Antisymmetric Higgs representation in SO(10) for neutrinos

Noriyuki Oshimo
Institute of Humanities and Sciences and Department of Physics
Ochanomizu University, Tokyo, 112-8610, Japan
(Dated: March 25, 2022)

A Model based on SO(10) grand unified theory (GUT) and supersymmetry is presented to describe observed phenomena for neutrinos. The large mixing angles among different generations, together with the small masses, are attributed to the Higgs boson structure at the GUT energy scale. Quantitative discussions for these observables are given, taking into account their energy evolution.

PACS numbers: 12.10.Dm, 12.15.Ff, 12.60.Jv, 14.60.Pq

Experiments for atmospheric and solar neutrinos suggest non-vanishing but extremely small masses for the neutrinos. Also observed are large mixing angles among different generations for the leptons. These results imply existence of physics beyond the standard model. In fact, the lightness of the neutrinos could be naturally accounted for by those models based on grand unified theory (GUT) which contain large Majorana masses for the right-handed neutrinos.

The GUT models, however, face one serious problem posed by the large generation-mixing angles of the leptons. Since the leptons stand on an equal footing with the quarks, in simple models the amount of their generation mixings becomes similar to that of the quarks, leading to small mixing angles. Some complication is thus necessary for accommodating the observed properties of the neutrinos within the framework of GUT. Aiming at construction of a plausible model, various works have been performed.

In this letter we propose an SO(10) GUT model coupled to supersymmetry, in which the lepton mixings and the neutrino masses can be described fairly simply. For group representations which contain the Higgs bosons responsible for the quark and lepton masses, our model includes \(120\), as well as \(10\) and \(126\). This \(120\) representation makes the mixing structures different between the quarks and the leptons, while the \(126\) representation yields large Majorana masses for the right-handed neutrinos. Below the GUT energy scale, the model is the same as the minimal supersymmetric standard model (MSSM) except that the dimension-five operators are generated for the SU(2) doublets of the leptons and the Higgs bosons. These terms give small Majorana masses to the left-handed neutrinos after electroweak symmetry is broken down. We make quantitative analyses for the neutrino properties at the electroweak energy scale, taking into account the energy evolution of the mass and mixing parameters for the quarks and leptons.

Experiments observe a solar neutrino deficit and an atmospheric neutrino anomaly, which could be understood as neutrino oscillations. For explaining the atmospheric neutrino problem, the mass-squared difference and the mixing angle between the \(\mu\)-neutrino and the other neutrino should be given by

\[
\Delta m^2_{\text{atm}} = (1 - 8) \times 10^{-3} \text{eV}^2, \\
\sin^2 2\theta_{\text{atm}} > 0.85. \tag{1}
\]

The solar neutrino problem is explained by several scenarios, among which most favored by experiments is a large mixing angle between the \(e\)-neutrino and the other neutrino under the Mikheyev-Smirnov-Wolfenstein (MSW) effect. Assuming this scenario, their mass-squared difference and mixing angle are observed as

\[
\Delta m^2_{\text{sol}} = (2 - 10) \times 10^{-5} \text{eV}^2, \\
\sin^2 2\theta_{\text{sol}} = 0.5 - 0.9. \tag{2}
\]

These two experimental results suggest that the leptons of three generations are fully mixed, contrary to the quark mixings. On the other hand, the reactor experiment by CHOOZ measures the mixing angle between the \(e\)-neutrino and the other neutrino. Non-observation of the mixing excludes the region

\[
\Delta m^2_{\text{chooz}} > 1 \times 10^{-3} \text{eV}^2, \\
\sin^2 2\theta_{\text{chooz}} > 0.2. \tag{3}
\]

The three experimental results provide a guide to constructing a model.

The minimal group for GUT which has a representation for the right-handed neutrinos is SO(10). The quark and lepton superfields of one generation, both left-handed and right-handed components, are all contained in one spinor \(16\) representation. The representations which can couple to \(16 \times 16\) are \(10, 126, \) and \(120\). The Higgs superfields for the masses of the quarks and leptons must be in these representations.

In the possible three representations for the Higgs superfields, the right-handed neutrinos can receive Majorana masses only from \(126\), provided that a non-vanishing vacuum expectation value (VEV) is generated for the scalar field of the SU(5) singlet component. If SO(10) is broken down to SU(5) by this VEV, its magnitude is as large as the GUT energy scale. Although
the representation also has SU(2) doublet components which could give Dirac masses to the quarks and leptons, the large mixing angles for the leptons are not yielded by themselves. Even if the 10 representation is introduced for the Higgs bosons, it is difficult to consistently accommodate the three experimental results. A scenario with a small mixing angle in the MSW effect for the solar neutrino oscillation could be provided by incorporating two superfields of 10 and the Majorana masses for the left-handed neutrinos from the VEV of the SU(2) triplet Higgs boson $\phi_{126}$. Large mixing angles for the leptons are not easily obtained by 10 and 126. This is because every SU(2) doublet Higgs boson in these representations gives the same contribution to the quark mixings and to the lepton mixings. On the other hand, the 120 representation has four SU(2) doublet components, in which one doublet gives Dirac masses only to the neutrinos and another only to the up-type quarks. These Higgs bosons could originate from the difference between the quarks and the leptons.

We introduce one superfield for each of 10, 120, and 126. All the possible Higgs couplings for the quark and lepton superfields at the GUT energy scale are written, in the framework of SU(3)×SU(2)×U(1), as

$$
\begin{align*}
&\eta^i \left[ \delta_\phi \left( Q^i D^{ij} + L^i E^{ij} \right) + \phi_{120} \left( Q^i U^{ij} + L^i N^{ij} \right) \right] \\
&\phi^{ij} \left[ \phi_{120} \left( Q^i D^{ij} + L^i E^{ij} \right) + \phi_{120} \left( Q^i U^{ij} + L^i N^{ij} \right) \\
&\phi_{126} \left( Q^i D^{ij} + L^i E^{ij} \right) + \phi_{126} \left( Q^i U^{ij} + L^i N^{ij} \right) \right].
\end{align*}
$$

(4)

Here, $\phi$'s stand for Higgs superfields with upper and lower indices showing transformation properties under SU(5) and SO(10), respectively. Superfields for the quarks and leptons are denoted in a self-explanatory notation by $Q^i$, $U^i$, $D^i$, $L^i$, $E^{ij}$, and $N^{ij}$, where the index $i$ represents the generation. The group indices are understood. The coupling constants $\eta^i$ and $\phi^{ij}$ are symmetric for the generation indices, while $\phi^{ij}$ are antisymmetric.

The scalar component of $\phi_{126}$ is assumed to have a VEV of order of a GUT energy scale. Large Majorana masses are then induced for the right-handed neutrinos through $\phi_{126} N^{ij} N^{ij}$. Exchanging the right-handed neutrinos, the couplings $\phi_{120} L^i N^{ij}$, $\phi_{120} L^i N^{ij}$, and $\phi_{126} L^i N^{ij}$ lead to dimension-five operators which are composed of SU(2) doublet fields for the left-handed leptons and Higgs bosons. At the electroweak energy scale where the SU(2) doublet Higgs bosons have non-vanishing VEVs, these dimension-five operators become tiny Majorana mass terms for the left-handed neutrinos. The left-handed neutrinos could also receive Majorana masses from $\phi_{126} L^i L^j$. However, the VEV of $\phi_{126}$ has to be as small as the observed neutrino masses, which necessitates an extreme fine-tuning of the Higgs potential. We therefore take $\phi_{126}$ for enough heavy not to develop a non-vanishing VEV.

The SU(2) doublet Higgs superfields for electroweak symmetry breaking are given by linear combinations of the superfields with the same quantum numbers in 10, 120, and 126, or other representations. The MSSM Higgs superfields $H_1$ and $H_2$ with hypercharges 1/2 and $-1/2$, respectively, are expressed by

$$
H_1 = C_1^1 \phi_{120} + C_2^1 \phi_{120} + C_3^1 \phi_{120} + C_4^1 \phi_{120} + \ldots
$$

(5)

$$
H_2 = C_1^2 \phi_{120} + C_2^2 \phi_{120} + C_3^2 \phi_{120} + C_4^2 \phi_{120} + \ldots
$$

(6)

where dots stand for possible components belonging to the representations different from 10, 120, and 126. For instance, one superfield of 126 is included to keep the VEV of the SO(10) D term small by cancellation between the VEVs for $\phi_{126}$ and $\phi_{126}$. This 126 representation contains SU(2) doublets which become components of $H_1$ or $H_2$. The gauge coupling unification of SU(3)×SU(2)×U(1) suggests that there should exist only one pair of light Higgs doublets. We assume that the other linear combinations of the SU(2) doublets have large masses and decouple from theory below the GUT energy scale.

The superpotential of our model relevant to the quark and lepton masses are given by

$$
W = \frac{\eta_{ij}}{2} H_1 Q^i D^{ij} + \frac{\eta_{ij}}{2} H_2 Q^i U^{ij} + \frac{\eta_{ij}}{2} H_1 L^i E^{ij} + \frac{\eta_{ij}}{2} H_2 L^i N^{ij} + \phi^{ij} \phi_{120} N^{ij} N^{ij}.
$$

(7)

Below the GUT energy scale, the right-handed neutrino superfields $N^{ij}$ decouple from theory, owing to their large masses. Instead of the last two terms in Eq. (6), the dimension-five operators are taken into consideration:

$$
\mathcal{L} = -\frac{1}{2} \kappa^{ij} \phi_{126} \psi_{126} \phi_{126} \psi_{126} + \text{h.c.,}
$$

(8)

where $\phi_{126}$ and $\psi_{126}$ represent the scalar component of $H_2$ and the fermion component of $L_i$, respectively. For definiteness, we define the Cabibbo-Kobayashi-Maskawa (CKM) matrix for the quarks and the Maki-Nakagawa-Sakata (MNS) matrix for the leptons as

$$
V_{CKM} = U_L U_L^d,
$$

(9)

$$
V_{MNS} = U_L U_L^e,
$$

(10)

where $\eta_{ij}^D$, $\eta_{ij}^D$, $\eta_{ij}^D$, and $\kappa^D$ denote diagonalized matrices, $U_L$'s and $U_R$'s being unitary matrices. Taking the VEVs of electroweak symmetry breaking for positive, the diagonal elements of $\eta_{ij}^D$, $\eta_{ij}^D$, and $\kappa^D$ should be positive, while those of $\eta_{ij}^D$ should be negative. The quarks and leptons then have positive masses.
A $3 \times 3$ unitary matrix has nine independent parameters. Although the numbers of physical parameters in the CKM matrix and the MNS matrix are respectively four and six for electroweak interactions, more parameters become physical for SO(10) interactions. For the expression of $V_{CKM}$ or $V_{MNS}$, we adopt the parametrization in which the energy evolution of the independent parameters can be traced explicitly:

$$V = P_A V_0 P_B$$

where

$$V_0 = \begin{pmatrix}
1 & \frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 1
\end{pmatrix}.$$  

The values of $\eta_u$, $\eta_d$, $\eta_e$, $\kappa_D$, $V_{CKM}$, and $V_{MNS}$ evolve depending on the energy scale. The renormalization group equations for these parameters and the gauge coupling constants of SU(3)$\times$SU(2)$\times$U(1) close on themselves at the one-loop level. Making use of the large mass differences among generations for the quarks and the charged leptons, the evolution equations for the independent parameters are obtained explicitly. Experimentally, the eigenvalues of $\eta_u$, $\eta_d$, and $\eta_e$ are known, if the ratio $\tan \beta$ of the vacuum expectation values for $H_1$ and $H_2$ is given. The CKM matrix elements have also been measured. Assuming $C_1^2 = C_2^2 = C_3^2 = 0$, therefore, unknown quantities at the GUT energy scale are $\nu_R/C_1^2$, $a$, $b$, $c_2\epsilon$, and the phase matrices $P_u$ and $P_d$ for $V_{CKM}$. If these quantities are given, the values of $\kappa_D$ and the MNS matrix are determined at the GUT energy scale and thus at the electroweak energy scale.

Our model is discussed quantitatively. We make an assumption that the mass differences among the neutrinos are very large, similarly to the quarks or charged leptons. Then, the measured quantities by the CHOOZ experiment are understood as

$$\sin^2 2\theta_{\text{chooz}} = 4|V_{31}|^2(1 - |V_{31}|^2),$$

$$\Delta m^2_{\text{chooz}} = m_{\nu_3}^2 - m_{\nu_1}^2,$$

where $m_{\nu_i}$ represents the mass eigenvalue for the neutrino of the $i$-th generation. For the atmospheric neutrino oscillation, the parameters are expressed by

$$\sin^2 2\theta_{\text{atm}} = 4|V_{32}|^2(1 - |V_{32}|^2),$$

$$\Delta m^2_{\text{atm}} = m_{\nu_3}^2 - m_{\nu_1}^2,$$

Combining Eqs. (3) and (4), the magnitude of $V_{31}$ should be small. This constraint make it possible to evaluate the parameters of the solar neutrino oscillation by

$$\sin^2 2\theta_{\text{sol}} = 4|V_{21}|^2(1 - |V_{21}|^2),$$

$$\Delta m^2_{\text{sol}} = m_{\nu_2}^2 - m_{\nu_1}^2.$$  

These evaluations would be sufficient for our present purpose to discuss plausibility of the model at the GUT energy scale.

### Table I: The masses of the quarks and leptons (in unit of GeV) and the CKM matrix parameters at the electroweak energy scale.

| $m_t$  | $m_e$  | $m_u$  |
|-------|-------|-------|
| 1.8$\times 10^2$ | 6.9$\times 10^{-1}$ | 1.8$\times 10^{-3}$ |
| $m_b$  | $m_s$  | $m_d$  |
| 2.9  | 8.5$\times 10^{-2}$ | 1.8$\times 10^{-3}$ |

| $\theta_1$ | $\theta_2$ | $\theta_3$ | $\delta$ |
|------------|------------|------------|---------|
| 3.2$\times 10^{-1}$ | 9.0$\times 10^{-2}$ | 4.0$\times 10^{-2}$ | 3.0$\times 10^{-2}$ |
This parameter set, together with appropriate values for the VEVs for the electroweak energy scale listed in Table I. The ratio of quark and charged lepton masses and the CKM matrix at $v_{\text{min}}$ determined by Eq. (12). Allowed regions for these parameters are very restricted. We take one example within the ranges of real values. The other parameters are put at $C_2^\ell_1\ell_2 = C_2^\ell_1\ell_3 = 0.02$. The value of $v_R/C_1^4$ determines the scale of $\kappa$ and does not affect the four quantities. The parameter ranges which give both $\sin^2 2\theta_{\text{atm}}$ and $\Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}}$ the values compatible with experiments are not wide. In Table I the resulting values are given for $C_2^\ell_2 = 0.27$. Putting at $v_R/C_1^4 = 5.0 \times 10^{15}$ GeV, the mass-squared difference for the atmospheric neutrino oscillation is given by $\Delta m^2_{\text{atm}} = 2.7 \times 10^{-3}$. Under the constraints from the CHOOZ experiment, the atmospheric and solar neutrino oscillations can be realized simultaneously for certain parameter values.

In conclusion, we have presented a model based on SO(10) GUT and supersymmetry, in which the quarks and leptons receive masses from the Higgs bosons in 10, 120, and 126. The antisymmetric 120 representation is the origin of the observed large generation mixings for the leptons. The small neutrino masses are traced back to large Majorana masses for the right-handed neutrinos generated by 126. Theoretical predictions are sensitive to the model parameters. All the experimental results can be described consistently in some regions of the parameter space.

This work is supported in part by the Grant-in-Aid for Scientific Research on Priority Areas (No. 14039204) from the Ministry of Education, Science and Culture, Japan.

The parameters $a$, $b$, $P_u$, and $P_d$ are constrained from Eq. (12). Allowed regions for these parameters are very restricted. We take one example within the ranges of real values, which is given by

$$a = -3.95, \quad b = -10.5, \quad P_u = \text{diag}(-1,-1,1), \quad P_d = \text{diag}(1,1,-1).$$

(18)

This parameter set, together with appropriate values for $\eta^D_d$, $\eta^D_u$, $V_{\text{CKM}}$ at the GUT energy scale, leads to the quark and charged lepton masses and the CKM matrix at the electroweak energy scale listed in Table I. The ratio of the VEVs for $H_1$ and $H_2$ is set for $\tan \beta = 30$. The obtained results are consistent with the values expected at the electroweak energy scale from experiments I. The $CP$-violating phase $\delta$ also lies in the range allowed by observed $CP$ violation in the $K^-\bar{K}^0$ and $B^0\bar{B}^0$ systems. For a smaller value of $\tan \beta$, the magnitudes of $a$ and $b$ become larger.

The coefficient matrix $\kappa$ in Eq. (13) is now determined by $v_R/C_1^4$ and $C_2^\ell \epsilon$. In Fig. I the mixing parameters $\sin^2 2\theta_{\text{atm}}$, $\sin^2 2\theta_{\text{sol}}$, $\sin^2 2\theta_{\text{chooz}}$, and the ratio of mass-squared differences $\Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}}$ at the electroweak energy scale are shown, corresponding respectively to curves (i), (ii), (iii), and (iv), as functions of $C_2^\ell 1_{23}(\equiv \kappa)$ within the range of real values. The other parameters are put at $C_2^\ell_1\ell_2 = C_2^\ell_1\ell_3 = 0.02$. The value of $v_R/C_2^4$ determines the scale of $\kappa$ and does not affect the four quantities. The parameter ranges which give both $\sin^2 2\theta_{\text{atm}}$ and $\Delta m^2_{\text{sol}}/\Delta m^2_{\text{atm}}$ the values compatible with experiments are not wide. In Table I the resulting values are given for $C_2^\ell_2 = 0.27$. Putting at $v_R/C_2^4 = 5.0 \times 10^{15}$ GeV, the mass-squared difference for the atmospheric neutrino oscillation is given by $\Delta m^2_{\text{atm}} = 2.7 \times 10^{-3}$. Under the constraints from the CHOOZ experiment, the atmospheric and solar neutrino oscillations can be realized simultaneously for certain parameter values.

In conclusion, we have presented a model based on SO(10) GUT and supersymmetry, in which the quarks and leptons receive masses from the Higgs bosons in 10, 120, and 126. The antisymmetric 120 representation is the origin of the observed large generation mixings for the leptons. The small neutrino masses are traced back to large Majorana masses for the right-handed neutrinos generated by 126. Theoretical predictions are sensitive to the model parameters. All the experimental results can be described consistently in some regions of the parameter space.

This work is supported in part by the Grant-in-Aid for Scientific Research on Priority Areas (No. 14039204) from the Ministry of Education, Science and Culture, Japan.

[1] S. Fukuda et al. (Super-Kamiokande Collaboration), Phys. Rev. Lett. 85, 3999 (2000).
[2] B.T. Cleveland et al., Astrophys. J. 496, 505 (1998);
W. Hampel et al. (GALLEX Collaboration), Phys. Lett. B447, 127 (1999);
J.N. Abdurashitov et al. (SAGE Collaboration), Phys. Rev. C60, 055801 (1999);
M. Altmann et al. (GNO Collaboration), Phys. Lett. B490, 16 (2000);
S. Fukuda et al. (Super-Kamiokande Collaboration), Phys. Rev. Lett. 86, 5656 (2001);
Q.R. Ahmad et al. (SNO Collaboration), Phys. Rev. Lett. 87, 071301 (2001).
[3] I. Dorsner and S.M. Barr, Nucl. Phys. B617, 493 (2001), and references therein.
[4] M. Apollonio et al., Phys. Lett. B466, 415 (1999).
[5] K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. 70, 2845 (1993);
T. Fukuyama and N. Okada, hep-ph/0205066 (2002).
[6] B. Brahmachari and R.N. Mohapatra, Phys. Rev. D58, 015001 (1998).
[7] S.G. Naculich, Phys. Rev. D48, 5293 (1993).
[8] N. Oshimo, in preparation.
[9] See e.g. H. Fusaoka and Y. Koide, Phys. Rev. D57, 3986 (1998).