A note on a sports league scheduling problem

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Abstract

Sports league scheduling is a difficult task in the general case. In this short note, we report two improvements to an existing enumerative search algorithm for a NP-hard sports league scheduling problem known as “prob026” in CSPLib. These improvements are based on additional rules to constraint and accelerate the enumeration process. The proposed approach is able to find a solution (schedule) for all prob026 instances for a number $T$ of teams ranging from 12 to 70, including several $T$ values for which a solution is reported for the first time.

Keywords: sports league scheduling, prob026 in CSPLib, balanced tournament design, enumerative search, constraints

1. Introduction

The sports league scheduling problem studied in this note, called “prob026” in CSPLib [1] and also known as the “balanced tournament design” problem in combinatorial design theory [2, pages 238-241], is a NP-hard problem [3] that seems to be first introduced in [4].

- There are $T = 2n$ teams (i.e., $T$ even). The season lasts $W = T - 1$ weeks. Weeks are partitioned into $P = T/2$ slots called “periods” or “stadiums”. Each week, one match is scheduled in every period;
- $c_H$ constraint: All teams play each other exactly once (Half competition);
- $c_P$ constraint: No team plays more than twice in a Period. This constraint may be motivated by the equal distribution of stadiums to teams;
- $c_W$ constraint: Every team plays exactly one game in every Week of the season, i.e., all teams are different in a week.

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The problem then is to schedule a tournament with respect to these definitions and constraints. A solution to prob026 is a complete assignment of \( D = \{(t, t' \leq T) \} \) items (couples of teams) to variables of \( X = \{ (p, w), 1 \leq p \leq P, 1 \leq w \leq W \} \) (couples of periods and weeks) verifying the constraint set \( C = \{c_H, c_P, c_W\}, (p, w) = (t, t') \) meaning that team \( t \) meets team \( t' \) in period \( p \) and week \( w \). Thus, a solution can be conveniently represented by a \( P \times W \) sized table, whose items are integer couples \((t, t')\), see Table 1 for an example of a valid schedule for \( T = 8 \). For \( T = 70 \) teams, this represents a problem with 2,415 variables and 2,415 values per variable. There are \( T(T-1)/2 \) matches to be scheduled. A valid schedule can be thought of as a particular permutation of these matches. So, for \( T \) teams, the search space size is \([T(T-1)/2]!\).

| Periods | Weeks 1 | Weeks 2 | Weeks 3 | Weeks 4 | Weeks 5 | Weeks 6 | Weeks 7 |
|---------|--------|--------|--------|--------|--------|--------|--------|
| 1       | 1.2    | 6.8    | 2.5    | 4.5    | 4.7    | 3.8    | 1.7    |
| 2       | 3.7    | 5.7    | 3.4    | 1.8    | 5.6    | 2.4    | 2.6    |
| 3       | 4.6    | 1.4    | 7.8    | 3.6    | 2.8    | 1.5    | 3.5    |
| 4       | 5.8    | 2.3    | 1.6    | 2.7    | 1.3    | 6.7    | 4.8    |

Direct construction methods exist when \((T - 1) \mod 3 \neq 0\) \([5, 6]\) or \(T/2\) is odd \([7, 8]\). However, finding a solution (schedule) in the general case for any arbitrary \( T \) remains a highly challenging task. Indeed, to our knowledge, the best performing search algorithm \([8]\) can solve all the instances for \( T \) up to 50, but only some cases when \( 50 < T \leq 70 \). Other representative solution approaches include integer programming \([10]\) (limited to \( T \leq 12 \)), transformation into the SAT problem \([11]\) \((T \leq 20)\), distributed approach \((T \leq 28\) according to \([12]\)), constraint programming \([13]\) and tabu search \([14]\) \((T \leq 40)\).

In this paper, we present two improvements to the Enumerative Algorithm (EnASS) proposed in \([9]\). With the proposed enhancements, all the instances for \( 12 \leq T \leq 70 \) can now be solved.

We provide in the next section a brief recall of the original EnASS method. We show then in the following sections a new EnASS variant that solves all instances up to \( T = 60 \) (including the problematic \( T \mod 4 = 0 \) cases) and another new variant that solves all the \( 12 \leq T \leq 70 \) instances.

2. A brief recall of the EnASS algorithm

EnASS starts with a complete \( \overline{a} \) conflicting assignment. \( \overline{a} \) is built, in linear-time complexity, to satisfy the \( c_W \) and \( c_H \) constraints (thanks to patterned one-factorization \( [2, \text{page } 662] \)). At this stage, the remaining \( c_P \) constraint is not verified in \( \overline{a} \), see Table 2 where team 8 appears more than twice in the 4th period.
Table 2: Initial conflicting $\mathfrak{x}$ schedule for 8 teams.

| Periods | Weeks |
|---------|-------|
| 1       | 1,2   |
| 2       | 3,7   |
| 3       | 4,6   |
| 4       | 5,8   |
|         | 2,3   |
|         | 1,4   |
|         | 2,5   |
|         | 3,6   |
|         | 4,7   |
|         | 1,5   |
|         | 2,6   |
|         | 3,5   |
|         | 1,6   |
|         | 2,7   |
|         | 1,3   |
|         | 2,4   |
|         | 3,5   |
|         | 1,7   |
|         | 6,7   |
|         | 1,7   |
|         | 5,6   |
|         | 4,7   |
|         | 1,5   |
|         | 2,6   |
|         | 3,5   |
|         | 1,8   |
|         | 2,8   |
|         | 3,8   |
|         | 4,8   |

Algorithm 1 EnASS: An overview.

Require: Two periods ($p$ and $\mathfrak{p}$) and a week ($w$)
1: if $p = P + 1$ then // A solution is obtained since all periods are filled and valid according to $\mathcal{R}$
   2: return true
3: end if
4: if $w = w_l + 1$ then // Period $p$ is filled and valid according to $\mathcal{R}$, try to fill next period
   5: return EnASS($p + 1, w_l, 1$)
6: end if
7: if $\mathfrak{p} = P + 1$ then // Backtrack since no match from week $w$ in $\mathfrak{x}$ can be scheduled in period $p$ of week $w$ without violating $\mathcal{R}$
   8: return false
9: end if
10: if $\exists 1 \leq p' < p : (p', w) = \mathfrak{x}(p, w)$ then // The $\mathfrak{x}(p, w)$ match is already scheduled, try next match
11: return EnASS($p, w, \mathfrak{p} + 1$)
12: end if
13: $(p, w) \leftarrow \mathfrak{x}(p, w)$ // Schedule the $\mathfrak{x}(p, w)$ match in period $p$ of week $w$
14: if $\mathcal{R}$ is locally verified and EnASS($p, w + 1, 1$) = true then // The previous assignment and next calls lead to a solution
15: return true
16: end if
17: // From this point, $\mathcal{R}$ is locally violated or next calls lead to a failure
18: Undo step 13 // Backtrack
19: return EnASS($p, w, \mathfrak{p} + 1$) // Try next value for $(p, w)$
Roughly speaking, EnASS uses \( \exists \) to search for a valid tournament by filling a \( P \times W \) table (initially empty) row by row, see Algorithm 1 where \( w_f \) and \( w_l \) are the first and last weeks EnASS considers when filling any period \( p (1 \leq w_f < w_l \leq W) \). \( \exists \langle p, w \rangle \) is the match in \( \exists \) scheduled in period \( p \) and week \( w \), and \( R \) is a set of properties (or “Requirements”) that (partial or full) solutions must verify. EnASS admits three integer parameters: \( p \) and \( w \) specify which \( \langle p, w \rangle \) variable is currently considered, \( \bar{p} \) specifies the value assignment tried (see step 13). The function returns TRUE if a solution has been found or FALSE otherwise. Backtracks are sometimes performed in the latter case. EnASS is called first, after the \( \exists \) initialization, with \( p = 1, w = w_f \) and \( \bar{p} = 1 \) meaning that it tries to fill the slot in the first period of week \( w_f \) with the \( \exists \langle 1, w_f \rangle \) match.

The basic EnASS skeleton presented in Algorithm 1 solves prob026 only up to \( T = 12 \) when the \( R \) set is restricted to \{cp\} while considering the first week as invariant with respect to \( \exists \) (i.e., \( \forall 1 \leq p \leq P, (p, 1) = \exists \langle p, 1 \rangle \) with \( w_f = 2 \) (since the first week is invariant) and \( w_l = W \)). Note that making the first week invariant helps to avoid some evident symmetries mentioned in \[9\] see Sect. 4 and 5.3.

To tackle larger-size problems, several EnASS variants were considered in \[4\]. EnASS1 solved prob026 up to \( T = 32 \), except the \( T = 24 \) case, including in \( R \) an implicit property (called “cp” in \[9\]) of all prob026 solutions: \( R_0 = \{cp, cp\} \). The \( cp \) property was not originally mentioned in the seminal definition of the problem \[4\] and seems to be first introduced in \[8\]. EnASS2, derived from EnASS1 by further including an “implied” requirement \( r_{\Rightarrow} \), solved all instances up to \( T = 50 \): \( R_1 = \{cp, cp, r_{\Rightarrow}\} \). Finally, EnASS3 solved some cases (when \( T \mod 4 \neq 0 \)) for \( T \) up to 70 with two additional invariants \( r_I \) and \( r_V \): \( R_2 = \{cp, cp, r_{\Rightarrow}, r_I, r_V\} \).

3. Solving all instances of prob026 up to \( T = 60 \)

The rule \( r_{\Rightarrow} \) used to solve all prob026 instances up to \( T = 60 \) resembles the original \( r_{\Rightarrow} \) requirement introduced in \[4\] Sect. 7. Like \( r_{\Rightarrow} \), \( r_{\Rightarrow} \) fixes more than one variable (two exactly, to be more precise) when exploring a new branch in the search tree. The difference between \( r_{\Rightarrow} \) and the new \( r_{\Rightarrow} \) rule is the weeks that are concerned: While \( r_{\Rightarrow} \) connects any week \( w_f \leq w \leq P \) to week \( T - w + 1 \), the \( r_{\Rightarrow} \) constraint links any week \( 1 \leq w \leq P - 1 \) together with week \( W - w + 1 \).

More formally, \( \forall 1 \leq w \leq P - 1, r_{\Rightarrow}(p, w) \iff (p, w) = \exists \langle p, w \rangle \Rightarrow (p, W - w + 1) = \exists \langle p, W - w + 1 \rangle \).

This leads to EnASS3 which comes from the EnASS1 algorithm from \[4\] by replacing in \( R_1 \) the \( r_{\Rightarrow} \) requirement with the new \( r_{\Rightarrow} \) rule: \( R_3 = \{cp, cp, r_{\Rightarrow}\} \). Like for EnASS1, step 13 in the basic EnASS description (see Algorithm 1) may be adapted since one additional variable has now to be instantiated and \( w_l \) has to be set to \( P - 1 \) before running EnASS3. Steps 14-16 have also to be modified since, when \( w = w_l + 1 \), the \( P \) week is not yet filled (so, the \( p \) period is not entirely filled either). Table 1 in Sect. 4 shows an example of a solution found by EnASS3 for \( T = 8 \): For instance, scheduling the \( (3, 4) \) match from week 3
in period 2 forces the (5, 6) match from week 5 (5 = 7 − 3 + 1) to be also in period 2.

In Table 3 we show for 6 ≤ T ≤ 50 comparisons of our new EnASS_3 variant (as well as another new EnASS_4 variant discussed in the next section), against the EnASS_1 algorithm which solves all the instances for T ≤ 50 within 3 hours per T value. The reported statistics include execution times (in seconds in all tables) and number of backtracks (columns labeled “|BT|”) needed to find a first solution.

In Table 4 we show for 52 ≤ T ≤ 70 comparisons between the new variant EnASS_3 (and EnASS_4) and the EnASS_2 algorithm from [9] which solves some instances with T ≤ 70 where T mod 4 ≠ 0. “–” marks in the “Time” (respectively “|BT|”) columns indicate that the method found no solution within 3 hours (resp. that |BT| exceeds the maximal integer value authorized by the compiler/system, i.e., 4 294 967 295). All EnASS variants were coded in C and all computational results were obtained on an Intel PIV processor (2 Ghz) Linux station with 2 Gb RAM.

Table 3: Solving all prob026 instances up to T = 50.

| T   | EnASS_1 [9] | EnASS_3 (Sect. 3) | EnASS_4 (Sect. 4) |
|-----|-------------|-------------------|-------------------|
|     | Time | | Time | | Time | |
| 6   | < 1  | 6   | < 1  | 1   | < 1  | 5   |
| 8   | < 1  | 16  | < 1  | 6   | < 1  | 111 |
| 10  | < 1  | 715 | < 1  | 350 | < 1  | 125 |
| 12  | < 1  | 86  | < 1  | 25  | < 1  | 560 |
| 14  | < 1  | 451 | < 1  | 65  | < 1  | 465 |
| 16  | < 1  | 557 | < 1  | 73  | < 1  | 1  |
| 18  | < 1  | 1099| < 1  | 772 | < 1  | 1  |
| 20  | < 1  | 2811| < 1  | 708 | < 1  | 1  |
| 22  | < 1  | 11615| < 1 | 1142 | < 1 | 3237 |
| 24  | < 1  | 12623| < 1 | 5332 | < 1 | 736 |
| 26  | < 1  | 37708| < 1 | 5313 | < 1 | 2311 |
| 28  | < 1  | 35530| < 1 | 16365| < 1 | 85315|
| 30  | < 1  | 650811| < 1 | 49620| < 1 | 68033|
| 32  | < 1  | 332306| < 1 | 91094| < 1 | 22407|
| 34  | < 1  | 1342216| < 1| 131169| < 1| 21696|
| 36  | < 1  | 2160102| < 1| 524491| < 1| 248184|
| 38  | 5.34 | 13469359| < 1| 763317| < 1| 83636|
| 40  | 6.25 | 16393039| 1.70| 7335775| < 1| 1720480|
| 42  | 107.69| 256686929| 2.74| 11575637| < 1| 612423|
| 44  | 876.91| 1944525360| 19.80| 79587812| 1.02| 2489017|
| 46  | 1573.31| 3565703651| 10.22| 38865293| 1.59| 343033|
| 48  | 542.79| 1231902706| 1112.55| 4289081568| 5.69| 12080931|
| 50  | 6418.52| – | 4018.20| – | 17.38| 34639665|

From Table 3–4 one observes that EnASS_3 solves more prob026 instances than EnASS_1 within 3 hours. Indeed, while EnASS_1 is limited to T ≤ 50, EnASS_3
Table 4: Solving all prob026 instances when $50 < T \leq 70$.

| $T$ | $\text{EnASS}_2$ [9] | $\text{EnASS}_3$ (Sect. [3]) | $\text{EnASS}_4$ (Sect. [4]) |
|-----|-----------------------|-------------------------------|-------------------------------|
|     | Time                  | Time                          | Time                          |
|     | $|BT|$                 | $|BT|$                         | $|BT|$                         |
| 52  | –                      | –                             | 377.84                        | 1345460512 | 50.11 | 101432823 |
| 54  | 10.59                  | 29767940                      | 763.08                        | 2802487580 | 101.74 | 196808595 |
| 56  | –                      | –                             | 2552.65                       | –          | 334.26 | 753747164 |
| 58  | 269.88                 | 827655311                     | 13715.33                      | –          | 878.96 | 1851547682 |
| 60  | –                      | –                             | 198250.44                     | –          | 2364.47 | –          |
| 62  | 279.38                 | 494071117                     | –                             | –          | 9866.51 | –          |
| 64  | –                      | –                             | –                             | –          | 32386.67 | –          |
| 66  | 7508.51                | 1614038658                    | –                             | –          | 85989.73 | –          |
| 68  | –                      | –                             | –                             | –          | 518194.31 | –          |
| 70  | 8985.05                | –                             | –                             | –          | 1512574.41 | –          |
4. Solving all prob026 instances when $50 < T \leq 70$

The rule $r_I'$ used to solve all prob026 instances for $50 < T \leq 70$ is similar to the original $r_I$ requirement introduced in \cite{b17} Sect. 7. Indeed, like $r_I$, $r_I'$ inverses two weeks and keeps them invariant during the search. The only difference between $r_I$ and the new $r_I'$ rule is the weeks that are concerned: While $r_I$ considers weeks 2 and $W$, the $r_I'$ constraint inverses weeks 2 and $W - 1$. More formally, $\forall w \in \{2, W - 1\}, r_I'(w) \iff \forall 1 \leq p \leq P, \langle p, w \rangle = \langle P - p + 1, w \rangle$.

This leads to $\text{EnASS}_4$ which comes from $\text{EnASS}_3$ by adding in $\mathcal{R}_3$ the new $r_I'$ rule: $\mathcal{R}_4 = \{c_P, c_P', r_I', r_{I'}\}$. Since the first two weeks are now invariant (and the last two due to $r_{I'}$), $w_f$ has to be set to 3 before running $\text{EnASS}_4$. Table 4 in Sect. 4 shows an example of a solution found by $\text{EnASS}_4$ (and $\text{EnASS}_3$) for $T = 8$: For instance, the first match in week 2 is $\mathcal{P}(4 - 1 + 1, 2)$, i.e., $\langle 1, 2 \rangle = (6, 8)$.

The computational performance of the $\text{EnASS}_4$ variant is provided in Table 4 for $6 \leq T \leq 50$ and in Table 4 for $50 < T \leq 70$\footnote{The first solution found by $\text{EnASS}_4$ for $50 < T \leq 70$ is available on-line from \url{http://www.info.univ-angers.fr/pub/hamiez/EnASS4/Sol52-70.html}.}. One notices that $\text{EnASS}_4$ is faster than $\text{EnASS}_3$ and $\text{EnASS}_1$ (see the “|BT|” columns in Table 4) to solve instances when $T \geq 12$ (and for $T = 8$), except for the $T \in \{12, 14, 16, 22, 28, 30\}$ cases. Furthermore, within 3 hours per $T$ value, $\text{EnASS}_4$ is capable of solving larger instances (up to $T = 62$, see Table 4) than $\text{EnASS}_1$ ($T \leq 50$) and $\text{EnASS}_3$ ($T \leq 56$). While $\text{EnASS}_2$ solves only some instances for $50 < T \leq 70$ (those verifying $T \text{mod} 4 \neq 0$, see Table 4), $\text{EnASS}_4$ finds solutions for all these cases. This is achieved within 3 hours for $T$ up to 62, but larger instances can require more execution time (about 18 days for $T = 70$). Finally, note that adding the new $r_I'$ rule excludes solutions for $T \in \{6, 10\}$.

5. Conclusion

We provided in this short note two enhancements to an Enumerative Algorithm for Sports Scheduling (EnASS) previously proposed in \cite{b17}. These enhancements are based on additional properties (identified in some solutions) as new constraints to reduce the search tree constructed by the algorithm. With these
enhancements, all prob026 instances with $T \leq 70$ can be solved for the first time. Since the main idea behind the enhancements is to add refined requirement rules in the EnASS method, we expect that the method can be further improved to solve prob026 instances for $T > 70$.

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