Charged currents, color dipoles and $xF_3$ at small $x$.

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Abstract

We develop the light-cone color dipole description of highly asymmetric diffractive interactions of left-handed and right-handed electroweak bosons. We identify the origin and estimate the strength of the left-right asymmetry effect in terms of the light-cone wave functions. We report an evaluation of the small-$x$ neutrino-nucleon DIS structure functions $xF_3$ and $2xF_1$ and present comparison with experimental data.

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At small Bjorken $x$ the driving term of the inclusive/diffractive excitation of charmed and (anti)strange quarks in the charged current (CC) neutrino deep inelastic scattering (DIS) is the $W^+$-gluon/Pomeron fusion,

$$W^+ g \rightarrow c\bar{s} \quad (1)$$

and

$$W^+ \mathbf{p} \rightarrow c\bar{s}. \quad (2)$$

Different aspects of the CC inclusive and diffractive DIS have been discussed in [1, 2].

In the color dipole approach [3, 4] (for the review see [5]) the small-$x$ DIS is treated in terms of the interaction of the $c\bar{s}$ color dipole of size $r$ with the target proton which is described by the beam- and flavor-independent color dipole cross section $\sigma(x, r)$. Once the light-cone wave function (LCWF) of a color dipole state is specified the evaluation of observable quantities becomes a routine quantum mechanical procedure. In this communication we extend the color dipole analysis onto the CC DIS with particular emphasis on the left-right asymmetry of diffractive interactions of electroweak bosons of different helicity. We derive the relevant LCWF and evaluate the structure functions $xF_3$, $\Delta xF_3$ and $2xF_1$. We focus on the vacuum exchange dominated leading $\log(1/x)$ region of $x \lesssim 0.01$.

At small $x$ the contribution of excitation of open charm/strangeness to the absorption cross section for scalar, ($\lambda = 0$), left-handed, ($\lambda = -1$), and right-handed, ($\lambda = +1$), $W$-boson of virtuality $Q^2$, is given by the color dipole factorization formula [6, 7]

$$\sigma_\lambda(x, Q^2) = \int dz d^2r \sum_{\lambda_1, \lambda_2} |\Psi_{\lambda_1, \lambda_2}(z, r)|^2 \sigma(x, r) . \quad (3)$$

In Eq. (3) $\Psi_{\lambda_1, \lambda_2}(z, r)$ is the LCWF of the $|c\bar{s}\rangle$ state with the $c$ quark carrying fraction $z$ of the $W^+$ light-cone momentum and $\bar{s}$ with momentum fraction $1-z$. The $c$- and $\bar{s}$-quark helicities are $\lambda_1 = \pm 1/2$ and $\lambda_2 = \pm 1/2$, respectively. The $W^+ \rightarrow c\bar{s}$-transition vertex is specified as follows:

$$gU_{cs} \bar{c}\gamma_\mu(1 - \gamma_5)s,$$

where $U_{cs}$ is an element of the CKM-matrix and the weak charge $g$ is related to the Fermi coupling constant $G_F$,

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{m_W^2}. \quad (4)$$
The polarization states of W-boson carrying the laboratory frame four-momentum
\[ q = (\nu, 0, 0, \sqrt{\nu^2 + Q^2}) \] (5)
are described by the four-vectors \( e_\lambda \), with
\[ e_0 = \frac{1}{Q}(\sqrt{\nu^2 + Q^2}, 0, 0, \nu) , \]
\[ e_\pm = \pm \frac{1}{\sqrt{2}}(0, 1, \pm i, 0) , \] (6)
the unit vectors \( \vec{e}_x \) and \( \vec{e}_y \) being in \( q_x \) - and \( q_y \) -direction, respectively. We find it convenient to use the basis of helicity spinors of Ref. [8]. Then, vector \((V)\) and axial-vector \((A)\) components of the LCWF
\[ \Psi_{\lambda_1,\lambda_2}(z, r) = V_{\lambda_1,\lambda_2}(z, r) - A_{\lambda_1,\lambda_2}(z, r) \] (7)
are as follows:
\[ V_{0,\lambda_1,\lambda_2}(z, r) = \frac{\sqrt{\alpha W N_c}}{2\pi Q} \left\{ \delta_{\lambda_1,\lambda_2} \left[ 2Q^2 z (1 - z) + (m - \mu) [(1 - z) m - z \mu] \right] K_0(\varepsilon r) - i \delta_{\lambda_1,\lambda_2}(2\lambda_1) e^{-i2\lambda_1\phi} (m - \mu) \varepsilon K_1(\varepsilon r) \right\} , \] (8)
\[ A_{0,\lambda_1,\lambda_2}(z, r) = \frac{\sqrt{\alpha W N_c}}{2\pi Q} \left\{ \delta_{\lambda_1,\lambda_2} \left[ 2Q^2 z (1 - z) + (m + \mu) [(1 - z) m + z \mu] \right] K_0(\varepsilon r) + i \delta_{\lambda_1,\lambda_2}(2\lambda_1) e^{-i2\lambda_1\phi} (m + \mu) \varepsilon K_1(\varepsilon r) \right\} . \] (9)
If \( \lambda = \pm 1 \)
\[ V_{\lambda,\lambda_2}(z, r) = -\frac{\sqrt{2\alpha W N_c}}{2\pi} \left\{ \delta_{\lambda_1,\lambda_2} \delta_{\lambda_2,2\lambda_1} [(1 - z) m + z \mu] K_0(\varepsilon r) - i (2\lambda_1) \delta_{\lambda_1,\lambda_2} e^{i\lambda\phi} [(1 - z) \delta_{\lambda_1,\lambda_2} + z \delta_{\lambda_2,2\lambda_1}] \varepsilon K_1(\varepsilon r) \right\} , \] (10)
\[ A_{\lambda,\lambda_2}(z, r) = \frac{\sqrt{2\alpha W N_c}}{2\pi} \left\{ \delta_{\lambda_1,\lambda_2} \delta_{\lambda_2,2\lambda_1} [(1 - z) m - z \mu] K_0(\varepsilon r) + i \delta_{\lambda_1,\lambda_2} e^{i\lambda\phi} [(1 - z) \delta_{\lambda_1,\lambda_2} + z \delta_{\lambda_2,2\lambda_1}] \varepsilon K_1(\varepsilon r) \right\} . \] (11)
where
\[ \varepsilon^2 = z(1-z)Q^2 + (1-z)m^2 + z\mu^2 \]  
and \( K_\nu(x) \) is the modified Bessel function. We do not consider Cabibbo-suppressed transitions and
\[ \alpha_W = g^2/4\pi. \]

The quark and antiquark masses are \( m \) and \( \mu \), respectively. The azimuthal angle of \( \mathbf{r} \) is denoted by \( \phi \). To switch \( W^+ \to W^- \) one should perform the replacement \( m \leftrightarrow \mu \) in the equations above.

The diagonal elements of density matrix
\[ \rho_{\lambda\lambda'} = \sum_{\lambda_1, \lambda_2} \Psi_\lambda^{\lambda_1, \lambda_2} (\Psi_{\lambda'}^{\lambda_1, \lambda_2})^* \]  
entering Eq. (3) are as follows:
\[ \rho_{00}(z, \mathbf{r}) = \sum_{\lambda_1, \lambda_2} \left( |V_0^{\lambda_1, \lambda_2}|^2 + |A_0^{\lambda_1, \lambda_2}|^2 \right) \]
\[ = \frac{2\alpha_W N_c}{(2\pi)^2 Q^2} \left\{ \left[ 2Q^2 z(1-z) + (m-\mu)[(1-z)m-z\mu] \right]^2 + \left[ 2Q^2 z(1-z) + (m+\mu)[(1-z)m+z\mu] \right]^2 \right\} \times K_0(\varepsilon r)^2 + [(m-\mu)^2 + (m+\mu)^2] \varepsilon^2 K_1(\varepsilon r)^2 \]  
\[ \rho_{+1-1}(z, \mathbf{r}) = |\Psi_{+1}^{1/2-1/2}|^2 + |\Psi_{-1}^{1/2+1/2}|^2 \]
\[ = \frac{8\alpha_W N_c}{(2\pi)^2} (1-z)^2 \left[ m^2 K_0(\varepsilon r)^2 + \varepsilon^2 K_1(\varepsilon r)^2 \right] \]  
\[ \rho_{-1+1}(z, \mathbf{r}) = |\Psi_{-1}^{1/2+1/2}|^2 + |\Psi_{-1}^{1/2-1/2}|^2 \]
\[ = \frac{8\alpha_W N_c}{(2\pi)^2} z^2 \left[ \mu^2 K_0(\varepsilon r)^2 + \varepsilon^2 K_1(\varepsilon r)^2 \right] . \]  

At \( Q^2 \to 0 \) the terms \( \sim m^2/Q^2, \mu^2/Q^2 \) in Eq. (14) remind us that \( W \) interacts with the current which is not conserved while the S-wave terms in Eqs. (15) and (16) proportional to \( m^2 \) and \( \mu^2 \) remind us that this current is the parity violating \( (V-A) \)-current.
The density of quark-antiquark $c\bar{s}$ states in the transversely polarized $W$-boson is

$$
\rho_{TT} = \frac{1}{2} (\rho_{+1+1} + \rho_{-1-1})
$$

$$
= \frac{4\alpha_W N_c}{(2\pi)^2} \left\{ \left[ (1 - z)^2 m^2 + z^2 \mu^2 \right] K_0(\varepsilon r)^2 
+ \left[ (1 - z)^2 + z^2 \right] \varepsilon^2 K_1(\varepsilon r)^2 \right\}.
$$

(17)

One can see that our $\rho_{00}$ and $\rho_{TT}$ coincide with the probability densities $|\Psi_L|^2$ and $|\Psi_T|^2$ of Ref. [1] (see also Ref. [9] where $z$-dependence of transverse and longitudinal CC cross sections has been discussed).

The momentum partition asymmetry of both $\rho_{-1-1}$ and $\rho_{+1+1}$ is striking, the left-handed quark in the decay of left-handed $W^+$ gets the lion’s share of the $W^+$ light-cone momentum. The nature of this phenomenon is very close to the nature of well known spin-spin correlations in the neutron $\beta$-decay. The observable which is strongly affected by this left-right asymmetry is the structure function of the neutrino-nucleon DIS named $F_3$. Its definition in terms of $\sigma_R$ and $\sigma_L$ of Eq. (3) is as follows:

$$
2xF_3(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_W} \left[ \sigma_L(x, Q^2) - \sigma_R(x, Q^2) \right].
$$

(18)

To estimate consequences of the left-right asymmetry for $F_3$ at high $Q^2$, such that

$$
\frac{m^2}{Q^2} \ll 1, \quad \frac{\mu^2}{Q^2} \ll 1,
$$

(19)

one should take into account that the dipole cross-section $\sigma(x, r)$ in Eq. (3) is related to the un-integrated gluon structure function $F(x, \kappa^2) = \partial G(x, \kappa^2)/\partial \log \kappa^2$, as follows [10]:

$$
\sigma(x, r) = \frac{\pi^2}{N_c} r^2 \alpha_S(r^2) \int \frac{d\kappa^2 \kappa^2}{(\kappa^2 + \mu_G^2)^2} \frac{4[1 - J_0(\kappa r)]}{\kappa^2 r^2} F(x_g, \kappa^2).
$$

(20)

In the Double Leading Logarithm Approximation (DLLA), i.e. for small dipoles,

$$
\sigma(x, r) \approx \frac{\pi^2}{N_c} r^2 \alpha_S(r^2) G(x_g, A/r^2),
$$

(21)

where $\mu_G = 1/R_c$ is the inverse correlation radius of perturbative gluons and $A \simeq 10$ comes from properties of the Bessel function $J_0(y)$. Because of scaling violation $G(x, Q^2)$ rises with $Q^2$, but the product $\alpha_S(r^2)G(x, A/r^2)$ is approximately flat in $r^2$. At large $Q^2$ the leading
contribution to $\sigma_\lambda(x, Q^2)$ comes from the P-wave term, \(\varepsilon^2 K_1(\varepsilon r)^2\), in Eqs. (15) and (16). The asymptotic behavior of the Bessel function, \(K_1(x) \simeq \exp(-x) / \sqrt{2\pi x}\) makes the r-integration rapidly convergent at \(\varepsilon r > 1\). Integration over r in Eq. (3) yields

\[
\sigma_L \propto \int_0^1 dz \frac{z^2}{\varepsilon^2} \alpha_S G \sim \frac{\alpha_S G}{Q^2} \log \frac{Q^2}{\mu^2} \log \frac{m^2}{\mu^2}
\]

and similarly

\[
\sigma_R \propto \int_0^1 dz \frac{(1 - z)^2}{\varepsilon^2} \alpha_S G \sim \frac{\alpha_S G}{Q^2} \log \frac{Q^2}{m^2} \log \frac{m^2}{\mu^2}.
\]

The left-right asymmetry certainly affects also the slowly varying product $\alpha_S G$ which for the purpose of crude estimate is taken at some rescaled virtuality \(\sim Q^2\) which is approximately/logarithmically the same for $\sigma_L$ and $\sigma_R$. Hence,

\[
\sigma_L - \sigma_R \propto \frac{\alpha_S G}{Q^2} \log \frac{m^2}{\mu^2}.
\]

Notice that in spite of the apparent asymmetry of the z-distribution both $\sigma_L$ and $\sigma_R$ get equal scaling contributions from the integration domains near by the peaks $z = 1$ and $z = 0$, respectively. Therefore, $xF_3$ is free of the end-point contributions.

At $Q^2 \to 0$ and $\mu^2/m^2 \ll 1$ the cross sections $\sigma_L$ and $\sigma_R$ are as follows:

\[
\sigma_L \propto \frac{\alpha_S G}{m^2} \log \frac{m^2}{\mu^2}, \quad \sigma_R \propto \frac{\alpha_S G}{m^2}.
\]

We evaluate $xF_3(x, Q^2)$ making use of Eqs. (3) and (20) with the differential gluon density function $F(xg, \kappa^2)$ determined in [11]. As reported in [11], the approach developed works very well in the perturbative region of high $Q^2$ and small $x$ ($x \ll 0.01$). Besides, a realistic extrapolation of $F(xg, \kappa^2)$ into the soft region allows calculations at lowest $Q^2$ also [11]. In our calculations for $Q^2 \ll M^2 = 2(m^2 + \mu^2)$ the gluon density $F(xg, \kappa^2)$ enters Eq. (20) at the gluon momentum fraction $x_g = x(1 + M^2/Q^2)$. For large virtualities, $Q^2 \gg M^2$, we put $x_g = 2x$. Direct evaluation of the proton DIS structure function $F_{2p}(x, Q^2)$ shows that this prescription corresponding to the collinear DLLA ensures a good description of experimental data on the light and heavy flavor electro-production in a wide range of the photon virtualities down to $Q^2 \sim 1$ GeV$^2$. The constituent quark masses are as follows $m_u = m_d = 0.2$ GeV, $m_s = 0.35$ GeV and $m_c = 1.3$ GeV.
The $xF_3$ data reported by the CCFR Collaboration are presented in Figure 1. Shown is the $Q^2$-dependence of $xF_3$ for several smallest values of $x$ [12]. It should be emphasized that we focus on the vacuum exchange contribution to $xF_3$ corresponding to the excitation of the $c\bar{s}$ state in the process (1). Therefore, the structure function $xF_3$ differs from zero only due to the strong left-right asymmetry of the light-cone $|c\bar{s}\rangle$ Fock state. Shown by the solid line in Fig. 1 is the Pomeron exchange contribution to $xF_3$. The latter can be interpreted in terms of parton densities as the sea-quark component of $xF_3$.

Looking at Figure 1 one should bear in mind that the smallest available values of $x$ are in fact only moderately small and there is also quite significant valence contribution to $xF_3$. The valence term, $xV$ is the same for both $\nu N$ and $\bar{\nu} N$ structure functions of an iso-scalar nucleon. The sea-quark term in the $xF_3^{\nu N}$ denoted by $xS(x, Q^2)$ has opposite sign for $xF_3^{\bar{\nu} N}$, the substitution $m \leftrightarrow \mu$ in Eqs. (15) and (16) entails $\sigma_L \leftrightarrow \sigma_R$. Therefore,

$$xF_3^{\nu N} = xV + xS,$$

(26)

and

$$xF_3^{\bar{\nu} N} = xV - xS.$$  

(27)
One can combine the $\nu N$ and $\bar{\nu} N$ structure functions to isolate the Pomeron exchange term,

$$\Delta xF_3 = xF_3^{\nu N} - xF_3^{\bar{\nu} N} = 2xS.$$  \hspace{1cm} (28)

The extraction of $\Delta xF_3$ from CCFR $\nu_\mu Fe$ and $\bar{\nu}_\mu Fe$ differential cross section in a model-independent way has been reported in [13]. Figure 2 shows the extracted values of $\Delta xF_3$ as a function of $Q^2$ for two smallest values of $x$. Also shown are the results of our calculations.

After evaluating the difference of left and right cross sections let us turn to their sum and, as a consistency check, evaluate the structure function

$$2xF_1(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_W} \sigma_T(x, Q^2),$$  \hspace{1cm} (29)

where

$$\sigma_T = \frac{1}{2} \left[ \sigma_L(x, Q^2) + \sigma_R(x, Q^2) \right].$$  \hspace{1cm} (30)

The CCFR Collaboration measurements [14] of the structure function $2xF_1$ as a function of $Q^2$ for three values of $x$ are shown in Fig. 3. Theory and experiment here are in qualitatively the same relations as in Fig. 1. In small-$x$ region, $x < 0.01$, dominated by the Pomeron exchange our estimates are in agreement with data. For larger $x$ the non-vacuum contributions enter the game and a certain divergence shows up. This divergence will increase if we take
Figure 3: CCFR measurements of $2xF_1(x, Q^2)$ [14] compared with our estimates. Curves show the vacuum exchange contribution to $2xF_1(x, Q^2)$.

into account the nuclear effects. Indeed, the CCFR/NuTeV structure functions $xF_3^{\nu N}$ and $xF_3^{\bar{\nu}N}$ are extracted from the $\nu Fe$ and $\bar{\nu}Fe$ data. The nuclear thickness factor, $T(b) = \int dz n(\sqrt{z^2 + b^2})$, where $b$ is the impact parameter and $n(r)$ is the nuclear matter density, $\int d^3r n(r) = A$, makes the nuclear cross section

$$\sigma_A^\lambda = A\langle \sigma_\lambda \rangle - \delta \sigma_A^\lambda,$$

(31)

with the nuclear shadowing term

$$\delta \sigma_A^\lambda \simeq \frac{\pi}{4} \langle \sigma_L^2 \rangle \int db^2 T(b)^2,$$

(32)

very sensitive to the left-right asymmetry of the $\nu$-nucleon cross sections. In Eqs. (31) and (32) $\langle \sigma_\lambda \rangle = \langle \Psi_\lambda | \sigma(x, r) | \Psi_\lambda \rangle$ and $\langle \sigma_\lambda^2 \rangle = \langle \Psi_\lambda | \sigma(x, r)^2 | \Psi_\lambda \rangle$. Hence, the nuclear shadowing correction

$$\delta xF_3 \simeq \frac{Q^2}{4\pi^2\alpha_W} \frac{\pi \langle \sigma_L^2 - \sigma_R^2 \rangle}{8A} \int db^2 T(b)^2,$$

(33)

which should be added to $xF_3$ extracted from the $\nu Fe$ data to get the “genuine” $xF_3$. Since $\langle \sigma_L^2 \rangle \propto 1/\mu^2$ and $\langle \sigma_R^2 \rangle \propto 1/m^2$, this correction is large, positive-valued and does increase $xF_3$ of the impulse approximation.

Summarizing, we developed the light-cone color dipole description of the left-right asymmetry effect in charged current DIS at small Bjorken $x$. We compared our results with experimental data and found a considerable vacuum exchange contribution to the structure functions.
$xF_3^{\nu N}$. This contribution is found to dominate the structure function $\Delta xF_3 = xF_3^{\nu} - xF_3^\bar{\nu}$ of an iso-scalar nucleon extracted from nuclear data. Theory is in reasonable agreement with data but the nuclear effects are shown can make this comparison a somewhat more complicated procedure. The color dipole analysis of nuclear effects in the CC DIS will be published elsewhere.

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