Evolving GANs: When Contradictions Turn into Compliance

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Abstract

Limited availability of labeled-data makes any supervised learning problem challenging. Alternative learning settings like semi-supervised and universum learning alleviate the dependency on labeled data, but still require a large amount of unlabeled data, which may be unavailable or expensive to acquire. GAN-based synthetic data generation methods have recently shown promise by generating synthetic samples to improve task at hand. However, these samples cannot be used for other purposes. In this paper, we propose a GAN game which provides improved discriminator accuracy under limited data settings, while generating realistic synthetic data. This provides the added advantage that now the generated data can be used for other similar tasks. We provide the theoretical guarantees and empirical results in support of our approach.

1 Introduction

Training deep learning algorithms under inductive settings is highly data intensive. This severely limits the adoption of these algorithms for domains such as healthcare, autonomous driving, and prognostics and health management, that are challenged in terms of labeled data availability. In such domains, labeling very large quantities of data is either extremely expensive, or entirely prohibitive due to the manual effort required. To alleviate this, researchers have adopted alternative learning paradigms including semi-supervised [18], universum [29, 8], transductive [9, 23] learning, etc., to train deep learning models. Such paradigms aim to harness the information available in additional unlabeled data sources. When unlabeled data are not naturally available, synthetic samples are generated using a priori domain information [27, 26, 3]. A more recent line of work utilizes GANs under semi-supervised settings to improve discriminator generalization. For example, virtual adversarial training (VAT) [17], smooths the output distribution of the classifier by using virtually adversarial samples and [6] adopts a complimentary generator to better detect the low-density boundaries instead of smoothing the loss function. Finally, Triple GAN architecture [14] seeks to improve upon GAN in the semi-supervised setting by modifying the GAN two-player formulation to include another player as a classifier. In that setting, the generator and classifier learns the conditional distributions between input and labels, while the discriminator learns to classify fake input-label pairs. In most of these approaches the underlying strategy is to provide improved discriminator generalization by harnessing the additional information in the unlabeled data as well as the generated samples using semi-supervised learning.

In this paper, we consider the scenario where no additional unlabeled samples are available. The goal is to provide improved discriminator generalization by only leveraging the additional information in the GAN generated data while also ensuring the quality of the generated data. While Feature Matching

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that leverages generated data can also be used. We demonstrate how our proposed approach yields improved generalization while additionally safeguarding against mode collapse (compared to FM), and generating better real-like synthetic data (compared to C-GAN). Our main idea pivots around evolving the discriminator network’s learning paradigm from universum [26] to semi-supervised [3] during different phases of the GAN game. To build this evolving framework we address a few gaps in the current approaches. The main contributions of this paper are the following,

1. We modify the traditional semi-supervised GAN game and introduce the notion of universum learning for the discriminator in the GAN game in Section 3. We refer to the generalized version of this approach as Universum GAN (U-GAN) and provide theoretical analysis for its consistency. We also formalize this approach for Hinge loss in (13).
2. We derive a unified loss through which we can evolve discriminator network’s universum hinge loss to semi-supervised hinge loss (see Section 2.4).
3. We justify evolving the discriminator loss with supporting pedagogical examples and introduce the new Evolving GAN algorithm in Section 4.
4. Finally, we empirically demonstrate the effectiveness of the proposed approach.

2 Preliminaries on Learning Settings

We first introduce the three main learning settings that will be used in this paper and exemplify them with the Crammer and Singer’s (C&S) hinge loss [5].

2.1 Inductive Learning

This is the most widely used learning setting for machine learning (or deep learning) algorithms. The goal here is to estimate a model using the labeled training data to predict on future test samples. The mathematical formalization of this setting is provided next,

**Definition 1. (Inductive Learning)** Given i.i.d training samples \( T = \{(x_i, y_i)\}_{i=1}^n \sim D_x^p \times D_y^q \), with \( x \in \mathcal{X} \subseteq \mathbb{R}^d \) and \( y \in \mathcal{Y} = \{1, \ldots, L\} \), estimate a hypothesis \( h^* : \mathcal{X} \to \mathcal{Y} \) from a hypothesis class \( \mathcal{H} \) which minimizes,

\[
\inf_{h \in \mathcal{H}} E_{D_T}[\mathbb{1}_{y \neq h(x)}],
\]

where \( D_T \) is the training distribution, \( E_{D_T}(\cdot) \) is the expectation under training distribution, and \( \mathbb{1}(\cdot) \) is the indicator function.

A popular mechanism under this setting is to estimate a multi-valued function \( \hat{f} = [f_1, \ldots, f_L] \) using the decision rule,

\[
\begin{cases}
\hat{h}(x) = k & \text{if } f_k(x) > f_\ell(x) ; \forall \ell \neq k \\
\hat{h}(x) \neq [1, \ldots, L] & \text{else}
\end{cases}
\]

(2)

There are several algorithms to estimate this multi-valued function. One popular approach is the Crammer & Singer (C&S) hinge loss [5] which solves

\[
\min_{w_1, \ldots, w_L, \xi} L_T(w_1, \ldots, w_L, \xi) = \sum_{i=1}^n \xi_i , \quad \text{s.t. } \xi_i = \max_{k \in \mathcal{Y}} \{1 - \delta_{ik} + w_k^T z_i - w_\ell^T z_i\}
\]

(3)

where \( \delta_{ik} = \mathbb{1}(y_i = \ell) \), \( z_i = \phi(x_i) \), and \( \phi(\cdot) \) is a function mapping to the feature space. The C&S-hinge loss minimizes the approximation error while keeping the estimation error small compared to other multi-class loss alternatives, and presents itself as a reliable choice for limited data settings. However, for high-dimensional problems, even such advanced hinge-based loss function may fail to provide good generalization. This necessitates the need for novel learning settings as discussed next.

2.2 Semi-Supervised Learning

Semi-supervised learning is a widely used advanced setting that leverages additional unlabeled data to improve generalization. It is formalized as follows,
With the above transformation in (9) we solve, we are provided with additional unlabeled samples which are known not to belong to any of the classes. Another advanced learning setting is the universum a.k.a contradiction learning setting. In this setting we create artificial samples belonging to all classes i.e. \( z^*_i \). As shown in [7], for the C&S formulation in (3) and the corresponding decision rule in (2), maximum contradiction on universum samples i.e. maximizes the following probability for \( x^* \in U^* \),

\[
\sup_{h \in H} \mathbb{P}_{D_{U^*}}[x^* \notin \text{any class}] = \sup_{h \in H} \mathbb{E}_{D_{U^*}}[\{1 - \cap_{k \in \{1, \ldots, L\}} h(z^*)\neq k\}]
\]

where, \( D_{U^*} \) is the universum distribution, \( \mathbb{P}_{D_{U^*}}(\cdot) \) and \( \mathbb{E}_{D_{U^*}}(\cdot) \) are the probability measure and expectation under universum distribution, respectively, and \( X^* \) is the domain of universum data.

**Proposition 1.** As shown in [7], for the C&S formulation in (3) and the corresponding decision rule in (2), maximum contradiction on universum samples \( x^* \in U^* \) can be achieved when,

\[
|w^T_k x^* - \max_{t \in Y} w^T_k x^*| = 0; \forall k \in Y
\]

Following Proposition 1, universum learning with C&S hinge loss involves solving (see [7]),

\[
\min_{w_1, \ldots, w_L, \xi} \sum_{i=1}^{n} \xi_i + C_U \sum_{i'=1}^{m} \sum_{k=1}^{L} \zeta_{i'k}
\text{s.t.} \quad \xi_i = \max_{k \in Y} \{1 - \delta_{ik} + w^T_k z_i - w^T_k y_i z_i\} \quad \forall i = 1 \ldots n.
\]

\[
\zeta_{i'k} = \max \{w^T_k z_{i'} - \max_{t \in Y} w^T_k z_{i'}| - \Delta, 0\} \quad \forall i' = 1 \ldots m.
\]

where, \( z = \phi(x) \quad z_{i'} = \phi(x_{i'}) \).

### 2.4 Unified Loss for Solving C&S Hinge Loss Under Different Learning Settings

In this paper we analyze the similarity in the optimization problems in (3) and (9), then provide a unified loss to solve both the problems. This is possible through extending Proposition 3 in [7] for the semi-supervised learning. First we perform the following transformation.

**Definition 4. (Transformation of Unlabeled Data)** For each unlabeled sample \( z_{i'} \) we create \( L \) artificial samples belonging to all classes i.e. \( (z_{i'}, y_{i'}^1 = 1), \ldots, (z_{i'}, y_{i'}^L = L) \) and define the transformation,

\[
(z_i, y_i, e_i, i) = \begin{cases} (z_i, y_i, e_i, i) & i = 1 \ldots n \\ (z_{i'}, y_{i'}^{i'}, e_{i'\ell}, i') & i = n + 1 \ldots n + mL, \ i' = 1 \ldots m, \ \ell = 1 \ldots L \end{cases}
\]

With the above transformation in (9) we solve,

\[
\min_{w_1, \ldots, w_L, \xi} \sum_{i=1}^{n} \xi_i + C_U \sum_{i=n+1}^{n+mL} \xi_i
\text{subject to} \quad \xi_i = \max_{k \in Y} \{1 - \delta_{ik} + w^T_k z_i - w^T_k y_i z_i\}, \quad i = 1 \ldots n.
\]

\[
\xi_i = \psi(\epsilon(1 - \delta_{ik} + w^T_k z_i - w^T_k y_i z_i)), \quad i = n + 1 \ldots n + mL.
\]

Appropriately selecting the \( \psi(\cdot) \) and \( \epsilon \) provides us the desired solutions for both (3) and (9).
Proposition 2. Solving (10) with $\epsilon = -\Delta$ and $\psi_\epsilon(x) = x$ provides the solution to (8).
Proposition 3. Solving (10) with $\epsilon = 1$ and $\psi_\epsilon(x) = \min\{x, \epsilon\}$ provides a solution to (4).

The advantages of this singular framework are two-fold,

- First, we can solve either of the formulations (4) or (8) using (10) by carefully tuning $\epsilon$ and $\psi(\cdot)$. In fact, this also provides us with the framework to transition the learning setting from universum to semi-supervised ($\epsilon = -\Delta \rightarrow 1$) when the data distribution of the unlabeled samples change from being contradictions (i.e. $x^* \notin \mathcal{Y}$) to being compliant (i.e. $x^* \in \mathcal{Y}$). This will be a very useful tool for the GAN game, when the generated data distribution changes (see section 4).

- Second, the Propositions 2, 3 and 6 allow us to harness the advanced optimization techniques that can be used to solve the standard C&S hinge loss. This property has already been established for universum settings [7]. For the semi-supervised setting, prior solvers [31, 2, 22] to (4) use a switching algorithm which incurs significant computation complexity. Through our transformation we can avoid such switching algorithms and still attain similar performance results (see additional results in Appendix B.1).

3 Universum GAN (U-GAN)

With the preliminaries on different learning settings in place, and a unified loss to solve the C&S loss for all these settings; next we introduce a new modified universum GAN game below (see Fig. 1).

Player 1: $\max_D L_D = \mathbb{E}_{x,y \sim \mathcal{D}_x \times \mathcal{D}_y} [\mathbf{1}_{y = D(x)}]$

\[+ C_G \mathbb{E}_z [\mathbf{1}_{D(G(z)) \notin \mathcal{Y}}] \quad (11)\]

Player 2: $\max_G L_G = \mathbb{E}_z [\mathbf{1}_{D(G(z)) \in \mathcal{Y}}] \quad (12)$

Note that the GAN game in (11) and (12) has the same intuition as the original semi-supervised GAN [21]. That is, Player 1 estimates a discriminator that explains the training samples (classes 1 through $L$) while simultaneously identifying the generated samples to not belong to any class. On the other hand, Player 2 confuses the discriminator by generating samples as belonging to one of the discriminator classes. However, different from [21] we do not assign all the generated samples to belong to one separate class (say $L + 1$). Rather, we utilize the universum setting and treat the generated samples as contradictions. This is a more desirable setting, as it does not make an overgeneralized assumption that all the generated samples belong to the same class $L + 1$.

Next we provide the theoretical justification behind our formulation in Proposition 4. Here we use a discriminator that estimates a multi-valued function $f = [f_1, \ldots, f_L]$ with parameterization $f_l = w_l^T \phi(x)$, and the decision rule as in [2]. To simplify the proof we use the following assumption,

Assumption 1. (Realizability) There exist a measurable function $h^*$ that achieves zero Bayes Risk on the training data distribution $R(h^*) = \mathbb{E}_{(x,y) \sim \mathcal{D}_x \times \mathcal{D}_y} [\mathbf{1}_{y=h^*(x)}] = 0$

Proposition 4. (Consistency) Under assumption 1 with $C_G \leq 1$, the optimal $(D^*, G^*)$ that solves the GAN game in (11) and (12) satisfies the following.

(i) $D^*$ achieves the Bayes Risk on $(x, y) \sim \mathcal{D}_x \times \mathcal{D}_y$, i.e., $R(D^*) = 0$.

(ii) The support of the generated data (i.e. support of $\mathbb{E}_{G^*}$) is contained in $\mathcal{X}$.

The proposition shows that the 2-player game in (11) and (12) indeed generates samples from the training data distribution; while achieving the best possible generalization performance for the discriminator. However, the proposition holds for a strong assumption. This assumption provides us with a mathematical construct that simplifies the proof significantly. We argue that the proposition holds even without the realizability assumption.

Claim 1. For appropriately selected $C_G$, the proposition 4 holds without the realizability assumption [7].

The Proposition 4 provides theoretical justification for U-GAN. To the best of our knowledge such a similar analysis is missing for most of the existing semi-supervised GAN research [21]. An immediate
advantage of the U-GAN compared to semi-supervised GAN \[21\] is that, it can provide an implicit regularization to increase the entropy of the predicted labels on generated samples. This intuition follows from the empirical results reported in \[7\]. Through the histogram of projections (HOP) visualization the authors (in \[7\]) demonstrated how univem model results to higher entropy on their predicted labels compared to inductive settings. In fact, for binary problems, \[24\] derives the connection between the hard-margin univem and maximum entropy solution. This maximum entropy property is highly desirable for GAN games as it alleviates the mode-collapse problem. Empirical results confirming this implicit regularization is also provided in Section 5. Note however, the loss functions in (11) and (12) are not differentiable. Hence, we use the hinge loss as the dominating surrogate loss for the discriminator and generator loss. We also add the feature matching loss to the generator loss. The final form of the discriminator and generator of the U-GAN game is given as,

\[
L_D = \sum_{i=1}^{n} z_i + C_U \sum_{i'=1}^{m} L_{k \in Y} \sum_{k=1}^{L} \xi_{i'k} + \ |\xi_{i'k}| - \mathbb{E}[\phi(G(n; \theta))] - \mathbb{E}[\phi(x)]| \tag{13}
\]

\[\forall i = 1 \ldots n; \forall i' = 1 \ldots m. \ 
\xi_i = \max_{k \in Y} \{1 - \delta_{ik} + w_k^T z_i - w_k^T z_i^*\}
\]

\[
\zeta_{i'k} = \max\{|w_k^T z_i^* - \max_{j \in Y} w_k^T z_j^*| - \Delta, 0\}; \ \exists y_i^*: \zeta_{i'k} = \max_{k \in Y} \{1 - \delta_{i'k} + w_k^T z_i^* - w_k^T z_i^*\}
\]

Here, \(z = \phi(x)\) \(z_i^* = \phi(x_i^*),\) and generator \(G(n; \theta)\) is parameterized with \(\theta,\) and its input random noise is \(n.\)

4 Evolving GANs: From Contradictions to Compliance

Although U-GAN admits a consistent solution (see Prop. 4) and guarantees advantages on the mode collapse problem seen for semi-supervised GAN \[21\], it still has the same caveats as discussed in \[6\]. Rightly so, since close to convergence the generated data follows a very similar distribution as the training data. This violates the Universum assumption in Definition 3 where the generated (universe) samples should act as contradictions and results in sub-optimal discriminator performance. Rather, a more apt setting for the discriminator when the generated data is in compliance with the training samples, is semi-supervised learning (see Def. 2). This can be better explained using the synthetic example in Fig. 2 which shows the performance of a linear model trained under different learning settings, i.e., inductive \[3\], semi-supervised \[4\] and universum \[8\] using the standard MNIST data \[13\]. Here, the goal is to build a multiclass ‘0’–‘9’ digit classifier using 50000 training samples to predict on 10000 test samples. Here, to simulate the changing distributions of the unlabeled data we randomly select any two training images \((x_i, y_i), (x_j, y_j)\) and perform a weighted average \(x^* = ax_i + (1 - a)x_j,\) with (mixing ratio \(a \in [0.5, 1.0]\)) to generate an unlabeled sample. Example of such a generated universum sample using a randomly selected digit ‘5’ and ‘9’ image for different mixing ratios is shown in Fig. 2 (b) - (f). As seen in Fig. 2 (a) universum outperforms the other approaches when the generated data act as contradictions \(a = 0.5\) (i.e. neither ‘5’ or ‘9’). However, as the mixing ratio increases \(a > 0.9,\) the performance under universum learning deteriorates. Rightly so, since with \(a > 0.9\) the generated data closely resembles the training data, training under semi-supervised setting is a more desirable choice. The main takeaway from this example is that, as the distribution of the unlabeled data changes from contradictions to compliance, it is favorable to evolve the discriminator loss from universum to semi-supervised setting. Doing so, may yield improved generalization.
Algorithm 1: Evolving GAN Algorithm

Initialize discriminator and generator networks;
Parameters: epsSet, evolvePeriod, numiter, and C_U;
for i ← 0 to numiter do
    Choose M samples from the dataset;
    Generate M samples by using the generator;
    \( \epsilon \leftarrow \text{epsSet}[\text{evolvePeriod}] \);
    if \( \epsilon < 0 \) then
        Update the discriminator with the assumption that generated samples are universum;
        Update the generator by minimizing the feature matching loss;
    else
        Update the discriminator with the semi-supervised setting assumption;
    end
end

performance. In this work, we adopt this intuition and modify (evolve) the discriminator loss for improved generalization. Note that mechanisms similar to C-GAN [6] could have been adopted, where we rather generate complimentary samples to boost the U-GAN’s performance. However, as discussed in section 1, such an approach will result to non-real-like generated samples and is contrary to our overall goal (see results section 5.1 in Fig 3).

To design our evolving mechanism we utilize the Propositions 2 and 3. The main advantage of the proposed unified loss function in (9) in Section 2.4, is the ability to seamlessly transition from a universum learning setting to a semi-supervised setting by changing \( \epsilon \) from \( -\Delta \) to 1. In this paper, we adopt this unified loss and update \( \epsilon \) in a staircase fashion. Specifically, we define a set of \( \epsilon \) values \( \text{epsSet} = [-0.05, -0.01, -0.005, -0.001, -0.0005, -0.0001, 1.0] \), start the training process with \( \epsilon = -0.05 \) (universum learning), and after each \( \text{evolvePeriod} = 5000 \) iteration, we select the next value from set \( \text{epsSet} \). Such a simple evolution routine may not be optimal, but has shown significant performance gains in our results (see Section 5). Although, there is a significant room for improvement using optimized evolution routines, we adopt this simple mechanism to illustrate our claim. Note that, \( \epsilon < 0 \) corresponds to universum learning setting, while \( \epsilon = 1 \) leads to semi-supervised loss. Also for this work, we stop training the generator as the discriminator switches to semi-supervised setting. A more advanced evolution mechanism and a detailed study on optimal mechanisms for training the generator even during the semi-supervised learning phase is still an open research problem.

5 Empirical Results

For our empirical results we use a similar setting as used in [6] with the same network architecture for both discriminator and generator. For all our experiments we use SVHN and CIFAR-10 datasets with randomly sampled 1,000 and 4,000 labeled samples as training data and do not use any sample as unlabeled data.

5.1 Effectiveness of U-GAN

Classification Accuracy: For our first set of experiments we provide the performance comparison of U-GAN with several state-of-the art GAN algorithms used for limited data settings in Table 1. Table 1 provides the mean ± standard deviation of the classification error over 10 random partitioning of the training data. Here, our main comparison is with FM [21] and C-GAN [6], which provides alternative mechanisms to improve discriminator’s generalization by utilizing the generated samples into the loss function. For this work, we do not use additional unlabeled data and it’s equivalent loss components during training for these methods. That is, in place of the unlabeled data, we rather use a randomly sampled labeled data to obtain feature statistics for the feature matching loss. Further to ensure fair comparison we also adopt the same network architectures for both generator and discriminator as used in [6]. As seen from Table 1, U-GAN significantly outperforms both FM and C-GAN approaches. Since the pull-away term and variational inference techniques used in [6] for increasing generator...
entropy are orthogonal to the U-GAN model, in this table we also include a flavor of U-GAN + PT/VI. Note, that U-GAN+VI/PT further improves the performance of U-GAN. This may be due to more diverse samples being generated by the U-GAN+VI/PT (also confirmed in Table 2). Since under universum setting the generated samples act as contradictions, this better constraints the search space in which the optimal model may lie (a.k.a. the complexity of the hypotheses class) and hence results in improvement of the discriminator generalization. Such a behavior is inline with previous research on Universum learning [26, 25, 7].

Finally, we also provide the performance comparison with methods ADGM [16], SDGM [16] and VAT large [17] also benchmarked in [6]. This is used to better place the results inline with existing literature. Note that, U-GAN+PT/VI matches the performance of the VAT large for SVHN data. For CIFAR-10, VAT large significantly performs better. However, as also mentioned in [6], VAT large adopts a larger discriminator architectures and adopts an orthogonal mechanism (using adversarial data) compared to our approach. We can expect further improvement by combining VAT with our presented approach.

**U-GAN Generated Data:** (Quality): Next we analyze the quality of the generated data using U-GAN compared to FM and C-GAN. Figure 6 provides a random set of images generated by U-GAN and the benchmark algorithms. As seen from figure 5 U-GAN provides more realistic images for both SVHN and CIFAR-10 datasets, while both FM and C-GAN in absence of unlabeled data perform poorly. Appendix B.4 provides a similar qualitative comparison of the generated data with FM and C-GAN even while using the additional unlabeled data provided in [6]. The results in Appendix B.4 further confirms the qualitative improvement in generated samples for U-GAN compared to the baseline algorithms.

(Diversity) One of the main issues in training a GAN game is the mode collapse. While C-GAN [6] aims to avoid mode collapse by including an entropy term into the generator cost function, U-GAN incorporates an implicit regularization to increase the generated data distribution, which in turn avoids mode collapse (also theoretically argued in Section 3). Here, we illustrate this property in Table 2. Table 2 provides the mean ± standard deviation of the entropy and FID scores of the generated data over 10 experiment runs. As seen from this table, U-GAN outperforms FM and C-GAN in terms of the entropy of the generated samples and FID scores. Such implicit mechanism is hugely desirable for 2-player games for avoiding mode collapse. In fact, adding explicit entropy terms in U-GAN+PT/VI further improved the generator entropy. Note that, for 10-class problems the maximum generator entropy is \( \log_2 10 = 3.32 \).
Figure 3: An example of data generated by U-GAN and state-of-the-art algorithms (Top row is the SVHN data and Bottom row is the CIFAR-10 data). The samples generated by the benchmark methods are based on our runs and do not use any unlabeled data which leads to significant degradation of the quality compared to the original papers (see Appendix B.4). Despite some low-quality, non-representative data generated by U-GAN, it generates the most realistic samples. Evolving-GAN provides similar quality generated data as we stop the generator during the semi-supervised learning phase and has hence not been shown. The VAT samples are the ones corresponding the the best-performing hyperparameter configuration, and the generated noise that has been added to the original image can be easily spotted.

Table 3: Comparison of discriminator accuracy between U-GAN and Evolving GAN setting for SVHN and CIFAR-10 datasets.

| Method           | SVHN       | CIFAR-10  |
|------------------|------------|-----------|
| U-GAN            | 84.96 ± 0.77 | 68.24 ± 0.85 |
| Evolving GAN     | 87.42 ± 0.53 | 70.46 ± 0.62 |

5.2 Effectiveness of Evolving GAN

For our final set of experiments we provide the performance comparisons between U-GAN vs. Evolving GAN (in Algorithm 1). In Fig. 7 and Table 3, we provide the mean ± std. of discriminator accuracy over 10 experimental runs for the SVHN and CIFAR-10 datasets. Table 3 illustrates that by evolving the discriminator network from universum → semi-supervised setting, as the generated data transitions from contradictions → compliance; we can expect performance gains in the discriminator accuracy. This confirms that as the GAN game converges the generated samples become more similar to the original training data and should be treated as being from the same distribution as the labeled training data. Hence, close to convergence transitioning to semi-supervised setting may yield significant performance gains. Another, interesting observation from Fig. 7 is that the training of evolving GAN is more stable than U-GAN. Note that, evolving GAN converges to a reasonable performance accuracy much faster for both SVHN and CIFAR-10 datasets. Secondly, the variation of the results in multiple runs is much smaller compared to U-GAN for both datasets. A complete set of model parameters for all our experiments is also provided in Appendix B.3 for reproducibility.
6 Future Research

There are two main directions for future research,

Evolution Routine: The current handling of the evolution process is hand tuned and may prove sub-optimal for different applications. A more systematic approach may be possible by connecting the existing theory in [7] (Theorem 2) to transition from contradiction into compliance or by hyperparameter optimization of $\text{epsSet}$ and $\text{evolutionPeriod}$ using tools like [15 10]. Identifying the optimal transition (from contradiction to compliance) point and the evolution mechanism is paramount for evolving GANs success and is an open research topic.

Extension to Advanced Learning Settings: U-GAN and evolving GAN can be extended to other advanced learning techniques such as semi-supervised learning (similar to FM [21] and C-GAN [6]). Such similar extensions to more advanced learning settings may yield additional performance improvements. Such extensions have not been explored in the current version and is a topic for future research.

7 Conclusion

This paper proposes building a GAN game with high discriminator accuracy for multiclass problems while generating high quality samples. Such solutions are highly desirable for domains where only limited labeled training data is available. While existing state-of-the-art approaches target the first objective but tend to be limited on the second one (see Fig. 3), we show that by using an evolving discriminator this can be achieved. Specifically, we evolve the discriminator loss to better account for the change in generated data distribution at different phases of GAN training. We presented the theoretical framework and the computation paradigm behind our evolving approach. Finally our empirical results confirm a) improved generalization performance of the discriminator network and b) generation of high quality diverse set of images for the evolving GAN compared to the state-of-the-art.

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A Proofs

A.1 Proof of Proposition 1

See Proposition 1 in [7].

A.2 Proof of Proposition 2

See Proposition 3 in [7].

A.3 Proof of Proposition 3

The proof follows from analyzing the contribution of each sample to the loss function. Note that for $\epsilon=1$ and $\psi_{\epsilon}(x)=\min(x, \epsilon)=\min(x, 1)$ gives,

$$\min_{w_1, \ldots, w_L} \sum_{i=1}^{n} \xi_i + C_U \sum_{i=n+1}^{n+mL} \xi_i$$

s.t. $\xi_i = \max_{k \in Y} \left( 1 - \delta_{ik} + w_k^T z_i - w_{y_i}^T z_i \right); \quad i = 1 \ldots n$

$\xi_i = \min \left\{ \max_{k \in Y} \{ (1 - \delta_{ik}) + w_k^T z_i - w_{y_i}^T z_i \}, 1 \right\}$

$\forall i = n + 1 \ldots n + mL$

Let the final solution be $w = [w_1, \ldots, w_L]$. We analyze the contribution of the unlabeled samples $(z_i, y_i)_{i=n+1}^{n+mL}$ i.e. $(z^*_i, y^*_i)_{i=1}^{m}$. Each $z^*_i$ is introduced multiple times with labels $y^*_i = 1 \ldots y^*_{iL} = L$ through the transformation in Definition (4). WLOG we assume with the final solution for the samples $(z^*_i, y^*_i = 1 \ldots y^*_i = L)$ we have,

$w^T_k z^*_i > w^T_l z^*_i; \quad \forall l \neq k \quad \text{i.e.} \quad y^*_i = k$

This gives,

$\zeta^*_i = 1$

$\vdots$

$\zeta^*_i = \max_{l \neq k} [1 + w^T_l z^*_i - w^T_k z^*_i]$

$\vdots$

$\zeta^*_i = 1$

Hence, the overall contribution becomes,

$\max_{l \neq k} [1 + w^T_l z^*_i - w^T_{y^*_i} z^*_i] + (L - 1)$

That is sum of the constraint in (4) and a constant. Hence the solution to the (10) also solves (4). □

A.4 Proof of Proposition 4

We divide the proof of this proposition into two parts as shown in Lemmas (1) and (2)

Lemma 1. Under assumption [7] for a fixed $G$, the optimal discriminator $D^*$ that solves (11) with any $C_G \leq \frac{P_X(x \in \Omega)}{P_X(z \in \Omega)}$; $\hat{x} = G(z)$ and $\Omega = X \cap G(z)$ satisfies,

$$D^*(x) = h^*(x); \quad \forall x \in X$$

$$\neq \left\{ 1, \ldots, L \right\}; \quad \forall x \notin X$$

(14)
This implies, For the decision rule in (2) with linear parameterization \( f_l = w_l^\top x \); \( \forall l \in \mathcal{Y} = \{1 \ldots L\} \) we have,

\[
\hat{D}(x^*) \Rightarrow \begin{cases} 
    w_y^\top x - \max_{k \neq y} w_k^\top x > 0 & \text{if } (x, y) \sim \mathcal{D}_X \times \mathcal{D}_Y \\
    w_k^\top x = w_l^\top x; & \forall k, l \in \{1, \ldots, L\} \text{ else }
\end{cases}
\] (15)

**Proof** The proof follows by partitioning the error probabilities in (11) into different event spaces. We define, \( A = \mathcal{X} - G(z) \); \( B = G(z) - \mathcal{X} \); \( \Omega = \mathcal{X} \cap G(z) \). Next, we rewrite,

\[
\Omega = \mathcal{X} \cap G(z)
\]

\[
A = \mathcal{X} - G(z) \quad B = G(z) - \mathcal{X}
\]

Figure 5: Partitioning the event space

\[
L_D = \mathbb{E}_{X,Y} [\mathbf{1}_{y = D(x)}] + C_G \mathbb{E}_z [\mathbf{1}_{D(G(z)) \notin \Omega}]
\]

\[
= \mathbb{E}_{X,Y} [\mathbf{1}_{y = D(x)} | x \in \mathcal{A}] \mathbb{P}_X (x \in \mathcal{A}) \quad \text{(Total probability)}
\]

\[
+ \mathbb{E}_{X,Y} [\mathbf{1}_{y = D(x)} | x \in \Omega] \mathbb{P}_X (x \in \Omega)
\]

\[
+ C_G \mathbb{E}_z [\mathbf{1}_{k \in \mathcal{Y}} D(G(z)) \neq k | G(z) \notin \mathcal{B}] \mathbb{P}_X (G(z) \in \mathcal{B})
\]

\[
+ C_G \mathbb{E}_z [\mathbf{1}_{k \in \mathcal{Y}} D(G(z)) \neq k | G(z) \in \Omega] \mathbb{P}_X (G(z) \in \Omega)
\]

\[
= (\mathbf{a}) + (\mathbf{b}) + (\mathbf{c})
\] (16)

Note that for the decision rule in (2), we have,

\[
\mathbf{1}_{y = D(x)} = \mathbf{1}_{w_y^\top x - \max_{k \neq y} w_k^\top x > 0}
\]

and from Proposition (1) we have,

\[
\mathbf{1}_{k \in \mathcal{Y}} D(G(z)) \neq k = \mathbf{1}_{[(w_y^\top x^* - \max_{i=1 \ldots L} w_i^\top x^*)]=0; \forall k \in \{1, \ldots, L\}}
\]

\[
= \mathbf{1}_{w_y^\top x^* = w_l^\top x^*; \forall (k,l) \in \mathcal{Y}}
\]

Hence, (16) translates to,

\[
\mathbf{(a)} = \mathbb{E}_{X,Y} [\mathbf{1}_{w_y^\top x - \max_{k \neq y} w_k^\top x > 0} | x \in \mathcal{A}] \mathbb{P}_X (x \in \mathcal{A})
\]

\[
\mathbf{(b)} = \mathbb{E}_{X,Y} [\mathbf{1}_{w_y^\top x - \max_{k \neq y} w_k^\top x > 0} | x \in \Omega] \mathbb{P}_X (x \in \Omega)
\]

\[
+ C_G \mathbb{E}_z [\mathbf{1}_{w_y^\top G(z) = w_l^\top G(z); \forall l \in \mathcal{Y}} | G(z) \in \Omega] \mathbb{P}_X (G(z) \in \Omega)
\]

\[
\mathbf{(c)} = C_G \mathbb{E}_z [\mathbf{1}_{w_y^\top G(z) = w_l^\top G(z); \forall l \in \mathcal{Y}} | G(z) \in \mathcal{B}] \mathbb{P}_X (G(z) \in \mathcal{B})
\]

Under assumption (1), the overall loss \( L_D \) (in 16) is maximized if \( D^*(x) \) follows (15). Why? Note that for such a \( D^* \), (16) translates to,

\[
\mathbf{(a)} = \mathbb{P}_X (x \in \mathcal{A}) \quad \text{(max. possible value)}
\]

\[
\mathbf{(b)} = \mathbb{P}_X (x \in \Omega) \quad \text{(max. possible value)}
\]

Since, \( \mathbf{1}_{w_y^\top x - \max_{k \neq y} w_k^\top x > 0} \) and \( \mathbf{1}_{w_y^\top x = w_l^\top x} \); \( \forall (k,l) \in \mathcal{Y} \) are mutually exclusive; only one event is triggered. For \( C_G \leq \frac{\mathbb{P}_X (x \in \{1\})}{\mathbb{P}_X (x \in \{1\})} \), the first term dominates and maximizes \( L_D \). Finally,

\[
\mathbf{(c)} = \mathbb{P}_X (G(z) \in \mathcal{B}) \quad \text{(max. possible value)}
\]
This justifies setting $D^*$ as in (13) to maximize $L_D$. It is straightforward to see (13) $\Rightarrow$ (14), under the above parameterization.

**Lemma 2.** For the fixed $D^*$ in Lemma (1), the optimal $G^*$ that maximizes (12) with $C_G \leq 1$ ensures $G(z) \subseteq X; \forall z \sim P_z$ i.e. support of $P_G^*$ is contained in $X$.

**Proof**

\[
L_G = \mathbb{E}_z[1_{D^*(G(z)) \in Y}|G(z) \in \Omega] \mathbb{P}(G(z) \in \Omega) + \mathbb{E}_z[1_{D^*(G(z)) \in Y}|G(z) \in B] \mathbb{P}(G(z) \in B)
\]

From (14), $\mathbb{E}_z[1_{D^*(G(z)) \in Y}|G(z) \in B] = 0$. Hence, $L_G$ is maximized if $\mathbb{P}(G(z) \in \Omega) = 1$. Further Lemma (1) holds for $C_G \leq 1$.

Finally combining Lemma (1) and (2) we get Proposition (4).

A.5 Proof of Claim 1

The proof follows by analyzing the weightage of the terms in (16). For the case without the Assumption (1), all that we need is to select $a$,  

\[
C_G \leq \frac{\mathbb{E}_z[1_{w^T_G x \geq \max_{k \neq y} w^T_k x > 0}|x \in \Omega] \mathbb{P}(x \in \Omega)}{\mathbb{E}_z[1_{w^T_G G(z) = w^T_G G(z)}; \forall l, k \in Y|G(z) \in \Omega] \mathbb{P}_z(G(z) \in \Omega)}
\]

For, such a selected $C_G$, the proposition (4) holds without Assumption (1).

B Additional Empirical results

B.1 Baseline comparisons for the Transductive C&S Loss solved using Proposition (3)

| Layer (Type) | Output Shape | Param # |
|-------------|--------------|--------|
| Conv2d-1   | [-1, 16, 28, 28] | 416    |
| ReLU-2     | [-1, 16, 28, 28] | 0      |
| MaxPool2d-3| [-1, 32, 14, 14] | 12,832 |
| Conv2d-4   | [-1, 16, 14, 14] | 0      |
| ReLU-5     | [-1, 32, 14, 14] | 0      |
| MaxPool2d-6| [-1, 32, 7, 7] | 0      |
| Linear-7   | [-1, 10] | 15,690 |

Figure 6: CNN architecture summary used for MNIST example.

In this work we solve the Transductive C&S problem in (4) using Proposition (3). This provides the following advantages,

1. Our approach is an extension to [4] for multiclass problems, and similarly scales to large problems. In addition, we can now avoid using switching algorithms typically adopted for multiclass Transductive C&S formulation [30, 22, 2], and adds significant computational load for solving the transductive C&S loss.

2. Further, now the formulation (4) can be easily implemented in most popular deep learning frameworks [19, 20, 1] and solved through the state-of-art first order solvers supported in these frameworks.

To validate the statistical performance of our approach we further baseline our implementation against existing T-SVM benchmarks [30, 28]. Table 4 provides the results on two datasets. Here we report the mean ± std. deviation of the test accuracies over 10 runs of the experimental setting discussed below.
Table 4: Mean (± standard deviation) of the test accuracies (in %) over 10 runs of the experimental setting.

| Dataset  | C&S Hinge | Transductive C&S (Ours) | Transductive C&S |
|----------|-----------|-------------------------|-------------------|
| Coil     | 73.39 ± 1.31 | 74.32 ± 1.11 | 74.58 ± 1.30 |
| MNIST    | 94.61 ± 0.8  | 96.7 ± 0.48  | 95.13 ± 0.38 |

**COIL DATASET** [3]: This is a 6 - class classification problem. We report the performance of the standard C&S [5] vs. Transductive C&S [4] losses over 10 random partitioning of the data. In each partition we randomly select \( n = 100 \) training samples (and remaining as test samples) following [30]. For this experiment we use linear parameterization. Further,

- **C&S loss** we use an Adam optimizer [11] with, batchSize = 100, No. of epochs = 5000, step size = 0.005. Further increase in epochs does not provide any improvement.
- **Transductive C&S** we use an Adam optimizer with, batchSize = 250, No. of epochs = 50000, step size = 0.005.

**MNIST DATASET** [12]: This is a 10 - class classification problem. For this experiment following [28] we use 1% (\( n = 600 \)) samples as training. Here we rather use a very simple CNN architecture shown in Fig 6. Further,

- **C&S loss** we use an Adam optimizer with, batchSize = 100, No. of epochs = 20000, step size = 0.001. Further increase in epochs does not provide any improvement.
- **Transductive C&S** we use an Adam optimizer with, batchSize = 250, No. of epochs = 25000, step size = 0.001.

As seen from the results in Table 4 the implementation through the transformation in Definition (4) and Proposition (3) we can obtain similar statistical performance. Here, in each experiment we randomly select the training samples in the same proportion as mentioned above. We use the complete test data. The results show that solving the transductive C &S loss using the transformation (in Definition 4) and Proposition (3) provides similar statistical performance as the existing benchmarks.

### B.2 Additional Analysis of the U-GAN formulation in section 3 using MNIST

This section further consolidates our U-GAN formulation in [3]. Note that, different from previous approaches used under multiclass settings [21] [6]; here we use Universum loss for the discriminator. Further for training the generator, we combine the Feature Matching loss with a dominating surrogate of the loss in (12). Slightly different from the U-GAN (hinge) formulation in (13), we use the following surrogate for the generator loss,

\[
\min_\theta \quad L_{FM}(\theta) + \lambda \cdot L_{Disc}(\theta) \\
\text{s.t.} \quad L_{FM}(\theta) = \|\mathbb{E}[\phi(G(z; \theta))] - \mathbb{E}[\phi(x)]\|, \\
L_{Disc}(\theta) = \sum_{i=1}^{m} \max \log D(G(z; \theta)) .
\]

We use the same discriminator loss as in (13). For this section we refer to this formulation using the (17) generator loss as U-GAN.

Our overall goal is to highlight,

- Improved diversity of the U-GAN generated data (high classification labels entropy on generated data) compared to [21].
- Effect of using the additional loss term \( L_{Disc} \) in the generator loss.

\[\text{publicly available at \url{http://olivier.chapelle.cc/ssl-book/benchmarks.html}}\]
Figure 7: Discriminator and Generator performance comparisons for U-GAN on MNIST data.

Firstly, we confirm that the U-GAN discriminator achieves similar (or better) generalization compared to standard inductive learning using a traditional C&S Hinge loss in (3) (see Fig 7a). For this experiment we do not see a significant improvement in discriminator generalization for U-GAN. Also provides similar performance. However, similar to the results reported in Table 2 we see significant improvement in the generated data diversity for U-GAN compared to [21]. Here, after convergence of the GAN games we generate 1000 samples and calculate the entropy of the classes by running the samples through the discriminator. For the U-GAN, we get entropy of 2.28 while for L + 1-class classifier [21] we have 2.09. Note that, the maximum entropy for a 10-class setting is \log_{10} = 2.3.

Next we explore the quality of the data generated by U-GAN and the effect of the additional loss component \( L_{\text{Disc}} \) in the generator loss. Using only the FM loss results to ‘salt’ noise in the generated images by the GAN’s generator (see Fig. 7b). Rather, adding the additional \( L_{\text{Disc}} \) component removes this ‘salt’ noise and provides near realistic digit data (Fig. 7c).

B.3 Experiment setup, Network Architecture and Model hyperparameters

The experiments were performed on Amazon AWS cloud servers, using a p3.8xlarge instance with 4 NVIDIA V100 Tensor Core GPUs. We use Auptimizer [15] library to analyze performance of different hyperparameters detailed below. The code for our research will be released shortly, pending an internal review on open source policies.

For the sections below we use \( C_U \) and \( C_{\text{gen}} \) \( (\hat{C}_U \text{ in eq.}(10)) \) to represent the loss multipliers during the unversum and semi-supervised setting respectively. From our analysis its clear that \( C_U \) value of 0.5 yields the ideal performance for both datasets, with the model being sensitive to it’s value. Optimal \( C_{\text{gen}} \) can vary for both the datasets, but the model performance remains fairly robust to it within a range of (0.1,1.0).

B.3.1 SVHN data

We analyze the performance of \( C_U \) for SVHN discriminator accuracy, with \( C_{\text{gen}} \) as 0 and \( C_{\text{gen}} \) as another hyperparameter in Fig. 8 and Fig. 9 respectively. For Fig. 8, \( C_U \) belongs to the set [5,2,1.0,0.5,0.2,0.1,0.01,0.001] and \( C_{\text{gen}} \) is fixed at 0. For Fig. 9, we use a Random search on \( C_U \) values [0.5,0.3,0.2,0.1] and with \( C_{\text{gen}} \) values [0.5,0.3,0.2,0.1,0.05]. Based on our analysis of the hyperparameter interaction graphs in Figs. 8 and 9 we fix \( C_U \) and \( C_{\text{gen}} \) values of 0.5 and 0.1 respectively for experiments in Table 3.

B.3.2 CIFAR-10 data

We analyze the performance of \( C_U \) for Cifar-10 discriminator accuracy, with \( C_{\text{gen}} \) as 0 and \( C_{\text{gen}} \) as another hyperparameter in Fig. 10 and Fig. 11 respectively. For Fig. 10, \( C_U \) belongs to the set [5,2,1.0,0.5,0.2,0.1,0.01,0.001] and \( C_{\text{gen}} \) is fixed at 0. For Fig. 11, we use a Random search on \( C_U \) values [1.0,0.75,0.5,0.3] and with \( C_{\text{gen}} \) values [1.0,0.5,0.3,0.1,0.05]. Based on our analysis of the
hyperparameter interaction graphs in Figs. 10 and 11, we fix $C_U$ and $C_{gen}$ values of 0.5 and 1.0 respectively for experiments in Table 3.

### B.3.3 Network Architecture

Finally we provide the Discriminator and Generator architectures for the models we used for both the datasets SVHN and Cifar. These architectures are the same as used in [6] and have been used for better comparisons with baseline models.

All our codes will be released publicly after internal approval.

### B.4 Comparison of the U-GAN generated data quality vs. state-of-the-art

Finally we also compare the quality of the U-GAN generated data in section ?? of the paper with those reported for FM-GAN [21], C-GAN [6] and VAT [17]. Note however, the results reported for FM-GAN [21], C-GAN [6] use an advanced semi-supervised setting and use additional unlabeled data. As seen from the results in Fig 16, the U-GAN generated data provides similar quality generated images, even without using any additional unlabeled samples. This sheds a very positive note for the proposed U-GAN approach. Since the Evolving GAN algorithm stops generator training during the semi-supervised learning phase when $\epsilon > 0$ (see Algorithm 1), it provides no additional improvement on the quality of the generated data. The evolving phase (when $\epsilon > 0$) mainly targets the discriminator performance at that stage.
Figure 10: Cifar performance based on Cu values

Figure 11: Cifar performance based on Cu and Cgen values

Figure 12: SVHN Generator Architecture
```
Discriminative(
    (core_net): Sequential(
        (0): Sequential(
            (0): GaussianNoise()
            (1): Dropout2d(p=0.15, inplace=False)
        )
        (1): WN_Conv2d(3, 64, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1))
        (2): LeakyReLU(negative_slope=0.2)
        (3): WN_Conv2d(64, 64, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1))
        (4): LeakyReLU(negative_slope=0.2)
        (5): WN_Conv2d(64, 64, kernel_size=(3, 3), stride=(2, 2), padding=(1, 1))
        (6): LeakyReLU(negative_slope=0.2)
        (7): Dropout2d(p=0.5, inplace=False)
        (8): WN_Conv2d(64, 128, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1))
        (9): LeakyReLU(negative_slope=0.2)
        (10): WN_Conv2d(128, 128, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1))
        (11): LeakyReLU(negative_slope=0.2)
        (12): WN_Conv2d(128, 128, kernel_size=(3, 3), stride=(2, 2), padding=(1, 1))
        (13): LeakyReLU(negative_slope=0.2)
        (14): Dropout2d(p=0.5, inplace=False)
        (15): WN_Conv2d(128, 128, kernel_size=(3, 3), stride=(1, 1))
        (16): LeakyReLU(negative_slope=0.2)
        (17): WN_Conv2d(128, 128, kernel_size=(1, 1), stride=(1, 1))
        (18): LeakyReLU(negative_slope=0.2)
        (19): WN_Conv2d(128, 128, kernel_size=(1, 1), stride=(1, 1))
        (20): LeakyReLU(negative_slope=0.2)
        (21): Expression()
    )
) (out_net): WN_Linear(in_features=128, out_features=10, bias=True)
)
```

**Figure 13: SVHN Discriminator Architecture**

```
Generator(
    (core_net): Sequential(
        (0): Linear(in_features=100, out_features=8192, bias=False)
        (1): BatchNorm2d(8192, eps=1e-05, momentum=0.1, affine=True, track_running_stats=True)
        (2): ReLU()
        (3): Expression()
        (4): ConvTranspose2d(512, 256, kernel_size=(5, 5), stride=(2, 2), padding=(2, 2), output_padding=(1, 1), bias=False)
        (5): BatchNorm2d(256, eps=1e-05, momentum=0.1, affine=True, track_running_stats=True)
        (6): ReLU()
        (7): ConvTranspose2d(256, 128, kernel_size=(5, 5), stride=(2, 2), padding=(2, 2), output_padding=(1, 1), bias=False)
        (8): BatchNorm2d(128, eps=1e-05, momentum=0.1, affine=True, track_running_stats=True)
        (9): ReLU()
        (10): WN_ConvTranspose2d(128, 3, kernel_size=(5, 5), stride=(2, 2), padding=(2, 2), output_padding=(1, 1))
        (11): Tanh()
    )
)
```

**Figure 14: Cifar Generator Architecture**
```python
Discriminative(
    (core_net): Sequential(
        (0): GaussianNoise()
        (1): Dropout2d(p=0.2, inplace=False)
        (1): WN_Conv2d(3, 96, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1))
        (2): LeakyReLU(negative_slope=0.2)
        (3): WN_Conv2d(96, 96, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1))
        (4): LeakyReLU(negative_slope=0.2)
        (5): WN_Conv2d(96, 96, kernel_size=(3, 3), stride=(2, 2), padding=(1, 1))
        (6): LeakyReLU(negative_slope=0.2)
        (7): Dropout(p=0.5, inplace=False)
        (8): WN_Conv2d(96, 192, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1))
        (9): LeakyReLU(negative_slope=0.2)
        (10): WN_Conv2d(192, 192, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1))
        (11): LeakyReLU(negative_slope=0.2)
        (12): WN_Conv2d(192, 192, kernel_size=(3, 3), stride=(2, 2), padding=(1, 1))
        (13): LeakyReLU(negative_slope=0.2)
        (14): Dropout(p=0.5, inplace=False)
        (15): WN_Conv2d(192, 192, kernel_size=(3, 3), stride=(1, 1))
        (16): LeakyReLU(negative_slope=0.2)
        (17): WN_Conv2d(192, 192, kernel_size=(1, 1), stride=(1, 1))
        (18): LeakyReLU(negative_slope=0.2)
        (19): WN_Conv2d(192, 192, kernel_size=(1, 1), stride=(1, 1))
        (20): LeakyReLU(negative_slope=0.2)
        (21): Expression()
    )
    (out_net): WN_Linear(in_features=192, out_features=10, bias=True)
)
```

Figure 15: Cifar Discriminator Architecture
Figure 16: An example of data generated by U-GAN and state-of-the-art algorithms (Top row is the SVHN data and Bottom row is the CIFAR-10 data). The samples generated by the benchmark methods are copied from the original papers and therefore use the unlabeled data as well. Despite some low-quality, non-representative data generated by U-GAN, similar to FM it generates the most realistic samples. The VAT samples are the ones corresponding the best-performing hyperparameter configuration, and the generated noise that has been added to the original image can be easily spotted.