Theoretical study of magnetism induced by proximity effect in a ferromagnetic Josephson junction with a normal metal

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We theoretically study the magnetism induced by the proximity effect in the normal metal of ferromagnetic Josephson junction composed of two s-wave superconductors separated by ferromagnetic metal/normal metal/ferromagnetic metal junction (S/F/N/F/S junction). We calculate the magnetization in the N by solving the Eilenberger equation. We show that the magnetization arises in the N when the product of anomalous Green’s functions of the spin-triplet even-frequency odd-parity Cooper pair and spin-singlet odd-frequency odd-parity Cooper pair in the N has a finite value. The induced magnetization $M(d, \theta)$ can be decomposed into two parts, $M(d, \theta) = M^I(d) + M^{II}(d, \theta)$, where $d$ is the thickness of N and $\theta$ is superconducting phase difference between two Ss. Therefore, $\theta$ dependence of $M(d, \theta)$ allows us to control the amplitude of magnetization by changing $\theta$. The variation of $M(d, \theta)$ with $\theta$ is indeed the good evidence of the magnetization induced by the proximity effect, since some methods of magnetization measurement pick up total magnetization in the S/F/N/F/S junction.

1. Introduction

In a superconductor/normal metal (S/N) junction, the pair amplitude of Cooper pair in the S penetrates into the N. This is called the proximity effect.\(^1\) One of crucial phenomena generated by the proximity effect is Josephson effect, which is characterized by DC current flowing without a voltage-drop between two Ss separated by normal metal (N).\(^1,\)\(^2\) The Josephson critical current in the S/N/S junction monotonically decreases with increasing the thickness of N .\(^1,\)\(^2\)

The proximity effect in s-wave superconductor/ferromagnetic metal (S/F) hybrid junctions provides interesting phenomena which are not observed in S/N hybrid junctions and thus has been extensively studied both theoretically and experimentally.\(^3\)\(-\)\(^22\) In a S/F junction, due to the proximity effect, spin-singlet Cooper pairs (SSCs) penetrate into the F. Because of the exchange splitting of the electronic density of states for up and down electrons, the SSC in the F acquires the finite center-of-mass momentum, and the pair amplitude shows a damped oscillatory behavior with the thickness of the F.\(^20\)\(-\)\(^22\) Interesting phenomenon induced by the damped oscillatory behavior of the pair amplitude is a $\pi$-state in a S/F/S junction, where the current-phase relation in the Josephson current is shifted by $\pi$ from that of the ordinary S/I/S and S/N/S junctions (called 0-state).\(^3\)\(-\)\(^22\)
In addition, the proximity effect in $S/F$ hybrid junctions also generates the spin-triplet odd-frequency even-parity Cooper pairs (STCs) in $Fs$, although the $S$ is an $s$-wave superconductor.\textsuperscript{22) Here, the anomalous Green's functions of spin-triplet components are odd functions with respect to the Fermion Matsubara frequency $\omega_n$, i.e., $f(\hat{p}_F, -\omega_n) = -f(\hat{p}_F, \omega_n)$ and even function with respect to the direction of Fermi momentum $\hat{p}_F = \hat{p}_F/|\hat{p}_F|$, i.e., $f(-\hat{p}_F, \omega_n) = f(\hat{p}_F, \omega_n)$ ($\hat{p}_F$ is Fermi momentum). It should be noted that the anomalous Green's functions in bulk $s$-wave superconductors are generally even functions with respect to $\omega_n$ and $\hat{p}_F$. When the magnetization in the $F$ is uniform in a $S/F$ junction, the STC composed of opposite spin electrons (i.e., total spin projection on $z$ axis being $S_z = 0$) and SSCs penetrates into the $F$ due to the proximity effect.\textsuperscript{22,23) The penetration length of STC with $S_z = 0$ and SSC as described above into the $F$ is very short and the amplitude of STC exhibits a damped oscillatory behavior inside the $F$ with increasing the thickness of $F$. The penetration length is determined by $\xi_F = \sqrt{\hbar D_F/h_{\text{ex}}}$, which is typically a order of few nanometers.\textsuperscript{3–17,20–22) Here, $D_F$ and $h_{\text{ex}}$ are the diffusion coefficient and the exchange field in the $F$, respectively.

Moreover, when the magnetization in the $F$ is non-uniform in a $S/F$ junction, the STC formed by electrons of equal spin ($|S_z| = 1$) can also be induced in the $F$. This includes cases, for instance, where the $F$ contains a magnetic domain wall,\textsuperscript{24–30) the junction consisting of multilayers of $Fs$,\textsuperscript{31–46) or spin active interface at the $S/F$ interface.\textsuperscript{47–54) The penetration length of STC with spin $|S_z| = 1$ is determined by $\xi_T = \sqrt{\hbar D_F/2\pi k_B T}$ in $Fs$ ($T$ is temperature).\textsuperscript{18) This is approximately 2 orders of magnitude larger than the propagation length of the SSC. Thus, the proximity effect of the STC with $|S_z| = 1$ is called the long-range proximity effect (LRPE).

The LRPE can be observed by the measurement of Josephson critical current in ferromagnetic Josephson junctions with $Fs$. Because the Josephson critical current carried by STCs with $|S_z| = 1$ shows monotonically decrease as a function of the thickness of $F$ and its decay length of STC is about determined by $\xi_T$. The Josephson current carried by the STCs firstly has been predicted theoretically\textsuperscript{24,33,47) and confirmed experimentally.\textsuperscript{55–58) Instead of measurement of Josephson current, earmark of STC has been observed by measuring the superconducting transition temperature ($T_C$) as a function of polar angle of magnetization between $F1$ and $F2$ in spin valves.\textsuperscript{59–61) As well as the detection of STC by measuring the Josephson critical current and $T_C$ as mentioned above, the spin-dependent transport of STC in $S/F$ hybrid junctions gives a direct evidence of spin in the STC.\textsuperscript{22,29,62–68) One of the simplest quantity induced by the spin of the STC is a magnetization induced by the proximity effect of STC.\textsuperscript{22,29,62,65,67) In $F/S$ junctions, the STC is also induced to $S$ by the inverse proximity effect.\textsuperscript{69–73) The magnetization induced by spin of STC is a good fingerprint to establish the existence of STC. $F/S/F$ junctions called
spin valve are a typical junction to obtain the magnetization induced by the STC.\textsuperscript{22,62,63)}

To detect the magnetization induced by the STC, recently, a rather complex ferromagnetic Josephson junction composed of two $S$'s separated by $F/F/S/F/F$ junction\textsuperscript{65)} is theoretically proposed in the diffusive transport region. The amplitude of the magnetization in this ferromagnetic Josephson junction can be freely controlled by tuning the superconducting phase difference ($\theta$). The variation of magnitude of magnetization by tuning $\theta$ can be detected by using the magnetic sensor based on SQUID.\textsuperscript{74)} On the other hand, in the ballistic transport region and weak non-magnetic impurity scattering transport region, the understanding of magnetic transport in the STC is also important in the ferromagnetic Josephson junction, since recent nanofabrication techniques have given the ballistic transport region to ferromagnetic Josephson junctions.\textsuperscript{8)}

In this paper, we calculate the magnetization induced by the proximity effect in the $N$ of a simple ferromagnetic Josephson junction composed of two $S$'s separated by $F1/N/F2$ junction in ballistic transport region and weak non-magnetic impurity scattering transport region. The present $S/F1/N/F2/S$ junction is indeed suitable to control the magnetization between two $F$s by using a weak external magnetic field, since the $N$ can reduce the exchange coupling between two $F$s.\textsuperscript{74)} For this purpose, we solve the Eilenberger equation in the quasiclassical Green’s function theory. Assuming that the magnetization in $F2$ is fixed along the $z$ direction perpendicular to the junction direction ($x$ direction), we show that i) the $x$ component of the magnetization in the $N$ is always zero, ii) the $y$ component becomes exactly zero when the magnetizations in $F1$ and $F2$ are collinear, and iii) the $z$ component is generally finite for any magnetization alignment between $F1$ and $F2$. We find that the magnetization in the $N$ is composed of $\theta$ dependent magnetization and $\theta$ independent magnetization. The origin of these magnetizations is due to the spin-triplet even-frequency odd-parity Cooper pair and spin-singlet odd-frequency odd-parity Cooper pair induced by the proximity effect. The magnetization is suppressed with decreasing the non-magnetic impurity scattering time. Moreover, the $\theta$ dependence of magnetization disappears with increasing the thickness ($d$) of $N$. The disappearance of $\theta$ dependence of magnetization is because $\theta$ dependent magnetization shows the rapid decrease with increasing $d$ compared with the $\theta$ independent magnetization.

The rest of this paper is organized as follows. In Sec. 2, we introduce a simple $S1/F1/N/F2/S2$ Josephson junction and formulate the magnetization induced by the proximity effect in the $N$ of this junction by solving Eilenberger equation in the ballistic transport region and the transport region in the case of presence of weak non-magnetic impurity scattering. The property of anomalous Green’s function in SSC and STC is discussed in the present system. In Sec. 3, the numerical results of magnetization are given. Moreover, the behavior of magnetization with thickness of $N$ is discussed. Finally, the magnetization induced by the proximity effect is estimated for a typical set of realistic parameters in Sec. 4. The summary
of this paper is given in Sec. 5.

2. Magnetization in normal metal induced by proximity effect in an S1/F1/N/F2/S2 junction

2.1 Set up of S1/F1/N/F2/S2 junction

As depicted in Fig. 1, we consider the S1/F1/N/F2/S2 junction made of normal metal (N) sandwiched by two layers of ferromagnetic metal (F1 and F2) attached to s-wave superconductors (Ss). We assume that the magnetization in F2 is fixed along the z direction, while the F1 is a free layer in which the magnetization can be controlled by an external magnetic field, pointing any direction in the yz plane, parallel to the interfaces, with ϕ being the polar angle of the magnetization. The uniform magnetizations are assumed in both F1 and F2 layers.

The thicknesses of S, F1, F2, and N, respectively, with L = d + dF1, LF = L + dF2, and LS = LF + dS.

2.2 Anomalous Green’s function in normal metal

To obtain the analytical form of anomalous Green’s function, we employ the linearized Eilenberger equation. The linearized Eilenberger equation can be applied to T/T_C ≈ 1, since the amplitude of anomalous Green’s function becomes small in this case. Moreover, it is expected that the amplitude of anomalous Green’s function becomes due to the mismatch of the Fermi surfaces between F and S, since F has the exchange splitting between up and down Fermi surfaces. Therefore, the linearization of Eilenberger equation can be suitable to the
present system.

In the ballistic transport region and the weak non-magnetic impurity scattering transport region, the magnetization inside the N induced by the proximity effect is evaluated by solving the linearized Eilenberger equation in each region $m$ (= $F1$, $N$, and $F2$):

$$i\hbar\tilde{p}_F V_F \partial_x \hat{f}^m(\vec{r}) + i2\text{sgn}(|\omega_n|)\hbar|\omega_n|\hat{f}^m(\vec{r}) - 2\hat{\Delta}(x) + \tilde{h}^y_{\text{ex}}(x) \begin{pmatrix} \sigma_y, \hat{f}^m(\vec{r}) \end{pmatrix} + \tilde{h}^z_{\text{ex}}(x) \begin{pmatrix} \sigma_z, \hat{f}^m(\vec{r}) \end{pmatrix},$$

+ isgn($\omega_n$)$\frac{\hbar}{\tau} \hat{f}^m(\vec{r}) = \hat{0}, \quad \text{(1)}$

where $\vec{r} = (x, \tilde{p}_F, \omega_n)$, $\tilde{p}_F = p_{F_x}/p_F = \cos \chi$, $\chi$ is the angle between Fermi momentum ($p_F$) and $x$ axis, $p_{F_x}$ is the $x$ component of Fermi momentum, and $\omega_n$ is the Fermion Matsubara frequency. $\tau$ is the non-magnetic impurity scattering time. We assume that $\tau$ has a finite value only inside the $N$. $\hat{f}^m(\vec{r})$ is given by

$$\hat{f}^m(\vec{r}) = \begin{pmatrix} f^m_{1\uparrow}(\vec{r}) & f^m_{1\downarrow}(\vec{r}) \\ f^m_{1\downarrow}(\vec{r}) & f^m_{1\uparrow}(\vec{r}) \end{pmatrix},$$

$$= \begin{pmatrix} -f^m_{1\downarrow}(\vec{r}) + if^m_{1\uparrow}(\vec{r}) & f^m_{1\uparrow}(\vec{r}) + f^m_{1\downarrow}(\vec{r}) \\ -f^m_{1\uparrow}(\vec{r}) + f^m_{1\downarrow}(\vec{r}) & f^m_{1\downarrow}(\vec{r}) + if^m_{1\uparrow}(\vec{r}) \end{pmatrix}. \quad \text{(2)}$$

The $s$-wave superconducting gap $\hat{\Delta}(x)$ is finite only in the $S$ and assume to be constant, i.e.,

$$\hat{\Delta}(x) = \begin{pmatrix} 0 & -\Delta_1 \\ \Delta_1 & 0 \end{pmatrix}, \quad -d_S < x < 0$$

$$\begin{pmatrix} 0 & -\Delta_2 \\ \Delta_2 & 0 \end{pmatrix}, \quad L_F < x < L_S$$

$$0, \quad \text{other} \quad \text{(3)}$$

where $\Delta_1(2) = \Delta e^{i\theta_1(2)}$ ($\Delta$: real) and $\theta_1(2)$ is the superconducting phase in the $S1$ ($S2$) (see Fig. 1). The exchange field $\tilde{h}_{\text{ex}}(x) = (h_{\text{ex}}^x(x), h_{\text{ex}}^y(x), h_{\text{ex}}^z(x))$ due to the ferromagnetic magnetizations in the $Fs$ is described by

$$\tilde{h}_{\text{ex}}(x) = \begin{pmatrix} h_{\text{ex}}^x \tilde{e}_y + h_{\text{ex}}^z \tilde{e}_z, \quad 0 < x < d_{F1} \\ h_{\text{ex}} 2 \tilde{e}_z, \quad L < x < L_F \quad \text{(4)}$$

$$0, \quad \text{other}$$

where $h_{\text{ex}}^y = h_{\text{ex}} \sin \varphi$, $h_{\text{ex}}^z = h_{\text{ex}} \cos \varphi$, and $\varphi$ is the polar angle of the magnetization in the $F1$ (see Fig. 1).

To obtain the solutions of Eq. (1), we impose appropriate boundary conditions:

$$\hat{f}^S(\vec{r}) \bigg|_{x=0} = \hat{f}^{F1}(\vec{r}) \bigg|_{x=0}, \quad \text{(5)}$$
Substituting Eq. (15) into Eq. (1), we can obtain the equations:

\[ f^F_1(\vec{r}) \bigg|_{x=d_{F_1}} = f^N(\vec{r}) \bigg|_{x=d_{F_1}}, \]

\[ f^N(\vec{r}) \bigg|_{x=L} = f^{F_2}(\vec{r}) \bigg|_{x=L}, \]

and

\[ f^{F_2}(\vec{r}) \bigg|_{x=L_{F_2}} = f^S(\vec{r}) \bigg|_{x=L_{F_2}}. \]

In the present calculation, we apply the rigid boundary condition \( \sigma_F/\sigma_S \ll \xi_{F_1(2)}/\xi_S \) (\( \sigma_S(F) \) is the conductivity of \( S(F) \) in the case of normal state, \( \xi_S = h \nu_F/2\pi k_B T \) is the superconducting coherence length, and \( \xi_{F_1(2)} = h \nu_F/2\hbar \Delta_{ex1(2)} \)) and we assume that \( d_S \) is much larger than \( \xi_S \). In this case, the anomalous Green’s function in the S1(S2) attached to the \( F_1(2) \) can be approximately given by

\[ f^S(\vec{r}) \big|_{x=0(L_F)} = -\frac{\Delta}{\sqrt{(\hbar \omega)^2 + \Delta^2}}, \]

Assuming \( d_{F_1}/\xi_{F_1} \ll 1 \), we can perform the Taylor expansion of \( f^F_1(\vec{r}) \) with \( x \) as follows,

\[ \hat{f}(\vec{r}) \approx f^F_1(d_{F_1}, \tilde{\nu}_F, \omega_n) + (x - d_{F_1})\partial_x f^F_1(d_{F_1}, \tilde{\nu}_F, \omega_n), \]

Applying the boundary condition of Eq. (5) to Eq. (12) and then substituting Eq. (12) into Eq. (1), we can approximately obtain \( f^F_1(\vec{r}) \) as

\[ f^F_1(\vec{r}) \approx f^F_1(d_{F_1}, \tilde{\nu}_F, \omega_n) + \frac{x - d_{F_1}}{d_{F_1}} \left[ f^F_1(d_{F_1}, \tilde{\nu}_F, \omega_n) - \hat{F}^{S_1} \right], \]

where,

\[ f^F_1(d_{F_1}, \tilde{\nu}_F, \omega_n) = \left[ 1 - \text{sgn}(\omega_n) \frac{2\hbar |\omega_n| d_{F_1}}{\tilde{\nu}_F \nu_F} \right] \hat{F}^{S_1} + \frac{\hbar \nu_F}{\tilde{\nu}_F \nu_F} \left\{ \sigma_{y, \Delta \nu} \hat{F}^{S_1} \right\}. \]

For \( f^{F_2}(\vec{r}) \), by performing the Taylor expansion with \( x \) to assume \( d_{F_2}/\xi_{F_2} \ll 1 \), applying Eq. (8) to \( f^{F_2}(\vec{r}) \), and then substituting the obtained \( f^{F_2}(\vec{r}) \) into Eq. (1), \( f^{F_2}(\vec{r}) \) is approximately given by

\[ f^{F_2}(\vec{r}) \approx f^{F_2}(L, \tilde{\nu}_F, \omega_n) - \frac{x - L}{d_{F_2}} \left[ f^{F_2}(L, \tilde{\nu}_F, \omega_n) - \hat{F}^{S_2} \right], \]

where

\[ f^{F_2}(L, \tilde{\nu}_F, \omega_n) = \left[ 1 + \text{sgn}(\omega_n) \frac{2\hbar |\omega_n| d_{F_2}}{\tilde{\nu}_F \nu_F} \right] \hat{F}^{S_2} - i \frac{\hbar \nu_F}{\tilde{\nu}_F \nu_F} \left\{ \sigma_z, \hat{F}^{S_2} \right\}. \]

Next, to solve Eq. (1) in the \( N \), we consider the solution \( f^N(\vec{r}) \) of Eq. (1) as a sum of symmetric \( (f^S_N(\vec{r})) \) and antisymmetric \( (f^A_N(\vec{r})) \) parts with \( \tilde{\nu}_F \) as follows

\[ f^N(\vec{r}) = f^A_N(\vec{r}) + f^S_N(\vec{r}). \]

Substituting Eq. (15) into Eq. (1), we can obtain the equations:

\[ f^S_N(\vec{r}) = -\text{sgn}(\omega_n) \frac{\nu_F}{2|\omega_n|} \partial_x \hat{f}^S_N(\vec{r}). \]
and
\[ \partial_s^2 \hat{f}_S^N(\vec{r}) - k_N^2 \hat{f}_S^N(\vec{r}) = \hat{0}, \tag{17} \]
where \( k_N = 2|\omega_n|/v_F|\hat{p}_F| \) and \(|\omega_n| = |\omega_n| + 1/2\tau \). By solving Eq. (17) and then using Eq. (16) to obtain \( \hat{f}_A^N(\vec{r}) \), the general solution of \( \hat{f}^N(\vec{r}) \) becomes
\[ \hat{f}^N(\vec{r}) = \left[ 1 - \text{sgn}(\omega_n)\hat{p}_F \frac{v_F k_N}{2|\omega_n|} \right] \hat{A}^N e^{k_N x} + \left[ 1 + \text{sgn}(\omega_n)\hat{p}_F \frac{v_F k_N}{2|\omega_n|} \right] \hat{B}^N e^{-k_N x}. \tag{18} \]

To determine arbitrary matrix coefficients \( \hat{A} \) and \( \hat{B} \), we apply Eqs. (6) and (7) to Eq. (18). As a result, anomalous Green’s functions in the \( N \) are found as
\[ f_x^N(\vec{r}) = f_1^N(\vec{r}) + f_2^N(\vec{r}), \tag{19} \]
\[ f_1^N(\vec{r}) = -\frac{i}{E_{\omega_n} \sinh(k_N d)} \sinh[k_N(x - L)] \]
\[ + \frac{\Delta_1}{E_{\omega_n} \sinh(k_N d)} \sinh[k_N(x - d_{F1})], \tag{20} \]
\[ f_2^N(\vec{r}) = \frac{\text{sgn}(\omega_n)}{\hat{p}_F} \frac{2\Delta_1 |\omega_n| d_{F1}}{v_F E_{\omega_n} \sinh(k_N d)} \sinh[k_N(x - L)] \]
\[ + \frac{\text{sgn}(\omega_n)}{\hat{p}_F} \frac{2\Delta_2 |\omega_n| d_{F2}}{v_F E_{\omega_n} \sinh(k_N d)} \sinh[k_N(x - d_{F1})], \tag{21} \]
\[ f_{1y}^N(\vec{r}) = \frac{1}{\hat{p}_F} \frac{2h_{ex}^y d_{F1} \Delta_1}{h v_F E_{\omega_n} \sinh(k_N d)} \sinh[k_N(x - L)], \tag{22} \]
and
\[ f_{1z}^N(\vec{r}) = \frac{1}{\hat{p}_F} \frac{2h_{ex}^z d_{F1} \Delta_1}{h v_F E_{\omega_n} \sinh(k_N d)} \sinh[k_N(x - L)] \]
\[ + \frac{1}{\hat{p}_F} \frac{2h_{ex}^z d_{F2} \Delta_2}{h v_F E_{\omega_n} \sinh(k_N d)} \sinh[k_N(x - d_{F1})]. \tag{23} \]
The anomalous Green’s function of spin-singlet Cooper pair in Eq. (19) is given by the sum of the spin-singlet even-frequency even-parity Cooper pair given by Eq. (20) and the spin-singlet odd-frequency odd-parity Cooper pair given by Eq. (21). It should be noticed that the spin-singlet odd-frequency odd-parity Cooper pair in Eq. (21) contributes to the magnetization \( (M_{y(z)}(d, \theta)) \) induced by the proximity effect, since \( M_{y(z)}(d, \theta) \propto \text{sgn}(\omega_n) \text{Im}[f_x^N(\vec{r})f_{y(z)}^N(\vec{r})] \) (see Eq. (25)). Moreover, it is immediately found that Eqs. (22) and (23) indicate the spin-triplet even-frequency odd-parity Cooper pair. These results are summarized in Table I. The general classification of symmetry of anomalous Green’s functions is given by Ref[78]. Here, it should be noted that \( f_{1y}^N(x) = 0 \) because the exchange field in the \( F1 \) does not have the \( x \) component. Therefore, the \( x \) component of the magnetization in the \( N \) is always zero, as discussed below.
Table I. The symmetry of anomalous Green’s function given by Eqs. (19)-(23) in the $N$ of the $S1/F1/N/F2/S2$ junction. Where $\omega_n$ is the Fermion Matsubara frequency and $\tilde{p}_F = \cos \chi$. $\chi$ is the angle between the Fermi momentum and the $x$ axis.

| Spin   | Frequency ($\omega_n$) | Parity ($\tilde{p}_F$) |
|--------|------------------------|------------------------|
| Singlet| Even                   | Even                   |
|        | Odd                    | Odd                    |
| Triplet| Even                   | Odd                    |

2.3 Magnetization in a normal metal

Within the quasiclassical Green’s function theory, the magnetization $M_{y(z)}(d, \theta)$ ($\theta : \text{superconducting phase difference between } S1 \text{ and } S2$) induced inside the $N$ is given by

$$\vec{M}(d, \theta) = (M_x(d, \theta), M_y(d, \theta), M_z(d, \theta)) = \frac{A}{V} \int_{dF_1}^{L} \vec{m}(x, \theta)dx,$$

where $\theta = \theta_2 - \theta_1$ is the superconducting phase difference between $S2$ and $S1$,

$$\vec{m}(x, \theta) = (m_x(x, \theta), m_y(x, \theta), m_z(x, \theta)) = g\mu_B \pi N_F k_B T \sum_{\omega_n} \text{sgn}(\omega_n) \int_{-1}^{1} d\tilde{p}_F \text{Im} \left[ f_s^N(\vec{r}) f_t^{N,*}(\vec{r}) \right],$$

and

$$f_t^N(x) = (f_{tx}^N(\vec{r}), -f_{ty}^N(\vec{r}), f_{tz}^N(\vec{r})).$$

Here, $\vec{m}(x, \theta)$ is the local magnetization density in the $N$ and $g$ is the $g$ factor of electron, $\mu_B$ is the Bohr magneton. $A$ and $V = Ad$ are the cross-section area of junction and the volume of $N$, respectively. In the quasiclassical Green’s function theory, the density of states $N_F$ per unit volume and per electron spin at the Fermi energy for up and down electrons in the spin polarized $N$ due to the proximity effect is assumed to be approximately the same.\(^{20-22}\) It should be noticed in Eq. (25) that $m_x(x, \theta)$ is always zero because $f_{tx}^N(x) = 0$ (see Sec. 2.2) and thus the $x$ component $M_x(d, \theta)$ of the magnetization is always zero. Therefore, in the following, we only consider the $y$ and $z$ components of the magnetization.

Substituting Eqs (19)-(23) into Eq. (25) and then performing the integration with respect to $x$ in Eq. (24), we can obtain the $y$ and $z$ components of magnetization in the $N$ induced by the proximity effect. The $y$ component of magnetization in the $N$ is given as

$$M_y(d, \theta) = M_y^I(d) + M_y^H(d, \theta),$$
where

\[ M^I_y(d) = 8g\mu_B\pi N_F \Delta^2 \frac{h_{ex}^y d_F^2}{(\hbar \nu_F)^2} \sum_{\omega_n > 0} \int_0^1 d\tilde{\nu}_F K_{\omega_n} (d, \tilde{\nu}_F), \] (28)

and

\[ M^{II}_y(d, \theta) = 8g\mu_B\pi N_F \Delta^2 \frac{h_{ex}^{y,1} d_F^2}{(\hbar \nu_F)^2} \sum_{\omega_n > 0} \int_0^1 d\tilde{\nu}_F R_{\omega_n} (d, \tilde{\nu}_F) \cos \theta. \] (29)

Here, we have introduced

\[ K_{\omega_n} (d, \mu) = \frac{h|\omega_n|}{|\tilde{\nu}_F E_{\omega_n} \sinh(k_N d)|^2} \frac{\sinh(2k_N d) - 2k_N d}{4k_N d} \] (30)

and

\[ R_{\omega_n} (d, \mu) = \frac{h|\omega_n|}{|\tilde{\nu}_F E_{\omega_n} \sinh(k_N d)|^2} \frac{\sinh(k_N d) - k_N d \cosh(k_N d)}{2k_N d}. \] (31)

From Eqs. (28) and (29), it is immediately found that \( y \) components \( M^I_y(d) \) and \( M^{II}_y(d, \theta) \) of magnetization is always zero for \( \varphi = 0 \) or \( \pi \), since \( M^I_y(d) \) and \( M^{II}_y(d, \theta) \) is proportional to \( h_{ex}^y = h_{ex1} \sin \varphi \). Similarly, the \( z \) component of magnetization in the \( N \) is decomposed into two parts,

\[ M_z(d, \theta) = M^I_z(d) + M^{II}_z(d, \theta), \] (32)

where

\[ M^I_z(d) = -8g\mu_B\pi N_F \frac{\Delta^2}{\beta(\hbar \nu_F)^2} \left[ h_{ex}^z d_F^2 + h_{ex2} d_F^2 \right] \sum_{\omega_n > 0} \int_0^1 d\tilde{\nu}_F K_{\omega_n} (d, \tilde{\nu}_F), \] (33)

and

\[ M^{II}_z(d, \theta) = -8g\mu_B\pi N_F \frac{\Delta^2}{\hbar \nu_F \hbar \nu_F} \left[h_{ex}^{z,1} d_F^2 + h_{ex2} d_F^2 \right] \sum_{\omega_n > 0} \int_0^1 d\tilde{\nu}_F R_{\omega_n} (d, \tilde{\nu}_F) \cos \theta. \] (34)

It is immediately found that from Eq. (33) \( M^I_z(d) \) is always zero when \( h_{ex}^z d_F^2 = -h_{ex2} d_F^2 \), whereas from Eq. (34), \( M^{II}_z(d, \theta) \) is always zero when \( h_{ex}^{z,1} = -h_{ex2} \).

3. Results

3.1 Magnetization-phase relation

Let us first numerically evaluate the amplitude of the magnetization in the \( N \) by using Eqs. (27) and (32), i.e.,

\[ M(d, \theta) = \sqrt{[M_y(d, \theta)]^2 + [M_z(d, \theta)]^2}. \] (35)

In order to perform the numerical calculation of \( M(d, \theta) \), the temperature dependence of \( \Delta \) is assumed to be \( \Delta = \Delta_0 \tanh(1.74 \sqrt{T_C/T - 1}) \), where \( \Delta_0 \) is the superconducting gap at zero temperature. The thickness of \( N, F1, \) and \( F2 \) is normalized by \( \xi_0 = \hbar \nu_F / 2\pi k_B T_C \). Figure 2
shows $M(d, \theta)$ normalized by $M_0 = g\mu_B N_F \Delta_0$ as a function of $\theta$ for different $\hbar/\tau$. It is found that $M(d, \theta)$ decreases with increasing $\hbar/\tau$ as shown in solid (black), dashed (red), and chain (blue) lines of Fig. 2. This reduction of $M(d, \theta)$ is due to the suppression of pair amplitude caused by the non-magnetic impurity scattering, i.e., finite $\hbar/\tau$ inside the $N$. The variation of $M(d, \theta)$ with respect to $\theta$ is the good fingerprint of magnetization induced by the proximity effect, since some experimental methods of magnetization measurement pick up magnetization of $F$s to be constant and $M(d, \theta)$. Moreover, it should be noticed that $M(d, \theta)$ has a periodicity of $2\pi$ as expressed by Eqs. (29) and (34).

Figure 3 shows the representative result of $M(d, \theta)$ normalized by $M(d, 0)$ as a function of $\theta$ for different thickness $d$ of $N$. From Fig. 3, it is immediately found that $\theta$ dependence of $M(d, \theta)$ gradually vanishes away with increasing $d$. For the thin thickness of $N$ as shown in the solid (black) line of Fig. 3, The amplitude of $M(d, \theta)$ exhibits the clear modulation as a
1.05

\( M(d, \theta) \) as a function of \( \theta \). With increasing \( d \) as shown in the dashed (red) line of Fig. 3, the amplitude of \( M(d, \theta) \) decreases but still we can find the modulation of \( M(d, \theta) \) with respect to \( \theta \). However, for the further increase of \( d \) as shown in the chain (blue) line of Fig. 3, the modulation of \( M(d, \theta) \) with respect to \( \theta \) is no longer acquired. Therefore, from these results, it is found that \( d \) is an important parameter to obtain \( M(d, \theta) \) controlled by changing \( \theta \). We will discuss \( d \) dependence of magnetization in the next subsection.

3.2 Thickness dependence of magnetization in normal metal

Let us now evaluate numerically the \( d \) dependence of magnetization induced by the proximity effect in the \( N \) by using Eqs. (27) and (32). Figure 4 shows the magnetization \( M_y(d, \theta) \) as a function of \( d \) for \( h/\tau = 0 \). Here, it should be noticed that we separately plot \( y \) and \( z \) component of magnetization (\( M_y(d, \theta) \) and \( M_z(d, \theta) \)) as a function of \( d \). At the same moment, \( M^{I}_{y(z)}(d) \) and \( M^{II}_{y(z)}(d, \theta) \), which are plotted by dashed (red) and chain (blue) lines are also indicated in Fig. 4. Figure 4 shows \( M_y(d, \theta) \) and \( M_z(d, \theta) \) as a function of \( d \) for \( h/\tau \Delta_0 = 0 \) and \( \varphi = \pi/2 \). From Figs. 4 (a) and (b), we find that \( M_y(d, \theta) \) and \( M_z(d, \theta) \) exhibits monotonically
It is also found that the damping rate of \( M_{y}^{I}(d, \theta) \) with \( d \) is remarkably weak compared with that of \( M_{y}^{II}(d, \theta) \) as shown in dashed (red) and chain (blue) lines of Figs. 4 (a) and (b). Therefore, when \( d/\xi_{0} \gg 1 \), the main contribution to \( M_{y}(d, \theta) \) arises from \( M_{y}^{I}(d, \theta) \), since \( M_{y}^{II}(d, \theta) \) is vanishingly small. This is the reason why the modulation of magnetization with \( \theta \) disappears for \( d/\xi_{0} \gg 1 \) as shown in Fig. 3. Moreover, it is immediately realized that the sign of \( M_{y}^{I}(d, \theta) \) and \( M_{y}^{II}(d, \theta) \) is different for any \( d \) from Fig. 4 (a) and (b).

Figure 5 shows \( M_{y}(d, \theta) \) and \( M_{z}(d, \theta) \) as a function of \( d \) for \( h/\tau\Delta_{0} = 5 \) and \( \varphi = \pi/2 \). As compared to Fig. 4 (a) and (b), Fig. 5 (a) and (b) indicate that the magnitude of magnetization is suppressed and the damping rate of magnetization with \( d \) is stronger as \( \tau \) decreases. For \( h/\tau\Delta_{0} = 5 \), \( M_{y}^{II}(d, \theta) \) is ignorable small around \( d/\xi_{0} = 1.5 \). Therefore, \( M_{y}(d, \theta) \) almost becomes independent with \( \theta \) for \( d/\xi_{0} > 1.5 \).
Fig. 5. (Color online) The magnetization of $y$ component $M_y(d, \theta)$ (a) and $z$ component $M_z(d, \theta)$ (b) as a function of $d$. Here we set $T/T_C = 0.3$, $\theta = 0$, $\varphi = \pi/2$, $h/\tau \Delta_0 = 5$, $d_{F1}/\xi_0 = 0.3$, $d_{F2}/\xi_0 = 0.2$, $h_{ex1}/\Delta_0 = 30$, $h_{ex2}/\Delta_0 = 20$. $\xi_0 = h v_F/2\pi k_B T_C$ and $M_0 = g \mu_B N_F \Delta_0$. We also plot $M_{y(z)}^I(d)$ and $M_{y(z)}^{II}(d, \theta)$, separately.

4. Discussions

Here, we shall discuss the $d$ dependence of $M_y(d, \theta)$ and $M_z(d, \theta)$ by using approximated formula of Eqs. (28)-(34). For $T \approx T_C$, $h/\tau = 0$, and $d/\xi_0 \gg 1$, the $y$ components $M_y^I(d)$ and $M_y^{II}(d, \theta)$ of magnetization are approximately given as

$$M_y^I(d) \approx 4g \mu_B N_F \Delta^2 \frac{d_{F1} k_B T y}{(h v_F)^2} \frac{\xi_0}{d}$$

and

$$M_y^{II}(d, \theta) \approx -4g \mu_B N_F \Delta^2 \frac{d_{F1} d_{F2} k_B T}{(h v_F)^2}
\times \frac{h_{ex}^y \left(1 - \frac{2\xi_0}{d}\right)}{d} \exp(-d/\xi_0) \cos \theta,$$

whereas the $z$ components $M_z^I(d)$ and $M_z(d, \theta)$ of magnetization are approximately given as

$$M_z^I(d) \approx -4g \mu_B N_F \Delta^2 \frac{k_B T}{(h v_F)^2}$$

$$M_z^{II}(d, \theta) \approx \frac{h_{ex}^z \left(1 - \frac{2\xi_0}{d}\right)}{d} \exp(-d/\xi_0) \cos \theta.$$
\[ \times \left[ h_{\text{ex}}^2 (d_{F1})^2 + h_{\text{ex}} (d_{F2})^2 \right] \frac{\xi_0}{d} \] (38)

and

\[ M_{z}^{\text{II}}(d, \theta) \approx 4g\mu_B \Delta^2 k_B T \left( \frac{d_{F1}d_{F2}}{(\hbar v_{F})^2} \right) \times \left( h_{\text{ex}}^2 + h_{\text{ex}} \right) \left( 1 - \frac{2\xi_0}{d} \right) \exp(-d/\xi_0) \cos \theta. \] (39)

From Eqs. (36)-(39), it is found that \( M_{y}^{\text{I}}(d) \) algebraically decreases as \( 1/d \), whereas \( M_{y}^{\text{II}}(d, \theta) \) exponentially decreases with \( d \). Moreover, from Eqs. (36)-(39), it is also found that the sign of \( M_{y}^{\text{I}}(d) \) and \( M_{y}^{\text{II}}(d, \theta) \) is always opposite. Here, it should be noticed that the coefficient \( (1-2\xi_0/d) \) of Eqs. (37) and (39) is always positive value, since \( \xi_0/d \) is much smaller than 1 in the present approximation. These results are indeed qualitatively consistent with numerical results as shown in Fig. 4 and 5.

Finally, we shall approximately estimate \( \xi_0 \) and the amplitude of magnetization in the \( N \). In clean normal metals, \( \xi_0 \) is in a range of several hundred nanometers. Therefore, the magnetization in the \( N \) induced by the proximity effect has a finite value in this length scale. The amplitude \( M \) of the magnetization is estimated to be of order \( g\mu_B N_F \Delta_0 \) (see Fig. 4 and 5). When we use a typical set of parameters, the amplitude \( M \) is approximately 100 A/m. It is expected that this value can be detected by the magnetization measurement by utilizing SQUID.

5. Summary

We have calculated the magnetization induced by the proximity effect in the \( N \) of the \( S1/F1/N/F2/S2 \) Josephson junction. Where it has been assumed that the magnetization of \( F1 \) is in the \( yz \) plane and the magnetization of \( F2 \) is fixed along with \( z \) axis. Based on the quasiclassical Green’s function theory in the ballistic transport region and the weak non-magnetic impurity scattering region, we have found that the magnetization in the \( N \) are induced by the emergence of the spin-triplet even-frequency odd-parity Cooper pair and spin-singlet odd frequency odd parity Cooper pair, which are induced by the proximity effect in the \( S1/F1/N/F2/S2 \) junction. We have shown that i) the \( x \) component of the magnetization in the \( N \) is always zero, ii) the \( y \) component is exactly zero when the magnetization direction between \( F1 \) and \( F2 \) is collinear, and iii) the \( z \) component is generally finite for any magnetization direction between \( F1 \) and \( F2 \).

We have found that the magnetization in the \( N \) can be controlled by tuning \( \theta \). This magnetization is suppressed with decreasing the relaxation time of the non-magnetic impurity scattering. Moreover, it has been found that \( \theta \) dependence of magnetization vanishes away when thickness \( d \) of \( N \) is much larger than \( \xi_0 \).
The magnetization induced in the $N$ can be decomposed into $\theta$ dependent and $\theta$ independent parts. We have shown that $\theta$ dependent magnetization rapidly decays with increasing $d$, whereas $\theta$ independent magnetization slowly decays with increasing the thickness of $N$. We have also found that the sign of $\theta$ independent and $\theta$ depend magnetizations are always opposite. This $\theta$ dependence of magnetization is important to confirm the existence of spin-triplet Cooper pair, since some experimental methods of magnetization measurement pick up net magnetization in the present ferromagnetic Josephson junction.

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