Parton saturation and $N_{\text{part}}$ scaling of semi–hard processes in QCD

Dmitri Kharzeev$^{a)}$, Eugene Levin$^{b),c)}$ and Larry McLerran$^{a)}$

$^{a)}$ Nuclear Theory Group, Physics Department, Brookhaven National Laboratory, Upton, NY 11973 - 5000, USA

$^{b)}$ HEP Department, School of Physics, Raymond and Beverly Sackler Faculty of Exact Science, Tel Aviv University, Tel Aviv 69978, Israel

$^{c)}$ DESY Theory Group 22603, Hamburg, Germany

Abstract

We argue that the suppression of high $p_t$ hadrons discovered recently in heavy ion collisions at RHIC may be a consequence of saturation in the Color Glass Condensate. We qualitatively and semi-quantitatively describe the data, in particular, the dependence upon the number of nucleon participants. We show that if parton saturation sets in at sufficiently small energy, then in nucleus–nucleus collisions at RHIC and LHC energies the cross sections of semi–hard processes should scale approximately with the number of participants, $N_{\text{part}}$. Our results provide a possible explanation of both the absence of apparent jet quenching at SPS energies and its presence at RHIC. Under the same assumption we predict that in semi–central and central $pA$ ($dA$) collisions at collider energies the dependence of semi–hard processes on the number of participating nucleons of the nucleus will change to $\sim (N_{\text{part}}^A)^{1/2}$. The forthcoming data on $dA$ collisions will provide a crucial test of this description.

The recent results from RHIC [1, 2, 3] suggest that relativistic heavy ion collisions at high energies probe QCD in the non–linear regime of high parton density and strong color fields [4, 5, 6, 7]. Parton dynamics in this new regime is quite different from what is expected in perturbative QCD, and is best described by parton saturation [3, 8, 9, 10, 11, 12] resulting in a formation of a new type of high density matter, the Colour Glass Condensate (CGC) [8, 9, 10, 11, 12, 13, 14].
In this high density regime, the transition amplitudes are dominated not by quantum fluctuations, but by the configurations of classical field containing large, $\sim 1/\alpha_s$, numbers of gluons. Even though the coupling $\alpha_s$ becomes small due to the high density of partons, the fields interact strongly due to the classical coherence. One thus uncovers new non-linear features of QCD, which cannot be investigated in a more traditional perturbative approach.

This new dynamics appears to be consistent \[14, 15, 16\] with the RHIC data on the multiplicity distributions of produced hadrons as a function of centrality, rapidity, and collision energy. In the Color Glass Condensate picture, the approximate scaling of hadron multiplicity with the number of participants $N_{\text{part}}$, and the logarithmic deviation from it, $\sim \ln N_{\text{part}}$ stemming from the running of the QCD coupling as a function of parton density, has been interpreted as a consequence of parton saturation \[14, 15, 16\]. In this letter we will show that QCD saturation gives a very substantial contribution to the suppression of the single particle inclusive distributions even at quite large transverse momenta, and can be responsible for (at least a part of) the observed approximate $N_{\text{part}}$ scaling of processes at high $p_t$.

The experiments on high $p_t$ hadron production show \[17, 18, 19\] that the inclusive cross section is strongly suppressed with respect to the scaling with the number of binary nucleon–nucleon collisions $N_{\text{coll}}$ expected on the basis of the factorization theorem \[23\] for hard processes in perturbative QCD (pQCD). An illuminating way to study this effect is to look at the dependence of hadron yields in a given $p_t$ bin as a function of centrality. The data seem to be closer to the approximate scaling with the number of participants, previously established for small $p_t$ particles dominating the total multiplicity. It is indeed a surprise that this behavior characteristic of soft processes persists up to the transverse momenta ($p_t \sim 5 \div 10$ GeV), where we expect the fragmentation of the incoherently produced jets to dominate.

The approximate $N_{\text{part}}$ scaling is of course a restatement of the observed earlier suppression of the high $p_t$ single particle inclusive distributions. One possible explanation of the observed suppression is the predicted quenching of the produced jets in hot quark–gluon matter \[20, 21, 22\]. The observation of approximate $N_{\text{part}}$ scaling in the entire range of hadron momenta up to $\sim 6$ GeV is however problematic for the scenario in which the jet energy loss is the only reason behind the suppression. Indeed, let us assume, following the pQCD approach, that most of the hadrons produced at RHIC originate from mini-jet fragmentation. If mini-jet production is a hard incoherent process, one expects the initial number of mini-jets to scale with the number of binary collisions $N_{\text{coll}}$. The energy loss in QCD is dominated by the induced gluon radiation and therefore as the produced mini-jets lose energy they will emit additional softer gluons. These emitted gluons in turn will fragment into hadrons, increasing further the total multiplicity of hadrons produced in the event. If the initial number of mini-jets were proportional to $N_{\text{coll}}$, the measured total multiplicity would then grow even faster with centrality, in a marked contradiction to the experimentally observed behavior. This contradiction can be avoided only if we assume that the initial production of moderate $p_t$ mini-jets, the fragmentation of which gives the dominant contribution to the measured multiplicity, is suppressed with respect to $N_{\text{coll}}$ scaling. (The jets with really high $p_t$ because of the small production cross section do not contribute to the total hadron multiplicity in any significant way, and their production according to $N_{\text{coll}}$ scaling, and subsequent energy loss, would not lead to any conflict with the
observed scaling of the total multiplicity).

The main goal of our letter is to show that the approximate $N_{\text{part}}$ scaling of mini-jet production at moderately high $p_t$ (up to $p_t \sim 6 \div 8$ GeV at RHIC energies) has a natural explanation in terms of parton saturation and the Color Glass Condensate. We also demonstrate that the $N_{\text{part}}$ scaling exhibits very non–trivial properties of dense partonic systems not only at momentum transfers around the saturation scale $Q_s$, but also at much higher momenta, naively as large as $\sim Q_s^2/\Lambda_{QCD}$. For heavy nuclei, this new momentum scale is $\mathcal{O}(5 - 10$ GeV) at RHIC energies, and even larger at the energies of the LHC, $\mathcal{O}(25 - 50$ GeV). We will in fact find that the correct scale for nuclei is somewhat reduced relative to this estimate, but is still large compared to $Q_{\text{sat}}$. We find this scale to be about 4 GeV at RHIC and of order 10 GeV at the LHC.

Our arguments are based on the following foundations. First, in the Color Glass Condensate, the saturated parton density provides a new dimensionful scale; as a consequence, the structure functions at momentum transfer $Q^2$ depend on a single variable $\tau \equiv Q^2_s/Q^2$ (“geometrical scaling”). This has been proven [5, 6, 8, 24, 25, 26, 27, 28] for $Q < Q_s$. Moreover, it has been observed that the deep inelastic scattering data at small $x \leq 0.01$ exhibit geometrical scaling in the entire kinematical region of $Q^2$ accessible at HERA, and it has been shown [29] that the scaling indeed holds in a wide range of $Q^2_s \leq Q^2 \leq Q^4_s/\Lambda_{QCD}^2$. Second, the properties of QCD evolution are modified in the domain of high parton density, where geometrical scaling holds. As we will discuss below, the anomalous dimension of the gluon density in this region is approximately equal to $1/2$, as was computed in [5, 24, 26, 29]. Third, in the range of $Q^2_s \leq Q^2 \leq Q^4_s/\Lambda_{QCD}^2$ the saturation scale for nuclear targets scales according to $Q^2_s \sim A^{1/3}$, a behavior which was argued to be responsible for the centrality dependence of hadron multiplicity at RHIC [14, 17].

Let us begin by considering an external probe with virtuality $Q^2$ interacting with a target of longitudinal size $L$ and an average (3-dimensional) parton density $n$. The probability of interaction in the target is determined by the ratio $\kappa \equiv L/\lambda$ of the target size to the mean free path of the probe $\lambda = (\sigma n)^{-1}$, where $\sigma \sim \alpha_s(Q^2)/Q^2$ is the cross section of the probe scattering off a parton. At small Bjorken $x$, when the coherence length $\sim 1/(mx)$ ($m$ is the nucleon mass) exceeds the target size, the probe can interact coherently with partons located at different longitudinal coordinates, and the only relevant parameter characterizing the target becomes the density of partons in the transverse plane, $\rho = nL$; for a nuclear target, this implies $\rho \sim A^{1/3}$. The interaction probability is then determined by a dimensionless “parton packing factor” $\kappa \sim \rho(x) \alpha_s(Q^2)/Q^2$. The boundary of the saturation region, and the corresponding value of “saturation scale” $Q^2_s(x)$, are given by the line in the $(x, Q^2)$ plane along which the interaction probability is of order one, $\kappa(x, Q^2) \simeq 1$.

We will now write the packing factor in the form [14] suited for nucleus–nucleus collisions, where a centrality cut can be used to select events with different transverse densities $\rho_{\text{part}}$ of nucleons participating in the collision (“participants”):

$$\kappa = \frac{8 \pi^2 N_c}{N_c^2 - 1} \frac{\alpha_s(Q^2)}{Q^2} xG(x, Q^2) \frac{\rho_{\text{part}}}{2}$$  (1)
where $xG(x, Q^2_s)$ is the gluon structure function; we assume that $x$ is sufficiently small for gluons to dominate the parton densities. Consider the kinematical regime of small $x$ and high $Q^2$ in which both $\alpha_s \ln Q^2$ and $\alpha_s \ln(1/x)$ are large; in this domain both logarithms have to be resummed, and the solution to DGLAP evolution equations \[8\] for $xG$ can be written in the “double log approximation” (DLA), which allows us to re-write Eq. \[1\] in the following naive form, neglecting the influence of saturation effects on the QCD evolution:

$$\kappa = \exp \left[ \sqrt{4\alpha_S} (y - y_0) \xi - \xi + \xi_{\text{part}} \right];$$

we have assumed that at the initial point $x_0$ of the evolution in $x$ the transverse momentum squared of partons is $Q^2_0$, and have introduced the following notations: $y$ is rapidity, with $y - y_0 = \ln(x_0/x)$, $\xi = \ln(Q^2/Q^2_0)$, $\xi_{\text{part}} = \ln(\rho_{\text{part}}/\rho_0) \approx \frac{1}{2} \ln A$, $A$ is the atomic number, $\rho_0$ is parton density in a nucleon, and $\bar{\alpha}_S = (N_c/\pi)\alpha_S$ in the large $N_c$ approximation.

We now investigate the effect of saturation on the function $\kappa$. Our treatment so far assumed, in the spirit of pQCD, that evolution starts at some initial fixed scale $Q^2_0 \sim \Lambda^2_{\text{QCD}}$ separating perturbative and non–perturbative domains, and that $Q^2 \gg Q^2_0$. Traditionally, one assumes that the dynamics at the scale $Q^2_0$ and below is driven by non–perturbative phenomena and should be described by a priori unspecified initial conditions for the structure functions, which are then constrained by experimental data. The parton saturation approach is different in that it provides theoretical tools to compute the parton distributions once their density becomes large enough. If saturation occurs in the nuclear wave function at some $x_0$ and at the corresponding scale $Q^2_s(x_0)$, in performing QCD evolution of parton densities to smaller values of $x$ and larger values of $Q^2$ we should therefore provide the saturated Color Glass Condensate distribution as the initial condition.

It is convenient to recall the well–known property of DGLAP equation \[30\] (see Ref.\[3\] for example) which allows to write its solutions in a form expressing their Green’s function–like properties:

$$\kappa(y, \xi) = \int^\xi d\xi' \kappa(y - y_0, \xi - \xi') \kappa(y_0, \xi').$$

Using the DLA form of the solution we see that the integral in \[3\] has a saddle point at $\xi' = \xi(y_0/y)$. If we consider $y \gg y_0$ ($x \ll x_0$), then the dominant contribution to the integral comes thus from $\xi' \rightarrow 0$, corresponding to the initial condition at $Q^2_0 \ll Q^2$. If, however, $x_0$ is sufficiently small for the saturation to set in, the initial condition will be provided by McLerran-Venugopalan model \[3\] at the scale $\xi' = \xi_{\text{sat}}(y_0) = \ln(Q^2_s(y_0)/Q^2_0)$. The integral over $\xi'$ will then be dominated by $\xi' \approx \xi_{\text{sat}}(y_0)$. This results in a different solution, which instead of Eq. \[2\] is now given by

$$\kappa = \exp \left[ \sqrt{4\bar{\alpha}_S} (y - y_0) (\xi - \xi_{\text{sat}}(y_0)) - \xi + \xi_{\text{sat}}(y_0) \right].$$

The saturation boundary can be found from the condition $\kappa = 1$. This leads to the following equation for the saturation scale at rapidity $y$:

$$4 \bar{\alpha}_S (\xi_{\text{sat}}(y) - \xi_{\text{sat}}(y_0)) (y - y_0) = (\xi_{\text{sat}}(y) - \xi_{\text{sat}}(y_0))^2.$$
This equation has a simple solution
\[ \xi_{\text{sat}}(y) = 4 \bar{\alpha}_S (y - y_0) + \xi_{\text{sat}}(y_0). \] (6)
Since \( Q_s^2(x_0) \) is proportional to \( A^{\frac{1}{3}} \) at relatively low energies, when \( y \approx y_0 \) (see Eq. (6) or the McLerran – Venugopalan formula [8]), one can still conclude that \( Q_s^2(y) \propto A^{\frac{1}{3}} \). For nuclear collisions, this means that the product of the nuclear overlap area \( S_A \) and the saturation scale is proportional to the number of participants, \( S_A Q_s^2 \propto N_{\text{part}} \) [14, 15].

Let us establish the leading behavior of the solution (4) at high \( Q^2 \). At very high \( \xi \gg 2\xi_{\text{sat}}(y) \), the solution tends to
\[ \kappa \sim \frac{Q_0^2}{Q^2}, \] (7)
modified by the double logarithmic factor of \( \exp \left( \sqrt{4\bar{\alpha}_S (y - y_0) \xi} \right) \). This result does not exhibit geometrical scaling, and is the same as the naive perturbative result. Consider, however, the region where
\[ \xi - \xi_s(y) \ll \xi_s(y) - \xi_s(y_0), \] (8)
which corresponds to a vast kinematical domain of
\[ Q^2 \ll \frac{Q_s^4(y)}{Q_s^4(y_0)}, \] (9)
note that this is precisely the condition for the observed geometrical scaling to hold in deep–inelastic scattering [29]. In this domain, we should expand the exponent of Eq.(6) in powers of the ratio \( \Delta \xi / (\xi_{\text{sat}}(y) - \xi_{\text{sat}}(y_0)) \ll 1 \), where we have introduced the notation \( \Delta \xi = \xi - \xi_{\text{sat}}(y) = \ln(Q^2/Q_s^2) \). The expansion to first order yields

\[
\sqrt{4\bar{\alpha}_S (y - y_0) \left( \xi - \xi_{\text{sat}}(y_0) \right)} - \xi + \xi_{\text{sat}}(y_0) = (\xi_{\text{sat}}(y) - \xi_{\text{sat}}(y_0)) \sqrt{1 + \frac{\Delta \xi}{\xi_{\text{sat}}(y) - \xi_{\text{sat}}(y_0)}} = \Delta \xi - (\xi_{\text{sat}}(y) - \xi_{\text{sat}}(y_0)) \approx -\frac{1}{2} \Delta \xi, \] (10)

where we have used Eq.(6). This result translates into the following \( Q^2 \) dependence of the packing factor:
\[ \kappa = \left( \frac{Q_{\text{sat}}^2(y)}{Q^2} \right)^{\frac{1}{2}}. \] (11)

The appearance of the square root clearly shows that the properties of QCD evolution are dramatically affected by the saturation phenomena in the entire region (9), much more broad than the originally anticipated \( Q^2 \sim Q_s^2(y) \). The comparison of (7) and (11) reveals that in the presence of Color Glass Condensate the gluon density acquires the anomalous dimension equal to 1/2.

Let us now turn to the discussion of semi–hard processes in hadron and nuclear collisions. Due to the AGK cancellation [31], the inclusive cross section for large \( p_t \) jet production is
described by the Mueller diagram shown in Fig. 1. It can be written as (see Refs. [3, 33, 34, 35, 36, 37, 38, 39, 28])

\[
E \frac{d\sigma}{d^3p} = \frac{4\pi N_c}{N_c^2 - 1} \frac{1}{p_t^2} \int dk_t^2 \alpha_s \varphi_{A_2}(Y - y, k_t^2) \varphi_{A_1}(y, (p - k_t)^2),
\]

where \( \varphi \)'s are the unintegrated parton distributions related to the packing factor \( \kappa \) discussed above by simple relation \( \varphi \sim S_A \kappa_A / \alpha_s \), and \( Y \) is the rapidity of the measured jet. Using the formula Eq. (11) for \( \varphi \sim S_A \kappa_A / \alpha_s \) in the domain (9), and noting that in this case \( Q^2 = p_t^2 \), we see that Eq. (12) leads to

\[
\frac{dN}{dyd^2p_t} = \frac{1}{S_A} E \frac{d\sigma}{d^3p} \propto S_A Q_s^2 / p_t^2 \rightarrow N_{part} / p_t^2
\]

where we have used that \( N_{part} \propto Q_s^2 S_A \). Note that the number of produced hard particles scales with the number of participants. The \( N_{part} \) scaling given by Eq. (13) can be formulated in the following way:

\[
\frac{1}{N_{part}} \frac{dN}{dyd^2p_t}
\]

is almost independent of centrality cut for \( \frac{Q_s^2(y)}{\Lambda} \geq p_t \geq Q_s(y) \), where \( \Lambda = Q_s(y_0) \) is the saturation scale at some \( y_0 < y \) marking the onset of saturation. This is the central result of our paper.

Of course, “genuine” hard processes still scale with the number of collisions. Indeed, consider \( p_t > Q_s^2 / \Lambda \); then according to Eq. (7), \( \varphi \propto Q_s^2 / p_t^2 \) and the formula Eq. (12) yields the number of hard particles proportional to \( S_A Q_s^4 / p_t^4 \sim N_{coll} / p_t^4 \) in accord with the pQCD factorization theorem.

We have demonstrated that at high energies corresponding to \( y \gg y_0 \) the effects of the Color Glass Condensate persist up to very large values of \( p_t \); how far do these effects extend at
RHIC and LHC energies? The answer depends on the energy at which the saturation effects set in. It seems reasonable to assume, on the basis of the analysis of centrality and energy dependence of hadron multiplicities \[14, 15, 16\], that the minimal value of saturation scale is about \( Q_s^2(y_0) \approx 0.5 \text{ GeV}^2 \). In central \( Au-Au \) collisions at RHIC energy, where \( Q_s^2(y) \approx 2 \text{ GeV}^2 \), the boundary of the kinematical domain fully dominated by saturation extends thus up to \( p_t \approx \sqrt{2 \frac{Q_s^2(y)}{Q_s^2(y_0)}} \approx 4 \text{ GeV} \). At LHC, where \( Q_s^2(y) \approx 4.5 \text{ GeV}^2 \) \[16\], this boundary extends up to \( p_t \approx 9 \text{ GeV} \). It is also important to note that in peripheral collisions, as \( Q_s^2(y) \) becomes small and approaches the value of \( Q_s^2(y_0) \), we return to the familiar situation in which the boundary of hard processes sets in at a rather small value of \( p_t \sim 1 \text{ GeV} \), and the number of produced mini-jets, and hadrons, thus scales proportionally to the number of binary collisions. However, as the centrality increases, and \( Q_s^2(y) \) grows, the number of the produced semi-hard particles begins to deviate from the binary collision scaling; the transition to the \( N_{\text{part}} \) scaling occurs.

While these arguments are well defined and grounded theoretically, reliable computations will require further improvement of the quantitative understanding of saturated parton distributions. Nevertheless, we will present a calculation of the semi-hard particle production using a simple input distribution stemming from Eq. (4):

\[
\varphi_A(y, p_{t}^2) = \exp \left[ \sqrt{4\bar{\alpha}_S (y - y_0) \xi(p_{t}^2)} \right] \varphi_0^0(y_0, p_{t}^2);
\]

where \( \xi(p_{t}^2) = \ln\left(\frac{p_{t}^2}{Q_s^2(y_0)}\right) \). To bring this formula in line with the standard description of the deep-inelastic scattering (DIS) data, we have to modify it by slightly changing the anomalous dimension. Indeed, it has been demonstrated \[11\] that the following simple form of the anomalous dimension in the Mellin moment variable \( \omega \) (in the leading order in \( \alpha_s \)):

\[
\gamma(\omega) = \alpha_s \left( \frac{1}{\omega} - 1 \right)
\]

describes the DIS data well. This “phenomenological” anomalous dimension leads to a multiplicative factor of \( (Q_s^2/Q_s^0)^{\alpha_s} \) in our formula in the \( x-p_t \) representation. The resulting anomalous dimension is consistent with the result found from analysis of the BFKL equation and saturation in leading and next to leading order \[10, 11, 12, 13, 14\]. We use the running coupling in \[13\], freezing its value at small virtualities at \( \alpha_s^{\text{max}} = 0.45 \); therefore, at very high energies, when \( \alpha_s < \alpha_s^{\text{max}} \) in the entire region \( \bar{\alpha} \), the additional multiplicative factor turns into a constant, and we return to the anomalous dimension of 1/2.

For the function \( \varphi_0^0(y_0, p_{t}^2) \), describing parton distributions in the Color Glass Condensate at the onset of saturation, we will use the formula

\[
\varphi_0^0(x, p_{t}^2) = \frac{S_A}{\alpha_s} \frac{d(\tau = \frac{p_{t}^2}{Q_s^2(x)})}{d(\tau)};
\]

\[
d(\tau) = \frac{(2\tau + 1)}{\sqrt{4\tau + 1}} \ln \frac{\sqrt{4\tau + 1} + 1}{\sqrt{4\tau + 1} - 1} - 1.
\]
where $S_A$ is the nuclear overlap area. At large $p_t$, this distribution falls off as $1/p_t^2$, corresponding to the classical bremsstrahlung, and at small $p_t$ it has the behavior $\sim \ln(Q_s^2/p_t^2)$, established previously for the Color Glass Condensate [26, 27]. It agrees well with the numerical solution of non-linear evolution equations [26]; the derivation of Eq. (16) and detailed discussion of its properties will be presented elsewhere [32]. The value of the initial rapidity $y_0$ should be chosen to correspond to the onset of coherent partonic phenomena; we choose $y_0 = y_b - \ln(1/x_0)$ ($y_b$ is the beam momentum) corresponding to the value of Bjorken $x_0 \simeq 0.1$; this marks the beginning of the regime in which the coherence length $\sim 1/(mx_0)$ starts to exceed the typical inter-nucleon distance.

The cross section of hadron production can then be obtained by convoluting the result of

Figure 2: The $p_t$-spectrum of charged hadrons in central ($0 - 10\%$ centrality cut) $Au - Au$ collisions at $\sqrt{s} = 130$ GeV. The data is from [17].
Figure 3: Centrality dependence of hadron yields per participant pair in $Au - Au$ collisions at $\sqrt{s} = 130$ and 200 GeV in different $p_t$ bins; the yields are normalized to the yield in peripheral collisions. Upper solid lines show the behavior expected in perturbative QCD.
Figure 4: Same as in Fig. 3 in the $p_t$ bins of 6 and 9 GeV.
calculation according to Eqs. (12, 14) with the jet fragmentation function $D_{\text{frag}}(z, p_t)$:

$$
\frac{d\sigma^{\text{hadron}}}{dy dp_t^2} = \int dz \frac{d\sigma^{\text{jet}}}{dy dq_t^2} \delta(p_t^2 - z^2 q_t^2) D_{\text{frag}}(z, p_t) ;
$$

(17)

we also introduce in the distribution functions the common $\sim (1 - x)^4$ factors enforcing the behavior at $x \to 1$ prescribed by the quark counting rules.

The result for the charged hadron spectrum in central $Au - Au$ collisions at $\sqrt{s} = 130$ GeV is compared in Fig. 2 to the experimental data from PHENIX Collaboration. While the agreement is not perfect, the calculated cross section does agree with the data within error bars (only statistical error bars are shown), which is remarkable in spite of the crude approximations we made and our use of the standard fragmentation functions [45] fitted for the use with pQCD calculations. In Fig. 3, we show also the dependence of particle yields in different $p_t$ bins on centrality. While the result admittedly depends on the approximations we made and is not very robust, it does show that up to quite large values of $p_t$ the yield of particles deviates strongly from the scaling with binary collisions, shown by the upper curve on Fig. 3.

Further work is clearly needed before a reliable calculation of high $p_t$ cross sections in the Color Glass Condensate approach can be performed. Nevertheless, it is already clear that high density QCD effects have to be taken into account in the interpretation of the discovered at RHIC suppression of high $p_t$ particles.

The mechanism proposed here may also explain the old puzzle [46]: the absence of apparent jet quenching in the SPS data [47], [48] on high $p_t$ neutral pion production. Indeed, at relatively small energy of the SPS ($\sqrt{s} \approx 17$ GeV) the transverse momentum spectra of the produced particles are rather steep, and even a small jet energy loss would induce large deviations from the perturbative scaling with the number of collisions. These deviations were not observed: the data appear consistent with the perturbative QCD calculations [46]. At the same time, many of the “soft” properties of the collisions, as well as the suppression of charmonium, point to the presence of strong collective effects (see, e.g., [49]).

Basing on the fact that the prediction of KLN model [16] for centrality dependence of hadron multiplicities in the $\sqrt{s} = 20$ GeV run at RHIC appeared to be successful [50], it may be possible to assume that saturation sets in around SPS energy. Since saturation provides favorable initial conditions for thermalization [51], [52], the emergence of collective effects in “soft” observables and charmonium suppression would thus be likely. However, since for $y \to y_0$ the domain (3) $Q_s^2(y) < Q^2 < Q_s^4(y)/Q_s^2(y_0)$ shrinks to zero, the hard processes above the saturation scale exhibit the usual perturbative behavior, and no apparent jet quenching should appear.

The effects of our suppression mechanism begin to fail around 6 GeV for the 130 GeV data and somewhat higher for the 200 GeV data. We might have been able to work harder and find some marginally acceptable parameterization of the gluon structure functions which would extend the agreement to larger $p_T$, but this would not be very natural. Part of the problem is at these large $p_T$ values, we become sensitive to the gluon structure function within

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*We do not believe that extrapolation to still lower energies would make sense since the coherence length becomes much smaller than nuclear size in that case.*
the nuclear fragmentation region $x > 0.1$, and this is a region which is more complicated than that for which our naive saturation model can accommodate. It is also true that there must be some effects of jet energy loss in the media produced in heavy ion collisions, and the degree to which our saturation mechanism is dominant or subdominant relative to this mechanism is impossible for us to assess a priori.

The $dA$ data at RHIC will allow to disentangle the effects of jet energy loss from gluon saturation. Saturation is a property of the nuclear wavefunction, and in so far as it modifies jet production in $AA$ collisions, there will be a corresponding effect in $dA$ collisions. Let us discuss this in more detail. In $dA$ collisions at RHIC energy, with the exception of peripheral interactions where the density of participants, and thus the saturation scale, is too small, the gluon distribution in the nucleus $A$ is saturated and in the domain $Q_s < Q < Q_s^2/\Lambda$ is thus given by

$$\varphi_A \sim \frac{S_A \kappa_A}{\alpha_s} \sim \frac{S_A}{\alpha_s} \left( \frac{Q_s^2}{Q^2} \right)^{1/2}. \tag{18}$$

The deuteron distribution is given by $\varphi_d \sim S_d \kappa_d/\alpha_s$; our conclusion on the scaling with the number of participants from the nucleus $A$ will be independent on the explicit assumption about $\varphi_d$. The dependence of semi–hard processes in the saturation region on the number of participants from the nucleus $A N_{\text{part}}^A$ will now be given by

$$\frac{dN_{dA}}{dyd^2p_t} = \frac{1}{S_A} E \frac{d\sigma_{dA}}{d^3p} \sim (Q_{s,A}^2)^{1/2} \sim \left( N_{\text{part}}^A \right)^{1/2}. \tag{19}$$

where we have used that in $dA$ collisions $Q_{s,A}^2 \sim N_{\text{part}}^A$. We predict the behavior (19) for semi–hard processes to hold in $dA$ collisions at RHIC with the exception of peripheral collisions, where the saturation scale $Q_{s,A}^2$ is too small. The yield of particles per participant from the nucleus $A$ in these collisions is thus predicted to decrease as $(N_{\text{part}}^A)^{-1/2}$, deviating from the scaling with the number of collisions expected in perturbative QCD. We expect the saturation effects to set in around $N_{\text{part}}^{Au} \simeq 6$, corresponding to the impact parameter of $dAu$ collision $b = 5 \div 6$ fm [53].

We thus predict that around $N_{\text{part}}^{Au} \simeq 6$ the yields of high $p_t$ particles would begin to deviate from the scaling with the number of collisions $N_{\text{coll}} \sim N_{\text{part}}^{Au}$; the yield per participant will start to decrease as $(N_{\text{part}}^{Au})^{-1/2}$. In $15\%$ most central $dAu$ events, where $N_{\text{part}}^{Au} \simeq 12$ [53], we therefore expect to see the normalized yield of $(6/12)^{1/2} \simeq 0.7$, corresponding to $\simeq 30\%$ suppression of high $p_t$ particles. The scaling of semi–hard processes in $dAu$ collisions with centrality at RHIC energy can thus expected to be drastically different from that observed previously at fixed target energies, where at high $p_t$ the yields of particles were proportional to the number of collisions.

On the other hand, jet energy loss is expected to be a minor effect for $dA$ collisions since the jets are produced in a region far from the fragments of the nucleus [54]; for recent predictions, see [55].

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1Numerical calculations according to the formulae given above give somewhat smaller, but close, $\simeq 25\%$ suppression effect.
In addition, the $dA$ collisions provide the opportunity of studying jet production in the fragmentation region of the deuteron, and this probes much smaller values of $x$ of the nucleus than is the case for centrally produced jets. One expects the saturation scale to grow as

$$ Q_{s,A}^2(y) = Q_{s,A}^2(0) \exp(\lambda y), \quad \text{with } \lambda \approx 0.25. $$

At 3 units of rapidity away from the central rapidity region toward the deuteron fragmentation region in $dA$ collisions at $\sqrt{s} = 200$ GeV, the saturation scale thus grows to about $Q_{s,A}^2(y = 3) \approx 4.5 \text{ GeV}^2$. The approximations we are forced to make for centrally produced jets in $AA$ collisions should be under much better control for jets arising in the fragmentation region of the deuteron; recently, this problem was addressed in [56], [57], [58], [59], [60], [61].

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