Implications of the fermion vacuum term in the extended SU(3) Quark Meson model on compact stars properties

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(Dated: February 18, 2022)

We study the impact of the fermion vacuum term in the SU(3) quark meson model on the equation of state and determine the vacuum parameters for various sigma meson masses. We examine its influence on the equation of state and on the resulting mass radius relations for compact stars. The tidal deformability $\Lambda$ of the stars is studied and compared to the results of the mean field approximation. Parameter sets which fulfill the tidal deformability bounds of GW170817 together with the observed two solar mass limit turn out to be restricted to a quite small parameter range in the mean field approximation. The extended version of the model does not yield solutions fulfilling both constraints. Furthermore, no first order chiral phase transition is found in the extended version of the model, not allowing for the twin star solutions found in the mean field approximation.

I. INTRODUCTION

The theory of the strong interaction, quantum chromodynamics (QCD), describes the interaction between quarks and gluons. The QCD Lagrangian possesses an exact color- and flavor symmetry for quarks and gluons. The QCD Lagrangian describes the interaction between quarks and gluons. The QCD Lagrangian possesses an exact color- and flavor symmetry for quarks and gluons. The QCD Lagrangian possesses an exact color- and flavor symmetry for quarks and gluons. The QCD Lagrangian possesses an exact color- and flavor symmetry for quarks and gluons. The QCD Lagrangian possesses an exact color- and flavor symmetry for quarks and gluons.

Since QCD can not be solved on the lattice at nonzero temperatures or densities chiral symmetry is expected to be restored[11]. The possible appearance of quark matter at high densities has interesting consequences for the properties of compact stars [11–18]. Since QCD can not be solved on the lattice at nonzero density, effective models are needed to study the features and interactions of high density matter [13, 22]. The chiral SU(3) quark meson model is a well established and studied framework [22, 34]. Its advantage in comparison to other chiral models like the Nambu-Jona-Lasinio model [54, 55] lies in its renormalizability [51, 52, 55] by taking into account vacuum fluctuations.

In this article we study the impact of the fermion vacuum term on the equation of state (EoS) in the SU(3) quark meson model in an extended mean field approximation (eMFA), see also [20, 28, 40, 13]. For that purpose we determine the vacuum parameters for different sigma meson masses $m_{\sigma}$. The dependence of the vacuum parameters on the renormalization scale parameter $\Lambda_r$ cancels with the dependence of the additional fermion vacuum term $\Omega^{vac}(\Lambda_r)$ in the grand potential, so that the whole grand potential is independent on the renormalization scale parameter $\Lambda_r$ [28, 51, 42, 43]. The resulting equations of state (EoSs) are investigated and subsequently used to solve the Tolman-Oppenheimer-Volkoff (TOV) equations [44] for various parameters of the model, that is the sigma meson mass $m_{\sigma}$, the repulsive vector coupling constant $g_{V}$ and the vacuum pressure constant $B^{1/4}$.

The GW170817 measurement on the tidal deformability deduces $\Lambda = 300^{+230}_{-420}$ for a 1.4M$_{\odot}$ star [49, 50]. Inferred from that measurement, the radius of a 1.4M$_{\odot}$ star cannot be larger than $R \geq 13.5\, \text{km}$ [51, 61].

The mass radius relations of the eMFA are compared to those of the standard mean field approximation (MFA). We find that the mass radius relations for a given parameter set in the eMFA are in general less compact compared to the MFA case. Less compact star configurations imply rather large values of the tidal deformability parameter $\Lambda$. Fulfilling all considered constraints on mass $M \geq 2M_{\odot}$, radius $R \leq 13.5\, \text{km}$ at 1.4M$_{\odot}$ and the tidal deformability parameter $\Lambda \leq 720$ for a 1.4M$_{\odot}$ star is possible in a narrow parameter space for the MFA. The parameters in the eMFA do not allow for solutions which satisfy the above mentioned constraints.

Further analysis of the parameter range of the SU(3) chiral quark meson model in the eMFA exhibits that the inclusion of the fermion vacuum term yields crossover transitions from a chirally broken phase to a restored phase exclusively. This feature consequently smoothen the EoSs compared to the MFA case not allowing for twin star solutions, that is two stable branches in the mass radius relation, as found for a certain parameter range in the MFA, see e.g. [18].

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II. THE SU(3) QUARK MESON MODEL

A chirally invariant model with three flavours \( N_f = 3 \) and with quarks as the active degrees of freedom is the SU(3) chiral quark meson model. Based on the theory of the strong interaction, an effective model must doubtless implement features of QCD, such as flavour symmetry and spontaneous- and explicit breaking of chiral symmetry. The \( N_f = 3 \) Lagrangian \([1, 13, 32, 62]\) respecting these symmetries and including vector meson interactions reads

\[
\mathcal{L} = \sum_{\alpha} \tilde{\Psi}_\alpha (i\gamma^\mu \partial_\mu - m_\alpha) \Psi_\alpha + \tilde{\Psi}_\alpha (i\gamma^\mu \partial_\mu - m_\alpha) \Psi_\alpha
\]

\[
+ \text{tr}[(\partial_\mu \Phi)(\partial^\mu \Phi)^\dagger - \lambda_1 |\text{tr}(\Phi^\dagger \Phi)|^2 - \lambda_2 \text{tr}(\Phi^\dagger \Phi)^2
\]

\[
+ \frac{1}{2} m_\sigma^2 (\text{tr}(\Phi^\dagger \Phi) - m_\sigma^2) - \frac{1}{4} m_\omega^2 (\text{tr}(\Phi^\dagger \Phi) - m_\omega^2)
\]

\[
+ \left[ \sum_{\alpha} \frac{(\phi_n + \phi_n^\dagger)}{\sqrt{2}} - \phi_n \right] (\phi_n^\dagger \phi_n - m_n^2)
\]

\[
+ \left[ \frac{\phi_s^\dagger \phi_s}{\sqrt{2}} - \phi_s \right] (\phi_s^\dagger \phi_s - m_s^2)
\]

\[
+ \text{det}(\Phi)
\]

(2)

with a Yukawa like coupling \( g_\alpha \) being the fields \( \sigma_n, \sigma_s, \omega, \rho \) and \( \phi \) involved, and the effective mass \( m_{n,s} \) to the spinors \( \Psi_n, \Psi_s \). The indices \( n=\)nonstrange and \( s=\)strange indicate the flavour content. All physical fields are arranged in the matrix \( \Phi \) \([1, 3]\).

\[
\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix}
\frac{(\phi_n + \phi_n^\dagger)}{\sqrt{2}} & \phi_n^\dagger & \phi_s^\dagger & K^+_n & K_0^+ & \sigma_n & \eta_n \\
\phi_n & \frac{(\phi_n + \phi_n^\dagger)}{\sqrt{2}} & \phi_n^\dagger & K_n^{-} & K_0^{-} & \sigma_n & \eta_n
\end{pmatrix}
\]

(3)

where we consider the condensed \( \sigma_{n,s} \) fields and the pions to determine the vacuum parameters \( \lambda_1, \lambda_2, m_0, c, h_n \) and \( h_s \) of the model, which are fixed at tree level \([27, 32, 62, 63]\), but change upon renormalization \([39, 42, 43]\), see Sec. II A). In thermal equilibrium the grand potential \( \Omega \) is calculated via the partition function \( Z \), which is defined as a path integral over the fermion fields.

\[
\Omega = -\frac{\ln Z}{\beta} \quad \text{with} \quad Z = \int \mathcal{D}\Phi \mathcal{D}\Psi \exp \left[ \int d^4x \mathcal{L} \right]
\]

(4)

Evaluated, the grand canonical potential reads

\[
\Omega_{qq} = V + \frac{\Omega_{qq}^{vac} + \Omega_{qq}^{th}}{16\pi^2} = V - \frac{3}{\pi^2 \beta} \int_0^\infty k^2 dk \cdot (R + N)
\]

(5)

where \( V \) is the tree level potential

\[
V = \frac{\lambda_1}{4} (\sigma_n^2 + \sigma_s^2) + \frac{\lambda_2}{8} (\sigma_n^4 + 2\sigma_s^4) + \frac{m_0^2}{2} (\sigma_n^2 + \sigma_s^2)
\]

\[
- \frac{h_n}{\sigma_n} \sigma_n - \frac{h_n}{\sigma_s} \sigma_s - \frac{c_0^2}{2\sqrt{2}}
\]

\[
- \frac{1}{2} \left( m_\omega^2 \omega^2 + m_\rho^2 \rho^2 + m_\phi^2 \phi^2 \right) + B^{1/4}
\]

(6)

\( B^{1/4} \) is a phenomenological vacuum pressure term \([64-67]\), which will stiffen or soften the EoS \( p(\epsilon) \), pressure \( p \) vs. energy density \( \epsilon \). The fermion vacuum term is

\[
\Omega_{qq}^{vac} = \mathcal{R} = \frac{E_{n,s}}{T}
\]

(7)

and the part from the mean field approximation is

\[
\Omega_{qq}^{th} = N = \ln \left( 1 + e^{-\beta(E_n + \mu_n)} \right) + \ln \left( 1 + e^{-\beta(E_n - \mu_n)} \right)
\]

\[+ \ln \left( 1 + e^{-\beta(E_s + \mu_s)} \right) + \ln \left( 1 + e^{-\beta(E_s - \mu_s)} \right)
\]

\[+ \ln \left( 1 + e^{-\beta(E_\omega + \mu_\omega)} \right) + \ln \left( 1 + e^{-\beta(E_\rho + \mu_\rho)} \right) + \ln \left( 1 + e^{-\beta(E_\phi + \mu_\phi)} \right)
\]

(8)

The dependence of \( \Omega_{qq}^{th} \) on the chiral condensates \( \sigma_{n,s} \) is implicit in the relativistic quasi-particle dispersion relation for the constituent quarks

\[
E_{n,s} = \sqrt{k^2 + \tilde{m}_{n,s}^2}
\]

(9)

The quantity \( \tilde{m}_{n,s} = g_n m_{n,s} \) is in the medium mass generated by the scalar fields. \( \mu_{n,s} \) in eq. \([8]\) are the respective chemical potentials

\[
\mu_{up} = \mu_n = g_n \omega + g_\rho \rho
\]

\[
\mu_{down} = \mu_s = g_s \omega - g_\rho \rho
\]

\[
\mu_{strange} = \mu_s = g_\phi \phi
\]

(10-12)

A. Renormalized vacuum parameters of the SU(3)

Quark Meson model

The implementation of the fermion vacuum term needs regularization schemes. In this work we employ the minimal subtraction scheme and follow the procedure as found in \([20, 28, 39, 43]\) to proper perform the regularization of the divergence. The diverging integral containing the fermion vacuum contribution in eq. \([10]\) is to lowest order just the one-loop effective potential at zero temperature \([41]\) and is dimensionally regularized via the corresponding counter term

\[
\delta \mathcal{L} = \frac{N_c N_f}{16\pi^2} \tilde{m}_{n,s}^4 \left[ \frac{1}{\epsilon} - \frac{1}{2} \left[ -3 + 2\gamma - 4 \ln (2\sqrt{\pi}) \right] \right]
\]

(13)

which gives

\[
\Omega_{qq}^{vac} = \frac{N_c N_f}{16\pi^2} \tilde{m}_{n,s}^4 \left[ \frac{1}{\epsilon} - \frac{1}{2} \left[ -3 + 2\gamma - 4 \ln \left( \frac{\tilde{m}_{n,s}}{2\sqrt{\pi} \Lambda_T} \right) \right] \right]
\]

(14)

where \( N_c = 3 \) is the number of colors, \( \lim \epsilon \rightarrow 0 \) from dimensional reasoning, \( \gamma \) is the Euler-Mascheroni constant and \( \Lambda_T \) the renormalization scale parameter. The dimensionally regularized fermion vacuum contribution eventually reads

\[
\Omega_{qq}^{vac} = -\frac{N_c N_f}{8\pi^2} \tilde{m}_{n,s}^4 \ln \left( \frac{\tilde{m}_{n,s}}{\Lambda_T} \right)
\]

(15)

As in mean field approximation, the six model parameters \( \lambda_1, \lambda_2, m_0^2, c, h_n \) and \( h_s \) are fixed by six experimentally known values \([32, 62, 63]\). As an input the pion mass \( m_\pi = 136 \text{ MeV} \), the kaon mass \( m_k = 496 \text{ MeV} \), the pion decay constant \( f_\pi = 92.4 \text{ MeV} \), the kaon decay constant \( f_k = 113 \text{ MeV} \), the masses of the eta meson \( m_\eta = 548 \text{ MeV} \), the mass of the eta-prime meson
occurrences in the equations (18). Rearranging the entries of eq. (18) gives

\[
\Phi = \frac{1}{2} \begin{pmatrix}
\sqrt{2} \sigma_0 + \sigma_3 + \frac{\sqrt{6}}{\sqrt{3}} & \sigma_1 - i \sigma_2 & \sigma_4 - i \sigma_5 \\
\sigma_1 + i \sigma_2 & \sqrt{2} \sigma_0 - \sigma_3 + \frac{\sqrt{6}}{\sqrt{3}} & \sigma_6 - i \sigma_7 \\
\sigma_4 + i \sigma_5 & \sigma_6 + i \sigma_7 & \sqrt{2} \sigma_0 - \frac{\sqrt{6}}{\sqrt{3}}
\end{pmatrix}
\]

(18)

and the transformation from the physical nonstrange-strange basis to the mathematical basis reads

\[
\begin{pmatrix}
\sigma_n \\
\sigma_s
\end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix}
\sqrt{2} & 1 & -\sqrt{2} \\
1 & 1 & -1
\end{pmatrix}
\begin{pmatrix}
\sigma_0 \\
\sigma_8
\end{pmatrix}.
\]

(19)

Separating the entries for the scalar and the pseudoscalar sector gives the potential in terms of the mathematical fields (18). The mass matrix \(m_{ij}\) is determined only by the mesonic part and by the fermionic vacuum term of the potential \(V\), eq. (16), because the quark contribution vanishes at \(T = \mu = 0\). Because of isospin symmetry some entries of \(m_{ij}\) are degenerate and furthermore \(m_{08}^2 = m_{80}^2\), so that

\[
m^2_{ij} = \frac{\partial^2 V}{\partial \Phi_i \partial \Phi_j} = \begin{pmatrix}
m_{00}^2 & \ldots & m_{08}^2 \\
\vdots & \ddots & \vdots \\
m_{80}^2 & \ldots & m_{88}^2
\end{pmatrix}
\]

(20)

This matrix needs to be diagonalized for \(m_{00}^2\) and \(m_{08}^2\) in the scalar sector, and for \(m_{00}^2\) and \(m_{08}^2\) in the pseudoscalar sector introducing a mixing angle \(\theta\). Eventually the mass of the kaon in the nonstrange-strange basis reads

\[
m_k^2 = m_0^2 - \frac{\sqrt{2}}{2} \lambda_2 (\sigma_0^2 + \sigma_8^2) + \frac{\lambda_2}{2} (\sigma_0^2 - \sqrt{2} \sigma_0 \sigma_8 + 2 \sigma_8^2)
\]

(21)

which determines the axial anomaly term to be

\[
c = -2 (m_k^2 - m_0^2) - \lambda_2 (\sqrt{2} \sigma_0 \sigma_8 - 2 \sigma_8^2).
\]

(22)

Using eq. (20), the sum of \(m_0\) and \(m_0'\) reads

\[
m_0 + m_0' = 2m_0^2 + 2\lambda_1 (\sigma_0^2 + \sigma_8^2) + \frac{\lambda_2}{2} (\sigma_0^2 + 2 \sigma_8^2) + \frac{\sqrt{2}}{4} \sigma_8
\]

\[
= 2m_0^2 - \frac{\lambda_2}{2} (\sigma_0^2 - 2 \sigma_8^2) + \frac{3 \sqrt{2}}{4} \sigma_8
\]

(23)

and inserting eq. (22) into eq. (23) to solve for \(\lambda_2\) gives

\[
\lambda_2 = \frac{m_0^2 + m_0'^2 - 2m_0^2 + \frac{\sqrt{2}}{4} \sigma_8}{\sigma_0^2 - \frac{3 \sqrt{2}}{4} \sigma_8} = \frac{\sigma_0^2 - \frac{3 \sqrt{2}}{4} \sigma_0 \sigma_8}{\sqrt{3} (\sigma_0 - \sqrt{2} \sigma_8)}.
\]

(24)

The further procedure in the mean field approximation is to determine \(\lambda_1(m_0^2)\) via \(m_0\) and \(m_0\). This is explained in the next section. The quantities obtained so far enter into the two condensate equations, eqs. (17), for the explicit symmetry breaking terms \(h_n\) and \(h_s\). Working in the extended version of the model, the vacuum contributing part from eq. (16)
The electron contribution reads
grand canonical potential it can be treated separately.
It is interesting to note that the grand canonical potential

\[ \mu \text{scale parameter } \Lambda \]

remains unaffected by the choice of the renormalization
These two parameters alone compensate the

\[ \Omega^{\text{tot}} = V - \frac{N_c N_f}{8\pi^2} \tilde{m}_f^4 \ln \left( \frac{\tilde{m}_f}{\Lambda} \right) - \frac{3}{\pi^2} \beta \int_0^\infty k^2 dk \cdot N \]

and the vacuum parameter sets for different sigma meson
mass \( m_\sigma \) are listed in Tab.\[1\]. The equations of motion
\[ \frac{\partial \Omega^{\text{tot}}}{\partial \sigma_n} = \frac{\partial \Omega^{\text{tot}}}{\partial \omega} = \frac{\partial \Omega^{\text{tot}}}{\partial \rho} = \frac{\partial \Omega^{\text{tot}}}{\partial \phi} = 0, \]

also known as the gap-equations, finally determine the
EoS, \( p(\epsilon) \), with the pressure \( p \) and the energy density \( \epsilon \).

III. COMPACT STARS

The observation of the binary neutron star merger
event GW170817 is used to constrain the EoSs of compact stars \([40, 41, 42, 43]\). During the inspiral phase, one stars quadrupole deformation \( Q_{ij} \) in response to the companions perturbing tidal field \( \mathcal{E}_{ij} \) is measured by the tidal polarizability \( \lambda \)

\[ Q_{ij} = -\lambda \mathcal{E}_{ij}. \]

\( \lambda \) depends on the EoS \([52, 53, 72]\) and is related to the stars quadrupolar tidal Love number \([51]\) \( k_2 \) via

\[ k_2 = \frac{3}{2} \lambda R^{-5}, \]

where \( R \) is the radius of the star.
The Love number \( k_2 \) is calculated as follows

\[ k_2 = \frac{8C^5}{5} (1 - 2C)^2 [2 + 2C(y_R - 1) - y_R] \times \]

\[ \{ 2C[6 - 3y_R + 3C(5y_R - 8)] + 4C^3[13 - 11y_R + C(3y_R - 2) + 2C^2(1 + y_R)] + 3(1 - 2C)^2[2 - y_R + 2C(y_R - 1)] \ln(1 - 2C) \}^{-1}. \]

with the compactness \( C = M/R \). The quantity \( y_R \equiv y(R) \) on the other hand is obtained by solving the differential equation for \( y(r) \) coming from the line element

B. Charge neutrality and the Gap equations

Since the lepton contribution decouples from the quark
grand canonical potential it can be treated separately.
The electron contribution reads

\[ \Omega_e = \frac{2}{\beta} \int \frac{d^3k}{(2\pi)^3} \ln \left( 1 + e^{-\frac{E_{k,e} + \mu_e}{T}} \right) \]

with \( E_{k,e} = \sqrt{k^2 + m_e^2} \), where \( m_e \) is the electron mass and \( \mu_e \) is the electron chemical potential.

A compact star is charge neutral, so that

\[ \sum_{f=u,d,s,e} Q_f n_f = \frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_e = 0 \]
of the linearized metric, described in greater detail in 
\[ \frac{\partial y'(r)}{\partial y(r)^2} + r^2 Q(r) \]
\[ + y(r) e^{\lambda(r)} \left[ 1 + \frac{4\pi r^2 (p(r) - e(r))}{c_2^2(r)} \right] = 0, \]
with
\[ Q(r) = 4\pi e^{\lambda(r)} \left( 5e(r) + 9p(r) + \frac{e(r) + p(r)}{c_2^2(r)} \right), \]
\[ = 6\frac{e^{\lambda(r)}}{r^2} - (y'(r))^2, \]
the metric functions from general relativity
\[ y' = 2e^{\lambda(r)} \frac{m(r) + 4\pi r^3 p(r)}{r^2}, \]
and \( c_2(r)^2 = dp/de \) as the speed of sound squared. The boundary condition of eq. (43) is \( y(0)=2 \), which implies no deformation at all in the center of the star. At the surface of a selfbound star \( c_2 \to 0 \) and the denominator in eq. (44) blows up. The density discontinuity leads to an extra expression just below the surface of the star. One has to subtract
\[ y_s = \frac{4\pi r^3 e_s(r)}{m(r)}, \]
from eq. (43) with \( e_s(r) \) being the value of the energy density just below the surface. The dimensionless tidal deformability \( \Lambda \) is
\[ \Lambda = \frac{2k_2}{3c_5^5}, \]
usually solved simultaneously with the TOV equations. The actual value of a 1.4\( M_\odot \) star has to be in a range \( \Lambda = 300 \pm 230 \). In this section we present our results for varying sigma meson mass \( m_\sigma \), repulsive vector coupling \( g_\omega \) and different values of the Bag constant \( B^{1/4} \). At the end of this section we compare the results from the extended mean field approximation (eMFA) with the mean field approximation (MFA). The constituent quark mass is held fixed at \( m_q = 300 \text{ MeV} \), which is roughly one third of the nucleon mass. The standard parameter set is \( m_\sigma = 300 \text{ MeV} \), \( m_\omega = 600 \text{ MeV} \), \( g_\omega = 3.5 \) and \( B^{1/4} = 80 \text{ MeV} \), in the following denoted as reference set.

### IV. RESULTS

#### A. Variation of the sigma meson mass

Figure 6 shows the solutions of the scalar condensate equations, eqs. (39) as a function of quark chemical potential \( \mu_q \) for the scalar fields \( \sigma_n \) and \( \sigma_s \). For all our choices of \( m_\sigma \) a crossover transition is found. For larger values of \( m_\sigma \) the restoration of chiral symmetry happens at larger quark chemical potential \( \mu_q \) than for a lower mass of the sigma meson. A first order phase transition in the eMFA within this parameter choice is not found. Chiral symmetry is not entirely restored, because of the explicit breaking of chiral symmetry. 

| \( m_\sigma^{vac} \) | \( \Lambda \) | \( \lambda_1 \) | \( \lambda_2 \) | \( c'[\text{MeV}] \) | \( m^2[\text{MeV}^2] \) | \( h_n[\text{MeV}] \) | \( h_s[\text{MeV}] \) |
|----------------|------|------|------|-------|------|------|------|
| 400 | -5.901 | 46.488 | 4807.245 | (494.272)^2 | (120.73)^3 | (336.41)^2 |
| 500 | -2.698 | 46.488 | 4807.245 | (434.541)^2 | (120.73)^3 | (336.41)^2 |
| 600 | 1.398  | 46.488 | 4807.245 | (342.496)^2 | (120.73)^3 | (336.41)^2 |
| 700 | 6.615  | 46.488 | 4807.245 | (161.918)^2 | (120.73)^3 | (336.41)^2 |
| 800 | 13.488 | 46.488 | 4807.245 | (306.289)^2 | (120.73)^3 | (336.41)^2 |
| 900 | 23.649 | 46.488 | 4807.245 | (520.82)^2 | (120.73)^3 | (336.41)^2 |

TABLE I. The vacuum parameters \( \lambda_1, \lambda_2, c, m^2, h_n \) and \( h_s \) for different values of the sigma meson mass \( m_\sigma^{vac} \) in mean field approximation (MFA-upper table) and with the inclusion of the fermion vacuum term for the renormalization scale parameter \( \Lambda = 200 \text{ MeV} \) (eMFA-lower table).
FIG. 1. The solutions of the condensate equations $\sigma_n$ and $\sigma_s$ in the extended mean field approximation as a function of the quark chemical potential $\mu_q$ for different values of $m_\sigma$. The other parameters are $m_q = 300$ MeV, $g_\omega = 3.5$ and $B^{1/4} = 80$ MeV.

The corresponding speed of sound $c_s^2$ for the different choices of $m_\sigma$ can be seen in Fig. 3. The inlaid figure displays that the speed of sound approaches $c_s^2 = 0.5$ at large energy densities. The EoS for $m_\sigma = 400$ MeV generates the highest values of the speed of sound $c_s^2$ at a given energy density for values of the energy density $\epsilon \geq 220$ MeV/fm$^3$. This feature results from the stiffness of the EoS, see also Fig. 2. $c_s^2$ influences the solution of the differential equation, eq. (43), via the quantity $Q(r)$, eq. (44). Thereby $k_2$ and eventually the tidal deformability $\Lambda$, eq. (48), are influenced by the EoS.

The mass radius relations can be seen in fig. 4, where for our parameter choices $2M_\odot$ are possible [45–48].
dicated by the upper horizontal line. The lower horizontal line indicates $1.4 M_\odot$. For the lowest value of $m_\sigma = 400$ MeV chosen in this article, the mass radius relation is more compact than for the other choices of $m_\sigma$. This feature is different in the MFA [16] and results from the nontrivial behaviour of the EoSs discussed for Fig. 2. At $m_\sigma = 400$ MeV the radius of a $1.4 M_\odot$ star is $R_{1.4 M_\odot} = 13.14$ km, whereas for $m_\sigma = 600$ MeV $R_{1.4 M_\odot} = 15.27$ km and for $m_\sigma = 800$ MeV $R_{1.4 M_\odot} = 16.22$ km, see also Tab. III. With increasing $m_\sigma$ the configurations become less compact, but nonetheless has the mass radius relation for $m_\sigma = 600$ MeV the smallest maximum mass of $2.02 M_\odot$. This value is at roughly the same radius $R = 12.12$ km as the mass radius relation for $m_\sigma = 400$ MeV with a maximum value of $2.24 M_\odot$. The maximum mass for $m_\sigma = 800$ MeV is $2.2 M_\odot$ at a radius $R = 15.65$ km. It is however not seen in the MFA that the maximum masses for two different values of $m_\sigma = 400$ MeV and $m_\sigma = 800$ MeV are nearly equal at different radii. This peculiarity can be explained with the nontrivial behaviour of the EoSs for $p \leq 100$ MeV/fm$^3$, see inlaid figure in Fig. 2.

![Mass Radius Relation](image)

**FIG. 4.** The mass radius relation for different values of the sigma meson mass $m_\sigma$. For $m_\sigma = 400$ MeV the maximum mass is $2.24 M_\odot$ at $12.12$ km, for $m_\sigma = 600$ MeV $2 M_\odot$ are reached at the same radius, whereas for $m_\sigma = 800$ MeV the maximum mass is $2.2 M_\odot$ at $15.65$ km radius. These values are also listed in Tab. III. The other parameters are $m_\omega = 300$ MeV, $g_\omega = 3.5$ and $B^{1/4} = 80$ MeV.

![Radial Profile](image)

**FIG. 5.** The radial profile of the maximum mass star with $M = 2.02 M_\odot$ at $R = 12.12$ km (the black triangle) from the standard parameter set $m_\sigma = 300$ MeV, $m_\omega = 600$ MeV, $g_\omega = 3.5$ and $B^{1/4} = 80$ MeV. The left figure displays the nonstrange- and the strange $\sigma$ condensate as a function of the stars radius $R$. The right figure shows the pressure $p$ and energy density $\epsilon$ as a function of the stars radius $R$.

B. Comparison with the mean field approximation

The stars on the mass radius relation for a larger value of the repulsive coupling parameter $g_\omega$ have larger maximum masses and also larger radii, which is qualitatively known from the MFA case [14, 16], see also the corresponding values in Tab. III for the eMFA and Tab. III for the MFA.

The vacuum pressure constant $B^{1/4}$ drops out in the equations of motion, eqs. [39] and [40] respectively. Smaller values of $B^{1/4}$ have essentially the same effect on the mass radius relation as a larger repulsive coupling $g_\omega$: Maximum masses and radii become larger.
Figure 6 shows contour lines of maximum masses of $2M_\odot$ in the $m_\sigma$ vs. $B^{1/4}$ plane for fixed $2.5 \leq g_\omega \leq 3.5$. Smaller values of the repulsive coupling $g_\omega$ accompanied with a relatively small value of the vacuum pressure $B^{1/4}$ yields a maximum mass of $2M_\odot$ in the mass radius relation. This holds for the MFA (dashed lines) and for the eMFA (continuous lines).

The contour lines of the MFA and the eMFA seem somehow to be shifted vertically with a difference of $m_\sigma \simeq 150$ MeV for $g_\omega = 2.5$, $m_\sigma \simeq 100$ MeV for $g_\omega = 3.0$ and $m_\sigma \simeq 50$ MeV for $g_\omega = 3.5$. The difference in the shift in $m_\sigma$ becomes smaller for larger values of $g_\omega$. For a certain $m_\sigma \leq 620$ MeV, larger values of $B^{1/4}$ are allowed for $2M_\odot$ in the MFA compared to the eMFA, resulting in more compact mass radius relations in the MFA. Recall that smaller values of $B^{1/4}$ generate rather larger radii, so that denser stars yield smaller values of the tidal deformability parameter $\Lambda \propto C^{-5}$, see eq. (5). For at least $2M_\odot$ in the eMFA with values $m_\sigma \geq 620$ MeV a rather large value of $B^{1/4}$ is necessary. The mass radius configurations however turn out to have already a too large radius for a small tidal deformability parameter $\Lambda \leq 720$, see also Tab. III for the MFA and Tab. III for the eMFA.

Figure 7 shows contour lines of maximum masses of $2M_\odot$ for the MFA and the eMFA cases at fixed $g_\omega$ plane at fixed $50$ MeV $\leq B^{1/4} \leq 110$ MeV. In the MFA case (dotted lines) no repulsive coupling is necessary to generate two solar masses for low $m_\sigma$. The configurations in MFA are more compact compared to the eMFA case (continuous lines).

Interesting to note is, that in the MFA case no repulsive coupling is necessary to generate two solar masses for rather small $m_\sigma$, making the configurations more compact compared to the eMFA case. As already mentioned, more compact configurations are in favour of a low value of the tidal deformability parameter $\Lambda$.

Figure 8 shows contour lines of maximum masses of $2M_\odot$ in the $m_\sigma$ vs. $g_\omega$ plane at fixed $2.5 \leq g_\omega \leq 3.5$. The mass radius configurations however turn out to have already a too large radius for a small tidal deformability parameter $\Lambda \leq 720$, see also Tab. III for the MFA and Tab. III for the eMFA.
star. The shaded area fulfills both constraints on either \( R \leq 13.5 \text{ km} \) [46] and \( \Lambda \leq 720 \) [50]. The \( c_2^2 \) curve corresponds to the MIT Bag model EoS \( p = c_2^2 \epsilon - 4B \), and is obtained for different values of the vacuum pressure constant \( B \). Interesting to note is that the curves for the MIT Bag model for \( c_2^2 = 1 \) and \( c_2^2 = 1/3 \) are relatively close together. This feature indicates that a particular function \( \Lambda_{1.4M_\odot}(R_{1.4M_\odot}) \) is rather independent on the speed of sound \( c_2^2 \), so that \( c_2^2 \) in eq. (14) plays a subdominant part. The curve for \( c_2^2 = 1 \) can also be seen as an upper limit due to causality.

The MFA and the eMFA cases are obtained by varying \( m_\sigma \) in the standard parameter set. In the eMFA case the \( \Lambda_{1.4M_\odot} \) values and the radii decrease linearly on the logarithmic scale with decreasing \( m_\sigma \) (see also Tab. III). In the MFA case a minimum value \( \Lambda = 680 \) at 12.51 km radius for \( m_\sigma = 600 \text{ MeV} \) is found (see also Tab. III). Smaller and larger values of \( m_\sigma = 600 \text{ MeV} \) lead to larger values of either \( \Lambda_{1.4M_\odot} \) and the corresponding radius. This feature may be explained due to a shift in the dominance of attractive and repulsive field contributions to the stiffness of the EoS, and has already be suspected and discussed in [16]. The only parameter set which respects the \( 2M_\odot \) limit, the \( R_{1.4M_\odot} \leq 13.5 \text{ km} \) bound and the \( \Lambda_{1.4M_\odot} \leq 720 \) constraint, is the reference set in the MFA, and is hence located within the shaded area in fig. 8.

The inclusion of the fermion vacuum term in the SU(3) chiral quark meson model seems not to be compatible with astrophysical measurements and constraints.

To sort our results for the SU(3) quark meson model within other approaches, a Skyrme parameter approach taken from Zhou et al. [77] and an relativistic mean field (RMF) model studied by Nandi et al. [78] are included in fig. 8. The RMF values correspond to the fit function \( \Lambda_{1.4M_\odot} = 1.53 \times 10^{-5}(R_{1.4M_\odot}/\text{km})^{6.83} \) [78]. The values of these two approaches are located at smaller \( \Lambda_{1.4M_\odot} \) at a given radius and lie well within the shaded area. Furthermore, the results for free Fermi gas EoSs \( p = k_T^4/4n \) for various constants \( K \), with \( n = 1 \) and \( n = 3/2 \) are also shown. The values of \( \Lambda_{1.4M_\odot}(R_{1.4M_\odot}) \) for the interaction dominated EoS with \( n = 1 \) lie in between the results of the quark matter EoSs and the Skyrme- and RMF approach. The results of the nonrelativistic EoS for \( n = 3/2 \) may be seen as an lower limit for the function \( \Lambda_{1.4M_\odot}(R_{1.4M_\odot}) \) in Fig. 8. The values for \( \Lambda_{1.4M_\odot} \) at a given radius of the hadronic EoSs are located above the values of the nonrelativistic EoS.

In general it seems, that stars composed of quark matter have larger values of the tidal deformability parameter \( \Lambda \) for a \( 1.4M_\odot \) star at a given radius than stars generated by hadronic EoSs.

V. CONCLUSIONS

We have studied the SU(3) quark meson model including the fermion vacuum term and have determined the vacuum parameters for different values of the sigma meson mass \( m_\sigma \). The whole potential is independent on any renormalization scale [28, 31, 42, 43]. For all scalar meson masses in the eMFA a crossover transition in the condensates is found. In general it seems that in the MFA a first order phase transition is possible in a larger parameter space. For larger values of \( m_\sigma \) the restoration of chiral symmetry happens at larger quark chemical potential \( \mu_q \) [29], which is also observed for larger values of \( g_\omega \). The Bag constant \( B^{1/4} \) on the other hand does not affect the condensates at all, since it drops out in the equations of motion.

The resulting equations of state (EoSs) have been used to calculate mass radius relations of compact stars. These mass radius relations have to respect the \( 2M_\odot \) limit [45, 48] and the constraints coming from the analysis of the tidal deformabilities of the GW170817 neutron star merger event [50]. We compare our results from the extended mean field approximation (eMFA) with the mean field approximation (MFA). Our finding is that the \( 2M_\odot \) constraint can be fulfilled in both approaches in a rather wide parameter space, see e.g. [10], and that in the eMFA the mass radius relations are generally less compact resulting in larger values of the tidal deformability parameter \( \Lambda \). This feature however implies that in the eMFA the constraints from GW170817 are not fulfilled, i.e. \( \Lambda_{1.4M_\odot} \geq 720 \). In the MFA a small parameter space is found where all considered restrictions are satisfied. Within our parameter choice in the eMFA a smaller value of the sigma meson mass \( m_\sigma \) is favoured for a rather compact mass radius relation. This is in contrast to the MFA, see e.g. [16], and may be explained via the additional term in the potential resulting from the fermion vacuum contribution.

A large repulsive coupling constant \( g_\omega \) stiffens the EoS and hence enables the star to generate more pressure against gravity. The maximum mass and also the radius become consequently larger. These features may then result in larger values for the tidal deformability parameter \( \Lambda \propto C^{-5} \), \( C \) being the compactness. Incidentally, this holds vice versa, i.e. small repulsive couplings \( g_\omega \) imply rather small tidal deformabilities, but the \( 2M_\odot \) constraint may not be fulfilled.

The EoSs substantially soften when increasing the vacuum pressure constant \( B^{1/4} \) so that for values \( B^{1/4} \geq 110 \text{ MeV} \) maximum masses of \( 2M_\odot \) are difficult to obtain. Smaller values of \( B^{1/4} \) on the other hand have essentially the same effect on the mass radius relation as a larger repulsive coupling \( g_\omega \), i.e. maximum masses and radii become larger and consequently also the tidal deformability parameter \( \Lambda \).

To sort our results we compare our findings for the tidal deformability parameter \( \Lambda_{1.4M_\odot} \), \( \Lambda_{1.4M_\odot}(R_{1.4M_\odot}) \), with the results for a constant speed of sound (linear) EoS with \( c_2^2 = 1 \), where \( M \propto R^3 \) and which can be seen as an upper limit due to causality. As a lower limit we introduce a nonrelativistic polytropic EoS with the polytropic index \( \Gamma = 5/3 \) where \( M \propto R^{-3} \). The
case $\Gamma = 2$ corresponds to an interaction dominated EoS where $R \approx \text{const.}$, independent on the mass of the stars on the mass radius relation. In between these results we find the results of the MFA and eMFA cases relatively close to the constant speed of sound EoSs and the polytropic $n = 2$. For further comparison we also took a Skyrme parameter approach taken from Zhou et al.\cite{17} and a relativistic mean field (RMF) model studied by Nandi et al.\cite{18}. Their results are located slightly below the values of the polytropic $\Gamma = 2$ case and centrally in between the results from the constant speed of sound- and the nonrelativistic polytropic EoS.

In general it seems, that quark matter stars have larger values of the tidal deformability parameter $\Lambda$ at $1.4M_\odot$ than hadronic stars. Taking into account more interactions among quarks could lower the value of the tidal deformability parameter $\Lambda_{1.4M_\odot}(R_{1.4M_\odot})$, because interactions among quarks may decrease the value of $\Lambda_{1.4M_\odot}$ at a given radius, see Fig.\ref{fig:18}.

A first order chiral phase transition yielding twin stars as in the MFA, see e.g. \cite{19}, was not found in the eMFA. Compared to the mean field case, the EoSs show a smooth behaviour due to the additional term in the potential coming from the fermion vacuum term. The MFA and the eMFA, however, yield stars with $\geq 2M_\odot$ and radii $\leq 13.5$ km at $1.4M_\odot$, but only in the MFA the additional $\Lambda \leq 720$ limit was fulfilled for one set of parameters, i.e. for $m_\pi = 600$ MeV. This is due to the fact that the MFA approach yields more compact mass radius relations than the eMFA case. In the eMFA the $2M_\odot$ limit is fulfilled for the compactest mass radius configuration at $m_\pi = 400$ MeV with a radius of 13.14 km, but the value of $\Lambda_{1.4M_\odot}(R_{1.4M_\odot}) = 1107$ is not compatible with GW170817.

ACKNOWLEDGMENTS

The authors thank Jan-Erik Christian for fruitful discussions on the tidal deformability. J.S. and A.Z. acknowledge support from the Hessian LOEWE initiative through the Helmholtz International Center for FAIR (HIC for FAIR).

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TABLE II. eMFA: The values of the radius at 1.4\,M\odot, the tidal deformability parameter \Lambda_{1.4M\odot}, the maximum radius, maximum mass and the central pressure and central energy density for the maximum mass stars for the parameters \(m_\epsilon\), \(g_\omega\) and \(B^{1/4}\).

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
m_\epsilon [\text{MeV}] & g_\omega & B^{1/4} \text{[MeV]} & R_{1.4M\odot} \text{[km]} & A_{1.4M\odot} & R_{max} \text{[km]} & M_{max} \text{[M\odot]} & p_{max} \text{[MeV/fm}^3] & \epsilon_{max} \text{[MeV/fm}^3]\n\hline
400 & 3.5 & 80 & 13.14 & 1107 & 12.12 & 2.24 & 320.02 & 999.55 \\
600 & 3.5 & 80 & 15.27 & 3253 & 12.12 & 2.02 & 343.21 & 1148.82 \\
800 & 3.5 & 80 & 16.22 & 5184 & 15.65 & 2.20 & 88.62 & 537.67 \\
600 & 1 & 80 & 9.32 & 47 & 8.87 & 1.44 & 409.65 & 1829.86 \\
600 & 6 & 80 & 18.06 & 1079 & 17.15 & 2.82 & 128.35 & 509.77 \\
600 & 3.5 & 50 & 22.57 & 5250 & 22.80 & 2.61 & 23.35 & 221.13 \\
600 & 3.5 & 110 & 10.10 & 66 & 9.36 & 1.79 & 578.83 & 1658.17 \\
\hline
\end{array}
\]

TABLE III. MFA: The values of the radius at 1.4\,M\odot, the tidal deformability parameter \Lambda_{1.4M\odot}, the maximum radius, maximum mass and the central pressure and central energy density for the maximum mass stars for the parameters \(m_\epsilon\), \(g_\omega\) and \(B^{1/4}\).

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
m_\epsilon [\text{MeV}] & g_\omega & B^{1/4} \text{[MeV]} & R_{1.4M\odot} \text{[km]} & A_{1.4M\odot} & R_{max} \text{[km]} & M_{max} \text{[M\odot]} & p_{max} \text{[MeV/fm}^3] & \epsilon_{max} \text{[MeV/fm}^3]\n\hline
400 & 3.5 & 80 & 14.38 & 2275 & 15.28 & 2.70 & 142.70 & 568.15 \\
600 & 3.5 & 80 & 12.51 & 680 & 11.47 & 2.03 & 330.56 & 1143.22 \\
800 & 3.5 & 80 & 16.10 & 4858 & 15.88 & 2.09 & 64.78 & 480.79 \\
600 & 1 & 80 & 10.74 & 389 & 10.45 & 1.71 & 206.77 & 1091.38 \\
600 & 6 & 80 & 18.00 & 9815 & 14.86 & 2.58 & 251.90 & 761.90 \\
600 & 3.5 & 50 & 21.92 & 25300 & 14.5 & 2.19 & 228.14 & 878.17 \\
600 & 3.5 & 110 & 10.08 & 243 & 9.61 & 1.82 & 467.07 & 1493.87 \\
\hline
\end{array}
\]
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