Non-iterative model based image reconstruction of diffuse optical tomography based on DE and RTE in quality control of agricultural product studies

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Abstract. In a previous study, we developed the non-iterative image reconstruction based on diffusion equation. Within this research, we applied the same non-iterative algorithm scheme using radiative transfer equation. Basically, the non-iterative image reconstruction is a development of model based image reconstruction that implemented truncated singular value decomposition and L-curve analysis to solve the ill-posed problem. This algorithm reduces the computation time to reconstruct the cross-sectional area. As part of continuous development on agricultural application, optical tomography based quality control is offered. An experiment was conducted on potato to evaluate these two non-iterative algorithms. The object was illuminated by the near infrared source from 8 positions on the object boundary. In this experiment, we vary the position and the amount of epoxy on the object then we analyze the residual value between measurement and reconstructed boundary data. The reconstructions were performed in continuous-wave domain. Furthermore, we compared the residual value from diffuse optical tomography using diffusion equation and radiative transfer equation. The result of this study indicates that these algorithms are promising to detect the presence of epoxy on potato which is useful for quality control of agricultural products.

1. Introduction
Several researchers have been developing the reconstruction methods to reduce the computation time in optical tomography (OT) with the regularization parameter selection method [7], a fast-forward solver of radiative transfer equation (RTE) [5], non-iterative algorithm [8], hardware development [14], and signal processing approach [1, 13, 15]. This paper focused on the study of non-iterative image reconstruction based on diffusion equation (DE) and RTE. To avoid the iterative step, L-curve based algorithm for automatic selection of optimal k value is performed to obtain truncated singular value.
decomposition (TVSD) matrix which will derive a new well-posed problem. In forward solving, the continuous wave signal processing approach was applied in order to get a faster boundary data calculation. Furthermore, a fast RTE solver that has been developed by H. Gao and H. Zhao was applied to solve the forward problem in OT based on RTE.

Usually, the development of non-destructive evaluation (NDE) based on optical tomography method has been focused on medical application [4, 6, 8, 9]. Nevertheless, NDE OT development on other application is also interesting and promising as described by some researcher on this field [3, 14]. This paper as well as our previous papers mainly study NDE based on OT method for agricultural product quality control [17, 18, 19]. Therefore, the experiment of internal defects monitoring on agricultural products using OT is described on this paper to validate our proposed non-iterative image reconstruction algorithm.

2. NIMoBIR OT (Non-Iterative Model Based Image Reconstruction of Diffuse Optical Tomography)

2.1. Principle of Optical Tomography (OT)

OT reconstruction system is divided into two essential parts. The first one is forward problem solving and the other one is inverse problem solving [13, 20], as shown in figure 1. The forward problem solving is used to calculate the boundary data of prediction model and/or simulation object when the optical properties of the medium and the amount of input light are given. In this paper, we used DE and RTE to describe the light propagation in biological material, which then are used as forward problem solving [1, 5, 16]. While inverse problem solving is an algorithm to predict an optical coefficient distribution inside a medium by comparing the boundary data of the “object” and those of prediction model.

![Figure 1. NIMoBIR algorithm](image-url)
A fundamental model for light transport in tissues is the radiative transfer equation. The continuous wave domain RTE is of the form

$$\delta V L(r, \delta) + (\mu_a + \mu_s) L(r, \delta) = \mu_a \int f(\delta, \delta') L(r, \delta') d\delta' + q(r, \delta)$$  \hspace{1cm} (1)

where $\mu_a$ and $\mu_s$ are absorption and scattering coefficient of the medium, respectively. Furthermore, $L(r, \delta)$ is the radiancy and $f(\delta, \delta')$ is the probability density function. The probability density function describes the probability of scattering from direction $\delta$ into direction $\delta'$. Even though RTE is a widely accepted model, DE is more commonly used because of its computational simplicity. In DE framework, the RTE is approximated as follows:

$$-\kappa \nabla^2 \Phi(r) + \mu_s \Phi(r) = q_0(r)$$  \hspace{1cm} (2)

where $\Phi$ is the diffuse photon fluency rate, $q_0$ is the integrated light source term, and $\kappa$ is the optical diffusion coefficient [1, 2, 15]. The optical diffusion coefficient can be represented as follows:

$$\kappa = \frac{1}{3[\mu_s + \mu_a]}$$  \hspace{1cm} (3)

where $\mu_s$ [m$^{-1}$] is the reduced scattering coefficient [13,16].

In order to simplify the boundary data calculation process using forward problem, discretization approach needs to be applied. In this paper FEM approach is used to divide the object or model into non-overlapping triangle elements as shown in figure 2.

Furthermore, FEM approach is applied to simulation object and prediction model to simplify the computational process. This FEM approach is dividing the object or model into non-overlapping triangle elements in order to discretize the diffusion equation as shown in figure 2.

**Figure 2.** Discretization object with FEM [20]

FEM is selected because it can provide fast, accurate and flexible modeling for complex geometries [17, 19]. By applying FEM to DE and RTE, the photon density $\Phi$ and the radiancy $L$ is approximated by a piecewise linear and continuous function. Definition of $\Phi^h$ and $L^h$ are as follows:

$$\Phi^h(r) = \sum_{j=1}^{N} \Phi_j(r) u_j(r)$$  \hspace{1cm} (4)

$$L^h(r) = \sum_{j=1}^{N} L_j(r) u_j(r)$$  \hspace{1cm} (5)

where $u_j$ are linear basis functions for which the values at element centers $n_j$ are defined as $u_i(n_j) = \delta_{ij}$ ($i,j = 1, …, N$). Either $\Phi^h$ or $L^h$ will be used as input for the inverse problem.

In image reconstruction algorithm of OT, the optical coefficient distribution of the “object” or/and predictive model are estimated based on boundary data and light propagation within the medium.
The boundary data from both DE and RTE can be described as a general vector \( b \) and the optical coefficient distribution can be described by a general vector \( \mu \). Then, a physical model that describes the relation between \( b \) and \( \mu \) can be written as follows:

\[
J = \frac{\partial b}{\partial \mu}
\]  

(6)

where \( J \) is the sensitivity matrix that defines sensitivity of \( F \) to perturbations in \( \mu \), also known as Jacobian matrix. Furthermore, the forward problem is defined as finding the values of data \( b \) from given \( \mu \), for which:

\[
b = J\mu
\]

An image representing optical coefficient distribution \( \mu \) from given data \( b \) needs to be estimated in inverse problem, which can be written as:

\[
\mu = J^{-1}b
\]  

(7)

Unfortunately, the Jacobian inversion process leads to ill-posed problem. Generally, it can be solved by applying iterative reconstruction. However, in this paper TSVD is used to solve this problem in a non-iterative way. The first step in TSVD algorithm is converting matrix \( J^{-1} \) to singular value decomposition matrix as follows:

\[
J^{-1} = VSD^T
\]  

(8)

where \( S = \text{diag}(s_1, \ldots, s_n) \) has non-negative diagonal elements appearing in non-increasing order [12]. In this case, \( V = (v_1, \ldots, v_n) \) and \( D = (d_1, \ldots, d_n) \) are matrices with orthonormal columns. Furthermore, to derive a new problem with well-posed rank deficient coefficient matrix, we need to truncate the \( S \) matrix.

The rank deficient matrix, \( S_k \), is obtained by truncating the \( S \) at \( k \) so that \( S_k \) is the best rank-\( k \) approximation to \( S \). \( S_k \) is given by

\[
S_k = \sum_{i=1}^{k} s_i, \quad k \leq n
\]  

(9)

L-curve analysis is the most convenient graphical tool for determining the optimum value of \( k \) precisely [11]. L-curve plot the norm \( \| \mu \|_2 \) of the SVD solution to the prediction error norm \( \| S_k \mu - b \|_2 \), where \( k \) minimizes these two quantities at the same time.

![Figure 3. L-curve analysis for TSVD](image-url)
3. Experiment of potato monitoring

Evaluation of the proposed non-iterative model based image reconstruction method was performed on a ring array optical tomography system used for quality control of potato. Measurement was done using cylindrical carrot with diameter and height of 30*10^{-3} m and 40*10^{-3} m, respectively. The experiment was done on four different circular objects with several defect conditions as shown in figure 5.

![Figure 4. Experimental configuration](image)

The experimental configuration is shown in figure 4. In this experiment, near-infrared laser emits continuous light having peak powers around 0.5 W at 759 nm. A beam of light is projected to the object through a single fiber at the center of fiber optic bundle. Then the scattered and absorbed light intensity is received by a photodetector via annular fibers within fiber optic bundle.

There are 8 test points on the object boundary that will be occupied by source and detector as shown in figure 4. The boundary data matrix is denoted as:

\[
b = \begin{bmatrix}
b_{1,1} & b_{2,1} & \cdots & b_{8,1} \\
b_{1,2} & b_{2,2} & \cdots & b_{8,2} \\
\vdots & \vdots & \ddots & \vdots \\
b_{1,8} & b_{2,8} & \cdots & b_{8,8}
\end{bmatrix}_{NS\times ND}
\]

where NS indicate number of sources and ND indicate number of detectors.

4. Results and analysis

The experiment objects with various target positions and the reconstructed images are shown in Figure 5. General conclusion based on visual analysis is that the NIMoBIR algorithm using DE succeeded to represent the position and the size of the objects. However, there are several white spots on reconstructed images that are difficult to distinguish from the presence of the target. Furthermore, this method is unable to show the separation between two targets on object. Meanwhile, as shown in Figure 5, the NIMoBIR using RTE has a better ability to separate two targets on object than the NIMoBIR using DE. However, this method still has weakness in reconstructing the target position. In order to carry out an objective analysis, the residual value is used to show the difference between measurement and reconstructed boundary data, as shown in table 1.
Figure 5. The results of CW-NIMoBIR reconstruction (a) the depiction of object condition (b) The reconstructed image using RTE (c) The reconstructed image using DE.

Table 1. Numerical analysis calculation results on NIMoBIR CW using DE and RTE

| Object | Residual Value   |
|--------|------------------|
|        | DE               | RTE              |
| 1      | 8.82*10^{-7}     | 6.5*10^{-12}     |
| 2      | 1.2*10^{-6}      | 8.2*10^{-9}      |
| 3      | 1.91*10^{-6}     | 7.49*10^{-12}    |
| 4      | 5.33*10^{-6}     | 2.87*10^{-12}    |
| 5      | 4.15*10^{-6}     | 3.36*10^{-13}    |
| 6      | 2.3*10^{-6}      | 1.08*10^{-9}     |
| 7      | 2.4*10^{-6}      | 6.34*10^{-12}    |
| 8      | 1.93*10^{-6}     | 8.36*10^{-10}    |
| 9      | 2.58*10^{-6}     | 3.73*10^{-12}    |

As shown in table 1, the averages of residual value on NIMoBIR CW DE and RTE are 2.52 * 10^{-6} and 1.13 * 10^{-9} respectively. These averages are relatively low, so it can be concluded that both algorithms are able to reconstruct the boundary data of the real object. The lower residual value obtained by applying NIMoBIR using RTE showed that this algorithm has a better performance to represent the measurement boundary data than the use of NIMoBIR using DE. Based on visual and residual analysis, it can be concluded that the algorithm NIMoBIR with continuous-wave domain using DE and RTE systems have the potential to reconstruct the absorption coefficient distribution of agricultural products, such as potato.

5. Conclusion
This article presents non-iterative model based image reconstruction for continuous-wave domain diffusion and radiative transport equations. Experiments using potato with the epoxy targets were conducted to demonstrate the proposed algorithm. The result of this experiment has shown that the
NIMoBIR using DE and RTE have high possibility to detect defects in agricultural products. Accuracy improvement of the reconstructed image will be conducted in future work.

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