Constraining the Minimum Luminosity of High Redshift Galaxies through Gravitational Lensing

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ABSTRACT

We simulate the effects of gravitational lensing on the source count of high redshift galaxies as projected to be observed by the Hubble Frontier Fields program and the James Webb Space Telescope (JWST) in the near future. Modeling the lens as a singular isothermal sphere residing at a redshift of \( z_L = 0.5 \), we explore the radial dependence of the resulting magnification bias and how it varies with the velocity dispersion of the lens, the photometric sensitivity of the instrument, the redshift of the background source population, and the minimum intrinsic luminosity \( (L_{\text{int}}) \) of the sources. We find that gravitational lensing enhances the number of galaxies with redshifts \( z \gtrsim 10 \) detected in the angular region \( \theta_E/2 \leq \theta \leq 2\theta_E \) by a factor of \( 5 \) and \( 200 \) when observations are made with JWST \((df/dv_0 \gtrsim 3.61 \text{ nJy})\) and HST \((df/dv_0 \gtrsim 9.12 \text{ nJy})\), respectively. Furthermore, we find that the bias is sensitive to the minimum intrinsic luminosity of galaxies, transitioning from positive to negative in certain cases when larger values of \( L_{\text{int}} \) are assumed. In those cases where a transition does take place, the upper bounds on \( L_{\text{int}} \) to detect a positive magnification bias are \( 5 \times 10^{27} \) and \( 3 \times 10^{27} \text{ erg s}^{-1} \text{ Hz}^{-1} \) when observing galaxies with redshifts \( z \gtrsim 8 \) and \( z \gtrsim 10 \) respectively. Gravitational lensing may therefore offer an alternative way of constraining the value of the minimum luminosity of high redshift galaxies by comparing source counts in blank fields against counts measured behind a foreground lens.

Key words: cosmology: theory – galaxies: high-redshift – gravitational lensing: strong.

1 INTRODUCTION

The characterization of the earliest galaxies in the universe remains one of the most important frontiers of observational cosmology, and also one of the most challenging (Loeb & Furlanetto 2012). High-redshift searches carried out with the Hubble Space Telescope (HST) have recently provided significant insights to the mass assembly and buildup of the earliest galaxies (\( z \gtrsim 6 \)) and the contribution of star formation to cosmic reionization (Bunker et al. 2004; Yan & Windhorst 2004; Bouwens et al. 2010; Ellis et al. 2013). However, because of their great distances and extreme faintness, as well as the high sky background, high redshift galaxies remain difficult to detect. Furthermore, those sources which are bright enough to be studied individually are drawn from the bright tail of the luminosity function (LF) of high redshift galaxies and are therefore not necessarily representative of the bulk of the population (Bayliss et al. 2010). Gravitational lensing by galaxy clusters has been highlighted as an efficient way of improving this situation, providing an opportunity to observe the high-redshift universe in unprecedented detail (Bradley et al. 2008; Zheng et al. 2012).

Light rays propagating through the inhomogeneous gravitational field of the Universe are often deflected by intervening clumps of matter, which cause most sources to appear slightly displaced and distorted in comparison with the way they would otherwise appear in a perfectly homogeneous and isotropic universe (Schneider, Ehlers, & Falco 1992; Bartelmann & Schneider 2001; Treu 2010). When the light from a distant galaxy is deflected by foreground mass concentrations such as galaxies, groups, and galaxy clusters, its angular size and brightness are increased and multiple images of the same source may form. This phenomenon, referred to as strong gravitational lensing, leads to a magnification bias that can have a significant effect on the observability of a population of galaxies. Magni-
fied sources, that would otherwise be too faint for detection without a huge investment of observing time, can be found, and unresolved substructure and morphological details in these intrinsically faint galaxies can be studied (Smail, Ivison, & Blair 1997. Petz et al. 2000; Ellis et al. 2001). The light magnification produced by nature’s “cosmic telescopes” can be exploited in the study of high-redshift galaxies which have greater probability of falling in alignment with, and therefore being lensed by, a foreground galaxy (Barkana & Loeb 2000). Zackrisson et al. (2012) explored the prospect of detecting a hypothetical population of population II galaxies via gravitational lensing by a particular galaxy cluster (J0717.5 + 3745) as the lens. Indeed, several highly magnified galaxy candidates at up to redshifts $z \sim 10$ have already been discovered behind massive clusters (Stark et al. 2003; Richard et al. 2006, 2008; Bradley et al. 2008; Bouwens et al. 2004; Biviss et al. 2010; Zheng et al. 2012; Coe et al. 2013). Beginning October 1, 2013, the Hubble Frontier Fields program is expected to lead to many more such discoveries. With its six deep fields centered on strong lensing galaxy clusters in parallel with six deep “blank fields”, the Hubble Frontier Fields will reveal previously inaccessible populations of $z \sim 10$ galaxies that are 10-50 times intrinsically fainter than any presently known. In the coming decade, the planned James Webb Space Telescope (JWST) promises to go even further by placing new constraints on the stellar initial mass function at high redshift, on the luminosity function of the first galaxies, and on the progress of the early stages of reionization with observations of galaxies at $z \gtrsim 10$ (Gardner et al. 2006; Windhorst et al. 2006; Haimar 2008).

While lenses magnify the observed flux and lift sources which are intrinsically too faint to be observed over the detection threshold, they simultaneously increase the solid angle within which sources are observed and thus reduce their number density and measured surface brightness in the sky (Refregier & Loeb 1997). Zemcov et al. (2013) recently reported measuring such a deficit of surface brightness within the central region of several massive galaxy clusters with the SPIRE instrument, and used the deficit to constrain the surface brightness of the cosmic infrared background. The outcome of this trade-off between depth and area depends on a variety of factors, such as the photometric sensitivity of the detecting device and the slope of the luminosity function of background sources. Given a photometric sensitivity capable of detecting faint sources even in the absence of any light amplification, the magnification effect leads to a negative magnification bias, reducing the apparent surface density behind lensing clusters. If, however, the fainter sources cannot be observed unless magnified, then whether the magnification bias leads to a surplus or deficit of observed sources depends on the effective slope of their luminosity function (Turner, Ostriker, & Gott 1984). At fainter magnitudes where the effective slope is shallow, there may not be enough faint sources in the lensed population to compensate for the increase in total surface area. However, in cases where the effective slope is greater than 2, the gain in depth due to apparent brightening may outweigh the loss in area; gravitational lensing will thus increase the apparent surface density behind the lensing object, boosting up the number of detected sources relative to that which would otherwise be observed in an unlensed field (Bouwens et al. 2009).

The observed number counts of galaxies residing at redshifts $z \gtrsim 8$ may also be sensitive to the minimum intrinsic luminosity chosen for the extrapolation of the galaxy LF. Theoretical and numerical investigations have established that a halo at $z \lesssim 10$ irradiated by a UV field comparable to the one required for reionization needs a mass $M_h \gtrsim (0.6 - 1.7) \times 10^8 M_\odot$, with a corresponding temperature $T_{vir} \gtrsim (1 - 2) \times 10^4 K$ at $z = 7$, in order to cool and form stars (Haimar, Thoul, & Loeb 1996; Tegmark et al. 1997; Munoz & Loeb 2011). Such claims have motivated models with cut-offs for the luminosity of the smallest halo capable of forming stars (Wyithe & Loeb 2006; Trenti et al. 2010). Gravitational lensing may provide an alternative avenue for constraining the value of this minimum luminosity given the fact that the predicted observed number count of high-redshift galaxies in a blank field versus behind a lensing cluster varies as a function of this intrinsic cut-off value.

In this paper, we predict the lensing rate of high-redshift objects that will be observed with both HST Frontier Fields in the upcoming months, and JWST within the next decade. In §2 we consider a simple single lens modeled as a singular isothermal sphere and examine its effect on the number count of the background lensed galaxy population. In addition to considering lensing clusters, we also consider galaxy-galaxy lensing as well as galaxy-group lensing and compute the lensing rates expected in each case given the velocity dispersion of the lensing object. We present our numerical results in §3 and show the transition from a positive to a negative magnification bias as a function of the minimum intrinsic luminosity, the photometric sensitivity, and the angular distance from the given lens. We conclude in §4 with a discussion of our findings and their implications for observations with the HST Frontier Fields and JWST in the near future.

2 THE LENSING MODEL

In this paper, we use a simple model for the matter distribution in a gravitational lens, namely that of a Singular Isothermal Sphere (SIS) (Schneider, Ehlers, & Falco 1992). This model provides a good first-order approximation to the projected mass distribution of known early-type galaxies and cluster lenses (Tyson & Fischer 1993; Narayan & Bartelmann 1996; Treu & Koopmans 2002; Koopman & Treu 2003; Rusin, Kochanek, & Keeton 2003). The surface mass density of a SIS is given by

\[
\Sigma(\xi) = \frac{\sigma_v^2}{2G\xi} \tag{1}
\]

where $G$ is Newton’s constant, $\sigma_v$ is the line-of-sight velocity dispersion of the lens, and $\xi$ is the distance from the center of the two-dimensional profile. A light ray which passes the
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The lens is deflected by a (reduced) angle

\[ \tilde{\alpha}(\xi) = -\frac{4G}{c^2} \frac{D_{ls}}{D_{os}} \int \frac{(\xi - \xi') \Sigma(\xi')}{|\xi - \xi'|^2} d^2 \xi' \]

\[ = 4\pi \frac{\sigma^2}{c^2} \frac{D_{ls}}{D_{os}} \]

where \( c \) is the speed of light, and \( D_{os} \) and \( D_{ls} \) are the comoving angular-diameter distances between the observer and the source, and the lens and the source, respectively. In the standard ΛCDM cosmology, the angular-diameter distance \( D_{ang}(z_1, z_2) \) of a source at redshift \( z_2 \) seen by an observer at redshift \( z_1 < z_2 \) is defined as

\[ D_{ang}(z_1, z_2) = \frac{c}{H_0} \left( \frac{1 + z_2}{1 + z_1} \right) \int_{z_1}^{z_2} \frac{dz}{E(z)} \]

where

\[ E(z) = \sqrt{\Omega_m(1 + z)^3 + \Omega_{\Lambda}} \]

\[ H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \] is the Hubble constant, and \( \Omega_m = 0.3 \) and \( \Omega_{\Lambda} = 0.7 \) are the present-day density parameters of matter and vacuum, respectively.

This reduced deflection angle is commonly referred to as the Einstein angle for the SIS mass distribution and is denoted as \( \theta_E \). The lensing effect causes the image of the source to be displaced, magnified, and sometimes split (Refregier & Loch 1997). The relation between the source position (described by the unlensed position angle \( \beta \)) and the positions of the images, \( \theta = \xi/D_{os} \) where \( D_{os} \) is the angular-diameter distance between the observer and the lens, is given by the lens equation, which in the case of a SIS lens, reduces to

\[ \beta = \tilde{\beta} - \tilde{\alpha}(\tilde{\theta}) = \tilde{\theta} - \frac{\theta}{|\theta|} \theta_E \]

where negative angles refer to positions on the opposite side of the lens center. When \( |\beta| < \theta_E \), the lens equation has two solutions, \( \theta_{\pm} = \beta \mp \theta_E \), and multiple images are obtained. Conversely, if the source lies outside the Einstein ring, i.e. \( |\beta| > \theta_E \), only one image is present at \( \theta = \theta_{\pm} = \beta \mp \theta_E \).

The corresponding magnification factor due to a SIS lens is given by

\[ \mu(\theta) = \frac{\theta}{\beta} \frac{d\beta}{d\theta} \left[ 1 - \frac{\theta_E}{|\theta|} \right]^{-1} \]

where negative values of \( \mu \) correspond to inverted images. For large values of \( \theta, \mu \approx 1 \) and the source is weakly affected by the lensing potential, while for \( \theta = 0 \) or \( \theta = \theta_E \), the magnification diverges. In practice, the maximum magnification is limited by the finite extent of the lensed source. Since we are considering primarily the lensing effect on compact, high-redshift galaxies (Oesch et al. 2011), we ignore the angular size of the sources and model the background as a collection of point sources.

We model the luminosity function of the background galaxy population as a Schechter function,

\[ \phi(z, L) dL = \phi_*(z) \left( \frac{L}{L_*(z)} \right)^{\alpha(z)} e^{-L/L_*(z)} \frac{dL}{L_*(z)} \]

where the parameters are the comoving number density of galaxies \( \phi_* \), the characteristic specific luminosity \( L_* \), and the faint-end slope \( \alpha \), all of which are taken to evolve with cosmic time. (Note that we denote specific luminosity in this paper as \( L \).) The evolution of \( \phi_*, L_*, \) and \( \alpha \) as functions of redshift in the interval \( z > 7 \) are taken as the central values of the fitting formulae provided by Bouwens et al. (2011),

\[ \phi_*(z) = (1.14 \pm 0.20) \times 10^{-3} \frac{(0.003 \pm 0.055)(z - 3.8)}{10^{4.45 + 4.65 \log D_{os}(z)}} \text{ Mpc}^{-3} \]

\[ L_*(z) = 9.52 \times 10^{20} \frac{4\pi D_{z}^2/(pc)^2}{(1 + z)^2} \text{ erg s}^{-1}\text{Hz}^{-1} \]

\[ \alpha(z) = (-1.73 \pm 0.05) + (0.01 \pm 0.04)(z - 3.8) \]

where

\[ M_*(z) = (-21.02 \pm 0.09) + (0.33 \pm 0.06)(z - 3.8) \]

and \( D_L = (1 + z)^2 D_{os} \) is the luminosity distance to a given source. Estimates of the number density of background galaxies obtained by applying these formulae serve as a conservative reference point; recent determinations of the LF at redshifts \( z \approx 7.9 \) from HUDF12 observations (Schenker et al. 2013; McLure et al. 2013) imply somewhat larger numbers. Furthermore, these formulae describing the evolution of the LF represent an extrapolation of the present LF results (\( z \approx 7.8 \)) to even higher redshifts (Bouwens et al. 2011). More recent studies address the uncertainty associated with the shape of the LF at these high redshifts and the fall-off in UV luminosity at \( z \approx 8 \) is still debated in the literature (Coe et al. 2013; McLure et al. 2013; Oesch et al. 2013).

The results in this paper may therefore change as the evolution of the LF parameters \( \phi_*, M_*, \) and \( \alpha \) as functions of redshift are modified in light of new observations.

In the absence of a lensing object (\( \mu = 1 \)), the number of sources with redshift in the range \( z_i < z < z_f \) seen within an angle \( \theta_\text{c} \) of the optical axis is simply the number which falls within \( \theta_\text{c} \) of \( \theta_\text{E} \), with \( \theta_\text{E} \) the angular separation of the source at redshift \( z_i \) and the detector.

\[ \Gamma \left( 1 + \alpha(z), \frac{1}{L_*} \text{Max}[L_{\text{int}}, L_{\text{det}}(z)] \right) \]

The limiting specific luminosity is set either by the minimum intrinsic brightness associated with a star-forming halo, \( L_{\text{int}} \), or, by what we denote as \( L_{\text{det}} \), the specific luminosity a source at redshift \( z_f \) must have to be above \( df/d\nu_0 \), the flux threshold set by the detector where

\[ L_{\text{det}} = 4\pi D_L^2 (1 + z) \frac{df}{d\nu_0} \]

The number of sources observed in a blank field with redshift \( z < z_f \) within an angle \( \theta_\text{c} \) thus reduces to

\[ N_{\text{unlensed}}(\theta_i, z_i, z_f) = \int_{z_i}^{z_f} \frac{dz}{E(z)} D_{os}^2(z) \phi_*(z) e^{-L/L_*(z)} \]

\[ \int_{L_{\text{int}}(z)}^{\infty} \frac{dL}{L} \]

\[ \phi_*(z) \left( \frac{L}{L_*(z)} \right)^{\alpha(z)} e^{-L/L_*(z)} \frac{dL}{L_*(z)} \]

\[ \frac{c}{H_0} \left( \frac{1}{\text{erg s}^{-1}\text{Hz}^{-1}} \right) \]

\[ \frac{dL}{d\nu_0} \]

\[ \phi_*(z) \left( \frac{L}{L_*(z)} \right)^{\alpha(z)} e^{-L/L_*(z)} \frac{dL}{L_*(z)} \]
In the presence of a lens, the magnification due to gravitational lensing has two effects on background point sources: their luminosities are magnified by a factor of $\mu$,

$$L_{\text{obs}}(\theta) = \mu(\theta) L,$$

(15)

and their surface number density is simultaneously diluted by the same factor,

$$n_{\text{obs}}(\theta) = n/\mu(\theta).$$

(16)

Consequently, the integrated source number count takes the following modified form,

$$N_{\text{sensed}}(\theta, z_i, z_f) = 2\pi c \int_{z_i}^{z_f} \frac{dz}{E(z)} D_{os,c}(z) \int_0^\theta d\theta' \frac{\theta'}{\mu^2(\theta')} \phi_*(z) \Gamma \left[ 1 + \alpha(z), \frac{1}{L_*} \max \left[ L_{\text{int}}, \frac{L_{\text{det}}(z)}{\mu(\theta')} \right] \right],$$

(17)

where the magnification factor in the case of a SIS lens is given by equation (6).

3 RESULTS

We now apply the general relations discussed above to lensing of background sources by a lens positioned at a redshift $z_L = 0.5$. This redshift is chosen to be consistent with the average redshift of the galaxy clusters centered in the six deep fields of the HST Frontier Fields program. While the Frontier Fields program, along with past searches for high-redshift galaxies, have relied solely on the lensing effect of galaxy clusters ($\sigma_e \approx 1000$ km/s), here we consider lensing by clusters as well as groups ($\sigma_v \approx 500$ km/s) and foreground galaxies ($\sigma_v \approx 200$ km/s). We restrict our attention to the flux thresholds set by the WFC3 aboard the HST and the NIRCam imager of JWST. The Frontier Fields program achieves an AB magnitude limit of $m_{AB} \approx 29$ with WFC3, corresponding to a flux limit of $df/d\nu_0 \sim 9.12$ nJy. NIRCam will be capable of detecting point source fluxes as low as $\sim 0.36$ nJy for a $10^5$ second exposure.

The radial dependence of the number of background sources with redshifts above $z = 8$ and $z = 10$ detected behind a lensing object at $z_L = 0.5$ is depicted in the left panels of Figures 2-3 and Figures 4-5, respectively. At large distances from the lens center ($\theta \gg 1$ arcmin), the magnification factor approaches unity and the number of detected sources per annular ring converges to the constant number that would be observed in the absence of a lens (dotted line). For small values of $\theta$, corresponding to regions closer to the lens, the behavior of the number of sources per solid angle, $dN/d\Omega$, reflects the angular dependence of the magnification factor associated with an SIS lens. For an image at $|\theta| < \theta_E/2$, $|\mu|$ is smaller than unity, and so the source luminosity is demagnified relative to the unlensed luminosity while the surface number density is amplified by the same factor. As $\theta$ approaches the Einstein angle, the magnification diverges, allowing small sources perfectly aligned with the lens center to form an "Einstein ring" and otherwise, causing the number density of observed sources to plummet. (This phenomenon corresponds to the sharp drop in $dN/d\Omega$ at $\theta \approx \theta_E$.) At image distances larger than the Einstein angle, $\mu$ converges back to unity, resulting in magnified luminosities and diluted number densities that gradually reduce to their unlensed values. The overall magnification bias depends on which of the two magnification effects wins out: under circumstances where the number of magnified sources lifted over the detection threshold outweighs the simultaneous dilution of the number density of sources in the sky, there is a positive magnification bias and $(dN/d\Omega)_{\text{lensed}} > (dN/d\Omega)_{\text{unlensed}}$. Conversely, when the reduction in number density dominates over luminosity magnifications, a negative magnification bias results and $(dN/d\Omega)_{\text{lensed}} < (dN/d\Omega)_{\text{unlensed}}$ in those regions.

The plots of $dN/d\Omega$ and $N(<\theta)$ (the cumulative number of sources observed within an angular radius $\theta$ of the lens center) in Figures 2-5 highlight the sensitivity of the magnification bias to the model parameters, $df/d\nu_0$ and $L_{\text{int}}$.  

Figure 1. Rest-frame UV Schechter Luminosity Function at $z = 8$ (left) and $z = 10$ (right). The dotted line represents the characteristic specific luminosity, $L_\star$, while the dashed and dash-dot lines represent the limiting detection specific luminosity, $L_{\text{det}}$, of the NIRCam aboard the JWST ($df/d\nu_0 \sim 3.61$ nJy) and the HST ($df/d\nu_0 \sim 9.12$ nJy) respectively.
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Figure 2. Predictions for observations by JWST’s NIRCam with a limiting flux sensitivity of \( \frac{df}{d\nu_0} = 3.61 \text{ nJy} \) and sources at \( z \gtrsim 8 \). Left panel: Radial dependence of the number of detected sources with redshifts above \( z = 8 \) in concentric annular cells centered on a lens at \( z_L = 0.5 \). Right panel: Cumulative number of detected sources with redshift \( z \gtrsim 8 \) within a circle of angular radius \( \theta = \xi/D_{\text{col}} \) centered on the lensing SIS. The dotted lines correspond to the numbers expected in the case where there is no lensing object. The dashed, solid, and dash-dot lines respectively show the effect of lensing by galaxies (dashed) depends strongly on the flux threshold of the instrument used for the survey, as well as on the redshifts and the minimum intrinsic specific luminosity of the background sources. A deep field survey carried out by the HST (\( \frac{df}{d\nu_0} \sim 9.12 \text{ nJy} \)) is expected to observe a surplus of galaxies with redshifts \( z \gtrsim 8 \) near a lensing object, assuming an intrinsic minimum luminosity of \( L_{\text{int}} = 10^{39} \text{ erg s}^{-1} \text{ Hz}^{-1} \) (top panel of Fig. 3). Deep field surveys conducted by the JWST (\( \frac{df}{d\nu_0} \sim 3.61 \text{ nJy} \)) are also expected to observe a surplus of galaxies with redshifts \( z \gtrsim 10 \) due to the lensing effect (top panel of Fig. 4). However, in the case where this limiting specific luminosity is an order-of-magnitude larger, \( L_{\text{int}} = 10^{40} \text{ erg s}^{-1} \text{ Hz}^{-1} \), the surveys will detect a deficit in source counts relative to the numbers observed in a blank field (Fig. 3 & Fig. 4, bottom panel).

This transition from a positive to a negative magnification bias as one varies the lower bound on the luminosity function can be seen explicitly in Figures 6-7. These plots depict the expected source count integrated over the range of angular distances \( \theta \in [\theta_E/2, 2\theta_E] \) from the lens center, as
Figure 3. Predictions for observations by HST’s WFC3, with a limiting flux sensitivity of $df/d
u_0 \sim 9.12$ nJy and sources at $z \gtrsim 8$. Left Panel: Radial dependence of the number of detected sources with redshifts above $z = 8$ in concentric annular cells centered on a lens at $z_L = 0.5$. Right panel: Cumulative number of detected sources with redshift $z \gtrsim 8$ within a circle of angular radius $\theta = \xi/D_{\text{ol}}$ centered on the lensing SIS. The dotted lines correspond to the numbers expected in the case where there is no lensing object. The dashed, solid, and dash-dot lines respectively show the effect of lensing by galaxies ($\sigma_v = 200$ km/s), groups ($\sigma_v = 500$ km/s), and clusters ($\sigma_v = 1000$ km/s) modeled as singular isothermal spheres with corresponding Einstein angles of $\theta_E \approx 0.015'$, $\theta_E \approx 0.095'$, and $\theta_E \approx 0.38'$. A minimum intrinsic specific brightness of $L_{\text{int}} = 10^{27}$ and $10^{28}$ erg s$^{-1}$ Hz$^{-1}$ is assumed in the top and bottom panels, respectively.

\[ \frac{df}{d\nu_0} = 9.12 \text{ nJy}, \quad L_{\text{int}} = 10^{27} \text{ erg s}^{-1} \text{ Hz}^{-1} \]

\[ \frac{df}{d\nu_0} = 9.12 \text{ nJy}, \quad L_{\text{int}} = 10^{28} \text{ erg s}^{-1} \text{ Hz}^{-1} \]
The dashed, solid, and dash-dot lines respectively show the effect of lensing by galaxies (clusters) at redshifts $z > 10$ within a circle of angular radius $\theta = \xi/D_{\text{L}}$ centered on the lensing SIS. The dotted lines correspond to the numbers expected in the case where there is no lensing object.

If focus is instead restricted to sources with redshifts $z > 10$, a slightly different trend emerges. Instruments such as the HST, operating with a flux threshold of 9.12 nJy, detect a surplus of sources in the region transitions from positive to negative at a value of $L_{\text{int}} \approx 5 \times 10^{27} \text{ erg s}^{-1} \text{Hz}^{-1}$ (left panel).

If focus is instead restricted to sources with redshifts $z > 10$ (Figure 7), a slightly different trend emerges. Instruments such as the HST, operating with a flux threshold of 9.12 nJy, detect a surplus of sources in the region transitions from positive to negative at a value of $L_{\text{int}} \approx 5 \times 10^{27} \text{ erg s}^{-1} \text{Hz}^{-1}$ (left panel).
(eq. 13), relative to that of the characteristic specific luminosity, \( L_\star(z) \) (Figure 1). In the Schechter function, \( L_\star(z) \) is the luminosity at which the power-law form of the function cuts off and the number of galaxies plummets exponentially with intrinsically brighter luminosities. If the instrumental detection threshold limits the observer to the exponential drop-off region of the function (\( L_{\text{det}} > L_\star \)), the magnification effect of gravitational lensing will drastically increase the number of observed sources when it pushes the detection threshold back to smaller values of \( L \). This is the case for observations made of galaxies at redshifts \( z > 8 \) and \( z > 10 \) by instruments such as the HST, \( \text{(df/d}\nu_0 \approx 9.12 \text{ nJy)} \); consequently, a surplus is detected as long as \( L_{\text{int}} \) is not too much larger than \( L_\star \) (Figures 6 & 7: right panel). This is also the case for instruments observing galaxies at redshifts \( z > 10 \) with a flux threshold that is a factor of three smaller, such as the JWST (\( \text{(df/d}\nu_0 \approx 3.61 \text{ nJy)} \) (left panel of Figure 7). On the other hand, if the instrumental detection threshold places the observer in the power-law region of the Schechter function, where the source count is flatter, pushing the threshold back to smaller values of \( L \) with gravitationally lensing does not result in the inclusion of a substantial population of sources that was otherwise undetectable. Therefore, in such instances, the diluting effect of
Figure 6. Expected number of sources with redshift \( z > 8 \) integrated over the region \( \theta_E/2 \leq \theta \leq 2\theta_E \) as a function of the minimum intrinsic brightness, \( L_{\text{int}} \), of the background sources. The solid and dashed lines respectively show the expected numbers with and without the lensing effect. The dash-dot line represents the difference in the detected number of sources in the lensed case vs. the unlensed case. The left panel shows the numbers expected to be observed given a flux threshold of 3.61 nJy in the case where galaxies (top), groups (middle), and clusters (bottom) are used as lenses with corresponding Einstein angles \( \theta_E \approx 0.015', 0.095', \) and \( 0.38' \). The right panel shows results given a flux threshold of 9.12 nJy.

\[
\frac{df}{d\nu_0} = 3.61 \text{ nJy, } \sigma_v = 200 \text{ km/s}
\]

\[
\frac{df}{d\nu_0} = 9.12 \text{ nJy, } \sigma_v = 200 \text{ km/s}
\]

\[
\frac{df}{d\nu_0} = 3.61 \text{ nJy, } \sigma_v = 500 \text{ km/s}
\]

\[
\frac{df}{d\nu_0} = 9.12 \text{ nJy, } \sigma_v = 500 \text{ km/s}
\]

\[
\frac{df}{d\nu_0} = 3.61 \text{ nJy, } \sigma_v = 1000 \text{ km/s}
\]

\[
\frac{df}{d\nu_0} = 9.12 \text{ nJy, } \sigma_v = 1000 \text{ km/s}
\]
Figure 7. Expected number of sources with redshift $z \geq 10$ integrated over the region $\theta_E/2 \leq \theta \leq 2\theta_E$ as a function of the minimum intrinsic brightness, $L_{int}$, of the background sources. The solid and dashed lines respectively show the expected numbers with and without the lensing effect. The dash-dot line represents the difference in the detected number of sources in the lensed case vs. the unlensed case. The left panel shows the numbers expected to be observed given a flux threshold of 3.61 nJy in the case where galaxies (top), groups (middle), and clusters (bottom) are used as lenses with corresponding Einstein angles $\theta_E \approx 0.015'$, 0.095', and 0.38'. The right panel shows results given a flux threshold of 9.12 nJy.
lensing wins out and a deficit in source counts is consistently observed (left panel of Figure 6).

In addition to the sensitivity of the magnification bias on the value of $L_{\text{int}}$, these plots also demonstrate the instances in which gravitational lensing improves the detection of high redshift sources as well as the magnitude of the bias in each case. When using JWST ($df/d\nu_0 \sim 3.61$ nJy), the number of detected sources with redshift $z \gtrsim 10$ is expected to increase by a factor of nearly 5 due to gravitational lensing by another galaxy, group, or cluster (assuming the limiting intrinsic luminosity is less than $\sim 3 \times 10^{27}$ erg s$^{-1}$ Hz$^{-1}$).

The lensing effect is expected to play an even greater role in enhancing the number of detected sources with redshift $z \gtrsim 8$ when the observations are made with an instrument such as the HST ($df/d\nu_0 \sim 9.12$ nJy), in which case, the numbers increase by a factor of nearly 200. The effect is slightly less pronounced when considering galaxies at redshifts $z \gtrsim 10$ when the observations are made with an instrument such as the HST ($df/d\nu_0 \sim 9.12$ nJy), in which case, the numbers increase by a factor of 3.5 for HST (assuming $L_{\text{int}} \lesssim 5 \times 10^{27}$ erg s$^{-1}$ Hz$^{-1}$) while reducing the number of sources detected by JWST relative to that of a blank field.

4 DISCUSSION

In this paper, we studied the effects of gravitational lensing on the source count of high redshift objects as observed by both the HST Frontier Fields and JWST. Although lensing magnifies the background sources, effectively lowering the flux threshold above which they can be detected, it simultaneously dilutes the apparent number density of sources on the sky. We found that the details of whether the number counts of distant background sources seen through a foreground gravitational lens are enhanced or reduced depends on several parameters characterizing the system. Modeling the lens as a singular isothermal sphere residing at a redshift $z_L = 0.5$, we explored how the magnification bias varied with the velocity dispersion of the lens ($\sigma_v$), the angular distance from the lens ($\theta$), the photometric sensitivity of the instrument ($df/d\nu_0$), the redshift of the background source population, and the minimum intrinsic specific luminosity characterizing the population ($L_{\text{int}}$). In the region of space lying between half and twice the Einstein angle, $\theta_E/2 \lesssim \theta \lesssim 2\theta_E$, (measured from the lens center), we found that the overall bias transitions from positive to negative in some instances as $L_{\text{int}}$ increases. In those cases where a transition does take place, the upper bounds on $L_{\text{int}}$ to detect a positive magnification bias are $5 \times 10^{27}$ (Fig. 6, right panel) and $3 \times 10^{27}$ erg s$^{-1}$ Hz$^{-1}$ (Fig. 7, left panel) when observing sources with redshifts $z \gtrsim 8$ and $z \gtrsim 10$ respectively. Gravitational lensing may therefore offer an alternative way of constraining the velocity dispersion of the lens ($\sigma_v$) or group ($\sigma_c$), with a photometric sensitivity of 9.12 nJy, in which case, the numbers are increased by a factor of nearly 5 due to gravitational lensing, effectively lowering the number of detectable sources by a factor of approximately 200.

Due to their large reservoirs of gravitationally bound dark matter, galaxy clusters ($\sigma_v = 1000$ km s$^{-1}$) proved to be the most effective among the cosmic lenses considered in this paper, significantly magnifying the brightness and sizes of background galaxies and resulting in a noticeable enhancement of the source count. Galaxy-group and galaxy-galaxy lensing also result in a positive magnification bias; however, given the low number of resolved sources behind a single one of these less massive cosmic lenses, source counts measured behind a foreground galaxy ($\sigma_v = 200$ km s$^{-1}$) or group ($\sigma_c = 500$ km s$^{-1}$) suffer from a weaker lensing signal. In these cases, stacking images of several foreground galaxies or groups together will significantly improve the lensing signal and increase the effective number of resolved sources. The measured distribution function of the velocity dispersion in galaxies (Bernardi et al. 2010) yields the relative number of SIS lenses with a given value of $\sigma_v$ that one would expect to find in a survey volume. This can in turn be used to determine the number of images that need to be stacked to obtain the target lensing rate. Out of a sample of a thousand galaxies, we expect a few hundred lensed images with redshifts $z \gtrsim 10$ to be observed with JWST and tens of lensed images with HST (Fig. 7, top panel).

5 ACKNOWLEDGEMENTS

We thank Dan Stark for helpful comments on the manuscript. This work was supported in part by NSF grant AST-0907890 and NASA grants NNX08AL43G and NNA09DB30A.

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