First-order phase transition from hypernuclear matter to deconfined quark matter obeying new constraints from compact star observations

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The number of nucleons is supposed to be infinite.

Coulomb interaction is disregarded because of the strong interaction between nucleons.

The density of nuclear matter is supposed to be finite.

\[ \rho = \lim_{N, V \to \infty} \frac{N}{V} \]

Nuclear Matter
M. Baldo, G. F. Burgio, and H. J. Schulze, Physical Review C 61, 055801 (2000)

| \( \rho_0 (fm^{-3}) \) | 0.1748 |
|--------------------------|-------|
| \( E_0/A (MeV) \)       | −15.58|
| \( E_{sym} (MeV) \)     | 39.9  |
| \( K_0 \)               | 295.77|
For PSRJ0740+6620

\[ M_{max} = 2.17^{+0.11}_{-0.10} M\odot \]

for the binary neutron star merger GW170817

\[ R(1.6M\odot) > 10.7 \text{ km} \]
\[ \& \]
\[ R(1.4M\odot) < 13.6 \text{ km} \]
Hyperon Puzzle
Phase Transition From Hyper Nuclear Matter to Deconfined Quark Matter as a Solution to the Hyperon Puzzle

Initiation of a new collaboration that joins different domains of state-of-the-art expertise

- **LOCV**
  For hadronic phase

- **nl-NJL**
  For quark phase
**Hamiltonian of nuclear matter**: 
\[ H = \sum_i \frac{p_i^2}{2m_i} + \sum_{i \neq j} V(ij) \]

**Trial wave function**: 
\[ \Psi(1 \ldots A) = F(1 \ldots A) \Phi(1 \ldots A) \]

**Energy**: 
\[ E = \langle H \rangle = \frac{1}{N} \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = E_1 + E_{MB} \approx E_1 + E_2 \]

**Characteristics**
- A pure variational method in configuration space
- Generalized to finite temperature
- Calculation of correlation functions
- Using both central and tensor correlation functions
- Energy per baryon and correlation functions are state-dependent
- Using normalization condition as the only constraint

**LOCV method**
Lowest Order Constrained Variational method
nl-NJL model
Nonlocal Nambu–Jona-Lasinio model

Characteristics

- nonlocal covariant extension of the NJL model
- quark fields interact via nonlocal (momentum dependent) vertices
- Nonlocal interactions regularize the model in such a way there is not need to introduce sharp cutoffs

nl-NJL model

- Constant coefficients (model A)
- density-dependent coefficients (model B)

G.A. Contrera, A. G. Grunfeld and D. Blaschke, EPJ A 52 (2016)
First Order Phase Transition (PT) by a Maxwell construction

\[ \mu_H = \mu_Q = \mu_c \]

\[ T_H = T_Q = T_c \]

\[ P_H(\mu_B, \mu_e) = P_Q(\mu_B, \mu_e) = p_c \]
Model A
Model B
Symmetric Matter

The graphs show the pressure ($P$) versus chemical potential ($\mu$) for different values of $\eta$ in the context of symmetric matter. The curves represent various nNIL models, with each line corresponding to a different $\eta$ value. The red line indicates the LOCVY symmetric matter model. The inset graph provides a closer view of the pressure at low chemical potentials.
Replacement Interpolation Method (RIM)  
A Mixed Phase Approach

\[ P_M(\mu) = a(\mu - \mu_c)^2 + b(\mu - \mu_c) + P_c + \Delta P. \]

\[ P_M(\mu_c) = P_c + \Delta P = P_M. \]

\[
\begin{align*}
P_M(\mu_{cH}) &= P_H(\mu_{cH}) = P_H, \\
P_M(\mu_{cQ}) &= P_Q(\mu_{cQ}) = P_Q, \\
n_M(\mu_{cH}) &= n_H(\mu_{cH}), \\
n_M(\mu_{cQ}) &= n_Q(\mu_{cQ}).
\end{align*}
\]
RIM for Model B
RIM for Model A
Main Results

1. model B with density dependent parameters allows for an intermediate hypernuclear matter phase in the hybrid star, between the nuclear and color superconducting quark matter phase, while in model A such a phase cannot be realized because the phase transition onset is at low densities, before the hyperon threshold density is passed.

2. for model A the cases with a sufficiently strong repulsive vector mean field ($\eta > 0.12$) which produce reasonable hybrid star EoS have also a phase transition under isospin-symmetric conditions. For $\eta = 0.12$ the critical density is $n_c = 0.79 \, fm^{-3}$ and for $\eta = 0.15$ it is $n_c = 0.98 \, fm^{-3}$. For the less repulsive vector mean fields ($\eta \leq 0.11$) there is no deconfinement transition in symmetric matter!

3. For model B in which a density-dependent bag pressure serves as a confining mechanism at low densities, a deconfinement phase transition under isospin-symmetric conditions is predicted for all considered parametrizations (set 1 - set 4) at densities between $2.2 \, n_0$ and $2.7 \, n_0$.

4. Using the RIM, for model A the low density problem (no confinement) is cured and model B is now more realistic with a mixed phase.
Thank you
LOCV Method: Lowest Order Constrained Variational Method

\[ f(ij) = \sum_{\alpha p=1}^{3} f^p_\alpha(ij) O^p_\alpha(ij) \]

\[ \alpha = \{ J, L, S, T, T_z \} \]

\[ O^p_\alpha(ij) = 1, \quad \frac{1}{6} (S_{12} + 4P_t), \quad \frac{1}{6} (2P_t - S_{12}) \]

\[ S_{12} = 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2 \]

\[ p=1 \text{ for } \begin{cases} s=0 \\ s=1 \text{ with } L=J \end{cases} \]

\[ p=2,3 \text{ for } s=1 \text{ with } J=L \pm 1 \]
The only constraint in LOCV method is renormalization condition of wave functions

\[ \langle \Psi | \Psi \rangle = 1 - \sum_{ij} \langle ij | F_p^2 - F^2 | ij - ji \rangle \quad : \quad \chi = \frac{1}{N} \sum_{ij} \langle ij | F_p^2 - F^2 | ij - ji \rangle = 0 \]

\[ F_p = \begin{cases} 
\left( 1 - \frac{9}{2} \left( \frac{I_1(Kf)}{Kf} \right)^2 \right)^{-\frac{1}{2}} & T_z = \pm 1 \\
1 & T_z = 0
\end{cases} \]

\[ E_2 = \int dr \left[ G \left( f'^2 (r) \right) + S(f(r)) - \lambda(f(r)) \right] = \int dr \left( f'(r), f(r) \right), \delta E_2 = 0 \]

\[ \frac{\partial L}{\partial f} - \frac{\partial}{\partial r} \frac{\partial L}{\partial f'} = 0 \]

H. Moshfegh and M. Modarres, Journal of Physics G: Nuclear and Particle Physics 24, 821 (1998)