Kinematic method of envelope points calculation when the
wraparound points are known

D A Babichev, D T Babichev and S Yu Lebedev
1Sibur Holding, ZapSibNeftekhim LLC, East Industrial District, Block 1, No. 6, Building 30, 626150 Tobolsk, Russia
2Industrial University of Tyumen, Institute of Transport, Melnikayte Street 72, 625039 Tyumen, Russia
E-mail: lebedevsergey1995@gmail.com

Abstract. There are two kinds of surfaces that are formed by the movable bodies: wraparound and envelope surfaces. In this work the authors develop velocity (kinematic) envelope points’ calculation method, when the envelope is known. The method is approximate; and based upon the usage of terms “speed and acceleration”; and lets reduce the amount of calculation using undifferentiated methods of envelope estimation.

1. Problem statement

Theory of surfaces formation by movable bodies distinguishes [1] two types of formed surfaces: wraparound and envelope – see Figure 1. Wraparound $\Sigma_1$ is a discrete family connected set of the reference gear tool surfaces pieces; envelope $\Sigma_2$ is a surface tangent to all the elements of this discrete set. Envelope $\Sigma_2$ is normally a smooth surface; wraparound $\Sigma_1$ – is always a faceted surface (with jags). Envelope estimation $\Sigma_2$ – one of the major objectives of gearing theory, and a large number of researches was focused on its resolution [2-11]. The above mentioned papers are based on differential methods: classic [2-3] and kinematic [4-11], which has been an essential method for gearing theory since the middle of 20-th century. Wraparound $\Sigma_1$ is estimated [1,12-13] by direct tracking of generating surface points situation $\Sigma_0$ relative to the blank – direct digital simulation method (PCM) as per [13]. Majority of specialists dealing with computer simulation of forming processes consider PCM as surer methods compared to kinematic ones and due to it they prefer to estimate wraparound rather than envelope. As a consequence, such profiling method is a basis of the of the most powerful Russian software solutions: a) EXPERT software for hypoid and spur gearing created by G.I. Shevelyova and her students; SPIDAL+ solution for worm type gearings created by V.I. Goldfarb and his students. As well as of many foreign solutions: a) PCM by S.V. Lunin (USA); b) Klingelnberg’s software solution (Germany) developed with contribution from famous Russian bevel and hypoid gearing specialist M.S.Segal. Though, estimation of wraparound points $\Sigma_1$ rather than envelope ones $\Sigma_2$ has a number of

![Figure 1.](image-url)
disadvantages. Firstly, it requires more calculations; overall total is more by 1 to 2 orders of magnitude compared to envelope points calculations.

Secondly, difficulties arise with respect to evaluation of generating surface curvature required for, specifically, assessment of teeth contact strength and oil film load take-up. These difficulties are caused by the following:

- Curvature of band and rippled surfaces differs fundamentally and materially from envelope curvature. For instance, curvature radius of the involute, as an envelope rectilinear-face rack, gradually changes along this curve: close to the base circle, at the tips diameter. However, for wraparound involute formed by the same rectilinear-face rack the curvature radii at all the points (as well as at the jags). Since wraparound of such involute is a jagged line, tangent to involute at each and every of its straight intervals.

- The points estimated using non-differential methods are often situated on band and rippled surfaces at the nodes of some irregular uv-grid, which is almost impossible to correlate with hollow centers or band throats. That is why the estimated set of points is rather ill-placed relative to the hollows and bands: any of the points may turn out to be situated: either in the center of the hollow, or at the jag, or next to the pyramid apex. It leads to errors, if the curvature is estimated upon point approximation by curves or surfaces.

Thirdly, contact problem solving through numerical techniques gets more complicated: the points alleet warpage values due to random situation at the jags and in the facet hollows. It causes mistakes in estimated contact stresses and results in ‘jagged’ boundaries of the contact areas.

Thirdly, contact problem solving through numerical techniques gets more complicated: the points alleet warpage values due to random situation at the jags and in the facet hollows. It causes mistakes in estimated contact stresses and results in ‘jagged’ boundaries of the contact areas.

Commonly used means of ‘approach’ of wraparound Σ1 and envelope Σ2 is a very fine-feed task for computer simulation of shape generation process. Definitely, it reduces ‘roughness’ of the simulated surface, resulting in reduction of errors while solving a contact problem. However, it is not a solution of the problem with envelope curvature estimation, if for no other reason than envelope’s curvature does not depend on the feed.

Work objective. To present a kinematic method of ‘approach’ of wraparound Σ1 and envelope Σ2, based on utilization of terms ‘indentation velocity VBH’ and ‘indentation acceleration aBH’. And assess its accuracy, reliability and amount of calculations. This method can be used instead of fine feed method or along with it. Prior to presentation of the method itself, let us look upon indentation velocity and acceleration, the basis of the method.

2. Indentation velocity

VBH, suggested by V.A. Shishkov [4] – also known as velocity of mutual approach and separation [4] – is a velocity of reference gear tool surface (RGTS) penetration into the blank volume. Indentation velocity is estimated as projection of a vector of gear tool’s movement velocity relative to machined element onto direction of a normal vector to the RGTS at this point:

\[ V_{BH} = V_{12} \cdot n = V_{12x} \cdot n_x + V_{12y} \cdot n_y + V_{12z} \cdot n_z \]  

where \( V_{12} \) – relative velocity vector at the point on RGTS: \( V_{12} = V_{tool} - V_{blank} \);

n – unit normal vector to RGTS at this point, always directed out of the gear tool body, e.g. out of grinding disc, but never inwards.

Velocity \( V_{BH} \), estimated through (1), is a scalar value which can be:

- positive – gear tool (Σ₁) indents into gear blank body (Σ₂), and removal of metal takes place, if it is available at this location of the gear blank;

- negative – Σ₁ moves off Σ₂, and machining at this location is impossible;

- zero – this point at the gear tool surface at the moment is forming the point at the surface which will become a result of machining: at the envelop of generating surfaces class.
Term ‘indention velocity’ is not only applicable to technological gearings, but also to working ones. Moreover, in [10] kinematic value “indention velocity” extends to all the possible types of generating elements: surfaces’ jags in the shape of edges or peaks; flat and space lines both with jags and smooth; points. And for all types of relative components’ motion: one-parameter, two-parameter, and multi-parameter.

3. Indentation acceleration

The $a_{BH}$ suggested by D.T. Babichev, is acceleration at which generating element (normally surface) indents (‘sinks in’) into the volume of the other body to form a mating element (normally a surface as well). I.e. $a_{BH}$ indicates how fast indentation velocity $V_{BH}$ varies with time at this particular point of the space where the formed element is situated (formed surface in space gearings, and formed line - in flat ones). Indentation acceleration is relative acceleration; as well as indentation velocity is a scalar value. For the first time $a_{BH}$ was used for estimation of cutting zones and calculation of thicknesses of the layers removed by cutting edges of generating gear tools [14]. Later on [10] it was found out that indentation acceleration $a_{BH}$ is an essential qualitative measure of shape-generation process: a) edge of regression at the surface formed by enveloping is a set of points where $a_{BH} = 0$ during shape generation; b) if $a_{BH}$ is positive, then envelope is formed inside of generating part field, that is impossible for metal and it means that undercutting takes place; c) contact of two mating surfaces is feasible for metal only in those points where $a_{BH} < 0$; d) through indentation acceleration $a_{BH}$, which ad hoc does not depend on section direction, a relative curvature is calculated [15] in any normal section and in one-parameter and multi-parameter gearings:

$$\frac{1}{R_p} = - \frac{\omega_k^2}{a_{BH}}, \quad \frac{1}{R_p} = \sum_{i=1}^{n} \left( \frac{\omega_{k_i}^2}{a_{BH_i}} \right) $$

(2)

where $\omega_k$ – angular velocity of bodies roll motion in the plain of the section in which curvature is estimated; $n$ – number of enveloping parameters. And this is far not complete list of the problems, where it is reasonable to use $a_{BH}$ [10] for resolution.

Let’s find $a_{BH}$ upon differentiation of $V_{BH}$ with respect to time $t$:

$$a_{BH} = \frac{dV_{BH}}{dt} = \frac{d}{dt}(V_{12} \cdot n) = a_{12} \cdot n + V_{12} \cdot \dot{n}$$

(3)

where $V_{12}$ – is relative velocity vector; $n$ – unit normal vector to the surface, directed out of the generating element field; $a_{12}$ – acceleration of the point situated on the generating surface $\Sigma_1$ and riding on it at a velocity “$-V_{12}$”;

It shall be noted that indentation acceleration differs from gearing equation derivative, normally [6] used for curvatures estimations. This difference is in calculation method of partial derivatives of curvilinear coordinates and of time parameter of enveloping; $\frac{\partial u}{\partial \tau}, \frac{\partial v}{\partial \tau}, \frac{\partial t}{\partial \tau}$ (below they will be marked as $\dot{u}, \dot{v}, \dot{t}$). In [6] they run over condition that the point remains action point (under gearing equation time derivative set to zero). Also here constraint equation $\dot{u}, \dot{v}, \dot{t}$ is presented differently:

$$C_u \cdot \dot{u} + C_v \cdot \dot{v} + V_{12} \cdot \dot{t} = n \cdot V_{BH}$$

(4)

i.e. point’s sliding velocity over generating element shall be selected (by means of assigning $\dot{u}, \dot{v}, \dot{t}$) in such a way that this point’s velocity vector would also be a vector of indentation velocity. This is a key...
statement in \( a_{BH} \) calculation method. It can be calculated at any point of generating element surface, not only at action points.

One more material remark: unit normal vector derivative \( \mathbf{n} \) included in (3) sometimes called as unit normal vector end-point velocity [6], is also an angular velocity of roll motion. And \( \omega_k \) and \( \omega_{k\perp} \) are projections of this vector \( \mathbf{n} \) on an axis of the right Darboux triad [16] with axes \( \{\mathbf{c}, \mathbf{c} \times \mathbf{n}, \mathbf{n}\} \). I.e. formula (3) for indention acceleration \( a_{BH} \) calculation may be presented as:

\[
a_{BH} = a_{12} \cdot \mathbf{n} + \mathbf{V}_{12} \cdot \mathbf{n} = a_{12} \cdot \mathbf{n} + \mathbf{V}_{12} \cdot \mathbf{c} \cdot \omega_k + \mathbf{V}_{12} \cdot (\mathbf{c} \times \mathbf{n}) \cdot \omega_{k\perp} \quad (5)
\]

4. Calculation of parameters required for estimation of indention acceleration

The Estimation of vectors \( \omega_k, \omega_{k\perp} \), and especially \( a_{12} \) included in (5), is a matter of considerable difficulties. Thus let’s give formulae for their estimation in generic gearing shown in Figure 2. Actually any specific transmission or generation may be refined in the systems of coordinates and motions of this gearing. That is why below specified formulae are applicable to any specific gearings. In Figure 2 both members making up a gearing perform helical motions around their axes, and the axes take part in helical motion around center-to-center perpendicular. Generating element is at the member 1, generated one – at the member 2. Type of members’ motion is not anyhow limited: motion parameters \((\omega_1, \omega_2, \omega_3, V_1, V_2, V_3)\) can be variable or constant in time, positive, negative or zero. All these parameters are assumed as curve of one, two, or \(n\) number of independent motion parameters \((t_1, t_2, ..., t_n)\), that will correspond to one-, two-, or \(n\)-parameter motion (enveloping). It shall be noted that in [16, 10] it is shown, also via examples, that: cut surfaces sometimes resulted by advance/retract of gear tools are envelopes belonging to the group of multi-parameter generating surfaces. For this reason analysis of multi-parameter gearings represents a problem of practical importance, not just an abstract theoretical research. Supposing that for the gearing in Figure 2 following is known:

- displacements, velocities, and accelerations for all the six motions \((x_1, y_1, z_1; ...; x_2, y_2, z_2)\);
- coordinates of the point on the generating element \((x_1, y_1, z_1; x_h, y_h, z_h; ...; x_2, y_2, z_2)\);
- unit normal vector \( \mathbf{n} \) to the generating element at this point \((n_{x1}, n_{y1}, n_{z1}; ...; n_{x2}, n_{y2}, n_{z2})\);
- curvatures of generating element at this point \((R_{1\perp}, R_{2\perp}, R_{k\perp}, c={c_{x1}, c_{y1}, c_{z1}})\).

Here: \( R_{1\perp}, R_{2\perp} \) – generating surface’s curvature radii in two mutually perpendicular normal sections; \( R_{k\perp} \) – radius of geodesic torsion of the line along unit tangent line vector \( \mathbf{c} \); \( \mathbf{c} \) – unit tangent line vector to the generating surface which sets first direction (curvature radius in it is equal to \( R_{1\perp} \)). It shall be noted that if unit vector \( \mathbf{c} \) follows one of the main directions at the generating surface, then \( R_{k\perp} = \infty \), i.e. geodesic curvature \( k_{k\perp} = 1/R_{k\perp} = 0 \). Hence it is strongly recommended to define a tangent line \( \mathbf{c} \) along one of the main directions at the generating surface: there will be no need to calculate \( R_{k\perp} \) or \( k_{k\perp} \).

Regarding direction of unit normal vector \( \mathbf{n} \) and signs of curvature and torsion radii:

- a) unit normal vector \( \mathbf{n} \) is always directed out of the generating element field;
- b) \( R_{1\perp} \) and \( R_{2\perp} \) are positive for convex surfaces; c) \( R_{k\perp} > 0 \), if while moving along unit vector \( \mathbf{c} \) unit normal vector \( \mathbf{n} \) creates a left-handed screw.

Below are formulae for calculation of all the parameters required for indentation acceleration calculation by formula (5). Though, the common known formulae for calculation of unit normal vector \( \mathbf{n} \) and curvature radii \( R_{1\perp} \) and \( R_{2\perp} \) through the given equation of generating surface are not shown. The coordinates conversion formulae for generic gearing in Figure 2 are not provided either.

4.1. Formulæ dependent on the type of gearing

Below see them for Figure 2.

Relative angular velocity vector \( \mathbf{\omega}_{12} \) in coordinates \( X_hY_hZ_h \):

\[
\mathbf{\omega}_{12} = \{\omega_1 \cdot \mathbf{i} - \omega_2 \cdot \sin \gamma \cdot \mathbf{j}, (\omega_0 - \omega_2 \cdot \cos \gamma) \cdot \mathbf{k}\} \quad (6)
\]

Relative linear velocity vector \( \mathbf{V}_{12} \) in coordinates \( X_hY_hZ_h \):
While analyzing two-parameter gearings, it is required to calculate twice by formulae (6)-(8), estimating velocities and accelerations upon each enveloping parameter. Accordingly, while analyzing \( n \)-parameter gearings one should estimate velocities and accelerations upon \( n \) number of values. It is possible to apply formulae (6) – (8) for particular gearings having removed items containing zero accelerations, velocities and displacements. E.g. for all the right-angle gearings (hypoid, worm, globe, spiroid, etc), under: \( \gamma = 90^\circ \), \( \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = a_1 = a_2 = a_3 = 0 \), \( \omega_1 = 1 \), \( \omega_1 = \pm \omega_2 \) (here sign is “-“) if direction of one of the velocities \( \omega_1 \) or \( \omega_2 \) does not follow the direction specified in Figure 2), \( \omega_2 = V_1 = V_2 = V_3 = 0 \), \( S_3 = a_w \), we will obtain, instead of equations (6)-(8), a simpler formulae:

\[
\begin{align*}
\omega_1^{(h)} &= [0 \cdot \text{i} \ 0 \cdot \text{j} \ i_{12} \cdot \text{k}] \\
\omega_2^{(h)} &= [0 \cdot \text{i} \ 1 \cdot \text{j} \ 0 \cdot \text{k}] \\
\omega_3^{(h)} &= [0 \cdot \text{i} \ -1 \cdot \text{j} \ i_{12} \cdot \text{k}]
\end{align*}
\]

\[
V_{12}^{(h)} = \{-i_{12} \cdot y + y_p \cdot \text{i} \ i_{12} \cdot x \cdot \text{j} \ x_p \cdot \text{k}\}
\]

\[
a_{12} = [\omega_1 \times (\omega_1 \times r_1) \ - \ \omega_2 \times (\omega_2 \times r_2)] - (\omega_1 + \omega_2) \times V_{12}
\]

here:

\[
r_1^{(h)} = \begin{bmatrix}
\cos \phi_1 & -\sin \phi_1 & 0 \\
\sin \phi_1 & \cos \phi_1 & 0 \\
0 & 0 & 1
\end{bmatrix} \\
r_2^{(h)} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

under \( \phi_2 = \phi_2 + \phi_1/i_{12} \); where \( \phi_1 \) – enveloping parameter.

Similarly it is possible to obtain simplified formulae for spur, rack and other gearings, as well as for cam control assemblies and machine-tool gearings. Thus, for flat rack gearing, assuming: \( \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = a_1 = a_2 = a_3 = 0 \), \( \gamma = 90^\circ \), \( \omega_2 = 1 \), \( V_1 = -a_w \), \( \omega_1 = \omega_3 = V_2 = V_3 = 0 \), \( S_3 = a_w \), we obtain:

\[
\omega_1^{(h)} = 0 \\
\omega_2^{(h)} = [0 \cdot \text{i} \ 1 \cdot \text{j} \ 0 \cdot \text{k}] \\
\omega_3^{(h)} = [0 \cdot \text{i} \ -1 \cdot \text{j} \ 0 \cdot \text{k}]
\]

\[
V_{12}^{(h)} = \{ y_p \cdot \text{i} \ 0 \cdot \text{j} \ (x_p - a_w) \cdot \text{k}\}
\]

\[
a_{12} = -\omega_2 \times (\omega_2 \times r_2) - \omega_2 \times V_{12}
\]

Also formulae (12) might be used assuming \( \phi_1=0 \); and setting in formulae coordinates conversion \( \varphi_2 = \varphi_2 + S_1/a_w \), where \( S_1 \) – enveloping parameter.

4.2. Formulae non-dependent on the type of gearing

By them we calculate roll motion angular velocities \( \omega_k \), and \( \omega_k^\bot \), parts of the equation (5). Roll motion angular velocity \( \omega_k \) is an angular velocity of unit normal vector \( \text{n} \) rotation to the generating surface relative to the generated surface measured in the plane set by vectors \( \text{n} \) and \( \text{c} \). It is calculated by formula:

\[
V^{(h)}_{12c} = V_3 - \omega_1 \cdot y_h + \omega_2 \cdot y_p; \\
V^{(h)}_{12y} = -V_2 \cdot \sin \gamma + \omega_1 \cdot x_h - \omega_2 \cdot x_p \cdot \cos \gamma - \omega_3 \cdot z_h; \\
V^{(h)}_{12z} = V_1 - V_2 \cdot \cos \gamma + \omega_2 \cdot x_p \cdot \sin \gamma + \omega_3 \cdot y_h.
\]
All three items in the central part of the formula (16) are rotations in the plane of the normal section passing vectors \( \mathbf{n} \) and \( \mathbf{c} \). First item is a relative angular velocity vector projection \( \mathbf{\omega}_{12} \) onto the normal to the sectional plane, i.e. normal rotation in this plane defined by relative rotation of the components. Second item considers normal rotation towards the surface caused by sliding of the bodies in the normal section plane due to surface curvature at this section. Second item considers normal rotation towards the surface caused by sliding of the bodies in the direction perpendicular to the normal section plane and defined by geodesic torsion of the line at the surface.

Roll motion angular velocity in the plane perpendicular to the unit vector \( \mathbf{c} \):

\[
\mathbf{\hat{\omega}^c} = \left( -\mathbf{\omega}_{12} + \frac{\mathbf{V}_{12}}{R_K} \right) \cdot \mathbf{c} + \frac{\mathbf{V}_{12}}{R_K} \cdot (\mathbf{c} \times \mathbf{n})
\]  

(17)

While analyzing two- and n-parameter gearings, it is required to calculate two or n times by formulae (16)-(17), estimating roll motion velocities upon each enveloping parameter.

5. Principle of envelope points estimation using kinematic method.

Indention Shape-generation is a process of interaction between generating element’s body surface \( \Sigma_0 \) and space \( g_2 \); this body is moving in. If another body – “item” is placed in such space \( g_2 \), then moving generating element running through the space will form on this body a surface \( \Sigma_2 \) which is an envelope belonging to surfaces group \( \Sigma_0 \) – boundaries of the generating element.

The points contained in generated surface \( \Sigma_2 \) are the points of surface \( \Sigma_0 \) of the generating body, in which indention acceleration \( V_{BH} = 0 \). Gearing equation has to be solved in order to estimate these points. If one of the curvilinear coordinate \( u \) or \( v \) is being estimated through gearing equation, it means calculation of the action point at the surface \( \Sigma_0 \) under the set value of enveloping parameter. If enveloping parameter \( t \) is being discovered, then it is estimation of surface \( \Sigma_0 \) displacement under which point given at \( \Sigma_0 \) would become action point. Nevertheless in any case estimation of the point at surface \( \Sigma_2 \) starts with solution of the gearing equation.

A markedly different method of envelope estimation, i.e. estimation of points at \( \Sigma_2 \), is considered in the current paper. Its specifics are as follows:

- Points at \( \Sigma_2 \) are estimated using direct method – without solving a gearing equation.
- For estimation of the “conjugate” point’s coordinates at \( \Sigma_2 \) one needs to know only: a) point’s coordinates at the surface \( \Sigma_1 \) (same as \( \Sigma_0 \)); b) unit normal vector projections to \( \Sigma_1 \) at this point; c) indention velocity and acceleration \( (V_{BH} \text{ and } a_{BH}) \) at this point.
- “Conjugate” points at the generated surface shall be found in the same system of coordinates where the parameters mentioned in the previous paragraph are set.
- The method is applicable to all the types of surface shape generations involving kinematic method: For one-, two-, and \( n \)-parameter motion of the generating element represented by surface, line or point.
- The method is approximate; the closer the datum point at the surface \( \Sigma_1 \) to the action point and to the generated surface \( \Sigma_2 \), the higher is accuracy of the method.

For the principle of point estimation at the surface \( \Sigma_2 \) formed by one-parameter enveloping see Figure 3. In this figure:

- points \( A_1 \) and \( B_1 \) are set at the generating surface \( \Sigma_1 \), and are situated close to the action point \( K \);
- points \( A_2 \) and \( B_2 \) are the desired points at the generated surface situated, accordingly, “opposite to” points \( A_1 \) and \( B_1 \);
- \( \mathbf{n}_1^{(A)} \) and \( \mathbf{n}_1^{(B)} \) – unit normal vectors to \( \Sigma_1 \);
- \( V_{BH} \) and \( a_{BH} \) – indention velocities and accelerations
- \( \delta_A \) and \( \delta_B \) – distances between the desired points \( A_2 \) and \( B_2 \) and datum points \( A_1 \) and \( B_1 \).
Near the point $B_1$ shape generation of the surface $\Sigma_2$ is ongoing by means of surface $\Sigma_0$ “indenting” towards normal vector ${\mathbf n}_1^{(B)}$. Point $A_2$ at $\Sigma_2$ is already generated, and surface $\Sigma_0$ remoted from the point $A_2$ at distance $\delta_A$ is moving away from it at the velocity $V_{BH}^{(a)}$. Let us consider (due to small distances $\delta_A$ and $\delta_B$) the “indenting” motion at the interval $\delta_B$ and separation motion at the interval $\delta_A$ as uniformly variable motions, i.e. $a_{BH} = \text{const}$. These intervals $\delta$, as well as time $\Delta t$ of passing them, are estimated by formulae for uniformly variable motion:

$$\delta = \frac{V_{BH}}{2 \cdot a_{BH}}$$

$$\Delta t = \frac{V_{BH}}{a_{BH}}$$

6. Algorithm of envelope points estimation using kinematic method.

Each In order to assess accuracy of the method, the process of spur gear involute profile generation by rectilinear-face rack was studied. Such gearing was selected as a study object because for involute it is not difficult to deduce formulae for estimation of distances measured from theoretical envelope represented by the involute itself. In the software engineered for simulation of this shape generation process the above mentioned algorithms and formulae have been implemented. Computations were made for the gear ring with reference diameter $d = \text{const} = 1000\ \text{mm}$. First stage involved analysis of three parameters’ influence over the height of the wraparound jags above the involute. $z$ – number of teeth; $k$ – number of motions (cuts), performed by the rack when the gear wheel is rotated by one tooth; $\rho_{Evol}$ – involute curvature radius defining point’s position at the gear wheel. The dependencies found are shown in the Figure 4.

It is apparent that: a) at the reference cylinder wraparound jags deflections from the involute are defined by dependence $\mu\text{m}$, under $d = 1000\text{mm}$; b) near the top circle under small teeth number it is four times more, and near the base circle it is 15 times less than at the reference circle.

Second stage involved study of the same three parameters ($z$, $k$, $\rho_{Evol}$)’ influence to the deflection of the points estimated using kinematic method from the involute. See Figure 5 for the results of this analysis.
Maximum deflections from involute, µm

accuracy of the proposed method, micron

Figure 5. Wraparound Σ1 deflections from the involute Σ2

Following is concluded based on this figure: a) maximum involute deflections of the estimated points almost do not depend on point situation at it; b) maximum involute deflections follow the dependency – see trends in the right part of the Figure 5; c) application of the above described kinematic method as a “postprocessor” after non-differential methods of envelop estimation (through wraparound) enables re-duction of maximum envelope deflections of the estimated points by 1-3 orders of mag-nitude, i.e. 10-1000 fold.

Regarding reliability of the suggested method: It complies with reliability of non-differential methods if one condition is met: there shall not be any elements (strips or hollows) with very little curvature radius at the wraparound surface. An element with a very little curvature radius is the element at the wraparound (and generating), on which between two neighboring edges or peaks the angle between normal lines toward this ele-ment exceeds 3-4 degrees. Elements with a very little curvature radius are represented, namely, by all the jags at generating surface. For this reason the method is not applicable to analysis of fillets formed by jags. It shall be noted that the estimation method for wraparounds formed by jags is described in [15, 17]. That method has been tested by Doctor of engineering sciences E.S. Trubachyov in the system “SPDIAL+”, and the re-sults confirmed high reliability with calculation time reduced by more than two orders of magnitude.

Acknowledgment

This work was supported by grant (project № 9.6355.2017/БЧ) of government order of Ministry of education and science of Russia Federation for the period 2017–2019 in Tyumen Industrial University.

References

[1] Sheveleva G I 1999 Theory of formation and contact of moving bodies, Mosstankin
[2] Gokhman H I 1886 Theory of gearing, generalized and developed by analysis, Diss. Master of Mechanics, Odessa
[3] Kolchin N I 1949 Analytical calculation of flat and spatial links (with application to the profiling of the cutting tool and the calculation of errors in the links), Mashgiz, 210 p
[4] Shishkov V A 1951 Formation of surfaces by cutting by generating method, Mashgiz
[5] Davydoiva Ya S 1950 Non-involute gearing, Mashgiz
[6] Litvin F L 1986 Theory of Gearing, Nauka
[7] Erikhov M L 1972 Principles of systematics, methods of analysis and issues of the synthesis of gearing schemes. Doctoral Thesis
[8] Zalgaller V A 1975 Envelope theory, Nauka
[9] Goldfarb V I 2001 Some exercises with gearing equations, Space of gears. Collection of reports of the scientific seminar of the Educational and Scientific Center of Gears and Gear Design, Izhevsk – Elektrostal 20–24
[10] Babichev D T 2005 Development of theory of gearing and shaping of surfaces on basis of new geometric-kinematic representations, Tyumen, Doctoral Thesis

[11] Rama Krishna K and Dibakar S 2019 Motion Space of Contacting Smooth Curves in Plane Using Screw Derivative, In book: Advances in Mechanism and Machine Science: Proceedings of the 15th IFToMM World Congress on Mechanism and Machine Science, Mechanisms and Machine Science 73 669-680

[12] Nesmelov I P, Goldfarb V I 1983 Non-differential approach to solving envelope problem, Mechanics of machines 61 3–10

[13] Goldfarb V I, Lunin S, Trubachov E.S 2003 Advanced computer modeling in gear engineering, In book: Proceedings of ASME International Power Transmission and Gearing Conference, Chicago Illinois USA, September 2–6

[14] Langofer A R, Babichev D T, Raikhman G N, Shunaev B K 1986 Study on the computer load on the cutting edges of the gear cutting tool, Machines and tools 1 18 -19

[15] Babichev D T 2008 Acceleration of penetration and curvature in gears, Theory and practice of gears and engineering, sat. report scientific and technical conference with international participation Izhevsk 157–161

[16] Dusev E E 1969 Connection between the curvatures of the mutually-bounded surfaces of the teeth of spatial links, Mechanical Engineering 3

[17] Babichev D T 2004 On use of multiparameter envelopes in computer simulation of the processes of shaping in workers and technological gears Theory and practice of gears, Coll. report scientific and technical conference with international participation, Izhevsk 302–315