Counterexamples to the B-spline conjecture for Gabor frames

Abstract

Frame set problems in Gabor analysis are classical problems that ask the question for which sampling and modulation rates the corresponding time-frequency shifts of a generating window allow for stable reproducing formulas of $L^2$-functions. In this talk we show that the frame set conjecture for B-splines of order two and greater is false. The arguments are based on properties of the Zak transform (also known as the Bloch-Floquet transform and Weil-Brezin transform). Our proof shows that, somewhat surprisingly, even nice Gabor windows in the Feichtinger algebra can have frame sets with a very complicated structure.