Multiphoton path entanglement by non-local bunching

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Multiphoton path entanglement is created without applying post-selection, by manipulating the state of stimulated parametric down-conversion. A specific measurement on one of the two output spatial modes leads to the non-local bunching of the photons of the other mode, forming the desired multiphoton path entangled state. We present experimental results for the case of a heralded two-photon path entangled state and show how to extend this scheme to higher photon numbers.

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Multiphoton path entangled states are superpositions of \( n \) photons in one out of two or more paths. Such states can be used to exceed the limitations imposed by the light wavelength. One example is quantum photolithography where the multiphoton interference of the different paths is used to define details on a special photoresist film which are \( 1/n \) finer than the diffraction limit. The photoresist should respond only to \( n \) photons or more, which is still an open challenge. Other uses are the enhancements of the resolution of interferometric measurements and atomic spectroscopy. In the context of interferometry, path entangled states are a subset of a more general group of usable photon-number correlated states.

Previously suggested methods to produce path entangled states require either large non-linearities, non-unitary operations, or include large statistical bottlenecks. Non-detection can be replaced by post-selection, that actually destroys the state. Furthermore, the schemes where \( n > 2 \) rely on the availability of various Fock states, which are difficult to produce. Two photons from parametric down-conversion can bunch to form a path entangled state, but this source is not expandable to larger photon numbers. Recently, a state of three path entangled photons was observed with post-selection through a bottleneck.

In this Letter we present a way to create multiphoton path entangled states without applying post-selection. The scheme relies on two unique quantum-mechanical phenomena: bunching (anti-bunching) of bosons (fermions) and non-locality. The former reflects the discreteness and symmetries of the quantum world. For example, it leads to the Hong-Ou-Mandel effect that two indistinguishable photons entering a beam-splitter simultaneously from both sides will always exit at the same output port. The latter implies that two (or more) distant particles can occupy a single quantum state and possess correlations which no classical theory can explain.

The scheme addressed in this Letter is based on the manipulation of multiphoton entangled states that originate from stimulated parametric down-conversion (PDC). By a specific measurement of one of the PDC output spatial modes, the photons of the other mode non-locally bunch and form the desired multiphoton path entangled state. It should be emphasized that because no detection is needed at the second PDC mode, the desired state is prepared by pre-selection. The detection in the first mode is used as a heralding signal that announces the creation of the path entangled state in the second mode.

We used non-collinear type-II parametric down-conversion with spatial and spectral filtering to create the following bi-partite state:

\[
|\psi\rangle = \frac{1}{\cosh^{2} \tau} \sum_{n=0}^{\infty} \sqrt{n+1} \tanh^{n} \tau |\psi_{n}^{+}\rangle , \quad (1a)
\]

\[
|\psi_{n}^{-}\rangle = \frac{1}{\sqrt{n+1}} \sum_{m=0}^{n} (-1)^{m} |m-n, m\rangle |m, n-m\rangle , \quad (1b)
\]

where \( |m, n\rangle \) represents \( m \) horizontally and \( n \) vertically polarized photons in mode \( i \). The magnitude of the interaction parameter \( \tau \) depends on the nonlinear coefficient of the crystal, its length and the intensity of the pump pulse. The state \( \psi \), as well as its individual terms \( \psi_{n}^{\pm} \) of different photon-pair number \( n \), are invariant under mutual rotations of the polarization basis of both spatial modes. The one-pair term \((n=1)\) is the familiar Bell state. We concentrate on the case when two indistinguishable photon pairs are produced \((n=2)\). It will be subsequently shown how the presented method extends to larger numbers of photons.

The two-pair normalized term contains three equally weighted elements:

\[
|\psi_{2}^{-}\rangle = \frac{1}{\sqrt{3}} (|2,0\rangle_{a}|0,2\rangle_{b} - |1,1\rangle_{a}|1,1\rangle_{b} + |0,2\rangle_{a}|2,0\rangle_{b} . \quad (2)
\]

The goal is to perform a measurement on mode \( a \) that will prepare mode \( b \) in a path entangled state. To achieve this, consider the setup presented in Fig. 4. The middle term of Eq. 2 has one horizontally and one vertically polarized photon in mode \( a \) as well as in mode \( b \). The photons in mode \( a \) are separated by a polarizing beam splitter (PBS). A \( \lambda/2 \) waveplate in one of the output
arms of the PBS rotates the polarization in that arm to the polarization of the other. The two photons from the middle term, now indistinguishable in polarization, bunch on a 50/50 beam splitter (BS), therefore they can not give rise to a coincidence detection between the two detectors[15]. Therefore, when such a coincidence is observed between the two detectors in mode \(a\), it could only have originated from the first and third terms of Eq. 2. The coherence between the two terms is preserved by the coincidence measurement, thus projecting the state of mode \(b\) to \(|\psi_b\rangle = (0, 2)_b + e^{i\phi} (2, 0)_b\) (we drop further normalization for simplicity). The phase \(\phi\) is determined by the difference between the lengths of the two arms after the PBS in mode \(a\). The coincidence detection heralds the successful production of a path entangled state of two photons in mode \(b\). Entanglement in photon numbers is created between two polarization modes rather than two paths. A polarization beam-splitter and a \(\lambda/2\) waveplate can translate between the two representations.

Using the equivalence between the operation of beam-splitters on two spatial modes and the operation of waveplates on two polarization modes[2], it is possible to considerably simplify the required coincidence measurement. As shown in Fig. 1b, the two modes \(a_h\) and \(a_v\) bunch when the polarization is rotated from the horizontal/vertical linear polarization basis \((hv)\) to either the plus/minus 45° linear \((pm)\) or right/left circular \((rl)\). With the same argument as above, it can be seen that coincidence at mode \(a\) results in the desired state \(\psi_a\) at mode \(b\). Actually, due to the rotational invariance of the state of Eq. 2, coincidence detection in mode \(a\) at any polarization state, implies bunching of mode \(b\) in the other two polarization bases. The difference between the bunched states in the two bases is the sign between their two terms.

The non-local bunching result can be understood from another point of view - starting from the detectors and propagating backwards along the photon paths through the optical elements. The detection operation is represented by annihilation operators, e.g. \(a_h\) for a detection of a horizontally polarized photon in mode \(a\). The two photon coincidence detection operator in mode \(a\) is the product \(a_h a_v\). This operator is transformed at a \(\lambda/2\) waveplate to \(a_h^2 - a_v^2\) and at a \(\lambda/4\) waveplate to \(a_h^2 + a_v^2\). Applying the transformed detection operator to mode \(a\) of Eq. 2, non-locally collapses mode \(b\) to the bunched state \(\psi_b\) with an efficiency of 1/3:

\[
|\psi\rangle = (a_h^2 + e^{i\theta} \cdot a_v^2) |\psi_2\rangle
\]

\[
= |0, 0\rangle_a \otimes |(2, 0)_b + e^{i\theta} \cdot |0, 2\rangle_b\rangle ,
\]

where \(\theta\) is a birefringent angle in the polarization representation that equals 0 or \(\pi\), depending on the choice of measurement basis of mode \(a\) \((rl\) or \(pm\), respectively).

In order to demonstrate non-local bunching, we down-converted 200 fs pulses at 390 nm in a BBO crystal with a double-pass configuration[17]. From the measured rates, the interaction parameter \(\tau\) was evaluated to be about 0.1, thus the production ratio of three-to-two pairs was

![FIG. 1: (a) A schematic setup for non-local bunching of two photons. The polarized photons from one of the spatial modes of PDC (mode \(a\)) are separated on a polarization beam-splitter (PBS), the polarization of one arm is rotated by a \(\lambda/2\) waveplate at 45° and the two arms are combined on a beam-splitter (BS). Coincidence detection is marked by connected single-photon avalanche photo-diodes (APD). (b) A simpler but equivalent scheme is to measure coincidence between the polarizations in a rotated polarization basis (\(\lambda/2\) at 22.5° or \(\lambda/4\) at 45°).]

![FIG. 2: Two-fold (squares) and four-fold (circles) visibilities and their fits (solid lines). (a) Mode \(a\) at the \(hv\) basis and mode \(b\) is scanned between \(hv\) and \(pm\). (b) Mode \(a\) at the \(pm\) basis and mode \(b\) is scanned between \(hv\) and \(pm\). Path entangled states are created at ±22.5° and ±67.5° for \(a\) and at 0°, ±45° and ±90° for \(b\) (dashed lines).]
Visibility measurements were taken by fixing the polarization basis of mode $a$ and recording various coincidences while rotating the polarization basis of mode $b$. When the polarization bases are different in the two modes, the two-fold coincidence on mode $a$ bunches the photons in mode $b$. The bunching prevents the $a_h a_v b_h b_v$ four-fold coincidence and corresponds to the minima points (dashed lines) in Fig. 2. Tsujino et al. [21] showed that the visibility of this four-fold coincidence is related to the content $\alpha$ of indistinguishable two photon-pairs, defined as

$$|\psi\rangle = \sqrt{\alpha} |\psi^-\rangle + \sqrt{1-\alpha} |\psi_{1,1}^+\rangle \otimes |\psi_{1,1}^-\rangle,$$

where Roman digits mark a distinguishing quantum number. In their experiment they evaluated the content of the indistinguishable state to be 37%. Figure 2 presents visibility curves for two polarization settings. From the measured four-fold visibility of $79 \pm 2\%$ we calculate $\alpha$ to be $83 \pm 1\%$.

The visibility measurements indicate that a two-photon path entangled state was produced in one of the two down-conversion modes. In order to observe the presence of the two terms of the state and their coherence, we interfered them on a beam-splitter. We used again the analogy between the two spatial modes of a beam-splitter and the two polarization modes and interfered the $b_h$ and $b_v$ modes with a $\lambda/2$ waveplate at $22.5^\circ$. Before the waveplate, a phase $\theta_b$ between the two polarization modes was introduced by tilting a birefringent crystal.

As the birefringent phase is scanned, $\theta$ of Eq. 3 varies as $2\theta_b$ and the state behind the $\lambda/2$ waveplate oscillates between $|2,0\rangle_h + |0,2\rangle_h$ and $|1,1\rangle_h$ at twice the induced phase. These oscillations were observed by detecting the four-fold coincidences of $b_h b_v$ conditioned on $a_h a_v$. They are compared in Fig. 3 to the oscillations of the $a_h b_v$ two-fold coincidence which follows $\theta_b$. The state of mode $b$ was bunched in the $hv$ basis, once by coincidence detection of mode $a$ in the $pm$ basis (resulting in $\theta = \pi$) and once in the $rl$ basis ($\theta = 0$). Thus, at zero birefringent phase the four-fold detection has a maximum in the first case and a minimum in the second.

In order to extend the scheme to higher photon numbers, operations that bunch more detection operators should be used. This is a generalization of the idea presented in Refs. [10, 11, 12], but it does not require non-detection or post-selection. In order to bunch $n$ photons at a certain polarization basis $p$ (defined by an arbitrary axis crossing the Poincaré sphere through its center, see Fig. 4), one should combine $n$ polarized photons $q_m$, residing equidistantly on the great circle whose plane is perpendicular to that axis. The product of the $n$ linear annihilation (or creation) operators $q_m$ is a different
The heralded four-photon path entangled state with an efficiency of $3/80$:

$$\psi = (a_a^4 - a_b^4)\psi^-$$

and different polarization bases. This heralding detection signal can be used to open a polarization insensitive switch in mode $b$, filtering out the lower photon number content. The generation of a heralded two-photon path entangled state was detected by observing interference at half the photon wavelength.

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