Study on the Modification on DS Theory of Evidence Based on Evidence Support Level

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Abstract. To avoid the unreasonable evidence theory caused by the high conflict evidence, the modification on DS Theory of Evidence (DSTE) based on evidence support level is proposed in this paper. First of all, various modifications on DS theory of evidence are generally analyzed. On this basement, the novel conflict information weighted allocation method based upon evidence support level is presented in detail. It realizes the unification of subjective objective diagnostic knowledge and the uncertain information, which is able to settle the fusion failure caused by the evidence conflict.

Introduction

The DS Theory of Evidence (DSTE), which is proposed by Dempster and Shafer, is a newborn method for uncertain information processing \cite{1}. DSTE is able to satisfy the axiomatics weaker than probability theory, therefore, it can distinguish the differences between uncertain and unknown, and make processing on the uncertainty caused by unknown. The typical character of DSTE is that it uses the interval estimation for description of uncertain information, so that the processing on incomplete data information is highly flexible and the inference form is easy, which has been applied in the field of information fusion diagnosis. DSTE can better describe the uncertain information and possess applicable inference rules, however, it also has some problems, which is shown as follows \cite{2}: (1) The adequacy and importance of evidence in inference are not considered, instead, it only simply make combination of basic confidence allocation function according to each evidence; (2) The condition of evidence synthesis is too strict. During the synthetizing, the focal element increases in exponential level so that the computation rises sharply. However, the synthetizing rule demands the mutual independent between evidences, which is hardly ensured in reality; (3) In the situation of completely conflict and highly conflict of evidence, the synthetizing rule in DSTE tends to be failed or to achieve unreasonable result leading to the failure of diagnosis.

To settle the above disadvantages, based on the deep study on tradition DSTE, the novel modification is presented based on evidence support level in this paper.

Traditional Modifications on DSTE

Since the synthetizing rule of DSTE has always been the key and difficult issues in evidence theory, it has close relations with the application of evidence theory. For the synthetizing of highly conflict and completely conflict evidences, simply denying all conflict evidences or denying some of the conflict evidences may cause the incorrect synthetizing result. Therefore, it is necessary to settle the problem of multiple evidence effective fusion in the situation of highly conflict and completely conflict. Generally speaking, the modifications can be divided into two kinds \cite{3-4}; one is to consider that the synthetizing rule is correct so that the evidence needs to be revised, such as Murphy method; The other is to consider that the synthetizing rule is incorrect so that the rule needs to be revised, such as Yager method. The key problem is the existence of conflict information. However, the conflict
information is too complicated. The above two categories actually process the conflict information from various starting point.

**Murphy Method**

The general thought of Murphy method is detailed as follows. Since the DSTE synthetizing rule has solid mathematic foundation, it is not wrong itself. In the situation of high conflict and complete conflict evidence, the conflict data needs to be preprocessing firstly. After this procedure, the DSTE synthetizing can be used. The Murphy method is the typical one that keeps the synthetizing rule and revising the evidence. In reality, it is an algorithm of average synthetizing of evidence. Assume that there are n evidences in the system. Firstly, the basic confidence allocation functions of the evidences are averaged. The new built evidences are then synthetized by DSTE synthetizing rule for n-1 times. Compared with traditional DSTE, Murphy is able to handle conflict evidence and the convergence rate is quickly. The result is better than that achieved by traditional DSTE. Meanwhile, this method remains the exchange law and binding law in the synthetizing rules. However, the key problem is how to reasonably divide the confidences provided by evidence to the proposition. Simply averaging the basic confidence allocation functions of multiple evidences may cause the following issues: (1) Theoretically, it is unreasonable to eliminate conflict by averaging basic confidence allocation functions of multiple mutual independent evidence, which may delete the differences and relations between each evidence. It is not accord with the existed conflict which is lack of conviction; (2) The averaging process is equal to each evidence, however, the importance and the contribution of every evidence during the real decision creation to the finally fused result are different; (3) The averaging process leads to the losing of various important performances during the synthetizing of independent evidence, such as confidence raising, uncertainty decreasing, information growing and so on. Meanwhile, this progress is not accorded with the cognitive logic and subjective judgment.

**Lefevre and Yager Method**

The second kind of method considers that the unreasonable result using the DSTE synthetizing rule in the situation of highly conflict evidence is caused by the normalization procedure of the rule. The keynote to make modification on the synthetizing rule is to make new allocation of conflict evidence. It always contains two aspects: (1) Which subsets should be provided allocated conflict information; (2) Which is the allocation rates for each subset after insurance of subsets. Comparably, the second kind of method cancels the normalization procedure in the synthetizing rule and only makes allocation of conflict information. Although it loses the exchange law and binding law, the differences and relations between evidences have been remained and the synthetizing of evidence is accorded with the human cognitive logic. However, the calculation of the weights is still a hard problem. Only depending on human subjective judgment can hardly fulfill the needs for multiple diagnosis Agent decision creation. Lefevre has presented a method for achieving weighs by gradient descent auto-learning algorithm, however, it is hard to get the accurate weights with limited evidences.

Yager has considered that the confidence provided by conflict information is not enough although the proposition confidence has been provided by various evidences, which seems to be non-meaningful for the synthetizing result. Therefore, Yager has canceled the normalization procedure during the synthetizing rule. The confidence provided by conflict information has been all brought to unknown fields for judging with new evidences. This synthetizing rule is suitable for the situation with completely conflict information and highly conflict information, but it still has some problems.

The basic mind of Yager modification method is to bring the conflict evidence into unknown fields, which leads to the low basic confidence allocation after synthetizing of the initial conflict evidence [5]. The initial basic confidence allocation which is almost equal to 1 has been allocated to $\Theta$, leading to the completely denying of conflict evidence. Although there are large amount of evidences, the fusion result is not satisfying. Meanwhile, similar to synthetizing rule in DSTE, Yager method also has a veto authority. During the real multi-Agent diagnosis, the evidence may have large error with the
real status of diagnosis object based on diagnosis theory because of the limit of knowledge and ability of single Agent diagnosis. Sometime, the result may even be opposite to the real status. It is not allowed to achieve a wrong result by the Agent diagnosis leading to the whole system hardly working.

Generally speaking, these methods cannot fundamentally handle the problem of conflict evidence, or bring new issues during settling the initial problems and cannot provide the obviously synthetizing process [6]. Based on the Yager, the distance function is introduced in this paper to evaluate the similarity between evidences for acquiring the support level for each evidence by others. And the support level rate is considered as the weight for conflict information allocation during the synthetizing evidence for realizing the reasonable allocation of conflict information.

**Modification of DSTE**

The modification proposed in this paper is detailed as follows. By the introduction of mutual support level between evidences distances and evidences, the weighted allocation method of conflict information based on evidence support level. The central idea is that when the every two evidences synthetizes, the conflict may be created. For improve the convergence rate and target identification rate, the basic confidence allocation function brought by conflict information is allocated within focal elements between two evidences. If the support level of some evidence provided by other evidences is higher, it is proved that this evidence is more confident and the weight of conflict information allocation is larger; If the support level of some evidence provided by other evidence is lower, it is proved that this evidence is less confident and the weight of conflict information allocation is smaller, so that it has little influence on the final result.

Assume that B and C are two independent evidences in the same framework Θ. The basic confidence allocation functions are $m_1$ and $m_2$. Focal elements are $B_1, \ldots, B_m$ and $C_1, \ldots, C_m$. The synthetizing rule of the weighted allocation method of conflict information based on evidence support level can be describe by Eq.(1) [7].

$$m(A) = \sum_{B_i \cap C_j} m_1(B_i)m_2(C_j) + \sum_{A_i \subseteq [\Theta \setminus \{B_i\} \cup \{C_j\}]_{\emptyset}} \left[ \omega m_1(A) m_2(C_j) + (1-\omega) m_1(B_i)m_2(A) \right]$$

where, $\omega = \frac{\sup(m_1)}{\sup(m_1) + \sup(m_2)}$, $1-\omega = \frac{\sup(m_2)}{\sup(m_1) + \sup(m_2)}$.

Proof: It only needs to proof that $m(A)$ is a basic confidence allocation.

It is easy to proof that $m(\emptyset) = 0$, and

$$\sum_{A \subseteq \Theta} m(A) = \sum_{A \subseteq \Theta} \sum_{B_i \cap C_j} m_1(B_i)m_2(C_j) + \sum_{A \subseteq \Theta} \sum_{A_i \subseteq [\Theta \setminus \{B_i\} \cup \{C_j\}]_{\emptyset}} \left[ \omega m_1(A) m_2(C_j) + (1-\omega) m_1(B_i)m_2(A) \right]$$

$$= \sum_{B_i \cap C_j} m_1(B_i)m_2(C_j) + \sum_{A \subseteq \Theta} \sum_{A_i \subseteq [\Theta \setminus \{B_i\} \cup \{C_j\}]_{\emptyset}} \left[ \omega m_1(A) m_2(C_j) + (1-\omega) m_1(B_i)m_2(A) \right]$$

$$= \sum_{B_i \cap C_j} m_1(B_i)m_2(C_j) + \sum_{A \subseteq \Theta} \sum_{A_i \subseteq [\Theta \setminus \{B_i\} \cup \{C_j\}]_{\emptyset}} \left( 1-\omega \right) m_1(B_i)m_2(A)$$

Since $A_i, A_j, B_i, C_j$ ergodic all values in $\Theta$, for $\forall i, j = 1,2,\ldots,|\Theta|$, $A_i$ is able to find $B_i$ for equal. Therefore, $A_i$ and $B_i$ is bijection. $B_i$ is used to instead $A_i$ in Eq.(2). Similarly, $C_j$ can also be used to instead $A_i$. Eq.(2) can be described by Eq.(3).

$$\sum_{A \subseteq \Theta} m(A) = \sum_{B_i \cap C_j} m_1(B_i)m_2(C_j) + \omega \sum_{A \subseteq \Theta \setminus \{B_i\} \cup \{C_j\}} m_1(A)m_2(C_j) + (1-\omega) \sum_{A \subseteq \Theta \setminus \{B_i\} \cup \{C_j\}} m_1(B_i)m_2(A)$$

$$= \sum_{B_i \cap C_j} m_1(B_i)m_2(C_j) + \sum_{A \subseteq \Theta \setminus \{B_i\} \cup \{C_j\}} m_1(A)m_2(C_j) + (1-\omega) \sum_{A \subseteq \Theta \setminus \{B_i\} \cup \{C_j\}} m_1(B_i)m_2(A)$$

$$= \sum_{B_i \cap C_j} m_1(B_i)m_2(C_j) + \sum_{A \subseteq \Theta \setminus \{B_i\} \cup \{C_j\}} \left( 1-\omega \right) m_1(B_i)m_2(A)$$

The definition of basic confidence allocation function, shown by Eq.(4):
\[
\sum_{B_i \subseteq \Theta} m_i(B_i) = 1, \quad \sum_{C_j \subseteq \Theta} m_j(C_j) = 1
\]  
(4)

Therefore, we can get Eq. (5).

\[
\sum_{A \subseteq \Theta} m(A) = \sum_{B_i \subseteq \Theta} m_i(B_i) m_j(C_j) = 1
\]  
(5)

As a result, the allocation function of the weighted allocation method of conflict information based on evidence support level is the basic confidence allocation function. The proof is finished.

Conclusions

To avoid the unreasonable evidence theory caused by the high conflict evidence, the modification on DS Theory of Evidence (DSTE) based on evidence support level is proposed in this paper. First of all, various modifications on DS theory of evidence are generally analyzed. On this basement, the initial DSTE is modified by the introducing of evidence distance, and the weighted allocation of conflict information based on evidence support level is detailed. The central idea is to introduce a distance function for evaluation the similarity between evidences for acquiring the similarity and the support level between evidences data and the support level rate is considered as the weight for conflict information allocation during evidence synthetizing. It realizes the unification of subjective objective diagnostic knowledge and the uncertain information, which is able to settle the fusion failure caused by the evidence conflict.

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