Dispersion of Light in the 1D Photonic Crystal

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Abstract. We propose a novel, Kurosawa-like model to evaluate the 1D (Bragg stack-like) mesoporous aluminium oxide photonic crystal. To do this, we analyze the internal potential of the photonic crystal superlattice and get it describing the set of the medium’s polar oscillators. Unlike the atomic oscillators for a common crystal, these ones are the abstract ones. This way, the real photonic crystal can be dealt as an abstract oscillators’ ensemble. The result is fully agreed with the thermodynamics, and makes the theory very powerful. To obtain the oscillators parameters, we compare the theory with the secondary emission spectrum of the crystal, and get the natural frequency and the force for each oscillator. This phenomenological approach allow us to calculate photonic crystal’s optical characteristics, such as the dispersion law for the light in the nanostructure, the secondary emission spectrum of the composite, the speed of light in the crystal and the effective mass of the speed quanta. We establish the room-temperature Bose-Einstein condensation of polaritons in crystal at the photonic bandgap edge. The results are important to the solid-state detection of paraphotons.

1. Introduction
Nowadays, there is a worldwide interest to the photonic crystals and films, the nanocomposites with photonic bandgaps in the visible and near-IR spectral range [1]. In the paper, we construct the novel phenomenological model to analyze its optical characteristics.

2. Analysis
To derive the dispersion law for the light in the 1D photonic crystal, let’s expand the internal photonic crystal potential \( V(x) \) near the equilibrium point \( x = 0 \):

\[
V(x) = V(0) + V'(0)x + \frac{V''(0)}{2}x^2 + \cdots = \frac{V''(0)}{2}x^2 + o(x^2) \equiv \frac{m_0\omega_0^2}{2}x^2 + o(x^2). \tag{1}
\]

Because of this, we can substitute the real material medium by a set of abstract material oscillators. In the optical spectral range, these are the electronic oscillators [2] with the following dynamics:

\[
x'' + m_0\omega_0^2x = q_0E_0e^{i\omega t}. \tag{2}
\]

Here \( x \) is the electron coordinate, \( m_0 \) is the mass of the oscillator, \( q_0 \) is its charge, \( \omega_0 \) is its natural frequency and \( E_0e^{i\omega t} \) is the external field (in general, \( \omega \) can differ from \( \omega_0 \)). The solution for (2) is

\[
x(t) = \frac{q_0E_0e^{i\omega t}}{m_0(\omega_0^2 - \omega^2)}. \tag{3}
\]
To get the dispersion equation, let's use the macroscopic parameters. For the material medium with \( n_0 \) polar oscillators per unit volume, we have the medium polarization

\[
P = n_0 q_0 x \equiv (\varepsilon - 1)\varepsilon_0 E.
\]  

(4)

By substitute (4) to (3), we get

\[
\varepsilon(\omega) = 1 + \frac{n_0 q_0^2}{m_0 \varepsilon_0 (\omega_0^2 - \omega^2)} \equiv 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2} \equiv \frac{\omega_0^2 - \omega^2}{\omega_0^2 - \omega^2}.
\]

(5)

Here we use the new designations, \( \omega_p \) (the plasma frequency) and \( \omega_l \) (the longitudinal-mode one), are clear from the formula.

Next, because of each oscillator is contribute to the total polarization, in the linear case (when its internal interactions are neglected), we have

\[
P = \sum_j P_j
\]

(6)

and therefore

\[
\varepsilon(\omega) = \sum_j \varepsilon_j(\omega) = 1 + \sum_j \frac{\omega_p^2}{\omega_0^2 - \omega^2} = \prod_j \frac{\omega_0^2 - \omega^2}{\omega_0^2 - \omega^2}.
\]

(7)

At solid, we should also take into account the crystal lattice potential [3]:

\[
\varepsilon(\omega) = \varepsilon_\infty + \sum_j \frac{\omega_p^2}{\omega_0^2 - \omega^2} = \varepsilon_\infty \prod_j \frac{\omega_0^2 - \omega^2}{\omega_0^2 - \omega^2}
\]

(8)

with \( \varepsilon_\infty \) is the medium dielectric constant at the very high frequencies.

So, at the second order of smallness (see (1)), the dispersion law for the light in composite is

\[
k(\omega) = \frac{\omega}{c_n(\omega)} = \frac{\omega}{c \sqrt{\varepsilon(\omega)\mu}} = \frac{\omega}{c} \sqrt{\varepsilon_\infty \prod_j \frac{\omega_0^2 - \omega^2}{\omega_0^2 - \omega^2} \mu}.
\]

(9)

The unknown parameters of the model, \( \{\omega_{lj}, \omega_{0j}\} \), can be obtained experimentally, for example, by the secondary emission spectrum. This way, at the normal incidence, the sample’s reflection [4] is

\[
R(\omega) = \left| \frac{n(\omega) - 1}{n(\omega) + 1} \right|^2 = \left| \frac{k(\omega) - \omega / c}{k(\omega) + \omega / c} \right|^2.
\]

(10)

The analyses of (10) gives the reflection peaks (\( R = 1 \)) at \( \omega = \omega_{lj} \) and \( \omega = \omega_{0j} \). So, the model parameters, \( \{\omega_{lj}, \omega_{0j}\} \), are the phenomenological ones, and these are the edges of the reflection bands in the spectra.

Note, that at the luminescence spectrum, these frequencies are the luminescence peaks because of the zero group speed of light:

\[
v(\omega) = \frac{d\omega}{dk} = \left[ \frac{dk(\omega)}{d\omega} \right]^{-1}.
\]

(11)

Herewith, the effective mass of the light quanta is

\[
m(\omega) = \left[ \frac{\partial^2 W}{\partial p^2} \right]^{-1} = \hbar \left[ \frac{\partial^2 \omega}{\partial k^2} \right]^{-1} = \hbar [v(\omega) \frac{dv(\omega)}{d\omega}]^{-1}
\]

(12)

and is non-zero to be prospective for the polariton Bose-condensation [5].
3. Simulation

We simulate the 1D aluminum oxide photonic crystal film synthesized by the aluminum anodizing during the acid etching [6-12]. The film is about 20 µm and made of regular alternated Al$_2$O$_3$ layers. All the odd layers are 193 nm long and 40% porous, all the even ones are 193 nm long and 60% porous. The simulation results are presented at the fig. 1.

Figure 1. The simulated characteristics of the aluminum oxide photonic crystal: (a) the light dispersion law, (b) the crystal reflectance (dots are the spectral data), (c) the group speed of light in the crystal, and (d) the effective mass of the speed quanta. At fig. 1a, dotted line marks the dispersion law for the vacuum $\omega = c \times k$. Gray lines mark the bandgaps.
4. Discussion

The simulation (see fig. 1) shows, that at the bandgaps edge, there is the room-temperature Bose-Einstein condensation of polaritons in the photonic crystal. In fact, at the bandgap edges, there is the giant density of states (see fig. 1c)

$$f = \frac{dp}{dW} = \frac{dk}{d\omega} = \frac{1}{v} = (v \to 0) = \infty.$$  \hspace{1cm} (13)

At the same time, the critical temperature of condensation is

$$T_c = \frac{2\pi h^2}{mk_B} \left(\frac{n}{\zeta(3/2)}\right)^2/3,$$ \hspace{1cm} (14)

where $m$ is the polaritons mass, $n$ is its concentration and constants: reduced Planck constant $\hbar = 1.055 \times 10^{-34}$ J$s$, Boltzmann constant $k = 1.38 \times 10^{-23}$ JK$^{-1}$ and Riemann zeta $\zeta(2/3) = 2.612$, and for $n \sim 10^{25}$ m$^{-3}$ (Loschmidt constant) and $m \sim 10^{-35}$ kg (see fig. 1d), is about $T_c \sim 10^7$ K. So, at room temperatures, at the bandgap edges, there always is the Bose-Einstein condensation of polaritons.

Let’s focus at the effect. Due to the energy flux density

$$S = w v$$ \hspace{1cm} (15)

conservation (w is the energy density, and v is its velocity), in the material medium, $S$ multiplies by

$$\frac{c}{v} = \frac{c}{d\omega/dk}^{-1} = c \frac{dk}{d\omega}$$ \hspace{1cm} (16)

times (compared to the void). So the high-dispersive media acts act the electromagnetic field amplifiers. This way, the probability of the two-photons process increase resonantly to observe the birth and the decay of the paraphoton, “dark matter” axion-like particle [13].

![Figure 2. The two photons – to – a paraphoton process.](image)

In the quantum field theory [14] this describes by the equation

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = -g E^2,$$ \hspace{1cm} (17)

where $\phi$ is the scalar paraphotonic field, $g$ is the gauge constant. The solution for (17) is [15]

$$\phi(\vec{r}, t) = \frac{1}{4\pi} \int \frac{g E^2(\vec{r}, t)}{|r - \vec{r}'|} d^3 r' = \frac{g l}{2k}[E_0 e^{i(kx - \omega t + \pi/4)}]^2.$$ \hspace{1cm} (18)

Here, $E = E_0 \exp[i(kx - \omega t)]$ is the excite electromagnetic wave, $l >> k^{-1}$ is the size of the conversion region and the geometry is the one dimensional.

So, the probability of the paraphoton processes is proportional to the energy density ($E^2$) which is proportional to the density of states ($f$) and, at the bandgaps, tends to the infinity (see (13)). Therefore, the fig. 2 process becomes resonant and paraphotons generate in the crystal.
5. Conclusion
The abstract medium oscillators model can deal with not only the normal but with the photonic crystal. This way, we get the correct results (see fig. 1) and can easily modify the model to evaluate the thermo-, the magneto-, the acousto-, the quantum optic- and the other effects. The very intriguing of them is the two photon process to perform the solid-state detection of paraphotons at a laboratory.

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