An Estimation of the White Dwarf Mass in the Dwarf Nova GK Persei with NuSTAR Observations of Two States

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ABSTRACT

We report on X-ray observations of the Dwarf Nova GK Persei performed by NuSTAR in 2015. GK Persei, behaving also as an Intermediate Polar, exhibited a Dwarf Nova outburst in 2015 March–April. The object was observed with NuSTAR during the outburst state, and again in a quiescent state wherein the 15–50 keV flux was 33 times lower. Using a multi-temperature plasma emission and reflection model, the highest plasma temperature in the accretion column was measured as 19.7±1.3 keV in outburst and 36.2±2.5 keV in quiescence. The significant change of the maximum temperature is considered to reflect an accretion-induced decrease of the inner-disk radius \( R_{in} \), where accreting gas is captured by the magnetosphere. Assuming this radius scales as \( R_{in} \propto M^{-2/7} \) where \( M \) is the mass accretion rate, we obtain \( R_{in} = 1.9^{+0.4}_{-0.2} \) \( R_{WD} \) and \( R_{WD} = 7.4^{+1.2}_{-1.0} \) \( R_{WD} \) in outburst and quiescence respectively, where \( R_{WD} \) is the white-dwarf radius of this system. Utilising the measured temperatures and fluxes, as well as the standard mass-radius relation of white dwarfs, we estimate the white-dwarf mass as \( M_{WD} = 0.87 \pm 0.08 \) \( M_\odot \) including typical systematic uncertainties by 7%. The surface magnetic field is also measured as \( B \sim 5 \times 10^9 \) G. These results exemplify a new X-ray method of estimating \( M_{WD} \) and \( B \) of white dwarfs by using large changes in \( M \).

Key words: stars: dwarf novae – X-rays: individual:GK Persei

1 INTRODUCTION

Cataclysmic Variables (CVs) are close binary systems consisting of a mass-accreting white-dwarf (WD) primary and a mass-donating companion. Gas overflowing from the Roche lobe of the companion accretes onto the WD surface, where gravitational energy of the gas is converted mainly into X-ray emission. CVs hosting a magnetised WD are further classified into “Polars” and “Intermediate Polars” (IPs), in which the WDs have magnetic-field strengths of \( B \sim 10^7-10^8 \) G and \( B \sim 10^8-10^9 \) G, respectively.

In an IP, the gas from the companion forms an accretion disk down to a radius \( R_{in} \) where the gravity working on the accreting matter is counter-balanced by the magnetic pressure. Then, the gas is captured by the WD’s magnetosphere, and accretes onto the WD surface to form a pair of accretion columns due to the strong magnetic field. In the accretion columns, the gas is heated to \( T \sim 10^8 \) K by a standing shock, and lands onto the WD surface after releasing most of its energies into thermal X-rays. If \( R_{in} \) is far enough from the WD surface, the temperature \( T_{in} \) just below the shock is proportional to the gravitational potential of the WD (Aizu 1973) as

\[
kT_{in} = \frac{3}{8} \mu m_p \frac{G M_{WD}}{R_{WD}^2},
\]

where \( \mu \) is the mean molecular weight, \( m_p \) is the proton mass, \( M_{WD} \) is the WD mass, and \( R_{WD} \) is its radius. Therefore, \( M_{WD} \) can be estimated by combining the measured \( T_{in} \) with...
the standard mass v.s. radius ($M_{\text{WD}}$-$R_{\text{WD}}$) relation of WDs (Nauenberg 1972)

$$R_{\text{WD}} = 7.8 \times 10^8 \text{ cm} \left[\left(\frac{1.44 M_{\odot}}{M_{\text{WD}}}\right)^{2/3} - \left(\frac{M_{\text{WD}}}{1.44 M_{\odot}}\right)^{2/3}\right]^{1/2}.$$  

(2)

An X-ray spectrum from an IP is a particular superposition of optically-thin thermal emissions of various temperatures, from $T_s$ downwards (Cropper et al. 1998). To determine $T_s$, it is hence important to accurately measure both the hard X-ray continuum (e.g. Suleimanov et al. 2005; Yuasa et al. 2016), and the ratio of Fe XXV and XXVI lines at $\sim 7$ keV (Fujimoto & Ishida 1997). This is because the former is sensitive to the hottest components (with temperature $\sim T_s$), whereas the latter tells us contributions from cooler components arising closer to the WD surface.

GK Persei, at an estimated distance of 4777$^{+28}_{-25}$ pc (Harrison et al. 2013a), interestingly exhibits three distinct aspects of CVs; it behaves as an IP, as a Dwarf Nova, and exhibited a classical Nova explosion in 1901 (Williams 1901; Hale 1901). It repeats Dwarf Nova outbursts every 2–3 years, each lasting for 2 months (e.g. Simon 2002). During outbursts, the optical and X-ray luminosities both increase by a factor of 10–20.

By optical observations, Reinsch (1994) and Morales-Rueda et al. (2002) obtained lower limits of the WD mass in GK Persei as $M_{\text{WD}} \geq 0.78 M_{\odot}$ and $M_{\text{WD}} \geq 0.55 M_{\odot}$ respectively, and upper limits of the inclination angle as $i \leq 73^\circ$ due to lack of eclipses. Through a model fitting to the Nova outburst light curve observed in 1901, Hachisu & Kato (2007) also derived $M_{\text{WD}} = 1.15 \pm 0.05 M_{\odot}$. Ezuka & Ishida (1999) and Suleimanov et al. (2005) measured the shock temperature in outbursts, and derived $M_{\text{WD}} = 0.52^{+0.14}_{-0.16} M_{\odot}$ with ASCA and $M_{\text{WD}} = 0.59 \pm 0.05 M_{\odot}$ with RXTE, respectively.

However, Suleimanov et al. (2005) pointed out that the WD mass based on the outburst observation could be underestimated by at least 20%. In fact, accretion onto the WD can occur only if $r_{\text{in}}$ is smaller than the co-rotation radius, defined as

$$\frac{R_{\text{in}}}{R_{\text{WD}}} = 2.3 \left(\frac{P}{1 \text{ min}}\right)^{2/3} \left(\frac{M_{\text{WD}}}{M_{\odot}}\right)^{1/3} \left(\frac{R_{\text{WD}}}{10^7 \text{ cm}}\right)^{-1}$$

(3)

e.g. Warner 1995), where the Keplerian rotation period is equal to the spin period $P$. When $P = 351$ sec of GK Persei (e.g. Watson et al. 1985) and $M_{\text{WD}} \sim 0.8 M_{\odot}$ are employed, $R_{\text{in}} \leq R_{\text{in}} \sim 10 R_{\text{WD}}$ should be required even in quiescence. Therefore, the condition $R_{\text{in}} > R_{\text{WD}}$ may not generally hold, particularly in outbursts. Actually, Brunschweiger et al. (2009) utilised the Swift/BAT survey data during quiescence and obtained $M_{\text{WD}} = 0.90 \pm 0.12 M_{\odot}$, which is higher than the estimates from the past X-ray results in outbursts.

A recent outburst from GK Persei started in March 2015, and continued for 2 months (Wilber et al. 2015). During this outburst, Zemko et al. (2017) triggered a Target of Opportunity (ToO) observation with NuSTAR and measured a high spin modulation even in a hard X-ray range. Suleimanov et al. (2016) also analysed the ToO data and constrained the WD mass as $M_{\text{WD}} = 0.86 \pm 0.02 M_{\odot}$. The onset of this outburst was serendipitously caught by Suzaku, and the obtained data allowed Yuasa et al. (2016) to study the accretion geometry at the beginning of the outburst.

With NuSTAR, we observed GK Persei again, after the object returned to its quiescence. Although previous observations of the object in quiescence were unable to detect the hard X-ray component, the high sensitivity of NuSTAR has for the first time allowed us to detect its hard X-rays (typically in energies above $\sim 20$ keV) in quiescence. The present

Table 1. The present observation log of GK Persei by NuSTAR.

| Observation ID | Start Date/Time | Stop Date/Time | Exposure$^a$ | Count rate$^b$ |
|---------------|-----------------|---------------|-------------|---------------|
| Outburst      | 90001008002     | 2015-04-04 02:46:07 | 2015-04-06 15:10:35 | 42, 18.09 ± 0.02 |
| Quiescence    | 30101021002     | 2015-09-08 15:46:08 | 2015-09-11 02:04:09 | 72, 1.080 ± 0.006 |

$^a$ A net exposure of each of FPMA and FPMB in ks.

$^b$ Averaged 3–50 keV combined count rates of FPMA plus FPMB in units of count s$^{-1}$.  

Figure 1. Optical light curves of GK Persei without a filter from AAVSO International Database, and the Swift/BAT (Krimm et al. 2013) 15–150 keV count rate history from Swift/BAT X-ray Transient Monitor web site. The shaded regions indicate the two NuSTAR observations.
paper describes a combined analysis of the outburst and quiescence data from NuSTAR, and presents a new method to determine $R_{\text{in}}$, $M_{\text{WD}}$, and $B$ of the WD in GK Persei utilising the large change in $M$.

2 OBSERVATIONS AND DATA REDUCTION

The 2015 outburst of GK Persei started on 2015 March 6.84 UT (Wilber et al. 2015). As shown in Figure 1, the ToO observation with NuSTAR (Harrison et al. 2013b) was conducted in the middle of the outburst from 2015 April 4 02:46:07 to April 6 15:10:35. The net exposures of FPMA and FPMB are 42 ks each. The second NuSTAR observation, in quiescence, was performed from 2015 September 8 15:46:08 to September 11 02:04:00 with a net exposure of 72 ks. The log of the two observations is given in Table 1.

We utilised the data analysis software package HEA- SOFT version 6.20 and a detector calibration database NuS- TAR CALDB version 20170222, both released and maintained by HEASARC at NASA Goddard Space Flight Center. Photon events in the data sets were extracted with an exclusive data reduction software for NuSTAR “nuproducts” version 0.4.6 and “nuproducts” version 0.3.0. The on-source events were accumulated from a circular region with a radius of 150′′ (in outburst) and 80′′ (in quiescence) centred on the source. The background data were accumulated over a region outside a circle of radius of 170′′ and 100′′ in outburst and quiescence, respectively. The X-ray spectra were analysed and fitted with XSPEC version 12.9.1 (Arnaud 1996).

Generally, the X-ray emission from an IP is pulsed at its $P$. In fact, pulsations of GK Persei in the X-ray band have been detected at $P = 351$ sec both in outbursts and quiescence (e.g. Watson et al. 1985; Norton et al. 1988; Ishida et al. 1992). In the 2015 outburst observation, Zemko et al. (2017) clearly detected the 351 sec pulsation both in the 3–10 keV and 10–79 keV ranges. In the quiescence observation by NuSTAR, we detected a faint pulsation with a modulation amplitude of ∼10% in the 3–50 keV range. In the present paper, we concentrate on spectral analysis and postpone the study of this pulsation for the next publication.

3 ANALYSIS AND RESULTS

Figure 2 shows 3–50 keV spectra of the outburst and quiescence observations. The background has been subtracted, but the instrumental response has not been removed. Data of FPMA and FPMB are separately plotted. As reported by Zemko et al. (2017) and Suleimanov et al. (2016), the hard X-ray continuum is detected up to 70 keV during the outburst. In quiescence, the source is detected up to 50 keV for the first time. The 3–50 keV count rate of FPFA plus FPMB was $18.09 \pm 0.02$ count s$^{-1}$ in outburst, and $1.080 \pm 0.006$ count s$^{-1}$ in quiescence. Thus, the outburst data have 17.5 times higher count rate than those in quiescence. The spectra, particularly the outburst data, exhibit Fe-K line complex at ~ 6.4 keV. From this energy, we regard the lines as mainly of fluorescence origin (from the WD surface and/or the accreting cold matter), rather than ionised lines from the accretion columns.

In the bottom panel, we show the ratio between the two spectra. It reveals three features of the outburst spectrum, in comparison with that in quiescence. Namely, a stronger low-energy absorption, the stronger Fe-K line, and a continuum break at ~ 20 keV.

To analyse the spectra, we employed a multi-temperature optically-thin plasma model cemekl (Done & Osborne 1997) based on a thermal plasma code mekal (Mewe et al. 1985, 1986; Liedahl et al. 1995; Kaastra et al. 1996). The differential emission measure of cemekl is proportional to the power law function of the plasma temperature $T$ as

$$d(\text{EM}) \propto (T/T_0)^{\alpha-1} \, dT,$$

where $\alpha$ is a positive parameter. When the accretion column has a cylindrical shape, $\alpha$ is theoretically calculated as 0.43 by Falanga et al. (2005), who used the spectral model computed by Suleimanov et al. (2005). We employed this value because high accretion rate systems such as GK Persei are considered to have nearly cylindrical accretion columns (Hayashi & Ishida 2014a,b). To imitate the reflection effect on the WD surface, reflect model (Magdziarz & Zdziarski 1995) was utilised. The solid angle of reflector from the irradiator was set to $2\pi$ assuming that the standing shock is formed near the WD surface. The abundances of the cemekl and reflect components were constrained to be the same assuming that the WD surface near the accretion column is covered by accreted material. A gaussian emission model was also added to represent Fe-K line.

With the model thus constructed, we first fitted the outburst spectrum in the 5–50 keV range, because the cemekl model is not available above 50 keV. A partial covering absorption model was applied to the spectral model in addition to a single column absorber. As shown in Figure 3a, this model approximately reproduced the spectrum, but the fit was formally not acceptable under a 90% confidence level, with the reduced chi-squared of $\chi^2/\nu = 2.26$ for 133 degrees of freedom even including 1% systematic error. In fact, as
shown in Figure 3b, significant residuals were seen in the low energy band (<10 keV).

The above fit failure to the outburst spectra is not surprising, because X-ray spectra of IPs are often subject to strong and complex absorption that is not modeled by partial absorption (e.g. Ezuka & Ishida 1999). Since refining the absorption model is beyond the scope of the present paper due to lack of constraining data other than the continuum shape, we have resorted to discarding low-energy ranges until the effects of complex absorption become negligible (see Ezuka & Ishida 1999 for a similar method utilised to avoid complex absorption from affecting the spectral fitting result). By limiting the fit range to 15–50 keV, the fit to the Ezuka & Ishida 1999 for a similar method utilised to avoid the effects of complex absorption become negligible (see due to lack of constraining data other than the continuum absorption model is beyond the scope of the present paper (though the fit was unacceptable).

The 5–50 keV quiescence spectrum has been reproduced successfully by the spectral model with a single column absorber. Therefore, we have finally conducted a simultaneous fitting using the 15–50 keV band of the outburst spectrum and the 5–50 keV band of the quiescence spectrum. The inclination angle and the abundance set were in common, while the other parameters were allowed to vary independently. This fitting including 1% systematic error has been acceptable. This fitting including 1% systematic error has become acceptable with $\chi^2/\nu = 1.01$ for 191 degrees of freedom. The fit result and the best-fit parameters are presented in Figure 3 and Table 2, respectively. Errors are at 90% confidence level. The shock temperature was constrained as $T_s = 19.4_{-0.8}^{+1.3}$ keV in outburst, and $T_s = 36_{-2}^{+3.5}$ keV in quiescence; thus, $T_s$ was significantly higher in the latter. The 15–50 keV absorbed flux in outburst was 33 times higher than that in quiescence. The 15–50 keV luminosity is also derived as $1.2_{-0.3}^{+0.1} \times 10^{34}$ erg s$^{-1}$ in outburst, and $3.5_{-0.5}^{+0.3} \times 10^{32}$ erg s$^{-1}$ in quiescence, with the estimated distance of 477 pc (Harisson et al. 2013a).

For our purpose, we need to calculate the total X-ray flux $F$ which is thought to represent the gravitational energy release from $R_m$ to $R_{WD}$. Starting from the absorbed 15–50 keV flux, $F$ was derived in the following way. First, the absorption and the reflection were removed. Second, the flux above 50 keV was included by extrapolating the best-fit model up to 100 keV. Finally, the contribution below 15 keV was incorporated by integrating the best-fit model down to 0.01 keV. The flux above 100 keV and below 0.01 keV are both estimated to be $\ll 0.01F$. The difference in $F$ between the two spectra amounts to a factor of 65.

Since our final fit to the outburst spectrum was obtained by discarding the data below 15 keV, obviously no information was obtained on the Fe-K line. Hence we fitted the 5–9 keV spectra of the outburst and quiescence to constrain the Fe-K line equivalent width (EW). With a simple model of a single-temperature bremsstrahlung, a gaussian, and a single column absorption model, the EW was measured to be $192 \pm 13$ eV and $52_{-24}^{+34}$ eV in outburst and quiescence respectively, as presented in Table 2. The latter is consistent with the value obtained by the 5–50 keV simultaneous fitting.

### 4 ESTIMATION OF THE WD MASS

As described in Section 1, the accreting matter is considered to be captured by the magnetic field at the inner-radius of the accretion disk $R_m$. In order to estimate the WD mass precisely, $R_m$ as well as the shock height $h$, has to be taken into account. Thus, Equation 1 is modified as

$$kT_s^{-3/2} = \frac{7}{8} \mu m_p \frac{GM_{WD}}{R_{WD}} \left( \frac{R_{WD}}{R_{WD} + h} \frac{R_{WD}}{R_m} \right),$$

and therefore $T_s$ is a function of $M_{WD}$, $R_{WD}$, $h$, and $R_m$. At the same time, $R_{WD}$ is related to $M_{WD}$ by the theoretical $M$–$R$ relation (Equation 2). Employing a numerical calculation of plasma emission in the accretion column, Suleimanov et al. (2008, 2016) theoretically calculated the behaviour of $h$ and $T_s$, and then reconstructed Equation 5 as the functional form,

$$\frac{kT_s}{1 \text{ keV}} \approx 23.4 \frac{M_{WD}}{M_{\odot}} \left( 1 - 0.59 \frac{M_{WD}}{M_{\odot}} \right)^{-1} \left( 1 - \frac{R_{WD}}{R_m} \right).$$

Based on the article, this expression is valid for $M_{WD}$ in the range of 0.4 $M_{\odot}$ to 1.2 $M_{\odot}$. Figure 4 presents, on the $M_{WD}$ v.s. $R_m$ plane, contours of $T_s$ implied by Equation 6; in this visualisation, we employed several representative values of $T_s$, including the present two measurements for outburst and quiescence. Thus, from the quiescence data, we can already set a lower limit as $M_{WD} > 0.77 M_{\odot}$ (for $R_m \rightarrow \infty$).

The inner-disc radius may be approximately identified as the Alfvén radius, which is determined by an equilibrium between the inward gravity and the outward magnetic pressure. Assuming spherical accretion and dipole magnetic field, Elsner & Lamb (1977) described it as

$$R_m / R_{WD} \approx 2.3 \left( \frac{\dot{M}}{10^{15} \text{ g/s}} \right)^{-2/7} \left( \frac{M_{WD}}{M_{\odot}} \right)^{-1/7} \left( \frac{R_{WD}}{10^9 \text{ cm}} \right)^{5/7} \left( \frac{B}{10^9 \text{ G}} \right)^{4/7},$$

where $\dot{M}$ is the accretion rate and $B$ is again the dipole magnetic-field strength on the WD surface. An extension of this formalism by Ghosh & Lamb (1979) has been shown to give a good explanation to the accretion-induced spin period changes in the binary X-ray pulsar 4U 1626–67 (Takagi et al. 2016). Since mass accretion that takes place in GK Persei shares similar geometry outside $R_m$, this formalism is employed here. With this formalisation, $R_m$ thus shrinks as $\dot{M}$ increases, $T_s$ must be lower in outbursts than in quiescence, in agreement with our result. Systematic uncertainties associated with this equation are discussed in Section 5.4.3.

In order to utilise our two observations in equal manner, let us introduce the ratio $\gamma$ between the two $R_m$ values as

$$\gamma = R_m^{\text{out}} / R_m^{\text{qui}} \approx \left( \frac{\dot{M}^{\text{out}}}{\dot{M}^{\text{qui}}} \right)^{-2/7},$$

where the superscripts “out” and “qui” indicate the outburst and quiescence values, respectively. Since $\dot{M}$ can be related to the total X-ray luminosity $L$ as

$$M = L \times \frac{GM_{WD}}{R_{WD}} \left( 1 - \frac{R_{WD}}{R_m} \right)^{-1},$$

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Figure 3. (a) The NuSTAR FPMA (black) and FPMB (red) spectra in outburst, and their best-fit models with a partial covering absorption. The thermal component, reflection component, and Fe-K line are indicated by the dash-dot, dash-dot-dot, and dashed lines, respectively. (b) The residuals from the 5–50 keV fit in panel (a). (c) The residuals of the 15–50 keV fit. (d) The quiescence spectrum and its best-fit model. (e) The residuals of the fit in panel (d).

Table 2. Best-fit parameters of the multi temperature plasma emission and reflection model to the time-averaged spectra.

|            | \(N_{\text{H}}\) | cos\(^b\) | \(T_c\) | \(Z\)\(^d\) | EW\(^e\) | EW\(^f\) | \(F_{15-50}\) | \(F_{0.01-0.00}\) | \(\chi^2/\nu\) |
|------------|-----------------|----------|---------|---------|-------|-------|---------------|----------------|-------------|
| Outburst   | 124 \(^{+14}_{-12}\) | <0.22 | 19.7\(^{+1.3}_{-1.0}\) | 0.10 \(\pm\) 0.04 | – | 192 \(\pm\) 13 | 4.3\(^{+0.6}_{-0.9}\) \(\times\) 10\(^{-10}\) | 3.6\(^{+0.5}_{-0.8}\) \(\times\) 10\(^{-9}\) | 1.01 |
| Quiescence | 10 \(^{+4}_{-3}\) | 36.2\(^{+3.5}_{-3.2}\) | 55\(^{+34}_{-26}\) | 52\(^{+34}_{-26}\) | 1.3\(^{+0.1}_{-0.2}\) \(\times\) 10\(^{-11}\) | 5.5\(^{+0.1}_{-0.1}\) \(\times\) 10\(^{-11}\) |

\(^a\) Column density of the single-column absorption.
\(^b\) Cosine of the inclination angle between the reflection surface and the observer’s line-of-sight.
\(^c\) The highest temperature of the accretion column.
\(^d\) Abundance relative to Solar.
\(^e\) Equivalent width of the Fe-K line by the 5–50 keV wide band fit.
\(^f\) Equivalent width of the Fe-K line by the 5–9 keV narrow band fit.
\(^g\) Obtained model fluxes in the 15–50 keV band.

we can combine Equation 6, 8, and 9 to obtain

\[
\gamma = \frac{\left(\frac{L_{\text{out}}}{L_{\text{in}}^{\text{out}}}ight)^{2/7}}{\left(\frac{T_{\text{out}}}{T_{\text{in}}^{\text{out}}}ight)^{2/7}}
\]

where \(F\) is the total X-ray flux. Then, the values of \(T_s\) and \(F\) measured in the two observations (Table 2) yield, via Equation 8,

\[
\gamma = 3.9 \pm 0.5.
\]

We can now determine the value of \(M_{\text{WD}}\) in Figure 4 so that the two \(R_{\text{in}}\) values satisfy equation 11 as indicated by a pair of horizontal lines in Figure 4; \(R_{\text{in}}^{\text{out}} = 1.9^{-0.4}_{+0.2} R_{\text{WD}}\).

\(R_{\text{in}} = 7.4^{+2.1}_{-1.2} R_{\text{WD}},\) and \(M_{\text{WD}} = 0.87 \pm 0.05 M_\odot\) are yielded with \(R_{\text{WD}} = (6.6 \pm 0.4) \times 10^4\) cm. The obtained \(R_{\text{in}}^{\text{out}}\) and \(R_{\text{in}}^{\text{out}}\) satisfy the accretion condition \(R_{\text{in}} < R_{\text{Q}} \sim 11 R_{\text{WD}}\) with \(P = 351\) sec, \(M_{\text{WD}} = 0.87 M_\odot\), and Equation 3. Thus, the clear increase in \(T_s\), observed in the transition from outburst to quiescence (Table 2), has been successfully explained as a factor \(\gamma \sim 4\) increase in \(R_{\text{in}}\), in response to the decrease in \(M\) by a factor of 120. \(M\) is obtained from Equation 9 and \(F\) as \(M_{\text{in}} = (1.2 \pm 0.4) \times 10^{18}\) g s\(^{-1}\) and \(M_{\text{out}} = (1.0 \pm 0.3) \times 10^{16}\) g s\(^{-1}\), respectively. An advantage of this method is that any systematic uncertainty involved in the coefficient of Equation 7 cancels by taking the ra-
tio of the two equations (Equation 10). Further discussion continues in Section 5.4.3.

5 DISCUSSION

5.1 Comparison between the Two Observations

In the present paper, we analysed a pair of NuSTAR spectra of GK Persei acquired in 2015. In the 5 months from the outburst observation on April 4 to the quiescence one on September 8, the optical emission from GK Persei diminished by ~3.2 magnitude or ~19 times (Figure 1). Meanwhile, the 3–50 keV FPMA+FPMB count rate decreased by a factor of 17, and the absorbed 15–50 keV flux by a factor of 33. The two factors become different because the outburst spectrum is more absorbed (Figure 2), and hence the count rate which is more weighted towards lower energies changed less than that of the flux which is more weighted towards higher energies. Correcting these spectra for the respective absorption, and extrapolating the best-fit models to >50 keV and <15 keV, the 0.01–100 keV unabsorbed total X-ray flux is inferred to have changed by 65 times.

In addition to the changes in the X-ray flux and absorption, we detected a clear increase in \( T_s \) from the outburst (~ 20 keV) to the quiescence (~ 36 keV) observations. Employing the disk-magnetosphere interaction model of Ghosh & Lamb (1979), the change in \( T_s \) has successfully been interpreted as due to a factor ~ 4 change in \( R_{\text{in}} \), in a negative correlation with \( M \). Considering that the gravitational potential drop available for the X-ray emission (from \( R_{\text{in}} \) to \( R_{\text{WD}} \)) thus became deeper in quiescence, the total X-ray luminosity change has been converted to a factor of 120 difference in \( M \). For reference, the temperature change we observed is qualitatively consistent with the report by Zemko et al. (2017), that the Swift/XRT light curve of the hardness ratio indicated a temperature decrease as the outburst proceeded towards its peak, and the very high value of \( T_s \) measured by Yuasa et al. (2016) at the outburst onset.

As presented in Table 2, the EW of the Fe-K line (nearby neutral component) was ~ 4 times higher in outburst. The line is usually ascribed to two emitting sources: the ambient matter as represented by \( M \) and \( M_{\text{H}} \), and the WD surface as represented by reflection. Evidently, both \( M \) and \( M_{\text{H}} \) were higher in outburst, so that the Fe-K line EW from the first source must be higher as well. Furthermore, as discussed later in Section 5.4.2, the standing shock is considered to come closer to the surface when \( M \) increases, because the higher density would increase the volume emissivity in the accretion columns and higher shock temperature can be dissipated within the boundary conditions. This will, in turn, increase the solid angle of reflection, and yield a high EW from the second source. The higher EW observed in outburst may be explained qualitatively as a combination of these two effects.

5.2 Comparison with Previous Optical Results

The WD mass we obtained, \( M_{\text{WD}} = 0.87 \pm 0.05 \, M_\odot \), is consistent with the optical results (\( \geq 0.78 \, M_\odot \): Reinsch 1994, \( \geq 0.63 \, M_\odot \): Morales-Rueda et al. 2002). These optical estimates gave only lower limits of \( M_{\text{WD}} \) because eclipses of the WD do not occur in the GK Persei system, and hence the inclination angle remains poorly constrained. In contrast, our method with X-rays can estimate \( M_{\text{WD}} \) without the knowledge of the inclination angle. When our \( M_{\text{WD}} \) determination is combined with the ratio of the WD mass and the companion mass \( M_K/M_{\text{WD}} = 0.55 \pm 0.21 \), the optically determined mass function

\[
\frac{M_{\text{WD}}}{M_\odot} = \frac{\sin^3 i}{(1 + M_K/M_{\text{WD}})^2} = 0.362
\]  

(Morales-Rueda et al. 2002), the companion mass is constrained as \( M_K = 0.48 \pm 0.18 \, M_\odot \), and then the lower limit of the inclination angle is derived as \( i \geq 63^\circ \). This lower limit on \( i \) is consistent with the optical upper limit, \( i \leq 73^\circ \), required by the lack of eclipses. Combining these results, \( 63^\circ \leq i \leq 73^\circ \) is obtained.

5.3 Comparison with Previous X-ray Results

Let us revisit the past X-ray result with the PCA and HEXTE onboard RXTE, obtained during an outburst by Suleimanov et al. (2005). They measured \( T_s = 21 \pm 3 \, \text{keV} \), and derived \( M_{\text{WD}} = 0.59 \pm 0.05 \, M_\odot \) assuming \( R_{\text{in}} \gg R_{\text{WD}} \) (the author noted that \( M_{\text{WD}} \) would be underestimated). At that time, the total X-ray flux in the 0.1–100.0 keV range was measured to be \( 8.86 \times 10^{-10} \, \text{erg cm}^{-2} \, \text{s}^{-1} \). When we use the present censecl model of the same \( T_s \), the 0.01–100 keV flux is re-estimated as \( 1.0 \times 10^{-9} \, \text{erg cm}^{-2} \, \text{s}^{-1} \), which falls in between the present two measurements (Table 2). Then, compared with them, the value of \( R_{\text{in}} \) during the RXTE observation is estimated, from Equation 8, as \( R_{\text{in}} \approx 1.5 \, R_{\text{WD}} \approx 0.37 \, R_{\text{in}} \approx 2.8 \, \text{R}_{\text{WD}} \). Substituting this value and \( T_s = 21 \, \text{keV} \) into Equation 6, or equivalently referring to Figure 4, \( M_{\text{WD}} \sim 0.8 \, M_\odot \) is derived. This revised mass is probably consistent with our result when various errors are taken in account. Our result is also consistent with the mass
estimation $M_{\text{WD}} = 0.90 \pm 0.12 \, M_\odot$ by Brunschwiger et al. (2009) with Swift/BAT within errors, as already referred to in Section 1.

The present outburst data were already analysed by Zemko et al. (2017). They derived $T_s = 16.2^{+0.5}_{-0.1} \, \text{keV}$ by the 3–50 keV broad band fitting, wherein the NuSTAR data are combined with those from the Chandra MEG and HETG. Their $T_s$ value is ~18% lower than our outburst result. This discrepancy may be caused by difference of emission models. The mckflow model they employed is a superposition of the mekal thermal emission model, like the cemekl model we used, but the emission measure of mckflow is weighted by the inverse of the bolometric luminosity at each temperature $T$. Because the bolometric flux is $\propto T^{1/2}$ when only the bremsstrahlung continuum is considered, the differential emission measure becomes $\propto (T/T_s)^{-1/2}$, and $\alpha = 0.5$ by Equation 4. It is slightly different from that of cemekl, $\alpha = 0.43$, and will make the composite spectrum more weighted towards higher temperatures. In the fit by Zemko et al. (2017), this effect is considered to be compensated by the lower $T_s$. (In the relevant temperature range, $\alpha$ would not change very much even considering the lines.)

Suleimanov et al. (2016) also analysed the same outburst data and obtained $M_{\text{WD}} = 0.86 \pm 0.02 \, M_\odot$. This is fully consistent with our estimate. In deriving this result, however, they employed a method that differs from ours in two points: $T_s$ and $R_{\text{in}}$. They fitted the 20–70 keV spectrum in outburst with their newly calculated spectral model (PSR model), and obtained $T_s \sim 26.3 \, \text{keV}$. They also employed, for an illustrative purpose, a single temperature bremsstrahlung model and obtained its temperature as $16.7 \pm 0.2 \, \text{keV}$; via Equation 2 in Suleimanov et al. (2016), this was converted to a consistent shock temperature of $T_s = 26.0 \pm 0.3 \, \text{keV}$. For consistency, we fitted the 20–70 keV spectrum with the bremsstrahlung model, and obtained the temperature as $16.6 \pm 0.3 \, \text{keV}$. Therefore, the present data analysis is consistent with theirs. They also derived $R_{\text{in}} = 2.8 \pm 0.2 \, R_{\odot}$ by the power density spectral analysis (Revnivtsev et al. 2009). After all, their $T_s$ is 1.3 times higher than our $T_s$, and their $R_{\text{in}}$ is 1.5 times larger than ours. These differences in $T_s$ and $R_{\text{in}}$ happened to cancel out, to yield the two $M_{\text{WD}}$ estimates which are very close to each other.

5.4 Systematic Uncertainties of the Mass Estimation

So far, we considered only statistic errors. Here, let us evaluate possible systematic errors that can affect our result.

5.4.1 Emission Models of the Accretion Column

In the present paper, we assumed the accretion columns to have a cylindrical shape, and hence employed the cemekl model with $\alpha = 0.43$ in Equation 4. Recently, emission models with a dipole geometry for the accretion column have been developed, including ACRAD model by Hayashi & Ishida (2014a,b), and the PSR model by Suleimanov et al. (2016). We thus refitted the spectra with the PSR model (ipolar model in XSPEC). The energy range below 7 keV in quiescence was ignored in this analysis because ipolar model has abundances fixed to 1 $Z_\odot$ and cannot reproduce the Fe emission lines which are well described with sub-solar Fe abundance. The shock temperature was then obtained as $20.4^{+0.6}_{-0.5} \, \text{keV}$ in outburst and $37.3^{+2.8}_{-2.2} \, \text{keV}$ in quiescence, and the WD mass was constrained as $M_{\text{WD}} = 0.88 \pm 0.05 M_\odot$. All these values agree well with the results in Section 3 within the statistic errors. Therefore, we consider that slight differences of the emission models, namely the detailed morphological and emissivity structures of the post-shock region, have insignificant impact on the mass estimation method presented above, at least, when applied to GK Persei.

5.4.2 Shock Height

As presented in Equation 5, $T_s$ depends on the shock height $h$, which is thought to negatively correlate with $M$. This effect was theoretically calculated and incorporated in Equation 6. However, even if the effect of $h$ is ignored (i.e. $h = 0$), the value of $M_{\text{WD}}$ changes by less than 0.5%, which is much smaller than the statistic error. Therefore, the value of $h$ itself would not affect the WD mass estimation in GK Persei.

In our spectral analysis, the reflection model was included to represent the reflection effect on the WD surface. The solid angle of the reflection was fixed to $2\pi$ simply assuming that the shock heating occurs just above the WD surface. In reality, $h$ are calculated as $0.014 \, R_{\text{WD}}$ and $0.026 \, R_{\text{WD}}$ by Equation 5, corresponding to the solid angle of $1.8\pi$ and $1.7\pi$ in outburst and quiescence, respectively. We thus repeated the spectral fitting using these solid angles, to find that neither $T_s^{\text{out}}$ nor $T_s^{\text{qui}}$ changes by more than 0.2% Therefore, the result of $M_{\text{WD}}$ is not affected either.

5.4.3 Alfvén Radius

As expressed by Equation 7, we assumed that $R_{\text{in}}$ is equal to the Alfvén radius. This formalism by Elsner & Lamb (1977), which considers spherical accretion, includes two possible uncertainties. One is the coefficient of Equation 7. In the case of disk accretion in rotating magnetic neutron star systems, Ghosh & Lamb (1979) argued that the actual inner-disk radius is almost half the Alfvén radius. Therefore, the estimated values of $R_{\text{in}}$ are probably subject to an uncertainty by a constant factor. However, this uncertainty cancels out in our work by taking the ratio $\gamma = R_{\text{in}}^{\text{qui}}/R_{\text{in}}^{\text{out}}$. Hence, our $M_{\text{WD}}$ estimation is free from the uncertainty, because in Figure 4 we utilise $\gamma$ rather than the actual values of $R_{\text{in}}$.

The other uncertainty is the power law index $\Gamma$ of $M$ in Equations 7, 8, and 10, for which we employ $\Gamma = -2/7$. A recent 3D simulation of accretion flows around a neutron star yields $\Gamma = -1/5$ (Kulkarni & Romanova 2013). In addition, Suleimanov et al. (2016) observationally evaluated $\Gamma = -0.2^{+1.0}_{-1.5}$. We thus quote a systematic uncertainty by $\Delta \Gamma \sim 0.1$, which translates into ~7% systematic errors in $M_{\text{WD}}$.

Adding up all these uncertainties, the overall systematic error in the WD mass estimate becomes comparable to the statistical error. Including this in quadrature, we quote our final mass determination as $M_{\text{WD}} = 0.87 \pm 0.08 \, M_\odot$, with $R_{\text{WD}} = (6.6 \pm 0.6) \times 10^8 \, \text{cm}$.
5.5 Strength of Magnetic Field on WD Surface

Since $M_{\text{WD}}$, $R_{\text{WD}}$, and $R_\text{m}$ were determined, the strength of the magnetic field on the WD surface can be now estimated to be $B \sim 5 \times 10^5$ G using Equation 7 and the distance of the object 477 pc (Harrison et al. 2013a). This value is consistent with the typical magnetic field strength of IPs. However, unlike $M_{\text{WD}}$, this result is subject to the uncertainties discussed in Section 5.4.3. If the coefficient of Equation 7 has an uncertainty by a factor of 2 for example, $B$ changes by a factor of 3–4.

Magnetic field measurements of Polars and strongly magnetised IPs ($\sim 10^7$ G) have been made by detecting spin-modulated polarisation in the near-ultraviolet to near-infrared bands (e.g. Piirla et al. 2008). However, the magnetic field of $\sim 10^5$ G on the WD surface cannot be measured at present. Therefore, the present method provides the only way to measure relatively weak magnetic field of IPs, even though it has a relatively poor accuracy due to the above-mentioned model uncertainty.

6 CONCLUSION

Analysing the outburst and quiescence data of GK Per obtained with NuSTAR, we found the 0.01–100 keV unabsorbed flux was $3.6^{+0.5}_{-0.8} \times 10^{-9}$ erg s$^{-1}$ cm$^{-2}$ and $5.5^{+0.5}_{-0.9} \times 10^{-11}$ erg s$^{-1}$ cm$^{-2}$, respectively, with a factor 65 difference. Analysing the 5–50 keV or 15–50 keV spectra using a multi-temperature spectral model, the shock temperature was determined as $19.7^{+1.3}_{-1.0}$ keV in outburst, and $36.2^{+3.5}_{-3.2}$ keV in quiescence. Assuming that this temperature difference is caused by a compression of the magnetosphere and the associated change in $R_\text{m}$, we determined the WD mass in GK Per as $M_{\text{WD}} = 0.87 \pm 0.08 M_{\odot}$, together with the radius as $R_{\text{WD}} = (6.6 \pm 0.6) \times 10^7$ cm (including a 7% systematic error). The values of $R_{\text{m}}$, relative to $R_{\text{WD}}$, was derived as $R_{\text{m}}/R_{\text{WD}} = 1.9^{+0.4}_{-0.2}$ in outburst and $R_{\text{m}}/R_{\text{WD}} = 7.4^{+2.1}_{-1.2}$ in quiescence, and the mass accretion rate is estimated to have changed by a factor of 120 between the two observations. Combined with optical observations, the inclination angle of GK Per is tightly constrained as $63^\circ \leq i \leq 73^\circ$. We also estimated the magnetic field of the WD as $\sim 5 \times 10^5$ G although it is subject to large systematic error uncertainty. The overall results demonstrate the power of our mass determination method using X-ray luminosity changes, wherein some major systematic uncertainties cancel out.

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