Atomic analysis of the \((n; t)\)-reaction of the helium-3 atoms with slow neutrons

Alexei M Frolov and David M Wardlaw

Department of Chemistry, University of Western Ontario, London, Ontario N6H 5B7, Canada
E-mail: afrilov@uwo.ca and dwardlaw@uwo.ca

Received 4 March 2011, in final form 6 March 2011
Published 10 May 2011
Online at stacks.iop.org/JPhysB/44/105005

Abstract

Probabilities of formation of various hydrogenic species during the exothermic nuclear \((n, \, ^3\text{He}; \, t, \, p)\)-reaction of atomic helium-3 with slow neutrons are determined. In particular, we have found that the probability of forming the tritium atom \(^3\text{H}\) in its ground state is \(\approx 55.192\,87\%\), while the analogous probability of forming the protium atom \(^1\text{H}\) in its ground state is \(\approx 1.023\,63\%\). Analogous probabilities of formation of the negatively charged hydrogen ions, i.e. the \(^3\text{H}^-\) and \(^1\text{H}^-\) ions, in the nuclear \((n, \, ^3\text{He}; \, t, \, p)\)-reaction with slow neutrons are \(\approx 7.8680\%\) and \(\approx 0.065\,83\%\), respectively. We also consider bremsstrahlung from fast fission-type reactions in atomic systems. The spectrum of emitted radiation is analysed.

1. Introduction

The reaction of the \(^3\text{He}\) nuclei with slow neutrons is written in the form \([1]\)

\[ ^3\text{He} + n = p + t + 0.764 \text{ MeV} \tag{1} \]

where the notations \(p\) and \(t\) stand for the protium \(^1\text{H}\) and tritium \(^3\text{H}\) nuclei, respectively. The cross-section \(\sigma\) of this nuclear reaction is extremely large for slow neutrons \([2]\) \((\sigma_{\text{max}} \approx 5330 \times 10^{-24} \text{ cm}^2\) or 5330 barn, for short). The velocities of the two nuclear fragments formed in the reaction, equation \((1)\), are \(\approx 1.596\,32\) au (tritium) and \(\approx 4.788\,97\) au (protium). All values in this study are given in atomic units, where \(\hbar = 1, \, m_e = 1, \, e = 1\). The unit of atomic velocity is \(v_e = ac \approx \frac{c}{137} \approx 2.188\,2661 \times 10^8 \text{ cm sec}^{-1}\), where \(c\) is the speed of light and \(a\) is the fine structure constant. The atomic velocity \(v_e\) is the velocity of the 1s electron in the hydrogen atom with the infinitely heavy nucleus \(^\infty\text{H}\).

In our earlier work \([3]\), we have determined the probabilities of detecting various final (bound) states in hydrogen atoms which arise during the exothermic nuclear \((n, \, ^3\text{He}; \, t, \, p)\)-reaction of the one-electron helium-3 ion \((^3\text{He}^+)\) with slow neutrons. In many actual experiments, however, the nuclear \((n, \, ^3\text{He}; \, t, \, p)\)-reaction involves the two-electron helium-3 atom rather than the bare helium-3 nucleus. Analogous nuclear reactions of the \(^6\text{Li}, \, ^{10}\text{B}, \, ^{14}\text{N}\) nuclei (see, e.g., \([1, \, 3]\)) with slow neutrons also involve few-electron atoms and ions, rather than the bare nuclei and/or one-electron ions. Consequently, the products of such reactions may also contain a few bound electrons. Our aim below is to evaluate the probabilities of formation of such few-electron species.

In order to determine the probabilities of formation of few-electron atoms and ions during nuclear reactions one needs to solve a number of problems. Some of these problems are related to the analysis of the basic principles of perturbation theory in quantum mechanics \([4]\). In particular, the theory of sudden approximations \([5, \, 6]\) plays a central role in our analysis below. Based on the sudden approximation we have developed a transparent and reliable method which is used to determine the final state probabilities for each possible system/state which can arise after the nuclear reaction, equation \((1)\). On the other hand, it is not clear what procedure can be used for actual computations of the final state probabilities. In \([3]\) we have described a general method based on the use of \(N_f\)-electron density matrices for the \(N_f\)-electron atomic systems (here \(N_f \leq N_i\)). This is a rigorous method, but in real applications it produces a very complex procedure. In many cases this procedure leads to the loss of numerical accuracy, e.g. in those cases when one uses highly accurate variational wavefunctions constructed for few-electron atomic systems. Another group of complications is related to the electron–electron correlation which cannot be ignored in the incident \(^3\text{He}\) atom and final \(^-\text{H}\) ion.

In this study we have re-considered all mentioned problems and developed an approach which can be used in actual calculations of the final probabilities in arbitrary atomic systems undergoing nuclear reactions. All details of our analysis will be published elsewhere \([7]\). Here we want to...
apply this method to obtain the actual data for the nuclear $(n, ^3\text{He}; t, p)$-reaction in the two-electron helium-3 atom. A successful test of our method for the $^3\text{He}$ atom with the exothermic nuclear $(n; t)$-reaction, equation (1), is extremely important. If our method works in this case, then it can be applied to other similar reactions in fast-electron atomic systems. Another goal of this study is to analyse radiation emitted during fast fission-type reactions in atomic systems. In particular, we consider the spectrum of bremsstrahlung emitted in the nuclear reaction, equation (1), of atomic helium-3 with slow neutrons.

2. Formation of the tritium/protonium atoms and ions

Note that all current evaluations of the final state probabilities in various atomic systems with nuclear reactions are based on the sudden approximation [5, 6]. For atomic systems with nuclear reactions the sudden approximation [5] means that the electron density of the incident atom does not change noticeably during the nuclear reaction in its nucleus. In addition to this, the sudden approximation for atomic systems means that the nucleus does not move substantially during the nuclear reaction. In general, this is true, if the nuclear reaction time $\tau_a$ is significantly shorter than the typical atomic time $\tau_0 = \frac{\hbar}{\Delta E} \approx 2.418 \times 10^{-17}$ sec. It can be shown that these conditions are always obeyed in the case of highly exothermic $(n; g)$-reactions of slow neutrons with light nuclei. This means that all changes of electron densities in arising atomic fragments can be described in terms of the sudden approximation.

Our method is also based on the use of the sudden approximation for atomic systems. In this work, we present only a few basic equations which are used to compute the final state probabilities for light atoms and ions. Other details and discussion can be found in [7]. In general, for few-electron atomic systems with $N$ electrons in the incident state we compute the following integral which is called the probability amplitude:

$$M_{in;fi} = \langle \Psi_{fi}(x_1, x_2, \ldots, x_K) \mid \prod_{i=1}^{K} \exp(i \mathbf{V} \cdot \mathbf{r}_i) \mid \Psi_{in}(x_1, x_2, \ldots, x_N) \rangle$$

(2)

where $K$ is the total number of bound electrons in the final nuclear system $K \leq N$. Here and below, the notation $x_i = (r_i, \sigma_i)$ means the complete set of spatial and spin coordinates of the $i$th electron. If the reaction (equation (1)) involves the two-electron He atom rather than the bare He nucleus, then for $K = 1$ one finds the $^3\text{H}$ atom (or $^1\text{H}$ atom) in the final state, while $K = 2$ corresponds to the two-electron $^3\text{He}^+$ ion (or $^1\text{H}^+$ ion) in the final state. For $K = 0$, the final system does not contain any bound electrons. The last case (i.e. when $K = 0$) is of the interest for this study.

In the case of reaction (equation (1)) in the He atom (where $N = 2$) with $K = 2$, one needs to compute the following two-electron matrix element (or probability amplitude):

$$M_{in;fi} = \langle \Psi_{fi}(x_1, x_2) \mid \exp(i \mathbf{V} \cdot \mathbf{r}_1 + i \mathbf{V} \cdot \mathbf{r}_2) \mid \Psi_{in}(x_1, x_2) \rangle$$

(3)

where $\mathbf{V}$ is the velocity of the final nucleus formed in the reaction (equation (1)). Analogously, for the reaction (equation (1)) in the He atom (where $N = 2$) with $K = 1$, we need to determine a slightly different two-electron integral (or probability amplitude):

$$M_{in;fi} = \langle \Psi_{fi}(x_1) \mid \exp(i \mathbf{V} \cdot \mathbf{r}_1) \mid \Psi_{in}(x_1, x_2) \rangle.$$  

(4)

In equations (3) and (4) the notation $\Psi_{fi}(x_1, x_2)$ stands for the two-electron wavefunction of the helium atom, while analogous notations $\Psi_{fi}(x_1)$ and/or $\Psi_{fi}(x_1)$ designate the two- and one-electron wavefunctions of the final atomic systems, respectively. Below, all two-electron wavefunctions are assumed to be properly antisymmetrized upon all electron variables.

Let us apply the formulas (equations (3) and (4)) to the actual nuclear $(n; t)$-reaction in the two-electron He atom. We assume that before such a reaction the He atom was in its ground $1^1\text{S}$ state. Also, for simplicity, in these calculations all nuclear masses are assumed to be infinite. The main interest is to compute the ground-state probabilities, since these values determine the scale of other similar probabilities. Moreover, if the incident and final atomic systems are in their ground states, then the corresponding probability amplitude is relatively large. In this case it is very easy to detect and correct possible numerical mistakes. The results of our calculations include the corresponding probability amplitudes $M_{in;fi}$, and the final state probabilities $p_{in;fi} = \mid M_{in;fi} \mid^2$ for both nuclei formed in the reaction (equation (1)), i.e. for the tritium $^3\text{H}$ and protium $^1\text{H}$ nuclei. Note that the velocities of these two final nuclei differ from each other by a factor of $\sim 3$. This explains very significant differences between the final state probabilities determined in calculations for the tritium and protium atoms/ions (see below).

As the starting point for numerical evaluations we can use the following approximate expression for the ground $1^1\text{S}$ state wavefunction $\Psi(x_1, x_2)$ of the helium atom (see, e.g., [8]):

$$\Psi(x_1, x_2) = \frac{A^3}{\pi} \exp(-Ar_1 - Ar_2)\frac{1}{\sqrt{\pi}}$$

(5)

where $A$ value is called the effective nuclear charge. For the actual $^\infty$He atom one finds $A = Q - \frac{2}{\pi^2}$, where $Q = 2$. By performing integration over all spin and angular variables one can produce the following formula for the corresponding probability amplitude (for the He $\rightarrow$ H transition during the nuclear reaction, equation (1)):

$$M_{in;fi} = 2A^\frac{3}{2} \int_0^{+\infty} \exp(-Ar_j)(\gamma r)^2 dr$$

(6)

where $j(x)$ is the Bessel function of the first kind (see, e.g., [9, 10]). $R_{q\gamma}(\gamma r)$ are the radial functions of the hydrogen-like atom/ion (see, e.g., [4]) formed in the final state. If the final one-electron atomic system is in its ground state, then we have from equation (6)

$$M_{in;fi} = 4(A\gamma)^\frac{3}{2} \int_0^{+\infty} \exp(-(A + \gamma)r)j_0(\gamma r)^2 dr$$

(7)

where $V$ is the velocity of the final fragment, i.e. the velocity of the hydrogen atom. The final state probability $P_{gg}$ is

$$P_{gg} = \mid M_{in;fi} \mid^2 = \frac{64(A\gamma)^3}{(A + \gamma)^3} \frac{1}{[1 + (\frac{V}{Q\gamma})^2]^2}.$$  

(8)
For the hydrogen atom the parameter \( \gamma = 1 \), while for the two-electron He atom \( \gamma = \frac{27}{16} \). In this case, for the tritium atom one finds from equation (8) \( P_{gg} \approx 24.378 \) 09%. This value corresponds to the process in which one bound \( \alpha \)-electron remains bound in the final state. Since we have two bound electrons (one \( \alpha \)-electron and one \( \beta \)-electron) in the incident He atom, the final state probability obtained above must be doubled, i.e. we have \( P_{gg} \approx 48.741 \) 75%. The use of the highly accurate wavefunction for the He atom (400 exponential basis wavefunctions [11]) produces for the \( P_{gg} \) probability \( \approx 55.192 \) 87%. The analogous probability for the proton atom \( ^1H \) is \( \approx 1.023 \) 63%. By using the formulas derived in [3] and the approximate wavefunction of the He atom, equation (5), one can evaluate the final state probabilities for some low-lying excited states with \( n \leq 3 \), where \( n \) is the principal quantum number, in the tritium and protium atoms from the nuclear reaction, equation (1). These values can be found in table 1.

It is interesting to evaluate the final state probability for the two-electron negatively charged hydrogen ion which can be formed during nuclear reaction, equation (1). To determine this probability we used the highly accurate variational wavefunctions known for the He atom and \(^1\)H ion [11]. Each of these two functions contains 400 exponents in the relative coordinates (basis functions). The total energies produced with such wavefunctions for the ground \(^1\)S states in these two systems are \(-2.903 \) 724 377 034 05 au for the \(^\infty\)He atom and \(-0.527 \) 751 016 544 308 au for the \(^\infty\)H ion, respectively. By performing numerical calculations with these variational wavefunctions we have evaluated the probability of forming the negatively charged tritium \(^3\)H\(^-\) ion in the nuclear \((n,^3\)He; \(t, p\))-reaction with slow neutrons as \( \approx 7.8680\% \). The analogous probability of forming the negatively charged proton \(^1\)H\(^-\) ion was evaluated as \( \approx 0.065 \) 83%. Such a relatively large probability for the newly formed tritium ions \(^3\)H\(^-\) means that these ions can be detected in modern experiments. Currently, these and other similar experiments to detect various atomic species formed during the nuclear reaction (equation (1)) in a two-electron helium-3 atom are critically needed to guide future theoretical development.

### 3. Bremsstrahlung from the fission-type process in atomic systems

Another group of problems is related to the analysis of radiation emitted during the reaction of the \(^3\)He nuclei with slow neutrons, equation (1). It is clear that such radiation must be similar to radiation emitted during an arbitrary fission-type reaction in few-electron atomic systems (atoms, ions, etc). First, let us consider a fast nuclear fission-type reaction in a one-electron atomic system. During a fission-type nuclear reaction the electron becomes free, and will interact with the rapidly moving nuclear fragments. Such an interaction will produce radiation which is, in fact, a breaking radiation or bremsstrahlung. Here we want to derive the explicit formulas for this radiation and investigate its spectrum. The electric charges and velocities of these fission fragments are \( Q_1 \), \( Q_2 \) and \( V_1 \), \( V_2 \), respectively. To simplify all formulas below, we assume that both the fission fragments move along the Z-axis. Furthermore, in this study we restrict ourselves to the consideration of the non-relativistic processes only. Briefly, this means that the two velocities \( V_1 \), \( V_2 \) are assumed to be significantly smaller than the speed of light c. The case of arbitrary velocities is discussed in [12].

The second time derivative of the dipole moment \( \mathbf{d} \) is \( \ddot{\mathbf{d}} = \mathbf{e} \ddot{\mathbf{r}} \), where \( \mathbf{r} = (x, y, z) \) is the radius vector of the accelerated electron. In the case of a fission-type reaction in atomic systems the explicit formula for an electron’s acceleration is

\[
\ddot{\mathbf{r}} = \frac{1}{m_e} \left[ \nabla \left( \frac{Q_1 e}{R_1} \right) + \nabla \left( \frac{Q_2 e}{R_2} \right) \right]
\]

\[
= \frac{Q_1 e}{m_e} \nabla \left( \frac{1}{R_1} \right) + \frac{Q_2 e}{m_e} \nabla \left( \frac{1}{R_2} \right)
\]

where \( R_1 = \sqrt{x^2 + y^2 + (z - V_1 t)^2} \) and \( R_2 = \sqrt{x^2 + y^2 + (z + V_2 t)^2} \). The notations \( V_1 \) and \( V_2 \) stand for the velocities of the first and second fission fragments.

The second time derivative of the dipole moment is

\[
\mathbf{d} = \frac{Q_1 e^2}{m_e} \left( \frac{1}{R_1^3} \right) + \frac{Q_2 e^2}{m_e} \left( \frac{1}{R_2^3} \right)
\]

\[
= -\frac{Q_1 e^2 \mathbf{R}_1}{R_1^3} - \frac{Q_2 e^2 \mathbf{R}_2}{R_2^3}
\]

where \( \mathbf{R}_1 = (x, y, z - V_1 t) \) and \( \mathbf{R}_2 = (x, y, z + V_2 t) \) are the three-dimensional vectors. The intensity of the non-relativistic bremsstrahlung from a fission-type reaction in a one-electron atomic system is

\[
dI = \frac{1}{4\pi e^3} \frac{(\mathbf{d} \times \mathbf{n})^2}{(\mathbf{d} \times \mathbf{n})^2} d\Omega
\]

\[
= \left( \frac{e^2}{m_e} \right)^2 \frac{Q^2}{4\pi e^3} \left[ \frac{Q_1 \mathbf{R}_1}{Q \mathbf{R}_1^3} + \frac{Q_2 \mathbf{R}_2}{Q \mathbf{R}_2^3} \right]^2 \sin^2 \theta d\Omega
\]

\[
dI = \left( \frac{e^2}{m_e} \right)^2 \frac{N \cdot Q^2}{4\pi e^3} \left[ \frac{Q_1 \mathbf{R}_1}{Q \mathbf{R}_1^3} + \frac{Q_2 \mathbf{R}_2}{Q \mathbf{R}_2^3} \right] \sin^2 \theta d\Omega
\]
where \( N_e \) is the number of electrons which become free after this nuclear reaction. The formula, equation (12), can be rewritten in the form

\[
\frac{dI}{d\omega} = \left( \frac{e^2}{mc^2} \right)^2 N_e Q^2 \frac{1}{4\pi c^3} \times \left[ \frac{Q_1^2}{Q_2^2 - R_1^2} + \frac{Q_2^2}{Q_1^2 - R_2^2} + \frac{Q_1 Q_2}{Q_1^2 - Q_2^2} \left( \frac{R_1}{R_2} \right)^2 \right]^2 \sin^2\theta \quad (13)
\]

for the differential cross-section. Our derivation of these formulas for few-electron systems is based on the fact that radiation emitted by different post-atomic electrons is non-coherent. Indeed, different free electrons move as independent (or ‘random’) quantum particles. Note that the bremsstrahlung from the fast fission-type process in atomic systems can be coherent at certain conditions, e.g. if two fission fragments move with very large velocities, while all post-atomic electrons almost do not move at the beginning of the process. In this case, one needs to introduce an additional factor \( N_e \) in the last formula, equation (13).

Let us discuss the spectrum of the emitted radiation. As follows from the formula, equation (13), the intensity of the bremsstrahlung from fission-type processes in atomic systems rapidly decreases with the time \( t \approx \tau^{-5} \) after the nuclear reaction which proceeds at \( t = 0 \). Such a behaviour is typical for atomic systems with fast nuclear reactions. The spectral resolution \( R(\omega) \) (or spectral function) of the intensity of dipole radiation is written in the form

\[
dR(\omega) = \frac{4}{3\omega} |d_\omega|^2 \frac{d\omega}{2\pi} = \frac{4\omega^4}{3\omega^3} |d_\omega|^2 \frac{d\omega}{2\pi} \quad (14)
\]

where \( d_\omega \) is the Fourier component of the dipole moment \( d \) introduced above. As follows from equation (14), to determine the spectral resolution \( R(\omega) \) of the bremsstrahlung from a fission-type process one needs to find the Fourier components of the dipole moment. In this study, we apply a different method and determine the Fourier components of the \( d_\omega \) vector, equation (9). Finally, the problem is reduced to the calculation of the following two Fourier transformations:

\[
I_1 \left( \omega; \frac{a}{V}, V, \cos \eta \right) = \int_0^{+\infty} \left( \frac{a^2}{V^2} \pm \frac{2a}{V} \cos \eta \cdot t + t^2 \right)^{\frac{\omega}{a} - \frac{\eta}{a}} \times \exp(\pm i\omega t) dt \quad (15)
\]

and

\[
I_2 \left( \omega; \frac{a}{V}, V, \cos \eta \right) = \int_0^{+\infty} \left( \frac{a^2}{V^2} \pm \frac{2a}{V} \cos \eta \cdot t + t^2 \right)^{\frac{\omega}{a} - \frac{\eta}{a}} \times t \exp(\pm i\omega t) dt = -i \frac{\partial}{\partial \omega} I_1 \left( \omega; \frac{a}{V}, V, \cos \eta \right) \quad (16)
\]

where \( a \) is the ‘effective’ radius of the original electron shell and \( \eta \) is the angle between the electron acceleration and Z-axis (or the line of the nuclear motion). Note that the spectrum of the emitted radiation depends explicitly upon \( \eta \). This dependence can be found for all fast fission-type reactions and processes. The lower limit in these formulas is zero, but, in reality, we cannot use times \( t \) which are shorter than \( \tau = \frac{\nu}{c} \). Formally, this means that the lower limits in equations (15) and (16) can slightly be changed and this can be used to simplify the explicit formulas for the corresponding Fourier components. The upper limits in equations (15) and (16) can also be chosen to be finite. For instance, it is possible to obtain a very good approximation for the integrals, equations (15) and (16), by using the upper limit \( \frac{\omega}{a} \). In actual situations one can use the formula \( \exp(\pm \omega t) = \cos \omega t + i \sin \omega t \) and then calculate all arising integrals numerically, or by using analytical formulas from the tables of Fourier sine/cosine transformations (see, e.g., [13]). A good analytical approximation for the integral, equation (15), is

\[
I_1 \left( \omega; \frac{a}{V}, V, \cos \eta \right) \approx \exp\left( -i\omega \frac{a}{V} \cos \eta \right) \int_0^{+\infty} \left[ b^2 + t^2 \right]^{-\frac{1}{2}} \times \exp(\pm i\omega t) dt = \exp\left( -i\omega \frac{a}{V} \cos \eta \right)
\]

\[
\times \left[ \frac{\omega}{b} K_1(b\omega) + \frac{1}{\omega} \ln \left( 1 + \frac{1}{b\omega} \right) - \frac{\pi}{4} \frac{\omega^2}{1 - \frac{1}{4} b^2 \omega^2} \right] \quad (17)
\]

where \( b = \frac{\omega}{2\eta} \) and \( K_1(x) \) is the MacDonald function (see, e.g., [14]). The derivative of equation (17) with respect to the frequency \( \omega \) allows one to determine the \( I_2(\omega; \frac{a}{V}, V, \cos \eta) \) integral, equation (16), and obtain the analytical formula for the spectral resolution \( R(\omega) \) of the intensity of the bremsstrahlung, equation (14), from the fast fission-type (fast) process in few- and many-electron atomic systems. The derivation of an explicit expression for the spectral resolution \( R(\omega) \) is straightforward, but the final formula is cumbersome and it is not given here. Note that only the term \( \approx Q_1 Q_2 \) in this formula represents interference of radiation which is emitted by the two rapidly moving fission fragments. In application to the nuclear fission of heavy nuclei, e.g. \( ^{239}\text{Pu}, ^{247}\text{Cm} \), etc, the spectral resolution \( R(\omega) \) must be averaged over the actual mass/charge distribution known for each fissionable nucleus.

4. Conclusion

We have considered the nuclear reaction, equation (1), involving the two-electron \( ^3\text{He} \) atom and slow neutrons. To determine the final state probabilities for one- and two-electron tritium and protium atoms/ions we have developed a new method which is based on direct computation of probability amplitudes. This method is much simpler than an alternative procedure based on the use of few-electron density matrices. Furthermore, it allows one to determine the final state probabilities for various bound states in the incident and final atomic systems. By using this method we have found that the probability of forming the tritium atom \( ^3\text{H} \) in its ground state is \( \approx 0.5.192.87\% \), while the analogous probability of forming the protium atom in its ground state \( ^1\text{H} \) is \( \approx 1.023.63\% \). Analogous probabilities for the negatively charged hydrogen ions, i.e. for the \( ^3\text{H}^+ \) and \( ^1\text{H}^+ \) ions to be formed in the nuclear \( (n, ^3\text{He}; t, p) \)-reaction with slow neutrons, are \( \approx 5.8680\% \) and \( \approx 0.665.83\% \), respectively. The corresponding uncertainties for such probabilities can be evaluated as \( \approx (0.3 - 1) \times 10^{-4}\% \) for the ground states and as \( \approx (0.5 - 1) \times 10^{-3}\% \) for the excited states. Another important problem is the analysis of bremsstrahlung from fast fission-type reactions in atomic
systems. In this study, such an analysis is performed for the non-relativistic case. Its generalization to the systems with relativistic velocities is quite complicated (see, e.g., [12]).

In the future it is necessary to study analogous nuclear reactions involving atoms with the $^6\text{Li}$, $^{10}\text{B}$, $^{14}\text{N}$ (and some other) nuclei. For such atomic systems we need to determine the final state probabilities of forming various few-electron atoms and ions in the reaction of these atoms with slow neutrons. These results are of interest in numerous applications of these nuclear reactions, e.g. in the Boron neutron capture therapy (BNCT) where the reaction ($n, ^{10}\text{B}$, $^7\text{Li}, ^4\text{He}$) is used [15]–[17]. Nuclear reaction (equation (1)) and the analogous reaction with the $^6\text{Li}$ nuclei are extensively used in thermonuclear explosive devices [3]. Also, these nuclear reactions are used in various detectors (see, e.g., [2, 18, 19]) of thermal and slow neutrons. By using our results for the final state probabilities of different atomic species formed in such reactions we can improve the overall sensitivity of such detectors.

**Acknowledgments**

It is a pleasure to thank Professor M Frederick Hawthorne for useful references and the University of Western Ontario for financial support.

**References**

[1] Kikoin I K (ed) 1976 *Tables of Physical Quantities: Handbook* (Moscow: Atomizdat) chapter 39 (in Russian)

[2] Weinberg A M and Wigner E P 1959 *The Physical Theory of Neutron Chain Reactors* (Chicago: University of Chicago Press)

[3] Frolov A M and Wardlaw D M 2009 *Phys. Rev. A* **79** 032703

[4] Landau L D and Lifshitz E M 1977 *Quantum Mechanics. Non-Relativistic Theory* 3rd edn (Oxford: Pergamon)

[5] Migdal A B 1941 *J. Phys. (USSR)* **4** 449

[6] Migdal A B and Kainov V 1969 *Approximation Methods in Quantum Mechanics* (New York: Benjamin)

[7] Frolov A M and Wardlaw D M 2007 *J. Chem. Phys.* **127** 234104

[8] March N H, Young W H and Sampanthere S 1995 *The Many-Body Problem in Quantum Mechanics* (New York: Dover) chapter 2

[9] Abramowitz M and Stegun I A (ed) 1972 *Handbook of Mathematical Functions* (New York: Dover)

[10] Gradstein I S and Ryzhik I M 2000 *Tables of Integrals, Series and Products* 6th revised edn (New York: Academic)

[11] Frolov A M 1998 *Phys. Rev. A* **57** 2436

[12] Frolov A M 2007 *Phys. Lett. A* **361** 346

[13] Erdelyi A (ed) 1954 *Bateman Manuscript Project (Tables of Integral Transforms)* vol 2 (New York: McGraw-Hill)

[14] Watson G N 1966 *Treatise on the Theory of Bessel Functions* 2nd edn (New York: Cambridge University Press)

[15] Watson-Clark R A, Banquerigo M L, Shelly K, Hawthorne M F and Brahn E 1998 *Proc. Natl Acad. Sci. USA* **95** 2531

[16] Hawthorne M F 1993 *Agnew. Chem. Int. Ed. Engl.* **32** 950

[17] Shelly K, Feakes D A, Hawthorne M F, Schmidt P G, Krisch T A and Bauer E 1992 *Proc. Natl Acad. Sci. USA* **89** 9039

[18] Yampolskii P A 1961 *Neutrons of Atomic Explosion* (Moscow: Atomizdat) (in Russian)

[19] Tsoulfanidis N 1995 *Measurement and Detection of Radiation* (Washington, DC: Taylor and Francis) pp 467–505