An Immunity Placement Method for Suppressing Spread of Viruses

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Abstract

The spread of viruses, such as flu and SARS, can be modeled using social networks. In this study, we modeled the spread of a virus in a social network and attempted to suppress the virus by immunizing nodes. Although a suppression method for virus diffusion using only local information has been proposed, we proposed a new suppression method using degree and the importance of lines centrality. We evaluated the performance of the proposed method for scale-free networks. In addition, we investigated whether our proposed method is valid for various types of social networks. In numerical experiments, the proposed method showed better performance than the conventional suppression method.

1. Introduction

In 2014, the Ebola hemorrhagic fever quickly spread and many people died. In order not to repeat this tragedy, a suppression method for virus diffusion is strongly desired to prevent the loss of our life. A virus can be suppressed using an immunity placement method, which controls the spread of the virus at a limited cost. In this study, we simulated the spread of a virus in a social network as above and attempted to suppress the virus. In [1], a social network was modeled by replacing people with nodes and connection between people with edges. In addition, the virus was modeled by discrete objects that flow on the social network, similarly to vehicles on a traffic network. In this study, we attempted to suppress a virus by placing immunizing nodes in a social network model. Actually, the placement of immunizing nodes using global network information, such as the degree centrality and betweenness centrality, is difficult because the computational complexity and volume of information are enormous. From these viewpoints, an improved acquaintance immunity placement method (IAM) [2], which uses local network information, has been proposed by Motoyoshi and Moriguchi. This method was evaluated using only a scale-free network. In this study, we first evaluated the performance of the IAM [2] for various network topologies. Next, we proposed a new immunity placement method using the DIL (degree and the importance of lines) centrality [3], which can obtain a betweenness-like centrality from local information. In numerical experiments, the proposed method showed better performance than the IAM.

2. Network Model

In this study, we used two topologies of the network model: the BA model proposed by Barabási and Albert [4] and the WS model proposed by Watts and Strogatz[5]. These models have some features observed in the real world.

- Scale-free property
  Most of the networks in the real world, such as friendship networks and the Internet, have a scale-free property [1]. The scale-free property states that some nodes have many edges; however, most nodes have only a few edges. In this study, we generated a scale-free network using the BA model [4].

- Small-world property
  The small-world property states that a network has a large clustering coefficient and a small average distance between nodes. In this study, we generated a small-world network using the WS model [5].

Figures 1(a) and 1(b) respectively show the degree distribution and a scatter plot of the degree and betweenness centrality of a scale-free network generated by the BA model.

![Figure 1: Network characteristics of the BA model](image)

From Fig. 1(a), most nodes have a few edges, and some nodes have many edges. This power-law relationship is called the scale-free property. As shown in Fig. 1(b), the betweenness centrality increases as the degree increases.

- Small-world property
  The small-world property states that a network has a large clustering coefficient and a small average distance between nodes. In this study, we generated a small-world network using the WS model [5].

Figures 2(a) and 2(b) respectively show the degree distribution and a scatter plot of the degree and betweenness centrality of a small-world network generated by the WS model. Comparing networks with rewiring probabilities $p=0$ (circular network) and $p=0.03$, the average distance in the network was smaller and the average cluster coefficient of the network was higher when $p$ was set to 0.03. Thus, the network topology whose rewiring probability was set to 0.03 had a small-world property. In Fig. 2(b), the betweenness centrality increases as the degree of the node increases.
3. Virus Spreading Model

The starting nodes for the spread of the virus were randomly determined using uniformly distributed random numbers. The virus spreading method is shown in Fig. 3. In Fig. 3, \( t \) is the starting time of virus spreading. As shown in Fig. 3(a), the virus spreads from the origin node to some of the adjacent nodes at \( t = 1 \). Here, we used the infection probability \( \beta_0 \), and the infection of the adjacent nodes was determined using this probability. We assumed that all the nodes have the same infection probability. \( \beta \) was a uniformly distributed random number from 0 to 1. If \( \beta \) is larger than \( \beta_0 \), the node is infected. One iteration is terminated after all the infected nodes have attempted to spread the virus to their adjacent nodes. In this study, we carried out 365 iterations. In addition, the infected node recovered its state after three iterations.

\[
\begin{align*}
\text{Betweenness centrality} & = \frac{1}{\text{number of nodes} - 1} \cdot \sum_{i=1}^{\text{number of nodes}} \sum_{j \neq i} \sum_{k \neq i, j} \frac{W_{ij} W_{kj}}{W_{ijk}} \\
\text{DIL centrality} & = \frac{1}{\text{number of nodes} - 1} \cdot \sum_{i=1}^{\text{number of nodes}} \sum_{j \neq i} \frac{W_{ij} W_{kj}}{W_{ijk}} + \frac{1}{\text{number of nodes} - 1} \cdot \sum_{i=1}^{\text{number of nodes}} \sum_{j \neq i} \sum_{k \neq i, j} \frac{W_{ij} W_{kj}}{W_{ijk}} + \frac{1}{\text{number of nodes} - 1} \cdot \sum_{i=1}^{\text{number of nodes}} \sum_{j \neq i} \sum_{k \neq i, j} \frac{W_{ij} W_{kj}}{W_{ijk}}
\end{align*}
\]

We then compared the node importance using the betweenness centrality and DIL centrality. Figure 4 shows the scatter plot of the betweenness centrality and DIL centrality for the BA model (Fig. 4(a)) and WS model (Fig. 4(b)).

4. Immunity Placement Method

In our social network model, we placed immunizing nodes, and a virus was removed from the network when it arrived at the immunizing nodes. Thus, the dispatch strategy of the immunizing nodes determines the virus infection density of the social network. In this study, we selected immunizing nodes as follows:

- **Degree strategy**
  First, we randomly selected one of the nodes from the network. Next, the adjacent node with the largest degree was immunizing.

- **DIL strategy** (proposed method)
  Similar to the degree strategy, we randomly selected one of the nodes from the network. Next, the adjacent node with the largest DIL centrality was immunizing.

5. DIL Centrality

The betweenness centrality is commonly used method for evaluating the importance of nodes in the network. Unfortunately, the calculation cost of the betweenness centrality is \( N^3 \), where \( N \) is the total number of nodes. Therefore, it is difficult to apply this centrality for large networks. In 2016, the DIL (degree and the importance of lines) centrality, which can obtain an approximate value of the betweenness centrality using only local information, was proposed [3]. In this study, we propose a new immunity placement method using the DIL centrality.

The DIL centrality is calculated by the following three equations.

\[
\begin{align*}
I_{ij} & = \frac{(k_i - p - 1)(k_j - p - 1)}{k_i + k_j - 2} \\
W_{n,n_j} & = I_{ij} \frac{k_i - 1}{k_i + k_j - 2} \\
L_{n_i} & = k_i + \sum_{n_j \in N_i} W_{n,n_j}
\end{align*}
\]
have a similar value. In addition, the cross-correlation coefficients between the DIL centrality and the degree centrality for the BA model and the WS model are 0.9267 and 0.4901, respectively.

6. Numerical Experiments

We evaluated the performance of the two immunity placement methods using the density of virus infection. We used the scale-free networks generated by the BA model and virus spreading model described in Sec. 3. In [2], the performance of the IAM was evaluated only for a scale-free network. Thus, we evaluated the performance of each immunity placement method not only for the BA model network but also for the WS model network.

Table 2: Characteristics of networks

| Network model | N  | k     | C   | L   |
|---------------|----|-------|-----|-----|
| BA            | 500| 5.976 | 0.056| 3.190|
| WS            | 500| 6.000 | 0.546| 8.311|

Table 2 lists typical characteristics of each network, where N is the total number of nodes, k is the average degree, C is the average cluster coefficient, and L is the average path length. From Table 2, k has a similar value in both network topologies. In the BA model, both C and L have lower values than whose in the WS model. On the other hand, the cluster coefficient is high and the average path length is low in the WS model.

The density of virus infection for the scale-free networks (BA model) is shown in Fig. 6.

In Fig. 6, the steep negative slope of the graph indicates good performance. As shown in Fig. 6, there is no significant difference between the IAM [2] and our proposed method. This is because the cross correlation between the degree and the DIL centrality was very high in the BA model, and similar immunizing nodes were selected by both methods.

Figure 7 shows the virus infection density and the density of immunizing nodes against the number of iterations. From Figs. 7(a) and 7(b), both methods suppressed the diffusion of the virus when the density of immunizing nodes was less than 0.5 compared with that of the IAM. This result suggests that the proposed method can place the immunizing nodes effectively to block the flows of virus on networks.

Figure 9 shows the virus infection density against the number of iterations. In Fig. 9(a), the density of immunizing nodes using the IAM is 0.3 when the virus infection density is less than 0.4. In Fig. 9(b), the virus infection density using the proposed method is suppressed when the density of immunizing nodes is less than 0.3. In the BA model (Fig. 6), the IAM and the proposed method showed similar performance. However, in the WS model, the proposed method showed better performance than the IAM. Therefore, the proposed method is efficient for placing immunizing nodes in various social networks.

7. Conclusion

In this study, we proposed a new method to suppress the spread of viruses in social networks. We first evaluated two types of immunity placement methods. The first one is a method with degree information, and the second one is a method with DIL centrality. In addition, these methods were evaluated using a scale-free network (BA model) and a small-world network (WS model). These network models were re-
The density of immunized nodes in network

Figure 8: Virus infection density against the density of immunizing nodes (WS model)

Figure 9: Virus infection density against the number of iterations (WS model)

In future works, we will attempt to evaluate the performance of each immunity placement method using more realistic virus diffusion models.

The research of T.K. was partially supported by a Grant-in-Aid for Young Scientists (B) from JSPS (No.16K21327).

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