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Analysis of Digital Competencies of 21st Century Teachers of Mathematics Education By Pentagonal Fuzzy Number And Some of Its Arithmetic Operations

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Abstract: Digital competencies is a collection of digital skills, categorized into five main areas of focus. They are designed as a tool for students to use to reflect on the digital skills and critical perspectives they develop while in school or college, in curricular and co-curricular experiences. The factors in digital competencies are digital survival skills, digital communication, data management and preservation, data analysis and presentation, critical making, design and development. In this paper we use pentagonal fuzzy number to find out the failure of digital competencies of 21st century teacher in mathematics education on the basis of the above criteria.

Keyword: Competencies, Pentagonal fuzzy number, Reliability, Fault tree.

INTRODUCTION
1.1 Digital Competencies

Each new stage in the development of society changes first of all the forms of learning. The school of the eighteenth to nineteenth centuries, the school of humanism of the Enlightenment, changed the attitude toward the child in accordance with the nature of their growing up, aimed learning on the study of nature. It was an era of great scientific research and geographical discoveries, travel, and knowledge of the evolution of life on earth. The school of the twentieth century, the school of the era of industrial society, became widespread and introduced into formal education scientific achievements, educational equipment, mathematics, as the language of science, and rationalism in cognition and design. It was a period of searching for new pedagogical methods, and as a result, the twentieth century became the century of universal literacy. The school of the twenty-first century, a digital school, made it possible to use the individualization of education in a global knowledge network that transformed the concept of school as a building where children study in classes. Network class, virtual network school, has become a reality. Digital forms of formal education in the open knowledge network have become part of a new educational environment, a smart education environment in which each student can build their own individual educational route outside formal education in accordance with their educational developmental interests. This new form of education is part of the social benefits of the digital age for everyone throughout their lives. The school of the twenty-first century is an open information resource of education for everyone; it is a school of universal education.
1.2 Fuzzy Sets and Number
In 1965, Lotfi A. Zadeh (Alefeld, G., and Herzberger, J., 1983), delivered new concept namely Fuzzy Sets theory. The theory of unsharp amounts has been applied with great success in many various fields. Chang and Zadeh introduced the concept of fuzzy numbers (Cheng, C.H. and Mon. D.L, 1993). Different mathematicians have been studying the theory (one-dimension or n-dimension fuzzy numbers, see for example Refs. (Cai. K.Y., Wen. C.Y. and Zhang. M.L, 1991; Cai. K.Y., Wen. C.Y. and Zhang. M.L; 1991). With the various improvement of theories and applications of fuzzy sets theory the topic become a topic of great interest.

![Figure 1 Flow Chart For Different Uncertain Parameters](image)

1.3 Pentagonal Fuzzy Number
Many researchers take pentagonal fuzzy number with different types of membership function. In this subsection we study on some published work which is associated with pentagonal fuzzy number:

**Table 1 Published Work**

| Author Information | Types of membership function | Main contribution | Application area |
|--------------------|-------------------------------|-------------------|------------------|
| Panda and pal      | Linear membership function with symmetry | Define arithmetic operation and exponent operation | Fuzzy matrix theory |
| Anitha and parvathi| Linear membership function | Find expected crisp value | Inventory control problem |
| Helen and Uma      | Linear membership function | Find the parametric from of pentagonal fuzzy number | Proof of all arithmetic operation using parametric from concept find the ranking of pentagonal fuzzy number |
1.4 Motivation
Fuzzy sets theory plays an important role in uncertainty modelling. Now the question is if we wish to take a fuzzy number then how its geometrical representations are. So, if decision maker takes a fuzzy number which can be graphically looks like a pentagon then how its membership function can be defined. From this point of view, we try to define different type of pentagonal fuzzy number which can be a better choice of a decision maker in different situation.

1.5 Novelties
There is various articles where pentagonal fuzzy sets and number are introduced and apply to different fields. But there are so many scopes to work on that topic. Some new interest and new work have done by our self which is mentioned below:

i. Try to utilise the properties of pentagonal fuzzy number to described the failure of digital competencies of 21st century teacher in mathematics education.
ii. Described the factors of digital competencies.
iii. Describe a numerical expression by pentagonal fuzzy number.
iv. We used all the allocation of number in the reliability system by pentagonal fuzzy number.
2. Preliminaries

Definition 2.1. Fuzzy Number: A fuzzy set \( \tilde{A} \), defined on the universal set of real number \( \mathbb{R} \), is said to be a fuzzy number if it possesses at least the following properties:

(i) \( \tilde{A} \) is convex.
(ii) \( \tilde{A} \) is normal i.e., \( \exists x_0 \in \mathbb{R} \) such that \( \mu_{\tilde{A}}(x_0) = 1 \)
(iii) \( \mu_{\tilde{A}}(x) \) is piecewise continuous.
(iv) \( A_\alpha \) must be closed interval for every \( \alpha \in [0,1] \).
(v) The support of \( \tilde{A} \), i.e., support (\( \tilde{A} \)) must be bounded.

3. Pentagonal fuzzy number and its variation:

In this section we develop different types of pentagonal fuzzy number in different viewpoint.

Definition 3.1. Pentagonal fuzzy number:

A pentagonal fuzzy number \( \tilde{A} = (a_1, a_2, a_3, a_4, a_5) \) should satisfy the following conditions:

1. \( \mu_{\tilde{A}}(x) \) is a continuous function in the interval \([0,1]\)
2. \( \mu_{\tilde{A}}(x) \) is strictly increasing and continuous function on \([a_1, a_2]\) and \([a_2, a_3]\)
3. \( \mu_{\tilde{A}}(x) \) is strictly decreasing and continuous function on \([a_3, a_4]\) and \([a_4, a_5]\)

Definition 3.2. Equality of two Pentagonal fuzzy number:

Two pentagonal fuzzy number \( \tilde{A} = (a_1, a_2, a_3, a_4, a_5) \) and \( \tilde{B} = (b_1, b_2, b_3, b_4, b_5) \) are equal if

\[
a_1 = b_1; \quad a_2 = b_2; \quad a_3 = b_3; \quad a_4 = b_4; \quad a_5 = b_5
\]

Now we try to define some new types of pentagonal fuzzy number in their different form.

3.1. Linear pentagonal fuzzy number with symmetry

Definition 3.3. Linear pentagonal fuzzy number with symmetry (LPFNS):

A linear pentagonal fuzzy number is written as \( \tilde{A}_{LS} = (a_1, a_2, a_3, a_4, a_5; r) \) whose membership function is written as

\[
\mu_{\tilde{A}_{LS}}(x) = \begin{cases} 
  r \frac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\
  1 - (1 - r) \frac{x - a_2}{a_3 - a_2} & \text{if } a_2 \leq x \leq a_3 \\
  1 & \text{if } x = a_3 \\
  1 - (1 - r) \frac{a_4 - x}{a_4 - a_3} & \text{if } a_3 \leq x \leq a_4 \\
  r \frac{a_5 - x}{a_5 - a_4} & \text{if } a_4 \leq x \leq a_5 \\
  0 & \text{if } x > a_5 
\end{cases}
\]
4. Arithmetic operation on Linear pentagonal fuzzy number with symmetry

i.e., $\tilde{A}_{LS} = (a_1, a_2, a_3, a_4, a_5; r)$

4.1. Multiplication by crisp number:

If k is a positive crisp number then $k \tilde{A}_{LS} = (ka_1, ka_2, ka_3, ka_4, ka_5; r)$ and k is a negative crisp number then $k \tilde{A}_{LS} = (ka_5, ka_4, ka_3, ka_2, ka_1; r)$

4.2. Addition of two pentagonal fuzzy numbers:

Consider two pentagonal fuzzy numbers $\tilde{A}_{LS} = (a_1, a_2, a_3, a_4, a_5; r_1)$ and $\tilde{B}_{LS} = (b_1, b_2, b_3, b_4, b_5; r_2)$ then the addition of the two numbers is given by

$C_{LS} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5; r)$ where $r = \min\{r_1, r_2\}$

4.3. Subtraction of two pentagonal fuzzy numbers:

Consider two pentagonal fuzzy numbers $\tilde{A}_{LS} = (a_1, a_2, a_3, a_4, a_5; r_1)$ and $\tilde{B}_{LS} = (b_1, b_2, b_3, b_4, b_5; r_2)$ then the subtraction of the two numbers is given by

$D_{LS} = (a_1 - b_5, a_2 - b_4, a_3 - b_3, a_4 - b_2, a_5 - b_1; r)$ where $r = \min\{r_1, r_2\}$

5. Representation of failure of digital competencies 21st century teacher in mathematics education depends on some factors:

Failure of digital competencies 21st century teacher in mathematics education depends on some factors. The facts are digital life competences, social-digital competences, professional and specialised digital competencies. There are two sub factors of each of facts.

- $F_{DC}$ represents the system failure of digital competencies.
- $F_{DL}$ represents the failure of digital competencies due to digital life competences.
- $F_{SDC}$ represents the failure of digital competencies due to social-digital competences.
- $F_{SPDC}$ represents the failure of digital competencies due to specialised digital competences.
- $F_{PD}$ represents the failure of digital competencies due to improper pedagogy.
- $F_{IB}$ represents the failure of digital competencies due to improper uses of board.
- $F_{E-G}$ represents the failure of digital competencies due to lack of E-gadgets.
- $F_{CN}$ represents the failure of digital competencies due to lack of connectivity.
- $F_{E-L}$ represents the failure of digital competencies due to lack of E-learning.
- $F_{TLM}$ represents the failure of digital competencies due to TLM.
The failure digital competencies 21\textsuperscript{st} century teacher in mathematics education can be calculated by pentagonal fuzzy number when the failures of the occurrence of basic fault events are known. Failure digital competencies 21\textsuperscript{st} century teacher in mathematics education can be evaluated by using the following steps.

\begin{itemize}
  \item [\textbf{Step 1.}]
  \begin{align*}
    F_{DLC} &= \Theta \left( F_{PD} \right) \Theta F_{IB} \\
    F_{S-DC} &= \Theta \left( F_{e-G} \right) \Theta F_{CN} \\
    F_{SpDC} &= \Theta \left( F_{e-L} \right) \Theta F_{TLM}
  \end{align*}
  \end{itemize}

\begin{itemize}
  \item [\textbf{Step 2.}]
  \begin{align*}
    F_{DC} &= \Theta \left( F_{DLC} \right) \Theta F_{S-DC} \Theta F_{SpDC}
  \end{align*}
  \end{itemize}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure3.png}
\caption{Fault-Tree of Failure of Digital Competencies}
\end{figure}

\textbf{6. Numerical Representation by Pentagonal Fuzzy Number:}

Numerical of failure of digital competencies using fault tree analysis with pentagonal fuzzy failure rate. The components failure rates as pentagonal fuzzy number are given by

\begin{align*}
  F_{hi} &= (0.01, 0.02, 0.03, 0.05, 0.06; 0.5) \\
  F_{lun} &= (0.02, 0.03, 0.04, 0.07, 0.08; 0.6) \\
  F_{vm} &= (0.02, 0.03, 0.04, 0.06, 0.07; 0.3) \\
  F_{ef} &= (0.02, 0.03, 0.04, 0.07, 0.08; 0.4) \\
  F_{veo} &= (0.04, 0.06, 0.07, 0.09, 0.1; 0.6) \\
  F_{hgf} &= (0.03, 0.04, 0.06, 0.08, 0.09; 0.5)
\end{align*}

Using (6.1.1) in the step-1 we have the following results.
\[ F_{hp} = (0.0298, 0.0494, 0.0688, 0.1165, 0.1352; 0.5), \]
\[ F_{vp} = (0.0396, 0.0591, 0.0784, 0.1258, 0.1444; 0.3), \]
\[ F_{vht} = (0.0688, 0.0976, 0.1258, 0.1628, 0.1810; 0.5). \]

using (6.1.2) in the second and final step, we get the calculated fuzzy failure of digital competencies represented by the following pentagonal fuzzy number
\[ F_{ftp} = (0.1323263895, 0.1928758071, 0.2497668751, 0.353383808, 0.3940031613; 0.3) \]

CONCLUSION
Different technological tools are need for the math students. Such as computer algebra system, dynamic geometry environments, interactive applets, handheld computation, data collection and idea to different devices. This system gives a student to explore, identify math concept perfectly and also relationships. But fuzzy system only gives the idea of uncertainty or where the data are insufficient. But we have to learn it to know the reality, because it is an obstruction in the modern e-learning world. We use fuzzy data so present the barrier of digital competencies of mathematics in reliability system. Our approaches and computational procedures may be efficient and simple to implement for calculation in a pentagonal fuzzy environment for all fields of engineering and science where impreciseness occur.

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