Bulk scalar field in brane-worlds with induced gravity
inspired by the $\mathcal{L}(R)$ term

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Abstract

We obtain the effective field equations in a brane-world scenario within the framework of a
DGP model where the action on the brane is an arbitrary function of the Ricci scalar, $\mathcal{L}(R)$, and the bulk action includes a scalar field in the matter Lagrangian. We obtain the Friedmann
equations and acceleration conditions in the presence of the bulk scalar field for the $R^n$ term in
four-dimensional gravity.

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1 Introduction

The type Ia supernovae (SNe Ia) [1] observations provide the first evidence for the accelerating expansion of the present universe. These results, when combined with the observations on the anisotropy spectrum of cosmic microwave background (CMB) [2] and the results on the power spectrum of the large scale structure (LSS) [3], strongly suggest that the universe is spatially flat and dominated by a component, though arguably exotic, with large negative pressure, referred to as dark energy [4]. The nature of such dark energy constitutes an open and tantalizing question connecting cosmology and particle physics. Different mechanisms have been suggested over the past few years to accommodate dark energy. The simplest form of dark energy is the cosmological constant. However, it suffers from serious problems such as the fine-tuning problem and the coincidence problem.

An interesting way of explaining the observed acceleration of the late time universe is to modify gravity at large scales. This scenario was proposed by Dvali, Gabadadze and Porrati (DGP) [5]. The DGP proposal is based on the key assumption of the presence of a four-dimensional (4D) Ricci scalar in the bulk action. There are two main reasons that make this model phenomenologically appealing. First, it predicts that 4D Newtonian gravity on a brane-world is regained at distances shorter than a given crossover scale $r_c$ (high energy limit), whereas 5D effects become manifest above that scale (low energy limit) [6]. Second, the model can explain the late time acceleration without having to invoke a cosmological constant or quintessential matter [29, 8]. For a recent and comprehensive review of the phenomenology of DGP cosmology, the reader is referred to [9]. Recently, it has been shown that a very tiny correction to the usual gravitational action of general relativity of the form $R^n$, with $n < 0$ could give rise to accelerating solutions of the field equations without dark energy [10]. In this framework, some attempts have been made to explain the observed cosmic acceleration by modifying the Einstein-Hilbert action.

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One important extension to brane-world models is to consider matter fields in the bulk space, such as scalar fields, see [11] and references therein. In fact, it is natural that the 5D theory is itself an effective theory which originates from a yet higher-dimensional theory and the 5D effective action includes some scalar fields of gravitational origin. It is believed that in the unified theory approach, a dilatonic gravitational scalar field term is required in the 5D Einstein-Hilbert action [12]. One of the first motivations for introducing a bulk scalar field has been to stabilize the distance between the two branes [13] in the context of the Randall-Sundrum type I brane model (RSI). A second motivation for studying scalar fields in the bulk is due to the possibility that such a setup could provide some clue to the solution of the famous cosmological constant problem. Interestingly, it has also been shown that inflation is caused solely by the dynamics of a 5D scalar field without introducing it in the brane universe [14]. The creation of a brane-world with a bulk scalar field using an instanton as the solution of the 5D Euclidean Einstein equations was considered in [15]. Bulk scalar fields in a DGP brane-world scenario without curvature corrections have been studied in [16].

The effective gravitational equations in a brane-world scenario with induced gravity have been derived in [17]. Very recently Saavedra and Vasquez [18] have obtained the effective equations on the brane for modified induced gravity, in this connection also see [19]. In this paper, we extend the effective equations derived in [18] to the case where the bulk space is endowed with a scalar field. In other words, we study a bulk scalar field in a DGP brane-world scenario with curvature correction and show that under certain conditions the universe undergoes a self-accelerating phase.

2 Bulk scalar field in DGP model with $\mathcal{L}(R)$ brane action

In this section we present a brief review of the model proposed in [18] and extend it to the case with a bulk scalar field. Consider a 5D space-time with a 4D brane, located at $Y(X^A) = 0$, where $X^A$, $(A = 0, 1, 2, 3, 4)$ are the 5D coordinates. The effective action is given by

$$S = \int d^5X \sqrt{-G} \left[ \frac{1}{2\kappa_5^2} \mathcal{R} + S_m^{(5)} \right] + \int_{Y=0} d^4x \sqrt{-g} \left[ \frac{1}{\kappa_5^2} K_\pm + S_{brane}(g_{\alpha\beta}, \psi) \right], \quad (1)$$

where $\kappa_5^2 = 8\pi G_5$ is the 5D gravitational constant, $\mathcal{R}$ and $S_m^{(5)}$ are the 5D scalar curvature and the matter Lagrangian in the bulk, respectively. Also, $x^\mu$ ($\mu = 0, 1, 2, 3$) are the induced 4D coordinates on the brane, $K_\pm$ is the trace of the extrinsic curvature on either side of the brane [20] and $S_{brane}(g_{\alpha\beta}, \psi)$ is the effective 4D Lagrangian, which is given by a generic functional of the brane metric $g_{\alpha\beta}$ and matter fields.

The 5D Einstein field equations are given by

$$\mathcal{R}_{AB} - \frac{1}{2} \mathcal{G}_{AB} = \kappa_5^2 \left[ T_{AB}^{(5)} + \delta(Y) \tau_{AB} \right], \quad (2)$$

where

$$T_{AB}^{(5)} = -2 \frac{\delta S_m^{(5)}}{\delta G_{AB}} + \mathcal{G}_{AB} S_m^{(5)}, \quad (3)$$

and

$$\tau_{\mu\nu} = -2 \frac{\delta S_{brane}}{\delta g_{\mu\nu}} + g_{\mu\nu} S_{brane}. \quad (4)$$

We study the case where the induced gravity scenario arises from higher-order corrections to the scalar curvature over the brane. The interaction between the bulk gravity and local matter induces gravity on the brane through its quantum effects. If we take into account quantum effects of the matter fields confined to the brane, the gravitational action on the brane is modified as

$$S_{brane}(g_{\alpha\beta}, \psi) = \frac{\mu^2}{2} \mathcal{L}(R) - \lambda + S_m, \quad (5)$$

2
where $\mu$ is a mass scale which may correspond to the 4D Planck mass, $\lambda$ is the tension of the brane and $S_m$ presents the Lagrangian of the matter fields on the brane. We note that for $\mathcal{L}(R) = R$, action (1) gives the DGP model if $\lambda = 0$ and $\Lambda^{(5)} = 0$ and gives the RSII model if $\mu = 0$.

We obtain the gravitational field equations on the brane-world as [21]

$$ G_{\mu \nu} = \frac{2\kappa_5^2}{3} \left[ T^{(5)}_{AB} g_\mu^A g_\nu^B + g_{\mu \nu} \left( T^{(5)}_{AB} n^A n^B - \frac{1}{4} T^{(5)} \right) \right] + \kappa_5^4 \pi_{\mu \nu} - \mathcal{E}_{\mu \nu}, $$

(6)

$$ \nabla_\nu \pi^\mu_{\nu} = -2T^{(5)}_{AB} g_\mu^A, $$

(7)

where $\nabla_\nu$ is the covariant derivative with respect to $g_{\mu \nu}$ and the quadratic correction has the form

$$ \pi_{\mu \nu} = -\frac{1}{4} \tau_{\mu \alpha} \tau^\alpha_{\nu} + \frac{1}{12} \tau_{\mu \nu} + \frac{1}{8} g_{\mu \nu} \tau_{\alpha \beta} \tau^{\alpha \beta} - \frac{1}{24} g_{\mu \nu} \tau^2, $$

(8)

and the projection of the bulk Weyl tensor to the surface orthogonal to $n^A$ is given by

$$ \mathcal{E}_{\mu \nu} = C^{(5)}_{ABCD} n^A n^B g^C_{\mu} g^D_{\nu}. $$

(9)

In order to find the basic field equations on the brane with induced gravity described by the $\mathcal{L}(R)$ term, we have to obtain the energy-momentum tensor of the brane $\tau_{\mu \nu}$, given by definition (4) from Lagrangian (5), yielding

$$ \tau_{\mu \nu} = -\Lambda(R) g_{\mu \nu} + T_{\mu \nu} - \Sigma(R) G_{\mu \nu} + D_{\mu \nu}, $$

(10)

where the functions $\Lambda(R)$, $\Sigma(R)$ and $D_{\mu \nu}$ are defined as

$$ \Lambda(R) = \frac{\mu^2}{2} \left[ R \frac{d \mathcal{L}(R)}{d R} - \mathcal{L}(R) + \frac{2 \lambda}{\mu^2} \right], $$

(11)

and

$$ \Sigma(R) = \mu^2 \frac{d \mathcal{L}(R)}{d R}, $$

(12)

and

$$ D_{\mu \nu} = \mu^2 \left[ \nabla_\mu \nabla_\nu \left( \frac{d \mathcal{L}(R)}{d R} \right) - g_{\mu \nu} \nabla_\beta \nabla_\beta \left( \frac{d \mathcal{L}(R)}{d R} \right) \right]. $$

(13)

Let us now introduce a scalar field in the bulk and assume that the cosmological constant is zero $\Lambda^{(5)} = 0$. The energy momentum tensor of the bulk scalar field is given by

$$ T^{(5)}_{AB} = \phi_{,A} \phi_{,B} - g_{AB} \left( \frac{1}{2} G^{CD} \phi_{,C} \phi_{,D} + V(\phi) \right). $$

(14)

Inserting equations (10) and (14) into equation (6), we find the effective field equations for the 4D metric $g_{\mu \nu}$ as

$$ [1 + \frac{1}{6} \kappa_5^4 \Lambda(R) \Sigma(R)] G_{\mu \nu} = \frac{1}{6} \kappa_5^4 \Lambda(R) T_{\mu \nu} + \frac{1}{6} \kappa_5^2 \tilde{T}_{\mu \nu} + \frac{1}{12} \kappa_5^2 \Lambda(R)^2 g_{\mu \nu} + \frac{1}{6} \kappa_5^4 D_{\mu \nu} + \kappa_5^4 \pi_{\mu \nu} + \mu^4 \pi^{(D)}_{\mu \nu}, $$

(15)

where

$$ \tilde{T}_{\mu \nu} = \frac{1}{6} \left[ 4 \phi_{,\mu} \phi_{,\nu} + \left( \frac{3}{2} (\phi_{,\mu})^2 - \frac{5}{6} g_{,\alpha \beta} \phi_{,\alpha} \phi_{,\beta} - 3V(\phi) \right) g_{\mu \nu} \right], $$

(16)

and

$$ \pi^{(T)}_{\mu \nu} = -\frac{1}{4} T_{\mu \alpha} T^\alpha_{\nu} + \frac{1}{12} T T_{\mu \nu} + \frac{1}{8} g_{\mu \nu} T_{\alpha \beta} T^{\alpha \beta} - \frac{1}{24} g_{\mu \nu} T^2, $$

(17)
\[
\pi^G_{\mu\nu} = -\frac{1}{4} G_{\mu\alpha} G^\alpha_{\nu} + \frac{1}{12} G G_{\mu\nu} + \frac{1}{8} g_{\mu\nu} G_{\alpha\beta} G^{\alpha\beta} - \frac{1}{24} g_{\mu\nu} G^2,
\]

(18)

\[
\pi^D_{\mu\nu} = -\frac{1}{4} D_{\mu\alpha} D^\alpha_{\nu} + \frac{1}{12} D D_{\mu\nu} + \frac{1}{8} g_{\mu\nu} D_{\alpha\beta} D^{\alpha\beta} - \frac{1}{24} g_{\mu\nu} D^2,
\]

(19)

and

\[
K_{\mu\nu\rho\sigma}^{(T)} = \frac{1}{4} (g_{\mu\rho} T_{\nu\sigma} - g_{\mu\sigma} T_{\nu\rho} + \frac{1}{12} T_{\mu\nu} g_{\rho\sigma} + T (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho})) ,
\]

(20)

\[
K_{\mu\nu\rho\sigma}^{(D)} = \frac{1}{4} (g_{\mu\nu} D_{\rho\sigma} - g_{\mu\rho} D_{\nu\sigma} - g_{\mu\sigma} D_{\nu\rho} + \frac{1}{12} (D_{\mu\nu} g_{\rho\sigma} + D (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho})),
\]

(21)

with \( T \) being the trace of the energy momentum tensor and \( D \) is

\[
D = g^{\mu\nu} D_{\mu\nu} = -3 \mu^2 \nabla^\alpha \nabla_\alpha \left( \frac{d\mathcal{L}(R)}{dR} \right).
\]

(22)

We note that these equations are exactly the same effective equations as presented in reference [18] in the absence of a bulk scalar field. In the next section, we discuss the effects of a bulk scalar field on the cosmology of our model.

3 Cosmology of the model

A brane-world model with induced gravity described by a \( \mathcal{L}(R) \) term has a self-accelerated branch in the low energy limit [18]. In the following we obtain the effective Friedmann equation for a Minkowski bulk \( \Lambda^{(5)} = 0 \), \( \varepsilon_{00} = 0 \), and investigate whether it is possible to have a late time accelerating phase on the brane when there is a scalar field in the bulk and the brane is empty.

Using equation (15), in the absence of the bulk matter field, the effective Friedmann equation for a Friedmann-Robertson-Walker (FRW) universe with zero tension is then given by [18]

\[
H^2 + \frac{k}{a^2} = \frac{1}{4r_c^2 \mathcal{L}'(R)} \left[ 1 + \sqrt{1 + \frac{4}{3\mu^2} r_c^2 \mathcal{L}'(R) \rho_m + \frac{2}{3} r_c^2 \mathcal{L}'(R) \left( R \mathcal{L}'(R) - \mathcal{L}(R) - 6 H \dot{R} \mathcal{L}''(R) \right) } \right] ^2,
\]

(23)

where \( \mathcal{L}'(R) \equiv \frac{d\mathcal{L}(R)}{dR} \), \( 2r_c \equiv \frac{\mu^2}{\kappa_5^2} = \kappa_5^2 \mu^2 \) and \( \rho_m \) is the energy density of the ordinary matter on the brane which has a perfect fluid form. The two different possible \( \varepsilon \) namely \( \varepsilon = \pm 1 \), correspond to two different embedding of the brane into the bulk spac-time. We define \( \hat{\tau}_c = r_c \mathcal{L}'(R) \) and \( \rho_{\text{tot}} = \hat{\rho}_m + \rho_{\text{curv}} \), thus equation (23) can be rewritten as

\[
H^2 + \frac{k}{a^2} = \left[ \frac{1}{2r_c} + \frac{\varepsilon}{2r_c} \sqrt{1 + \frac{4}{3\mu^2} r_c^2 \mathcal{L}'(R) \rho_m} \right] ^2,
\]

(24)

where

\[
\rho_{\text{curv}} = \frac{\mu^2}{2 \mathcal{L}'(R)} \left( R \mathcal{L}'(R) - \mathcal{L}(R) - 6 H \dot{R} \mathcal{L}''(R) \right),
\]

(25)

and

\[
\hat{\rho}_m = \frac{\rho_m}{\mathcal{L}'(R)}.
\]

(26)

Equation (24) has a similar form to the Friedmann equation in the DGP model [29], differing only in replacing \( \hat{\tau}_c \) by \( r_c \). Since \( \mathcal{L}(R) \) is an arbitrary function of the Ricci scalar on the DGP brane, different choices of \( \mathcal{L}(R) \) lead to different forms of \( \hat{\tau}_c \). Also for a specific function of \( \mathcal{L}(R) \) when it varies from point to point on the DGP brane, the crossover scale takes different values. By using the most recent
Supernovae observations, the best-fit value for the crossover scale \( r_c \) in terms of the Hubble radius is given by [22]
\[
    r_c \simeq 1.09 H_0^{-1}. \tag{27}
\]

Since \( \mathcal{L}'(R) = \frac{2\kappa_5^2}{r_c^2} \), if we choose \( \hat{r}_c \sim 1.09 H_0^{-1} \) it allows us to put constraints on such a \( \mathcal{L}(R) \) scenario, in agreement with observational data. Using equation (15) the effective Friedmann equation with a bulk scalar field is given by
\[
H^2 + \frac{k}{a^2} = \frac{1}{2r_c^2 \mathcal{L}'(R)^2} \left[ 1 + \frac{2}{3 \mu^2} \hat{r}_c^2 \mathcal{L}'(R) \rho_m + \frac{1}{3} \hat{r}_c^2 \mathcal{L}'(R) \left( R \mathcal{L}'(R) - \mathcal{L}(R) - 6H \hat{r}_c \mathcal{L}''(R) \right) \right]
+ \varepsilon \sqrt{1 - \frac{4}{3} \kappa_5^2 \hat{r}_c^2 \mathcal{L}'(R)^2 \rho_\phi + \frac{2}{3} \hat{r}_c^2 \mathcal{L}'(R) \left( R \mathcal{L}'(R) - \mathcal{L}(R) - 6H \hat{r}_c \mathcal{L}''(R) \right) } \right]. \tag{28}
\]

The energy density and pressure of the bulk scalar field on the brane are given by
\[
    \rho_\phi = \frac{1}{2} \left[ \frac{1}{2} \hat{\phi}^2 + V(\phi) \right], \tag{29}
\]
\[
    P_\phi = \frac{1}{2} \left[ \frac{5}{6} \hat{\phi}^2 - V(\phi) \right], \tag{30}
\]
which are obtained using equation (16) under the assumption that the bulk scalar field is constant with respect to \( y \) on the brane. In other words, it satisfies the following boundary condition
\[
    \phi|_{y=0} = 0. \tag{31}
\]

Now let us focus attention on the low energies limit \( \rho_m \to 0 \). Equation (28) thus becomes
\[
H^2 + \frac{k}{a^2} = \frac{1}{2r_c^2 \mathcal{L}'(R)^2} \left[ 1 + \frac{1}{3} \hat{r}_c^2 \mathcal{L}'(R) \left( R \mathcal{L}'(R) - \mathcal{L}(R) - 6H \hat{r}_c \mathcal{L}''(R) \right) \right]
+ \frac{\varepsilon}{3 \mu^2} \sqrt{1 - \frac{4}{3} \kappa_5^2 \hat{r}_c^2 \mathcal{L}'(R)^2 \rho_\phi + \frac{2}{3} \hat{r}_c^2 \mathcal{L}'(R) \left( R \mathcal{L}'(R) - \mathcal{L}(R) - 6H \hat{r}_c \mathcal{L}''(R) \right) } \right]. \tag{32}
\]

Defining \( \hat{r}_c = r_c \mathcal{L}'(R) \) and the curvature energy density as
\[
    \rho_{\text{curv}} = \frac{\mu^2}{2 \mathcal{L}'(R)} \left( R \mathcal{L}'(R) - \mathcal{L}(R) - 6H \hat{r}_c \mathcal{L}''(R) \right), \tag{33}
\]
the DGP Friedmann equation (32) with \( k = 0 \) can be rewritten as
\[
    H^2 = \frac{1}{2\hat{r}_c} + \frac{1}{3 \mu^2} \rho_{\text{curv}} + \frac{\varepsilon}{2\hat{r}_c} \sqrt{1 - \frac{4}{3} \kappa_5^2 \hat{r}_c^2 \rho_\phi + \frac{4}{3} \hat{r}_c^2 \rho_{\text{curv}}}. \tag{34}
\]

In order to investigate the behavior of the solutions in this modified DGP brane in the presence of the bulk scalar field, we consider two branches of the solutions under the condition that \( \kappa_5^2 \rho_\phi \ll \frac{1}{\hat{r}_c^2} \). After obtaining these solutions, we will check whether this condition is valid or not. Under such a condition equation (34) reduces to
\[
    H^2 = \frac{1}{2\hat{r}_c} + \frac{1}{3 \mu^2} \rho_{\text{curv}} + \frac{\varepsilon}{2\hat{r}_c} \sqrt{1 + \frac{4}{3} \hat{r}_c^2 \rho_{\text{curv}}}. \tag{35}
\]

This is as far as one could go without specifying the form of \( \mathcal{L}(R) \). For ease of exposition and clarity, let us focus attention on theories where the term \( R^n \) is present in the brane action and write
\[
    \mathcal{L}(R) = R^n. \tag{36}
\]
For having an accelerated expansion we choose the brane scale factor as

\[ a(t) = a_0 e^{\alpha(t - t_0)}, \]

(37)

where \( \alpha \) is a positive constant. The solutions with \( \alpha = 0 \) are not interesting since they provide static cosmologies with a non-evolving scale factor on the brane. The Ricci scalar on the brane for a spatially flat FRW geometry is given by

\[ R = 12H^2 + 6\dot{H} = 12\alpha^2. \]

(38)

Substituting equations (36) and (37) into equation (35) and using equation (38) we obtain

\[ \alpha^2 \dot{r}_c^2 (2 - n) = \pm \alpha n. \]

(39)

where \( \dot{r}_c = r_c n 12^{n-1} \alpha^{2(n-1)} \). First let us consider the case \( n = 1 \), namely \( \mathcal{L}(R) = R \). In this case \( \rho_{\text{curv}} = 0 \), \( \dot{r}_c = r_c \) and we obtain two branches as

- \( \varepsilon = +1 \)
  \[ H = \alpha = \frac{1}{r_c}, \]

(40)

- \( \varepsilon = -1 \)
  \[ H = \alpha = 0. \]

(41)

Therefore, the positive branch is the self-accelerating solution [29]. In this case the above condition reduces to \( \kappa_5^2 \rho_\phi \ll 1/\dot{r}_c^2 \) in agreement with [16]. In the case \( n = 2 \), we obtain \( H = \alpha = 0 \) in both branches, that is, a static universe. For \( n > 2 \) case, the two branches are

- \( \varepsilon = +1 \)
  \[ H = \alpha = 0, \]

(42)

- \( \varepsilon = -1 \)
  \[ H = \alpha = \left[ \frac{1}{(2 - n)r_c^{12n-1}} \right]^{(2n-1)}. \]

(43)

Therefore, under the condition \( \kappa_5^2 \rho_\phi \ll \left[ 1/n^{2n-1}(n - 2)^2 - 2n r_c 12^{n-1} \right]^{2/(2n-1)} \) the negative branch has a self-accelerating phase. Since the bulk scalar field in the absence of brane matter satisfies the usual momentum conservation law on the brane, we have \( \rho_\phi \propto e^{-3\alpha(w_\phi + 1)t} \). Thus the energy density of matter goes to zero for late times and reaches a regime where it is small in comparison with \( \left[ 1/n^{2n-1}(n - 2)^2 - 2n r_c 12^{n-1} \right]^{2/(2n-1)} \). For the case \( n < 0 \), we have two branches as

- \( \varepsilon = +1 \)
  \[ H = \alpha = \left[ \frac{1}{(2 - n)r_c^{12n-1}} \right]^{(2n-1)}, \]

(44)

- \( \varepsilon = -1 \)
  \[ H = \alpha = 0. \]

(45)

Therefore, under the condition \( \kappa_5^2 \rho_\phi \ll \left[ 1/n^{2n-1}(2 - n)^2 - 2n r_c 12^{n-1} \right]^{2/(2n-1)} \), the positive branch has a self-accelerating phase. We have summarized the results in table 1.

In most brane-world models, the 5D bulk space-time only includes a cosmological constant, and the matter fields on the brane are regarded as responsible for the dynamics of the brane. In this paper we have shown that in the framework of a modified DGP brane the late time behavior of the universe does not change even if we ignore the local matter fields and consider a model of the universe filled with a bulk scalar field.

An interesting feature of DGP models is the existence of the ghost-like excitations [23, 24, 25]. It would therefore be interesting to mention recent results relevant to the present work. The study of the spectrum of gravitational perturbations without matter perturbation about a de Sitter brane shows that for the positive tension brane the massive spin-2 perturbations contain a helicity-0 mode.
that becomes a ghost if the mass is in the range $0 < m^2 < 2H^2$, while for a negative tension brane the spin-0 mode becomes a ghost if $Hr_c > 1/2$. In the self-accelerating universe without tension, the mass of the discrete mode of the spin-2 perturbations becomes $2H^2$. This is a special mass in Pauli-Fierz massive gravity theory because there exists an enhanced symmetry that eliminates the helicity-0 mode. However, in DGP models, there is a spin-0 perturbation of the same mass and this breaks the symmetry, leading to a ghost from the mixing between the spin-0 and spin-2 perturbations [24]. It should be mentioned that this is consistent with the result obtained by the boundary effective action in [23] where a scalar mode is found to be a ghost if $Hr_c > 1/2$.

The ghost carries negative energy density and it leads to the instability of the space-time. The self-accelerating branch of solutions turns out to be plagued by the ghost instability. In order to avoid the instability some authors have attempted to construct a ghost-free model by modifying the self-accelerating branch of the DGP model [26]. In [27], the authors have studied the possibility of avoiding the appearance of ghosts by first modifying the model via the introduction of a second brane in the bulk and then stabilizing the brane separation by introducing a bulk scalar field. It has been shown that it is easy to remove the spin-2 ghost by putting a second brane in the bulk and make the distance between the two brane small. However it was found that the spin-0 perturbation, the radion, becomes a ghost. By stabilizing the radion, the perturbations of the scalar field which is necessary to stabilize the radion becomes a ghost. The most interesting finding is that it is impossible to remove the spin-2 ghost and spin-0 ghost simultaneously [27, 28]. However, several interesting and different ways to avoid the appearance of ghosts is proposed in [29], where it is argued that claims of instability of the self-accelerating solutions of the DGP model that are based on linearized calculations are unwarranted. Finally, another possibility may be to consider altering the theory at the level of the action itself, for example by adding extra terms in the bulk or on the brane [30]. It was also shown that the introduction of Gauss-Bonnet term in the bulk does not help [31]. In our model the presence of the $\mathcal{L}(R)$ term, instead of $R$ in the brane action and the presence of the bulk scalar field provide the new self-accelerating solutions. Since such a brane effective theory is definitely a higher derivative theory, there should be a new degree of freedom which could be a ghost. Probably these new solutions also suffer from a ghost instability and this could be the subject of a separate investigation.

| $\mathcal{L}(R) = R^n$ | $\varepsilon = +1$ |  | $\varepsilon = -1$ |  |
|------------------------|-------------------|-------------------|-------------------|
| $n < 0$                | accelerating universe | static universe |  |  |
| $n = 1$                | accelerating universe | static universe |  |  |
| $n = 2$                | static universe     | static universe   |  |  |
| $n > 2$                | static universe     | accelerating universe |  |  |

Table 1: The late time behavior of the universe for two branches, positive ($\varepsilon = +1$) and negative ($\varepsilon = -1$), under the condition $\kappa_5^2 \rho_\phi \ll \frac{1}{r_c}$.

4 Conclusions

In this letter we have derived the effective Einstein field equations on the brane in the framework of the DGP model where the action on the brane is an arbitrary function of the Ricci scalar, $\mathcal{L}(R)$, and the bulk action includes a scalar field in the matter Lagrangian. We have shown that in a DGP model with curvature correction, $\mathcal{L}(R) = R^n$, and a scalar field in the bulk space, one can obtain an asymptotically static universe and a self accelerating solution at late times respectively for two
different embeddings of the brane $\varepsilon = -1$ and $\varepsilon = +1$ for $n = 1$ and $n < 0$. The role of the two branches is exchanged if we consider $n > 2$. The study of this scenario when the quintom field is considered as the bulk matter field will be the subject of a future investigation.

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