Elastic modulus measurement of thermoplastic composites through modal analysis

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Abstract. Conventional tensile testing of materials employ pulling of a specimen till it breaks to determine the elastic modulus and other tensile properties. Modal analysis is one of the ways to measure the elastic modulus without destructing the sample. The present study was carried out to evaluate the elastic modulus of short carbon fiber (SCF) reinforced polyethersulfone (PES) composites filled with potassium titanate whisker (PTW) and ultra high molecular weight polyethylene (UHMWPE) through modal analysis. Specimens were prepared by extrusion and injection moulding processes. Modal analysis was done with cantilever type free-vibration test rig on the developed composites using an accelerometer sensor. Based on the acceleration response of the composites, their natural frequencies and damping values were determined. Further, elastic moduli of the composites were computed and compared with that of actual tensile test. Elastic modulus of PES/SCF composites determined by free vibration test was well in agreement with that of tensile test results. In addition, numerical modal analysis simulation was performed for by considering the tensile modulus and density of each of the composite material. Natural undamped frequencies of the composites found by numerical modal analysis were closer to that of the experimental values.

1. Introduction
Carbon fiber reinforced composites are widely used in automobile, marine and aerospace industries due their high load carrying capacity[1]–[3]. However, high stiffness and high strength materials exhibit poor damping properties. Damping properties of fiber reinforced composites can be enhanced by hybridization with filler materials. Fillers will introduce more number of internal interfaces and forms a way to dissipate the vibration energy [4]. Vibration techniques can be used as non-destructive tests to detect and locate the flaws in fiber reinforced composites [5]. Modal analysis is an important tool to determine the vibration parameter such as natural frequency and corresponding mode shapes of a system [6]. Neglecting air damping, a vibrating system is expected to attain the zero velocity if the material constituting the system absorbs all the vibration energy and dissipates it to surrounding. Based on the amount of reduction in amplitude of the vibrating body in the successive cycles, logarithmic decrement can be evaluated. Logarithmic decrement will be useful in computing the damping ratio[7]–[9]. Natural damped frequency is determined by taking the reciprocal of the time period from the acceleration-time plot. Natural undamped frequency is computed through the relation between damping ratio and damped frequency.

Modal analysis was done with cantilever type free-vibration test rig on the developed composites using an accelerometer sensor. Acceleration response during the vibration of the composites due to
the initial applied impact was captured till amplitude of acceleration decay to zero. Damped time period is measured from the time plot. For cantilever beam system, its natural frequency [10] is determined using the relation (1).

$$f_n = \frac{1}{2\pi} \sqrt{\frac{3EI}{\rho l^4}}$$

$$140$$

where $E$ is the tensile modulus of the material, $I$ is the moment of inertia of beam cross section in bending, $l$ is the length of the beam, $m$ is the mass of beam and $M$ is the mass of the accelerometer. Equation (1) can be further used to evaluate the Young’s modulus of the composite material [11]. Numerical modal analysis can assist in understanding the experimental results better[12], [13].

2. Materials and methods

2.1. Materials

The polyethersulfone (PES) was procured from the company Solvay, USA and was used as matrix material. The short carbon fiber (SCF) based on polyacrylonitrile (PAN) was purchased from the company Teijin, Japan and was used as reinforcement material. Potassium titanate whisker (PTW) was supplied by Nobel Alchem, India and Ultra high molecular weight polyethylene (UHMWPE) was procured from Spectra Plast, India. PTW and UHMWPE were used as fillers to provide the self lubrication. The properties of PES, SCF, PTW and UHMWPE are stated in the previous work [14]. Materials formulation and designations adopted in the present work are presented in table 1. All the composites were fabricated for a dimension of 120 mm x 10 mm x 3.85 mm by extrusion process followed by injection moulding as stated in elsewhere[14].

| Composites | Matrix wt. % | Fiber wt. % | Filler wt. % | Filler wt. % |
|------------|--------------|-------------|--------------|--------------|
| C00P       | 100          | 00          | -            | -            |
| C25P       | 75.0         | 25          | -            | -            |
| C25WP2.5P  | 72.5         | 25          | 2.5          | -            |
| C25WP5.0P  | 70.0         | 25          | 5.0          | -            |
| C25U2.5P   | 72.5         | 25          | -            | 2.5          |
| C25U5.0P   | 70.0         | 25          | -            | 5.0          |

2.2. Methods

2.2.1. Free vibration test. Damping properties of the PES/SCF composites were determined by single cantilever free vibration decaying test [10]. The developed samples were mounted as cantilever beam and 10 mm length was used for clamping purpose. Effective length of the beam was taken as 110 mm for analytical calculations and numerical simulations.

![Figure 1](image-url)
Piezoelectric accelerometer, model PCB 356A15 was used to capture the response of the vibrating beam as indicated in figure 1. Mass of the accelerometer used is 10.5 grams and it is capable to measure the frequency in the range 2 to 5000 Hz with ± 5 % accuracy. Accelerometer was mounted at the free end of cantilever beam arrangement with a temporary adhesive (petro wax) as in figure 1. Analog response captured by accelerometer was then digitalized with the help of NI 9234 IEPE input module. Digitalized response was then transferred to the computer system through NI compact data acquisition module cDAQ 9178. Data received by the computer was then analyzed using NI Labview software package.

2.2.2. Numerical simulation of modal analysis. Numerical modal analysis was performed with Abaqus CAE 2017 finite element software. Steps involved in the simulation process are illustrated in figure 2. Geometry of the beam was modeled as rectangular cross section bar (10 mm x 3.85 mm) of 125 mm length and properties obtained from the experiments were assigned to created beam geometry. Accelerometer sensor was modeled as cube of 13 mm and cube was assigned with customized material of density 4560 kg/m$^3$ so that mass of the sensor equals to its actual value of 10.5 gram.

![Figure 2. Numerical simulation steps (a) Geometry (b) Materials assigned (c) Assembly (d) Meshing](image)

Beam and cube were assembled together followed by applying the tie constraint between them. Both the geometries were meshed with C3D8R elements. Mesh convergence was achieved with minimum of 500 elements. Linear perturbation frequency analysis was carried to extract the first natural frequency and mode shapes.

3. Results and discussion

3.1. Test results

3.1.1. Free vibration test. Responses of the composites during the free vibration are presented with respect to time as shown in figure 3. From the acceleration of vibration versus time plots, time taken for five successive cycles of vibration and corresponding reduction in amplitudes were noted as presented in table 2.

![Table 2. Free vibration test results of PES/SCF composites](table)

| Parameters                  | Composites       |
|-----------------------------|------------------|
|                             | $C_{00}P$ | $C_{25}P$ | $C_{25}W_{12}P$ | $C_{25}W_{5,0}P$ | $C_{25}U_{12}P$ | $C_{25}U_{5,0}P$ |
| Time $t_1$ (s)              | 0.014   | 0.003   | 0.003  | 0.015  | 0.004  | 0.019  |
| Time $t_2$ (s)              | 0.215   | 0.113   | 0.112  | 0.121  | 0.123  | 0.148  |
| Amplitude at time $t_1$ (m/s$^2$) | 65.7    | 101.6   | 94.0   | 124.2  | 87.8   | 128.4  |
| Amplitude at time $t_2$ (m/s$^2$) | 1.7     | 21.8    | 19.5   | 29.4   | 16.7   | 20.1   |
| Logarithmic decrement, $\delta$ | 0.731   | 0.308   | 0.315  | 0.288  | 0.332  | 0.371  |
| Damping ratio, $\xi$       | 0.116   | 0.049   | 0.050  | 0.046  | 0.053  | 0.059  |
| Time period (s)            | 0.040   | 0.022   | 0.022  | 0.021  | 0.024  | 0.026  |
| Damped natural frequency, $f_d$ (Hz) | 24.88   | 45.45   | 45.87  | 47.17  | 42.02  | 38.76  |
| Undamped natural frequency, $f_n$ (Hz) | 25.04   | 45.51   | 45.93  | 47.22  | 42.08  | 38.83  |
Damping ratio of each composite was determined based on vibration amplitude decayed in mode-1 vibration. Addition of SCF into neat PES resulted in reduction of damping by 57.8% due to stiffening of the C25P composite. Addition of 2.5 wt.% of PTW did not exhibit much change in the damping with respect to C25P composite. However, damping value of C25P composite reduced further by 6.1 % with addition of 5 wt. % of PTW. This attributed to the slight improvement in the tensile modulus due to PTW addition.

Figure 3. Vibration response of composites in time domain (a) C0P (b) C25P (c) C25W25P (d) C25W50P (e) C25U25P (f) C25U50P

UHMWPE filled composites showed improvement in damping ratio by 8.2 % and 20.4 % with addition of 2.5 and 5 wt. % of UHMWPE into C25P composite. This indicated slight softening and enhanced ductility in the UHMWPE filled composites.

Neat PES showed maximum time period of vibration of 0.04 seconds and minimum damped frequency of 24.88 Hz. With addition of SCF and PTW fillers, lowering of time period and enhancement of natural frequency was observed. However, time periods of UHMWPE filled composites found higher than PTW filled composites and represented decrease in their natural frequencies.

Figure 4. Elastic modulus of PES/SCF composites
As natural frequency is directly proportional to the stiffness which in turn depends on elastic modulus of the material, tensile modulus of all the developed composites were determined as higher than unfilled PES due to their vibrations at higher natural frequencies. Computed elastic modulus of PES/SCF composites filled with PTW and UHMWPE measured by free vibration test are printed in figure 4 and also comparison with that of tensile test results are shown. Instead of computing the natural frequencies of the composites based on time period consideration, Fast Fourier Transform (FFT) analysis was also performed in which acceleration signal in time domain was transformed to acceleration in frequency domain as shown in figure 5. Frequency corresponding to the maximum peak in the frequency domain curves represents the natural frequencies of the composites which were exactly same as reciprocal of the time period.

![Figure 5. Acceleration-time plots of PES/SCF composites](image)

3.1.2. Numerical modal analysis. Simulated mode-1 shapes of each composite along with their natural frequencies were presented in figure 6. Error between the numerical and experimental natural frequencies of the PES/SCF composites was around 5% as reported in table 3. This error was due to the assumption of homogeneous and elastic material model. More accurate simulation results can be achieved by assigning the geometry with visco-elastic material model parameters obtained from stress relaxation and/or creep test.

| Composites | Experimental | Numerical | Error (%) |
|------------|--------------|-----------|-----------|
| C_{0}P     | 25.04        | 26.33     | 5.1       |
| C_{25}P    | 45.51        | 47.80     | 5.0       |
| C_{25}W_{2.5}P | 45.93    | 48.31     | 5.2       |
| C_{25}W_{5.0}P | 47.22     | 49.59     | 5.0       |
| C_{25}U_{2.5}P | 42.08     | 44.23     | 5.1       |
| C_{25}U_{5.0}P | 38.83     | 40.78     | 5.0       |
4. Conclusion

Elastic moduli of PES/SCF composites determined by free vibration test were well in agreement with that of tensile test results. Error between modulus obtained by free vibration test and tensile test was attributed to the anisotropic nature of composites due to random distribution and orientation of fibers in the matrix material. In conventional tensile test, even if five replicate tests were done on each composite as per ASTM standard, usually ±10% variation in tensile properties from their mean is observed. Hence, when tensile modulus is the property of interest for designing, free vibration test can be used as an alternate method to estimate the tensile modulus through natural frequencies of composite materials without any material destruction. Numerical simulation results were validated with the experimental values with minimum error.

5. References

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