Final State Interaction Effects in Semi-inclusive Deep Inelastic processes $A(e, e'p)X$ off the deuteron and complex nuclei

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Abstract

The effects of the final state interaction (FSI) in semi inclusive deep inelastic electron scattering processes $A(e, e'p)X$ off nuclei are investigated in details. Proton production is described within the spectator and the target fragmentation mechanisms whose relevance to the experimental study of the deep inelastic structure functions of bound nucleons and the non perturbative hadronization process is analyzed. Particular attention is paid to the deuteron target within kinematical conditions corresponding to the available and forthcoming experimental data at Jlab. We argue that there are kinematical regions where FSI effects are minimized, allowing for a reliable investigation of the DIS structure functions, and regions where the interaction of the quark-gluon debris with nucleons is maximized, which makes it possible to study hadronization mechanisms. Nuclear structure has been described by means of realistic wave functions and spectral functions and the final state interaction has been treated within an eikonal approximation approach which takes into account the rescattering of the quark-gluon debris with the residual nucleus and, in the case of complex nuclei, within an optical potential approach to account for the FSI of the struck proton.

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I. INTRODUCTION

Semi inclusive Deep Inelastic Scattering (SIDIS) of leptons off nuclei can provide relevant information on: (i) possible modification of the nucleon structure function in medium (EMC-like effects), (ii) the relevance of exotic configurations at short NN distances; (iii) the mechanism of quark hadronization (see e.g. \[1\]). A process which attracted much interest from both the theoretical \[2, 3\] and experimental \[4\] points of view is the production of slow protons, i.e. the process \(A(l, l'p)X\), where a slow proton is detected in coincidence with the scattered lepton. In plane wave impulse approximation (PWIA), after the hard collision of \(\gamma^*\) with a quark of a bound nucleon, two main mechanisms of production of slow protons have been considered, namely the spectator mechanism \[1\], and target fragmentation \[2, 3\]. In the target fragmentation slow protons originate from the spectator diquark which captures a quark from the vacuum creating the recoiling proton. In case of fragmentation of free nucleons by highly virtual photons this is the only source of slow nucleons. In SIDIS off nuclei both target fragmentation and the spectator processes can occur, the latter originating from nucleon-nucleon (NN) short range correlations. In the past the spectator mechanism has been intensively investigated, though most calculations either completely disregarded the effects of the final state interaction, or considered them within simple models. In this paper the results of calculations of both the spectator and fragmentation mechanisms, taking into account FSI effects will be presented. Our paper is organized as follows: the results for the deuteron are presented in Section II, complex nuclei are treated in Section III, the Conclusions are presented in Section IV.

II. PROTON PRODUCTION FROM THE DEUTERON

We analyze the process of proton production from the deuteron target of the type

\[ e + D = e' + p + X, \]  

(1)

which not only has been considered in several theoretical papers \[5, 6, 7\], but is at present under experimental investigation at JLab \[8\]. This process, if the spectator mechanism is valid, can provide unique information on the DIS structure functions of bound nucleons. In ref. \[9\] the theoretical treatment of the spectator mechanism approach has been implemented by considering the effects of the FSI between the hadrons (mostly mesons), created during the
hadronization of the quark-gluon debris, and the spectator proton. Note that hadronization is basically a QCD nonperturbative process, and, consequently, any experimental information on its effects on the reaction (1) would be a rather valuable one. It has been observed in ref. [9] that in the kinematical range where the FSI effects are relevant, the process (1) is essentially governed by the hadronization cross section. This opens a new and important aspect of these reactions, namely the possibility, through them, to investigate hadronization mechanisms by choosing a proper kinematics where FSI effects are maximized. Herebelow we improve our previous analysis of the process (1) by also considering the possibility of proton production due to target fragmentation. For the sake of simplicity, we consider here DIS kinematics in the Bjorken limit ($Q^2 \rightarrow \infty; \nu \rightarrow \infty; x = Q^2/2m\nu$ finite); the generalization to finite values of $Q^2$ is straightforward.

In the one-photon-exchange approximation the cross section for the process given by (1) can be written as follows

$$\frac{d\sigma}{dx dQ^2 dp_2} = \frac{4\alpha_{em}^2 \pi \nu}{Q^4 x} \left[ \frac{1}{1 - y - \frac{Q^2}{4E^2}} \tilde{l}_{\mu\nu} L_{\mu\nu}^D \right]$$

$$\frac{4\alpha_{em}^2 \pi \nu}{Q^4 x} \left[ 1 - y - \frac{Q^2}{4E^2} \right] \left[ \tilde{l}_L W_L + \tilde{l}_T W_T + \tilde{l}_{TL} W_{LT} \cos \phi_s + \tilde{l}_{TT} W_{TT} \cos(2\phi_s) \right]$$

where $\alpha_{em}$ is the fine-structure constant, $Q^2 = -q^2 = -(k - k')^2 = q^2 - \nu^2 = 4EE'\sin^2\theta_2/2$ the four-momentum transfer (with $q = k - k'$, $\nu = E - E'$ and $\theta \equiv \theta_{kk'}$), $x = Q^2/2M\nu$ the Bjorken scaling variable, $y = \nu/E$, $p_2$ the momentum of the detected recoiling nucleon, and $\tilde{l}_{\mu\nu}$ and $L_{\mu\nu}^D$ the electron and deuteron electromagnetic tensors, respectively; the former has the well known standard form [10], whereas the latter can be written as follows

$$L_{\mu\nu}^D = \sum_X \langle P_D | J_\mu | P_f \rangle \langle P_f | J_\nu | P_D \rangle (2\pi)^4 \delta^{(4)}(k + P_D - k' - p_X - p_2) d\tau_X,$$

where $J_\mu$ is the operator of the deuteron electromagnetic current, and $P_D$ and $P_f = p_X + p_2$ denote the three-momentum of the initial deuteron and the final hadron system, respectively, with $p_X$ being the momentum of the undetected hadronic state created by the DIS process on the active nucleon. In Eq. (3) the various $W_i$ represent the nuclear response functions and the quantities $\tilde{l}_i$ are the corresponding components of the photon spin density matrix [10]. It is well known, that within the PWIA, i.e., when FSI effects are disregarded, the four response functions in Eq. (3) can be expressed in terms of two independent structure functions, viz $W_L$ and $W_T$. Moreover, in the DIS kinematics, when the Callan-Gross relation
holds, $2xF_1^N(x) = F_2^N(x)$, the semi inclusive cross section (3) will depend only upon one DIS structure function, e.g. $F_2^N(x)$. In the presence of FSI all four responses contribute to the cross section (3); however if FSI effects are not too large, nucleon momenta are sufficiently small ($|p_2|^2 < m^2$) and the momentum transfer $|q|$ large enough, one can expect that the additional two structure functions are small corrections, so that the SIDIS cross section can still be described by one, effective structure function $F_{2A}^{s.i.}(Q^2, x, p_2)$ [5, 6, 7, 9]. Note, that in the considered SIDIS processes the final proton can originate from two different mechanisms, i) a nucleon from the target (different from the detected proton, i.e., the neutron in the deuteron case) is struck by the (highly) virtual photon producing the final hadron debris, while the detected proton acts merely as a spectator (the so-called spectator mechanism) ii) the highly excited quark-gluon system, produced by the interaction of the photon with an active proton hadronizes into a final detected proton and a number of (undetected) mesons, which together with the nuclear remnants, form the undetected debris $X$ (the so called target fragmentation mechanism).

The magnitude of the momentum of the proton from the target fragmentation is similar to the momentum of protons resulting from the spectator mechanism, therefore we do not consider here protons arising from current fragmentation which have a much higher value of the momentum.

A. The spectator mechanism

As already mentioned, according to the spectator mechanism, the deep-inelastic electromagnetic process occurs on the active nucleon, e.g. nucleon ”1”, while the second one (the spectator) recoils and is detected in coincidence with the scattered electron. At high values of the 3-momentum transfer the produced hadron debris propagates mostly along the $q$ direction and re-interacts with the spectator nucleon. The wave function of such a state can be written in the form

$$\Psi_f(\{\xi\}, r_X, r_s) = \phi_{\beta_f}(\{\xi\})\psi_{p_X,p_s}(r_X, r_s), \quad (5)$$

where $r_X$ and $r_s$ are the coordinates of the center-of-mass of system $X$ and the spectator nucleon, respectively, $\{\xi\}$ denotes the set of the internal coordinates of system $X$, described by the internal wave function $\phi_{\beta_f}(\{\xi\})$, $\beta_f$ denoting all quantum numbers of the final state.
the wave function $\psi_{p_x,p_s}(r_x,r_s)$ describes the relative motion of system $X$ interacting elastically with the spectator $s$. The matrix elements in Eq. (4) can easily be computed, provided the contribution of the two-body part of the deuteron electro-magnetic current can be disregarded, which means that the deuteron current can be represented as a sum of electromagnetic currents of individual nucleons, $J_\mu(Q^2,X) = j_\mu^N_1 + j_\mu^N_2$. Introducing complete sets of plane wave states $|k_1',k_2'\rangle$ and $|k_1,k_2\rangle$ in intermediate states, one obtains

$$
\langle P_D|j_\mu^N|\beta_f, P_f = p_X + p_2\rangle = 
\sum_{\beta,k_1',k_2',k_1,k_2} \langle P_D|k_1',k_2'\rangle \langle k_1',k_2'|j_\mu^N|\beta, k_1,k_2\rangle \langle \beta, k_1,k_2| \beta_f, k_X, k_2\rangle = 
\int \frac{d^3k}{(2\pi)^3} \psi_D(k_1) \langle k_1|j_\mu^N(Q^2,k\cdot q)|\beta_f, k_1 + q\rangle \psi_{\kappa_f}(q/2 - p_2),
$$

where $\kappa_f = (p_X - p_2)/2$. In Eq. (6) the matrix element $\langle k_1|j_\mu^N(Q^2,k\cdot q)|\beta_f, k_1 + q\rangle$ describes the electromagnetic transition from a moving nucleon in the initial state to a final hadronic system $X$ in a quantum state $\beta_f$. The sum over all final state $\beta_f$ of the square of this matrix element times the corresponding energy conservation $\delta$-function defines the deep inelastic nucleon hadronic tensor for a moving nucleon.

1. The PWIA

In PWIA, $\gamma^*$ interacts with a quark of the neutron, a nucleon debris is formed and the proton recoils without interacting with the debris. The relative motion debris-proton is thus described by a plane wave

$$
\psi_{\kappa_f}(q/2 - k_2) \sim (2\pi)^3\delta^{(3)}(q/2 - k_2 - \kappa_f) = (2\pi)^3\delta^{(3)}(k_2 - p_2)
$$

and the well known result

$$
\frac{d\sigma_{sp}^{PWIA}}{dx dQ^2 dp_2} = K(x,Q^2,p_2) n_D(|k_1|) F_2^{N/A}(x,p_2),
$$

is obtained, where the kinematical factor $K(x,Q^2,p_2)$, in the Bjorken limit is (see, e.g. ref. [2])

$$
K(x,Q^2,p_2) = \frac{4\pi\alpha^2_{em}}{xQ^4} \left[ 1 - y + \frac{y^2}{2} \right],
$$
$F_2^{N/A}(x, p_2) = 2 x F_1^{N/A}(x, p_2)$ is the DIS structure function of the active nucleon, and $n_D$ the momentum distribution of the hit nucleon with $k_1 = -p_2$, viz

$$n_D(|k_1|) = \frac{1}{3} \frac{1}{(2\pi)^3} \sum_{M_D} \left| \int d^3 r \Psi_{1,M_D}(r) \exp(-i k_1 r / 2) \right|^2. \quad (10)$$

2. **FSI**

Consider now FSI effects within the kinematics when the momentum of the spectator is low and the momentum transfer is large enough, so that the rescattering process of the fast system $X$ off the spectator nucleon could be considered as a high-energy soft hadronic interaction. In this case the momentum of the detected spectator $p_2$ only slightly differs from the momentum $k_2$ before rescattering, so that in the matrix element $\langle -k_2 | j_N^N(Q^2, p \cdot q) | \beta_f, -k_2 + q \rangle$ one can take $k_2 \sim p_2$, obtaining in co-ordinate space

$$\langle P_D | j^{N}_N \rangle \approx j^{N}_N(Q^2, x, p_2) \int d^3 r \psi_D(r) \psi^{+}_f(r) \exp(i r q / 2). \quad (11)$$

The cross section then becomes

$$\frac{d\sigma^{FSI}_{sp}}{dx dQ^2 dp_2} = K(x, Q^2, p_2) n^{FSI}_D(p_2, q) F_2^{N/A}(x, p_2), \quad (12)$$

where

$$n^{FSI}_D(p_2, q) = \frac{1}{3} \frac{1}{(2\pi)^3} \sum_{M_D} \left| \int d^3 r \Psi_{1,M_D}(r) \psi^{+}_f(r) \exp(i r q / 2) \right|^2, \quad (13)$$

is the distorted momentum distribution, which coincides with the momentum distribution of the hit nucleon (Eq. (10)) when $\psi^{+}_f(r) \sim \exp(-i \kappa_f r)$.

In our case, when the relative momentum $\kappa_f \sim q$ is rather large and the rescattering processes occur with low momentum transfers, the wave function $\psi^{+}_f(r)$ can be replaced by its eikonal form describing the propagation of the nucleon debris formed after $\gamma^*$ absorption by a target quark, followed by hadronization processes and interactions of the newly produced hadrons with the spectator nucleon. This series of soft interactions with the spectator can be characterized by an effective cross section $\sigma_{sff}(z, Q^2, x) \quad (14)$ depending upon time (or the distance $z$ traveled by the system $X$). Thus the distorted nucleon momentum distribution, Eq. (13), becomes
\[ n^{FSI}_{D}(p_s, q) = \frac{1}{3(2\pi)^{3}} \sum_{M_D} \int d\mathbf{r} \Psi_{1,M_D}(\mathbf{r}) S(\mathbf{r}, q) \chi_f^+ \exp(-ip_s \mathbf{r})^2, \]  
\[ (14) \]

where \( \chi_f \) is the spin function of the spectator nucleon and \( S(\mathbf{r}, q) \) the \( S \)-matrix describing the final state interaction between the debris and the spectator, viz.

\[ S(\mathbf{r}, q) = 1 - \theta(z) \frac{\sigma_{eff}(z, Q^2, x)(1 - i\alpha)}{4\pi b_0^2} \exp(-b^2/2b_0^2). \]  
\[ (15) \]

where the \( z \) axis is directed along \( q \), i.e. \( r = z \frac{q}{|q|} + b \).

**B. Target fragmentation**

The target fragmentation mechanism is rather different from the spectator one (c.f. Fig. [1]). However, by introducing the notion of fragmentation functions \( H_{1,2}(Q^2, x, p_2) \) [12], the theoretical analysis of both target fragmentation and spectator mechanisms becomes similar and a common theoretical framework can be used. The only difference consists in replacing the nuclear DIS structure function \( F_{s.i.}^{s,i}(Q^2, x, p_2) \) by the nuclear fragmentation function \( H_{2A}(Q^2, x, p_2) \). Then in the Bjorken limit the corresponding cross section reads as follows

\[ \frac{d\sigma_{t.f.}}{dxdQ^2 dp_s} = K(x, Q^2, p_2) H^D_2(x, z_s, p_{s\perp}^2) \]  
\[ (16) \]

where the kinematical factor \( K(x, Q^2, p_2) \) is given by Eq. (9). In Eq. (16) the deuteron target fragmentation structure function \( H^D_2(x, z_s, p_{s\perp}^2) \) can be expressed as a convolution of the nucleon distribution with the corresponding nucleon structure function \( H^N_2(x, z_s, p_{s\perp}^2) \) as follows

\[ H^D_2(x, z_s, p_{s\perp}^2) = \int_{x+z_p}^{M_D/m_N} dz_1 f(z_1) H^N_2 \left( \frac{x}{z_1}, z_s \frac{1-x}{z_1-x}, p_{s\perp}^2 \right), \]  
\[ (17) \]

where

\[ f(z_1) = 2\pi m_N z_1 \int_p^\infty d|p| |p| n_D(p) \]  
\[ (18) \]

with \( z_s = (p_2q)/M_N\nu, z_p = z_s(1-x) \) and \( p_{min} = \left| [(m_N z_1 - M_D)^2 - m_N^2] / [2(m_N z_1 - M_D)] \right| \). The nucleon structure function \( H^N_2(x, z_s, p_{s\perp}^2) \) is

\[ H^N_2 \left( x, z_s, p_{s\perp}^2 \right) = x \frac{\rho(p_{s\perp})}{E_h} z_s \left[ \sum_q e_q^2 f_q(x) D_{qq}(z_s) \right], \]  
\[ (19) \]
where $\rho(p_{s\perp})$ is the transverse momentum distribution of the nucleon, $f_q(x)$ is the parton distribution function, and $D_{qq}^p(z_s)$ is the diquark fragmentation function representing the probability to produce a proton with energy fraction $z_s$ from a diquark. The nucleon functions $H^N_2(x, z_s, p_{s\perp}^2)$, likewise the DIS structure functions $F^N_2(x, Q^2)$, are well known experimentally and the parametrized form of $\rho(p_{s\perp})$ and $D_{qq}^p(z_s)$ in Eq. (19) can be found, e.g. in Refs. [13, 14].

C. Results for the deuteron target

In order to analyze the kinematical conditions under which the effects of FSI are minimized or maximized, the ratio of the PWIA cross section (Eq. (8)) to the cross section including FSI (Eq. (12)) has been considered. Obviously, such a quantity reflects the effects of FSI in the distorted momentum distribution $n_D^{FSI}(p_s, q)$ (Eq. (14)). The results of calculations are presented in Fig. 2 where the angular dependence (left panel) and the dependence upon the value of the spectator momentum (right panel), are shown at $x = 0.6$. Kinematics has been chosen so as to correspond to the one considered at the Jlab experiments at 12$GeV$. The shaded area reflects the uncertainties in the choice of the parameters for $\sigma_{eff}$ [11], $b_0$ and $\alpha$ in Eq. (15). It is clearly seen that at low momenta and at parallel kinematics the effects of FSI are minimized, so that in this region the process $D(e, e'p)X$ could be successfully used to extract the DIS structure function of a bound nucleon. Contrarily, at the perpendicular kinematics the FSI effects are rather important and essentially depend upon the process of hadronization of the quark-gluon debris. Therefore, in this region, the processes $D(e, e'p)X$ can serve as a source of unique information about nonperturbative QCD mechanisms in DIS. Actually, a systematic experimental study of the processes $D(e, e'p)X$ has started at Jlab [8] and first experimental data at initial electron energy $E = 5.765GeV$ are already available [17]. It should be however noted, that our approach corresponds to the Bjorken limit, so that a direct comparison with the experimental data of ref. [17] requires the generalization of our formulae to finite values of $Q^2$ and $\nu$, when the Callan-Gross relation is violated and a proper modification of the hadronization mechanism is required. Such an analysis will be presented in details elsewhere [18]. In order to minimize the statistical errors, it is common in the literature to present the ratio of the experimental cross section to a properly chosen kinematical factor, which, within the PWIA spectator mechanism, corresponds to a product
of the neutron DIS structure function $F^\text{n}_2(x,Q^2)$ and the deuteron momentum distribution \cite{10}. In such a way, one expects that any deviation of this ratio from the corresponding theoretical ratio, would represent a measure of FSI effects. In Fig. 3 we present the results of calculations of such a reduced cross section obtained within the spectator mechanism. The dashed curves represent the PWIA results, whereas the solid ones correspond to the calculations with FSI taken into account. One can conclude from Fig. 3, that the spectator mechanism within the PWIA does not explain the data in the whole kinematical range, whereas an overall better agreement can be achieved when FSI effects are taken into account.

In order to estimate the role of the target fragmentation mechanism, we have calculated the ratio

\[
R_{\text{tf}} = \frac{d\sigma_{\text{tf}} + d\sigma_{\text{PWIA}}^{\text{sp}}}{d\sigma_{\text{PWIA}}^{\text{sp}}},
\]

which, obviously, characterizes the relative contribution of the fragmentation cross section to the total cross section. In our calculations the transverse hadron momentum distribution has been parametrized in the form \cite{13}

\[
\rho(p_{s\perp}) = \beta \frac{\pi}{\beta} \exp(-\beta p_{s\perp}^2),
\]

while the fragmentation function $D_{qq}$ has been taken from ref. \cite{14}. The results of calculations are shown in Fig. 4, where $R_{\text{tf}}$ is presented as a function of the emission angle of the detected proton at several fixed values of the momentum (left panel) and the spectator momentum at fixed emission angles (right panel). As expected, the fragmentation mechanism contributes only in a very narrow forward direction and for large values of the spectator momentum.

III. COMPLEX NUCLEI

A. The spectator mechanism within the PWIA

In this section our approach is generalized to complex nuclei in the same way as it has been done in ref. \cite{3}. The only essential difference with respect to the formalism used in those papers is our consideration of the FSI of the quark-gluon debris with the spectator nucleons. The basic nuclear ingredient in these calculations is the two-nucleon spectral function, which has been chosen according to the nucleon-correlation model. In the simplest
version of this model, the Center-of-Mass motion of correlated NN pair is disregarded \[1\], whereas in the extended 2NC model \[16\] it is taken into account.

A calculation of the PWIA diagram of Fig. 5(a) yields

\[
\frac{d\sigma}{dx dQ^2 dp_2} = 4\pi \alpha_{em}^2 \left[ 1 - y + \frac{y^2}{2} \right] F_2^A(x, p_2),
\]

where the nuclear structure function \( F_2^A(x, p_2) \) is defined via the following convolution integral \[16\]

\[
F_2^A(x, p_2) = \int_x^{M_A/m_N - z_2} d\bar{z}_1 z_1 F_2^N \left( \frac{x}{\bar{z}_1} \right) \int d\vec{k}_{cm} dE^{(2)} S(\vec{k}_{cm} - \vec{k}_2, \vec{k}_2, E^{(2)}) \times m_N \delta(M_A - m_N(z_1 + z_2) - M_{A-2}^f z_{A-2})
\]

where \( \vec{k}_{cm} = \vec{k}_1 + \vec{k}_2 = -P_{A-2} \), \( F_2^N(x/z_1) \) is the structure function of the hit nucleon, \( z_1 = (k_1 q)/m_N \nu \), \( z_2 = [(m_N^2 + p_2^2)^{1/2} - |p_2| \cos \Theta_2]/m_N \), and \( z_{A-2} = [((M_{A-2}^f)^2 + k_{cm}^2)^{1/2} + (k q)/|q|]/M_{A-2}^f \) are the light-cone momentum fractions of the hit nucleon, the detected nucleon and the recoiling spectator nucleus \( A-2 \), respectively, \( E^{(2)} \) denotes the two nucleon removal energy and the two nucleon spectral function \( S(\vec{k}_1 = \vec{k}_{cm} - \vec{k}_2, \vec{k}_2, E^{(2)}) \) is given by

\[
S(\vec{k}_1, \vec{k}_2, E^{(2)}) = n_{rel}(\vec{k}_1 - \vec{k}_2)/2 n_{cm}(\vec{k}_1 + \vec{k}_2) \delta(E^{(2)} - E_{th}^{(2)})
\]

where \( E_{th}^{(2)} \) is the two nucleon emission threshold.

**B. The Spectator Mechanism with FSI**

Contrarily to the deuteron case, the effects of the FSI in complex nuclei are much more complicated to treat since in this case the structure of the Spectral Function \[24\] implies that \((A-2)\) is in the ground state; in this case, after \( \gamma^* \) absorption, the final state consists of at least three different interacting systems (c.f. Fig. 5(b) and 5(c)): the undetected debris \( X \), the undetected \( A-2 \) nucleus and, eventually, the detected proton \( p_2 \). Correspondingly, the FSI can formally be divided into three classes \[15\], namely: i) the FSI of the quark-gluon debris with the spectator \( A-2 \) system; ii) the interaction of the recoiling nucleon with the \( A-2 \) system; iii) the interaction of the debris with the recoiling proton.
1. **FSI of the debris with the spectator nucleons**

The FSI of the debris and $A-2$ system is treated in the same way as in the deuteron case: the quark-gluon debris rescatters off the recoiling proton and the $A-2$ spectator system. Then in Eq. (23) the spectral function $S(p_1, p_2, E^{(2)})$ has to be replaced by the *Distorted spectral function*, which can be written in the following way

$$ S_{N_1N_2}^D(p_1, p_2, E^{(2)}) = \sum_f |T_{fi}|^2 \delta(E^{(2)} - E_{th}^{(2)}) = \sum_f \left|\langle p_X, p_2; \Psi_{A-2}^f, \hat{S}_{FSI} | q, \Psi_A^0 \rangle \right|^2 \delta(E^{(2)} - E_{th}^{(2)}), \quad (25) $$

where $\hat{S}_{FSI}$ is the FSI operator and $T_{fi}$ is the transition matrix element of the process having the following form

$$ T_{fi} = \frac{1}{(2\pi)^6} \int \prod_{i=1}^A dr_i e^{-i p_X \cdot r_1} e^{-i q \cdot r_1} e^{-i p_2 \cdot r_2} \times $$

$$ \times \Psi_{A-2}^{x_f}(r_3, \ldots, r_A) \hat{S}_{FSI}^+(r_1, \ldots, r_A) \Psi_A^0(r_1, \ldots, r_A). \quad (26) $$

According to our classification of the FSI effects, the operator $\hat{S}_{FSI}$ will read as follows

$$ \hat{S}_{FSI}(r_1, r_2, \ldots, r_A) = D_{p_2}(r_2) G(r_1, r_2) \prod_{i=3}^A G(r_1, r_i), \quad (27) $$

where $D_{p_2}(r_2)$ and $G(r_1, r_2)$ take into account the interaction of the slow recoiling proton with $A-2$ and with the fast nucleon debris, whereas $\prod_{i=3}^A G(r_1, r_i)$ takes into account the interaction of the nucleon debris with $A-2$. Using momentum conservation $p_X = q - p_2 - P_{A-2}$, the transition matrix element of the process $A(e, e'p)X$ becomes:

$$ T_{fi} = \frac{1}{(2\pi)^6} \int \prod_{i=1}^A dr_i e^{i(P_{A-2} + p_2) \cdot r_1} e^{-i p_2 \cdot r_2} \times $$

$$ \times \Psi_{A-2}^{x_f}(r_3, \ldots, r_A) \hat{S}_{FSI}^+(r_1, \ldots, r_A) \Psi_A^0(r_1, \ldots, r_A) = \frac{1}{(2\pi)^6} \int dr_1 dr_2 e^{i(P_{A-2} + p_2) \cdot r_1} e^{-i p_2 \cdot r_2} I_{FSI}(r_1, r_2), \quad (28) $$

where

$$ I_{FSI}(r_1, r_2) = \int \prod_{i=3}^A dr_i \Psi_{A-2}^{x_f}(r_3, \ldots, r_A) \hat{S}_{FSI}^+(r_1, \ldots, r_A) \Psi_A^0(r_1, \ldots, r_A) \quad (29) $$

is the distorted two body overlap integral.

We reiterate, that in the present approach we consider protons with relatively large, at the average fermi motion scale, momenta which originate from a short-range correlated pair.
in the parent nucleus. Then for such kinematics the nuclear wave function can be written as follows [16]

$$\Psi_A^0(r_1, \ldots, r_A) = \sum_{\alpha \beta} \Phi_\alpha(r_1, r_2) \otimes \Psi_{A-2}^\beta(r_3, \ldots, r_A),$$  \hspace{1cm} (30)

where $\Phi_\alpha(r_1, r_2)$ and $\Psi_{A-2}^\beta(r_3, \ldots, r_A)$ describe the correlated pair and the $A-2$ remnants, respectively. In Eq. (30) the symbol $\otimes$ is used for a short-hand notation of the corresponding Clebsh-Gordon coefficients. The wave function of the correlated pair can be expanded over a complete set of wave functions describing the intrinsic state of the pair and its motion relative to the $A-2$ kernel, viz.

$$\Phi_\alpha(r_1, r_2) = \sum_{mn} c_{mn} \phi_m(r) \chi_n(R),$$ \hspace{1cm} (31)

where $R = \frac{1}{2}(r_1 + r_2)$ and $r = r_1 - r_2$ are the center of mass and relative coordinate of the pair. As mentioned, in the 2NC model is assumed that the correlated pair carries out the most part of the nuclear momentum, while the momentum of the relative motion of the pair and $A-2$ nucleus is small [16]. This allows one to treat the CM motion in its lowest $^1S_0$ quantum state (in what follows denoted, for the sake of brevity, as os-state). We can therefore write

$$\Phi_\alpha(r_1, r_2) \simeq \chi_{os}(R) \sum_m c_{mo} \phi_m(r) = \chi_{os}(R) \varphi(r) \equiv \phi(r_1, r_2),$$ \hspace{1cm} (32)

with

$$\varphi(r) = \sum_m c_{mo} \phi_m(r)$$ \hspace{1cm} (33)

Finally we have

$$\Psi_A^0(r_1, \ldots, r_A) \simeq \chi_{os}(R) \varphi(r) \Psi_{A-2}^0(r_3, \ldots, r_A).$$ \hspace{1cm} (34)

Placing this expression in Eq. (29) we get:

$$I_{FSI}^{FSI}(r_1, r_2) = \int \prod_{i=3}^A dr_i \chi_{os}(R) \varphi(r) \hat{S}_{FSI}^+(r_1, \ldots, r_A) |\Psi_{A-2}^0(r_3, \ldots, r_A)|^2$$ \hspace{1cm} (35)

and, disregarding correlations in the $A-2$ system, one can write [19, 20]:

$$|\Psi_{A-2}^0(r_3, \ldots, r_A)|^2 \simeq \prod_{i=3}^A \rho(r_i),$$ \hspace{1cm} (36)

with $\int \rho(r_i)dr_i = 1$, so that, eventually, the distorted overlap integral becomes:

$$I_{FSI}^{FSI}(r_1, r_2) = \int \prod_{i=3}^A dr_i \phi(r_1, r_2) \prod_{i=3}^A \rho(r_i) D_{p_2}(r_2) G(r_1, r_2) \prod_{i=3}^A G(r_1, r_i) =$$

$$= \phi(r_1, r_2) G(r_1, r_2) D_{p_2}(r_2) \left[ \int dr \rho(r) G(r_1, r) \right]^{A-2},$$ \hspace{1cm} (37)

and
where \( \phi(r_1, r_2) = \chi_{os}(R) \varphi(r) \) (cf. Eq. 32). In our calculations the function \( \phi(r_1, r_2) \) has been chosen phenomenologically in such a way, that in PWIA to provide the same high momentum components of the two-nucleon spectral function, as reported in ref. [16].

The explicit form of the operator \( G(r_1, r_i) \), describing the FSI of debris with spectators reads as [11]

\[
\prod_{i=2}^{A} G(r_1, r_i) = \prod_{i=2}^{A} \left[ 1 - \theta(z_i - z_1) \Gamma(b_1 - b_i, z_i - z_1) \right]
\]

(38)

where \( b_i \) and \( z_i \) are, the transverse and longitudinal components of the coordinates of nucleon “i”, and the function \( \theta(z_i - z_1) \) describes forward debris propagation. The profile function in this case is determined by an effective cross section, which depends up on the distance, traveled by the string from the creation to the corresponding hadronization points.

\[
\Gamma(b_1 - b_i, z_i - z_1) = (1 - i \alpha) \frac{\sigma_{eff}(z_i - z_1)}{4 \pi \beta} e^{-\frac{(b_1 - b_i)^2}{2 \beta}}
\]

(39)

where the cross section for the effective cross section \( \sigma_{eff}(z) \) consists on a sum of cross sections of nucleon-nucleon and meson-nucleon interaction at the given point \( z \), \( \sigma_{eff}(z) = \sigma_{tot}^{NN} + \sigma_{tot}^{\pi N} [n_M(z) + n_G(z)] \), where \( n_M(z) \) and \( n_G(z) \) are the effective numbers of mesons produced by color string and gluon radiation, respectively. As demonstrated in ref. [21], the considered hadronization model for the \( \sigma_{eff}(z) \) provides a good description of grey track production in DIS off nuclei. Now, assuming that the single particle density is normalized as \( \int dr \rho(r) = 1 \) and is almost constant for complex nuclei, we can write:

\[
\left[ \int dr \rho(r) G(r_1, r) \right]^{A-2} = \left[ \int dr \rho(r) - \frac{1}{2} \int db \int_{z_1}^{\infty} dz \rho(b, z) \Gamma(b_1 - b, z - z_1) \right]^{A-2} \approx
\]

\[
\approx \left[ 1 - \frac{1}{2} \int_{z_1}^{\infty} dz \rho(b_1, z) \sigma_{eff}(z - z_1) \right]^{A-2} \approx
\]

\[
\approx \exp \left( -\frac{1}{2} A \int_{z_1}^{\infty} dz \rho(b_1, z) \sigma_{eff}(z - z_1) \right)
\]

(40)

and finally we can write the transition matrix element in the following way:

\[
T_{fi} = \frac{1}{(2\pi)^6} \int dr_1 dr_2 e^{i(P_{A-2} + P_2 \cdot r_1)} e^{-iP_2 \cdot r_2} \phi(r_1, r_2) \times
\]

\[
\times G(r_1, r_2) D_{p_2}(r_2) \exp \left( -\frac{1}{2} A \int_{z_1}^{\infty} dz \rho(b_1, z) \sigma_{eff}(z - z_1) \right)
\]

(41)
Here the effect of the interaction of the recoiling spectator nucleon with the $A - 2$ residual nucleus will be estimated. In order to describe the motion of the spectator in the field of $A - 2$ system, we use an Optical Potential approach, also known as the conventional Distorted Wave Impulse Approximation (DWIA). This approach is normally used to describe nucleon-nucleus interaction at low energies [10]. In the DWIA approach the interaction is considered as due to a mean field, a distorting potential $V(x, y, z)$, which, in our framework, takes into account the interaction between the residual nucleus and the emitted nucleon. The outgoing nucleon plane wave becomes distorted as a consequence of the eikonal phase factor:

$$e^{-i\mathbf{p}_2 \cdot \mathbf{r}_2} \rightarrow e^{-i\mathbf{p}_2 \cdot \mathbf{r}_2} D_{p_2}(\mathbf{r}_2),$$

where

$$D_{p_2}(\mathbf{r}_2) = \exp \left( i \frac{E_2}{p_2} \int_{z_2}^{\infty} dz V(x_2, y_2, z) \right).$$

The optical potential $V(x, y, z)$ does not depend on the individual coordinates of the spectator nucleons and embodies an effective description of how the nuclear medium influences the wave function of the propagating nucleon. We use an energy dependent optical potential:

$$V(\mathbf{r}) = V(\mathbf{r}) + iW(\mathbf{r})$$

in which the real part $V$ represents elastic re-scattering and the imaginary part $W$, reducing the proton flux, describes absorption processes. The energy dependence of the potential is given by the well known choice

$$V(|p_2|) = -\rho \frac{v(i + \alpha)}{2} \sigma_{tot}^{NN}(|p_2|),$$

where $\rho$ is the nuclear density, $v$ the struck nucleon velocity, $\alpha$ the ratio of the real to the imaginary parts of the forward scattering amplitude $f_{NN}(0)$, and $\sigma_{tot}^{NN}(p)$ the total nucleon nucleon scattering cross section, related to $f_{NN}(0)$ by the optical potential $\sigma_{tot}^{NN} = \frac{4\pi}{p}Imf_{NN}(0)$. The parameters of $\alpha$ and $\sigma_{tot}^{NN}$ have been taken from [22]. When the energy of the propagating proton is small, each rescattering causes a considerable loss of energy-momentum and the flux of the outgoing proton plane wave is suppressed: this effect is modeled in the DWIA by the imaginary part of the potential. Numerical calculations show that the effect of the distortion is mainly determined by the depth of the imaginary part of the optical potential and depends weakly on the real part, so we can safely neglect it.
C. The target fragmentation mechanism

Let us now consider proton production from the target fragmentation mechanism in which the quark-gluon debris originates from current fragmentation and the proton arising from target fragmentation (c.f., Fig. 5(c)). The corresponding cross section can be expressed in terms of two structure functions $H^A_1$ and $H^A_2$ as follows

$$\frac{d\sigma^A}{dx dy dp_2} = \frac{8\pi\alpha^2 ME}{Q^4} \left[ xy^2 H^A_1 \left( x, z_s, p^2_{s\perp} \right) + (1 - y) H^A_2 \left( x, z_s, p^2_{s\perp} \right) \right], \quad (46)$$

where

$$z_s = \frac{(p_sq)/\nu}{m_N(1 - x)} \quad (47)$$

is the light-cone momentum fraction of the detected proton. The structure functions can be written as convolutions of the nucleon structure functions with the familiar one-body nuclear spectral function $P(p, E_r)$ as

$$H^A_1 \left( x, z_s, p^2_{s\perp} \right) = \int dz_1 f(z_1) \frac{1}{z_1} H^N_1 \left( x, z_1, \frac{1 - x}{z_1 - x}, p^2_{s\perp} \right), \quad (48)$$

$$H^A_2 \left( x, z_s, p^2_{s\perp} \right) = \int dz_1 f(z_1) H^N_2 \left( x, z_1, \frac{1 - x}{z_1 - x}, p^2_{s\perp} \right), \quad (49)$$

where $H^N_1$ and $H^N_2$ are the structure functions for the free nucleon and $f(z_1)$ is given by

$$f(z_1) = \int dp dE_r P(p, E_r) z_1 \delta \left( z_1 - \frac{x}{x_N} \right). \quad (50)$$

Within the quark-parton model, the nucleon structure functions have the following form

$$H^N_1 \left( x, z_s, p^2_{s\perp} \right) = \frac{z_s}{E_2} \rho(p^2_{s\perp}) \frac{1}{2} \sum_q e^2_q f_q(x) D^h_{qq}(z_s) \quad (51)$$

$$H^N_2 \left( x, z_s, p^2_{s\perp} \right) = \frac{z_s}{E_2} \rho(p^2_{s\perp}) x \sum_q e^2_q f_q(x) D^h_{qq}(z_s), \quad (52)$$

where $E_2$ is the energy of the detected proton and $f_q(x)$ and $\rho(p^2_{s\perp})$ are the parallel and transverse momentum distributions, respectively.

D. Results of calculations

We have calculated the differential cross section $d\sigma^A/dE'd\Omega_1dT_2d\Omega_2$ for the $^{12}C(e,e'p)X$ process with full FSI described by the operator $\hat{S}_{FSI}$ of Eq. [27]. The results of our
calculations are presented in Figs. 6-9, where the separate contributions of the various FSI’s are shown as a function of the detected proton kinetic energy $T_2$. Calculations have been performed assuming an incident electron energy of $E = 20\, GeV$ and an electron scattering angle of $\theta_e = 15^0$, with values of the Bjorken scaling variable equal to $x = 0.2$ and 0.6; the proton emission angle has been fixed at the two values of $\theta_2 = 25^0$ (forward) and $\theta_2 = 140^0$ (backward). It can be seen that the most relevant contribution to FSI comes from hadronization of the hit quark, in forward as well as in backward nucleon emission. The effects of FSI between the recoiling nucleon and $A - 2$ amounts to an attenuation factor which, in the analyzed proton momentum $|p_2|$ range, decreases the cross section by a factor of $\sim 0.5 - 0.7$; as expected, this contribution is more relevant for low values of the proton kinetic energy. We checked the sensitivity of the process upon the nucleon debris effective cross section using a constant cross section $\sigma_{eff} = 20\, mb$ [2]; the results presented in Fig. 8 show an appreciable difference with respect to the calculation performed using the time-dependent effective cross section of ref. [23]. The results for the differential cross section with the full FSI are presented in Fig. 9 for the detection of the slow proton both in the forward ($\theta_2 = 25^0$, $z_2 < 1$) and backward ($\theta_2 = 140^0$, $z_2 > 1$) hemispheres. We analyzed the role of the fragmentation mechanism in the considered SIDIS processes. The results for the $^{12}C$ target are presented in Fig. 10; it can be seen that, as in the deuteron case, the target fragmentation mechanism contributes to nucleon emission in the forward direction and becomes noticeable at high values of $T_2$ ($T_2 > 600\, MeV$). It should be noted that such large kinetic energy are beyond of applicability of our approach. In the region $50\, MeV < T_2 < 250\, MeV$, where the use of a non relativistic spectral function is well grounded, the effects of target fragmentation play a minor role and the contribution from final state interaction of the spectator nucleon with $A - 2$ practically reduce to an attenuation factor.

IV. SUMMARY AND CONCLUSIONS

We have considered proton production in semi inclusive processes $A(e, e'p)X$ in the deep inelastic limit within the spectator and the target fragmentation mechanisms taking all kind of FSI into account. A systematic study of this process is of great relevance in hadronic physics. As a matter of fact, in case of deuteron targets detailed information on the DIS neutron structure function could in principle be obtained by performing experiments in the
kinematical region where FSI are minimized (backward production); at the same time if the experiment is performed when FSI are maximized (perpendicular kinematics) the non perturbative QCD phenomenon of hadronization could be investigated. Experimental study of SIDIS off deuteron targets has already started at Jlab and is planned to be extended to higher energies at the upgraded facility.

In case of complex nuclei SIDIS could also represent a tool to investigate short-range correlations in nuclei. As a matter of fact the main sources of slow backward protons in SIDIS originates from a correlated pair.

In the present paper we have investigated in details the effects of FSI. In particular, the interaction of the quark-gluon debris with the recoiling proton and the residual $A-2$ nucleus, treated within the eikonal approximation, and the interaction of the proton with the $A-2$ treated within the distorted wave impulse approximation. We have demonstrated, that the FSI due to the quark-gluon debris off the spectator $A-2$ nucleons appreciably attenuates the cross section, reflecting the fact that the rescattering of the quark-gluon debris in the final state strongly decreases the survival probability of the $(A-2)$ nucleus [21].

To main results obtained in the present paper can be summarized as follows:

1. in case of SIDIS off the deuteron, FSI can be minimized in the parallel kinematics and maximized in the perpendicular one. In the former case the bound nucleon structure function can be investigated, whereas in the second case information on QCD hadronization mechanisms could be obtained;

2. for complex nuclei, FSI appreciably decreases the cross section casting doubts as to the possibility to perform experiments of the type we have considered, where the underlying mechanism is almost fully exclusive, being the unobserved $(A-2)$ nucleus in a well defined energy state. A more realistic case would be to consider a really semi-inclusive process by summing over all energy states of $(A-1)$. Calculations of this type are in progress and will be presented elsewhere [24];

3. as in ref. [3], we found that the interaction of the recoiling proton with the $A-2$ system is relevant only at low proton kinetic energies, leading to an overall small reduction of the cross section;

4. the target fragmentation mechanism plays a minor role in slow proton production in
SIDIS except at the extreme forward direction and large values of the proton momentum.

In conclusion, slow hadron production in SIDIS appears to be a powerful tool to investigate both the properties of bound nucleons and the hadronization mechanisms.

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FIG. 1: The Feynman diagrams of the process $D(e, e'p)X$ corresponding to the spectator (left panel) and to target fragmentation (right panel) mechanisms.
FIG. 2: An illustration of the role of FSI in $D(e,e'p)X$ processes within the spectator mechanism. Left panel: angular dependence of the ratio $\sigma_{FSI}/\sigma_{PWIA}$ at several fixed values of the spectator momentum. Right panel: dependence of the ratio $\sigma_{FSI}/\sigma_{PWIA}$ upon the momentum of the spectator proton at parallel ($\theta = 0^\circ$ and $\theta = 180^\circ$) and perpendicular ($\theta = 90^\circ$) kinematics. Since the actual parametrizations of the nucleon DIS structure functions $F_2(x,Q^2)$ depend upon $Q^2$, in our calculations we put $Q^2 = 12 GeV^2/c^2$. The chosen kinematics is close to the one planned in the future experiments at JLAB at $E \sim 12 GeV$. The shaded area is due to the uncertainties in the input parameters for $\sigma_{eff}$ in Eq. (15).
FIG. 3: The reduced cross section \( F_{2n}(x, Q^2) n_D(p_s, \cos \theta_{pq}) \) \( [(\text{GeV/c})^{-3}] \), representing, within the PWIA, the product of the neutron DIS structure function \( F_{2n}(x/z_1, Q^2) \) and the deuteron momentum distribution \( n_D(|p_s|) \), vs. the proton emission angle for different values of \(|p_s|\) and fixed value of the invariant mass of the debris \( X \), \( W = \sqrt{(P_D - p_2 + q)^2} \) and \( Q^2 = 1.8 \text{ GeV}^2/c^2 \). The dashed curves correspond to PWIA results, whereas the full ones include the effects of FSI. Experimental data from ref. [17].
FIG. 4: Nucleon emission by target fragmentation. The ratio \( R_{tf} = (\sigma_{tf} + \sigma_{PWIA})/\sigma_{PWIA} \) is plotted vs. \( \cos\theta_2 \) (\( \theta_2 \) is the proton emission angle) and vs. \( |p_s| \).

FIG. 5: Proton production in \( A(e, e'p)X \) processes: (a) spectator mechanism within the PWIA; (b) various contribution to the FSI within the spectator mechanism; (c) proton production from target fragmentation.
FIG. 6: The SIDIS differential cross section for the process $^{12}\text{C}(e,e'p)X$ versus the kinetic energy $T_2$ of the detected proton, emitted forward at $\theta_2 = 25^0$ in correspondence of two values of the Bjorken variable. Solid curve: PWIA results; dashed curve: PWIA plus FSI of the debris $X$ with the recoiling proton; dotted curve: PWIA plus FSI of the proton with $A-2$; dot-dashed curve are the resulting FSI effects from all the interactions.

FIG. 7: The SIDIS differential cross section for the process $^{12}\text{C}(e,e'p)X$ versus the kinetic energy $T_2$ of the detected proton, emitted backward at $\theta_2 = 140^0$ for two values of the Bjorken variable. The labeling of the different curves is as in Fig. 6.
FIG. 8: The SIDIS differential cross section for the process $^{12}\text{C}(e,e'p)X$ versus the kinetic energy $T_2$ of the detected proton, emitted forward at $\theta_2 = 25^0$ and $x = 0.2$. The FSI between the quark-gluon debris and the spectator nucleon is calculated with the time dependent $\sigma_{eff}(z - z')$ (11) (dashed curve) and with a constant $\sigma_{eff} = 20mb$ (dotted curve). The PWIA is given by the full curve.

FIG. 9: The SIDIS differential cross section for the process $^{12}\text{C}(e,e'p)X$ versus the kinetic energy $T_2$ of the detected proton, emitted forward ($\theta_2 = 25^0$) and backward ($\theta_2 = 140^0$) at $x = 0.2$. The dot-dashed curve includes the FSI between the quark-gluon debris and the spectator nucleon and between the latter and $A - 2$. The dashed curve includes also the FSI between quark-gluon debris and $A - 2$. The PWIA is given by the solid curves.
FIG. 10: The SIDIS differential cross section for the process $^{12}\text{C}(e,e'p)X$ versus the kinetic energy $T_2$ of the detected proton, emitted forward at $\theta_2 = 25^0$ and $x = 1.5$. Solid curve: spectator mechanism within the PWIA; dashed curve: spectator mechanism within the PWIA plus FSI of the spectator nucleon with $A - 2$; dot-dashed curve: spectator mechanism within the PWIA plus FSI of the spectator nucleon with $A - 2$ and target fragmentation.
$^{12}\text{C}(e,e'p)X$

$x = 0.4$

$\theta_2 = 25^0$

$\frac{d^4\sigma}{dE'd\Omega'd\Omega''} = \frac{\text{nbarn}^2}{\text{sr}^2 / \text{GeV}^2}$

**Graph:**

- **Black line:** PWIA
- **Red line:** FSI - $D(p_2)$
- **Green dashed line:** FSI + target fragmentation

**Axes:**
- **y-axis:** $10^0$ to $10^{-6}$
- **x-axis:** $250$ to $1000$ [MeV]

**Legend:**
- PWIA
- FSI - $D(p_2)$
- FSI + target fragmentation