INTRODUCTION

Here we present results from three papers:

1. The requirements of sufficient inflation and microwave background anisotropy limits on the generation of density fluctuations place important constraints on inflationary models. The fine-tuning of parameters required in inflationary models can be quantified (Adams, Freese, and Guth 1991). Rolling fields need flat potentials: the ratio of height to (width) of the potential must be \( \chi = \left[ \Delta V / (\Delta \psi)^4 \right] \leq O(10^{-8}) \); for extended inflation, \( \chi \leq O(10^{-15}) \).

2. Double field inflation (Adams and Freese 1991) couples two scalar fields to obtain a time-dependent nucleation rate of true vacuum bubbles and thus a successful end to (old) inflation. Cosmic strings responsible for large-scale structure may form at the end of the inflationary phase transition, and bubble collisions may give rise to interesting structure.

3. We show (Freese, Frieman, and Olinto 1990) that a pseudo-Nambu–Goldstone boson, with a potential of the form \( V(\phi) = \Lambda^4 \left[ 1 + \cos \left( \phi / f \right) \right] \), can naturally give rise to an epoch of inflation in the early universe. The potential is flat, as required in Adams et al. (1991) without any fine-tuning. Successful inflation can be achieved if \( f \sim m_p \) and \( \Lambda \sim m_{\text{GUT}} \). Such mass scales arise in particle physics models with a large gauge group that becomes strongly interacting at a scale \( \sim \Lambda \), for example, as can happen in the hidden sector of superstring theories. The density fluctuation spectrum is non-scale-invariant, with more power on large length scales.

The inflationary universe model was proposed (Guth 1981) to solve several cosmological puzzles, notably the horizon, flatness, and monopole problems. During the inflationary epoch, the energy density of the universe is dominated by a (nearly constant) false vacuum energy term \( \rho = \rho_{\text{vac}} \) and the scale factor \( R(t) \) of the Universe expands exponentially: \( R(t) = R(t_i) e^{H(t_i - t)} \), where \( H = R/R \) is the Hubble parameter, \( H^2 = 8\pi G \rho / 3 - k / R^2 \) (\( = 8\pi G \rho_{\text{vac}} / 3 \) during inflation), and \( t_i \) is the time at the beginning of inflation. If the interval of exponential expansion satisfies \( t_{\text{end}} - t_i \geq 65H^{-1} \), a small causally connected region of the universe grows to a sufficiently large size to explain the observed homogeneity and isotropy of the universe today. In the process, any overdensity of magnetic monopoles produced at an epoch of grand unification is diluted to acceptable levels. The predicted grand unified theory (GUT)
abundance of monopoles is $\Omega_{\text{mon}} \approx 10^{12}$, whereas the energy density of our universe is observed to be within an order of magnitude of $\Omega = 1$; here the excess monopoles are simply "inflated away" beyond our visible horizon. Inflation predicts a geometrically flat universe ($k = 0$), $\Omega \equiv 8\pi G\rho_p/3H^2 \rightarrow 1$.

I begin with a discussion of the proposals and problems of the earliest inflationary models, now known as “old” inflation (Guth 1981) and “new” inflation (Linde 1982; Albrecht and Steinhardt 1982). The problems with these original models then lead into a discussion of new ideas proposed to circumvent some of these problems.

OLD INFLATION

In the model of old inflation, the universe passes through a first-order phase transition at a critical temperature $T_c$, for example, at the GUT epoch $T_c \approx 10^{14}$ GeV. Figure 1 plots the effective potential for the scalar field at three different temperatures. Above the critical temperature, that is, for $T > T_c$, the potential has only one minimum, at point A. The (expectation value of the) scalar field is situated at this minimum. At $T = T_c$, there are two equally energetic minima. Once the temperature drops below $T_c$, however, there is only one true minimum, at point B. The old minimum at point A is now called the false vacuum, and the new minimum at point B is now called the true vacuum. The universe wants to go from A to B, since B has lower energy, but is prevented from doing so by an energy barrier. The universe can get from the false vacuum to the true vacuum by either thermal fluctuations or vacuum tunneling. As illustrated in Figure 2, bubbles of true vacuum nucleate in a sea of false vacuum. In the inflationary model, the nucleation rate for vacuum tunneling (Voloshin, Kobzarev, and Okun 1975; Coleman 1977) is arranged to be very slow, so that the universe is trapped in the metastable false vacuum for a long time. The difference in energy density between points A and B is the vacuum energy density $\rho_{\text{vac}}$. While the universe is trapped in the false vacuum, this energy density dominates over matter and radiation energy, and the universe expands exponentially: $H^2 = 8\pi G\rho_{\text{vac}}/3 = \text{const}$ has the solution $R \propto e^{Ht}$. Sufficient inflation to solve the flatness, horizon, and monopole problems requires that the scale factor at the
FIGURE 2. Nucleation of true vacuum bubbles at first-order phase transition. (a) For $T > T_c$, the Universe is entirely in the false vacuum phase. (b) For $T < T_c$, bubbles of true vacuum start to nucleate in the sea of false vacuum.

end of inflation satisfy $R_{\text{end}} = 10^{27} R_{\text{begin}} = e^{65} R_{\text{begin}}$, where $R_{\text{begin}} = R(t_i)$ is the scale factor at the beginning of inflation; that is, approximately 65 e-foldings of inflation are required.

Although old inflation can successfully obtain these 65 e-foldings of expansion, the original model has the problem that it cannot reheat, that is, after the inflationary phase, the energy in the vacuum cannot be converted to ordinary radiation (Guth and Weinberg 1983). In a successful inflationary model, the period of exponential expansion must be followed by a return to a radiation-dominated epoch, so that the universe can subsequently evolve according to the standard model.

In old inflation, all of the latent heat of the phase transition is converted into kinetic energy of the bubble walls. In order to end up with a universe like ours, the energy somehow has to be extracted from the bubble walls into thermalizing the interior. One might hope that bubble collisions could achieve this thermalization. However, although the bubbles expand with the speed of light, the false vacuum background expands exponentially, and the bubbles cannot find one another and do not percolate and thermalize. Occasionally finite clusters of a few bubbles form; in this case, the largest bubble dominates the dynamics and again thermalization does not take place. Thus the final result of old inflation is the “Swiss cheese universe” of Figure 2(b), with many empty bubbles of true vacuum in a false vacuum sea. This is not our universe.

NEW INFLATION

Shortly after the realization that old inflation fails to reheat, “new” inflation was proposed (Linde 1982; Albrecht and Steinhardt 1982). In this model, it is assumed
that the diagram of the effective potential (or free energy) of the inflation field $\psi$ has a very flat plateau, and the field evolves by "slowly rolling" off the plateau. Figure 3 illustrates a typical new inflationary potential. The phase transition can be second order or only weakly first order. This model is conceptually different in that a coherent region of the universe can be described as rolling down the hill described by the potential. While the scalar field $\phi$ rolls along the flat portion of the potential, the value of the potential is almost constant and vacuum energy density dominates the energy density of the universe. As before, $R \propto e^N$, where $H^2 = 8\pi G\rho_{\text{vac}}/3$; the universe expands exponentially. Then, once the field approaches the minimum of the potential, the $\phi$ field decays into particles and radiation, and reheating takes place to an ordinary radiation-dominated epoch. In this way a "graceful exit" from inflation is achieved.

![Diagram of effective potential](image)

**FIGURE 3.** A typical new inflationary potential. The universe inflates while $\phi$ "rolls down" the flat portion of the potential and reheats while $\phi$ oscillates around the minimum.

**CONSTRAINTS ON INFLATIONARY POTENTIALS**

There are several requirements on potentials in inflationary models (Steinhardt and Turner 1984). The two most restrictive are:

1. Sufficient inflation: the scale-factor must grow by at least 65 e-foldings

$$N_s(\phi_1, \phi_2, f) \equiv \ln \left( \frac{R_{\text{end}}}{R_{\text{begin}}} \right) = \int_{t_1}^{t_2} H \, dt = \frac{8\pi}{m_{\text{pl}}^2} \int_{\phi_1}^{\phi_2} \frac{V(\phi)}{V'(\phi)} \, d\phi \geq 65. \tag{1}$$

Here, $N_s$ is the number of e-foldings of the scale factor, $m_{\text{pl}}$ is the Planck mass, $\phi_1$ and $t_1$ are the value of the scalar field and the time at the beginning of inflation, $\phi_2$ and $t_2$ are the value of the scalar field and the time at the end of inflation, and prime denotes derivative with respect to $\phi$.

2. Amplitude of density fluctuations: quantum fluctuations in the scalar field give rise to density perturbations of amplitude (Guth and Pi 1982; Hawking 1982; Starobinskii 1982; Bardeen, Steinhardt, and Turner 1983)

$$\left. \frac{\delta \rho}{\rho} \right|_{\text{horizon}} = \frac{H^2}{\dot{\phi}}. \tag{2a}$$

The amplitude of these fluctuations is constrained not to exceed the observed limits on the anisotropy of the microwave background (see, e.g., Meinhold and
Lubin 1991; Readhead et al. 1989; Uson and Wilkinson 1985)

\[ \frac{\delta \phi}{\rho} = \frac{\delta T}{T} \leq O(10^{-5}). \]  

(2b)

There are two nice features of inflationary density perturbations: (1) causal processes can explain their origin, and (2) during inflation, \( H \) and \( \dot{\phi} \) vary slowly, and thus, from (2a), we can see that the amplitude of the fluctuations at the time they enter the horizon is nearly the same on all scales (for many inflationary models), that is, one obtains a Harrison-Zel'dovich spectrum. The problem, however, is that inflationary models typically overproduce the amplitude of the fluctuations unless one fine-tunes the parameters of the potential.

In order to obtain sufficient inflation and to satisfy microwave background anisotropy limits on density fluctuations, the potential of the field responsible for inflation (the inflaton) must be very flat. Paper I (Adams, Freese, and Guth 1991) shows that, for a general class of inflation models involving a single slowly rolling field [including new, chaotic (Linde 1983), and double field inflation (Adams and Freese 1991; Linde 1990)], the ratio of the height to the (width)\(^4\) of the potential must satisfy the constraint (Adams, Freese, and Guth 1991):\[ \chi \equiv \frac{\Delta V}{(\Delta \phi)^4} \leq O(10^{-6} - 10^{-8}), \]  

(3)

where \( \Delta V \) is the change in the potential \( V(\phi) \) and \( \Delta \phi \) is the change in the field \( \phi \) during the slowly rolling portion of the inflationary epoch. [For extended inflation, a model described below (La and Steinhardt 1989), \( \chi \leq O(10^{-15}) \) (Adams, Freese, and Guth 1991).] Thus, the inflaton must be extremely weakly self-coupled, with effective quartic self-coupling constant \( \lambda_\phi < O(\chi) \) (e.g., \( V(\phi) = \frac{1}{4} \lambda_\phi \phi^4 \); in realistic models, \( \lambda_\phi < 10^{-12} \)). Naturally, if there is only one mass scale in the problem, one expects \( \chi = O(1) \).

**NEW IDEAS WITH FIRST-ORDER PHASE TRANSITIONS**

In order to circumvent the reheating problems of old inflation and the fine-tuning in new inflation, several new ideas have been proposed. Some of these involve returning to first-order phase transitions (as in old inflation), where bubbles of true vacuum (T) nucleate in a sea of false vacuum (F).

Consider the nucleation efficiency \( \beta = \Gamma_\nu / H^4 \) (during a period of exponential expansion, the probability of remaining in the false vacuum is approximately \( p(t) = e^{-4\nu / 3\hbar H t} \)). Here \( \Gamma_\nu \) is the nucleation rate for true vacuum bubbles and the Hubble parameter \( H \) characterizes the expansion rate of the universe. A successful inflationary model must have \( \beta \) small initially, so that the universe is trapped in the metastable false vacuum for a long time and there is a sufficiently long period of inflation. Then, \( \beta \) must become very large, that is, nucleation must abruptly become very efficient. In this case, all of the universe can go from false to true vacuum at once. The bubbles of true vacuum are all of the same size, they can find one another and percolate, and presumably bubble collisions can thermalize the interiors. Thus one is not left with a Swiss cheese universe as in old inflation; instead, a thermal
Friedmann–Robertson–Walker universe results. This abrupt change of nucleation efficiency from very small to very large is illustrated in FIGURE 4. Guth first pointed out that a time-dependent nucleation rate would solve the reheating problems of old inflation in 1981.

One can achieve a time-dependent nucleation efficiency $\beta \equiv \Gamma_n / H^4$ by varying either the numerator or the denominator. Extended inflation (La and Steinhardt 1989), which uses Brans–Dicke gravity, has a time-dependent denominator $H = H(t)$. The scale factor initially expands exponentially, but quickly develops a power-law dependence on time. True vacuum bubbles can percolate in the sea of false vacuum, which is only expanding as power law (not exponentially as in old inflation). However, $\chi$ as defined in (3) must be small (Adams, Freese, and Guth 1991): $\chi \leq O(10^{-15})$. Hyperextended inflation (Steinhardt and Accetta 1990) uses more complicated couplings to gravity.

Another way to achieve a time-dependent nucleation efficiency $\beta$ is to modify the numerator, that is, to have a time-dependent nucleation rate. Double-field inflation is the model described in Paper II (Adams and Freese 1991) to implement a time-dependent nucleation rate (a similar idea has been proposed by Linde 1990). In this model, there are two coupled scalar fields. The potential $V_1(\phi)$ of one field, in the absence of coupling, is much like that of old inflation (see FIG. 5). The universe passes through a first-order phase transition, and expands exponentially while it is trapped in the false vacuum. This first field $\phi$ is the inflaton. The potential of the second field $V_2(\psi)$, in the absence of coupling, describes a rolling field as in new inflation. The sole purpose of this second $\psi$ field is to catalyze a change in the nucleation rate for the inflaton $\phi$ field. The total potential for the system is

$$V_{\text{tot}} = V_1(\phi) + V_2(\psi) + V_{\text{int}}(\phi, \psi),$$

where, for calculational simplicity, we take $V_{\text{int}}(\phi, \psi) = -\gamma(\phi - a)\psi^3$ for the interaction term between the two fields, although the results should hold for more generic interaction terms as well. Here, $a$ is the minimum of the potential $V_1(\phi)$ and $\gamma$ is the coupling constant between the two fields. It can be shown that the nucleation rate $\Gamma_n \propto e^{-S} \propto e^{-\text{cons} \cdot \epsilon_{\text{eff}}}$, where $S$ is the Euclidean action and $\epsilon_{\text{eff}} = \rho_{\text{vac}} + 2a\gamma\psi^3$. In other words, the nucleation rate of the $\phi$ field has a term proportional to the value of the $\psi$ field. When the $\psi$ field is near the top of its potential and has a value close to

![FIGURE 4. Required time dependence of nucleation efficiency $\beta$ of true vacuum bubbles at first-order phase transition (for successful inflation).](image-url)
FIGURE 5. Potentials required for double field model. (a) The potential $V_1(\phi)$, in the absence of coupling, for the inflaton field $\phi$, is much like that of old inflation. (b) The potential $V_2(\psi)$, in the absence of coupling, describes a rolling field as in new inflation. This purpose of this $\psi$ field is to catalyze a change in the nucleation rate for the inflaton $\phi$ field.

zero, the nucleation rate of the inflation field $\phi$ is very small. When the $\psi$ field nears the minimum of the potential and has a large value, then the nucleation rate suddenly gets very large and the phase transition completes. Thus one has the required $\beta$ as in Figure 4, and the universe successfully reheats. Double field inflation obtains a "graceful exit" from inflation at a first-order phase transition with bubble nucleation.

There are several implications for large-scale structure. The same field that produces inflation can give rise to cosmic strings at the end of inflation. Inflation dilutes any preexisting cosmic strings. It is a nice feature of the double field model that cosmic strings, which may be important for structure formation, may be formed at the end of the inflationary phase transition. In addition, the collisions of true vacuum bubbles may give rise to interesting large-scale structure. (Both of these features, strings and bubble structures, also arise from extended inflation.)

However, the rolling fields in the double field model must have flat potentials [small values of $\chi$ in (3)] to avoid overproduction of density fluctuations.

**NATURAL INFLATION WITH PSEUDO-NAMBU–GOLDSTONE BOSONS**

Paper III (Freese, Frieman, and Olinto 1990) describes a model that naturally provides the flat potentials that are required for rolling fields in inflation [for either an inflationary model with only a single scalar field, which is a rolling field; or for the
rolling field in the context of a double field model. This paper takes advantage of pseudo-Nambu–Goldstone bosons [particles such as axions (Weinberg 1978; Wilczek 1978) and schizons (Hill and Ross 1988)] in particle physics that can naturally have potentials with two widely disparate mass scales for height and width.

For the past ten years, people have realized that rolling fields in inflation require flat potentials and that the parameters of the potentials must typically be fine-tuned. To reiterate, Paper I quantifies this statement, \( \chi = [\Delta V/(\Delta \phi)^4] \leq 10^{-8} \). Most particle physics models require \( \chi = O(1) \). But we know of a particle with a small ratio of scales: the “invisible” axion has self-coupling \( \lambda_{\chi} = [\Lambda_{\text{QCD}}/f_{\text{QCD}}]^4 \approx 10^{-64} \) (Dine, Fischler, and Srednicki 1981; Wise, Georgi, and Glashow 1981). Paper III uses a potential similar to that for axions in inflation; we obtain “natural” inflation, without any fine-tuning.

The potential we obtain is of the form

\[
V(\phi) = \Lambda^4 [1 + \cos (\phi/f)],
\]

as in Figure 6 (the potential takes this form for temperatures \( T \leq \Lambda \)). The height of the potential is \( 2\Lambda^4 \), and the width of the potential is \( \pi f \). Thus, the height and the width are given by two different mass scales, \( \Lambda \) and \( f \). As explained below, \( f \) is the scale of spontaneous symmetry breaking of some global symmetry, and \( \Lambda \) is the scale at which a gauge group becomes strong. For example, for the quantum chromodynamics (QCD) axion, \( f \) is given by the Peccei–Quinn scale, \( f_{\text{QCD}} \sim 10^{15} \text{ GeV} \), and \( \Lambda \) by the QCD scale, \( \Lambda_{\text{QCD}} \sim 100 \text{ MeV} \). The ratio of these two scales to the fourth power is a very small number, \( \chi \sim 10^{-64} \). For inflation, we do not need a ratio quite this small, but, in order to satisfy various constraints on the model, we will need the mass scales to be higher. We can use a particle similar to the QCD axion (but not the QCD axion itself). Inflation needs \( \Lambda \sim m_{\text{GUT}} \) and \( f \sim m_{\text{pl}} \).

I will now briefly illustrate in what sense these potentials satisfy the criterion of

![FIGURE 6. Potential of (5) for natural PNGB inflation; also, axion potential \( V(\phi) \) for temperatures \( T \leq \Lambda \).]
naturalness. I will use the definition of naturalness proposed by 't Hooft (1979): a small parameter $\alpha$ is natural if, in the limit $\alpha \to 0$, the symmetry of the system increases. I will show how the axion satisfies this criterion. [For references on axion cosmology, see Preskill, Wise, and Wilczek 1983; Abbott and Sikivie 1983; and Dine and Fischler 1983]. In order to solve the strong CP problem of QCD, Quinn and Peccei (1977) introduced a global $U(1)$ symmetry that is broken at a scale $f$, that is, for $T \leq f$, the potential of the PQ (Peccei-Quinn) field is as illustrated in FIGURE 7(a). For $T \ll f$, the radial modes are frozen out because they are very massive, and the only remaining degree of freedom is the angle $\phi/f$ around the bottom of the Mexican hat-shaped potential. This angular degree of freedom $\phi$ is the axion field (Weinberg 1978; Wilczek 1978).

One can plot the value of the potential around the bottom of the Mexican hat, that is, $V$ as a function of the angular variable $\phi/f$. For temperatures $f \geq T \geq \Lambda$, the potential has the same value for any choice of $\phi$. This flat potential is plotted in FIGURE 7(b). The Lagrangian for the $\phi$ degree of freedom is just $L_\phi = \frac{1}{2} (\partial_\mu \phi)^2$, which is invariant under the transformation $\phi \to \phi + \text{const}$. In this sense, the $U(1)$ symmetry is said to be nonlinearly realized (any point around the bottom of the Mexican hat is equivalent).

For the axion, however, this is not the entire story because of the chiral anomaly in QCD. The axion part of the QCD Lagrangian is

$$L_\phi = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{g^2}{32\pi^2} \frac{\phi}{f} tr (F \tilde{F}).$$

At finite temperature, the free energy density path integral is dominated by instanton configurations.

As the temperature drops below $T \leq \Lambda$, instanton effects turn on, and the bottom
of the Mexican hat develops ripples in it. Not all points in the bottom are equivalent any more. One can think of this as placing a block under one side (or under several points) of the hat, so that the potential as a function of the angular variable $\phi$ is as in Figure 6. This is the cosine potential given in (5). This cosine potential is invariant under the transformation $\phi \rightarrow \phi + 2\pi f N$, where $N$ is the number of minima (ripples) of the potential [in (5) I have set $N = 1$]. These ripples around the bottom of the Mexican hat break the nonlinearly realized symmetry from continuous to discrete. If one were to set $\Lambda = 0$, then the potential would always retain the flat form of Figure 7(b) for all temperatures, rather than picking up the cosine shape from instantons. Thus, taking the limit $\Lambda \rightarrow 0$ restores a continuous symmetry. By the definition of naturalness given earlier, small $\Lambda$ is therefore natural. As mentioned before, for the case of QCD axions, $(\Lambda_{\text{QCD}}/f)^4 \sim 10^{-64}$.

For the inflation, we used a scalar field with small self-coupling $\chi$, similar to the case of axions, although the ratio of parameters does not need to be as small. Thus, we considered an axionlike model with scales $\Lambda$ and $f$ as free parameters; we found that inflation is successful for $f \sim m_{\text{pl}}$ and $\Lambda \sim m_{\text{GUT}} \sim 10^{15}$ GeV. Here, $f$ is the scale at which a global symmetry is spontaneously broken, and $\Lambda$ is the scale at which a gauge group becomes strongly interacting. These mass scales can arise naturally in particle physics models. For example, in the hidden sector of superstring theories, if a large non-Abelian group remains unbroken, the running gauge coupling can become strong at the GUT scale, that is, $\Lambda \sim m_{\text{GUT}}$; a hidden sector group that becomes strong at the GUT scale is suggested as a way to break supersymmetry in a phenomenologically viable way. In this case, the role of the PNGB inflaton might be played, for example, by the model-independent axion (Witten 1984).

Paper III considered several requirements on inflationary models with naturally small couplings provided by PNGBs. Since the potential as a function of $\phi$ is flat for temperatures $T \geq \Lambda$, we assumed that the value of the $\phi$ field is initially laid down at random anywhere in the range $0 \leq \phi \leq 2\pi f$ in different causally connected regions. Once the cosine potential appears, the value of $\phi$ has an equal probability of starting anywhere on the potential; subsequently, $\phi$ rolls down the hill. We estimated the probability of success of our scenario by calculating (as a function of $f$) the probability of $\phi$ being close enough to the top of the potential to have sufficient inflation. The requirement that sufficient inflation occurred with a reasonable probability drives the value of $f$ to be near the Planck mass $m_{\text{pl}}$.

We considered only the overdamped (slowly rolling) regime, $(\ddot{\phi} \ll 3H\dot{\phi})$. We required at least 65 e-foldings of inflation; we checked that we did not need to specify the initial value of the $\phi$ field to greater accuracy than allowed by quantum fluctuations; we checked that reheating to radiation can be successful; and we checked that spatial gradients and topological defects do not prevent the onset of inflation.

In addition, we ensured that density fluctuations are not overproduced; this requires $\Lambda \leq m_{\text{GUT}}$ [as expected, since then $\chi$ as defined in (3) is $\leq 10^{-12}$]. We found that the density fluctuation spectrum is non-scale-invariant, with extra power on large length scales: in Fourier space, $|\delta_k|^2 \sim k^n$, where $n = 1 - (m_{\text{pl}}^2/8\pi f^2)$, for $f \leq 3m_{\text{pl}}/4$. 
CONCLUSIONS

Inflation can explain the homogeneity, isotropy, flatness, oldness, and low flux of monopoles of our universe by having a period of exponential expansion in the early universe. Old inflation, which involves nucleation of true vacuum bubbles at a first-order phase transition, suffers from the problem that there is no thermalization of bubble interiors, and thus no return to the ordinary radiation-dominated universe. New inflation suffers from fine-tuning of parameters. The quest for a successful inflationary model without these problems had led to new ideas in the past few years.

In this paper, I reported on three papers:

1. The fine-tuning of parameters required in inflationary models can be quantified (Adams, Freese, and Guth 1991). Rolling fields need flat potentials; the ratio of height to (width)\(^4\) of the potential must be \(\chi = [\Delta V/(\Delta \Psi)^4] \leq O(10^{-20})\).
   For extended inflation, the constraint is somewhat more restrictive: \(\chi \leq O(10^{-15})\).

2. Double field inflation (Adams and Freese 1991) couples two scalar fields to obtain a time-dependent nucleation rate of true vacuum bubbles, and thus a successful end to inflation with a first-order nucleating transition (as in old inflation).

3. Natural inflation with PNGBs (Freese, Frieman, and Olinto 1990) provides flat potentials (as required by Paper I) without any fine-tuning. A pseudo-Nambu–Goldstone boson (e.g., an axion), with a potential \(V\) that arises naturally from particle physics models, can lead to successful inflation if the global symmetry breaking scale \(f = m_\ast\) and \(\Lambda = m_{\text{GUT}}\).

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