The weak decays of $\Xi_c^{(')} \rightarrow \Xi$ in the light-front quark model

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Abstract

Without contamination from the final state interactions, the calculation of the branching ratios of semileptonic decays $\Xi_c^{(')} \rightarrow \Xi + e^+ \nu_e$ may provide us more information about the inner structure of charmed baryons. Moreover, by studying those processes, one can better determine the form factors of $\Xi_c \rightarrow \Xi$ which can be further applied to relevant estimates. In this work, we use the light-front quark model to carry out the computations where the three-body vertex functions for $\Xi_c$ and $\Xi$ are employed. To fit the new data of the Belle II, we re-adjust the model parameters and obtain $\beta_{s[\bar{s}q]} = 1.07$ GeV which is 2.9 times larger than $\beta_{s\bar{s}} = 0.366$ GeV. This value may imply that the $ss$ pair in $\Xi$ constitutes a more compact subsystem. Furthermore, we also investigate the non-leptonic decays of $\Xi_c^{(')} \rightarrow \Xi$ which will be experimentally measured soon, so our model would be tested by consistency with the new data.

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I. INTRODUCTION

Recently the Belle Collaboration measured the branching fraction of the semi-leptonic decay of the charmed baryon $\Xi_c$ as $B(\Xi^0_c \to \Xi^- e^+ \nu_e) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50)\%$[1] where the first and second uncertainties are statistical and systematic whereas the third one arises from the uncertainty of $B(\Xi^{0}_c \to \Xi^{-} \pi^{+}) = (1.80 \pm 0.52)\%$ measured also by the Belle collaboration[2]. In 1993 the ARGUS Collaboration[3] first observed the decay $\Xi^0_c \to \Xi^- e^+ \nu_e$ and in 1995 the CLEO Collaboration[4] also found $\Xi^0_c \to \Xi^- e^+ \nu_e$ events and the ratio $B(\Xi^0_c \to \Xi^- e^+ \nu_e)/B(\Xi^{0}_c \to \Xi^{-} \pi^{+})$ has been measured to be $0.96 \pm 0.43 \pm 0.18$ by the ARGUS and $3.1 \pm 1.0^{+0.3}_{-0.5}$ by the CLEO, respectively. With the data $B(\Xi^0_c \to \Xi^- e^+ \nu_e)$ is more precise at present. Recently a lattice calculation on $B(\Xi^0_c \to \Xi^- e^+ \nu_e)$ has been finished in Ref.[5] and the result is $(2.29 \pm 0.29 \pm 0.31)\%$. Some theoretical predictions based on different phenomenological models are given as $(3.4 \pm 0.7)\%$[7], $(3.49 \pm 0.95)\%$[8], $(7.26 \pm 2.54)\%$[9], $1.35\%$[10], $(4.87 \pm 1.74)\%$[11], $(2.4 \pm 0.3)\%$[12], and $2.38\%$[13]. Definitely the precision of theoretical evaluations should be further improved.

In this work we will employ the light-front quark model to study the weak decays of $\Xi^0_c \to \Xi$. The light-front quark model (LFQM) is a relativistic quark model which has been applied to study transitions among mesons and the relevant theoretical predictions with this model agree with the data within reasonable error tolerance [14–28]. Later the model was extended to study the decays of pentaquark within the diquark-diquark-antiquark picture[29,30] and the weak decay of baryons in the quark-diquark picture[31–37]. With this model we employed the three-body vertex function to explore the decays of $\Lambda_b$ and $\Sigma_b$[38] where three individual quarks are concerned, the related picture is somewhat different from the structure where one-diquark and one-quark stand as basic constituents. Geng et. al.[8] also studied the decay of baryons under the three-quark picture in the light front quark model where they took different approaches from ours to deal with the flavor-spin wave function of baryons. It is noted that in the three-body structure, even though two quarks might be loosely bound as a subsystem with definite spin and color, unlike the diquark which is a relatively stable subject, the subsystem may break and its components would undergo a re-combination with other quarks to constitute different sub-systems, especially after a hadronic transition.

In the three-body vertex function of baryon two quarks join together into a subsystem which has a definite spin and then the subsystem couples with the rest quark to form a baryon with the defined spin. In order to evaluate the transition between two baryons we first need to know the inner spin structures of the concerned baryons. For single charmed-baryons $\Xi_c$ and $\Xi'_c$, the two light quarks $sq$ where $q$ denotes $u$ or $d$ quark can be regarded as a subsystem with definite spin (in analog to a diquark)[39,40]. By contrast, there are three light quarks in $\Xi$ where two $s$ quarks possess definite spin 1 due to the antisymmetry of total wave function, so that can be regarded as a subsystem. In the transition of $\Xi^{0}_c \to \Xi$ the $c$ quark in the initial state would transit into an $s$ quark by emitting a gauge boson.
and the original $sq$ pair can be regarded as a spectator which does not undergo any changes during the hadronic transition. However in the final state the newly emerged quark $s$ would couple to the $s$ quark which is the decay product of the initial $c$ quark to form a physical $ss$ subsystem with definite spin, thus the $sq$ from initial state is no longer a physical subsystem. Namely the picture is that the old subsystem $sq$ is broken and a new subsystem $ss$ emerges during the hadronic transition. Definitely, we would account the changes of constituents in the subsystems as caused by a non-perturbative QCD effect. Anyhow, the $ss$ subsystem isn’t a spectator because one of the $s$ quarks originates from weak decay of the initial $c$ quark. Therefore the simple quark-diquark picture does not apply to these processes. In order to use the spectator proximation, one needs to rearrange the $(ss)-(q)$ structure into the $s-(sq)$ structure by a Racah transformation.

This paper is organized as follows: after the introduction, in section II we deduce the transition amplitude for $\Xi^c(1) \to \Xi$ in the light-front quark model and provide the expressions of the form factors, then we present our numerical results for $\Xi^c(1) \to \Xi$ in section III. Section IV is devoted to the conclusion and discussions.

II. $\Xi_c \to \Xi$ AND $\Xi'_c \to \Xi$ IN THE LIGHT-FRONT QUARK MODEL

A. The vertex functions of $\Xi_c$, $\Xi'_c$ and $\Xi$

Enlightened by Ref.[41] in our previous work[38] we constructed the vertex functions of baryons under the three-quark picture where two quarks constitute a subsystem with definite spin, and then the subsystem couples with the rest quark to form a baryon. We have employed the vertex functions to study the decays of $\Lambda_b$ and $\Sigma_b$[38]. Later we employed the three-quark vertex functions to study the transition $\Xi_{cc} \to \Xi_c$[42]. The success encourages us to apply this picture to further study relevant processes.

In analog to Refs.[38] the vertex functions of $\Xi_c$, $\Xi'_c$ and $\Xi$ with total spin $S = 1/2$ and momentum $P$ are

$$\begin{align*}
|\Xi^c(1)(P,S,S_z)\rangle &= \int \{d^3\bar{p}_1\} \{d^3\bar{p}_2\} \{d^3\bar{p}_3\} 2(2\pi)^3\delta^3(\bar{P} - \bar{p}_1 - \bar{p}_2 - \bar{p}_3) \\
&\times \sum_{\lambda_1,\lambda_2,\lambda_3} \Psi_{S\xi}^{S\xi}(\bar{p}_1, \lambda_1)\mathcal{C}^{\alpha\beta\gamma}\mathcal{F}_{csq} \langle c_\alpha(p_1, \lambda_1)s_\beta(p_2, \lambda_2)q_\gamma(p_3, \lambda_3)\rangle, \\
|\Xi(P,S,S_z)\rangle &= \int \{d^3\bar{p}_1\} \{d^3\bar{p}_2\} \{d^3\bar{p}_3\} 2(2\pi)^3\delta^3(\bar{P} - \bar{p}_1 - \bar{p}_2 - \bar{p}_3) \\
&\times \sum_{\lambda_1,\lambda_2,\lambda_3} \Psi_{\Xi}^{SS}(\bar{p}_1, \lambda_1)\mathcal{C}^{\alpha\beta\gamma}\mathcal{F}_{ssu} \langle s_\alpha(p_1, \lambda_1)s_\beta(p_2, \lambda_2)q_\gamma(p_3, \lambda_3)\rangle.
\end{align*}$$

(1) (2)

Let us repeat some details about the three-body picture because it plays the key role in our calculations. In order to obtain the expression of $\Psi_{\Xi}^{SS}$ and $\Psi_{\Xi}^{S\xi}$ one needs to know their inner spin-flavor structure. In Ref.[40] the $sq$ in $\Xi_c$ is considered as a scalar subsystem whereas in $\Xi'_c$ it is a vector. In the decay process $\Xi^c(1) \to \Xi$ the $c$ quark transits into an $s$ quark via weak interaction and the original $sq$ subsystem can be approximately regarded as a spectator because it does not undergo any change during the hadronic transition. However
the two strange quarks in $\Xi$ compose a physical subsystem whose spin is 1. To apply the spectator approximation for the transition, we rearrange the quark structure of $(ss)$-$q$ into a sum of $\sum_i s-(sq)_i$ where $i$ runs over all possible spin projections by a Racah transformation. With the rearrangement of quark spin-flavor the physical structure $(ss)$-$q$ in $\Xi$ is rewritten into a sum over the effective structures of $s-(sq)$. The detailed transformations are

$$[s^1 s^2]_1[q] = -\frac{\sqrt{3}}{2}[s^1][s^2]_0 + \frac{1}{2}[s^1][s^2]_1,$$

and then

$$\psi_{\Xi}^{ss}(\bar{p}_i, \lambda_i) = -\frac{\sqrt{3}}{2} \psi_0^{ss}(\bar{p}_i, \lambda_i) + \frac{1}{2} \psi_1^{ss}(\bar{p}_i, \lambda_i),$$

with

$$\psi_0^{ss}(\bar{p}_i, \lambda_i) = A_0 \bar{U}(p_3, \lambda_3)[(\bar{P} + M_0)\gamma_5]V(p_2, \lambda_2)\bar{U}(p_1, \lambda_1)U(\bar{P}, S)\varphi(x_i, k_{i\perp}),$$

$$A_0 = \frac{1}{4\sqrt{P + M_0^2}(m_1 + e_1)(m_2 + e_2)(m_3 + e_3)},$$

$$\psi_1^{ss}(\bar{p}_i, \lambda_i) = A_1 \bar{U}(p_3, \lambda_3)[(\bar{P} + M_0)\gamma_\alpha\gamma_5]V(p_2, \lambda_2)\bar{U}(p_1, \lambda_1)\gamma_{\perp\alpha}U(\bar{P}, S)\varphi(x_i, k_{i\perp}),$$

$$A_1 = \frac{1}{4\sqrt{3P + M_0^2}(m_1 + e_1)(m_2 + e_2)(m_3 + e_3)},$$

where $p_1$ is the momentum of the newly emerged $s$ quark in the transition, $p_2, p_3$ are the momenta of the spectator quarks $s$ and $q$, $U$ and $V$ are spinors and $\lambda_1, \lambda_2, \lambda_3$ are the helicities of the constituents. Since the spin of $sq$ subsystem is 0 (1), the expressions of $\psi_{\Xi}^{ss}$ ($\psi_{\Xi'}^{ss}$) is the same as $\psi_0^{ss}$ ($\psi_1^{ss}$) except that $p_1$ is the momentum of the $c$ quark.

The spatial wave function is

$$\varphi(x_i, k_{i\perp}) = \frac{e_1 e_2 e_3}{x_1 x_2 x_3 M_0} \varphi(\vec{k}_1, \beta_1) \varphi(\frac{\vec{k}_2 - \vec{k}_3}{2}, \beta_{23}),$$

with $\varphi(\vec{k}, \beta) = 4(\frac{x}{2\beta})^{3/4}\exp(-\frac{k^2 - k_j^2}{2\beta^2})$.

**B. Calculating the form factors of $\Xi_c^{(*)} \to \Xi_c$ in LFQM**

First we consider the weak decay $\Xi_c \to \Xi$. The leading order Feynman diagram is depicted in Fig. 1. Following the procedures given in Ref. [29–32] the transition matrix element can be evaluated with the vertex functions of $|\Xi_c(P, S, S_x)\rangle$ and $|\Xi'/P', S', S_x'\rangle\rangle$. The $sq$ subsystem stands as a spectator, i.e. its spin-flavor configuration does not change during the transition, so only the first term in Eq. (3) can contribute to the transition. The hadronic matrix element can be:
FIG. 1: The Feynman diagram for $\Xi_c^+(\Omega) \rightarrow \Xi$ transition, where $\bullet$ and $q$ denote $V-A$ current vertex and $u$ or $d$ quark respectively.

\[
\langle \Xi(P', S'_2) \mid \bar{s} \gamma^\mu (1 - \gamma_5) c \mid \Xi_c(P, S_2) \rangle = -\frac{\sqrt{3}}{2} \int \frac{d^4 \vec{p}_2}{d^4 \vec{p}_3} \phi_\Xi(x', k'_1) \phi_\Xi(x, k_1) \text{Tr}[(\vec{P}' - M_0')\gamma_5 (p_2 + m_2)(\vec{P} + M_0)\gamma_5 (p_3 - m_3)]
\]

\[
\times \bar{u}(\vec{P}', S'_1)(\gamma^\mu (1 - \gamma_5) (p_1 + m_1) u(\vec{P}, S_2), \tag{8}
\]

where

\[
m_1 = m_c, \quad m_1' = m_s, \quad m_2 = m_s, \quad m_3 = m_d, \quad c \rightarrow \gamma_\alpha - \gamma_\beta = \gamma_\beta - \vec{P}P/\gamma^2 \tag{9}
\]

with $P(P')$ is the four-momentum of $\Xi_c(\Xi)$ and $M(M')$ is the mass of $\Xi_c(\Xi)$. Setting $\vec{p}_1 = \vec{p}_1' + \vec{Q}, \vec{p}_2 = \vec{p}_2'$ and $\vec{p}_3 = \vec{p}_3'$ one has

\[
x_{1,2,3}' = x_{1,2,3} - (1 - x_1)Q_\perp, \quad k_{1\perp}' = k_{1\perp} + x_2Q_\perp, \quad k_{2\perp}' = k_{2\perp} + x_3Q_\perp. \tag{10}
\]

The form factors for the weak transition $\Xi_c \rightarrow \Xi$ are defined in the standard way as

\[
\langle \Xi(P', S', S'_2) \mid \bar{s} \gamma^\mu (1 - \gamma_5) c \mid \Xi_c(P, S, S_2) \rangle = \bar{u}_\Xi(P', S'_2) \left[ \gamma^\mu f_1^s - i\sigma^\mu_\nu Q^\nu M_\Xi f_2^s + \frac{Q^\mu}{M_\Xi} f_3^s \right] u_\Xi(P, S_2)
\]

\[
-\bar{u}_\Xi(P', S'_2) \left[ \gamma^\mu g_1^s - i\sigma^\mu_\nu Q^\nu M_\Xi g_2^s + \frac{q_\mu}{M_\Xi} g_3^s \right] \gamma_5 u_\Xi(P, S_2). \tag{11}
\]

where $Q \equiv P - P'$. Similarly one can write out the transition matrix of $\Xi_c' \rightarrow \Xi$ whose form factors are denoted as $f_i^c$ and $g_i^c$.

The detailed expressions of the form factors are

\[
f_1^s = -\frac{\sqrt{3}}{2} \int \frac{dx_2 dx_2 k_{2\perp}^2 dx_3 dx_3 k_{3\perp}^2}{2(2\pi)^3} \frac{\text{Tr}[(\vec{P}' - M_0')(\vec{P} + M_0)\gamma_5 (p_3 - m_3)]}{\sqrt{M_0^3(m_1 + e_1)(m_2 + e_2)(m_3 + e_3)(m_1' + e_1')(m_2' + e_2')(m_3' + e_3')}}
\]
These form factors are the same as those in our earlier paper\cite{38} except for an additional factor \(-\frac{\gamma}{2}\) or \(\frac{1}{2}\) which exist in corresponding equations of the equation group (4).
TABLE I: The quark mass and the parameter $\beta$ (in units of GeV).

| $m_c$  | $m_s$  | $m_u$  | $\beta_{c[ss]}$ | $\beta_{s[ss]}$ | $\beta_{[cu]}$ |
|-------|-------|-------|-----------------|-----------------|--------------|
| 1.5   | 0.5   | 0.25  | 0.699           | 1.07            | 0.546        |

### III. NUMERICAL RESULTS

#### A. The form factors for $\Xi_c \to \Xi$ and $\Xi'_c \to \Xi$

In order to study the transitions $\Xi_c \to \Xi$ and $\Xi'_c \to \Xi$ one needs to calculate aforementioned form factors numerically where the parameters in the model need to be predetermined. The lifetime of $\Xi_c$ and the masses of $\Xi_c$, $\Xi'_c$ and $\Xi$ are taken from the data book of particle data group [5]. The masses of quarks given in Ref. [43] are collected in Table I. In fact, we still know little about the parameters $\bar{\alpha}, \bar{\beta}, \bar{\nu}$ for the initial and final baryons. Generally the reciprocal of $\beta$ is related to the electrical radius of meson with two constituents. Since the strong coupling between $q$ and $q'q''$ is half of that between $qq''$, for a Coulomb-like interaction if the two interactions are equal one can expect the electrical separation of $q$ and $q''$ to be $1/\sqrt{2}$ times that of $q$ and $q''$ i.e. $\beta_{qq''} \approx \sqrt{2}\beta_{qq'}$. By considering the binding energy the authors of Ref. [44] obtained the same results. In our early paper for a compact $qq''$ system we find $\beta_{qq''} = 2.9\beta_{qq''}$. Since the $sq$ subsystem is easy to be broken we think it is not a compact system and we estimate $\beta_{c[ss]} \approx \sqrt{2}\beta_{cs}$ and $\beta_{[ss]} \approx \sqrt{2}\beta_{sq}$ where $\beta_{cs}$ and $\beta_{sq}$ were obtained for the meson case [43]. As for $ss$ we don’t know if it is a compact system so we let $\beta_{[ss]}$ be a free parameter to fix the data of $\Xi_0^0 \to \Xi^{-}e^+\nu_e$. With these parameters we calculate the form factors and the transition rates theoretically.

These form factors $f_i^s$, $g_i^s$, $f_i^v$ and $g_i^v$ ($i = 1, 2$) are evaluated in the space-like region($Q^+ = 0$ i.e. $Q^2 = -Q_{\perp}^2 \leq 0$) so should be analytically extrapolated to the time-like region. In Ref. [30] the authors employed a three-parameter form

$$F(Q^2) = \frac{F(0)}{1 - a \left( \frac{Q^2}{M_{\Xi_c}^2} \right) + b \left( \frac{Q^2}{M_{\Xi_c}^2} \right)^2},$$

(13)

where $F(Q^2)$ represents the form factors $f_i^s$, $g_i^s$, $f_i^v$ and $g_i^v$ ($i = 1, 2$). Using the numerical form factors evaluated in the space-like region we may fix the parameters $a$, $b$ and $F(0)$ in the un-physical region. When one uses Eq. (13) at $Q^2 \geq 0$ region these form factors are extended into the physical region. The values of $a$, $b$ and $F(0)$ for the form factors $f_1, f_1, g_1$ and $g_2$ are listed in Table II. The dependence of the form factors on $Q^2$ is depicted in Fig. [24]. The values of $F(0)$ in different works are listed in Table III and they deviate from each other as noted. The differences of $f_1^s$, $f_2^s$ and $g_2^s$ in Ref. [7, 8, 10, 13] are not very significant but that of $g_2^s$ is. The values of $f_1^v$, $f_2^v$, $g_1^v$ and $g_2^v$ at $Q^2 = 0$ can be found in Ref. [8] and they are very different from ours.
TABLE II: The form factors given in the three-parameter forms.

| $F$  | $F(0)$  | $a$    | $b$    |
|------|---------|--------|--------|
| $f_1^s$ | -0.640  | 0.711  | 0.0981 |
| $f_2^s$ | -0.366  | 1.07   | 0.295  |
| $g_1^s$ | -0.515  | 0.471  | 0.0839 |
| $g_2^s$ | -0.117  | 1.31   | 0.381  |
| $f_1^v$ | 0.373   | 1.29   | 0.499  |
| $f_2^v$ | -0.259  | 1.26   | 0.425  |
| $g_1^v$ | -0.0990 | 0.458  | 0.312  |
| $g_2^v$ | 0.0214  | 0.989  | 0.938  |

TABLE III: The form factors at $Q^2 = 0$ given in different works.

| $F(0)$ | this work | Ref.[7] | Ref.[8] | Ref.[9] | Ref.[10] | Ref.[13] |
|--------|-----------|---------|---------|---------|---------|---------|
| $f_1^s(0)$ | -0.640   | -0.71  | 0.77    | 0.194   | -0.567  | 0.590   |
| $f_2^s(0)$ | -0.366   | -0.46  | 0.96    | 0.356   | -0.305  | 0.441   |
| $g_1^s(0)$ | -0.515   | -0.71  | 0.69    | 0.311   | -0.491  | 0.582   |
| $g_2^s(0)$ | -0.117   | -0.14  | 0.0068  | 0.151   | -0.046  | -0.184  |
| $f_1^v(0)$ | 0.373    | -      | -       | 0.577   | -       | -       |
| $f_2^v(0)$ | -0.259   | -      | -       | 0.501   | -       | -       |
| $g_1^v(0)$ | -0.0990  | -      | -       | 0.451   | -       | -       |
| $g_2^v(0)$ | 0.0214   | -      | -       | 0.341   | -       | -       |

B. Semi-leptonic decays of $\Xi_c \to \Xi + l\bar{\nu}_l$ and $\Xi'_c \to \Xi + l\bar{\nu}_l$

Using the central value of the data $\Xi^0_c \to \Xi^- e^+ \bar{\nu}_e$ from the Belle measurements [5], we fix the parameter $\beta_{s[sd]}$. Concretely, we first assign $\beta_{s[sd]}$ a value and then calculate the form factors $f_1^s$, $f_2^s$, $g_1^s$ and $g_2^s$ to get the theoretical prediction on the rate for $\Xi^0_c \to \Xi^- e^+ \bar{\nu}_e$. Then compare the theoretical estimates of $BR(\Xi^0_c \to \Xi^- e^+ \bar{\nu}_e)$ with the data, a deviation would require us to modify the parameter, repeating the procedure further and further until they are equal to each other, thus we fix $\beta_{s[sd]}$ to be 1.07 GeV, which is 2.9 times larger than $\beta_{s\bar{s}} = 0.366$ GeV. The value may imply that the $ss$ pair in $\Xi$ is a more compact subsystem. With the same parameters the form factors $f_1^v$, $f_2^v$, $g_1^v$ and $g_2^v$ can be obtained and we evaluate the rate of $\Xi'_c \to \Xi l\bar{\nu}_l$. The differential decay widths $d\Gamma/d\omega$ ($\omega = \frac{P_{l\bar{\nu}_l}P_{MM'}}{M_{MM'}}$) are shown in Fig. 3.

Our numerical results on the decay widths and the ratio of the longitudinal versus transverse decay rates $R$ (see its expression in Appendix) of $\Xi^0_c \to \Xi^- e^+ \bar{\nu}_e$ and $\Xi^+ \to \Xi^0 e^+ \bar{\nu}_e$ are all presented in table IV. Letting the model parameters (the quark masses and all $\beta$s) fluctuate up to $\pm 10\%$, we estimate the theoretical uncertainties in our numerical results. Some predictions on the same channels are also presented in table IV. One may notice the prediction on $\Gamma(\Xi^0_c \to \Xi^- e^+ \bar{\nu}_e)$ given by the author of Ref.[10] is close to our result but his
FIG. 2: The dependence of form factors $f_1^s$, $f_2^s$, $g_1^s$ and $g_2^s$ in a three-parameter form on $Q^2$ (a) and the dependence of the form factors $f_1^v$, $f_2^v$, $g_1^v$ and $g_2^v$ on $Q^2$ (b).

FIG. 3: Differential decay rates $d\Gamma/d\omega$ for the decay $\Xi_c \to \Xi l\bar{\nu}_l$ (a) and $\Xi'_c \to \Xi l\bar{\nu}_l$ (b)

prediction on the branching ratio is lower than the central value of data because the measured lifetime of $\Xi_0^{-}$ [45] has been significantly modified since the work [10] was done. The values of $R$ given in Refs. [7, 10] is lower than ours. In table V the predictions on $\Xi'_c \to \Xi l\bar{\nu}_l$ are presented. Since the form factors $f_1^v$, $f_2^v$, $g_1^v$ and $g_2^v$ in Ref. [9] deviate from ours very significantly the width of $\Xi'_c \to \Xi l\bar{\nu}_l$ they obtained is two orders of magnitude larger than ours. This should also be tested in more precise measurements in the future.

C. Non-leptonic decays of $\Xi_c \to \Xi + M$ and $\Xi'_c \to \Xi + M$

As a matter of fact, a theoretical exploration for the non-leptonic decays is more complicated than for semi-leptonic processes. Based on the factorization assumption which may be the lowest order approximation by omitting possible final-state-interaction effects, the hadronic transition matrix element is factorized into a product of two independent hadronic
matrix elements,

\[
\langle \Xi(P', S'_z) M | \mathcal{H} | \Xi'_c(P, S_z) \rangle = \frac{G_F V_{cs} V_{qc}^*}{\sqrt{2}} \langle M | q' \gamma^\mu(1 - \gamma_5)q | 0 \rangle \langle \Xi(P', S'_z) | s \gamma^\mu(1 - \gamma_5)c | \Xi'_c(P, S_z) \rangle ,
\]

where the first hadronic matrix element \( \langle M | q' \gamma^\mu(1 - \gamma_5)q | 0 \rangle \) is determined by a well-fixed decay constant and the second one \( \langle \Xi(P', S'_z) | s \gamma^\mu(1 - \gamma_5)c | \Xi'_c(P, S_z) \rangle \) is evaluated in the previous sections. For the decay \( \Xi'_c \rightarrow \Xi + M \) which is a color-favored channel, the factorization should be a plausible approximation. The results on these non-leptonic decays can be checked in the future measurements and the validity degree of the obtained form factors would be confirmed (within an error range) or suggests serious modifications.

With the theoretical width \( \Gamma(\Xi_c^0 \rightarrow \Xi^- \pi^+) \) and the lifetime of \( \Xi_c^0 \) one can obtain that branching ratio as \( (1.87 \pm 0.28)\% \) which is consistent with the measured value of the Belle collaboration[2] and the up-down asymmetry is consistent with the present data[3]. From the results shown in Table VI we find that \( \Gamma(\Xi_c^0 \rightarrow \Xi^- \pi^+) \) and \( \Gamma(\Xi_c^0 \rightarrow \Xi^- \rho^+) \) are very close to each other, but there exists an obvious gap between their up-down asymmetries. There is a similar situation for the decays of \( \Gamma(\Xi_c^0 \rightarrow \Xi^- K^+) \) and \( \Gamma(\Xi_c^0 \rightarrow \Xi^- K^+) \). One also notices that \( \Gamma(\Xi_c^0 \rightarrow \Xi^- M^+) \) is three or four times smaller than \( \Gamma(\Xi_c^0 \rightarrow \Xi^- M^+) \). The up-down asymmetry for a different channel \( \Gamma(\Xi_c^0 \rightarrow \Xi^- M^+) \) is close to 0.5. For \( \Gamma(\Xi_c^+ \rightarrow \Xi^0 M^+) \) and \( \Gamma(\Xi_c^+ \rightarrow \Xi^0 M^+) \) their decay rates and up-down asymmetry are very close to those of \( \Gamma(\Xi_c^0 \rightarrow \Xi^- M^+) \) and \( \Gamma(\Xi_c^0 \rightarrow \Xi^0 M^+) \) which we omit here. We estimate the branching ratio of \( \Gamma(\Xi_c^+ \rightarrow \Xi^0 \pi^+) \) to be \( (5.56 \pm 0.80)\% \).

**TABLE IV: The theoretical results of \( \Xi^0 \rightarrow \Xi^- \bar{\nu}_\tau \) (left) and \( \Xi^+_c \rightarrow \Xi^0 \bar{\nu}_\tau \) (right).**

|                | \( \Gamma(10^{-12}\text{GeV}) \) | \( B \)     | \( R \)     | \( \Gamma(10^{-12}\text{GeV}) \) | \( B \)     | \( R \)     |
|----------------|-----------------|------------|------------|-----------------|------------|------------|
| this work      | 0.0740±0.015    | (1.72±0.35) | 2.85±0.14  | 0.0750±0.016    | (5.20±1.02) | 2.85±0.15  |
| Ref.[7]        | 0.145 ± 0.031   | (3.4 ± 0.7) | 1.90 ± 0.39 | 0.147 ± 0.032   | (10.2 ± 2.2) | 1.90 ± 0.38 |
| Ref.[8]        | -               | (3.49 ± 9.5) | -         | -               | (11.3 ± 3.3) | -         |
| Ref.[9]        | 0.4264±1.490    | (7.26 ± 2.54) | -         | 0.4264±1.490    | -         | -         |
| Ref.[10]       | 0.0791          | 1.35%      | 1.98       | 0.0803          | 5.39%      | 1.98       |
| Ref.[13]       | -               | 2.38%      | -          | -               | 9.40%      | -          |

**TABLE V: The theoretical results of \( \Xi'_c \rightarrow \Xi l\bar{\nu}_l \).**

|                | \( \Gamma(\text{in unit } 10^{-12}\text{GeV}) \) | \( R \)     |
|----------------|-----------------|------------|
| this work      | 0.0187±0.0035   | 10.39±1.00 |
| Ref.[9]        | 1.109           | -          |
TABLE VI: Our predictions on widths (in unit $10^{-14}\text{GeV}$) and up-down asymmetry of non-leptonic decays $\Xi_0^{(c)} \rightarrow \Xi M$.

| mode | width   | up-down asymmetry | mode | width   | up-down asymmetry |
|------|---------|-------------------|------|---------|-------------------|
| $\Xi_0^0 \rightarrow \Xi^- \pi^+$ | 8.03±1.15 | -0.975±0.006       | $\Xi_0^{(c)} \rightarrow \Xi^- \pi^+$ | 2.24±0.36 | 0.493±0.014   |
| $\Xi_0^0 \rightarrow \Xi^- \rho^+$ | 8.53±1.25 | -0.397±0.013       | $\Xi_0^{(c)} \rightarrow \Xi^- \rho^+$ | 1.93±0.35 | 0.557±0.158   |
| $\Xi_0^{(c)} \rightarrow \Xi^- K^+$ | 0.558±0.086 | -0.951±0.009       | $\Xi_0^{(c)} \rightarrow \Xi^- K^+$ | 0.174±0.030 | 0.476±0.014   |
| $\Xi_0^0 \rightarrow \Xi^- K^{*+}$ | 0.349±0.079 | -0.252±0.014       | $\Xi_0^{(c)} \rightarrow \Xi^- K^{*+}$ | 0.0774±0.0141 | 0.582±0.017 |

IV. CONCLUSIONS AND DISCUSSIONS

In this paper we calculate the transition rate of $\Xi_0^{(c)} \rightarrow \Xi$ in the light front quark model. For the baryons $\Xi_0^{(c)}$ and $\Xi$ we employ the three-quark picture instead of the quark-diquark one to carry out the calculation. For $\Xi_0^{(c)}$, the $sq$ content constitutes a physical subsystem which has a definite spin, instead, in $\Xi ss$ constitutes a physical subsystem which has also a definite spin. In the transition $\Xi_0^{(c)} \rightarrow \Xi$, the physical subsystem in the initial state is different from that in the final state, so that the diquark picture which is considered as an unchanged spectator cannot be directly applied in this case. However in the process the strange quark which does not undergo a change and meanwhile the $q$ ($u$ or $d$) quark is also approximatively to be a spectator as long as higher order non-perturbative QCD effects are neglected, so the $sq$ pair is regarded as an effective subsystem in the final state. Baryon is a three-body system whose total spin can be obtained via different combinations among that of all three constituents. By a Racah transformation we can convert one configuration into another. The Racah coefficients determines the correlation between the two configuration $(ss)$-$q$ and $s$-$(sq)$. However, one is noted that the subsystem of $(sq)$ is not regarded as a diquark and moreover there exists a relative momentum between the two constituents. Thus in the vertex function of the three-quark picture there exists an inner degree of freedom for every quark which just manifests by the relative momentum. In the three-body vertex function of baryon two quarks are bound to a subsystem which has a definite spin and then the subsystem couples with the rest quark to form a baryon with the required spin.

Using the central value of the data $\Xi_0^0 \rightarrow \Xi^- e^+ \bar{\nu}_e$ from the Belle, we fix the parameter $\beta_{[sqd]}$ and calculate the form factors $f_1^s$, $f_2^s$, $g_1^s$, $g_2^s$. The differences of $f_1^s$, $f_2^s$ and $g_1^s$ given in all the literatures Ref.[, 8, 10, 13] are not very significant but that for $g_2^s$ are. We calculate the ratios of the longitudinal to transverse decay rates $R$ for the decay $\Xi_0^0 \rightarrow \Xi e^+ \bar{\nu}_e$ where the definition of $R$ is given in the appendix, It is noted that the width obtained by Zhao et al. [10] is close to present data but their estimated value of $R$ is smaller than ours.

With the same parameters the form factors $f_1^v$, $f_2^v$, $g_1^v$ and $g_2^v$ can be obtained and they are very different from those in Ref.[9]. We evaluate the decay rate of $\Xi_0^{(c)} \rightarrow \Xi l \bar{\nu}_l$ and the ratios of the longitudinal to transverse decay rates $R$. The width we obtained is two order of magnitude smaller than that in Ref.[9].

Under the factorization assumption, we also evaluate the rates of several non-leptonic
decays. Our numerical results indicate that $\Gamma(\Xi^0_c \to \Xi^- \pi^+)$ and $\Gamma(\Xi^0_c \to \Xi^- \rho^+)$ are close to each other but the up-down asymmetries are very different. The situation for $\Gamma(\Xi^0_c \to \Xi^- K^+)$ and $\Gamma(\Xi^0_c \to \Xi^- K^{*+})$ is very similar. $\Gamma(\Xi^0_c \to \Xi^- M^+)$ is three or four times smaller than $\Gamma(\Xi^0_c \to \Xi^- M^+)$ but the up-down asymmetry for a different channel $\Gamma(\Xi^0_c \to \Xi^- M^+)$ is close to 0.5.

Since there exists a large uncertainty in data, so that none of the different theoretical approaches for exploring the semi-leptonic decay $B(\Xi^0_c \to \Xi^- e^+ \nu_e)$ can be ruled out so far. We suggest the experimentalists to make more accurate measurements on this channel and other non-leptonic decay modes, thus the data would tell us which approach can work better. Definitely, the theoretical studies on the baryons are helpful to understand the quark model and the non-perturbative QCD effects.

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Appendix A: Semi-leptonic decays of $B_1 \to B_2 l \bar{\nu}_l$

The helicity amplitudes are expressed with the form factors for $B_1 \to B_2 l \bar{\nu}_l$ through the following expressions

\[
\begin{align*}
H^{V,0}_{2,0} &= \frac{\sqrt{T_+}}{\sqrt{Q^2}} \left( (M_{B_1} + M_{B_2}) f_1 - \frac{Q^2}{M_{B_1}} f_2 \right), \\
H^{V,1}_{2,1} &= \frac{\sqrt{T_+}}{\sqrt{Q^2}} \left( -f_1 + \frac{M_{B_1} + M_{B_2}}{M_{B_1}} f_2 \right), \\
H^{A,0}_{2,0} &= \frac{\sqrt{T^+}}{\sqrt{Q^2}} \left( (M_{B_1} - M_{B_2}) g_1 + \frac{Q^2}{M_{B_1}} g_2 \right), \\
H^{A,1}_{2,1} &= \frac{\sqrt{T^+}}{\sqrt{Q^2}} \left( -g_1 + \frac{M_{B_1} - M_{B_2}}{M_{B_1}} g_2 \right),
\end{align*}
\]

where $T_\pm = 2(P \cdot P' \pm M_{B_1} M_{B_2})$ and $M_{B_1} (M_{B_2})$ represents $M_{\Xi_c} (M_{\Xi})$. The amplitudes for the negative helicities are obtained in terms of the relation

\[
H^{V,A}_{-\lambda',-\lambda_W} = \pm H^{V,A}_{\lambda',\lambda_W},
\]

where the upper (lower) index corresponds to $V (A)$. The helicity amplitudes are

\[
H_{\lambda',\lambda_W} = H^{V}_{\lambda',\lambda_W} - H^{A}_{\lambda',\lambda_W}.
\]

$\lambda_W$ is the helicities of the $W$-boson which can be either 0 or 1, which correspond to the longitudinally and transversely polarized states, respectively. The longitudinally (L) and transversely (T) polarized states are obtained by the relations

\[
H^{L}_{\lambda'} = H^{V}_{\lambda'} + H^{A}_{\lambda'}, \quad H^{T}_{\lambda'} = H^{V}_{\lambda'} - H^{A}_{\lambda'}.
\]
(T) polarized rates are respectively

\[ \frac{d\Gamma_L}{d\omega} = \frac{G_F^2 |V_{cb}|^2}{(2\pi)^3} \frac{Q^2 p_c M_{B_2}}{12 M_{B_1}} \left[ |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right], \]

\[ \frac{d\Gamma_T}{d\omega} = \frac{G_F^2 |V_{cb}|^2}{(2\pi)^3} \frac{Q^2 p_c M_{B_2}}{12 M_{B_1}} \left[ |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right]. \] (A4)

where \( p_c \) is the momentum of \( B_2 \) in the rest frame of \( B_1 \).

The ratio of the longitudinal to transverse decay rates \( R \) is defined by

\[ R = \frac{\Gamma_L}{\Gamma_T} = \frac{\int_{\omega_{1\text{max}}}^{\omega_{\text{max}}} d\omega Q^2 p_c \left[ |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right]}{\int_{\omega_{2\text{max}}}^{\omega_{\text{max}}} d\omega Q^2 p_c \left[ |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right]} \] (A5)

Appendix B: \( B_1 \rightarrow B_2 M \)

In general, the transition amplitude of \( B_1 \rightarrow B_2 M \) can be written as

\[ \mathcal{M}(B_1 \rightarrow B_2 P) = \bar{u}_{A_c} (A + B \gamma_5) u_{B}, \]

\[ \mathcal{M}(B_1 \rightarrow B_2 V) = \bar{u}_{A_c} \epsilon^{\mu} [A_1 \gamma_\mu \gamma_5 + A_2 (p_c)_{\mu} \gamma_5 + B_1 \gamma_\mu + B_2 (p_c)_{\mu}] u_{B}, \] (B1)

where \( \epsilon^{\mu} \) is the polarization vector of the final vector or axial-vector mesons. Including the effective Wilson coefficient \( a_1 = c_1 + c_2/N_c \), the form factors in the factorization approximation are \[39, 49\]

\[ A = \lambda f_P (M_{B_1} - M_{B_2}) f_1 (M^2), \]

\[ B = \lambda f_P (M_{B_1} + M_{B_2}) g_1 (M^2), \]

\[ A_1 = -\lambda f_V M \left[ g_1 (M^2) + g_2 (M^2) \frac{M_{B_1} - M_{B_2}}{M_{B_1}} \right], \]

\[ A_2 = -2\lambda f_V M \frac{g_2 (M^2)}{M_{B_1}}, \]

\[ B_1 = \lambda f_V M \left[ f_1 (M^2) - f_2 (M^2) \frac{M_{B_1} + M_{B_2}}{M_{B_1}} \right], \]

\[ B_2 = 2\lambda f_V M \frac{f_2 (M^2)}{M_{B_1}}, \] (B2)

where \( \lambda = \frac{G_F}{\sqrt{2}} V_{cs} V_{q_1 q_2}^\ast a_1 \) and \( M \) is the meson mass. Replacing \( P, V \) by \( S \) and \( A \) in the above expressions, one can easily obtain similar expressions for scalar and axial-vector mesons.

The decay rates of \( B_1 \rightarrow B_2 P(S) \) and up-down asymmetries are \[49\]

\[ \Gamma = \frac{p_c}{8\pi} \left| \left( \frac{(M_{B_1} + M_{B_2})^2 - M^2}{M_{B_1}^2} \right) |A|^2 + \left( \frac{(M_{B_1} - M_{B_2})^2 - m^2}{M_{B_1}^2} \right) |B|^2 \right|, \]

\[ \alpha = -\frac{2\kappa \text{Re}(A^*B)}{|A|^2 + \kappa^2 |B|^2}, \] (B3)
where $p_c$ is the $B_2$ momentum in the rest frame of $B_1$ and $m$ is the mass of pseudoscalar (scalar). For $B_1 \to B_2 V (A)$ decays, the decay rate and up-down asymmetries are

$$
\Gamma = \frac{p_c (E_{B_2} + M_{B_2})}{4\pi M_{B_1}} \left[ \frac{2 \left( |S|^2 + |P_2|^2 \right)}{m^2} + \frac{e_v^2}{m^2} \left( |S + D|^2 + |P_1|^2 \right) \right],
$$

and

$$
\alpha = \frac{4m^2 \text{Re}(S^* P_2) + 2e_v^2 \text{Re}(S + D)^* P_1}{2m^2 \left( |S|^2 + |P_2|^2 \right) + \varepsilon^2 \left( |S + D|^2 + |P_1|^2 \right)},
$$

where $e_v$ ($m$) is energy (mass) of the vector (axial vector) meson, and

$$
S = -A_1,
$$

$$
P_1 = \frac{p_c}{e_v} \left( \frac{M_{B_1} + M_{B_2}}{E_{B_2} + M_{B_2}} B_1 + M_{B_1} B_2 \right),
$$

$$
P_2 = \frac{p_c}{E_{B_2} + M_{B_2}} B_1,
$$

$$
D = \frac{p_c^2}{e_v (E_{B_2} + M_{B_2})} (A_1 - M_{B_1} A_2).
$$

\[\text{(B4)}\]

\[\text{(B5)}\]

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