YAOPBM — II: Extension to Higher Degrees and to Shorter Time Series

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Abstract.
In 2005, I presented a new fitting methodology (Yet AnOther Peak Bagging Method - YAOPBM), derived for very-long time series (2088-day-long) and applied it to low degree modes, \( \ell \leq 25 \). That very-long time series was also sub-divided into shorter segments (728-day-long) that were each fitted over the same range of degrees, to estimate changes with solar activity levels.

I present here the extension of this method in several “directions”: a) to substantially higher degrees (\( \ell \leq 125 \)); b) to shorter time series (364- and 182-day-long); and c) to additional 728-day-long segments, covering now some 10 years of observations.

I discuss issues with the fitting, namely the leakage matrix, and the \( f \)- and \( p_1 \) mode at very low frequencies, and I present some of the characteristics of the observed temporal changes.

1. Introduction
I presented in [1] a new fitting methodology, developed initially for very-long time series. It was applied only to low degree modes, \( \ell \leq 25 \), using the 2088-day-long time series, but also five overlapping 728-day-long ones.

The key elements of this fitting method are:

(i) It uses an optimal sine-multi-taper as power spectrum estimator. The number of tapers is selected from a preset list as to be the largest value for which the spectral resolution is 5 times smaller than the mode FWHM, whenever possible.

(ii) It fits an asymmetric profile.

(iii) It uses the complete leakage matrix, namely the horizontal and vertical components.

(iv) It fits individual modes for each \( m \), fitting all \( m \) for a given \((n, \ell)\) simultaneously. The fitting window is selected so as to always include the closest spatial leak (\( \delta \ell = 0, \delta m = 2 \)), but so as not to extend beyond the mid point to the nearby mode. The asymmetry and the FWHM are set to be independent of \( m \).

(v) Mode contamination is included, i.e. \((n', \ell')\) modes present in the fitting window of the \((n, \ell)\) mode are taken into account. The method iterates to include a better estimate of these leaks from having fitted them at the previous iteration.

(vi) The amplitude of all the leaks are not adjusted, but set by the amplitude of the mode, attenuated by the appropriate leakage matrix ratios.
Figure 1. Extended coverage of the fitting in the $\nu$–$\ell$ plane. Each individual multiplet fitted is shown as a dot. The vertical line, at $\ell = 25$, shows the extent of the results presented in [1]. A horizontal line is drawn at $\nu = 1$ mHz; below that frequency, the fit is deemed still unreliable.

This method was applied to GONG and MDI data, for one 2088-day-long time series and 5 overlapping 728-day-long time series. Additional details can be found in [1].

2. Extension of the Method

2.1. The 2088-day-long Time Series

I have since extended the fit to the 2088-day-long time series up to $\ell \leq 125$, for MDI observations only. Figure 1 illustrates the coverage in the $\ell$–$\nu$ plane of the fit.

Figures 2 and 3 compare these results to those obtained by Schou (private communication) using the same 2088-day-long time series. Coverage in the $\ell$–$\nu$ plane is similar, although my fit at very low frequencies is still tentative (see below). At high frequencies my coverage is more extensive and/or consistent, except for a few low degrees.

The frequency differences present the same double humped profile, as already discussed at length in [1]. The major contributor to this difference is the inclusion (or not) of the profile asymmetry\(^1\). But this difference is not fully accounted for by a simple model of the offset introduced by fitting an asymmetric profile with a symmetric one (again see [1]), shown as the green points in the top left panel of Figure 3. The extension to substantially higher degrees has not changed the nature of this comparison; the differences are only a function of frequency.

Comparisons of mode FWHM and mode power are intriguing. In one case (my fit) the FWHM shows a smooth variation with frequency with no hint of dependence on the order, $n$, while the mode power shows a dependence on $n$. In the other case (Schou’s fit), we observe the opposite. While the values themselves agree rather well, this contrast is somewhat puzzling. Comparison of the background power levels also shows the same contrasting behavior.

\(^1\) Schou’s fit uses a symmetric profile.
Figure 2. Comparison with Schou’s 2088-day-long fit, singlets: respective coverage in the $\ell$–$\nu$ plane (top); respective error bars estimate comparison (bottom left) and frequency differences (bottom right).
Figure 3. Comparison with Schou’s fit, singlets. Top left panel compares the frequencies (blue) and a model of the offset resulting from fitting an asymmetric profile with a symmetric one (green). Corrected frequency differences—after accounting for the asymmetry—and relative differences are shown in the lower left panels. The FWHM, mode power and background power levels are compared in the panels on the right.
Table 1. Time range for the nine 728-day-long segments

| Seg no. | Start (UT) | End (UT)          |
|---------|------------|-------------------|
| 1       | 1996.04.30 23:59:30 | 1998.04.29 04:15:30 |
| 2       | 1997.04.30 02:07:30 | 1999.04.28 06:23:30 |
| 3       | 1998.04.29 04:15:30 | 2000.04.26 08:31:30 |
| 4       | 1999.04.28 06:23:29 | 2001.04.25 10:39:30 |
| 5       | 2000.04.26 08:31:29 | 2002.04.24 12:47:29 |
| 6       | 2001.04.25 10:39:29 | 2003.04.23 14:55:29 |
| 7       | 2002.04.24 12:47:29 | 2004.04.21 17:03:29 |
| 8       | 2003.04.23 14:55:29 | 2005.04.20 19:11:29 |
| 9       | 2004.04.21 17:03:29 | 2006.04.19 21:19:29 |

2.2. The 728-day-long Time Series

The 728-day-long segments have since been fitted up to $\ell \leq 50$, and the fitting extended to 9 overlapping segments (see table 1) to cover some 3,600 days of observations.

2.3. Shorter Time Series

Since there is nothing intrinsic to the method that limits it to long or very-long time series, I have applied it to shorter time series as well. I have fitted 19 overlapping 364-day-long segments and 39 overlapping 184-day-long segments that span the exact same range as the 728-day-long segments listed in table 1.

This fit has been thus far carried out up to $\ell \leq 25$. The coverage in the $\ell$–$\nu$ plane (see figure 10), especially at low frequency, is—as one would expect— not as extended.

3. Problems: Low Frequency and Leakage at Higher Degrees

3.1. Very Low Frequency ($n=0 \& 1$)

To fit the f-modes ($n=0$), I had to reduce the rejection factor of spurious peaks (the sanity check, see [1]) to a lower threshold. The visibility of spherical harmonics, apodized by the line-of-sight projection of radial velocity, plus the one included in MDI’s spatial decomposition, is a strong function of $\frac{m}{\ell}$. At low frequencies, the multi-tapering is not anymore optimal (i.e., a minimum of 5 tapers is always used), so the narrow peaks barely emerge above the background. This is illustrated in figure 4. The side effect of reducing this rejection factor is a poor immunity to noise, and the fitting can latch to spurious peaks. To avoid this, one needs a very good first guess of the frequency splittings.

One possible solution under consideration is to reduce the number of tapers. However, this will increase the realization noise (i.e., the background will not be smoothed) and might turn out to be a zero sum gain; the fitting itself might have to be modified to use a maximum likelihood (see discussion in [1]).

Example of fit for $p_1$-modes ($n=1$) are shown in figure 5, where the rejection factor was not changed. As a result of the strong dependence on $\frac{m}{\ell}$ of the visibility function, only the near sectoral modes rise above the noise at very low frequencies. When individual modes are fitted, as they are here, this is less of a problem than merely a fact (i.e., an observational selection effect). But when the frequency of each $m$ is expanded as a polynomial expansion, the examples above clearly show that this polynomial expansion is poorly constrained and the resulting splitting coefficients are themselves potentially suspicious.
Figure 4. Example of fit, $n = 0$, $\ell = 125, 100, 90$ and 80 (left to right). Top: data, middle: model, as a function of $m$ and shifted frequency; bottom: $m$-averaged data and model.
Figure 5. Example of fit, $n = 1$, $\ell = 60, 45, 41$ and 37 (left to right), as in figure 4
3.2. Leakage at Higher Degrees

Example of fit are shown in figure 6, for $\ell = 25, 50, 75$, and, 100. Note in the $m$-averaged profiles how the quality of leakage model diminishes at higher degrees, especially for near sectoral modes. High degree analysis, see [2], has shown that the wrong plate scale was used in the spatial decomposition. The intermediate degrees are affected too and other effects, like the image distortion and the distortion of the spherical harmonics by differential rotation, also have some impact on the leakage matrix at these degrees. Hence the leakage matrix used here is not optimal. This is likely to be the cause of the observed discrepancy.

4. Results: Changes with Activity Levels

4.1. 728-day-long Time Series

The coverage in $\ell$--$\nu$ plane and the frequency changes for the nine 728-day-long segments are shown in figure 7, as well as scaled frequency changes (frequency differences divided by a power law in $\nu$), shown as a function of frequency and $\nu/L$, a proxy for the depth of region sampled by each mode.

These scaled frequency changes were binned over the ($\nu, m/\ell$) space, for each segment (see figure 8a). The resulting quantities can be modeled as:

$$F(t, \nu, m/\ell) = <\delta\nu_{n,\ell,m}(t)\nu_{n,\ell,m}^q> = A(t)\Sigma_i\Sigma_j C_{i,j}\nu^i (m/\ell)^j$$

(1)

where $A(t)$ is the amplitude of a simple and constant function of $\nu$ and $m/\ell$. The corresponding model is shown in figure 8b while the difference between the binned data and the fitted model are shown in figure 8c, demonstrating that these changes are well modeled by equation 1. The resulting amplitude $A(t)$ is compared to solar activity indices in figure 8.

4.2. 364- and 182-day-long Time Series

Results from fitting the 364- and 182-day-long segments are shown in figure 10. A fit to the same model as for the 728-day-long segments (equation 1) was carried out, and the resulting amplitudes are compared to solar activity indices in figure 11.

5. Conclusions

The fitting methodology developed for very long time series can be expanded both to higher degrees and to shorter time series. At higher degrees the leakage matrix is clearly not as good as one would like, and one should either use re-decomposed spherical harmonic coefficients, where the instantaneous plate scale is used, or recompute the leakage matrix to account for the known error in the spatial decomposition. Results for the f- and p_1 mode at very low frequency (below 1mHz) are still tentative. When fitting shorter time series the coverage in the $\ell$--$\nu$ plane does not extend as far towards low frequencies, due to the resulting lower SNR.

Changes with solar activity are very well modeled with a simple constant function of frequency and $\nu/\ell$ modulated by a time varying amplitude. That amplitude correlates very well with solar activity indices, at all three temporal resolutions (728, 364 & 182 days), with a somewhat better correlation when using the 10.7cm radio flux, rather than the Sunspot number or the Mg II core-to-wing ratio. The cycle hysteresis observed when using longer time series disappears when using shorted ones for the radio flux index, but not for the Sunspot number.

Acknowledgments

The Solar Oscillations Investigation (SOI) involving MDI is supported by NASA grant NNG05GH14G at Stanford University. SOHO is a mission of international cooperation between ESA and NASA. SGK is supported by NASA grant NNG05GD58G.
Figure 6. Example of fit, \((n, \ell) = (7, 25), (5, 50), (3, 75), (2, 100)\) (left to right). Similar to figure 4, except for one extra row that shows the residuals (model – data).
Figure 7. (a): Coverage in the $\ell$–$\nu$ plane for the 728-day-long segments; (b): frequency changes versus frequency; (c) scaled frequency changes versus frequency; (d) scaled frequency change versus $\nu L$.

References
[1] Korzennik, S. G. 2005, Ap. J., 626, 585.
[2] Korzennik, S. G., Rabello-Soares, M. C., & Schou, J. 2004, Ap. J., 602, 481.
Figure 8. (a): Binned frequency changes versus frequency and $\frac{m}{T}$; (b) fitted model (equation 1); (c) residuals (data – model).

Figure 9. (a): Frequency change model amplitude versus time, $A(t)$ (black), compared to scaled solar activity indexes (Sunspot number, Mg II core-to-wing ratio, and 10.7cm radio flux; in red, green, and, blue resp.), and (b) the corresponding regressions.
Figure 10. Coverage in the $\ell$–$\nu$ plane (a & c) and frequency changes (b & d), for 364- and 182-day-long segments (a,b & c,d resp.).
Figure 11. Frequency change model amplitude versus time compared to solar activity indices, for the 364- and 184-day-long segments, as in figure 9. That amplitude correlates very well with solar activity indices, at all temporal resolutions, with a somewhat better correlation when using the 10.7cm radio flux. The cycle hysteresis observed when using longer time series disappears when using shorted ones for the radio flux index, but not for the Sunspot number.