Penguins, Trees and Final State Interactions in B Decays in Broken SU3

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ABSTRACT

The availability of data on $B_s$ decays to strange quasi-two-body final states, either with or without charmonium opens new possibilities for understanding different contributions of weak diagrams and in particular the relative contributions of tree and penguin diagrams. Corresponding $B_d$ and $B_s$ decays to charge conjugate final states are equal in the SU(3) symmetry limit and the dominant $SU(3)$ breaking mechanism is given by ratios of CKM matrix elements. Final State Interactions effects should be small, because strong interactions conserve $C$ and should tend to cancel in ratios between charge conjugate states. Particularly interesting implications of decays into final states containing $\eta$ and $\eta'$ are discussed.
A large number of relations between ratios of $B^o$ and $B_s$ amplitudes to charge conjugate final states are obtainable by extending the general SU(3) symmetry relations for $B \to PP$ decays found by Gronau et al\cite{1} beyond the two-pseudoscalar case to all quasi-two-body charmless strange decays and charmonium strange decays. These relations are of particular interest because: (1) they relate a large number of decay ratios in the SU(3) symmetry limit, and (2) strong final state interactions should cancel out in ratios between decay amplitudes to charge conjugate final states which have the same strong final state rescattering. We extend the treatment of ref. \cite{1} by noting the following points:

1. Many relations are obtainable with the U spin SU(2) subgroup of SU(3) and in particular the discrete transformation (Weyl reflection) $d \leftrightarrow s$ which simply interchanges the $d$ and $s$ flavors.

2. U spin relations can be valid also for contributions from the electroweak penguin diagrams which break SU(3) because the photon and the $Z$ are both singlets under U spin (they couple equally to $d$ and $s$ quarks) while they contain octet components in SU(3) (their couplings to $u$ quarks differs from those to $d$ and $s$).

3. Relations obtained from the discrete $d \leftrightarrow s$ transformation do not require that both final hadrons are in the same SU(3) octet. Thus they apply equally well to other channels than PP.

4. Ratios of amplitudes that go into one another under the $d \leftrightarrow s$ transformations and have final states which are charge conjugates of one another should be insensitive to strong final state interactions which are invariant under charge conjugation.

With this approach, we find the following relations

\[
\frac{A(B^o \to \pi^- K^{(*)+})}{A(B_s \to \pi^+ K^{(*)-})} = \frac{A(B^o \to \pi^0 K^{(*)0})}{A(B_s \to \pi^0 K^{(*)0})} = \frac{A(B^o \to \rho^- K^{(*)+})}{A(B_s \to \rho^+ K^{(*)-})} = \frac{A(B^o \to \rho^0 K^{(*)0})}{A(B_s \to \rho^0 K^{(*)0})} = \frac{A(B^o \to \omega K^{(*)0})}{A(B_s \to \omega K^{(*)0})} = \frac{A(B^o \to a_2^- K^{(*)+})}{A(B_s \to a_2^+ K^{(*)-})} = \frac{A(B^o \to a_2^0 K^{(*)0})}{A(B_s \to a_2^0 K^{(*)0})} = \frac{A(B^o \to f_2 K^{(*)0})}{A(B_s \to f_2 K^{(*)0})} = \frac{A(B_s \to \psi \bar{K}^{(*)})}{A(B_s \to \psi \bar{K}^{(*)})} = \frac{A(B_s \to \psi \bar{K}^{(*)})}{A(B_s \to \psi \bar{K}^{(*)})} = F_{SU3} \quad (1a)
\]

where $K^{(*)}$ denotes $K$ or any $K^*$ resonance, $D^{(*)}$ denotes $D$ or any $D^*$ resonance, $\psi^{(*)}$ denotes any charmonium state and $F_{SU3}$ denotes an SU(3)-breaking parameter which may be different
for different final states. Similarly for the charge conjugate states,

\[
\frac{A(\bar{B}^o \to \pi^+ K^{(*)-})}{A(\bar{B}_s \to \pi^- K^{(*)+})} = \frac{A(\bar{B}^o \to \pi^o \bar{K}^{(*)o})}{A(\bar{B}_s \to \pi^o K^{(*)o})} = \frac{A(\bar{B}^o \to \rho^+ K^{(*)-})}{A(\bar{B}_s \to \rho^- K^{(*)+})} = \\
\frac{A(\bar{B}_s \to \rho^o \bar{K}^{(*)o})}{A(\bar{B}_s \to \rho^o K^{(*)o})} = \frac{A(\bar{B}^o \to \omega \bar{K}^{(*)o})}{A(\bar{B}_s \to \omega K^{(*)o})} = \frac{A(\bar{B}^o \to a_2^+ K^{(*)-})}{A(\bar{B}_s \to a_2^- K^{(*)+})} = \\
\frac{A(\bar{B}_s \to a_2^o \bar{K}^{(*)o})}{A(\bar{B}_s \to a_2^o K^{(*)o})} = \frac{A(\bar{B}^o \to f_2 \bar{K}^{(*)o})}{A(\bar{B}_s \to f_2 K^{(*)o})} = \frac{A(\bar{B}_s \to \psi K^{(*)})}{A(\bar{B}^o \to \psi K^{(*)})} = \\
= \frac{A(\bar{B}_s \to \psi^{(*)} K^{(*)o})}{A(\bar{B}^o \to \psi^{(*)} \bar{K}^{(*)o})} = \frac{A(\bar{B}_s \to D^- D_s)}{A(\bar{B}^o \to D^+ D_s)} = \frac{A(\bar{B}_s \to D^{(*)-} D_s^{(*)})}{A(\bar{B}^o \to D^{(*)+} \bar{D}_s^{(*)})} = F_{SU3} \quad (1b)
\]

Note that only tree and penguin diagrams contribute to these transitions and that the individual tree and penguin diagrams, including both gluonic and electroweak penguins, also go into one another under this transformation.

The final states in the numerator and denominator of each ratio go into one another under charge conjugation. Thus final state strong interactions which conserve C should be the same and therefore not disturb the equalities. These ratios may then give information about the relative contributions of different weak diagrams without the usual caveats about unknown strong phases.

In the limit of exact SU(3) symmetry \(F_{SU(3)} = 1\). Thus the relations (1) hold when SU(3) is broken by the same factor in all cases. This is not expected to be valid everywhere. Thus the relations (1) provide a means for selecting groups of related decay modes which all have the same SU(3) breaking factor.

Since experimental data generally quote branching ratios rather than partial widths or amplitudes, we note that the relations (1b) can be rearranged to give ratios of branching ratios from the same initial state; e.g.

\[
BR(B^o \to \pi^- K^{(*)+})/BR(B^o \to \pi^o K^{(*)o})/BR(B^o \to \rho^- K^{(*)+})/BR(B^o \to \rho^o K^{(*)o}) = \\
= BR(B_s \to \pi^+ K^{(*)-})/BR(B_s \to \pi^o \bar{K}^{(*)o})/BR(B_s \to \rho^+ K^{(*)-})/BR(B_s \to \rho^o \bar{K}^{(*)o}) \quad (2a)
\]
\[ BR(B^o \to \omega K^{(*)0}) / BR(B^o \to a_2^- K^{(*)+}) / BR(B^o \to a_2^0 K^{(*)0}) / BR(B^o \to f_2 K^{(*)0}) = \]

\[ = BR(B_s \to \omega K^{(*)0}) / BR(B_s \to a_2^+ K^{(*)-}) BR(B_s \to a_2^0 \bar{K}^{(*)0}) / BR(B_s \to f_2 \bar{K}^{(*)0}) \quad (2b) \]

These relations can be used to distinguish between decays having the same SU(3) breaking factor and those having different SU(3) breaking factors.

The dominant SU(3) breaking effect is in the difference between the weak strangeness-conserving and strangeness-changing vertices. For the charmless tree diagrams this breaking introduces a common factor \( F_{SU3} = r_{T(uds)} = V_{us}/V_{ud} \approx 0.23 \) into each ratio, thereby leaving all the ratios \( (1a) \) and \( (1b) \) equal to one another and only changing the value to \( V_{us}/V_{ud} \) instead of unity. For the charmonium and charmed pair tree diagrams the appropriate breaking factor \( F_{SU3} = r_{T(cds)} \approx V_{cd}/V_{cs} \approx r_{T(uds)} \) which is nearly the same as that of the charmless tree diagrams.

If only tree diagrams contribute, relations for the charmonium branching ratios analogous to eqs. (2) can also be written.

Penguin contributions will have a different SU(3) breaking factor \( F_{SU3} = r_P > 1 > r_{T(uds)} \approx 0.23 \); e.g. \( V_{cs}/V_{cd} \) or \( V_{ts}/V_{td} \).

The penguin diagram is expected to dominate in the charmless \( B^o \) decays, and perhaps also in the charmless \( B_d \) decays, since the charmless tree diagram is Cabibbo suppressed. The tree diagram is expected to dominate in the charmonium and charmed pair decays, where the tree is Cabibbo favored while the penguin requires the creation of a heavy quark pair by gluons from the vacuum. These features can be checked out by experimental tests of the relations (1). The most interesting cases are those in which both penguin and tree contributions are appreciable and CP violation can be observed in the interference. These decay modes can be identified by violations of the relations (1). The most favorable candidates seem to be the \( B_s \) decays where one of the two weak vertices is Cabibbo favored and will have a better chance to compete with the penguin.

We can correct the relations (1) for the difference between penguin and tree SU(3) breaking by writing for example:

\[ \frac{A(B^o \to \rho^0 K^{(*)0})}{A(B_s \to \rho^0 K^{(*)0})} = \frac{r_{T(uds)} \cdot T_s + r_P \cdot P_s}{T_s + P_s}; \quad \frac{A(\bar{B}^o \to \rho^0 \bar{K}^{(*)0})}{A(B_s \to \rho^0 \bar{K}^{(*)0})} = \frac{r_{T(uds)} \cdot \bar{T}_s + r_P \cdot \bar{P}_s}{T_s + P_s} \quad (3a) \]

where \( T, P, \bar{T} \) and \( \bar{P} \) denote respectively the contributions to the decay amplitude \( A(B^o \to \rho K^{(*)0}) \) and to the charge conjugate decay amplitude \( A(\bar{B}^o \to \rho \bar{K}^{(*)0}) \) from tree and penguin diagrams and \( T_s, P_s, \bar{T}_s \) and \( \bar{P}_s \) denote respectively the analogous contributions to the corresponding \( B_s \) decay amplitudes. We can obtain similar relations for final states containing
the $\omega$ instead of the $\rho^o$ by noting that the corresponding $\rho^o$ and $\omega$ decay modes are related if electroweak penguins are neglected, because the tree diagram produces both $\rho^o$ and $\omega$ via their common $(u \bar{u})$ component and the penguin produces both via their common $(d \bar{d})$ component[2].

\[
\frac{A(B^o \rightarrow \omega K^{(*)o})}{A(B_s \rightarrow \omega K^{(*)o})} = \frac{r_{T(uds)} \cdot T_s - r_p \cdot P_s}{T_s - P_s}; \quad \frac{A(\bar{B}^o \rightarrow \omega \bar{K}^{(*)o})}{A(\bar{B}_s \rightarrow \omega \bar{K}^{(*)o})} = \frac{r_{\bar{T}(uds)} \cdot \bar{T}_s - r_{\bar{P}} \cdot \bar{P}_s}{\bar{T}_s - \bar{P}_s} \tag{3b}
\]

\[
\frac{BR(B^o \rightarrow K^{(*)o} \rho^o)}{BR(B^o \rightarrow K^{(*)o} \omega)} = \frac{|T + \bar{P}|^2}{|T - \bar{P}|}; \quad \frac{BR(\bar{B}^o \rightarrow \bar{K}^{(*)o} \rho^o)}{BR(\bar{B}^o \rightarrow \bar{K}^{(*)o} \omega)} = \frac{|\bar{T} + \bar{P}|^2}{|\bar{T} - \bar{P}|} \tag{4a}
\]

\[
\frac{BR(B_s \rightarrow \bar{K}^{(*)o} \rho^o)}{BR(B_s \rightarrow \bar{K}^{(*)o} \omega)} = \frac{|T_s + P_s|^2}{|T_s - P_s|^2}; \quad \frac{BR(\bar{B}_s \rightarrow \bar{K}^{(*)o} \rho^o)}{BR(\bar{B}_s \rightarrow \bar{K}^{(*)o} \omega)} = \frac{|\bar{T}_s + \bar{P}_s|^2}{|\bar{T}_s - \bar{P}_s|^2} \tag{4b}
\]

We can also consider linear combinations that project out direct and interference terms:

\[
\frac{|A(B^o \rightarrow \rho^o K^{(*)o})|^2 + |A(B^o \rightarrow \omega K^{(*)o})|^2}{|A(B_s \rightarrow \rho^o K^{(*)o})|^2 + |A(B_s \rightarrow \omega K^{(*)o})|^2} = \frac{|r_{T(uds)}^2 T_s^2| + |r_{P}^2 P_s^2|}{|T|^2 + |P_s|^2} \tag{5a}
\]

\[
\frac{|A(B^o \rightarrow \rho^o K^{(*)o})|^2 - |A(B^o \rightarrow \omega K^{(*)o})|^2}{|A(B_s \rightarrow \rho^o K^{(*)o})|^2 - |A(B_s \rightarrow \omega K^{(*)o})|^2} = r_{T(uds)} r_P \tag{5b}
\]

\[
\frac{|A(B^o \rightarrow \rho^o \bar{K}^{(*)o})|^2 - |A(B^o \rightarrow \omega \bar{K}^{(*)o})|^2}{|A(B_s \rightarrow \rho^o \bar{K}^{(*)o})|^2 - |A(B_s \rightarrow \omega \bar{K}^{(*)o})|^2} = r_{\bar{T}(uds)} r_{\bar{P}} \tag{5c}
\]

where we have noted that $|T_s| = |\bar{T}_s|$ and $|P_s| = |\bar{P}_s|$. Any violation of the relations (1) could indicate existence of both tree and penguin contributions and also offer the possibility of measuring their relative phase. Since the penguin and tree can have different weak phases, $CP$ violation can be observable as asymmetries in decays of charge-conjugate $B$ mesons into charge-conjugate final states and also in differences between the charge-conjugate $\rho/\omega$ ratios (4a) and (4b).
The relations (4) also provide additional input from $B \to K\omega$ decays that can be combined with isospin analyses of $B \to K\rho$ decays to separate penguin and tree contributions[3]. A similar additional input is obtainable from combining $\omega$ decay modes with isospin analyses of other $\rho$ decay modes[4].

Similar relations, with different values of T and P hold for the other ratios. Before extending this result to other cases, we note that additional SU(3) breaking can arise from differences in hadronic form factors. This can be seen at the quark level by noting the quark couplings in the color-favored and color-suppressed tree diagrams and penguin diagrams:

\begin{align}
B^o(b\bar{d}) &\rightarrow (ud)_{c\text{f}had}(\bar{u}s)_{c\text{f}pt}; & B^s(b\bar{s}) &\rightarrow (u\bar{s})_{c\text{f}had}(\bar{u}d)_{c\text{f}pt} & (6a) \\
B^o(b\bar{d}) &\rightarrow (c\bar{d})_{c\text{f}had}(\bar{c}s)_{c\text{f}pt}; & B^s(b\bar{s}) &\rightarrow (c\bar{s})_{c\text{f}had}(\bar{c}d)_{c\text{f}pt} & (6b) \\
B^o(b\bar{d}) &\rightarrow (u\bar{u})_{c\text{s}pt}(\bar{d}s)_{c\text{shad}}; & B^s(b\bar{s}) &\rightarrow (u\bar{u})_{c\text{s}pt}(\bar{s}d)_{c\text{shad}} & (7a) \\
B^o(b\bar{d}) &\rightarrow (c\bar{c})_{c\text{s}pt}(\bar{d}s)_{c\text{shad}}; & B^s(b\bar{s}) &\rightarrow (c\bar{c})_{c\text{s}pt}(\bar{s}d)_{c\text{shad}} & (7b) \\
B^o(b\bar{d}) &\rightarrow \text{penguin} (\bar{d}s) \rightarrow \text{Hadrons}; & B^s(b\bar{s}) &\rightarrow \text{penguin} (\bar{s}d) \rightarrow \text{Hadrons} & (7c)
\end{align}

where $c\text{f}pt$ and $c\text{s}pt$ denote respectively color-favored and color suppressed form factors which are point-like and proportional to wave functions at the origin; e.g. to factors like $f_\pi$ or $f_K$, while $c\text{f}had$ and $c\text{shad}$ denote respectively color-favored and color suppressed form factors which involve overlap integrals on a hadronic scale. The pairs of color favored transitions (6) are seen to involve different form factors. One has a hadronic nonstrange form factor and a pointlike strange form factor; the other has a hadronic strange form factor and a pointlike nonstrange form factor. This form factor difference has been recently pointed out12[512] as possibly responsible for a reversal of relative phase of the two contributions for exclusive decay modes where there are nodes in wave functions.

The color suppressed tree and the penguin transitions (7) are seen to involve identical form factors in both cases. The trees have the same $u\bar{u}$ or $c\bar{c}$ form factor and charge-conjugate hadronic $sd$ and $ds$ form factors. Pairs of penguin diagrams always have the same form factors, since the hadronization into the final state occurs from charge conjugate intermediate states of a single $q\bar{q}$ pair and a gluon or electroweak boson. The only possible difference arises from the slight difference in the hadronic scale of the $B_s$ and $B^o$ wave functions.
We therefore extend the result to all cases where form-factor corrections are expected to be small: those having no color-favored tree contribution and no $s\bar{s}$ component in the wave function as in $\eta$ and $\eta'$.

\[
\frac{A(B^o \to \pi^0 K^{*0})}{A(B_s \to \pi^0 K^{*0})} \approx \frac{A(B^o \to \rho^0 K^{*0})}{A(B_s \to \rho^0 K^{*0})} \approx \frac{A(B^o \to \omega K^{*0})}{A(B_s \to \omega K^{*0})} \approx \frac{A(B^o \to M^0K^{*0})}{A(B_s \to M^0K^{*0})} \approx \frac{r_T}{T_s + P_s};
\]

where $M^0$ can denote any neutral isovector or ideally mixed nonstrange isoscalar meson; e.g. $\pi^0$, $\rho^0$, $a_2^0$ or $\omega$. Similarly $M^\pm$ will denote any charged meson pair; e.g. $\pi^\pm$, $\rho^\pm$ or $a_2^\pm$. Each ratio is equal to an expression analogous to the right hand side of (3) with appropriate different values for $T$ and $P$.

The color-favored transitions to charged final states may have form factor corrections. Let $F_{AB}$ denote this form factor correction, where $A$ and $B$ denote the two particles in the final state. Then

\[
\frac{A(B^o \to \pi^- K^{*+})}{A(B_s \to \pi^+ K^{*-})} \approx \frac{A(B^o \to \rho^- K^{*+})}{A(B_s \to \rho^+ K^{*-})} \approx \frac{A(B^o \to \omega^- K^{*+})}{A(B_s \to \omega^+ K^{*-})} \approx F_{AB} \cdot r_P
\]

Here the approximate equalities are exact if the tree contribution is negligible, and will be violated where both contributions are appreciable. In the latter case, each ratio is again equal to an expression analogous to the right hand side of (3) with appropriate different values for $T$ and $P$.

The SU(3)-breaking effect is different in decays to final states containing $\eta$ and $\eta'$ because the discrete transformation (Weyl reflection) $d \leftrightarrow s$ which simply interchanges the d and s flavors interchanges the $d\bar{d}$ and $s\bar{s}$ components in the $\eta$ and $\eta'$ system, which we denote by $P_d$ and $P_s$. These decays are of particular interest since recently reported high branching ratios[6] for strange $B$ decays to $\eta'$ final states has led to suggestions for new types of diagrams[7].

We first note that the Cabibbo-favored tree diagram is expected to be dominant in $\eta$ and $\eta'$ decays with charmonium and that in these decay modes they are produced via $P_d$ in $B^o$ decay and via $P_s$ in $B_s$ decay. Thus these decays immediately provide a measure of the $\eta-\eta'$ mixing. We first obtain the SU(3) symmetry result

\[
\frac{A(B^o \to \psi^{(s)}P_d)}{A(B_s \to \psi^{(s)}P_s)} = r_{T(cds)}; \quad A(B^o \to \psi^{(s)}P_s) = A(B_s \to \psi^{(s)}P_d) = 0
\]

This immediately gives the values of the strange and nonstrange components in the $\eta$ and $\eta'$ and a condition which must be satisfied if the $\eta-\eta'$ mixing is described by a $2 \times 2$ matrix.
\[
\frac{A(B^0 \to \psi^*(\eta'))}{A(B^0 \to \psi^*(\eta))} = \frac{\langle P_d | \eta' \rangle}{\langle P_d | \eta \rangle}; \quad \frac{A(B_s \to \psi^*(\eta'))}{A(B_s \to \psi^*(\eta))} = \frac{\langle P_s | \eta' \rangle}{\langle P_s | \eta \rangle}
\] (11a)

\[
\frac{A(B^0 \to \psi^*(\eta'))}{A(B^0 \to \psi^*(\eta))} = -\frac{A(B_s \to \psi^*(\eta))}{A(B_s \to \psi^*(\eta'))}
\] (11b)

A failure of the relation (11b) would indicate a breakdown of the simple mixing model.

We now investigate the decays into strange final states with \(\eta\) and \(\eta'\). The standard penguin diagram predicts\(^2\),\(^8\)

\[
\tilde{\Gamma}(B^\pm \to K^\pm \eta') : \tilde{\Gamma}(B^\pm \to K^\pm \eta) : \tilde{\Gamma}(B^\pm \to K^\pm \pi^o) = 3 : 0 : 1
\] (12a)

\[
\tilde{\Gamma}(B^\pm \to K^{*\pm}(890)\eta') : \tilde{\Gamma}(B^\pm \to K^{*\pm} \eta) : \tilde{\Gamma}(B^\pm \to K^{*\pm} \pi^o) = (1/3) : (8/3) : 1
\] (12b)

\[
\frac{\tilde{\Gamma}(B^\pm \to K^\pm \eta') + \tilde{\Gamma}(B^\pm \to K^\pm \eta)}{\tilde{\Gamma}(B^\pm \to K^\pm \pi^o)} \leq 3
\] (12c)

where \(\tilde{\Gamma}\) denotes the theoretical partial width without phase space corrections. We have assumed SU(3) symmetry with one of the standard mixings:

\[
|\eta\rangle = \frac{1}{\sqrt{3}} \cdot (|P_u\rangle + |P_d\rangle - |P_s\rangle); \quad |\eta'\rangle = \frac{1}{\sqrt{6}} \cdot (|P_u\rangle + |P_d\rangle + 2|P_s\rangle)
\] (13)

and noted that the penguin diagram creates the two states \(K^\pm P_u\) and \(K^\pm P_s\), with a relative phase depending upon the orbital angular momentum \(L\) of the final state.

\[
A(B^\pm \to K^\pm P_s) = (-1)^L \cdot (1 - \epsilon) \cdot A(B^\pm \to K^\pm P_u)
\] (14)

where \(\epsilon\) is a parameter describing SU(3) symmetry breaking and \(K^\pm\) can also denote any \(K^*\) resonance. The sum rule inequality (12c) holds generally for all mixing angles and for all positive values of \(\epsilon\).
The dramatic reversal of the $\eta'/\pi^o/\eta$ ratio in the final states with $K^{*\pm}(890)$ occurs naturally in this penguin interference model and does not occur in any other suggestion for enhancing the $\eta'$. Present data indicate $K^{*\pm}(890)\eta'$ suppression. Better data showing significant suppression will rule out most other $\eta'$ enhancement mechanisms.

A violation of the inequality (12c) would require an additional contribution. The Cabibbo favored charmed tree diagram $A(B^\pm \to K^\pm P_c \to K^\pm \eta')$ can contribute via hidden or intrinsic charm in the $\eta'$ wave function and may contribute appreciably even though the charm in the $\eta'$ is quite small.

We now estimate the effect of an additional contribution from the production of the $\eta$ and $\eta'$ via an additional diagram which in the SU(3) symmetry limit produces the states $|P_u\rangle$, $|P_d\rangle$ and $|P_s\rangle$ with equal amplitudes.

\[
A(B^\pm \to K^\pm \eta) = \sqrt{2/3} \cdot \xi \cdot A(B^\pm \to K^\pm \pi^o) \tag{15a}
\]
\[
A(B^\pm \to K^\mp \eta') = \sqrt{1/3} \cdot (3 + 4\xi) \cdot A(B^\pm \to K^\pm \pi^o) \tag{15b}
\]
\[
A(B^\pm \to K^{*\pm}(890)\eta) = \sqrt{2/3} \cdot (2 - \xi) \cdot A(B^\pm \to K^{*\pm} \pi^o) \tag{15c}
\]
\[
A(B^\pm \to K^{*\pm}(890)\eta') = -\sqrt{1/3} \cdot (1 + 4\xi) \cdot A(B^\pm \to K^{*\pm} \pi^o) \tag{15d}
\]

where $\xi$ defines the extra contribution strength. Consider for example the case $\xi = 0.5$

\[
\tilde{\Gamma}(B^\pm \to K^\pm \eta') : \tilde{\Gamma}(B^\pm \to K^\pm \eta) : \tilde{\Gamma}(B^\pm \to K^\pm \pi^o) = (25/3) : (1/6) : 1 \tag{16a}
\]
\[
\tilde{\Gamma}(B^\pm \to K^{*\pm}(890)\eta') : \tilde{\Gamma}(B^\pm \to K^{*\pm} \eta) : \tilde{\Gamma}(B^\pm \to K^{*\pm} \pi^o) = 3 : (1.5) : 1 \tag{16b}
\]
\[
\tilde{\Gamma}(B^\pm \to K^\pm \eta') + \tilde{\Gamma}(B^\pm \to K^\pm \eta)/\tilde{\Gamma}(B^\pm \to K^\pm \pi^o) \leq (17/2) \tag{16c}
\]

The inequality (16c) holds for all mixing angles and all $\epsilon \geq 0$. Thus a comparatively small contribution interfering constructively with the dominant penguin can give an appreciable enhancement. With $\xi$ sufficiently large to give $(25/3)$ for $(\tilde{\Gamma}(B^\pm \to K^\pm \eta') : \tilde{\Gamma}(B^\pm \to K^\pm \pi^o)$ and a 50:1 ratio favoring $\eta'$ over $\eta$, the enhancement of $\eta'$ over $\eta$ is only a factor of two for the $K^*$ final state. The drastic difference between the $K$ and $K^*$ branching ratios still persists if both the penguin and the extra contribution are present, in contrast to the case where the extra contribution is dominant. Thus the $K^*$ data are important for determining the exact mechanism for the $\eta'$ enhancement.
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