Analysis of students difficulties in mathematical abstraction thinking in the mathematics statistic course

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Abstract. This study aims to analyze and describe student difficulties in solving mathematical abstraction thinking problems in the Mathematics Statistics subject. This research is qualitative research with the research subjects being students of the mathematics education study program at the University of Indraprasta PGRI Jakarta. The sampling technique used in this study was purposive sampling and the instruments used were documents in the form of the results of the Initial Mathematical Capability test, tests of mathematical abstraction thinking ability, observation sheets, interview guidelines, then analyzed using the triangulation technique. The results showed that for all indicators of students' mathematical abstraction thinking ability, average overall achievement and based on Initial Mathematical Capability level, on the indicator ‘problem transformation into symbol form’ and ‘process of manipulating symbols’ lower than the indicator ‘making generalizations’ and ‘the formation of mathematical concepts related to other concepts’.

1. Introduction
For the college level, mathematics learning requires high order thinking skills because the subject taught is already abstract and requires a formal framework of the axiomatic system as well as a mathematical proof. One of the courses that require a high level of mathematical thinking is Mathematics Statistics. Mathematics Statistics is a must-study course for students of mathematics and mathematics education. The expected learning achievements after studying this course are students can understand probability, expectation, variance, the transformation of two random variables, and sequence statistics.

For the learning achievement of this Math Statistics course to be achieved, then a student needs a high level of thinking ability, one of them is an abstraction. Abstraction is a fundamental process in mathematics because abstraction ability allows students to construct a mathematics concept in their mind by using their initial knowledge [1-2]. In the learning process, the existence of abstraction is a must, because it has an important role in forming mathematical concepts. In the abstraction, the process requires a lot of attention because, through this process, a teacher or lecturer can measure the extent to which students understand a concept and how students construct previous concepts with new concepts to find a relationship [3]. Abstraction is a construction process, in which a mental framework is designed from a mathematical framework and vice versa [4]. Wiryanto emphasizes that abstraction is an activity of mental processes in constructing mathematical concepts which consist of relationships between mathematical structures or objects [5]. The abstraction is rooted in deep constructivist views that the individual’s initial knowledge is the main source for acquiring new knowledge [6].
Based on the above explanation, the process of abstraction will happen through one's experiences, i.e. the student's experience in constructing an initial mathematical knowledge and the concept. Here, the concept in mathematics feels very important, because the concepts are interrelated and mutually required. So, students need to master the process of abstraction [7]. A German philosopher, namely Ernst Cassirer explained that abstraction is projected into different objects by seeing it as related from a certain point of view, focuses on the relationship of all objects that are concerned, and, in the same way, ignoring all aspects which are outside this focus. Words, pictures, and signs help students to focus on the relationships they find relevant for representation [8].

But in reality, what happened to students of mathematics education at the Indraprasta PGRI University, students felt that studying mathematics-statistics material was difficult because the concepts included in the course were too abstract. From the results of observations on students of Unindra Mathematics Education, the ability of mathematical abstraction has not been as expected. One of the obstacles in the course of mathematical statistics is students still have difficulty in solving questions whose concepts are abstract. Especially if they lack an understanding of pre-material such as integral calculus, it will be difficult for them to solve the problem. When students are given questions that match the example, they can work easily. But, if the question has given is a little different and in the form of a story, They are confused to solve it. This can be seen from the test results of several students below the minimum criteria in the mathematics statistics course.

Based on previous research, it was found that in statistics learning, student involvement has been highlighted as the main cause of their unsatisfactory performance and the learning process becomes meaningless because there is no interest shown by students so they consider learning statistics as a burden and knowledge that is irrelevant for studied [9]. Besides that, Mathematical abstraction is often taken seriously constraints in mathematics education [10]. Other research also tells that students have difficulty in making conclusions. The results on the inference determine that the imperfections in other components such as data representations, central tendency, and size of the spread prevent students from making conclusion; insufficient time is devoted to conclusions and predictions from the data, and manufacture conclusion from the data requires higher-level skills [11].

Indicators of abstraction thinking abilities according to Piaget, distinguish three types of abstractions namely: Empirical Abstraction, Pseudo-Empirical Abstraction, and Reflective Abstraction [1]. Another indicator regarding the ability of mathematical abstraction is identifying the manipulated object and transforming the problem (idea) in symbol formula [12]. Hong & Kim Hong in their research used indicators of mathematical abstraction ability, among others: 1) level I, is the introduction of mathematical structures through perceptual abstraction; 2) level II, is the application of mathematical structures through internalization; 3) level III, is the development of new mathematical structures through interiorization [13]. So, indicators of the ability to think mathematically abstraction used in this study include 1) the transformation of the problem into symbols, 2) the formation of mathematical concepts related to other concepts, 3) generalization and 4) the symbol manipulation process.

Based on the above background, the title of this research is “Analysis of Students Difficulties in Mathematical Abstraction Thinking in The Mathematics Statistic Course”. This research aims to find out: 1) how students' mathematical abstraction skills on math statistics courses overall and reviewed from the Initial Mathematical Capability level; 2) What are the difficulties of students in resolving the problem of mathematical abstraction in the course of mathematical statistics; dan 3) What treatment needs to be given to students to solve the problem of mathematical abstraction in the course of mathematics statistics.

2. Methods
This research was conducted on students of mathematics education, University of Indraprasta PGRI Jakarta addressed in the Raya Tengah street, Gedong, Jakarta. The type of research used is qualitative research with case study methods, where researchers try to analyze the difficulties experienced by students in solving the problem of mathematical abstraction thinking skills in mathematics statistics courses. Research subjects are selected using purposive sampling which data source comes from
students. The subjects used in this study were 29 students and then took six students for further analysis. The six research subjects each represent the Initial Mathematical Capability level for example Initial Mathematical Capability high, medium, and low.

Instruments used in this study include test documents (results of Initial Mathematical Capability and mathematical abstraction thinking abilities), observation sheets, interview guidelines, and researcher. For Initial Mathematical Capability’s test instruments and mathematical abstraction thinking abilities have been validated first so that it is ready to be tested to the research subject. The methods of data collection used are triangulation method, while its data analysis techniques use miled and Huberman models. Data analysis activities in the model include data reduction, data display, and conclusion/verification). To test the validity of the data, researchers use credibility tests (through triangulation), transferability tests, dependability tests, and confirmability test.

3. Results and Discussion
Before the research, researchers analyzed student’s Initial Mathematical Capability data obtained in the previous semester. This data aims to group Initial Mathematical Capability into three levels: high, medium, and low. The student's Initial Mathematical Capability grouping uses Noer rules with modified. For Initial Mathematical Capability test results students are given in the following table 1.

| Initial Mathematical Capability score (X) | Category | Sum |
|------------------------------------------|----------|-----|
| X ≥ 70%                                  | High     | 9   |
| 60% ≤ X < 70%                            | Medium   | 15  |
| X < 60%                                  | Low      | 5   |
| Total                                    |          | 29  |

By table 1, the number of students at the high Initial Mathematical Capability level is 9 people or 31.03%; medium Initial Mathematical Capability level is 9 people or 51.72% and low Initial Mathematical Capability level are 5 people or 17.24%. From the data in table 1, it can be concluded that students' Initial Mathematical Capability in the class are more at a medium level. After dividing students' Initial Mathematical Capability into three levels, then researchers took data on the thinking skills of mathematical abstraction by providing tests on mathematical statistics. The problem consists of 4 numbers and represents each indicator of mathematical abstraction thinking ability and based on indicators are 1) the transformation of the problem into symbols, 2) the formation of mathematical concepts related to other concepts, 3) generalization and 4) the symbol manipulation process. Data is provided in table 2 below.

| Types of Abstraction | Indicator | Initial Mathematical Capability score | Postest (X̄) | Average per indicator (%) |
|----------------------|-----------|---------------------------------------|-------------|---------------------------|
| Abstraction reflective| problem transformation into symbol form (ideal score = 100) | High | 66,67 | 41,11% |
|                      |           | Medium | 41,67 | 81,85% |
|                      |           | Low    | 15    | 81,85% |
| Empirical abstraction | making generalizations (ideal score = 100) | High | 97,22 | 88,70% |
|                      |           | Medium | 88,33 | 88,70% |
|                      |           | Low    | 60    | 88,70% |
| Theoretical abstraction | the formation of mathematical concepts related to other concepts (ideal score = 100) | High | 94,44 | 88,70% |
|                      |           | Medium | 96,67 | 88,70% |
|                      |           | Low    | 75    | 88,70% |
|                      | process of manipulating symbols (ideal score = 100) | High | 72,22 | 56,30% |
|                      |           | Medium | 66,67 | 56,30% |
|                      |           | Low    | 30    | 56,30% |
Based on table 2, the research found that for all indicators of students' mathematical abstraction thinking ability, average overall achievement and based on Initial Mathematical Capability level, on the indicator 'problem transformation into symbol form' (41.11%) and 'process of manipulating symbols' (56.30%) lower than the indicator ‘making generalizations’ (81.85) and ‘the formation of mathematical concepts related to other concepts’ (88.70%).

In solving the question of thinking mathematical abstraction in the course of Mathematical Statistics, results that in general students are still having difficulty. To find out more about students' ability to solve the question of mathematical abstraction thinking skills in mathematics statistics courses, the following will be further outlined about the difficulties experienced by students. The analysis will be done per indicator by taking each of the two students at each Initial Mathematical Capability level. Here's an example of a question that presents a mathematical abstraction problem in the indicator with the lowest percentage or the indicator with the highest level of difficulty experienced by students (Figure 1 and Figure 2).

3.1. First indicator ‘problem transformation into symbol form’

Abstraction thinking skills that reveal the first indicator are as follows:

Runi buys fruits at a market and in the fruit basket is filled with 3 oranges, 2 mangoes, and 3 apples. Then, 2 pieces will be taken randomly from the basket.

a. How does the combined probability function of the large number of oranges taken and the number of mangoes taken? Explain!

b. How the probability of orange and mango taking if the taken does not exceed two?

Figure 1. Answer to question no. 1 (medium Initial Mathematical Capability)

The above question aims to uncover the ability of students in transforming problems into symbols related to the probability of an event. The results of document analysis (test answers based on Initial Mathematical Capability level), observations, and interviews related to the indicator are as follows: For students with high Initial Mathematical Capability, generally, they do not have significant difficulty in problem transformation into symbol form'; For students with medium Initial Mathematical Capability, have little difficulty in transforming the problem into the form of the symbol used, as seen in figure 1. This is evident from their reasoning in the answer sheet, concepts used in resolving such questions that are less relevant. Based on the results of an interview with one of the subjects in the high Initial Mathematical Capability, to determine X and Y scores is not difficult because she already understands the steps to solve them. Based on the results of an interview with one of the subjects in the medium Initial Mathematical Capability group, he is still confused in determining the X and Y values of the two random variables; For students with low Initial Mathematical Capability, difficulty in transforming the problem into the form of symbols used. Based on the results of an interview with one of the subjects in
the low Initial Mathematical Capability group, still confused when given about the story and hesitant in completing its calculations.

3.2. Fourth indicator ‘process of manipulating symbols’

The question of abstraction thinking ability that reveals the fourth indicator is as follows:

A combined density function consists of random variables X and Y:

\[ f(x, y) = e^{-(x+y)}; x > 0, y > 0 \]

\[ = 0; \text{ other} \]

If \( U = X + Y \) and \( V = \frac{x}{y} \), then specify the combined density function of the U and V

![Figure 2. Answer to question no. 1 (low Initial Mathematical Capability)](image)

The above question aims to identify the characteristics of objects manipulated or imagined to the transformation of two continuous random variables. To find the transformation, must identify the characteristics of the random variable and manipulate it. The results of document analysis (test answers based on Initial Mathematical Capability level), observations, and interviews related to these indicators are as follows: For students with high Initial Mathematical Capability, generally, they have no significant difficulty in resolving the question of the transformation of two continuous random variables, but still not precise in describing and determining the limits of the transformed density function. Similar to students at the high Initial Mathematical Capability Level, for students with medium Initial Mathematical Capability are also having difficulty and mistakes in determining the limits. Similarly for students with low Initial Mathematical Capability can't even solve the problem, as seen in figure 2. The difficulties experienced by students at each Initial Mathematical Capability level on indicator ‘process of manipulating symbols’ related to the transformation of two continuous random variables are almost the same, in the example not precise in determining of limits.

In general, based on triangulated analysis (documentation, observations, and interviews), obtained the results of that students are still having difficulty in solving the question of mathematical abstraction skills in the course of Mathematics Statistics, especially on indicators ‘problem transformation into symbol form’ and ‘process of manipulating symbols’. These findings can be seen from the results of descriptive analysis, both based on Table 2 and the results of analysis per indicator of the mathematical abstraction capabilities of each Initial Mathematical Capability level.

The most difficulty experienced by students based on abstraction ability test results and interview results, in general, include: 1) difficulty understanding the story problem so they don't know what concept is used to solve it. At this point the result confirms that it happens because the student's way of thinking is focusing on the steps of the problem-solving procedure without knowing why the step is used [14-15]; 2) still wrong in describe and define the limits of the transformed density function on the
indicator process of manipulating symbols. These results are similar to previous research that the cause of the low ability of student abstraction in mathematics statistics courses is lack of student ability to master basic concepts in pre-requisite courses, namely calculus and basic statistics. Then to help students overcome learning difficulties in maths statistics courses, it takes time to understand mathematics as a language and symbol. Moreover, it takes good mathematical insight into contextual issues to have good abstracting skills. How students understand and learn depends on cognitive processing ability and abstract thinking level [16].

4. Conclusion
The research found that for all indicators of students' mathematical abstraction thinking ability, average overall achievement and based on Initial Mathematical Capability level, on the indicator ‘problem transformation into symbol form’ and ‘process of manipulating symbols’ lower than the indicator ‘making generalizations’ and ‘the formation of mathematical concepts related to other concepts’. Most difficulty experienced by students based on abstraction ability test results and interview results, in general, include: 1) difficulty understanding the story problem so they don't know what concept is used to solve it; 2) still wrong in describe and define the limits of the transformed density function on the indicator process of manipulating symbols. Then, to help students overcome learning difficulties in maths statistics courses, it takes time to understand mathematics as a language and symbol. Moreover, it takes good mathematical insight into contextual issues to have good abstracting abilities.

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