An effective interaction at the Planck mass and the Planck length

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Abstract

A simple model is suggested by assuming an effective interaction at the crossing point of the two curves, the Schwarzschild radius and the Compton wavelength, that is at the Planck length and Planck mass. It is argued that there would be a physical effect that may be measurable at, say, nucleon mass and size, while the Schwarzschild radius would remain unaffected for macroscopic lengths and masses.

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It is well known that the Planck mass and the Planck length are determined as the intersection point of the two curves: the Schwarzschild radius and the Compton wave length when the respective lengths are plotted versus the mass. It is also known that the Planck length is too minuscule to be amenable to observation while the Planck mass is macroscopic. For the following we prefer to view these two curves with the axes interchanged, i.e. we view the two curves with the mass plotted as a function of the length. The character of the curves does not change: the straight line (Schwarzschild radius) remains a straight line and the hyperbola (Compton wave length) remains a hyperbola. Note that in this redefined plot the observable physics is at the very far right from the intersection point, in fact, the Planck length being about $10^{-35}[m]$ is 20 orders of magnitude smaller than the size of a nucleon being about $10^{-15}[m]$.

The masses, now given at the ordinate, are a form of energy that can interact (we may think of multiplying the mass by $c^2$). When we assume an effective interaction at the intersection point, the crossing point will become a level repulsion \[1\]. Our interest here is exclusively focused upon the physical consequences of such an interaction rather than upon the precise nature giving rise to such an effective interaction. Also, whether or not the two levels cross or repel is at this stage beyond observation even though it might be of fundamental interest. Somewhat related in spirit, but using a different starting point, is a discussion about 'The Black Hole Uncertainty Principle Correspondence' as presented in \[2\].

The two curves, the Schwarzschild radius and the Compton wave length, are presented in the diagonal of the matrix

\[
H = \begin{pmatrix} x/a & m_P f \\ m_P f & b/x \end{pmatrix}
\]

where $a = G/c^2$ and $b = \hbar/c$. The coupling term has been chosen to be a (small) multiple of the Planck mass $m_P = \sqrt{b/a}$ as the coupling term must also have the dimension of a mass. The dimensionless constant $f$ with $0 \leq f \ll 1$ denotes the strength of the coupling. We are aware that an effective interaction as used here will probably involve higher order terms of the Planck mass but this would require more parameters which we do not consider in this simple treatment. In order to ensure that the gravitational mass is always largest we consider the two curves only for $x \geq l_{\text{Planck}}$ with $l_{\text{Planck}} = \sqrt{ab}$ being the Planck length.
FIG. 1: The two sets of curves, the one for $f = 0$ where the straight line and the hyperbola are joining at the left at $l_{\text{Planck}}$, and the corresponding curves with $f = 0.3$ as chosen here for illustration. The rough orders of magnitudes as given in the text for $a$ and $b$ are used here.

The eigenvalues of $H$ are

$$m_{1,2}(x) = \frac{b}{2x} + \frac{x}{2a}(1 \pm \sqrt{1 + \frac{a^2b^2}{x^4} - \frac{2ab}{x^2} + \frac{4abf^2}{x^2}}).$$

(2)

As noted above we now focus our attention upon large values of $x$, that is we expand (2) in powers of $x$. The leading term of the first eigenvalue is $x/a$, it is independent of the coupling. The next order $bf^2/x$ is minuscule as such and even more so for large values of $x$. In other words, the Schwarzschild radius remains essentially unaffected by the coupling.

However, the second eigenvalue is affected by the coupling. The expansion yields for the leading term

$$m_2(x) = \frac{b}{x}(1 - f^2) + O(1/x^3).$$

(3)

The higher orders have the form $b l_{\text{Planck}}^{2n}/x^{2n+1} P_{n+1}(f^2), n = 1, 2, \ldots$ where $P_n$ is a polynomial that vanishes at $f = 0$ (and at $f = 1$). These terms again are minuscule. Note that the higher orders depend explicitly on $G$ while (3) appears to be independent of $G$, but this is only apparent, since $f$ is a fraction of the Planck mass which does depend on $G$.

Observable physics happens some twenty orders of magnitude further to the right of the display in Fig.1. Inserting simply orders of magnitudes for $a = G/c^2 \approx 10^{-27}$ and $b = \hbar/c \approx 10^{-42}$ (in [m,kg,s]) we find for $x = 10^{-15}$ and $f = 0$ the mass $10^{-27}$ which
becomes smaller by the factor \((1 - f^2)\) when \(f\) is switched on. Moving towards smaller values of \(x\) by the same factor the original mass value is retrieved.

In other words, the interpretation of (3) is that the product of the Compton wave length (denoted here by \(x\)) of, say, a proton and its mass is diminished by the factor \((1 - f^2)\). Fixing the mass as measured by experiment the associated Compton wave length will appear smaller by the factor \((1 - f^2)\).

It is not obvious from this consideration whether the interaction at the Planck length as assumed in this paper diminishes the mass or the Compton wave length or both of, say, a nucleon as only the product is affected. If the factor \(f\) is not too small, the deviation should be measurable. In addition, as the consideration shows, the mere assumption of an interaction at the Planck length will have a physical effect in the realms of quantum mechanics whereas the effect would be virtually zero for the gravitational attraction between masses.

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[1] W.D. Heiss, J. Phys. A: Math. Theor. 45 444016 (2012)

The paper discusses in detail the connection of Exceptional Points (EP) and level repulsion. In the present context the positions of the EP guarantee that the closest approach of the repelling levels is at \(l_{\text{Planck}}\) for sufficiently small coupling.

[2] BJ Carr: arXiv:1402.1427 [gr-qc]