Energy conditions in $f(T)$ gravity with non-minimal torsion-matter coupling

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Abstract

The present paper examines the validity of energy bounds in a modified theory of gravity involving non-minimal coupling of torsion scalar and perfect fluid matter. In this respect, we formulate the general inequalities of energy conditions by assuming the flat FRW universe. For the application of these bounds, we particularly focus on two specific models that are recently proposed in literature and also choose the power law cosmology. We find the feasible constraints on the involved free parameters and evaluate their possible ranges graphically for the consistency of these energy bounds.

Keywords: $f(T)$ gravity; Raychaudhuri equation; Energy conditions.

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1 Introduction

The recent speedy expansion of our cosmos is one of the most attractive advances on the observational landscape of fundamental physics. This expanding paradigm has been corroborated by the observational probes of numerous

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mounting astronomical evidences like type Ia supernovae (SNe Ia), the cosmic microwave background (CMB), large scale structure surveys (LSS) and Wilkinson Microwave Anisotropy Probe (WMAP) (Riess et al. 1998; Perlmutter 1999; Bennett 2003; Allen et al. 2004; Tegmark et al. 2004; Spergel et al. 2007). In evolution picture of cosmos, two stages of cosmic acceleration has been put forward by the researchers, the primordial inflationary stage (before radiation state of cosmos) and the recent as well as the final cosmic phases (after the matter dominated state). Inspite of all fascinating aspects of general relativity (GR), it gives rise to decelerated phases of cosmos as the nature of gravitational force is attractive and hence turns out to be incompatible with this primal fact. Thus it leads to the speculation that there is some other mysterious anti-gravitational source with some unusual particulars causing this cosmic expansion faster and is labeled as dark energy (DE).

The investigation of the complete evolutionary cosmic picture, from the Big Bang era to its final fate, has been widespread among the scientists. For this purpose, numerous efforts have been made based upon different strategies. These efforts are mainly grouped into two kinds: modifications in the matter sector of the Einstein-Hilbert Lagrangian density and the extension of the gravitational framework of GR by introducing some terms representing the DE source. The cosmological constant (Peebles and Ratra 2003), Chaplygin gas matter with its different modified versions (Bento et al. 2002; Benaoum 2012), scalar field models like quintessence (Caldwell et al. 1998; Steinhardt et al. 1999) are some leading examples of the candidates of the first group, while the modified theories of gravity are the representatives of the second group (Lobo 2008; Clifton et al. 2012). Some well-motivated examples of such theories include $f(R)$ gravity (the Ricci scalar of the Einstein-Hilbert action functional is replaced by a generic function $f(R)$) (Nojiri and Odintsov 2011; Bamba et al. 2012a), Gauss-Bonnet gravity (including Gauss-Bonnet invariant term) (Cognola et al. 2006; Li et al. 2007), $f(T)$ theory (based on the torsion tensor as well as its corresponding scalar) (Ferraro and Fiorini 2007; Bamba et al. 2012b; Setare and Darabi 2012; Setare and Mohammadi-pour 2012, 2013), $f(R, T)$ gravity (where $T$ is the trace of the energy-momentum tensor) (Harko et al. 2011; Sharif and Zubair 2014a, 2014b, 2014c, 2014d) and the scalar-tensor theories (based on both scalar and tensor fields) (Fujii and Maeda 2004; Brans 2005; Faraoni 2004) etc.

Recently, the introduction of non-minimal coupling between matter and
curvature in the context of modified theories has become a center of interest for the researchers. Bertolami et al. (2007) derived the dynamical equation for massive particles in $f(R)$ gravity by assuming an explicit interaction between the scalar curvature and density of matter. They concluded that the presence of this coupling yields an extra force. Bertolami and Sequeira (2009) discussed the energy condition bounds in $f(R)$ gravity involving a non-minimal interaction of curvature and matter and investigated the stability via Dolgov-Kawasaki criterion. Bertolami and Pramos (2014) studied the modification of Friedmann equation due to the inclusion of non-minimal coupling and provided two ways to handle the corresponding situation. Furthermore, they addressed the cosmological constant problem in such theory.

Another important and conceptually rich class consists of gravitational modifications involving torsion description of gravity. It is interesting to mention here that teleparallel equivalent of GR has been constructed by Einstein himself by including torsionless Levi-Civita connection instead of curvatureless Weitzenbck connection and the vierbein as the fundamental ingredient for the theory (Moller 1961; Pellegrini and Plebanski 1963; Hayashi and Shirafuji 1979; Maluf 2013). Consequently, the corresponding formulation replaces the Ricci tensor and Ricci scalar by the torsion tensor and torsion scalar respectively. Further, its modified form has been proposed and discussed by numerous authors like (Ferraro and Fiorini 2007; Bengochea and Ferraro 2009; Bamba and Geng 2011). Harko et al. (2014) constructed a more general type of $f(T)$ gravity by introducing a non-minimal interaction of torsion with matter in the Lagrangian density. They discussed the cosmological implications of this theory and concluded that the universe model may correspond to de Sitter, dark-energy-dominated, accelerating phase when model parameters are assigned to large values.

The energy bounds have been widespread to investigate various issues in GR and cosmology (Visser 1997; Santos and Alcaniz 2006; Santos et al. 2006; Gong, Y. et al. 2007; Gong and Wang 2007). Energy bounds have been explored in different modified theories to constrain the free variables like scalar-tensor theory (Sharif and Waheed 2013), modified Gauss-Bonnet gravity (Garcia 2011; Zhao 2012), $f(R)$ gravity (Santos et al. 2007; Santos et al. 2010), $f(T)$ gravity where $T$ is torsion scalar (Liu and Reboucas 2012), $f(R)$ gravity with nonminimal coupling to matter (Wang et al. 2010; Bertolami and Sequeira 2009), $f(R, \mathcal{L}_m)$ gravity (Wang and Liao 2012), $f(R, T)$ gravity (Sharif and Zubair 2013a) and $f(R, T, R_{\mu\nu}T^{\mu\nu})$ gravity (Sharif and Zubair 2013b). Sharif and Waheed (2013) explored the energy condition bounds in
the most general scalar-tensor theory involving second-order derivatives of scalar field and then discuss these conditions for different cases. Sharif and Zubair (2013a) have investigated the energy bounds in $f(R, T)$ gravity and by selecting a particular class of models, they studied the stability of power law solutions. In another paper (2013b), the same authors discussed the validity of the energy bounds in a newly modified gravity labeled as $f(R, T, R_{\mu\nu}T^{\mu\nu})$ gravity and also explore Dolgov-Kawasaki instability for two specific $f(R, T)$ models.

In the present work, we deal with the energy conditions bounds in a modified theory of gravity involving a non-minimal coupling of torsion scalar and matter by taking flat FRW model filled with perfect fluid. The paper has been designed in the following outline. In the next section, we provide the basic formulation of the field equations of this gravity. Section 3 provides a brief description of the energy bounds in the context of GR and also their extension to modified frameworks of gravity. In the same section, we analyze the obtained inequalities by choosing two recently proposed models. Section 4 concludes the whole discussion.

## 2 Modified $f(T)$ gravity with non-minimal torsion-matter coupling

A more general $f(T)$ gravity involving non-minimal coupling between torsion scalar and matter Lagrangian is defined by the action (Harko et al. 2014)

$$A = \frac{1}{2\kappa^2} \int dx^4 e \{ T + f_1(T) + [1 + \lambda f_2(T)]\mathcal{L}_m \}, \quad (1)$$

where $\kappa^2 = 8\pi G$, $f_i(T)$ ($i=1,2$) are arbitrary functions of torsion scalar, $\lambda$ is the coupling parameter and $\mathcal{L}_m$ denotes the Lagrangian density of matter part. The field equations in non-minimal $f(T)$ theory can be determined by varying the action with respect to the tetrad $e_i^\mu$ as

$$
\begin{align*}
(1 + f_1' + \lambda f_2' \mathcal{L}_m) \left[ e^{-1}\partial_\mu (e e_i^\sigma S_\sigma^{\mu\rho}) - e_i^\sigma T_\mu^{\mu\rho} S_\rho^{\mu}\right] + (f_1'' + \lambda f_2'' \mathcal{L}_m) \\
\times \partial_\mu T e_i^\sigma S_\sigma^{\rho\mu} + \frac{1}{4} e_i^\rho (f_1 + T) - \frac{1}{4} \lambda f_2' \partial_\mu T e_i^\sigma S_\sigma^{(m)\rho\mu} + \lambda f_2' e_i^\sigma S_\sigma^{\rho\mu} \partial_\mu \mathcal{L}_m \\
= 4\pi G (1 + \lambda f_2) e_i^\sigma T_\sigma^{(m)\rho},
\end{align*}
$$
\quad (2)
where prime indicates differentiation with respect to torsion scalar and $S_i^{(m)\rho\mu}$ is defined as
\begin{equation}
S_i^{(m)\rho\mu} = \frac{\partial L_m}{\partial \partial_{\rho} e_\mu^i}.
\end{equation}

Equation (2) reduces to the field equations in $f(T)$ theory of gravity for $\lambda = 0$ or $f_2(T) = 0$. The contribution to the energy momentum tensor of matter is defined as
\begin{equation}
T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu},
\end{equation}
where energy density and pressure are denoted by $\rho$ and $p$. We set the matter Lagrangian density as $L_m = -\rho$, which implies $S_i^{(m)\rho\mu} = 0$. We take the homogeneous and isotropic flat FRW metric defined as
\begin{equation}
ds^2 = dt^2 - a^2(t)dx^2,
\end{equation}
where $a(t)$ represents the scale factor and $dx^2$ contains the spatial part of the metric and corresponding tetrad components are $e_\mu^i = (1, a(t), a(t), a(t))$. In the FRW background, the field equations may be written as
\begin{align}
3H^2 &= 8\pi G[1 + \lambda(f_2 + 12H^2f'_{2})]\rho - \frac{1}{6}(f_1 + 12H^2f'_{1}), \quad (6) \\
\dot{H} &= -\frac{4\pi G(\rho + p)[1 + \lambda(f_2 + 12H^2f'_{2})]}{1 + f_1' - 12H^2f''_1 - 16\pi G\lambda \rho(f_2' - 12H^2f''_{2})}, \quad (7)
\end{align}
where $H = \dot{a}/a$ is the Hubble parameter and dot denotes the derivative with respect to cosmic time $t$. Equations (6) and (7) can be expressed as
\begin{align}
3H^2 &= 8\pi G\rho_{eff}, \quad -(2\dot{H} + 3H^2) = p_{eff}, \quad (8)
\end{align}
where $\rho_{eff}$ and $p_{eff}$ are the energy density and pressure respectively, defined by
\begin{align}
\rho_{eff} &= [1 + \lambda(f_2 + 12H^2f'_{2})]\rho - \frac{1}{16\pi G}(f_1 + 12H^2f'_{1}), \quad (9) \\
p_{eff} &= (\rho + p)\frac{[1 + \lambda(f_2 + 12H^2f'_{2})]}{1 + f_1' - 12H^2f''_1 - 16\pi G\lambda \rho(f_2' - 12H^2f''_{2})} \quad + \frac{1}{16\pi G}(f_1 + 12H^2f'_{1}) - [1 + \lambda(f_2 + 12H^2f'_{2})]\rho. \quad (10)
\end{align}
3 Energy Conditions

Here firstly, we will discuss the general procedure for energy condition bounds in GR and then extend it to modified gravity theories. Then by following the outlined procedure, we concentrate on flat FRW model filled with perfect fluid matter. Further we investigate these bounds by focusing on two different models of \( f(T) \) gravity.

3.1 Raychaudhuri Equation

In order to comprehend various cosmological geometries and some general results associated with the strong gravitational fields, energy bounds are of great interest. In GR, there are four explicit forms of energy conditions namely: the strong (SEC), null (NEC), dominant (DEC) and weak energy conditions (WEC) (Hawking and Ellis 1973; Carroll 2004). Basically, the SEC and NEC arise from the fundamental characteristic of gravitational force that it is attractive along with a well-known geometrical result describing the dynamics of nearby matter bits known as Raychaudhari equation. The Raychaudhari equation specifies the temporal evolution of expansion scalar \( \theta \) in terms of some tensorial quantities like Ricci tensor, shear tensor \( \sigma^{\mu\nu} \) and rotation \( \omega^{\mu\nu} \) for both the time and lightlike curves. Mathematically, it is given by relations

\[
\frac{d\theta}{d\tau} = -\frac{1}{3} \theta^2 - \sigma_{\mu\nu} \sigma^{\mu\nu} + \omega_{\mu\nu} \omega^{\mu\nu} - R_{\mu\nu} u^\mu u^\nu, \quad (11) \\
\frac{d\theta}{d\tau} = -\frac{1}{3} \theta^2 - \sigma_{\mu\nu} \sigma^{\mu\nu} + \omega_{\mu\nu} \omega^{\mu\nu} - R_{\mu\nu} k^\mu k^\nu. \quad (12)
\]

Due to attractive nature of gravity, geodesics become closer to each other satisfying \( \frac{d\theta}{d\tau} < 0 \) and hence yields converging time and lightlike congruences. To simplify the resulting inequalities, we can ignore the quadratic terms by taking the assumptions that there are infinitesimal distortions in geodesics (time or null) which is hypersurface orthogonal as well, i.e., \( \omega_{\mu\nu} = 0 \) (no rotation). Consequently, integration of the simplified Raychaudhari equations leads to \( \theta = -\tau R_{\mu\nu} u^\mu u^\nu = -\tau R_{\mu\nu} k^\mu k^\nu \) for timelike and null geodesics, respectively. Using \( \frac{d\theta}{d\tau} < 0 \), this can also be rearranged to

\[
R_{\mu\nu} u^\mu u^\nu \geq 0, \quad R_{\mu\nu} k^\mu k^\nu \geq 0.
\]
Since a relation of Ricci tensor in terms of energy-momentum tensor and its trace can be found by inverting the gravitational field equations (which interrelates both curvature (Ricci tensor) and matter sectors) as follows

\[ R_{\mu\nu} = T_{\mu\nu} - \frac{T}{2} g_{\mu\nu}. \]  \hspace{1cm} (13)

Therefore the inequalities of energy bounds take the following forms

\[ R_{\mu\nu} u^\mu u^\nu = (T_{\mu\nu} - \frac{T}{2} g_{\mu\nu}) u^\mu u^\nu \geq 0, \]  \hspace{1cm} (14)

\[ R_{\mu\nu} k^\mu k^\nu = (T_{\mu\nu} - \frac{T}{2} g_{\mu\nu}) k^\mu k^\nu \geq 0. \]  \hspace{1cm} (15)

In the case of perfect fluid matter, the SEC and NEC given by (14) and (15) impose the following constraints \( \rho + 3p \geq 0 \) and \( \rho + p \geq 0 \) to be satisfied, while the WEC and DEC require these bounds \( \rho \geq 0 \) and \( \rho \pm p \geq 0 \), respectively for consistency purposes.

The concept of energy conditions can be extended to the case of modified theories of gravity using Raychaudhari equation, a purely geometrical relation. Thus, its interesting particulars like focussing of geodesic congruences along with the attractive nature of gravity can be used to formulate these bounds in any modified gravitational framework. In such cases, we take the total matter contents of the universe behaving as perfect fluid and consequently, the respective conditions can be defined by simply replacing the energy density and pressure, respectively by an effective energy density and effective pressure as follows

\[ \text{NEC :} \quad \rho^{\text{eff}} + p^{\text{eff}} \geq 0, \]
\[ \text{SEC :} \quad \rho^{\text{eff}} + p^{\text{eff}} \geq 0, \quad \rho^{\text{eff}} + 3p^{\text{eff}} \geq 0, \]
\[ \text{WEC :} \quad \rho^{\text{eff}} \geq 0, \quad \rho^{\text{eff}} + p^{\text{eff}} \geq 0, \]
\[ \text{DEC :} \quad \rho^{\text{eff}} \geq 0, \quad \rho^{\text{eff}} \pm p^{\text{eff}} \geq 0. \]  \hspace{1cm} (16)

It is also interesting to mention here that the violation of these energy bounds ensure the existence of the ghost instabilities (some interesting feature of modified gravity that support the cosmic acceleration due to DE).
3.2 Energy Conditions in modified $f(T)$ Gravity

Using Eqs. (9) and (10), the energy conditions for $f(T)$ gravity with non-minimal torsion-matter coupling are obtained as

**NEC**:  
\[ \rho_{\text{eff}} + p_{\text{eff}} = \frac{(\rho + p)(1 + \lambda(f_2 + 12H^2 f'_2))}{1 + f'_1 - 12H^2 f''_1 - 16\pi G\lambda \rho(f'_2 - 12H^2 f''_2)} \geq 0, \quad (17) \]

**WEC**:  
\[ \rho_{\text{eff}} = [1 + \lambda(f_2 + 12H^2 f'_2)]\rho - \frac{1}{16\pi G}(f_1 + 12H^2 f'_1) \geq 0, \quad (18) \]
\[ \rho_{\text{eff}} + p_{\text{eff}} \geq 0, \]

**SEC**:  
\[ \rho_{\text{eff}} + 3p_{\text{eff}} = -2[1 + \lambda(f_2 + 12H^2 f'_2)]\rho + \frac{1}{8\pi G}(f_1 + 12H^2 f'_1) + \frac{3(\rho + p)(1 + \lambda(f_2 + 12H^2 f'_2))}{1 + f'_1 - 12H^2 f''_1 - 16\pi G\lambda \rho(f'_2 - 12H^2 f''_2)} \geq 0, \quad (19) \]
\[ \rho_{\text{eff}} + p_{\text{eff}} \geq 0, \]

**DEC**:  
\[ \rho_{\text{eff}} - p_{\text{eff}} = 2[1 + \lambda(f_2 + 12H^2 f'_2)]\rho - \frac{1}{8\pi G}(f_1 + 12H^2 f'_1) - \frac{(\rho + p)(1 + \lambda(f_2 + 12H^2 f'_2))}{1 + f'_1 - 12H^2 f''_1 - 16\pi G\lambda \rho(f'_2 - 12H^2 f''_2)} \geq 0, \quad (20) \]
\[ \rho_{\text{eff}} + p_{\text{eff}} \geq 0, \quad \rho_{\text{eff}} \geq 0. \]

The inequalities (17)-(20) represent the null, weak, strong and dominant energy conditions in the context of $f(T)$ theory with nonminimal torsion-matter coupling for FRW spacetime. In the following, we consider some specific functional forms for the Lagrangian (1) to develop constraints under these conditions. Harko et al. (2014) presented some viable models of $f(T)$ gravity with nonminimal torsion-matter coupling and discuss different evolutionary phases depending on the coupling and free parameters. It is shown that different model parameters can result in de Sitter, DE dominated and accelerating phases.

### 3.3 $f_1(T) = -\Lambda + \alpha_1 T^2$, $f_2(T) = \beta_1 T^2$

In the first place, we consider the model (Harko et al. 2014)
\[ f_1(T) = -\Lambda + \alpha_1 T^2, \quad f_2(T) = \beta_1 T^2, \quad (21) \]
where $\alpha_1$, $\beta_1$ are model parameters and $\Lambda > 0$ is a constant. These functions involve quadratic contribution from $T$ and appear as corrections to teleparallel theory. The derivatives of these functions are defined as $f_1' = 2\alpha_1 T$, $f_2' = 2\beta_1 T$ and $f_2'' = 2\beta_1$. Since torsion is defined in terms of Hubble parameter so we can change the functional dependence from $T$ to $H$ as $f_1(T) \equiv f_1(H) = -\Lambda + \alpha H^4$ and $f_2(T) \equiv f_2(H) = \beta H^4$, where $\alpha = 36\alpha_1$, $\beta = 36\beta_1$. For the derivatives of $f_1$ and $f_2$, we have $f_1'(H) = -\alpha H^2/3$, $f_1''(H) = \alpha/18$, $f_2'(H) = -\beta H^2/3$ and $f_2''(H) = \beta/18$.

In FRW background, the constraints to fulfill the WEC energy condition ($\rho_{\text{eff}} \geq 0$, $\rho_{\text{eff}} + p_{\text{eff}} \geq 0$) for such model can be represented as

$$\rho (1 - 3\lambda \beta H^4) + \frac{1}{2}(\Lambda + 3\alpha H^4) \geq 0, \quad (22)$$

$$\rho + p\{(1 - 3\lambda \beta H^4)/(1 + (2\lambda \beta \rho - \alpha) H^2)\} \geq 0. \quad (23)$$

In terms of present day value of $H$, the inequality (22) can be satisfied if we set $\beta < 0$ with $(\Lambda, \alpha, \lambda) > 0$. For the inequality (23), if one set $\beta < 0$ then it requires $(2\lambda \beta \rho + \alpha) < 1$. It can also be satisfied by choosing the parameters such that $2\lambda \beta \rho > \alpha$ and $1 > 3\lambda \beta H^4$.

To be more explicit about the validity of these inequalities, we consider the power law cosmology

$$a(t) = a_0 t^m, \quad (24)$$

where $m$ is a positive real number. If $0 < m < 1$, then the required power law solution is decelerating, while for $m > 1$, it exhibits accelerating behavior. We set $m > 1$ with $\rho = \rho_0 t^{-3m}$. To explore the validity of inequalities (22) and (23), we present the evolution of WEC for various values of parameters. In Figure 1, we show the variation of inequalities (22) and (23) versus $t$ and $\lambda$. We fix the model parameters $\alpha$ and $\beta$ to show the evolution for different values of $\lambda$. It is shown that WEC can be satisfied if $\lambda > 0$ with $\alpha = 0.01$ and $\beta = 0.02$. In Figure 2, we fix $\lambda$ and vary the parameter $\beta$, the plots show that WEC can be met for both positive and negative values of $\beta$. We also present the evolution of WEC for various values of parameter $\alpha$ in Figure 3. The left plot shows that inequality (22) is satisfied only for positive values of parameter $\alpha$ and violates for negative values whereas in right plot it can be met for all values of $\alpha$. The inequality (23) represents the NEC in non-minimally coupled $f(T)$ gravity for the model (21). The constraints to fulfill the SEC and DEC in power law cosmology for above model can be found from inequalities (19) and (20). We also show the evolution of SEC.
Figure 1: Evolution of WEC versus $t$ and $\lambda$ for $\alpha = 0.01$, $\beta = 0.02$, $\Lambda = .5$ and $m = 2$. The left and right plots correspond to inequalities (22) and (23) respectively.

Figure 2: Evolution of WEC versus $t$ and $\beta$ for $\alpha = 0.01$, $\lambda = .1$, $\Lambda = .5$ and $m = 2$. The left and right plots correspond to inequalities (22) and (23) respectively.

Figure 3: Evolution of WEC versus $t$ and $\alpha$ for $\beta = .02$, $\lambda = .1$, $\Lambda = .5$ and $m = 2$. The left and right plots correspond to inequalities (22) and (23) respectively.
Figure 4: Evolution of SEC (a) versus $t$ and $\lambda$ for $\alpha = -0.06$, $\beta = 0.02$, $\Lambda = 0.1$ and $m = 2$ (b) versus $t$ and $\alpha$ for $\beta = 0.02$ (c) versus $t$ and $\beta$ for $\alpha = -0.06$. and DEC in Figures 4 and 5 for the $f(T)$ model (21). In plot 4(a), we show the variation of SEC versus $\lambda$ and $t$. Here, SEC can be met for $\lambda > 0$ if $\alpha < 0$. The evolution of SEC is also presented for various values of $\alpha$ and $\beta$. We find that SEC holds for $\alpha < 0$ with fixed values of $\lambda$, $\beta$ and $\Lambda$ as shown in Figure 4(b). In Figure 4(c), we vary $\beta$ for fixed values of other parameters and find that SEC can be satisfied for any value of $\beta$. We also present the evolution of DEC for different values of parameters in Figure 5. In this case DEC can be satisfied for $\lambda > 0$ with $\alpha > 0$. The evolution for the parameter $\alpha$ and $\beta$ is also shown in Figures 5(b) and 5(c).

3.4 $f_1(T) = -\Lambda, \quad f_2(T) = \alpha_2 T + \beta_2 T^2$

In second example, we consider the model defined by the following functions (Harko et al. 2014)

$$f_1(T) = -\Lambda, \quad f_2(T) = \alpha_2 T + \beta_2 T^2,$$  \quad (25)
Figure 5: Evolution of DEC (a) versus $t$ and $\lambda$ for $\alpha = 0.01$, $\beta = 0.02$, $\Lambda = .1$ and $m = 2$ (b) versus $t$ and $\alpha$ for $\beta = 0.02$ and $\lambda = 0.1$ (c) versus $t$ and $\beta$ for $\alpha = 0.01$. 
where $\alpha_2$ and $\beta_2$ are parameters for the model (25). We express the functions $f_1$ and $f_2$ in terms of $H$ as $f_1(H) = -\Lambda$, $f_2(H) = \gamma T + \delta T^2$, where $\gamma = -6\alpha_2$ and $\delta = 36\beta_2$. Similarly, the derivatives of $f_1$ and $f_2$ are given by $f_1'(H) = 0$, $f_2'(H) = -\gamma/6 - \delta H^2/3$ and $f_2''(H) = \delta/18$.

The WEC ($\rho_{eff} \geq 0$, $\rho_{eff} + p_{eff} \geq 0$) for the model (25) requires the following inequalities to be satisfied

$$\rho + \Lambda/2 - \lambda \rho(\gamma H^2 + 3\delta H^4) \geq 0,$$ (26)

$$(\rho + p)[1 - \lambda(\gamma H^2 + 3\delta H^4)]/[1 + 2\lambda(\gamma/6 + \delta H^2)] \geq 0.$$ (27)

It can be seen that above inequalities require $(\gamma, \delta) < 0$ to satisfy the WEC in this case. We present the evolution of constraints (26) and (27) for different choices of parameters in Figures 6-8. In Figure 6, we develop the range for coupling parameter $\lambda$ to satisfy the WEC. WEC can be satisfied for $\lambda > 0$ if $\gamma = -0.1$, $\delta = -0.2$, $\Lambda = 1$ and $m = 2$. In Figure 7, evolution of WEC is shown for different values of parameter $\gamma$. WEC can be met for $\gamma > 0$ but it needs some specific value for the parameter $\delta$, like $\gamma = \{0, \ldots, 10\}$ requires $\delta = 160$ as shown in Figure 8. We also explore different possibilities to validate the SEC and DEC for the model (25). Figure 9 shows the evolution of SEC representing the variation for the parameters $\lambda$, $\gamma$ and $\delta$. The SEC can be satisfied for $\lambda > 0$ if $\gamma = 1$ and $\delta = 2$ as shown in Figure 9(a). We find that SEC can hold for $\gamma > 0$ but it needs some specific value for the parameter $\delta$, like $\gamma = \{0, \ldots, 10\}$ requires $\delta = 160$ as shown in Figure 9(b). Similarly,
Figure 7: Evolution of WEC versus $t$ and $\gamma$ for $\delta = -0.2$, $\lambda = 0.01$ and $m = 2$ for the model (25). The left and right plots correspond to constraints (26) with $\Lambda = 1$ and (27) respectively.

Figure 8: Evolution of WEC versus $t$ and $\delta$ for $\gamma = -0.1$, $\lambda = 0.01$ and $m = 2$ for the model (25). The left and right plots correspond to constraints (26) with $\Lambda = 1$ and (27) respectively.
Figure 9: Evolution of SEC (a) versus $t$ and $\lambda$ for $\gamma = 1$, $\delta = 2$, $\Lambda = 1$ and $m = 2$ (b) versus $t$ and $\gamma$ for $\delta = 160$, $\lambda = 2$ (c) versus $t$ and $\delta$ for $\gamma = 20$.

one can fix $\gamma$ to constrain $\delta$ for the validity of SEC. Figure 9(c) depicts the validity of SEC for $\delta = \{0, \ldots, 10\}$ with $\gamma = 20$. We also explore the validity of DEC in Figure 10 and present the respective constraints. One need to set negative values for $\gamma$ and $\delta$ to validate the DEC for $\lambda > 0$ as shown in plot 10(a). We find that DEC can be met for negative values of parameters $\gamma$ and $\delta$. We also test the positive values these parameters as shown in plots 10(b) and 10(c). These plots show that DEC is satisfied for $\gamma = \{-10, \ldots, 10\}$ if $\delta = -12$ and similarly for $\delta = \{-10, \ldots, 10\}$ it requires $\gamma = -24$. 

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Figure 10: Evolution of DEC (a) versus $t$ and $\lambda$ for $\gamma = -1$, $\delta = -2$, $\Lambda = 1$ and $m = 2$ (b) versus $t$ and $\gamma$ for $\delta = -12$, $\lambda = 2$ (c) versus $t$ and $\delta$ for $\gamma = -24$. 
4 Conclusions

Modified theories of gravity have attained significant attention to explain the observed accelerated cosmic expansion. In this perspective, $f(T)$ gravity has appeared as a handy candidate and many interesting results have been discussed in this theory (Ferraro and Fiorini 2007; Bamba et al. 2012b; Setare and Darabi 2012; Setare and Mohammadi-pour 2012, 2013; Rodrigues et al. 2012, 2013, 2014; Karami et al. 2013). Recently, an extension of $f(T)$ gravity involving non-minimal matter torsion coupling is presented in (Harko et al. 2014) which introduces two independent functions of torsion. In such theories, one can develop different cosmological models depending on the choice of these functions. The various forms of Lagrangian raises a question how to constrain such theory on physical grounds. In this paper, we have developed constraints on specific forms $f(T)$ by examining the respective energy conditions. We have derived the energy conditions directly from the effective energy momentum tensor under the transformation $\rho \rightarrow \rho_{eff}$ and $p_{eff} \rightarrow p_{eff}$.

To illustrate how these conditions can constrain the $f(T)$ gravity with non-minimal matter torsion coupling, we consider two particular forms of Lagrangian namely, (i) $f_1(T) = -\Lambda + \alpha_1 T^2$, $f_2(T) = \beta_1 T^2$ (ii) $f_1(T) = -\Lambda$, $f_2(T) = \alpha_2 T + \beta_2 T^2$. We have set the power law cosmology and developed some constraints on coupling parameter $\lambda$ and model parameters $\alpha_1$, $\alpha_2$, $\beta_1$ and $\beta_2$. We have also analyzed the role of these model parameters graphically in Figures (1)-(10) by exploring the evolution of WEC, SEC and DEC for both selected models. For the first model, we have discussed the WEC, DEC, SEC conditions for different cases of these parameters with $m > 1$, $0 < \Lambda < 1$. It is found that WEC can be satisfied for $\lambda > 0$, where we take $0 < \alpha$, $\beta < 1$. Further, it can also be satisfied for both negative and positive $\beta$ values if we fix $0 < \lambda$ while $0 < \alpha < 1$. Also, if we fix both $\beta$ and $\lambda$ as $0 < \beta$, $\lambda < 1$, then WEC can be satisfied only for $\alpha > 0$. Further, it is found that SEC will be satisfied for this model if $\alpha < 0$ and $\lambda > 0$ by fixing other parameters. It is also noticed that DEC will be satisfied for this model for $\alpha$, $\lambda > 0$ with fixed values of other parameters.

For the second model, the involved parameters are $m$, $\gamma$, $\delta$ and $\Lambda$ and we have found the appropriate ranges of these parameters via these bounds. Firstly, we have fixed $m > 1$ and $\Lambda = 1$ with $-1 < \gamma, \delta < 0$ and it is found that WEC may be satisfied only for $\lambda > 0$. Further, if we set $0 < \lambda < 1$, $\delta < 0$ then WEC remains valid for $-10 \leq \gamma \leq 10$. Likewise, if $0 < \lambda < 1$, $\gamma < 0$, $\delta < 0$, and $\lambda > 0$
then this will be satisfied for $-10 \leq \delta \leq 10$. We have found that DEC can be met for negative values of parameters $\gamma$ and $\delta$ while the SEC can be satisfied for $\lambda > 0$, if $\gamma = 1$, $\delta > 1$. It would be interesting to check these energy bounds for other models of $f(T)$ gravity and develop the constraints on the corresponding parameters.

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