Cross-Referencing Method for Scalable Public Blockchain

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Abstract

We previously proposed a cross-referencing method for enabling multiple peer-to-peer network domains to manage their own public blockchains and periodically exchanging the state of the latest fixed block in the blockchain with hysteresis signatures among all the domains via an upper network layer. In this study, we evaluated the effectiveness of our method from three theoretical viewpoints: decentralization, scalability, and tamper resistance. We show that the performance of the entire system can be improved because transactions and blocks are distributed only inside the domain. We argue that the transaction processing capacity will increase to 56,000 transactions per second, which is as much as that of a VISA credit card system. The capacity is also evaluated by multiplying the number of domains by the average reduction in transaction-processing time due to the increase in block size and reduction in the block-generation-time interval by domain partition. For tamper resistance, each domain has evidence of the hysteresis signatures of the other domains in the blockchain. We introduce two types of tamper-resistance-improvement ratios as evaluation measures of tamper resistance for a blockchain and theoretically explain how tamper resistance is improved using our cross-referencing method. With our method, tamper resistance improves as the number of domains increases. The proposed system of 1,000 domains are 3-10 times more tamper-resistant than that of 100 domains, and the capacity is 10 times higher. We conclude that our method enables a more scalable and tamper-resistant public blockchain balanced with decentralization.

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1. Introduction

Since Bitcoin [1] appeared in 2008, blockchain technology has been gaining considerable public attention. The words “block chain” were born from a dialogue between the Bitcoin inventor, Satoshi Nakamoto, and a cryptographer, Hal Finney, on the cryptography mailing list [2]. A blockchain is a chain of block-structured databases where each block is connected in a time-series order by a cryptographic hash function, and works as a timestamp server. A blockchain is not a new idea, as the citing of Massias et al.’s paper published in 1999 [3] in Nakamoto’s white paper suggests. However, it has become an iconic image for related technology.

All transactions in Bitcoin, including the issuance and exchange of electronic cash, are disclosed on the blockchain. Therefore, nodes participating in the peer-to-peer (P2P) network can verify which address (not user) holds how many bitcoins by tracking transaction records. However, since users can keep as many bitcoins addresses as they want, it is difficult for others to know how many bitcoins one owns in total. Therefore, a minimum level of users’ privacy is considered. Personal transaction information has generally been kept private due to privacy concerns. However, Bitcoin has made all transaction records public with privacy in mind (although anonymity is not guaranteed). This makes it possible for an unspecified number of nodes to reach a (extended) consensus about transaction records on the blockchain on the basis of the longest chain rule. Because this consensus algorithm was essentially a new idea, compared with known ones in distributed systems, it is called Nakamoto Consensus.

Transaction processing is completed when a transaction is saved to the blockchain through a high-load computational process called proof of work (PoW) [4]. Any node can become an authority that connects a block with several transactions to the blockchain by presenting evidence to other nodes that it is able to execute the PoW correctly and quickly. Therefore, the system is designed so that no particular node can remain as an unjustified authority. In addition, PoW makes transaction records on the
blockchain tamper-resistant.

The essential value of Bitcoin is *micropayment*, which is possible by reducing the transaction fee to an extremely low level, such as less than one cent or one yen. Micropayment has the potential to create a new decentralized economy, as it enables charging for various operations performed on the Internet. For example, Wikipedia, which is always struggling to collect donations, may be able to collect server maintenance fees from very small donations from some article readers by introducing micropayment. However, the current blockchain technology is practically incapable of supporting micropayment because the transaction processing capacity is restricted by the block transfer speed between nodes and the limit of block size, which is known as the *Bitcoin scalability problem*.

Several technologies have been studied to solve this problem. Off-chain scaling technologies, such as Lightning Network [6], are currently attracting attention. Off-chain technologies leave less transaction records in the blockchain. Bitcoin has been used as a payment means of conducting illegal transactions in darknet markets, and there have been many reports on the managers and users of these markets being arrested [7, 8, 9]. These arrests are attributable to Bitcoin’s disclosure of all transaction records with tamper resistance, which are available as legal evidence. If off-chain scaling technologies become widespread, transactions that cannot be audited by our societies and governments can be easily created and off-chain services might become hotbeds of illegal transactions in darknet markets for money laundering and financing of terrorism by criminal organizations. Thus, when we consider the use of blockchain technology on the basis of law and ethics, it is ultimately necessary to solve the scalability problem on-chain. Existing blockchains, such as Bitcoin, are based on the premise that only one blockchain can be globally integrated and managed. As a result, blocks are transferred and shared among nodes all over the world, which slows down transaction processing.

As a way of speeding up transactions, we have proposed a mechanism for allocating domains to geographically close nodes, similar to the country code top-level domain in the Domain Name System, and managing blockchains in each domain [10, 11, 12]. Through this mechanism, it is possible to distinguish between the inside and outside of a domain by the communication speed with the central node, which is the entrance.
to the P2P network. In addition, our mechanism allows domain-specific geographic information to be handled by connecting secure Internet-of-Things devices to some of the nodes. However, the domain partition entailed by our mechanism reduces the number of nodes participating in each domain, which degrades the decentralization and tamper resistance of the blockchain. It is possible to distinguish between the inside and outside of a domain by the communication speed with the central node which is the entrance to the P2P network. In addition, domain-specific geographic information can be handled by connecting secure Internet-of-Things devices to some of the nodes. However, domain partition reduces the number of nodes participating in each domain, which degrades the decentralization and tamper-resistance of the blockchain.

To solve this problem, we previously proposed a cross-referencing method for periodically exchanging the state of the latest fixed block in the blockchain with hysteresis signatures \[13\] among all the domains via the upper network layer \[14\]. We also designed a communication protocol to autonomously execute our cross-referencing method among domains. We explained the effectiveness of this method only in terms of a definition of tamper resistance.

In this study, we evaluated the effectiveness of our method from the viewpoints of decentralization and scalability, as well as tamper resistance. We define decentralization as the ability of every participating node to affect the entire system by creating blocks. With our method, in-domain decentralization is equivalent to that of usual public blockchains, such as Bitcoin, while out-domain decentralization is preserved by hysteresis signatures, which affect some of the blocks in other domains. Regarding scalability of transaction processing capacity, the performance of the entire system is improved because transactions and blocks are distributed only inside the domain. We theoretically demonstrate that the transaction processing capacity should increase to that of a typical credit card system, such as VISA, meaning that the capacity can be evaluated by multiplying the number of domains by the average improvement ratio of transaction processing speed due to the increase in block size and reduction in the block-generation-time interval by domain partition. For tamper resistance, each domain has evidence of the hysteresis signatures of the other domains in the blockchain. Therefore, to tamper with a block in a domain, one must also tamper with the blocks
containing the relevant hysteresis signatures in all domains. We introduce two types of tamper-resistance-improvement ratios $R$ and $R'$ and theoretically show that, in the terms of both measures, tamper resistance improves when using our cross-referencing method through Monte Carlo simulations. We conclude that it is possible to achieve a more scalable and tamper-resistant public blockchain balanced with decentralization by using our cross-referencing method.

The main contributions of this paper are summarized as follows.

- We formulate the Bitcoin scalability problem to show an upper limit of transaction processing capacity.

- We clarify the definition of decentralization and elaborate that the scalability of public blockchains potential to dramatically improved to increase the transaction processing capacity to that of a VISA credit card system.

- We define two tamper-resistance-improvement ratios, $R$ and $R'$, to explain that our cross-referencing method can also improve the tamper-resistance of public blockchains. We also discuss how the ratios change when some nodes fail.

2. Related Work

There are naive methods for solving the problem on-chain: increasing the block size and shortening the block-generation-time interval. Both are possible by increasing the communication speed between nodes, but it is difficult for nodes with slow communication speed to join the network. The former method is being evaluated on the Bitcoin Scaling Test Network (STN) [15]. While the maximum block size of Bitcoin Core is 1 MB, STN has eliminated the block size limit and achieved an average processing speed of 2,596 transactions per second in 24 hours [15] (Accessed 13 April 2021). The latter method is being evaluated by bloXroute [16], a project that proposes to alleviate the scalability problem by introducing a backbone network to propagate large blocks in a shorter time.

Hysteresis signatures [13] enhance the tamper resistance of ordinary electronic signatures by adding a nested structure with other past electronic signatures. A typical
structure of a hysteresis signature is shown in Fig. 2. A nested structure is naturally created by signing the content including the previous signature.

By repeating the nested structure, the electronic signatures are chained to create a time-series context. For example, let the previous hysteresis signature be $S_{n-1}$ and the content to be signed be the block data of $m$ domains in total ($B_{D_1}, \cdots, B_{D_m}$). In this case, the hysteresis signature is created by signing the concatenation of the summary of the previous hysteresis signature $H(S_{n-1})$ and hashed contents $H(B_{D_1}), \cdots, H(B_{D_m})$. The created signature $S_n$ is also added to the hysteresis signature, as shown in Fig. 2.

Transaction signatures in Bitcoin was used to prove the legitimacy of a transaction by having the holder of the Bitcoin sign the transaction using his or her private key. In hysteresis signature, the central core node of the domain signs the latest confirmed block in order to prove the validity of the block. In addition, the summary finalized blocks of all past domains are aggregated and stored in the cross-referencing part, which enables tamper detection and correction among domains.

Ordinary electronic signatures can be tampered if the private key is leaked, and it is often impossible to detect tampering. In hysteresis signatures, since the signature is signed in the nested structure, an attacker would have to tamper with all nested signatures after the tampered content, which makes tampering significantly more difficult. This is similar to the block structure in the public blockchain. If a contradiction in a hysteresis signature is found, it is almost certain that the content has been tampered with.

BBc-1 [17] is a distributed ledger technology, with which a distributed system manages a consistent ledger, but it does not handle any public blockchain. The system stores private transaction records in a tamper-resistant manner by using hysteresis signatures. There is a reference implementation of the node, and the effectiveness of the system can be confirmed. With BBc-1, all the transaction data have a hysteresis signature, which is exchanged across domains to increase tamper resistance. The energy cost is also low compared with a typical public blockchain, such as Bitcoin, because of the absence of the PoW. However, it is difficult to estimate how much tamper resistance is improved by introducing the hysteresis signature.

Atomic Swap is a technique that allows the exchange of cryptocurrencies recorded
on different blockchains without needing to trust any third party\textsuperscript{[18]}. This technique is also useful for coexisting multiple blockchains and exchanging their native cryptocurrencies.

3. Cross-referencing Method

The structure of the P2P network used in this study is shown in Fig. 1. The whole network consists of two P2P layers, \textit{i.e.}, Layers 0 and 1. Layer 1 assumes that there are multiple P2P networks of typical public blockchains which share transaction and block data. Each P2P network in Layer 1 is called a \textit{domain}, and it is assumed that there is a set of core nodes in each domain. There are two types of core node: central core nodes (CCNs) and peripheral core nodes (PCNs). It is assumed that at least one CCN is selected as a leader in each domain beforehand. In this study, we assumed that the number of domains is \(m\) and the CCNs of multiple domains \((D_1, D_2, \cdots, D_m)\) have a

\[
\begin{array}{c}
\text{Layer 0} \\
\text{Domain 1 (D_1)} \quad \text{Domain 2 (D_2)} \quad \text{Domain 3 (D_3)} \quad \cdots \quad \text{Domain m (D_m)} \\
\text{Layer 1} \\
\end{array}
\]

\[H(S_{n-1}) \mid H(B_{D_1}) \mid \cdots \mid H(B_{D_m}) \mid S_n = \text{Sig}(H(S_{n-1})||H(B_{D_1})||\cdots||H(B_{D_m}))\]

Figure 1: Two-layer P2P network used in this study

Figure 2: Hysteresis signature
prior agreement to share their block records and domain hysteresis signatures to use the cross-referencing method. In Layer 0, the CCNs are connected to each other to form another consortium-type P2P network, which is disconnected from that of Layer 1. For simplicity, we consider the case in which the number of CCNs for each domain is one, but it is also possible to generalize the case in which the number of CCNs is more than one. Note that the addition of Layer 0 over Layer 1 is common with bloXroute [16], but the difference is that Layer 0 is also another P2P network with our method, while Layer 0 in bloXroute is a faster network transport layer for both transaction and block records.

The block structure of our cross-referencing method is shown in Fig. 3. The difference between the block structure in ordinary public blockchains and that with our method is the cross-reference part. In this part, a hysteresis signature is created by signing the concatenation of the summary of the previous hysteresis signature $H(S_{n-1})$ and hashed contents $H(B_{D_1}), \ldots, H(B_{D_m})$. The created signature $S_n$ is the same as that shown in Fig. 2. The block records with the cross-reference part are shared between CCNs via the P2P network in Layer 0.

The timeline of our cross-referencing method in a normal case, meaning that no
stop failure occurs in CCNs, is shown in Fig. 4. The timeline is divided into three phases. The details of each phase are explained as follows.

1. In Phase 1, a CCN in a domain notifies the other CCNs to start cross referencing by sending a message. Then, each CCN transfers a message including the \( l \)-confirmed block to the other CCNs, where \( l \) is a positive integer and \( l \)-confirmed means the block approved \( l \) blocks before the latest block. Phase 1 finishes if all the CCNs collect all the \( l \)-confirmed block records by sharing them with each other.

2. In Phase 2, each CCN first generates a hysteresis signature, as shown in Fig. 2. The CCN then sends a request message with the hysteresis signature to PCNs in the same domain to mine the block having the cross-reference component. After independently mining the latest block, a PCN sends a message with the mined block back to the CCN, and the CCN checks whether the block has been properly mined. If the block has not been properly mined, the CCN waits until a properly mined block is received and then Phase 2 finishes.

3. In Phase 3, each CCN broadcasts the mined block to announce that the cross-
referencing was successful.

We designed a distributed algorithm [19] for our cross-referencing method. The details of which are shown in Fig. 5. We also considered a distributed algorithm that is tolerant of \( t \)-stop failures (CCNs in \( t \) domains do not respond because they experienced a stop failure or refuse to execute cross referencing) as shown in Fig. 6.

Figure 5: Flowchart 1 (Phase 2 is omitted because it consists of usual mining process on Layer 1)
There are three assumptions under which these flowchart work properly.

1. The P2P network in Layer 0 is synchronous and its structure should be a complete graph.
2. All the CCNs are reliable, meaning that they execute the cross-referencing method following the flowchart to share the requested block data with each other.
3. In Flowchart 1, none of the CCNs exhibit any experience stop failure and in Flowchart 2, The CCNs allow $t$-stop failures, which means that our cross-referencing method works even if at most $t$ CCNs do not cooperate to share their block records.

Note that we can assume that the number of nodes having experiencing stop failure is small because each domain needs to strengthen tamper resistance with the cross-
referencing method. If it takes a long time to repair a failed CCN, the blockchain in its domain becomes vulnerable to malicious tampering attacks. Therefore, each domain should recover from the stop failure as soon as possible.

The efficiency of these distributed algorithms can be evaluated by measuring both communication and time complexity. The communication complexity of Flowchart 1 is $O(m^2 \cdot b)$, where $m$ is the number of CCNs (equivalent to the number of domains), and $b$ is the bit size of the message to send between CCNs. The communication complexity of Flowchart 2 is $O((t+1)(m^2 \cdot b))$. Assume that the time complexity of Flowchart 1 is $T_1 + T_2 + T_3$, where $T_i (i = 1, 2, 3)$ is the waiting time taken for Phase $i$ in Flowchart 1. Then, the time complexity of Flowchart 2 is $(t + 1)T_1 + T_2 + T_3$.

4. Theoretical Evaluation

Our cross-referencing method can improve the scalability and tamper resistance of the blockchain while preventing degradation of decentralization. We theoretically evaluated the effectiveness of the proposed method from three viewpoints: decentralization, scalability, and tamper resistance.

4.1. Evaluation of decentralization

The definition of decentralization is often ambiguous, so, we will define it as the ability of every participating node to affect the entire system by creating blocks. With our method, in-domain decentralization is equivalent to that of usual public blockchains, such as Bitcoin. Out-domain decentralization is preserved by hysteresis signatures, which affect some of the blocks in other domains. Note that this governance structure between in-domain and out-domain decentralization is similar to modern democratic systems in our society.

4.2. Theoretical formulation of Bitcoin scalability problem

The block-generation-time interval of blockchain $T$ is a stochastic variable, and $T$ obeys the exponential distribution, i.e.,

$$F(t) = P(T \leq t) = \int_0^t \lambda e^{-\lambda t'} dt' = 1 - e^{-\lambda t}, \quad (1)$$
where $\lambda$ is the inverse of the average block-generation-time interval \cite{5}. In Bitcoin Core, the average interval $1/\lambda$ is 10 minutes (600 seconds). The latency required to transfer a block of size $b = 1$ MB in Bitcoin Core to 90\% of the nodes on the P2P network was experimentally evaluated as $t = \tau_{fork}(b) \approx 12$ seconds \cite{16}. Therefore, the probability of a blockchain fork (branching) is estimated as

$$F(\tau_{fork}) = P(\tau_{fork}) = 1 - e^{-\lambda \tau_{fork}} \approx \lambda \tau_{fork} = 12/600 = 0.02.$$  \hfill (2)

If the blockchain forks, some of the block-generation capacities are wasted because one of the branches will be rejected. Therefore, the fork probability is kept small enough by using a difficulty adjustment algorithm \cite{20}.

There are two simple ways of increasing the transaction processing capacity: (1) increasing the block size $b$ and (2) shortening the average block-generation-time interval $1/\lambda$. As shown in Eq. (2), however, the fork probability increase with both methods, which means the transaction processing capacity is reduced. Therefore, it is generally difficult to resolve this tradeoff of transaction processing capacity, which is called the scalability problem in public blockchains.

It is estimated that the transaction processing capacity of Bitcoin is 5-7 transactions per second. However, it is known that the credit card company VISA, Inc. has a capacity of 56,000 transactions per second. We explain below that it is difficult for Bitcoin Core to reach this level of the capacity. From Eq. (2), the unfork probability, which only contributes to creating valid blocks, is calculated as

$$P_{unfork} = 1 - P(\tau_{fork}) = e^{-\lambda \tau_{fork}} = e^{-\tau_{fork}/\tau},$$  \hfill (3)

where $\tau = 1/\lambda$ is the average block-generation-time interval. In this case, the blockchain does not fork and it is effective on the transaction processing capacity. Therefore, the transaction processing capacity per second is roughly

$$G(\tau) = \frac{C}{\tau} e^{-\tau_{fork}/\tau},$$  \hfill (4)

where $C$ is the average number of transactions in a block. In Bitcoin Core, $\tau = 600$ seconds, $\tau_{fork} = 12$ seconds, the unfork probability is $e^{-\tau_{fork}/\tau} = 1 - 0.02 = 0.98$, and $G(\tau = 600) = 7$ transactions per second at maximum. Substituting these values into Eq. (4) the average number of transactions in a block is about $C = 4,286$ transactions.
We can further consider the optimal transaction processing capacity $\tau_{opt}$. To this end, we estimate the maximum $G(\tau)$ by changing $\tau$, i.e.,

$$\max_{\tau} G(\tau) = C \max_{\tau} (e^{-\tau_{fork}/\tau} / \tau).$$  \hspace{1cm} (5)

It is easily calculated that $\tau_{opt} = \tau_{fork} = 12$ seconds. Therefore, the optimal transaction processing capacity per second is calculated as

$$\max_{\tau} G(\tau) = G(\tau_{fork}) = Ce^{-1}/\tau_{fork}. \hspace{1cm} (6)$$

Substituting $C = 4286$ transactions and $\tau_{fork} = 12$ seconds into Eq. (6), the optimal $G$ turns out to be about 132 transactions per second, which is considered to be the upper bound of the transaction processing capacity in the Bitcoin Core blockchain.

4.3. Improvement in scalability of transaction processing capacity

As we explained in the previous subsection, an upper bound to the transaction processing capacity seems to exist, which causes the scalability problem. Existing blockchains including Bitcoin are based on the premise that only one chain is globally integrated and managed. Therefore, it is necessary to transfer and share blocks among nodes located all over the world, which dramatically reduces the transaction processing capacity. Therefore, we considered a mechanism for allocating domains to geographically close nodes and managing one chain for each domain.

Regarding scalability in transaction processing speed, the performance of the entire system can be improved because transactions and blocks are distributed only inside the domain. For example, if the block size is increased from 1 to 10 MB, block-generation-time interval from 10 to 2 minutes, and number of domains from 1 to 200, the transaction processing capacity can reach an equal or better level compared with that of VISA, Ltd.

$$5 \text{tx./sec.} \times \left( 10 \times \frac{10}{2} \times \frac{200}{1} \right) = 50,000 \text{tx./sec.} \simeq 56,000 \text{tx./sec.} \hspace{1cm} (7)$$

$$7 \text{tx./sec.} \times \left( 10 \times \frac{10}{2} \times \frac{200}{1} \right) = 70,000 \text{tx./sec.} > 56,000 \text{tx./sec.} \hspace{1cm} (8)$$

It is highly possible that the transaction processing capacity will exceed that of VISA credit card system, but the actual performance needs to be evaluated experimentally, which is left for our future work.
4.4. Evaluation of tamper resistance

Our cross-referencing method can improve the tamper resistance of blockchains. We will use two tamper-resistance-improvement ratios $R$ and $R'$ to evaluate tamper resistance. Let the total number of core nodes in the distributed system be $N$ and the hash rate of core node $i$ be $h_i$. Note that the hash rate is the number of times that a cryptographic hash function can be computed per unit time.

With $R$, tamper resistance can be estimated by the maximum hash rate because we assume that the node with the highest hash rate has the highest probability of generating blocks, so it continues mining. On the other hand, most nodes with relatively smaller hash rates have a smaller probability of generating blocks, so they tend to quit mining. Therefore, tamper resistance is proportional to the following value, i.e.,

$$\max\{h_1, h_2, \cdots, h_N\}.$$ (9)

Suppose that the total number of core nodes in the $m$-th domain is $D_m$. The tamper resistance of each domain is given by the maximum hash power of the nodes for each domain, i.e.,

$$A_1 = \max\{h_{11}, \cdots, h_{1D_1}\},$$ (10)
$$A_2 = \max\{h_{21}, \cdots, h_{2D_2}\},$$ (11)
$$\vdots$$
$$A_m = \max\{h_{m1}, \cdots, h_{mD_m}\}.$$ (12)

When only one domain has $N = D_1 + D_2 + \cdots + D_m$ core nodes, the tamper resistance of the domain without the cross-referencing method can be estimated as

$$A = \max\{A_1, A_2, \cdots, A_m\}.$$ (13)

With $R'$, tamper resistance can be estimated by the accumulated hash rates of the top $X\%$ nodes with high hash rates. Tamper resistance is then proportional to the following value, i.e.,

$$A' = \sum_{i \in \text{nodes with top } X\% \text{ hash rate}} h_i.$$ (14)
Equation (13) evaluates the maximum hash rate as tamper resistance, and Eq. (14) is an expression that evaluates the sum of the hash rates of the top $X\%$ as tamper resistance.

By applying our cross-referencing method among $m$ domains, tamper resistance can be estimated by the sum of all the highest hash rates in the domains, i.e.,

$$B = \sum_{i=1}^{m} A_i,$$  \hspace{1cm} (15)

$$B' = \sum_{i=1}^{m} \sum_{j \in \text{nodes with top } X\% \text{ hash rate}} A_{ij},$$  \hspace{1cm} (16)

Similarly, Eq. (15) is an expression that evaluates the maximum hash rate in the domain as tamper resistance. Equation (16) evaluates the sum of the hash rates of the top $X\%$ that contributes to tamper resistance.

Therefore, $R$ is defined as

$$R = \frac{B}{A} (> 1),$$  \hspace{1cm} (17)

and, $R'$ is defined as

$$R' = \frac{B'}{A'}.\hspace{1cm} (18)$$

To estimate typical values of $R$ and $R'$, we conducted Monte Carlo simulations in which the hash rate $h_{ij}$ ($i$ is the domain number, and $j$ is the serial number of nodes in a domain) was randomly assigned in accordance with a Pareto distribution, i.e.,

$$P(h_{ij}) = \frac{\alpha}{h_{ij}^{1+\alpha}} (h_{ij} > 1),$$  \hspace{1cm} (19)

where $\alpha$ is a scale parameter. This distribution is often used to explain wealth distribution in economics. The rate represents the total amount of computational resources, which is proportional to the amount of capital of the miner, so using the Pareto distribution is considered appropriate.

It is known that the tamper resistance of a blockchain for PoW is determined by the hash rate of the mining nodes on the P2P network. This hash rate is generally considered to follow the Pareto distribution in Eq. (19) because computing resources of nodes are unequal. Therefore, the hash rate depends on the financial strength of the miner who owns the nodes. Also, since only a handful of miners succeed in mining on
a regular basis, we defined the tamper resistance improvement ratio as shown in Eqs. (17) and (18).

We assume that the total number of core nodes is $N = 10,000$ and that the number of core nodes in each domain is uniform, i.e., 10, 100, and 1000 nodes when the number of domains is 1000, 100, and 10, respectively. Typical simulation results are shown below.

Fig. 7 and 8 show the results when the maximum hash rate is used as tamper resistance. Fig. 9 and 10 show the results of $R'$ with the top $X = 10\%$. Fig. 11 and 12 show the results of $R'$ with the top $X = 30\%$. A comparison of Figs. 9, 10, 11 and 12 indicates that the peak of the distribution of $R'$ is the position slightly higher than one. As $m$ increases, the peak of the histograms in both $R$ and $R'$ generally shifts to a higher position. As $m$ increases, the tamper resistance gets higher. In Figs. 7 and 8, the tamper resistance with $m=1,000$ is about 3-5 times higher than that with $m=100$. In Figs. 9 and 11, comparing the peak positions of the distribution of $R'$, we can see that the tamper resistance with $m=1,000$ is 10 times higher that that with $m=100$. In Figs. 9 and 11, there is no significant difference between them in both the top 10% and 30%. There is similar tendency in Figs. 10 and 12. In addition, these figure show that the variance of the distribution increases as $m$ increases.
Figure 7: Simulation results of probability density function of tamper-resistance-improvement ratio $R$ when scale parameter $\alpha = 2$

Figure 8: Simulation results of probability density function of $R$ when $\alpha = 3$
Figure 9: Simulation results of probability density function of $R'$ when $\alpha = 2$ and $X = 10\%$

Figure 10: Simulation results of probability density function of $R'$ when $\alpha = 3$ and $X = 10\%$
Figure 11: Simulation results of probability density function of $R'$ when $\alpha = 2$ and $X = 30\%$

Figure 12: Simulation results of probability density function of $R'$ when $\alpha = 3$ and $X = 30\%$
4.5. Effect of stop failures on tamper resistance

We also estimated the effect of stop failures on tamper resistance in the entire system. We show Monte Carlo simulation results when CCNs in $f = 1, 3, 5$ domains become stop failure. As shown in Figs. 13, 14, 15, 16, 17, and 18, the number of failed CCNs is sufficiently large ($f = 3$ or 5), the tamper resistance in the entire system deteriorates significantly. However, $R$ and $R'$ shifts higher as $m$ increases indicating that the effect of stop failures on tamper resistance is relatively small. If failure occurs, the $R$ improved as well. As $f$ increases, the peak of the histograms in $R$ generally shifts toward a higher position.

However, $R'$ did not improve compared with $R$. If $f$ is small and the number of stop-failed CCNs is sufficiently large, the tamper resistance in the entire system can degrade significantly. However, as $m$ increases, $R'$ shifts toward the higher position, and the effects on tamper resistance can be relatively small.

Each domain has a demand to strengthen tamper-resistance by joining the system to execute our cross-referencing method. Therefore, CCNs with stop failure are assumed to be repaired quickly. It is reasonable to assume that only a small number of CCNs experience stop failure.
Figure 13: Simulation results of probability density function of $R$ when $m = 10$ and $\alpha = 2$

Figure 14: Simulation results of probability density function of $R$ when $m = 100$ and $\alpha = 2$
Figure 15: Simulation results of probability density function of $R'$ when $m = 10$, $X = 10\%$, and $\alpha = 2$

Figure 16: Simulation results of probability density function of $R'$ when $m = 100$, $X = 10\%$, and $\alpha = 2$
Figure 17: Simulation results of probability density function of $R'$ when $m = 10$, $X = 30\%$, and $\alpha = 2$

Figure 18: Simulation results of probability density function of $R'$ when $m = 100$, $X = 30\%$, and $\alpha = 2$
5. Conclusion

We explained our previously proposed cross-referencing method for enabling multiple domains of P2P networks to manage their own blockchains and periodically exchange among each other the state of the latest confirmed block in the blockchain with the hysteresis signatures. We also discussed the design of a communication protocol that autonomously executes our cross-referencing method among the domains.

We theoretically evaluated the effectiveness of our method from three viewpoints: decentralization, scalability, and tamper-resistance. The method improved the scalability and tamper-resistance of a blockchain while reducing the degradation of decentralization. With our method, in-domain decentralization is equivalent to a usual public blockchain, such as Bitcoin, while out-domain decentralization is preserved by hysteresis signatures, which affects some blocks.

We theoretically formulated the Bitcoin scalability problem by estimating the transaction processing capacity per second $G(\tau)$. We found that $G(\tau_{fork})$ is the optimal upper bound of transaction processing capacity in Bitcoin-type blockchain systems, which is the cause of the scalability problem. We demonstrated that our cross-referencing method can break through the limit of the upper bound and capacity can reach the same level as that of the credit card company VISA, Ltd. in theory. It is highly possible that the transaction processing capacity will exceed that of VISA credit card system, but the actual performance needs to be evaluated experimentally, which is left for our future work. The effectiveness of our method was examined from the theoretical perspective by defining two tamper-resistance-improvement ratios $R$ and $R'$. As the number of domains increases, the peaks of the distributions of $R$ and $R'$ generally shift toward the higher position. We confirmed that the dispersion of this distribution increases as the number of domains increases. We assumed that the hash rate obeys a Pareto distribution, and the comparison of the scale parameter of the Pareto distribution $\alpha = 2, 3$ showed that as $\alpha$ decreases, the peak of the distribution of $R$ shifts to the smaller position. The proposed system of 1,000 domains are 3-10 times more tamper-resistant than that of 100 domains, and the capacity is 10 times higher. We also estimated the effect of stop failures on tamper resistance in the entire system. We showed that $R$ improves but
R′ does not. However, as the number of domains increases, the peak of the histograms in R and R′ shift toward the higher position, so performance degradation due to stop failures is relatively small.

We are currently developing a program of CCNs for conducting experimental evaluations on our cross-referencing method as a reference implementation of the communication protocol between CCNs. The program is open to the public at our Github website [22]. We will present experimental results elsewhere.

For future work, we will determine if our cross-referencing method can be successfully applied to various situations, for example, when the starting CCN for cross-referencing is not fixed and frequently changes and when multiple cross-referencing requests are sent from multiple domains at the same time. It is also important to consider security issues in Layer 0 where CCNs can share block information securely with each other.

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