A linear realization of the BRST symmetry

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Abstract

In this Letter, we propose a model which is equivalent to the Yang-Mills theory at long distances but for which all symmetries are realized linearly. On top of the gauge and Fadeev-Popov ghosts fields, the model presents several massive fields, all of which can be merged in a unique vector field with support in a curved superspace. The equivalence of this model with the gauge-fixed Yang-Mills theory is shown to result from the decoupling of these extra massive modes at small momenta. Most of the symmetries of Yang-Mills theory, including the famous BRST symmetry, can be interpreted in this way as isometries of the superspace.

1 Introduction

Nonlinear realizations of symmetries are notoriously difficult to handle. In a linear realization, the effective action (the generating functional for proper vertices) has the same symmetries as the action if the regularization procedure respects the symmetries. This is not the case for non-linear realizations for which the radiative corrections change the form of the symmetry transformation. On top of this, in the linear realizations, the symmetries manifest themselves at the level of the effective action as linear Ward identities, while one needs to invoke more involved Slavnov-Taylor identities in the non-linear case.

Nonlinear realizations of symmetry groups appear in very different physical situations, both in condensed matter and particle physics. They play a major role, for example, for the analysis of the low momentum behavior of theories where some symmetry is spontaneously broken (as in magnetic systems in the low-temperature phase or in strong interactions physics where chiral symmetry is spontaneously broken). They appear also in the formulation of many supersymmetric models (see for instance [1]). A third example

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of symmetry realized non-linearly take place in Quantum Chromodynamics (QCD). For this theory, most of the theoretical approaches use the Fadeev-Popov procedure to fix the gauge. In a seminal work, Becchi, Rouet, Stora [2,3] (and independently Tyutin [4]) observed that the resulting theory presents a symmetry, referred to as BRST symmetry. Today, BRST symmetry is a cornerstone of our understanding of nonabelian gauge theories. It simplifies considerably the proofs of renormalizability and unitarity of the theory. Moreover, the Fadeev-Popov procedure enlarges the ket space and the BRST symmetry is fundamental to reduce it to the physical Hilbert space. As we shall discuss in more details below, the BRST transformation is nonlinear: some fields have variations that are quadratic.

There is, however, a point where BRST is at odds with most nonlinearly realized symmetries. In all other cases one knows how to construct a model where the symmetry is realized linearly and that behaves in the same way (at least at large distances) as the non-linearly realized one. This is the case for any continuous bosonic internal symmetry [5,6], and is also the case for standard supersymmetries [1]. However, the construction of such a model was missing for the BRST symmetry and this is the subject of the present letter.

In all cases where such a linear realization of the symmetries is available, it proves to be extremely clarifying on the conceptual level and very useful in practical calculations. For example, when the nonlinear symmetry takes its origin in a spontaneously broken symmetry, the linear realization is essential in order to understand the restoration of the symmetry induced by fluctuations. Moreover, even if nonlinear realization of symmetries can be handled at a perturbative level, they become intractable nonperturbatively. These difficulties are particularly problematic for two functional approaches that try to formulate approximation schemes that go beyond perturbation theory: the Non-Perturbative Renormalization Group (NPRG) [7,8] and the 2-PI formalism [9,10,11]. Both approaches rely on introducing in the action a term quadratic in the fields that is interpreted as a regularizing term in the framework of the NPRG and as a source for composite operators in the 2-PI formalism. The variation of this quadratic term under non-linear symmetries is not quadratic itself so this term explicitly breaks the symmetry. Actually, in both approaches, the regularizing function/sources are put to zero at the end of the calculation so one could, at least in principle [12,13], recover the symmetry in the ‘physical’ limit, but this requires in general to impose fine-tuning conditions.

In many cases, this difficulty has been avoided with great success by exploiting the equivalence of such a model with another where the symmetry is realized linearly [8]. However, in absence of a linear realization of the BRST symmetry for Yang-Mills theory, this strategy has been beyond reach for QCD. The construction of a linear realization of BRST which is addressed in this letter is then an interesting starting point for formulating functional methods for QCD where the BRST symmetry is respected at all intermediate steps.

The letter is organized as follows. In Sect. 2 we discuss the linear and nonlinear versions
of a simple bosonic model. This is used to illustrate the ideas developed in the following
of the article. In Sect. 3, we briefly present the Curci-Ferrari-Delbourgo-Jarvis (CFDJ)
gauge-fixing of Yang-Mills (YM) theory. The associated gauge-fixed action has a large
group of symmetries which simplify our discussion. In Sect. 4 we propose a model which
is shown to coincide with the YM theory in the CFDJ gauge fixing at large distances but
for which the BRST symmetry is realized linearly. We give our conclusions in Sect. 5.

2 The case of the NL\(\sigma\)

Before discussing the construction of a linear realization of BRST symmetry, we will
review first a similar construction for a much simpler field theory: the Nonlinear \(\sigma\) (NL\(\sigma\))
model in two dimensions. As is well known, Quantum Chromodynamics (QCD) shares
many illuminating similarities with it that go beyond the nonlinear realization of symme-
tries. To cite the most striking similarities, both theories present asymptotic freedom: the
effective coupling constant vanishes logarithmically at high momentum scales. In the in-
frared regime, both theories present a scale at which the spectrum changes drastically, this
scale being generically (without fine tuning of the microscopic parameters) much smaller
than the relevant ultraviolet scales. For QCD, this momentum scale is \(\Lambda_{\text{QCD}} \sim 200\, \text{MeV}\)
which is 19 orders of magnitude smaller than the Planck scale. For momenta larger than
\(\Lambda_{\text{QCD}}\), the relevant degrees of freedom are quarks and massless gluons, while the low
energy spectrum has a gap and is made of hadrons. The NL\(\sigma\) model presents a similar
property. The infrared scale is now \(\Lambda_{\text{NL}\sigma}\) which is generically found to be orders of magni-
tude smaller than the inverse lattice spacing. For higher momentum scales, the theory is
described in terms of massless Goldstone modes while, in the opposite regime, the spec-
trum presents massive modes. A surprising property of the NL\(\sigma\) model is that the number
of Goldstone modes in the ultraviolet does not coincide with the number of massive modes
in the infrared.

Let us describe in more details this model in the case of the \(O(N) \rightarrow O(N-1)\)
symmetry-breaking scheme. It is parametrized in terms of a \(N-1\) component field \(\pi_i\)
\((i = 1, \ldots, N-1)\) that interacts via a euclidean action

\[
S = \frac{1}{2g^2} \int d^2x \left( (\nabla \pi_i)^2 + \frac{(\pi_i \nabla \pi_i)^2}{1-\pi^2} \right).
\]  

On top of the \(O(N-1)\) linear symmetry group that consists in rotating the \(\pi\)'s \((\pi_i \rightarrow \pi_i + \epsilon_{ij} \pi_j\) with \(\epsilon_{ij}\) antisymmetric), this action is invariant under the non-linear transformations:
\(\pi_i \rightarrow \pi_i + \epsilon_{ij} \sqrt{1-\pi^2}\). These \(N-1\) symmetries together with the linearly realized ones
generate the \(O(N)\)-symmetry group of the action \(S\).

Nonlinear symmetries translate into Slavnov-Taylor [14][15] equations that are hard to
handle because they are nonlinear in the effective action \(\Gamma\). A crucial observation is that it
is possible to construct a theory which is equivalent to the NL\(\sigma\) model at large distances,
but in which all symmetries are realized linearly. To do so, one adds-up a massive field $\sigma$ that completes the standard $N$-multiplet of the linear $O(N)$ model. The action is the standard Landau-Ginsburg one

$$S = \frac{1}{2g^2} \int d^2x \left[ (\nabla \sigma)^2 + (\nabla \pi_i)^2 + M^2(\sigma^2 + \pi_i^2 - 1)^2 \right]. \quad (2)$$

In the limit of low momenta (that is for momenta much smaller than $M$), the $\sigma$ mode is frozen. The potential term can be seen as a constraint for the field $\sigma$, which can therefore be replaced by the solution of its equation of motion, that is $\sigma \rightarrow \sqrt{1 - \pi_i^2}$. One then recovers the non-linear version of the model (1).

The linear realization of the symmetry has many virtues. At a technical level, the linearly-realized symmetries impose Ward identities (that are linear in $\Gamma$) which are much simpler to handle than Slavnov-Taylor identities in actual calculations. Another virtue, which is probably more important, is that it helps understanding the infrared sector of the theory and the way in which the spectrum of the theory changes at the scale $\Lambda_{\text{NL}\sigma}$. Upon renormalization, the minimum of the effective potential is shifted toward the origin. At a scale of the order of $\Lambda_{\text{NL}\sigma}$ and for all smaller momentum scales, the effective potential has actually a single minimum at the origin. The symmetry which was spontaneously broken in the ultraviolet is restored by fluctuations. Consequently, instead of having $N - 1$ massless Goldstone bosons as one could naively guess from Eq. (1), the spectrum is actually composed of $N$ degenerate massive fields. It is interesting to note that $\Lambda_{\text{NL}\sigma} \sim M \exp(-\text{cte}/g^2)$ which shows that one need not perform a fine-tuning of the parameters to have a UV scale $M$ much larger than $\Lambda_{\text{NL}\sigma}$.

The strong analogies between QCD and the NL$\sigma$ model call for a parallel treatment of these theories \textsuperscript{3}. In particular, since the linear realization of the symmetry in the NL$\sigma$ model is so useful for understanding its infrared regime, one can expect that a similar construction for QCD would give a clarifying viewpoint on this theory in the infrared sector.

3 Yang-Mills theory in Curci-Ferrari-Delbourgo-Jarvis gauge

In the next sections, we will present a model where such a linear realization takes place. We will show that it behaves at large distances as the YM theory in the CFDJ gauge fixing \textsuperscript{16,17}. This class of gauge fixing admits the Landau gauge as a particular case. Moreover, as we recall later, it presents a large symmetry group that considerably simplifies the analysis. We show that, very much as for the NL$\sigma$ model, one can add massive fields (of typical mass $M$) to those of the gauge-fixed YM theory (called light

\textsuperscript{3} We should stress, however, a strong difference between the two models: there is no equivalent in the NL$\sigma$ model of the confinement phenomenon that shows up in QCD.
fields in the following) such that the BRST symmetry is realized linearly. In the low-energy limit, where the massive fields decouple, one recovers the YM theory for the light fields.

Before presenting that model, it is convenient to describe first the CFDJ gauge fixing and its symmetries. The action in the euclidean space reads:

$$S = \int d^d x (\mathcal{L}_{YM} + \mathcal{L}_{GF}).$$

(3)

$\mathcal{L}_{YM}$ is the YM Lagrangian:

$$\mathcal{L}_{YM} = \frac{1}{4} F_{\mu \nu}^\alpha F_{\mu \nu}^\alpha,$$

(4)

$F_{\mu \nu}^\alpha = \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha + g f^{\alpha\beta\gamma} A_\mu^\beta A_\nu^\gamma$ is the field strength, $g$ is the gauge coupling, $A_\mu$ is the gauge field, and $f^{\alpha\beta\gamma}$ denotes the structure constants of the gauge group that are chosen completely antisymmetric. $\mathcal{L}_{GF}$ is the gauge fixing term, which includes a ghost sector. It takes the form:

$$\mathcal{L}_{GF} = \frac{1}{2} \partial_\mu c^\alpha (D_\mu c)^\alpha + \frac{1}{2} (D_\mu \bar{c})^\alpha \partial_\mu \bar{c}^\alpha + \frac{\xi}{2} h^\alpha h^\alpha + i h^\alpha \partial_\mu A_\mu^\alpha - \frac{g^2}{8} (f^{\alpha\beta\gamma} \bar{c}^\beta c^\gamma)^2.$$

(5)

Here, $c$ and $\bar{c}$ are ghost and antighosts fields respectively, $h$ is the Lagrange multiplier field and $(D_\mu \varphi)^\alpha = \partial_\mu \varphi^\alpha + g f^{\alpha\beta\gamma} A_\mu^\beta \varphi^\gamma$ is the covariant derivative for any field $\varphi$ in the adjoint representation.

The gauge fixing Lagrangian is invariant under a) the euclidean symmetries of the spacetime; b) the global color symmetry; c) the ghost conjugation symmetry: $c^\alpha \rightarrow \bar{c}^\alpha$, $\bar{c}^\alpha \rightarrow - c^\alpha$; d) the continuous symplectic group $SP(2, \mathbb{R})$ [18] with generators $N$, $t$ and $\bar{t}$ that act only on the ghost sector, and defined by:

$$tc^\alpha = \bar{t} \bar{c}^\alpha = 0$$

$$t \bar{c}^\alpha = Nc^\alpha = c^\alpha$$

$$\bar{t}c^\alpha = N\bar{c}^\alpha = -\bar{c}^\alpha;$$

(6)

e) the model is also invariant under the nonlinear BRST symmetry:

$$s A_\mu^\alpha = (D_\mu c)^\alpha,$$

$$sc^\alpha = -\frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma,$$

$$s \bar{c}^\alpha = i h^\alpha - \frac{g}{2} f^{\alpha\beta\gamma} \bar{c}^\beta c^\gamma,$$

$$si h^\alpha = \frac{g}{2} f^{\alpha\beta\gamma} (ih^\beta c^\gamma + \frac{g}{4} f^{\gamma\delta\epsilon} \bar{c}^\beta c^\delta c^\epsilon).$$

(7)
By virtue of the ghost conjugation symmetry, the action is also invariant under an anti-
BRST symmetry $\bar{s}$. These symmetries satisfy the standard nilpotency property 
$(s^2 = \bar{s}^2 = \bar{s}s + s\bar{s} = 0)$; f) recently, we showed that the CFDJ action is also invariant under four 
gauge supersymmetries [19]. One corresponds to a local shift of the field $h$. A second one 
corresponds to a shift of the ghost field by an infinitesimal local parameter simultaneously 
with a color transformation of the field $h$ with the same parameter. A third one is just the 
ghost conjugated of the previous one. The last one corresponds to a gauge transformation 
of the fields. These gauge transformations are not truly symmetries but the associated 
variations of the action are linear in the fields so that one can write a simple Ward identity 
associated with them, that in particular imply some non renormalization theorems [19][20].

4 Realizing linearly the BRST symmetry

Having described the symmetries of the CFDJ gauge fixing, let us now present a model 
in which those symmetries are realized linearly. The arena of the model is a superspace 
with $d$ bosonic coordinates $x^\mu$ and two grassmanian anticommuting coordinates $\theta$ and 
$\bar{\theta}$ ($\theta^2 = \bar{\theta}^2 = \theta\bar{\theta} + \bar{\theta}\theta = 0$). It is convenient to work in such a superspace because, 
as well known since the work of Tonin and Bonora [21][22] (see also [23][17]), the BRST 
symmetry appears then as an invariance under translation in the $\theta$ direction. Similarly 
the translation in the $\bar{\theta}$ direction yields the symmetry $\bar{s}$. The geometry of the superspace 
is now dictated by the symmetries of the CFDJ action. A first guess would be to take 
a flat superspace. However this superspace admits superrotations that mix the bosonic 
and fermionic coordinates, which have no equivalent in the CFDJ action. In what follows, 
we therefore choose the geometry of the superspace in such a way that these unwanted 
symmetries are explicitly broken. To implement this idea, we consider a Riemannian 
superspace with metric:

$$g_{AB} = \begin{cases} 
\delta_{\mu\nu} & \text{if } A = \mu, \ B = \nu \\
-(1 + M^2\theta\bar{\theta}) & \text{if } A = \theta, \ B = \bar{\theta} \\
(1 + M^2\bar{\theta}\theta) & \text{if } A = \bar{\theta}, \ B = \theta \\
0 & \text{otherwise.}
\end{cases}$$

(8)

Here and below, the uppercase indices run over bosonic and fermionic components of the 
superspace. The formulas of the Riemannian geometry in the superspace are essentially 
identical to those in bosonic spaces, except for signs as explained in [24][25]. Here, contrarily 
to [24][25] we use left derivatives.

The isometries of this superspace have been discussed in details in [19]. Let us here 
comment on some of its properties. This curved superspace has a typical length scale $M^{-1}$. At larger length scales, the grassmanian directions are wrapped around and the 
space is similar to a purely bosonic space. In the other limit, at length scales much smaller
than $M^{-1}$, the curvature is irrelevant and the space is equivalent to a flat superspace with $d$ bosonic and 2 fermionic directions. As is well known since the pioneering work of Parisi and Sourlas [26] (see also [27]), theories defined in this kind of flat superspace present the property of dimensional reduction. This means that a theory in this superspace has the same correlation functions as the equivalent theory in a purely bosonic space with $d - 2$ dimensions if their isometries are not spontaneously broken. For this reason, it is often said that the Grassmannian directions count as negative dimensions. In a sense, the superspace considered here realizes the standard idea of adding extra confined dimensions of space as possible sources of new physics. However in the case considered here, the wrapped directions are fermionic so that there are a finite number of excitations associated with these extra dimensions [28].

Having described the superspace and its isometries, let us now come to the field content of the theory. We consider a vectorial superfield $A^A = \{A^\mu, A^\theta, \bar{A}^\theta\}$ in this space where the last two components are fermionic. The action for this field is chosen to respect the isometries of the superspace. We therefore contract the superspace indices with the tensors associated with the space, that are in our case the metric, the Riemann and the Ricci tensors. In the superspace we are considering, the Riemann tensor can be expressed in terms of the Ricci tensor so we do not consider it as an independent tensor in what follows. Moreover, the Ricci tensor $R_{AB}$ is equal to $M^2 g_{AB}$ in the fermionic sector, and zero in the bosonic one. In order to satisfy the isometries of the superspace, it is, as usual, necessary to use an invariant measure $I = \int \sqrt{s \det g} \, dx d\theta d\bar{\theta}$ [25] (where $s \det$ is the superdeterminant). There is obviously an ambiguity in the sign of the square root. For later convenience, we choose $\sqrt{s \det g} = -1 + M^2 \theta \bar{\theta}$.

As discussed above and more thoroughly in [19], the CFDJ presents a gauge symmetry up to some non renormalized terms. We therefore require the theory to be invariant under the gauge transformation $A^\alpha_A \rightarrow A^\alpha_A + \partial_A \Lambda^\alpha + gf^{\alpha\beta\gamma} A^\beta_A \Lambda^\gamma$ with $\Lambda^\alpha = \Lambda^\alpha(x, \theta, \bar{\theta})$ an arbitrary function of the superspace coordinates. In the following, in order to respect the gauge invariance, we write the action in terms of the field strength

$$F_{AB}^\alpha = (-1)^b \partial_A A_B^\alpha - (-1)^{ab} \partial_B A_A^\alpha + gf^{\alpha\beta\gamma} A_B^\beta A_A^\gamma$$

where the lowercase letters are 1 if the associated uppercase letters are fermionic, and 0 otherwise. The action therefore has a Yang-Mills-like term

$$S_{YM} = -\frac{1}{4} \int (-1)^a F_{AB}^\alpha g^{BC} F_{CD}^\alpha g^{DA}$$

which is clearly gauge invariant and invariant under the isometry group, thanks to the $(-1)^a$. Of course other terms with the same symmetries can be constructed but we only consider the simplest one here.

If we had only this term in the action, we would be in deep trouble since we could not invert the 2-point function to build the propagator. Fortunately it is possible, as in
QED, to add a mass term in the theory that breaks gauge invariance in a controlled way. Actually, because of the structure of the curved superspace there are two independent mass-like terms invariant under the isometries of the space. We therefore add two mass terms to the action:

\[ S_{\text{mass}} = \int x \left( \frac{m^2}{2} g_{AB} + \frac{\nu}{2} R_{AB} \right) A^A A^B. \] (11)

The mass term breaks the super-gauge invariance in a simple way. Actually its variation under gauge transformation is linear in the fields so one can write a linear Ward identity for this transformation, that reads:

\[ D_B \left( g^{AB} \delta \left( \Gamma - S \right) \delta A^A \right) = 0. \] (12)

This identity directly shows that the part of the action that breaks gauge-invariance is not renormalized. The breaking of the gauge invariance induced by these mass terms has a counterpart in the CFDJ model. As mentioned in the previous section (see discussion of the symmetry f) above) the CFDJ action is not invariant under the super-gauge transformation but the variation of the action under this transformation is also linear in the fields.

Let us now discuss how the decoupling of the massive modes works. To do so, we expand the field components in a Taylor series in the Grassmannian coordinates \( \theta \) and \( \bar{\theta} \):

\[
A^\alpha_{\mu}(x, \theta, \bar{\theta}) = A^\alpha_{\mu}(x) + \theta B^\alpha_{\mu}(x) - \theta \bar{B}^\alpha_{\mu}(x) + \bar{\theta} E^\alpha_{\mu}(x), \\
A^\alpha_{\theta}(x, \theta, \bar{\theta}) = -c^\alpha(x) + \theta \bar{c}^\alpha(x) - \bar{\theta} b^\alpha(x) - \bar{\theta} \bar{c}^\alpha(x), \\
A^\alpha_{\bar{\theta}}(x, \theta, \bar{\theta}) = c^\alpha(x) + \theta d^\alpha(x) + \theta \bar{b}^\alpha(x) + \theta \bar{d}^\alpha(x). \] (13)

The fields \( A, B, \bar{B}, \) etc. are standard fields in the \( d \)-dimensional Euclidean space. As we now show, most of them are massive, and therefore are decoupled in the infrared. It is straightforward to write down the action \( S_{\text{YM}} + S_{\text{mass}} \) in terms of these fields. This leads to a lengthy expression that need not be given here. We just reproduce the leading order of \( S_{\text{YM}} \) at large \( M^2 \), which reads

\[
S_{\text{YM}}^{\text{large}M} = M^2 \int d^d x \left[ \frac{3}{2} (b^\alpha - \bar{b}^\alpha + g f^{\alpha\beta\gamma} c^\beta c^\gamma)^2 - 6 (\bar{d}^\alpha + \frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma)(\bar{d}^\alpha + \frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma) + 2 (\bar{B}^\alpha_{\mu} - D_{\mu} b^\alpha)(B^\alpha_{\mu} - D_{\mu} \bar{b}^\alpha) + \frac{1}{4} (F^\alpha_{\mu\nu})^2 \right]. \] (14)

The relevant regime of parameters and momenta that allows us to recover the usual YM theory is \( M \gg p \gg m \). The first three terms play the role of constraints for the fields \( b - \bar{b}, B_{\mu}, \bar{B}_{\mu}, d, \bar{d} \), which can be replaced in the rest of the expression by their classical
values \(-g f^{\alpha\beta\gamma} \bar{c} \beta c^\gamma, D_{\mu} c, D_{\mu} \bar{c}, -\frac{g}{2} f^{\alpha\beta\gamma} \bar{c} \beta c^\gamma\) respectively. One then obtains an action for the remaining fields that is actually quadratic in \(E_{\mu}, F\) and \(\bar{F}\). One can then integrate over these fields and get an action for \(A_{\mu}, c, \bar{c}\) and \(b + \bar{b}\). These are, up to renormalization factors, the 4 light fields that appear in the gauge-fixed CFDJ action. It is important to note that the equations of motion for the massive modes give the same expression as the transversality conditions in \([21,22,23,17]\). Eliminating the massive modes is therefore equivalent to imposing these transversality conditions, except for the important fact that these relations need not be imposed externally here. Actually, in order to retrieve the Lagrangian \((3,4,5)\), one has to make the replacements \(A_{\mu} \rightarrow A_{\mu}/M, c \rightarrow c/m, g \rightarrow gM, b + \bar{b} \rightarrow -2ihM/m^2\) and \(\xi = (m^2 + M^2 \nu)M^2/m^4\). This concludes the proof that the gauge theory in the curved superspace is equivalent to the CFDJ gauged-fixed theory in the low energy limit. In fact, it is interesting to note that in the context of the transversality conditions formalism it has already been noted \([17]\) that the gauge-fixing term is obtained by imposing the transversality conditions to the mass term \((11)\).

5 Conclusions and perspectives

To conclude, we have presented a model in which the BRST symmetries are realized linearly and which reduces to the YM theory in the CFDJ gauge at long distances. This model treats on an equal footing the gauge and ghosts fields. At distances much smaller than the inverse of the mass of some massive fields, the model behaves as the massive YM theory in two dimensions, which is renormalizable (see for example \([29]\)).

With a model where BRST symmetry is realized linearly, we are for the YM theory in a position very similar to that of the \(\sigma\) model. We recall that in this case, the existence of the linear version of the \(\sigma\) model plays a fundamental role in order to exploit functional methods (non-perturbative renormalization group equations, and 2-PI formalism) respecting in each steps all the symmetries of the model. It is easy to show that the present model gives a 2-PI functional respecting all the symmetries of the model in a linear way. In what concerns the non-perturbative renormalization group formalism, things are more involved. All isometries (including the BRST symmetry) are respected trivially along the flow but the control of the supergauge symmetries requires further analysis. This work is in progress.

Another aspect where the linear version of the model is helpful for the \(\sigma\) model is in clarifying the appearance of the mass gap and we can wonder if the present model has the same virtue for YM theory. Of course, we have no definitive answer, but we speculate that the infrared fluctuations effectively flatten the superspace. If this is the case, at distances much larger than \(\Lambda_{\text{QCD}}^{-1}\), all fields should be treated on an equal footing. Thanks to the dimensional reduction property \([26,27]\), the theory would then behave in the infrared as YM in two dimensions, which is expected to have a physical mass gap, at least for large \(N\) \([30]\).
This work opens the way to several developments. In light of the previous discussion, it is important to study the loop corrections to the present analysis. One should also determine how to reduce the state space to the physical Hilbert space, probably in a similar way as in the standard BRST formalism. It would also be interesting to investigate if similar constructions can be made to recover at low energy other gauge fixings and to introduce matter fields. Moreover, in this Letter, we focused on the case of YM theory, but we plan to generalize the present work to other gauge theories, in particular to gravity. If what was done here translates to that case, one would recover in the ultraviolet the quantum gravity in 2 dimensions [31,32], which is known to be renormalizable.

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References

[1] S. Weinberg, “The quantum theory of fields. Vol. 3: Supersymmetry,” Cambridge, UK: Univ. Pr. (2000) 419 p.
[2] C. Becchi, A. Rouet and R. Stora, Commun. Math. Phys. 42, 127 (1975).
[3] C. Becchi, A. Rouet and R. Stora, Annals Phys. 98 (1976) 287.
[4] I. V. Tyutin, LEBEDEV-75-39.
[5] R. S. Palais, J. Math. Mech. 6 (1957) 673.
[6] G. D. Mostow, Annals of Math. 65 (1957) 773.
[7] C. Wetterich, Phys. Lett. B 301, 90 (1993).
[8] J. Berges, N. Tetradis, and C. Wetterich, Phys. Rep. 363, 223 (2002).
[9] J. M. Cornwall, R. Jackiw, E. Tomboulis, Phys. Rev. D 10 (1974) 2428;
[10] J. Berges, J. Cox, Phys. Lett. B 517 (2001) 369;
[11] J. Berges and J. Serreau, arXiv:hep-ph/0410330.
[12] C. Becchi, in Elementary Particles, Field Theory and Statistical Mechanics, eds. M. Bonini, G. Marchesini and E. Onofri, Parma University 1993.
[13] U. Ellwanger, Phys. Lett. B 335 (1994) 364 [arXiv:hep-th/9402077].
[14] A. A. Slavnov, Theor. Math. Phys. 10, 99 (1972) [Teor. Mat. Fiz. 10, 153 (1972)].
[15] J. C. Taylor, Nucl. Phys. B 33 (1971) 436.
[16] G. Curci and R. Ferrari, Nuovo Cim. A 32, 151 (1976).
[17] R. Delbourgo and P. D. Jarvis, J. Phys. A 15 (1982) 611.
[18] F. Delduc and S. P. Sorella, Phys. Lett. B 231, 408 (1989).
[19] M. Tissier and N. Wschebor, arXiv:0809.1880 [hep-th].
[20] N. Wschebor, Int. J. Mod. Phys. A 23 (2008) 2961 [arXiv:hep-th/0701127].
[21] L. Bonora and M. Tonin, Phys. Lett. B 98 (1981) 48.
[22] L. Bonora, P. Pasti and M. Tonin, Nuovo Cim. A 64 (1981) 307.
[23] L. Baulieu and J. Thierry-Mieg, Nucl. Phys. B 197 (1982) 477.
[24] P. Nath and R. L. Arnowitt, Phys. Lett. B 56 (1975) 177.
[25] R. L. Arnowitt, P. Nath and B. Zumino, Phys. Lett. B 56 (1975) 81.
[26] G. Parisi and N. Sourlas, Phys. Rev. Lett. 43 (1979) 744.
[27] B. McClain, A. Niemi, C. Taylor and L. C. R. Wijewardhana, Phys. Rev. Lett. 49 (1982) 252.
[28] R. Delbourgo, AIP Conf. Proc. 917 (2007) 122.
[29] J. Zinn-Justin, “Quantum field theory and critical phenomena”, fourth edition, Oxford science publications (2002) p. 536.
[30] G. ’t Hooft, Nucl. Phys. B 72 (1974) 461.
[31] J. Ambjorn, J. Jurkiewicz and R. Loll, Phys. Rev. Lett. 95 (2005) 171301
[32] O. Lauscher and M. Reuter, JHEP 0510 (2005) 050