Time-dependent Dalitz plot analysis of $B^0 \to D\mp K^0\pi^\pm$ decays.

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We present for the first time a measurement of the weak phase \(2\beta+\gamma\) obtained from a time-dependent Dalitz plot analysis of \(B^0 \rightarrow D^+ K^- \pi^\pm\) decays. Using a sample of approximately \(347 \times 10^6\) \(B\bar{B}\) pairs collected by the \(\text{BaBar}\) detector at the PEP-II asymmetric-energy storage rings, we obtain \(2\beta+\gamma = (83 \pm 53 \pm 20)^\circ\) and \((263 \pm 53 \pm 20)^\circ\) assuming the ratio \(r\) of the \(b \rightarrow u\) and \(b \rightarrow c\) decay amplitudes to be 0.3. The magnitudes and phases for the resonances associated with the \(b \rightarrow c\) transitions are also extracted from the fit.
The weak phase $\gamma \equiv \arg \left( -\frac{V_{cd} V_{cb}^*}{V_{ud} V_{ub}^*} \right)$, where $V_{ij}$ are elements of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix [1], is the least constrained angle of the unitarity triangle [2]. Over the past few years, several methods [3] have been employed to measure $\gamma$ directly in charged $B \to D^{(*)} K^{(*)}$ decays [4], where sensitivity to the weak phase arises from interference between the $b \to c$ (flavored) and $b \to u$ (suppressed) transitions. In addition, decays to two-body final states containing charm have been studied, such as $B^0 \to D^{(*)} \pi \pm$ and $B^0 \to D^\mp \rho^\pm$ [4], which are sensitive to the weak phase $2\beta + \gamma$ due to $B^0$-$\bar{B}^0$ mixing. The phase $\beta \equiv \arg \left( -\frac{V_{ub} V_{cd}^*}{V_{cd} V_{ub}^*} \right)$ is well measured in neutral $B$ decays to charmonium final states. The sensitivity of this method is limited by the ratio $r$ between the $b \to u$ and $b \to c$ transitions, which is expected to be very small ($\sim 0.02$). Three-body $B$ decays have been suggested [5] as a way to avoid this limitation, since $r$ in these decays could be as large as 0.4 in some regions of the Dalitz plot.

In this paper we report on the first measurement of the weak phase $2\beta + \gamma$ obtained from a time-dependent Dalitz plot analysis of the decay $B^0 \to D^{(*)} K^0 \pi^\pm$ [4] (charge conjugation is implied throughout the paper). In the decay $B^0 \to D^\mp K^0 \pi^\pm$ the three body final state is reached predominantly through intermediate $B^0 \to D^{(*)} K^0_S$ and $B^0 \to D^{(*)} K^0_{S\bar{S}}$ decays. In the first case, $D^{(*)} K^0_S \to K^0_S (2400)$ or a $D^{(*)} K^0_{S\bar{S}}$ state produced through $b \to u$ and $b \to c$ color-suppressed transitions. In the second case, $K^* (892), K^0_S (1430), K^0_S (1430)$ and $K^* (1680)$ are produced through $b \to c$ tree-level transitions. A small contribution from the $b \to u$ decay $B^0 \to D^{(*)}_s (2573) \pi^-$ is also expected.

Defining $\vec{x}$ as the vector of the two invariant masses squared $m^2 (K^0 \pi^\pm)$ and $m^2 (D^{\mp} \pi^\pm)$, the amplitude $A$ at each point $\vec{x}$ of the Dalitz plot can be parameterized as a coherent sum of two-body decay matrix elements according to the isobar model [4]:

$$A_{c(u)}(\vec{x}) e^{i\phi_{c(u)}(\vec{x})} = \sum_j a_j e^{i\delta_j} BW(\vec{x}; M_j, \Gamma_j, s_j),$$

where $c \ (u)$ indicates the $b \to c \ (b \to u)$ transition and $\phi$ is the total strong phase. Each resonance $j$ is parameterized by a magnitude $a_j$, a phase $\delta_j$, and a factor $BW(\vec{x}; M_j, \Gamma_j, s_j)$ giving the Lorentz invariant expression for the matrix element of the resonance as a function of the position $\vec{x}$, the spin $s$, the mass $M$, and the decay width $\Gamma$.

The time-dependent probability of a $B^0$ or $\bar{B}^0$ initial state to decay to a final state with a $D^+$ or $D^-$ can be expressed as:

$$P(\vec{x}, \Delta t, \xi, \eta) = \frac{A_c(\vec{x})^2 + A_u(\vec{x})^2}{2} \times e^{-\frac{i\phi_{c(u)}(\vec{x})}{4\tau_B}} \times (1 - \eta \xi C(\vec{x}) \cos(\Delta m_d \Delta t) + \xi S_\eta(\vec{x}) \sin(\Delta m_d \Delta t)).$$

Here:

$$S_\eta(\vec{x}) = \frac{2 \Im(A_c(\vec{x}) A_u(\vec{x}) e^{i(2\beta + \gamma) + \eta (\phi_c(\vec{x}) - \phi_u(\vec{x}))})}{A_c(\vec{x})^2 + A_u(\vec{x})^2},$$

$$C(\vec{x}) = \frac{A_c(\vec{x})^2 - A_u(\vec{x})^2}{A_c(\vec{x})^2 + A_u(\vec{x})^2},$$

where $\Delta t$ is the difference in proper decay times of the reconstructed meson $B_{rec}$ and the flavor-tagging meson $B_{tag}$, $\xi = +1(-1)$ if the flavor of the $B_{rec}$ is a $B^0 (\bar{B}^0)$ and $\eta = +1(-1)$ if the final state contains a $D^+ (D^-)$. We use the world averages for the $B^0$ lifetime $\tau_B$ and the mass-eigenstate difference $\Delta m_d$ [10].

Because Eq. (2) contains the terms $BW^j(\vec{x}, m, \Gamma, s)$, which vary over the Dalitz plot, we can fit the magnitudes $a_j$ and the phases $\delta_j$ of Eq. (1) to determine $2\beta + \gamma$ with only a two-fold ambiguity [7]. Most of the sensitivity to $2\beta + \gamma$ is expected to come from the interference between $b \to u$ and $b \to c$ transitions leading to $D^{*+} K^0_S$ final states (with expected $r \sim 0.4$), and from the interference of the former with the $b \to c$ transition of the decay $B^0 \to D^- K^{*+}$.

The analysis is based on $347 \times 10^6 B \bar{B}$ pairs collected at the T(4S) resonance by the BABAR detector at the PEP-II storage rings. The detector is described in detail elsewhere [11]. In order to estimate signal selection efficiencies and to study physics backgrounds, a Monte Carlo (MC) simulation based on GEANT4 [12] is used.

We reconstruct $D^+$ mesons in the decay mode $K^- \pi^+ \pi^+$. The tracks from $D^+$ decay are required to originate from a common vertex, and the kaon is selected using a likelihood based particle identification (PID) algorithm. The $D^+$ candidates are required to have a mass within $\pm 12$ MeV/$c^2$ (2$\sigma$) of the nominal $D^+$ mass [10], where $\sigma$ is the experimental resolution. Oppositely charged tracks from a common vertex are recognized as $K^0_L$ candidates if they have an invariant mass within $\pm 7$ MeV/$c^2$ (3$\sigma$) of the nominal $K^0_L$ mass [10] and a transverse flight-length significance $4\sigma$ greater than zero. The $\pi^-$ candidate is a track for which the PID is inconsistent with its being a kaon or an electron.

To form $B^0$ candidates, each $D^+$ candidate is combined with a $K^0_L$ candidate and a $\pi^-$ candidate requiring that the three particles originate from a common vertex. We reject $B^0$ candidates with $m^2 (K^0_L \pi^-)$ in the window [3.40, 3.95] GeV/$c^2$ in order to remove backgrounds with non-zero CP content arising from $B^0 \to D^{(*)} D^\mp$ decays. Using the beam energy in the $e^+e^-$ center-of-mass
(CM frame, two kinematic variables are constructed: the beam-energy substituted mass \( m_{ES} = \sqrt{s/4 - p_B^*} \), and the difference between the measured \( B^0 \) candidate energy and the beam energy, \( \Delta E = E_B^* - \sqrt{s}/2 \). Here \( p_B^* \) and \( E_B^* \) are the momentum and the energy of the \( B_{rec} \) in the CM frame respectively. Candidates with \( \Delta E \) in the range \([-0.1, 0.1]\) GeV and \( m_{ES} \) in the range \([5.24, 5.29]\) GeV/c\(^2\) are selected. We require \(|\cos \theta_B|\), the absolute value of the cosine of the angle between the \( B_{rec} \) momentum and the beam axis, be less than 0.85, and \(|\cos \theta_B^*|\), the absolute value of the cosine of the angle between the thrust axis of the \( B_{rec} \) decay products and the thrust axis of the rest of the event (ROE), be less than 0.95, both in the CM frame \([13]\).

The difference of proper-time \( \Delta t \) of the two \( B \)s in the event is calculated from the measured separation \( \Delta z \) between the vertices of the \( B_{rec} \) and the \( B_{tag} \) along the beam direction \([6]\). We accept events with calculated \( \Delta t \) uncertainty less than 2.5 ps and \( |\Delta t| < 20 \) ps. The average \( \Delta t \) resolution is approximately 1.1 ps. The flavor of the \( B_{tag} \) is identified from particles that do not belong to the \( B_{rec} \), using a multivariate algorithm \([3]\). The effective efficiency of the tagging algorithm, defined as \( Q = \sum_k \epsilon_k (1 - 2w_k)^2 \), is \((30.1 \pm 0.5)\%\), where \( \epsilon_k \) and \( w_k \) are the efficiency and the mistag probability, respectively, for each of the six tagging categories \( k \). Untagged events contribute to the determination of magnitudes and phases of the resonances and they are grouped in a separate seventh category corresponding to the case \( \xi = 0 \) in Eq. \([2]\) and containing about 38% of the events.

To further suppress the dominant continuum background, which have a more jet-like shape than \( B\bar{B} \) events, we use a linear combination \( \mathcal{F} \) of five variables: \( L_0 = \sum_i |p_i|, L_2 = \sum_i p_i |\cos \theta_i|^2 \), the global thrust of the event, \(|\cos \theta_F|\), and \(|\cos \theta_B|\). Here, \( p_i \) is the momentum and \( \theta_i \) is the angle, with respect to the thrust axis of the \( B_{rec} \), of the tracks and clusters of the ROE in the CM frame. The coefficients of \( \mathcal{F} \) are chosen to maximize the separation between the distributions obtained from Monte Carlo simulated signal events and 28 fb\(^{-1}\) of continuum events collected at a CM energy 40 MeV below that of the \( T(4S) \) resonance (off-resonance), whose energy is rescaled to the energy of the beams. The correlations among the set of measured values of the variables \( (m_{ES}, \Delta E, \mathcal{F}) \) are negligible. Since both \( \mathcal{F} \) and the flavor-tagging utilize the ROE information, the distribution of \( \mathcal{F} \) is correlated with the tagging category. To take into account this correlation, we parameterize the \( \mathcal{F} \) distribution for each tagging category separately.

Approximately 7% of selected events contain more than one reconstructed signal candidate, arising primarily from multiple \( D^+ \) candidates. We select the one having the \( D \)-candidate mass nearest to the nominal value \([10]\). For simulated signal events, the entire selection chain has an efficiency of \((9.9 \pm 0.1)\%\), where the error is statistical only.

To separate signal from background and to determine their yields, we first perform an unbinned extended maximum likelihood fit to the selected on-resonance data sample in the variables \( m_{ES}, \Delta E, \) and \( \mathcal{F} \). The role of this first step fit is to extract all the shape parameters, the fractions of events by tagging category, and the overall yields, which will then be fixed in the subsequent time-dependent fit to the Dalitz plot. We define the logarithm of the likelihood:

\[
\ln \mathcal{L} = \sum_{k=1}^{7} \left( \sum_{j=1}^{N_{tot}} \ln \left( \sum_{i=1}^{5} N_{jk} Y_{jk}^i \right) - \sum_{i=1}^{5} N_{jk} \right),
\]

where \( Y_{jk}^i \) is the product of the PDFs of \( m_{ES}, \Delta E, \) and \( \mathcal{F}_k \) for the event \( i \) in the tagging category \( k \). \( N_{tot} \) is the total number of events and \( N_{jk} \) is the number of events of each sample component \( j \): signal (Sig), continuum (Cont), combinatoric \( B\bar{B} \) decays (\( B\bar{B} \)) and \( B\bar{B} \) events that peak in \( m_{ES} \) but not in the \( \Delta E \) signal region (Peak).

The signal is described by a Gaussian function for the \( m_{ES} \) distribution, two Gaussian functions with common mean for the \( \Delta E \) distribution, and a Gaussian function with different widths on each side of the mean \( \text{("bifurcated Gaussian function") for the} \mathcal{F} \text{ distribution. The signal model parameters are obtained from a fit to a high-statistics data control sample of} B^0 \rightarrow D^+ a_1^\pm \text{ decays. The selection of these events is similar to signal, except that no} K_S^0 \text{ candidate is required. The decay chain} a_1^+ \rightarrow \rho^0 \pi^\pm \text{ with} \rho^0 \rightarrow \pi^\pm \pi^\mp \text{ is reconstructed requiring the dipion invariant mass be within} \pm 150 \text{ MeV/c}^2 \text{ of the nominal} \rho^0 \text{ mass} \([10]\), and the invariant mass of the} \rho \text{ candidate with the third pion be within} \pm 250 \text{ MeV/c}^2 \text{ of the nominal} a_1^+ \text{ mass} \([10]\).

The \( m_{ES} \) distributions of the continuum and combinatoric \( B\bar{B} \) backgrounds are described by empirical threshold functions \([14]\), while for \( \Delta E \) distributions linear functions are used. The \( \mathcal{F} \) distributions are parameterized by a bifurcated Gaussian function for the continuum background and a sum of two Gaussian functions for the \( B\bar{B} \) combinatoric background. For the latter the parameters are determined by \( B\bar{B} \) Monte Carlo simulation. All the shape parameters of the continuum background are taken from fitting the off-resonance data. The \( m_{ES} \) distribution of the Peak background is parameterized by a Gaussian function with the same mean as the signal and a width fixed to the value obtained from Monte Carlo simulation. The \( \Delta E \) distribution is described by an exponential function. The \( \mathcal{F} \) distribution of Peak is described using the same PDF as for \( B\bar{B} \) background.

The yields and the fraction of events for each tagging category are fitted together with the free shape parameters. The yields obtained for each component are \( N_{Sig} = 558 \pm 34, N_{Cont} = 13222 \pm 226, N_{background} = 5647 \pm 213 \) and \( N_{peak} = 183 \pm 41 \), in agreement with the previous
composed of three Gaussian distributions. Incorrectly
tagged events are described by convolving Eq. 2 with a resolution function
together with the magnitude values obtained in Ref. \cite{6}. The $D_{s}^{+}(2573)$
magnitude and phase are fixed to the values given in \cite{5}. Despite the fact that the $b \rightarrow u$ phases cannot be precisely
determined they are left free in the fit. All the $b \rightarrow c$ magnitudes and phases together with $2\beta + \gamma$ are free parameters. The whole fitting procedure has been validated using high statistic parameterized (toy) Monte
Carlo samples.

The fit is performed on events satisfying $m_{ES} > 5.27$ GeV/$c^2$, $|\Delta E| < 50$ MeV and $\mathcal{F} > -2$. Results are shown in Table II. Figure 2 shows the projections of the on-resonance data sample on the two Dalitz plot variables $m^2(K_{S}^{0}\pi^{\pm})$ and $m^2(D^{\pm}\pi^{\mp})$ with the fitted PDFs superimposed. Figure 2 shows the $m_{ES}$ distribution and the fitted PDFs for each component, after applying additional requirements on $\Delta E$ and $\mathcal{F}$. Besides the value of $2\beta + \gamma$, an important outcome of the analysis is the fit of the resonance contributions to the $b \rightarrow c$ part of the Dalitz plot. Biases related to the small sample size are observed in the measurement of the magnitudes. They are estimated using a large number of toy experiments generated with the magnitudes values obtained in the fit to the on-resonance data sample.

The expression for the time-dependent Dalitz plot likelihood function is then:

$$
\ln L = \sum_{k=1}^{7} \sum_{\mathcal{B}^{0}_{tag}} \ln L_{+,k} + \sum_{\mathcal{B}^{0}_{tag}} \ln L_{-,k},
$$

The likelihood function $L_{+,k}$ ($L_{-,k}$) for an event in the tagging category $k$ with $B_{tag} = B^{0}$ ($B_{tag} = B^{0}$) is:

$$
L_{+,k} = N_{\text{Sig}}^{k} \mathcal{P}_{\text{Sig}}^{k} V_{\text{ES}}^{k} + N_{\text{Bkgd}}^{k} \mathcal{D}_{\text{Bkgd}}^{k} T_{\text{Bkgd}}^{k} B_{\text{Bkgd}}^{k} T_{\text{Bkgd}}^{k} V_{\text{ES}}^{k}
$$

Here $Y$ indicates the product of PDFs for $m_{ES}$, $\Delta E$, and $\mathcal{F}$, and $\mathcal{P}_{\text{Sig}}$ is the time-dependent Dalitz plot PDF for

The systematic errors are summarized in Table II. The main contribution is related to the parameterization of the background Dalitz plot. This effect has been estimated by repeating the fit with a parameterization obtained from off-resonance data and $BB$ generic Monte Carlo simulation. The systematic uncertainty due to the
efficiency variation over the Dalitz plot has been evaluated assuming a flat efficiency. The effect of potential CP content of the $\bar{B}B$ peaking background is taken into account assuming the same CP violation structure as in the signal with a value $r_{\text{eff}} = 0.4$. The systematic uncertainties on the signal Dalitz plot come from the variation of the $r$ factor ($0.3 \pm 0.1$), of the $D_{s+}^{(2573)}$ magnitude ($0.02 \pm 0.01$) and from the introduction of up to 7% of a non resonant component. In addition, the masses and widths of the resonances have been varied by one standard deviation [10]. We obtain the systematic uncertainty arising from imperfect knowledge of the $Y$ shape parameters and the yields by varying all fixed parameters within their uncertainties. Similar variations are applied to the signal and background fractions in each tagging category as well as for the $\Delta t$ resolution parameters, the effective lifetimes, the B lifetime and the mixing frequency. The systematic uncertainties due to the dependence of the tagging efficiency on the $B$ flavor, the beam spot position and the SVT alignment have been obtained following the procedure described in [6].

Figure 2 shows the dependence of the measurement of $2\beta + \gamma$ on $r$. For each fixed value of $r$, a point in the plot represents the result of the fit on $2\beta + \gamma$ with its statistical error. The error decreases, as expected, for increasing $r$ and the central value remains stable. The projection on $2\beta + \gamma$ of the negative logarithm of the likelihood in Figure 2 clearly shows the minimum corresponding to the result of the fit and the expected mirror solution at $+\pi$ rad. Having fixed some magnitudes and strong phases, the second solution is disfavored, but it should be regarded as equivalent.

In summary, we present the first results of a time-dependent Dalitz plot analysis of the decay $B^0 \rightarrow D^{\pm} K^0 \pi^\pm$ to determine the Dalitz plot model parameters and the weak phase $2\beta + \gamma$. Assuming $r = 0.3$ we find $2\beta + \gamma = (83 \pm 53 \pm 20)^\circ$ and $(263 \pm 53 \pm 20)^\circ$, where the first error is statistical and the second is systematic.

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### TABLE I: Results for the $b \rightarrow c$ transitions magnitudes and phases and for $2\beta + \gamma$ assuming $r = 0.3$. The first uncertainty is statistical, the second is systematic.

| Resonance  | Bias correction for the magnitude | $V_{cb}$ magnitude after bias correction | Phase ($\circ$) |
|------------|-----------------------------------|-----------------------------------------|----------------|
| $K^*(892)$ | $0.001$                           | $0.290 \pm 0.048 \pm 0.067$             | $267 \pm 22 \pm 35$ |
| $D_0^*(2400)$ | $+0.003$                           | $0.042 \pm 0.050 \pm 0.048$             | $325 \pm 46 \pm 20$ |
| $K^0(1430)$ | $-0.033$                           | $0.135 \pm 0.056 \pm 0.099$             | $284 \pm 30 \pm 11$ |
| $K^*_{2}(1430)$ | $-0.025$                           | $0.108 \pm 0.056 \pm 0.051$             | $221 \pm 30 \pm 14$ |
| $K^*(1680)$ | $-0.017$                           | $0.404 \pm 0.047 \pm 0.046$             | $128 \pm 22 \pm 24$ |

$2\beta + \gamma$ assumed: $(83 \pm 53 \pm 20)$ and $(263 \pm 53 \pm 20)$

### TABLE II: Sources and sizes of systematic errors.

| Systematic         | $2\beta + \gamma$ | $D_0^*(2400)$ | $D_2^*(2460)$ | $K^0(1430)$ | $K^*_{2}(1430)$ | $K^*(1680)$ |
|--------------------|-------------------|---------------|---------------|-------------|----------------|-------------|
| Bkgd Dalitz plot param. |                   |               |               |             |                 |             |
| Eff. over the Dalitz plot |                   |               |               |             |                 |             |
| CP content of bkgd   |                   |               |               |             |                 |             |
| $r$                 |                   |               |               |             |                 |             |
| $\alpha(D_0^+(2573))$ |                   |               |               |             |                 |             |
| $m, \Gamma$         |                   |               |               |             |                 |             |
| $Y$ PDF param.       |                   |               |               |             |                 |             |
| Signal and bkgd frac.|                   |               |               |             |                 |             |
| Yields              |                   |               |               |             |                 |             |
| Tagging and time param. |                 |               |               |             |                 |             |

$\pm$ 2

1. $\pm$ is the statistical uncertainty.
2. $\pm$ is the total uncertainty.
3. The first uncertainty is statistical, the second is systematic.
4. For the magnitude of $2\beta + \gamma$, the first uncertainty is statistical, the second is systematic.
5. The first uncertainty is statistical, the second is systematic.
6. The first uncertainty is statistical, the second is systematic.
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