Signatures of tunable Majorana-fermion edge states

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Abstract

Chiral Majorana-fermion modes are shown to emerge as edge excitations in a superconductor–topological-insulator hybrid structure that is subject to a magnetic field. The velocity of this mode is tunable by changing the magnetic-field magnitude and/or the superconductor’s chemical potential. We discuss how quantum-transport measurements can yield experimental signatures of these modes. A normal lead coupled to the Majorana-fermion edge state through electron tunneling induces resonant Andreev reflections from the lead to the grounded superconductor, resulting in a distinctive pattern of differential-conductance peaks.

1. Introduction

The possibility to create and study Majorana quasi-particles in condensed-matter systems has become a focus of intense attention in recent years [1–6]. Spatially localized versions of such excitations have been predicted to exist, e.g. in the $\nu = 5/2$ quantum-Hall state [7, 8], p-wave superconductors [9] such as strontium ruthenate [10–12] and semiconductor–superconductor heterostructures [13–15]. Zero-bias conductance anomalies [16–19] associated with Majorana quasi-particles have been measured recently [20–23] (see, however, [24]), and observations of an unconventional Josephson effect [25] mediated by these excitations have also been reported [26, 27].

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Chiral Majorana modes (CMMs) can also be realized as edge states in hybrid structures formed from a topological insulator (TI) [28, 29], an s-wave superconductor (S) and a ferromagnetic insulator [30–32]. The requirement of broken time-reversal symmetry and gapped excitation spectrum for the surface states in the TI is fulfilled by proximity to a ferromagnetic insulator [31, 32] or Zeeman splitting due to a magnetic field [30]. In an alternative realization, Landau quantization of the surface states’ orbital motion in a uniform perpendicular magnetic field could be the origin of the gap and breaking of time-reversal symmetry [33]. This setup avoids materials-science challenges associated with the fabrication of the hybrid structures involving three different kinds of materials and having new features enabling the manipulation of the Majorana excitation’s properties. It is shown in [33] that the velocity of the CMM can be tuned by changing the magnitude of the external magnetic field. In this paper, we further explore the properties of this tunable Majorana excitation and its signatures in typical transport experiments. We show that a normal lead coupled to this tunable CMM through electron tunneling would measure a differential conductance that oscillates as the magnitude of the external magnetic field is changed. The oscillations in the conductance arise due to the velocity tunability of this CMM. Recently, it has been proposed that this velocity tunability could be used for adiabatic quantum pumping induced by Majorana fermions revealing the chiral nature of these modes [34]. Some crucial aspects of interferometry with these CMMs are highlighted in [35].

The remainder of this paper is organized as follows. In section 2, we describe the specific interferometer-like sample geometry where Landau-quantization-induced CMMs could be probed by quantum-transport experiments. In section 3, we discuss the electronic properties of a S–TI interface that is subject to a magnetic field, showing the existence of Andreev edge states [36, 37] and emergence of a Majorana excitation among them [33]. In section 4, we apply the results of section 3 to elucidate the properties of the CMM that is present in the particular sample geometry considered here. We then present numerical results for the conductance of the system in section 5, revealing the signatures of the CMM. The final section 6 gives a brief discussion of experimental parameters and the conclusions.

2. Setup of the S–TI hybrid structure

Figure 1 illustrates the proposed sample geometry. The massless-Dirac-like surface states of a bulk three-dimensional TI material occupy the $xy$ plane. A planar contact with a superconductor (indicated by the red region in the center of the TI surface) induces a pair potential for this part of the TI surface (henceforth called the S region), while a uniform perpendicular magnetic field $B = B \hat{z}$ is present on the rest of the surface (the N region of the TI surface). We assume the magnetic field to create a finite number of vortices in the S part and neglect Zeeman splitting throughout. (A more detailed theoretical treatment of effects due to the finite magnetic field in S can be developed along the lines of previous studies [38, 39].) The dimensions of the S and N regions are denoted by $l_S \times w_S$ and $l_{TI} \times w_{TI}$ respectively. We assume

$$\{l_{TI}, w_{TI}, l_S, w_S, l_{TI} - l_S, w_{TI} - w_S\} \gg l_B,$$

where $l_B = \sqrt{\hbar/|eB|}$ denotes the magnetic length. The cyclotron motion of charge carriers in the N region hybridizes with Andreev reflection from the interface with the S region. As shown in [33], this results in the formation of chiral Dirac–Andreev edge states, similar to the ones discussed previously for an S-graphene hybrid structure [36]. The quantum description of these
edge channels reveals that one of them is associated with a chiral Majorana fermion mode with tunable velocity and guiding-center-dependent electric charge.

We now consider the situation where this CMM is tunnel coupled to an ideal normal electronic lead coming from the $z$-direction, e.g. as shown in figure 1. The interior of the S region may accommodate $n_v$ number of vortices, each permitting a magnetic flux quantum threading through and containing a Majorana bound state (MBS). The chiral Majorana fermion traveling around the boundary of the S region picks up a phase that contains information about the number of vortices and the velocity of the mode before scattering into the normal lead. Before calculating the differential conductance in our setup, we compute its excitation spectrum and demonstrate the presence of the CMM along the boundary of the S region.

3. Theoretical description of the S–TI interface in a magnetic field

Single-particle excitations in the S–N heterostructure made from TI surface states can be described by the Dirac–Bogoliubov–de Gennes (DBdG) equation [30, 33, 36]

$$\begin{pmatrix} H_D(r) - \mu & \Delta(r) \sigma_0 \\ \Delta^*(r) \sigma_0 & \mu - TH_D(r) T^{-1} \end{pmatrix} \Psi(r) = \varepsilon \Psi(r), \quad (2)$$

where the pair potential $\Delta(r) = e^{i\theta} \Delta_0$ is finite only in the S region, $\sigma_0 \equiv I_{2 \times 2}$ denoted the two-dimensional identity matrix and $H_D(r) = v_F (p + eA(r)) \cdot \sigma$ is the massless-Dirac Hamiltonian for the TI surface states. The position $r \equiv (x, y)$ and momentum $p \equiv -i\hbar(\partial_x, \partial_y)$ are restricted to the TI surface. $\sigma$ is the vector of Pauli matrices acting in spin space. Furthermore, $T$ denotes the time-reversal operator, $-e$ denotes the electron charge and $A$ denotes the vector potential associated with the magnetic field $B = \nabla \times A$. The excitation energy $\varepsilon$ is measured relative to the chemical potential $\mu$ of the superconductor, with the absolute zero of the energy set to be at the Dirac (i.e. neutrality) point of the TI surface states. The wave function $\Psi$ in equation (2) is a spinor in Dirac–Nambu space, which can be expressed explicitly in terms of spin-resolved
amplitudes as $\Psi = (u_+, u_-, v_+, -v_-)^T$. As we will confirm later, the zero-energy solution of the DBdG equation localized at the boundary between the S and the N regions constitutes the chiral Majorana excitation in our system.

To describe the uniform perpendicular magnetic field in the N region, we adopt the Landau gauge $A = B \times \hat{y}$. To explicitly verify the presence of these Majorana excitations at the boundary of our S–N heterostructure, we restrict ourselves to the right boundary of the system. Assuming that the various lengths in our system satisfy equation (1), we model this right boundary as a one-dimensional (1D) edge ($x = 0$) between two half planes. The left half-plane ($x < 0$) represents the S region and the right half plane ($x > 0$) represents the N region. Then the momentum $hq$ parallel to the interface (i.e. in $\hat{y}$ direction) is a good quantum number of the DBdG Hamiltonian, and a general eigenspinor is of the form

$$\Psi_{nq}(r) = e^{-\frac{i}{\hbar}\phi/2} e^{i q y} \Phi_{nq}(x).$$

(3)

Here $\tau_\zeta$ is a Pauli matrix acting in Nambu space, $\sigma_0$ is the identity in spin space and $n$ enumerates the energy (Landau) levels for a fixed $q$. The spinors $\Phi_{nq}(x)$ are solutions of the 1D DBdG equation $H(q) \Phi_{nq}(x) = \varepsilon \Phi_{nq}(x)$, with

$$H(q) = \hbar v_F \left\{ \sigma_x \otimes \tau_\zeta(-i) \delta_x + \sigma_y \otimes \left[ \tau_\zeta q + \tau_0 \frac{eB}{\hbar} x \Theta(x) \right] \right\}$$

$$- \mu \sigma_0 \otimes \tau_\zeta + \Delta_0 \Theta(-x) \sigma_0 \otimes \tau_x,$$

(4)

where the $\tau_j$ are Pauli matrices acting in Nambu space and $\tau_0$ is the identity matrix in Nambu space. The spectrum of LL eigenenergies $\varepsilon_{nq}$ and the explicit expressions for $\Phi_{nq}(x)$ in the N and S regions can then be obtained by demanding the continuity of the wavefunction at the S–N interface.

The functional form of the solutions to this 1D DBdG equation decaying for $x \to \infty$ (in the region $x > 0$) can be expressed as

$$\Psi(x, y) = e^{i q y} \begin{pmatrix} -i C_e e^{-(x+q)^2/2} (\mu + \varepsilon) H_{(\mu+\varepsilon)^2/2-1}(x+q) e^{i \phi/2} \\ C_e e^{-(x+q)^2/2} H_{(\mu+\varepsilon)^2/2}(x+q) e^{i \phi/2} \\ C_h e^{-(x-q)^2/2} H_{(\mu-\varepsilon)^2/2}(x-q) e^{-i \phi/2} \\ -i C_h e^{-(x-q)^2/2} (\mu - \varepsilon) H_{(\mu-\varepsilon)^2/2-1}(x-q) e^{-i \phi/2} \end{pmatrix}$$

(5)

with $H_s(x)$ denoting the Hermite function [36]. The complete solution is then obtained from the requirement of particle-current conservation across the interface. Similar to [36] we find

$$C_e = \frac{-i \Delta_0 C_h (\mu - \varepsilon) H_{(\mu-\varepsilon)^2/2-1}(-q)}{H_{(\mu+\varepsilon)^2/2}(q) \varepsilon + (\mu + \varepsilon) H_{(\mu+\varepsilon)^2/2-1}(q) \sqrt{\Delta_0^2 - \varepsilon^2}}$$

(6)

and the dispersion relationship is given by the solutions of

$$f_{\mu+\varepsilon}(q) - f_{\mu-\varepsilon}(-q) = \frac{\varepsilon \left[ f_{\mu+\varepsilon}(q) f_{\mu-\varepsilon}(-q) + 1 \right]}{\sqrt{\Delta_0^2 - \varepsilon^2}},$$

(7)
Figure 2. Dispersion relationship \( \varepsilon_{\eta q} \) of single-particle excitations at the S–N junction on a TI surface that is subject to a strong perpendicular magnetic field. Only the zeroth Landau level (LL) is shown for various values of \( \mu \) as indicated in the legend. The magnitude of the pair potential is \( \Delta_0 = 5\hbar v_F/l_B \), and \( l_B \equiv \sqrt{\hbar/|eB|} \) denotes the magnetic length.

where

\[
f_\alpha(q) = \frac{H_{\alpha^2/2}(q)}{\alpha H_{\alpha^2/2-1}(q)}. \tag{8}\]

The solutions \( \varepsilon_n(q) \) of equation (7) can be labeled with a (LL) index \( n = 0, \pm 1, \pm 2, \ldots \). Figure 2 shows the zeroth LL \( (n = 0) \) for \( \mu = 0.25, 0.5, 1.0 \) and 1.5 \( \hbar v_F/l_B \) for \( \Delta_0 = 5\hbar v_F/l_B \). For \( \mu = 0 \), we obtain the familiar dispersionless LLs at \( 0, \pm \sqrt{2}\hbar v_F/l_B, \ldots \) [36]. Away from the edge the various LLs saturate at \( \sqrt{2}(\hbar v_F/l_B)\text{sgn}(n)\sqrt{|n| - \mu} \) for \( q \to -\infty \) and \( \sqrt{2}(\hbar v_F/l_B)\text{sgn}(n)\sqrt{|n| + \mu} \) for \( q \to \infty \). This suggests that an interesting regime can be reached by increasing \( \mu \), so that \( n = \pm 1 \) levels start contributing to the low-energy excitations of the system (as shown in [33]). When the chemical potential \( \mu \) is finite (as measured from the charge-neutrality point of the free Dirac system), the LLs acquire a dispersion around \( q = 0 \) that signals the existence of Andreev edge excitations [36, 37].

For the special case of \( \varepsilon = 0 \) and \( q = 0 \), we obtain \( C_h = iC_e \), and this zero-energy state can then be expressed as

\[
\Psi_{00}(x) = C_e e^{- \frac{x^2}{\sigma}} \begin{pmatrix}
-i\mu H_{\mu^2/2-1}(x) e^{i\phi/2} \\
H_{\mu^2/2}(x) e^{i\phi/2} \\
iH_{\mu^2/2}(x) e^{-i\phi/2} \\
\mu H_{\mu^2/2-1}(x) e^{-i\phi/2}
\end{pmatrix}. \tag{9}\]

The particle–hole-conjugation operator is given by \( \Xi = \sigma_y \tau_y K \) [30], where \( \sigma_j \) and \( \tau_j \) are again the Pauli matrices acting on spin and particle–hole space, respectively, and \( K \) symbolizes complex conjugation. Straightforward verification establishes \( \Xi \Psi_{00} = -i\Psi_{00} \), and \( \Xi \Xi \Psi_{00} = \Psi_{00} \). Hence the state \( \Psi_{00} \) is a Majorana fermion.
4. CMM in our sample geometry

A general symmetry property [40] of the DBdG equation mandates that, for any eigenstate $\Psi_{nq}$ with the excitation energy $\varepsilon_{nq}$, its particle–hole conjugate $\Xi \Psi_{nq}$ in Nambu space is also an eigenstate and has the excitation energy $-\varepsilon_{nq}$. This symmetry implies that the zero-energy state with quantum numbers $n = 0$ and $q = 0$ is its own particle–hole conjugate in our system, thus exhibiting the defining property of a Majorana fermion [1, 2]. While the Majorana state $\Psi_{00}(r)$ has a localized spatial profile in the direction perpendicular to the S–N junction (i.e. along the $x$–axis), it is completely delocalized in the direction parallel to the S–N interface. Similarly, one can show the existence of the chiral Majorana edge excitation all around the boundary of the S region, as is required by the particle–hole symmetry at zero energy. Thus the CMM encloses the entire S region that is created on the TI surface via the proximity effect. In principle, the S region may include $n_v$ vortices, each supporting a MBS at zero energy. The coupling between these MBSs and the CMM discussed above decays exponentially as a function of their spatial separation. Thus the presence of vortices will not affect the Majorana edge mode in the typical situation where the vortices are located far from the edge.

The chiral Majorana edge excitation is characterized by its velocity $v_M$. As the chemical potential approaches zero, the edge dispersion of the zeroth LL flattens out, implying a very small Majorana-mode velocity ($v_M \sim 0$). On the other hand, increasing the chemical potential sharpens the edge dispersion and increases $v_M$ (see figure 2). In previously considered situations where CMMs emerge [31], the Majorana-mode velocity could be adjusted by changing the magnitude of the magnetization in a ferromagnetic insulator. However this is rather difficult to perform experimentally. Our setup offers a more controllable route to tune this velocity by changing the magnitude of the external magnetic field.

The semiclassical cyclotron trajectories of electrons and Andreev reflected holes for this system are shown in figure 3, where we consider a finite positive value for $\mu$. Figure 3(a) shows the conical dispersion relationship describing the topologically protected surface states of the TI. Due to the proximity effect a gap of magnitude $2\Delta_0$ opens up at $\mu$. First we consider $\Delta_0 > \mu$. In this regime, there exist two kinds of cyclotron trajectories. In the first case $\epsilon > \mu$ as shown in figure 3(b), the electron and its conjugate Andreev-reflected hole are from different (conduction and valence) bands defined with respect to the Dirac point. The semiclassical cyclotron trajectories for the electrons and the Andreev-reflected holes in this case travel in opposite directions. In the second case $\epsilon < \mu$, both the electron and its conjugate Andreev-reflected hole belong to the same (e.g. the conduction) band and move in the same direction. The chiral modes we discuss belong to the latter situation and are present even if $\Delta_0 < \mu$. However, by increasing $\Delta_0$, we can realize a regime with multiple LLs within the gap. In this regime the coupling between the nearest LLs enriches the dynamics of the system. The low-energy excitations (close to the chemical potential) can then be described by an effective envelope-function Hamiltonian including $n = 0$ and $\pm 1$, as discussed in [33].

5. Conductance of the CMM interferometer

The differential conductance of our setup (as shown in figure 1) can be calculated within the scattering matrix formalism [41, 42]. Ignoring the coupling between the MBSs and the chiral Majorana edge state enclosing the S region, the effect of vortices is to account for an additional contribution $\phi_v = n_v \pi$ to the total phase $\phi$ that the chiral Majorana edge fermion picks up after
Figure 3. Semiclassical picture of Dirac–Andreev edge states [36]. (a) Conical dispersion relationship representing the surface states of the TI, consisting of a top cone (conduction band) and a symmetric bottom cone (valence band), which touch at the Dirac point. A proximity-induced superconducting gap of $2\Delta_0$ opens up at the chemical potential $\mu$. (b) If the excitation energy $\varepsilon > \mu$, the electron and the Andreev-reflected hole belong to the conduction and valence band, respectively, and the edge channel is not chiral. (Solid and dashed lines represent electron and hole trajectories, respectively.) (c) If $\varepsilon < \mu$, electrons and Andreev-reflected holes belong to the same band, and their skipping orbits form a chiral Andreev edge channel.

traversing the entire boundary of the S region. Including an additional phase of $\pi$ coming from the Berry phase and the dynamic phase, the total phase acquired during a round trip is

$$\phi = n_\nu \pi + \pi + \frac{E L}{\hbar v_M},$$

where $L$ is the circumference of the S region and $E$ is the energy of the propagating CMM state. The normal lead in this setup plays the role of both electron and hole lead. The tunneling current from the leads to the grounded superconductor can be calculated straightforwardly by working on the Majorana basis [42]. An incoming electron or hole from the normal lead can be expressed in terms of artificial Majorana modes $\eta_1$ and $\eta_2$. On the Majorana basis, only one of the Majorana modes in the normal lead ($\eta_1$ or $\eta_2$) is coupled to the CMM going around the S region, the other is reflected with amplitude 1 [42]. Accounting for multiple reflections of the Majorana mode in the lead, the scattering matrix in the Majorana basis is [42]

$$S_M = \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix},$$

where $r_2 = 1$ and

$$r_1 = \frac{r_0 - e^{i\phi}}{1 - r_0 e^{i\phi}}.$$
Figure 4. Contour plot of the differential conductance \((dI/dV)/(e^2/h)\) as a function of the dimensionless magnetic field denoted by \(\beta = \sqrt{\hbar |eB|v_F/\Delta_0}\) and \(eV\) measured in units of \(\pi \hbar v_F/L\). The reflection amplitude \(r_0\) of the locally coupled normal lead is set to 0.9, the chemical potential is chosen to be \(\mu = 0.5\Delta_0\), and the S region encloses an even number of vortices.

We calculate the differential conductance of our setup by numerically evaluating the velocity of the CMM in the zeroth LL
\[
v_M = \frac{\partial \varepsilon_n(q)}{\partial q} \bigg|_{q \to 0}. \tag{14}
\]
We assume that \(v_M\) is constant all around the S region and determine it from the solution of equation (7). Figure 4 shows a contour plot of the differential conductance \((dI/dV)\) in units of \(e^2/h\) as a function of the dimensionless magnetic field \(\beta = \sqrt{\hbar |eB|v_F/\Delta_0}\), and \(eV\) measured in units of \(\pi \hbar v_F/L\). The chemical potential is chosen to be \(\mu = 0.5\Delta_0\), \(r_0 = 0.9\), and we consider an even number of vortices. The apparent highly nonlinear behavior arises from the Majorana-fermion-induced resonant Andreev reflection (MIRAR) \([18]\) in conjunction with the variation of \(v_M\) as the magnitude of magnetic field changes. The differential conductance is periodic in \(eV\), and the period depends on the CMM velocity. The nonlinear behavior due to MIRAR along the \(eV\) axis was investigated in detail in \([18]\). In our setup, \(dI/dV\) shows a nonlinear oscillating behavior as a function of the magnetic-field-dependent parameter \(\beta\) also. The origin of these oscillations can be traced back to the magnetic-field dependence of \(v_M\) as shown in \([33]\). Recent studies \([34]\) have indicated that such a velocity modulation could be used as a pump parameter in an adiabatic-quantum-pumping setup where the pumped current is induced by the CMM.

We have also calculated the differential conductance for the case of an odd number of vortices present in the S region, see figure 5. In the zero-bias limit, \(dI/dV|_{eV \to 0} \to 2e^2/h\). In contrast, for the case of an even number of vortices shown in figure 4, \(dI/dV|_{eV \to 0} \to 0\). It has
been suggested that this jump in conductance with the parity of the number of vortices is a clear signature of the Majorana mode [18].

Along similar lines we calculate the differential conductance of our setup for a fixed value of magnetic field $\beta = \sqrt{\hbar|eB|v_F}/\Delta_0 = 0.5$, as a function of $\mu$ and $eV$. The results are shown in figure 6 for an even number of vortices and in figure 7 for an odd number of vortices. In the zero-bias limit, we once again observe a jump in the differential conductance from $2e^2/h$ to 0 when the number of vortices changes from odd to even. We also observe oscillations as the chemical potential changes which can be traced back to oscillations in $\nu_M$ [33].
6. Discussion and summary

Finally, we would like to comment upon the experimental feasibility of realizing the device setup suggested here. Using typical values such as $\Delta_0 = 0.7 \text{ meV}$ [43], $v_F = 5 \times 10^5 \text{ m s}^{-1}$ [43], and a magnetic field of 1 T, we obtain $\hbar v_F / l_B \sim 10 \text{ meV}$, implying that the observation of the CMM discussed here is within the reach of current experimental efforts. However, realizing a regime with multiple LLs within the gap necessitates an increase in the proximity-induced superconducting gap $\Delta_0$. To ensure that a finite temperature $T$ does not completely smear out the effects we discussed, we have to ensure that $k_B T \ll \{\Delta_0, \hbar v_F / l_B, \mu\}$, where $k_B$ is the Boltzmann constant. Therefore, measurements at sub-Kelvin temperatures will be required for these experiments.

We have ignored disorder in our calculation. Disorder in the system will result in fluctuations of the Dirac-point energy which will translate into a spatially varying chemical potential. Majorana excitations will still exist but will be subject to a fluctuating velocity. Due to the topological nature of the chiral Majorana edge excitation we expect its largely unimpeded propagation even under such circumstances. In the limit of strong disorder the fluctuations in the Majorana mode velocity may suppress the oscillations described in figures 4–7.

In addition to creating the S region via the proximity effect, the planar contact with a superconducting material on top of the TI surface is likely to also induce band bending in the TI surface state beneath it. In the likely case where the resulting potential gradient is smooth on the scale of the magnetic length, the electronic structure of the Majorana mode discussed here will be largely unaffected. Furthermore, as normal reflection is suppressed for states with guiding centers close to the interface because of Klein tunneling [44], the Majorana-mode velocity will, in general, be most dominantly determined by Andreev reflection.

Our theory applies to a situation without vortices or a small number of them, i.e. for magnetic fields below or just above the first critical field $B_{c1}$ of the superconductor. For $B < B_{c1}$, there are no vortices in the superconducting region, and the conductance for this case corresponds to that shown in figures 4 and 6. For fields just above $B_{c1}$, the vortices will
be separated by distances larger than the magnetic penetration depth $\lambda$. For large $\kappa = \lambda / \xi$, where $\xi$ is the coherence length of the superconductor, the coupling between the MBSs at the vortices can be safely ignored. However, if $\kappa \sim 1$, the hybridization between MBSs within the vortices can change our results significantly. From the basic relationships between $B_{c1}$ and the thermodynamical critical field $B_c$, in the large-$\kappa$-limit, and between $B_c$ and the condensation energy [45], the above considerations impose a limiting condition on the dimensionless magnetic-field-dependent parameter $\beta$ used in our calculations

$$\beta \lesssim \frac{\ln \kappa}{\kappa} \frac{\mathcal{N}[(eV \text{ nm}^3)^{-1}]}{B_{c1}[T]}.$$  \hspace{1cm} (15)

Here $\mathcal{N}$ is the normal-state density of states at the Fermi energy for the superconducting material. In available material systems, the right-hand side of equation (15) can be larger than 1. Thus our theoretical description is valid for the range of values of $\beta$ shown in figures 4 and 5.

In conclusion, we have studied quantum transport in a S–TI hybrid structure in the presence of a perpendicular magnetic field. We have shown that Landau quantization results in the emergence of a tunable CMM at the edge of the superconducting region induced on the surface of the TI. We find that the velocity of this mode can be tuned by changing the magnitude of the external magnetic field and/or the chemical potential of the superconductor. The velocity tunability gives rise to unique signatures in the differential conductance of the system when the Majorana edge mode is coupled to a normal electronic lead. Experimental verification of the tunability of the velocity and the detailed structure of the differential conductance will provide a new platform to explore Majorana physics.

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