Contextual completeness and a classification scheme for theories

George Jaroszkiewicz
School of Mathematical Sciences, University of Nottingham, Nottingham, UK

January 14, 2015

Abstract
We discuss the role of propositions, truth, context and observers in scientific theories. We introduce the concept of generalized proposition and use it to define an algorithm for the classification of any scientific theory. The algorithm assigns a number 0, 1, 2 or 3 to a given theory, thereby classifying it as of metaphysical, mathematical, classical or quantum class. The objective is to provide an impartial method of assessing the scientific status of any theory.

1 Introduction
At a time when the empirical sciences are making ever greater advances in quantum physics, it is a paradox that there should remain uncertainty and fruitless debate regarding the interpretation of quantum mechanics (QM). This paradox has been intensified by the speed and ease of modern communication. Everyone thinks they are entitled to have an opinion that carries the same weight as any other: some say time is one of the dimensions of the block universe, others say it is a process; some say quantum wave functions collapse, others say they do not; some say there is but one reality, others say there is an infinity of parallel worlds; some say there are hidden variables, others say there are not; some say the universe started as a quantum fluctuation, others ask of what? and so on. The great scientific principle of the Royal Society of London, Nullius in verba (take no-one's word for it) seems to have been abandoned in many quarters in favour of unsupported opinion and pseudo-scientific assertion. Hard won principles of science are routinely flouted in the media by experts who should know the difference between real science and science fiction.

It seems timely to assert that science is not a democracy but a dictatorship: that of the laboratory. It is metaphysics and no business of theorists to propagate as science unsubstantiated beliefs in undetectable parallel worlds, or to advocate mathematical elegance as a scientific principle outweighing lack of empirical evidence, or to assert that the universe originated in a quantum fluctuation, or to claim that mathematical truth has the same status as physical observation [27]. These are all baseless ideologies serving to breed a complacent train of thought, no more useful in science than fairy stories.

2 Truth values
The problem as we see it lies in the meaning of truth. Whenever any statement is made in whatever discipline, be it scientific, philosophical or mathematical we should immediately ask several questions and insist on an answer:

1. Has this statement been validated, i.e., assigned a truth value?
2. Can it be validated? If so, by what actual means?
3. Who has or will validate that statement, i.e., for whom or what is this statement intended to be meaningful?
4. What would observers do with this statement if validated?

Outside of science, it has been fashionable for some time to abandon belief in absolute truth and to regard truth as relative. However, relative truth in science seems at face value to be inconsistent with empiricism: we want to believe that observations have a meaning beyond the narrow field of view of the observer. It is our intention in this article to emphasize that the principle of relativity of truth can indeed be placed on a proper scientific footing, fully in accordance with empiricism. But it requires us to abandon belief in absolute scientific truth.

A by-product of our analysis is an effective principle, a methodology, a weapon, that allows the expert and the non-expert to relatively easily identify and classify assertions as either metaphysical, mathematical, classical or quantum physical with a degree of impartiality and certainty. We will propose such a principle, which we call contextual completeness.

Contextual completeness allows us to examine any statement in a range of subjects and assign a classification integer $k$ to that statement. If $k = 0$, then we can say we are dealing with a metaphysical statement; if $k = 1$ we are dealing with a mathematical or logical statement; if $k = 2$ we are dealing with a classical mechanical or classical relativistic statement; and if $k = 3$ we are dealing with a quantum mechanical statement. For example, we find $k = 0$ for most statements in the Multiverse paradigm and the original version of decoherence theory. Consistent histories scores 2 at best. Interestingly, frequentist probability theory scores 1 whilst Bayesian probability scores 3.

The principle of contextual completeness requires some explanation, or context of its own. Therefore to understand our approach, we first clarify our terms of reference, bearing in mind that what we are doing should be thought of as metascience, or how to do science. It is not precise in the preliminary form given here but should serve as a useful tool in discussions about specific scientific theories. Hopefully it encourage a more careful approach to the writing of science. Of course, our approach will be the subject of criticism itself, but that is to be welcomed. Our test for contextual completeness can be applied to comments as readily as to theories.

3 Terms of reference

3.1 Systems under observation

It is a powerful principle in science that the universe can be divided into systems under observation (SUOs) and observers. There are two ways to think about SUOs. The traditional and intuitive view is that SUOs “exist”, have physical properties and experiments can determine these properties. The other view is the one expounded by Wheeler in his “participatory principle” [29]:

"Stronger than the anthropic principle is what I might call the participatory principle. According to it we could not even imagine a universe that did not somewhere and for some stretch of time contain observers because the very building materials of the universe are these acts of observer-participancy. You wouldn’t have the stuff out of which to build the universe otherwise. This participatory principle takes for its foundation the absolutely central point of the quantum: No elementary phenomenon is a phenomenon until it is an observed (or registered) phenomenon.”

J. A. Wheeler

From this perspective, SUOs cannot be discussed without the context of observation: truth values have no meaning without observers to register them. The Kochen-Specker theorem emphasises this point clearly [19].

3.2 Observers

A fundamental question in physics is: what is the essential difference between SUOs and observers? We shall refer to this as the difference question. The problem we have here is that according to the principle of the unity of physics, SUOs and observers are described by the same laws of physics. According to Feynman [12]:

2
“...we have an illusion that we can do any experiment that we want. We all, however, come from the same universe, have evolved with it, and don’t really have any real freedom. For we obey certain laws and have come from a certain past.”

R. P. Feynman

There is, according to this principle, no intrinsic difference between observers and SUOs. Whilst we entirely agree with this principle, it seems unsatisfactory because it does not take into account the fact that observers and SUOs play very different roles in physics. We may reconcile the difference question and the unity of physics principle if as before we revise our notion of truth and make it contextual. Given the difference question, we should ask the crucially related question, who or what is interested in the difference? To whom is the difference question addressed?

The only answer that makes sense to us is: the difference question is meaningful to the observer and to no one or no thing else. The difference between an observer and an SUO is contextual. An observer observes an SUO but that SUO does not observe the observer, relative to that observer. It is perfectly possible however to imagine that an SUO $S_1$ being observed by an observer $O_1$ could act as an observer $O_2$ of $O_1$, in which case $O_1$ now plays the role of an SUO $S_2$, but only relative to $O_2$.

When contextuality is respected, observers have characteristics that SUOs do not have. Observers have memories and an internal sense of time, they build apparatus that can prepare states of SUOs, extract and record information about those states, and eventually use it in some way that is meaningful relative to those observers for purposes defined by those observers. In context, SUOs do none of those things.

### 3.3 Complete and partial observation

Suppose two observers $O_1$ and $O_2$ observe an SUO $S$. There may well be aspects of $S$ that $O_1$ observes that $O_2$ does not, and vice-versa. In such a case each observer is observing $S$ partially. One of the differences between classical mechanics (CM) and quantum mechanics (QM) is that the principles of CM do not rule out complete observation, that is, the notion that an observer can have available to them all possible information about an SUO in principle. QM on the other hand rules out complete observation: there is no context in which an observer can observe both the position and the momentum of a particle in one outcome. The famous debate between Einstein, Podolsky and Rosen (EPR) on one side and Bohr on the other boils down eventually to this difference between QM and CM [10, 4]. Since QM does not allow complete, context-free observation, there is no requirement for physicists to believe that it means anything: we do not have to permit the EPR phrase “element of reality” into the debate as legitimate.

### 3.4 Primary observers

It is an empirical fact that observers do not have infinite lifetimes: they come and go. Suppose we attempted to discuss the origin of some physical observer $A$. This would be meaningful only from the perspective of some earlier physical observer $B$ that had observed the creation or formation of $A$. But this would immediately raise the same question as to the origin of $B$, requiring some yet earlier observer $C$, and so in. If we allowed this argument to continue indefinitely, we would be led to an infinite regress. Such things are generally regarded as unsatisfactory.

To avoid such an infinite regress, we should stop somewhere, placing a veto on questions about the origin of our chosen primary observer. We stop at some point convenient to us, accepting the existence of that observer for granted and defining all contexts relative to them only and no further in. This suggests that we may have to scale down our expectations of physics and accept the possibility that we may never have a complete TOE (theory of everything) or an understanding of the origin of the laws of quantum mechanics.

This infinite regress problem was understood in antiquity. To avoid it, Aristotle invoked the concept of an absolute primary observer known as a first mover [1]:

\[\text{1 The essential question as to who notices this is answered contextually: the observer themselves.}\]
“Since motion must be everlasting and must never fail, there must be some everlasting first mover, one or more than one.”

Aristotle

By definition, primary observers have a sense of time, memory and purpose, for without any of these attributes they could not be regarded as observers. At any moment of their time they hold data in their memories that they interpret in terms of a hypothesized past relative to that moment. With sufficient data, observers can even attempt to account for their own origins. But that past is a map of the past and should not be confused with it \(^{22}\). Different primary observers might construct different relative pasts and there is nothing in physics, apart from the need for consistency should they exchange information, to prevent that. This means that observer’s past may be as uncertain for them as their future is. It is metaphysics to assert that “the past” is unique and absolutely fixed.

It was the notable achievement of Hugh Everett III to enhance the discussion about observers in QM. Specifically, he discussed the possibility of one observer \(O_1\) observing other observer \(O_0\) who is performing observations on some SUO \(S_0\) \(^{11}\). All the rules of standard QM are assumed to apply to the observations as made by \(O_0\) on \(S_0\). By the principle of the unity of physics, the same rules should apply to the observations made by \(O_1\) on the combined SUO \(S_1\) consisting of \(S_0\) and \(O_0\).

How \(S_1\) is related to \(S_0\) and \(O_0\) is a deep and interesting question. Typically, \(O_0\) describes \(S_1\) in terms of a pure or mixed quantum state. Also typically, \(O_1\) describes \(S_1\) as a pure or mixed state involving tensor products of state vectors in a tensor product Hilbert space.

Everett asserted that \(O_1\) would/should regard \(O_0\) and \(S_0\) together as a single SUO, described by a wavefunction describing both as a single SUO. Then, to avoid an infinite regress, Everett postulated that there is a fundamental wavefunction, not just for our universe but for a plethora of all possible universes. In addition, he asserted that only his “Process 2” had intrinsic significance \(^{11}\):

“The continuous, deterministic change of state of an isolated system with time according to a wave equation \(\partial \psi / \partial t = A \psi\), where \(A\) is a linear operator.”

H. Everett III

Everett gave no context of validity for this assertion, which therefore represents a return to an absolutist perspective. This accounts for the fierceness of the debate between those who believe that the Many Worlds/Multiverse paradigm \(^{17}\) is physics and those who see it as metaphysics.

The concept of primary observer raises the question “what then is the objective of physics?” If we are not allowed to ask questions about a primary observer, surely that means that we have an incomplete understanding of the universe. That is of course true. But our response is: does it matter? Who said in the first place that quantum mechanics could explain everything? Quantum mechanics only ever was a theory of observation, not a description of SUOs. The history of quantum mechanics is littered with the debris of this debate. It was the realist Einstein who attributed Planck’s quanta to properties of the radiation field \(^{23}\), whereas Planck himself only ever discussed the responses of atomic oscillators in the walls of black body containers \(^{25}\). Although we think of Heisenberg’s matrix mechanics \(^{16}\) as formally equivalent to Schrödinger’s wave mechanics \(^{26}\), nothing could be further apart than their respective views about what they had discovered.

At this point we emphasise that a primary observer is a contextual construct: there is not a unique, ‘final’ primary observer in the sense of Aristotle above.

### 3.5 Propositions

Arguably, the proper business of physics is to establish the truth status of propositions or statements, denoted here generically by the symbol \(P\). Examples are \(P_1 \equiv ‘\text{energy is always conserved}'\) and \(P_2 \equiv ‘\text{the mass of the electron is } 9.10938291 \times 10^{31}\) kilograms’. Propositions in the sense of the word employed here can be elementary, in that they need not contain any conditionals such as ‘if’, as in the above examples \(P_1\) and \(P_2\). They can also be compound, such as \(P_3 \equiv ‘\text{If it is Tuesday and it is eleven o’clock then if we turn on this magnet, the electron beam will be deflected}'\).

### 3.6 The validation function \(\mathbb{V}\)

We introduce the validation function \(\mathbb{V}\), which maps all propositions into the discrete set \(\{0, 1\}\). If we know \(\mathbb{V}P = 1\) then we know that \(P\) is true, whereas if \(\mathbb{V}P = 0\) then \(P\) is false.
3.7 Context

The validation function $V$ introduced above raises a fundamental problem: we have used it without any information about the circumstances that gave a truth value 0 or 1. Also, there has been no reference to any observer for whom the truth value is meaningful. Addressing this issue is the subject matter of this article.

What is missing is context. In this article, a context is all the information that gives meaning to the validation process $V$. Context should include apparatus when discussing physical propositions such as $P_3 \equiv \text{“the momentum of this particle is } p\text{”}$.

Some reflection leads us to the conclusion that all propositions and laws of physics are contextual. For instance, the proposition ‘energy is conserved’ is not an absolute truth: it does not hold in those situations where energy can leak out of an otherwise closed system. It is metaphysics to assert that energy is conserved for open systems: how could we know? The truth status of the law of conservation of energy therefore is contextual.

Even the “constants of physics” are contextual. For instance, Planck’s constant lay undiscovered until the appropriate technology was available to detect it. Even now, many observations in science can be explained in terms of classical mechanics (CM), for which we set $h = 0$ for convenience. We do not, as a rule, use QM to discuss the orbit of the Moon. Surprisingly, even the proposition $P_2$ above is contextual: the effective mass of an electron in vacuo is measurably different to the effective mass of an electron in a crystal [14]. We point out that physicists such as Dirac have explored the notion that physical ‘constants’ depend on time. If this is indeed the case, then these constants are contextual in a deeper sense than simply contextual on our current technology and mathematical modelling.

This raises an interesting question: can we create “laws of physics” by devising contexts never hitherto seen in nature? For example, in Lagrangian mechanics, we usually choose the Lagrangian $L$ to reflect the conditions in the laboratory. For instance, if we have no external electromagnetic fields, we might not have any electromagnetic potentials in $L$. Similarly, when the search for the Higgs particle was under way, the presence of the Higgs field in the Lagrange density was validated by the construction of very special apparatus, which contributes to context. Another example is early universe cosmology, where the effective ‘fundamental’ Lagrangian changes radically as the universe evolves from the Big Bang to the current epoch.

This line of argumentation does not lead to the conclusion that we can create any law of physics that we like. Contrary to the Multiverse conjecture that all things are possible in some universe, the universe we live in does appear to have some rules. Empirical physics is the search for those rules, for contexts that will validate certain physical propositions and invalidate other. Given an arbitrary physical proposition, we cannot in general always create contexts that will validate that proposition: currently we cannot travel in time or exceed the speed of light.

3.8 Varieties of proposition

Further consideration lead us to distinguish three classes of proposition: absolute, contextual and empirical.

3.8.1 Absolute propositions

An absolute proposition $P$ is one given without any context. Such propositions occur throughout metaphysics and philosophy and, unfortunately, can be found in what is asserted to be science. The problem here is that, for an absolute proposition to have a truth value, it has to have that truth value relative to all conceivable contexts. But physicists cannot validate any proposition relative to more than a finite number of contexts, simply because every validation takes a finite time: the search for the Higgs boson took many years. Therefore absolute propositions are meaningless in physics because their truth status cannot be established.

---

$^2$The question for whom is this meaningful? will to be addressed presently.
3.8.2 Contextual propositions

A contextual proposition consists of a proposition \( P \) that is asserted relative to a specific context \( C \). We denote such a context \( (P|C) \). A contextual proposition can be true or false. If true we write \( V(P,C) = 1 \) whilst if false we write \( V(P,C) = 0 \).

In physics, a given proposition is generally true relative to more than a single context. For example, let \( P \) be the proposition \( P \equiv 'a \text{ free particle moves uniformly along a straight line}' \) (Galileo’s law, Newton’s first law of motion), relative to the contexts \( C_1 \equiv '\text{As observed in inertial frame } F_1' \) and \( C_2 \equiv '\text{As observed in inertial frame } F_2' \). Then we have

\[
V(P,C_1) = V(P,C_2) = 1.
\]

(1)

Given a proposition \( P \), we define its domain of validity \( C_P \) to be the set of all contexts such that

\[
C \in C_P \Rightarrow V(P,C) = 1.
\]

(2)

We define \( C^* \) to be the contextual universe, the class of all possible contexts\(^3\). Then given \( C_p \), we define \( \overline{C_p} \) to be the complement of \( C_p \) relative to \( C^* \), i.e.

\[
C \in \overline{C_p} \Rightarrow V(P,C) = 0.
\]

(3)

By definition, we have \( C^* = C_P \cup \overline{C_p} \) and \( C_P \cap \overline{C_p} = \phi \).

Suppose \( C_1, C_2, \ldots, C_n \) are elements of the contextual universe \( C^* \). Then we denote the set of simultaneous contextual propositions

\[
(P|C_1, C_2, \ldots, C_n) \equiv (P|C_1) \land (P|C_2) \land \ldots \land (P|C_n),
\]

(4)

where each of the terms \( (P|C_i), i = 1, 2, \ldots, n \), is a contextual proposition and the symbol \( \land \) is the logical ‘and’. Then we have the rule

\[
V(P,C_1, C_2, \ldots, C_n) = \prod_{i=1}^{n} V(P,C_i)
\]

(5)

The notation in (4) does not mean that an experiment can be done with two or more contexts simultaneously. Each factor on the right hand side of (4) refers to a separate contextual proposition. Contexts are mutually exclusive: the factors that make up one context cannot be added to a second context without creating a new context. This is one of the fundamental rules of quantum mechanics that does not apply in classical mechanics. Therefore, care has to be taken when contexts depend on continuous parameters, for if a contextual parameter\(^4\) is changed by what seems to be an infinitesimal amount in some apparatus, the context and therefore the experiment changes.

We defined an absolute proposition above as one that is independent of context. Equivalently, its truth status is unaffected by choice of context, so its domain of validity is the contextual universe. Therefore, if \( P \) is an absolute proposition, we may write

\[
V(P,C) = 1 \quad \forall C \in C^*,
\]

(6)

or equivalently,

\[
P \equiv (P,C^*).
\]

(7)

3.8.3 Empirical propositions

An empirical proposition is a contextual proposition that has been validated under a sufficiently large number of contexts that it is regarded as a law of physics, for all practical purposes (FPP, [23]).

---

\(^3\)We are not concerned here whether \( C^* \) is a set or a class.

\(^4\)A contextual parameter is a parameter involved in the description of a context, such as the vertical angle of orientation of the main magnetic field in a Stern-Gerlach experiment.
Any law of physics $L$ is necessarily an empirical proposition, because its context set $C_L \equiv \{C_1, C_2, \ldots, C_n\}$ is always necessarily a finite discrete set in physics, simply because every validation experiment takes non-zero time. Assertions that involve continuous parameters, such as ‘In each run of a Stern-Gerlach experiment, the electron spin value in any arbitrary orientation of the main magnet will always be plus or minus one half’, are theoretical abstractions from a finite number of validated empirical propositions.

4 Generalized propositions

We assert that absolute propositions are meaningless *per se* in physics: we should always supply a context for any physical proposition. However, that is still not enough. For the truth value of a proposition $P$ to have a physical meaning, two conditions have to be satisfied, not one. These are i) as stated above, the proposition must have a context $C_P$ relative to which that proposition might or might not be true, and ii) the contextual proposition $(P, C_P)$ must be *contextually complete*. By this we mean that there should be a statement identifying the primary observer for whom that contextual proposition’s truth value is meaningful.

Our approach to classification of theories is to write every proposition in a given theory as a generalized proposition. If $P$ is a proposition, $C_I$ its context, and $O$ an observer, then its associated generalized proposition is of the form $(P, C_I|O, C_E)$. Here $C_E$ is the relative external context that defines the observer $O$, relative to whom the validity status of the proposition may be established and $C_I$ is the relative internal context that includes details of the apparatus used to validate the proposition [18]. If a generalized proposition is true we write $V(P, C_I|O, C_E) = 1$; if it is false we write $V(P, C_I|O, C_E) = 0$. Truth and falsity is defined contextually by the observer and have no meaning outside of that context.

4.1 Contextual completeness

The concept of context is a deep one and we have to deal with it on an intuitive, heuristic basis. Relative internal context is generally easy to deal with: if we were observing electron spin in a Stern-Gerlach experiment we would need reasonably well-specified apparatus for instance. Relative internal context may well involve abstract concepts such as mathematical axioms, mathematical modelling of an SUO, and principles such as wave-function superposition. Relative external context is generally more familiar, as it will contain classical information that the observer has about themselves, such as what sort of spacetime they are sitting in.

4.2 Heisenberg cuts

A *Heisenberg cut* is a hypothetical line dividing the worlds of quantum mechanics and classical mechanics. Heisenberg wrote [17]:

"The dividing line between the system to be observed and the measuring apparatus is immediately defined by the nature of the problem but it obviously signifies no discontinuity of the physical process. For this reason there must, within certain limits, exist complete freedom in choosing the position of the dividing line.”

W. Heisenberg

Although frequently discussed as if there an absolute divide between classical and quantum worlds, Heisenberg cuts are contextual, separating relative internal context and relative external context in any given quantum mechanical generalized proposition. Given a contextually complete quantum proposition $(P, C_I|O, C_E)$, the corresponding Heisenberg cut is represented by the vertical bar in our notation. Heisenberg cuts are heuristic and impossible to define precisely, so occasionally there may be some uncertainty in or debate about our classification of a given theory.
4.3 Exophysics versus endophysics

An exophysical observer stands outside the SUO they are observing whilst an endophysical observer is within a greater system. Some theories such as Newtonian mechanics posit an absolute observer standing outside of space and time, and is therefore an example of an exophysical observer. Exophysical observers may be absolute, in that there is no relative external context specified for them, or there may be such a context. For example, in classical mechanics, we may associate a given inertial frame with an exophysical observer at rest in that frame. Exophysical observers will be denoted by the generic symbol Ω whilst endophysical observers are denoted by ω. Endophysical observers invariably have an associated relative external context: otherwise, we would not know that they were endophysical.

In a given generalized proposition an absence of any context or reference to an observer will be denoted by the empty set symbol ∅. For example, an absolute proposition is denoted by (P, ∅|∅, ∅).

5 The classification of theories

The various branches of knowledge such as metaphysics and philosophy, mathematics and science can be classified in terms of degree of contextual completeness. We can go so far as to assign a numerical value to this classification as follows.

5.1 The classification function

We define the classification function $K$ that acts on generalized propositions according to the rule

$$K(P, C_I|O, C_E) = \alpha + 2\beta,$$

where

$$\alpha = \begin{cases} 0 & \text{if } C_I = \emptyset, \\ 1 & \text{if } C_I \neq \emptyset, \end{cases} \quad \beta = \begin{cases} 0 & \text{if } C_E = \emptyset, \\ 1 & \text{if } C_I \neq \emptyset. \end{cases}$$

The value $k \equiv \alpha + 2\beta$ so obtained will be called the classification of the generalized proposition. We shall call generalized propositions with classification 0, 1, 2 or 3 metaphysical or philosophical, mathematical or logical, classical or standard, and quantum or complete respectively.

A given theory may have generalized propositions of various classifications. In such a case, the theory should be classified according to the classification generally taken to characterize that theory. For example, Hugh Everett’s relative state theory has to be given a classification of zero because Everett’s core axiom “Proposition 2” above makes no reference to any observer: the ‘Many Worlds’ wavefunction is asserted to exist in an absolute sense. Indeed, B. de Witt, a leading advocate of Everett’s work, explicitly referred to the paradigm as metaphysical [5].

6 Classifications

In this section we apply our classification scheme to a number of disciplines and theories.

6.1 Metaphysics

In metaphysics, typical generalized propositions are unverifiable assertions of the form (P, ∅|∅, ∅). Typical metaphysical discussions involve the words existence, universals, and so on without giving any details as to how to validate such terms. Hence typically $\alpha = \beta = 0$, giving a classification $k = 0$. Some branches of metaphysics such as idealism do attempt to relate to observers, but not in any way that can be validated according to scientific principles. Indeed, the classification of something as metaphysical is generally regarded as synonymous with the impossibility of validation, and therefore, unscientific. This is an important criticism of the Many worlds/Multiverse paradigm discussed below.
We note that a generalized proposition that has a truth value of zero is not a metaphysical one but a false mathematical, classical or contextually complete proposition. Metaphysical generalized propositions have no truth values.

6.2 Philosophy

In a Philosophy not based on a specific moral code, typical generalized propositions are contextually unsupported assertions relating to some unspecified exophysical primary observer, which we write in the form \((P, \emptyset|\Omega, \emptyset)\). Hence \(\alpha = \beta = 0\), giving classification \(k = 0\).

In a Moral Philosophy based on a moral code \(\text{Code}\), typical generalized propositions are contextually supported assertions relating to some unspecified primary observer, which we write in the form \((P, \text{Code}|\Omega, \emptyset)\).

Such propositions can have contextual truth values: the function of \(\text{Code}\) in this case is to provide a validation mechanism for a truth value of the proposition \(P\), “truth” here being equated to being in accordance with the dictats of the code. Hence \(\alpha = 1, \beta = 0\), giving classification \(k = 1\).

6.3 Logical Philosophy

In this discipline, a proposition \(P\) is often a consequent of an antecedent \(A\), i.e., we encounter statements of the form \(A \Rightarrow P\). If we interpret the antecedent \(A\) as a sufficient condition for the truth of \(P\) then \(A\) is part of the internal context. No statement is made concerning any observer however, so generalized propositions in this subject are usually of the form \((P, A|\Omega, \emptyset)\). Hence \(\alpha = 1, \beta = 0\), giving \(k = 1\).

6.4 Mathematics and Formal Logic

In the traditional approach to mathematics, the truth value of a proposition \(P\) is determined by the axioms \(\text{Axioms}\), i.e. we have

\[
\forall (\text{Axioms} \Rightarrow P) = 1.
\] (10)

The axioms are a necessary part of the relative internal context. Standard mathematics usually makes no explicit reference to any observer, although of course mathematicians are aware that they are doing mathematics. At best we have generalized propositions of the form \((P, \text{Axioms}|\Omega, \emptyset)\), so for standard mathematics, we find \(k = 1\). The same analysis applies to most formal logic systems. Some branches of logic take some care in the formal rules of the axioms, such as modal logic, but completely ignore the question of who or what can take meaning from the results.

Plato's theory of forms explicitly demotes the role of any observer, be it exophysical or endophysical, to an inessential role. Moreover, axioms are not regarded as subject to any choice: they are just assumed to ‘be there’ as part of some higher universe of truth, whatever that means. This automatically gives this interpretation of mathematics the metaphysical classification of \(k = 0\). Validation when viewed as a decision process cannot be applied to a proposition that is postulated to be true in the first instance. This may explain why many working mathematicians are reluctant to say whether they are Platonists or not: the mathematician who is seen to merely stumble upon something already there there seems not so creative as the mathematician who invents a branch of mathematics.

There are several interesting exceptions to the classification \(k = 1\) for mathematics:

1. Set theory: Generally acknowledged as the Father of set theory, Cantor gave the following definition of a set \([5]\):

'A set is a gathering together into a whole of definite, distinct objects of our perception or of our thought—which are called elements of the set.'

Clearly, reference to perception and our thought relate to the mathematician describing the set, not to the actual set itself. The mathematician is at this point playing the role of an endophysical observer with a human mind, so we give Cantor’s proposition a classification \(k = 2\).
Clearly, if we rule out any reference to any thought processes, then Cantor’s definition becomes contextually incomplete. Nowadays, mathematicians have generally discarded Cantor’s approach and accepted that sets can be approached from the $k = 0$ level, without any need for an observer. Apart from his definition of a set, Cantor’s work on cardinality (the size of sets) rapidly reverted to $k = 1$ mathematics, dealing with concepts of infinity that transcend any possible real observer to deal with or comprehend on an intuitive level. Semi-constructivists such as Kronecker found this profoundly objectionable.

2. **Constructivist/intuitionist mathematics**

The mathematicians who created both of these approaches to mathematics came to the conclusion that class $k = 1$ mathematics begged certain questions. In the case of the constructivists, they believed that abstraction without the possibility of implementation is inadequate: they insisted that an observer has to have a model that implements the axioms of the mathematical system being studied.

The intuitionists believed that the truth of a mathematical statement is a subjective claim, corresponding to a mental construction of the mathematician. We identify such a mathematician as an observer.

Since both of these related approaches require some external context, such as a model or a sense of intuition, we classify them as class 3 approaches to mathematics.

3. **The learning of mathematics:** our classification scheme plays a significant explicitly unstated role in the learning of mathematics, a role that is nevertheless well understood intuitively by teachers of the subject. Generally, students learning a new branch of mathematics find it hard to deal with a presentation at the $k = 1$ level, as that is at a level of abstraction that the untrained human mind is not used to. The typical human tends to operate mentally at the solipsist level of thinking, for which propositions are dealt with at the $k = 0$ level and the meaning of truth is debatable. Mathematics is more effectively taught by examples, which are usually presented as a chain of steps taken by an observer. Students learn a new mathematical subject by copying steps from given examples, with the teacher assuming the role of an exophysical observer with experience. With sufficient practice, students then make a transition from a $k = 2$ approach to mathematics to the more abstract $k = 1$ approach, i.e., a transition of the form

$$(P_1 \Rightarrow P_2, \emptyset|\text{teacher, worked examples}) \rightarrow (P_2, \text{axioms}|\text{student, }\emptyset). \quad (11)$$

4. **Non-euclidean geometry:** for two thousand years, mathematicians believed that propositions in geometry were of the form $(P, E|\Omega, \emptyset)$, where $E$ contains the context set of Euclid’s five axioms and five postulates. Less than two hundred years ago, Gauss, Bolyai and Lobachewsky independently showed that generalized propositions in geometry could take the form $(P, E_i|\Omega, C_i)$, where $i = 0$ corresponds to the choice of Euclid’s axioms and postulates, describing flat space, $i = 1$ corresponds to the observer’s choice of spherical geometry and $i = 3$ corresponds to hyperbolic geometry. The freedom to choose different geometries gives $k = 3$. Once the choice is made, we revert to $k = 1$.

5. **Gödel’s incompleteness theorems:** Gödel showed that for some axiomatic mathematical systems, an endophysical observer could add propositions that were apparently true but could not be validated from the axioms alone. There would be no way to disprove such propositions by finding counter examples, for instance. In our terms, we write

$$\left\{ \forall(P, \text{axioms}|\Omega, \text{choice of } P) \Rightarrow 1 \right\} \not\Rightarrow \left\{ \forall(P, \text{axioms}|\Omega, \emptyset) = 1 \right\} \quad (12)$$

Gödel’s theorems implicitly involve an exophysical observer making choices, which is an attribute of a primary observer.
6.5 Computers, Church’s $\lambda$ calculus and Turing machines

The advent of real computers illustrates the significance of our classification system. An early approach to computation (in a formal sense of the word) was that of the mathematician Alonzo Church, who developed an approach to the study of computable functions involving a notation developed by Church known as the $\lambda$-calculus [24]. Since Church’s approach makes no reference to any observer, the $\lambda$-calculus has to be given the classification $k = 1$, i.e., a purely formal, mathematical one.

At about the same time, the theorist Alan Turing was developing what turned out to be the same ideas, but he expressed them differently. Turing’s approach was to imagine an exophysical observer with a hypothetical machine, now called a Turing machine. Turing imagined that the observer processed the same propositions that Church had considered, but via such a machine. This elevates Turing’s approach to a $k = 3$ level. Moreover, the Second World War stimulated the development of physical implementations of Turin’s concept, leading to the recent development of computers as we know them now. If the Turing machine concept had not been developed, it is likely that the world would have followed a radically different temporal development in the previous century.

6.6 Probability

Historically, there have been two conflicting approaches to probability: the Frequentist approach and the Bayesian approach. Our classification gives a different $k$ value to each.

The Frequentist approach takes the view that probability is an objective attribute of an SUO and the measurement protocol. The Frequentist approach does not deny the role of the latter: a fair dice that is dropped carefully onto a table without any initial angular momentum and with its heads side initially up should eventually come to rest with its heads side always up. There is a role for the observer, who is assumed exophysical, but beyond that, the observer is regarded as inessential. As far as probability is concerned, the observer’s role is to perform a number of runs or trials of a basic procedure and count outcome values. In particular, the number of trials or counts of frequency performed by the observer is assumed to be sufficiently large so as to home in, in some way, on the perceived ‘absolute probability’ associated with the SUO and the sampling protocol. At that point the observer becomes irrelevant. Hence with Frequentism we are dealing with generalized propositions of the form ($P$, sampling protocol | $\Omega$, $\emptyset$), giving a classification of $k = 1$. This approach to probability became more popular with the advent of Kolmogoroff’s axioms of probability, as these reinforce the mathematical side of probability.

On the other hand, the Bayesian approach to probability gives the observer and their prior information a central role in the calculation of probabilities. This does not rule out an intrinsic (aleatoric) component to a probability calculation. A typical generalized proposition in this approach would take the form ($P$, aleatoric context | $\Omega$, observer’s state of knowledge), giving a classification $k = 3$.

The Bayesian approach to probability has on occasion been compared to the way QM generates conditional probabilities. Attempts to re-interpret QM in terms of a Bayesian approach to CM have not been successful because, whilst the observer’s ignorance is classical in both disciplines, relative internal context in QM involves quantum mechanical rules. These cannot in general be duplicated by CM, as Bell inequalities have proven empirically [2].

We note that contextual hidden variables theory attempts to link relative external context with relative internal context, but this has not led to any clear verdict apart from seeming to be a tortuous way to avoid QM.

6.7 Classical Mechanics

CM emerged from the mists of antiquity and persisted into Renaissance times in the form of Aristotelian mechanics. Generalized propositions in that approach to motion would have been of the form ($P$, $\emptyset$ | $\Omega$, $\emptyset$) relative to some unidentified absolute observer $\Omega$, giving a classification of $k = 0$, i.e., metaphysics. Aristotelian propositions about motion are generally contextually incomplete and
conceptually flawed. Where they can be tested, such as the flight of projectiles, they are empirically incorrect.

Aristotelians appear to have been reluctant to contemplate any empirical validation of their propositions on the grounds that their objective was to understand the universe as it is. According to that philosophy, experiments are intrinsically unnatural and any conclusions drawn from them would not reflect the true universe.

Galileo is often regarded as the father of science on account of his attitude towards empirical validation. He identified the significance of the observer in the description of mechanics, and his ideas were encoded into the first of Newton’s laws of motion. Newton made considered statements about absolute space and time and relative space and time in the Principia, so generalized propositions in Galilean-Newtonian mechanics are of the form \((P, \emptyset | \Omega, \text{Absolute frame context})\). This gives a classification \(k = 2\).

A further refinement was the realization that the absolute frame could not be identified, so CM then became described in terms of exophysical observers each at rest in their individual inertial frames. This leads to generalized propositions of the form \((P, \emptyset | \Omega, \text{inertial frame context})\), also giving \(k = 2\).

In general, varieties of CM have this classification, there being no real discussion of the apparatus used to validate propositions. Indeed, the core principles of CM are those of realism, a belief in the objective existence and reality of SUOs and their physical properties, all of which are asserted to exist independently of any observers or processes of observation.

The same realist agenda runs throughout Special Relativity and General Relativity, so we give them the same CM classification \(k = 2\) when they are used to discuss physics. In case there is any doubt about this, we need only look at spacetime diagrams of black holes: these usually depict regions inside and outside the Schwarzschild radius using the same coordinate patch, as if physics could be discussed from the perspective of a superobserver standing outside spacetime. That is certainly a metaphysical perspective. When they are described as branches of mathematics, SR and GR have the classification \(k = 1\).

A proper quantum theory of gravity will only emerge when the role of observers and their apparatus is fully incorporated, and then we would expect a classification \(k = 3\), but the prospects at this time seem remote.

6.8 Old Quantum Mechanics

We stated above that when in 1900 Planck postulated the quantization of energy, he was referring to the oscillators in the walls of the black-body containers. Therefore, he was discussing a crude model of apparatus, so generalized propositions are of the form \((P, \text{detectors} | \Omega, \text{inertial frame})\), giving a classification \(k = 3\). The same analysis can be made of the Bohr-Sommerfeld atom.

Despite its ultimate failure to explain relative internal context, i.e., the relationship between SUOs and apparatus, Old Quantum Mechanics (OQM) played a pivotal role in the ultimate emergence of QM, as the older theory made a break with the realistic classical principles that led to \(k = 2\) for CM. When Born proposed the probabilistic interpretation of Schrödinger’s wave function, he was already familiar with the application of probabilities to OQM.

6.9 Einstein’s photon theory

We also commented above that Einstein’s association of quanta with the electromagnetic field was a realist one, with no dependence on detectors. Therefore, we classify his theory as \(k = 2\), i.e., classical. Einstein was inherently a realist at heart despite his enormous contributions to the development of quantum mechanics.

6.10 Quantum mechanics

Bohr, Born, Heisenberg and Jordan cut their teeth on Old Quantum Mechanics and this left its mark on their attitudes towards QM when it arrived in 1925. Bohr went on to develop the Cor-
correspondence Principle, which links the quantum numbers associated with relative internal context and the classical parameters of relative external context. Therefore, we give the Correspondence Principle a classification $k = 3$.

Heisenberg developed Matrix mechanics on the basis that only what could be observed was physically meaningful. Accordingly, we give a classification of $k = 3$ to Matrix mechanics, Heisenberg’s version of QM. On the other hand, Schrödinger’s initial interpretation of his wave mechanics has to be given a classification $k = 2$ on account of his realist interpretation of the wave function. The same classification gets assigned to hidden variables theories such as Bohmian mechanics [3].

Schrödinger’s formalism was quickly interpreted in probabilistic terms by Born, at which point it merits a classification of $k = 3$. The non-classical aspect of QM known as state reduction (wave function collapse) involves changes in both relative internal and relative external context, and not just the former and so the projection postulates associated with von Neumann and Lüders get classified as $k = 3$.

At this point our classification scheme may be inadequate to do full justice to certain quantum theories. We refer to those applications of quantum theory that attempt to model both SUOs and apparatus specifically. We have in mind particularly the notable paper by Mott on the detection of alpha particles in a cloud chamber [21] and Unruh’s paper on accelerating detectors in Minkowski spacetime [28], as well as the work of Schwinger, Glauber and others on localized detection models. We would assign a classification $K = 3^*$ to those theories.

### 6.11 Thermodynamics and statistical mechanics

Classical theoretical thermodynamics views entropy as a property of an SUO, which would on that basis give the subject a classification $k = 0$. However, the subject also discusses irreversible changes in entropy, which involves the physical time associated with the external observer. Therefore, we give the subject a classification $k = 2$.

Classical statistical mechanics introduces probability into the observer’s relative external context, but does not require a discussion of apparatus in the truth values of its generalized propositions. Therefore we assign it a classification of $K = 2$ also.

Quantum statistical mechanics involves both internal relative context and relative external context for its truth values, giving a classification $k = 3$.

### 6.12 Quantum field theory

As an application of quantum principles to SUOs with continuously many degrees of freedom, relativistic quantum field theories based on Minkowski spacetime have a classification of $k = 3$. Several perspectives support this conclusion, although apparatus is rarely explicitly modelled. One important feature that helped establish the empirical validity of quantum electrodynamics was the success of the renormalization programme. This depends on a heuristic bridge between the divergences associated with the relative internal context (involving metaphysical concepts such as bare mass and bare fields) and the relative external context associated with physical values of particle mass and other observables. Another feature is the explicit reference to the in and out states of the LSZ scattering formalism [15], interpreted here as part of the external relative context. Yet another feature supporting the classification $k = 3$ is the embedding of Lorentz and other symmetries into the unitary evolution formalism associated with the relative internal context and the frame transformation properties associated with the relative external context.

Attempts to extend the formalism to more general background spacetimes runs into technical problems on all fronts: the renormalization program is poorly defined and there is no natural concept of in and out states. Even the quantized particle concept has issues in curved spacetimes [6].

### 6.13 Quantum gravity and quantum cosmology

The ease and frequency with which quantum gravity and quantum cosmology are discussed in the imaginary time formulation suggests that these topics are not manifestly based on empiricism.
Indeed, quantum gravity has had a long history during which the identification of the correct observables to quantize had no consensus and there is currently no way that any of its concepts can be validated empirically. Quantum cosmology has gone even further into the realms of metaphysics: the Wheeler-de Witt equation suggests that the quantum state of the universe does not evolve in time (assuming such a concept has any more than a metaphysical significance).

We assign a classification $k = 0$ to both of these subjects because whilst they are mathematical in general form, they purport to be theories of physics.

### 6.14 M-theory

This is manifestly metaphysics, with barely comprehensible propositions asserted with no reference to any relative internal or external context or observer whatsoever. Even the origin of the latter “M” is uncertain, so we would propose it to be pinned down as *Metaphysics*, in line with its classification.

### 6.15 Experimental reports

Experiments of all types represent the ultimate in proper scientific activity. A complete report of an experiment will be contextually complete, documenting all relevant validation contexts, such as a statement of the problem, a description of the apparatus and the experimental protocol, an identification of the individuals carrying out the experiment and tables of all results. Important additional information that should always be given is the time and place at which the experiment was done, because this can have a significant effect on outcome, as the following examples illustrate:

1. In a rotating frame of reference such as the earth, any application of Newtonian mechanics should take into account frame-induced forces such as ‘centrifugal’ force;
2. In the Large Hadron Collider, the position of the moon affects the calibration of the detectors and this has to be adjusted for [13];
3. The universe is expanding, according to consensus. Therefore, there would have been an epoch in the early universe when atoms did not exist. Discussions of early universe big bang temperatures, pressures and particle densities are therefore necessarily contextually incomplete on that account. Therefore, early universe cosmology should be carefully discussed so as to emphasize the fact that all its propositions are contextual on current observations and theoretical retrodiction based on them. This is reasonable as far as classical mechanics is concerned but in the case of quantum processes goes against the Wheeler’s dictum, the last sentence in the quote from Wheeler given above. This warns us not to use counterfactuality when discussing quantum mechanics. This will be particularly relevant when discussing hypothetical objects such as “quantum black holes” in the early universe.

### 7 Concluding remarks

We emphasize that relative internal context does not refer to a description of a proposition, but to the mechanism, apparatus, or set of axioms relative to which that proposition can be validated. For example, if we assert that “the pressure in this box is one atmosphere”, the box would not be listed as part of the relative internal context. To give that context we would have to explain how we could establish the truth of that proposition, i.e., we would have to define the pressure meter involved and details of where it was placed in the box, and so on.

Our classification of theories is at best a guide to the significance of various theories. Undoubtedly it could be improved on. However, it may have a value in identifying research initiatives that may be “not even wrong”. For example, GR is a perfectly respectable and reputable classical theory with $k = 2$. Generalized propositions in GR are of the form $(P, \emptyset | \Omega, \text{metric})$. Attempts to quantize GR would require generalized propositions of the form $(P, \text{observables} | \omega, \text{localized laboratory context})$ with $k = 3$. It has proven difficult to see how to make the transition from classical to quantum in this context in a convincing or meaningful way. Although the technical problems seem insurmountable,
the conceptual issues in quantum gravity are the real problem. A classification approach such as the one we have outlined may be a useful tool to have.

Finally, we give a useful rule. In the application of the contextual completeness test of any proposition, the most useful questions to ask of the author of that proposition are: *how do you know this to be true?* and *who are you talking to?* It is interesting to discover how many propositions are made in nominally scientific articles where no answers are forthcoming.

References

[1] Aristotle: Physica (The Physics). The Clarendon Press, Oxford (1930)

[2] Bell, J.: Speakable and Unspeakable in Quantum Mechanics. CUP (1988)

[3] Bohm, D.: A suggested interpretation of the quantum theory in terms of ‘Hidden Variables’, I and II. Phys. Rev. 85, 166–193 (1952)

[4] Bohr, N.: Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. 48, 696–702 (1935)

[5] Cantor, G.: Beiträge zur Begründung der transfiniten Mengenlehre (Contributions to the founding of the theory of transfinite numbers). Mathematische Annalen pp. 481–512 (1869)

[6] Colosi, D., Rovelli, C.: What is a particle? Classical and Quantum Gravity 26, 025,002 (22pp) (2009)

[7] Deutsch, D.: The Fabric of Reality. The Penguin Press (1997)

[8] DeWitt, B.S., Graham, N.: The Many-Worlds Interpretation of Quantum Mechanics. Princeton University Press (1973)

[9] Einstein, A.: ber einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt (Concerning an Heuristic Point of View toward the Emission and Transformation of Light). Annalen der Physik 17, 132–148 (1905). Translation into English American Journal of Physics, v. 33, n. 5, May 1965

[10] Einstein, A., Podolsky, B., Rosen, N.: Can quantum-mechanical description of physical reality be considered complete? Phys. Rev. 47, 777–780 (1935)

[11] Everett, H.: ‘Relative State’ formulation of quantum mechanics. Rev. Mod. Phys. 29(3), 454–462 (1957)

[12] Feynman, R.P.: Simulating physics with computers. Int. Journal. Theor. Phys. 21(6/7), 467–488 (1982)

[13] Gagnon, P.: Is the moon full? just ask the LHC operators. http://www.quantumdiaries.org/2012/06/07/is-the-moon-full-just-ask-the-lhc-operators/ (2012)

[14] Gainutdinov, R.K., Khamadeev, M.A., Salakhov, M.K.: Electron rest mass and energy levels of atoms in the photonic crystal medium. Phys. Rev. A 85 (2012).

[15] H. Lehmann, K.S., Zimmermann, W.: Zur Formulierung quantisierter Feldtheorien. Il Nuovo Cimento 1(1), 205–225 (1955)

[16] Heisenberg, W.: Über Quantentheoretische Umdeutung Kinematischer und Mechanischer Beziehungen. Zeits. Physik A Hadrons and Nuclei 33(1), 879 – 893 (1925). Quantum-theoretical re-interpretation of kinematic and mechanical relations

[17] Heisenberg, W.: Questions of principle in modern physics. In: Philosophic Problems in Nuclear Science, p. pp. 4152. Faber and Faber, London (1952)
[18] Jaroszkiewicz, G.: Towards a dynamical theory of observation. Proc. Roy. Soc. A \textbf{466}(2124), 3715–3739 (2010).

[19] Kochen, S., Specker, E.: The problem of hidden variables in quantum mechanics. Journal of Mathematics and Mechanics \textbf{17}, 59–87 (1967)

[20] Korzybski, A.: Science and Sanity: An Introduction to Non-Aristotelian Systems and General Semantics, 5th edn. Institute of General Semantics (1994)

[21] Mott, N.: The Wave Mechanics of Alpha-Ray Tracks. Proceedings of the Royal Society \textbf{A126}, 79–84 (1929)

[22] Newton, I.: The Principia (Philosophiae Naturalis Principia Mathematica) (1687). New translation by I. B. Cohen and Anne Whitman, University of California Press (1999)

[23] Penrose, R.: The Emperor’s New Mind. Oxford University Press (1990)

[24] Petzold, C.: The Annotated Turing. Wiley (2008)

[25] Planck, M.: On an improvement of Wein’s equation for the spectrum. Verhandl. Dtsch. Phys. Ges. \textbf{2}, 202–204 (1900)

[26] Schrödinger, E.: Quantisierung als eigenwertproblem (erste mitteilung). Ann. Phys. \textbf{79}, 361–376 (1926)

[27] Tegmark, M.: Our Mathematical Universe: My Quest for the Ultimate Nature of Reality. Alfred A. Knopf, New York (2014)

[28] Unruh, W. G.: Notes on Black-Hole Evaporation Phys. Rev. D \textbf{14}, 870–892 (1976)

[29] Wheeler, J.A.: From the Big Bang to the Big Crunch. Cosmic Search Magazine \textbf{1}(4) (1979).