DARK ENERGY MAY PROBE STRING THEORY

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The problem of dark energy arises due to its self-gravitating properties. Therefore explaining vacuum energy may become a question for the realm of quantum gravity, that can be addressed within string theory context. In this talk I concentrate on a recent, string-inspired model, that relies on nonlinear physics of short-distance perturbation modes, for explaining dark energy without any fine-tuning. Dark energy can be observationally probed by its equation of state, \( w \). Different models predict different types of equations of state and string-inspired ones have a time dependent \( w(z) \) as their unique signature. Exploring the link between dark energy and string theory may provide indirect evidence for the latter, by means of precision cosmology data.

1 Introduction

There is still no fundamental physical theory of the very early universe which addresses issues that arise from the regime of transplanckian physics. It is the lack of a fundamental theory, valid at all energies, that makes the model building of the transplanckian regime very interesting. The main issue is how much are the known observables affected by the unknown theory. The apparently \textit{ad hoc} modification of the dispersion relation at high energies is constrained by the criterion that its low energy predictions do no conflict the observables.

In this talk we address the possibility that transplanckian regime can contribute to the observed dark energy of the universe through a counterintuitive UV/IR mixing\textsuperscript{[1]}. The physics mechanism that gives rise to dark energy is the freeze-out of ultralow frequency short distance modes by the expansion of the background universe. This transplanckian dark energy (TDE) model is motivated from closed string theory in a toroidal geometry by invoking superstring duality.

The transition from string theory to conventional cosmology is becoming increasingly important to theoretical physics. As we describe in \textsuperscript{[2]} corrections to short distance physics due to the nonlocal nature of strings contribute to dark energy. The possibility to detect their signature observationally is thus very intriguing.

A potential way for detecting dark energy and distinguishing it from a pure vacuum energy \( \Lambda \) is through its equation of state. I will briefly touch upon the calculation of the stress-energy tensor, in order to address the puzzle of “cosmic coincidence” (see \textsuperscript{[3]} for details), and the time dependence in the equation of state \( w(z) \) predicted by this model.
2 The TDE Model

Let us start with the generalized Friedmann-Lemaître-Robertson-Walker (FLRW) line-element in the presence of scalar and tensor perturbations. For simplicity, we will take the class of inflationary scenarios that has a power law solution for the scale factor \( a(\eta) \), \( a(\eta) = |\eta_c/\eta|^\beta \), where \( \beta \geq 1 \) and \(|\eta_c| = \beta/H(\eta_c)\). The initial power spectrum of the perturbations can be computed once we solve the time-dependent equations in the scalar and tensor sector. The mode equations for both sectors reduce \([4, 6]\) to a Klein-Gordon equation of the form

\[
\mu_n'' + \left( n^2 - \frac{a''}{a} \right) \mu_n = 0,
\]

where the prime denotes derivative with respect to conformal time, and \( n \) is the comoving wavenumber. Therefore, studying perturbations in a FLRW background is equivalent to solving the mode equations for a scalar field \( \mu \) related (through Bardeen variables) to the perturbation field in the expanding background. The above equation represents a linear dispersion relation for the frequency \( \omega \),

\[
\omega^2 = p^2 = \frac{n^2}{a^2}.
\]

The dispersion relation of Eq. \( (2) \) holds for values of momentum smaller than the Planck scale \( M_P \). There is no reason to believe that it remains linear at ultra-high energies larger than \( M_P \).

In what follows, we replace the linear relation \( \omega^2(p) = p^2 = n^2/a(\eta)^2 \) with a nonlinear dispersion relation \( \omega(p) = F(p) \). Therefore, in Eq. \( (1) \), \( n^2 \) should be replaced by:

\[
\begin{align*}
n_{\text{eff}}^2 &= a(\eta)^2 F(p)^2 = a(\eta)^2 F[n/a(\eta)]^2.
\end{align*}
\]

Let us consider the following Epstein function \([5]\) for the dispersion relation, in the squared bracket in Eq. \( (1) \):

\[
\begin{align*}
\omega^2(p) - \frac{a''}{a} &= F^2(p) = p^2 \left( \frac{\epsilon_1}{1 + e^x} + \frac{\epsilon_2 e^x}{1 + e^x} + \frac{\epsilon_3 e^x}{(1 + e^x)^2} \right), \\
n_{\text{eff}}^2 - \frac{a''}{a} &= a^2(\eta) F^2(n, \eta) = n^2 \left( \frac{\epsilon_1}{1 + e^x} + \frac{\epsilon_2 e^x}{1 + e^x} + \frac{\epsilon_3 e^x}{(1 + e^x)^2} \right),
\end{align*}
\]

where \( x = (p/p_C)^{1/\beta} = A|\eta| \), with \( A = (1/|\eta_c|)(n/p_C)^{1/\beta} \), in general for power-law inflation with \( \beta \geq 1 \). This is the most general expression for this family of functions. For our purposes, we will constrain some of the parameters of the Epstein family in order to satisfy the features required for the dispersion relation as follows.

Imposing the requirement of superstring duality fixes \( \epsilon_2 \), and the condition of a nearly linear dispersion relation for \( p < p_C \) requires that

\[
\epsilon_2 = 0, \quad \frac{\epsilon_1}{2} + \frac{\epsilon_3}{4} = 1.
\]

Still we will have a whole family of functions parametrised by the constant \( \epsilon_1 \), as can be
seen in Fig. [1].

The initial vacuum state is well-defined in this model and it is a Bunch-Davis vacuum, due to the dispersion function going asymptotically flat at early times. At late times the solution becomes a squeezed state by mixing of positive and negative frequencies:

\[
\mu_n \rightarrow \eta \rightarrow +\infty \quad \frac{\alpha_n}{\sqrt{2\Omega_{\text{out}}^n}} e^{-i\Omega_{\text{out}}^n \eta} + \frac{\beta_n}{\sqrt{2\Omega_{\text{out}}^n}} e^{+i\Omega_{\text{out}}^n \eta},
\]

with \( |\beta_n|^2 \) being the Bogoliubov coefficient equal to the particle creation number per mode \( n \), and \( \Omega_{\text{out}}^n \approx \sqrt{\epsilon_1} n \). Using the linear transformation properties of hypergeometric functions we obtain:

\[
|\beta_n|^2 = \frac{e^{-2\pi\sqrt{\epsilon_1}}}{2 \sinh 2\pi \sqrt{\epsilon_1}}.
\]

Clearly the spectrum of created particles is nearly thermal to high accuracy.

We define the tail as the range of those transplanckian modes in fig.1 whose frequency is less or at most equal to the present Hubble rate, \( H_0 \) (see Fig. 2). Since \( H \) has been a decreasing function of time, many modes, even those in the ultralow frequency range, have become dynamic and redshifted away one by one, everytime the above condition is broken, i.e. when the expansion rate \( H \) dropped below their frequency. Clearly, what is left from the tail modes at the present Hubble scale have always been frozen. They contain vacuum energy of very short distance, hence of very low energy. The last mode in the tail (at infinite momenta) would decay when and only if \( H = 0 \) by acquiring a kinetic term. The tail starts from some value \( p_H \) which must be found by solving the equation

\[
\omega^2(p_H) = H_0^2.
\]
The range of modes defining the tail is then for $p_H < p < \infty$. Their equation of state depends on the evolution of $H$ and is a complicated tracking solution because they contribute to the expansion rate for $H$ and their equations of motion are coupled to the Friedmann equation for expansion.

Figure 2: The range of modes in the tail, $p_H < p < \infty$, defined by Eq. (9). $H_0$ is the present value of the Hubble constant.

The energy for the tail is given by:

$$\langle \rho_{\text{tail}} \rangle = \frac{\beta_n^2}{2\pi^2} \int_{p_H}^{\infty} \frac{\omega(p)}{p} \omega(p) p^2 dp,$$

while the expression for the total energy is:

$$\langle \rho_{\text{total}} \rangle = \frac{\beta_n^2}{2\pi^2} \int_{0}^{\infty} \frac{\omega(p)}{p} \omega(p) p^2 dp.$$

The numerical calculation of the tail energy produced the following result: for random different values of the free parameters, the dark energy of the tail is $\rho_{\text{tail}} = 10^{-122} f(\epsilon_1)$, times less than the total energy during inflation, i.e. $\frac{\rho_{\text{tail}}}{\rho_{\text{total}}} = 10^{-122} f(\epsilon_1)$ at Planck time. The prefactor $f(\epsilon_1)$, which depends weakly on the parameter of the dispersion family $\epsilon_1$, is a small number between 1 to 9, which clearly can contribute at the most by 1 order of magnitude. *This is an amazing result!* The lack of fine-tuning in obtaining the right amount of dark energy can be understood from the following feature: due to the decaying exponential, the main contribution to the energy integral in Eq. (10) comes from the highest value of this exponentially decaying frequency, which is the value of the integrand at the tail starting point, $p_H \sim O(M_P)$, i.e.,

$$\langle \frac{\rho_{\text{tail}}}{\rho_{\text{total}}} \rangle \approx \frac{p_H^2}{M_P^2} \omega^2(p_H) \approx \frac{H_0^2}{M_P^2} \approx 10^{-122}.$$
From the physical requirement that the tail modes must have always been frozen, the tail starting frequency $\omega(p_H)$ is then proportional to the current value of Hubble rate $H_0$ (Eq. (3)).

3 TDE Dispersion Relation from Closed Strings in Toroidal Cosmology

Here we attempt to motivate the exponentially supressed transplanckian dispersion function from T-duality of the following string theory model. Let us consider the Brandenberger-Vafa model of a D-dimensional anisotropic torus with radius $R_i$, by including the dynamics of both modes: momentum modes, $p_{1,i} = m/R_i$ (where $m$ is the wavenumber), and winding modes with momenta $p_{2,i} = wR_i/\alpha'$. The dimensionless quantity for the radius is $R_i = R_i/\sqrt{\alpha'}$, where $\alpha'$ is the string scale. Based on the arguments presented in [8], we choose a cosmology with three toroidal radii equal and large $R\gg 1$ in units of the string or Planckian scale, with the other $(D-3)$ toroidal radii equal and small $R_C \ll 1$. Here the subscript $C$ refers to compactified dimensions. Then, $R(t)$ becomes the scale factor for the 3+1 metric in conventional FRW cosmology $R(t) = a(t)$, while $R_C$ corresponds to the radius, in this factorizable metric, of the $D-3$ compact dimensions $z_j$ that decrease with time,

$$ds^2_D = -dt^2 + 4\pi R^2(t)dx_i^2 + 4\pi R_C^2(t)dz_j^2 = a(\eta)^2[du^2 + dv^2] + ds^2_{D-3}.$$  

(13)

These solutions were found by Mueller, Ref. [9], by ignoring the backreaction of string matter on the geometry.

The partition function for this system was calculated, from first principles, by summing up over their momenta in [11]:

$$Z = \sum_\sigma e^{-n_\sigma \epsilon_\sigma},$$  

(14)

where $n_\sigma$ is the number of strings in state $\sigma$ with energy $\epsilon_\sigma$

$$\epsilon_\sigma = p_0 = \sqrt{\left(\frac{m}{R}\right)^2 + (wR)^2 + N + \tilde{N} - 2},$$  

(15)

and $\sigma$ counts over $(m, w)$, with the constraint $N - \tilde{N} = mw$ for closed strings where $N$ and $\tilde{N}$ are the sums over the left- and right- mover string excitations, respectively. By now, in Eq.(15), we are considering only the large 3 spatial dimensions. The string state can also be described by its left and right momenta, $k_L = p_1 + p_2$, $k_R = p_1 - p_2$. The string state for left and right modes can be expanded in terms of the creation and annihilation operators $\alpha_m, \tilde{\alpha}_n$, with higher excitation string states given by $N = \sum_{n=1}^{\infty} \alpha_{-n} \alpha_n$ (similarly for $\tilde{N}$), and string energy $L_0 + \tilde{L}_0 = p_1^2 + p_2^2 + (N + \tilde{N} - 2)/\alpha'$.

We would like to write the path integral for this configuration in terms of quantum fields[1]. The path integral is calculated from the hamiltonian density. The hamiltonian density over the fields in configuration space is extracted from the string spectrum in such

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1Below we use quantum string equations under the assumption that the dilaton is massive and stable.
a way that its Fourier transform in \( k \)-space corresponds to the string energy expression Eq. (13).

Our system of winding and momentum modes is described by nonequilibrium dynamics due to the expanding background spacetime. All the information about the evolution of these modes will be contained in the effective action. The kinetic terms are unambiguous while for the interaction terms we must appeal to simplicity and the requirement of T-duality.

Correlation functions are obtained by using the correspondence between the Euclidean path integral of the persistence vacuum amplitude \( \langle \text{in} | \text{out} \rangle \) and the partition function \( Z \). All the string quantum operators below are promoted to quantum field operators with the corresponding hamiltonian density \( H(t, x) \) in configuration space derived from the quantum string hamiltonian \( \mathcal{H}(t) \).

The following calculations are done in the conformally flat background through the scaling of the fields and operators with the conformal factor \( a(\eta) \). The momentum field \( \phi_1(R, x) \) and the winding field \( \phi_2(R, x) \) are defined by the relation:

\[
\phi_i(x) = \int e^{ip_i x} \phi_i(p_i) d^3p_i , \quad \int |\nabla \phi_i|^2 d^3x = \int d^3p_i p_i^2 \phi_i(p_i) , \tag{16}
\]

where

\[
\nabla = R \partial / \partial x = \partial / \partial y , \tag{17}
\]

and \( p_i = p_1, p_2 \). Let us also define two new fields, \( \psi_L(R, x) \) and \( \psi_R(R, x) \), with momenta \( k_L, k_R \) that are the left and right combinations of the Kaluza Klein momentum and winding modes

\[
\begin{align*}
\psi_L(R, x) &= \phi_1(R, x) + \phi_2(R, x) , \tag{18} \\
\psi_R(R, x) &= \phi_1(R, x) - \phi_2(R, x) . \tag{19}
\end{align*}
\]

These fields live in the expanding \((3+1)\) spacetime dimensions.

The Hamiltonian density ansatz that would describe the energy of our two string states in the \( D = 3 \) expanding dimensions with energy \( H = L_0 + \bar{L}_0 \), including the oscillators from string’s higher excitations \((N + \bar{N} - 2)/\alpha'\), is similar to the hamiltonian of spin waves in a periodic lattice\(^2\). Our lattice spacing is given by the string scale \( \sqrt{\alpha'} \). Therefore the hamiltonian density can be written for this dual lattice in terms of wave functional “spin” fields \( \psi_L(R, x), \psi_R(R, x) \) of Eqs. \((18), (19)\) as follows

\[
\mathcal{H}_3 = |\nabla \psi_L|^2 + |\nabla \psi_R|^2 + |\nabla \psi_L| |\nabla \psi_R| + m_0^2 (|\psi_L|^2 + |\psi_R|^2) + g_1(|\psi_L|^4 + |\psi_R|^4) + g_2 |\psi_L|^2 |\psi_R|^2 , \tag{20}
\]

where the fields \( \psi_L, \psi_R \) are expanded in terms of the mode functions \( u_n, \tilde{u}_n, \)

\[
\psi_L = \sum u_n b_n + u_n^* b_n^+ , \quad \psi_R = \sum \tilde{u}_n \tilde{b}_n + \tilde{u}_n^* \tilde{b}_n^+ , \tag{21}
\]

\(^2\)Torus is obtained by identifying the first and the last lattice sites, thus the periodicity.
and $b_n, \tilde{b}_n$ are the normalised quantum creation and annihilation operators of $\alpha_n, \tilde{\alpha}_n$. The commutation relation for the unnormalised operators are such that $[\alpha_n, \alpha^+_m] = \omega_{\pm}\delta_{nm}$ with $\omega_{\pm}$ the frequency of left and right moving modes.

The periodic lattice condition $N - \tilde{N} = mw$ introduces an interaction term in the Hamiltonian $H_3$ of the form $\nabla\psi_L\nabla\psi_R$. In terms of the 2-component state $\Psi_{EN} = (\psi_L, \psi_R)$, the Hamiltonian reads

$$H_3 = |\nabla\Psi_{EN}|^2 + \nabla\Psi_{EN}\tilde{X}\nabla\Psi_{EN} + m_0^2|\Psi_{EN}|^2 + g_1|\Psi_{EN}|^4 + (g_2 - 2g_1)|\Psi_{EN}\tilde{X}\Psi_{EN}|^2,$$  \hspace{1cm} (22)

with

$$\tilde{X} = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix}$$  \hspace{1cm} (23)

The system is known as the dual momentum-space lattice, and for $g_2 = 2g_1$ reduces to the XYZ model of condensed matter. Let us for simplicity limit to the XYZ model case, $g_2 = 2g_1$, for the rest of this talk.

These periodic lattice systems studied in 3+1 dimensions in terms of Bloch wavefunctions have a solution which respect lattice translation invariance, $\exp(-pl)$, with the lattice spacing “l” equal to the string scale $\sqrt{\alpha'}$. The interaction term, in the tight-binding approximation, lifts the degeneracy between the energy eigenstates due to the leakage/tunnelling of the wavefunction from one lattice site to the neighbour site. As a result the gap energy produced between the ground (bound) state and higher excitation states is

$$p^2 \Delta_p = p^2 \cos(2\theta) = p^2 |2\cos^2(\theta) - 1|,$$  \hspace{1cm} (24)

in which

$$pl = p\sqrt{\alpha'} = \sqrt{\alpha'(p_1^2 + p_2^2)} = \sqrt{(m/R)^2 + (wR)^2},$$  \hspace{1cm} (25)

and $\theta \rightarrow \theta + ipl$. Therefore,

$$\Delta_p \leq 2\cosh^2(pl) - 1.$$  \hspace{1cm} (26)

The first term in $\theta$ is a pure phase of rotation of the “spin-wave” in the dual lattice, but the second term describes the tunnelling of the wavefunction to the nearest neighbour. The gap energy of Eq. (24) introduces a correction to the kinetic energy, such that in momentum space the Hamiltonian reads

$$H_3 = z_p p^2|\Psi_{EN}|^2 + m_0^2|\Psi_{EN}|^2 + g_1|\Psi_{EN}|^4,$$  \hspace{1cm} (27)

with $z_p = 1 + \Delta_p$. One can evaluate the reduced 3 dimensional partition function $Z$ from the above Hamiltonian[2]. We want to separate our modes into system (S) + environment (E) degrees of freedom, and coarse grain by integrating out the degrees of freedom for the environment. This amounts to finding out the backreaction of the coarse grained environment on the system, and eventually leads to the RGE’s[4]. We will consider as environment all the short wavelength modes with momenta

$$(E) : \frac{\Lambda}{b} < p^E = \frac{1}{\sqrt{\alpha'}}[(m/R)^2 + (wR)^2]^{1/2} < \Lambda,$$  \hspace{1cm} (28)

\[3\] In condensed matter this is known as Coulomb dipole type of vortex interaction.

\[4\] The running of the coupling constants with time depends on how one selects the environment and the system.
where the cutoff $\Lambda = (\alpha')^{-1/2}$ is the string scale because $(\alpha')^{1/2}$ is identified with the lattice spacing $l$, and $b = a(t)/a(t_0)$ is the coarse grain scaling parameter, where $t_0$ is the initial time. The scale factor $a(t)$ plays the role of the collective coordinate describing the environmental degrees of freedom. Time in this procedure is playing the role of a scaling parameter and dynamics is being replaced by scaling, an artificial procedure, known as Kadanoff-Migdal transform, that relates the microscopic and macroscopic properties of a system based on the existence of scaling properties of the system in the infrared limit.

The system modes are the ones with:

$$(S) : \quad p^S < \frac{\Lambda}{b},$$

From the above definitions of system and environment, Eqs. (29) and (28), at initial times when $b \approx 1$ we have $p \leq \Lambda/b$ thus all our modes, momentum and winding, are in the system; but at later times when $b \geq 1$ more and more winding modes systematically transfer to the environment because the condition of Eq. (29), $p = \frac{1}{\sqrt{\alpha'}}[(m/R)^2 + (wR)^2]^{1/2} \leq \Lambda/b$ is satisfied only for vanishingly small winding numbers $w \to 0$. As $t$ becomes large, the system contains $m \leq RA, w = 0$, i.e. all the modes except $m \leq RA, w = 0$ have been transferred to the environment.

After integrating out the high energy environmental modes in the above action, one can extrapolate the correlation function from the coarse grained effective action of the system. The canonical two-point correlation function at high energy for system-environment interaction is calculated from the path integral of the canonical fields $\tilde{\Psi}_{S,E}$ in momentum space (Fourier transform of $G^{A/b}$). It is related to the correlation function of the original fields $\Psi_E$ (which decreases at high energy) as follows

$$\langle \Psi_E \Psi_E \rangle = \frac{\langle \tilde{\Psi}_E \tilde{\Psi}_E \rangle}{z_p}. \quad (30)$$

where $z_p = 1 + \Delta_p$ and $\Delta_p$ is given in Eq.(26). This is the crucial result for the interpretation of the cosmological dark energy. Because of the mass gap, the correlation function is suppressed exponentially in p-space. It is very familiar that a mass gap leads to an exponential fall off in x-space, but here for the dual lattice the exponential fall off is in momentum space. This may be traced to the T-duality of the closed strings and the resultant interchange of the IR/UV limits. Thus the two point function of the original fields (by dividing with the $z_p$ normalisations factor), goes as an exponential for large momentum $p$, and as a polynomial for low momenta.

The observed small value $\Lambda \sim 10^{-120}$ in natural units has an explanation in the toroidal cosmology of closed strings and thus the dark energy provides an exciting opportunity to connect string theory to precision cosmology. We may argue that numerically the size of the cosmological constant in the present approach is a combination of the string scale and the Hubble expansion rate in the sense that $\Lambda/M_{\text{Planck}}^4 \simeq 10^{-120} \simeq (H_0/M_{\text{Planck}})^2$. Therefore the correct amount of dark energy obtained by this frequency dispersion function does not require any fine tuning and relies, besides a physical mechanism (such as freeze-out), only on the string scale as the parameter of the theory.
4 TDE Equation of State

Nonlinearity of short distance physics and the breaking of Lorentz invariance results in a violation of Bianchi identity for transplanckian models. Therefore one needs to modify the stress-energy tensor ($T_{\mu\nu}$) in the Einstein equations, such that the modified ones satisfy Bianchi identity.

Based on the equation of motion as our sole information for short-distance physics, we therefore use a kinetic theory approach for modifying Einstein equations in the absence of an effective lagrangian description. The assumption made is that a kinetic theory description of the cosmological fluid is valid even in the transplanckian regime. Despite its nonlinear behavior at short distances, this imperfect fluid shares the same symmetries, namely homogeneity and isotropy, as the background FRW universe. Then the corrections $\tau_{\mu\nu}$ to the stress energy tensor $T_{\mu\nu}$ will also be of a diagonal form

$$\tau_{\mu\nu} = (\bar{\epsilon} + \Pi)u_\mu u_\nu + \Pi g_{\mu\nu}$$  \hspace{1cm} (31)

In a similar manner to particle creation cases in imperfect fluids [12], the highly nontrivial time-dependence of the mode $p_H$ and the transfer of energy between regions, due to the defrosting of this mode across the boundary $p_H$, gives rise to pressure corrections in the fluid energy conservation law. The defrosting of the modes results in a time-dependent “particle number” for regions near $p_H$. From kinetic theory we know that this “particle creation”, (the defrosting of the modes), gives rise to effective viscous pressure modifications [12]. The term $\Pi$ denotes the effective viscous pressure modification to the “bare” pressure, $\langle \bar{p} \rangle$.

The criteria we will use for modifying $T_{\mu\nu}$ is that Bianchi identity must be satisfied [13] with the new expressions for pressure, $P$,

$$\Sigma[\dot{\rho} + 3H(\rho + \bar{p} + \Pi)] = \Sigma_i[\dot{\rho}_i + 3H(\rho_i + P_i)] = 0 , $$  \hspace{1cm} (32)

The presence of pressure corrections, $\Pi_i$, (where $i=II,H$ denotes transplanckian defrosted and tail modes respectively) is due to the exchange of energy between the two regions, from the defrosting of the modes $p_H$ at the boundary. This is directly related to the time dependence of the boundary $p_H$, which in turn is going to be controlled by the Hubble parameter $H$. In essence, there is an exchange of modes between the two regions. Although the number of particles\(^5\) in each of these regions, $N_{II}$ and $N_H$, is not conserved, their rate of change, in the physical FRW Universe, is related through the conservation of the total number of particles which contains both of these components

$$\dot{N}_T = 0 , $$  \hspace{1cm} (33)

The contribution terms to pressure, $\Pi_i$, are related to decay rates $\Gamma_i$ through [12]

$$3H\Pi_i = - (\rho_H + \bar{p}_H)\Gamma_H , $$  \hspace{1cm} (34)

\(^5\)We are loosely using the term particle here to refer to the wavepackets of the transplanckian modes, centered around a momenta $p_i$.\n
Details of the calculation can be found in[3]. The value of the tail’s decay rate $\Gamma_H$ is:

$$\Gamma_H \simeq 3H + \frac{\dot{H}}{H} \simeq 3H\left(1 - \frac{w_{\text{total}}}{2}\right),$$

where $w_{\text{total}} = \frac{\bar{p}_{\text{total}}}{\rho_{\text{total}}}$. Notice that $\Gamma_H$ is positive for all equations of state $w_{\text{total}} \leq 1$ and thus it slows down the dilution of the tail with the scale factor.

The effective equation of state resulting from the modification $\Gamma_H$ of stress-energy tensor becomes:

$$w_H = \left(\frac{w_{\text{total}} - 1}{2}\right).$$

This is a tracking solution. During a radiation dominated universe, $w_H = -1/3$ and only after matter domination, the tail starts acquiring a strongly negative equation of state until it asymptotically approaches $w_H = -1$ when it becomes the only component left in the expansion equation. Therefore cosmic coincidence is explained naturally by the intrinsic time evolution of the tail which tracks the background equation of state $w_{\text{total}}$ through its coupling to the Hubble parameter.

Careful analysis of the highly anticipated new data from CMB, LSS and SN1a will reveal more scrutiny between various models[14].

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