New Results on Perturbative Color Transparency in Quasi-Exclusive Electroproduction

Pankaj Jain\textsuperscript{a}, Bijoy Kundu\textsuperscript{a}, John Ralston\textsuperscript{b} and Jim Samuelsson\textsuperscript{a}

\textsuperscript{a}Physics Department, I.I.T. Kanpur, 208016, India
\textsuperscript{b}Department of Physics & Astronomy, University of Kansas, Lawrence, KS 66045, USA

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Abstract

We review the perturbative QCD formalism of hadronic electromagnetic form factors and the color transparency ratio for quasi-exclusive electroproduction of the proton and pion from nuclear targets. We have completed the first full calculations including all leading order quark subprocesses and integrations over distribution amplitudes, including Sudakov effects. For the case of the proton, the calculated result shows scaling beyond $Q^2 = 10 \text{ GeV}^2$. The calculation incorporating filtering due to the nuclear medium is cleaner than the corresponding calculation in free space because of attenuation of large distance amplitudes. We find that the color transparency ratio is rather insensitive to theoretical uncertainties inherent in the perturbative formalism, such as the choice of the hadron distribution amplitude.

1 Introduction

The practical applicability of perturbative QCD to exclusive processes\cite{1,2,3} such as hadronic electromagnetic form factors cannot yet considered to be settled. It has been argued that even at the highest momenta explored so far in the laboratory, the dominant contribution to form factors comes from the end point regions of the wave function, where the perturbative treatment fails\cite{4,5}. In the case of hadron-hadron scattering there exist further difficulties, such as the common failure of the helicity conservation selection rules to agree with experimental data. However in nearly all experiments one finds that the naive prediction for quark counting scaling laws tend to agree very well with
data. In view of the problems listed, this is quite mysterious, since so far there
does not exist any alternate mechanism which can explain these scaling laws.

An interesting prediction of perturbative QCD is color transparency [6]. At
large momentum transfers only the short distance components of the hadron
wave function can contribute to exclusive processes. Since the total cross section
of hadrons $\sigma$ is inversely proportional to their area $b^2$, the strong interac-
tions of these hadrons is expected to be reduced. If we consider quasi-exclusive electron-
nucleus scattering, $eA \to e'p(A-1)$, where $A$ is nuclear number, then the nucleus
is predicted to be transparent to all protons participating in this process. This
is an asymptotic argument applicable for fixed $A$ as $Q^2 \to \infty$. Experimentally,
however, we can only take the limit of large $A$ and moderately large $Q^2$, in which
such processes appear to be more complicated and much more interesting. One
picture that is emerging [7] is that exclusive processes in free space get significant
contribution from perturbatively calculable hard amplitudes but also have non-
negligible soft contamination. The corresponding nuclear processes, however,
may be much cleaner [8, 9] because the large quark separations will be strongly
attenuated in nuclear medium. This phenomenon, called nuclear filtering, has
some experimental support. Experimentally one finds that the fixed-angle free
space process $pp' \to p'p''$ [10] shows significant oscillations at 90 degrees as a
function of energy. These oscillations are not a small effect, but roughly 50% of
the $1/s^{10}$ behavior, and are interpreted as coming from interference of long and
short distance amplitudes. The corresponding process in a nuclear environment
$pA \to p'p''(A-1)$ shows no oscillations, and obeys the pQCD scaling power
law better than the free-space data [8, 9]. The $A$ dependence, when analyzed
at fixed $Q^2$, shows statistically significant evidence of reduced attenuation [11].

2 Formalism

We briefly review the framework for calculation of hadronic form factors following Li and Sterman [12]. It has long been known that the transverse separation of
quarks in free space reactions is controlled by effects known as the Sudakov
form factor. The pion form factor is the simplest example. Li and Sterman
included Sudakov effects here, arguing that a perturbative treatment becomes
fairly reliable at momenta of the order of 5 GeV. As low as 2 GeV, it was found
that less than 50 % of the contribution comes from the soft region.

Let $b_{ij}$ be the transverse separation between quarks $i$ and $j$, or $b$ the cor-
responding quantity for a single pair of quarks. An essential feature is the
inclusion of $\exp(-S)$, a Sudakov form factor which suppresses the large $b$
region. Including the $b$ dependence, the pion electromagnetic form factor can be
written as,

$$F_\pi(Q^2) = \int dx_1 dx_2 \frac{d\vec{b}}{(2\pi)^2} \mathcal{P}(x_2, \vec{b}, P', \mu) T_H(x_1, x_2, \vec{b}, Q, \mu) \mathcal{P}(x_2, \vec{b}, P, \mu). \quad (1)$$
where
\[ \mathcal{P}(x, b, P, \mu) = \exp(-S) \times \phi(x, 1/b) + O(\alpha_s(1/b)) , \]
plays the role of the hadron wave function, \( \phi(x, 1/b) \) is the meson distribution amplitude, \( P \) and \( P' \) are the incident and outgoing pion momenta respectively, and \( S \) is the Sudakov form factor. The improved factorization used in Ref. [12] retains the intrinsic transverse momentum \( k_T \) dependence in the gluon propagator, since \( k_T \) need not be small compared to \( \sqrt{x_1 x_2 Q} \), if one of the \( x_i \) get close to zero. The variable \( b \) in Eq. [3] is conjugate to \( k_T^1 - k_T^2 \), where \( k_T^1 \) and \( k_T^2 \) are the transverse momenta of the incident and outgoing pions. As long as \( x_1 \) and \( x_2 \) are not close to their endpoints, the dominant scale in the scattering is \( \sqrt{x_1 x_2 Q} \) and the small \( b \) region dominates the amplitude. Close to the end points of \( x_1 \) or \( x_2 \), \( \sqrt{x_1 x_2 Q} \) may become very small. However, the dominant scale in this region is \( 1/b \), which is again not too small since the large \( b \) region is strongly damped by the Sudakov form factor. The results for the free space form factor for pion using this procedure are given in Ref. [13]. The authors show that at \( Q^2 = 5 \) \( \text{GeV}^2 \), something like 90% of the contribution comes from a region where \( \alpha_s/\pi \) is less than 0.7 and hence could be regarded as perturbative.

The nuclear medium modifies the quark wave function such that
\[ \mathcal{P}_A(x, b, P, \mu) = f_A(Q^2, b)\mathcal{P}(x, b, P, \mu), \]
where \( \mathcal{P}_A \) is the wave function inside the medium and \( f_A \) is the nuclear filtering amplitude. We use a simple model for \( f_A \),
\[ f_A = \exp(- \int dz \sigma \rho) . \]
The effective inelastic cross section \( \sigma \) is known to scale like \( b^2 \) in QCD, where \( b \) is the size of the hadron. We parametrize it as \( k b^2 \) and adjust the value of \( k \) to find a reasonable fit to the experimental data.

The situation for the proton form factor [15] is somewhat more complicated than that of the pion; we do not have the space for all details here which are given in Ref. [16, 17]. There has been some controversy regarding the proper choice of the infrared cutoff in the Sudakov exponent. In the case of pion this was simply the quark-antiquark separation \( b \). The choice proposed in Ref. [18] uses the largest distance between the three quarks as the cutoff. It was found that this gave results about 50% smaller than experiments. Perhaps this is the right direction, if indeed other wave functions (and in particular, non-zero quark angular momentum) contribute heavily in free space. On the other hand, in Ref. [16] it was observed that the largest distance does not correspond to a physical size of the three quark system. A more appropriate choice might be obtained by considering the triplet of valence quarks as a quark-diquark system. This choice takes the maximum value of the distance between quark and diquark as the effective cutoff in the Sudakov exponent. This essentially amounts to using
Figure 1: The $Q^2$ dependence of the proton form factor $Q^4 F_1$ using the KS wave function ($c = 1.14$, solid line; $c = 1$, dense-dot line) and for the CZ wave function ($c = 1.14$, dashed line; $c = 1$, dotted line). The experimental data are also shown.

a scale $cw$ for infrared cutoff, with $c \approx 1.14$, where $w$ is the inverse of the largest distance between any two valence quarks in proton. Remarkably, this small modification leads to results in good agreement with the experiment[16].

From investigations of the proton form factor in free space, it seems that Sudakov effects eliminate about 50% of the contribution from the soft region. The Sudakov filtering in free space does something useful, but does not seem to be sufficient to make present free-space calculations totally reliable. The same diagrams for Sudakov effects of course occur in a nuclear environment. In addition, there are much stronger interactions with the nuclear target, when one goes from pure “vacuum filtering” by Sudakov to nuclear filtering. We find that nuclear medium eliminates much of the remaining 50% of the soft region. These are the first full calculations of these ideas within perturbative QCD. We find that the main uncertainty in the nuclear calculation arises from uncertainties in nuclear medium itself, in particular, in uncertainties in the nuclear spectral functions and correlations. With standard assumptions one can proceed with the calculation essentially using zero parameters and no model dependence. However, we find that numerical differences between models of nuclear matter are large enough to cause significant uncertainties. Indeed, comparison with data shows that the uncertainties in the nuclear spectral functions and the nuclear correlations now dominate the theoretical uncertainties, and are larger effects than, for example, the dependence on the hadron distribution amplitude.
3 Results and Discussions

The results for free space proton form factor are shown in Fig. 1. An important feature of this result, which is independent of the details of the wave function, is that it shows scaling for $Q^2$ larger than about 10 GeV$^2$. This is a nontrivial confirmation that $Q^2$ indeed dominates over the intrinsic momentum $k^2_T$.

In Fig. 2, we show results for color transparency for electroproduction of pions for different nuclei using the CZ wave function. Here we adjust the value of $k$, corresponding to the pion attenuation cross-section of 25-30 mb for a pion size of about 0.8 fm. The predicted results are shown for $k=4$. The precise value of $k$ might best obtained by making a fit to the data for color transparency after it becomes available, or perhaps by detailed comparison with diffractive calculations. Compared to the asymptotic wave functions, the results for $T$ change by less than 3% for $Q^2$ larger than 10 GeV$^2$.

The results for the proton transparency ratio are given in Fig. 3. The parameter $k$ in the attenuation cross section $\sigma = kb^2$ was chosen so as to provide a reasonable fit to the experimental data $^{19, 20}$. We find that a value of $k = 6$ gives a reasonable fit. Taking the attenuation cross section of normal protons to be 36 mb, this corresponds to a typical $b$ of about 0.77 fm, which is a reasonable estimate of the proton size. Since the data for $T$ is available only in the region where the calculated free space form factor is in disagreement with the experimental result, the value of $k$ obtained by this procedure cannot be taken too seriously. In fact, parameter $k$ would be best obtained by fitting to the experimental value of $T$ after it is measured at higher energies. A reasonable range of $k$ values, which we take to be $k = 5$ and $k = 6$, corresponds to $b$ values of 0.85 fm and 0.77 fm respectively, and has been used in the figure.
We have also checked the dependence of our result on the infrared cutoff parameter $c$ and the choice of the wave function. We find in Fig. 4 that the results for transparency ratio change very little if we use the CZ wave function instead of the KS. This is a surprising result, and one of the basis of our claim that the dominant uncertainty in transparency ratio may be due to the nuclear model itself.

4 Conclusion

We have reviewed the calculation of hadronic electromagnetic form factors and color transparency using perturbative QCD. We find a slow rise in the transparency ratio for energies that can be probed in the future at CEBAF and ELFE. As discussed elsewhere [11, 7], precision experiments can discover color transparency even with a slow rise in $Q^2$ by measuring the $A$ dependence at fixed moderately large $Q^2$. Due to filtering of long distance components in the medium, the nuclear calculation is considerably cleaner compared to the free space calculation. We also find rather remarkable insensitivity of the transparency ratio to present theoretical uncertainties in the perturbative QCD treatment, such as the choice of the distribution amplitude. To further improve the accuracy of predictions for color transparency ratio, it is necessary to improve the modelling of nuclear medium which now appears to be the dominant source of error.
Figure 4: The sensitivity of the calculated transparency ratio to different proton wave functions. Slight oscillations are an artifact of the Monte Carlo integrations. The solid curve is calculated with the KS wave function, as in Fig. 3, and the dotted curve is calculated with the CZ wave function; both curves use the infrared cutoff parameter \( c = 1.14 \). For reference, the dashed curve shows the result for the cutoff proposed in Ref. 18, which amounts to setting \( c = 1.0 \), using the KS wave function. The calculations are shown for \( A = 197 \).

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