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ABSTRACT
We experimentally study the spatiotemporal dynamics of self-excited shedding of millimeter-sized water drops acoustically levitated in a single-node standing wave cavity. By decreasing the sound intensity below the threshold, the interplay of drop motion and its perturbed acoustic wave field lead to the transition from stable self-excited drop oscillation to chaotic drop oscillation with growing fluctuations and intermittent droplet shedding. Using azimuthal Fourier transform, the top-view drop shape can be decomposed into zonal and sectoral modes with varying amplitudes. The shedding is led by the increasing amplitudes of the low order sectoral modes (azimuthal mode number \( m = 2 \) and \( 3 \)), which cause the strongest amplitude in the zonal mode (\( m = 0 \)) in the re-expansion stage after the shrinking of the side lobes in the low order modes. It in turn causes synchronized excitations of high order sectoral modes with \( m > 3 \). Their constructive superposition at certain points along the flattened thin edge of the re-expanding drop leads to sharp protrusions, where the surface tension cannot hold the thin rapid expanding jets, and shedding occurs.

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I. INTRODUCTION

Small droplet shedding from a liquid drop is important for a wide range of applications such as ink-jet printing, spray combustion in the engine, and spray cooling. Shedding usually happens in the highly stretched lobes of the drop where surface tension no longer holds the rapid expanding flow. The most common mechanism involved in the formation of highly stretch lobes is often explained by the growth of perturbation on the surface through the Rayleigh and Kelvin-Helmholtz type of instabilities. Conventionally, shedding can be induced by strong external perturbations such as mechanically vibrating the drop, exposing to surface acoustic waves, or shearing the drop using high-speed gas jets. The abovementioned three driving methods require contact between a surface and the drop. However, only a few studies focus on droplet shedding in a contact-free environment.

The acoustically levitated drop is an interesting contactless nonlinear system exhibiting rich dynamical behaviors under the interplay of drop inertia, surface tension, viscosity, background acoustic field, and acoustic streaming. The drop can be suspended at the pressure node of the standing acoustic field between an acoustic transducer and a reflecting plate. The gravitation force and surface tension of the drop are balanced by the strong acoustic pressure from the nonlinear effect of intensive sound. The benefit of fewer restrictions on the drop material and the simple setup in the acoustic levitation systems make it a good platform for studying the spatiotemporal dynamics of a free drop. For example, recent studies showed that zonal and sectoral modes (modes with azimuthal mode number \( m = 0 \) and \( m \neq 0 \), respectively) of drops can be externally excited through parametric instability by periodically modulating the sound intensity. The dynamics of bubble formation via acoustic resonance under increasing sound intensity which causes the formation of a liquid film was also studied. In a steady acoustic field, a drop associated with shape oscillation after external perturbation exhibits damped vertical oscillation of its center of mass. In another recent experiment under steady weak sound intensity in a small chamber, the drop exhibits self-excited oscillations and shedding of small droplets due to the growth of oscillation instability through the...
interplay of drop motion, drop shape variation, and the perturbation on the background sound field by drop motion. The time-delay response of the acoustic field to the moving levitated drop is the source which gives rise to the velocity-dependent negative damping term for sustaining the self-excited oscillations.\textsuperscript{10–20}

In this work, a more detailed experimental investigation on the spatiotemporal dynamics of self-excited droplet shedding in a weak acoustic levitation field is conducted. The temporal evolution and the spectrogram of the drop radius at a fixed azimuthal angle, the temporal evolutions of the amplitudes of the Fourier decomposed azimuthal modes, and shedding events are correlated. It is found that as the oscillation instability grows, the self-excited low-order sectoral modes change from alternate emerging to collective excitation, which is associated with the growth of the shedding occurring rate. Shedding events occur when the amplitude of the zonal mode reaches the temporal maximum, accompanied with synchronized excitation of high-order sectoral modes with \textit{m} > 3. The induced thinning and protrusion spike formation of the drop edge are key for droplet shedding.

II. EXPERIMENT

The experiment is conducted in a single-axis levitator with a single node, as shown in Fig. 1(a). The drop injected by a microsyringe can be suspended at the center pressure node of the standing wave between a top flat glass reflector and an aluminum cylinder connected to a piezoelectric transducer (operated at 29.1 kHz). The gap width is 5.84 mm (0.5 ultrasound wavelength). The aluminum cylinder has a conical top surface (apex angle 178°) for radial confinement. The acoustic wave amplitude is linearly proportional to the amplitude of the driving voltage \( V_0 \). In this experiment, the water drop volume is fixed at 20 \( \mu \)l, and \( V_0 \) is maintained at a steady value after tuning. The side and the top view images of the drop are captured by two high-speed cameras at a 600 Hz frame rate.

III. RESULT AND DISCUSSION

A. Self-excited stable oscillations

Figure 1(b) shows the side and top views of a static drop levitated and flattened at \( V_0 = 448 \) V. Squeezed by the acoustic and gravitational fields, the side view drop shape is oblate and sustains the circular top view shape. Decreasing \( V_0 \) to 352 V changes the acoustic Bond number from 1.86 to 1.19.\textsuperscript{23} It leads to the transition to the stable self-excited oscillating state dominated by the sectoral mode with an azimuthal angle \( \theta = 0 \). Figure 2(a) shows the sequential top and side view images in an oscillation cycle. Figure 2(b) shows the temporal evolutions of \( R_{\text{pp}} \) (the distance from the drop center to the drop boundary at an azimuthal angle \( \theta = 0 \)) and \( |A_2| \) (instantaneous amplitudes of the azimuthally decomposed mode with \( m = 2 \)). Here \( |A_2(t)| \) is obtained from the azimuthal Fourier transform of \( R_{\text{pp}} \), i.e., \( R(\theta, t) = \sum_m A_m(t) \exp(i m \theta) + R \), where \( R \) is the temporal and azimuthal average of the distance \( R \) from the drop boundary to the drop center. Figure 2(c) further shows the spectrograph of \( R_{\text{pp}}(t) \) using the Morlet wavelet.\textsuperscript{22} It means that the amplitude of shape oscillation is stable and there are two oscillation modes.

Stable drop oscillation is associated with the periodic oscillation of the top view drop shape aspect ratio and the alternate change of the drop elongation direction by \( 90^\circ \) to its conjugate phase.\textsuperscript{24} Namely, a stable standing azimuthal wave with \( m = 2 \) and constant oscillation amplitude dominates. It causes the stable periodic oscillations of \( |A_2| \) and induces a straight frequency ridge at \( f = 48 \) Hz in the spectrograph of \( R_{\text{pp}}(t) \), as shown in Fig. 2(c). Note that in the previous study shown by parametrically modulating the levitating acoustic field, this eigenmode with \( m = 2 \) can also be passively excited.\textsuperscript{25} Also note that the ridge at \( f = 96 \) Hz in the spectrogram is from the weak second harmonic (\( m = 4 \)) of the fundamental mode with \( m = 2 \). It can be further confirmed from the temporal power spectra of \( A_2 \) and \( A_4 \).

B. Self-excited chaotic oscillations and shedding

Decreasing \( V_0 \) to 320 V further weakens drop vertical confinement and leads to chaotic drop shape oscillation. Figures 3(a) and 3(b) show the long-time temporal evolution of \( R_{\text{pp}} \) and its wavelet spectrogram. The fluctuation with chaotic modulation of the oscillating \( R_{\text{pp}} \) gradually grows, which is associated with the slow increase of temporally coarse-grained \( R_{\text{pp}} \), indicated by the orange line in Fig. 3(a), followed by the fast decrease starting from 8 s due to the volume reduction through shedding. There are a few distinct bands in the wavelet spectrogram. The first two bands at 50 and 75 Hz are contributed by the zonal mode with \( m = 2 \) and 3, respectively, which alternately emerge in the first 5 s, while the oscillation amplitude of \( R_{\text{pp}} \) are not very large. The next two bands at 100 and 150 Hz are contributed by the zonal mode with \( m = 0 \) (see Ref. 9). Their emergence is phase-locked with the emergence of

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**FIG. 1.** (a) Sketch of the experimental setup and (b) top and side views of the static drops that are 20 \( \mu \)l in volume at \( V_0 = 440 \) V. [reprinted with permission from P. C. Lin and L. I, Phys. Rev. E 93, 021101(R) (2016). Copyright 2016, American Physical Society]. The bottom and top edges of the side-view images are the boundaries of the transducer and the top reflector, respectively.
FIG. 2. (a) Temporal evolutions of $R_{θ}=0$ and $|A_{2}|$ of the drop 20 $μl$ in volume at $V_{0}=352$ V, exhibiting stable oscillation; the dots in the curves are the digitally sampled points, (b) typical sequential top- and side-view images of the drop in one oscillating period, demonstrating the alternating switching of the two side lobes to the conjugated phase with $90°$ azimuthal angle difference, (c) spectrogram of the temporal oscillation of $R_{θ}$, and (d) temporal power spectra of $A_{2}$ and $A_{4}$. The two frequency ridges at 48 and 96 Hz shown in (c) are contributed by modes with $m=2$ and 4, respectively. The stable shape oscillation in the weak acoustic levitation field is self-excited through the interplay of the perturbed acoustic wave field made by drop motion and shape variation.

the modes with $m=2$ and 3. Figure 3(c) shows the temporal evolutions of $|A_{m}|$ with $m=0–8$ in two different time regions (from 1.3 to 1.5 s of the left panel and from 7.6 to 7.8 s of the right panel). In the first region, modes 0, 2, and 3 dominate and alternate appear [see Fig. 4(a) of Ref. 9 and also see the alternate growth of the modulation envelopes of these two modes in the left part of Fig. 3(c)]. The envelopes of $|A_{m}|$ of those modes exhibit irregular long-time modulations. In the second region, the envelopes of $|A_{m}|$ with $m=0$ and 2–8 all exhibit large amplitude irregular fluctuations, which cause the increase in widely spread spectra [Fig. 3(b)] in this region.

As indicated by the green dots in Fig. 3(a), shedding events occur intermittently. The occurring rate increases with increasing

FIG. 3. [(a) and (b)] Typical long-time evolution of $R_{θ}$ and its wavelet spectrogram of the drop 20 $μl$ in volume at $V_{0}=320$ V [reprinted with permission from P. C. Lin and L. I, Phys. Rev. E 93, 021101(R) (2016), Copyright 2016, American Physical Society]; the orange line in (a) is the temporally coarse-grained $R_{θ}$ and (c) temporal oscillations of $|A_{m}|$ in two different time regions. The gray lines are the lines indicating the zero levels; the dots in (a) and (c) indicate shedding event locations; the chaotic radial fluctuation is mainly contributed by the strongly coupled low-order sectoral and zonal modes with mode numbers $m=0, 2,$ and 3. The fluctuation levels of $|A_{m}|$ grow with an increasing shedding rate at the later stage, especially for modes with $m>3$. 

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FIG. 4. Color-coded plot of $\langle |A_m| \rangle / \Delta |A_m|$ in the $m$-$\tau$ space averaged over all the shedding events (from 5 to 8.8 s) with event times reset at $\tau = 0$, where $\Delta |A_m|$ is the standard deviation of $|A_m|$ from 5 to 8.8 s. Shedding occurs at high $|A_0|$, which is preceded by a temporal maximum that is $-4$ ms ahead and is immediately trailed by the sudden rises and falls of $|A_4|$ to $|A_{16}|$ peaked about 1 ms later between the temporal local maxima of $|A_2|$ and $|A_3|$. The oscillations of $|A_4|$ to $|A_{16}|$ are phase locked, sharing the same oscillation period of $|A_3|$ but with phase shifts. Namely, they are strongly correlated with the standing wave with $m = 3$. Note that, on average, the temporal envelopes of all $|A_m|$ gradually rise and reach their temporal maxima around the shedding event. Only the temporal local maxima of $|A_2|$ and $|A_3|$ do not occur right at shedding. Also, temporal spline interpolation with 10 times the sampling rate (6000 Hz) from digitally sampled data (600 Hz sampling frequency) is used for obtaining data, as shown in Fig. 4.

Let us construct a kinetic picture of shedding, especially for explaining what is shown in Fig. 4. Figure 5(a) shows the typical sequential side and top view images around the shedding event at point A and $t = 7708.3$ ms, the corresponding azimuthal variations of $R$ and $|A_m|$ spectra; the boundaries of the top view images are labeled by red lines. (b) azimuthal variation of $R$ and its Fourier decomposed modes $R_m(\theta)$ at 7706.7 and 7708.3 ms, and (c) other two examples showing the shedding events occurring at the sharp protrusions and $t = 8266.7$ and 8273.3 ms. The re-expansion after shrinking of the side lobes leads to the formation of sharp protrusions around the flattened drop edge where the shedding of small droplets occurs, which is associated with $|A_0|$ of the zonal mode ($m = 0$) and high $m$ sectoral modes, all spatiotemporally synchronized and reaching their temporal local maxima.
of $R$, and the $|A_m|$ spectra. Figure 5(b) shows $R(θ)$ and various azimuthally Fourier decomposed modes $R_m(θ)$ of $R$ at 7706.7 and 7708.3 ms (before and at the shedding event, respectively). Here, $R_m(θ, t) = |A_m(t)| \exp[i m \theta + \theta_0(t)]$, where $\theta_0(t)$ is the initial azimuthal angle of the $m$ mode at each time. At 7705 ms, the top view drop shape has three distorted lobes with one extended lobe, mainly contributed by the superposition of large amplitude sectoral modes with $m = 2$ and $m = 3$. Then, the three lobes shrink radically, which causes the increase in the drop vertical width at the center (see the images at $t = 7705$ ms). In the subsequent re-expansion stage, similar to the change of the drop elongation direction by $90^\circ$, as shown in Fig. 2(b), the three lobes change the expanding direction by $60^\circ$ (see images at $t = 7706.7$ ms). Namely, the standing azimuthal wave enters the conjugate phase. The bottom row of Fig. 5(a) further indicates that at shedding (7708.3 ms), $|A_3|$ is relatively small compared to $|A_2|$ at 7705 and 7711.6 ms, while $|A_0|$ and other $|A_m|$ with $m > 3$ reach their temporal local maxima after shedding.

In the middle of the re-expanding process ($t = 7708.3$ ms), the vertical squeezing of the drop and rapid expanding leads to the development of the thin liquid sheet with the largest top-view area and the irregular boundary with sharp protrusions at the edges. They are associated with $|A_m|$ of the zonal mode ($m = 0$) reaching temporal local maxima and immediately trailed by the high $m$ sectoral modes reaching their $|A_m|$ temporal local maxima, respectively (see Fig. 5(b) and the bottom two rows of Fig. 5(a)). Note that at the local azimuthal extrema of $R$ caused by the local sharp protrusions, e.g., at points A and B, the local maxima of most modes are spatiotemporally synchronized. Shedding of small droplets occurs at these sharp protrusions, where the surface tension cannot hold the thin rapid expanding jets. After the shedding event, the drop switches to the conjugate phase of the distorted three-lobe state and waits for the recurrence of shedding. Namely, the large $m$ modes rapidly decrease their intensity associated with the re-emergence of the low order $m$ mode.

The above physical picture clearly indicates that the re-expansion after the shedding of the three large amplitude lobes causes the thinning of the peripheral region and sharp irregular protrusion is key for shedding as $|A_0|$ and other $|A_m|$ with $m > 3$ reach their temporal local maxima in the re-expansion of the three conjugate lobes. Therefore, the alternate switching between the two conjugate phases of the three distorted lobes is a major clock controlling the oscillation frequency of higher $m$ sectoral modes, which leads to frequency locking between high $m$ modes and low $m$ modes.

The sequential side and top view images shown in Fig. 5(c) with two sheddings occurring at 8267 and 8273.3 ms give other examples showing a similar dynamical rule for shedding.

IV. CONCLUSION

We experimentally investigate the spatiotemporal dynamics of self-excited intermittent shedding of millimeter sized water drops levitated by a weak standing acoustic wave in a single-node cavity. The drop exhibits self-excited stable oscillation under the interplay of drop motion and perturbed weak acoustic field. Further decrease in the acoustic pressure leads to chaotic drop shape fluctuations, which are associated with intermittent drop shedding. Led by increasing amplitudes of low order sectoral modes with $m = 2$ and 3, the drop re-expansion after shrinking of the side lobes of those modes causes the formation of a thin liquid sheet with the growing amplitude of the zonal mode. As its amplitude reaches the temporal maximum, the high order sectoral modes with $m > 3$ of the thin drop expanding sheet rapidly emerge. The spatiotemporal phase synchronization of those modes leads to the formation of sharp protrusions at the flattened thin edge of the sheet, where the surface tension cannot hold the rapid expanding liquid jet, and shedding occurs.

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21. The acoustic Bond number is defined as $B_a = \frac{R_0 P^2}{(\rho_0 c^2 \sigma)}$, where $R_0$ is the drop radius, $P$ is the local acoustic pressure amplitude, $\rho_0$ is the air mass density, $c$ is the sound speed, and $\sigma$ is the surface tension coefficient. It is proportional to the ratio of acoustic stress to drop surface tension.

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