Using Conditional Measurements to Combat Decoherence

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With the help of some remarkable examples, it is shown that conditional measurements performed on two-level atoms just after they have interacted with a resonant cavity field mode are able to recover the coherence of number-state superpositions, which is lost due to dissipation.

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I. INTRODUCTION: DECOHERENCE OF NON-CLASSICAL STATES

The phenomenon of decoherence [1] is a fundamental aspect of the dynamics of open quantum systems, since it rapidly destroys the characteristic feature of non-classical states of a quantum “object”, the so-called quantum coherence between the components of a superposition state, leading to the corresponding classical mixture. Recently, decoherence has also acquired a great applied importance, because it determines the feasibility of quantum information storage, encoding (encrypting) and computing [2]. Many proposals have been made in the last years to combat the irreversibility of decoherence processes in quantum computing. One consists of the filtering out of the part of the ensemble which has not decohered [3]. The other proposal is encoding the state (qubit) by means of the part of the ensemble which has not decohered [4]. The ability to approximately restore any mixture to any pure state is the advantage of our post-selection CM approach, compared to the non-selective measurement (tracing) approach.

II. RECOVERING COHERENCE BY OPTIMAL CONDITIONAL MEASUREMENTS

Let us consider a single-mode cavity in which the quantized electromagnetic field is initially prepared in a finite superposition of Fock states,

\[ |\phi(0)\rangle = \sum_{n=0}^{N} c_n |n\rangle. \]  

We assume that the cavity field is in interaction with a (zero-temperature, for the sake of simplicity) heat bath, to take into account the effect of dissipation. The resulting master equation describing such coupling, in the interaction picture, is

\[ \dot{\rho}_F = \gamma (2\hat{a}\rho_F \hat{a}^\dagger - \hat{a}^\dagger \hat{a} \rho_F - w_P \hat{a}^\dagger \hat{a} \rho_F), \]  

where \( w_F(t) \) is the density operator of the field mode, \( \hat{a} \) and \( \hat{a}^\dagger \) are the annihilation and creation operators of the field, and \( \gamma \) is the damping constant of the cavity.

The solution of Eq. (2) after dissipation over time \( t > 0 \) is given by

\[ w_{n,m}(t) = \sum_{k=0}^{\infty} w_{n+k,m+k}(0) \sqrt{\binom{n+k}{m+k}(e^{-2\gamma t})^n (1 - e^{-2\gamma t})^k} \times \sqrt{\binom{m+k}{m}(e^{-2\gamma t})^m (1 - e^{-2\gamma t})^k}, \]  

where \( w_{n,m}(t) \) is the density operator of the field mode, \( \hat{a} \) and \( \hat{a}^\dagger \) are the annihilation and creation operators of the field, and \( \gamma \) is the damping constant of the cavity.
written here in Fock basis, \( w_{n,m}(t) = \langle n|w_{r}(t)|m \rangle \).

In order to recover the original state of the field we propose to apply an optimized CM (or a sequence thereof) to the cavity: Using a classical field we prepare a two-level atom in a chosen superposition \([8,10]\)

\[
|\psi^{(i)}\rangle = \alpha^{(i)}|a\rangle + \beta^{(i)}|b\rangle
\]

of its ground \(|b\rangle\) and excited \(|a\rangle\) states, and let it interact with the field for a time \(\tau\) by sending it through the cavity with controlled speed. The field-atom interaction is adequately described by the resonant Jaynes-Cummings (JC) model \([11]\). We assume the field-atom interaction time \(\tau\) to be much shorter than the cavity lifetime, \(\gamma\tau \ll 1\), so that we are allowed to neglect dissipation during each CM. Upon exiting the cavity the atom is conditionally measured, using a second classical field, to be in a state

\[
|\psi^{(f)}\rangle = \alpha^{(f)}|a\rangle + \beta^{(f)}|b\rangle
\]

which is in general different from the initial atomic state \(|\psi^{(i)}\rangle\). This means that we post-select, using the same setup as in Ref. \([8]\), the atomic superposition state \((5)\) which is correlated to a cavity field state that is as close as possible to the original state \((4)\).

The effect of the applied CM on the cavity field is then calculated as follows: Initially, at the time the atom enters the cavity, the density matrix of the field-atom system is

\[
w_{F,A}(t) = w_{F}(t) \otimes |\psi^{(i)}\rangle\langle\psi^{(i)}|.
\]

It then evolves unitarily by the JC interaction of duration \(\tau\) into

\[
w_{F,A}(\bar{t} + \tau) = \bar{U}(\tau)w_{F,A}(\bar{t})\bar{U}^\dagger(\tau),
\]

where \(\bar{U}(\tau)\) is the (interaction picture) time evolution operator

\[
\bar{U}(\tau)|n\rangle|a\rangle = C_n|n\rangle|a\rangle - iS_n|n+1\rangle|b\rangle
\]

\[
\bar{U}(\tau)|n\rangle|b\rangle = C_{n-1}|n\rangle|b\rangle - iS_{n-1}|n-1\rangle|a\rangle,
\]

where \(C_n = \cos(g\tau\sqrt{n+1})\) and \(S_n = \sin(g\tau\sqrt{n+1})\), and \(g\) equals the field-atom coupling strength, i.e. the vacuum Rabi frequency. Finally, the conditional measurement of the atom in the state \(|\psi^{(f)}\rangle\) results in a density matrix of the field given by

\[
w_{F}(\bar{t} + \tau) = \text{Tr}_A \left[ w_{F,A}(\bar{t} + \tau)|\psi^{(f)}\rangle\langle\psi^{(f)}| \right] / P,
\]

where

\[
P = \text{Tr}_A \left[ w_{F,A}(\bar{t} + \tau)|\psi^{(f)}\rangle\langle\psi^{(f)}| \right]
\]

is the success probability of the CM.

In order to approximately recover the initial state of the field, we use the dependence of \(w_{F}(\bar{t} + \tau)\) on the initial and final atomic states and the field-atom interaction time, choosing optimal parameters \(\alpha^{(i)}, \beta^{(i)}, \alpha^{(f)}, \beta^{(f)}\) and \(\tau\) such that the relation

\[
w_{F}(\bar{t} + \tau) \approx w_{F}(0)
\]

is satisfied. These optimal CM parameters are found by minimizing the cost function \([8]\)

\[
G = \frac{d(w_{F}(\bar{t} + \tau), w_{F}(0))}{P},
\]

where \(d\) is a distance function between two density operators, defined as

\[
d(w_{F}^{(1)}, w_{F}^{(2)}) = \sqrt{\sum_{nm}(w_{nm}^{(1)} - w_{nm}^{(2)})^2},
\]

\(P\) is the CM success probability \([11]\), and the tunable exponent \(r > 0\) determines the relative importance of the two factors in \(G\). If this CM does not bring us as close to the original state as our experimental accuracy permits, we can repeat the process over and over again, as long as the distance to the original state keeps decreasing, while the CM success probability remains high. The specific form of the atomic states \((4)\) and \((5)\) is chosen by minimizing Eq. \((12)\) at each step. We emphasize that the application of each CM may introduce widening of the photon-number distribution by one photon, and yet the optimized CMs are able of preventing this widening and, moreover, of restoring the field to its initial pure state. This ability amounts to an effective control of a large Fock-state subspace.

### III. EXAMPLE

In this section we illustrate our proposal with the help of an example, making use of the Husimi Q-function defined as \(Q_{w_{F}}(\beta, \beta^{*}) = \langle \beta|w_{F}|\beta^{*} \rangle\), where \(|\beta\rangle\) represents a coherent state of complex amplitude \(\beta\), to display the error-correction process.

Let us take as the original field state a symmetric superposition of the vacuum and one-photon state,

\[
|\phi(0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\pi/3}|1\rangle),
\]

whose Q-function is shown in Fig. 1(a). Dissipation by \(\gamma\bar{t} = 0.3\) makes the error matrix \(w_{F}(\bar{t}) - w_{F}(0)\) of considerable magnitude, as seen in Fig. 1(b). After the application of one CM \(|\psi^{(f)}\rangle = \cos(3\pi/8)|a\rangle + \sin(3\pi/8)e^{i\pi/4}|b\rangle\), \(g\tau = 37.95\), \(|\psi^{(f)}\rangle = \cos(3\pi/8)|a\rangle + \sin(3\pi/8)e^{i\pi/4}|b\rangle\), optimized to yield high success probability \((r = 2)\), the resulting error matrix \(w_{F}(\bar{t} + \tau) - w_{F}(0)\) is reduced by a factor of about 3, as it is visible in Fig. 1(c). The success probability of the CM is markedly high (74%). Subsequent CMs can further reduce the distance to 1/6 (one sixth) its original magnitude, with 62% success probability for the full CM sequence. Stronger
error reduction is obtainable at the expense of success probability: the application of 4 CMs optimized for \( r = 1 \) (respectively \( r = 0 \)) yields an error reduction factor of 11 (respectively 28) with sequence probability of 33% (respectively 16%).

We emphasize that the application of our approach is not limited to equal-amplitude superposition states: indeed the error correction is even better achieved for strongly unequal superpositions.

A full analysis of the distance \( d_K = d(u^K, w_F(0)) \) [Eq. (13)] between the recovered state and the original state and of the CM sequence probability \( P_{\text{seq},K} = \prod_{k=1}^{K} P_k \), with \( P_k \) given by (14), as a function of the number of CMs performed shows that the first CMs achieve a strong reduction of such a distance, whereas after a few successive CMs saturation sets on, in terms of both distance and success probability.

It is very interesting to note that the success probability in our approach is often larger (and sometimes even much larger) than the theoretical probability to find the original state in the dissipation-spoilt state, namely, \( \text{Tr}_F[w_F(0)w_F(\bar{0})] \), which we call the filtering probability.

### IV. CONCLUSIONS

We have shown the ability of simple JC-dynamics CMs as an effective means of reversing the unwanted effect of dissipation on coherent superpositions of Fock-states of a cavity field: the successive application of a small number of optimized CMs recovers the original (pure) state of the field with high success probability, which is comparable or even surpasses the filtering probability. The simplest tactics may employ a single highly-probable trial to achieve nearly-complete error correction. Surprisingly, even though we have only five control parameters at our disposal for each CM, our optimization procedure is able to effectively control the amplitudes in a large Fock-state subspace [14].

Among the practical difficulties an experimenter might encounter in the application of any CM approach [13,14], realistic atomic velocity fluctuations (of 1%) and cavity-temperature effects (below 1°C) are relatively unimportant, and especially so in the present scheme which makes use of a single or few CMs so that the effect of experimental imperfections is linear in the input errors. Only atomic detection efficiency is an experimental challenge [14]. Although the detection efficiency is currently low, it is expected to rise considerably in the coming future.

In conclusion, we would like to stress that extensions of this approach to more complex field-atom interaction Hamiltonians can make this correction procedure effective even with fewer CMs and for highly complicated states, encoding many qubits of information. However, even in its current simple form the suggested approach has undoubted merits: (i) it can yield higher success probabilities than the filtering approach; (ii) it is not limited to small errors as “high level” unitary-transformation approaches are; (iii) it corrects errors after their occurrence, with no reliance on ideal continuous monitoring of the dissipation channel and on instantaneous feedback; and (iv) it is realistic in that it can counter combined phase-amplitude errors which arise in cavity dissipation, and is of general applicability, that is, it is not restricted to specific models of dissipation.

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FIG. 1. $Q$-function description of (a) original field, with $w_F(0) = |\phi(0)\rangle\langle \phi(0)|$, $|\phi(0)\rangle = (|0\rangle + e^{i\pi/3}|1\rangle)/\sqrt{2}$; (b) error after dissipation, $w_F(\bar{t}) - w_F(0)$; (c) reduced error after 1 optimized CM [minimizing Eq. (12)], $w_F(\bar{t} + \tau) - w_F(0)$. 