Atomic quantum transistor based on swapping operation

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We propose an atomic quantum transistor based on exchange by virtual photons between two atomic systems through the control gate-atom. The quantum transistor is realized in two QED cavities coupled in nano-optical scheme. We have found novel effect in quantum dynamics of coupled three-node atomic system which provides control-SWAP(θ) – processes in quantum transistor operation. New possibilities of quantum entanglement in an example of bright and dark qubit states have been demonstrated for quantum transport in the atomic chain. Potentialities of the proposed nano-optical design for quantum computing and fundamental issues of multi-atomic physics are also discussed.

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Introduction: One of the basic elements of microelectronic devices is the transistor - a three-port device where the electron flux through two ports is controlled by the third port. The same is with optoelectronics where light is controlled by light via electrons or atomic medium in integrated design. Moor’s law dictates increasing of elements number per chip. But confining of light on the subwavelength scale is complicated. Therefore excitonic transistor where exciton fluxes of medium excitation are controlled directly by a gate voltage without using light itself for logic operations seems to be a promising approach. As for quantum logics rapidly developing last decades, it also strongly demands for creation of transistors like in the case of its classical counterpart. Several quantum transistor designs were proposed. Among them, spintronics approach is interesting from dense integration point of view. It is useful for creation of NMR quantum computer in microwave range. Here, we propose quantum transistor in atomtronics approach based on SWAP – operation using exchange between atoms by virtual photons that was proposed initially in and observed experimentally for neutral atoms in traps recently. This transistor does not use real photons and therefore permits subwavelength integration extremely urgent for quantum technologies dealing with single atoms or mesoscopic atomic ensembles. We show that such kind of transistors can be used for realization of control SWAP– and √SWAP– processes. Finally we discuss possible applications of the atomic transistor in optical solid state quantum computer and fundamental issues of quantum nature revealed in multi-particle systems.

Basic scheme: The atomic quantum transistor (AQT) contains pair of resonant multi-atomic ensembles (or pair of two-level atoms, we call it ME1 and ME2-nodes) and a single three-level gate-atom. ME1 and ME2-nodes are characterized by long-lived coherence and coupled with each other via the gate-atom which could be a natural or artificial atom with appropriate energy levels and transition dipole moments. Depending on the initial state, gate-atom controls (switches on in the state |0⟩ or blocks in the state |1⟩) an exchange by virtual photons between the two atomic nodes providing AQT operation in two interesting regimes. In the first regime of AQT, only control-SWAP operation can be implemented, the second regime of AQT provides besides a control-√SWAP operation for quantum states of the atomic nodes.

The proposed AQT exploits nano-optical scheme of the light-interface depicted in Fig. ME1 and ME2-nodes are situated in different single mode resonators. The resonators can be fabricated with nanoptical waveguide technology. Such type of the nanoptical fiber with enhanced evanescent field has been demonstrated recently as a promising method for strong coupling of evanescent light field with single atoms situated at small distance (≈200 nm) from the fiber. There, the authors also observed that the trapped atoms do not acquire essential line broadening. Thus ME1 and ME2 nodes could be the atomic systems trapped close to the surface of the waveguide resonators or it could be rare-earth ions in the inorganic crystal (and NV centers in nano-diamond) embedded in a body of the waveguide...
resonators. We put the gate-atom in the crossing point of the resonator axis as depicted in Fig. 1. Here, gate-atom will interact intensively with each field mode. At the same time the field modes will be decoupled from each other for non-collinear axis orientation of the QED-cavities.

Let us take that working frequencies of the atoms shifted from the frequency \( \omega_c \) of cavity field modes where we also assume equalized frequencies of the field modes \( \omega_{\alpha} = \omega_c \) (\( \alpha = 1, 2 \) is an index of the modes, \( k_\alpha \) are wave vectors of the field modes). The effective Hamiltonian describing the process can be obtained by unitary transformation of initial James-Cummings model Hamiltonian that excludes non-resonant atomic-photon interaction in the first perturbation order [17, 23]. This procedure yields the following Hamiltonian of the ME1 and ME2-nodes and gate-atom in a photon vacuum state:

\[
H_s = H_{(s,o)} + V,
\]

\[
H_{(s,o)} = \hbar \omega_0 S_c^z + \hbar \omega_1 \sum_{j_1} N_1 S_{j_1}^z + \hbar \omega_2 \sum_{j_2} N_2 S_{j_2}^z + \hbar \sum_{\alpha=1,2} \sum_{j_\alpha} \frac{g_{j_\alpha} g_{j_\alpha}^*}{\Delta_\alpha} \exp(i r_{(j_\alpha,j_{\alpha}^{'})} S_{j_\alpha}^+ S_{j_{\alpha}^{'}}^-),
\]

\[
V = \hbar \sum_{\alpha=1,2} \sum_{j_\alpha} \left\{ \frac{g_{j_\alpha} g_{j_\alpha}^*}{\Delta_\alpha} \left[ 1 + \frac{1}{\Delta_0} \right] \exp(i r_{(j_\alpha,j_{\alpha}^{'})} S_{j_\alpha}^+ S_{j_{\alpha}^{'}}^-) + \text{h.c.} \right\},
\]

where \( H_{(s,o)} \) describes Hamiltonian of ME1, ME2-nodes and gate-atom, while \( V \) determines an exchange processes between these three atomic groups, \( \tilde{\omega}_0 = \omega_0 + \frac{1}{\Delta_0} \left( |\tilde{g}_1|^2 + |\tilde{g}_2|^2 \right) \); \( \omega_0 \) and \( \omega_1, \omega_2 \) are the initial transition frequencies of gate-atom and the atoms in two nodes; \( N_1 \) and \( N_2 \) are the number of atoms in ME1 and ME2-nodes; \( S_{j_1}, S_{j_2} \) and \( S_{j_\alpha} \) are \( z \)-components of effective 1/2-spin in sites \( j_1, j_2 \) of ME1, ME2 and for gate-atom; \( S_{j_\alpha}^+, S_{j_\alpha}^\pm \) and \( S_{j_\alpha}^\pm \) are an appropriate raising and lowering 1/2-spin operators; \( g_{j_\alpha} \) are the photon-atom coupling for \( j_\alpha \) atom in \( \alpha \) atomic node, (\( \alpha = 1,2 \) \( \tilde{g}_{\alpha} \) are the coupling constants of gate-atom with photon in \( \alpha \)-evanescent mode; \( \Delta_\alpha = \omega_\alpha - \omega_c, \Delta_0 = \omega_0 - \omega_c \) are frequency offsets; \( r_{ij} \) is radius-vector connecting sites \( i \) and \( j \), where \( r_{00} = 0 \) is used.

\textit{Swap-dynamics:} Let’s consider double SWAP-processes. We assume that ME1-node is prepared in the single atomic excitation state, while the second ME2 node and gate-atom stay in the ground states that gives the following initial quantum state \( |\Psi_1\rangle = |0_0\rangle |1_1\rangle |0_2\rangle \equiv |0_0\rangle |1_1\rangle |0_2\rangle \) (where \( \langle a|b|...|f\rangle_n \equiv \langle ab...f\rangle \), \( |j\rangle = \frac{1}{\sqrt{N}} \sum_{j_\alpha} g_{j_\alpha} \exp(i r_{j_{\alpha} j_\alpha} ) |0_1\rangle |0_2\rangle |...|0_{N_\alpha} \rangle \) is a the single atomic excitation state where \( A_\alpha = \sum_{j_\alpha} |g_{j_\alpha}|^2 \) and \( |0\rangle = |0_1\rangle |0_2\rangle ... |0_{N_\alpha} \rangle \) is a ground state in \( \alpha \)-atomic node. The studied quantum dynamics of the atoms described by the effective Hamiltonian \( H \) yields the wave function

\[
|\Psi(t)\rangle = c_0(t)|\Psi_0\rangle + c_1(t)|\Psi_1\rangle + c_2(t)|\Psi_2\rangle,
\]

decomposed into the quantum superposition of single atomic excitation states in the system of gate-atom plus ME1 and ME2-nodes, where \( |\Psi_0\rangle = |1_0\rangle |0_1\rangle |0_2\rangle \), \( |\Psi_2\rangle = |0_0\rangle |0_1\rangle |1_2\rangle \) and \( |\Psi_1\rangle = |0_0\rangle |1_1\rangle |0_2\rangle \) are the ground and excited states of the gate-atom.

Let us assume equal numbers \( N_1 = N_2 = N \) and resonant frequencies \( \omega_1 = \omega_2 = \omega \) of atoms in ME1 and ME2 nodes. By taking into account \( H_{(so)} |\Psi_0\rangle = (E_0 + \hbar \omega_0)|\Psi_0\rangle \), \( H_{(so)} |\Psi_1\rangle = (E_0 + \hbar \omega_0)|\Psi_1\rangle \) and \( \langle \Psi_0|V|\Psi_1\rangle = \sqrt{N}\hbar \Omega_{ac} \) (where \( E_0 = -\hbar \omega_0 - N\hbar \omega \) and \( \omega = \omega + N\Omega_{\alpha}, \Omega_{ac} = \frac{\sqrt{N}\hbar}{\omega_0} |\langle g_2 |g_2\rangle| \left( \frac{1}{\Delta_0} + \frac{1}{\Delta_\alpha} \right), \Omega_{\alpha} = \frac{1}{\Delta_\alpha} \sum_{j_\alpha} |g_{j_\alpha}|^2 \) ) we find a solution for the amplitudes

\[
c_{0,1,2}(t) = e^{-i E_0 t / \hbar} c_{0,1,2}(t) \text{ using the initial quantum state } c_0(t = 0) = c_2(t = 0) = 1 \text{ and } c_1(t = 0) = 1:
\]

\[
\tilde{c}_0(t) = 2i \sqrt{N} \hbar \Omega_{ac} \sin \frac{St}{2},
\]

\[
\tilde{c}_1(t) = \frac{1}{2} e^{-i \omega t} \left[ 1 + e^{-i \frac{\Delta}{2} \sin \left( \frac{St}{2} \right)} \right],
\]

\[
\tilde{c}_2(t) = \frac{1}{2} e^{-i \omega t} \left[ -1 + e^{-i \frac{\Delta}{2} \sin \left( \frac{St}{2} \right)} \right],
\]

where \( S = \sqrt{8N\Omega_{ac}^2 + \Delta^2}, \Delta = \tilde{\omega}_0 - \tilde{\omega} \).

The solution reveals an interesting dynamics in two particular cases.

\textit{Resonant swapping} (\( \Delta = \tilde{\omega}_0 - \tilde{\omega} = 0 \)): We assume that it is possible to control the atomic resonant frequencies \( \tilde{\omega}_0 \) and \( \tilde{\omega} \) by using Stark or Zeeman effects in external electric or magnetic fields applied to the gate-atom and ME1, ME2-nodes. By equalizing the frequencies \( \tilde{\omega}_0 = \tilde{\omega} \) in Eq. (5), we find a nutation with a maximum oscillation amplitude in a system of three atomic systems:

\[
\tilde{c}_0(t) = \frac{i}{\sqrt{2}} e^{-i \omega t} \sin \frac{S_o t}{4},
\]

\[
\tilde{c}_1(t) = e^{-i \omega t} \cos \frac{S_o t}{4},
\]

\[
\tilde{c}_2(t) = -e^{-i \omega t} \sin \frac{S_o t}{4},
\]

where \( S_o = 2 \sqrt{2} N \Omega_{ac} \).

Eq. (6) demonstrates a 100% transfer of excitation from ME1 to ME2-node at the moment of time \( t = t_{swap}^{(r)} = 2\pi / S_o \) where \( c_0 = c_1 = 0 \) and \( c_2 = 1 \). We note that \( |c_0(t)| \) gets a maximum \( |c_0| = 1 / \sqrt{2} \) at \( t' = t_{swap}^{(r)} / 2 \).
that corresponds to 50% of probability to find the excitation on gate-atom. At this moment of time \(|c_1(t^*)| = |c_2(t^*)| = 1/2\), i.e. other 50% of the excitation are equally between the ME1 and ME2-nodes.

For arbitrary initial state of ME1-node \(|\psi(\phi_1)\rangle = \alpha_1 |0\rangle_1 + \beta_1 e^{i\phi_1} |1\rangle_1\), we get

\[
|0\rangle_0 |\psi(\phi_1)\rangle_1 |0\rangle_2 \xrightarrow{SWAP} |0\rangle_0 |\psi(\phi_1 - \tilde{\omega}(r_{\text{swap}}) t^{(r_{\text{swap}})} + \pi)\rangle_2,
\]

(7)

with phase transformation \(\phi_1 \rightarrow \phi_1 - \tilde{\omega}(r_{\text{swap}}) t^{(r_{\text{swap}})} + \pi\).

Nonresonant swapping: More general scenarios of the excitation transfer occurs for nonresonant interaction with spectral detuning \(\Delta = 2\sqrt{2N/3\Omega_{ac}} = S/2\) between the resonant frequencies of gate-atom and the atomic nodes. By putting \(t^* = 2\pi/S = \frac{2\pi}{2\sqrt{3}\sqrt{2}} = t_{\text{swap}}\sqrt{2}\) in Eq. (3), we get at this moment of time

\[
|\Psi(t^*)\rangle = e^{-i\delta\phi} \left(|\Psi_1\rangle + i|\Psi_2\rangle\right) / \sqrt{2},
\]

(8)

where \(\delta\phi = (E_c/h + \tilde{\omega} + \Delta/4)t_{\text{swap}} - \pi/2\), \(t_{\text{swap}}\) corresponds to \(\sqrt{2}\) operation (equal redistribution of the initial excitation in ME1-node over the ME1 and ME2-nodes).

Succeeding atomic evolution yields a complete transfer of the state between ME1 and ME2-nodes for the moment of time only \(\sqrt{3}\) times longer than resonant swapping time \(t_{\text{swap}}^{(r)} = \sqrt{M_{\text{swap}}}\). Temporal behavior of excitation probabilities for gate-atom and ME1, ME2-nodes is depicted in Fig. 2. At the point \(tS/\pi = 2\), excitation probability density at central atom is zero when the excitation probabilities of ME1 and ME2-nodes are equal each other and equal 1/2 that is we are in purely entangled state of spatially separated ensembles ME1 and ME2. It had become possible due to the quantum interference of the excitations in distant nodes at the special choice of parameters \(\Delta = S/2\) when intranode detuning oscillation frequency and internode swapping frequency coincide. On the other hand there are also two points in Fig. 2 \((tS/\pi \approx 0.8 \text{ and } tS/\pi \approx 3.2)\) where excitation probabilities are equal each other and equal 1/3 that is we are in purely entangled state of three spatially separated atomic systems - gate-atom, ME1 and ME2-nodes.

By using the initial state \(|\psi(\phi_1)\rangle_1\), we get

\[
|0\rangle_0 |\psi(\phi_1)\rangle_1 |0\rangle_2 \xrightarrow{SWAP} |0\rangle_0 |\Phi_{\text{swap}}^{(nr)}(\psi)\rangle,
\]

(9)

\[
|\Phi_{\text{swap}}^{(nr)}(\psi)\rangle = \alpha_1 |00\rangle + \beta_1 e^{i(\phi_1 - \delta\phi)} |10\rangle + i|01\rangle / \sqrt{2}.
\]

(10)

Nonresonant SWAP-gate is realized with the same phase transformation \(\phi_1 \rightarrow \phi_1 - \tilde{\omega}(r_{\text{swap}}) t^{(r_{\text{swap}})} + \pi\) as in resonant swapping. Moreover \(\sqrt{2}\) gate is directly generalized to SWAP\((\theta_{n,m})\)-gate if we use a swapping time \(t = 2m\pi/S\) and swapping phase \(\theta_{n,m} = \pi m/(4n)\) with spectral detuning \(\Delta = S/(2n)\) where \(m, n\) are integers. Here, SWAP-process is realized for \(m = 2n\) and \(\sqrt{2}\) occurs for \(n = m\).

Control-SWAP\((\theta_{n,m})\)-operations: Here, we take into account that gate-atoms is a three level system and excitation in it can be transferred from the states \(|0\rangle_0, |1\rangle_0\) to state \(|b\rangle_0\) highly decoupled from ME1 and ME2-nodes. Availability of the states \(|0\rangle_0\) and \(|b\rangle_0\) in the gate-atom opens a possibility for realization of control-SWAP\((\theta_{n,m})\)-gates. Below we demonstrate this AQT working in a quantum fashion for control-SWAP- and control-\(\sqrt{2}\) processes.

Initially we transfer the quantum state of control ME3-node \(|\Psi_c(\phi_c)\rangle_3 = \alpha_c |0\rangle_3 + \beta_c e^{i\phi_c} |1\rangle_3\) to the gate-atom. For this step we equalize resonant frequency \(\tilde{\omega}_0\) of the gate-atom with the frequency of control two-level atoms \(\tilde{\omega}_0\) providing a transfer of \(|\Psi_c(\phi_c)\rangle_3\text{ to gate-atom}\)

\[
|0\rangle_0 |\psi(\phi_1)\rangle_1 |0\rangle_2 \xrightarrow{(g+ME3)\text{SWAP}} |0\rangle_0 |\psi(\phi_1 + \pi/2)\rangle_1 |0\rangle_2 |3\rangle.
\]

(11)

In the second step we apply an auxiliary laser \(\pi\)-pulse to gate-atom which transforms the state component \(\beta_c e^{i\phi_c} |1\rangle_0\) in Eq. (11) to the blockade state \(-\beta_c e^{i\phi_c} |b\rangle_0\). By taking into account the initial state of ME1 and ME2-nodes, we get

\[
|0\rangle_0 |\psi(\phi_1)\rangle_1 |0\rangle_2 |\Psi_c(\phi_c + \pi)\rangle_3 \xrightarrow{(g+ME3)\text{SWAP}} |0\rangle_0 |\psi(\phi_1)\rangle_1 |0\rangle_2 |3\rangle.
\]

(12)

where \(|\Psi_c(\phi_c + \pi)\rangle_0 = \{\alpha_c |0\rangle_0 - \beta_c e^{i\phi_c} |b\rangle_0\}\). Now we realize the scenario of resonant (or nonresonant) swapping for the state (12)

\[
|\tilde{\Psi}_c(\phi_c + \pi)\rangle_0 |\psi(\phi_1)\rangle_1 |0\rangle_2 |3\rangle \xrightarrow{(g+ME3)\text{SWAP}} \{\alpha_c |0\rangle_0 |1\rangle_1 |\psi(\phi_1 - \tilde{\omega}(r_{\text{swap}}) t^{(r_{\text{swap}})} + \pi)\rangle_2 + \beta_c e^{i\phi_c} |b\rangle_0 |\psi(\phi_1)\rangle_1 |0\rangle_2 |3\rangle\}.
\]

(13)

After subsequent using of the laser \(\pi\)-pulse on the transition \(|1\rangle_0 \leftrightarrow |b\rangle_0\) and resonant transfer between ME3-node
and gate-atoms, we terminate a realization of SWAP-gate controlled by the qubit state of ME3-node

\[
|0\rangle_0|\psi(\phi_1)\rangle_1|0\rangle_2|\Psi_c(\phi_c + \pi)_3 \xrightarrow{\text{control-SWAP}} |0\rangle_0|\alpha_c\rangle_1|\psi(\phi_1) - i\omega t_{\text{SWAP}}(\phi_1,\phi_2) + \pi\rangle_2|0\rangle_3 + \beta_e e^{i\phi_c}|\psi(\phi_1)\rangle_1|0\rangle_2|1\rangle_3.
\]

As seen here, AQT operation leads to entanglement of ME3-node with ME1 and ME2-nodes coupled via SWAP-gate where the entanglement gets a maximum at \(|\alpha_c| = |\beta_c| = 1/\sqrt{2}\). Similarly we realize a \(\sqrt{\text{SWAP}}\)-gate controlled by ME3-node

\[
|0\rangle_0|\psi(\phi_1)\rangle_1|0\rangle_2|\Psi_c(\phi_c + \pi)_3 \xrightarrow{\text{control-\sqrt{SWAP}}} |0\rangle_0|\alpha_c|\Psi^{(nr)}_{\text{SWAP}}(\psi)\rangle_1|0\rangle_2|0\rangle_3 + \beta_e e^{i\phi_c}|\psi(\phi_1)\rangle_1|0\rangle_2|1\rangle_3.
\]

and another control-SWAP(\(\theta_{n,m}\)) gate that yields more sophisticated quantum superposition coupling the entangled state caused by SWAP(\(\theta_{n,m}\)) gate (for example, state (11) for \(\sqrt{\text{SWAP}}\) and initial qubit state of ME1-node.

**Discussion and Conclusion:** We have described main properties of the proposed atomic quantum transistor (AQT). Here, we have found novel effect of quantum transport between resonant atomic nodes providing a perfect realization of control-SWAP(\(\theta_{n,m}\)) processes. The elaborated model of the quantum transport can be implemented with various quantum systems (natural and artificial atoms and molecules) by using nano-optical and nano-plasmonic schemes interesting for quantum computing and communication. It is worth noting that by using the swapping processes, one can realize a complete set of single and two-qubit gates in a system of many atomic nodes where SWAP- and control-SWAP processes will play a role of single- and two-qubit gates. Detailed study of these quantum protocols will be a subject of further investigation. From all set of possible quantum states, here we briefly discuss only one specific eigenstate (we call it spatial dark multi-atomic (SDMA-) state \(|\Psi_{\text{sDMA}}\rangle\) of gate-atom and ME1, ME2-nodes coupled by the effective Hamiltonian (11). SDMA-state can be prepared via \(\sqrt{\text{SWAP}}\)-process in Eq. (10) with additional phase factor of atoms in ME2-node caused by using a local Stark or Zeeman shift of the atomic levels during short enough time. This procedure leads to the state \(|\Psi_{\text{sDMA}}\rangle = (|\Psi_1\rangle - |\Psi_2\rangle)/\sqrt{2} = |0\rangle_0(|10\rangle - |01\rangle)/\sqrt{2}\) which demonstrates an immunity to the exchange by virtual photons between three atomic groups in the case of usual atomic dark state (23) that demonstrates a complete decoupling from the interaction with two (or more) coherent light field in the electromagnetically induced transparency (EIT) effect. By taking into account that SDMA-state belongs to the decoherence free subspace for quantum dynamics of the atoms, this points out an important resource for realization of long-lived quantum coherence in this kind of multi-atomic systems evolving in nano-optical scheme. SDMA-state and state (8) (which is a spatial bright multi-atomic state in contrast to SDMA-state) are a clear demonstration of the entanglement in many atomic systems situated in the distant nodes. The states reveal themselves in a new effect of spatial quantum interference at coherent transport through our three-node chain similarly to dark and bright states in quantum transport based on EIT effect.

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