Smoothed Particle Magnetohydrodynamics: Some shocking results...

D.J. Price¹, J.J. Monaghan²
¹Institute of Astronomy, Madingley Rd, Cambridge, CB3 0HA, UK
²School of Mathematical Sciences, Monash University, Clayton 3800, Australia

Abstract. There have been some issues in the past in attempts to simulate magnetic fields using the Smoothed Particle Hydrodynamics (SPH) method. SPH is well suited to star formation problems because of its Lagrangian nature. We present new, stable and conservative methods for magnetohydrodynamics (MHD) in SPH and present numerical tests on both waves and shocks in one dimension to show that it gives robust and accurate results.

Keywords: numerical methods, magnetohydrodynamics (MHD)

1. Introduction

Smoothed Particle Hydrodynamics (SPH) is a unique method of solving the equations of gas dynamics in that it involves no spatial grid. For this reason SPH is ideally suited to star formation type problems. However there have been some issues in the past with attempts to simulate magnetic fields in SPH, namely that when a conservative formulation of the momentum equation was used an instability developed which causes particles to clump together unphysically (Phillips and Monaghan, 1985).

We resolve this instability by adding a short range repulsive force to prevent particles from clumping (Monaghan, 2000). In addition we formulate the equations of Smoothed Particle Magnetohydrodynamics (SPMHD) using the continuum equations of Janhunen (2000) and Dellar (2001) which are consistent even when the divergence of the magnetic field is non zero. Consequently, even though non zero \( \nabla \cdot \mathbf{B} \) may be produced during the simulation, it is treated consistently.

Shocks are captured within SPMHD by use of artificial dissipation terms. The formulation of these terms follows naturally by requiring dissipation in the total energy across a shock front and then demanding that these terms result in positive definite changes to the entropy. The artificial dissipation is effectively turned off away from shocks by use of the switch proposed by Morris and Monaghan (1997).

The resulting equations, when implemented with a simple predictor corrector scheme for the timestepping, give good results for a wide range of shock tube problems (Price and Monaghan, 2003a; Price and Mon-
While we have yet to apply our algorithm to problems in two and three dimensions the present results encourage us to believe that our SPMHD code will provide a secure basis for astrophysical MHD problems.

2. Numerical tests in one dimension

The numerical scheme has been tested on a wide variety of one dimensional problems. We present results here on three standard tests which have been used to test many grid-based astrophysical MHD codes (e.g. Stone et al., 1992; Dai and Woodward, 1994; Ryu and Jones, 1995; Balsara, 1998). For one dimensional MHD problems the magnetic field and velocity are allowed to vary in three dimensions.

The first test involves a fast MHD wave propagating in a periodic 1D domain and is taken from Dai and Woodward (1998). The wave is evolved for 10 periods, corresponding to 10 crossings of the domain. The velocity profile in the SPMHD solution is shown in Figure 1, at resolutions of 32, 64, 128, 256 and 512 particles (solid points indicate the particles). Note that the numerical solutions are in phase with the initial conditions (dashed line), demonstrating that the speed at which the wave travels is correctly captured by our method. This improvement over previous SPH results (e.g. Morris, 1996) is obtained by allowing the smoothing length to vary with local particle density and deriving self-consistently (via a Lagrangian variational principle) the additional terms which should therefore be included in the SPH (Springel and Hernquist, 2002) and correspondingly the SPMHD (Price and Monaghan, 2003b) equations.

The second test (Figure 2) was first described by Brio and Wu (1988) and is the MHD analog of the Sod (1978) shock tube problem. The problem consists of a discontinuity in pressure, density, transverse magnetic field and internal energy initially located at the origin. As time develops complex shock structures develop which only occur in MHD because of the different wave types. The results shown compare well with the solution computed by Balsara (1998) using a grid-based MHD code incorporating a Riemann solver, indicated by the solid lines.

The third example is a test of the code for isothermal MHD and is compared with the solution computed by Balsara (1998) in Figure 3. The problem illustrates the formation of six discontinuities in the same problem.
Figure 1. A travelling MHD fast wave propagating in a periodic 1D domain. Initial conditions are indicated by the dashed line whilst the SPMHD solution is given by the solid points after 10 periods (corresponding to 10 crossings of the domain), at resolutions of 32, 64, 128, 256 and 512 particles. The numerical solutions are in phase with the initial conditions, demonstrating that the speed at which the wave travels is correctly captured by our method. There is a small amount of steepening present due to non-linear effects, which is in accordance with the solution presented in Dai and Woodward (1998).

3. Future work

We hope to apply the algorithm to a large simulation of star formation similar to that performed by Bate et al. (2003), including the effects of magnetic fields. This should provide significant insight into the role that magnetic fields play in the star formation process.

Acknowledgements

DJP acknowledges the support of the Association of Commonwealth Universities and the Cambridge Commonwealth Trust. He is supported by a Commonwealth Scholarship and Fellowship Plan. We also thank the referee, Enrique Vazquez-Semadeni, for useful suggestions which have helped to improve this paper.
References

Balsara, D. S.: 1998, ‘Total Variation Diminishing Scheme for Adiabatic and Isothermal Magnetohydrodynamics’. ApJs 116, 133–+.
Bate, M. R., I. A. Bonnell, and V. Bromm: 2003, ‘The formation of a star cluster: predicting the properties of stars and brown dwarfs’. MNRAS 339, 577–599.
Brio, M. and C. C. Wu: 1988, ‘An upwind differencing scheme for the equations of ideal magnetohydrodynamics’. J. Comp. Phys. 75, 400–422.
Dai, W. and P. R. Woodward: 1994, ‘Extension of the Piecewise Parabolic Method to Multidimensional Ideal Magnetohydrodynamics’. J. Comp. Phys. 115, 485–514.
Dai, W. and P. R. Woodward: 1998, ‘On the Divergence-free Condition and Conservation Laws in Numerical Simulations for Supersonic Magnetohydrodynamic Flows’. ApJ 494, 317–+.
Dellar, P. J.: 2001, ‘A Note on Magnetic Monopoles and the One-Dimensional MHD Riemann Problem’. J. Comp. Phys. 172, 392–398.
Janhunen, P.: 2000, ‘A Positive Conservative Method for Magnetohydrodynamics Based on HLL and Roe Methods’. J. Comp. Phys. 160, 649–661.
Monaghan, J. J.: 2000, ‘SPH without a Tensile Instability’. J. Comp. Phys. 159, 290–311.
Morris, J. P.: 1996, ‘Analysis of smoothed particle hydrodynamics with applications’. Ph.D. thesis, Monash University, Melbourne, Australia.
Morris, J. P. and J. J. Monaghan: 1997, ‘A switch to reduce SPH viscosity’. J. Comp. Phys. 136, 41–50.
Phillips, G. J. and J. J. Monaghan: 1985, ‘A numerical method for three-dimensional simulations of collapsing, isothermal, magnetic gas clouds’. MNRAS 216, 883–895.
Price, D. J. and J. J. Monaghan: 2003a, ‘Smoothed Particle Magnetohydrodynamics I. Algorithms and tests in one dimension’. MNRAS, submitted.
Price, D. J. and J. J. Monaghan: 2003b, ‘Smoothed Particle Magnetohydrodynamics II. Variational principles and variable smoothing length terms’. MNRAS, submitted.
Ryu, D. and T. W. Jones: 1995, ‘Numerical magnetohydrodynamics in astrophysics: Algorithm and tests for one-dimensional flow’. ApJ 442, 228–258.
Sod, G. A.: 1978, ‘A Survey of Several Finite Difference Methods for Systems of Nonlinear Hyperbolic Conservation Laws’. J. Comp. Phys. 27, 1–31.
Springel, V. and L. Hernquist: 2002, ‘Cosmological smoothed particle hydrodynamics simulations: the entropy equation’. MNRAS 333, 649–664.
Stone, J. M., J. F. Hawley, C. R. Evans, and M. L. Norman: 1992, ‘A test suite for magnetohydrodynamical simulations’. ApJ 388, 415–437.
Figure 2. Results of the Brio and Wu (1988) shock tube test. The problem consists of an initial discontinuity at the origin, with two phases of gas brought into contact at $t = 0$. The initial conditions to the left of the shock are $(\rho, P, v_x, v_y, B_y) = [1, 1, 0, 0, 1]$, whilst to the right the conditions are $(\rho, P, v_x, v_y, B_y) = [0.125, 0.1, 0, 0, -1]$ with $B_x = 0.75$ everywhere and $\gamma = 2.0$. These initial conditions are shown by the dashed lines, whilst the solid points, corresponding to the SPH particles, indicate the profiles of density, pressure, $v_x$, $v_y$, transverse magnetic field $B_y$ and thermal energy $u$ at time $t = 0.1$. The simulation uses 800 particles in the one dimensional domain $x = [-0.5, 0.5]$. The dissipation switch has not been used in this case. The solution corresponds well to that computed using a grid-based code incorporating a Riemann solver (Balsara 1998 - solid lines).
Figure 3. Results of the isothermal MHD shock tube test. This problem illustrates the formation of six discontinuities in isothermal MHD. The setup is similar to that in Figure 2 with gas left of the origin initially in the state $(\rho, v_x, v_y, v_z, B_y, B_z) = [1.08, 1.2, 0.01, 0.5, 3.6/(4\pi)^{1/2}, 2/(4\pi)^{1/2}]$ and gas to the right in the state $(\rho, v_x, v_y, v_z, B_y, B_z) = [1, 0, 0, 0, 4/(4\pi)^{1/2}, 2/(4\pi)^{1/2}]$ with $B_x = 2/(4\pi)^{1/2}$ everywhere. These initial conditions are shown by the dashed lines, whilst points indicate the positions of the SPH particles at time $t = 0.2$ which are shown to agree well with the numerical solution computed by Balsara (1998) using a grid-based code (solid lines).