Transformation between Coleman-Weinberg and MS schemes and phenomenological implications

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The Coleman-Weinberg (CW) renormalization scheme for the effective potential is particularly valuable for CW symmetry-breaking mechanisms, which in general more easily lead to the strong first-order phase transition necessary for stochastic gravitational wave signals. A full understanding of the CW-MS scheme transformation thus becomes important in the era of gravitational wave detection and precision coupling measurements. A generalized Coleman-Weinberg (GCW) renormalization scheme is formulated and methods for transforming scalar self-couplings between the GCW and MS (minimal-subtraction) renormalization schemes are developed. Scalar $\lambda \Phi^4$ theory with global $O(4)$ symmetry is explicitly studied up to six-loop order to explore the magnitude of this scheme transformation effect. The dynamical rescaling of renormalization scales between the GCW and MS schemes can lead to significant (order of 10%) differences in the coupling at any order, and consequently GCW-MS scheme transformation effects must be considered within precision measurements of scalar couplings. Preprint: CP³-Origins-2020-03 DNRF90

The era of gravitational wave detection and precision coupling measurements represents promising opportunities for observing new physics beyond the Standard model. Detection of stochastic gravitational wave signals typically require a strong first-order phase transition \cite{1} (see also \cite{2}). Models with Coleman-Weinberg (CW) symmetry breaking, typically in the CW renormalization scheme, more easily lead to this necessary strong first-order phase transition \cite{3}. Thus, CW symmetry-breaking is a crucial element in the study of stochastic gravitational waves.

On the other hand, Higgs cubic and quartic coupling measurements are extremely important in exploring the underlying mechanism of electroweak symmetry breaking (see e.g., typical Coleman-Weinberg type cases \cite{4-11} and implications of various forms of the effective potential \cite{12}) and the nature of the electroweak phase transition \cite{13}. The targeted sensitivity in future collider experiments should have sufficient accuracy to distinguish the conventional SM with most of the beyond-SM new physics (see e.g., Refs. \cite{14, 15}). Thus it is important to understand how the CW renormalization scheme can be related to MS (minimal subtraction) scheme observables and couplings, and assess the magnitude of these effects.

In the CW symmetry breaking mechanism, the associated CW renormalization scheme \cite{16, 17} provides a valuable framework for the effective potential. In particular, advantages of this renormalization scheme include simple renormalization-group (RG) improvement properties \cite{4, 18, 19}, the absence of kinetic term corrections in the effective action, and the ability to uniquely specify the effective potential from the RG functions \cite{20}. Interesting phenomenological models involving CW effective potentials include (conformal) two-Higgs doublet model \cite{4, 19}, and various hidden-sector models such as the real/complex singlet model \cite{10, 21} and the $U(1)'$ model \cite{22}. However, the CW renormalization scheme affects the RG functions of the theory \cite{23} and hence there are scheme-transformation effects on the CW-scheme couplings that must be considered when comparing to MS benchmark values of the couplings.

In this paper we formulate a generalization of the CW renormalization scheme that provides greater flexibility for model building and develop methods for transformation of scalar couplings between the generalized-Coleman-Weinberg (GCW) and MS schemes. Numerical effects of the scheme transformations are studied in detail for $\lambda \Phi^4$ theory with global $O(4)$ symmetry (the scalar sector of the SM) up to six-loop order in the MS-scheme RG functions \cite{24, 25}. The availability of RG functions to this high-loop order enables a systematic study of loop effects in the scheme transformation. We find that the numerical effects of scheme transformation can be significant (order of 10%) within the available parameter space, and hence for accurate phenomenology it is important to account for scheme transformation effects in models that employ the CW renormalization scheme.

I. GENERALIZED COLEMAN-WEINBERG RENORMALIZATION SCHEME

In its original form, the Coleman-Weinberg (CW) renormalization scheme is defined from the following condition for the effective potential for $O(N)$ globally-symmetric scalar field theory \cite{16, 17}

$$
\frac{d^4 V_{\text{eff}}}{d\Phi^4} \bigg|_{\mu^2 = \phi^2} = 24 \lambda, \quad \phi^2 = \sum_{i=1}^{N} \phi_i \phi_i, \quad (1)
$$
where $\mu$ is the CW renormalization scale and condition (1) is chosen to align with the definition of the tree-level Lagrangian. However, as first noted in Ref. [23], the CW renormalization scale $\mu$ can be related to the minimal-subtraction (MS) renormalization scale $\tilde{\mu}$ via

$$\lambda(\tilde{\mu})/\tilde{\mu}^2 = 1/\mu^2,$$

where $\lambda(\tilde{\mu})$ is the MS-scheme running coupling. Eq. (2) allows conversion between MS and CW renormalization schemes. It is evident that (2) will map typical MS-scheme effective potential logarithms $\log(\lambda\Phi^2/\mu^2)$ into CW-scheme effective potential logarithms $\log(\Phi^2/\mu^2)$. However, (2) also implies that the RG functions in the CW and MS schemes will be different [20, 23]:

$$\mu \frac{d\lambda}{d\mu} = \beta(\lambda) = \frac{\tilde{\beta}(\lambda)}{1 - \tilde{\beta}(\lambda)}, \mu \frac{d\lambda}{d\tilde{\mu}} = \tilde{\beta}(\lambda),$$

where $\tilde{\beta}$ denotes the MS-scheme beta function and $\beta$ denotes the CW-scheme. A perturbative expansion of (3) provides the relation between the coefficients of the RG functions in the MS and CW schemes

$$\tilde{\beta}(\lambda) = \sum_{k=2}^{\infty} b_k \lambda^k, \beta(\lambda) = \sum_{k=2}^{\infty} b_k \lambda^k,$$

with the first few terms given by [20]

$$b_2 = \tilde{b}_2, b_3 = \tilde{b}_3 + \frac{1}{2} \tilde{b}_2^2, b_4 = \tilde{b}_4 + \tilde{b}_2 \tilde{b}_3 + \frac{1}{4} \tilde{b}_2^3.$$  (5)

Relations similar to (3) and (5) exist for the anomalous field dimension RG function [20]. It can be verified that the two-loop CW-scheme effective potential [17] satisfies the RG equation containing the CW-scheme RG functions [20].

The CW-MS scheme transformation expression (2) represents a dynamical rescaling between the renormalization scale in the two schemes governed by the MS running coupling. Given an MS scale $\tilde{\mu}$, the corresponding CW-scale $\mu$ can be calculated via (2) and the corresponding CW coupling is then given by $\lambda(\mu)$. In principle the process can be inverted: given a CW-scale $\mu$ (2) can be solved for $\tilde{\mu}$ and then the corresponding MS coupling is given by $\lambda(\tilde{\mu})$. The couplings in the two schemes match at a special scale $\mu_*$ where

$$\lambda(\mu_*) = 1, \mu = \tilde{\mu} = \mu_* \rightarrow \lambda(\mu) = \lambda(\tilde{\mu}) = 1.$$  (6)

Since the MS-scheme scalar couplings increase with increasing energy scale in the perturbative regime $\lambda(\tilde{\mu}) < 1$, Eq. (2) implies that for $\mu < \mu_*$, $\mu > \tilde{\mu}$ and therefore the CW-scheme coupling is naturally enhanced compared to the MS-scheme coupling. It is also clear that the difference between the scales could be significant when $\lambda(\tilde{\mu}) \ll 1$, corresponding to a large dynamical rescaling between $\mu$ and $\tilde{\mu}$.

The alignment of the two schemes at the scale $\mu_*$ represented by (6) suggests a natural extension of the CW scheme (2)

$$\lambda(\tilde{\mu})/\tilde{\mu}^2 = \lambda_0/\mu^2,$$  (7)

defining the generalized Coleman-Weinberg (GCW) scheme transformation. This generalization does not alter the relationship between the beta functions in the two schemes given in (3) and will still map MS-scheme effective potential logarithms into CW-scheme logarithms. The parameter $\lambda_0$ then characterizes the matching between the two schemes:

$$\lambda(\mu_*) = \lambda_0, \mu = \tilde{\mu} = \mu_* \rightarrow \lambda(\mu) = \lambda(\tilde{\mu}) = \lambda_0,$$  (8)

providing greater flexibility for model building where it may be desirable to match the CW and MS schemes at a different scale than would emerge from (6). Similar to the case where $\lambda_0 = 1$, for scalar couplings that increase with increasing energy scale, there will be a natural enhancement of the CW coupling compared to the MS coupling for $\mu < \mu_*$ and a natural suppression for $\mu > \mu_*$. There are a variety of possibilities for fixing the model-dependent parameter $\lambda_0$. For example, Eq. (3) implies that the MS and GCW schemes will share the same fixed points (both UV and IR) that could define $\lambda_0$. Similarly, the anomalous dimension will have the same zeroes in the two schemes thereby providing a value for $\lambda_0$. Because the GCW scheme transformation maps MS to the CW forms of the effective potentials, $\lambda_0$ can be constrained by matching the effective potentials at a particular loop order. A UV boundary condition on $\lambda_0$ could also emerge from a UV completion (e.g., asymptotic safety [26]). Finally, $\lambda_0$ could be determined through a method such as principle of minimal sensitivity for a particular observable [27].

Dimensionless CW and MS scales associated with the GCW matching condition (8) are defined by

$$\xi = \mu/\mu_*, \tilde{\xi} = \tilde{\mu}/\mu_*, \lambda(\xi = 1) = \lambda(\tilde{\xi} = 1) = \lambda_0,$$  (9)

providing a boundary condition for the RG equations (3) expressed in terms of the dimensionless scales

$$\xi \frac{d\lambda}{d\xi} = \tilde{\beta}(\lambda), \lambda(\xi = 1) = \lambda_0$$  (10)

$$\tilde{\xi} \frac{d\lambda}{d\tilde{\xi}} = \tilde{\beta}(\lambda), \lambda(\tilde{\xi} = 1) = \lambda_0.$$  (11)

Similarly, the GCW scheme transformation (7) expressed in terms of dimensionless scales is

$$\lambda(\xi)/\xi^2 = \lambda_0/\xi^2.$$  (12)

In the analysis below, the solution to the MS-scheme RG equation (10) will be denoted by $\lambda_{\text{MSRG}}(\xi, \lambda_0)$ and the solution to the CW-scheme RG equation will be denoted...
by \( \lambda_{\text{CWRG}}(\xi, \lambda_0) \). Thus the GCW scheme transformation is founded on the MS-scheme running coupling through the dynamical scale transformation

\[
\xi = c(\tilde{\xi}) = \frac{\lambda_0}{\sqrt{\lambda_{\text{MSRG}}(\xi, \lambda_0)}}. \quad (13)
\]

II. SCHEME CONVERSION OF COUPLINGS

For a given value of the MS scale \( \tilde{\xi} \), the corresponding GCW-scheme coupling is obtained by calculating the corresponding CW-scale \( \xi \) (13), and the resulting GCW coupling \( \lambda_{\text{CW}} \) is then given by

\[
\lambda_{\text{CW}}(\tilde{\xi}, \lambda_0) = \lambda_{\text{MSRG}}(\xi, \lambda_0).
\]

The inverse of this expression has an interesting symmetric form. The MS coupling \( \lambda_{\text{MS}} \) corresponding to a CW scale \( \xi \) is given by the \( \tilde{x} \) solution of

\[
\lambda_{\text{MS}}(\xi) = \lambda_{\text{MSRG}}(\tilde{x}, \lambda_0), \quad (15)
\]

\[
\lambda_{\text{CW}}(\tilde{x}) = \lambda_{\text{CWRG}}(\xi, \lambda_0). \quad (16)
\]

However, (7) and (14) can be used to re-express (16) as

\[
\lambda_{\text{MSRG}}(c(\tilde{x}), \lambda_0) = \lambda_{\text{CW}}(\tilde{x}) = \lambda_{\text{CWRG}}(\xi, \lambda_0) = \lambda_{\text{MSRG}}(c^{-1}(\xi), \lambda_0) \rightarrow c(x) = c^{-1}(\xi)
\]

where \( c^{-1}(\xi) \) is understood as the \( \xi \) root of (13) associated with the scale \( \xi \). Using (17) and (7) leads to the final result for \( \lambda_{\text{MS}} \)

\[
\lambda_{\text{MS}}(\xi, \lambda_0) = \lambda_{\text{MSRG}}(\tilde{x}, \lambda_0) = \lambda_{\text{CWRG}}(c(\tilde{x}), \lambda_0) = \lambda_{\text{CWRG}}(c^{-1}(\xi), \lambda_0), \quad (18)
\]

which has a form symmetric to (14). Thus, Eq. (16) performs the mapping of the MS to GCW coupling and Eq. (18) performs the opposite.

In principle, the transformation of couplings between the GCW and MS schemes can be performed via a numerical solution for the MS-coupling within (14) in cases of higher-loop beta functions where an analytic solution does not exist. However, this purely numerical approach has some disadvantages. First, it can be difficult to sample extreme ranges of the coupling parameter space. For small couplings, (12) leads to large dynamical hierarchies in \( \xi \) and \( \xi \), and hence a small change in \( \xi \) leads to a large change in \( \xi \). For large \( \lambda \) the perturbative series may begin to have poor convergence and because it is necessary to do a numerical solution from the boundary condition \( \lambda_0 \), the coupling could enter a non-perturbative regime. Thus it is necessary to go beyond purely numerical solutions and develop other methodologies for performing GCW-MS scheme transformation.

The one-loop analysis for \( \lambda \phi^4 \) theory with \( O(4) \) global symmetry provides a phenomenologically relevant exactly solvable case that illustrates the main features of the GCW-MS scheme transformation and motivates the methodology that will be developed for the higher-loop cases. Using the one-loop \( O(4) \) beta function \( [24] \), the solution to (10) is

\[
\lambda_{\text{MSRG}}(\tilde{\xi}, \lambda_0) = \frac{\lambda_0}{1 - \tilde{b}_2 \lambda_0 \log \tilde{\xi}}, \quad \tilde{b}_2 = \frac{6}{\pi^2}, \quad (19)
\]

which combined with (14) leads to the relationship between \( \lambda_{\text{CW}} \) and \( \lambda_{\text{MSRG}} \) shown in Figure 1 for different choices of \( \lambda_0 \). The curves intersect at \( \xi = 1 \) as required by the boundary condition (9), and as discussed above, the GCW-MS scheme transformation leads to a natural enhancement of the CW coupling below \( \xi < 1 \) and a suppression for \( \xi > 1 \). Depending on \( \lambda_0 \), the enhancement can be numerically significant and it is clear that a naive assumption that the CW and MS coupling are identical could introduce an error of up to order of 10%. Thus depending on the desired phenomenological precision of the couplings, a careful consideration of GCW-MS scheme transformation effects may be needed.

![FIG. 1. The ratio of \( \lambda_{\text{CW}} \) and \( \lambda_{\text{MSRG}} \) is shown at one-loop order in \( O(4) \) \( \lambda \phi^4 \) as a function of the dimensionless scale \( \tilde{\xi} \).](image-url)

Figure 2 shows the underlying relation (13) between the GCW scale \( \xi \) and MS scale \( \tilde{\xi} \) for different choices of \( \lambda_0 \). As discussed above, \( \xi \) is enhanced compared to \( \tilde{\xi} \) leading to the natural enhancement of \( \lambda_{\text{CW}} \). However, although \( \xi > \tilde{\xi} \), it is apparent that \( \xi < 1 \) for \( \tilde{\xi} < 1 \) so that it is not necessary to evolve the MS couplings above \( \lambda_0 \), providing some control over perturbative convergence. The non-linear dynamical rescaling between \( \xi \) and \( \xi \) in Figure 2 illustrates why the GCW-MS scheme transformation can become significant.

Figure 2 illustrates one of the challenges of the direct application of the GCW scheme transformation (14). For \( \xi \ll 1 \), a small variation in \( \xi \) leads to a large change in \( \xi \), thus making it difficult to sample a full range of coupling parameter space. However, the one-loop case provides a way forward by solving the MS RG equation (10) to relate the MS-scheme coupling at two scales

\[
\lambda(\xi_1) = \lambda(\tilde{\xi}_2) = \frac{\lambda(\tilde{\xi}_2)}{1 - \tilde{b}_2 \lambda(\tilde{\xi}_2) \log (\xi_1/\xi_2)}, \quad (20)
\]
where the notation has been compressed so that \( \lambda(\tilde{\xi}_1) = \lambda_{\text{MSRG}}(\tilde{\xi}_1, \lambda_0) \) in (20). By choosing \( \tilde{\xi}_1 = \xi = c(\xi) \) and \( \tilde{\xi}_2 = \xi \) related through the GCW scheme transformation (12). Eq. (20) provides a direct relation between \( \lambda_{\text{CW}} \) and \( \lambda_{\text{MSRG}} \):

\[
\lambda_{\text{CW}} = \lambda_{\text{MSRG}} S_0(w), \quad S_0(w) = 1/w, \quad w = 1 - \frac{\tilde{b}_2}{2} \lambda_{\text{MSRG}} \log \left( \frac{\lambda_0}{\lambda_{\text{MSRG}}} \right) \tag{21}
\]

where the functional arguments of \( \lambda_{\text{CW}} \) and \( \lambda_{\text{MSRG}} \) have been suppressed for simplicity. Eq. (21) provides a direct relation between the CW and MS couplings without requiring a solution of the MS RG equation, addressing the challenges of the direct approach outlined above. Furthermore, because we do not to solve (10), \( \lambda_0 \) could be in a non-perturbative regime, and the applicability of (21) is simply constrained by perturbative convergence of higher-loop contributions (e.g., \( \lambda \beta_{n+1}/\beta_n \lesssim 1 \)). Figure 3 shows how the relation (21) between the GCW- and MS-scheme couplings can now be examined across a wider range of coupling parameter space (e.g., compare with Fig. 2) and the scale has been extended into a region of slow perturbative convergence to show the required \( \lambda_{\text{CW}}/\lambda_{\text{MSRG}} = 1 \) intersection of each curve at \( \lambda_{\text{MSRG}} = \lambda_0 \) from the matching condition (9). A natural enhancement of the CW coupling now occurs for \( \lambda_{\text{MSRG}} < \lambda_0 \) and a natural suppression for \( \lambda_{\text{MSRG}} > \lambda_0 \).

If the expression (20) is expanded as a series in \( \lambda \left( \tilde{\xi}_2 \right) \) then it has the form of a leading-logarithm \((LL)\) summation, and hence following Ref. [28] a higher-loop extension of (20) can be found by including sub-leading logarithms in the series solution

\[
\lambda(\tilde{\xi}_1) = \lambda_2 \left[ 1 + T_{1,1} L \lambda_2 + \lambda_2^2 \left( T_{2,1} L + T_{2,2} L^2 \right) + \ldots \right],
\]

\[
L = \log \left( \frac{\tilde{\xi}_1}{\tilde{\xi}_2} \right), \quad \lambda_2 = \lambda \left( \tilde{\xi}_2 \right), \quad \tag{22}
\]

which can be rearranged as sums of \( N^n LL \) terms as

\[
\lambda(\tilde{\xi}_1) = \sum_{n=0}^{\infty} \lambda_2^{n+1} S_n(\lambda_2 L), \quad S_n(u) = \sum_{k=1}^{\infty} T_{n+k,k} u^k.
\]

The notation has again been compressed so that \( \lambda(\tilde{\xi}_1) = \lambda_{\text{MSRG}}(\tilde{\xi}_1, \lambda_0) \) and will be restored later for clarity. The requirement that (23) is independent of the scale \( \tilde{\xi}_2 [28] \)

\[
0 = \frac{d}{d\tilde{\xi}_2} \lambda(\tilde{\xi}_1) = \left( \tilde{\xi}_2 \frac{\partial}{\partial \tilde{\xi}_2} + \tilde{b}(\lambda_2) \frac{\partial}{\partial \lambda_2} \right) \lambda(\tilde{\xi}_1)
\]

provides an RG equation defining the \( S_n \). For example, at \( LL \) order the RG equation for \( S_0 \) is

\[
0 = (1 - \tilde{b}_2 u) \frac{dS_0}{du} - \tilde{b}_2 S_0, \quad S_0(0) = 1
\]

where the boundary condition for \( S_0 \) ensures that (23) is self-consistent when \( \tilde{\xi}_1 = \tilde{\xi}_2 \). The solution to (25) is

\[
S_0(w) = 1/w, \quad w = 1 - \tilde{b}_2 u
\]

and hence

\[
\lambda(\tilde{\xi}_1) = \lambda_2 S_0(\lambda_2 L) = \frac{\lambda_2}{1 - \tilde{b}_2 \lambda_2 L}, \quad \tag{27}
\]

identical to the two-loop result (20). At \( N^n LL \) order the generalization of (25) to \( n \geq 0 \) is

\[
0 = - \left( 1 - \tilde{b}_2 u \right) \frac{dS_n}{du} + (n + 1) \tilde{b}_2 S_n + \sum_{k=0}^{n-1} \tilde{b}_{n+2-k} \left[ (k + 1) S_k + u \frac{dS_k}{du} \right], \quad S_n(0) = 0.
\]

Since the MS beta function is known to six-loop order (i.e., \( b_7 \)) [25] (28) can be iteratively solved up to \( S_5(u) \). The next two solutions are

\[
S_1(w) = - \frac{\tilde{b}_3 \log w}{\tilde{b}_2 w^2}, \quad w = 1 - \tilde{b}_2 u \tag{29}
\]

\[
S_2(w) = \left( \tilde{b}_2 \tilde{b}_4 - \tilde{b}_5^2 \right) \left( 1 - w \right) - \tilde{b}_3^2 \log w + \tilde{b}_4 \log^2 w/\tilde{b}_2 w^3,
\]
Efficient methodologies have been developed to per-
sance of
Effects of scheme transformation can be large (or-
Dynamical rescaling of renormalization scales
theory \[ (\lambda) \]
the MS-scheme coupling at the scales $\tilde{\xi}_1$ and $\tilde{\xi}_2$ is

$$
\lambda^{(n)} \left( \tilde{\xi}_1 \right) = \sum_{k=0}^{n} \lambda_2^{k+1} S_k(w), \ w = 1 - b_2 \lambda \left( \tilde{\xi}_2 \right) \log \left( \tilde{\xi}_1 / \tilde{\xi}_2 \right). 
$$

(30)

The accuracy of the $N^n LL$ approximations (30) can be
can be checked by setting $\tilde{\xi}_2 = 1$, $\lambda_2 = \lambda_0$, and then comparing
against numerical solution of MS-scheme coupling at the
same order. As expected, agreement tends to worsen as
$\lambda_0$ increases because the truncated perturbative expan-
sion will have slower convergence. The resulting limita-
tions on $\lambda_0$ are discussed below.

Figure 4 compares the GCW- and MS-scheme couplings
at successively higher-loop $N^n LL$ orders arising from (31) for different choices of $\lambda_0$. As in the exact
one-loop case, a natural enhancement of the CW coupling
occurs for $\lambda_{MSRG} < \lambda_0$ and a natural suppression for $\lambda_{MSRG} > \lambda_0$. As $\lambda_0$ is decreased, the enhancement
for $\lambda < \lambda_0$ tends to decrease, while the suppression for $\lambda > \lambda_0$ tends to increase. As noted before, the scheme-
transformation effects can lead to significant (order of
10%) differences in the coupling, which could be further
magnified by subsequent large-distance RG running to the
desired phenomenological scale.

An interesting feature of Figure 4 is the persistence of the
relative enhancement (or suppression) as loop order
is increased, emphasizing that the effects of the GCW-
MS scheme transformation are fundamentally related to
the non-linear dynamical rescaling of the renormalization
scales (12) and cannot be avoided by going to higher-loop
order.

The ideas presented in this paper can be extended
to systems with multiple scalar fields and multiple couplings. In cases where the couplings satisfy the flatness
condition, the Gildener-Weinberg method [29, 30] can be
implemented and the single-coupling analysis of this
paper can be applied without modification. Otherwise,
multi-scale RG methods [31–33] are required to generalize
Eq. (7) to introduce an additional renormalization scale and associated parameter $\lambda_0$ for each required coupling.

III. CONCLUSIONS

In this paper, a generalization of the Coleman-
Weinberg (CW) renormalization scheme [16, 17, 23]
has been developed and analyzed for $O(4)$ globally-
symmetric $\lambda \Phi^4$ theory up to six-loop order. The key
messages of our paper are:

- Dynamical rescaling of renormalization scales
leads to the transformation between generalized
Coleman-Weinberg (GCW) and MS schemes
- Effects of scheme transformation can be large (or-
der of 10%) and can therefore have important phe-
nomenological consequences
- Efficient methodologies have been developed to per-
form the GCW-MS scheme transformation, where the key parameter in the GCW-MS scheme conver-
sion is $\lambda_0$ where the couplings in the two schemes
align at a common energy scale $\mu_*$

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