Steady state numerical solutions for determining the location of MEMS on projectile

K Abiprayu, M F F Abdigusna and P H Gunawan
School of Computing, Telkom University, Jalan Telekomunikasi No. 1 Terusan Buah Batu, Bandung 40257, Indonesia.
E-mail: krishajarr@gmail.com; firly.feisal@gmail.com; phgunawan@telkomuniversity.ac.id

Abstract. This paper is devoted to compare the numerical solutions for the steady and unsteady state heat distribution model on projectile. Here, the best location for installing of the MEMS on the projectile based on the surface temperature is investigated. Numerical iteration methods, Jacobi and Gauss-Seidel have been elaborated to solve the steady state heat distribution model on projectile. The results using Jacobi and Gauss-Seidel are shown identical but the discrepancy iteration cost for each methods is gained. Using Jacobi’s method, the iteration cost is 350 iterations. Meanwhile, using Gauss-Seidel 188 iterations are obtained, faster than the Jacobi’s method. The comparison of the simulation by steady state model and the unsteady state model by a reference is shown satisfying. Moreover, the best candidate for installing MEMS on projectile is observed at point $T(10,0)$ which has the lowest temperature for the other points. The temperature using Jacobi and Gauss-Seidel for scenario 1 and 2 at $T(10,0)$ are 307 and 309 Kelvin respectively.

1. Introduction
Microelectromechanical systems or MEMS is a combination of electrical and mechanical components in a miniature sized device. Generally, this technology has been developed since the last few decades and its limited applications have been used. For instance, several applications of MEMS can be found in MIT microengine, electrostatic micromotor, thermal inkjet printhead, digital micromirror device, and etc [1, 2]. Moreover, this technology is used currently for military purposes. One of examples is MEMS can be integrated on projectile for obtaining the accuracy of a shot to the target. Indeed, the technology MEMS brings high level technology in military forces [3, 4, 5].

Several research and studies about the installation MEMS on projectile have been done [6, 7]. The fact that, the technology MEMS can work properly in a maximum temperature $71^\circ\text{C}$ or 344 K. Meanwhile, the temperature of projectile can reach a maximum temperature $267^\circ\text{C}$ or 540 K [8, 9]. Therefore, the installation of MEMS on the projectile is not an easy task. Since the MEMS might malfunction if the installation place on the surface of projectile has a temperature more than 344 K.

In the papers of [8, 9], the numerical simulation of unsteady heat distribution is elaborated. The numerical method for solving the unsteady model is quite complicated since the discrete time and space should be considered. For instance, in the paper of [8], the unsteady model is approximated by finite element method which has complicated discrete property. Nevertheless,
this method is known as a robust method for complex geometry domain. In this paper, the steady state heat distribution model will be considered on fix domain of projectile. This idea appears to be emphasized to reduce the complexity of numerical scheme such as a stability from the relation of discrete time and space.

Here, steady model will be discretized by a finite difference method which has uniform grid spacing. This method is simple and straightforward method. Thus, no complicated discrete property will be considered. By using the steady state model, the system algebraic equations will be obtained. Indeed, the approximation of solution for linear system equations should be calculated. Here, two numerical iterative methods for solving linear system equations will be used. The methods are known as Jacobi and Gauss-Seidel method. The comparison of results by unsteady model in a reference and steady model also will be considered in this paper.

2. Mathematical Model

In this paper, the steady state of temperature distribution model will be considered. The 2D mathematical model of steady state distribution of heat over entire domain of projectile $\Omega + \partial \Omega$ is given as

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = 0, \quad (x,y) \in \Omega,$$

$$T(x,y) = \beta(x,y), \quad (x,y) \in \partial \Omega$$

where $T(x,y)$ is the temperature distribution function, $\beta(x,y)$ is the known boundary function, $x$ and $y$ are the spatial coordinate of domain. Here, the $\partial \Omega$ denotes the boundary of domain. The illustration of projectile domain can be seen in Fig 1.

![Figure 1. The configuration of projectile domain.](image-url)

In order to discretize (1-2), the domain (including boundary) can be discretized by several points in a mesh (see Figure 2 for more detail). Assume a point in a mesh is in a set $\mathcal{M} = \{1,2,\cdots,N\}$, $N \in \mathbb{Z}^+$, thus the discrete form of equation (1) can be given as

$$\frac{(T_a - 2T_i + T_b)}{\Delta x^2} + \frac{(T_c - 2T_i + T_d)}{\Delta y^2} = 0, \quad i, a, b, c, d \in \mathcal{M}$$

The grids spacing $\Delta x$ and $\Delta y$ are set to be uniform and similar. Thus the equation can be rewritten as

$$T_i = \frac{T_a + T_b + T_c + T_d}{4}$$
Thus from Figure 2, the system of equations can be formed as follow,

\[ T_1 = \frac{325 + 330 + T_2 + T_6}{4}, \]
\[ T_3 = \frac{315 + T_2 + T_4 + T_8}{4}, \]
\[ T_5 = \frac{310 + T_4 + 295 + T_{10}}{4}, \]
\[ T_7 = \frac{T_2 + T_6 + T_8 + T_{13}}{4}, \]

\[ T_2 = \frac{315 + T_1 + T_3 + T_7}{4}, \]
\[ T_4 = \frac{310 + T_3 + T_5 + T_9}{4}, \]
\[ T_6 = \frac{330 + T_1 + T_7 + T_{12}}{4}, \]
\[ T_N = \cdots + \cdots + \cdots + \cdots. \]  

(5)

Indeed now the problem is to solve this system of equations (5). In this paper, two numerical methods Jacobi and Gauss-Seidel iterative method for approximating the solutions of (5) will be used.

3. Numerical Methods and Algorithms

3.1. Jacobi iterative method

Jacobi iterative method is a method for solving the linear system of equations \( Ax = b \). Indeed the system (5) can be transformed into \( AT = b \). The formula of Jacobi iterative method is given as

\[ T_i = \frac{1}{a_{ii}} \left[ b_i - \sum_{j=1,j\neq i}^{N} (a_{ij}T_j) \right], \quad i \in M. \]  

(6)

The equation (6) will be solved iteratively using the initial guess \( T_i^{(0)} \) of \( T^{(0)} \). The algorithm of Jacobi is given in Algorithm 1. In the Algorithm 1, the iterative method can be interrupted using the relation of \( ||T^{(k)} - T^{(k-1)}|| < TOL \). Here, TOL is the tolerance number which indicates the steady state is tolerated at a fix distance. The second stopping criteria in this iterative is used the maximum number of iteration \( Kmax \). Since in some cases, the distance \( ||T^{(k)} - T^{(k-1)}|| \) is never reach the TOL value due to the initial guess \( T^{(0)} \).

3.2. Gauss-Seidel iterative method

\[ T_i^{(k)} = \frac{1}{a_{ii}} \left[ b_i - \sum_{j=1}^{i-1} (a_{ij}T_j^{(k)}) - \sum_{j=i+1}^{N} (a_{ij}T_j^{(k-1)}) \right]. \]  

(7)
Algorithm 1 The algorithm of Jacobi’s method.

Step 0. Start.

Step 1. Set $k = 1$, $TOL$: tolerance, $Kmax$: maximum number of iterations, $T^0$: initial approximation.

Step 2. While $(k \leq Kmax)$ do Steps 3-6

Step 3. For $i \in \mathcal{M}$

$$T_i^{(k)} = \frac{1}{a_{ii}} \left[ b_i - \sum_{j=1, j \neq i}^{N} \left( a_{ij} T_j^{(k-1)} \right) \right].$$

Step 4. If $||T^{(k)} - T^{(k-1)}|| < TOL$
the OUTPUT $(T^{(k)})$.

(The procedure was successful.) STOP

Step 5. Set $T^{(k-1)} = T^{(k)}$

Step 6. Set $k = k + 1$

Step 7. OUTPUT ('Maximum iterations exceeded.')</n
Step 8. End.

Algorithm 2 The algorithm of Gauss-Seidel method.

Step 0. Start.

Step 1. Set $k = 1$, $TOL$: tolerance, $Kmax$: maximum number of iterations, $T^0$: initial approximation.

Step 2. While $(k \leq Kmax)$ do Steps 3-6

Step 3. For $i \in \mathcal{M}$

$$T_i^{(k)} = \frac{1}{a_{ii}} \left[ b_i - \sum_{j=1}^{i-1} \left( a_{ij} T_j^{(k)} \right) - \sum_{j=i+1}^{N} \left( a_{ij} T_j^{(k-1)} \right) \right].$$

Step 4. If $||T^{(k)} - T^{(k-1)}|| < TOL$
the OUTPUT $(T^{(k)})$.

(The procedure was successful.) STOP

Step 5. Set $T^{(k-1)} = T^{(k)}$

Step 6. Set $k = k + 1$

Step 7. OUTPUT ('Maximum iterations exceeded.')</n
Step 8. End.

The second method is called the Gauss-Seidel iterative method. This method is a modification of Jacobi’s method [10]. The modification on acting on the iterative process. The modified iteration method is given in Equation (7), for all $i \in \mathcal{M}$.

Further, Algorithm 2 shows the algorithm of Gauss-Seidel method. Here, the modification to calculate $a_{ij} T_j^{(k-1)}$ in Jacobi’s method is considered. Indeed, for $i > 1$, the solutions $T_1^{(k)}, \cdots, T_{i-1}^{(k)}$ have been calculated and thus the remain $T_{i+1}^{(k-1)}, \cdots, T_N^{(k-1)}$ should be used.
4. Numerical simulations

In this paper, the investigation of finding the best position for installing MEMS on projectile will be elaborated. The investigation of the observation points on projectile can be seen in Figure 3. This observation points can be used as a candidate for installing MEMS on projectile.

![Figure 3. The candidate places of points on MEMS. Four points are chosen in order to validate the proposed methods.](image)

Here, two scenarios are elaborated based on the input of initial data at boundary. First scenario, the boundary condition is set as the results of unsteady heat distribution model in [8]. The example of the application interface for Jacobi and Gauss-Seidel can be seen in Figure 4.

![Figure 4. The example of application interface with Jacobi results.](image)

In this simulation, the other conditions are given as follow

\[ T^{(0)} = 280 \text{ Kelvin}, \text{(for inner domain)}, \]
\[ TOL = 10^{-6}, \]
\[ K_{max} = 10^4. \]

The boundary conditions of this simulation in the area \( X_1, X_2, X_3, X_4, X_5 \) and \( X_6 \) are described in Figure 4. These values of boundary conditions are obtained from the paper of Akbar A, et al., [8].

The results of first scenario can be seen in Figure 5. From Figure 5, the heat map shows the highest temperature is located on the back side of projectile (330 K). Meanwhile the lowest temperature is shown in the front of projectile (290). Using previous initial conditions, the iteration by Jacobi to solve this scenario is 350 iterations. Meanwhile, by Gauss-Seidel the final iteration is obtained 188 iterations.
For the second scenario, the modification of initial boundary condition from first scenario is made. Here, the boundary conditions is obtained from [8], but in more detail than the approximation as shown in Figure 4. The result of second scenario is shown in Figure 6. In this results, the highest temperature is not on entire of back side projectile. However, the highest temperature is located at the back corners of projectile only.

![Figure 5.](image1.png) ![Figure 6.](image2.png)

**Figure 5.** The results of steady state heat distribution on projectile for first scenario.

**Figure 6.** The results of steady state heat distribution on projectile for second scenario.

In addition, the comparison of numerical result of steady state heat distribution model using Jacobi and Gauss-Seidel methods and unsteady model using finite element method is shown in Figure 7.

From Figure 7, the result using scenario 2 is close to the numerical result by [8]. Indeed, the close solution to the reference is depend on the initial boundary conditions. As in Figure 3, the candidate for installing MEMS on projectile should be investigated. The investigation results for each observation points are shown in Table. 1.

| Points   | Scenario 1 | Scenario 2 | Results in [8] |
|----------|------------|------------|----------------|
| T1(5,0)  | 318        | 317        | 315            |
| T2(10,0) | 307        | 309        | 305            |
| T3(2,2)  | 325        | 320        | 318            |
| T4(7,2)  | 314        | 315        | 315            |

From Table 1, it appears that temperature for four points are still under heat tolerance of MEMS which is 344 Kelvin. And it is concluded that the location placement of MEMS can be done in every observation points. Moreover, from Table 1, the best location to install MEMS is obtained at point T2 which has a lowest temperature than the other points.
Figure 7. The results of steady state heat distribution using Jacobi’s (a) and Gauss-Seidel (b) method and simulation by [8] (c) on projectile.

5. Conclusion
The Jacobi and Gauss-Seidel iteration methods have been elaborated to solve the steady state heat distribution model on projectile. The results using Jacobi and Gauss-Seidel is similar but they have different iteration cost. Using Jacobi’s method, the steady state is obtained by 350 iterations. Meanwhile, using Gauss-Seidel the steady state is obtained by 188 iterations. The satisfied comparison by the simulation result in [8] is achieved. Moreover, from two scenarios, the best candidate for installing MEMS on projectile is observed at point $T_2(10, 0)$ which has the lowest temperature.

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