Impact of Pitot tube calibration on the uncertainty of water flow rate measurement

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Abstract. Water utility companies often use Cole type Pitot tubes to map velocity profiles and thus measure flow rate. Frequent monitoring and measurement of flow rate is an important step in identifying leaks and other types of losses. In Brazil losses as high as 42% are common and in some places even higher values are found. When using Cole type Pitot tubes to measure the flow rate, the uncertainty of the calibration coefficient ($C_d$) is a major component of the overall flow rate measurement uncertainty. A common practice is to employ the usual value $C_d = 0.869$, in use since Cole proposed his Pitot tube in 1896. Analysis of 414 calibrations of Cole type Pitot tubes show that $C_d$ varies considerably and values as high as 0.020 for the expanded uncertainty are common. Combined with other uncertainty sources, the overall velocity measurement uncertainty is 0.02, increasing flowrate measurement uncertainty by 1.5% which, for the Sao Paulo metropolitan area (Brazil) corresponds to $3.5 \times 10^7$ m$^3$/year.

1. Introduction
Loss of treated water is large in Brazil. While there are no official measurements, the losses are estimated to vary between 37 and 42%. These losses are usually a result of leaks and illegal water connections. To improve this picture, it is necessary to implement an efficient measurement/monitoring in all stages of the supply network.

Since flow rate is an important parameter in quantifying the losses, it is important to use calibrated flow meters with low uncertainty. Usually, large flow meters are calibrated in field by mapping the velocity profile [1] using Cole type Pitot tubes (Figure 1). Thus, flow rate uncertainty is directly dependent upon the uncertainty of the velocity measurement. High flow rate uncertainty not only, potentially, corresponds to the large monetary value of unmeasured water flow but also makes it difficult to identify losses in the water supply system, whether they are physical in nature such as leaks or non-physical such as illegal connections to the supply network, for instance.

The advantage of using a Cole type Pitot tube when mapping the velocity profile is that it can be used in water pipes with with a wide range of sizes, from diameters as low as 300 mm up to a few meters. While the mean value of the calibration coefficient or discharge coefficient ($C_d$) of a Cole type Pitot tube is 0.869 [2], its use has a large impact on flow rate uncertainty as shown here, where 414 calibrations were carried out between 2004 and 2013 at the anemometry lab of the Instituto de Pesquisas Tecnológicas (IPT), Brazil.
The impact of the uncertainty of \( C_d \) was estimated for the metropolitan area of São Paulo (RMSP). RMSP encompasses 8 distribution systems of treated water: Alto Cotia (1.2 \( m^3/s \)), Alto Tietê (15 \( m^3/s \)), Cantareira (33 \( m^3/s \)), Guarapiranga (14 \( m^3/s \)), Rio Claro (4 \( m^3/s \)), Rio Grande (5 \( m^3/s \)), Baixo Cotia (0.9 \( m^3/s \)) and Ribeirão da Estiva (0.1 \( m^3/s \)). This amounts to 73.2 \( m^3 \) or \( 2.3 \times 10^9 \) \( m^3/year \) of clean water. It should be noted that even with small changes, on the order of 1 \%\, of the calibration coefficient, can lead to errors of several million cubic meters at the end of one year.

Analyzing this database of calibrations shows that the periodic calibration of Cole type Pitot tubes is important. This is not a simple metrological quality system requirement [3] but a fundamental step if improvement of flow rate uncertainty is desired.

2. Flow rate measurement using Cole type Pitot tube

The Cole type Pitot tube is a differential pressure device designed by Edward Cole in 1896 [2], that has a total pressure tap (tip A, faces upstream) and a wake pressure tap (tip B, faces downstream) as shown in Figure 1.

When Cole type Pitot tubes are used for measuring water flow rate, the velocity profile is measured using parameters established in the Standard ISO 3966 [1]. Figure 2 shows an example of velocity profile including the measurement points.

The flow rate \( (Q) \) is determined using the mean velocity \( (V) \) obtained from the velocity profile along one or two orthogonal diameters in a cross section of the pipe with area \( A \):

\[
Q = V \cdot A
\]  

(1)

The mean velocity \( (V) \) is obtained from point measurements of velocity \( (V_i) \) at positions specified by the Standard ISO 3966 [1]. The velocity \( (V_i) \) in each point is obtained from the differential pressure across the Cole type Pitot tube according to the following equation:

\[
V_i = C_d \sqrt{\frac{2 \Delta P}{\rho}}
\]  

(2)

where \( C_d \) is the calibration coefficient of the Cole type Pitot tube, \( \Delta P_i \) is the pressure differential measured in each point \( i \) of the profile and \( \rho = 998.202 \, kg/m^3 \) is the density of water at 20 °C and 101.325 kPa.

The mean calibration coefficient recommended by the literature for conventional Cole type Pitot tubes (a slight modification of the usual Cole type Pitot tube will also be analyzed), including corrections is 0.869 [2]. Figure 3 presents a set of measurements that are commonly used to calibrate Cole type Pitot tubes.
used by water utility companies in Brazil [4]. This figure shows the dependence of $C_d$ with Reynolds number. The Reynolds number is defined as:

$$Re = \frac{VL}{\nu}$$

(3)

where $L$ is a characteristic length, here fixed as 1 m, $\nu = 1.004 \times 10^{-6} \text{m/s}^2$ is the kinematic viscosity of water at 20°C.

The results in figure 3 for $0.5 \times 10^6 \leq Re \leq 3.0 \times 10^6$, correspond to a velocity range of $0.5 \leq V \leq 3.0 \text{ m/s}$ in water. In this velocity range, the calibration coefficient of the Cole type Pitot tube varies between 0.883 for 0.5 m/s and 0.861 for 3.0 m/s with a mean value of 0.867.

The modified Cole type Pitot tubes tested in this work possess a safety pin between the tips (figure 4) to protect the tips from damage when internal walls of the pipes are hit. Calibrations of modified Cole type Pitot tubes over the years resulted in a mean calibration coefficient of 0.886.

Table 1 presents the sources of uncertainty of the flow rate measurement using Pitot tubes. The standard uncertainties associated with these sources are estimated from instrument calibration and measurement record. These uncertainties are combined using the following equation:

$$u^2_c(y) = \sum_{i=1}^{n} \left| c_i \right|^2 u^2(x_i) = \sum_{i=1}^{n} \left[ \frac{\partial f}{\partial x_i} \right]^2 u^2(x_i)$$

(4)

where $x_i$ is an uncertainty source, $c_i$ is its sensitivity coefficient and $u(x_i)$ is the corresponding standard uncertainty. The uncertainty of the calibration coefficient is one of the major contributions to the overall uncertainty of the the velocity [5]. Thus, the use of a single mean value for $C_d$ can have a large effect on the uncertainty of flow rate.
Figure 3. Reynolds number dependence of calibration coefficient ($C_d$) according to [4].

At IPT, the Cole type Pitot tubes are calibrated using an aerodynamic wind tunnel (figure 5) [6]. Calibrations in a wind tunnel of Cole type Pitot tubes to be used in water agree with calibrations in Pipes and towing tank (both executed at IPT) [7].

During calibration, the Cole type Pitot tubes are positioned at the discharge of the wind tunnel. A static Pitot tube is used as a standard. Both Pitot tubes are connected to manometers and two rising sequences of points consisting of 10 air velocities between 5 and 36 m/s are compared. Using Reynolds similarity, where

$$Re_{water} = Re_{air}$$

these velocities correspond to 0.3 and 2.4 m/s in water respectively.
Table 1. Uncertainty sources and its sub-components in flow rate measurement using Pitot tubes.

| Uncertainty Source | Components                        |
|--------------------|-----------------------------------|
| Pressure differential | Density                        |
| Density            | Calibration coefficient          |
| Local velocity     | Velocity time fluctuations | Velocity gradient |
|                    | Pitot tube orientation          | Pitot tube blockage |
| Mean velocity      | Local velocity                  |
|                    | Pitot tube position             |
|                    | Numerical integration           |
| Pipe cross section | Diameter                        |

Figure 5. Calibration of Cole type Pitot tube at the aerodynamic wind tunnel.

The calibration coefficient of the Cole type Pitot tube is obtained using the following equation:

\[ C_d = C_p \sqrt{\left( \frac{\Delta P_P}{\Delta P_C} \right)} \]  

(5)

where \( C_p = 0.997 \) is the calibration coefficient of the static Pitot tube used as a standard [8], \( \Delta P_P \) and \( \Delta P_C \) are, respectively, the pressure differential of the static and Cole type Pitot tubes.
3. Methodology
The variation of calibration coefficients of Cole type Pitot tubes were analyzed from a total of 414 calibrations consisting of 212 conventional Cole type Pitot tubes and 202 modified Cole type Pitot tubes (with a protective pin between tips). These calibrations date from 2004 until 2013.

Initially, the mean and standard deviation of calibrations were obtained for both modified and conventional Cole type Pitot tubes. Then, the successive calibrations of four individual Pitot tubes were analyzed. Each Pitot tube analyzed was calibrated 5 times and the conventional Cole type Pitot tubes are identified by C1 and C2, while the modified Pitot tubes are identified by M1 and M2.

While the calibration coefficient \( C_d \) varies with Reynolds, usually most variation happens at lower Reynolds numbers (low speeds). The calibration coefficient is calculated for each calibration using the mean \( C_d \) of calibration points in the region of lower dependence on Reynolds number (larger velocities). The calibration coefficients obtained from every calibration was used to estimate mean value and standard deviation of \( C_d \) for both conventional and modified Cole Pitot tubes. This information was used to determine the expanded uncertainty of \( C_d \) with a confidence level of approximately 95 %. This overall uncertainty was compared to the individual uncertainty of each calibration. Finally, the impact on the flow rate from using a single generic value for \( C_d \) instead of a calibrated value was estimated.

4. Results
4.1. Individual calibrations
For some Pitot tubes, several calibrations over a period of time where available. With this information it was possible to verify how \( C_d \) changed over the lifetime of the Pitot tube due to typical wear and damage. This change is mostly due to deformations of the tips that usually occur when the tips hit the inside walls of pipes.

Two conventional (C1 and C2) and two modified (M1 and M2) Cole type Pitot tubes which were calibrated 5 times over the span of 10 years were analyzed. Figure 6 shows the mean \( C_d \) for Pitot tubes C1 and C2 for different Reynolds number (Re). Each point corresponds to the average of 5 calibrations and the vertical bars represent the range of \( C_d \) values obtained considering every calibration of the same Pitot tube at the same Re.

Ideally, the Cole type Pitot tube is symmetrical so that either tip can be used to measure the total pressure. Unfortunately manufacturing problems and wear cause differences on the tips and both sides are calibrated (A and B). In figure 6(b), the differences between sides A and B are considerable and therefore, special care should be taken to identify the sides during calibration. Figure 7 corresponds to modified Pitot tubes M1 and M2.

Figures 6 and 7 show that even for a single Cole type Pitot tube there is significant standard deviation of the order of 0.007 of \( C_d \) during its life time. Variations of the same order are also observed when comparing calibrations of both sides of the Cole type Pitot tubes. In the case of Pitot tube C2 this difference is close to 0.013 while for M2 this difference is 0.007, lower but still considerable. These differences are close to the uncertainty itself: different sides of a same Pitot tube are equivalent to completely different Pitot tubes.

4.2. Historical database of Cole type Pitot tube calibration coefficients
The 212 conventional Cole type Pitot tubes presented a mean value of \( \bar{C}_d = 0.8696 \) with a standard deviation of 0.0094 as shown in figure 8(a). The mean calibration coefficient is close to the value of 0.869 originally suggested by Cole [2]. However, this historic mean has a large uncertainty that can not be neglected.

The modified Cole type Pitot presents a slightly higher calibration coefficient of 0.883 and standard deviation of 0.0084 as shown in figure 8(b). This standard deviation is similar to the conventional Pitot tube.
(a) C1

(b) C2

Figure 6. Mean value of $C_d$ and variation during 5 calibrations for sides A and B of Cole type Pitot tubes C1 and C2.

(a) M1

(b) M2

Figure 7. Mean value of $C_d$ and variation during 5 calibrations for sides A and B of Cole type Pitot tubes M1 and M2.
This historical database of modified and conventional Cole type Pitot tubes include periodical calibrations of a large number of distinct Pitot tubes. Considering only Pitot tubes that were calibrated at least 3 times, and calculating the standard deviation of \( C_d \) for each Cole type Pitot tube, we obtain the histogram in figure 9(a).

This histogram has a peak near 0.005 and falls off rapidly standard deviations larger than 0.01. Clearly this distribution is not normal. It is interesting to remember that one of the situations when the Pitot tube is calibrated occurs when the tip is damaged and the specific Pitot tube is effectively a different Pitot tube. This histogram includes data from Pitot tubes that were repaired and Pitot tubes that suffered normal wear. Without further information on the history of each individual Pitot tube, no further assessments of the causes of the peculiar shape of the histogram can be made.

Figure 9(b) obtained by integrating the histogram of figure 9(a) shows the probability that a given Pitot tube will have a standard deviation of \( C_d \) during its lifetime lower than a certain value. For a probability of 95 \%, this value corresponds to 0.105. It is worthy of note that this value is very close the standard deviation of all calibration coefficients (figure 8).

4.3. Impact on flow rate measurement
When the velocity profile is well behaved as shown in figure 2 and considering all sources of uncertainty in table 1, equation (4) can be used to obtain the expanded uncertainty of flow rate.

If a calibrated \( C_d \), with expanded uncertainty of 0.008, is used, the expanded uncertainty of the flow rate is \( U_Q = 2.5 \% \). On the other hand, if a generic mean value for \( C_d \) is used with expanded uncertainty of 0.02, the overall expanded flow rate uncertainty is \( U_Q = 4.0 \% \).

Therefore, the use of a calibrated Pitot tube reduces the overall uncertainty of the flow rate by approximately 1.5 \%. This reduction is still expected, approximately, for poor velocity profiles that present a higher overall uncertainty.
Figure 9. (a) Histogram of standard deviation over the life of single Cole type Pitot tube calibration coefficients of tubes calibrated at least 3 times; and (b) probability that a given Pitot tube will have, over several calibrations, a standard deviation lower than a given value, calculated by integrating the histogram in part (a).

The contribution of calibration coefficient uncertainty is important but still limits the uncertainty of flow rate measurement to values above 2%. However by using a non calibrated Cole type Pitot tube an additional 1.5% is to be expected. While some may consider this value, in general, acceptable, it is large enough to affect water loss management and the financial impact on supplier or consumer cannot be dismissed. As an example, the city of São Paulo distributes 73.2 m$^3$/s of treated water. An additional uncertainty of 1.5% represents $3.5 \times 10^7$ m$^3$/year.

5. Conclusions
While the use of mean calibration coefficient for Cole Pitot tubes seems reasonable since this mean doesn’t change over a period of decades, when comparing individual Cole type Pitot tubes, the historical database with 414 calibrations carried out from 2004 until 2013 shows that the expanded combined uncertainty of the mean calibration coefficient can be over 0.02.

This uncertainty contributes to an additional 1.5% uncertainty which corresponds to $3.5 \times 10^7$ m$^3$/year in the metropolitan region of São Paulo. Periodic calibration of the Pitot tube reduces this uncertainty to 0.008 and is an essential step if lower flow rate uncertainties are desired.
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