Collaborative multi-aggregator electric vehicle charge scheduling with PV-assisted charging stations under variable solar profiles

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Abstract: Electric vehicles (EVs) are on the path to becoming a solution to the emissions released by the internal combustion engine vehicles that are on the road. EV charging management integration requires a smart grid platform that allows for communication and control between the aggregator, consumer and grid. This study presents an operational strategy for PV-assisted charging stations (PVCSs) that allows the EV to be charged primarily by PV energy, followed by the EV station’s battery storage (BS) and the grid. Multi-Aggregator collaborative scheduling is considered that includes a monetary penalty on the aggregator for any unscheduled EVs. The impact of the PVCS is compared to the case with no PV/BS is included. A variation in the PV profile is included in the evaluation to assess its impact on total profits. Profit results are compared in cases of minimum, average and maximum PV energy output. The results indicate that the inclusion of penalties due to unscheduled EVs resulted in lowered profits. Further, the profits experienced an increase as the number of EVs scheduled through PV/BS increased, implying that a lesser percentage of EVs are scheduled by the grid when a greater amount of PV and battery energy are available.

Nomenclature

- \( i, j, k \): subscripts indicating the different aggregators
- \( c_{en} \): cost of purchasing energy from the grid
- \( e_{en} \): energy purchased from the grid
- \( e_{en_pv} \): energy used from PV
- \( e_{en_batt} \): energy used from the battery
- \( e_{pv} \): peak energy
- \( e_{pv_gp} \): grid purchased energy in PV assistance
- \( e_{en} \): total energy consumed for charging all EVs
- \( G \): total number of aggregators
- \( G_j \): ith aggregator from \( G \) aggregators
- \( V_{i}, V_{j}, V_{k} \): number of scheduling requests received by the ith, jth and kth aggregator, respectively
- \( C_{S} \): set of charging stations under the ith aggregator
- \( V_{i}, V_{c}, V_{i} \): number of EVs charged using \( e_{en_pv}, e_{en_batt}, \) and \( e_{en_batt} \) respectively
- \( q \): charging rate of the battery
- \( \text{batt} \text{cap} \): EV battery capacity
- \( \text{SOC}_D \): desired state-of-charge (SOC) of battery
- \( \text{SOC}_{\text{ind}} \): initial SOC of the battery
- \( \text{SOC}_{\text{max}} \): minimum SOC for EV to reach the destination
- \( T_D \): time to charge to \( \text{SOC}_D \)
- \( T_M \): time to charge to \( \text{SOC}_{\text{max}} \)
- \( e_{en_{pv}}, e_{en_{M}} \): energy required to charge up to \( \text{SOC}_{D} / \text{SOC}_{M} \)
- \( T_{\text{charge}} \): time to charge \( v \in V_{i} \)
- \( t_{A} \): arrival time of \( v \in V_{i} \) at c ∈ \( C_{S} \)
- \( t_{b} \): time to start charging
- \( t_{\text{slot}} \): time in one slot taken as 15 min
- \( m_{\text{eq}} \): number of charging slots required by \( v \in V_{i} \)
- \( m_{b} \): slot in which charging begins
- \( m_{t} \): next filled slot after \( m_{b} \)
- \( e_{V_{i}}, e_{V_{d,i}} \): revenue received from the ith EV to charge discounted revenue received from collaborative scheduling for \( v \in V_{i} \)
- \( V_{st} \): total scheduled EVs
- \( x_{e} \): product of \( x_{i} \) and \( x_{e} \)
- \( \text{cost}_{en} \): cost of energy
- \( a_{g_{i}}, a_{g_{S}} \): incentive received/given by aggregator in collaborative scheduling
- \( P_{a_{g_{i}}} \): profit of the ith aggregator
- \( P_{a_{g_{NSC,i}}} \): profit of the ith aggregator in NC scheduling
- \( P_{i} \): sum of profits of all aggregators
- \( P_{a_{g_{C,i}}} \): profit of the ith aggregator in C scheduling
- \( e_{en_{batt_{min}}} \): minimum battery energy
- \( e_{en_{bat}} \): required energy to charge the battery
- \( T_{charge} \): time to charge
- \( c_{st} \): cost of energy per slot
- \( e_{en_{sl}} \): energy to be used in one slot
- \( p_{f} \): unscheduled EV penalty factor
- \( e_{en_{sd}} \): energy source indicator = 1 for using \( e_{en_{p}} \)
- \( S \): binary indicator of whether the EV has been scheduled \((S = 1)\) or not \((S = 0)\)
- \( C \): collaboration indicator

1 Introduction

With the ever-increasing growth of industries and the overall development of countries, large amounts of harmful emissions are being released into the environment. A number of measures to reduce these harmful emissions are being implemented. The use of renewable resources, such as solar and wind, for power generation instead of fossil fuels, is one such measure. In addition to industrial development, a major contributor to unwanted emissions into the environment, is the increasing number of internal combustion engine vehicles (ICEVs) that are on the road. This number has been observing a steady growth in the past few years in most countries. For example, in India, the total number of personal use two and four-wheelers (cars and motorcycles) have seen a ~75% increase between 2001 and 2016 [1] while evaluating the increase in total vehicles. Electric vehicles (EVs) provide a solution to the problem of emission release from ICEVs. EVs require battery
recharging just as ICEVs require refuelling but unlike ICEVs, the EV charging time is greater than the 2–5 min it takes to refuel the ICEV. Furthermore, charging of an EV adds a small load on the grid when a single EV is charging. However, as most countries are now proposing a complete transition from their existing ICEVs to EVs, a large number of EVs charging in an unplanned manner at the same time (uncontrolled charging) might result in a larger load on the grid. A large number of EVs charging in an unplanned manner at the same time (uncontrolled charging) might result in a larger load on the grid. Generally, for residential scheduling of EVs to charge in a manner that results in minimal peak load, residents, service providers and the transportation network [3] in a microgrid, where EVs can be charged without having to subscribe to a charging station, an empty slot may not be available, in which case the EVs have to wait to be charged. Using the flexibility envelope strategy, the authors have demonstrated that their approach results in reduced waiting time for the EVs and an increase in the overall profit for the FCS. However, the problem is not considered on a large scale as this scheduling is attempted for one charging station only and not for a grid-scale situation. Even with the presence of renewable energy sources on the grid, the FCS may be charged with less stress on the grid. A number of analyses have been done to evaluate the benefits of the installation of rooftop solar generation at the workplace and public charging stations as in [6], where the potentially reduced carbon footprint of the renewable assisted charging station is also evaluated. With the implementation of PVCS at business premises, the authors in [11] present a PVCS that allows the management of up to 1000 EVs at a university where employees and students are able to access the facility. The research work evaluates the scheduling for five different PV profiles to show the ability of the PVCS. The authors in [12] present the charging management of a workplace PVCS that considers discharging from EV as an option. While classifying the EVs into different categories based on the flexibility in their discharging/charging parameters, the charging is attempted. It was observed that such a configuration allows for reducing the impact of the EV charging on the grid.

PVCSs may also be explored for PCS. The work presented in [13] attempts the scheduling at a PV assisted FCS that is a public charging station and services the EVs on a first come-first served basis. Even with the presence of renewable energy sources on the grid, the FCS may be charged with less stress on the grid. A number of analyses have been done to evaluate the benefits of the installation of rooftop solar generation at the workplace and public charging stations as in [6], where the potentially reduced carbon footprint of the renewable assisted charging station is also evaluated. With the implementation of PVCS at business premises, the authors in [11] present a PVCS that allows the management of up to 1000 EVs at a university where employees and students are able to access the facility. The research work evaluates the scheduling for five different PV profiles to show the ability of the PVCS. 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the BS provides an additional alternative source for charging EVs in the absence of availability of energy from PV [11, 18].

In [19], a model for EV charging management in a microgrid is developed where the efficiency of the PV assist is evaluated for scheduling a large number of scheduling requests. The schedule is found to be efficient in increasing the PV energy consumption and reducing the grid consumption during peak hours. Workplace PVCS is evaluated in [20] for different output conditions, where the PV output is low. The presented system in [20] attempts to manage the scheduling so that the average work hours coincide with the charging requests while considering a single charger. In [21], PV assist is utilised in the case of DG fluctuation and market price variations. The scheduling is implemented using Lyapunov optimisation and the overall operation costs observed a decrease with the inclusion of the PV assist in the scheduling.

Considering an FCS, García-Triviño et al. [22] develop a control strategy for a PVCS which chooses the source of EV charging by controlling the common medium-voltage DC (MVDC) bus. The scheduling is attempted for different operating conditions for a limited number of EVs, resulting in the proper operation of the FCS. The work presented by the authors in [18] attempts to schedule EVs using PV, battery or grid based on the availability of sufficient energy in a multiple aggregator scenario. This scheduling is attempted for greater than 500 EV requests to evaluate the capability of the PVCSs to accommodate a large number of EVs while optimising the profits of the aggregators. It was found that PV assistance resulted in lower consumption from the grid, contributing to greater profits for the aggregators. Other works such as [23], evaluate the performance of the PVCS in summer and wintertime for a workplace charging station. Even though the generation is less during winters, it was deemed sufficient for charging EVs.

Based on the survey of the existing literature on PVCSs, certain key issues may be ascertained: (i) The PVCS scheduling presented attempts to accommodate a limited number of EVs, which may not be realistic when considering a large-scale switch over from ICEVs to EVs. (ii) Centralised scheduling via a single aggregator has been implemented in the majority of the research, which leaves room to explore multi-aggregator non-collaborative/collaborative scheduling. (iii) Few research works consider the variation in PV profiles in multi-aggregator scheduling. (iv) To the best of the author's knowledge, mobility parameters of the EV driver have been considered by only a few research works. In other words, a comprehensive system that evaluates these factors and their impacts on the performance of the scheduling has not been considered.

The research work presented in this paper elaborates EV charging scheduling in a multi-aggregator collaborative scheduling environment with PV and battery aware requests. The requested time by the EV user determines the time slots where the EV may be scheduled. It is assumed that appropriate communication exists between all involved entities. Scheduling is considered at PVCSs with storage capabilities. The aggregator may schedule the EV to charge via the grid, PV or BS. The aim of the scheduling is to maximise the profits of the aggregator while evaluating the impacts of the PV assist on the grid energy consumption as well as the scheduling efficiency. As mentioned in [23], PV output varies with a change in the weather and seasons. This variation will also impact the utilisation of PV as a source of EV charging. In addition to PVCS scheduling, the impact of variation of the PV output is explored for four scenarios -- no PV in the system, maximum PV output, average PV output, minimum PV output. The impact of these variations on the grid energy consumption, along with profits, is also assessed. The presented work adds contributions to the existing literature on public FCS and PV assisted FCS in the following manners:

i. The research presents an evaluation of a multi-aggregator strategy while considering penalties on the aggregators for the EVs that remain unscheduled. The evaluation of the multi-aggregator scheduling is shown in the case of no-collaboration and collaboration between aggregators for charging stations with and without PV assistance. This evaluation is presented while considering the aggregator profits and the consumption of energy from the grid.

The evaluation for a varying number of possible scheduling requests is considered, providing an analysis of the limit of EVs that may be accommodated by the scheduling system in both the PVCSs and non-PVCS.

The primary objective of the scheduling remains to optimise the profits of the aggregators. However, the influence of the scheduling on the other stakeholders, i.e. the consumer and the grid, in terms of their respective benefits, are also evaluated to obtain a comprehensive impact analysis of the multi-aggregator collaborative scheduling with PV assistance. The benefits of the grid are evaluated in terms of the energy consumed from the grid and the benefits of the consumer are evaluated in terms of the potential discounts due to unscheduled EVs and collaborative scheduling.

To the best of researcher's knowledge, the multi-aggregator collaborative strategy has not been explored for the PVCSs.

The PV profile variation is an important aspect to consider as solar is a source of energy that not only fluctuates with change in seasons but may fluctuate on an every day/hourly basis. This results in a variation in the amount of energy that may be generated and utilised. The historical data on variation in the availability of solar radiation may be obtained through various web platforms such as [24]. It becomes important to evaluate the performance of the proposed system under varying solar conditions as it (i) defines the limitations of the PVCS and (ii) allows for a more optimal sizing consideration for the BS. Although the research work presented in this paper does not primarily focus on sizing, the evaluation of the PVCS for varying PV profiles provides an indication of the consideration for sizing.

The remainder of the paper is organised as follows: Section 2 presents the problem development for PVCS along with introduction of the unscheduled EV penalty; Section 3 presents the implementation of the scheduling algorithm; Section 4 describes the simulation conditions, network and parameters; Section 5 presents and discusses the results for the scheduling in different PV output conditions; and finally, Section 6 concludes the presented work.

2 PVCS system and problem formulation

The PVCS aims at enhancing the functioning of the EV charging stations while reducing the load on the grid. A list of variables that are used in the development of the objective is described in the Nomenclature section.

The PVCS problem is developed for the configuration where both PV and battery energy storage (BES) are available. At any PVCS, the system flow consists of an energy network flow and a communication flow. The energy flow and communication for a single aggregator at a specific charging station are shown in Fig. 1a. Each aggregator monitors the use of the different sources by which an EV may be charged. The system is designed so that primarily PV and BES are utilised for scheduling. If neither of those options is capable of satisfying the requirements of the EV, then the aggregator allows the respective energy amount to be fulfilled using the grid. The source that is chosen at the charging station is communicated to the aggregator so that the necessary updates may be made in terms of the total energy available to be utilised from the grid, and the respective PV and BES sources. It is to note that the PV and BES are localised, i.e. each charging station has its own PV and BES and, therefore, communicates with them directly. The PV energy ($e_{PV}$) may be utilised for charging the EV or for recharging the BS, in the case, it is below the minimum value ($e_{min} < e_{BSmin}$). Even though each charging station is equipped to obtain the required amount of energy from the grid, direct control on the amount of grid energy that may be utilised is ultimately decided by the aggregator. Here the aggregator may instead choose to schedule the EV through collaboration.

In addition to the localised control/communication at the charging station for PV/BES and that between the different charging stations and the aggregator, communication also exists
between the different aggregators. When considering collaborative scheduling, the aggregators share lists of unscheduled EVs amongst themselves so as to increase the chances of scheduling a greater number of EVs. The overall communication flow is shown in Fig. 1b. It is to note that the charging stations do not communicate with each other, but only respond to the potential scheduling ability for the particular EV that the respective aggregator is requesting to schedule.

The formulation of the charging scheduling problem at the PVCS is shown for the system mentioned in Fig. 1. The process of EV scheduling is carried out in distinct phases, where the first phase deals with the scheduling of EVs, followed by a collaboration between aggregators where needed, in the second phase. As a mobility aware system is considered, the EV user specifies the state-of-charge (SOC) at the time the request is made, the source and destination of the EV, the route to be taken, the desired SOC and the estimated time of departure from the source [5]. Based on these parameters, the aggregator attempts to schedule the EV first at its own charging station and then through collaboration if necessary. The following subsections detail the formulation of the objective for the scheduling while considering these parameters.

2.1 Non-collaborative and collaborative scheduling set-up

Initially, a system is considered that attempts to schedule the EVs with grid energy only, i.e. the charging stations are not equipped with PV/BES facilities. In a set of total \( G \) aggregators, the \( i \)th aggregator receives \( V_i \) charging requests to be scheduled at \( CS_i \) charging stations. Upon the receipt of the request, the aggregator determines the charging station(s) on the route of the EV and estimates the respective arrival times \( (t_a) \) at each of those charging stations. In addition, the estimated time of charging \( T_D \) and \( T_M \) are calculated on whether the EV is being charged to the desired SOC \( (SOC_D) \) or to the minimum SOC required to reach the destination \( (SOC_M) \). The respective energy requirements are calculated based on the charging rate and the values of \( T_D \) and \( T_M \) [5]

\[
\begin{align*}
T_D &= \frac{(SOC_D - SOC_M) \times batt_{cap}}{\rho} \\
T_M &= \frac{(SOC_M - SOC_{\min}) \times batt_{cap}}{\rho}
\end{align*}
\]

The respective energies are termed as \( en_{D} \) and \( en_{M} \) [5]. The revenue for charging any \( v \in V_i \) is calculated for the \( M \) time slots as follows:

\[
ev_v = \sum_{m=1}^{M} c_{ev,m} \times en_{D,m} = \sum_{m=1}^{M} c_{ev,m} \times en_{M}
\]

Each time slot is considered of a 15 min duration and \( en_{D,m} \) is the energy calculated for each slot on \( en_{D} \) and \( en_{M} \). The profit for the \( i \)th aggregator \( (P_{ag,i}) \) in non-collaborative (NC) scheduling, may be defined by the sum of revenues from all \( v \in V_i \) and by the amount paid for purchasing the total energy that is consumed from the grid to recharge the EVs

\[
P_{ag,i} = \sum_{v \in V_i, v \in CS_i} ev_v - cost_{en}
\]

where \( cost_{en} \) is the cost of purchasing energy from the grid and \( ev_v \) is the revenue generated by the scheduling of all EVs. For this scenario, no contribution of PV/battery is included in the scheduling and this is called the \( no PV \) scheduling or the \( no PV \) scenario.

In collaborative (C) scheduling, the aggregators attempt to schedule the EVs of another aggregator at their own charging station. As an incentive for collaboration, \( G_i \) provides a portion of the revenue received from the \( v \in V_i \) to \( G_j \) for scheduling the EV at \( c \in CS_j \). The revenue given and received is termed as \( ags_j \) and \( ags_i \), respectively. The aggregator determines which charging stations are close to the route of the EV and attempts to collaborate with the respective aggregator for possible scheduling. The collaborative profit is calculated in [5] and is given as

\[
P_{agC,i} = \sum_{v \in V_i, v \in CS_i} ev_v + S \times \sum_{v \in V_i, v \in CS_i} ags_j - C \times \sum_{v \in V_i, v \in CS_i} ev_{dr,j} - cost_{en}
\]

The customer is given a discounted charging rate \( (en_{rd}) \) to accommodate the potential inconvenience of charging at a station that is out of its route. In both NC and C scheduling here, the total energy consumed \( (en_{tot}) \) is consumed from the grid, i.e. \( en_{tot} = en_{g} \).

2.2 PV assistance at EV charging station

With the inclusion of PV and BS at the charging station, there now exists an option to utilise a source other than the grid to charge the EV. In the case of PVCS, each \( v \in V_i \) may be scheduled to charge either by PV energy, BS or by the grid. A number of EVs scheduled to charge using grid energy \( (en_{g}) \) are represented by \( V_{1,i} \in V_i \). Similarly, let \( V_{2,i} \in V_i \) and \( V_{3,i} \in V_i \) represent the total number of EVs that are scheduled through PV \( (en_{pv}) \) and the battery \( (en_{bat}) \), respectively. It is to note that \( V_{1,i} + V_{2,i} + V_{3,i} = V_{i} \) which is the total number of EVs that are scheduled. In all cases where \( v \in V_{2,i} \) or \( v \in V_{3,i} \), then, the amount of energy required by the respective EVs will not be taken as a load on the grid. This implies that the total energy consumed from the grid will be less as compared to the \( no PV \) scheduling, resulting in lower costs. The total energy used in charging all EVs is given as \( en_{tot} \). In the case of PVCS, the energy consumed from the grid is defined as \( en_{g} \) and may be related with \( en_{pv} \) and \( en_{bat} \) as follows:

\[
en_{pv} = en_{tot} - en_{g} - en_{bat}
\]
where the use of $en_{gpv}$ and $en_{batt}$ results in reduced use of the grid energy as compared with the no PV, i.e. $en_{gpv} < en_{c}$. With PV assistance in consideration, the profit of the $i$th aggregator is modified from (3) as follows [18]: (see (6)). In this work, the cost of charging does not vary based on the source of charging for the EV user. Therefore, when there is a reduction in the use of the energy from the grid, a consequent decrease is observed in the cost without any decrease in the revenue for that EV. Collaborative scheduling in the case of PVCS works in the same manner and the profit equation (4) is modified as follows [18]:

$$P_{agCPV,i} = S(1 - C) \sum_{v \in V_i, c \in CS} ev_{t,v} - S(1 - C) \sum_{v \in V_i, c \in CS} en_{gpv} \cdot c_{en}$$

$$= \sum_{v \in V_i, c \in CS} en_{gpv} \cdot c_{en} + SC \sum_{v \in V_i, c \in CS} ev_{dr,i} + SC \sum_{v \in V_i, c \in CS} ev_{t,v}$$

The profit equations (6) and (7) are focused only on the EVs that are scheduled by the aggregator(s). EVs that remain unscheduled in both NC/C scheduling are not accommodated in these equations. The inclusion of unscheduled EVs is discussed in the next section.

### 2.3 Unscheduled penalty

In order to accommodate feedback for the EVs that are unscheduled, a monetary penalty is introduced in the profit of the aggregators. The penalty levied on the aggregator is a percentage of the cost in the slot where the EV was to be scheduled with respect to the peak cost during the day [25]. Other penalty functions such as a quadratic or logarithmic function may also be used to model the penalty function for unscheduled EVs, however, the function in [25] offers a direct relation with the cost at the time of consideration. The penalty $u_{n_p}$ is calculated as follows:

$$u_{n_p} = \sum_{m=1}^{M} s_{m,m} \cdot c_{ev,m}$$

(8)

Here $s_{m,m}$ is a product of $s_{m}$ and a constant that represents the amount of energy that is to be considered for the penalty ($c_{ev,m}$). The profit equations are modified to include the penalty for unscheduled EVs. The equations are modified as follows:

$$P_{agCPV,i} = P_{agNC,i} - \sum_{v \in V_m} en_{gpv} \cdot c_{en} + SC \sum_{v \in V_i, c \in CS} ev_{t,v}$$

$$- \sum_{v \in V_i, c \in CS} en_{gpv} \cdot c_{en} - \sum_{v \in V_m} en_{gpv} \cdot c_{en}$$

(9)

The collaborative profit equation is modified as

$$P_{agNC,i} = P_{agC,i} - \sum_{v \in V_m} u_{n_p}$$

(10)

Though the inclusion of the penalty should result in lesser profits for the aggregators, it reflects the feedback of the customers. The penalty $u_{n_p}$ will be included on profit equations for the PVCS as well.

### 2.4 Objective function

The objective is to maximise the total profits of the aggregators. The total profit ($P_i$) is the sum of the individual aggregator profits. The objective of the work may be defined as follows:

$$\text{maximise } P_i = \sum_{j \in i} P_{ag,j}$$

(11)

Subject to the following constraints:

$$en_{en} \leq en_{req} \leq en_{en}$$

(12a)

$$T_M \leq T_{charge} \leq T_D$$

(12b)

$$t_a \leq t_b \leq t_a + t_{slot}$$

(12c)

$$m_i - m_i \geq m_{eq}$$

(12d)

$$en_{ch} + en_{en,1} \leq en_{en}$$

(12e)

$$en_{ch} \leq en_{en}$$

(12f)

$$en_{ch} \leq en_{bh}$$

(12g)

As in (12b), the required energy to recharge the EV must be between the maximum and minimum energies required to achieve SOC$_{D}$ and SOC$_{A}$, respectively. Constraint (12d) ensures that the EV charging for a $v \in V_i$ begins within a certain duration post arrival at the charging station. This time is taken as 15 min (duration of a time slot). Constraint (12f) limits the use of grid energy for charging to be less than the peak limit $en_{p}$ of the hour. Constraints (12f) and (12g) ensure that $v \in V_i$ is charged with PV or battery based on the availability of sufficient energy. The scheduling problem is discrete in nature, where mixed integer non-linear programming (MINLP) is employed.

### 3 Methodology

A distributed algorithm is used for scheduling the EVs, which is performed in distinct phases [5]. An overview of the scheduling process is shown in Fig. 2.a.

The first part of the scheduling is done by the aggregator $G$, for $v \in V_i, i.e. vehicles subscribed under it. A list of unscheduled EVs is obtained at the end of this scheduling. Similarly, the other aggregators attempt to schedule their respective EVs and obtain their lists of unscheduled EVs. The next phase is the collaborative section of the scheduling, where the aggregators share the lists of unscheduled EVs and attempt to schedule EVs subscribed to another aggregator at their own charging stations. It is to observe from Fig. 2a that the NC and C profits and unscheduled EVs are calculated separately. However, the unscheduled EVs from the NC stage are sent to Phase 2. Both NC and C scheduling have Phase 1 whereas Phase 2 is only observed in collaborative scheduling. Fig. 2b and 2c detail the scheduling process in the first phase of the scheduling where the PV assist has been included.

$$P_{agCPV,i} = S \sum_{v \in V_i, c \in CS} ev_{t,v} - S \sum_{v \in V_i, c \in CS} en_{gpv}$$

$$= S \sum_{v \in V_i, c \in CS} ev_{t,v} + S \sum_{v \in V_{2,i}, c \in CS} ev_{t,v}$$

$$- S \sum_{v \in V_{1,i}, c \in CS} ev_{t,v}$$

(6)
If condition (i) is satisfied, then the appropriate source of energy for charging the EV is selected (condition (ii)). The priority is given to scheduling using PV. If PV is not available, then the EV is attempted to be charged using the battery. If the battery power is also not sufficient to charge the EV, then the EV is attempted to be scheduled from the grid. A limit exists on the total amount of energy that may be consumed from the grid. In this scheduling, in order to show a clear variation in the profits when compared to the PV assisted scheduling, a maximum number of scheduling requests have been considered such that the requests may be accommodated by available grid energy.

The scheduling algorithm checks to see if \( e_{ch} < e_{pv} \) for the required time slots. If not, then it checks if \( e_{ch} < e_{batt} \). If this is not available, then it is checked \( e_{ch} < e_{g} \). If \( e_{batt} < e_{batt\_min} \), then the battery is scheduled to be charged for that time slot. The battery is primarily charged using the PV output. When scheduling the battery to be charged, it is checked to see if either 100, 50, 10 or 5% capacity may be charged by the PV.

Depending on the time at which the EV request is coming in, the battery availability will be variable. Therefore, the scheduling algorithm ensures that the correct value of the battery energy available is used for \( e_{batt} \).

### 4 Simulation parameters

Solar radiation profile will vary based on location and season [26]. The variation in solar radiation would consequently impact the output of the PV system, thereby resulting in a variation in the availability of energy for charging EVs. Solar radiation profile has been taken for the city of Jaipur (26.9124°N, 75.7873°E) from [24] and the maximum, average and minimum energy outputs are shown in Fig. 3a.

For the simulation of the PVCS and its comparison with the non-PVCS, a different number of total charging requests that have to be managed for charging in one scheduling period have been considered and their performance has been evaluated. Each charging slot is of a 15 min duration. The maximum battery capacity is taken to be 30 kWh. The initial parameters that are communicated by the EV owner to the aggregator are given in Table 1. Here \( D \) is the maximum distance where a charging station may be considered for collaborative scheduling. The cost of purchasing energy is obtained from [27] and is shown for 3 August 2018 in Fig. 3b. It is to note that whether the EV is charged by the grid or by the PV, the cost of charging is followed, as shown in Fig. 3b. In the case with no PV/battery assist, the aggregator has maximum energy, which it may purchase from the grid. Day-ahead scheduling has been considered for a 24-h scheduling period.

The network under consideration is shown in Fig. 3c. In this network, the source and destination nodes are chosen in a manner so as to ensure that a minimum of one charging station falls on the route of the EV. Nodes 1–15 represent charging stations and the remaining nodes (16–30) are considered as source/destinations. It is to note that the charging station nodes may not be considered as either source or destination.

### 5 Results and discussion

The performance of the PVCS is evaluated in terms of the total profit of the aggregators, the impact on the number of scheduled EVs, and the energy consumption from PV and the grid. The results are presented for the base case (without PV/BS), and for variable PV outputs. The scheduling has been attempted for a variable number of EVs where each aggregator has a total of five EVs. Depending on the time at which the EV request is coming in, the battery availability will be variable. Therefore, the scheduling algorithm ensures that the correct value of the battery energy available is used for \( e_{batt} \).

![Flow of PV scheduling](image)

**Fig. 2** Flow of PV scheduling (a) Phases with C and NC scheduling, (b) Detailed phase 1 scheduling, (c) Battery charging
The number of EV requests is varied to see how the network responds with respect to each of the four cases. In the results, the number of EVs scheduled from the grid, PV and BS may be different in each case. As both these conditions act as limiting factors, a difference in the number of EVs scheduled was not observed for Scenarios 1–4. In scheduling where PV assistance is being used, the number of EVs scheduled from the grid, PV and the battery may be reduced, depending on the grid consumption. The profits in Scenario 4 are the lowest in comparison to the maximum PV output, resulting in greater profits. Overall, in the presence of PV variation, the EVs are able to be scheduled by through PV or through the battery. The battery may only be charged via PV. The exception is when the PV output is not sufficient to charge the EVs. However, it is possible to charge the battery, which is able to then charge EVs. The variation in the PV output plays a significant role in the scheduling of EVs. The profits in Scenario 4 are presented in Table 7 for cases 1–4. The profits in Scenario 4 are the lowest in comparison to the maximum PV output. The solar profile from Fig. 3a for ‘Max PV’ is considered. Table 2 further shows the cases the percentage EV scheduled and consumed by the grid, PV and the battery. Table 4 provides a comparison of the profit that is observed in Scenario 1 for all cases. From Fig. 3, it is observed that a majority of the scheduled EV load was taken by PV and battery for each of the total number of requests presented. Since the number of charging points, time slots and energy are limited, for the higher number of requests, a greater number of EVs remain unscheduled.

5.1 Scenario 1 – maximum PV output

The solar profile from Fig. 3a for ‘Max PV’ is considered. Table 2 further shows the cases the percentage EV scheduled and consumed by the grid, PV and the battery. Table 4 provides a comparison of the profit that is observed in Scenario 1 for all cases. From Fig. 3, it is observed that a majority of the scheduled EV load was taken by PV and battery for each of the total number of requests presented. Since the number of charging points, time slots and energy are limited, for the higher number of requests, a greater number of EVs remain unscheduled.

5.2 Scenario 2 – average PV output

The solar profile given in Fig. 3a for ‘Avg PV’ is considered. Table 2 further shows the cases the percentage EV scheduled and consumed by the grid, PV and the battery. Table 4 provides a comparison of the profit that is observed in Scenario 1 for all cases. From Fig. 3, it is observed that a majority of the scheduled EV load was taken by PV and battery for each of the total number of requests presented. Since the number of charging points, time slots and energy are limited, for the higher number of requests, a greater number of EVs remain unscheduled.

5.3 Scenario 3 – minimum PV output

Minimum PV output is observed usually in the winter. The solar profile from Fig. 3a for ‘Min PV’ is considered. The PV output, though minimum, may still be utilised for charging the BS. Fig. 7 shows the output of the percentage EVs scheduled and load utilised from the grid, PV and battery. Table 6 shows the profits in Scenario 3 for cases 1–4. It can be seen from Fig. 7, that no EVs are scheduled using PV as the PV output is not sufficient to charge the EVs. However, it is used to charge the battery, which is able to then charge EVs. The profits may be seen to be the least in comparison to the maximum PV and average PV outputs.

5.4 Scenario 4 – no PV

In Scenario 4, no PV or battery assistance is considered for scheduling. This implies that all of the EVs that are scheduled, are dependent on the grid. The profits in Scenario 4 are presented in Table 7 for cases 1–4. The profits in Scenario 4 are the lowest in the case where a higher number of requests are considered and the unscheduled EV penalty is also implemented.

5.5 Impact of PV variation

From the data in Tables 4–7, it may be clearly observed that the variation in the PV output plays a significant role in the implementation of the scheduling using the PVCS. The consumption of grid energy decreases with an increase in PV output, resulting in greater profits. Overall, in the presence of PV output, the EVs are able to be scheduled by through PV or through a battery. The battery may only be charged via PV.
to this rule occurs when the battery is charged at the end of the scheduling period, in which case it is charged by the grid. This is included in the consumption of the grid energy calculations. This allows for the use of a fully charged storage available at the start of the scheduling period. A slot-wise scheduling comparison for all scenarios is shown in Fig. 8.

Comparing Fig. 8 with the solar profiles in Fig. 3a, the performance of the scheduling with respect to the PV output may be observed. In Fig. 8d, Scenario 4 is shown in the case where all EVs are scheduled by the grid. In comparison, from Fig. 8a for maximum PV output, it may be observed that during the availability of the PV output, the majority of the EVs are scheduled

### Table 3 Number of requests and EVs scheduled in the C and NC cases

| Requests | NC | C |
|----------|----|---|
| 235      | 140| 156 |
| 599      | 359| 397 |
| 942      | 568| 602 |
| 1433     | 822| 856 |
| 2380     | 1156| 1177|
| 3376     | 1335| 1354|
| 3867     | 1396| 1413|
| 4612     | 1479| 1489|

### Table 4 Scenario 1 – Profit

| Req. | Case 1 Profit(INR) | Case 2 Profit(INR) | Case 3 Profit(INR) | Case 4 Profit(INR) |
|------|--------------------|--------------------|--------------------|--------------------|
| 235  | 8269               | 7795               | 9456.6             | 9060.4             |
| 490  | 13,763             | 12,760             | 15,367             | 14,539             |
| 599  | 16,690             | 15,351             | 18,321             | 17,235             |
| 942  | 22,255             | 20,049             | 23,518             | 21,500             |
| 1433 | 29,244             | 25,896             | 30,379             | 27,183             |
| 2380 | 37,582             | 30,161             | 38,299             | 30,978             |
| 3376 | 43,931             | 31,216             | 44,504             | 31,894             |
| 3867 | 46,084             | 30,685             | 46,627             | 31,317             |
| 4232 | 47,564             | 30,504             | 48,011             | 31,021             |
| 4612 | 47,214             | 27,903             | 47,531             | 28,274             |

### Fig. 5 Max PV

(a), (b) % EVs scheduled in NC and C, (c), (d) % EV load taken by PV, grid and battery in NC and C

### Fig. 6 Avg PV

(a), (b) % EVs scheduled by grid, PV and battery, (c), (d) % EV load and taken by PV, grid and battery

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using $en_{PV}$. If PV is not available, then $en_{batt}$ is considered. In Fig. 8b, even though lesser PV output is available (Fig. 3a), the EVs are primarily scheduled for using PV energy, when available. In the absence of sufficient PV output in Fig. 8c, the EVs are scheduled by the grid and the battery. In all of these cases, it may be observed that the PV and battery assist aids in reducing the consumption from the grid to varying degrees. The average energy consumption from the grid, PV and BS is shown in Fig. 9. This is

### Table 5  Scenario 2 – Profit

| Req. | Case 1 | Case 2 | Case 3 | Case 4 |
|------|--------|--------|--------|--------|
| 235  | 8269   | 7795   | 9456.6 | 9060.4 |
| 490  | 12,834 | 11,832 | 14,439 | 13,610 |
| 599  | 15,288 | 13,948 | 16,918 | 15,832 |
| 942  | 18,942 | 16,735 | 20,204 | 18,186 |
| 1433 | 24,047 | 20,699 | 25,182 | 21,986 |
| 2380 | 31,585 | 24,164 | 32,303 | 24,982 |
| 3376 | 34,381 | 21,666 | 34,954 | 22,344 |
| 3867 | 35,793 | 20,394 | 36,336 | 21,026 |
| 4232 | 39,870 | 22,810 | 40,317 | 23,327 |
| 4612 | 37,321 | 18,010 | 37,637 | 18,380 |

### Table 6  Scenario 3 – Profit

| Req. | Case 1 | Case 2 | Case 3 | Case 4 |
|------|--------|--------|--------|--------|
| 235  | 8269   | 7795   | 9456.6 | 9060.4 |
| 490  | 11,603 | 10,601 | 13,208 | 12,380 |
| 599  | 12,380 | 11,041 | 14,011 | 12,925 |
| 942  | 14,479 | 12,273 | 15,741 | 13,723 |
| 1433 | 18,880 | 15,532 | 20,015 | 16,820 |
| 2380 | 25,906 | 18,485 | 26,624 | 19,303 |
| 3376 | 25,605 | 12,890 | 26,178 | 13,569 |
| 3867 | 27,606 | 12,206 | 28,148 | 12,839 |
| 4232 | 30,013 | 12,953 | 30,013 | 12,953 |
| 4612 | 27,712 | 8401   | 28,029 | 8772  |

### Table 7  Scenario 4 – Profit

| Req. | Case 1 | Case 2 | Case 3 | Case 4 |
|------|--------|--------|--------|--------|
| 235  | 1769   | 1295   | 6106.7 | 5710.5 |
| 490  | 3967.1 | 2965   | 7593.9 | 6765.1 |
| 599  | 4720.5 | 3381.2 | 7330.7 | 6213.5 |
| 942  | 7588.9 | 5382.6 | 9786.6 | 7769  |
| 1433 | 11,432 | 8084   | 13,463 | 10,276 |
| 2380 | 15,817 | 8396   | 16,548 | 9262  |
| 3376 | 18,872 | 6158   | 19,540 | 6934  |
| 3867 | 19,217 | 3817   | 19,782 | 4486  |
| 4232 | 20,253 | 3192   | 20,699 | 3707  |
| 4612 | 20,157 | 846    | 20,486 | 1245  |
shown for each 15 min time slot. This is further illustrated by a direct comparison of the % EV scheduled in Scenarios 1–3 for NC and C cases in Tables 8 and 9, respectively.

From these tables, as the PV output decreases, the number of EVs scheduled from PV decreases. However, the BS may be charged from the PV energy. This is seen in Scenarios 2 and 3, where the % of EVs scheduled from PV are less, but the contribution of the BS is significantly greater as compared with that of Scenario 1. This implies that even though the PV energy may not be able to provide sufficient energy for charging the EV, it provides enough to charge the BS.

As discussed in Section 1 of this paper, the variation in PV profile and the consequent simulations for scheduling in various conditions, elaborate the requirements for the sizing of the PV and the BS. As the results indicated, the least amount of PV generation in the current configuration results in no EVs being accommodated directly by PV (Fig. 9c). However, the PV energy is utilised in charging the BS. Based on these simulations, a few conclusions may be drawn regarding the considerations for the sizing of the BS.

• The PV and battery sizing require the consideration of the maximum number of EVs that are to be serviced at the charging stations under the aggregators.
• Based on the lowest amount of PV generation observed, it becomes crucial to have a greater BS so that it is possible to schedule the EVs such that they draw limited power from the grid.

5.5.1 Profits: From Tables 4–7, a comparison is drawn to evaluate the profits in each case with respect to all scenarios. This is presented in Fig. 10. The unscheduled EV penalty results in an overall decrease in the profits for NC and C scheduling. From Fig. 10, the NP profits in all scenarios are higher as compared with...
collaborative scheduling between aggregators along with PV/battery assistance for energy supply is implemented. Charging stations are equipped with the capability to utilise the available PV energy and/or switch to the grid when required. Simulations are conducted for a larger number of scheduling requests in day-ahead scheduling to explore the impact of PV and battery assist on the amount of energy consumed from the grid, and the total profits of the aggregator. Furthermore, the impact of the inclusion of penalties for unscheduled EVs is included. As solar profiles observe a variation in the output due to seasons/weather, simulations have been carried out to evaluate the relationship between the PV profile variation and the profits. The presented results highlight reduced profits in case of inclusion of penalties for unscheduled EVs. Further, the variation in PV profiles results in an inverse impact on the amount of energy consumed from the grid. As a result, the profits observe an increase as the PV energy output increases.

Further extensions of this work may include the evaluation of a system that considers a greater number of EV requests at charging stations with larger charging infrastructure. Variable charging rates may also be explored in the case of PVCSs. An analysis of variation in aggregator profits may also be considered while implementing different penalty functions.

7 References

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8 Appendix

8.1 Proof of concavity of the discrete profit function

The CV charge scheduling problem while considering the profits of the aggregator in multi-aggregator collaborative scheduling at a PVCS is considered here with the objective given by (11), i.e. the research work attempts to maximise the total profits of the aggregators. The profit for each aggregator is given by (3), (4) and (6), (7) in this paper for no PV and PV conditions, respectively.

When considering the total profits, i.e. the scheduling/non-scheduling of all subscribed EVs, it can be seen that the problem is discrete in nature as the scheduling occurs at distinct times that are not connected for two vehicles. This implies that the profit calculation is also discrete at this instant. In this case, since a maximisation formulation has been considered, the proof of concavity is discussed here.

As the given function is discrete, the concavity is proven by establishing that the second-order difference matrix is semi-negative definite. The profit equation (3) along with the inclusion of the unscheduled penalty is given as

\[
Pr_{\text{ScS}}(cst, en) = S \sum_{v \in V} \sum_{c_i \in c_{\text{im}}} \sum_{m = 1}^{M} cst_{im} \ast en_{m} - \text{cost}_{en} - (1 - S) \sum_{v \in \tilde{V}} \sum_{c_i \in c_{\text{im}}} \sum_{m = 1}^{M} (cst_{im} \ast en_{m}) \ast \frac{cst_{im}}{\max} + \text{en}_{m}
\]  

(13)

From this point, the Pr is used instead of PrScS. For this equation, the finite second-order difference matrix is developed as follows:

\[
H = \begin{bmatrix}
Pr_{\text{cst, cst}} & Pr_{\text{cst, en}} \\
Pr_{\text{cst, en}} & Pr_{\text{en, en}}
\end{bmatrix}
\]  

(14)

Considering that the second difference for any discrete function involves the evaluation at ±h from cst and at ±k from en. It is to note that both values, handk are very small and represent a minor change in the discrete values.

The values of each term in (14) are solved by the following set of expansions for finite differences in multiple variables:

\[
Pr_{\text{cst, cst}} = \frac{\Pr (cst + h, en) - \Pr (cst - h, en)}{2h}
\]  

(15)

\[
Pr_{\text{cst, en}} = \frac{\Pr (cst, en + k) - \Pr (cst, en - k)}{2k}
\]  

(16)

\[
Pr_{\text{en, en}} = \frac{\Pr (en + k, en + k) - \Pr (en - k, en - k)}{4hk}
\]  

(17)

where (17) is given as

\[
\frac{Pr_{\text{cst, en}}}{4hk} = \frac{\Pr (cst + h, en + k) - \Pr (cst + h, en - k) - \Pr (cst - h, en + k) + \Pr (cst - h, en - k)}{4hk}
\]  

(18)

Evaluating (13) for the values given in (14) by using (15)–(18). On solving these, the following expressions are obtained for the functions:

\[
Pr_{\text{cst, en}} = \frac{-2 \ast (1 - S)cst^2}{cmax}
\]  

(19)

\[
Pr_{\text{en, en}} = \frac{-2 \ast (1 - S)cst^2}{cmax}
\]  

(20)

\[
Pr_{\text{cst, en}} = \frac{4hk\left(-2S\ast cst\ast en + 5.6en - \frac{2(1 - S)cst^2}{cmax}\right)}{8(1 - S)cst^2 \ast en + h \ast k}
\]  

(21)

Using these values in (14), and calculating the determinant, the following reduced values are obtained: (see (22)) . On further solving this equation and evaluating for the two conditions of S, i.e. S = 1 and S = 0, the following results are obtained.

8.1.1 S = 1, i.e. v \in V, is scheduled: In this case, in (22), on including the value of S, it can be seen that the equation reduces to

\[
H_{\text{ScS}}^{S = 1} = \frac{1}{16h^2k^2}
\]  

22Acst + en^2 - 4 \ast cst^2en^2 - 31.36en^2
\]  

(22)

It may be observed that \forall cst, en > 0, the negative terms will dominate ⇒ |H_{ScS}|_{S = 1} < 0, which indicates that the problem is concave.

8.1.2 S = 0, i.e. v \in V was not scheduled: Evaluating (22) for S = 0, the following is obtained:

\[
H_{\text{ScS}}^{S = 0} = \frac{4en^2cst^2}{cmax} + 22.4en^2h^2k^2 + \frac{89.6cst \ast en \ast hk}{cmax \ast (16h^2k^2)} - \frac{4hk^2}{cmax \ast (16h^2k^2)}
\]  

+ \frac{32 \ast cst \ast h \ast k}{cmax^2 \ast (16h^2k^2)} - \frac{64cst^2en^2h^2k^2}{cmax^2 \ast (16h^2k^2)}
\]  

\Rightarrow |H_{ScS}|_{S = 0} = \frac{4en^2cst^2}{cmax} + 22.4en^2 \ast (16h^2k^2)
\]  

- \frac{89.6cst \ast en^2}{cmax \ast (16h^2k^2)} - \frac{4hk^2}{cmax \ast (16k^2)}
\]  

+ \frac{32 \ast cst \ast h \ast k}{cmax^2 \ast 16} - \frac{64cst^2en^2}{cmax^2 \ast 16}
\]  

(24)

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\[ H_{\text{NC}} k = - \frac{1.96en^2}{(hk)^2} + \frac{1.4en}{c_{\text{max}}} - \frac{5.6c_{\text{st}} * en^2}{c_{\text{max}} * (hk)} \]

As the values of \( h, k \) are small, their higher-order terms may be neglected. Again, considering the equation \( \forall c_{\text{st}}, en > 0 \) it may be seen that the negative terms will dominate the outcomes and, therefore \( H_{\text{NC}} k = 0 < 0 \), implying the concavity of the function.

With this analysis, the concavity of the initial profit equation with the inclusion of the unscheduled penalty is proven. A similar analysis may be conducted to obtain the concavity of the objective functions in collaborative scheduling. While considering the inclusion of PV at the charging station along with BS, modifications are made to the term that evaluates the cost of purchasing the energy from the grid. While considering the utilisation of PV and/or battery for recharging of \( v \in V_i \), the revenue scheme, i.e. charging cost calculation, remains the same as that for when no PV/BS is considered. Therefore, the concavity may be proven in a similar manner as is presented here for \( P_{\text{NC}} \).