Near-threshold pion production with radioactive beams
at the Rare Isotope Accelerator

Bao-An Li∗

Department of Chemistry and Physics,
P.O. Box 419, Arkansas State University,
State University, Arkansas 72467-0419, USA

Gao-Chan Yong and Wei Zuo

Institute of Modern Physics, Chinese Academy of Science, Lanzhou 730000, P.R. China and
Graduate School, Chinese Academy of Science, Beijing 100039, P.R. China

(Dated: March 30, 2022)

Abstract

Using an isospin- and momentum-dependent transport model we study near-threshold pion production in heavy-ion collisions induced by radioactive beams at the planned Rare Isotope Accelerator (RIA). We revisit the question of probing the high density behavior of nuclear symmetry energy $E_{sym}(\rho)$ using the $\pi^-/\pi^+$ ratio. It is found that both the total and differential $\pi^-/\pi^+$ ratios remain sensitive to the $E_{sym}(\rho)$ when the momentum-dependence of both the isoscalar and isovector potentials are consistently taken into account. Moreover, the multiplicity and spectrum of $\pi^-$ mesons are found more sensitive to the $E_{sym}(\rho)$ than those of $\pi^+$ mesons. Finally, effects of the Coulomb potential on the pion spectra and $\pi^-/\pi^+$ ratio are also discussed.

PACS numbers: 25.70.-z, 25.70.Pq., 24.10.Lx

∗Electronic address: bali@astate.edu
I. INTRODUCTION

The isospin asymmetry of pions produced in nuclear reactions was shown to be useful for extracting interesting information about the structure of radioactive nuclei and neutron skins of heavy stable nuclei within the Glauber model \[1, 2, 3, 4, 5\]. Within hadronic transport models the isospin asymmetry of pions was found useful also for studying the equation of state (EOS) of isospin asymmetric nuclear matter \[6, 7, 8\]. In particular, the $\pi^-/\pi^+$ ratio was proposed as a sensitive probe of the high density behavior of nuclear symmetry energy $E_{sym}(\rho)$ \[7\]. The latter is an important part of the nucleon specific energy $E(\rho, \delta)$

$$E(\rho, \delta) = E(\rho, \delta = 0) + E_{sym}(\rho)\delta^2 + O(\delta^4)$$

in asymmetric matter of isospin asymmetry $\delta \equiv (\rho_n - \rho_p)/(\rho_p + \rho_n)$. The density dependence of symmetry energy is very important for many interesting astrophysical problems \[9, 10, 11, 12, 13\], the structure of radioactive nuclei \[14, 15, 16, 17\] and heavy-ion reactions \[4, 11\]. Unfortunately, the density dependence of symmetry energy, especially at supranormal densities, is still very poorly known. Predictions based on various many-body theories diverge widely at both low and high densities. In fact, even the sign of the symmetry energy above $3\rho_0$ is still very uncertain \[12\]. Since nuclear reactions induced by high energy radioactive beams can produce transiently dense neutron-rich matter, the fast fragmentation beams from RIA and the new accelerator facility at GSI provide the first opportunity in terrestrial laboratories to explore experimentally the EOS of dense neutron-rich matter \[4, 7, 18, 19\]. Crucial to the extraction of critical information about the $E_{sym}(\rho)$ is to compare experimental data with transport model calculations. Based on isospin-dependent transport model calculations, several experimental observables have been identified as promising probes of the $E_{sym}(\rho)$, such as, the neutron/proton ratio \[20\], isoscaling in nuclear multifragmentation \[21, 22, 23\], the neutron-proton differential flow \[24, 25, 26, 27\], the neutron-proton correlation function \[28\] and the isobaric yield ratios of light clusters \[29\]. The first experimental constraint on the density dependence of symmetry energy at subnormal densities was recently obtained by analyzing the isospin diffusion data from heavy-ion reactions \[30, 31\]. While it is much more challenging to constrain the symmetry energy at supranormal densities. In anticipation of coming experiments with high energy radioactive beams at RIA and GSI, more theoretical studies identifying experimental observables sensitive to the symmetry energy at supranormal densities are needed.
The single nucleon potential is an important input to the transport models. It includes an isovector part (symmetry potential) and an isoscalar part, and both of them are momentum dependent due to the non-locality of strong interactions and the Pauli exchange effects in many-fermion systems. However, in all transport models for heavy-ion collisions the momentum-dependence of the symmetry potential was seldom taken into account until very recently. It was found that the momentum dependence of the symmetry potential affects many experimental observables that were known to be sensitive to the symmetry energy\cite{31,32,33}. In this work we study near-threshold pion production at RIA using a momentum and isospin dependent transport model\cite{32}. We revisit the question of whether the high density behavior of nuclear symmetry energy $E_{sym}(\rho)$ can be probed using the $\pi^-/\pi^+$ ratio. We found that both the total and differential $\pi^-/\pi^+$ ratios remain sensitive to the $E_{sym}(\rho)$ when the momentum-dependence of both the isoscalar and isovector potentials are consistently taken into account. Moreover, the multiplicity and spectrum of $\pi^-$ mesons are found more sensitive to the $E_{sym}(\rho)$ than those of $\pi^+$ mesons.

II. A BRIEF SUMMARY OF THE ISOSPIN AND MOMENTUM DEPENDENT TRANSPORT MODEL IBUU04

In this work, we use the isospin and momentum dependent transport model for heavy-ion collisions induced by neutron-rich nuclei\cite{32}. In the latest version of this model IBUU04 we use a single nucleon potential\cite{34}

$$U(\rho, \delta, \vec{p}, \tau, x) = A_u(x) \frac{\rho_x}{\rho_0} + A_l(x) \frac{\rho_x}{\rho_0}$$

$$+ B \left( \frac{\rho_0}{\rho_0} \right)^2 (1 - x^2) - 8 \tau x \frac{B}{\sigma + 1} \frac{\rho^{\sigma - 1}}{\rho_0^0} \delta \rho_{x}$$

$$+ \frac{2C_{\tau,\tau}}{\rho_0} \int d^3p' \frac{f_\tau(\vec{r},\vec{p}')}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2}$$

$$+ \frac{2C_{\tau',\tau'}}{\rho_0} \int d^3p' \frac{f_{\tau'}(\vec{r},\vec{p}')}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2}. \tag{2}$$

In the above $\tau = 1/2 (-1/2)$ for neutrons (protons) and $\tau \neq \tau'$; $\sigma = 4/3$; $f_{\tau}(\vec{r},\vec{p})$ is the phase space distribution function at coordinate $\vec{r}$ and momentum $\vec{p}$. The parameters $A_u(x), A_l(x), B, C_{\tau,\tau}, C_{\tau',\tau'}$ and $\Lambda$ were obtained by fitting the momentum-dependence of the $U(\rho, \delta, \vec{p}, \tau, x)$ predicted by the Gogny Hartree-Fock and/or the Brueckner-Hartree-Fock calculations, saturation properties of symmetric nuclear matter and the symmetry energy.
of 30 MeV at normal nuclear matter density $\rho_0 = 0.16/fm^3$. The compressibility of symmetric nuclear matter $K_0$ is set to be 211 MeV.

The last two terms contain the momentum-dependence of the single particle potential. The momentum dependence of the symmetry potential stems from the different interaction strength parameters $C_{\tau,\tau'}$ and $C_{\tau,\tau}$ for a nucleon of isospin $\tau$ interacting, respectively, with unlike and like nucleons in the background fields. More specifically, we use $C_{\text{unlike}} = -103.4$ MeV and $C_{\text{like}} = -11.7$ MeV. One characteristic of the momentum dependence of the symmetry potential is the different effective masses for neutrons and protons in isospin asymmetric nuclear matter. With the above potential, we found that the neutron effective mass is higher than the proton effective mass and the splitting between them increases with both the density and isospin asymmetry of the medium. Moreover, with the potential of eq.2 both the isoscalar and isovector potentials at $\rho_0$ are in agreement with the corresponding nucleon optical potentials extracted from nucleon-nucleus scattering data.

The parameters $A_u(x)$ and $A_l(x)$ depend on the $x$ parameter according to

$$A_u(x) = -95.98 - \frac{2B}{\sigma + 1}$$

(3)

$$A_l(x) = -120.57 + \frac{2B}{\sigma + 1}$$

(4)

The parameter $x$ can be adjusted to mimic predictions on the density dependence of the symmetry energy $E_{\text{sym}}(\rho)$ by microscopic and/or phenomenological many-body theories. In this work we choose a range of the $x$ parameter from 1 to -2. With this choice the density dependence of the symmetry energy samples a wide range of theoretical predictions as shown in Fig. 1.

Other details of the IBUU04 model can be found in ref.32. The details of modeling pion production can be found in our earlier publications, e.g., ref.7, 35. Pion production was also studied previously by many other people within transport models. However, none of the previous studies has taken into account the momentum dependence of the symmetry potential. Only very recently the latter was recognized as an important issue in connection with probing the high density behavior of symmetry energy using the $\pi^-/\pi^+$ ratio.
III. RESULTS AND DISCUSSIONS

We study the reaction of $^{132}\text{Sn} + ^{124}\text{Sn}$ at a beam energy of 400 MeV/nucleon and an impact parameter of 1 fm as an example of high energy central reactions at RIA. This particular reaction will be carried out using the fast fragmentation beam line at RIA. The beam energy selected here is about the highest one to be available at RIA. A Time Projection Chamber (TPC) has been proposed to study the EOS of isospin asymmetric nuclear matter by detecting charged particles including pions [36]. In the following we discuss several features of near-threshold pion production at RIA. The emphasis will be on examining their sensitivities to the density dependence of symmetry energy.

A. Formation of high density isospin asymmetric nuclear matter at RIA

What are the maximum baryon density and isospin asymmetry that can be achieved in central heavy-ion collisions at RIA? This is an interesting question relevant to the study of the EOS of asymmetric nuclear matter. To answer this question we show in Fig. 2 the central baryon density (upper window) and the average $(n/p)_{\rho \geq \rho_0}$ ratio (lower window) of all regions with baryon densities higher than $\rho_0$. It is seen that the maximum baryon density is about 2 times normal nuclear matter density. Moreover, the compression is rather insensitive to the symmetry energy because the latter is relatively small compared to the EOS of symmetric matter around this density. The high density phase lasts for about 15 fm/c from 5 to 20 fm/c for this reaction. It is interesting to see that the isospin asymmetry of the high density region is quite sensitive to the symmetry energy. The soft (e.g., $x = 1$) symmetry energy leads to a significantly higher value of $(n/p)_{\rho \geq \rho_0}$ than the stiff one (e.g., $x = -2$). This is consistent with the well-known isospin fractionation phenomenon. Because of the $E_{\text{sym}}(\rho)\delta^2$ term in the EOS of asymmetric nuclear matter, it is energetically more favorable to have a higher isospin asymmetry $\delta$ in the high density region with a softer symmetry energy functional $E_{\text{sym}}(\rho)$. In the supranormal density region, as shown in Fig. 1, the symmetry energy changes from being soft to stiff when the parameter $x$ varies from 1 to -2. Thus the value of $(n/p)_{\rho \geq \rho_0}$ becomes lower as the parameter $x$ changes from 1 to -2. It is worth mentioning that the initial value of the quantity $(n/p)_{\rho \geq \rho_0}$ is about 1.4 which is less than the average n/p ratio of 1.56 of the reaction system. This is because of the neutron-skins of...
the colliding nuclei, especially that of the projectile \( ^{132}Sn \). In the neutron-rich nuclei, the n/p ratio on the low-density surface is much higher than that in their interior. It is clearly seen that the dense region can become either neutron-richer or neutron-poorer with respect to the initial state depending on the symmetry energy functional \( E_{\text{sym}}(\rho) \) used.

**B. Dynamics of pion productions in heavy-ion collisions induced by radioactive beams**

To understand the dynamics of pion production and its dependence on the symmetry energy, we show in Fig. 3 the multiplicity of \( \pi^+ \), \( \pi^- \) and \( \Delta(1232) \) as a function of time. The multiplicity of \( \Delta(1232) \) resonances shown in the figure includes all four charge states while in the model we do treat and follow separately different charge states of the \( \Delta(1232) \) and \( N^*(1440) \) resonances. At a beam energy of 400 MeV/nucleon which is just about 100 MeV above the pion production threshold in nucleon-nucleon scatterings, almost all pions are produced through the decay of \( \Delta(1232) \) resonances. The contribution due to \( N^* \) resonances is negligible. By comparing Fig. 2 and Fig. 3 one can notice that most of the \( \Delta \) resonances are produced in the high density region. Pions from the decay of these resonances thus carry useful information about the high density phase. It is interesting to see that the \( \pi^- \) multiplicity depends more sensitively on the symmetry energy. This feature is seen more clearly in Fig. 4 where the \( \pi^- \) and \( \pi^+ \) multiplicities at the freeze-out are shown as a function of the \( x \) parameter. While the \( \pi^+ \) multiplicity remains about the same the \( \pi^- \) multiplicity increases by about 20% by varying the \( x \) parameter from -2 to 1. The multiplicity of \( \pi^- \) is about 2 to 3 times that of \( \pi^+ \). This is because the \( \pi^- \) mesons are mostly produced from neutron-neutron collisions. The \( \pi^- \) mesons are thus more sensitive to the isospin asymmetry of the reaction system and the symmetry energy. In fact, assuming pions are all produced through \( \Delta(1232) \) resonances in the first chance nucleon-nucleon scatterings and neglecting the influence of subsequent pion rescatterings and reabsorptions, the \( \pi^-/\pi^+ \) ratio is expected to scale with the N/Z ratio of the participant region according to

\[
\pi^-/\pi^+ = (5N^2 + NZ)/(5Z^2 + NZ) \approx (N/Z)^2.
\]

In reactions induced by neutron-rich nuclei one thus expects to see more \( \pi^- \) than \( \pi^+ \) mesons. For the reaction considered here, the average \( (N/Z)^2 \) of the reaction system is about 2.4.
The observed $\pi^-/\pi^+$ ratio is somewhat different from this value depending on the symmetry energy used. It indicates the effects of the different $N/Z$ ratios of the participant regions with the different $x$ parameters due to the isospin fractionation. Moreover, it also indicates the importance of pion reabsorption and rescatterings which are both taken into account properly in our transport model investigations.

Our finding that $\pi^-$ mesons are more sensitive to the symmetry energy has interesting implications for the experimental studies. With the TPC, for instance, the $\pi^-$ mesons can be easily identified because of their negative charges. While the $\pi^+$ mesons will be bent to the same direction in the magnetic field as protons and are thus difficulty to be separated out cleanly. To investigate further the possibility of exploring the symmetry energy using $\pi^-$ mesons alone, we show in Fig. 5 the $\pi^-$ and $\pi^+$ kinetic energy spectra. It is seen that the $\pi^-$ spectra with different $x$ parameters can indeed be clearly separated. It thus raises the interesting possibility of probing the density dependence of symmetry energy by studying the $\pi^-$ spectrum.

\section{The $\pi^-/\pi^+$ ratio probe of the high density behavior of symmetry energy}

We now turn to the $\pi^-/\pi^+$ ratio as a probe of the high density behavior of symmetry energy. The advantage of using the $\pi^-/\pi^+$ ratio over the $\pi^-$ spectrum itself is that the ratio can reduce largely the systematic errors involved in the experiments. Moreover, within the statistical model for pion production\cite{38, 39}, the $\pi^-/\pi^+$ ratio is proportional to $\exp\left[\left(\mu_n - \mu_p\right)/T\right]$, where $T$ is the temperature, $\mu_n$ and $\mu_p$ are the chemical potentials of neutrons and protons, respectively. At modestly high temperatures ($T \geq 4$ MeV), the difference in the neutron and proton chemical potentials can be written as\cite{40}

$$
\mu_n - \mu_p = U^n_{asy} - U^p_{asy} - U_{Coulomb} + T \left[ \ln \frac{\rho_n}{\rho_p} + \sum_m \frac{m+1}{m} B_m \left( \frac{\lambda T}{2} \right)^m \left( \rho_n^m - \rho_p^m \right) \right], \quad (6)
$$

where $U_{Coulomb}$ is the Coulomb potential for protons, $\lambda T$ is the thermal wavelength of a nucleon and $B'_m$ are the inversion coefficients of the Fermi distribution function\cite{40}. The difference in neutron and proton symmetry potentials $U^n_{asy} - U^p_{asy} \approx 2U_{asy}\delta$, where the function $U_{asy}$ is the strength of the symmetry potential, depends on the $x$ parameter, the density $\rho$ and the nucleon momentum. It can be estimated readily from the single nucleon potential of eq.2. It is also seen that the kinetic part of the difference $\mu_n - \mu_p$ relates directly
to the isospin asymmetry $\rho_n/\rho_p$ or $\rho_n - \rho_p$. Thus from the statistical point of view the $\pi^-/\pi^+$ ratio is theoretically a good probe of the symmetry energy. However, to verify this expectation and study more realistically the $\pi^-/\pi^+$ ratio one has to rely on the transport models.

Shown in Fig. 6 is the quantity $(\pi^-/\pi^+)_{like}$ as a function of time. Taking into account the dynamics of resonance production and decays we define

$$
(\pi^-/\pi^+)_{like} \equiv \frac{\pi^- + \Delta^- + \frac{1}{3}\Delta^0}{\pi^+ + \Delta^{++} + \frac{1}{3}\Delta^+}. \tag{7}
$$

This ratio naturally becomes the final $\pi^-/\pi^+$ ratio after all resonances have decayed. First, it is seen that the $(\pi^-/\pi^+)_{like}$ ratio reaches a very high value in the early stage of the reaction. This is due to the abundant neutron-neutron scatterings when the two neutron-skins start overlapping at the beginning of the reaction. Secondly, the $(\pi^-/\pi^+)_{like}$ ratio saturates after about 25 fm/c indicating that a chemical freeze-out stage has been reached. Finally, the sensitivity to the symmetry energy is clearly shown in the final $\pi^-/\pi^+$ ratio. One can notice that the sensitivity of the $\pi^-/\pi^+$ ratio to the $x$ parameter is quantitatively about the same as the $\pi^-$ multiplicity shown in Fig. 4. Shown in Fig. 7 and Fig. 8 are the differential $\pi^-/\pi^+$ ratios versus the kinetic energy and transverse momentum, respectively. In the low energy ($E_{kin} \leq 120$ MeV) or transverse momentum ($p_t \leq 200$ MeV/c) region the $\pi^-/\pi^+$ ratio is clearly separable with the $x$ parameter varying from 1 to -2. It indicates that it is sufficient to measure accurately the low energy (transverse momentum) pions, instead of the whole spectrum, in order to constrain the density dependence of the symmetry energy.

**D. Coulomb effects on the $\pi^-/\pi^+$ ratio**

First of all, it is necessary to mention that the Coulomb effects on $\pi^-/\pi^+$ ratio is well known, in particular from experiments at the BEVALAC and SIS/GSI, see, e.g., ref. [41] and references therein. A number of models have been used in analysing the $\pi^-/\pi^+$ ratio in heavy-ion reactions from low to ultra-relativistic energies. However, in previous studies the effects of the symmetry energy was generally neglected. From eq. (6) it is seen that the $\pi^-/\pi^+$ ratio depends on both the Coulomb and symmetry potentials. Here we are interested in understanding the relative effects of the symmetry and Coulomb potentials. As an example, we compare in Fig. 9 the multiplicities of $\pi^-$ and $\pi^+$ mesons as a function of transverse
momentum calculated with and without the Coulomb potential with the parameter \(x = 0\). It is seen that the Coulomb potential is to shift \(\pi^- (\pi^+)\) to lower (higher) \(p_t\) as one expects. The \(\pi^-/\pi^+\) ratio with and without the Coulomb potential is shown in Fig. 10. It is seen that the \(\pi^-/\pi^+\) ratio is quite flat without the Coulomb potential. The Coulomb effect increases the \(\pi^-/\pi^+\) ratio at \(p_t = 0\) for about 30% while suppresses it at higher \(p_t\). What is the relative effect of the symmetry potential with respect to that of the Coulomb potential? The answer to this question can be obtained from inspecting Fig. 7 and Fig. 8. With different \(x\) parameters, the reaction dynamics is about the same as indicated by the almost identical evolution of central density shown in Fig. 2. Thus the Coulomb effect should also be about the same in calculations with the different \(x\) parameters. From the variation of the \(\pi^-/\pi^+\) ratio by varying the \(x\) parameter as shown in Figs. 7 and 8, we can conclude that the Coulomb effect is stronger than the symmetry potential. Of course, we should stress that the symmetry potential has an indirect effect on the \(\pi^-/\pi^+\) ratio through its interactions on nucleons, while the Coulomb potential acts directly on charged pions. Since the Coulomb effect is well-known, our results discussed above indicate that it is very promising to extract useful information about the symmetry potential from studying the \(\pi^-/\pi^+\) ratio.

IV. SUMMARY

In summary, using an isospin- and momentum-dependent transport model we have studied near-threshold pion production in heavy-ion collisions induced by radioactive beams at the planned Rare Isotope Accelerator (RIA). We studied properties of the high density matter formed in high energy central reactions at RIA. It is found that the isospin asymmetry of the high density hadronic matter is very sensitive to the symmetry energy used. With the soft symmetry energy the n/p ratio of the high density region can be significantly higher than that of the reaction system due to the isospin fractionation. We also examined the multiplicities and spectra of both \(\pi^-\) and \(\pi^+\) mesons. We found that the \(\pi^-\) mesons carry more sensitive information about the symmetry energy than the \(\pi^+\) mesons. Both the kinetic energy and transverse momentum spectra of \(\pi^-\) mesons can be useful for studying the symmetry energy at high densities. Moreover, we revisited the question of probing the high density behavior of nuclear symmetry energy \(E_{\text{sym}}(\rho)\) using the \(\pi^-/\pi^+\) ratio. It was found that both the total and differential \(\pi^-/\pi^+\) ratios remain sensitive to the \(E_{\text{sym}}(\rho)\) when
the momentum-dependence of both the isoscalar and isovector potentials are consistently taken into account. Furthermore, we found that the effects of the Coulomb potential on the $\pi^-/\pi^+$ ratio are important. In fact, the Coulomb effect is stronger than that of the symmetry potential. Nevertheless, taking into account the Coulomb effect and the momentum dependence of the symmetry potential the $\pi^-/\pi^+$ ratio, especially in the low energy or transverse momentum region, remains very sensitive to the variation of the symmetry energy. Our study thus confirms that the isospin asymmetry of pions from heavy-ion reactions induced by radioactive beams is a very promising tool for studying the EOS of dense neutron-rich matter, especially the high density behavior of symmetry energy. Our findings here are expected to be useful for designing the TPC and planning experiments at RIA.

V. ACKNOWLEDGMENTS

We would like to thank Scott Pratt, Jianye Liu and Xiguo Lee for helpful discussions. The work of B.A. Li is supported in part by the National Science Foundation of the United States under grant No. PHYS-0243571 and PHYS0354572. The work of G.C. Yong and W. Zuo is supported in part by the Chinese Academy of Science Knowledge Innovation Project (KECK2-SW-N02), Major State Basic Research Development Program (G2000077400), the National Natural Science Foundation of China (10235030) and the Important Pare-Research Project (2002CAB00200) of the Chinese Ministry of Science and Technology.

[1] A. Tellez, R. J. Lombard and J.P. Maillet, J. Phys. G13, 311 (1987).
[2] R.J. Lombard and J.P. Maillet, Europhys Lett. 6, 323 (1988).
[3] B.A. Li, M.S. Hussein and W. Bauer, Nucl. Phys. A533, 749 (1991).
[4] B.A. Li, C.M. K, and W. Bauer, topical review, Int. Jour. Mod. Phys. E 7, 147 (1998).
[5] A. Szcurek and P. Pawlowski, Int. J. of Modern Phys. A (2004) in press, nucl-th/0409035
    nucl-th/0402029
[6] V.S. Uma Maheswari at al., Phys. Rev. C57, 922 (1998).
[7] B.A. Li, Phys. Rev. Lett. 88, 192701 (2002); Nucl. Phys. A708, 365 (2003); Phys. Rev. C67, 017601 (2003).
References

[8] T. Gatitanos et al., Nucl. Phys. A732, 24 (2004).
[9] H.A. Bethe, Rev. Mod. Phys. 62, 801 (1990).
[10] J.M. Lattimer and M. Prakash, Phys. Rep., 333, 121 (2000); Astr. Phys. Jour. 550, 426 (2001); Science Vol. 304, 536 (2004).
[11] Isospin Physics in Heavy-Ion Collisions at Intermediate Energies, Eds. B. A. Li and W. Udo Schröder (Nova Science Publishers, Inc, New York, 2001).
[12] I. Bombs, in ref. [11], p.35.
[13] A. W. Steiner, M. Prakash, J.M. Lattimer and P.J. Ellis, nucl-th/0410066, submitted to Phys. Rep. (2004).
[14] B.A. Brown, Phys. Rev. Let. 85, 5296 (2000).
[15] C.J. Horowitz and J. Piekarewicz, Phys. Rev. Let. 86, 5647 (2001); Phys. Rev. C64, 062802 (2001); ibid 66, 055803 (2002).
[16] R.J. Furnstahl, Nucl. Phys. A706, 85 (2002).
[17] J.R. Stone at al., Phys. Rev. C68, 034324 (2004).
[18] P. Danielewicz, R. Lacey and W.G. Lynch, Science 298, 1592 (2002).
[19] The DOE/NS Nuclear Science Advisory Committee, Opportunities in Nuclear Science, April (2002).
[20] B.A. Li, C.M. Ko, and Z.Z. Ren, Phys. Rev. Let. 78, 1644 (1997).
[21] M.B. Tsang et al., Phys. Rev. Let. 86, 5023 (2001).
[22] W.P. Tan et al., Phys. Rev. C 64, 051901(R) (2001).
[23] V. Baran, M. Colonna, M. Di Toro, V. Greco, M. Zielinkska-Pfabe and H.H. Wolter, Nucl. Phys. A 703, 603 (2002).
[24] B.A. Li, Phys. Rev. Let. 85, 4221 (2000).
[25] V. Greco, M. Colonna, M. Di Toro and F. Matera, Phys. Rev. C67, 015203 (2003).
[26] L. Scalone, M. Colonna and M. Di Toro, Phys. Let. B461, 9 (1999).
[27] J. Rizzo, M. Colonna, M. Di Toro and V. Greco, Nucl. Phys. A732 (2004) 202.
[28] L.W. Chen, V. Greco, C.M. Ko and B.A. Li, Phys. Rev. Let. 90, 162701 (2003); Phys. Rev. C68, 014605 (2003).
[29] L.W. Chen, C.M. Ko and B.A. Li, Phys. Rev. C68, 017601 (2003); Nucl. Phys. A729, 809 (2003).
[30] M.B. Tsang et al., Phys. Rev. Let. 92, 062701 (2004).
[31] L.W. Chen, C.M. Ko and B.A. Li, nucl-th/0407032.

[32] B.A. Li, C.B. Das, S. Das Gupta and C. Gale, Phys. Rev. C69, 011603 (2004); Nucl. Phys. A735, 563 (2004).

[33] L.W. Chen, C.M. Ko and B.A. Li, Phys. Rev. C69, 054606 (2004).

[34] C.B. Das, S. Das Gupta, C. Gale and B.A. Li, Phys. Rev. C67, 034611 (2003).

[35] B.A. Li and W. Bauer, Phys. Rev. C44, 450, (1991); B.A. Li, W. Bauer and G.F. Bertsch, ibid, C44, 2095 (1991).

[36] W.G. Lynch, L.G. Sobotka and G.D. Westfall, private communications.

[37] R. Stock, Phys. Rep., 135, 259 (1986).

[38] G.F. Bertsch, Nature 283, 280 (1980).

[39] A. Bonasera and G.F. Bertsch, Phys. Let. B195, 521 (1987).

[40] H.R. Jaqaman, A.Z. Mekjian and L. Zamick, Phys. Rev. C27, 2782 (1983); ibid, C29, 2067 (1984); H.R. Jaqaman, Phys. Rev. C39, 169 (1988).

[41] B.A. Li, Phys. Let. B346, 5 (1995) and references therein.
FIG. 1: (Color on line) Nuclear symmetry energy as a function of density with different $x$ parameter.
FIG. 2: (Color on line) Central baryon density (upper window) and isospin asymmetry (lower window) of high density region for the reaction of $^{132}$Sn + $^{124}$Sn at a beam energy of 400 MeV/nucleon and an impact parameter of 1 fm.
FIG. 3: (Color on line) Evolution of the pion and $\Delta(1232)$ multiplicity in the reaction of $^{132}$Sn+$^{124}$Sn at a beam energy of 400 MeV/nucleon and an impact parameter of 1 fm.
FIG. 4: (Color on line) The average multiplicity of $\pi^+$ and $\pi^-$ as a function of the $x$ parameter for the reaction of $^{132}\text{Sn} + ^{124}\text{Sn}$ at a beam energy of 400 MeV/nucleon and an impact parameter of 1 fm.
FIG. 5: (Color on line) The kinetic energy spectra of $\pi^-$ and $\pi^+$ in the reaction of $^{132}Sn + ^{124}Sn$ at a beam energy of 400 MeV/nucleon and an impact parameter of 1 fm.
FIG. 6: (Color on line) Evolution of the $\pi^-/\pi^+$ ratio in the reaction of $^{132}$Sn + $^{124}$Sn at a beam energy of 400 MeV/nucleon and an impact parameter of 1 fm.
FIG. 7: (Color on line) The $\pi^-/\pi^+$ ratio as a function of pion kinetic energy in the reaction of $^{132}\text{Sn} + ^{124}\text{Sn}$ at a beam energy of 400 MeV/nucleon and an impact parameter of 1 fm.
FIG. 8: (Color on line) The $\pi^-/\pi^+$ ratio as a function of transverse momentum in the reaction of $^{132}\text{Sn} + ^{124}\text{Sn}$ at a beam energy of 400 MeV/nucleon and an impact parameter of 1 fm.
FIG. 9: (Color on line) $\pi^-$ and $\pi^+$ transverse momentum spectra calculated with and without the Coulomb potential for the reaction of $^{132}$Sn + $^{124}$Sn at a beam energy of 400 MeV/nucleon and an impact parameter of 1 fm.
FIG. 10: (Color on line) $\pi^-/\pi^+$ ratios as a function of transverse momentum calculated with and without the Coulomb potential in the reaction of $^{132}\text{Sn} + ^{124}\text{Sn}$ at a beam energy of 400 MeV/nucleon and an impact parameter of 1 fm.