Application of topological optimisation methodology to finitely wide slider bearings operating under incompressible flow

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Abstract
The search for the optimal bearing geometry has been on for over a century. In a publication from 1918, Lord Rayleigh revealed the infinitely wide bearing geometry that maximises the load carrying capacity under incompressible flow, i.e. the Rayleigh step bearing. Four decades ago, Rohde, who continued on the same path, revealed the finitely wide bearing geometry that maximises the load carrying capacity, referred to as the Rayleigh-pocket bearing. Since then, the numerical results have been perfected with highly refined meshes, all converging to the same Rayleigh-pocket bearing. During recent years new methods for performing topology optimisations have been developed and one of those is the method of moving asymptotes, frequently used in the area of structural mechanics. In this work, the method of moving asymptotes is employed to find optimal bearing geometries under incompressible flow, for three different objectives. Among the results obtained are (i) show new bearing geometries that maximise the load carrying capacity, which performs better than the ones available, (ii) new bearing geometries minimising the coefficient of friction and (iii) new bearing geometries minimising the friction force for a given load carrying capacity are presented as well.

Keywords
Hydrodynamic lubrication, topological optimisation, Reynolds equation, slider bearing, pocket bearing, Method of moving asymptotes

Introduction
The bearing geometry is of highest importance for optimal performance, such as the film thickness and efficiency of the bearing. Indeed, an increased load carrying capacity (LCC) will result in an increased film thickness, thereby decreasing the risk of wear. In turn, less wear increase useful life and the need for replacements, thereby reducing costs. A reduced coefficient of friction (COF) decreases friction losses, thereby improving the efficiency of the bearing, and ultimately increasing the productivity or decreasing resources needed for running the system, thereby also reducing costs.

The quest of finding the optimal bearing geometry started decades ago with an early publication by Rayleigh, dating back to 1918. By application of calculus of variation, Rayleigh analytically optimised the bearing geometry of an infinitely wide slider operating under incompressible flow with the objective of maximising the LCC. The result was a step-bearing geometry, nowadays referred to as the Rayleigh step bearing. The drive of finding the optimum bearing geometry continued and half a decade later, Maday developed a new method and optimised the geometry of a bearing operating under (compressible) ideal gas flow. The resulting bearing geometry that Maday found, also featured the characteristic and flat trailing land of the Rayleigh step bearing, but with and but with a converging section instead of a flat leading land. Rohde was the first one to consider a new type of objective when optimising the bearing geometry, with the goal set to find the bearing geometry that under incompressible flow minimises the COF. The result was a bearing geometry with a flat leading- and trailing lands, but with a tapered instead of the vertical step in the Rayleigh step-bearing geometry. Later, Auloge et al. optimised a bearing geometry
for a non-Newtonian fluid flow, this resulted in a step-bear- ing geometry, with the step location and height different from the Rayleigh step bearing. Boldyrev later optimised a bearing geometry subjected to a compressible ideal gas flow using a model with periodic boundary conditions, the result showed similarities with the one found by Maday. These findings on infinitely wide sliders are especially interesting where application is suitable, i.e. where no side leakage occurs.

For many applications, the side leakage is too large to be disregard and optimisation has naturally shifted focus to infinitely wide sliders. Kettleborough presented a finitely wide pocketed geometry, performing well in terms of LCC. In the same article he also presented a geometry with an internal fluid entrapment, which reduces the shear stresses in the fluid film. Rohde, optimised parameterised pocket bearings with piece-wise constant film thickness for maximised LCC, while considering different types of pockets. A few years later, Rohde and McAllister continued the work on optimising for maximum LCC using an FDM approach, the method allowed every discrete point on the mesh to take any value of the film thickness, independently. They found the bearing geometry that maximise the LCC under incompressible flow, i.e. the Rayleigh-pocket bearing. Robert optimised bearing geometries with piece-wise constant film thickness operating under ideal gas flow. The results were pocketed type of sliders. The same year, Boldyrev et al. maximised the LCC, considering ideal gas flow. This resulted in a pocketed type of bearing, having an increased tapering on the leading edge compared to the Rayleigh-pocket bearing. Later, Buscaglia et al. found the bearing geometry that maximises LCC for a bearing exhibiting Burgdorfer gas flow, where the method was also demonstrated to work on journal bearings. Buscaglia et al. implemented a new method for optimising bearing geometries for maximum LCC and demonstrated its applicability by replicating the work by Rohde and McAllister. The result was obtained using a mesh having about 50 times more elements than in 1976, the generated optimal bearing geometry was smoother and the LCC was also slightly increased both as a result from the increased mesh resolution. Just a couple of years later, van Ostayen developed a new method for obtaining the optimal bearing geometry. As a benchmark, he too replicated the work by Rohde and McAllister, but with a mesh having 750 times more elements. As expected, he found that with a refined mesh the LCC further increased. It was also demonstrated that the method could be used to obtain bearing geometries for different objectives and changes made in the computational domain. A few years later, Fesanghary and Khonsari found the axial bearing geometry that maximises the LCC for an incompressible fluid. Results based both on having a piece-wise constant- and a variable film thickness were shown, and the geometries found exhibited a cylindrically shaped Rayleigh-pocket bearing characteristics. The aforementioned works show that there are various methods for fining optimal bearing geometries.

Topology optimisation comprises a class of methods within the field of structural mechanics, frequently used for obtaining light weight structures while minimising strain energy. An advantage with topology optimisation is that the control variable can be more or less arbitrary defined, when maximising or minimising an objective function. In the field of tribology and more specifically in the branch of hydrodynamically lubricated slider bearings, topology optimisation methodology has not been applied that frequently. There are several different topology optimisation methods, e.g. in the field of structural mechanics Jakiela et al. demonstrated the use of a genetic algorithm, and Di Cesare and Domaszewski used a particle swarm optimisation method, both considering the same type of problem. The topology optimisation method globally convergent method of moving asymptotes (GCMMA) is available in COMSOL Multiphysics and was developed by Svanberg. This method can be used in structural analysis and recently, Kalliorinne et al. used it to optimise the bearing geometry of infinitely wide sliders for maximum LCC as well as minimum COF. Kalliorinne et al. numerically replicated the works in literature verifying a new method for obtaining existing optimal bearing geometries, and used it to find the bearing geometries that minimise the COF when exhibiting compressible ideal gas flow. They also unveiled the bearing geometries that maximise the LCC and the bearing geometries that minimises the COF, for fluids exhibiting a constant bulk modulus behaviour. In parallel an analytic approach was performed by Almqvist et al. for fining the bearing geometries that maximises the LCC for fluids exhibiting a constant bulk modulus.

It has come to the authors’ knowledge that there is yet no solution for the finite-bearing geometry that minimises the COF under incompressible flow. The authors have also discovered that when applying GCMMA method for optimising finite bearing geometries under incompressible flow, obtained bearings’ geometries maximising LCC outperform previously found bearing geometries. The obtained bearing geometry also has substantial differences to the previous bearing geometry. Here optimisation of bearing geometries which is subject to a incompressible flow formulated as the classical Reynold’s equation using the GCMMA method inside COMSOL Multiphysics, the objective will be to either maximise the LCC or minimise the COF.

**Governing equations**

In this work, the Reynolds equation is adopted to govern the thin film flow of an incompressible fluid...
under steady conditions, inside a rectangular 2D computational domain $\Omega$ bounded by $\partial \Omega$ and reads

$$\nabla \cdot \left( \frac{h^3}{12 \eta} \nabla p - \frac{f u}{2} \right) = 0 \quad \text{in} \quad \Omega$$

$$p = p_a \quad \text{on} \quad \partial \Omega$$

where $h$ is the film thickness, $\eta$ is the lubricant viscosity, $p$ is the pressure with ambient pressure $p_a$ at the boundaries and $u$ is the speed of the moving surface. The friction force (FF) $f$ acting on the bearing surfaces can be obtained by integrating the shear stresses and reads

$$f = \int \int_{\Omega} \frac{h}{2} \nabla p \pm \frac{\eta u}{h} d\Omega$$

(2)

where the plus and minus sign indicates that the force is evaluated on the lower moving surface or the upper stationary surface. The LCC $w$ of the bearing reads

$$w = \int \int_{\Omega} p - p_a d\Omega$$

(3)

where the ambient pressure $p_a$ is subtracted to obtain the net LCC. Using equations (2) and (3), COF $\mu$ can be calculated as

$$\mu = \frac{f}{w}$$

(4)

The computational domain can be non-dimensionalised as $\hat{\Omega}$, and in terms of the dimensionless variables $\hat{x} = x/x_r$ and $\hat{y} = y/x_r$, the bearing surface that will be considered is bounded by $-0.5 \leq \hat{x} \leq 0.5$ and $0 \leq \hat{y} \leq 1$. By also scaling $\hat{h} = h/h_r$ and $\hat{p} = (p - p_a)/p_r$, the governing Reynolds equation (1) can now be non-dimensionalised and stated as

$$\nabla \cdot (\hat{h}^2 \nabla \hat{p} - \hat{h} \Lambda) = 0 \quad \text{in} \quad \hat{\Omega}$$

$$\hat{p} = 0 \quad \text{on} \quad \partial \hat{\Omega}$$

(5)

where $\hat{h}$ is the dimensionless film thickness, $\hat{p}$ is the dimensionless pressure and the bearing number $\Lambda = 6 \eta u x_r/(p_r h_r^2)$. In this study, the flat surface will be considered to be moving in the $x$-direction only. Thereby, the velocity can be expressed as $u = [0, U]$. The LCC $w$ formulated in equation (3) can also be non-dimensionalised and reads

$$\hat{w} = \frac{w}{p_R x_r^2} = \int \int_{\hat{\Omega}} \hat{p} d\hat{\Omega}$$

(6)

The FF $f$ formulated in equation (2) also needs to be non-dimensionalised and in terms of the present scaling it becomes

$$\hat{f} = \frac{f}{h_p x_r} = \int \int_{\hat{\Omega}} \frac{\hat{h}}{2} \nabla \hat{p} + \frac{\Lambda}{6 \hat{h}} d\hat{\Omega}$$

(7)

Now the scaled COF $\hat{\mu}$ can be written in terms of $\mu$ and it reads

$$\hat{\mu} = \frac{x_r}{\hat{h}_r} \mu = \frac{\hat{f}}{w}$$

(8)

Explicitly, it can be expressed as

$$\hat{\mu} = \frac{\int \int_{\hat{\Omega}} \frac{\hat{h}}{2} \nabla \hat{p} + \frac{\Lambda}{6} d\hat{\Omega}}{\int \int_{\hat{\Omega}} \hat{p} d\hat{\Omega}}$$

(9)

### Description of the numerical approach

Prior to this study, Kalliorinne et al.\(^{22}\) used the finite element (FE) software COMSOL Multiphysics® to conduct topology optimisations on infinitely wide sliders’ geometries. In Kalliorinne et al.,\(^{22}\) the Reynolds equation was modelled using the coefficient form partial differential equation physics interface, and the GCMAA-based optimisation algorithm was setup inside the optimisation physics interface, with the film thickness as the control variable. Here, the governing equation (5) and the optimisation will be setup in a similar way. To this end, the control variable will be specified as the dimensionless film thickness $\hat{h}$, bounded by a lower and an upper limit, i.e.

$$\hat{h}_{\text{lower}} \leq \hat{h} \leq \hat{h}_{\text{upper}}$$

(10)

where $\hat{h}_{\text{lower}} = 1$ and $\hat{h}_{\text{upper}}$ is selected to be about 10 times larger than the thickest point of the film thickness inlet. The upper limit can be readily obtained from running the optimisation with a coarse mesh. However, both when maximising the LCC and minimising the COF, the result will include areas where the film thickness saturates at the upper limit $\hat{h}_{\text{upper}}$ regardless of the value specified.

The optimisation starts from a specification of an initial bearing geometry $\hat{h}_0$, that was specified as a tapered bearing and it was found using a tapered bearing geometry

$$\hat{h}_0 = 2 - \frac{y}{\hat{y}}$$

(11)

as initial design was more stable than using a step-bearing geometry, which was previously used by Kalliorinne et al.\(^{22}\). The symmetry that appears about the $y$-axis can be used to reduce the computational expenses. More precisely, this is done by taking $0 \leq \hat{x} \leq 0.5$ and $0 \leq \hat{y} \leq 1$ as the domain of symmetry, $\Omega^*$, and by adding a Neumann boundary condition along the $y$-axis denoted by $\partial \Sigma$. The boundary conditions for equation (5) now read

$${\partial \hat{h} \over \partial \hat{y}} = 0 \quad \text{on} \quad \partial \Sigma^*$$

(12)
Note that the symmetry needs to be considered before calculating the LCC and the FF, as specified by equations (6) and (7), respectively.

Results and discussion

Next, the results will be presented and discussed. It starts with optimisation for maximum LCC, then for minimum COF, followed by a comparison between the different objectives. In the end, results for minimising the FF are presented. In the next subsection, results consisting of bearing geometries, pressure distributions and shear stresses from Poiseuille- and Couette flow, for three different meshes when maximising the LCC will be presented. In the subsequent subsection, results from minimising the COF will be presented in a similar way. It will be continued in the following subsection, by a comparison of using the two different objectives, i.e. maximising LCC and minimising COF, presented as a mesh convergence study. In connection to this results from the previous works, \(9,14,15\) where maximisation of LCC as been addressed, are also included in the discussion. Lastly, in the final subsection, results for minimising the FF, for three markedly different choices of LCC, will be presented.

Maximising the LCC

Here, the results from maximising LCC are presented. The results are partitioned into three differently refined meshes with number of elements \(E\), i.e. \(E = 2^{11}\), \(E = 2^{15}\) and \(E = 2^{19}\). Depicted in Figure 1 are the bearing geometries that maximise LCC. The colour legend shows the film thickness and it applies to all three of the geometries. Notice that, the areas in white are regions above the limit of the colour legend. The reason for this is that the film thickness in these regions saturates at the value of the upper limit \(h_{\text{upper}}\), irrespective of the value chosen for it. Looking at the bearing geometry in Figure 1(a), one can see a geometry which resembles the classical solution for a Rayleigh-pocket bearing. That is, a geometry with large fluid intake converging in width and depth towards the trailing edge, surrounded by a step down to the flat land, which was also found in references.\(^9,14,15\) In Figure 1(b) and (c), much more details are revealed when the mesh is refined, and a shark-fin like pattern appears at the back end of the pocket. These ‘shark fins’ direct the fluid towards the middle and it can be seen that the number of ‘shark fins’ depends on the mesh resolution. Mesh dependence is a known phenomenon in topology optimisation, which has been addressed before, see e.g. Sigmund and Petersson.\(^{24}\)

Depicted in Figure 2 are the same bearing geometries as in Figure 1, but here with contour lines included to elucidate upon the height variations within the pocket. Here we can see that the mesh refinement only marginally affects the large scale geometry features of the pocket. When looking at the region where the shark fins are located there is, however, significant differences. Indeed, expect for the fine details of the shark fins themselves, the film thickness increases within the shark fins as well. Moreover, the optimal shape actually assumes the upper limiting value \(h_{\text{upper}}\), at the tip of the fins.

Depicted in Figure 3 are the pressure distributions corresponding to the geometries in Figures 1 and 2. The figure shows that, the maximum pressure increases as the mesh is refined, and the contribution from the shark fins can clearly be seen as well.

Figure 4 depicts the Poiseuille-flow contribution to the shear stress distributions, i.e. the contribution from the first term in the expression for the dimensionless FF (7), for the bearing geometries in

![Figure 1. The bearing geometries that maximise the LCC for three meshes with increasing resolution from left to right. (a) \(E = 2^{11}\); (b) \(E = 2^{15}\); (c) \(E = 2^{19}\).](image)
Figure 2. The bearing geometries that maximise the LCC for three meshes with increasing resolution from left to right, with contours levels. (a) $E = 2^{11}$; (b) $E = 2^{15}$; (c) $E = 2^{19}$.

Figure 3. The pressure distribution corresponding to the bearing geometries depicted in Figures 1 and 2, which maximises the LCC for three meshes with increasing resolution from left to right. (a) $E = 2^{11}$; (b) $E = 2^{15}$; (c) $E = 2^{19}$.

Figure 4. The Poiseuille-flow shear stress contribution corresponding to the bearing geometries depicted in Figures 1 and 2, which maximises the LCC for three meshes with increasing resolution from left to right. (a) $E = 2^{11}$; (b) $E = 2^{15}$; (c) $E = 2^{19}$. 
Figures 1 and 2. As expected, the largest contribution comes from the pocket, where the Poiseuille-flow shear stress also takes a more or less constant value. Moreover, it shows that the shear stress at the trailing edge is negative, as the fluid here is pushed out by the pressure in the same direction as the speed of the slider. The large negative values act over a very small area around the edges of the shark fins.

Figure 5 depicts the Couette-flow contribution to the shear stress distributions, i.e. the contribution from the second term in the expression for the dimensionless FF (7), for the bearing geometries in Figures 1 and 2. The figure clearly shows that the major part of the Couette-flow shear stress component is generated at the flat trailing land area. It is also clear that the Couette- dominates over the Poiseuille flow, when comparing the amplitude of the corresponding shear stress distributions.

Minimising the COF

Here, the results from minimising the COF are presented. The results are given for the same three mesh resolutions that were used when maximising the LCC, i.e. \( E = 2^{11}, E = 2^{15} \) and \( E = 2^{19} \). Depicted in Figure 6 are the bearing geometries that minimise the COF. The colour legend shows the film thickness and it applies to all three of the geometries. Once again, the areas in white are regions above the limit of the colour legend. The bearing geometries depicted in Figure 6 have pockets, and they show some similarity to the geometries obtained when maximising the LCC. The key differences are (i) the redesigned pocket topology and that the step down to the trailing land at the far end of the pocket is now tapered, (ii) the trailing land without any shark fins and (iii) that material has been removed from the corners at the back end of the bearing. What is also an
obvious difference when increasing the mesh resolution is the appearance of an internal fluid entrapment area at the back end of the pocket, and the enlargement and refinement of the cutout corners. It is also observed that the pit at the corner of the leading edge is more pronounced in this geometry, compared to the bearing geometries maximising the LCC. The depth of the internal fluid entrapment area is limited by the upper limit ($h_{\text{upper}}$) of the control variable. Moreover, compared to the geometries maximising the LCC, the deep corners at the pocket’s leading edge become more pronounced and the deepest part also assumes the $h_{\text{upper}}$ value.

The bearing geometries which are minimising the COF are also depicted in Figure 7, but here with height contours added. When comparing the geometry in Figure 7(a) (obtained using the coarsest mesh) with geometries in Figure 7(b) and (c), one can see that there are less contours crossing the y-axis in the former, which exhibits a tapered type of pocket without the internal fluid entrapment area at the back end. This tells us that the pocket becomes less steep, as the internal fluid entrapment area develops. For the intermediate mesh resolution, $E = 2^{15}$, there is a feature connecting the back end corner of the pocket, with the pocket’s leading edge. When the resolution is increased further, this feature vanishes and the geometry converges to the one depicted in Figure 7(c).

The pressure distributions corresponding to the geometries in Figures 6 and 7 are depicted in Figure 8. What stands out the most here is the area of the uniform pressure which appears as the internal fluid entrapment region develops when increasing the mesh resolution. The effect associated with the material removal of the upper corners can also be seen. It is expressed by the pressure being zero there, thus effectively moving the boundary condition to the edge of the cutout region.

Figure 9 depicts the Poiseuille-flow shear stress distributions for the bearing geometries in Figures 6 and 7. As for the bearing geometries maximising LCC, the largest contribution comes from the pocket, and the Poiseuille-flow shear stress takes a more or less constant value for the COF minimised geometries as well. Moreover, it shows that the shear stress at the trailing edge is negative here too. Here the benefits of having an internal fluid entrapment area are also shown. This is demonstrated by the shear stress being zero, as a result of that the pressure derivative is zero. This is also true for the corners, where materials have been removed.

Figure 10 depicts the Couette-flow shear stress distributions for the bearing geometries in Figures 6 and 7. The figure clearly shows that the major part of the Couette-flow shear stress component comes from the flat land. Having the internal fluid entrapment area and the material removal at the corners are clearly shown beneficial here, as the Couette-flow shear stress almost vanishes, due to the substantial film thickness increase.

**Comparison between LCC and COF as objectives**

In this section, the performance of the bearing geometries will be presented as a mesh-refinement study. A comparison will be made between results obtained with geometries that either maximise the LCC or minimise the COF. Results available in the literature will also be included in the comparison. The results depicted in Figure 11(a) are the LCC of the bearings optimised for maximum LCC – blue line with circle markers and minimum COF – red line with triangular markers. In addition, results presented in the previous works,9,14,15 where maximisation of LCC were performed with other methods, are also included – black circles. The optimisation method used by Rohde and McAllister9 does result in a better
Figure 8. The pressure distributions corresponding to the bearing geometries depicted in Figures 6 and 7, which minimise the COF for three meshes with increasing resolution from left to right. (a) $E = 2^{11}$; (b) $E = 2^{13}$; (c) $E = 2^{19}$.

Figure 9. The Poiseuille-flow shear stress distribution corresponding to the bearing geometries depicted in Figures 6 and 7, which minimise the COF for three meshes with increasing resolution from left to right. (a) $E = 2^{11}$; (b) $E = 2^{13}$; (c) $E = 2^{19}$.

Figure 10. The Couette-flow shear stress distribution corresponding to the bearing geometries depicted in Figures 6 and 7, which minimise the COF for three meshes with increasing resolution from left to right. (a) $E = 2^{11}$; (b) $E = 2^{13}$; (c) $E = 2^{19}$. 
performing bearing when the same amounts of elements are used. However, when the mesh is further refined, the LCC of the bearing geometry maximising LCC eventually surpasses all the previous works. To be noted, is that a uniform mesh was used in this study, while van Ostayen\textsuperscript{15} was using an adaptive meshing technique, expected to perform better, since it has the ability to resolve critical areas of the mesh more accurately. With similar element count, the present method does, however, render a geometry that performs better. Looking at the LCC for the bearing geometry minimising the COF, it is clear that the appearance of the internal fluid entrapment area results in a significant decrease in the LCC. The maximum pressures, as functions of the number of mesh elements, obtained with each of the optimisation setups are depicted in Figure 11(b). The maximum pressure for the both LCC-maximising and the COF-minimising geometries shows similar trends as the LCC in Figure 11(a). That is, the LCC-maximising geometries give rise to an increased maximum pressure as the mesh is refined, and the effect of the formation of the internal fluid entrapment area is also clearly visible in the results for the COF-minimising geometries.

A comparison between the performance of the absolute FF and the COF for geometries obtained

![Figure 11](image1.png)

**Figure 11.** Showing the mesh dependence of $\sqrt{C_2^2 w}$ and $\sqrt{C_2^2 p_{\text{max}}}$. For the LCC comparison, previous result obtained by Rohde and McAllister,\textsuperscript{9} Buscaglia et al.\textsuperscript{14} and van Ostayen\textsuperscript{15} are included. (a) Comparison of dimensionless load carrying capacity. (b) Comparison of dimensionless maximum pressure.

![Figure 12](image2.png)

**Figure 12.** Showing the impact on $|f|$ and $|\gamma|$ when the mesh is refined. (a) Comparison of dimensionless friction force. (b) Comparison of dimensionless coefficient of friction.
with optimisation for LCC and COF as objectives is depicted in Figure 12. As in Figure 11, the results are presented as a mesh-refinement study. Figure 12(a) shows that no obvious mesh dependence of the absolute FF can be observed for the geometries maximising the LCC (blue line with circle markers). The absolute FF for the bearing geometries minimising the COF does, however, decrease when the mesh is refined. This is expected, and due to the formation of the internal fluid entrapment region. In Figure 12(b), the mesh dependence of the predicted COF is shown, and here both the LCC-maximising and the COF-minimising geometries exhibit a decreasing trend. Again, the effect of the formation of the internal fluid entrapment region is clearly visible in the results for the COF-minimising geometries.

The data graphically illustrated in Figures 11 and 12, are presented in the Tables 1 and 2, respectively.

### Table 1. Results obtained when optimising for maximum LCC.

| E  | \( \tilde{\omega} \) | \( |\tilde{f}| \) | \( \bar{p} \) | \( \hat{p}_{\text{max}} \) |
|----|----------------|-------------|-----|-------------|
| 27 | 0.020796 | 0.13473 | 6.4786 | 0.054054 |
| 29 | 0.021051 | 0.13576 | 6.4147 | 0.055382 |
| 29 | 0.021164 | 0.13524 | 6.3901 | 0.056170 |
| 27 | 0.021217 | 0.13543 | 6.3831 | 0.056650 |
| 29 | 0.021246 | 0.13539 | 6.3725 | 0.057150 |
| 2 | 0.021268 | 0.13547 | 6.3697 | 0.058156 |
| 19 | 0.021277 | 0.13544 | 6.3656 | 0.058472 |

LCC: load carrying capacity.

### Table 2. Results obtained when optimising for minimum COF.

| E  | \( \tilde{\omega} \) | \( |\tilde{f}| \) | \( \bar{p} \) | \( \hat{p}_{\text{max}} \) |
|----|----------------|-------------|-----|-------------|
| 27 | 0.019738 | 0.12361 | 6.2625 | 0.049248 |
| 29 | 0.019932 | 0.12295 | 6.1685 | 0.050032 |
| 21 | 0.020032 | 0.12263 | 6.1217 | 0.050134 |
| 23 | 0.020747 | 0.12241 | 6.0979 | 0.050182 |
| 17 | 0.019696 | 0.11228 | 5.7006 | 0.059785 |
| 27 | 0.019583 | 0.11081 | 5.6585 | 0.058830 |
| 19 | 0.019542 | 0.11012 | 5.6350 | 0.058377 |

COF: coefficient of friction.

### Table 3. Results obtained when optimising for minimum friction force, for three different choices of LCC.

| \( \tilde{\omega} \) | E  | \( |\tilde{f}| \) | \( \bar{p} \) | \( \hat{p}_{\text{max}} \) | \( \hat{h}_{\text{min}} \) |
|-----|----|-------------|-----|-------------|-------------|
| 10^{-2} | 2^{15} | 1.9702 \times 10^{-2} | 1.9702 | 5.6446 \times 10^{-2} | 6.4995 \times 10^{-2} |
| 10^{-3} | 2^{15} | 4.5249 \times 10^{-3} | 6.2012 | 4.2417 \times 10^{-3} | 2.4274 \times 10^{-1} |
| 10^{-4} | 2^{15} | 2.3178 \times 10^{-3} | 23.178 | 5.0784 \times 10^{-4} | 7.6994 \times 10^{-1} |

LCC: load carrying capacity.

### Minimising the friction force

The results obtained when minimising the FF are presented here. Three markedly different values of the dimensionless LCC, i.e. \( \tilde{\omega} = 10^{-2}, \tilde{\omega} = 10^{-3} \) and \( \tilde{\omega} = 10^{-4} \), were used to partition the results. The results, in terms of dimensionless FF, COF, maximum pressure and minimum film thickness, obtained for the bearing geometries that minimise the FF, are listed in Table 3. Figure 13 depicts the bearing geometries that minimise the FF. The colour legends show the film thickness and each bearing geometry has an individual colour legend, where the white regions exceed the upper limit. Looking at the bearing geometries one can clearly see that a substantial amount of material has been removed and only a fraction of the initial bearing area is actively contributing to bearing performance. The large portion of material removal from the leading edge effectively shortens the bearing and the width is actually controlling the bearing length. All bearing geometries have internal fluid entrapment areas, and decreasing the LCC leads to sectioning of the internal fluid entrapment areas. These fluid entrapment areas are surrounded by a very narrow land constituting a very small bearing area. Moreover, the film thickness at the land is very thin. In fact, the land surrounding the internal fluid entrapment takes the width of one mesh element, making the land area a highly mesh-dependent quantity. The apparent bearing geometries have a leading edge, functioning as a fluid feeder for the internal fluid entrapment. The thickness of the leading edge is more or less constant in the sliding direction, but varies with the width of the slider.

Figure 14 depicts the pressure distributions corresponding to the bearing geometries in Figure 13. The figure shows that the pressure inside the internal fluid entrapment region is constant and this is also where the absolute majority of the LCC is generated. As in the case of the COF minimisation, the material removal effectively moves the pressure boundary conditions to the apparent bearing geometry.

Figure 15 depicts the Poiseuille-flow shear stress distributions corresponding to the bearing geometries in Figure 13. From each of the depictions, it can be seen that the shear stress is zero on the majority of the domain. However, the leading edge of the apparent bearing has a positive- and the trailing edge a negative contribution, to the shear stress.
Figure 13. The bearing geometries that minimise the FF with three values of LCC, decreasing from left to right. (a) $\bar{w} = 10^{-2}$; (b) $\bar{w} = 10^{-3}$; (c) $\bar{w} = 10^{-4}$.

Figure 14. The pressure distribution corresponding to the bearing geometries depicted in Figure 13, which minimise the FF. The LCC decreases from left to right. (a) $\bar{w} = 10^{-2}$; (b) $\bar{w} = 10^{-3}$; (c) $\bar{w} = 10^{-4}$.

Figure 15. The Poiseuille-flow shear stresses corresponding to the bearing geometries depicted in Figure 13, which minimise the FF. The LCC decreases from left to right. (a) $\bar{w} = 10^{-2}$; (b) $\bar{w} = 10^{-3}$; (c) $\bar{w} = 10^{-4}$. 
Figure 16 depicts the Couette-flow shear stress distributions corresponding to the bearing geometries in Figure 13, which are non zero on the land of the apparent bearing. It can also be seen that the trailing land has a larger contribution to the shear stress component than the leading edge. The largest Couette-flow shear stress is, however, observed at the sides of the narrow land, where the film thickness is the thinnest.

The performance data obtained from the bearing geometries minimising the FF are shown in Table 3.

Concluding remarks

The performance of hydrodynamically lubricated bearings operating under incompressible flow was studied by means of a Reynolds equation based model implemented within the FE-based simulation software COMSOL Multiphysics. The optimisation physics interface inside the program was set to optimise the bearing geometries using the globally convergent method of moving asymptotes, developed in literature. The objectives were to find the bearing geometry that either maximises the LCC, minimises the COF or minimises the FF. Mesh convergence studies were conducted for the LCC and the COF objectives. The bearing geometries maximising the LCC obtained when using coarse meshes were observed to closely resemble the Rayleigh-pocket bearing previously addressed in references. However, when the mesh was refined, shark-fin like features appeared in the transition zone between the pocket’s trailing edge and the flat land. The shark fins were also found to contribute to an increased LCC. The geometries minimising the COF were also of pocket type, but without shark fins at the back end of the pocket. However, when the mesh was refined, an internal fluid entrapment region appeared there instead, and the corners of the flat trailing edge land were also removed by the optimisation procedure.

A study on minimising the FF with the LCC prescribed by three markedly different values was also performed. The results were three optimum bearing geometries, that all featured internal fluid entrapment regions. However, the land of the bearing turned out to be extremely narrow with a very limited amount surface area, thus not suitable in practice. The knowledge gained from this part of the work, is that minimisation of FF with a prescribed LCC, requires a lower limit for the film thickness. This would thereby generate bearing geometries which would depend on this lower limit, thus it would not be as generally applicable as the LCC-maximising- and the COF-minimising geometries.

It has been shown that the present method can be used to obtain bearing geometries with better performance than the bearing geometries previously found. For very rough meshes the present method produces results that to the eye are identical to the geometries available in the literature, but even with fairly small mesh refinements the present method produces geometries that are different than those. The present method also provides us with new bearing geometries that minimise the COF.

All in all, the findings presented herein, suggest that the present approach could be used to improve the design of hydrodynamically lubricated bearings under incompressible flow.

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**Appendix**

**Notation**

- **E**: number of elements in mesh
- **f**: friction force
- **i**: dimensionless friction force
- **h**: film thickness
- **hr**: characteristic film thickness
- **h**: dimensionless film thickness
- **p**: fluid pressure
- **pa**: ambient pressure
- **pc**: cavitation pressure
- **pr**: characteristic pressure
- **p**: dimensionless pressure
- **u**: speed of moving surface (m/s)
- **w**: load carrying capacity
- **w**: dimensionless load carrying capacity
- **x**: Cartesian coordinate
- **xr**: bearing length
- **x**: dimensionless coordinate
- **y**: Cartesian coordinate
- **y**: dimensionless coordinate
- **η**: lubricant viscosity (Pas)
- **Λ**: bearing number $6/[u_0/(ph^2)]$
- **μ**: coefficient of friction
- **μ**: scaled coefficient of friction
- **ρ**: lubricant density (kg/m$^3$)