Evidence for maximality of strong interactions from LHC forward data

E. Martynova, B. Nicolescu

aBogolyubov Institute for Theoretical Physics, Metrologichna 14b, Kiev, 03680 Ukraine
bFaculty of European Studies, Babes-Bolyai University, Emmanuel de Martonne Street 1, 400090 Cluj-Napoca, Romania

Abstract

It is important to check if the Froissaron-Maximal Odderon (FMO) approach is the only model in agreement with the LHC data. We therefore generalized the FMO approach by relaxing the $\ln^2 s$ constraints both in the even- and odd-under-crossing amplitude. We show that, in spite of a considerable freedom of a large class of amplitudes, the best fits bring us back to the maximality of strong interaction. Moreover, if we leave Odderon Regge pole intercept $\alpha_O(0)$ completely free we find a very good solution for $\alpha_O(0)$ near -1 in agreement with the result of oddballs spectroscopy in QCD based on AdS/CFT correspondence.

Keywords: Froissaron, Maximal Odderon, total cross sections, the phase of the forward amplitude, Odderon intercept.

1. Introduction

In a previous paper [1], we showed that the Froissaron-Maximal Odderon (FMO) approach is in agreement with both the $\ln^2 s$ behaviour of total cross sections at high energies and the surprisingly low TOTEM $\rho_{pp}$ datum at 13 TeV [2].

The even-under-crossing amplitude in the FMO approach is constituted by a 2-component Pomeron (the Froissaron and the Pomeron Regge pole located at $j = 1$) to which are added the secondary Regge poles located around $j = 1/2$ of even signature. In its turn, the odd-under-crossing amplitude is constituted by a 2-component Odderon (the Maximal Odderon and the Odderon Regge pole located at $j = 1$) to which are added secondary Regge poles located around $j = 1/2$ of odd signature. The Odderon was introduced in 1973 on the theoretical basis of asymptotic theorems [3]. It has to be noted that, when it was introduced, there were no experimental indications for its existence.

The Froissaron (or Maximal Pomeron) and the Maximal Odderon represent the asymptotic contributions (at $s \to \infty$), while the Regge poles describe the low and medium range of energies. In other words, the FMO approach describes the finite-energy effects of asymptotic theorems, by a nice and simple interplay between asymptotic and non-asymptotic contributions.

The Froissaron corresponds to the maximal behavior $\ln^2 s$ of total cross sections allowed by general principles. It was first introduced by Heisenberg in 1952 [4] and a rigorous demonstration was given nine years later by Froissart [5].

In its turn, the Maximal Odderon corresponds to the maximal behavior $\ln^2 s$ of the real part of the odd-under-crossing amplitude and to the maximal behaviour $\ln s$ of the difference of the antihadron-hadron and hadron-hadron total cross sections [3,6].

The FMO approach embodies a new form of the old principle of maximum strength of the strong interactions [7]: both the even and the odd-under-crossing amplitudes saturate functionally the asymptotic bounds.

In the present paper we investigate the following question: is the maximality of strong interactions not only in agreement with experimental data but it is even required by them? For that we are relaxing the $\ln^2 s$ constraint by a generalization of the FMO approach.

2. Generalization of the FMO approach

Let us consider the following form of the amplitudes:

$$F^H_+(z) = i(s - 2m^2)[H_1 \ln^{\beta^+}(-iz)^+ + H_2 \ln^{\beta^+ - 1}(-iz)^+ + H_3]$$

(1)

$$F^{MO}_+(z) = (s - 2m^2)[O_1 \ln^{\beta^{MO}}(-iz)^+ + O_2 \ln^{\beta^{MO} - 1}(-iz)^+ + O_3]$$

(2)

$$F^R_(\pm) \equiv \left\{ \begin{array}{ll}
P, R_+ \quad & O, R_- \end{array} \right\}

(3)$$

The Froissaron corresponds to the maximal behavior $\ln^2 s$ of total cross sections allowed by general principles. It was first introduced by Heisenberg in 1952 [4] and a rigorous demonstration was given nine years later by Froissart [5].

In its turn, the Maximal Odderon corresponds to the maximal behavior $\ln^2 s$ of the real part of the odd-under-crossing amplitude and to the maximal behaviour $\ln s$ of the difference of the antihadron-hadron and hadron-hadron total cross sections [3,6].

The FMO approach embodies a new form of the old principle of maximum strength of the strong interactions [7]: both the even and the odd-under-crossing amplitudes saturate functionally the asymptotic bounds.

In the present paper we investigate the following question: is the maximality of strong interactions not only in agreement with experimental data but it is even required by them? For that we are relaxing the $\ln^2 s$ constraint by a generalization of the FMO approach.

2. Generalization of the FMO approach

Let us consider the following form of the amplitudes:

$$F^H_+(z) = i(s - 2m^2)[H_1 \ln^{\beta^+}(-iz)^+ + H_2 \ln^{\beta^+ - 1}(-iz)^+ + H_3]$$

(1)

$$F^{MO}_+(z) = (s - 2m^2)[O_1 \ln^{\beta^{MO}}(-iz)^+ + O_2 \ln^{\beta^{MO} - 1}(-iz)^+ + O_3]$$

(2)

$$F^R_(\pm) \equiv \left\{ \begin{array}{ll}
P, R_+ \quad & O, R_- \end{array} \right\}

(3)$$
where
\[ z = (s - 2m^2)/2m^2 \]  
and \( m \) is mass of proton. The elastic \( pp \) and \( \bar{p}p \) forward scattering amplitudes are defined through the relations
\[ F_{\pm}(z, 0) = (1/2)(F_{pp}(z, 0) \pm F_{\bar{p}p}(z, 0)). \]
The observables are
\[ \sigma_{\text{tot}}(s) = \text{Im} F(z)/\sqrt{(s - 4m^2)} \]  
\[ \rho(s) = \text{Re} F(z)/\text{Im} F(z). \]
The proton-proton forward scattering amplitude is
\[ F_{pp}(z) = F^{H}(z) + F^{MO}(z) + F^{P}(z) + F^{O}(z) + F^{R}(z) + F^{E}(z) \]
and the amplitude of antiproton-proton scattering is
\[ F_{\bar{p}p}(z) = F^{H}(z) - F^{MO}(z) + F^{P}(z) - F^{O}(z) + F^{R}(z) - F^{E}(z). \]

For \( \beta_F = \beta_{MO} = 2 \) and \( \alpha_F(0) = \alpha_{MO}(0) = 1 \) we get exactly the FMO model of Ref. [1]. Our aim is to verify our analysis of the data on \( \sigma_{\text{tot}}(s) \) and \( \rho(s) \).

The parameters \( \beta_F \) and \( \beta_{MO} \) are not arbitrary. They are constrained by analyticity, unitarity, crossing-symmetry and positivity of cross sections [2].

\[ \beta_F \leq 2, \quad \text{if} \quad \beta_F \geq 0, \]
\[ \beta_{MO} \leq \beta_F/2 + 1, \quad \text{if} \quad \beta_F \geq 0, \]
\[ \beta_{MO} \leq \beta_F + 1, \quad \text{if} \quad \beta_F \leq 0. \]

These three constraints can be visualized in the Cornille’s plot shown in Fig. 1 where we also show the different behaviors of \( \Delta \sigma = |\sigma_{\text{pp}} - \sigma_{\text{tot}}| \) and \( \rho \) in the different subregions allowed by general principles. It can be noted that the overall allowed region is strongly constrained, a fact which shows the power of general principles. It can be also be noted that the "conventional" region (where \( \Delta \sigma \to 0 \) and \( \rho \to 0 \)) is only a part (the hatched area of Fig. 1) of the allowed domain. The LN point in Fig. 1 is precisely the point corresponding to the FMO approach (\( \beta_F = \beta_{MO} = 2 \)).

It is also interesting to note that there are an entire class of Odderon which are different from the Maximal Odderon and might present interest on experimental level:
\[ 0 \leq \beta_{MO} < 2. \]

In particular, the case \( \beta_{MO} = 0 \) corresponds to the Odderon Regge pole, partner of the usual Pomeron Regge pole and which leads to the behavior
\[ F_{-}(s, 0) \to \text{const} \cdot s. \]

3. Numerical analysis

For the analysis of the models we use the standard set of the data on \( \sigma_{\text{pp}}^{\text{TOT}}, \sigma_{\text{pp}}^{\text{LO}}, \rho_{\text{pp}}, \rho_{\text{pp}} \) from PDG [3]. We also add the latest data of TOTEM at \( \sqrt{s} = 13 \text{ GeV} \) [2]. The two ATLAS data for \( \sigma_{\text{pp}}^{\text{TOT}} \) [11]: 95.35 ± 1.3065 mb at 7 TeV and 96.07 ± 0.82 mb at 8 TeV are not included in PDG set, though they were published before PDG Review. The data form the PDG set were taken at energies \( \sqrt{s} > 5 \text{ GeV} \): 248 points without the two ATLAS data and 250 points with the ATLAS data.

In the Ref. [11] we excluded these two ATLAS \( \sigma_{\text{pp}}^{\text{TOT}} \) data because they are not compatible with the TOTEM data. However in the present paper we include them in order to see their effects on the fits. We made separate fits in the first of considered models: one excluding them \( (M_1) \) - 248 data points and one including them \( (M'_1) \) - 250 data points.

We expand first the model \( M_1 \), defined by leaving \( \beta_F \) and \( \beta_{MO} \) free but keeping fixed \( \alpha_F(0) = 1 \) and \( \alpha_{MO}(0) = 1 \).

The results are quite spectacular (see the Table 1): the values of \( \beta_F \) and \( \beta_{MO} \) come back to the saturation values \( \beta_F = 2 \) and \( \beta_{MO} = 2 \). The parameters are near those of Ref. [1].

In spite of the supplementary two parameters as compared with Ref. [1], the value of \( \chi^2/\text{dof} = 1.075 \) is
slightly bigger than 1.070, the value get in Ref. [11]. When ATLAS data are included the $\chi^2$/dof is, of course, worse ($\approx 1.10$) but the best fit parameters are again not far from those of Ref. [11].

The resulting curve for pp total cross section is between TOTEM and ATLAS points with ATLAS points included. (parameter $H_1$ is slightly less than in the case without ATLAS points). However the most important fact is that ATLAS points have no influence on the value of $\rho^{pp}$ at 13 TeV. It means that the Odderon effect is present even if the total $pp$ cross section is a little bit lower at LHC energies.

In Figs. 2 and 3 we show the best fit Models $M_1$ and $M_1'$ vs. experimental data.

We made a lot of other explorations by leaving more parameters or all parameters free (with $\alpha_F(0)$ and $\alpha_O(0)$ near 1) but there are no surprises: the best fits always go back to the saturation values $\beta_F = \beta_{MO} = 2$. An interesting fact is that variants with $\alpha_O(0) < 1$ (but not far from $\alpha_O(0) = 1$) lead to an intercept $\alpha_\chi(0)$ closer to the spectroscopic data, but the value of $\chi^2$/dof is worse.

### Table 1

| Parameter | $M_0$ | $M_1$ | $M_1'$ | $M_2$ |
|-----------|-------|-------|--------|-------|
| $\beta_F$ | 2.0   | 2.0   | 2.0    | 2.0   |
| $\delta_{MO}$ | 0.0   | 0.28 $\cdot 10^{-1}$ | 0.108 $\cdot 10^{-8}$ | 0.0   |
| $\alpha_F(0)$ | 1.0   | fixed | 1.0    | fixed |
| $\alpha_O(0)$ | 1.0   | fixed | 1.0    | fixed |
| $\alpha_\chi(0)$ | 0.640 | $\pm 0.047$ | 0.674 $\pm 0.039$ | 0.5294 $\pm 0.006$ |
| $H_1$(mb) | $-0.272$ | $\pm 0.070$ | 0.290 $\pm 0.067$ | 0.202 $\pm 0.211$ |
| $H_2$(mb) | 28.496 $\pm 5.394$ | 22.237 $\pm 6.069$ | 35.938 $\pm 0.177$ |
| $\rho_1$(mb) | $-0.0484$ | $\pm 0.011$ | $-0.0415$ | $\pm 0.054$ |
| $\rho_2$(mb) | 0.985 $\pm 0.212$ | 0.877 $\pm 0.213$ | 1.108 $\pm 0.074$ |
| $\rho_3$(mb) | $-4.758$ | $\pm 1.030$ | $-4.328$ | $\pm 3.349$ |
| $C_F$(mb) | 48.289 $\pm 3.367$ | 53.835 $\pm 3.551$ | 52.107 $\pm 1.293$ |
| $C_F^2$(mb) | 35.931 $\pm 4.592$ | 35.469 $\pm 4.138$ | 42.867 $\pm 4.619$ |
| $N$ of free parameters | 10    | 12    | 12    | 12    |
| $\chi^2$/dof | 1.070 | 1.075 | 1.00 | 1.046 |

Table 1: Here $\delta_{MO} = 1 + \beta_F/2 - \beta_{MO}$. $M_0$ denotes the model of Ref. [11]. $M_1$ is defined in the text. The parameters of $M_1$ are get in fitting without the two ATLAS points, while those of $M_1'$ are get when they are included. $M_2$ is the variant where $\alpha_O(0)$ is left free.

4. An interesting solution for Odderon Regge pole intercept $\alpha_O(0)$

Intrigued by several calculations of the oddballs Regge trajectory in different theoretical contexts, indicating an Odderon intercept which is small and even negative (around -1) we decided to make a careful analysis of the case in which we leave $\alpha_O(0)$ completely free.
The agreement between these values (13) and (15) can be hardly interpreted as a numerical coincidence. It may point to a basic structure of the 2-component Odderon: the Maximal Odderon corresponding to a dipole in imaginary part of Maximal Odderon amplitude located at \( j = 1 \) and a simple pole located at \( j = -1 \). The Maximal Odderon is dominating at very high energy, while the Odderon Regge pole is important at low energy.

This reminds us about the two solutions found in the LLA of QCD: the Lipatov et al. solution [13], located precisely at \( j = 1 \) and corresponding to a complicated cut in the \( j \)-plane and the Janik-Wosiek solution [14] located at \( j < 1 \). It is therefore tempting to suppose that the Lipatov et al. solution corresponds to the Maximal Odderon in non-perturbative regime, while Janik-Wosiek solution - to the Odderon Regge pole. Search in this direction would be key important.

5. Conclusion

In the recent paper we showed that the LHC data strongly favor the maximality of strong interaction and, in particular, give evidence for Maximal Odderon. In our formulation of the maximality the Maximal Odderon needs not to saturate unitarity. Saturation of asymptotic bounds does not mean saturation of unitarity. This is simple to understand from physical point of view. Unitarity means the sum over all possible channels. But at infinite energies, there are a lot of non-asymptotical channels, which are not covered by Maximal Odderon. Troshin indeed verified that the Maximal Odderon does not saturate unitarity [15]. Saturation of unitarity was imposed by Chew in his version of a maximality of strong interaction [7] because Chew considered, at that moment of time, Regge physics as the final theory of strong interactions. But in the presence of singularities other than Regge poles, this assumption is no more valid.

However, the Maximal Odderon satisfies indeed unitarity [16]. The contrary claim made recently by Khoze et al. [17] is based on a too strong assumption: factorization in \( s,t \)-representation is not valid for contributions corresponding to dipole, tripole and any singularity different of simple pole in \( j \)-plane. The Pomeron interaction vertices should have a complicated dependence on \( j \) and momenta with zero at \( j - 1 \sim t \sim 0 \). Besides, the arguments considered in [17] are valid for models with the Black Disk Limit rather than with unitarity limit.

Beyond maximality, an interesting result when considering LHC data in conjunction with low and medium energy data is that the additional trajectory has a very low and negative intercept, in agreement with the
AdS/CFT correspondence. The $t \neq 0$ data at LHC will reserve to us certainly several other surprises.

**Acknowledgment.** One of us (E.M.) thanks the Department of Nuclear Physics and Power Engineering of the National Academy of Sciences of Ukraine for support.

**References**

[1] E. Martynov, B. Nicolescu, Phys. Lett. B 777(2018)414.
[2] TOTEM Collaboration, G. Antchev et al., CERN-EP-2017-321: CERN-EP-2017-335, submitted to Phys. Rev. D.
[3] L. Lukaszuk, B. Nicolescu, Lett. Nuovo Cim. 8(1973)405.
[4] W. Heisenberg, Z. Phys. 131(1952)65.
[5] M. Froissart, Phys. Rev. 123(1961)1053.
[6] J. Fischer, Phys. Reports, 73(1981)157.
[7] G.F. Chew, Rev. Mod. Phys. 34(1962)394.
[8] C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40(2016)100001 and 2017 update. The data tables are given at [http://pdg.lbl.gov/2017/hadronic-xsections/hadron.html](http://pdg.lbl.gov/2017/hadronic-xsections/hadron.html).
[9] H. Cornille and R.E. Hendrick, Phys. Rev. D10(1974)3805. See Fig. 2 in this reference.
[10] D. Joynson, E. Leader, B. Nicolescu and C. Lopez, Nuovo Cim. A30(1975) 345.
[11] ATLAS Collaboration, G. Aad et al., Nucl. Phys. B889(2014)486; ATLAS Collaboration, M. Aabod et al., Phys Lett. B761(2016)150.
[12] J.Sadeghi, B. Pourhassan, H. Farahani, S.Gh. Hashemi, J. Theor. Phys. 56(2017)2271.
[13] J. Bartels, L.N. Lipatov, G.P. Vacca, Phys. Lett. B447(2000)178.
[14] R.A. Janik and J. Wosiek, Phys. Rev. Lett. 82(1999)1092.
[15] S.M. Troshin, Phys. Lett. B682(2009)40.
[16] P. Gauron, L. Lukaszuk and B. Nicolescu, Phys. Lett. B 294(1992)298.
[17] V.A. Khoze, A.D. Martin, M.G. Ryskin, Phys. Lett. B780(2018)352.