 Manufacture and delivery scheduling for customers on one manufacturing machine with an availability constraint

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Abstract. We consider a scheduling problem with the joint consideration of manufacture and delivery for \( m \) customers in different locations. In the manufacture period, there must be some interrupted job because of the arranged maintenance of the machine, and that job can be resumable when the machine becomes available again. In the delivery period, amounts of vehicles without limits of load capacity deliver completed jobs belonging to the same customer to customers in batches. The cost of each delivery is proportional to the distance between the customer and the manufacturer. Our goal is to minimize the sum of total departure time and total delivery cost. The problem is solved optimally by a dynamic programming algorithm with polynomial time \( O(m^2 n^3) \), where \( n \) is the number of all jobs.

1. Introduction

From 1990’s Cheng [1] started to study integrated manufacture and delivery scheduling problems, many results [2-5] have been achieved by researchers. They usually focus on such manufacturing machines that are continuously available throughout the production period.

But in the real industrial settings, the machine may become unavailable due to arranged maintenance during the manufacturing period. Lee [6] showed the SPT sequence can solve the resumable problem optimally to minimize the total completion times of jobs. And more reviews can be referred to Ying Ma et al. [7].

It is notable that the literature on the associated manufacture and delivery scheduling with an availability constraint is not so much. Wang and Cheng[8] studied the objective of minimizing the arrival time of the last delivery batch and the total delivery cost when there is only one vehicle with capacity limit to deliver jobs within a fixed time. If jobs are resumable and need to be processed on a single machine, they provided an optimal polynomial time algorithm. Fan et al. [9] solved optimally the integrated scheduling of manufacture and delivery for one customer on a single machine with an availability constraint by a dynamic programming algorithm if the jobs are resumable and the vehicles are homogeneous and without capacity limits. Fan [10] developed an optimal polynomial time algorithm to minimize the sum of total departure time and total delivery cost using vehicles with capacity limits. Fan [11] also studied a scheduling problem on the machine with periodic unavailability constraint.

In this paper, we study an associated manufacture and distribution scheduling problem, in which there is a unavailable interval on the machine, jobs are resumable and there is no limit to the capacity of the vehicle.
2. Problem Description
In this paper, we study such a scheduling problem that some job may be interrupted by the unavailable interval and may be resumable after the machine turns into available again. In the delivery period, homogeneous vehicles without limits of load capacity deliver jobs to the customer. A completed job is permitted waiting until some other jobs of the same customer are finished processing because only jobs belonging to the same customer can be delivered together in the same shipment. The cost of each transportation task is always calculated according to the distance from the manufacturer to the customer. Hence, the manager of the manufacturer must consider not only how to finish processing products of all customers as early as possible, but also how to decrease the expenses during deliveries as many as he could.

The problem we study can be described as follows. Given a set of \( n \) jobs \( J = \{ J_1^1, J_1^2, \ldots, J_1^m, J_2^1, J_2^2, \ldots, J_2^m, \ldots, J_n^1, J_n^2, \ldots, J_n^m \} \), in which the subset of jobs \( J(i) = \{ J_i^1, J_i^2, \ldots, J_i^m \} \) belong to the \( i \)-th customer for \( i = 1, 2, \ldots, m \). Every job is ready to be processed at time 0 on a manufacturing machine, which is unavailable within a given time interval \([S, E]\), and \( S < E \). The processing time of job \( J_i^j \) is \( p_i^j \). Once there is a completed job, it can be delivered by one of vehicles. And one or more jobs belonging to the same customer can be transported as one batch because of no capacity limits to vehicles. The distance from the \( i \)-th customer to the manufacture is \( d_i \). The cost per unit length of distance for each delivery is \( c \), and the total delivery cost is denoted by \( T \). The departure time of job \( J_i^j \), denoted by \( D_i^j \), is defined as the time by which it is delivered from the manufacturer to the customer. The objective is to minimize the sum of total departure time and total delivery cost. By the notation of Chen[5], the problem is represented by

\[
(P): 1| r-a | V(\infty, \infty), direct | m | \sum \sum D_i^j + T,
\]

where ‘r-a’ denotes the job is resumable, ‘\( V(\infty, \infty) \)’ describes the situation of vehicles, ‘direct’ denotes that only jobs belonging to the same customer can be delivered together in the same delivery, and ‘\( \sum \sum D_i^j + T \)’ is the objective function.

Without loss of generality, we suppose that \( \sum_i \sum_j p_i^j > S \).

3. Analysis and Algorithm
In this section, we firstly analyze the optimal properties of the problem \((P): 1| r-a | V(\infty, \infty), direct | m | \sum \sum D_i^j + T\), then we present an optimal dynamic programming algorithm for it.

3.1. The optimal properties
Now we analyze some properties of the optimal schedule to the problem \((P)\).

**Lemma** For the problem \((P): 1| r-a | V(\infty, \infty), direct | m | \sum \sum D_i^j + T\), there exists an optimal schedule that satisfies the following conditions:

1. There is no idle time on the manufacturing machine before all jobs are completed except the unavailable interval;
2. In the manufacture period, jobs of each customer are processed in the shortest processing time first (SPT) order on the machine;
3. In the delivery period, the departure time of a batch of jobs belonging to \( i \)-th customer is the completion time of the last completed job in that batch;
4. For every customer, the job which is completed earlier can be delivered no later than the other jobs.
It is obvious that (1), (3) and (4) of Lemma are correct. Hence, we will prove that (2) is hold by adjacent pairwise interchanging method.

**Proof**

Let $\sigma^*$ be an optimal schedule for the problem (P), in which jobs of each customer are processed in the shortest processing time first (SPT) order on the machine. We will prove the conclusion by contradiction.

Suppose that in $\sigma^*$ job $J_{i+1}$ is immediately followed by $J_i$ and $p_{i+1} < p_i$. Then we interchange job $J_i$ and job $J_{i+1}$ to obtain a new schedule, denoted by $\sigma^*$, in which the number of batches are the same as in $\sigma^*$.

If $J_i$ and $J_{i+1}$ are delivered in the same batch in $\sigma^*$, and they are still delivered in the same batch in $\sigma^*$, the objective function value does not change.

If $J_i$ and $J_{i+1}$ are delivered in the same batch in $\sigma^*$, but they are delivered in the different batches in $\sigma^*$, there must be a delivery when $J_{i+1}$ is completed in $\sigma^*$. Hence, the change of objective function value must be caused by the arrangement of the deliveries of $i$-th customer. Now if the schedule $\sigma^*$ is the optimal schedule, we can get a new better schedule than $\sigma^*$ by interchanging the position of $J_i$ and $J_{i+1}$ again and keeping the number of jobs in each batch, because the delivery of the batch is earlier when $J_i$ is completed earlier. It is a contradiction.

Note that in the optimal schedule, jobs of all customers will not be processed in the shortest processing time first (SPT) order on the machine. For example, there are three jobs from two customers. The information required is listed as follows:

$p_1 = 1, p_2 = 3, p_3 = 2, d_1 = 10, d_2 = 1$, and $S = 2, E = 3, c_1 = 1$.

If we process all of these jobs in SPT order, that is, $J_1$ is the first, $J_2$ the second and $J_3$ the third, job $J_2$ is interrupted by the unavailable interval [2,3]. Hence, the completion time of three jobs is 1, 4 and 7, respectively. And it is easy to obtain the delivery time is $D_1 = 1, D_2 = 2, D_3 = 7$, and the total delivery cost is $c(d_1 + d_2) = 11$. As a result, the objective function is 29.

But it is not the optimal solution. If we process $J_1$ in the first position, and $J_2$ the second followed by $J_3$, there are no interrupted job. The completion time of three jobs is 2, 4 and 7, respectively. Then the delivery time is $D_1 = 2, D_2 = 4, D_3 = 7$, and the total delivery cost is still 11. Now the objective function is 27, which is the better.

3.2. **Algorithm**

To obtain the optimal schedule of the problem (P), we index jobs of each customer in the SPT order according to (2) of Lemma, so that $p_1 < p_2 < \ldots < p_n$ for $i = 1, 2, \ldots, m$.

Since Lemma indicates the SPT order of jobs of each customer can be used when they are manufactured, we develop the following dynamic programming algorithm concerned with the processing position and the delivery arrangement.
In the dynamic programming algorithm, we need 0-1 variables as follows for the $i$-th customer:

\[ x_{jk}^{i} = \begin{cases} 
1, & \text{if } J_j^i \text{ is assigned in the } k\text{-th position} \\
0, & \text{otherwise} 
\end{cases} \quad (j \in \{1, 2, \ldots, n_i\}, k = 1, 2, \ldots, n).
\]

**Algorithm DPm**

Define $F(i, l)$ as the minimum objective function value for jobs $J_1^i, J_2^i, \ldots, J_{m_i}^i$ of the $i$-th customer.

The initial condition: $F(i, 0) = 0$.

The recurrence relation:

For $1 \leq l \leq n_i (i = 1, 2, \ldots, m)$,

\[ F(i, l) = \min \{ F(i, l - h) + h \cdot D_j^i + c \cdot d_j \mid h = 1, 2, \ldots, j \}; \]

The optimal value: $\sum_{i=1}^{m} F(i, n_i)$.

Here,

\[ D_j^i = \begin{cases} 
p_j^i + \sum_{i=1}^{m} \sum_{j \in J(i)} \sum_{k < K} x_{jk}^{i} p_j^i, & \text{if } p_j^i + \sum_{i=1}^{m} \sum_{j \in J(i)} \sum_{k < K} x_{jk}^{i} p_j^i \leq S  \\
\left( E - S \right) + p_j^i + \sum_{i=1}^{m} \sum_{j \in J(i)} \sum_{k < K} x_{jk}^{i} p_j^i, & \text{if } p_j^i + \sum_{i=1}^{m} \sum_{j \in J(i)} \sum_{k < K} x_{jk}^{i} p_j^i > S  
\end{cases} \]

and ‘$K$’ is denoted the position that job $J_j^i$ is manufactured.

The optimality of Algorithm DPm can be demonstrated in the following theorem.

**Theorem**

For the problem $1 \mid r - a \mid V(\infty, \infty), direct \mid m \mid \Sigma D_j^i + T$, the output of Algorithm DPm must be optimal and the running time is $O(m^2 n^4)$.

**Proof**

By (2) of Lemma, there exists an optimal schedule in which jobs of each customer are processed in SPT order. We use 0-1 variables $x_{jk}^i$ to record the position of every job that is indexed in SPT sequence and manufactured on the machine. Moreover, the value of $F(i, l)$ is calculated by comparing all possible delivery batches including job $J_j^i$ for the $i$-th customer. In consideration of the correspondence between one delivery batch and one customer, we can think about the deliveries of jobs of different customers separately. Thus, $\sum_{i=1}^{m} F(i, n_i)$ is the optimal objective function.

In Algorithm DPm, there are at most $n^3$ states for each customer because $\sum_{i=1}^{m} n_i = n$ and the total number of processing positions is also $n$. And it takes no more than $O(mn^3)$ time to calculate the value for each state. Therefore, the time complexity of Algorithm DPm is bounded by $O(m^2 n^4)$.

4. Conclusion

Based on the results and discussions presented above, the conclusions are obtained as below:
(1) This paper studies an associated scheduling problem, i.e., the manufacture and delivery. In the manufacture stage, jobs of different customers need to be processed on a single machine. The processing of the job is resumable if it is interrupted by an unavailable interval on that machine. In the delivery stage, vehicles without capacity limitation are arranged to deliver completed jobs of the same customer in batches to that customer.

(2) We focus on minimizing the sum of total departure time and total delivery cost. A dynamic programming algorithm -Algorithm DPm is presented and we prove that this algorithm is optimal with time complexity $O(m^2n^3)$.

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References
[1] Cheng T.C.E., Gordon V.S. Batch delivery scheduling on a single machine[J]. Journal of the Operational Research Society, 1994, 45 (10): 1211–1216.
[2] Cheng T.C.E., Gordon V.S., Kovalyov M.Y. Single machine scheduling with batch deliveries[J]. European Journal of Operational Research, 1996, 94 (2): 277–283.
[3] Cheng T.C.E., Kovalyov M.Y., Lin B.M.T. Single machine scheduling to minimize batch delivery and job earliness penalties[J]. SIAM Journal on Optimization, 1997, 7(2): 547–559.
[4] Hall Nicholas G., Potts Chris N. Supply chain scheduling: batching and delivery[J]. Operations Research, 2003, 51(4): 566–584.
[5] Chen Zhilong. Integrated production and outbound distribution scheduling: Review and extensions[J]. Operations Research, 2010, 58(1) : 130–148.
[6] Lee Chung-Yee. Machine scheduling with an availability constraint[J]. Journal of Global Optimization, 1996, 9(3-4): 395-416.
[7] Ma Ying, Chu Chengbin and Zuo Chunrong. A survey of scheduling with deterministic machine availability constraints[J]. Computers Industrial Engineering, 2010, 58(2): 199-211.
[8] Wang Xiuli and Cheng T. C. Edwin. Machine scheduling with an availability constraint and job delivery coordination[J]. Naval Research Logistics, 2007, 54(1): 11-20.
[9] Fan Jing, Lu Xiwen and Liu Peihai. Integrated scheduling of production and delivery on a single machine with availability constraint[J]. Theoretical Computer Science, 2015, 562(1): 581-589.
[10] Fan Jing. Manufacture and Delivery Scheduling for Multiple Customers on a Single Machine with Availability Constraint[J]. Iwama-16-Advances in Economics, Business and Management Research, Manchester UK, 2016, 273-276.
[11] Fan Jing, Lu Xiwen. Supply chain scheduling problem in the hospital with periodic working time on a single machine[J]. Journal of Combinatorial Optimization, 2015, 30 (4): 892-905.