Quantum vorticity in nature∗

Kerson Huang
Physics Department
Massachusetts Institute of Technology
Cambridge, MA 02139 USA

August 25, 2015

Abstract
Quantum vorticity occurs in superfluidity, which arises from a spatial variation of the quantum phase. As such, it can occur in diverse systems over a wide range of scales, from the electroweak sector and QCD of the standard model of particle theory, through the everyday world, to the cosmos. I review the observable manifestations, and their unified description in terms of an order parameter that is a complex scalar field.

1 Overview
Quantum vorticity is a manifestation of superfluidity, which arises from a spatial variation of the quantum phase. This can occur in different types of physical systems, but a unified description can be given in terms of an order parameter that is a complex scalar field:

\[ \phi(x) = F(x) e^{i\sigma(x)} \]  

(1)

Its existence signals the spontaneous breaking of a U(1) gauge symmetry.

In atomic systems with Bose-Einstein condensation [1], the broken gauge symmetry is global. In this case, \( \phi \) represents the condensate wave function, which can be described by the Gross-Pitaevskii equation, a nonlinear Schrödinger equation (NLSE):

\[- \frac{\hbar^2}{2m} \nabla^2 \phi + (g\phi^* \phi - \mu_0) \phi = i\hbar \frac{\partial \phi}{\partial t} \]  

(2)

where \( m \) is the mass of the atoms, \( g \) an interaction parameter, and \( \mu_0 \) is the chemical potential.

In superconductivity [2], the broken gauge symmetry is local, and \( \phi \) obeys a nonlinear Schrödinger equation with gauge coupling:

\[- \frac{\hbar^2}{2m} \left( \nabla - \frac{i\hbar q}{c} A \right)^2 \phi + (g\phi^* \phi - \mu_0) \phi = i\hbar \frac{\partial \phi}{\partial t} \]  

(3)

∗Invited contribution to The Proceedings of a Conference on Sixty Years of Yang-Mills Gauge Field, Institute of Advanced Studies, Nanyang Technological University, Singapore, May 26-28, 2015.
where \( q = 2e \) is the charge of Cooper pairs, and \( \mathbf{A} \) is a magnetic field. This is called the Ginsburg-Landau (GL) equation.

In relativistic systems \([3]\), these equations generalize to a nonlinear Klein-Gordon (NLKG) equation:

\[
\Box \phi + V' \phi = J
\]

where \( \Box \) is the d'Alembertian in curved spacetime, which may contain gauge couplings, and \( J \) denotes a possible external current. The self-interaction potential may be chosen to be a phenomenological \( \phi^4 \) potential:

\[
V = \frac{\lambda}{2} (\phi^* \phi - F_0)^2 + V_0
\]

Thus

\[
V' = \lambda (\phi^* \phi - F_0)
\]

where a prime denotes differentiation with respect to \( \phi^* \phi \). Physically, \( \phi \) may correspond to a component of the Higgs field in the standard model, the order parameter associated with chiral symmetry breaking in QCD, or Higgs-like fields in grand-unified or supersymmetric theories.

All these systems exhibit superfluidity, with superfluid velocity given by \([4]\)

\[
v_s = \kappa \nabla \sigma
\]

where \( \kappa = -c (\partial \sigma / \partial t)^{-1} \). In the relativistic case, the requirement \( |v_s| < c \) is guaranteed, when \( \partial^\mu \sigma \) is time-like, i.e., \( \partial^\mu \sigma \partial_\mu \sigma < 0 \). In the nonrelativistic limit, \( \sigma \to -\left(mc^2/h\right) t + v_s \) and \( \kappa \to \hbar/m \).

The equations of motions can be rewritten in terms of \( F \) and \( \sigma \), and the equation for \( \sigma \) yields an equation for \( v_s \) similar to the classical Euler equation of hydrodynamics, with quantum corrections. However, for many purposes it is simpler to stay with the original forms, particularly numerical analysis.

Quantum vorticity arises from a quantization of the circulation, by virtue of the continuity of \( \phi \):

\[
\oint_C dx \cdot \nabla \sigma = 2\pi n \quad (n = 0, \pm 1, \pm 2, \ldots)
\]

where \( C \) is a closed loop. If \( n \neq 0 \), the loop encircles a vortex line on which \( \phi = 0 \). As illustrated in in Fig.1, the field rises from zero at the line to an asymptotic value, with a characteristic healing length \( \xi \). The vortex line can thus be thought of as a tube of radius \( \xi \) in which symmetry is restored, with expulsion of of the order parameter. The superfluid velocity \( v_s \) drops off with distance \( r \) from the line center like \( r^{-1} \). Since the energy density of the superfluid flow is proportional to \( v_s^2 \), the energy per unit length of a single straight vortex line diverges with the radius of the container. The existence of vorticity depends on a vacuum field, hence on the nonlinearity; it is absent in the usual Schrödinger equation, or Klein-Gordon equation, where the field goes to zero at infinity.

A vortex line cannot terminate, except on boundaries, or upon itself, forming a closed curve. The simplest example of the latter is vortex ring of radius \( R \), as illustrated
in Fig.2. The ring moves with a translational velocity normal to the plane of the ring \( v \sim R^{-1} \ln R \), with energy \( E \sim R \ln R \). Thus, roughly speaking, \( E \sim v^{-1} \). A vortex line of arbitrary shape will move in such a manner that, at any point of the line, the local velocity is normal to the tangent circle, with velocity inversely proportional to the local radius of curvature. Thus, the line will generally execute a self-driven writhing motion, as illustrated in Fig.2.

When two vortex lines cross, they will reconnect, as illustrated in Fig.3. In the final configuration, there appear two cusps, which spring away from each other at great speed, because of the smallness of the radii of curvature. A reconnection thus creates two jets, which represents an efficient way to convert potential energy into kinetic energy in a short time. The reconnection of magnetic flux lines in the sun’s corona is believed to create jets of solar flares, as shown in Fig.3.

Vortex rings can be created in a superfluid with a heat source. They would expand, intersect and reconnect, and reach a steady-state vortex tangle called quantum turbulence. Fig.4 [6] shows a computer simulation of the process. The vortex tangle is a new geometrical object, with a fractal of dimension 1.6 [7]. When the heat source is removed, the vortex tangle will decay. The dynamics of growth and decay can be described phenomenologically by Vinen’s equation [5]

\[
\frac{d\ell}{dt} = A\ell^{3/2} - B\ell^2
\]

(9)

where \( \ell \) is the average vortex-line density \( \ell \) (length per unit volume), and \( A \) and \( B \) are phenomenological parameters, with \( A \) governing the growth, and \( B \) the decay of the vortex tangle.

In the following, we briefly survey quantum vorticity in diverse physical systems.
Figure 2: Left panel: The vortex ring has a translational velocity $v$ approximately inversely proportional to its radius $R$. Right panel: A general vortex line has local velocity normal to the tangential vortex ring, with magnitude inversely proportional to the radius of curvature.

Figure 3: Left panel: Reconnection of vortex lines creates two cusps that spring away from each other at high velocity, creating high-velocity jets of flow. Right panel: Jets in magnetic reconnections in the sun’s corona creates solar flares.
2 Liquid helium and cold trapped atomic gases

Quantized vortices in superfluid helium and atomic gases can be described by the NLSE \( \text{[2]} \), with superfluid density given by \( n = \mu_0/g \), and healing length given by

\[
\xi = \frac{\sqrt{g}}{\mu_0} \tag{10}
\]

By shooting alpha particles or electrons into superfluid helium, vortex rings can be created, which trap the projectiles \( \text{[8]} \). The velocity-energy curve of the composite object can be obtained by dragging it cross the liquid against an electric field. The results fit that of a quantized vortex ring, as shown in Fig.5.

In superfluid helium and cold condensed atomic gases, vortices can be created by rotating the container. When the angular velocity of the container exceeds a critical value, the superfluid responds by developing vortex lines parallel to the axis of rotation \( \text{[9]} \) \( \text{[10]} \). Experimental results in a trapped atomic gas are shown in Fig.6, showing the development of a vortex lattice as the angular frequency increases \( \text{[11]} \).

Quantum turbulence \( \text{[13]} \), as well as vortex reconnections \( \text{[12]} \), have been studied in superfluid helium. The velocity distribution in quantum turbulence is found to have a power-law tail \( v^3 \), as shown in Fig.7. This is quite different from the Gaussian distribution of classical turbulence, and is due to the occurrence of large velocities in vortex reconnections. Computer simulations using the NLSE have reproduced this distribution \( \text{[14]} \).

Vortex lines in liquid helium have been made visible by coating them with nano-sized metallic dust, as shown in Fig.8 \( \text{[15]} \). The metallic dust adheres to the surface of the vortex tube because of the Bernoulli pressure arising from a decrease of superfluid velocity away from the the core.

The computer simulations in Fig.9 \( \text{[16]} \) demonstrates that a Bose condensate far from equilibrium creates quantum turbulence, and relaxes through its decay.

Figure 4: Emergence of quantum turbulence in computer simulation. Numbers under each picture indicate the number of reconnections. From Ref.\[6\].
Figure 5: Velocity-energy curve of vortex ring created by ions shot into liquid helium. The ion (alpha particle or electron) gets trapped in the ring it created. The theory curve is that for a vortex with one unit of quantized circulation.

Figure 6: Lattice of vortices in rotating cold trapped atomic gas, at increasing angular velocity of rotation. From Ref. [11].

Figure 7: Velocity distribution in quantum turbulence in superfluid helium (data points), as compared with the Gaussian distribution of classical turbulence (solid curves). The former has a $v^{-3}$ tail due to the jets of large velocities accompanying vortex reconnections, which are essential for the maintenance of quantum turbulence. From Ref. [13].
Figure 8: Vortices in superfluid helium made visible by coating the surface of the vortex with metallic dust of nano size. The adhesion is due to Bernoulli pressure. From Ref. [15].

Figure 9: Computer simulation of the evolution of a Bose condensate far from equilibrium, showing that it equilibrates through the creation and decay of quantum turbulence. These pictures show the decay sequence. From Ref. [16].


3 Superconductivity

In superconductivity, as described by the GL equation (3), there is another length besides the healing length $\xi$, the penetration depth $d$. This arises from the conserved current

$$J = \frac{\hbar d}{2mi} (\phi \nabla \phi^* - \phi^* \nabla \phi) - \frac{q^2}{mc} \phi^* \phi A$$  \hspace{1cm} (11)

When this is substituted into Maxwell’s equation $\nabla \times \nabla \times A = 4\pi c J$, we obtain, in Coulomb gauge $\nabla \cdot A = 0$,

$$\left(\nabla^2 + \frac{4\pi nq^2}{mc^2}\right)A = 0$$  \hspace{1cm} (12)

for uniform $\phi^* \phi = n$. The second term in the brackets is the photon’s squared mass inside a superconductor. An external magnetic field can penetrate a superconductor only to a depth $d$ corresponding to the inverse mass:

$$d = \sqrt{\frac{mc^2}{4\pi nq^2}}$$  \hspace{1cm} (13)

This is the Meissner effect, a simple example of the Higgs mechanism, i.e., spontaneous breaking of local gauge invariance gives mass to the gauge particle.

The presence of two characteristic lengths leads two types of superconductors: If $\xi > d$, we have type I behavior: the magnetic field is completely repelled from the interior. If $\xi < d$ we have type II behavior: the magnetic field penetrates the body in vortex tubes containing quantized magnetic flux $\hbar c/e$, arranged in a regular lattice known as the Abrikosov lattice. We can understand these behaviors by referring to Fig.10, which depicts the interface between magnetic field and order parameter. In the type I case, there is little overlap between magnetic field and order parameter. If a flux tube were formed, the system can lower the energy by expelling it outside, and filling in the hole in the order parameter. For Type II, an overlap between magnetic field and order parameter lowers the energy, creating a negative surface tension. Thus, flux tubes are formed, penetrating the superconducting body, and maintained by solenoidal supercurrents. If the medium is infinite, the flux cannot be expelled, and finite-energy configurations are flux tube of finite length, terminated by magnetic monopoles. In this regard, see Fig.11 for a comparison between the magnetic field due to two monopoles in vacuum and in a superconductor. If the tube is cut, one does not get free monopoles, but more flux tubes terminating in $N$ and $S$. In this sense, a superconductor is a medium of magnetic-monopole confinement.

4 “Vorticons” in the Higgs field

In the electroweak sector of the standard model, the local gauge symmetry is spontaneously broken by of the multi-component Higgs field, which gives mass to the gauge bosons $W_\pm$ and $Z_0$, while leaving the photon massless. We can envisage a “vorticon” [17], a microscopic vortex ring in the Higgs field, in the shape of a donut, containing a gauge boson that is massless, as illustrated in Fig.12. It is a microscopic donut-shaped
Figure 10: Upper panels show the behavior of the magnetic and order parameter at an interface, for type I and type II superconductors. Lower panels illustrate the expulsion of magnet flux in type I superconductors, and the formation of a quantized-flux lattice in type II case.

Figure 11: The magnetic field of a magnetic dipole in empty space (left) and in a superconductor (right). In the latter the magnetic flux is squeezed into a flux tube by circulation supercurrents. The situation suggests that a superconductor would exhibit magnetic monopole confinement.
Figure 12: A \( Z_0 \) vector boson created in a high-energy collision induces superfluid flow in the background Higgs field, creating, and is trapped by, of a vortex ring, resulting in an unstable particle dubbed the “vorticon”, with a mass of approximately 3 TeV, and a lifetime of the order of \( 3 \times 10^{-25} \) s.

waveguide, which could be created when an energetic \( Z_0 \), say, tears through the Higgs field, much as a projectile can create and be trapped by a vortex ring in liquid helium.

The mass of a vorticon can be estimated by constructing normal modes of the \( Z_0 \) field inside a torus cut out from the background Higgs field, and minimizing the energy of the lowest mode \([17]\). The standard-model Hamiltonian in the \( Z_0 \) sector is, with \( \hbar = c = 1 \),

\[
H = \int d^3x \left[ \frac{1}{2} (\mathbf{B}^2 + \mathbf{E}^2) + |(\nabla - iq\mathbf{Z}) \phi|^2 + V(\phi) \right]
\]  

(14)

Here, \( \phi \) is the Higgs field, \( \mathbf{Z} \) the vector potential in Coulomb gauge (\( \nabla \cdot \mathbf{Z} = 0 \)), and \( \mathbf{B} = \nabla \times \mathbf{Z}, \mathbf{E} = -\partial \mathbf{Z}/\partial t \). The Higgs potential is given by

\[
V(\phi) = \frac{\lambda}{2} (\phi^* \phi - F_0^2)^2
\]  

(15)

with

\[
F_0 = 174 \text{ GeV}
\]

\[
\sqrt{2\lambda} F_0 = m_H = 125 \text{ GeV}
\]  

(16)

where \( m_H \) is the Higgs mass. Thus \( \lambda = 0.256 \). The gauge coupling constant \( q \) and \( Z_0 \) mass are given by

\[
q = -\frac{e}{\sin 2\theta_W}
\]  

(17)

\[
m_Z = \frac{g F_0}{\sqrt{2} \cos \theta_W}
\]  

(18)

where \( e \) is given through the fine-structure constant \( e^2/4\pi \approx 1/137 \), and \( \theta_W \) is the Weinberg angle given through \( \sin^2 \theta_W \approx 1/4 \).

There are two types of vorticons: magnetic and electric, with the magnetic (electric) field pointing along the toroidal direction. As in electromagnetic wave guides, there is
no completely transverse mode. The masses are found to be

\[ \frac{M_{\text{mag}}}{m_Z} = 35.3 + 6.42 \left( \frac{m_H}{m_Z} \right)^{2/3} - 1.03 \left( \frac{m_H}{m_Z} \right)^{1/2} \]

\[ \frac{M_{\text{elec}}}{m_Z} = 27.7 + 5.66 \left( \frac{m_H}{m_Z} \right)^{2/3} + 0.504 \left( \frac{m_H}{m_Z} \right)^{1/2} \]  

(19)

With the experimental values \( m_Z = 91 \text{ GeV} \) and \( m_H = 125 \text{ GeV} \), we have

\[ M_{\text{mag}} = 3.47 \text{ TeV} \]

\[ M_{\text{elec}} = 2.88 \text{ TeV} \]  

(20)

The size of these vorticons are of order \( m_Z^{-1} \approx 10^{-12} \text{ cm} \). They are unstable, with lifetimes of the order the \( Z_0 \) lifetime \( 3 \times 10^{-25} \text{s} \).

5 QCD strings

A phenomenological description of quark confinement in QCD can be modeled after the magnetic monopole confinement in a superconductor. The difference is that quarks generate Yang-Mills color-electric fields instead of magnetic fields, so the QCD vacuum confines electric flux instead of magnetic flux. (In the nonlinear Yang-Mills gauge theory, there is no electric-magnetic duality as in Maxwell theory.) A meson consists of a quark and antiquark pair, belonging respectively to the representations \( 3 \) and \( \bar{3} \) of color SU(3), connected by a flux tube. This can be represented by the picture in Fig.11., if we substitute \( 3 \) for \( N \), \( \bar{3} \) for \( S \). The order parameter corresponds to a condensate of quark-antiquark pairs, and the electric flux tube is maintained by solenoidal color-magnetic currents. A baryon made of three quarks can be represented by a flux tube with one quark at one end, at two at the other. The two-quark system has color representations \( 3 \times 3 = \bar{3} + 6 \), and we can use the \( \bar{3} \) irreducible combination. Alternatively, the baryon could be represented by Y-shaped flux tubes each terminated by \( 3 \).

One can approximate the flux tube by a massless string terminated at both ends by quarks, with the string rotating about a perpendicular axis, under tension \( T_0 \) [18], as illustrated in the upper panel of Fig.13. In the limit of small quark masses, one obtains a relation between between \( J \) and \( M \):

\[ J = a_0 + a' M^2 \]  

(21)

where \( a_0 \) is a model parameter, and

\[ a' = \frac{1}{2\pi T_0} \]  

(22)

As shown in Fig.13, \( J-M^2 \) plots of meson and baryons do yield straight lines known as Regge trajectories, with a universal slope \( a' \approx 0.9 \text{ GeV}^{-2} \). This corresponds to a string tension of approximately 16 tons.
Figure 13: Hadrons modelled as strings terminated in quarks. The strings are idealization of vortex tubes containing color-electric flux. Their rotational angular momentum \( J \) and squared mass \( M^2 \) bear a linear relation that agrees with the observed Regge trajectory of particles, and show the string has a tension of about 16 tons.

It would be desirable to set up an analog of the GL equation for QCD, but the situation here is more complicated than superconductivity, because of the intrinsic nonlinearity of Yang-Mills gauge fields. Besides quark confinement, there is chiral symmetry breaking. The condensate could involve topological object such as monopoles. We refer the reader to [19], [20], [21], for works related to that goal.

6 Cosmology

The Higgs field of particle theory has found experiment support with the discovery of the Higgs boson [22], [23]. Grand-unified or supersymmetric theories call for more Higgs-like fields. Any long-range order, such as the phase coherence of these fields, will persist to infinite distances in spatial dimension greater than two. In two or fewer dimensions, the long-range order will be destroyed by the long-wavelength fluctuations of the Goldstone modes [24]. The Higgs field, and other possible Higgs-like fields, thus make the whole universe a superposition of different types of superfluid, (like a mixture of \(^4\)He and \(^3\)He below \(10^{-3}\)K.) We discuss the manifestations of superfluidity in terms of a generic complex scalar field, in three different epochs of the universe: the big-bang era, the CMB (cosmic microwave background) era, and the present.

The scalar field presumably emerges during the big bang, but how it does this depends on model. A quantum field needs a high-momentum cutoff \( \Lambda \), which should be infinite at the big bang. If the potential emerges from zero at that moment, it must be asymptotically free. This rules out all polynomial forms, and admits only the Halpern-Huang potential [25], with exponential behavior for large fields. A big-bang model has been constructed based on this potential [26] [27], with uniform Robertson-Walker metric whose scale is tied to the cutoff: \( a = \Lambda^{-3} \). Solving Einstein equation numeri-
where $c_0$ and $p < 1$ depend on model parameters and initial data. The result is equivalent to having a cosmological constant that decays in time according to a power law. If this behavior persists to later times, it could explain dark energy without "fine-tuning". Phenomenological studies so based are referred to as "intermediate inflation" [28].

During the big bang, the scalar field would emerge far from equilibrium, and, like the Bose gas illustrated in Fig.9, equilibrate by going through a period of quantum turbulence. The vortex reconnections that maintain the vortex tangle produce jets of energy that can create matter, like the solar flares created by magnetic reconnections illustrated in Fig.3. This provides a new scenario for the inflation era, in which all matter in the present universe were created in quantum turbulence.

A detailed analysis of the turbulent era based on the coupled Einstein-scalar equations, unfortunately, faces the formidable problem of a non-uniform metric. We can bypass this difficulty with a phenomenological approach using Vinen’s equation, which deals with a uniform distribution of vortex lines. It is then possible to construct a scenario for the growth and decay of a vortex tangle, as illustrated in Fig.14. Parameters can be so chosen that the lifetime of the vortex tangle is of the order of $10^{-26}$s, during which time the universe expanded by a factor of $10^{27}$, and a total amount of matter was created equal to what is presently observed, roughly $10^{22}$ suns. This presents a new picture of inflation [27].

The big-bang era, including inflation and the creation of all matter through quantum turbulence, lasted about $10^{-26}$s. After this, nonuniformities in the universe become important, and the Robertson-Walker metric ceases to be valid. The Halpern-Huang...
potential, which is a high-cutoff approximation, may need corrections. Also, while there was only one scale in the big-bang era, the Planck scale of $10^{22}$ GeV, a new scale appears with the emergence of matter, the QCD scale of 1 GeV. Vortices created during the big bang era, including those in the vortex tangle, have core sizes of the order of the Planck length of $10^{-33}$ cm, and they will co-expand with the universe. Those created after the emergence of the matter, however, will have much smaller fixed core sizes that do not expand with the universe.

After the big bang era, our model will be replaced by a cosmic perturbation theory, which must include the scalar field, and this will govern the creation of the CMB. The widely used $\Lambda$CDM model would be modified with inclusion of a vacuum complex scalar field, which supplies a cosmological-constant ($\Lambda$) via its energy-momentum tensor, and generates cold dark matter (CDM) through density variations of the scalar field. A complete reformulation is yet to be done, but we expect that the results of the usual cosmic perturbation theories will be largely preserved. The only new element would be quantum vorticity, which will make contributions to the tensor mode, in competition with gravitational waves.

In the present universe, galaxies will attract superfluid to form halos of denser-than-vacuum superfluid around it, and these will be perceived as dark matter, through gravitational lensing [4]. When galaxy clusters collide, their haloes undergo distortions according to superfluid hydrodynamics, as observed in the "bullet cluster" [29]. Computer simulations based on the NLKG may be found in [4]. The literature contains studies of cosmic vorticity under the name "cosmic strings" [32] [33].

A fast-rotating body such as a black hole will drag the surrounding superfluid into rotation through the creation of vortices. The so-called "non-thermal" filaments [34] observed near the center of the Milky Way could be vortex lines, as shown in the upper panel of Fig.15. The lower panels shows 3D simulations based on the NLKG [3].

The core sizes of remnant vortices from quantum vortex turbulence in the big bang era, originally of order of the Planck size of $10^{-33}$ cm, will co-expand with the universe, and in the 13.7 billion years since, they could reach sizes of the order of $10^7$ light years. Since matter was created in the scalar field, these cores are devoid of matter, and show up as voids in the galactic distribution. The so-called "stick man" configuration [35] is shown in Fig.16, with a simulation by superposition of vortex tubes.

### 7 Galaxy formation

The CMB was formed about $10^5$ yrs after the big bang. Between that and the present, there was a long period of galactic formation, in which quantum vorticity may play an important role. A speculation of Lathrop [36] is that a galaxy can form from a large vortex ring with accretion of dust, which gravitates to form a central mass, squeezing the vortex ring into a shape with spiral arms, as illustrated in Fig.17. Can one find any hint of this mechanism in currently-observed galactic properties?

The central mass would correspond to the black hole observed at the center of all galaxies, whose mass $M$ bears power-law relations to other galactic properties [37]:

$$M \sim X^\beta$$

(24)
Figure 15: Upper panel: The "non-thermal filaments" observed near the center of the Milky Way, Ref.[34], could be vortex lines surrounding black holes. Lower panel: 3D simulations of a vortex-ring assembly arising from a rotating body at the center, based on the NLKG, in various perspectives, Ref.[3].

Figure 16: Left: Voids in galaxy distribution in the "stick man" configuration, Ref. [35]. Right: Simulation by the superposition of three primordial vortex tubes, which have co-expanded with the universe to gigantic sizes. The inside of the tubes are devoid of galaxies, which cling to the outside due to hydrodynamic pressure, like the metallic dust clinging to vortex tubes in superfluid helium shown in Fig.8.
Figure 17: Lathrop’s suggestion for galaxy formation: dust particles accrete onto a vortex ring, gravitate and clump, squeezing the ring into a spiral shape with a central mass, which would become a black hole.

as indicated in the following:

|        |        |
|--------|--------|
|        |        |
|        |        |

\[
X \quad \beta
\]

Stellar mass \( m \) 1.05 ± 0.11
Luminosity \( L \) 1.11 ± 0.13
Stellar velocity \( v \) 5.57 ± 0.33

Assume that the dust particles initially accrete onto the vortex ring uniformly, and then clump up under gravitation. Assume further that a fixed fraction forms the central mass, which becomes a black hole and goes dark, while the rest remains luminous. This would mean \( m \propto M \), and \( L \propto m \), which are consistent with the relevant exponents being unity.

The initial vortex ring may be generated by self-avoiding random walk (SAW). (The fact that it is a close ring matters little for the arguments here.) Thus, the dust particles, which initially adhere uniformly to the vortex ring, form a SAW sequence, with the relation \( N \sim R^{5/3} \), where \( R \) is the spatial extension, and \( N \) the number of steps. In our case, \( R \) corresponds to the size of the galaxy, and \( N \) is proportional to \( M + m \), hence to \( M \). This means that \( M \) scales like \( R^{5/3} \). Now assume that the total angular momentum \( J \), which is a constant of the motion, scales as the galactic volume:

\[
M \sim R^{5/3}
\]

\[
J \sim R^3
\]

Defining the stellar velocity \( v \) through \( J = Rmv \propto RMv \), we obtain

\[
M \sim v^5
\]

which is not inconsistent with observations. As an interesting note, the SAW exponent 5/3 is the same as the Kolmogorov exponent in classical turbulence, and the Flory exponent for polymers \([38]\).
References

[1] F. Dafolvo, S. Giorgini, L.D. Pitaevskii, and S. Stringari, Rev. Mod. Phys. 71, 463 (1999).
[2] P.G. DeGenne, Superconductivity of Metals and Alloys (Westview Press, Boulder, 1999).
[3] C. Xiong, M. Good, X. Liu, and K. Huang, Phys. Rev. D90, 125019 (2014); arXiv:1408.0779.
[4] K. Huang, C. Xiong, and X. Zhao, J. Mod. Phys. A, 29, 1450074 (2014); arXiv:1304.1595.
[5] S.K. Nemirovskii and W. Fizdon, Rev. Mod. Phys. 67, 37 (1995).013603 (2002).
[6] K.W. Schwarz, Phys. Rev. B38, 2398 (1988).
[7] D. Kivotides, C.F. Barenghi, and D.C. Samuels, Phys. Rev. Lett. 87, 155301 (2001).
[8] G.W. Rayfield and F. Reif, Phys. Rev. 136, A1194 (1964).
[9] R.J. Donelly, Quantized vortices in helium II (Cambridge University Press, 1991).
[10] A.L. Fetter, Rev. Mod. Phys. 81, 647 (2009).
[11] Abo-Shaeer, I. Raman, and W. Ketterle, Phys. Rev. Lett. 88, 070409 (2002).
[12] M.S. Paoletti, M.E. Fisher, and D.P.Lathrop, Physica D239, 1367 (2010)
[13] M.S. Paoletti, M.E. Fisher, K.R. Sreenivasan, and D.P. Lathrop, Phys. Rev. Lett. 101,154501 (2008).
[14] A.C. White, C.F. Barenghi, N.P. Proukakis, A.L. Youd, and D.H. Hawks, Phys. Rev. Lett. 104, 075301 (2010).
[15] V. Lebedev, P. Moroshkin, B. Grobey, and A. Weis, J. Low Temp. Phys.165, 166 (2011).
[16] N.G. Berlof and B.V. Svistunov, Phys. Rev. Rev. B66, 013603 (2002).
[17] K. Huang and R. Tipton, Phys. Rev. 23, 3050 (1981).
[18] K. Johnson and C. Nohl, Phys. Rev. D19, 291 (1979).
[19] K. Huang, Quarks, leptons and gauge fields, 2nd ed (World Scientific, Singapore, 1992), Chap.14.
[20] V.Gusynin and V.A. Miranskii, Zh. Eksp.Teor.Fiz. 101, 414 (1992) [Sov. Phys. JETP 74, 216 (1992).]
[21] C. Xiong, Phys. Rev. D88, 025042 (2013).
[22] ATLAS collaboration, Phys. Lett. B 716, 1 (2012).
[23] CMS collaboration, Phys. Lett. B 716, 30 (2012).
[24] K. Huang, *Statistical Mechanics*, 2nd ed (Wiley, New York, 1987), Sec.16.7.
[25] K. Halpern and K. Huang, Phys. Rev. 53, 3252 (1996).
[26] K. Huang, H.B. Low, and R.S. Tung, Class. Quantum Grav. 29, 155014 (2012); arXiv:1011.4012
[27] K. Huang, H.B. Low, and R.S. Tung, Int. J. Mod. Phys. A 27, 1250154 (2012); arXiv:1106.5283.
[28] J. Barrow, Phys. Rev. D 51, 2729 (1995).
[29] D. Crowe et al, Astrophys. J. 648, 806 (2006).
[30] B. Kain and H.Y. Ling, Phys. Rev. D 82, 064042 (2010).
[31] V.C. Rubin, Thorndard, and W.K.J. Ford, Astrophys. J. 238, 471 (1980).
[32] B. Gradwohl, Kalbermann, T. Piran, and E. Bertschinger, Nucl. Phys. B 338, 371 (1990).
[33] A. Villenkin and E.P.S. Shellard, *Cosmological strings and other topological defects* (Cambridge University Press, 1994).
[34] T.N. LaRosa et al, Astrophys. J. 607, 302 (2004).
[35] V. deLapparent, M.J. Geller, and J.P. Huchra, Astrophys. J. 302, L1 (2004).
[36] D.P. Lathrop, to be published.
[37] J.M. Nicolas and C.P. Ma, arXiv:1211.2816 (2012).
[38] K. Huang, *Lectures on statistical physics and protein folding* (World Scientific, Singapore, 2005), Chaps.14,15.