Analytical Design of Dual-Band Negative Group Delay Circuit With Multi-Coupled Lines

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This work was supported in part by the NSFC under Grant 61971230 and Grant 61601233, in part by the Jiangsu Distinguished Professor program and Six Major Talents Summit of Jiangsu Province under Grant 2019-DZXX-022, in part by the Postgraduate Research and Practice Innovation Program of Jiangsu Province under Grant SJKY19_0974, and in part by the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD) fund.

ABSTRACT This paper develops an innovative design method of a multi-coupled line (CL) topology of low-loss dual band negative group delay (NGD) circuit. The proposed is conceived using three transmission lines (TLs) with different length in which integrate two couplers. The three traveling paths are designed to generate the dual-band NGD effect, where the CLs allow lowering the signal attenuation. The NGD circuit S-parameter model as a function of the TLs physical lengths and coupling coefficient are established. To validate the NGD circuit proof-of-concept, it is fabricated and measured. The NGD values of approximately –4.06 ns and –3.83 ns are measured at center frequencies of 2.43 GHz and 3.02 GHz, respectively. The measured transmission coefficient is better than –2.9 dB and the measured reflection one is better than –12 dB into the NGD band. The measurement results of the NGD circuit are in good agreement with the simulations results.

INDEX TERMS Low-loss, dual-band, negative group delay (NGD), S-parameter model, coupled lines.

I. INTRODUCTION

Theoretical analysis and experimental researches in the microwave field demonstrated that the existence of negative group delay (NGD) phenomenon was reasonable at certain passive circuits [1]–[5]. This abnormal physical phenomenon can be proven to occur around the resonance frequency of the absorptive media.

The NGD special effect with respect to time advancement without violation of causality and relativity [1], [6] usually plays an important role in designing high performance microwave phase shifters [7]–[9] or delay equalizers [10]–[12], feedforward and feedback amplifiers [13]–[15], antennas [16]–[19], and non-Foster elements [20]–[23]. For example, to resolve signal distortion caused by delay fluctuations in wireless communication systems, an equalization solution to the delay compensation and signal recovery was introduced in [10]. In addition, further novel methods of higher performance microwave circuits based on the NGD circuit for designing feedforward [13], [14] and feedback amplifier [15] topologies were developed. These innovative NGD circuits were designed with competitive size reduction, enhanced efficiency, and significant improvement on the bandwidth, as introduced in [13]–[15]. As stressed in [16] and [19], non-foster elements synthesized by NGD networks are employed to overcome beam squinting of series-fed antenna arrays. Furthermore, in order to exhibit unconditional stability, a novel type of non-Foster element by using a microwave transversal-filter-based NGD circuit is presented in [21].

In recent years, with the orthodox development of its research and exploration, the NGD circuits can be divided into two categories: active and passive NGD circuits. The NGD functional characteristic can be implemented easily by
using some simple active microwave circuits that initially depended only on passive lumped elements, such as resistor R, inductor L, and capacitor C [24]–[26]. The low-pass, high-pass, bandpass, and stop-band NGD circuit cells were designed to understand the basic NGD properties and synthesis methodology in [26]. Most of these existing active NGD microwave circuits based on the use of low noise amplifiers (LNAs) and field effect transistors (FETs) associated with lumped RLC resonant networks have recently been proposed and developed [27]–[30]. It was found that these active NGD circuits would unavoidably suffer from design inflexibility restrictions on lumped components as well as increasing design difficulties in the microwave band.

To overcome these limitations, some topologies of the passive NGD circuits implemented by distributed elements such as transmission line (TL) and coupled line (CL) were developed [31]–[34]. In [31] and [34], the passive NGD circuits have unavoidable large signal attenuation in the desired NGD bandwidth. To obtain low signal attenuation, two short-circuited coupled lines with high characteristic impedance and small coupling coefficient. The circuits’ losses in the case of types-I and type-II NGD networks were decreased to 7.43 dB and 9.23 dB, respectively, [35]. Furthermore, a general-purpose gain amplifier can be used to compensate signal attenuation of the passive NGD networks [16], [36]. However, it can increase the out-of-band noise thus making the circuit more complicated.

Compared to single-band NGD microwave devices [16], [31]–[36], the design of dual-band NGD circuits [37]–[39] remains a challenging task, notably for RF non-specialist and microwave design engineers. Similarly to most of RF and microwave functions (such as filters, phase shifters, amplifiers, power combiners/dividers), the NGD functions should be susceptible to operate in different frequency bands. However, so far, few studies focused on the design of dual-band NGD circuits are available. As reported in the literature [37], a single-band reflection type NGD circuit composed of two short stubs are employed to achieve two separate frequency bands, where the attenuation has exceeded 30 dB. Among the existing dual-band NGD circuit, one can cite a miniaturized dual-band NGD passive device proposed in [38]. The design of this miniature circuit was carried out by utilizing dual-plane U-shaped defected structures (attenuation > 35 dB). An 18 dB attenuation was obtained using a compact dual-band NGD circuit composed of an open-circuited TL and two resistors connected by two TLs, as proposed in [39]. The drawbacks in the design of NGD circuit applications is the severe signal attenuation, which profoundly affects the NGD performances and limit their possibilities. Therefore, it becomes important to design low-loss dual-band NGD circuit. An active dual band is presented in [40] and an arbitrary frequency ratio dual band is presented in [41].

In this paper, a topology of passive circuit based on fully distributed elements is proposed. The design does not require any external RF amplifier. The proposed NGD topology consists of two identical coupled lines (CLs) and three different transmission lines’ length (TLs).

The paper is mainly organized in four different sections. In Section II, the topological analysis and S-parameter model are introduced. Based on the input and output wave powers analysis, the microwave circuit theory established from S-parameters model of the innovative NGD topology is developed. In Section III, the analytical expression of the group delay derived from transmission coefficient is presented. The NGD analysis as a function of frequencies related to the constituting NGD topology TL parameters is also given. To verify the feasibility of the design, the investigation of NGD circuit prototypes are described in Section IV. Afterwards, the theoretical, simulated, and measured results are compared and discussed. Finally, this paper is ended by a conclusion in Section V.

**II. TOPOLOGICAL ANALYSIS AND S-PARAMETER MODEL**

This section mainly describes the theoretical analysis of the multi-coupled line NGD topology. After description of the topological structure, the NGD circuit design is conceived from the combination of the CL and TL theories, based on the S-parameter model. The group delay analytical expression is also provided. Then, the NGD analysis will be introduced.

**A. TOPOLOGICAL ANALYSIS**

Fig. 1 shows the topology of the proposed NGD circuit under study. This circuit topology behaves as a two-port network without using any lossy lumped elements. The circuit consists of two identical CLs with ① - ③ - ⑥- ⑦ and ② - ④ - ⑤ - ⑧ connection ports. As depicted in Fig. 1, the left CL port ① and the right CL port ② serve as main input and output for overall NGD cell, respectively. Furthermore, the left CL port ③ (resp. ⑦, ⑧) and the right CL port ④ (resp. ⑤, ⑥) are interconnected through a lossy TL1 (resp. TL2, TL3) where the characteristic impedance is $Z = 50 \, \Omega$, $d_1$ (resp. $d_2$, $d_3$) is the physical length, $\tau_1$ (resp. $\tau_2$, $\tau_3$) is the delay, and $\alpha_1$ (resp. $\alpha_2$, $\alpha_3$) is the attenuation loss.

It is worth emphasizing that the TL length $d_m$ and the CL length $d_0$ have the following relationship:

\[
\begin{align*}
    d_1 &= 2d_0 \\
    d_2 &= 6d_0 \\
    d_3 &= 10d_0.
\end{align*}
\]
The expected S-matrix will be determined by the mean of the introduced in Fig. 1. To do this, a S-matrix model is needed. The main purpose of the present theoretical recall is to establish the group delay analytical expression of the topology introduced in Fig. 1. To do this, a S-matrix model is needed. The expected S-matrix will be determined by the mean of the TL and CL combined theories.

1) RECALL ON CL THEORY
According to the microwave theory, the CL S-matrix can be expressed as [32]:

\[
[S]_{CL} = \begin{bmatrix}
0 & k_1 & k & 0 \\
k_1 & 0 & 0 & k \\
k & 0 & 0 & k_1 \\
0 & k_1 & 0 & 0 \\
\end{bmatrix},
\]

where \( k \) is the CL coupling coefficient and:

\[
k_1 = -j\sqrt{1-k^2},
\]

and \( j^2 = -1 \). Here, the coupling coefficient \( k \) must satisfy the condition \( 0 < k < 1 \). In the present study, the CL propagation delay is assumed to be negligible.

2) RECALL ON TL THEORY
One assumes that the TLs is lossy, matched, and can be constituted by the topology shown in Fig. 1. Let us denote \( \tau_m \) the propagation delay and \( a_m \) the attenuation loss of the elementary transmission line, TL\( _m \) with \( m = \{1, 2, 3\} \). According to the TL theory, the TL\( _m \) S- matrices are given by

\[
[S]_{TLm} = \begin{bmatrix}
S_{11TLm} & S_{12TLm} \\
S_{21TLm} & S_{22TLm} \\
\end{bmatrix},
\]

with

\[
\begin{align*}
S_{11TLm} &= S_{22TLm} = 0 \\
S_{12TLm} &= S_{21TLm} = \alpha_m e^{-j\omega \tau_m}.
\end{align*}
\]

The parameters \( \tau_0 \) and \( a_0 \) represent the delay and attenuation loss of a microstrip line with a length of \( d_0 \), respectively. In addition, \( a_m \) and \( b_m \) (\( m = \{1, \ldots, 8\} \)) represent the input and output wave powers.

B. S-PARAMETER MODEL FROM THE TL AND CL COMBINED THEORIES
The main purpose of the present theoretical recall is to establish the group delay analytical expression of the topology introduced in Fig. 1. To do this, a S-matrix model is needed. The expected S-matrix will be determined by the mean of the TL and CL combined theories.

The parameters \( \tau_0 \) and \( a_0 \) represent the delay and attenuation loss of a microstrip line with a length of \( d_0 \), respectively. In addition, \( a_m \) and \( b_m \) (\( m = \{1, \ldots, 8\} \)) represent the input and output wave powers.
The proposed NGD topology transmission phase \(\varphi\) with detailed expression is written as

\[
\begin{align*}
S_{11}(j\omega) &= S_{22}(j\omega) = 0 \\
S_{21}(j\omega) &= S_{12}(j\omega) = \frac{k_1^2x_1 + k_2^2x_3 - x_1x_2x_3}{1 - k_1^2x_2x_3 - k_2^2x_1x_2}
\end{align*}
\]  

(15)

Consequently, the S-parameter element magnitude and group delay expressions will be analytically explored in the next section.

III. NGD THEORY OF THE MULTI-COUPLED LINE NGD TOPOLOGY

Based on the previously expressed S-parameter model, the NGD theory of the multi-CL based topology will be established in this section. Before the NGD analysis, the magnitude and phase of the transmission coefficient will be presented.

A. TRANSMISSION COEFFICIENT MAGNITUDE AND PHASE

The magnitude \(S_{21}(\omega) = |S_{21}(j\omega)|\) of the transmission coefficient formulated in (15) is given by

\[
S_{21}(\omega) = \frac{\sqrt{R_1^2(\omega) + R_2^2(\omega)}}{\sqrt{R_3^2(\omega) + R_4^2(\omega)}}.
\]

(16)

with

\[
\begin{align*}
R_1(\omega) &= \begin{cases} 
  k_2^2\alpha_1 \cos(\omega\tau_1) + k_2^2\alpha_3 \cos(\omega\tau_3) \\
  -\alpha_1\alpha_2\alpha_3 \cos[\omega(\tau_1 + \tau_2 + \tau_3)] \\
  \alpha_1\alpha_2\alpha_3 \sin[\omega(\tau_1 + \tau_2 + \tau_3)] \\
  -\alpha_1\alpha_2\alpha_3 \sin[\omega(\tau_1 + \tau_2 + \tau_3)] \\
  \alpha_1\alpha_2\alpha_3 \sin[\omega(\tau_1 + \tau_2 + \tau_3)] \\
  \alpha_1\alpha_2\alpha_3 \sin[\omega(\tau_1 + \tau_2 + \tau_3)] \\
  -k_2^2\alpha_1\alpha_2\cos[\omega(\tau_1 + \tau_2)] \\
  k_2^2\alpha_1\alpha_2\sin[\omega(\tau_1 + \tau_2 + \tau_3)] \\
  +k^2\alpha_1\alpha_2\sin[\omega(\tau_1 + \tau_2 + \tau_3)] \\
\end{cases}
\end{align*}
\]

(17)

The expected group delay calculated from previously expressed transmission phase (18) is defined by

\[
\tau(\omega) = -\frac{\partial\varphi(\omega)}{\partial\omega} = \tau_d(\omega) - \tau_n(\omega),
\]

(21)

with

\[
\begin{align*}
\tau_d(\omega) &= -\frac{\partial\varphi_d(\omega)}{\partial\omega} \\
\tau_n(\omega) &= -\frac{\partial\varphi_n(\omega)}{\partial\omega}.
\end{align*}
\]

(22)

Knowing the quantities expressed previously in (21) and (22), one has

\[
\tau_d(\omega) = \frac{R_1'(\omega)R_3(\omega) - R_2'(\omega)R_4(\omega)}{R_3^2(\omega) + R_4^2(\omega)}.
\]

(23)

with

\[
\begin{align*}
R_3 &= \begin{cases} 
  (\tau_2 + \tau_3)k_2^2\alpha_2\alpha_3 \sin[\omega(\tau_2 + \tau_3)] \\
  +k_1^2\alpha_2\alpha_3 \sin[\omega(\tau_1 + \tau_2)] \\
  +k_1^2\alpha_2\alpha_3 \cos[\omega(\tau_1 + \tau_2)] \\
  +k^2\alpha_1\alpha_2\sin[\omega(\tau_1 + \tau_2 + \tau_3)] \\
  +k^2\alpha_1\alpha_2\cos[\omega(\tau_1 + \tau_2 + \tau_3)] \\
\end{cases}
\end{align*}
\]

(24)

and

\[
\tau_n(\omega) = \frac{R_1'(\omega)R_1(\omega) - R_2'(\omega)R_4(\omega)}{R_1^2(\omega) + R_2^2(\omega)}.
\]

(25)

with

\[
\begin{align*}
R_1'(\omega) &= \begin{cases} 
  -\tau_1k_2^2\alpha_1 \sin(\omega\tau_1) + \tau_3k_2^2\alpha_3 \sin(\omega\tau_3) \\
  +k_1^2\alpha_2\alpha_3 \sin[\omega(\tau_1 + \tau_2 + \tau_3)] \\
  +k_1^2\alpha_2\alpha_3 \cos[\omega(\tau_1 + \tau_2 + \tau_3)] \\
  -k_2^2\alpha_1\alpha_2\sin[\omega(\tau_1 + \tau_2)] \\
  -k_2^2\alpha_1\alpha_2\cos[\omega(\tau_1 + \tau_2 + \tau_3)] \\
  -k_2^2\alpha_1\alpha_2\sin[\omega(\tau_1 + \tau_2 + \tau_3)] \\
  -k_2^2\alpha_1\alpha_2\cos[\omega(\tau_1 + \tau_2 + \tau_3)] \\
\end{cases}
\end{align*}
\]

(26)

To validate this multi-CL NGD theory, a circuit is designed to achieve a low loss dual-band NGD circuit. To materialize the feasibility of the NGD function, the next section will present the validation results from simulations and measurements.

IV. SIMULATED AND MEASURED BEHAVIOR OF A DUAL-BAND NGD TOPOLOGY

The present section discusses the validations of the developed NGD topology. The validation is originally focused on the proof-of-concept (POC) showing dual-band NGD behavior by means of S-parameter theory, simulations, and measurements. The simulations are carried out with the HFSS simulation tool while the measurements are realized using a Vector Network Analyzer (VNA).

A. POC DESIGN DESCRIPTION

Figs. 3 presents the design of the NGD POC. The NGD layout realized in the HFSS 3-D EM environment is shown in Fig. 3(a) and the optimized final physical dimensions are presented in Table 1. The picture of the fabricated NGD circuit is shown in Fig. 3(b).
The overall size of the circuit is 54 mm × 63 mm. The NGD prototype was built as a microstrip planar circuit, on the Rogers 3210 substrate. The TLs and CLs electrical and physical parameters are shown in Table 1. The curved TL2 and TL3 lines are designed to reduce the circuit size and generate a simple layout. It is noteworthy that to design the dual-band NGD circuits based on the topology under study with required arbitrary frequencies, we can proceed with:

- The calculations of delays $\tau_1$, $\tau_2$ and $\tau_3$ by controlling the physical lengths $d_1$, ($L_1$, $D_1$) and ($L_2$, $D_2$), respectively of the structure shown in Fig. 3 in function of the required frequencies.
- To control the frequency $f_1$, we can modify the physical length $d_1$.
- And acting as a distributed microwave circuit depending on TL design, the ratio between the dual NGD center frequencies $f_2/f_1$, can be chosen as the ratio between the fundamental and harmonic frequencies.

To confirm the theoretical hypothesis presented in Subsection II-B, it is noteworthy that the CL propagation delay associated to 10.56 mm is approximately 0.18 ns. This value can be neglected when compared to NGD absolute value, which is approximately $-4$ ns.

### B. PARAMETRIC ANALYSES OF $d_0$ AND $s$

Parametric analyses with respect to the physical length $d_0$ and coupling space $s$, have been conducted. The parametric analyses enable to explain more explicitly the length and coupling coefficient effects onto the NGD function. Two cases of S-parameter parametric simulations are carried out with HFSS® simulator from ANSYS® by changing $d_0$ and $s$.

#### 1) PARAMETRIC ANALYSIS OF THE LENGTH $d_0$

The current parametric analysis is mainly to explore the effect of physical length $d_0$ on NGD function. As indicated in Section II.A, the length of CLs and TLs are all related to $d_0$. By increasing $d_0$ will increase the length of CLs and the three TLs. Figs. 4 displays the obtained group delay GD, transmission loss $S_{21}$, and return loss $S_{11}$.

As highlighted in Fig. 4(a), by changing $d_0$ from 10 mm to 11 mm, it can be seen that the first and second center frequencies show a clear shift from 2.56 GHz to 2.33 GHz and 3.19 GHz to 2.91 GHz, respectively. The maximum delay varies less than 0.2 ns, $S_{21}$ varies less than 0.1 dB, while $S_{11}$ variation is less than 0.2 dB for the first and second center frequencies. Moreover, as depicted in Fig. 4(b) and Fig. 4(c), the multi coupler topology illustrates transmission coefficient better than 3 dB and reflection coefficient better than 15 dB.

#### 2) PARAMETRIC ANALYSIS OF CL SPACE $s$

The CL effect was investigated by varying coupling space $s$ from 3 mm to 4 mm during the S-parameter simulations.

| Components       | Description | Parameter | Value |
|------------------|-------------|-----------|-------|
| Dielectric substrate | Material    | Rogers 3210 | -     |
|                  | Relative permittivity | $\varepsilon_r$ | 10.2  |
|                  | Loss tangent | $\tan(\delta)$ | 0.0027 |
|                  | Thickness    | $h$       | 1.27 mm |
| Metallization    | Material     | Copper (Cu) | -     |
|                  | Conductivity | $\sigma$  | 58 MS/m |
| TL1              | Total length | $d_1$     | 21.12 mm |
|                  | width        | $w$       | 1.18 mm |
|                  | attenuation loss | $\alpha_1$ | -0.05 dB |
|                  | delay        | $\tau_1$  | 0.184 ns |
| TL2              | Total length | $d_2$     | 63.36 mm |
|                  | width        | $w$       | 1.18 mm |
|                  | attenuation loss | $\alpha_2$ | -0.15 dB |
|                  | delay        | $\tau_2$  | 0.552 ns |
|                  | partial length | $L_1$    | 15.092 mm |
|                  | diameter     | $D_1$     | 18.76 mm |
| TL3              | Total length | $d_3$     | 105.6 mm |
|                  | width        | $w$       | 1.18 mm |
|                  | attenuation loss | $\alpha_3$ | -0.25 dB |
|                  | delay        | $\tau_3$  | 0.92 ns |
|                  | partial length | $L_2$    | 17.771 mm |
|                  | diameter     | $D_2$     | 42.24 mm |
| CL               | length       | $d_4$     | 10.56 mm |
|                  | width        | $w$       | 1.18 mm |
|                  | Interspace   | $s$       | 3 mm |
|                  | Coupling coefficient | $k$    | -25.64 dB |

| Access line      | length       | $d_5$     | 10 mm |
|                  | width        | $w$       | 1.18 m |

FIGURE 3. NGD circuit prototype: (a) layout and (b) prototype.
C. DISCUSSION ON THE DUAL-BAND NGD RESULTS OF THE TESTED MULTI-COUPLED LINE CIRCUIT

The experimental validation of the proposed NGD topology will be explored in the present subsection. Comparisons of results from theory, HFSS® simulation from ANSYS®, and measurements are discussed.

Similar to classical microwave circuits, the NGD tests, which are based on S-parameter analysis, were realized with a Vector Network Analyzer (VNA). The fabricated NGD prototype, previously introduced in Fig. 3(b), was measured with VNA Rohde & Schwarz (ZNB-20, frequency band 100 kHz to 20 GHz).

Comparisons between the theory, HFSS simulated (“Simu.”), and measured (“Meas.”) results have been carried out in the frequency band from 2.2 GHz to 3.2 GHz. The theory, simulated, and measured GDs and S-parameters are plotted in Figs. 6. It can be pointed out that these comparative results agree well with the simulation results, which validates the design of the NGD circuit.
TABLE 2. The proposed circuit NGD specifications.

| Validation Method | Center frequency | GD (ns) | BW (MHz) | S21 (dB) | S11 (dB) |
|-------------------|------------------|---------|----------|----------|----------|
| Simu. | f1=2.436 GHz | -5 | 12 | -2.2 | -12 |
| | f2=3.022 GHz | -4.2 | 13 | -2.8 | -14.02 |
| Meas. | f1=2.436 GHz | -4.06 | 12 | -2.13 | -12.61 |
| | f2=3.022 GHz | -3.83 | 13 | -2.86 | -12.40 |
| Calc. | f1=2.382 GHz | -3.55 | 10 | -1.08 | - |
| | f2=3.062 GHz | -3.60 | 10 | -1.08 | - |

results present a very good agreement. As expected, in the working frequency band, the tested prototype behaves as a bandpass NGD behaviors around first and second center frequencies of approximately \( f_1 = 2.43 \, \text{GHz} \) and \( f_2 = 3.02 \, \text{GHz} \).

Table 2 summarizes the differences between theory, simulated, and measured NGD parameters. Figs. 6 illustrates the good performances of the multi coupler NGD topology in terms of group delay, transmission coefficient, and reflection coefficient at \( f_1 \) and \( f_2 \). As depicted in Fig. 6(a), the tested NGD circuit has NGD of approximately \(-4.06\) ns over 12 MHz bandwidth at \( f_1 \) and has NGD of approximately \(-3.83\) ns over 13 MHz bandwidth at \( f_2 \). It can be seen from Fig. 6(b) and 6(c) that in the test frequency band, \( S_{21} \) and \( S_{11} \) are globally better than \(-3\) dB and \(-12\) dB, respectively. The frequency shift, between the theory, simulation, and measurement results are mainly due to the dielectric substrate effective permittivity dispersion and the right-angles connection between TL and CLs. The theory results neglect the influence of these two factors, but the influences are included in HFSS simulation and test results. The slight difference in NGD, \( S_{21} \), and \( S_{11} \) values are mainly due to fabrication inaccuracies, dielectric substrate effective permittivity, and losses versus the aided computational method accuracy.

D. DISCUSSION ON THE PERFORMANCE OF THE PROPOSED NGD CIRCUIT

The basic performance comparative results between the proposed NGD topology and the existing NGD circuit topologies are summarized in Table 3. The introduced NGD topology presents the following advantages:

- Considerable fully distributed elements without lossy lumped component,
- Low signal attenuation, which is less than 2.86 dB,
- A reflection coefficient better than \(-12\) dB and without the need for external matching networks in the NGD bandwidth.

TABLE 3. Performance comparison.

| Ref. | \( f \) (GHz) | GD (ns) | \( S_{11} \) (dB) | \( S_{21} \) (dB) | FOM |
|------|---------------|---------|------------------|------------------|-----|
| [39] | \( f_1=2.14 \) | -3 | -34.2 | -17 | 0.0150 |
| & \( f_2=3.5 \) | -3.1 | -34.9 | -17 | 0.0100 |
| [40] | \( f_1=3.5 \) | -4.54 | -47.4 | - | 0.0023 |
| & \( f_2=5.15 \) | -4.2 | -38.8 | - | 0.0048 |
| [41] | \( f_1=0.667 \) | -1.19 | -18.2 | -24.8 | 0.0338 |
| & \( f_2=1.377 \) | -1.19 | -18.2 | -24.7 | 0.0332 |
| This work | \( f_1=2.436 \) | -4.06 | -2.13 | -12.61 | 0.0381 |
| & \( f_2=3.022 \) | -3.83 | -2.86 | -12.40 | 0.0358 |

The designed NGD has a better NGD performance and FOM, which is defined in [39] as

\[
FOM = |GD(f_0)| \times BW \times |S_{21}(f_0)|
\]  

It is noteworthy that the NGD frequencies are limited by:

- The substrate dispersions which are defined by the manufacturers,
- And the fabrication accuracies of the TL physical sizes and spaces.

V. CONCLUSION

An innovative analytical design of low-loss dual-band NGD circuit is investigated. The NGD topology consisting of two CLs and three TLs with different length is proposed. The difference in the physical length of the three TLs is used to generate the dual-band NGD phenomenon. By combining the CL and TL theories, the circuit’s S-parameters and group delay model can be developed. It is analytically certificated that this novel circuit can be susceptible to generate the NGD phenomenon.

To confirm the feasibility of the NGD analytical design, a microstrip prototype of the multi-CL circuit topology was designed, simulated, fabricated, and measured. As expected, the measured results highlight the dual-band NGD behaviors. It was shown that the tested prototype generates the dual-band NGD phenomenon with minimal values better than \(-4\) ns and \(-3.8\)ns around the first and second center frequencies, respectively. As expected, the theory and measurement and simulation results were found to be in very good correlation. The NGD circuits respect satisfy all the criteria of RF/microwave circuits. Compared to the most of existing dual band NGD topologies [37]–[39], the proposed one presents huge advantages in low transmission coefficient and using fully distributed elements without extra lumped components. The topology can also be potentially used for multi-band NGD function.

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