Inter-layer modified $d_{y^2 - z^2}$-wave superconductivity in an effectively doped spin-1 ladder

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Abstract

We construct a four-leg spin-1/2 $t$–$J$ type model to simulate a doped two-leg spin-1 antiferromagnetic Heisenberg ladder. Employing renormalized mean-field theory with simple Gutzwiller factors, we obtain three degenerate superconducting states with different pairing symmetry. Through improving the Gutzwiller factors, we find that the state C with inter-layer modified $d_{y^2 - z^2}$-wave pairing has the lowest energy in a large doping range. Besides, we use the density matrix renormalization group method to solve the model. The negative binding energy reveals the pairing tendency, and the pair--pair correlation functions exhibit a slowly decaying behavior on certain types of bonds. From the pair correlations, we confirm the inter-layer modified $d_{y^2 - z^2}$-wave superconducting state as the ground state of the model.

1. Introduction

Doped spin systems have been attracting intense attention since Anderson proposed that high-temperature superconductivity may arise from a doped resonating valence bond (RVB) spin liquid [1, 2]. In general, the undoped spin system is a Mott insulator with localized spin-1/2 magnetic moments at all sites. Various kinds of interactions can generate intriguing states without long-range magnetic order, such as quantum spin liquid states [3]. Doping these magnetic models will give rise to large quantities of exotic states [4, 5], in particular the unconventional superconducting states [2, 6–8]. After charge carriers doping, the system is usually described by the so-called $t$–$J$ model [9], where $t$ term represents the hopping of electrons, and $J$ term includes magnetic interactions. In this model, double occupancy of two electrons on the same site is prohibited from taking account of the strong on-site Coulomb repulsion.

Most of the previous works are focused on doped spin-1/2 systems. However, in real materials, the local magnetic moment can be $S = 1$ [10, 11]. In iron pnictides and chalcogenides, models in which each Fe$^{2+}$ possesses a localized spin-1 have been widely employed considering the existence of Hund’s coupling [12–16]. The recent discovery of superconductivity at high pressure in the two-leg ladder compounds BaFe$_2$X$_3$ ($X = S, Se$) [17, 18] motivates us to explore possible superconducting states in a doped two-leg spin-1 model given that the high pressure may alter the band structure such that the ladder becomes hole-doped. Besides, doped spin-1 systems have their theoretical significance. The ground state of an antiferromagnetic Heisenberg spin-1 chain is a symmetry protected topological phase [19]. Doping this spin chain leads to a symmetry-protected topological Luttinger liquid with $d_{xy}$-wave superconductivity, and it can be understood in the framework of RVB theory [20–22]. The ladder systems can be regarded as a crossover between one and two dimensionalities and are also of great interest [23–25]. The ground state of the undoped spin-1 two-leg ladder is a plaquette singlet solid-state without magnetic order or topological property [26, 27]. It can be seen as a spin liquid because of its purely short-
range spin correlation\textsuperscript{[28]}. Some novel superconducting states are expected to emerge from doping a spin-1 ladder model, and this may shed some light on our understanding of a doped spin liquid.

In this paper, we use a doped four-leg spin-1/2 system to simulate a doped two-leg spin-1 antiferromagnetic Heisenberg ladder and solve the four-leg $t$-$J$ model by both renormalized mean-field theory (RMFT)\textsuperscript{[29–32]} and density matrix renormalization group (DMRG) methods\textsuperscript{[33–35]}. The undoped four-leg spin-1/2 system is an effective model to mimic the spin-1 ladder in the low-energy regime\textsuperscript{[26]}. Here we generalize this mapping to the doped case, where spin-1/2 electrons can hop between nearest neighboring sites constrained by that doubly occupied sites are strictly forbidden. This scheme has been applied to the doped spin-1 chain\textsuperscript{[22]}. With the simplest version of RMFT method, we find three degenerate superconducting ground state with distinct pairing symmetries. An improved version with modified Gutzwiller factors may prefer the inter-layer modified $d_{x^2-y^2}$-wave state as the ground state in a broad range of doping levels. To unambiguously reveal the ground state properties, we perform DMRG calculations and confirm that the inter-layer modified $d_{x^2-y^2}$-wave pairing has a slowly decaying correlation function, which is consistent with RMFT results. The negative binding energy also supports the pairing tendency. Besides, the ground state hosts no edge mode, and there is no double degeneracy of the entanglement spectrum, showing a topological trivial feature. The spin–spin correlation functions exhibit an exponential decay, which indicates the spin sector is gapped. Compared with the spin-1 Heisenberg ladder, doping will induce superconductivity and does not change the property of spin correlations.

2. Model Hamiltonian

The mapping from a two-leg spin-1 ladder to a four-leg spin-1/2 model is proposed in\textsuperscript{[26]} for the convenience of calculations as the low-energy spectra are the same. This mapping may be also a more physical approach as spin-1 is often formed from spin-1/2 by Hund’s coupling or ferromagnetic interactions in many materials. When a spin-1 system is doped, we expect the physical degrees of freedom are still spin-1/2 electrons. Our four-leg $t$-$J$ model is shown in figure 1 and its Hamiltonian is

\begin{equation}
H = -t \sum_{\langle ji \rangle, \alpha, \sigma} (c_{i, \alpha, \sigma}^+ c_{j, \alpha, \sigma} + \text{h.c.}) + J \sum_{\langle ji \rangle, \alpha, \alpha'} S_{\alpha, \sigma} \cdot S_{\alpha', \sigma'}.
\end{equation}

Here the operator $c_{i, \alpha, \sigma}^+$ creates a fermion with spin $\sigma$ at site $i$ on layer $\alpha$. $S_{\alpha, \sigma}$ denotes the spin-1/2 operator. $\alpha = 1$ and 2 represent the front layer and the back layer respectively (the $x$ coordinate of the front layer is larger than the back layer). Each layer is a two-leg ladder system with lattice sites $i = (i_x, i_y), i_x = 1...L_x, i_y = 1, 2$ ($\langle ij \rangle$) in the sum means $i$ and $j$ satisfy $i_x = j_x, i_y = j_y \pm 1$ or $i_y = j_y \pm 1, i_x = j_x$. This model has 4$L_x$ sites and 12$L_x$L_y bonds in total. An important Hilbert space constraint in the model is that there is at most one electron per site.

The undoped magnetic system has no $t$ term. The $J$ term includes intra-layer ($\alpha = \alpha'$, blue bonds in figure 1) and inter-layer ($\alpha = \alpha'$, red bonds in figure 1) antiferromagnetic Heisenberg interactions. In each oval, the two spin-1/2 with the same $i$ and different $\alpha$ form an effective $S = 1$ spin, and the whole system is a two-leg spin-1 antiferromagnetic Heisenberg ladder effectively. The leg direction is $y$, and the rung direction is $z$. The triplet part of two coupled spin-1/2 in the oval corresponds to the three basis states of a spin-1, and the singlet part is beyond the Hilbert space of spin-1. As a result, the mapping is theoretically not exact. However, the two spin-1/2 have strong effective ferromagnetic interaction because they both antiferromagnetically interact with the same sites, which makes the probability of the singlet considerably small. We have calculated the spin correlation between two sites in each oval. It is found that the difference between the spin correlation and 0.25 is less than

![Figure 1](image-url)
10^{-10}, which confirms the validity of the mapping from spin-1 to spin-1/2. This undoped spin model has only a short-range magnetic order with a finite spin gap [26].

When the holes are doped into the system, the $t$ term starts to play an important role. We allow the electrons to hop between nearest-neighbor sites at the same layer (blue bonds in figure 1). This kind of hopping is designed in the same spirit of the doped spin-1 chain [22]. We set $t = 1$ as an energy unit and $J = 0.3$ in this paper. The doping level is defined as $\delta = N_h/N$. $N_h$ is the number of holes, and $N = 2 \times L \times 2$ is the number of the total lattice sites.

3. Renormalized mean-field theory

3.1. Method

The strong coupling constraint of no double occupancy constraint is difficult to handle analytically. RMFT is based on the Gutzwiller approximation of the projected Bardeen–Cooper–Schrieffer wavefunction and has been proven to be efficient to obtain low-lying states in $t$–$J$ type models [36–38]. The Gutzwiller renormalization factors $g^t$ and $g^s$ are added before $t$ term and $J$ term to approximate the effects of constraint. The renormalized Hamiltonian with no double occupancy constraint can be written as

$$H = - \sum_{\langle ij \rangle, \alpha, \sigma} g^t_{\alpha, \sigma} t (c^+_{ij, \alpha, \sigma} c_{ij, \alpha, \sigma} + \text{h.c.}) + \sum_{\langle ij \rangle, \alpha, \sigma} g^s_{\alpha, \sigma} 1/2 \Delta_{\alpha, \sigma} S_{\alpha, i} \cdot S_{\alpha, j}.$$

Then we define the pair and bond mean-fields between two sites at layer $\alpha$ and $\alpha'$ by

$$\Delta_{\alpha, \sigma} = \sigma \langle \Psi_0 | c_{\alpha, i, \sigma} c_{\alpha', j, -\sigma} | \Psi_0 \rangle, \quad \chi_{\alpha, i, \sigma} = \langle \Psi_0 | c^+_{\alpha, i, \sigma} c_{\alpha', j, -\sigma} | \Psi_0 \rangle.$$  

Here $\langle \Psi_0 \rangle$ is the unprojected ground state wavefunction of Hamiltonian (2). The mean-field Hamiltonian is written as

$$H_{\text{MF}} = - \sum_{\langle ij \rangle, \alpha, \sigma} g^t_{\alpha, \sigma} t (c^+_{ij, \alpha, \sigma} c_{ij, \alpha, \sigma} + \text{h.c.}) + \frac{3}{4} \sum_{\langle ij \rangle, \alpha, \sigma} \frac{1}{2} \Delta_{\alpha, \sigma} \chi_{\alpha, i, \sigma}^+ \chi_{\alpha', j, -\sigma} + \text{h.c.} - \mu \sum_{\alpha, i, \sigma} c^+_{\alpha, i, \sigma} c_{\alpha, i, \sigma},$$

where $\mu$ is the chemical potential to control the hole density. The calculations are performed by solving the mean-field Bogoliubov de-Gennes (BdG) self-consistently. For a certain chemical potential, first we start from some mean-fields value according to our guesses. Then we solve the BdG equation iteratively. When the values of pair fields, bond fields and hole density $\delta$ change less than $10^{-3}$ between two successive iterations, we obtain a converged solution. The system size is $2 \times 80 \times 2$ and the periodic boundary condition is used in $y$ direction. Here a state with finite local magnetic moments is not taken into account. Therefore, the renormalization factors and the mean-fields $\chi$ and $\Delta$ are spin independent.

3.2. Results with simple Gutzwiller factors

First, we use the simplest version of Gutzwiller factors, which only depends on the doping level of $\delta$ [29]. The factors are given as

$$g^t_{\alpha, \sigma} = \frac{2 \delta}{1 + \delta}, \quad g^s_{\alpha, \sigma} = \frac{4}{(1 + \delta)^2}.$$  

Using these renormalization factors, we obtain three degenerate states shown in figure 2 in a large doping range with different pairing symmetries as candidates for the real ground state. Although we cannot exclude the possibility of the existence of other states satisfying the BdG equation, we believe there is no state with lower energy than these three states. The mean-field solutions are characterized by the values of mean-fields $\chi$ and $\Delta$ at all bonds with Heisenberg interactions $J$. Our solutions are uniform in the $y$ direction and have five types of bonds shown in figure 2(a). Bond 1(2) connects two neighboring sites in $y$($z$) direction at the same layer. Bond 3, 4 and 5 connect two sites, $i$ and $j$ at a different layer. Suppose $i$ is at the front layer and $j$ is at the back layer, then $i_y = j_y + 1$, $i_z = j_z$ at bond 3 and $i_y = j_y + 1$, $i_z = j_z$ at bond 4. Bond 5 satisfies $i_y = j_y$, $i_z = j_z \pm 1$. Each bond has its $\chi$ field and $\Delta$ field. The $\chi$ fields satisfy $\chi_1 > 0$, $\chi_2 > 0$, $\chi_3 = \chi_4 = \chi_5 = 0$ in all of the three states. This feature is easy to understand and similar to the bond fields in an effectively doped spin-1 chain. If the hopping term appears at the bond, the $\chi$ field will get a non-zero value. The pair fields $\Delta$ determine the pairing symmetry and are emphasized in figure 2(b) by the colors. The black color means $\Delta = 0$ at this bond. The red bond has $\Delta > 0$, and the green bond has $\Delta < 0$.

The state A shown in figure 2(a) has $\Delta_3 = - \Delta_4 > 0$, $\Delta_2 = \Delta_5 = 0$. The bond 1, 3 and 4 in the upper layer constitute an effectively doped spin-1 chain because the behavior of the mean-fields $\chi_1, \chi_3, \chi_4, \Delta_2, \Delta_5, \Delta_4$ is exactly same as the doped spin-1 chain [22]. $\Delta_3 = - \Delta_4$ is the feature of $d_{xy}$-wave and consistent with previous
doped spin-1 chain result. The lower layer constitute another doped spin-1 chain. Therefore, in state A the doped spin-1 ladder is just two doped spin-1 chains connected by the bond field $\chi_2$.

The state B shown in figure 2 (b) has $\Delta_1 > 0$, $\Delta_2 < 0$, $\Delta_3 = \Delta_4 = 0$. The bond 1 and 2 in the front layer form a two-leg spin-1/2 $t$-$J$ ladder with modified $dyz$-pairing symmetry, which has been widely explored by various methods [39, 40]. It is a modified $d$-wave because the pairing amplitudes on the bonds along the $y$-axis are not equal to those along the $z$-axis. The back layer is just a replica of the front layer. At the inter-layer bonds 3, 4 and 5 with $\chi = 0$ and $\Delta = 0$, these results indicate that the state B can be seen as two decoupled doped two-leg square ladder.

The state C shown in figure 2 (c) has $\Delta_3 = \Delta_4 > 0$, $\Delta_2 < 0$, $\Delta_1 = \Delta_5 = 0$. In this state, all of the bonds connecting the front and back layer have finite pair fields. If the two sites connected by the bond have the same $i$($j_i$), there is a positive (negative) pair field at this bond. To a certain extent, the pairing symmetry can be seen as a variant of modified $d_{j_1j_2}$, namely the inter-layer version of the state B. Compared with the state B, the state C connects the two spin-1/2 layers by the pair fields and abandons the intra-layer pairing. In this state, all of the bonds have a finite $\Delta$ or $\chi$ field, which is very different from the state A and B.

The energies of the three states as a function of doping are shown in figure 3 (a). The state A disappears at around $\delta = 0.35$ while the state B and C exist up to about $\delta = 0.5$. The three states have the same energy in the present numerical accuracy. It is the initial condition that determines which state is the converged solution. Therefore, the real ground state of the model is undetermined.

3.3. Results with improved Gutzwiller factors
The Gutzwiller factors can be improved to take account of mean-fields $\chi$ and $\Delta$ [41]. The modified version is

$$g'_{\alpha_i;j_\alpha} = \frac{2\delta}{1 + \delta} \left( 1 + \delta \right)^2 - \frac{4(\chi_{\alpha_i,j_\alpha}^2 + \Delta_{\alpha_i,j_\alpha}^2)}{(1 + \delta)^2} \right) - \beta g_{\alpha_i;j_\alpha} = \frac{4}{(1 + \delta)^2} a^{-7},$$

Figure 2. Schematic illustration of three mean-field states. (a)–(c) are the state A, state B and state C, respectively. The $\Delta$ field of each bond is shown by its color. The red, green and black bonds stand for $\Delta > 0$, $\Delta < 0$ and $\Delta = 0$ respectively. There are 5 kinds of bonds in the model and their positions are shown in panel (a). In state A $\Delta_5 = -\Delta_4 > 0$, showing a $d_{xy}$-wave like pairing symmetry. In state B the intra-layer fields $\Delta_1 > 0$, $\Delta_2 < 0$. It can be seen as two decoupled spin-1/2 $t$-$J$ ladders with modified $dyz$-pairing symmetry. State C has only inter-layer pair fields $\Delta_3 = \Delta_4 > 0$, $\Delta_5 < 0$. The pairing symmetry can be seen as the inter-layer modified $d_{j_1j_2}$-wave.
where
\[
\alpha = 1 + \frac{8\delta^2 (\Delta_{m,n_2}^2 - \chi_{m,n_2}^2) + 16 (\Delta_{m,n_2}^2 + \chi_{m,n_2}^2)^2}{(1 - \delta^2)^2}.
\] (7)

Using these modified factors we find that the energies of the three states split in the undoped and low doping range. Figure 3(b) shows their energies as a function of the hole doping level \(\delta\). The state C has the lowest energy among the three states. The state A is higher and the state B is the highest. The difference is decreasing when \(\delta\) is increased. When \(\delta > 0.35\), they have almost same energy. This result shows that the state C is probably the ground state of this model when hole doping is not very large. This state has non-vanishing mean-fields at all bonds. Therefore, it can take advantage of all antiferromagnetic interactions \(J\) to lower its energy.

In figure 3(c) we show the pair fields \(\Delta_3, \Delta_5\) and bond fields \(\chi_1, \chi_2\) as a function of \(\delta\) in state C. The pair fields decrease with doping and drop to 0 at around \(\delta = 0.49\). The behavior of the four mean-fields is qualitatively similar to the mean-fields \(\Delta_3, \Delta_5, \chi_1, \chi_2\) in the doped two-leg spin-1/2 Heisenberg ladder [39]. In this sense, the superconducting states of the two-leg spin-1/2 and two-leg spin-1 ladder have intrinsic connections. Figure 3(d) shows the values of superconducting order parameter \(\Delta_3^{SC}\) and \(\Delta_5^{SC}\) as a function of \(\delta\). They are related to the pair fields by \(\Delta_3^{SC} = g^2 \Delta_3\), \(\Delta_5^{SC} = g^2 \Delta_5\) where \(g^2 = \frac{2\delta}{1 - \delta} \left(1 + \delta^2 + 4 (\chi_{m,n_2}^2 + \Delta_{m,n_2}^2)\right)^{1/2} [41].\) The superconducting order first increases, then decreases with doping as expected. The magnitude of \(\Delta_5^{SC}\) is larger than \(\Delta_3^{SC}\). We conclude that the most possible ground state of the effectively doped two-leg spin-1 ladder is the inter-layer modified \(d_{ij} - z\)-wave superconducting state with a similar doping dependence to a two-leg spin-1/2 ladder.

4. DMRG studies

DMRG method has been widely used to explore doped spin systems [5, 42, 43]. To determine the ground state of the doped two-leg spin-1 ladder more accurately, we use the DMRG method to solve the model with ITensor Library [44]. We keep up to \(m = 4000\) states with the truncation error \(\varepsilon < 10^{-6}\). The hole density is fixed at
\[ \delta = 0.025 \] for most calculations, and the system size is up to \( L = 80 \). An open boundary condition (OBC) is always employed.

In the ground state the local electron density \( n_{i\alpha} \) is found to be independent of \( \alpha \) and \( i_z \). We sum over \( \alpha \) and \( i_z \) and show the hole density \( n_{i\alpha}^h = \sum_{\alpha,i_z} (1 - n_{i\alpha,i_z}) \) as a function of \( i_y \) with system sizes \( L = 40, 60, 80 \) in figure 4(a). The hole density shows an oscillating behavior because of the OBC. The wave-length is \( \lambda = 1/2\delta \) and the wave-vector is \( Q = \pi/10 = 4\pi\delta \). This feature is very similar to the doped \( t-J \) model on a four-leg cylinder with power-law superconducting correlations [45]. A recent study on the doped Hubbard model [46] exhibits a similar result. The \( L = 40, 60, 80 \) system has 4, 6, 8 holes respectively. From the figure we find that the distribution of their hole densities has 2, 3, 4 peaks, indicating that each two holes are bound together and form a peak. We also calculated the pair binding energy \( E_b = E_{g\delta}(N) + E_{g\delta}(N-2) - 2E_{g\delta}(N-1) \) in the \( L = 40 \) system with \( N = 4L(1 - \delta) = 156 \). \( E_{g\delta}(M) \) denotes the ground state energy of the system with \( M \) electrons. We obtain a negative pair binding energy, \( E_b = -0.37 \), showing the pairing tendency.

The undoped two-leg spin-1 antiferromagnetic Heisenberg ladder has a topologically trivial ground state [27]. In our doped model, the expectation value of \( S^z \) is 0 at all sites, indicating that there is no edge state in this
system. Besides, we use the entanglement spectrum to detect the possible topological properties [19]. The system is divided into two subblocks and we calculate the eigenvalues $w_l$ of the reduced density matrix $\rho_l = \text{Tr}_r \rho$ with $l = L/2$. Figure 4(b) shows the entanglement spectrum $\xi_\alpha = -\ln w_\alpha$ with different $L$ at $\delta = 0.025$. The absence of double degeneracy suggests that the effectively doped two-leg spin-1 ladder is topologically trivial. The undoped or doped spin-1 ladder is intrinsically different from the spin-1 chain, which is topologically nontrivial [22].

The undoped spin model has a finite spin gap without long-range magnetic order [26]. It is expected that this property still holds in our doped model (1). In figure 4(c) we show the absolute values of the spin–spin correlation functions between two sites along $y$ direction as a function of $r$ in the $L = 40, 60, 80$ systems. The correlation is defined by $F(r) = \langle \hat{S}_{i_x,i_x} \cdot \hat{S}_{i_x,i_x+r,y} \rangle$ with $i_x = 1$. $\hat{S}_{i_x,i_y}$ is the spin operator at site $(i_x,i_y)$ of layer $\alpha$. From the figure, we find that $F(r)$ oscillates, and the envelope is nearly a straight line, indicating that the spin–spin correlation is exponentially decaying with $r$ mediated by a density wave. This result provides evidence that the spin correlations are short-ranged, and a spin gap exists in the effectively doped two-leg spin-1 ladder, which is the same as the undoped ladder.

To further investigate the superconducting properties, we calculate the pair–pair correlations $\Phi_{ab}(r)$ by

$$\Phi_{ab}(r) = \langle \hat{\Delta}_a(i_a) \hat{\Delta}_b(i_a + r) \rangle.$$  

(8)

Here $i_a$ is the index of the reference bond and $r$ is the distance between bond $a$ and $b$ along the $y$ direction. The annihilation operator of a singlet pair at bond $a(b = 1–5$, shown in figure 2(a)) is defined by

$$\hat{\Delta}_a(i_a) = \frac{1}{\sqrt{2}} (\hat{c}_{i_a,1,\uparrow} \hat{c}_{i_a,1,1,\downarrow} - \hat{c}_{i_a,1,\downarrow} \hat{c}_{i_a,1,1,\uparrow}),$$

$$\hat{\Delta}_b(i_a) = \frac{1}{\sqrt{2}} (\hat{c}_{i_a,1,\uparrow} \hat{c}_{i_a,1,2,\downarrow} - \hat{c}_{i_a,1,\downarrow} \hat{c}_{i_a,1,2,\uparrow}),$$

$$\hat{\Delta}_3(i_a) = \frac{1}{\sqrt{2}} (\hat{c}_{i_a,1,\uparrow} \hat{c}_{i_a,1,1,\uparrow} - \hat{c}_{i_a,1,\downarrow} \hat{c}_{i_a,1,1,\downarrow}),$$

$$\hat{\Delta}_4(i_a) = \frac{1}{\sqrt{2}} (\hat{c}_{i_a,2,\uparrow} \hat{c}_{i_a,1,1,\downarrow} - \hat{c}_{i_a,2,\downarrow} \hat{c}_{i_a,1,1,\uparrow}),$$

$$\hat{\Delta}_5(i_a) = \frac{1}{\sqrt{2}} (\hat{c}_{i_a,2,\uparrow} \hat{c}_{i_a,1,1,\downarrow} - \hat{c}_{i_a,2,\downarrow} \hat{c}_{i_a,1,1,\uparrow}),$$

where $c_{i_x,i_y,\sigma}$ annihilate a fermion with spin $\sigma$ at site $(i_x,i_y)$ of layer $\alpha$. According to figure 2(a) there exist other choices for a certain $\hat{\Delta}_a(i_a)$. For example, we can choose $\hat{\Delta}_a(i_a) = \frac{1}{\sqrt{2}} (\hat{c}_{i_a,1,\uparrow} \hat{c}_{i_a,1,1,\downarrow} - \hat{c}_{i_a,1,\downarrow} \hat{c}_{i_a,1,1,\uparrow}).$

However, we find that the results below are independent of the choices. Figure 4(d) shows the correlation functions $\Phi_{55}, \Phi_{35}, \Phi_{54}, -\Phi_{55}, -\Phi_{35}$ as a function of $r$ in the $L = 40, \delta = 0.025$ system with $i_x = 11$. They exhibit a slowly decaying behavior and all of the other pair–pair correlation functions drop to $10^{-10}$ at several sites. The non-vanishing pair correlations have three different magnitudes. $\Phi_{55}$ is the largest of all. The finite pairing at bond 5 excludes the existence of the state A and B obtained by RMFT. $\Phi_{35}$ and $\Phi_{54}$ are smallest and have almost same value after $r > 5$, indicating that the pairing at bond 3 and bond 4 are same. We also calculate $\Phi_{44}$ (not shown) and it is same with $\Phi_{55}, \Phi_{54}$ and $\Phi_{35}$ are negative and have nearly same values, which suggests that the pairing at bond 3 and bond 4 or 5 have opposite sign. The pairing symmetry is exact same as the state C obtained in the last section.

In addition, we calculate $\sqrt{\Phi_{55} \times \Phi_{35}}$ and show its value in figure 4(d). On the whole, it is very close to $-\Phi_{54}$ and $-\Phi_{35}$. Pair–pair correlations $\Phi_{ab}$ can be seen as the product of the pairing at bond $a$ and bond $b$. Therefore, $\sqrt{\Phi_{55}}$ and $\sqrt{\Phi_{35}}$ are actually the absolute value of the pairing at bond 5 and bond 3. $\sqrt{\Phi_{55} \times \Phi_{35}}$ naturally measures the pair–pair correlations $\Phi_{55}$, which explains the above result. $\Phi_{55} > \Phi_{35}$ is consistent with the RMFT result where $\Delta_5^{SC} > \Delta_3^{SC}$. In figure 4(e) we show $\Phi_{35}(r)$ by keeping $m = 2000–4000$ states in the double-logarithmic plot. The nearly straight line with $m = 4000$ indicates that correlation is probably power-law decaying. We also calculate the correlation functions in the $L = 60$ and 80 systems with $\delta = 0.025$ and in the $L = 40$ system with $\delta = 0.05$. The pairing symmetry is same as above. Combining the RMFT and DMRG results, we conclude that state C is the ground state of the effectively doped two-leg spin-1 ladder.

5. Conclusion

In summary, we construct and solve a $t-J$ type model to simulate a doped two-leg spin-1 antiferromagnetic Heisenberg ladder. RMFT method obtains three degenerate superconducting states with different pairing symmetry with simple renormalization factors. The state A with $d_{xy}$–wave pairing can be seen as two coupled doped spin-1 chain. The state B can be regarded as two decoupled $S = 1/2$ two-leg $t-J$ ladders with modified $d_{x^2−y^2}$–wave pairing symmetry. The state C has a non-zero pairing at all bonds connecting two spin-1/2 layers,
which can be seen as an inter-layer modified $d_{xy}$–$d_{xz}$–wave. Using improved Gutzwiller factors, we find that state C has the lowest energy among the three states in a large doping range.

In DMRG calculations, we find that the pair binding energy is negative, and every two holes are bound in the charge density profile, indicating the pairing tendency. Given the sign and magnitude of the slowly decaying pair–pair correlation functions, we confirm that the pairing symmetry is the same as that of state C. Besides, the absences of edge states and double degeneracy of the entanglement spectrum suggest a topologically trivial ground state. The exponentially decaying spin–spin correlation functions indicate the existence of a spin gap. These results show that the effectively doped two-leg spin-1 ladder has an inter-layer modified $d_{xy}$–$d_{xz}$–wave superconducting ground state without the topological feature or long-range magnetic order. From the doped spin-1 chain and ladder calculations, we infer that the Gutzwiller-RVB method can capture the right pairing tendency in doped spin-1 systems.

In terms of real materials, we notice that the two-layered ladder system can be regarded as a two-orbital ladder system with Hund’s interaction. The pairing in our system appears between two electrons at the nearest neighboring sites with different orbitals, which is similar to the previous studies on iron-based ladder compounds [47]. Besides, this work can also be helpful for future studies of the doped spin-1 Heisenberg model in two-dimension.

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References

[1] Anderson P W 1987 Science 235 1196–8
[2] Lee P A, Nagaosa N and Wen X-G 2006 Rev. Mod. Phys. 78 17
[3] Zhou Y, Kanoda K and Ng T-K 2017 Rev. Mod. Phys. 89 025003
[4] Kivelson S A, Fradkin E and Emery V J 1998 Nature 393 550
[5] Jiang H-C, Devereaux T and Kivelson S A 2017 Phys. Rev. Lett. 119 066402
[6] You Y Z, Kimchi I and Vishwanath A 2012 Phys. Rev. B 86 085145
[7] Li K, Yu S-L and Li J-X 2015 New J. Phys. 17 043032
[8] Schmidt J, Scherer D D and Black-Schaffer A M 2018 Phys. Rev. B 98 014504
[9] Ogata M and Fukuyama H 2008 Rep. Prog. Phys. 71 036501
[10] Xu G et al 1996 Phys. Rev. B 54 R6827–30
[11] Wang Q et al 2016 Nat. Commun. 7 12182
[12] Wang F, Kivelson S A and Lee D H 2015 Nat. Phys. 11 959
[13] Wang Z, Hu W-J and Nevidomskyy A H 2016 Phys. Rev. Lett. 116 247203
[14] Lai H H, Hu W-J, Nica E M, Yu R and Si Q 2017 Phys. Rev. Lett. 118 176401
[15] Yu R et al 2012 Phys. Rev. B 86 085148
[16] Gong S S, Zhu W, Sheng D N and Yang K 2017 Phys. Rev. B 95 205132
[17] Takahashi H et al 2015 Nat. Mater. 14 1008
[18] Ying J J et al 2017 Phys. Rev. B 95 241109(R)
[19] Pollmann F, Turner A M, Berg E and Oshikawa M 2010 Phys. Rev. B 81 064439
[20] Jiang H-C, Li Z X, Seidel S and Lee D H 2018 Sci. Bull. 63 753–8
[21] Zhu Z, Sheng D N and Weng Z Y 2018 Phys. Rev. B 97 115144
[22] Hou J, Lee T K, Lou J and Chen Y 2019 Phys. Rev. B 99 094510
[23] Rice T M 1997 Z. Phys. B 103 165–72
[24] Berg E, Kivelson S A and Scalapino D J 2009 New J. Phys. 11 085007
[25] Tohyama T, Mori M and Sota S 2018 Phys. Rev. B 97 235137
[26] Todo S, Matsumoto M, Yasuda C and Takayama H 2001 Phys. Rev. B 64 242412
[27] Wierschem K and Sengupta P 2014 Phys. Rev. Lett. 112 247203
[28] Dagotto E and Rice T M 1996 Science 271 618
[29] Zhang F C, Gross C, Rice T M and Shibai H 1988 Supercond. Sci. Technol. 1 56
[30] Gan J Y, Chen Y, Su Z B and Zhang F C 2005 Phys. Rev. Lett. 94 067005
[31] Edegger B, Muthukumar V N and Gros C 2007 Adv. Phys. 56 927
[32] Chen C, Chen Y, Wang Z D and Ting C S 2010 Phys. Rev. B 82 174502
[33] White S R 1992 Phys. Rev. Lett. 69 2863
[34] Schollwock U 2005 Rev. Mod. Phys. 77 259
[35] He Y C, Sheng D N and Chen Y 2014 Phys. Rev. Lett. 112 137202
[36] Yang K Y, Chen W-Q, Rice T M, Sigrist M and Zhang F C 2009 New J. Phys. 11 055053
[37] Choubey P, Tu W-L, Lee T-K and Hirschfeld P J 2017 New J. Phys. 19 013028
[38] Tu W-L, Schindler F, Neupert T and Poilblanc D 2018 Phys. Rev. B 97 035154
[39] Sigrist M, Rice T M and Zhang F C 1994 Phys. Rev. B 49 12058
[40] Hayward C A, Poilblanc D, Noack R M, Scalapino D J and Hanke W 1995 Phys. Rev. Lett. 75 926
[41] Ogata M and Himeda A 2003 J. Phys. Soc. Jpn. 72 374
[42] White S R and Scalapino D J 1997 Phys. Rev. B 55 6504
[43] White S R, Scalapino D J and Kivelson S A 2015 Phys. Rev. Lett. 115 056401
[44] ITensor c++ library available at http://itensor.org/
[45] Jiang H C, Weng Z Y and Kivelson S A 2018 Phys. Rev. B 98 140505(R)
[46] Jiang H C and Devereaux T P 2019 Science 365 1424
[47] Patel N D, Nocera A, Alvarez G, Moreo A and Dagotto E 2017 Phys. Rev. B 96 024520