Model-independent measurement of the Hubble Constant and the absolute magnitude of Type Ia Supernovae

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Abstract. In this work, we propose a cosmological model-independent and non-local method to constrain the Hubble Constant $H_0$. Inspired by the quasi cosmological model-independent and $H_0$-free properties of the ‘shifted’ Hubble diagram of HII galaxies (HIIGx) defined by Wei et al. (2016), we joint analyze it with the parametric type Ia supernova (SN Ia) Hubble diagram (e.g. the joint-lightcurves-analysis sample, JLA) and get a Bayesian Inference of Hubble constant, $H_0 = 71 \pm 20 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Although with large uncertainty, we find that $H_0$ is only strongly degenerate with the B-band absolute magnitude ($M_B$) of SN Ia but almost independent on other nuisance parameters. Therefore the accuracy can be simultaneously improved by a tight constraint of $M_B$ through a cosmological and $H_0$ independent way. This method can be extended further to get more-literally non-local results of $H_0$ by using other Hubble diagrams at higher redshifts.
1 Introduction

The value of the Hubble constant $H_0$, which is defined as the current expansion rate of the universe, has not yet reached consensus in this era of the precision cosmology. The two major measurement methods are cosmic distance ladders (the direct measurements) in the local universe (over which the effects of cosmic evolution are small) and the cosmic microwave background (CMB) inference in the epoch of the recombination. The former provides the cosmological model-independent and local measurements of the $H_0$, while the latter is in the opposite. The principle of the distance ladders measurements are based on the Hubble’s law $v = H_0 D$, which tells us that the receding speeds $v$ of galaxies are proportional to their distances $D$ from us, and the coefficient $H_0$ is constant. To measure the $H_0$ more accurately, we need to improve the precisions of both receding speeds and distances. Different from this approach, the CMB inference method measures the imprints of the baryon acoustic oscillation (BAO) on the anisotropic power spectrum of CMB and combines with the assumption that our universe evolves as a Flat-$\Lambda$CDM model to constrain the $H_0$.

Recently, collaborators in the SH0ES team used 70 long-period Cepheids in the Large Magellanic Cloud (LMC) as the standard candle and combined Type Ia Supernovae (SN Ia) as secondary distance indicator to finally make the most precise local result as $H_0 = 74.03 \pm 1.42 \text{km s}^{-1} \text{Mpc}^{-1}$ [1]. But the latest observational CMB result from the Plank collaboration ($H_0 = 67.36 \pm 0.54 \text{km s}^{-1} \text{Mpc}^{-1}$ [2]) is in $4.4\sigma$ difference with the latest SH0ES result. Such difference between the early and late universe measurements of $H_0$ is an urgent crisis named the Hubble-Tension in modern cosmology.

To deal with this tension, we need to both estimate all systematic effects which might be included in these measurements and try some new physics that beyond the $\Lambda$CDM framework. Besides, new independent measurements of the $H_0$ are particularly important to verify this tension. Collaborators in the H0LiCOW ($H_0$ Lenses in COSMOGRAIL’s Wellspring) team apply a strong-lensing cosmography and get their latest result as $H_0 = 73.3^{+1.7}_{-1.8} \text{km s}^{-1} \text{Mpc}^{-1}$ [3]. This method is fully independent of all rungs of the distance ladders, and the result is in agreement with the latest SH0ES result, but is in $3.1\sigma$ tension with the latest Planck result. In combination of the latest SH0ES result, this tension is intensified up to $5.3\sigma$. However, a cosmological model-independent non-local measurement of the $H_0$ is even more important in determining whether we need to go beyond the standard $\Lambda$CDM framework or not.
In this paper, we propose such a Bayesian approach which jointly analyzes the Hubble-free quasi cosmology-independent shifted HIIGx Hubble diagram with the JLA [4] SN Ia Hubble diagram to constrain $H_0$. In section 2, we illuminate the methodology which makes our intents workable. In section 3, we describe the data used in this paper and present our results. In section 4, we draw our conclusions and make some discussions.

2 Methodology

The Hubble diagrams, which show the correlation between distances (luminosity distances $D_L$ or angular diameter distances $D_A$) and redshifts, are used to depict the evolutionary history of the universe. Mathematically, the $D_L$ as a function of redshift can be expanded into Taylor series as

$$D_L(z) = \frac{cz}{H_0} \left[ D^0_L + D^1_L z + D^2_L z^2 + O(z^3) \right], \quad (2.1)$$

where

$$\begin{align*}
D^0_L &= 1 \\
D^1_L &= -\frac{1}{2} (1 - q_0) \\
D^2_L &= -\frac{1}{6} (1 - q_0 - 3q^2_0 + j_0). 
\end{align*} \quad (2.2)$$

The $q_0$ and $j_0$ in above equation are the deceleration parameter and jerk parameter respectively, which are the dimensionless second and third derivative of the scale factor $a(t)$ with respect to cosmic time $t_0$. The first-order approximation at low redshifts of the formula (2.1) is the famous Hubble’s law. Theoretically, the $D_L(z)$ can be expressed as

$$D_L(z) = \frac{c \frac{1}{H_0} \frac{1}{\sqrt{\Omega_k}} \sinh \left[ \sqrt{\Omega_k} \int_0^z \frac{dz'}{E(z')} \right]}{1 + z} \quad \text{for } \Omega_k > 0$$

$$D_L(z) = \frac{c \frac{1}{H_0} \frac{1}{\sqrt{\Omega_k}} \sin \left[ \sqrt{|\Omega_k|} \int_0^z \frac{dz'}{E(z')} \right]}{1 + z} \quad \text{for } \Omega_k = 0 ,$$

$$D_L(z) = \frac{c \frac{1}{H_0} \frac{1}{\sqrt{|\Omega_k|}} \sin \left[ \sqrt{|\Omega_k|} \int_0^z \frac{dz'}{E(z')} \right]}{1 + z} \quad \text{for } \Omega_k < 0 \quad (2.3)$$

where the dimensionless Hubble parameter $E(z)$ is model-dependent, i.e. in $\Lambda$CDM model

$$E(z) = \sqrt{\Omega_A + \Omega_k (1 + z)^2 + \Omega_m (1 + z)^3}. \quad (2.4)$$

The direct measurements of $H_0$ by Hubble’s Law are intrinsically local, while all the constraints of $H_0$ by Hubble diagrams applying Equation (2.3) are inevitably model-dependent. Currently, we could only conclude that the Hubble tension is existed between the local direct and non-local model-dependent measurements. To develop a cosmological model-independent non-local approach which can be applied to measure $H_0$ is quite promising. Fortunately, the ‘shifted’ Hubble-free HIIGx Hubble diagram makes achieving this aspiration becoming pragmatic.

2.1 SN Ia Hubble diagram

SN Ia are widely used as secondary standard candles to measure luminosity distances $D_L$ because the peak luminosities of light curves of all SN Ia are nearly identical. Finding a SN Ia which shares the same host galaxy with a Cepheid variable, one can measure the $D_L$ of
the host galaxy through this Cepheid variable. Principally, combining peak magnitude \( m_{\text{max}} \) of the with the formula for distance modulus,

\[
\mu = 5 \log \frac{D_L}{\text{Mpc}} + 25 = m_{\text{max}} - M_{\text{max}},
\]

(2.5)

the peak absolute magnitude \( M_{\text{max}} \) of all SN Ia can be obtained. Therefore, the \( D_L \) of an arbitrary SN Ia can be easily obtained by measuring its \( m_{\text{max}} \).

However, the \( M_{\text{max}} \) of SN Ia are not exactly the same but also related to the shapes can colors of the light curves. Considering this, the formula for SN Ia distance modulus should be modified by adding perturbations of shapes \( x \) and color \( c \) as

\[
\mu_B(\alpha, \beta, M_B) = m_{\text{max}}^B - M_B + \alpha x - \beta c,
\]

(2.6)

where the subscript \( B \) stands for B band, while \( \alpha \) and \( \beta \) are nuisance parameters for modification. It is well known that \( M_B \) is degenerate with \( H_0 \), and inevitably, its value and also the SN Ia Hubble diagram are commonly based on an assumed value of \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \). This makes it illogical to constrain \( H_0 \) using the SN Ia Hubble diagram which consists of higher redshifts (non-local) observations. But if there exists a method which can be used to calibrate the SN Ia Hubble diagram in a cosmological model-independent way, the two interested parameters will be constrained simultaneously.

### 2.2 ‘shifted’ Hubble-free HIIGx Hubble diagram

The luminosity of Balmer H\( \beta \) lines that emitted in HIIGx is strongly correlated with the ionized gas velocity dispersion \( \sigma_v \) [6], because both the intensity of ionizing radiation and \( \sigma_v \) increase with the starburst mass [7]. This correlation can be approximated as [8]

\[
\log \left[ \frac{L(\text{H}\beta)}{\text{erg s}^{-1}} \right] = \alpha \log \left[ \frac{\sigma_v(\text{H}\beta)}{\text{km s}^{-1}} \right] + \kappa,
\]

(2.7)

where \( \alpha \) and \( \kappa \) are constants. The relatively small scatter in the relationship between \( L(\text{H}\beta) \) and \( \sigma_v \), allows these galaxies and local HII regions to be used as standard candles [9–12].

With the selecting criteria that guarantee the selected HIIGx are comprised of systems in which the luminosity is dominated by single and very young starbursts (less than 5 Myr in age) [9], the bolometric flux of the HIIGx may thereby be regarded as constituting principally the \( \text{H}\beta \) line. Therefore the luminosity distance of an HIIGx can be approximated with the luminosity \( L(\text{H}\beta) \) and flux \( F(\text{H}\beta) \) pertaining to the \( \text{H}\beta \) line,

\[
D_L = \left[ \frac{L(\text{H}\beta)}{4\pi F(\text{H}\beta)} \right]^{1/2},
\]

(2.8)

where \( F(\text{H}\beta) \) is the reddening corrected \( \text{H}\beta \) flux.

Wei et al. [10] defined the ‘shifted’ distance modulus as

\[
\eta = 5 \log \left[ \frac{H_0 D_L}{\text{km s}^{-1}} \right],
\]

(2.9)

which is shifted by a constant difference from the conventional distance modulus as the relation

\[
\mu = \eta - 5 \log \left[ \frac{H_0}{\text{km s}^{-1} \text{ Mpc}^{-1}} \right] + 25.
\]

(2.10)
Combining with the “$H_0$-free” logarithmic luminosity defined as
\[ \delta \equiv -2.5\kappa - 5 \log \left( \frac{H_0}{\text{km s}^{-1} \text{Mpc}^{-1}} \right) + 125.2, \]  
(2.11)
thus the $\eta$ can be obtained as
\[ \eta = -\delta + 2.5 \left[ \alpha_0 \log \sigma(H\beta) - \log F(H\beta) \right]. \]  
(2.12)

Authors in Ref. [10, 13, 14] adopt the maximum likelihood estimation to constrain the two ‘nuisance’ parameters $\alpha_0$ and $\delta$ within three cosmological models ($R\Lambda = ct$, $\Lambda$CDM and $\omega$CDM). They have found that the two parameters are very insensitive to these cosmological models (see results in the Table 2 of Ref. [10]). This critical property allows us to use the HIIGx Hubble diagram in a quasi cosmological model-independent way (maybe some exotic models are excluded [15]). Here we adopt the average value of these parameters as a reasonable representation, i.e., $\alpha_0 = 4.87^{+0.11}_{-0.08}$ and $\delta = 32.42^{+0.42}_{-0.33}$. Then given the flux and gas velocity dispersion (along with their uncertainties) of HIIGx and GEHR, we can get the observed ‘shifted’ distance modulus $\eta_{\text{obs}}$ using Equation (2.12).

2.3 Juxtaposition of Hubble diagrams

Known that the shifted amount of HIIGx Hubble diagram is $H_0$-only related, an optimized value of $H_0$ can be extracted in a cosmological model-independent fashion by jointly fitting the HIIGx and SN Ia Hubble diagrams with these nuisance parameters ($M_B$, $\alpha$ and $\beta$) and $H_0$. We name this analysis procedure as juxtaposition of Hubble diagrams. In this article, we perform the Bayesian statistical methods and the Markov Chain Monte Carlo (MCMC) technique to calculate their joint posterior probability density function (PDF).

We use a $\chi^2$ minimization to constrain $H_0$,
\[ \chi^2(H_0, M_B, \alpha, \beta) = \sum \frac{[\mu_{\text{HII}}(H_0; z_i) - \mu_B(\alpha, \beta, M_B; z_i)]^2}{\sigma_{\mu_{\text{HII}}}(z_i) + \sigma_{\mu_B}^2(z_i)}, \]  
(2.13)
where $\mu_{\text{HII}}(H_0; z_i)$ of HIIGx comes from Equation (2.10) and (2.12), $\mu_B(\alpha, \beta, M_B; z_i)$ of the SN Ia sample is given by Equation (2.6), and the total uncertainty comes from the uncertainty of $\mu_{\text{HII}}$ and $\mu_B$. The error propagation of $\sigma_{\mu_{\text{HII}}}(z_i)$ is given by
\[ \sigma_{\mu_{\text{HII}}}^2(z_i) = \sigma_0^2 + 2.5^2[\alpha_0^2 \sigma_{\log \sigma}^2 + \sigma_{\alpha_0}^2 \log \sigma + \sigma_{\log F}^2], \]  
(2.14)
where $\sigma_{\alpha_0}$, $\sigma_0$, $\sigma_{\log \sigma}$ and $\sigma_{\log F}$ are the errors of $\alpha_0$, $\delta$, $\log \sigma(H\beta)$ and $\log F(H\beta)$ respectively. Meanwhile, the error propagation of $\sigma_{\mu_B}(z_i)$ is given by
\[ \sigma_{\mu_B}^2(z_i) = \sigma_m^2(z_i) + \alpha^2 \sigma_x^2(z_i) + \beta^2 \sigma_c^2(z_i) + 2\alpha\beta \text{Cov}(x, c) + 2\sigma \text{Cov}(m_B^{\text{max}}, x) + 2\beta \text{Cov}(m_B^{\text{max}}, c), \]  
(2.15)
where $\sigma_m$, $\sigma_x$ and $\sigma_c$ are the errors of $m_B^{\text{max}}$, $x$ and $c$ respectively, while $\text{Cov}(m_B^{\text{max}}, x)$, $\text{Cov}(m_B^{\text{max}}, c)$ $\text{Cov}(x, c)$ are the covariances of them.

In principle, the $\chi^2$ can only be calculated for those $\mu_{\text{HII}}$ and $\mu_B$ that share the same redshift. However, this condition usually cannot be satisfied for the two different targeted observations. So we need to reconstruct one of the Hubble diagrams in a model-independent
way. In this article, we choose to reconstruct the "shifted" HIIGx Hubble diagram by using the Gaussian Process (GP), which is a machine learning algorithm without any cosmological nor astrophysical assumptions, therefore are widely used in recent researches. Once we get the GP reconstructed data points \( \{ \eta, \sigma_\eta \} \) from the observed HIIGx data, we can combine the observational SN Ia light curve data \( \{ m_B^{\text{max}}, x, c, \sigma_m, \sigma_x, \sigma_c, \text{Cov}(m_B^{\text{max}}, x), \text{Cov}(m_B^{\text{max}}, c), \text{Cov}(x, c) \} \) to calculate the \( \chi^2 \) that is defined in Eq. (2.13). Then, the joint PDF of these parameters can be obtained, \( P(H_0, M_B, \alpha, \beta) = A \exp(-\chi^2/2) \), where \( A \) is a normalised coefficient, which makes the total probability \( \iiint P \, dH_0 \, dM_B \, d\alpha \, d\beta = 1 \). Finally, by integrating over \( (M_B, \alpha \text{ and } \beta) \) the PDF of \( H_0 \) is obtained, i.e., \( P(H_0) = \iint P \, dM_B \, d\alpha \, d\beta \).

3 Data and Results

In this paper, we implement the above procedure by combining the shifted HIIGx Hubble diagram with the JLA SDSS-II/SNLS3 sample. The data we used are listed in detail as the following.

(1) HIIGx. This catalog [10] contains 156 objects in the range of \( 10^{-5} < z < 2.315 \). It includes 25 high-z HII galaxies, 107 local HII galaxies and 24 giant HII regions from...
Figure 2. One-dimensional and two-dimensional marginalized distributions with 1σ and 2σ contours for $H_0$, $M_B$, $\alpha$ and $\beta$ using JLA + HIIGx samples. The titles of each sub PDF plot show the best fit values and 1σ uncertainties of each parameters respectively. The $H_0$ is in the unit of km s$^{-1}$ Mpc$^{-1}$, while the others are unitless.

the catalog from the observational works in Ref. [9, 16–20]. The fluxes and gas velocity dispersions (along with their uncertainties) of HIIGx and GEHR that we use in this paper are all referred from this catalog.

(2) JLA. This catalog is composed by Betoule et al. [4] from observations obtained by the SDSS-II and SNLS collaborations. The data set includes several low-redshift samples ($z < 0.1$), all three seasons from the SDSS-II (0.05 < $z$ < 0.4), and three years from SNLS (0.2 < $z$ < 1), and it totals 740 spectroscopically confirmed type Ia supernovae with high quality light curves.
The shifted distance modulus data of HIIGx and its GP reconstruction (generated from the Python module GaPP\textsuperscript{1} [21]) are shown together in Figure 1. As we can see in this figure, the error of the reconstructed $\eta(z)$ function is smaller than that of the original distance modulus data. This is a common case when there is a large correlation between the dataset points $\{z_i\}$ and the reconstructed point $z_*$ [11, 15, 21].

We use the Python module emcee\textsuperscript{2} [22] to sample from the posterior distribution of $H_0$ and nuisance parameters $M_B$, $\alpha$ and $\beta$, and then obtain its optimized value and 1$\sigma$ error. The corresponding joint PDF plot is shown in Figure 2. As we can see, the nuisance parameters $\alpha$ and $\beta$ are almost mutually independent and also independent on $H_0$ and $M_B$. More importantly, the degeneracy between $H_0$ and $M_B$ is strongly re-affirmed, which means the value of $H_0$ almost only dependents on the value of the B-band absolute magnitude $M_B$ but not the other nuisance parameters. Our result shows $H_0 = 71 \pm 20$ km s$^{-1}$ Mpc$^{-1}$ and $M_B = -18.83^{+0.78}_{-0.37}$, which even though with large uncertainty but are all consistent with their present values from the community. But the coefficients $\alpha$ and $\beta$ of the light curves are not identical to those from Ref. [4] due to our juxtaposition of the two Hubble diagrams. The GP reconstruction underestimates the error of the shifted HIIGx distance modulus, therefore systematical offset of the two data sets can cause such difference. The good point is that this difference almost doesn’t affect the result of $H_0$. Therefore, it will not underrate the proof-of-principle of our approach.

4 Conclusion and Discussion

With the joint analysis of HIIGx Hubble diagram and JLA SN Ia Hubble diagram, we prove that our juxtaposition of the Hubble diagrams method to constrain $H_0$ is feasible. In our proof-of-principle example, the value of $H_0$ is strongly degenerate with the B-band absolute magnitude of SN Ia $M_B$ but almost independent with the other nuisance parameters. Although our result gives a large uncertainty in $H_0$ constraint, the accuracy can be simultaneously improved by a tight constraint of $M_B$ through a cosmological and $H_0$ independent way. The self-calibrated distance of GW sources can be treated as a standard siren to independently construct the Hubble diagram, therefore, to circumvent the $H_0 - M_B$ degeneracy and measure the $H_0$ directly. The third generation GW detection can also model-independently measure $M_B$ with uncertainty more than one order of magnitude smaller than that from the present cepheid calibration [23].

This method can also be applied to the juxtaposition of HIIGx Hubble diagram with other Hubble diagrams, which might be a larger sample or consist of higher redshift objects such as quasars and FRBs. With higher redshift samples, we expect to obtain more-literally cosmological model-independent non-local constraint on $H_0$, which will be constructive to fill in the gaps in its measurement from the long evolutionary history between the early- and late- Universe, thereby adjudicate whether there is an an early time transition or late time transition in the expansion rate that drives us to go beyond the $\Lambda$CDM model.

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\textsuperscript{1}http://www.acgc.uct.ac.za/~seikel/GAPP/index.html
\textsuperscript{2}http://dfm.io/emcee/current/
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