A CONVEX DECOMPOSITION THEOREM FOR FOUR-MANIFOLDS

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ABSTRACT. In this article we show that every smooth closed oriented four-manifold admits a decomposition into two submanifolds along common boundary. Each of these submanifolds is a complex manifold with pseudo-convex boundary. This imply, in particular, that every smooth closed simply-connected four-manifold is a Stein domain in the complement of a certain contractible 2-complex.

1. Introduction

Exact manifold with pseudo-convex boundary (PC manifold, for short) is a compact complex manifold $X$, which admits strictly pluri-subharmonic Morse function $\psi$, such that set of maximum points of $\psi$ coincides with the boundary $\partial X$. We prefer the term PC manifold, since combination of words “compact Stein manifold” is likely to precipitate heart palpitations in some mathematicians.

Such manifold admits a symplectic structure $\omega = \frac{i}{2} \partial \psi$ and serves as an analogue of closed symplectic manifold. Boundary $\partial X$ of PC manifold $X$ inherits a contact structure $\xi$, which, in this case, is a distribution of maximal complex subspaces in $TX$ tangent to $\partial X$. In dimension four analogy between PC manifolds and closed symplectic manifolds is further illustrated by the following two theorems in terms of Seiberg-Witten invariants.

Theorem 1a. (C. Taubes, [T]) Let $(X, \omega)$ be a closed, symplectic four-manifold and $K$ be Chern class of the canonical bundle of an almost complex structure compatible with $\omega$. Then $SW_X(K) = \pm 1$.

In the relative case Kronheimer and Mrowka proved the following theorem:

Theorem 1b. (P. Kronheimer, T. Mrowka, [KM]) Let $(X, \omega)$ be compact, symplectic manifold and $\xi$ be a positive contact structure on $\partial X$ compatible with $\omega$. Then $SW_X(K) = 1$, where $K$ is the canonical class of $\omega$.

In particular, it was shown by P. Kronheimer and T. Mrowka that properly embedded surface in PC manifold satisfies Eliashberg-Bennequin inequality analogous to adjunction inequality in the case of closed symplectic manifold, once proper conditions on the boundary of the surface are imposed. Namely, if $F \subset X$ is a properly embedded surface, such that $\alpha = \partial F \subset \partial X$ is a Legendrian knot with respect to induced contact structure on $\partial X$ and $f$ is framing on $\alpha$ induced by a trivialization.

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of the normal bundle of \( F \) in \( X \), then
\[
[tb(\alpha) - f] + |\text{rot}(\alpha, F)| \leq -\chi(F),
\]
where \( tb(\alpha) \) is Thurston-Bennequin framing defined by the vector field along \( \alpha \) transversal to \( \alpha \) and tangent to contact distribution. To define rotation number observe that \( TX|_{\partial X} \cong \xi \oplus \mathbb{C} \) and thus \( \xi \cong \Lambda^2 TX|_{\partial X} \). This allows us to view \( tb(\alpha) \) as a section of \( \Lambda^2 TX|_{\partial X} \). Then \( \text{rot}(\alpha, F) \) of Legendrian knot \( \alpha \) bounding oriented proper surface \( F \subset X \) is defined to be an obstruction to extend vector field \( tb(\alpha) \) to a non-vanishing section of \( \Lambda^2 TX|_F \), i.e.
\[
\text{rot}(\alpha, F) = c_1(\Lambda^2 TX|_F, tb(\alpha)) \cdot [F, \partial F].
\]
Details could be found, for example, in [AM].

2. Whitehead multiple of a knot

Suppose \((K, f)\) is a framed knot in an oriented 3-manifold \( M \). Positive Whitehead multiple \( P_n(K, f) \) of knot \( K \) is a band connected sum of \( n \) parallel (according to framing \( f \)) copies of \( K \) equipped with alternating orientations as on Figure 1 (we assume usual orientation of \( \mathbb{R}^3 \)). We shall omit framing \( f \) from the notation when

\[
\begin{align*}
\text{Figure 1. Positive Whitehead double of knot } K
\end{align*}
\]

it is clear from the context or irrelevant to the discussion.

This construction is a generalization of Whitehead double of a knot, namely \( P_2(K) = Wh(K) \).

Here we list some properties of the Whitehead multiple \( P_n(K, f) \).

1. The homology class of \( P_n(K, f) \) is either that of \( K \) if \( n \) is odd, or zero if \( n \) is even. In fact, for odd values of \( n \), \( P_n(K, f) \) is homotopic to \( K \). In both cases, \( P_n(K, f) \) can be naturally equipped with a framing, which we also call \( f \). It is framing corresponding to the given framing of \( K \) in case of odd \( n \) (this is defined correctly because \( K \) and \( P_n(K, f) \) are in the same homology class). Canonical framing of \( P_n(K, f) \) for even \( n \) is just a zero framing, which is well-defined for the closed curve homologous to zero.

2. The key property of the above operation is that given a Legendrian knot \( K \) in tight contact 3-manifold with zero Thurston-Bennequin invariant we can produce another Legendrian knot in small neighborhood of \( K \) homotopic to \( K \) and with an arbitrary large Thurston-Bennequin invariant. More precisely, let \((K, f)\) be framed Legendrian knot in tight contact manifold \( M \). Let \( K_1, \ldots, K_n \) be parallel copies of \( K \) corresponding to framing \( f \). Isotopy class of link \((K_1, \ldots, K_n)\) has a Legendrian representative with \( K_1 = K \) and \( tb(K_i) - f = -|tb(K) - f| \),
This is obvious for $K$ being Legendrian unknot with $tb(K) = 1$ and $rot(K) = 0$. The general case follows from the fact that any two Legendrian knots have contactomorphic neighborhoods. Different proof is given in [AM1].

Each band in the construction of $P_n(K, tb(K))$ contributes 1 into $tb(P_n(K, tb(K)))$, thus we have

$$tb(P_n(K, tb(K))) - tb(K) = n - 1.$$ 

One has to choose $n$ to be odd to make $P_n(K, f)$ homotopic to $K$.

3. Suppose knot $K \subset M^3$ is a boundary of properly embedded disc $D$ in smooth 4-manifold $X$ with $\partial X = M^3$. Taking a ribbon sum of $n$ copies of disc $D$, using the same ribbons, which are used to construct $P_n(K)$, we obtain another disc $P_n(D)$. It is bounded by $P_n(K, f)$, where $f$ is the framing induced by trivialization of the normal bundle $\nu_X(D)$ of $D$ in $X$. Note also, that disc $P_n(D)$ can be constructed in arbitrary small neighborhood of $D$.

4. If $(l_1, l_2)$ is a Hopf link then link $(P_n(l_1, 0), l_2)$ in $S^3$ is symmetric (i.e. there is a diffeomorphism of $S^3$ interchanging components of the link). Moreover, two triads $(B^4, P_n(D_1), D_2)$ and $(B^4, D_1, P_n(D_2))$ are diffeomorphic. Here, $D_1$ and $D_2$ is a pair of linear 2-discs in $B^4$ intersecting at one point.

Paying homage to the popularity of physics terminology first author suggested to call manifold $W_n$ on Figure 2 a positron. Manifold $W_n$ is obtained from $B^4$ by removing open regular neighborhood of $D_1$ (this has an effect of turning $B^4$ to $S^1 \times B^3$) and then attaching a 2-handle to the framed knot $P_n(\partial D_2)$. It is contractible if $n$ is odd and has PC structure if $n \geq 2$.

3. **Handlebodies of PC manifolds**

Handlebodies of four-dimensional PC manifolds are characterized by the following theorem of Ya. Eliashberg.

**Theorem 2.** (Ya. Eliashberg, [E]; see also [E]) Let $X = B^4 \cup (1\text{-handles}) \cup (2\text{-handles})$ be four-dimensional handlebody with one 0-handle and no 3- or 4-handles. Then:

- The standard PC structure on $B^4$ can be extended over 1-handles, so that manifold $X_1 = B^4 \cup (1\text{-handles})$ has pseudo-convex boundary.
• If each 2-handle is attached to $\partial X_1$ along a Legendrian knot with framing one less than Thurston-Bennequin framing of this knot, then the complex structure on $X_1$ can be extended over 2-handles to a complex structure on $X$, which makes $X$ a PC manifold.

In this section we shall study “partial handlebodies” obtained by attaching 2-handles on top of a PC manifold. Let $Z$ be a PC manifold and $h$ be a two handle attached to $\partial Z$ along a Legendrian knot $K \subset \partial Z$ with framing $f$. If $tb(K) \geq f + 1$ then by $C^0$-small smooth isotopy of $K$ we can decrease Thurston-Bennequin invariant of $K$ and make it equal to $f + 1$. Therefore, by a theorem of Eliashberg, manifold $Z \cup h$ possesses PC structure.

However, in general, it is not possible to increase Thurston-Bennequin invariant of $K$ by isotopy, and equip $Z \cup h$ with the structure of PC manifold. Thus, we make the following definition: the defect $D(h^2)$ of a 2-handle $h^2$ attached to a Legendrian knot $K$ on the boundary of PC manifold with framing $f$ is a number $\max\{f + 1 - tb(K), 0\}$. If we have several 2-handles $h_i^2, \ldots, h_n^2$ attached to a Legendrian link on the boundary of PC manifold $Z$, then the defect $D(Z \cup \cup_i h_i^2)$ of this partial handlebody built on top of $Z$ is a sum of the defects of individual handles. So if the defect is zero, the PC structure extends over 2-handles. Given such partial handlebody $Z \cup \cup_i h_i$ with a base $Z$ being a PC manifold, we can build another $Z' \cup \cup_i h'_i$ with lesser defect, so that $Z$ and $Z \cup \cup_i h_i$ are homotopy equivalent to $Z'$ and $Z' \cup \cup_i h'_i$, respectively. To see this, let $h_i$ be a handle with a non-zero defect. Let $D_i$ be a cocore of handle $h_i$ and $m_i$ be a meridian of $P_n(\partial D_i)$ in $\partial Z$, see Figure 3. We may view $D_i$ as a disk in $Z$. Note that $(P_n(\partial D_i), m_i)$ is a small (in a chart) Hopf link in $\partial Z$. Manifold $Z'$ is obtained by removing $Nd_Z(P_n(D_i))$ — a tubular neighborhood of $P_n(D_i)$ in $Z$, from $Z$ and attaching a 2-handle to $P_n(m_i)$. We assume that $k$ is odd and $k \geq 3$. Manifold $Z'$ is boundary connected sum of $Z$ and a positron $W_k$, hence it is PC manifold homotopy equivalent to $Z$. New handle $h'_i$ is attached to the connected sum of attaching circle of $h_i$ and $P_n(\text{core of 1-handle in positron})$, thus it’s defect is $n$ less than defect of $h_i$. Manifold $Z' \cup h'_i$ is shown on Figure 4.

![Figure 3. Manifold $Z \cup h_i$](image-url)
4. Convex Decomposition Theorem

We will use above construction to prove the following convex decomposition theorem.

Theorem 3. Let $X = X_1 \cup \partial X_2$ be a decomposition of a closed smooth oriented 4-manifold into a union of two compact, smooth, codimension zero submanifolds $X_1$ and $X_2$ along common boundary. Suppose each $X_i$, $i = 1, 2$, has a handlebody without 3- and 4-handles. Then there exist another decomposition $X = \tilde{X}_1 \cup \partial \tilde{X}_2$, such that manifolds $\tilde{X}_1$ and $-\tilde{X}_2$ admit structures of PC manifolds and each $\tilde{X}_i$ is homotopy equivalent to $X_i$, $i = 1, 2$.

Proof: Consider handlebodies of $X_i$, $i = 1, 2$, with the properties stated in the assumption of the theorem. Let $Y_i$ be a union of 0- and 1-handles in $X_i$. According to theorem of Eliashberg (Theorem 2, above) $Y_i$ is a PC manifold. Complex structures on $Y_i$ are chosen so that complex orientation coincides with the induced orientation on $Y_1 \subset X$ and is opposite on $Y_2 \subset X$. Since every curve in contact manifold is isotopic to a Legendrian curve via smooth $C^0$-small isotopy, we can assume that 2-handles in handlebodies of $X_i$ are attached to Legendrian knots in $\partial Y_i$. Let $h$ be a 2-handle in $X_1$ with a non-zero defect, $D$ be the cocore of $h$, $d = \partial D$, $m$ be the meridian of $P_n(d)$ and $F$ be a trivial embedded disc in $X_2$ bounded by $m$.

As in the construction in previous section, we remove $P_n(D)$ from $X_1$, and attach handle to $P_k(m)$ with framing 0, reducing defect of $h$ and, therefore, total defect of $X_1$ by $n$. Manifold $X_1'$ can be built inside of $X$, namely

$$X_1' = [X_1 \setminus Nd_{X_1}(P_nD)] \cup Nd_{X_2}(P_kF).$$

Here $Nd_X(Y)$ stands for a tubular neighborhood of $Y$ in $X$. Its complement $X_2'$ is obtained from $X_2$ by attaching a new 2-handle $g = Nd_{X_1}(P_nD)$ and removing neighborhood of $P_k(F)$. Since $X_1$ and $-X_2$ induce the same orientation on their common boundary, positive Whitehead multiple is the same whether it is considered in $\partial X_1$ or $\partial(-X_2)$. If we choose $n$ to be defect of $h$ and $k$ to be odd and greater then or equal to defect of $g$ (after Legendrianization of attaching circle), than total defect of $X_1$ is reduced by $n$ and defect of $X_2$ is not increased. By applying this procedure to every 2-handle of $X_1$ with non-zero defect, we obtain manifold $\tilde{X}_1$ with pseudo-convex boundary, and it’s complement $\tilde{X}_2$ has a defect less then or equal
to the defect of $X_2$. To finish the proof one has to apply the same procedure to decomposition $-X = (-X_2) \cup (-X_1)$ to obtain decomposition $X = X_1 \cup_{\partial} X_2$, with $X_1$ and $-X_2$ being PC manifolds homotopy equivalent to $X_1$ and $-X_2$, respectively.

\[ \square \]

**Corollary.** Every closed simply-connected four-manifold $X$ possesses a structure of complex manifold with pseudo-convex boundary in the complement of some compact contractible submanifold (which is also PC manifold).

**Proof:** Consider arbitrary handle decomposition of $X$. Let $Y_1$ be a union of all 0- and 1-handles in $X$. The fundamental group of $Y_1$ is free and the natural map $\pi_1(\partial Y_1) \to \pi_1(Y_1)$ is an isomorphism.

We shall rearrange handlebody of $X$ by introducing pairs of dual 2- and 3-handles and handle addition, so that among 2-handles of new handlebody of $X$ it is possible to choose a set $\{h_i\}$ such that $X_1 = Y_1 \cup \cup_i h_i$ is a contractible manifold.

More detailed description of this construction is as follows: Let $\{x_1, \ldots, x_l\}$ be a free basis of $\pi_1(\partial Y_1)$. If we fix paths from a base-point to attaching spheres of 2-handles then they represent elements of $\pi_1(\partial Y_1)$, say $y_1, \ldots, y_L$. Since $X$ is simply-connected, $\{y_1, \ldots, y_L\}$ normally generate $\pi_1(\partial Y_1) \cong \pi_1(Y_1)$. Thus, each $\tilde{x}_i$, $i = 1, \ldots, l$ is a product of elements adjoint to $y_1, \ldots, y_L$. Introduce a pair of canceling 2- and 3-handles. We can slide this new 2-handle over handles corresponding to the elements $y_1, \ldots, y_L$ in the decomposition of $\tilde{x}_i$, so that its attaching circle become homotopic to $\tilde{x}_i$. Union $X_1$ of manifold $Y_1$ and 2-handles obtained by construction above for every element in $\{\tilde{x}_1, \ldots, \tilde{x}_l\}$ is a contractible submanifold of $X$.

Manifolds $X_1$ and $X_2 = X \setminus X_1$ have only handles of indexes less then 3 in their handle decompositions. Thus, Theorem 3 can be applied to decomposition $X = X_1 \cup_{\partial} X_2$, which finishes proof of the corollary. \[ \square \]

Above corollary implies that every closed simply-connected manifold possesses structure of Stein domain in the complement of certain contractible two-dimensional complex, hence every closed embedded surface $F$ in the complement of this complex satisfies adjunction inequality:

\[ -\chi(F) \geq F \cdot F + K \cdot F. \]

5. Corks with pseudo-convex boundary

**Theorem 4.** (See [CH], [M]) Let $M_1$, $M_2$ be two closed, smooth, simply-connected, $h$-cobordant 4-manifolds. Then:

\[ M_1 = N \cup_{\varphi_1} A_1, \quad M_2 = N \cup_{\varphi_2} A_2, \]

where $N$ is simply-connected, $A_1$ and $A_2$ are contractible and diffeomorphic to each other and $\varphi_i : \partial A_i \to \partial N$, $i = 1, 2$, are some diffeomorphisms.

Manifolds $A_1$ and $A_2$ have come to be known as corks, [K].

The proof of Theorem 3 can be adapted to show that the corks and manifold $N$ in the theorem above can be made pseudo-convex.

**Theorem 5.** Decompositions in Theorem 4 can be made pseudo-convex.
Proof: Start with decompositions provided by Theorem 4. Note that by the construction, see [3], manifolds $N$, $A_1$ and $A_2$ have handlebodies without handles of indexes 3 and 4. We divide the rest of the proof into three steps:

1. Make $N$ pseudo-convex.

Apply procedure from the proof of Theorem 3 to both decompositions $M_1 = N \cup \varphi_1 A_1$, and $M_2 = N \cup \varphi_2 A_2$, simultaneously. We obtain new decompositions $M_1 = N' \cup \varphi_1' A_1'$, and $M_2 = N' \cup \varphi_2' A_2'$, where $N'$ is pseudo-convex and homotopy equivalent to $N$; $A_1'$, $A_2'$ are contractible, but not necessarily diffeomorphic to each other.

2. Make $A_1'$ and $A_2'$ pseudo-convex.

Consider decompositions resulting from step 1 “up side down”:

$$-M_1 = (-A_1') \cup \varphi_1'^{-1} (-N') \text{ and } -M_2 = (-A_2') \cup \varphi_2'^{-1} (-N').$$

Suppose $h$ is a 2-handle in handlebody of $-A_1'$ with non-zero defect. Let $D$ be a cocore of $h$, $d = \partial D$, $m$ be a meridian of $P_n(d)$ and $F$ be a trivial embedded disk in $N$ bounded by $\varphi_1'(m)$ (note that $m$ is unknot in $\partial A_1 \cong \partial N$). We set

$$A_1'' = [A_1' \setminus \text{Nd}(P_n D)] \cup \text{Nd}(P_k F),$$

$$N'' = [N' \setminus \text{Nd}(P_k F)] \cup \text{Nd}(P_n D).$$

Hence $M_1 = N'' \cup A_1''$. Now we have to find manifold $A_2''$, so that $N'' \cup A_2'' = M_2$. Note that $N''$ is obtained from $N'$ by attaching a 2-handle along $\varphi_1'(P_n(d))$ and then removing Whitehead multiple of its cocore. Thus, $\partial N''$ is the result of the surgery of $\partial N'$ along the link $(l_1, l_2)$, where $l_1 = \varphi_1'(P_n(d))$ and $l_2 = P_k($meridian of $l_1$). Take

$$A_2'' = [A_2' \cup h'] \setminus \text{Nd}(P_k (\text{cocore of } h')), $$

where $h'$ is a 2-handle attached along $\varphi_2'^{-1} \circ \varphi_1'(P_n(d))$. Boundary of $A_2''$ is obtained by the surgery of $\partial A_2'$ along link $(\varphi_2'^{-1}(l_1), \varphi_2'^{-1}(l_1))$, therefore $\varphi_2''$ extends to $\varphi_2'': \partial A_2' \rightarrow \partial N''$. Manifold $N'' \cup \varphi_2'' A_2''$ is diffeomorphic to connected sum of $N'' \cup \varphi_2' A_2'$ and the double of positron $W_k$. It is easy to see that the double of a positron is diffeomorphic to $S^4$, which implies that $N'' \cup \varphi_2'' A_2'' \cong N_2$. Now, if we choose $k$ greater than $n$ then $\max\{D(\text{Nd}(P_k F)), D(h')\}$ (here $\text{Nd}(P_k F)$ is considered as a 2-handles attached to $N''$) and take $n$ to be the defect of $h$, then defect of $A_1'$ is reduced by $n$ and defects of $A_2'$ and $N_2'$ are not increased. Apply the above construction to every 2-handle with non-zero defect in $A_1'$ and $A_2'$. The resulting decompositions

$$M_1 = N'' \cup \varphi_1'' A_1'',$$

$$M_2 = N'' \cup \varphi_2'' A_2$$

are pseudo-convex, but corks are not diffeomorphic to each other. To fix that is the subject of the next step.

3. Make $A''_1$ diffeomorphic to $A''_2$.

It is shown in [3] that manifolds $A_1$, $A_2$ have the property that their doubles $A_i \cup \partial (-A_i)$ as well as their union $A_1 \cup \varphi_1^{-1} \circ \varphi_2 (-A_2)$ are diffeomorphic to $S^4$. Not difficult but rather technical calculation shows that this properties are preserved
under modifications from steps 1 and 2 above. Thus we have
\[ A''_i \cup_{\partial} (-A''_i) \cong S^4, \ i = 1, 2; \]
\[ A''_i \cup_{\varphi_1^{-1} \circ \varphi_2} (-A''_2) \cong S^4. \]

We define
\[ \tilde{N} = N'' \sharp (-A''_1), \]
\[ \tilde{A}_1 = A''_1 \sharp A''_2, \]
\[ \tilde{A}_2 = A''_2 \sharp A''_1, \]
where \( X \sharp Y \) stands for boundary connected sum of \( X \) and \( Y \). Since boundary connected sum of PC manifolds is a PC manifold, \( \tilde{N}, \tilde{A}_1 \) and \( \tilde{A}_2 \) have pseudo-convex boundary and, obviously, \( \tilde{A}_1 \cong \tilde{A}_2 \). Now we calculate
\[
\tilde{N} \cup \tilde{A}_1 = [N'' \sharp (-A''_1)] \cup_{\varphi_1 \sharp \varphi_1^{-1} \circ \varphi_2} [A''_1 \sharp A''_2] \\
\cong [N'' \cup_{\varphi_1} A_1] \# [(-A''_1) \cup_{\varphi_1^{-1} \circ \varphi_2} A''_1] \\
\cong N_1 \# S^4 \cong N_1.
\]

Analogously,
\[
\tilde{N} \cup \tilde{A}_2 = [N'' \sharp (-A''_2)] \cup_{\varphi_2 \sharp \varphi_2} [A''_2 \sharp A''_1] \\
\cong [N'' \cup_{\varphi_2} A_2] \# [(-A''_2) \cup_{id} A''_1] \\
\cong N_2 \# S^4 \cong N_2.
\]

This gives us convex decomposition of \( N_1 \) and \( N_2 \) with diffeomorphic corks \( \tilde{A}_1 \) and \( \tilde{A}_2 \) and finishes the proof of Theorem 5.

\[ \square \]

6. Questions and remarks

We would like to conclude with some questions and remarks.

**Question 1.** Is it always possible to find convex decomposition as in Theorem 3, so that contact structures on \( \partial \tilde{X}_1 \) and \( \partial (-\tilde{X}_2) \) coincide. (Authors can show that contact structures on \( \partial \tilde{X}_1 \) and \( \partial (-\tilde{X}_2) \) can be made homotopic.)

**Question 2.** Does Theorem 3 (and possibly positive answer to Question 1) pose any restriction to the genera of embedded surfaces in four-manifold.
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