Application Research on DOA Estimation Based on Software-Defined Radio Receiver

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Abstract—Using a KerberosSDR, with four omnidirectional antennas and a raspberry pi, a four-element system of direction of arrival estimation system is formed. The four antennas are arranged in accordance with a uniform linear array and a uniform circular array respectively. Four algorithms including Bartlett, Capon, MEM, and MUSIC are used. Finally, the MUSIC algorithm is used to estimate the direction of arrival of the static target signal source, and the results are displayed on the map.

1. INTRODUCTION
Direction of arrival estimation is an active field in array signal processing. It has broad application value in the fields of communication [1-3,7], radar [4-7], exploration [8] and navigation[9-10]. However, most of the researchers innovated the DOA estimation algorithms, and most of these algorithms were verified on the simulation platform. As we all know, the experimental results of the simulation platform deviate from the results in engineering applications [11-12]. This article focuses on the engineering application of DOA estimation, using a KerberosSDR device and four omnidirectional antennas as a signal receiver, and using a Raspberry Pi as a data processor to implement a system with a simple structure and reliable DOA estimation performance.

KerberosSDR is a new 4-input Coherent RTL-SDR. RTL-SDR is a very cheap software-defined radio receiver. Each RTL-SDR is composed of an RTL2832U chip and an R820T tuner. It can receive radio frequency signals from 25MHz to 1.75GHz in space and convert it to baseband. Finally, the digital 8-bit sampling signal is output from the USB port. There is a noise source module inside KerberosSDR, which can realize the sampling time synchronization and phase synchronization of the four signal receiving channels. The four signal receiving channels share a clock source, and the four digital signals communicate with the Raspberry Pi through a USB HUB. Run the signal processing algorithm on the Raspberry Pi and display the DOA estimation result and signal strength in real time through the web page.

The communication frequency between the UAV and the remote control in this experiment is 2.400-2.4835 GHz, which is not in the RTL-SDR receiving frequency range, Therefore, a small FM transceiver is fixed on the UAV as a signal source, The transmission frequency of the FM transceiver is 446.0063MHz. The UAV equipped with a FM transceiver hovers in the air, so as to ensure that the signal sent by the FM transceiver is not blocked by obstacles, Use this signal source to verify the DOA estimation accuracy of the system.
2. THEORETICAL ANALYSIS

2.1 Hardware Structure

1) Data Processing: The composition of the system is shown in Fig. 1.

![System block diagram](image)

Figure 1. System block diagram.

The UAV is equipped with a FM transceiver as the target signal source. The center frequency of the transmitted signal is 446.0063MHz and the frequency bandwidth is 25KHz. The four antennas are placed in a uniform line array and a uniform circular array. After demodulated to digital baseband by KerberosSDR, it is transferred to the Raspberry Pi for data processing. The Raspberry Pi connects to the smartphone through wifi and displays the processing result on the mobile phone.

2) Array model

a) Uniform linear array: The uniform linear array is to arrange the four sensor array elements into a straight line in an equidistant manner, the interval between adjacent array elements is d, and the sensor array element distributed at the coordinate origin is set as reference array element. The uniform linear array model is shown in Fig. 2. Because the uniform linear array realizes the estimation of one-dimensional DOA, that is to say, it can realize the estimation of the heading angle or pitch angle of the target signal source, and the uniform linear array only has a resolution of 180 degrees.

![Uniform linear array model](image)

Figure 2. Uniform linear array model

b) Uniform circular array: In the three-dimensional coordinate system, in the plane formed by the X axis and the Y axis, the center of the circle is the origin, the radius is R, and the isotropic sensor array elements are evenly and uniformly distributed on the circumference to form a circular antenna. In the array, the linear distance between adjacent sensor array elements is d. The uniform circular array model is shown in Figure. 3.
2.2 DOA Estimation Algorithm

In this paper, four commonly used algorithms in the field of DOA estimation are used. Among them, the Bartlett algorithm, Capon algorithm, and MEM algorithm are all linear DOA estimation algorithms, and the MUSIC algorithm is an algorithm based on decomposition of subspaces.

After an array is weighted and summed, the directional gain received by the array can be focused in one direction, which is equivalent to forming a beam. Different weight vectors can direct the formed beam to different directions. Some weight vectors maximize the signal output power, and the direction corresponding to these weight vectors is the direction of arrival.

Assuming that there is an array of \( M \) array elements in the space, the received signal vector of the array element is \( \mathbf{x}(t) \), and the weight vector of each array element is:

\[
\mathbf{w} = [w_1 \ w_2 \ \cdots \ \ w_M]^T
\]

The output of the array signal is:

\[
y(t) = \mathbf{w}^H \mathbf{x}(t) = \sum_{i=1}^{M} w_i^* x_i(t)
\]

The average power of the entire array signal output is:

\[
P(\mathbf{w}) = \frac{1}{L} \sum_{t=1}^{L} |y(t)|^2 = \mathbf{w}^H E\{x(t)x^H(t)\} \mathbf{w} = \mathbf{w}^H \mathbf{R} \mathbf{w}
\]

1) Bartlett Algorithm: CBF (Conventional Beam Forming) method is recognized as the earliest proposed and classic DOA estimation algorithm based on narrow-band arrays. This method is also commonly known as the Bartlett beamforming method. This algorithm maximizes the output power of the beamformer relative to a certain direction. This maximization relationship can be expressed as:

\[
\theta_{Bartlett} = \arg \max_{\mathbf{w}} [P(\mathbf{w})]
\]

When \( \mathbf{w} = \mathbf{a}(\theta) \), The output power is the largest. The spatial spectral of Bartlett algorithm can be obtained:

\[
P_{Bartlett} = \mathbf{a}^H(\theta) \mathbf{R} \mathbf{a}(\theta)
\]

2) Capon Algorithm: Under the framework of beamforming, the Capon algorithm keeps the array gain unchanged in the DOA direction. By suppressing noise and interference signals in other directions, the array output power is minimized, and the following optimization problem is proposed:

\[
\theta_{Capon} = \arg \min_{\mathbf{w}} P(\mathbf{w}) \ \ s.t. \ \ \mathbf{w}^H \mathbf{a}(\theta) = 1
\]

Using the Lagrangian multiplier method to obtain:

\[
\mathbf{w} = \frac{\mathbf{R}^{-1} \mathbf{a}(\theta)}{\mathbf{a}^H(\theta) \mathbf{R}^{-1} \mathbf{a}(\theta)}
\]

The spatial spectral of Capon algorithm can be obtained:

\[
P_{Capon} = \frac{1}{\mathbf{a}^H(\theta) \mathbf{R}^{-1} \mathbf{a}(\theta)}
\]
3) **MEM Algorithm**: The MEM algorithm is also called the maximum entropy algorithm. The optimal weight constraint conditions of the MEM algorithm and the Capon algorithm are different. The optimization problem of the MEM algorithm is proposed by the following formula:

$$\theta_{MEM} = \arg \min_w P(w) \quad \text{s.t.} \quad u_0^Tw = 1$$

(9)

The optimal weight of the MEM algorithm is:

$$w = \mu \hat{R}^{-1}u_0$$

(10)

Where $\mu$ is a constant:

$$\mu = \frac{1}{(R^{-1})_{11}}$$

(11)

After ignoring the constant $\mu$, The spatial spectral of MEM algorithm is:

$$P_{MEM} = \frac{1}{\left|a^H(\theta)R^{-1}u_0\right|^2}$$

(12)

4) **MUSIC Algorithm**: The MUSIC algorithm is also called the decomposition subspace algorithm [13]. The MUSIC algorithm has good angle measurement performance when performing DOA estimation on non-coherent signal sources. Since the MUSIC algorithm breaks through the performance bottleneck of the linear prediction algorithm, it can distinguish multiple target signal sources existing in a beam.

The mathematical model of the target signal source is:

$$X(t) = A(\theta)S(t) + N(t)$$

(13)

Assuming that the noise is spatially ideal white noise and the noise power is $\sigma^2$, the received data covariance matrix $R$ of the antenna array can be obtained from (13):

$$R = E[XX^H] = AE[SS^H]A^H + \sigma^2I = ARSA^H + \sigma^2I$$

(14)

Eigenvalue decomposition of $R$:

$$R = U_S\Sigma_SU_S^H + U_N\Sigma_NU_N^H$$

(15)

In (15), $U_S$ is a subspace formed by eigenvector corresponding to large eigenvalues, which also becomes a signal subspace, and $U_N$ is a subspace formed by eigenvector corresponding to small eigenvalues, and also becomes a noise subspace. Under ideal conditions, the steering vector in the signal subspace is orthogonal to the noise subspace:

$$a^H(\theta)U_N = 0$$

(16)

Considering that the actual received data matrix is limited, the maximum likelihood estimate of the covariance matrix is:

$$\hat{R} = \frac{1}{L}\sum_{l=1}^{L}X_lX_l^H$$

(17)

The MUSIC algorithm is implemented with minimum optimized search:

$$\theta_{MUSIC} = \arg \min_{\theta} a^H(\theta)\hat{U}_N\hat{U}_N^Ha(\theta)$$

(18)

The spatial spectral of MUSIC algorithm is:

$$P_{MUSIC} = \frac{1}{\left|a^H(\theta)\hat{U}_N\hat{U}_N^Ha(\theta)\right|^2}$$

(19)

5) **FB Average**: In the actual environment, due to the complexity and variability of the propagation environment in space, such as the effect of multipath, the independence of the target signal in the space under test is destroyed, resulting in the generation of coherent target signal sources. To solve this problem, the classic spatial algorithm is an effective method to solve the coherent target signal source [13]. The central idea of the algorithm is to divide ULA into a series of identical sub-arrays that overlap each other, and then calculate the $R$ of each sub-array and perform an average operation on them to achieve decoherence to the coherent target signal source. On this basis, the conjugate reverse operation is performed on each sub-array to reconstruct another sub-array, and the obtained forward $U_f$ and
backward $U_b$ are averaged to form the final $R$ to achieve decoherence to the coherent target signal source. In this paper, if the array elements are arranged uniformly and linearly, the FB average algorithm will be applied.

3. EXPERIMENTAL VERIFICATION

Set the center frequency of KerberosSDR to 446.0063MHz, set the sampling rate to 0.9MHz, the gain of the four receiving channels to 28dB, set the FIR filter bandwidth to 25kHz, set the FIR tap size to 100, and set the Decimation to 4. Display processing results on the web.

3.1 Hardware Device Connection

Since the system does not have a strong ability to resist multipath effects, the antenna should be erected as high as possible. The experimental equipment and experimental scene are shown in Fig. 4.

![Figure 4. The signal receiving device is connected.](image)

In order to make the UAV easy to carry, the experiment uses a miniature FM transceiver. The total power of the FM transceiver is only 0.5W. This also determines that the UAV cannot fly too far from the antenna array, otherwise the signal strength will be greatly attenuated, which will seriously affect the DOA estimation ability. UAV is equipped with FM transceiver as shown in Fig. 5.

![Figure 5. UAV carries FM transceiver as signal source](image)

Fourier transform the signals received by the four channels and display them together. It can be seen from Fig. 6 that all four channels can receive the target signal with a center frequency of 446.0063 MHz. It is a civilian frequency band, and the rational use of this frequency band will not affect the
communication safety of others. There is no crosstalk of other signal sources in this frequency band near the experimental site, which is more conducive to the DOA estimation of the target signal source.

3.2 Phase Synchronization
The phase synchronization between multiple channels is a necessary condition for accurate DOA estimation. The four signal receiving channels in KerberosSDR share a clock, which is used to synchronize the four signal receiving channels through local noise sources. Because the frequency synthesizer will generate random phase shift, so phase synchronization is required after each tuning. The phase synchronization process is shown in Fig. 7.

During T1, the four channels maintain sampling time synchronization, but at this time, the phase difference between channels is a non-zero constant. During T2, the four channel phase synchronization is maintained, and the phase difference between channels is zero.

3.3 Algorithm Comparison in Uniform Linear Array
The array elements are arranged in a uniform linear array. The distance between adjacent array elements is $d$.

$$d = 0.33 \times \lambda = 0.33 \times 0.67 = 0.22 \text{m}$$

(20)

Make the UAV hover in the direction of 0 degrees directly in front of the antenna array, and simultaneously run the four algorithms of Bartlett, Capon, MEM, and MUSIC, where the purple curve
is the average of the four algorithms, and the spatial spectrum of the four algorithms are shown in Fig. 8.

![Figure 8. Spatial spectrum of four algorithms for uniform linear arrays.](image)

3.4 Algorithm Comparison in Uniform Circular Array
The straight line distance between adjacent array elements is d. The value of d is the same as (20). Make the UAV hover 180 degrees directly behind the antenna array, and simultaneously run the four algorithms of Bartlett, Capon, MEM, and MUSIC. The purple curve is the average of the four algorithms. The spatial spectrum of the four algorithms is shown in Fig. 9.

![Figure 9. Spatial spectrum of four algorithms for uniform circular array.](image)

4. Static Statistical Experiment
The experiment site was selected on the west side of the rooftop on the fourth floor of the Information Building of East China Normal University. The UAV was used to carry the FM transceiver and hovered over the red, yellow, and green points in the Fig. 10 and Fig. 11. The array was a uniform linear array and a uniform circular array, The receiver parameters and array element spacing d are the same as in (20). Adopt MUSIC algorithm. Plot the DOA estimation results on the map.

4.1 Uniform Linear Array
Using MUSIC algorithm and applying FB average algorithm, Continuous sampling of 20 points, The results of the DOA estimation are shown in Fig. 10.
Figure 10. MUSIC algorithm uniform linear array DOA results.

4.2 Uniform Circular Array.
Using MUSIC algorithm, continuous sampling of 20 points, the result is shown in Fig. 11.

Figure 11. MUSIC algorithm uniform circular array DOA results.

5. CONCLUSION
The use of KerberosSDR and four omnidirectional antennas can complete the DOA estimation. The spatial forward and backward smoothing algorithm can reduce the impact of multipath effects. When the MUSIC algorithm is applied, the DOA error of the uniform linear array is less than the uniform circular array.
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REFERENCES
[1] Hongji H, Jie Y, Yiwei S, et al. Deep Learning for Super-Resolution Channel Estimation and DOA Estimation based Massive MIMO System[J]. IEEE Transactions on Vehicular Technology, 2018:1-1.
[2] Wang X, Wan L, Huang M, et al. Polarization Channel Estimation for Circular and Non-Circular Signals in Massive MIMO Systems[J]. IEEE Journal of Selected Topics in Signal Processing, 2019, PP(99):1-1.
[3] Fan D, Gao F, Wang G, et al. Angle Domain Signal Processing aided Channel Estimation for Indoor 60GHz TDD/FDD Massive MIMO Systems[J]. IEEE Journal on Selected Areas in Communications, 2017:1-1.
[4] Li J, Jiang D, Zhang X. DOA Estimation Based on Combined Unitary ESPRIT for Coprime MIMO Radar[J]. IEEE communications letters, 2017, 21(1):96-99.
[5] Wen F, Xiong X, Su J, et al. Angle estimation for bistatic MIMO radar in the presence of spatial colored noise[J]. Signal processing, 2017, 134(may):261-267.
[6] Si Qin, Yimin D. Zhang, Moeness G. Amin. DOA estimation of mixed coherent and uncorrelated targets exploiting coprime MIMO radar[M]. Academic Press, Inc. 2017.
[7] Hassanien A, Zhang Y D, Ahmad F, et al. Phase-modulation based dual-function radar-communications[J]. 2017.
[8] Shahzad Ahmad Butt, Luciano Lavagno. Design space exploration and synthesis for digital signal processing algorithms from Simulink models[J]. 2012.
[9] Shamaei K, Khalife J, Kassas Z M. A Joint TOA and DOA Approach for Positioning with LTE Signals[C]// 2018 IEEE/ION Position, Location and Navigation Symposium (PLANS). IEEE, 2018.
[10] Amin M G, Wang X, Zhang Y D, et al. Sparse Arrays and Sampling for Interference Mitigation and DOA Estimation in GNSS[J]. Proceedings of the IEEE, 2016, 104(6):1302-1317.
[11] Unlersen, Fahri, M,. FPGA Based Fast Bartlett DoA Estimator for ULA Antenna Using Parallel Computing[J]. Applied Computational Electromagnetics Society Journal, 2018.
[12] Hussain A A, Tayem N, Soliman A H, et al. FPGA-based Hardware Implementation of Computationally Efficient Multi-Source DOA Estimation Algorithms[J]. IEEE Access, 2019, PP(99):1-1.
[13] Pal P, Vaidyanathan P P. Coprime sampling and the music algorithm[C]// Digital Signal Processing Workshop & IEEE Signal Processing Education Workshop. IEEE, 2011.