Streambank Erosion Prediction

Azlinda Saadon1,*  Jazuri Abdullah2, Junaidah Ariffin3

1Faculty of Civil Engineering, Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia.
2Faculty of Civil Engineering, Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia.
3Faculty of Civil Engineering, Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia.

*Corresponding author: azlindasaadon@uitm.edu.my

Abstract. Bank instability as a result of flow fluctuations may lead to massive bank erosions and subsequent damage of adjacent properties. Continuous erosion process promotes change in the river morphology, sedimentation problems due to the presence of secondary currents and local scouring at piers downstream of the erosion point. Knowledge on the extent of erosion should facilitate river engineers to resolve issues on river training works and river sedimentation problems. A study has been carried out in the field to quantify the amount of eroded materials using erosion pins that were driven into the groundnormal to the bank surface. The erosion pins consist of 6 mm diameter metal rods and 60 cm – 80 cm in length. A spatial variation profile for rates of erosion has been identified with units expressed in unit cm per day. The defining parameters for bank erosion rates have included near-bank velocity, Ub, water depth, Y, stream bank geometry and soil bearing capacity. Development of empirical equations had used multiple linear and nonlinear regression techniques to determine the significant erosion predictors. It takes into consideration the coefficient of determination (r-squared) and Root-mean square error (RSME) as determinants for best predictors. Accuracy of developed equations is measured using the discrepancy ratio, D.R. This is the ratio of predicted to measured erosion rate. Analysis suggest that the equation derived using polynomial function (order-2) gave better accuracy compared to the equation derived using linear and power functions. An accuracy of 75% has been obtained. Scatter plots of the predicted to the measured erosion rates have shown to be between 0.5 – 2.0 within the line of good agreement.

1. Introduction
Streambank erosion is the intense loss of bank surface layer under the influence of the hydrodynamic force of water impacting the bank surface. The major factors of streambank erosion are climatic, hydraulic, vegetative, geologic, and human [1]. Bank erosion studies are key processes in river morphodynamics affecting channel morphology, floodplain evolution and associated habitat development [2] causing damage to riparian lands and infrastructure [3]; and mobilizes sediments that impacts turbidity, nutrient level and became source of contaminants to the water ecosystem. The morphological change in the river planform involves complex interaction between fluid dynamics and bank erosion [4]. The dynamic process that takes place in a channel involves flow convergence onto the concave banks and flow divergence near the convex banks cause banks to retreat or advance. Bed degradation near concave banks is due to the momentum of the flow redistribution while deposition stabilizes the convex banks. The occurrences of erosion result in bank retreat while the convex bank
advances with the building-up of point bars. These processes of aggradation and degradation of banks will continue until the banks reach critical bank heights and enter the threshold stage through mass wasting and channel widening. The process of bank collapses due to the instability of the bank material commonly treated under bank erosion studies consist of both fluvial entrainment and geotechnical instability, while the deposition and transportation of the collapsed bank material are often treated in sediment transport studies. Further to this, continuous process of bank erosion could be a crucial threat to major scouring of hydraulic structures located at the upstream, downstream, or adjacent point of erosion.

Streambank erosion promotes river morphological change and recognized as one of the main causes for serious flooding. The excessive deposition of materials may result in the shallowing of rivers that reduces the river capacity. Overbank flow results with intense and continuous rainfall. Hence, the monitoring and prediction of streambank erosion is essential in managing and sustaining the whole river system. The aim of this study is to derive a streambank erosion prediction equation using data measured in the field that makes use of erosion pin method of measurements. Development of the empirical equations had used two techniques: multiple linear regression and non-linear regression techniques. Accuracy of prediction is measured using the discrepancy ratio, which can be defined as the ratio of predicted to the measured value.

2. Bank erosion studies

Analytical approach provides an alternative method for streambank erosion prediction. Pioneer work related to riverbank erosion by [5] uses a simple two-dimensional shallow water flow model to derive an equation to predict channel shift. The rate of bank erosion adopted as proportional to the excess of near-bank depth-averaged stream wise flow velocity over the mean value of the cross-section. This study suggests that bank retreats when the excess near-bank velocity exceeded zero; otherwise, the bank advances. The same approach has been adopted and improved by [6]. An equation for bank erosion inclusive of channel migration rate derived theoretically from sediment continuity equation accounting various processes associated with bank erosion. This is the same theory derived by [5] of which bank erosion assumed proportional to the excess of near-bank depth-averaged streamwise flow velocity over the mean cross-sectional velocity. As a result, a generalized universal bank erosion coefficient derived based on the channel shift data of several Japanese rivers. The universal bank erosion coefficient based on the type of bank material (sandy or clayey).

An approach quantifying the rate of bank erosion incorporating both basal erosion and bank failures attempted using integration includes the effects of hydraulic force, bank geometry, bank material properties and probability of bank failure [7]. Theoretically, any stable channel undergoes bank erosion as their natural adjustment in dynamic equilibrium. Alluvial channels adjust themselves to reach regime conditions through degradation, aggregation of riverbed, width adjustment and planform evolution. The newly derived analytical model includes the analyses of basal erosion, bank failure and probability of bank failure. This model classified as a detail model compared to others and it accounts both basal erosion and bank failure in quantifying the actual rate of bank erosion. A methodology predicting the probability of the presence and absence in a river section derived by quantifying the amount eroded at the locations vulnerable to erosion [8]. The statistical tool based on a series of independent local variables employing logistic regression showing results of potential eroded areas in probability forms. Empirical relationships of streambank erosion prediction based on hydraulic characteristic and streambank geometry derived by [9]. Uniformity of the dimension and classification of the independent variables to the dependent variable using dimensional analysis aid in identifying the most significant parameters to bank erosion. Ten (10) equations were initially derived using multiple nonlinear regression (power function). The developed models yield model correlation coefficient (r-squared) values ranging from 0.365 to 0.479 with the best performing equation yielded 60% accuracy. Further analysis conducted to enhance the previous developed equations, using multiple linear, nonlinear and logarithmic transformation function [9]. The best predictor yields model r-squared of 0.513. However, the logarithmic transformation model yields accuracy of 93.5%. More advance method using Nonlinear Autoregressive eXogenous using QR factorization method (NARX-QR) employed in the development on riverbank erosion rate predictor for natural channels [10].
Nonlinear analysis was employed based on the time-series NARX-QR simulation. The sensitivity analysis between independent variables and dependent variable yields significant correlation for the nonlinear functions.

2.1. Measurement methods using erosion pins.

The erosion pin method is one of the most reliable methods to observe small changes in the ground surface level resulting from erosion and deposition. This method is suitable for short timescales of up to a year measurement. The use of pins is economical compared to other methods and is suitable for the measurement of bank retreat caused by fluvial actions and soil failure [11]. The erosion pins must be installed, and monitoring must be made at least after every flood event or if there is evidence of high flows. Measurement process can be challenging in conditions when the water level is high. Generally, any decrease in erosion pin length termed as ‘Ground Advance’ while any increase in erosion pin length termed as ‘Ground Retreat’. It is to note that the advances and retreats of the ground may occur from recurring expansion and contraction of the soil surface due to heating, cooling, wetting and drying. Most studies encourage the use of metal pins, but metals have some disadvantages in their uses under humid region as they are easily corroded. However, they offer several benefits with regards to their economic benefits for use in large fluvial environment with acceptable accuracy. A schematic diagram on the arrangement of erosion pins for erosion measurements shown in Figure 1. Arrangement of the erosion pins modified after Kuehn (2015) is as shown in Figure 2 below. The accuracy of fieldwork measurements can be further improved following the precautions and preventions below [12]:

i. Erosion pins should be in small diameter (around 5-6mm) metal rods with lengths between 500-600 mm. Shorter pins are not recommended as they may be subjected to disturbance from soil creep and trampling. On the other hand, shorter pins cause less disturbance while nailing them into the soil especially in erodible areas.

ii. Erosion pins should not be driven deep below the bank surface to avoid being buried as a result of deposition especially in areas where there is rapid erosion of the streambank area.

iii. Record of the eroded length should be at least semi-annually and after every flood and rainfall event. Care should be observed not to trample or touch the erosion pins and the surrounding area.

3. Study area

Location of the study area lies between 3°40’45.9” N (latitude), 101°31’24.16” E (longitude) and 3°40’44.46” N (latitude), 101°31’22.8” E (longitude) along Bernam River (Sg. Bernam). The river flows through Tanjung Malim, Perak along the border of Selangor (north of Sabak Bernam District) and south of Perak. The fieldwork site is located approximately 200 m upstream of the confluence of Sg. Inki. The area identified as actively undergoing the process of erosion evidenced through observation in the field. Basis of selection is based on the following criteria: (1) steep, high and nearly vertical banks; (2) absence of woody bank vegetation; (3) proximity to the streamflow, water level and rainfall measuring station; and (4) evidence of bank erosion. Through field observation, Sg. Bernam categorized as banks with active erosion. Various modes of streambank erosions observed at the left and right banks. Physical observation indicates that the reach underwent streambank erosion processes due to hydraulic action from fluvial entrainment and mass failure categorized by undercutting and scouring. The reach length for the study area is 50 meters with six transects at every interval of ten meters.
Figure 1. Schematic diagram on the arrangement of erosion pins for erosion measurements.

Figure 2. Erosion pins installed on each vertical streambank, modified from [11].

3.1. Fieldwork investigation on streambank erosion
The fieldwork investigations on streambank erosion uses conventional erosion pins each of 60 cm in length and 6 mm in diameter, are driven perpendicular to the streambank face, for about 50 cm deep, leaving 10 cm of the rod exposed and coated, on the streambank face. Six transect locations have been identified for the erosion pin array installation. More number of pins installed at critical points for better representation of the eroded profiles. Records of erosion depth measurements recorded for two weeks. The values of erosion depths represented as erosion rates by dividing the erosion depth with number of days. Figure 3 illustrates the array of erosion pins installation at study site.
3.2. Sensitivity analysis on streambank erosion predictor

The streambank erosion predictors comprise of three main classes namely the hydraulic characteristics (near-bank velocity $U_b$, water depth $Y$ and flow $Q$), and streambank geometry (side slope angle $\beta$, length $HL$ and height $H_b$). The sensitivity of each variable tested against the streambank erosion rate $\xi$ (cm/day) as the dependent variable. Significance of each independent variable was measured using the correlation coefficient ($r$-squared) and Root-mean square error (RSME). Table 1 shows the list of dependent variable and other independent variables used in this study.

| Variables           | Description                  | Symbol | Unit   |
|---------------------|------------------------------|--------|--------|
| Dependent variable  | Streambank erosion rate      | $\xi$  | cm/day |
|                     | Near-bank velocity $U_b$      | $U_b$  | m/s    |
|                     | Water depth                  | $Y$    | m      |
| Independent variable| Streambank angle $\beta$      | $\beta$|        |
|                     | Streambank length $HL$       | $HL$   | m      |
|                     | Streambank height $H_b$      | $H_b$  | m      |

Table 2 provides the summary of the observed erosion depths (cm), average erosion depth (cm), average erosion rates (cm/day) and near-bank velocity (m/s). Observation in week 1 (11/9/2018) on the average erosion rates features a parabolic function (polynomial relationship) with respect to the near-bank velocity. The erosion depth at transect 5 yields the highest amount of erosion depth at 4.90 cm with a magnitude of 0.11 cm per day from the day of the pin installation. Physical observation confirmed mass failure occurred at transect 5. This transect has a curve-shaped outer bank which promotes the acceleration of hydraulic pulses hitting on the streambank. The maximum near-bank velocity recorded at this transect is 0.90 m/s. Transect 4 exhibits erosion depth amounting to 4.50 cm with a magnitude of 0.20 cm per day. Transect 4 is exposed to cracks on the streambank face. Transect 2 recorded 3.5 cm of erosion depth with a magnitude of 0.08 cm per day. Scouring was not apparent at the outer bank of this transect.
Table 2. Summary of erosion depth (cm), average erosion depth (cm), and erosion rates (cm/day) to the near-bank velocity (m/s) for week 1 dated 11/9/2018.

| Date    | Transect | Erosion Depth, (cm) | Average Erosion Depth (cm) | Erosion rate, $\zeta$ (cm/day) | Near-bank Velocity, $U_b$ (m/s) |
|---------|----------|----------------------|----------------------------|-------------------------------|---------------------------------|
| 11/9/2018 | 1        | 1.80                 | 0.60                        | 0.05                          | 0.70                            |
|         | 2        | 3.50                 | 0.88                        | 0.08                          | 0.60                            |
|         | 3        | 0.40                 | 0.20                        | 0.02                          | 0.50                            |
|         | 4        | 4.50                 | 2.25                        | 0.20                          | 0.90                            |
|         | 5        | 4.90                 | 1.23                        | 0.11                          | 0.20                            |
|         | 6        | 0.50                 | 0.50                        | 0.05                          | 0.15                            |

The polynomial relationship yields a degree of determination ($r^2$-squared) value of 0.746 for the prediction of erosion rate within this bank. The scatter plots in Fig. 4 suggest a polynomial relationship as follows:

$$y = 0.7345 x^2 - 0.6223 x + 0.1605$$  

where, $y$ is the erosion rate in cm/day; $x$ is the near-bank velocity in m/s.

Figure 4. Average erosion rate against near-bank velocity for week 1.

Table 4 shows the relationship between erosion depths (cm), average erosion depth (cm), average erosion rate (cm/day) and near-bank velocity (m/s) for two weeks of fieldwork observations for week 2. Based on the field observation on week 2 (23/9/2018), the erosion rate shows a declining parabolic function (polynomial relationship) to the near-bank velocity.

Table 3. Summary of erosion depth (cm), average erosion depth (cm), and erosion rates (cm/day) to the near-bank velocity (m/s) for week 2 dated 23/9/2018.

| Date    | Transect | Erosion Depth, (cm) | Average Erosion Depth (cm) | Erosion rate, $\zeta$ (cm/day) | Near-bank Velocity, $U_b$ (m/s) |
|---------|----------|----------------------|----------------------------|-------------------------------|---------------------------------|
| 23/9/2018 | 1        | 7.70                 | 1.93                        | 0.16                          | 0.56                            |
The erosion depth at transect 2 yields the highest amount of erosion depth at 13.10 cm with a magnitude of 0.27 cm per day from the day of the pin installation. Through physical observation, prolonged scouring at this transect could be one of the factors in the incremental value of erosion depth. The variation of water depths during high flow (rainy day) to low flow (normal condition) could be the factor influencing the erosion depth. Transect 5 recorded 11.90 cm depth of erosion, with a magnitude of 0.33 cm per day. The erosion depth at transect 5 increased from 4.90 cm to 11.90 cm. The depth of erosion increased by 7 cm (58.9%). This transect have a curve shaped outer bank which promotes the acceleration of hydraulic pulses hitting on the streambank. The bank deposited and the edge of the bank have collapsed. Transect 1 recorded value of 7.70 cm of erosion depth and Transect 4 recorded a value of 6.60 cm of erosion depth. The erosion depth at Transect 4 increased from week 1 (4.5 cm) to 6.6 cm in week 2. Highest value of near-bank velocity is recorded at Transect 2 (0.62 m/s). This is due to the scouring at this transect which promotes the flow in vortex motion. This can also influence the scouring of the outer bank hence the value of erosion depth. The polynomial relationship yields degree of determination (r-squared) value of 0.746 for the prediction of erosion rate within this bank. The scatter plot in Figure 5 develops a polynomial relationship which can be used as a predictor to streambank erosion rate (cm/day) as follows:

\[ y = -2.6765x^2 + 2.2425x - 0.2085 \]  

(2)

Figure 6 to Figure 13 provide results of the sensitivity analysis for water depth, \( Y \), streambank angle, \( \beta \), streambank length, \( H_l \), and streambank height, \( H_b \). Table 4 shows the coefficient correlation and the mathematical function between the independent variables to the dependent variable. Based on the correlation coefficient through sensitivity analysis, the streambank angle, \( \beta \) yields highest positive correlation to the streambank erosion rates at 0.822, followed by near-bank velocity, \( U_b \) at accuracy of 0.746, and water depth, \( Y \) at accuracy of 0.725.

| Transact | Erosion Depth (cm) | Magnitude (cm/day) |
|----------|--------------------|--------------------|
| 2        | 13.10              | 0.27               |
| 3        | 3.20               | 0.07               |
| 4        | 6.60               | 0.18               |
| 5        | 11.90              | 0.33               |
| 6        | 5.80               | 0.24               |

Figure 5. Average erosion rate against near-bank velocity for week 2.
Figure 6. Average erosion rate against water depth for week 1.

Figure 7. Average erosion rate against water depth for week 2.

Figure 8. Average erosion rate against bank angle for week 1.

Figure 9. Average erosion rate against bank angle for week 2.

Figure 10. Average erosion rate against streambank length for week 1.

Figure 11. Average erosion rate against streambank length for week 2.
streambank length for week 1.

![Average Erosion Rate (cm/day) against Streambank Height (m) for Week 1](image1)

**Figure 12.** Average erosion rate against streambank height for week 1.

streambank length for week 2.

![Average Erosion Rate (cm/day) against Streambank Height (m) for Week 2](image2)

**Figure 13.** Average erosion rate against streambank height for week 2.

**Table 4.** Summary of sensitivity analysis between independent variables and dependent variable.

| Dependent Variable | Independent Variables | Symbol | Correlation Coefficient |
|--------------------|-----------------------|--------|-------------------------|
| Streambank erosion Rates, $\xi$ | Near-bank velocity | $U_b$ | Week 1: 0.746 Week 2: 0.222 |
| Streambank erosion Rates, $\xi$ | Water depth | $Y$ | Week 1: 0.725 Week 2: 0.650 |
| Streambank erosion Rates, $\xi$ | Streambank angle | $\beta$ | Week 1: 0.822 Week 2: 0.237 |
| Streambank erosion Rates, $\xi$ | Streambank length | $H_L$ | Week 1: 0.309 Week 2: 0.296 |
| Streambank erosion Rates, $\xi$ | Streambank height | $H_b$ | Week 1: 0.404 Week 2: 0.239 |

4. Streambank erosion prediction using statistical method

Two regression models are employed in the development of a new streambank equation, namely, multiple linear regression and multiple nonlinear regression. The multiple nonlinear regression used to develop equations in the form of polynomial-order-2 function and power function. The polynomial-order-two is a function similar to a quadratic function with the highest order of two. Equation 3 shows the general form of a linear relationship between independent variables to the dependent variable.

$$ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_i x_i $$  \hspace{1cm} (3)

where, $y$ is the dependent variable; $x_1, x_2, x_3$ and $x_i$ are independent variables; $\beta_0, \beta_1, \beta_2, \beta_3$ and $\beta_i$ and constants generated from regression analysis.

Equation 4 shows the general form of a polynomial-order-2 (quadratic) function between independent variables to the dependent variable.

$$ y = \beta_0 x_1^2 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_2^2 + \beta_4 x_2 + \beta_5 + \ldots + \beta_i x_i^2 + \beta_j x_i + \beta_k $$ \hspace{1cm} (4)

where, $y$ is the dependent variable; $x_1, x_2$, and $x_i$ are independent variables; $\beta_0, \beta_1, \beta_2, \beta_3$ and $\beta_i$ and constants generated from polynomial-order-2 regression analysis.

Equation 5 shows the general form of a power function between independent variables to the dependent variable.

$$ y = \beta_0 \times x_1^{\beta_1} \times x_2^{\beta_2} \times x_3^{\beta_3} \times \ldots \times x_i^{\beta_i} $$ \hspace{1cm} (5)
where, \( y \) is the dependent variable; \( x_1, x_2, \) and \( x_i \) are independent variables; \( \beta_0, \beta_1, \beta_2, \beta_3 \) and \( \beta_i \) and constants generated from power regression analysis.

### 4.1. Multiple linear regression

Using data obtained from fieldwork observation, regression analysis performed to obtain a new equation that yields value of correlation coefficient (\( r^2 \)) of 0.602. Table 5 shows the model summary for multiple regression analysis and Table 6 shows the coefficients of independent variables.

#### Table 5. Model summary for multiple linear regression analysis

| Model | R  | R Square | Adjusted R-Square | Std. Error of the Estimate |
|-------|----|----------|-------------------|---------------------------|
| 1     | 0.776\( ^a \) | 0.602 | 0.271 | 0.08462 |

The multiple linear regression analysis yields the following equation in a linear form (Equation 6):

\[
y = 0.178 + (0.252 \times U_b) + (0.602 \times Y) + (0.004 \times \beta) + (0.143 \times H_L) + (-0.333 \times H_b)
\]

(6)

where, \( y \) is streambank erosion rate in cm/day; \( U_b \) is near-bank velocity in m/s; \( Y \) is the water depth in m, \( \beta \) is the streambank angle; \( H_L \) is the streambank length in m; \( H_b \) is the streambank height in m.

#### Table 6. Coefficients of independent variables

| Model | Unstandardized Coefficients | Standardized Coefficients | t | Sig. |
|-------|-----------------------------|---------------------------|---|-----|
|       | \( \beta \) Standard Error | \( \beta \) Standard Error |     |     |
| (Constant) | 0.178 | 1.020 | - | 0.174 | 0.867 |
| \( U_b \) | 0.252 | 0.153 | 0.555 | 1.640 | 0.152 |
| \( Y \) | 0.602 | 0.311 | 0.557 | 1.939 | 0.101 |
| \( \beta \) | 0.004 | 0.008 | 0.306 | 0.453 | 0.667 |
| \( H_L \) | 0.143 | 0.814 | 0.492 | 0.176 | 0.866 |
| \( H_b \) | -0.333 | 0.751 | -1.128 | -0.444 | 0.673 |

### 4.2. Multiple nonlinear regression using Polynomial-Order-2 function

Multiple nonlinear regression using polynomial-order-2 function has been proposed due to the high value of correlation coefficient in the previous analysis. The \( r^2 \) generated from this model is relatively higher compare to the multiple linear regression model. However, only 3 independent variables are selected for this analysis, namely near-bank velocity, water depth and streambank angle. Table 7 shows the model summary for the nonlinear regression using polynomial-degree-two function. Table 8 shows the list of coefficients generated by SPSS as the weights to each of the independent variables.

#### Table 7. Model summary for multiple nonlinear regression (Polynomial-order-2 function) using SPSS

| Source | Sum of Squares | df | Mean Squares |
|--------|---------------|----|-------------|
| Regression | 0.351 | 9 | 0.039 |
| Residual | 0.016 | 3 | 0.005 |
| Uncorrected Total | 0.366 | 12 |
Corrected Total: 0.108

Dependent variable: DV

R squared = 1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = 0.855.

**Table 8.** List of coefficients for independent variables.

| Parameter         | Estimate | Standard Error | Lower Bound | Upper Bound |
|-------------------|----------|----------------|-------------|-------------|
| Near-bank velocity, $U_b$ | $a$ | -0.320 | 3.061 | -10.062 | 9.422 |
|                    | $b$ | 0.231 | 1.130 | -7.60 | 1.60 |
|                    | $c$ | 163265.5  | 0.055 | 163265.3 | 163265.7 |
| Water depth, $Y$  | $d$ | 7.210 | 0.892 | 4.371 | 10.049 |
|                    | $e$ | -0.5917 | 0.000 | -5.918 | -5.916 |
|                    | $f$ | -97249.4 | 1.130 | -97249 | 97249 |
| Streambank angle, $\beta$ | $g$ | 0.001 | 3.013 | -9.588 | 9.589 |
|                    | $i$ | -66011.2 | 0.000 | -66011.2 | -66011.2 |

The multiple nonlinear regression analysis yields the following equation in a polynomial-order-2 form (Equation 7):

\[
y = -0.320 U_b^2 + 0.231 U_b + 163265.6 + 7.210 Y^2 - 5.917 Y - 97249.5 + 0.001 \beta^2 - 0.099 \beta - 66011.2
\]

where, $y$ is streambank erosion rate in cm/day; $U_b$ is near-bank velocity in m/s; $Y$ is the water depth in m and $\beta$ is the streambank angle.

**4.3. Multiple nonlinear regression using Power function.**

The r-squared generated from this model was relatively low compare to the model using multiple linear regression and polynomial-order-2 function with an r-squared value greater than 50%. Table 9 shows the model summary for the nonlinear regression using power function. Table 10 shows the coefficients of all independent variables.

**Table 9.** Model summary for multiple nonlinear regression (Power function).

| Source               | Sum of Squares | df | Mean Squares |
|----------------------|----------------|----|--------------|
| Regression           | 0.316          | 6  | 0.053        |
| Residual             | 0.050          | 6  | 0.008        |
| Uncorrected Total    | 0.366          | 12 |              |
| Corrected Total      | 0.108          | 11 |              |

Dependent variable: DV

R squared = 1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = 0.537.

**Table 10.** List of coefficients for independent variables.

| Parameter         | Estimate | Standard Error | Lower Bound | Upper Bound |
|-------------------|----------|----------------|-------------|-------------|
| Near-bank velocity, $U_b$ | $a$ | -0.320 | 3.061 | -10.062 | 9.422 |
|                    | $b$ | 0.231 | 1.130 | -7.60 | 1.60 |
|                    | $c$ | 163265.5  | 0.055 | 163265.3 | 163265.7 |
| Water depth, $Y$  | $d$ | 7.210 | 0.892 | 4.371 | 10.049 |
|                    | $e$ | -0.5917 | 0.000 | -5.918 | -5.916 |
|                    | $f$ | -97249.4 | 1.130 | -97249 | 97249 |
| Streambank angle, $\beta$ | $g$ | 0.001 | 3.013 | -9.588 | 9.589 |
|                    | $i$ | -66011.2 | 0.000 | -66011.2 | -66011.2 |
The multiple nonlinear regression analysis using power function yields the following equation (Equation 8).

\[ y = 0.001 \times (U_b)^{0.743} \times (Y)^{1.140} \times (\beta)^{2.641} \times (H_L)^{11.818} \times (H_b)^{-16.032} \]

where, \( y \) is streambank erosion rate in cm/day; \( U_b \) is near-bank velocity in m/s; \( Y \) is the water depth in m; \( \beta \) is the streambank angle; \( H_L \) is the streambank length in m; \( H_b \) is the streambank height in m.

5. Model Accuracy and Performance

The accuracy of the developed equations is measured using discrepancy ratio. Discrepancy ratio (D.R.) is the ratio of the predicted values to the measured values. These values are deemed accurate if the data lie within 0.5 – 2.0 limit. Figure 14, Figure 15 and Figure 16 show the plots of measured erosion rate (cm/day) to the predicted erosion rate (cm/day) for both regression models. Table 9 shows the predicted values and discrepancy ratio calculated using both Equation 1 and Equation 2. The D.R. is translated into percentage (%) and was assigned as yes if the D.R. value lie within the range of 0.5 – 2.0. The number of “Yes” indicates the accuracy of the predicted value in comparison to the measured value. Based on the validation, Equation 1 yields accuracy at 67%. The total number of data lies within 0.5 to 2.0 if 8 out of 12 data. This equation predicted well and in good standing having accuracy of more than 60%. The scatter plot of predicted value against measured values is shown in Figure 14. Three data over predicted the streambank erosion rate and one data under predict the streambank erosion rate. The trend appears as spasmodic but acceptable at 60% accuracy.

Three models generated from statistical analysis using multiple linear regression method and multiple nonlinear regression method. Model 1 (Equation 6) yields an empirical equation in the form of linear function. The performance of this model tested using D.R. and the model predicted the streambank erosion rates of 67% accuracy when compared to the measured data. Model 2 (Equation 7) yields accuracy of 75%, which relatively higher than Model 1. Trend of the scatter plots (Figure 15) are congruent and in good agreement with the measured data. Meanwhile the performance of Model 3 (Equation 8) developed using multiple nonlinear regression (power function) had also yielded accuracy of 75% when compared to the measured data. Figure 17 shows the trend of scatter plots of Model 3 (power function). It can be concluded that Model 2 and Model 3 yield better prediction than Model 1. The validation analysis is very important for future model testing to reaffirm the capability of the equations to predict streambank erosion rates at different river sampling.
Figure 14. Plot of predicted streambank erosion rate (cm/day) and measured streambank erosion rate (cm/day) for Model 1 using multiple linear regression analysis.

Figure 15. Plot of predicted streambank erosion rate (cm/day) and measured streambank erosion rate (cm/day) for Model 2 using multiple nonlinear regression analysis (polynomial-order-2 function).
Based on the visual evaluation of the scatter plots for all 3 models, Model 2 gives a better representation of the predicted values against the measured values. The scatter plot for Model 2 is congruent and in good agreement with the measured data. Hence, the model correlation coefficient (r-squared) of the model yields 0.855, the highest among all models. Table 11 shows the summary of the best predictor and its performances based on discrepancy ratio.

Table 11. Summary of model performance for Model 2.

| Regression Method | Model Correlation Coefficient | Model Performance | D.R (%) |
|-------------------|-------------------------------|-------------------|---------|
| Multiple nonlinear (Polynomial order-2 function) | 0.855 | $y = -0.320 U_b^2 + 0.231 U_b + 163265.6 + 7.210 Y^2 - 5.917Y - 97249.5 + 0.001 \theta^2 - 0.099 \theta - 66011.2$ | 75 % |

6. Conclusions
The newly empirical expressions to predict streambank erosion rates have been successfully developed in this study using multiple linear and nonlinear regression approaches. The defining parameters for streambank erosion rates have included near-bank velocity, water depth, streambank angle, streambank length, and streambank height. The significance of the three mathematical models were evaluated using the correlation coefficient (r-squared) and mean-squared errors. Models 1, 2 and 3 yield r-squared value of 0.6, 0.855 and 0.537 respectively. Analysis suggest that Model 2 that was derived using polynomial functions (order-2) gave better accuracy compared to the other two models which were derived using linear and power functions. Accuracy of 75% obtained for both Model 2
using polynomial functions (order-2) and power functions (Model 3) however, the graphical presentation of the scatter plots suggest that Model 2 provides better accuracy with prediction within the line of agreement. Scatter plot of Model 3 is more spasmodic in comparison to Model 2. The empirical models in this study were derived based on local streambank erosion data, which can be further improved with more observations. The sensitivity analysis and model accuracy confirmed that this prediction model can be used at another site to predict the streambank erosion. The relationships between measured variables yield strong positive correlation. Nevertheless, the empirical equation was limited to five (5) measured variables in quantifying the streambank erosion rate, namely, near-bank velocity, water depth, streambank angle, streambank length and streambank height. Further studies needed to evaluate various factors affecting the streambank erosion rate.

References

[1] Klausmeyer KJ 2016 Streambank Erosion retrieved from Natural Resources Conservation.
[2] Rinaldi M, Mengoni B, Luppi L, Darby SE, and Mosselman E 2008 Numerical simulation of hydrodynamics and bank erosion in a river bend, Water Resources Research 44(9).
[3] Simon A, Curini A, Darby SE, and Langendoen EJ 2000 Bank and near-bank processes in an incised channel, Geomorphology 35(3-4) pp. 193–217.
[4] Duan JG and Julien PY 2010 Numerical simulation of meandering evolution, Journal of Hydrology 391(1-2) pp. 34–46.
[5] Ikeda S, Parker G, Sawai K 1981 Bend theory of river meanders; Part I, linear development, Fluid Mech J 112:363-377.
[6] Hasegawa K 1989 Universal bank erosion coefficient for meandering rivers, Hydra Eng J ASCE 115:744-765.
[7] Duan JG 2005 Analytical approach to calculate rate of bank erosion, Hydra Eng J 131(11):980–990.
[8] Varouchakis EA, Giannakis GV, Lilli MA, Ioannidou E, Nikolaidis NP, and Karatzas GP 2016 Development of a statistical tool for the estimation of riverbank erosion probability, Soil J 2:1-11.
[9] Saadon, A., Abdullah, J., Ariffin, J. 2016 Dimensional analysis relationships of streambank erosion rates, Jurnal Teknologi (Sciences and Engineering) 78:5-5 pp. 79 - 85.
[10] Saadon A, Abdullah J, Muhammad NS, and Ariffin J 2020 Development of riverbank erosion rate predictor for natural channels using NARX-QR factorization model: a case study of Sg. Bernam, Selangor, Malaysia, Neural Computing and Applications, in press.
[11] Saadon A, Abdullah J, and Ariffin J 2016 Streambank erosion prediction for natural river channels, International Journal of Applied Environmental Sciences, 11, 5, pp. 1273 - 1284.
[12] Kuehn E 2015 Erosion pin array monitoring, Stream bank erosion trends and sediment contributions in a south-western Missouri river, 1-143.

Acknowledgments
The authors appreciate the support from the Ministry of Science, Technology and Innovation (MOSTI), through Science Fund (Project Number: 06-01-01-SF0773). Additional funding to the second author, provided by the Ministry of Education, through the Fundamental Research Grant Scheme (Grant No. FRGS/1/2018/TK10/UITM/02/5) is also acknowledged.