Medium effects of magnetic moments of baryons on neutron stars under strong magnetic fields

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Abstract

We investigate medium effects due to density-dependent magnetic moments of baryons on neutron stars under strong magnetic fields. If we allow the variation of anomalous magnetic moments (AMMs) of baryons in dense matter under strong magnetic fields, AMMs of nucleons are enhanced to be larger than those of hyperons. The enhancement naturally affects the chemical potentials of baryons to be large and leads to the increase of a proton fraction. Consequently, it causes the suppression of hyperons, resulting in the stiffness of the equation of state. Under the presumed strong magnetic fields, we evaluate relevant particles’ population, the equation of state and the maximum masses of neutron stars by including density-dependent AMMs and compare them with those obtained from AMMs in free space.

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I. INTRODUCTION

Recently, strong magnetic fields were observed at the surface of soft gamma ray repeaters, called magnetar. The magnitude of the fields was estimated as an order of $10^{14} - 10^{15}$ G [1]. In the interior of neutron stars, according to the scalar virial theorem, the magnetic field strength could be about $10^{18}$ G. Such strong magnetic fields may affect the structure of a neutron star such as the populations of particles, the equation of state (EoS) and mass-radius relations. Many studies for neutron stars with strong magnetic fields have been reported by several papers, which included the electromagnetic interaction, the Landau quantization of charged particles, and anomalous magnetic moments (AMMs) of baryons [2–8]. But roles of relevant particles’ AMMs in a strong magnetic field are still uncertain because properties of the AMMs in nuclear matter are not fully scrutinized yet.

On the other hand, medium effects of the electromagnetic (EM) form factors for nucleons have been mainly investigated on the electron scattering in both experimental [9–11] and theoretical aspects [12, 13]. From their results, one may expect the swollen effect of the EM form factors by about $20 \sim 40\%$. In particular, various possible variations of the AMMs of baryons in nuclear matter have been studied extensively by many different theoretical models [14–21]. However, there are still remained some ambiguities on the density dependence of the AMMs stemming from the model dependence of baryons in nuclear matter. Furthermore, experimental data also show large error bars. For example, the AMM of $\Lambda$ hyperon in $^7\Lambda$Li nucleus recently measured at BNL [22] still showed large error bars. Further experiments are expected to deduce more clearly the AMM properties in nuclear matter.

Authors in Ref. [21] studied the medium dependence of the AMMs of baryons in symmetric nuclear matter by using both different models, the quark-meson coupling (QMC) [23] and the modified quark-meson coupling (MQMC) models [24]. In the QMC model, the density dependence of the AMMs of baryons is very small, while the AMM values of a proton and a $\Lambda$ hyperon in the MQMC model are enhanced by about 25% and about 10%, respectively, at saturation density. Such large enhancements in the MQMC model are quite feasible because the AMM of a baryon generally depends strongly on the bag radius.

In the sense, the MQMC model could effectively take the swollen effect of nucleons into account, by increasing the bag radius about 20% at saturation density. But in the QMC model, the bag radius is rarely changed to make the change of AMMs very small. Therefore,
the MQMC model can provide us with a theoretical framework to discuss medium effects of the AMMs.

In this work, under the assumption that the AMM values of baryons may considerably depend on medium, we apply the effects to a neutron star. The calculation of the AMMs of baryons in medium is done by considering only SU(6) quark wave functions obtained by using the MQMC model. Further possible consequences of the effects under strong magnetic fields are also discussed from observational quantities on neutron stars.

Since the quantum hadrodynamics (QHD), which is a systematically developed model for finite nuclei and nuclear matter, provides us with results very similar to those by the MQMC model for the structure of a neutron star, we employ the QHD model for a neutron star under strong magnetic fields by including the electromagnetic potential, the Landau quantization of charged particles, and the AMM values of baryons [3, 5, 7, 8]. But, to extract the density dependence of the AMMs, we adopt the MQMC model because the model can be easily applied to describe the AMMs in nuclear matter rather than the QHD model and generate successfully the AMM values of baryon octets in nuclear matter.

This paper is organized as follows. In Sec. II, the QHD model for dense matter under a strong magnetic field is briefly introduced by focusing on the roles of the AMM in the magnetic field. Results and discussions are presented in Sec. III. Summary and conclusions are given at Sec. IV.

II. THEORY

The lagrangian density of the QHD model for dense matter in the presence of strong magnetic fields, which is introduced by the vector potential $A^\mu$ due to magnetic fields, can be represented in terms of octet baryons, leptons, and five meson fields as follows

$$
\mathcal{L} = \sum_b \bar{\psi}_b \left[ i\gamma_\mu \partial^\mu - q_b \gamma_\mu A^\mu - M_b^* (\sigma, \sigma^*) - g_{\omega b} \gamma_\mu \omega^\mu - g_{\phi b} \gamma_\mu \phi^\mu 
\right. 
- g_{\rho b} \gamma_\mu \vec{\tau} \cdot \rho^\mu - \frac{1}{2} \kappa_{b\sigma} F^\mu\nu \left. \right] \psi_b + \sum_l \bar{\psi}_l \left[ i\gamma_\mu \partial^\mu - q_l \gamma_\mu A^\mu - m_l \right] \psi_l 
+ \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - U(\sigma) + \frac{1}{2} \partial_\mu \sigma^* \partial^\mu \sigma^* - \frac{1}{2} m_{\sigma^*}^2 \sigma^{*2} - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} 
+ \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \Phi_{\mu\nu} \Phi^{\mu\nu} + \frac{1}{4} m_\phi^2 \phi_\mu \phi^\mu - \frac{1}{4} R_{i\mu\nu} R^{i\mu\nu} + \frac{1}{2} m_{\rho}^2 \rho_\mu \rho^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \tag{1}
$$
where $b$ and $l$ denote the octet baryons and the leptons ($e^-$ and $\mu^-$), respectively. The effective mass of a baryon, $M_b^*$, is simply given by $M_b^* = M_b - g_{\sigma b}\sigma - g_{\sigma^* b}\sigma^*$, where $M_b$ is the free mass of a baryon in vacuum. The $\sigma$, $\omega$ and $\rho$ meson fields describe interactions of nucleon-nucleon ($N-N$) and nucleon-hyperon ($N-Y$). Interaction of $Y-Y$ is mediated by $\sigma^*$ and $\phi$ meson fields. $U(\sigma)$ is the self interaction of the $\sigma$ field given by

$$
U(\sigma) = \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4.
$$

$W_{\mu\nu}$, $R_{i\mu\nu}$, $\Phi_{\mu\nu}$, and $F_{\mu\nu}$ represent the field tensors of $\omega$, $\rho$, $\phi$ and photon fields, respectively. The AMMs of baryons interact with an external magnetic field in the form of $\kappa_b\mu_N\sigma_{\mu\nu}F^{\mu\nu}$ where $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ and $\kappa_b$ is the strength of AMM of a baryon, i.e. $\kappa_p = 1.7928\mu_N$ for a proton in a vacuum where $\mu_N$ is the nucleon magneton defined as $\mu_N = e/2m_p$. Since the medium dependence of the AMM on the density is considered, the ratios of $\kappa_b$ of baryon octet are taken from the results of the MQMC model in Ref. [21]. In the MQMC model, baryons are treated as MIT bags and $\kappa_b$ is calculated from SU(6) quark wave functions and the bag radius depending on medium. For $\mu_N = e/2m_p$, $\mu_N$ is always defined with the mass of a proton in free space, $m_p$. Thus in this work, $\kappa_b$ depends on the density but $\mu_N$ does not.

The Dirac equations of octet baryons and leptons in the mean field approximation are given by

$$
\left[ i\gamma_\mu \partial^\mu - q_b\gamma_\mu A^\mu - M_b^* (\sigma, \sigma^*) - g_{\omega b}\gamma^0 \omega_0 - g_{\rho b}\gamma^0 \phi_0 \\
- g_{\rho b}\gamma^0 \tau_3 \rho_{30} - \frac{1}{2}\kappa_b \sigma_{\mu\nu} F^{\mu\nu} \right] \psi_b = 0, \quad (2)
$$

$$
(i\gamma_\mu \partial^\mu - q_l\gamma_\mu A^\mu - m_l) \psi_l = 0, \quad (3)
$$

where $A_\mu = (0, 0, Bx, 0)$ refers to the constant magnetic field $B$, which is assumed as along the $z$-axis. The energy spectra of baryons and leptons are given by

$$
E_b^C = \sqrt{k_z^2 + \left( \sqrt{M_b^*^2 + 2\nu|q_b|B - s\kappa_b B} \right)^2 + g_{\omega b}\omega_0 + g_{\phi b}\phi_0 + g_{\rho b}\tau_3^0 \rho_{30}},
$$

$$
E_b^N = \sqrt{k_z^2 + \left( \sqrt{M_b^*^2 + k_x^2 + k_y^2 - s\kappa_b B} \right)^2 + g_{\omega b}\omega_0 + g_{\phi b}\phi_0 + g_{\rho b}\tau_3^0 \rho_{30}},
$$

$$
E_l = \sqrt{k_z^2 + m_l^2 + 2\nu|q_l|B}, \quad (4)
$$

where $E_b^C$ and $E_b^N$ represent energies of a charged baryon and a neutral baryon, respectively. The Landau quantization of a charged particle due to magnetic fields is denoted as $\nu = \ldots
\[ n + 1/2 - \text{sgn}(q)s/2 = 0, 1, 2 \cdots \] with electric charge \( q \) and spin up (down) \( s = 1(-1) \).

Equations of meson fields are given by

\[
\begin{align*}
& m^2_\sigma \sigma + \frac{\partial U(\sigma)}{\partial \sigma} = g_{\sigma b} \sum_b \rho_s^b, \\
& m^2_{\sigma^*} \sigma^* = g_{\sigma^* b} \sum_b \rho_s^b, \\
& m^2_\omega \omega = g_{\omega b} \sum_b \rho_v^b, \\
& m^2_\phi \phi = g_{\phi b} \sum_b \rho_v^b, \\
& m^2_\rho \rho = g_{\rho b} \sum_b \rho_v^b,
\end{align*}
\]

(5)

where \( \rho_s \) and \( \rho_v \) are the scalar and the vector densities under magnetic fields, respectively.

Detail expressions for these quantities are given in Ref. [3, 7]. The chemical potentials of baryons and leptons are, respectively, given by

\[
\begin{align*}
\mu_b &= E^b_f + g_{\omega b} \omega_0 + g_{\phi b} \phi_0 + g_{\rho b} I^b_3 \rho_30, \\
\mu_l &= \sqrt{k^2_f + m^2_l + 2\nu|q_l|B},
\end{align*}
\]

(6, 7)

where \( E^b_f \) is the Fermi energy of a baryon and \( k_f \) is the Fermi momentum of a lepton. For charged particles, the \( E^b_f \) is written as

\[
E^b_f = k^2_f + \left( \sqrt{m^2_b + 2\nu|q_b|B} - s\kappa_B B \right)^2,
\]

(8)

where \( k^b_f \) is the Fermi momentum of a baryon. Since the Landau quantization does not appear for neutral baryons, the Fermi energy is simply given by

\[
E^b_f = k^2_f + (m_b^* - s\kappa_B B)^2.
\]

We exploit three constraints for calculating properties of a neutron star: baryon number conservation, charge neutrality, and chemical equilibrium. The meson field equations in Eq. (5) are solved with the chemical potentials of baryons and leptons under the above three constraints. Total energy density is given by

\[
\varepsilon_{\text{tot}} = \varepsilon_m + \varepsilon_f,
\]

where the energy density for matter fields is given by

\[
\varepsilon_m = \sum_b \varepsilon_b + \sum_l \varepsilon_l + \frac{1}{2} m^2_\sigma \sigma^2 + \frac{1}{2} m^2_{\sigma^*} \sigma^{*2} + \frac{1}{2} m^2_\omega \omega^2 + \frac{1}{2} m^2_\phi \phi^2 + \frac{1}{2} m^2_\rho \rho^2 + U(\sigma),
\]

(10)
and the energy density due to the magnetic field is given by $\varepsilon_f = B^2 / 2$. The total pressure can also be written as

$$P_{\text{tot}} = P_m + \frac{1}{2} B^2,$$

(11)

where the pressure due to matter fields is obtained from $P_m = \sum_i \mu_i \rho_i^0 - \varepsilon_m$. The relation between mass and radius for a static and spherical symmetric neutron star is generated by calculating the Tolman-Oppenheimer-Volkoff (TOV) equations with the equation of state (EoS) above.

III. RESULTS AND DISCUSSION

We use the parameter set in Ref. [25] for the coupling constants, $g_{\sigma N}$, $g_{\omega N}$ and $g_{\rho N}$, where $N$ denotes the nucleon. For the coupling constants of hyperons in nuclear medium, $g_{\omega Y}$ is determined by the quark counting rule, and $g_{\sigma Y}$ is fitted to reproduce the potential of each hyperon at saturation density, whose strengths are given by $U_\Lambda = -30$ MeV, $U_\Sigma = 30$ MeV and $U_\Xi = -15$ MeV. For density-dependent AMM values of baryons, we use the values obtained from our previous calculation done by the MQMC model [21]. Since the magnetic fields may also depend on density, we take density-dependent magnetic fields used in Refs. [2, 8]

$$B(\rho/\rho_0) = B_{\text{surf}} + B_0 [1 - \exp\{-\beta (\rho/\rho_0)^\gamma\}],$$

(12)

where $B_{\text{surf}}$ is the magnetic field at the surface of a neutron star, which is taken as $10^{15}$ G from observations and $B_0$ represents the magnetic field saturated at high densities.

In the present work, we use two different sets, slow ($\beta = 0.05$ and $\gamma = 2$) and fast ($\beta = 0.02$ and $\gamma = 3$) varying magnetic fields. Since the magnetic field is usually written in a unit of the critical field for the electron $B_c^e = 4.414 \times 10^{13}$ G, the $B$ and the $B_0$ in Eq. (12) can be written as $B^* = B / B_c^e$ and $B_0^* = B_0 / B_c^e$. Here, we regard the $B_0^*$ as a free parameter and investigate medium effects of AMMs in a neutron star for three different magnetic fields given by $B_0^* = 1 \times 10^5$, $2 \times 10^5$, and $3 \times 10^5$. 
FIG. 1: (Color online) Populations of particles in a neutron star for the slow varying magnetic field ($\beta = 0.05$ and $\gamma = 2$). Left panels denote results for constant AMMs in free space and right panels are for density-dependent AMMs obtained from the MQMC model. For more direct comparison, all results for $p$ and $\Xi^-$ are summarized in the left hand side (LHS) of Fig.3.

A. Medium effects on the populations of particles

Before presenting medium effects by density-dependent AMMs, we shortly discuss effects of a magnetic field on a neutron star. The strong magnetic field affects charged particles
FIG. 2: (Color online) Same as Fig.1 but for the fast varying magnetic field ($\beta = 0.02$ and $\gamma = 3$). For more direct comparison, all results for $p$ and $\Xi^-$ are summarized in the right hand side (RHS) of Fig.3.

through the EM interaction term ($eB$), which leads to the Landau quantization, and all baryons by the AMM term ($\kappa_b B$).

The quantum numbers for the Landau levels have positive values $\nu = 0, 1, 2 \cdots$, so that the magnetic field increases energies of charged particles. Consequently, the chemical potentials of charged particles are increased by the magnetic field.
FIG. 3: (Color online) Populations of $p$ and $\Xi^-$ in a neutron star for both slow (LHS) and fast (RHS) varying magnetic field. Thick lines represent results for density-dependent AMM and thin lines are for constant AMM. $B_0^*$ values are given by a unit of $10^5$.

On the other hand, the AMM term gives rise to the spin splitting, so that the energy level is divided into two levels: one is higher level and the other is lower one. Since the chemical potential means the Fermi surface energy of a particle, the AMM term with increasing magnetic fields enlarges the chemical potential of a baryon.

If we allow the variations of AMMs in a nuclear medium, AMM values of relevant baryons are usually swollen. According to our previous results by the MQMC model [21], for example, the AMM enhancements of a proton, $\Lambda$, and $\Xi$ are about 25%, 10% and 5%, respectively, at saturation density. Therefore, medium effects due to density-dependent AMMs cause the chemical potentials of relevant baryons to become larger in addition to the enlargement by the effect of magnetic fields.

In Figs. 1 and 2, populations of baryons and leptons for the slow (Fig. 1) and the fast (Fig. 2) varying magnetic fields are presented for various $B_0^*$ values. Left panels are results of constant AMMs and right panels are those of density-dependent AMMs. Populations of protons and electrons are enhanced with the higher $B^*$ from upper to lower figures. If we notice the electron population at $\rho/\rho_0 = 10$, the enhancement is easily discerned. In particular, the population of electrons is larger than that of protons because the Bohr magneton $\mu_e$ is about 2000 times larger than the nucleon magneton $\mu_N$. This effect is fully ascribed to the increased magnetic fields.

The difference between left and right panels shows medium effects due to density-dependent AMMs. One can notice the increase of electron population from left to right
panels. The higher magnetic field is given, the larger medium effect appears.

In order to clearly demonstrate both effects, i.e., the magnetic field effects and medium effects due to density-dependent AMMs, we present both effects in a sheet in Fig. 3. We showed populations of protons and a Ξ− for two different $B_0^*$ fields, and for the constant AMM and the density-dependent AMM values. Since both effects increase chemical potentials of charged particles, populations of both particles are clearly increased.

The magnetic field effect seems to play a major role of increasing populations compared to the medium effect. But, in the $\rho/\rho_0 = 6 \sim 8$ region, the medium effect due to density-dependent AMMs can be competing with the magnetic field effect. The medium effect is almost same as that by the magnetic field increased by one unit.

The enhancement of the proton fraction gives rise to the suppression of other baryons because of baryon number conservation. It means that there appears the suppression of neutrons and Λ hyperons as shown in Figs. 1 and 2.

On the other hand, the threshold density for Ξ− is pushed to the higher density with the stronger magnetic field as shown in Fig. 3. However, the abundance of Ξ− is not changed so much in comparison with Λ as shown in Figs. 1 and 2. Since the Ξ− hyperon is a charged particle, the population is increased by the magnetic field, while the baryon number conservation and the charge neutrality lead to suppress the population. Therefore, the behavior of Ξ− population is balanced by the effects of the magnetic field and the conditions of a neutron star.

The difference between the slow (Fig. 1) and fast (Fig. 2) varying magnetic fields is the slope of magnetic field in the region of middle densities. Therefore, this difference just corresponds to the increase of the magnetic field strength $B_0^*$ at the same density. However, the effect due to the difference is not remarkable because the exponential term is small by comparing with the effect of the $B_0^*$ term, irrespective of $\gamma$ and $\beta$ values used here.

**B. Medium Effects on the EoS, Mass and Radius**

Magnetic fields and density-dependent AMMs also affect the EoS and the maximum mass of a neutron star. As shown in Fig. 4 the EoS in dense matter becomes stiffer with the increase of chemical potentials and the suppression of hyperons by the magnetic field. As a result, maximum masses of neutron stars are increased. The pressure due to matter fields
FIG. 4: (Color online) Equation of state (slow in LHS and fast in RHS). Thick lines represent results for density-dependent AMM and thin lines are for constant AMM. $B_0^*$ values are given by a unit of $10^5$.

FIG. 5: (Color online) Mass-radius relations. Thick lines represent results for density-dependent AMM and thin lines are for constant AMM. LHS is for the slow varying magnetic field and RHS is for the fast one. $B_0^*$ values are given by a unit of $10^5$.

also strongly depends on the strength of magnetic fields, but weakly depends on the density-dependent AMMs as shown in Fig. 4. For the fast magnetic field, slopes of EoS between $\rho/\rho_0 = 3$ and 5 are rapidly changed because magnetic fields cause the EoS to be fast stiffer.

But the effects of density-dependent AMMs are smaller than those of the magnetic field strength similar to the case of the populations. In a relatively small magnetic field ($B_0^* = 1 \times 10^5$ G), the density-dependence of AMMs rarely affects the EoS. However, in the strong magnetic fields ($B_0^* \geq 2 \times 10^5$), the contribution of the density-dependent AMMs in nuclear medium appears explicitly. For example, the increase of the pressure $P_m$ is about $37$ MeV
fm$^{-3}$ at $\rho = 6\rho_0$ for the fast case in $B_0^* = 3 \times 10^5$.

Mass-radius relations of neutron star obtained from TOV equations are shown in Fig. Masses of neutron stars, $M_{\text{star}}$, which are obtained from total energy density and total pressure ($\varepsilon_{\text{tot}}, P_{\text{tot}}$), depend very strongly on the strength of magnetic fields. But the contribution of density-dependent AMMs is indiscernible, which is about $0.1M_\odot$, maximally, even in the largest magnetic fields. Since there are no direct data for mass-radius relation of magnetars, we compare our results with the observed neutron stars in next section.

C. Comparison with observations

Neutron stars and heavy ion collisions may provide valuable constraints for the nuclear EoS [26]. Recent data reported higher masses and larger radii for neutron stars. For instance, $M = 2.0 \pm 0.1M_\odot$ for 4U 1636-536 was reported in Ref. [27] and authors in Ref. [28] recently investigated seven neutron stars, six binaries and a isolated neutron star (RX J1865-3754), showing $M = 1.9 - 2.3M_\odot$ and $R = 11 - 13$ km. Pulsar I of the globular cluster Terzan 5 (Ter 5 I) shows a lower mass limit $M \geq 1.68M_\odot$ at 95% confidence level [29]. Another constraint deduced independently of given models is obtained from XTE J1739-285 [30], which presents a constrained curve for the ratio between mass and radius.

Thus we compare our results with Ter 5 I and XTE J1739-285 in Fig. In the hyperonic star without magnetic fields (‘no B’ in Fig. [5]), the maximum mass is about $1.59M_\odot$ which does not satisfy the mass limit $(1.68M_\odot)$ by Ter 5 I. In addition, the constraint by XTE J1739-285 runs through an unstable region. When magnetic fields are introduced, the LHS in Fig. for the slow varying field shows that the line by XTE J1739-285 also goes through the unstable region. However, results for the fast varying magnetic field can satisfy the constraint of XTE J1739-285 and explain masses of neutron stars as $2 - 3M_\odot$ with magnetic fields for hyperonic stars.

In order to detail density-dependent AMMs effects, in Tab. I the central density ($\rho_c$), maximum masses, and magnetic fields at central density ($B_c^*$) for the fast varying magnetic field are tabulated for both constant and density-dependent AMMs cases. The effects of density-dependent AMMs are negligible in small magnetic fields. But as magnetic fields increase, the effect also increases and then the maximum mass is increased by about $0.07M_\odot$ for $B_0^* = 3 \times 10^5$ in the fast varying magnetic fields.
Finally, one can derive the limit of magnetic fields in the interior of a neutron star and the limit of density-dependent AMMs in medium. The allowed strength of magnetic fields is usually constrained by the scalar virial theorem \[4, 31\]. It is given by the following approximate relation, \( B \sim 2 \times 10^8 \frac{M}{M_\odot} \left(\frac{R_\odot}{R}\right)^2 G \) for the non-rotating star. For the star with \( R \approx 10 \) km and \( M \sim M_\odot \), we obtain \( B \sim 10^{18} G \) from the above relation.

In the calculation of model independent method for the maximum mass of neutron star, the limit of maximum mass is about \( M = 3 \sim 5 M_\odot \) \[31\]. Furthermore the observations show that there is no any neutron star in large mass region, which exceeds \( 3 M_\odot \). In this results, for fast case in \( B_0^* = 3 \times 10^5 G \), the maximum mass of the star is \( 2.96(2.89) M_\odot \) for density-dependent (constant) AMM and the central magnetic fields is about \( B = 2.75(2.71) \times 10^5 B_c^e G \). We can thus conclude that the upper limit of magnetic fields might be \( B \approx 3 \times 10^5 B_c^e G \) in neutron star with hyperons in this work, although detailed numbers depend on the model and parameters.

According to the model dependence of the AMM in other calculations \[14–20\], the largest enhancement is by about 40% for nucleons at saturation density \[15\], but the other models show the enhancement of about 10 ~ 25%. Thus the enhancement of 25% in this work, corresponds to the maximum enhancement except Ref. \[15\]. If we employ much larger enhancement for the AMM like the value in Ref. \[15\], the contribution of density-dependent

| \( B_0^* \) | \( \rho_c \) | \( \frac{M_{\text{star}}}{M_\odot} \) | \( B_c^* \) |
|----------|--------|----------------|--------|
| \( 5 \times 10^4 \) | 6.05 | 2.08 | \( 4.94 \times 10^4 \) |
| \( 1 \times 10^5 \) | 6.05 | 2.43 | \( 9.88 \times 10^4 \) |
| \( 2 \times 10^5 \) | 5.55 | 2.71 | \( 1.93 \times 10^5 \) |
| \( 3 \times 10^5 \) | 4.90 | 2.88 | \( 2.71 \times 10^5 \) |
| \( 5 \times 10^4 \) | 6.05 | 2.08 | \( 4.94 \times 10^4 \) |
| \( 1 \times 10^5 \) | 6.05 | 2.42 | \( 9.88 \times 10^4 \) |
| \( 2 \times 10^5 \) | 5.75 | 2.76 | \( 1.95 \times 10^5 \) |
| \( 3 \times 10^5 \) | 5.00 | 2.96 | \( 2.75 \times 10^5 \) |

TABLE I: The central density \( (\rho_c) \), maximum masses \( (M/M_\odot) \) and central magnetic field \( (B_c^*) \) for various \( B_0^* \) in both constant and changing AMMs. The results are obtained from fast varying magnetic fields.
AMM in medium may become larger. However, all populations, EoS, and maximum mass should depend on the strength of magnetic field very strongly, so that the contribution due to varying the AMM is still remained as a subsidiary role. Thus the effect of density-dependent AMM might be maximally around $0.1 M_\odot$.

IV. SUMMARY

We investigate the effect of the density-dependent AMM of baryons in neutron star under strong magnetic fields by using the QHD model, which includes baryon octet and leptons. By exploiting the density-dependent AMM values of baryons obtained from the MQMC model, we calculate the populations of particles, EoS, and the mass-radius relations for the slow and the fast varying magnetic fields. The strength of magnetic field is expressed as EM interaction of all charged particles and their AMM of baryon octet.

In the populations of particles, all charged particles experience Landau quantization and its effect depends severely on the strength of magnetic fields. The increase of the magnetic fields enhances the chemical potentials of all charged particles. In particular, since a proton is the lightest particle among baryons, the fraction of protons is enlarged by the magnetic field. As a result, it gives to rise the suppression of hyperons to satisfy the conservation of baryon number. The EoS becomes stiffer and then maximum mass of neutrons star also becomes larger.

The mass-radius relations of neutron star obtained from magnetic fields are compared with observational data. The mass-radius relation by fast varying magnetic fields satisfy the constraint by XTE J1739-285. The effect of density dependent AMM appears in very high magnetic fields, causing the increase of maximum mass of the star with about $0.1 M_\odot$ in $B_0^* = 3 \times 10^5$.

We assume the constant magnetic field along z-axis for non-rotating star. However, the real neutron star under the strong magnetic field rotates very rapidly and the magnetic fields may be taken place by the rotation of matter fields [32]. Thus the calculation should be self-consistent with each other, that is, the matter fields in rotating star create magnetic field and the produced magnetic field affects matter fields. This self-consistent approach for the magnetic field will be our next work.
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