Updated results on neutrino mass and mass hierarchy from cosmology with Planck 2018 likelihoods

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Abstract. Current upper bounds on the sum of 3 active neutrino masses, $\sum m_\nu$ from analyses of cosmological data in the backdrop of $\Lambda$CDM+$\sum m_\nu$ model are close to the minimum sum of neutrino masses required by the inverted hierarchy, which is around 0.1 eV. However, these analyses are usually done with the assumption of degenerate masses, which is not a good approximation any more since the bounds are strong enough that the neutrino mass-squared splittings can no longer be considered negligible. In this work we update the bounds on $\sum m_\nu$ from latest publicly available cosmological data and likelihoods while explicitly considering particular neutrino mass hierarchies. In the minimal $\Lambda$CDM+$\sum m_\nu$ model with most recent CMB data from Planck 2018 TT,TE,EE, lowE, and lensing; and BAO data from BOSS DR12, MGS, and 6dFGS, we find that at 95\% C.L. the bounds are: $\sum m_\nu < 0.121$ eV (degenerate), $\sum m_\nu < 0.146$ eV (normal), $\sum m_\nu < 0.172$ eV (inverted); i.e., the bounds vary significantly across the different mass orderings. Also, we find that the normal hierarchy is very mildly preferred relative to the inverted: $\Delta \chi^2 \equiv \chi^2_{\text{NH}} - \chi^2_{\text{IH}} = -0.95$ (best-fit). In this paper we also provide bounds on $\sum m_\nu$ considering different hierarchies in various extended cosmological models: $\Lambda$CDM+$\sum m_\nu + r$, $\omega$CDM+$\sum m_\nu$, $w_0w_a$CDM+$\sum m_\nu$, $w_0w_a$CDM+$\sum m_\nu$ with $w(z) \geq -1$, $\Lambda$CDM+$\sum m_\nu + \Omega_k$, and $\Lambda$CDM+$\sum m_\nu + A_{\text{Lens}}$. We do not find any strong evidence of normal hierarchy over inverted hierarchy from looking at the $\chi^2$ values in the extended models either. However the mass bounds do differ across different hierarchies in the extended models also. In particular, using the (unphysical) degenerate approximation leads to more aggressive constraints than in the normal or inverted hierarchies, and gives a wrong notion about how strong the bounds really are.
1 Introduction

Earth based neutrino oscillation experiments [1–8] have confirmed that neutrinos are massive, which is the first known departure from the Standard Model of particle physics where neutrinos were considered to be massless. The 3 neutrino flavor states (ν_e, ν_μ, ν_τ) are quantum superpositions of the 3 mass eigenstates (ν_i, with respective distinct masses, m_i for i = 1, 2, 3). However, because oscillation experiments use ultra relativistic neutrinos they are only sensitive to the mass-squared splittings (Δm^2_{ij} = m_i^2 - m_j^2) and not the absolute masses, thus keeping the mass of the lightest neutrino unbounded. On the other hand, while magnitudes of Δm^2_{21} and Δm^2_{31} are known to considerable accuracy from the current oscillation data, sign of Δm^2_{31} is unknown. This leads to two possible hierarchies of neutrino masses: m_1 < m_2 ≪ m_3 (normal hierarchy or NH) and m_3 ≪ m_1 < m_2 (inverted hierarchy or IH) depending on whether Δm^2_{31} is positive or negative, respectively. As per the latest NuFit 4.0 [9] global analysis of oscillations data, the values of the mass-squared splittings (in units of eV^2) are (limits are given at 1σ):

\[ \Delta m^2_{21} = 7.39^{+0.21}_{-0.20} \times 10^{-5}, \quad \Delta m^2_{31} = 2.525^{+0.03}_{-0.032} \times 10^{-3} \text{(NH)}; \quad \Delta m^2_{32} = -2.512^{+0.034}_{-0.032} \times 10^{-3} \text{(IH)}. \] (1.1)

Here, the value for Δm^2_{21} is applicable to both NH and IH, while the other values are for the particular hierarchies as mentioned in the brackets. It is to be noted that for IH, Δm^2_{32} is provided (whose value is negative) instead of Δm^2_{31}, but since Δm^2_{31} ≪ Δm^2_{32}, sign of Δm^2_{31} for IH is also negative (since Δm^2_{31} = Δm^2_{32} + Δm^2_{21}). See [10–15] for results from other global analyses.

A solution to the neutrino mass hierarchy problem may come from cosmology, which currently provides the strongest bounds on the absolute neutrino mass scale, defined as the sum of the three active neutrino masses,

\[ \sum m_\nu = m_1 + m_2 + m_3. \] (1.2)

As far as known physics goes, at temperatures \( T \gg \text{MeV} \), neutrinos remain in equilibrium with the primordial plasma through the standard model weak interactions. At around \( T \sim \text{MeV} \) neutrinos decouple from the plasma and start free streaming. When neutrinos are relativistic (\( T \gg m_\nu \)) they contribute to the radiation energy density. This continues until
much later when they turn non-relativistic at temperatures $T \sim m_\nu$, and then they contribute to the matter energy density. Effects of massive neutrinos on cosmological observables have been extensively studied in the literature [16–22] and these effects help in constraining the sum of neutrino masses. If we consider neutrinos with masses $\ll 1$ eV, at the time of photon decoupling they are still relativistic, and their mass has very limited effect on the photon perturbations and their evolution. Hence, for the primary CMB signal, the effect of small neutrino masses can only be seen through the background evolution, and secondary anisotropies like Integrated Sachs-Wolfe (ISW) effect, and these too can be compensated partially by varying other free parameters in the $\Lambda$CDM model. Thus, strong bounds on $\sum m_\nu$ cannot be obtained with CMB power spectrum data alone. On the other hand, at late times, neutrinos affect the evolution of matter perturbations to a large extent. Due to the free-streaming effect of neutrinos, i.e. large thermal velocities, neutrinos do not cluster on small length scales, and this causes increasing suppression of small scale matter power spectrum with increasing fraction of neutrino energy density with respect to the total matter density [20]. Thus if we augment CMB anisotropy data with data coming from Baryon Acoustic Oscillations (BAO), Large Scale Structure (LSS), CMB lensing (which is the weak lensing effect on CMB photons due to LSS) measurements etc, strong bounds on $\sum m_\nu$ can be obtained. Even so, currently it is only possible to get an upper bound on $\sum m_\nu$ from cosmological data alone.

Let us define the mass of the lightest neutrino mass eigenstate to be $m_0$. For normal hierarchy, $m_0 = m_1$, whereas for inverted hierarchy, $m_0 = m_3$. In terms of $m_0$, the sum of the neutrino masses can be defined as,

$$\sum m_\nu = m_0 + \sqrt{\Delta m_{21}^2 + m_0^2} + \sqrt{\Delta m_{31}^2 + m_0^2}$$

(NH), \hfill (1.3)

and

$$\sum m_\nu = m_0 + \sqrt{|\Delta m_{32}^2| + m_0^2} + \sqrt{|\Delta m_{32}^2| - \Delta m_{21}^2 + m_0^2}$$

(IH). \hfill (1.4)

Putting $m_0 = 0$, one can obtain the minimum neutrino mass sums allowed by the two possible hierarchies, and these are $\sum m_\nu = 0.05885^{+0.00045}_{-0.00044}$ eV ($1\sigma$(NH)) and $\sum m_\nu = 0.09950^{+0.00070}_{-0.00067}$ eV ($1\sigma$(IH)). Assuming that the normal hierarchy is the true one, the way cosmology can help is by constraining the $\sum m_\nu$ below the minimum mass required by inverted hierarchy with reasonable statistical significance, e.g., a determination of $\sum m_\nu = 0.058 \pm 0.008$ eV ($1\sigma$) will exclude inverted hierarchy at 5$\sigma$ and also provide cosmological evidence for non-zero neutrino masses at 7.25 $\sigma$. Currently, the most recent and strongest bounds on $\sum m_\nu$ in the minimal $\Lambda$CDM+$\sum m_\nu$ model are around $\sum m_\nu < 0.12$ eV (95% C.L.) [23, 24] with CMB and BAO data, whereas some other studies had reported bounds around $\sum m_\nu < 0.15$ eV (95%) or better [15, 25–41]. The strongest bounds are very close to the minimum $\sum m_\nu$ required for inverted hierarchy and thus it seems that inverted hierarchy is starting to get under pressure from cosmological data. These bounds are obtained with the assumption that all the three neutrino masses are equal ($m_i = \sum m_\nu/3$ for $i = 1, 2, 3$), an approximation we denote as degenerate hierarchy (DH). There are also studies covering interesting aspects of measurement of neutrino hierarchy from cosmology [25, 42–50]. The current cosmological data, however, is not yet sensitive enough to make the distinction between the two hierarchies on a level that can be considered statistically significant, but there is a small preference for normal hierarchy [25, 42]. It is to be noted that the bounds depend on the underlying cosmological model, and any extensions to the minimal $\Lambda$CDM+$\sum m_\nu$ model will usually lead to a more relaxed bound on $\sum m_\nu$ [23, 25, 34, 35, 51–57]. However, it can happen that
the neutrino mass bound improves when the extension to ΛCDM is done in such a way that
the allowed parameter space of the new parameters prefers neutrino masses which are smaller
than what we get in ΛCDM. In fact this is the case when one incorporates dynamical dark
energy in the cosmological model but restricts the parameter space to non-phantom dark
energy only, i.e. neutrino mass bounds in a cosmology with non-phantom dynamical dark
energy are stronger than that in ΛCDM [23, 41, 58]. Another such model where neutrino
mass bounds are stronger than that in ΛCDM is holographic dark energy (HDE) [59, 60].

While the degenerate hierarchy approximation has predominant in the cosmological pa-
rameter estimation literature, and makes sense when neutrino masses are relatively large
compared to the square-root of the mass-squared splittings (i.e. \( m_i \gg \sqrt{\Delta m^2_{ij}} \)), the
cosmological neutrino mass bound is becoming strong enough that it should be replaced by
a treatment using either the normal or the inverted hierarchy. Hence, in this paper we
have updated the bounds on the \( \sum m_\nu \) while explicitly considering three different hierar-
chies (degenerate, normal and inverted), using latest datasets and likelihoods that are publicly
available, for the minimal ΛCDM + \( \sum m_\nu \) and some of its extensions. Except in the case of
extension with the tensor-to-scalar ratio (\( r \)) parameter, all the other extensions studied in
this paper includes new parameters which are considerably correlated with the sum of neu-
trino masses in the datasets considered. Details of the models are given in the next section.
The neutrino mass bounds are supposed to relax in most of the extended models, and the
difference between the upper limits obtained for the three hierarchies is supposed to diminish
as the individual masses become much larger than the square root of mass-squared splittings.
But still, our motivation to study these extended models is to see whether the latest datasets
can make a difference.

To implement the normal and inverted hierarchy, we use the mean values of the mass
squared splittings given in eq.(1.1) along with the lightest neutrino mass \( m_0 \) to define \( m_1 \),
\( m_2 \), and \( m_3 \), and use \( m_0 \) as a free parameter and \( \sum m_\nu \) as a derived parameter. We ignore
the errors in the measurement of the mass-squared splittings from oscillations data since
they are small compared to the mean values and incorporating them would have a very small
effect on the bounds on \( \sum m_\nu \). For degenerate hierarchy, we simply have \( \sum m_\nu = 3m_0 \).

For CMB anisotropies we use the most recently released Planck 2018 likelihoods for the
data on temperature, E mode polarisation and their cross-correlation. Other than Planck
CMB anisotropies, we use latest data from measurements of Planck lensing, CMB B mode,
BAO, and SNe Ia luminosity distance; dark energy survey (includes galaxy clustering and
weak lensing); and Hubble parameter measurements. Also from now on, we shall use the
abbreviations DH, NH, and IH for degenerate, normal and inverted hierarchies respectively.

This paper is structured as follows. In section 2 we provide brief details about the
cosmological models and datasets used in this paper. In section 3 we provide and explain
the results of our analyses. In section 4 we conclude.

## 2 Methodology: Models and datasets

### 2.1 Models

In this work we have performed our analyses using a variety of different cosmological models
which we shall describe below. Note that when we label models we use the term \( \sum m_\nu \) to
refer to the neutrino mass. This is done in order to conform to the standard labelling used
in cosmological parameter analyses. In fact all our runs use a flat prior on \( m_0 \), the mass of
the lightest mass state, rather than a flat prior on $\sum m_{\nu}$. In total we have investigated the following 7 sets of cosmological models:

- **i)** The minimal $\Lambda$CDM + $\sum m_{\nu}$ model: Below we list the vector of varying parameters in this model.

\[
\theta \equiv [\omega_c, \omega_b, \Theta_s, \tau, n_s, \ln[10^{10} A_s], m_0].
\]  

Here the first six parameters correspond to the $\Lambda$CDM model. $\omega_c = \Omega_c h^2$ and $\omega_b = \Omega_b h^2$ are the present cold dark matter and baryon energy densities respectively. $\Theta_s$ is the ratio between sound horizon $r_s$ and angular diameter distance $D_A$ at the time of photon decoupling. $\tau$ is the optical depth to re-ionization of the universe at late times. $n_s$ and $A_s$, on the other hand, relate to early universe cosmology. They are the power-law spectral index and power of the primordial scalar perturbations respectively, evaluated at the pivot scale of $k_* = 0.05 h \text{ Mpc}^{-1}$. As defined in the previous section, the seventh parameter, $m_0$ is the mass of the lightest neutrino and it is the parameter of primary concern in this paper.

- **ii)** The $\Lambda$CDM + $\sum m_{\nu} + r$ model: Apart from the main 7 parameters, in this model we also include the evolution of tensor perturbations in the analysis along with scalar perturbations, and add an additional free parameter $r$, which is the tensor-to-scalar ratio evaluated at the same pivot scale as $n_s$ and $A_s$.

- **iii)** The $w$CDM + $\sum m_{\nu}$ model: Here instead of a cosmological constant with a dark energy equation of state (DE EoS hereafter) fixed at $w(z) = -1$ we opt for a DE EoS $w$ which varies as a free parameter, but does not vary in time (i.e. $w$ can assume different values but the values are fixed throughout the evolution history of the universe). Here $z$ denotes the cosmological redshift ($z = 1/a - 1$, where $a$ is the scale factor of FRW metric). Here, as well as in all models including non-$\Lambda$ dark energy, we use the PPF prescription [61] for incorporating dark energy perturbations.

- **iv)** The $w_0w_a$CDM + $\sum m_{\nu}$ model: In this case we incorporate a dynamically varying DE EoS, i.e. $w(z)$ also varies with time. We parametrize the EoS with the $w_0 - w_a$ approach. This is the Chevallier-Polarski-Linder (CPL) parametrization [62, 63]:

\[
w(z) = w_0 + w_a(1 - a) = w_0 + w_a \frac{z}{1 + z}. \tag{2.2}
\]

We hereafter may simply refer to this model as DDE.

- **v)** The $w_0w_a$CDM + $\sum m_{\nu}$ model with $w(z) \geq -1$: This model has the same parametrization as the previous model, but the dynamical dark energy is forced to stay in the non-phantom range, i.e. $w(z) \geq -1$. This is achieved by noticing that $w(z)$ in eq. (2.2) is a monotonic function, that at present day $w(z) = w_0$ (since $a = 1$ is the present value of the scale factor by convention), and that at the very early universe ($a \to 0$) we had $w(z) \to w_0 + w_a$. It thus suffices to have the following hard priors applied on the CPL parameters to keep them from crossing the phantom barrier [23, 41, 58]:

\[
w_0 \geq -1; \quad w_0 + w_a \geq -1 \tag{2.3}
\]

We hereafter may simply refer to this model as NPDDE.
Table 1. Flat priors on the main cosmological parameters constrained in this paper.

| Parameter | Prior             |
|-----------|-------------------|
| \( \omega_c \) | [0.005, 0.1]    |
| \( \omega_b \) | [0.001, 0.99]    |
| \( \Theta_s \) | [0.5, 10]        |
| \( \tau \) | [0.01, 0.8]      |
| \( n_s \) | [0.8, 1.2]       |
| \( \ln[10^{10} A_s] \) | [2, 4]        |
| \( m_0 \) (eV) | [0, 1]           |
| \( r \) | [0, 1]           |
| \( w \) | [-3, -0.33]      |
| \( w_0 \) | [-3, -0.33]      |
| \( w_a \) | [-2, 2]          |
| \( \Omega_k \) | [-0.3, 0.3]      |
| \( A_{\text{Lens}} \) | [0, 3]          |

- vi) The \( \Lambda \text{CDM} + \sum m_\nu + A_{\text{Lens}} \) model: In this extended model we include \( A_{\text{Lens}} \), which is the scaling of the lensing amplitude. In a particular model, the theoretical prediction for the gravitational potential (which generates the weak lensing of the CMB) corresponds to \( A_{\text{Lens}} = 1 \). When \( A_{\text{Lens}} \) is varied, the weak lensing is uncoupled from the primary anisotropies which cause it, and then scaled by the value of \( A_{\text{Lens}} \) [64]. \( A_{\text{Lens}} \) acts as a consistency check parameter. In a particular model, if the data prefers \( A_{\text{Lens}} > 1 \), it means it prefers more smoothing of its acoustic peaks in the power spectra (typically caused by lensing) than what theoretically should be. The physical reason for this extra smoothing (if it can’t be accounted for statistical fluctuation in the data) may not be extra lensing but may be any new effect that mimics lensing [24].

- vii) The \( \Lambda \text{CDM} + \sum m_\nu + \Omega_k \) model: Here we go away from a flat universe to the one which can be curved. The curvature of the universe is parametrized by \( \Omega_k \), which is called the curvature energy density, and we allow it to vary freely in this model.

We use the publicly available Markov Chain Monte-Carlo package CosmoMC [65] (which uses the Boltzmann solver CAMB [66]) to perform a Bayesian analysis of cosmological datasets and derive constraints on \( \sum m_\nu \) and other cosmological parameters. We use the Gelman and Rubin statistics [67] to estimate the convergence of the chains. All our chains had reached a \( R - 1 < 0.01 \). We use flat priors on all the the parameters that are varied in a particular model. The priors are listed in table 1.

2.2 Datasets

**CMB: Planck 2018.** We use the high-\( l \) (30 \( \leq l \) \( \leq 2508 \)) and low-\( l \) (2 \( \leq l \) \( \leq 29 \)) CMB TT likelihood along with high-\( l \) E mode polarization and temperature-polarisation cross correlation likelihood from the recent Planck 2018 public release [68] and we call this combination “TTTEEE”. We use the Planck low-\( l \) E mode polarization data, and in the text we mention it as “lowE.”

**CMB: Planck 2018 lensing.** While the CMB anisotropy power spectra is determined
from 2-point correlation functions (TT, TE, EE), the power spectra of the lensing potential is proportional to the 4-point correlation functions such as TTTT, TTEB and so on [24]. We use it in our analyses as an additional CMB probe to ascertain neutrino physics properties, as lensing of CMB photons is produced by the gravitational potential of large scale structure which in turn is affected greatly by the free streaming massive neutrinos. We will refer to this dataset simply as “lensing” from now on.

**CMB: BICEP2/Keck array data.** While running an MCMC analysis for the ΛCDM + \( \sum m_\nu + r \) model, we also use the latest publicly available data (taken up to and including 2015) on the CMB BB spectra from the BICEP2/Keck collaboration (spanning the range: 20 < \( l \) < 330) [69]. This dataset is referred to as “BK15” in the paper.

**Baryon acoustic oscillations (BAO) Measurements.** In this paper we use the latest measurements of the BAO signal from different galaxy surveys: SDSS-III BOSS DR12 (LOWZ and CMASS galaxy samples at \( z_{\text{eff}} = 0.38, 0.51 \) and 0.61) [70], the DR7 Main Galaxy Sample (MGS) at the effective redshift of \( z_{\text{eff}} = 0.15 \) [71], and the Six-degree-Field Galaxy Survey (6dFGS) survey at \( z_{\text{eff}} = 0.106 \) [72]). We simply name this combined dataset as “BAO”.

All the CMB and BAO data together constitute our “base” dataset:

\[
\text{Base} \equiv \text{Planck 2018 TTTEEE + lowE + lensing + BAO}.
\]

For the ΛCDM + \( \sum m_\nu + r \) model, the “base” dataset shall also include BK15.

The other datasets that are used in this paper are as follows:

**Supernovae luminosity distance measurements.** We use the most recent Supernovae Type-Ia (SNe Ia) luminosity distance measurements from the Pantheon Sample [73], which consists of distance information of 1048 SNe Ia (0.01 < \( z \) < 2.3), largest till date. Out of the 1048, 279 are from the Pan-STARRS1 (PS1) Medium Deep Survey (0.03 < \( z \) < 0.68) and rest of them from SDSS, SNLS, various low-\( z \) and HST samples. We call this dataset “SNe”.

**Hubble parameter measurements.** We use a Gaussian prior of \( H_0 = 74.03 \pm 1.42 \) km/sec/Mpc (68%) on the Hubble constant, which is the most recent measurement of \( H_0 \) by Riess et al [74], a 1.91% determination based on a local distance ladder calibrated with Cepheids and Detached Eclipsing Binaries (DEB) in Large Magellanic Cloud(LMC), masers in NGC 4258, and Milky Way parallaxes. We call this prior “R19”. This measurement is in a 4.4 \( \sigma \) tension with the Planck 2018 measurement of \( H_0 = 67.36 \pm 0.54 \) km/sec/Mpc (68%) in the base ΛCDM model with two massless and one massive neutrino with mass of 0.06 eV [24]. However a separate measurement very recently provided a bound of \( H_0 = 73.1^{+2.3}_{-2.4} \) km/sec/Mpc (68%), using an inverse distance ladder method using data from SNe Ia and strong lensing time delays in quasar imaging [75], which corroborates the findings of [74]. Thus it seems that the “Hubble tension” is here to stay and it is thus important to take this most recent bound on \( H_0 \) into account to see how the bounds on neutrino masses change.

**Dark Energy Survey.** We use the latest publicly available galaxy clustering and weak gravitational lensing data from the first year of the Dark Energy Survey collaboration [76]. The data combines three two point functions: cosmic shear correlation, galaxy angular auto-
considering different hierarchies. With the Base data recover the following 95% bound on the

Table 2 provides confidence limits on cosmological parameters for the

datasets, where Base

considering three different hierarchies (degenerate, normal, and inverted) for four dataset combi-

In this subsection we provide results of our analyses in the ΛCDM +

models and datasets are given in section 2.

In this section we provide the results of our analyses on the bounds on neutrino masses

correlation, and galaxy-shear cross-correlation. We call this dataset as “DES” hereafter.

3 Results

In this section we provide the results of our analyses on the bounds on neutrino masses

0.1191 ± 0.0009

0.1192 ± 0.0009

0.1191 ± 0.0009

0.1184 ± 0.0009

0.1182 ± 0.0009

0.1189 ± 0.0009

0.02243 ± 0.00013

0.02244 ± 0.00014

0.02243 ± 0.00013

0.02253 ± 0.00013

0.02254 ± 0.00013

0.02255 ± 0.00013

1.04100 ± 0.00029

1.04100 ± 0.00029

1.04100 ± 0.00029

1.04116 ± 0.00029

1.04116 ± 0.00029

1.04117 ± 0.00029

\[ \omega_\nu \begin{pmatrix} 0.1191 \pm 0.0009 \\ 0.1192 \pm 0.0009 \\ 0.1191 \pm 0.0009 \end{pmatrix} \]

\[ \sigma_8 \begin{pmatrix} 0.814^{+0.010}_{-0.007} \\ 0.806^{+0.009}_{-0.006} \\ 0.799^{+0.008}_{-0.006} \end{pmatrix} \]

\[ H_0 \begin{pmatrix} 67.81^{+0.54}_{-0.44} \\ 67.50^{+0.49}_{-0.44} \\ 67.22 \pm 0.45 \end{pmatrix} \]

\[ \ln[10^{10} A_s] \begin{pmatrix} 3.048^{+0.014}_{-0.015} \\ 3.051^{+0.014}_{-0.015} \\ 3.053 \pm 0.015 \end{pmatrix} \]

\[ \Theta_x \begin{pmatrix} 0.0554^{+0.0006}_{-0.0006} \\ 0.0556^{+0.0006}_{-0.0007} \\ 0.0558^{+0.0006}_{-0.0007} \end{pmatrix} \]

\[ m_\nu (eV) \begin{pmatrix} < 0.040 \\ < 0.040 \\ < 0.04 \end{pmatrix} \]

\[ \sum m_\nu (eV) \begin{pmatrix} < 0.121 \\ < 0.121 \\ < 0.12 \end{pmatrix} \]

\[ H_0 \begin{pmatrix} 68.46 \pm 0.44 \\ 68.46 \pm 0.44 \\ 68.46 \pm 0.44 \end{pmatrix} \]

\[ \sigma_8 \begin{pmatrix} 0.817_{-0.007}^{+0.008} \\ 0.807_{-0.006}^{+0.007} \\ 0.799 \pm 0.007 \end{pmatrix} \]

\[ S_8 \begin{pmatrix} 0.818 \pm 0.011 \\ 0.823 \pm 0.011 \\ 0.820 \pm 0.011 \end{pmatrix} \]

\[ \Delta \chi^2 = \chi^2 - \chi^2_{\text{Base}} \begin{pmatrix} -2.89 \\ -0.95 \end{pmatrix} \]

\[ \Delta \chi^2 = \chi^2 - \chi^2_{\text{Base}} \begin{pmatrix} -2.89 \\ -0.95 \end{pmatrix} \]

Table 2. Constraints on the cosmological parameters in the minimal ΛCDM + \( \sum m_\nu \) model considering three different hierarchies (degenerate, normal, and inverted) with the Base and Base+R19 datasets, where Base \( \equiv \) Planck 2018 TTTEEE+lowE+lensing+BAO. Here \( m_\nu \) is the mass of the lightest neutrino in a particular hierarchy and a freely varying parameter in the model, whereas \( \sum m_\nu \), \( H_0 \), \( \sigma_8 \), and \( S_8 \) are derived parameters. Marginalized constraints are given at 1\( \sigma \) whereas upper or lower bounds are given at 2\( \sigma \). The \( \chi^2 \) differences are calculated at best-fit points. Details about models and datasets are given in section 2.

3.1 Results in the minimal ΛCDM + \( \sum m_\nu \) model

In this subsection we provide results of our analyses in the ΛCDM + \( \sum m_\nu \) model, considering three different hierarchies (degenerate, normal, and inverted) for four dataset combinations, namely Base, Base+R19, Base+SNe+R19, and Base+SNe+R19+DES where Base \( \equiv \) Planck 2018 TTTEEE+lowE+lensing+BAO. The main results are contained in tables 2 and 3 respectively. Table 2 provides confidence limits on cosmological parameters for the Base, and Base+R19 combinations, whereas table 3 provides the same for Base+SNe+R19 and Base+SNe+R19+DES.

In figure 1 we depict the 1-D posterior distributions of \( m_\nu \) (mass of the lightest neutrino in a hierarchy) and \( \sum m_\nu \) in the ΛCDM + \( \sum m_\nu \) model for Base and Base+R19 datasets considering different hierarchies. With the Base data recover the following 95% bound on the
mass sum: $\sum m_\nu < 0.121$ eV which is almost same as the bound of $\sum m_\nu < 0.120$ eV quoted by the Planck 2018 collaboration [69] with the same data. The Planck 2015 result with similar data was $\sum m_\nu < 0.17$ eV (95\%, Planck 2015 TT,TE,EE+lowP+BAO). The main reason this improvement happens with Planck 2018 is because of improved measurement of $\tau$. Planck 2015 TT,TE,EE+lowP likelihoods produced a bound of $\tau = 0.079 \pm 0.017$ (68\%) in the base $\Lambda$CDM model [77], which improved to $\tau = 0.054^{+0.0070}_{-0.0081}$ (68\%) with Planck 2018 TT,TE,EE+lowE [24]. The parameters $\tau$ and $\sum m_\nu$ are strongly correlated in the Planck TT data and high-$l$ polarization data [23], and this degeneracy can be broken through better and better measurement of $\tau$ from the low-$l$ polarization data. Since an increase in $\sum m_\nu$ leads to suppression of the matter power spectrum which in turn leads to less gravitational lensing of the CMB photons, and hence increase in $\sum m_\nu$ leads to a decrease in the smearing of the CMB acoustic peaks (smearing happens due to lensing). This effect due to increasing neutrino masses, however, can be countered with increasing $\tau$, which exponentially suppresses power in the CMB anisotropy spectra (given other parameters are kept fixed). However, the effect of reionization is also found in the low-$l$ polarization data (TE, EE, BB) in the form of a “reionization bump” whose amplitude is proportional to $\tau^2$ in the EE and BB spectra, and to $\tau$ in the TE spectra, and this change can’t be compensated by varying other parameters in the model [78]. Hence the improved removal of systematics in the low-$l$ polarization data with Planck 2018 helps in breaking the degeneracy between $\tau$ and $\sum m_\nu$, and this leads to stronger upper bounds on $\sum m_\nu$. However, not only $\tau$, Planck 2018 also largely corrects various systematic effects previously present in the high-$l$ polarization spectra of Planck 2015, and that also contributes towards obtaining a better bound on the $\sum m_\nu$.

In our Base dataset combination, BAO is another effective tool in constraining neutrino masses. BAO data is useful in breaking the strong anti-correlation between the Hubble constant $H_0$ and $\sum m_\nu$ present in the Planck data. The comoving distance to the last scattering

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Comparison of 1-D marginalized posterior distributions for $m_0$ (eV) and $\sum m_\nu$ (eV) for Base and Base+R19 datasets in the $\Lambda$CDM + $\sum m_\nu$ model for the three hierarchies. The two dashed vertical lines are at $\sum m_\nu = 0.0589$ and 0.0995 eV representing the minimum mass required for normal and inverted hierarchy respectively.}
\end{figure}
surface in a flat $\Lambda$CDM + $\sum m_\nu$ universe is defined as: $\chi(z_{\text{dec}}) = \int_0^{z_{\text{dec}}} dz / H(z)$, where $z_{\text{dec}}$ is the redshift of photon decoupling, and $H(z) = \sqrt{\omega_\gamma (1 + z)^4 + (\omega_c + \omega_b) (1 + z)^4 + \omega_\Lambda + \rho_\nu(z) h^2 / \rho_{cr,0}}$ (note: $\omega_i = \Omega_i h^2$, and $i = \gamma, c, b, \Lambda$ with $\gamma \equiv$ photons, $c \equiv$ CDM, $b \equiv$ baryons, $\Lambda \equiv$ cosmological constant; $\rho_\nu(z)$ is the neutrino energy density at a redshift $z$, $\rho_{cr,0} = 3H_0^2/8\pi G$ is the critical density today). In a flat universe $\Omega_\Lambda = 1 - \Omega_\gamma - (\Omega_c + \Omega_b) - \Omega_\nu$, and at late times $\rho_\nu(z) h^2 / \rho_{cr,0} = \Omega_\nu h^2 \propto \sum m_\nu$. Now, given that early universe physics remains unchanged, $\chi(z_{\text{dec}})$ is very well constrained through $\Theta_s$ which is the most well constrained parameter by the Planck CMB anisotropies. On the other hand, $\Omega_\gamma$ and $\langle \omega_c + \omega_b \rangle$ are also well constrained by the data. Thus any change in $\chi(z_{\text{dec}})$ due to an increase in $H_0$ (or $h = H_0 / 100$ km/sec/Mpc) has to be compensated by a decrease in $\sum m_\nu$ and vice versa, and hence there is a large anti-correlation between $H_0$ and $\sum m_\nu$. Thus in the $\Lambda$CDM + $\sum m_\nu$ model, lower values of $H_0$ correspond to higher values of $\sum m_\nu$ and higher values of $H_0$ correspond to lower $\sum m_\nu$. BAO data breaks this degeneracy partially by rejecting the low $H_0$ values preferred by Planck data, well studied in previous literature [23, 25, 79]. For instance, in $\Lambda$CDM + $\sum m_\nu$, Planck 2015 TT,TE,EE+lowP+BAO prefers a value of $H_0 = 66.17^{+1.96}_{-0.81}$ km/sec/Mpc, whereas TT,TE,EE+lowP+BAO prefers $H_0 = 67.67^{+0.54}_{-0.51}$ km/sec/Mpc [23].

Apart from CMB power spectra and BAO, our Base dataset also contains Planck 2018 lensing likelihoods, which as per Planck 2018 collaboration [24] prefers a slightly increased lensing power spectrum amplitude compared to Planck 2015, and thus leads to a slightly tighter neutrino mass constraints, contrary to Planck 2015 lensing likelihoods which used to relax the constraints. In our case, without the Planck 2018 lensing likelihoods, we obtained a bound of $\sum m_\nu < 0.128$ eV (95%, Planck 2018 TT,TE,EE+lowE+BAO), i.e., Planck 2018 lensing has only a small effect in this $\Lambda$CDM + $\sum m_\nu$ model when used along with Planck 2018 CMB anisotropies, and BAO.

We see that the bounds on $\sum m_\nu$ do differ significantly across the different hierarchies. We see that with the Base dataset we get the following bounds: $\sum m_\nu < 0.146$ eV (NH), $\sum m_\nu < 0.172$ eV (IH) at 95% C.L. and thus the degenerate approximation with a prior $\sum m_\nu \geq 0$ is indeed not a good approximation to go with, any more. As previously stated, we use the mass of the lightest neutrino, $m_0$ as the varying parameter and then use the mass-squared splittings given in eq. 1.1 to determine the other masses in a particular hierarchy, and this implicitly puts a prior on the $\sum m_\nu$, i.e., $\sum m_\nu \geq 0.0589$ eV for normal hierarchy.
and $\sum m_\nu \geq 0.0995$ eV for inverted hierarchy. The reason the bound on $\sum m_\nu$ differs significantly across the three hierarchies is possibly the priors imposed on $\sum m_\nu$ (see figure 1 for the visualization of the same). We cross-check this by running MCMC chains with the degenerate hierarchy, but with the priors: i) $\sum m_\nu \geq 0.0589$ eV, and ii) $\sum m_\nu \geq 0.0995$ eV. We found a 95% upper bound of $\sum m_\nu < 0.149$ eV in the first case, and in the second case it was $\sum m_\nu < 0.179$ eV. It seems evident that the priors do play an important role in relaxing the bounds. However it is to be noted that the method we have used in this paper for the normal and inverted hierarchy (i.e. with lightest mass $m_0$ and the mass-squared splittings from oscillations experiments) produces bounds which are slightly stronger than the degenerate case with the priors $\sum m_\nu \geq 0.0589$ eV and $\sum m_\nu \geq 0.0995$ eV respectively, i.e. the two methods are not completely equivalent. The difference is clearer with Planck 2018 TT,TE,EE+lowE data, with which the NH case leads to a 95% bound of $\sum m_\nu < 0.291$ eV, whereas the DH case with the $\sum m_\nu \geq 0.0589$ eV prior gives us a bound of $\sum m_\nu < 0.320$ eV. For the same data, the IH case produces a 95% bound of $\sum m_\nu < 0.332$ eV, whereas the DH case with the $\sum m_\nu \geq 0.0995$ eV prior obtains a bound of $\sum m_\nu < 0.353$ eV. Hence we think that it is better to implement the normal and inverted hierarchy properly instead of a simplified degenerate mass approximation which does not correspond to a physical model.

From table 2 we can see an important trend: in the $\Lambda$CDM + $\sum m_\nu$ model with Base dataset, as we go from degenerate to normal to inverted hierarchy there appears a decrease in the preferred values of $H_0$ and $\sigma_8$. This is also visualized in figure 2. The reason for the decrease in $H_0$ is simply the anti-correlation between $H_0$ and $\sum m_\nu$ as described above, and the fact that the normal hierarchy prefers neutrino masses which are larger than the degenerate case, and the inverted hierarchy prefers masses which are larger than the normal hierarchy. There is, again, a strong anti-correlation present between $\sigma_8$ and $\sum m_\nu$, since $\sigma_8$ is the amplitude of the linear matter power spectrum at a length scale of $8h^{-1}$Mpc, and increasing $\sum m_\nu$ increases the neutrino energy density and that increases the suppression of the small scale matter power-spectrum leading to a lower $\sigma_8$. Thus, from degenerate to normal to inverted hierarchy, the $H_0$-tension between Planck and local measurements increases, whereas the $\sigma_8$-tension [80] between Planck and cosmic shear measurements decreases. This trend remains true for the other datasets we have considered in this model (as can be seen from tables 2 and 3). From figure 2 we also observe that the parameter $S_8 = \sigma_8 \sqrt{\Omega_m/0.3}$ also goes downward, but not as strongly as $\sigma_8$, since increasing neutrino masses also cause $\Omega_m$ to increase which compensates for the decreased $\sigma_8$, i.e. $S_8$ is defined such that its correlation with $\sum m_\nu$ is small in this model. Again, while $H_0$ and $\sigma_8$ both are strongly anti-correlated with $\sum m_\nu$, the magnitude of the correlation coefficients ($R_{ij} = C_{ij}/\sqrt{C_{ii}C_{jj}}$ where $i,j$ are the two parameters, and $C$ is the covariance matrix of the cosmological parameters) decrease from degenerate to normal to inverted hierarchy because of the implicit priors on $\sum m_\nu$, i.e the priors help in partially breaking the degeneracies. We find that $R_{H_0,\sum m_\nu} = -0.55$ (DH), $-0.47$ (NH), $-0.43$ (IH); and $R_{\sigma_8,\sum m_\nu} = -0.76$ (DH), $-0.72$ (NH), $-0.67$ (IH).

Apart from the results with the Base dataset, in table 2, we provide results with the Base+R19 dataset also. The R19 prior: $H_0 = 74.03\pm1.42$ km/sec/Mpc breaks the degeneracy between $H_0$ and $\sum m_\nu$ far more strongly than BAO data, leading to very strong 95% upper bounds on $\sum m_\nu$. We get $\sum m_\nu < 0.076$ eV for the degenerate case, $\sum m_\nu < 0.112$ eV for the normal ordering, and $\sum m_\nu < 0.146$ eV for the inverted ordering. The preference for lower neutrino masses can be visualized in figure 1. The relative difference between the
given in tables 2 and 3, adding SNe data to the Base+R19 combination has almost no effect stronger than the bound without the SNe data. However, as can be observed from the results

\[ \sum H \]

SNe data is able to constrain \( \Omega \) and Base+SNe+R19+DES. In the absence of a varying dark energy equation of state, the

Such degradation in the fit is indicative of the large tension present between the Base and R19 datasets, and hence the results obtained with the R19 prior should be interpreted with care.

In table 3 we provide results with two other dataset combinations: Base+SNe+R19 and Base+SNe+R19+DES. In the absence of a varying dark energy equation of state, the SNe data is able to constrain \( \Omega_m \) effectively [73], while the Planck CMB data can constrain \( \Omega_m h^2 \) well. Thus together they can effectively constrain \( H_0 \), and as found out in [23], the Planck+SNe combination actually prefers \( H_0 \) values which are higher than Planck alone, and thus the SNe data can help in breaking the degeneracy between \( H_0 \) and \( \sum m_\nu \) partially. BAO data is however much more efficient in breaking the degeneracy and with Base+SNe (note that Base already contains BAO). In the DH case, we find the following 95% bounds on the neutrino mass sum in this \( \Lambda \)CDM + \( \sum m_\nu \) model, \( \sum m_\nu < 0.115 \) eV, which is only slightly stronger than the bound without the SNe data. However, as can be observed from the results given in tables 2 and 3, adding SNe data to the Base+R19 combination has almost no effect on the bounds, because the R19 prior dominates over the SNe as far as the \( H_0 - \sum m_\nu \) degeneracy breaking is considered. Addition of the DES data to the Base+SNe+R19 combination

| Table 3. Constraints on the cosmological parameters in the minimal \( \Lambda \)CDM + \( \sum m_\nu \) model considering three different hierarchies (degenerate, normal, and inverted) with the Base+SNe+R19 and Base+SNe+R19+DES datasets, where Base \( \equiv \) Planck 2018 TTTEEE+lowE+lensing+BAO. Here \( m_0 \) is the mass of the lightest neutrino in a particular hierarchy and a freely varying parameter in the model, whereas \( \sum m_\nu \), \( H_0 \), \( \sigma_8 \), and \( S_8 \) are derived parameters. Marginalized constraints are given at 1\( \sigma \) whereas upper or lower bounds are given at 2\( \sigma \). The \( \chi^2 \) differences are calculated at best-fit points. Details about models and datasets are given in section 2. | Base+SN+R19 | Base+SN+R19+DES |
|---|---|---|
| | DH | NH | HI | DH | NH | HI |
| \( \Lambda \)CDM + \( \sum m_\nu \) | | | | | | |
| \( \omega_c \) | 0.1118 ± 0.0009 | 0.1116 ± 0.0009 | 0.1117 ± 0.0009 | 0.1117 ± 0.0008 | 0.1137 ± 0.0008 | 0.1171 ± 0.0008 |
| \( \omega_b \) | 0.02252 ± 0.00012 | 0.02255 ± 0.00014 | 0.02255 ± 0.00013 | 0.02259 ± 0.00011 | 0.02261 ± 0.00013 | 0.02261 ± 0.00013 |
| \( \theta_s \) | 1.04116 ± 0.0029 | 1.04119 ± 0.0029 | 1.04118 ± 0.0029 | 1.04121 ± 0.0029 | 1.04124 ± 0.0029 | 1.04125 ± 0.0029 |
| \( \tau \) | 0.0577 ± 0.0008 | 0.0601 ± 0.0009 | 0.0620 ± 0.0009 | 0.0579 ± 0.0006 | 0.0605 ± 0.0008 | 0.0622 ± 0.0009 |
| \( n_0 \) | 0.9091 ± 0.0036 | 0.9096 ± 0.0036 | 0.9070 ± 0.0037 | 0.9705 ± 0.0037 | 0.9711 ± 0.0036 | 0.9615 ± 0.0036 |
| \( \ln(10^{10} A_s) \) | 3.048 ± 0.014 | 3.053 ± 0.015 | 3.056 ± 0.015 | 3.047 ± 0.014 | 3.054 ± 0.014 | 3.055 ± 0.014 |
| \( m_\nu \) (eV) | < 0.025 | < 0.026 | < 0.028 | < 0.028 | < 0.029 | < 0.030 |
| \( \sum m_\nu \) (eV) | < 0.074 | < 0.109 | < 0.146 | < 0.086 | < 0.116 | < 0.147 |
| \( H_0 \) (km/s/Mpc) | 68.48 ± 0.42 | 68.17 ± 0.41 | 67.90 ± 0.42 | 68.83 ± 0.41 | 68.52 ± 0.40 | 68.24 ± 0.38 |
| \( \sigma_8 \) | 0.816 ± 0.006 | 0.807 ± 0.007 | 0.799 ± 0.007 | 0.812 ± 0.006 | 0.803 ± 0.007 | 0.796 ± 0.006 |
| \( S_8 \) | 0.818 ± 0.010 | 0.813 ± 0.010 | 0.809 ± 0.010 | 0.807 ± 0.009 | 0.803 ± 0.009 | 0.800 ± 0.009 |
| \( \Delta \chi^2 = \chi^2 - \chi^2_{DH} \) | -7.91 | -3.28 | 0 | -6.22 | -2.17 | 0 |
however relaxes the bounds on $\sum m_\nu$ slightly, since DES prefers lower $\sigma_8$ values.

**Akaike information criterion (AIC).** To compare the goodness of fit of different models to the same data a popular method is to compute the Akaike information criterion (AIC) [81] for each model, defined as:

$$\text{AIC} = \chi^2_{\text{best-fit}} + 2k$$  \hspace{1cm} (3.2)

where $k$ is the number of parameters in the model. Comparison with another model is done by computing the difference: $\Delta \text{AIC} = \Delta \chi^2_{\text{best-fit}} + 2\Delta k$. Between two models the model with a lower AIC is considered the preferred model. The $2\Delta k$ term penalizes the model with greater number of parameters as it is usually able to provide better fit because of the larger parameter space being available to it. In this work however, we are interested in the comparison of the quality of fits between the neutrino mass hierarchies and since we have the same number of parameters in all the different cases, $2\Delta k = 0$, and $\Delta \text{AIC} = \Delta \chi^2_{\text{best-fit}}$. We have provided the $\Delta \text{AIC} = \chi^2_{\text{best-fit}} - \chi^2_{\text{IH, best-fit}}$ values for each neutrino hierarchy in tables 2 and 3. We find that cosmological datasets slightly prefer the normal hierarchy over the inverted one. In this $\Lambda$CDM + $\sum m_\nu$ model, with the Base data, we find that the NH is preferred to the IH by a $\Delta \text{AIC} = -0.95$, which is a very mild result.

This showcases that the current cosmological data is not sensitive enough to differentiate between the two hierarchies, and is completely consistent with the findings in e.g. [42, 82] which both find that a formal sensitivity to $\sum m_\nu$ in the 0.01-0.02 eV range is required to guarantee a distinction between the two hierarchies. This is not possible using current data, but will become possible in the near future using high precision data from e.g. EUCLID (see e.g. [83, 84] for a discussion). With the other dataset combinations also the $\Delta \text{AIC}$ remains mild between NH and IH. The (unphysical) degenerate hierarchy on the other hand, produces a better fit to the data compared to both NH and IH in case of all the dataset combinations we have studied.

### 3.2 Results in the extended models

In this section we present the results in the extended cosmologies. Table 4 contains the constraints on selected cosmological parameters in the extended models with Base and Base+SNe+R19 datasets. Marginalized constraints are given at 1$\sigma$ whereas upper or lower bounds are given at 2$\sigma$. Details about models and datasets are given in section 2. Note that the Base data also contains BK15 when we consider the $\Lambda$CDM + $\sum m_\nu + r$ model.

- **i) Results in the $\Lambda$CDM + $\sum m_\nu + r$ model:** In this model we allow the tensor perturbations to vary as well, along with scalar ones. The CMB B-mode polarization comes from two sources: i) primordial gravitational waves, ii) gravitational weak lensing. With the Base data (which also includes BK15) we get the following 95% bounds: $\sum m_\nu < 0.115$ eV (DH), $\sum m_\nu < 0.142$ eV (NH), and $\sum m_\nu < 0.167$ eV (IH). With the Base+SNe+R19 data, these bounds strengthen, largely due to the R19 prior. We do not see any big changes on the mass bounds compared to $\Lambda$CDM + $\sum m_\nu$. The bounds are, however, slightly tighter in the $\Lambda$CDM + $\sum m_\nu + r$ model with the BK15 data than without, most likely due to the additional lensing information encoded in the B mode data BK15, since $r$ and $\sum m_\nu$ are not correlated, as can be seen from the left panel of figure 3. For instance, in $\Lambda$CDM + $\sum m_\nu + r$, with the degenerate neutrino masses, and Base+SNe+R19 data combination we get a bound of $\sum m_\nu < 0.072$ eV, and it
| Parameter | DH | NH | IH | DH | NH | IH |
|-----------|----|----|----|----|----|----|
| $\sum m_{\nu}$ (eV) | < 0.0626 | < 0.0632 | < 0.0618 | < 0.0645 | < 0.0650 | < 0.0666 |
| $w$ | -1.042 ± 0.072 | -1.066 ± 0.071 | -1.089 ± 0.070 | -1.072 ± 0.054 | -1.084 ± 0.031 | -1.096 ± 0.031 |
| $H_0$ (km/s/Mpc) | 67.78 ± 0.52 | 67.48 ± 0.48 | 67.21 ± 0.44 | 68.43 ± 0.44 | 68.18 ± 0.41 | 67.87 ± 0.41 |
| $\sigma_8$ | < 0.815 ± 0.007 | 0.807 ± 0.010 | 0.800 ± 0.006 | 0.818 ± 0.006 | 0.808 ± 0.007 | 0.801 ± 0.007 |
| $S_8$ | 0.823 ± 0.011 | 0.827 ± 0.010 | 0.822 ± 0.011 | 0.820 ± 0.010 | 0.815 ± 0.010 | 0.811 ± 0.010 |

Table 4. Constraints on selected cosmological parameters in the extended models considering three different hierarchies (degenerate, normal, and inverted) with the Base and Base+SNe+R19 datasets. In the $\Lambda$CDM + $\sum m_{\nu} + r$ model Base data also includes BK15 data.
gets slightly relaxed to $\sum m_{\nu} < 0.075$ eV if we don’t use the BK15 data. The effect of the neutrino mass bounds getting slightly tighter with B mode data was previously noticed in [23, 41, 85] with BK14 data [86], predecessor of BK15.

• ii) **Results in the $w$CDM + $\sum m_{\nu}$ model:** In this model we go away from the cosmological constant, and consider the DE EoS $w$ as a varying parameter. A well-known degeneracy exists between $w$ and $\sum m_{\nu}$ [51], through their mutual degeneracy with $\Omega_m$, which considerably relaxes the bounds on $\sum m_{\nu}$ in this model compared to $\Lambda$CDM + $\sum m_{\nu}$. The visualization of the anti-correlation between the two parameters is available in the right panel of figure 3 for the Base and Base+SNe+R19 data. We find a correlation coefficient of $R_{w,\sum m_{\nu}} = -0.57$ (DH), $-0.52$ (NH), $-0.47$ (IH) with the Base data, whereas $R_{w,\sum m_{\nu}} = -0.46$ (DH), $-0.38$ (NH), $-0.34$ (IH) with Base+SNe+R19. The SNe+R19 combination when used with the Base data, reduces the magnitude of the anti-correlation with better constraints on $w$. This happens, since in the SNe data $w$ and $\Omega_m$ are strongly anti-correlated, whereas in the CMB data $w$ and $\Omega_m$ are strongly correlated (for instance see figure 20 of [73]). So including the SNe data with Base leads to a large decrease in the $w$-$\Omega_m$ correlation, which in turn leads to a decrease in magnitude of $R_{w,\sum m_{\nu}}$. Also from figure 3, we see that due to the anti-correlation lower values of $w$ prefer higher $\sum m_{\nu}$ and higher values of $w$ prefer lower $\sum m_{\nu}$. The Base+SNe+R19 combination constrains $w$ better than Base such that the lowest values of $w$ allowed by Base data is rejected, and this leads to stronger bounds on $\sum m_{\nu}$ with Base+SNe+R19, as can be seen from table 4. The SNe+R19 combination also rejects high $w$ values, but those regions prefer low $\sum m_{\nu}$ values and hence rejecting the high $w$ region does not help in strengthening the upper bound on $\sum m_{\nu}$.
Figure 4. 68% and 95% marginalized contours for $w_0$ vs $w_a$ in the $w_0+w_a \Lambda$CDM + $\sum m_\nu$ model using Base and Base+SNe+R19 datasets, for DH (left panel), NH (middle panel), and IH (right panel). The region at the right of the vertical dashed blue line and above the slanted dashed blue line is the non-phantom DE region.

There is also a strong degeneracy between $w$ and $H_0$, that exists since lower values of $w$ correspond to higher present day expansion rate and hence higher $H_0$ values. This happens since changing $w$ can change the comoving distance to the last scattering surface $\chi(z_{dec}) = \int_0^{z_{dec}} dz/H(z)$, which is well-constrained by the CMB data, and thus any change in $\chi(z_{dec})$ needs to be compensated with another parameter. We have,

$$H(z) = \sqrt{\omega_\gamma (1+z)^4 + (\omega_c + \omega_b)(1+z)^3 + \Omega_{DE}(z) h^2 + \rho_c(z) h^2/\rho_{cr,0}}. \quad (3.3)$$

Here $\Omega_{DE}(z) = \Omega_{DE}(0)(1+z)^{3(1+w)}$ is the energy density of dark energy with EoS $w$, and at late times when dark energy is a dominant component, decreasing the value of $w$ leads to a decrease in $H(z)$, which can be compensated by increasing either $h$ (or $H_0$) or $\sum m_\nu$ or both. This is why in the $w$CDM + $\sum m_\nu$ model, with Base data, we find that not only $w$ and $H_0$ are anti-correlated ($R_{w,H_0} = -0.91$) (DH), the correlation between $H_0$ and $\sum m_\nu$ is inverted from what it was in $\Lambda$CDM + $\sum m_\nu$, i.e. here $H_0$ and $\sum m_\nu$ are positively correlated with $R_{H_0,\sum m_\nu} = +0.30$ (DH). The SNe+R19 data constrains $w$ and $H_0$ and breaks the degeneracy present between $H_0$ and $\sum m_\nu$, and we find that $R_{H_0,\sum m_\nu} = -0.021$ (DH) with Base+SNe+R19. This is why, in table 4 with the Base data, as we go from DH to NH to IH, the preference for higher and higher neutrino masses and the positive correlation between $H_0$ and $\sum m_\nu$ leads to slight increase in the preferred values of $H_0$, but with Base+SNe+R19, the almost negligible correlation explains the lack of any big change in the preferred $H_0$ values across the different hierarchies. On the other hand, the strong degeneracy between $w$ and $H_0$ survives, and becomes slightly weaker at $R_{w,H_0} = -0.71$ (DH). This leads to the $w$CDM + $\sum m_\nu$ model predominantly preferring values of $w < -1$, i.e. the phantom DE region.

- iii) Results in the $w_0 + w_a \Lambda$CDM + $\sum m_\nu$ model: In this model the DE EoS $w(z) = w_0 + w_a \frac{1}{1+z}$ is dynamical in nature, but remains close to the value of $w_0 + w_a$ for high redshift values, and only changes significantly during the late times. Like the previous model, there is degeneracy between $w(z)$ and $H_0$, and $w(z)$ and $\sum m_\nu$. However now
we have two parameters $w_0$ and $w_a$ instead of $w$, with

$$\Omega_{DE}(z) = \Omega_{DE}(0)(1+z)^3(1+w_0+w_a)\exp \left(-3w_a \frac{z}{1+z}\right).$$ (3.4)

Any change in $\chi(z_{dec})$ due to $H_0$ can be readily compensated with $w_0$ and $w_a$, and hence, in this model the correlation between $H_0$ and $\sum m_\nu$ is very small even with Base data ($R_{H_0,\sum m_\nu} = +0.08$ (DH)). Again, $w_0$ and $w_a$ are anti-correlated between themselves, since the change in $\chi(z_{dec})$ due to an increase in $w_0$ can be countered with a decrease in $w_a$. The anti-correlation can be seen clearly in figure 4. As it can be seen from the figure, neither the Base data nor Base+SNe+R19 combination prefers the non-phantom DE region much. The data prefers regions where the DE is currently phantom or has been phantom in the past. Since the phantom DE region, $w(z) < -1$ prefers neutrino masses which are larger, the mass bounds are essentially very relaxed in this model, as can be seen from table 4. We have $\sum m_\nu < 0.249$ eV (DH), 0.256 eV (NH), 0.276 eV (IH) with Base data, whereas with Base+SNe+R19, we get $\sum m_\nu < 0.227$ eV (DH), 0.250 eV (NH), 0.253 eV (IH). The SNe+R19 combination produces better constraints on the DE parameter, leading to slightly better bounds on $\sum m_\nu$. In figure 4, another important thing to notice is that as we go from DH to NH to IH, the 2D contours shift away from the non-phantom DE region due to the preference for higher and higher masses, and in the IH case, both Base and Base+SNe+R19 data reject the non-phantom region at more than $2\sigma$.

- iv) Results in the $w_0w_a\Lambda$CDM $+ \sum m_\nu$ model with $w(z) \geq -1$ : This is essentially the same model as in the previous case, but the parameter space is restricted to the non-phantom range only, i.e. $w(z) \geq -1$. This parameter space corresponds to dark energy field theories modelled with a single scalar field, like quintessence [87], which cannot cross the phantom barrier (the $w(z) = -1$ line). Due to the degeneracy between $w$ and $\sum m_\nu$ (which we have discussed for the last two models) the cosmological data actually prefers smaller and smaller neutrino masses as we go deeper in the non-phantom region in the $w_0 - w_a$ parameter space. Thus the bounds on $\sum m_\nu$ obtained in this model are even tighter than the $\Lambda$CDM $+ \sum m_\nu$ model. This was first noticed in [58], and subsequently in [23, 41]. From table 4, with the base data we find $\sum m_\nu < 0.096$ eV (DH), 0.129 eV (NH), 0.157 eV (IH). With the Base+SNe+R19 combination, the bounds stand at $\sum m_\nu < 0.067$ eV (DH), 0.108 eV (NH), 0.141 eV (IH). These are the tightest bounds on $\sum m_\nu$ reported in this paper for the datasets considered.

- v) Results in the $\Lambda$CDM $+ \sum m_\nu + A_{\text{Lens}}$ model : As mentioned in section 2.1, the $A_{\text{Lens}}$ parameter is used to artificially scale the lensing amplitude predicted by the underlying theoretical model. There is a well-known $A_{\text{Lens}}$-issue in the CMB anisotropy data which corresponds to preference for $A_{\text{Lens}}$ values which are more than $2\sigma$ away from the theoretical expectation of $A_{\text{Lens}} = 1$. For instance Planck 2018 TT,TE,EE+lowE data yields an $A_{\text{Lens}} = 1.180 \pm 0.065$ (68%) in the $\Lambda$CDM $+ A_{\text{Lens}}$ model, showing a $2.8\sigma$ tension [24]. The $\Lambda$CDM $+ A_{\text{Lens}}$ also provides significantly better fit to the Planck power spectrum data compare to $\Lambda$CDM. Planck power spectrum constrains $A_{\text{Lens}}$ by measuring the smoothing of the CMB acoustic peaks due to the gravitational lensing of the CMB photons from large scale structures. Planck lensing data, however, constrains $A_{\text{Lens}}$ directly, and adding CMB lensing data to the anisotropy power spectrum usually brings back the $A_{\text{Lens}}$ values closer to the theoretically expected result.
Figure 5. 68% and 95% marginalized contours for $\Omega_k$ vs $\sum m_\nu$ in the $\Lambda$CDM + $\sum m_\nu + \Omega_k$ model on the left, and for $A_{\text{Lens}}$ vs $\sum m_\nu$ in the $\Lambda$CDM + $\sum m_\nu + A_{\text{Lens}}$ model on the right, using Base and Base+SNe+R19 datasets and considering degenerate hierarchy.

We find it true in this model as well. With Base data, for the DH case, we have $A_{\text{Lens}} = 1.100^{+0.046}_{-0.056}$, which is consistent with $A_{\text{Lens}} = 1$ at 2$\sigma$.

In this $\Lambda$CDM + $\sum m_\nu + A_{\text{Lens}}$ model, $\sum m_\nu$ and $A_{\text{Lens}}$ are strongly correlated, since increasing $\sum m_\nu$ has an effect of reducing lensing induced smearing of the acoustic peaks since increasing $\sum m_\nu$ caused increasing suppression to the small scale matter power. The positive correlation between these two parameters can be seen in figure 5. This degeneracy causes the bounds on $\sum m_\nu$ to relax considerably compared to the $\Lambda$CDM + $\sum m_\nu$ model, as can be seen from table 4. Compared to a 95% bound of $\sum m_\nu < 0.121$ eV (DH) with Base data in the minimal $\Lambda$CDM + $\sum m_\nu$ model, here we get a bound of $\sum m_\nu < 0.293$ eV with the same data. Rather this $A_{\text{Lens}}$ issue actually explains why the neutrino mass bounds are so strong in the $\Lambda$CDM + $\sum m_\nu$ model, especially with the degenerate hierarchy. The CMB power spectrum data simply doesn’t like parameters which decrease lensing of CMB photons, and $\sum m_\nu$ is one such parameter. From DH to NH to IH, while using same data, the preference for higher $A_{\text{Lens}}$ values occurs with higher $\sum m_\nu$ values, and the discrepancy with $A_{\text{Lens}} = 1$ increases.

In the absence of varying dark energy EoS, however, $H_0$ and $\sum m_\nu$ are strongly correlated in this model. Thus compared to the Base data, Base+SNe+R19 data produces much stronger bounds by breaking the degeneracy between them. The model also produces lower $\sigma_8$ and $S_8$ values compared to other models considered in the paper, leading to a decrease in the $\sigma_8$ tension present between Planck and cosmic shear experiments like CFHTLenS [88], KiDS-450 [89] etc.

• v) Results in the $\Lambda$CDM + $\sum m_\nu + \Omega_k$ model: In this model we consider the possibility of a non-flat background geometry of the universe. $\Omega_k$ and $\sum m_\nu$ are correlated
positively and strongly with both Base and Base+SNe+R19 combinations, as can be seen from the right panel of figure 5. This causes the bounds on $\sum m_\nu$ to degrade compared to the $\Lambda$CDM + $\sum m_\nu$ model. With Base data we have: $\sum m_\nu < 0.150$ eV (DH), $\sum m_\nu < 0.173$ eV (DH), $\sum m_\nu < 0.198$ eV (IH). The origin the degeneracy again lies in the tightly constrained comoving distance to the last scattering surface, i.e. $\chi(z_{\text{dec}})$, leading to a three way degeneracy between $H_0$, $\Omega_k$ and $\sum m_\nu$ [90]. It is to be noted however, that the Planck data actually prefers values of $\Omega_k < 0$ [24], since closed universe models produce more lensing amplitude compared to a flat universe. However, inclusion of lensing data usually brings the parameters back closer to a flat universe. BAO data helps in partially breaking the three way degeneracy by constraining $H_0$. We find that with Base data, in the DH case $\Omega_k = 0.0004 \pm 0.0020$, which is perfectly consistent with $\Omega_k = 0$. The correlation coefficients are $R_{\Omega_k, \sum m_\nu} = +0.41$, $R_{H_0, \sum m_\nu} = -0.18$, and $R_{H_0, H_0} = +0.60$ for DH, i.e. the three parameters still remain considerably correlated with each other. From DH to NH to IH, preference for higher neutrino masses leads to preference for higher values of $\Omega_k$, although with the Base data $\Omega_k = 0$ is included within 2$\sigma$ ranges of $\Omega_k$, for any of the hierarchies. Things change with the inclusion of SNe+R19 data, which drive both $H_0$ and $\Omega_k$ to higher values, resulting in the rejection of a flat universe at more than 2$\sigma$ in case of IH.

### Akaike information criterion (AIC).

Here we compare the goodness of fit of normal and inverted hierarchy scenarios in the extended models that we have studied in this section. As in previous section, we use AIC as a measure of goodness of fit. AIC for the normal hierarchy case is denoted as $\text{AIC}_{\text{NH}}$, whereas for the inverted hierarchy it is $\text{AIC}_{\text{IH}}$. Since the NH and IH cases both have the same number of parameters, $\Delta \text{AIC} = \text{AIC}_{\text{NH}} - \text{AIC}_{\text{IH}} = \Delta \chi^2$, where $\Delta \chi^2$ is the $\chi^2$ difference between the NH and IH case at the best-fit points. The $\Delta \text{AIC}$ values for each of the extended models has been listed in table 5, for the Base and Base+SNe+R19 datasets. We find that in most scenarios $\Delta \text{AIC} < 0$, i.e. the fit due to NH is better than IH, whereas in a few cases $\Delta \text{AIC} > 0$. But in none of the models do we see any statistically significant difference. Rather in some cases $\Delta \text{AIC}$ is almost negligible. It is however, important to point out that also in most of the extended models, the bounds on $\sum m_\nu$, produced using the degenerate mass approximation and the $\sum m_\nu \geq 0$ prior, are considerably stronger than the bounds obtained with normal or inverted hierarchy of neutrino masses. Thus, using the unphysical degenerate case instead of properly incorporating the neutrino hierarchy might lead to a wrong notion of how strong the bounds on $\sum m_\nu$ really are, from cosmological data.

### 4 Discussion and conclusions

Presently the upper bounds on the sum of 3 active neutrino masses, $\sum m_\nu$ from analyses of cosmological data in the backdrop of $\Lambda$CDM + $\sum m_\nu$ model are bordering on the minimum sum of neutrino masses required by the inverted hierarchy, which is around 0.1 eV. However, these analyses are usually done with the assumption of 3 degenerate neutrino masses, which is not a good approximation any more since the bounds are strong enough that the neutrino mass-squared splittings ($\Delta m^2_{ij} = m^2_i - m^2_j$) can no longer be considered negligible. Thus in this paper we update the bounds on $\sum m_\nu$ from latest publicly available cosmological data while explicitly considering particular neutrino mass hierarchies using the results on mass-squared splittings from a global analysis of neutrino oscillations data, NuFit 4.0 [9]. For
implementing the normal and inverted hierarchy scenarios, we use the mass of the lightest mass eigenstate, denoted with \( m_0 \), as the varying parameter, and use the mass-squared splittings from NuFit 4.0 to determine the other masses in a particular hierarchy, and thus use the total mass sum, i.e. \( \sum m_\nu \), as a derived parameter. This approach puts some implicit priors on the neutrino mass sum: \( \sum m_\nu \geq 0.0589 \) eV for the normal hierarchy (NH) case, \( \sum m_\nu \geq 0.0995 \) eV for the inverted hierarchy (IH) case.

In the minimal \( \Lambda \)CDM + \( \sum m_\nu \) model with Planck 2018 TT,TE,EE, lowE, lensing, and the latest BAO data from various galaxy surveys, we find that at 95% C.L. \( \sum m_\nu < 0.121 \) eV in the case of degenerate mass approximation. We call this dataset “Base.” This is almost similar to the bound of \( \sum m_\nu < 0.120 \) eV quoted by the Planck 2018 collaboration using the same data, in the same model. We also find that in the same model we have \( \sum m_\nu < 0.146 \) eV in case of NH and \( \sum m_\nu < 0.172 \) eV in case of IH; i.e., the bounds vary significantly across the different mass orderings. The main reason for these differences in the bounds is the implicit priors on \( \sum m_\nu \) when we assume a particular hierarchy. However, we have verified that the case of degenerate neutrino masses with a prior \( \sum m_\nu \geq 0.0589 \) eV or \( \sum m_\nu \geq 0.0995 \) eV does not produce the same bounds as the NH or IH case with the lightest mass \( m_0 \) parametrization we have used in the paper. The NH and IH case produce bounds which are slightly stronger than the degenerate case with priors of \( \sum m_\nu \geq 0.0589 \) eV and \( \sum m_\nu \geq 0.0995 \) eV respectively, i.e. the two methods are not equivalent. Also, we find that the normal hierarchy is very mildly preferred to the inverted: \( \Delta \chi^2 \equiv \chi^2_{\text{NH}} - \chi^2_{\text{IH}} = -0.95 \) (best-fit). We also study this model against other dataset combinations like Base+R19, Base+SNe+R19, and Base+SNe+R19+DES, where SNe is the largest and latest type Ia supernovae sample (Pantheon sample), R19 is a Gaussian prior on \( H_0 = 74.03 \pm 1.42 \) km/sec/Mpc from local universe, and DES is the year 1 data from the Dark Energy Survey. For all the datasets the \( \chi^2 \) differences between the NH and IH case remain statistically mild, but the bounds on \( \sum m_\nu \) across the three different mass orderings vary considerably.

In this paper, we also provide bounds on \( \sum m_\nu \) considering different hierarchies in various extended models: \( \Lambda \)CDM + \( \sum m_\nu + r \), \( w \)CDM + \( \sum m_\nu \), \( w_0 w_a \)CDM + \( \sum m_\nu \), \( w_0 w_a \)CDM + \( \sum m_\nu \) model with \( w(z) \geq -1 \), \( \Lambda \)CDM + \( \sum m_\nu + A_{\text{Lens}} \), and \( \Lambda \)CDM + \( \sum m_\nu + \Omega_k \). Here \( r \) is the tensor-to-scalar ratio, \( w \) is the dark energy equation of state (DE EoS) parameter, \( w_0 \) and \( w_a \) again parametrize the DE EoS but in a dynamical manner: \( w(z) = w_0 + w_a z/(1 + z) \) (CPL parametrization), \( A_{\text{Lens}} \) is the parameter for artificial scaling of the lensing amplitude, and \( \Omega_k \) is the curvature energy density. We used the Base and Base+SNe+R19 dataset

| Models | Base | Base+SNe+R19 |
|--------|------|--------------|
| \( \Lambda \)CDM + \( \sum m_\nu + r \) | -1.39 | -3.30 |
| \( w \)CDM + \( \sum m_\nu \) | -1.59 | -0.36 |
| \( w_0 w_a \)CDM + \( \sum m_\nu \) | +2.08 | -1.19 |
| \( w_0 w_a \)CDM + \( \sum m_\nu \) with \( w(z) \geq -1 \) | -0.69 | -3.75 |
| \( \Lambda \)CDM + \( \sum m_\nu + A_{\text{Lens}} \) | +0.20 | -0.40 |
| \( \Lambda \)CDM + \( \sum m_\nu + \Omega_k \) | -1.96 | -1.02 |

Table 5. The values of \( \Delta \text{AIC} = \text{AIC}_{\text{NH}} - \text{AIC}_{\text{IH}} \) for various extended models studied in this paper, with Base and Base+SNe+R19 dataset combinations.
combinations to constrain these models.

Consistent with other studies (see e.g. [91] for a very recent example) we found that in some cases the formal bound $\sum m_\nu$ could be loosened by up to factor of two. However, in none of the extended models could we find any statistically significant difference in the quality of fit to the data between NH and IH, i.e. the current cosmological data is not sufficiently strong to demarcate the two hierarchies. This finding is consistent with previous work showing that a formal sensitivity to $\sum m_\nu$ of 0.01-0.02 eV is required to guarantee a conclusive distinction between the two hierarchies [42, 82]. However, the bounds on $\sum m_\nu$ do vary across different hierarchies in each model with the degenerate case always producing the strongest upper bounds. Thus we think that it is important to properly implement neutrino mass hierarchies. Otherwise, with the unphysical degenerate approximation we shall end up with neutrino mass bounds which are possibly unrealistically low, and might lead to a wrong notion of how strong the bounds on $\sum m_\nu$ really are, from cosmological data.

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References

[1] T2K Collaboration, K. Abe et al., Observation of Electron Neutrino Appearance in a Muon Neutrino Beam, Phys. Rev. Lett. 112 (2014) 061802, [arXiv:1311.4750].

[2] RENO Collaboration, J. K. Ahn et al., Observation of Reactor Electron Antineutrino Disappearance in the RENO Experiment, Phys. Rev. Lett. 108 (2012) 191802, [arXiv:1204.0626].

[3] Double Chooz Collaboration, Y. Abe et al., Reactor electron antineutrino disappearance in the Double Chooz experiment, Phys. Rev. D86 (2012) 052008, [arXiv:1207.6632].

[4] Daya Bay Collaboration, F. P. An et al., Observation of electron-antineutrino disappearance at Daya Bay, Phys. Rev. Lett. 108 (2012) 171803, [arXiv:1203.1669].

[5] KamLAND Collaboration, T. Araki et al., Measurement of neutrino oscillation with KamLAND: Evidence of spectral distortion, Phys. Rev. Lett. 94 (2005) 081801, [hep-ex/0406035].

[6] MINOS Collaboration, P. Adamson et al., Measurement of Neutrino Oscillations with the MINOS Detectors in the NuMI Beam, Phys. Rev. Lett. 101 (2008) 131802, [arXiv:0806.2237].

[7] Super-Kamiokande Collaboration, Y. Fukuda et al., Evidence for oscillation of atmospheric neutrinos, Phys. Rev. Lett. 81 (1998) 1562–1567, [hep-ex/9807003].

[8] SNO Collaboration, Q. R. Ahmad et al., Direct evidence for neutrino flavor transformation from neutral current interactions in the Sudbury Neutrino Observatory, Phys. Rev. Lett. 89 (2002) 011301, [nucl-ex/0204008].
[9] I. Esteban, M. C. Gonzalez-Garcia, A. Hernandez-Cabezudo, M. Maltoni, and T. Schwetz, *Global analysis of three-flavour neutrino oscillations: synergies and tensions in the determination of $\theta_{23}, \delta_{CP}$, and the mass ordering*, JHEP 01 (2019) 106, [arXiv:1811.0548].

[10] D. V. Forero, M. Tortola, and J. W. F. Valle, *Neutrino oscillations refitted*, Phys. Rev. D90 (2014), no. 9 093006, [arXiv:1405.7540].

[11] M. C. Gonzalez-Garcia, M. Maltoni, and T. Schwetz, *Updated fit to three neutrino mixing: status of leptonic CP violation*, JHEP 11 (2014) 052, [arXiv:1409.5439].

[12] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, and T. Schwetz, *Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity*, JHEP 01 (2017) 087, [arXiv:1611.0151].

[13] F. Capozzi, E. Lisi, A. Marrone, D. Montanino, and A. Palazzo, *Neutrino masses and mixings: Status of known and unknown $3\nu$ parameters*, Nucl. Phys. B908 (2016) 218–234, [arXiv:1601.0777].

[14] F. Capozzi, E. Di Valentino, E. Lisi, A. Marrone, A. Melchiorri, and A. Palazzo, *Global constraints on absolute neutrino masses and their ordering*, Phys. Rev. D95 (2017), no. 9 096014, [arXiv:1703.0447].

[15] A. Caldwell, A. Merle, O. Schulz, and M. Totzauer, *Global Bayesian analysis of neutrino mass data*, Phys. Rev. D96 (2017), no. 7 073001, [arXiv:1705.0194].

[16] J. Lesgourgues and S. Pastor, *Massive neutrinos and cosmology*, Phys. Rept. 429 (2006) 307–379, [astro-ph/0603494].

[17] Y. Y. Y. Wong, *Neutrino mass in cosmology: status and prospects*, Ann. Rev. Nucl. Part. Sci. 61 (2011) 69–98, [arXiv:1111.1436].

[18] J. Lesgourgues and S. Pastor, *Neutrino mass from Cosmology*, Adv. High Energy Phys. 2012 (2012) 608515, [arXiv:1212.6154].

[19] Topical Conveners: K.N. Abazajian, J.E. Carlstrom, A.T. Lee Collaboration, K. N. Abazajian et al., *Neutrino Physics from the Cosmic Microwave Background and Large Scale Structure*, Astropart. Phys. 63 (2015) 66–80, [arXiv:1309.5383].

[20] J. Lesgourgues and S. Pastor, *Neutrino cosology and Planck*, New J. Phys. 16 (2014) 065002, [arXiv:1404.1740].

[21] M. Archidiacono, T. Brinckmann, J. Lesgourgues, and V. Poulin, *Physical effects involved in the measurements of neutrino masses with future cosmological data*, JCAP 1702 (2017), no. 02 052, [arXiv:1610.0985].

[22] M. Lattanzi and M. Gerbino, *Status of neutrino properties and future prospects - Cosmological and astrophysical constraints*, Front.in Phys. 5 (2018) 70, [arXiv:1712.0710].

[23] S. Roy Choudhury and S. Choubey, *Updated Bounds on Sum of Neutrino Masses in Various Cosmological Scenarios*, JCAP 1809 (2018), no. 09 017, [arXiv:1806.1083].

[24] Plank Collaboration, N. Aghanim et al., *Planck 2018 results. VI. Cosmological parameters*, arXiv:1807.0620.

[25] S. Vagnozzi, E. Giusarma, O. Mena, K. Freese, M. Gerbino, S. Ho, and M. Lattanzi, *Unveiling $\nu$ secrets with cosmological data: neutrino masses and mass hierarchy*, Phys. Rev. D96 (2017), no. 12 123503, [arXiv:1701.0817].

[26] N. Palanque-Delabrouille et al., *Neutrino masses and cosmology with Lyman-alpha forest power spectrum*, JCAP 1511 (2015), no. 11 011, [arXiv:1506.0597].

[27] E. Di Valentino, E. Giusarma, M. Lattanzi, O. Mena, A. Melchiorri, and J. Silk, *Cosmological Axion and neutrino mass constraints from Planck 2015 temperature and polarization data*, Phys. Lett. B752 (2016) 182–185, [arXiv:1507.0866].
A. J. Cuesta, V. Niro, and L. Verde, *Neutrino mass limits: robust information from the power spectrum of galaxy surveys*, Phys. Dark Univ. 13 (2016) 77–86, [arXiv:1511.0598](https://arxiv.org/abs/1511.0598).

Q.-G. Huang, K. Wang, and S. Wang, *Constraints on the neutrino mass and mass hierarchy from cosmological observations*, Eur. Phys. J. C76 (2016), no. 9 489, [arXiv:1512.0589](https://arxiv.org/abs/1512.0589).

M. Moresco, R. Jimenez, L. Verde, A. Cimatti, L. Pozzetti, C. Maraston, and D. Thomas, *Constraining the time evolution of dark energy, curvature and neutrino properties with cosmic chronometers*, JCAP 1612 (2016), no. 12 039, [arXiv:1604.0018](https://arxiv.org/abs/1604.0018).

E. Giusarma, M. Gerbino, O. Mena, S. Vagnozzi, S. Ho, and K. Freese, *Improvement of cosmological neutrino mass bounds*, Phys. Rev. D94 (2016), no. 8 083522, [arXiv:1605.0432](https://arxiv.org/abs/1605.0432).

F. Couchot, S. Henrot-Versill, O. Perdereau, S. Plaszczynski, B. Rouillé d’Orfeuil, M. Spinelli, and M. Tristram, *Cosmological constraints on the neutrino mass including systematic uncertainties*, Astron. Astrophys. 606 (2017) A104, [arXiv:1703.1082](https://arxiv.org/abs/1703.1082).

C. Doux, M. Penna-Lima, S. D. P. Vitenti, J. Triguero, E. Aubourg, and K. Ganga, *Cosmological constraints from a joint analysis of cosmic microwave background and spectroscopic tracers of the large-scale structure*, Mon. Not. Roy. Astron. Soc. 480 (2018), no. 4 5386–5411, [arXiv:1706.0458](https://arxiv.org/abs/1706.0458).

S. Wang, Y.-F. Wang, and D.-M. Xia, *Constraints on the sum of neutrino masses using cosmological data including the latest extended Baryon Oscillation Spectroscopic Survey DR14 quasar sample*, Chin. Phys. C77 (2017), no. 11 762, [arXiv:1707.0274](https://arxiv.org/abs/1707.0274).

A. Upadhye, *Neutrino mass and dark energy constraints from redshift-space distortions*, JCAP 1905 (2019), no. 05 041, [arXiv:1707.0935](https://arxiv.org/abs/1707.0935).

L. Salvati, M. Douspis, and N. Aghanim, *Constraints from thermal Sunyaev-Zeldovich cluster counts and power spectrum combined with CMB*, Astron. Astrophys. 614 (2018) A13, [arXiv:1708.0069](https://arxiv.org/abs/1708.0069).

R. C. Nunes and A. Bonilla, *Probing the properties of relic neutrinos using the cosmic microwave background, the Hubble Space Telescope and galaxy clusters*, Mon. Not. Roy. Astron. Soc. 473 (2018), no. 4 4404–4409, [arXiv:1710.1026](https://arxiv.org/abs/1710.1026).

L.-F. Wang, X.-N. Zhang, J.-F. Zhang, and X. Zhang, *Impacts of gravitational-wave standard siren observation of the Einstein Telescope on weighing neutrinos in cosmology*, Phys. Lett. B782 (2018) 87–93, [arXiv:1802.0472](https://arxiv.org/abs/1802.0472).

S. Roy Choudhury and A. Naskar, *Strong Bounds on Sum of Neutrino Masses in a 12 Parameter Extended Scenario with Non-Phantom Dynamical Dark Energy (w(z) ≥ −1) with CPL parameterization*, Eur. Phys. J. C79 (2019), no. 3 262, [arXiv:1807.0286](https://arxiv.org/abs/1807.0286).

S. Hannestad and T. Schwetz, *Cosmology and the neutrino mass ordering*, JCAP 1611 (2016), no. 11 035, [arXiv:1606.0469](https://arxiv.org/abs/1606.0469).

L. Xu and Q.-G. Huang, *Detecting the Neutrinos Mass Hierarchy from Cosmological Data*, Sci. China Phys. Mech. Astron. 61 (2018), no. 3 039521, [arXiv:1611.0517](https://arxiv.org/abs/1611.0517).

M. Gerbino, M. Lattanzi, O. Mena, and K. Freese, *A novel approach to quantifying the sensitivity of current and future cosmological datasets to the neutrino mass ordering through Bayesian hierarchical modeling*, Phys. Lett. B775 (2017) 239–250, [arXiv:1611.0784](https://arxiv.org/abs/1611.0784).
[45] F. Simpson, R. Jimenez, C. Pena-Garay, and L. Verde, Strong Bayesian Evidence for the Normal Neutrino Hierarchy, JCAP 1706 (2017), no. 06 029, [arXiv:1703.0342].

[46] T. Schwetz, K. Freese, M. Gerbino, E. Giusarma, S. Hannestad, M. Lattanzi, O. Mena, and S. Vagnozzi, Comment on "Strong Evidence for the Normal Neutrino Hierarchy", arXiv:1703.0458.

[47] A. J. Long, M. Raveri, W. Hu, and S. Dodelson, Neutrino Mass Priors for Cosmology from Random Matrices, Phys. Rev. D97 (2018), no. 4 043510, [arXiv:1711.0843].

[48] S. Gariazzo, M. Archidiacono, P. F. de Salas, O. Mena, C. A. Ternes, and M. Trtola, Neutrino masses and their ordering: Global Data, Priors and Models, JCAP 1803 (2018), no. 03 011, [arXiv:1801.0494].

[49] A. F. Heavens and E. Sellentin, Objective Bayesian analysis of neutrino masses and hierarchy, JCAP 1804 (2018), no. 04 047, [arXiv:1802.0945].

[50] P. F. De Salas, S. Gariazzo, O. Mena, C. A. Ternes, and M. Trtola, Neutrino Mass Ordering from Oscillations and Beyond: 2018 Status and Future Prospects, Front. Astron. Space Sci. 5 (2018) 36, [arXiv:1806.1105].

[51] S. Hannestad, Neutrino masses and the dark energy equation of state - Relaxing the cosmological neutrino mass bound, Phys. Rev. Lett. 95 (2005) 221301, [astro-ph/0505551].

[52] S. Joudaki, Constraints on Neutrino Mass and Light Degrees of Freedom in Extended Cosmological Parameter Spaces, Phys. Rev. D87 (2013) 083523, [arXiv:1202.0005].

[53] W. Yang, R. C. Nunes, S. Pan, and D. F. Mota, Effects of neutrino mass hierarchies on dynamical dark energy models, Phys. Rev. D95 (2017), no. 10 103522, [arXiv:1703.0255].

[54] C. S. Lorenz, E. Calabrese, and D. Alonso, Distinguishing between Neutrinos and time-varying Dark Energy through Cosmic Time, Phys. Rev. D96 (2017), no. 4 043510, [arXiv:1706.0073].

[55] W. Sutherland, The CMB neutrino mass/vacuum energy degeneracy: a simple derivation of the degeneracy slopes, Mon. Not. Roy. Astron. Soc. 477 (2018), no. 2 1913–1920, [arXiv:1803.0229].

[56] M. Sahln, Cluster-Void Degeneracy Breaking: Neutrino Properties and Dark Energy, Phys. Rev. D99 (2019), no. 6 063525, [arXiv:1807.0247].

[57] E. Di Valentino, A. Melchiorri, E. V. Linder, and J. Silk, Constraining Dark Energy Dynamics in Extended Parameter Space, Phys. Rev. D96 (2017), no. 2 023523, [arXiv:1704.0076].

[58] S. Vagnozzi, S. Dhawan, M. Gerbino, K. Freese, A. Goobar, and O. Mena, Constraints on the sum of the neutrino masses in dynamical dark energy models with $w(z) \geq -1$ are tighter than those obtained in $\Lambda$CDM, Phys. Rev. D98 (2018), no. 8 083501, [arXiv:1801.0855].

[59] X. Zhang, Impacts of dark energy on weighing neutrinos after Planck 2015, Phys. Rev. D93 (2016), no. 8 083011, [arXiv:1511.0265].

[60] S. Wang, Y.-F. Wang, D.-M. Xia, and X. Zhang, Impacts of dark energy on weighing neutrinos: mass hierarchies considered, Phys. Rev. D94 (2016), no. 8 083519, [arXiv:1608.0067].

[61] W. Hu and I. Sawicki, A Parameterized Post-Friedmann Framework for Modified Gravity, Phys. Rev. D76 (2007) 104043, [arXiv:0708.1190].

[62] M. Chevallier and D. Polarski, Accelerating universes with scaling dark matter, Int. J. Mod. Phys. D10 (2001) 213–224, [gr-qc/0009008].

[63] E. V. Linder, Exploring the expansion history of the universe, Phys. Rev. Lett. 90 (2003) 091301, [astro-ph/0208512].

[64] E. Calabrese, A. Slosar, A. Melchiorri, G. F. Smoot, and O. Zahn, Cosmic Microwave Weak lensing data as a test for the dark universe, Phys. Rev. D77 (2008) 123531, [arXiv:0803.2309].
A. Lewis and S. Bridle, Cosmological parameters from CMB and other data: A Monte Carlo approach, Phys. Rev. D66 (2002) 103511, [astro-ph/0205436].

A. Lewis, A. Challinor, and A. Lasenby, Efficient computation of CMB anisotropies in closed FRW models, Astrophys. J. 538 (2000) 473–476, [astro-ph/9911177].

S. P. Brooks and A. Gelman, General methods for monitoring convergence of iterative simulations, Journal of Computational and Graphical Statistics 7 (1998), no. 4 434–455, [https://www.tandfonline.com/doi/pdf/10.1080/10618600.1998.10474787].

Planck Collaboration, N. Aghanim et al., Planck 2018 results. V. CMB power spectra and likelihoods, arXiv:1907.1287.

BOSS Collaboration, S. Alam et al., The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological analysis of the DR12 galaxy sample, Mon. Not. Roy. Astron. Soc. 470 (2017), no. 3 2617–2652, [arXiv:1607.0315].

A. J. Ross, L. Samushia, C. Howlett, W. J. Percival, A. Burden, and M. Manera, The clustering of the SDSS DR7 main Galaxy sample I. A 4 per cent distance measure at z = 0.15, Mon. Not. Roy. Astron. Soc. 449 (2015), no. 1 835–847, [arXiv:1409.3242].

F. Beutler, C. Blake, M. Colless, D. H. Jones, L. Staveley-Smith, L. Campbell, Q. Parker, W. Saunders, and F. Watson, The 6dF Galaxy Survey: Baryon Acoustic Oscillations and the Local Hubble Constant, Mon. Not. Roy. Astron. Soc. 416 (2011) 3017–3032, [arXiv:1106.3366].

D. M. Scolnic et al., The Complete Light-curve Sample of Spectroscopically Confirmed SNe Ia from Pan-STARRS1 and Cosmological Constraints from the Combined Pantheon Sample, Astrophys. J. 859 (2018), no. 2 101, [arXiv:1710.0084].

A. G. Riess, S. Casertano, W. Yuan, L. M. Macri, and D. Scolnic, Large Magellanic Cloud Cepheid Standards Provide a 1% Foundation for the Determination of the Hubble Constant and Stronger Evidence for Physics Beyond LambdaCDM, Astrophys. J. 876 (2019), no. 1 85, [arXiv:1903.0760].

S. Taubenberger, S. H. Suyu, E. Komatsu, I. Jee, S. Birrer, V. Bonvin, F. Courbin, C. E. Rusu, A. J. Shajib, and K. C. Wong, The Hubble Constant determined through an inverse distance ladder including quasar time delays and Type Ia supernovae, arXiv:1905.1249.

DES Collaboration, T. M. C. Abbott et al., Dark Energy Survey year 1 results: Cosmological constraints from galaxy clustering and weak lensing, Phys. Rev. D98 (2018), no. 4 043526, [arXiv:1708.0153].

Planck Collaboration, P. A. R. Ade et al., Planck 2015 results. XIII. Cosmological parameters, Astron. Astrophys. 594 (2016) A13, [arXiv:1502.0158].

C. L. Reichardt, Understanding the Epoch of Cosmic Reionization: Challenges and Progress, pp. 227–245. Springer International Publishing, Cham, Switzerland, 2016.

Z. Hou et al., Constraints on Cosmology from the Cosmic Microwave Background Power Spectrum of the 2500 deg^2 SPT-SZ Survey, Astrophys. J. 782 (2014) 74, [arXiv:1212.6267].

T. D. Kitaching, L. Verde, A. F. Heavens, and R. Jimenez, Discrepancies between CFHTLenS cosmic shear and Planck: new physics or systematic effects?, Mon. Not. Roy. Astron. Soc. 459 (2016), no. 1 971–981, [arXiv:1602.0296].

H. Akaike, A new look at the statistical model identification, IEEE Transactions on Automatic Control 19 (December, 1974) 716–723.
[82] C. Mahony, B. Leistedt, H. V. Peiris, J. Braden, B. Joachimi, A. Korn, L. Cremonesi, and R. Nichol, Target Neutrino Mass Precision for Determining the Neutrino Hierarchy, \textit{arXiv:1907.0433}.

[83] J. Hamann, S. Hannestad, and Y. Y. Y. Wong, Measuring neutrino masses with a future galaxy survey, \textit{JCAP 1211} (2012) 052, [\textit{arXiv:1209.1043}].

[84] T. Brinckmann, D. C. Hooper, M. Archidiacono, J. Lesgourgues, and T. Sprenger, The promising future of a robust cosmological neutrino mass measurement, \textit{JCAP 1901} (2019) 059, [\textit{arXiv:1808.0595}].

[85] S. Roy Choudhury and S. Choubey, Constraining light sterile neutrino mass with the BICEP2/Keck Array 2014 B-mode polarization data, \textit{Eur. Phys. J. C79} (2019), no. 7 557, [\textit{arXiv:1807.1029}].

[86] \textbf{BICEP2, Keck Array} Collaboration, P. A. R. Ade et al., \textit{BICEP2 / Keck Array VIII: Measurement of gravitational lensing from large-scale B-mode polarization}, \textit{Astrophys. J. 833} (2016), no. 2 228, [\textit{arXiv:1606.0196}].

[87] E. V. Linder, \textit{The Dynamics of Quintessence, The Quintessence of Dynamics}, \textit{Gen. Rel. Grav. 40} (2008) 329–356, [\textit{arXiv:0704.2064}].

[88] T. Erben et al., \textit{CFHTLenS: The Canada-France-Hawaii Telescope Lensing Survey - Imaging Data and Catalogue Products}, \textit{Mon. Not. Roy. Astron. Soc. 433} (2013) 2545, [\textit{arXiv:1210.8156}].

[89] H. Hildebrandt et al., \textit{KiDS-450: Cosmological parameter constraints from tomographic weak gravitational lensing}, \textit{Mon. Not. Roy. Astron. Soc. 465} (2017) 1454, [\textit{arXiv:1606.0533}].

[90] C. Howlett, A. Lewis, A. Hall, and A. Challinor, CMB power spectrum parameter degeneracies in the era of precision cosmology, \textit{JCAP 1204} (2012) 027, [\textit{arXiv:1201.3654}].

[91] E. Di Valentino, A. Melchiorri, and J. Silk, \textit{Cosmological constraints in extended parameter space from the Planck 2018 Legacy release}, \textit{arXiv:1908.0139}.