Multi-objective Optimization of Pneumatic Mixing Systems for Anaerobic Digesters: A Hybrid Technique of Statistical Modeling and Numerical Simulations

Mahmood Mahmoodi-Eshkaftaki1 · Hossein Rahmanian-Koushkaki1

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Abstract
Development a suitable mixing system for biogas plants is one of the most important factors to degrade organic matters in digesters. Therefore, pneumatic and mechanical mixing systems were integrated with their optimum conditions to produce a suitable strategy for mixing slurry. Pneumatic mixing parameters were investigated using computational fluid dynamics and image processing techniques. The mixing characteristics and biogas compounds were modeled according to biogas injection parameters, and integrated into the desirability function to determine the optimum conditions. The models indicated that for lower injection repetition in different injection positions, a higher mass flow rate was needed to improve the mixing characteristics and CH4 content. Optimization of these parameters showed that the best amounts of injection mass rate, injection pressure and injection repetition were 0.02 g/s, 2.84 bar and 3 times, respectively, and their optimum ranges were 0.015–0.028 g/s, 1.5–4.7 bar and 1–4 times. However, the best injection was injection through mixer and injection in floor of digester for 2 or 3 times in 24 h. From the optimum ranges, some amounts were investigated in the digesters using the computational fluid dynamics, and the determined mixing characteristics were in agreement with findings of the image processing method.

Graphical Abstract

Keywords Biogas injection parameters · Computational fluid dynamics · Image processing technique · Optimization · Pneumatic mixer

Extended author information available on the last page of the article
List of Symbols

| Symbol | Description                  |
|--------|------------------------------|
| AR²    | Adjusted R²                  |
| AP     | Adequate precision           |
| CFD    | Computational fluid dynamics |
| Colc   | Color rating                 |
| CD     | Drag coefficient             |
| d      | Diameter of bubbles or droplets |
| df     | Degree of freedom            |
| DPM    | Discrete particle method     |
| f      | Drag function                |
| G      | Generation                   |
| g      | Represents gravity           |
| LF     | Lack of fit                  |
| mi     | Mass transfer                |
| P      | Static pressure shared by all phases |
| PR²    | Predicted R²                 |
| R²     | Coefficient of determination |
| ri     | Response’s relative importance |
| R      | Interaction force            |
| Re     | Reynolds number              |
| RSM    | Response surface methodology |
| SE     | Standard error               |
| tmix   | Mixing time                  |
| TFM    | Two-fluid model             |
| x      | Independent variable         |
| y      | Dependent variable           |
| a      | Volume fraction              |
| γ      | Shear rate                   |
| μ      | Viscosity                    |
| v      | Velocity                     |
| σ      | Turbulent Prandtl numbers    |
| στ     | Stress–strain tensor         |
| τp     | Particulate relaxation time  |
| subscript | Description |
| k      | Turbulent kinetic energy     |
| m      | Mixture                      |
| p      | Phase p                      |
| pq     | Phase p to phase q           |
| q      | Phase q                      |
| qp     | Phase q to phase p           |
| t      | Turbulent                    |
| ε      | Dissipation rate of turbulent kinetic energy |

Statement of Novelty

Nowadays, the researchers in bio-system engineering focus on development of biogas plants to enrich biogas production and increase their performance. One of the most important factors for this goal is the mixing system. The novelty of this article is to optimize the pneumatic mixing process in several constraint stages based on optimal values of injection mass rate, injection pressure, injection repetition and injection position, in which CFD simulations and calorimetric method base image processing techniques are utilized to study the mixing process.

Introduction

The development of biogas plants to enrich CH₄ production and increase their energy performance is very important. One of the most important factors for this goal is the mixing system. Mixing the slurries in the digesters helps to degrade organic matters and increase biogas output [1]. The purpose of the mixing process is to distribute the nutrients in the digester uniformly [2], to shape a suitable suspension of liquid and solid part [3], to avoid sedimentation of particles [1, 3], to ensure uniform heat distribution [2], to prevent foam formation and to enable gas lift from the digestion substrate [2]. There are several mixer technologies commercially known as mechanical, hydraulic or pneumatic mixing systems [4]. Mechanical mixing is the most common mixing type being used today, and uses different types of impellers and agitators. As it has been revealed, the mechanical mixers have the highest power efficiency per volume unit mixed while the pneumatic mixers have the lowest [4]. The common pneumatic mixing systems in the biogas plants work with biogas recirculation through the digester. The use of these two mixer types together leads to the benefits of both systems.

Different studies have been done on the mechanical mixers to determine the suitable impeller type, impeller position in the digester and impeller dimension in order to increase the plant performance energetically [4, 5], and improve the biodegradation of organic matters [5, 6]. As reported in the literature, integrating the mechanical and pneumatic mixing systems can increase the efficiency of biogas digesters in low energy consumption [7]. However, the optimization of pneumatic mixers has not been investigated, especially in co-using with mechanical mixers. The pneumatic mixers in biogas plants can influence of different parameters such as biogas injection mass flow rate, injection pressure, injection position in the digester, injection repetition, etc. A mixing process in optimum conditions improves the biodegradation of organic matters in the digesters, decreases mixing time (tmix) and purifies the biogas. Determination of the injection parameters during the digestion is very difficult, computed simulation models of computational fluid dynamics (CFD) are appropriate approaches to determine the injection parameters [8]. The mixing characteristics of the slurry in the digesters have been studied using CFD analyses [9–11]. According to the CFD analyses, the mixing process is affected by digester size [12], digester type [9], mixer type [10], etc. Two mixer types, pneumatic and mechanical mixers, were studied by Wu [10] using a CFD-based model. The performance of pneumatic mixing system was evaluated in a digester with four gas mixing designs. Eulerian multiphase flow approach was considered during the modeling process, and the indices of average velocity and a uniformity index.
for velocity were selected to determine the best design of the gas mixer. Two different approaches of CFD modeling are available for predicting multiphase flow in the digesters, namely Eulerian-Eulerian (two-fluid model or TFM) and Eulerian–Lagrangian (discrete particle method or DPM). Because of TFM modeling technique requires fewer computational resources compared with the DPM method, it is selected by the researchers for simulation of multiphase flow in the digesters [9, 10].

Biodegradation improvement of the organic matters in the biogas digester leading to decrease impurities such as H₂S, H₂O and CO₂ from the biogas and increase CH₄ [13–15]. The pneumatic mixing systems using biogas recirculation not only helping to mix the slurry but also can purify the biogas biologically [7, 16, 17]. This shows the pneumatic mixing process can improve the biodegradation of organic matters in the biogas digesters. It remains to consider the optimum conditions of biogas injection for the pneumatic mixers. An integrated linear and non-linear optimization method such as response surface methodology (RSM) shows good accuracy for similar problems [18]. However, to optimize multiple responses, the RSM alone is not very accurate, especially when optimum ranges of parameters should be determined. Thus, a desirability function can be integrated by RSM to determine the optimum values and ranges. Therefore, the goals of the present work are to (i) utilize the CFD simulations and calorimetric method base image processing techniques for studying the mixing process, and (ii) employ a hybrid optimization technique, integrating RSM and desirability analysis, to determine the optimum conditions of biogas injection in the cases of optimum absolute values and optimum ranges.

Materials and Methods

Biogas Setup

The diameter, height, and wall thickness of the digesters were 0.57, 0.9 and 0.01 m, respectively, and their volumes were 0.22 m³. A mechanical mixer with four blade marine impeller stirred material in each digester. Impeller dimensions including impeller diameter, blade thickness, blade length and blade width were 0.32, 0.004, 0.1 and 0.04 m, respectively. These stirred digesters were constructed according to rules stated in Mahmoodi-Eshkaftaki and Ebrahimi [5, 15]. Three similar stirred digesters were constructed for the experiments, and just their biogas injection position was different. Figure 1 shows a digester with the pneumatic mixing process via biogas recirculation process from the bottom of digester. For biogas recirculation, a compression system equipped by a 0.1 kW compressor with delivery pressure up to 3000 kPa was used to compress the biogas and recirculate it into the digester. The compressed biogas was injected into the digester through one of these three positions, floor of digester, through the mixer, and lateral wall at a height of 10 cm from the floor. Temperature of the digesters was kept at 37 °C using a solar heating system.

![Fig. 1 Schematic illustration of the anaerobic digester with both mechanical and pneumatic mixing systems](image-url)
Slurry Preparation

A mixture of cow manure, municipal waste and fruit waste diluted with tap water was prepared to be used in the anaerobic digestion experiments. According to our previous work in laboratory scale, the optimum anaerobic digestion was acquired with the amounts of 42, 56 and 2% of cow manure, municipal waste and kitchen waste pretreated with 3 and 1% of NaOH and H$_2$O$_2$, respectively, in which CH$_4$ production reached to the most amount [15]. To produce the slurry for each state, these amounts of the substrates were grinded using a force mill and mixer part, diluted with 40% water, enriched with 20% inoculum (from the biogas plant of Jahrom university), and pretreated with 3 and 1% of NaOH and H$_2$O$_2$, respectively. The headspace of the digesters was purged using N$_2$. The mixing process was done using both pneumatic and mechanical mixers. Different parameters of the pneumatic mixing system would be optimized in this research to improve the mixing process and enrich CH$_4$. For this purpose, an optimal experimental design based on RSM was used.

Experimental Design

To determine the best strategy for the pneumatic mixing system using biogas recirculation, four effective parameters of inlet biogas were selected. The parameters were mass flow rate (0.009, 0.017 and 0.025 g/s), biogas injection pressure (1, 2.5 and 5 bar), repeat of injection in each 24 h (2, 3 and 4 times), and injection position (0: lateral wall at a height of 10 cm from the floor, 1: floor of digester and 2: through the mixer). The mass rates were measured according to biogas inlet velocities of 0.044, 0.084 and 0.123 m/s, respectively, with the slurry density of 1080 kg/m$^3$ and nozzle cross section of 0.000177 m$^2$. The ranges of biogas injection parameters of the experiment points were selected according to the previous studies [15–17]. The experiments of anaerobic digestion were done based on a randomized optimal design using RSM in the biogas digesters. Summary of the experiments is reported in Table 1. As shown the independent variables are the biogas injection parameters and the responses are the biogas compounds and mixing characteristics.

In the experimental design, 15 points were required to develop models, three points to estimate the lack of fit (LF), and three points to replicate the experiments [19]. For multivariate modeling of the responses, a quadratic equation (Eq. 1) was used as the basic form of the models.

$$y(x) = a_0 + \sum_{i=0}^{N} a_i x_i + \sum_{i<j}^{N} a_{ij} x_i x_j, \ i = 1, 2, ..., N$$  \hspace{1cm} (1)

where $y$ is transformation cases of the responses (dependent variables), $x_i$ related to the amounts of the biogas injection parameters (independent variables), $a_i$ is model coefficients, and $N$ is the number of independent variables ($N = 4$) described in Table 1 [20].

Measured Parameters

Biogas Compounds

Five biogas compounds were measured for experiment points. CH$_4$, H$_2$S, CO and O$_2$ were measured according to ppm and %VOL using a multifunction gas detector brand GMI Ltd model GT43 in temperature range of −20 to 50 °C and relative humidity of 0–95% [5, 15]. CO$_2$ compound was measured using CO$_2$ meter model TESTO 535 in range of 0–100%VOL for temperature range of 0–50 °C with a non-dispersive infrared detector [7].

| Table 1 | Statistical information of the experiments including independent variables and responses |
|---------|---------------------------------------------------------------|
| Factor  | Independent variables | Type | Range  | Mean  | Std. Dev |
| $x_1$   | Injection mass rate (g/s) | Numeric | 0.009–0.025 | 0.017 | 0.006 |
| $x_2$   | Injection pressure (bar) | Numeric | 1–5 | 2.595 | 1.530 |
| $x_3$   | Injection repetition | Numeric | 2–4 | 2.762 | 0.700 |
| $x_4$   | Injection position | Numeric | 0–2 | 0.905 | 0.944 |
| Response | Dependent variables | Model trans | Range | Mean  | Std. Dev |
| $y_1$   | CH$_4$ (%) | Power | 5–74 | 39.524 | 18.745 |
| $y_2$   | H$_2$S (%) | Power | 0.1–4.0 | 1.562 | 1.048 |
| $y_3$   | CO (%) | None | 0.005–0.050 | 0.022 | 0.010 |
| $y_4$   | O$_2$ (%) | None | 7.10–19.10 | 11.786 | 2.588 |
| $y_5$   | CO$_2$ (%) | Power | 6.79–57.23 | 34.974 | 15.215 |
| $y_6$   | Color rating (%) | Power | 1.9–5.2 | 3.362 | 1.077 |
| $y_7$   | Mixing time (s) | Power | 123.5–150.1 | 136.143 | 8.225 |
Mixing Characteristics

The slurry mixing characteristics including mixing time ($t_{mix}$) and color rating ($Col_r$) were determined using a single indicator system developed by image processing techniques using MATLAB (Ver. 7.8) software. To determine the $t_{mix}$ and $Col_r$ for the digesters, water was used as the basic liquid in the colorimetric method because not only its mixing pattern remains constant through visual inspection [21] but also its density is close to the slurry used in this research. For this purpose, potassium permanganate solution was titrated to the digesters according to the experiment points, and the color changes in the digesters were captured using cameras located in the center over the digesters. Their image sequences were saved for color analysis. The input images were improved by preprocessing techniques, then a coupled multilevel thresholding-based Otsu’s method and label connected components was used to detect the color changes (Fig. 2). The detected colored regions were tracked in the image sequences as reported by Mahmoodi-Eshkaftaki and Ebrahimi [5]. The $t_{mix}$ and $Col_r$ were automatically determined using the software based on how long it takes the distracted objects get 99.5% of the digester surface.

Optimization Process

Desirability function was used to simultaneously optimize all affecting parameters to achieve the best biogas injection conditions. This method finds operating conditions $x$ providing the most desirable response values [22]. For each response $y_i(x)$, the desirability function $d_i(y_i)$ got values between 0 to 1 to the possible values of $y_i$; $d_i(y_i) = 0$ represented an undesirable value of $y_i$, and $d_i(y_i) = 1$ represented a highly desirable value. The overall desirability was calculated using the geometric mean of the individual desirabilities (Eq. 2). The $y_i$ values were replaced by fitted response values $\hat{y}_i$ to find optimal values outside the evaluation range or values other than the values of experiment points as suggested in the literature [18].

$$D = \left[ \prod_{i=1}^{n} d_i(\hat{y}_i) \right]^{1/n}$$
In this equation, \( N \) is the number of responses (7), \( r_i \) is response’s relative importance to the other responses. \( L_i, U_i \) and \( T_i \) are the lower, upper and target values, respectively, which are desired for response \( y_i \) (\( L_i \leq T_i \leq U_i \)) as described in the literature [18]. In this study, the \( r_i \) varied from the least important (2) to the highest important (5) for the responses. The optimization process was done using Design Expert software (Ver. 10) according to the modified model.

Development of CFD Multiphase Flow Model

Because of the multiphase and turbulent nature of pneumatic and mechanical mixing of the slurry in the digesters, governing equations of the current CFD simulations are mass and momentum conservations, interphase momentum transfer and turbulence transport.

\[
\frac{\partial (\alpha_q \rho_q \vec{v}_q)}{\partial t} + \nabla \cdot (\alpha_q \rho_q \vec{v}_q \vec{v}_q) = - \alpha_q \nabla P + \nabla \cdot \tau_q + \alpha_q \rho_q \vec{g} + \sum_{p=1}^{n} \left( \frac{\vec{R}_{qp}}{\rho_{pq}} + \frac{m_{pq}}{\rho_{pq} \vec{v}_{qp}} \right) \quad \text{(3)}
\]

The concept of volume fraction for each phase is used to define the volume of that phase defined as Eq. (4). It should be noted that sum of the volume fractions of the phases is equal to one.

\( V_q = \int \alpha_q \, dV \quad \text{(4)} \)

The momentum conservation equation is expressed as Eq. (5).

In this equation, \( P \) denotes as static pressure shared by all phases, \( \tau_q \) is stress–strain tensor for phase \( q \), \( \vec{g} \) represents gravity, \( \vec{R} \) is the interaction force between phases, \( \vec{v}_q \) and \( \vec{v}_p \) are interphase velocities. The interaction force can be presented as Eq. (6) [23].

\[
\sum_{p=1}^{n} \vec{R}_{qp} = \sum_{p=1}^{n} \frac{\alpha_p \rho_p f}{\tau_p} \left( \vec{v}_p - \vec{v}_q \right) \quad \text{(6)}
\]

where \( \tau_p \) is the particulate relaxation time and \( f \) is the drag function and. These two parameters were calculated with Eqs. (7) and (8).
\[ \tau_p = \frac{\rho_p d_p^2}{18 \mu_p} \]

(7)

\[ f = \frac{C_D \text{Re}}{24} \]

(8)

In the mentioned equations, \( C_D \) is the drag coefficient which is a function of the relative Reynolds number (Re) (Eq. 9), \( d_p \) is the diameter of bubbles or droplets of phase p, and \( \mu_p \) is the viscosity of phase p.

\[
C_D = \begin{cases} 
24(1 + 0.15\text{Re}^{0.657})/\text{Re} & \text{Re} \leq 1000 \\
0.44 & \text{Re} > 1000 
\end{cases}
\]

(9)

The mixture \( k - \varepsilon \) turbulence model was used to describe the effect of turbulence fluctuations with standard wall function near walls as bellow:

\[
\frac{\partial}{\partial t} (\rho_m k) + \nabla \cdot (\rho_m \nabla k) = \nabla \cdot \left( \frac{\mu_{t,m}}{\sigma_k} \nabla \mu \right) + G_{k,m} - \rho_m \varepsilon
\]

(10)

\[
\frac{\partial}{\partial t} (\rho_m \varepsilon) + \nabla \cdot (\rho_m \nabla \varepsilon) = \nabla \cdot \left( \frac{\mu_{t,m}}{\sigma_\varepsilon} \nabla \varepsilon \right) + \frac{\varepsilon}{k} (C_{1,k} G_{k,m} - C_{2,k} \rho_m \varepsilon)
\]

(11)

where \( \rho_m, \nabla, \mu_t \) and \( \mu_{t,m} \) are mixture density, mixture velocity and turbulent viscosity of the mixture, respectively. In addition, \( G_{k,m} \) is the generation of turbulent kinetic energy, \( \sigma_k \) and \( \sigma_\varepsilon \) are turbulent Prandtl numbers for \( k \) and \( \varepsilon \), and \( C_{1,k} \) and \( C_{2,k} \) are constants.

According to the literature, the manure slurry exhibits non-Newtonian pseudo-plastic fluid behavior [24].

\[ \mu = k \gamma^n \]

(12)

In above equation, \( k \) is the consistency coefficient, \( \gamma \) is the shear rate, and \( n \) is the power-law index.

The commercial CFD code ANSYS-Fluent 18.0 was used for simulations. A two-dimensional, pressure-based, implicit, double precision and transient algorithm were chosen to solve the governing differential equations. The TFM under the Eulerian-Eulerian model was active, in which the slurry was considered as the primary phase while the biogas was treated as the secondary phase. The drag function between the phases was adjusted as Schiller-Naumann. The boundary and initial conditions for the phases were considered as below:

a. No-slip boundary conditions were imposed on all wall surfaces.

b. The mass flow inlet boundary condition was used for the injection of the biogas.

c. The moving wall boundary condition was used for the impeller.

d. At the initial time, the height of slurry in the digester was set as 60 cm.

e. At the moment of biogas injection, no gas existed in the slurry zone.

The phase coupled SIMPLE scheme was set for pressure-velocity coupling. Also, discretization schemes were QUICK for volume fraction and momentum and first-order upwind for the other variables. A time step of 0.01 s was chosen. Under-relaxation factors were 0.3, 0.5, 0.5, 0.5, 0.3, 0.2, 0.2 and 0.2 for pressure, density, body force, momentum, volume fraction, turbulent kinetic energy, turbulent dissipation rate and turbulent viscosity, respectively. The convergence criteria were chosen as \( 10^{-6} \) for all the variables and a final convergence was achieved when the monitored value remained almost constant.

### Grid Independence and Time Step Study

A high quality grid has been used for the simulation process. The calculation domain (57 cm width \( \times \) 90 cm height) was divided into structured tetrahedral grid. The grid was created in two dimensions with 179,600 cells. Several grids with different mesh resolutions were generated to perform the grid-independence study. Finally, the element number 179600 was chosen for numerical simulations. The results of the grid study are not shown for brevity. The grid quality was examined to ensure that the high skewness and high aspect ratio was below 0.9 and 5, respectively.

### Results and Discussion

As shown in Table 1, all the biogas injection factors were created as a numeric type (coded: \(-1 \) to \( 1 \)). The response factors produced relatively wide ranges of the biogas compounds and mixing characteristics that were suitable for model development and optimization process. For this purpose, effect of the biogas injection factors on the responses were studied using the multivariate regression analysis. To model the responses in the regression analysis, different transformations were considered, and the best ones were selected as shown in Table 1. To optimize the parameters, the desirability function developed by overlaying the responses would be maximized.

### Models

For the multivariate regression analysis, different functions by varying transformation \( \gamma \) as square root, natural log, base
Table 2: Mathematical models to estimate the biogas compounds and mixing characteristics

| Parameters | Eq. No | Mathematical models |
|-----------|--------|---------------------|
| CH₄ (%)  | 13     | \((\text{CH}_4 + 5)^{0.77} = 22.52 - 3.09x_1 + 0.33x_2 + 0.77x_3 + 4.28x_4 - 5.81x_5 - 0.4x_6x_7 - 6.45x_8^2\) |
| H₂S (%)  | 14     | \((\text{H}_2\text{S} + 5)^{0.67} = 3.44 + 0.18x_1 + 0.061x_2 - 0.24x_4 + 0.275x_5 - 0.219x_6x_7 - 0.346x_8x_9\) |
| CO₂ (%)  | 15     | \((\text{CO}_2 + 3)^{0.73} = 10.3 - 0.995x_1 - 0.605x_2 - 2.67x_4 + 2.91x_5x_6 - 0.665x_7x_8 + 3.177x_9^2\) |
| CO (%)   | 16     | \(\text{CO} = 0.022 \times 2.7 \times 10^{-3}x_1 + 3.17 \times 10^{-3}x_2 - 5.8 \times 10^{-3}x_3 + 5 \times 10^{-3}x_4 - 5 \times 10^{-3}x_5x_6\) |
| O₂ (%)   | 17     | \(\text{O}_2 = 1.7 + 0.0057x_1 + 0.0055x_2 - 0.04x_6x_7 - 0.04x_8x_9\) |
| Colr (%) | 18     | \((\text{Colr} + 1)^{0.56} = 2.185 + 0.274x_1 + 0.302x_2 + 0.089x_4 + 0.104x_6x_7 + 0.118x_8^2 + 0.093x_9^2\) |
| Tₘix (s) | 19     | \(t_{\text{mix}} = 3 \times 10^{-2} + 8.06 \times 10^{-3}x_1 + 8.03 \times 10^{-3}x_2 + 2.90 \times 10^{-2}x_4 + 1.91 \times 10^{-2}x_6 + 3.48 \times 10^{-2}x_8^2\) |

**Significant at 0.01
*Significant at 0.05
**Not significant
Equations (18) and (19), power transformation of the improved multivariate regression equations, could estimate the $Col_r$ and $t_{mix}$ with high accuracy (their $R^2 > 0.92$, $AR^2 > 0.89$, $PR^2 > 0.85$ and $AP > 17.41$). The model F values higher than 34.97 implied that the models were significant. LF F values higher than 1.21 denoted that they were not significant relative to the pure errors. The AP values of the models were higher than 17.41 indicating adequate signals for these models to be used to navigate the design space. These results indicated that the models had high accuracies for estimation the mixing characteristics. The larger coefficients of the model terms including $x_1$ indicated a higher effect of biogas injection mass rate on color rating and mixing time. The terms including $x_3$ were eliminated from these two models because the effect of injection repetition was not investigated for the mixing characteristics.

Totally, these models had high accuracies for estimation of the biogas compounds and mixing characteristics confirming that they can be successfully used in the optimization process. The high accuracy of the models is better seen in Fig. 3. Figure 3 shows predicted values of the biogas compounds and the mixing characteristics versus actual values. As shown, the dispersion of predicted values versus actual values was suitable.

The effects of the significant interactions of injection parameters on the biogas compounds and the mixing characteristics are graphically represented in Fig. 4. As shown the effect of biogas pressure was not illustrated because the
Fig. 4 Response graphs of more important biogas compounds and mixing characteristics as a function of significant interactions detected by the developed models
mass rate was calculated according to biogas pressure (using CFD method), and therefore their trends were nearly similar. It is shown in Fig. 4 that for biogas injection in similar mass rates, CH$_4$ was more enriched for injection position through the mechanical mixer. Furthermore, for lower injection repetition, we needed a higher mass rate to produce high CH$_4$ content. With increasing the injection mass rate, H$_2$S and CO$_2$ contents of the biogas output increased. Biogas injection from the lateral wall of the digesters could not eliminate H$_2$S and CO$_2$ from the biogas significantly. With increasing the mass rate for different injection positions, Col$_i$ increased rapidly and thus t$_{mix}$ decreased. As shown in the figure, injection from the lateral wall of digesters and through the mixer had a similar effect on mixing characteristics, and injection from the floor of digesters produced the least Col$_i$ (or the most t$_{mix}$). These regression analyses show the importance of having a suitable strategy for biogas injection in the pneumatic mixers.

**Optimization Process**

Slurry can best be mixed using the pneumatic mixer when the injection parameters are in optimum conditions. As described above different injection parameters have influence on the mixing process. The optimization of these parameters is essential for the successful operation of the anaerobic digestion, though it is difficult to carry out [26].

For desirable digestion, the biogas compounds and the mixing characteristics should be optimized. The optimization process was done with definition of constraints for the biogas compounds and mixing characteristics. As described for the constraints, CH$_4$, Col$_i$ and t$_{mix}$ were maximized with $r_i$ equal to 5, 4 and 4, respectively, while the other responses were minimized.

Eight runs were obtained by running the models together in order of high desirability and the first three runs have been reported in Table 3. These runs were used to illustrate the effect of changes in the biogas injection factors on the biogas compounds and mixing characteristics. The best run (Run 1) revealed the highest desirability, and was selected to report the optimum absolute amounts of the parameters. Accordingly, the best amounts of injection mass rate, injection pressure, injection repetition and injection position were 0.02 g/s, 2.84 bar, 2.64 (rounded 3 times) and 1.95 (rounded 2), respectively. Considering the eight runs specified that the best injection repetition were 2 or 3 times in 24 h. These optimum runs were precise and reliable because they were determined by a hybrid optimization technique, integrating accurate models and desirability analysis. Similar optimization methods were developed in other studies for different parameters [18, 27–30].

The overall desirability function D(x) for the best run (Run 1) according to the biogas injection factors is illustrated in Fig. 5. The function D(x) was not quite flat in the vicinity of the optimum absolute amounts indicating that variations around the maximum desirability (0.809) could change the overall desirability drastically. As shown in Fig. 5, the highest desirability value (highlighted in the plots) is within the tested ranges of the factors; and thus, the optimum absolute amounts of the injection parameters can be understood from the plots reliably.

The optimum regions of the injection parameters are illustrated in Fig. 6. These regions were determined using Design Expert software (Ver. 10) by overlaying all the responses according to defined constraints. The suitable ranges of biogas injection factors were extracted from the optimum regions. Figure 6a shows the optimum region in which CH$_4$ and Col$_i$ were maximized and the other responses were minimized (predefined constraints). The optimum regions detected in Fig. 6b and c were determined using supplementary constraints in addition to predefined constraints. Optimum region in Fig. 6b produced by overlaying all the responses in which CH$_4$ and Col$_i$ got at and above 90% of their maximums from the tested values, and the other responses got values lower 90% of their minimums. Using this limitation, the optimum region was not found.

**Table 3** Optimal quantities of the biogas compounds and mixing characteristics for optimum amounts of input factors

| Response | CH$_4$ (%) | H$_2$S (%) | CO$_2$ (%) | CO (%) | O$_2$ (%) | Col$_i$ (%) | t$_{mix}$ (s) |
|----------|------------|------------|------------|--------|-----------|-------------|-------------|
| Run 1    | 63.323     | 0.685      | 15.477     | 0.018  | 9.602     | 4.041       | 128.974     |
|          | 12.779     | 0.559      | 7.842      | 0.007  | 1.358     | 0.286       | 0.096       |
|          | [49.83–77.44] | [0.21–1.17] | [7.76–24.47] | [0.01–0.02] | [8.52–10.76] | [3.77–4.31] | [127.13–130.91] |
| Mean     | Std. Dev   | 95% PI     | Mean       | Std. Dev | 95% PI   | Mean       | Std. Dev   |
| Run 2    | 63.307     | 0.687      | 15.510     | 0.018  | 9.605     | 4.035       | 128.977     |
|          | 12.779     | 0.559      | 7.845      | 0.007  | 1.357     | 0.286       | 0.096       |
|          | [49.83–77.44] | [0.22–1.17] | [7.78–24.51] | [0.01–0.02] | [8.52–10.76] | [3.77–4.31] | [127.16–130.93] |
| Mean     | Std. Dev   | 95% PI     | Mean       | Std. Dev | 95% PI   | Mean       | Std. Dev   |
| Run 3    | 44.797     | 1.825      | 30.911     | 0.027  | 12.585    | 3.138       | 138.650     |
|          | 11.789     | 0.594      | 9.431      | 0.007  | 1.608     | 0.262       | 0.920       |
|          | [34.44–55.70] | [1.37–2.29] | [23.03–39.41] | [0.02–0.03] | [11.52–13.71] | [2.86–3.42] | [133.72–138.65] |
| Mean     | Std. Dev   | 95% PI     | Mean       | Std. Dev | 95% PI   | Mean       | Std. Dev   |

$x_i$ injection mass rate (g/s), $x_j$ injection pressure (bar), $x_k$ injection repetition, $x_l$ injection position

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Figure 6c shows the optimum region in which CH₄ and Col got at and above 70% of their maximums from the tested values, and the other responses got values lower 70% of their minimums. All the models and their standard error were overlayed according to the above limitations to determine the optimum regions. As shown from Fig. 6a, suitable mass rates were ranged from bellow 0.009 to after 0.025 g/s or injection repetition 1–5 times in 24 h which were higher than the tested ranges. Therefore, this optimum region was not reliable. To determine a more suitable region, the range of each response was further limited until attached to the best region as shown in Fig. 6c. The best range for mass flow rate was 0.015–0.028 g/s, injection pressure was 1.5–4.7 bar, injection position was 2, and injection repetition was 1–4 times. With further limitation, CH₄ and Col were higher than 0.5 their maximums and the other responses were lower than 0.5 their minimums, the injection position of 1 was also suitable for the pneumatic mixing system.

To investigate in detail the mixing process of the slurry with optimum conditions of the pneumatic mixer, the volume fraction contours of the slurries from CFD modeling were studied. The CFD was used for some of the optimum conditions in the vertical cross-section of the plane parallel to the feeding pipes to get an overall picture of the flow pattern. Figure 7 shows the volume fraction contours of the slurry for biogas injection from the floor of the digester with optimum mass rate of 0.015 g/s. The volume fraction contours of the slurry differed as follows: Firstly, the injected biogas messed up the region below the impeller and risen quickly in the slurry (tₘᵢₓ = 10 s). Secondly, the slurry upper the impeller mixed significantly with both injected biogas and mechanical mixer (tₘᵢₓ = 40 s). Then the region below the impeller started to mess up powerfully (tₘᵢₓ = 80 s). As the end of the stirring time approaches, unmixed region below the impeller increased (tₘᵢₓ = 120 s). Therefore, tₘᵢₓ = 120 s was enough to mix the slurry. This time was near to the times determined by the image processing method. A mixing process, pausing time of 5 h and mixing time of 300 s, has been suggested by other researchers [5, 7] to produce the most efficiency for anaerobic digesters. These illustrations indicated a strong influence of the biogas recirculation on the flow pattern in agreement with previous studies [8]. Moreover, the biogas injection disrupted the uniform patterns made by the impeller in the digester which is very suitable for a mixing system in the biogas digesters, especially at a low rotation speed. These two reasons, tₘᵢₓ reduction and pattern disruption, could improve the biodegradation of organic matters, and thus increase biogas production. However, the
Fig. 6 Optimum region of the injection factors by overlaying the responses; a Primary optimum region; b Optimum region in 90% of the best amount of each response (an optimum region was not found); c Optimum region in 70% of the best amount of each response.
CFD analysis in Fig. 7 indicates that an excessive injection speed and injection pressure or long injection time may have the opposite effect on pneumatic mixing process. This is in agreement with determined optimum amounts of the biogas injection factors. According to literature [8, 31, 32], the theoretical findings are adopted for the operation of a real-life anaerobic digester.

It is practically impossible to keep all the injection parameters at constant levels for the pneumatic mixers. Even though it is possible to control the mixing system using equipment to a certain extent, this would considerably increase the cost of operations. The range and region optimization based on the overlaying method would then be a handy tool for the operators in such situations. These facts point to the importance of desirability function and overlaying method in this study to clarify the optimum conditions of the pneumatic mixing systems.

**Conclusion**

In this study, the best strategy for pneumatic mixing systems using biogas recirculation was determined. For this purpose, mixing characteristics including Col and tmix (using image processing base image segmentation technique), and anaerobic digestion quality (based on biogas
compounds) were studied using multivariate modeling according to biogas injection parameters. The biogas injection parameters were carefully considered using the CFD. The regression analysis indicated that for a similar mass rate, biogas injection through the mixer produced more CH$_4$ than the other positions. Furthermore, for a low injection repetition, a higher mass flow rate was needed to increase the CH$_4$ content and decrease $t_{mix}$. These different interaction effects between the parameters show the importance of having a suitable strategy for biogas injection in the pneumatic mixing systems. All of the models were integrated in the desirability function to determine the best amounts of injection parameters related to the highest desirability value. Accordingly, the best amounts of mass flow rate, injection pressure and injection repetition were 4.7 bar and 1–4 times in 24 h. Further, the best injection position was the injection through the mixer and after that was the injection from floor of digester. The CFD analysis showed that an excessive injection speed and pressure or long injection time may have the opposite effect on pneumatic mixing process. This confirms the importance of region optimization or point optimization. However, determination the optimum ranges is more valuable and more commonplace for practical use in plants than point optimization.

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Data Availability The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Authors and Affiliations

Mahmood Mahmoodi-Eshkaftaki1, Hossein Rahmanian-Koushkaki1

Mahmood Mahmoodi-Eshkaftaki
m.mahmoodi5@gmail.com; m.mahmoodi5@jahromu.ac.ir

1 Department of Mechanical Engineering of Biosystems, Jahrom University, P.O. Box 74135-111, Jahrom, Iran