Element matrix formulation for family of hybrid quadrilateral transition elements in finite element analysis

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Abstract. The main aim of the project is to find the Element matrix formulation for family of quadrilateral transition element using the standard Gauss Quadrature method. The formulated method is coded in MATLAB to verify the results by solving standard problems. The project deals with the development of a MATLAB code to generate element stiffness matrix. The MATLAB program is developed on the basis of hybrid meshes containing lower order, higher order and transition elements. The CPU run time for Hybrid meshes and pure eight-noded element is computed. The results from MATLAB are compared with ANSYS.

1. Introduction

An element is a specific domain of the region being simulated. A Node is a keypoint located at the corner of the element and nodes together form the elements. There are different types of elements for different types of application. For example, triangular, quadrilateral (two dimensional), tetrahedral and hexahedral element (three dimensional elements). Elements can be of different elemental equations on the basis of their polynomial order. Uday S. Dixit [1], for example, two dimensional quadrilateral element can be of bilinear order if element contains four-nodes and can be cubic if it contains eight-noded element. As shown in Fig. 1, Jeyakarthikeyan, P. V [2], Mesh contains the regular and different
type of transition elements. This type of mesh combinations are used in different types of applications. While applying in finite element analysis, it decreases the requirement of prototypes and product testing, but it doesn’t eliminate the mesh. While it gives the very high approximation for the solutions to any type of problems, it cannot give the perfect or exact solution. Finite Element model is very sensitive to the boundary conditions. High refinement of the model is sometimes necessary to get a better solution. Sometimes, FEA solutions may not be correct, because of inappropriate choice of elements or boundary conditions. For this reason, a high level of proficiency is necessary in the required mathematical way to reach the solution using FEA as shown in above figure. 2.

1.1 TRANSITION ELEMENT

Most of the higher order elements are quadrilateral such as nine-noded form. These are converted to the 5-noded, 6-noded and 7-noded transition elements to form the hybrid-stress (EAS) formation for the elasticity problems. Improved version of the elements are written by modifying the shape functions as shown in below Fig. 3. For verifying the results, numerical test is carried out. It shows high accuracy of the results after modifying the shape functions of the elements. The output procured after modifying the nodal results are of high accuracy and are not affecting the accuracy of the results. Important guidelines for the mesh transitions are as given below Wu, D., [3].

• Never ever place a mesh transition in an area where there is large variation in stress.
• Mesh transitions should be located at some distance from the area of interest in a region, Lim [4].

2. Isoparametric displacement element Review

There are different type of elements involved in the meshing process, but some special type of elements are also involved. Those are transition elements which in cases as shown in figure. 4, Nodes 1, 2 or 3, mid-point nodes are in extra addition to the four corner nodes. For the sake of reaching compatibility, different elements are shown in this paper, Lim [4].

2.1 2-D transition elements shape functions.

It should be intuitive that the more elements we use, more accurate we get. This however results in increased computations. In other words, we must assess relationship between accuracy and complexity. Shape functions can take variety of forms –Linear (Bar/Truss element), Quadratic (Higher-order truss element), Bi-linear (Quadrilateral elements). Shape functions varies linearly along the sides between its nodes and other nodes as shown in Fig. 4, Shan [5].

2.2 FOUR-NODED QUADRILATERAL ELEMENTS

Four-noded quadrilateral element is considered as in below figure. 5, and is transformed from the co-ordinates $r-z$ to $\xi-\eta$ coordinates, Dörfler [6].
2.3 EIGHT-NODED QUADRILATERAL ELEMENTS

 Eight-noded quadrilateral element is considered as in below figure 6 and is transformed from the co-ordinates r-z to $\xi-\eta$ coordinates, Luo [7].

\begin{align}
N_1 &= \frac{1}{4} (1 - \eta)(1 - \xi)
N_2 &= \frac{1}{4} (1 - \eta)(1 + \xi)
N_3 &= \frac{1}{4} (1 + \eta)(1 + \xi)
N_4 &= \frac{1}{4} (1 + \eta)(1 - \xi)
N_5 &= \frac{1}{2} (1 - \eta)^2(1 - \xi)
N_6 &= \frac{1}{2} (1 - \eta)^2(1 + \xi)
N_7 &= \frac{1}{2} (1 + \eta)^2(1 - \xi)
N_8 &= \frac{1}{2} (1 + \eta)^2(1 + \xi)
\end{align}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure5}
\caption{4-Noded Element}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure6}
\caption{The above Element contains 8 nodes}
\end{figure}
2.4 SIX-NODED TRANSITION QUADRILATERAL ELEMENTS

Figure 7. Transition element

Above shown figure 7, contains 6-nodes which is called as transition element. These transition element is derived from the normal 9 noded element, Band [8].

\[
\begin{align*}
N1 &= \frac{1}{4}(\eta-1)(\xi-1)(\eta+1)(\xi+1) \\
N2 &= \frac{1}{4}(\eta-1)(\xi+1)(\eta+1)(\xi+1) \\
N3 &= \frac{1}{4}(\eta+1)(\xi-1)(\eta+1)(\xi+1) \\
N4 &= \frac{1}{4}(\eta+1)(\xi+1)(\eta+1)(\xi+1) \\
N5 &= -(\xi+1)(\xi-1)/4(\eta+1) \\
N6 &= -(\xi+1)(\xi-1)/4(\eta+1) \\
\end{align*}
\]

2.5 FIVE-NODED TRANSITION QUADRILATERAL ELEMENTS

\[
\begin{align*}
N1 &= (\xi-1)(\eta-1)/4((\xi+1)(\eta+1)+\xi(\eta+1)) \\
N2 &= (\xi+1)(\eta-1)/4((\xi+1)(\eta+1)+\xi(\eta+1)) \\
N3 &= (\xi+1)(\eta-1)/4((\xi+1)(\eta+1)+\xi(\eta+1)) \\
N4 &= (\xi-1)(\eta+1)/4((\xi+1)(\eta+1)+\xi(\eta+1)) \\
N5 &= n/2(\xi-1)(\eta-1)(\xi+1) \\
\end{align*}
\]

Figure 8. Transition element of 5 noded

2.6 MATLAB coding for the transition element
This MATLAB code flowchart is for the transition element assembly. Here number of elements types are four-noded, eight-noded and six-noded transition element as shown in figure. 9.

Figure 9. Flow chart of transition element for MATLAB code

3. Numerical Example

A 2-D flat plate of dimension 31*11 units and the entire plate is now meshed with 4-noded, 8-noded and transition element (8-6-4 noded).

Given values : Elastic modulus $E = 1000 \, \text{N/mm}^2$, Poisson ratio is 0.3 and is subjected to the 1000 KN and displacement along the left hand side is fully fixed as shown in figure, 10.

Figure 10. Plate example problem

As above shown in the figure, 10, it is meshed with the help of transition element as six-noded element and remaining area is meshed with the help of eight-noded and four noded elements. For connecting the higher order to lower order, the transition element is used.

3.1 Ansys results
4 Results and Discussion

4.1 CPU TIME comparison of 6-node transition element

The CPU time comparison of 6-noded transition element is conducted using hybrid mesh generation only. Fig. 13 shows the plot of CPU time comparison of transition element. The results show that, for 10000 elements the time required for hybrid mesh is 12.56478 seconds and for pure eight-noded element result is 14.68534 seconds. For 300 elements, the time comparison between eight-noded and transition is elements output is 1.117234 sec and 1.084650 sec respectively. Table. 1 shows the CPU comparison results of transition elements.

![Figure 13](image_url)
Table 1: 6-noded transition element time comparison (CPU).

| No. of elements | Run time 8-node | Run time 4-node | Run time transition |
|-----------------|-----------------|-----------------|---------------------|
| 300             | 1.17324         | 0.81526         | 1.08461             |
| 3000            | 9.4947          | 6.3815          | 8.3305              |
| 10000           | 14.685          | 10.632          | 12.56147            |

4.2 Displacement results of 8-node and transition element.

Table 2: 8-node and transition element displacement results (CPU).

| Node No | 8-Node element UX | 8-Node element UY | Transition element UX | Transition element UY |
|---------|-------------------|-------------------|-----------------------|-----------------------|
| 1       | 0                 | 0                 | 1                     | 0                     |
| 2       | 0.016532          | 0.001491          | 2                     | 0.01646               | 0.001501          |
| 3       | 0.000341          | 0.000221          | 3                     | 0.000351              | 0.000231          |
| 4       | 0.000631          | 0.00037           | 4                     | 0.00064               | 0.000361          |
| 5       | 0.000871          | 0.00048           | 5                     | 0.000861              | 0.000481          |
| 6       | 0.0011            | 0.000551          | 6                     | 0.001152              | 0.000561          |
| 7       | 0.001333          | 0.00061           | 7                     | 0.00132               | 0.00067           |
| 8       | 0.00156           | 0.00065           | 8                     | 0.00157               | 0.00064           |

5. CONCLUSION

In this paper, the formulation of stiffness matrix for the family of quadrilateral transition element is discussed. Places where stress concentration exist is meshed with higher order elements (8 noded quadrilateral element) and the remaining area is meshed using lower order element (4 noded quadrilateral element), transition elements is used to connect these two elements. The results obtained are compared with the same domain meshed with the individual elements. It is evident from the results obtained that transition elements can be used to combine higher and lower order elements to get a hybrid mesh, thus, taking advantage of both the elements. The presence of higher order elements assures better meshing and approximation while the lower order elements significantly reduce the computation time.

6. References

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