The security of Ping-Pong protocol

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Ping-Pong protocol is a type of quantum key distribution which makes use of two entangled photons in the EPR state. Its security is based on the randomization of the operations that Alice performs on the travel photon (qubit), and on the anti-correlation between the two photons in the EPR state. In this paper, we study the security of this protocol against some known quantum attacks, and present a scheme that may enhance its security to some degree.

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I. INTRODUCTION

In the early of 1990s, A. K. Ekert proposed a conception of realizing quantum cryptography based on Bell’s theorem [1], and C. H. Bennett and S. J. Wiesner brought forward a scheme for communicating via one- and two-particle operators on EPR states [2]. Since then, how to use EPR pairs to distribute a key has become a significant field of quantum key distribution (QKD) and has drawn physicists’s attention. In 2002, Kim Boström and Timo Felbinger proposed a novel QKD protocol called ‘Ping-Pong’ protocol [3], a number of works have been done in this aspect of QKD by far, some of them suggested improving security level of it, while others aimed at proposing eavesdropping schemes to attack it. In this letter, we discuss its robustness to some known quantum attacks, and present a scheme that may enhance its security to some degree.

For the purpose, let us recapitulate the ‘Ping-Pong’ protocol: Bob prepares two photons in an entangled state \(|\psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle |1\rangle + |1\rangle |0\rangle)\). He keeps one of them (home photon), and sends the other (travel photon), to Alice through a quantum channel. After receiving the travel photon, Alice randomly switches between a control mode and a message mode. In the control mode, Alice measures the travel photon with basis \(B_z = \{ |0\rangle, |1\rangle \}\) and then announces her measurement result through a classical public channel. After receiving the public announcement from Alice, Bob also switches to the control mode and measures the home photon with the same basis. In the absence of an eavesdropper, Eve, both the results should be anti-correlated, otherwise, it is an evidence that Eve is in line, the QKD process should be stopped. In the message mode, Alice performs a unitary operation \(Z^j\) to encode her message \(j \in \{0, 1\}\) on the travel photon, where \(Z^j = |0\rangle \langle 0| + (-1)^j |1\rangle \langle 1|\). Then Alice sends the travel photon to Bob. Bob performs a measurement with a Bell basis to draw the information Alice encoded. If the measurement result is \(|\psi^+\rangle\), Bob knows that \(j = 0\). Likewise, if the result is \(|\psi^-\rangle\), Bob knows that \(j = 1\). Repeating the process above could transmit the classic bits that Alice would like to share with Bob, so the QKD is done. The security of Ping-Pong protocol is based on the randomization of the operations that Alice performs on the travel photon (qubit), and on the anti-correlation between the two photons in the EPR state.

Compared to BB84 [4] and B92 [5], Ping-Pong protocol possesses a remarkable advantage: in the QKD process, it is unnecessary for Alice and Bob to discard some (may be a considerable amount of) unsuitable bits, so the efficiency of Ping-Pong protocol was ever thought of to be higher than BB84 and B92 by some researchers. However, ‘how safe is it’ is still a problem that needs to be solved. In the next three sections, I’ll discuss this problem.

II. TO OPAQUE EA VESDROPPING

The opaque eavesdropping is the simplest attack, which is also called ‘intercept-resend attack’. In this eavesdropping, Eve intercepts the quantum carrier on its way from Alice to Bob and/or from Bob to Alice and performs a measurement to get information about what state is sent and which operation Alice performs to the travel photon. Fig.1 demonstrates Eve’s eavesdropping process.

FIG. 1: The process of Eve’s eavesdropping.

Ping-Pong protocol itself plus this opaque eavesdropping could be described as follows:

1) At first, Bob prepares a pair of photons in \(|\psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle |1\rangle + |1\rangle |0\rangle)\). Assuming that Eve is absent, then after Alice’s operation on the travel photon \(Z^0\) with the probability \(p_o\), and \(Z^1\) with \(p_1\), the state of the pair...
respectively, the statistical QBER is $q_{Bob}$.

When Eve is in line, she captures the travel photon from Bob to Alice, and performs a projective measurement on it. Assuming Eve gets $|0\rangle$, the home qubit at Bob’s hand becomes $|1\rangle$. Then Eve prepares another $|0\rangle$ and sends it to Alice. Alice operates $Z^0$ (with $p_0$) or $Z^1$ (with $p_1$) to the qubit. However, neither $Z^0$ nor $Z^1$ could change $|0\rangle$. At last, Bob receives the travel qubit and the final state of the pair reads $|1\rangle_h |0\rangle_t$. In this case, the capacity of the channel, i.e. the maximal information between Alice and Bob, can be calculated to be

$$I_{AB} = 1 + q \log_2 q + (1 - q) \log_2 (1 - q) = 0.$$

From the analysis above, one can come to a conclusion that opaque eavesdropping is unskilled to Ping-Pong protocol, because it could make neither Eve nor Bob obtain any information, and in addition, it may cause a QBER up to 50%. As a result, a wise eavesdropper would not use it, so Ping-Pong protocol is robust to opaque attack.

### III. TO TRANSLUCENT EAVESDROPPING

We will still follow the process in Sec.II (see Fig.1). In the following analysis, we make an assumption that measurements do not make photons disappear, although with current technology, a photon disappears after it is measured. In fact, if the photon disappears, the analysis in this section would degenerate to that in Sec.II.

1. Bob prepares a pair of qubits in $|\psi^{(0)}\rangle = |\psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_h |1\rangle_t + |1\rangle_h |0\rangle_t)$, and sends one of them to Alice.

2. Eve captures the travel qubit, and makes it interact with an ancilla $|\chi\rangle$, obtaining

$$|\psi^{(1)}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_h (\sqrt{F} |1\rangle_t |\chi_1\rangle + \sqrt{D} |0\rangle_t |\chi_0\rangle) + |1\rangle_h (\sqrt{F} |0\rangle_t |\chi_0\rangle + \sqrt{D} |1\rangle_t |\chi_1\rangle)),$$

$$= (\sqrt{\frac{D}{2}} |0\rangle_h + \sqrt{\frac{F}{2}} |1\rangle_h) |0\rangle_t |\chi_0\rangle + (\sqrt{\frac{F}{2}} |0\rangle_h + \sqrt{\frac{D}{2}} |1\rangle_h) |1\rangle_t |\chi_1\rangle).$$

in which $D$ is the probability of error, and $F + D = 1$. Thus the reduced density matrix of the ancilla and travel qubit reads

$$\rho_{at}^{(1)} = tr_h |\psi^{(1)}\rangle \langle \psi^{(1)}| = \frac{1}{2} \begin{pmatrix}
1 & 0 & 0 & 2\sqrt{D(1-D)} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
2\sqrt{D(1-D)} & 0 & 0 & 1
\end{pmatrix}.$$ 

3. Eve continues to pass the travel qubit to Alice. If Alice chooses ‘control mode’, she would detect out Eve with a probability $D$. Else, if Alice chooses ‘message mode’, performing $Z^0$ with $p_0$ and $Z^1$ with $p_1 = 1 - p_0$, the state of the ancilla and travel qubit would become

$$|\psi^{(2)}\rangle = \frac{\sqrt{D}\langle 0|h + \sqrt{F}\langle 1|h)(\sqrt{p_0} + \sqrt{p_1}) |0\rangle_t |\chi_0\rangle + \sqrt{F}\langle 0|h + \sqrt{D}\langle 1|h)(\sqrt{p_0} - \sqrt{p_1}) |1\rangle_t |\chi_1\rangle}{\sqrt{\frac{D}{2}}(\sqrt{p_0} + \sqrt{p_1}) |0\rangle_t |\chi_0\rangle + \sqrt{\frac{F}{2}}(\sqrt{p_0} - \sqrt{p_1}) |1\rangle_t |\chi_1\rangle} + \frac{\sqrt{D}\langle 0|h + \sqrt{F}\langle 1|h)(\sqrt{p_0} + \sqrt{p_1}) |0\rangle_t |\chi_0\rangle + \sqrt{F}\langle 0|h + \sqrt{D}\langle 1|h)(\sqrt{p_0} - \sqrt{p_1}) |1\rangle_t |\chi_1\rangle}{\sqrt{\frac{D}{2}}(\sqrt{p_0} + \sqrt{p_1}) |0\rangle_t |\chi_0\rangle + \sqrt{\frac{F}{2}}(\sqrt{p_0} - \sqrt{p_1}) |1\rangle_t |\chi_1\rangle}.$$
The density matrix is

$$\rho_{at}^{(2)} = \begin{pmatrix} \frac{1}{2} + \sqrt{p_0 p_1} & 0 & 0 & (p_0 - p_1) \sqrt{D(1 - D)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (p_0 - p_1) \sqrt{D(1 - D)} & 0 & 0 & \frac{1}{2} - \sqrt{p_0 p_1} \end{pmatrix},$$

(6)

whose eigenvalues are as follows

$$\lambda_1 = \lambda_2 = 0,$$
$$\lambda_3 = \frac{1}{2} + \sqrt{p_0 p_1 + (p_0 - p_1)^2 D(1 - D)},$$
$$\lambda_4 = \frac{1}{2} - \sqrt{p_0 p_1 + (p_0 - p_1)^2 D(1 - D)}.$$  

Thus the maximum information Eve could get can be calculated as

$$I_{AE} = -\sum_{i=1}^{4} \lambda_i \log_2 \lambda_i = -\lambda_3 \log_2 \lambda_3 - \lambda_4 \log_2 \lambda_4.$$  

(8)

In fact, the density matrix of the whole system (ancilla, home and travel qubit) reads

$$\rho^{(2)} = \frac{1}{2} \begin{pmatrix} D & 0 & 0 & (p_0 - p_1) \sqrt{D(1 - D)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (p_0 - p_1) \sqrt{D(1 - D)} & 0 & 0 & \frac{1}{2} - \sqrt{p_0 p_1} \end{pmatrix} \begin{pmatrix} \sqrt{D(1 - D)} & 0 & 0 & (p_0 - p_1) \sqrt{D(1 - D)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (p_0 - p_1)(1 - D) & 0 & 0 & \frac{1}{2} - \sqrt{p_0 p_1} \sqrt{D(1 - D)} \end{pmatrix}.$$  

(9)

(4) The travel qubit is sent back by Alice and captured again by Eve. Now, Eve could perform a measurement on the two qubits: ancilla and the travel one. In this case, this qubit pair is in the EPR state in the subspace of the two qubits, thus a Bell measurement could help Eve get the information about which operation Alice has performed on the travel one. Eve makes use of two Bell basis-vectors for the measurement:

$$|\phi_{at}^I\rangle = \frac{1}{\sqrt{2}} (|0\rangle_t |\chi_0\rangle + |1\rangle_t |\chi_1\rangle),$$
$$|\phi_{at}^Z\rangle = \frac{1}{\sqrt{2}} (|0\rangle_t |\chi_0\rangle - |1\rangle_t |\chi_1\rangle).$$  

(10)

So the probability of Alice’s operation that Eve obtains could be calculated as

$$p(I) = \langle \phi_{at}^I | \rho_{at}^{(2)} | \phi_{at}^I \rangle = \frac{1}{2} + (p_0 - p_1) \sqrt{D(1 - D)}$$
$$p(Z) = \langle \phi_{at}^Z | \rho_{at}^{(2)} | \phi_{at}^Z \rangle = \frac{1}{2} - (p_0 - p_1) \sqrt{D(1 - D)}.$$  

(11)

After Eve’s Bell measurement, the subsystem of home and travel qubits becomes

$$\rho_{ht}^{(3)} = \begin{pmatrix} \frac{1}{2} + \frac{1}{2} \sqrt{p_0 (1 - p_0)} (2D - 1) & 0 & \frac{1}{2} \sqrt{D(1 - D)} & 0 \\ 0 & \frac{1}{2} + \frac{1}{2} \sqrt{p_0 (1 - p_0)} (2D - 1) & 0 & \frac{1}{2} \sqrt{D(1 - D)} \\ \frac{1}{2} \sqrt{D(1 - D)} & 0 & \frac{1}{2} + \frac{1}{2} \sqrt{p_0 (1 - p_0)} (2D - 1) & 0 \\ 0 & \frac{1}{2} \sqrt{D(1 - D)} & 0 & \frac{1}{2} + \frac{1}{2} \sqrt{p_0 (1 - p_0)} (2D - 1) \end{pmatrix}.$$  

(12)

Note that $D + F = 1$ and $p_0 + p_1 = 1$.  

(5) Eve sends the travel qubit back to Bob. Statist-
tically, Bob uses performs measurements on both home and travel qubits, and the QBER of Bob’s measurements is
\[ q = 1 - \langle \psi'' \rangle |p_0^{(3)} | \psi'' \rangle = \frac{3}{4} - p_0(1 - p_0)(2D - 1). \] (13)
Thus, the maximum mutual information between Alice and Bob (i.e. the capacity of this quantum channel) is
\[ I_{AB} = 1 + q \log_2 q + (1 - q) \log_2(1 - q). \] (14)
We plot \( I_{AE} \) and \( I_{AB} \) in the figure below:

![Graph](image)

**FIG. 2:** The comparison of \( I_{AB} \) and \( I_{AE} \).

From the Fig.2, we can come to a conclusion that when Alice conducts an equiprobable coding, that is to say, \( p_0 = p_1 = \frac{1}{2} \), which is also the security requirement of classical cryptography. Bob could always gets more information than Eve, especially in the case of \( D = 0 \) (Eve’s optimal eavesdropping), Alice and Bob can share the maximum information (\( I_{AB} = 1 \)), and Eve gets the minimum information (\( I_{AE} = 0 \)). Thus, in short, as long as Alice codes the travel qubits equiprobably, the Ping-Pong protocol is secure.

**IV. TO WÓJCIK’S ATTACK**

In 2003, Antoni Wójcik proposed a novel eavesdropping scheme to attack Ping-Pong protocol, claiming that if the quantum channel transmission efficiency \( \eta \) is no more than 60%, Eve could get more information than \( I_{AB} \) without being detected. By far, there is no effective preventing method against this attack. We propose a so-called Disguising Photon Detecting (DPD) method to implement this task, because in our scheme, we use some single photons in the state \( |+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \) disguising the travel photons in the original Ping-Pong protocol. The disguising photon can be called a ‘false photon’, and correspondingly, the travel photon entangled with another one in the state \( |\psi^+\rangle = \frac{1}{\sqrt{2}} (|0\rangle |1\rangle + |1\rangle |0\rangle) \) in the original Ping-Pong protocol \( \mathcal{E} \) is called a ‘true photon’. In the DPD method, the original Ping-Pong protocol has to be modified: Bob randomly sends Alice a travel photon that is a true or false photon. After receiving the travel photon, Alice switches between control mode and message mode, and then goes ahead just as in the original Ping-Pong protocol. But after Bob receiving the traveling-back photon, what he should do is somewhat different from that in the original Ping-Pong protocol. If he sends a true photon, he then take the same action on the traveling-back photon just as what he should do in the original Ping-Pong protocol; else, if he sends a false photon and Alice chooses the message mode, after receiving the traveling-back photon, he asks Alice which operation she performed on the photon, \( Z^0 \) or \( Z^1 \), if Alice performed \( Z^0 \), Bob does nothing to the traveling-back photon and discard it, while if Alice performed \( Z^1 \), Bob performs a projective measurement on the traveling-back photon with the projector \( P_+ = |+\rangle \langle +| = \frac{1}{2}(|0\rangle + |1\rangle)\langle 0| + |1\rangle \langle 1| \), which could be done by using some optical devices \([7, 8, 9]\).; else, if Bob sends a false photon and Alice chooses the control mode, Bob tells Alice to discard this bit in the authentication step after sending all the photons.

Theoretically, if Eve is absent, the false photon after being performed \( Z^1 \) must be in the state \( |\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \), so the outcome of the measurement must be zero, because \( |\rangle \) is orthogonal to \( |+\rangle \). But if the outcome is not zero, it could be an evidence that Eve is eavesdropping the communication between Alice and Bob, thus the QKD process must be stopped.

To demonstrate this method is feasible, let us now analyze the states that Bob sends and receives. The initial state that Bob sends is \( |+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \), whose density matrix is
\[
\rho_t = |+\rangle \langle +| = \frac{1}{2} (|0\rangle \langle 0| + |1\rangle \langle 1| + |1\rangle \langle 0| + |0\rangle \langle 1|) = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.
\] (15)

There are two situations that should be considered:
(1) When Wójcik’s Eve is in line. After Eve’s \( B\)-A attack, the state becomes \( |B\rangle = Q_{txy} (1 + |\psi\rangle |\psi\rangle_0 y = \frac{1}{2} (|0\rangle |0\rangle |0\rangle_0 y + |\psi\rangle_0 y + |\psi\rangle_0 y + |\psi\rangle_0 y) \). If Alice performs \( Z^0 \) (that is, an identical operation \( I \)), the state maintains in \( \mathcal{B}\)-A, then Alice sends it back and Eve commits the A-B attack \( Q_{txy}^{-1} \) (according to transformations (3) in Wójcik’s paper \([10]\), \( |B\rangle = Q_{txy}^{-1} I |\psi\rangle = Q_{txy}^{-1} (1 + |\psi\rangle |\psi\rangle_0 y + |\psi\rangle_0 y) \), where the subscript \( I \) indicates that Alice performs \( I = Z^0 \). So the travel photon Bob receives would be still in the state \( |+\rangle \). Else, if Alice performs \( Z^1 \) (that is, the Pauli \( \sigma_z \) operation), the state becomes \( |\psi\rangle = \frac{1}{2} (|0\rangle |0\rangle |\psi\rangle_0 y + |\psi\rangle_0 y + |\psi\rangle_0 y + |\psi\rangle_0 y) \). Then Alice sends the photon back and Eve commits the A-B attack \( Q_{txy}^{-1}: |\psi\rangle = \frac{1}{2} (|0\rangle |0\rangle |\psi\rangle_0 y + |\psi\rangle_0 y + |\psi\rangle_0 y + |\psi\rangle_0 y) \).
\[
Q_{txy}\sigma_z |B-A\rangle = \frac{1}{\sqrt{2}} |0\rangle_x |\text{vac}\rangle_y |0\rangle_y + \frac{1}{\sqrt{2}} |1\rangle_x |\text{vac}\rangle_y |1\rangle_y,
\]
where the subscript \(Z\) indicates that Alice performs \(Z = Z^1\). So the density matrix of the false photon is
\[
\rho_{zt} = Tr_{x,y} |A-B\rangle_Z Z (A-B) = \frac{1}{2} |0\rangle \langle 0| + \frac{1}{2} |1\rangle \langle 1| \quad (16)
\]
where the subscript ‘\(t\)’ denotes the travel photon. This means that the false photon Bob receives is in either \(|0\rangle\) or \(|1\rangle\) with the probability of \(1/2\) respectively.

(2) When Wójcik’s Eve is absent. If Alice performs \(Z^0\) on the false photon and sends it back, Bob would receives the photon in \(|+\rangle\). Else, if Alice performs \(Z^1\), the photon Bob would receive would be in \(Z^1 |+\rangle = |−\rangle\).

It could be concluded from the analysis above that if Alice performs \(Z^0\) on the false photon, no matter Wójcik’s Eve is in line or not, Bob would receive the photon in \(|+\rangle\). But if Alice performs \(Z^1\), the photon Bob would receive would be in \(Z^1 |+\rangle = |−\rangle\). With this difference, to detect Wójcik’s Eve is possible, and we propose a projector \(P_+\) could fulfill this task.

V. DISCUSSION AND COMMENT

In 2003, Qing-yu Cai published his comment claiming that the Ping-Pong protocol can be attacked without eavesdropping [10]. In the comment, Cai proposed that Eve could attack the communication between Alice and Bob with the following method: ‘In every message mode, Eve captures the travel back qubit Alice sent to Bob and perform a measurement in the basis \(B_z\) and forwards to Bob this qubit. Alice and Bob have zero probability to find Eve’s attack. Then Bob lets this communication continue. But every one of Bob’s measurement results is meaningless since the two qubits become independent of each other after Eve’s attack measurement.’... When the communication is terminated, Bob has learned nothing but a sequence of nonsense random bits.’ However, we think this attack would not work as well as claimed for at least two reasons: (1) when Eve performs a measurement on the qubit travelling from Alice to Bob, the entanglement between the home qubit and the travel qubit is destroyed, it is no longer \(|ψ^+\rangle = \frac{1}{\sqrt{2}} (|0\rangle |1\rangle + |1\rangle |0\rangle)\) or \(|ψ^−\rangle = \frac{1}{\sqrt{2}} (|0\rangle |1\rangle − |1\rangle |0\rangle)\), but simply \(|01\rangle\) or \(|10\rangle\), which could be detected by a Bell measurement, and Eve could not gain any useful information about what operation Alice performs on the travel qubit; (2) after terminating the QKD process, Alice and Bob would pick out a part of the key established in the process to make a classic authentication, if the attack makes Bob’s measurement results meaningless, it would be found that Eve is in line in the classic authentication. As a result, Cai’s claim that the Ping-Pong protocol can be attacked without eavesdropping is open to doubt.

VI. CONCLUSION

In this letter, we analyze the robustness of Ping-Pong protocol to some known quantum attacks, from the analysis, we can come to the conclusion that, to opaque and translucent attacks, Ping-Pong protocol is robust and secure, and to Wójcik’s attack, as long as Bob sends sufficient disguising photons, he could make this attack useless. In summary, the Ping-Pong protocol is secure as long as it is modified to use the DPD method. We call the Ping-Pong protocol associated with the DPD method a modified Ping-Pong protocol, of which process may not be depicted clearly in words, so it would be necessary and beneficial to describe it in a chart. Thus, we draw a flow chart to make the modified Ping-Ping protocol more clear to be understood. See Fig.3

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BEGIN

Bob sends a travel photon to Alice.

True or False

Control
Mode
Message

Alice measures the travel photon in the basis $B_j = \{ |0\rangle, |1\rangle \}$ and tells Bob the result.

Bob measures the home photon in the same basis.

Are their results identical?

No

Yes

Eve is in line. Stop the QKD process.

Alice first performs a $Z^j$ operation on the travel photon ($j = 0$ or $1$) and sends it back to Bob.

Bob receives the travel photon and performs a Bell-measurement on both the home and travel photons to gain $j$.

Are their results identical?

No

Yes

Continue the QKD process.

Alice measures the travel photon in the basis $B_j = \{ |0\rangle, |1\rangle \}$ and tells Bob the result.

Bob knows that the photon is false, so ignores the result.

Bob receives the travel photon and asks Alice about $j$ through a classical channel. Alice replies Bob.

$j = 0$

$j = 0$ or $1$?

Yes

Bob performs a projective measurement on the travel photon with $P_x = |0\rangle\langle 0|$. Is the result 0?

Eve is in line. Stop the QKD process.

Yes

Is the result 0?

No

Is the number of bits enough to establish a key string?

No

Eve is in line. Stop the QKD process.

Yes

END

FIG. 3: The flow chart of the modified Ping-Pong protocol.