Multifunctional Space-Time Metasurfaces

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Abstract

Integrating multiple functionalities into a single metasurface is becoming of great interest for future intelligent communication systems. While such devices have been extensively explored for reciprocal functionalities, in this work, we integrate a wide variety of nonreciprocal applications into a single platform. The proposed structure is based on spatiotemporally modulated impedance sheets supported by a grounded dielectric substrate. We show that, by engineering the excitation of evanescent modes, nonreciprocal interactions with impinging waves can be configured at will. We demonstrate a plethora of nonreciprocal components such as wave isolators, phase shifters, and circulators, on the same metasurface. This platform allows switching between different functionalities only by modifying the pumping signals (harmonic or non-harmonic), without changing the main body of the metasurface structure. This solution opens the door for future real-time reconfigurable and environment-adaptive nonreciprocal wave controllers.

I. INTRODUCTION

The next generations of communication systems will need multifunctional devices that adapt to different usage and environment requirements, enhancing user experiences and providing better information exchange. In this emerging paradigm, due to their integration capabilities and the unprecedented opportunities for controlling waves, metasurfaces are positioned as the platform of choice to implement multifunctional responses. By varying the local properties of each composing element in space and time, one can create metasurfaces with different scattering responses. Metasurfaces can be treated as multi-port devices that are able to simultaneously control waves incoming and outgoing from/to a number of propagation directions, for example, waves coming from oppositely tilted angles [1, 2]. Following the classical notion of multi-port networks, the properties of a metasurface can be modeled by the scattering matrix, where each element $S_{ij}$ represents the ratio of the flux amplitudes between the outgoing waves to $i$-port and the incoming wave from $j$-port. Assume that there is no frequency transformations during scattering, the functionalities of metasurfaces can be classified into two groups: reciprocal, characterized by symmetric scattering matrices ($\widehat{S} = \overline{S}$), and non-reciprocal, where scattering matrices are asymmetric ($\widehat{S} \neq \overline{S}$).

Reciprocal metasurfaces have been intensively studied in the last decade. In practice, one can make use of gradient or space-modulated metasurfaces for tailoring the scattering properties between different ports and design anomalous reflectors [3–5], beam splitters, multichannel retroreflectors [1] or asymmetric absorbers [2]. Recent advances in reciprocal metasurfaces have shown that one metasurface can perform a multitude of functionalities, switching from one application to another by reconfiguring electromagnetic parameters of the individual metamats. For example, in [6], independent tunability of resistance and reactance of each unit cell using embedded tunable circuits has been discussed. In this way, by locally modifying the impedance of each unit cell, the metasurface can act as a perfect absorber or anomalous reflector without changing.

Reciprocity imposes certain restrictions in the design of metasurfaces, and there are practical applications where it is necessary to go beyond them. Traditionally, this goal has been achieved with magneto-optical materials, such as ferrites [7] and graphene [8–10], biased by external static or slowly-varying magnetic fields. But the use of bulky magnets makes these devices incompatible with integrated technologies. In addition, the weak magnetic response and high losses in terahertz and optical ranges further inhibit their use in future optoelectronic and nanophotonic systems. One magnetless alternative is to embed nonlinear materials in resonant cavities [11–13]. By creating spatial asymmetries in the cavity, the distribution of electromagnetic fields can be made significantly different for opposite illuminations. However, this solution is restricted to strong excitation intensities, poor isolation, and nonsimultaneous excitation from different ports [14–15].

In recent years, dynamic modulation of material properties has attracted considerable attentions in both physics and engineering communities, due to its great potentials to induce extraordinary wave phenomena [16–21]. Time-varying materials break Lorentz reciprocity in linear and magnet-free devices. Spatiotemporal modulation can impart synthetic linear or angular momentum in the operating systems that emulate high-speed mechanical motion, and therefore produce Doppler-like nonreciprocity [22–24]. Based on this principle, many interesting phenomena have been explored [25], such as wave isolation [26, 33] and circulation [35, 36], one-way beam splitting [37], frequency conversion [38–40], and nonreciprocal antennas [23, 41–43]. However, the nonreciprocal applications are usually fixed and unadaptable to changing environments or application requirements. While the
existing works mostly focus on a specific nonreciprocal effect realized with a fixed modulation scheme, in this paper, we show that it is possible to develop robust and general means for reconfigurable nonreciprocal functionalities on a metasurface platform.

In particular, we demonstrate that by altering the space-time modulation functions it is possible to implement various canonical nonreciprocal devices on a universal platform. This multifunctional platform is based on a tunable surface impedance sheet supported by a metalized substrate. We assume that the resistance and reactance of the surface impedance can be independently and locally controlled in real time [6]. We first develop a generalized circuit theory to characterize the scattering harmonics of surface impedances modulated with arbitrary traveling waveforms (Section II). The theoretical formulation allows not only direct computations of scattering harmonics for a known modulated surface impedance, but also finding the optimal surface impedance that ensures excitation of a prescribed set of scattering harmonics. By incorporating evanescent fields into the picture, the optimized surfaces exhibit strong and controllable nonreciprocity. Based on this principle, we assign optimal modulation functions to a single metasurface with fixed dimensions and demonstrate possibilities to realize isolators, phase shifters, quasi-isolators, and circulators (Section III). In addition, we propose a viable practical implementation scheme based on graphene, showing the realizability of the proposed multifunctional and adaptable platform (Section IV).

II. GENERALIZED CIRCUIT THEORY FOR SPACE-TIME MODULATED SURFACES

In this section, we present the theoretical formulation that describes a reconfigurable platform for nonreciprocal devices. We consider the structure shown in Fig. 1(a) an electrically thin sheet mounted on a grounded substrate. The main goal is to allow arbitrary reconfiguration of the nonreciprocal functionality using spatiotemporal modulations of the electric response of the sheet. Its electrical properties are defined by the parallel connection of a conductance, \( G(z,t) \), and an inductance, \( L(z,t) \), periodically modulated in space and time [see Fig. 1(b)]. We assume that \( G(z,t) \) and \( B(z,t) \) are modulated with an arbitrary traveling waveform (the inverse of inductance, \( B(z,t) \), is introduced in this paper for mathematical simplicity).

To obtain the general expressions of the travelling-wave modulation, let us first assume that both \( G \) and \( B \) are spatially varying with periodicity \( D \). Generally, these periodic functions can be expanded in Fourier series

\[
\Psi(z) = \sum_{m=-\infty}^{+\infty} \psi_m e^{-j m \beta_M z}, \tag{1}
\]

where \( \Psi(z) \) denotes either \( G(z) \) or \( B(z) \), \( \psi_m \) represents \( y_m \) and \( b_m \) which are the harmonic coefficients of \( G(z) \) and \( B(z) \), respectively, and \( \beta_M = 2\pi/D \) is the modulation wavenumber. It is important to notice that \( \psi^*_m = \psi_{-m} \) to ensure that both \( G(z) \) and \( L(z) \) are real-valued functions. This expansion allows engineering the conductance and the inductance with arbitrary periodic waveforms.

Next, we assume that the waveforms \( G(z) \) and \( B(z) \) are moving along the z-axis with a constant velocity \( v_M = D/T \) (\( T \) denotes the temporal periodicity). In this case, \( G \) and \( B \) are periodical functions of both space and time, and their Fourier expansions can be obtained by replacing \( z \) in Eq. (1) by \( z - v_M t \):

\[
\Psi(z, t) = \Psi(z - v_M t) = \sum_{m=-\infty}^{+\infty} \psi_m e^{-j m (\beta_M z - \omega_M t)}, \tag{2}
\]

where \( \omega_M = v_M \beta_M = 2\pi/T \) is the modulation frequency.

![FIG. 1. (a) General scattering scenario of space-time modulated impenetrable surface and (b) its equivalent circuit model.](image)

Then, if a plane wave (wavenumber \( k_0 \) and frequency \( \omega_0 \)) impinges on the structure at the incidence angle \( \theta = \theta_i \), according to the Floquet-Bloch theorem, the stationary scattered fields have the form

\[
K(z,t) = e^{-j(k_z z - \omega_0 t)} P(z') \tag{3}
\]

with \( k_z = k_0 \sin \theta_i \) being the tangential wavenumber and \( z' = z - v_M t \). Here, \( K \) represents both electric, \( E \), and magnetic, \( H \), fields. Notice that \( P(z') \) is a periodical function with the spatial periodicity \( D \), \( P(z') = P(z'+n D) \), and can be expanded in Fourier series:

\[
P(z') = P(z - v_M t) = \sum_{n=-\infty}^{+\infty} p_n e^{-j n \beta_M (z - v_M t)}. \tag{4}
\]

After substituting Eq. (4) into Eq. (3) the scattered fields can be written as

\[
K(z,t) = \sum_{n=-\infty}^{+\infty} p_n e^{-j (k_z + n \beta_M) z - (\omega_0 + n \omega_M) t}. \tag{5}
\]

From Eq. (5), we can see that the scattered fields contain an infinite number of Floquet harmonics. The \( n \)-th harmonic propagates with the transverse wavenumber \( k_{zn} = k_z + n \beta_M \) and has the frequency \( \omega_n = \omega_0 + n \omega_M \).
Knowing the general expressions for the scattered harmonics, we use the mode-matching method to calculate their amplitudes and phases. We start by writing the tangential components of the total electrical and magnetic fields on the surface as a sum of the incident wave and all the scattered harmonics:

\[
E_{\text{tot}}^t = E_1^t + E_2^t = \sum_{n=-\infty}^{+\infty} E_n^t e^{-j(k_2 z - \omega_n t)}
\]

\[
H_{\text{tot}}^t = H_1^t + H_2^t = \sum_{n=-\infty}^{+\infty} H_n^t e^{-j(k_2 z - \omega_n t)}.
\]

The coefficients \(E_n^t\) and \(H_n^t\) can be uniquely determined by enforcing the boundary conditions on all the interfaces of the structure [see Fig. 1(a)]. However, this could be a cumbersome task, especially for structures with multiple constitutive layers. To simplify the mathematical derivations, we analyze the metasurface using the transmission-line model shown in Fig. 1(b). In this model, the spatiotemporally varying surface conductance and susceptance form a shunt connected between two transmission lines, which represent free space and the substrate, respectively. The tangential components of the total electric and magnetic fields on the surface are analogous to the total voltage \(V_{\text{tot}}\) and current \(I_{\text{tot}}\) in the circuit \(E_{\text{tot}}^t \to V_{\text{tot}}\) and \(H_{\text{tot}}^t \to I_{\text{tot}}\). From (6) we see that the voltage is composed of infinitely many harmonics:

\[
V_{\text{tot}}(z,t) = \sum_{n=-\infty}^{+\infty} v_n e^{-j(k_2 z - \omega_n t)}.
\]

The total current is a superposition of all currents flowing in the conductance, inductance, and absorption, and the shunted transmission line: \(I_{\text{tot}} = I_G + I_L + I_D\). The partial current in each circuit component is also a sum of an infinite number of Floquet harmonics:

\[
I_q(z,t) = \sum_{n=-\infty}^{+\infty} i_{q,n} e^{-j(k_2 z - \omega_n t)}, \quad q \in \{G, L, D\}.
\]

Unlike the stationary scenario (without spatiotemporal modulation), in space-time modulated systems, \(V_{\text{tot}}(z,t)\) and \(I_q(z,t)\) cannot be simply related by a scalar impedance or admittance. Instead, one must consider all the harmonics from \(n = -\infty\) to \(n = +\infty\). Practically, we only take a finite number of harmonics from \(n = -N\) to \(n = +N\) into consideration (\(N\) is large enough to ensure the convergence of harmonics). The voltage and current at each element can be written as 2\(N\) + 1 dimensional vectors: \(\vec{v} = [v_{-N}, v_{-N+1}, \ldots, v_N]^T\) and \(\vec{i}_q = [i_{q,-N}, i_{q,-N+1}, \ldots, i_{q,N}]^T\). These voltage and current vectors can be related by an \((2N+1) \times (2N+1)\) admittance matrix \(Y_q\)

\[
\vec{i}_q = Y_q \vec{v}.
\]

Next, using the linear relations between voltage and current vectors, we find the admittance matrix for each lumped component in the circuit. The current flowing in the conductance reads

\[
I_G(z,t) = G(z,t) V_{\text{tot}}(z,t).
\]

After substituting Eqs. (2), (7), and (8) into Eq. (10), we have

\[
\sum_{n=-\infty}^{+\infty} i_{G,n} e^{-j(k_2 z - \omega_n t)} = \sum_{\ell,m=-\infty}^{+\infty} g_m v_{\ell} e^{-j(k_2 \ell + m z - \omega_{\ell + m} t)},
\]

Using the change of variable \(\ell \to \ell - m\), the \(n\)-th current harmonic can be written as

\[
i_{G,n} = \sum_{m=-\infty}^{+\infty} g_m v_{n-m}.
\]

We can see that the current harmonics are not only induced by the \(n\)-th voltage harmonic, but also contributed by coupling with other harmonics. Equation (12) must be satisfied for all harmonics, generating a system of equations formed by \(2N + 1\) linear equations \((n \in [-N,N])\). The equations can be written in the matrix form \(\vec{i}_G = Y_G \vec{v}\), which can be expanded as

\[
\begin{pmatrix}
i_{G,-N} \\
i_{G,1-N} \\
i_{G,N}
\end{pmatrix} =
\begin{pmatrix}
g_0 & g_{-1} & \cdots & g_{-2N} \\
g_1 & g_0 & \cdots & g_{-1-N} \\
g_{2N} & g_{2N-1} & \cdots & g_0
\end{pmatrix}
\begin{pmatrix}
v_{-N} \\
v_{-N+1} \\
v_{N}
\end{pmatrix}.
\]

Here, \(Y_G\) is called admittance matrix. It should be noted that \(Y_G\) is a Toeplitz matrix with elements \(Y_G(s,t) = g_{s-t}\), where \(s,t \in [-N,N]\) are the row and column indexes of the matrix, respectively.

Following a similar approach, we derive the admittance matrix for the space-time modulated inductance. The time-domain relation between the current and voltage across a time-varying inductor reads

\[
I_L(z,t)L(z,t) = \int_{t}^{t'} V_{\text{tot}}(z,t') dt'.
\]

After substituting Eq. (2) and Eq. (8) into Eq. (14) and using the same mathematical treatments in Eq. (11), the \(n\)-th harmonic of the current in the inductor can be written as

\[
i_{L,n} = \sum_{m=-\infty}^{+\infty} b_m v_{n-m}.
\]

The system of equations defined by Eq. (15) can be written also in the matrix form \(\vec{i}_L = Y_L \vec{v}\), where \(Y_L\) is the time-modulated inductor admittance matrix filled by elements \(Y_L(s,t) = b_{s-t}/j\omega t\).

Finally, we need to calculate the admittance matrix of the shunted transmission line, \(Y_D\). One must notice that different Floquet modes have different frequencies.
and propagation constants in the substrate. Since the substrate is not modulated, there are no coupling elements in \( Y_D \). Therefore, \( Y_D \) is a diagonal matrix with elements \( Y_D(n,n) = \gamma_D(n) \), where \( n \in [-N, +N] \). For TM-polarized waves, the elements of the matrix can be written as

\[
y_D, n = \frac{1}{z_{TM,n}} \tanh(jk_{z,n}d),
\]

where \( k_{z,n} = \frac{\omega_n^2 \varepsilon_0 \mu_0 - k_D^2}{\varepsilon_n^2} \) is the normal component of the propagation constant in the dielectric substrate and \( z_{TM,n} = k_{TM,n}/(\varepsilon_n \omega_n) \) is the characteristic impedance of dielectric substrate.

After the admittance matrices for all the circuit components are determined, the total admittance is defined as \( Y_{tot} = Y_G + Y_L + Y_D \). This matrix relates the total current and voltage (summation of the incident and reflected harmonics) of the circuit:

\[
\tilde{v}_i - \tilde{v}_o = \frac{1}{Y_{tot}}(\tilde{v}_i + \tilde{v}_o),
\]

where the superscripts ‘in’ and ‘re’ stay for the incident and reflected harmonics, respectively. The tangential electrical and magnetic fields of the incident and scattered harmonics are related by the characteristic impedance matrix of free space \( Z_0 \). For TM incidence, the impedance \( Z_0 \) is a diagonal matrix filled with \( Z_0(n,n) = z_{TM,n} = k_{z,n}/(\varepsilon_n \omega_n) \), where \( k_{z,n} = \sqrt{\omega_n^2 \varepsilon_0 \mu_0 - k_D^2} \) is the vertical wavenumber of \( n \)-th harmonic. Therefore, we have

\[
\tilde{v}_i^{in} = Z_0 \tilde{v}_i^{in}, \quad \tilde{v}_o^{re} = Z_0 \tilde{v}_o^{re}.
\]

Substituting (18) into (17), we can find \( \tilde{v}_o^{re} = -\Gamma \tilde{v}_i^{in} \). The parameter \( \Gamma \) is called the reflection matrix and is given by

\[
\Gamma = (Y_{tot}Z_0 + I)^{-1}(Y_{tot}Z_0 - I),
\]

where \( I \) is the \((2N + 1) \times (2N + 1)\) unity matrix. The source vector is defined as \( \tilde{v}_i^{in} = [0, \ldots, 0, 1, 0, \ldots, 0]^T \). We can see that the \( n \)-th reflected harmonic is equal to the matrix element located at the \( n \)-th row and 0-th column of the reflection matrix \( \Gamma \), \( \tilde{v}_o^{re} = \Gamma(n,0) \).

The developed theory not only works for shunt-connected modulation components in Fig. (b) but can be generalized to arbitrary connection forms, for example, series connection of RLC components, which will be discussed in Section VII. The basic idea is to find the admittance matrix of a circuit branch relating the total voltage and current. The theory is applicable also to TE incidence. In fact, the polarization states only affect \( Y_D \) and \( Z_0 \). The method can be easily adapted for TE-polarized wave after replacing \( z_{TM,n} \) with \( z_{TE,n} = \mu_0 \omega_n/k_{z,n}^D \), and \( z_{TM,n} \) with \( z_{TE,n} = \mu_0 \omega_n/k_{x,n}^D \).

If the surface conductance and inductance in Fig. (b) and the substrate properties are known, for a specific plane wave excitation the scattering harmonics are uniquely determined by Eq. (19). Alternatively, for desired scattering responses, it is also possible to find properly modulated conductance and inductance that correspond to the generation of the desired scattering harmonics by the metasurface. In the following sections, we focus on controlling scattered fields (both propagating and evanescent) by engineering \( G(z,t) \) and \( B(z,t) \), with the goal to realize desired nonreciprocal effects.

III. ENGINEERING NONRECIPROCITY OF SPACE-TIME MODULATED METASURFACES

In this section, we show that the generalized circuit model developed in Section II can be used to realize different nonreciprocal devices and optimize their performances. We will present the design methodology to realize isolators, phase shifters, quasi-isolators, and circulators.

A. Metasurface Isolators

An isolator is a lossy, passive, and matched two-port device that allows unidirectional wave transmission [40]. For a perfect isolator, waves incident on one port are fully absorbed, while the waves incident on the second port are fully transmitted without frequency conversion. The scattering matrix of an ideal isolator is defined as

\[
\Xi = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.
\]

Figure 2 shows the design principle of the metasurface isolator. Plane waves incident at \( \theta = +\theta_i \) (Port 1) are completely absorbed, while waves incident at \( \theta = -\theta_i \) (Port 2) all are specularly reflected into Port 1 \((n = 0)\). It is important to notice that, according to this definition, there is no frequency conversion between ports.

![FIG. 2. Schematic representation of the metasurface platform for implementing isolators.](image)
the incident wave from the attenuation port is not absorbed due to material losses but converted to higher-order harmonics and transmitted, requiring absorptive filters to remove the generated harmonics. In another work \cite{33}, although there is no frequency conversion of the scattered propagating waves, the waves incident from the attenuation port are not actually absorbed but leak away through other ports (transmission side). Here, we propose a solution based on a lossy gradient surface impedance that allows us to design isolators according to the classical definition shown in Eq. (20). The design methodology consists of three steps.

**Step 1:** Making use of the multi-channel metasurface concept, the first step in the design is to find the period of the metasurface space modulation that allows the required propagating channels. In this case, the two-port system will be implemented in the metasurface platform by the incident direction and the specular reflection, as it is shown in Fig. 2. This condition can be fulfilled by restricting the spatial period as \( D < \lambda_0/(1 + \sin \theta_1) \). In addition, this periodicity ensures that both \( S_{11} \) and \( S_{22} \) are zero (no retroreflection or other higher order diffraction modes).

**Step 2:** Once the spatial period of the metasurface has been chosen, the second step is to find a proper traveling-wave modulation profile that can realize perfect absorption for plane-wave incident from \( \theta = +\theta_1 \) (Port 1). We define the frequency of incidence as \( \omega_0 = \omega_0 \), and the modulation frequency is \( \omega_M = \omega_0/10^3 \). For total absorption at Port 1, the scattered field should have no specular reflection which means

\[
|\Gamma(0,0)| = 0 \quad (21)
\]

should be satisfied for incidence of \( \theta = +\theta_1 \). Next, we ensure strong evanescent fields are excited simultaneously, which is the key point of this design method. The reason for the excitation of evanescent fields includes: (1) the wavelength of these fields are much smaller than the spatial modulation periodicity, and they are generated in order to locally satisfy the gradient impedance boundary condition. Therefore, they tend to be sensitive to the surface boundary perturbation, as well as the incident angles; (2) the amplitudes of evanescent fields can control the resonance strength. To warrant the excitation of evanescent fields, we impose,

\[
|\Gamma(1,0)| = A_1. \quad (22)
\]

Notice that we only need to ensure the desired amplitude of the first-order evanescent mode \((n = 1)\). Eq. (21) and Eq. (22) are called objective functions that guarantee evanescent fields induced absorption at Port 1.

Next we aim to find the optimal surface impedance that can satisfy these two objectives. From the results of Section II, we know that \( \Gamma(n,0) \) are determined by the Fourier coefficients of \( G(z,t) \) and \( B(z,t) \). In order to find \( g_m \) and \( b_m \) that satisfy the objectives, we can introduce unlimited numbers of \( g_m \) and \( b_m \) in Eq. (22). However, we assume that only three Fourier terms are enough to adequately describe each component \(|g_{0,\pm 1}| \) for \( G(z,t) \) and \( |b_{0,\pm 1}| \) for \( B(z,t) \). Without loosening generality, we also assume that \( g_m = g_{-m} \) and \( b_m = b_{-m} \) (this assumption always holds in the following text), such that the Fourier terms are real numbers and both \( G(z,t) \) and \( B(z,t) \) are even functions with respect to the \( z \)-axis. Under these assumptions, the Fourier series for \( G(z,t) \) and \( B(z,t) \) in Eq. (11) can be reduced to

\[
G(z,t) = g_0 + 2g_1 \cos(\beta_M z - \omega_M t) \quad (23)
\]

and

\[
B(z,t) = b_0 + 2b_1 \cos(\beta_M z - \omega_M t) \quad (24)
\]

At this point, in order to preserve the physical meaning of the model, one must ensure that the conductance and inductance are always positive along the \( z \)-axis. To do that, two additional constraints are introduced, \( g_0 - 2|g_1| > 0 \) and \( b_0 - 2|b_1| > 0 \).

In total, there are two objective equations and two linear constraints to be considered with four unknown Fourier coefficients. To solve this optimization problem, we use MATLAB Optimization Toolbox to find the solutions of \( |g_0| \) and \( |b_0| \), using the optimization function \( fmincons \). Table I shows the optimized \( b_m \) and \( g_m \) for different evanescent mode amplitudes.

**Step 3:** Once the surface impedance satisfying the objective functions at \( \theta = +\theta_1 \) has been optimized, we examine its scattering fields for \( \theta = -\theta_1 \) using the mode matching method presented in Section II. For incidence from Port 2, the resonance shifts to higher frequency (dashed line in Fig. 3). The space-time modulated surface emulates a moving slab with velocity of \( v_M \) along the +\( z \)-direction. Effective motion of the metasurface compromises the established phase matching of the incident plane wave and one of the surface modes (as designed at Step 2). For incidence from Port 2, the phase matching condition holds at lower frequencies. This effect is similar to the phenomenon in \cite{34}, based on asymmetric distortion of the dispersion diagram under travelling-wave modulation. For the case of \( A_1 = 1 \), although nonreciprocity is achieved, the isolation performance at the defined frequency \( \omega_0 = \omega_0 \) is not good, since the wave incident from port 2 is almost absorbed

| Evansenct field \((n=1)\) | \(g_0\) \(g_1\) \(b_0\) \(b_1\) \(|S_{12}|^2\) \(|S_{21}|^2\)| (dB) |
|---|---|---|---|---|---|---|
| \(A_1 = 1\) | 208.15 | -38.1 | 43.36 | -11.42 | -21.6 | -83 |
| \(A_1 = 3\) | 26.17 | -5.50 | 36.03 | -3.46 | -5.37 | -64 |
| \(A_1 = 10\) | 2.29 | -0.67 | 35.25 | -1.03 | -6.08 | -43.7 |
Similarly to the design of metasurface isolators, next, we ensure proper excitation of resonant modes. The proposed design methodology consists of the following steps.

Step 1: Similarly to the design of metasurface isolators, the period of the metasurface phase shifter is selected within the range $D < \lambda_0/(1 + \sin \theta_i)$ to allow only specular reflection. This condition will warrant $S_{11} = S_{22} = 0$.

Step 2: Next, we ensure proper excitation of resonant evanescent fields for incidence of $\theta = +\theta_i$. As we know from the design principle of the metasurface isolator (Section III A), such resonance is very sensitive to the transverse unidirectional modulation momentum and the resonance frequency is expected to shift at the opposite incidence ($\theta = -\theta_i$). Following this idea, we introduce one objective function $|\Gamma(1, 0)| = A_1$ at $\theta = +\theta_i$, which guarantees that the first order evanescent field is excited with amplitude $A_1$. Since the system is required to be lossless, the surface conductance is removed ($G = 0$), and only the inductance is subject to modulation. We assume a simple harmonic modulation waveform on $B(z, t)$

$$B(z, t) = b_0 + 2b_1 \cos(\beta_M z - \omega_M t),$$

where $\omega_M = 0.01\omega_0$. Notice that the constraint $b_0 - 2|b_1| > 0$ should be imposed to warrant that the inductance values remain positive. Using optimization, a proper set of $b_0$ and $b_1$ can be found to make sure that $A_1 = 3$ (as an example). We can see from Fig. 5 (blue dashed line) that the transmission phase has an abrupt change around the defined resonant frequency $\omega_d$.

Step 3: After obtaining the optimal surface impedance, we investigate its transmission phase for the opposite incidence. Similar to the case of metasurface isolators (see Section III A), sharp resonance in the phase spectrum shifts to higher frequency, creating a large phase difference at a fixed frequency. For example, at $\omega_0 = 1.003\omega_d$ and $\omega_0 = 1.023\omega_d$, the devices exhibits $-\pi$ and $+\pi$ phase shifts for waves traveling in the opposite directions, which can be practically used as a gyrator. The phase shift is continuously tunable by increasing or decreasing the modulation frequency, which adjusts spectral distances of the two resonances. The frequency bandwidth of such phase shifters depends on the quality factor of the resonance which is determined by $A_1$. Decreasing $A_1$ and increasing the modulation frequency we can obtain a more flat phase curve which effectively expands the frequency bandwidth.
The first step is to define the spatial period of the metasurfaces. The studied surface impedance is synthesized with \( b_0 = 39.85 \times 10^{10} \) and \( b_1 = -7.50 \times 10^{10} \). The spatial period is \( D = 0.419\lambda_0 \).

### C. Metasurface Quasi-Isolators

In Section II-A we have shown how one can realize wave isolators using lossy modulated impedance sheets and demonstrated near perfect isolation without the need of frequency filters. Here, we show that wave isolation can be realized also in lossless two-port systems, as schematized in Fig. 6. Instead of being absorbed, as in usual isolators, the energy from Port 1 is fully reflected black (retro-reflection) with a frequency conversion \( \omega_1 \rightarrow \omega_0 - \omega_M \), while the energy from Port 2 is fully delivered to Port 1 (specular reflection) without frequency conversion \( \omega_0 \rightarrow \omega_0 \). It is important to notice that, although frequency conversion occurs when the system is illuminated from Port 1, this higher-order harmonic is not delivered to Port 2 and, for this reason, frequency filters are still not necessary. These devices provide the same functionality as ideal isolators if one considers the signal frequency, and therefore we call them as quasi-isolators. Note that ideal lossless isolators without frequency conversion or conversion to some other mode or channel are forbidden by energy conservation. The design of these devices has two steps.

**Step 1:** The first step is to define the spatial period of the metasurface that allows not only specular reflection but also retro-reflection. This requirement can be satisfied choosing \( D = \lambda_0 / (2 \sin \theta_0) \). In this case, we still create a two-port system, but the incident waves are allowed to reflect back into the same port (retro-reflection). Notice that the difference with the lossy isolator where the condition \( S_{11} = S_{22} = 0 \) was automatically fulfilled by choosing the period which does not allow propagation of any diffracted mode, including the retro-reflected one.

**Step 2:** The second step is to realize perfect retro-reflection for incidence from Port 1 (\( \theta = +\theta_1 \)) at the target frequency \( \omega_1 \). Notice that in the case of ideal retro-reflection the energy transmitted to Port 2 is zero due to energy conservation. In addition, the evanescent fields should be specified to control the \( Q \)-factor of the system. Taking these factors into considerations, we design these devices in two steps.

Combining these constraints with the linear condition that warrants only positive values of the inductance, \( B(z,t) > 0 \), one can see that at least three degrees of freedom are required for specifying the system. For this reason, three unknowns \( (b_0, b_1, b_2) \) are introduced to synthesize \( B(z,t) \). We can write the expression for \( B(z,t) \) as

\[
B(z,t) = b_0 + \sum_{m=1}^{m=2} 2b_m \cos[m(\beta_M z - \omega_M t)].
\]

where modulation frequency \( \omega_M = 2\omega_A / 10^4 \) is assumed in this example. Using optimization tools, we find the Fourier coefficients for three different values of \( A_1 \), shown in Table III

### TABLE II. Optimized amplitudes of the Fourier harmonics for different values of the evanescent-field amplitude.

| Evanescent field \((n = 1)\) | \(b_0 \times 10^{10}\) | \(b_1 \times 10^{10}\) | \(b_2 \times 10^{10}\) | \(|S_{12}|^2\) (dB) | \(|S_{21}|^2\) (dB) |
|---|---|---|---|---|---|
| \(A_1 = 1\) | 229.83 | 30.19 | -37.70 | -15.73 | -81.87 |
| \(A_1 = 3\) | 192.52 | 12.34 | -13.86 | -2.41 | -46.02 |
| \(A_1 = 10\) | 187.79 | -6.30 | 6.13 | -0.04 | -42.42 |

Figure II presents the scattered harmonics at two opposite incidence for three optimal surfaces. The first panel shows the results for \( A_1 = 1 \). We can see that the harmonics excited by plane waves at Ports 1 and 2 (incidence angles +45° and -45°) are almost identical, meaning that if the amplitude of the first-order evanescent harmonic is so small, there is no significant phase mismatch for excitation from the other port. Therefore, waves incident on both ports are reflected back into the same port. However, as \( A_1 \) increases, the enhanced resonance of evanescent fields becomes more sensitive to the angle of illumination. For the case when \( A_1 = 10 \), the excited evanescent modes are completely different for illuminations from the opposite angles, which leads to enhanced transmission from Port 2 to Port 1 and realization of the...
First, we increase the spatial period of the metasurface to \( D = \lambda_0 / \sin \theta_{i} \). This periodicity allows propagation of two additional diffracted modes, so that the metasurface can be viewed as a three-port system (see Figure 8).

**D. Metasurface Circulators**

With the proper choice of spatial period and using more complex time-modulation function, it is possible to control nonreciprocal wave propagation between more ports. As a particular example, we use the design method explained above to realize three-ports metasurface circulators. Figure 8 shows the schematics of the proposed metasurface circulator, where the incident wave travels unidirectionally between three ports: 1 → 3, 3 → 2, and 2 → 1. It is important to note that the device introduces frequency down-conversion (\( \omega_0 \rightarrow \omega_0 - \omega_M \)) for propagation from 1 → 3, and up-conversion (\( \omega_0 - \omega_M \rightarrow \omega_0 \)) for propagation from 3 → 2. A similar functionality has been realized in a recent work [25] exciting two independent optical modes with distance (\( \beta_M, \omega_M \)) in the dispersion diagrams and creating a high-quality resonator system. However, the known design method needs careful numerical optimizations of the band structures to ensure proper support of multiple optical modes. In addition, these optical modes should be tailored to be strongly resonant (high \( Q \)), to spectrally resolve them.

Here, we show that using the mode-matching method and mathematical optimization one can easily find surface impedances that realize perfect wave circulation, thus avoiding complicated band structure engineering. The design methodology for metasurface circulators consists of two steps.

**Step 1:** First, we increase the spatial period of the metasurface to \( D = \lambda_0 / \sin \theta_{i} \). This periodicity allows propagation of two additional diffracted modes, so that the metasurface can be viewed as a three-port system (see Figure 8).

**Step 2:** The second step is to find the constraint functions for the metasurface circulator design. Enforcing that waves incident from Port 1 are fully directed to Port 3 and considering that the system is space-time modulated (\( \omega_M \neq 0 \)), the amplitude of the \( n = -1 \) diffraction mode should be \( |\Gamma(1, 0)| = 1 / \cos \theta_{i} \) due to the energy conservation principle [4]. This condition automatically ensures photonic transition between two different optical modes as [26] aims to achieve. Then we prescribe the amplitude of the first-order evanescent field harmonic as \( |\Gamma(1, 0)| = A_1 \), to control the quality factor of the system.

In the system optimization we define two constraints. However, we should notice that, as the number of ports increases, the positive-inductance condition is more difficult to satisfy if we only introduce a few Fourier terms in the modulation function. For this reason, more Fourier harmonics are included in \( B(z, t) \) to provide more freedoms in the construction of a purely inductive modulated sheet impedance. In particular, we introduce five unknowns in \( B(z, t) \):

\[
B(z, t) = b_0 + \sum_{m=1}^{m=4} 2b_m \cos\{m(\beta_M z - \omega_M t)\} \quad (29)
\]

Next, the Fourier harmonics of \( B(z, t) \) are optimized, and the resulting values for the harmonics are summarized in the caption of Figure 8.

Now, we examine the scattering harmonics when the circuit is illuminated from each of the three ports. The results are shown in Figure 8. In the first scenario (left panel in Figure 8), the wave incident from \( \theta = +\theta_i \) is fully delivered to the normal direction (Port 2) with strong evanescent fields excited on the surface. In the second scenario (middle panel in Figure 8), the wave incident from Port 3 goes to Port 2 and excites the same sets of evanescent fields with scenario 1. This behavior is ensured by the phase matching condition established for scenario 1. Finally, for the wave incident from \( \theta = -\theta_i \) (Port 3), the incident wave propagates in the opposite direction with the modulation wave. Due to the strong phase mismatch, the resonance disappears and the device behaves as a normal mirror with full transmission to Port 1.

As the number of channels increases, more Fourier harmonics should be incorporated in the design of the mod-
ulation function, which opens up the possibility to realize wave circulation between five or more ports.

![Graph showing harmonic order n vs amplitude (dB)](image)

**FIG. 9.** Calculated harmonics for excitation from (a) \( \theta = +45^\circ \), (b) \( \theta = 0^\circ \) and (c) \( \theta = -45^\circ \). The modulation frequency is \( \omega_M = \omega_d/10^3 \). The blue squares represents for propagating modes and the yellow circles means evanescent modes. The optimized harmonic amplitudes of \( B(z,t) \) are \( b_0 = 198.54 \times 10^{10} \), \( b_1 = 22.71 \times 10^{10} \), \( b_2 = 8.77 \times 10^{10} \), \( b_3 = 7.18 \times 10^{10} \) and \( b_4 = 1.47 \times 10^{10} \). The prescribed evanescent field amplitude is \( A_1 = 3 \).

### IV. IMPLEMENTATIONS BASED ON GRAPHENE PLATFORM

In Section III we have demonstrated that various non-reciprocal functionalities can be realized within the proposed space-time modulated surface impedance platform. For actual implementations, the modulations of surface impedance can be achieved in many means. In the microwave band, integrated circuits (ICs) with incorporated varactors and varistors can be embedded into the metasurface, in order to independently control the local resistance and reactance of the meta-atoms. In this case, it is possible to implement all the functionalities discussed in Section III and more. It is worth to mention that in practice it is usually easier to realize temporal modulation of capacitance than inductance. For this reason, it may be reasonable to mimic time-varying inductance using a large fixed inductance in series with a time-modulated capacitance. In this way, the total reactance is inductive and modulated in time, producing the same effect as time-varying inductance. In the terahertz and mid-infrared band, graphene is a good candidate to implement the tunable component of space-time modulated metasurfaces due to its tunable electrical conductivity and the compatibility with the advanced nanofabrication technologies [29, 30, 38, 39, 42]. Considering that graphene is a lossy material, in this section we show how one can use space-time modulated graphene sheets to create tunable wave isolators in the terahertz range.

It is known that the sheet conductivity of graphene can be effectively modified via electrical bias or optical pumping. This provides the possibility of synthesizing space-time modulated surface impedances by locally varying the bias voltage applied on graphene. The sheet conductivity of graphene in the low terahertz range can be modeled as

\[
\sigma_s = \frac{\sigma_0}{1 + j\omega\tau},
\]

where

\[
\sigma_0 = \frac{e^2\tau k_B T}{\pi\hbar^2} \left[ \frac{E_{F0}}{k_B T} + 2 \ln \left( e^{\frac{E_{F0}}{k_B T}} + 1 \right) \right]
\]

is the DC conductivity, \( \tau \) is the electron scattering time, \( e \) is the electron charge, and \( E_{F0} \) is the Fermi level of graphene which can be controlled by the external voltage. As can be seen from Eqs. (30) and (31), graphene is naturally resistive and inductive in the terahertz range. If spatiotemporally varying gate voltage is exerted on a graphene strip, both of its sheet resistance \( (R_s = 1/\sigma_0) \) and inductance \( (L_s = \tau/\sigma_0) \) can be harmonically modulated with the same pumping scheme.

![Schematic representation of a practical design](image)

**FIG. 10.** Schematic representation of a practical design. The surface impedance of one unit cell is implemented by \( k \) periodically arranged graphene strips (sub-cells). The size of one sub-cell is \( P = 2 \mu m \), and the gap width between two neighboring strips is \( g = 100 \text{ nm} \). The permittivity of SiO2 substrate is \( \epsilon_r = 4 \), and its thickness equals \( d = 4 \mu m \). The local gate voltages on graphene are changing with time. When a wave is incident from \( \theta = +\theta_i \), it is transformed into evanescent fields and eventually dissipated in graphene (red wave). For the incidence at \( \theta = -\theta_i \), it is reflected in the specular direction (purple wave).

Figure 10 shows the schematic representation of the proposed design where graphene strips are periodically positioned on top of a metalized substrate. For the convenience of gating, a pair of graphene strips are separated by a thin dielectric film (e.g., Al2O3) to form a self capacitor [29]. Due to the stacked structure, the effective sheet impedance of graphene is reduced to half of the
single-layer graphene impedance. Under TM polarized incidence, the electrical response of graphene patterns can be described as a series of resistance, inductance and capacitance (all of them are effective values). The static resistance and inductance of graphene without modulation are

\[
R_0 = \frac{R_g}{2} \left( \frac{P - g}{g} \right), \quad L_0 = \frac{L_g}{2} \left( \frac{P - g}{g} \right),
\]

where \( P \) is the period of the graphene strip array, and \((P - g)/g\) is the structural factor which takes into account the enlargement of resistance and inductance due to the patterning of graphene \[48\]. The capacitance \( C_0 \) is contributed by capacitive coupling of the adjacent strips \[49\].

\[
C_0 = \frac{2}{\pi} \varepsilon_0 \varepsilon_r P \ln \left[ \frac{\sin \left( \frac{\pi g}{2P} \right)}{\sin \left( \frac{\pi g}{2P} \right)} \right]
\]

with the effective permittivity \( \varepsilon_{\text{eff}} = (\varepsilon_r + 1)/2 \). Since this capacitance is determined only by the structuring of graphene and the substrate permittivity, it does not change during the modulation. A pump voltage is exerted on graphene, and the resistance and inductance of graphene are modulated around their static values. We assume the modulation function in form

\[
f_M(z, t) = a_0 + \sum_{m=1}^{+\infty} 2a_m \cos[m(\beta_M z - \omega_M t)]
\]

with \( a_0 = 1 \). Thus, the space-time lumped values are \( R(z, t) = R_0 f_M(z, t) \) and \( L(z, t) = L_0 f_M(z, t) \).

The current \( I_S(z, t) \) and voltage \( V_{\text{tot}}(z, t) \) of the series circuit are related as

\[
V_{\text{tot}}(z, t) = R(z, t) I_S(z, t) + L(z, t) \frac{dI_S(z, t)}{dt} + I_S(z, t) \frac{dL(z, t)}{dt} + C_0^{-1} \int_{t'}^t I_S(z, t') dt'.
\]

Following the same mathematical treatment as in Section II we obtain the coupling equation

\[
\sum_{m=\pm \infty} \left( R_0 + j\omega_n L_0 \right) a_m i_{S,n} - m + \frac{1}{j\omega_n C_0} i_{S,n} = v_n,
\]

where \( i_{S,n} \) and \( v_n \) are the nth harmonic components of \( I_S \) and \( V_{\text{tot}} \), respectively. The voltage and current vectors are related by the impedance matrix, \( Z_{S} v_S = v \). \( Z_{S} \) is a \((2N + 1) \times (2N + 1)\) matrix filled with elements

\[
Z_S(s, t) = (R_0 + j\omega_n L_0) a_{s-t} + \frac{\delta(s - t)}{j\omega_n C_0},
\]

where \( \delta(x) \) is the Dirac delta function. After \( Z_S \) is known, the admittance matrix is found by inversion: \( Y_S = Z_S^{-1} \).

Interestingly, at other frequencies, we can always find an optimal sets of \( a_1 \), \( a_2 \) and \( E_{F0} \) to reconstruct the isolation functionality. Figure 12 shows that the absorption frequency (from \( \theta = +45^\circ \)) decreases as we lower the medium Fermi level of graphene \( E_{F0} \). In addition, in order to achieve good isolation performance, the modulation function \( f_M(z, t) \) should be re-optimized to maximize the absorption level. Due to the modifications of the modulation function, the isolator can operate over a wide band from 5.5 THz to 12 THz without sacrificing the isolation performance.

The isolation is determined by forward attenuation as well as by reversed insertion loss. After perfect absorption is ensured in the forward direction (\( S_{21} = 0 \)), the transmission for the opposite incidence \( S_{12} \) is affected by the modulation speed which in fact determines the spectral distance between the two transmission minima.

For this reason, all-optical modulation schemes may be adopted for fast modulation \[50\]. On the other hand,
enhancing the quality factor of the resonance also alleviates the reverse insertion loss. This requires reduction of the effective sheet resistance of graphene, which can be achieved either by improving graphene quality (increasing $\tau$) in the fabrication process or employing metal-graphene hybrid structures [51].

V. CONCLUSIONS

In this paper, we have introduced a general approach to realize nonreciprocal metasurface devices to control waves in single or multiple scattering channels in free space. Importantly, we have shown that a variety functionalities can be realized using the same physical platform by adjusting the space-time modulation of the single tunable element: an impedance sheet. As examples, the proposed technique has been applied to design nonreciprocal metasurfaces which act as isolators, nonreciprocal phase shifters, quasi-isolators, and circulators. In addition, a graphene-based modulation platform has been proposed, where the isolation frequency can be dynamically tuned in a wide frequency range by changing the modulation function. We expect that the introduced technique can be useful in the developments of future computer-controlled intelligent nonreciprocal metasurfaces for various applications.

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