Excitation spectra of heavy baryons in diquark models

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The excitation energy spectra of heavy baryons consisting of a heavy quark and two light quarks are investigated by using diquark models in order to examine the nature of the diquark as a constituent of single heavy baryons. We consider two diquark models; in model A the diquark is treated as a point-like particle, while it has a spatial size in model B. We determine the masses of scalar and axial vector diquarks by the mass difference of the ground state charmed baryons to the \(\Lambda_c\) baryon, while the mass of the \(ud\) scalar diquark in the \(\Lambda_c\) baryon is assumed to be 500 MeV as a reference. The parameters of these models are fixed by the \(1p\) excitation energy of \(\Lambda_c\). We find that model A reproduces well the excitation energy spectra of the charmed and bottomed baryons, although the string tension of the confinement potential in model A should be a half of that of the charmonium, while Model B suggests degeneracy of the \(2s\) and \(1d\) states, which is not seen in the \(\Lambda_c\) spectrum.

1. Introduction

Understanding the hadron structure in terms of dynamics of quarks and gluons is one of the most important issues in hadron physics. For this purpose, the quest of effective constituents of hadrons is a clue to understand the structure of the hadrons. The constituent quarks are such effective degrees of freedom, explaining, for instance, the fine structure of heavy quarkonia \([1, 2]\) and the static properties, such as the magnetic moments of the light baryons \([3, 4]\). Along this line we try to find the possibility that the diquark also can be a constituent of hadrons.

The diquark is a pair of quarks and was already mentioned in Ref. \([5]\) at the same time when quarks were introduced to explain the spectra of hadrons \([5, 6]\). The diquark is a colored object and cannot be observed directly in experiments. Since introduced as a constituent of baryons \([7, 8]\), the diquark correlations have been pointed out so far phenomenologically as summarized in Ref. \([9, 10]\). Among various diquarks, one expects stronger attraction between two quarks in flavor, spin and color antisymmetric configurations, which is so-called good diquark, owing to the most attractive color magnetic interaction in perturbative calculation and instanton-induced interaction \([10]\). The good diquark is also favored in lattice calculation \([11–14]\). The characteristic size of the diquark was suggested to be about \(1.1 \pm 0.2\) fm in lattice calculation \([14]\) and to be smaller than 1 fm in the Schwinger-Dyson approach \([15]\). It
has been reported that light scalar mesons are described well in a diquark-antidiquark picture [16–19] and their strong decays are reasonably reproduced in Refs. [18, 19]. The diquark has been also studied extensively in the quark model approaches [20–28], and recently it has been found in Refs. [29, 30] that the confinement potential for the heavy quark and the light diquark should be weaker than that for the heavy quark and antiquark in quarkonia in order to produce the first excitation energy of the Λc baryon. A diquark QCD sum rule was proposed in Ref. [31] and found the constituent ud diquark mass to be around 0.4 GeV for the Λ, Λc and Λb baryons. Recently a symmetry between the s quark and the ud diquark was introduced in Ref. [32] based on the V(3) symmetry [33, 34] and proposed a new classification scheme of the heavy hadrons. Based on this symmetry, a sum for the weak decay rates of $B^0_s$ and $Λ_b$ was derived in Ref. [36]. A quark-diquark symmetry for heavy systems was also discussed in Ref. [37].

Thanks to the strong correlation between light quarks, the diquark may play an important role of the structure of baryons. If so, diquarks can be effective constituents of hadron. Nevertheless, as discussed in Ref. [38], tetraquark meson wavefunctions are found to be dominated by quark-antiquark configurations, which tells us that diquark correlations may be rather suppressed even in multiquark hadrons. This also implies that the diquark correlation is possibly fragile and the reformation of the quark-antiquark pairs from the diquark and anti-diquark can be important in light quark systems. Therefore, baryons with a single heavy quark are good systems to investigate the effectiveness of diquark as a constituent of hadrons. If the diquark picture works for the single heavy baryons, their excitation spectra should be explained by two-body dynamics of a heavy quark and a diquark.

In this paper, we investigate the properties of the diquark constituents in the hadron structure and examine whether the constituent diquark picture works well. For this purpose, we perform a phenomenological investigation of the excitation energy spectra of the single heavy baryons, $Λ_c$, $Ξ_c$, $Σ_c$, $Ω_c$ and the counterparts of the bottomed baryons. Here we assume that these baryons are composed of a light diquark and a heavy quark. It is not our intention that we would reproduce the precise energy spectra of these baryons grounded on fundamental theories, but we would rather grasp the global feature of the heavy baryons from the observed spectra using simple models. For this reason, we focus on the orbital excitations in the confinement potential and pay no attention to the fine structure induced by the spin dependent forces. Here we take two diquark models; Model A considers a point-like diquark with a weaker string tension than the heavy quarkonium systems as suggested in Ref. [29], while Model B introduces a spatial size of the diquark for the color interaction between the heavy quark and the diquark by following Ref. [30].

The paper is organized as follows. In Sec. 2, we explain our models, and construct the two-body potential of the heavy quark and diquark system. In Sec. 3 we determine the model parameters by the first excitation energy of the $Λ_c$ baryon and the diquark masses other than the ud scalar diquark by the ground state masses of the charmed baryons. In Sec. 4, we show our numerical results. After showing the excitation spectrum of $Λ_c$ in Sec. 4.1, we show the calculated excitation spectra of the heavy baryons in Sec. 4.2. We also discuss the Regge trajectory of the obtained mass spectra in Sec. 4.3 and the excitation energy of the doubly charmed baryon $Ξ_{cc}$ in Sec. 4.4. Finally, Sec. 5 is devoted to summary and conclusion.
2. Formulation

In this paper, we treat heavy baryons as two-body systems of a light diquark and a heavy quark. The light diquarks that we consider here are composed of two quarks out of the \( u, d \) and \( s \) quarks and form color anti-triplet. The spin of the diquark is either singlet (\( S = 0 \)) for the flavor antisymmetric configuration or triplet (\( S = 1 \)) for the flavor symmetric case. We treat the diquark as a fundamental degree of freedom and do not consider radial excitations of two quarks in the diquark nor spin transitions of the diquark induced by rotational excitations of the diquark. For the heavy quark \( h \) we consider the charm and bottom quarks.

In Table 1, we summarize the heavy baryons that we consider in this work and also show the quantum numbers of the diquark appearing in the heavy baryon. The \( \Lambda_h \) baryon has isospin \( I = 0 \) and is composed of a \([ud]\) isoscalar-scalar diquark and a heavy quark. The \( \Xi_h \) baryon has isospin \( I = 1/2 \) with one strange quark, and is made up with a \( qs \) diquark and a heavy quark. Here we call \( u \) and \( d \) quarks by \( q \) collectively. The spin of the \( qs \) diquark can be singlet or triplet, and thus we distinguish the \( \Xi_h \) baryon by the diquark spin: \( \Xi_h \) has the spin 0 diquark \([qs]\) of which flavor configuration is antisymmetric under the quark exchange, while \( \Xi'_c \) has the spin 1 \{qs\} diquark with symmetric flavor configuration. These \( \Xi_h \) baryons have the same quark contents. To distinguish them experimentally, one has to understand their diquark structure. The \( \Sigma_h \) baryon has isospin \( I = 1 \) and is constructed by a spin 1 \{qq\} diquark and a heavy quark. Finally \( \Omega_h \) has two strange quarks and is composed of a spin 1 strange diquark, \{ss\}, and a heavy quark.

### Table 1  Diquark quantum numbers in the heavy baryons. The light quark \( q \) denotes the up or down quark. The bracket \([\cdots]\) and brace \{\cdots\} in the flavor row stand for symmetric and antisymmetric under the exchange of two quarks, respectively.

| heavy baryon | \( \Lambda_h \) | \( \Xi_h \) | \( \Sigma_h \) | \( \Xi'_h \) | \( \Omega_h \) |
|--------------|----------------|------------|------------|------------|-----------|
| isospin \( I \) | 0 | 1/2 | 1 | 1/2 | 0 |
| spin \( S \) | 0 | 0 | 1 | 1 | 1 |
| flavor \( f \) | \([ud]\) | \{qs\} | \{qq\} | \{qs\} | \{ss\} |

2.1. Schrödinger equation

We compare two diquark models. In Model A, the diquark is assumed to be a point-like particle. This model was investigated in Ref. [29] for the \( \Lambda_c \) baryon. In the second model, Model B, we consider the size of the diquark to evaluate the interaction between the heavy quark and diquark. Reference [30] introduced the size of the diquark by treating the diquark as a rigid rotor made up of two quarks spatially separated with a fix distance. Here we use a Gaussian distribution for the size of the diquark, which is more suitable for quantum systems. In both models, we consider the heavy baryons as a two-body system of a heavy quark and a diquark. The Hamiltonian in the center of mass system is given as

\[
\hat{H} = \hat{m}_h + \hat{m}_d - \frac{1}{2 \mu} \frac{d^2}{dr^2} + \frac{\hat{L}^2}{2\mu r^2} + V(r),
\]

where \( r \) is the distance between the diquark and heavy quark, \( \hat{m}_h \) and \( \hat{m}_d \) are the masses of the heavy quark and the diquark, respectively, \( \mu \) is the reduced mass given by
\( \mu = m_h m_d / (m_h + m_d) \), \( \hat{L}_r \) is the orbital angular momentum operator of the two-body system, and \( V(r) \) is the two-body potential for the heavy quark and the diquark. The potential is commonly used in the whole calculations for the heavy baryon spectra once the model parameters are fixed. The explicit form of the potential depends on the model. We take a central potential and do not consider fine structure splittings induced by the spin-spin, spin-orbit ant tensor interactions in order to investigate global structure of excitation spectra.

The total angular momentum of the system reads \( J = S + L \) with the total spin \( S = S_h + S_d \) given by the heavy quark spin \( S_h \) and the diquark spin \( S_d \) and the total orbital angular momentum \( L = L_r + L_\rho \) where \( L_\rho \) is the diquark rotational angular moment for Model B. The spin of the heavy quark is 1/2, while that of the diquark is 0 or 1 depending on the flavor of diquark. Because we do not consider the fine structure of the energy spectra in the present work, the baryon masses can be labeled by the orbital angular momentum \( L \). Thus, the angular part of the Schrödinger equation with the Hamiltonian (1) is solved with the spherical harmonics \( Y_{LM} \), where \( M \) is the magnetic quantum number for the orbital angular momentum \( L \).

To obtain the energy of each state with the angular momentum \( L, E_L \), we just solve the radial differential equation:

\[
\left[ -\frac{1}{2\mu} \frac{d^2}{dr^2} + \frac{L(L+1)}{2\mu r^2} + V(r) \right] \chi_L(r) = E_L \chi_L(r),
\]

where the radial wavefunction \( \chi_L(r) \) is normalized as

\[
\int |\chi_L(r)|^2 dr = 1.
\]

We assume the flavor symmetry among the diquarks and use a common interaction potential \( V(r) \) for the heavy baryons. The symmetry breaking appears in the diquark mass.

2.2. Model A

In Model A the diquark is treated as a point-like particle. The main component of the confinement potential may be brought about by the color electric interaction, which primarily depends on the color charge and is insensitive to the masses. Thanks to the same color structure of the quark-diquark systems as that of the quark-antiquark systems, we take the interaction potential from the quarkonium systems. Following Ref. [29], here we use the so-called Coulomb-plus-linear potential [1] given by

\[
V_A(r) = -\frac{4}{3} \frac{\alpha_s}{r} + kr + V_0
\]

with the strength of the Coulomb potential, \( \alpha_s \), and the string tension \( k \) are the model parameters. The parameter \( V_0 \) determines the absolute value of the baryon mass. According to the universality of the color interaction, the color electric potential \( V(r) \) should be common in the quark-antiquark and quark-diquark systems. The values which reproduce the global structure of the excitation spectra of charmonium and bottomonium were found to be \( \alpha_s \simeq 0.4 \) and \( k \simeq 0.9 \) GeV/fm in Ref. [29, 39]. This parameterization reproduces well the excitation spectra of the open charm and bottom systems [29]. Contrary to our expectation, it was found in Ref. [29] that these values do not reproduce the \( \Lambda_c \) excitation spectrum and we have to use a half strength of the string tension. Thus, we determine the strength of the string tension later by the excitation energy of \( \Lambda_c \).
2.3. Model B

Also in Model B, we treat the diquark as a kinematically point-like particle, but we introduce the size and deformation of the diquark. We consider the diquark as a rigid rotor with a size \( \rho \) and calculate the interaction between the heavy quark and the rigid rotor. Folding this interaction with a Gaussian distribution for the diquark size we obtain an effective potential between the heavy quark and the diquark. With the size of the diquark, we consider its rotational excitations. According to symmetry of quarks in the diquark, the rotational angular momentum of the diquark must be an even number. This is because the diquark has color anti-triplet, which is antisymmetric under the quark exchange, and we consider symmetric spin-flavor configurations for the diquarks. Thus, the spacial configuration of the quarks in the diquark must be symmetric under the quark exchange. Because the parity of the states with angular momentum \( \ell \) under spacial inversion is given as \((-)^\ell\), the rotational angular momentum of the diquark considered here must be an even number. We consider the rotational excitation with \( L_\rho = 2 \) for low-lying states. The diquark with \( L_\rho = 2 \) has deformation. One might consider that \( L_\rho = 1 \) excitation with the spin rearrangement of the diquark would have a comparable excitation energy with that of the \( L_\rho = 2 \) without the spin rearrangement. These kinds of excitation can be described fully in quark models in which light quarks are treated individual particles. Since we consider a diquark model in order to examine the nature of the diquark as an effective constituent of hadrons here, we do not consider the spin rearrangement of the diquark.

The radial coordinate \( r \) in the Schödinger equation (2) denotes the distance between the heavy quark and the center of the diquark. The total orbital angular momentum of the system is given by \( \mathbf{L} = \mathbf{L}_r + \mathbf{L}_\rho \). In general, a state with total orbital angular momentum \( L \) is composed of several combinations of \( L_r \) and \( L_\rho \). According to Ref. [30], the lowest lying \( L = 0 \) and \( L = 1 \) states are found to be constructed essentially by the states with \((L_\rho, L_r) = (0, 0)\) and \((0, 1)\), respectively. This is because the rotational excitation needs two quanta and costs more than the orbital excitation \( L_r \). Thus, we take the \((L_\rho, L_r) = (0, 0)\) and \((0, 1)\) configurations for the lowest \( S \) and \( P \) states, respectively. For the \( D \) state \((L = 2)\), the states with \((L_\rho, L_r) = (0, 2)\) and \((2, 0)\) may have a comparable energy depending on the size of the diquark. As we will find later, with the diquark size appropriate to reproduce the \( 1p \) excitation energy, these two states have a very similar energy and can be mixed. Here, we consider the \((0, 2)\) and \((2, 0)\) configurations for \( L = 2 \) and diagonalize their admixture. For these states we call the one which has a larger contribution of the \((0, 2)\) configuration the \( d_\lambda \) state and the other the \( d_\rho \).

We follow Ref. [30] to calculate the effective potential \( v(r; \rho) \) for the interaction between a heavy quark and a fixed-size diquark. The interaction between the quarks forming the color anti-triplet configuration, \( v_{qq}(r) \), is assumed to be half of the color singlet quark-antiquark interaction \( v_{\bar{q}q}(r) \) and the \( \bar{q}q \) potential is described by the Coulomb-plus-linear type form:

\[
v_{\bar{q}q}(r) = \frac{1}{2} v_{qq}(r) = \frac{1}{2} \left[ -\frac{4}{3} \frac{\alpha_s}{r} + k' r \right].
\]

The parameters, \( \alpha_s \) and \( k' \), are determined so as to reproduce the charmonium and bottomonium spectra [29] as

\[
\alpha_s = 0.4, \quad k' = 0.9 \text{ [GeV/fm].}
\]
The light quarks in the diquark are apart off distance $\rho$. The interaction between the heavy quark and the light quarks is written as

$$v_{hd}(r, \rho) = v_{qq}(r - \frac{1}{2} \rho) + v_{qq}(r + \frac{1}{2} \rho),$$

where we take the geometrical middle point between two light quarks as the center of the diquark. Projecting out the component of the total orbital angular momentum $L$ composed of $L_r$ and $L_\rho$ for the potential $v_{hd}(r, \rho)$, we obtain

$$v(r; \rho) = \int d\Omega_r d\Omega_\rho \left[ Y_{L_r}^{M_r}(\Omega_r)Y_{L_\rho}^{M_\rho}(\Omega_\rho)\right]^* \left[ Y_{L_r}^{M_r}(\Omega_r)Y_{L_\rho}^{M_\rho}(\Omega_\rho)\right] L_{v_{hd}}(r, \rho),$$

where $\cdot \cdot \cdot_L$ is understood to take the appropriate linear combination to compose the total angular momentum $L$ in terms of $L_r$ and $L_\rho$. The details of the derivation of the potential (7) is explained in Ref. [30].

We consider a Gaussian distribution for the size of the diquark. The effective potential between the heavy quark and the diquark for model B, $V_B(r)$, is obtained by folding the potential $v(r; \rho)$ calculated in Eq. (7) with the Gaussian form of the size distribution of the diquark:

$$V_B(r) = \int_0^\infty v(r; \rho) \sigma(\rho; \beta) \rho^2 d\rho + V_{rot} + V_0,$$

where a constant $V_{rot}$ represents the rotational energy of the diquark, the parameter $V_0$ determines the absolute value of the baryon mass. The size distribution $\sigma(\rho; \beta)$ with a Gaussian parameter $\beta$ is given by

$$\sigma(\rho; \beta) = \begin{cases} 
\frac{4}{\beta^3 \sqrt{\pi}} \exp\left(-\frac{\rho^2}{\beta^2}\right) & \text{for } L_\rho = 0, \\
\frac{16}{15 \beta^7 \sqrt{\pi}} \rho^4 \exp\left(-\frac{\rho^2}{\beta^2}\right) & \text{for } L_\rho = 2.
\end{cases}$$

The rotational energy of the diquark is evaluated as

$$V_{rot} = \int_0^\infty \frac{L_\rho (L_\rho + 1)}{2I(\rho)} \sigma(\rho; \beta) \rho^2 d\rho,$$

with the moment of inertia $I(\rho) = \rho^2 m_d/4$. For $L_\rho = 2$, the rotational energy is calculated as $V_{rot} = 24/(5\beta^2 m_d)$.

### 3. Parameter determination

In this section, we first determine the model parameters appearing in the potential of each model by reproducing the $1p$ excitation energy of the $\Lambda_c$ baryon. In the determination of the model parameters, we use the masses of the $ud$ diquark and the charm quark as 0.5 GeV/$c^2$ and 1.5 GeV/$c^2$ as an example for the purpose to discuss the global feature of the heavy baryon spectra in the diquark models. For more detailed discussion, one can perform fine-tuning of the model parameters including these masses so as to reproduce all of the baryon masses. After fixing the model parameters by the $\Lambda_c$ spectrum, we apply the same potential for the other heavy baryons, because we consider the interaction between the heavy quark and the diquark to be universal. The diquark masses are determined so as to produce the masses of the ground state charmed baryons with the potential determined by the $\Lambda_c$ spectrum.
3.1. Model parameter determination by $\Lambda_c$ spectrum

The model parameters are determined so as to reproduce the $1p$ excitation energy of the $\Lambda_c$ baryon. To calculate the $1p$ excitation energy, we use the spin-weighted average mass of the first excited states, $\Lambda_c(2595)$ with spin $1/2^-$ and $\Lambda_c(2625)$ with spin $3/2^-$, because we do not consider the fine structure induced by the spin dependent interactions. This average removes the effect of the spin-orbit interaction perturbatively. The $1p$ excitation energy is given by

$$E_{1p}^{\text{obs}} = \frac{m_{\Lambda_c(2595)} + 2m_{\Lambda_c(2625)}}{3} - m_{\Lambda_c} = 0.33 \text{ GeV}. \quad (11)$$

3.1.1. Model A. It has been reported that the string tension which reproduces the spectra of the charmonium and bottomonium is not suitable for $\Lambda_c$ and $\Lambda_b$ and their excitation energies are obtained with a half strength of the string tension [29]. In this work we take the value of the string tension as $k = 0.5 \text{ GeV/fm}$. A recent lattice calculation [40] has suggested that the Coulomb attraction should be considerably smaller in the diquark-quark potential than that in the quark-antiquark system and this might be attributed to the size of the diquark.

One may wonder whether there is room to reproduce the $\Lambda_c$ excitation energy by adjusting the masses of the diquark and charm quark within their ambiguities. The value of the scalar $ud$ diquark mass may be in the range of 0.3 to 0.6 GeV and the possible value of the charm quark mass in quark models may be within 1.27 to 1.6 GeV. For these values the reduced mass of the diquark and charm quark is in the range of 0.2 to 0.5 GeV. We calculate the $1p$ excited energies both for $k = 0.9$ and 0.5 GeV/fm as functions of the reduced mass $\mu$. Figure 1 shows the values of the excited energies measured from the $1s$ ground state energy. The horizontal dotted line stands for the spin averaged excitation energy given in Eq. (11).

The figure shows that the calculation with the string tension $k = 0.9 \text{ GeV/fm}$ overestimates largely the observed excitation energy in the wide range of the reduced mass. Therefore, the string tension which reproduces the quarkonium spectra is not suitable for the quark-diquark interaction in the charmed baryon. In addition, the decay property of $\Lambda_c$ also prefers $k = 0.5 \text{ GeV/fm}$ [29].

Determining the absolute value of the potential energy so as to reproduce the mass of the ground state of the $\Lambda_c$ baryon, we obtain

$$V_0 = -0.054 \text{ GeV}, \quad (12)$$

for Model A.

3.1.2. Model B. Model B has the Gaussian parameter $\beta$ appearing in the size distribution of the diquark (9). This parameter is determined so as to reproduce the $1p$ excitation energy of the $\Lambda_c$ baryon. In Fig. 2, we show the excited energies of the $\Lambda_c$ baryon calculated in Model B as functions of the Gaussian parameter $\beta$. This figure shows that the calculation with $\beta = 1.0 \text{ fm}$ reproduces the observed value of the spin average $1p$ excitation energy (11). Hereafter we take this value of the $\beta$ parameter for Model B. The mean squared value of the diquark is calculated as

$$\langle r^2 \rangle = \int_0^\infty \rho^2 \sigma(\rho; \beta) \rho^2 d\rho. \quad (13)$$
Excitation energies of the 1p state of the \( \Lambda_c \) baryon calculated by Model A with \( k = 0.9 \) and \( k = 0.5 \) GeV/fm as functions of the reduced mass \( \mu \) of the scalar \( ud \) diquark and the charm quark. The horizontal dotted line stands for the spin averaged excitation energy given in Eq. (11). The energies shown in the figure are measured from the 1s ground state.

The root mean squared value of the diquark distribution with the Gaussian parameter \( \beta = 1.0 \) fm corresponds to

\[
\sqrt{\langle r^2 \rangle} = 1.2 \text{ [fm]},
\]  

for \( L_\rho = 0 \), which agrees with the Lattice QCD calculation [14]. This value implies that the diquark in Model B has an object with a radius of 0.6 fm.

We fix the absolute value of the effective potential, \( V_0 \), using the mass of the \( \Lambda_c \) ground state and obtain

\[
V_0 = -0.565 \text{ GeV}.
\]  

As explained in Sec. 2.3, there are two 1d states; one is called 1d\( _\lambda \) and has a larger component of the orbital excitation of the diquark-quark system, \( (L_\rho, L_r) = (0, 2) \), while the other is called 1d\( _\rho \) and have a larger component of the rotational excitation of the diquark with size, \( (L_\rho, L_r) = (2, 0) \). In Fig. 2, we call the energetically lower state 1d\( _1 \) and the higher 1d\( _2 \). For smaller \( \beta \), the lower state is 1d\( _\lambda \), because the rotational excitation costs larger energy for a smaller diquark. With larger \( \beta \) the rotational excitation energy gets smaller and level crossing between 1d\( _\lambda \) and 1d\( _\rho \) takes place. Thus, for larger \( \beta \) the 1d\( _\rho \) state turns to be the lower state. The diquark size \( \beta \) at which the level crossing takes place depends on the system. For the \( \Lambda_c \) case, the level crossing is found to take place at \( \beta = 0.95 \) fm.

Another characteristic feature of Model B is that the 2s and 1d\( _\lambda \) are found to have a similar energy and tend to degenerate. This is because the effective potential between the diquark and heavy quark of Model B becomes moderate at the origin due to the size effect and resembles the harmonic oscillator potential at the short distances as shown in Fig. 3. The degeneracy of the 2s and 1d states is one of the general features of the harmonic oscillator potential. This tendency is also seen in the quark model calculation of the \( \Lambda_c \) spectrum performed in Ref. [41], where the \( \Lambda_c \) baryon is treated as a three-body system of the up,
Fig. 2 Excitation energies of the $\Lambda_c$ baryon in Model B as functions of the Gaussian parameter $\beta$. The energies are measured from the corresponding $1s$ ground state. The horizontal dotted line denotes the spin weighted averaged value of the observed masses of the $1p$ excited $\Lambda_c$ states, which is 0.33 GeV. For the $1d$ states here we call the energetically lower state $1d_1$ and the higher $1d_2$.

down and charm quarks interacting with two-body potentials. The finite distance effect in the $ud$ quark system may push up the $2s$ state close to the $1d$ state.

3.2. Diquark masses

In this section, we determine the diquark masses from the ground state masses of the charmed baryons. The single heavy baryons are characterized by the type of the diquarks as listed in Table 1. Each diquark has its own spin-flavor content and mass. Due to the absence of the interactions inducing the fine structure, the spin structure of the charmed baryon does not make difference. In Fig. 4, we show the mass of the $1s$ ground state of the singly charmed baryon as a function of the diquark mass $m_d$ for both Model A and B. We show also the charmed baryon masses as the horizontal dotted lines. For the $\Sigma_c$, $\Xi'_c$ and $\Omega_c$ baryons, which have a diquark with spin 1, we take the spin averaged masses of the spin $1/2^+$ and $3/2^+$ states as their ground states. From Fig. 4 one can read the value of the diquark mass for each charmed baryon. The obtained diquark masses are summarized in Table 2. Both models lead to similar values of the diquark masses. It is interesting that the mass differences between the $qq$ and $qs$ diquarks for spin 0 and spin 1 are 0.25 GeV and 0.15 GeV, respectively. This implies that the flavor SU(3) breaking effect is different in the spin 0 and spin 1 diquark.
Fig. 3  Effective potentials of Model B with $\beta = 0, 0.5, 1.0$ and 1.5 fm.

Table 2  Diquark masses determined by the ground states of the single charmed baryons in units of GeV. The numbers with an asterisk are input values. The diquark is specified by its spin $S$ and flavor $f$. For flavor $f$, $q$ denotes either up or down quark, and $[\cdots]$ and $\{\cdots\}$ imply the antisymmetric and symmetric configurations under the quark exchange, respectively.

| $m^S_f$ | $m^0_{[ud]}$ | $m^0_{[qs]}$ | $m^1_{[q\bar{q}]}$ | $m^1_{[qs]}$ | $m^1_{ss}$ |
|---------|--------------|--------------|-------------------|--------------|-------------|
| Model A | 0.50*        | 0.76         | 0.80              | 0.96         | 1.10        |
| Model B | 0.50*        | 0.75         | 0.79              | 0.94         | 1.07        |

4. Results

In this section we show the numerical result of the heavy baryon spectra calculated in Model A and B. The parameters introduced as inputs in this work are summarized in Table 3. The parameters determined with the $\Lambda_c$ spectrum in the previous section are summarized in Table 4. We use the interaction potential with the same parameters for the calculation of the other heavy baryon spectra, while the diquark and heavy quark masses are different in each system.

4.1. $\Lambda_c$ excitation spectrum

First of all, we show the results of the $\Lambda_c$ baryon calculated with Models A and B in Fig. 5. Both models reproduce the $1p$ excitation energy, as the parameter in each model is adjusted by the $1p$ excitation energy. Here we should compare the calculated values with spin-averaged values of the experimental data due to the absence of the spin dependent force in the calculation. For Model A, we also show the calculation with $k = 0.9$ GeV/fm.
Fig. 4  Ground state masses of the charmed heavy baryons in Model A and B as functions of the diquark mass. The horizontal dotted lines represent the ground state masses of the charmed baryons. For the $\Xi'_c$, $\Sigma_c$ and $\Omega_c$ baryons, we take the spin-weighted average of the masses of the spin $1/2$ and $3/2$ states.

Fig. 5  Excitation spectrum of the $\Lambda_c$ baryon calculated with Model A and B together with the experimental observation. The masses are measured from the ground state in units of GeV. For Model A, we show also the result with $k = 0.9$ GeV/fm for comparison. The experimental data are taken from Ref. [42].

that reproduces the charmonium and bottomonium spectra well, which overestimates the $\Lambda_c$ excitation energies.

Figure 5 shows that Model A with $k = 0.5$ GeV/fm reproduces the excitation spectrum of $\Lambda_c$ well. The spin unknown $\Lambda_c(2765)$ is reproduced by Model A as a $2s$ state, while it is not
Table 3  Input model parameters used in this work. The masses $m^0_{ud}$, $m_c$ and $m_b$ stand for the masses of the scalar $ud$ diquark, the charm quark and the bottom quark, respectively. The coupling constant $\alpha_s$ appears in Eqs. (3) and (4) for the Coulomb part in the quark-diquark interaction of Model A and in the quark-quark interaction of Model B, respectively. The string tension $k'$ is for the quark-quark interaction of Model B, which reproduces the charmonium and bottomonium spectra.

| $m^0_{ud}$ | $m_c$ | $m_b$ | $\alpha_s$ | $k'$ |
|------------|-------|-------|------------|------|
| 0.5 GeV    | 1.5 GeV | 4.0 GeV | 0.4        | 0.9 GeV/fm |

Table 4  Model parameters of Model A and B. The string tension $k$ is for the quark-diquark interaction of Model A (3), while the Gaussian parameter $\beta$ determines the diquark size distribution (9) for Model B. These parameters are determined by the $\Lambda_c$ excitation energy in Sec. 3. The absolute value of the effective potential in each model, $V_0$, is fixed by the mass of the $\Lambda_c$ ground state.

| Model   | $k$ | $\beta$ | $V_0$ |
|---------|-----|---------|-------|
| Model A | 0.5 GeV/fm |          | $-0.054$ GeV |
| Model B | 1.0 fm |         | $-0.565$ GeV |

obtained by Model B. According to Particle Data Group [42] the quantum number $\Lambda_c(2765)$ state is not known yet including its isospin and the observed state could be $\Sigma_c(2765)$, while a recent experimental analysis [43] have found no isospin partners and concluded the isospin of $\Lambda_c(2765)$ to be zero. For further determination of the $\Lambda_c(2765)$ quantum number, a theoretical analysis [44] have suggested that the ratio of the decays of $\Lambda_c(2765)$ to $\Sigma_c(2520)\pi$ and $\Sigma_c(2455)\pi$ and their angular correlations are sensitive to the spin and parity of $\Lambda_c(2765)$. In Model B the $2s$ state appears 100 MeV higher than that of Model A. The $2s$ excitation energy is one of the important differences between Models A and B.

Model A obtains the $1d$ state around the energies of the $\Lambda_c(2860)$ with $J^P = (3/2)^+$ and $\Lambda_c(2880)$ with $J^P = (5/2)^+$, and thus these resonances are interpreted as $d$-wave excitations of a quark-diquark system. In Model B, the $1d$ states have larger energies than that of Model A. As discussed in Sec. 3.1.2, the level crossing between the $1d_\lambda$ and $1d_\rho$ states takes place at $\beta = 0.95$ fm for $\Lambda_c$ and the $1d_\rho$ becomes the lower state for a larger size of the diquark. Thus, the lower state is found to be $1d_\rho$ for $\beta = 1.0$ fm. Providing this state close to the observed $\Lambda_c(2860)$ and $\Lambda_c(2880)$, Model B interprets the main component of these $\Lambda_c$ states to be states having a rotational excitation of the diquark with size.

4.2. Excitation spectra of single heavy baryons

With the diquark masses determined in Sec. 3.2, we calculate the mass spectra of the $\Xi_c$, $\Sigma_c$, $\Xi'_c$ and $\Omega_c$ charmed baryons in Model A and B. Once we have determined the potential parameters and the diquark masses, we have no adjustable parameters in the excitation spectra of these baryons.

First we show the excitation spectra of the $\Xi_c$ baryon in Fig. 6 together with the experimental observation. In this figure we do not show the $\Xi'_c$ baryons with spin $1/2$ and $3/2$, because we suppose these baryons to have a $qs$ diquark with spin 1. We find that both models reproduce the $1p$ excitation energy, although Model B underestimates the $1p$ energy a
bit, which may be reproduced by fine-tuning the model parameters. Thus, we conclude that both models work for the \( \Xi_c \) baryon. Similarly to the \( \Lambda_c \) spectrum, there is the characteristic difference between Models A and B in the higher excited states. In particular, the 2\( s \) state in Model B is higher than that in Model A. The two 1\( d \) states in Model B appears close to the 2\( s \) state. For \( \Xi_c \) the level crossing between 1\( d_\lambda \) and 1\( d_\rho \) is found to take place at \( \beta = 0.77 \) fm. There are several excited states observed in experiments, of which spin-parity is not determined yet. Some of them should be described by the spin 1 \( q_s \) diquark, and such states are categorized into \( \Xi'_c \) in this work. In order to identify the structure of these baryons, definitely the information on the spin-parity is necessary.

In Figs. 7, 8 and 9, we show the excitation spectra of the \( \Sigma_c \), \( \Xi'_c \) and \( \Omega_c \) baryons, respectively. These baryons are composed of a charm quark and a diquark with spin 1. The splitting of the ground states is induced by the spin-spin interaction between the charm quark and the spin 1 diquark. In the figures the excitation energies of the observed states are measured from the spin-weighted average of the ground state masses.

These figures show that the first excited states of these baryons are reproduced well by both models on the whole. In Fig. 8, we do not show the \( \Xi_c(2790) \) and \( \Xi_c(2815) \) baryons with spin 1\( p^- \) and 3\( /2^- \), because we assume these resonances to be 1\( p \) excited states of the \( \Xi_c \) baryon having the \( q_s \) diquark with spin 0. The observed \( \Xi_c \) baryons having the mass of around 2.9 GeV are reproduced as a 1\( p \) state in both models. For the \( p \) state, the fine structure splittings are induced by both the spin-spin and the spin-orbit interactions. For further comparison we need to include such fine structure interactions and to know the spin-parity of the observed states.

It is also interesting to mention that the excited energies obtained in Model A are less sensitive to the value of the diquark mass than those in Model B. In particular, the excitation energies in Model B are reduced as the diquark mass increases more than those in Model A. In Model B, between two 1\( d \) states the lower states are found as the 1\( d_\rho \) state in these baryons. The level crossings of 1\( d_\lambda \) and 1\( d_\rho \) are found at \( \beta = 0.75 \) fm for \( \Sigma_c \), \( \beta = 0.68 \) fm for \( \Xi'_c \) and \( \beta = 0.63 \) fm for \( \Omega_c \). It is interesting that the level crossing point appears at the smaller size...
parameter for the heavier diquark mass. In Table 5, we summarize the excitation energies obtained in Models A and B in comparison with those of the candidates in the observed baryons.

![Excitation Energies for \( \Sigma_c \) Baryon](image)

**Fig. 7**  Same as in Fig. 5 but for the \( \Sigma_c \) baryon. The excitation energy of the \( \Sigma_c(2800) \) state is measured from the spin average of the masses of \( \Sigma_c(2455) \) and \( \Sigma_c(2520) \).

![Excitation Energies for \( \Xi'_c \) Baryon](image)

**Fig. 8**  Same as in Fig. 5 but for the \( \Xi'_c \) baryon. The excitation energies of the observed states are measured from the spin average of the masses of \( \Xi'_c(2580) \) and \( \Xi'_c(2645) \). The \( \Xi_c(2790) \) and \( \Xi_c(2815) \) baryon with spin 1/2− and 3/2− are not shown here, because they are assumed as excited states of the \( \Xi_c \) baryon having the \( qs \) diquark with spin 0.

In Fig. 10 and Table 6, we show the excitation spectra of the heavy baryons with one bottom quark in the same way as the charmed baryons. Here we assume the bottom quark mass to be 4.0 GeV. All of the model parameters are already fixed, and the excitation energies of the bottomed baryons are calculated without adjustable parameters. The bottom quark mass used here may be smaller than one used for phenomenological studies, for instance in Ref. [45]. We have also performed a calculation with the bottom quark mass 5.0 GeV and found that the results do not change much. The differences in the excitation energies are
Table 5  Excitation energies of the charmed baryons obtained in the calculation of Models A and B in units of GeV. The observation candidates corresponding to the calculation are also shown.

|     | Model A | Model B | candidates | spin | Exp. |
|-----|---------|---------|------------|------|------|
| $\Lambda_c$ | 1s | 0 | 0 | $\Lambda_c$ | 1/2+ | 0 |
| 1p | 0.330 | 0.334 | $\Lambda_c(2595)$ | 1/2− | 0.306 |
|     |       |       | $\Lambda_c(2625)$ | 3/2− | 0.342 |
| 2s | 0.484 | 0.595 | $\Lambda_c(2765)$ | ? | 0.480 |
| 1d | 0.569 | 0.599 (ρ) | $\Lambda_c(2860)$ | 3/2+ | 0.570 |
|     |       |       | 0.659 (λ) $\Lambda_c(2880)$ | 5/2+ | 0.595 |
| $\Xi_c$ | 1s | 0 | 0 | $\Xi_c$ | 1/2+ | 0 |
| 1p | 0.322 | 0.299 | $\Xi_c(2790)$ | 1/2− | 0.324 |
|     |       |       | $\Xi_c(2815)$ | 3/2− | 0.349 |
| 2s | 0.467 | 0.536 |       |       |       |
| 1d | 0.555 | 0.497 (ρ) |       |       |       |
|     |       |       | 0.586 (λ) |       |       |
| $\Sigma_c$ | 1s | 0 | 0 | $\Sigma_c(2455)$ | 1/2+ | −0.043 |
|     |       |       | $\Sigma_c(2520)$ | 3/2+ | 0.022 |
| 1p | 0.321 | 0.295 | $\Sigma_c(2800)$ | ? | 0.303 |
| 2s | 0.455 | 0.530 |       |       |       |
| 1d | 0.543 | 0.486 (ρ) |       |       |       |
|     |       |       | 0.578 (λ) |       |       |
| $\Xi'_c$ | 1s | 0 | 0 | $\Xi'_c$ | 1/2+ | −0.045 |
|     |       |       | $\Xi'_c(2645)$ | 3/2+ | 0.022 |
| 1p | 0.319 | 0.282 | $\Xi'_c(2930)$ | ? | 0.312 |
|     |       |       | $\Xi'_c(2970)$ | ? | 0.345 |
| 2s | 0.446 | 0.509 |       |       |       |
| 1d | 0.536 | 0.454 (ρ) |       |       |       |
|     |       |       | 0.555 (λ) |       |       |
| $\Omega'_c$ | 1s | 0 | 0 | $\Omega'_c$ | 1/2+ | −0.047 |
|     |       |       | $\Omega'_c(2770)$ | 3/2+ | 0.024 |
| 1p | 0.319 | 0.273 | $\Omega'_c(3000)$ | ? | 0.258 |
|     |       |       | $\Omega'_c(3050)$ | ? | 0.308 |
|     |       |       | $\Omega'_c(3065)$ | ? | 0.323 |
|     |       |       | $\Omega'_c(3090)$ | ? | 0.348 |
|     |       |       | $\Omega'_c(3120)$ | ? | 0.377 |
| 2s | 0.441 | 0.494 |       |       |       |
| 1d | 0.532 | 0.433 (ρ) |       |       |       |
|     |       |       | 0.539 (λ) |       |       |
Fig. 9  Same as in Fig. 5 but for the $\Omega_c$ baryon. The excitation energies of the observed states are measured from the spin average of the masses of $\Omega_c(2695)$ and $\Omega_c(2770)$.

less than 2%. This is because the reduced masses of the bottom quark and the diquark are very similar in both cases for such heavy quark masses.

The first excitation energies of the $\Lambda_b$ and $\Sigma_b$ baryons are found to be reproduced well in both models. For the heavier bottom baryons, the excitation energies are reduced generally. Nevertheless, the excitation energies in Model A are less sensitive to the diquark mass and the $1p$ excitation energies are slightly enhanced in the $\Xi_b'$ and $\Omega_b$ baryons. We find again the characteristic difference between two models in the value of the $2s$ excitation energy. Model B predicts larger $2s$ excitation energies and the $2s$ state almost degenerates with one of the $1d$ states. In Model B, unlike the charmed baryons, the level crossing does not take place for $\Lambda_b$, $\Xi_b'$ and $\Sigma_b$, while it takes place twice at $\beta = 0.85$ fm and $\beta = 1.03$ fm for $\Xi_b'$ and at $\beta = 0.75$ fm and $\beta = 1.03$ fm for $\Omega_b$. For further comparison, we need the detailed information of the quantum numbers of the observed states and the observation of missing states.

4.3. Regge trajectories

Model A has a smaller string tension than that of our common knowledge, while in Model B the string tension that reproduces the heavy quarkonia is used. It is interesting to visualize the difference of the global features of the quark-diquark interaction. For this purpose, we show the Regge trajectories of the heavy baryons [46, 47]. We take the Regge trajectories of the spin $J$ and mass squared $M^2$ given as

$$J = aM^2 + a_0. \quad (16)$$

In our calculation without the fine structure interactions, the total spin $J$ corresponds to the relative angular momentum $L$ for the heavy quark and diquark. For the absolute values of the bottom baryon masses, we fix the potential parameter $V_0$ so as to reproduce the observed ground state $\Lambda_b$ mass. Then we obtain $V_0 = 0.824$ GeV for Model A and $V_0 = 0.310$ GeV for Model B. We take the lower state of two $1d$ states in the calculation of Model B.

We fit the calculated baryon spectra using the Regge trajectory (16). The obtained slope and intersect parameters, $a$ and $a_0$, are listed in Table 7 for the charmed and bottomed
Excitation spectra of the heavy baryons with one bottom quark calculated with Model A and B together with the experimental observation. The masses are measured from the ground states in units of GeV. For the ground states of the Σ_b and Ξ_b baryons, the spin-weighted average of the masses of the states with spin 1/2^+ and 3/2^+ are taken. For Model A, we show also the result with k = 0.9 GeV/fm for comparison. The experimental data are taken from Ref. [42].

Baryons, and fitted trajectories are shown in Fig. 11 for the charmed baryons and Fig. 12 for the bottomed baryons. It is interesting to find that the slope parameter a obtained in Model A for the Λ_c baryon has a slightly larger value than that of the Ω_c while Model B has the opposite tendency. The flavor dependence of the slope parameter is opposite in Models A and B.

4.4. Doubly charmed baryon

We have obtained a smaller string tension for the systems of a point-like diquark and a heavy quark than those of a quark and an antiquark in order to reproduce the excitation energies of the single heavy baryons. It is interesting to examine whether this conclusion is valid only for the light diquarks or a heavy diquark system also has similarly such a smaller string tension. For this purpose, we calculate the excitation spectra of the doubly charmed baryon Ξ_{cc} using the quark-diquark model (Model A) with the string tensions k = 0.9 GeV/fm and k = 0.5 GeV/fm, and we compare the excitation energy spectra obtained by these calculations.
Table 6  Excitation energies of the bottomed baryons obtained in the calculation of Model A and B in units of GeV. The observation candidates corresponding to the calculation are also shown.

|      | L   | Model A | Model B | candidates | spin | Exp. |
|------|-----|---------|---------|------------|------|------|
| Λ_b | 1s  | 0       | 0       | Λ_b        | 1/2+ | 0    |
|     | 1p  | 0.325   | 0.313   | Λ_b(5912)  | 1/2− | 0.293|
|     |     |         |         | Λ_b(5920)  | 3/2− | 0.300|
|     | 2s  | 0.468   | 0.560   | Λ_b(6070)  | ?    | 0.453|
|     | 1d  | 0.554   | 0.577 (λ) | Λ_b(6146)  | 3/2+ | 0.527|
|     |     |         |         | 0.650 (ρ) | Λ_b(6152) | 5/2+ | 0.533|
| Ξ_b | 1s  | 0       | 0       | Ξ_b        | 1/2+ | 0    |
|     | 1p  | 0.319   | 0.272   |            |      |      |
|     | 2s  | 0.441   | 0.492   |            |      |      |
|     | 1d  | 0.532   | 0.486 (λ) |          | 0.560 (ρ) |      |      |
| Σ_b | 1s  | 0       | 0       | Σ_b        | 1/2+ | −0.013|
|     |     |         |         | Σ_b        | 3/2+ | 0.006|
|     | 1p  | 0.319   | 0.268   | Σ_b(6097)  | ?    | 0.271|
|     | 2s  | 0.438   | 0.484   |            |      |      |
|     | 1d  | 0.530   | 0.475 (λ) |          | 0.551 (ρ) |      |      |
| Ξ_b | 1s  | 0       | 0       | Ξ_b(5935)  | 1/2+ | −0.013|
|     |     |         |         | Ξ_b(5945)  | 3/2+ | 0.006|
|     | 1p  | 0.320   | 0.253   | Ξ_b(6227)  | ?    | 0.279|
|     | 2s  | 0.431   | 0.460   |            |      |      |
|     | 1d  | 0.525   | 0.444 (ρ) |          | 0.523 (λ) |      |      |
| Ω_b | 1s  | 0       | 0       | Ω_b        | 1/2+ | 0    |
|     |     |         |         | Ω_b(6316)  | ?    | 0.270|
|     | 1p  | 0.321   | 0.242   | Ω_b(6316)  | ?    | 0.270|
|     |     |         |         | Ω_b(6330)  | ?    | 0.284|
|     |     |         |         | Ω_b(6340)  | ?    | 0.294|
|     |     |         |         | Ω_b(6350)  | ?    | 0.304|

The Ξ_{cc} is composed of a light quark (up or down quark) and two charm quarks. In the calculation we treat the two charmed quarks as a point-like diquark. This might be a good approximation, because the light quark spreads over more widely than the charm quarks. The potential for the quark-diquark system is given by Eq. (3). For the string tension we use $k = 0.9$ GeV/fm and $k = 0.5$ GeV/fm and $\alpha_s = 0.4$ for the Coulomb part. The masses of the light quark and the charm diquark are assumed to be $m_q = 0.3$ GeV and $m_{cc} = 3.0$ GeV.
The excitation spectra obtained by these calculations are shown in Fig. 13. As seen in the figure, the obtained spectra are obviously different: the excitation energy obtained by using $k = 0.9$ GeV/fm is about 40% larger than that obtained with $k = 0.5$ GeV/fm. Recently the doubly charmed baryon has been found at LHCb [48], but its excited states have not been observed yet. It would be very interesting if one could observe the first excited state of the $\Xi_{cc}$ baryon and measure its excitation energy. Such observation would reveal the nature of the two quark correlation.

5. Summary

We have investigated the single heavy quark baryons using the quark-diquark models, in which two light quarks are treated as a diquark and the system is reduced to a two-body problem, in order to investigate the properties of the diquark constituent in the hadron structure. We consider two models: In Model A the diquark is a point-like object and the confinement
Table 7  Fitted values of the slope and intersect parameters given in Eq. (16) for the charmed baryons (upper table) and the bottomed baryons (lower table).

| Model | $a$ [GeV$^2$] | $a_0$ | $a$ [GeV$^2$] | $a_0$ |
|-------|---------------|-------|---------------|-------|
|       | Model A       |       | Model B       |       |
| $\Lambda_c$ | 0.681 ± 0.042 | −3.09 ± 0.28 | 0.645 ± 0.021 | −2.89 ± 0.02 |
| $\Xi_c$ | 0.666 ± 0.051 | −3.59 ± 0.39 | 0.734 ± 0.067 | −4.01 ± 0.51 |
| $\Sigma_c$ | 0.661 ± 0.052 | −3.66 ± 0.41 | 0.742 ± 0.073 | −4.19 ± 0.56 |
| $\Xi'_c$ | 0.640 ± 0.054 | −3.96 ± 0.47 | 0.761 ± 0.090 | −4.80 ± 0.75 |
| $\Omega_c$ | 0.619 ± 0.056 | −4.21 ± 0.52 | 0.767 ± 0.101 | −5.31 ± 0.91 |
| $\Lambda_b$ | 0.304 ± 0.026 | −9.13 ± 0.92 | 0.293 ± 0.010 | −8.77 ± 0.35 |
| $\Xi_b$ | 0.308 ± 0.032 | −9.83 ± 1.17 | 0.340 ± 0.020 | −10.93 ± 0.73 |
| $\Sigma_b$ | 0.307 ± 0.032 | −9.92 ± 1.21 | 0.343 ± 0.021 | −11.26 ± 0.79 |
| $\Xi'_b$ | 0.304 ± 0.035 | −10.23 ± 1.34 | 0.364 ± 0.026 | −12.39 ± 0.98 |
| $\Omega_b$ | 0.300 ± 0.036 | −10.46 ± 1.45 | 0.375 ± 0.028 | −13.30 ± 1.12 |

Fig. 13  Excitation spectrum of the doubly charmed baryon $\Xi_{cc}$ calculated by the quark-diquark model. The left and right spectra are obtained using the string tensions $k = 0.9$ GeV/fm and $k = 0.5$ GeV/fm, respectively.

potential for the quark-diquark system has a smaller string tension $k = 0.5$ GeV/fm to reproduce the excitation energies of the $\Lambda_c$ baryon as suggested in Ref. [29, 30]. In Model B the two quarks in the diquark have a distance given by the Gaussian distribution and the interquark potential is consistent with the quark-antiquark system in which the string tension is found
to be $k = 0.9$ GeV/fm. The potential for the quark-diquark system is obtained by folding the interquark potential obtained with a fixed distance over the Gaussian distribution of the distance as given in Eq. (8). The size of the Gaussian distribution is determined so as to reproduce the spin averaged excitation energy of the first excited states of $\Lambda_c$ and is found to be $\beta = 1.0$ fm which corresponds to a mean root square distance $\sqrt{\langle r^2 \rangle} = 1.2$ fm. The scalar $ud$ diquark mass is fixed as 0.5 GeV as an input, while the other diquark masses are determined by the ground state masses of the charmed baryons.

Model A reproduces well the spectra of the single heavy baryons on the whole, while Model B overestimates the $2s$ state and obtains its energy close to the $1d$ states. This tendency is also found in a three-body calculation of heavy baryons in Ref. [41]. If Model A is the case, the string tension between the heavy quark and the light diquark is as weak as half of that for the quark-antiquark potential appearing in mesonic systems, although the color configurations are same. It would be very interesting if the nature of the $2s$ states of the single heavy baryons could be clarified in experimental observations because it reveals the properties of the diquark in the heavy baryons. We have also examined the Regge trajectories to understand the global feature of the heavy baryon spectra. We have found the difference of two models in the flavor dependence of the slopes of the trajectories. In order to confirm the strength of the quark-diquark interaction, we propose to observe the excitation energy of the double charm quark $\Xi_{cc}$.

In conclusion, the diquark models reproduce the orbital excitation energies of the single heavy baryons and the diquarks can work very well as their constituent. The investigation of the decay properties of the heavy baryons based on this picture will be one of the further confirmations. It would be also interesting if one could understand the favorable outcomes of the diquark model from much more sophisticated calculations.

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