Primordial Perturbations During a Slow Expansion

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Recently, it has been shown that a slow expansion, which is asymptotically a static state in infinite past and may be described as an evolution with $\epsilon \ll -1$, of early universe may lead to the generation of primordial perturbation responsible for the structure formation of observable universe. However, its feasibility depends on whether the growing mode of Bardeen potential before phase transition can be inherited by the constant mode of curvature perturbation after phase transition. In this note, we phenomenally regard this slow expansion as that driven by multi NEC violating scalar fields. We calculate the curvature perturbation induced by the entropy perturbation before phase transition, and find that the spectrum is naturally scale invariant with a slight red tilt. The result has an interesting similarity to that of slow roll inflation.

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The results of recent observations \cite{1} are consistent with an adiabatic and nearly scale invariant spectrum of primordial perturbations, as predicted by the simplest models of inflation. The inflation stage is supposed to have taken place at the earlier moments of the universe \cite{2,3,4}, which superluminally stretched a tiny patch to become our observable universe today. During the inflation the quantum fluctuations in the horizon will be able to leave the horizon and become the primordial perturbations responsible for the formation of cosmological structure \cite{5,6}. This is one of remarkable successes of inflation. However, this success is also shared by an expansion with the null energy condition (NEC) violation, which corresponds to e.g. the phantom inflation \cite{7,8,9,10,11}, see also Ref. \cite{12} for comments. The reason is that the inflation can be generally regarded as an accelerated or superaccelerated stage, and so may defined as an epoch when the slow Hubble length decreases, which occurs equally during the NEC violating expansion. It is the shrinking of this comoving Hubble length that leads the causal generation of primordial perturbations.

The primordial perturbation generated during the NEC violating evolution has been studied in Ref. \cite{7,8}. In Ref. \cite{7}, it was firstly noticed that there is an interesting limit case in which $\epsilon \ll -1$, where $\epsilon$ is defined as $-\dot{h}/h^2$ and $h$ is the Hubble parameter, which corresponds to that the scale factor grows very slowly but the Hubble length rapidly shrinks. During the slow expansion the primordial perturbation can be generated, see Fig.1. The end of slow expanding phase may be regarded as a reheating process or phase transition that the fields dominating the background decay into usual radiation, which then will be followed by a FRW evolution of standard cosmology. We found that the spectrum of Bardeen potential $\Phi$ before the transition is dominated by an increasing mode and is nearly scale invariant \cite{7}. Though during this period the spectrum of comoving curvature perturbation $\xi$ is strong blue, if the growing mode of spectrum of Bardeen potential before the transition may be inherited by the constant mode of $\xi$ after the transition, which is similar to the case \cite{13,14,15} of the ekpyrotic/cyclic scenario \cite{16,17}, the spectrum of result-

ing adiabatic fluctuations appearing at late time will be scale invariant. However, it is obvious that the result is severely dependent of whether this inheriting can occur, which is actually determined by the physics at the epoch of phase transition. Thus there is generally an uncertainty. In the simple and conventional scenario it seems that the growing mode of $\Phi$ can hardly be matched to the constant model after the transition \cite{18,19,20}, which has been shown by some numerical studies \cite{21,22,23}. Further, it has been illuminated \cite{12,24} that whether the final spectrum is that of the growing mode before the transition depends on whether there is a direct relation between the comoving pressure perturbation and $\Phi$ in the energy momentum tensor, in which the new physics mastering the transition might be encoded. Thus with these points it seems that though whether the nearly scale invariant primordial perturbation may be generated during a slow expansion of early universe is still open, the possibility remains.

The slow expansion with $\epsilon \ll -1$ may have some interesting applications in cosmology. For example, the semiclassical studies of island universe model, in which initially the universe is in a cosmological constant sea, then the local quantum fluctuations with the NEC violation will create some islands with matter and radiation, which under certain conditions might correspond to our observable universe \cite{25,26,27}. Thus with the debate whether the scale invariant spectrum of curvature perturbation may be obtained during such a slow expansion, the study of relevant issues is quite interesting. Note that in Ref. \cite{7}, we adopt the work hypothesis that the NEC violating phase with $\epsilon \ll -1$ is implemented by a scale field with the NEC violation, in which the scalar field has a reverse sign in its dynamical terms. Thus it may be conceivable that our hypothesis and simplified operation in the calculations of primordial perturbation spectrum might have missed what. In this paper, we will study a slight nontrivial case, in which the slow expansion with $\epsilon \ll -1$ is simulated phenomenally by that driven by multi scalar fields with the reverse sign in their dynamical terms. We find that the spectrum of entropy perturbation is scale invariant with a slight red tilt. The
curvature perturbation under certain conditions may be induced by the entropy perturbation, and thus may has the same spectral index with the entropy perturbation. We show that the spectrum and amplitude of curvature perturbation induced by the entropy perturbation at the end epoch of the NEC violating phase can be related to those of inflation by a dual invariance.

Firstly, let us briefly review the results of Ref. [7]. For a slow expansion with the NEC violation, the evolution of scale factor $a(t)$ may be simply taken as

$$a(t) \sim \frac{1}{(-t)^n} \sim (-\eta)^{-\frac{n}{n+1}},$$

(1)

where $n < 1$ is a positive constant. When $t$ is initially from $-\infty$ to $0_-$, it corresponds to a slow expansion. The Hubble parameter is

$$h = \frac{n}{(-t)^n}, \quad \dot{h} = \frac{n}{(-t)^{n+1}},$$

(2)

thus $\epsilon = -1/n < -1$. The $\epsilon$ can be rewritten as $\epsilon \approx \frac{1}{n} \frac{\partial V}{\partial \dot{\phi}}$, thus in some sense $\epsilon$ actually describes the change of $h$ in unit of Hubble time and depicts the abrupt degree of background background. From Eq. (2), during the slow expansion, though the scale factor is hardly changed, the Hubble parameter will rapidly increase, which means an abrupt change of background. In Ref. [7], it was shown that when the slow expansion is implemented by a scalar field with a reverse sign in its dynamical term, the spectral index of Bardeen potential $\Phi$ is given by

$$n_{\Phi} - 1 \simeq 2n,$$

(3)

which is nearly scale invariant with a slightly blue tilt. When the optimistic matching of the growing mode of $\Phi$ before the phase transition to the constant model of $\xi$ after the phase transition can be made, the amplitude after the end of slow expanding phase is given by [26]

$$P(\Phi - \xi) \simeq \frac{1}{n} \left( \frac{h_e}{2\pi} \right)^2$$

(4)

where $G = 1$ has been set and the subscript ‘e’ denotes the end epoch of slow expansion.

Then let us see what occurs when the slow expansion is simulated phenomenally by that driven by two or more NEC violating scalar fields with the reverse sign in their dynamical terms. In this case there is not only the curvature perturbation but the entropy perturbation. No loose generality, we will study the case with two scalar fields $\varphi_1$ and $\varphi_2$. Note that there exists a scale solution in which $\dot{\varphi}_1/\dot{\varphi}_2$ is a constant. In this case, the background values of all relevant quantities of fields can be determined simply. We may write $\dot{\varphi}_1$ and $\dot{\varphi}_2$ as

$$\dot{\varphi}_1 = \sqrt{\frac{n_1}{4\pi}} \frac{1}{(-t)} \dot{\varphi}_2 = \sqrt{\frac{n_2}{4\pi}} \frac{1}{(-t)} \dot{\varphi}_2,$$

(5)

where both $n_1$ and $n_2$ are positive constants. When $n_1 + n_2 = n$ is taken, where $n$ is given by Eq. (1), we may have

$$V(\varphi_1, \varphi_2) = \frac{n(3n+1)}{8\pi} \frac{1}{(-t)^2},$$

(6)

which can be obtained by combining Eqs. (2) and (5) and Friedmann equation. We see that for arbitrary value $n > 0$, $V(\varphi)$ is always positive, which is different from that of the usual scalar field, in which when $n < 1/3$, the potential must be negative [32, 33]. The reason is that here what we use is the scalar fields with the reverse sign in their dynamical terms. Integrating (5), and substituting the result obtained into (6), we can split the effective potential (6) into two parts for $\varphi_1$ and $\varphi_2$, respectively,

$$V(\varphi_1) = \frac{n_1(3n+1)}{8\pi} \exp \left(-\sqrt{\frac{16\pi}{n_1}} \varphi_1 \right),$$

(7)

$$V(\varphi_2) = \frac{n_2(3n+1)}{8\pi} \exp \left(-\sqrt{\frac{16\pi}{n_2}} \varphi_2 \right).$$

(8)

Thus both fields are decoupled. Note that $n \ll 1$, thus $n_1, n_2 \ll 1$, Eqs. (7) and (8) suggests that the potential

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1. When $t$ approaches negative infinity, we have $h \rightarrow 0$, which means that the universe is asymptotically a static state in infinite past. This in some sense is similar to an emergent universe studied in Ref. [28], see also [24, 34], in which the initial static state is constructed by introducing a positive curvature. However, here it corresponds to be implemented by using a scalar field with the NEC violation, in which the initial kinetic energy of scalar field just approximately sets off its potential energy.
of both $\varphi_1$ and $\varphi_2$ are very steep. During the slow expansion, they will climb up along their potentials, which is determined by the property of the NEC violating field, e.g. 34, 35. In this case, it may be showed that this scale solution is an attractor, e.g. see Ref. 36.

Before calculating the primordial perturbation, we need to decompose these two fields into the field $\varphi$ along the field trajectory, and the field $s$ orthogonal to the trajectory by making a rotation in the field space as follows

$$\varphi = \sqrt{n_1} \varphi_1 + \sqrt{n_2} \varphi_2, \quad s = \sqrt{n_2} \varphi_2 - \sqrt{n_1} \varphi_1, \quad (9)$$

as has been done in Ref. 37. In this case, the potential perturbation amplitude is negligible on large scale. While the entropy and (4), or the spectrum will be strong blue, whose amplitude can be given by

$$\delta s_k \equiv v_k/a$$

has been defined and the prime denotes the derivative with respect to the conformal time, and $f(\eta)$ is generally given by

$$f(\eta) = \frac{\alpha''}{a} + \mu^2(s)a^2$$

$$\simeq \frac{2 + 3n}{\eta^2}, \quad \text{for } n \simeq 0, \quad (13)$$

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where Eqs. (11) and $1/\eta = (1 + 1/n)\alpha$ has been used. Note that the right side of the first line in Eq. (13) is the plus between two terms, but not minus as usual, which is actually the result using the fields with the reverse sign in their dynamical terms.

The solutions of Eq. (12) are Hankel functions. In the regime $k\eta \rightarrow \infty$, all interesting modes are very deep inside the horizon of the slow expanding phase, thus Eq. (12) can be reduced to the equation of a simple harmonic oscillator, in which $v_k \sim e^{-i k \eta / (2k)^{1/2}}$, which in some sense suggests that the initial condition can be taken as usual Minkowski vacuum. In the superhorizon scale, i.e. $k \eta \rightarrow 0$, in which the modes become unstable and grow, the expansion of Hankel functions to the leading term of $k$ gives

$$v_k \sim \frac{1}{\sqrt{2k}} (-k \eta)^{\pm v}, \quad (14)$$

where $v \simeq 3/2 + n$, which may be deduced from Eq. (13), and here the phase factor and the constant with order one have been neglected. During the slow expansion the change of Hubble parameter $h$ is quite abrupt, as has been pointed out. Thus it may be expected that the perturbation amplitude of $v_k$ will continue to change after the corresponding perturbations leaving the horizon, up to the end of the slow expanding phase. This can also be explained as follows. When $k \eta \rightarrow 0$, which corresponds to the superhorizon scale, we have $v_k' - (2 + 3n)v_k/\eta^2 \simeq 0$. This equation has one growing solution and one decay solution. The growing solution is given by $v_k \sim a^{1/n}$, where Eq. (11) has been used. The scale factor $a$ is nearly unchanged, but since $n \simeq 0$, the change of $v_k$ has to be significant, thus generally one can not obtain that the $|\delta s_k| = |v_k/a| \sim a^{1/n}$ is constant, which actually occurs only for the slow roll inflation in which approximately $n \rightarrow \infty$. The details can also be seen in Ref. 38, in which the spectrum of a test scalar field not affecting the evolution of background was calculated, which in some sense corresponds to the case of $n_2 = 0$ here. This suggests that in principle we should take the value of $v_k$ at the time when the slow expansion ends to calculate the amplitude of perturbations. Thus the perturbation spectrum is

$$k^{3/2} \left| \frac{v_k(\eta_0)}{a} \right| \sim k^{3/2 - v}, \quad (15)$$

which suggests that the spectrum index is given by $n_s - 1 \equiv 3 - 2v$. This leads to

$$n_s - 1 \simeq -2n, \quad (16)$$
which means that during the slow expansion the spectrum of entropy perturbation is nearly scale invariant with a slightly red tilt, since \( n \simeq 0.1 \). This result is only determined by the evolution of background during the slow expansion, but not dependent of other details.

We can see that if \( |\epsilon| \sim 10^4 \), the spectrum of entropy perturbation may be very naturally matched to recent observation \([1]\), since \( n \equiv 1/|\epsilon| \sim 0.01 \). Thus it may be interesting to consider how these entropy perturbations can be responsible for the structure formation of observable universe. To do so, we need to make the curvature perturbation at late time have an opportunity to inherit the characters of entropy perturbation generated during the slow expansion. This can be accomplished by noting that the entropy perturbation sources the curvature perturbation

\[
|\xi| \simeq \frac{h_\epsilon}{\dot{\varphi}} \delta s \quad (17)
\]

on large scale \([37]\), where \( \theta \equiv \arctg \sqrt{\frac{n_{s}}{n_{1}}} \) depicts the motion trajectory of fields in field space of \( \varphi_1 \) and \( \varphi_2 \), see Eq. (10). When \( \theta \) is a constant, the trajectory is a straight line. In this case, \( \dot{\theta} = 0 \), thus the entropy perturbation is decoupled from the curvature perturbation, which also assures the validation of Eq. (12), or there will be some terms such as \( \sim \dot{\theta}^2 \) and \( \sim \dot{\theta} \Phi \). However, if there is a sharp change of field trajectory, \( \dot{\theta} \) must not be equal to 0, in this case \( \xi \) will inevitably obtain a corresponding change induced by \( \delta s \) by Eq. (17), as has been pointed out and applied in ekpyrotic model \([39, 40]\), see also earlier Refs. \([41, 42]\) and recent studies \([43]\) on the ekpyrotic collapse with multiple fields. It may be expected that at the end epoch of slow expanding phase the scale solution will generally be broken, which also actually may be constructed by modifying the potential of fields around the end. For example, around the end epoch, instead of being steep, the potential of one of fields will has a maximum or a plateau, which will lead to the rapid stopping of up climbing of corresponding field, while the up climbing of another field remains, note that here the motion of field is mainly managed by its potential, see e.g. Refs. \([34, 35]\). In this case, the entropy perturbation will be able to source the curvature perturbation.

We assume, for a brief analysis, that at split second before the end of slow expanding phase the motion of \( \varphi_2 \) rapidly stops while \( \varphi_1 \) remains, and then the phase transition occurs and the universe quickly thermalizes into a radiation phase and evolve with standard FRW cosmology. Following Ref. \([39, 40]\), this corresponds to a sharp change from initial fixed value \( \theta_\epsilon = \arctg \sqrt{\frac{n_{s}}{n_{1}}} \) to \( \theta \simeq 0 \).

It is this change that leads that \( \xi \) acquires a jump induced by the entropy perturbation and thus inherits the nearly scale invariant spectrum of the entropy perturbation. In the rapid transition approximation, one has obtained

\[
|\xi| \simeq \frac{h_\epsilon}{\dot{\varphi}} \delta s \simeq \frac{h_\epsilon}{\dot{\varphi}} \delta s, \quad (18)
\]

where the constant factor with order one have been neglected. From Eq. (15), the amplitude of entropy perturbation can be calculated at the time when the slow expansion ends and given by

\[
k^{3/2} | \frac{v_k(\eta_e)}{a} | \simeq \frac{1}{n} \left( \frac{h_\epsilon}{2\pi} \right)^2, \quad (19)
\]

where \( n \ll 1 \) has been used. The calculations are similar to that done in Ref. \([33]\). The prefactor \( 1/n \) is from the relation \( 1/\eta_e = (1 + 1/n)a_0h_\epsilon \), which corresponds the \( g \) factor introduced and discussed in Ref. \([38]\). Note that \( h_\epsilon^2/\dot{\varphi}^2 \simeq -1/\epsilon = n \), thus we have the amplitude of curvature perturbation

\[
P_{\xi} \simeq \left( \frac{h_\epsilon}{\dot{\varphi}} \right)^2 k^3 \left| \frac{v_k(\eta_e)}{a} \right|^2 \simeq \frac{1}{n} \left( \frac{h_\epsilon}{2\pi} \right)^2. \quad (20)
\]

We can see that this result is the same as Eq. (4) in form, only up to a numerical factor with unite order. Thus for the slow expanding phase with \( n \ll 1 \) or equally \( \epsilon \ll -1 \), it seems that whether induced by the increasing mode of Bardeen potential, or by the entropy perturbation before the phase transition, the resulting curvature perturbations after phase transition is nearly scale invariant with the amplitude described by the same equation at least in form. Though this dose not means that the scalar spectrum of slow expansion must be scale invariant, it seems at least that there are some convinced possibilities that it may be.

The amplitude of usual slow roll inflation models with \( \epsilon \simeq 0 \) may generally written as

\[
P_{\xi} \simeq \left( \frac{1}{\epsilon} \left( \frac{h_\epsilon}{2\pi} \right)^2 \right) \quad (21)
\]

For \( \epsilon \) approximately being a constant, which corresponds to the case of scale solution in which the inflation is driven by the scalar field with an exponent potential, the spectra index is given by \( n_s - 1 \simeq -2\epsilon \). Thus we can see that they can be related to Eqs. (20) and (10) by making a replacement \( |\epsilon| \rightarrow n \). During the slow expansion the spectral index of curvature perturbation induced by the increasing mode of Bardeen potential is given in Eq. (4), which is slightly blue tilt. Thus for this case, it is also suitable for above replacement. This replacement may be regarded as a dual transformation between their background evolution, i.e. between the nearly expony expanding expansion with \( n \rightarrow \infty \), since here \( |\epsilon| \equiv 1/n \), and the slow expansion \( n \simeq 0 \). This extends the studies on the dualities of the primordial density perturbation in Refs. \([44, 45, 46, 47]\) and see also recent Ref. \([48]\).

In summary, we phenomenally regard the slow expansion with \( \epsilon \ll -1 \) as that driven by multi NEC violating

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2 see also Refs. \([44, 50, 51, 52]\) for the discussions on the dualities of scale factor.
fields. We calculate the curvature perturbation induced by the entropy perturbation before phase transition, and find that the spectrum is naturally scale invariant with a slight red tilt, which may fit recent observations well. This result to some extent highlights the fact again that a slow expansion, which may be described as an evolution in infinite past, before usual FRW evolution may be feasible for seeding of the primordial perturbation responsible for the structure formation of observable universe. Though we still lack of understandings on some aspects of the phenomena with the NEC violation, which might be quantum, we think that this work, regarded as a semiclassical and effective description, might in some sense have captured some basic ingredients of the NEC violating evolution of early universe, which may be interesting and significant for many applications in cosmology.

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