Calculating stress intensity factor for high-velocity crack

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Abstract. In this paper the earlier elaborated finite-element method of calculating the stress intensity factor for stationary cracks at dynamic loading by means of cohesion finite elements and on the basis of the method of lines is generalized for high-velocity cracks. The method in question does not require rearranging the finite-element mesh. The crack growth is modeled by consecutively releasing the nodes located on the crack line. The calculated data are compared with the analytical solutions of other researchers and with the experimental data.

1. Introduction

The experimental investigations of high-velocity crack propagation allow concluding that the regularities followed by this process differ from the regularities of quasi-static crack growth [1]. This is why, the elaboration of new theoretical approaches is a relevant topic. Their indispensable component is the modeling of high-velocity crack growth, taking account of inertial forces. The available analytical methods [2] are complicated and stifled with restrictions, which is why their applicability confines to model solutions of test problems for numerical methods. The number of the latter is high and continues to rise. The ones to be distinguished are mathematically correct methods based on the classical finite elements method. First of all, these are Nilsson’s method [3], the crack growth in which is modeled by consecutively releasing the finite-elements mesh nodes found on the crack extension. The method’s strength is that it does not require rearranging the finite-element mesh at crack growth, whereas the weakness is that it does not consider the peculiarity of the stress state of the crack tip. The alternatives, e.g., Atluri and Nishioka’s method [4], allow modeling the crack tip peculiarity by means of special finite elements; however, crack growth makes it necessary to rearrange the mesh, which results in significant inaccuracies due to multiple interpolations. There are also a lot of methods elaborated as modifications of Galerkin’s [5, 6] or the finite element method [7–9]. They have one weakness in common, i.e., imperfect mathematical correctness. The additional coordinate functions introduced in these methods to make more accurate calculations result in non-physical strong discontinuities in the displacement field, and this defect devalues the attained results.

A method able to provide satisfactorily accurate results must, first of all, be mathematically correct, secondly, take account of the crack tip peculiarity, and, thirdly, not require rearranging the finite-element mesh with crack growth. This method and some of the results of using it are exposed below. Its main constituents have been elaborated earlier and used to create the method of solving dynamic fracture mechanics problems for stationary cracks [10–12].
2. Statement of the problem
Assume that a body with an isolated crack is exposed to plane deformation. The body’s cross section is region $S$, bounded by the contour $L$ (figure 1). It is supposed that the crack is straight and found on the abscissa axis. Region $S$ and surface load distribution $p(t)$, where $t$ is the time (like other similar studies, this one does not take account of mass forces), are symmetrical relative to the abscissa axis. In this case, the crack shown in figure 1 will be an in-plane opening crack (mode I). Since the abscissa axis is the problem’s axis of symmetry, only the upper cross section part $x_2 \geq 0$ is considered below. The fracture is supposed to be quasi-brittle. This means that the strains are small and the material is linearly elastic right to fracture. The problem for consideration is described below. The crack remains stationary until moment $T$, i.e., the crack length is $a = a_0 = \text{const}$. At $t > T$ the crack propagates with defined velocity $\dot{a} = V(t)$. The crack length and (or) load changes so rapidly that the inertial forces cannot be ignored. It is required to calculate function $K_i(t)$ defined as the time relation of the stress intensity factor.

The mathematical model is based on the known relations from the elasticity theory:

\[
\begin{align*}
\varepsilon_{im} &= (\partial_i u_m + \partial_m u_i)/2; \quad \sigma_{im} = \lambda \varepsilon_{im} + 2\mu \varepsilon_{in}; \quad i, k, m = 1, 2; \quad \lambda = \partial_i \partial_i = \partial/\partial x_i; \\
\lambda &= \nu E/[(1 + \nu)(1 - 2\nu)] \quad \mu = E/[2(1 + \nu)]; \quad c_d = \sqrt{\lambda + 2\mu}/\rho; \quad c_s = \sqrt{\mu}/\rho
\end{align*}
\]

(1)

where $u_k(x_m, t)$ is the displacement field, $\varepsilon_{im}$ is the strain tensor, $\sigma_{im}$ is the stress tensor, $\lambda, \mu$ are the Lamé elastic constants, $E$ is Young’s modulus, $\nu$ is Poisson’s ratio, $\delta_{im}$ is Kronecker’s delta, $\rho$ is the density, $c_d$ is the elastic dilatational wave velocity, $c_s$ is the elastic shear wave velocity.

The motion equations fulfilled in region $S$ are

\[
\begin{align*}
\partial_t \varepsilon_{km} &= \rho \partial_t v_k, \quad \varepsilon_{km} = \partial_i u_k, \quad \partial_i = \partial/\partial t
\end{align*}
\]

(2)

where $v_k$ is the velocity field. The solution of equation (2) must meet the initial and boundary conditions. The boundary conditions can be either principal (kinematic) or natural (dynamic).

3. The method of lines
The problem’s weak solution is derived from the virtual work principle as

\[
\int_S \left( \rho \partial_i v_k \delta u_k + \sigma_{im} \delta \varepsilon_{im} \right) dS = \int_L p_i \delta u_i dL
\]

(3)

where $\delta$ is the variation symbol, $p_i$ is the external load (figure 1). Equation (3) is equivalent to motion equations and the natural boundary conditions. The solution of (3) is sought in the class of functions satisfying the principal boundary conditions. In this case, the variations of the displacements observed in the boundary contour’s sections, where the principal boundary conditions are specified, are equal to zero.
The problem is solved by the method of lines [13, 14]. The solution’s first step is to convert equation (3) to a finite-difference equation by time. In this paper this is fulfilled by the Crank–Nicolson implicit scheme. Assume that \( \Delta t \) is the time value of the integration step, \( n \) is the integration step number. The finite-difference form of the derivative by time at the \( n \)th step is
\[
\frac{\partial_n y}{\Delta t} = \left( y^n - y^{n-1} \right) / \Delta t
\]
(4)
where \( y^{n-1}, y^n \) are the values of \( y \) at the time range boundaries. The quantities without time derivatives are recorded at the \( n \)th time integration step as
\[
\begin{align*}
\frac{1}{2}(v_k^n + v_k^{n-1}) &= \frac{u_k^n - u_k^{n-1}}{\Delta t}, \\
\end{align*}
\]
(5)
Taking account of velocity expression (2), equation (3) is recorded as
\[
\int_s \left[ \frac{4 \rho}{\Delta t} \left( u_k^n \right) + \sigma_{km}^{n-1} \delta e_{km} \right] \delta u_k \, dS = \int_L \left( p_k^n + p_k^{n-1} \right) \delta u_k \, dL;
\]
(6)
The values with indices \( n-1 \) are known from the solution for the preceding step. System (6) is divided in two equations as
\[
\int_s \left[ \frac{4 \rho}{\Delta t} \left( u_k^n \right) + \sigma_{km}^{n-1} \delta e_{km} \right] \delta u_k \, dS = \int_L \left( p_k^n + p_k^{n-1} \right) \delta u_k \, dL
\]
(7)
\[
\int_s \frac{4 \rho}{\Delta t} \left( \frac{u_k^{n-1}}{\Delta t} + v_k^{n-1} \right) \delta u_k - \sigma_{km}^{n-1} \delta e_{km} \, dS
\]
\[
v_k^n = \frac{2}{\Delta t} \left( u_k^n - u_k^{n-1} \right) - v_k^{n-1}
\]
(8)
First of all, displacements \( u_k^n \) are found from equation (7); then, velocities \( v_k^n \) are found from equation (8).

The finite element method is used to solve equation (7) at each time integration step. Assume that \( U \) is the column matrix of nodal unknowns, \( V \) is the column matrix of their velocities, \( E \) is the stiffness matrix, \( M \) is the mass matrix, \( P \) is the load column matrix conditioned by the first summand in the right part of equation (7). The system of linear algebraic equations relative to unknowns \( U^n \) and equivalent to variation equation (7), is recorded as
\[
\begin{bmatrix}
4 \rho / (\Delta t)^2 & M + E \\
\end{bmatrix}
\begin{bmatrix}
U^n \\
\end{bmatrix}
= \begin{bmatrix}
P^n + P^{n-1} \\
\end{bmatrix}
+ \begin{bmatrix}
4 \rho / (\Delta t)^2 \\
\end{bmatrix}
\begin{bmatrix}
(U^{n-1} + \Delta V^{n-1}) - E U^{n-1} \\
\end{bmatrix}
\]
(9)
The usual finite element procedure, reducing the solution of the problem to solving a system of linear algebraic equations [15], is applied next. Principal (kinematic) boundary conditions (in this class of problems they can be zero only) are treated as follows [15]. Let \( j \) be number (in the global numbering) of nodal displacements that should be zero. Row \( j \) and column \( j \) of the matrix of the resolving system of equations (the last is so as to preserve the symmetry of the matrix) are zeroed except the diagonal element. Element \( j \) of the free term column is zeroed also.

The formula for determining nodal velocities stems from equation (8) is recorded as
\[ V^n = \frac{2}{\Delta t} \left( U^n - U^{n-1} \right) - V^{n-1} \]  

(10)

4. Finite elements
The finite element mesh consists of two types of elements: the first one is common isoparametric elements, and the second one is special cohesive elements. They allow taking account of the specificity introduced by the crack to the boundary problems of the elasticity theory. The crack line adjoins a layer of cohesive finite elements (figure 2). The rest of the cross section is covered with a mesh of first-order isoparametric finite elements. Each such element in local coordinates is square-shaped (figure 3).

![Figure 2](image1.png)

**Figure 2.** Finite-element mesh (basic diagram): 1 is the isoparametric element, 2 is the cohesive element.

![Figure 3](image2.png)

**Figure 3.** Finite element in local coordinates with double numbered nodes.

The global coordinates of an element’s points and the displacements in these points are determined as

\[ x^I = L_I(\xi)L_J(\eta)X^{IJ,\mu}; \quad u^I = L_I(\xi)L_J(\eta)U^{IJ,\mu}; \quad I, J = 1, 2; \quad \xi, \eta \in [-1, 1] \]  

(11)

where \( X^{IJ,\mu} \) are the desired global coordinates of a node numbered as \( IJ, U^{IJ,\mu} \) are the displacements of node \( IJ \), \( L_I(z) \) are the Lagrange interpolation polynomials recorded, in this case, as

\[ L_I(z) = 0.5(1-z); \quad L_2(z) = 0.5(1+z) \]  

(12)

The global nodal coordinates are specified. Nodal displacements \( U^{IJ,\mu} \) are the problem’s principal unknowns.

It is supposed that the crack tip vicinity contains an cohesion zone with length \( \Delta \), within which the crack’s opposite edges are attracted to each other. The forces of this attraction are called cohesion forces. The crack’s strained upper edge is shown in figure 4. The image for the lower edge is symmetrical.

The exposed theory relies on Barenblatt’s postulates [16]. According to the first of these, the distribution of cohesive forces provides zero singularity to the stress field in the crack tip vicinity. This is equivalent to the requirement that the crack’s edges smoothly join together in its tip (figure 4), which is recorded as

\[ x_1 = a; \quad x_2 = 0; \quad \partial_1 u_2 = 0 \]  

(13)

According to the second postulate, the cohesive zone is much shorter than the crack. The third postulate implies that the stress state of the crack tip’s immediate vicinity will not change as the tip
moves forward. It follows from these postulates [2] that the energy release rate with the advancement of the crack tip and its relation to the stress intensity factor $K_I$ is determined as

$$ G = 2 \int_{a}^{a+\Delta} q_2 \frac{\partial}{\partial x} u_2 \, dx_1 = \frac{1-v^2}{E} A_I(V) K_I^2 $$

(14)

where function $A_I(V)$ is determined as

$$
A_I(V) = \frac{V^2 \alpha_d}{(1-v)c_d^2 R(V)}; \quad R(V) = 4\alpha_d \alpha_s - (1+\alpha_s^2)^2;
$$

$$
\alpha_d = \sqrt{1-(V/c_d)^2}; \quad \alpha_s = \sqrt{1-(V/c_s)^2}
$$

(15)

Here $R(V)$ is Rayleigh’s function. Formula (14) is true for both, stationary crack ($A_I(0) = 1$) and at the movement of the crack tip. Sought quantity $K_I$ is found from equation (14).

Figure 4. Upper crack edge at strain; $\Delta$ is the cohesion region length; $q(x_i)$ are the cohesion forces.

Figure 5. Straining of the crack tip vicinity. The crack tip is found in node $B$. $\Delta x_i$ is the distance between the neighboring nodes found on the crack line.

Cohesive finite elements ensure the smooth joining of the crack’s edges in its tip and, therefore, zero singularity in the stress and strain fields. Cohesive elements are not isoparametric, and in global coordinates they are shaped as rectangles. That said, the formulas for the connection between the local and global coordinates are simplified as

$$
x_i = a_i + b_i \xi; \quad x_2 = a_2 + b_2 \eta; \quad a_i = 0.5( X_{i,22} + X_{i,11} );
$$

$$
b_i = 0.5( X_{i,22} - X_{i,11} ); \quad X_{1,1} = X_{1,12}; \quad X_{2,2} = X_{2,22}
$$

(16)

The displacements of the points of a cohesive finite element are determined [16] as

$$
u_i = L_i(\xi) L_j(\eta) U_{i,j};
$$

$$
u_2 = H_1(\xi) L_j(\eta) U_{2,j} + L_i(\xi) L_2(\eta) U_{2,2j} + 0.5\Delta x_i H_4(\xi) L_1(\eta) U_{3,1j}
$$

(17)

where $\Delta x_i$ is the element size along the abscissa axis, $U_{k,ij}$ are the displacements of node $IJ$, $U_{3,1j}$ are the values of derivative $\frac{\partial u_2}{\partial x_1}$ in the $11^{th}$ node (these quantities are determined only for the nodes located along the abscissa axis – a straight line, where the crack is found), $H_m(z)$ are the Hermite interpolation polynomials:

$$
H_1(z) = 0.25(2 - 3z + z^2); \quad H_2(z) = 0.25(2 + 3z - z^3);
$$

$$
H_3(z) = 0.25(1 - z - z^2 + z^3); \quad H_4(z) = 0.25(-1 - z + z^2 + z^3)
$$

(18)
The need for introducing a whole range of cohesive elements is conditioned by the required continuity of displacements between both, cohesive elements and cohesive and common isoparametric elements. In fact, the result obtained at \( \xi = \pm 1 \) (the left and right part of the element) is

\[
\begin{align*}
\xi &= -1: \quad u_k = L_j(\eta)U_{k,1,1}; \\
\xi &= 1: \quad u_k = L_j(\eta)U_{k,2,1}
\end{align*}
\]  

(19)

This means that the displacements of the points on one side of the element are unambiguously determined by the displacements of the nodes found on this side. The results obtained using formulas (17) for the upper part of the cohesive element, where it borders on the isoparametric element, are

\[
\eta = 1: \quad u_k = L_j(\xi)U_{k,1,2}
\]  

(20)

The displacements of the points on one side of the element are determined by the displacements of the nodes, found on this side, according to the same law as for isoparametric elements. This ensures continuous interelement displacements.

To ensure the convergence to a precise solution with the rising number of elements, it is also necessary to meet the completeness condition formulated as below. As the elements become smaller, the strains within their limits will tend to constant values and the displacements – to the linear relation to the coordinates. The element shape’s functions must ensure the consistency of limit distributions of strains and displacements. If the displacements are distributed according to the linear law recorded as

\[
u_k = \alpha_k + \beta_k x_1 + \gamma_k x_2
\]  

(21)

where \( \alpha_k, \beta_k, \gamma_k \) are the constants, the strains will be recorded as

\[
\begin{align*}
\varepsilon_{11} &= \beta_1; \\
\varepsilon_{12} &= 0.5(\beta_1 + \gamma_1); \\
\varepsilon_{22} &= \gamma_2
\end{align*}
\]  

(22)

Assume that the nodal displacements are distributed according to law (21) as

\[
U_{k,1,1} = \alpha_k + \beta_k X_{1,1} + \gamma_k X_{2,1}; \quad U_{3,1,1} = \beta_2
\]  

(23)

In this case, the strains in all of the element’s points, calculated using (17), must concur with expressions (22). The direct insertion, taking account of (16), allows making sure that this requirement is met by cohesive elements.

The mesh dimensions are selected so that the tip of the crack with length \( a_0 \) falls into the node. That said, condition (5) is met by means of cohesive finite elements: it is assumed that \( U_{3,1,1} = 0 \) in the node coincident with the crack tip. If no restrictions are imposed on the nodal displacements of a cohesive finite element, there will be no cohesion forces along its boundary. This is why, the scope of cohesion forces (cohesion region) is localized within the finite element adjoining the crack tip. Thus, the finer is the finite-element mesh, the more precisely will Barenblatt’s theory requirement be met that the region of cohesion be considerably shorter than the crack. The cohesion region’s length is equal to the element’s length in the line of the abscissa axis. If the cohesion region is short, the result of solving the problem, i.e., function \( K_i(t) \) will not depend on the distribution of forces along the crack edge. This is why, this distribution can be replaced with its integral characteristics, including the resultant vector and the resultant moment relative to the crack tip. They are determined by the boundary conditions in the crack tip (symmetry condition and condition (13)) and, thus, shaped as constraint reactions. Their quantities \( Q_a \) and \( M_a \) (figure 5) are easy to find by inserting the nodal displacement values to the respective non-transformed equations of the resolving system of equations.

When the crack moves forward by finite element length \( \Delta x_1 \), the constraint reactions weaken by module to zero and respective displacements \( u_2 \) and degree of rotation \( \partial u_2/\partial x_1 \) take on finite values. According to Barenblatt’s third postulate, the crack growth does not change the tip region configuration. This means that the displacements obtained by node \( B \), when the crack propagates by
\( \Delta x_i \), will be equal to the displacements of node \( A \) at the coincidence of the crack tip with node \( B \). In addition, the energy release rate is determined by the apparent formula, taking into account, that the crack has two edges:

\[
G = -\frac{1}{\Delta x_i} (Q_{a} u_{z_a} + M_{a} \partial_{t} u_{z_a})
\]  

(24)

The method’s efficiency is confirmed by papers \([10, 12]\), where the calculated results are compared with the experimental data for a stationary crack.

The specificity of the problems for the growing crack shows in changes in the problem’s principal boundary conditions. As soon as the crack tip moves forward relative to the node, where it was located, this node is released from constraints. As noted above, however, the crack tip must be located in some node of the mesh. This means that, according to the exposed method, the crack can grow only in leaps; the leap size is equal to finite element length \( \Delta x_i \). If to consider that this leap is made in one time step \( \Delta t \), this step turns out too big \( (\Delta t = \Delta x_i / V) \), whereas, according to Courant’s condition, inequation \( \Delta t < \Delta x_i / c_{d} \) must be fulfilled. The emerging contradiction can be resolved only roughly, by relying on certain assumptions. Let us do as follows. Assume that \( \Delta t_o \) is the chosen time step. It is supposed that, when the crack extends by \( \Delta x_i \), crack growth velocity \( V \) remains unchanged. Let us find the value of \( \Delta x_i / (V \Delta t_o) \) are round it to the nearest integer which will be \( m \) defined as the number of steps in time for which the crack tip covers, so to speak, the finite element length. In this case, the time step is \( \Delta t = \Delta x_i / (m V) \). Obviously, this value is close to \( \Delta t_o \).

Assume that the crack tip is found in node \( A \) (figure 5); in this node \( u_z \) and \( \partial u_z / \partial x_1 \) are zero. Constraint reactions \( Q_A \) and \( M_A \) are known. Then the crack tip is carried to the next node \( B \) (figure 5), whereas node \( A \) is disengaged from constraints. However, \( Q_A \) and \( M_A \) do not fade away but convert from constraint reactions to active forces. This operation does not change the stress state. Then generalized forces \( Q_A \) and \( M_A \) gradually weaken by module with each next time step and become zero for \( m \) steps. It appears that the crack tip has moved forward with velocity \( V \) by one node of the finite-element mesh for \( m \) time steps.

The question arises, what is the law under which \( Q_A \) and \( M_A \) become weaker? Only their initial and final values are known. This issue was discussed in paper \([3]\) but no justified solution was derived. Nor was it derived in the course of this study. In any case, the linear law that can be adopted at a first approximation is

\[
Q_A(t) = Q_{A0} \left(1 - \frac{t - t_0}{m \Delta t} \right); \quad M_A(t) = M_{A0} \left(1 - \frac{t - t_0}{m \Delta t} \right)
\]  

(25)

where \( t_0 \) is the moment when the crack tip begins to leave node \( A \); \( Q_{A0}, M_{A0} \) are the initial values of the generalized forces (constraint reactions). This law was the one used in this paper.

5. Calculation results

Let us consider some of the results of the calculation made by the elaborated method. Assume that the strip with an edge crack is loaded like it is shown in figure 6. The problem has a symmetry axis, which is why the design pattern is one half of the cross section (figure 7). The boundary conditions are set as exposed below. The load in boundary sections \( AB, BC \) and \( CD \) is zero: \( p_1 = 0; p_2 = 0 \). The symmetry conditions fulfilled in section \( DE \) are \( u_z = 0; p_1 = 0 \); the constant time-dependent load distributed by length on section \( EA \) (crack tip) is \( p_1 = 0; p_2 = q(t) \).
This design pattern models Ravi-Chandar and Knauss’s experiments [18]. The test specimens were made from optically transparent polymer Homalite-100 with the following mechanical characteristics [1]: \( E = 4550 \text{ MPa}, \ \nu = 0.31, \ \rho = 1230 \text{ kg/m}^3 \). The specimen dimensions were \( W = 500 \text{ mm}, \ H = 150 \text{ mm}, \ a_0/W = 0.63 \ [19] \). At first, test load \( q(t) \) increased under the linear law up to the value of \( q_M \) (at \( t = t_M \)), and then it remained constant. In addition to dynamic loading, the specimens were exposed to quasi-static loading because it was necessary to lay the crack edges open to some degree so as to place in between the electrodes to generate dynamic loading. During the calculation the stress intensity factor from static loading was added to the stress intensity factor conditioned by dynamic loading. The calculations were made at \( \Delta t/t = 10^{-3} \). The chosen time step was \( \Delta t_0 = 0.5 \cdot 10^{-3}W\sqrt{\rho/E} \), which was in line with the Courant condition. The two considered variants of calculation (table 1) differ by the value of \( q_M \). The value of \( t_M \) is 25 \( \mu \text{s} \) for both design cases.
Table 1. Maximal loading values, initial cracking moment and crack growth velocity [20]

| Variant | $q_w$ (MPa) | $T$ (µs) | $V$ (m/s) |
|---------|-------------|----------|-----------|
| 1       | 1.10        | 56       | 240       |
| 2       | 5.55        | 18       | 410       |

The calculated and the experimental data are shown in figure 8. It follows from the plots that the numerical and the analytical solutions coincide when the crack is stationary; when the crack begins to propagate, a minor difference appears and then increases with the rising crack propagation velocity. The theory and the experimental data reach a satisfactory agreement at a comparatively low crack growth velocity (figure 8a). The crack grows, like it follows from linear fracture mechanics, at an almost constant value of $K_I$ close to fracture toughness $K_{IC}$, equal to $0.44 \text{ MPa} \cdot \text{m}^{1/2}$ for the considered material [1]. A significant difference between the calculated and the experimental data is observed at high velocities (figure 8b) and is given a possible explanation by Ma and Freund [20]. The explanation consists in the fact that the crack tip vicinity is known to always contain a region of nonlinear deformation. At low velocities this region is negligibly small, which is why the analysis based on linear fracture mechanics is acceptable. As the velocity rises, however, this region sharply expands and nonlinearity must be taken into account.

6. Conclusion
The new method suggested in this paper for solving dynamic fracture mechanics problems is based on the method of lines and the finite element method. The method allows finding the time relation of the stress intensity factor for plane-strained bodies with a stationary in-plane opening crack. The method consists in using special cohesive elements. They allow finding the solution of the fracture mechanics problem for a Barenblatt’s model cohesive crack. Constraint reaction forces and nodal parameters are used to find the energy release rate. Since the theories of Griffith and Barenblatt are equivalent, this energy release rate is used to find the stress intensity factor. Cohesive elements meet the completeness requirement and the required interelement continuity of displacements.

The method’s strengths are simplicity and zero need for rearranging the finite-element mesh in case of crack growth. This allows using the method to mathematically model the crack growth process and is confirmed by comparing the calculated data with the analytical solutions.

However, the comparison with the experimental data does not allow making a certain conclusion. The theory and the experimental data reach a satisfactory agreement at a moderate loading rate and low crack growth velocities. At high velocities, however, the theory does not agree with the test data: this can be explained by the nonlinearity of constitutive material equations. The potential of the analytical method are restricted by linear problems. The elaborated numerical method allows considering various kinds of nonlinearities at straining. This consideration is the goal of further studies.

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