A comparative study of effect of heater orientation on flow and heat transfer characteristics in buoyancy driven flow in cryogenic liquid

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Abstract. The present study aims to analyse the buoyancy driven fluid flow and heat transfer characteristics using different heater orientation in liquid nitrogen for cooling superconductor. For this purpose, a localized circular heater (analogous to superconductor) generating heat at constant rate submerged in liquid nitrogen enclosure is analysed. The study is performed with two orientations of heater; one is vertical and another is horizontal. The entropy generation associated with heat transfer and fluid flow is also discussed in the present study. The Rayleigh number, average Nusselt number, maximum vertical velocity, entropy generation and Bejan number are used as comparing parameters for two orientations of heater. Under same heat generation rate, Rayleigh number is comparatively higher for vertically oriented heater whereas average Nusselt number is comparatively higher for horizontally oriented heater. The maximum vertical velocity of fluid is higher when heater is vertically oriented. The horizontally oriented heater has comparatively less value of thermal entropy generation than vertically oriented heater but horizontally oriented heater has higher value of fluid friction irreversibility. In summary, horizontally oriented heater has comparatively higher value of total entropy generation than vertically oriented heater. Bejan number is higher for horizontally oriented heater. In overall study, the value of Bejan number is less than 0.5; therefore, it can be said that fluid friction irreversibility is more dominated than heat transfer irreversibility.

1. Introduction
Superconductors are used in many applications like superconducting transformers, motors, generators, magnetic bearings, MRI and NMR machine, etc. Cryogenic fluid performs important role in convective cooling of a superconductor so that superconductor can work efficiently; but at the same time orientation of superconductor can also affect the heat transfer from its surface and flow characteristics of cryogenic liquid during convective cooling. Therefore, it is important to have an idea of how the orientation of superconductor affects the heat transfer and flow characteristics in cryogenic liquid during natural convective cooling.

Because of immersion of a localized heater, the fluid flow caused in a liquid is studied by Satpathy et al. [1]. They have noticed a characteristic 3D flow and two convective cells which are symmetric in nature. Further, they continued their study by comparing different silicon oils with liquid nitrogen for different heating conditions. Dubois et al. [2] experimentally presented the convective heat transfer process in laminar regime caused by local heater surrounded by liquid. In their study, they depicted
that the motion of fluid caused by the Rayleigh-Bénard instability. They observed two revolving cells which were rotating in a counter-clockwise manner in the experimental chamber. In their investigation, they compared the data obtained from experiments conducted at room temperature for different gradients of temperature in nitrogen liquid accompanying silicon oils of various viscosities. Satpathy et al. [3] have presented a numerical analysis on cooling through natural convection in a fluid operating at cryogenic temperature. The influence of the ratio of width to the height of heater on the rate of heat transfer has been investigated in terms of Nusselt number. They have resolved that the rate of heat transfer will be more efficient when heater length is smaller. A complete study about transient natural convection and flow of fluid within an enclosure for a heat producing and heat carrying body is presented numerically by Ha et al. [4]. They have concluded that flow of fluid in the enclosure increases with increase in Rayleigh number, thereby increasing the rate of heat transfer. They have also concluded that, the shape of transient streamlines and isotherms in the enclosure is influenced by different Rayleigh number. The matter of entropy generation in rectangular cavities because of natural convective heat transfer is presented by Oliveski et al. [5]. They have concluded that the problem of entropy generation because of viscous effects increases when Rayleigh number increases under same ratio of the width to the height. They have also concluded that as the Rayleigh number increases, the total generation of entropy also increases in a linear manner. Problem of entropy generation due to natural convective heat transfer in the enclosures of right-angled triangular shapes has been studied numerically by Basak et al. [6]. They have observed that when Rayleigh number is low, the transfer of heat is mainly because of conduction and when the vertex angle at top is 15° the higher value of entropy generation is detected. Also, for increase of total irreversibility, the contribution of fluid friction entropy generation is higher. Ngo and Byon [7] presented a study on natural convective heat transfer inside the square cavity using finite element method. They analyzed the influence of size of heater and location of heater so that internal heat transfer due to free convection can be improved. A review on natural convective heat transfer in non-square enclosures is presented by Das et al. [8]. Sarris et al. [9] have carried out a 2D and 3D numerical study on influence of heated strips oriented horizontally at the lower wall of melting gas. In their study, the parameters that are influencing the rate of heat transfer are Rayleigh number, aspect ratio of tank and width of heated strip. They have concluded that when width of strip and aspect ratio of tank increases, fluid flow and heat transfer increases. Study on natural convective heat transfer inside an enclosure of rectangular shape filled with air was presented numerically by Cianfrini et al. [10]. In their study the bottom side of enclosure was separately heated and one side was cooled. Their study was presented with various values of the Rayleigh number, the aspect ratio of the enclosure, and size of heater. They had noticed a single roll cell in the flow structure. They had concluded that when size of heater and non-dimensional Rayleigh number increases, the effectiveness of heat removal from heater also increases. Koizumi [11] has presented an experimental and numerical study on effects of Grashof number and height of container in a rectangular container locally heated from below. Author has mentioned that thermal boundary conditions of the bottom wall can significantly influence the pattern of flow and process of transition to chaos. The author has experimentally found that when Grashof number is equal to 1800000, 3-dimensional form of roll cell changes from time dependent periodic motion to chaotic fluid flow directly. In most of the studies related to natural convection, appreciable care has been given to Rayleigh-Bénard convection from vertical enclosure or horizontal enclosure clearly and explicitly stated either with constant temperature or heat flux condition [12]; whereas, Satpathy et al. [1,3] and Dubois et al. [3] performed study which is distinctly separate from Rayleigh-Bénard convection.

As the orientation of heater (analogous to superconductor) is changed; the heat transfer characteristics of fluid flow and entropy generation in the domain will also change. It is not reported in the literature. Therefore, effect of horizontal and vertical orientation of heater on characteristics of fluid flow and heat transfer in buoyancy driven flow is presented in the present paper.

2. Problem Definition
For vertical orientation of heater, the 3D domain is symmetrical about x-axis; whereas for horizontal orientation of heater, the 3D domain is symmetrical about y-axis. Therefore, one-half portion of the domain is simulated to save the computational time as demonstrated in Fig. 1. The dimensions of domain which is used for computation are 100 mm (length) along x-axis, 100 mm (height) along y-axis and 30 mm (width) along z-axis. The length of heater is 50 mm and material used for heater is copper. For vertical orientation of heater, the distance between top surface of enclosure and top surface of heater is 50 mm whereas for horizontal orientation of heater, the distance between top surface of enclosure and central axis of heater is 52.5 mm. Height above the heater surface \( h = 50 \text{mm} \) is taken as characteristics length. The diameter of heater is taken as 5mm.

![Figure 1. Computational domain for vertical orientation of heater (left) and computational domain for horizontal orientation of heater (right)](image)

3. Governing equations

1) Governing differential equations for flow of fluid and transfer of heat in 3-Dimensional, steady state and laminar form are as follows;

a) Mass conservation equation

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1a}
\]

b) Momentum conservation equations

- Momentum conservation equation in x-direction

\[
\frac{u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \tag{1b}
\]

- Momentum conservation equation in y-direction

\[
\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + v \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + g\beta(T - T_0) \tag{1c}
\]

- Momentum conservation equation in z-direction

\[
\frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + v \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)
\]
Energy conservation equation

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial z} + v \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$

(1d)

c) Energy conservation equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{Q_g}{\rho C_p}$$

(1e)

Here, $u$, $v$, and $w$ are the velocity components in $x$, $y$ and $z$ directions, respectively. The symbols $T$ and $T_0$ stand for localized temperature and operating temperature (i.e., temperature of surrounding fluid) respectively. Furthermore, $\rho$ is the density of liquid ($\text{kg/m}^3$) at temperature $T_0$ and $p$ is the pressure ($\text{N/m}^2$), $Q_g$ ($\text{W/m}^3$) is heat generation rate and $g$ is the acceleration due to gravity ($\text{m/s}^2$) in negative $y$-direction. Moreover, $\alpha$, $\nu$ and $\beta$ are thermal diffusivity ($\text{m}^2/\text{s}$) of fluid, kinematic viscosity ($\text{m}^2/\text{s}$) of fluid and thermal expansion coefficient (1/K) of fluid respectively.

2) Governing equations for entropy generation are as follows

In buoyancy driven or free convection flows, entropy is generated because of presence of temperature gradients (i.e., due to heat transfer) and velocity gradients (i.e., due to fluid flow friction) [13].

$$S_T = S_{HT} + S_{FF}$$

(2a)

$$S_{HT} = \int_v S_h \, dv$$

(2b)

$$S_{FF} = \int_v S_f \, dv$$

(2c)

Where, $S_{HT}$ is the generation of entropy ($\text{W/K}$) in the domain due to transfer of heat, $S_{FF}$ is the generation of entropy ($\text{W/K}$) in the domain due to fluid flow friction and $S_T$ is the total entropy generation ($\text{W/K}$) in the domain. Furthermore, $S_h$ and $S_f$ are localized generation of entropy due to transfer of heat and localized generation of entropy due to fluid flow friction respectively. Moreover, $K_f$ ($\text{W/m-K}$) and $\mu$ ($\text{kg/m-s}$) are thermal conductivity and dynamic viscosity of the fluid respectively.

$$S_h = \frac{K_f}{T_0} \left( \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 + \left( \frac{\partial T}{\partial z} \right)^2 \right)$$

(2d)

$$S_f = \frac{2\mu}{T_0} \left( \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right) + \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 + \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)^2 + \left( \frac{\partial w}{\partial z} + \frac{\partial u}{\partial x} \right)^2$$

(2e)

Bejan number ($Be$)

Bejan number is the ratio of generation of entropy due to transfer of heat to the total generation of entropy.

$$B_e = \frac{S_{HT}}{S_T} = \frac{S_{HT}}{S_{HT} + S_{FF}}$$

(3)

If $B_e > 0.5$, then irreversibility due to transfer of heat will be higher.

If $B_e < 0.5$, then fluid friction irreversibility will be higher.

4. Boundary conditions and solution methodology

For horizontal orientation of heater: Enclosure top surface is kept at a constant temperature $T_0 = 75$ K and free slip velocity ($\tau_v = 0$) condition is applied. Zero heat flux and no slip velocity condition are
applied to bottom wall, left wall and side walls of the enclosure. A constant heat generation rate ($Q_g$ in W/m$^3$) is applied for heater. Symmetry boundary condition is applied on right wall attached to heater.

For vertical orientation of heater: Enclosure top surface is kept at a constant temperature $T_0 = 75$ K and free slip velocity ($\tau_w = 0$) condition is applied. Zero heat flux and no slip velocity condition are applied to right wall, left wall and side walls of the enclosure. A constant heat generation rate ($Q_g$ in W/m$^3$) is applied for heater. Symmetry boundary condition is applied on bottom wall attached to heater.

The equations of mass, momentum and energy conservation are solved using finite-volume method with the help of commercial Ansys Fluent software. The coupling between pressure and velocity is obtained by the SIMPLE algorithm [14]. To discretize the momentum and energy equations, 1$^{st}$ order upwind scheme has been used. The local entropy generation equations are solved using custom field function in Ansys Fluent and the generation of total entropy is calculated through the integration of the localized generation of entropy in a computational domain. The convergence criteria for mass and momentum conservation equation are taken as $10^{-4}$, whereas for energy conservation, it is taken as $10^{-9}$. The operating temperature is taken as 75K.

5. Validation study and grid independence test
In the present study, the effect of heater orientation on flow characteristics of fluid and heat transfer in a buoyancy driven flow is discussed. The computational results of Satpathy et al. [1] and experimental results of Dubois et al. [2] are used to validate the present numerical results. So, we compared distribution of normalized velocity and non-dimensional temperature on the height over the surface of heater with [1] and [2]. In general, satisfactory agreement has been established between the existing simulations and the benchmark results as shown in Figure 2.

Study of grid independence is executed with three mesh sizes viz. Mesh C1, Mesh C2, Mesh C3 as shown in Table 1. It is noticed that comparative changes in the computed temperature difference $\Delta T$ ($T_H - T_0$) for vertical orientation and horizontal orientation of heater are 0.20% and 1% respectively, whereas comparative changes in computed maximum velocity of fluid ($V_{max}$) for vertical orientation and horizontal orientation of heater are 2% and 0.5% respectively when moved from Mesh C2 to Mesh C3. Therefore, Mesh C2 is selected for the existing simulation in order to preserve computational cost.

**Table 1.** Grid independence study for vertical and horizontal orientation of heater generating heat ($Q_g$) at a rate of 19978W/m$^3$

| Orientation | Mesh  | Number of control volumes | $\Delta T$ (K) | $V_{max}$ (m/s) |
|-------------|-------|---------------------------|----------------|----------------|
| Vertical    | C1    | 6,76,101                  | 0.1401 (75.1249-75) | 0.0150         |
|             | C2    | 11,56,529                 | 0.1507 (75.1507-75) | 0.0165         |
|             | C3    | 15,60,324                 | 0.1510 (75.1510-75) | 0.0168         |
| Horizontal  | C1    | 6,75,204                  | 0.1092 (75.0958-75) | 0.0058         |
|             | C2    | 11,56,204                 | 0.1266 (75.1266-75) | 0.0064         |
|             | C3    | 15,52,803                 | 0.1278 (75.1278-75) | 0.006432       |
6. Dimensionless numbers

Average Nusselt number \((Nu_{Avg})\): It is the ratio between convective transfer of heat and the conductive heat transfer.

\[
Nu_{Avg} = \frac{q_{Avg} h}{(T_h - T_b) K_f}
\]  
\(4\)

Rayleigh Number \((Ra)\): It is the ratio between buoyancy force and inertia force. Higher the Rayleigh number, more will be the dominance of buoyancy force.

\[
Ra = \frac{g \beta (T_h - T_b) h^3}{\nu \alpha}
\]  
\(5\)

7. Results and discussion

Fig. 3 (left) shows the velocity vector above heater surface for vertical orientation of heater. In Fig. 3 (left) we can see that, the fluid in contact with heater surface, gets hot, gets lighter and move towards the top surface of enclosure because of buoyancy action. When fluid reaches the top surface of enclosure, buoyancy effect ends. But because of inertia effect in fluid, the fluid gets diverted along both sides of heater and arrive at their respective top corners of enclosure. Now because of gravity action, fluid starts moving downward and hence in this way two symmetric rotating structures are formed in the fluid domain. The maximum vertical velocity of fluid when heater is vertically oriented is 0.0165 m/s. Figure 3 (right) shows the velocity vector above heater surface for horizontal orientation of heater. The fluid in contact with heater gets hot, gets lighter, moves upward, reaches top surface of enclosure. In this case also, the fluid gets diverted along both sides of heater, arrive at their respective top corners and due to gravity action again starts moving downward. But the rotating structures that are formed are not symmetric; also, their size and location is not same. One rotating structure is at left side and two rotating structures at right side of heater. The maximum vertical velocity of fluid when heater is horizontally oriented is 0.0064 m/s.
Figure 3. Vector of velocity showed in y-z plane for vertical orientation of heater (left) and vector of velocity in y-z plane for horizontal orientation of heater (right)

Figure 4 shows temperature contour in the domain when heater is vertically oriented. From Fig. 4 it is observed that the fluid portion which is in-line with vertical heater is more influenced by heat generated in heater (shown by yellow and green plume). The portion of fluid which is just below the top surface of enclosure is also affected by heat generation in heater. From Fig. 4 it is also clear that, fluid temperature which is not in-line with heater and away from top surface of enclosure is not increased i.e., it is not affected by heat generated in heater (shown by blue plume). The maximum temperature in the domain when heater is vertically oriented is 75.1507 K. Figure 5 shows temperature contour in the domain when the heater is horizontally oriented. Just above the heater surface, the fluid has uniform temperature along heater length (shown by continuous red and yellow plume). The temperature is not uniform above and away from heater surface. The fluid below heater is comparatively less affected by heat and hence it has lower temperature. The maximum temperature in the domain when heater is horizontally oriented is 75.1266 K.

Figure 6 shows contour for local Bejan number distribution in the domain when heater is vertically and horizontally oriented. In the solid heater, the value of Bejan number is equal to unity (shown by red colour) and hence zero fluid friction irreversibility in the heater. In the fluid domain (i.e., outside the heater), irreversibility due to transfer of heat and fluid flow friction both comes into picture.
Figure 4. Temperature contour in x-y plane (left) and in y-z plane (right) for vertical orientation of heater

The blue plume designates dominance of irreversibility due to fluid friction; whereas red plume designates dominance of irreversibility due to transfer of heat. On the other hand, yellow plume indicates equal dominance of irreversibility due to transfer of heat and fluid flow friction. Local Bejan number effect is not significant for horizontal orientation of heater because it has lower heat transfer irreversibility. The computed values of average velocity of fluid ($V_{avg}$), non-dimensional Rayleigh number ($Ra$), non-dimensional average Nusselt number ($Nu_{avg}$), total generation of entropy or total irreversibility ($S$) and Bejan number ($Be$) are shown in Table 2. Difference between average temperature of heater surface and bulk mean temperature of fluid ($T_h - T_b$) is higher when the heater is vertically oriented. Therefore, $Ra$ number is higher (i.e., dominance of buoyancy force in the fluid domain will be higher) when heater is vertically oriented. The average velocity $V_{avg}$ in fluid domain is higher when heater is horizontally oriented. Difference between average temperature of heater surface and bulk mean temperature of fluid ($T_h - T_b$) is lower when heater is horizontally oriented and hence horizontally oriented heater has higher value of average Nusselt number on its surface. Temperature gradients in the domain are higher when heater is vertically oriented. Therefore, heat transfer irreversibility and hence Bejan number are higher when heater is vertically oriented. Velocity gradients in the domain are higher when heater is horizontally oriented, which results in comparatively higher irreversibility due to fluid friction and lower Bejan number.

Table 2. Analysis on heater orientation

| Orientation | ($T_h - T_b$) K | $V_{avg}$ (m/s) | Ra | $Nu_{avg}$ | $S_T$ (W/K) | $Be$ |
|-------------|-----------------|-----------------|----|------------|-------------|------|
| Vertical    | 0.13553         | 0.000508        | 51935519.53 | 60.1851    | 0.009743    | 0.14820 |
| Horizontal  | 0.10377         | 0.000859        | 39764988.28 | 78.6055    | 0.041486    | 0.02979 |
8. Conclusion

3D computational study on effect of heater orientation on flow characteristics of fluid and heat transfer in a buoyancy driven flow using cryogenic liquid is presented. Two symmetric rotating structures of approximately same size are formed in the fluid domain when heater is vertically oriented; whereas, non-symmetric rotating structures of non-identical size are formed in the fluid domain when heater is horizontally oriented. The maximum velocity of the fluid is found to be decreased when heater orientation is changed from vertical to horizontal. When heater is vertically oriented, only some portion of fluid is affected by heat generation in heater; whereas, when heater is horizontally oriented, complete portion of fluid is influenced by heat generation in heater. Rayleigh number (dominance of buoyancy force) is higher in the fluid domain when heater is vertically oriented, because difference between average temperature of heater surface and bulk mean temperature of fluid is higher for vertical orientation of heater. Higher average velocity of the fluid is obtained when heater orientation is changed from vertical to horizontal. Velocity gradients in the domain are higher when heater is horizontally oriented.
horizontally oriented and hence fluid friction irreversibility in the domain is higher (i.e., lower \( Be \)). Temperature gradients in the domain are higher when heater is vertically oriented and hence heat transfer irreversibility is higher (i.e., higher \( Be \)). Difference between average temperature of heater surface and bulk mean temperature of fluid is lower when heater is horizontally oriented. Therefore, \( Nu_{avg} \) on heater surface is higher when heater is horizontally oriented. The vertical orientation of heater involves lower total entropy generation in the domain.

**Nomenclature**

- \( T_b \): Bulk mean temperature of fluid
- \( T_{H} \): Maximum temperature in the domain
- \( h \): Characteristic length
- \( Ra \): Rayleigh number
- \( q_{avg} \): Average heat flux from heater surface (W/m\(^2\))
- \( Be \): Bejan Number
- \( S_T \): Total entropy generation
- \( T_h \): Average temperature of heater surface
- \( V_{avg} \): Average velocity of fluid
- \( Nu_{avg} \): Average Nusselt number
- \( K_f \): Thermal conductivity of fluid (W/m K)

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