Effect of throttling orifice head loss on dynamic behavior of hydro-turbine governing system with air cushion surge chamber

W C Guo 1,2, J Yang 2,3, J B Yang 4
1 School of Hydropower and Information Engineering, Huazhong University of Science and Technology, Wuhan, 430074, China
2 Division of Hydraulic Engineering, Royal Institute of Technology, Stockholm, SE-100 44, Sweden
3 R&D Laboratories, Vattenfall AB, Alvkarleby, SE-814 26, Sweden
4 State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, Wuhan, 430072, China

Corresponding author: W C Guo, E-mail: wench@whu.edu.cn

Abstract: This paper aims to study the effect of throttling orifice head loss on the dynamic behavior of the hydro-turbine governing system with air cushion surge chamber. Firstly, a nonlinear mathematical model of hydro-turbine governing system considering the nonlinear head loss of throttling orifice is established. Then, the nonlinear dynamic behavior of the hydro-turbine governing system is investigated by using the Hopf bifurcation theory and direct numerical integration. The stability and dynamic response of the system are detailed described. Finally, the effect mechanism of throttling orifice head loss on the dynamic behavior of system is revealed based on the comparison between the cases with nonlinear head loss and no head loss. The results indicate that the mathematical model considering the nonlinear head loss of throttling orifice for the hydro-turbine governing system is a fifth-order nonlinear state equation. The system goes through Hopf bifurcation at the bifurcation points and the type of Hopf bifurcation is supercritical. The domain at the bottom of the bifurcation line is the stable domain. The throttling orifice cannot affect the system stability through its head loss. The throttling orifice can affect the system dynamic response through its head loss. The throttling orifice head loss is favorable for the damping of the water level oscillation in the surge chamber.

1. Introduction

For the hydropower station, a chamber with its top closed and with compressed air between the water surface and the top of the chamber is called an air cushion surge chamber [1,2]. Air cushion surge chamber is an important pressure reduction facility in hydropower station. Compared with open surge tank, air cushion surge chamber is more efficient to decrease the water hammer pressure and improve the operating condition of hydro-turbine unit [3].

The throttling orifice is frequently used in air cushion surge chamber to reduce the surge amplitudes. The presence of the throttling orifice adds a nonlinear term of head loss to the hydro-turbine governing system, and its effect becomes predominant for dynamic behavior of the system. Because of the throttling orifice head loss, the modeling and characteristic analysis of the hydro-turbine governing system become more complicated. About the dynamic behavior of hydro-turbine governing system...
with air cushion surge chamber of throttling orifice, representative achievements are stated as follows: Chaudhry et al. [4] studied the stability of closed surge tanks using the phase plane method, which allows inclusion of nonlinear effects in the analyses. Yang et al. [5] investigated the surge stability of closed surge chamber by the direct method of Liapunov, which introduced the Liapunov function from the energy consideration. Yang et al. [6] studied the stability of air cushion surge tanks with throttling by means of linearization and direct numerical integration, and identified the type of singularities in the phase plane and their stability criteria in case of small oscillations. Vereide et al. [7] presented the design and results from a hydraulic scale model of mass oscillations in a hydropower plant with a closed surge tank constructed as an underground rock cavern. The summary of the above literatures indicates that the previous researches mainly focus on the pipeline system, and the hydro-turbine, generator and governor are always neglected or simplified. So the obtained results are always unilateral and limited. Guo et al. [8] analyzed the stability of waterpower-speed control system for hydropower station with air cushion surge chamber, in which the system contains the hydro-turbine, generator and governor. But the throttling orifice is neglected.

This paper aims to study the effect of throttling orifice head loss on the dynamic behavior of hydro-turbine governing system with air cushion surge chamber. The motivation and innovation are as follows: (1) Establish a complete nonlinear mathematical model of hydro-turbine governing system, which contains the pipeline system, power generation system and throttling orifice. (2) Investigate the stability and dynamic response of the system, and reveal the effect of throttling orifice head loss on dynamic behavior. The paper is organized as follows. In Section 2, for the hydro-turbine governing system with air cushion surge chamber, the nonlinear mathematical model considering the nonlinear head loss of throttling orifice is established. In Section 3, the nonlinear dynamic behavior of the hydro-turbine governing system is investigated by using the Hopf bifurcation theory and direct numerical integration. In Section 4, the effect mechanism of throttling orifice head loss on the dynamic behavior of system is revealed based on the comparison between the cases with nonlinear head loss and no head loss.

2. Mathematical formulation

The pipeline and power generation system of hydropower station with air cushion surge chamber is shown in Figure 1.

![Figure 1. Schematic diagram of pipeline and power generation system of hydropower station with air cushion surge chamber.](image)

The basic equations for all subsystems of the hydro-turbine governing system shown in Figure 1, i.e. headrace tunnel, air cushion surge chamber, penstock, hydro-turbine, generator and governor, are presented as follows. Note that the definition and explanation for the notations and variables in this section are presented in Appendix A.

Dynamic equations of headrace tunnel and penstock:

When the throttling orifice head loss of air cushion surge chamber is not considered, we have the dynamic equations of headrace tunnel and penstock as follows [2,8]:

\[
\text{(equations...)}
\]
The throttling orifice head loss of air cushion surge chamber is denoted as $h_r$. If $h_r$ is considered, the equation (1) and equation (2) can be rewritten as

$$
\frac{d q_H}{dt} = -h_d - \frac{2 h_{l0}}{H_0} q_H - \frac{h_r - h_{l0}}{H_0}
$$

(3)

$$
\frac{d q_P}{dt} = h_d - h - \frac{2 h_{p0}}{H_0} q_P - \frac{h_r - h_{l0}}{H_0}
$$

(4)

The head loss coefficient of throttling orifice is denoted as $\alpha_T$. Then we have

$$
\frac{h_r - h_{l0}}{H_0} = \frac{\alpha_T (Q_p - Q_H)}{H_0}^2
$$

(5)

For $Q_p - Q_H$, we have

$$
Q_p - Q_H = Q_o \left( \frac{Q_p - Q_H}{Q_o} - \frac{Q_p - Q_{p0}}{Q_{p0}} \right) = Q_o \left( q_p - q_H \right).
$$

Then equation (5) can be transferred to the following form:

$$
\frac{h_r - h_{l0}}{H_0} = \frac{\alpha_T Q_o^2}{H_0} \left( q_p - q_H \right)^2
$$

(6)

Substitution of equation (6) into equation (3) and equation (4) gives the dynamic equations of headrace tunnel and penstock considering the throttling orifice head loss of air cushion surge chamber as follows:

$$
\frac{d q_H}{dt} = -h_d - \frac{2 h_{l0}}{H_0} q_H - \frac{\alpha_T Q_o^2}{H_0} \left( q_p - q_H \right)^2
$$

(7)

$$
\frac{d q_P}{dt} = h_d - h - \frac{2 h_{p0}}{H_0} q_P - \frac{\alpha_T Q_o^2}{H_0} \left( q_p - q_H \right)^2
$$

(8)

Continuity equation of air cushion surge chamber [2,8]:

$$
q_H = \frac{1}{F H_0} \frac{d h}{dt} + q_P
$$

(9)

Moment equation and discharge equation of hydro-turbine [9,10]:

$$
m_i = e_{h1} h + e_{x1} x + e_{y1} y
$$

(10)

$$
q_p = e_{h2} h + e_{x2} x + e_{y2} y
$$

(11)

Equation of generator [9,10]:

$$
T_a \frac{dx}{dt} = m_i - (m_g + e_{g1} x)
$$

(12)

Equation of governor [9,10]:

$$
\frac{dy}{dt} = -K_r \frac{dx}{dt} - K_i x
$$

(13)

For the hydro-turbine governing system considering the throttling orifice head loss of air cushion surge chamber, the mathematical model is composed by equation (7) - equation (13). By integrating equation (7) - equation (13), we obtain the following fifth-order state equation for the hydro-turbine governing system.
\[
\begin{align*}
q_H &= \frac{1}{T_{\mu,H}} \left[ -h_i - \frac{2h_{p0}}{H_0} q_H - \frac{\alpha_f Q_0^2}{H_0} (q_p - q_H)^2 \right] \\
\dot{q}_p &= \frac{1}{T_{\mu,P}} \left[ h_j - \frac{2h_{p0}}{H_0} + \frac{1}{e_p} q_p - \frac{\alpha_f Q_0^2}{H_0} (q_p - q_H)^2 + \frac{e_w}{e_p} x + \frac{e_w}{e_p} y \right] \\
\dot{h}_j &= \frac{(1+mF_{p0}/\nabla_e)Q_0}{FH_0} (q_H - q_p) \\
\dot{x} &= \frac{K_p e_h}{T_e} e_p q_p + \left[ \frac{K_e}{T_e} e_e - \frac{e_p}{e_e} e_q - e_g \right] x + \left[ e_g - e_p e_q \right] y - m_g \\
\dot{y} &= \frac{K_p e_h}{T_e} e_p q_p + \left[ \frac{K_e}{T_e} e_e - \frac{e_p}{e_e} e_q - e_g \right] x + \frac{K_p m_g}{T_e} y + \frac{K_p}{T_e} m_g 
\end{align*}
\] (14)

Because of the existence of the nonlinear term \((q_p - q_H)^2\), the hydro-turbine governing system equation (14) is a nonlinear dynamic system. The operating condition studied in this paper is that the unit goes through a load variation during normal output operation. \(m_g\) is regarded as the load variation, i.e. the external disturbance. Therefore, the input signal for the nonlinear hydro-turbine governing system is \(m_g\). The state variables in equation (14), i.e. \(q_H\), \(q_p\), \(h_j\), \(x\) and \(y\), are the output signals for the nonlinear hydro-turbine governing system. Except for the state variables and \(m_g\), the parameters in equation (14) are the characteristic parameters of the system.

3. Nonlinear dynamic behavior analysis

In this section, the nonlinear dynamic behavior of the hydro-turbine governing system with air cushion surge chamber is investigated by using the Hopf bifurcation theory and direct numerical integration. The stability and dynamic response of the system are detailed described and analyzed.

As one of the nonlinear mathematical theories, the Hopf bifurcation theory is adopted to study the nonlinear dynamic behavior of the hydro-turbine governing system equation (14). Hopf bifurcation theory is widely used in the area of the regulation and control of hydropower system. The principles and application procedures are detailed introduced in [11–13]. In this paper, the Hopf bifurcation theory is directly applied without multifarious presentations of the basic methodology.

For the hydro-turbine governing system, equation (14) can be transferred into the form of \(\dot{x} = f(x, \mu)\), where \(x = (q_H, q_p, h_j, x, y)^T\) is the state vector, and \(\mu\) is the bifurcation parameter. The equilibrium point \(x_e\) of the system can be determined from \(f(x_e, \mu) = 0\). The Jacobian matrix of the system at \(x_e\) is \(J(\mu) = DF_{x_e}(x_e, \mu)\). Then the characteristic equation of \(J(\mu)\) is \(\det(J(\mu) - \chi I) = 0\). By expanding \(\det(J(\mu) - \chi I) = 0\) yields

\[
\chi^5 + a_4 \chi^4 + a_3 \chi^3 + a_2 \chi^2 + a_1 \chi + a_0 = 0
\] (15)

When \(\mu = \mu_c\), we have

(i) \(a_i(\mu_c) > 0\) \((i = 1, 2, 3, 4, 5)\), \(\Delta_2 = \begin{bmatrix} a_1 & 1 \\ a_2 & a_3 \end{bmatrix} > 0, \ \Delta_3 = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix} > 0, \ \Delta_4 = \begin{bmatrix} a_1 & 1 & 0 & 0 \\ a_3 & a_4 & a_5 & a_6 \\ 0 & 0 & a_7 & a_8 \end{bmatrix} = 0\).

(ii) The traversal coefficient \(\sigma'(\mu_c) = \text{Re}\left(\frac{\partial \chi}{\partial \mu} \big|_{\mu=\mu_c} \right) \neq 0\).

Then at \(\mu = \mu_c\), equation (15) has a pair of purely imaginary eigenvalues \(\chi_{1,2} = \pm i \omega\), and the
system (14) goes through Hopf bifurcation. Note that the expressions for \( x_E \) and \( a_i \) \((i=1, 2, 3, 4, 5)\) are presented in Appendix B.

A hydropower station with air cushion surge chamber is selected as example for the nonlinear dynamic behavior analysis. The basic data of the example are: \( Q_0=13.3 \text{m}^3/\text{s} \), \( H_0=140 \text{m} \), \( T_{\text{rot}}=6.68 \text{s} \), \( T_{\text{up}}=0.55 \text{s} \), \( h_{\text{up}}=3.91 \text{m} \), \( h_{\text{ro}}=0.97 \text{m} \), \( \alpha_r=0.0004 \text{s}^2/\text{m}^5 \), \( p_0=140 \text{m} \), \( m=1.4 \), \( l_1=10 \text{m} \), \( F=850 \text{m}^2 \), \( V_0=I_0F \), \( T_a=5.54 \text{s} \), \( e_0=0 \), \( e_1=1.5 \), \( e_2=1 \), \( e_3=0.5 \), \( e_4=0 \), \( e_5=1 \), \( m_g=-0.1 \). \( K_i \) is selected as the bifurcation parameter, and the corresponding bifurcation point is denoted as \( \mu^*_c=K^*_c \).

By using \( a_i(\mu_c)>0\) \((i=1, 2, 3, 4, 5)\), \( \Delta_2>0\), \( \Delta_3>0\) and \( \Delta_4=0\), we can obtain the bifurcation line of the system on the \( K_p-K_i \) coordinate system, which is composed of \((K_p^*,K_i^*)\). The result of bifurcation line is shown in Figure 2. For all the points on the bifurcation line, the corresponding traversal coefficient can be determined by \( \sigma'(\mu_c)=\text{Re}(\frac{d\eta}{d\mu} \bigg|_{\mu=\mu_c}) \) and the result is shown in Figure 3.

![Figure 2. Bifurcation line and stable domain of the system.](image)

![Figure 3. \( \sigma'(\mu_c) \) corresponding to all bifurcation points.](image)

Figure 2 shows that the bifurcation line is a smooth curve and divides the whole \( K_p-K_i \) plane into two parts. For all the points on the bifurcation line, we always have \( \sigma'(\mu_c)>0 \) from Figure 3. Therefore, the system (14) goes through Hopf bifurcation at the bifurcation points and the type of Hopf bifurcation is supercritical. As a result, for a \( K_p \), the system is stable when \( K_i<K_i^* \). The dynamic response of the system stabilizes at an equilibrium point. Then we can get that the state points at the bottom of the bifurcation line can make the system stable, which indicates that the domain at the bottom of the bifurcation line is the stable domain. Accordingly, the domain at the top of the bifurcation line is the unstable domain. For the state points in the unstable domain, the dynamic response of the system stabilizes at a limit cycle. The graphical representation of the stable domain and unstable domain is shown in Figure 2.

Based on the stable domain, we can determine the dynamic states of the system. For different states points on the \( K_p-K_i \) plane, the dynamic behaviors are different. From the dynamic response processes of the system, we can evaluate the dynamic characteristics of the system. The dynamic response processes of the system can be obtained from equation (14) by using the direct numerical integration. In this section, three state points in Figure 2, i.e. \( S_1 \), \( S_2 \) and \( S_3 \), are selected for numerical simulation. \( S_1 \) locates in the stable domain and its coordinate value is \((2.564, 3.5)\). \( S_2 \) locates on the bifurcation line and its coordinate value is \((2.564, 4)\). \( S_3 \) locates in the unstable domain and its coordinate value is \((2.564, 4.2)\). The function ODE45 in MATLAB [14] is adopted to solve equation (14). The results for the dynamic response processes of \( x \) and phase space trajectories of \((q_F,x,y)\) are shown in Figure 4.
Figure 4. Phase space trajectories and dynamic response processes of the system for $S_1$, $S_2$ and $S_3$.

Figure 4 shows that, for the state point $S_1$, the phase space trajectory of $(q_P, x, y)$ stabilizes at an equilibrium point after several rounds of movement. Accordingly, the dynamic response process of $x$ is damped and enters a new steady value after several periods of oscillation. For the state point $S_2$, the phase space trajectory of $(q_P, x, y)$ stabilizes at a limit cycle. The dynamic response process of $x$ is a persistent oscillation. For the state point $S_3$, the phase space trajectory of $(q_P, x, y)$ firstly shows an emanative movement and then enters a stable limit cycle. Accordingly, the dynamic response process of $x$ is emanative at the initial stage.

4. Effect of throttling orifice head loss on dynamic behavior

In this section, the effect mechanism of throttling orifice head loss on the dynamic behavior of system is revealed based on the comparison between the cases with nonlinear head loss and no head loss. Another hydropower station with air cushion surge chamber is selected as example for the analysis. The basic data are as follows: $Q_0=228.6m^3/s$, $h_0=288.0m$, $T_{u0}=23.84s$, $T_{u0}=1.26s$, $h_{h0}=12.92m$, $h_{p0}=2.91m$, $\alpha_T=0.0004s^2/m^5$, $p_p=200m$, $m=1.4$, $l_0=20m$, $F=7000m^2$, $\nabla_0=l_0F$, $T_a=9.46s$. The other data and operating condition are the same with those of the example in Section 3.

The throttling orifice head loss can be evaluated by $\alpha_T$. Therefore, $\alpha_T$ is selected as the control parameter for the effect analysis on dynamic behavior. Firstly, for the selected hydropower station with air cushion surge chamber under $m_g=-0.1$, the stable domains under $\alpha_T (s^2/m^5)$ of 0, 0.0004, 0.0014 and 0.0024 are drawn, respectively. The results are shown in Figure 5. Then, a state point $S_4 (2, 1)$ is selected to determine the phase space trajectory of $(q_P, x, y)$ and the dynamic response process of $h_j$ under $\alpha_T (s^2/m^5)$ of 0, 0.0004, 0.0014 and 0.0024, respectively. The results are shown in Figure 6.

Figure 5. Effect of throttling orifice head loss $\alpha_T (s^2/m^5)$ on stability of the system.
The dynamic response of the system is revealed based on the Hopf bifurcation analysis, the characteristics of hydro turbine governing system with air cushion surge chamber is investigated by us. Based on the Hopf bifurcation theory, the water level oscillation of system is described and analyzed. The effect mechanism of throttling orifice head loss on the dynamic behavior of the hydro-turbine governing system with air cushion surge chamber is as follows:

From Figure 6(b) we can get that, with the increase of $\alpha_r$, the amplitude of $h_d$ decreases and the attenuation rate of $h_d$ increases. Based on the definition of $h_d$ we can know that the oscillation of $h_d$ is consistent with the water level oscillation in the surge chamber. $h_d$ can be adopted to evaluate the dynamic process characteristics of the water level oscillation in the surge chamber. Therefore, the following conclusion can be obtained: The throttling orifice head loss is favorable for the damping of the water level oscillation in the surge chamber.

5. Summary and conclusions
For the hydro-turbine governing system with air cushion surge chamber, the nonlinear mathematical model considering the nonlinear head loss of throttling orifice is established. The nonlinear dynamic behavior of the hydro-turbine governing system is investigated by using the Hopf bifurcation theory and direct numerical integration. The stability and dynamic response of the system are detailed described and analyzed. The effect mechanism of throttling orifice head loss on the dynamic behavior of system is revealed based on the comparison between the cases with nonlinear head loss and no head loss. Several conclusions can be drawn from this study:

(1) The Hopf bifurcation theory provides a practical approach for analyzing the nonlinear dynamic characteristics of hydro-turbine governing system with air cushion surge chamber. Based on the Hopf bifurcation analysis, the stability and dynamic response of the system can be detailed described.

(2) The mathematical model considering the nonlinear head loss of throttling orifice for the hydro-turbine governing system is a fifth-order nonlinear state equation. The system goes through Hopf bifurcation at the bifurcation points and the type of Hopf bifurcation is supercritical. The domain at the bottom of the bifurcation line is the stable domain. For the state points in the stable domain and unstable domain, the dynamic response of the system stabilizes at an equilibrium point and a limit.
cycle, respectively.

(3) The throttling orifice cannot affect the system stability through its head loss. The stable domain of the system is the same under different $\alpha_T$. The throttling orifice can affect the system dynamic response through its head loss. The dynamic response of the system is the different under different $\alpha_T$. The throttling orifice head loss is favorable for the damping of the water level oscillation in the surge chamber.

Acknowledgement
This work was supported by the J. Gust. Richert stiftelse in Sweden (Project No. 2017-00317).

Appendix A

| Nomenclature                                    |
|-----------------------------------------------|
| $Q_{H}$ discharge in headrace tunnel, m$^3$/s |
| $L_H$ length of headrace tunnel, m            |
| $f_H$ sectional area of headrace tunnel, m$^2$|
| $h_H$ head loss of headrace tunnel, m          |
| $T_{wH}$ flow inertia time constant of headrace tunnel, s |
| $H$ turbine net head, m                       |
| $F$ sectional area of surge tank, m$^2$       |
| $\alpha_T$ head loss coefficient of throttling orifice, s$^2$/m$^5$ |
| $m$ gas polytropic exponent                   |
| $p$ absolute air pressure in surge chamber, m |
| $\nabla$ air volume in surge chamber, m$^3$   |
| $Q_0$ initial discharge in pipeline, m$^3$/s  |
| $M_t$ kinetic moment, N-m                    |
| $M_g$ resisting moment, N-m                   |
| $N$ turbine unit frequency, Hz                |
| $Y$ guide vane opening, mm                    |
| $e_{h_0}$ moment transfer coefficients of turbine |
| $e_{v_0}$ discharge transfer coefficients of turbine |
| $T_a$ hydro-turbine unit inertia time constant, s |
| $K_p$ proportional gain                       |
| $K_i$ integral gain, s$^{-1}$                |
| $g$ acceleration of gravity, m/s$^2$         |

$$h = \frac{H - H_0}{H_0}, \quad h_i = \frac{H_i - H}{H_{10}}, \quad q_H = \frac{Q_{H0} - Q_{H00}}{Q_{H0}}, \quad q_P = \frac{Q_P - Q_{P00}}{Q_{P0}}, \quad m_i = \frac{M_i - M_{i0}}{M_{i0}}, \quad m_g = \frac{M_g - M_{g0}}{M_{g0}},$$
$$x = \frac{N - N_0}{N_0}, \quad y = \frac{Y - Y_0}{Y_0},$$
$$T_{wH0} = \frac{L_HQ_{H00}}{gH_0f_H}, \quad T_{wP0} = \frac{L_PF_{P0}}{gH_0f_P}, \quad e_h = \frac{\partial m}{\partial h}, \quad e_i = \frac{\partial m}{\partial i}, \quad e'_h = \frac{\partial q_P}{\partial h}, \quad e'_i = \frac{\partial q_P}{\partial i}.$$ (The subscript '0' refers to the initial value.)

Appendix B

(1) Equilibrium point
\[ x_e = (q_{HE}, q_{PE}, h_{AE}, x_e, y_e)^T, \]

where

\[ q_{HE} = q_{PE} = \frac{m_e}{2h_{H0} + 2h_{pE} e_{ph} (e_y - e_{ph} e_{ph}) + e_y} \]

\[ h_{AE} = -\frac{2h_{H0} + 2h_{pE} e_{ph} (e_y - e_{ph} e_{ph}) + e_y}{H_0 m_e} \]

\[ x_e = 0 \]

\[ y_e = \frac{2h_{H0} + 2h_{pE} e_{ph} (e_y - e_{ph} e_{ph}) + e_y}{H_0 e_y} \]

(2) Expressions of coefficients

\[ a_1 = \left( \frac{\partial q_{HE}}{\partial h_M} + \frac{\partial q_{AE}}{\partial h_M} + \frac{\partial h}{\partial x} + \frac{\partial h}{\partial y} \right) \]

\[ a_2 = \left( \frac{\partial q_{HE}}{\partial h_M} \frac{\partial h}{\partial q_M} \frac{\partial h}{\partial q_M} + \frac{\partial q_{AE}}{\partial h_M} \frac{\partial h}{\partial q_M} \frac{\partial h}{\partial q_M} + \frac{\partial h}{\partial x} \frac{\partial h}{\partial y} + \frac{\partial h}{\partial x} \frac{\partial h}{\partial y} \right) \]

\[ a_3 = \left( \frac{\partial q_{HE}}{\partial h_M} \frac{\partial h}{\partial q_M} \frac{\partial h}{\partial q_M} + \frac{\partial q_{AE}}{\partial h_M} \frac{\partial h}{\partial q_M} \frac{\partial h}{\partial q_M} + \frac{\partial h}{\partial x} \frac{\partial h}{\partial y} + \frac{\partial h}{\partial x} \frac{\partial h}{\partial y} \right) \]

\[ a_4 = \left( \frac{\partial q_{HE}}{\partial h_M} \frac{\partial h}{\partial q_M} \frac{\partial h}{\partial q_M} + \frac{\partial q_{AE}}{\partial h_M} \frac{\partial h}{\partial q_M} \frac{\partial h}{\partial q_M} + \frac{\partial h}{\partial x} \frac{\partial h}{\partial y} + \frac{\partial h}{\partial x} \frac{\partial h}{\partial y} \right) \]

\[ a_5 = \left( \frac{\partial q_{HE}}{\partial h_M} \frac{\partial h}{\partial q_M} \frac{\partial h}{\partial q_M} + \frac{\partial q_{AE}}{\partial h_M} \frac{\partial h}{\partial q_M} \frac{\partial h}{\partial q_M} + \frac{\partial h}{\partial x} \frac{\partial h}{\partial y} + \frac{\partial h}{\partial x} \frac{\partial h}{\partial y} \right) \]

where

\[ \frac{\partial q_{HE}}{\partial h_M} = -\frac{1}{T_{H0} H_0}, \quad \frac{\partial q_{HE}}{\partial h_M} = 0, \quad \frac{\partial q_{HE}}{\partial h_M} = -\frac{1}{T_{H0}}, \quad \frac{\partial q_{HE}}{\partial h_M} = 0, \quad \frac{\partial q_{HE}}{\partial h_M} = 0. \]
\[
\frac{\partial h}{\partial q_H} = \frac{(1 + mFp_0 / \nabla o)Q_0}{FH_0}, \quad \frac{\partial h}{\partial q_F} = \frac{-(1 + mFp_0 / \nabla o)Q_0}{FH_0}, \quad \frac{\partial h}{\partial h_{ax}} = 0, \quad \frac{\partial h}{\partial x} = 0, \quad \frac{\partial h}{\partial y} = 0,
\]

\[
\frac{\partial \dot{x}}{\partial q_H} = 0, \quad \frac{\partial \dot{x}}{\partial q_F} = \frac{1}{T_a} e_h, \quad \frac{\partial \dot{x}}{\partial h_{ax}} = 0, \quad \frac{\partial \dot{x}}{\partial x} = \frac{1}{T_a} (e_s - e_q e_p - e_p), \quad \frac{\partial \dot{x}}{\partial y} = \frac{1}{T_a} (e_s - e_q e_p),
\]

\[
\frac{\partial \dot{y}}{\partial q_H} = 0, \quad \frac{\partial \dot{y}}{\partial q_F} = \frac{K_p}{T_a} e_h, \quad \frac{\partial \dot{y}}{\partial h_{ax}} = 0, \quad \frac{\partial \dot{y}}{\partial x} = \frac{K_p}{T_a} (e_s - e_q e_p - e_p) - K_i, \quad \frac{\partial \dot{y}}{\partial y} = -\frac{K_p}{T_a} (e_s - e_q e_p).
\]

References

[1] Chaudhry M H 2014 Applied Hydraulic Transients (New York: Van Nostrand)

[2] Liu Q Z and Peng S Z 1995 Surge Tank of Hydropower Station (Beijing: China Waterpower Press)

[3] Hu J Y, Zhang J, Suo L S and Fang J 2007 Advance in research on air cushion surge chamber in hydropower plant ASME International Mechanical Engineering Congress and Exposition 6 Paper No. IMECE2007-42800 259-264

[4] Chaudhry M H, Sabbah M A and Fowler J E 1985 Analysis and stability of closed surge tanks J Hydraul. Eng. 111(7) 1079-1096

[5] Yang X L, Cederwall K and Kung C S 1992 Large-amplitude oscillations in closed surge chamber J Hydraul. Res. 30(3) 311-325

[6] Yang X L and Kung C S 1992 Stability of air-cushion surge tanks with throttling J Hydraul. Res. 30(6) 835-850

[7] Vereide K, Lia L and Nielsen T K 2015 Hydraulic scale modelling and thermodynamics of mass oscillations in closed surge tank J Hydraul. Res. 53(4) 519-524

[8] Guo W C, Yang J D, Chen J P and Teng Y 2014 Study on the stability of waterpower-speed control system for hydropower station with air cushion surge chamber 27th IAHR Symposium on Hydraulic Machinery and Systems, IOP Conference Series: Earth and Environmental Science 22 Article Number: UNSP 042004 DOI: 10.1088/1755-1315/22/4/042004

[9] IEEE Working Group 1992 Hydraulic turbine and turbine control model for system dynamic studies IEEE Trans Power Syst. 7 167-179

[10] Wei S P 2009 Hydraulic Turbine Regulation (Wuhan: Huazhong University of Science and Technology Press)

[11] Wiggins S 2013 Global Bifurcations and Chaos: Analytical Methods (New York: Springer Science & Business Media)

[12] Strogatz S H 2018 Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering (Boca Raton: CRC Press)

[13] Guo W C and Yang J D 2017 Hopf bifurcation control of hydro-turbine governing system with sloping ceiling tailrace tunnel using nonlinear state feedback Chaos Soliton Fract. 104 426-434.

[14] Hanselman D C and Littlefield B 2005 Mastering Matlab 7 (New Jersey: Pearson/Prentice Hall)