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Dual-station Passive Positioning Accuracy Analysis Based on TDOA/AOA

Huang Xin¹, Xiong Xin¹, Zhu Zhen-bo²
¹Graduate Department, Air Force Early Warning Academy, China
²No. 1 Department, Air Force Early Warning Academy, China

Abstract. To solve the resource allocation problem of aircraft-based multi-platform passive positioning system, we study dual-station passive positioning system model. Through the simulation analysis, the positioning accuracy distribution graphs of different measurement parameter sets are obtained, and the influences of azimuth angle, elevation angle and time difference on the positioning accuracy are analyzed in detail. The positioning ability of different measurement sets is obtained, which is reasonable for utilizing resources in the multi-aircraft co-location system.

1. Introduction
With the rapid development of modern science and technology, a large number of electronic information equipment have been put in use and electronic countermeasures have gotten to a new stage in term of way and scale. In the future wars, it is quite important to covertly and accurately locate the enemy position when facing those challenges, such as the battlefield environment is becoming more complex, the battlefield response time is gradually shortening, the battlefield threats are diverse, and uncertain factors are difficult to control [1,2]. To this end, this paper proposed a multi-platform passive positioning technology.

In passive positioning systems, TDOA-based source localization technology plays an important role [3]. TDOA-based source localization technology is mainly a method of positioning by using the time difference of signals arriving at different receiver stations [4]. The positioning technology only needs to measure the time difference, so its biggest feature is that the system is relatively simple and the positioning accuracy is high. In the passive positioning technology, AOA-based source localization technology has a wide range of applications, based on AOA-based source localization technology, mainly by obtaining the angle information of the target radiation source signal to estimate the target position [5,6]. This article is researching the hybrid TDOA and AOA positioning technology. By comparing the positioning accuracy of different measurement sets, the accuracy impact caused by the change of parameter information is studied, which provides a theoretical basis for the selection of parameters and the optimal utilization of resources in multi-platform passive positioning systems.

2. Positioning principle and system model

2.1 Positioning principle
Single-station passive positioning technology uses an observation platform to locate the target. Because the amount of information acquired is relatively small, parameter measurement requires time
accumulation, and positioning is difficult to implement[7]. Multi-platform passive positioning technology makes full use of space advantages for positioning. By measuring the target angle information (AOA) or time difference information (TDOA), we can obtain some trajectories, and then multiple trajectories are cross-positioned to determine the target position. In the dual-station passive positioning system studied in this paper, the angle information (two elevation angles and two azimuth angles) measured by each station and the time difference of the joint measurement are used to perform the hybridization cross-positioning of the direction-to-time difference.

2.2 Geometrical model
The airborne dual-station passive positioning system configuration is shown in Figure 1. The system consists of two receivers (respectively referred to as $R_1$ and $R_2$, which only accept signals), one target (denoted as $T$, which is the signal radiation source), and information obtained by the receiver is transmitted and exchanged through the data link. The spatial geometric relationship of the system and the meaning of each symbol are shown in Figure 1. Now assume that the coordinates of the two receiving stations are $R_1(x_1, y_1, z_1)$ and $R_2(x_2, y_2, z_2)$, and the target coordinates are $T(x, y, z)$. Elevation angle $\theta_1$ and azimuth angle $\phi_1$ are observed by the receiving station $R_1$, elevation angle $\theta_2$ and azimuth angle $\phi_2$ are observed by the receiving station $R_2$, and $r_1$ is the distance from the target $T$ to the receiving station $R_1$, $r_2$ is the distance from the target $T$ to the receiving station $R_2$.

![Figure 1. Geometric model of dual-station passive positioning system](image)

2.3 Model Hypothesis
On the basis of no loss of generality, in order to facilitate the analysis, taking into account the particularity of airborne radar, the following assumptions are made on the model:

1. Since the receiving station is an air-based platform, the two receiving stations remain relatively stationary during flight, meaning that they have the same direction and speed.
2. The multipath effect of radar is not considered.

3. Positioning method
When the dual-station passive positioning system performs target positioning, the station $R_1$ observes elevation angle $\theta_1$ and azimuth angle $\phi_1$, the station $R_2$ observes elevation angle $\theta_2$ and azimuth angle $\phi_2$, the station $R_1$ and $R_2$ observe time difference $\Delta r$, and the observation information $\Delta r$ of $R_1$ is transmitted to $R_2$ through the data link[8,9]. Referring to the geometric relationship in the geometric model of Figure 1, it can be obtained that.
\begin{equation}
\begin{aligned}
\sin \theta_1 &= \frac{z-z_1}{r_1} \\
\sin \theta_2 &= \frac{z-z_2}{r_2} \\
\tan \varphi_1 &= \frac{y-y_1}{x-x_1} \\
\tan \varphi_2 &= \frac{y-y_2}{x-x_2} \\
\Delta r &= r_1 - r_2
\end{aligned}
\end{equation}

(1)

where, \( \theta_1 \) is the pitch angle and \( \varphi_1 \) is azimuth angle measured by the receiver \( R_1 \), \( \theta_2 \) is the pitch angle and \( \varphi_2 \) is azimuth angle measured by the receiver \( R_2 \), \( \Delta r \) is the distance difference between the target to the receivers \( R_1 \) and \( R_2 \), \( r_1 \) is the distance between the target and the receiver \( R_1 \), \( r_2 \) is the distance between the target and the receiver \( R_2 \). In it,

\begin{equation}
\begin{aligned}
r_1 &= \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2} \\
r_2 &= \sqrt{(x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2}
\end{aligned}
\end{equation}

(2)

Differentiate the two sides of the positioning equation of the formula (1), and combine the formula (2), then

\begin{equation}
\begin{aligned}
d\theta_1 &= -\frac{(x-x_1)(z-z_1)}{r_1^2 \cos \theta_1} (dx - dx_1) - \frac{(y-y_1)(z-z_1)}{r_1^2 \cos \theta_1} (dy - dy_1) \\
d\theta_2 &= -\frac{(x-x_2)(z-z_2)}{r_2^2 \cos \theta_2} (dx - dx_2) - \frac{(y-y_2)(z-z_2)}{r_2^2 \cos \theta_2} (dy - dy_2) \\
d\varphi_1 &= -\frac{\sin \varphi_1}{y-y_1} (dy - dy_1) \cos \frac{\theta_1}{x-x_1} \\
d\varphi_2 &= -\frac{\sin \varphi_2}{y-y_2} (dy - dy_2) \cos \frac{\theta_2}{x-x_2} \\
d\Delta r &= \left( \frac{x-x_1}{r_1} - \frac{x-x_2}{r_2} \right) dx + \left( \frac{y-y_1}{r_1} - \frac{y-y_2}{r_2} \right) dy + \left( \frac{z-z_1}{r_1} - \frac{z-z_2}{r_2} \right) dz \\
&= \frac{y-y_1}{r_1} dy_1 - \frac{z-z_1}{r_1} dz_1 + \frac{x-x_1}{r_1} dx_1 + \frac{y-y_2}{r_2} dy_2 + \frac{z-z_2}{r_2} dz_2 + \frac{x-x_2}{r_2} dx_2
\end{aligned}
\end{equation}

(3)

convert equation (3) into a matrix, then

\begin{equation}
\begin{aligned}
dV &= C dX + dX_s
\end{aligned}
\end{equation}

(4)

In the equation (4), \( dV \) is observation error vector, \( C \) is coefficient matrix, \( dX \) is target position error vector, \( dX_s \) is vector related to station error. It is expressed as follows

\begin{equation}
\begin{aligned}
dV &= [d\theta_1 \quad d\theta_2 \quad d\varphi_1 \quad d\varphi_2 \quad d\Delta r]^T
\end{aligned}
\end{equation}

(5)
\[
C = \begin{bmatrix}
\frac{(x-x_i)(z-z_i)}{r_i' \cos \theta_i} & \frac{(y-y_i)(z-z_i)}{r_i' \cos \theta_i} & \frac{z-z_i}{r_i'} \\
\frac{(x-x_i)(x-x_i)}{r_i' \cos \theta_i} & \frac{(y-y_i)(x-x_i)}{r_i' \cos \theta_i} & \frac{z-z_i}{r_i'} \\
\frac{\sin \phi_i}{y-y_i} & \frac{\cos \phi_i}{x-x_i} & 0 \\
\frac{\sin \phi_i}{y-y_i} & \frac{\cos \phi_i}{x-x_i} & 0 \\
\frac{x-x_i}{r_i} & \frac{x-x_i}{r_i} & \frac{z-z_i}{r_i} \\
\end{bmatrix}
\]  \hspace{1cm} (6)

\[
dX = \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}
\]  \hspace{1cm} (7)

\[
dx_i = \begin{bmatrix}
\frac{(x-x_i)(z-z_i)}{r_i' \cos \theta_i} dx_i + \frac{(y-y_i)(z-z_i)}{r_i' \cos \theta_i} dy_i - \frac{r_i^2 - (z-z_i)^2}{r_i' \cos \theta_i} dz_i \\
\frac{(x-x_i)(x-x_i)}{r_i' \cos \theta_i} dx_i + \frac{(y-y_i)(x-x_i)}{r_i' \cos \theta_i} dy_i - \frac{r_i^2 - (z-z_i)^2}{r_i' \cos \theta_i} dz_i \\
\frac{\sin \phi_i}{y-y_i} dx_i - \frac{\cos \phi_i}{x-x_i} dy_i \\
\frac{\sin \phi_i}{y-y_i} dx_i - \frac{\cos \phi_i}{x-x_i} dy_i \\
- \frac{x-x_i}{r_i} dx_i - \frac{x-x_i}{r_i} dy_i - \frac{z-z_i}{r_i} dz_i + \\
x-x_i \frac{z-z_i}{r_i} dz_i + \frac{z-z_i}{r_i} dz_i \\
\end{bmatrix}
\]  \hspace{1cm} (8)

then

\[
dX = (C^T C)^{-1} C^T (dV - dX_i) \hspace{1cm} (9)
\]

The covariance matrix of the error is

\[
P_{dX} = E\left[ dXdV^T \right] = \begin{bmatrix}
(C^T C)^{-1} C^T (dV - dX_i) (dV^T - dX_i^T) C (C^T C)^{-1} \\
\end{bmatrix}
\]  \hspace{1cm} (10)

For the convenience of analysis, it is assumed that the observations measured by the two receiving stations are not related to each other, and the measurement error of the elevation angle is \( \sigma_{\theta} \), the measurement error of the azimuth angle is \( \sigma_{\phi} \), the measurement error of the time difference is \( \sigma_{\Delta} \). Assume that the station errors of the three stations are equal (considering the same airborne platform), and the site errors are not related to each other, all of them are \( \sigma_s \), then

\[
E\left[ dVdV^T \right] = \text{diag}\left( \left[ \sigma_{\theta}^2 \sigma_{\phi}^2 \sigma_{\phi}^2 \sigma_{\Delta}^2 \right] \right) \hspace{1cm} (11)
\]

\[
E\left[ dX_i dX_i^T \right] = \begin{bmatrix}
\frac{1}{r_i} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{r_i^2} & 0 & 0 & 0 \\
0 & 0 & \frac{\cos^2 \phi_i}{(x-x_i)^2} & 0 & 0 \\
0 & 0 & 0 & \frac{\cos^2 \phi_i}{(x-x_i)^2} & 0 \\
0 & 0 & 0 & 0 & 2
\end{bmatrix} \hspace{1cm} \sigma_s^2 \hspace{1cm} (12)
\]
It can be seen from the above formulas that the positioning error of the target is related to the angle measurement error, the time measurement error, and the station error of the receivers. GDOP is now introduced to describe the positioning accuracy of passive positioning, and its expression is

$$GDOP = \sqrt{\text{trace}(P_0^2)} = \sqrt{P_0(1,1) + P_0(2,2) + P_0(3,3)}$$  \hspace{1cm} (13)$$

GDOP reflects the positioning accuracy \[10\]. The larger the GDOP value, the worse the positioning accuracy. The smaller the GDOP value, the better the positioning accuracy.

4. Positioning accuracy simulation analysis

4.1 Parameter settings

Suppose the target area is a square with a side length of 200km on the z = 10km plane, with the center of the square as the coordinate origin. For the convenience of analysis, the receiver stations \( R_1 \) and \( R_2 \) of this system are located at (-20,0,0), (20,0,0), then the baseline length is 40km. The measurement error of azimuth and elevation angle is \( \pm 1^\circ \), the time measurement error is 50ns, and the receiver and target site errors are both 10m. The measurement set is shown in Table 1.

| set number | measurement set |
|------------|-----------------|
| 1          | \((\varphi_1, \varphi_2, \theta_1, \theta_2, \Delta t)\) |
| 2          | \((\varphi_1, \theta_1, \Delta t)\) |
| 3          | \((\varphi_1, \theta_1, \Delta t)\) |
| 4          | \((\theta_1, \Delta t)\) |
| 5          | \((\varphi_1, \theta_1, \Delta t)\) |

4.2 Simulation analysis

(1) measurement set 1

The parameters of the measurement set 1 in the table 1 are the full parameters of the TDOA/AOA joint positioning system, and The 2D result graph of the GDOP is shown in the figure 2(a). The 3D result graph is shown in the figure 2(b).
The figure 2 shows that the dual-station passive positioning system utilizes only two azimuth angles and two elevation angles of the GDOP distribution. Compared with the full-parameter simulation results, as shown in Figure 2(a), the positioning accuracy decreases near the baseline of the two observatories, and the system positioning accuracy decreases more rapidly on the baseline extension line. Good positioning accuracy is maintained on the vertical and peripheral portions of the baseline. As shown in Figure 2(b), the GDOP is reduced at the far area of the receiver baseline extension line, and the receiver baseline extension line has fewer positioning blind spots than the full parameters. After the time difference parameter is removed, positioning accuracy is still good, which can be considered in resource allocation and spatial configuration.

(2) measurement set 2

The figure 3 shows that the dual-station passive positioning system utilizes only two azimuth angles and two elevation angles of the GDOP distribution. Compared with the full-parameter simulation
results, as shown in Figure 3(a), the positioning accuracy decreases near the baseline of the two receivers, and the system positioning accuracy decreases more rapidly on the baseline extension line. Good positioning accuracy is maintained on the vertical and peripheral portions of the baseline. As shown in Figure 3(b), the GDOP is reduced at the far area of the receiver baseline extension line, and the receiver baseline extension line has fewer positioning blind spots than the full parameters. After the time difference parameter is removed, the drop in positioning accuracy is not obvious, and even improvement is made on the baseline extension line, which can be considered in resource allocation and spatial configuration.

(3) measurement set 3

Figure 4. GDOP distribution

The figure 4 shows that the dual station passive positioning system using the GDOP distribution of the azimuth measured by \( R_1 \) and \( R_2 \) and time difference parameter. Compared with the full-parameter simulation results, as shown in Figure 4(a), the positioning accuracy decreases near the baseline of the two observatories, and the system positioning accuracy decreases greatly on the vertical line in the baseline, and shows a rapid downward trend on the baseline extension line. The peripheral part can barely maintain better positioning accuracy. As shown in Figure 4(b), the GDOP is reduced at the far end of the receiver baseline extension line, and the positioning accuracy decreases sharply on the vertical line in the baseline. The GDOP is 500 times higher than the GDOP near the observation station, and there is a serious positioning blind zone.

(4) measurement set 4
The figure 5 shows that the GDOP distribution of the dual-station passive positioning system using the pitch angles measured by $R_1$ and $R_2$ and the time difference parameter. Compared with the full-parameter simulation results, as shown in Figure 5(a), the positioning accuracy is greatly reduced throughout the target area. In the vertical line of the baseline, the rate of decline is the fastest. As shown in Figure 5(b), the GDOP at the far end of the receiver is 50,000 times higher than the GDOP near the station, and the dual-station positioning system is almost impossible to locate.

(5) measurement set 5

The figure 6 shows the GDOP distribution of the dual-station passive positioning system using the azimuth angle, elevation angle of the $R_1$ and time difference parameter. Compared with the full-parameter simulation results, as shown in Figure 6(a), the positioning accuracy is only greatly reduced on the $R_1$ side of the receiver baseline extension line, and the positioning accuracy of the remaining areas remains good. As shown in Figure 6(b), the far-area GDOP on the $R_1$ side of the receiver baseline
baseline extension line is 40,000 times higher than the GDOP near the station, and the dual-station positioning system is almost impossible to locate on this side.

5. Conclusion
In this paper, based on the rational allocation problem of multi-aircraft cooperative working resources, a system model of dual-station TDOA/AOA joint positioning is established, and the influence of measurement set on positioning accuracy is studied. Through the simulation analysis of different measurement sets, the following conclusions are obtained: when the angle (azimuth and elevation angle) information is complete, the positioning accuracy is very good, and the time difference information can be added to improve the positioning near the baseline and near the vertical line in the baseline. Precision. If a certain type of angle (azimuth or elevation angle) information is missing, the positioning accuracy is seriously degraded, and there is a serious positioning dead zone near the vertical line in the baseline. When the time difference data is integrated by using all angle information of a single station, in addition to the vicinity of the station and the baseline extension line on the side, good positioning accuracy can be maintained for other areas. This study provides a theoretical basis for the rational allocation of multi-aircraft location system resources and the selection of measurement parameters.

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