MODELLING BOLTED JOINTS USING A SIMPLIFIED BOLT MODEL

The paper presents a part of research on bolted connections based on the system approach to the problem of their modelling and calculations. With this approach it is possible to individual consideration of each system element in order to find the best model of the element. The aim of this study is to develop a model of the single-bolted joint separated from the bolted connection. An analysis is conducted for the spider bolt model which is an equivalent model corresponding to the spatial bolt model. The effect of preload distribution in the spider bolt model on the stiffness value of the flange element fastened to a rigid support has been examined. The result of research is a proposal of the way of simplified bolt model preloading, which gives the best matching this model to the spatial bolt model.

Keywords: bolted joint, preloaded bolt model, screw

1. INTRODUCTION

The fundamental task of the machine modelling phase is to find a compromise between the level of its model simplification and the expected accuracy of its modelling. This is particularly meaningful for modelling systems with many different elements being in a contact [Zimmerman and Śnieżek 2009, Restivo, Marannano and Isaicu 2010, Molnár et al. 2014, Szulc, Malujda and Talaśka 2015].

An example of the multi-complex systems are bolted connections. In practice, the following types of such joints are most used:

− bolted angle connections [Wang and Menzemer 2005, Daryan, Ziaei and Sadrnejad 2011, Liu, Tan and Fung 2015],

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bolted end-plate connections [Diaz et al. 2011, Saberi, Gerami and Kheyrod-din 2014, Yang and Eatherton 2014],
- bolted flange connections [Radhakrishnan et al. 2014, Henriksen et al. 2015, Mourya, Banerjee and Sreedhar 2015].

Fig. 1. Example of a bolted flange connection

The bolted connection can be considered as a system composed of subsystems which are the elements of this connection [Grzejda 2015]. In the case of the bolted flange connection (Fig. 1), these subsystems include: the bolts (subsystem $B$), the flange element (subsystem $F$) and the contact layer between the flange element and the rigid support (subsystem $C$). Then, the equation of equilibrium of the system can be written as:

$$\begin{bmatrix}
K_{BB} & K_{BF} & 0 \\
K_{FB} & K_{FF} & K_{FC} \\
0 & K_{CF} & K_{CC}
\end{bmatrix} \cdot \mathbf{q} = \mathbf{p}$$

(1)

where:
$K_{BB}, K_{FF}, K_{CC}$ – stiffness matrices of subsystems $B$, $F$, $C$,
$K_{BF}, K_{FB}, K_{FC}, K_{CF}$ – matrices of elastic couplings among subsystems $B$, $F$, $C$,
$q$ – displacement vector,
$p$ – load vector.

Due to the system approach to the problem, each of the bolted connection subsystems can be developed, studied and modeled independently using for this purpose various modelling methods. Thus, the components of the stiffness matrix of the system may take different forms depending on the adopted modelling method. The object of research in this work is the subsystem of bolts, as a part of the bolted flange connection defined above.
The most common method of modelling and simulation of complex structures is currently the finite element method (FEM) [Wunderlich and Pilkey 2003]. In the works [Montgomery 2002, Kim, Yoon and Kang 2007, Grzejda 2014] several different FEM-based models of bolts are presented, which can be applied for modelling bolted connections. Among them are:  
- models without explicit occurrence of bolts, but with the influence of the preload,  
- plain models,  
- beam models,  
- spatial models.  

The best accurate results of modelling bolted connections can be obtained applying spatial models [Wang et al. 2013]. In some cases, it is worth to use simplified models of bolts and bolted joints which are substitutes for reference spatial models [Grzejda 2014]. In the present study, the spider bolt model (named as the SB model) is assumed as the model of the bolt. This is an equivalent model corresponding to the reference spatial bolt model (named as the 3DB model). As a criterion to carry out a comparative analysis of the bolt models compliance of stiffness of elements combined in the bolted joint is selected.

2. MODELS OF THE BOLTED JOINT

The tests were performed on the example of the bolted flange connection shown schematically in Fig. 1.

The considered joint consists of a deformable flange element fastened to a rigid support with a single bold M10 made in the mechanical property class 10.9. Thickness of the flange element $h$ is equal to 30 mm. The preload of the bolt $F_m$ is equal to 17.2 kN and it was set down based on [PN-EN 1591-1]. The total surface area of preload acting $A_m$ is equal to $69.75\pi$ mm$^2$ and it was set down on the base of [PN-EN ISO 7091].

For the construction of discrete models the following standard finite elements have been used:  
- spatial elements, in the case of the joined elements and the 3DB model of the bolt,  
- beam elements, in the case of the SB model of the bolt.

The spider bolt model is formed of two parts. Both the plain part of the bolt and its head are modeled with use of beam elements but the total volume of beam elements for the head is assumed to be equal to the volume of the head of the bolt in the 3DB model. Between the flange element and the support (in the 3DB model and in the SB model) and between the bolt and the flange element (in the 3DB model) standard contact elements are introduced. Developed discrete models of the bolted joint are presented in Fig. 2.
Methods of load models are shown in Fig. 3. In this figure the following new designations are used:

- $F_h$ – part of the preload $F_m$ attached to the head of the bolt [kN],
- $F_p$ – part of the preload $F_m$ attached to the plain part of the bolt [kN].

Between $F_h$, $F_p$ and $F_m$ the following dependences are true:

$$F_m = F_h + F_p$$  \hspace{1cm} (2)
Modelling bolted joints using a simplified bolt model

\[ F_h = \alpha \cdot F_m \]  
\[ F_p = \beta \cdot F_m \]

where:
\( \alpha \) – constant of load proportionality for the head of the bolt,
\( \beta \) – constant of load proportionality for the plain part of the bolt,

wherein:
\[ \alpha + \beta = 1 \]

3. RESULTS OF CALCULATIONS

Treating the bolt as a linear element, its stiffness can be easily and correctly determined with use of the Hooke's law. Such a proceeding is well known and widely used [Grudziński 2014, Pedersen and Pedersen 2008, Williams et al. 2009]. According to this law, stiffness of the bolt \( k_b \) can be designated from the relation:

\[ k_b = \frac{E \cdot A}{l} \]

where:
\( E \) – modulus of elasticity [MPa],
\( A \) – cross-sectional area of the bolt [mm\(^2\)],
\( l \) – length of the bolt [mm].

| \( \alpha \) | \( \beta \) | \( k_{f,SB} \) [MN/mm] | \( k_{f,1DB} \) [MN/mm] |
|---|---|---|---|
| 1.0 | 0 | 2.15 | |
| 0.9 | 0.1 | 2.39 | |
| 0.85 | 0.15 | 2.53 | |
| 0.8 | 0.2 | 2.68 | 2.89 |
| 0.75 | 0.25 | 2.86 | |
| 0.7 | 0.3 | 3.06 | |
| 0.6 | 0.4 | 3.57 | |

There is no simple formulas for calculating stiffness of elements connected in the bolted joint. At present, to determine it the most frequently the finite element method is used. Then, stiffness of the joined flange element \( k_{f,j} \) can be defined based on the relationship:
\[ k_{f,j} = \frac{F_m}{\delta_{sum}} \]  

(7)

where:
\( \delta_{sum} \) – average normal displacement of nodes lying in the total surface area \( A_m \), under the action of forces \( F_m \) [mm],
\( j \) – symbol of the bolted joint model, \( j \in \{ SB, 3DB \} \).

The stiffness values of the joined flange element for both models are reported in Tab. 1. The relative difference between the \( k_{f,SB} \) and \( k_{f,3DB} \) values can be analyzed based on the \( W \) index:

\[ W = \frac{k_{f,SB} - k_{f,3DB}}{k_{f,3DB}} \times 100 \]  

(8)

Table 2

| \( \alpha \) | \( \beta \) | \( W \) [%] |
|---|---|---|
| 1.0 | 0 | -25.57 |
| 0.9 | 0.1 | -17.38 |
| 0.85 | 0.15 | -12.55 |
| 0.8 | 0.2 | -7.11 |
| 0.75 | 0.25 | -0.95 |
| 0.7 | 0.3 | 6.10 |
| 0.6 | 0.4 | 23.75 |

Fig. 4. Distribution of \( W \) index values as a function of the bolt load
Calculated $W$ index values are given in Tab. 2. They are presented also as a graph shown in Fig. 4. Based on these results it can be concluded that the spatial bolt model in the best way may be replaced by the spider bolt model, when:

$$\frac{\alpha}{\beta} \geq 3 \quad (9)$$

Additional quantitative comparison of the used bolted joint models is presented in Tab. 3.

### Table 3
Quantitative comparison of the bolted joint models

| Parameter                          | Bolted joint with the SB model | Bolted joint with the 3DB model |
|------------------------------------|--------------------------------|--------------------------------|
| Number of nodes                    | 12 755                         | 14 828                         |
| Number of elements                 | 11 553                         | 14 012                         |
| Number of degrees of freedom       | 40 665                         | 46 881                         |
| Number of equations                | 39 120                         | 45 336                         |
| CPU time [s]                       | 46                             | 245                             |

## 4. CONCLUSIONS

In the paper an analysis of the single-bolted joint separated from the bolted connection with use of the finite element method is presented. In considerations two types of the joint were adopted: the bolted joint with the simplified bolt model modeled using beam elements (named as the spider bolt model) and the bolted joint with the reference bolt model modeled using spatial elements. It has been shown that for the stiffness analysis of elements combined in the bolted joint the spider bolt model can be successfully applied as a substitute model for the spatial model by adopting the adequate preload distribution. It should be noted that in a case of the SB model using, it is not possible to test the contact joint between the bolt head and the flange element.

In order to generalize the achieved results additional tests should be performed. They could for example concern a study the impact of the used material or the size of the joint.

A comparative analysis is carried out on the basis of stiffness of the joined elements. It is however also possible to conduct a similar analysis according to the strength criterion of these elements.
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Modelling bolted joints using a simplified bolt model

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MODELOWANIE ZŁĄCZY ŚRUBOWYCH ZA POMOCĄ UPROSZCZONEGO MODELU ŚRUBY

Streszczenie

W pracy przedstawiono część badań nad połączeniami śrubowymi bazującymi na systemowym podejściu do zagadnienia ich modelowania i obliczeń. Przy tym podejściu możliwe jest indywidualne rozpatrywanie każdego z elementów systemu w celu określenia jego najlepszego modelu. Celem pracy jest rozwój modelu pojedynczego złącza śrubowego wydzielonego z połączenia śrubowego. Analizę przeprowadzono dla modelu typu „spider bolt”, który jest modelem zastępczym dla odpowiadającego mu modelu przestrzennego. Zbadano wpływ rozmieszczenia napięcia wstępnego w modelu typu „spider bolt” na wartość sztywności elementu kołnierzowego łączonego z nieokrągłą ostoją. Wynikiem prac jest propozycja sposobu napinania uproszczonego modelu śruby, dzięki któremu uzyskuje się jego najlepsze dopasowanie do przestrzennego modelu śruby.

Słowa kluczowe: połączenie śrubowe, model śruby z napięciem wstępnym, śruba
