The Ω(2012) as a hadronic molecule

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Recently the Belle collaboration has discovered a narrow $S = -3$ baryon, the Ω(2012). We explore the possibility that the Ω(2012) is a Ξ(1530) $\bar{K}$ molecule, where the binding mechanism is the coupled channel dynamics with the Ωη channel. The characteristic signature of a molecular Ω(2012) will be its decay into the three body channel $\Xi\pi\bar{K}$, for which we expect a partial decay width of $2 - 3$ MeV. The partial decay width into the $\Xi\bar{K}$ channel should lie in the range of $1 - 11$ MeV, a figure compatible with experiment and which we have deduced from the assumption that the coupling involved in this decay is of natural size. For comparison purposes the decay of a purely compact Ω(2012) into the $\Xi\bar{K}$ and $\Xi\pi\bar{K}$ channels is of the same order of magnitude as and one order of magnitude smaller than in the molecular scenario, respectively. This comparison indicates that the current experimental information is insufficient to distinguish between a compact and a molecular Ω(2012) and further experiments will be required to determine its nature. A molecular Ω(2012) will also imply the existence of two- and three-body molecular partners. The two-body partners comprise two Λ hyperons located at 1740 and 1950 MeV respectively, the first of which might correspond to the Λ(1800) while the second to the Λ(2000) or the Λ(2050). The three-body partners include a Ξ(1530) $\bar{K}$ and a Ξ(1530)η $\bar{K}$ molecule, with masses of $M = 2385 - 2415$ MeV and $M = 2434 - 2503$ MeV respectively. We might be tempted to identify the first with the Ξ(2370) and the latter with the Ω(2470) listed in the PDG.

The discovery of the Ω(1673) baryon\textsuperscript{1} confirmed the SU(3)-flavour symmetry of Gell-Mann\textsuperscript{2} and Ne'eman\textsuperscript{3} as the ordering principle of baryon spectroscopy. After five decades the Ω(1673) remains to be the only four star $S = -3$ baryon resonance. Only other three $S = -3$ baryons are listed in the PDG\textsuperscript{4}, the three star Ω(2250) and the two-star Ω(2380) and Ω(2470). Recently the Belle collaboration discovered a new addition to the family, a narrow Ω baryon with a mass $M = 2012.4 \pm 0.7 \pm 0.6$ and a width $\Gamma = 6.4^{+2.5}_{-2.0} \pm 1.6$\textsuperscript{3}.

The most prosaic explanation for the nature of the new Ω(2012) baryon is that of a decuplet $2^- \Omega$ compact state, as indicated by the Belle collaboration itself\textsuperscript{5} and predicted for instance in the Isgur-Karl model at 2020 MeV\textsuperscript{6}. Recent theoretical works have explored this idea further from the point of view of the chiral quark model\textsuperscript{7}, QCD sum rules\textsuperscript{8} and SU(3) flavour symmetry\textsuperscript{9}, in all cases suggesting the quantum numbers $J^P = 2^-$. As a matter of fact the existence of a $2^- \Omega$ baryon in the 2.0 – 2.1 GeV region was already expected from the quark model\textsuperscript{10}, large $N_c$\textsuperscript{11}, the Skyrme model\textsuperscript{12} and lattice QCD\textsuperscript{13}. Here we consider the possibility that the Ω(2012) is molecular instead of fundamental, as proposed in Ref.\textsuperscript{9}. Molecular hadrons are a prolific theoretical concept, which indeed explain the properties of a few hadrons that do not fit into the quark model, see Refs.\textsuperscript{14,15} for recent reviews. But there are a few drawbacks to this idea too: for instance there is no uniform description of hadronic molecules, a few of the approaches are phenomenological and lack a clear connection with QCD and what constitutes a molecular state is sometimes a nebulous concept. In this regard it is important to work in detail, if possible, how the theoretical description of a specific molecule relates to other molecular candidates, whether their internal dynamics can be related to the low energy manifestations of QCD (for instance, chiral symmetry) and how the predictions for fundamental and compound hadrons differ.

Here by a compound Ω(2012) we refer to a hadron in the line of the Λ(1405) or $D_{10}(2317)$, which are suspected to be $NK$\textsuperscript{17,19} and $DK$\textsuperscript{20,21} molecules respectively (the list of possible molecular candidates is long and growing, see Ref.\textsuperscript{14} for instance). Analogously to the two previous examples, the mechanism responsible for the binding of the Ω(2012) will be the Weinberg-Tomozawa (WT) interaction between a baryon and a pseudo Nambu-Goldstone boson. The difference lies in the requirement of coupled channel dynamics: besides the $\Xi(1530)\bar{K}$, which is a natural molecular explanation for the Ω(2012)\textsuperscript{9}, the Ωη channel will also be involved. In fact the WT interaction in the $\Xi(1530)\bar{K}$-Ωη system is\textsuperscript{22,23}

$$V = -\frac{\omega + \omega'}{2f^2} \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix},$$

(1)

with $\omega$ and $\omega'$ the incoming and outgoing energies of the pseudo Nambu-Goldstone bosons and where we are taking the normalization $f = f_\pi = 132$ MeV. This interaction is attractive and particularly strong in the eigen-
\[ \left[ \frac{1}{\sqrt{2}} \left( |\Xi(1530)\bar{K}| + |\Omega\eta| \right) \right], \]  

where its strength indeed matches that of the \( N\bar{K} \) interaction generating the \( \Lambda(1405) \) and surpasses that of the \( DK \) potential that gives rise to the \( D_{s0}(2317) \). The \( \Xi(1530)\bar{K} - \Omega\eta \) interaction is known to be able to generate a pole \( \Xi(1530)\bar{K} \) in the same channel as a consequence of the existence of an \( \Omega\eta \) component in the wave function and that the antikaon takes out momentum of the pion, further reducing the partial decay width. The details of the calculation are analogous to those of the \( \Xi(1530)\bar{K} \) channel. Concrete calculations yield a partial decay width of

\[ \Gamma(\Omega^* \rightarrow \Xi\pi K) \approx 2 - 3 \text{ MeV}, \]

FIG. 1. Feynman diagrams involved in the decays of a molecular \( \Omega^*(1208) \): (a) represents the three body decay into \( \Xi\pi \bar{K} \), which is the characteristic signature of a molecular \( \Omega^*(1208) \), while (b) represents the two body decay into \( \Xi K \), which happens via a short-range operator.

in the regularization we are using here, respectively. The absence of experimental information about the location of a candidate \( \Omega^* \) pole, the previous predictions are completely plausible.

Yet this by itself is not enough to decide whether the \( \Omega(1208) \) is really compatible with the molecular hypothesis. For this it is also necessary to compute its decays, if possible with theoretical uncertainties. We consider the decays into \( \Xi\pi\bar{K} \) and \( \Xi K \) via the mechanisms shown in Fig. 1. For the theoretical uncertainties we will modify the WT interaction as follows

\[ V = - \frac{\omega + \omega'}{2f^2} c(\Lambda) g(p''_\Lambda)g(p'_\Lambda) \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}, \]

that is, we will consider that the coupling runs with the cut-off, instead of being fixed. We then determine \( c(\Lambda) \) from the condition of reproducing the \( \Omega(1208) \) pole. The cut-off will vary in the \( \Lambda = 0.5 - 1.0 \text{ GeV} \) window, which comprises the privileged cut-off for which \( c(\Lambda) = 1 \), i.e. \( \Lambda = 721 \text{ MeV} \). The cut-off window is wide enough as to accommodate the changes of this privileged cut-off owing to subleading order corrections to the baryon-meson interaction. Finally we will interpret the variation of the results with the cut-off as the uncertainty of our calculations.

The most defining feature of a molecular \( \Omega(1208) \) will be the three body decay \( \Omega(1208) \rightarrow \Xi\pi\bar{K} \), see Fig. 1. The partial width of this process can indeed be directly deduced from: (i) the decay \( \Xi(1530) \rightarrow \Xi\pi \) where \( \Gamma \approx 9 - 10 \text{ MeV} \) and (ii) the wave function of the \( \Omega(1208) \), which determines the coupling to the \( \Xi(1530)\bar{K} \) channel. Concrete calculations yield a partial decay width of

\[ \Gamma(\Omega^* \rightarrow \Xi\pi K) \approx 2 - 3 \text{ MeV}, \]

where \( \Omega^* \) refers to the \( \Omega(1208) \). This width is smaller than that of the \( \Xi(1530) \) in the same channel as a consequence of the existence of an \( \Omega\eta \) component in the wave function and that the antikaon takes out momentum of the pion, further reducing the partial decay width. The details of the calculation are analogous to those of the \( X(3872) \rightarrow D\pi \) partial decay width in the molecular description of the \( X(3872) \), which is deduced from the \( D^* \rightarrow D\pi \) amplitude. The only difference with the

1 Here we will follow the formalism of Refs. \[24, 25\] for the two- and three-body integral equations (latter we will show three-body results). For the two-body system, the eigenvalue equation we are using is

\[ \phi_i(p) = \int \frac{d^3q}{(2\pi)^3} \frac{m_i}{\omega(q)} \langle i| V_{ij} |q \rangle \psi_j(q), \]

where \( i = 1, 2 \) refers to the \( \Xi(1530)\bar{K} \) and \( \Omega\eta \) channels, \( m_i, m_\eta, M_\eta, M_{\Xi\pi} \) are the pseudo Goldstone-Boson and Baryon masses, \( E_i \) the energy with respect to the \( i \)-channel threshold, \( \omega(q) = \sqrt{m_i^2 + q^2} \) and \( \xi_i(q) = \omega(q) - m_i \). That is, we will treat the baryons as non-relativistic and the pseudo Nambu-Goldstone bosons as relativistic. The dynamical equation in the two-body sector will be equivalent to the Kadyshevsky equation \[25\], except for the detail of the non-relativistic baryon.

2 If we consider \( f_K \) instead of \( f_s \) for the strength of the Weinberg-Tomozawa, the cut-off at which the \( \Omega(1208) \) binds will be 890 MeV instead, to be compared with 762 MeV for the \( \Lambda(1405) \) and 1042 MeV for the \( D_{s0}(2317) \). Other subleading order corrections will modify the cut-off too, likely within the range of the previous estimates.
work of Ref. [27] is that here we do not include $\Xi K$ rescattering effects, as the $\Xi K$ WT term vanishes. Subleading order corrections to the $(1530)\bar{K}$-$\Omega\eta$ potential will have a moderate impact in the $\Xi\pi\bar{K}$ decay width: there will be a diagonal term in the potential that will change the probability of the $(1530)\bar{K}$ component in the wave function. However taking into account the large uncertainty of the lowest order result, the inclusion of subleading effects is probably not justified. Finally if the $\Omega(2012)$ is a compact state, the three body $\Xi\pi\bar{K}$ decay width is probably of the order of $50 - 100$ KeV. This figure, which we have deduced from phase space, the angular momentum of the final three body state and the size of the decay coupling as estimated from naive dimensional analysis [28], is remarkably smaller than in the molecular scenario.

Yet the decay that is experimentally known is $\Omega(2012) \rightarrow \Xi\bar{K}$, which for a molecular $\Omega(2012)$ happen via the mechanism depicted in the right panel of Fig. [1]. The short-range potential involved in this decay is a $(1530)\bar{K} \rightarrow \Xi\bar{K}$ vertex of the type

$$\langle \Xi K(\vec{p}') | V | \Xi^* \bar{K}(\vec{p}) \rangle = C_D \vec{S} \cdot \vec{q} \vec{S} \cdot \vec{q},$$

(6)

where $\Xi^*$ refers to the $(1530)$, $\vec{S}$ stands for the spin-$\frac{3}{2}$ matrices of the $\Xi^*$, $\vec{S}$ are spin-$\frac{3}{2}$ to -$\frac{1}{2}$ transition matrices and $\vec{q} = \vec{p}' - \vec{p}$ is the momentum transfer $\vec{p}$. The size of the coupling can be estimated from naive dimensional analysis [28]

$$C_D \sim \frac{1}{f^2\Lambda_X},$$

(7)

with $f$ the pion decay constant and $\Lambda_X \sim 1$ GeV the chiral symmetry breaking scale. This gives us

$$\Gamma(\Omega^* \to \Xi\bar{K}) \sim 2 - 11 \text{ MeV} \quad (1 - 5 \text{ MeV}),$$

(8)

for $f = f_\pi$ ($f = f_K \simeq 160$ MeV). The details of the calculation are not presented here, but they are again analogous to the decay of the theoretical X(4012) molecule into $D\bar{D}$, which were presented in Ref. [31] (the only difference is that the decay mechanism in this case is a contact-range operator, instead of one pion exchange).

A few things are worth noticing: the diagram leading to the $\Xi\bar{K}$ decay is linearly divergent, which partly explains the spread of the width in the $\Lambda = 0.5 - 1.0$ GeV range. This is not crucial for the previous estimation, which is there to give a sense of whether the molecular hypothesis is compatible with experiment. But from the point of view of an effective field theory description the interpretation is as follows [32 33]: (i) the coupling $C_D$ is actually a running coupling with scales as the inverse of the cut-off and (ii) as a consequence of this scaling the previous contribution is enhanced with respect to naive dimensional analysis. Notice too that there are also long-range contributions to the two-body decay: if we consider the original three body decay, there is the possibility that the pion rescatters with the antikaon with the latter absorption of the pion by the final $\Xi$ cascade, i.e. a triangle diagram. However this contribution is strongly divergent and cannot be evaluated without the inclusion of higher order counterterms.

This is to be compared with a compact, non-molecular $\Omega(2012)$, in which case the $\Omega(2012)$ will belong to a decuplet with a $\Delta$ isobar partner. As a consequence, the $\Omega^* \rightarrow \Xi\bar{K}$ decay width can be related to the $\Delta^* \rightarrow N\pi$ decay width. The concrete relation can be worked out easily from $SU(3)$-flavour symmetry

$$\Gamma(\Omega^* \to \Xi\bar{K}) \sim 2 \left( \frac{M_{\Omega^*}}{M_{\Delta^*}} \right)^{\frac{32}{21} + 1} \Gamma(\Delta^* \to N\pi),$$

(9)

where the factor 2 is a consequence of the different $SU(3)$ Clebsch-Gordan coefficients, $M_{\Omega^*}$, $M_{\Delta^*}$ are the masses of the baryons involved and $p_\pi$ and $p_\pi$ are the decay momenta of the antikaon and pion respectively. If we assume the quantum numbers of the $\Omega(2012)$ to be $J^P = \frac{3}{2}^+$, $\frac{1}{2}^+$ and $\frac{3}{2}^-$, then it will belong to the same decuplet as the $\Delta(1620)$, $\Delta(1600)$ and $\Delta(1700)$ respectively. Now we can make predictions for the partial decay width to $\Xi\bar{K}$, which can be consulted in Table 1. From this the only identification that is within the experimental limits for the width is the $J^P = \frac{3}{2}^+$, for which the partial decay width is $5 - 14$ MeV. This range is on average broader than, but still compatible with, the experimental value. From this comparison the compact and molecular hadron hypotheses are equivalent in what regards the total decay width. Besides, the theoretical calculation of the $\Omega^* \rightarrow \Xi\bar{K}$ decay width is subjected to large uncertainties independently of whether the $\Omega(2012)$ is a three quark or a molecular state. For a three quark $\Omega^*$ it is worth notic-
ing that flavour symmetry is not expected to work as well for excited baryons as it does for the fundamental ones, particularly regarding masses (yet from Ref. 34, where flavour symmetry relations were successfully applied to excited baryons, SU(3)-flavour seems to work well enough for our purposes). For a molecular $\Omega^*$ the estimate for the $\Xi K$ decay width hinges on dimensional analysis estimations, which can be perfectly off by a numerical factor of order one (explaining, for instance, the smaller partial decay width obtained in Ref. 33). The admixture of the compact and molecular components cannot be ruled out either, as this might be already happening to its $\Delta$ isobar partner: the $\frac{3}{2}^-$ $\Delta(1700)$ can indeed be reproduced from the chiral interaction among the $\Delta\pi\Sigma K$-$\Delta\eta$ baryon-meson channels [22]. Other example is the $\frac{1}{2}^+$ octet, which can be viewed as dynamically generated [23] comprising the $N(1535)$, $\Lambda(1670)$, $\Sigma(1620)$ and $\Xi(1620)$, or as a standard SU(3) octet [31] comprising $N(1535)$, $\Lambda(1670)$, $\Sigma(1620)$ and $\Xi(1620)$, though in this case there is a mismatch about which $\Sigma^*$ baryon completes the multiplet.

A molecular $\Omega(2012)$ also implies the existence of partner states that share the same binding mechanism. The $\Xi K$ and $\Sigma^* K$ WT terms are strong in the $I = 0$ channel

$$V = -3 \frac{\omega + \omega'}{2f^2} c(\Lambda) g(\frac{p'_\ell}{\Lambda}) g(\frac{p}{\Lambda}),$$

(10)

which probably implies the existence of $\Xi K$ and $\Sigma^* K$ bound states with the quantum numbers of a $\Lambda$ baryon ($I = 0, S = 1$). We find two states located at 1740 and 1950 MeV with quantum numbers $\frac{1}{2}^-$ and $\frac{3}{2}^+$ respectively, see Table II for details. The previous calculations assume that the same cut-off can be used for the $\Xi$ and $\Sigma^*$ cascades. They also ignore the $\Sigma \pi$ and $\Sigma^* \pi$ channels: they are expected to move the location of the poles, giving a finite width in the process. If the experience with the $\Lambda(1405)$ is of any help, where the location of the pole moves from 1427 MeV to 1428$-17i$ MeV after the inclusion of the $\Sigma \pi$ channel [19], we will expect moderate correction at most in the location of the previous two $\Lambda$ baryons.

In addition, the existence of both $\Sigma^* K$ and $\Xi^* K$ molecules suggest the existence of a $\Xi^* \Sigma K$ bound state, which can be computed for instance with the formalism of Refs. 21, 22. For this type of three body calculation we have to take into account the $\Xi^* K$ and $\Xi^* K$ WT interaction in the $I = 1$ channel, which is one third of that in the $I = 0$ case. The strength of the $K K$ interaction in the isoscalar channel is determined from the condition of reproducing the $f_0(980)$ pole, which is thought to be molecular [35], while in the isovector channel we set it to one third of the strength in the isoscalar channel as expected for a WT term [10]. From this the position of the $\Xi^* K K$ three-body bound state is about 2385$-2445$ MeV, where the spread reflects the cut-off variation. Notice however that we did not include widths in the calculation: this molecule will have a sizable width from the $K K \to \pi \pi$ transition. This three body exotic $\Xi^*$ might be identified with the $\Xi(2370)$ or maybe with the $\Xi(2500)$ that are included in the PDG [1], but of which little is known. Other possible three body partner is a $\Xi^* K \eta$-$\Omega \eta$ bound state, which binds owing to the coupled channel dynamics of the $\Omega(2012)$. If we assume the $\eta$ to be non-interacting (which greatly simplify the calculations) the location of this state is about 2434$-2503$ MeV. We might identify this $\Omega^*$ with the $\Omega(2470)$ of the PDG. The results for the molecular partners of the $\Omega(2012)$ are summarized in Table II. However the $\frac{3}{2}^+$ $\Xi^*$ and $\Omega^*$ excited baryons we predict in Table II are not necessarily a signature of a molecular $\Omega(2012)$: the diagonal and non-diagonal WT interactions of a compact $\Omega(2012)$ with the kaon and the $\eta$ are strong enough as to generate these $\frac{3}{2}^+$ $\Xi^*$ and $\Omega^*$ states.

To summarize, the molecular hypothesis for the $\Omega(2012)$ is compatible with the experimentally known information about this baryon and might be able to explain a few $\Lambda^*$, $\Xi^*$ and $\Omega^*$ baryons listed in the PDG. The comparison of the molecular and compact baryon scenarios indicates that the total decay width is roughly identical in both cases, the only difference being that a molecular $\Omega(2012)$ is expected to have a sizable branching ratio into $\Xi K K$ of the order of 30$-50%$. This branching ratio is at least one order of magnitude smaller for a compact $\Omega(2012)$, which indicates that this is the experimental quantity to look for if we want to determine the nature of this baryon (a point which has been stressed in the recent literature, see Refs. 8, 35, 36, 40). Other defining feature is what partner states are to be expected in each case. A compact $\Omega(2012)$ belongs to the $\frac{3}{2}^-$ decuplet, which probably comprises the $\Delta(1700)$ and two other $\Sigma$ and $\Xi$ baryons that have not been detected yet with masses of $M_{\Sigma} = 1805 \pm 40$ and $M_{\Xi} = 1910 \pm 40$ MeV respectively [8]. This decuplet pattern is mostly a consequence of SU(3) flavour symmetry rather than of the nature of the baryons and it is also dynamically reproduced by chiral interactions [22]. Maybe the defining difference is that a molecular $\Omega(2012)$ will have additional partners that are dictated by the sign and strength of the Weinberg-Tomozawa interaction. In the two-body sector (baryon-meson) these partners comprise two $\Lambda$ hyperons, a $J^P = \frac{3}{2}^-$ one with a mass of 1740 MeV and another with $J^P = \frac{3}{2}^-$ at 1950 MeV, which might be identified with the $\Lambda(1800)$ and maybe the $\Lambda(2000)$ or $\Lambda(2050)$ respectively. In the three-body sector (baryon-meson-meson) we find a $J^P = \frac{3}{2}^+$ $\Xi(2400)$ cascade and

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Notice that in the isovector channel the $a_0(980)$ is also expected to have a large $K K$ molecular component [35], but probably not so large as to determine the isovector $K K$ interaction from it. If taken into account as in the isoscalar case, it will make the three body states a bit more bound.
TABLE II. Predicted two- and three-body molecular partners of the Ω(2012) as a Ξ∗K−Ωη bound state. We indicate their particle content, their isospin, spin and parity, the relative strength of the WT term (for the two-body case), the binding energy and mass (MeV), plus the prospective candidates among experimentally known baryons in the PDG [4]. The relative strength of the WT interaction is defined such that $V = C_{WT} (\omega + \omega^\prime)/2 f^2 c(\Lambda) g(p/\Lambda) g(p/\Lambda)$, where we always use the same regulator. The bands in the binding energies and masses are a consequence of the cut-off variation in the range $\Lambda = 0.5 - 1.0$ GeV and are interpreted as the theoretical uncertainty of the calculations. The binding energy of the three body bound states is calculated with respect to the three body thresholds $Ξ∗K K$ and $Ξ∗K η$, respectively.

| Molecule | $I(J^P)$ | $C_{WT}$ | $B$ | $M$ | Candidate |
|----------|-----------|-----------|-----|-----|-----------|
| $Ξ^*K−Ωη$ | $0(\frac{3}{2}^-)$ | $(0 \ -3 \ (-3 \ 0)$ | 16 | 2012 | Ω(2012) |
| $ΞK$ | $0(\frac{3}{2}^-)$ | -3 | 69 - 73 | 1739 - 1743 | Λ(1800) |
| $Ξ^*K$ | $0(\frac{3}{2}^-)$ | -3 | 77 - 80 | 1948 - 1951 | Λ(2000), Λ(2050) |
| $Ξ^*KK−ΩηK \ (\frac{1}{2}^+$) | -3 | 78 - 138 | 2385 - 2445 | Ξ(2370), Ξ(2500) |
| $Ξ^*Kη−Ωηη \ 0(\frac{3}{2}^-)$ | -3 | 57 - 126 | 2434 - 2503 | Ω(2470) |

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