Vacuum structure in a chiral $\mathcal{R} + \mathcal{R}^n$ modification of pure supergravity

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Abstract

We discuss an $\mathcal{R} + \mathcal{R}^n$ class of modified $\mathcal{N} = 1$, $D = 4$ supergravity models where the deformation is a monomial $\mathcal{R}^n|_F$ in the chiral scalar curvature multiplet $\mathcal{R}$ of the “old minimal” auxiliary field formulation. The scalaron and goldstino multiplets are dual to each other in this theory. Since one of them is not dynamical, this theory, as recently shown, cannot be used as the supersymmetric completion of $\mathcal{R} + \mathcal{R}^n$ gravity. This is confirmed by investigating the scalar potential and its critical points in the dual standard supergravity formulation with a single chiral multiplet with specific Kähler potential and superpotential. We study the vacuum structure of this dual theory and we find that there is always a supersymmetric Minkowski critical point which however is pathological for $n \geq 3$ as it corresponds to a corner ($n = 3$) and a cusp ($n > 3$) point of the potential. For $n > 3$ an anti-de Sitter regular supersymmetric vacuum emerges. As a result, this class of models are not appropriate to describe inflation. We also find the mass spectrum and we provide a general formula for the masses of the scalars of a chiral multiplet around the anti-de Sitter critical point and their relation to $osp(1, 4)$ unitary representations.

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1 Introduction

Motivated by the latest Planck mission data \cite{1,2}, there has been recently a renewed interest in $R + R^2$ bosonic theories, which realize the Starobinsky model of inflation. The supersymmetric extension of such theories depends on the off-shell degrees of freedom of supergravity. As there are two such minimal extensions, old and new minimal supergravity, there are two inequivalent ways to supersymmetrize the bosonic $R + R^2$ theory. This has been done originally in \cite{3} in the old minimal supergravity framework and in \cite{4} in new minimal formulation. These theories have the common feature of adding to pure supergravity four bosonic and four fermionic degrees of freedom, two chiral multiplets in old minimal \cite{3} and a massive vector multiplet in new minimal \cite{4}, in accordance with the linearized analysis given in \cite{14}. Recently, these theories have been considered \cite{5–11} in the light of the new constraints set by the Planck mission on inflation \cite{2}.

In the same spirit, there was an effort to supersymmetrize bosonic $f(R)$ gravity theories, called “$F(R)$ supergravity” \cite{12,13}. On the gravity side, one may eliminate either the vector auxiliary $A_\mu$ or the complex scalar auxiliary $X$ of the off-shell gravity multiplet depending on an integration by parts. Integrating out $A_\mu$ one gets a gravity theory with propagating $X$ \cite{15}. Integrating $X$, one gets a non-linear theory for $R$ and $A_\mu$ where both the scalaron and $D^\mu A_\mu$ are propagating. This theory when $A_\mu$ is neglected reduces to theory considered in \cite{12,13}. Both supergravities are dual to standard supergravity coupled to a chiral multiplet. The latter is dual to two propagating bosonic degrees of freedom on the supergravity side, depending on which one of the auxiliary fields has been integrated out. Here we will investigate the dual theory for models with higher powers of the chiral superfield curvature multiplet. We will see that for this class of models, the dual theory is standard supergravity coupled to a single chiral multiplet. The latter is dual to two propagating bosonic degrees of freedom on the supergravity side, depending on which one of the auxiliary fields has been integrated out. Here we will investigate the dual theory for models with higher powers of the chiral superfield curvature multiplet. We will see that for this class of models, the dual theory is standard supergravity coupled to a single chiral multiplet with a no-scale Kähler potential and a given superpotential. The induced scalar potential fails to have a de Sitter asymptotic regime as it has already been observed in \cite{8}. In addition, we will find the vacuum structure of the resulting supergravity and we will provide a general formula for the masses of the scalars on supersymmetric AdS vacuua.

Theories without two gravitationally generated chiral multiplets fail to reproduce the linearized analysis of \cite{14} and, as shown in \cite{15}, $R + R^2$ or $R^n$ power modifications of Einstein supergravity. Models which are the supersymmetric completion of $R + R^2$ and reproduce the Starobinsky model are not unique since they allow a Kähler potential and superpotential for the goldstino multiplet \cite{3}. Actually, as shown in \cite{6} and \cite{15}, non minimal Kähler potential modifications for the goldstino are required for the inflaton flow to be a stable direction in field space. The same class of models have been recently revisited and further investigated by the proponents of $F(R)$ supergravity \cite{16}.

In the next section 2, we review modifications of gravity by higher curvature terms in the old
minimal formulation. In section 3, we discuss particular $\mathcal{R} + \mathcal{R}^n$ modifications and the structure of the vacuum of the dual standard supergravity. In section 4, we calculate the mass spectrum of a supergravity theory coupled to a single chiral multiplet around a supersymmetric AdS vacuum and we identify it with unitary representations of $osp(1,4)$. Finally, we conclude in section 5.

2 Modified Supergravity By Higher Curvature Terms

The $R + R^2$ theory, as is it revealed by a linearized analysis [14] contains the degrees of freedom of two chiral multiplets. Three of them come from the Einstein supergravity auxiliaries $X, \partial^\mu A_\mu$ where [17–19]

$$X = \frac{1}{3}u = \frac{1}{3}(S - iP), \quad (1)$$

and the fourth comes from the scalar curvature $R$. The minimal $R + R^2$ theory is given by

$$L_{R+R^2} = \gamma [S_0 \bar{S}_0]_D + \alpha [\mathcal{R} \bar{\mathcal{R}}]_D, \quad (2)$$

where the two D-terms above are the supersymmetric extension of the $R + R^2$ bosonic theory. Here $S_0$ is the compensator chiral superfield, with scaling weight and chiral weight equal to 1, the curvature chiral superfield $\mathcal{R}$ has scaling and chiral weight equal to 1 as well, and $[O]_{D,F}$ are the standard D- and F-term density formulae of conformal supergravity, where $O$ is a real superfield with scaling weight 2 and vanishing chiral weight. The bosonic components of the curvature chiral scalar multiplet $\mathcal{R}$ are

$$\mathcal{R} = \bar{X} + \cdots + \theta^2 \mathcal{F}_R, \quad (3)$$

where

$$\mathcal{F}_R = -\frac{1}{2}R - 3A_\mu^2 + 3i D^\mu A_\mu. \quad (4)$$

Let us also note that $\mathcal{R}/S_0$ is of zero chiral and Weyl weight and its bosonic content is

$$\mathcal{R}/S_0 = \bar{X} + \cdots + \theta^2(\mathcal{F}_R - 18X\bar{X}). \quad (5)$$

There is an alternative modified supersymmetric action, considered in the literature [12] as an alternative “$f(R)$” action, given by the F-term
\[ \mathcal{L}_{f(R)} = [F\left(\mathcal{R}/S_0\right)S_0^3]_F, \]  

whose linear and constant terms are representing the Einstein term and a cosmological constant. The other higher order terms make only one of the two chiral multiplet degrees of freedom propagating and therefore cannot describe \( R + R^2 \) gravity. As shown in [15], the bosonic part of this action (including all auxiliary fields \( X, A_\mu \)) is

\[ \mathcal{L}_{\text{bos}} = -\frac{1}{2} \sqrt{-g} \left\{ 27XF(\bar{X}) - 18F'(\bar{X})XX + F'(\bar{X})F_R \right\} + \text{h.c.}. \]

By noticing that \( \text{Im} \mathcal{F}_R = 3\partial^\mu A_\mu \) and integrating by parts this term we can solve for \( A_\mu \). Then in eq. (7) we find that the scalar \( X \) is propagating [15]

\[ \mathcal{L}_{\text{bos}} \bigg|_{\delta \mathcal{L}/\delta A_\mu = 0} = \frac{1}{2} \sqrt{-g} \left[ \mathcal{R} - \frac{3}{4} \frac{1}{\text{Re} F'(\bar{X})} \left\{ \left( \partial_\mu \text{Re} F'(\bar{X}) \right)^2 + \left( \partial_\mu \text{Im} F'(\bar{X}) \right)^2 \right\} \right. 
\left. + \sqrt{-g} \left( -27 \text{Re}(XF(\bar{X})) + 18 \text{Re}(F'(\bar{X}))XX \right) \right]. \]  

This Lagrangian has a gravity dual with a seemingly different action involving non-linear \( R \) terms and the propagating \( \partial^\mu A_\mu \) auxiliary scalar. The dual action is obtained by integrating \( X \) in eq. (7) and leads to

\[ \mathcal{L}_{\text{bos}} \bigg|_{\delta \mathcal{L}/\delta X = 0} = \mathcal{L}_{\text{bos}}^D(\mathcal{F}_R, \bar{\mathcal{F}}_R). \]

If we set \( A_\mu = 0 \) so that \( \mathcal{F}_R = -R/2 \), we get the non-linear R-theory constructed in [12, 13]. The dual gravity theories described by eqs. (8,9) are both dual to a standard supergravity of a self interacting chiral multiplet with a superpotential term. We are going to investigate the vacuum of the latter for a particular class of models where the superpotential can easily be computed.

From eq. (8) it is obvious that the physical chiral multiplet is \( \Lambda = F'(\bar{X}) \) so a Legendre transform can be performed to express the theory in \( \Lambda \) (rather that \( \bar{X} \)) variables, and to find its superpotential. To explicitly show this, one may consider an action in superconformal calculus and in the old minimal supergravity context, of the form

\[ \mathcal{L} = -[S_0 \bar{S}_0]_D + [S^3_0 f(\mathcal{R}/S_0)]_F. \]

This theory can be obtained from

\[ \mathcal{L}_D = -[S_0 \bar{S}_0]_D + [\Lambda \left( A - \frac{\mathcal{R}}{S_0} \right) S^3_0]_F - [S^3_0 f(A)]_F. \]
Indeed, integrating out the Lagrange multiplier superfield $\Lambda$ in (11), we get back the original theory (10). However, by using the identity \[3,15\]

\[
[\Lambda RS_0^2]_F = [(\Lambda + \bar{\Lambda})S_0\bar{S}_0]_D,
\]

we may write (11) as

\[
L_D = -(1 + \Lambda + \bar{\Lambda})S_0\bar{S}_0]_D + [(\Lambda A - f(A))S_0^3]_F.
\]

By integrating out the chiral Lagrange multiplier $A$, we obtain the dual action

\[
L_D\bigg|_{\frac{\delta L_D}{\delta A}} = -(1 + \Lambda + \bar{\Lambda})S_0\bar{S}_0]_D + [W(\Lambda)S_0^3]_F,
\]

where

\[
W(\Lambda) = (Af'(A) - f(A))\bigg|_{f'(A) = \Lambda}.
\]

3 $R^n$ Modification of Supergravity

As we have seen above, the theory (6) can be described in a dual formulation by standard supergravity coupled to a single chiral multiplet. Although the discussion could be kept general, we will consider here the case

\[
f(A) = \varepsilon_n A^n,
\]

which corresponds to the choice $(F = -1 - f)$

\[
F(R/S_0) = -R/S_0 + \varepsilon_n(R/S_0)^n.
\]

In this case we get the superpotential

\[
W(\Lambda) = \lambda_n \Lambda^{\frac{n}{n-1}}, \quad \lambda_n = \varepsilon_n^{\frac{1}{n-1}}(n-1)n^{\frac{n}{n-1}}
\]

and after defining $\Lambda$ as

\[
\Lambda = T - \frac{1}{2},
\]

the theory is described by

\[
\mathcal{L} = -[(T + \bar{T})S_0\bar{S}_0]_D + \lambda_n[(T - \frac{1}{2})^{\frac{n}{n-1}}S_0^3]_F.
\]
In other words, the theory has been turned into standard supergravity with a no-scale Kähler potential \[ K = -3 \log(T + \bar{T}), \quad \text{Re} \, T > 0 \] (21)

and superpotential

\[ W(T) = \lambda_n \left( T - \frac{1}{2} \right)^{\frac{n}{n-1}} = \frac{\lambda_n}{2^{\frac{n}{n-1}}} (C - 1 + iB)^{\frac{n}{n-1}}, \]

(22)

where we have parametrized \( T \) as \( T = (C + iB)/2 \) so \( C > 0 \). It is straightforward now to calculate the potential using the standard formula

\[ V = \frac{1}{(T + \bar{T})^2} \left\{ \frac{1}{3} |T + \bar{T}| |W_T|^2 - W\bar{W}_T - \bar{W}W_T \right\}. \]

(23)

Explicitly, we have

\[ V = \frac{\lambda_n^2}{(T + \bar{T})^2} \frac{n}{n-1} |T - \frac{1}{2}|^{\frac{2}{n-1}} \left\{ \frac{n}{n-1} \frac{T + \bar{T}}{3} - (T + \bar{T}) + 1 \right\}, \]

(24)

and in terms of \( C, B \), the potential \( V \) is

\[ V = \frac{\lambda_n^2}{4 \pi^2} \frac{n}{n-1} \frac{1}{C^2} \left\{ (C - 1)^2 + B^2 \right\} \frac{1}{n-1} \left\{ C \frac{3 - 2n}{3(n-1)} + 1 \right\}. \]

(25)

The form of the potential for \( n = 2, 3 \) and \( n > 3 \) has been plotted in Figure 1. Note that the canonically normalized scalar is \( \phi \) defined by \( C = 2 \text{Re} \, T = e^{\sqrt{2} \phi} \).

In order to find the supersymmetric vacua of the theory, one should look for the solutions of

\[ D_TW = \partial_TW + K_TW = 0, \]

(26)

which are \( B = 0 \) and

\[ \frac{1}{2^{\frac{1}{n-1}}} \left( C - 1 \right)^{\frac{1}{n-1}} \left\{ \frac{n}{n-1} - \frac{3}{2C} (C - 1) \right\} = 0. \]

(27)

Eq. (27) has two solutions:

\[ W = \partial_TW = 0, \quad (V = 0), \quad C = 1, \quad B = 0, \quad (\text{Minkowski}), \]

(28)
Figure 1: The scalar potential $V(C)$ at $B = 0$ and $n = 2, 3$ and $n > 3$ is given in the left figure. The right figure magnifies the region near $C = 1$, where the “corner” and the cusp are easily recognized for $n = 3$ and $n > 3$, respectively.

and

$$W \neq 0, \quad (V = -3e^G) \quad G = K + \log |W|^2, \quad (\text{AdS}). \quad (29)$$

A solution to eq. (29) exists for $n > 3$ and it is explicitly given by

$$C = \frac{3(n - 1)}{n - 3}, \quad B = 0, \quad n > 3, \quad \text{AdS vacuum.} \quad (30)$$

Note that the zeros of the potential $V = 0$ are at

$$C = 1, \quad B = 0 \quad \text{and} \quad C = \frac{3(n - 1)}{2n - 3}, \quad B = 0. \quad (31)$$

In order to explicitly study the vacuum structure of the theory, we should distinguish three cases according to the asymptotic behaviour of the potential for large values of the fields $C$. As $C \to \infty$ we may have: I) $V \to -\infty$, II) $V \to -\frac{3}{8}$ and III) $V \to 0^-$. These cases correspond to: I) $n = 2$, II) $n = 3$ and III) $n > 3$, respectively. In the case I), there exists a local minimum at $C = 1$ where the potential vanishes and corresponds, as we will see, to a supersymmetric Minkowski vacuum. There is also a maximum at $C = 2$ which is not supersymmetric. In the $n = 3$ case, there exists a supersymmetric Minkowski vacuum at $C = 1$, which is now a “corner” i.e. a point where the first derivative has a finite discontinuity. There exists also a non-supersymmetric maximum which is at $C = 4/3$. Finally, for $n > 3$ there exists the supersymmetric Minkowski vacuum at $C = 1$, which is now a cusp, the non-supersymmetric maximum at

$$C_1 = \frac{2(n - 1)}{2n - 3}, \quad B_1 = 0, \quad (32)$$
and a supersymmetric one at

\[ C_2 = \frac{3(n-1)}{n-3}, \quad B_2 = 0. \]  

(33)

The above vacuum structure has been tabulated in Table 1.

| \( n \) | SUSY mimima \( V \leq 0 \) | non-SUSY maxima \( V > 0 \) |
|---------|-----------------|-----------------|
| \( n = 2 \) | \( C = 1 \), Minkowski | \( C = 2 \) |
| \( n = 3 \) | \( C = 1 \), Minkowski (corner) | \( C = \frac{4}{3} \) |
| \( n > 3 \) | \( C = 1 \), Minkowski (cusp), \( C = \frac{2(n-1)}{2n-3} \) | \( C = \frac{2(n-1)}{2n-3} \) |

Table 1: Supersymmetric and non-supersymmetric critical points of the potential \( V \).

The masses of the \( C, B \) fields are given in the two cases as (with \( \lambda_n = 1 \))

\[
m^2_{C_1} = 2K_{TT}^{-1}V_{CC}\bigg|_{C_1,B_1} = -\frac{2^{\frac{2}{n-1}}n(4n-3)(2n-3)^{\frac{2(n-2)}{n-1}}}{9(n-1)^3} < 0, \tag{34}
\]

\[
m^2_{B_1} = 2K_{TT}^{-1}V_{BB}\bigg|_{C_1,B_1} = \frac{2^{\frac{2}{n-1}}n(2n-3)^{\frac{2(n-2)}{n-1}}}{9(n-1)^2} > 0, \tag{35}
\]

and

\[
m^2_{C_2} = 2K_{TT}^{-1}V_{CC}\bigg|_{C_2,B_2} = \frac{n^{\frac{2}{n-1}}}{27(n-1)^3}(4n-3)(n-3)^{\frac{2(n-2)}{n-1}} > 0, \tag{36}
\]

\[
m^2_{B_2} = 2K_{TT}^{-1}V_{BB}\bigg|_{C_2,B_2} = -\frac{n^{\frac{2}{n-1}}}{3(n-1)^2}(n-3)^{\frac{n-2}{n-1}} < 0. \tag{37}
\]

Note that we have multiplied the second derivatives of the potential by \( 2K_{TT}^{-1} \) in order to canonically normalize the kinetic terms of \( C, B \).

We should mention here that the \( C = 1 \) Minkowski point \([28]\) is quite particular. Namely, although it is a normal local minimum for \( n = 2 \), it is a point with discontinuous first derivative (corner) for \( n = 3 \) and singular first derivative for \( n > 3 \) (cusp). This can explicitly be seen in Figure 1, where the potential is depicted around \( C = 1 \) for the three cases. As a result, the scalar equations are not satisfied at this point for \( n \geq 3 \) although it is a supersymmetric critical point.

The values of the potential \( V \) and its first derivative \( V_C \) at the supersymmetric points have been tabulated in the following Table 2.
|           | Minkowski | AdS         |
|-----------|-----------|-------------|
| $n = 2$   | $n = 3$   | $n > 3$     |
| $V$       | 0         | 0           |
| $V_C$     | 0         | $\pm \frac{3}{8}$ | $\infty$ | 0 |

Table 2: The value of the potential $V$ and its first derivative $V_C$ at the supersymmetric critical points

On the other hand, (30) corresponds to an AdS vacuum since at this point $W \neq 0$ and $V < 0$. Therefore, we expect that (36,37) to be the masses of unitary representations of the $AdS_4$ simple superalgebra $osp(1,4)$. To see this, we will discuss in the next section the more general case of a single chiral multiplet in supergravity.

4 Supersymmetric AdS vacua and masses of unitary multiplets of $osp(1,4)$

Let us consider the general form of the $\mathcal{N} = 1$ scalar potential of a single multiplet $z$

$$V = e^G \left( G_z G_{\bar{z}} G_z^{-1} - 3 \right).$$  \hspace{1cm} (38)

Since

$$V_z = e^G G_z^2 G_{\bar{z}} G_z^{-1} + e^G G_{\bar{z}} G_z G_{\bar{z}}^{-1} + e^G G_z + e^G G_z G_{\bar{z}} (G_{\bar{z}}^{-1})_{\bar{z}} - 3 e^G G_z,$$  \hspace{1cm} (39)

it is easy to see that critical points of the potential are points where

$$G_z = G_{\bar{z}} = 0.$$  \hspace{1cm} (40)

These correspond to supersymmetric $AdS_4$ vacua with cosmological constant

$$\Lambda = V \bigg|_{G_z = G_{\bar{z}} = 0} = -3 e^G.$$  \hspace{1cm} (41)

The $AdS_4$ scalar curvature is $R = -12 L_{AdS}^{-2}$ where the $AdS_4$ radius $L_{AdS}$ is

$$L_{AdS}^2 = -\frac{3}{\Lambda} = e^{-G}.$$  \hspace{1cm} (42)
and the Breitenlohner-Freedman bound in $d+1$-spacetime dimensions

$$m^2 L^2_{AdS} \geq -\frac{d^2}{4}$$

is written in our case as

$$m^2 \geq -\frac{9}{4},$$

in units of $e^G$. At the critical points (40), we find that

$$V_{zz} = -e^G G_{zz}, \quad V_{\bar{z}z} = -2e^G G_{\bar{z}z} + e^G G_{zz} G_{\bar{z}z} G^{-1}_{z\bar{z}}$$

and therefore, after multiplying with $G^{-1}_{z\bar{z}}$ we get

$$V_{zz} G^{-1}_{z\bar{z}} = -e^G A,$$

$$V_{\bar{z}z} G^{-1}_{z\bar{z}} = e^G (-2 + |A|^2),$$

where

$$A = G_{zz} G^{-1}_{z\bar{z}}.$$

In terms of the real and imaginary parts of $z$ ($C = \sqrt{2} \Re z, \ B = \sqrt{2} \Im z$) we may write

$$e^{-G} \frac{1}{2} \left( V_{CC} + V_{BB} \right) = -2 + |A|^2,$$

$$e^{-G} \frac{1}{2} \left( V_{CC} - V_{BB} \right) = -\Re A,$$

$$e^{-G} V_{CB} = -\Im A$$

and thus, the mass matrix turns out to be, in $e^G$ units

$$\mathcal{M}^2 = \begin{pmatrix} V_{CC} & V_{CB} \\ V_{CB} & V_{BB} \end{pmatrix} = \begin{pmatrix} -2 + |A|^2 - \Re A & -\Im A \\ -\Im A & -2 + |A|^2 + \Re A \end{pmatrix}.$$ (52)

By diagonalizing the mass matrix, we find that the mass eigenvalues are ($m^2_C > m^2_B$)

$$m^2_B = (|A| + 1)(|A| - 2),$$ (53)

$$m^2_C = (|A| - 1)(|A| + 2),$$ (54)

($m^2_B > m^2_C$, eq.(53) and eq.(54) are interchanged) so that

$$|A| \geq 2, \quad m^2_B \geq 0, \quad m^2_C > 0, \quad 1 \leq |A| < 2, \quad m^2_B < 0, \quad m^2_C \geq 0, \quad |A| < 1, \quad m^2_C, m^2_B < 0.$$ (55)
Particular values are

\[ \begin{align*}
|A| &= 2, \quad m_B^2 = 0, \quad m_C^2 = 4, \\
|A| &= 1, \quad m_B^2 = -2, \quad m_C^2 = 0, \quad G_{zz} = G_{\bar{z}\bar{z}} = G_{\bar{z}z}, \\
|A| &= 0, \quad m_B^2 = -2, \quad m_C^2 = -2, \quad G_{zz} = 0, \quad G = z\bar{z}.
\end{align*} \]  

(56)

By introducing

\[ E_0 = |A| + 1, \]

(57)

the mass spectrum may be expressed as

\[ \begin{align*}
m_B^2 &= E_0(E_0 - 3), \\
m_C^2 &= (E_0 - 2)(E_0 + 1).
\end{align*} \]  

(58)

We recognize in (58) the masses of the scalars in the Wess-Zumino unitary representation of \( osp(1,4) \)

\[ D(E_0, J) \oplus D(E_0 + 1, 0), \]

(59)

where \( D(E_0, J) \) are unitary representations of \( so(2,3) \) with energy \( E_0 \) and spin \( J \).

It can easily be checked that eqs. (36, 37) are given by (53), respectively. In particular, the \( AdS_4 \) vacuum at \( C = \frac{3(n-1)}{n-3} \) for \( n > 3 \) has \( |A| = (2n - 3)/n \). Therefore \( 1 < |A| < 2 \) in this case and thus \( m_B^2 < 0, \quad m_C^2 > 0 \) in accordance with (36, 37). Note also that although \( m_B^2 < 0 \) we have

\[ m_B^2 = -\frac{9(n-1)}{n^2} > -\frac{9}{4}, \quad n > 3 \]

(60)

and thus, the Breitenlohner-Freedman bound is satisfied.

## 5 Conclusions

We have discuss here a particular \( F(R) \) supergravity [12, 13], namely the \( F(R) = R + R^n \) class of supergravity models. We found that this theory is not the supersymmetric completion of \( R + R^n \), since it does not contain two chiral multiplets. In the old minimal formulation, such theory contains extra degrees of freedom for \( n > 2 \) [3]. By introducing appropriate Lagrange multiplier chiral superfields, we found the dual theory, which describes a single chiral superfield coupled to supergravity with no-scale Kähler potential and a superpotential term. We discussed the vacuum structure of this theory and we found that it has always a supersymmetric Minkowski local minimum for any \( n > 1 \) and a anti-de Sitter vacuum for \( n > 3 \). However, only for \( n = 2 \)
this local minimum corresponds, strictly speaking, to a vacuum. The reason is that for $n = 3$ this local minimum is a “corner” of the potential whereas, for $n > 3$, it is a cusp point. As a result, the second derivative of the potential, which enters the classical equations of motion for the scalar, has either a delta-function peak ($n = 3$), or, it is not defined at all ($n > 3$). This makes the interpretation of this point as a vacuum questionable. On the other hand, we found that the theory possess a global supersymmetric Anti de Sitter minimum for $n > 3$\footnote{It is a minimum because all scalar masses obey the Breitenlohner-Friedmann bound.}. We calculated the masses of the scalar fluctuations around the anti-de Sitter vacuum and we found that they agree with the masses of the scalars of the Wess-Zumino unitary representation of the simple superalgebra $osp(1,4)$.

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