É um facto empírico que países de baixo rendimento produzem bens de baixo preço, utilizando tecnologias simples, sem investimento em Investigação e Desenvolvimento. Tal constitui uma armadilha de pobreza onde não há vantagem para os trabalhadores obter formação adicional e os trabalhadores com fracas habilitações não são capazes de ingressar no sector de I&D. Apresentamos um modelo com dois sectores, o sector dos bens e da I&D, que utilizam o factor trabalho como input. Aplicando técnicas computacionais investigamos o impacto da escolaridade obrigatória. Concluímos que a imposição de um mínimo legal de escolaridade tornaria possível aos países menos desenvolvidos escaparem a armadilha de pobreza. Este resultado é também relevante para países de desenvolvimento intermédio como Portugal onde a extensão do período de escolaridade obrigatória permitiria ultrapassar o ciclo vicioso de baixa qualificação/ baixa inovação onde algumas indústrias e regiões parecem estar bloqueadas.

C'est un fait empirique que des pays dont le revenu est faible produisent des biens à bas prix, en utilisant une technologie simple, sans investissement dans la Recherche et le Développement. Ceci constitue un piège de pauvreté où il n'est pas avantageux pour les travailleurs d'obtenir une formation additionnelle et où ceux ayant un faible niveau d'études ne sont pas capables de s'insérer dans le secteur de R&D. Nous présentons un modèle avec deux secteurs, le secteur des biens et celui de R&D qui utilisent comme input le travail. Par l'application des techniques de calcul numérique, nous avons étudié l'impact de la scolarité obligatoire. Nous en avons conclu que l'imposition d'un niveau minimum de scolarité permettrait aux pays les moins développés d'échapper au piège de pauvreté. Ce résultat est également approprié aux pays dont le développement est intermédiaire comme le Portugal où l'allongement du niveau minimum de la scolarité obligatoire permettrait de surmonter le cercle vicieux de qualification faible / innovation faible, cercle dans lequel semblent enfermées quelques industries et régions.

It is an empirical fact that low-level income countries manufacture low price commodities, using simple technology, with no investment in Research and Development. This constitutes a poverty trap where it is not advantageous for workers to get further training and poorly educated/trained workers are not productive to the R&D sector. We present a model with two sectors, the Commodities sector and the R&D sector, which use labour as input. Applying computation techniques we investigate the impact of compulsory schooling on development. We conclude that the imposition of a minimum level of schooling would make it possible for poorly developed countries to escape from the poverty trap. This result is also relevant for intermediate developed countries such as Portugal where the extension of the existing minimum level of compulsory schooling would permit to overcome the vicious cycle of low-skills-low-innovation that some industries and regions are stuck.
1. Introduction

It is an empirical fact that low-level income countries manufacture low price commodities using simple technology, with little or no investment in Research and Development (R&D) with a view to producing new or improved commodities whose prices would be higher (Ashton and Green, 1996).

This situation represents a poverty trap (Azariadis and Drazen, 1990; Azariadis, 1996), which has two sides. First, the use of simple technology and lack of investment in R&D makes it unprofitable for workers to allocate a significant fraction of their time to schooling and, second, on the firms side, it is unproductive to engage workers with a low level of education in research and development activities.

Most of the studies concerning the issue of poverty traps focus mainly on underdeveloped countries (Dasgupta, 1998; Jalan and Ravallion, 2002; Hung and Makedissi, 2004), aiming at stress the lack of development derived from the absence of basic education, which prevent the performance of routine, basic productive tasks. To our knowledge, few studies on poverty traps exist addressing the issue of higher levels of education and non-routine and complex productive tasks, that is, the complementarity between human capital and R&D output (Redding, 1996). New growth theory stressed that R&D has a significant effect upon growth of developed countries (Coe and Helpman, 1997; Aghion and Howit, 1998) and that human capital is a key growth enhancement factor (Lucas, 1988). Such complementarity is empirically validated. For instance, Steedman and Wangner (1989), in a comparative study of clothing manufacture, and at a more macroeconomic level, Teixeira and Fortuna (2004), indicate that human capital is one of the most important explanations for the innovativeness of German firms and the Portuguese economy, respectively.

The existence of poverty traps in this context (of developed economies) might be seen as providing a theoretical rationalization for state intervention on schooling, namely through the imposing of a high compulsory schooling level (e.g., twelve years).

In order to formally address this question, we present a two-sector theoretical neoclassical model with a Commodities sector and an R&D sector, using labour as input, and assuming perfectly open and competitive markets. In the Commodities sector, we assume that its output is consumption goods and that there is no increase in productivity when workers increase the time they devote to schooling. For the R&D sector, we assume that its output is the invention of techniques, which allow production of higher quality consumption goods (vertical differentiation). Additionally, as this sector is more technology intensive, its productivity increases when workers increase the time spent in school. Notice that, without loss of generality, we assume that countries where goods have a higher quality level are more developed (Grossman and Helpman, 1992).

The model is derived using simulation methods, and confirms the existence of the poverty trap: there is a development threshold above which workers increase their utility by attending school and which, in turn, makes it feasible to develop an R&D sector. Below that threshold, countries do not develop, being trapped in a situation of poverty.

As the market alone is incapable of overcoming this poverty trap, public authorities should intervene (Teixeira, 1997) for instance, by providing some kind of financial incentive for R&D investment or by imposing a minimum period of schooling. We investigate here the impact of public intervention in terms of compulsory schooling whereby workers are made to dedicate a certain number of hours a week to formal schooling. The results of this investigation demonstrate that measures taken to increase education/training will allow a country to escape from the ‘poverty trap’. 
2. General assumptions of the theory
Assuming that countries are in a situation of equilibrium, we first focus our attention on a single country and then present a cross-country comparative analysis. Let us assume that in the single country under analysis, workers, technology and firms are characterised in the following way:

2.1. Characteristics of Workers
In our model workers characteristics are framed in a neoclassical paradigm.
A1. There are $M$ identical workers in perfect competition.
A2. Workers aggregated behaviour results from the maximisation of a utility function, $U(L, S, C | K)$, by a representative household, where $L$ represents the time devoted to work, $S$ represents the acquired skills level, $C$ represents the consumption level and $K$ represents the quality level of consumed goods.

We assume that the functional form of the "one period" utility function is:

$$U(L, S, C | K) = c_0 - (C + L + e \cdot S)^{\lambda}, \lambda > 1$$ (1)

This function is decreasing with $L$ and $S$ (see Assumption A4) and increasing with $C$ and $K$.

Being so, there must be in the model an incentive so that workers supply labour and acquire a non-zero skill level.

Reporting to Assumption A8 (the good has different degrees of quality), we assume for the sake of simplicity that each worker consumes only one good type. This has no loss of generality as the existence of several workers implies on the aggregated that several good types may be consumed. This assumption is standard in the literature (e.g., Grossman and Helpman, 1992).

A3. Workers have rational expectations, assuming that present period behaviour have impact in the future periods. This assumption encompasses an incentive to the worker acquiring a non-zero skill level: when a worker takes a private investment in schooling there is an increase in the probability of occurring an improvement in goods quality.

A4. Workers spend their time working, receiving a salary; studying, acquiring the degree of skills $S$; or being inactive.

The utility function is decreasing with the skills level $S$ because the worker has to spend a certain time period in school to acquire it.

A5. Workers spend their wages and profits on consumption (they are the owners of firms).

A6. Workers are "price takers".

2.2. Characteristics of the technology
A7. There is in the economy only one good.

A8. The good has different degrees of quality (vertical differentiation – following Aghion and Howitt, 1992).

A9. The production of the good uses as input undifferentiated labour: $y_i = y_i(l)$.

A10. The output quantity is independent of the output quality.

A11. For it to be possible to produce a good with quality $K$, the firm must have a certain amount of know-how. Technology does not save labour.

A12. Firm acquires know-how by investing in $R&D$. 

A13. The R&D uses as input labour, which is more productive when the workers' skills are higher: \( g_i = g(l, S) \). It is a stochastic process with \( g_i \) being the probability of quality improvement by firm \( i \).

### 2.3. Characteristics of Firms

A14. There are in the market \( N \) firms that manufacture one good.

A15. The firm that possesses the technology to produce with the highest quality is a Stakelberg leader and other firms compete \( \text{a la Cournot} \).

A16. Firms are optimisers and have rational expectations.

A17. On aggregate, firms hire \( L_Y \) labour in production and \( L_{RD} \) labour in R&D activities.

### 3. Formalisation of the theory

Here we formalise the model so that it can be computed by numerical methods in a way that we may assess alternative policy measures' impact.

We assume time is discrete in periods with unitary duration. Utility is discounted with the constant \( B \) and profits are discounted with the constant \( R \).

#### 3.1. Formalisation of workers' market side

As is standard in economic neo-classical theory, workers' behaviour results from the maximization of utility subjected to restrictions. To achieve the skill level \( S \) it is necessary to spend the time \( e \cdot S \) in school, being the consumption expenditure, \( C - P \), equal to the salary, \( b \cdot W \).

The working time is used both in production and R&D activities: on the aggregate \( L = L_Y + L_{RD} \).

The function \( G(L_{RD}, S) \) quantifies on aggregate the probability, during the present time period, of being discovered by any of the firms how to improve the quality one step ahead. Being assumed that workers know this probability and that they have rational expectations, the workers' look forward expected utility function is (see, Muth, 1961):

\[
V(K) = \max_{L,Y,C} \{ U(L, S, C | K) + \beta \cdot [ V(K+1) \cdot G + V(K) \cdot (1 - G)] \} 
\]

\( s = a \cdot C \cdot P = L \cdot W; G = G(L_{RD}, S) \)

Workers have an incentive to acquire a non-zero skill level by assuming that it increases the probability that consumed goods quality improves.

By maximizing this utility function assuming \( L_{RD} \) and \( W \) exogenous (half market partial equilibrium), one obtains the labour supply function, \( L \), and its skill level \( S \):

\[
L(K, L_{RD}, W), S(K, L_{RD}, W) 
\]

#### 3.2. Formalisation of the technology

The degree of quality improves in regular steps (quality ladder) (cf. Grossman and Helpman, 1992) being that a good with quality \( K \) has a perceived marginal utility, which is a monotonic transformation of \( \lambda^K, \lambda > 1 \) (see the assumed workers' utility function, expression (1)). This functional form is an assumption that, without generality loss, simplifies the algebraic manipulation.

Being \( K \) the highest quality that the good can be manufactured in the present period, it will be
possible for a firm, if it discovers the appropriate technology, to produce in the next period the good with quality $K+1$.

The "high standard" technology is not open to imitation, e.g., because of copyrights. Nevertheless, there are knowledge spillovers. First, imitation of the "second best" technology is possible and costless. Second, the objective of R&D investment by any firm is to improve market "high standard" technology one-step up. These assumptions are standard in the literature, (e.g., Aghion and Howitt, 1998).

The R&D output of a firm, $g(l, S)$, quantifies the probability of a step improvement over the "high standard" technology by a certain firm if it hires $l$ quantity of labour with the skill level $S$ devoted to R&D activities. The $g(l, S)$ probability is non-correlated between firms and in time. This function is both an increasing function with quantity and skills of workers. In order to may be easily aggregated we assume, without generality loss, an exponential functional form:

$$g(l, S) = 1 - \exp(-cJS)$$

Let us assume also that the production function of each firm is linear increasing with hired labour: $y(l) = d \cdot l$.

### 3.3. Formalisation of firms’ market side

In each period the "high standard" firm is able to produce the good with quality $K$, and the other "low standard" firms are able to produce the good with quality $K-1$.

It results straight from the utility function (1) that workers, as consumers, are indifferent about good quality if the price ratio between the high quality and the low quality goods is $\lambda$. Being so, we can normalise the high quality good price to 1 becoming the low quality good price $1/\lambda$. As the price of lower quality goods is smaller than its higher quality counterparts and its production uses the same amount of labour, no firm will produce goods with quality lower than $K-1$ (recall that "second best" technology is public and free). Thus, in the market there is one firm whose output quality is $K$ and there are $N-1$ firms whose output quality is $K-1$.

Considering only the present time period, being $l_y$ and $l_{RD}$ the labour hired to production and to R&D, respectively, the profits of the "high standard" firm and the other "low standard" firms are, respectively:

$$\pi(y, l_{RD} | \text{high}) = y(l_y) - (l_y + l_{RD}) \cdot W$$
$$\pi(l_y, l_{RD} | \text{low}) = y(l_y) \cdot 1/\lambda - (l_y + l_{RD}) \cdot W$$

Assuming that the "high standard" firm is a Stakelberg leader, it means that the firm will take into account the effect of hiring more labour on the "low standard" firms profit function. Moreover, assuming that firms are risk averse, the "low standard" firms’ output will be zero when its price is equal to the marginal cost. Being so, "high standard" firms will hire labour to increase wages till the "low standard" marginal cost equals to $1/\lambda$ and the "low standard" firms’ output is zero (limit strategy): $W = f(l)/\lambda$.

As in the present period the "low standard" firm’s output is zero, it will only have revenues by becoming a "high standard" firm, which only happens when the firm discovers how to produce a higher quality product. This result seems at odds with empirical evidence as in the generality of markets there are more than one firm producing goods. In the literature, this theoretical zero-output is interpreted as an short-term output level, which is less than satisfactory to cover fixed costs (e.g., Holmstrom and Tirole, 1989).

Assuming that $g$ is the probability that it does so, $G^*$ is the probability that one of other firms makes the discovery (firms are Cournot contestants), and $1 - G$ the probability that no firm makes the discovery, the "low standard" firm’s forward looking expected profit will be:
\[ I_{RD} : E \left[ \pi (K) | \text{low} \right] = \max \{ -I_{RD} \cdot W(K) + \]
\[ R \left[ E \left[ \pi (K + 1) | \text{high} \right] \cdot g + E \left[ \pi (K + 1) | \text{low} \right] \cdot G^* + E \left[ \pi (K) | \text{low} \right] \cdot (1 - G) \} \]  
\[ \text{s.t.} \quad g = g \left( I_{RD}, S \right) \]  

This expression formalises that in the next period there are three possibilities for a “low standard” firm: it becomes the “high standard” firm, it continues to be “low standard”, but the “standard” increases to \( K + 1 \), or the “standard” does not increase.

It is implicit that when an improvement is made on the product, instantaneously all “low standard” firms improve their output quality, the price of the previous high quality good decreases to \( \frac{1}{\lambda} \), and \( 1 \) becomes the new higher quality good price.

In relation to the “high standard” firm, although in the next period it may become a “low standard” firm it has no incentive to invest in R&D in the present period because if it innovates to \( K + 1 \) quality, other firms will imitate its quality:

\[ I_J : E \left[ \pi (K) | \text{high} \right] = \max \{ y(I_J) - I_J \cdot W(K) + E \left[ \pi (K + 1) | \text{high} \right] \cdot (1 - G) + E \left[ \pi (K) | \text{low} \right] \} \]  

This lack of incentives results directly from assuming that the technology that turns possible the production of the high quality goods becomes instantaneously public and free as it is discovered a better technology. Although the empirical evidence shows a gradual diffusion-imitation pattern of the former ‘new’ technology that encompasses a certain incentive to the “high standard” firm to invest in R&D, here (as Grossman and Helpman, 1992; Aghion and Howitt, 1998) we concentrate on the instantaneous diffusion-imitation as the engine of economic development in opposition to the Schumpeterian appropriateness. This zero-investment in R&D should be red as less-than-socically-acceptable level of R&D.

Given that \( g \) is non-correlated between firms and independent of time, and the “high standard” firm does not invest in R&D and all other firms are identical, the probability that a discover occurs \((G)\) and the probability that this discover is undertaken by other firm that not the reference firm \((G^*)\), comes:

\[ G = 1 - (1 - G)^{N-1} \quad \text{and} \quad G^* = 1 - (1 - G)^{N-2} \]  

The leader firm’s strategy will force wages to increase till “low standard” firms’ marginal costs of production are equal to price. As \( y(I_J) = d \cdot I_J \), the “low standard” firms output is zero when \( W \geq d / \lambda \). Notice that wages do not change with the increase in R&D activity (neither prices) because the leader firm will always impose that \( W = d / \lambda \) (this limit wage maximises the “high standard” firm’s profit) and prices equal to 1.

Thus, wages do not increase with increases in technological level, but workers improve their living standard because goods of higher quality imply higher utility.

### 4. Computation of market equilibrium

In a neoclassical theoretical non-tatonnement framework, the market is closed while economic agents compute the market equilibrium prices and quantities and then they enforce the computed equilibrium, (e.g. Arrow and Debreu, 1954). Being so, theoretically the market equilibrium is the algebraic solution of our system of non-linear equations with three endogenous aggregated economic variables \(-I_{RD}(K), S(K)\) and \(L(I)(K)\) – that are only dependent on economy’s development degree (the technological level, \( K \)).
But our model, being non-linear and recursive, is algebraically intractable. To overpass this difficulty, instead of simplifying it even more, we use a computational backward iterative procedure. Notice that this procedure does not model the way market equilibrium is computed by economic agents, being rather a computational algorithm.

In this computational iterative algorithm, we assume initially a distant future where the quality level is $K = K + 1$, much higher than what we conjecture for the present level (e.g. 1000).

Assuming these values $K$ and $K+1$ and being given firms' aggregated $L_{RD}$ level, workers set the total supply of labour and the skill level $S$ (half equilibrium analysis). Then, the leader firm announces the wage. Finally, "low standard" firms choose their optimal labour hiring in $R&D$, $L_{RD}$.

These three steps are repeated till it converges to a point $(L, S, L_{RD})$, which is the solution of our non-linear equation model for quality level $K$. Then, we compute the model backward for the quality level $K-1$, $K-2$, ..., until reaching quality level 1.

Being the solution of the model a point $(L, S, L_{RD})$ for each quality level, it is difficult to visualise how market forces act toward equilibrium. We present two "partial equilibrium" snapshots (figures 1 and 2) and two "general equilibrium" situations (figures 3 and 4) that are useful for the understanding of the dynamics of the poverty trap.

Calibrating the $R&D$ technology as $G = 1 - \exp(-0.1 S_{LRD})$, the utility function as $U(L, S, C|K) = (\frac{1}{2}W) \cdot 1.1^K \cdot \{L + 0.1 \cdot S\}^2$, and setting $\beta = 0.9$, the school attendance function becomes (workers' half market equilibrium):

\[
U(L, S, C|K) = (\frac{1}{2}W) \cdot 1.1^K \cdot \{L + 0.1 \cdot S\}^2
\]

It may be seen in the figure 1 that when firms do not recruit labour to the $R&D$ sector above a certain level, workers do not spend time at school. This minimum level is decreasing with $K$, being required a high level of $R&D$ in the low development stage for workers being motivated to attend school (if $K=10$, $L_{RD}$ should be higher than 0.75).

In firms' half market equilibrium, knowing $S$ and $L$ for each $L_{RD}$, firms adopt the strategy that maximizes their expected profits. Assuming $R = 0.9$ and $d = 1$, for different concentration scenarios the looking-forward expected profit of a "low standard" firm becomes:
Although in a situation of “negative profit” a firm will get out of the market, this negative profit is the optimal “no zero” value (a second best attainable when there are public subsidies):

Figure 3 shows that without state intervention (e.g. subsidy to the R&D sector), there will be no development in “low quality” countries. Notwithstanding, as in more concentrated situations ($N = 2$) firms’ expected profit is always higher, without state intervention this later situation is more favourable for the development of “low quality” countries than less concentrated market structures. This outcome reflects and justifies the “lei de condicionamento industrial” implemented in Portugal in the post Second World War.

As the efficient distribution of subsidies tends to be difficult to implement and to limit the entrance into the market is a questionable measure (it may induce dynamic inefficiency), an alternative policy is proposed here. Instead of a R&D subsidy, we investigate the impact of introducing (or enlarging) compulsory schooling on “low quality” countries.

Our models predicts that (see figure 4) in those “low quality” countries it is necessary a minimum of 30% of time in school to guarantee that it is profitable for firms to invest in the R&D sector:
Conclusion

Based on an economy with two sectors – the Commodities sector and the R&D sector – we built a theoretical model that shows that in low-level income countries, firms do not invest in R&D because workers have a low level of schooling and workers do not allocate any significant proportion of their time to training because the use of simple technology and the lack of investment in R&D does not require this.

This result is a poverty trap, which the market is incapable of overcoming: there is a development threshold above which workers increase their utility by attending school, which, in turn, makes it possible to sustain an R&D sector. Below that threshold, countries do not develop into high quality standards, and remain in a situation of relative poverty and of low quality standards.

Because the market cannot get over the barrier of this ‘poverty trap’, public authorities must intervene. An obvious possible focus for public policy is a subsidised R&D sector.

However, in the generality of countries efficient distribution of subsidies is not easy. Thus, an alternative policy is proposed here: public intervention through workers’ compulsory schooling.

We conclude that, the imposition by public authorities of a minimum level of schooling for workers would make it possible for less developed countries to escape from their ‘quality poverty trap’.

This result seems to be crucial for countries with intermediate levels of development, such as Portugal, where the issue is not so much of the existence of a compulsory minimum level of schooling but instead of enlargement the existing minimum in order to overcome the vicious cycle of low-skills, low-quality, low-innovation that some industries and regions seems to be stuck (Teixeira, 2004).
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