Quantum dynamics of quasicharge in an ultrahigh-impedance superconducting circuit

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Josephson effect is usually taken for granted because quantum fluctuations of the superconducting phase-difference are stabilized by the low-impedance embedding circuit. To realize the opposite regime, we shunt a weak Josephson junction with a nearly ideal kinetic inductance, whose microwave impedance largely exceeds the resistance quantum, reaching above 160 kΩ. Such an extraordinary value is achieved with an optimally designed Josephson junction chain released off the substrate to minimize the stray capacitance. The low-energy spectrum of the resulting free-standing superconducting loop spectacularly loses magnetic flux sensitivity, explained by replacing the junction with a 2e-periodic in charge capacitance. This long-predicted quantum non-linearity dramatically expands the superconducting electronics toolbox with applications to metrology and quantum information.

A Josephson junction between two superconductors can be viewed phenomenologically as a non-linear inductive circuit element (Fig. 1A). The voltage $V$ across the junction and its energy $E$ are given in terms of the superconducting phase-difference variable $\varphi$ according to $V = (\hbar/2e) \times \varphi$ and $E = -E_J \cos \varphi [1]$. These relations define an inductance that carries flux $\hbar/2e \times \varphi$ and has its energy oscillating with a period of the flux quantum $\hbar/2e$ (here $\hbar$ is the Planck constant; $\hbar = \hbar/2\pi$; $2e$ is the Cooper pair charge; and $E_J$ is the Josephson energy). Such a profound form of electrodynamic non-linearity has found numerous applications in classical electronics, such as the Josephson voltage standard [2]. Moreover, unlike the case of a diode, the Josephson non-linearity is fundamentally non-dissipative. This unique combination enables quantum-mechanical behavior of macroscopic electrical circuits [3, 4], currently a leading platform for quantum information science [5].

Paradoxically, one of the first macroscopic quantum effect predictions was that the Josephson effect should be completely destroyed by quantum fluctuations of the phase-difference [6]. This prediction comes from analyzing the low-energy behavior of a junction shunted by a capacitance (Fig. 1B). The circuit equations are analogous to those of an electron in a periodic potential: the variable $\hbar/2e \times \varphi$ is the position; the capacitance is the mass; the capacitor’s charge $Q$ is the momentum; and the Josephson energy corresponds to a periodic crystal field with a lattice constant $\hbar/2e$ (Fig. 1C). Leaving aside, for the moment, the influence of the external circuit connected to the junction, the quantum dynamics of the phase-difference could be described by Bloch waves with a spectrum of continuous Bloch bands. The energy $E_B(q)$ within the lowest band is determined by the circuit analog of quasimomentum — the quasicharge $q$ (Fig. 1C). Connecting a Cooper pair transfer to a Bragg reflection, the quasicharge must be the externally supplied charge. Therefore, the Josephson effect is gone in the sense that the circuit in Fig. 1B no longer responds inductively at low frequencies. Instead, it turns into a non-linear Bloch capacitance (the effective mass), defined by the 2e-periodic charging energy $E_B(q)$. Realization of this new kind of quantum non-linearity in an electrical circuit is what we report here.

In most experiments, however, the phase-difference $\varphi$ is either compact at the interval $(-\pi, \pi]$ or it is localized at a scale much smaller than $\pi$. The first scenario takes place for galvanically isolated junctions, e.g. Cooper pair box [7], where the conjugate variable is the discrete number of tunneled Cooper pairs. The periodic bound-

FIG. 1. (A) Josephson junction is a non-linear inductance with the energy that is a $2\pi$-periodic function of the phase-difference $\varphi$. (B) Junction shunted by a small capacitance: the charge at the capacitor is $Q$ and the total (quasi)charge supplied by the external circuit is $q$ (see text). (C) Quantum hoping of $\varphi$ in the periodic Josephson potential (blue) gives rise to continuous Bloch bands in the spectrum. The lowest band energy $E_B(q)$ is a 2e-periodic function of $q$ (magenta), i.e. the junction effectively becomes a non-linear capacitance.
FIG. 2. (A) Josephson junction with the Josephson energy $E_J$ and the charging energy $E_C$, shunted by an inductance $L$. The resulting superconducting loop is pierced by an external flux $\varphi_{\text{ext}} \times h/2e$. (B) Implementation of the circuit in (A) using a chain of Josephson junctions (dark blue) as a shunting inductance. Each junction in the chain has its own sufficiently large oxide capacitance to locally stabilize the Josephson effect. The stray capacitance between the oppositely-facing islands (red) is enhanced by the substrate permittivity. (C) Scanning electron micrographs of a fabricated circuit released from the silicon substrate to reduce the stray capacitance. Insets zoom in on the small junction elevated a few tens of microns above the substrate and on the section of the chain near the connection to the readout circuit. (D, E) Examples of released circuits with different degrees of curling. In all micrographs, the distance between the opposite leads of the chain is $10 \, \mu m$.

ary condition on Bloch waves restricts the quasicharge to a constant known as the offset charge $\bar{q}$ [5, 6]. In the second scenario, an external circuit dresses the junction by a sufficiently large capacitance such that the tunneling across the Josephson barrier is exponentially suppressed, which is the case in phase and transmon qubits [10, 11]. An extended phase-difference $\varphi$ has long been predicted for a junction connected to a resistance $R \gg R_Q$, where $R_Q = h/(2e)^2 \approx 6.5 \, \text{k}\Omega$ is the resistance quantum for Cooper pairs [12, 14]. Transport experiments on such devices found initial evidence of Bloch oscillations, where a dc current $I = \dot{q}$ fed through the Bloch capacitance generates an ac signal at the frequency $I/2e$ [15, 18]. However, circuits with explicitly resistive components proved unsuitable for observing fully-developed quantum effects, and the extended nature of the phase-difference remains a controversy to date [19].

In this experiment, we shunt the junction by a large-valued linear inductance, i.e. our device has a topology of a superconducting loop (Fig. 2A). The periodic boundary condition does not apply here because a translation of $\varphi$ by $2\pi$ changes the inductor’s energy. Yet, if this energy change is sufficiently small, the phase-difference is free to spread over multiple Josephson wells. The inductance $L$ must satisfy $L\omega \gg R_Q$, where the frequency $\omega$ is of the same order as the Bloch band width. For a smaller $L$ the loop reduces to a flux/fluxonium qubit, where tunneling is only possible at a half-integer flux frustration [20, 21]. Thus, the fundamental question of extended phase-difference is reduced in our approach to engineering the highest value inductance operating at typical microwave frequencies of Josephson circuits.

To maximize the inductance, we make it out of a Josephson junction chain (Fig. 2B), originally demonstrated with a fluxonium qubit [21]. With optimally chosen parameters, the phase-difference across chain junctions is locally stabilized by their oxide capacitances (Fig. 2B, blue circuit elements), allowing the linearized Josephson inductance per unit length to exceed the geometric one by a factor of $10^4$. Condensing inductance
any further would rapidly destroy the superconducting order via quantum phase-slips [22]. Alternatively, on increasing the chain length, one faces a more basic problem: stray capacitive coupling between the opposite parts of the chain (Fig. 2B, capacitors shown in red). This results in parasitic resonances at frequencies scaling inversely with the total inductance [23]. Therefore, going beyond the fluxonium benchmark $L \approx 0.3 \, \mu H$ requires a proportional reduction of the stray capacitance.

Coincidentally, the stray capacitance is unnecessarily large in superconducting circuits because of the high relative dielectric permittivity ($\epsilon \approx 10 - 12$) of silicon or sapphire substrates. We minimized this effect by releasing the entire circuit off the substrate (Fig. 2C-E). In a two-step process, devices with up to 460 chain junctions are first fabricated out of Al on Si using the standard Dolan bridge technique [24]. In the second step, a gentle burst of isotropic silicon etch is applied, with the oxidized Al film acting as a natural mask [25]. Because etching is more efficient underneath the skinner leads, the small junction end of the chain (labeled ‘1’ in Fig. 2C) detaches from the substrate prior to the other parts and immediately curls upwards by strain relaxation. This peculiar curling effect is robust and it is even possible to vary the amount of curling (Fig. 2D,E). Experiments were done on devices with a nearly vertically standing chain (Fig. 2C), where all parasitic capacitances are minimized.

The loop is inductively coupled to a readout circuit using the method described in Ref. [26]. The transition spectrum was obtained as a function of flux through the loop using a traditional two-tone rf-spectroscopy technique (Fig. 3A). To start, we compare the data (Fig. 3A, markers) to the spectrum of an effective three-element circuit Hamiltonian:

$$H_1 = 4EC(Q/2e)^2 + \frac{1}{2}EL\dot{\varphi}^2 - EJ\cos(\varphi - \varphi_{\text{ext}}), \quad (1)$$

where $E_J$ is the Josephson energy of the small junction, $EC = \epsilon^2/2C$ is the charging energy of the total capacitance $C$ across the junction, and $EL = (h/2e)^2/L$ is the inductive energy of the loop. The variables $\varphi$ and $Q$ obey the position-momentum type commutation relation $[\varphi, Q] = i \times 2e$. This model accurately fits the transitions to the first five excited states (Fig. 3, dashed lines), yielding $E_J/h = 4.70 \, \text{GHz}$, $EC/h = 7.07 \, \text{GHz}$, and $EL/h = 66.5 \, \text{MHz}$. The capacitance $C \approx 2.7 \, \text{fF}$ can be accounted for by the junction’s area and intrinsic oxide capacitance. The inductance $L \approx 2.5 \, \mu H$ exceeds that of a typical fluxonium by nearly an order of magnitude with no parasitic modes below 10 GHz.

The most striking feature of the measured spectrum is a rapid suppression of flux-sensitivity at low energies. It has no analogs with previously reported superconducting quantum interference devices, including fluxoniums. In fact, the data in Fig. 3 resembles spectroscopy of a transmon but with the offset charge axis replaced by $\varphi_{\text{ext}}$ [27].

This motivates a reinterpretation of the circuit in Fig. 2B as an inductance $L$ shunting a non-linear Bloch capacitance (Fig. 1B) that is characterized by the $2e$-periodic charging energy $E_B(q)$. Expressing the inductor’s energy as $Lq^2/2$, we get a Hamiltonian that is conceptually different from $H_1$:

$$H_2 = \frac{1}{2}EL(\varphi - \varphi_{\text{ext}})^2 + E_B(q). \quad (2)$$

By construction, the quasicharge $q$ is a compact variable at the interval $(-e, e)$. The conjugate momentum $\varphi$ is accordingly quantized in units of $2\pi$, such that $\exp(-i\pi q/e)\varphi \exp(i\pi q/e) = \varphi + 2\pi$. The external flux in this model couples to momentum like a gauge field. The function $E_B(q)$ can be obtained from values $E_J$ and $E_C$ extracted from the fit using Hamiltonian (1) [28].

If the higher harmonics of $E_B(q)$ are neglected in its Fourier expansion, Hamiltonian (2) matches the one of a Cooper pair box. The transmission-like regime of Hamiltonian (2) appears when $\pi^2 EL/h$ (about 650 MHz) is much smaller than the Bloch band width (about 6 GHz), i.e. the role of the transmon’s large capacitance is now played by the large inductance of the loop. This model was previously used to describe an insulating state of superconducting nanorings due to the proliferation of quantum phase-slips [29]. The higher harmonics of $E_B(q)$ generate
translations of $\varphi$ by $4\pi$, $6\pi$, etc. and from that perspective the Bloch capacitance should be seen as a generalization of the phase-slip junction circuit element [30].

Armed with the transmon analogy, we discuss the fate of the Josephson effect for the circuit parameters realized in our experiment (Fig. 4). Spectacularly, the loop in Fig. 2 becomes an insulator, since its ground state energy is nearly insensitive to $\varphi_{\text{ext}}$ (Fig. 4A). The quasicharge localizes near the bottom of the Bloch band (Fig. 4B), while the phase-difference spreads out such that the probability of $|\varphi| > \pi$ is significant (Fig. 4C). The transition to the first excited state corresponds to semi-classical oscillations of quasicharge near $q = 0$, nicknamed a "metaplasmom" [28]. Already the third excited state lies above the band edge. In this case, the quasicharge spreads over the entire interval $(-e, e)$ while the phase-difference localizes, and this is how the loop recovers a superconducting response at high energies. From the circuits perspective, the metaplasmom is simply an $LCB$-resonance, where $C_B = (d^2 E_B/dq^2)^{-1}$ is the linearized Bloch capacitance evaluated at $q = 0$. Curiously, for $E_J \sim E_C$ we get $C_B \approx C$ and hence the insulating behavior of the loop can be explained by literally removing the Josephson cross element from the circuit in Fig. 2B. The anharmonicity and flux-modulation of the excited states reflect the unusual non-linearity of the Bloch capacitance.

Towards higher frequencies the spectra of Hamiltonians (1) and (2) deviate from each other (Fig. 4A). Moreover, the high energy part of the experimental spectrum fits better to the spectrum of Hamiltonian (1). This is the only proof that the test device physically contains a Josephson junction. In the Bloch bands picture, the observed discrepancy stems from the influence of higher bands [28]. Thus, the remarkable transformation of Josephson inductance into a Bloch capacitance in a junction with $E_J \sim E_C$ manifests only if the quasicharge dynamics is sufficiently slowed down by a proper external circuit. The degree to which this was achieved in our high-inductance loop introduces a new regime of quantum fluctuations in superconductors. Namely, the phase is decompactified beyond the interval $(-\pi, \pi)$ but it remains localized on a larger scale (Fig. 4C). Simultaneously to this, the conjugate (quasi)charge variable is localized but not quantized (Fig. 4B), maintaining the circuit immunity to charge offsets. Besides its relevance for stabilizing Bloch oscillations in metrological standards of electrical current [31][32], this new regime is a foundation behind the proposed topologically protected superconducting qubits [34] and the grid-states autonomous quantum error correction [35][36]. More generally, the insensitivity of a metaplasmom to both charge and flux noise can be a useful resource for improving coherence and control of traditional superconducting qubits.

In closing, our experiment was made possible by a noteworthy development of its own: a linear superconducting inductance $L \approx 2.5$ $\mu$H operating up to $\omega/2\pi = 10$ GHz such that $L\omega > 160$ k$\Omega \approx 25R_Q$. This is close to the highest impedance an electrical circuit can possibly have in the broadband. Because the term superinductance is reserved for inductors satisfying a much weaker condition $L\omega \sim R_Q$, we colloquially call the circuit element realized here hyperinductance. The energy relaxation time $T_1 \approx 10 - 20$ $\mu$s measured for the metaplasmom translates to a hyperinductance quality factor $Q_L = \text{Re}[L]/\text{Im}[L] > 10^5$, which is promising for a broad range of hybrid quantum technology applications.

R.A.M. fabricated devices and performed measurements guided by I.V.P. I.V.P. analyzed the data and co-wrote the manuscript with V.E.M. L.B.N. and Y.-H.L. built the low-temperature microwave measurement setup. V.E.M. managed the project. All authors contributed to discussions of the results. We acknowledge funding from NSF-DMR, NSF PFC, and ARO HiPS.

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