LES OF THERMALLY STRATIFIED TURBULENT CHANNEL FLOW AT LOW PRANDTL NUMBER

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Abstract. In this paper Large Eddy Simulation (LES) is carried out for turbulent channel flow to assess the role the Prandtl and Richardson number on the turbulent heat transport. Fully developed turbulent channel flow configuration is used in the present study with top and bottom walls kept at constant temperature of $T_h$ and $T_c$. The Reynolds number calculated on the basis of friction velocity and channel half width are kept as 180, the molecular Prandtl number is 0.1 and Richardson number are 0, 20, 40, 60, 120 and 240. Wall Adaptive Local Eddy viscosity (WALE) model is used for modelling sub-grid stress tensor and Dynamic thermal model is used for modelling sub-grid heat flux. Mean velocity and temperature statistics as well as its variance, turbulent heat flux, turbulent Prandtl number are carried out and the role of Prandtl and Richardson number are examined. Also the instantaneous flow structures and temperature contours are visualized in order to understand the inherent physics of of streaks and vortices.

1. Introduction

1.1. Background and Motivation
Most of the flows appearing in everyday life and engineering applications are turbulent. In some case turbulence is favourable, in other not. Whatever the case is, a high level of knowledge about the physics of turbulence and related phenomena is necessary. Turbulent mixed convection is a mode of heat transfer, which combines both natural and forced convection. It is frequently encountered in variety of engineering applications, e.g. nuclear reactor cooling systems, turbine blades, cooling of electronic equipment, heat exchangers, solar panels etc. as well as in the nature, e.g. dynamic of oceanic and atmospheric circulations. As a consequence of density or temperature gradients, the buoyancy force arise, changing the structure and intensity of both mean and turbulence fields. Effects of buoyancy enter the Navier-Stokes equation through a body force. Temperature acts as an active scalar so that the momentum and energy transport are closely coupled. Because the density variations directly depend on temperature, it is possible to use only the temperature differences to represent the buoyancy force. This approach is known as the Boussinesq approximation. Many studies pertaining to experimental and computational work of heat transfer between a solid wall and the adjacent turbulent fluid flow at various Reynolds and Prandtl numbers were carried out by researchers. Results concerning the effects of Richardson number on the heat transfer in stably stratified wall-bounded flows were reported in literature for Prandtl number of 0.71. As per the knowledge of author none of the researcher have studied the effect of Richardson number on the heat transfer at low Prandtl
number. The basic mechanism and fluctuation characteristics of the temperature and its effect on thermal transport need to be investigated in detail.

1.2 Literature Review
Thermally stratified channel flows are difficult to simulate in the laboratory because of technical issue of maintaining the walls at higher temperatures, the influence of sidewalls, etc. Thus, the available experimental studies are very few and at low Reynolds number. Fukui, et al.[1] experimentally studied the stratification effects on thermal transport process in the atmospheric surface layer or in the wall region. They reached a fully developed stratified condition in turbulent flow between horizontal parallel plates maintained at different temperatures for both stable and unstable situation and provided elaborate data on stratification effects. Most laboratory experiments on stratified wall turbulence have been made for other configurations such as open channels or boundary layers. All these experiments have shown that when the class level is stable, and it is increased, the skin friction and the heat transmission decreases and suppressed the turbulence as the potential energy toll is required to yield larger mean shear. Kim & Moin [2] were the first who carried out a DNS for Prandtl numbers $Pr = 0.1, 0.7$ and $2.0$ at a moderate Reynolds number of $Re_f = 180$. They assumed generation of heat within the fluid and removal of heat at both walls. They examined the computed scalar fields for structures and found that the temperature fields were highly correlated with the streamwise velocity. They observed that streaky structures in the wall region were also present in the temperature field with the same non-dimensional mean spacing, almost independent of the molecular Prandtl numbers. Kasagi & Ohtusubo[3] used a constant heat flux at both walls at Prandtl numbers of $0.71$ and $0.025$ at $Re_f = 150$ and allowed the temperature to vary with time and position. They observed that as the Prandtl number becomes very low, the temperature variance and the turbulent heat fluxes are markedly decreased with their peak values moving away from the wall. They also showed that the turbulent Prandtl number at $Pr=0.025$ is much larger that for the moderate Prandtl number fluids.

The earliest studies of homogeneous shear flows with stratification was performed by Rohr[4] who showed, that at a threshold gradient Richardson number (i.e, 0.25), turbulence neither grows nor decays. However, at low Richardson numbers, the turbulent kinetic energy increases & at higher values, the turbulence suppressed. Iida & Kasagi[5] performed DNS of turbulent channel flow under unstable density stratification. They reported that at increased Grashoff numbers the large-scale buoyant thermal plumes diminish the quasi-coherent streamwise vortices which result in increased bulk mean velocity and a decreased turbulent friction coefficient. Consequently, the vertical fluid motion of thermal plumes drastically changes the transport mechanism of the Reynolds shear stress. In addition, found that the thermal plumes are spatially aligned in the direction of flow, and the low-speed streaks and vortical structures are concentrated in the region where the thermal plume begins to rise.

1.3 Problem description
The main phenomenon we want to study is the effect of Prandtl number on the turbulent transport of scalar filed under moderate to high stratification. We consider a flow in a plane channel with constant pressure gradient driven flow in streamwise direction, periodic boundary conditions in streamwise as well as in spanwise direction and no slip condition at the wall. Top wall is heated and bottom wall is cooled. We chose a large enough domain which does not alter the characteristic of flow through boundary conditions having size $L_x = 4\pi\delta$, $L_y = \frac{4}{3}\pi\delta$, $L_z = 2\delta$, in the x, y, and z direction respectively. We studied for molecular Prandtl number of 0.1 with friction Richardson number of 0, 20, 40, 60, 120 and 240. Shear Reynolds number based on friction velocity and channel half width is fixed as 180 for all the simulations [6,7].
2. Governing equations and numerical methodology

An incompressible, homogenous, Newtonian fluid with constant thermo-physical properties is considered. Assuming constant kinematic viscosity and a reference fluid density $\rho_{ref}$ at a reference temperature $T = T_{ref}$. The equations of motion can be written in the following dimensional form in terms primitive variables of velocity $u_i$, pressure $p$, and temperature $T$. These fields are solution of the equations of conservation of mass, momentum and energy. They can be written in the following dimensional form[8]

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (2.1)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho_{ref}} \frac{\partial p}{\partial x_i} + \frac{1}{\rho_{ref}} \frac{\partial (\tau_{ij})}{\partial x_j} + B_T \delta_{ij} + F \delta_{il} \quad (2.2)$$

With $\tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$. These partial differential equations (PDEs) (equation (2.1) and equation (2.2)) represent the conservation of mass and momentum respectively, applied to an infinitesimal region of space along with a constitutive relation called Stokes Law which relates the velocity gradients to the stresses in a fluid particle. In thermally coupled flow problems, the governing equations are fully coupled. First, as density is temperature sensitive, temperature variations may lead to density gradients. This can result in buoyancy forces due to gravitational effects. In equation (2.2), $B_T$ is the thermally buoyancy force which is equal to $g (\rho_{ref} - \rho_{ref})$, where $g$ is the gravitational acceleration. $F$ is a constant pressure gradient needed to maintain the flow in the stream-wise direction. In non-isothermal flow situation, the fluid density $\rho$ and viscosity $\mu$ generally depends upon temperature and pressure. The variations of density are highly important because of the production of buoyancy forces. The buoyancy forces appear automatically when an equation of state is inserted to the equation of motion. For the cases that density as coincident with the reference value $\rho_{ref}$ .

Expanding a Taylor series for $\rho(T)$ at constant pressure $p$ and keeping only first two terms of the series, we can write

$$\rho(T) = \rho_{ref} - \rho_{ref} \beta (T - T_{ref}) \quad (2.3)$$

Where $\beta = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_p$ is the isobaric coefficient of thermal expansion, evaluated at the reference temperature $T_{ref}$.

Figure 2.1. Schematics of a channel flow with the adopted coordinate system
3. Validation

A schematic sketch of the flow configuration is shown in figure 3.1. The distance between the two-parallel walls of the channel is $2\delta$ where $\delta$ is referred to as the channel half-height. Validation is presented in subsequent figures. Mean velocity profile non-dimensionalized by friction velocity is shown in fig. 3.3. Also shown is the mean velocity profiles from other references of DNS and LES database as given by Armenio & Sarkar[9]. As can be seen in small differences are present when comparing solutions with each other. A maximum deviation of 2% occur at center of channel while comparing with DNS database of Kawamura [10]. The present neutral profiles collapse perfectly with those calculated by Armenio & Sarkar [9]. The profiles are symmetric about channel’s vertical center plane and thus are only presented for half of channel height. The small discrepancies seen in the velocity profiles are most likely a result of numerical error, and a consequence of different numerical methods being employed.

![Figure 3.1. Mean velocity profile $\bar{u}_r$ as a function of wall normal height](image1)

![Figure 3.2. RMS of fluctuations in stream-wise direction non-dimensionalized by friction velocity](image2)

![Figure 3.3. RMS of velocity fluctuations in wall-normal direction](image3)

![Figure 3.4. Reynolds shear stress as a function of wall normal height](image4)
4. Results and discussions

In this section, we present detailed numerical results of the considered test problems in order to check the accuracy and durability of the chosen algorithm. This is the case of large-eddy simulation (LES) of turbulent channel flow at $Re_x = 180$, and $Pr = 0.1$. The main purpose of this study is to examine how the turbulent structures are going to change in a stably stratified channel flow for different Prandtl number under temperature stratification. This is done by analyzing turbulence statistics, heat fluxes, relevant non-dimensional quantities and flow visualization to examine the structure of streaks and vortices.

4.1 Effect of stable stratification on the mean flow

In figure 4.1 the mean stream wise velocity profiles, $u^+$ are shown as a function of the dimensionless wall-normal coordinate $z^+$. As in the previous investigations, of Armenio & Sarkar [9], Garcia-Villalba & del Alamo [11], stable stratification resulted in a systematic decrease in both friction coefficient and Nusselt number. Since the driving force is pressure gradient which is held constant for all simulations, the wall shear stress and therefore the slope of the mean velocity profile in the case of stable stratification should be invariant compared to those of the non-buoyant case (solid line). It can also be seen that mean velocity increases compared to the non-buoyant simulation and so does the mass flow rate.

![Figure 4.1. Comparison of mean velocity profile as a function of wall-normal height](image)

![Figure 4.2. Comparison of mean streamwise velocity in logscale](image)

![Figure 4.3. Comparison of mean temperature profile as a function of wall-normal height](image)
4.2 Effect of stratification on Reynolds stress

The root mean square (rms) of fluctuations (Fig. 4.4-4.5) in streamwise velocity field and Reynolds shear stress distribution for different stratification is analyzed here. The scaling factor chosen here is bulk velocity and in the subsequent section we will see that the second order statistics of turbulence for this simulation condition is better described by bulk velocity rather than shear velocity, also Amenio & Sarkar [9] has chosen similar type of scaling factor. We have seen a kink type formation at the core which is nothing but symbolize the IGW at the centre and fluctuations get suppressed on both the walls. Same is the case with wall shear stress.

Figure 4.4. Comparison of streamwise fluctuation of velocity field upon stable stratification

Figure 4.5. Comparison of wall shear stress upon stable stratification

4.3 Visualization of temperature contour

The temperature contour is plotted after a sufficient time interval so that the fluid properties reach a statistical equilibrium (Fig.4.6). The plane of contour is taken at half of channel i.e. mid y-plane [12]. As stable stratification is imposed we see internal gravity waves at the core of the channel. As we increase the stratification relaminarization is observed at the hot wall and the major portion of heat transfer is through molecular diffusion rather than turbulent heat flux. In Fig.4.7 iso-surfaces of non-dimensional temperature is shown. We can see clearly from figure that there exist some turbulent mixing at the core of the channel which further get dampened at higher Richardson number confirms the laminarizing behaviour of overall flow.

Ri_e = 20

Figure 4.6. Temperature contours for Pr = 0.1 at different stratification at mid y-plane

Ri_e = 240
5. Conclusions
A large eddy simulation (LES) of a turbulent channel flow with thermal buoyancy was carried out. The Reynolds number ($Re$) based on the friction velocity and half-channel height is 180 and the molecular Prandtl number is taken as 0.1. We have seen a kink type formation (IGW) at the centre and fluctuations are suppressed on both the walls. As we increase the stratification relaminarization is observed at the hot wall and the major portion of heat transfer is through molecular diffusion rather than turbulent heat flux. There exists some turbulent mixing at the core of the channel which further get dampened at higher Richardson number confirms the laminarizing behaviour of overall flow.

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