MUSCO: MULTI-STAGE COMPRESSION OF NEURAL NETWORKS

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Abstract

The low-rank tensor approximation is very promising for the compression of deep neural networks. We propose a new simple and efficient iterative approach, which alternates low-rank factorization with a smart rank selection and fine-tuning. We demonstrate the efficiency of our method comparing to non-iterative ones. Our approach improves the compression rate while maintaining the accuracy for a variety of tasks.

1 Introduction

The development of deeper and more complex networks in order to achieve higher performance has become commonplace. However such networks contain tens of millions of parameters and often cannot be efficiently deployed on embedded systems and mobile devices due to their computational power and memory limitations.

Low-rank matrix and tensor approximations provide excellent compression of neural network layers \cite{9,10,11,23}. In these methods, factorization of weight tensors yields an approximate compressed network. For example, when we approximate a 4-dimensional convolutional kernel by a tensor, whose factorized form has three components, we can replace a corresponding layer with three consecutive convolutional layers (Figure 4). However, approaches based on a low-rank tensor factorization are built on the same scheme: compression followed by fine-tuning to compensate for a significant loss of the quality of the model. The main benefit of this approach is that the compressed version provides an initial approximation, which leads to a better quality after fine-tuning than if the same architecture is learned directly from a random initialization.

In our paper, we propose a way to substantially improve the abovementioned scheme by applying low-rank decomposition and fine-tuning iteratively several times (Algorithm 1). It turns out that such simple idea can significantly improve the quality of neural networks compression. For example, for Faster R-CNN with ResNet-50 backbone, we achieve a better compressed model for the same quality than can be obtained with non-iterative algorithm (Section 6).

We introduce Multi-Stage Compression method (MUSCO) for automated network compression (Sections 2, 3). The algorithm consists of two repetitive steps: compression and fine-tuning (Figure 1). During the compression step model weights of selected layers are approximated using tensor decomposition with automatically selected rank values (Section 5). At this step, the redundancy present in the weight parameters is partially reduced. The next step allows to recover the quality of the model by performing fine-tuning. By repeating these two steps several times we can gradually compress the model by substantially reducing the number of parameters in the selected layers.
In comparison with other approaches, MUSCO does not lose quality significantly during more aggressive parameters reduction. In practice, MUSCO allows achieving higher compression ratio than state-of-the-art non-iterative approaches with the same quality of the model. Our main contributions are:

- We propose an iterative low-rank approximation algorithm to efficiently compress neural networks that outperform non-iterative methods for the desired accuracy.
- We introduce a method for automatic tensor rank selection for tensor approximations performed at each compression step.
- We validate and demonstrate the high efficiency of our approach in a series of extensive computational experiments for object detection and classification problems.

2 Problem statement

In this section we introduce a formal description of a model compression in terms of transitions from one class of models to another.

Each neural network can be described as a pair \((f, \theta)\), where \(\theta\) denotes model parameters and \(f\) defines network architecture (i.e., graph structure). Given \(\theta\), a continuous function \(f\) assigns to each input \(X\) a result of its propagation through the whole network, \(f(X, \theta)\).

Let \(M\) be our pre-trained neural network model. We denote the class of all neural networks with the same architecture as \(M\) by \(\mathcal{M} = \{(f, \theta) | \theta \in \Theta\}\). Here \(\theta\) is an array of weight tensors that parametrize individual layers of the network architecture \(f\), and \(\Theta\) defines a set of all possible parametrizations.

We perform a network compression via low-rank tensor approximation of its weight tensors \(\theta\). The concept of rank can be defined for any tensor. We use \(\text{rank}(\theta)\) to determine an array of ranks corresponding to tensors in \(\theta\). The expression \(\text{rank}(\theta) \leq R\), where \(R\) is an array of constant values, is used to describe elementwise constraints applied to tensors from \(\theta\).

To apply a rank-\(R\) factorization to the weights \(\theta \in \Theta\) is to find an array of \(\hat{\theta}\) from \(\Theta^R\),

\[
\Theta^R = \{\theta \in \Theta | \text{rank}(\theta) \leq R\},
\]

which approximates \(\theta\) in a certain norm and can be represented in a factorized format \(\hat{\theta}^{\text{fact}} \in \Theta^R_{\text{fact}}\) (i.e., \(\hat{\theta}^{\text{fact}}\) is an array, where each element corresponds to one weight tensor from \(\theta\) and represented by a tuple of factors).

We introduce operators \(\mathcal{F}_{\text{fact}}\) and \(\mathcal{F}_{\text{full}}\) that perform these mappings from \(\Theta^R\) to \(\Theta^R_{\text{fact}}\) and vice versa, i.e.,

\[
\Theta^R_{\text{fact}} = \{\mathcal{F}_{\text{fact}}(\theta) | \theta \in \Theta^R\}
\]

and \(\mathcal{F}_{\text{full}}(\mathcal{F}_{\text{fact}}(\theta)) = \theta\) for \(\theta \in \Theta^R\).

When we compress a network \((f, \theta) \in \mathcal{M}\) using a rank-\(R\) weight factorization (Figure 2), firstly, we obtain a model \((f, \hat{\theta})\) with the same architecture by projecting \(\theta\) to the parameter set \(\Theta^R \subseteq \Theta\). Secondly, we get a compressed model \((f^R, \hat{\theta}^{\text{fact}}) \in \mathcal{M}^R\) with a new architecture \(f^R\) by replacing \(\hat{\theta}\) with its factorized version \(\hat{\theta}^{\text{fact}}\), where

\[
\mathcal{M}^R = \{(f^R, \theta) | \theta \in \Theta^R_{\text{fact}}\}
\]

and \(f^R\) is a modification of \(f\), which contains decomposed linear layers instead of original ones. A decomposed layer is a sequence of linear layers, each of which is represented by one factor (component) from the factorization of the original layer weight tensor.

Figure 2: Iterative compression (first iteration): pre-trained \((f, \theta)\) is factorized \((f, \hat{\theta}^{\text{R}})\) and then compressed \(\hat{\theta}^{\text{R}}\) to \(\hat{\theta}^{\text{fact}}\) and fine-tuned \((f^R, \hat{\theta}^{\text{fact}})\).

After fine-tuning a network \((f^R, \hat{\theta}^{\text{fact}})\), we obtain a model \((f^R, \hat{\theta}^{\text{fact}}) \in \mathcal{M}^R\), which attains a local minimum of the loss function calculated on training samples.

We propose an iterative low-rank factorization to the parameter set \(\Theta^R \subseteq \Theta\). We use rank(\(\theta\)) to determine an array of ranks corresponding to tensors in \(\theta\). The expression rank(\(\theta\)) \(\leq R\), where \(R\) is an array of constant values, is used to describe elementwise constraints applied to tensors from \(\theta\).

To apply a rank-\(R\) factorization to the weights \(\theta \in \Theta\) is to find an array of \(\hat{\theta}\) from \(\Theta^R\),

\[
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\]

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\Theta^R_{\text{fact}} = \{\mathcal{F}_{\text{fact}}(\theta) | \theta \in \Theta^R\}
\]

and \(\mathcal{F}_{\text{full}}(\mathcal{F}_{\text{fact}}(\theta)) = \theta\) for \(\theta \in \Theta^R\).

When we compress a network \((f, \theta) \in \mathcal{M}\) using a rank-\(R\) weight factorization (Figure 2), firstly, we obtain a model \((f, \hat{\theta})\) with the same architecture by projecting \(\theta\) to the parameter set \(\Theta^R \subseteq \Theta\). Secondly, we get a compressed model \((f^R, \hat{\theta}^{\text{fact}}) \in \mathcal{M}^R\) with a new architecture \(f^R\) by replacing \(\hat{\theta}\) with its factorized version \(\hat{\theta}^{\text{fact}}\), where

\[
\mathcal{M}^R = \{(f^R, \theta) | \theta \in \Theta^R_{\text{fact}}\}
\]

and \(f^R\) is a modification of \(f\), which contains decomposed linear layers instead of original ones. A decomposed layer is a sequence of linear layers, each of which is represented by one factor (component) from the factorization of the original layer weight tensor.

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After fine-tuning a network \((f^R, \hat{\theta}^{\text{fact}})\), we obtain a model \((f^R, \hat{\theta}^{\text{fact}}) \in \mathcal{M}^R\), which attains a local minimum of the loss function calculated on training samples.

When we further compress an already decomposed network \((f^R, \theta^{\text{fact}})\), we apply the mentioned rank-\(R'\) compression procedure to the model \((f, \theta) \in \mathcal{M}\), where \(\theta \in \Theta^R\) (Figure 3). In Section 4 we show how this step can be optimized for different types of tensor factorizations.

Figure 3: Iterative compression (next iteration): the rank of factorized weights \(\theta^{\text{fact}}\) is further reduced. The architecture is changed but the number of linear layers remains the same.

Thus, when we gradually compress the pre-trained model \(M\) over \(K\) iterations (each iteration contains compression and fine-tuning steps), we sequentially obtain models from classes \(\mathcal{M}_{R_1}, \ldots, \mathcal{M}_{R_K}\), namely,

\[
M \rightarrow \tilde{M}_1 \rightarrow M_1 \rightarrow \ldots \rightarrow \tilde{M}_K \rightarrow M_K,
\]

s.t. \(\tilde{M}_k, M_k \in \mathcal{M}_{R_k}\),

where \(R_1 \geq \ldots \geq R_K\). Transitions \(M_{k-1} \rightarrow \tilde{M}_k\) and \(\tilde{M}_k \rightarrow M_k\) correspond to the compression and fine-tuning steps respectively, \(k = 1 \ldots K\). To compare,
for non-iterative approach the similar path looks like
\( M \to \tilde{M}_K \to M_K \), i.e. architecture \( f^{R_K} \) of resulting
compressed model is determined at the first (and the only)
compression step.

If we instead train the architecture \( f^{R_K} \) from scratch, it is
often impossible to achieve the quality comparable to
the initial model \( M \). If non-iterative approach is applied
and we compress \( M \) directly to the model from \( \mathcal{M}_{R_K} \)
(as it is done in [11, 10]), after fine-tuning we end up with
a good baseline. We show that our iterative approach
(Algorithm 1) beats the baseline in terms of compression
rate with the same quality drop. Moreover, our approach allows to perform compression by
automatically selecting ranks for low-rank
approximations. Thus, every time we approximate weights (i.e. project
weights to the smaller parameter subspace), we make a
step away from the local minimum on the loss surface.
Due to the continuity of the model, the more we reduce
the weight rank, the bigger is the step and, hence, the more
difficult it is to get back to a local optimum during the
subsequent fine-tuning (because of the non-convexity of
the optimization problem). In the iterative approach, in
contrast to the non-iterative, the ranks decrease smoothly
and gradually, and that allows to obtain a higher model
compression rate with the same quality drop.

Moreover, our approach allows to perform compression by
automatically searching for the best rank values \( R_k, k = 1, \ldots, K \) (see Section 5 for the details).

3 Compression algorithm

In this section, we formulate the optimization problems
which need to be solved during one iteration of the compression
algorithm, and we provide the detailed procedure
for iterative neural network model compression.

3.1 One iteration of the algorithm

To compress a layer with a weight tensor \( \theta \), for the first
time, given rank \( R \) we solve a problem of minimizing
Frobenius norm, given rank \( R \):

\[
\min_{\hat{\theta}^R_1, \ldots, \hat{\theta}^R_N} ||\theta - \hat{\theta}^R||, \quad \text{such that} \quad \mathcal{F}_{\text{fact}}(\hat{\theta}^R) = (\hat{\theta}^R_1, \ldots, \hat{\theta}^R_N),
\]

where \( \hat{\theta}^R_1, \ldots, \hat{\theta}^R_N \) denote components in a factorized form
of the tensor and define weights of \( N \) layers into which
initial layer is decomposed during the rank-\( R \) factorization.

To further compress an already decomposed layer, we
update its fine-tuned weights \( \{\theta^{R_n}_n\}_{n=1}^N \). Namely, given
rank \( R' < R \), we solve the following minimization problem:

\[
\min_{\theta^{R'}_1, \ldots, \theta^{R'}_N} ||\mathcal{F}_{\text{fact}}(\hat{\theta}^{R'}_1, \ldots, \hat{\theta}^{R'}_N) - \hat{\theta}^{R'}||,
\]

where \( \theta^{R'}_1, \ldots, \theta^{R'}_N \) denote updated weights (factor matrices) of the decomposed layer.

During the fine-tuning step we minimize the loss function \( \ell \) given training data \( \{(X_j, Y_j)\}_{j=1}^J \), where \( X_j \) is an
input sample and \( Y_j \) is a corresponding target value. Thus, we solve the following optimization problem:

\[
\mathcal{L}(\theta) \rightarrow \min_{\theta \in \Theta^{R_{\text{fact}}}_1} \ell(\mathcal{F}(X_j, \theta), Y_j),
\]

where \( \mathcal{F}(\cdot) \) is our compressed architecture and \( \Theta^{R_{\text{fact}}}_1 \) is the set of all possible model parameters.

3.2 Iterative procedure

Our proposed algorithm is an alternation of compression and fine-tuning steps with automatically selected ranks for the weights approximation (see Algorithm 1 for the details).

At the compression step of each iteration for each of the
selected layers, we solve the optimization problem (5) if a
layer has not been compressed yet, and the optimization problem (6) otherwise. The fine-tuning step is the same
for all iterations.

Algorithm 1 Iterative low-rank approximation algorithm
for automated network compression

\begin{itemize}
  \item \textbf{Input:} Pre-trained original model, \( M \)
  \item \textbf{Output:} Fine-tuned compressed model, \( M^* \).
  \item 1: \( M^* \leftarrow M \)
  \item 2: \textbf{while} desired compression rate is not attained or
  \textbf{automatically selected ranks have not stabilized} \textbf{do}
  \item 3: \( R \leftarrow \) automatically selected ranks for low-rank
  tensor approximations of convolutional and fully-
  connected weight tensors.
  \item 4: \( \hat{M} \leftarrow (\text{further}) \) compressed model obtained from
  \( M \) by replacing layer weights with their rank-\( R \) tensor
  approximations.
  \item 5: \( M^* \leftarrow \text{fine-tuned model } \hat{M} \).
  \item 6: \textbf{end while}
\end{itemize}
4 Layer compression in details

In this paper, we focus on Tucker and HOSVD (High Order Singular Value Decomposition) decompositions \[^2]\[^3\]. Our framework can be used for other decompositions as well, and supplementary results using CP-based and SVD-based compressions are given in Appendix.

Since the definition of a tensor rank is not unified, we use a multilinear rank for Tucker (see definition below) and a CPD tensor rank for CP (Appendix).

A Tucker decomposition of an \( N \)-dimensional tensor is a factorization into a small size core tensor and \( N \) factor matrices. For a convolutional kernel \( \theta \in \mathbb{R}^{d \times d \times C_{in} \times C_{out}} \), with \( C_{in} \) input channels, \( C_{out} \) output channels, and a \( d \times d \) spacial filter, it can be written as

\[
\theta \approx \theta_C \times_h \theta_h \times_w \theta_w \times_{in} \theta_{in} \times_{out} \theta_{out},
\]

where \( \theta_C \) is a 4-dimensional core tensor, \( \theta_h, \theta_w, \theta_{in}, \theta_{out} \) are matrices to be multiplied along each dimension of the core tensor. Symbols \( \times_h, \times_w \) and \( \times_{in}, \times_{out} \) denote multilinear products along spacial and channel dimensions respectively.

If decomposition \[^8\] holds exactly, the multilinear rank of the tensor \( \theta \) is defined as a tuple \( (R_h, R_w, R_{in}, R_{out}) \), where the \( n \)-th element, \( n = 1, \ldots, 4 \), is a rank of the dimension-\( n \) unfolding of the tensor \[^7\].

In convolutional kernels spacial dimensions usually are quite small. Therefore, similar to \[^10\], we factorize only two channel related dimensions, i.e. apply Tucker-2 decomposition, which is a specific form of the Tensor Train decomposition \[^17\]:

\[
\theta \approx \theta_C \times_{in} \theta_{in} \times_{out} \theta_{out}.
\]

The corresponding multilinear rank equals \((d \times d, R_{in}, R_{out})\), everywhere later we refer to it as \((R_{in}, R_{out})\).

4.1 First-time layer compression using Tucker-2 kernel approximation

Let \( \hat{\theta} \in \mathbb{R}^{d \times d \times C_{out} \times C_{in}} \) be a kernel approximation obtained via Tucker-2 decomposition with rank \( R = (R_{out}, R_{in}) \). Since decomposition methods search directly for a factorized representation, we introduce an operator \( \mathcal{F}_{dec} \), which performs rank-\( R \) approximation and factorization simultaneously, i.e.

\[
\mathcal{F}_{dec}(\theta) = \mathcal{F}_{fact}(\hat{\theta}) = (\theta_C, \theta_{out}, \theta_{in}),
\]

where \( \theta_{out} \in \mathbb{R}^{C_{out} \times R_{out}}, \theta_{in} \in \mathbb{R}^{C_{in} \times R_{in}} \) are factor matrices, and \( \theta_C \in \mathbb{R}^{d \times d \times R_{out} \times R_{in}} \) is a core tensor.

An output tensor \( Y \in \mathbb{R}^{H' \times W' \times C_{out}} \) given a layer input \( X \in \mathbb{R}^{H \times W \times C_{in}} \) can be calculated in a consecutive way as follows \[^10\]:

\[
Z_1 = \theta_{in} \ast X, \quad Z_2 = \theta_C \ast Z_1, \quad Y = \theta_{out} \ast Z_2,
\]

where operation \( \ast \) denotes a convolution over all common dimensions.

Therefore, the initial convolutional layer with kernel \( \theta \) can be replaced by three convolutional layers (Figure 4). Indeed, we obtain \( Z_1, Z_2, Y \) by sequentially propagating \( X \) through the layers with convolutions of spacial sizes \( 1 \times 1, d \times d \) and \( 1 \times 1 \) respectively. Thus, for the decomposed layer we get \( O(C_{in}R_{in} + d^2R_{out}R_{in} + C_{out}R_{out}) \) layer parameters, and propogation through this layer requires \( O(HWC_{in}R_{in} + H'W'(d^2R_{out}R_{in} + C_{out}R_{out})) \) operations.

Figure 4: Decomposition of a convolutional layer into 3 new ones using Tucker-2 kernel approximation. The top row shows an approximation of a 3D weight tensor with a low-rank tensor, which can be represented in Tucker-2 format (\( \times_{in}, \times_{out} \) denote multilinear products along channel dimensions). The bottom row depicts how the initial layer is replaced with a sequence of 3 convolutional layers. Weights of new layers are the reshaped components of the factorized format for the low-rank tensor. The notation \( \otimes C_{out} \) means that the 4D weight tensor has \( C_{out} \) output channels.

4.2 Further compression of a Tucker-2 decomposed layer

To perform further compression we need to update weights \( \theta_{fact} = (\theta_C; \theta_{out}; \theta_{in}) \) of the decomposed layer, i.e. to find \( \theta'_{fact} = (\theta'_C; \theta'_{out}; \theta'_{in}) \) such that \( \mathcal{F}_{full}(\theta'_{fact}) \) has multilinear rank \( R' = (R_{out}, R_{in}), R' \leq R \) (elementwise comparison).

The naive way to do that is to obtain a new core and factor matrices by approximating tensor \( \mathcal{F}_{full}(\theta_{fact}) \) with Tucker-2 decomposition (the path along dashed arrows in Figure 5).
We propose to use a more efficient update based on the properties of Tucker decomposition (Figure 5). Namely, we approximate the core \( \theta_C \) using Tucker-2 and then update the weights in the following way

\[
\mathcal{F}_{\text{dec}}(\theta_C) = (\theta_C^*, \theta_{out}^*, \theta_{in}^*),
\]

\[
\theta_C' = \theta_C, \quad \theta_{in}' = \theta_{in} \theta_{in}^*, \quad \theta_{out} = \theta_{out} \theta_{out}^*.
\]

(12)

The GAS of EVBMF can automatically find matrix rank by performing Bayesian inference, however, it provides a suboptimal solution. Unlike authors of [10], we use GAS of EVBMF not to set a rank for a weight approximation (i.e. \( R = R_{\text{evbmf}} \)), but only to determine extreme rank (i.e. \( R_{\text{extr}} = R_{\text{evbmf}} \)).

To determine the extreme rank for Tucker-2 approximation using the GAS of EVBMF, we apply it to the unfoldings of the weight tensor associated with channel dimensions [10]. That is, at the \((k+1)\)-th iteration we apply it to matrices of sizes \( R_{in}^k \times d^2 R_{out}^k \) and \( R_{out}^k \times d^2 R_{in}^k \).

The weakened rank \( R_{\text{weak}} \) depends linearly on the extreme rank and serves to preserve more redundancy in the decomposed tensor. Setting \( R = R_{\text{weak}} \) facilitates fine-tuning and yields a compression step with better accuracy.

The weakened rank is defined as follows:

\[
R_{\text{weak}} = R_{\text{init}} - w(R_{\text{init}} - R_{\text{extr}}),
\]

(13)

where \( w \) is a hyperparameter called weakening factor, \( 0 < w < 1 \), and \( R_{\text{init}} \) stands for initial rank. This results in \( R_{\text{extr}} \leq R_{\text{weak}} \leq R_{\text{init}} \). Our experiments show that the optimal value for \( w \) is in the range: \( 0.5 \leq w \leq 0.9 \). If the initial rank is less than 21, our algorithm considers such kernels as small enough and does not compress them.

5 Automatic rank selection

In Section 2 we have shown that iterative architecture compression and model fine-tuning is equivalent to iterative parameter set reduction \((\Theta \supset \Theta_1 \supset \Theta_2 \supset \ldots)\) for the initial architecture \( f \) and fine-tuning with parameter constraints.

To automatically choose gradually decreasing ranks \( R_1 > R_2 > \ldots \) for low-rank approximation of \( \theta \), we experiment with two different scenarios: Bayesian approach and constant compression rate. Bayesian approach based on rank estimation in order to remove redundancy from the weight tensor. Constant compression rate approach based on rank calculation in order to obtain the desired parameter reduction rate after several iterations.

5.1 Bayesian approach

For convenience, we introduce two notations: an extreme rank and a weakened rank. The extreme rank is the value at which almost all redundancy is eliminated from the tensor after decomposition. The weakened rank is the value at which a certain amount of redundancy is preserved in the tensor after decomposition.

In Bayesian approach, firstly, extreme rank \( R_e \) is found via GAS of EVBMF (Global Analytic Solution of Empirical Variational Bayesian Matrix Factorization [10]), and secondly, a rank weakening is performed.

The weakened rank is defined as follows:

\[
R_{\text{weak}} = R_{\text{init}} - w(R_{\text{init}} - R_{\text{extr}}),
\]

where \( w \) is a hyperparameter called weakening factor, \( 0 < w < 1 \), and \( R_{\text{init}} \) stands for initial rank. This results in \( R_{\text{extr}} \leq R_{\text{weak}} \leq R_{\text{init}} \). Our experiments show that the optimal value for \( w \) is in the range: \( 0.5 \leq w \leq 0.9 \). If the initial rank is less than 21, our algorithm considers such kernels as small enough and does not compress them.

5.2 Constant compression rate

Ranks for tensor approximations can be chosen based on parameter reduction rate that we want to achieve at each compression step. By choosing rank in such a way, we can control the speed-up of each convolutional layer.

Suppose we want to reduce the number of kernel parameters in initial convolutional layer, \( d^2 C_{in} C_{out} \) times \( \alpha \). In Tucker-2 case, having \( R_{in} C_{in} + R_{out} C_{out} + R_{in} d^2 + R_{out} C_{out} \) parameters in the decomposed layer, and assuming the multilinear rank has the form \((\beta R, R)\), \( \beta > 0 \), we can derive

\[
R \leq \frac{-C_{in} + \beta C_{out}}{\beta \alpha} + \sqrt{\left(\frac{C_{in} + \beta C_{out}}{\beta \alpha} - \frac{2 C_{in} C_{out}}{\beta \alpha}\right)^2 + \frac{4 C_{in} C_{out}}{\beta \alpha}}.
\]

(14)

Therefore, to achieve times \( \alpha \) parameters reduction using Tucker-2 tensor approximation, we choose ranks according to the inequality (14).

6 Experiments

In this paper, we focus on compressing object detection neural network models. We apply our Multi-Stage Compression (MUSCO) approach on Faster R-CNN [13] and Faster R-CNN [20] with ResNet-50 and VGG-16 backbones. To show the applicability of our method to the
variety of tasks, we apply it to compress several classifications and other object detection benchmarks. We have made the code publicly available.\(^5\)

In this section, we report results on model compression obtained by MUSCO based on Tucker-2 decomposition with an automatic rank selection. To one iteration of MUSCO we further refer as Tucker2-iter.

### 6.1 Compression of Faster R-CNN

This section demonstrates the effectiveness of MUSCO algorithm in compressing Faster R-CNN with VGG-16 backbone\(^6\) Faster R-CNN C4 (used for PASCAL VOC dataset) and Faster R-CNN FPN (used for COCO dataset) with ResNet-50 backbone.\(^7\) In our experiments, we focus on backbone compression since tensor methods can reduce the parameter redundancy, which usually occurs in convolutional layers. The quality of object detection tasks is evaluated using mAP (mean Average Precision) metrics.

#### 6.1.1 Faster R-CNN with VGG-16 backbone

Faster R-CNN with VGG-16 backbone has been fine-tuned and evaluated on Pascal VOC 2007\(^3\) train and test datasets respectively.

At the bottom part of Table 1 we provide compressed models obtained via several compression iterations along with models compressed at one step. Model parameter $nx$ means that we select ranks for Tucker-2 decomposition based on constant compression rate strategy. For example, MUSCO(nx, 3.16, 2) is a compressed model obtained after two compression steps using $3.16 \times$ parameter reduction at each step.

You can see from the table that the iterative approach allows obtaining higher FLOPs reduction than non-iterative one at a similar mAP level. For example, in this sense MUSCO(nx, 1.77, 2) and MUSCO(nx, 2, 2) outperform Tucker2-iter (nx, 3.16), MUSCO(nx, 3.16, 2) and MUSCO(nx, 1.77, 4) is better than Tucker2 (nx, 10).

Comparing to the state-of-the-art methods from Table 1, the MUSCO approach outperforms all models except the one from \(^6\). However, our model MUSCO(nx, 3.16, 2) gives $10.49 \times$ FLOPs reduction comparing to $4 \times$ from \(^6\) and outperforms the latter in terms of absolute mAP value. The FLOPS are computed in the same way as in \(^7\).

#### 6.1.2 Faster R-CNN with ResNet-50 backbone

To the best of our knowledge, there are currently no sufficiently good compression methods for ResNet50-based Faster R-CNN and lack of methods, evaluated on COCO\(^14\) dataset, while both aspects are currently standard for the object detection research area.

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\(^5\)https://github.com/juliagusak/musco

\(^6\)https://github.com/chenyuntc/simple-faster-rcnn-pytorch

\(^7\)https://github.com/facebookresearch/maskrcnn-benchmark

### Table 1: Comparison of Faster R-CNN (with VGG-16 backbone) compressed models on VOC2007 evaluation dataset.

| Model | FLOPs | mAP |
|-------|-------|-----|
| FASTER R-CNN (VGG-16) @ VOC2007 | | |
| baseline | $1 \times$ | 68.7 |
| Channel Pruning \(^21\) | $4 \times$ | 66.9 (-1.8) |
| Accelerating VD \(^23\) | $4 \times$ | 67.8 (-0.9) |
| AutoML Compression \(^6\) | $4 \times$ | 68.8 (+0.1) |

### Table 2: Comparison of Faster R-CNN (with ResNet-50 backbone) compressed models on VOC2007 evaluation dataset.

| Model | FLOPs | mAP |
|-------|-------|-----|
| FASTER R-CNN C4 (RESNET-50) @ VOC2007 | | |
| Used baseline | $1.0 \times$ | 75.0 |
| Tucker2-iter (nx, 1.4) | $1.17 \times$ | 76.8 (+1.8) |
| MUSCO(nx, 1.4, 2) | $1.39 \times$ | 77.0 (+2.0) |
| MUSCO(nx, 1.4, 3) | $1.57 \times$ | 75.4 (+0.4) |
| Tucker2-iter (nx, 3.16) | $1.49 \times$ | 75.0 (+0.0) |

### 6.2 Compression of other models

To demonstrate the difference between our approach and non-repetitive one, we performed compression using the GAS of EVBMF as the only rank selector. Results are shown in Table 4.

Comparing the results of compression ratio and speed up in two tables, we can say that iterative approach showed itself better for almost all networks. We can also see that the highest speedup is achieved on CPUs as follows from

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\(^4\)https://github.com/AutoML/Compression

\(^5\)https://github.com/AutoML/Compression

\(^6\)https://github.com/AutoML/Compression

\(^7\)https://github.com/AutoML/Compression

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6
There are several works devoted to the deep convolutional neural networks compression. Authors of [4] proposed a pipeline that consists of three different methods: pruning, trained quantization, and Huffman coding. They demonstrated the possibility of significantly reducing storage requirements by combining different techniques. Our method differs since we focus not only on compression ratio but also on speedup and seamless integration into any framework.

Table 5: Results of iterative low-rank approximation for AlexNet, VGG-16, YOLOv2, and Tiny YOLO. These tests were performed on CPUs of two different series and on GPU: Intel Core i5-7600K, Intel Core i7-7700K and NVIDIA GeForce GTX 1080 Ti respectively.

Table 4: Quality drop after iterative compression and one-time compression. For AlexNet and VGG-16 metric is $\Delta$ Top-5 accuracy, for YOLO - $\Delta$ mAP

Table 3: Comparison of Faster R-CNN (with ResNet-50 backbone) compressed models on COCO2014 dataset. MUSCO (vbm, 0.7, 2) corresponds to the two-iteration compression approach. For AlexNet and VGG we measured Top-5 accuracy and for YOLOv2 and Tiny YOLO we measured mAP.

7 Related work

There are several experiments on training low precision networks [19]. Their methods allow to use only 2 bits for weight storage but accuracy is much lower than in full precision networks, and it is not a compression algorithm because such networks have to be trained from scratch.

Several approaches based on different algorithms of low-rank approximation were proposed in [11, 2]. The authors of [2] have demonstrated the successful application of singular value decomposition (SVD) to fully connected layers. Further, the authors of [11] found a way to decompose 4-dimensional convolutional kernel tensor by applying canonical polyadic (CP) decomposition. But these approaches are able to compress only one or a couple of layers. Moreover, for each layer the rank is unique and the process of rank selection has to be performed manually every time.

Another way to compress a whole network was introduced in [10]. The approach used in their work is automated. Authors combined two different decompositions to be able to compress both fully connected and convolutional layers. To compress fully connected layers they adopted the approach used in [2] and applied SVD. For convolutional layers, the authors applied a Tucker decomposition [22]. Unlike [11, 2], the authors have found a way to automatically select ranks without any manual search. Ranks are determined by a global analytic solution of variational Bayesian matrix factorization (VBMF) [16]. We found that the global analytic VBMF provides ranks for which it is difficult to restore the initial accuracy by fine-tuning for deep networks. In our algorithm, we use the global analytic VBMF but to select the extremal ranks which will be weakened afterward.

CP decomposition which was used by [11] is a special case of Tucker decomposition, where the core tensor is constrained to be superdiagonal. In our approach, we use Tucker-2 decomposition. To compress fully connected layers we adopt SVD as it was proposed in [2].

Our approach is different from these methods because all of them apply decomposition algorithms only one time per layer, and ranks provided by the global analytic VBMF can be considered as upper bounds for compression. Our algorithm is iterative, and decomposition algorithms can be applied multiple times for the same layer. Moreover, we can achieve a higher compression rate because we do not have such boundary.
8 Conclusion

In this paper, we addressed the problem of compression of deep convolutional neural networks. We proposed a multi stage compression algorithm MUSCO for neural network compression, which performs gradual redundancy reduction. Our method consists of two repetitive steps: compression and fine-tuning. Compression step includes automatic rank selection and low-rank tensor factorization according to the selected rank. We evaluated our approach on the following deep networks used for object detection: Faster R-CNN with VGG-16 and ResNet-50 backbones, YOLOv2, Tiny YOLO and classification: AlexNet, VGG-16. Experimental results show that our iterative approach outperforms non-repetitive ones in the compression ratio providing less accuracy drop.

Our method is designed to compress any neural network architecture with convolutional and fully connected layers using Tucker-2, CP or SVD decomposition with two different strategies of automatic rank selection. As future work we plan to increase the variety of matrix/tensor decom-positions used by MUSCO approach at the compression step. Also we will investigate the effect of combining our approach with hardware-dependent approaches such as quantization and channel pruning approaches.

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9 Appendix

9.1 Compression using CP decomposition

CP decomposition (CPD) is a special case of Tucker decomposition when the cube core tensor has nonzero elements only on the main diagonal. Thus, for an \( N \)-dimensional tensor CPD (we call it CPD-\( N \) hereinafter) is defined by \( N \) factor matrices.

Unlike [11], instead of applying CPD-4 to a 4-dimensional kernel tensor \( \theta \), we propose to apply CPD-3 to a reshaped tensor of size \( d^2 \times C_{out} \times C_{in} \). The advantage of CPD-3 over CPD-4 is that it allows catching dependencies within spatial dimensions. Moreover, it yields faster convergence during approximation because of a decreased number of factors.

9.1.1 First time layer compression using CP-format kernel approximation

Using CPD-3 decomposition with CP-rank \( R \) (here CP-rank is defined as the minimum number of rank-one tensors required to yield its exact CP decomposition), a factorized form of the target kernel approximation \( \hat{\theta} \in \mathbb{R}^{d^2 \times C_{out} \times C_{in}} \) is equal to

\[
\mathcal{F}_{\text{fact}}(\hat{\theta}) = (\theta_{d^2 \times R}, \theta_{out \times R}, \theta_{in \times R}),
\]

where \( \theta_{d^2 \times R} \in \mathbb{R}^{d^2 \times R} \), \( \theta_{out \times R} \in \mathbb{R}^{C_{out} \times R} \), \( \theta_{in \times R} \in \mathbb{R}^{C_{in} \times R} \) are factor matrices (components).

Similar to the Tucker-2 case, we replace one convolutional layer with a sequence of three layers when we apply CPD-3 to approximate a kernel tensor. The only difference is that the second convolutional layer is determined by grouped convolutions (with \( R \) groups), not by a standard one. Therefore, if \( X \in \mathbb{R}^{H \times W \times C_{in}} \) and \( Y \in \mathbb{R}^{H' \times W' \times C_{out}} \) are layer input and output respectively, we have \( O(R(C_{in} + d^2 + C_{out})) \) layer parameters, and the computational cost equals \( O(R(HW'C_{in} + H'W'd^2 R + H'W'C_{out})) \).

\[
\text{weights update} \quad \approx \quad \text{dual tensor of CPD-3-format.} \quad \circ \quad \text{outer product operation.}
\]

9.1.2 Further compression of CP - decomposed layer

For further compression we update weights \( \theta_{\text{fact}} = (\theta_{d^2 \times R}, \theta_{out \times R}, \theta_{in \times R}) \) of the decomposed layer, i.e. find \( \hat{\theta}_{\text{fact}} = (\theta'_{d^2 \times R'}, \theta'_{out \times R'}, \theta'_{in \times R'}) \), such that \( \mathcal{F}_{\text{fact}}(\hat{\theta}_{\text{fact}}) \) has CP-rank \( R' \leq R \).

A naive way to update weights of a decomposed layer is to construct a full tensor from fine-tuned weights, approximate it with CPD-3 with reduced CP-rank and update the weights using new factors. However, that might not guarantee an appropriate approximation error.

To improve the convergence, we approximate current fine-tuned weights with CP factors of lower rank directly, using an ALS-type algorithm.

9.1.3 Rank selection

Suppose we want to reduce \( \alpha \) times the number of kernel parameters in initial convolutional layer, \( d^2 C_{in} C_{out} \). Having \( R(C_{in} + d^2 + C_{out}) \) kernel parameters in a CPD-3 decomposed layer, the estimated CP-rank \( R \) should satisfy the following inequality

\[
R \leq \frac{d^2 C_{in} C_{out}}{\alpha(C_{in} + d^2 + C_{out})}.
\]

Therefore, to achieve times \( \alpha \) parameters reduction using CPD-3 tensor approximation, we choose ranks according to the inequality [16]. For CP case, please, see Appendix.

9.1.4 Experiments

We tested CPD-3 based compression on Faster R-CNN model with ResNet-50 backbone. Compressing a part of layers at each step, we achieve the following results.

| Model | FLOPs | mAP |
|-------|-------|-----|
| FASTER R-CNN C4 (RESNET-50) @ VOC2007 | | |
| Used baseline | 1.0× | 75.0 |
| MUSCO (nx, 5, 3 × 1/3) | 1.63× | 74.79 (-0.21) |
| MUSCO (nx, 10, 3 × 1/3) | 1.77× | 69.20 (-5.80) |

Table 6: Compressed Faster R-CNN (with ResNet-50 backbone) models on Pascal VOC2007 evaluation dataset. CPD-3 based compression is used. MUSCO(nx, k, 3 × 1/3) is a compressed model obtained after 3 compression steps (where 1/3 of layers is compressed per one step) using \( k \times \) parameter reduction at each step.

9.2 Compression using SVD

To compress a fully connected layer, we approximate its weight tensor \( \theta \in \mathbb{R}^{l_{in} \times l_{out}} \) using rank-\( R \) singular value decomposition (SVD). Namely, an approximation \( \hat{\theta} \) can be represented as \( \hat{\theta} = USV^T \), where \( U \in \mathbb{R}^{l_{in} \times R} \) and \( V \in \mathbb{R}^{l_{out} \times R} \) are orthogonal matrices and \( S \) is a diagonal matrix.

Thus, defining \( \theta_{in} = US \) and \( \theta_{out} = V^T \) we obtain

\[
\mathcal{F}_{\text{fact}}(\hat{\theta}) = (\theta_{out}, \theta_{in}),
\]

and hence, the fully connected layer is replaced by two consecutive fully connected layers with weights \( \theta_{in} \in \mathbb{R}^{l_{in} \times R} \) and \( \theta_{out} \in \mathbb{R}^{R \times l_{out}} \).
\[ \mathbb{R}^{l_{in} \times R} \text{ and } \theta_{out} \in \mathbb{R}^{R \times l_{out}}. \] Automatic rank selection for rank-\( R \) SVD is performed using GAS of EVBMF.

All models provided in Section 6.2 were iteratively compressed using Tucker-2 based compression for convolutional layers and SVD based for fully connected ones.

### 9.3 Layer-by-layer statistics

In Tables 8 and 7 there are shown floating points operation, required by each part of the initial and compressed models for both VGG and ResNet based models.

| Layer Block |   | Original | MUSCO (nx, 1.4, 3) |
|-------------|---|----------|-------------------|
|             |   | In/out channels of (3x3) kernel | MFLOPs* | In/out channels of (3x3) kernel | MFLOPs* | MFLOPs × |
| stem        | 0 | 3x64 (7x7) | 118 | 3x64 (7x7) | 118 | 1.00 |
| layer1      | 0 | 64x64 | 231 | 25x25 | 143 | 1.61 |
|             | 1 | 64x64 | 218 | 25x25 | 130 | 1.67 |
|             | 2 | 64x64 | 218 | 25x25 | 130 | 1.67 |
| layer2      | 0 | 128x128 | 295 | 51x51 | 208 | 1.42 |
|             | 1 | 128x128 | 218 | 51x51 | 131 | 1.66 |
|             | 2 | 128x128 | 218 | 51x51 | 131 | 1.66 |
|             | 3 | 128x128 | 218 | 51x51 | 131 | 1.66 |
| layer3      | 0 | 256x256 | 295 | 103x103 | 209 | 1.41 |
|             | 1 | 256x256 | 218 | 103x103 | 132 | 1.66 |
|             | 2 | 256x256 | 218 | 103x103 | 132 | 1.66 |
|             | 3 | 256x256 | 218 | 103x103 | 132 | 1.66 |
|             | 4 | 256x256 | 218 | 103x103 | 132 | 1.66 |
|             | 5 | 256x256 | 218 | 103x103 | 132 | 1.66 |

Table 7: Faster R-CNN C4 with ResNet backbone fine-tuned on VOC2007+2012. In the table we represent each residual block by its conv2 layer with a kernel of spacial size 3 × 3. MFLOPs* are computed for all layers in a residual block, given an input image of size 244 × 244 × 3.
| Layer | Original shape | MUSCO (nx, 1.77, 4) |
|-------|----------------|---------------------|
|       | In/out channels of (3x3) kernel | MFLOPs | In/out channels, (1x1) (3x3) (1x1) kernels | MFLOPs | MFLOPs × |
| 1     | 3x64           | 87                 | 3x64           | 87                 | 1.00   |
| 2     | 64x64          | 1850               | 16x16 16x64    | 32                 | 16.91  |
| 3     | 64x128         | 925                | 64x17 17x27 27x128 | 8                 | 17.86  |
| 4     | 128x128        | 1850               | 128x34 34x34 34x128 | 35                 | 14.76  |
| 5     | 128x256        | 925                | 128x36 36x57 57x256 | 8                 | 15.76  |
| 6     | 256x256        | 1850               | 256x69 69x69 69x256 | 35                 | 14.76  |
| 7     | 256x256        | 1850               | 256x69 69x69 69x256 | 35                 | 14.76  |
| 8     | 256x512        | 925                | 256x73 73x116 116x512 | 9                 | 14.86  |
| 9     | 512x512        | 1850               | 512x139 139x139 139x512 | 36                 | 14.29  |
| 10    | 512x512        | 1850               | 512x139 139x139 139x512 | 36                 | 14.29  |
| 11    | 512x512        | 462                | 512x139 139x139 139x512 | 9                 | 14.29  |
| 12    | 512x512        | 462                | 512x139 139x139 139x512 | 9                 | 14.29  |
| 13    | 512x512        | 462                | 512x139 139x139 139x512 | 9                 | 14.29  |

Table 8: Faster R-CNN with VGG-16 backbone fine-tuned on VOC2007. In the table MFLOPs are computed for an input image of size $244 \times 244 \times 3$. MFLOPs $\times$ denotes flops reduction for each convolutional layer from the original model. MFLOPs $\times$ is calculated as MFLOPs of original layer divided by MFLOPs of decomposed layer (i.e., a sum of MFLOPs of all three convolutional layers, substituting the original one). For example, for the original layer with a weight of size $512 \times 512 \times 3 \times 3$, a decomposed layer consists of three layers with weights $139 \times 512 \times 1 \times 1$, $139 \times 139 \times 3 \times 3$, $512 \times 139 \times 1 \times 1$, respectively.