Breakdown of quantisation in a Hubbard-Thouless pump

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The fate of topological transport in the strongly correlated regime raises fundamental questions on the role of geometry in quantum many-body physics [1, 2]. Recently, different platforms have emerged as possible probes for many-body transport beyond the traditional Hall response [3]; these include ultracold quantum gases [4–9], Rydberg atoms [10–12], and photonics [13–15]. A paradigm of quantised transport is the topological Thouless pump, which represents the one-dimensional, dynamic analogue of the quantum Hall effect [16]. A few experiments have explored the effects of interactions on Thouless pumping in two-body [6] and optical mean-field [14, 15] systems, but the strongly correlated regime has so far remained out of reach [17–23]. Here, we experimentally detect the breakdown of topological transport due to strong Hubbard interactions in a fermionic Thouless pump, facilitated by a dynamical optical superlattice. We observe the deviation from quantised pumping for repulsive Hubbard $U$, which we attribute to pinned atoms in a Mott insulator. On the attractive side another mechanism is reducing the pumping efficiency: a smaller energy gap for pair pumping makes the evolution less adiabatic. The dynamical superlattice operated at a single frequency establishes a novel platform for studying topology in the presence of strong correlations in one, two, and possibly three dimensions, including avenues to fractional transport.

Ultracold atoms in optical lattices constitute a quantum many-body system where interparticle interactions can be tuned and different lattice configurations are accessible. In our experiment, strongly correlated states emerge via the atomic contact interaction between fermions of different spin. We create a dynamical superlattice by overlaying phase-controlled standing waves with an additional running wave component and study topological charge pumping in the interacting Rice-Mele model [24],

$$\hat{H}(\tau) = -\sum_{j,\sigma} \left[ \delta + (-1)^j \delta(\tau) \right] (\hat{c}_{j\sigma}^\dagger \hat{c}_{j+1\sigma} + \text{h.c.}) + \Delta(\tau) \sum_{j,\sigma} (-1)^j \hat{c}_{j\sigma}^\dagger \hat{c}_{j\sigma} + U \sum_{j} \hat{n}_{j\uparrow} \hat{n}_{j\downarrow}. $$

The interactions enter as the Hubbard $U$ for two fermions of opposite spin $\sigma \in \{\uparrow, \downarrow\}$ occupying the same lattice site $j$. The fermionic annihilation and number operators are denoted by $\hat{c}_{j\sigma}$ and $\hat{n}_{j\sigma}$, respectively. The parameter $\delta(\tau) = \delta_0 \cos(2\pi \tau/T)$ defines the alternating bond dimerisation, whereas $\Delta(\tau) = \Delta_0 \sin(2\pi \tau/T)$ is the sublattice offset. Both $\delta(\tau)$ and $\Delta(\tau)$ are periodic in time $\tau$ with period $T$. The bipartite lattice structure in Eq. 1 leads to a two-band model characterised by a gap. Typical parameters are $\Delta_0 \approx 2t$, $\delta_0 \approx t$, leading to small variations of the single-particle band gap between

FIG. 1. Experimental setup and the role of interactions for topological charge pumping. a, schematic of the dynamical optical superlattice setup. The interfering lattice (yellow) is imbalanced along the $x$-direction, leading to a movement of the ‘long’ lattice with respect to the ‘short’ lattice (non-interfering, red arrows) when ramping the phase $\varphi$ of the incoming light. The running wave is due to a rotated polarisation of the retro-reflected laser beam ($\lambda/4$-plate). The $y$-direction is not shown for clarity. $d = \lambda = 1064\text{ nm}$ is the size of one unit cell. b, the resulting lattice structure along $x$ corresponds to the Rice-Mele Hamiltonian (1), including the Hubbard repulsion $U$. c, bond-centred inversion symmetry (grey shading) in the non-interacting Rice-Mele model is present whenever the site offset is zero ($\Delta = 0$, SSH model), protecting two distinct topological phases, separated by a gap closing (red dot). Quantised pumping can be achieved by adiabatically connecting the two protected ground states (black line). d, strong repulsive interactions ($U \gg t, |\delta|, |\Delta|$) extend the regions of preserved inversion symmetry, separated by a line of gap closing (red dashed line). Therefore, the cyclic pumping orbit necessarily crosses a gap and the quantisation of pumping is no longer guaranteed.
FIG. 2. Microscopic mechanisms for topological pumping in the interacting Rice Mele model. a, schematic atom occupations in the superlattice for strongly attractive (left), noninteracting (middle), and strongly repulsive systems (right) at two representative points during pumping. The upper row corresponds to the dimerised case (SSH model, no site offset) whereas the lower row shows the ionic Hubbard model (maximum site offset, no dimerisation). In the attractive and noninteracting cases, finite density fluctuations are indicated by showing several possible atom occupations. b, evolution of the in-situ centre-of-mass (c.m.) over half a pumping cycle, measured for $T = 25\hbar/t$ (i.e. 25 tunnelling times). The grey points and error bars correspond to the average and standard error of 20 individual measurements for a noninteracting system, while the lines denote numerical simulations of the many-body ground state with a density matrix renormalisation group (DMRG) algorithm (Methods). The second data point is an experimental outlier, most likely due to an instability of the absolute position of the atom cloud. c, measured double occupancy fraction over half a pumping cycle for $U/\Delta_0 = \{-3.0, 0, 3.0\}$ (grey points, blue diamonds, and red squares, respectively). Data points correspond to the average of two separate sets of measurements for two opposite pumping directions each made up of 3 repetitions. Errors bars correspond to the propagated error estimated from standard deviation. The second half of the pumping cycle is just a repetition of the first half and we omit it for clarity.

2.0 $\Delta_0$ and 2.2 $\Delta_0$ over one period (Methods, Fig. ED1).

In the experiment we load a balanced spin-mixture ($\uparrow, \downarrow$) of ultracold potassium-40 atoms into a three-dimensional optical lattice (Fig. 1, Fig. ED1, and Methods). We modified our previous setup [25] to enable dynamical control over the lattice potential by including an intensity imbalance (i.e. a running wave) along the x-axis. The lattice comprises interfering laser beams in the $x$–$z$ plane and additional non-interfering standing waves in all three spatial directions, $x$, $y$, and $z$. These potentials combine to form one-dimensional superlattices along $x$. An intensity imbalance enables dynamical phase control ($\varphi$) over the interfering (‘long’) lattice with respect to the non-interfering (‘short’) lattice, inspired by the self-oscillating mechanism of ref. [23]. The sliding potential landscape traces an elliptical path of the Rice-Mele parameters $\delta$ and $\Delta$ around the origin. In contrast to previous realisations of the Rice-Mele pump in optical lattices [18, 19], our setup uses a single laser source at $\lambda = 1064\text{nm}$ for all lattice beams, avoiding wavelength-dependent phase shifts in the optical path. The light phase $\varphi$ is actively stabilised using a Michelson interferometer (Methods and Fig. ED2), allowing almost arbitrary control over $\varphi$ in time.

In the absence of interactions ($U = 0$) a cyclic and adiabatic modulation of dimerisation $\delta$ and site offset $\Delta$ around the origin of the $\Delta$–$\delta$ plane leads to a drift of polarisation or, equivalently, the Wannier centre. For an insulator or a homogeneously filled band this drift is quantised, realising a Thouless pump [16]. In the experiment, a change of polarisation manifests as a displacement of the centre-of-mass (c.m.) of the atomic cloud [18, 19]. The topological nature of this transport phenomenon can be understood from the perspective of symmetry: the Hamiltonian (1) for $U = 0$ and $\Delta(\tau) = 0$ describes the Su-Schrieffer-Heeger (SSH) model [26], featuring bond-centred inversion symmetry (grey shading in Fig. 1c). The presence of inversion symmetry ensures a topological gap closing between two protected ground states for $\delta < 0$ and $\delta > 0$, each featuring quantised polarisation. Topological charge pumping requires a parameter orbit which adiabatically connects the two ground states while keeping the energy gap open. This trajectory must therefore break inversion symmetry by deviating from $\Delta = 0$ and encircling the origin (Fig. 1c).
The presence of interactions can profoundly affect topological charge pumps and their symmetries, which has sparked significant and ongoing theoretical interest [27–43]. In the strongly repulsive limit of the Rice-Mele model \( (U \gg t, |\Delta|, |\delta|) \) the region of bond-centred inversion symmetry extends across the plane in Fig. 1c which can be seen from a mapping of Eq. 1 to a spin chain with alternating exchange couplings [35]. The topological gap then closes along the horizontal axis of equal dimerisation \( (\delta = 0) \) and precludes adiabaticity along any topological orbit. Consequently, strong interparticle interactions can destroy quantised charge pumping.

The many-body ground state of Eq. 1 passes through a rich phase diagram during one pumping cycle [44]. In particular, we can distinguish the dimerised lattice [26] (zero site offset, SSH) and the ionic Hubbard model [45, 46] (maximal site offset, no dimerisation). In the following, we briefly describe the charge distributions relevant for topological pumping in the different cases, shown in Fig. 2a. On the one hand, the strongly attractive case \( (−U \gg t, |\delta|, |\Delta|) \), left panel in Fig. 2a), as well as the band insulator \( (U \sim 0, \text{middle panel}), \) both allow for an imbalanced charge distribution within a unit cell. Consequently, the many-body polarisation, defined as the expectation value of the position operator \([47, 48]\), can change when ramping from the dimerised to the ionic Hubbard lattice. On the other hand, the strongly repulsive case \( (U \gg t, |\delta|, |\Delta|) \), right panel in Fig. 2a) results in a Mott insulator for which the charge distribution is pinned. An asymmetric charge distribution is prohibited for both lattice configurations, precluding a shift of many-body polarisation during pumping.

In a first experiment, we track the many-body polarisation in the free-fermion case \( (U = 0) \) for half a pump cycle and fit a gaussian to the in-situ image of the atomic cloud. The centre-of-mass (c.m.) position is plotted in units of a unit cell \( d \) in Fig. 2b, following the expected behaviour for quantised pumping. Residual discrepancies can result from finite temperature, leading to spurious population in the second band, inadmissibility, as well as the non-uniform quasimomentum distribution in the ground band due to harmonic confinement. The experimental data is compared to a numerical simulation of the many-body ground state for half-filling (lines in Fig. 2b), performed with a density matrix renormalisation group (DMRG) algorithm on 64 lattice sites (Methods). The experimental data and numerical simulation agree well, suggesting that the temperature in the lattice is sufficiently low to neglect population in the excited band. In addition, we support our microscopic picture of pumping in the presence of interactions (Fig. 2a) with DMRG simulations in the limiting cases of strongly attractive \( (U/\Delta_0 = −3.0, \text{red dashed line}) \) and strongly repulsive interactions \( (U/\Delta_0 = +3.0, \text{blue dashed-dotted line}) \). The numerics for the attractive case predict the same overall drift of many-body polarisation as in the noninteracting system. In contrast, the calculations on the repulsive side suggest a complete breakdown of pumping, i.e. no c.m. displacement, in agreement with ref. [35]. While symmetry arguments (Fig. 1c) account for the many-body gap closing and explain a deviation from unit efficiency, they do not predict the stand-still of polarisation. Instead, the complete breakdown can be understood as the freezing of charges in the Mott regime (Fig. 2a).

To corroborate our understanding of the behaviour of pumping in the presence of strong Hubbard interactions, we measure the double occupancy fraction in our system during half a pumping cycle for \( U/\Delta_0 \in \{-3.0, 0, 3.0\} \) (Fig. 2c). To this end, we load the atoms into the optical lattice at fixed scattering length, corresponding to the targeted \( U \) during pumping. For different values of Hubbard \( U \), the ground states in the lattice feature different initial double occupancy fractions at \( \tau = 0 \) prior to pumping. For \( U = 0 \) and \( U/\Delta_0 = −3.0 \) (grey points and red squares, respectively) the double occupancy varies significantly over one half-cycle of the pump. It reaches a maximum for \( \tau = 0.25T \), corresponding to the ionic Hubbard point, at which the atoms are localised to the deeper wells. This observation suggests that topological pumping is possible, since two atoms can simultaneously populate the same site in a unit cell. The suppression of double occupancy modulation for \( U/\Delta_0 = +3.0 \) (blue diamonds) indicates that the charge distribution is essentially pinned for large repulsive \( U \). In general, the double occupancy measurements are in good qualitative agreement with DMRG simulations (Extended data Fig. ED3). The differing absolute values of double occupancy are a result of averaging over different fillings in the trap.

We now turn to the measurement of pumping efficiency as function of Hubbard \( U \). The efficiency is extracted by fitting a line to c.m. data such as Fig. 2b, measured for a fixed period of \( T = 8 \hbar/\Delta_0 = 25\hbar/t \) (i.e. 25 tunnelling times). Unit efficiency corresponds to quantised pumping, expected for a perfect band insulator in the adiabatic limit. In the \( U = 0 \) case and the weakly interacting limit \( (|U| \lesssim |\Delta|) \), we observe near-quantised efficiencies above \( 0.9 \times d/T, \) consistent with half-filling in our model (1). Small deviations from unity are expected, since we work at non-zero temperature, finite system size (Fig. ED4), and finite pump period, similar to refs. [18, 19].

For larger values of the interaction, both attractive and repulsive, we observe a striking deviation from unit pumping efficiency. The largest absolute values of Hubbard interaction lead to measured efficiencies of \( 0.62(3) \times d/T \) and \( 0.69(3) \times d/T \) for \( U = +3.0 \Delta_0 \) and \( −3.0 \Delta_0 \), respectively. The lowest pumping efficiency for repulsive interactions is observed for values of Hubbard \( U \) that are larger than \( 2 \Delta_0 \), corresponding to the single-particle band gap at the ionic Hubbard point. The breakdown of topological pumping on the repulsive side is con-
FIG. 3. Breakdown of topological pumping in the interacting Rice-Mele model. Measured pumping efficiency for a fixed period of $T = 25\hbar/t$ as function of Hubbard $U$. Attractive interactions result in hard-core bosons that experience a smaller gap due to renormalised pair tunnelling of $2t^2/|U|$, leading to reduced efficiency at finite pumping period (left cartoon). Strong repulsive interactions cause atoms to be pinned to their lattice sites, preventing the displacement of the cloud (right cartoon). Residual pumping efficiency on the repulsive side can be explained by regions of low density surrounding a Mott-insulating core. The gap in the data for repulsive $U$ is due to a change in spin mixture (Methods). Data points correspond to the fitted slopes to the c.m. drift over three pumping cycles averaged over the pumping direction. Error bars correspond to the propagated error estimated from the uncertainty of the linear fit.

FIG. 4. Adiabatic timescale for the topological pump in the presence of attractive and repulsive Hubbard interactions. The measured efficiency is plotted versus pumping period $T$ in units of tunnelling times for the non-interacting ($U = 0$), strongly attractive ($U/\Delta_0 = -3.0$), and strongly repulsive ($U/\Delta_0 = 3.0$) system. Each data point is the mean and error of the fitted slope to the cloud’s c.m. drift from at least 50 individual measurements within $\tau \in [0, 76] \times \hbar/t$ (1.5 to 8 pumping cycles). The longest periods, which are influenced by technical noise, are excluded from the exponential fit (shaded region).

lattice, affecting quantised transport.

The different mechanisms for breaking quantisation on the attractive and the repulsive sides are highlighted by measuring pumping efficiency versus the period $T$ for three values of Hubbard $U$. We plot the results in Fig. 4 (points), together with a heuristic exponential fit (lines). First, the noninteracting case (grey points) shows a departure from the adiabatic limit for rapid pumping, as previously observed in ref. [19]. The fitted $1/e$ time constant of $2.4(1) t^2/h$ is on the order of the inverse gap, confirming that we can reach the deeply adiabatic regime for a near-insulating state of ultracold fermions. Second, we observe the same adiabatic timescale of $2.4(1)$ tunnelling times for strongly repulsive atoms ($U/\Delta_0 = 3.0$, blue diamonds) while the saturation value of the efficiency for long periods is $0.644(3) d/T$, significantly lower than in the $U = 0$ case. This behaviour is consistent with a fundamental breakdown of pumping, independent of $T$, due to a change in protecting symmetry and a many-body gap closing in the Mott core. The residual pumping efficiency of 0.64 unit cells per period is attributed to the outer (less than half-filled) regions of the trap where the effects of interactions are reduced. On the attractive side (red squares), the observed adiabiatic timescale is measured to be $8(2) t^2/h$, a factor of three slower than the repulsive and noninteracting cases, which is consistent with the numerical calculation in Fig. ED6. This confirms our hypothesis that the deviation from quantised pumping on the attractive side is not due to a fundamental breakdown, but rather a man-
ifestation of nonadiabtic processes in the effective pair pumping regime [38], resulting from a reduced energy gap on the order of $2t^2/|U|$ (see also Fig. ED5). In principle, the pumping efficiency for attractive $U$ should saturate to unity for even longer pumping periods. However, residual phase noise in the lattice limits us to probing periods shorter than approximately 40 tunnelling times, indicated by the drop in efficiency at $T = 50\,\hbar/t$ for all measured values of $U$ (shaded region in Fig. 4). The exponential fit (red dashed line) suggests that in the absence of technical noise the efficiency for attractive $U$ would approach higher values, compared to the repulsive case. We attribute the residual discrepancy to unit efficiency to imperfect loading due to a reduced many-body case. In conclusion, we have experimentally observed the breakdown of topological Thouless pumping in the Fermi-Hubbard regime. Compared to recent results [14], where an optical nonlinearity localises the mean-field wavefunction, the breakdown in our experiment relies on the many-body nature of the fermionic ensemble. Our measurements not only set the stage for interaction-induced $[39, 40]$ or fractional pumping $[27, 30, 31, 37, 49]$ of charge and spin $[35, 50]$, but also enable the investigation of the ‘spontaneously dimerised phase’ (predicted for Eq. 1 around $U = 2.5\,\Delta_0$ $[35, 36, 44]$) and its topological properties. Finally, the dimensionality of our system can be increased by lowering the transverse confinement, enabling studies of interaction effects on two-dimensional topology $[51]$.

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Author contributions

The data was measured and analysed by A.-S.W., Z.Z., M.G., J.M., and K.V. A.-S.W. and Z.Z. performed the numerical calculations. K.V. and T.E. supervised the work. All authors contributed to planning the experiment, discussions and the preparation of the manuscript.

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FIG. ED1. Optical lattice setup and Rice-Mele parameters. a, schematic of the optical setup in the x–z plane. The λ/4 waveplate produces an intensity imbalance between the different contributing beams in such a way that the phase ϕ can be used to move the chequerboard over the square lattice, realising pumping. b, idealised cut through the lattice potential in x-direction, corresponding to the two points along the pump cycle shown in d. The site-offset case (2) is exaggerated for clarity. c, tight-binding parameters during one pump cycle. The phase ϕ (defined in Eq. M1) is ramped from 0 to 2π. In the main text the time dependence of the average tunnelling t has been dropped for clarity. The trajectories for Δ, δ, and t are fitted by sinusoids (solid lines) and the fit parameters are summarised in table I. d, elliptical trajectory of δ and Δ over one pump cycle. The solid line corresponds to the fitted curves in c. Single-particle band gap (e) and bandwidth (f) in units of Δ₀ over a pump half-cycle (the second half is symmetric).
FIG. ED2. **Schematic of the phase control.** The back-reflected lattice beam forms a Michelson interferometer together with the reference path before the optical fibre. A linear increase in the interference phase can be realized either by linearly ramping the set-point (a) or by using a square waveform as input for the frequency modulation, which will also result in a linear phase ramp (b).
FIG. ED3. **Numerical simulation of double occupancy fraction evolution over half a pumping cycle.** The numerical simulations assume $L = 64$ lattice sites and half-filling. Compared to the experimental data (Fig. 2c), the absolute value of double occupancy is different, resulting from various fillings in the harmonic trap. Except for the absolute value, the measured data and the numerical simulation, including the small dip for $U = 3\Delta_0$, agree very well.
FIG. ED4. Effect of finite system size and finite pump period. a, pumping efficiency vs. Hubbard $U$ at different system size $L$ in unit of lattice site, calculated with DMRG. b, pumping efficiency vs. Hubbard $U$ at different pump period on 64 lattice sites, simulated with TEBD. Both calculations assume half-filling in OBC. All curves converge around $U = 2.5\Delta_0$, where the so-called ’spontaneously dimerised phase’ is predicted to occur [44]. Pumping through this parameter region could lead to interesting transport phenomena.
FIG. ED5. Internal gap as a function of Hubbard $U$.
The internal gap is calculated with exact diagonalisation in a system with $L = 8$ at half filling and PBC. The solid and dashed curves correspond to the internal gap at $\tau = 0$ (dimerised) and $\tau = 0.25T$ (ionic Hubbard), respectively.
FIG. ED6. Numerical simulation of adiabatic timescale for the topological pump at different Hubbard $U$. The numerical simulations assume $L = 64$ lattice sites and half-filling in OBC. The solid and dashed line are exponential fits for $U = 0$ and $U/\Delta_0 = -3.0$. The fitted $1/e$ time constant is $1.23(2) \times h/t$ and $7.2(3) \times h/t$, respectively.
METHODS

Experimental sequence

We start by evaporatively pre-cooling a cloud of fermionic potassium $^{40}$K in the magnetic state $F = 9/2$, $m_F = -9/2$ and confining it to a crossed dipole trap. To gain access to the interacting regime, we create a spin mixture of either $m_F = \{-9/2,-7/2\}$ for strongly attractive, non-interacting and weakly repulsive interactions ($U/\Delta_0 < 2$) or $m_F = \{-9/2,-5/2\}$ for strongly repulsive systems ($U/\Delta_0 > 2$). Afterwards the spin mixture is further evaporatively cooled, yielding $47\,000 (4'000)$ atoms at a temperature of $0.11(3)T/T_F$. Values in brackets correspond to the standard deviation over all measured data points; the atom number is calibrated within a systematic error of 10%. The atoms are then loaded within 200 ms into the lowest band of a three-dimensional optical lattice at the scattering length yielding the targeted Hubbard $U$ in Eq. 1. The resulting trapping frequencies are $91.6(1.7)$, $81.8(1.2)$, $120.0(1.3)$Hz in the $x$, $y$, and $z$-directions.

After pumping the system for varying times, we either measure the in-situ position of our cloud or detect the double occupancy fraction. The in-situ image is taken directly after after the ramp of the phase in the presence of the dipole trap, optical lattice and homogeneous magnetic field. For the double-occupancy fraction we first freeze the dynamics of the atoms by quenching into a deep cubic lattice within 100 $\mu$s. We then sweep the magnetic field over the $-7/2$, $-9/2$ Feshbach resonance and spectroscopically resolve the interaction shift with radio-frequency radiation, transferring atoms in the $-7/2(-5/2)$ state in doubly occupied sites to the $-5/2(-7/2)$ state. The Zeeman sublevels are then separated by applying a magnetic field gradient and 8 ms time-of-flight [52].

Optical lattice

The lattice is made up of four retro-reflected beams at a wavelength of 1064 nm. The non-interfering beams in $x$, $y$, and $z$-direction create a cubic lattice to which the interfering beams in the $x$-$z$ plane superimpose a chequerboard lattice. The resulting potential as seen by the atoms is given by

$$V(x, y, z) = -V_X I_{self} \cos^2(kx + \vartheta/2)$$

$$- V_{Xint} I_{self} \cos^2(kx)$$

$$- V_y \cos^2(ky)$$

$$- V_z \cos^2(kz)$$

$$- \sqrt{V_{Xint} V_Z} \cos(kz) \cos(kx + \varphi)$$

$$- I_{XZ} \sqrt{V_{Xint} V_Z} \cos(kz) \cos(kx - \varphi),$$

where $k = 2\pi/\lambda$. The lattice depths $[V_X, V_{Xint}, V_Y, V_Z]$ used in this paper are given by $[5.40(5), 0.09(2), 15.02(6), 17.04(8)]E_R$, measured in units of recoil energy $E_R = \hbar^2/2m\lambda^2$, where $m$ the mass of the atoms. The phase $\varphi$, which is the relative phase between the incoming lattice beams in $x$- and $z$-direction, governs the depth and the relative position of the chequerboard with respect to the square lattice. The angle $\vartheta$, defining the relative position between the one-dimensional sinusoidal lattice formed by $V_X$ and that formed by $V_{Xint}$, is controlled by the difference in light frequency of the two beams. We calibrate $\vartheta$ to $1.000(2)\pi$ by minimising the double occupancy during splitting of a chequerboard into a dimerised lattice at $U = 0$. The imbalance factors $I_{self}$ and $I_{XZ}$ are due to the $\lambda/4$ waveplate in the retro-path (Fig. 1 and Fig. ED1). The factor $I_{XZ}$ plays a crucial role in our pumping scheme which is based on sliding a varying chequerboard lattice over a square lattice. The sliding is achieved by ramping the relative phase $\varphi$ which is stabilized using a locking scheme, detailed in the next section. Without the imbalance (i.e. $I_{XZ} = 1$), as was the case in our previous work [52], the phase $\varphi$ would enter as an overall amplitude $\cos(\varphi)$. However, in case of $I_{XZ} < 1$ the interference terms proportional to $\sqrt{V_{Xint} V_Z}$ in Eq. M1 acquire a $\varphi$-dependent position, explaining the ability to slide the chequerboard using $\varphi$. We rotated the $\lambda/4$ waveplate such that the incoming, linearly polarised light is rotated by $26^\circ$ after passing the plate twice. This results in imbalance factors of $I_{self} = 0.98(2)$ and $I_{XZ} = 0.81(2)$, which are independently calibrated using lattice modulation spectroscopy.

The Rice-Mele parameters in Eq. 1 are calculated via the basis of maximally localised Wannier states, spanning the space of solutions to the single particle Hamiltonian M1. Overlap integrals between these Wannier states yield the relevant tight-binding tunneling elements, on-site energies, as well as interactions $U$. The values of $\Delta$, $\delta$, and $f$ are plotted in Fig. ED1c as function of $\varphi \in [0,2\pi]$. Sinusoidal fits to this data simplify the theoretical description; the resulting fit parameters are listed in Table I. Due to the strong confinement along $y$ and $z$ the tunnelings along those directions $t_{Y,Z}$ are below 30 Hz over the whole pump cycle. The onsite interaction $U$ is given by $980$Hz for a reference scattering length of 100
Bohr radii, which varies by about 1% over the pump cycle, and the interaction between neighbouring sites is always below 15 Hz.

### Phase lock

Topological pumping is realised by shifting the interference phase $\varphi$ in time. The scheme for controlling the phase $\varphi$ is illustrated in Fig. ED2, taking the $x$-direction as an example. The setup is replicated on the $z$-axis, which is not shown in Fig. ED2 for clarity. Active stabilisation of the light phase is necessary since the optical fibre introduces significant phase noise. In short, the back-reflection from the optical lattice forms a Michelson-interferometer together with a reference beam, which does not pass through an optical fibre. In this manner, the absolute phase of the lattice can be measured, assuming a perfectly stable reference arm. We shift the phase of the lattice beam by using the frequency modulation input (‘FM in’) of a Rohde & Schwarz (‘RS’) function generator (SMC100A) creating the RF-frequency for the acousto-optic modulator (AOM). A small frequency shift will result in a phase shift of the laser beam at the position of the atoms (red cloud in Fig. ED2). The set-point of the phase can now be varied in two different ways: For long pumping periods (longer than 5 ms) an Arbitrary- Waveform-Generator (AWG) (Keysight 33500B) generates a sawtooth signal as the set-point of the phase lock, which results in a linear phase ramp. For short pumping cycles (less than 10 ms) the bandwidth of the phase lock is not large enough to follow the set-point. In this case the AWG creates a square signal, which is added to the feedback signal from the phase lock before the frequency modulation input using a power splitter. The square waveform after integration also results in a linear phase shift of the lattice beam. For example, a frequency shift of 400 Hz on the RF signal of the AOM leads to a pumping slope of $\Delta \varphi = 2\pi/5$ ms.

The data in Fig. 4 was measured with either of the two techniques, including two data points ($13h/t$ and $25h/t$) that were measured with and averaged over both methods. These overlapping points agreed with the averaged ones within error bars.

### Numerical simulation

**Exact diagonalisation.** We numerically determine the many-body gap of the model described by Eq. 1 using the QuSpin python package [53] based on exact diagonalization. We assume a system of $L = 8$ sites at half filling ($N_{\uparrow} = N_{\downarrow} = 4$, i.e. $\hat{S}_z = 0$) and periodic boundary conditions (PBC). The internal gap is defined as the energy difference between ground state and the first excited state. It changes over the course of a pumping trajectory, as shown in Fig. ED5 at $\tau = 0$ (SSH model) and $\tau/T = 0.25$ (ionic Hubbard model) for varying interactions $U$.

**DMRG and TEBD calculations.** Numerical results of pumping displacement and double occupancy dynamics presented in the main text are calculated with density matrix renormalisation group (DMRG) and time evolving block decimation (TEBD) methods using the TeNPy python package [54]. The polarization and double-occupancy dynamics shown in Fig. 2b and Fig. 2c are calculated with DMRG using open boundary conditions (OBC), where we assume $L = 64$ and half filling. The polarisation, i.e., the center of mass of the ground state $\langle \Psi(t) \rangle$ is defined by

$$P_{\text{open}}(t) = \frac{1}{L} \sum_{j=0}^{L-1} \langle \Psi(t) | (j - j_0) \hat{n}_{j\sigma} | \Psi(t) \rangle,$$

and the double occupancy fraction $D$ is defined as the fraction of atoms on doubly occupied lattice sites

$$D = \frac{2}{N} \sum_{j} \langle \hat{n}_{j\uparrow} \hat{n}_{j\downarrow} \rangle,$$

where $N$ is the total atom number. Since DMRG cannot faithfully predict the time evolution in the presence of a gap closing, we compare the DMRG results to TEBD algorithm and find agreement for long enough pump periods.

The numerical result of pumping efficiency vs. Hubbard $U$ at different system size in Fig. ED4a is computed with DMRG (OBC) assuming half filling, where we use the symmetry of the pumping trajectory and only calculate the displacement in the first quarter cycle. The result of efficiency vs. Hubbard $U$ in Fig. ED4b is calculated with TEBD for half a cycle, starting from an initial state calculated with DMRG given the tight-binding parameters at $\tau = 0$. The results in Fig. ED6 are calculated in the same way. Open boundary conditions with $L = 64$ and half filling are assumed in both.

| parameter | offset | ampl. | freq. | phase offset |
|-----------|--------|-------|-------|--------------|
| $B$ [Hz]  | $t$    | $A$ [Hz] | $\nu$ | $\kappa$    |
| $\Delta$  | 403    | 118   | 2     | $\pi/2$     |
| $\delta$  | 0      | 847   | 1     | $\pi$       |

TABLE I. Rice-Mele parameters for the fitted sines in Fig. ED1. The parameters correspond to the expression $B + A \sin(2\pi \nu \tau / T + \kappa)$, where $\tau$ is time and $T$ is the pump period.