We present a 3-D hydro + cascade model, in which viscosity and a realistic freeze-out process for the hadronic phase are taken into account. We compare our results to experimental data and discuss the final state interaction effects on physical observables.

1. FREEZEOUT PROCESS AND VISCOSITY IN HYDRODYNAMICS

Hydrodynamic models have been very successful in describing the collective behavior of matter at RHIC, such as single particle spectra and elliptic flow. In particular the strong elliptic flow which, for the first time, reaches the hydrodynamic limit at RHIC, provides us with a new understanding of the nature of the quark-gluon plasma (QGP) created at RHIC as strongly interacting or correlated QGP [1]. However there exist a number of experimental observations that contradict perfect hydrodynamic models: transverse momentum spectra above 2 GeV, elliptic flow at large pseudo-rapdities $\eta$ and Hanbury Brown - Twiss (HBT) interferometry. These observations suggest that there exist limitations to the application of a simple perfect hydrodynamic model to RHIC physics and that an improvement on a perfect hydrodynamic model is needed in order to obtain a comprehensive and unified description of the data from the point of view of hydrodynamics.

In general, hydrodynamic models require initial conditions, an equations of state (EoS) and freezeout conditions as parameters besides the relativistic hydrodynamic equation. One of the main advantages of the hydrodynamic model lies in its ability to investigate the relation between the EoS and physical observables via comparison to experimental data. However, it has been pointed out repeatedly, e.g. by Hirano [2], that details of the treatment of the freeze-out process can have large effects on physical observables. For example, the data on $P_T$ spectra and elliptic flow show that the assumption of chemical equilibrium [3] or partial chemical equilibrium [4] in the freezeout process is not realistic [2]. In addition, studies of collective flow at AGS, SPS and RHIC based on both of hydrodynamic models and cascade models suggest that the effects of viscosity are not negligible [5]. Therefore we construct a hybrid 3-D hydro + cascade model to include a realistic treatment of the freezeout process and viscosity in the hadronic phase. Such hybrid models have been implemented in the past, however with reduced dimensionality in the hydrodynamic component and thus with restrictions to observables at mid-rapidity.

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†This work was supported by DOE grants DE-FG02-05ER41367 and DE-FG02-03ER41239.
As a cascade model we use UrQMD in which final state interactions are included correctly.  

2. 3-D HYDRO + CASCADE MODEL

2.1. 3-D hydrodynamic model

We solve the relativistic hydrodynamic equation,
\[ \partial_\mu T^{\mu\nu} = 0, \]
where \( T^{\mu\nu} \) is energy momentum tensor, including baryon number conservation,
\[ \partial_\mu (n_B(T, \mu) u^\mu) = 0. \]
We modify our original code for the hydrodynamic expansion in Cartesian coordinates to that in light-cone coordinates \((\tau, x, y, \eta)\) \((\tau = \sqrt{t^2 - z^2})\), in order to optimize our hydrodynamic model for ultra-relativistic high energy heavy collisions. We employ Lagrangian hydrodynamics which has the following advantages over Eulerian hydrodynamics: 1) We can trace the adiabatic path of each volume element of fluid on phase diagram. 2) We can easily investigate effects of phase transition on physical observables.

We assume that hydrodynamic expansion starts at \( \tau_0 = 0.6 \) fm. Initial energy density and baryon number density are parameterized by
\[ \epsilon(x, y, \eta) = \epsilon_{\text{max}} W(x, y; b) H(\eta), \]
\[ n_B(x, y, \eta) = n_{B_{\text{max}}} W(x, y; b) H(\eta), \]
where \( b \) and \( \epsilon_{\text{max}} \) \((n_{B_{\text{max}}})\) are the impact parameter and the maximum value of energy density (baryon number density), respectively. \( W(x, y; b) \) is given by a combination of wounded nuclear model and binary collision model and \( H(\eta) \) is given by \( H(\eta) = \exp \left[ - (|\eta| - \eta_0)^2 / 2\sigma_\eta^2 \cdot \theta(|\eta| - \eta_0) \right] \). Parameters \( \epsilon_{\text{max}}, n_{B_{\text{max}}}, \eta_0 \) and \( \sigma_\eta \) are listed in Table 1. We set initial flow in the longitudinal direction to \( v_L = \eta \) (Bjorken’s solution) and \( v_T = 0 \) in the transverse plane. We use an equation of state with a 1st order phase transition, namely a Bag model EoS with and excluded volume correction. The thermal freezeout temperature is 110 MeV.

2.2. 3-D hydro + cascade model

The key improvements of the hybrid model approach compared to the 3-D hydrodynamic model are the inclusion of viscosity in the hadronic phase as well a realistic treatment of final state interactions and a species dependent freezeout process. For the transition from hydrodynamic model to UrQMD we introduce the switching temperature \( T_{SW} \) which should be chosen below but near the critical temperature \((T_c = 160 \) MeV\)). We set the switching temperature to 150 MeV. The calculated procedure is as follows: first hadron distributions from the hydrodynamic model are calculated at the switching temperature via the Cooper-Frye formula. Second, initial conditions for UrQMD on an event by event basis are generated from the hadron distributions via the hydrodynamic expansion through a Monte Carlo implementation. Finally the UrQMD calculation starts with these initial conditions. In this scheme we neglect the reverse contribution from UrQMD to the hydrodynamic model. For an initial condition for 3-D hydro + UrQMD we assume the same parameterization as that of pure hydrodynamic model except the
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value of maximum energy density, which we have adjusted to achieve a better agreement to experimental data. The parameters are summarized in Table 1.

Table 1
Parameters for initial conditions of pure hydro and hybrid model.

|              | $\tau_0$ (fm/c) | max. of $\epsilon$ (GeV/fm$^3$) | max. of $n_{B\text{max}}$ (fm$^{-3}$) | $\eta$ | $\sigma_\eta$ |
|--------------|-----------------|-------------------------------|-----------------|--------|-------------|
| pure hydro   | 0.6             | 55                            | 0.15            | 0.5    | 1.5         |
| hydro + UrQMD| 0.6             | 43                            | 0.15            | 0.5    | 1.5         |

3. RESULTS and DISCUSSIONS

Figure 1 shows the $P_T$ spectra of $\pi$, $K$, and $p$ at Au + Au $\sqrt{s} = 200$ GeV central collisions based on 3-D hydro + UrQMD. Our calculated results are very close to PHENIX data [13] up to $P_T \sim 2$ GeV. In 3-D hydro + UrQMD model the hadron ratio $\pi/p$ and $\pi/K$ are obtained correctly. In the case of the pure 3-D hydrodynamic model with low thermal freezeout temperature under the assumption of chemical equilibrium, we fail to obtain the correct hadron ratios [9]. This observation emphasizes the importance of the realistic treatment for the freezeout process, but further studies are necessary to investigate the effects of cross-over transition as well.

Figure 2 shows the centrality dependence of $P_T$ spectra of $\pi^+$. We can see that in peripheral collisions the difference between experimental results and calculated results appears at lower $P_T$ compared to central collisions. These deviations are indicative of the diminished importance of the soft, collective, physics described by hydrodynamics compared to the contribution of jet-physics in these peripheral events.

![Figure 1. $P_T$ spectra for $\pi^+$, $K^+$, $p$ at central collisions with PHENIX data [13].](image1)

![Figure 2. Centrality dependence of $P_T$ spectra of $\pi^+$ with PHENIX data [13].](image2)

Figure 3 shows the centrality dependence of the pseudorapidity distribution of charged hadrons together with PHOBOS data [14]. The impact parameters are set to $b = 2.4, 4.5, 6.3$ and $7.9$ fm for 0-6 %, 6-15 %, 15-25 % and 25-35 %, respectively. Our results agree with the experimental data not only at mid rapidity but also at large rapidity.

Figure 4 shows the mean $P_T$ as a function of rapidity for $\pi$, $K$ and $p$. The difference between open symbols (hadronic decays without rescattering) and solid symbols (full
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Figure 3. Centrality dependence of rapidity distribution with PHOBOS data [14].

hadronic rescattering) determines the reaction dynamics of the final state interactions. We find that the average transverse momentum $\langle P_T \rangle$ for $\pi$ decreases whereas the proton $P_T$ increases. Since the number of $\pi$ is larger compared to that of $p$, the effect on the protons per particle is larger than for the pions. This transfer of transverse momentum is commonly referred to as the "pion-wind".

In summary, we have presented a fully 3-D implementation of a hybrid hydro+micro transport model. The model includes a realistic treatment of hadronic freeze-out as well as an implementation of viscous effects in the hadronic phase. We have compared single-particle spectra and pseudo-rapidity distributions to data and have studied the reaction dynamics of protons, pions and kaons in the hadronic phase of heavy-ion collisions at RHIC.

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