Faraday Rotation Limits on a Primordial Magnetic Field from WMAP Five-Year Data

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A primordial magnetic field in the early universe will cause Faraday rotation of the linear polarization of the cosmic microwave background generated via Compton scattering at the surface of last scattering. This rotation induces a non-zero parity-odd (B-mode) polarization component. The Wilkinson Microwave Anisotropy Probe (WMAP) 5-year data puts an upper limit on the magnitude of the B-polarization power spectrum; assuming that the B-polarization signal is totally due to the Faraday rotation effect, the upper limits on the comoving amplitude of a primordial stochastic magnetic field range from $6 \times 10^{-8}$ to $2 \times 10^{-6}$ G on a comoving length scale of 1 Mpc, depending on the power spectrum of the magnetic field.

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I. INTRODUCTION

The cosmic microwave background temperature and polarization anisotropy measurements presented in the 5-year data of the Wilkinson Microwave Anisotropy Probe (WMAP) provide the most comprehensive characterization to date of the perturbations present in the early universe at the time of photon-baryon decoupling [1]. The microwave background polarization field is conventionally decomposed into a sum of parity-even (“E-type”) and parity-odd (“B-type”) parts; since B-type polarization is absent in a cosmological scenario involving only scalar density perturbations, it serves as a powerful test of other cosmological physics, including primordial tensor and vector metric perturbations [2]. Another source of B-type polarization is Faraday rotation of the orientation of linear polarization due to the presence of a cosmological magnetic field: if a purely E-type polarization pattern is everywhere rotated by $\pi/4$, it becomes a purely B-type polarization. Smaller rotation angles generate a component of B-type polarization from an initial pure E-type polarization field.

Faraday rotation of polarized radiation is in general a powerful tool to estimate magnetic field amplitudes and spectra (see [3] for a recent review). In the cosmological context, Kosowsky and Loeb [4] proposed microwave background Faraday rotation to probe a homogeneous primordial magnetic field; this issue has been re-addressed by several authors for the homogeneous field [5-6], as well as for a stochastic field [7-9-10]. For a homogeneous field, the same polarization rotation effect additionally induces non-zero parity-odd cross correlations between microwave temperature and B-polarization anisotropies and E-polarization and B-polarizations anisotropies [11] and correlations between different multipoles [12] which are absent for standard cosmological scenarios. Primordial magnetic fields are interesting because they could serve as seed fields for the observed magnetic fields in galaxies and galaxy clusters [13].

Similar effects (microwave background birefringence and parity-odd anisotropy spectra) may also have non-magnetic explanations. In particular, microwave background polarization plane rotation occurs in models with violation of parity symmetry and/or Lorentz invariance in the early universe [14]. Parity-odd polarization anisotropy spectra appear due to gravitational lensing [15] or non-zero primordial magnetic helicity [16].

Here we use WMAP 5-year B-polarization power spectrum upper limits [17] to place limits from Faraday rotation on a stochastic primordial magnetic field. We assume that the cosmological magnetic field was generated during or prior to the early radiation-dominated epoch. The high conductivity of the primordial plasma results in a “frozen-in”
magnetic field, fixing the temporal dependence to be the simple scaling $\mathbf{B}(x, \eta) = \mathbf{B}(x)/a^2$ where $a$ is the scale factor and $\eta$ conformal time. Throughout this paper, $\mathbf{B}$ represents the comoving value of the magnetic field. We also normalize the scale factor $a$ by setting $a_0 = 1$ today, and we employ cosmological units with $\hbar = 1 = c$ and gaussian units for electromagnetic quantities.

II. CMB FARADAY ROTATION EFFECT

Electromagnetic waves propagating in a magnetized medium have the plane of their linear polarization rotated (see, e.g., [18]). A linearly polarized wave can be expressed as a superposition of left and right circularly polarized waves. The magnetic field induces a phase velocity difference between the two circular polarization, resulting in rotation of the polarization plane. The rotation angle $\alpha$ for a plane wave with comoving frequency $\nu_0$ propagating in the direction $\mathbf{n}$ satisfies

$$\frac{d\alpha}{d\eta} = \frac{3}{(4\pi)^2\nu_0 q} \hat{\tau}(x) \mathbf{n} \cdot \mathbf{B}(x)$$

(1)

where $\hat{\tau} = x_e n_e \sigma_T a$ is the differential optical depth, $n_e$ and $x_e$ are the comoving electron number density and ionization fraction, $\sigma_T$ is the Thomson cross section, and $q$ is the magnitude of the electron charge. Faraday rotation of the microwave background is a subtle problem, because the polarization is generated and rotated simultaneously in the region of the last scattering surface. However, as long as the total rotation is small compared to $\pi/2$, the total rotated polarization angle can be expressed simply as an average of the rotated polarization angle from each infinitesimal piece of path length through the surface of last scattering, neglecting depolarization effects [8].

When considering a stochastic magnetic field, we make the simplifying approximation that any magnetic field component with a wavelength shorter than the thickness of the surface of last scattering is neglected: for these components, the rotation of polarization generated at different optical depths will tend to cancel, leaving little net rotation. Then we can treat the magnetic field as constant throughout the rotation region, so that the total rotation, which is the sum over the rotations of each infinitesimal piece of generated polarization, can be expressed as the total rotation incurred by the polarization generated at some particular effective optical depth (for details see [8].)

A Gaussian random magnetic field is described by the two-point correlation function in wavenumber space as

$$\langle B_i^*(k)B_j(k') \rangle = (2\pi)^3 \delta^{(3)}(k - k') P_{ij} P_B(k),$$

(2)

where $P_B(k)$ is the magnetic field power spectrum, vanishing for all wavenumber larger than the damping scale $k_D$, $P_{ij} = \delta_{ij} - \hat{k}_i \hat{k}_j$ is the plane projector, and $\hat{k}_i = k_i/k$. We use the Fourier transform convention $B_j(k) = \int d^3xe^{i \mathbf{k} \cdot \mathbf{x}} B_j(x)$. (We neglect nonzero magnetic helicity since it does not affect the polarization plane rotation [7, 19].) The power spectrum $P_B(k)$ is related to the total energy density $E_B$ of the magnetic field through $E_B = \int_0^k k^2 d^3k P_B(k)$, and it is given by simple power law $P_B(k) = A_B k^{n_B}$. Smoothing the field on a comoving scale $\lambda > \lambda_D = 2\pi/k_D$ by convolving with a Gaussian smoothing kernel, the smoothed magnetic field amplitude $B_\lambda$ is [8]

$$B_\lambda^{n_B+3} = \frac{A_B}{2\pi^2} \Gamma (n_B/2 + 3/2), \quad \lambda > \lambda_D.$$

(3)

We assume that the magnetic field cut-off scale is determined by the Alfen wave damping scale [20],

$$\left( \frac{k_D}{\text{Mpc}^{-1}} \right)^{n_B+5} = 2 \times 10^4 \left( \frac{B_\lambda}{10^{-9} \text{G}} \right)^{-2} \left( \frac{k_\lambda}{\text{Mpc}^{-1}} \right)^{n_B+3},$$

(4)

which will always be a much smaller scale than the Silk damping scale (thickness of the last scattering surface) for standard cosmological models.

For a given comoving radiation frequency $\nu_0$, consider the rotation angle $\alpha(\mathbf{n})$ of the microwave background linear polarization as a function of sky direction $\mathbf{n}$. For a stochastic magnetic field, it is obvious that the rotation angle averaged over the sky is zero. Expanding the rotation angle in spherical harmonics leads to the definition of the angular power spectrum $C_\ell$ via

$$\langle \alpha(\mathbf{n})\alpha(\mathbf{n}') \rangle = \sum_l \frac{2l+1}{4\pi} C_\ell P_l(\mathbf{n} \cdot \mathbf{n}'),$$

(5)

where $P_l(x)$ are Legendre polynomials. The rotation angle power spectrum due to a stochastic magnetic field is [8]

$$C_\ell^{\alpha} \approx \frac{9(l+1)}{(4\pi)^3\eta_0^2 \nu_0^2} \frac{B_\lambda^2}{\Gamma (n_B/2 + 3/2)} \int_0^{x_D} dx x^{n_B} j_l^2(x),$$

(6)
where $x_D = k_D n_0$, $j_l(x)$ are spherical Bessel functions. This rotation multipole expression contains a sharp short-wavelength cutoff $k_D$ in the magnetic field; in reality, the effective cutoff will be smoothly spread over a range of scales. Given this rotation angle power spectrum, we need to compute the B-polarization power spectrum induced by the rotation field from the primordial E-polarization. This is given by Eq. (44) of Ref. [8],

$$C_l^{BB} = N_l^2 \sum_{l_1 l_2} \frac{(2l_1 + 1)(2l_2 + 1)}{4\pi(2l + 1)} N_{l_1}^2 K(l, l_1, l_2)^2 C_l^{EE} C_l^{\text{rot}} (C_{l_1 l_2}^{0})^2$$

where $C_{l_1 l_2}^{0}$ are Clebsch-Gordan coefficients (with techniques for numerical evaluation in Appendix B of Ref. [8]), the normalization factor $N_l = (2(l - 2)!/(l + 2)!)^{1/2}$ and the function $K(l, l_1, l_2) = \frac{1}{2} (L^2 + L_1^2 + L_2^2 - 2L_1 L_2 - 2L_1 L - 2L_1 L - 2L_2 - 2L)$ with $L \equiv l(l + 1)$, $L_1 \equiv l_1(l_1 + 1)$, and $L_2 \equiv l_2(l_2 + 1)$. The cross-correlations between temperature and B-polarization and between E and B-polarization are zero for a stochastic magnetic field with zero helicity. The modifications to the existing E-polarization power spectrum and the cross-correlation between E-polarization and temperature are negligible for small rotation angles, and we ignore this second-order effect in our analysis.

III. RESULTS AND DISCUSSION

To compare the data with the theory prediction, we calculate $C_l^{BB}$ for $B_\lambda = 1$ nG, $\lambda = 1$ Mpc, $n_B$ between -2.9 and -1 in steps of 0.005, and $l$ values from 2 to 1000. Our range of spectral indices is based on plausible magnetic field generation mechanisms [21] and spectral indices which are not excluded by other bounds; see Refs. [22, 23]. The input $C_l^{EE}$ values up to $l = 4000$ were calculated using CMBFAST [24] for the best-fit WMAP cosmological model. The obtained set of $C_l^{BB}$ is then further rescaled as $B_\lambda^2$ and $\lambda^{n_B+3}$ to obtain theoretical $C_l^{BB}$ for different values of magnetic field amplitude and scale, respectively. WMAP data [25] covers five frequency bands with central frequencies ranging from 23 GHz to 94 GHz. We use the measured B-polarization power spectrum limits for the three highest-frequency WMAP bands centered at 41, 61, and 94 GHz (Q, V, and W); our limits could be improved by including information from the lowest two frequency channels, but these are dominated by synchrotron foregrounds. We use only data for $l > 32$, for which we can treat measurement errors for different $l$ values as uncorrelated. Most of the constraining power of the data comes from multipoles between $l = 200$ and $l = 500$, as displayed in Fig. 1, where the theoretical signal begins to be comparable to the errors on the data points; neglecting the data with $l < 32$ only marginally affects the limits obtained here.

The theoretical $C_l^{BB}$ power spectra are compared with the WMAP data using $\chi^2$ statistics. We conservatively include an extra 40% error on theoretical $C_l^{BB}$ values to compensate for using the nominal frequency center of each WMAP band rather than the detailed frequency response of each band. Theoretical uncertainties on calculated values of $C_l^{EE}$ are neglected, since these are much smaller than the experimental uncertainties on WMAP data. As the data
is consistent with the hypothesis of $C^{BB} = 0$ (see Fig. 1), we set limits on values of $B_\lambda$ as a function of $n_B$ and $\lambda$. The 68% and 95% confidence limit bands on $B_\lambda$ as a function of $n_B$ are given in Fig. 2. We also display in Fig. 3 the 95% confidence upper limits on $B_\lambda$ for fixed values of $n_B$ ranging from -2.9 to -1.0 as functions of $\lambda$.

These results presume that B-polarization signal is totally due to the Faraday rotation effect. Other possible source of B-polarization, such as inflationary gravitational waves \cite{2}, gravitational lensing \cite{15} or non-zero primordial magnetic helicity \cite{16}, will strengthen the magnetic field limits derived here.

A primordial magnetic field on a scale $\lambda = 100$ Mpc must have an amplitude $B_\lambda$ of less than $10^{-7}$ G for any power spectrum between $n_B = -2.9$ and $n_B = -1$. These limits are weaker than those obtained recently in Refs. \cite{10}. Improvements on this limit using the Faraday rotation signal will be challenging, requiring substantially more sensitive polarization measurements at frequencies of 50 GHz or below; most current or planned high-sensitivity polarization experiments use bolometer detection technology, which tend to lose sensitivity in this frequency range. Upcoming measurements from the LFI instrument on Planck will be of roughly comparable polarization sensitivity to the WMAP measurements at similar frequencies. One notable effort is the QUIET experiment \cite{29}, which uses coherent detector technology at frequencies of 40 and 90 GHz. It is eventually anticipated to measure the BB polarization power spectrum with an improvement in sensitivity over WMAP by a factor of $10^4$. The lowest frequency of 40 GHz, compared to 23 GHz for WMAP, makes the Faraday polarization power spectrum smaller by a factor of 7.3 for the same magnetic field; such an experiment could thus place limits on the amplitude of a primordial field which are a factor of approximately $(10^4/7.3)^{1/2} = 37$ more stringent than those here. A detailed analysis for this experiment \cite{30} projects limits on a homogeneous primordial field below $10^{-10}$ G; however, primordial field limits will always be stronger than the stochastic field considered here, because Faraday rotation of a primordial field also generates nonzero TB and EB polarization cross-correlations which are larger than the corresponding BB power spectrum. Polarized foreground emission is also a potentially difficult systematic limit to these measurements. Other upcoming...
polarization experiments aimed at detecting the BB polarization from inflation are likely to improve on the Faraday rotation limits here, though the Faraday rotation power spectrum amplitude decreases by a factor of 230 between WMAP’s 23 GHz channel and a likely 90 GHz lowest-frequency bolometer channel.

The cosmological magnetic field itself generates microwave background anisotropies, particularly via the tensor and vector perturbations it induces \(^{26}\) (see Refs. \(^{23}\) for overviews of magnetized cosmological perturbations). Here we have neglected these perturbations, limiting ourself to considering the Faraday rotation effect alone, which provides a distinctive signature of magnetic fields nearly independent of other properties of the universe. We use the direct observational data without any priors. Our magnetic field limits are weaker than those arising from CMB temperature maps \(^{10, 27}\) or current bounds on a homogeneous magnetic field due to correlations between different \(l\) modes \(^{28}\) and CMB temperature non-gaussianity \(^{31}\). The strongest future limits on magnetic fields in the early universe will likely come from limits on the temperature and polarization fluctuations generated by magnetic field-induced vector perturbations \(^{26}\); we will address these current limits from WMAP data elsewhere.

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