Stress-strain state of a reinforced stepped bimodular column

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Abstract. The article discusses the reinforced stepped column under the action of forces applied along its axis. The column is clamped at the ends and made of a bimodular material, that is, a material having different elastic moduli in tension and compression. Formulas for normal stresses are obtained, depending on the mechanical characteristics of the column material, the geometrical values of the column, and the reinforcement bars. Numerical calculations were carried out for a column made of fibro-foam concrete, in which the elastic modulus of tensile strength is greater than the elastic modulus of compression. The influence of different modulus of the material on the stressed state of the column in the compressed and stretched parts was investigated. The graphs of the normal stresses as a function of the number of reinforcement bars of the stretched zone with a constant number of reinforcement bars in the compressed zone with and without bimodularity are given. It is shown that in some cases normal tensile stresses without bimodularity are 46% less than with bimodularity, and compressive normal stresses are 29% more than with bimodularity.

1. Introduction

A sufficiently large number of materials have the property of bimodularity, i.e., different values of the values of elastic moduli for tensile and compression. So, for example, bimodularity is established for numerous alloys: iron, bronze, and steel. Steel has a bimodularity that is insignificant, the difference in the values of Young's modulus during tension and compression is no more than 3-5%, in cast iron it can reach 30% or more [1,2]. Some structural materials, in particular, reinforced and non-reinforced polymers, possess the property of bimodularity. Such a widespread building material as concrete, which is the main building material, has a strongly pronounced property of bimodularity. For some types of fine-grained concrete, the modulus of tensile elasticity is two to three times less than under compression, for example, AFB-1 concrete: $E_c = 1.75 - 104$ MPa, $E_p = 0.75 - 104$ MPa. Fibro-concrete used in this work refers to types of light concretes, whose compressive modulus of elasticity is 2.2 times less than the tensile modulus of elasticity: $E_c = 2250$ MPa, $E_p = 5000$ MPa.

The founder of the calculation of different moduli of isotropic material in the theory of elasticity is Ambartsumyan S.A. [3]. His ideas were developed in the works of Russian scientists [4-7]. The range of problems in the theory of elasticity, in which it is necessary to take into account the multi-modularity of the material when calculating the strength, is rather wide. For example, in [7,8], the analysis of the elastoplastic stress–strain state of flexible layered shells of revolution under axisymmetric loading is investigated. It was shown that both the difference between the modules of stretching and compression of the outer layer, and the flexibility of the shell have a significant impact on the results.

Significant differences in the values of elastic parameters, of course, lead to significant discrepancies in the results of deformation calculations without taking into account the bimodularity property. Thus,
obtaining an accurate picture of the stress-strain state is extremely important and important in the calculations of many structures and elements of building structures.

2. Materials and methods

In this work, a stepped reinforced column, rigidly embedded at the ends (figure 1a), and loaded with a concentrated force \( F \), made of fibro-concrete.

The increase in the bearing capacity of a building structure depends on many factors, including the mechanical characteristics of the material. When static uncertainty is disclosed, the equation of strain compatibility includes a stiffness value that depends on the mechanical characteristics of the material from which the elements of this structure are made, therefore, the resulting support reactions, as a result, internal forces and stresses also depend on the mechanical characteristics of the material from which the structural element is made.

The purpose of this work is to study the effect of bimodularity of the material, and the number of reinforced rods in the stretched and compressed zones on the values of the stress-strain state in a statically indefinable stepped concrete column.

To determine the internal forces and stresses in concrete and reinforcement, we consider the static, geometric, and physical aspects of the problem. The static side of the problem is the equilibrium equation of a column

\[
\sum F_x = 0: \quad X_A + X_B = F
\]  

\( (1) \)

![Figure 1. Scheme stepped reinforced rod.](image)

The geometric side of the problem is the deformation compatibility equations

\[
\Delta l = \Delta l_I + \Delta l_{II} = 0
\]  

\( (2) \)

\[
\Delta l_I = \Delta l'_I = \Delta l''_I; \quad \Delta l_{II} = \Delta l''_{II} = \Delta l''_{II}
\]  

\( (3) \)
where $\Delta l_1$, $\Delta l_2$ are the absolute elongations of the column on the first and second sections, respectively; $\Delta l'_1$, $\Delta l''_2$ are the absolute elongations of the reinforcement bars, respectively, on the first and second sections.

The physical side of the problem is Hooke’s law for absolute deformations

$$\Delta l_1' = \frac{N_1^a l_1}{E_1^a A_1^a}; \Delta l_2' = \frac{N_2^a l_2}{E_2^a A_2^a}; \Delta l'_1 = \frac{N_1^b}{E_1^b A_1^b}; \Delta l''_2 = \frac{N_2^b}{E_2^b A_2^b}$$  \hspace{1cm} (4)

where $N_1^a, N_2^a$ are the forces in reinforcement on I and II sections; $N_1^b, N_2^b$ are the forces in concrete on I and II sections; $l_1 = a; l_2 = b$ the length of I and II sections; $E_1^a, E_2^a$ are the moduli of reinforcement of reinforcement in sections I and II; $E_1^b, E_2^b$ are the moduli of concrete at I and II; $A_1^a, A_2^a$ are areas reinforcement cross-sections in I and II sections; $A_1^b = A_1, A_2^b = A_2$ concrete cross-sections area on I and II sections.

Using the equilibrium equations of the upper part (I segment) and the lower part (II segment) (figure 1b), we express the internal efforts through the support reactions

$$N_1^0 + N_1^a = X_A; N_2^0 + N_2^a = -X_B$$ \hspace{1cm} (5)

The system of equations (1), (2), (3), and (5) concerning expressions (4) –system 6 equations with respect to $N_1^0, N_1^a, X_A; N_2^0, N_2^a, X_B$. The solution

$$N_1^b = \frac{SF}{(1+K_1)+S(1+K_1)}; N_1^a = \frac{K_1SF}{(1+K_2)+S(1+K_1)};$$ \hspace{1cm} (6)

$$N_2^b = \frac{F}{(1+K_2)+S(1+K_1)}; N_2^a = -\frac{K_2F}{(1+K_2)+S(1+K_1)};$$ \hspace{1cm} (7)

$$S = \frac{bE_1^b A_1^b}{aE_2^a A_2^a}; K_1 = \frac{E_1^a A_1^a}{E_1^b A_1^b}; K_2 = \frac{E_2^b A_2^b}{E_2^a A_2^a}$$

It is obvious that the formulas can determine the stress in concrete and reinforcement

$$\sigma_1^b = \frac{N_1^b}{A_1^b}; \sigma_1^a = \frac{N_1^a}{A_1^a}; \sigma_2^a = \frac{N_2^a}{A_2^a};$$ \hspace{1cm} (8)

3. Results and discussions

The calculation was carried out with the following initial data. Fibro-concrete concrete column material $a=1$ m; $b=2$ m; $A_1^b = 1600$ cm$^2$; $A_2^b = 3600$ cm$^2$; $d_0=1.2$ cm;

In the problem, the first section is stretched, the second is compressed; therefore, for greater clarity, it can be denoted $E_1^b = E_p$, $E_2^b = E_c$.

$$F = 300$ kN; $E_1^a = E_2^a = 2.06\times10^5$ MPa ; $A_1^a = n_p \pi (d_a)^2 / 4; A_2^a = n_c \pi (d_a)^2 / 4.$
Here, \( d \) is the diameter of the reinforcement; \( n_p \) is the number of reinforcement bars in the tension zone, \( n_c \) is the number of reinforcement bars in the compression zone.

**Figure 2.** Dependences of tension stresses \( \sigma_p \) and compression stresses \( \sigma_c \) on the number of reinforcement bars in the extension zone \( n_p \) in the absence of reinforcement bars in the compression zone, taking into account the different modulus of the material.

**Figure 3.** Dependences of tension stresses \( \sigma_p \) and compression stresses \( \sigma_c \) on the number of reinforcement bars in the tension zone \( n_p \) with the reinforcement bars \( n_c = 4 \) in the compression zone taking into account the different modulus of the material.
Figure 4. Dependences of tension stresses $\sigma_p$ and compression stresses $\sigma_c$ on the number of reinforcement bars in the extension zone $n_p$ in the absence of reinforcement bars in the compression zone, taking into account the different modulus of the material.

Figure 5. Dependences of tension stresses $\sigma_p$ and compression stresses $\sigma_c$ on the number of reinforcement bars in the extension zone $n_p$ with the presence of reinforcement bars $n_c = 4$ in the compression zone without taking into account the different modulus of the material.

As can be seen from the graphs (taking into account the bimodularity of the material), presented in figure 2, figure 3, the normal tensile stress when adding reinforcement bars only to the compressed zone decreases by 4%, the compressive normal stress decreases by 2%. With an increase in the number of reinforcing bars in the stretched zone, the normal tensile stress decreases by 1.8%; the normal compressive stress decreases by 4.5%.

If the calculation does not take into account the bimodularity of the material (figure 4, figure 5), adding reinforcing bars to the compressed zone only reduces the normal tensile stress by 5.6%, and the compressive normal stress decreases by 4.2%. With an increase in the number of reinforcing bars in the
stretched zone, the normal tensile stress decreases by 16%; the compressive normal stress decreases by 9.5%.

Calculations for the column with the same number of reinforcing bars in the stretched and compressed zones, but with and without taking into account the multi-modulus of the material (figure 3, figure 5) showed that tensile normal stresses without bimodularity are 46% less than with bimodularity, and normal compressive stresses are 29% more than with bimodularity.

With the same length of sections and with the same modulus of elasticity in tension and compression, the normal stresses in both sections will be (6) - (8), (figure 5).

Figure 6. Dependences of tension stresses $\sigma_p$ and compression stresses $\sigma_c$ on the number of reinforcing bars in the stretched zone, excluding bimodularity at $n_c = 4$ and equal lengths of sections

Figure 7. Dependences of tension stresses $\sigma_p$ and compression stresses $\sigma_c$ on the number of reinforcing bars in the stretched zone, taking into account the bimodularity at $n_c = 4$ and equal lengths of sections

For the same case, considering for bimodularity in figure 7 shows that tensile and compressive stresses are different in magnitude: excluding bimodularity, normal tensile stress is less by 63% than...
normal tensile stress taking into account bimodularity, and normal compressive stress without bimodularity is more than 27% than bimodularity.

From the graphs presented in figure 6 and figure 7, it can be seen that the dependence of normal stresses on the number of reinforcing bars located in the stretched zone, taking into account bimodularity, differs from the same dependence without bimodularity. Without bimodularity, tensile and compressive stresses (with \( n_c = 4, n_p = 7 \)) decreased by 12% simultaneously, and with bimodule, normal tensile stress decreases by 9.9%, and normal compressive stress decreases by 7.5%. Using information about the bimodularity of materials for the calculation of structural elements allows to increase the accuracy of calculations and apply these data to complex diagnosis of buildings and structures [9], measuring the elastic moduli in tension and compression, based on the influence of different modularity on the strength of structural elements [10,11].

4. Conclusions
The results obtained in the work show that the formulas for normal stresses arising in a statically indefinable stepped reinforced column taking into account the different modulus of the material from which the column is made, make it possible to investigate the stress state depending on the location of the reinforcement in the compressed and stretched zones with and without taking into account the bimodularity of the material with different parameters of the column.

As can be seen, from the numerical studies taking into account the bimodularity of the material affects not only the magnitudes of normal stresses but also qualitatively changes the picture of the stress state, which is clearly expressed for a column with sections of equal length. Accounting for the bimodularity of the material also affects the dependence of tensile and compressive normal stresses on the number and location of reinforcement bars in the first and second sections of the column.

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