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A Capacity of OQAM/OFDM System Calculation Method Under the Effect of HPA

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Abstract. In the process of signal transmission of offset quadrature amplitude modulation/orthogonal frequency division multiplexing (OQAM/OFDM) system, high power amplifier (HPA) causes signal distortion due to the limited amplitude of high power signal, which lead to a decrease of capacity of the system. To calculate the system capacity under the effect of HPA, according to the Bussagang theorem, the equivalent signal-to-noise ratio (ESNR) of the signal under HPA is derived. The validity of the derivation is verified by the bit error rate (BER) performance of the system. The capacity mean and capacity variance with the effect of HPA of the system are obtained according to the derivation in the Rayleigh fading channel. This method provides reference for performance analysis and peak to average power ratio (PAPR) reduction of OQAM/OFDM system.

1. Introduction

Orthogonal frequency division multiplexing (OFDM) [1] is a kind of multi-carrier modulation (MCM). It transfers high-speed data to multiple orthogonal subcarriers to reduce the symbol rate on each subcarrier. Therefore, OFDM technology has strong resistance to multipath fading [2] compared with single carrier modulation technology. However, in order to resist inter symbol interference (ISI), OFDM technology needs to introduce cyclic prefix (CP) leads to reducing the spectrum efficiency. In addition, the OFDM technology uses a rectangular window filter to reshaping, so the external radiation is more serious and makes OFDM system very sensitive to carrier frequency offset and Doppler frequency offset, which is easy to produce inter carrier interference (ICI).

The lack of OFDM technology is caused by its own inherent properties. Even if some measures can be taken to improve its performance a little, it cannot solve the problem fundamentally. The introduction of OQAM/OFDM brings MCM technology into a new direction. OQAM/OFDM technology is also a kind of MCM. Due to the introduction of a good time-frequency focusing filter, OQAM/OFDM system has better ability to resist ICI and ISI without CP. It also improved the spectrum efficiency [3]. The cost is to relax the orthogonality between symbols, so that they only satisfy the orthogonal condition of real number field. This characteristic brings virtual partial interference between adjacent symbols and subcarriers, which will have a great impact on the system synchronization, channel estimation and equalization. [4] In addition, as an MCM technology, high peak-to-average power ratio (PAPR) is also one of the inherent disadvantages of OQAM/OFDM. The higher PAPR of signal, the more clipping effect of HPA, the more performance reduction of the system. [5]
2. System model of OQAM/OFDM

In the OFDM system, the sub carriers transfer complex symbols. OQAM/OFDM technology is to dismantle the real and virtual parts of the complex symbols transmitted into two real numbers in OFDM system, and then offset half a symbol period after each other and transmit it in the real number domain. Considering CP, OQAM/OFDM system has higher efficiency than OFDM system. Moreover, the implementation of OQAM/OFDM system is similar to that of OFDM system, and it can be implemented based on fast Fourier transformation/inverse fast Fourier transformation (FFT/IFFT).

The time-domain baseband form of OQAM/OFDM signal is:

$$ s(t) = \sum_{n=-\infty}^{\infty} \sum_{m=0}^{M-1} a_{m,n} g_{m,n}(t) $$

(1)

Where, $a_{m,n}$ is the real-valued symbol at the time-frequency position $(m,n)$, $g_{m,n}(t)$ is the base function generated by the prototype filter:

$$ g_{m,n}(t) = g\left(t-n\tau_0\right)e^{j2\pi mF_0t}e^{j\phi_{m,n}} $$

(2)

Where $\tau_0$ is the time interval between symbols, $F_0$ is the interval between subcarriers, and:

$$ \tau\Delta = 1/2 $$

(3)

Where $\phi_{m,n}$ is the phase of a symbol, we adopt:

$$ \phi_{m,n} = \frac{\pi}{2}(m+n) $$

(4)

After sending the signal through the wireless channel, it is received by the receiving end. The received signal is:

$$ r(t) = h(t,\tau) \otimes s(t) + \eta(t) = \int_{0}^{\Delta} h(t,\tau)s(t-\tau)d\tau + \eta(t) $$

(5)

Where $h(t,\tau)$ is the impulse response, $\eta(t)$ is the white Gaussian noise, $\otimes$ is convolution, $\Delta$ is the channel maximum time delay. We assume that $g_{m,n}(t)$ satisfies the orthogonal condition:

$$ \langle g_{m,n}g_{p,q} \rangle = \Re\left\{ \int_{-\infty}^{\infty} g_{m,n}(t)g^{*}_{p,q}(t)dt \right\} = \delta_{m,p}\delta_{n,q} $$

(6)

Where $\delta$ is the Kronecker function:

$$ \delta_{i,j} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} $$

(7)

Where $\Re\langle \cdot \rangle$ is the real inner product operation, $\Re\langle \cdot \rangle$ is the real option. We assume that $\tau_0 \gg \Delta$, the expression of the signal at receiver is:

$$ r(t) = \sum_{n=-\infty}^{\infty} \sum_{m=0}^{M-1} a_{m,n}e^{j2\pi m\tau_0}e^{j2\pi mF_0t}g(t-n\tau_0) \times \int_{0}^{\Delta} h(t,\tau)e^{-j2\pi mF_0\tau}d\tau + \eta(t) $$

$$ = \sum_{n=-\infty}^{\infty} \sum_{m=0}^{M-1} a_{m,n}g_{m,n}(t)H_{m,n} + \eta(t) $$

(8)

Where $H_{m,n}$ is the channel frequency response at $(m,n)$. The demodulation symbols are:
\[
\hat{a}_{m',n'} = \left\langle r, g_{m',n'} \right\rangle = 
\sum_{m=-M}^{M-1} \sum_{n=-M}^{M-1} a_{m,n} H_{m,n} \left\langle g_{m,n}, g_{m',n'} \right\rangle + 
\left\langle \eta(t), g_{m',n'} \right\rangle
\]  
(9)

If the estimated value of frequency response is equal to the true value, the zero forcing equalization can be used to demodulate the symbols correctly:

\[
\hat{a}_{m',n'} = \hat{H}_{m',n'} a_{m',n'} + 
\sum_{m,m',n \neq m',n'} a_{m,n} \hat{H}_{m,n} \left\langle g_{m,n}, g_{m',n'} \right\rangle + 
\left\langle \eta(t), g_{m',n'} \right\rangle
\]  
(10)

Where \( I \) is the inherent virtual part interference of the system.

The system block diagram of the OQAM/OFDM system is shown in Figure 1.

3. Influence of HPA to OQAM/OFDM system

During the transmission of OQAM/OFDM signal, if the amplitude of the signal exceeds the linear amplification range of HPA, the signal will get clipped by HPA. Typical HPA models \([6]\) include soft limiter (SL), solid state power amplifier (SSPA), cubic polynomial (CP) and traveling-wave tube (TWT). The SL model is a widely used HPA model in literature, so in this paper adopts this model. Its amplification characteristic is:

\[
\Psi(x(t)) = \begin{cases} 
  x(t), & |x| \leq A \\
  Ae^{j\phi(t)}, & |x| > A
\end{cases}
\]  
(11)

Where \( \Psi(x(t)) \) is the output signal through HPA. \( x(t) \) is the time domain waveform of the input signal, \( A \) is the threshold of HPA, \( \phi(t) \) is the signal phase at time \( t \). It can be seen that the SL model of HPA only changes the amplitude of signal, and does not change the phase of signal.

Figure 1. OQAM/OFDM system model
3.1. Clipping ratio

The clipping ratio (CR) is a parameter that describes the magnitude of signal limiting, and it depends on the limiting threshold. The smaller the CR is, the lower the limiting threshold is, the higher the degree of signal distortion. We set \( \gamma \) represents CR, \( \sigma \) is the root mean square (RMS) of signal \( s(n) \), then \( \gamma \) can be expressed as:

\[
\gamma = \frac{A}{\sigma}
\]  

(12)

Where

\[
\sigma = \sqrt{E[(s(n))^2]}
\]  

(13)

In the actual communication system, to ensure the performance of a certain communication system, \( \gamma \) cannot be set too low, the appropriate \( \gamma \) can be selected through the size of the error vector amplitude (error vector magnitude, EVM)

3.2. EVM

EVM can be used to describe the size of the noise introduced. When the output signal is completely undistorted, the EVM value is 0. The definition of EVM is:

\[
\text{EVM} = \sqrt{\frac{\sum_{i} |s^i(n) - s(n)|^2}{\sum_{i} |s(n)|^2}} = \frac{\|s^i - s\|_2}{\|s\|_2}
\]

(14)

Where \( s^i(n) \) and \( s(n) \) denote the \( n \)th sampling points of the output and input signals respectively, and \( \|\cdot\|_2 \) is the 2-norm.

In practical communication systems, distortion is usually allowed to a certain extent. The 802.11a/g standard stipulates that the transmitter EVM below -25dB supports 54Mbps, and the transmitter EVM below -22dB supports 48Mbps [7]. On the other hand, the EVM value of the output signal directly affects the bit error rate (BER) performance of the system [8]. The lower the limiting threshold, the greater the impact on the signal, the higher the signal EVM is. We set OQAM/OFDM signals of 2048 subcarriers, 40 symbol numbers and 4 IOTA filters are selected to calculate the curve of EVM with the clipping rate as figure 2:

![Figure 2. EVM with \( \gamma \) about OQAM/OFDM signal](image-url)
It can be seen that with the increase of $\gamma$, the signal EVM decreases. When $\gamma$ is 0.8, 1.2 and 1.6, the signal EVM is 19.6%, 7.5% and 3.4%, respectively.

3.3. The influence of HPA on SNR

\[
\hat{x}_n = \begin{cases} 
  x_n, & |x_n| \leq A \\
  A e^{j\phi_n}, & |x_n| > A
\end{cases}
\]  

(15)

Where $\phi_n$ is the phase of $x_n$, $\hat{x}_n$ is the output signal, its power is:

\[
\sigma^2_s = E\left(|x_n|^2\right) = \int_0^A x^2 f_{|x|}(x) \, dx + \int_A^{\infty} A^2 f_{|x|}(x) \, dx = \sigma_s^2 \left(1 - e^{-\gamma^2}\right)
\]  

(16)

According to the Bussgang theorem [9], if $x_n$ and $\hat{x}_n$ are Gauss signals, $\hat{x}_n$ can be expressed as a scaling of the proportional factor $\alpha$ plus a nonlinear distortion signal that is not related to the input signal $x_n$:

\[
\hat{x}_n = \alpha x_n + d_n
\]  

(17)

Another equivalent form of the Bussgang theorem is that the intercorrelation function $R_{xx}$ of the amplifier number input signal $x_n$ and the output signal $\hat{x}_n$ is the autocorrelation function of the attenuated input signal, in which the attenuation factor is $\alpha$, then $\alpha$ can be expressed as:

\[
\alpha = \frac{R_{\hat{x}x}}{R_{xx}} = E\left(\hat{x}_n x_n^*\right) / E\left(|x_n|^2\right) = \frac{1}{\sigma_s^2} \int_0^A x^2 f_{|x|}(x) \, dx + \frac{1}{\sigma_s^2} \int_A^{\infty} A x f_{|x|}(x) \, dx = 1 - e^{-\gamma^2} + \frac{1}{2} \pi \gamma \text{erfc}(\gamma)
\]  

(18)

When the number of non-zero values in the nonlinear distortion signal $d_n$ is large enough, it is considered that $d_n$ of the power is evenly distributed in the signal bandwidth, and the power is:

\[
\sigma^2_d = E\left(|d_n|^2\right) - |\alpha|^2 E\left(|x_n|^2\right) = \left(1 - e^{-\gamma^2}\right) \sigma_s^2 - |\alpha|^2 \sigma_s^2 = \left(1 - e^{-\gamma^2} - |\alpha|^2\right) \sigma_s^2
\]  

(19)

When the signal $\hat{x}_n$ passes through the additive Gauss white noise (AWGN) channel, the received signal is:

\[
r_n = \hat{x}_n + \omega_n = \alpha x_n + d_n + \omega_n
\]  

(20)

Where $\omega_n$ is AWGN, and the variance is $\sigma^2_\omega$. Assuming that the receiver is fully synchronous, the received signal's $\text{SNR}'$ (ESNR) is:

\[
\text{SNR}' = \frac{\alpha^2 \sigma_s^2}{\sigma_\omega^2 + \sigma_d^2}
\]  

(21)

And there is:

\[
\text{SNR} = \frac{\sigma_s^2}{\sigma_\omega^2}
\]  

(22)

Take (21) into (22):
\[ SNR' = |\alpha|^2 \left[ SNR^{-1} + \left(1 - e^{-\gamma^2} - |\alpha|^2 \right) \right]^{-1} \]  

(23)

According (18), \( \alpha \) expressed by \( \gamma \):

\[ SNR' = \frac{1 - e^{-\gamma^2} + \frac{1}{2} \sqrt{\pi \gamma \text{erfc}(\gamma)}}{\left[ SNR^{-1} + 1 - e^{-\gamma^2} - \left(1 - e^{-\gamma^2} + \frac{1}{2} \sqrt{\pi \gamma \text{erfc}(\gamma)} \right) \right]} \]  

(24)

Where \( \text{erfc}(\cdot) \) is the Gauss error function. \( SNR' \) in different \( \gamma \) is as figure 3.

Figure 3. \( SNR' \) in different \( \gamma \)

It can be seen from Figure 3 that the clipping affects the SNR of the signal, reducing the SNR of the signal, and the smaller the \( \gamma \), the greater the SNR decrease. When the \( \gamma \) is 1.6, 1.2 and 0.8, as \( SNR \) is 10, \( SNR' \) is 8, 5.54 and 3.39, respectively. As \( SNR \) is 20, \( SNR' \) is 14.6, 9 and 5.2.

4. BER performance

This section will simulate the derivation and the actual limiting effect under the Rayleigh multipath fading channel, and verify the accuracy of the derivation.

4.1. Simulation parameters

We assume that the channel parameters are known, the sub carrier number \( M=2048 \), symbol number \( n=80 \) and IOTA filter are simulated. Filter length \( L=4\tau_0 \), number of IFFT \( F_s=2048 \), forward error correction (FEC) code, and the rate is 0.5, the sampling frequency is 9.14MHz, the channel model is the Rayleigh multipath fading model, the multipath number is 6, the time delay vector of each diameter is [-3, 0, 2, 4, 7, 11] \( \mu s \), and the signal gain vector is [-6, -7, -22, -16, -20]dB. The limiting rate is 1.6, 1.2 and 0.8, \( SNR' \) is calculated by the upper section calculation method.

4.2. Simulation results

According to the parameters of the section 4.1, the BER of the system is as figure 4.
5. Capacity calculation of OQAM/OFDM system

5.1. Calculation formula of capacity

The capacity of the OQAM/OFDM system[10] can be expressed as:

\[ C_{m,n} = \log_2 \left( 1 + \text{SNR} |H(m,n)|^2 \right) \]  

(25)

Where \( H(m,n) \) is the channel frequency response of the \( n^{th} \) symbol on the \( m^{th} \) sub carrier.

The mean of the capacity of the system can be expressed as:

\[
\mu_c = E \left[ \frac{1}{N} \sum_{n=0}^{N-1} C_{m,n} \right] 
= E \left[ \frac{1}{N} \sum_{n=0}^{N-1} \log(1 + \text{SNR} |H(m,n)|^2) \right] 
= \frac{1}{N} \sum_{n=0}^{N-1} \int_{-\infty}^{\infty} \log(1 + \text{SNR} \cdot x^2) f_{\hat{p}(m,n)}(x) \, dx
\]

(26)

The variance of capacity can be expressed as:

\[
\sigma_c^2 = E \left[ \left( \frac{1}{N} \sum_{n=0}^{N-1} C_{m,n} \right)^2 \right] - \left[ E \left( \frac{1}{N} \sum_{n=0}^{N-1} C_{m,n} \right) \right]^2 
= \frac{1}{N^2} \sum_{n=0}^{N-1} E(C_{m,n}^2) + \frac{1}{N^2} \sum_{n=0}^{N-1} \sum_{n'=0}^{N-1} E(C_{m,n} C_{m,n'}) - \mu_c^2 
= \frac{1}{N^2} \sum_{n=0}^{N-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \log(1 + \text{SNR} \cdot x_1^2) f_{\hat{p}(m,n)}(x_1) \, dx_1 dx_2
\]

(27)
Where $f_{H_{m,n}}(x)$ and $f_{H_{m,n}}(y|x_1,x_2)$ are Rayleigh channel probability density function and joint probability density function respectively [11]. Its expression is:

$$
\begin{align*}
    f_{H_{m,n}}(x) &= \frac{1}{\pi \sigma_1 \sigma_2 (1-\rho_{xx}^2 - \rho_{yy}^2) \prod_{i=1}^{L_i} x_i \prod_{j=1}^{L_j} y_j} \\
    &\times \exp \left[ \frac{-1}{\prod_{i=1}^{L_i} x_i \prod_{j=1}^{L_j} y_j} \left( (1-\rho_{xx})^{\frac{L_i}{2}} \frac{\sum_{i=1}^{L_i} x_i^2}{\sigma_1^2} + (1-\rho_{yy})^{\frac{L_j}{2}} \frac{\sum_{j=1}^{L_j} y_j^2}{\sigma_2^2} \right) \right] \\
    &\times \prod_{i=1}^{L_i} \prod_{j=1}^{L_j} \left[ \frac{\rho_{xx}^2 + \rho_{yy}^2}{\prod_{i=1}^{L_i} x_i \prod_{j=1}^{L_j} y_j} \right]^{\frac{L_i}{2}} \\
    &\times \prod_{i=1}^{L_i} \prod_{j=1}^{L_j} \left[ \frac{1}{\prod_{i=1}^{L_i} x_i \prod_{j=1}^{L_j} y_j} \right]^{\frac{L_i}{2}} (1-\rho_{xx}^2 - \rho_{yy}^2) \\
    &\times \exp \left[ \frac{-1}{\prod_{i=1}^{L_i} x_i \prod_{j=1}^{L_j} y_j} \left( (1-\rho_{xx})^{\frac{L_i}{2}} \frac{\sum_{i=1}^{L_i} x_i^2}{\sigma_1^2} + (1-\rho_{yy})^{\frac{L_j}{2}} \frac{\sum_{j=1}^{L_j} y_j^2}{\sigma_2^2} \right) \right], x_1,x_2 \geq 0
\end{align*}
$$

Among them, $I_0(\cdot)$ is a zero order Bessel function modified by the first class, and $\rho_{xx}, \rho_{yy}$ represents the correlation coefficient of the real and imaginary Gauss random variables in the two correlated channels, and $\rho_{xy}, \rho_{yx}$ represents the correlation coefficient between the real and the imaginary numbers of the Gauss random variables.

The $\text{SNR}$ in the (26) sum (27) is replaced by $\text{SNR}'$. The capacity mean and capacity variance of the system after limiting are obtained.

5.2. Simulation results and analysis

According to the calculation formula of the mean and capacity variance of the system capacity in the 5.1 section Rayleigh fading channel and the signal to noise ratio calculated by the 3.3 section, the mean and volume variance of the system capacity are calculated.

![Figure 5](image-url)

**Figure 5.** Capacity mean of OQAM/OFDM system with the limiting rate under Rayleigh fading channel
In Figure 5, we can see that the mean of OQAM/OFDM system capacity decreases with the decrease of clipping rate. In the original SNR=5, the mean value of system capacity in $\gamma = 1.6$, 1.2 and 0.8 decreased by 7.3%, 18.9% and 35.3%, respectively; while the original SNR=10, $\gamma = 1.6$, 1.2 and 0.8, reduced the system capacity averages by 8%, 20.8% and 36.5%, respectively.

In Figure 6, it can be seen that when the original signal SNR is lower (lower than 6), the system capacity variance increases with the increase of SNR, and decreases with the decrease of the limiting rate.

6. Conclusion
Aiming at the limiting effect of HPA on signal, a method of calculating the capacity of OQAM/OFDM system under Rayleigh channel is proposed. First, according to the Bussagang theorem, we deduce the change of signal ESNR after clipping. The effectiveness of the derivation is verified by the performance of system BER. Then, according to the SNR after limiting, we calculate the capacity mean and capacity variance of system capacity under Rayleigh multipath channel by using the capacity formula of OQAM/OFDM system. The simulation results show that the average capacity of the system decreases with the decrease of the limiting rate, and the capacity variance of the system is flat with the increase of the signal to noise ratio.

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