Should one minimize the expected Bellman residual or maximize the mean value?

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Abstract

We study reinforcement learning from an optimization perspective. We consider maximizing the mean value (the predominant approach in policy search) and minimizing the expected Bellman residual (the Bellman residual being prevalent in approximate dynamic programming). For doing so, we introduce a new approach that consists in minimizing the mean residual $\nu(T^*v_\pi - v_\pi)$ for a class of parameterized policies. We prove that this method enjoys a performance bound that is better than the sole known bound for maximizing the mean value and that matches the best known bounds in approximate dynamic programming. We also conduct experiments on randomly generated generic Markov decision processes to compare both approaches empirically. It appears that maximizing the mean value is much more efficient, and that the Bellman residual is actually not a such good proxy for optimizing a value function. This suggests to envision maximizing the mean value for designing new reinforcement learning approaches, and that much remains to be done regarding its theoretical analysis.

1 Introduction

Generally speaking, Reinforcement Learning (RL) aims at estimating a policy close to the optimal one, in the sense that $\|v_\pi - v^*\|$ is small, for some norm. The idea of controlling the residual $\|T^*v - v\|$ is a classic of RL, and especially of Approximate Dynamic Programming (ADP). This is due to the fact that controlling this residual allows controlling the distance to the optimal value function: generally speaking, we have that $\|v_\pi - v^*\| \leq \frac{2}{1-\gamma} \|T^*v - v\|$, with the policy $\pi$ being greedy respectively to $v$ [Munos, 2007].
Moreover, the principle behind some classical ADP approaches is indeed to minimize a projected Bellman residual, \( \| \Pi(T_*v - v) \| \), where \( \Pi \) is the projection onto the hypothesis space to which the value functions \( v \) belongs to: Approximate Value Iteration (AVI) \cite{Gordon1995,Ernst2005} tries to minimize this using a fixed-point approach, \( v_{k+1} = \Pi T_*v_k \), and it has been shown recently by \cite{Perolat2016} that Least-Squares Policy Iteration (LSPI) \cite{Lagoudakis2003} tries to minimize it using a Newton approach\(^1\). Notice that in this case (projected residual), there is no general performance bound\(^2\) for controlling \( \| v_* - v_\pi \| \).

Despite the fact that (unprojected) residual approaches come easily with performance guarantees, they are not extensively studied in the (value-based) literature (one can mention \cite{Baird1995} that consider a subgradient descent or \cite{Piot2014} who frame it as a difference of convex functions, for example). A reason for this is that they lead to biased estimates when the Markovian transition kernel is stochastic and unknown \cite{Antos2008}, which is a rather standard case.

An alternative approach consists in maximizing directly the mean value \( \nu v_\pi \), for a user defined state distribution \( \nu \), see Sec. 2. This suggests defining a class of parameterized policies and optimizing over them, which is the predominant approach in policy search \cite{Deisenroth2013}.

This article aims at comparing these two approaches, maximizing the mean value and minimizing the residual. For doing so, we introduce a new policy search method that minimizes the mean residual \( \nu (T_*v_\pi - v_\pi) \), the optimization being done over policies. We present this approach in Sec. 3 and provide a performance guarantee that matches the best known bounds in ADP \cite{Scherrer2014}, better that the sole known bound for maximizing the mean value \cite{Scherrer2014}.

In Sec. 4, we compare both approaches on a bunch of randomly generated Markov Decision Processes (MDPs). It appears that maximizing the mean value is much more efficient, and that the Bellman residual might actually not be such a good proxy for optimizing a value function. This suggests to envision maximizing the mean value for designing new reinforcement learning approaches, and that much remains to be done regarding its theoretical analysis.

\(^1\) (Exact) policy iteration actually minimizes \( \| T_*v - v \| \) using a Newton descent \cite{Filar1991}.

\(^2\) With a single action, this approaches reduces to LSTD (Least-Squares Temporal Differences) \cite{Bradtke1996}, that can be arbitrarily bad in an off-policy setting \cite{Scherrer2010}. 

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2 Background

2.1 Notations

Let $\Delta_X$ be the set of probability distributions over a finite set $X$ and $Y^X$ the set of applications from $X$ to the set $Y$. By convention, all vectors are column vectors, except distributions (for left multiplication). An MDP is a tuple $\{S, A, P, R, \gamma\}$, where $S$ is the finite state space, $A$ is the finite action space, $P \in (\Delta_S)^{S \times A}$ is the Markovian transition kernel ($P(s'|s,a)$ denotes the probability of transiting to $s'$ when action $a$ is applied in state $s$), $R \in \mathbb{R}^{S \times A}$ is the bounded reward function and $\gamma \in (0,1)$ is the discount factor.

A stochastic policy $\pi \in (\Delta_A)^S$ associates to each state a distribution over actions. The policy-induced reward and transition kernel, $R^\pi \in \mathbb{R}^S$ and $P^\pi \in (\Delta_S)^S$, are defined as

$$R^\pi(s) = \mathbb{E}_{\pi(.|s)}[R(s, A)] \quad \text{and} \quad P^\pi(s'|s) = \mathbb{E}_{\pi(.|s)}[P(s'|s, A)].$$

The quality of a policy is quantified by the associated value function $v^\pi \in \mathbb{R}^S$:

$$v^\pi(s) = \mathbb{E} \left[ \sum_{t \geq 0} \gamma^t R^\pi(S_t) | S_0 = s, S_{t+1} \sim P^\pi(.|S_t) \right].$$

The value $v^\pi$ is the unique fixed point of the Bellman operator $T^\pi$, defined as $T^\pi v = R^\pi + \gamma P^\pi v$ for any $v \in \mathbb{R}^S$.

Let define the second Bellman operator $T_*$ as, for any $v \in \mathbb{R}^S$,

$$T_* v = \max_{\pi \in (\Delta_A)^S} T^\pi v.$$

A policy $\pi$ is greedy respectively to $v \in \mathbb{R}^S$, denoted $\pi \in \mathcal{G}(v)$ if $T^\pi v = T_* v$. There exists an optimal policy $\pi_*$ that satisfies componentwise $v_{\pi_*} \geq v_\pi$, for all $\pi \in (\Delta_A)^S$. Moreover, we have that $\pi_* \in \mathcal{G}(v_*)$, with $v_*$ being the unique fixed point of $T_*$.

Finally, for any distribution $\mu \in \Delta_S$, the $\gamma$-weighted occupancy measure induced by the policy $\pi$ when the initial state is sampled from $\mu$ is defined as

$$d_{\mu,\pi} = (1-\gamma)\mu \sum_{t \geq 0} \gamma^t P^\mu_\pi = (1-\gamma)\mu(I-\gamma P^\pi)^{-1} \in \Delta_S.$$

For two distributions $\mu$ and $\nu$, we write $\|\frac{\mu}{\nu}\|_{\infty}$ the smallest constant $C$ satisfying, for all $s \in S$, $\mu(s) \leq C \nu(s)$. This quantity measures the mismatch between the two distributions.

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3 This choice is done for ease and clarity of exposition, the following results can be extended.
2.2 Maximizing the mean value

Let $\mathcal{P}$ be a space of parameterized stochastic policies and let $\mu$ be a distribution of interest. The optimal policy has a higher value than any other policy, for any state. If the MDP is too large, satisfying this condition is not reasonable. Therefore, a natural idea consists in searching for a policy such that the associated value function is as close as possible to the optimal one, in expectation. More formally, this means minimizing $\mu(v^* - v_\pi)$. The optimal value function being unknown, one cannot address this problem directly, but it is equivalent to maximizing $\mu v_\pi$.

This is the basic principle of policy search:

$$\max_{\pi \in \mathcal{P}} J_\nu(\pi) \text{ with } J_\nu(\pi) = \nu v_\pi = \mathbb{E}_\nu[v_\pi(S)].$$

Notice that we used a distribution $\nu$ here, possibly different from the distribution of interest $\mu$. Policy search methods differ notably by the considered criterion (e.g., it can be the mean reward rather than the $\gamma$-discounted cumulative reward considered here) and by how the corresponding optimization problem is solved. We refer to Deisenroth et al. [2013] for a quite exhaustive survey on the subject.

Contrary to ADP, the theoretical efficiency of this kind of approach has not been studied a lot. Indeed, as far as we know, there is a sole performance bound for policy search.

**Theorem 1** (Performance bound of Scherrer and Geist [2014]). Assume that the policy space $\mathcal{P}$ is stable by stochastic mixture, that is

$$\forall \pi, \pi' \in \mathcal{P}, \forall \alpha \in (0, 1), \quad (1 - \alpha)\pi + \alpha\pi' \in \mathcal{P}.$$

Define the $\nu$-greedy-complexity of the policy space $\mathcal{P}$ as

$$\mathcal{E}_\nu(\mathcal{P}) = \max_{\pi \in \mathcal{P}} \min_{\pi' \in \mathcal{P}} d_{\nu,\pi}(Tv_\pi - T_{\pi'}v_\pi).$$

Then, any policy $\pi$ that is an $\epsilon$-local optimum of $J_\nu$, in the sense that

$$\forall \pi' \in \Pi, \quad \lim_{\alpha \to 0} \frac{\nu((1-\alpha)\pi + \alpha\pi') - v_\pi}{\alpha} \leq \epsilon,$$

enjoys the following global performance guarantee:

$$\mu(v^* - v_\pi) \leq \frac{1}{(1 - \gamma)^2} \left\| d_{\mu,\pi^*_{\nu}} \right\|_\infty (\mathcal{E}_\nu(\mathcal{P}) + \epsilon).$$

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This bound (as all bounds of this kind) has three terms: an horizon term, a concentrability term and an error term. The term $\frac{1}{1-\gamma}$ is the average optimization horizon. The concentrability coefficient ($\|d_{\mu,\pi^*}\|_{\infty}$) measures the mismatch between the used distribution $\nu$ and the $\gamma$-weighted occupancy measure induced by the optimal policy $\pi^*$ when the initial state is sampled from the distribution of interest $\mu$. This tells that if $\mu$ is the distribution of interest, one should optimize $J_{d_{\mu,\pi^*}}$, which is not feasible (as $\pi^*$ is unknown). However, it is also the best concentrability coefficient according to Scherrer [2014], who provides a theoretical and empirical comparison of Approximate Policy Iteration (API) schemes. The error term is $\mathcal{E}_{\nu}(\mathcal{P}) + \epsilon$, where $\mathcal{E}_{\nu}(\mathcal{P})$ measures the capacity of the policy space to represent the policies being greedy respectively to the value of any policy in $\mathcal{P}$ and $\epsilon$ tells how close the computed policy $\pi$ is close to a local optimum of $J_{\nu}$.

This bound matches the kind of bounds that exist for ADP (with a comparable horizon term and a better concentrability coefficient), see the discussion of Scherrer and Geist [2014] for more details. However, it is based on a very strong assumption (stability by stochastic mixture of the policy space) which is not satisfied by all commonly used policy parameterizations.

3 Minimizing the expected Bellman residual

For comparing the direct maximization of the mean value to a residual approach, we introduce a policy search method that minimizes the mean residual. We also prove that it benefits from a performance bound better than the one of Th. [1] and that applies under much weaker conditions for $\mathcal{P}$.

3.1 Optimization problem

We propose to search a policy in $\mathcal{P}$ that minimizes the expected Bellman residual:

$$\min_{\pi \in \mathcal{P}} \mathcal{J}_{\nu}(\pi) \text{ with } \mathcal{J}_{\nu}(\pi) = \nu(T_{*}v_{\pi} - v_{\pi}).$$

Notice that we have always $J_{\nu}(\pi) \geq 0$, as $T_{*}v_{\pi} \geq v_{\pi}$. As a consequence, it is indeed a weighted $\ell_1$-norm, $\mathcal{J}_{\nu}(\pi) = \|T_{*}v_{\pi} - v_{\pi}\|_{1,\nu}$. Notice also that there is no bias problem for considering a residual here, contrary to value-based approaches.

Any optimization method can be envisioned to minimize $\mathcal{J}_{\nu}$. Here, we simply propose to do a subgradient descent. This requires computing the subgradient.
Recall that given the considered notations, the distribution $\nu P_{G(v)}$ is the state distribution obtained by sampling the initial state according to $\nu$, applying the action being greedy respectively to $v_\pi$ and following the dynamics to the next state. This being said, the subgradient of $J$ is given by

$$-\nabla J(\pi) = \frac{1}{1-\gamma} \sum_{s \in S} \left( d_{\nu,\pi}(s) - \gamma d_{\nu P_{G(v)}}(s) \right) \sum_a \pi(a|s) \nabla \ln \pi(a|s) q_\pi(s,a).$$

**Proof.** Let $q_\pi$ be the state-action value function of $\pi$, defined as $q_\pi(s,a) = R(s,a) + \gamma \sum_{s' \in S} P(s'|s,a) v_\pi(s')$. The considered objective function can be rewritten as

$$J_\nu(\pi) = \sum_{s \in S} \nu(s) \left( \max_{a \in A} q_\pi(s,a) - v_\pi(s) \right).$$

The classic policy gradient theorem [Sutton et al., 1999] states that

$$\nabla (\nu v_\pi) = \sum_{s \in S} \nu(s) \nabla v_\pi(s) = \frac{1}{1-\gamma} \sum_{s \in S} d_{\nu,\pi}(s) \sum_{a \in A} \nabla \pi(a|s) q_\pi(s,a).$$

On the other hand, we have that $\nabla \max_{a \in A} q_\pi(s,a) = \nabla q_\pi(s,a^*_s)$, with $a^*_s \in \argmax_{a \in A} q_\pi(s,a)$). Therefore:

$$\nabla \sum_{s \in S} \nu(s) \max_{a \in A} q_\pi(s,a) = \sum_{s \in S} \nu(s) \nabla q_\pi(s,a^*_s),$$

$$= \sum_{s \in S} \nu(s) \nabla \left( R(s,a^*_s) + \gamma \sum_{s' \in S} P(s'|s,a^*_s) v_\pi(s') \right),$$

$$= \gamma \sum_{s \in S} \nu(s) \sum_{s' \in S} P(s'|s,a^*_s) \nabla v_\pi(s').$$

By noticing that $\nu(s) \sum_{s' \in S} P(s'|s,a^*_s) = [\nu P_{G(v)}](s)$ and that $\nabla \pi(a|s) = \pi(a|s) \nabla \ln \pi(a|s)$, we obtain the stated result. 

We have two terms in the negative subgradient $-\nabla J_\nu$: the first one corresponds to the gradient of $J_\nu$, the second one (up to the multiplication by $-\gamma$) is the gradient of $J_{\nu P_{G(v)}}$ and acts as a kind of correction. This subgradient can be estimated using Monte Carlo rollouts, but doing so is heavier than for classic policy search (as it requires additionally sampling from $\nu P_{G(v)}$, which requires estimating the state-action value function). Also, this gradient involves computing the maximum over actions (as it requires sampling from $\nu P_{G(v)}$, that comes from explicitly considering the Bellman optimality operator), which prevents from extending easily this approach to continuous
actions, contrary to classic policy search. Thus, from an algorithmic point of view, the proposed approach has some drawbacks. Yet, it comes with a nicer and more general performance guarantee.

3.2 Analysis

In general, $J_\nu(\pi)$ is not convex (nor is $J_\nu(\pi))$, so there is no guarantee to find the global optimum. Yet, after computing a solution (with a subgradient descent or any other approach), we can tell how small is $J_\nu$. We relate this to the mean distance to the optimal value function.

**Theorem 3** (Performance bound for residual policy search). If we have that

$$\nu(T_*v_\pi - v_\pi) \leq e,$$

then

$$\mu(v_* - v_\pi) \leq \frac{1}{1 - \gamma} \left\| \frac{d_{\mu,\pi}}{\nu} \right\|_\infty e.$$

**Proof.** The proof is a direct consequence of the analysis of Scherrer and Geist [2014]. Their Lemma 1 states that for any policies $\pi$ and $\pi'$, we have

$$v_{\pi'} - v_\pi = (I - \gamma P_{\pi'})^{-1}(T_{\pi'}v_\pi - v_\pi).$$

Using this and the fact that $T_*v_\pi \geq T_{\pi'}v_\pi$, we have that

$$\mu(v_{\pi'} - v_\pi) = \mu(I - \gamma P_{\pi'})^{-1}(T_{\pi'}v_\pi - v_\pi) = \frac{1}{1 - \gamma} d_{\mu,\pi'}(T_{\pi'}v_\pi - v_\pi) \leq \frac{1}{1 - \gamma} d_{\mu,\pi'}(T_*v_\pi - v_\pi).$$

By the definition of the concentrability coefficient, $d_{\mu,\pi'} \leq \nu \left\| \frac{d_{\mu,\pi'}}{\nu} \right\|_\infty$, and as we assumed that $\nu(T_*v_\pi - v_\pi) \leq e$, we have

$$\mu(v_{\pi'} - v_\pi) \leq \frac{1}{1 - \gamma} \left\| \frac{d_{\mu,\pi'}}{\nu} \right\|_\infty \nu(T_*v_\pi - v_\pi) \leq \frac{1}{1 - \gamma} \left\| \frac{d_{\mu,\pi'}}{\nu} \right\|_\infty e.$$

Choosing $\pi' = \pi_*$ gives the stated bound. □

Contrary to Th.1, no assumption is made on the hypothesis space. Notably, it is not assumed that it is stable by stochastic mixture. Also, this bound has a better dependency on the horizon term (linear dependency instead of square dependency). The concentrability coefficients are the same and the
error term ($e$ for residual policy search, $\mathcal{E}_\nu(P) + \epsilon$ for policy search) are not easily comparable.

Actually, this result matches the best known bounds in approximate policy iteration, as far as we know, at least regarding the horizon term and the concentrability coefficient. We refer here to Table 1 of [Scherrer 2014]. The sole algorithms that have only a linear dependency on the horizon term compute non-stationary policies: PSPD (Policy Search by Dynamic Programming) [Bagnell et al., 2003] and NSPI (Non-Stationary Policy Iteration) [Scherrer and Lesner, 2012]. Either they have a linear dependency on the horizon term and a worst concentrability coefficient, or they have the same concentrability coefficient but a quadratic dependency on the horizon. Again, the error terms are not comparable.

To sum up, we obtain a rather nice bound for minimizing the mean residual, better than the one for maximizing the mean value, and that matches ADP best known bounds.

4 Experiments

We consider Garnet problems [Archibald et al., 1995, Bhatnagar et al., 2009]. They are a class of randomly built MDPs meant to be totally abstract while remaining representative of the problems that might be encountered in practice. Here, a Garnet $G(|\mathcal{S}|, |\mathcal{A}|, b)$ is specified by the number of states, the number of actions and the branching factor. For each $(s, a)$ couple, $b$ different next states are chosen randomly and the associated probabilities are set by randomly partitioning the unit interval. The reward is null, except for 10% of states where it is set to a random value, uniform in $(1, 2)$. The discount factor is always set to $\gamma = 0.99$.

For the policy space, we consider a Gibbs parameterization:

$$\mathcal{P} = \left\{ \pi_w : \pi_w(a|s) = \frac{e^{w^\top \phi(s,a)}}{\sum_{b} e^{w^\top \phi(s,b)}}, w \in \mathbb{R}^{d|\mathcal{A}|} \right\}.$$  

The features are also randomly generated, $F(d, l)$. First, we generate binary state-features $\varphi(s)$ of dimension $d$, such that $l$ components are set to 1 (the others are thus 0). The positions of the 1’s are selected randomly such that no two states have the same feature. Then, the state action features, of dimension $d|\mathcal{A}|$, are classically defined as

$$\phi(s, a) = (0 \ldots 0 \varphi(s) 0 \ldots 0)^\top,$$

\[4\]Actually, the variations studied by [Scherrer 2014].
the position of the zeros depending on the action. Notice that $P$ is not stable by stochastic mixture in this case, so the bound for policy search does not formally apply.

We compare classic policy search (denoted as PS($\nu$)), that maximizes the mean value, and residual policy search (denoted as RPS($\nu$)), that minimizes the mean residual. We optimize the relative objective functions with a normalized gradient ascent (resp. normalized subgradient descent) with a constant learning rate $\alpha = 0.1$. The gradient are computed analytically (as we have access to the model), so the following results represent an ideal case, when one can do an infinity of rollouts. In the sequel, the distribution $\mu \in \Delta_S$ of interest is the uniform distribution.

First, we consider $\nu = \mu$. We generate randomly 100 Garnets $G(30, 4, 2)$ and 100 features $F(8, 3)$. For each Garnet-feature couple, we run both algorithms for $T = 1000$ iterations. For each algorithm, we measure two things, the (normalized) error $\frac{\mu(v_* - v_{\pi})}{\mu v_*}$ and the mean Bellman residual $\mu(T_* v_{\pi} - v_{\pi})$, where $\pi$ depends on the algorithm and on the iteration. We show the results

Figure 1: Results on the Garnet problems, when $\nu = \mu$. 
One can observe that PS is much better at decreasing the error than RPS (Fig. 1a vs Fig. 1b). However, RPS is better at decreasing the residual (Fig. 1c vs Fig. 1d). Actually, each algorithm does its job, and this let think that the bound of policy search might not be tight. Yet, by observing how the error and the residual evolves for each algorithm (Fig. 1a vs Fig. 1c for PS, Fig. 1b vs Fig. 1d for RPS), we can make the following comments. For RPS, we can see the bound. However, for PS, we see that the Bellman residual increases while the error decreases. This let us think that the Bellman residual might not be the best proxy for optimizing a value, and that much remains to be done for understanding why maximizing the expected value works so well.

Yet, the bounds in Th. 1 and 2 tells that one should choose $\nu = d_{\mu, \pi_*}$. This is an unrealistic setting, but as we work with Garnets, we can do it. The results are presented in Fig. 2. Here, we see that there is no significant difference between PS and RPS regarding the decrease of the error (Fig. 2a vs Fig. 2b), and that the behavior of the residual is similar (Fig. 2c vs Fig. 2d).
Figure 3: Results on the Garnet problems, when $\nu = \mu$ and with tabular features.

Note that it is normal that the residual in Fig. 2.d does not decrease, as we measure $\mu(T_\pi v_\pi - v_\pi)$ while minimizing $J_{d_\mu,\pi_*}$. One can also remark that PS($\mu$) does as well as PS($d_\mu,\pi_*$) for decreasing the error (Fig. 1.a vs Fig 2.a). This is undoubtedly in favor of maximizing the mean value, and much better that what we can tell from the bound.

We did the same experiments for the same problems ($G(30, 4, 2)$), but with a perfect, tabular, representation (which amounts to choose $F(|S|, 1)$). In this case, the bound of policy search (Th. 1) applies: the policy space $\mathcal{P}$ contains all stochastic policies, so it is stable by stochastic mixtures. Related results are reported in Fig. 3 and 4 and the observations are the same. Notably, one can see that even if RPS leads to a small residual (Fig. 3.d), the related error remains high (Fig. 3.b, this does not contradict Th. 3), while maximizing the mean value is effective (Fig. 3.a).
Figure 4: Results on the Garnet problems, when $\nu = d_{\mu,\pi_*}$, and with tabular features.
5 Conclusion

In this paper, we have studied two optimization approaches to reinforcement learning, namely the maximization of the mean value (classic policy search) and the minimization of the mean Bellman residual (such residuals being prevalent in ADP), both theoretically and empirically. To do so, we introduced a new policy search approach based on the minimization of the residual.

We derived a rather nice bound for the resulting algorithm. As far as we know, there is no other bound in the reinforcement learning literature that has a linear dependency on the horizon as well as a concentrability coefficient as nice as ours. Yet, when comparing empirically this two optimization approaches to RL, it appears that directly maximizing the mean value is much more efficient. One cannot make general conclusions from these specific experiments, but they do not contradict the studied bounds, while showing that the Bellman optimality operator might not be a so good proxy for minimizing the mean error $\mu(v_* - v_\pi)$ (as the residual is increased for decreasing the error). Moreover, in these experiments, the results are much less sensitive to the choice of the distribution defining the objective function when maximizing the mean value.

This suggests some lines of work: studying the calibration of the residual as a proxy (in the spirit of what exists for classification [Bartlett et al., 2006]), seeking for possibly better proxys, starting the design of new algorithms from the maximization of the mean value, and seeking for a better theoretical understanding of why maximizing the mean value works so well.

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