A simple and rapid approach for the numerical simulation of non-linear elements and examples of its application

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Abstract. In earthquake design philosophy non-linear inelastic damage is concentrated at predetermined locations in a structure, with the aim being to keep structural members critical for collapse avoidance, elastic. Non-linear elements can include the beam column joints in steel and reinforced concrete moment resisting frames, the nail connections of timber sheathing-to-framing shear walls, and the hold-down connectors of shear walls. To numerically model the seismic behaviour of these structures, it is critical to be able to quickly and simply model their respective hysteretic behaviours. It is particularly important to identify the parameters that will allow for the accurate replication of the degrading and pinching qualities of the force-displacement relationships. The authors summarise the use of a proprietary multi-linear plastic link, available in most finite element packages, in order to achieve this. A step-by-step process to model the links, from previous research, is described. This process considers the initial elastic stiffness, the force-displacement curve up to ultimate-strength, the post ultimate-strength degrading stiffness, and the reversing stiffness. The procedure can be adopted for both linear and rotational cyclic excitation. The modelled individual elements are subjected to a series of displacement-controlled time-history schedules and validated against the results from experimental tests. These elements, representing non-linear behaviour, are in turn implemented in various numerical models of shear walls. The authors apply cyclic loading to a model wall, and then for a separately designed wall apply seismic ground accelerations associated with the 2011 Canterbury earthquakes in New Zealand, and the results are presented. While the examples provided in this paper involve timber shear walls, the numerical modelling method will be equally useful for a variety of structures of different material types and configuration.

1. Introduction

This paper provides an overview of a methodology developed by which allows for the rapid numerical modelling of a common form of non-linear relationship under seismic excitation - pinched hysteresis loops. Pinched hysteresis loops arise when a structural element, subject to cyclic displacement regime of successively increasing displacements degrades in both strength and stiffness. Pinched hysteretic behaviour can be observed at the reinforced concrete beam column joints [1], reinforced concrete shear walls [2], and are the characteristic behaviour of structures with compressive members that buckle, such as structures with conventional steel bracing [2]. In timber structures, it is the interaction of the steel connectors, primarily nail connections, with wood that provides ductility (wood in itself being brittle) [3], and in the case of timber shear walls it is the sheathing-to-framing-nail connections that govern the overall force-displacement behaviour. The methodology described here was originally developed by Loo, Quenneville, and Chouw [4, 5], and in this paper the authors adopt it to investigate the behaviour of a structure subject to ground accelerations from the 2011 Canterbury earthquake. While the examples presented are timber based, the methodology can nevertheless readily be applied to any type of structure or structural element with pinched hysteretic behaviour. The paper is organised as follows:

- A mechanics-based method to determine the ultimate lateral strength of nail connections in shear walls.
- Modelling of non-linear behaviour
- Implementation of the concept in shear walls
• Implementation of the concept in shear walls subjected to seismic loading

2. Yield Model theory and the determination of lateral connection strength
European Yield Model (EYM) theory provides a mechanics-based approach to determine the lateral resistance of timber members connected by dowel elements – nails, bolts, spikes etc. [6]. EYM theory (developed in the 1940s) assumes that when a dowel connector, such as a nail or a bolt is placed under load, failure will occur either through bearing failure of the wood material, or when one or more plastic hinges develops in the dowel fastener. This gives rise to various failure modes [7]. Using equations derived from basic mechanics theory, the lateral strength associated with each of the possible modes of failure can be calculated, for dowels in both single shear and double shear. The lowest calculated lateral strength of the possible modes of failure will be that connection’s ultimate lateral strength.

The EYM theory identifies six possible failure modes for connections in single shear and four possible failure modes for connections in double shear. These modes are designated as Im, Is, II, IIIm, IIIIs, and IV. All six of these modes can apply to single shear connections, but only Im, Is, IIIIs, and IV, are possible for double shear connections. The reader is referred to [6, 7] for detailed descriptions of these failure modes. Using the formulas associated with each of the modes, ultimate loads can be calculated and plotted for various sheathing thicknesses and for standard nail penetrations. The example shown below is for nails in double shear connecting pine plywood sheathing to pine framing (Figure 1). The procedure is described in detail in [4, 8].

![Figure 1](image_url)

**Figure 1.** Ultimate nail loads from General dowel formulae – nails in double shear (for OSB sheathing SG=0.64, and for SPF framing, SG=0.42).

The strengths calculated from EYM, while useful from a design perspective, appear to be conservative. From a review of the actual data from lateral strength tests, the EYM consistently underestimates the actual ultimate lateral load. This is possibly because the EYM ignores end fixity of the dowel, inter-member friction between sheathing and framing, and string resistance which develops from nail tension. A discussion of the influence of each of these factors is provided in [4]. While the EYM values would be suitable for design, they would not be realistic enough for numerical modelling, where the aim of modelling is to replicate as closely as possible actual wall behaviour. Following a review of the literature on actual testing [9-11], an equation relating the measured values of ultimate lateral strength to the theoretical EYM values was proposed for single shear [4]:

\[ F_{ult} = 994 \ln(F_{ult(EYM)}) - 5536 \]  

(1)

For double shear, the Loo, Quenneville, and Chouw [4], proposed:

\[ F_{ult} = F_{ult(EYM)} + 3976 \ln(0.5F_{ult(EYM)}) - 22144 \]  

(2)
In summary, to determine the lateral strength, the EYM is first used to obtain an initial value, and then adjusted using equations (1) single shear or (2) double shear.

3. Modelling the non-linear behaviour
This discussion is based on detailed one found in [4, 8]. Non-linear springs (multi-linear plastic link elements) can be used to model the sheathing to the nailed connections. The multi-linear plastic link allows definition of a nonlinear load slip curve for shear deformation in two perpendicular directions. Strength and stiffness degradation can be modelled and hysteretic behaviour observed. In SAP2000 [12] link elements connecting two nodes consists of six springs. Three springs relate to translation (U1, U2, and U3), and three springs relate to rotation (R1, R2, and R3) - refer Figure 2.

![Figure 2. Non-linear element connecting two nodes.](image)

For modelling of nail connections, only the U2 and U3 directions which relate to shear (Figure 3.16) are considered when defining the force displacement relationship. U2 is the vertical shear direction, and U3 the horizontal shear direction. So, for vertical stud framing members, U2 is parallel to the grain and U3 perpendicular to the grain. For top and bottom plates, these switch around – U3 is parallel to the grain and U2 is perpendicular to grain. The load displacement relationship for both U2 and U3 are assumed to be the same (Dolan and Madsen [9] found the force-displacement behaviour of nails in sheathing to be largely directionally independent). All the nail connections will be oriented horizontally along their length i.e. the orientation of U1 will lie in the horizontal direction. Note that any nail withdrawal effect is considered represented by the U2 and U3 defined relationships – the force displacement behaviour (as with actual testing), simply records laterally applied force against the observed displacement – the displacement itself could be the sum of various parts – nail withdrawal included. The load displacement relationship for the hysteretic envelope curve (or simply the monotonic load displacement curve) is input for the U2 and U3 directions (assumed same for both directions). This can be carried out rapidly – SAP2000 allows the upload of the data sets from an Excel Spreadsheet. The Multilinear plastic element has three available hysteresis types for selection; Kinematic, Takeda, and Pivot. For hysteretic behaviour with pinching, the pivot type is selected, and there are parameters which can be adjusted to account for the shape of the envelope curve, the degrading stiffness, and the pinching strength. The steps described in [4, 8] are used to model the element. An example is an element modelling the behaviour through wood of a 3 mm nail. It is set up as shown in Figure 3(a), subject to the displacement schedule of Figure 3(b), to produce the hysteretic relationship of Figure 3(c). It is shown in [4] that this result closely matches the result from actual testing carried out by Dinehart et al. [13].
4. Implementation in a timber shear wall

In order to explore the force-displacement characteristics of sheathing-to-framing timber shear walls, a 2.44 x 2.44 m sheathing to framing shear wall was numerically modelled in SAP2000 [12], with the nail connections represented by multi-linear plastic links modelled in the manner described in the preceding sections. The sheathing was modelled as 12 mm thick F11 grade plywood, and the framing studs were of 38 mm x 89 mm Spruce Pine Fir with an MOE of 9500 MPa and density of 420 kg/m³. The studs were spaced at 406 mm, and the top beam was represented by a steel beam (to replicate the experimental conditions in tests carried out on shear walls by Varoglu et al. [14]). One nail element was used to represent the behaviour of four actual nails, each of ultimate lateral strength 1500 N, and an assumed yield strength of 750 N (half of the ultimate strength). The model wall was subjected to a cyclic load protocol with a maximum horizontal displacement applied at the top corner of 125 mm. The wall was structurally designed for a yield strength of 13 kN, and an ultimate strength of 26 kN (twice the yield strength). Figure 4(a) shows the model wall in SAP2000, while Fig 4(b) shows the hysteretic behaviour after application of the displacement cycles. The numerically obtained hysteretic behaviour (Figure 4(b)) replicates the form of the typical pinched behaviour of timber shear walls from experimental studies [14, 15].
5. Earthquake loading

The authors modelled a shear wall using the methodology of the preceding section and subjected it to a sequence of simulated earthquake loadings. Earthquake motions from the destructive 22nd February 2011 Christchurch earthquake (magnitude 6.3 ML) were applied. The data used were from three different sites in the Christchurch central business district: Christchurch Cathedral College, Christchurch Hospital, and Christchurch Resthaven. From the response spectra shown in Figure 5, it is seen that the seismic demands of this earthquake significantly exceeded the code required level of design at the time.

![Spectral accelerations for the February 2011 Christchurch earthquake at three sites, and the ULS design spectrum (500 yr return period) for NZS1170.5 [16], Type D (soft) soils (Spectra produced from data provided by GeoNet NZ [17])](image)

The first set of simulations involved scaled down accelerations corresponding to a design level event. The second set of simulations applied the actual, un-scaled acceleration records. Details are shown in Table 1.

| Site                  | Component | PGA (g) | ULS scale factor | Scaled PGA (g) |
|-----------------------|-----------|---------|-----------------|---------------|
| Christchurch Cathedral College | N96E     | 0.483   | 0.7             | 0.338         |
| Christchurch Hospital  | S89W     | 0.359   | 0.6             | 0.215         |
| Christchurch Resthaven | S88E     | 0.718   | 0.7             | 0.503         |

1Data from GeoNet NZ [17]
2Event 10 km south-east of Christchurch 2011 Feb 21, 23:51:42 UT Dist 8km Depth 5Rkm MI 6.30
3500 year return period, NZS1170.5 soil type ‘D’ (deep and soft soils) [16]

The model wall was of a 2.44 x 2.44 m midply wall (design ductility, $\mu=3$) designed for an ultimate limit state event in Christchurch (for a description of midply walls refer [14]). It consisted of two vertical 12 mm thick plywood sheathing panels, sandwiched between 90 x 45 mm laminated-veneer-lumber studs. Finite elements were used to model 4 mm diameter nails in double shear, at required spacings of 500 mm. The wall was assigned a seismic mass of 4550 kg at its top, which corresponded to a design base shear from NZS1170.5 [16] of about 9600 kN (assuming a fundamental period of vibration of less than 0.4 seconds).

The hysteretic response of the walls to the scaled ULS events, and then the actual, un-scaled events are shown in Figure 6.
Figure 6. Response of model wall to (a) Christchurch Cathedral ULS event, and (b) actual event, and (c) Christchurch Hospital ULS event, and (d) actual event, and (e) Resthaven ULS event, and (f) actual event.

From Figure 6, it is seen that the wall survived the three design level events, maintaining its lateral strength in each case, while incurring some inelastic damage. This response was in line with the intention of the ductile design - which allows the possibility of yielding, but without collapse of the structure during an earthquake.

The ultimate lateral strength of the wall was found to be a little over 20 kN. From inspection of the force-displacement records, the yield strength is approximately 10 kN, which corresponds closely to the intended design lateral strength of 9.6 kN. All three of the walls completely failed when placed under the actual, un-scaled earthquake loadings. This was as expected because the demands of this earthquake, as previously mentioned, significantly exceeded the relevant code design requirements.

6. Conclusions

A numerical method to model the behaviour of non-linear elements with pinching hysteretic behaviour is introduced. The method requires little if any programming but rather simply the configuration of proprietary elements that are available in most finite element software packages. While pinched hysteretic behaviour can be associated with braced frames, concrete shear walls, and the beam-column connection of reinforced concrete structures, it almost invariably occurs in timber structures with slender steel connectors (most typically nails). Therefore, the example of nail connections is presented. The first step is to apply a mechanics-based approach to determine the theoretical lateral ultimate strength of a nail connection. This method appears to apply a value that is overly conservative, which is acceptable for design, but not for numerical simulation, in which the actual behaviour should be approximated as closely as possible. The relationship between theoretical and actual strengths (collated from
experimental results carried out by other researchers) is presented. An example of a model nail connection is subjected to cyclic loading, with the resulting hysteresis behaviour providing an excellent representation of an experimentally obtained result. These elements are then implemented in a timber shear wall and subject to simulations of cyclic loading. The pinched hysteretic behaviour reflects that of actual timber shear walls, in which the force-displacement behaviour is largely governed by the behaviour of the connectors. Finally, the method is used to test a wall subject to ground accelerations from the 2011 Canterbury earthquakes. The ground accelerations are first scaled to a ULS level earthquake and the model walls behave as expected, with non-linear damage but no collapse. The actual unscaled accelerations are next applied, and the walls experience complete collapse after a sequence of cycles of pinched hysteretic shape. The method presented to model pinched hysteretic behaviour has potential not only in the research of timber structures but will also find applicability in concrete and steel structures, as well as rigid timber structures which are held down by nail brackets.

7. References

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