DC Transformer and DC Josephson(-like) Effects in Quantum Hall Bilayers

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Received December 4, 2001; accepted January 22, 2002

PACS ref: 73.43.-f, 73.43.Lp, 73.43.Nq

Abstract

In the early days of superconductivity, Ivar Giaver discovered that it was possible to make a novel DC transformer by using one superconductor to drag vortices through another. An analogous effect was predicted to exist in quantum Hall bilayers and has recently been discovered experimentally by Eisenstein’s group at Caltech. Similarly, new experiments from the Caltech group have demonstrated the existence of a Josephson-like ‘supercurrent’ branch for electrons coherently tunneling between the two layers.

1. Introduction

The various quantum Hall effects are among the most remarkable many-body phenomena discovered in the second half of the twentieth century [1–4]. The fractional effect has yielded fractional charge, spin and statistics, as well as unprecedented order parameters [5]. There are beautiful connections with a variety of topological and conformal field theories of interest in nuclear and high energy physics.

The quantum Hall effect (QHE) takes place in a two-dimensional electron gas formed in a quantum well in a semiconductor host material and subjected to a very high magnetic field. In essence it is a result of a commensuration between the number of electrons, N, and the number of flux quanta, NΦ, in the applied magnetic field. The electrons condense into distinct and highly non-trivial ground states (‘vacua’) formed at each rational fractional value of the filling factor v ≡ N/NΦ.

The essential feature of (most) of these exotic states is the existence of an excitation gap. The electron fluid is incompressible and flows rigidly past obstacles (impurities in the sample) with no dissipation. A weak external electric field will cause the fluid to move, but the excitation gap prevents the fluid from absorbing any energy from the electric field. Hence the current flow must be exactly at right angles to the field and the conductivity tensor takes the remarkable universal form

\[ \sigma^{xx} = \sigma^{yy} = 0; \quad \sigma^{yx} = -\sigma^{xy} = \frac{e^2}{h}. \]

(1)

Ironically, this ideal behavior occurs because of imperfections and disorder in the samples which localize topological defects (vortices) whose motion would otherwise dissipate energy. In a two-dimensional superconductor, such vortices undergo a confinement phase transition at the Kosterlitz-Thouless temperature and dissipation ceases. In most cases in the QHE, an analog of the Anderson-Higgs mechanism causes the vortices to be deconfined [5] so that dissipation is strictly zero only at zero temperature. In practice, values of \( \sigma^{xx}/\sigma^{yy} \) as small as \( 10^{-13} \) are not difficult to obtain at dilution refrigerator temperatures.

Recent technological progress in molecular beam epitaxy techniques has led to the ability to produce pairs of closely spaced two-dimensional electron gases. Strong correlations between the electrons in different layers lead to a great deal of completely new physics involving spontaneous interlayer phase coherence [3,6–20]. This is the first example of a QHE system with a finite-temperature phase transition. This transition is in fact a Kosterlitz-Thouless transition into a broken symmetry state which is closely analogous to that of a 2D superfluid. Recent remarkable tunneling experiments by Eisenstein’s group at Caltech [21,22] have observed something closely akin to the Josephson effect in superconducting tunnel junctions and have measured the dispersion of the superfluid Goldstone mode. It was predicted some years ago that the broken symmetry state should exhibit quantized drag [9,23], and the Caltech group has recently observed this effect [24]. This experiment is analogous to the classic DC transformer effect discovered by Ivar Giaver [25] in which one superconductor is used to drag vortices across another thereby creating a 1:1 voltage transformer that works with DC rather than AC currents.

2. Giaver DC flux transformer

Let us begin our analysis with a review of dissipation in a superconducting film in the presence of a weak magnetic field. The magnetic field will induce vortices so that the superconducting order parameter is approximately given by

\[ \Psi(z) \sim \prod_j \frac{z - Z_j}{|z - Z_j|} = e^{i\phi(x,y)} \]

(2)

where \( z = x + iy \) is a complex number representing the position vector \((x, y)\) in the 2D film and \( Z_j \) is the position of the \( j \)th vortex in the same complex notation. (I have neglected the variation in the magnitude of the order parameter in the core region of the vortices.) The phase \( \phi \) of the order parameter winds by \( 2\pi \) in circling a vortex

\[ \oint \mathbf{r} \cdot d\mathbf{r} = 2\pi. \]

(3)

Under typical circumstances, the vortices in a superconductor should be viewed as ‘heavy’ classical objects with strong dissipation due to the normal region in the core. Hence these objects are strongly coupled to the lattice and tend to remain at rest. However in the absence of actual pinning of the vortices to disorder in the lattice, a bias current applied to

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the film will cause the vortices to drift perpendicular to the current due to the Magnus force. For each vortex that drifts across the sample, the phase difference along the direction of the current slips by $2\pi$

$$\frac{d\phi_2 - \phi_1}{dt} = 2\pi \nu_v$$  \hspace{1cm} (4)

where $\nu_v$ is the flux of vortices drifting across the sample. Using the Josephson relation

$$h \dot{\phi} = 2eV$$  \hspace{1cm} (5)

we conclude that the dissipative voltage drop along the current direction is

$$V_2 - V_1 = \frac{h}{2e} \dot{\phi}_v.$$  \hspace{1cm} (6)

The Giaever flux transformer consists of two superconducting films separated by a thin insulating layer. Provided that the magnetic penetration depth is not too large, the magnetic field will be inhomogeneously concentrated in the vicinity of the vortices. As a result there will be a magnetic coupling energy which will prefer for the vortices in the two layers to be bound together. When this binding force is strong enough, current applied to one layer (the ‘primary’) will drag the vortices together across both the ‘primary’ and the ‘secondary’ layers so that

$$\dot{\phi}_{\text{secondary}} = \dot{\phi}_{\text{primary}}$$  \hspace{1cm} (7)

and the voltage drop will be identical in both layers. Hence the system acts as a precise 1:1 voltage transformer [25].

3. Coulomb drag DC transformer

In addition to this magnetic coupling mechanism, one might imagine that Coulomb interactions between the electrons in the two superconducting films might lead to a drag effect. For layer separation $d$, the interlayer Coulomb interaction has strength

$$U_{\text{inter}} = \frac{2\pi}{q} e^{-qd}$$  \hspace{1cm} (8)

for momentum transfer $hq$ between the layers. However because the layer spacing (~3–10 nm) is considerably larger than the spacing between electrons within a layer, $k_F d \gg 1$ and the interaction is negligible. Each electron gas sees the other as an essentially structureless continuum.

Coulomb interactions are however very important if we consider drag in a pair of closely spaced two-dimensional electron gases (2DEGs) created in semiconductor quantum wells. Here the layer spacing is also on the scale of 10 nm, but the electron density is vastly lower so that $k_F d \sim 1$ and the interlayer Coulomb interactions are comparable in strength to the intralayer interactions. Consider first the force balance in a single layer. Newton’s law for the carrier drift velocity in the presence of an electric field $E$ is

$$\dot{v} = -\frac{e}{m} E - \left(\frac{1}{\tau_L} + \frac{1}{\tau_{\text{ee}}}\right)v. \hspace{1cm} (9)$$

where $1/\tau_L$ is the relaxation rate due to collisions with impurities in the lattice and $1/\tau_{\text{ee}}$ is the relaxation rate due to electron-electron collisions. However due to galilean invariance, electron-electron collisions conserve the center of mass momentum of the colliding particles and hence can not relax the current and so $1/\tau_{\text{ee}} = 0$. Assuming steady state ($\dot{v} = 0$) and relating the drift velocity to the current density via $J = -ne\dot{\phi}$ we obtain the standard result:

$$J = \frac{ne^2 \tau_L}{m} E.$$  \hspace{1cm} (10)

Thus even though Coulomb interactions produce strong correlations and collisions, their effect is invisible in ordinary transport measurements.

Drag measurements do not suffer from this problem and are an excellent probe of Coulomb interactions [26, 27]. To see how this works, consider the force balance in a pair of closely spaced layers denoted by $\uparrow, \downarrow$:

$$\dot{v}_\uparrow = -\frac{e}{m} E \frac{1}{\tau_L} \frac{1}{\tau_{\text{D}}} (v_\uparrow - v_\downarrow),$$

$$\dot{v}_\downarrow = -\frac{e}{m} E \frac{1}{\tau_L} \frac{1}{\tau_{\text{D}}} (v_\downarrow - v_\uparrow).$$  \hspace{1cm} (11)

Here $1/\tau_{\text{D}}$ is the rate of momentum transfer between the layers due to electron-electron interactions which attempt to relax the two layers towards a common center of mass velocity. Galilean invariance only applies to the sum of the velocities in the two layers, not the difference, and hence this relaxation term does not vanish. Assuming steady state and requiring that there be no current flowing in the secondary layer ($\downarrow$; say) we can readily solve for the drag induced electric field

$$E_\downarrow = -\rho_D J \uparrow.$$  \hspace{1cm} (12)

where $\rho_D$ is the drag or transresistance given by

$$\rho_D = +\frac{1}{m \tau_0}.$$  \hspace{1cm} (13)

Equivalently

$$E_\downarrow = -\frac{\tau_{\text{D}}^{-1}}{\tau_0^{-1} + \tau_{\text{D}}^{-1}} E \uparrow.$$  \hspace{1cm} (14)

The minus sign indicates that the electric field induced in the drag layer is opposite in direction to that in the drive layer. (I have chosen the standard sign convention in Eq. (12) which makes the transresistance positive.) The sign is readily understood from the fact that collisions with the electrons in the drive layer tend to push electrons in the passive layer to flow in the same direction as those in the drive layer. However because we are insisting on zero current in the drive layer, charge builds up until the opposing electric field stops the current flow. The electric field force on the drag layer is thus opposite to that in the drive layer.

Despite the Coulomb interactions, if the density is not too low, the electrons in the two layers constitute fermi liquids in which Pauli blocking severely limits the phase space available for collisions. Only electrons that lie within $k_F T$ of the fermi energy can participate in collisions and so we anticipate that

$$\frac{1}{\tau_{\text{D}}} \sim \frac{1}{\tau_0} \left(\frac{k_F T}{T_F}\right)^2.$$  \hspace{1cm} (15)
where $\tau_0$ is some microscopic collision rate in the absence of Pauli blocking. Hence interlayer collisions become less and less important as the temperature decreases and we expect the drag resistance to be small and ultimately vanish [26,27]

$$\rho_D \sim + T^2. \quad (16)$$

This is precisely what was observed by Gramila et al. [26]. At $T = 1K$, they observed a drag resistance of only a few milli-ohms per square in a pair of 2DEGs.

4. Quantum Hall coherent states

Non-fermi liquids are of great current interest in the study of strongly correlated systems. It is easy to destroy 2DEG fermi liquids by applying a strong external magnetic field which quenches the kinetic energy and places the system in the quantum Hall regime [4]. As shown in Fig. 1, the drag in this case rises by some 6 orders of magnitude and for certain values of the magnetic field, the transverse (Hall) drag resistance takes on a universal value

$$\rho_{xy} = \frac{h}{e^2} \sim -25,813 \Omega. \quad (17)$$

This is very different from the fermi liquid result because the electric field is at right angles to the current, does not vanish at low temperatures, and is in the same, not opposite, direction in both layers.

We begin our analysis of the QHE regime with the simplest example of the integer QHE in a single layer system of spinless electrons at Landau level filling factor $\nu = 1$. The strong magnetic field quantizes the kinetic energy into discrete Landau levels [4] separated in energy by the cyclotron energy $h\omega_c \sim 100K$. Each level has a macroscopic degeneracy equal to $N_\Phi$. This degeneracy in the kinetic energy means that interactions are enormously important and have non-perturbative effects at fractional filling factors. However for $\nu = \frac{N}{\Phi} = 1$, every state of the lowest Landau level (LLL) is occupied and, since there is a large kinetic energy gap to the next Landau level, interactions are (relatively) unimportant. It is this gap which makes the system incompressible. Since the lowest LLL is completely full, the state is a simple Slater determinant. In first-quantized form the state is most easily expressed in the symmetric gauge [4]

$$\Psi(z_1, z_2, \ldots, z_N) = \prod_{i<j}^{N} (z_i - z_j)e^{-\frac{1}{2}\sum_{n=1}^{N}(\mid n_i - n_j \mid)^2} \quad (18)$$

where $z_j = (x_j + iy_j)/\ell$ is a dimensionless complex number representing the 2D position vector of the $j$th particle in units of the magnetic length $\ell$. The vandermonde polynomial factor in this Laughlin state is totally antisymmetric and is equivalent to a single Slater determinant filling all the orbitals in the LLL.

The meaning of the Laughlin wave function can be seen by comparison with the superconducting vortex problem. In the QHE regime vortices are not heavy classical objects with normal cores, but rather highly quantum objects which attach themselves to the electrons. Equation (18) tells us that each electron sees a complex zero of the wave function (i.e., a vortex) at the position of each and every other electron. Thus as current flows through the device, exactly one vortex passes through for each electron that passes through. The vortex flux is therefore simply related to the total current $I$

$$\mathcal{N}_V = \frac{I}{e} \quad (19)$$

and the Josephson relation (now for charge $e$, not $2e$) yields the universal result

$$V = \frac{h}{e} \phi = \frac{h}{e} 2\pi \mathcal{N}_V = \frac{h}{e^2} I. \quad (20)$$

Since the vortex motion is now parallel rather than perpendicular to the current, the (Hall) voltage drop is perpendicular to the current and the flow is dissipationless. Moving the magnetic field away from the point which gives filling factor $\nu$ exactly unity introduces extra vortices (or antivortices) into the ground state, but in the presence of random disorder these are pinned (just as in a disordered superconductor) and do not affect the transport. This is what allows the quantized plateau to exist over a finite range of magnetic fields rather than just at a single unique value of $B$.

We see this very wide plateau in the QHE drag data in the left portion of Fig. 1 corresponding to $\nu = 1$ in each layer. In this regime the drag voltage vanishes exponentially with temperature because each layer has an excitation gap and so it is not possible at low temperatures to have any excitations produced by interaction between the layers. In the language of vortices we can understand this result using the cartoon representation of the state shown in Fig. 2. At filling factor $\nu = 1$ in each layer ($\nu_T = 2$), there are exactly as many vortices in each layer as there are electrons. The quantum state satisfies this condition by having each electron see only vortices attached to electrons in the same layer. Hence the electrons in one layer do not see the vortices in the other layer. (If they did, then they would see a total of too many vortices.) As a result, when current flows in the drive layer, the electrons

Fig. 1. Hall resistance (upper set of curves) and Hall drag resistance (lower set of curves) in a QHE bilayer system as a function of the inverse total filling factor $\nu^{-1} = 1/(\nu_1 + \nu_2) \propto B$. For small layer separation $d$ relative to the magnetic length $\ell$, there is a Hall plateau at $\nu_1 = \nu_2 = 1/2$ in which the Hall field is identical in drive and drag layers even though no net current is flowing in the drag layer. After Kellogg et al. Ref. [23].
in the drag layer do not see any vortices moving by and the drag voltage vanishes.

At filling factor \( v = 1/2 \) in each layer, the magnetic field is twice as large and there are now twice as many vortices as electrons in each layer. The Coulomb energy favors the state shown in the lower panel of Fig. 2 in which each electron sees a vortex attached to every electron whether they are in the same or different layers. Because the vortices are complex zeros of the wave function, the electrons strongly avoid each other, independent of whether they are in the same or different layers. This state turns out to be the exact ground state for zero layer spacing and is a good approximation for small \( d/\ell \).

Because electrons see vortices from all the other electrons, a current in the drive layer drags vortices through the lower layer at a rate given by Eq. (19). Again applying the Josephson relation as in Eq. (20) yields a universal quantized Hall transresistance:

\[
\rho_{\text{D}}^{xy} = -\frac{h}{e^2}.
\]

The electric field is perpendicular to the current and has exactly the same sign and magnitude as the field in the drive layer. This prediction [9,23,28,29] is beautiful and precisely verified in the recent experiment of Kellogg et al. shown in the right portion of Fig. 1.

Because every electron sees a vortex attached to every other electron, the microscopic wave function for this special state is simply that given by Eq. (18). We first wrote this down above for a single layer, but it applies here because the wave function is completely independent of which of the two layers any electron is in! [6,7,9,23,28–30]. This is an explicit manifestation of the strange fact that, even though tunnelling between the layers might be forbidden, quantum mechanics still allows for the possibility of states in which we are uncertain which layer the electrons are in. It is very useful to introduce a pseudospin 1/2 to represent the layer index, \( | \uparrow \rangle \) representing the electron being in the upper layer and \( | \downarrow \rangle \) corresponding to the lower layer. [We assume that the real spin is frozen out by the applied magnetic field.] In this language the microscopic wave function becomes

\[
\Psi(z_1, z_2, \ldots, z_N) = \prod_{i<j}^{N} (z_i - z_j)e^{\pm \frac{i}{\hbar} \sum_n |z_n|^2} |\rightarrow| \rightarrow | \rightarrow \ldots |\rangle,
\]

where the arrows represent the coherent spinors

\[
| \rightarrow \rangle = \frac{1}{\sqrt{2}} (| \uparrow \rangle + e^{i\phi} | \downarrow \rangle).
\]

This corresponds to the pseudospin lying in the \( xy \) plane at an angle \( \phi \) away from the \( x \) axis. Because the spatial part of the wave function is fully antisymmetric (which optimizes the Coulomb energy), the pseudospin part must be fully symmetric implying that this is a pseudospin ferromagnet. Each electron is in a coherent superposition of both layers. The microscopic wave function in Eq. (18) actually corresponds to the special case of \( \phi = 0 \). However because tunnelling is absent, there is a global symmetry (corresponding to the conservation of \( N_1 - N_2 \)) which tells us that there is actually an entire family of degenerate states with different values of \( \phi \). That is, there is a spontaneously broken \( U(1) \) symmetry [6,7] in which the order parameter

\[
\langle \sigma_z + i\sigma_y \rangle = \langle \psi_1^\dagger \psi_2 \rangle \sim e^{i\phi}
\]

condenses.

While the energy can not depend on the global value of \( \phi \) it can depend on spatial gradients

\[
H = \frac{1}{2}\rho_s \int d^2 r |\nabla \phi|^2
\]

where the pseudospin stiffness \( \rho_s \) represents the loss in Coulomb exchange energy between the two layers when \( \phi \) varies with position. The gradient energy is stored in a ‘supercurrent’

\[
J_\perp = \rho_s \nabla \phi.
\]

Because the ‘charge’ conjugate to \( \phi \) is \( \sigma_z \sim n_1 - n_2 \), this supercurrent is oppositely directed in the two layers. With this knowledge in hand we can reanalyze the drag experiment in terms of the symmetric and antisymmetric currents

\[
J_\parallel = J_1 \pm J_1.
\]

The symmetric channel transport is that of a \( v = 1 \) quantized Hall plateau with \( \rho_{\text{D}}^{xy} = h/e^2 \). The antisymmetric channel transport is that of a superfluid with \( \sigma_{\text{AS}}^{xy} = \infty \). In the drag experiment there is only current in the drive layer so that

\[
J_\parallel = J_\perp = J_1.
\]

The symmetric current produces a quantized Hall electric field which is identical in both layers and the superfluidity means that the antisymmetric current produces no field at
all. This is precisely the effect observed in the drag experiment by Kellogg et al. and provides strong evidence for the existence of superfluidity in this special interlayer phase coherent state.

Further strong evidence for phase coherence was discovered in a remarkable tunnelling experiment by Spielman et al. [21] in samples in which an extremely weak tunnelling amplitude between the layers was used as a sensitive probe. Because the order parameter in Eq. (24) is the tunnelling operator itself, it is possible to tunnel an electron from one layer to the other and still be in the same quantum state! This paradox arises from the fact that we were uncertain which layer the electron was in originally. If the quantum state is unchanged then energy is conserved only at zero bias voltage. The Caltech group indeed observed an enormous and extremely narrow (half width < 5 \( \mu \)V) zero bias anomaly in the tunnelling as shown in Fig. 3. Unlike the true Josephson effect the dissipation is not infinitesimal on the supercurrent branch. Various proposals involving a finite phase coherence time have been made to explain the finite height and width of the differential conductance peak [13–15,17]. But this is a question which is still poorly understood and is a subject of current study.

If the layer separation is increased, the system undergoes a quantum phase transition to a quantum disordered state in which the zero bias anomaly disappears and is replaced by a Coulomb pseudogap as shown in the right hand panel of Fig. 3.

Further evidence for the broken symmetry comes from observation of the goldstone mode associated with the superfluidity. By applying a magnetic field in the plane of the 2DEGs, the tunnelling electron picks up a finite in-plane momentum due to the Lorentz force. This momentum \( hq \) goes into exciting a goldstone boson with energy \( \hbar \omega \sim hq \). The energy required to produce the boson should cause a small feature in the tunnel \( I-V \) characteristic at bias voltage \( eV = \hbar \omega \) which shifts continuously with applied magnetic field [13–15]. This goldstone mode feature has been found by the Caltech group [22]. The left panel of Fig. 4 shows that application of the parallel magnetic field fairly quickly kills the central peak and a small side feature appears which disperses outward with increasing \( B_y \). Spielman et al. identify the inflection point as the center of this derivative feature and plot the resulting dispersion curve as shown in the right panel of Fig. 4. The dispersion is indeed linear and agrees to within about a factor of two of the predicted mode velocity of \( \sim 10^4 \) m/s. It is perhaps not surprising that the measured mode velocity is somewhat lower since quantum fluctuations neglected in the Hartree-Fock approximation will lower the spin stiffness.

5. Open issues

At the present time, there are still a variety of open issues. Experimentally there is now very strong evidence for the interlayer phase coherent state and the corresponding superfluidity in the antisymmetric channel. The Kosterlitz-Thouless phase transition seems to be occurring as expected because the coherent tunnelling peak at zero bias appears at temperature scales which are roughly consistent with the predicted value of the pseudospin stiffness as well as the observed and predicted goldstone mode velocity. However no direct measurement of the universal jump in superfluid stiffness has been possible so far. An open theoretical issue is that we do not have a complete microscopic understanding of the dissipation/decoherence mechanism which gives a finite width and height to the tunnelling peak [13–15,17]. We do not understand why the central peak is not destroyed more rapidly with the addition of \( B_y \). The peak is still visible even when the Goldstone feature has moved out far enough to be distinct from it. (Most likely this is due to disorder but a quantitative model is lacking.) Finally, we do not have a good understanding of the nature of the quantum phase transition or transitions that occur as the layer spacing is increased. Various scenarios have been suggested theoretically [16,19,31] and there is some numerical evidence.

![Fig. 3](image1.png)

**Fig. 3.** Upper left panel: Differential conductance of a QHE bilayer system in the phase coherent state at filling factor \( v = 1 \) and layer spacing \( d/l = 1.61 \). The central peak is remarkably narrow with a HWHM of only about 6 \( \mu \)eV. Lower left panel: \( IV \) curve showing the nearly vertical ‘supercurrent’ branch and a remnant of the Coulomb gap feature at larger voltages. After Spielman et al. Ref. [23]. Right panel: Differential conductance at low density (small \( d/l \)) in the phase coherent state and high density (large \( d/l \)) where the layers are uncorrelated. In the latter case the tunnel current vanishes at small voltages due to the Coulomb gap. There is a peak in the current at a voltage corresponding to the scale of the Coulomb interactions in the system. Figure courtesy of J. P. Eisenstein.

![Fig. 4](image2.png)

**Fig. 4.** Left panel: Differential conductance vs voltage for a variety of values of the parallel B magnetic \( B_y \). Inset: Magnified view showing the Goldstone feature dispersing outward in voltage with increasing \( B_y \). Black dots indicate inflection points which are used to determine the mode dispersion shown in the right panel. The velocity agrees to within a factor of two of the value \( \sqrt{m^*} \sim 10^4 \) m/s predicted from Hartree-Fock estimates of the spin stiffness and the compressibility parameters \( \rho_s \) and \( \Gamma \). After Spielman et al. Ref. [22].
hinting that there might be a single weakly first order transition [11].

Acknowledgements

This work was supported by NSF DMR-0087133 and DMR-096503, and represents long-standing collaborations with many colleagues including Allan MacDonald, Ady Stern, I. Schliemann, Ning Ma, K. Moon, and Kun Yang. I also would like to thank J. P. Eisenstein and his group for numerous helpful discussions of their experiments.

Dedication

I would like to dedicate this paper to the memory of William Caswell who was tragically killed on September 11, 2001. Bill was a couple years ahead of me in graduate school and gave me invaluable help in my effort to become a physicist.

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