Positron storage in micro-traps with long aspect ratio: results of computer simulations*

Paola Folegati$^{1,2}$, Jia Xu$^2$, Marc H Weber$^2$ and Kelvin G Lynn$^2$

$^1$Polo Regionale di Como, Politecnico di Milano, Via Anzani 42, 22100 Como, Italy
$^2$Washington State University, Pullman, WA 99164-2711

paola.folegati@wsu.edu, kgl@wsu.edu

Abstract. A feasibility study of a non–conventional Penning-Malmberg trap is presented. The study is based on simulations performed by the WARP code. [1,2] Comparisons with previous simulations done with the Charged Particle Optics (CPO) programs [3] are presented under homogeneous magnetic field conditions; however, real B fields do not have a significant effect. The analysis has been carried out on micro-traps of radii ranging from 10 to 100 micrometers and length of 126 mm. Different configurations of the applied electric fields have been considered and an axial uniform magnetic field of 7 T was used in most of the simulations. For the final device, the microtraps should be combined into a multiple parallel tube array. Results from the simulations with the WARP and CPO codes indicate that decreasing the microtrap radius allows an increase in the total number of positrons per macrotrap. This is due to the increased number of micro-tubes when reducing the beam radius.

1. Introduction
During the last years, positron physics is becoming increasingly more important from the technological point of view for applications in materials analysis [4]. Many of these applications require the use of a significant number of positrons and sometimes require the possibility of tuning their energy to test different depths inside a material (defect profiling). Slow positron beams have been constructed to meet these requirements and have been used for many years [5]. The possibility of storing large numbers of positrons may have other applications: (1) allow the study of Bose condensation of positronium atoms [6]: (2) provide a means for studying electron–positron plasmas in parameter regimes in astrophysics [7] (3) allow the creation of antihydrogen [8]; and (4) production of intense pulses of positronium atoms [9]. Storing positrons in “portable” devices could provide an attractive alternative to radioactive positron sources for these and other uses[10,15]. Various techniques have been considered for storing large numbers of positrons including the use of electron–positron plasmas [10] and confinement in specially designed Penning–Malmberg traps [11]. The advanced technology that can be used for the accumulation of positrons in Penning–Malmberg traps makes it one of the most attractive candidates for the accumulation of a large numbers of positrons [12]. The unavoidable limitations imposed on the number of particles that can be stored in such devices has led to the proposal of a new type of trap with long aspect ratio and portability [14,15,16,17,18].
2. Objectives
Charged plasmas can reach thermodynamic equilibrium and be confined in Penning-Malmberg type trap by static magnetic and electric fields for very long times – for an infinite time in principle. Losses arise because of trap imperfections associated with the trap components [13]. Our goal is to evaluate the maximum number of positrons that can be stored in a trap with large length-to-diameter aspect ratio and micrometer diameter (Fig. 1). A large number $N_{\text{tubes}}$ of these microtraps, each with conducting walls, are than stacked in parallel to fill a macrotrap of volume $V_{\text{macro}}$ inside a 7 Tesla superconducting magnet. The diameter is shrink to micrometers, while keeping the length fixed. $N_{\text{tubes}}$ is increased to fill $V_{\text{macro}}$ (of 50 mm diameter and 126 mm length) with a 75% fill factor. Gain is achieved when the total number of positrons $N = N_t \times N_{\text{tubes}}$ exceeds the number of positrons that can be stored in a single trap of the size of $V_{\text{macro}}$. $N_t$ is the number of positrons in a single microtrap. In addition, the conducting walls of the microtraps generate image potentials that shield positrons in one tube from those in all other tubes. This lowers the required electric potential at the end caps, bringing portable traps closer to reality.

The schematic diagram for our simulations is shown in Fig. 1. It is a cylindrical microtrap $L_t = 126$ mm length and variable radii $R_t$ from 5 to 50 microns (containing a positron plasma source $L_S = 70$ mm in length, centered in the trap cylinder, and radius $R_S = R_t/\sqrt{3}$ to keep constant a scale factor of 3 between the cross sectional areas). Static electric and magnetic fields are always used. A uniform magnetic field, parallel to the trap axis, is used for the radial confinement and an electric field for the azimuthal confinement. The model device has a central grounded cylinder, three thin (1mm length) electrodes at increasing potential (2.5, 5 and 7.5 V respectively) on each side and two end-caps at 10 V. The trap simulated in CPO code does not have the grids on the end caps, but the result is the same when added.

This objective is approached within standard physics equations or codes however careful attention is paid to the aspect ratio and trap dimensions. This means limitations on the number of positrons that can be stored in the trap come essentially from two factors:

1) the Brillouin limit, which is an upper limit on the density that can be stored in a given magnetic field:

$$n_B = \frac{B^2}{8 \mu_0 mc^2}$$

where $B$ is the magnetic field in the trap, $m$ is the mass of the plasma particles, and

2) the space-charge limit,

$$V_s \approx \left(1.4 \times 10^{-7}\right) \frac{N_t}{L} \left[1 + 2 \epsilon n \frac{R_p}{R_c}\right]$$

where $R_p$ and $R_c$ are the trap radius and source radius, respectively, $N_t$ is the total number of particles in the microtrap, $L$ is the source length, $V_s$ represents the maximum potential generated by the charge itself, and if $L$ is in meters the result is in volts.

![Fig.1. Schematic diagram of the trap used in WARP simulations](image)

3. Computational tools
Two software tools were used and compared to model the positrons stored in microtraps. Charged Particle Optics (CPO) builds in single charge particle optics and adds space charge effects. Warp is designed to include space charge of high density beams and can was used for plasmas. Here, the code is used at lower density and very narrow trap dimensions. Results from each are compared.
3.1 CPO

The CPO program package uses Boundary Element Method or surface charge method to simulate the motions of charged particles in the electromagnetic fields. It has proven to agree with the experimental results based on the simulations of low-energy ion dynamics. To evaluate the effects of space charge in the tube, the space charge tube method [4] is utilized in the ray tracing. After several test runs, a reasonable number of rays (normally 36) have been found to reach a stable result for the simulations. The code uses several iterations to evaluate the space charge effect by applying the space charge effect of the current iteration upon the particles for the next iteration. A number of rays can be traced representing the motions of charged particles in the beam. The mechanics of space charge tube method is when the program processes the space charge effect of one ray, a small tube surrounding the trajectories of each ray and the space charge of the ray will be deposited uniformly in the tube. The space charge deposited in the tube will be used for next iteration and the radius of the tube can be specified by the user.

In our case, the magnetic attraction between the rays is excluded by the CPO program. The formation of positronium (Ps) on background impurities in the vacuum system is not accounted in our simulations as the vacuum is planned to be better than $10^{-11}$ torr and the positron energy below the threshold for Ps formation.

In CPO all rays start at the vertical middle plane of the microtrap with randomized initial angles with respect to the horizontal axis of the tube to spread the rays across the length of the trap to simulate a plasma. Each ray simulation finishes a complete oscillation – an iteration – ending at the middle plane. A shell program\(^1\) based on CPO program was written using C++ language to feed the final current and averaged energy of all rays from the current iteration back into CPO for the next iteration. In every iteration all rays are uniformly distributed within a circle with the radius with the circle radius $\frac{3}{1}$ of the radius of the tube. The energy of the rays for the next iteration is based on the averaged final energy for the current iteration. The radius of the beam for the next iteration is set as 0.7$\times$ the maximum radius of rays of the current iteration. Then, the number of trapped particles is the flight time of rays staying the tube multiplying the current that the rays carry for all the rays.

3.2 WARP

The WARP code in its latest three-dimensional version, WARP3d, is a multidimensional particle-in-cell originally developed to simulate creation and propagation of high-current, space-charge dominated beams and also allows the study of plasmas. It has been used in many applications and was used for comparing the electron trapping experiments[20]. Geometry of the trap, materials constituting the trap walls and fields can be set by the user. Different boundary conditions can be applied for the solution of Poisson’s equation (Dirichlet, Neumann). The WARP code allows for the use of multipile fields, time-varying fields and periodic structures necessary to simulate particle accelerators (linacs or colliders). Not only including the space-charge effects, the WARP code used Langevin collision operator to treat the Coulomb collision during the simulation[21,22].

4. Results

4.1 CPO simulations

4.1.1 Energy increase of positrons: In Fig.2, a typical example shows the averaged energy of particles increases as a function of number of iterations.

\(^1\) A loop program was constructed by setting up the initial conditions of each ray in one iteration based on the final conditions of each ray in previous iterations, but the CPO program does not read in the initial conditions of all rays in each iteration, and the ray tracing time will take very unpractical long times.

Fig.2 The energy increase as iterations proceeds with the particles trapped in a microtube with a radius of 50µm. After 1.2µs (8 iterations), the energy reaches about 5.3eV which is much smaller than 10V of end cap potentials. Initial conditions for the beam shown in this figure are 25µm and 30µm and denotes the initial radius of the beam.
of time in a 50µm microtrap. In Fig.2, the maximum averaged energy of the beam reaches up to 5.3eV, much lower than the end cap potentials 10V. In all cases of CPO code, regardless of the radius of the microtrap and the initial conditions of the beam, the averaged energy of the beam never exceeds about 6.4eV after about 0.003ms (20 iterations). The end cap potentials can trap the positrons with 6.4eV as well since they are much higher than the final energies of positrons. In all CPO results, no positrons are lost nearby the end caps of the tube, which confirms the increase of the energy is not a major issue for the loss of the positrons within the current time duration. The results also show the positrons always hit the electrode radically. This evidence is also found in WARP code results.

4.1.2 The number of trapped particles as a function of time: The number of particles that can be trapped in a microtrap as a function of time is shown in Fig.3. There are three phases for the behaviors of the beam: (1) At first the radius of the beam expands due to the space charge effect but does not reach the radius of the tube. Thus, there is no particle loss in this phase. (2) Then the radius of the beam reaches up to the radius of the tube, the beam starts to significantly lose positrons based on the slope of the curve shown in Fig. 2. (3) Finally, the beam reaches an equilibrium state and there is no further positron loss. In all the cases from CPO and WARP codes, the beam or plasma arrives at equilibrium status.

As the radius of one microtrap is decreased, the total number of positrons that can be trapped in one macrotrap increases as shown in Table 1. The CPO program has difficulty doing the ray tracing when the size of the tube is narrowed down to around 10µm and smaller. The number of trapped positrons in a 10µm microtrap in Table 1 shows preliminary results. More simulation work will be done.

4.2 WARP simulations

The WARP code [1,2,18] requires the definition of the number of physical particles (positrons in this case) per “simulation particle” which is named “positron weight”. For this weight, a reasonable compromise has to be chosen at 100 positrons per simulation particle to limit computation time. The fraction of positrons lost in trap wall collisions was evaluated for different “weights” from 10 to 100, and the results suggest that a positron weight equal to 100 can be a reasonable choice. In all cases of WARP code, the positrons with 0.025eV of initial energy start from the source of cylindrical in the microtrap with random initial angle with respect to magnetic field ranging from 0 degree to 180 degree, while in CPO code all the positrons with 5eV start from a plane with the initial angle from 0 to 90 degree with respect to the magnetic field.

Simulations at fixed density and changing the trap size have been made to compare with CPO results, and they resulted in good agreement. Increasing the density and decreasing the trap radius up to values that are more difficult to be reached with the CPO code were simulated. The choice of parameters was based on the assumption that the behavior of a trap should be the same as positron containment for a constant value of the ratio between electrostatic and thermal energy. The electrostatic energy is indeed a space charge effect affecting the radial containment and increases as the square of the trap radius. This is the reason for going to smaller radii which allows an increase the density. The results of the simulations

---

2 The Coulomb scattering time is estimated to be around 2ps for 2.0*10^{17}m^{-3} of plasma density and 0.025eV of positrons as used in WARP simulations, and the simulation time is larger than 100ns much longer than Coulomb scattering time. With the increased plasma density, the Coulomb scattering time will be reduced further. Thus the positron loss from Coulomb scattering is not a major factor for positron loss.
are also shown in Table 1. In the case of WARP the densities have been chosen keeping fixed the ratio between electrostatic and thermal energy.

Table 1. The number of trapped positrons with radii of microtraps. The macrotrap has a 2.5cm radius,70 mm source length and about126 mm trap length. A fill factor of 75% is accounted for in evaluating the number of microtraps contained in a macrotrap, and therefore the total number of positrons that can be stored. The first four rows refer to CPO results, the last four to WARP results.

| Trap type          | R=10µm | R=30µm | R=50µm |
|--------------------|--------|--------|--------|
| Density n (m⁻³)    | 4.8×10¹⁶ | 5×10¹⁸ | 4.5×10¹⁶ | 5×10¹⁷ | 2.3×10¹⁶ | 2×10¹⁷ |
| Loss % from initial total number of positrons | 16.7% from 1.8×10⁶ | 0% from 3.7×10⁷ | 14.3% from 1.4×10⁷ | 0% from 3.3×10⁷ | 0% from 1.9×10⁷ | 0% from 3.7×10⁷ |
| Number of positrons per Microtrap Nₜ | 1.5×10⁶ | 3.7×10⁹ | 1.2×10⁷ | 3.3×10⁷ | 1.9×10⁷ | 3.7×10⁷ |
| microtraps in a macrotrap Nₕ | 4.7×10⁶ | 4.7×10⁹ | 5×10⁶ | 5.2×10⁷ | 1.8×10⁷ | 1.8×10⁷ |
| Total number of positrons in macrotrap | 7.5×10¹² | 1.7×10¹⁴ | 6×10¹² | 1.7×10¹³ | 3.4×10¹² | 6.6×10¹² |

5. Conclusions
From these preliminary modelling results, we can confirm the advantage of smaller trap radii for increasing the total number of trapped positrons. The discrepancies between CPO and WARP are less than an order of magnitude but the overall conclusions remain the same that with decreased size of one microtrap more positrons can be trapped in one macrotrap. In the future, we will work on the simulations of positrons in a tube with a radius even smaller than 10µm and with longer simulation time.

6. Acknowledgements
The authors acknowledge very productive discussions and help from David Grote for the WARP simulations and Frank Read for the CPO simulations. *Funding* for this research provided by the Office of Naval Research Award No. N00014-10-1-0543, Dr. Harold S. Coombe

7. References
[1] D P Grote et al, Proc. of the 1995 International Symposium on Heavy Ion fusion, Fusion Engineering and Design, 32-33 (1996) 193-200
[2] D P Grote, A Friedman, I Huber, W M Fawley, J L Vay, Nucl. Instrum. Methods Phys. Res. A 415, 428 (1998)
[3] CPO programs, free versions available at http://www.electronoptics.com/
[4] F H Read, A Chalupka and N J Bowring, “The charge-tube method for space-charge simulations.” The International Journal for computation and Mathematics in Electrical and Electronic Engineering 18, no. 4 (1999): 548-555.
[5] P J Schultz. and K G Lynn, Rev. Mod. Phys. 60 (1988) 701.
[6] Paul Coleman, Positron Beams and its applications, ed. World Scientific Books, 2004
[7] P M Platzman, A P Mills Jr, Phys. Rev. B 49,454-458, 1994.
[8] R G Greaves, C M Surko, 2002. In: Anderegg, F., et al. (Ed.), Non-Neutral Plasma Physics, Vol. IV. American Institute of Physics, Melville, NY, pp. 10–23.
[9] M Amoretti, C Amster, G Bonomi, et al., 2002. Nature 419, 456.
[10] C M Surko, M Leventhal, W S Crane, et al., 1986. Rev. Sci. Instrum. 57, 1862–1867.
[11] R G Greaves, S J Gilbert and C M Surko, Appl. Surf. Sci. 194 (2002) 56.
[12] R G Greaves, C M Surko, 1997. Phys. Plasmas 4, 1528–1543.
[13] C F Driscoll, J K Malmberg, Phys. Rev. Lett. 50 (1983) 167.
[14] R G Greaves, S J Gilbert, C M Surko, Appl. Surf. Sci. 194 (2002) 56.
[15] J Xu, M H Weber and K G Lynn Proc. of the Int. Enrico Fermi School 2009, “Physics with many Positrons”, in press
[16] C M Surko and R G Greaves, Radiation Physics and Chemistry, 68, 419 (2003)
[17] J R Danielson, T R Weber and C M Surko, Phys. Plasma, 13, 123502 (1996)
[18] R.G. Greaves, C.M. Surko, 2002. In: Anderegg, F., et al. (Ed.), Non-Neutral Plasma Physics, Vol. IV. American Institute of Physics, Melville, NY, pp. 10–23.
[19] A Friedman and S P Auerbach. Numerically induced stochasticity. Journal of Computations Physics, 93: 171, 1991
[20] K. Gomberoff, J S Wurtele, A Friedman, D P Grote and J.L. Vay J. Comput. Phys. 225 1736-1752(2007)
[21] W M Manheimer, M Lampe and G Joyce J. Comput. Phys. 138 563(1997)
[22] B I Cohen L Divol A B Langdon and E A Williams Phys. Plasma, 13, 022705 (2006)