PION INTERFEROMETRY USING WAVEPACKETS *

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Abstract

The conditions for the HBT interferometry from stars are compared to those encountered in heavy ion reactions. As a consequence the formalism using wavepackets is developed. The modifications relative to the standard formalism with plane waves are presented, and the effects of the residual Coulomb interactions between charged pions and with the charged source are calculated.

1 Introduction

Originally the interferometric method of Hanbury Brown, Twiss (HBT) was developed in astronomy and used to determine the angular radii of stars [1]. Shortly afterwards it was recognized that a similar procedure can also be applied to pions emitted in high energy reactions to measure the size of the pion emitting source [2]. However, it is clear that the different conditions prevailing in high-energy or heavy-ion reactions ask for certain modifications of the original formalism. The table 1 lists the relevant parameters encountered in the study of stars or hot nuclear matter. The correlation areas $\Delta \Omega$ are given by the solid angle necessary to observe the speckle pattern in the emission of a large number of identical bosons as displayed in Fig.1 of ref. [3]. Their difference by 30. order of magnitudes is compensated by the equivalent difference in the distance between the star, respectively the nucleus and the interferometer. The large difference between the source size $R$ and

Table 1: Parameters relevant to stars and nuclei.

|                      | star (photon) | nucleus (pion) |
|----------------------|---------------|----------------|
| source size $R$      | $10^9$ m      | $10^{-14}$ m   |
| momentum $p$         | 2 eV/c        | $2 \cdot 10^8$ eV/c |
| correlation area $\Delta \Omega = \pi/(pR)^2$ | $3 \cdot 10^{-32}$ sr | $3 \cdot 10^{-2}$ sr |
| de Broglie wavelength $\lambda = h/p$ | $6 \cdot 10^{-7}$ m | $6 \cdot 10^{-15}$ m |
| coherence length $\Lambda$ | $ct \approx 3$ m | $\lambda^2/\Delta \lambda \approx 10^{-14}$ m |

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the coherence length $\Lambda$ in case of a star ensures that the star can be treated as a chaotic radiator. In general one finds for stars $R \gg \Lambda \gg \lambda$ and the emitted photons may be described by plane waves. In case of the nucleus one finds $R \approx \Lambda \approx \lambda$, and wavepackets are the more appropriate description of the emitted pions.

2 Wavepacket Formalism

The width of a Gaussian wavepacket at time $t=0$ is given by $\sigma_0$ which from the principles of quantum mechanics is related to the pion coherence length: $\sigma_0 \approx \Lambda/4\pi \approx 1$ fm. At later times $t$ the free, i.e. non-interacting wavepacket in coordinate space is

$$\Psi(r, t) = \left(2\pi s^2\right)^{-\frac{3}{4}} \exp\left\{\frac{i}{\hbar} \left(Pr - Et\right) - \frac{(r - R(t))^2}{4s\sigma_0}\right\}, \quad (1)$$

where $P, E$ are the centre momentum, energy of the wavepacket, $R(t)$ is its spatial coordinate which changes with time according to $R(t) = R + (P/m) \cdot t$, and the dispersion of the width is given by

$$\sigma = |s| = \sigma_0 \sqrt{1 + \frac{\hbar^2}{4m^2s^2\sigma_0^2} t^2}. \quad (2)$$

Similarly the wavepacket may be described in momentum space, where it turns out to be stationary:

$$\Phi(p, t) = \left(2\pi \sigma_p^2\right)^{-\frac{3}{4}} \exp\left\{\frac{i}{\hbar} \left(R (p - P) + \frac{p^2}{2m} t\right) - \frac{(p - P)^2}{4\sigma_p^2}\right\}, \quad (3)$$

with $\sigma_p = \hbar/2\sigma_0$. It is obvious that the wavepacket representation of the pions fulfills Heisenberg’s uncertainty relation

$$\sigma_p \sigma \geq \frac{\hbar}{2}. \quad (4)$$

In the following we will restrict our discussion to SIS energies, i.e. the like-charge pion multiplicities are in central collisions of very heavy systems $m_\pi \approx 20$, and multi pion effects can be neglected. We parametrize the pion source function by $g(R, P) = \rho(R)f(P)$ with

$$\rho(R) = \left(\pi R_s^2\right)^{-\frac{3}{2}} \exp\left\{-\frac{R^2}{R_s^2}\right\} \quad (5)$$

$$f(P) = \left(2\pi mT\right)^{-\frac{3}{2}} \exp\left\{-\frac{P^2}{2mT}\right\} \quad (6)$$

and with the temperature $T$. The single pion observables in spatial and momentum space are then easy to calculate and yield the identical forms as in eqs.(5,6) except that the relevant parameters have to be replaced by $\bar{R}_s^2 = R_s^2 + 2\sigma_0^2$ and
Figure 1: Left panel: The exact and 1. order correlation functions. Right panel: The dependence of the chaoticity $\lambda$ on the localization $\sigma_0$.

$$T_{\text{eff}} = T + T_{qm}$$ where $T_{qm} = \frac{\hbar^2}{4m\sigma_0}$. This implies that for small pion multiplicities the localization $\sigma_0$ of the pions in the source does not change the single pion distributions, but the relevant parameters are changed: The effective source radius is modified by the pion localization, and the effective temperature is modified by a quantum contribution which takes into account the zero point energy of the pions.

Similarly the two pion correlation function $C_2(p_1, p_2) = \frac{T_2(p_1, p_2)}{P_1(p_1)P_2(p_2)}$ may be calculated. This calculation is more involved than in case of the single pion observables, the exact result was published in [4]. As an approximate expression one obtains

$$C_2(p_1, p_2) = 1 + \lambda \exp \left\{ -\frac{R_{\text{eff}}^2}{2\hbar^2} q^2 \right\}, \quad q = p_1 - p_2 \quad (7)$$

with an effective radius parameter $R_{\text{eff}}^2 = R_s^2 + 2\sigma_0^2 \frac{T}{T_{\text{eff}}}$, so that $R_s < R_{\text{eff}} < \tilde{R_s}$. The chaoticity parameter $\lambda$ is a function of the ratios $R_s^2/(2\sigma_0^2)$ and $T/T_{\text{eff}}$ and can be expanded into the series

$$\lambda = 1 + \sum_{k=1}^{\infty} (-)^k \left( 1 - k \frac{R_s^2}{2\sigma_0^2} \right)^{-\frac{3}{2}} \left( 1 - k \frac{T}{T_{\text{eff}}} \right)^{-\frac{3}{2}}. \quad (8)$$

The Fig. 1 displays on the left side the exact and first order ($\lambda = 1$) results for $C_2(q)$
using specific values for $R_s$, $\sigma_0$ and $T$, and on the right side the dependence of $\lambda$ on $\sigma_0$ is shown for 3 different values of $R_s$ and the temperature $T = 50$ MeV. From this dependence we infer that $\lambda$ approaches 0 only when $\sigma_0$ becomes very large and $T$ goes to zero.

The wavepacket formalism for pions introduces a new parameter $\sigma_0$ which was called localization because it determines the probability to localize the pion within a certain volume which is usually assumed to be within the hot nuclear matter. Naively the value of $\sigma_0$ is therefore experimentally bounded by the two limits $R_{eff} = \sqrt{2}\sigma_0$ and $T_{eff} = T_{qm}$ which yields $0.8$ fm $< \sigma_0 < 4$ fm under SIS conditions. However it is more appropriate to relate $\sigma_0$ to the coherence length $\Lambda$ of the pion, which in the case that the pion is emitted in the decay of the $\Delta(1232)$ resonance is coupled to the lifetime of that resonance in hot nuclear matter. Alternatively one may also consider the mean free path of pions in hot nuclear matter as the appropriate quantity. We have used this latter conjecture in our calculations and have assumed $\sigma_0 = 1.8$ fm. This implies a quantum contribution of $T_{qm} = 21$ MeV to the effective temperature of pions. The plane wave limits are obtained in two ways: $\sigma_0 \to \infty$ (sharp momentum states) yields $C_2(p_1, p_2) = 1 + \delta(p_1 - p_2)$, in contradiction to experiment, $\sigma_0 \to 0$ (sharp position states) yields an infinite zero point energy, also in contradiction to experiment. This again demonstrates the inherent difficulties to describe the HBT interferometry with pions in a plane wave formalism. The wavepacket formalism was applied in other recent papers to the HBT interferometry with pions [4, 5, 6, 7, 8]. In cases where $\sigma_0$ was specified its value was set to $\sigma_0 \approx 0.7$ fm which yields $T_{qm} \approx 100$ MeV, a value which appears to be too high for the SIS energy range.

3 The Coulomb Residual Interactions

HBT interferometric studies in relativistic heavy-ion reactions are usually performed with charged pions. In this case the pions after emission cannot be described by free wavepackets but experience residual interactions from the charge of the source and from their own charges. As a result the centre motion of the wavepacket is modified and the wavepacket gets deformed. There are two ways to study these effects with respect to HBT interferometry, both ways are based on the time dependent Schrödinger equation: The multiconfigurational method [9], and the molecular dynamic method [4].

3.1 Coulomb Interaction with the Source

The second method was used to study the modifications of the two-pion correlation function when both pions interact with the source via the Coulomb force. Depending on the charge of the source the width of the correlation function is decreased for $(\pi^-, \pi^-)$ but increased for $(\pi^+, \pi^+)$ pairs, yielding a larger effective source radius from the former pairs, and a smaller radius from the latter pairs. The situation is depicted in the left panel of Fig.2 for a source with radius $R_s = 5.5$ fm and temperature $T = 50$ MeV. Surprisingly the increase respectively decrease are not
of same size, but stronger for the $(\pi^-,\pi^-)$ pairs. The reason for this asymmetry is found in the right panel of Fig. 2 where the average time is shown which a charged pion spends within the escape radius $R_{\text{esc}} = 20$ fm of the source. In case of the $(\pi^+,\pi^+)$ pairs the correlation signal develops to a large fraction after the $\pi^+$ have left the interaction region, and it is therefore only little influenced by this interaction. In case of the $(\pi^-,\pi^-)$ pairs the times the $\pi^-$ spend inside the interaction region are longer and therefore this interaction disturbs the correlation signal more strongly. The asymmetry in the radius distortions is a direct proof that the appearance of the correlation does not occur instantaneously but is a process in time.

### 3.2 Coulomb Interaction between Charged Pions

The first and the second methods were used to study the effects of their mutual Coulomb interaction onto the two-pion correlation function. The results were similar: The Coulomb interaction between like-charge pions alters the correlation by such a small amount that it becomes unobservable under normal experimental conditions [9]. This conclusion is in apparent contradiction to experimental results from $(\pi^+,\pi^-)$ pairs [10], we will therefore focus our discussion on this type of pairs.

The reason why the suppression of the correlation signal for very small values of $q$ is not observed for like-charge pion pairs is connected to the zero point energy $E_0 = \frac{2}{\hbar^2} T_{\text{qm}}$ of the pion which is almost two orders of magnitude larger than the Coulomb energy between two pions with localization $\sigma_0 = 1.8$ fm. This is particularly easy to see in the case of two pions with different charges where this Coulomb energy attains its maximum value at complete overlap and is, for particles with charge
number $Z$, of size $E_{\text{Coul}} = -\frac{Z^2e^2}{\sqrt{\pi}\sigma_0}$. One may define the parameter

$$\xi = \frac{|E_{\text{Coul}}|}{E_0} = \frac{8e^2}{3\sqrt{\pi}\hbar^2} Z^2 m\sigma_0$$

the value of which determines whether or not the Coulomb interaction becomes discernible. For $\sigma_0 = 1.8$ fm and $Z = 1$ one obtains $\xi = 1.4 \cdot 10^{-2}$, but $\xi = 5.6$ in case of a hypothetical charge number $Z = 20$. The time development of the pion wavepacket in relative coordinates is shown in Fig.3 for both cases, the contour lines give the shape of the wavepacket, the dark curves the movement of its centre. It is evident that in the first case the wavepacket movement and its deformation is not modified by the Coulomb interaction whereas it is strongly modified in the second case with respect to the centre as well as to the deformation. In order to observe the Coulomb interaction the $\xi$ parameter has to be of order 1. This condition cannot only be met by an increase of the charge number $Z$ but also by increasing the reduced mass of the particle pair or by increasing the localization $\sigma_0$. Finally one may also test the validity of the present procedure by approaching the classical limit $\hbar \to 0$. These dependences were studied in detail, as examples the Fig.4 displays in the left panel the ($\pi^+\pi^-$) correlation functions for three different values of $\sigma_0$ and for the case $\hbar \to \hbar/10$. In this latter case the calculated result almost agrees with the classical expectation shown by the continuous curve. The dependence on $\sigma_0$ is as predicted, as a cross check, shown in the right panel, we have also performed the calculation for $\sigma_0 = 1.8$ fm using the second method.

We conclude that the Coulomb interaction only modifies the correlation function at small values of $q$ when the $\xi$ parameter is of order one. Whereas this is not to be expected at SIS energies in case of pion pairs, it can occur for such pairs at much higher energies. At high energies pions are also emitted from the decay of long-living resonances which were produced by the nucleus - nucleus collision. Such pions have a large coherence length with the related extended localization. The
strength of the Coulomb distortion depends on the relative abundance of pions from the interaction zone to those from the long-living resonances, only the former will contribute to the observation of the HBT signal. The strength of the HBT signal at $q \to 0$ also depends on the pion localization and on the source temperature, dependences which experimentally have not been explored up to now. Our results illustrate the important role the pion coherence length plays in describing the HBT interferometry with pions.

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