Synthesis of control of the aircraft angular velocities based on algorithm with a predictive model

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Abstract. An urgent issue is to obtain optimal control laws for nonlinear dynamic objects. The article presents a numerical algorithm to control stabilization of the angular velocities of spacecraft with various axial moments of inertia based on optimization with respect to the Krasovsky functional using the predictive model method. The simulation results are presented. The character of the curves of angular velocity dynamics and control moments obtained in the simulation corresponds to the curves of angular velocity dynamics and control moments obtained in the simulation of control using the analytical law for an axisymmetric spacecraft, which confirms the adequacy of the synthesized algorithm.

1. Introduction

Air and space crafts belong to nonlinear dynamic systems. Linear models for control of dynamic systems are used when changes in operating conditions are small. Thus, the use of linear models for the synthesis of a craft controls in real operating conditions leads to unacceptable errors in achieved control goals. Therefore, the problem of synthesizing the control laws for nonlinear objects is relevant and scientists pay constant attention to it, for example [1-6]. A large group of optimal control synthesis algorithms is based on the dynamic programming method [7], which application is limited by the need to solve the nonlinear functional Bellman equation in partial derivatives [8].

The difficulties of synthesis based on the method of dynamic programming with minimization of classical functionals [9] can be avoided using nonclassical semidefinite functionals (functionals of “generalized work”) presented in the works of Academician N.N. Krasovsky [10, 13]. The semidefiniteness of the functionals of the generalized work is due to the fact that it contains an unknown optimal control, which is determined during the synthesis.

Set of papers [6, 14, 15, 16, 17, 18, 19] presents a solution to the problem of synthesizing the optimal control of angular movement of an axisymmetric craft based on the Krasovsky functional using the predictive model method. A positive feature of the algorithms presented in these works is the analytical form of the control laws. This form of synthesized algorithms is obtained by means of analytical solutions to the fundamental matrix and to the free motion of an axisymmetric craft.

But many crafts have a shape differ from axisymmetric. Therefore, it is important to obtain the optimal control law for a non-axisymmetric craft.

This paper presents the results of synthesis and modeling the optimal control law for a non-axisymmetric spacecraft.
To synthesize the stabilization law, it is convenient to represent the craft in the form of a rigid body with the corresponding axial moments of inertia. The dynamics of the rotational motion of a rigid body is described by the following equations [11]:

\[
\begin{align*}
\dot{\omega}_1 + A_1\omega_2\omega_3 &= u_1, \\
\dot{\omega}_2 - A_2\omega_1\omega_3 &= u_2, \\
\dot{\omega}_3 + A_3\omega_1\omega_2 &= u_3,
\end{align*}
\]

where \( A_1 = (I_{yy} - I_{zz})/I_{xx} \), \( A_2 = (I_{zz} - I_{xx})/I_{yy} \), \( A_3 = (I_{xx} - I_{yy})/I_{zz} \) are the reduced moments of inertia with respect to the corresponding axes, \( I_{xx}, I_{yy}, I_{zz} \) are the moments of inertia with respect to the axes \( x, y, z \); \( \omega_1, \omega_2, \omega_3 \) are the angular velocities; \( u_1, u_2, u_3 \) are the reduced control moments constituting vectors \( \omega \) and \( u \) respectively.

According to [14], for the Krasovsky functional

\[
J = \int_{t_0}^{t_1} \Psi(\omega) dt + \frac{1}{2} \int_{t_0}^{t_1} (u^T K^{-1} u + u_0^T K^{-1} u_0) dt
\]

the corresponding optimal control from the predictive model method is determined by the formula:

\[
u_0(t) = -K(t) \int_{t}^{t_1} G^T(s, t) \frac{\partial}{\partial \omega_m} \left( \Psi(\omega_m, s) \right) ds + \frac{\partial}{\partial \omega_m} \left( \Psi(\omega_m, s) \right) = Q \omega_m(t, s),
\]

where \( \Psi(\omega) = \omega^T Q \omega \); \( Q, K \) are the positive definite diagonal weight matrices; \( G(s, t) \) is the fundamental matrix determined on the free motions of system (1) from the equation

\[
\frac{\partial}{\partial s} G(s, t) = F^* G(s, t), \quad G(s, t) \Bigg|_{s=t} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};
\]

\( \omega_m(s) = [\omega_{m1}, \omega_{m2}, \omega_{m3}]^T \) is the vector of free motion, determined from the motion equations of system (5) with zero right-hand side:

\[
\begin{align*}
\frac{d\omega_{m1}}{ds} + A_1\omega_{m2}\omega_{m3} &= 0, \\
\omega_{m1}(s) \Bigg|_{s=t} &= \omega_1(t), \\
\frac{d\omega_{m2}}{ds} - A_2\omega_{m1}\omega_{m3} &= 0, \\
\omega_{m2}(s) \Bigg|_{s=t} &= \omega_2(t), \\
\frac{d\omega_{m3}}{ds} + A_3\omega_{m1}\omega_{m2} &= 0, \\
\omega_{m3}(s) \Bigg|_{s=t} &= \omega_3(t).
\end{align*}
\]

The Jacobi matrix \( F \) for system (5) can be defined by the following expression:

\[
F = \begin{bmatrix}
0 & -A_1 & -A_1 \\
A_2 & 0 & A_2 \\
-A_3 & -A_3 & 0
\end{bmatrix}
\]
The matrix equation (4) is a system of 9 first-order differential equations:

\[
\begin{align*}
\frac{\partial G_{11}}{\partial s} &= -A_1\omega_1 G_{21} - A_1\omega_2 G_{31}, \\
\frac{\partial G_{12}}{\partial s} &= -A_1\omega_3 G_{22} - A_1\omega_4 G_{33}, \\
\frac{\partial G_{13}}{\partial s} &= -A_1\omega_5 G_{23} - A_1\omega_6 G_{33}, \\
\frac{\partial G_{21}}{\partial s} &= A_2\omega_1 G_{11} + A_2\omega_3 G_{21}, \\
\frac{\partial G_{22}}{\partial s} &= A_2\omega_4 G_{12} + A_2\omega_5 G_{22}, \\
\frac{\partial G_{23}}{\partial s} &= A_2\omega_6 G_{13} + A_2\omega_7 G_{23}, \\
\frac{\partial G_{31}}{\partial s} &= -A_3\omega_1 G_{11} - A_3\omega_4 G_{21}, \\
\frac{\partial G_{32}}{\partial s} &= -A_3\omega_2 G_{12} - A_3\omega_5 G_{22}, \\
\frac{\partial G_{33}}{\partial s} &= -A_3\omega_3 G_{13} - A_3\omega_6 G_{23},
\end{align*}
\]

(7)

Numerical integration of systems for the fundamental matrix (7) and for the free motion (5) are the basis for calculating controls in accordance with expression (3).

It should be noted that the matrix (7) for an axisymmetric craft “splits” into three independent systems of differential equations, for which analytical solutions are found in [14].

2. Simulation modeling

To check the adequacy of the synthesized algorithm, we carry out calculations for an axisymmetric spacecraft (\(A_1=A_2=1, A_3=0\)) with initial velocities \(\omega_1(0) = 2.5\), \(\omega_2(0) = -1.7\), \(\omega_3(0) = 1.2\). The dynamics of angular velocities and control moments are shown in Fig. 1 and 2.

**Figure 1.** Graphs of changes in angular velocities

**Figure 2.** Graphs of changes in control action
3. Conclusion
This paper presents a numerical algorithm for angular stabilization of a non-axisymmetric craft based on a predictive model along with the results of control modeling. The nature of the curves of the dynamics of angular velocities and control moments obtained by simulating the control using the synthesized algorithm corresponds to the curves of the dynamics of angular velocities and control moments obtained by simulating the control using the analytical law for an axisymmetric craft [14], which confirms the adequacy of the synthesized algorithm.

Thus, the obtained numerical control algorithm can be implemented in a craft on-board computer to stabilize the angular velocities.

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