A path-based many-to-many assignment game to model Mobility-as-a-Service market networks

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ABSTRACT
As Mobility as a Service (MaaS) systems become increasingly popular, travel is changing from unimodal trips to personalized services offered by a market of mobility operators. Traditional traffic assignment models ignore the interaction of different operators. However, a key characteristic of MaaS markets is that urban trip decisions depend on both user route decisions as well as operator service and pricing decisions. We adopt a new paradigm for traffic assignment in a MaaS network of multiple operators using the concept of stable matching to allocate costs and determine prices offered by operators corresponding to user route choices and operator service choices without resorting to nonconvex bilevel programming formulations. Unlike our prior work, the proposed model allows travelers to make multimodal, multi-operator trips, resulting in stable cost allocations between competing network operators to provide MaaS for users. Algorithms are proposed to generate stability conditions for the stable outcome pricing model. Extensive computational experiments demonstrate the use of the model, and effectiveness of the proposed algorithm, to handling pricing responses of MaaS operators in technological and capacity changes, government acquisition, consolidation, and firm entry, using the classic Sioux Falls network.

Keywords: Mobility-as-a-Service, network design, traffic assignment, stable matching, assignment game

1. INTRODUCTION
There is a growing need to focus on managing the capacities, allocation, and pricing of mobility services in a Mobility as a Service (MaaS) (Hensher, 2017; Djavadian and Chow, 2017) ecosystem. Under this ecosystem, city agencies play a key role as facilitators in either economic deregulation through relationships with suppliers, or through government contracting with the operators, as illustrated in Figure 1 (Wong et al., 2019). As such, city agencies need to be able to assess the impact that a new mobility operator entering the market, or an existing one changing their service capacity, routing algorithm, or pricing mechanism, can have on other mobility operators and travelers’ welfare. A new mobility service or change to an existing one can cause travelers to switch routes or combine the service with other services to fulfill their trips or lead to certain routes becoming unstable to operate. Changes in algorithms (Stiglic et al., 2015) or government policies like ride surcharges (Hu, 2019) can alter the allocation of costs between travelers and operators. Any MaaS market equilibrium model needs to be sensitive to both traveler (multimodal/multi-operator routes) and operator (service coverage, fleet, pricing) decisions.
Figure 1. Evolution from current service delivery model (A) to two alternative models under MaaS (B, C) (source: Wong et al., 2019).

For this purpose, classic traffic assignment models that emphasize only traveler route decision-making are not effective tools. Instead, a new class of tools is needed to capture both travelers’ and operators’ decisions. Rasulkhani and Chow (2019) proposed such a method for evaluating multimodal systems, where each operator acts a set of service routes and travelers match “many-to-one” to each route while ensuring the line capacities are not violated and stability conditions from the core (Shapley and Shubik, 1971) of the model are met. The model is computationally tractable and can be solved using classic algorithms for capacitated assignment and linear programming (for linear pricing mechanisms). Unlike network flow games (e.g. Bird,
form coalitions between operators, the model matches between travelers and operators so that it explicitly captures both operator and travel behavior in a network of mobility markets.

The model from Rasulkhani and Chow (2019) matches traveler origin-destination pairs with single operator service routes. As such, it does not handle matching of travelers’ paths to multiple operators which is necessary for modeling MaaS. The challenge of matching multiple links of different traveler paths to multiple operators is a many-to-many assignment game. In such a game, the stability conditions become more complex because they need to be considered from both a user’s path level as well as an operator’s level in serving that user. Consider Figure 2 as an illustration. Traditional assignment models would simply assign travelers along the shortest cost paths between (1,6) and (2,5). In a MaaS market, however, each link is owned by an operator and may exit the market if there is insufficient incentive to provide service there. Each link is capacitated (for MaaS markets capacities are more appropriate than nonlinear congestion functions). Furthermore, each operator can choose a price to charge for using their link to the users. How should competing operators sharing different legs of a traveler’s trip set their prices? For example, travelers of (1,6) may opt to use only BLUE-SOLID operator. Travelers along (2,5), however, may take either ORANGE-DASH, GREEN-DOT, or a mix of all three (via node sequence (2,3,4,5)). The choice of the user-path depends on demand for OD (1,6), capacity on link (3,4), travel costs including travel time and prices set by all three operators as either of the other operators may reduce price to make the user switch. These more complex “blocking pairs” that can form need to be considered at a path level.

**ODs: (1,6), (2,5)**

![Figure 2](image-url) Illustration of the complexity of a MaaS market network.

We propose a model for this many-to-many assignment game and show how to derive an optimal assignment flow and corresponding stable outcome space between the operators and the travelers or users. If a stable space exists, it provides boundaries over which a city agency can work with competing operators to allocate costs between users and each operator to set prices. For example, travelers taking path (2,3,4,5) in Figure 2 would have to pay fares to all three operators. By knowing the stable range, the city agency can facilitate an agreement to offer bundled fares for such users across the three operators. The model does not assume any particular cost allocation mechanism or policy used by each operator; it only determines the thresholds within which such mechanisms would be stable. An empty set implies the unsustainability of the market structure as defined, which would warrant further planning (e.g. changing travel costs through infrastructure
investments or policies, adding further capacities, or introducing additional candidate routes for operators to serve). MaaS policy-makers, including government agencies and transport providers, can use the solution of the model to make trade-offs in cost allocation policies and algorithms, link capacities, and determine negotiating power of different operators for the purpose of forming coalitions or justifying subsidies between operators to match with traveler paths. This work is a companion piece to Rasulkhani et al. (2020). That study focuses on designing a solution method for a special case of this problem where the market is composed entirely of decentralized link operators.

2. LITERATURE REVIEW

While several studies have examined demand for MaaS services (Strömberg et al., 2018; Matyas and Kamargianni, 2019a,b), these studies have not sought to quantify or structure the relationships between decisions made by operators and users in providing and consuming routes in a MaaS market. The objective of modeling MaaS markets as networks is to identify feasible cost allocations that can support an optimal assignment of users. Earlier research on coexisting operators (see Chow and Sayarshad, 2014, for a review) consider noncooperative games, including a generalized Nash equilibrium for a duopolistic market of private mass transit operators (Harker, 1988) and toll pricing operators (Yang and Woo, 2000; Zhang et al., 2011). Cooperative game research, on the other hand, determines the cost allocations necessary to allow cooperation to take place as a coalition formation problem. Examples of cooperative games include Agarwal and Ergun (2008) and Lu and Quadrifoglio (2019).

The basis for this latter type of model is a stable matching model that forms coalitions between users and operators that no user or operator has incentive to break. Stable matching models in which utilities are transferable between operators and users are called assignment games, first formulated as linear programs (LPs) by Shapley and Shubik (1971).

In the basic model, there is a set of buyers $P$ and sellers $Q$. A buyer $i \in P$ that matches with a seller $j \in Q$ providing the product at cost $c_j$ earns a utility of $U_{ij}$. The difference between the utility and cost of production is the payoff $a_{ij} = \max(0, U_{ij} - c_j)$. A successful match means the seller transfers the utility to the buyer with a price $p$. The buyer earns utility equal to $u_i = U_{ij} - p$ while the seller profits $v_j = p - c_j$. The basic assignment game as an LP is shown in Eq. (1).

\[
\begin{align*}
\max & \sum_{i \in P} \sum_{j \in Q} a_{ij} x_{ij} \\
\text{Subject to} & \\
\sum_{i \in P} x_{ij} & \leq q_j, \quad \forall j \in Q \\
\sum_{j \in Q} x_{ij} & \leq w_i, \quad \forall i \in P
\end{align*}
\]
where \( x_{ij} \) is a binary variable whether a match occurs, and \( q_j \) and \( w_i \) are quotas for each side. If \( q_j \) and \( w_i \) are equal to one, the assignment game is one-to-one. Nonsingular integers reflect many-to-one or many-to-many games. An outcome \((u,v); x\) of the game is feasible if \( u_i \geq 0 \) and \( v_j \geq 0 \) and satisfies the constraints (1b) – (1d) as well as \( \sum_{i \in P} u_i + \sum_{j \in Q} v_j = \sum_{i \in P} \sum_{j \in Q} a_{ij} x_{ij} \). A feasible payoff is stable if \( u_i + v_j = a_{ij} \) when \( x_{ij} = 1 \) and \( u_i + v_j \geq a_{ij} \) when \( x_{ij} = 0 \). The core of the assignment game corresponds to the solutions of the dual of the LP.

In the multiple partner assignment game (Sotomayor, 1992), the values to the buyers and sellers are distributed to different matching partners: \( u_{ij} \) is the utility gained by buyer \( i \) when matched to seller \( j \) and \( v_{ij} \) is the profit gained by seller \( j \) when matched to buyer \( i \). A feasible outcome \((u,v); x\) is stable if \( u_i + v_j = a_{ij} \) when \( x_{ij} = 1 \) and \( u_i + v_j \geq a_{ij} \) when \( x_{ij} = 0 \), where \( \min_j \{u_{ij}\} \geq 0 \) and \( \min_i \{v_{ij}\} \geq 0 \). Under these games, two extreme vertices of the stable outcome space can be identified as the “buyer-optimal” and the “seller-optimal” outcomes reflecting ideal outcomes for each side between which cost allocation mechanisms can be negotiated. If the outcome space is empty, it means the optimal assignment is not stable.

Several studies have been conducted using stable matching (e.g. Cseh and Skutella, 2014; Wang et al., 2017; Lin et al., 2018; Peng et al., 2018; Zhang and Zhao, 2018; Lu and Quadrifoglio, 2019; Yang et al., 2019) generally as a mechanism for optimizing ridesharing services, in a normative sense. There is, however, also a need to use the stable matching theory as a basis for descriptive modeling of a MaaS system.

Rasulkhani and Chow (2019) proposed such a descriptive assignment game model where buyers are traveler OD pairs and sellers are bundles of service routes. In this case, each operator \( f \in F \) owns a set of one or more routes \( R_f \), where \( R = \bigcup_{f \in F} R_f \). Each route \( r \) consists of a set of links \( A_r \). The resulting many-to-one assignment game between multiple user OD pairs and different links of each service route is characterized by Eq. (2) for a set of users \( S \), demand \( d_s \), and a service route capacity \( w_r \) defined as the load that cannot be exceeded anywhere along a route.

The index \( \{k\} \) is used to refer to a dummy user that is matched to unused routes. The parameter \( \delta_{asr} \) is set to 1 if a match between user \( s \) and route \( r \) uses link \( a \) and 0 otherwise. \( M \) is a big constant. \( a_{sr} = \max(0, U_{sr} - t_{sr}) \), where \( t_{sr} \) is the generalized travel disutility for user \( s \) matched to route \( r \).

\[
\max \sum \sum a_{sr} x_{sr} \tag{2a}
\]

Subject to

\[
\sum_{r \in R} x_{sr} \leq d_s, \quad \forall s \in S \setminus \{k\} \tag{2b}
\]

\[
\sum_{s \in S \setminus \{k\}} \delta_{asr} x_{sr} \leq w_r, \quad \forall a \in A_r, r \in R \tag{2c}
\]
\[ \sum_{s \in S \setminus \{k\}} x_{sr} \leq M(1 - x_{kr}), \quad \forall r \in R \quad (2d) \]

\[ x_{sr} \in \{0, \mathbb{Z}_+\}, \quad \forall s \in S, r \in R \quad (2e) \]

A fare \( p_{sr} \) is charged to each member of user group \( s \) for matching to route \( r \). The operating cost of a route \( r \) is set to \( C_r \) and it is distributed to each user as \( c_{sr} \). \( B(r, x) \) \((B(s, x))\) is the set of users (routes) matched to route \( r \) (user \( s \)) in assignment \( x \) and \( \bar{R} \) is the set of routes matched to at least one user. \( G_r \) is the set of user groups that can be feasibly matched to route \( r \). \( v_r = \sum_{s \in B(r, x)} v_{rs} \) is the total benefit that route \( r \) gains from matches in assignment \( x \). The stable outcome space for this assignment game is defined by the following set of constraints in Eq. (3), where Eq. (3a) – (3d) represent the feasibility conditions and Eq. (3e) is the stability condition.

\[ \sum_{s \in B(r, x)} u_s + v_r = \sum_{s \in B(r, x)} a_{sr} - C_r, \quad \forall r \in \bar{R} \quad (3a) \]

\[ v_r = 0, \quad \forall r \in R \setminus \bar{R} \quad (3b) \]

\[ u_s = 0, \quad \forall s: B(s, x) = \emptyset \quad (3c) \]

\[ u_s \geq 0, v_{sr} \geq 0, \quad \forall r \in \bar{R}, s \in S \quad (3d) \]

\[ \sum_{s \in G_r} u_s + v_r \geq \sum_{s \in G_r} a_{sr} - C_r, \quad \forall G_r: r' \notin R_f, r \in R_f, f \in F \quad (3e) \]

The buyer-optimal and seller-optimal vertices of the space can be found by maximizing \( Z = \sum_{s \in S} u_s \) (buyer-optimal) or \( Z = \sum_{r \in R} v_r \) (seller-optimal) as LPs, and any solution within the space can be interpolated since the space is convex. Specific cost allocation mechanisms can also be sought by setting the appropriate objective \( Z \). While mechanism design can be incorporated, the scope of this work is on defining the stable outcome space so a review of such studies is not provided. Readers are referred to Rasulkhani and Chow (2019) for a synthesis instead.

As shown in the formulations, each user \( s \) is treated as an OD pair as opposed to a path of multiple operator routes with transfers. This therefore would not be able to model MaaS setting. To handle a MaaS setting, the assignment game needs to be redefined from a link-based perspective, where each operator owns a set of links (each link connecting two stops, zone centroids, or transfer points) where users are now distributed over different paths consisting of links involving transfers from one operator to another.
3. PROPOSED METHODOLOGY

3.1. Model formulation

We present a many-to-many assignment game model to be used to evaluate MaaS networks. We define two disjoint sets of players: the first set includes all network users \( s \in S \) that represent distinct O-D demand pairs and the second set includes all the operators \( f \in F \) that provide transportation services on network links. In this assignment users are matched to a feasible user path \( r \) from origin node \( O(s) \) to destination node \( D(s) \) that contains a set of operators \( F_r \). Unlike in Rasulkhani and Chow (2019), operators’ service routes are modeled as links in this network and the paths are defined from users’ perspective. In a capacitated network, a user group \( s \) may be distributed over a path set \( R_s^* \) among a feasible set \( R_s \). The network is defined as \( G(N, A) \) where \( G \) is composed of a set of mutually exclusive operator-owned subgraphs \( G_f, F \in F \), and \( G = \bigcup_{f \in F} G_f \). Let \( N_i(+) \) and \( N_i(-) \) respectively be the sets of inbound and outbound links from node \( i \in N \). The difference between the prior work and the proposed is explicitly illustrated in Figure 3.

![Diagram](a) Three operator lines \{(1,2,4,6), (1,3,4), (2,5,6)\} matched to ODs
(1,4) and (2,6) may result in each OD taking one line

![Diagram](b) Three operators own 7 links: Red \{(1,3), (3,4)\}, Blue \{(1,2), (2,4), (4,6)\}, Green \{(2,5), (5,6)\}, which are matched to user paths \{(1,2,4,6), (1,3,4,6), (1,2,5,6)\} for OD (1,6)

Figure 3. Illustration of differences in representation in (a) Rasulkhani and Chow (2019) and (b) the proposed model.

The output for a given set of operator service links and users along with operating costs \( c_{ij} \) and link capacities \( w_{ij} \) for each link \( (i, j) \in A \) and user travel disutilities \( t_{ij} \) is a set of link flows \( x_{ij}^s \) per user group \( s \), identity of unmatched operator links, and corresponding stable outcome \((u, v, x)\) space for user value \( u \), operator profits \( v \), and fares \( p_{rf} \) paid by each user-path \( r \) to each operator \( f \). The output is divided into two components: (1) the optimal flow assignment and (2) the corresponding stable cost allocations of user value and operator profit.

The first component, the flow assignment, can be modeled using a familiar formulation from network design (Magnanti and Wong, 1984; Gendron and Larose, 2014): the multicommodity capacitated fixed-charge network design problem (MCND). Each commodity corresponds to user group \( s \). Each user group \( s \) is characterized by demand \( d_s \) for that unique O-D pair. A binary variable \( y_{ij} = 1 \) if link \( (i, j) \) is operated by its owner and 0 otherwise. Unlike the interpretation of the model in a conventional use of the MCND (which is to find a subset of new links within a budget to build out), in this use case it is determining which operator-links should enter the market.
under the optimal assignment so that stable outcomes for that solution can be determined. For example, conventional use of MCND assumes some existing network upon which a subset of new candidate links is being considered. In our use of the model, all links are treated as candidate links and are each owned by one operator.

\[
\phi(N) = \min \sum_{(i,j) \in A} \sum_{s \in S} t_{ij} x_{ij} + \sum_{(i,j) \in A} c_{ij} y_{ij} \quad (4a)
\]

Subject to

\[
\sum_{j \in N_i(+)} x_{ij}^s - \sum_{j \in N_i(-)} x_{ji}^s = \begin{cases} 
    d_s, & \text{if } i = O(s) \\
    -d_s, & \text{if } i = D(s), \\
    0, & \text{otherwise}
\end{cases} \quad \forall i \in N, s \in S \quad (4b)
\]

\[
\sum_{s \in S} x_{ij}^s \leq w_{ij} y_{ij}, \quad \forall (i,j) \in A \quad (4c)
\]

\[
x_{ij}^s \in \{0, Z_+\}, \quad y_{ij} \in \{0, 1\}, \quad \forall (i,j) \in A \quad (4d)
\]

The objective function (4a) minimizes total costs which includes the travel costs of passenger and operation costs of operators. Constraint (4b) ensures the feasibility of flow in the network. Constraint (4c) is the capacity constraint for each link and constraints (4d) are the integral constraints. MCND is a well-defined problem in the literature. The problem can be solved using conventional MCND methods like the branch-and-bound-and-cut algorithms (Gendron and Larose, 2014). The MCND belongs in a broader category of network design problems where demand consolidation is encouraged, by combining shortest path savings for users (or commodities) and link operating costs. The main goal is to provide efficient routes for user OD demand by operating only the most efficient parts of the network. This problem outputs an optimal matching of users to transportation systems \(x_{ij}\) and whether links are unused, \(y_{ij}\). The optimal solution also has a corresponding set of dual variables for the capacitated links as \(\mu_{ij}\). Links with \(x_{ij}^* < w_{ij}\) have \(\mu_{ij}^* = 0\) while links with \(x_{ij}^* = w_{ij}\) have \(\mu_{ij}^* \geq 0\).

The second component of the model is the stable pricing problem. Our goal is to determine the range of stable payoff allocations corresponding to the core value \(\phi(N)\) obtained from the MCND between operators and users. To do so we employ a path-based formulation for stability. For a user flow assignment and link operation solution to Eq. (4) there exists a corresponding path flow \(\{z_{r}^s\}_{r \in R}\) and we define the outcomes based on this flow, \(((u, p), z)\). The binary parameter \(\delta_{ijr}\) indicates whether a route \(r \in R\) is incident on link \((i,j) \in A\). The link set \(A_r \subseteq A\) consists of links included in path \(r \in R_s\) and each user \(s\) generates payoff (trip utility) from using route \(r\) to realize their trip, \(U_{sr}\). Let \(R_f\) be the set of routes in which operator \(f\) serves. If we assume the utility of a user reaching their destination is separate from the route taken since the travel disutility is separately accounted for, then \(U_{sr} = U_s, \forall s \in S, r \in R\). For each user \(s\) that is matched to a route \(r \in R_s\) there are travel costs that further reduce the core payoff: \(U_s - \sum_{(i,j) \in A_r}(t_{ij})\).

The stability constraints are derived as follows. For each user \(s \in S\) the core payoff is denoted as \(u_s\), the utility surplus from making a trip, while for each operator \(f \in F\) the payoff is the total
revenue collected from network users $\sum_{r \in R_f} p_{rf} z_r$ based on the price $p_{rf}$ charged to user-route $r$ by operator $f$. This revenue formulation assumes if a user traverses two separate links owned by the same operator along a route, they would only pay $p_{rf}$ to that operator once. Stability conditions need to ensure that no player in a coalition has incentive to generate a higher payoff by forming another coalition, whether it is a user with another feasible route or an operator with other users in the network. In our case we ensure stability for both operators and users:

- For each user $s$ using path $r \in R_s^*$ there is no incentive to switch to another path $r \in R_s$; they should be indifferent between paths in $R_s^*$;
- For each operator $f \in F$ there is no incentive to break their coalition with a user by closing the link or matching with another user;
- The total ticket price that is paid to each operator should cover the operation costs of all his links that are operational, keeping in mind that even for subsidized systems (e.g. public transit routes serving at a loss) there exists a maximum operating cost threshold.

For the third condition we assume without loss of generality that government subsidies for all mobility operators are set to zero. A feasible outcome is defined in **Definition 1**.

**Definition 1.** The outcome $(u, p; z)$ is feasible if:

\[
\begin{align}
    u_s + \sum_{f \in F_r} p_{rf} &= U_s - \sum_{(i, j) \in A_r} t_{ij}, \forall r \in R_s^*, s \in S \quad (5a) \\
    u_s &\geq 0, \quad \forall s \in S \quad (5b) \\
    p_{rf} &\geq 0, \quad \forall r \in R, f \in F \quad (5d) \\
    \sum_{r \in R_f} p_{rf} z_r &\geq \sum_{(i, j) \in A_f} c_{ij} y_{ij}, \quad \forall f \in F \quad (5e)
\end{align}
\]

We define the stable outcome space as a subset of the feasible outcome space described in Eq. (5a) – (5e).

**Proposition 1.** The stability condition for the assignment in Eq. (4) is expressed as Eq. (6).

\[
\begin{align}
    \sum_{f \in (F_r \cap F_{r'})} p_{rf} + u_s &\geq U_s - \sum_{(i, j) \in A_{r'}} \left( t_{ij} + \mu_{ij} + c_{ij} (1 - y_{ij}) \right), \\
    \forall r \in R_s^*, s \in S, \forall r' \in R_s \setminus R_s^*
\end{align}
\]

**Proof.**
A stable set of prices ensures that no users or operators have incentive to break their current coalitions and form others that will yield larger payoffs. For a user, the stable path is such that they achieve maximum user-payoff $u_s$. We can rewrite Eq. (5a) to represent the user-payoff in terms of
prices paid to the operators of a stable route: \( u_s = U_s - \sum_{(i,j) \in A_r} (t_{ij} - \sum_{f \in F_r} p_{rf} f \in R_s, s \in S, \text{where } u_s \text{ is the same value for all used paths. A user-path is operator-stable when operators in alternative feasible paths } r' \text{ do not have an incentive to break their coalition with current users of that alternative path to accommodate the first. In other words, coalitions are considered operator-stable when they generate the maximum payoff that each operator can achieve: } \sum_{r \in R_f} p_{rf} z_r \text{. All the above are described in Eq. (6).}

The first term \( \sum_{f \in (F_r \cap F_r)} p_{rf} \) ensures that an operator is not double-counted on the two routes being compared. The right hand term \( \sum_{(i,j) \in A_r} \left( t_{ij} + \mu_{ij} + c_{ij}(1 - y_{ij}) \right) \) captures the different cases considered for stability: when links are unused in a route the stability needs to include the cost of operation; when links are used but capacity is binding it increases the travel cost by the amount needed to take the next available route, which is quantified by the dual variable \( \mu_{ij} \) (see Xu et al., 2018). The term \( \mu_{ij} \) represents the added cost that deviating users will face on links \((i,j) \in A_r\) of the unused feasible path \( r' \in R_s \setminus R_s^* \) (which may have flow from other routes on the links). The multipliers \( \mu_{ij} \) can be found by fixing the values \( y_{ij}^* \) in Eq. (4a) – (4d) and finding the capacity dual variables in the LP-relaxed subproblem. If a link is neither unused nor capacitated, then there is no added cost incurred by adding an additional unit of flow to that path.

The stable pricing problem is then defined as a mathematical program with constraints formed by Eq. (5) – (6). The objective can be set to maximize user cost allocation (seller-optimal: \( \max Z = \sum_f \sum_{r \in R} p_{rf} z_r \) or maximize operator cost allocation (buyer-optimal: \( \max Z = \sum_s u_s \)) to define the full stable outcome space in between the two vertices. Alternatively, one can also run the stable pricing problem with a known cost allocation mechanism which should output a price set within this range. The first two objectives are linear; determining the stable outcome space can therefore be done using linear programming.

An outcome depends on a path flow assignment. However, a path flow solution is non-unique to Eq. (4). Due to the non-uniqueness of paths in the assignment solution, the set of operator-routes that enter the market at equilibrium, and the corresponding cost allocations, is not unique. There is a concern that a pricing policy may result in a different set of path flows if two or more path flow solutions share the same pricing solution, which can be problematic when implementing a pricing strategy expecting one path flow and instead getting another.

However, multiple path flow solutions only happen if there are two or more paths that share the same path cost, and neither are at binding capacity. If the solution in Eq. (4) makes use of only one of the multiple paths, the other ones will have \( y_{ij} = 0 \). In that case, the pricing will make sure that users do not have incentive to switch to those unused links. On the other hand, if all the paths are used, then a pricing solution may diverge to a flow assignment where the path flows differ. When this happens, the aggregate revenues of the operators are impacted, resulting in non-unique aggregate revenues. Nonetheless, the pricing outcome \( p_{rf} \) and user path utilities remain the same.

3.2. Solution algorithm

Since the MCND component of the model is a standard model, we simply adopt existing solution methods to solve that part. The determination of the stability conditions for a user path is more complex because it needs to consider multiple operators deciding on network-wide prices. The number of stability conditions in Eq. (6) depends on the number of feasible paths in the
network and grows exponentially. We propose the following Algorithm 1 for the overall model and elaborate further on the constraint generation step in a subsequent Algorithm 2.

**Algorithm 1. Solution method for path-based many-to-many assignment game**

1. Solve MCND using an existing solution algorithm, e.g. path-based column generation algorithm (Gendron and Larose, 2014), to obtain a set of user paths \( R_s^* \) (non-unique solution) and optimal solution \( z_{r_{ij}}^*, y_{ij}^* \) (unique).
2. Fix the values of \( y_{ij}^* \) and solve the LP-relaxed formulation of Eq. (4) to obtain \( \mu_{ij}^* \).
3. For the path-based solution, identify all acyclic feasible paths for each O-D pair \( s \) and add to set \( R_s \).
4. Construct the stable pricing problem depending on objective function. The constraints of the program include the linear feasibility conditions in Eq. (5) and the linear stability conditions in Eq. (6). The latter constraints are generated using Algorithm 2.

A computationally efficient constraint generation method is proposed to reduce the number of path constraints of Eq. (6). Since paths increase the problem size exponentially, we seek to generate only those feasible paths that could become binding while avoiding explicit enumeration. To achieve this, we seek to generate all non-dominated alternative feasible path constraints that will lead to the same convex polytope as Algorithm 1. These paths are derived from the best-response dynamics of network users (O-D pairs) by removing distinct subsets from the optimal set \( F_r \) \( \forall r \in R^*_s \). The best-response path constraints are then computed on the subgraph: \( G' := \bigcup_{f \in F - F'^*} G_f \), where \( F'^*[r] \in \mathbb{S}(F_r) \). The steps of the solution algorithm are summarized in Algorithm 2.

**Algorithm 2. Constraint generation for Eq. (6) without explicit path enumeration**

1. Let \( r \in R_s \) be an optimal path from an origin to a destination node of a directed graph \( G(N,A) \), obtained from the assignment outcome \( ((u,p),z) \) and \( F_r \) is the set of operators on path \( r \). We will also define the set \( \mathbb{S}(F_r) \) that contains all the possible operator combinations.
2. We will generate \( |\mathbb{S}(F_r)| \) stability constraints by identifying the best response shortest path \( r' \) of user \( s \), by removing each distinct sub-coalition formed by operators in optimal route \( r \).
3. For constraint \( i \), we compute the shortest path \( r'^* \) on subgraph \( G' := \bigcup_{f \in F - F'^*[r]} G_f \), where \( F'^*[r] \in \mathbb{S}(F_r) \).
4. We define the cost for each link in the subgraph as: \( \omega_{ij} = t_{ij} + \mu_{ij} + c_{ij}(1 - y_{ij}) \)

**Proposition 2.** The generation of stability constraint set \( \mathbb{S}(F_r) \) for each optimal path \( r \) using Algorithm 2 is equivalent to the explicit path enumeration constraint (6).

**Proof.**

For every user \( s \in S \) if \( F_r \cap F'^*[r] \) for any \( F'^*[r] \in \mathbb{S}(F_r) \) where \( r' \) is the solution of a shortest path subproblem:
a) If $F_r \cap F_{r'} \neq \emptyset$ then $\sum_{f \in (F_r \cap F_{r'})} p_{rf} + u_s \geq U_s - \sum_{(i,j) \in A_{r'}} \omega_{ij}$ could be binding for that shortest path. For any other feasible path $r''$:

1. If $\sum_{f \in (F_r \cap F_{r'})} p_{rf} + u_s \geq U_s - \sum_{(i,j) \in A_{r'}} \omega_{ij}$ is binding for any $F_{r''} \neq F_{r'}$ then $F_{r''} \in \mathcal{S}(F_r)$ and that cut will be generated by another subproblem.

2. If $u_s \geq U_s - \sum_{(i,j) \in A_{r'}} \omega_{ij}$ is binding then for $\sum_{(i,j) \in A_{r'}} \omega_{ij} < \sum_{(i,j) \in A_{r''}} \omega_{ij}$ the LP becomes infeasible and for $\sum_{(i,j) \in A_{r'}} \omega_{ij} = \sum_{(i,j) \in A_{r''}} \omega_{ij}$ then $\sum_{f \in (F_r \cap F_{r''})} p_{rf} = 0$.

b) If $F_r \cap F_{r'} = \emptyset$ for any $F_{r'} \in \mathcal{S}(F_r)$ then $u_s \geq U_s - \sum_{(i,j) \in A_{r'}} \omega_{ij}$ could be binding. For any other feasible path $r''$, if $\sum_{f \in (F_r \cap F_{r''})} p_{rf} + u_s \geq U_s - \sum_{(i,j) \in A_{r''}} \omega_{ij}$ is binding then case 1 applies. For the case where $u_s \geq U_s - \sum_{(i,j) \in A_{r'}} \omega_{ij}$, case 2 applies.

The computational savings when using the constraint generation method instead of path enumeration are highly significant. The problem’s solution time is reduced on average by 98% for the instances tested. More detailed reports on the computation times are summarized on Table 3 in the computation tests in Section 4.

4. NUMERICAL TESTS

4.1. Illustrative instance

An example of the model is shown in Figure 4 where the link costs are shown, both OD pairs have gain utilities of $U_s = 20$, and there is demand $d_{13} = 1000$ and $d_{14} = 500$. Operator A (blue) owns links $\{(1,3), (1,2)\}$, operator B (orange) owns $(2,3)$, operator C (green) owns $(2,4)$, and operator D (black) owns $(1,4)$. The optimal assignment has 1000 flow on $(1,3)$, 200 flow on $(1,2,4)$, and 300 flow on $(1,4)$. The capacity at link $(1,2)$ is binding with a corresponding dual variable $\mu_{12} = 4$.

Based on this the model solution would be as shown in Table 1. Operator B does not enter the market. Since there are no identical non-binding, used paths in this example, the path flow solution is unique, and we can therefore get revenues per operator. For this mix of operated links, Operator A stands to gain net profits up to 12,600, while Operator C gains net profit between 333.33 to 2,600 and Operator D up to 2,800. As seen from these results, Operator A has the most profit to gain, whereas Operator C has the most to gain even in the buyer-optimal setting due to negotiating power from the binding capacity in link $(1,2)$. At this stage, the operators and users (through the city agency as proxy) can work out the cost allocation mechanism that falls within the convex outcome range.

From this example, we can extract sensitivity of each operator’s performance and consumer surplus based on changes in link capacities, OD demand, utility per OD pair, operating cost, generalized travel disutility (alterations in-vehicle time, access time, transfer time, wait time due to external factors or operator policies), and addition/removal of operator routes.
Figure 4. Example network with 4 nodes, 4 operators, and 2 OD pairs assigned to 3 paths.

Table 1. Solution to example M2M problem

| Operator,user-route | Flow $\sum_{r \in R_f} z_r$ | Buyer-optimal | Seller-optimal |
|---------------------|-------------------------------|----------------|----------------|
| (A, (1,3))          | 1000                          | $0$            | $13$           |
| (A, (1,2,4))        | 200                           | $2$            | $0$            |
| (C, (1,2,4))        | 200                           | $2.67$         | $14$           |
| (D, (1,4))          | 300                           | $0.67$         | $10$           |

User group-route

| User group-route | $U_s - \sum_{(i,j) \in A_r} (t_{ij})$ | $U_s - \sum_{(i,j) \in A_r} (t_{ij}) - \sum_{f \in F_r} p_{rf}$ | $U_s - \sum_{(i,j) \in A_r} (t_{ij}) - \sum_{f \in F_r} p_{rf}$ |
|------------------|----------------------------------------|---------------------------------------------------------------|---------------------------------------------------------------|
| ((1,3),(1,3))    | $13$                                   | $13$                                                         | $0$                                                           |
| ((1,4),(1,2,4))  | $14$                                   | $9.67$                                                      | $0$                                                           |
| ((1,4),(1,4))    | $10$                                   | $9.67$                                                      | $0$                                                           |

Operator

| Operator | $\sum_{r \in R_f} p_{rf} z_r - \sum_{(i,j) \in L_f} C_{ij} y_{ij}$ | $\sum_{r \in R_f} p_{rf} z_r - \sum_{(i,j) \in L_f} C_{ij} y_{ij}$ |
|----------|------------------------------------------------------------------|------------------------------------------------------------------|
| A        | $0$                                                              | $12,600$                                                         |
| B        | $0$                                                              | $0$                                                              |
| C        | $333.33$                                                         | $2,600$                                                         |
| D        | $0$                                                              | $2,800$                                                         |

4.2. Sioux Falls test instance

For this more realistic small-sized network instance, we consider two operators in a base scenario. For the base scenario, we assume that both operators have revenue maximization: $\max Z = \sum_{r \in R_f} P_{rf} z_r$ as their objective. Relative to this baseline, we seek several experimental objectives:

1) Compute the consumer surplus and profit when each link is operated by a different operator (fully decentralized market) versus one with operators owning multiple links;
2) Assessing the new prevailing market conditions and payoff allocations when one of the two operators is acquired by a government agency and becomes welfare maximizing.

3) Evaluate the impact of one operator increasing its binding capacity on the consumer surplus and market revenues of other operators, or from the effects of network degradation;

4) Evaluate the effect of technology improvement (e.g. improved matching algorithms) that reduce operating costs $c_{ij}$ for privately operated links while increasing travel times $t_{ij}$ for the whole network;

5) Evaluate the entry of a new operator to the consumer surplus and market revenues of existing operators;

6) Evaluate the impact of multiple independent operators choosing to merge together to become one operator.

For the purpose of conducting various analyses on the impact of different operator ownerships in urban networks we use the 24-node Sioux Falls test network (Stabler, 2019) shown in Figure 5. The free flow travel times are used as $t_{ij}$. The other parameters ($w_{ij}, c_{ij}$) are listed in Table 2. For this illustration we select only four origin-destination pairs and appropriate demand quantities, as shown in Figure 5, to capture the effects of binding capacity on competition and stability of network pricing.

### Table 2. Additional parameter for Sioux Falls example

| Link | $w_{ij}$ | $c_{ij}$ | Link | $w_{ij}$ | $c_{ij}$ | Link | $w_{ij}$ | $c_{ij}$ | Link | $w_{ij}$ | $c_{ij}$ |
|------|----------|----------|------|----------|----------|------|----------|----------|------|----------|----------|
| 1    | 25901    | 6        | 20   | 7842     | 3        | 39   | 5092     | 4        | 58   | 4824     | 2        |
| 2    | 23404    | 4        | 21   | 5051     | 10       | 40   | 4877     | 4        | 59   | 5003     | 4        |
| 3    | 25901    | 6        | 22   | 5046     | 5        | 41   | 5128     | 5        | 60   | 23404    | 4        |
| 4    | 4959     | 5        | 23   | 10000    | 5        | 42   | 4925     | 4        | 61   | 5003     | 4        |
| 5    | 23404    | 4        | 24   | 5051     | 10       | 43   | 13513    | 6        | 62   | 5060     | 6        |
| 6    | 17111    | 4        | 25   | 13916    | 3        | 44   | 5128     | 5        | 63   | 5076     | 5        |
| 7    | 23404    | 4        | 26   | 13916    | 3        | 45   | 14565    | 3        | 64   | 5060     | 6        |
| 8    | 17111    | 4        | 27   | 10000    | 5        | 46   | 9600     | 3        | 65   | 5230     | 2        |
| 9    | 17783    | 2        | 28   | 13513    | 6        | 47   | 5046     | 5        | 66   | 4886     | 3        |
| 10   | 4909     | 6        | 29   | 4855     | 4        | 48   | 4855     | 4        | 67   | 9600     | 3        |
| 11   | 17783    | 2        | 30   | 4994     | 8        | 49   | 5230     | 2        | 68   | 5076     | 5        |
| 12   | 4948     | 4        | 31   | 4909     | 6        | 50   | 19680    | 3        | 69   | 5230     | 2        |
| 13   | 10000    | 5        | 32   | 10000    | 5        | 51   | 4994     | 8        | 70   | 5000     | 4        |
| 14   | 4959     | 5        | 33   | 4909     | 6        | 52   | 5230     | 2        | 71   | 4925     | 4        |
| 15   | 4948     | 4        | 34   | 4877     | 4        | 53   | 4824     | 2        | 72   | 5000     | 4        |
| 16   | 4899     | 2        | 35   | 23404    | 4        | 54   | 23404    | 2        | 73   | 5079     | 2        |
| 17   | 7842     | 3        | 36   | 4909     | 6        | 55   | 19680    | 3        | 74   | 5092     | 4        |
| 18   | 23404    | 2        | 37   | 25901    | 3        | 56   | 23404    | 4        | 75   | 4886     | 3        |
| 19   | 4899     | 2        | 38   | 25901    | 3        | 57   | 14565    | 3        | 76   | 5079     | 2        |

The solution to Eq. (4) is shown in Figure 5, where the red links are the ones that would need to be operated. From this solution, only link 58 (node 19 to 17) is at binding capacity resulting in a
dual variable of $\mu_{50}^* = 1$. From this assignment the following sections examine different operator scenarios as well as variations to the scenarios. Table 3 summarizes all these results which are discussed subsequently in each section. Note that the last two columns refer to the run times via explicit path enumeration versus the Algorithm 2 approach. Two numbers are included in each cell; the first number is the time it takes to construct the constraints for the stable pricing model, and the latter is the solution time.

| Origin | Destination | Demand |
|--------|-------------|--------|
| 1      | 24          | 4000   |
| 4      | 22          | 3000   |
| 11     | 18          | 200    |
| 14     | 8           | 5000   |

Figure 5. Sioux Falls example with modified OD demand.

4.2.1. Rail-flexible transit duopoly

The network ownership regime shown in Fig. 3 is treated as the “base scenario” in this experiment. The competition in the network is formed as a duopoly of operators where links in blue in Figure 6 represent a private mobility service (let’s call Operator 1) and the orange links a privately-owned rail service (let’s call Operator 2) that operates two main lines (13-12-3-1 and 20-19-17-16-8-6-2). We assume that the rail service sets a unique price for all O-D pairs. This “cash fare” policy results in a more constrained price setting than the stability conditions in Eq. (5) – (6) where pricing could vary by user route.
Table 3. Comparison of aggregate measures of different scenarios with $U_S = 20$

| Parameters: | Revenues ($) [$f = 1, 2, 3$] | Avg. operator fare ($) [$f = 1, 2, 3$] | Avg. operated link revenue ($) | Operator ridership $\sum_{r \in R_f} z_r$ [$f = 1, 2, 3$] | Runtime: Model generation / Solution (msec) (original LP) | Runtime: Model generation / Solution (msec) (constraint generation) |
|-------------|-------------------------------|---------------------------------|-------------------------------|---------------------------------|---------------------------------------------------|---------------------------------------------------|
| Scenarios:  |                               |                                 |                               |                                 |                                                   |                                                   |
| Network duopoly (Base scenario) | [24424,18000] | [2,2]                           | 2497                          | [12200,9000]                    | 6259.2 / 20.4                                     | 28.8 / 0.3                                       |
| Network monopoly | [42424] | [3.47]                         | 2497                          | [12200]                        | 6216.2 / 17.7                                     | 22.9 / 0.2                                       |
| Decentralized market | 2497 (per operator) | 1 (per operator)               | 2497                          | 2812 (per operator)             | 6061.2 / 17.9                                     | 178.4 / 0.7                                      |
| Government rail acquisition | [42417.7] | [3.47,0.0008]                  | 2497                          | [12200,9000]                    | 6259.2 / 20.4                                     | 28.8 / 0.3                                       |
| Firm entry | [60422,10000,0.75] | [5.2,0.0002]                  | 3912                          | [12200,5000,4000]               | 12095.6 / 35.6                                    | 32.8 / 0.2                                       |
| Binding capacity increase ($w_{58} = 4900$) | [24500,18000] | [2,2]                           | 2497                          | [12200,9000]                    | 6259.2 / 20.4                                     | 28.8 / 0.3                                       |
| Binding capacity increase ($w_{58} = 5000$) | [15600,27000] | [1.3,3]                         | 2663                          | [12200,9000]                    | 5300.8 / 15.5                                     | 27.8 / 0.2                                       |
| Technological change | [27506,40500] | [2.25,4.5]                     | 3400                          | [12200,9000]                    | 6259.2 / 20.4                                     | 28.8 / 0.3                                       |

The cost allocation mechanism assumes that both operators seek to maximize their revenues. This translates to an objective value of: $Z = \sum_{r \in R_1} p_{r_1} z_r$. The resulting revenue allocation for this example is $R_1 = $24,424 for the private mobility service and $R_2 = $18,000 for the public rail. The average fares, average revenue gains per link and passenger volumes per operator are summarized in Table 3. In this instance, travelers use 1.6 services on average to reach their destinations for the 4 OD pairs.
Figure 6. Sioux Falls scenario with 2 operators: a public operator (orange) and a private operator (blue).

4.2.2. Case of network monopoly

For this scenario we assume that the entire network is owned by a monopoly operator and thus the absence of competition allows the provider to diminish consumer payoff entirely under the seller-optimal case. In this case we introduce the concept of a single network fare for all travelers flowing through the network for the seller-optimal case for $U_s = 20$ in Table 3. We see the revenue can increase significantly.

We also investigate the existence of a stable network fare range for all users that enter the network when utility increases up to $U_s = 85$. Table 4a summarizes the solution results of the problem for the range of prices between buyer- and seller-optimal mechanisms. Due to binding capacity there are two optimal paths for O-D pairs 14-8 and their prices are set to ensure stability between those users. This does not imply that all operator payoff allocations that belong in that range are stable but a subset of that range.

4.2.3. Comparison of fully decentralized to oligopoly market

In this scenario we assume that each link is owned by a different operator and thus pricing is determined at the link level. This ownership scheme represents an outlier case where the effects of competition between different operators/links strongly influences revenues and consumer payoff. For this scenario we assume that all operators are revenue-maximizing agents employing seller-optimal policies with $U_s = 20$ as reported in Table 3.
Table 4a. Assignment and stable pricing solutions for the monopoly system with $U_s = 85$

| Origin | Destination | Demand | Used path       | Travel time | Stable price range | Network fare |
|--------|-------------|--------|-----------------|-------------|--------------------|--------------|
| 1      | 24          | 4000   | [1, 3, 12, 13, 24] | 15          | [1, 70]            | [1, 67]      |
| 4      | 22          | 3000   | [4, 11, 14, 23, 22] | 18          | [1, 67]            |              |
| 11     | 18          | 200    | [11, 10, 16, 18]  | 12          | [1, 73]            |              |
| 14     | 8           | 177    | [14, 11, 10, 16, 8] | 18          | [0, 67]            |              |
| 14     | 8           | 4823   | [14, 15, 19, 17, 16, 8] | 17          | [1, 68]            |              |

Table 4b. Assignment and stable pricing solutions for the decentralized system with $U_s = 85$

| Origin | Destination | Demand | Used path       | Travel time | Stable price range |
|--------|-------------|--------|-----------------|-------------|--------------------|
| 1      | 24          | 4000   | [1, 3, 12, 13, 24] | 15          | [0.004 , 37]       |
| 4      | 22          | 3000   | [4, 11, 14, 23, 22] | 18          | [0.006 , 11]       |
| 11     | 18          | 200    | [11, 10, 16, 18]  | 12          | [0.039 , 16]       |
| 14     | 8           | 177    | [14, 11, 10, 16, 8] | 18          | [0.023 , 8]        |
| 14     | 8           | 4823   | [14, 15, 19, 17, 16, 8] | 17          | [1.023 , 9]       |

We perform a sensitivity analysis on consumer surplus and revenues when passenger utility is increased. This analysis reveals the strong effects of parallel operator competition on revenues. As shown in Figure 7 when $U_s \geq 50$, operators are unable to increase their prices due to the existence of competing services that could potentially satisfy the network users. An antisymmetrical relationship is revealed between the complete decentralized case and the network monopoly at the utility $U_s = 50$. In Table 4b we show that a decentralized market will lead to significantly lower payoffs per operator. In this setting users have to use 4 services on average to conduct their trip.

Figure 7. Sioux Falls scenario with 2 operators.
4.3. Government rail acquisition

When a government agency is deciding whether to buy a transit service it is important to forecast its future revenues under a new pricing policy. Since most public transit services are heavily subsidized, they offer low ticket fares to users in order to maximize consumer surplus. Thus, for the first type of ownership the objective will be revenue maximization: \( \max Z = \sum_{r \in R_f} p_{rf} z_r \) while for government-owned agencies the goal is maximizing consumer surplus: \( \max Z = \sum_{s \in S} u_s (p_{rf}) \). In some cases (like our base duopoly network) a private agent will re-adapt its pricing strategy to the new market conditions. The cost allocation mechanism assumes the private operator seeks seller-optimal pricing while the public operator prices via buyer-optimal pricing. This means the objective value is \( Z = \sum_{r \in R_1} p_{r1} z_r + \sum_{s \in S} u_s (p_{r2}) \). In this scenario ride-hailing revenues will increase while those of the public rail service will be diminished. In our test case all O-D pairs will at some point use the services of the ride-hailing operator to conduct their trips and thus that operator can always set prices at the upper bound of the feasible area. This fact is established in the first term of Eq. (6). As we can see in Table 3, changes are only observed in the payoff allocations between operators.

4.4. Firm entry

In this section, we make modifications to the base scenario. What could be the effect of a new operator entering the market on users and on the other transit services? We assume that this new operator is a direct competitor to the services offered by the western rail line (13-12-3-1) and operates parallel links connecting the same nodes. However, this new operator only carries 25% of the rail capacities for those links. This new service has significantly lower operating costs and travel times (travel times and operating cost are set to 25% of the competing service links), which will make it a strong competitor to the rail service. The new scenario MaaS network is shown in Figure 8.

The results of this analysis are surprising. Table 3 shows that the base flows among the two operators is 12000 and 9000. When the new operator joins, the 9000 is split to 5000 and 4000 for the new operator, which corresponds to all the demand that was originally using the western line. In other words, the new operator obtains the entire passenger volume of the western line but cannot generate any profit. The effect of strong competition also ends up diminishing the rail line’s revenues, while the payoff of the ride-hailing operator is maximized. This can be attributed to the geometry of this network: since all O-D pairs need to use the services of the flexible operator it is completely logical that this will be the only service to benefit from this situation.
4.5. Evaluation of capacity effects on MaaS market

Capacity is considered one of the most critical attributes of urban networks. The model allows planners to evaluate the effects of changes in capacity of one operator’s link to the performance of operator operators and the users. Such capacity changes may refer to roadway capacity, frequency changes in urban rail lines (effective line capacities), station capacity or queueing operations, fleet size of an on-demand service, or a rebalancing strategy for a bike sharing service that can effectively increase flow capacity along different corridors.

4.5.1 Binding capacity increase

We refer again to our base duopoly scenario in order to quantify the benefits of capacity increases on users and operators. From the assignment game solution described in Eq. (4) we see that link 58 capacity is binding. We test two capacity increases. In this first scenario, we increase capacity of link 58 from 4824 to 4900; in the second case, we go further up to 5000. With a change in capacity, the assignment solution to Eq. (4) may change, and that also impacts the stable outcomes.
While the first scenario does not significantly affect our measures, increasing capacity to 5000 significantly favors the rail operator. The average rail fare increases from $2 to $3, while the private mobility operator is forced to reduce its average fare from $2 to $1.3. All key comparisons are listed in Table 3.

4.5.2 Line capacity degradation

We study the impact of service degradation on rail service revenues by gradually decreasing the capacity of all links that belong to the eastern rail line (s20-19-17-16-8-6-2), which have average link capacities of 4993.5, by reducing the capacity line-wide. This analysis does not aim to simulate the effects of short-term service degradation (such as delays) but rather longer-term net-effective capacity changes (e.g. frequency decisions, or degradation of signal switches that reduce service reliability) that may impact the network. For this purpose we conduct a sensitivity analysis on consumer utility to increasing line delays.

![Capacity reduction effects on welfare](image)

**Figure 9.** Capacity reduction effects on consumer surplus.

As shown in Figure 9, consumer utility remains constant until the effective capacity reaches 60% of its initial value. In the degraded region between 60% and 50% $u_s$ does not decrease linearly but instantaneously when effective capacity becomes half of the original, due to changes in the assignment solution. Impact of capacity changes on travelers depends on availability of other mobility options.

4.6. Technological change

In the age of smartphones and big data on-demand transportation services have increased their market share significantly, due to technological breakthroughs that led to large improvements in user interface. For this reason, we lastly examine a scenario where study the impact of such technological improvements as new routing or matching algorithms that can reduce operator cost and travel disutility on the stable outcome of the MaaS market.

In this new scenario, we assume that a new matching algorithm is deployed by Operator 1 that reduces their links’ operating costs by 50% and travel times on the same links by 20%. This may represent savings in users’ wait times, reduction in detours reducing vehicles miles traveled or
empty trips, etc. Under this new technology, the results are quite surprising. While the revenues for the private Operator 1 increase by 12.5% as expected, we also see the rail service gaining 125% in revenue. This fact is attributed to the existing mutualistic relationship (see Chow and Sayarshad, 2014) between the two operators, as no operator alone can provide a complete service to users in the market setting. The improved performance of the private “feeder” service results in allowing the public service to respond with higher fares (from $2 to $4.50) and increase revenue.

4.7. Results summary

These computational experiments with Sioux Falls demonstrate the potential of this new modeling framework. It allows us to evaluate MaaS markets with a much more expanded scope beyond route choice and link loading. Scenarios related to operator interactions and dependencies can be captured. Pricing and changes in generalized travel costs can also be evaluated, allowing us to quantify the costs of new algorithmic developments not only on its own operator, but also on other operators and travelers in the market. We summarize several key lessons learned.

- The number of operators in the market impact the pricing, but only to an extent (up to $3.47 for a single operator from $1 each per single private operator per link);
- Having a firm enter to compete directly with Operator 2 can result in significant advantages to a third party – in this case, Operator 1 can benefit greatly by allowing them to increase their prices;
- Capacity increases even for single links have nonlinear effects, where exceeding some threshold improvement can lead to a significant shift in assignment and stable outcomes, as we see going from 4824 to 4900 to 5000 for link 58. These impacts effect the revenues of other operators as well, so this model allows us to consider the importance of each operator monitoring the route capacities of their competitors;
- Technological changes that impact either system-wide (or operator-wide) operating costs, travel disutilities, and capacities can be evaluated for the whole market. Depending on the type of relationship between the operators, it can be beneficial for multiple parties (if mutualistic) or detrimental to other parties (if parasitic).

5. CONCLUSION

With the emergence of MaaS ecosystems, public agencies need modeling tools to consider trade-offs when facilitating markets with private operators. For the most part, such modeling tools do not exist. Recent work from Rasulkhani and Chow (2019) sought to rectify this but only capture line-level interactions between operators and users without allowing users to make multimodal trips. In this work and its companion piece (Rasulkhani et al., 2019), a new modeling framework is proposed using the MCND as the link-based assignment tool to fully capture network effects and user paths. Unlike Rasulkhani et al. (2019) which focuses on the innovation of solving the stable outcomes without path enumeration in the case of a fully link-decentralized operator market, this work looks at the more general case where operators may own sets of links.

Under this setting, we propose path-based stability conditions and solution algorithms for deriving stable outcomes corresponding to path flows obtained from an MCND along with link capacity dual variables. The modeling framework is tested on an illustrative network as well as in a series of comprehensive experiments on the Sioux Falls network to demonstrate the model’s capabilities. As we can see from the lessons learned, the strength of this model lies in using stability
conditions to link network design decisions and algorithmic policies (effects on link capacities, operating costs, travel disutilities) as well as market dynamics (firm entry, market consolidation, subsidies, and bundled pricing) to capture market performance for both operators and users.

There are several directions for future research. The current formulation of stability conditions correspond to planning-level decisions. However, we are also looking into dynamic stability conditions to address day-to-day operations based on probabilistic user route preferences (analogy of stochastic user equilibrium for the MaaS market), flow-dependent travel disutility functions for nonlinear congestion effects and time-dependent demand patterns. Empirical studies using the model and calibrating the model to real data are also important. There are also many other fields that this work can be applied to beyond urban transportation: freight, airlines, other two-sided markets, and other network flow games where user preferences are of import.

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REFERENCES

1. Agarwal, R., & Ergun, Ö. (2008). Mechanism design for a multicommodity flow game in service network alliances. Operations Research Letters, 36(5), 520-524.
2. Bird, C. G. (1976). On cost allocation for a spanning tree: a game theoretic approach. Networks, 6(4), 335-350.
3. Chow, J.Y.J., & Sayarshad, H.R. (2014). Symbiotic network design strategies in the presence of coexisting transportation networks. Transportation Research Part B: Methodological, 62, 13-34.
4. Cseh, Á., & Skutella, M. (2014, September). Paths to stable allocations. In International Symposium on Algorithmic Game Theory (pp. 61-73). Springer, Berlin, Heidelberg.
5. Derks, J. J. M., & Tijs, S. H. (1985). Stable outcomes for multi-commodity flow games (No. f643e6a4-cf4e-4892-8f00-ca8a1a337dd9). Tilburg University, School of Economics and Management.
6. Djavadian, S., & Chow, J.Y.J. (2017). An agent-based day-to-day adjustment process for modeling ‘Mobility as a Service’ with a two-sided flexible transport market. Transportation Research Part B 104, 36-57.
7. Gale, D., & Shapley, L. S. (1962). College admissions and the stability of marriage. The American Mathematical Monthly, 69(1), 9-15.
8. Gendron, B., & Larose, M. (2014). Branch-and-price-and-cut for large-scale multicommodity capacitated fixed-charge network design. EURO Journal on Computational Optimization, 2(1-2), 55-75.
9. Harker, P. T. (1988). Private market participation in urban mass transportation: application of computable equilibrium models of network competition. Transportation science, 22(2), 96-111.
10. Hensher, D. A. (2017). Future bus transport contracts under a mobility as a service (MaaS) regime in the digital age: Are they likely to change?. Transportation Research Part A: Policy and Practice, 98, 86-96.
11. Hu, W. (2019). ‘Suicide surcharge’ or crucial fee to fix the subway? Taxi drivers brace for battle over $2.50 charge. New York Times, January 17.
12. Lin, Y., Wang, Y. M., & Chin, K. S. (2018). An enhanced approach for two-sided matching with 2-tuple linguistic multi-attribute preference. Soft Computing, 1-14.
13. Lu, W., & Quadrifoglio, L. (2019). Fair cost allocation for ridesharing services—modeling, mathematical programming and an algorithm to find the nucleolus. Transportation Research Part B: Methodological, 121, 41-55.
14. Magnanti, T. L., & Wong, R. T. (1984). Network design and transportation planning: Models and algorithms. Transportation science, 18(1), 1-55.
15. Matyas, M., & Kamargianni, M. (2019a). Survey design for exploring demand for Mobility as a Service plans. Transportation, 46(5), 1525-1558.
16. Matyas, M., & Kamargianni, M. (2019b). The potential of mobility as a service bundles as a mobility management tool. Transportation, 46(5), 1951-1968.
17. Peng, Z., Shan, W., Jia, P., Yu, B., Jiang, Y., & Yao, B. (2018). Stable ride-sharing matching for the commuters with payment design. Transportation, 1-21.
18. Rasulkhani, S., & Chow, J.Y.J. (2019). Route-cost-assignment with joint user and operator behavior as a many-to-one stable matching assignment game. Transportation Research Part B, 124, 60-81.
19. Rasulkhani, S., Pantelidis, T., & Chow, J.Y.J. (2020). A many-to-many assignment game method to evaluate cost allocations of link operators in a Mobility-as-a-Service market without route enumeration, 99th Annual Meeting of the TRB, Washington, DC.
20. Shapley, L. S., & Shubik, M. (1971). The assignment game I: The core. International Journal of Game Theory, 1(1), 111-130.
21. Sotomayor, M. (1992). The multiple partners game. In Equilibrium and Dynamics (pp. 322-354). Palgrave Macmillan, London.
22. Stabler, B. (2019). Transportation Networks for Research. https://github.com/bstabler/TransportationNetworks, last accessed November 7, 2019.
23. Stiglic, M., Agatz, N., Savelsbergh, M., & Gradisar, M. (2015). The benefits of meeting points in ride-sharing systems. Transportation Research Part B, 82, 36-53.
24. Strömberg, H., Karlsson, I. M., & Sochor, J. (2018). Inviting travelers to the smorgasbord of sustainable urban transport: evidence from a MaaS field trial. Transportation, 45(6), 1655-1670.
25. Wang, X., Agatz, N., & Erera, A. (2017). Stable matching for dynamic ride-sharing systems. Transportation Science, 52(4), 850-867.
26. Wong, Y. Z., Hensher, D. A., & Mulley, C. (2019). Mobility as a service (MaaS): Charting a future context. Transportation Research Part A: Policy and Practice.
27. Xu, G., Yang, H., Liu, W., & Shi, F. (2018). Itinerary choice and advance ticket booking for high-speed-railway network services. Transportation Research Part C: Emerging Technologies, 95, 82-104.
28. Yang, H., Qin, X., Ke, J., & Ye, J. (2019). Optimizing Matching Time Interval and Matching Radius In On-Demand Matching of a Ride-Sourcing Market. Available at SSRN 3372349.
29. Yang, H., & Woo, K. (2000). Competition and equilibria of private toll roads in a traffic network. Transportation Research Record, 1733(1), 15-22.
30. Zhang, H., & Zhao, J. (2018). Mobility sharing as a preference matching problem. IEEE Transactions on Intelligent Transportation Systems.
31. Zhang, X., Zhang, H. M., Huang, H. J., Sun, L., & Tang, T. Q. (2011). Competitive, cooperative and Stackelberg congestion pricing for multiple regions in transportation networks. Transportmetrica, 7(4), 297-320.