Mass spectra of heavy mesons with instanton effects

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We investigate the mass spectra of ordinary heavy mesons, based on a nonrelativistic potential approach. The heavy-light quark potential contains the Coulomb-type potential arising from one-gluon exchange, the confining potential, and the instanton-induced nonperturbative local heavy-light quark potential. All parameters are theoretically constrained and fixed. We carefully examine the effects from the instanton vacuum. Within the present form of the local potential from the instanton vacuum, we conclude that the instanton effects are rather marginal on the charmed mesons.

Keywords: Heavy mesons, Instanton-induced heavy-light quark interactions

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I. INTRODUCTION

The structure of hadrons containing a heavy quark is systematically understood when the mass of the heavy quark is taken to infinity. This is valid, since the heavy-quark mass $m_Q$ is much larger than the $\Lambda_{\text{QCD}}$, i.e. $m_Q \gg \Lambda_{\text{QCD}}$. Then a new type of symmetry arises: the physics is not changed by the exchange of the heavy-quark flavor. This is called heavy-quark flavor symmetry. In this limit, the spin of the heavy quark $S_Q$ is conserved, which brings about the spin conservation of the light degrees of freedom $S_L$. So, the spin of a heavy hadron is also conserved in this limit: $S = S_L + S_Q$. This is often called heavy-quark spin symmetry \cite{1,2}. The heavy quark is entirely decoupled from the internal dynamics of a heavy hadron in the limit of $m_Q \to \infty$ and the interaction among light degrees of freedom becomes spin-independent. The infinitely heavy-quark mass limit allows one to use the inverse of the heavy-quark mass, $1/m_Q$, as an expansion parameter. The spin-dependent part of the interaction appears as the next-to-leading order in the $1/m_Q$ expansion, which is proportional to $1/m_Q$ and stems from the chromomagnetic moment of the quark (see, for example, reviews \cite{4,5} and books \cite{6,7}).

In the limit of $m_Q \to \infty$, the classification of conventional heavy meson states $Q\bar{q}$ with a single heavy quark $Q$ is rather simple, where $\bar{q}$ denotes the light anti-quark constituting the heavy meson. Since the heavy quark is decoupled in the $m_Q \to \infty$ limit, the flavor structure is solely governed by the light quarks. Thus the lowest-lying states of the heavy meson is classified as the antitriplet meson $3$. Moreover, the mesons with spin $s = 0$ and those with $s = 1$ are found to be degenerate, so that the pseudoscalar and vector heavy mesons consist of the doublets in the limit of $m_Q \to \infty$. This degeneracy is lifted by introducing the spin-dependent interactions coming from $1/m_Q$ order. Based on this heavy-quark flavor-spin symmetry, there has been a great deal of theoretical works on properties of both the lowest-lying and excited heavy mesons: lattice QCD \cite{10,15}, the nonrelativistic and relativistic quark models \cite{16,20}, potential models \cite{21,22}, QCD sum rules \cite{27,29}, holographic QCD \cite{30}, and so on.

The potential models for the heavy mesons are usually based on two important physics: the quark confinement and the perturbative one-gluon exchange. While these two ingredients of the potentials describe successfully both properties of quarkonia and heavy mesons, certain nonperturbative effects need to be considered. Diakonov et al. derived the central part of the heavy-quark potential from the instanton vacuum, using the Wilson loop \cite{31}. The spin-dependent part can be easily constructed by employing the Eichten-Feinberg formalism \cite{32}. The effects of the heavy-quark potential from the instanton were examined only very recently by computing the quarkonium spectra \cite{33}. The results showed that the effects of the instanton turn out to be rather small on the quarkonium spectra. Chernyshev et al. investigated the effects of a random gas of instantons and anti-instantons on mesons and baryons containing one or several heavy quarks \cite{34}. They first derived the local effective interactions from the random instanton-gas model (RIGM) and then employed them to estimate the heavy-hadron mass spectra within a simple variational method, including the harmonic oscillator potential as a simple expression of the quark confinement. They obtained results in qualitative agreement with the experimental data on the low-lying heavy mesons. However, it is of great importance to examine cautiously such nonperturbative effects on the heavy hadron spectra in a quantitative manner.

In the present work, we aim at exploring carefully the heavy-light quark potentials, which were derived from the RIGM, examining their effects on the mass spectra of the heavy mesons. For simplicity and convenience, we will use the nonrelativistic framework in dealing with the heavy-light quark interactions from the RIGM. In any potential models for describing the quarkonia and heavy mesons, there are two essential components: the quark confinement and the one-gluon exchange contribution, which we want to introduce in addition to the interaction from the instantons. Instead of a simple variational method used in Ref. \cite{34}, we employ a more elaborated and sophisticated framework, i.e. the Gaussian expansion method (GEM), which is well known for the successful description of two- and few-body systems \cite{35,36}, so that we reduce numerical uncertainties arising from the simple variational method. As will be shown in this work, the present form of the heavy-light quark interaction based on the RIGM has only marginal effects on the mass spectra of the heavy mesons. The quark potentials of one-gluon exchange and the quark confinement already reproduce approximately the experimental data on the spectra of the low-lying heavy mesons. However, since the heavy-mesons contain a light quark, we still expect that certain nonperturbative effects will come into play. We will discuss them also in the present work.

This paper is organized as follows: In Section II, we define the heavy-light quark potentials arising from one-gluon exchange and the quark confinement. We then introduce the effective potential coming from the nonperturbative heavy-light quark interactions based on the RIGM. In Section III, we show how to solve the nonrelativistic Schrödinger equation with the heavy-light quark potential within the framework of the GEM. That will be the framework for numerical calculations in the present work. In Section IV, we present the results and discuss them in comparison with the experimental data. The final Section is devoted to summary and conclusion. We also discuss a possible future outlook.
The general structure of the heavy-light quark potentials is expressed as

\[ V(r) = V_c(r) + V_{SS}(r)(S_Q \cdot S_q) + V_{LS}(r)(L \cdot S) + V_T(r)[3(S_Q \cdot \hat{n})(S_q \cdot \hat{n}) - S_1 \cdot S_2], \]  

(1)

where \( V_c \) is the central part of the potential. The \( V_{SS}, V_{LS}, \) and \( V_T \) are called respectively the spin-spin, the LS term that shows the coupling between the orbital angular momentum and the spin angular momentum, and the tensor term. Following Ref. [32], the spin-dependent potential is derived from the central potential. \( S_Q \) and \( S_q \) denote the spin operators for the heavy and light quarks, respectively. \( L \) and \( S \) represent respectively the operator of the relative orbital angular momentum and the total spin operator defined as \( S = S_Q + S_q \). In a nonrelativistic constituent-quark potential model, the heavy-light quark potential consists of two different contributions: the confining linear potential

\[ V_{\text{conf}}(r) = \kappa r \]  

(2)

with the parameter of the string tension \( \kappa \) and the Coulomb-like interaction arising from one-gluon exchange

\[ V_{\text{Coul}}(r) = -\frac{4\alpha_s}{3r}, \]  

(3)

where \( \alpha_s \) is the strong running coupling constant at the one-loop level

\[ \alpha_s(\mu) = \frac{1}{\beta_0 \ln(\mu^2/A_{QCD}^2)}. \]  

(4)

The one-loop \( \beta \) function is given as \( \beta_0 = (33 - 2N_f)/(12\pi) \). The dimensional transmutation parameter are taken from the Particle Data Group (PDG) [40], i.e. \( A_{QCD} = 0.217 \text{GeV} \). Since we include the charmed quark, the number of flavor is given by \( N_f = 4 \). The scale parameter \( \mu \) will be set equal to the mass of the charmed quark.

\[ V_c(r) = V_{\text{conf}}(r) + V_{\text{Coul}}(r) \]  

(5)

and the spin-dependent parts are generated from this central potential and are expressed as

\[ V_{SS}(r) = \frac{32\pi \alpha_s}{9M_Qm_q}\delta(r), \]

\[ V_{LS}(r) = \frac{1}{2M_Qm_q}\left(\frac{4\alpha_s}{r^3} - \frac{\kappa}{r}\right), \]

\[ V_T(r) = \frac{4\alpha_s}{3M_Qm_q} \frac{1}{r^3}, \]  

(6)

where \( M_Q \) and \( m_q \) are stand for the dynamical heavy and light quark masses, respectively, which will be discussed shortly.

In a practical calculation, the point-like spin-spin interaction is required to be smeared by using the exponential form

\[ \delta_\sigma(r) = \left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r^2}, \]  

(7)

where \( \sigma \) stands for the smearing factor. Thus, one has a given set of parameters \( \kappa \) and \( \sigma \) which are fit to the spectra of mesons. In order to reduce the number of free parameters in the present work, we fix the strong running coupling constant \( \alpha_s = 0.4106 \) defined in Eq. (4) at the the scale of the charmed quark mass: \( \mu = M_Q = m_{\text{current}} + \Delta M_Q \) with \( m_{\text{current}} = 1.275 \text{GeV} \) and \( \Delta M_Q = 0.086 \text{GeV} \). Here \( \Delta M_Q \) is the shift of the heavy quark mass caused by the heavy-light quark interactions that arise from a random instanton gas of the QCD vacuum. Its numerical value used here is determined in Ref. [33] (see also discussions in Ref. [32]). The dynamical mass of the light quark arises from the spontaneous breakdown of chiral symmetry (SB\(\chi\)). The QCD instanton vacuum explains quantitatively the mechanism of the \( \text{SB}\chi\) (41) (see also reviews [42, 43]). In the present work, we take the value of \( M_{u,d} = 340 \text{MeV} \). The strange dynamical quark mass is taken to be \( M_s = m_{s} + \Delta m_{s} = (150 + 340) \text{MeV} = 490 \text{MeV} \).

Since the main purpose of the present work is to consider the contribution of the nonperturbative heavy-light quark interaction from the instanton vacuum, we will introduce the effective instanton-induced heavy-light quark potential.
For simplicity, we follow Ref. 34, where the local effective interactions between the heavy and light quarks due to instantons were derived in terms of the heavy and light quark operators $Q$ and $q$

$$\mathcal{L}_{qQ} = - \left( \frac{M_q \Delta M_Q}{2n N_c} \right) \left( \frac{1}{Q^2} + \frac{1}{4} Q \gamma^\alpha Q \gamma \right) \left( \frac{\gamma^\alpha}{2} - \frac{\gamma^0}{2} \lambda^\alpha \sigma \right),$$

$$\mathcal{L}_{qQ}^{\text{spin}} = - \left( \frac{M_q \Delta M_Q^{\text{spin}}}{2n N_c} \right) \left( \frac{1}{4} \gamma^\alpha + \frac{1}{2} \lambda^\alpha \sigma^{\mu \nu} Q \gamma \lambda^\alpha \sigma_{\mu \nu} q. \right.$$  \hspace{1cm} (8)

The density parameter $n$ of the random instanton gas is defined by $N/2Vq N_c$, where $N/V_q \sim 1 \text{ fm}^{-4}$ is the instanton density with the four-dimensional volume $V_q$ and $N_c$ denotes the number of colors. $\Delta M_Q$ is the mass shift of the heavy quark caused by the instantons. $\Delta M_Q^{\text{spin}}$ arises from the $M_Q^{-1}$-order chromomagnetic interaction and, therefore, its value is different from that of $\Delta M_Q$. In Ref. 34, the numerical value of $\Delta M_Q^{\text{spin}}$ is determined to be 3 MeV for the charmed quark. Other standard quantities in the Lagrangian are the Gell-Mann matrices for color space and the combinations from the Dirac matrices. Consequently, the relevant two-body instanton-induced central and spin-spin potentials are expressed as

$$V_1^c(r) = \left( \frac{M_q \Delta M_Q}{2n N_c} \right) \left( 1 + \frac{1}{4} \lambda_q^\alpha \lambda_q \right) \delta^3(r),$$

$$V_1^{\text{spin}}(r) = - \left( \frac{M_q \Delta M_Q^{\text{spin}}}{2n N_c} \right) \left( 1 + \frac{1}{4} \lambda_q^\alpha \lambda_q \right) S_q \cdot S_q \lambda_q^\alpha \lambda_q \delta^3(r),$$  \hspace{1cm} (9, 10)

where $r$ designates the relative coordinates $r = r_q - r_Q$.

Yet another spin-dependent potentials 32 are derived from the central potential from the instanton vacuum as follows:

$$V_{SS}^J(r) = \frac{1}{3M_Q M_q} \nabla^2 V_1(r),$$

$$V_{LS}^J(r) = \frac{1}{2M_Q M_q} \frac{1}{r} \frac{d V_1(r)}{d r},$$

$$V_{TT}^J(r) = \frac{1}{3M_Q M_q} \left( \frac{1}{r} \frac{d V_1(r)}{d r} - \frac{d^2 V_1(r)}{d r^2} \right).$$  \hspace{1cm} (11)

Since the central and spin-spin potentials are given as the Dirac delta functions, we need to introduce here also a smearing function to remove any divergence that would be caused by them. So, we introduce the Gaussian type of the smearing function

$$\delta_{\sigma}(r) = \left( \frac{\sigma_f}{\sqrt{\pi}} \right)^3 e^{-\sigma_f^2 r^2}$$  \hspace{1cm} (12)

in both central and spin-spin potentials. Here $\sigma_f$ stands for the another smearing factor, of which the numerical value will not be much changed from that of $\sigma$ to avoid any additional uncertainty. The explicit forms of the spin-dependent potentials are obtained as

$$V_{SS}^J(r) = \left( \frac{\Delta M_Q}{6n N_c M_Q} \right) \left( 1 + \frac{1}{4} \lambda_q^\alpha \lambda_q \right) \left( -6\sigma_f^2 + 4\sigma_f^4 r^2 \right) \delta_{\sigma}(r),$$

$$V_{LS}^J(r) = \left( \frac{\Delta M_Q}{4n N_c M_Q} \right) \left( 1 + \frac{1}{4} \lambda_q^\alpha \lambda_q \right) \left( -2\sigma_f^2 \right) \delta_{\sigma}(r),$$

$$V_{ TT}^J(r) = \left( \frac{\Delta M_Q}{6n N_c M_Q} \right) \left( 1 + \frac{1}{4} \lambda_q^\alpha \lambda_q \right) \left( -4\sigma_f^4 r^2 \right) \delta_{\sigma}(r).$$  \hspace{1cm} (13)

The total potential can be constructed by combining the potentials from the instanton vacuum given in Eqs. 9, 10, and 13 with those from the confining and Coulomb-like potentials in Eqs. 5 and 6

$$V_{Qq}(r) = V(r) + V_1(r).$$  \hspace{1cm} (14)

where $V_1(r)$ is defined as

$$V_1(r) = V_1^c(r) + V_1^{\text{spin}}(r) + V_{SS}^J(r)(S_q \cdot S_q) + V_{LS}^J(r)(L \cdot S) + V_{ TT}^J(r)[3(S_q \cdot \hat{n}) (S_q \cdot \hat{n}) - S_1 \cdot S_2].$$  \hspace{1cm} (15)
The matrix element of the potential in the $^{2S+1}L_J$ basis is given by

$$\langle ^{2S+1}L_J | V_{Qq}(r) | ^{2S+1}L_J \rangle = \tilde{V}_c(r) + \left[ \frac{1}{2} S(S + 1) - \frac{3}{4} \right] \tilde{V}_{SS}(r) + \frac{1}{2} (L \cdot S) \tilde{V}_{LS}(r)$$

$$+ \left[ \frac{2(L \cdot S)(2(L \cdot S) + 1)}{4(2L - 1)(2L + 3)} + \frac{S(S + 1)L(L + 1)}{3(2L - 1)(2L + 3)} \right] \tilde{V}_T(r),$$

(16)

where

$$\langle L \cdot S \rangle = [J(J + 1) - L(L + 1) - S(S + 1)]/2.$$  

(17)

Here we have taken the conventional spectroscopic notation $^{2S+1}L_J$ given in terms of the total spin $S$, the orbital angular momentum $L$, and the total angular momentum $J$ with the addition of the angular momenta, $J = L + S$. The corresponding terms $\tilde{V}_c(r)$, $\tilde{V}_{SS}(r)$, $\tilde{V}_{LS}(r)$ and $\tilde{V}_T(r)$ denote generically the central, spin-spin, spin-orbit, and tensor parts of the total potential.

### III. CALCULATIONS AND RESULTS

In Ref. [34], the mass spectra of the heavy mesons were already studied within a simple variational method, the potential from the instanton vacuum and the potential of the simple harmonic oscillator being combined. The results from Ref. [34] were in qualitative agreement with the experimental data. However, it is essential to consider more realistic contributions such as the confining potential and the Coulomb-like potential from one-gluon exchange in order to understand the effects of the instantons on the mass spectra of the heavy mesons in a quantitative manner. In the present work, we will include all the potentials mentioned in the previous section.

A nonrelativistic potential approach for a heavy-light quark system is represented by the time-independent Schrödinger equation with the static potential $V_{Qq}(r)$

$$\left[ -\frac{\hbar^2}{\mu} \nabla^2 + V_{Qq}(r) - E \right] \Psi_{JM}(r) = 0,$$

(18)

where $\mu$ denotes the reduced mass of the heavy meson system and $\Psi_{JM}$ stands for the wavefunction of the state with the total angular momentum $J$ and its third component $M$. To solve the Schrödinger equation numerically, we employ the GEM which was successfully applied to describe few-body systems such as light nuclei (see a review [36] and references therein).

In the GEM the wavefunction is expanded in terms of a set of $L^2$-integrable basis functions $\{ \Phi_{JM,k}^L \}$

$$\Psi_{JM}(r) = \sum_{k=1}^{k_{\text{max}}} C_{k,LS}^J \Phi_{JM,k}^L(r)$$

(19)

and the Rayleigh-Ritz variational method is used. So, we are able to formulate a generalized eigenvalue problem given as

$$\sum_{m=1}^{k_{\text{max}}} \left[ \Phi_{JM,k}^L \left| -\frac{\hbar^2}{\mu} \nabla^2 + V_{Qq}(r) - E \right| \Phi_{JM,m}^L \right] C_{m,LS}^J = 0.$$  

(20)

The angular part of the basis function $\Phi_{JM,k}^L$ is expressed in terms of standard spherical harmonics and the normalized radial part $\phi_k^L(r)$ is written in terms of the Gaussian basis functions

$$\phi_k^L(r) = \left( \frac{2^{2L+2} \pi r_k^{2L+3}}{\sqrt{5}(2L+1)!!} \right)^{1/2} r^L e^{-r^2/r_k^2},$$

(21)

where $r_k$, $k = 1, 2, ..., k_{\text{max}}$ designate variational parameters. When it comes to the case of a two-body problem, the total number of the variational parameters is reduced by using the geometric progression in the form of $r_k = r_1 d^{k-1}$, which provides a good convergence of the results. Thus, in the two-body problem, we need only three variational
parameters, i.e. \( r_1, a \) and \( k_{\text{max}} \). Once the Schrödinger equation is solved, the energy eigenvalue \( E_N \) is found and the mass of the heavy meson is determined by

\[
M = M_Q + M_q + E_N + \Delta E_q,
\]

(22)

where \( \Delta E_q \) is the overall energy shift in the spectra depending on the light-quark content of the meson and plays a role of a simple tuning parameter. As mentioned already, \( M_Q \) and \( M_q \) are the dynamical masses of the heavy and light quarks, respectively. Note that \( M_Q \) contains also the mass shift arising from the instanton vacuum. In this work we will slightly vary the total mass of the strange quark mass \( M_s \) and try to analyze the corresponding effects.

Since, some of remaining parameters cannot be determined theoretically, we construct several sets of the parameters and call them Model I', Model I, Model II, and Model III, respectively. The numerical values of model parameters are listed in Table I and we use them to calculate the spectra of the heavy mesons.

### Table I. Free parameters of the model: \( m_s \) denote the dynamical mass of the strange quark, \( \kappa \) stands for the string tension, \( \sigma \) and \( \sigma_I \) designate the smearing parameters corresponding to point like interactions in Eqs. (7) and (22), \( \Delta E_{u,d} \) and \( \Delta E_s \) are the constant overall energy shifts of mesons corresponding to the up (down) and strange quark constituents, and \( n \) is the density of instanton medium.

| Model | \( m_s \) [GeV] | \( \kappa \) [GeV \(^{-1}\)] | \( \sigma \) [GeV] | \( \sigma_I \) [GeV] | \( \Delta E_{u,d} \) [GeV] | \( \Delta E_s \) [GeV] | \( n \) [fm \(^{-2}\)] |
|-------|-----------------|------------------|------------|----------------|-----------------|----------------|----------------|
| I     | 0.450           | 0.169            | 1.43       | –              | -0.365          | -0.299         | –              |
| I'    | 0.450           | 0.169            | 1.43       | 1.18           | -0.365          | -0.299         | 1.0            |
| II    | 0.490           | 0.165            | 0.95       | 1.19           | -0.347          | -0.287         | 1.0            |
| III   | 0.470           | 0.163            | 0.93       | 1.17           | -0.339          | -0.274         | 0.9            |

The results of the charmed meson masses corresponding to the different models are listed in Table II in comparison with the experimental data taken from the Particle Data Group (PDG) [40]. In the second column, the results without instanton-induced quark-quark interactions are presented. It is called Model I' that is obtained by including only the confining and Coulomb-like type interactions. One can assume that in this model the nonperturbative effects are only taken into account by means of dynamically generated masses of the corresponding light quarks. It is seen that the results are relatively in good agreement with the experimental data. It indicates that a nonrelativistic approach to the heavy-light quark system works even quantitatively at least for the mass spectra of the conventional heavy mesons.

Model I has the same parameter set as Model I' except for the instanton-induced potentials, which means that the parameters are not tuned but the instanton-induced heavy-light quark interactions are taken into account. By doing this, we can examine how the instanton-induced quark-quark interactions affect the mass of each charmed meson. The effects of instanton-induced quark-quark interactions are clearly seen in the ground state \( D^+ \) meson, while they are rather tiny on other charmed mesons. In particular, the effects are almost negligible on the \( P \)-wave charmed meson spectra. One can conclude that in general instanton-induced interactions do not affect much the spectra of heavy mesons and play only a role in the fine-tuning level.

Thus, we present the results of Model II in which the free parameters are fitted to the experimental data. One can see that the results slightly change in comparison with the Model I' and shows that the instanton-induced quark-quark interactions are seem to be important in the fine-tuning level. In Model III, we change also the density of the instanton medium is slightly changed, considering it as an input parameter. This is allowed, as was already discussed in Ref. [33] in detail. All other parameters are fitted to the experimental data as in the case of Model II.

### Table II. The results of the charmed \( D \)-meson masses in units of MeV. The second column lists the results without the instanton-induced quark-quark interactions and is coined as Model I'. The third, fourth, and fifth columns list those of Models I, II, and III. The last column shows the corresponding experimental data taken from PDG [40].

| Model | I' | I | II | III | Exp. |
|-------|----|---|----|-----|-----|
| \( D^+ (1^+ S_0) \) | 1867.7 | 1787.0 | 1868.3 | 1868.0 | 1869.65 \( \pm \) 0.05 |
| \( D^{*+} (2^+ S_0) \) | 2013.5 | 2006.4 | 2009.7 | 2010.2 | 2010.26 \( \pm \) 0.05 |
| \( D_1 (1^+ P_1) \) | 2461.2 | 2461.5 | 2458.7 | 2456.7 | 2432.3 \( \pm \) 2.4 |
| \( D_2 (1^+ P_2) \) | 2462.2 | 2461.2 | 2461.7 | 2460.1 | 2465.4 \( \pm \) 1.3 |
| \( D^{*+} (1^+ S_1) \) | 2639.0 | 2593.4 | 2634.1 | 2630.4 | 2637 \( \pm \) 2 \( \pm \) 6 |
| \( (2^+ S_1) \) | 2737.0 | 2732.6 | 2724.0 | 2719.8 | – |

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1. For more details, see Refs. [33][34].
2. The corresponding explanation of model parameters will be given hereafter in the text.
The results of Model III are slightly better than those of Model II. As expected from the comparison of Model I with Model I’, the prediction of Model III is not much different from that of Model I’. Thus the potential from the instanton vacuum in the present form change slightly the mass spectrum of the charmed mesons and does not affect quantitatively the results from the calculation without instanton-induced quark-quark interactions.

### Table IV

**The results of the instanton effects on the low-lying charmed heavy mesons in units of MeV.** The values of the potentials from the instanton vacuum on the conventional heavy mesons.

| Model      | I II III Exp. |
|------------|--------------|
| $D_{s}^{+}(1^{1}S_{0})$ | 1969.1 1887.9 1969.0 1968.9 1968.34 ± 0.07 |
| $D_{s}^{*+}(2^{1}S_{0})$ | 2108.3 2100.8 2113.2 2111.5 2112.2 ± 0.4 |
| $D_{s}^{+}(1^{1}P_{1})$ | 2538.3 2538.1 2543.1 2540.5 2535.10 ± 0.06 |
| $D_{s}^{*+}(1^{1}P_{2})$ | 2546.2 2545.1 2555.2 2551.8 2569.1 ± 0.8 |
| $D_{s}^{*+}(1^{3}S_{1})$ | 2703.7 2661.6 2697.4 2696.0 2708.3±4.0 |
| (2$^{1}S_{1}$) | 2792.6 2788.2 2780.5 2778.5 |

Table III lists the results of the charmed strange $D_{s}$-meson masses. As done in Table III, we first compute the masses of the charmed strange mesons without the instanton contributions, which are listed in the second column of Table III. Then we include the instanton-induced quark-quark interactions, of which the results are presented in the other columns. The effects of the instantons are similar to the case of the charmed mesons, that is, the instanton effects are noticeable only on the ground state $D_{s}^{+}$ meson whereas they are negligibly small on the $P$-wave charmed strange mesons. Though the results of Model III seem slightly better than those of Model I’, for the quark-quark potential from the instanton vacuum, at least in the present form, the improvement is marginal in the charmed strange meson mass spectrum. Moreover, the effects of the instanton-induced potential on the charmed strange mesons are even smaller than on the charmed nonstrange ones.

Finally, we would like to note that although we have changed the density of instanton medium $n$ in Model III in comparison with Model II the mass contribution $\Delta M_Q$ is unchanged and kept in both cases equal to 0.086 GeV. However, $\Delta M_Q$ is proportional to $n$ and therefore it must be also modified if the value of $n$ changes. As a result, eigenfunctions and eigenvalues of the Hamiltonian should be also altered. Consequently, a better fine-fitting of the whole mass spectra can be achieved by means of changes of instanton parameters in a self-consistent manner. Though these selfconsistent changes of parameters are expected to improve the present results further, we do not perform it because in the present work we aim at examining the effects of the existing nonperturbative heavy-light quark potentials from the instanton vacuum on the conventional heavy mesons.

### Table IV

**The results of the instanton effects on the low-lying charmed heavy mesons in units of MeV.** The values of the relevant parameters are taken from those for Model I.

| Heavy meson | Instanton contribution [MeV] | Exp. [MeV] |
|-------------|-----------------------------|------------|
| $D^{+}(1^{1}S_{0})$ | 80.7 | 1869.65 ± 0.05 |
| $D^{+}(1^{1}S_{1})$ | 7.1 | 2010.26 ± 0.05 |
| $D_{1}(1^{1}P_{1})$ | -0.3 | 2423.2 ± 2.4 |
| $D_{1}(1^{3}P_{2})$ | 0.1 | 2465.4 ± 1.3 |
| $D^{+}(2^{1}S_{0})$ | 45.6 | 2637 ± 2 ± 6 |
| (2$^{1}S_{1}$) | 4.4 | |
| $D_{s}^{+}(1^{1}S_{0})$ | 81.2 | 1968.34 ± 0.07 |
| $D_{s}^{+}(1^{3}S_{1})$ | 7.5 | 2112.2 ± 0.4 |
| $D_{s}^{*+}(1^{1}P_{1})$ | 0.2 | 2535.10 ± 0.06 |
| $D_{s}^{*+}(1^{3}P_{2})$ | 1.1 | 2569.1 ± 0.8 |
| $D_{s}^{*+}(2^{1}S_{0})$ | 42.1 | 2708.3±4.0 |
| (2$^{1}S_{1}$) | 4.4 | |

In Table IV we list the results of the contributions from the instanton-induced potentials. While they have visible effects on the masses of the $D^{+}$ and $D_{s}^{+}$ mesons, and marginal contributions to the radially excited $S$-wave $D^{+}(2^{1}S_{0})$ and $D_{s}^{*+}(2^{1}S_{0})$ mesons, they have almost no impact on other excited $D$ and $D_{s}$ mesons. Thus, in conclusion, the present form of the instanton-induced potentials contributes to some of the $D$ and $D_{s}$ mesons as explicitly shown in Tables III and IV: its overall effects turn out to be marginal. Possible ways of improving the present results will be mentioned in the next Section.
IV. SUMMARY AND OUTLOOK

In the present work, we have investigate the effects of the heavy-light quark potential from the instanton vacuum on the mass spectra of the conventional charmed mesons. First, we have considered the confining potential that is proportional to the relative distance between the heavy and light quarks. The Coulomb-like potential, which arises from one-gluon exchange, has been included. The spin-dependent potentials were generated from the central part. Then we have computed the mass spectra of the charmed mesons, employing the Gaussian expansion method to solve the nonrelativistic Schrödinger equation. The results are in good agreement with the experimental data even without the potential from the instanton vacuum included. Then, we have introduced the central and spin-dependent potentials from the instanton vacuum. The additional spin part of the potential was obtained from the central part of the instanton-induced potential. While the instanton effects are noticeable on the $S$-wave charmed and charmed strange heavy mesons, the contribution from the instanton-induced potential is rather tiny to their masses.

Though the present form of the instanton-induced potential does not give any significant contribution to the heavy meson masses, there are some possible ways of elaborating the present analysis:

- The present work is based on the nonrelativistic Schrödinger equation, since we aim mainly at investigating the effects of the instanton-induced potential. However, once the light quark is involved, it is inevitable to include certain relativistic effects.

- The instanton-induced potentials used in the present work was derived from the random instanton gas model and are given as local ones. However, if one uses the instanton liquid model, the interaction between the heavy and light quarks turn out to be nonlocal [44]. This nonlocality will have certain effects on the mass spectra of the heavy mesons.

- Recently, Ref. [45] showed that rescattering of gluons with instantons generates dynamically the effective momentum-dependent gluon mass that will cause the screened heavy-quark potential. It indicates that certain nonperturbative effects from the instanton vacuum will contribute also to the heavy-light quark system.

Thus, one needs to study systematically nonperturbative effects on both heavy mesons and heavy baryons, arising from the instanton vacuum. The corresponding investigations are under way.

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