Moore-Read Fractional Quantum Hall wavefunctions and $SU(2)$ quiver gauge theories

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We identify Moore-Read wavefunctions, describing non-abelian statistics in fractional quantum Hall systems, with the instanton partition of $\mathcal{N} = 2$ superconformal quiver gauge theories at suitable values of masses and $\Omega$-background parameters. This is obtained by extending to rational conformal field theories the $SU(2)$ gauge quiver/Liouville field theory duality recently found by Alday-Gaiotto-Tachikawa. A direct link between the Moore-Read Hall $n$-body wavefunctions and $Z_n$-equivariant Donaldson polynomials is pointed out.

Introduction- There are now several proposed experiments aimed at identifying the existence of non-Abelian statistics in nature \cite{1}. The evidence of such statistics may be in fact observed for the first time in fractional quantum Hall (FQH) systems occurring at filling fraction $\nu = 5/2$ \cite{2}. Non-Abelian phases are gapped phases of matter in which the adiabatic transport of one excitation around another implies a unitary transformation within a subspace of degenerate wavefunctions which differ from each other only globally \cite{3}. Using this property it has been shown that systems exhibiting non-Abelian statistics can store topologically protected qubits and are therefore interesting for topological quantum computation \cite{4}.

Much of the comprehension of non-Abelian quantum Hall states relies on the conformal field theory (CFT) approach \cite{7} \cite{8}. The FQH-CFT connection was suggested by the relation between $(2+1)$ Chern-Simons theory and rational CFTs \cite{9}. The corresponding conformal blocks form higher-dimensional representations of the braiding group which can describe quasi-particle statistics in two dimensions. In the CFT approach \cite{7} \cite{8}, this link is extended to interpret conformal blocks as the analytic part of trial wavefunctions for the underlying particles. The analytic properties of the conformal blocks are then directly related to the universal properties characterizing a topological phase, such as the quantum numbers of the ground state and of the excitations. In particular the effects on adiabatic exchange of excitations, and especially their non-Abelian nature, should be encoded in the monodromy properties of these conformal blocks \cite{3} \cite{10}.

In this Letter we consider the Moore-Read (MR) states \cite{4}. There is now ample (numerical) evidence that the physics of the $\nu = 5/2$ plateau is well captured by one particular realization of the MR states which describes a (p-wave) pairing of electrons occurring at the first excited Landau level. The non-Abelian nature of these states is well understood in terms of the Ising CFT. We show that a recent relation found by Alday-Gaiotto-Tachikawa (AGT) between conformal blocks in Liouville field theory and instanton partition functions in $SU(2)$ quiver gauge theories \cite{5} can be generalized to the case of conformal blocks of the Ising CFT. A nice geometrical description of $\mathcal{N} = 2$ superconformal quiver gauge theories was provided in \cite{6} by using M-theory compactification. It was shown that the data of the quiver are encoded in the extended Teichmuller space of a suitable Riemann surface with punctures. AGT duality identify the instanton partition function of $SU(2)$ quiver gauge theories with the conformal blocks of a Liouville field theory on this Riemann surface. By specializing this duality to Ising CFT, we are able to provide a direct link with MR wavefunctions. As explained in detail later, there is a precise dictionary between the quantities characterizing a MR wavefunction and the ones defining an $SU(2)$ quiver gauge theory. For a given number $N_p$ of particles and $n_{qh}$ quasi-holes, we show that: \textit{i}) there is a corresponding $SU(2)$ quiver gauge theory with gauge group $G = \prod_i^{N_p} SU(2)^{n_{qh}}$ coupled to a total number of $N_{t} = N_p + n_{qh}$ (bi-)fundamental hypermultiplets with a suitable chosen set of mass parameters; \textit{ii}) the values of the gauge couplings associated to this quiver gauge theory are fixed by the positions of the particles and of the quasi-holes; \textit{iii}) each degenerate quasi-hole wavefunction is related to a specific Coulomb branch of the quiver gauge theory. In this Letter we will give an illustrative and explicit example of this relation by considering the most simple MR wavefunctions exhibiting non-Abelian statistics.

In earlier works other connections between the FQH and certain (Chern-Simons) effective field theories were mainly based on universality arguments \cite{11}. The relation discussed here is instead directly manifest in the specific form of the MR many-body wavefunctions. A consequence of our result is that $\mathbb{Z}_n$-equivariant Donaldson polynomials are encoded in the MR wavefunctions, suggesting the appearance of new unexpected topological features in the FQH.

Moore-Read states- In symmetric gauge, the many-body wavefunction $\tilde{\Psi}$ describing $N_p$ particle states in the lowest Landau level takes the general form:

$$\tilde{\Psi}(\{z_i, \bar{z}_i\}) = P_{N_p}(z_1, \cdots, z_N) \prod_{i=1}^{N_p} \mu(z_i, \bar{z}_i),$$

(1)

where $z_i$ is a complex variable which represents the particle coordinates, $P(z_1, \cdots, z_N)$ is a polynomial in the $N_p$
variables \(z_i\) and \(\mu(z_i, \bar{z}_i)\), which is the non-analytic part of the wavefunction, is the measure corresponding to the surface where the particles live. As \(\prod \mu(z_i, \bar{z}_i)\) is a simple one body-term, we can drop it for simplifying notations.

MR wavefunctions are given by the conformal blocks of the minimal CFT with central charge \(c = 1/2\) \cite{13}. This is a rational CFT with two primary fields, \(\Psi\) and \(\sigma\), which, together with the identity \(\text{Id}\) field, close under the operator algebra. The field \(\Psi\), with conformal dimension \(\Delta_\Psi = 1/2\), is a free fermion field and fuses with itself into the identity, \(\Psi \times \Psi \rightarrow \text{Id}\). This field is identified with the particle operator: the MR ground state \(P_{N_\Psi}\) is given by the following \(N_\Psi\)-point correlation function:

\[
P_{N_\Psi}^{\text{gs}}(\{z_i\}) = \langle \Psi(z_1) \cdots \Psi(z_{N_\Psi}) \rangle = \frac{1}{\prod_{i<j} z_i z_j^{N_\Psi+1}} \text{Pf}(z_{ij}^{-1}) \prod_{i<j} z_i^{M+1} \prod_{i<j} z_j^{M+1},
\]

where \(z_{ij} = z_i - z_j\) and \(\text{Pf}(A_{ij})\) is the Pfaffian of the \(N_\Psi \times N_\Psi\) matrix \(A_{ij} = z_{ij}^{-1}\). The r.h.s of (2) is obtained by using Wick theorem to compute the \(N_\Psi\)–free fermions correlation function. For even (odd) values of \(M\), the state (2) describes bosons (fermions) at filling \(\nu = 1/(1 + M)\). Hereafter, without any loss of generality, we can focus on the bosonic \(M = 0\) MR states. The field \(\sigma\) has conformal dimension \(\Delta_\sigma = 1/16\) and represents the elementary quasi-hole operator. The subspace of degenerate wavefunctions with the \(n_q\) quasi-holes at fixed positions \(\{w_i\}, P_{N_\Psi}^{\text{gh}}(\{w_i\}, \{z_i\})\), is set by the correlator:

\[
P_{N_\Psi}^{\text{gh}}(\{w_i\}, \{z_i\}) = F_{(a)}(\{w_i\}, \{z_i\}) \times \prod_{i<j} z_{ij}^{n_q} \prod_{i=1}^{N_\Psi} (z_i - w_j)^{1/2} \prod_{i<j} w_{ij}^{1/2},
\]

where

\[
F_{(a)}(\{w_i\}, \{z_i\}) = \langle \sigma(w_1) \cdots \sigma(w_{n_q}) \Psi(z_1) \cdots \Psi(z_{N_\Psi}) \rangle_{(a)}.
\]

In the above equations \(w_{ij} = w_i - w_j\) while the index \(a\) runs over the the possible conformal blocks. From the fusion between the particle \(\Psi\) and the quasi-hole \(\sigma\) operators, \(\Psi \times \sigma \rightarrow \sigma\), one can verify that the particles and quasi-holes are mutually local, as required. The dimension of the degenerate space of the excited wavefunctions \(\{\}\) is given by the number \(2^n - 1\) of conformal blocks corresponding to \(\{\}\). This number can be derived from the number of all possible conformal blocks consistent with the fusions \(\sigma \times \sigma = \text{Id} + \Psi\) and \(\Psi \times \sigma = \sigma\), as shown in Fig. 1.

Nekrasov functions and AGT relation

It has been known for a long time that supersymmetric gauge theories in four dimensions display a topological sector describing Donaldson polynomials, defined by integration of suitable chiral observables on the instanton moduli space \cite{15}. Nekrasov proposed \cite{19} a deformation of the four-dimensional space-time manifold, the so-called \(\Omega\)-background, which paved the way to an explicit evaluation of these integrals via localisation techniques \cite{19, 20, 21}. The idea is to exploit the rotational symmetries \(U(1) \times U(1) \subset SO(4)\) of \(\mathbb{R}^4 \sim \mathbb{C}^2\), \((z_1, z_2) \rightarrow (e^{i\epsilon_1} z_1, e^{i\epsilon_2} z_2)\) in order to reduce the integral to a discrete sum over fixed points. From the mathematical viewpoint, this amounts to consider equivariant Donaldson polynomials \cite{22}. More precisely, Nekrasov’s partition function for a \(U(r)\) gauge theory is the generating functional of the integral of the fundamental equivariant cohomology class \(1 \in H^2(M_{k,r})\), where \(M_{k,r}\) is the moduli space of \(U(r)\) framed instantons with second Chern class \(k\), and \(T\) the \((r+2)\)-dimensional torus associated to the above mentioned rotational symmetry and to the change of framing, i.e. \(U(1)^r\) global gauge transformations. In further generality, given two multiplicative classes \(E, F\) one defines

\[
Z^r_{\text{inst}}(\tilde{a}, m, \mu; q) = \sum_{k=0}^{\infty} q^k \int_{M_{k,r}} E_{T_m}(T_M) F_{T_\mu}(V)
\]

where \(T_M\) is the tangent bundle and \(V\) is a vector bundle on \(M_{k,r}\), while \(q^k = e^{2\pi i k}\) is the \(k\)-instanton action, \(\tau\) being the complex coupling of the gauge theory, and \(\epsilon_1, \epsilon_2, \tilde{a} = (a_1, \ldots, a_r)\) are the parameters associated to the torus action \(T\). Indeed, in presence of matter multiplets, one has further global symmetries, and one has to include their contribution to the localisation formula. For one adjoint hypermultiplet with mass \(m\), which has \(U(1)_m\) global symmetry, the contribution is given by the \(T_m\) equivariant Euler class of the tangent bundle \(E_{T_m}(T_M)\). For \(N_f\) fundamental hyprops with masses \(\{\mu_f\} = \{\mu_1, \ldots, \mu_{N_f}\}\) one has a \(U(N_f)\) global flavour symmetry, and one has to multiply by the \(T_\mu\)-equivariant Euler class \(F_{T_\mu}(V)\) associated to the fundamental representation of \(U(N_f)\), \(T_\mu\) being its maximal torus \cite{21}.

We now propose a mathematical interpretation of the Nekrasov partition functions for quiver gauge theories which are relevant for our problem. Quiver gauge theories with gauge group \(\prod_{j=1}^n U(r_j)\), \(\sum_{j=1}^n r_j = r\) arise naturally by considering a finite group action \(Z_n\) on the equivariant parameters \(a_\alpha \rightarrow a_\alpha + 2\pi i \frac{a_\alpha}{n}, \alpha = 1, \ldots, r\). The integers \(r_j\) are the number of times that the \(j\)-th
irreducible representation of the $\mathbb{Z}_n$ group appears in the decomposition. In particular, in order to obtain the product gauge group of our interest one has to consider $r = 2n$ and set $r_j = 2$ for any $j$. The specialization to $SU(2)\,j$ factors come simply by picking the relevant Cartan subgroup $\tilde{a} = (a_1, -a_1, \ldots, a_n, -a_n)$. The quiver has matter fields with masses $m = (m_1, \ldots, m_n)$ transforming in the bifundamental $(r_j, r_{j+1})$ representation. The finite group acts accordingly on the associated equivariant parameters as $m_j \to m_j + 2\pi i n$. In the localization formulæ, one needs to restrict only to the fixed point set which is invariant under the finite group action, namely one consider the $\mathbb{Z}_n$-equivariant classes $1^n$, $E_T^{Z_n} (T_M)$ associated to the $\mathbb{Z}_n$ action on $M_{k,r}$. One thus get $\mathbb{Z}_n$-equivariant Donaldson polynomials

$$Z_{\text{inst}}^{\alpha,\xi} (\mathbf{a}, \mathbf{m}, \mu; \mathbf{q}) = \sum_{k=0}^{\infty} q^k \int_{M_{k,r}} 1^n E_T^{Z_n} (T_M) \prod_{j=1}^{4} F_{T_{r_j}} (V)$$

where $q^k = \prod_{i=1}^n q_i^{k_i}$, $q_i$ encodes the gauge coupling of the $i$-th node and $k_i$ is the associated Chern class, with $\sum_i k_i = k$. The contribution $\prod_j F_j$ of the two fundamental and two antifundamental multiplets at the ends.

The general conformal block

$$\mathcal{G}(\xi_1, ..., \xi_{N-3}) \equiv \langle \phi^{a_1}(0) \prod_{i=2}^{N-2} \phi^{a_{i-1}}(1) \phi^{a_{N-1}}(1) \phi^{a_N}(\infty) \rangle$$

depends on $N - 3$ variables and it is fully characterized by the $N$ charges $a_i$ of the "external" operators together with the $N - 3$ charges $a_{in}$ of the internal operators, i.e. the ones appearing in the fusion channels. Consider the conformal block $\mathcal{G}(\xi_1, ..., \xi_{N-3})$ represented by the diagram in Fig. 3 and define $\delta_i = \Delta_{a_{in}} - \Delta_{a_1} - \Delta_{a_2}$, $\delta_i = \Delta_{a_{in}} - \Delta_{a_{i-1}} - \Delta_{a_{i+1}}$ for $i = 2, ..., N-4$ and $\delta_{N-3} = \Delta_{a_N} - \Delta_{a_{N-1}} - \Delta_{a_{N-3}}$. Using the following mapping:

$$q_i = \frac{\xi_i / \xi_{i+1}}{\xi_{N-3}} \quad q_{N-3} = \xi_{N-3} \quad i = 1, ..., N-4$$
$$\epsilon_1 = b \quad \epsilon_2 = 1/b \quad a_1 = a_{in} - Q/2 (i = 1, ..., N - 3)$$
$$\mu_1 = \alpha_2 + \alpha_1 - Q/2 \quad \mu_2 = \alpha_2 - \alpha_1 + Q/2$$
$$\mu_3 = \alpha_{N-1} - \alpha_N - Q/2 \quad \mu_4 = \alpha_{N-1} - \alpha_N + Q/2$$
$$m_i = \alpha_{i+2} (i = 0, ..., N-1),$$

the AGT relation states that:

$$\frac{\mathcal{G}(\xi_1, ..., \xi_{N-3})}{\prod_{i=1}^{N-3} q_i^{\delta_i}} = Z_{\text{inst}}^{\alpha,\xi} (\mathbf{a}, \mathbf{m}, \mu; \mathbf{q}) Z_{U(1)}^{\epsilon_1,\epsilon_2} (\mathbf{m}, \mathbf{q}),$$

with

$$Z_{U(1)}^{\epsilon_1,\epsilon_2} (\mathbf{m}, \mathbf{q}) = \prod_{C_{ij}} \left(1 - q_i q_{i+1} \cdots q_{j} \right)^{2m_i \left(Q - m_{j+1} \right)}.$$
the corresponding Verma module a null vector at level \( nm \). The degenerate fields form a subset of fields which closes under the operator algebra. For certain values of the parameter \( b \) there is within the set of degenerate fields a finite subset which closes under operator algebra. This is what happen for \( b = 2i/\sqrt{3} \) (or equivalently \( b = -i\sqrt{2}/3 \)), corresponding to \( c = 1/2 \): the operator \( V_{21} \) and \( V_{12} \), with dimension \( \Delta_{21} = 1/2 \) and \( \Delta_{12} = 1/16 \), form together with the Identity the Ising CFT:

\[
b = 2i/\sqrt{3} : \quad \Psi \equiv V_{21} \quad \sigma \equiv V_{12}. \quad (13)
\]

The analytical properties of the Nekrasov partition function with respect to the charges \( \alpha_i \) and \( \alpha_i^{in} \) (Fig.[4]) entering in the corresponding conformal block have been discussed in [21, 25], showing in particular the appearance of poles when the charges \( \alpha_i^{in} \) correspond to the degenerate fields, see (12). From this we can argue that the AGT relation (9)-(10) is still valid for computing the corresponding conformal block (4) where all operators are degenerate fields. Indeed, as it has been discussed in [26] for the Liouville conformal block, the residues at the poles corresponding to degenerate internal operators should vanish when the external operators are degenerate fields too. This is the reason why we expect the AGT relation to be valid also for correlation functions of rational CFTs, and then for quantum Hall wavefunctions. To be more concrete, we consider as an illustrative and non trivial example a six-point conformal block [4] with \( n = 4 \) and \( N = 2 \). These are the simplest wavefunctions exhibiting non-Abelian statistics. There are two possible conformal blocks \( \mathcal{F}_{0,1/2}(w_1, w_2, w_3, w_4, z_1, z_2) \), represented by the diagram in Fig.[4]. The explicit expression of these functions can be found in Eq.(7.16) of [14] and we do not report it here. Using a conformal map, we can send the points \( w_1 \rightarrow 0, z_1 \rightarrow 1 \) and \( z_2 \rightarrow \infty \), which means, in the sphere geometry, that we put one quasi-hole at the South Pole and one particle at the North Pole. We thus study the function

\[
\mathcal{F}_{0,1/2}(w_2, w_3, w_4) \equiv \lim_{R \rightarrow \infty} \mathcal{F}_{0,1/2}(0, w_2, w_3, w_4, 1, R), \quad (14)
\]

and we derive its expansions for small \( q_i \). This expansion, at the second order, has the following form:

\[
\frac{\mathcal{F}_0(q_1 q_2 q_3, q_2 q_3, q_3)}{q_1^{-1/8} q_2^{-1/8} q_3^{-1/4}} \sim 1 + \frac{q_2}{8} + \frac{2 q_2^2 + 9 q_2^3 + 16 q_2^4}{128} + .. \quad (15)
\]

\[
\frac{\mathcal{F}_2(q_1 q_2 q_3, q_2 q_3, q_3)}{q_1^{-3/8} q_2^{-1/8} q_3^{-1/4}} \sim 1 + \frac{2 q_1 - 3 q_2}{8} + \frac{18 q_1^2 - 15 q_1^3 + 16 q_1^4 + 20 q_1 q_2}{128} + .. \quad (16)
\]

In order to check (10), we consider general \( b \) and we study the Liouville conformal block (8) with \( N = 6 \) and:

\[
\alpha_i = -\frac{1}{2b} \quad (i = 1, ..., 4) \quad \alpha_5 = \alpha_6 = -\frac{b}{2}. \quad (17)
\]

We are thus considering the Liouville correlator \( \langle V_{12} V_{12} V_{12} V_{21} V_{21} \rangle \). In this case, there are two possible conformal blocks corresponding to the fusion channels \( V_{12} \times V_{12} \rightarrow \text{Id} \) and \( V_{12} \times V_{12} \rightarrow \Psi \). The internal charges appearing in Fig.[3] are then set to

\[
\text{Id channel:} \quad \alpha_i^{in} = 0 ; \quad \alpha_i^{in} = -\frac{1}{2b} ; \quad \alpha_i^{in} = 0 \quad (18)
\]

\[
\text{V}_{13} \text{ channel:} \quad \alpha_i^{in} = -\frac{1}{b} ; \quad \alpha_i^{in} = -\frac{1}{b} ; \quad \alpha_i^{in} = 0. \quad (19)
\]

For \( b \rightarrow 2i/\sqrt{3} \) the above two channels correspond respectively to the conformal blocks \( \mathcal{F}_0 \) and \( \mathcal{F}_{1/2} \). Indeed, for \( b = 2i/\sqrt{3} \), i.e. for \( c = 1/2 \), one can show that the operator \( V_{13} \) can be identified with \( \Psi \).

An alternative route to the computation of the expansions (15,16) in terms of the instanton partition function (6) is provided by the dictionary (4) which determine the set of parameters \( \hat{a}, \hat{m}, \mu \) as a function of \( \alpha_3 \) and \( \alpha_3^{in} \). The explicit formula to compute (6) can be found in [21, 27] and are resumed in [3]. Here we show for instance the case corresponding to (18):

\[
Z_{\text{inst}}^{b,1/6} \sim 1 + \frac{3}{2b^2} q_1 + \left( \frac{\frac{3}{2} + \frac{1}{b^2}}{2} \right) q_3 + \frac{9 + 32b^2 + 66b^4 + 12b^6}{8b^4 + 12b^6} q_1^2 + \left( \frac{2 + \frac{2}{b^2} + \frac{1}{b^2}}{4b^4} \right) q_2^2 + \left( \frac{3 + \frac{1}{b^2}}{2b^2} \right) q_1 q_2 + \left( \frac{\frac{3}{2} + \frac{1}{b^2}}{2b^2} \right) q_2 q_3 + \frac{6 + 13b^2 + 6b^4}{4b^4} q_1 q_3 + .. \quad (20)
\]

Taking into account the function (11), that in the case under consideration takes the form:

\[
Z_{U(1)}^{b,1/6}(q_1, q_2, q_3) = \left[ (1 - q_1)(1 - q_2)(1 - q_1 q_2) \right]^{2 + \frac{b}{2}} \quad (21)
\]

we find, by setting \( b \rightarrow 2i/\sqrt{3} \), the expansion (15). The same can be shown also for the expansion (19). The

FIG. 4: Conformal blocks corresponding to the functions computed in [14]
expansions \cite{15,16} can then be directly related to the one given in \cite{6}.

In this Letter we have shown a neat connection between the theory of the Moore-Read non-Abelian quantum Hall states and four dimensional superconformal $SU(2)$ quiver gauge theories. We provided a mathematical interpretation linking the Moore-Read wavefunctions to generating functionals of $Z_{nm}$-equivariant Donaldson polynomials. Let us notice that our analysis unveil some special properties of the submanifold \cite{12} of the equivariant parameter space. Indeed, it is well known that correlation functions of $V_{nm}$ degenerate fields satisfy order $nm$ partial differential equations: it would be interesting to investigate the interpretation of this result in Donaldson theory, trying to make a connection with the results of \cite{28}.

Our result opens a bridge between different theories that can provide new insights in their comprehension; nonetheless, the origin of this relation is still mostly unclear. Some arguments for the AGT correspondence have been proposed in the context of topological strings and M-theory \cite{15-17}. One line of arguments that could be relevant for our problem is that the four dimensional gauge theories in question can be realized as superstring compactifications in presence of D-branes. In particular \cite{14} one can consider B model topological branes in the local geometry of a resolved $A_1$ singularity fibered over a Riemann surface $\Sigma$. The dynamics of such branes is governed by a holomorphic Chern-Simons theory, which upon reduction to the blown-up $\mathbb{P}^1$ provides a matrix model with a $\beta$-ensemble measure induced by the non-triviality of the fibration. The collective field describing the large $N$ limit of this matrix model is precisely a Liouville field on the double covering of $\Sigma$. It is thus tempting to suppose that this matrix model captures at least some aspects of the microscopic description of the non-Abelian Fractional Quantum Hall systems \cite{22}.

There are several interesting aspects that are worth to be investigated further. For instance, it would be interesting to study the consequences of the vanishing of the Berry connection, proven for Moore-Read states \cite{10}, in terms of $SU(2)$ quiver gauge theories. As a final remark, we mention that AGT relation \cite{10} can be extended to a relation between the $SU(k)$ quiver gauge theories and $W_{2k-1}$ Toda theories \cite{29}. The results presented in this Letter are expected to be generalized to the states based to the affine $W_{2k-1}$ Toda theories \cite{30}. These states include as a special case the Moore-Read states and their most direct generalization, the so-called $Z_k$ Read-Rezayi states \cite{8}. The ground state wavefunctions of these states can be written as particular Jack polynomials \cite{30}. Via the AGT relation, these Jack polynomials are related to Nekrasov partition functions. It is compelling to gain further insights on the origin of all these relations.

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