A two-phase-five-phase transformer calculating features

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Abstract. The paper describes a new approach to constructing a transformer with a rotating magnetic field. Such a transformer mathematical description with a rotating magnetic field is given. A transformer calculating method for a primary and secondary windings phases different number is considered. A two-phase-five-phase transformer with a rotating magnetic field calculation is shown as an example. Based on a transformer with a rotating magnetic field calculation, its simulation model is built. The output currents oscillograms at the sinusoidal and rectangular shape input voltage were obtained.

1. Introduction
In recent years, transformers with a rotating magnetic field (TRMF) are increasingly being used in converter technology. This is due to such transformers higher integral indicators number in comparison with traditional ones, such as smaller weight and size indicators at the same specific power, the phases obtaining an arbitrary number possibility at the output. There are two known approaches to the TRMF implementation: based on an asynchronous machine with various schemes for connecting the secondary windings and on a toroidal core basis.

2. The TRMF structure
The TRMF peculiarity is that the magnetic connection between the primary windings' system and the secondary windings' system is provided not due to a pulsating magnetic field, as is implemented in traditional transformers, but due to a rotating magnetic field [1, 2].

One of the conditions required to create a rotating field is to provide a certain spatial angular shift between the primary windings magnetic axes, depending on the transformer phase. The spatial angular displacement must also be implemented between the secondary windings magnetic axes.

The second prerequisite is to ensure a phase shift between the one system (primary winding system) winding currents. In this case, the phase shift must be equal to the spatial shift between the windings axes. The relationship between the phases number \( m \) and the angular displacement \( \alpha \) is given below (1).

\[
\alpha = \begin{cases} 
\frac{2\pi m}{m} & \text{at } m=3,5,7,9,... \\
\frac{\pi m}{m} & \text{at } m=2,4,6,8,... 
\end{cases}
\] (1)

Figure 1 shows a two-phase-five-phase TRMF diagram. Primary and secondary windings are located on the transformer core 3. The voltages phase shift on the primary windings 1 and 2 is equal to \( \alpha_1 = \pi/2 \), therefore, the voltages functions \( U_1 \) and \( U_2 \), respectively) supplying the primary windings will take the form:
\[ U_1 = U_m \sin(\omega t), \]
\[ U_2 = U_m \sin(\omega t + \pi/2). \]

According to expression (1), the secondary windings are located in space so that the angle between their magnetic axes is equal \( \alpha_2 = 2\pi/5 \). The secondary windings beginnings are collected at the zero points. In this case, the secondary windings 4-8 ends to generate a five-phase voltage at the transformer output.

\[
\begin{align*}
U_1 &= U_m \sin(\omega t), \\
U_2 &= U_m \sin(\omega t + \pi/2).
\end{align*}
\]  \hspace{1cm} (2)

Figure 1. Two-phase-five-phase TRMF schematic diagram.

In addition to an ideal sinusoidal voltage, square-wave voltage pulses can be applied to the transformer primary windings [3], with a phase shift \( \alpha_1 = \pi/2 \) (figure 2). Rectangular pulses \( U_{y1}, U_{y3} \) timing diagram - form positive alternating voltage half-waves applied to windings 1 and 2 (figure 1), respectively, \( U_{y2}, U_{y4} \) - form negative sinusoids half-waves. This supply voltage form can be obtained, for example, from a constant voltage source by periodic switching.

Figure 2. Supply voltage control pulses.

A lot of works are known on the TRMF implementation and control method [4-9]. TRMF implementation and control can be used as a frequency converter main element or used as a voltage rectifier.

3. TRMF mathematical description
Currently, there are many methods and approaches to calculating TRMF [10-13]. To calculate the transformer parameters, we will use the matrix transformations method. For this purpose, it is necessary to select the transformer main elements and establish a connection between them through mathematical relations.

An abstract TRMF structure includes a primary windings system, a magnetic circuit (transformer core), a secondary windings system.
The TRMF general mathematical description for the primary and secondary windings phases any number is presented in a differential equations system form (3). Electrical balance equations are drawn up for each winding. The primary winding system phases number is taken as \( m \), the secondary winding system phases number - as \( n \). The primary windings electric parameters are denoted with one stroke, the secondary windings parameters with two strokes.

\[
\begin{align*}
U_1 &= r'_1 i'_1 + L'_1 \frac{di'_1}{dt} + L'_2 \frac{di'_2}{dt} + \cdots + L'_{1m} \frac{di'_{1m}}{dt} + \\
&+ M_{11} \frac{di''_1}{dt} + M_{12} \frac{di''_2}{dt} + \cdots + M_{1n} \frac{di''_n}{dt} \\
U_2 &= r'_2 i'_2 + L'_2 \frac{di'_2}{dt} + L'_2 \frac{di'_2}{dt} + \cdots + L'_{2m} \frac{di'_{2m}}{dt} + \\
&+ M_{21} \frac{di''_1}{dt} + M_{22} \frac{di''_2}{dt} + \cdots + M_{2n} \frac{di''_n}{dt} \\
&\vdots \\
U_m &= r'_m i'_m + L'_m \frac{di'_m}{dt} + L'_m \frac{di'_m}{dt} + \cdots + L'_{mm} \frac{di'_{mm}}{dt} + \\
&+ M_{m1} \frac{di''_1}{dt} + M_{m2} \frac{di''_2}{dt} + \cdots + M_{mn} \frac{di''_n}{dt} \\
&- R_{11} \frac{di''_1}{dt} = r''_1 i''_1 + L''_1 \frac{di''_1}{dt} + L''_2 \frac{di''_2}{dt} + \cdots + L''_{1n} \frac{di''_{1n}}{dt} + \\
&+ M_{11} \frac{di''_1}{dt} + M_{12} \frac{di''_2}{dt} + \cdots + M_{1n} \frac{di''_n}{dt} \\
&- R_{22} \frac{di''_2}{dt} = r''_2 i''_2 + L''_2 \frac{di''_2}{dt} + L''_2 \frac{di''_2}{dt} + \cdots + L''_{2n} \frac{di''_{2n}}{dt} + \\
&+ M_{21} \frac{di''_1}{dt} + M_{22} \frac{di''_2}{dt} + \cdots + M_{2n} \frac{di''_n}{dt} \\
&\vdots \\
&- R_{nn} \frac{di''_n}{dt} = r''_n i''_n + L''_n \frac{di''_n}{dt} + L''_{n1} \frac{di''_{1n}}{dt} + \cdots + L''_{nn} \frac{di''_{nn}}{dt} + \\
&+ M_{1n} \frac{di''_1}{dt} + M_{2n} \frac{di''_2}{dt} + \cdots + M_{mn} \frac{di''_m}{dt} 
\end{align*}
\]

We will assume that the primary and secondary windings are evenly distributed in the slots symmetrically about the transformer central axis. Let's take the secondary winding load active and symmetrical. In this case, the primary windings number ranges from 1 to \( (a = 1, 2, 3, \ldots, m) \), and the secondary windings number from 1 to \( n \) \( \left( b = 1, 2, 3, \ldots, n \right) \); \( a \neq b \); then:

- \( L'_{11}, \ldots, L'_{mm} \) - mutual inductances between the primary windings, here the numbers in the index indicate the corresponding primary windings numbers, therefore \( L'_{ab} = L'_{ba} \), etc.;
- \( M_{11}, \ldots, M_{mn} \) - mutual inductances between primary and secondary windings, here the first digit in the index is the primary winding number, and the second digit is the secondary winding number, therefore \( M_{ab} \neq M_{ba} \);

- \( L''_{11}, \ldots, L''_{nn} \) - mutual inductance between the secondary windings, and \( L''_{ab} = L''_{ba} \), etc.;
- \( U_1, \ldots, U_m \) - voltages applied to the primary windings;
- \( r'_1, \ldots, r'_m, r''_1, \ldots, r''_n \) - primary and secondary windings active resistances, respectively;
- \( R_1, \ldots, R_n \) - active load resistances on the secondary windings. With symmetrical load \( R_1 = R_2 = \cdots = R_n \);
- \( i'_1, \ldots, i'_m, i''_1, \ldots, i''_n \) - primary and secondary windings currents, respectively.

Let us consider the proposed approach implementation using the calculating a two-phase-five-phase TRMF example, i.e. \( m = 2, n = 5 \). In this case, the equations system (3) will contain 7 equations and take the form (4)
The left matrix each column elements in equation (5) are represented by the coefficients (inductance) of each current derivatives, respectively. This recording form makes it quite easy to implement the TRMF simulation model. To reduce the simulation model complexity, we will perform a minor transformation above the equations present system, namely, we will solve it relative to each derivative. To solve this problem, you should bring the equation left matrix (5) to the diagonal view. This transformation can be done in the Mathcad mathematical computing system. As a conversion result, the equation (5) takes the form of:

\[
\begin{bmatrix}
U_1 = r'_{1}i'_{1} + L'_{11}\frac{di'_{1}}{dt} + L'_{12}\frac{di'_{2}}{dt} + M_{11}\frac{di''_{1}}{dt} + M_{12}\frac{di''_{2}}{dt} + \\
+M_{13}\frac{di''_{3}}{dt} + M_{14}\frac{di''_{4}}{dt} + M_{15}\frac{di''_{5}}{dt} \\
U_2 = r'_{2}i'_{2} + L'_{21}\frac{di'_{1}}{dt} + L'_{22}\frac{di'_{2}}{dt} + M_{21}\frac{di''_{1}}{dt} + M_{22}\frac{di''_{2}}{dt} + \\
+M_{23}\frac{di''_{3}}{dt} + M_{24}\frac{di''_{4}}{dt} + M_{25}\frac{di''_{5}}{dt} \\
-R_1i''_{1} = r''_{1}i''_{1} + L''_{11}\frac{di''_{1}}{dt} + L''_{12}\frac{di''_{2}}{dt} + L''_{13}\frac{di''_{3}}{dt} + \\
+L''_{14}\frac{di''_{4}}{dt} + L''_{15}\frac{di''_{5}}{dt} + M_{11}\frac{di''_{1}}{dt} + M_{21}\frac{di''_{2}}{dt} \\
-R_2i''_{2} = r''_{2}i''_{2} + L''_{21}\frac{di''_{1}}{dt} + L''_{22}\frac{di''_{2}}{dt} + L''_{23}\frac{di''_{3}}{dt} + \\
+L''_{24}\frac{di''_{4}}{dt} + L''_{25}\frac{di''_{5}}{dt} + M_{12}\frac{di''_{1}}{dt} + M_{22}\frac{di''_{2}}{dt} \\
-R_3i''_{3} = r''_{3}i''_{3} + L''_{31}\frac{di''_{1}}{dt} + L''_{32}\frac{di''_{2}}{dt} + L''_{33}\frac{di''_{3}}{dt} + \\
+L''_{34}\frac{di''_{4}}{dt} + L''_{35}\frac{di''_{5}}{dt} + M_{13}\frac{di''_{1}}{dt} + M_{23}\frac{di''_{2}}{dt} \\
-R_4i''_{4} = r''_{4}i''_{4} + L''_{41}\frac{di''_{1}}{dt} + L''_{42}\frac{di''_{2}}{dt} + L''_{43}\frac{di''_{3}}{dt} + \\
+L''_{44}\frac{di''_{4}}{dt} + L''_{45}\frac{di''_{5}}{dt} + M_{14}\frac{di''_{1}}{dt} + M_{24}\frac{di''_{2}}{dt} \\
-R_5i''_{5} = r''_{5}i''_{5} + L''_{51}\frac{di''_{1}}{dt} + L''_{52}\frac{di''_{2}}{dt} + L''_{53}\frac{di''_{3}}{dt} + \\
+L''_{54}\frac{di''_{4}}{dt} + L''_{55}\frac{di''_{5}}{dt} + M_{15}\frac{di''_{1}}{dt} + M_{25}\frac{di''_{2}}{dt}
\end{bmatrix}
\]

\[
\begin{bmatrix}
U_1 = r'_{1}i'_{1} \\
U_2 = r'_{2}i'_{2} \\
-R_1i''_{1} \\
-R_2i''_{2} \\
-R_3i''_{3} \\
-R_4i''_{4} \\
-R_5i''_{5}
\end{bmatrix} =
\begin{bmatrix}
U_1 - r'_{1}i'_{1} \\
U_2 - r'_{2}i'_{2} \\
-(R_1 + r''_{1})i''_{1} \\
-(R_2 + r''_{2})i''_{2} \\
-(R_3 + r''_{3})i''_{3} \\
-(R_4 + r''_{4})i''_{4} \\
-(R_5 + r''_{5})i''_{5}
\end{bmatrix}
\]
inductances and mutual inductances formula [14].

The windings' inductivity, placed in a given form, can be approximately calculated according to the shape has the most common shape, in a rectangle form with the primary windings m and secondary n number, the p total number are equal to:

\[
N = \frac{(m+n)^2 + (m+n)}{2}.
\]

The windings coil shape, as a rule, adapts to the core type. To be specific, let's assume that the coil shape has the most common shape, in a rectangle form shown in figure 3.

The windings' inductivity, placed in a given form, can be approximately calculated according to the formula [14].

\[
L = \frac{\mu_0}{\pi} w^2 (b + c) \left[ \ln \frac{2bc}{a+r} - \frac{c}{b+c} \ln(c + \sqrt{b^2 + c^2}) - \frac{b}{b+c} \ln(b + \sqrt{b^2 + c^2}) + \frac{2\sqrt{b^2 + c^2}}{b+c} - \frac{1}{2} + 0.447 \times \frac{a+r}{b+c} \right].
\]
where \( w \) - the coil turns number, \( \mu \) - the TRMF core material magnetic permeability, \( \mu_0 \) - the magnetic constant.

The primary windings consist of several coaxial rectangular coils. Since such a system mutual inductance calculation seems too complicated, we present for simplicity the following transformations. Select the main rectangular contour in each rectangular coil. According to figure 3, its dimensions will be \( b \times c \). Then we replace this rectangular contour with an equivalent area circular contour, that is, such a contour radius will be (9).

\[
R = \sqrt{\frac{b \times c}{2}}.
\]

The coaxial coils mutual inductance can be calculated using the following formula [14]:

\[
M = w_1 w_2 \mu_0 \frac{\mu}{4\pi} F \sqrt{R_1 R_2},
\]

where \( R_1, R_2 \) - the equivalent circular contours radii for the first and second coils, respectively; \( w_1, w_2 \) - the first winding and the second turns number, respectively; \( F \) - a hypergeometric function that depends on the geometric mean distance \( m \) between the contours and is determined approximately according to Table 5-5 of the source [14]. The geometric mean distance square is found by the formula [14].

\[
m^2 = \frac{(R_1 - R_2)^2 + x^2}{(R_1 + R_2)^2 + x^2},
\]

where \( x \) - the distance between the circular contours centres.

The two-phase-five-phase TRMF secondary windings coils magnetic axes are located at an angle of \( \alpha = 2\pi/5 \) relative to each other. Concerning the primary windings coils, this angle takes on different values, depending on the location of a particular secondary winding relative to the primary. To calculate such windings mutual inductances, one should take into account the angle \( \theta \) between the coils magnetic axes, as well as the distance between the coils' centres \( x \). Taking into account the above transformations, such coils mutual inductances' calculation is reduced to the following formula (12):

\[
M = M_0 q \cos \theta,
\]

where \( q \) - a coefficient depending on the contours relative position, which can be determined from table 5-9 of the source [14]; \( M_0 \) - mutual inductance between these coaxial circuits, which can be found by formula (10).

5. TRMF simulation model

After calculating the TRMF inductances on the equations (5) matrix system basis and converting it to the form (6), we implement the TRMF simulation model (figure 4). In the developed model, the coefficients numerical values \( A_{ij}, B_{ij} \) correspond to the equations' matrix system coefficients. Proportional links Gain1-Gain5 allow setting the transformer secondary windings active resistance and the load resistance applied to these windings on each phase, respectively [15].
The primary and secondary windings currents curves oscillograms are presented in figures 5 and 6. The TRMF load on the secondary windings is active and symmetrical, i.e. $R_1 = R_2 = R_3 = R_4 = R_5$. The oscillograms shown in figure 5 correspond to the case when a two-phase sinusoidal voltage is applied to the primary windings (2). And the oscillograms in figure 6 are the case when the alternating rectangular signal shown in figure 2 is applied to the primary windings. The secondary windings currents analysis clearly shows that the phase shift between the winding currents is equal to $2\pi/5$, that is, 72° the output current frequency is equal to the magnetic field rotation frequency. The existing mismatch in the secondary winding currents' amplitude is explained by calculating the model coefficients inaccuracy and does not exceed 10%. The secondary windings currents curves shape when a rectangular signal is applied to the primary windings indicates the higher harmonics present in the current curve, which ultimately will affect the transformer efficiency. Therefore, it is advisable to subject a rectangular signal to pulse-width modulation.

Figure 4. TRMF simulation model.
6. Conclusion

Based on the above, we can state:

- proposed a method for calculating a two-phase-multiphase transformer with a rotating magnetic field, taking into account the electromagnetic connections between the windings and the core geometric dimensions;
- a two-phase-multiphase transformer with a rotating magnetic field mathematical description was developed, which served as the basis for creating a TRMF simulation model;

The outlined approach to calculating and constructing a two-phase-multiphase TRMF will expand the using direct-coupled frequency converters functionality implemented on such a transformer basis in electric drive systems for various purposes, including electric drive systems based on double-powered machines.

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