On the fourth $P_{11}$ resonance predicted by the constituent quark model

L. Theußl*
Institut des Sciences Nucléaires,
(Unité Mixte de Recherche CNRS-IN2P3, UJF), F-38026 Grenoble Cedex, France

R.F. Wagenbrunn†
Institut für Theoretische Physik,
Universität Graz, Universitätsplatz 5, A-8010 Graz, Austria

We point out a distinguishing difference between constituent quark models based on one-gluon exchange and one-boson exchange dynamics. In the latter one, the $P_{11}$ nucleon resonance with predominantly symmetrical spatial wave function in the $N = 4$ band gets strongly attracted such that it drops below some states in the $N = 2$ band. Calculations of strong decay widths are presented in order to establish an identification with experimental states. Our results are relevant for the interpretation of the fourth $P_{11}$ resonance that was found in the partial wave analysis of the Zagreb group and recently discussed by Capstick et. al. in the framework of a model based on one-gluon exchange.

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Constituent quark models (CQM’s) are useful for interpreting the results of partial wave analyses (PWA) that are usually employed to extract the properties of nucleon resonances from the experimental data of pion-nucleon scattering [1]. Generally, such models predict nucleon resonances from the experimental data of pion-nucleon scattering that are usually employed to extract the properties of nucleon resonances from the experimental data of pion-nucleon scattering [1]. Generally, such models predict nucleon resonances from the experimental data of pion-nucleon scattering [1]. Generally, such models predict nucleon resonances from the experimental data of pion-nucleon scattering [1]. Generally, such models predict nucleon resonances from the experimental data of pion-nucleon scattering [1]. Generally, such models predict nucleon resonances from the experimental data of pion-nucleon scattering [1]. Generally, such models predict nucleon resonances from the experimental data of pion-nucleon scattering [1]. Generally, such models predict nucleon resonances from the experimental data of pion-nucleon scattering [1].

The fourth state at $M = 1975$ MeV is the totally antisymmetrical resonance of the $N_{1710}$ resonance, an identification that is confirmed by studies of decay amplitudes with a specific decay operator [2]. The third state predicted is an orbital momentum $L = 2$ state with quark spin $\frac{1}{2}$ and a mixed symmetrical spatial wave function while the fourth is a $L = 1$ state with quark spin $\frac{1}{2}$. The latter one has a totally antisymmetrical spatial wave function and is therefore naturally expected to be the heaviest of these resonances with a weak coupling to the $\pi N$ decay channel.

The last two states were not found yet by any partial wave analysis, a fact that constitutes part of the problem of “missing states” [1], i.e. the fact that constituent quark models generally predict more states than observed. In ref. [1] it was argued that the lighter of these missing states in the $N = 2$ band can be identified with the fourth $P_{11}$ resonance that was discovered by the Zagreb group in their partial wave analysis based on a multi-channel, multi-resonance, unitary model [2]. It was pointed out that the crucial element for finding this resonance is the fit to the $\pi N \rightarrow \eta N$ data, which was carried out with great care in ref. [3]. Results for partial decay widths in the strong decay channels $\pi N$, $\eta N$ and $\pi \pi N$, as calculated in ref. [3] in a relativized version of a model based on OGE dynamics, were given to confirm the identification. We redisplay these results in Table I together with the experimental results from ref. [4].

It can be seen, that the first OGE state that could correspond to the new solution found by the Zagreb group at $M \simeq 1740$ MeV is the state at $M = 1880$ MeV. A feature that was not discussed in ref. [4] is the fact that the model based on OGE predicts this state to decay more strongly into $N \eta$ than into $N \pi$, a situation that seems to be contradicted by the experimental results (even though one should consider the large error bars in this case).

The fourth state at $M = 1975$ MeV is the totally antisymmetrical and the heaviest one in the $N = 2$ band, as expected. Its weak coupling to the decay channels was used in particular to explain why it has not been seen in any partial wave analysis yet.

The first OGE state in the $N = 4$ band appears at $M = 2065$ MeV and it was assigned to the $P_{11}(N_{2100})$ state of the Particle Data Group [5], which, however, has the status of a 1-star resonance only. Its decay properties are very similar to those of the Roper resonance which indicates the same structure for these two states. However, as for the relative strength of decay amplitudes in the $\pi N$ and $\eta N$ channels, the wrong ordering is again predicted by the OGE as compared to the solution of the Zagreb analysis. In fact, just from the decay widths, one would rather guess that this state corresponds to the new solution of the Zagreb group at $M \simeq 1740$ MeV, an identification that is hardly possible in this model because of the large mass of this state in the OGE.

We have calculated the spectrum and wave functions that follow from a model based on Goldstone-boson ex-

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*e-mail address: lukas@isn.in2p3.fr
†e-mail address: Robert.Wagenbrunn@kfunigraz.ac.at
TABLE I. Resonance parameters from different quark models. The first column gives the assignment of the Particle Data Group (PDG, ref. [6]), OGE is from ref. [7], GBE are present results and the experimental data correspond to the four $P_{11}$-resonance solution of ref. [6]. For the latter ones we also give the uncertainties in the total width that have to be added to the uncertainty in the partial decay width (the numbers in parentheses). All numbers in MeV.

| PDG state | OGE | GBE | exp. |
|-----------|-----|-----|------|
| $P_{11}(1440)$ | 1540 | 1443 | 1439 $\pm$ 19 $(271 \pm 18)^{+1+4}_{-1-2}$ $(0 \pm 0)^{+0}_{-0}$ |
| $P_{11}(1710)$ | 1770 $^{+8}_{-7}$ 67 | 1770 $^{+8}_{-7}$ 15 | 1729 $\pm$ 16 $(40 \pm 40)^{+1+13}_{-1-24}$ $(11 \pm 11)^{+6}_{-6}$ |
| $P_{11}$ | 1880 | 1874 $^{+70}_{-67}$ | 1740 $^{+11}_{-11} (39 \pm 39)^{+24}_{-24}$ $(17 \pm 13)^{+2}_{-2}$ |
| $P_{11}$ | 1975 $^{+4}_{-4}$ 50 | 1970 $^{+5}_{-5}$ 26 | $-$ $-$ $-$ $-$ |
| $P_{11}(2100)$ | 2065 $^{+59}_{-59}$ 3 | 2104 $^{+4}_{-4}$ 14 | 2157 $\pm$ 42 $(57 \pm 17)^{+1+9}_{-2-11}$ $(295 \pm 18)^{+6}_{-69}$ |

change (GBE) dynamics [10]. Here we use a slightly different parameterization where the tensor components of the pseudoscalar meson-exchange interaction are included. Those tensor forces were dropped in [10] and only the spin-spin components of the pseudoscalar meson-exchange interaction were taken into account.

Partial decay widths are then calculated in a microscopic decay model of the $^3P_0$ type, which differs from the one used in ref. [10] by two main points:

- First we use a $^3P_0$ decay operator in a modified form as introduced in ref. [10]. It has been checked that the qualitative features of decay predictions are not influenced by this choice.

- Second, and more important, for all the baryon resonances we use the theoretical masses as they follow from the solution of the Schrödinger equation as input in our decay calculation. This would result in different phase space factors for states whose masses are predicted far off their experimental values, which is not the case in the GBE for any of the resonances considered here. However it may influence the relative ordering of $\pi N$ and $\eta N$ widths due to the “structure dependence” [10] of the $P_{11}$ resonances [10].

First results in this approach were presented in ref. [10], where also a more thorough discussion regarding constituent quark- and decay models may be found. The masses of the lightest resonances predicted by the GBE in the $P_{11}$ channel are given in Table I and it can be seen that, apart from the Roper resonance, which is better described by the GBE, they correspond almost exactly to the masses of the OGE model. The decay properties are rather different however.

First we note that for the $N_{1710}$ resonance the GBE model predicts $\Gamma_\eta < \Gamma_\pi$, contrary to the OGE model but in qualitative agreement with the experimental result of the Zagreb analysis. A similar difference appears for the next $P_{11}$ resonance, where again the GBE reproduces roughly the experimental pattern of decay amplitudes, whereas the OGE predicts the opposite ordering.

In order to explain these qualitative differences we display in Table II the $LS$ - components of the GBE wave functions in question.

TABLE II. Probabilities (in %) of all possible $LS$ - components of $N^*$ wave functions for $P_{11}$ resonances from the GBE model with tensor force.

| $N^*$ | Mass | $S = 1/2$ | $S = 3/2$ |
|-------|------|-----------|-----------|
| $N_{1710}$ | 1770 | 98.9 | 3.1 |
| $N_{1440}$ | 1443 | 98.8 | 1.1 |
| $N_{939}$ | 939 | 98.7 | 1.3 |
| $N_{2100}$ | 2104 | 54.7 | 0.8 |
| $N_{1874}$ | 2104 | 8.5 | 42.7 |

It is seen immediately, that the GBE state at $M = 1874$ MeV cannot be identified with any of the remaining states in the $N = 0$ band, since it is an almost pure $L = 0$ state. It rather corresponds to the predominantly symmetrical solution in the $N = 4$ band, i.e. the second radial excitation of the nucleon after the Roper resonance. The properties of the Goldstone-boson exchange dynamics lead to a strong attraction in symmetrical components of the wave function, a property that explained already the low mass of the Roper resonance in this model.

We see also from Table I that the decay properties correspond almost exactly to those of the $N = 4$ state in the GBE model. In particular, we predict the fourth $P_{11}$ resonance to decay more strongly into $N\pi$ than into $N\eta$, as it was found for the new resonance in the Zagreb analysis.
with the corresponding states. A qualitative difference between the models again occurs when comparing the \( N\eta \) with the \( N\pi \) widths: both these resonances give \( \Gamma_\eta > \Gamma_\pi \) in the GBE, a relation that is also satisfied by the state at \( \sim 2157 \) MeV found in the Zagreb analysis, but contradicted by the predictions of the OGE. This feature is a consequence of the strong mixing of these two states, as evidenced in Table II. In fact, the main contribution to the decay widths for both of them comes from the \( L = 2, S = \frac{3}{2} \) component which explains their similar decay properties as opposed to the OGE model.

We would finally like to stress that the inversion of states discussed above is also found in different parameterizations of the GBE interaction. We have checked in particular that in the models of ref. [11,12] as well as ref. [13], it is always the mainly symmetrical state that follows the \( N\eta \) resonance. A good description of the Roper resonance has also been obtained in a model including a three-body force [14]. Since this one acts mainly on the nucleon and its radial excitations while producing essentially no effect on states with mixed symmetry, one may expect a similar inversion in the upper part of the spectrum as in the GBE model. Unfortunately, calculations of decay widths as presented in ref. [13] do not include the states of interest here.

In addition to \( \pi N \) and \( \eta N \) widths, also results for the \( \pi\pi N \) channel were given in ref. [11] and compared to the results of the Zagreb group. We did not repeat these calculations in the GBE model because of some theoretical ambiguities associated with the determination of a decay operator for quasi-two-body decays [11]. Furthermore, the branching fractions extracted from the PWA depend sensitively on the input data of \( \pi N \rightarrow \pi\pi N \) reactions, but the inelastic data in channels like \( \pi\Delta \), \( \rho N \), etc., were not explicitly included in the analysis of the Zagreb group [11].

In all the considerations presented above, one has to bear in mind the rather large uncertainties of experimental data which prevents one from drawing definite conclusions about the quality of predictions from any CQM for the moment. However, the inversion of states in the GBE model is a unique and distinguishing feature leading to qualitatively different predictions of decay properties. This would permit a definite discrimination of different models, once the experimental data are determined with sufficient accuracy.

In summary, we have pointed out that the states that are predicted by models based on OGE and GBE dynamics at almost the same mass of \( M \approx 1880 \) MeV do not correspond to the same \( SU(6) \otimes O(3) \) state in a harmonic oscillator basis. As a consequence they show very different decay properties. In particular, predictions for decay widths using wave functions stemming from GBE dynamics are in qualitative agreement with the recent PWA of ref. [9]. Differences with predictions from the OGE model may allow a discrimination between the two models, which is not possible at the moment due to rather large experimental uncertainties. Clearly, a more precise determination of the resonance parameters discussed in this paper is needed to definitely settle the issue.

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