A vector leptoquark interpretation of the muon $g - 2$ and $B$ anomalies

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We show that a single vector leptoquark can explain both the muon $g - 2$ anomaly recently measured by the Muon g-2 experiment at Fermilab, and the various $B$ decay anomalies, including the $R_{D^{(*)}}$ and $R_{K^{(*)}}$ anomalies which have been recently reported by the LHCb experiment. In order to provide sizeable positive new physics contributions to the muon $g - 2$, we assume that the vector leptoquark particle couples to both left-handed and right-handed fermions with equal strength. Our model is found to satisfy the experimental constraints from the large hadron collider.

Introduction:– Recently, the Muon g-2 experiment at Fermilab, E989, has announced its new measurement on the anomalous magnetic moment of muon

$$a_{\mu} = \frac{g_{\mu} - 2}{2}.$$  

(1)

The result of the previous muon g-2 experiment at BNL, E821, is $3.7\sigma$ from the standard model (SM) expectation. The new E989 Run 1 data give a smaller $a_{\mu}$ with better accuracy which is $3.4\sigma$ away from the SM value. The combined results of the Fermilab E989 Run 1 data and the BNL E821 data give rise to a slightly smaller $a_{\mu}$ with a better precision, yielding a $4.24\sigma$ deviation from the SM value [1, 2]

$$\Delta a_{\mu} = a_{\mu}^{\text{Exp}} - a_{\mu}^{\text{SM}} = (251 \pm 59) \times 10^{-11}.$$  

(2)

Recently, various $B$ meson decays have shown significant deviations from the SM predictions, most of which are related to muon final states. The lepton flavour universality in $B$ meson decays can be tested by measuring the ratios of the $b \to s\ell\bar{\ell}$ transitions

$$R_K = \frac{\text{BR}(B^+ \to K^+\mu^+\mu^-)}{\text{BR}(B^+ \to K^+\mu^+\mu^-)};$$  

(3)

$$R_{K^{(*)}} = \frac{\text{BR}(B^0 \to K^{(*)}\mu^+\mu^-)}{\text{BR}(B^0 \to K^{(*)}\mu^+\mu^-)};$$  

(4)

and the ratios of the $b \to c\ell\bar{\nu}_{\ell}$ decays

$$R_{D^{(*)}} = \frac{\text{BR}(B \to D^{(*)}\ell\bar{\nu}_{\ell})}{\text{BR}(B \to D^{(*)}\ell\bar{\nu}_{\ell})},$$  

(5)

where $\ell = e, \mu$. Recently, LHCb has updated the measurement on $R_K$ [3]

$$R_{K}^{\text{Exp}} = 0.846^{+0.042+0.013}_{-0.039-0.012},$$  

(6)

in the region of $q^2 = [0.045, 1.1]$ GeV$^2$ and $q^2 = [1.1, 6]$ GeV$^2$ respectively, indicating $2.2\sigma$ and $2.4\sigma$ deviations from the SM predictions, which are $R_{K}^{\text{SM}} = 0.92 \pm 0.02$ and $R_{K}^{\text{SM}} = 1.00 \pm 0.01$ in these two regions. Together with the recent results from Belle [9], the world averages for $R_{D^{(*)}}$ measurements are $R_{D}^{\text{Exp}} = 0.340 \pm 0.030$ and $R_{D}^{\text{Exp}} = 0.295 \pm 0.014$, whereas the SM expectations are $R_{D}^{\text{SM}} = 0.299 \pm 0.003$ and $R_{D}^{\text{SM}} = 0.258 \pm 0.005$, yielding a $\sim 3\sigma$ deviation when these two are combined.

Recently, LHCb has also released its measurement on $B_s \to \mu^+\mu^-$ with the full run 2 data [11]

$$\text{BR}(B_s \to \mu^+\mu^-) = (3.09^{+0.46+0.15}_{-0.43-0.11}) \times 10^{-9}.$$  

(9)

By updating the world average of $B_s \to \mu^+\mu^-$ branching ratio and the correlated $B^0 \to \mu^+\mu^-$, Ref. [12] found a $2.3\sigma$ deviation from the SM prediction.

In this paper, we use a single vector leptoquark (LQ) to explain both the muon $g - 2$ anomaly and the various $B$ decay anomalies, in particular the $R_{K^{(*)}}$ and $R_{D^{(*)}}$ anomalies, which have been measured more precisely recently and shown significant deviations from the SM. LQs are new physics particles that couple simultaneously to a lepton and a quark; see e.g. [13] for a recent review. LQ models have been proposed to explain various $B$ anomalies. A single scalar LQ has been proposed to explain both the muon $g - 2$ and the $B$ anomalies [14]1. However, recently, Ref. [17] shows that it is difficult for a single scalar LQ to explain both $R_{K^{(*)}}$ and $R_{D^{(*)}}$ anomalies because the measured $R_{K^{(*)}}$ values are smaller than the SM expectations whereas $R_{D^{(*)}}$ are larger. It is also found that a vector LQ that transforms as $(3, 1, 2/3)$ under the standard model gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ can explain both $R_{K^{(*)}}$ and $R_{D^{(*)}}$ anomalies [17]. In this study, we further show that such a vector LQ model can also explain the muon $g - 2$ anomaly, in addition to the $B$ meson decay anomalies which satisfying various LHC constraints. Our model is different from that considered in Ref. [17] because we consider the couplings

1 Models with multiple scalar leptoquarks have also been explored [15, 16].
to both left-handed and right-handed fermions, whereas only left-handed couplings are assumed in Ref. [17]. Our analysis shows that both left-handed and right-handed couplings between the $U_1$ boson and the SM fermions are essential for a sizeable new physics contribution to explain the muon $g-2$ anomaly.

**Leptoquark model:**—We consider the vector LQ $U_1 = (3, 1, 2/3)$ that couples to both left-handed and right-handed standard model fermions. The interaction Lagrangian between the vector LQ $U_1^T$ and SM fermions in the weak basis is given by

$$\mathcal{L} = x_{ij}^L Q_L^\gamma \mu U_1^\mu + x_{ij}^R \bar{Q}_R^\gamma \ell_R^\mu U_1^\mu + h.c., \quad (10)$$

where $Q_L^\gamma = (u_L, d_L)$ is the left-handed quark doublet, $L_L^\gamma = (\nu_L, \ell_L)$ is the left-handed lepton doublet, $d_R (\ell_R)$ is the right-handed quark (lepton), $x_{ij}^L$ and $x_{ij}^R$ are couplings with $i$ and $j$ as the generation index of the SM fermions. Here we assume that the down quarks and the charged leptons are diagonal. Rotating the up quark fields to the mass basis, the interaction Lagrangian becomes

$$\mathcal{L} = (V x^L)^i_j \bar{u}_L^\gamma \nu_L^i U_1^\mu + (V x^\gamma)^i_j \bar{\nu}_L^\gamma \ell_L^i U_1^\mu + (V x^R)^\gamma_i \bar{d}_R^\gamma \ell_R^i U_1^\mu + h.c., \quad (11)$$

where $V$ is the CKM matrix. We do not consider couplings with right-handed neutrinos. We assume that the coupling between the vector LQ $U_1$ and the SM gauge bosons (photon and gluon) are standard gauge couplings. The $U_1$ coupling to photon contributes to the muon $g-2$.

The $U_1$ coupling to gluons is important for the production cross section of $U_1$ at the LHC.

Figure 1. New contributions to the muon $g-2$ anomaly from the leptoquark model.

Muon $g-2$ anomaly:—In our vector LQ model, the main new physics contributions to the muon $g-2$ arise from the loop diagrams shown in Fig. 1. The vector LQ contributions are given by [18–21]

$$\Delta a_\mu = -\frac{N_c Q_q}{8\pi^2} \int_0^1 dx f_a \frac{F(v, x) + F_a(x, x)}{(1-x)(\lambda^2 - x) + \epsilon^2 x} + \frac{N_c Q_{U_1}}{8\pi^2} \int_0^1 dx f_a \frac{\epsilon F(v, x) + \epsilon F_a(x, x)}{(1-x)(\epsilon^2 - x) + \lambda^2 x}, \quad (12)$$

where $N_c$ is color number, $Q_q$ is the electric charge of the SM quark, and $Q_{U_1} = 2/3$ is electric charge of $U_1$, $\epsilon = m_\mu/m_{U_1}$, $f_v = (x^R + x^L)/2$, $f_a = (x^R - x^L)/2$, $F_1(x, x) = 2x(1-x)(x-2(1-\epsilon)) + \lambda^2(1-\epsilon)^2x^2(1+\epsilon-x)$, $F_a(x, x) = -2\lambda^2(1+x-2\epsilon) + \lambda^2(1-\epsilon)^2x(1-x)(x+\epsilon)$, $F_a(x, x) = F_a(x, x)$.

We consider four different types of couplings: $x^L = x^R$, $x^L = -x^R$, $x^L = 0$, and $x^L = 0$; we find that only the $x^L = x^R$ case can give rise to a sizeable positive new physics contribution to the muon $g-2$. Thus we assume $x^L = x^R$ hereafter. We denote $x = x^L = x^R$. We note that the $U_1$ boson has pure vector couplings to fermions in the $x^L = x^R$ case. To explain the muon $g-2$, $R_{K^{(*)}}$ and $R_{D^{(*)}}$ anomalies, we assume nonzero $x_{b\mu}$, $x_{b\mu}$ and $x_{b\tau}$ values.

Figure 2. New contributions to the $B^+ \to K^+ \ell^+ \ell^-$ process due to the vector LQ $U_1$.

$$R_{K^{(*)}} \text{ anomaly:}$$—The semileptonic process $b \to s\ell^+\ell^-$ is responsible for the $B^+ \to K^+ \ell^+ \ell^-$ decay. The new physics contributions due to the new vector LQ $U_1$ are shown in Fig. 2. The effective Hamiltonian describing the $b \to s\ell^+\ell^-$ process can be parameterized as follows [22–25]

$$\mathcal{H} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i \mathcal{O}_i + h.c., \quad (13)$$

where $\mathcal{O}_i$ are the four-fermion operators, $\mathcal{C}_i$ are the Wilson coefficients, $G_F$ is the Fermi constant, $\alpha \sim 1/137$, $V_{tb}$ and $V_{ts}$ are the CKM matrix element. The operators that are of importance to the $R_{K^{(*)}}$ analysis in our model include $\mathcal{O}_{l\ell}^{9} = \bar{s} \gamma_{\mu} P_L R b (\bar{\ell} \gamma^\mu \ell)$, and $\mathcal{O}_{l\ell}^{10,10'} = \bar{s} \gamma_{\mu} P_L R b (\bar{\ell} \gamma^\mu \gamma_5 \ell)$, where $\ell = e, \mu, \tau$ and $P_{R,L} = (1 \pm \gamma_5)/2$. The Wilson coefficients can be parameterized as $\mathcal{C}_i = \mathcal{C}_{i}^{SM} + \Delta \mathcal{C}_i$ where $\mathcal{C}_{i}^{SM}$ represents the SM contribution and $\Delta \mathcal{C}_i$ denotes the new physics contributions. For the vector LQ $U_1$ model where nonzero $x_{b\mu}$ and $x_{b\mu}$ values are introduced, the new physics contributions to the Wilson coefficients are given by

$$\Delta \mathcal{C} \equiv \Delta \mathcal{C}_{9}^{\mu} = \Delta \mathcal{C}_{9}^{\nu} = \Delta \mathcal{C}_{10}^{\mu} = -\Delta \mathcal{C}_{10}^{\nu}, \quad (14)$$

$^2$ Operators such as $\mathcal{O}_{S_{s}\ell\ell', p_{p'}}$ also arise in the $x^L = x^R$ case. These operators are neglected in our analysis, because they do not give rise to significant contributions to $R_{K^{(*)}}$ as the $\mathcal{O}_{S_{s}\ell\ell', 10,10'}$ operators, as shown in Ref. [26].
where
\[ \Delta C = -\frac{\pi v^2}{V_{tb}V_{ts}^*} \frac{x_{3\mu}x_{3\tau}^*}{m_{U_1}^2}. \] (15)

where \( v = 246 \text{ GeV} \).

For the \( R_K \) calculations, operators \( O_i \) and \( O_{i'} \) yield nearly the same effects, for \( i = 9,10 \). Because our model predicts \( \Delta C_{9}^{\mu} = \Delta C_{10}^{\mu} \) and \( \Delta C_{10}^{\mu} = -\Delta C_{9}^{\mu} \) in the \( x^L = x^R \) case, the effects on \( R_K \) due to \( O_{10} \) cancel with \( O_{10}^{'} \), and we only need to consider the effects due to the operator \( O_9 \) with the coefficient \( -2\Delta C \). We determine the \( \Delta C \) value needed to interpret the recent LHCb \( R_K \) result [3] using the theoretical analysis on \( R_K \) in Ref. [26] and find that
\[ \Delta C(R_K) \approx -0.35 \pm 0.11 \] (16)

Thus one can explain both \( R_K \) and \( R_{K^*} \) anomalies within the 1 \( \sigma \) (2 \( \sigma \)) error corridor in the range \(-0.46 < \Delta C < -0.35 \) (\(-0.57 < \Delta C < -0.13 \)).

\( R_{D^(*)} \) anomaly:— Here we compute \( R_{D^(*)} \) for the LQ model we consider. The relevant low-energy EFT Lagrangian describing the \( b \rightarrow c\ell\nu \) is given by [27-29]
\[ \mathcal{L} = -2\sqrt{2}G_F V_{cb} \left[ (1 + y_{\ell V_L}) O_{\ell L} + y_{\ell S_R}^* O_{S_R} \right] + h.c., \] (18)

where \( y_{\ell V_L} \) and \( y_{\ell S_R} \) are the Wilson coefficients and the fermion-operators are given by
\[ O_{\ell L} = (\bar{c}_L \gamma^\mu b_L) (\bar{L}^* \gamma^\mu \nu_L), \quad O_{S_R} = (\bar{c}_L b_R) (\bar{L} \nu_L). \] (19)

The new physics contributions to the Wilson coefficients at tree level from the vector LQ model can be obtained as
\[ y_{\ell V_L} = -\frac{1}{2} y_{\ell S_R}^* \frac{v^2}{2V_{cb}} \left( V_{x}x_{x'} \right)_\ell (x_{bd})^*, \]
where the sum over \( \ell \) for generation of the neutrino is implicit on the right hand side. We assume that the non-zero matrix elements in our model are \( x_{3\mu}, x_{3\mu} \), and \( x_{3\tau} \) values. In our numerical analysis, we set \( x_{3\mu} = 5x_{3\mu} \) so that the new physics contributions to the \( b \rightarrow c\ell\nu \) transitions are much smaller than the new physics contributions to the \( b \rightarrow c\ell\nu \) transitions. Thus We neglect the new physics contributions to the \( b \rightarrow c\ell\nu \) transitions and adopt the analysis in Ref. [27] to obtain
\[ y_{\ell V_L} = 0.08_{-0.03}^{+0.03}, \quad y_{\ell S_R} = -0.05_{-0.10}^{+0.09}, \] (21)

where the two fits are combined. In order to explain the relation \( R_{K^*}^\text{Exp} < R_{K^*}^\text{SM} \), the matrix elements \( x_{3\mu} \) and \( x_{3\mu} \) have to be opposite in sign. Taking all matrix elements to be real, and setting both \( x_{3\mu} \) and \( x_{3\mu} \) positive and \( x_{3\mu} \) negative, in order to explain \( R_{D^(*)}^\text{Exp} > R_{D^(*)}^\text{SM} \), the condition \( |x_{3\mu}| < (x_{3\mu} + x_{3\mu})V_{cb}/V_{cs} \) has to be satisfied. Thus in our vector LQ model, both \( R_{K^*} \) and \( R_{D^(*)} \) anomalies can be explained simultaneously.

\textbf{LHC constraints}—The LQs can be either singly produced or pair-produced at hadron colliders [30, 31]. The singly produced LQ is accompanied by a lepton in the final state, \( qg \rightarrow q U_1 \ell \); the pair-production process can occur via either the gluon–gluon fusion process or the quark-antiquark annihilation process (mediated by either a t-channel lepton or an s-channel photon/gluon).

Searches for these direct productions of leptoquarks have been carried out at the LHC for various final states [32-36]. From these searches, one can derive the collider exclusion bounds on a given leptoquark mass as a function of its branching ratio (denoted by \( \beta \)) into a specific fermion final state.

Using the recent LHC data, a recasting result from Ref. [17] shows the current limits for the vector LQs. In particular, for pair-produced LQs decaying into the final states of \( b\tau\bar{\tau}, t\ell\bar{\nu}, jj\mu\mu, bb\mu\mu, jj\tau\nu, bb\tau\nu, \) and \( tt\nu\nu \), the lower limits on its mass are 1.5 TeV, 2.0 TeV, 2.3 TeV, 2.3 TeV, 2.0 TeV, 1.8 TeV, 1.8 TeV and 1.8 TeV, respectively, assuming \( \beta = 1 \) [17]. The limits from \( b\tau\bar{\tau}, jj\mu\mu, bb\mu\mu, jj\tau\nu, \) and \( tt\nu\nu \) final states can be applied for the \( U_1 \) model. The most stringent constraints are from \( jj\mu\mu \) and \( bb\mu\mu \) final states. For the pair of LQs decaying into different quark-lepton final states case, i.e., \( U_1 \rightarrow b\tau, t\ell \), the lower limit on the vector LQ is about 1.7 TeV [37].

However, these limits are under the assumption that the branching fraction \( \beta = 1 \) and the interaction between vector leptoquark and gluons is described by \( ggU_1^\dagger G_{\mu\nu}U_{1\nu} \) with \( g_b \) is the strong coupling and \( \kappa = 1 \). Thus if one tunes \( \beta \) and \( \kappa \) to be small, the limits from LHC direct production searches can be significantly weakened. Indeed, once switching off the the interaction between the vector LQ and gluons, i.e., \( \kappa = 0 \), the
quark–antiquark annihilation process becomes dominant; because the quark–antiquark annihilation for the vector LQ pair production is typically smaller than the gluon fusion, the limit on the vector LQ mass is reduced to $\sim 1.3$ TeV for the $U_1 \to b\bar{t}, t\bar{t}$ searches [37].

The searches for the singly produced LQs have also been performed at CMS using data with $L = 137$ fb$^{-1}$ [37]. The limits on the LQ mass as a function of the LQ-quark-lepton coupling $x$ have been derived; for LQs coupled to the third-generation fermions with coupling $x = 1.5$, the LQ mass $m_{U_1} \lesssim 1.2$ TeV is excluded.

LQs can also be searched for in the $qg \to \ell\ell$ process at the LHC in which the LQ particle serves as a t-channel mediator. Thus LQs are constrained by the high $p_T$ resonance searches via $pp \to \ell\ell^{(*)}$ and $pp \to t\bar{t}$ recently performed at the LHC [17, 38–44]. A recent analysis from Ref. [17] used data from [45] and [46] to set upper bounds on the couplings $x_{b\mu} \lesssim 0.7$, $x_{s\mu} \lesssim 0.5$, $x_{b\tau} \lesssim 1.0$ and $x_{s\tau} \lesssim 0.7$ for the vector LQ mass below 1 TeV in $U_1$ model. The limit on $x_{b\mu}$ as a function of $m_{U_1}$ is shown in Fig. 3.

![Figure 3](image_url)

**Figure 3.** The 2$\sigma$ favored regions for the muon $g-2$ (blue band), $R_{K^{(*)}}$ (pink band) and $R_{D^{(*)}}$ (gray band) on the $(m_{U_1}, x_{b\mu})$ plane. Here we set $x_{s\mu} = -0.005$ and $x_{b\tau} = 5x_{b\mu}$. The black solid line indicates the LHC high $p_T$ searches, and the vertical purple solid line represents the leptoquark searches via pair-production at the LHC [17].

**Numerical results:** Here we present our numerical results. We choose $x_{b\mu} = 5x_{b\mu}$ so that the new physics contributions to the $b \to c\ell\nu$ process are larger than those to the $b \to c\mu\nu$ process, leading to $R_{D^{(*)}} > R_{D^{(*)}}^{SM}$. Note that varying the $x_{b\mu}$ coupling does not affect the $R_{K^{(*)}}$ and $\Delta a_\mu$ computations. We choose $x_{s\mu} = -0.005$ which has an opposite sign to $x_{b\mu}$ in order to satisfy the $R_{K^{(*)}}$ anomaly which is required to obtain $R_{K^{(*)}}^{Exp} < R_{K^{(*)}}^{SM}$. Such a small value of $x_{s\mu}$ also satisfies the condition $|x_{s\mu}| < (x_{b\mu} + x_{b\tau})V_{cb}/V_{cs}$ needed for a positive contribution to the $R_{D^{(*)}}$.

Fig. 3 shows the 2$\sigma$ regions of the parameter space of our vector LQ model, for the muon $g-2$, $R_{K^{(*)}}$, and $R_{D^{(*)}}$ anomalies. It is remarkable that there exists a large parameter space ranging from 1 TeV to above 4 TeV, in which one can simultaneously explain the muon $g-2$, $R_{K^{(*)}}$ and $R_{D^{(*)}}$ anomalies. The LHC constraints including the high $p_T$ di-lepton searches and the pair-produced LQ searches are also displayed on Fig. 3; both LHC constraints are adopted from Ref. [17]. Because we have $x_{b\tau} = 5x_{b\mu}$, the branching fraction of $(U_1 \to b\tau)$ dominates, namely $BR(U_1 \to b\tau) \sim 1$. Thus we adopt the limits in the $b\tau\tau\tau$ final state searches for pair-produced LQs at the LHC [17, 32], which rule out the $U_1$ boson with a mass below 1.5 TeV. The LHC high $p_T$ data [32] exclude the $x_{b\mu} > 1$ region which have already put constraints on the 2$\sigma$ region for $R_{K^{(*)}}$ interpretation when $m_{U_1} > 3.5$ TeV. We note that the large parameter space in our model, from 1.5 TeV to 4 TeV, in which both the muon $g-2$ anomaly and the various $B$ anomalies can be explained are allowed by the current LHC searches on LQs.

**Conclusions:** We have shown that the vector LQ $U_1 = (3, 1, 2/3)$ can simultaneously explain the muon $g-2$ anomaly and the $B$ decay anomalies, including the $R_{K^{(*)}}$ and $R_{D^{(*)}}$ anomalies, while satisfying the experimental constraints from the large hadron collider, including searches on singly produced and pair-produced LQs, and the high $p_T$ dilepton searches.

We found that in order to provide a sizeable positive new physics contribution to the muon $g-2$ anomaly while satisfying various LHC constraints, the vector LQ $U_1$ has to couple to left-handed and right-handed fermions equally, namely $x_L = x_R$, so that only vector couplings are present in our model. Unlike the scalar LQ model, the new physics contributions from the vector LQ to $R_{K^{(*)}}$ and $R_{D^{(*)}}$ can be somewhat adjusted independently in the parameter space of our model. Thus conditions $R_{K^{(*)}}^{Exp} < R_{K^{(*)}}^{SM}$ and $R_{D^{(*)}}^{Exp} > R_{D^{(*)}}^{SM}$ can be satisfied with ease. The LHC searches for LQ decays set stringent constraints on the LQ mass, $m_{U_1} \gtrsim O(1)$ TeV. The couplings of LQs to quark and lepton are strongly constrained to be $x \lesssim O(1)$ in the LHC high-$p_T$ searches for LQ mass extending to several TeV. Taking into account all the LHC constraints, there still exists a large parameter space in which the muon $g-2$ anomaly recently reported by the Fermilab g-2 experiment and the various $B$ decay anomalies including $R_{K^{(*)}}$ and $R_{D^{(*)}}$ can be simultaneously explained in the vector leptoquark model in the mass range from 1.5 TeV to several TeV.

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