INTUITIONISTIC FUZZY $\Psi$-CONTINUOUS MAPPINGS

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ABSTRACT
In this paper we introduce intuitionistic fuzzy $\Psi$-continuous mappings and intuitionistic fuzzy $\Psi$-irresolute mappings. Some of their properties are studied.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy $\Psi$-closed set, intuitionistic fuzzy $\Psi$-continuous mappings and intuitionistic fuzzy $\Psi$-irresolute mappings.

1. INTRODUCTION
The concept of fuzzy sets was introduced by Zadeh (1965), is a framework to encounter uncertainty, vagueness and partial truth and it represents a degree of membership for each element of the universe of discourse to a subset of it. By adding the degree of non-membership (namely $\mu$), intuitionistic fuzzy sets. In this paper we introduced fuzzy topological spaces using the notation of opportunity uncertainity quantification and provides the opportunity to precisely model the problem, based on the existing knowledge and observations. On the other hand Coker (1997) introduced intuitionistic fuzzy topological spaces using the notation of intuitionistic fuzzy sets. In this paper we introduced intuitionistic fuzzy $\Psi$-continuous mappings and studied some of their basic properties. We provide some characterizations of intuitionistic fuzzy $\Psi$-continuous.

2. PRELIMINARIES
2.1. Definition (Atanassov, 1986)
Let $X$ be a non empty fixed set and $I$ be the closed interval $[0,1]$. In intuitionistic fuzzy set (IFS) $A$ is an object of the following form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$$

Where the mapping $\mu_A : X \rightarrow I$ and $\nu_A : X \rightarrow I$ denote the degree of membership (namely $\mu_A(x))$ and the degree of non membership (namely $\nu_A(x)$) for each element $x \in X$ to the set $A$, respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Obviously, every fuzzy set $A$ on a nonempty set $X$ is an IFS of the following form

$$A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle : x \in X \}$$

2.2. Definition (Atanassov, 1986)
Let $A$ and $B$ be IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ and $A = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X \}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$;
(ii) $\overline{A} = \{ \langle x, \nu_A(x), \mu_A(x) \rangle : x \in X \}$;
(iii) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X \}$
(iv) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X \}$
(v) $A = B$ if $A \subseteq B$ and $B \subseteq A$;
(vii) $\langle \rangle A = \{ \langle x, 1 - \nu_A(x), \mu_A(x) \rangle : x \in X \}$
(viii) $\{ \langle x, 1, 0 \rangle, x \in X \}$ and $\{ \langle x, 1, 0 \rangle, x \in X \}

We will use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$.
The intuitionistic fuzzy sets

\[ 0_\alpha = \left\{ (x, 1, 0), x \in X \right\} \] and

\[ 1_\alpha = \left\{ (x, 1, 0), x \in X \right\} \] are respectively the empty set and the whole set of \( X \).

2.3. Definition (Coker, 1997)

An intuitionistic fuzzy topology (IFT in short) on \( X \) is a family \( \tau \) of IFSs in \( X \) satisfying the following axioms.

(i) \( 0_\alpha, 1_\alpha \in \tau \)

(ii) \( G_1 \cap G_2 \in \tau \) for any \( G_1, G_2 \in \tau \)

(iii) \( G_i \in \tau \) for any family \( \{ G_i \mid i \in J \} \subseteq \tau \).

Note that for any IFS \( A \) in \( cl(A) = int(A) = int(\alpha) = \alpha \), the family of all IFOS (respectively IFSOS, IF \( \alpha \) OS, IFSPOS, IFPOS, IFROS) of an IFTS \( (X, \tau) \) is denoted by IFO(X) (respectively IFSO(X), IF \( \alpha \) O(X), IFSPO(X), IFPO(X), IFRO(X)).

2.4. Definition (Coker, 1997)

Let \( (X, \tau) \) be an IFTS and \( A = \{ x, \mu_A, \mu_A \} \) be an IFS in \( X \). Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

\[ int(A) = \bigcup\{ G \mid G \text{ is an IFS in } X \text{ and } G \subseteq A \} \]

\[ cl(A) = \bigcap\{ K \mid K \text{ is an IFS in } X \text{ and } A \subseteq K \} \]

Note that for any IFS \( A \) in \( (X, \tau) \), we have \( cl(A^C) = [int(A)]^C \) and \( int(A^C) = [cl(A)]^C \).

2.5. Definition

An IFS \( A = \{ \mu_A, \mu_A \} \) in an IFTS \( (X, \tau) \) is said to be an

(i) Intuitionistic fuzzy semi open set (Joug Kon et al., 2005) (IFSOS in short) if \( A \subseteq cl(int(A)) \),

(ii) Intuitionistic fuzzy \( \alpha - \) open set (Joug Kon et al., 2005) (IF \( \alpha \) OS in short) if \( A \subseteq int(cl(int(A))) \),

(iii) Intuitionistic fuzzy semi pre open set (Young Bae and Seok-Zun, 2005) (IFSPOS in short) if \( A \subseteq cl(int(cI(A))) \),

(iv) Intuitionistic fuzzy pre open set (Young Bae and Seok-Zun, 2005) (IFPOS in short) if \( A \subseteq int(cl(A)) \).

(v) Intuitionistic fuzzy regular open set (Joug Kon et al., 2005) (IFROS in short) if \( A = int(cl(A)) \).

The family of all IFOS (respectively IFSOS, IF \( \alpha \) OS, IFSPOS, IFPOS, IFROS) of an IFTS \( (X, \tau) \) is denoted by IFO(X) (respectively IFSO(X), IF \( \alpha \) O(X), IFSPO(X), IFPO(X), IFRO(X)).

2.6. Definition

An IFS \( A = \{ \mu_A, \mu_A \} \) in an IFTS \( (X, \tau) \) is said to be an

(i) Intuitionistic fuzzy semi closed set (Joug Kon et al., 2005) (IFS in short) if \( A \subseteq cl(A) \),

(ii) Intuitionistic fuzzy \( \alpha - \) closed set (Joug Kon et al., 2005) (IF \( \alpha \) CS in short) if \( cl(int(cI(A))) \subseteq A \),

(iii) Intuitionistic fuzzy semi pre closed set (Young Bae and Seok-Zun, 2005) (IFSPCS in short) if \( int(cl(A)) \subseteq A \),

(iv) Intuitionistic fuzzy pre closed set (Young Bae and Seok-Zun, 2005) (IFPCS in short) if \( cl(int(A)) \subseteq A \).

(v) Intuitionistic fuzzy regular closed set (Joug Kon et al., 2005) (IFRCS in short) if \( A = cl(int(A)) \).

The family of all IFCS (respectively IFCSs, IF \( \alpha \) CS, IFSPCS, IFPCS, IFRCS) of an IFTS \( (X, \tau) \) is denoted by IFCS(X) (respectively IFCS(X), IF \( \alpha \) CS(X), IFSPCS(X), IFPCS(X), IFRCS(X)).

2.7. Definition (Young Bae and Seok-Zun, 2005)

Let \( A \) be an IFS in an IFTS \( (X, \tau) \). Then

\[ \text{sint}(A) = \bigcup\{ G \mid G \text{ is an IFPOS in } X \text{ and } G \subseteq A \} \]

\[ \text{scl}(A) = \bigcap\{ K \mid K \text{ is an IFSPCS in } X \text{ and } A \subseteq K \} \]

Note that for any IFS \( A \) in \( (X, \tau) \), we have \( scl(A) = (\text{sint}(A))^C \) and \( sint(A) = (\text{scl}(A))^C \).

2.8. Definition

An IFS \( A \) in an IFTS \( (X, \tau) \) is an

(i) Intuitionistic fuzzy generalised closed set (Thakur and Rekha, 2006) (IFGCS in short) if \( cl(A) \subseteq U \) whenever \( A \subseteq U \).
2.9. Definition

Let \( f \) be a mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \( f \) is said to be

(i) Intuitionistic fuzzy semi continuous (Joung Kon et al., 2005) (IFS continuous in short) if \( f^{-1}(B) \in IFSO(X) \) for every \( B \in \sigma \).

(ii) Intuitionistic fuzzy \( \alpha \) continuous (Joung Kon et al., 2005) (IF \( \alpha \) continuous in short) if \( f^{-1}(B) \in IF\alpha O(X) \) for every \( B \in \sigma \).

(iii) Intuitionistic fuzzy pre continuous (Joung Kon et al., 2005) (IFP continuous in short) if \( f^{-1}(B) \in IFPO(X) \) for every \( B \in \sigma \).

(iv) Intuitionistic fuzzy semi pre continuous (Young Bae and Seok-Zun, 2005) (IFSP continuous in short) if \( f^{-1}(B) \in IFSPO(X) \) for every \( B \in \sigma \).

2.10. Definition (Thakur and Rekha, 2006)

Let \( f \) be a mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \( f \) is said to be

(i) Intuitionistic fuzzy generalised continuous (IFG continuous in short) if \( f^{-1}(B) \in IFGCS(X) \) for every \( B \in \sigma \).

(ii) Intuitionistic fuzzy semi generalised continuous (IFGSCS continuous in short) if \( f^{-1}(B) \in IFGSCS(X) \) for every \( B \in \sigma \).

(iii) Intuitionistic fuzzy generalised semi open (IFGSO continuous in short) if \( f^{-1}(B) \in IFGSO(X) \) for every \( B \in \sigma \).

(iv) Intuitionistic fuzzy semi open (IFISO continuous in short) if \( f^{-1}(B) \in IFISO(X) \) for every \( B \in \sigma \).

2.11. Theorem (Parimala et al.)

Let \((X, \tau)\) be an intuitionistic fuzzy topological space. Then the following are hold

(i) Every IFCS in \( X \) is an IF\( \psi \)CS in \( X \).

(ii) Every IFRCS in \( X \) is an IF\( \psi \)CS in \( X \).

(iii) Every IF \( \alpha \)CS and hence IFSCS in \( X \) is an IF\( \psi \)CS in \( X \).

(iv) Every IF\( \psi \)CS in \( X \) is an IFSPCS in \( X \).

(v) Every IF\( \psi \)CS in \( X \) is an IFGSPCS in \( X \).

(vi) Every IF\( \psi \)CS in \( X \) is an IFGSCS and hence IFSGCS in \( X \).

**INTUITIONISTIC FUZZY \( \psi \)-CONTINUOUS MAPPINGS**

In this section we introduce intuitionistic fuzzy \( \psi \)-continuous mapping and studied some of its properties.

3.1. Definition

A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) function intuitionistic fuzzy \( \psi \)-continuous (IF\( \psi \)-continuous in short) if \( f^{-1}(B) \) is an IF\( \psi \)CS in \((X, \tau)\) for every IFCS \( B \) of \((Y, \sigma)\).
3.2. Example
Let \( X = \{a, b\}, Y = \{u, v\} \) and
\[ T_1 = \langle x, (0.5, 0.3), (0.4, 0.3) \rangle, \]
\[ T_2 = \langle y, (0.5, 0.7), (0.5, 0.3) \rangle. \]
Then \( \tau = \{0, T_1, 1\} \) and \( \sigma = \{0, T_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IF \( \psi \)-continuous mapping.

3.3. Theorem
Every IF continuous mapping is an IF \( \psi \)-continuous mapping but not conversely.

3.3.1. Proof
Let \( f : (X, \tau) \to (Y, \sigma) \) be an IF continuous mapping. Let \( A \) be an IFCS in \( Y \). Since every IF continuous mapping is an IF \( \psi \)-continuous mapping but not conversely.

3.4. Example
Let \( X = \{a, b\}, Y = \{u, v\} \) and
\[ T_1 = \langle x, (0.5, 0.3), (0.4, 0.3) \rangle, \]
\[ T_2 = \langle y, (0.5, 0.7), (0.5, 0.3) \rangle. \]
Then \( \tau = \{0, T_1, 1\} \) and \( \sigma = \{0, T_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( A = \langle y, (0.5, 0.3), (0.5, 0.7) \rangle \) is IFCS in \( Y \).

3.5. Theorem
Every IF semi continuous mapping is an IF \( \psi \)-continuous mapping but not conversely.

3.5.1. Proof
Let \( f : (X, \tau) \to (Y, \sigma) \) be an IF semi continuous mapping. Let \( A \) be an IFCS in \( Y \). Since \( f \) is an IF semi-continuous mapping, then \( f^{-1}(A) \) is an IFSCS in \( X \) by Theorem 2.11. Since every IFSCS is an IFCS in \( X \). Therefore \( f \) is an IF \( \psi \)-continuous mapping.

3.6. Example
Let \( X = \{a, b\}, Y = \{u, v\} \) and
\[ T_1 = \langle x, (0.2, 0.2), (0.4, 0.5) \rangle, \]
\[ T_2 = \langle y, (0.5, 0.5), (0.1, 0.1) \rangle. \]
Then \( \tau = \{0, T_1, 1\} \) and \( \sigma = \{0, T_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( A = \langle y, (0.1, 0.1), (0.5, 0.5) \rangle \) is IFCS in \( Y \).

3.7. Theorem
Every IF \( \alpha \)-continuous mapping is an IF \( \psi \)-continuous mapping but not conversely.

3.7.1. Proof
Let \( f : (X, \tau) \to (Y, \sigma) \) be an IF \( \alpha \)-continuous mapping. Let \( A \) be an IFCS in \( Y \). Then by hypothesis \( f^{-1}(A) \) is an IF \( \alpha \) CS in \( X \). Since every IF \( \alpha \) CS is an IF \( \psi \) CS in \( X \) by Theorem 2.11. Therefore \( f \) is an IF \( \psi \)-continuous mapping.

3.8. Example
Let \( X = \{a, b\}, Y = \{u, v\} \) and
\[ T_1 = \langle x, (0.2, 0.2), (0.4, 0.5) \rangle, \]
\[ T_2 = \langle y, (0.5, 0.5), (0.1, 0.1) \rangle. \]
Then \( \tau = \{0, T_1, 1\} \) and \( \sigma = \{0, T_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( A = \langle y, (0.1, 0.1), (0.5, 0.5) \rangle \) is IFSCS in \( Y \).

3.9. Theorem
Every IF \( \psi \)-continuous mapping is an IFSP continuous mapping but not conversely.

3.9.1. Proof
Let \( f : (X, \tau) \to (Y, \sigma) \) be an IF \( \psi \)-continuous mapping. Let \( A \) be an IFCS in \( Y \). Then by hypothesis \( f^{-1}(A) \) is an IF \( \psi \) CS in \( X \). Since every IF \( \psi \) CS is an IFSPCS by Theorem 2.11, \( f^{-1}(A) \) is an IFSPCS in \( X \). Therefore \( f \) is an IFSP continuous mapping.
3.12. Example

Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \) and
\[
T_1 = \langle x, (0.5, 0.7), (0.4, 0.3) \rangle, \\
T_2 = \langle y, (0.5, 0.3), (0.5, 0.7) \rangle .
\]
Then \( \tau = \{ 0, T_1, 1 \} \) and \( \sigma = \{ 0, T_2, 1 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( A = \langle y, (0.5, 0.7), (0.5, 0.3) \rangle \) is IFCS in \( Y \). Then \( f^{-1}(A) \) is IFSPCS in \( X \) but not IF \( \psi \) CS in \( X \).

Then \( f \) is an IFSP-continuous mapping but not IF \( \psi \) -continuous mapping.

3.10. Theorem

Every IF \( \psi \) -continuous mapping is an IFSG continuous mapping but not conversely.

3.10.1. Proof.

Let \( f : (X, \tau) \to (Y, \sigma) \) be an IF \( \psi \) -continuous mapping. Let \( A \) be an IFCS in \( Y \). Then by hypothesis \( f^{-1}(A) \) is an IF \( \psi \) CS in \( X \). Since every IF \( \psi \) CS is an IFSGCS by Theorem 2.11, \( f^{-1}(A) \) is an IFGCS in \( X \).

Therefore \( f \) is an IFGCS continuous mapping.

3.11. Example

Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \) and
\[
T_1 = \langle x, (0.6, 0.5), (0.3, 0.4) \rangle, \\
T_2 = \langle y, (0.2, 0.2), (0.6, 0.6) \rangle .
\]
Then \( \tau = \{ 0, T_1, 1 \} \) and \( \sigma = \{ 0, T_2, 1 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( A = \langle y, (0.6, 0.6), (0.2, 0.2) \rangle \) is IFCS in \( Y \). Then \( f^{-1}(A) \) is IFSPCS in \( X \) but not IF \( \psi \) CS in \( X \).

Therefore \( f \) is an IFGCS continuous mapping but not IF \( \psi \) -continuous mapping.

3.13. Example

Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \) and
\[
T_1 = \langle x, (0.6, 0.5), (0.3, 0.4) \rangle, \\
T_2 = \langle y, (0.2, 0.2), (0.6, 0.6) \rangle .
\]
Then \( \tau = \{ 0, T_1, 1 \} \) and \( \sigma = \{ 0, T_2, 1 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( A = \langle y, (0.6, 0.6), (0.2, 0.2) \rangle \) is IFCS in \( Y \). Here IFOS \( G = \langle x, (0.7, 0.6), (0.2, 0.2) \rangle \), clearly \( A \subseteq G \). Therefore \( A \) is an IFGSCS in \( X \). Then \( f^{-1}(A) \) is IFGCS in \( X \) but not IF \( \psi \) CS in \( X \).

Then \( f \) is an IFGCS continuous mapping but not IF \( \psi \) -continuous mapping.

3.14. Theorem

Every IF \( \psi \) -continuous mapping is an IFGSP continuous mapping but not conversely.

3.14.1. Proof.

Let \( f : (X, \tau) \to (Y, \sigma) \) be an IF \( \psi \) -continuous mapping. Let \( A \) be an IFCS in \( Y \). Then by hypothesis \( f^{-1}(A) \) is an IF \( \psi \) CS in \( X \). Since every IF \( \psi \) CS is an IFGSPCS by Theorem 2.11, \( f^{-1}(A) \) is an IFGSPCS in \( X \).

Therefore \( f \) is an IFGSCS continuous mapping but not IF \( \psi \) -continuous mapping.

3.15. Example

Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \) and
\[
T_1 = \langle x, (0.6, 0.5), (0.3, 0.4) \rangle, \\
T_2 = \langle y, (0.3, 0.4), (0.6, 0.5) \rangle .
\]
Then \( \tau = \{ 0, T_1, 1 \} \) and \( \sigma = \{ 0, T_2, 1 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( A = \langle y, (0.6, 0.5), (0.3, 0.4) \rangle \) is IFCS in \( Y \). Here IFOS \( G = \langle x, (0.7, 0.6), (0.3, 0.2) \rangle \), clearly \( A \subseteq G \). Therefore \( A \) is an IFGPCS in \( X \). Then \( f^{-1}(A) \) is IFGSPCS in \( X \) but not IF \( \psi \) CS in \( X \).

Then \( f \) is an IFGSP continuous mapping but not IF \( \psi \) -continuous mapping.
3.16. Remark
If $\psi$-continuity and IFG-continuity are independent of each other.

3.17. Example
Let $X = \{a, b\}$, $Y = \{u, v\}$ and
$T_1 = \langle x, (0.4, 0.5), (0.5, 0.6) \rangle$,
$T_2 = \langle y, (0.4, 0.7), (0.2, 0.3) \rangle$. Then
$\tau = \{0 \leq T_1 \leq 1\}$ and $\sigma = \{0 \leq T_2 \leq 1\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping
$f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then
$f$ is an IFG-continuous but not an IF $\psi$-continuous mapping since $A = \langle y, (0.2, 0.3), (0.4, 0.7) \rangle$ is
an IFC in $Y$ but $f^{-1}(A) = \langle x, (0.2, 0.3), (0.4, 0.7) \rangle$ is not IFGCS in $X$.

3.18. Example
Let $X = \{a, b\}$, $Y = \{u, v\}$ and
$T = \langle x, (0.2, 0.2), (0.4, 0.4) \rangle$,
$T_2 = \langle y, (0.5, 0.5), (0.1, 0.1) \rangle$. Then
$\tau = \{0 \leq T_1 \leq 1\}$ and $\sigma = \{0 \leq T_2 \leq 1\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping
$f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then
$f$ is an IF $\psi$-continuous but not an IFG-continuous mapping since $A = \langle y, (0.1, 0.1), (0.5, 0.5) \rangle$ is
an IFCS in $Y$ but $f^{-1}(A) = \langle x, (0.1, 0.1), (0.5, 0.5) \rangle$ is not IFGCS in $X$.

3.19. Remark
If $\psi$-continuity is independent from IF $\alpha$-continuity, IFG $\alpha$-continuity and pre-continuity.

3.20. Example
Let $X = \{a, b\}$, $Y = \{u, v\}$ and
$T_1 = \langle x, (0.4, 0.5), (0.5, 0.6) \rangle$,
$T_2 = \langle y, (0.2, 0.3), (0.4, 0.7) \rangle$. Then
$\tau = \{0 \leq T_1 \leq 1\}$ and $\sigma = \{0 \leq T_2 \leq 1\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping
$f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then
$f$ is an IF $\alpha$-continuous but not an IF $\psi$-continuous mapping since $A = \langle y, (0.4, 0.7), (0.2, 0.3) \rangle$ is an IFCS in $Y$ but
$f^{-1}(A) = \langle x, (0.4, 0.7), (0.2, 0.3) \rangle$ is not IF $\psi$-CS in $X$.

3.21. Example
Let $X = \{a, b\}$, $Y = \{u, v\}$ and
$T_1 = \langle x, (0.2, 0.2), (0.3, 0.4) \rangle$,
$T_2 = \langle y, (0.5, 0.4), (0.2, 0.2) \rangle$. Then
$\tau = \{0 \leq T_1 \leq 1\}$ and $\sigma = \{0 \leq T_2 \leq 1\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping
$f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then
$f$ is an IF $\psi$-continuous but not an IF $\alpha$-continuous mapping since
$A = \langle y, (0.2, 0.2), (0.5, 0.4) \rangle$ is an IFCS in $Y$ but
$f^{-1}(A) = \langle x, (0.2, 0.2), (0.5, 0.4) \rangle$ is not IF $\alpha$-CS in $X$.

3.22. Example
Let $X = \{a, b\}$, $Y = \{u, v\}$ and
$T_1 = \langle x, (0.6, 0.5), (0.3, 0.4) \rangle$,
$T_2 = \langle y, (0.2, 0.2), (0.6, 0.6) \rangle$. Then
$\tau = \{0 \leq T_1 \leq 1\}$ and $\sigma = \{0 \leq T_2 \leq 1\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping
$f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Let
$A = \langle y, (0.6, 0.6), (0.2, 0.2) \rangle$ is an IFCS in $Y$.
Here IFOS $G = \langle x, (0.9, 0.9), (0.1, 0.1) \rangle$, clearly
$A \subseteq G$. Therefore $A$ is an IF $\alpha$-CS in $X$. Then $f$ is
an IF $\alpha$-continuous but not an IF $\psi$-continuous mapping since but $f^{-1}(A) = \langle x, (0.6, 0.6), (0.2, 0.2) \rangle$ is not IF $\psi$-CS in $X$.

3.23. Example
Let $X = \{a, b\}$, $Y = \{u, v\}$ and
$T_1 = \langle x, (0.8, 0.8), (0.2, 0.1) \rangle$,
$T_2 = \langle y, (0.1, 0.3), (0.9, 0.7) \rangle$. Then
$\tau = \{0 \leq T_1 \leq 1\}$ and $\sigma = \{0 \leq T_2 \leq 1\}$ are IFTs on $X$ and $Y$ respectively. Define a mapping
$f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then
$f$ is an IF $\psi$-continuous but not an IF $\alpha$-continuous mapping since
$A = \langle y, (0.9, 0.7), (0.1, 0.3) \rangle$ is an IFCS in $Y$ but
$\alpha$-CS in $X$.

3.24. Example
Let $X = \{a, b\}$, $Y = \{u, v\}$ and
$T_1 = \langle x, (0.4, 0.5), (0.5, 0.6) \rangle$ .
\( T_2 = \langle y, (0.4, 0.7), (0.2, 0.3) \rangle \). Then 
\( \tau = \{0_, T_1, 1_\} \) and \( \sigma = \{0_, T_2, 1_\} \) are IFTs on 
\( X \) and \( Y \) respectively. Define a mapping 
\( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then 
\( f \) is an IF\( \psi \)-continuous but not an IF\( \psi \)-continuous 
mapping since \( A = \langle y, (0.2, 0.3), (0.4, 0.7) \rangle \) is 
an IFCS in \( Y \) but \( f^{-1}(A) = \langle x, (0.2, 0.3), (0.4, 0.7) \rangle \) is not IFPCS in \( X \).

3.25. Example

Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and 
\( T_1 = \langle x, (0.3, 0.4), (0.7, 0.6) \rangle \), 
\( T_2 = \langle y, (0.4, 0.5), (0.6, 0.5) \rangle \). Then 
\( \tau = \{0_, T_1, 1_\} \) and \( \sigma = \{0_, T_2, 1_\} \) are IFTs on 
\( X \) and \( Y \) respectively. Define a mapping 
\( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then 
\( f \) is an IF\( \psi \)-continuous but not an IF-continuous 
mapping since \( A = \langle y, (0.6, 0.5), (0.4, 0.5) \rangle \) is 
an IFCS in \( Y \) but \( f^{-1}(A) = \langle x, (0.6, 0.5), (0.4, 0.5) \rangle \) is not IFPCS in \( X \).

3.26. Theorem

A mapping \( f : X \to Y \) is an IF\( \psi \)-continuous if and 
only if the inverse image of each IFOS in \( Y \) is an IF\( \psi \) 
OS in \( X \).

3.26.1. Proof.

Let \( A \) be an IFOS in \( Y \). This implies \( A^C \) is an IFCS in \( Y \). 
Since \( f \) is an IF\( \psi \)-continuous, \( f^{-1}(A^C) \) is IF\( \psi \) CS 
in \( X \). Since \( f^{-1}(A^C) = (f^{-1}(A))^C \), \( f^{-1}(A) \) is an 
IF\( \psi \) OS in \( X \).

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