Decays and Lifetime of $B_c$ in QCD Sum Rules

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Abstract: In the framework of three-point QCD sum rules, the form factors for the semileptonic decays of $B_c^+ \to B_s(B_s^*)l^+\nu_l$ are calculated with account for the Coulomb-like $\alpha_s/v$-corrections in the heavy quarkonium. The generalized relations due to the spin symmetry of HQET/NRQCD for the form factors are derived at the recoil momentum close to zero. The nonleptonic decays are studied using the assumption on the factorization. The $B_c$ meson lifetime is estimated by summing up the dominating exclusive modes in the $c \to s$ transition combining the current calculations with the previous analysis of $b \to c$ decays in the sum rules of QCD and NRQCD.

Keywords: QCD sum rules, NRQCD, weak decays, HQET, spin symmetry.

1. Introduction

For better understanding and precise measuring the weak-action properties of heavy quarks, governed by the QCD forces, we need as wide as possible collection of snapshots with hadrons, containing the heavy quarks. Then we can provide the study of heavy quarks dynamics by testing the various conditions, determining the forming of bound states as well as the entering of strong interactions into the weak processes. So, a new lab for such investigations is a doubly heavy long-lived quarkonium $B_c$ recently observed by the CDF Collaboration [1] for the first time.

This meson is similar to the charmonium and bottomonium in the spectroscopy, since it is composed by two nonrelativistic heavy quarks, so that the NRQCD approach [2] is well justified to the system. The modern predictions for the mass spectra of $\bar{b}c$ levels were obtained in refs. [3] in the framework of potential models and lattice simulations. The measured value of $B_c$ mass yet has a large uncertainty $M_{B_c} = 6.40 \pm 0.39 \pm 0.13$ GeV, in agreement with the theoretical expectations.

The measured $B_c$ lifetime

$$\tau[B_c] = 0.46^{+0.18}_{-0.16} \pm 0.03 \text{ ps},$$

agrees with the estimates obtained in the framework of both the OPE combined with the evaluation of hadronic matrix elements in NRQCD [4, 5, 6] and potential quark models, where one has to sum up the dominating exclusive modes to calculate the total $B_c$ width $\tau_{\text{OPE,PM}}[B_c] = 0.55 \pm 0.15 \text{ ps}$. The accurate measurement of $B_c$ lifetime could allow one to distinguish various parameter dependencies such as the optimal heavy quark masses, which basically determine the theoretical uncertainties in OPE.

At present, the calculations of $B_c$ decays in the framework of QCD sum rules were performed in [9, 10, 11, 12]. The authors of [9, 10] got the results, where the form factors are about 3 times less than the values expected in the potential quark models, and the semileptonic and hadronic widths of $B_c$ are one order of magnitude less than those in OPE. The reason for such the disagreement was pointed out in [11] and studied in [12]: in the QCD sum rules for the heavy quarkonia the Coulomb-like corrections are significant, since they correspond to summing up the ladder diagrams, where $\alpha_s/v$ is not a small parameter, as the heavy quarks move nonrelativistically, $v \ll 1$. The Coulomb rescaling of quark-quarkonium vertex enhances the estimates of form factors in the QCD sum rules for the $B_c^+ \to \psi(\eta_c)l^+\nu_l$ decays. In [12] the soft limit
\( v_1 \cdot v_2 \to 1 \) at \( v_1 \neq v_2 \), where \( v_{1,2} \) denote the four-velocities of initial and recoil mesons, was considered, and the generalized spin symmetry relations were obtained for the \( B_c \to \psi(q_c) \) transitions: four equations, including that of [13]. Moreover, the gluon condensate term was calculated in both QCD and NRQCD, so that it enforced a convergency of the method.

In the present paper we calculate the \( B_c \) decays due to the \( c \to s \) weak transition in the framework of QCD sum rules, taking into account the Coulomb-like \( \alpha_s/v \)-corrections for the heavy quarkonium in the initial state. In the semileptonic decays the hadronic final state is saturated by the pseudoscalar \( B_s \) and vector \( B_s^* \) mesons, so that we need the values of their leptonic constants entering the sum rules and determining the normalization of form factors. For this purpose, we reanalyze the two-point sum rules for the \( B \) mesons to take into account the product of quark and gluon condensates in addition to the previous consideration of terms with the quark and mixed condensates. We demonstrate the significant role of the product term for the convergency of method and reevaluate the constants \( f_B \) as well as \( f_{B_s} \). Taking into account the dependence on the threshold energy \( E_c \) of hadronic continuum in the \( bs \) system in both the value of \( f_{B_s} \), extracted from the two-point sum rules and the form factors in the three-point sum rules, we observe the stability of form factors versus \( E_c \), which indicates the convergency of sum rules.

The spin symmetries of leading terms in the lagrangians of HQET [13] for the singly heavy hadrons (here \( B_s^{(*)} \)) and NRQCD [3] for the doubly heavy mesons (here \( B_s \)) result in the relations between the form factors of semileptonic \( B_c \to B_s^{(*)} \) decays. We derive two generalized relations in the soft limit \( v_1 \cdot v_2 \to 1 \): one equation in addition to what was found previously in ref. [13]. The relations are in a good agreement with the sum rules calculations up to the accuracy better than 10\%, that shows a low contribution of next-to-leading \( 1/m_Q \)-terms.

We perform the numerical estimates of semileptonic \( B_c \) widths and use the factorization approach [13] to evaluate the nonleptonic modes. Summing up the dominating exclusive modes, we calculate the lifetime of \( B_c \), which agree with the experimental data and the predictions of OPE and quark models. We discuss the preferable prescription for the normalization point of nonleptonic weak lagrangian for the charmed quark and present our optimal estimate of total \( B_c \) width. We stress that in the QCD sum rules to the given order in \( \alpha_s \), the uncertainty in the values of heavy quark masses is much less than in OPE. This fact leads to a more definite prediction on the \( B_c \) lifetime.

2. Three-point sum rules

The hadronic matrix elements for the semileptonic \( B_c(p_1) \to B_s(p_2) \) decays can be written down as follows:

\[
\langle B_s | V_\mu | B_c \rangle = f_+(p_1 + p_2)_\mu + f_+ - q_\mu, \tag{2.1}
\]

\[
\frac{1}{i} \langle B_s^{*} | V_\mu | B_c \rangle = i F_V \epsilon_{\mu \nu \alpha \beta} \epsilon^{*\nu}(p_1 + p_2)\alpha \beta q, \tag{2.2}
\]

\[
\frac{1}{i} \langle B_s^{*} | A_\mu | B_c \rangle = F_0 \epsilon_\mu + F_+^{A}(\epsilon^* \cdot p_1)(p_1 + p_2)_\mu + F_+^{A}(\epsilon^* \cdot p_1)q_\mu,
\]

where \( q_\mu = (p_1 - p_2)_\mu \) and \( \epsilon^\mu = \epsilon^\nu(p_2) \) is the polarization vector of \( B_s^* \) meson. \( V_\mu \) and \( A_\mu \) are the flavour changing vector and axial electroweak currents. Following the standard procedure for the evaluation of form factors in the framework of QCD sum rules [16], we consider the three-point functions, say,

\[
\Pi_\mu(p_1, p_2, q^2) = i^2 \int dx dy e^{i(p_2 - x - p_1 - y)} \langle 0 | T\{ \bar{q}_1(x)\gamma_5 q_2(x), V_\mu(0), \bar{b}(y)\gamma_5 c(y) \} | 0 \rangle,
\]

where \( \bar{q}_1(x)\gamma_5 q_2(x) \) and \( \bar{q}_1(x)\gamma_\mu q_2(x) \) denote interpolating currents for \( B_s \) and \( B_s^* \), correspondingly.

The Lorentz structures in the correlators can be written down as \( \Pi_\mu = \Pi_+(p_1 + p_2)_\mu + \Pi_- q_\mu \). The form factors \( f_\pm \) are determined from the amplitudes \( \Pi_\pm \), respectively.

The leading QCD term is a triangle quark-loop diagram, for which we can write down the double dispersion representation at \( q^2 \leq 0 \)

\[
\Pi_\mu^{pert}(p_1^2, p_2^2, q^2) = -\frac{1}{(2\pi)^2} \int \frac{d^4 s_1 d^4 s_2}{(s_1 - p_1^2)(s_2 - p_2^2)} ds_1 ds_2 + \text{subtractions},
\]
where the limits of integration region and the spectral densities are given in [13].

The physical spectral functions are generally saturated by the ground hadronic states and a continuum starting at some effective thresholds.

The ladder diagram of the Coulomb-like interaction.

For the heavy quarkonium $\bar{b}c$, where the relative velocity of quark movement is small, an essential role is taken by the Coulomb-like $\alpha_s/v$-corrections. They are caused by the ladder diagram, shown in Fig. 1. This leads to the finite renormalization for $\rho_1$ [12], so that $\rho_1^c = C\rho_1$,

$$C = \frac{|\Psi_{bc}^C(0)|}{|\Psi_{bc}^{\text{free}}(0)|} = \sqrt{\frac{4\pi\alpha_s}{3v}(1 - \exp\{-\frac{4\pi\alpha_s}{3v}\})^{-1}},$$

where $v$ is the relative velocity of quarks in the $\bar{b}c$-system, $v = \sqrt{1 - \frac{4m_{\bar{b}}m_c}{E_{\bar{b}c}^2 - (m_{\bar{b}} - m_c)^2}}$.

3. Numerical estimates

We evaluate the form factors in the scheme of spectral density moments. This scheme is not strongly sensitive to the value of the $\bar{b}c$-system threshold energy, and we put $E_{\bar{b}c}^0 = 1.2$ GeV. The two-point sum rules for the $B_c$ meson with account for the Coulomb-like corrections give

$$\alpha_s^c(\bar{b}c) = 0.45,$$

which corresponds to $f_{B_c} = 400$ MeV [17]. The quark masses are fixed by the calculations of leptonic constants $f_\Psi$ and $f_\Gamma$ in the same order over $\alpha_s$. The requirement of stability in the sum rules including the contributions of higher excitations, results in quite an accurate determination of masses $m_c = 1.40 \pm 0.03$ GeV and $m_b = 4.60 \pm 0.02$ GeV, which are in a good agreement with the recent estimates in [18], where the quark masses free off a renormalon ambiguity were introduced.

The leptonic constant for the $B_s$ meson is extracted from the two-point sum rules. The Borel improved sum rules for the $B$ meson leptonic constant [19] have the following form:

$$f_B^2 M_B e^{\Lambda^\text{ren} (\mu) \tau} = K^2 \frac{3}{\pi^2} C(\mu) \int_0^\infty d\omega \, \omega^2 e^{-\omega\tau}$$

$$+ \langle \bar{q}q \rangle (1 - \frac{m_b^2}{16} + \frac{\pi^2\tau^4}{288} \frac{\alpha_s}{\pi} G^2),$$

where the K-factor is due to $\alpha_s$-corrections [19]. We find that NLO corrections to the leptonic constant are about 40%. Using the Padé approximation, we find that higher orders corrections can be about 30%. So, we hold the K factor in conservative limits $1.4 \div 1.7$. It is quite reasonable to suppose its cancellation in evaluating the semileptonic form factors due to the renormalization of heavy-light vertex in the triangle diagram.

In the limit of semi-local duality [20, 21] $\tau \to 0$ we get the relation: $\Lambda(\mu) = \frac{3}{4} \omega_0(\mu)$. We introduce the renormalization invariant quantities

$$\omega_{\text{ren},\text{dual}} = C^{-1/3}(\mu) \omega_0(\mu), \quad \Lambda_{\text{ren},\text{dual}} = \frac{3}{4} \omega_{\text{ren},\text{dual}}^\text{ren}.$$ For $\Lambda_{\text{ren},\text{dual}}$ we have $\Lambda_{\text{ren}} = M_B - m_b = 0.63$ GeV, and we obtain that in the semi-local duality the threshold energy $\omega_{\text{ren},\text{dual}} = 0.84$ GeV. Neglecting the quark condensate term in the leptonic constant we have $f_B^2 M_B = K^2 \frac{3}{\pi^2} (\omega_{\text{ren},\text{dual}}^\text{ren})^3$. In the general Borel scheme for $f_B$ we have to consider the stability at $\tau \neq 0$ with the extended region of resonance contribution. We expect, that the sum rules with the redefined $\omega_{\text{ren}}$ and $\Lambda_{\text{ren}}$ have a stability point at $\tau = \frac{1}{2}$. The results are in a good agreement with the semi-local duality if the threshold energy of continuum equals $E_c = 1.1 \div 1.3$ GeV (see Fig. 2, where the overall K-factor was ignored). So, we find the $E_\omega^{1/2}$-dependence of $f_B \sqrt{M_B}$, whereas the contribution of condensate is numerically suppressed, as expected from the semi-local duality. Multiplying the result taken from Fig. 2 by the K-factor we find the value $f_B = 140 \div 170$ MeV, which is in a good agreement with the recent lattice results [22] and the estimates in the QCD SR by other authors [23].

For the vector $B^*$ meson constant $f_{B^*}$ we put $\frac{\Lambda_{\text{ren}}}{f_{B^*}} = 1.11$ (see [24, 23]). For the leptonic...
constant of $B_s$ meson we explore the following relation $f_{B_s}/f_B = 1.16$, which expresses the flavor SU(3)-symmetry violation for B mesons [21].

We have investigated the dependence of form factors on the $\bar{b}s$ threshold energy of continuum in the range $E_c = 1.1 \div 1.3$ GeV, so that the optimal choice for the $\bar{b}s$ system threshold energy is 1.2 GeV. In Table 1 we present the results of sum rules for the form factors. Comparing with the estimates in the framework of potential models [8, 25], we find a good agreement of estimates in the QCD sum rules with the values in the quark model.

| mode       | $\Gamma$, $10^{-14}$ GeV | BR, % |
|------------|--------------------------|-------|
| $B_s e^+ \nu_e$ | 5.8                     | 4.0   |
| $B_s^* e^+ \nu_e$ | 7.2                     | 5.0   |

Table 2: The widths of semileptonic $B_c$ decay modes and the branching fractions calculated at $\tau_{B_c} = 0.46$ ps.

4. The symmetry relations

At the recoil momentum close to zero, the heavy quarks in both the initial and final states have small relative velocities inside the hadrons, so that the dynamics of heavy quarks is essentially nonrelativistic. This allows us to use the combined NRQCD/HQET approximation in the study of mesonic form factors. The expansion in the small relative velocities leads to various relations between the form factors due to the spin symmetry of effective lagrangians to the leading order. Solving these relations results in the introduction of an universal form factor (an analogue of the Isgur-Wise function) at $q^2 \to q^2_{\text{max}}$. 

\[ f_{B}\sqrt{M}, \text{GeV}^{3/2} \]

\[ \bar{\Lambda}, \text{GeV} \]

Figure 2: The leptonic constant of B meson and the b-quark binding energy in the semi-local duality sum rules (dashed curve) and in the general Borel scheme (solid line) with the corrected value of $\bar{\Lambda}$, which improves the stability of result obtained in the semi-local duality.
We have derived the symmetry relations for the following form factors:

\[ f_+(e_1^P \cdot M_2 - e_2^P \cdot M_1) - f_-(e_1^P \cdot M_2 + e_2^P \cdot M_1) = 0, \]

\[ F_0^A \cdot C_V - 2c_e \cdot F_V M_1 M_2 = 0, \]  

(4.1)

\[ F_0^A P + c_e \cdot M_1 (f_+ + f_-) = 0, \]

where \( M_1 = m_c + m_b, M_2 = m_s + m_b, \) and

\[
\begin{align*}
  c_e &= -2, \\
  c_V &= -1 - \tilde{B} - \frac{m_b}{2m_c}, \\
  c_1^P &= 1 - \tilde{B} + \frac{m_b}{2m_c}, \quad (4.2) \\
  c_2^P &= 1 + \tilde{B} - \frac{m_b}{2m_c}. 
\end{align*}
\]

Equating the second relation in (4.1), for example, we obtain

\[ \tilde{B} = \frac{-2m_c + m_b + 4m_b(m_c + m_b) F_V}{F_0} \approx 10.0, \]

where all form factors are taken at \( q^2_{\text{max}}. \) Substituting \( \tilde{B} \) in first and third relations, we get \( f_+ \approx 2.0 \) and \( f_- \approx -8.3. \) These values have to be compared with the corresponding form factors obtained in the QCD sum rules: \( f_+(q^2_{\text{max}}) = 1.8 \) and \( f_-(q^2_{\text{max}}) = -8.1, \) where we suppose the pole like behaviour of form factors. Thus, we find that in the QCD sum rules, relations (4.1) are valid with the accuracy better than 10% at \( q^2 = q^2_{\text{max}}. \) The deviation could increase at \( q^2 < q^2_{\text{max}} \) because of variations in the pole masses governing the evolution of form factors. However, in \( B_c^+ \to B_s^{(*)} \eta' \nu \) decays the phase space is restricted, so that the changes of form factors are about 50%, while their ratios develop more slowly.

5. Nonleptonic decays and the lifetime

The hadronic decay widths can be obtained on the basis of assumption on the factorization for the weak transition between the quarkonia and the final two-body hadronic states. For the nonleptonic decay modes the effective Hamiltonian can be written down as

\[ H_{\text{eff}} = \frac{G_F}{2\sqrt{2}} V_{cs} V_{ud}^* \{ C_+ (\mu) O_+ + C_- (\mu) O_- \}, \]

where \( O_+ = (\bar{u} \gamma_{\mu} (1 - \gamma_5) d_i)(\bar{s}_j \gamma^\nu (1 - \gamma_5) c_j) \pm (\bar{u} \gamma_{\mu} (1 - \gamma_5) d_i)(\bar{s}_j \gamma^\nu (1 - \gamma_5) c_j), \) and the factors \( C_\pm (\mu) \) account for the strong corrections to the corresponding four-fermion operators caused by hard gluons. The review on the evaluation of \( C_\pm (\mu) \) can be found in [2]. The results are collected in Table 3.

| mode     | \( \Gamma, 10^{-14} \text{ GeV} \) | BR, % |
|----------|-----------------------------------|-------|
| \( B_c \pi^+ \) | 15.8 \( a_1^2 \) | 17.5  |
| \( B_c \rho^+ \) | 6.7 \( a_1^2 \) | 7.4   |
| \( B_c^* \pi^+ \) | 6.2 \( a_1^2 \) | 6.9   |
| \( B_c^* \rho^+ \) | 20.0 \( a_1^2 \) | 22.2  |

Table 3: The widths of dominant nonleptonic \( B_c \) decay modes due to \( c \to s \) transition and the branching fractions calculated at \( \tau_{B_c} = 0.46 \) ps. We put \( a_1 = 1.26. \)

In the parton approximation we could expect \( \Gamma [B_c^+ \to B_s^{(*)} + \text{light hadrons}] = (2C_2^2 (\mu) + C_2^2 (\mu) \Gamma [B_c^+ \to B_s^{(*)} e^+ \nu_e], \) which results in the estimate very close to the value obtained as the sum of exclusive modes at \( \mu > 0.9 \) GeV. The deviation between these two estimates slightly increase at \( \frac{m_d^2}{2} < \mu < 0.9 \) GeV. Concerning the comparison of hadronic width summing up the exclusive decay modes with the estimate based on the quark-hadron duality, we insist that the deviation between these two estimates is unessential since it is less than 10%.

We estimate the lifetime using the fact that the dominant modes of the \( B_c \) meson decays are the \( c \to s, b \to c \) transitions with the \( B_s^{(*)} \) and \( J/\psi, \eta_c \) final states respectively, and the electroweak annihilation.

The method for the calculation of multi-particle branching fractions was offered by Bjorken in his pioneering paper devoted to the decays of hadrons containing heavy quarks [27]. In order to estimate the contribution of non-resonant \( 3\pi \) modes of \( B_c \) decays into \( B_s^{(*)} \) we use this technique, i.e. the Poisson distribution with the average value corrected to agree with the non-resonant \( 3\pi \)-modes in the decays of \( D \) mesons. We have found \( \text{BR}(B_c^+ \to B_s^{(*)} (3\pi)^+) \approx 0.2, \)

\footnote{The \( \bar{b} \to \bar{b} c \bar{s} \) transition is negligibly small in the \( B_c \) decays because of destructive Pauli interference for the charged quark in the initial state and the product of decay.}
while $\text{BR}(B_c^+ \rightarrow B_s^+(2\pi)^+ |_{\text{non-resonant}}) \approx 3\%$. We see that the neglected modes contribute to the total width of $B_c$ as a small fraction in the limits of uncertainty involved.

We have investigated the semileptonic decays of $B_c$ meson due to the weak decays of charmed quark in the framework of three-point sum rules in QCD. We have pointed out the important role played by the Coulomb-like $\alpha_s/v$-corrections. As in the case of two-point sum rules, the form factors are about three times enhanced due to the Coulomb renormalization of quark-meson vertex for the heavy quarkonium $B_c$. We have studied the dependence of form factors on the threshold energy, which determines the continuum region of $\bar{b}s$ system. The obtained dependence has the stability region, serving as the test of convergence for the sum rule method. The HQET two-point sum rules for the leptonic constant $f_{B_s}$ and $f_{B_s'}$ have been reanalyzed to introduce the term caused by the product of quark and gluon condensates. This contribution essentially improves the stability of SR results for the leptonic constants of $B$ mesons, yielding: $f_B = 140 \div 170$ MeV.

We have studied the soft limit for the form factors in combined HQET/NRQCD technique at the recoil momentum close to zero, which allows us to derive the generalized relations due to the spin symmetry of effective lagrangian. The relations are in a good agreement with the full QCD results, which means that the corrections to the form factors in both relative velocity of heavy quarks inside the $\bar{b}c$ quarkonium and the inverse heavy quark masses are small within the accuracy of the method.

Next, we have studied the nonleptonic decays, using the assumption on the factorization of the weak transition. The results on the widths and branching fractions for various decay modes of $B_c$ are collected in Tables.

Finally, we have estimated the $B_c$ meson lifetime, and showed the dependence on the scale for the hadronic weak lagrangian in decays of charmed quark $\tau(B_c) = 0.48 \pm 0.05$ ps. Our estimates are in a good agreement with the theoretical predictions for the lifetime in both the

6. Conclusion

We have investigated the semileptonic decays of $B_c$ meson due to the weak decays of charmed quark in the framework of three-point sum rules in QCD. We have pointed out the important role played by the Coulomb-like $\alpha_s/v$-corrections. As in the case of two-point sum rules, the form factors are about three times enhanced due to the Coulomb renormalization of quark-meson vertex for the heavy quarkonium $B_c$. We have studied the dependence of form factors on the threshold energy, which determines the continuum region of $\bar{b}s$ system. The obtained dependence has the stability region, serving as the test of convergence for the sum rule method. The HQET two-point sum rules for the leptonic constant $f_{B_s}$ and $f_{B_s'}$ have been reanalyzed to introduce the term caused by the product of quark and gluon condensates. This contribution essentially improves the stability of SR results for the leptonic constants of $B$ mesons, yielding: $f_B = 140 \div 170$ MeV.

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potential models and OPE as well as with the experimental data.

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