Communication through measurements and unitary transformations

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Abstract

It is generally accepted that no ‘faster than light signalling’ (FTLS) using two entangled spin 1/2 particles is possible because of indeterminism in a quantum measurement and linearity of standard quantum mechanics. We show how in principle one bit of information could be transmitted using local measurements and a global unitary transformation of the state of two entangled spatially separated spin 1/2 particles. Assuming that the postulate of a state collapse due to measurement is valid, the no FTLS condition is saved if we do not have physical access to the required global unitary transformation. This means that the no FTLS condition is also present on the operational level, namely as imposing a physical restriction on the possible realizable unitary transformations, in this case of two entangled but spatially separated spin 1/2.

1 Introduction

The implications of an instantaneous collapse of the state of a quantum system of two entangled particles due to a measurement has been a matter of debate from the very beginning of quantum mechanics \cite{1, 2, 3, 4, 5}. If two spin 1/2 particles are in a singlet state, a local measurement on one of the particles provokes an immediate change of the state (of the composite system and therefore also the state) of the other particle, even if the two particles are spatially separated. However, no ‘faster than light signalling’ (FTLS) using local measurements on a pair of two entangled spin 1/2 particles is possible because one does not control the collapse of the spin. Gisin \cite{6, 7}, reacting on the proposal of a nonlinear quantum mechanics by Weinberg \cite{8, 9}, put forward a clever way of sending signals by a system of entangled spins in the case a nonlinear evolution would be available, by using the entanglement of the reduced density states and by coding

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one bit of information in the chosen measurement direction. In linear quantum theory, this is not possible because the reduced density states are independent of the chosen measurement direction such that the receiver cannot decide by a local measurement which choice the sender has made. Therefore, only in non linear modifications of quantum theory superluminal signalling could be possible. This suggests that the no FTLS condition could be used as a physical motivation for the linear structure of quantum mechanics [10,11,12,13,14]. However, non linear generalizations of quantum theory are possible in which the no FTLS condition does hold [15]. Also, choosing a suitable nonlinear gauge transformation [16] one can always ‘disguise’ a linear evolution equation into a nonlinear one. Therefore, non linearity is identified as a necessary but not a sufficient condition for FTLS.

In this paper we present a thought experiment which applies standard (linear) quantum mechanics on a system of two entangled spin 1/2, with a state evolution described by unitary transformations, and an instantaneous collapse of the state if a measurement is performed. The thought experiment uses two unitary transformations which are well-known in the theory of quantum computation, namely the Hadamard gate and the Controlled NOT gate. Hence it is natural to present our thought experiment with the concepts of quantum computation (e.g. [17]) and to talk about ‘qubits’ rather than ‘spin 1/2 particles’. First, we briefly recall some basic properties of the Hadamard and the Controlled NOT gate to keep this paper self-contained. Next, we show how to transmit the value of a bit by performing local measurements and a unitary transformation (Controlled NOT) on a system of two entangled (but possibly spatially separated) qubits.

2 Signaling via CNOT and local measurements

2.1 Some unitary transformations used in quantum computation

The Hadamard gate $H$ is a unitary operation acting on a single qubit, mapping the vector $|0\rangle$, respectively $|1\rangle$, into the superposition vector $H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$, respectively $H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$. The Controlled NOT gate $CNOT$ is a unitary operation acting on two qubits mapping $|i,j\rangle$ onto $CNOT|i,j\rangle = |i,j \oplus i\rangle$, with $\oplus$ addition modulo 2. An intuitive view of this gate is that the value $j$ of the ‘target’ (second) bit is changed into its inverse $j \oplus 1$ whenever the ‘control’ bit $i$ has value 1, and is left unchanged if the control bit $i$ has value 0. Nevertheless, one should keep in mind that this is only an intuitive view of a unitary operation which acts on the two qubit system as a whole. Indeed, in the basis $\{ |0\rangle' = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, |1\rangle' = \frac{|0\rangle - |1\rangle}{\sqrt{2}\rangle} \}$ the roles of ‘control’ and ‘target’ qubit are switched, such that $CNOT|i',j\rangle = |i' \oplus j',j\rangle$; e.g. $CNOT|0',1\rangle = |1',1\rangle$ and so on, showing that only for a fixed choice of basis we can interpret the action of the CNOT gate in terms of a control and a target qubit. Finally, we
remark that $H = H^{-1}$ and $CNOT = CNOT^{-1}$.

2.2 FTLS thought experiment

Let the system of two qubits be prepared in the ground state $|00\rangle$. After a Hadamard transformation is applied to the first qubit, a $CNOT$ is applied with the first qubit as ‘control’ bit and the second qubit as ‘target’ bit. The state of the two entangled qubits is given by $\psi_A$:

$$\psi_A = \text{CNOT} \left( (H \otimes 1) |00\rangle \right)$$

$$= \text{CNOT} \left( \frac{|00\rangle + |10\rangle}{\sqrt{2}} \right)$$

$$= \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

which is a maximally entangled state. Next, we assume that the two qubits are spatially separated but stay in the entangled spin state $\psi_A = \frac{|0_B0_A\rangle + |1_B1_A\rangle}{\sqrt{2}}$ such that Alice, the sender, has access to the second qubit, and Bob, the receiver, has access to the first qubit. If Alice wants to send a bit value 1 she performs a spin measurement in the computational basis $\{|0_A\rangle, |1_A\rangle\}$. If Alice wants to send a bit value 0 she does no measurement at all (hence not provoking a state collapse). Let us denote the state after Alice has made her choice by $\psi_A'$. Next, a ‘restoring procedure’ is established by applying $CNOT^{-1} = CNOT$ and $H^{-1} \otimes 1 = H \otimes 1$ on the two qubits system (we assume that this is possible, and discuss its validity and consequences in next section). This state we denote by $\psi_B$. Finally, Bob who has access to the second qubit performs a spin measurement in his computational basis $\{|0_B\rangle, |1_B\rangle\}$. There are three possible events:

1) Alice does not perform a measurement (sending a bit value 0) and the ‘restoring procedure’ maps the state $\psi_A = \psi_A'$ back into the original initial state $\psi_B = |00\rangle$:

$$\psi_B = (H \otimes 1) \left( \text{CNOT} \left( \frac{|00\rangle + |11\rangle}{\sqrt{2}} \right) \right)$$

$$= \frac{(H \otimes 1) |00\rangle + |10\rangle}{\sqrt{2}} = |00\rangle$$

such that Bob obtains with certainty the outcome ‘0’ in his (local) measurement and the outcome 1 has zero probability to occur.

2) Alice does perform a measurement (sending a bit value 1) and has observed the outcome 1. The state $\psi_A$ of the two entangled qubits has collapsed in the state $\psi_A' = |11\rangle$. After applying the ‘restoring procedure’, the state $\psi_B$ prior to the measurement by Bob is given by:

$$\psi_B = (H \otimes 1) \text{CNOT} (|11\rangle)$$

$$= \frac{|00\rangle - |10\rangle}{\sqrt{2}}$$


such that Bob observes the outcome 0 with probability 1/2 or the outcome 1 with probability 1/2.

3) Alice does perform a measurement (sending a bit value 1) and has observed the outcome 0. The state $\psi_A$ of the two entangled qubits has collapsed in the state $\psi_A' = |00\rangle$. After applying the ‘restoring procedure’, the state prior to the measurement by Bob is given by:

$$\psi_B = (H \otimes 1) CNOT (|00\rangle)$$

$$= (H \otimes 1) |00\rangle$$

$$= \frac{|00\rangle + |10\rangle}{\sqrt{2}}$$

such that again Bob observes the outcome 0 with probability 1/2 or the outcome 1 with probability 1/2.

To conclude, if Bob observes an outcome 1, he knows with certainty that Alice wanted to send a bit value 1. If Bob observes the outcome 0, Bob cannot decide with certainty which of the three events has happened (i.e., whether Alice has performed a measurement or not). However, Alice and Bob could use this procedure on a number $N$ of pairs of entangled qubits. E.g., for $N = 10$, the probability that Alice performs a measurement (wanting to send bit value 1) but Bob observes an outcome 0 in each of his 10 spin measurements, causing him to assume that actually a bit value 0 was sent, is $(0.5)^{10} \approx 0.1\%$. Combining this redundancy technique with classical bit correction techniques, this procedure could be used to transmit bits of information with probability of Bob correctly receiving the bit arbitrary close to unity.

2.3 Discussion

The thought experiment shows how it is in principle (mathematically) possible using unitary transformations and state collapse due to measurement to transmit one bit of information with probability of Bob correctly receiving the bit arbitrary close to unity. Hence, if one wants to maintain the no FTLS condition in physical reality, one of the assumptions made in the thought experiment has to be physically impossible. One possibility is to drop the assumption that the collapse of the state due to measurement is real. Another possibility is that not all unitary transformations used in the thought experiment can be performed in reality. Since the Hadamard gates are local unitary gates, working on a single qubit, the only ‘impossible’ unitary transformation should be the CNOT gate acting on two spatially separated qubits. This means that the no FTLS condition is not only determined by the linearity of standard quantum mechanics (equipped with the state collapse postulate), but is also present on the operational level, namely as imposing a physical restriction on the possible realizable unitary transformations of a quantum system. It means that certain (which appear to be mathematically in principle possible) unitary transformations are physically impossible.
3 Conclusions

We have shown that in theory bits of information could be transmitted using local measurements and a global unitary transformation on a system of two entangled (but spatially separated) qubits, following the rules of standard linear quantum mechanics with an instantaneous collapse of the state due to measurement. Although it is impossible to control the collapse of the state in a quantum measurement, whether a collapse has actually occurred or not does make a difference. Therefore, the thought experiment shows that the no FTLS condition is not just a consequence and possible physical justification of the linearity of quantum mechanics, but also translates onto the operational level as the impossibility to perform a certain global unitary transformation (in this case the CNOT) on a pair of spatially separated qubits.

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