Divergence of the effective mass near a density wave instability in a MOSFET system.

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We study the renormalization of the Fermi-liquid parameters in the vicinity of a density wave quantum phase transition, which should occur in MOSFET systems at low densities. First, using a perturbative RPA treatment of fluctuations, we calculate the electronic self-energy and show that the effective mass diverges at the density wave transition point. Second, we go beyond perturbation theory, making use of the exact Pitaevskii identities. Within this exact analysis, we also find a divergence of the effective mass, which occurs at higher densities in the fluctuation region, as compared to the perturbation theory. This result signals the break-down of conventional Fermi-liquid description in the vicinity of the transition point. The divergence of the effective mass gives rise to a singular behavior of the electronic compressibility. We suggest that the experimentally observed enhancement of the effective mass is a precursor to a second order thermodynamic phase transition into a glassy density wave state.

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I. INTRODUCTION

Novel experiments in two-dimensional high-mobility electron systems have provided a great variety of remarkable results which do not fit into the framework of the conventional theory of fermion systems. The most prominent effect is indeed the metal-insulator crossover observed in such systems, which clearly contradicts to the scaling theory of localization and a lot of attempts have been made to reveal the underlying nature of the phenomenon. Apart from the unexpected metallic regime, there is a number of other surprising experimental results, such as the sharply increasing effective mass at low electron densities in silicon metal-oxide-semiconductor field-effect transistors (MOSFETs). There are also experiments clearly showing some unusual behavior in thermodynamic properties such as the local electronic compressibility suggesting the existence of a second order thermodynamic phase transition in the system, which, if indeed exists, should be related to the quantum crossover observed in the transport behavior.

A successful theory describing the peculiar physics of the high-mobility electron systems must be based on the correct choice of the quantum ground state of the system. It is clear now that traditional paramagnetic Landau Fermi liquid theory in its perturbative form does not provide an adequate explanation of the observed physics. One is forced to assume the existence of a density-driven quantum phase transition(s) in the two-dimensional electron system.

The competition between the potential and kinetic energies of a pure electron system yields the first order phase transition into the Wigner solid phase at very low densities. This fact was realized a very long time ago but the theoretical description of the liquid-solid phase transition is still missing and only numerical studies are available. These numerical studies derive extremely high values of the Wigner transition RPA parameter: $r_s \approx 110$ and $r_s \approx 37$ in three and two dimensions correspondingly. There are also numerical works which suggest that in between Wigner crystal and usual Landau Fermi gas there may be an intermediate phase, the charge or spin-density wave phase, in which a stable charge (spin) density modulation exists in the ground state with some wave-vector $Q \lesssim 2p_F$:

$$n_{\uparrow\downarrow}(r) = n_0 \left[ 1 + \Delta \cos (Q r + \delta_{\uparrow\downarrow}) \right],$$

where for the charge-density wave: $\delta_{\uparrow} = \delta_{\downarrow}$ and for the spin-density wave: $\delta_{\uparrow} - \delta_{\downarrow} = \pi$. Parameter $\Delta$ serves as the order parameter for the phase transition. Usually, a density wave transition is associated with the Peierls instability in effectively one-dimensional systems which occurs due to the strong electron-phonon coupling. Nevertheless, such a state may appear in higher dimensional electron systems within a jellium-like model, as was first pointed out in Ref. 13. At low electron densities, there is a region where a density wave state is energetically more favorable compared both to the uniform density distribution corresponding to usual Landau Fermi gas and to Wigner crystal. Numerical simulations of Świerkowski et al showed that in double-layer and multi-layer systems, the transition into the charge-density wave state occurs at $r_s \sim 10-20$, i.e. much earlier than the Wigner crystal phase transition is expected. The charge-density wave state in an isotropic system is possible only if the neutralizing “jellium” is polarizable, while the spin-density wave should occur for any elasticity of the positive background. In a MOSFET system, the only possible candidate for such a polarizable background is the charge distributed on the metal-gates, which is separated from the two-dimensional system of interest by an oxide dielectric barrier, usually SiO$_2$, of some finite thickness $d$. Indeed, if the thickness is very large compared to the typical inter-electron distance, the “jellium” on the gates does not “see” any changes of the electron density at the length-scale of $1/p_F$. 

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and any charge-density modulations should become unstable. However, if the oxide layer is thin enough, there should exist a critical density at which a phase transition into the charge-density phase occurs. This corresponds to the experiments of Tsui et al. where a divergence of the effective mass was reported for a thin-oxide MOSFET. In the majority of other experiments, the thickness of the oxide is about \( d \sim 10^3 \text{Å} \), which is much greater than the typical inter-electron distance, and the transition into the spin-density wave phase is anticipated. Thus, independently of the geometry of a two-dimensional structure, a density-wave phase may exist separating the paramagnetic Fermi liquid and Wigner solid phases.

The subject matter of the present paper is to explore the behavior of the Landau Fermi-liquid theory parameters as one approaches a density wave transition. Our main finding is that the Fermi-liquid theory apparently breaks down in vicinity of the transition point as the effective mass diverges, which leads to the instability of the usual Fermi-step density distribution even at \( T = 0 \). We suggest that the observed enhancement of the effective mass is a precursor of a density wave transition in the two-dimensional systems.

Our paper is structured as follows: In Sec. II, we study the renormalization of the quasiparticle Z-factor and the effective mass within the self-consistent RPA approximation in the vicinity of the charge-density wave transition. This method of treating fluctuations is in the spirit of the classical paper of Doniach and Engleberg, who considered the ferromagnetic transition case, and also similar to Ref. 13, where the self-energy function was studied near the antiferromagnetic transition in connection with the physics of high-\( T_c \) cuprates. First, we derive general expressions for the self-energy function. Second, we discuss the structure of the appropriate RPA density-wave fluctuation propagator. The actual calculations are done for the charge density wave case and it is shown that within the RPA approach the quasiparticle Z-factor vanishes at criticality (c. f., Ref. 17) and the effective mass diverges as \( 1/Z \) at the critical point. In Sec. III, we study the fluctuation effects going beyond conventional RPA perturbation scheme, by using some exact formulae from the Landau Fermi-liquid theory. Namely, on the basis of the Pitaevskii identities, we prove that for a density-wave phase transition (and more generally, for any phase transition, which breaks the translational invariance of an otherwise isotropic system) the renormalization of the derivative (\( \partial \Sigma / \partial \omega \)) is never singular and the Z-factor formally never vanishes as distinct of the RPA result. The effective mass does diverge in the fluctuation region near the phase transition, but not at the criticality itself, again, opposed to the RPA conclusion. At this point, the group velocity of quasiparticles changes sign and the conventional Fermi-liquid theory breaks down. Moreover, at this point some thermodynamic quantities, such as the electronic compressibility, experience a singular behavior. In Sec. IV, which is the conclusive part, we summarize our findings and briefly discuss our understanding of possible physics in the density-wave phase.

II. RPA-TREATMENT OF THE DENSITY-WAVE FLUCTUATIONS

A. Fermi-liquid theory distilled

The effective Fermi-liquid parameters follow from the quasiparticle Green function, defined as usual,

\[
G_c (t_1, r_1; t_2, r_2) = -i \langle T \psi (t_1, r_1) \psi^\dagger (t_2, r_2) \rangle ,
\]

where \( T \) is the time-ordering operator and \( \psi \) and \( \psi^\dagger \) are electron field operators. In what follows, we will use subscript “c” to denote a time-ordered quantity and subscripts “R” and “A” to denote a retarded and advanced quantities correspondingly. The Green function is in turn determined by the self-energy function \( \Sigma (\varepsilon, p) \) as follows:

\[
G_c (\varepsilon, p) = \frac{1}{\varepsilon - \xi (0) (p) - \Sigma_c (\varepsilon, p)}.
\]

In the vicinity of the Fermi-momentum, Eq. 3 can be re-written as

\[
G_c (\varepsilon, p) = \frac{Z}{\varepsilon - \xi (p) \pm i \gamma (\varepsilon, p)} ,
\]

where \( Z \in [0, 1] \) is the jump in the quasiparticle distribution function at the Fermi level, which is determined by the following identity:

\[
Z^{-1} = 1 - \left[ \frac{\partial}{\partial \varepsilon} \text{Re} \Sigma_R (\varepsilon, p_F) \right]_{\varepsilon=0} \tag{5}
\]

and \( \xi (p) = \langle |p| - p_F \rangle / m_c \) is the quasiparticle spectrum, with the effective mass defined via the following relation

\[
\frac{m_c}{m} = \frac{1}{Z} \left\{ 1 - \left[ \frac{\partial}{\partial \xi_c} \text{Re} \Sigma_R (0, p) \right]_{p=p_F} \right\}^{-1} .
\]

The parameter \( \gamma = Z \text{Im} \Sigma_c (\varepsilon, p_F) \) determines the quasiparticle life-time and should be small compared to the energy of a quasiparticle, for the latter to be well-defined. Under this assumption, we can write the retarded Green function in the following form, which shall be used in further actual calculations:

\[
G_R (\varepsilon, p) = P \frac{Z}{\varepsilon - \xi (p)} - i \pi Z \delta (\varepsilon - \xi (p)) \tag{7},
\]

where symbol \( P \) stands for the principal value of the corresponding integral.

Within the perturbative treatment of fluctuations, only the first order diagram (containing one fluctuation propagator) for the self-energy is retained. The
corresponding analytically continued retarded self-energy function reads (for \( \varepsilon > 0 \))\(^{21}\):

\[
\Sigma_R (\varepsilon, \mathbf{p}) = -\frac{1}{(2\pi)^3} \int d^2 q \int d\omega \mathcal{G} (\omega + \varepsilon, \mathbf{p} - \mathbf{q}) D_R (\omega, \mathbf{q}) \tanh \frac{\omega}{2T} + G_R (\omega + \varepsilon, \mathbf{p} - \mathbf{q}) \text{Im} D_R (\omega, \mathbf{q}) \coth \frac{\omega}{2T} \right\}.
\]

(8)

The interaction \( D (\omega, \mathbf{q}) \) changes the properties of the system and renormalizes the Fermi-liquid theory parameters. In the case under discussion, the density wave fluctuation propagator will be used as the appropriate effective interaction term.

From now on, we will consider zero temperature case only, so that the tangent and cotangent factors in Eq. (8) reduce to theta-functions. Moreover, using Eqs. (4) and (5), we can evaluate the integral over the directions of \( \mathbf{q} \) exactly (provided that the interaction is isotropic) and obtain the following expression for the real part of the self-energy, which is quite involved:

\[
\text{Re} \Sigma_R (\varepsilon, \mathbf{p}) = \Sigma_1 (\mathbf{p}, \varepsilon) + \Sigma_2 (\mathbf{p}, \varepsilon),
\]

(9)

where

\[
\Sigma_1 (\varepsilon, \mathbf{p}) = \frac{Z \nu}{\pi} \int d\omega \text{sgn} (\omega + \varepsilon) \int dq q_1 (\Omega) \frac{q \text{Re} D_R (\omega, q)}{\sqrt{[q^2 - q_1^2 (\Omega)][q_2^2 (\Omega) - q^2]}}
\]

(10)

and

\[
\Sigma_2 (\varepsilon, \mathbf{p}) = -\frac{Z \nu}{\pi} \int d\omega \text{sgn} \int dq q_1 (\Omega) \text{Im} D_R (\omega, q) \frac{\text{Im} D_R (\omega, q)}{\sqrt{[q^2 - q_1^2 (\Omega)][q_2^2 (\Omega) - q^2]}}
\]

(11)

where

\[
q_{1,2} (\Omega) = \sqrt{2} p_F \left[ 1 + \frac{\Omega}{2E_F} \right]^{1/2} \left[ 1 + \frac{\Omega}{E_F} \right]^{1/2}
\]

(12)

and \( \Omega = (\omega + \varepsilon - \xi_p) \), with \( \nu \) being the density of states at the Fermi-line and \( E_F \) the Fermi energy.

Using Eqs. (7), (10), and (11), one can prove the following useful identity:

\[
\left[ \frac{\partial}{\partial \varepsilon} + \frac{\partial}{\partial \xi_p} \right] \Sigma_R (\varepsilon, \mathbf{p}) \bigg|_{p = p_F \xi_p = 0} = \frac{2Z \nu}{\pi} \int_0^{2p_F} dq D (0, q) \frac{(2p_F^2 - q^2)}{\sqrt{(2p_F^2 - q^2)}}.
\]

(13)

Let us note, that the right-hand side of Eq. (13) is very large, provided that the effective propagator is singular in the static limit.

### B. Charge-density wave fluctuation propagator.

Now let us consider a thin-oxide MOSFET in which the two-dimensional electron system is separated from the metallic gates by a barrier of width \( d \), which we suppose to be not too large compared to the typical inter-electron distance. In this case, the charge on the metallic gates serves an elastic (not rigid) neutralizing background, which can compensate the energy cost produced by inhomogeneities in the electron density distribution in the two-dimensional electron system. At sufficiently low electron densities, one anticipates a transition into the charge-density wave state \( \text{CDW} \) which should be energetically more favorable than the homogeneous distribution. Below the critical density \( n_c \) the initial Galilean invariance of the system is broken. However, above the transition the system is still isotropic and only fluctuating density waves can exist with no preferable direction. The appropriate static fluctuation propagator, quite generally, has the form:

\[
D (\omega, q) \propto -\frac{1}{\xi^2 + (q - Q)^2},
\]

(14)

where \( \xi \) is the charge-density wave coherence length, which diverges at the transition and \( Q \sim 2p_F \). The static part of the propagator \( \text{CDW} \) has a typical Lorentzian form (see, e.g., Ref. [22]) and, as we noted, should be still isotropic above the transition as the Galilean invariance is preserved until the transition point: \( n = n_c \). The transition itself is thermodynamic in nature and is about the sign of the difference between the two energies of the homogeneous distribution and distribution \( \text{CDW} \).

The RPA philosophy assumes that the dynamic part of the fluctuation propagator, which is not crucial for the transition itself, can be obtained by summing up a series of diagrams containing the RPA polarizability bubbles, which generate the required frequency dependence. Taking into account these considerations, we write the RPA charge-density wave fluctuation propagator in the following form:

\[
D (\omega, q) = -\frac{\lambda}{\nu \alpha + [1 - q/Q]^2} + iP (\omega, q),
\]

(15)

where \( \alpha \sim (n - n_c)/n_c \) is the Ginzburg-Landau-like coefficient, which changes sign at the transition point, \( \nu \) is the bare density of states at the Fermi-line and the imaginary part \( iP \) has the form

\[
P (\omega, q) = \frac{\Lambda}{\nu \alpha} \text{Im} \Pi (\omega, q)
\]

where \( \Pi (\omega, q) \) is the polarizability bubble and \( \lambda \) and \( \Lambda \) are some dimensionless constants, which may depend on \( r_s \) only. The polarizability is defined as usual (here, we use Matsubara notations):

\[
\Pi (\omega_m, q) = 2 \sum_{\varepsilon_n} \int \frac{d^2 k}{(2\pi)^2} \mathcal{G} (\varepsilon_n, k) \mathcal{G} (\varepsilon_n+\omega_m, k + q).
\]

(16)
For a two-dimensional electron gas at $T = 0$, this quantity was calculated by Stern.\textsuperscript{23} We do not need its exact expression. Let us only emphasize, that the case $Q \sim 2p_F$ may be different from the case $Q \equiv 2p_F$, since the polarizability is a non-analytic function of frequency at $q = 2p_F$ and the mixture of the two non-analyticities makes the problem technically more cumbersome. (for Stern’s polarizability\textsuperscript{23} $\Im \Pi (\omega, 2p_F) \sim \sqrt{\omega} \sgn \omega$). Quite generally, $Q < p_F$\textsuperscript{10,11} in which case we can expand the propagator at low frequencies to obtain:

$$P(\omega) \approx \nu Z^2 \left( \frac{\omega}{\omega_0} \right), \quad \text{as } \omega \to 0,$$

where $\omega_0$ is a constant, which does not depend on the closeness to the critical point, $Z$ is the quasiparticle $Z$-factor, and $\nu = m_\ast / (2\pi)$ is the density of states, both to be found.

In the case of a spin-density wave transition, which may occur in a MOSFET with a large oxide barrier, the $\nu$ factor, and $\nu$ in which case we can expand the propagator at low frequencies to obtain:

$$D^{\text{SDW}}_{\alpha\beta} (\omega, q) \propto \frac{\sigma_{\alpha\gamma} \sigma_{\gamma\beta}}{\xi^{-2} + (q - Q)^2 + iP(\omega, q)},$$

In what follows, we will do explicit calculations for the spin-density wave case is straightforward and the qualitative results are identical.

C. Renormalized $Z$-factor and effective mass

Now, we are at the position to calculate the renormalization of the Fermi liquid parameters due to charge-density fluctuations within the RPA-approach. Identity\textsuperscript{13} allows us to calculate only one derivative, the other will be automatically extracted from Eq. (13). Let us focus on the $\varepsilon$-derivative, which determines the renormalization of the quasiparticle $Z$-factor. From Eq. (10), we have

$$\frac{\partial}{\partial \varepsilon} \Lambda (\varepsilon, p) = \frac{Z\nu}{\pi} \int_{-\infty}^{\infty} dq \int_{q_1(\omega)} dq \frac{dq}{q_1(\omega)} \int \frac{dq}{(q^2 - q_1^2)(q_2^2 - q^2)} \times \frac{\partial}{\partial \varepsilon} \Re D_R(\omega - \varepsilon, q),$$

where functions $q_{1,2}(\omega)$ are defined in Eq. (12). Let us consider the region in the immediate vicinity of the transition only so that $\alpha \ll (2p_F - Q) / 2p_F$. Then, the integrals in Eq. (10) and (11) are determined by low frequencies $\omega \sim \alpha / Z^2 \to 0$. Using the following properties of any physical propagator

$$\Im D_R(\omega, q) = - \Im D_R(-\omega, q)$$

and

$$\Re D_R(\omega, q) = \Re D_R(-\omega, q),$$

one can see that expression (10) is identical to the right-hand side of Eq. (13). Eq. (11) is exactly zero as the principal value of the corresponding integral. In the leading order, we obtain the following results:

$$\frac{\partial}{\partial \varepsilon} \Re \Lambda (\varepsilon, p) \bigg|_{\varepsilon = 0} = - \frac{2\lambda}{\sqrt{1 - (Q / 2p_F)^2} \sqrt{\alpha} Z} \frac{Z^2(\alpha)}{Z - 1} = 0,$$

and

$$\frac{\partial}{\partial \varepsilon} \Re \Lambda (0, p) \bigg|_{p = p_F} = \Re \frac{Z(\alpha)}{Z} \to 0,$$

where we remind that $\alpha = (n - n_c) / n_c$ is a small deviation from the charge-density wave transition point.

Let us mention that besides the singular fluctuation effects we studied so far there are other contributions coming from the $q \to 0$ channel. Such corrections have been considered previously by many authors (see e.g., Ref. \textsuperscript{24}) and it was found that they do increase the effective mass by a factor of two or so (Ting et al.\textsuperscript{25} predict $m_\ast / m \approx 2.0$ for $r_s = 5$) but do not lead to any divergent behavior. Let us denote the corresponding renormalized effective mass as $\tilde{m}$ and the quasiparticle $Z$-factor as $Z$. In these notations we obtain the following self-consistent equation for the quasiparticle $Z$-factor within the RPA approach [we consider here the limit $\alpha \ll (2p_F / Q - 1)^2$]:

$$\frac{2\lambda}{\sqrt{1 - (Q / 2p_F)^2} \sqrt{\alpha} Z^2(\alpha) + Z(\alpha)} Z^{-1} = 0.$$

This quadratic equation can be easily solved exactly, but we are mostly interested in the behavior in the closest vicinity of the transition, which leads to (c. f., Ref. \textsuperscript{17}):

$$Z (\alpha \to 0) = \frac{1}{\sqrt{2\lambda}} \left[ 1 - \left( \frac{Q}{2p_F} \right)^2 \right] \frac{\alpha}{\alpha^{1/4} \times (n - n_c)^{1/4}}.$$

Thus, the effective mass diverges as $1 / Z$, and we have

$$m_\ast = \frac{\tilde{m}}{Z(\alpha)} \propto \frac{1}{(n - n_c)^{1/4}}, \quad Q < 2p_F.$$

We see that within the RPA-treatment of charge-density wave fluctuations, the effective mass diverges at criticality, while the $Z$-factor vanishes.

Let us mention that the case $Q = 2p_F$ is somewhat pathological, since the pole of the propagator coincides with the point of the non-analyticity of the polarizability function and also with the point where the inverse square-root term appearing in the two-dimensional integrals (10) and (11) diverges. To obtain the correct numerical factors and to avoid unphysical divergences, one should use the exact form of the propagator in the hole range of the $(\omega, q)$-space. If we are not interested in exact numbers, we can easily estimate the scaling law of the
Thus, the effective mass diverges as

\[ m_\ast \propto \frac{1}{(n - n_\ast)^{3/8}} \]  

(25)

Let us emphasize that results (22—25) take into account only one diagram, which alone yields very divergent results.

### III. NON-PERTURBATIVE ANALYSIS.

Although, the RPA treatment is widely-accepted and often leads to reasonable results, we believe that in some cases there are serious doubts concerning its reliability. Unlike in the electron-phonon problem, where the Migdal theorem holds and circumvents the necessity to study complex vertex corrections, some other problems (such as, e. g., paramagnon coupling near a magnetic phase transition, charge density wave transition, spin density wave transition, etc.) require correct account for the vertex corrections, which are not small in the vicinity of the critical point and the Migdal theorem is apparently violated. The exact account for the vertex corrections seems an insurmountable problem at the moment and the usual practice is to cover the underlying difficulties by saying that the domain of applicability of the RPA-like treatment may be extended and that the perturbative approach should qualitatively explain the key physics of the transition. In the present section, we prove that such an arguing is quite dangerous and may lead to some qualitatively incorrect conclusions. Fortunately, the density-wave transition case may uncover the underlying problem, as one can obtain some very important results based on very general grounds. As we show below, in the vicinity of any phase-transition, which breaks the Galilean invariance of an initially isotropic system, some important effects are hidden in higher order diagrams. The main qualitative result we are going to derive in the present section is that the effective mass diverges before the phase transition occurs and this divergence leads to the break-down of the Fermi-liquid theory. Apart from the perturbative result, predicting \( Z = 0 \) at the criticality, we show that the \( Z \)-factor formally always remains finite within the non-perturbative treatment.

Paradoxically, we will use the essence of the Fermi-liquid theory construction to justify its break-down. Let us introduce such a standard element as the irreducible four-vertex function, \( \Gamma (p_1; p_2 | p_1 + k; p_2 + k) \), where we introduce notations \( p_i = (\varepsilon_i, p_i) \) for brevity, with \( p_1 \) and \( p_2 \) being the momenta of incoming and outgoing particles correspondingly. In the limit \( k = (\omega, k) \to 0 \) the four-vertex has a singular structure and we, following Ref. [21], introduce the following standard function:

\[ \Gamma^\omega (p_1; p_2) = \lim_{|k|/\omega \to 0} \Gamma (p_1; p_2 | p_1 + k; p_2 + k). \]  

(26)

We also decompose the product of two Green functions involved in the calculation of the four-vertex \( \Gamma \) into a singular and regular parts:

\[ G(p)G(p + k) = \frac{2\pi i Z^2 \nu k}{\omega - \nu k} \delta (\varepsilon) \delta (|p| - p_F) + g_{\text{reg}}(p), \]

(27)

where \( g_{\text{reg}}(p) \) is a regular function in the limit \( k, \omega \to 0 \). One of the central identities of the formal Landau Fermi liquid theory construction reads

\[ \frac{\partial G^{-1}(p)}{\partial \varepsilon} = \frac{1}{Z} = 1 + \frac{1}{2} \int \Gamma^\omega (p, p') g_{\text{reg}}(p') d^3p'/i (2\pi)^3. \]  

(28)

The other identities, which we refer to as the Pitaevskii identities, can be obtained from Eq. (28) by considering symmetries of the initial Hamiltonian (in our case isotropic electron system with Coulomb interactions between electrons). The Galilean invariance, which is broken in a density wave state, but is preserved above the phase transition yields the second Pitaevskii identity which reads

\[ \frac{\partial G^{-1}(p)}{\partial \varepsilon} = p + \frac{1}{2} \int \Gamma^\omega (p; p') g_{\text{reg}}(p') d^3p'/i (2\pi)^3. \]  

(29)

From Eqs. (28) and (29), the following relations follow immediately:

\[ \frac{\partial \Sigma (\varepsilon, p_F)}{\partial \varepsilon} \bigg|_{\varepsilon = 0} = \frac{1}{2} \int \Gamma^\omega (p; p') g_{\text{reg}}(p') d^3p'/i (2\pi)^3 \]  

(30)

and

\[ \frac{\partial \Sigma (\varepsilon, p_F)}{\partial \varepsilon} \bigg|_{\varepsilon = 0} = \frac{1}{2} \int \Gamma^\omega (p; p') \frac{(pp')}{p_F^2} g_{\text{reg}}(p') d^3p'/i (2\pi)^3. \]  

(31)

In the case of a phase transition, preserving the Galilean invariance of the system, the four-vertex function should be peaked at \( |p - p'| = 0 \) and the relations (30) and (31) become identical in the leading order, since \( (pp') = p_F^2 \). In the case of a second order phase transition, which does break the translational symmetry (of which a density-wave phase transition is an excellent example indeed) we anticipate the four-vertex to diverge at the transition point at some finite wave-vector \( Q \). I. e., in the fluctuation region, the major contribution should comes from the domain \( |p - p'| = Q \). Therefore, relations (30) and (31) lead to an obvious contradiction, which is especially pronounced in the case \( Q = 2p_F \), when \( (pp')/p_F^2 = -1 \). The only possible resolution of the paradox is to accept that

\[ \frac{\partial \Sigma (\varepsilon, p_F)}{\partial \varepsilon} \bigg|_{\varepsilon = 0} = 0 \]  

(32)
in the leading order in the closeness to the phase transition. We see that this exact result is in some sense opposite to the one obtained within the perturbative treatment [see Eq. (22)], which proves that the RPA-approach contains a serious deficiency. Obviously, the singularity can not just disappear from the theory, since the general Landau relation ensures that at least one of the derivatives \((d\Sigma/d\epsilon) \) and \((d\Sigma/dp) \) is singular \([c. f., \text{ Eq. (19)}]\).

The relation reads
\[
\left[ \frac{\partial}{\partial \epsilon} + \frac{\partial}{\partial \xi_p} \right] \Sigma(\epsilon, p) \bigg|_{\epsilon = 0} = \frac{m}{Z} \int \frac{d^2p}{(2\pi)^2} \Sigma(p, p') \times \frac{\partial n(p')}{\partial p} \frac{d^2p}{(2\pi)^2}, \tag{33}
\]
where we introduced the full scattering amplitude (Landau function), which is the static four-vertex function:
\[
f(p, p') = \Gamma^\omega(0; 0, p', p). \tag{17}\]

In the perturbative approach, this singularity arose in the \(\epsilon\)-derivative of the self-energy. We see, that within the very general non-perturbative analysis the singularity is “transferred” into the \(p\)-derivative, which automatically means that the quasiparticle \(Z\)-factor never vanishes and the only quantity which acquires singular contributions and eventually diverges is the effective mass. The equation determining effective mass renormalization can be derived from Eq. (28) and the gauge invariance of the system, and in the most convenient form can be written as:
\[
\frac{p}{m} = \frac{\partial \xi_p[n]}{\partial p} - \int f(p - p') p \frac{d^2p'}{(2\pi)^2}, \tag{34}
\]
where \(n(p)\) is the electron distribution, which does not have to be the Fermi-distribution function, in general. \(\xi_p[n]\) is the electron spectrum, which in general is a complex functional of the distribution function. Ideally, the electron distribution function should be self-consistently determined from the Landau energy functional, as its absolute minimum and must be subject to the stability constraint \(\partial \xi_p[n]/\partial p > 0 \) \((i. e., m_+ > 0)\). However, as long as the group velocity of the quasiparticles (the inverse effective mass) is positive, the familiar Fermi-step solution is the correct distribution function at \(T = 0\).

The scattering amplitude \(f\) is a singular quantity in the vicinity of the transition and quite generally can be written as:
\[
f(p - p') = -\frac{\lambda}{\nu_0} \left\{ \alpha + \frac{\left| p - p' \right|}{Q} \right\}^{-1}, \tag{35}
\]
where again \(\nu_0 = m/(2\pi)\) is the “bare” density of states at the Fermi line, \(\alpha \propto (n - n_c)\) is a small parameter, which changes sign at the transition, \(\lambda\) is a dimensionless constant, which may depend on \(r_s\) only, and \((2\pi/Q)\) is the spatial period of the density wave modulations. Eq. (34) leads to the following formula for the effective mass:
\[
\frac{1}{m} = \frac{1}{m_+} - \frac{2\lambda}{m_+} \int_0^{\pi} \left[ \alpha + \frac{2\pi f(2\pi)}{Q} \right]^{-1} \cos \chi d\chi. \tag{36}
\]

After elementary integration we obtain the following result for \(\sqrt{Q} < Q < 2\pi\):
\[
m_+ = m \left[ 1 - 2\lambda \left| \frac{Q^2/2\pi^2 - 1}{\sqrt{(2\pi/Q)^2 - 1}} \right|^{-1} \right] \tag{37}
\]
and for \(Q = 2\pi\):
\[
m_+ = m \left( 1 - 2\lambda \alpha^{-3/4} \right)^{-1}. \tag{38}
\]

We see that in the vicinity of the transition \(\alpha \to 0\), the effective mass diverges, independently on the value of \(\lambda\), and formally changes sign after this point in the fluctuation region. This immediately signals that the Fermi-step distribution function is not stable\(^{18,19}\) and conventional Fermi liquid theory breaks down.

Another quantity, which can be easily estimated from the general considerations of the Landau Fermi liquid theory is the electronic compressibility, which can be expressed as\(^{26}\)
\[
\left( \frac{\delta \mu}{\delta n} \right) \propto \frac{1}{m_+} \left( 1 + \nu f(\chi) \right),
\]
where \(\nu = m_+/(2\pi)\) is the density of states and \(f(\chi)\) is a singular quantity in the vicinity of the phase transition \([\text{see Eq. (35)}]\). Therefore, we conclude that the electronic compressibility \(\partial \mu/\partial n\) must show a divergent behavior in the vicinity of the phase transition (as \(m_+ \to \infty\)). Clearly, other thermodynamic quantities such as specific heat, susceptibility (\(g\)-factor), etc. may also acquire a singular behavior. A detailed analysis of thermodynamics in the vicinity of the charge- and spin-density wave transitions will be reported elsewhere.

IV. CONCLUSION

Summarizing, we suggest that the metal-insulator transition may be actually a signature of a thermodynamic phase transition in a dilute two-dimensional electron system. Namely, the charge- or spin-density wave transition. Which of the two phases prevails is a tentative question and the answer strongly depends on the geometry of the system and can be determined only on the basis of an accurate numerical analysis comparing the energies of the two states. Experimentally, the spin-density wave case may be distinguished from the charge-density wave one by measuring the electron \(g\)-factor which should be singular in the former scenario and
unremarkable in the latter. Our expectation is that in thin-oxide MOSFET’s the charge-density wave should be more energetically favorable, while the spin-density wave should take over if the metallic gates are located far from the two-dimensional electron gas. In both scenarios, we proved that the density-wave fluctuations lead to the divergence of the electron effective mass and to a singular behavior of the electron compressibility. Both effects have been observed experimentally. The density corresponding to \( n_\ast = \infty \) is the point, where the Fermi-liquid description in its conventional form apparently breaks down. In the immediate vicinity of the density-wave transition, the problem directly maps on the model of so-called fermion condensation introduced earlier by Khodel and Shaginyan and considered later by Nozières and also by Volovik. The issue of such a non-Fermi-liquid state is far from its closure, but the main consequence is that the Fermi-distribution function can not be a stable solution of the Landau energy functional and to the pinning of density waves (according to the general theorem due to A. I. Larkin and D. E. Khmelnitski ı, Sov. Phys. JETP 30, 228 (1979)). Possible model distributions were suggested all characterized by a very high density of states at the chemical potential \( \nu(\mu) \) (with the singularity smeared out only by temperature), which may correspond to the sharp increase of the conductivity.

Below the phase transition, i.e., in a density wave state, disorder should play the key role leading to the destruction of the long-range order and to the pinning of density waves (according to the general theorem due to A. I. Larkin). The glassy nature of the density wave state should reveal itself via a noisy behavior of linear response quantities (such as conductivity), which may show some real-time glassy behavior. The physics here should be quite similar to the physics of superconducting vortices in the presence of disorder. Let us mention experiments of Jaroszyński et al. and of Bogdanovich and Popović in which the low-frequency resistance noise was measured clearly showing the glassy freezing of the electronic system.

Finally, let us emphasize an important technical discovery concerning the reliability of a perturbative RPA-like treatment of strong fluctuations and the importance of vertex corrections. The problem is quite general and is related, in particular, to the physics of fluctuation phenomena near the antiferromagnetic phase transition in high-\( T_c \) cuprates. We have shown that the vertex correction, which are usually excluded from consideration, yield qualitatively important changes and ensure that the quasiparticle \( Z \)-factor does not vanish. Our current understanding, based on studying a model example allowing parquet technique treatment (to be published elsewhere), is that the effect of the vertex corrections is mostly the renormalization (suppression) of the imaginary part of the initial RPA-like propagator. This result seems quite reasonable, since the imaginary part of the propagator is related to the life-time of fluctuations, which should become infinite at the transition point.

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1. E. Abrahams, S. V. Kravchenko, and M. P. Sarachik, Rev. Mod. Phys. 73, 251 (2001).
2. E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. 42, 673 (1979); L. P. Gor’kov, A. I. Larkin, and D. E. Khmelnitskiı, Sov. Phys. JETP 30, 228 (1979).
3. A. Shashkin, M. Rahimi, S. Anissimova, S. Kravchenko, V. Dolgopolov, and T. Klapwijk, Phys. Rev. Lett. 91, 046403 (2003).
4. A. A. Shashkin, S. V. Kravchenko, V. T. Dolgopolov, and T. M. Klapwijk, Phys. Rev. Lett. 66, 073303 (2002).
5. W. Pan, D. C. Tsui, and B. L. Draper, Phys. Rev. B 59, 10208 (1999).
6. S. C. Dultz and H. W. Jiang, Phys. Rev. Lett. 84, 4689 (2000); S. Ilani, A. Yacoby, D. Mahalu, and H. Shtrikman, ibid. 84, 3133 (2000).
7. E. Wigner, Phys. Rev. 46, 1002 (1934).
8. D. M. Ceperley and B. Adler, Phys. Rev. Lett. 45, 566 (1980).
9. B. Tanatar and D. M. Ceperley, Phys. Rev. B 39, 5005 (1989).
10. L. Świerkowski, D. Neilson, and J. Szymański, Phys. Rev. Lett. 67, 240 (1991).
11. S. Orozco, R. M. Méndez-Moreno, and M. Moreno, Phys. Rev. Lett. 75, 4650 (1995).
12. A. Göld and L. Calmels, Phys. Rev. B 48, 11622 (1993).
13. G. Grüner, Rev. Mod. Phys. 60, 1129 (1988).
14. G. Grün, Rev. Mod. Phys. 66, 1 (1994).
15. A. W. Overhauser, Adv. Phys. 27, 343 (1978).
16. S. Doniach and S. Engelsberg, Phys. Rev. Lett. 17, 750 (1966).
17. A. Abanov, A. V. Chubukov, and J. Schmalian, Adv. Phys. 52, 119 (2003).
18. V. A. Khodel and V. R. Shaginyan, JETP Lett. 51, 553 (1990); V. A. Khodel, V. R. Shaginyan, and V. V. Khodel, Phys. Rep. 249, 1 (1994); V. A. Khodel, V. R. Shaginyan, and M. V. Zverev, JETP Lett. 65, 254 (1997).
19. P. Nozières, J. Phys. I France 2, 443 (1992).
20. G. D. Mahan, Many-Particle Physics (Plenum Press, New York, 1965).
21. A. A. Abran, L. P. Gor’koy, and I. Y. Dzyaloshinskii, Quantum field theoretical methods in statistical physics.
(Pergamon Press, New York, 1990).

22 A. M. Dyugaev, Sov. Phys. JETP 43, 1247 (1976).
23 F. Stern, Phys. Rev. Lett. 18, 546 (1967).
24 C. S. Ting, T. K. Lee, and J. J. Quinn, Phys. Rev. Lett. 34, 870 (1975).
25 L. P. Pitaevskii, Sov. Phys. JETP 37, 1267 (1960).
26 E. M. Lifshitz and L. P. Pitaevskii, Landau Course: Statistical Physics, Part II (Pergamon Press, Oxford, 1980).
27 G. E. Volovik, JETP Lett. 59, 830 (1994); ibid. 63, 763 (1996).
28 A. I. Larkin, Sov. Phys. JETP 31, 784 (1970).
29 G. Blatter, M. V. Feigel’man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. 66, 1125 (1994).
30 D. P. J. Jaroszyński and T. M. Klapwijk, Phys. Rev. Lett. 89, 276401 (2002).
31 S. Bogdanovich and D. Popović, Phys. Rev. Lett. 88, 236401 (2002).