Constructing Non-Abelian Vortices with Arbitrary Gauge Groups

Tokyo Institute of Technology  Toshiaki Fujimori
E-mail: fujimori@th.phys.titech.ac.jp

Recently there has been a significant progress in the understanding of non-Abelian vortices in $SU(N) \times U(1)$ gauge theories. Although many interesting features have been extensively explored, most studies have been restricted to the gauge group $SU(N) \times U(1)$. We proposed a simple framework for writing the most general non-Abelian BPS vortex solutions in theories with an arbitrary gauge group of the type $G = G' \times U(1)$ [1]. Our model is a $(3+1)$-dimensional $\mathcal{N} = 2$ supersymmetric gauge theory coupled to $N_F$ Higgs fields $H$ in a representation $R$ of a rank $r$ simple Lie group $G'$ and charged under $U(1)$. If the VEV of the Higgs fields completely breaks the gauge symmetry, the topological charges of the vortex configurations are classified by $\pi_1((U(1) \times G')/\mathbb{Z})$, where $\mathbb{Z}$ is the center of $G'$. The boundary condition for the Higgs field is $H \to e^{i\alpha(\theta)}g(\theta)H$, $e^{i\alpha} \in U(1)$, $g \in G'$, where $\langle H \rangle$ is a VEV of the Higgs fields and $\theta$ is the angular coordinate parameterizing large $S^1$ at spatial infinity. In general, the elements of the gauge groups $e^{i\alpha}$ and $g$ can be written as $e^{i\alpha}g = \exp[i(k/n_0 + \vec{\nu} \cdot H)\theta]$, where $k$ is the vortex number, $n_0$ is the order of the center $Z$ and $\vec{H}$ is an $r$-vector of the generators of the Cartan subalgebra of $G'$. The $r$-vector $\vec{\nu}$ should satisfy the condition $k/n_0 + \vec{\nu} \cdot \vec{\mu} \in \mathbb{Z}$ with $\vec{\mu}$ being the weight vectors of the representation $R$. That is, $\vec{\nu}$ should be an element of the coweight lattice, which is identified with the weight lattice of the GNO (or Langlands) dual group $L^{\dagger}G'$.

The BPS equations for the gauge fields and the Higgs fields $H$ can be rewritten in terms of new variables $S_e(z, \bar{z}), S'_e(z, \bar{z})$ and $H_0(z)$, where $S_e$ and $S'_e$ are elements of the complexified gauge groups $U(1)^C$ and $G'^C$ respectively and $H_0$ is related to $H$ as $H_0 = S_eS'H$. From one of those equations, we find that $H_0$ should be holomorphic in the complex coordinate $z$ parameterizing the plane perpendicular to the vortex string. Once $H_0(z)$ is given, the other equations are specified and $S_e$ and $S'_e$ can be determined by solving those equations. Then, the BPS solution can be obtained by the inverse transformation to the original fields. The complexified transformations $V_e(z) \in U(1)^C$ and $V'_e(z) \in G'^C$ which act on $S_e$, $S'_e$, $H_0(z)$ do not change the original fields. Therefore, sufficient and necessary information for specifying the BPS vortex configuration is the equivalence class of $H_0(z)$ and the parameters contained in $H_0(z)$ parameterize the moduli space of vortices. Our method gives a powerful tool to study the moduli space of vortices.

References

[1] M. Eto, T. Fujimori, S. B. Gudnason, K. Konishi, M. Nitta, K. Ohashi and W. Vinci, arXiv:0802.1020 [hep-th].