Approximate calculation of the dislocation oscillators generalized susceptibility matrix elements

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Abstract. In the present work, an approximate calculation of the dislocation oscillators generalized susceptibilities matrix elements is carried out. The calculation of this matrix on its previously established quasi-diagonal form is based. When finding the elements, the matrix inversion lemma is used. First, the diagonal element in the k-th row is calculated. The number k from the condition of the initial matrix diagonal element minimum modulus is chosen. The second expression for the elements in the k-th row and the k-th column, except for the diagonal, is obtained. The third expression for the diagonal elements, except for the number k, is obtained. The fourth expression for the remaining elements is obtained. To verify the expressions obtained, a computational experiment with a model matrix was conducted. The numerical calculation of the matrix elements according to the obtained formulas showed good agreement with the direct calculation of the inverse matrix.

1. Introduction

The determining role of dislocations in the formation of mechanical, thermodynamic, electromagnetic and other properties of crystalline solids is known. This explains the interest in the problem of vibrations of a crystal with a dislocation. The oscillations of a single dislocation segment were first investigated by Koehler on the basis of a string model [1]. This model was further developed by Granato and Lucke [2, 3]. They did not consider the specific mechanism of energy dissipation by an oscillating dislocation. These mechanisms in reviews [4, 5] were considered. Various problems of oscillations of dislocation segments in later works [6-11] have been investigated on the basis of a string model. In article [6], the oscillation spectrum of interacting dislocation segments system was found in the long-wave approximation. In [7] the problem of electromagnetic emission of an edge dislocation segment moving in an ionic lattice with a NaCl-type structure is considered. In [8] the amplitude-independent dislocation absorption (internal friction) is investigated under the joint action of constant and random external forces on the dislocation. In [9] is proposed the new model based on sin-Gordon equation for a dislocation segment displacement, average energy losses owing to internal friction of dislocation oscillations are calculated, dislocation segment dynamics under external both constant and harmonic strength is considered. In [10, 11], the interaction of elastic waves with one fixed dislocation and with prismatic dislocation loops was considered. The nonlocality of the dislocation characteristics in the string model was analyzed in [12, 13]. In papers [14, 15], the inverse generalized susceptibility matrix \( B \) of the dislocation segment was found. In [16], a numerical calculation of the dislocation oscillators generalized susceptibilities matrix \( B^{-1} \) was performed, corresponding graphs were plotted, and it was established that these matrices are quasi-diagonal. To
find the effect of dislocation segments on the properties of crystals, an analytical expression of the generalized susceptibility matrix elements is necessary. Therefore, in this article, based on the results of works [14-16], an approximate calculation of the generalized susceptibility matrix elements is carried out.

2. General expression for inverse matrix $B^{-1}$ elements

Consider the matrix elements $B_{mn}$ that form the matrix $B$ of size $N \times N$ [14, 15]. The expression for the elements of inverse matrix generalized susceptibility in case of $m \neq n$:

$$B_{mn} = (-1)^{[m/2]+[n/2]} \frac{\mu \eta_{mn}}{4L} \left[ 3b_{s}^{2} L_{mn}^{11} + 2\gamma (b_{c}^{2} - 2b_{s}^{2}) L_{mn}^{1} - b_{c}^{2} L_{mn}^{10} - \gamma b_{c}^{2} q_{l}^{2} L_{mn}^{10} - 4b_{c}^{2} - b_{c}^{2} L_{mn}^{(2)} / q_{l}^{2} - \Delta_{mn} \right],$$

where

$$\Delta_{mn} \approx \frac{1}{2} \frac{1}{q_{n}^{2} - q_{m}^{2}} \left[ -(3 - 4\gamma) b_{c}^{2} + 2\gamma b_{c}^{2} \left( q_{n} \arctg \frac{2q_{n}}{k_{D}L} - q_{m} \arctg \frac{2q_{m}}{k_{D}L} \right) + (b_{c}^{2} + \gamma^{2} b_{c}^{2}) q_{l}^{2} \left( \frac{1}{q_{n}} \arctg \frac{2q_{n}}{k_{D}L} - \frac{1}{q_{m}} \arctg \frac{2q_{m}}{k_{D}L} \right) \right].$$

The expression for the elements of the generalized susceptibility inverse matrix in case of $m = n$:

$$B_{nn} = \frac{\pi^{2}}{4L} \mu \left[ 3b_{s}^{2} L_{nn}^{11} + 2\gamma (b_{c}^{2} - 2b_{s}^{2}) L_{nn}^{1} - b_{c}^{2} L_{nn}^{10} - \gamma b_{c}^{2} q_{l}^{2} L_{nn}^{10} - 4b_{c}^{2} - b_{c}^{2} L_{nn}^{(2)} / q_{l}^{2} - \Delta_{nn} \right],$$

where

$$\Delta_{nn} \approx \frac{1}{2} \left[ (3 - 4\gamma) b_{c}^{2} + 2\gamma b_{c}^{2} \left( \frac{1}{2q_{n}} \arctg \frac{2q_{n}}{k_{D}L} + \frac{k_{D}L}{(k_{D}L)^{2} + q_{n}^{2}} \right) + (b_{c}^{2} + \gamma^{2} b_{c}^{2}) q_{l}^{2} \left( \frac{1}{2q_{n}} \arctg \frac{2q_{n}}{k_{D}L} - \frac{k_{D}L}{(k_{D}L)^{2} + q_{n}^{2}} \right) \right].$$

Here $L$ is the length of a dislocation segment, $\mu$ is the shear modulus of the crystal, $b$ is Burgers vector of dislocation, $b_{c}$ and $b_{s}$ are edge and screw components of the Burgers vector, $q_{l} = \omega L / c_{l}$, $q_{s} = \omega L / c_{s}$, $\omega$ is frequency, $c_{l}$ and $c_{s}$ are the transverse and longitudinal sound velocities, $\gamma = c_{l}^{2} / c_{s}^{2}$, $q_{m} = m\pi$, $q_{n} = n\pi$, $k_{D}$ is the Debye wave number,

$$L_{mn}^{nk} = \frac{1}{2} \frac{i}{q_{n}^{2} - q_{m}^{2}} \left[ q_{m}^{2k-1} \left[ \text{Ein}(-i(q_{m} - q_{n})) - \text{Ein}(-i(q_{n} + q_{m})) \right] - q_{n}^{2k-1} \left[ \text{Ein}(-i(q_{n} - q_{m})) - \text{Ein}(-i(q_{n} + q_{m})) \right] - (-1)^{m} \exp(iq_{m}) \delta_{k,2} \right].$$
\[ L_{mn}^{(2k-2)} = \frac{q_n^{2k-2}}{4} \left\{ 2C + 2 \ln(k_0 L) + 2i q_n \alpha \frac{1-(-1)^n \exp(i q_n)}{q_n^2 - q_n^2} \right\} \]
\[ + \frac{1-2k}{4} i q_n^{2k-3} \left[ \text{Ein}(-i(q_n + q_n)) - \text{Ein}(-i(q_n - q_n)) \right], \]
\[ L_{mn}^{(2)} = l_{mn}^{(2)} - l_{mn}^{(2)} + i(q_i - q_i), \quad L_{mn}^{(2)} = l_{mn}^{(2)} - l_{mn}^{(2)} + i(q_i - q_i) - (-1)^n \left[ \exp(i q_i) - \exp(i q_i) \right], \]
\[ \text{Ein}(z) = \int_0^z (1 - \exp(-t)) dt \text{ is the integer part of the integral exponential function} \ [17], \ \delta_{ik} \text{ is Kronecker’s symbol, } C \approx 0.577 \text{ is Euler's constant.} \]

The matrix \( B \) is symmetric, i.e. \( B_{mn} = B_{nm} \) [14, 15]. Define the vector \( b = (B_{1k}/\beta \ B_{2k}/\beta \ \ldots \ \ B_{nk}/\beta)^T \) and express the matrix elements through the components of this vector:

\[ b = b_n b_n + B_{mn} \text{ or } B = bb^T + B'. \] (1)

Here the number \( k \) is chosen from the condition of the matrix \( B \) diagonal element minimum modulus, i.e. from the condition of resonance. Then we obtain:

\[ b = \left( \begin{array}{cccc}
B_{1k}/\beta^2 & B_{1k}B_{2k}/\beta^2 & \ldots & B_{1k}B_{nk}/\beta^2 \\
B_{2k}B_{1k}/\beta^2 & B_{2k}/\beta^2 & \ldots & B_{2k}B_{nk}/\beta^2 \\
& \ddots & \ddots & \ddots \\
& & B_{nk}B_{nk}/\beta^2 & \ldots & B_{nk}/\beta^2
\end{array} \right) \]

\[ B' = B - bb^T = \left( \begin{array}{cccc}
B_{11} - \frac{B_{1k}^2}{\beta^2} & B_{12} - \frac{B_{1k}B_{2k}}{\beta^2} & \ldots & 0 \\
B_{12} - \frac{B_{2k}B_{1k}}{\beta^2} & B_{22} - \frac{B_{2k}^2}{\beta^2} & \ldots & 0 \\
& \ddots & \ddots & \ddots \\
& & 0 & \ldots & B_{kk} - \beta^2 \\
& & & \ddots & \ddots \\
B_{nn} - \frac{B_{nk}B_{nk}}{\beta^2} & B_{2n} - \frac{B_{nk}B_{2k}}{\beta^2} & \ldots & 0 \\
& & & & \ddots \\
& & & & & B_{nn} - \frac{B_{nk}^2}{\beta^2}
\end{array} \right) \]

The number \( \beta \) is defined as \( \beta = \sqrt{B_{kk} - \alpha} \), then \( B_{kk} - \beta^2 = \alpha \). Note that when \( \alpha \to -\infty, \beta \to \infty \), we obtain
Let us write the matrix inverse to the original matrix in the form of the sum of the matrix, inverse to the $B'$, and the additional term, similarly to the matrix inversion lemma [18, 19]:

$$B^{-1} = (B')^{-1} + \gamma C C^T$$

or

$$B_{mn}^{-1} = B_{mn}^{-1} + \gamma C_m C_n.$$  \hfill (2)

Considering the formulas (1) and (2), we obtain

$$BB^{-1} = (bb^T + B') (B'^{-1} + \gamma C C^T) = (bb^T) B'^{-1} + \gamma (bb^T)(CC^T) + B' B'^{-1} + \gamma B' (CC^T)$$

$$= b (b^T B'^{-1}) + \gamma b C (b^T C) + E + \gamma (B' C) C^T = E,$$  \hfill (3)

where $E$ is the identity matrix. From (3) we obtain the equation

$$b (b^T B'^{-1}) + \gamma b C (b^T C) + \gamma (B' C) C^T = 0.$$  \hfill (4)

Let us choose the vector $C$ in the form of $C^T = \varepsilon \left( b^T B'^{-1} \right)$, where $\varepsilon$ is an unknown number so far.

Then $B'C = \varepsilon B' (b^T B'^{-1})^T = \varepsilon B' (B'^{-1}) b = \varepsilon (B' B'^{-1}) b = \varepsilon b$. Here take into account the symmetry of the matrix $B'^{-1}$. Substituting $b^T B'^{-1} = \frac{1}{\varepsilon} C^T$ and $B'C = \varepsilon b$ into equation (4), we obtain

$$\frac{1}{\varepsilon} b C^T + \gamma (b C)(b^T C) + \gamma \varepsilon b C^T = 0.$$  \hfill (5)

From here we have an equation for determining the parameter $\varepsilon$:

$$\frac{1}{\varepsilon} + \gamma b C + \gamma \varepsilon = 0.$$  \hfill (5)

Next we take into account the matrix equality $(b C) = C^T b = \varepsilon \left( b^T B'^{-1} \right)b$. Substituting $b C = \varepsilon (b^T B'^{-1}) b$ in equation (5), we obtain

$$\frac{1}{\varepsilon} + \gamma \varepsilon (b^T B'^{-1}) b + \gamma \varepsilon = 0.$$  \hfill (5)

Therefore $\gamma \varepsilon = \frac{1}{\varepsilon} \left[ 1 + (b^T B'^{-1}) b \right]$. Considering this expression, we have

$$\gamma C C^T = \gamma \varepsilon^2 (B'^{-1}) b (b^T B'^{-1}) = \frac{(B'^{-1} b)(b^T B'^{-1})}{1 + (b^T B'^{-1}) b}.$$
Substituting this formula in (2), we obtain

\[ B^{-1} = B'^{-1} - \frac{(B'^{-1}b)(B'^{-1}b)^T}{1 + (b^T B'^{-1})b} \quad \text{or} \quad B^{-1}_{mn} = B'^{-1}_{mn} - \frac{(B'^{-1}b)_m (B'^{-1}b)^T}{1 + (b^T B'^{-1})b}. \] (6)

3. Calculation of the matrix $B'^{-1}$ elements

3.1. Calculation of the element $B^{-1}_{kk}$

We calculate the diagonal element of the inverse matrix in the row $k$ and the column $k$. In this case, the expression (6) is written in the form

\[ B^{-1}_{kk} = B'^{-1}_{kk} - \frac{(B'^{-1}b)_k (B'^{-1}b)^T}{1 + (b^T B'^{-1})b}. \] (7)

We find the individual expressions included in the formula (7):

\[ B'^{-1}_{kk} = \frac{M_{kk}}{\det B'} = \frac{M_{kk}}{\alpha M_{kk}} = \frac{1}{\alpha}, \] (8)

where $M_{kk}$ is the corresponding minor;

\[ (b^T B'^{-1})b = (b^T B'^{-1}b)_1 = \sum_{i,j} h_i B'^{-1}_{ij} b_j = \frac{B^2}{\alpha} + \frac{1}{\beta^2} \sum_j B_{jk} \sum_i B_{ik} B'^{-1}_{ij} \]

\[ = \frac{B_{kk} - \alpha}{\alpha} + \frac{1}{B_{kk} - \alpha} \sum_{i,j \neq k} B_{kl} B'^{-1}_{lj} B_{jk}, \] (9)

\[ (B'^{-1}b)_k = \left( (B'^{-1}b)^T \right)_k = \frac{B_k}{\alpha} \frac{\sqrt{B_{kk} - \alpha}}{\alpha}. \] (10)

When writing these expressions, it is taken into account that the matrix $B'^{-1}$ has the same structure as the matrix $B'$, i.e. contains zeros in the $k$-th row and $k$-th column. Substituting expressions (8)-(10) into the formula (7), we obtain

\[ B^{-1}_{kk} = 1 - \frac{B_{kk} - \alpha}{\alpha B_{kk} + \alpha \sum_{i,j \neq k} B_{kl} B'^{-1}_{lj} B_{jk}}. \] (11)

Note that the $\alpha$ parameter introduced above has no limitations other than $\alpha \neq 0$, therefore, we consider the limit of expression (11) with $\alpha \to -\infty$. Then we obtain the following formula

\[ B^{-1}_{kk} = \frac{1}{B_{kk} - \sum_{l,j \neq k} B_{kl} B'^{-1}_{lj} B_{jk}}. \] (12)
Formula (12) has a flaw - it contains elements of the inverse matrix $B^{-1}$, which are unknown. Let us find approximate expressions for these elements, for which we divide the $B'$ into two matrices: $B' = B'^{(d)} + B'^{(nd)}$. In this case, the $B'^{(d)}$ matrix consists of matrix $B'$ the diagonal elements, and the $B'^{(nd)}$ matrix consists of off-diagonal. From the numerical calculations [16], it can be seen that the diagonal elements of our matrix have significantly prevail over non-diagonal in absolute value. Therefore, the inverse to it we are looking for as a sum of a diagonal matrix with elements inverse to the corresponding elements of the $B'$ matrix and the $D$ matrix with elements that are significantly smaller in absolute value:

$$B'_{ij}^{-1} = \delta_{ij} / B'_{ii} + D_{ij}.$$  

Consider the product $B'B^{-1}$ and equate it with the identity matrix:

$$\left( \begin{array}{ccc}
\delta_{ij} & \ldots & \delta_{ij} \\
\ldots & \ldots & \ldots \\
\delta_{ij} & \ldots & \delta_{ij}
\end{array} \right) = \left( \begin{array}{ccc}
\delta_{ij} & \ldots & \delta_{ij} \\
\ldots & \ldots & \ldots \\
\delta_{ij} & \ldots & \delta_{ij}
\end{array} \right).$$

the fourth term is neglected, considering it to be small. From here we obtain $D_{mj} = -B'^{(nd)}_{mj} / (B'^{(nd)}_{nm} B_{ji})$.

Thus, the elements of the inverse matrix

$$B'_{ij}^{-1} \approx \delta_{ij} / B'_{ii} B_{ji}' .$$

(13) Substituting expression (13) into formula (12), we obtain

$$B_{kk}^{-1} \approx \frac{1}{B_{kk} - \sum_{i,j \neq k} B_{ij} B_{jk} / B_{ii}' \delta_{ij} + \sum_{i,j \neq k} B_{ij} B'^{(nd)}_{jk} / B_{ii}' B_{jj}' \delta_{ij} - B_{kk}' + \sum_{i,j \neq k} (B_{ij})^2 / B_{ii}' + \sum_{i,j \neq k} B_{ij} B'^{(nd)}_{ij} / B_{ii}' B_{jj}' .}$$

(14) In formula (14), it is taken into account that $B_{ii}' = B_{ii}$ and $B'^{(nd)}_{ij} = B'^{(nd)}_{ij}$ at $i, j \neq k$ in the limit $\alpha \to -\infty$.

3.2. Calculation of elements standing in the $k$-th column and the $k$-th row, except for the diagonal

In this case, the expression (6) is written in the form

$$B_{ik}^{-1} = B_{ik}^{-1} - \frac{(B^{-1} b)_{i} (B^{-1} b)^{T}_{i}}{1 + (B^{-1} b)_{i} b}, \quad i \neq k.$$  

(15) We find the individual expressions included in the formula (15):

$$B_{ik}^{-1} = 0,$$

(16) since the matrices $B$ and $B'$ have the same structure, as noted above;

$$(B^{-1} b)_{i} = \sum_{j} b_{j} B_{ij}^{-1} = \sum_{j} \frac{B_{ij}}{B_{ij}'} B_{ij}^{-1} + \beta B_{ik}^{-1} = \frac{1}{\beta} \sum_{j \neq k} B_{ij} B_{ij}^{-1} ;$$

(17) the denominator in the formula (15) is the same as in the formula (7). Substituting the expressions (16), (17), (10) into formula (15), we obtain
\[ B_{ik}^{-1} = -\frac{1}{\alpha} \sum_{j \neq k} B_{kj} B_{ji}^{-1} + \frac{1}{\alpha B_{kk} - \alpha B_{mj} B_{jk}^{-1}}. \quad (18) \]

In the limit \( \alpha \to -\infty \), the expression (18) takes the following form:

\[ B_{ik}^{-1} = -\frac{1}{\alpha} \sum_{j \neq k} B_{kj} B_{ji}^{-1} + \frac{1}{\alpha B_{kk} - \alpha B_{mj} B_{jk}^{-1}}, \quad i \neq k. \quad (19) \]

Substituting expression (13) into formula (19), we finally obtain

\[ B_{ik}^{-1} \approx \frac{1}{B_{ii}} \left( -B_{ik} + \frac{B_{ij} B_{ji}^{(nd)}}{B_{jj}} \right) \left( B_{kk} - \sum_{m,j} \frac{(B_{km})^2}{B_{mm}} + \sum_{m,j} \frac{B_{km} B_{mj}^{(nd)} B_{jk}}{B_{mm} B_{jj}} \right), \quad i \neq k. \quad (20) \]

In formula (20), it is taken into account that \( B_{ii}^{(nd)} = B_{ij}^{nd} = B_{ij}^{nd} \) at \( i, j \neq k \) in the limit \( \alpha \to -\infty \).

3.3. Calculation of the inverse matrix diagonal elements \( B_{ii}^{-1} \ (i \neq k) \)

In this case, the expression (6) is written in the form

\[ B_{ii}^{-1} = B_{ii}^{-1} - \frac{(B_{ii}^{-1} b)(B_{ii}^{-1} b)\left( B_{ii}^{-1} \right)^\top}{1 + (B_{ii}^{-1} b)^2}, \quad i \neq k. \quad (21) \]

We find the individual expressions included in the formula (21):

\[ B_{ii}^{-1} \approx 1/B_{ii} \]

– from formula (13); the denominator in the formula (21) is the same as in the formula (7).

Substituting the expressions (22) and (17) into formula (21), we obtain

\[ B_{ii}^{-1} = \frac{1}{B_{ii}} + \left( \sum_{j \neq k} B_{kj} B_{ji}^{-1} \right)^2 \frac{1}{B_{kk} - \alpha \sum_{m,j} B_{km} B_{mj}^{-1} B_{jk}}. \quad (23) \]

In the limit \( \alpha \to -\infty \), the expression (23) takes the following form:

\[ B_{ii}^{-1} = \frac{1}{B_{ii}} + \left( \sum_{j \neq k} B_{kj} B_{ji}^{-1} \right)^2 \frac{1}{B_{kk} - \sum_{m,j} B_{km} B_{mj}^{-1} B_{jk}}. \quad (24) \]

Substituting expression (13) into formula (24), we finally obtain
In formula (25), it is taken into account that \( B_{ij}' = B_{ij} \) and \( B_{ij}'^{(nd)} = B_{ij}' \) at \( i, j \neq k \) in the limit \( \alpha \to -\infty \).

### 3.4. Calculation of the inverse matrix remaining elements \( B_{ij}^{-1} \) (\( i, j \neq k \) and \( i \neq j \))

In this case, the expression (6) is written in the form

\[
B_{ij}^{-1} = B_{ij}'^{-1} - \frac{(B^{-1}b)_i((B^{-1}b)^T)_j}{1 + (b^TB^{-1}b)} , \quad i, j \neq k, \ i \neq j.
\]

We find the individual expressions included in the formula (26):

\[
(B^{-1}b)_i = \sum_{m} b_mB_{im}^{-1} = \sum_{(m \neq k)} B_{ik}^{-1} + \frac{1}{\beta} \sum_{(m \neq k)} B_{km}B_{mi}^{-1}, \quad \text{(27)}
\]

\[
(B^{-1}b)_j = \sum_{l} b_MB_{jl}^{-1} = \sum_{(l \neq k)} B_{kj}^{-1} + \frac{1}{\beta} \sum_{(l \neq k)} B_{kl}B_{lj}^{-1}; \quad \text{(28)}
\]

the denominator in the formula (26) is the same as in the formula (7). Substituting the expressions (13), (27) and (28) into formula (26), we obtain

\[
B_{ij}^{-1} = \frac{-B_{ij}'^{(nd)}}{B_{ij}' B_{ij}'} - \frac{\alpha}{B_{kk} - \alpha} \sum_{m} B_{km}B_{ml}^{-1} \sum_{(l \neq k)} B_{kj}^{-1} - \frac{1}{B_{kk} - \alpha} \sum_{m,l} B_{km}B_{ml}^{-1} B_{lk}. \quad \text{(29)}
\]

In the limit \( \alpha \to -\infty \), the expression (29) takes the following form:

\[
B_{ij}^{-1} = \frac{-B_{ij}'^{(nd)}}{B_{ij}' B_{ij}'} + \sum_{(m \neq k)} B_{km}B_{ml}^{-1} \sum_{(l \neq k)} B_{kj}^{-1} - \frac{1}{B_{kk}} \sum_{m,l} B_{km}B_{ml}^{-1} B_{lk}. \quad \text{(30)}
\]

Substituting expression (13) into formula (30), we finally obtain
for indices \(i, j \neq k\) and \(i \neq j\). In formula (31), it is taken into account that \(B'_{ij} = B_{ij}\) and \(B'^{(nd)}_{ij} = B_{ij}^{(nd)}\) at \(i, j \neq k\) in the limit \(\alpha \to -\infty\).

### 4. Computational experiment

To verify the applicability of the obtained formulas (14), (20), (25) and (31), a model matrix \(B\) of size \(N \times N\) was considered. For calculations, the value \(N = 40\) was used. The elements of the matrix were set taking into account numerical calculations [16].

\[
B_{ij} = \frac{(-1)^{[(i/2)+|j/2|]}}{(i-j)^2 + 1 + i + j}. 
\]  

(32)

In expression (32) takes into account the elements decrease with distance from the main diagonal. The sign of the element was set in accordance with the real expression for the off-diagonal elements of the dislocation oscillators inverse generalized susceptibilities matrix [15]. The matrix symmetrization was performed by the operation \(B_{ij} = B_{ji}\). For the diagonal elements used formula

\[
B_{ii} = i^2 - 80. 
\]  

(33)

With this choice, the diagonal elements take values from -79 to 1520, i.e. they are significantly larger than non-diagonal elements [16]. The smallest in absolute value element is \(B_{9,9} = 1\) (\(k = 9\)). Calculations of the inverse matrix elements were carried out according to the formulas:

\[
B^{-1}_{kk} \approx \frac{1}{2B_{kk} - \sum_{j=1}^{N} \frac{(B_{kj})^2}{B_{jj}} + \sum_{i=1}^{N} \left[ \frac{\sum_{j=1}^{N} B_{kj} B^{(nd)}_{ij} B_{jk}}{B_{jj} B_{ii}} \right] - \sum_{j=1}^{N} \frac{B^{(nd)}_{kj} B_{jk}}{B_{jj}}} , \tag{34}
\]

in contrast to formula (14), formula (34) explicitly takes into account the conditions \(i \neq k\) and \(i, j \neq k\); 

\[
B^{-1}_{ik} \approx \frac{1}{B_{ii}} \left( B_{ik} + \sum_{j=1}^{N} \frac{B_{kj} B^{(nd)}_{ij}}{B_{jj}} - B^{(nd)}_{ki} \right) , \tag{35}
\]

in contrast to formula (20), formula (35) explicitly takes into account the conditions \(j \neq k, m \neq k\) and \(m, j \neq k\);
in contrast to formula (25), formula (36) explicitly takes into account the conditions \( j \neq k \) and \( m, j \neq k \);

\[
B_{ii}^{-1} \approx \frac{1}{B_{ii}} \left[ 1 + \frac{1}{2B_{ii}} \left\{ B_{ij} - \frac{\sum_{m=1}^{N} (B_{m9})^2}{B_{mm}} + \frac{\sum_{j=1}^{N} B_{mi} B_{mj} (B_{ij})^{(nd)}}{B_{mm} B_{jj}} - \frac{\sum_{j=1}^{N} B_{mj} B_{m9} (B_{ij})^{(nd)}}{B_{mm} B_{jj}} \right\} \right], \quad i \neq k, (36)
\]

in contrast to formula (31), formula (37) explicitly takes into account the conditions \( m \neq k \), \( l \neq k \) and \( m, l \neq k \). Then, the inverse matrix \( A = B^{-1} \) was calculated by direct calculation and compared with the \( B^{-1} \) matrix consisting of the elements found by formulas (34) - (37). The maximum error of the calculation of the element

\[
\max_{i,j} \left| \frac{A_{ij} - B_{ij}^{-1}}{A_{ij}} \right| = 0.017
\]

is determined. Ratios of the norms

\[
\frac{\| A - B^{-1} \|_F}{\| A \|_F} = 1.86 \cdot 10^{-6}, \quad \frac{\| A - B^{-1} \|_1}{\| A \|_1} = 2.77 \cdot 10^{-6}, \quad \frac{\| A - B^{-1} \|_2}{\| A \|_2} = 1.74 \cdot 10^{-6}, \quad \frac{\| A - B^{-1} \|_\infty}{\| A \|_\infty} = 2.33 \cdot 10^{-6}
\]

are also found. From the obtained results it can be seen that numerical calculation using formulas (14), (20), (25) and (31) showed good agreement with the direct calculation.

5. Conclusion

In this paper, we performed an approximate calculation of the dislocation segment generalized susceptibility matrix elements in order to obtain analytical formulas. Four expressions for different matrix elements are found. The first expression for the diagonal element \( B_{kk}^{-1} \) is obtained, where the number of \( k \) is chosen from the condition of minimum modulus of the diagonal matrix element \( B_{kk} \). Other expressions for the remaining elements are obtained. The computational experiment with the model matrix showed good agreement with the direct calculation of the inverse matrix. This confirms the correctness of the obtained approximate formulas. Thus, the developed method of calculating the inverse matrix can be used to calculate the matrix elements of the dislocation oscillators generalized susceptibility. The results obtained in this paper can be used to determine the orientation and size dependence of the dislocation segment vibrational spectrum, to find the oscillation eigenfrequencies of dislocation segment and their damping coefficients, for the calculation of dislocation segment dynamic characteristics and the dislocation internal friction, to find the dependence of the dislocation segment vibrational spectrum on the Poisson's ratio.
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