The influence on casing stress for shale gas fracturing wells considering thermo-pressure coupling effect

Abstract

During the hydraulic fracturing process for shale gas wells, the fracturing fluid is injected into the wellbore with large pump rate. The bottom-hole temperature will decrease sharply, which increasing the risk of casing failure. Based on the actual data of shale gas wells, the influence of rheological parameters of fracturing fluid on heat transfer coefficient is analyzed. A transient thermo-pressure coupling model of casing-cement sheath-formation combination is established. Sensitivity analysis is conducted for different pump rates, pressures and injection temperatures. The results indicate that:

i. The rheological parameters of fracturing fluid can affect the heat transfer coefficient between the fluid and borehole wall, which in turn affects the temperature distribution at the bottom of the wellbore.

ii. The increasing of pump rate will drastically reduce the bottom-hole temperature, then thermal stress generating on the casing.

iii. The higher the original reservoir temperature, the greater the influence of the temperature.

iv. The casing stress decreases then increases with the increasing of fracturing pressure. Therefore, the rheological parameters of fracturing fluid should be chosen reasonably to reduce the heat transfer coefficient. What is more, it is crucial to choose the appropriate fracturing pump rate, fracturing pressure and injection temperature, as far as possible to reduce the bottom-hole temperature difference, which avoiding excessive casing stress to enhance the casing safety for fracturing shale gas wells.

Keywords: rheological parameter thermo-pressure coupling, transient temperature field, casing stress, hydraulic fracturing

Introduction

The casing deformation problems are presented over 36 wells (among 112 horizontal wells by 2016) during fracturing processes in Weyuan-Changning shale gas play in China. Subsequent tools could be blocked in the wellbore. The segments with serious deformation had to be abandoned before completing fracturing operations. The casing deformation issues make a great influence on production of shale gas wells.

Shale gas fracturing is carried out with large pump rate and high pressure. When the fracturing fluid is injected into the wellbore, it will have a great influence on the downhole temperature. Zhao et al. calculated the distribution of wellbore temperature field during fluid injection or fluid production by implicit difference model. Wang et al. proposed a new method of heat transfer in unsteady state of wellbore and took a detailed calculation of the wellbore temperature field during fracturing process. Yang et al. established a mathematical model of heat conduction in porous media, giving the temperature analysis solutions of well in the case of injection and production operations through the mathematical method. Wang et al. considered the friction heat of fluid during fracturing process, and calculated the wellbore temperature field. He held the point that the friction heat generated on the fracturing fluid temperature could not be ignored. Satman et al. conducted an in-depth discussion of the wellbore heat transfer theory and clarified the essence of static and transient heat transfer. You et al. established a fully implicit algorithm for the wellbore to gain the unsteady temperature field of surrounding rock. Cai et al. established a coupled model to study the temperature field distribution during the production of fractured horizontal wells. To sum up, scholars have done a lot of research on the calculation of wellbore temperature field, but most of them only focused on the calculation of temperature field under the condition of tubing injection. Rare researches were conducted on transient temperature field and thermal stress during fracturing process for shale gas wells. Based on rheology and thermo-elasticity theory, a transient temperature-pressure coupling model of casing-cement sheath-formation combination is established in this paper. Considering the influence of fracturing fluid rheological parameters, emphasis is put on the influences of fracturing pump rate, injection temperature and fracturing pressure on casing stress. Corresponding countermeasures are proposed according to the law of the impact.

Keywords: rheological parameter thermo-pressure coupling, transient temperature field, casing stress, hydraulic fracturing
Materials and methods

Heat transfer coefficient calculation

The fracturing fluid used in the fracturing process is generally non-Newtonian fluid\textsuperscript{13}, while its rheological property equation is:

\[ \tau = K' \gamma^n \quad \text{(1)} \]

During fracturing process, the pump rate is usually very large, being in turbulent flow state. The heat transfer coefficient between the fracturing fluid and the inner wall of the casing can be calculated using the Marshall model\textsuperscript{14} in Eq. (2-7):

\[ \mu_{w,app} = K \left( \frac{3n+1}{4n} \right)^{\frac{1}{n}} \left( \frac{8V}{D} \right)^{1-n} \quad \text{(2)} \]

\[ R_{eg} = \frac{\Delta D_{eg} \times}{\mu_{w,app}} \quad \text{(3)} \]

\[ P_r = \frac{\mu_{w,app} C_n}{\kappa_n} \quad \text{(4)} \]

\[ v = \frac{q}{15 \pi D^2} \quad \text{(5)} \]

Finite element model establishment

After cementing slurry solidification, the casing, cement sheath and surrounding rock wall will be consolidated as a whole. Assuming that the contact part of the three cementing good, non-slip generated, three parts are uniform homogeneous thermo-elastic\textsuperscript{15,16} based on the thermo-elastic theory, a temperature - pressure coupling combined model is established, shown in Figure 1.

Since the axial dimension of the assembly is much larger than the radial dimension, the assembly can be simplified as a plane strain model.\textsuperscript{17} It means that the strain exists only in the radial direction but no strain in the axial direction.

Thermal conduction equations

The transient heat transfer equation is obtained as time-dependent temperature distribution by Eq. (8):

\[ k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + Q_i = C_r \frac{\partial T}{\partial t} \quad \text{(8)} \]

Initial condition: \( T^i = T^i(x, y, z) \big|_{t=0} \)

Boundary conditions

Surface S1 boundary condition: \( T^i = T^i(S, t) \big|_{S_1} \)

Surface S2 boundary condition: \( \frac{\partial T}{\partial n} |_{S_2} = q_i(S, t) \big|_{S_2} \)

Based on the Galerkin’s weak form of equivalent integrals, the heat conduction equations can be written as Eq. (A-3)

\[ \int \left( \nabla \cdot \left( \kappa_{ij} \nabla T \right) \right) \delta T \, dx \, dy \, dz = \int Q_i \delta T \, dx \, dy \, dz \quad \text{(9)} \]

The regions are divided into finite elements. Specific element nodes temperatures are used to represent the whole elements temperatures through the appropriate shape function. Finite element equations about nodes temperature are obtained using the boundary conditions. The nodes temperatures can be obtained by solving these finite equations, shown in Eq. (10-13).

\[ C_T + K_T = P \quad \text{(10)} \]

\[ C_y = \sum_i C_{ij} \quad \text{(11)} \]

\[ K_y = \sum_i K_{ij} + \sum_i H_{ij} \quad \text{(12)} \]

\[ P_i = \sum_i P^i_{qk} + \sum_i P^i_{ik} + \sum_i P^i_{jk} \quad \text{(13)} \]

Thermal-pressure coupling equations

The basic equations of mechanics are derived from the theory of elasticity.

Balance equations:

\[ \mathbf{L} \cdot \sigma + f = 0 \quad \text{(14)} \]

Geometric equations:
constitutive equations:
\[ \sigma = D(\varepsilon - \varepsilon_0) = DBa = S = s \quad \text{(16)} \]

boundary conditions in \( \Sigma \varepsilon \):
\[ \varepsilon n = T \quad \text{(17)} \]

degree of freedom conversion equations
\[ a = Ga \quad \text{(18)} \]

there would be no free space to deform because the casing, cement sheath, and formation consolidated as a whole. Thermal strain would be generated when the temperature changed. The strains in transverse isotropic elastic body are:
\[ \varepsilon_0 = \left[ \begin{array}{ccc}
\alpha & \alpha & 0 \\
\alpha & \alpha & 0 \\
0 & 0 & 0
\end{array} \right] T \quad \text{(19)} \]

the functional total of the minimum potential energy is:
\[ \Pi = \int_{\Omega} \frac{1}{2} \frac{T}{\varepsilon} D \varepsilon d \Omega - \int_{\Omega} \frac{1}{2} E \varepsilon \varepsilon d \Omega - \int_{\Omega} f d \Omega - \int_{\Gamma} t_{\varepsilon} u^T T d s \quad \text{(20)} \]

solved domains should be discrete firstly. The system potential energy can be represented by the node displacements through the elements total potential energies. According to the variation principle, one order variation of functional is set zero. The available value of the function is the displacement vector sought. Then finite element solution equations can be obtained as the following:

\[ K a = P \quad \text{(21)} \]

\[ K = \sum_{\varepsilon} G^T \left[ \int_{\Omega_{\varepsilon}} B^T D B d \Omega \right] G \quad \text{(22)} \]

\[ P = \sum_{\varepsilon} G^T \left[ \int_{\Omega_{\varepsilon}} N^T f d \Omega + \int_{\Sigma} N^T T d s \right] + \sum_{\Gamma_{\varepsilon}} B^T D e d \Omega \quad \text{(23)} \]

since the thermal parameters do not change with time within the considered temperature range, it can be superimposed with the mechanical analysis step after the entire transient temperature field analysis is completed. In this way, the stress can be calculated under the condition of thermal-pressure coupling.

**Parameter setting**

according to the theory of elastic mechanics, stress concentration phenomenon occurs near the circular hole in an infinite plate, but when the boundary size exceeds 5-6 times than that of well-hole diameter, the influence of stress concentration is very small. The material and thermal parameters are shown in Table 1.

| Table 1 Basic parameters of the FEM model |
| Parameters | Value         | Parameters | Value         |
|---------------|--------------|------------|--------------|
| Wellbore diameter /mm | 215.9         | Well depth /m | 1500         |
| Casing diameter/mm | 139.7         | Minimum horizontal stress \( \sigma_0 \)/MPa | 29           |
| Casing thickness/mm | 9.17          | Maximum horizontal stress \( \sigma_0 \)/MPa | 48           |
| Boundary geometry/mm | 3000         | Vertical stress gradient /MPa/m | 0.023        |
| Elastic modulus \( E \)/GPa (i=0,1,2,3) | 210, 10, 22 | Geothermal gradient /K/m | 0.025        |
| Poisson’s ratio \( \nu \) (i=0,1,2,3) | 0.3, 1.17, 1.23 | Pump rate \( Q \)/m³/min | 20-Jan        |
| Coefficient of thermal expansion \( \alpha_\varepsilon \)/×10⁻⁵/K (i=0,1,2,3) | 1.06, 1.0, 1.02 | Fluid consistency coefficient \( K \)/ Pa·sⁿ | 0.01-1       |
| Specific heat capacity \( C_\varepsilon \)/J/(kg·K)(i=0,1,2,3) | 393,546,018,301,043 | Fluid rheological behavior index \( n \)/ sⁿ | 0.1-1        |
| Heat conduction coefficient \( K_\varepsilon \)/ W/(m·K) (i=0,1,2,3) | 1.73,58.2,1.74,1.0 | Fluid temperature | 100,150      |
| Density \( \rho \)/ kg/m³ (i=0,1,2,3) | 1,080,785,018,002,500 | i=0, 1, 2, 3 represented the fluid, casing, cement sheath, and formation |

**Results and discussion**

**The heat convection coefficient**

The rheological parameters and pump rate of fracturing fluid will affect the heat transfer coefficient²¹. Using equations (2-7), the variation law of heat transfer coefficient under different rheological parameters and pump rates can be calculated, shown in Figure 2.

As shown in Figure 2 when the rheological parameters \( n \) or \( K \) keeps constant, the heat transfer coefficient increases with the increasing of the pump rate. When the pump rate stays the same, the heat transfer coefficient firstly decreases dramatically then slowly with the increasing of rheological parameters. Therefore, the small heat transfer coefficient can be gained by appropriate rheological parameters and low pump rate.

**The transient temperature of fracturing fluid**

Setting the rheological parameters of fracturing fluid \( K=0.08, n=0.8 \), pump rate \( Q=0/3/6/20\text{m}^³/\text{min} \), then calculating the corresponding heat transfer coefficient. The results are shown in Table 2.

| Table 2 Heat transfer coefficient for different pump rate |
| Pump Rate \( Q \)/m³/min | Heat transfer coefficient \( h \)/ W/(m²·K) |
|----------------------|------------------------------------------|
| 0                    | 50                                       |
| 3                    | 107                                      |
| 6                    | 777                                      |
| 20                   | 1890                                     |

From the heat transfer coefficient calculated above, the transient bottom-hole temperature change can be calculated by establishing the
heat conduction model (10) to calculate the reservoir temperature at 100 and 150, respectively. The results are shown in Figure 3.

It can be seen from Figure 3 that when the pump rate of fracturing fluid is zero, the bottom hole temperature changes more slowly, and when the pump rate increases, the bottom hole temperature will decrease drastically. With a certain pump rate, the higher the temperature at the bottom of the well, the higher the temperature decreases. For example, the maximum temperature difference at the bottomhole temperature of 100°C is close to 80°C and the maximum temperature difference at the bottomhole temperature of 150°C is close to 120°C. Therefore, using large pump rate parameter will have a larger temperature difference.

The influence of pump rate on casing stress

Based on the basic equations of temperature-pressure coupling model, the transient temperature field changes calculated above are substituted into the stress model of the composite model. The temperature-pressure coupling calculation is carried out to calculate the stress variation of the inner wall of the casing body under different pump rates. The results are shown in Figure 4.

It can be seen from the Figure 4 that the fracturing pump rate will exert a certain influence on the casing stress. With the increase of rate, the bottom hole temperature difference increases, then the casing stress increases. At the same time, the higher the formation temperature, the greater the temperature difference of the bottom hole and the larger the casing stress will be. This indicates that during fracturing process, large pump rate will lead to dramatic changes in temperature, thereby increasing the casing stress.

The influence of fluid injection temperature on casing stress

The fracturing fluid temperature is crucial to guarantee the casing safety. The fluid temperature in the model are set as 0/20/40/60/80/100°C. The regularity of the influences of different fluid temperature on casing stress are investigated and the results are shown in Figure 5.

The transient internal casing temperature drops dramatically in the first 5 minutes, then keeps almost the same with the lapse
of time. The lower the fluid temperature is, the greater the bottom casing temperature reduction. At the same time, the casing stress will increase with the decreasing of fluid temperature. In view of this, the fracturing fluid temperature is important for the safety of casing. During fracturing, the high temperature fracturing fluid can be adopted to reduce the risk of casing failure.

The influence of the pump rate on casing stress

The fracturing fluid with high pressure is adopted to fracture the shale formation. The larger the tectonic stress is, the higher the pressures are. Meanwhile, the high fracturing pressure can pose a great potential challenge to casing deformation. The influences of different pressures on casing stress were simulated. The results are shown in Figure 6.

It can be seen from Figure 6 that the casing stress decreases firstly, then increases with the increasing of fracturing pressure. The maximum stress changes from 90° to 0°. In order to fracture the shale formation, the fracturing pressure must be high enough. But too large pressure will increase casing stress. That can tell us that some measures needs to be adopted to decrease the fracturing stress of shale formation. Only in this way, can the casing be a safe place.

Conclusion

During the hydraulic fracturing process for shale gas wells, temperature drops dramatically in the bottom of the well due to the large pump rate. To evaluate the casing stress during volume fracturing, the heat transfer coefficient was calculated. A finite element model considering the transient thermal-pressure coupling was established. Sensitivity analyses of the influences of pump rate, injection temperature, and fracturing pressure on casing stress were investigated. Conclusions can be drawn as the following:
i. A small convective heat transfer coefficient could be gained by appropriate rheological parameters and low pump rate. Downhole temperature decreased quickly with the increasing of the pump rate.

ii. Casing stress increased with the increasing of pump rate. The higher the initial reservoir temperature, the greater the casing temperature decreasing.

iii. The bottom-hole temperature decreased with the increasing of fluid injection temperature, then decreasing the casing stress.

iv. The casing stress firstly decreased, and then increased with the increasing of fracturing pressure.

During fracturing operation, the warm injection fracturing fluid, appropriate pump rate and fracturing pressure could be adopted to reduce the casing stress.

**Nomenclature**

$K$ is the consistency factor, Pa·s$^{-n}$;  
$n$ is the fluidity index, $n<1$ presented the pseudo-plastic fluid, $n>1$ presented the expansive fluid, for the fracturing fluid $n<1$, meaning the pseudo-plastic fluid;  
$\gamma$ is shear Rate, s$^{-1}$;  
$h$ is the heat transfer coefficient. W/(m$^2$·°C)  
$Nu$ is the Nusselt number  
$Pr$ is the Prandtl number  
$Re$ is the Reynolds number  
$\mu_{app}$ is the fluid apparent viscosity  
$D$ is the inner diameter, m,  
$D_{eff}$ is the equivalent diameter, m  
$\rho$ is the fluid density, kg/m$^3$.  
$v$ is the fluid velocity.  
$Q$ is the fracturing pump rate, m$^3$/min,  
$k_n$ is the coefficient of heat conductivity. W/(m·°C)  
$C_m$ is the fluid specific heat capacity, J/(kg).  
$k$ is the material coefficient of heat conductivity, W/(m·°C).  
$C_p$ is the material specific heat capacity, J/(kg·°C).  
$\rho$ is the material density, kg/m$^3$.  
$Q_h$ is the heat source density, W/m$^3$.  
$q_i$ is the boundary heat source density. W/m$^2$  
$T$ is the temperature, °C  
$T_i$ is the fracturing fluid temperature, °C  
$t$ is the time, s,  
$s$, $n_x$, $n_y$, $n_z$ are the normal vector of $S_i$ in $x$, $y$, $z$ directions.  
i=1, 2, 3 are represented the casing, cement sheath, and formation respectively.

$\alpha$ is the coefficient of thermal expansion, °C$^{-1}$,  
$\Delta T$ is the temperature difference.  
$L$ is the differential operator.  
$\sigma$ is the stress matrix.  
$\varepsilon$ is the strain matrix.  
$\varepsilon_s$ is the thermal strain matrix.  
$f$ is the body force array.  
$u$ is the displacement array.  
$N$ is the shape function matrix.  
a is the element displacement array.  
$D$ is the elastic matrix.  
$B$ is the strain matrix.  
$S$ is the stress matrix.  
$T$ is the boundary surface force array.  
$n$ is the surface normal vector.  
a is the structure displacement array  
$G$ is the transformation matrix of degree of freedom between element nodes and structure nodes.  
$K$ is the Structure overall stiffness matrix.  
$P$ is the structural equivalent node load array.  
$C$ is the heat capacity matrix, $C_{ij}$ is any element of $C$.

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**Conflict of interest**

The authors declares no conflict of interest.

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