Method of calculating the shear stiffness of the flooring under the action of horizontal loads

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Abstract. This article deals with the issue of calculating combined plywood-metal flooring as a horizontal disk of stiffness, taking into account the structural non-linearity. An analytically obtained method calculating the shear stiffness of the flooring under the action of horizontal loads is proposed.

The task was to derive a three-line correlation - relationship for the fictitious connection operation, based on the constructively nonlinear operation of the flooring. Based on the proposed analytical method of calculating the shear stiffness of the flooring under the influence of horizontal loads, the work of the flooring is modeled as part of the design scheme shown in the figure. The resulting diagram of fictitious relationships was introduced as a characteristic of nonlinear work. In manual calculation in the derivation of the analytical formulas, the compound was considered as a pure hinge. We can conclude that the proposed analytical method of calculating the shear stiffness of the flooring when it is working on a horizontal load, taking into account the structural non-linearity.

Keywords: stiffness, modular collapsible structures, characteristic of nonlinear work, work of the flooring, horizontal loads, finite element method.

1 Introduction

In the system of modular collapsible structures based on wedge-type scaffolding, different types of flooring are used:

1. Boardwalk (at least 40 mm thick);
2. Steel flooring of complex cross section made of thin sheet metal with a perforated bearing surface in various sizes and types;
3. Aluminum flooring (similar to steel) - for lightweight construction;
4. Combined plywood-metal flooring consisting of a load-bearing steel frame, a support surface made of multi-layer bakelized water-resistant plywood and steel stiffeners [1-5].

It should be noted that decking (flooring) of type 1 has some degree of shear stiffness, although not so significant [6-7]. The problem is that, based on the conditions of the support of this flooring on the supporting structures of the scaffolding, it is not able to join in the spatial work of the entire structure. In addition, this type of flooring is currently replaced by other more advanced types (2-4) and is not used [8-12].

Next, we will consider the issue of taking into account the shear stiffness for decking types 2-4, which are widely used at present in structures [13-15].

Flooring of type 4 is mainly used in the stands, podiums and stages. This type of flooring was used at the Kazanka facility for the 2015 world aquatics Championships in Kazan [16-19].

It is assumed that the strut nodes that are not anchored to the facade (in the case of facade scaffolding) can be elastically supported by the scaffolding floor due to its shear stiffness in the horizontal direction perpendicular to the facade. This elastic "support" should be taken into account between the nodes attached to the facade and represents essential supports for sections of flooring that work in a horizontal plane [20-21].

According to the norms in force in our country, scaffolding racks are calculated with the estimated length between the points of attachment to the facade of the building (as a rule, fixing to the facade on...
50 m$^2$ of area). That is, no effect from the flooring is taken into account. The situation is similar with foreign standards.

2 Materials and methods

The article considers the results of field tests of flooring for the impact of horizontal loads in two mutually perpendicular directions.

As a result of cyclic loading, a force-displacement diagram is obtained, characterized by the development of hysteresis loops.

![Figure 1](image1.png)

**Figure 1.** Force-displacement diagram based on test results of flooring on the shear.

On the basis of experiments two characteristic features of the flooring operation were established:

1. The work of the flooring on the shift in two mutually perpendicular directions is significantly different and should be taken into account separately and independently;

2. It is difficult to determine the reason for the failure of the flooring. However, in most tests, it comes down to the welds of the flooring frame, specifically the near-seam zones of the material that are subject to thermal effects during welding.

Averaged values of the stiffness of imaginary diagonal deformable elements are obtained when considering a mechanical model of a structure with several boards of flooring. It is worth noting that these dependencies are structurally linear.

![Figure 2](image2.png)

**Figure 2.** Mechanical model of the construction in question.

All the above calculations regarding the shear stiffness of the flooring can be applied to free-standing structures based on modular scaffolding (stands, towers, walls, etc.), which include flooring.

However, depending on the structural schemes of facilities, the role of supports for ensuring the work of the flooring on the shift can be played by "free" cross frames (frames with connecting
diagonals). The rest of the frames ("non-free"), located between the "free" frames, will be elastically loosened in their plane from the action of horizontal loads due to the work of the flooring on the shift. Thus, we can count on the spatial operation of the entire frame and, as a result, on reducing the metal content on the diagonal elements of "non-free" frames or increasing the step of "free frames".

3 Results

Let's consider a flat system of three transverse frames of width B, installed with a standard step L. We consider the case of the flooring working in the direction of its least shear stiffness - in the direction parallel to the plane of the transverse frames of the system’s framework.

![Figure 3. Calculation scheme for determining the rigidity of "fictitious" links.](image)

It is known that the cross frames consist of two standard racks and crossbars, which generally support the load-bearing flooring. The frames are connected to each other by longitudinal crossbars (beams) that play the role of longitudinal struts in the system. The connection of such beam to the post when turning the beam in the plane of the flooring works as a conditional hinge with a finite stiffness.

Characteristics of elastic supports, for modeling the work of the flooring in the form of a three-line diagram "force-displacement" are given.

![Figure 4. Three-line diagram of pliability of elastic supports.](image)

The first section of the diagram is characterized by slippage ("slack") – a kind of gap in the work of the flooring on a horizontal load.
Further, to simplify the output of formulas, we assume that the connection of beams with posts in the plane of the flooring is hinged, that is, in fact, we consider a hinge-supported truss of two panels.

We consider a cross lattice of “fictitious” connections, since the flooring can work for a shift in both directions along the orientation of the cross frames. However, only two connections are taken into account, which only work for tension when a horizontal load is applied in a particular direction (flexible compressed connections lose their stability and do not participate in the work). Longitudinal deformations of beams are neglected due to their greater rigidity in comparison with fictitious connections.

The problem is to deduce a three-line relationship for the fictitious link operation, based on the constructively nonlinear operation of the flooring. In the future, the resulting diagram of the fictitious connection can be entered into any software package that implements the finite element method.

For small rotation angles $\varphi$ (since the displacements $f$ are also small compared to $L, B$) we can assume that

$$\Delta l = l - l_0 = f \times \cos \alpha$$

where $f$ is the relative displacement of the “non-free” frame relative to free frames (deflection of the truss).

Then the relative longitudinal strain of the fictitious connection corresponding to the frame movement $f$ will be determined by the formula

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{f \times \cos \alpha}{l_0} = \frac{f \cdot B}{l_0^2}$$

The stresses in the cross section of the fictitious connection are equal to

$$\sigma = \frac{N_{\text{con}}}{A_{\text{con}}}$$

where $N_{\text{con}}$ is the force in the dummy link; $A_{\text{con}}$ is the cross-sectional area of the fictitious link.

The force that occurs in a fictitious connection from the load is determined using the cross-section method.
Equation of moments relative to a point B ($\sum M_B = 0$):

$$R_A \cdot L + N_1 \cdot B = 0$$

$$N_1 = -\frac{R_A \cdot L}{B} = -\frac{P \cdot L}{2 \cdot B}$$

The sum of all forces on the axis X ($\sum X = 0$):

$$N_2 \cdot \sin \alpha + N_1 = 0$$

$$N_2 = -\frac{N_1}{\sin \alpha} = \frac{P \cdot L}{2 \cdot B \cdot \sin \alpha} = \frac{P \cdot l_0}{2 \cdot B}$$

Thus, the stresses in the fictitious link section are defined as

$$\sigma = \frac{P \cdot l_0}{2 \cdot B \cdot A_{con}}$$

Then we create a diagram of operation of a fictitious connection $\sigma - \varepsilon$ to count the shift stiffness plywood flooring with a width of 0.61 m and length $L = 2.57$ m, installed on scaffolding with the passage width of $B = 0.73$ m. According to the figure, the shear stiffness is the derivative (tangent of the angle of inclination of a straight line to the axis of displacement $f$) of the first order function of the applied horizontal load $P$ from displacements and is determined from the expression:

$$C_{i,d} = \frac{P_i - P_{i-1}}{f_i - f_{i-1}}, i = 1...2.$$  (9)

From here we get an expression for displacement $f$

$$f_i = f_{i-1} + \frac{P_i - P_{i-1}}{C_{i,d}}, i = 1...2.$$  (10)

The relative deformations will be equal to

$$\varepsilon_j = \frac{f_i \cdot B}{l_0^2}, j = 1...3, i = 1...2.$$  (11)
The elastic modulus of a fictitious connection is defined from the expression

\[ E_i = \frac{\sigma_{i+1} - \sigma_i}{\varepsilon_{i+1} - \varepsilon_i} = \frac{l_0^3 \cdot (P_i - P_{i-1})}{2 \cdot B^2 \cdot A \cdot (f_i - f_{i-1})}, i = 1 \ldots 2. \tag{12} \]

Knowing, that

\[ P_0 = 0 \text{ kg}; \]
\[ P_1 = 2.27 \text{ kN} = 231.397 \text{ kg}; \]
\[ P_2 = P_{R,d} = 2.91 \text{ kN} = 296.636 \text{ kg}; \]
\[ C_0 = 0 \text{ kg/cm}; \]
\[ C_1 = 0.58 \text{ kN/cm} = 59.123 \text{ kg/cm}; \]
\[ C_2 = C_{R,d} = 0.30 \text{ kN/cm} = 30.581 \text{ kg/cm}, \]

We get the original diagram of the flooring work on shear and the diagram of operation of a fictitious connection when taking into account structural nonlinearity.

![Figure 7. Diagram of the work of the flooring on the shear.](image-url)
3.1 Modeling of the work of the flooring on the shear in the PC LIRA-CAD

Based on the proposed analytical method of calculating the shear stiffness of the flooring under the influence of horizontal loads, we will simulate the operation of the flooring as part of the calculation scheme shown in the figure (see Figure 9). The resulting diagram of fictitious relationships is introduced as a characteristic of the nonlinear operation of FE201.

![Figure 8](image1)

**Figure 8.** Diagram of the work of a fictitious connection on tensile.

![Figure 9](image2)

**Figure 9.** Calculation scheme in the LIRA - CAD PC.

![Figure 10](image3)

**Figure 10.** FE201 work diagram in LIRA-CAD PC.
Data on stress and strain values were obtained using analytical formulas. However, giving the specifics of the LIRA-CAD algorithm, it is impossible to enter the initial fully horizontal section of the diagram (the section with a gap of \( f_0 \)). Therefore, the stress values in the cross-section of the fictitious connection for deformations corresponding to the gap of 4.9 cm are slightly overestimated in comparison with the zero value and are 0.0265 kg/cm\(^2\). This value is obtained by successive approximations, taking into account the specifics of the algorithm of the LIRA – CAD PC operation and the minimum error.

The comparison of the obtained results of calculations using the analytical (theoretical) method and the results of the FEM calculations is shown in the figure 11.

![Diagram comparison f – P.](image)

**Figure 11.** Diagram comparison f – P.

| P, kg | \( F \) (theor), cm | \( f \) (FEM), cm | Inaccuracy |
|-------|---------------------|------------------|------------|
| 30    | 5.4074138           | 5.05004          | 7 %*       |
| 231.397 | 8.813793267         | 8.58829          | 3 %        |
| 296.636 | 10.94712686         | 10.4784          | 4 %        |

Inaccuracies in the first section of the flooring (fictitious connections), as indicated earlier, are not taken into account. Inaccuracies in the values of movement of the “non-free” frame on the second and third sections of the structurally nonlinear work of the flooring on the shear do not exceed 5 %.

Inaccuracies in the values of forces and displacements are due to the fact that when calculating in the LIRA – CAD PC, the connection of the beam with the post was modeled as a connection of finite stiffness. In manual calculation in the derivation of the analytical formulas, the connection was considered as a pure hinge. Therefore, when calculating the FEM, there was a redistribution of forces in the elements of the design scheme and reduced movement of the “non-free” frame (beams were included in the work).
Thus, we can make conclusion about the proposed analytical method of calculating the shear stiffness of the flooring when it is working on a horizontal load, taking into account the structural non-linearity.

4 Discussions

A brief analysis of existing theoretical methods for calculating the influence of shear stiffness of flooring on the spatial work of the frame was carried out. A method of calculating the shear stiffness of the flooring when it works on a horizontal load is proposed by introducing fictitious connections with the cross grid and their operation, taking into account the structural non-linearity of the flooring.

Based on the data on the shear stiffness of combined plywood – metal flooring, a three-line diagram of fictitious connections is obtained analytically.

The resulting diagram is introduced into the LIRA – CAD software package for modeling the actual operation of combined flooring. Thus, the proposed analytical method was verified by comparing the numerical values of forces in fictitious connections and movements of a “non-free” frame. The inaccuracies in the displacement values did not exceed 5%.

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