Mathematical model of the extraction of vegetable oil by an organic solvent

Yu P Barmetov¹, E D Chertov¹, V G Matveykin², V K Bityukov¹, Yu V Bugaev¹ and A A Bush¹

¹ Voronezh State University of Engineering Technologies, 19, Revolutzii Ave., Voronezh, 394036, Russian Federation
² Tambov State Technical University, 106, Sovetskaya st., Tambov, 392000, Russian Federation

E-mail: barmetovu@mail.ru

Abstract. A mathematical model of the process of extraction of vegetable oil by an organic solvent for a separate section of the extractor with irrigation has been developed. The model is intended for the synthesis of an automatic control system. A description of the change in the oil concentration in the oilcake along the length of the extraction section and in the miscella for a steady-state operation is obtained. And a system of independent differential equations with a delayed argument is obtained that connects deviations from the nominal oil concentrations in the oilcake and miscella at the outlet of the section with flow deviations and oil concentrations at the inlet of the section. The transition from partial differential equations to equations with a delayed argument will reduce the time spent on calculating the adjustable parameters of the control system during its synthesis.

1. Introduction

One method of increasing the degree of extraction of vegetable oil from raw materials is its extraction from the oilcake by an organic solvent which is benzine or hexane. One applies several schemes of extraction by immersion of oilcake into a solvent or its irrigation [1] – [3]. In this paper we consider the scheme with irrigation, in accordance with which the oilcake is evenly distributed on the mesh conveyor belt and is moved into the zone of extraction, represented by a sequence of separate sections [4], [5]. The irrigation of oilcake happens in each section with a solution of oil in organic solvent, called miscella. Miscella for irrigation is taken by a circulating pump from the miscella storage tank in the same section, it is heated and mixed with fresh miscella with a lower concentration of oil or solvent. With the passage of the miscella through the oilcake layer it absorbs oil at the expense of diffusion process and gets into the storage tank. Part of the miscella from the storage tank is poured in the next section. During the movement oilcake is agitating with the stirring mass in the vertical section.

Management objectives are to decrease the concentration of residue oil in the oilcake and increasing the oil concentration in the miscella at the exit of the extractor, while ensuring the necessary performance for the treated oilcake [4]. Since these requirements are contradictory, management is conducted either by the criterion of minimum losses of oil when the minimum concentration is in the miscella at the exit of the extractor and productivity, or by the criterion of maximum oil concentration in the miscella while limiting the loss of oil and oilcake. The process
control is carried out by changing the solvent consumption and the speed of the conveyor. The use of the multivariable control system with the expenses management of added fresh miscella or solvent in each partition will improve the quality of the process [6]. For the synthesis of such system, the mathematical model of the mass transfer processes in sections is needed.

2. Writing equations in general form

The section is a section of the conveyor on which the cake is moistened with miscella and irrigated with a mixture of miscella from the collection of the same section and fresh, with a known oil concentration. It is necessary to write down a system of differential equations relating the oil concentrations in miscella and oil cake at the outlet of the section to the consumption of oilcake and fresh miscella added for irrigation, as well as the oil concentrations in them at the inlet of the section.

2.1. Key Assumptions

The cross-sectional area of the cake layer on the conveyor, the relative volume of cake in its mixture with miscella on the conveyor, the specific surface area of granules or petals per unit volume of cake is constant; temperatures, product densities and diffusion coefficients of oil from oilcake to miscella are constant; the longitudinal movement of the cake corresponds to ideal displacement; the oil concentration in the cake and miscella over the height of the cake layer as a result of loosening are constant.

2.2. Differential equations of mass transfer

Based on the equations of material balance for vegetable oil in the cake for a small time interval $dt$ and a small section of the conveyor $dz$, we write the differential equations:

$$ F \cdot \frac{\partial}{\partial t} x_1(t, z) + Q_1 \cdot \frac{\partial}{\partial z} x_1(t, z) + F \cdot f \cdot k \cdot x_1(t, z) = F \cdot f \cdot k \cdot x_2(t) , \quad (1) $$

$$ [F \cdot L \cdot (1 - \varepsilon) + V_c] \frac{dx_2}{dt} + [Q_2 + Q_1 \cdot (1 - \varepsilon) + F \cdot f \cdot k \cdot L] x_2(t) = $$

$$ = Q_1 \cdot (1 - \varepsilon) x_{2,1} (t) + Q_2 \cdot x_{2,2} (t) + F \cdot f \cdot k \cdot \int_0^L x_1(t, z) dz . \quad (2) $$

Here $Q_1$ – olumetric flow of a mixture of oilcake and miscella on the conveyor; $Q_2$ – olumetric flow of a mixture of oilcake and miscella on the conveyor; $x$ – concentration of oil in the product, moreover, index 1 corresponds to the parameters of oilcake, index 2 corresponds to miscella; $x_{2,1}$, $x_{2,2}$ – the concentration of oil in miscella contained in the cake at the inlet of the section and fresh miscella added for irrigation; $F$ – cake cross-sectional area; $\varepsilon$ – relative volume of oilcake in a mixture of it with miscella on the conveyor; $f$ – specific area of granules or petals mass transfer of one cubic meter of cake; $t$ – time; $z$ – section longitudinal coordinate; $k$ – mass transfer coefficient; $V_c$ – volume if the miscella in the collection; $L$ – section length.

3. Solving equations for steady state

In the steady state, at constant flow rates of oilcake and fresh miscella, the system of equations (1), (2) takes the form:

$$ Q_1 \cdot \varepsilon \cdot \frac{d}{dz} x_1(z) + F \cdot f \cdot k \cdot x_1(z) = F \cdot f \cdot k \cdot x_2 , \quad (3) $$
Expressing \( x_2 \) from the equation (4):

\[
x_2 = \frac{Q_1 \cdot (1-\varepsilon)x_2 + Q_2 x_{2,2} + F \cdot f \cdot k \cdot L \cdot x_2}{Q_2 + Q_1 \cdot (1-\varepsilon) + F \cdot f \cdot k \cdot L} \int_0^L x_1(z)dz.
\]

and substituting in (3), we obtain the Fredholm integro-differential equation for the function \( x_1(z) \). It can be written in a general form as:

\[
\frac{1}{A} \frac{dx_1}{dz} + x_1(z) = A_1 \int x_1(z)dz + D_1 \cdot x_{2,1} + D_2 \cdot x_{2,2},
\]

where \( A = \frac{F \cdot f \cdot k}{Q_1 \cdot (1-\varepsilon)} \), \( A_1 = \frac{F \cdot f \cdot k}{Q_2 + Q_1 \cdot (1-\varepsilon) + F \cdot f \cdot k \cdot L} \), \( D_1 = \frac{Q_1 \cdot (1-\varepsilon)}{Q_2 + Q_1 \cdot (1-\varepsilon) + F \cdot f \cdot k \cdot L} \),
\[
D_2 = \frac{Q_2}{Q_2 + Q_1 \cdot (1-\varepsilon) + F \cdot f \cdot k \cdot L}.
\]

Since the coefficients \( A, A_1, D_1, D_2 \) and the initial concentrations \( x_{2,1}, x_{2,2} \) are constant in equation (6), we seek its solution in the form:

\[
x_1(z) = C_1 \cdot e^{-Az} + C_2.
\]

The constants \( C_1 \) and \( C_2 \) are found taking into account the fulfillment of equation (6) and the initial conditions \( z = 0, x_1(0) = x_{1,0} \):

\[
A_1 \cdot \exp(-A \cdot L) \cdot C_1 + A \cdot (1-A_1 \cdot L) \cdot C_2 = A \cdot D_1 \cdot x_{2,1} + A \cdot D_2 \cdot x_{2,2},
\]

\[
C_1 + C_2 = x_{1,0},
\]

where \( x_{1,0} \) - oil concentration in the cake at the inlet of the section.

From system (8), (9) we obtain:

\[
C_1 = \frac{A_1}{A}, \quad C_2 = \frac{A_2}{A},
\]

\[
A = A_1 \cdot \exp(-A \cdot L) - A \cdot (1-A_1 \cdot L), \quad A_1 = A \cdot D_1 \cdot x_{2,1} + D_2 \cdot x_{2,2} - (1-A_1 \cdot L) \cdot x_{1,0},
\]

\[
A_2 = A_1 \cdot \exp(-A \cdot L) - A \cdot (D_1 \cdot x_{2,1} + D_2 \cdot x_{2,2}).
\]

The concentration of oil in the miscella collection in accordance with (5) and the expressions for \( A, A_1, D \) will be equal to:

\[
x_2 = \frac{A_1 \cdot C_1}{A} \left[ 1 - \exp(-A \cdot L) \right] + A_1 \cdot L \cdot C_2 + D_1 \cdot x_{2,1} + D_2 \cdot x_{2,2}.
\]
4. Dynamic equation conversion

4.1. Writing equations in deviations of variables from nominal values
We linearize equations (1), (2) to obtain a dynamic linear model for variable flow rates and initial oil concentration. We go to the deviations in time for concentrations and flows, relative to their nominal values at the initial instant and we get:

\[ F \cdot \varepsilon \cdot \frac{\partial}{\partial t} \Delta x_1(t, z) + Q_1 \cdot \varepsilon \cdot \frac{\partial}{\partial z} \Delta x_1(t, z) + F \cdot f \cdot k \cdot \Delta x_1(t, z) = \]
\[ = F \cdot f \cdot k \cdot \Delta x_2(t) - \Delta Q_1(t) \cdot \varepsilon \cdot \frac{\partial}{\partial z} x_1(t, z) \bigg|_{t=0}, \]

\[ [F \cdot L \cdot (1-\varepsilon) + Vc] \frac{d \Delta x_2}{dt} + [Q_2 + Q_1 \cdot (1-\varepsilon) + F \cdot f \cdot k \cdot L] \Delta x_2(t) = \Delta Q_2(t)(x_{2,2} - x_2) + \]
\[ + \Delta Q_1(t) \cdot [(1-\varepsilon)(x_{2,1} - x_2)] + Q_1 \cdot (1-\varepsilon) \Delta x_{2,1}(t) + Q_2 \Delta x_{2,2}(t) + F \cdot f \cdot k \cdot \int_0^L \Delta x_1(t, z) dz, \]

where \( Q_1, Q_2 \) - nominal values of expenses at the initial time; \( \Delta Q_1(t), \Delta Q_2(t) \) - deviations of expenses in time from nominal; \( \Delta x_1(t, z), \Delta x_{2,1}(t), \Delta x_{2,2}(t) \) - deviations of oil concentrations from nominal; \( \frac{\partial x_1(t, z)}{\partial z} \bigg|_{t=0} \) - the profile of the derivative concentration of oil in the cake along the length of the section at the initial time; \( x_{2,1}, x_{2,2} \) - nominal values of the concentration of oil in the miscella that came with the cake and added for irrigation.

4.2. Using Laplace transforms to simplify equations.
We will simplify the system of equations using Laplace transforms. Since the values of the increments at zero time are equal to zero, in the Laplace transforms, equations (10), (11) can be written as:

\[ \frac{\partial}{\partial z} \Delta X_1(s, z) + \frac{F \cdot (s \cdot \varepsilon + f \cdot k)}{Q_1 \cdot \varepsilon} \Delta X_1(s, z) = \frac{F \cdot f \cdot k}{Q_1 \cdot \varepsilon} \Delta X_2(s) - \frac{\Delta Q_1(s)}{Q_1} \frac{\partial}{\partial z} x_1(t, z) \bigg|_{t=0}, \]

\[ \Delta X_2(s) = \left[ F \cdot L \cdot (1-\varepsilon) + M \right] s + Q_2 + Q_1 \cdot (1-\varepsilon) + F \cdot f \cdot k \cdot L \left[ x_{2,2} - x_2 \right] \Delta Q_2(s) + \]
\[ + \left[ (1-\varepsilon)(x_{2,1} - x_2) \right] \Delta Q_1(s) + Q_1 \cdot (1-\varepsilon) \Delta X_{2,1}(s) + Q_2 \Delta X_{2,2}(s) + F \cdot f \cdot k \int_0^L \Delta x_1(s, z) dz, \]

where \( \Delta X_1(s, z), \Delta X_2(s), \Delta X_{2,1}(s), \Delta X_{2,2}(s), \Delta Q_1(s), \Delta Q_2(s) \) - Laplace images of deviations of oil concentrations and costs from nominal, \( s \) – Laplace variable.

We substitute the expression for \( \Delta X_2(s) \) from (13) into (12) and write the equations in the general form:

\[ \frac{\partial}{\partial z} \Delta X_1(s, z) + A_d(s) \cdot \Delta X_1(s, z) = a_0 \cdot [A_d(s) \int_0^L \Delta X_1(s, z) dz + B_d(s) \cdot \Delta G_2(s) + D_{d,1}(s) \Delta X_{2,1}(s) + \]
\[ + D_{d,2}(s) \Delta X_{2,2}(s) + D_{d,3}(s) \cdot \Delta Q_1(s) - \frac{\Delta Q_1(s)}{Q_1} \frac{\partial}{\partial z} x_1(t, z) \bigg|_{t=0}, \]

\[ \Delta X_2(s) \]
\[ \Delta X_2(s) = A_{di}(s) \int_0^L \Delta X_1(s, z) dz + B_d(s) \cdot \Delta Q_2(s) + D_{d,1}(s) \Delta X_{2,1}(s) + D_{d,2}(s) \Delta X_{2,2}(s) + D_{d,3}(s) \Delta Q_1(s). \]  

(15)

where \( A_d(s) = a_1 s + a_0 \), \( A_{di}(s) = \frac{b_1}{b_0 s + 1} \), \( B_d(s) = \frac{b_2}{b_0 s + 1} \), \( D_{d,1}(s) = \frac{d_1}{b_0 s + 1} \), \( D_{d,2}(s) = \frac{d_2}{b_0 s + 1} \),

\[ D_{d,3}(s) = \frac{d_3}{b_0 s + 1} \cdot a_0 = \frac{F \cdot f \cdot k}{Q_1 \cdot \varepsilon}, \quad a_1 = \frac{F}{Q_1}, \quad a_2 = Q_2 + Q_1 \cdot (1 - \varepsilon) + F \cdot f \cdot k \cdot L, \]

\[ b_0 = \frac{F \cdot L \cdot (1 - \varepsilon) + V_c}{a_2}, \quad b_1 = \frac{F \cdot f \cdot k}{a_2}, \quad b_2 = \frac{x_2 + x_{2,2}}{a_2}, \quad d_1 = \frac{Q_1 \cdot (1 - \varepsilon)}{a_2}, \quad d_2 = \frac{Q_2}{a_2}, \]

\[ d_3 = \frac{(1 - \varepsilon)x_2 + (1 - \varepsilon)x_{2,1}}{a_2}. \]

As can be seen, in addition to the integral component, the right-hand side of equation (14) contains four terms, independent of \( z \), and one term dependent on \( z \).

If we consider the extraction section as a control object, then \( \Delta x_1(t) \) and \( \Delta x_2(t) \) are controlled state variables, \( \Delta Q_2(t) \) - control action, \( \Delta x_{2,2}(t) \) can be both a control action and a disturbance; \( \Delta Q_1(t) \) and \( \Delta x_{2,1}(t) \) - disturbances.

4.3. Converting equations at a constant flow of cake

We consider the case when the flow rate of the mixture of cake and miscella on the conveyor is constant, i.e. \( \Delta Q_1(s) = 0 \). We seek the solution of equation (14) in the absence of a term depending on the coordinate \( z \) in the right-hand side, similarly to the solution of equation (6) under initial conditions: \( z = 0, \Delta X_1(0) = \Delta X_{1,0}(s) \) in the form of:

\[ \Delta X_1(s, z) = C_1(s) \exp[-A_d(s) \cdot z] + C_2(s). \]  

(16)

To determine \( C_1(s) \) and \( C_2(s) \) we substitute solution (16) into equation (14) and obtain one equation, and for \( z = 0 \) from solution (16) taking into account \( \Delta X_1(s, 0) = \Delta X_{1,0}(s) \), we write the second equation:

\[ \frac{A_{di}(s)}{A_d(s)} \left( \exp[-A_d(s) \cdot L] - 1 \right) C_1(s) + \left[ \frac{A_d(s)}{a_0} - A_{di}(s) L \right] C_2(s) = B_d(s) \cdot \Delta Q_2(s) + D_{d,1}(s) \Delta X_{2,1}(s) + D_{d,2}(s) \Delta X_{2,2}(s), \]

\[ C_1(s) + C_2(s) = \Delta X_{1,0}(s). \]

\( C_1(s) \) and \( C_2(s) \) can be defined as:

\[ C_1(s) = \frac{\Delta_1(s)}{\Delta(s)}, \quad C_2(s) = \frac{\Delta_2(s)}{\Delta(s)}, \]

where

\[ \Delta(s) = \frac{A_{di}(s)}{A_d(s)} \left( \exp[-A_d(s) \cdot L] - 1 \right) - \frac{A_d(s)}{a_0} + A_{di}(s) L. \]
The Laplace image of the distribution of oil concentration in the cake along the length of the conveyor depends on the change in the consumption of fresh miscella for irrigation, the oil concentration in the miscella, and the cake at the inlet of the section:

\[
\Delta X_1(s) = \frac{a_0 B_d(s) A_d(s)}{Z(s)} \Delta Q_2(s) + \frac{a_0 D_{d,1}(s) A_d(s)}{Z(s)} \Delta X_{2,1}(s) + \frac{a_0 D_{d,2}(s) A_d(s)}{Z(s)} \Delta X_{2,2}(s) + \frac{a_0 A_{d,1}(s)}{Z(s)} \left[ \exp[-A_d(s) \cdot L] - 1 \right] \Delta X_{1,0}(s),
\]

where

\[
Z(s) = A_{d,1}(s) a_0 \left[ \exp[-A_d(s) \cdot L] - 1 \right] - A_d^2(s) + A_{d,1}(s) \cdot A_d(s) \cdot L a_0.
\]

Multipliers before \(\Delta Q_2(s), \Delta X_{2,1}(s), \Delta X_{2,2}(s)\) and \(\Delta X_{1,0}(s)\) are the transfer functions from the corresponding input to the concentration of oil in the cake. At \(z=L\) we get an image of the oil concentration in the cake at the outlet of the section. We obtain the solution for \(\Delta X_2(s)\) by substituting from (17) into equation (15):

\[
\Delta X_2(s) = B_d \left[ \frac{H(s)}{Z(s)} + 1 \right] \Delta Q_2(s) + D_{d,1} \left[ \frac{H(s)}{Z(s)} + 1 \right] \Delta X_{2,1}(s) + D_{d,2} \left[ \frac{H(s)}{Z(s)} + 1 \right] \Delta X_{2,2}(s) + \frac{A_d(s) \exp[-A_d(s) \cdot L] - 1}{Z(s)} \Delta X_{1,0}(s),
\]

where

\[
H(s) = A_{d,1}(s) \cdot A_d(s) \cdot a_0 \int_0^L \left[ \exp[-A_d(s) \cdot z] - 1 \right] dz = A_{d,1}(s) \cdot a_0 \left[ 1 - \exp[-A_d(s) \cdot L] - A_d(s) \cdot L \right].
\]

### 4.4. Return to the time domain

To go from the Laplace transforms to the time domain, it is necessary to bring the right-hand sides of the equations to fractional rational functions. We expand the expression for \(Z(s)\) and group the terms in powers of \(s\):

\[
Z(s) = -\frac{\alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0 - \alpha \cdot e^{-a_0 L} s}{b_0 s + 1},
\]

where

\[
\alpha_0 = \left( a_0 - b_1 \cdot L a_0 + b_1 \right) a_0, \quad \alpha_1 = \left( 2a_1 + b_0 - b_1 \cdot a_1 \cdot L \right) a_0, \quad \alpha_2 = 2a_1 a_0 b_0 + a_1^2,
\]

\[
\alpha_3 = a_1^2 b_0, \quad \alpha = b_1 a_0 e^{-a_0 L}.
\]

We multiply the left and right sides of equation (17) by \(-Z(s)(b_0 s + 1)\). Then the left side of the equation in the time domain is written as:
\[\alpha_3 \frac{d^3 \Delta x_1}{dt^3} + \alpha_2 \frac{d^2 \Delta x_1}{dt^2} + \alpha_1 \frac{d \Delta x_1}{dt} + \alpha_0 \cdot \Delta x_1(t) - \alpha \cdot \Delta x_1(t - \tau), \tag{19}\]

where \(\tau = a_1 L\) - time-lag [7].

The first three terms on the right side of equation (17) after multiplying by \(-Z(s)(b_0 s + 1)\) will have a common factor

\[a_0 \left[(a_1 s + a_0) - (a_1 s + a_0)e^{-a_0 L} \cdot e^{-a_1 L s}\right]\]

and when going into the time domain for these three terms, we can write expressions of the form:

\[a_0 \beta \left[a_1 \frac{dy}{dt} + a_0 y(t) - \left(a_1 \frac{d}{dt} y(t - \tau) + a_0 y(t - \tau)\right)e^{-a_0 L}\right], \tag{20}\]

where \(\beta\) and \(y\) change in pairs to \(b_2\) and \(\Delta Q_2(t)\), \(d_1\) and \(\Delta x_{1,1}(t)\), \(d_2\) and \(\Delta x_{2,2}(t)\).

Multiplier before \(\Delta X_{1,0}(s)\), the fourth term on the right side of equation (17) after multiplying by \(-Z(s)(b_0 s + 1)\), can be converted to:

\[
\left[(a_1 s + a_0)^2(b_0 s + 1) - b_1 L a_0 (a_1 s + a_0)\right]e^{-a_0 L} \cdot e^{-a_1 L s} + a_0 b_1 - a_0 b_1 \cdot e^{-a_0 L} \cdot e^{-a_1 L s} = \\
\alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + a_0 = 2a_0 b_1 \right]e^{-a_0 L} \cdot e^{-a_1 L s} + a_0 b_1 ,
\]

and this term in the time domain will be equal to:

\[
\left[\alpha_3 \frac{d^3 \Delta x_{1,0}(t - \tau)}{dt^3} + \alpha_2 \frac{d^2 \Delta x_{1,0}(t - \tau)}{dt^2} + \alpha_1 \frac{d \Delta x_{1,0}(t - \tau)}{dt} + \alpha_0 \cdot \Delta x_{1,0}(t - \tau)\right]e^{-a_0 L} + a_0 b_1 \Delta x_{1,0}(t). \tag{21}\]

In equation (18), the right-hand side contains common factors of the form \(\frac{H(s)}{Z(s)} + 1\), in which connection:

\[H(s) = \frac{-b_1 a_0 a_1 L \cdot s + b_1 a_0 (1 - a_0 L) - b_1 a_0 \cdot e^{-a_0 L} \cdot e^{-a_1 L s}}{b_0 s + 1}.
\]

Simplifying the expression \(\frac{H(s)}{Z(s)} + 1\), we get

\[\frac{H(s)}{Z(s)} + 1 = \frac{\alpha_3 s^3 + \alpha_2 s^2 + (2a_1 + b_0)a_0 \cdot s + a_0^2}{\alpha_3 s^3 + \alpha_2 s^2 + a_1 \cdot s + a_0 - \alpha \cdot e^{-a_1 L s}}.
\]

Since the first factors of the three terms on the right-hand side of equation (18) contain the denominator \(b_0 s + 1\) not to increase the order of the equation, we introduce an intermediate variable \(\Delta X(s)\) related to \(\Delta X_2(s)\), the dependence \((b_0 s + 1)\Delta X_2(s) = \Delta X(s)\) or in the time domain:

\[b_0 \frac{d \Delta x_2}{dt} + \Delta x_2(t) = \Delta x(t) .
\]

Multiplying the left and right sides of equation (21) by \(Z(s)(b_0 s + 1)\) and taking into account the dependence \((b_0 s + 1)\Delta X_2(s) = \Delta X(s)\), we write the left side of the differential equation for \(\Delta x(t)\) :
The right side of the equation for the first three terms will be represented by the sum of terms of the form:

\[ \beta \left( \alpha_3 \frac{d^3 y}{dt^3} + \alpha_2 \frac{d^2 y}{dt^2} + \alpha_1 \frac{dy}{dt} + \alpha_0 \cdot y(t) \right), \]  

where \( \beta \) and \( y \) change in pairs to \( b_2 \) and \( \Delta Q_2(t) \), \( d_1 \) and \( \Delta x_{1,1}(t) \), \( d_2 \) and \( \Delta x_{2,2}(t) \).

The fourth term in the time domain is:

\[
b_0 \frac{d}{dt} \Delta x_{1,1}(t) - \left[ b_0 a_1 \frac{d^2}{dt^2} \Delta x_{1,1}(t - \tau) + (b_0 + a_1) \frac{d}{dt} \Delta x_{1,1}(t - \tau) + \alpha_0 \cdot y(t) \right] e^{-\alpha_0 L}.
\]

Thus, a system of equations in ordinary derivatives is obtained.

5. Results

The transformed equations (19) – (23) were solved numerically for approximate values of the parameters of the «De Smet» oil extractor, given in [4], [8] – [10]: \( Q_1 = 0.006 \text{ m}^3/\text{s}; \) \( Q_2 = 0.0015 \text{ m}^3/\text{s}; \)

\( x_{1,0} = 0.2; \) \( x_{2,1} = 0.19; \) \( x_{2,2} = 0.15; \) \( F = 2 \text{ m}^2; \) \( \varepsilon = 0.8; \) \( k = 1.4 \text{ 10}^{-7} \text{ m/s}; \)

\( V_c = 1 \text{ m}^3; \) \( L = 1.2 \text{ m.} \) The solutions are obtained by the finite difference method with constant steps. The initial profile of the oil concentration in the cake along the conveyor length was calculated using the formula (7). The results of the solution shown in Fig. 1 correspond to physical representations of ongoing processes, close to experimental data. Unlike the original equations (1) and (2), equations (19) - (23) are easy to program and require little machine time.

![Figure 1](image-url)

Figure 1. Transients for the concentration of oil in the cake when increasing the concentration of oil by 0.01 at the entrances of the section: 1 - \( \Delta x_{1,0} \), 2 - \( \Delta x_{2,2} \), 3 - \( \Delta x_{2,1} \); 4 – transition process with an increase in the consumption of fresh miscella for irrigation by 0.001 m\(^3\)/s

6. Conclusions

A mathematical model of the vegetable oil extraction process was proposed for a separate section of a horizontal vegetable oil extractor with irrigation in the form of a system from a partial differential equation and an integro-differential equation.

The solution of a system of equations for a stationary process is obtained.

A transformed dynamic model of the process is obtained in the form of a system of differential equations with a lagging argument, the solution of which by the numerical method requires 10-100 times less machine time compared to that of the original system (1), (2). The model can be useful for...
the synthesis of control systems for similar extractors with appropriate refinement of some experimental parameters, such as $F$, $\varepsilon$, $f$, $k$, $M$, which can be obtained in laboratory conditions or directly during experimental operation of the extractor.

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