Response analysis of 3D braided two-stage gear system excited by different frequency signals

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Abstract
The knitting principle of 3D braided gear was studied, and the dynamic model of the two-stage gear system was established. The fourth-order Runge-Kutta method was used to numerically simulate the dynamic characteristics of common gear and 3D braided gear. The results showed that the fundamental frequency \(v_1\) of the static transmission error excitation had the greatest effect on the speed and frequency characteristics of the first-stage gear along the meshing line. The research on frequency characteristics of common gear and 3D braided gear shows that the fundamental frequency \(v_1\) of the static transmission error excitation has a large effect on the speed and frequency characteristics of the first-stage gear along the meshing line. With the reduction of the gear mass and moment of inertia, the amplitude in the low-frequency band increases. The vibration resonance of the system is studied by defining the amplitude gain of the response of the system output at the low-frequency signal \(v_3\). The results show that with the reduction of gear mass and moment of inertia, when the input stage torque fluctuation frequency is \(\Omega > 5\), the fluctuation of amplitude gain \(Q\) disappears, which indicates that the vibration resistance of the 3D braided gear to high-frequency input stage torque fluctuation frequency is greatly improved.

Keywords
Two-stage gear system, dynamic model, 3D braided gear; vibration resonance

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Introduction
Textile composite material is a kind of composite material formed by weaving fiber bundles into the required structural shapes of parts by using textile technology, and then impregnating and curing them by using resin film infiltration and other processes. In recent years, with the needs of aerospace and other fields for the properties of impact resistance, interlaminar shear and light weight, 3D woven composite materials have been widely used and studied\(^1\)\(^-\)\(^4\).

In rectangular braiding and mechanical properties, Xu et al. analyzed the advantages, uses, disadvantages and deficiencies of 3D braided composites. By selecting a representative volume element (RVC) to study the

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influence of different braiding angles on the mechanical properties of braided composites.\textsuperscript{5} Chen et al. studied the microstructure of 3D braided preform produced by four-step method, analyzed the structural geometry of preform, and derived the mathematical relationship among structural parameters such as yarn stacking factor, fiber orientation, fiber volume fraction, braiding spacing, etc. It was verified that the calculated values of geometric characteristics of braided composite samples were in good agreement with the measured values.\textsuperscript{6} Zhang et al. established three different solid structure models of inner, surface and corner elements of 3D rectangular braided composites, simulated the mechanical properties of 3D rectangular braided composites with finite element method, and gave the deformation and stress distribution of the three element models, and studied in detail the influence of braiding angle and fiber volume fraction on elastic constants of 3D braided composites, which laid a foundation for the application of braided composites in aerospace field.\textsuperscript{7–9} In the aspect of ring knitting and mechanical properties, because the macroscopic structure of yarn is different in different radial positions, the microscopic geometry of 3D braided ring preform is more complex than that of rectangular preform.\textsuperscript{10,11} Wang et al. carried out finite element analysis on 3D circular braided composite pipe, and established a circular element model by geometric drawing method. The deformation and stress distribution are obtained by finite element analysis of the annular element, and the influence of weaving parameters on the longitudinal modulus is studied by comparing the predicted longitudinal modulus with the experimental results.\textsuperscript{12} Ma et al. divided the circular braiding into different regions according to the different movement modes of yarns in the circular braiding, and established the unit geometric model corresponding to different regions. By using this model, the mechanical characteristics of braiding can be accurately described, which laid a theoretical foundation for the structural design of three-dimensional and five-directional circular braiding.\textsuperscript{13} Hwan et al. predicted the elastic modulus of three-dimensional braided tube by using space spring model, and studied the influence of braiding parameters on the effective elastic modulus of 3D braided tube. At the same time, relevant compression tests were carried out to verify the calculation results.\textsuperscript{14} At present, 3D multi-directional braided composite materials have been used in the fields of satellite load-bearing space truss structure, high-temperature and ablation-resistant missile nose cone, rocket engine nozzle and so on.\textsuperscript{7,8}

The paper has been divided into five sections. In Sec. 2, the forming principle of 3D braided gear is analyzed. In Sec. 3, based on the consideration of backlash, time-varying meshing stiffness and meshing error, the differential equation of motion of the two-stage tooth transmission system is established by using the Lagrange equation. The characteristics of the system are studied by the speed time history diagram, phase plane, Poincaré map and FFT spectrum of the main and driven gears along the meshing line. In Sec. 4, the system vibration resonance is studied by defining the response amplitude gain of the system output at the low-frequency signal. In Sec. 5, we state the main conclusions. Through the research in this paper, it is found that with the decrease of mass and inertia of 3D braided gears, the amplitude and frequency of gears change during rotation, which indicates to some extent that the vibration resistance of 3D braided gears is better than that of ordinary gears.

**Forming principle of 3D braided gear**

The common compiling method used for 3D braided gear is four-step circular knitting, the circular knitting method is similar to the rectangular knitting method. The difference lies in that the coordinate system of circular knitting is the polar coordinate system, the working principle of four-step circular knitting is illustrated in Figure 1 with the example of $3 \times 18$ ($M = 3$, $N = 18$, $M$ is the number of circumferentially woven fiber layers, and $N$ is the number of radially woven fiber columns). In the first step, the row yarn carriers move one position in a staggered way in the radial direction. In the second step, the row yarn carriers move alternately by one position in the circumferential direction. In the third step, the row yarn carriers move one position in a staggered way in the radial direction. The fourth step, the row yarn carriers move alternately by one position in the circumferential direction. According to the above

![Figure 1. Working principle of four-step circular knitting.](image-url)
movement, the yarn carrier forms a machine cycle after four steps, and the length of the prefabricated part obtained after one cycle is called a knitting knuckle length $h$, and the woven prefabricated part with the required length can be obtained by circulating in turn.\textsuperscript{10,14}

In the four-step circular knitting, there is no additional yarn in the circumferential direction for the number of yarns, so the calculation formula of the total number of yarns $S$ is as follows:\textsuperscript{5}

$$S = (M + 1) \times N$$ \hspace{1cm} (1)

In the equation (1), $M$ is the number of circumferentially woven fiber layers, and $N$ is the number of radially woven fiber columns and must be an even number, otherwise, there must be two adjacent columns of yarn carriers with the same motion trajectory. In Figure 1, the total number of yarns is $S = (3 + 1) \times 18 = 72$.

In the four-step circular weaving, the weaving grain angle $\alpha$ formed on the surface of the preform is defined as the weaving angle. The weaving angle $\alpha$ is an important weaving process parameter, and the tangent of the weaving angle is equal to the ratio of the flower node width to the flower node height on the surface of the preform (as shown in Figure 2)\textsuperscript{3}:

$$\tan \alpha_{\text{in}} = \frac{2\pi d_{\text{in}}}{Nh}$$

$$\tan \alpha_{\text{out}} = \frac{2\pi d_{\text{out}}}{Nh}$$

(2)

In the equation (2), $\alpha_{\text{in}}$ is the inner surface weaving angle, $\alpha_{\text{out}}$ is the outer surface weaving angle, and $N$ is the number of columns. As the yarns in the column $i$ move to the adjacent columns after one cycle, the plane of the curve traversed is projected as an arc, and the length of the arc is $\frac{2\pi d_i}{N}$, $h$ is the length of braided knuckles. Due to $d_{\text{out}} > d_{\text{in}}$, it can be known from equation (2) $\alpha_{\text{out}} > \alpha_{\text{in}}$, and the weaving angle gradually increases from inside to outside.

The yarn carrier continuously circulates the four-step motion and gradually weaves into a required braided yarn preform; the braided yarn preform is dipped, pressed and cured in a liquid matrix to form an annular braided composite material; Figure 2 is a diagram of a braided fiber bundle model obtained by a four-step motion; Figure 3 is a gear matrix model; and Figure 4 is a braided gear model composed of a matrix and fibers.

The 3D braided gear is composed of high-strength and high-modulus carbon fiber and matrix. Carbon fiber provides structural rigidity and strength of composite material, while matrix fixes carbon fiber and disperses and transmits load between carbon fibers.\textsuperscript{7,8} Compared with traditional metal gear, 3D braided gear has many advantages such as good comprehensive mechanical properties, light weight and good shock absorption. In this paper, T300 carbon fiber is used for knitting yarn, and its density is 1.8 g/cm$^3$, the matrix is TDE-86 epoxy resin with a density of 0.980 g/cm$^3$. The
mass calculation formula of 3D braided gear is as follows:

\[ m_{3D} = V_1 \rho_1 + V_2 \rho_2 \]  

(3)

In equation (3), \( m_{3D} \) is the gear mass of 3D braided gear, \( V_1 \) is the volume of carbon fiber, \( V_2 \) is the matrix volume, \( \rho_1 \) is carbon fiber density, \( \rho_2 \) is the matrix density.

The moment of inertia of the 3D braided gear can be calculated by equation (4):

\[ J_i = \frac{\rho \pi B d_i^4}{32} \]  

(4)

In equation (4), \( \rho \) is gear density, \( \rho = v_1 \rho_1 + (1-v_1) \rho_2 \), \( v_1 \) is the filling factor of carbon fiber in 3D braided gear, \( B \) is the gear width, \( d_i \) is the diameter of gear base circle.

**Dynamic analysis of two-stage braid gear system**

The dynamic model of the two-stage gear system is shown in Figure 5. Considering the backlash, time-varying meshing stiffness and meshing error, the dimensionless differential equation of motion of the two-stage gear transmission system is obtained according to Lagrange equation as shown in equation (5)\(^{15-17}\):

\[
\begin{align*}
J_1 \ddot{\theta}_1 + r_1 P_1 &= T_{in} \\
(J_2 + J_3) \ddot{\theta}_2 - r_2 P_1 + r_3 P_2 &= 0 \\
J_2 \ddot{\theta}_2 - r_3 P_1 &= -T_{out}
\end{align*}
\]  

(5)

In equation (5), \( P_1 = c_1 \dot{x}_1 + k_1(t)g(x_1) \), \( P_2 = c_2 \dot{x}_2 + k_2(t)g(x_2) \), \( c_1 \) and \( c_2 \) are equivalent damping of meshing gears, \( c_1 = 2\xi_1 \sqrt{k_{av1} J_1 J_2/(r_1^2 J_1 + r_2^2 J_2)} \), \( c_2 = 2\xi_2 \sqrt{k_{av2} J_3 J_4/(r_3^2 J_3 + r_4^2 J_4)} \), \( x_1 = r_1 \theta_1 - r_2 \theta_2 - e_1(t) \), \( x_2 = r_3 \theta_3 - r_4 \theta_4 - e_2(t) \). \( r_i \) is the base circle radius of the \( i \) gear, \( J_i \) is the moment of inertia of the \( i \) gear, \( \xi_i \) is the meshing damping ratio of gear pair, and the value range is 0.03–0.17, \( k_{av1} \) and \( k_{av2} \) are the average meshing stiffness.\(^{15,16}\)

\( k_1 \) and \( k_2 \) are time-varying meshing stiffness, which is expanded by Fourier series at meshing frequency:

\[ k_i(t) = k_{ari} + \sum_{r=-1}^{\infty} k_{mir} \cos(r \omega_i t + \phi_i) \]  

(6)

In equation (6), \( k_{ari} \) is the average meshing stiffness of the \( i \) stage meshing gear, \( k_{mir} = C_i h_i \), \( C_i \) is the average value of the total tooth stiffness of \( i \) stage, \( h_i \) is the gear tooth width of \( i \) stage, \( k_{mir} \) is the variable stiffness amplitude of the meshing gear, \( \omega_i = \omega Z \), \( Z \) is gear speed, \( Z \) is the number of teeth, \( \phi_i \) is the initial phase of meshing gear variable stiffness, only the fundamental frequency in the series is considered, \( r = 1 \).

\( e_1(t) \) and \( e_2(t) \) are integrated meshing errors, \( e_i(t) = c_{mi} + A_i \sin(\omega_i t + \phi_i) \), \( c_{mi} \) is the steady-state value of static transfer error of the \( i \) stage meshing gear, \( c_{mi} = 0 \), \( A_i \) is the static transmission error amplitude of the \( i \) stage meshing gear. Fluctuation coefficient \( \kappa_{ii} = \frac{\Delta_{emi}}{c_{emi}} \), \( \kappa_{ii} = 0.3 \), \( \omega_i \) is the internal excitation fundamental frequency of the static transmission error of the \( i \) stage meshing gear, \( \phi_i \) is the initial phase of static transmission error of the \( i \) stage meshing gear.

In equation (5), \( T_{in} \) and \( T_{out} \) as input, output torque. Due to geometric deviation, misalignment error between mass center and geometric center, motor driving torque fluctuation, load torque fluctuation and bearing clearance, the actual working conditions are complicated. In order to facilitate the analysis, this paper only considers the factors that have great influence on the dynamic behavior of the system, such as the imbalance of rotating mass, the fluctuation of motor driving torque and the fluctuation of load torque, ignoring the influence of other factors on the system. At this time, the expressions of input and output torque are:

\[
\begin{align*}
T_{in}(t) &= T_{avep} + T_{iap} \cos(\Omega t + \varphi_{iap}) \\
T_{out}(t) &= T_{aveq} + T_{outq} \cos(\omega_3 t + \varphi_{outq})
\end{align*}
\]  

(7)

In equation (7), \( T_{avep} \) and \( T_{aveq} \) are the average torque of active and driven gears, units: N·m, \( T_{iap} \) and \( T_{outq} \) are torque amplitude of active and driven gears, units: N·m, \( \Omega \) is the torque ripple frequency of input stage, \( \omega_3 \) is the torque wave frequency of output stage,
Table 1. Common gear parameters.

| Quality/kg | Moment of inertia/kg.m² | Modulus | Number of teeth | Tooth width/mm | Base circle radius/mm |
|------------|-------------------------|---------|-----------------|----------------|-----------------------|
| m₁ = 0.15  | J₁ = 0.048e-3           | 1.5     | Z₁ = 30         | 20             | r₁ = 22.5             |
| m₂ = 1.5   | J₂ = 6.2e-3             | 1.5     | Z₂ = 100        | 20             | r₂ = 75               |
| m₃ = 0.25  | J₃ = 0.15e-3            | 1.5     | Z₃ = 40         | 20             | r₃ = 30               |
| m₄ = 1.2   | J₄ = 4.1e-3             | 1.5     | Z₄ = 90         | 20             | r₄ = 67.5             |

\( \varphi_{inp} \) and \( \varphi_{outq} \) are the initial phase units of active and driven gears, units: rad.

Because the physical quantities in the equation (5) differ greatly in order of magnitude, differential equations such as equation (5) are generally treated as dimensionless in mathematics. Define the time scale \( \omega_{b3} \), order \( \tau = \omega_{b3} \). Let the dimensionless displacement be \( \bar{x}_i = x_i / b_c \), to dimensionless processing equation (5), get equation (8).\(^{15, 16}\)

\[
\begin{align*}
\bar{x}_1 &= \frac{c_1}{m_{c1}\omega_b} \bar{x}_1 + \frac{k_1(\tau)}{m_{c1}} f_1(\bar{x}_1) - \frac{c_2}{m_{c2}\omega_b} \bar{x}_2 - \frac{k_2(\tau)}{m_{c2}\omega_b} f_2(\bar{x}_2) = Q_1 \\
\bar{x}_2 &= \frac{c_2}{m_{c2}\omega_b} \bar{x}_2 + \frac{k_2(\tau)}{m_{c2}\omega_b} f_2(\bar{x}_2) - \frac{c_1}{m_{c1}\omega_b} \bar{x}_1 - \frac{k_1(\tau)}{m_{c1}\omega_b} f_1(\bar{x}_1) = Q_2 \\
\end{align*}
\]

In equation (8), \( m_{c1}, m_{c2}, m_{c3} \) are equivalent mass of gear, \( m_{c1} = \frac{\rho_{c1} b_c}{\rho_{c1} + \rho_{c2}} \), \( m_{c2} = \frac{\rho_{c2} b_c}{\rho_{c1} + \rho_{c2}} \), \( m_{c3} = \frac{\rho_{c1} \rho_{c2} b_c}{\rho_{c1} + \rho_{c2}} \)

\( Q_1 = \frac{\tau_{inc}}{J_{inc}} + \frac{A_1}{\Omega_1^2} \sin(\Omega_1 \tau + \varphi_1) \), \( Q_2 = \frac{\tau_{outq}}{J_{outq}} + \frac{A_2}{\Omega_2^2} \sin(\Omega_2 \tau + \varphi_2) \).

In equation (8), \( f_i(\bar{x}_i) \) is the nonlinear function of the meshing gear side clearance of the \( i \) stage, which is defined as\(^{17}\):

\[
\begin{align*}
f_1(\bar{x}_1) &= \begin{cases} 
(r_1\theta_1 + r_2\theta_2 + e_1(t))/b_c - b_1/b_c, & (r_1\theta_1 + r_2\theta_2 + e_1(t))/b_c > b_1/b_c \\
0, & -b_1/b_c \leq (r_1\theta_1 + r_2\theta_2 + e_1(t))/b_c \leq b_1/b_c \\
(r_1\theta_1 + r_2\theta_2 + e_1(t))/b_c + b/b_c, & (r_1\theta_1 + r_2\theta_2 + e_1(t))/b_c < -b_1/b_c 
\end{cases} \\
f_2(\bar{x}_2) &= \begin{cases} 
(r_3\theta_2 + r_4\theta_4 + e_2(t))/b_c - b_2/b_c, & (r_3\theta_2 + r_4\theta_4 + e_2(t))/b_c > b_2/b_c \\
0, & -b_2/b_c \leq (r_3\theta_2 + r_4\theta_4 + e_2(t))/b_c \leq b_2/b_c \\
(r_3\theta_2 + r_4\theta_4 + e_2(t))/b_c + b_2/b_c, & (r_3\theta_2 + r_4\theta_4 + e_2(t))/b_c < -b_2/b_c 
\end{cases}
\end{align*}
\]

In the equations (9) and (10), \( b_1 \) is one half of the gear side gap. When the relative displacement of the gear tooth on the meshing line is \( f_i(\bar{x}_i) > b_1/b_c \), the gear is in normal meshing state, also known as non-impact state. When \( -b_1/b_c \leq f_i(\bar{x}_i) \leq b_1/b_c \), the two gears lose teeth, that is, the tooth surfaces are separated, and the gear teeth contact again after losing teeth, which produces impact, so they are in a unilateral impact state. When \( f_i(\bar{x}_i) < -b_1/b_c \), the tooth profile of driving face and non-driving face of the driving wheel are in contact with the driven wheel, which is called bilateral impact state.

In equation (8), \( y_i = \bar{x}_i, z_i = \bar{x}_i, X = \left( \begin{array}{c} y_i \\ z_i \end{array} \right) = \left( \begin{array}{c} \bar{x}_i \\ \bar{x}_i \end{array} \right), \) to reduce the second derivative to the first order, there is \( \dot{X} = \left( \begin{array}{c} \dot{y}_i \\ \dot{z}_i \end{array} \right) = \left( \begin{array}{c} \bar{x}_i \\ \bar{x}_i \end{array} \right), \) the initial conditions are

\[
X_0 = \left( \begin{array}{c} \bar{x}_i(0) \\ \dot{x}_i(0) \end{array} \right) = \left( \begin{array}{c} 0 \\ 0 \end{array} \right).
\]

\[
\begin{align*}
k_1 &= f(t(i), y(i), z(i)) \\
L_1 &= g(t(i), y(i), z(i)) \\
k_2 &= f(t(i) + h/2, y(i) + h/2 \times k_1, z(i) + h/2 \times L_1) \\
L_2 &= g(t(i) + h/2, y(i) + h/2 \times k_1, z(i) + h/2 \times L_1) \\
k_3 &= f(t(i) + h/2, y(i) + h/2 \times k_2, z(i) + h/2 \times L_2) \\
L_3 &= g(t(i) + h/2, y(i) + h/2 \times k_2, z(i) + h/2 \times L_2) \\
k_4 &= f(t(i) + h, y(i) + h \times k_3, z(i) + h \times L_3) \\
L_4 &= g(t(i) + h, y(i) + h \times k_3, z(i) + h \times L_3) \\
\end{align*}
\]

\[
\begin{align*}
\gamma(i + 1) &= y(i) + h/6 \times (k_1 + 2 \times k_2 + 2 \times k_3 + k_4) \\
z(i + 1) &= z(i) + h/6 \times (L_1 + 2 \times L_2 + 2 \times L_3 + L_4) 
\end{align*}
\]

(11)

With the simulation parameters shown in Tables 1 and 2, the numerical solution of differential equation (8) is obtained by using the fourth-order Runge-Kutta method shown in equation (11), and the speed-time history diagram (Figures 6(a) and 7(a)), phase plan diagram (Figures 6(b) and 7(b)), Poincaré map (Figures 6(c) and 7(c)), and FFT spectrum diagram (Figures 6(d) and 7(d)) of the first-stage and second-stage gears along the meshing line direction are obtained.
In Figures 6 and 7, the initial conditions are $x_1 = 0$, $\dot{x}_1 = 0$, $x_2 = 0$, $\dot{x}_2 = 0$.

As can be seen from Figures 6(a) and 7(a), the velocity time series curve of the first and second gears along the meshing line is composed of signals of many different frequencies superimposed together. As can be seen from Figures 6(c) and 7(c), Poincaré diagram presents irregular discrete points.

It can be seen from Figures 6(b) and 7(b) that the phase diagram is roughly a quasi-periodic motion zone. It can be seen from Figures 6(d) and 7(d), the first-stage excitation fundamental frequency $\omega_1$, input signal fluctuation frequency $\Omega$ and first-stage time-varying stiffness fundamental frequency $\omega_{11}$ have great influence on the speed and frequency characteristics of the first-stage gear along the meshing line, among which the first-stage static transmission error excitation fundamental frequency $\omega_1$ has the greatest influence, while the second-stage excitation fundamental frequency $\omega_2$, output signal fluctuation frequency $\omega_3$ and second-stage time-varying stiffness fundamental frequency $\omega_{22}$ have great influence on the speed and frequency characteristics of the second-stage gear along the meshing line, among which the second-stage static transmission error excitation fundamental frequency $\omega_2$ has the greatest influence.

Since the mass of 3D braided gear is much lower than that of traditional metal gear, it is necessary to analyze the dynamic characteristics of gears with different masses. Figure 8 for quality affects the amplitude frequency characteristics, Figure 8 is the first-stage in the direction along the meshing gear speed frequency characteristic figure, Figure 8(a) shows that the static transmission error excitation fundamental frequency $\omega_2$, the output signal fluctuation frequency $\omega_3$ and the secondary fundamental frequency time-varying stiffness $\omega_{22}$, on the first-stage gear along the line of action have little impact on the direction of the velocity amplitude frequency characteristic curve. The first-stage excitation fundamental frequency $\omega_1$, input signal fluctuation frequency $\Omega$, first-stage time-varying stiffness fundamental frequency $\omega_{11}$, on the first-stage gear along the line of action have a great influence on the direction of the velocity amplitude frequency characteristic curve, with the decrease of gear mass, the amplitude increases.

Figure 8(b) shows the static transmission error excitation fundamental frequency $\omega_2$, the output signal fluctuation frequency $\omega_3$ and the secondary fundamental frequency time-varying stiffness $\omega_{22}$ have a great influence.
on the amplitude of the velocity and frequency characteristics curve of the second-stage gear along the meshing line. The first-stage excitation fundamental frequency $v_1$, the input signal frequency fluctuations $\Omega$ and first-stage time-varying stiffness fundamental frequency $v_{11}$ have little impact on the amplitude of the velocity and frequency characteristics curve of the second-stage gear along the meshing line. With the decrease of the quality of gear, low frequency amplitude increases, the high frequency amplitude increase is not obvious, this suggests that the 3D braided gear quality is light, not less than ordinary metal gear vibration resistance.

**Vibration resonance analysis of two-stage braided gear system**

Nonlinear systems under different frequency signal excitation vibration resonance phenomenon is first put

Figure 7. Simulation results of the speed of the second-stage gear along the meshing line: (a) time series, (b) phase diagram, (c) poincaré diagram, and (d) spectrum diagram.

Figure 8. Influence of mass on frequency characteristic amplitude.
forward by Landa and McClintock,\textsuperscript{18,19} they, in the study of stochastic resonance noise with high frequency excitation signal, and find that the amplitude of the system response to low frequency signal and high frequency signal amplitude changes to present a nonlinear relation, namely resonance vibration. The complex equation (8) frequency components input stage in high speed, torque ripple frequency $\Omega$ can be regarded as the high frequency signal.\textsuperscript{20–22} The output stage torque wave frequency $\omega_3$ is in the low speed stage, torque ripple frequency $\omega_3$ can be regarded as the low frequency signal. The equation (12) is used to define the response amplitude gain $Q(\omega_3)$ of $\omega_3$ output by the system at the low frequency signal, which is used to study the degree of vibration resonance.\textsuperscript{22,23}

$$Q(\Omega) = \sqrt{B_s^2 + B_c^2}/T_{outq}$$ \hspace{1cm} (12)

In equation (12), $B_s = \frac{2}{\pi^2} \int_{0}^{\pi} \tau_2(t) \sin(\omega_3 t) dt$, $B_c = \frac{2}{\pi^2} \int_{0}^{\pi} \tau_2(t) \cos(\omega_3 t) dt$, $m$ is a positive integer.\textsuperscript{24}

No interference is considered

Based on the model of equation (8), when the torque amplitude of the driving gear $T_{inp}$ and the torque fluctuation frequency $\Omega$ of the input stage change, the influence on the amplitude gain $Q$ is shown in Figure 9, where $T_{outq} = 0.002$, $\omega_3 = 25$, $\omega_1 = 2$, $\omega_2 = 1.6$, $\omega_{11} = 20$, and $\omega_{22} = 25$. The mass and moment of inertia of each gear in Figure 9(a) are shown in Table 1, and Figure 9(b) as the corresponding Figure 9(a) projection drawing. Figure 9(a) and (b) shows that when the input torque fluctuation frequency $\Omega$ is equal to 2, 20 and 25, near extremum amplitude gain $Q$, this is because $\omega_3$, $\omega_1$, $\omega_{11}$, $\omega_{22}$, and $\Omega$ overlay, which resulted in increased amplitude gain $Q$, and the driving gear torque amplitude $T_{inp}$ increases, the amplitude gain $Q$ increases.

In Figure 9(c), $J_1 = 0.0365e-3$ kg.m$^2$, $J_2 = 4.38e-3$ kg.m$^2$, $J_3 = 0.1095e-3$ kg.m$^2$, $J_4 = 2.92e-3$ kg.m$^2$, Figure 9(d) is a projection diagram corresponding to Figure 9(c). Comparing (a)–(d) in Figure 9, it can be seen that with the decrease of gear moment of inertia, the amplitude gain $Q$ increases, but the number of weak vibrations decreases.

The mass and moment of inertia of each gear in Figure 9(e) are shown in Table 3, and Figure 9(f) is a projection diagram corresponding to Figure 9(f). By comparing (c)–(f) in Figure 9, it can be seen that with the reduction of gear mass and moment of inertia, the fluctuation of amplitude gain $Q > 5$ disappears, indicating that the vibration resistance has been greatly improved.
Consider noise interference

On the basis of equation (8) model, noise interference with amplitude of 0.000001 is added. The mass and moment of inertia of each gear in Figure 10(a) are shown in Table 1, Figure 10(b) is a projection drawing corresponding to Figure 10(a). The mass and moment of inertia of each gear in Figure 10(c) are shown in Table 2, Figure 10(d) is a projection drawing corresponding to Figure 10(c).

Comparing the four figures, it can be seen that when the torque amplitude of the driving gear $T_{inp}$ and the torque amplitude of the driven gear $T_{outq}$ change in equation (7), the amplitude gain $Q$ of the ordinary gear changes slightly, which is $0.00004$, while the amplitude gain $Q$ of the 3D braided gear changes greatly, which is $0.00006$. However, comparing the Figure 10(b) and (d), it is found that the extreme value of the amplitude gain $Q$ of the ordinary gear (red area in figures b and d) is much larger than that of the 3D braided gear. When the torque amplitude of the driving gear $T_{inp}$ and the torque amplitude of the driven gear $T_{outq}$ change, the vibration resistance of the 3D braided gear is better than that of the ordinary gear, which is mainly due to the light weight of the 3D braided gear, so the vibration amplitude is slightly higher, but the vibration resistance is better.

On the basis of equation (8) model, noise interference with amplitude of 0.000001 is added. The mass and moment of inertia of each gear in Figure 11(a) are shown in Table 1, and Figure 11(b) is the projection diagram corresponding to Figure 11(a). Comparing the four figures, it can be seen that when the torque amplitude of the driving gear and the fluctuation frequency of the input stage torque change in equation (6), the amplitude gain $Q$ of 3D braided gear varies greatly, with the magnitude of 0.00006. However, comparing Figure 11(b) and (d), it is found that the extreme value of amplitude gain $Q$ of ordinary gear (red area in b and d) is still far larger than that of 3D braided gear. Therefore, when the torque amplitude of driving gear and the fluctuation frequency of input stage torque change, the vibration resistance of 3D braided gear is better than that of ordinary gear.

### Table 3. 3D braided gear parameters.

| Quality/kg | moment of inertia/kg.m² | Modulus | Number of teeth | Tooth width/mm | Base circle radius/mm |
|------------|-------------------------|---------|-----------------|----------------|-----------------------|
| $m_1 = 0.045$ | $J_1 = 0.0365e-3$ | 1.5     | $Z_1 = 30$      | 20             | $r_1 = 22.5$       |
| $m_2 = 0.45$  | $J_2 = 4.38e-3$      | 1.5     | $Z_2 = 100$     | 20             | $r_2 = 75$         |
| $m_3 = 0.075$ | $J_3 = 0.1095e-3$   | 1.5     | $Z_3 = 40$      | 20             | $r_3 = 30$         |
| $m_4 = 0.36$  | $J_4 = 2.92e-3$      | 1.5     | $Z_4 = 90$      | 20             | $r_4 = 67.5$       |

![Figure 10](image_url)
Conclusion

On the basis of studying the knitting forming principle of 3D braided gear, this paper establishes the motion differential equation of two-stage gear transmission system by using Lagrange equation, taking into account the factors such as the backlash, time-varying meshing stiffness and meshing error, and uses the fourth-order Runge-Kutta method to numerically analyze the system differential equation. It is found that the first-stage excitation fundamental frequency $v_1$, the input signal fluctuation frequency $\Omega$ and the first-stage time-varying stiffness fundamental frequency $v_{11}$ have great influence on the speed and frequency characteristics of the first-stage gear along the meshing line. Therefore, the speed and frequency characteristics of the first-stage gear along the meshing line can be changed through three signal frequencies: $v_1$, $\Omega$, and $v_{11}$. The second-stage excitation fundamental frequency $\omega_2$, the output signal fluctuation frequency $\omega_3$ and the second-stage time-varying stiffness fundamental frequency $\omega_{22}$ have great influence on the speed and frequency characteristics of the second-stage gear along the meshing line. Therefore, the speed and frequency characteristics of the second-stage gear along the meshing line can be changed through three signal frequencies: $\omega_2$, $\omega_3$, and $\omega_{22}$. The vibration resonance analysis of the system without considering noise factors shows that with the decrease of gear mass and moment of inertia, the vibration resistance of the gear is improved. The vibration resonance analysis of the system considering the noise factor with the amplitude of 0.000001 shows that the amplitude gain Q of the 3D braided gear changes greatly with the change of the torque amplitude of the driving gear $T_{inp}$, the fluctuation frequency of the input stage torque $\Omega$ and the torque amplitude of the driven gear $T_{outq}$, but the number of extreme value of amplitude gain Q decreases obviously, which shows that with the decrease of gear mass and moment of inertia, the number of extreme value in gear rotation process decreases, that is, under the influence of noise factors, the vibration resistance of 3D braided gear is better than that of ordinary gear.

Declaration of conflicting interests

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