SU(3) Flavor Symmetry for Weak Hadronic Decays of $B_{bc}$ Baryons

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Baryons with a heavy c-quark or a heavy b-quark and also two c-quarks have been discovered. These states are expected in QCD and therefore provide a test for the theory. There should be double beauty baryons, and also an intriguing possibility that baryons $B_{bc}$ with a c-quark, a b-quark. These states are yet to be discovered. The main decay modes of $B_{bc}$ are expected to be weak processes from theoretical understanding of their mass spectrum. These decay modes can provide crucial information about these heavy baryons $B_{bc}$. We analyze two body hadronic weak decays for $B_{bc}$ using SU(3) flavor symmetry. Any one of the $c$ and $b$ decays will induce $B_{bc}$ to decay. We find that the Cabibbo allowed decays $B_{bc} \to B_b + M$ due to $c \to su\bar{d}$ can be crucial for exploration. The LHC may have the sensitivity to discover such decays. Other $B_{bc}$ decays due to $b \to cq\bar{q}$ are sub-leading. Several relations among branching ratios are obtained which can be used to test SU(3) flavor symmetry.

I. EFFECTIVE HAMILTONIAN OF WEAK $B_{bc}$ DECAYS

Heavy $b$ and $c$ quarks can be baryon constituents as long as they form color singlets with other constituents inside the baryon according to QCD. Baryons with one $c$-quark and also baryons with one $b$-quark have been found a long time ago [1]. Two years ago, a baryon with two $c$-quarks, the $\Xi_{cc}^{++}$, had also been discovered [2]. They expected baryons with two $b$-quarks, or a $c$-quark and a $b$-quark yet await to be discovered experimentally [3–6]. With LHCb continuing to collect more and more data, it is hoped that these states will eventually be discovered.

Although there are no experimental data on ground states of $B_{bc}$ baryons with constituent quarks ($bcu$, $bcd$, $bcs$), there are theoretical estimates on their masses. The masses of these baryons are all estimated to be below the thresholds of strong decays (less than $\sim 7.3$ GeV [7, 8]), the mass splittings between them are also below the possible strong decay of a heavier ground state $B_{bc}$ decaying into a lighter one plus a pion. There may be some radiative decays from heavier to lighter states. The lowest state will decay through weak interactions. After being produced, the identification of $B_{bc}$ ground states will be through their dominant weak decays. These weak decay modes can provide crucial information about these heavy baryons $B_{bc}$. In this work we will analyze two body weak hadronic decays to obtain the main decay modes and also to find relations among different decays as tests for SU(3) flavor symmetry where $u$, $d$ and $s$ form a fundamental representation 3. SU(3) flavor symmetry has been used to study hadronic decays for hadrons with light quarks and with heavy quarks [9–17]. In the lack of results from established method of first principle calculation from QCD, such as lattice calculation, SU(3) studies have given many interesting results for hadrons that contain a heavy $b$ quark, and in many cases are quantitatively in agreement with experimental data. One expects that SU(3) studies may also provide some useful information for understanding $B_{bc}$ properties and for experimental search.

There are three $B_{bc}$ baryons,

$$B_{bc}(B_{bc} \ i) = (bcu, bcd, bcs) = (\Xi_{bc}^{++}, \Xi_{bc}^{0}, \Omega_{bc}^{0}).$$

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These three states form a fundamental representation 3 of SU(3). The subscript “i” taking the values 1, 2, 3 in \( B_{bc \ i} \) is the SU(3) representation index. Changing it to a superscript, it becomes the anti-representation index. The subscript \( bc \) is related to the naming of the particle. We will use the same notions for other particles and also Hamiltonian in our later discussions.

The weak decays of \( B_{bc} \) baryons can be induced by the constituent b-quark or c-quark decays. The dominant c-quark decay is induced by \( c \rightarrow \bar{u}q'q \) with \( q' \) and \( q \) taking the values \( d \) and \( s \). These decays are proportional to \( \lambda_{q'q} = V_{uq'}V_{cq}^\ast \) with Wilson coefficients of order one. These decays are classified as Cabibbo allowed (\( \lambda_{ds} = V_{uq'}V_{cq}^\ast \)), Cabibbo suppressed (\( \lambda_{dd} = V_{uq'}V_{cq}^\ast \) or \( \lambda_{ss} = V_{uq'}V_{cq}^\ast \)), and doubly Cabibbo suppressed (\( \lambda_{sd} = V_{uq'}V_{cq}^\ast \) decays). The lifetimes of the two \( B_{bc} \) states are estimated to be an order of a few times \( 10^{-13} \) s [18–20].

The \( b \) can decay in several ways at tree level, a) \( b \rightarrow c\bar{c}q \), b) \( b \rightarrow c\bar{u}q \), c) \( b \rightarrow u\bar{c}q \), d) \( b \rightarrow u\bar{u}q \). Among them a) and b) are dominating ones which are proportional to \( \lambda_{q'q} \). c) and d) are proportional to \( \lambda_{q'q} = V_{uq'}V_{cq}^\ast \) and \( \lambda_{q'q} = V_{uq'}V_{cq}^\ast \) with Wilson coefficients of order one. c) and d) are proportional to \( \lambda_{q'q} \). At one loop level, there are also the penguin decay modes \( b \rightarrow q \sum_{q'=u,d,s,c} q'q' \). These decays are, however, suppressed by loop induced small Wilson coefficients. In our later discussions we will only keep decays with \( q=q=d \) for a) and b), respectively, and neglect the suppressed parts [21]. Note that even the two largest classes of b-quark induced decays are suppressed by a factor of \( |V_{ub}/V_{cd}|^2 \sim 1.7 \times 10^{-3} \) compared with Cabibbo allowed c-quark decays. But they are different from c-quark decay induced processes and we hope experiments may find some different favorable search strategies. For instance, the b-quark induced \( B_{bc} \) decays can have a displaced baryon for subsequent decays, which is able to travel a \( 10^3 \) longer distance before decaying than that from the c-quark decay induced ones. One can hence consider a displaced vertex as a signal of the b-quark’s decay vertex is displaced from the prompt production vertex, which has been used as a technique of searching for doubly-bottom hadrons [22].

For the main \( B_{bc} \) baryon weak decays described above, the effective Hamiltonians are given by [23]

\[
\mathcal{H}_{c\,\text{eff}}^i = \frac{G_F}{\sqrt{2}} \lambda_{q'q} \left[ c_1(\bar{u}q')(\bar{q}c) + c_2(\bar{u}\beta q')(\bar{q}\alpha c\beta) \right], \\
\mathcal{H}_{b\,\text{eff}}^i = \frac{G_F}{\sqrt{2}} \lambda_{q'q} \left[ c_1(\bar{q}a)(\bar{c}b) + c_2(\bar{q}\beta a)(\bar{c}\alpha b\beta) \right],
\]

for the c and b-quark decays, respectively. \( G_F \) is the Fermi constant. \( a \) takes the values \( c \) and \( u \). In the above \( (\bar{q}q_2) = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2 \), and the subscripts \((\alpha, \beta)\) denote the color indices. The Wilson coefficients \( c_i \) are scale \((\mu)\)-dependent, with \( \mu = m_{q(c)} \) for the b(c) decays.

We now specify the notation of SU(3) group tensor properties for the effective Hamiltonians, omitting Lorentz structure, for \( c \rightarrow q_j \bar{q}_k q_k \) and \( b \rightarrow c\bar{q}_i q_j \). We will denote them by \( H_{j\,i}^{jk} \) for \( H_{c\,\text{eff}}^i \), \( H_i^f \) for \( H_{b\,\text{eff}}^i \) with \( a = u \), and \( H_i^3 \) for \( H_{b\,\text{eff}}^i \) with \( a = c \), respectively. Their nonzero entries are given by [15, 24]

\[
H_{2\,3}^{31} = \lambda_{ds}, \quad H_{2\,2}^{21} = \lambda_{dd}, \quad H_{3\,3}^{31} = \lambda_{ss}, \quad H_{3\,2}^{21} = \lambda_{sd} \\
H_{1\,3}^{31} = \lambda_{ud}, \quad H_{3\,1}^{31} = \lambda_{us}, \quad H_{2\,2} = \lambda_{cd}, \quad H_{3\,3} = \lambda_{cs}.
\]

\( H_{i\,j}^{jk} \) contains a \( 3_H \), a \( 6_H \) and a \( \overline{15}_H \) [13]. Here the subscript for the representations indicates where they come from, and the \( H \) shows that the given representation comes from the effective Hamiltonian. When calculating various decays, the results for \( 3_H \) will be proportional to \( \lambda_{dd} + \lambda_{ss} \). Using the unitarity property of the CKM matrix, this combination is equal to \( -V_{ub}V_{cd}^\ast \) which is much smaller than any of the \( \lambda_{q'q} \) and can be safely neglected. In other words, \( \lambda_{ss} = -\lambda_{dd} \) will be a good approximation for our purpose. \( H_2^3 \) and \( H_3^2 \) transform as a \( 5_H \) and \( 3_H \), respectively.

The above effective Hamiltonians can induce various hadronic \( B_{bc} \) decays, including two and multi-body channels [5]. We will concentrate on the two-body hadronic decays which may provide the most promising chances for experimental measurements.
II. THE TWO BODY B_{bc} DECAY MODES

We now list some of the interesting two body decay modes of B_{bc}. The effective Hamiltonian $H_i^{jk}$ can induce B_{bc} to decay into an octet-8$_B$ (decuplet-10$_B^*$) baryon B (B') and a 3$_{M_b}$ b-meson $M_b$ and also into a 3$_{B_c}$ (6$_{B^*_c}$) b-baryon B$_b$ (B'$_b$) plus an octet-8$_M$ meson $M$,

$$B_{bc} \rightarrow B^{(i)} + M_b, \quad B_{bc} \rightarrow B^{(i)}_b + M,$$

(4)

where B$^{(i)}$ stands for the octet (decuplet) baryon and B$^{(i)}_b$ the anti-triplet (sextet) Q-baryon with $Q = (b,c)$.

$H_j^i$ can induce B_{bc} to decay into a triplet-3$_{B_{cc}}$ double charmed baryon B$_{cc}$ and a meson $M$, and also into a 3$_{B_c}$ (6$_{B^*_c}$) c-baryon plus 3$_M$ c-meson $M_c$

$$B_{bc} \rightarrow B_{cc} + M, \quad B_{bc} \rightarrow B^{(i)}_c + M_c.$$

(5)

$H^i$ can induce the following decay modes

$$B_{bc} \rightarrow B_{cc} + M_c, \quad B_{bc} \rightarrow B^{(i)}_c + M_{c\bar{c}},$$

(6)

where $M_{c\bar{c}}$ denotes the anti-particle of $M_c$, and $M_{c\bar{c}} = (\eta_c, J/\psi)$.

The usual octet-8 B baryon has components B$^i$: (n, p, $\Sigma^{\pm,0}, \Xi^{\pm,0}, \Lambda$). The usual decuplet-10 B' has components B'$^{ijk}$: ($\Delta^{++,\pm,0}, \Sigma'^{\pm,0}, \Xi'^{-,0}, \Omega^-$) with the subscripts $ijk$ being totally symmetric. The usual octet-8 pseudoscalar meson $M$ has components $M^i$: ($\pi^{\pm,0}, K^\pm, K^0, \bar{K}^0, \eta$). The baryon states containing heavy quarks are indicated by

$$B_{cc}(B_{cc}^i) = (\Xi^+_cc, \Xi^-cc, \Omega^+_cc),$$

$$B_b(B_b^{ij}) = \left( \begin{array}{ccc} 0 & \Lambda^0_b & \Xi^0_b \\ -\Lambda^0_b & 0 & \Xi^0_b \\ -\Xi^0_b & -\Xi^0_b & 0 \end{array} \right), \quad B'_b(B'_b^{ij}) = \left( \begin{array}{ccc} \frac{1}{\sqrt{2}}\Sigma^+_b & \frac{1}{\sqrt{2}}\Sigma^0_b & \frac{1}{\sqrt{2}}\Xi^0_b \\ -\frac{1}{\sqrt{2}}\Sigma^+_b & \frac{1}{\sqrt{2}}\Sigma^0_b & \frac{1}{\sqrt{2}}\Xi^0_b \\ -\frac{1}{\sqrt{2}}\Xi^0_b & -\Xi^0_b & 0 \end{array} \right),$$

$$B_c(B_c^{ij}) = \left( \begin{array}{ccc} 0 & \Lambda^+_c & \Xi^+_c \\ -\Lambda^+_c & 0 & \Xi^+_c \\ -\Xi^-_c & -\Xi^-_c & 0 \end{array} \right), \quad B'_c(B'_c^{ij}) = \left( \begin{array}{ccc} \frac{1}{\sqrt{2}}\Sigma^+_c & \frac{1}{\sqrt{2}}\Sigma^0_c & \frac{1}{\sqrt{2}}\Xi^0_c \\ -\frac{1}{\sqrt{2}}\Sigma^+_c & \frac{1}{\sqrt{2}}\Sigma^0_c & \frac{1}{\sqrt{2}}\Xi^0_c \\ -\frac{1}{\sqrt{2}}\Xi^0_c & -\Xi^0_c & 0 \end{array} \right),$$

(7)

and the component meson states in $M_{b,c}$ read

$$M_b(M_b^i) = (B^-, \bar{B}_0, B_0^0), \quad M_c(M_c^i) = (D^0, D^+, D_s^+).$$

(8)

The octet baryon can also be written as $B_{ijk} = \epsilon_{ijk}B^i_b$. This form will be particularly useful in drawing topological diagrams for the decay amplitude at quark level.

III. SU(3) INVARIANT AMPLITUDES

We now provide some details for the SU(3) invariant decay amplitudes for the decays mentioned in previous sections. To obtain SU(3) amplitudes, one just needs to contract all upper and lower indices of the hadrons and the Hamiltonian to form all possible SU(3) singlets and associate each with a parameter which lumps up the Wilson coefficients and unknown hadronization effects. These parameters can be determined theoretically and experimentally. Our emphasis will not be on how to determine these hadronic parameters, but to identify the dominant modes and some relations for experimental search. We will normalize the decay amplitudes as $A \equiv (G_F/\sqrt{2})m$ and leave the spinor and Lorentz structure out.
The $H_i^{jk}$ induced $B_{bc}$ decays are given by

$$B_{bc} \rightarrow B^{(i)} + M_b, \quad B_{bc} \rightarrow B^{(j)}_b + M. \quad (9)$$

For $B_{bc} \rightarrow B^{(i)} M_b$, we have

$$\mathcal{M}(B_{bc} \rightarrow B M_b) = d_1 B_{bc}^{i} H_i^{jk} B_{ij} M^i_b + d_2 B_{bc}^{i} H_i^{jk} B_{ijkl} M^i_b + d'_3 B_{bc}^{i} H_i^{jk} B_{ij} M^i_b + d'_4 B_{bc}^{i} H_i^{jk} B_{ij} M^i_b,$$

$$\mathcal{M}(B_{bc} \rightarrow B' M_b) = d'_1 B_{bc}^{i} H_i^{jk} B_{ij} M^i_b + d'_2 B_{bc}^{i} H_i^{jk} B^{*}_{ij} M^i_b. \quad (10)$$

We show the corresponding topological diagrams for these two classes of decays in Fig. 1.

In the above, we have neglected terms needing to contract two indices of the Hamiltonian $H_i^{ij}$. Because this will result in the combination of $\lambda_{dd} + \lambda_{ss}$, leading to $-V_{ub} V_{cb}^*$, whose absolute value $\sim 10^{-4}$ is very small compared to $|\lambda_{dd,ss}| \approx 0.22$.

One needs to make sure if the above amplitudes are all independent. This can be checked by group theoretical considerations. The number of independent amplitudes for $B_{bc} \rightarrow B^{(i)} + M_b$ is equivalent to the number of $SU(3)$ singlets in the production of $3_{B_{bc}} \times (\bar{3}_H, 6_H, \bar{15}_H) \times \bar{8}_{10}(B') \times \bar{3}_{M_b}$. With $\bar{3}_H$ for $B_{bc} \rightarrow B + M_b$, 2 singlets can be formed which correspond to 2 invariant amplitudes. However, to get a 3 requires to contract two indices and to have $H_i^{ij}$ whose contributions are small, proportional to $\lambda_{dd} + \lambda_{ss}$ and can be neglected. For $6_H$ and $\bar{15}_H$, each produces 2 singlets. Therefore there are total 4 independent invariant amplitudes. Naively there are 6 terms as can be seen in the above equation. However using the identity $B_{ij} = B_{jki} + B_{ij} = 0$, one can rewrite $d' B_{ijkl} = -d' B_{ijkl} - d' B_{ijkl}$. Then $d'$ term can be absorbed by replacing $d_1 - d'$ and $d_1 - d'$ by the $d_1$ and $d_2$ terms, respectively. Similarly, $d''$ can be absorbed into $d_3$ and $d_4$ terms. We will choose $d_1$, $d_2$, $d_3$, $d_4$ as independent invariant amplitudes in our later discussions.

For $B_{bc} \rightarrow B' + M_b$, neglecting 1 invariant amplitude from $3_H$ for the same reason as before, there are 2 independent amplitudes with $6_H$ and $\bar{15}_H$ contributing to 0 and 2, respectively. We use the notations in the second equation of eq. (10).

For $B_{bc} \rightarrow B^{(i)}_b + M$, we can naively write down the following terms

$$\mathcal{M}(B_{bc} \rightarrow B^{(i)}_b M) = e_1 B_{bc}^{i} H_i^{jk} B_{ji} M^i_b + e_2 B_{bc}^{i} H_i^{jk} B_{ijkl} M^i_b + e_3 B_{bc}^{i} H_i^{jk} B_{ji} M^i_b + e_4 B_{bc}^{i} H_i^{jk} B_{ijkl} M^i_b,$$

$$\mathcal{M}(B_{bc} \rightarrow B^{(j)}_b M) = e'_1 B_{bc}^{i} H_i^{jk} B_{ji} M^i_b + e'_2 B_{bc}^{i} H_i^{jk} B_{ijkl} M^i_b + e'_3 B_{bc}^{i} H_i^{jk} B_{ji} M^i_b + e'_4 B_{bc}^{i} H_i^{jk} B_{ijkl} M^i_b. \quad (11)$$

The corresponding topological diagrams for these decays are shown in Fig. 2.

The $B_{bc} \rightarrow B + M$ decay has the same group structure as $B_{bc} \rightarrow B + M_b$, therefore this class of decay has only 4 independent amplitudes. One of the parameters can be absorbed into others. For example, with the appearances of $(e_1 - e_5), (e_2 + e_5), (e_3 - e_5)$ and $(e_4 + e_5)$ in the full expansion of $\mathcal{M}(B_{bc} \rightarrow B M)$, the $e_5$ term is redundant. We choose to work with the convention $e_5 = 0$. But for $B_{bc} \rightarrow B'_b + M$, neglecting $3_H$ contribution, there are total 5 independent invariant amplitudes with $6_H$ and $\bar{15}_H$ contributing to 2 and 3, respectively. We use the above terms associated with $e'_i$.

For $H_i^{ij}$ induced $B_{bc}$ decays, $B_{bc} \rightarrow B_{cc} + M$ and $B_{bc} \rightarrow B^{(i)}_c + M_c$, we have

$$\mathcal{M}(B_{bc} \rightarrow B_{cc} M) = a_1 B_{bc}^{i} H_i^{jk} B_{cc} i M^i_j + a_2 B_{bc}^{i} H_i^{jk} B_{cc} i M^i_j + a_3 B_{bc}^{i} H_i^{jk} B_{cc} j M^i_j,$$

$$\mathcal{M}(B_{bc} \rightarrow B^{(i)}_c M) = b_1 B_{bc}^{i} H_i^{jk} B_{c} jk M^i_c + b_2 B_{bc}^{i} H_i^{jk} B_{c} jk M^i_c. \quad (12)$$

The corresponding topological diagrams are shown in Fig. 3. Similar group theoretical analysis shows that the above invariant amplitudes are all independent ones.
FIG. 1: Topological diagrams for $B_{bc} \to B^{(i)} + M_b$. The two quarks next to "|" are anti-symmetric, and the three quarks next to "\{" are totally symmetric.

Finally, for $H^i$ induced $B_{bc}$ decays, $B_{bc} \to B_{cc} + M_{\bar{c}}$ and $B_{bc} \to B_{c}^{(i)} + M_{\bar{c}c}$, we have

\[
\mathcal{M}(B_{bc} \to B_{cc} M_{\bar{c}}) = f_1 B_{bc}^i H^j B_{cc}^i M_{\bar{c}c} j + f_2 B_{bc}^i H^j B_{cc}^i M_{\bar{c}c} i,
\]

\[
\mathcal{M}(B_{bc} \to B_{c}^{(i)} M_{\bar{c}c}) = g_{c}^{(i)} B_{bc}^i H^j B_{c}^{(i)} M_{\bar{c}c} j.
\] (13)

The corresponding topological diagrams are shown in Fig. 4.

IV. DECAY MODES FOR EXPERIMENTAL ANALYSIS

For experimental discovery of $B_{bc}$, the most favored decay modes will certainly be those with large branching ratios, and at the same time particles in the final state can be easily identified and analyzed. For the decay modes discussed in previous sections, there are two classes of decays, one is decay induced by $c$ decays and another induced by $b$ decays. We discuss $c$ induced decays below,

\[
B_{bc} \to B^{(i)} + M_b, \quad B_{bc} \to B_{b}^{(i)} + M_b.
\] (14)

Among $B_{bc} \to B^{(i)} + M_b$ and $B_{bc} \to B_{b}^{(i)} + M$, the first one has advantages compared with the second one because the final states $B^{(i)}$ and $M_b$ are all well studied experimentally. The further decay branching ratios for many of them are known to good precisions. For the second one although properties of $M$ are well known, the final baryon
$B_{bc} \rightarrow B_b^{(i)} + M$

FIG. 2: Topological diagrams for $B_{bc} \rightarrow B_b^{(i)} + M$. The two quarks next to ""] are anti-symmetric (symmetric) for $B_b$ ($B_b'$), respectively. For $B_{bc} \rightarrow B_b + M$, one should remove the $e_5$ term.

$B_b^{(i)}$ properties are not as well known as $B^{(i)}$ baryons and therefore this class of decays may not be as easy as $B_{bc} \rightarrow B^{(i)} + M_b$ for analysis. But still many decays modes of $B_b^{(i)}$ have been measured. The second one can also serve to confirm the discovery of $B_{bc}$ and study its detailed properties and may even luckily become the discovered modes if identification of $B_b^{(i)}$ can be optimized.

From Tables I and II, we can identify the following Cabibbo allowed decays as

$B_{bc} \rightarrow B + M_b$ type:

$\Xi_{bc}^0 \rightarrow \Xi^0 \bar{B}_s^0$, $\Omega_{bc}^0 \rightarrow \Xi^0 \bar{B}_s^0$, $\Xi_{bc}^+ \rightarrow \Sigma^+ \bar{B}_s^0$, $\Xi_{bc}^0 \rightarrow \Lambda \bar{B}_s^0$, $\Xi_{bc}^0 \rightarrow \Sigma^0 \bar{B}_s^0$.

$B_{bc} \rightarrow B' + M_b$ type:

$\Xi_{bc}^0 \rightarrow \Sigma^{*+} \bar{B}_s^0$, $\Xi_{bc}^0 \rightarrow \Xi^0 \bar{B}_s^0$, $\Omega_{bc}^0 \rightarrow \Xi^0 \bar{B}_s^0$, $\Xi_{bc}^0 \rightarrow \Sigma^0 \bar{B}_s^0$.

$B_{bc} \rightarrow B_b + M$ type:

$\Xi_{bc}^0 \rightarrow \Xi_b^- \pi^+$, $\Xi_{bc}^0 \rightarrow \Xi_b^0 \pi^+$, $\Xi_{bc}^0 \rightarrow \Xi_b^0 \pi^0$, $\Omega_{bc}^0 \rightarrow \Xi_b^0 \bar{K}_0$, $\Xi_{bc}^0 \rightarrow \Xi_b^0 \bar{\eta}$, $\Xi_{bc}^0 \rightarrow \Lambda_b \bar{K}_0$.

$B_{bc} \rightarrow B_b' + M$ type:

$\Xi_{bc}^0 \rightarrow \Omega_b \bar{K}_0$, $\Xi_{bc}^0 \rightarrow \Xi_b^- \bar{K}^-$, $\Xi_{bc}^0 \rightarrow \Xi_b^0 \bar{\eta}$, $\Xi_{bc}^0 \rightarrow \Xi_b^0 \bar{\eta}$, $\Xi_{bc}^0 \rightarrow \Xi_b^0 \bar{\eta}$, $\Xi_{bc}^0 \rightarrow \Xi_b^0 \bar{\eta}$, $\Xi_{bc}^0 \rightarrow \Xi_b^0 \bar{\eta}$.

(15)

We expect the above decay modes to be the dominant ones which are some of the most likely to-be-discovered decay modes. Since the lifetime of $B_{bc}$ is mainly determined by $c \rightarrow s u d$, the lifetime of $\Xi_{bc}^0$ would be similar to $\Xi_{cc}^{++}$ and the decay phase spaces and the particle masses are different and therefore may deviate from each other. This expectation is in agreement with some theoretical estimates. For example, in Ref. [20] it is estimated to be a few times of $10^{-13}$ s. A
\[ B_{bc} \rightarrow B_{cc} + M \]

FIG. 3: Topological diagrams for \( B_{bc} \rightarrow B_{cc} M \) (top panel) and \( B_{bc} \rightarrow B_{cc}^{(i)} M \) (bottom panel), where the two quarks next to "\( a \)" are anti-symmetric (symmetric) for \( B_c \) (\( B_c' \)).

\[ B_{bc} \rightarrow B_{cc}^{(i)} + M \]

FIG. 4: Topological diagrams for \( B_{bc} \rightarrow B_{cc} M \bar{c} \) (top panel) and \( B_{bc} \rightarrow B_{cc}^{(i)} M \bar{c} \) (bottom panel). The two quarks next to "\( a \)" are anti-symmetric (symmetric) for \( B_c \) (\( B_c' \)).
similar argument would lead to the expectation that the decay width of $\Xi_{bc}^0 \rightarrow \Xi_b^{0+} + \pi^+$ is similar to $\Xi_{bc}^{0+} \rightarrow \Xi_b^{0+} + \pi^+$ up to phase space modifications. Taking factorizable contributions as the example for an order of magnitude estimate, we derive that $\mathcal{M}(\Xi_{Qc} \rightarrow \Xi_Q\pi^+) = \lambda_{d_s a_1 f}\bar{q}^u (\Xi_Q)(\bar{s}c)\mathcal{M}_{Qc}$, where the decay constant $f_x$ is from $\langle \pi^+ | (\bar{u}d) | \Xi_Q \rangle = i f_x q^u$, and $a_1 = c_1 + c_2 / N_c$ with $N_c$ the color number. With $\langle \Xi_{Q}(\bar{s}c)|\mathcal{M}_{Qc} \rangle \simeq \bar{u}_{\Xi_Q}(f_1 \gamma_\mu - g_1 \gamma_\mu \gamma_5) u_{\Xi_Q}$, we obtain

$$\Gamma(\Xi_{Qc} \rightarrow \Xi_Q\pi^+) = \frac{G_F^2}{32\pi} (\lambda_{d_s a_1 f_x})^2 m_{\Xi_Q}^3 \left( 1 - \frac{m_{\Xi_Q}^2}{m_{\Xi_Q}^2} \right)^3 (f_1^2 + g_1^2), \quad (17)$$

where $m_{\pi} \simeq 0$ has been used. We can hence have $\Gamma(\Xi_{bc}^0 \rightarrow \Xi_b^{0+} + \pi^+)/\Gamma(\Xi_{bc}^{0+} \rightarrow \Xi_b^{0+} + \pi^+) \sim (m_{\Xi_{bc}^{0+}}/m_{\Xi_{bc}^{0+}})^3(1 - m_{\Xi_{bc}^{0+}}^2/m_{\Xi_{bc}^{0+}}^2)/(1 - m_{\Xi_{bc}^{0+}}^2/m_{\Xi_{bc}^{0+}}^2) \approx 1.4\times(m_{\Xi_{bc}^{0+}}/m_{\Xi_{bc}^{0+}}, m_{\Xi_{bc}^{0+}}/m_{\Xi_{bc}^{0+}}, m_{\Xi_{bc}^{0+}}/m_{\Xi_{bc}^{0+}}) \approx (1.93, 0.83, 0.68) [1, 7]$. This leads to $B(\Xi_{bc}^{0+} \rightarrow \Xi_b^{0+} + \pi^+) \approx 10^{-2}$ [25], in agreement with the calculations in Refs. [26-29]. One expects other Cabibbo-allowed $B_{bc} \rightarrow B^{(i)}_c + M$ and also $B_{bc} \rightarrow B^{(i)} + M_b$ branching fractions to have similar order of magnitudes [26-29].

According to the gluon-gluon fusion mechanism that dominantly produces the baryons with two heavy quarks [3, 30, 31], the cross section ($X$) for the $\Xi_{bc}$ production is estimated to be $(17 \pm 3)$ and $(35 \pm 7)$ nb at LHC for the center-of-mass (c.m.) energy $\sqrt{s} = 7$ GeV and $\sqrt{s} = 14$ GeV, respectively, where the theoretical errors mainly consider the uncertainties from the quark masses $m_b$ and $m_c$, together with the non-perturbative effects and the factorization scale. Since the LHC luminosity ($L$) of the c.m. energy $\sqrt{s} = (7,14)$ GeV can be $(10,100)$ fb$^{-1}/yr$ [3], using $X \cdot L$ we obtain the $\Xi_{bc}$ events about $(1.7 \pm 0.3) \times 10^8$ and $(3.5 \pm 0.7) \times 10^9$ per year. Note that the uncertainties of around 20% come from the cross sections, which demonstrates the reliability of the theoretical estimates. On the other hand, to discover $\Xi_{bc}$ and determine its mass, the decay products have to be fully constructed, involving the branching ratios of the discovery channels and the subsequent decays. The most promising discovery channels should be the Cabibbo-allowed ones with $B$ or $B_c$ in Eqs. (15) and (16), which cause less subsequent decays compared to those with $B'$ and $B'_c$. Let us take $\Xi_{bc}^0 \rightarrow \Lambda + \bar{B}^0$ and $\Xi_{bc}^+ \rightarrow \Xi_b^{0+} + \pi^+$ as our illustrations. For $\Xi_{bc}^0 \rightarrow \Lambda \bar{B}^0$, since $\Lambda \rightarrow p\pi^-$ and $\bar{B}^0 \rightarrow D^+\pi^-$ can be suitable subsequent decays, with $B(\Xi_{bc}^0 \rightarrow \Lambda \bar{B}^0) \sim 10^{-2}$, $B(\Lambda \rightarrow p\pi^-) \simeq 64\%$, $B(\bar{B}^0 \rightarrow D^+\pi^-) \simeq 2.5 \times 10^{-3}$ and $B(\bar{D}^+ \rightarrow \pi^+\pi^0) \sim 1.2 \times 10^{-3}$, we estimate that at least $10^8$ $\Xi_{bc}$ events are needed. For $\Xi_{bc}^+ \rightarrow \Xi_b^{0+} \pi^+$, we have $\Xi_{bc}^0 \rightarrow \Xi_{c}^0 J/\Psi$ as the subsequent decay with $B = 10^{-4} - 10^{-3}$ [32]. In addition to $B(\Xi_{bc}^0 \rightarrow \Lambda \pi^0, \Lambda \rightarrow p\pi^-) \simeq 64\%$ and $B(J/\Psi \rightarrow \mu^+\mu^- + e^+e^-) \simeq 12\% [1]$, it leads to the estimation of the $10^6 \sim 7$ $\Xi_{bc}$ events at least [33]. There can be other subsequent processes, such as $\Xi_{bc}^0 \rightarrow \Xi_{c}^0 + (D^+_c, \pi^+)$, whose needed $\Xi_{bc}^+$ events are of the same orders. Similarly, we estimate the other most promising discovery channels, and summarize the needed $\Xi_{bc}$ events in Table V. Therefore, the LHC may be able to discover $B_{bc}$ given their excellent capabilities of identifying $B^{(i)}_c$, $M_b$ and also reasonably good identifications for $B_{bc}$ baryons. We strongly urge our experimental colleagues to search for those $B_{bc}$ decays.

One also sees from the Tables I and II that there are several relations among decay amplitudes induced by $c \rightarrow u\bar{q}'\bar{q}$. Those Cabibbo allowed decays offer good chances to be tested experimentally, given by

$$\mathcal{M}(\Xi_{bc}^0 \rightarrow \Sigma^{+} B^-) = \mathcal{M}(\Xi_{bc}^0 \rightarrow \Xi_{bc}^{0} B_{s}^0) = \frac{1}{\sqrt{3}} \lambda_{d_s} d_1^2,$$
$$\mathcal{M}(\Omega_{bc}^0 \rightarrow \Xi_{bc}^{0} B_{s}^0) = \mathcal{M}(\Xi_{bc}^0 \rightarrow \Sigma^{+} B^-) = \frac{1}{\sqrt{3}} \lambda_{d_s} d_2^2, \quad (18)$$

and triangle relations

$$\mathcal{M}(\Xi_{bc}^0 \rightarrow \Sigma^{+} B^-) + \mathcal{M}(\Xi_{bc}^+ \rightarrow \Sigma^{+} B^-) - \sqrt{2} \mathcal{M}(\Xi_{bc}^0 \rightarrow \Sigma_{bc}^{0} B_{s}^0) = 0,$$
$$\mathcal{M}(\Xi_{bc}^+ \rightarrow \Xi_{bc}^{0} \pi^+) - \mathcal{M}(\Xi_{bc}^+ \rightarrow \Xi_{bc}^{0} \pi^+) + \sqrt{2} \mathcal{M}(\Xi_{bc}^0 \rightarrow \Xi_{bc}^{0} \pi^0) = 0,$$
$$\mathcal{M}(\Xi_{bc}^+ \rightarrow \Sigma_{bc}^{+} K^0) + \mathcal{M}(\Xi_{bc}^+ \rightarrow \Sigma_{bc}^{+} K^-) - \sqrt{2} \mathcal{M}(\Xi_{bc}^0 \rightarrow \Sigma_{bc}^{0} K_{s}^0) = 0 \quad (19).$$

There are also some relations between Cabibbo allowed, Cabibbo suppressed and doubly Cabibbo suppressed decay modes which can be read off from Tables I and II. With more and more data being collected, these relations can also serve further examinations.
For $b$-decay induced decays, we have

$$
\begin{align*}
B_{bc} & \rightarrow B_{cc} + M, \\
B_{bc} & \rightarrow B_{c}^{(f)} + M_c, \\
B_{bc} & \rightarrow B_{cc} + M_{\bar{c}}, \\
B_{bc} & \rightarrow B_{c}^{(f)} + M_{\bar{c}}.
\end{align*}
$$

(20)

As mentioned before that $b$-decay induced modes are suppressed by a factor of $|V_{cb}/V_{cs}| \simeq 0.04$ compared with the Cabibbo allowed ones in $c$-decay induced modes. Nonetheless, their branching fractions are not necessarily small. Using the $B_{bc} \rightarrow B_{c}$. transition form factors obtained in Refs. [34], we estimate that $B(\Xi_{bc}^{+} \rightarrow \Sigma_{cc}^{+} D^{0}) \sim 10^{-6}$, $B(\Xi_{bc}^{+} \rightarrow \Xi_{cc}^{+} \eta_{c}) \sim 10^{-5}$, $B(\Xi_{bc}^{+} \rightarrow \Xi_{cc}^{+} \pi^{-}) \sim 10^{-4}$ and $B(\Xi_{bc}^{+} \rightarrow \Xi_{cc}^{+} D_{s}^{-}) \sim 10^{-3}$. Therefore, we expect that the branching fractions of $B_{bc} \rightarrow B_{c}^{(f)} + (M_c, M_{\bar{c}})$ and $B_{bc} \rightarrow B_{cc} + (M, M_{\bar{c}})$ can be as large as $(10^{-6}, 10^{-5}, 10^{-4}, 10^{-3})$, respectively.

According to the current luminosity at LHCb, they are much more difficult to be measured experimentally. The decayed meson particles are all easy to be identified. In particular, by adopting the situation in Ref. [22], where the weakly decaying double beauty hadrons have been discussed, we take $B_{bc}^{(f)}$ and $B_{cc}$ as the displaced baryons. They travel sizeable distances before decaying, which helps to distinguish the signal from the prompt background that might come from the $c$-quark induced $B_{bc} \rightarrow B_{c} M(B_{cb})$ decays [22]. There may be some chances when more and more data are collected. For example, with the subsequent $\Xi_{cc}^{+}$ decay, the needed $\Xi_{bc}$ events to discovery $\Xi_{bc}^{+} \rightarrow \Xi_{cc}^{+} \pi^{−}$ are estimated to be around $10^{9}$. We list the $SU(3)$ invariant decay amplitudes in Tables III and IV for completeness. There are also several relations among different decay modes.

In conclusion, we have studied the two-body $B_{bc}$ weak decays using the $SU(3)$ flavor symmetry aiming to provide the most promising decay channels to discover $B_{bc}$. With the branching fractions estimated as a few times $10^{-2}$ for the Cabibbo-allowed $c$-quark weak decays, the LHC may be able to discover $B_{bc}$ given their excellent capabilities of identifying $B_{c}^{(f)}$, $M_b$ and also reasonably good identifications for $B_{b}$ baryons. The decay modes $B_{bc} \rightarrow B_{c}^{(f)} + (M_c, M_{\bar{c}})$ and $B_{bc} \rightarrow B_{cc} + (M, M_{\bar{c}})$ induced by $b$-quark weak decay have suppressed decay branching ratios as they are suppressed by CKM factor of $|V_{cb}/V_{cs}| \simeq 0.04$. Nonetheless, their branching fractions are not necessarily too much smaller, such as $B(\Xi_{bc}^{+} \rightarrow \Xi_{cc}^{+} D_{s}^{-}) \sim 10^{-3}$. They may have some chance to be eventually detected at the LHCb. We strongly urge our experimental colleagues to search for $B_{bc}$ using two-body weak decays.

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### TABLE I: Amplitudes of $B_{bc} \to B^{(*)} M_b$ with $\lambda_{(d, d, d, s)} = (V_d V_{cd}, V_d V_{cd}, V_d V_{cd})$, where $\lambda_{dd} = -\lambda_{ds}$ has been used.

| Decay modes | Amplitudes | Decay modes | Amplitudes |
|-------------|------------|-------------|------------|
| $B_{bc} \to \Xi^0 B^0$ | $\lambda_{dd} (d_1 - d_2)$ | $B_{bc} \to \Xi^0 B^0$ | $\lambda_{dd} (d_1 - d_2)$ |
| $B_{bc} \to \Xi^0 B^0$ | $\lambda_{dd} (d_1 - d_2)$ | $B_{bc} \to \Xi^0 B^0$ | $\lambda_{dd} (d_1 - d_2)$ |
| $B_{bc} \to \Xi^0 B^0$ | $\lambda_{dd} (d_1 - d_2)$ | $B_{bc} \to \Xi^0 B^0$ | $\lambda_{dd} (d_1 - d_2)$ |
| $B_{bc} \to \Xi^0 B^0$ | $\lambda_{dd} (d_1 - d_2)$ | $B_{bc} \to \Xi^0 B^0$ | $\lambda_{dd} (d_1 - d_2)$ |
| $B_{bc} \to \Xi^0 B^0$ | $\lambda_{dd} (d_1 - d_2)$ | $B_{bc} \to \Xi^0 B^0$ | $\lambda_{dd} (d_1 - d_2)$ |
| $B_{bc} \to \Xi^0 B^0$ | $\lambda_{dd} (d_1 - d_2)$ | $B_{bc} \to \Xi^0 B^0$ | $\lambda_{dd} (d_1 - d_2)$ |
### TABLE II: Amplitudes of $B_{bc} \to B^{(0)}_c M$.

| Decay modes | Amplitudes |
|-------------|------------|
| $\Xi_{bc} \to \Xi_{c}^{+}$ | $-\lambda_{dd} e_1$ |
| $\Xi_{bc} \to \Xi_{c}^{-}$ | $-\lambda_{dd} e_3$ |
| $\Omega_{bc} \to \Xi_{c}^{+} K^{0}$ | $\lambda_{dd} \frac{e_1 - e_3}{2}$ |
| $\Xi_{bc} \to \Xi_{c}^{-} K^{0}$ | $-\lambda_{dd} (e_3 + e_4)$ |
| $\Omega_{bc} \to \Xi_{c}^{+} \eta$ | $\lambda_{dd} (e_1 + 2e_3 + e_4)$ |
| $\Xi_{bc} \to \Lambda_{b} K^{0}$ | $\lambda_{dd} (e_2 - e_4)$ |

| Decay modes | Amplitudes |
|-------------|------------|
| $\Xi_{bc} \to \Xi_{c}^{+}$ | $\lambda_{dd} e_1$ |
| $\Xi_{bc} \to \Xi_{c}^{-}$ | $\lambda_{dd} e_3$ |
| $\Omega_{bc} \to \Xi_{c}^{+} K^{0}$ | $\lambda_{dd} (e_2 + e_3)$ |
| $\Xi_{bc} \to \Xi_{c}^{-} K^{0}$ | $-\lambda_{dd} (e_3 + e_4)$ |
| $\Omega_{bc} \to \Xi_{c}^{+} \eta$ | $\lambda_{dd} (e_1 + 2e_3 + e_4)$ |
| $\Xi_{bc} \to \Lambda_{b} K^{0}$ | $\lambda_{dd} (e_2 - e_4)$ |

### TABLE III: Amplitudes of $B_{bc} \to B_{cc} M$ and $B_{bc} \to B^{(0)}_c M$ with $\lambda^{cc}_{ad(s)} = V_{cb} V^{*}_{ud(s)}$.

| Decay modes | Amplitudes |
|-------------|------------|
| $\Xi_{bc} \to \Omega_{c}^{+}$ | $\lambda_{ad} a_1$ |
| $\Xi_{bc} \to \Xi_{c}^{+} K^{0}$ | $\lambda_{ad} a_1$ |
| $\Omega_{bc} \to \Xi_{c}^{+} K^{0}$ | $\lambda_{ad} a_2$ |
| $\Omega_{bc} \to \Xi_{c}^{+} K^{-}$ | $\lambda_{ad} a_3$ |
| $\Omega_{bc} \to \Xi_{c}^{-} K^{+}$ | $\lambda_{ad} a_3$ |
| $\Xi_{bc} \to \Xi_{c}^{+} \eta$ | $\lambda_{ad} a_3$ |
| $\Xi_{bc} \to \Xi_{c}^{-} \eta$ | $\lambda_{ad} a_3$ |
| $\Omega_{bc} \to \Xi_{c}^{+} \eta$ | $\lambda_{ad} a_3$ |

| Decay modes | Amplitudes |
|-------------|------------|
| $\Xi_{bc} \to \Omega_{c}^{+}$ | $\lambda_{ad} b_1$ |
| $\Xi_{bc} \to \Xi_{c}^{+} K^{0}$ | $-\lambda_{ad} b_1$ |
| $\Omega_{bc} \to \Xi_{c}^{+} K^{0}$ | $\lambda_{ad} b_2$ |
| $\Omega_{bc} \to \Xi_{c}^{+} K^{-}$ | $-\lambda_{ad} b_2$ |
| $\Xi_{bc} \to \Xi_{c}^{+} \eta$ | $\lambda_{ad} b_3$ |
| $\Xi_{bc} \to \Xi_{c}^{-} \eta$ | $\lambda_{ad} b_3$ |
| $\Omega_{bc} \to \Xi_{c}^{+} \eta$ | $\lambda_{ad} b_3$ |

Note: $\lambda_{ad(s)}$ are the appropriate amplitudes for the respective decay modes.
TABLE IV: Amplitudes of $B_{bc} \rightarrow B_{bc} M_c$ and $B_{bc} \rightarrow B_c^{(i)} M_c$ with $\lambda_{cd} = V_{cb} V_{cd}^\ast$.

| Decay modes | Amplitudes | Decay modes | Amplitudes |
|-------------|------------|-------------|------------|
| $\Xi^+_{bc} \rightarrow \Xi^+_c D^0_c$ | $\lambda^c_{cd} f_1$ | $\Xi^+_{bc} \rightarrow \Lambda^+_c M_c$ | $\lambda^c_{cd} g_{bc} f_1$ |
| $\Omega^0_{bc} \rightarrow \Omega^0_c D^-$ | $\lambda^c_{cd} f_1$ | $\Omega^0_{bc} \rightarrow \Xi^0_c M_c$ | $\lambda^c_{cd} g_{bc} f_1$ |
| $\psi^+ \rightarrow \Xi^{+0}_{bc} D^0_c$ | $\lambda^c_{cd} f_1$ | $\Xi^+_{bc} \rightarrow \Xi^+ M_c$ | $\lambda^c_{cd} g_{bc} f_1$ |
| $\psi^0_{bc} \rightarrow \Xi^{00}_{bc} D^0_c$ | $\lambda^c_{cd} f_1$ | $\Xi^+_{bc} \rightarrow \Xi^0 M_c$ | $\lambda^c_{cd} g_{bc} f_1$ |
| $\Xi^+_{bc} \rightarrow \Xi^+_c D^0_c$ | $\lambda^c_{cd} f_1$ | $\Xi^+_{bc} \rightarrow \Xi^0 M_c$ | $\lambda^c_{cd} g_{bc} f_1$ |
| $\Omega^0_{bc} \rightarrow \Omega^0_c D^0_c$ | $\lambda^c_{cd} f_1$ | $\Xi^+_{bc} \rightarrow \Xi^0 M_c$ | $\lambda^c_{cd} g_{bc} f_1$ |
| $\psi^+ \rightarrow \Xi^{+0}_{bc} D^0_c$ | $\lambda^c_{cd} f_1$ | $\Xi^+_{bc} \rightarrow \Xi^0 M_c$ | $\lambda^c_{cd} g_{bc} f_1$ |
| $\psi^0_{bc} \rightarrow \Xi^{00}_{bc} D^0_c$ | $\lambda^c_{cd} f_1$ | $\Xi^+_{bc} \rightarrow \Xi^0 M_c$ | $\lambda^c_{cd} g_{bc} f_1$ |
| $\psi^+ \rightarrow \Xi^{+0}_{bc} D^0_c$ | $\lambda^c_{cd} f_1$ | $\Xi^+_{bc} \rightarrow \Xi^0 M_c$ | $\lambda^c_{cd} g_{bc} f_1$ |
| $\psi^0_{bc} \rightarrow \Xi^{00}_{bc} D^0_c$ | $\lambda^c_{cd} f_1$ | $\Xi^+_{bc} \rightarrow \Xi^0 M_c$ | $\lambda^c_{cd} g_{bc} f_1$ |

TABLE V: Needed $\Xi_{bc}$ events for the Cabibbo-allowed $\Xi_{bc} \rightarrow B M_b$ and $\Xi_{bc} \rightarrow B_b M_b$ decays.

| Decay modes | Subsequent decays | needed $\Xi_{bc}$ events |
|-------------|-------------------|-------------------------|
| $\Xi^+_{bc} \rightarrow \Xi^+ D^0_c$ | $\mathcal{B}(\Xi^+ \rightarrow \pi^0 (\Lambda \rightarrow p \pi^-)) \approx 64\%$, $\mathcal{B}(\bar{B}^0 \rightarrow \rho^- (D^0 \rightarrow \pi^+ \pi^-)) \approx 7.5 \times 10^{-5}$ | $10^7$ |
| $\Xi^0_{bc} \rightarrow \Xi^0 D^0_c$ | $\mathcal{B}(\Xi^0 \rightarrow \pi^0 (\Lambda \rightarrow p \pi^-)) \approx 52\%$, $\mathcal{B}(\bar{B}^0 \rightarrow \rho^- (D^0 \rightarrow \pi^+ \pi^-)) \approx 2 \times 10^{-5}$ | $10^7$ |
| $\psi^+ \rightarrow \psi^+ D^0_c$ | $\mathcal{B}(\psi^+ \rightarrow \rho^- (D^0 \rightarrow \pi^+ \pi^-)) \approx 52\%$, $\mathcal{B}(\bar{B}^0 \rightarrow \rho^- (D^0 \rightarrow \pi^+ \pi^-)) \approx 3 \times 10^{-6}$ | $10^8$ |
| $\psi^0_{bc} \rightarrow \psi^0 D^0_c$ | $\mathcal{B}(\Lambda \rightarrow p \pi^-) \approx 64\%$, $\mathcal{B}(\bar{B}^0 \rightarrow \rho^- (D^0 \rightarrow \pi^+ \pi^-)) \approx 3 \times 10^{-6}$ | $10^8$ |
| $\Xi^+_{bc} \rightarrow \Lambda B^+$ | $\mathcal{B}(\Lambda \rightarrow p \pi^-) \approx 64\%$, $\mathcal{B}(\bar{B}^0 \rightarrow \rho^- (D^0 \rightarrow \pi^+ \pi^-)) \approx 3 \times 10^{-6}$ | $10^8$ |
| $\Xi^0_{bc} \rightarrow \Xi^0 D^0_c$ | $\mathcal{B}(\Xi^0 \rightarrow \pi^0 (\Lambda \rightarrow p \pi^-)) \approx 64\%$, $\mathcal{B}(\bar{B}^0 \rightarrow \rho^- (D^0 \rightarrow \pi^+ \pi^-)) \approx 3 \times 10^{-6}$ | $10^8$ |
| $\psi^+ \rightarrow \psi^+ D^0_c$ | $\mathcal{B}(\psi^+ \rightarrow \rho^- (D^0 \rightarrow \pi^+ \pi^-)) \approx 64\%$, $\mathcal{B}(\bar{B}^0 \rightarrow \rho^- (D^0 \rightarrow \pi^+ \pi^-)) \approx 3 \times 10^{-6}$ | $10^8$ |
| $\psi^0_{bc} \rightarrow \psi^0 D^0_c$ | $\mathcal{B}(\psi^0 \rightarrow \rho^- (D^0 \rightarrow \pi^+ \pi^-)) \approx 64\%$, $\mathcal{B}(\bar{B}^0 \rightarrow \rho^- (D^0 \rightarrow \pi^+ \pi^-)) \approx 3 \times 10^{-6}$ | $10^8$ |
| $\psi^+ \rightarrow \psi^+ D^0_c$ | $\mathcal{B}(\psi^+ \rightarrow \rho^- (D^0 \rightarrow \pi^+ \pi^-)) \approx 64\%$, $\mathcal{B}(\bar{B}^0 \rightarrow \rho^- (D^0 \rightarrow \pi^+ \pi^-)) \approx 3 \times 10^{-6}$ | $10^8$ |
| $\Xi^0_{bc} \rightarrow \Xi^0 D^0_c$ | $\mathcal{B}(\Xi^0 \rightarrow \pi^0 (\Lambda \rightarrow p \pi^-)) \approx 64\%$, $\mathcal{B}(\bar{B}^0 \rightarrow \rho^- (D^0 \rightarrow \pi^+ \pi^-)) \approx 3 \times 10^{-6}$ | $10^8$ |