Quantized Black Holes, Correspondence Principle, and Holographic Bound

I.B. Khriplovich

Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia, and Novosibirsk University
e-mail: khriplovich@inp.nsk.su

Abstract
An equidistant spectrum of the horizon area of a quantized black hole does not follow from the correspondence principle or from general statistical arguments. Such a spectrum obtained earlier in loop quantum gravity (LQG) does not comply with the holographic bound. This bound fixes the Barbero-Immirzi parameter of LQG, and thus leads to the unique result for the spectrum of horizon area.

Keywords: Quantum gravity, horizon area, holographic bound

1 Is Spectrum of Quantized Black Hole Equidistant?

The idea of quantizing the horizon area of black holes was put forward many years ago by Bekenstein in the pioneering article [1]. It was based on the intriguing observation, made by Christodoulou and Ruffini [2, 3]: the horizon area of a nonextremal black hole behaves as an adiabatic invariant. Of course, the quantization of an adiabatic invariant is perfectly natural, in accordance with the correspondence principle.

One more conjecture made in [1] is that the spectrum of a quantized horizon area is equidistant. The argument therein was that a periodic system is quantized by equating its adiabatic invariant to $2\pi \hbar n$, $n = 0, 1, 2, ...$

Later it was pointed out by Bekenstein [4] that the classical adiabatic invariance does not guarantee by itself the equidistance of the spectrum, at least because any function of an adiabatic invariant is itself an adiabatic invariant. However, up to now articles on the subject abound in assertions that the form

$$A = \beta l_p^2 n, \quad n = 1, 2, ...$$

(1)

for the horizon area spectrum $^1$ is dictated by the respectable correspondence prin-

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$^1$Here and below $l_p^2 = \hbar k/c^3$ is the Planck length squared, $l_p = 1.6 \cdot 10^{-33}$ cm, $k$ is the Newton gravitational constant; $\beta$ is here some numerical factor.
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circle. The list of these references is too lengthy to be presented here.

Let us consider an instructive example of the situation when a nonequidistant spectrum arises in spite of the classical adiabatic invariance. We start with a classical spherical top of an angular momentum $J$. Of course, the projection $J_z$ of $J$ is an adiabatic invariant. If the $z$-axis is chosen along $J$, the value of $J_z$ is maximum, $J$, or $\hbar j$ in the quantum case. The classical angular momentum squared $J^2$ is also an adiabatic invariant, with eigenvalues $\hbar^2 (j+1)$ when quantized. Let us try now to use the operator $\hat{J}^2$ for the area quantization in quite natural units of $l_p^2$. For the horizon area $A$ to be finite in the classical limit, the power of the quantum number $j$ in the result for $j \gg 1$ should be the same as that of $\hbar$ in $l_p^2$ [5]. With $l_p^2 \sim \hbar$, we arrive in this way at

$$A \sim l_p^2 \sqrt{j(j+1)}.$$  

Since $\sqrt{j(j+1)} \rightarrow j + 1/2$ for $j \gg 1$, we have come back again to the equidistant spectrum in the classical limit. However, the equidistance can be avoided in the following way. Let us assume that the horizon area consists of sites with area on the order of $l_p^2$, and ascribe to each site $i$ its own quantum number $j_i$ and the contribution $\sqrt{j_i(j_i+1)}$ to the area. Then the above formula changes to

$$A \sim l_p^2 \sum_i \sqrt{j_i(j_i+1)}$$  

(2)

(in fact, this formula for a quantized area arises as a special case in loop quantum gravity, see below). Of course, to retain a finite classical limit for $A$, we should require that $\sum \sqrt{j_i(j_i+1)} \gg 1$. However, any of $j_i$ can be well comparable with unity. So, in spite of the adiabatic invariance of $A$, its quantum spectrum (2) is not equidistant, though of course discrete.

One more argument in favour of the equidistant spectrum (1) is as follows [4, 6, 7]. On the one hand, the entropy $S$ of a horizon is related to its area $A$ through the Bekenstein-Hawking relation

$$S = \frac{A}{4l_p^2}.$$  

(3)

On the other hand, the entropy is nothing but $\ln g(n)$ where the statistical weight $g(n)$ of any state $n$ is an integer. In [4, 6, 7] the requirement of integer $g(n)$ is taken literally, and results after simple reasoning not only in equidistant spectrum (1), but also in the following allowed values for the numerical factor $\beta$ in this spectrum:

$$\beta = 4 \ln k, \quad k = 2, 3, ....$$

Let us imagine however that with some model for $S$ one obtains for $g(n)$, instead of an integer value $K$, a noninteger one $K + \delta, \quad 0 < \delta < 1$. Then, the entropy will be

$$S = \ln(K + \delta) = \ln K + \delta/K.$$  

Now, the typical value of the black hole entropy $S = \ln K = A/4l_p^2$ is huge, something like $10^{76}$. So, the correction $\delta/K$ is absolutely negligible as compared to $S = \ln K$. Moreover, it is far below any conceivable accuracy of a description of entropy in general, and of the semiclassical Bekenstein-Hawking formula (3) relating
the black hole entropy to the quantized horizon area in particular. Therefore, this correction can be safely omitted and forgotten. As usual for macroscopic objects, the fact that \( g(n) \) is an integer has no consequences for entropy.

Thus, contrary to the popular belief, the equidistance of the spectrum for the horizon area does not follow from the correspondence principle and/or from general statistical arguments [8].

2 “It from Bit”, Loop Quantum Gravity, and Holographic Bound

It does not mean however that any model leading to an equidistant spectrum for the quantized horizon area should be automatically abandoned. Quite simple and elegant version of such a model, so called “it from bit”, for a Schwarzschild black hole was formulated by Wheeler [9]. The assumption is that the horizon surface consists of \( \nu \) sites, each of them supplied with an “angular momentum” quantum number \( j \) with two possible projections \( \pm 1/2 \). The total number \( K \) of degenerate quantum states of this system is obviously

\[
K = 2^\nu. \tag{4}
\]

Then the entropy of the black hole is

\[
S_{1/2} = \ln K = \nu \ln 2. \tag{5}
\]

And finally, with the Bekenstein-Hawking relation [8] one obtains for the area spectrum the following equidistant formula:

\[
A_{1/2} = 4 \ln 2 l_p^2 \nu. \tag{6}
\]

This model of the quantized Schwarzschild black hole looks quite OK by itself.

Later this result was derived in Ref. [10] in the framework of loop quantum gravity (LQG) [11-15]. We discuss below whether the “it from bit” picture, if considered as a special case of the area quantization in LQG, can be reconciled with the holographic bound [16-18].

More generally, a quantized surface in LQG is described as follows. One ascribes to it a set of punctures. Each puncture is supplied with two integer or half-integer “angular momenta” \( j^u \) and \( j^d \):

\[
j^u, j^d = 0, 1/2, 1, 3/2, \ldots; \tag{7}
\]

at least one of them should not vanish. \( j^u \) and \( j^d \) are related to edges directed up and down the normal to the surface, respectively, and add up into an angular momentum \( j^{ud} \):

\[
j^{ud} = j^u + j^d; \quad |j^u - j^d| \leq j^{ud} \leq j^u + j^d. \tag{8}
\]

The area of a surface is

\[
A = 8\pi\gamma l_p^2 \sum_i \sqrt{2j_i^u(j_i^u + 1) + 2j_i^d(j_i^d + 1) - j_i^{ud}(j_i^{ud} + 1)}. \tag{9}
\]
The numerical factor $\gamma$ in (9) cannot be determined without an additional physical input. This ambiguity originates from a free (so called Barbero-Immirzi) parameter [19, 20] which corresponds to a family of inequivalent quantum theories, all of them being viable without such an input. Once the general structure of the horizon area in LQG is fixed, the only problem left is to determine this overall factor, i.e., the Barbero-Immirzi parameter.

To make the discussion more clear and concrete, we confine from now on to the simplified version (quite popular now) of general formula (9) when $j^{d(u)} = 0$, so that this formula for a surface area reduces to

$$A = 8\pi \gamma l_p^2 \sum_{i} \sqrt{j_i(j_i + 1)}, \quad j = j^{u(d)}. \quad (10)$$

It is worth mentioning that this particular case of general formula (9) coincides with the naive model [2].

The result (6) was obtained in [10] under an additional condition that the gravitational field on the horizon is described by the $U(1)$ Chern-Simons theory. Formula (6) is a special case of general one (10) when all $j = 1/2$. As to the overall factor $\gamma$, its value here is

$$\gamma = \frac{\ln 2}{\pi \sqrt{3}}. \quad (11)$$

Let us turn now to the holographic bound [16-18]. According to it, the entropy $S$ of any spherical nonrotating system confined inside a sphere of area $A$ is bounded as follows:

$$S \leq \frac{A}{4l_p^2}, \quad (12)$$

with the equality attained only for a system that is a black hole.

A simple intuitive argument confirming this bound is as follows [18]. Let us allow the discussed system to collapse into a black hole. During the collapse the entropy increases from $S$ to $S_{bh}$, and the resulting horizon area $A_{bh}$ is certainly smaller than the initial confining one $A$. Now, with the account for the Bekenstein-Hawking relation (3) for a black hole, we arrive through the obvious chain of (in)equalities

$$S \leq S_{bh} = \frac{A_{bh}}{4l_p^2} \leq \frac{A}{4l_p^2}$$

at the discussed bound (12).

The result (12) can be formulated otherwise. Among the spherical surfaces of a given area, it is the surface of a black hole horizon that has the largest entropy.

The last statement was used as an assumption by Vaz and Witten [23] in a model of the quantum black hole as originating from a dust collapse. Then this assumption was employed by us [24] in the problem of quantizing the horizon of a black hole in LQG. In particular, the coefficient $\gamma$ was calculated in Ref. [24] in the case when the area of a black hole horizon is given by the general formula (9) of LQG, as well as under some more special assumptions on the values of $j^{u}$, $j^{d}$.

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2Earlier attempts to calculate the black hole entropy in LQG go back at least to Refs. [21, 22]. They did not lead however to concrete quantitative results.
Moreover, it was demonstrated in [24] for a rather general class of the horizon quantization schemes that it is the maximum entropy of a quantized surface that is proportional to its area.

3 Problem of Distinguishability of Edges

We consider in fact the “microcanonical” entropy $S$ of a quantized surface defined as the logarithm of the number of states of this surface for a fixed area $A$ (instead of fixed energy in common problems). Obviously, this number of states $K$ depends on the assumption made on the distinguishability of the edges. So, let us discuss first of all which of a priori possible assumptions is reasonable from the physical point of view [8].

It is instructive to start the discussion with a somewhat more detailed analysis of the “it from bit” model. The natural result (4) for the total number of states in this model corresponds in fact to the assumption that among all the sites with $j = 1/2$, those with same $j_z$, +1/2 or −1/2 (their numbers are $\nu_+$ and $\nu_-$, respectively), are indistinguishable, and those with different projections are distinguishable. Indeed, in this case the number of states with given $\nu_+$, $\nu_-$ is obviously $\nu!/(\nu_+!\nu_-!)$, and the total number of states and entropy,

$$K = \sum_{\nu_+, \nu_-} \frac{\nu!}{\nu_+!\nu_-!} = \sum_{\nu_+=0}^{\nu} \frac{\nu!}{\nu_+!\nu_+!} = 2^\nu \quad \text{and} \quad S = \nu \ln 2,$$

coincide with (4) and 5).

It should be noted that the configuration with $\nu_+ = \nu_- = \nu/2$ not only gives here the maximum contribution to $K$ and $S$. For $\nu \gg 1$ both the “it from bit” statistical weight and entropy are strongly dominated by this configuration [27]. Indeed,

$$\ln \frac{\nu!}{(\nu/2)!^2} = \nu \ln 2 - \frac{3}{2} \ln \nu.$$

Let us also note in passing that if we confine here to the states with the total angular momentum $J = 0$, the entropy is still about the same:

$$\nu \ln 2 - \frac{3}{2} \ln \nu.$$

Now, we are going over to the analysis of the more general case with various $j$’s.

One possibility, which might look quite appealing here, is that of complete indistinguishability of edges. It means that no permutation of any edges results in new states. To analyze this situation, let us rewrite formula (10) as follows:

$$A = 8\pi\gamma l_p^2 \sum_{jm} \sqrt{j(j+1)} \nu_{jm}.$$

Here the projections $m$ of an angular momentum $j$ run as usual from $-j$ to $j$; $\nu_{jm}$ is the number of edges with given $j$ and $m$. Under the assumption of complete
indistinguishability, the total number of angular momentum states created by \( \nu_j = \sum_m \nu_{jm} \) edges of a given \( j \) with all \( 2j + 1 \) projections allowed, is

\[
K_j = \frac{(\nu_j + 2j)!}{\nu_j! (2j)!}. \tag{16}
\]

The partial contributions \( s_j = \ln K_j \) to the black hole entropy \( S = \sum_j s_j \) that can potentially dominate the numerically large entropy, may correspond to the three cases: \( j \ll \nu_j \), \( j \gg \nu_j \), and \( j \sim \nu_j \gg 1 \). These contributions are as follows:

- \( j \ll \nu_j \), \( s_j \approx 2j \ln \nu_j \);
- \( j \gg \nu_j \), \( s_j \approx \nu_j \ln j \);
- \( j \sim \nu_j \gg 1 \), \( s_j \sim 4j \ln 2 \).

In all the three cases the partial contributions to the entropy \( S \) are much smaller parametrically than the corresponding contributions

\[
a_j \sim j \nu_j
\]

to the area \( A = \sum_j a_j \). Thus, in all these cases \( S \ll A \), so that with indistinguishable edges of the same \( j \), one cannot make the entropy of a black hole proportional to its area \[25, 27\].

Let us consider now the opposite assumption, that of completely distinguishable edges. In this case the total number of states is \( K = \nu! \), with the microcanonical entropy \( S = \nu \ln \nu \). In principle, this entropy can be made proportional to the black hole area \( A \). The model (though not looking natural) could be as follows. We choose a large quantum number \( J \gg 1 \), and assume that the horizon area \( A \) is saturated by the edges with \( j \) in the interval \( J < j < 2J \), and with “occupation numbers” \( \nu_j \sim \ln J \). Then, the estimates both for \( S \) and \( A \) are \( \sim J \ln J \), and the proportionality between the entropy and the area can be attained.

However, though under the assumption of complete distinguishability the entropy can be proportional to the area, the maximum entropy for a given area is much larger than the area itself. Obviously, here the maximum entropy for fixed \( A \sim \sum_j \sqrt{j(j + 1)} \nu_j \) is attained with all \( j \)‘s being as small as possible, say, \( 1/2 \) or \( 1 \). Then, in the classical limit \( \nu \gg 1 \), the entropy of a black hole grows faster than its area: while \( A \sim \nu \), \( S = \nu \ln \nu \sim A \ln A \). Thus, the assumption of complete distinguishability is in conflict with the holographic bound, and therefore should be discarded \[26\].

Let us go over to the third conceivable possibility, which is quite popular (see,
for instance, Ref. [29]. Here the assumption is that the total number of states is
\[ K = \prod_j (2j + 1)^{\nu_j}, \] (17)
with the entropy of the horizon surface
\[ S = \sum_j \nu_j \ln(2j + 1). \] (18)

Obviously, in this case the maximum entropy \( S_{\text{max}} \sim A \) is reached with the smallest possible value of \( j \) for each edge. Thus, one arrives here effectively to the “it from bit” picture which may look attractive.

However, it can be easily demonstrated that this scheme corresponds to the following assumption on the distinguishability of edges:

- nonequal \( j \), any \( m \) → indistinguishable;
- equal \( j \), nonequal \( m \) → distinguishable;
- equal \( j \), equal \( m \) → indistinguishable.

Comparison of the first two lines in this table demonstrates that this assumption looks strange and unnatural.

Thus, the only reasonable assumption on the distinguishability of edges that may result in acceptable physical predictions (i.e., may comply both with the Bekenstein-Hawking relation and with the holographic bound) is as follows:

- nonequal \( j \), any \( m \) → distinguishable;
- equal \( j \), nonequal \( m \) → distinguishable;
- equal \( j \), equal \( m \) → indistinguishable.

4 Microcanonical Entropy of Black Hole

Under the last assumption, the number of states of the horizon surface for a given number \( \nu_{jm} \) of edges with momenta \( j \) and their projections \( j_z = m \), is obviously
\[ K = \nu! \prod_j \frac{1}{\nu_{jm}!}, \quad \nu = \sum_j \nu_j = \sum_j \nu_{jm}, \] (19)
and the corresponding entropy equals
\[ S = \ln K = \ln(\nu!) - \sum_{jm} \ln(\nu_{jm}!). \] (20)

The structures of the last expression and of formula (15) are so different that in a general case the entropy certainly cannot be proportional to the area. However, this is the case for the maximum entropy in the classical limit.
In the classical limit, with all effective “occupation numbers” large, $\nu_{jm} \gg 1$, the entropy in the Stirling approximation is

$$S = \sum_{jm} \nu_{jm} \times \ln \left( \sum_{j'm'} \nu_{j'm'} \right) - \sum_{jm} \nu_{jm} \ln \nu_{jm}. \quad (21)$$

We calculate its maximum for a fixed area $A$, i.e., for a fixed sum

$$N = \sum_{jm} \sqrt{j(j+1)} \nu_{jm} = \text{const}. \quad (22)$$

The problem reduces to the solution of the system of equations

$$\ln \left( \sum_{j'm'} \nu_{j'm'} \right) - \ln \nu_{jm} = \mu \sqrt{j(j+1)}, \quad (23)$$

or

$$\nu_j = (2j+1) e^{-\mu \sqrt{j(j+1)}} \nu. \quad (24)$$

Here $\mu$ is the Lagrange multiplier for the constraining relation (22). Summing expressions (24) over $j$, and recalling that $\sum_j \nu_j = \nu$, we arrive at the equation for $\mu$:

$$\sum_{j=1/2}^{\infty} (2j+1) e^{-\mu \sqrt{j(j+1)}} = 1, \quad (25)$$

with the solution

$$\mu = 1.722. \quad (26)$$

On the other hand, when multiplying equation (24) by $\nu_{jm}$ and summing over $jm$, we arrive with the constraint (22) at the following result for the maximum entropy for a given value of $N$:

$$S_{\text{max}} = 1.722 N. \quad (27)$$

Now, with the Bekenstein-Hawking relation we find the value of the Barbero-Immirzi parameter:

$$\gamma = 0.274. \quad (28)$$

The above derivation follows closely Ref. [24] (see also [8]).

It should be emphasized that this calculation is not special for LQG only, but applies (with obvious modifications) to a more general class of approaches to the quantization of surfaces. The following assumption is really necessary here: the surface should consist of sites of different sorts, so that there are $\nu_i$ sites of each sort $i$, with a generalized effective quantum number $r_i$ (here $\sqrt{j(j+1)}$), and a statistical weight $g_i$ (here $2j+1$). Then in the classical limit, the maximum entropy of a surface can be found, at least numerically, and it is certainly proportional to the area of the surface.

Few more comments are appropriate here.
A nice feature of the obtained picture is that here the occupation numbers $\nu_j$ have a sort of Boltzmann distribution (see (24)), where the partial contributions $\sqrt{j(j+1)}$ to the horizon area correspond to energies, and $\mu$ is the analogue of the inverse temperature.

The next point worth mentioning here is as follows. By substituting the occupation numbers given by formula (24) into expression (22), one arrives at the conclusion that both the effective quantum number $N$ and with it the horizon area $A$ are proportional to an integer $\nu$. It may create the impression of the equidistant area spectrum for a black hole. However, one should keep in mind that relation (24) for occupation numbers is an approximate one, it is valid only for $\nu_j \gg 1$ in the leading approximation to the Stirling formula. In fact, all $\nu_j$'s are integers, and therefore the exact formula (22) in no way corresponds to an equidistant spectrum.

And at last, let us note that the leading correction to our simplest version (21) of the Stirling formula results in the correction $\sim \ln^2 A$ to the Bekenstein-Hawking relation (3) [24].

5 Again Loop Quantum Gravity and Holographic Bound

We come back now to the result of Ref. [10]. If one assumes that the value (11) of the parameter $\gamma$ is the universal one (i. e., it is not special to black holes, but refers to any quantized spherical surface), then the value (5) is not the maximum one in LQG for a surface of the area (6). This looks quite natural: with the transition from the unique choice made in [10], $j^{(d)} = 1/2$, $j^{(u)} = 0$, to more extended and rich one, the number of quantum states should, generally speaking, increase. And together with this number, its logarithm, which is the entropy of a quantized surface, increases as well.

To prove this statement we rewrite formula (5) as follows:

$$S_{1/2} = \ln 2 \sqrt{\frac{4}{3}} N = 0.800 N, \quad N = \sqrt{\frac{3}{4}} \nu,$$

(29)

and consider this value of $N$, together with the horizon area $A$, as a fixed one. This is certainly less than the above result (27) $1.722 N$.

As expected, in the general case, with $N$ given by formula (22) with all values of $j_i^u, j_i^d, j_i^{ud}$ allowed and $g_i = 2j_i^{ud} + 1$, the maximum entropy is even larger (24)

$$S_{\text{max}} = 3.120 N.$$  

(30)

The corresponding value of the Barbero-Immirzi parameter in this case is

$$\gamma = 0.497.$$  

(31)

Thus, the conflict is obvious between the holographic bound and the result advocated in [10].

One might try to avoid the conflict by assuming that the value (11) for the Immirzi parameter $\gamma$ is special for black holes only, while for other quantized surfaces $\gamma$ is smaller. However, such a way out would be unattractive and unnatural.
Quite recently the result of \cite{10} was also criticized and revised in \cite{30, 31}. It is instructive to compare the starting points of Refs. \cite{30, 31} with ours. What both approaches have in common, is the LQG expression \cite{10} for the quantized horizon area $A$. But in other respects the difference between them is drastic.

Our approach is based on the natural and transparent physical requirement that the “occupation numbers” $\nu_{jm}$ (see formulae (19)–(22)) should guarantee the maximum black hole entropy $S$ for given horizon area $A$. Our additional assumption, formulated in section 3, is also natural and physically sound: only those edges that have same $jm$ are indistinguishable. Then the problem reduces to a straightforward and simple calculation presented in section 4.

On the other hand, the approach of Refs. \cite{30, 31} is based on the equidistance assumption, going back to \cite{32, 33} and formulated in \cite{30} as follows: “the fixed classical area $a$ is quantized in the following way,

$$a = 4\pi \gamma l_p^2 k, \quad k \in \mathbb{N},$$

where $k$ is arbitrary.” However, as has been demonstrated already in section 1, there are no physical arguments in favor of an equidistant spectrum. Thus, this assumption is in fact an \textit{ad hoc} one.

Nevertheless, the equation derived in \cite{31} for the Barbero-Immirzi parameter (rewritten in our notations)

$$2 \sum_{j=1/2}^{\infty} e^{-2\pi \gamma \sqrt{j(j+1)}} = 1$$

is rather close to ours \cite{26} ($\mu$ in \cite{26} is equal to $2\pi \gamma$). But I do not see any reasons why the number of states for a given $j$ should be here 2, instead of the usual $2j+1^5$. Still, since the sum is strongly dominated by the first term, with $j = 1/2$, the result 0.238 for $\gamma$, obtained in \cite{31}, is close to ours 0.274.

The conclusions can be summarized as follows.

1. The equidistant result of Ref. \cite{10} is not true.
2. The value of the Barbero-Immirzi parameter, $\gamma = 0.274$, and thus the spectrum of a quantized horizon in LQG (with expression \cite{10} for area), are fixed uniquely by the holographic bound, i.e., by the requirement that among the spherical surfaces of a given area, it is the horizon surface that has the maximum entropy.

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\footnote{The comment on formula \cite{33} in \cite{31} (also rewritten in our notations) is: \textit{"... if the number of states for a given spin $j$ was $2j+1$ instead of 2 one would have in \cite{33} $2j+1$ instead of 2 in front of the exponential and the appropriate $\gamma$ would be equal to 0.273985635..."}.}
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