New Physics in $B_s^0 \to J/\psi\phi$: a General Analysis

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Abstract: Recently, the CDF and D0 collaborations measured indirect CP violation in $B_s^0 \to J/\psi\phi$ and found a hint of a signal. If taken at face value, this can be interpreted as a nonzero phase of $B_s^0\bar{B}_s^0$ mixing ($\beta_s$), in disagreement with the standard model, which predicts that $\beta_s \approx 0$. In this paper, we argue that this analysis may be incomplete. In particular, there can be new physics (NP) in the $\bar{b} \to \bar{s}c\bar{c}$ decay. If so, the value of $\beta_s$ is different than for the case in which NP is assumed to be present only in the mixing. We have examined several models of NP and found that, indeed, there can be significant contributions to the decay. These effects are consistent with measurements in $B \to J/\psi K^*$ and $B_d^0 \to J/\psi K_s$. Due to the NP in the decay, polarization-dependent indirect CP asymmetries and triple-product asymmetries are predicted in $B_s^0 \to J/\psi\phi$.

Keywords: $B$ decays, CP violation, Flavor-changing neutral current.
1. Introduction

Over the past several years, the measurements of several quantities in a number of $B$ decays differ from the predictions of the Standard Model (SM) by $\sim 2\sigma$. For example, in $B \to \pi K$, it is difficult to account for all the experimental measurements within the SM [1]. Also, the SM predicts that the measured indirect (mixing-induced) CP asymmetry in $\bar{b} \to \bar{s}$ penguin decays should generally be equal to that in $B_s^0 \to J/\psi K^*$. However, it is found that these two quantities are not identical for several decays [2]. In addition, the measurement of the lepton forward-backward asymmetry in $\bar{B} \to \bar{K}^*\mu^+\mu^-$ ($A_{FB}$) is not in perfect agreement with the predictions of the SM [3]. Finally, in $B \to \phi K^*$, the final-state particles are vector mesons, so that this decay is in fact three separate decays, one for each polarization (one longitudinal, two transverse). Naively, one expects the fraction of transverse decays, $f_T$, to be much less than the fraction of longitudinal decays, $f_L$. However, it is observed that these two fractions are roughly equal: $f_T/f_L (B \to \phi K^*) \simeq 1$ [4].

In most cases, the individual “disagreements” with the SM are not statistically significant. And it may be possible to explain the value of $f_T/f_L$ within the SM, though this is not certain. Thus, no real discrepancy with the SM can be claimed. Still, the differences are intriguing since (i) there are several $B$ decays involved and a number of different effects, and (ii) they all appear in $\bar{b} \to \bar{s}$ transitions. (Indeed, the Belle experiment itself has claimed that the disagreement in $A_{FB}$ shows a clear hint
of physics beyond the SM. New-physics (NP) scenarios have been considered to explain the possible problems. Various models have been proposed, all of which contain new contributions to the decay $\bar{b} \rightarrow \bar{s}q\bar{q}$ $(q = u,d, or s)$.

Recently, a new effect has been seen in $B^0_s \rightarrow J/\psi\phi$. In the SM, the decay of $B^0_d \rightarrow J/\psi K_S$ is dominated by the tree-level transition $\bar{b} \rightarrow \bar{c}c\bar{s}$ and is real, so that indirect CP violation probes only the phase of $B^0_d-\bar{B}^0_d$ mixing, $\beta$. This mixing phase has been measured: $\beta = (21.58^{+0.90}_{-0.81})^\circ$ [6]. The same logic can be applied to the $B^0_s$ system. The decay of $B^0_s \rightarrow J/\psi\phi$ is similar to that of $B^0_d \rightarrow J/\psi K_S$. Thus, indirect CP violation in $B^0_s \rightarrow J/\psi\phi$ can be used to probe the phase of $B^0_s-\bar{B}^0_s$ mixing, $\beta_s$ (which is $\simeq 0$ in the SM). The CDF [7] and D0 [8] collaborations have presented 2-dimensional correlations of $\beta_s$ vs. $\Delta\Gamma$. In a recent conference proceeding [9], the results of the two experiments were combined, and the CDF and D0 collaborations claimed a 2.2$\sigma$ deviation from the prediction of the SM. This could hint at NP in $B^0_s \rightarrow J/\psi\phi$, and this is the assumption we will use in the rest of the paper.

The phase of $B^0_s-\bar{B}^0_s$ mixing therefore appears to differ from 0 by more than 2$\sigma$. Many theoretical papers have been written exploring the prediction for $\beta_s$ of various NP models [10, 11, 12, 13, 14, 15, 16, 17]. The main purpose of this paper is to point out that this analysis may be incomplete. In particular, the possibility of new physics in the decay of $B^0_s \rightarrow J/\psi\phi$ has not been included. However, this is important – most NP which gives a phase in $B^0_s-\bar{B}^0_s$ mixing also contributes to the decay. As an example, consider the model with $Z$-mediated flavor-changing neutral currents [10]. A $Z\bar{b}s$ coupling will lead to $B^0_s-\bar{B}^0_s$ mixing at tree level. However, this same coupling will give the (tree-level) decay $\bar{b} \rightarrow \bar{s}c\bar{c}$, mediated by an off-shell $Z$. (In fact, in certain models, there is little contribution to $\beta_s$; the main NP effect is in the decay [13].) It is therefore quite natural to consider the possibility of NP in the decay $B^0_s \rightarrow J/\psi\phi$.

However, once one does this, the conclusion regarding the phase of $B^0_s-\bar{B}^0_s$ mixing is no longer justified. While there is NP in $B^0_s \rightarrow J/\psi\phi$ (our assumption), it need not be only in the mixing. It could also be in the decay, and $\beta_s$ can be different from the case where NP contributes only to the mixing.

As we noted above, for quantities in several $B$ decays, there are differences between the SM predictions and the central values of the measurements, and there are NP models which account for these differences by having new contributions to the decay $\bar{b} \rightarrow \bar{s}q\bar{q}$ $(q = u,d,s)$. In light of this, it would not be a surprise to also find NP in $\bar{b} \rightarrow \bar{s}c\bar{c}$.

Now, $B^0_s \rightarrow J/\psi\phi$ is similar to $B \rightarrow J/\psi K^*$ (here, $B = B^0_d$ or $B^+$), as both decays contain final-state vector mesons related by flavor SU(3) symmetry. This symmetry is assumed to hold approximately even in the presence of NP. (Although the $\phi$ meson has a flavor-singlet component, this does not cause a serious problem in

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1 The idea of NP in the decay $\bar{b} \rightarrow \bar{s}c\bar{c}$ is not new (for example, see Ref. [18]). However, its application to $B^0_s \rightarrow J/\psi\phi$ has not been considered previously.
relating the two decays by SU(3). It was shown in Ref. [19] that the singlet diagrams in $B^0_s \rightarrow J/\psi \phi$ suffer either Cabibbo suppression or suppression by the Okubo-Zweig-Iizuka (OZI) rule [20]. Thus, if NP contributes to $B^0_s \rightarrow J/\psi \phi$, it will also be present in $B \rightarrow J/\psi K^*$, and this will lead to new CP-violating effects (assuming that there is at least one new weak phase). Of course, there will be flavor SU(3) breaking in relating the NP (and even the SM) contributions in these decays. However, we do not expect this effect to be very large since the masses of the $K^*$ and $\phi$ mesons are similar: $m_{K^*} = 892$ MeV, $m_\phi = 1020$ MeV.

In the decay $B \rightarrow J/\psi K^*$, CP-violating effects have been looked for experimentally, but none has been found. This can put constraints on any NP contribution to the decay in $B^0_s \rightarrow J/\psi \phi$. There are two possible conclusions for a given model. First, the constraints might be so strong that any significant contribution of the model to the decay is ruled out. Second, the constraints might be weak enough that a NP contribution to the decay of $B^0_s \rightarrow J/\psi \phi$ is still possible. In this case, if the contribution is sizeable, the CDF/D0 analysis will have to take into account NP in both the mixing and the decay. Alternatively, if this contribution is small, the CDF/D0 analysis applies directly. In both cases, it is important to ascertain if there are any other predictions of the model. From this, we see that it is necessary to examine all NP models to see which of these possibilities occurs.

In Sec. 2, we present the results of the experimental searches for CP violation in $B \rightarrow J/\psi K^*$. The absence of evidence for such CP violation can lead to constraints on NP in the decay of $B^0_s \rightarrow J/\psi \phi$. We also discuss possible future measurements. These can be used to confirm the presence of NP and to distinguish different models. In Sec. 3 we look at several models which lead to NP in the mixing and/or the decay of $B^0_s \rightarrow J/\psi \phi$. We examine the effect of the constraints from $B \rightarrow J/\psi K^*$, and look at predictions of further effects. We conclude in Sec. 4.

2. CP Violation in $B \rightarrow J/\psi K^*$

The decay $B \rightarrow J/\psi K^*$ is really three separate decays, one for each polarization state $\lambda$ of the final-state vector particles; longitudinal: $\lambda = 0$, transverse: $\lambda = \{||, \perp\}$.

Suppose now that there are several new-physics amplitudes, each with a different weak phase, that contribute to the decay. In Ref. [21], it is argued that all strong phases associated with NP amplitudes are negligible. The reason is that the strong phases are generated by rescattering, and this costs a factor of about 25. The strong phase of the SM color-suppressed $\bar{b} \rightarrow \bar{c}c\bar{s}$ diagram $C$ is generated by rescattering of the color-allowed $\bar{b} \rightarrow \bar{c}c\bar{s}$ tree diagram $T$. Since $|C/T|$ is expected to be in the range 0.2-0.6, the SM strong phase is on the small side, but is not negligible. On the other hand, the strong phases of NP amplitudes can only be generated by rescattering of the NP diagrams themselves (i.e., self-rescattering) at low energies. They are therefore very small, and can be neglected. In this case, for each polarization one
can combine all NP matrix elements into a single NP amplitude, with a single weak phase $\varphi_\lambda$:

$$\sum \langle (J/\psi K^*)_\lambda | O_{NP} | B \rangle = b_\lambda e^{i \varphi_\lambda}.$$  (2.1)

It must be emphasized that the above argument for negligible strong rescattering phases associated with NP amplitudes complies with the operator product expansion formalism, and does not rely on factorization. Moreover, the validity of the argument can be checked experimentally by carefully measuring and comparing direct CP asymmetries and triple-product correlations in $B^0_s \to J/\psi \phi$ and $B \to J/\psi K^*$, because of their different dependences on the weak and strong phases.

We now assume that this single NP amplitude contributes to the decay. The decay amplitude for each of the three possible polarization states may then be written as

$$A_\lambda \equiv \text{Amp}(B \to J/\psi K^*)_\lambda = a_\lambda e^{i (\delta^\|_\lambda - \delta^0_\lambda)} + b_\lambda e^{i \varphi_\lambda} e^{-i \delta^\|_\lambda},$$

$$\bar{A}_\lambda \equiv \text{Amp}(\bar{B} \to J/\psi \bar{K}^*)_\lambda = a_\lambda e^{i (\delta^\|_\lambda - \delta^0_\lambda)} + b_\lambda e^{-i \varphi_\lambda} e^{-i \delta^\|_\lambda},$$  (2.2)

where $a_\lambda$ and $b_\lambda$ represent the SM and NP amplitudes, respectively, $\varphi_\lambda$ is the new-physics weak phase, and the $\delta^0_\lambda$ are the SM strong phases. All strong phases are given relative to $\delta^\|_\lambda$. $a_\lambda$ is defined to be positive for every polarization. $b_\lambda$ can also be taken to be positive: if it is negative, the minus sign can be absorbed in the weak phase by redefining $\varphi_\lambda \to \varphi_\lambda + \pi$. We emphasize this fact by writing the ratio $b_\lambda/a_\lambda$ as the positive-definite quantity $|r_\lambda|$.

Note: if there is only one NP operator, the weak phase can be taken to be polarization-independent. We write it simply as $\varphi$. (Several of the models studied in the next section are of this type.) However, one has to be careful here: in this case the $b_\lambda$’s cannot be taken to be positive since, if one $b_\lambda$ is negative, the minus sign cannot be removed by redefining $\varphi$.

The polarization amplitudes can be extracted experimentally by performing an angular analysis of $B \to J/\psi K^*$, in which the $J/\psi$ and $K^*$ are detected through their decays to $\ell^+ \ell^-$ and $P_1 \bar{P}_2$, respectively (the $P_i$ are pseudoscalars) [22]. We take the $K^*$ linear polarization vector to lie in the $x$-$y$ plane. We then define the angle $\psi$ to be that of the $P_1$ in the $K^*$ rest frame relative to the polarization axis (the negative of the direction of the $J/\psi$ in that frame). The $K^*$ has a single linear polarization state $\varepsilon$ for each amplitude: in the $J/\psi$ rest frame,

$$A_\parallel : \quad \varepsilon = \hat{y} ; \quad A_0 : \quad \varepsilon = \hat{x} ; \quad A_\perp : \quad \varepsilon = \hat{z}.$$  (2.3)

A unit vector $\hat{n}$ in the direction of the $\ell^+$ in $J/\psi$ decay is defined to have components

$$(n_x, n_y, n_z) = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta),$$  (2.4)

where $\varphi$ is the angle between the projection of the $\ell^+$ on the $P_1 \bar{P}_2$ plane in the $J/\psi$ rest frame and the $x$ axis.
The angular distribution is then
\[
\frac{d^4 \Gamma[B \to (\ell^+ \ell^-)_{J/\psi}(P_1 \bar{P}_2)_{K^*}]}{d \cos \theta \, d \phi \, d \cos \psi \, dt} = \frac{9}{32\pi} \left[ 2|A_0|^2 \cos^2 \psi (1 - \sin^2 \theta \cos^2 \varphi) + |A_\parallel|^2 \sin^2 \theta - \text{Im}(A_\parallel^* A_\perp) \sin 2 \theta \sin \varphi \right] \\
+ \sin^2 \psi \left\{ |A_\parallel|^2 (1 - \sin^2 \theta \sin^2 \varphi) + |A_\perp|^2 \sin^2 \theta - \text{Im}(A_\parallel^* A_\perp) \sin 2 \theta \cos \varphi \right\}.
\]
(2.5)

For $\bar{B}$ decays, the interference terms involving the $A_\perp$ amplitude are of opposite sign and all other terms are unchanged. The angular distribution is the same for $B^0_s \to J/\psi \phi$, where the $\phi$ is detected via its decay to $K^+ K^-$. In order to probe the NP amplitudes $b_\lambda$ of Eq. (2.2), one has to measure CP-violating observables. The most obvious of these is the direct CP asymmetry, in which one compares the rates for process and anti-process. Ideally, this would be measured for each of the three polarization states individually. Unfortunately, this has not been done at BaBar or Belle (although they have both measured the angular distribution). BaBar has measured the direct CP asymmetry for the entire process, combining $\lambda = 0, \parallel, \perp$. They find a result of $(2.5 \pm 8.3 \pm 5.4)\%$ \cite{23}, which is consistent with zero. This could suggest that each of the $b_\lambda$’s is tiny, i.e., that there is essentially no NP in $\bar{b} \to \bar{s}c\bar{c}$. However, the direct CP asymmetry is proportional to the sine of the difference of the strong phases of the interfering amplitudes, i.e., $\sin \delta_\lambda$. As we have argued above, this is expected to be small. As a result, the direct CP asymmetry will also be small, not because the NP amplitudes are absent, but rather due to the size of the strong phases. Thus, the direct CP asymmetry cannot be used to constrain the $b_\lambda$’s.

It should be noted that BaBar finds a rather large strong-phase difference: $\delta_\parallel - \delta_\perp = (25.8 \pm 2.9 \pm 1.1)\degree$ \cite{24}. This is confirmed by Belle \cite{25}. However, this is misleading. BaBar has implicitly assumed that there is no NP in the decay – this is indicated by the presentation of results as a “strong-phase difference.” What really is measured is the difference $\text{Arg}(A_\parallel) - \text{Arg}(A_\perp)$ (which is how Belle presents its results), and this could have a contribution from NP. In addition, because the SM is assumed by both collaborations, the data from the decay and charge-conjugate decay are added together. But in the presence of NP with a new weak phase, these are different. The upshot is that this measurement does not take into account the possibility of NP in the decay, and so is not relevant to our analysis.

Now, even though the direct CP asymmetry is not used here, there is fortunately another CP-violating observable which is pertinent. It involves the triple-product correlation (TP) \cite{26}. In the rest frame of the $B$, the TP takes the form $\vec{q} \cdot (\vec{\varepsilon}_{J/\psi} \times \vec{\varepsilon}_{K^*})$, where $\vec{q}$ is the momentum of one of the final vector mesons, and $\vec{\varepsilon}_{J/\psi}$ and $\vec{\varepsilon}_{K^*}$ are the polarizations of the $J/\psi$ and $K^*$. TP asymmetries are similar to direct CP asymmetries in that both are obtained by comparing a signal in the process with the corresponding signal in the anti-process,
and both are nonzero only if there are two interfering decay amplitudes. However, whereas the direct CP asymmetry is $A_{\text{CP}}^{\text{dir}} \propto \sin \varphi \sin \delta^\lambda$, the TP asymmetry $A_{\text{TP}}$ involves the product $\sin \varphi \cos \delta^\lambda$. That is, while the direct CP asymmetry is produced only if there is a nonzero strong-phase difference between the two decay amplitudes, the TP asymmetry is maximal when the strong-phase difference vanishes. Thus, given that the strong phases are expected to be small, TP asymmetries are particularly promising for $B$ decays.

There are two TPs in $B \to J/\psi K^*$; they are proportional to $\text{Im}(A\perp A^*_0)$ and $\text{Im}(A\perp A^*_\parallel)$ [26]. These two terms appear in the angular distribution [Eq. (2.5)], and can thus be obtained through this measurement. The TPs are defined as

$$A^{(1)}_T \equiv \frac{\text{Im}(A\perp A^*_0)}{|A_0|^2 + |A_\parallel|^2 + |A\perp|^2},$$

$$A^{(2)}_T \equiv \frac{\text{Im}(A\perp A^*_\parallel)}{|A_0|^2 + |A_\parallel|^2 + |A\perp|^2}. \quad (2.6)$$

The corresponding quantities for the charge-conjugate process, $\bar{A}^{(1)}_T$ and $\bar{A}^{(2)}_T$, are defined similarly. The TP asymmetries are obtained by calculating the difference of each of the above TPs and that in the anti-process.

As noted above, both BaBar and Belle have measured the angular distribution. As such, they have obtained the TP asymmetries for a variety of $K^*$ decays [24, 25]. Averaging the results, they find

$$A^{(1)}_T = 0.017 \pm 0.033 \quad (B^0_d),$$

$$A^{(2)}_T = -0.004 \pm 0.025 \quad (B^0_s),$$

$$A^{(1)}_T = 0.013 \pm 0.053 \quad (B^+) ,$$

$$A^{(2)}_T = -0.014 \pm 0.030 \quad (B^+). \quad (2.7)$$

All the measured TP asymmetries are consistent with zero. In contrast to the direct CP asymmetry, these measurements cannot be explained by small strong phases.

In the introduction, we noted that any NP in the decay of $B_s^0 \to J/\psi \phi$ would also be present in $B \to J/\psi K^*$. Thus, results in $B \to J/\psi K^*$ can be used to constrain the NP. We conservatively incorporate the measurements of Eq. (2.7) by requiring that the predictions of TPs in $B_s^0 \to J/\psi \phi$ obey $|A_{TP}^{(1,2)}| \leq 10\%$ (this value takes into account the errors in Eq. (2.7), as well as a possible SU(3)-breaking effect in relating the two decays).

Above, we have argued that the strong phases are expected to be small, and will therefore not produce a small TP asymmetry since this is proportional to $\cos \delta^\lambda$. On the other hand, one might wonder whether nonperturbative effects might play a role, and lead to $\delta^\lambda \simeq \pi/2$. If so, they would lead to $A_{TP}^{(1)} \simeq 0$. However, one can also produce a “fake TP asymmetry,” which is calculated from the sum of each of
the TPs in Eq. (2.6) and that in the anti-process. This quantity $A_{TP}^{\text{fake}}$ involves the product $\cos \varphi \sin \delta_a$, and has been measured by Belle [25]:

$$A_{TP}^{\text{fake},(1)} = 0.138 \pm 0.046 \ (B^0_d),$$
$$A_{TP}^{\text{fake},(2)} = -0.187 \pm 0.043 \ (B^0_d),$$

(2.8)

The fake TP asymmetries are nonzero, pointing to a nonzero value of the strong phase $\delta_a$. But these are all fairly small, so that $\delta_a \simeq \pi/2$ is not allowed. This rules out the possibility of nonperturbative effects leading to large strong phases.

Finally, a full time-dependent angular analysis of $B_0^d \to J/\psi K^*0$ ($K^*0 \to K_S\pi^0$) has not yet been done. However, if it can be performed, there are many more tests for NP in the decay. This is discussed in detail in Ref. [27]; we summarize the results briefly below.

The time-dependent decay rates can be written as

$$\Gamma(B_0^d(t) \to J/\psi K^*0) = e^{-\Gamma t} \sum_{\lambda \leq \sigma} \left( \Lambda_{\lambda\sigma} + \Sigma_{\lambda\sigma} \cos(\Delta Mt) - \rho_{\lambda\sigma} \sin(\Delta Mt) \right) g_{\lambda} g_{\sigma},$$

$$\Gamma(\bar{B}_0^d(t) \to J/\psi \bar{K}^*0) = e^{-\Gamma t} \sum_{\lambda \leq \sigma} \left( \Lambda_{\lambda\sigma} + \Sigma_{\lambda\sigma} \cos(\Delta Mt) + \rho_{\lambda\sigma} \sin(\Delta Mt) \right) g_{\lambda} g_{\sigma},$$

(2.9)

where the $g_{\lambda}$ are the coefficients of the helicity amplitudes written in the linear polarization basis. The $g_{\lambda}$ depend only on the angles describing the kinematics [23].

By performing the above time-dependent angular analyses, one can measure the 18 observables $\Lambda_{\lambda\sigma}, \Sigma_{\lambda\sigma}$ and $\rho_{\lambda\sigma}$ ($\lambda \leq \sigma$). Not all of these are independent. There are a total of six amplitudes describing $B_0^d, \bar{B}_0^d \to J/\psi K^*0$ decays. At best one can measure the magnitudes and relative phases of these six amplitudes, giving 11 independent measurements.

In the absence of NP, the $b_{\lambda}$ are zero in Eq. (2.2). The number of parameters is then reduced from 13 to 6: three $a_{\lambda}$'s, two strong-phase differences, and the weak phase of $B_0^d$-$\bar{B}_0^d$ mixing, $\beta$. Given that there are 18 observables, there must exist 12 relations among the observables in the absence of NP. These are:

$$\Sigma_{\lambda\lambda} = \Sigma_{\parallel0} = 0, \quad \Lambda_{\perp i} = 0, \quad \rho_{\parallel0} = 0,$$

$$\frac{\Lambda_{\parallel0}}{\Lambda_{\perp\perp}} = \frac{\rho_{\perp\perp}}{\Lambda_{\parallel0}},$$

$$\Lambda_{\parallel0} = \frac{1}{2\Lambda_{\perp\perp}} \left[ \Lambda_{\perp\perp}^2 + \Sigma_{\parallel0}^2 - \Sigma_{\perp\perp}^2 \right],$$

$$\frac{\rho_{\perp\perp}^2}{4\Lambda_{\perp\perp} \Lambda_{\parallel0} - \Sigma_{\perp\perp}^2} = \frac{\Lambda_{\perp\perp}^2 - \rho_{\perp\perp}^2}{\Lambda_{\perp\perp}^2},$$

(2.10)
where \( i = \{0, \parallel\} \). The violation of any of the above relations is a smoking-gun signal of NP in the decay.

The first line in Eq. (2.10) simply states that, if NP is absent, the direct CP asymmetries \((\Sigma_{\lambda\lambda}, \Sigma_{\parallel 0})\) and the triple-product asymmetries \((\Lambda_{\perp i})\) all vanish. This has been discussed earlier. It is the second line which is particularly interesting. It states that the indirect CP asymmetry for each polarization state should be the same within the SM (all giving \(\sin 2\beta\) in \(B \to J/\psi K^*\), modulo an overall sign for the \(\perp\) polarization). If a different result for different polarizations is found, this would be a clear sign of NP in the decay. Although this measurement has not yet been made, it might be done in the future at LHCb.

As we will see in the next section, some models of NP in \(B_s^0 \to J/\psi \phi\) do predict polarization-dependent indirect CP asymmetries in both \(B \to J/\psi K^*\) and \(B_s^0 \to J/\psi \phi\). This property of the NP should be taken into account when analyzing the \(B_s^0 \to J/\psi \phi\) data. In analyzing the CDF/D0 data, it is important not to average over polarizations. Thus, not only should any analysis take into account the possibility of NP in the decay, it should also consider the case where the NP is polarization-dependent.

Note: in Eq. (2.9) and elsewhere throughout the paper, we neglect the width difference in the \(B_s^0\) system, \(\Delta \Gamma_s\). Now, there are theoretical estimates of \(\Delta \Gamma_s\) in the SM [29, 30]. For example, in Ref. [29] it is found that, for a typical set of parameters, \(\Delta \Gamma_s / \Gamma_s = 0.15 \pm 0.06\). The central value is significant, suggesting that \(\Delta \Gamma_s\) might not be negligible. However, the theoretical error is also large, so that the prediction of a sizeable width difference is not certain. In principle, this uncertainty can be resolved by an experimental measurement. Unfortunately, here too the errors are very large [31]. Thus, at present, there is no clear experimental or theoretical result suggesting a large value of \(\Delta \Gamma_s\), so that our neglect of this quantity is justified. However, should this change in the future, for example through a direct measurement at LHCb, it will be necessary to take \(\Delta \Gamma_s\) into account in the calculations.

3. Models of New Physics in \(B_s^0 \to J/\psi \phi\)

In this section, we examine several models of NP which have been proposed to produce large \(B_s^0 - \bar{B}_s^0\) mixing. In all cases, the aim is to see whether significant contributions to the decay \(\bar{b} \to \bar{s} c \bar{c}\) are also possible, consistent with constraints from \(B \to J/\psi K^*\).

In order to determine whether the contribution to the decay is “significant,” we need to compare its apparent effect on \(B_s^0 - \bar{B}_s^0\) mixing (as deduced from the indirect CP asymmetry) with the best-fit measured value, \(\beta_s^{\text{meas}}\). Unfortunately, though the CDF and D0 collaborations noted that there is a 2.2\(\sigma\) effect in indirect CP violation in \(B_s^0 \to J/\psi \phi\) [3], they never gave a central value for \(\beta_s^{\text{meas}}\). There is, however, an alternative. The UTfit Collaboration [32] analyzed the CDF/D0 data, and found...
favored values for $\beta_s^{\text{meas}}$. It should be noted that the UTfit analysis is not universally accepted. They found an effect larger than $3\sigma$, which is at odds with that found by the experimental collaborations themselves. (And, in obtaining $\beta_s^{\text{meas}}$, the UTfit group averaged over polarizations, which, as explained above, is not completely general.) Still, we need only a preferred value for $\beta_s^{\text{meas}}$, not the error. For this reason, in our numerical analysis below, we use the UTfit best-fit value. But the analysis can be straightforwardly repeated for any other value of $\beta_s^{\text{meas}}$. The UTfit analysis finds that $B^0_s-\bar{B}^0_s$ mixing obeys $\sin 2\beta_s^{\text{meas}} = -(0.6 \pm 0.2)$ [S1: $\beta_s^{\text{meas}} = (-19.9 \pm 5.6) ^\circ$] or $-(0.7 \pm 0.2)$ [S2: $\beta_s^{\text{meas}} = (-68.2 \pm 4.9) ^\circ$].

If there is NP in the decay, it will contribute to the indirect CP asymmetry. In the presence of a nonzero $b_\lambda$ [Eq. 2.3], the general expression for the result of the indirect CP asymmetry in $B^0_s \rightarrow J/\psi K_s$ is

$$\sin 2\beta_s^{\text{meas}} = \frac{\sin 2\beta_s + 2|r_\lambda| \cos \delta_\lambda \sin (2\beta_s + \varphi_\lambda) + |r_\lambda|^2 \sin (2\beta_s + \varphi_\lambda)}{1 + 2|r_\lambda| \cos \delta_\lambda \cos \varphi_\lambda + |r_\lambda|^2}, \quad (3.1)$$

where $|r_\lambda| \equiv b_\lambda/a_\lambda$ (although we retain the index $\lambda$, in the following we ignore the polarization dependence). (There is an additional overall minus sign for the $\perp$ polarization.)

For small $|r_\lambda|$ [neglecting $O(|r_\lambda|^2$)], we have

$$\sin 2\beta_s^{\text{meas}} = \sin 2\beta_s + 2|r_\lambda| \cos \delta_\lambda \sin (2\beta_s + \varphi_\lambda) \cos \varphi_\lambda \equiv \sin 2\beta_s + 2|r_\lambda| \cos 2\beta_s \sin \varphi_\lambda \cos \delta_\lambda. \quad (3.2)$$

In the SM, $\beta_s \simeq 0$. Taking $\delta_\lambda \simeq 0$, we see that NP in the decay can reproduce the observed value of $\sin 2\beta_s^{\text{meas}}$ if $|r_\lambda| \simeq 30\%$ (and $\sin \varphi_\lambda \simeq 1$). However, even if $B^0_s-\bar{B}^0_s$ mixing arises from NP contributions (so that $\beta_s \rightarrow \beta_s^{NP} \neq 0$ and $\cos 2\beta_s^{NP} < 1$), the contribution from the decay is still a non-negligible fraction of $\sin 2\beta_s^{\text{meas}} = 0.6-0.7$ if $|r_\lambda| \gtrsim 10\%$. If it is found that $|r_\lambda|$ satisfies this limit, we consider the contribution to the decay significant.

Given that there is NP in the decay $\bar{b} \rightarrow \bar{s}c\bar{c}$, it is important to check that the measurement of $\beta$ in $B^0_d \rightarrow J/\psi K_s$ is still consistent. In the presence of NP in the decay ($|r| \neq 0$), the effective measured $\sin 2\beta$ in $B^0_d \rightarrow J/\psi K_s$ is given by

$$\sin 2\beta^{\text{meas}} = \sin 2\beta + 2|r| \cos 2\beta \sin \varphi \cos \delta_\lambda. \quad (3.3)$$

The (true) value of $\sin 2\beta$ can be taken from the fit to the sides of the unitarity triangle: $\sin 2\beta = 0.731 \pm 0.038$, while the experimental measurement gives $\sin 2\beta = 0.668 \pm 0.028$ [3]. Taking $\delta_\lambda \simeq 0$ and $\sin \varphi \simeq -1$, we obtain

$$|r| = \frac{\sin 2\beta^{\text{meas}} - \sin 2\beta}{-2 \cos 2\beta} = (4.6 \pm 3.5)\%. \quad (3.4)$$

Allowing up to a $3\sigma$ variation, we see that $|r| \leq 15\%$ is permitted.
In a recent article [33], it was suggested that, based on lattice calculations, the true value of $\sin 2 \beta$ is even larger, perhaps up to 0.87. In this case, even bigger values of $|r|$ are allowed.

The point here is that some authors have claimed that there is an apparent discrepancy between the measured value of $\sin 2 \beta$ and the true, underlying value, and that this calls for NP in $B_d^0 - \bar{B}_d^0$ mixing. What we have seen above is that this discrepancy might be due to NP in the $\bar{b} \to \bar{s} c \bar{c}$ decay.

For all models, we will compare their contributions to the decay to those of the SM. The SM operator is $LL$ and is given by

$$G_F \sqrt{2} \left( c_2 + \frac{c_1}{N_c} \right) V^*_{cb} V_{cs} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{c} \gamma^\mu (1 - \gamma_5) c ,$$

(3.5)

where $c_{1,2}$ are Wilson coefficients. We have $c_1(m_b) = 1.081$, $c_2(m_b) = -0.190$, so that $c_2 + c_1/N_c = 0.17$. Also, $|V^*_{cb} V_{cs}| = 0.041$. Thus, the coefficient of the SM amplitude is 0.007 (here and below, we ignore the factor $G_F/\sqrt{2}$).

Within factorization, the SM matrix elements are given by $\langle K^* | \bar{s} \gamma_\mu (1 - \gamma_5) b | B \rangle$ and $\langle J/\psi | \bar{c} \gamma^\mu (1 - \gamma_5) c | 0 \rangle$. Since the $J/\psi$ is a vector meson, $\langle J/\psi | \bar{c} \gamma^\mu \gamma_5 c | 0 \rangle = 0$, i.e., the axial-vector piece vanishes. Thus, the SM operator is really $LV$. The calculation of $|r_\lambda|$ includes the ratio of NP and SM matrix elements.

Now, throughout the paper, we perform the calculations in the context of factorization. But there may be some concerns about the use of factorization in decays in which $J/\psi$ mesons are involved. For example, the decay $B \to J/\psi K^*$ was studied in QCD factorization in Ref. [34]. It was found that naive factorization is unable to explain the branching ratio and the various polarization fractions in this decay. In addition, the quantity $a_2 = c_2 + c_1/N_c$ can be extracted from experiment [34], but it is found to be dependent on the polarization. This could lead to errors in the predictions of various NP models for the polarization-dependent indirect CP asymmetries. However, in all cases, the effects are small. Still, the results of our studies should be understood as estimates rather than precise calculations.

Finally, if there is NP in the decay $\bar{b} \to \bar{s} c \bar{c}$, one might be concerned about its effect on the width difference $\Delta \Gamma_s$ in the $B_s^0$ system. If this were significant, the NP could be detected by the measurement of this difference. Fortunately, the NP discussed in this paper does not contribute significantly to $\Delta \Gamma_s$. The NP contribution to $\bar{b} \to \bar{s} c \bar{c}$ is at most 15% that of the SM. But the SM contribution in this case comes from the color-suppressed diagram, $C$. On the other hand, the main contribution to $\Delta \Gamma_s$ is due to the color-allowed diagram, $T$. Since $|C/T|$ is about 20%, the NP contribution to $\Delta \Gamma_s$ is only at the percent level, and is negligible.

### 3.1 Z-mediated FCNC’s

In the model with Z-mediated FCNC’s (ZFCNC), it is assumed that a new vector-like isosinglet down-type quark $d'$ is present. Such quarks appear in $E_6$ GUT theories,
for example. The ordinary quarks mix with the d'. As a result, FCNC’s appear at tree level in the left-handed sector. In particular, a Zbs coupling can be generated:

\[ \mathcal{L}_{FCNC}^{Z} = -\frac{g}{2 \cos \theta_W} U_{sb} \bar{s} L \gamma_{\mu} b L Z^\mu + \text{h.c.} \] (3.6)

This coupling leads to a NP contribution to $B^0_s - \bar{B}^0_s$ mixing at tree level. In Ref. [10], it is found that one can reproduce the measured value of $\Delta M_s$ if $|U_{sb}| \approx 0.002$.

This coupling will also lead to the decay $\bar{b} \to \bar{s}c\bar{c}$ at tree level, mediated by a virtual Z. The amplitude is

\[ \frac{G_F}{\sqrt{2}} U_{sb} \bar{s} \gamma_{\mu} (1 - \gamma_5) b \bar{c} \gamma_{\mu} (I_3 - Q \sin^2 \theta_W) (1 \mp \gamma_5) c . \] (3.7)

There are thus two types of NP operators, $O_{LL}$ and $O_{LR}$, depending on whether the c quark is left- or right-handed.

However, above we have noted that the matrix element \( \langle J/\psi | \bar{c} \gamma_{\mu} \gamma_5 c | 0 \rangle \) vanishes, so that it is only the $\gamma_{\mu}$ piece of both of these operators which contributes. In other words, both operators are proportional to $O_{LV}$ (as in the SM), and they can therefore be combined. In addition, as we have noted previously, since there is only one operator, the NP weak phase \( \varphi \) can be taken to be polarization-independent.

In summary, within factorization, the total Hamiltonian can be written as

\[ H_{\text{eff}}^{\text{tot}} = H_{\text{eff}}^{\text{SM}} + H_{\text{eff}}^{\text{NP,Z}} , \]
\[ H_{\text{eff}}^{\text{SM}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_2 V_{LV} + \text{h.c.} , \]
\[ H_{\text{eff}}^{\text{NP,Z}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_2 V_{LV} + \text{h.c.} , \]
\[ V_{LV} = \bar{s} \gamma_{\mu} (1 - \gamma_5) b \bar{c} \gamma_{\mu} c . \] (3.8)

where $a_2 = c_2 + c_1/N_c = 0.17$, and

\[ |a_Z| = \left| \left( \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \right) \frac{U_{sb}}{V_{cb} V_{cs}^* a_2} \right| \approx 0.06 . \] (3.9)

Thus, compared to the SM, the contribution of the ZFCNC model is about 6%. This is not very large. We therefore see that the model with Z-mediated FCNC’s does not lead to significant new effects in the decay.

Since this contribution is proportional to $V_{LV}$, as in the SM, the ratio $b_\lambda/a_\lambda$ is in fact independent of $\lambda$; we simply refer to it as $r$. Furthermore, the NP and SM matrix elements cancel in this ratio.

We can now calculate various quantities with the above effective Hamiltonian. However, before doing so, it is useful to re-examine the amplitudes in some detail. Within factorization, there is no rescattering, and the strong phases are zero. Since the NP contribution has the same form as that of the SM, there is a relative + sign
between the two. Also, due to form factors, etc., the amplitude $A_0$ has an additional $-\,\text{sign}$ [34]. When one includes nonfactorizable effects, the SM strong phases become nonzero. Dropping the global phase, the polarization amplitudes then take the form

$$
A_0 = a_0[e^{i\delta_0} - |r|e^{i\varphi}],
$$

$$
A_{\perp} = a_{\perp}[e^{i\delta_{\perp}} + |r|e^{i\varphi}],
$$

$$
A_{\parallel} = a_{\parallel}[e^{i\delta_{\parallel}} + |r|e^{i\varphi}],
$$

(3.10)

where $\delta_0$ is near $\pi$, and $\delta_{\perp}$ and $\delta_{\parallel}$ are near 0.

The experiments have measured differences of phases and give the results in terms of strong-phase differences: $\delta_{\perp} - \delta_0 \simeq 180^\circ$, $\delta_{\parallel} - \delta_{\perp} \simeq 21^\circ$ in $B^0_s \to J/\psi\phi$ [7, 8], with similar results for $B \to J/\psi K^*$ [24, 25]. As discussed earlier, what is really measured is the total phase difference between amplitudes, and this could, in principle, have a contribution from NP. However, because $|r|$ is small, the effect of NP is also small, and so the phase differences between total amplitudes are approximately equal to the strong-phase differences between the SM amplitudes $\delta_0$. Thus, the experimental measurements confirm the approximate factorization results that $\delta_0 \simeq \pi$, $\delta_{\perp}$, $\delta_{\parallel} \simeq 0$.

We now turn to the triple products. As noted earlier, both the SM and NP contributions are proportional to $O_{LV}$. Thus, in the factorization limit (no strong phases), the interfering amplitudes are kinematically identical, and so the TPs in $B \to J/\psi K^*$ vanish. When the SM strong phases are included, one can in fact generate TPs. However, the TP asymmetries are proportional to $(\cos \delta_{\perp} \mp \cos \delta_{\parallel})$ [the $-$ (+) sign applies to $i = \parallel$ ($i = 0$)]. While this is now nonzero, it is small. Thus, the TP asymmetries in the ZFCNC model are small, and there are no constraints from the measurements of TP asymmetries in $B \to J/\psi K^*$.

Since there is a contribution to the $\bar{b} \to \bar{s}c\bar{c}$ decay, there could in principle be a polarization-dependent indirect CP asymmetry in $B^0_s \to J/\psi\phi$ or $B \to J/\psi K^*$. However, as noted above, the ratio $|r_\lambda|$ and the weak phase $\varphi$ are in fact polarization-independent, so that no such effect is predicted.

This is confirmed by explicit calculation. As shown previously, in the presence of NP in the decay, the indirect CP asymmetry in $B^0_s \to J/\psi\phi$ is given by

$$
\sin 2\beta^{\text{meas}}_s = \sin 2\beta^{NP}_s \pm 2|r|\cos 2\beta^{NP}_s \sin \varphi \cos \delta^{NP}_\lambda ,
$$

(3.11)

where the $+$ (−) sign applies to $\lambda = \perp, \parallel$ ($\lambda = 0$). Taking $\delta_0 \simeq \pi$, $\delta_{\perp}$, $\delta_{\parallel} \simeq 0$, we see that the correction to $\sin 2\beta^{NP}_s$ is $2|r|\cos 2\beta^{NP}_s \sin \varphi$ for all polarizations.

In summary, the ZFCNC model does include a contribution to the $\bar{b} \to \bar{s}c\bar{c}$ decay. However, this effect is not large. In addition, because the NP operator is proportional to that of the SM, any TPs in $B \to J/\psi K^*$ are small, and no polarization-dependent indirect CP asymmetries in $B^0_s \to J/\psi\phi$ or $B \to J/\psi K^*$ are predicted.
3.2 $Z'$-mediated FCNC’s

In this subsection, we describe the model with $Z'$-mediated FCNC’s ($Z'$FCNC). We assume that the gauge group contains an additional $U(1)'$, which leads to a $Z'$. Suppose that, in the gauge basis, the $U(1)'$ currents are

$$J_{Z'}^\mu = g' \sum_i \bar{\psi}_i \gamma^\mu \left[ \epsilon_i^{\psi_L} P_L + \epsilon_i^{\psi_R} P_R \right] \psi_i,$$

where $i$ is the family index, $\psi$ labels the fermions (up- or down-type quarks, or charged or neutral leptons), and $P_{L,R} = (1 \mp \gamma_5)/2$. According to some string-construction or GUT models such as $E_6$, it is possible to have family non-universal $Z'$ couplings. That is, even though $\epsilon_{L,R}^i$ are diagonal, the couplings are not family universal. After rotating to the physical basis, FCNC’s generally appear at tree level in both the left-handed (LH) and right-handed (RH) sectors. Explicitly,

$$B_{\psi_L} = V_{\psi_L} \epsilon_{\psi_L} V_{\psi_L}^\dagger, \quad B_{\psi_R} = V_{\psi_R} \epsilon_{\psi_R} V_{\psi_R}^\dagger.$$  

Moreover, these couplings may contain CP-violating phases beyond that of the SM. In particular, $Z'\bar{b}s$ couplings can be generated:

$$\mathcal{L}_{FCNC}^{Z'} = -g' \left( B_{sb}^L \bar{s}_L \gamma_{\mu} b_L + B_{sb}^R \bar{s}_R \gamma_{\mu} b_R \right) Z'^\mu + \text{h.c.}$$

These couplings lead to a NP contribution to $B_{s}^0 - \bar{B}_s^0$ mixing at tree level. We define

$$\rho_{L,R}^{s_{LL}} = \left| \frac{g' M_Z}{g M_{Z'}} B_{f_{LL}}^{L,R} \right|,$$

where $g$ is the coupling of $SU(2)_L$ in the SM. In Refs. [11, 36], it is assumed that only the LH sector of quarks has family non-universal $U(1)'$ couplings. Thus, only the LH interaction above contributes to $B_{s}^0 - \bar{B}_s^0$ mixing. It is found that one can reproduce the measured value of $\Delta M_s$ if

$$\rho_{sb}^{L} \sim 10^{-3}.$$  

If one assumes that $g' M_Z/g M_{Z'} \sim 0.1$, then this implies that $|B_{sb}^L| \sim 10^{-2}$. Note that if one wants to include FCNC in the RH sector as well, then the constraints on $B_{sb}^{L,R}$ would be different and more uncertain because of more free parameters in the model.

The couplings in Eq. (3.14) will also lead to the decay $b \to s\bar{c}c$ at tree level, mediated by a virtual $Z'$. The amplitude is

$$\frac{g'^2}{M_{Z'}^2} \left( B_{sb}^L \bar{s}_L \gamma_{\mu} b_L + B_{sb}^R \bar{s}_R \gamma_{\mu} b_R \right) \left( B_{cc}^L \bar{c}_L \gamma^\mu c_L + B_{cc}^R \bar{c}_R \gamma^\mu c_R \right).$$

There are thus four types of operators, $O_{LL}$, $O_{LR}$, $O_{RL}$, and $O_{RR}$.

However, above we have noted that the matrix element $\langle J/\psi |\bar{c}\gamma^\mu \gamma_5 c|0 \rangle$ vanishes, so that it is only the $\gamma^\mu$ piece of these operators which contributes. In other words,
we are left with two kinds of operators — $O_{LV}$ (as in the SM) and $O_{RV}$ — and they can therefore be partially combined. (If we make the same assumption as in Refs. [11, 36], then there is no $O_{RV}$ operator.) In the following, we still present the general formulae with both LH and RH FCNC interactions. But when computing numerical values, we restrict ourselves to LH only.

In summary, within factorization, the total Hamiltonian can be written as

$$H_{\text{tot}}^{\text{eff}} = H_{\text{SM}}^{\text{eff}} + H_{\text{NP,Z}}^{\text{eff}},$$

$$H_{\text{NP,Z}}^{\text{eff}} = 2G_F \sqrt{\frac{2}{2}} (\rho_{cc}^L + \rho_{cc}^R) (\rho_{sb}^L V_{LV} + \rho_{sb}^R V_{RV}) + \text{h.c.},$$

$$V_{RV} = \bar{s}_\mu (1 + \gamma_5) b \bar{c}_\mu c,$$  \hspace{1cm} (3.18)

where $H_{\text{eff}}^{\text{SM}}$ and $V_{LV}$ have been defined in Eq. (3.8). If we suppose that $B_{sb}^R = 0$, then we can combine the SM and $Z'$ FCNC contributions. The ratio of the $Z'$ contribution to the SM for the $LV$ operator only is

$$\frac{2(\rho_{cc}^L + \rho_{cc}^R)\rho_{sb}^L}{|V_{sb}V_{cb}^*a_2|} \sim \frac{2 \times 0.1 |B_{cc}^L + B_{cc}^R| \times 0.001}{0.041 \times 0.17} = 0.03 |B_{cc}^L + B_{ce}^R|. \hspace{1cm} (3.19)$$

(The NP and SM matrix elements are the same, so they cancel in the ratio.) If $|B_{cc}^L + B_{cc}^R| < 3$, then the NP contribution to the $\bar{b} \to \bar{s}c\bar{c}$ decay is not significant (as in the $ZFCNC$ model). But if $|B_{cc}^L + B_{cc}^R| \gtrsim 3$, then the NP does contribute significantly to the decay, and the analysis of $B_s^0 \to J/\psi \phi$ should be modified to take this into account. In either case, the NP contribution has the same form as that of the SM, so that the TPs in $B \to J/\psi K^*$ and the corrections leading to polarization-dependent indirect CP asymmetries are small.

However, this can change if $B_{sb}^R \neq 0$. In this case, the NP contribution is not proportional to that of the SM. Depending on what the flavor-changing $Z'$ couplings are, the contribution to the decay can be significant, and there can be non-negligible contributions to TPs and polarization-dependent indirect CP asymmetries. (Of course, in the case of TPs, constraints from $B \to J/\psi K^*$ must be taken into account.)

### 3.3 Two-Higgs-Doublet Model

Here we examine the model with two Higgs doublets (2HDM) [12]. In this model, the $\bar{b} \to \bar{s}c\bar{c}$ decay occurs at tree level, as in the SM, because of the presence of the charged Higgs boson. The Lagrangian for the $H^\pm ff'$ interaction is given by

$$\mathcal{L}_{2HDM}^{\pm ff'} = \frac{g}{2\sqrt{2}M_W} H^+ \bar{f}_i (A_{ij} + B_{ij} \gamma_5) f'_j + \text{h.c.}, \hspace{1cm} (3.20)$$

where $f$ and $f'$ correspond to the up-type and the down-type quarks, respectively. The couplings $A$ and $B$ depend on the set of underlying assumptions of the particular model. In our analysis, we study the following two scenarios: (a) the Lagrangian for
the $\bar{b} \to \bar{s}c\bar{c}$ transition has a similar structure to that of the 2HDM of type II [37], and (b) the Lagrangian for the $\bar{b} \to \bar{s}c\bar{c}$ transition has a similar structure to that of the so-called top-quark 2HDM [38]. Note: the scenarios we consider are not identical to either the 2HDM of type II or to the top-quark 2HDM, and so the constraints on these models do not necessarily apply here. In both cases, CP-violating phases are added to the operators in the effective Hamiltonian.

In the most general model with new scalars, there is a flavor-changing $H^0\bar{b}s$ coupling. A tree-level NP contribution to $B^0_s - \bar{B}^0_s$ mixing is then produced by $H^0$ exchange. This can be competitive with the SM and, if the coupling includes a CP-violating phase, can lead to a significant $B^0_s - \bar{B}^0_s$ mixing phase. This scenario occurs in little-Higgs [14] and unparticle [15] models, both of which have been proposed to explain the CDF/D0 $B^0_s \to J/\psi \phi$ data.

Of course, the exchanged particle also contributes to the decay $\bar{b} \to \bar{s}c\bar{c}$. However, here the $c\bar{c}$ quark pair forms a spin-1 $J/\psi$, which cannot be produced by the spin-0 $H^0$. Thus, the $H^0\bar{b}s$ coupling does not lead to a contribution to the decay $B^0_s \to J/\psi \phi$. On the other hand, if the model contains other particles, such a contribution may be possible. For example, the unparticle model includes flavor-changing vector particles. In this case, the size of the contribution to the decay $B^0_s \to J/\psi \phi$ must be estimated to determine whether the analysis of $B^0_s \to J/\psi \phi$ needs to be modified.

Now, in the 2HDM scenarios considered in this paper, there is no flavor-changing $H^0\bar{b}s$ coupling. As a result, the only NP contribution to $B^0_s - \bar{B}^0_s$ mixing comes from a box diagram which involves charged-Higgs exchange. Unless the couplings to quarks are taken to be very large, these diagrams are smaller than the SM contribution, so that we still have $\beta_s \simeq 0$. Therefore, the 2HDM cannot explain the CDF/D0 $B^0_s \to J/\psi \phi$ data through new effects in $B^0_s - \bar{B}^0_s$ mixing.

However, as we will see below, there can be a significant contribution to $\bar{b} \to \bar{s}c\bar{c}$ decay. We find $|r_\lambda|$ in the range of 10-15%, which leads to $\sin 2\beta_s^{\text{meas}} = 0.2$-0.3. Although this cannot account for the central values of the current data, if future measurements find that the indirect CP asymmetry in $B^0_s \to J/\psi \phi$ is smaller than is presently found, but is still nonzero, the 2HDM could be the explanation. Thus, this is an example of a NP model which affects $B^0_s \to J/\psi \phi$ through new effects in the decay. The $B^0_s - \bar{B}^0_s$ mixing phase is still quite small, and the analysis of $B^0_s \to J/\psi \phi$ must be redone to take this into account.

In cases (a) and (b), we have

\begin{align}
\text{case (a)} & : \quad A_{ij}^{(a)} = V_{ij}(m_j \tan \beta + m_i \cot \beta) , \\
& \quad B_{ij}^{(a)} = V_{ij}(m_j \tan \beta - m_i \cot \beta) , \\
\text{case (b)} & : \quad A_{ij}^{(b)} = V_{ij}(m_j - m_i) \tan \beta , \\
& \quad B_{ij}^{(b)} = V_{ij}(m_j + m_i) \tan \beta ,
\end{align}

where $\tan \beta$ is the ratio of the two vacuum expectation values of the Higgs doublets.
$V_{ij}$ is the $ij$ CKM matrix element. Performing a Fierz transformation, we find that, within factorization, the total effective Hamiltonian is given by

$$H_{\text{eff}} = H_{\text{eff}}^{\text{SM}} + H_{\text{eff}}^{\text{NP,2HDM}},$$

$$H_{\text{eff}}^{\text{NP,2HDM}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* [a_{RR} T_{RR} + a_{LL} T_{LL} + a_{RV} V_{RV} + a_{LV} V_{LV}] + \text{h.c.},$$

$$T_{RR} = \frac{1}{4} \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) b \bar{c} \sigma^{\mu\nu} (1 + \gamma_5) c,$$

$$T_{LL} = \frac{1}{4} \bar{s} \sigma_{\mu\nu} (1 - \gamma_5) b \bar{c} \sigma^{\mu\nu} (1 - \gamma_5) c,$$

$$V_{RV} = \bar{s} \gamma_\mu (1 + \gamma_5) b \bar{c} \gamma_\mu c,$$

$$V_{LV} = \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{c} \gamma_\mu c,$$

(3.23)

where $H_{\text{eff}}^{\text{SM}}$ is defined in Eq. (3.8), and we have dropped the scalar operators which do not contribute to this decay within factorization.

Keeping only the contributions expected to be dominant, we then find that

$$\text{case (a)} : \quad H_{\text{eff}}^{2\text{HDM}(a)} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_{RV} V_{RV},$$

$$a_{RV} = -\frac{1}{2 N_c} \frac{m_b m_s}{m_{H^\pm}^2} \tan^2 \beta e^{i \varphi^{(a)}},$$

(3.24)

where we have assumed a large $\tan \beta$, and

$$\text{case (b)} : \quad H_{\text{eff}}^{2\text{HDM}(b)} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_{RR} T_{RR},$$

$$a_{RR} = \frac{1}{2 N_c} \frac{m_b m_c}{m_{H^\pm}^2} \tan^2 \beta e^{i \varphi^{(b)}},$$

(3.25)

The new CP-violating phases $\varphi^{(a),(b)}$ have been added to the operators in the effective Hamiltonian. In both cases, there is one NP operator, so these weak phases can be taken to be polarization-independent.

In order to estimate the contributions to the indirect CP-asymmetry and to the TP asymmetries in $B^0_s \to J/\psi \phi$, we have taken $M_B^2 \tan^2 \beta / m_{H^\pm}^2 \simeq 0.5 (0.05)$ for the first (second) case [39], where $M_B$ is the $B$-meson mass. With the known form factors, this allows us to calculate the allowed values for the ratios $r_\lambda$ that we use in the following analysis. In order to evaluate the form factors, we consider the Melikhov-Stech [40] and Ball-Zwicky [41] models (see Appendix for details). However, no significant difference is found between the two models.

We begin with case (a). The first important observation is that the ratios $r_\lambda$ depend on $\lambda$, in contrast to the ZFCNC and Z′FCNC models. This is because $V_{RV}$ (2HDM) and $V_{LV}$ (SM) have opposite chiralities. Although their matrix elements are the same in magnitude for each amplitude in the polarization basis, there is a relative
sign between the $\lambda = \{0, \parallel\}$ and $\lambda = \perp$ contributions. This leads to a polarization-dependent prediction for $\sin 2\beta_s^{meas}$. In addition, the different chiral structures of the SM and 2HDM operators may generate large TPs.

The expression for $\sin 2\beta_s^{meas}$ is still given by Eq. (3.11) (with $\sin 2\beta_s^{NP} = 0$). However, there is a change in the sign assignments in front of $|r|$. Now the $+ (-)$ sign applies to $\lambda = \parallel$ ($\lambda = 0, \perp$). (An explicit sign in the Wilson coefficient for $a_{RV}$ [Eq. (3.24)] has been included here.) We find $|r| \simeq 0.12$. Setting $\delta^a_\pi \simeq \pi$, $\delta^a_\perp, \delta^a_\parallel \simeq 0$, $\sin 2\beta_s^{meas}$ is then $\pm 0.24 \sin \varphi^{(a)}$ for $\lambda = 0, \parallel$ ($+$ sign) and $\lambda = \perp$ ($-$ sign).

As mentioned above, TPs are expected to be large. We compute the two TP asymmetries, $A_{TP}^{(1,2)}$. Taking the same values for the ratio $|r|$ and the strong phases $\delta^a_\lambda$ as in the previous paragraph, we obtain $|A_{TP}^{(1)}| \lesssim 0.14$ and $|A_{TP}^{(2)}| \lesssim 0.07$, where the maximum values occur at $\varphi^{(a)} = \pm \pi/2$.

In this case, the maximum value of $|A_{TP}^{(1)}|$ is in conflict with the data from $B \to J/\psi K^*$ [Eq. (2.7)]. In order to resolve this, we must take $M_d^2 \tan^2 \beta/m_{H^\pm}^2 \simeq 0.34$ in case (a). This leads to $|r| \simeq 0.08$, which in turn yields predictions of $\sin 2\beta_s^{meas} = \pm 0.16 \sin \varphi^{(a)}$ (polarization-dependent indirect CP-asymmetry), $|A_{TP}^{(1)}| \lesssim 0.09$ and $|A_{TP}^{(2)}| \lesssim 0.04$. We therefore see that, in this case, the constraints from $B \to J/\psi K^*$ reduce the effect of NP in the decay.

Now consider case (b). In this scenario, the ratios $r_\lambda$ also depend on $\lambda$. The reason is that the matrix elements of the operator $T_{RR}$ entering a given polarization amplitude are different from those of the corresponding SM contributions. This in turn implies that there is a polarization-dependent prediction for $\sin 2\beta_s^{meas}$. In addition, due to the different Lorentz structures of $T_{RR}$ (2HDM) and $V_{LV}$ (SM), we may expect to have large TP asymmetries.

Here, $\sin 2\beta_s^{meas}$ is given by Eq. (3.2) (with $\sin 2\beta_s = 0$). For $\delta^a_\pi \simeq \pi, \delta^a_\perp, \delta^a_\parallel \simeq 0$, the NP corrections to $\sin 2\beta_s^{meas}$ are $2|r_\lambda| \sin \varphi^{(b)}$, where $|r_\lambda| = 0.01, 0.10, 0.11$ for $\lambda = 0, \parallel, \perp$, respectively. We therefore see that there is a polarization-dependent prediction for $\sin 2\beta_s^{meas}$ in this scenario.

For the TP asymmetries, we find that $|A_{TP}^{(1)}| \lesssim 0.06$ and $|A_{TP}^{(2)}| \lesssim 0.004$, where the maximum values occur at $\varphi^{(b)} = \pm \pi/2$. The suppression in $A_{TP}^{(2)}$ is due to the similar sizes of $r_\parallel$ and $r_\perp$, and because $\delta^a_\perp, \delta^a_\parallel \simeq 0$. In this case, we have checked that the measured TP asymmetries in $B \to J/\psi K^*$ [Eq. (2.7)] do not reduce the effects of NP in this decay.

Now, the presence of NP in $b \to s c \bar{c}$ can produce deviations in the measurement of $\beta$ in $B_d^0 \to J/\psi K_s$. We have therefore examined whether the constraints imposed by Eq. (3.2) restrict the NP effects in $B_s^0 \to J/\psi \phi$. In case (a), $r$ is simply the ratio between the 2HDM and SM Wilson coefficients, as the relevant matrix elements are the same in both models and thus cancel. We find $|r| \simeq 0.08$, which is allowed within a $1\sigma$ variation. In case (b), by following the analysis given in Refs. [12, 13, 14], we obtain $|r| \simeq 6 \times 10^{-3}$. We therefore see that the present measurement of $\beta$ in $B_d^0 \to J/\psi K_s$ does not reduce the effect of the NP in $B_s^0 \to J/\psi \phi$ in either case.
In summary, we have seen that, in both cases (a) and (b) of the 2HDM, the NP in the decay is small, but still significant, with $|\rho_{\lambda}| = O(10\%)$. Both predict a polarization-dependent indirect CP asymmetry in $B_s^0 \rightarrow J/\psi\phi$, with $\sin 2\beta_{\lambda}^{\text{meas}}$ varying by 0.2-0.3 for different values of $\lambda$. Small but nonzero TP asymmetries are also expected. $A_{\nu}^{(1)}$ is found to be at most of order (5-10)% in both scenarios, whereas $A_{\nu}^{(2)}$ is this size only in the first.

### 3.4 Supersymmetry

We now examine the effect of supersymmetry (SUSY) on $B_s^0 \rightarrow J/\psi\phi$. The discussion is based on the analysis in Ref. [13]. There it was shown that the experimental measurements of the mass difference $\Delta M_{B_s}$ and the mercury electric dipole moment significantly constrain the SUSY contribution to $B_s^0 - \bar{B}_s^0$ mixing: the total effect is just $\sin 2\beta_s^{\text{NP}} \lesssim 0.1$. Thus, the only way to explain the CDF/D0 $B_s^0 \rightarrow J/\psi\phi$ data is through a SUSY contribution to the decay. This occurs principally through the one-loop correction to $\bar{b} \rightarrow \bar{s}$ from gluino exchange.

Here we extend the previous analysis in several ways. The focus of Ref. [13] was mainly on the transverse amplitudes, as the predictions for the indirect CP asymmetries were found to be independent of hadronic form factors. (For completeness, we repeat these results below.) In this paper we also calculate the indirect CP asymmetry for the longitudinal amplitude, which does depend on the hadronic form factors. (However, the effect of SUSY on the longitudinal amplitude is found to be small). We also compute the TP asymmetries in $B \rightarrow J/\psi K^*$ and $B_s^0 \rightarrow J/\psi\phi$ in the presence of SUSY, as well as its contribution to $B_0^d \rightarrow J/\psi K_S$.

There are two main operators that can contribute to the transition $\bar{b} \rightarrow \bar{s}c\bar{c}$, both of dipole type. After a Fierz transformation (and neglecting the color-octet pieces), they are

$$
H_{\text{SUSY}} = C_g O_g + \tilde{C}_g \tilde{O}_g,
$$

$$
O_g = Y_g \left[ -\frac{2}{N_c} \left( \bar{s}_a \gamma_\mu \frac{d}{m_b} (1 + \gamma_5) b_\alpha \right) (c_\beta \gamma_\mu c_\beta) \right],
$$

$$
\tilde{O}_g = Y_g \left[ -\frac{2}{N_c} \left( \bar{s}_a \gamma_\mu \frac{d}{m_b} (1 - \gamma_5) b_\alpha \right) (c_\beta \gamma_\mu c_\beta) \right].
$$

(3.26)

Here, $Y_g = -\frac{m_b^2}{4\pi m_{j/\psi}}$; $q$ is the momentum transfer. The coefficients $C_g$ and $\tilde{C}_g$ can be found in Ref. [13] and depend on the mass insertions $(\delta_{LR}^d)_{23}$ and $(\delta_{RL}^d)_{23}$.

With the operators above, we can compute the ratio of the SUSY and SM contributions for the transverse polarizations [13]:

$$
|r_\parallel| e^{ix_\parallel_{\text{SUSY}}} = \frac{(Y - \bar{Y})}{X}, \quad |r_\perp| e^{ix_\perp_{\text{SUSY}}} = \frac{(Y + \bar{Y})}{X},
$$

(3.27)
where
\[ Y = \frac{\sqrt{2} C_\phi}{G_F} y_g \left[ -\frac{2}{N_c} \right], \quad \tilde{Y} = \frac{\sqrt{2} C_\phi}{G_F} y_g \left[ -\frac{2}{N_c} \right], \quad X = V_{cb} V_{cs}^* (c_2 + \frac{c_1}{N_c}) \approx 0.007. \] 

(3.28)

As is clear from these expressions, the form factors and other hadronic quantities cancel in the ratios. We therefore obtain clean predictions for \( r_\parallel \) and \( r_\perp \) in SUSY.

We take the following values for the masses: \( m_\bar{q} = m_\bar{q} = 500 \text{ GeV} \) and \( m_b(m_b) = 4.5 \text{ GeV} \), obtaining
\[ Y \approx 2.13 (\delta_{LR}^d)_{23} \left[ -\frac{2}{N_c} y_g \right] = 0.05 (\delta_{LR}^d)_{23}, \]
\[ \tilde{Y} \approx 2.13 (\delta_{RL}^d)_{23} \left[ -\frac{2}{N_c} y_g \right] = 0.05 (\delta_{RL}^d)_{23}. \]

(3.29)

Eq. (3.27) then gives
\[ |r_\parallel| \approx 7 \left[ |(\delta_{LR}^d)_{23}|^2 \right] \left[ |(\delta_{RL}^d)_{23}|^2 - 2 |(\delta_{LR}^d)_{23}| |(\delta_{RL}^d)_{23}| \cos(\varphi_{LR} - \varphi_{RL}) \right]^{1/2}, \]
\[ |r_\perp| \approx 7 \left[ |(\delta_{LR}^d)_{23}|^2 + |(\delta_{RL}^d)_{23}|^2 + 2 |(\delta_{LR}^d)_{23}| |(\delta_{RL}^d)_{23}| \cos(\varphi_{LR} - \varphi_{RL}) \right]^{1/2}, \]

(3.30)

where \( \varphi_{LR} \) and \( \varphi_{RL} \) are the phases of \( (\delta_{LR}^d)_{23} \) and \( (\delta_{RL}^d)_{23} \), respectively. The phases of the helicity amplitudes can be obtained from
\[ |r_{\parallel}| \sin \varphi_{\parallel \text{SUSY}} \approx 7 \left[ |(\delta_{LR}^d)_{23}| \sin \varphi_{LR} - |(\delta_{RL}^d)_{23}| \sin \varphi_{RL} \right], \]
\[ |r_{\perp}| \sin \varphi_{\perp \text{SUSY}} \approx 7 \left[ |(\delta_{LR}^d)_{23}| \sin \varphi_{LR} + |(\delta_{RL}^d)_{23}| \sin \varphi_{RL} \right]. \]

(3.31)

In order to calculate the quantities \( |r_\parallel| \) and \( |r_\perp| \), we need values for the square roots of Eq. (3.30). Below, we consider several different realistic scenarios for the magnitudes and phases of \( (\delta_{LR}^d)_{23} \) and \( (\delta_{RL}^d)_{23} \). In all cases, each of the square roots takes a value between 0 and 0.02, so that \( |r_\parallel| \) and \( |r_\perp| \) can be (independently) in the range 0-14%. We therefore see that these ratios can be significant (\( \gtrsim 10\% \)) within SUSY.

We now turn to the longitudinal amplitude. In this case, the ratio \( |r_0| \) will depend on form factors. The SUSY amplitude for the longitudinal polarization is
\[ A_{\text{SUSY}}^0 = \frac{G_F}{\sqrt{2}} m_{J/\psi} g_{J/\psi} (Y - \tilde{Y}) \left[ \xi_1 + \xi_2 \right], \]

(3.32)

where
\[ \xi_1 = \left[ (m_{B_s} + m_{\phi}) A_1(m_{J/\psi}^2) x - \frac{2m_{J/\psi} m_{\phi}}{(m_{B_s} + m_{\phi})} A_2(m_{J/\psi}^2) (x^2 - 1) \right], \]
\[ \xi_2 = \frac{2m_{J/\psi} m_{\phi}}{m_b^2} (x^2 - 1) \]

(3.33)
\[
\times \left[ - (m_{B_s} + m_\phi) A_1 (m_{J/\psi}^2) + \frac{A_2 (m_{J/\psi}^2)}{m_{B_s} + m_\phi} \left( m_{B_s}^2 + (m_{B_s}^2 + m_\phi^2 - m_{J/\psi}^2)/2 \right) \right] \\
+ \frac{(m_{B_s}^2 + m_{J/\psi}^2 - m_\phi^2)}{2m_{J/\psi}^2} \left[ (m_{B_s} + m_\phi) A_1 (m_{J/\psi}^2) - A_2 (m_{J/\psi}^2)(m_{B_s} - m_\phi) \right. \\
- \left. 2m_\phi A_0 (m_{J/\psi}^2) \right]. \quad (3.34)
\]

The various hadronic form factors in the expression above are defined in the Appendix. We find that \(|r_0|\) is given by

\[
|r_0| = \left| \frac{A_{0,\text{SUSY}}}{A_{0,\text{SM}}} \right| = \left| 1 + \frac{\xi_2}{\xi_1} \right| \left( \frac{Y - \bar{Y}}{X} \right). \quad (3.35)
\]

In order to compute \(|r_0|\), we have to use a model to calculate the form factors. We consider the models by Melikhov-Stech (MS) [40] and Ball-Zwicky (BZ) [41]. Using the results of the form factors (see Appendix for details) one can obtain predictions for \(|r_0|\):

\[
|r_0| \approx C \times \left| (\delta_{LR}^d)_{23}^2 + (\delta_{RL}^d)_{23}^2 - 2 |(\delta_{LR}^d)_{23}| |(\delta_{RL}^d)_{23}| \cos (\varphi_{LR} - \varphi_{RL}) \right|^{1/2}, \quad (3.36)
\]

where \(C = 0.8 \) (MS) or 0.9 (BZ). Since, as noted above, the square root takes a maximum value of 0.02, \(|r_0|\) is quite small, due to a cancellation between the various NP amplitudes. We therefore see that, although the SUSY contribution to the transverse-amplitude ratios \(|r_\|\) and \(|r_\perp|\) can be significant, the contribution to the longitudinal-amplitude ratio \(|r_0|\) is negligible.

We can now estimate the contributions to the indirect CP asymmetry in \(B_s^0 \to J/\psi\phi\) and to the TP asymmetries. Since one has different contributions to the three \(r_\lambda\)'s in SUSY, these effects may be important. We consider the following four scenarios for the magnitudes and phases of \((\delta_{LR}^d)_{23}\) and \((\delta_{RL}^d)_{23}\):

1. \(|(\delta_{LR}^d)_{23}| = |(\delta_{RL}^d)_{23}| = 0.01\) and \((\varphi_{LR} - \varphi_{RL}) = 0\),
2. \(|(\delta_{LR}^d)_{23}| = |(\delta_{RL}^d)_{23}| = 0.01\) and \((\varphi_{RL} - \varphi_{LR}) = \pi\),
3. \(|(\delta_{LR}^d)_{23}| = 0.01\), \((\delta_{RL}^d)_{23} = 0\),
4. \((\delta_{LR}^d)_{23} = 0\), \(|(\delta_{RL}^d)_{23}| = 0.01\).

(The value of 0.01 for \(|(\delta_{LR}^d)_{23}|\) and/or \(|(\delta_{RL}^d)_{23}|\) is consistent with the constraint from \(b \to s\gamma\) [43].)

For the indirect CP asymmetry in \(B_s^0 \to J/\psi\phi\), one obtains the following result:

\[
\sin 2\beta_{s,\text{meas}}^a \big|_0 = \sin 2\beta_{s,\text{NP}}^a, \\
\sin 2\beta_{s,\text{meas}}^\| = \sin 2\beta_{s,\text{NP}}^a + 2|r_\| \cos 2\beta_{s,\text{NP}}^a \sin \varphi_{\text{SUSY}} \cos \delta^a, \\
\sin 2\beta_{s,\text{meas}}^\perp = \sin 2\beta_{s,\text{NP}}^a + 2|r_\perp| \cos 2\beta_{s,\text{NP}}^a \sin \varphi_{\text{SUSY}} \cos \delta^a. \quad (3.37)
\]
In the above, we have set the SM $\beta_s = 0$. $\beta_s^{NP}$ is the SUSY contribution to $B^0_s \rightarrow \bar{B}_s^0$ mixing. $\varphi_{\text{SUSY}}^{\parallel, \perp}$ are the SUSY weak phases of the $\parallel$ and $\perp$ amplitudes, and $\delta_{\parallel, \perp}$ are the SM strong phases (both $\approx 0$). The values of $|r_{\parallel}|$, $|r_{\perp}|$, $\varphi_{\text{SUSY}}^{\parallel}$ and $\varphi_{\text{SUSY}}^{\perp}$ are given in Table 1 using Eqs. (3.30) and (3.31). We see that SUSY does indeed predict a polarization-dependent indirect CP asymmetry in $B_0^0 \rightarrow J/\psi \phi$, with the value of $\sin 2\beta_{s,\text{meas}}^{\parallel}$ in different polarizations varying by as much as 0.28. Note: the effect is largest in scenarios (1) and (2). However, even in scenarios (3) and (4), for which the contributions to $|r_{\parallel}|$ and $|r_{\perp}|$ are not considered significant, one has a maximal difference of 0.14 for $\sin 2\beta_{s,\text{meas}}^{\parallel}$ in the longitudinal and transverse polarizations. This may be measurable.

| scenario | $|r_{\parallel}|$ | $|r_{\perp}|$ | $\varphi_{\text{SUSY}}^{\parallel}$ | $\varphi_{\text{SUSY}}^{\perp}$ |
|----------|-----------------|-----------------|-----------------|-----------------|
| (1)      | 0               | 0.14            | 0               | $\varphi_{LR}$  |
| (2)      | 0.14            | 0               | $\varphi_{LR}$  | 0               |
| (3)      | 0.07            | 0.07            | $\varphi_{LR}$  | $\varphi_{LR}$  |
| (4)      | 0.07            | 0.07            | $-\varphi_{RL}$ | $\varphi_{RL}$  |

Table 1: SUSY predictions for $|r_{\parallel}|$, $|r_{\perp}|$, $\varphi_{\text{SUSY}}^{\parallel}$ and $\varphi_{\text{SUSY}}^{\perp}$ in the four scenarios for the magnitudes and phases of $(\delta_{LR}^d)_{23}$ and $(\delta_{RL}^d)_{23}$ described in the text.

We now move to the predictions of triple-product asymmetries in $B^0_s \rightarrow J/\psi \phi$. Using the experimental strong phases $\delta_0 \approx \pi$, $\delta_0^{\perp, \parallel} \approx 0$ and the values of the form factors given in the Appendix, the TP asymmetries can be calculated. They are shown in Table 2 (there is no difference between the MS and BZ predictions). We see that effects of up to 5-10% are allowed. (We have explicitly calculated the TP asymmetries in $B \rightarrow J/\psi K^*$, and find that they are equal to those in $B^0_s \rightarrow J/\psi \phi$. There is therefore no conflict with the $B \rightarrow J/\psi K^*$ data.)

| scenario | $A_{TP}^{(1)}$ | $A_{TP}^{(2)}$ |
|----------|----------------|----------------|
| (1)      | $\approx 0.09 \sin \varphi_{LR}$ | $\approx 0.08 \sin \varphi_{LR}$ |
| (2)      | 0              | $\approx -0.08 \sin \varphi_{LR}$ |
| (3)      | $\approx 0.04 \sin \varphi_{LR}$ | 0              |
| (4)      | $\approx 0.04 \sin \varphi_{RL}$ | $\approx 0.08 \sin \varphi_{RL}$ |

Table 2: SUSY predictions for the triple-product asymmetries $A_{TP}^{(1)}$ and $A_{TP}^{(2)}$ in the four scenarios for the magnitudes and phases of $(\delta_{LR}^d)_{23}$ and $(\delta_{RL}^d)_{23}$ described in the text.

Now, if there is new physics in the decay $\bar{b} \rightarrow \bar{s}c\bar{c}$, it can affect the measurement of $\beta$ in $B^0_d \rightarrow J/\psi K_s$. Earlier we said that a NP contribution of $\leq 15\%$ is still permitted. However, in the case of SUSY, we can do an explicit calculation. We find
that the ratio of SUSY and SM amplitudes in $B_d^0 \to J/\psi K_s$ is

$$|r| = \left| \frac{A_{\text{SUSY}}}{A_{\text{SM}}} \right| = \left| \frac{(Y + \tilde{Y})}{X} \right| \left( 1 - \frac{\xi^{BK}(m_{J/\psi})^2}{m_b^2 F_1^{BK}(m_{J/\psi})} \right),$$  

(3.38)

where

$$\xi^{BK}(m_{J/\psi}) = F_1(m_{J/\psi}) \left( m_B^2 + \frac{m_B^2 - m_K^2 - m_{J/\psi}^2}{2} \right) - \left( \frac{m_B^2 - m_K^2}{m_{J/\psi}^2} \right) \left( m_B^2 + \frac{m_{J/\psi}^2 - m_K^2}{2} \right) \left( F_1(m_{J/\psi}) - F_0(m_{J/\psi}) \right),$$  

(3.39)

and the form factors $F_{0,1}$ are defined in the Appendix. Using $m_b(m_b) = 4.5$ GeV and the form-factor values \cite{form factors},

$$F_1^{BK}(m_{J/\psi}) = 0.70, \quad F_0^{BK}(m_{J/\psi}) = 0.50,$$

(3.40)

one finds

$$|r| = -0.5 \left[ |(\delta_{LR})_{23}|^2 + |(\delta_{RL})_{23}|^2 + 2 |(\delta_{LR})_{23}||(\delta_{RL})_{23}| \cos(\varphi_{LR} - \varphi_{RL}) \right]^{1/2}.$$  

(3.41)

The square root takes a maximum value of 0.02, so that $|r|$ is quite small. We therefore see that the SUSY contribution to $B_d^0 \to J/\psi K_s$ is suppressed due to a cancellation of the various contributing amplitudes.

4. Conclusions

The CDF and D0 collaborations recently made a measurement of indirect CP violation in $B_d^0 \to J/\psi \phi$ and found a $2.2\sigma$ deviation from the standard model (SM). This suggests a nonzero value of $\beta_s$, the phase of $B_s^0-\bar{B}_s^0$ mixing. Since the SM predicts $\beta_s \simeq 0$, we assume the CDF/D0 result may be due to new physics (NP). In this paper, we have argued that any analysis of NP in $B_s^0-\bar{B}_s^0$ mixing is incomplete if NP in the decay $\bar{b} \to \bar{s}c\bar{c}$ is not considered.

In fact, most models that produce new effects in $B_s^0-\bar{B}_s^0$ mixing also contribute to $\bar{b} \to \bar{s}c\bar{c}$. We have analyzed a number of such models and find that, indeed, there can be NP effects in the decay. In general, the effect is not enormous. However, it may not be insignificant either – we find that the ratio of NP to SM contributions can be as large as $O(10-15\%)$. If this ratio is big in a given model, then even if the NP contribution to $B_s^0-\bar{B}_s^0$ mixing is not large enough to reproduce the CDF/D0 measurement, the addition of the new effects in $\bar{b} \to \bar{s}c\bar{c}$ may be sufficient. Similarly, there are certain models which do not contribute significantly to $\beta_s$, and hence cannot account for the current data. However, if future measurements find a smaller (nonzero) value for the indirect CP asymmetry in $B_s^0 \to J/\psi \phi$, these models might be able to explain the data through NP contributions to the decay.
Specifically, we have examined four NP models. (In all cases, constraints from $B^0_d \to J/\psi K_s$ have been taken into account.) We find that the model with $Z$-mediated FCNC’s does not lead to big effects in the $\bar{b} \to \bar{s}c\bar{c}$ decay. On the other hand, the model with $Z'$-mediated FCNC’s may do so if certain of the $Z'$ couplings are sufficiently large. The two-Higgs-doublet model (2HDM) contributes very little to $B^0_s - \bar{B}^0_s$ mixing. However, it can give significant contributions to the decay, so that the 2HDM can account for somewhat smaller values of the indirect CP asymmetry in $B^0_s \to J/\psi \phi$. Supersymmetry is similar – the contribution to $\beta_s$ is small (though nonzero), but it can give large contributions to the decay.

The models which contribute significantly to the decay $\bar{b} \to \bar{s}c\bar{c}$ typically also have other effects. In general, they predict polarization-dependent indirect CP asymmetries in both $B^0_s \to J/\psi \phi$ and $B \to J/\psi K^*$. And they also predict nonzero, triple-product (TP) asymmetries in $B^0_s \to J/\psi \phi$ ($\leq 10\%$), consistent with the constraints from TP asymmetries in $B \to J/\psi K^*$.

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A. Form Factors and Matrix Elements

For a general effective four-quark operator $O \sim X \otimes Y$ ($X, Y = \bar{q}\gamma_\mu q$, $\bar{q}\gamma_\mu \gamma_5 q$, $\bar{q}\gamma_\mu \gamma_5 q$ or $\bar{q}\sigma_{\mu\nu} \gamma_5 q$), the matrix element is factorized as

$$\langle J/\psi \phi | O | B^0_s \rangle \to \langle \phi | X | B^0_s \rangle \langle J/\psi | Y | 0 \rangle \ ,$$

(A.1)

where $\langle \phi | X | B^0_s \rangle$ is calculable using known form factors and $\langle J/\psi | Y | 0 \rangle$ is proportional to the $J/\psi$ decay constant.

For the $B^0_s \to \phi$ form factors, we follow the definitions in Ref. [2]:

$$\langle \phi(p, \epsilon) | \bar{s}\gamma_\mu (1 \pm \gamma_5) b \left| B^0_s(p_{B_s}) \right\rangle = \pm i\epsilon^*_\mu (m_{B_s} + m_\phi) A_1(s)$$

$$\mp i(p_{B_s} + p)_\mu (\epsilon^* \cdot p_{B_s}) \frac{A_2(s)}{m_{B_s} + m_\phi}$$

$$\mp i\epsilon_\mu (\epsilon^* \cdot p_{B_s}) \frac{2m_\phi}{s} \left( A_3(s) - A_0(s) \right)$$

$$+ \epsilon_{\mu\nu\rho\sigma} \epsilon^{\nu\rho} p_{B_s}^\sigma \frac{2V(s)}{m_{B_s} + m_\phi}$$

$$\langle \phi(p, \epsilon) | \bar{s}\sigma_{\mu\nu} b \left| B^0_s(p_{B_s}) \right\rangle = -i\epsilon_{\mu\nu\rho\sigma} \left[ g_+(s) \epsilon^{\rho\sigma}(p_{B_s} + p)^\sigma + g_-(s) \epsilon^{\rho\sigma} q^\sigma \right]$$

$$+ h(s) \frac{\epsilon^* \cdot p_{B_s}}{m_{B_s}^2 - m_\phi^2} (p_{B_s} + p)^\rho q^\rho \right] \ ,$$

$$\frac{m_{B_s}^2 - m_\phi^2}{m_{B_s} - m_\phi (p_{B_s} + p)^\rho q^\rho} \right] \ .$$
\[ \langle \phi(p, \epsilon) | \bar{s} \sigma_{\mu \nu} \gamma_5 b | B_s^0(p_{B_s}) \rangle = g_+(s) [\epsilon_\mu^* (p_{B_s} + p)_\nu - \epsilon_\nu^* (p_{B_s} + p)_\mu] \\
+ g_-(s) [\epsilon_\mu^* q_\nu - \epsilon_\nu^* q_\mu] \\
+ h(s) \frac{\epsilon_\mu^* \cdot p_{B_s}}{m_{B_s}^2 - m_\gamma^2} [(p_{B_s} + p)_\mu q_\nu - (p_{B_s} + p)_\nu q_\mu], \quad (A.2) \]

where \( q = p_{B_s} - p \) and \( s = q^2 \). We consider two scenarios for the form factors – the Melikhov-Stech [40] and Ball-Zwicky [41] models. The results are shown in Table 3.

| FF   | \( A_1 \) | \( A_2 \) | \( A_0 \) | \( V \) | \( g_+ \) | \( g_- \) | \( h \) |
|------|------|------|------|------|------|------|------|
| MS   | 0.42 | 0.49 | 0.76 | 0.80 | 0.69 | -0.66 | 0.18 |
| BZ   | 0.42 | 0.38 | 0.89 | 0.82 | 0.70 | -0.68 | 0.30 |

Table 3: Predictions for vector (\( A_1, A_2, A_0, V \)) and tensor (\( g_+, g_-, h \)) form factors in the Melikhov-Stech and Ball-Zwicky models, all evaluated at \( s = m_{J/\psi}^2 \). The tensor form factors are computed in the heavy-quark effective theory at maximum recoil [42].

We define the \( J/\psi \) decay constants according to Ref. [41]:

\[ \langle J/\psi(q, \epsilon) | \bar{c} \gamma^\mu c | 0 \rangle = f_{J/\psi} m_{J/\psi} \epsilon^\mu, \]

\[ \langle J/\psi(q, \epsilon) | \bar{c} \sigma^{\mu\nu} c | 0 \rangle = -i f_{J/\psi}^\perp (\epsilon^\mu q^\nu - \epsilon^\nu q^\mu), \quad (A.3) \]

which implies

\[ \langle J/\psi(q, \epsilon) | \bar{c} \sigma^{\mu\nu} \gamma_5 c | 0 \rangle = -\frac{1}{2} f_{J/\psi}^\perp \epsilon^{\mu\nu\rho\sigma} (\epsilon_\rho^* q_\sigma - \epsilon_\sigma^* q_\rho). \quad (A.4) \]

We take \( f_{J/\psi}^\perp \sim f_{J/\psi} = 405 \text{ MeV} \) [34].

The form factors relevant for \( B_d^0 \to J/\psi K_S \) are defined as [43]

\[ \langle K(k_2) | \bar{s} \gamma_\mu (1 - \gamma_5) b | B(p) \rangle = R_\mu F_1^{BK}(r^2) + Q_\mu F_0^{BK}(r^2), \]

\[ R_\mu = \frac{p_\mu + k_{2\mu} - m_{B_S}^2 - m_{K_\mu}^2}{r^2}, \]

\[ Q_\mu = \frac{m_{B_S}^2 - m_{K_\mu}^2}{r^2}. \quad (A.5) \]

In order to calculate the matrix elements, we define in the \( B \)-meson rest frame the four-momenta

\[ p_{B_s} = (m_{B_s}, 0, 0, 0), \]

\[ p_{\phi} = (E_{\phi}, 0, 0, -p_c), \]

\[ p_{J/\psi} = (E_{J/\psi}, 0, 0, p_c), \quad (A.6) \]

and polarization four-vectors

\[ \epsilon_0^\phi = \frac{1}{m_\phi} (p_c, 0, 0, -E_\phi), \]
\[ \epsilon^\pm_\phi = \frac{1}{\sqrt{2}} \left( 0, \mp 1, \pm i, 0 \right), \]
\[ \epsilon^0_{J/\psi} = \frac{1}{m_{J/\psi}} (p_c, 0, E_{J/\psi}), \]
\[ \epsilon^\pm_{J/\psi} = \frac{1}{\sqrt{2}} \left( 0, \mp 1, -i, 0 \right), \]

where \( p_c = [(m_{B_s}^2 - (m_{J/\psi} + m_\phi)^2)(m_{B_s}^2 - (m_{J/\psi} - m_\phi)^2)]^{1/2} / 2 m_{B_s}. \)

The resulting matrix elements for a given helicity state \((\lambda = 0, \pm)\) are

\[ \langle \phi | \bar{s} \sigma_{\mu \nu} b | B_s^0 \rangle \langle J/\psi | \bar{c} \sigma^{\mu \nu} c | 0 \rangle \big|_{\lambda = \pm} = \langle \phi | \bar{s} \sigma_{\mu \nu} \gamma_5 b | B_s \rangle \langle J/\psi | \bar{c} \sigma^{\mu \nu} \gamma_5 c | 0 \rangle \big|_{\lambda = \pm} \]
\[ = \mp 4 i f_{J/\psi} g_{+} (m_{J/\psi}^2) m_{B_s} p_c, \]
\[ \langle \phi | \bar{s} \sigma_{\mu \nu} b | B_s^0 \rangle \langle J/\psi | \bar{c} \sigma^{\mu \nu} \gamma_5 c | 0 \rangle \big|_{\lambda = 0} = \langle \phi | \bar{s} \sigma_{\mu \nu} \gamma_5 b | B_s \rangle \langle J/\psi | \bar{c} \sigma^{\mu \nu} c | 0 \rangle \big|_{\lambda = 0} \]
\[ = - 4 p_c^2 m_{B_s}^2 \left[ g_{+}(m_{J/\psi}^2) - h(m_{J/\psi}^2)m_{J/\psi}^2/(m_{B_s}^2 - m_\phi^2) \right] \]
\[ - i \frac{f_{J/\psi}^+}{m_{J/\psi}^2 m_\phi^2} \left[ g_{+}(m_{J/\psi}^2)(m_{B_s}^2 - m_{J/\psi}^2 - m_\phi^2)(m_{B_s}^2 - m_\phi^2) \right], \]
\[ \langle \phi | \bar{s} \gamma_\mu b | B_s^0 \rangle \langle J/\psi | \bar{c} \gamma^\mu c | 0 \rangle \big|_{\lambda = \pm} = \langle \phi | \bar{s} \gamma_\mu \gamma_5 b | B_s^0 \rangle \langle J/\psi | \bar{c} \gamma^\mu \gamma_5 c | 0 \rangle \big|_{\lambda = \pm} \]
\[ = 2 i f_{J/\psi} g_{+}(m_{J/\psi}^2)[m_{B_s}^2 - m_\phi^2] + \frac{g_{-}(m_{J/\psi}^2)}{g_{+}(m_{J/\psi}^2)} m_{J/\psi}^2, \]
\[ \langle \phi | \bar{s} \gamma_\mu \gamma_5 b | B_s^0 \rangle \langle J/\psi | \bar{c} \gamma^\mu c | 0 \rangle \big|_{\lambda = 0} = i f_{J/\psi} \frac{m_{B_s}^2 + m_\phi^2}{2 m_\phi} \times \]
\[ \left\{ (m_{B_s}^2 - m_{J/\psi}^2 - m_\phi^2) A_1 - \frac{4 m_{B_s}^2 p_c^2 A_2}{(m_{B_s}^2 + m_\phi^2)^2} \right\}, \]
\[ \langle \phi | \bar{s} \gamma_\mu \gamma_5 b | B_s^0 \rangle \langle J/\psi | \bar{c} \gamma^\mu | 0 \rangle \big|_{\lambda = \pm} = - i f_{J/\psi} m_{J/\psi}(m_{B_s} + m_\phi) A_1. \]

The remaining matrix elements are zero.

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