Probing in-medium nucleon-nucleon inelastic scattering cross section by using energetic n/p ratio

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The frequently mentioned meson in heavy-ion collisions at intermediate energies is \( \pi \), which is the lightest meson produced via nucleon-nucleon inelastic scatterings. Nowadays consensus has been reached on free nucleon-nucleon inelastic scatterings while its in-medium form is still in arguments and rarely constrained. Based on the Isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model, the in-medium nucleon-nucleon inelastic scattering is explored. It is found that the in-medium modification of nucleon-nucleon inelastic scatterings evidently reduces the neutron to proton ratio \( n/p \) at higher kinetic energies. Though the in-medium modification of nucleon-nucleon inelastic scatterings, as expected, affects the value of \( \pi^-/\pi^+ \) ratio, considering a series of undetermined properties of delta resonance and \( \pi \) in medium, the energetic neutron to proton ratio \( n/p \) is more suitable to be used to probe the in-medium correction of nucleon-nucleon inelastic scatterings.

Dynamical \( \pi \) production in heavy-ion collisions at intermediate energies was involved into transport models more than three decades ago \cite{1,2}. In the models, \( \pi \)'s are usually treated via resonance decay while the resonance experiences formation and reabsorption in the whole collision process.\cite{3,12} And channels relating to resonance or pion productions are all implemented in free space, this is because in-medium corrections of nucleon-nucleon inelastic cross section are still undetermined \cite{13,22}. Uncertainties of in-medium nucleon-nucleon inelastic cross section in heavy-ion collisions would affect \( \pi^-/\pi^+ \) ratio \cite{19,21}, an observable which may be used to constrain the high-density symmetry energy \cite{28,29}, the latter has been considered to play crucial role in many aspects including nuclear physics and astrophysics.\cite{26}.

The reduction of in-medium elastic cross section of nucleon-nucleon scattering has currently been relatively well determined, see, e.g., Refs. \cite{18,27,33}. Although great efforts have been made to find experimental observables constraining the symmetry energy, little work has been done so far to probe the in-medium effects of nucleon or resonance inelastic scattering cross sections, especially in isospin asymmetric nuclear matter. Because the in-medium reductions of baryon-baryon scattering cross section are not only moment and density dependent, but isospin-dependent \cite{18,20,22}, double ratio of \( \pi^-/\pi^+ \) from neutron-rich and neutron-deficient reactions with the same isotopes thus cannot cancel out uncertainties of the in-medium effects of baryon-baryon scattering cross section \cite{34}. What is more, for heavy system, such as Au+Au system, the counterpart with the same isotopes is not easily to find. Therefore, it is meaningful to find a way to probe and constrain the in-medium baryon-baryon inelastic scattering cross section. Unfortunately, till now there are rare studies on this subject. \( \pi \) production in heavy-ion collisions is thought to be not only affected by the in-medium nucleon-nucleon inelastic scattering cross section but also the unconstrained symmetry energy \cite{35}. And \( \pi \) production in heavy-ion collisions is related to a series of undetermined properties of delta resonance in medium, which complicates the question. So the straightforward and good way is to use nucleon observable. In heavy-ion collisions at intermediate energies, most energetic nucleons in fact experience inelastic process, such process may shed interesting light on the exploration of the nucleon-nucleon inelastic scattering cross section in medium. In this study, the in-medium effect of nucleon-nucleon inelastic cross section on the neutron to proton ratio \( n/p \) at higher kinetic energies is first investigated. It is shown that the energetic \( n/p \) ratio is indeed sensitive to the in-medium effects of nucleon-nucleon inelastic cross section.

In the present study, the used Isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model includes nucleon-proton short-range correlations, isospin-dependent in-medium elastic and free/in-medium inelastic baryon-baryon cross sections as well as the momentum-dependent isoscalar and isovector pion potentials \cite{34,35}. The isospin- and momentum-dependent single nucleon potential reads \cite{39,41}

\begin{equation}
U(\rho, \delta, \bar{\rho}, \tau) = A_u(x) \frac{\rho^\tau}{\rho_0} + A_l(x) \frac{\rho^\tau}{\rho_0} \\
+ B \left( \frac{\rho}{\rho_0} \right)^8 (1 - x \delta^2) - 8 \pi \rho \frac{\rho_{\delta^2}}{\sigma + 1} \delta \rho_{\tau'}
+ 2 C \tau \int d^3 \vec{p} \frac{f_{\tau}(\vec{r}, \vec{p})}{1 + (\bar{\rho} - \rho)^2 / \Lambda^2}
+ 2 C \tau' \int d^3 \vec{p}' \frac{f_{\tau'}(\vec{r}, \vec{p}')}{1 + (\bar{\rho} - \rho)^2 / \Lambda^2},
\end{equation}

where \( \rho_0 \) stands for saturation density, \( \tau, \tau' = 1/2(-1/2) \), for neutron (proton) and \( \delta \) is the isospin asymmetry of...
medium. Detailed parameter values can be found in Ref. [40]. The symmetry energy’s stiffness parameter $x$ is optional to mimic different forms of the symmetry energy predicted by various many-body theories without changing any property of the symmetric nuclear matter and the symmetry energy at normal density. In this study, we fix $x = 1$ [40] since the effects of symmetry energy are not the question focused here.

We assume the potential for $\Delta$ resonances is a weighted average of those for neutron and proton. The weighted factor depending on the charge state of the resonance is the square of the Clebsch-Gordon coefficients for isospin coupling in the processes $\Delta \leftrightarrow \pi N$ [11], i.e.,

$$
U_{\Delta^-} = U_n, \\
U_{\Delta^0} = \frac{2}{3} U_n + \frac{1}{3} U_p, \\
U_{\Delta^+} = \frac{1}{3} U_n + \frac{2}{3} U_p, \\
U_{\Delta^{++}} = U_p.
$$

The nucleon-nucleon free inelastic isospin decomposition cross sections

$$
\sigma^{pp \rightarrow n\Delta^{++}} = \sigma^{nn \rightarrow p\Delta^-} = \sigma_{10} + \frac{1}{2} \sigma_{11}, \\
\sigma^{pp \rightarrow p\Delta^+} = \sigma^{nn \rightarrow n\Delta^0} = \frac{3}{2} \sigma_{11}, \\
\sigma^{np \rightarrow p\Delta^0} = \sigma^{np \rightarrow n\Delta^+} = \frac{1}{4} \sigma_{11} + \frac{1}{4} \sigma_{10}
$$

are parameterized via

$$
\sigma_{II'}(\sqrt{s}) = \frac{\pi (\hbar c)^2 \alpha(p_0)^2}{2p^2} \frac{m_0^2 \Gamma^2 (q/p_0)^3}{(s^* - m_0^*)^2 + m_0^* \Gamma^2} \quad \text{(4)}
$$

with $I$ and $I'$ being the initial state and final state isospins of two nucleons. The parameters $\alpha, \beta, m_0, \Gamma$ as well as other kinematic quantities can be found in Ref. [42]. The cross section for the two-body free inverse reaction is calculated by the modified detailed balance, which takes into account the finite width of baryon resonances [6, 6].

$$
\sigma_{N\Delta \rightarrow NN} = \frac{m_\Delta \sqrt{m} \sigma_{NN \rightarrow N\Delta}}{2(1 + \delta) p_i} \int d\Delta \frac{1}{2\pi} P(m_\Delta).
$$

We extend the effective mass scaling model for in-medium elastic cross sections to inelastic case in isospin asymmetric matter [18, 40]. Compared with the free-space baryon-baryon scattering cross section $\sigma_{free}$, the baryon-baryon scattering cross section in medium $\sigma_{medium}$ is reduced by a factor

$$
R_{medium}(\rho, \delta, \tilde{p}) \equiv \sigma_{medium}/\sigma_{free} = \left(\frac{\mu_{BB}^*}{\mu_{BB}}\right)^2, \quad \text{(6)}
$$

where $\mu_{BB}$ and $\mu_{BB}^*$ are the reduced masses of the colliding baryon pairs in free and medium cases, respectively.

The effective mass of baryon in medium reads [43]

$$
\frac{m_B^*}{m_B} = 1/(1 + \frac{m_B dU}{p dp}), \quad \text{(7)}
$$

Since it has been argued that the in-medium reduction of baryon-baryon inelastic cross sections are also final-state dependent [20, 44, 45], similar reduction factors connecting with the effective mass of final state particles, e.g., as proposed in Refs. [44, 45], are checked in our model. It is found that both forms of the reduction factor of the in-medium inelastic baryon-baryon cross section are in fact less sensitive to different final-state outgoing channels. Compared with our present used form, the reduction factor from Ref. [44] is too small and the form from Ref. [45] is quite close to our present used form. So in the present study, for different scattering outgoing channels, we do not consider the splitting of the reduction factor.

Fig. 1 shows reduction factors of nucleon-nucleon scattering cross section in medium as a function of nucleon momentum with different densities. Compared with free-space values, it is seen that the in-medium cross sections are not only reduced, but the reduction factors $R_{medium}$ of $nn$ and $pp$ split. The reduction factor of neutron involved is overall larger than that of proton involved. To see the in-medium effects of inelastic cross section on some observables, we in the following make comparative studies of relevant observables with free or in-medium baryon-baryon inelastic cross sections.

With increase of baryon-baryon colliding energy, more and more inelastic scatterings occur, thus energetic nucleon spectrum should be affected by the in-medium baryon-baryon inelastic cross sections more or less. Fig. 2 shows the effects of the in-medium inelastic scattering cross section on the n/p ratio in the central collision.
\textsuperscript{132}Sn+\textsuperscript{124}Sn at beam energies of 400 MeV/nucleon and 270 MeV/nucleon, respectively. In left and right panels of Fig. 2 since elastic baryon-baryon scatterings dominate at lower colliding energies, one can see that at lower kinetic energies, effects of the in-medium baryon-baryon inelastic scattering cross section on the neutron to proton ratio n/p are less pronounced. While the n/p ratio of emitted energetic nucleons is clearly affected by the in-medium inelastic baryon-baryon cross section. With increase of nucleon-nucleon colliding energy, inelastic scatterings gradually become dominant, it is thus not surprising to see a larger effect of in-medium inelastic cross section on the energetic neutron to proton ratio n/p. As the reduction of scattering cross section, less nucleons accumulate more energies through many-time scatterings, thus numbers of energetic nucleons decrease. And positively charged particles like proton suffer from the Coulomb interactions which, to some extend, cancel out the reduction of proton involved scattering processes. Therefore relatively the decrease of the number of energetic neutron is more than that of proton in the final stage of reaction. So the reduction of in-medium inelastic cross section lowers the value of energetic neutron to proton ratio. From both panels of Fig. 2 it is seen that the effects of the in-medium baryon-baryon inelastic scattering cross section on the energetic n/p ratio reach about measurable 15%.

It is known that the squeezed-out energetic neutron to proton ratio n/p is very sensitive to the symmetry energy \cite{16,17}. To see whether the present analyzed energetic n/p ratio is sensitive to the symmetry energy or not, for the in-medium cases in Fig. 2 the symmetry energy parameter from x= 1 (soft) to x= 0 (stiff) is switched.

FIG. 2: (Color online) Effects of the in-medium baryon-baryon inelastic scattering cross section on the n/p ratio in the central collision \textsuperscript{132}Sn+\textsuperscript{124}Sn at beam energies of 400 (a) and 270 (b) MeV/nucleon. For the in-medium cases, to see the effects of symmetry energy, the parameter x in Eq. (1) is switched from x=1 (soft) to x=0 (stiff).

One can clearly see that the symmetry energy really plays negligible role in energetic non-squeezed-out neutron to proton ratio n/p.

To further demonstrate the in-medium baryon-baryon inelastic scattering cross section on particle production, we plot Fig. 3 energetic nucleons, resonances and the ratio of charged resonances at maximum compression stage in the central \textsuperscript{132}Sn+\textsuperscript{124}Sn reactions at 400 MeV/nucleon. As discussed in Fig. 2 it is seen from the left panel of Fig. 3 that the in-medium baryon-baryon inelastic scattering cross section decreases energetic nucleon, especially energetic neutron productions. This is the reason why one sees the in-medium baryon-baryon inelastic scattering cross section reduces the ratio of energetic neutron to proton ratio as shown in Fig. 2. Since energetic nucleon-nucleon collision produce delta resonances and proton-proton (neutron-neutron) collision produces $\Delta^+/(\Delta^-)$, one sees similar in-medium baryon-baryon inelastic scattering cross section on energetic resonance production as shown in the middle panel of Fig. 3. In the right panel of Fig. 3 effect of in-medium baryon-baryon inelastic scattering cross section on the ratio of $\Delta^-/\Delta^+$ is clearly shown. Because the productions of energetic $\Delta^-/\Delta^+$ directly connect with the energetic neutron-neutron/proton-proton collisions and the number of energetic nucleons reduces more than that of energetic proton, the reduction of in-medium baryon-baryon inelastic scattering cross section thus decrease the ratio of $\Delta^-/\Delta^+$. Note here that the energetic nucleons studied here is not the result of iso-fractionation, but the result of nucleon-nucleon scatterings. So the energetic proton to proton ratio is a direct reflection of energetic nucleon in dense matter, rather than the other way round.

Since $\Delta^+/(\Delta^-)$ decays into $\pi^+/(\pi^-)$, it is expected that the reduction of inelastic baryon-baryon scattering
FIG. 4: (Color online) Effects of in-medium baryon-baryon inelastic scattering cross section on the kinetic energy distribution of the $\pi^-/\pi^+$ ratio in the central collision $^{132}$Sn+$^{124}$Sn at the beam energy of 400 MeV/nucleon. For the in-medium case, to see the effects of symmetry energy, the parameter $x$ in Eq. 1 is switched from $x=1$ (soft) to $x=0$ (stiff).

The ratio of energetic $\pi^-/\pi^+$ cross section in medium also decreases the ratio of energetic $\pi^-/\pi^+$. Fig. 4 shows the kinetic energy distribution of energetic $\pi^-/\pi^+$ ratio in the central reaction $^{132}$Sn+$^{124}$Sn at 400 MeV/nucleon incident beam energy with free or in-medium baryon-baryon inelastic scattering cross sections. It is seen that the ratio of energetic $\pi^-/\pi^+$ can reduce about 15% when employing in-medium inelastic baryon-baryon scattering cross section. Although the $\pi^-/\pi^+$ ratio is frequently mentioned in probing the high-density symmetry energy, its kinetic distribution at high energy shown here is not sensitive to the symmetry energy. This result in fact does not conflict with that shown in Ref. [48]. The latter is for squeezed-out pion emission whereas our present analysis is for general pion emission. As it is mentioned before that pion production in heavy-ion collision relates to a series of undetermined properties of delta resonance and pion in medium, thus it is not suitable to probe the in-medium baryon-baryon inelastic scattering cross section.

In summary, based on the transport model, effects of the in-medium baryon-baryon inelastic scattering cross section on energetic nucleons, delta resonances and pion mesons are studied. It is found that in $^{132}$Sn+$^{124}$Sn reactions at intermediate energies the in-medium baryon-baryon inelastic scattering cross section reduces emitting numbers of energetic nucleon, especially energetic neutron. Therefore the in-medium baryon-baryon inelastic scattering cross section decreases the energetic neutron to proton ratio $n/p$. As energetic nucleon in the reaction directly relates to energetic delta resonance production, resonance yields and ratio are also sensitive to the in-medium baryon-baryon inelastic scattering cross section.

Because pion production is closely connected with delta resonance, the ratio of $\pi^-/\pi^+$ is also sensitive to the in-medium baryon-baryon inelastic scattering cross section. In the study, it is also seen that, due to non-squeezed-out emissions, both the energetic neutron to proton ratio $n/p$ and the energetic charged pion ratio $\pi^-/\pi^+$ are not very sensitive to the symmetry energy. Since pion production in heavy-ion collision relates to a series of undetermined properties of delta and pion in medium, energetic neutron to proton ratio $n/p$ is more suitable than energetic $\pi^-/\pi^+$ ratio to be used to probe the in-medium baryon-baryon inelastic scattering cross section.

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