Role of dynamics of the population inversion in attosecond pulse amplification in the active medium of a plasma-based X-ray laser dressed by an optical laser field

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Abstract. We study the role of dynamics of the population inversion of an active medium of an X-ray laser dressed by a strong infrared (IR) laser field in the amplification of an attosecond pulse train produced via high-harmonic (HH) generation by the same IR field. We derive an analytical solution for the case of a recombination hydrogen-like X-ray laser, which shows that the time dependence of the population inversion strongly affects the level of the HH amplification and the envelope of the amplified attosecond pulse train. The possibilities for experimental implementation are discussed for the case of C⁵⁺ X-ray laser with the transition wavelength ~3.4 nm in the "water window" range.

1. Introduction

High-harmonic generation (HHG) by optical laser fields is a powerful tool of the attosecond physics and X-ray spectroscopy. In particular, it has enabled formation of pulses with durations of tens of attoseconds [1, 2] and generation of coherent spectra spanning over multiple absorption edges of matter [3, 4]. Such radiation sources enable study of various ultrafast processes in atoms, molecules and solids [4-7]. However, the power of high-order harmonics (HHs) is quite limited, especially in the X-ray range, which limits their practical applications. This makes amplification of attosecond high-harmonic signals an important task.

Recently, we have shown the possibility to amplify an attosecond pulse train produced via HHG of an IR laser field in the active medium of a plasma-based X-ray laser dressed by a replica of the IR field used for HHG with linear polarization (along z axis) [8]. As an active medium, we considered the plasma of hydrogen-like ions with the population inversion at the transition from the first excited energy level, \( n=2 \), to the ground state, \( n=1 \) (where \( n \) is the principal quantum number). Under the action of the IR field, the excited energy level of the ions is split into three sublevels. The two of them oscillate in time and space along with the IR field strength due to the linear Stark effect (and undergo a quadratic Stark shift). These sublevels correspond to the states \( |2\rangle = (|2s\rangle + |2p, m=0\rangle)/\sqrt{2} \) and \( |3\rangle = (|2s\rangle - |2p, m=0\rangle)/\sqrt{2} \), which are the eigenstates of hydrogen-like ions in parabolic coordinates. The third sublevel remains degenerate and corresponds to the states \( |4\rangle = |2p, m=1\rangle \) and \( |5\rangle = |2p, m=-1\rangle \), which experience only a quadratic Stark shift under the action of the IR field. The sub-IR-field cycle variation of the frequencies of the transitions \( |2\rangle \leftrightarrow |1\rangle \) and \( |3\rangle \leftrightarrow |1\rangle \) (whose dipole moments are oriented...
along z axis), where $|1\rangle = |1s\rangle$ is the ground state of the ions, results in redistribution of the gain of the active medium for the X-ray field with z-polarization from the resonance frequency to the sidebands separated from the resonance by even multiples of the frequency of the IR field. If the same IR field is used both for HHG and for the modulation of the active medium, then the separation between the neighboring harmonics is the same as between the induced gain lines, which enables amplification of the multifrequency HH fields. In turn, the transitions $|4\rangle \leftrightarrow |1\rangle$ and $|5\rangle \leftrightarrow |1\rangle$ (with dipole moments along $y$ axis) are the source of amplified spontaneous emission of $y$-polarization.

In [8] it was assumed that the population inversions at the transitions of the ions are created instantly. In fact, the population inversion (i) appears as a result of pumping, which takes a finite time, and (ii) decays because of the radiative transitions and inelastic collisions. In the case of multiply charged ions with the inverted transitions in the X-ray range, the radiative decay of the population inversion occurs on the sub-picosecond time scale and may considerably affect the level of HH amplification and the envelope of the amplified attosecond pulse train.

In the present contribution, we derive an analytical solution for the amplified HH field, which takes into account the gradual growth of the population inversion due to pumping and its subsequent decay. We study the role of dynamics of the population inversion in the amplification of attosecond pulse trains and discuss the possibilities for experimental implementation in the plasma of $C^{5+}$ hydrogen-like ions with the transition wavelength $\sim 3.4$ nm in the "water window" range.

2. Basic concept

The population inversion at the transition $n=2 \leftrightarrow n=1$ of hydrogen-like ions is created as a result of (i) complete ionization of the active medium by a femtosecond laser pulse, (ii) recombination of the electrons to the Rydberg states of the ions, and (iii) their subsequent radiative decay into the states with $n=2$ [9]. In order to derive an analytical theory, we substitute the higher lying states of the ions by a single effective state $|6\rangle$, which decays into each of the lasing states $|2\rangle$-$|5\rangle$ with the rate $\gamma_{\text{pump}}$. In turn, the states $|2\rangle$-$|5\rangle$ decay to the ground state $|1\rangle$ with the rate $\gamma_{\text{decay}}$, see figure 1. If initially (at local time $\tau = t - z/c = 0$) all the ions are in the state $|6\rangle$ and the populations of the states change only due to the spontaneous and non-radiative transitions, then the population difference at each of the transitions $|i\rangle \leftrightarrow |1\rangle$, $i=2,3,4,5$ has a form

$$n_{i\gamma}(\tau) = e^{-\gamma_{\text{coherence}} \tau} - 1 + \frac{\gamma_{\text{pump}} + \gamma_{\text{decay}}}{4\gamma_{\text{pump}} - \gamma_{\text{decay}}} \left(e^{-\gamma_{\text{coherence}} \tau} - e^{-\gamma_{\text{pump}} \tau}\right).$$

![Figure 1. The relevant energy level scheme of the hydrogen-like ions.](image)

![Figure 2. The population difference (1) for different values $\gamma_{\text{pump}}$ and $\gamma_{\text{decay}}$ (see the text).](image)

The local-time dependencies of the population difference (1) for different values of $\gamma_{\text{pump}}$ and $\gamma_{\text{decay}}$ are plotted in figure 2. Red solid curve corresponds to $\gamma_{\text{decay}}/\gamma_{\text{coherence}}=0.016$ and $\gamma_{\text{pump}}/\gamma_{\text{decay}}=125$, where $\gamma_{\text{coherence}}$ is the decoherence rate at the transition $n=2 \leftrightarrow n=1$. Blue dashed curve is plotted for the case $\gamma_{\text{decay}}/\gamma_{\text{coherence}}=0.016$ and $\gamma_{\text{pump}}/\gamma_{\text{decay}}=10$; black dotted curve shows the case of $\gamma_{\text{decay}}/\gamma_{\text{coherence}}=0.5$ and...
The four panels in figure 5 correspond to the same four conditions and parameter values as considered in [8]. Figure 4 shows the time dependencies of the population difference \( \gamma_{\text{coh}} \) between the states \( \gamma_{\text{coh}} \). The effective gain coefficients for the harmonics reach the maximum values if both the peak value of the population inversion \( \gamma_{\text{coh}} \) is large and the decay \( \gamma_{\text{coh}} \) is small even in the case of fast pumping.

If the decay rate is large, the gain of the active medium is small even in the case of fast pumping.

In figure 5 we plot the time dependencies of the IR field, which corresponds to the population difference \( \gamma_{\text{coh}} \). If the amplitude of any harmonic in the medium doesn't change considerably on the time scale of the linear Stark shift of the states \( \Omega \), then one can find the amplified HH field at the exit from the medium in the form

\[
\tilde{E}_n(x,\tau) = \sum_{n=-N_{\text{max}}}^{N_{\text{max}}} E_n^{(\text{inc})} \exp\left\{ g_n(\tau) x \right\} e^{-i2\Omega x} , \quad \text{where} \quad g_n(\tau) = g_{\text{max}} J^2_{\text{coh}}(P_{\Omega}) f(\tau),
\]

\( E_n^{(\text{inc})} \) is the amplitude of the \( n \)-th harmonic at the entrance to the medium (2\( n \Omega \)) its detuning from the resonance, \( \Omega \) is the frequency of the IR field, \( x \) is the propagation coordinate, \( g_n(\tau) \) is the effective gain coefficient for the \( n \)-th harmonic, \( g_{\text{max}} \) is the maximum possible gain of the active medium in the absence of the IR field, which corresponds to the population difference \( n_{\text{max}} = 1/4 \), \( J_{\text{coh}}(P_{\Omega}) \) is the Bessel function of the first kind of order \( 2n \), \( P_{\Omega}=\Delta\Omega/\Omega \) is the modulation index, \( \Delta\Omega \) is the amplitude of the linear Stark shift of the states \([2,3]\) under the action of the IR field, and \( f(\tau) \) has a form

\[
f(\tau) = \frac{1}{n_{\text{max}}} \left[ e^{-\gamma_{\text{coh}}\tau} - 1 + \frac{5\gamma_{\text{pump}}}{4\gamma_{\text{pump}} - \gamma_{\text{decay}}} e^{-\gamma_{\text{coh}}\tau} - \frac{\gamma_{\text{pump}}}{4\gamma_{\text{pump}} - \gamma_{\text{decay}}} e^{-\gamma_{\text{coh}}\tau} - \gamma_{\text{coh}} \left( 1 - \frac{\gamma_{\text{pump}}}{\gamma_{\text{coh}}} \right) \right].
\]

If \( \gamma_{\text{coh}}/\gamma_{\text{coh}} << 1 \), \( \gamma_{\text{pump}}/\gamma_{\text{coh}} << 1 \), and \( \gamma_{\text{coh}}/\gamma_{\text{coh}} >> 1 \), then \( f(\tau) = n_{\text{p}}(\tau)/n_{\text{p}}^{\text{max}} \); otherwise, there is an integral relation between \( f(\tau) \) and \( n_{\text{p}}(\tau) \). The maximum value of \( f(\tau) \) as a function of the parameters \( \gamma_{\text{coh}}/\gamma_{\text{coh}} \) and \( \gamma_{\text{pump}}/\gamma_{\text{coh}} \) is plotted in figure 3. Figure 4 shows the time dependencies of \( f(\tau) \), which correspond to the time dependencies of the population inversion \( n_{\text{p}}(\tau) \) in figure 2.

**Figure 3.** Peak value of the normalized effective gain coefficients, (of \( f(\tau) \), see (3)), as a function of \( \gamma_{\text{coh}}/\gamma_{\text{ coh}} \) and \( \gamma_{\text{pump}}/\gamma_{\text{coh}} \).

**Figure 4.** The time dependencies of \( f(\tau) \). Line styles and colours correspond to the same parameter values as in figure 2.

As follows from figures 3 and 4, the effective gain coefficients for the harmonics reach the maximum values if both \( \gamma_{\text{pump}} >> \gamma_{\text{coh}} \) (the peak value of the population inversion (1) is large) and \( \gamma_{\text{ coh}} << \gamma_{\text{coh}} \) (the resonant polarization settles up before the population inversion decays). On the other hand, if the decay rate is large, the gain of the active medium is small even in the case of fast pumping.

In figure 5 we plot the time dependencies of intensity of the attosecond pulse train, amplified in a 1 mm long active medium of C\(^{5+}\) hydrogen-like ions (the ion and free-electron densities are \( 10^{19} \) cm\(^{-3} \) and \( 1.5 \times 10^{29} \) cm\(^{-3} \), respectively; \( g_{\text{max}} \approx 545 \) cm\(^{-1} \)), dressed by the IR field of the fundamental frequency. The four panels in figure 5 correspond to the same four combinations of \( \gamma_{\text{coh}} \) and \( \gamma_{\text{coh}} \), as considered in previous sections.
in figures 2 and 4. The pulses are constituted by 7 in-phase HHs of the IR field with the central wavelength of 3.38 nm. The pulse duration is 150 as. The modulation index is $P_0=6.4$.

![Figure 5](image)

**Figure 5.** The amplified train of attosecond pulses. $I_0$ is intensity of the seeding field. (a) $\gamma_{\text{decay}}/\gamma_{\text{coh}}=0.016$ and $\gamma_{\text{pump}}/\gamma_{\text{decay}}=10$; (b) $\gamma_{\text{decay}}/\gamma_{\text{coh}}=0.016$ and $\gamma_{\text{pump}}/\gamma_{\text{decay}}=125$; (c) $\gamma_{\text{decay}}/\gamma_{\text{coh}}=0.5$ and $\gamma_{\text{pump}}/\gamma_{\text{decay}}=0.32$; (d) $\gamma_{\text{decay}}/\gamma_{\text{coh}}=0.5$ and $\gamma_{\text{pump}}/\gamma_{\text{decay}}=4$. Colours are the same as in figures 2, 4.

As follows from figure 5, the time dependence of the population inversion strongly affects the level of amplification of the attosecond pulse train and the envelope of the amplified signal. However, the pulse shape and duration are almost the same in all the panels, which shows that this mechanism of the attosecond pulse amplification is robust with respect to the variation of the population difference.

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