Correlations between Aharonov-Bohm effects and one-dimensional subband populations in GaAs/Al$_x$Ga$_{1-x}$As rings

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The Aharonov-Bohm (AB) interference patterns in ring-shaped conductors are usually dominated by random features. The amplitude of the oscillations is random from sample to sample and from point to point on the magnetic field axis owing to random scattering of the electron trajectories by impurities within the wires. We report experiments on new devices made with wet etching and global gates, which have shown major progress towards removing the random features. In loops that exhibit ballistic conductance plateaux and cyclotron orbit trapping at 4.2$K$, the random pattern of AB oscillations (observed for $T < 0.1K$) can be replaced by much more ordered one – especially if only a few transverse modes are populated in the ring. The amplitude and shape of the oscillation envelope function change systematically as subbands are populated in the wires forming the loops. Mechanisms governing the AB effect in the ballistic regime are discussed. Correlation has been found between the $G(V_g, B = 0)$ staircase and the “beating period” of the envelope functions.
Quantum oscillations in \( G(V_g, B = 0) \) are consistent with direct interference of paths of unequal length. Both the correlations and the quantum oscillations in gate voltage are signatures of ballistic transport.

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Introduction

The Aharonov-Bohm (AB) interference effects\(^1\) have been studied extensively in small metallic rings.\(^2\) In these devices, electrons encounter large amounts of elastic scattering and move diffusively. The elastic mean free path is typically \( l_e \sim 10 - 100\,\text{Å} \). The AB effects are observable in these systems because at low temperatures the quantum phase information is retained over a much longer distance \( l_\phi \), the phase coherence length,\(^3\) which can be 3 to 4 orders of magnitude greater than \( l_e \) at temperatures below \( T = 1\,\text{K} \). In metals the electrons are highly degenerate and the Fermi wave length is much smaller than the diameters of wires that form the loop (typically 3 orders of magnitude), hence there are a large number of states on Fermi surface contributing to the electron transport, and there is no quantization of electron motion in the direction transverse to the wire.

Following the discovery of the quantized conductance in semiconductor heterostructure point contacts\(^4\), much work has been devoted to the study of one-dimensional (1-d) electron subbands (or modes) in electron transport.\(^6\)\(^7\) The subbands arise due to the low carrier density in semiconductor interfaces, which leads to a long Fermi wavelength, which is in turn comparable to the width \( t \) of the wire. Both theory and experiments\(^4\)\(^6\)\(^8\) have shown that, in a straight channel, the conductance contributed by each subband is \( 2e^2/h \) (2 from spin degeneracy). If the carrier density is controlled by a gate voltage, the conductance
will change in increments of the basic conductance quanta $2Ne^2/h$, where $N = 0, 1, 2...$ is the number of occupied subbands. Recently, AB experiments have been carried out in loops fabricated on high electron mobility GaAs/Al$_x$Ga$_{1-x}$As heterostructures. In this type of device, large amplitude AB oscillations have been reported by several groups. [9–14] Since electron conduction is via the subbands, the subband population ought to affect the AB interference pattern dramatically. If no scattering occurs, and electrons are guided only by the electrostatic confinement that defines the shape of the device and the Lorentz force, we would expect the electron transmission to be very similar to microwave transmission through waveguides, and that changing the electron density would be analogous to tuning the microwave frequency. In a pure ballistic situation, such a device would be a true solid state interferometer in a magnetic field.

Consider what happens to the interference when the mode population changes. At zero magnetic field, conductance $G(V_g, B = 0)$ is a mode-counting staircase in $V_g$. In the plateaux (the tread of the “staircase”), the modes are well defined (transverse momentum is the conserved quantum number), and in the transition regions or the risers, new modes are just turning on. At low enough fields, we may suppose that the magnetic field does not alter the subband population substantially. Correlations should be expected between AB oscillations at fixed gate voltage $G(V_g = \text{const}, B)$ and the mode-counting staircase $G(V_g, B = 0)$: for $V_g$ on the plateaux of the mode-counting staircase, the AB oscillations are larger and more ordered than $V_g$ in the risers. As a result, the large ordered and somewhat small and disordered patterns will appear alternatingly in the AB oscillations as we sweep the gate voltage. At the same time, we also expect that as more and more modes are populated the structure of AB effect will become more and more complicated. This can be understood in analogy with a waveguide operated in a single-mode (or few-mode) transmission and in multi-mode transmission. Earlier experiments [14], however, did not find the expected correlations. Apparently the interference pattern was dominated by random scattering from donors and surface defects, [7,6] and our ballistic picture simply does not apply. (An exception occurred at high magnetic fields in the regime of the quantized Hall
effect, where beautiful regular oscillations were observed from a single point contact in the
transition between two successive Hall plateaux \cite{7}.

This paper will report the results from devices in which scattering has been eliminated
to a large degree. Correlations have been found between AB interference patterns and
subband population. In addition, quantum oscillations were seen in $G(V_g)$ at $B = 0$, which
are consistent with direct interference of paths of unequal length. These are signatures of
pure ballistic transport, and so our results contain an encouraging step toward the ultimate
goal of completely ballistic devices.

**Experiment**

Our samples were fabricated on a standard high-mobility GaAs/Al$_x$Ga$_{1-x}$As modulation-
doped layer ($x = 0.3$, carrier density $n_s = 2 \times 10^{15}/m^2$ and mobility $\mu = 90m^2/V\text{sec}$) grown
by molecular beam epitaxy. The ring geometries was defined by shallow mesas formed
through a wet etching technique. Metal gates cover all active portions of the devices. We
attribute the excellent quality of these devices directly to the softness of the etching and
screening effect of the global gate. The effect of the gate in reducing the long range in-
teraction of ionized dopants should not be underestimated. More details of the fabrication
are reported elsewhere. \cite{15} An earlier paper by the authors discussed an experiment on
a similar single ring device 4.2K. Although that temperature was too high to see any AB
oscillations, cyclotron orbit trapping was observed, which in turn is convincing evidence of
substantial ballistic transport in the device. \cite{16}

The samples we used are single ring and coupled two rings and four rings. The litho-
graphic geometry for the coupled two rings is illustrated in of Figure 1. The two parallel
rings are “joined at the hip”. The four ring sample are the obvious extension of the series
with four parallel rings. The rings in all the sample are of the same size. The lithographic
pattern has the center radius of $r = 0.80\mu m$ and line width of $t = 0.4\mu m$. Previous analysis
of the data in similar samples leads to a more realistic estimate for width of the conduction
channel to be \( t = 0.3\mu m \). [13]. The temperature at which the data was taken was 0.04K unless specified. This temperature is in the regime of large phase coherence length, where interference effects are quite apparent. The resistance was measured through standard low frequency ac technique with PAR 124A Lock-in amplifiers and Ithaco 1211 or 1212 current amplifiers. The excitation voltage ranged from 1\( \mu V \) to 4\( \mu V \) at the frequency of 13 or 278.5Hz. During the measurements, the samples were immersed in the mixing chamber of the dilution refrigerator to ensure good thermal contact.

**Analysis of the magnetoresistance**

In Figure 2 the lower curve displays a magnetoresistance data for the single ring in a “random” state. At this \( V_g \) there are 5.5 subbands are filled (judged by the conductance \( G(V_g, B = 0) \), which is not shown) in the ring. The interference pattern in this state is similar to data from earlier experiments in heterojunctions [3,4,12,13], and also from diffusive metallic loops [2]. The phase and amplitude of the oscillations are uncorrelated from point to point along the magnetic field axis. The \( h/je \) (j is an integer) frequencies are mixed with considerable amounts of other “frequencies” of oscillation yielding a random amplitude for the AB oscillations. In the “random” state, no correlations among the data at different \( V_g \) or between the AB oscillations and the \( G(V_g, B = 0) \) staircase are apparent.

In Figure 2 the top curve displays a typical magnetoresistance data for the single ring sample in the “ordered” state. \( V_g = 0.15V \), which corresponds to having 2 subbands filled in the ring. In contrast, it is dramatically different from the bottom curve, and from what has been observed typically in the metal rings by many authors, [2,17], and also from the previously reported results in GaAs/Al\(_x\)Ga\(_{1-x}\)As heterostructure rings. [4,11,13]. There is a strong correlation in the oscillation phase and amplitude throughout the magnetic field range. This implies that the scattering has been eliminated considerably in these devices, or that the scattering event are “gentler”. The latter prospect is consistent with the disappearance of noise in filled subbands. [18,6] Different sweeps at the same gate voltages
are in good agreement. The reproducibility of two traces at a fixed $V_g$ is 99% with a time delay of 5 hours between the up and down trace in magnetic field, which is comparable to earlier work on GaAs/Al$_x$Ga$_{1-x}$As \cite{4}. If we use the same data and subtract the smooth background (see below), the envelope of the $h/e$ oscillations, which account for $\sim 5\text{-}15\%$ of the total conductance, has a reproducibility of 70-90\%. In the following we will concentrate on the data which is ordered and reproducible.

The exact reason that the devices get into one or the other state is not clear yet. A more systematic study of device processing and low temperature transport is needed. It is suggestive that under conditions where good plateaux exist in $G(V_g)$ the AB effect is cleaner. For two of the samples used in this paper, the initial cooling (300K to 0.04K) obtained noisy data, one with very active time dependent conductance fluctuations on the time scale of seconds, which almost totally buried the AB effect (nevertheless reasonably good conductance plateaux were seen at $T \sim 1K$ in the same cool-down). After bringing the samples to 300K \textit{in the dark} and recooling back to 0.04K, the \textit{same} sample was in quiet, ordered state, and clean AB oscillations were then observed. Once the sample is in the ordered state, it is fairly stable unless it is raised to a very high temperature ($\gg 4.2K$) or suffers an electrical shock. Based on the theory and experiments on universal conductance fluctuations, this is not so surprising, because only few active impurities are enough to kill the correlations and even the AB effect itself, \cite{19} even when most of the conductance channel is ballistic. The regions near the ports are especially critical and an inconvenient impurity configuration there can have a huge effect, \cite{20} and this just the region where the strongest electric fields appear during transport experiments. So we speculate that above randomness and changes from ordered to random “states” is related to the movement of a few impurities or defects, probably near the ports. With the view towards the ultimate goal of pure quantum waveguides, these results imply that the task is even more formidable. Not only a very clean channel is needed, but also the elimination of all back-scattering within $\sim l_\phi$ of the ports.

Each trace (random or ordered) can be described by a field dependent smooth background
resistance summed with Aharonov-Bohm oscillations of frequencies $\hbar/e$, $\hbar/2e$, and so on. The background resistance (the smooth line through the oscillations of curve a in Figure 2) is calculated by averaging the original data in every $\Delta B = 0.005T$ interval of magnetic field. This background resistance can be attributed to two parts. One is the overall parabolic component, which we believe to be the magnetic steering of electron away from head-on collisions with the inner wall of the ring. Other physics might contribute, however, for instance, the electron-electron interactions can cause a similar negative magnetoresistance [21]. The electron-electron interaction should be strongly enhanced by the 1D nature of the transport, but (perhaps) suppressed by the screening from the gate and complicated by the donor charges. [22] However, the cause of the large negative magnetoresistance will not affect the analysis to follow. The second component of the smooth background resistance comprises the broad peaks at $\sim \pm 0.07T$, which agree well with a model of trapped classical orbits in the loop. [10] The peaks in resistance arise when the cyclotron orbit of a individual mode matches the size of the ring, so that the electrons in that mode are guided away from the outlet port and remain in the ring. A peak appears for each individual value of forward momentum, i.e. for each occupied transverse mode, so at the same Fermi energy more than one trapping peak could be seen. In this figure the trapping peaks, except for being dressed by the AB oscillations, are not different from the equivalent data taken at high temperatures. The lack of temperature dependence provides further support for our semiclassical trapping picture, and rules out interference-related models for the resistance enhancement.

To be able to see the AB oscillations better, the smooth background $G_0$ (reciprocal of the dark line) has been subtracted from the original conductance in figure 2. Since we will compare the AB effect from a wide range of $V_g$ and $B$, in which the background itself changes (typically) by a factor of 5, we will study the relative conductance change $\Delta G/G_0$ (which is also in line with the usual perturbative treatment of AB effect), and this relative conductance oscillation for the top curve in Figure 2 is shown in Figure 3a (Figure 2 is cut to $\pm 0.1T$ to show the detailed structure of oscillations). The envelope function here is mainly from the contribution of $\hbar/e$ frequency. The average spacing between two adjacent nodes is
0.07T, spanning about 34 fundamental AB oscillations.

It is straightforward to realize that even in the ideal case when absolutely no random scattering is involved, the envelope function will not be featureless; instead it will be governed by some physics which was not emphasized in the diffusive case. First, due to the finite width of the ring arms, different modes with different spatial distribution across the width of the wire will encircle different amounts of flux and, therefore, have different AB frequencies. A simple calculation of the flux difference between the first mode and second mode in a square well leads to a result $\sim \pi (r_o - r_i)^2 B/4$, where $r_o$ and $r_i$ stand for the outer and inner radius of the ring. For our geometry we estimate a 4% of difference between the frequencies, and for higher modes our simple argument will give somewhat smaller result. The result of the composition of two close frequencies is periodic beating of the oscillation amplitude rather like that in Fig. 3a. If we attribute the change in amplitude to such a mechanism, then typical h/e frequencies differ only by $\sim 3\%$, which is in reasonable agreement with our estimate. From this point of view, Fig. 3a is very close to what we can see from an ideal solid state interferemeter. We note, however, that even when only one mode is populated there is still some beating of the interference amplitude, although over a longer field scale. Another more plausible explanation is directly related to the cyclotron orbit trapping. When the electron is guided away from the outlet, the amplitude of the oscillations, which result from interference of trajectories escaping the ring, will be suppressed too. As a result, a node will develop in the AB oscillation amplitude accompanying a trapping peak in the resistance. This naive model is not completely supported by curve a in Figure 2a where the envelope nodes do not always line up with the trapping peaks, but the average period is never-the-less on the right ballpark.

Ordered data like curve a in Figure 2 occur at specific values of $V_g$, while at other $V_g$ the data are not as satisfying, but as long as the sample stays in the quiet state, the typical data are much more ordered than the random case (curve b) in Figure 2. So we generally characterize our “ordered” data as in an intermediate scattering regime, where analysis of the envelope functions based purely on ballistic transport is not entirely adequate. But, as
we will see, statistical methods are appropriate and useful in this special regime.

Comparison with mode counting steps

Before using more sophisticated methods, we will directly look at the Fourier transform (FT). Figure 3b shows the FT amplitude of the data in Figure 2a. Since the smooth background has been subtracted, the zero frequency component in FT spectrum is not present. The effective cut-off “frequency” \( (1/\Delta B) \) due to this filtering is \( \sim 200 T^{-1} \). To compare the FT spectrum with the mode counting steps, systematic measurements have been conducted for the double ring sample.

In Fig.4 the zero-field conductance \( G \) versus \( V_g \) at the temperatures \( 2K, 0.74K \) and \( 60mK \) is shown. The conductance staircase is clearest at \( 0.74K \), and at \( 60mK \) an oscillatory feature dresses the staircase. The oscillations in \( G(V_g) \) always accompany the AB effects. At temperatures where the staircase was smooth \( (T > 1K) \), no (or very small) AB oscillations were observed. In this figure, we also show the \( 60mK \) data after subtracting the \( 0.74K \) sweep. The average magnitude of the \( G(V_g) \) oscillations agrees with the magnitude of the AB effects at a given temperature. This leads us to relate the \( G(V_g) \) oscillations to quantum interference. \[3\] These features differ from the AB effects in that they are not caused by the magnetic flux, they are related instead to the change of the Fermi wavelength with changing electron density. From the AB effects, we know that the dominant contribution to the interference oscillations is from the fundamental \( h/e \) signal. This frequency results from partial waves from the two ring arms interfering at the outlet. If there is a difference in the two arm lengths, when the Fermi vector changes, the phases accumulated on the trajectories through each arm will be different, the difference being \( \Delta l \Delta k_F \). \[3,14\] The average period of the coherence oscillations in \( G(V_g) \) is \( \Delta V_g = 3mV \). Using our previous Shubnikov-de Haas measurements of Fermi energy in similar samples, \[16\] we have estimated that the corresponding change in Fermi wavelength is \( \Delta k_F = 40 \mu m^{-1} \). From \( \Delta l \Delta k_F = 2\pi \), we have \( \Delta l/l = 2\pi/(\Delta k_F \pi r) \sim 6\% \), or \( \Delta l = 0.15 \mu m \). This is certainly plausible. The existence of so
many oscillations (about 30 total), however, implies that the simple picture above must be modified, since it would account for about 10 oscillations at most. If the modes are distinct within the wire then each subband can contribute separate oscillations, which is consistent with the increase in the number of oscillations per plateau as more subbands are populated. Another possibility is that of Fabry-Perot interference between the inlet and outlet ports, which has been reported in similar structures [23]. Further experiments are required to sort out the details as well as the rest of the phenomenology.

In Figure 5a and 5b we show the Fourier spectrum of the relative AB oscillations at a series of gate voltages. The surface represents 32 magnetoconductance measurements equally spaced between $V_g = 0.5975V$ to 0.6700V and interpolation between successive curves. Three peaks ($h/e$, $h/2e$ and $h/3e$) are clearly seen throughout the range of gate voltage, and the $h/4e$ peak can be seen at high $V_g$. The average frequencies for the first three peaks are $506.6 \pm 6.4T^{-1}$, $1011 \pm 11T^{-1}$ and $1511 \pm 18T^{-1}$ respectively, which scale fairly linearly with the harmonic order as expected. To obtain the frequencies, each harmonic peak at a specific $V_g$ is fitted with a Gaussian. There is no systematic dependence of the frequencies on $V_g$ for any of the peaks. These frequencies are typically larger than corresponding frequencies from the single ring, where $h/e$ oscillation is at $f = 482.2T^{-1}$. We speculate this is due to the widening of the ring at the “hip” region.

In the same figure we include a panel at the left containing $G(V_g)$ at 0.06K and for comparison, the (negative of the) transconductance $-g_m = -dG/dV_g$ has also been plotted in the same frame. In $-g_m$ the plateaux of $G$ correspond to plateaux between -50 to 50, and the risers appear as valleys. In this range of $V_g$ the conductance $G(V_g)$ increases from $\sim 0.94e^2/h$ to $\sim 4.2e^2/h$, corresponding to switching-on of modes 1 to 4. We can see that, despite the large change in $G(V_g)$, the average relative AB oscillation $\Delta G/G$ for $h/e$ is approximately constant (although there are order of magnitude fluctuations) implying that the contributions to $h/e$ frequency from each mode are about the same. In contrast, there is an slight increase in $\Delta G/G$ with $V_g$ for the higher harmonics, especially $h/4e$. In principle, this is not expected. At low $V_g$ our resolution in $\Delta G$ decreased because the detected current
was smaller, so it is possible that the apparent increase in the fourth peak is an instrumental artifact. Other possible explanations include a longer phase coherence at higher $V_g$ or an effect of the intermode scattering when many modes are populated.

For the $h/e$ frequency, there is an obvious correlation between the spectral density and the population of the modes. When the $B = 0$ conductance is at the center of a conductance plateau, the AB oscillations are stronger, and on the risers weaker.

Now examine the scattering processes when a new mode is turned on. Because the subband bottom mainly comprises small $k$ states, it is sensitive to imperfections of the conducting channel. As a result, the scattering is stronger when a mode is newly populated, which is the reason for the finite width of the risers in $V_g$. According to the Büttiker-Landauer formula, small angle intra-band scattering has very little effect on the conductance. So we can think the gradual risers are merely manifestations of heavy inter-band or large-angle intra-band scattering (generally are called back-scattering). Both of these scattering mechanisms will reduce the $h/e$ AB oscillation size (their roles in general, and for $h/2e$ will be discussed later). Following this line of thinking, a good staircase in $G(V_g, B = 0)$ (not counting the fine interference pattern in curve d in Figure 4) foretells observation of a periodic oscillation of the scattering strength, and an observation of a correlation between $G(V_g)$ and the AB magnitude is thus inevitable. The lack of observation of such correlations in earlier experiments in heterojunctions is consistent with the absence of a clear zero B staircase in their experiments. The above analysis is based on a static scattering potential, and not time dependent events which may serve as a phase-breaking source, but not a conductance killer (e.g. spin-spin interactions). In this case we may see a staircase at $T \sim 1K$, but at lower temperature might not be able to observe any correlations in the AB effect (or in fact any phase coherent response at all). This may be the reason for poor AB oscillations in the “random” state (see above).

The discussion in the last paragraph implies that the quality of correlation closely depends on how clean the staircase is. We note that the low temperature oscillations at $B = 0$ (curve d in Figure 4) contribute a significant amount to the conductance ($\sim 10\%$). The
non-ideal correlation may be attributed the substructures in the curves of Figure 4. One should in principle self-consistently model the AB interference patterns with the dressed $B = 0$ transmission coefficient. [3]

As a final remark we note that, if, starting in Figure 5 at a peak in the FT at $h/e$ frequency, we move towards either side of $V_g$, the peak height decreases smoothly while the peak frequency drifts. As a result, rather than uncorrelated random spikes, hills of significant footprint are seen. The gate voltage correlation range among the hills is $\sim 20mV$, i.e. a range in which almost a full mode is turned on. This correlation with gate voltage correlation may be a manifestation of the response of a particular mode evolving its detailed charge distribution as $V_g$ change, but more detailed investigation of this physics is required. Another consequence of the frequency shift is that the peaks move as $V_g$ changes, so if we show a cross-section at any particular $f$, the global correlation seen in Figure 5 will not be apparent. For the higher harmonics ($h/2e$, $h/3e$ and $h/4e$), no obvious correlations can be seen between the peak heights and the plateaux in $G(V_g, B = 0)$.

**Analysis of the envelope functions**

Besides studying the magnitude of the AB oscillations, we also investigated the patterns of the envelope functions with respect to the subband populations. Similar data has been discussed previously for the disordered (metallic) limit. [2] One expects the interference contribution to the conductance to be of the form [3,24]

$$\Delta G = \sum_{j=0}^{\infty} G_j(B, V_g) \cos \left( \frac{2\pi j \Phi}{(h/e)} + \alpha_j(B, V_g) \right),$$

where $\Phi$ is the average amount of magnetic flux enclosed by the electron trajectories encircling the ring, while $G_j$ and $\alpha_j$ are the envelope function and phase that account for individual harmonics. Information about the details of the scattering are contained in $G_j$ and $\alpha_j$ while the oscillatory components contain only the size of the ring. For metal samples these functions have been random in B. The correlation scale for a envelope function $G_j$ is
$B_c$, which in the diffusive case, is just the field scale to introduce a flux h/e into the sample (i.e. the ports and the arms of the ring). But this diffusive description for $B_c$ do not apply to our data. Because even though the effective width of the wire does increase with $V_g$, but it is too small, only 40% \[40\%\], to account for what we have observed in our samples; in addition for our data the $B_c$ changes periodically rather than monotonically (see below). Here we will borrow this vocabulary, such as envelope function $G_j$ and correlation field $B_c$, to describe our data. And for the ensuing analysis of the data we will explore the degree of “order” in the envelope functions $G_j$, through their autocorrelation functions.

Now consider the two contrasting curves in Figure 2 first. In order to analyze the envelope $G_1$ for the h/e oscillations, we use a Gaussian filter and reverse Fourier transform each h/e peak back into $B$ space. Figure 6a illustrates the result for h/e oscillations from spectrum in Figure 3b. The h/e signal accounts for about 80% of the total oscillation magnitude in Figure 3a. To ensure the information in the chosen peaks are fully included, generous filters of half-width $> 100 T^{-1}$ around the center frequencies were used. These are at least 3 times bigger than the widths (15$T^{-1}$ and 30$T^{-1}$) of the h/e and h/2e peaks calculated by fitting Gaussians to the peaks. To reassure the validity of the filter, we have done a FT for Figure 6a. If we plot the result in the same picture with 3b, no difference could be seen between these two curves in the region $\pm 100 T^{-1}$ around the center frequency. The same filter width was used throughout the analysis. The same operation for the bottom curve in Figure 2 obtains the result in Figure 6b. Both curves contain AB oscillations of the same average frequency. The differences between the two curves are in the envelope function $G_1$ (dark curves that bound the oscillations), in both the amplitude (a factor of $\sim 5$ even in the relative conductance) and in the correlation scales.

In Figure 7 we have calculated the autocorrelation functions (solid lines) for $G_1$ in Figure 6. The huge difference between the two envelopes is reflected here. For envelope 6b, the oscillation amplitudes at a particular value of B are not correlated with those at other places on the B axis, so its autocorrelation function for the envelope (7b) is mainly Gaussian-like: a single monotonically decaying peak signifying random correlations. The long-dashed line
is the autocorrelation function calculated from the average value of the envelope, which is
consistent with the constant offset. In contrast for $V_g = 0.15V$ a much more regular envelope
function seems to prevail. The autocorrelation function for this envelope contains a decaying
peak and a regular oscillatory “tail”, which indicates a non-random pattern in the envelope
function.

In general, the shape of the correlation function can be understood as follows. Because
the envelope function basically consists of the summation of two parts, $G_1(B) = P(B) + Q(B)$, where $P(B)$ is a periodic function, and $Q(B)$ a random function. The autocorrelation
function of $G_1(B)$ can then be calculated as,

$$C(G_1) = \int [P(B) + Q(B)] [P(B + \Delta B) + Q(B + \Delta B)] dB$$

$$= C(P) + C(Q)$$

$$+ \int [Q(B)P(B + \Delta B) + P(B)Q(B + \Delta B)] dB$$

$$= a \cos(2\pi B/B_{c1}) + c \exp(-B^2/B_{c2}^2) + \text{constant}$$

In the derivation, we have assumed that $P(B)$ and $Q(B)$ are uncorrelated, so the cross-
term only contribute a insignificant constant. The autocorrelation for $P(B)$ is simply a $\cos$
function, $C(P) = a \cos(2\pi B/B_{c1})$; and for the random function $Q(B)$ a Gaussian $C(Q) =
c \exp(-B^2/B_{c2}^2)$. The final constant is from all three terms (cross-term, $C(P)$ and $C(Q)$),
since both $P(B)$ and $Q(B)$ has nonzero average values. $B_{c1}$ is the “beating” period, and
$B_{c2}$ the random correlation scale. We acknowledge that there are some problems associated
with performing the calculation in a finite field range. The consequence is that none of
the above three terms would be ideal, the random function $Q(B)$ will produce oscillatory
features in both the cross-term and in Gaussian term. This is clearly seen in Figure 7b;
there are oscillatory features in the “tail”, which would disappear if the field range were
long enough. But we also realize that if the correlation $B_{c2}$ does not exceed the long range
correlation field $B_{c1}$ (which is what always assumed for the experimental data anyway), only
$C(P)$ gives the long-range correlation field $B_{c1}$, which is what we are primarily interested in
the analysis.
If we use eq. (2) to fit the two correlation functions in Figure 7 (dash lines), it yields $B_{c1} = 0.069T$, $B_{c2} = 0.015T$ for curve a, and $B_{c1} = 0.012T$, $B_{c2} = 0.0024T$ for curve b. $C(Q)$ is restricted to small $\Delta B$; on the other hand, $C(P)$ persists to large $\Delta B$. Here beating period $B_{c1}$ represents the long range order of the envelope function. For both curves we can see the fitted $B_{c1}$ are consistent with the periods we get from the original envelope functions in Figure 6. So the beating period $B_{c1}$ can be used as a parameter to characterize the “order” of the envelope function quantitatively. Even for the metallic case 7b, which is outside the realm of validity for formula (2), the fit yields correctly, at least qualitatively, a very small value of $B_{c1}$, but the fit quality is rather poor, indicating that there is no long range order in this correlation function. In the very ordered case such as 7a, there might be an even longer $B_{c1}$ if the measurement were extended to wider range of field, because the window size $\pm 0.2T$ puts a upper limit of $0.2T$ on $B_{c1}$. Unfortunately, this cannot be done because depopulation of the subbands and Shubnikov-de Haas oscillations and finally the quantized Hall effect set in, and they cause dramatic changes in the physics behind the AB oscillations, which goes beyond the physics that this paper set out to to study.

We have performed the same calculation for all three samples. We notice that for coupled rings (2 rings and 4 rings), equation (2) does not always fit the data, and the discrepancy worsens as more modes are occupied. Representative data from 2-ring sample at $V_g = 0.660V$ data shown in Figure 8. The autocorrelation function (solid line) contains two oscillation periods. Obviously a single beat frequency is not adequate in these cases. Instead of invoking a more complicated model, we will keep using (2) to fit the original data by seeking a local minimum in the sum of squares of differences starting from different initial conditions. This method handily yields the two beat frequencies as shown (dotted and dashed lines) in Figure 8, where the two periods are $0.081T$ and $0.019T$, respectively. The average beating periods $B_{c1}$ for $G_1$ of the three samples are collected in Table 1. In cases where two frequencies exist, they were considered with equal weight in the statistics.
| SAMPLE  | $B_{c1}$ | # of $V_g$ | mode range |
|---------|----------|-----------|------------|
| 1 ring  | 0.078T   | 8         | 1 $\rightarrow$ 2 |
| 2 rings | 0.045T   | 15        | 1 $\rightarrow$ 2 |
|         | 0.038T   | 32        | 1 $\rightarrow$ 4 |
| 4 rings | 0.037T   | 8         | 1 $\rightarrow$ 2 |

From this table, we can see that, in the same mode population range, the average beat period tends to decrease as the number of rings increases. For the 2-ring sample, the only sample for which a relatively wide mode population range was measured, the average beat period tends to decrease as more modes are populated. Among all the samples, only the data from 2-ring sample allows us to make a detailed comparison between the AB effect and $V_g$. The beating periods for $h/e$ oscillations are summarized in Figure 9b. In the calculation the fitting results are constantly checked “by eye” against the original envelopes to make sure that the fit results are reasonable. The biggest and the smallest period differ by almost an order of magnitude. In cases where two periods exist, the extra one is plotted as a ‘x’. The dashed line connects all points (in cases where there are two points, the average is used) together, and the solid line is a smoothed rendition of the dashed line. Again $G(V_g)$ and $g_m$ are shown in Figure 9a. On the plateaux, the correlation functions have relatively long periods $B_{c1}$, and shorter periods on the risers, which coincides with the relationship between FT peak amplitude and $G(V_g)$ in Figure 5.

We have also performed the reverse Fourier transform for $h/2e$ oscillations, and the resulting $B_{c1}$ is shown in Figure 9c. However, the overall trend seems to be about the opposite of that for $h/e$ (There is no similar correlation for the FT amplitude as a function of $V_g$ in Figure 5.). This result came a little surprising to us at beginning, we do not rule out the possibility of an artifact. But it is also plausible that the trend is an indication of real transport physics. We have discussed the role of back-scattering in quenching the $h/e$ oscillations. Now consider the case of large-angle, intra-band scattering, if a $k$ is reflected
into the $-k$ state in the same subband, the phase coherence is retained. This kind of scattering kills the $h/e$ oscillation if the scattering takes place either inside the ring or in the ports. If the scattering occurs in the ports it could enhance the $h/2e$ oscillation amplitude, but the overall effect of back-scattering on $h/2e$ is, at least, not clear. The simple characterization of scattering as “elastic” and “inelastic” (the latter usually synonymous with “phase-breaking”), which so far has guided our thought on diffusive transport, is no longer sufficient. Instead more detailed study of the roles of specific back-scattering mechanisms on different AB oscillation components is necessary.

Summary

We have performed a careful experiment on GaAs/Al$_x$Ga$_{1-x}$As rings to study the correlation between the $B = 0$ subband population and the AB oscillation amplitude at low magnetic fields. Strong correlations are observed between the one-dimensional subband populations and the Fourier amplitude of the oscillations. There is also a correlation with the degree of order in the envelope function as judged through its autocorrelation function. These samples have shown improvements on eliminating most of the impurity scattering, pointing towards the possibility of purely ballistic solid state interferometers. Questions for further theoretical and experimental investigation include whether or not the modes contribute independently to the interference patterns and how classical mechanisms (such as scattering in the port junctions and orbit trapping) affect the envelope function.

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FIGURES

FIG. 1. Schematic drawing with lithographic dimensions for a two-ring sample. The loops
and ports are to scale, but the large area regions are not. The region comprising the rings and the
ports is covered by a Ti/Au gate.

FIG. 2. The magnetoresistance for the single ring at the gate voltage of \( V_g = 0.15V \) (a) and
\( V_g = 0.3V \) (b). The smooth line through the 0.15V data is the background resistance (its reciprocal
is \( G_0 \) which will be used later), calculated by averaging the original data in 0.005\( T \) intervals. The
peaks near \( \pm 0.07T \) are due to trapping of cyclotron orbits.

FIG. 3. (a) \( \Delta G/G_0 \) is the relative conductance oscillations calculated from the subtrating
curve a in Figure 2 from the averaged (heavy) line. (b) The Fourier transform amplitude of \( \Delta G/G_0 \)
from (a).

FIG. 4. \( G(V_g) \) at several temperatures and \( B = 0 \) recorded upon a second cool-down with
a considerably shifted threshold voltage. The temperatures for the curves are: a: \( T = 2K \), b:
\( T = 0.74K \), and c: \( T = 0.06K \). The difference between c and b, which is attributed to interference
effects from changing the Fermi vector \( k_F \), is plotted as d.

FIG. 5. The Fourier transform of \( \Delta G/G_0 \) as a function of gate voltage for the double-ring
sample, in frequency ranges from 300\( T^{-1} \) to 1300\( T^{-1} \) (a), and 1300\( T^{-1} \) to 2300\( T^{-1} \) (b). The
vertical axis in (b) is the same as for (a). The surfaces are constructed from 32 equally-spaced
measurements between \( V_g = 0.59V \) and \( V_g = 0.67V \) recorded on the second cool-down. Four peaks
are seen at the frequencies 506.6 \( \pm 6.4T^{-1} \), 1011 \( \pm 11T^{-1} \), 1511 \( \pm 18T^{-1} \) and \( \sim 2020T^{-1} \). The
\( G(V_g, B = 0) \) and (the negative of) its derivative \(-dG/dV_g \) versus \( V_g \) are drawn on the z-y panel.
The correlation between the mode population and the height of the \( h/e \) peak is obvious; there are
four main peaks in \( h/e \) and the same number “hills” in \(-dG/dV_g \). The strongest \( h/e \) oscillations
are found around the conductance plateaux, i.e. the zeros (“hills”) of \(-dG/dV_g \).
FIG. 6. Inverse Fourier transforms of the $h/e$ peaks are calculated to obtain the oscillation patterns for the upper (a) and lower (b) curves in Figure 2. The two patterns are defined by different amplitude envelope functions $G_1/G_0(B)$, which are drawn as the smooth dark lines along the oscillation maxima.

FIG. 7. The measured autocorrelation function (solid line) of the envelope functions $G_1/G_0$ for the single ring for (a) the upper curve and (b) the lower curve in Figure 6. The dashed line in (b) is the autocorrelation function calculated from the average value of the envelope, which is consistent with the constant background offset. They are fitted with eq. (2) (dotted line) to obtain the beat periods $B_{c1}$ and decay scales $B_{c2}$.

FIG. 8. The autocorrelation of $G_1/G_0$ for the double ring (solid line) at $V_g = 0.660V$ (second cool-down). It contains more than one beat frequency. It was fitted with formula (2) with different initial conditions to obtain the periods $0.081T$ (dotted line) and $0.019T$ (dashed line).

FIG. 9. Comparison between the mode counting staircase $G(V_g)$ (solid line) and transconductance (dotted line) recorded at $B = 0$ (a) and the beat periods for the $h/e$ (b) and $h/2e$ (c) peaks from all of the measurements from Figure 5. In (b) when two periods are found for a single correlation function as in Figure 4, the extra one is shown as ‘x’. The dashed lines connect the beat periods (the average in cases there are two). The solid line is a smooth (Gaussian filtered) version of the dashed line. In (b) there is a correlation to the staircase similar to that seen in figure 5 for the average amplitude of the oscillations. In (c) the opposite trend to (b) is seen.