Masses of fully heavy tetraquarks $QQ\bar{Q}\bar{Q}$ in an extended relativized quark model

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Inspired by recent measurement of possible fully charmed tetraquarks in LHCb Collaboration, we investigate the mass spectra of fully heavy tetraquarks $QQ\bar{Q}\bar{Q}$ in an extended relativized quark model. Our estimations indicate that the broad structure around 6.4 GeV should contain one or more ground $cc\bar{c}\bar{c}$ tetraquark states, while the narrow structure near 6.9 GeV can be categorized as the first radial excitation of $cc\bar{c}\bar{c}$ system. Moreover, with the wave functions of the tetraquarks and mesons, the strong decays of tetraquarks into heavy quarkonium pair are quantitatively discussed, which can be further checked by the LHCb and CMS Collaborations.

I. INTRODUCTION

Since the observation of $X(3872)$ in 2003 [1], the searching for hadrons beyond the conventional mesons and baryons have become one of intriguing topics in the past decades. On the experimental side, a growing number of new hadron states have been observed experimentally. Some of these states cannot be accommodated into the traditional mesons or baryons, which can be good candidates of molecular or tetraquark states. Recent experimental and theoretical status can be found in the literature reviews [2–11].

Among the observed new hadron states, those with heavy quark components are particularly interesting, since the spectroscopy of traditional mesons and baryons with heavy quarks are much clearer than the light hadrons. Moreover, the interactions involved heavy quarks are supposed to be dominated by the short range one gluon exchange potential rather than the long range potential resulted from light meson exchanges. Thus, the new hadron states composed by four heavy quarks should be good candidates of compact tetraquark states rather than deuteron-like molecular states.

In 2017, the CMS Collaboration reported their measurement of exotic tetraquark state in four lepton channel and found an excess in $18.4 \pm 0.1\text{(stat.)} \pm 0.2\text{(syst.) GeV}/c^2$ with a global significance of 3.6 $\sigma$ [12]. This structure indicates a possible $bb\bar{b}\bar{b}$ tetraquark state [12–14]. It should be noticed that this structure is below the threshold of bottomonium meson pair, which demonstrates that the decays into bottomonium meson pair through quark rearrangement should be hindered. Later, the LHCb and CMS Collaborations analyzed the invariant mass distributions of $\Upsilon(1S)f^*\mu^+\mu^-$, but no evident structure was observed [15, 16].

On the theoretical side, the compact tetraquark states composed of $bb\bar{b}\bar{b}$ have been investigated extensively, but the conclusions are model dependent. In Refs. [17–25], the lowest $bb\bar{b}\bar{b}$ tetraquark state is estimated to be below the threshold of bottomonium meson pair, while in Refs. [26–34], all the $bb\bar{b}\bar{b}$ tetraquark states are above the threshold. To further distinguish different model and reveal the underlying dynamics of fully heavy tetraquark states, more efforts are needed, especially from the experimental side.

Very recently, the LHCb Collaboration reported their measurement of the $J/\psi \psi$ pair invariant mass spectrum and a structure near 6.9 $\text{GeV}/c^2$ was observed with the significance greater than 5$\sigma$ [35]. The resonance parameters are fitted to be

$$m = 6905 \pm 11(\text{stat.}) \pm 7(\text{syst.}) \text{MeV}/c^2,$$

$$\Gamma = 80 \pm 19(\text{stat.}) \pm 33(\text{syst.}) \text{MeV}/c^2,$$  \hspace{1cm} (1)

in a no-interference scenario, or

$$m = 6886 \pm 11(\text{stat.}) \pm 11(\text{syst.}) \text{MeV}/c^2,$$

$$\Gamma = 168 \pm 33(\text{stat.}) \pm 69(\text{syst.}) \text{MeV}/c^2,$$  \hspace{1cm} (2)

in an interference scenario. Besides the structure near 6.9 GeV, the experimental data also indicated another two structures in the vicinity of 6.4 GeV and 7.2 GeV, respectively [35]. These structures may be the evidence of compact tetraquark states composed by $cc\bar{c}\bar{c}$, which can be a criterion for different models.

After the observation of the LHCb Collaboration, the state around 6.9 GeV has been investigated in different models. In Ref. [36], this state was interpreted as a $P$–wave tetraquark state in a nonrelativistic quark model, while the QCD sum rule estimations indicated that it could be a second radial excited $S$–wave tetraquark state [37]. The results in Refs. [38, 39] suggested that the resonances with $J^P = 0^+$ and $1^+$ are about 6.4 $\sim$ 6.6 GeV, while the $2^+$ state is about 7.0 GeV, which are consistent with the structures reported by LHCb Collaboration [35].

In Ref. [40], we extended the relativized quark model proposed by Godfrey and Isgur to investigate the doubly heavy tetraquarks with the same model parameters. With such an extension, the tetraquarks and conventional mesons can be described in a uniform frame. In the present work, we further

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study the full heavy tetraquarks $QQQQ$ in the extended relativized quark model and give possible interpretation of the newly observed state around 6.9 GeV. Moreover, the newly observed structures are above the threshold of heavy quarkonium pair, thus, these states can decay into heavy quarkonium pair by quark rearrangement. For simplicity, the decay amplitude should be proportional to the overlap of wave functions of the initial and final states, thus, we can qualitatively discuss the decay behaviors of tetraquarks with the wave functions estimated from the relativized quark model.

This work is organized as follows. In section II, we present a review of the extended relativized quark model used in the present work. The numerical results of the masses and decays for the tetraquarks are given in Section III. The last section is devoted to a brief summary.

II. EXTENDED RELATIVIZED QUARK MODEL

To investigate the masses of fully heavy tetraquarks $Q_1Q_2Q_3Q_4$, we employ an extended relativized quark model, which has been developed very recently for the tetraquark states [40]. It is an extension of the relativized quark model to deal with the four-body systems. The Hamiltonian for a $Q_1Q_2Q_3Q_4$ state can be expressed as

$$ H = H_0 + \sum_{i<j} V_{ij}^{\text{qeg}} + \sum_i V_{ij}^{\text{conf}},$$  

where

$$H_0 = \sum_{i=1}^{4} \left( p_i^2 + m_i^2 \right)^{1/2}$$

is the relativistic kinetic energy, $V_{ij}^{\text{qeg}}$ is the one gluon exchange potential including the spin-spin interaction, and $V_{ij}^{\text{conf}}$ stands for the confining part. The explicit formula and parameters of relativized potentials can be found in Refs. [40, 41].

The wave function of a $Q_1Q_2Q_3Q_4$ state is composed of color, flavor, spin, and spatial parts. In the color space, two types of colorless states with determinate permutation properties exist

$$|\bar{3}3\rangle = |(Q_1Q_2)^3(\bar{Q}_3\bar{Q}_4)^3\rangle,$$

$$|6\bar{6}\rangle = |(Q_1Q_2)^6(\bar{Q}_3\bar{Q}_4)^6\rangle,$$

where the $|\bar{3}3\rangle$ and $|6\bar{6}\rangle$ are antisymmetric and symmetric under the exchange of $Q_1Q_2$ or $\bar{Q}_3\bar{Q}_4$, respectively. In the flavor space, the combinations of $[cc]$, $[c\bar{c}]$, $[bb]$, and $[b\bar{b}]$ are always symmetric, where the braces $\{\}$ are adopted to stand for symmetric flavor wave functions.

For the spin part, the six spin bases can be written as,

$$\chi_0^{00} = |(Q_1Q_2)0(\bar{Q}_3\bar{Q}_4)0\rangle,$$

$$\chi_0^{11} = |(Q_1Q_2)1(\bar{Q}_3\bar{Q}_4)1\rangle,$$

where $(Q_1Q_2)0$ and $(\bar{Q}_3\bar{Q}_4)0$ are antisymmetric and the $(Q_1Q_2)1$ and $(\bar{Q}_3\bar{Q}_4)1$ are symmetric for the two fermions under permutations. The matrix elements of the color and spin parts are same as the doubly heavy tetraquarks [40].

In the spatial space, the Jacobi coordinates are presented in Figure 1. For the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ systems, we can define

$$r_{12} = r_1 - r_2,$$

$$r_{34} = r_3 - r_4,$$

and

$$r = \frac{r_1 + r_2 + r_3 + r_4}{4}.$$

Then, other relevant coordinates of this system can be obtained in terms of $r_{12}$, $r_{34}$, and $r$. For a $S$–wave state, we adopt a set of Gaussian functions to approach its realistic spatial wave function [42]

$$\psi(r_{12}, r_{34}, r) = \sum_{n_{12}, n_{34}} C_{n_{12}n_{34}} \psi_{n_{12}}(r_{12}) \psi_{n_{34}}(r_{34}) \psi_n(r),$$

where $C_{n_{12}n_{34}}$ are the expansion coefficients. The $\psi_{n_{12}}(r_{12})\psi_{n_{34}}(r_{34})\psi_n(r)$ is the position representation of the basis $|n_{12}n_{34}n\rangle$, where

$$\psi_n(r) = \frac{2^3/4\pi^{3/4}}{\sqrt{\gamma_n}} e^{-\gamma_n r^2} Y_0(\hat{r}) = \left(\frac{2\nu_n}{\pi}\right)^{3/4} e^{-\nu_n r^2},$$

$$\gamma_n = \frac{1}{r_{12}^{2(\alpha-1)}} , \quad (n = 1 - N_{\text{max}}).$$

It should be stressed that our final results are independent on geometric Gaussian size parameters $r_1$, $a$, and $N_{\text{max}}$ when sufficiently large bases are chosen [42]. The $\psi_{n_{12}}(r_{12})$ and $\psi_{n_{34}}(r_{34})$ can be written in a similar way, and the momentum representation the basis $|n_{12}n_{34}n\rangle$ can be obtained by the Fourier transformation.

According to the Pauli exclusion principle, the total wave function of a tetraquark should be antisymmetric, and possible configurations for $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ systems are presented in Table I. With the full wave functions, all the matrix elements of the Hamiltonian can be worked out. Then, the masses can
be obtained by solving the following generalized eigenvalue problem
\[ \sum_{j=1}^{N_{\text{max}}} (H_{ij} - E N_{ij}) C_j = 0, \quad (i = 1 - N_{\text{max}}^3), \] (20)

where the \( H_{ij} \) are the matrix elements in the total bases, \( N_{ij} \) is the overlap matrix elements of the Guassian functions arising form the nonorthogonality of bases, \( E \) stands for the mass, and \( C_j \) are the eigenvector corresponding to the coefficients \( C_{ni,j\kappa\lambda} \) of spatial wave function. Moreover, for a given system, different configurations with same \( j^{PC} \) can mix with each other. The mixing effects are taken into account by diagonalizing the mass matrix of these configurations.

### III. Results and Discussions

In present work, we adopt \( N_{\text{max}}^3 = 10^3 \) Gaussian bases to estimate the \( S - \)wave \( QQQQ \) spectra. With these large bases, the numerical results are stable enough for our quark model calculations. The predicted masses of ground states for \( ccc\bar{c} \) and \( b\bar{b}b\bar{b} \) systems are presented in Table II. For the \( ccc\bar{c} \) system, the masses of four ground states lie in the range \( 6435 \sim 6543 \) MeV, which are higher than the \( J/\psi \) \& \( J/\psi \) threshold. Compared with the experimental data, we expect that these states should correspond to the broad structure in the vicinity of 6.4 GeV. This broad structure may be one state or an overlap of several states from current data, and more experimental information are needed to clarify its nature. For the \( b\bar{b}b\bar{b} \) system, the masses are also above the relevant \( 't''t' \) thresholds. These results for the ground states are consistent with nonrelativistic quark model calculations where the pairwise potentials are adopted properly [31–34, 43].

Besides the masses, we can also calculate the proportions of hidden color components and the root mean square radii. In addition to \((33)\) and \((66)\), other sets of color representations can be defined as
\[ |11\rangle = |Q_1\bar{Q}_3\rangle^1(Q_2\bar{Q}_4\rangle^1), \] (21)
\[ |88\rangle = |Q_1\bar{Q}_3\rangle^8(Q_2\bar{Q}_4\rangle^8), \] (22)
and
\[ |1'1'\rangle = |Q_1\bar{Q}_3\rangle^1(Q_2\bar{Q}_4\rangle^1), \] (23)
\[ |8'8'\rangle = |Q_1\bar{Q}_3\rangle^8(Q_2\bar{Q}_4\rangle^8). \] (24)

Then, the relations among three sets of color representations can be expressed as follows,
\[ |11\rangle = \sqrt{\frac{2}{3}} |33\rangle + \sqrt{\frac{2}{3}} |66\rangle, \] (25)
\[ |88\rangle = -\sqrt{\frac{2}{3}} |33\rangle + \sqrt{\frac{1}{3}} |66\rangle, \] (26)
and
\[ |1'1'\rangle = -\sqrt{\frac{1}{3}} |33\rangle + \sqrt{\frac{2}{3}} |66\rangle, \] (27)
\[ |8'8'\rangle = \sqrt{\frac{2}{3}} |33\rangle + \sqrt{\frac{1}{3}} |66\rangle. \] (28)

In present work, we adopt the \( |11\rangle \) and \( |88\rangle \) representations to stand for the neutral color and hidden color components, respectively. The color proportions and root mean square radii of the calculated ground states are displayed in Table III. For the \( ccc\bar{c} \) and \( b\bar{b}b\bar{b} \) systems, the expectations satisfy the following relations
\[ \langle r_{12}^2 \rangle^{1/2} = \langle r_{34}^2 \rangle^{1/2}, \] (29)
we find that another set of excitations are around 7050 MeV, which can be adopte

d to describe the relative magnitudes between $J/\psi J/\psi$ and $\psi(2S)J/\psi$ final states. For simplicity, one can assume the decay amplitudes are proportional to the overlap of initial and final states, and the proportional coefficient can be canceled in the final ratios. Here, the wave functions for initial tetraquarks have been obtained by solving the generaliz
ed eigenvalue problem, and the wave functions of $J/\psi, \psi(2S), \Upsilon$, and $\Upsilon(2S)$ can be got within the relativized quark model as well. With these wave functions, the ratios for $0^{++}$ and $2^{++}$ states can be estimated to be

$$R[cc\bar{c}\bar{c}(6849)] = 0.113,$$  \hspace{1cm} (33)

$$R[cc\bar{c}\bar{c}(6940)] = 0.122,$$  \hspace{1cm} (34)

$$R[cc\bar{c}\bar{c}(6948)] = 0.075.$$  \hspace{1cm} (35)

Combined with the branching ratios of $J/\psi \to \mu^+\mu^-$ and $\psi(2S) \to \mu^+\mu^-$, one can further define

$$R_{4\mu} = \frac{\Gamma[cc\bar{c}\bar{c} \to J/\psi J/\psi \to \mu^+\mu^-\mu^+\mu^-]}{\Gamma[cc\bar{c}\bar{c} \to \psi(2S)J/\psi \to \mu^+\mu^-\mu^+\mu^-]}.$$  \hspace{1cm} (36)

Then, the ratios $R_{4\mu}$ are predicted to be

$$R_{4\mu}[cc\bar{c}\bar{c}(6849)] = 0.843,$$  \hspace{1cm} (37)

$$R_{4\mu}[cc\bar{c}\bar{c}(6940)] = 0.910,$$  \hspace{1cm} (38)

$$R_{4\mu}[cc\bar{c}\bar{c}(6948)] = 0.559.$$  \hspace{1cm} (39)

It can be found that the $\psi(2S)J/\psi$ channel for the excited states is important even though the phase spaces are smaller. The similar situation occurs for the lower excited $bb\bar{b}\bar{b}$ states, where the $R_{4\mu}$ of $bb\bar{b}\bar{b}(19567), bb\bar{b}\bar{b}(19625)$, and $bb\bar{b}\bar{b}(19633)$ states are 0.113, 0.111, and 0.084, respectively. These ratios indicate that the lower excited $bb\bar{b}\bar{b}$ states can decay to $\mu^+\mu^-\mu^+\mu^-$ final states through $\Upsilon(2S)\Upsilon$ more easily than $\Upsilon\Upsilon$ mode. Future experiments can search for these states in $\psi(2S)J/\psi$ and $\Upsilon(2S)\Upsilon$ final states.
TABLE III: The color proportions and the root mean square radii of the ground states for $c^2c^2$ and $b^2b^2$ systems. The units of masses and root mean square radii are in MeV and fm, respectively.

| System | $J^P$ | Mass | $|33⟩$ | $|66⟩$ | $|11⟩$ | $|88⟩$ | $⟨r_{+}^2⟩^{1/2}$ | $⟨r_{-}^2⟩^{1/2}$ | $⟨r_{0}^2⟩^{1/2}$ | $⟨r_{2}^2⟩^{1/2}$ |
|--------|-------|------|-------|-------|-------|-------|----------------|----------------|----------------|----------------|
| $c^2c^2$ | $0^{++}$ | 6435  | 38.1% | 61.9% | 54.0% | 46.0% | 0.433          | 0.265          | 0.405          | 0.306          |
|         | $0^{++}$ | 6542  | 61.9% | 38.1% | 46.0% | 54.0% | 0.415          | 0.281          | 0.406          | 0.293          |
|         | $1^{−−}$ | 6515  | 100%  | 0%    | 33.3% | 66.7% | 0.387          | 0.310          | 0.414          | 0.274          |
|         | $2^{++}$ | 6543  | 100%  | 0%    | 33.3% | 66.7% | 0.394          | 0.321          | 0.425          | 0.278          |
| $b^2b^2$ | $0^{++}$ | 19201 | 16.8% | 83.2% | 61.1% | 38.9% | 0.253          | 0.206          | 0.269          | 0.177          |
|         | $0^{++}$ | 19255 | 83.2% | 16.8% | 38.9% | 61.1% | 0.257          | 0.194          | 0.266          | 0.182          |
|         | $1^{−−}$ | 19251 | 100%  | 0%    | 33.3% | 66.7% | 0.251          | 0.203          | 0.269          | 0.177          |
|         | $2^{++}$ | 19262 | 100%  | 0%    | 33.3% | 66.7% | 0.253          | 0.206          | 0.272          | 0.179          |

TABLE IV: Predicted masses of radial excitations for $c^2c^2$ and $b^2b^2$ systems.

| System | $J^P$ | Configuration | $(H)$ (MeV) | Mass (MeV) | Eigenvector |
|--------|-------|---------------|-------------|------------|-------------|
| $c^2c^2$ (I) | $0^{++}$ | $|cc⟩^0[|cc⟩^0]_0$ | 6917 $−39$ | 6849       | $|0.500, 0.866⟩$ |
|         | $1^{−−}$ | $|cc⟩^1[|cc⟩^1]_1$ | 6928        | 6928       | $|0⟩$        |
|         | $2^{++}$ | $|cc⟩^1[|cc⟩^1]_2$ | 6948        | 6948       | $|0⟩$        |
| $c^2c^2$ (II) | $0^{++}$ | $|cc⟩^0[|cc⟩^0]_0$ | 7046 $−19$ | 7025       | $|0.664, 0.748⟩$ |
|         | $1^{−−}$ | $|cc⟩^1[|cc⟩^1]_1$ | 7052        | 7052       | $|0⟩$        |
|         | $2^{++}$ | $|cc⟩^1[|cc⟩^1]_2$ | 7064        | 7064       | $|0⟩$        |
| $b^2b^2$ (I) | $0^{++}$ | $|bb⟩^0[|bb⟩^0]_0$ | 19621 $−14$ | 19567      | $|0.966, 0.258⟩$ |
|         | $1^{−−}$ | $|bb⟩^1[|bb⟩^1]_1$ | 19625       | 19625      | $|0⟩$        |
|         | $2^{++}$ | $|bb⟩^1[|bb⟩^1]_2$ | 19633       | 19633      | $|0⟩$        |
| $b^2b^2$ (II) | $0^{++}$ | $|bb⟩^0[|bb⟩^0]_0$ | 19731 $−6$  | 19726      | $|0.570, 0.822⟩$ |
|         | $1^{−−}$ | $|bb⟩^1[|bb⟩^1]_1$ | 19736       | 19740      | $|0⟩$        |
|         | $2^{++}$ | $|bb⟩^1[|bb⟩^1]_2$ | 19736       | 19736      | $|0⟩$        |

FIG. 2: The predicted masses of $c^2c^2$ and $b^2b^2$ systems.

IV. SUMMARY

In this work, we investigate the masses of fully heavy tetraquarks $c^2c^2$ and $b^2b^2$ in an extended relativized quark model. The four-body Hamiltonian including the Coulomb
TABLE V: The color proportions and root mean square radii of the radial excited states for $cc\bar{c}$ and $b\bar{b}b$ systems. The units of masses and root mean square radii are in MeV and fm, respectively.

| System | $J^P$ | Mass | $|33\rangle$ | $|66\rangle$ | $|11\rangle$ | $|88\rangle$ | $(r_{11}^2)^{1/2}$ | $(r_{66}^2)^{1/2}$ | $(r_{33}^2)^{1/2}$ | $(r_{88}^2)^{1/2}$ |
|--------|-------|------|----------|----------|----------|----------|----------------|----------------|----------------|----------------|
| $cc\bar{c}$ (I) | 0$^{++}$ | 6849 | 25.0% | 75.0% | 58.3% | 41.7% | 0.630 | 0.351 | 0.567 | 0.445 |
|     | 0$^{++}$ | 6940 | 75.0% | 25.0% | 41.7% | 58.3% | 0.530 | 0.452 | 0.587 | 0.374 |
|     | 1$^{-+}$ | 6928 | 100% | 0% | 33.3% | 66.7% | 0.471 | 0.504 | 0.604 | 0.333 |
|     | 2$^{++}$ | 6948 | 100% | 0% | 33.3% | 66.7% | 0.468 | 0.520 | 0.616 | 0.331 |
| $cc\bar{c}$ (II) | 0$^{++}$ | 7025 | 44.0% | 56.0% | 52.0% | 48.0% | 0.642 | 0.320 | 0.556 | 0.454 |
|     | 0$^{++}$ | 7063 | 56.0% | 44.0% | 48.0% | 52.0% | 0.631 | 0.329 | 0.554 | 0.446 |
|     | 1$^{-+}$ | 7052 | 100% | 0% | 33.3% | 66.7% | 0.592 | 0.361 | 0.553 | 0.419 |
|     | 2$^{++}$ | 7064 | 100% | 0% | 33.3% | 66.7% | 0.598 | 0.365 | 0.558 | 0.423 |
| $b\bar{b}b$ (I) | 0$^{++}$ | 19567 | 6.7% | 93.3% | 64.4% | 35.6% | 0.436 | 0.208 | 0.372 | 0.309 |
|     | 0$^{++}$ | 19625 | 93.3% | 6.7% | 35.6% | 64.4% | 0.324 | 0.334 | 0.405 | 0.229 |
|     | 1$^{-+}$ | 19635 | 100% | 0% | 33.3% | 66.7% | 0.313 | 0.345 | 0.410 | 0.221 |
|     | 2$^{++}$ | 19633 | 100% | 0% | 33.3% | 66.7% | 0.312 | 0.351 | 0.414 | 0.221 |
| $b\bar{b}b$ (II) | 0$^{++}$ | 19726 | 67.6% | 32.4% | 44.1% | 55.9% | 0.413 | 0.228 | 0.370 | 0.292 |
|     | 0$^{++}$ | 19740 | 32.4% | 67.6% | 55.9% | 44.1% | 0.428 | 0.211 | 0.369 | 0.303 |
|     | 1$^{-+}$ | 19733 | 100% | 0% | 33.3% | 66.7% | 0.398 | 0.244 | 0.372 | 0.282 |
|     | 2$^{++}$ | 19736 | 100% | 0% | 33.3% | 66.7% | 0.399 | 0.245 | 0.374 | 0.282 |

TABLE VI: The decay channels of the $cc\bar{c}$ and $b\bar{b}b$ tetraquarks via fall-apart mechanism.

| System | $J^P$ | $S$-wave | $P$-wave |
|--------|-------|----------|----------|
| $cc\bar{c}$ | 0$^{++}$ | $\eta_1\eta_8$, $J^P|J/\psi|\eta_1(2S)\eta_8$, $\eta_1(2S)\eta_8(2S)J/\psi$, $h_1h_0$, $\chi_0\chi_0$, $\chi_1\chi_1$, $\chi_2\chi_2$ | $\eta_1\chi_1$, $J^P|\psi|\chi_1(2S)$, $\psi(2S)\chi_1$ |
|     | 1$^{--}$ | $\eta_1J^P|\psi|\eta_1(2S)\eta_8$, $\eta_1(2S)\eta_8(2S)J/\psi$, $h_1h_0$, $\chi_0\chi_0$, $\chi_1\chi_1$, $\chi_2\chi_2$ | $\eta_1\chi_1$, $J^P|\psi|\chi_1(2S)$, $\psi(2S)\chi_1$ |
|     | 2$^{++}$ | $J/\psi|\psi|\eta_1(2S)\eta_8$, $\eta_1(2S)\eta_8(2S)J/\psi$, $h_1h_0$, $\chi_0\chi_0$, $\chi_1\chi_1$, $\chi_2\chi_2$ | $\eta_1\chi_1$, $J^P|\psi|\chi_1(2S)$, $\psi(2S)\chi_1$ |
| $b\bar{b}b$ | 0$^{++}$ | $\eta_1\eta_b$, $T^T\eta$, $\eta_b(2S)\eta_b$, $\eta_b(2S)T^T$, $h_bh_0$, $\chi_0\chi_0$, $\chi_0\chi_1$, $\chi_0\chi_2$, $\chi_2\chi_0$, $\chi_2\chi_1$ | $\eta_1\chi_0$, $T\chi_0$, $\eta_b(2S)\chi_0$, $\eta_b(2S)\chi_1$, $\eta_b(2S)\chi_2$, $\eta_b(2S)\chi_0$, $\eta_b(2S)\chi_1$, $\eta_b(2S)\chi_2$ |
|     | 1$^{--}$ | $\eta_b\eta_b$, $T^T\eta_b$, $\eta_b(2S)\eta_b$, $\eta_b(2S)T^T$, $h_bh_0$, $\chi_0\chi_0$, $\chi_0\chi_1$, $\chi_0\chi_2$, $\chi_2\chi_0$, $\chi_2\chi_1$ | $\eta_b\chi_0$, $T\chi_0$, $\eta_b(2S)\chi_0$, $\eta_b(2S)\chi_1$, $\eta_b(2S)\chi_2$, $\eta_b(2S)\chi_0$, $\eta_b(2S)\chi_1$, $\eta_b(2S)\chi_2$ |
|     | 2$^{++}$ | $T^T, T^T|2S\eta_b, h_bh_0, \chi_0\chi_0, \chi_0\chi_1, \chi_0\chi_2$ | $\eta_b\chi_0$, $T\chi_0$, $\eta_b(2S)\chi_0$, $\eta_b(2S)\chi_1$, $\eta_b(2S)\chi_2$, $\eta_b(2S)\chi_0$, $\eta_b(2S)\chi_1$, $\eta_b(2S)\chi_2$ |

FIG. 3: The narrow $cc\bar{c}$ structure near 6.9 GeV.

potential, confining potential, spin-spin interactions, and relativistic corrections are solved within the variational method. Our estimations indicate that the broad structure around 6.4 GeV should contain one or more ground $cc\bar{c}$ tetraquark states, while the narrow structure near 6.9 GeV can be categorized as the first radial excitation of $cc\bar{c}$ system. The significant hidden color component and small root mean square radii demonstrate that these states are compact tetraquarks. For the radial excited states, the decay ratios between the $J/\psi|J/\psi$ and $\psi(2S)|J/\psi$ or $\Upsilon(2S)|\Upsilon(2S)$ modes are also qualitatively discussed with the wave functions of the tetraquarks and mesons. Our results show that the $\psi(2S)|J/\psi$ or $\Upsilon(2S)|\Upsilon(2S)$ channel is significant for these excited tetraquarks. We hope our sophisticated calculations of the fully heavy tetraquarks may provide valuable information for future experimental searches.

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[1] S. Choi et al. [Belle], Observation of a narrow charmonium -like state in exclusive $B^+ \to K^+ \pi^+ \pi^- J/\psi$ decays, Phys. Rev. Lett. 91, 262001 (2003).

[2] E. Klempt and A. Zaitsev, Glueballs, Hybrids, Multiquarks. Experimental facts versus QCD inspired concepts, Phys. Rept. 454, 1 (2007).

[3] N. Brambilla et al., Heavy Quarkonium: Progress, Puzzles, and Opportunities, Eur. J. Phys. C 71, 1534 (2011).

[4] H. X. Chen, W. Chen, X. Liu and S. L. Zhu, The hidden-charm pentaquark and tetraquark states, Phys. Rept. 639, 1 (2016).

[5] R. F. Lebed, R. E. Mitchell and E. S. Swanson, Heavy-Quark QCD Exotica, Prog. Part. Nucl. Phys. 93, 143 (2017).

[6] F. K. Guo, C. Hanhart, U. G. Meiñner, Q. Wang, Q. Zhao and B. S. Zou, Hadronic molecules, Rev. Mod. Phys. 90, 015004 (2018).

[7] A. Esposito, A. Pilloni and A. D. Polosa, Multiquark Resonances, Phys. Rept. 668, 1 (2017).

[8] A. Ali, J. S. Lange and S. Stone, Exotics: Heavy Pentaquarks and Tetraquarks, Prog. Part. Nucl. Phys. 97, 123 (2017).

[9] Y. R. Liu, H. X. Chen, W. Chen, X. Liu and S. L. Zhu, Pentaquark and Tetraquark states, Prog. Part. Nucl. Phys. 107, 237 (2019).

[10] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo and C. Z. Yuan, The XYZ states: experimental and theoretical status and perspectives, arXiv:1907.07583.

[11] Y. Dong, A. Faessler and V. E. Lyubovitskii, Description of heavy exotic resonances as molecular states using phenomenological Lagrangians, Prog. Part. Nucl. Phys. 94, 282 (2017).

[12] V. Khachatryan et al. (CMS Collaboration), Observation of $\Upsilon(1S)$ pair production in proton-proton collisions at $\sqrt{s} = 8$ TeV, J. High Energy Phys. 05, 013 (2017).

[13] CMS Collaboration Collaboration, S. Duttag for the collaboration. https://meetings.aps.org/Meeting/APR18/Session/U09.6

[14] K. Yi, Things that go bump in the night: From $J/\psi$ to other mass spectrum, Int. J. Mod. Phys. A 33, 1850224 (2019).

[15] R. Aaij et al. (LHCb Collaboration), Search for beautiful tetraquarks in the $\Upsilon(1S)\mu\mu$ invariant-mass spectrum, J. High Energy Phys. 10, 086 (2018).

[16] A. M. Sirunyan et al. (CMS Collaboration), Measurement of the $\Upsilon(1S)$ pair production cross section and search for resonances decaying to $\Upsilon(1S)\mu^+\mu^-$ in proton-proton collisions at $\sqrt{s} = 13$ TeV, arXiv:2002.06393.

[17] Z. G. Wang, Analysis of the $QQQQ$ tetraquark states with QCD sum rules, Eur. Phys. J. C 77, 432 (2017).

[18] M. Karliner, S. Nussinov, and J. L. Rosner, $QQQQ$ states: Masses, production, and decays, Phys. Rev. D 95, 034011 (2017).

[19] A. V. Berezhnov, A. V. Luchinsky and A. A. Novoselov, Tetraquarks composed of 4 heavy quarks, Phys. Rev. D 86, 034004 (2012).

[20] Y. Bai, S. Lu and J. Osborne, Beauty-full Tetraquarks, Phys. Lett. B 798, 134930 (2019).

[21] M. N. Anwar, J. Ferretti, F. K. Guo, E. Santopinto, and B. S. Zou, Spectroscopy and decays of the fully-heavy tetraquarks, Eur. Phys. J. C 78, 647 (2018).

[22] A. Esposito and A. D. Polosa, A $bb\bar{b}\bar{b}$ di-bottomium at the LHC, Eur Phys J.C 78, 782 (2018).

[23] W. Chen, H. X. Chen, X. Liu, T. G. Steele and S. L. Zhu, Hunting for exotic doubly hidden-charm/bottom tetraquark states, Phys. Lett. B 773, 247 (2017).

[24] V. R. Debastiani and F. S. Navarra, A non-relativistic model for the $[cc][\bar{c}\bar{c}]$ tetraquark, Chin.Phys.C 43, 013105 (2018).

[25] Z. G. Wang and Z. Y. Di, Analysis of the vector and axial-vector $QQQQ$ tetraquark states with QCD sum rules, Acta Phys. Polon. B 50, 1335 (2019).

[26] J. Wu, Y. R. Liu, K. Chen, X. Liu, and S. L. Zhu, Heavy-flavored tetraquark states with the $QQQQ$ configuration, Phys. Rev. D 97, 094015 (2018).

[27] R. J. Lloyd and J. P. Vary, All charm tetraquarks, Phys. Rev. D 70, 014009 (2004).

[28] J. P. Adler, J. M. Richard and P. Taxil, Do narrow heavy multi - quark states exist, Phys. Rev. D 25, 2370 (1982).

[29] C. Hughes, E. Eichten, and C. T. H. Davies, Searching for beauty-fully bound tetraquarks using lattice nonrelativistic QCD, Phys. Rev. D 97, 054505 (2018).

[30] J. M. Richard, A. Valcarce, and J. Vijande, Few-body quark dynamics for doubly heavy baryons and tetraquarks, Phys. Rev. C 97, 035211 (2018).

[31] M. S. Liu, Q. F. Lú, X. H. Zhong and Q. Zhao, All-heavy tetraquarks, Phys. Rev. D 100, 016006 (2019).

[32] G. J. Wang, L. Meng and S. L. Zhu, Spectrum of the fully-heavy tetraquark state $QQQQ^-$, Phys. Rev. D 100, 096013 (2019).

[33] X. Chen, Analysis of hidden-bottom $bb\bar{b}\bar{b}$ states, Eur. Phys. J. A 55, 106 (2019).

[34] C. R. Deng, H. Chen and J. L. Ping, Towards the understanding of fully-heavy tetraquark states from various models, arXiv:2003.01514.

[35] L. An, LHC Seminar, https://indico.cern.ch/event/900972/.

[36] M. S. Liu, F. X. Liu, X. H. Zhong and Q. Zhao, Full-heavy tetraquark states and their evidences in the LHCb di-$J/\psi$ spectrum, arXiv:2006.11952.

[37] Z. G. Wang, Tetraquark candidates in the LHCb’s di-$J/\psi$ mass spectrum, arXiv:2006.13028.

[38] X. Jin, Y. Xue, H. Huang and J. Ping, Full-heavy tetraquarks in constituent quark models, arXiv:2006.13745.

[39] G. Yang, J. Ping, L. He and Q. Wang, A potential model prediction of fully-heavy tetraquarks $QQQQ$ ($O = c,b$), arXiv:2006.13756.

[40] Q. F. Liu, D. Y. Chen and Y. B. Dong, Masses of doubly heavy tetraquarks $T_{QQ}$ in a relativized quark model, arXiv:2006.08087.

[41] S. Godfrey and N. Isgur, Mesons in a relativized quark model with chromodynamics, Phys. Rev. D 32, 189 (1985).

[42] E. Hiyama, Y. Kino, and M. Kamimura, Gaussian expansion method for few-body systems, Prog. Part. Nucl. Phys. 51, 223 (2003).

[43] J. M. Richard, A. Valcarce and J. Vijande, Hall-Post inequalities: Review and application to molecules and tetraquarks, Annals Phys. 412, 168009 (2020).