The evolution of the magnetic inclination angle as an explanation of the long term red timing-noise of pulsars

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ABSTRACT

We study the possibility that the long term red timing-noise in pulsars originates from the evolution of the magnetic inclination angle $\chi$. The braking torque under consideration is a combination of the dipole radiation and the current loss. We find that the evolution of $\chi$ can give rise to extra cubic and fourth-order polynomial terms in the timing residuals. These two terms are determined by the efficiency of the dipole radiation, the relative electric-current density in the pulsar tube and $\chi$. The following observation facts can be explained with this model: a) young pulsars have positive $\nu$; b) old pulsars can have both positive and negative $\nu$; c) the absolute values of $\nu$ are proportional to $-\dot{\nu}$; d) the absolute values of the braking indices are proportional to the characteristic ages of pulsars. If the evolution of $\chi$ is purely due to rotation kinematics, then it cannot explain the pulsars with braking index less than 3, and thus the intrinsic change of the magnetic field is needed in this case. Comparing the model with observations, we conclude that the drift direction of $\chi$ might oscillate many times during the lifetime of a pulsar. The evolution of $\chi$ is not sufficient to explain the rotation behavior of the Crab pulsar, because the observed $\chi$ and $\dot{\chi}$ are inconsistent with the values indicated from the timing residuals using this model.

Keywords: Pulsars: General

1 INTRODUCTION

Temporally correlated residuals are very common in the pulse time of arrival (TOA) of pulsars, after accounting for the standard model of spin down, astrometric variations, interstellar medium (ISM) effects and potential binary motion. The systematic deviations from the rotation model are referred as red timing-noise, since the TOA residuals have more power in the low Fourier frequencies than in the high frequencies. In order to find the signals of gravitational waves (GW) hidden in timing data (Sazhin 1978; Detweiler 1971), we need to model other sources of red noise as best as possible.

Most mechanisms which are proposed to explain the red timing-noise can be sorted into two classes: a) random transfer of angular momentum, either between the neutron star and the interior (Jones 1990), or between the neutron star and the fallback matter (Boynton et al. 1972); b) variance of the braking torque (Kramer et al. 2006; Lyne et al. 2014). A new attempt was made by Shannon et al. (2013), who attributed the red noise in the timing residuals of PSR B1937+21 to an unknown belt of asteroids. A swarm of orbiting objects around the pulsar is equivalent in mathematics to introducing many sinusoid waves components into the timing residuals. In fact, the timing residuals of PSR B1937+21 in the time span of the study of Shannon et al. (2013) can be more naturally fitted to a single cubic polynomial rather than to a series of sinusoids (Lyne et al. 2014).

In the sample of Hobbs et al. (2010), a considerable number of pulsars’ timing residuals show the shape of a cubic polynomial, and some of the other pulsars exhibit the forth-order polynomial residuals or quasi-periodic structures, after fitting the pulse arrival phases to the spin frequency derivative ($\dot{\nu}$). If the braking of pulsars is mainly contributed by the dipole radiation of a constant magnetic field, which is a simplest assumption and is widely used, the timing model needs only to include up to the $\dot{\nu}$ term. However, the unexpectedly significant cubic and higher order terms in the timing residuals compel us to revisit such assumption.

Assume that the braking of a pulsar can be expressed as follows:

$$ I\nu \dot{\nu} = -\frac{2\pi^2 B_0^2 \nu^4 R^6}{3c^2}, \quad (1) $$

where $I$ and $R$ are the momentum of inertia and the radius of the pulsar, and $B_0$ is the equivalent magnetic field. We do not presume the $B_0$ to be constant, and as long as the relative change of the equivalent magnetic field $B_0/B_*$ is small, the following expression is valid:

$$ B_* = B_0 \nu (1 + b(t)), \quad (2) $$
where \( b(t) \) is the dimensionless time variant part of \( B_\ast \).

Integration of Equation (1) twice gives the pulse arrival phase:

\[
\Phi(t) = \Phi_0 + \nu_0 t - AB_\ast^2 \nu_0^2 \frac{t^2}{2} + 3A^2 B_\ast^4 \nu_0^3 \frac{t^3}{3!} - 2AB_\ast^2 \nu_0^3 \int b(t)dt^2, \tag{3}
\]

where \( \nu_0 \) is \( \nu \) at the beginning of the observation \( (t = 0) \), \( \Phi_0 \) is a constant phase offset and \( A \equiv 2R^6/(3c^31) \).

From Equation (3) one can clearly see that the linear term in \( b(t) \) will give rise to an extra cubic term in the timing residuals, and the \( n \)-th order polynomial term in \( b(t) \) will produce the \( n + 2 \)-th order polynomial term in the timing residuals. In other words:

\[
b(t) = -\frac{\nu_0}{2\nu_0} \frac{d^2 R(t)}{dt^2}, \tag{4}
\]

where \( R(t) \) is the timing residuals defined as:

\[
R(t) = \left( \Phi(t) - (\Phi_0 + \nu_0 t + \nu_0^2 \frac{t^2}{2} + 3\nu_0^2 \frac{t^3}{3!}) \right) / \nu_0, \tag{5}
\]

and \( \nu_0 \equiv -AB_\ast^2 \nu_0^3 \).

As for the evolution of \( B_\ast \), one proposed mechanism is the decay of \( B \) through Ohmic dissipation (Haensel et al. 1996). The monotonically decreasing of \( B_\ast \) can only count for a positive \( \dot{\nu} \), whereas in observations, the number of positive-\( \dot{\nu} \)-pulsars does not overwhelm that of negative \( \dot{\nu} \). In order to solve that problem, Zhang & Xie (2012a) proposed that the long term decay of \( B \) is modulated by short term oscillations. Therefore, in a time span shorter than the oscillation period, the value of \( B \) can be both increasing and decreasing, and the sign of \( \dot{\nu} \) can be both positive and negative.

Another possibility was proposed by Lyne et al. (2013) that the magnetic inclination angle \( \chi \) drifts, and they argued that the consequent change of the equivalent magnetic field can take account for the 45 years of timing residuals of the Crab pulsar. In this work we further study this possibility. In Section 2 and 3, we derive how the evolution of \( \chi \) will affect the timing residuals, given that the braking torque is the combination of the dipole radiation and the current loss, and how this model can explain observations. In section 4, we discuss the origins of the \( \chi \) evolution, and what we can learn from the observed braking index. In section 5, we discuss how our theory is related to observations, and what cannot be explained by this mechanism. We conclude our work in the final section.

## 2 THE EQUIVALENT MAGNETIC FIELD WHEN THE CURRENT LOSS TORQUE IS PRESENT

The magnetic dipole radiation braking mechanism predicts that \( \chi \) of a pulsar approaches 0 or 180°, and if the braking is dominated by the current loss, \( \chi \) approaches 90° (Barsukov et al. 2009). As was pointed out by many authors (Beskin et al. 1993; Mestel et al. 1999; Beskin & Nokhrina 2007) that the dipole braking is not efficient when the pulsar is surrounded by plasma, and the observed profile evolution of the Crab pulsar implies that \( \chi \) is drifting towards 90° as expected by the current loss mechanism (Lyne et al. 2013).

Therefore, we consider the total braking torque of a pulsar as a combination of both mechanisms (Jones 1979):

\[
K = \alpha K_{\text{dip}} + K_{\text{cur}}, \tag{6}
\]

where \( 0 < \alpha < 1 \) is to take account the inefficiency of dipole radiation braking. \( K_{\text{dip}} \) is parallel with the spin axis, and its value is:

\[
K_{\text{dip}} = -2\pi \nu^3 B_\ast^2 A \sin^2 \chi; \tag{7}
\]

\( K_{\text{cur}} \) is in the direction of the magnetic dipole; the component parallel to the spin axis contributes to the braking of the pulsar:

\[
K_{\text{cur}} = -2\pi B_\ast^2 A \cos^2 \chi, \tag{8}
\]

where \( \beta \) is a scaling factor proportional to the ratio between the electric-current density in the pulsar tube and the Goldreich-Julian current (see Barsukov et al. 2009, Equation (12)).

As a result of the combination, the equivalent magnetic field of Equation (1) is:

\[
B_\ast^2 = B_\ast^2 (\alpha \sin^2 \chi + \beta \cos^2 \chi). \tag{9}
\]

Then Equation (9) can be rewritten as:

\[
B_\ast^2 = B_\ast^2 (1 + \frac{\beta - \alpha}{\beta + \alpha} \cos(2\chi)), \tag{10}
\]

where \( B_\ast^2 \equiv [(\alpha + \beta)/2] B_\ast^2 \).

## 3 THE MAGNETIC INCLINATION ANGLE EVOLUTION AND THE TIMING RESIDUALS

It is reasonable to assume that during the observation time span, the change rate of \( \chi \) can be treated as a constant \( \dot{\chi}_0 \), therefore

\[
\chi(t) = \chi_0 + \dot{\chi}_0 t. \tag{11}
\]

Equation (10) becomes:

\[
B_\ast^2 = B_\ast^2 (1 + \frac{\beta - \alpha}{\beta + \alpha} \cos(2\chi_0 + 2\dot{\chi}_0 t)). \tag{12}
\]

Suppose the change of \( \chi \) is small, thus Equation (12) can be expanded as:

\[
B_\ast^2 = B_\ast^2 (1 - 2\dot{\chi}_0 t \sin(2\chi_0) - 2\dot{\chi}_0^2 t^2 \cos(2\chi_0)). \tag{13}
\]

In Equation (13), \( B_\ast^2 \) is once again redefined to absorb the factor \((\beta - \alpha)/(\beta + \alpha) \cos(2\chi_0))\), and

\[
\gamma \equiv (\beta - \alpha)/(\beta + \alpha), \tag{14}
\]

which is a time-independent constant. Comparing with Equation (2) we obtain:

\[
b(t) = -\gamma \dot{\chi}_0 t \sin(2\chi_0) - \gamma \dot{\chi}_0^2 t^2 \cos(2\chi_0). \tag{15}
\]

As a result, from Equation (3) we expect to see the fourth-order polynomial as the pulse arrival phase:

\[
\Phi(t) = \Phi_0 + \nu_0 t + \nu_0^2 \frac{t^2}{2} + \nu_0^3 \frac{t^3}{3!} + \nu_0^4 \frac{t^4}{4!}, \tag{16}
\]
then the first equation in Equations (17) can be written as

\[ \ddot{\nu}_0 = 3\nu_0^2 - 2i\nu_0\gamma\chi_0 \sin(2\chi_0). \]  

(17)

\[ \nu_0 = -4\nu_0\gamma^2 \cos(2\chi_0). \]

If we use the concept of characteristic age, \( \tau \equiv \nu_0/2\dot{\nu}_0 \), then the first equation in Equations (17) can be written as

\[ \dot{\nu}_0 = -i\nu_0(\frac{3}{2\tau} + 2\gamma\chi_0 \sin(2\chi_0)). \]  

(18)

Several conclusions can be drawn from Equation (18):

a) for young pulsars whose \( \tau \) are small, the first term in the bracket of Equation (18) dominates and therefore have \( \dot{\nu}_0 > 0 \); b) for old pulsars, the second term in the bracket of Equation (18) dominate. Since \( \chi_0 \sin(2\chi_0) \) can be either positive and negative, \( \dot{\nu}_0 \) can be either \( > 0 \) or \( < 0 \); c): the absolute values of \( \dot{\nu}_0 \) are proportional to \( \dot{\nu}_0 \) statistically, as long as the quantity \( \gamma\chi_0 \sin(2\chi_0) \) distributes in a small range for all the pulsars. All of the three predictions have been confirmed by observations (see Zhang & Xie 2012a, Figure1).

The braking index is defined as \( n = \dot{\nu}/\nu^2 \). From Equation (17) we know that:

\[ n - 3 = -\frac{2\nu_0}{\nu_0}\gamma\chi_0 \sin(2\chi_0). \]  

(19)

Since \( n \) is widely distributed over eight order of magnitude (see Zhang & Xie 2012a, Figure10)), we can infer that \( \chi_0 \neq 0 \) for most of the pulsars.

Equation (19) can also be rewritten in terms of the characteristic age:

\[ n = 3 + 4\tau\gamma\chi_0 \sin(2\chi_0). \]  

(20)

Equation (20) can be used to explain the observation fact that the absolute values of \( n \) are proportional to \( \tau \) (see Zhang & Xie 2012a, Figure 2).

We use Equation (16) and (17) to simulate the TOAs, and fit a second degree polynomial to the TOAs. The resulting timing residuals are plotted in Figure 1, under different simulating parameters (see the caption of the figure). The shapes of four panels of Figure 1 include most of that of the pulsars’ timing residuals in the sample of Hobbs et al. (2010). The variety of the shapes of observed timing residuals is due to various values of \( \chi_0 \) and \( \dot{\chi}_0 \) at different stages of the evolution.

Figure 1. Timing residuals of simulated TOAs after fitting a polynomial to the second degree. In all the simulation, \( \nu_0 = 30.0 \) Hz, \( \dot{\nu}_0 = -3 \times 10^{-11} \) s\(^{-2} \) and \( \gamma = 1 \). (a): \( \chi_0 = 0.75 \), \( \dot{\chi}_0 = 3 \times 10^{-12} \) s\(^{-1} \); (b): \( \chi_0 = 0.75 \), \( \dot{\chi}_0 = -3 \times 10^{-12} \) s\(^{-1} \); (c): \( \chi_0 = 0.1 \), \( \dot{\chi}_0 = -7.55 \times 10^{-12} \) s\(^{-1} \); (d): \( \chi_0 = 1.4 \), \( \dot{\chi}_0 = -4.47 \times 10^{-12} \) s\(^{-1} \).

4 THE ORIGIN OF THE \( \chi \) EVOLUTION AND THE BRAKING INDEX

The evolution of \( \chi \) can come from either the intrinsic change of the magnetic field originating from the interior of the neutron star, or a kinematic process due to the rotation of the pulsar. As mentioned above, both the magnetic dipole braking and the current loss predict a kinematic \( \chi \) evolution, and the combination of them gives (Barsukov et al. 2004):

\[ \dot{\chi} = \frac{AB\nu_0^2}{2} (\beta - \alpha) \sin(2\chi). \]  

(21)

Therefore,

\[ \ddot{\chi}_0 = \frac{AB\nu_0^2}{2} (\beta - \alpha) \sin(2\chi_0). \]  

(22)

From Equation (22) we see that if \( \beta > \alpha \) then the magnetic dipole moves towards the equator of the pulsar, and when \( \alpha > \beta \) the magnetic dipole evolves towards alignment with the spin axis.

In Equation (22), if \( \dot{\chi}_0 \) is determined by Equation (22), then

\[ \gamma\chi_0 \sin(2\chi_0) = \frac{AB\nu_0^2}{2} \frac{(\beta - \alpha)^2/(\beta + \alpha)}{1 + (\beta - \alpha)/(\beta + \alpha) \cos(2\chi_0)} \sin^2(2\chi_0). \]  

(23)

Therefore, as long as \( \beta \) is positive (as widely supposed), \( \gamma\chi_0 \sin(2\chi_0) > 0 \) and \( n \) is always larger than 3. It is intuitive since the kinematic processes will simultaneously adjust the configuration of the pulsar so that the braking torque decreases. As a result, a pulsar with \( n < 3 \) requires that the intrinsic magnetic field of the neutron star is evolving. If we do not assume \( \beta > 0 \), then \( n < 3 \) requires that (from Equation (23))

\[ \frac{\alpha + \beta}{\alpha - \beta} < \cos(2\chi_0). \]  

(24)
braking index of 2.5 of the Crab pulsar (PSR J0534+2200). From Table 1 we see that $\psi_{\text{break}} = 2.70 \times 10^{-10}$ rad s$^{-1}$ for the Crab pulsar. However the observation suggests that $\chi_0 = \pm 3 \times 10^{-10}$ rad s$^{-1}$ towards the equator, and $45^\circ < \chi_0 < 70^\circ$ or $110^\circ < \chi_0 < 135^\circ$ (Weisskopf & Rudak 2003; Harding et al. 2008; Watts et al. (2009; Du et al. 2012). As a result, we have $-7 \times 10^{-12} < \dot{\chi} < 0$ rad s$^{-1}$. The inconsistency between $\psi_{\text{break}}$ and $\chi_0$ indicates that the evolution of the magnetic field of the Crab pulsar is required for the rotation behavior of the Crab pulsar. From section 3 we learnt that since the braking index of the Crab pulsar is less than 3, the rotation of the Crab pulsar has an intrinsic change of the magnetic field. Therefore the strength and the magnetic field of the Crab pulsar might evolve together. Thus, the $B$ field strength evolution mechanism (Lin & Zhang 2004; Chen & Li 2006; Espinoza 2013; Zhang & Xie 2012) can not be replaced by the pure $\chi$ evolution. The pulsar wind braking ($Xu & Qiao 2001$; Wu et al. 2003; Contopoulos & Spitkovsky 2006) is another competitive explanation for the abnormal rotational behavior of the Crab pulsar, which has been studied by Kou & Tong (2015).

### 5.3 Correction for the red timing-noise

The evolution of $\chi$ induces the extra cubic and fourth-order terms in the timing residuals, therefore a fitting to the fourth-order polynomial can remove the effects. However an arbitrary fitting of polynomial may also remove other useful signals in the timing residuals, e.g., GW. Although $\chi$ and $\chi_0$ can be constrained by observations, we can not yet constrain $\dot{\nu}_0$ and $\ddot{\nu}_0$ since $\gamma$ is unknown in Equations (17). Nevertheless, these two terms are coupled with each other by Equation (25). Therefore if $\chi$ and $\dot{\chi}$ (and thus $\chi_0$) are determined by independent methods, we can reduce the number of free parameters as shown in

$$\Phi(t) = \Phi_0 + \nu_0 t + \frac{\nu_0 t^2}{2} + \frac{\dot{\nu}_0 t^3}{3} + \chi_0 (\dot{\nu}_0 - 3 \ddot{\nu}_0 \frac{t^2}{2}) + \chi (\dot{\nu}_0 - 3 \ddot{\nu}_0 \frac{t^2}{2})$$

and thus alleviate the removal of other useful signals while fitting a fourth-order polynomial to the timing residuals.

### 6 SUMMARY AND CONCLUSION

We modeled the braking torque of the pulsar spin-down as a combination of the dipole radiation and the current loss, and derived the red timing-noise caused by the evolution of the magnetic inclination angle $\chi$ of pulsars. With this model, we reproduced the four typical types of timing residuals observed in the sample of Hobbs et al. (2010). Comparing with observations, we calculated the quantity $2\chi \cot(2\chi)$ for nine pulsars in the ATNF catalogue.

Our conclusions are as follows:

(i) The cubic and the fourth-order polynomial terms in the timing residuals can be explained as a result of the evolution of magnetic inclination angle (see Equations (17)), if we consider the spin-down mechanism of pulsars to be a combination of the magnetic dipole radiation and the current loss. The variety of shapes of the timing residuals originate from the various values of $\chi$ and $\dot{\chi}$ at different stages of the evolution (see Figure 1).
(ii) The evolution of magnetic inclination angle can explain the following observation facts: a) young pulsars (small $\tau$) have $\dot{\nu}_0 > 0$; b) old pulsars can have either $\dot{\nu}_0 < 0$ or $\dot{\nu}_0 > 0$; c) $|\dot{\nu}_0|$ are proportional to $-\dot{\nu}_0$ among pulsars; d) $|n|$ are proportional to $\tau$.

(iii) The evolution of $\chi$ purely due to rotation kinematics can not explain the pulsars with braking index less than 3. The intrinsic change of the magnetic field (either strength or $\chi$) is needed.

(iv) The sign of $\dot{\chi}$ changes many times during the lifetime of a pulsar, which might be responsible for the proposed oscillation of the magnetic field of neutron stars in [Zhang & Xie (2012a)]

(v) The evolution of $\chi$ is not sufficient to explain the rotation behavior of the Crab pulsar.

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REFERENCES

Barsukov, D. P., Polyakova, P. I., & Tsygan, A. I. 2009, Astronomy Reports, 53, 1146
Beskin, V. S., Gurevich, A. V., & Istomin, Y. N. Physics of the pulsar magnetosphere, Cambridge University Press (1993)
Beskin, V. S., & Nokhrina, E. E. 2007, Astrophysics and Space Science, 308, 569
Boynton, P. E., Groth, E. J., Hutchinson, D. P., et al. 1972, ApJ, 175, 217
Contopoulos, I., & Spitkovsky, A. 2006, ApJ, 643, 1139
Chen, W. C., & Li, X. D. 2006, A&A, 450, L1
Detweiler, S. 1979, ApJ, 234, 1100
Du, Y. J., Qiao, G. J., & Wang, W. 2012, ApJ, 748, 84
Dyks, J., & Rudak, B. 2003, ApJ, 598, 1201
Espinoza, C. M. 2013, IAU Symposium, 291, 195
Haensel, P., Urpin, V. A., & Iakovlev, D. G. 1990, A&A, 229, 133
Harding, A. K., Stern, J. V., Dyks, J., & Frackowiak, M. 2008, ApJ, 680, 1378
Hobbs, G., Lyne, A. G., & Kramer, M. 2010, MNRAS, 402, 1027
Jones, P. B. 1976, ApJ, 209, 602
Jones, P. B. 1990, MNRAS, 246, 364
Kou, F. F., & Tong, H. 2015, MNRAS, 450, 1990
Kramer, M., Lyne, A. G., O’Brien, J. T., Jordan, C. A., & Lorimer, D. R. 2006, Science, 312, 549
Lin, J. R., & Zhang, S. N. 2004, ApJL, 615, L133
Lyne, A., Hobbs, G., Kramer, M., Stairs, I., & Stappers, B. 2010, Science, 329, 408
Lyne, A., Graham-Smith, F., Weltevrede, P., et al. 2013, Science, 342, 598
Lyne, A. G., Jordan, C. A., Graham-Smith, F., et al. 2015, MNRAS, 446, 857
Manchester, R. N., Hobbs, G. B., Teoh, A., & Hobbs, M. 2005, The Astronomical Journal, 129, 1993
Mestel, L., Panagi, P., & Shibata, S. 1999, MNRAS, 309, 388
Radhakrishnan, V., & Cooke, D. J. 1969, Astrophys. Lett., 3, 225
Rookyard, S. C., Weltevrede, P., & Johnston, S. 2015, MNRAS, 446, 3367
Sazhin, M. V. 1978, Soviet Astronomy, 22, 36
Shannon, R. M., Cordes, J. M., Metcalfe, T. S., et al. 2013, ApJ, 766, 5
Watters, K. P., Romani, R. W., Weltevrede, P., & Johnston, S. 2009, ApJ, 695, 1289
Wu, F., Xu, R. X., & Gil, J. 2003, A&A, 409, 641
Xu, R. X., & Qiao, G. J. 2001, ApJL, 561, L85
Zhang, S.-N., & Xie, Y. 2012a, ApJ, 757, 153
Zhang, S.-N., & Xie, Y. 2012b, ApJ, 761, 102