Proposal for the Open String Tachyon Effective Action in the Linear Dilaton Background

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ABSTRACT: In this paper we propose tachyon effective actions for unstable D-branes in superstring and bosonic string theories in the presence of the linear dilaton background.

KEYWORDS: D-branes
1. Introduction

It is well known that bosonic and Type IIA and Type IIB string theories contain in their spectrum unstable D-branes. These unstable D-branes are characterised by having a single tachyonic mode of the mass\(^2 \mu_{\text{super}}^2 = -\frac{1}{2}\) in case of supersymmetric non-BPS D-branes or a tachyonic mode with \(\mu_{\text{bos}}^2 = -1\) (in \(\alpha' = 1\) unit) living on their world-volume. The tachyon field has local maximum at \(T = 0\) and pair of global minima at \(\pm \infty\) where the negative contribution from the potential exactly cancels the tension of non-BPS D-brane in supersymmetric theory, in bosonic theory the story is basically the same with the exception that there is only one global minimum at \(T = \infty\). As a result the configuration where the tachyon potential is at its minimum corresponds to vacuum without any D-branes and fluctuations of the open string field around these minima do not contain any perturbative open string states in the spectrum\(^1\).

One can also study the time-dependent tachyon solution where the tachyon rolls away from near the top of the potential towards the minimum of the potential at \(T = \infty\) \(\cite{17, 18, 19, 21, 22, 23}\). This time-dependent tachyon condensation is described be marginal deformation on the boundary of the world sheet \(T(X^0) = e^{\mu X^0}\). It was shown in \(\cite{34, 21}\) that this time-dependent tachyon condensation has nice effective

\(^1\)For review of the tachyon condensation in string theory, see \(\cite{1, 4, 3, 2, 5, 6, 7, 8}\). Some papers, where the effective field theory descriptions of the tachyon dynamics can be found, are \(\cite{9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 37}\).
field theory description based on the superstring tachyon effective action \[21\]

\[ S = - \int d^D x \mathcal{L}, \quad \mathcal{L} = \frac{1}{g_s(1 + \frac{T^2}{2})} \sqrt{1 + \frac{T^2}{2} + \eta^{\mu\nu} \partial_\mu T \partial_\nu T}, \quad (1.1) \]

where \( g_s \) is constant string coupling constant related to the constant dilaton \( \Phi_0 \) as \( g_s = e^{\Phi_0} \). It was shown that this action correctly captures the physics around the marginal tachyon condensation in case of constant dilaton and the flat Minkowski space-time \[21, 37\]. This action was reconstructed around the vicinity of the conformal point corresponding to the time-dependent background which should represent the exact boundary conformal field theory

\[ T = T_0 e^{\sqrt{2} \Phi_0} + T_- e^{-\sqrt{2} \Phi_0}. \quad (1.2) \]

The remarkable fact about the action (1.1) is that if we demand that the generic first order tachyon Lagrangian

\[ \mathcal{L} = V(T)K((\partial T)^2) \quad (1.3) \]

has (1.2) as its exact solution fixes its time-dependent part to be

\[ \mathcal{L} = \frac{1}{g_s(1 + \frac{T^2}{2})} \sqrt{1 + \frac{T^2}{2} - (\partial_0 T)^2}. \quad (1.4) \]

Then (1.1) arises from (1.4) by Lorenz-covariant generalisation which was extensively discussed in \[29\]. Note also that after field redefinition

\[ \frac{T}{\sqrt{2}} = \sinh \frac{T}{\sqrt{2}} \quad (1.5) \]

the Lagrangian (1.1) becomes “tachyon DBI” like Lagrangian

\[ \mathcal{L} = e^{-\Phi_0} V(\tilde{T}) \sqrt{-\det(\eta_{\mu\nu} + \partial_\mu \tilde{T} \partial_\nu \tilde{T})} = e^{-\Phi_0} V(\tilde{T}) \sqrt{1 + \eta^{\mu\nu} \partial_\mu \tilde{T} \partial_\nu \tilde{T}}. \quad (1.6) \]

Further support for the validity of the (1.6) in the description of the tachyon dynamics around the marginal deformation was given in \[35\].

Even if the relation between the action (1.1) and the tachyon effective action calculated from the string theory partition function is not completely clear \[29\] we mean that the fact that the tachyon marginal perturbation is an exact solution of the equation of motion arising from (1.1) is very attractive and deserve further study. In particular, we can ask the question whether there are another marginal tachyon profiles that are solutions of the equation of motion arising from (1.1). Some

\(^2\)Our convention is \( \eta_{\mu\nu} = \text{diag}(-1,1,\ldots,1) \), \( \mu, \nu = 0,\ldots,D-1 \), \( i, j = 1,\ldots,D-1 \) where \( D \) is dimension of the space time.
interesting comments and suggestions considering these problems were also presented in [29] and we hope to return to them in near future. For next purposes we also present the version of the D-brane tachyon effective action in bosonic string theory that we proposed in [27] 3

$$\mathcal{L} = \frac{1}{g_s(1+T)} \sqrt{1 + T + \frac{\eta^{\mu\nu} \partial_\mu T \partial_\nu T}{T}}$$  \quad (1.7)

We have shown in [27] that the tachyon profile

$$T = e^{\beta \mu x^\mu}, \beta_\mu \eta^{\mu\nu} \beta_\nu + 1 = 0$$  \quad (1.8)

is exact solution of the equation of motion arising from (1.7). The pure time-dependent case $\beta_0 = 1, \beta_i = 0$ corresponds to the decay of unstable brane where in the far past D-brane is in its unstable maximum $T = 0$ and rolls to the stable minimum $T = \infty$ at far future.

Since the tachyon effective actions (1.1), (1.6) and (1.7) are useful in the description of the tachyon dynamics near the marginal tachyon profile in flat space time one can ask the question whether their description of the tachyon condensation in nontrivial closed background is as successful as in flat space time. An example of such a closed string background that has simple conformal field theory description and where one can easily find marginal boundary interaction [36] is linear dilaton background. The results given there were used recently in [40] where we have shown that the tachyon effective Lagrangian obtained from the string partition function is different from the tachyon effective Lagrangian (1.7). Despite this fact we mean that it is useful to try to find the generalisation of the action (1.1) to the linear dilaton background that will not be directly related to the Lagrangian evaluated from the string partition function but which will have the rolling tachyon profile in the linear dilaton background as its exact solution. This is the question which we will address in this paper. We find such a form of the tachyon action that reduces to (1.1) for constant dilaton and that the rolling tachyon profile in the linear dilaton background [36] is the exact solution of the equation of motion that arises from it. We will also calculate the component of the stress energy tensor and the dilaton source. Unfortunately we will find that their asymptotic behaviour is different from the exact results given in [36]. We will discuss this issue more extensively in conclusion and suggest its possible resolution.

The organisation of this paper is as follows. In section (2) we present the main proposal for the tachyon effective action in superstring theory. In section (3) we will generalise the action (1.7) to the case of linear dilaton background as well. And finally in conclusion (4) we outline our results and suggest possible extension of this work.

3More detailed discussion of the tachyon effective action in bosonic theory can be found in [28].
2. Proposal for supersymmetric tachyon effective action in the linear dilaton background

In this section we propose such a form of the tachyon effective action in superstring theory which has the rolling tachyon profile in the linear dilaton background as its exact solution. The linear dilaton background is characterised with the spacelike vector $V_\mu$ so that the dilaton is

$$\Phi = V_\mu x^\mu + \Phi_0,$$  \hspace{1cm} (2.1)

where $\Phi_0$ is constant part of the dilaton and which coincides with the constant string coupling constant $g_s$ in case of vanishing gradient $V_\mu = 0$. Since it is not completely clear how to include the massless scalars that parametrise transverse position of unstable D-brane into the action (1.1) we restrict ourselves to the case of D-brane that fills the whole space-time.

The motivation for our proposal is that the straightforward generalisation of (1.1) and (1.7) to the case of the arbitrary dilaton $\Phi$

$$L_{\text{super}} = \frac{e^{-\Phi}}{1 + \frac{T^2}{2}} \sqrt{1 + \frac{T^2}{2} + \eta^{\mu\nu} \partial_\mu T \partial_\nu T}, \quad L_{\text{bos}} = \frac{e^{-\Phi}}{1 + T} \sqrt{1 + T + \frac{\eta^{\mu\nu} \partial_\mu T \partial_\nu T}{T}}$$  \hspace{1cm} (2.2)

does not correctly captures the physics of the tachyon condensation in the linear dilaton background (1.7). In particular, we have shown that there exists the rolling tachyon solution $T \sim e^{\beta t}$ only for large $T$, however the relation between $\beta$ and $V_0$ is not the same as the relation coming from the condition of the marginality of the boundary interaction $T = e^{\beta t}$. Moreover we have also shown that the behaviour of components of the stress energy tensor calculated from (2.2) differs from the results given in [36] in rather dramatic way. We meant that these facts suggest that in order to find the tachyon effective action in the linear dilaton background that would have the rolling tachyon as its exact solution we should allow more general insertion of $\Phi$.

The second leading point in our proposal is the requirement that for constant $\Phi$ our action reduces to (1.1). For that reason we conjecture that the factor in the square root of (1.1) should be generalised in case of the linear dilaton background as

$$B = 1 + g_s e^{-\Phi} \left( \frac{T^2}{2} + \eta^{\mu\nu} \partial_\mu T \partial_\nu T - T \eta^{\mu\nu} \partial_\mu T \partial_\nu \Phi \right).$$  \hspace{1cm} (2.3)

First of all we see that this term reduces to the standard one given in (1.1) for $\Phi = \Phi_0$. On the other hand for the half S-brane solution

$$T = e^{\beta_\mu x^\mu}, \quad \eta^{\mu\nu} \beta_\mu \beta_\nu - \eta^{\mu\nu} \beta_\mu V_\nu = \frac{1}{2}$$  \hspace{1cm} (2.4)

(2.3) is constant which according to our observations given in [27] is an important fact for the search of the exact solution with the correct marginal condition. We
have also included the exponential factor $e^{-\Phi}$ in (2.3) from the following reason. If we demand that the action should have the tachyon profile (2.4) as its exact solution it is possible that some terms in the equation of motion will be equal for $\sqrt{B} = 1$ which, however need not to be true for any $B = \text{const} \neq 1$. It would then be useful to have such an exact solution which will lead to any constant value of $B$. In case of the constant dilaton background such a solution is full S-brane profile. Then the requirement that the tachyon effective action should have this profile as its exact solution was used in the determination of the tachyon effective action in $^4$[21, 28, 37].

An analogue such a solution in case of the linear dilaton background is

$$T = T_+ e^{\beta^+_\mu x^\mu} + T_- e^{\beta^-_\mu x^\mu}, \beta^\pm_\mu \eta^{\mu \nu} \beta^\pm_\nu - \eta^{\mu \nu} \beta^\pm_\mu V_\nu = \frac{1}{2},$$

where now

$$(\beta^\pm_0)^2 - \beta^\pm_0 V_0 = \frac{1}{2} \Rightarrow \beta^\pm_0 = V_0 \pm \sqrt{V_0^2 + 2}, \beta^+_i = V_i, \beta^-_i = 0.$$ (2.6)

In this case we have

$$B = 1 + g_s e^{-\Phi} \left( \frac{1}{2} \right) T_+^2 e^{2\beta^+_\mu x^\mu} + 2T_+ T_- e^{(\beta^+_\mu + \beta^-_\mu)x^\mu} + T_-^2 e^{2\beta^-_\mu x^\mu} + T_+ T_- e^{(\beta^+_\mu + \beta^-_\mu)x^\mu} (2\beta^+_\mu \eta^{\mu \nu} \beta^-_\nu - (\beta^+_\mu + \beta^-_\mu) \eta^{\mu \nu} V_\nu) + T_+ T_- e^{(\beta^+_\mu + \beta^-_\mu)x^\mu} (2\beta^+_\mu \eta^{\mu \nu} \beta^-_\nu - (\beta^+_\mu + \beta^-_\mu) \eta^{\mu \nu} V_\nu)) = 1 + 2g_s T_+ T_- (1 - \frac{V_\mu V_\mu}{2})$$ (2.7)

using

$$(\beta^+_\mu + \beta^-_\mu)x^\mu = V_0 x^0 + V_i x^i = \Phi,$$

$$\beta^+_\mu \eta^{\mu \nu} \beta^-_\nu = -\beta^+_0 \beta^-_0 = \frac{1}{2},$$

$$(\beta^+_\mu + \beta^-_\mu) \eta^{\mu \nu} V_\nu = -V_0^2 + V_i^2 = V_\mu V_\mu.$$ (2.8)

Now we come to proposal considering tachyon effective action in the linear dilaton background. We require that for constant dilaton this Lagrangian reduces to the (1.1) and also we would like to have a connection with the formulation of the Lagrangian evaluated on the tachyon marginal profile given in [40]. For that reason we propose following form of the supersymmetric tachyon effective Lagrangian

$$\mathcal{L} = \frac{1}{g_s} \sqrt{B} \int_0^\infty ds e^{-s \frac{1}{2} g_s e^{-\Phi} T^2 \frac{e}{x}} \equiv \frac{1}{g_s} \sqrt{B} \int_0^\infty ds e^{-D(x,s)},$$

$$D(x,s) = s + x s^{F(V)} G(V), x \equiv \frac{g_s e^{-\Phi} T^2}{2}.$$ (2.9)
where $F, G$ are unknown functions of $V_\mu$ that should obey following conditions: 
$\lim_{V \to 0} F(V) = 1$, $\lim_{V \to 0} G(V) = 1$ that imply that (2.3) reduces to (1.1). Our goal is to determine these functions $F, G$ so that the general tachyon profile (2.5) will be solution of the equation of motion that arises from (2.9)

$$
-\sqrt{B}g_s e^{-\Phi} T \int_0^\infty dse^{-D}s^F \frac{g_s e^{-\Phi} T}{\sqrt{B}} \int_0^\infty dse^{-D} - \frac{g_s e^{-\Phi} \eta^{\mu\nu} \partial_\mu T \partial_\nu \Phi}{2\sqrt{B}} \int_0^\infty dse^{-D} - 
$$

$$
-\partial_\mu \left[ \frac{g_s e^{-\Phi} \eta^{\mu\nu} \partial_\nu T}{\sqrt{B}} \int_0^\infty dse^{-D} \right] + \partial_\mu \left[ \frac{g_s e^{-\Phi} T \eta^{\mu\nu} \partial_\nu \Phi}{2\sqrt{B}} \int_0^\infty dse^{-D} \right] = 0.
$$

(2.10)

Using the fact that $\sqrt{B} = \text{const}$ and also

$$
g_s e^{-\Phi} [\eta^{\mu\nu} \partial_\mu T \partial_\nu T - T \eta^{\mu\nu} \partial_\mu T \partial_\nu \Phi] = B - 1 - \frac{g_s e^{-\Phi} T^2}{2}.
$$

(2.11)

the equation of motion (2.9) after some calculation takes the form

$$
\int dse^{-D}s^F \frac{1}{G} \left[ 1 + x \left( 1 - \frac{V_\mu V_\mu}{2} \right) \right] = \int dse^{-D} \left( 1 - \frac{V_\mu V_\mu}{2} \right).
$$

(2.12)

The last equation is obeyed if we take

$$
F = 1, \frac{1}{G} = (1 - \frac{V_\mu V_\mu}{2})
$$

(2.13)

as can be seen from following integrals

$$
\int_0^\infty dse^{-D}s^F \frac{1}{G} \left[ 1 + x \left( 1 - \frac{V_\mu V_\mu}{2} \right) \right] = \frac{(1 - \frac{V_\mu V_\mu}{2})}{1 + x(1 - \frac{V_\mu V_\mu}{2})},
$$

$$
\left( 1 - \frac{V_\mu V_\mu}{2} \right) \int_0^\infty dse^{-D} = \frac{(1 - \frac{V_\mu V_\mu}{2})}{1 + x(1 - \frac{V_\mu V_\mu}{2})}.
$$

(2.14)

We see that the generalisation of (1.1) to the case of linear dilaton background takes remarkable simple form

$$
\mathcal{L} = \frac{1}{g_s(1 + \frac{g_s e^{-\Phi} T^2}{2}(1 - \frac{V_\mu V_\mu}{2}))} \sqrt{B}.
$$

(2.15)

Note that for small $T$ (2.15) is equal to

$$
\mathcal{L} = \frac{1}{g_s} - \frac{e^{-\Phi} T^2}{4}(1 - V_\mu V_\mu) + \frac{e^{-\Phi} \eta^{\mu\nu} \partial_\mu T \partial_\nu T}{2} - \frac{e^{-\Phi} T^2 \partial_\mu \Phi \partial_\nu \Phi}{4} = 
$$

$$
= \frac{1}{g_s} + \frac{e^{-\Phi}}{2} \left[ \eta^{\mu\nu} \partial_\mu T \partial_\nu T - \frac{T^2}{2} \right].
$$

(2.16)
which is correct (up to constant term) tachyon effective Lagrangian around the point $T = 0$. In the previous equation we have used the integration by parts in the D-brane effective action that implies

$$\frac{1}{2} \int d^Dxe^{-\Phi}T \eta^{\mu\nu} \partial_\mu T \partial_\nu \Phi = -\frac{1}{4} \int d^Dxe^{-\Phi}T^2 \eta^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi \ . \ (2.17)$$

Tachyon effective action (2.15) also implies one remarkable fact that could be useful for further research. As is well known the norm of the dilaton field $V_\mu V^\mu$ is related to the number of space-time dimensions as $[52, 53]$}

$$V_\mu \eta^{\mu\nu} V^\nu = \frac{10 - D}{4} \ . \ (2.18)$$

Then we immediately get that the potential term in (2.15) vanishes in two dimensions since $1 - \frac{V_\mu V^\mu}{2} = \frac{D - 2}{8}$. We can interpret this property as a consequence of the absence of the perturbative instability in two dimensional field theory where the tachyon is massless.

As a next step we will calculate the stress energy tensor and the dilaton source from (2.15). Since it is convenient to have the stress energy tensor symmetric we rewrite the term containing gradient of the dilaton in (2.15) as

$$g_s e^{-\Phi} \eta^{\mu\nu} \partial_\mu T \partial_\nu \Phi = g_s e^{-\Phi} \eta^{\mu\nu} \left( \partial_\mu T \partial_\nu \Phi + \partial_\mu \Phi \partial_\nu T \right) \ . \ (2.19)$$

Before explicit calculation of the stress energy tensor and dilaton charge we should discuss one important issue. The effective tachyon action contains the term $1 - \frac{V_\mu V^\mu}{2}$ that for linear dilaton background can be written as

$$1 - \frac{\partial_\mu \Phi \eta^{\mu\nu} \partial_\nu \Phi}{2} \ . \ (2.20)$$

We mean that this is the right expression in the action that we should use if we calculate the stress energy tensor and dilaton charge by variation of the action with respect to $g^{\mu\nu}$ and $\Phi$. For example, when we calculate the stress energy tensor in flat space-time we replace the Minkowski metric $\eta^{\mu\nu}$ with arbitrary $g^{\mu\nu}$, perform the variation with respect to $g^{\mu\nu}$ and then we again introduce the flat metric $\eta^{\mu\nu}$. With analogue with the previous example we mean that it is appropriate to replace $V_\mu V^\mu$ with $g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi$, perform the variation with respect to $\Phi$ and $g^{\mu\nu}$ and then insert the values of metric and dilaton corresponding to the flat space-time and linear dilaton background $^5$. As a result of this calculation we get components of the stress energy

$^5$There could be potential subtlety with this approach. Since we have obtained the action (2.17) in fixed linear dilaton background this off-shell continuation could be ambiguous.
tensor for unstable D-brane in linear dilaton background

\[ T_{\mu\nu} = -\eta_{\mu\nu} \mathcal{L} + \frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} = \]

\[ -\frac{\eta_{\mu\nu} \sqrt{B}}{g_s(1 + \frac{g_s e^{-\Phi} T^2}{2} (1 - \frac{V_0^\nu}{2}))} + \frac{g_s e^{-\Phi} \partial_\mu T \partial_\nu T - \frac{g_s e^{-\Phi} T^2}{2} (\partial_\mu TV_\nu + V_\mu \partial_\nu T)}{g_s(1 + \frac{g_s e^{-\Phi} T^2}{2} (1 - \frac{V_0^\nu}{2})) \sqrt{B}} + \]

\[ \frac{g_s e^{-\Phi} T^2 V_\mu V_\nu \sqrt{B}}{2 g_s(1 + g_s e^{-\Phi} T^2 (1 - \frac{V_0^\nu}{2}))^2}. \]

(2.21)

For the half S-brane solution with \( T = e^{\beta x^0} \) we obtain following components

\[ T_{00} = \frac{1 + \frac{g_s e^{-\Phi} T^2}{2} (1 - \frac{V_0^\nu}{2})}{g_s(1 + \frac{g_s e^{-\Phi} T^2}{2} (1 - \frac{V_0^\nu}{2}))} \]

\[ + \frac{g_s e^{-\Phi} T^2 V_0 V_0}{2 g_s(1 + g_s e^{-\Phi} T^2 (1 - \frac{V_0^\nu}{2}))^2}, \]

\[ T_{ij} = -\delta_{ij} \frac{1}{g_s(1 + \frac{g_s e^{-\Phi} T^2}{2} (1 - \frac{V_0^\nu}{2}))} + \frac{g_s e^{-\Phi} T^2 V_i V_j}{2 g_s(1 + g_s e^{-\Phi} T^2 (1 - \frac{V_0^\nu}{2}))^2}, \]

\[ T_{0i} = T_{i0} = -\frac{g_s e^{-\Phi} \beta V_i T^2}{2 g_s(1 + \frac{g_s e^{-\Phi} T^2}{2} (1 - \frac{V_0^\nu}{2}))} + \frac{g_s e^{-\Phi} T^2 V_0 V_i}{2 g_s(1 + g_s e^{-\Phi} T^2 (1 - \frac{V_0^\nu}{2}))^2}. \]

(2.22)

In the same way we get the dilaton source

\[ J_\Phi = -\frac{\delta \mathcal{L}}{\delta \Phi} = -\frac{g_s e^{-\Phi} T^2 (1 - \frac{V_0^\nu}{2}) \sqrt{B}}{2 g_s(1 + \frac{g_s e^{-\Phi} T^2}{2} (1 - \frac{V_0^\nu}{2}))^2} + \frac{g_s e^{-\Phi} \left( \frac{T^2}{2} + \eta^\mu\nu \partial_\mu T \partial_\nu T - \eta^\mu\nu \partial_\mu TV_\nu \right)}{2 g_s \sqrt{B}(1 + \frac{g_s e^{-\Phi} T^2}{2} (1 - \frac{V_0^\nu}{2}))} - \]

\[ -\partial_\mu \left[ \frac{g_s e^{-\Phi} T \eta^\mu\nu \partial_\nu T}{2 g_s(1 + \frac{g_s e^{-\Phi} T^2}{2} (1 - \frac{V_0^\nu}{2})) \sqrt{B}} \right] + \partial_\mu \left[ \frac{\eta^\mu\nu V_\nu g_s e^{-\Phi} T^2 \sqrt{B}}{2 g_s(1 + g_s e^{-\Phi} T^2 (1 - \frac{V_0^\nu}{2}))^2} \right] \]

(2.23)

that for the tachyon profile \( T = e^{\beta x^0} \) is equal to

\[ J_\Phi = -\frac{g_s e^{-\Phi} T^2 (- \frac{V_0^\nu}{2} - \beta V_0)}{2 g_s(1 + \frac{g_s e^{-\Phi} T^2}{2} (1 - \frac{V_0^\nu}{2}))^2} + \frac{(-\beta V_0 - \frac{V_0^\nu}{2}) g_s e^{-\Phi} T^2 (1 - \frac{g_s e^{-\Phi} T^2}{2} (1 - \frac{V_0^\nu}{2}))}{g_s(1 + g_s e^{-\Phi} T^2 (1 - \frac{V_0^\nu}{2}))^3}. \]

(2.24)

One can easily confirm that \( T_{\mu\nu} \) and \( J_\Phi \) are related through the conservation law

\[ \partial^\mu T_{\mu\nu} = V_\nu J_\Phi. \]

(2.25)

From (2.22) we determine the asymptotic behaviour of the components of the stress energy tensor at far future and far past. Using the fact that for the half S-brane \( e^{-\Phi} T^2 = e^{-V_0 x^0 + t \sqrt{\alpha^2 + 2}} \) we obtain the asymptotic behaviour of \( T_{\mu\nu} \) at far future

\[ T_{00} \sim \frac{1}{g_s \left(1 - \frac{V_0^\nu}{2}\right)}, T_{ij} \sim 0, T_{0i} \sim -\frac{\beta V_i}{g_s \left(1 - \frac{V_0^\nu}{2}\right)}. \]

(2.26)
together with
\[ J_\Phi \to 0 \, . \] (2.27)

We see that at far future the energy density is independent on time while \( T_{ij} \) scales to zero. It would be nice to interpret this result as a fact, that the the tachyon dust is compression of gas of massive closed string whose energy is unaffected in a way by change of the dilaton. This behaviour is in agreement with the CFT calculation given in [36]. On the other hand CFT analysis implies that off-diagonal components of the stress energy tensor are zero while we have got nonzero components \( T_{0i} \) whose meaning is at present not completely clear to us. However the fact that they are proportional to \(-V_i\) suggests that tachyon dust slides towards weak coupling which would be in agreement with the interpretation given in [36]. To see more clearly this correspondence we should study the fluctuations around different tachyon rolling solution. We hope to return to these problems in future. However much more serious problem arises in the opposite limit \( t \to -\infty \). In this case we again find

\[ T_{00} \sim \frac{1}{g_s} + \frac{g_s e^{-\Phi} T^2}{g_s^4} V_\mu V^\mu + \frac{g_s e^{-\Phi} T^2 V_0^2}{g_s^2} \to \frac{1}{g_s} , \]

\[ T_{ij} \sim -\delta_{ij} \left( 1 - \frac{g_s e^{-\Phi} T^2}{g_s^2} \left( 1 - \frac{V_\mu V^\mu}{2} \right) \right) + \frac{g_s e^{-\Phi} T^2 V_i V_j}{g_s^2} \to -\delta_{ij} \frac{1}{g_s} , \]

\[ T_{0i} = -\frac{g_s e^{-\Phi} T^2 V_i}{g_s^2} (\beta - V_0) \to 0 , \]

\[ J_\Phi \sim \frac{g_s e^{-\Phi} T^2}{g_s} \to 0 . \] (2.28)

Even if the components of the stress energy tensors scale as \( \frac{1}{g_s} \) this result is not in agreement with the calculations given in [36] where it was shown that components of the stress energy tensors scale at far past as \( \sim e^{-\Phi} \).

We mean that this could be indication that our proposed effective action, even if it has the tachyon rolling solution for every values of \( T \), should be modified at small \( T \) where we could expect that terms with higher derivatives could be important. For example, let us presume that some additional term in the action contains following combination \( C = 1 + \frac{T^2}{2} + T \partial_\mu \eta^{\mu\nu} \partial_\nu T - T \eta^{\mu\nu} \partial_\mu T \partial_\nu \Phi \). Clearly this combination is equal to one for any marginal tachyon profile and hence the term in the tachyon effective Lagrangian that contains such a combination should come with the standard factor \( e^{-\Phi} \) and consequently could give significant contribution to the tachyon dynamics in weak coupling. On the other hand when we include terms containing the second derivatives to the action there is no reason why we should not include terms containing derivative of higher orders. For that reason we restrict ourselves in our proposal to the action containing first derivatives only and leave the more general case for future work.
3. Proposal for tachyon effective action in bosonic string theory in linear dilaton background

In this section we will propose the generalised form of the tachyon effective action in bosonic string theory in linear dilaton background. Recall that in the flat space-time the tachyon effective action has form given in (1.7). Following the results given in case of the supersymmetric action we should include some powers of the factor $g_s e^{-\Phi}$ in front of the tachyon $T$. Since in the effective action (1.7) tachyon components scales as $T$ we propose that we should include everywhere the factor $g_s^{1/2} e^{-\Phi/2}$. In other words our proposal for the tachyon effective action in bosonic string theory has the form

$$L = \frac{1}{g_s} \sqrt{B} \int_0^\infty ds e^{-s - g_s^{1/2} e^{-\Phi/2} T} \approx \sqrt{B} \int_0^\infty ds e^{-D},$$

$$B = 1 + g_s^{1/2} e^{-\Phi/2} \left[ \frac{\eta_{\mu\nu} \partial_\mu T \partial_\nu T}{T} - \eta_{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi \right],$$

$$D = s + x \frac{F}{G}, x \equiv g_s^{1/2} e^{-\Phi/2} T.$$  \hspace{1cm} (3.1)

with the property that for $V \to 0$ we have $F, G \to 1$. The equation of motion that arises from (3.1) is

$$-g_s^{1/2} e^{-\Phi/2} \sqrt{B} \int_0^\infty ds e^{-D} \frac{F}{G} + \frac{g_s^{1/2} e^{-\Phi/2}}{2 \sqrt{B}} \int_0^\infty ds e^{-D} - g_s^{1/2} e^{-\Phi/2} \eta_{\mu\nu} \partial_\mu T \partial_\nu T \frac{2T^2 \sqrt{B}}{D} \int_0^\infty ds e^{-D} -$$

$$-\partial_\mu \left[ \frac{g_s^{1/2} e^{-\Phi/2} \eta_{\mu\nu} \partial_\nu T}{T \sqrt{B}} \right] \int_0^\infty ds e^{-D} + \partial_\mu \left[ \frac{g_s^{1/2} e^{-\Phi/2} \eta_{\mu\nu} \partial_\nu \Phi}{2 \sqrt{B}} \right] \int_0^\infty ds e^{-D} = 0$$

that for the tachyon profile $T = e^{\beta_\mu x^\mu}, B = 1$ is equal to

$$g_s^{1/2} e^{-\Phi/2} \int dse^{-D} \frac{F}{G} \left[ 1 + x \left( 1 - \frac{V^2}{4} \right) \right] = g_s^{1/2} e^{-\Phi/2} \int dse^{-D} \left( 1 - \frac{V^2}{4} \right).$$ \hspace{1cm} (3.2)

The last formulation suggests that $F, G$ should be equal to

$$F = 1, \quad \frac{1}{G} = (1 - \frac{V_\mu V^\mu}{4})$$ \hspace{1cm} (3.4)

as follows from the same arguments as were given in the previous section. Since in the linear dilaton background we have $\partial_\mu \Phi = V_\mu$ we can also write the tachyon effective action as

$$L = \frac{\sqrt{B}}{g_s} \int_0^\infty ds e^{-s(1 + \Phi)} = \frac{1}{g_s \left( 1 + g_s^{1/2} e^{-\Phi/2} T \left( 1 - \frac{\eta_{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi}{4} \right) \right)} \cdot \sqrt{B}$$ \hspace{1cm} (3.5)
Now we will calculate the stress energy tensor from \((3.3)\). This calculation can be performed completely in the same way as in previous section with the result

\[
T_{\mu\nu} = -\eta_{\mu\nu}\mathcal{L} + 2\frac{\delta \mathcal{L}}{\delta g^{\mu\nu}} =
\]

\[
= -\eta_{\mu\nu}\frac{1}{g_s(1 + g_s^{1/2} e^{-\Phi/2} T(1 - \frac{V_\mu V_\nu}{4}))}\sqrt{B} +
\]

\[
+ \frac{g_s^{1/2} e^{-\Phi/2}\partial_\mu T \partial_\nu T - \frac{1}{2} g_s^{1/2} e^{-\Phi/2} }{g_s(1 + g_s^{1/2} e^{-\Phi/2} T(1 - \frac{V_\mu V_\nu}{4}))}\sqrt{B} +
\]

\[
= \frac{g_s e^{-\Phi/2} TV_\mu V_\nu \sqrt{B}}{2 g_s(1 + g_s^{1/2} e^{-\Phi/2} T(1 - \frac{V_\mu V_\nu}{4}))^2}
\]

(3.6)

that for the rolling tachyon solution \(T = e^{\beta x_0}\) gives

\[
T_{00} = \frac{1 + g_s^{1/2} e^{-\Phi/2} T}{g_s(1 + g_s^{1/2} e^{-\Phi/2} T(1 - \frac{V_\mu V_\nu}{4}))} + \frac{g_s^{1/2} e^{-\Phi/2} TV_0^2}{2 g_s(1 + g_s^{1/2} e^{-\Phi/2} T(1 - \frac{V_\mu V_\nu}{4}))^2},
\]

\[
T_{ij} = -\delta_{ij} \frac{1}{g_s(1 + g_s^{1/2} e^{-\Phi/2} T(1 - \frac{V_\mu V_\nu}{4}))} + \frac{g_s^{1/2} e^{-\Phi/2} TV_i V_j}{2 g_s(1 + g_s^{1/2} e^{-\Phi/2} T(1 - \frac{V_\mu V_\nu}{4}))^2},
\]

\[
T_{0i} = T_{i0} = -\frac{\beta_0 V_i g_s^{1/2} e^{-\Phi/2} T}{2 g_s(1 + g_s^{1/2} e^{-\Phi/2} T(1 - \frac{V_\mu V_\nu}{4}))} + \frac{g_s^{1/2} e^{-\Phi/2} TV_0 V_i}{2 g_s(1 + g_s^{1/2} e^{-\Phi/2} T(1 - \frac{V_\mu V_\nu}{4}))^2}.
\]

(3.7)

For \(t \to \infty\) we have

\[
T_{00} \to \frac{1}{g_s(1 - \frac{V_\mu V_\nu}{4})}, T_{ij} \to 0, T_{0i} \to -\frac{\beta_0 V_i}{g_s(1 - \frac{V_\mu V_\nu}{4})},
\]

(3.8)

while for \(t \to -\infty\) we again obtain the same behaviour as in supersymmetric case given in previous section

\[
T_{\mu\nu} \sim -\eta_{\mu\nu} \frac{1}{g_s}.
\]

(3.9)

After some straightforward calculation we can show that the stress energy tensor obeys the conserved law

\[
\partial^\mu T_{\mu\nu} = V_\nu J_\Phi,
\]

(3.10)

where the dilaton source \(J_\Phi = -\frac{\beta_0}{\sqrt{5}}\) is equal to

\[
J_\Phi = \frac{e^{-\Phi/2} T(1 - \frac{V_\mu V_\nu}{4})}{2(1 + e^{-\Phi/2} T(1 - \frac{V_\mu V_\nu}{4}))^2}\sqrt{B} - \frac{e^{-\Phi/2}(T + \frac{\eta_{\mu\nu}\partial_\mu T \partial_\nu T - \eta_{\mu\nu}\partial_\mu T \partial_\nu \Phi)}{4(1 + e^{-\Phi/2} T(1 - \frac{V_\mu V_\nu}{4}))}\sqrt{B}} +
\]

\[
+ \partial_\mu \left[ \frac{e^{-\Phi/2} \eta_{\mu\nu} \partial_\nu T}{2(1 + e^{-\Phi/2} T(1 - \frac{V_\mu V_\nu}{4}))}\sqrt{B} \right] - \partial_\mu \left[ \frac{T e^{-\Phi/2} \eta_{\mu\nu} \partial_\nu \Phi \sqrt{B}}{2(1 + e^{-\Phi/2} T(1 - \frac{V_\mu V_\nu}{4}))^2} \right].
\]

(3.11)
In this section we have performed the generalisation of the tachyon effective action \((1.7)\) to the case of linear dilaton field. As in supersymmetric case we have got the tachyon effective action that has remarkable simple form and that has the rolling tachyon in the linear background as its exact solution. On the other hand we have also seen that there are problems with the asymptotic behaviour of the stress energy tensors whose origin is probably the same as in the supersymmetric case.

4. Conclusion

In this paper we proposed the tachyon effective actions for unstable space-time filling D-branes in superstring and bosonic theories in the linear dilaton background. Our proposal was based on requirements that for constant \(\Phi\) these tachyon effective actions reduce to the tachyon effective actions \((1.1)\) and \((1.7)\). The second condition was that in the case of unstable D-brane in superstring theory the rolling tachyon profiles known as half S-brane and full S-brane are solutions of the equation of motion that arises from it. Using these conditions we were able to obtain the tachyon effective action for unstable space-time filling D-brane in linear dilaton background. Then we apply the similar procedure to the case of tachyon effective action in bosonic theory and we have found the action that has half S-brane in the linear dilaton background as its exact solution \(^6\). We have also calculated the stress energy tensors from these actions and then we have studied their behaviour for half S-brane tachyon profile with emphasise to their asymptotic form at far past and future. In the far future we have found that the energy remains constant and the pressure goes to zero together with the observation that the tachyon dust seems to drift to the weak coupling region. We mean that these results are in rough coincidence with the calculations performed in \([36]\) even if the stress energy tensor calculated here is different from the stress energy tensor given in \([36]\). This fact is not surprising since as was shown recently in \([29, 40]\) tachyon effective actions \((1.1)\) and \((1.7)\) are not directly determined from the string partition function. On the other hand we have seen that in the far past the behaviour of the stress energy tensor is completely different from results presented in \([36]\) that show that in the asymptotic past the components of the stress energy tensor scale as \(1/g_s(t)\). On the other hand we have found that in this limit the diagonal components of the stress energy tensor approach constant values and the off-diagonal ones vanish. It seems to us that this result is uncomfortable and suggests that something important is missing in our analysis. We mean that in order to correctly describe the tachyon dynamics near the point \(T = 0\) we should include into the effective Lagrangians terms that contain higher order derivatives of tachyon and which should give significant contributions around the point \(T = 0\). In

\(^6\)We have restricted ourselves to the half S-brane tachyon profile since the action \((1.7)\) has not solution corresponding to the full S-brane, for more general approach to the tachyon effective actions in bosonic theory, see \([28]\).
fact there is no reason why we should restrict ourselves to the action with the first
derivatives only since as one can see all higher derivatives scale as the first order one
in case of rolling tachyon profile. We must also stress that the proposed form of the
tachyon effective action was not determined directly from the string theory analysis
that should be based on the calculation using the powerful machinery of background
independent string field theory [11] or alternatively using the sigma model approach [12].

In spite of these comments we still hope that our proposed actions could be use-
ful for the study of some aspects of D-branes in two dimensions [13] [16, 17, 18, 19].
For that reason we mean that it deserves further study, at least as a toy model for
the description of the tachyon condensation on D-brane in nontrivial dilaton back-
ground. So that let us mention some possibilities for further research. We mean
that it would be interesting to study how the tachyon effective action given above
could be applied in the tachyon cosmology (For recent interesting papers discussing
this subject with detailed list of references, see [50, 51].) It would be also nice to see
whether our proposal could be useful in the study of D-branes decay in two dimen-
sions [14, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65] even if there are limitations in appli-
cation of our action since it does not contain the coupling between the closed string
tachyon and D-brane. We also believe that our action could be useful in recent anal-
ysis of two dimensional Type OA and Type 0B theories [10, 67, 68, 69, 70, 71, 72, 73].
We hope to return to these problems in near future.

Acknowledgement This work was supported by the Czech Ministry of Education
under Contract No. 14310006.

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