A Field Theoretic Investigation of Spin in QCD

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Abstract

Utilizing the kinematical boost in light-front formalism one can address the issue of relativistic spin operators in an arbitrary reference frame. In the gauge $A^+ = 0$, the interaction dependent transverse spin operators can be separated into three parts. In analogy with the helicity sum rule, we propose a transverse spin sum rule. We perform a one loop renormalization of the transverse spin operator and show that the counterterm needed is the same as the linear mass counterterm in the light-front QCD Hamiltonian.

1 Introduction

The issue of the relativistic spin operators in quantum field theory in an arbitrary reference frame is quite complex. The spin operators can be constructed from the Pauli-Lubanski operators, however, they are interaction dependent in an arbitrary frame and it is quite difficult to separate the center of mass motion from internal motion for a composite system in the usual equal time formulation [1]. Light-front formulation of quantum field theory is more suitable for the study of relativistic spin operators since boost is kinematical in this formulation. The interaction dependence of the light-front transverse spin operators come from the transverse rotation generators, which are dynamical (interaction dependent) in light-front theories in contrast to the equal-time case. The separation of the center of mass motion from the internal motion in this case is as simple as in non-relativistic theory.
2 Light-Front Spin Operators

In terms of Poincare generators, light-front spin operators can be written as [2],

\[ MT^i = W^i - P^i T^3, \quad i = 1, 2 \]
\[ = \epsilon^{ij} \left( \frac{1}{2} F^j P^+ - \frac{1}{2} E^j P^- + K^3 P^j \right) - P^i T^3, \]

(1)

where \( W^\mu \) is the Pauli-Lubanski operator, \( W^\mu = -\frac{1}{2} \epsilon^{\mu\nu\rho\sigma} M_{\nu\rho} P_{\sigma} \) with \( \epsilon^{+12} = -2 \), \( M \) is the mass operator; \( M^{\mu\nu} \) are the generalized angular momentum and \( P^\mu \) is the momentum 4-vector. The boost operators are \( M^{+-} = 2 K^3, M^{+i} = E^i \). Rotation operators are, \( M^{12} = J^3, M^{-i} = F^i \). The third component of the spin operator is the helicity operator which is defined as,

\[ T^3 = \frac{W^+}{P^+} = J^3 + \frac{1}{P^+} (E^1 P^2 - E^2 P^1). \]

(2)

In addition to the light-front Hamiltonian \( P^- \), the transverse rotation operators \( F^i \) are dynamical (interaction dependent). As a result, the transverse spin operators are also interaction dependent.

In the light-front gauge \( A^+ = 0 \), the helicity operator is equal to its naive canonical form, independent of interaction, provided the fields vanish at infinity. This situation can be contrasted with the third component of the spin operator in the equal-time case, which is interaction dependent. Explicitly, the light-front helicity operator can be separated into quark and gluon orbital and intrinsic parts at the operator level and the helicity sum rule for the nucleon is given by [3,4],

\[ \langle PS^\parallel | T^3_{f(i)} + T^3_{f(o)} + T^3_{g(o)} + T^3_{g(i)} | PS^\parallel \rangle = \pm \frac{1}{2}. \]

(3)

It can be verified that the matrix element of \( T^3_{f(i)} \) is directly related to the first moment of the flavor singlet part of the longitudinally polarized structure function \( g_1(x, Q^2) \).

In order to construct the transverse spin operators, one has to calculate the dynamical transverse rotation operators. The transverse spin operators cannot be separated into orbital and spin parts. We have shown that [5,6], in the light-front gauge, there exists a physically interesting decomposition for them,

\[ T^i = T^i_I + T^i_{II} + T^i_{III}, \]

(4)
where $T^{i}_{II}$ and $T^{i}_{III}$ do not depend on the coordinates explicitly and arise from the fermionic and gluonic parts of the gauge invariant, symmetric energy momentum tensor respectively. In analogy with the helicity sum rule, we propose a transverse spin sum rule for the nucleon,

$$\langle PS_{\perp} | T^{i}_{I} + T^{i}_{II} + T^{i}_{III} | PS_{\perp} \rangle = \frac{1}{2}. \tag{5}$$

The intrinsic fermionic part $T^{i}_{II}$ is directly related to the first moment of the flavor singlet part of the structure function $g_{T}$ measured in transverse polarized scattering. The difference from the helicity case is the non-trivial interaction dependence of the transverse spin operators, because of which the above relation has no partonic interpretation.

### 3 Renormalization of the Transverse Spin Operators

The transverse spin operators acquire divergences in perturbation theory and have to be renormalized. We have performed one loop renormalization of the full transverse spin operator by evaluating its matrix element for a quark state dressed with a gluon in light-front Hamiltonian perturbation theory [7]. Explicit evaluation in an arbitrary reference frame shows that all the center of mass momentum dependence get canceled. Also the contribution from $T^{i}_{I}$ exactly cancels the contribution from $T^{i}_{III}$ and the entire contribution comes from $T^{i}_{II}$, which is given by,

$$\langle P, S^{i} | MT^{i} | P, S^{i} \rangle = \frac{m}{2} \left( 1 + \frac{3\alpha_{s}}{4\pi} C_{F} \ln \frac{Q^{2}}{\mu^{2}} \right), \tag{6}$$

where $m$ is the bare mass of the quark, $C_{F}$ is the color factor and $\mu$ is the hadronic factorization scale. The details of the calculation can be found in [7]. Using the expression of the renormalized mass of the quark in terms of the bare mass [8],

$$m_{R} = m \left( 1 + \frac{3\alpha_{s}}{4\pi} \ln \frac{Q^{2}}{\mu^{2}} \right), \tag{7}$$

we verify the sum rule (5). This shows that only one counterterm is needed to renormalize the transverse spin operator, and that is the same as the linear mass counterterm in the light-front QCD Hamiltonian. Also, the mass of the quark plays a very important role in this case since the transverse spin operator is responsible for helicity flip and it is the terms linear in mass that causes helicity flip in light-front theory.
In summary, we have studied the relativistic spin operators for a composite system in an arbitrary reference frame in light-front QCD. The transverse spin operators are interaction dependent and in the light-front gauge, they can be separated into three physically interesting parts, provided the fields vanish at the boundary. The helicity operator in the same gauge is interaction independent. In analogy with the helicity sum rule, we have proposed a transverse spin sum rule. We have renormalized the transverse spin operators upto one loop and shown that the counterterm that has to be added is the same as the linear mass counterterm in light-front QCD Hamiltonian.

4 Acknowledgments

This is based on the work done in collaboration with A. Harindranath and R. Ratabole. I would also like to thank the organisers of X-th International Light-Cone Meeting on Non-Perturbative QCD and Hadron Phenomenology for giving me the chance to present my work.

References

[1] See, for example, B. Bakamjian and L. H. Thomas, Phys. Rev. 92 1300 (1955); H. Osborn, Phys. Rev. 176 1514 (1968).

[2] K. Bardakci and M. B. Halpern, Phys. Rev. 176, 1686 (1968); D. E. Soper, Field Theories in the Infinite Momentum Frame, Ph. D. Thesis, Stanford University, (1971), SLAC-137; H. Leutwyler and J. Stern, Ann. Phy. 112, 94 (1978).

[3] A. Harindranath and Rajen Kundu, Phys. Rev. D 59, 116013 (1999).

[4] S. J. Brodsky, D. S. Hwang, B. Q. Ma and I. Schmidt, [hep-ph/0003082].

[5] A. Harindranath, Asmita Mukherjee and Raghunath Ratabole, [hep-ph/990842]. Phys. Lett. B476, 471 (2000).

[6] A. Harindranath, Asmita Mukherjee and Raghunath Ratabole, [hep-th/0004192].

[7] A. Harindranath, Asmita Mukherjee and Raghunath Ratabole, [hep-th/0004193].

[8] R.J. Perry, Phys. Lett. B 300, 8 (1993); A. Harindranath and W.-M. Zhang, Phys. Rev. D 48, 4903(1993).