A new interval differential equation for edge detection and determining breast cancer regions in mammography images

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ABSTRACT
Breast cancer is the most common form of cancer in women. The importance of diagnosing breast cancer is one of the important issues in medical science. Diagnosis of benign or malignant cancer is of great importance in addition to reducing costs in the direction of treatment. A non-destructive test method for early detection of breast cancer is image processing. Image processing has various uncertainties which are generated by different reasons such as sampling to noise, initial digitalization, intensity, and special domain. In this study, a strong image segmentation method based on interval uncertainties is proposed for the breast cancer images diagnosis. The main purpose of this paper is to improve the ordinary Sobel filter based on interval analysis by considering the intensity uncertainties. Simulation results have been implemented on MIAS which is an applicable database for breast cancer detection. The results of the proposed method have been compared with some state of the art methods such as LoG, Prewitt and canny filters. Final results showed that using the proposed method gives better achievement than the others by considering some kinds of uncertainties like Gaussian noise and salt and pepper noise.

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1. Introduction
Breast cancer usually begins with the formation of small tumours in the form of a gland or accumulation of calcium in breast tissue. In cancer, cells grow uncontrollably. Women are more likely to develop breast cancer than men.

Indeed, breast cancer is the most common cancer in women and the second leading cause of death in women after lung cancer. One in four women develops breast cancer throughout their lives (Costa et al., 2018). Breast cancer usually occurs in women over the age of 50, but younger women are not immune. Unfortunately, global statistics show that breast cancer is rising in young women, affecting about 7 percent of women under 40.

Advances in diagnosis and treatment since 1989 have reduced the rate of cancer-related deaths. In 2017, approximately 25,2710 breast cancer cases were diagnosed in women, with approximately 40,610 women dying from the disease. Timely awareness of symptoms and diagnosis is an important way to reduce the risk (Almurshidi & Abu-Naser, 2018).

Increasing age, genetics, and the presence of a person with breast cancer in a family increase the risk of breast cancer development. Other factors such as unhealthy diet and hormone therapy for menopausal problems are also involved (Gallego-Ortiz & Martel, 2019; Hamidinekoo, Denton, Rampun, Honnor, & Zwiggelaar, 2018; Sachs et al., 2018).

A research project on 280,000 American women shows that medical imaging, mammography, clinical examinations, and (BSE) breast cancer screening provide an early detection ability for the system, whereas more than 41% of cancerous tissues were diagnosed with mammography (Razmjooy, Ramezani, & Ghadimi, 2017; Skaane et al., 2019). Generally, early detection of breast cancer enhances the curative percentage. Several studies have been done for solving the problem of precise detection of breast cancer tissues from the mammography (Liu, Wang, & Ghadimi, 2017).

Recently, by the development of artificial intelligence applications in technology, useful tools have been improved for automatic cancer detection. The applications of image processing and machine learning for automatically diagnosis of cancer tissues increase the detection speed and decrease human errors. Furthermore, medical image processing can be used as an auxiliary tool for radiologists and physicians for increasing the diagnosis accuracy (Hashemi, Ghadimi, &
Intensity sampling includes numbers of intensities where the image intensity as a part of sampling uncertainty. This study considers image (Lopez-Molina et al., 2016).

Image segmentation is one of the important tasks of machine vision and image processing for pattern recognition and recognition. By grouping the pixels of an image, it helps to understand an image (Eskandari Nasab, Maleksaeedi, Mohammadi, & Ghadimi, 2014; Hosseini, Jafari, Mirzaei, Asghari, & Akhavan, 2015; Mohammadi & Ghadimi, 2015; Razmjooy, Naghibzadeh, & Mousavi, 2014). Therefore, a good way for precise detection of the tumour area is to use image segmentation (Wan et al., 2017). The first case before image segmentation is to quantize and to sample the input image by discretizing the image from the spatial domain (Zeng et al., 2014).

Image sampling is the process of discretizing the input image to turn it understandable for the computer by eliminating some information. This process makes some intensity uncertainties for the image which have an impact on the next step (here image segmentation). This uncertainty hardens image segmentation to achieve an agreement to select the correct boundary for the objects (Zeng, Wang, Zhang, Liu, & Alsaaedi, 2016). From the literature, it is clear that there are several researchers that have been worked on uncertainties, such as fuzzy methods (Ghadiimi, 2015a, 2015b; Gollou & Ghadimi, 2017; Hosseini Firouz & Ghadimi, 2016; Yin, Gao, Qiu, & Kaynak, 2016), statistical methods (Jassim & AbdulNaby, 2017) and interval methods (Zhang, Chen, Ngan, Yang, & Hua, 2017).

Among the determined methods, interval analysis is a proper technique which needs only lower and upper bounds for modelling uncertainties. In order to the aforementioned cases, in this study, interval analysis is utilized for robust handling of the uncertainties in digital images for breast tumour detection.

2. Image representation based on interval analysis

An image can be defined by matrices of $i$ rows and $j$ columns, that is the image $p = [1, \ldots, i] \times [1, \ldots, j]$ is the set of the positions. By considering image $A$ and its pixel value as $A(p)$ which is placed in a certain position $\alpha \in p$, such that $n(\alpha) \subset p$ is defined as the set of positions in a $3 \times 3$ neighbourhood centred at $\alpha$, containing themselves. $|n(\alpha)| = 9$ unless $\alpha$ belongs to the margin of the image (Lopez-Molina et al., 2016).

An image can be discretized by two techniques including quantizing and sampling. This study considers the image intensity as a part of sampling uncertainty. Intensity sampling includes numbers of intensities where there are some limitations about the image intensity accuracy.

Using different numbers of brightness for an image makes some uncertainties to the image which can be defined by an error measurement of $\pm \delta$.

An extended definition for the image brightness based on interval analysis by considering its uncertainties is given in the following.

$$IA = [\max (0, \min (A - \delta)), \min (M, \min (A + \delta))]$$  \hspace{1cm} (1)

where, $\delta$ describes the intensity uncertainty for the image, $M$ represents the highest value of the image intensity based on its class ($L = 1$ for double class and $L = 255$ for uint8 class). In the following, an example of breast cancer image with its interval representation is shown (Figure 1).

3. Pre-processing step

3.1. Histogram equalization

The histogram is a graphical image in which the number of pixels of each brightness level in the input image is specified. Suppose the input image is a Grayscale image with 256 brightness levels, so each image pixel can have a brightness in the range of $[0, 255]$. To obtain the image histogram, we calculate the number of pixels of each brightness level by scrolling the pixels of the image.

The normal histogram is also obtained by dividing the histogram values by the total number of pixels in the image. Histogram equalization is a method for normalizing the histogram of an image by improving its contrast. The low contrast in an image is derived by less difference between the lowest and the highest brightness of an image.

For instance, consider an $i \times j$ image $(im)$ with intensity value in the interval $[0, L - 1]$ that is 1 (for double class) and is 256 (for uint8 class). Here, $h$ is a normalized histogram of $im$ with a bin for the possible intensities:
where, \( h_n = \frac{N_n}{N_T} \), \( n = 0, 1, \ldots, L - 1 \) (2)

where, \( N_n \) is the number of pixels with intensity \( n \), and \( N_T \) describes the total number of pixels.

By considering the above definition, the histogram equalization is obtained as follows:

\[
eq_{(m,n)} = \text{floor}\left( L - \sum_{n=0}^{im(m,n)} h_n \right) \quad (3)
\]

where, floor(.) represents the flooring round of the given integer into the nearest value.

\( Y \) is in the interval \([0, L - 1]\) and is achieved by the following formula:

\[
Y = (L - 1) \times \int_0^x h_{im}(x)dx \quad (4)
\]

where, \( h_{im} \) is the image histogram.

Figure 2 shows an example of histogram equalization on the low contrast image. The left-hand side shows the images and the right-hand side shows their intensity probability density function (histogram). In the histogram, whatever the distribution of the intensity fold in the left, the image intensity is darker, and inverse, the image is lighter and if the distribution is balanced in all parts, the image histogram is equalized.

From Figure 2, it is obvious that the first input image has unbalanced histogram in the left which makes it dark. Instead, after applying the histogram equalization, the histogram is distributed in all intensities.

### 3.2. Noise reduction

Noise reduction from images is always one of the key steps in algorithms presented for automatic segmentation of the objects. Image noise reduction is a process of eliminating noise from an image signal. All recording devices have features that expose them to noise. Noise can be random or white noise (a signal that is said to be uniformly distributed across frequencies as a function of its power density).

The purpose of noise reduction from the signal is to reconstruct the distorted image and restore it to its original state based on ideal models in which we investigate how this process is performed. In some cases, the noise has a significant effect on the image, especially during image edge detection that requires differentiation; the reason for this is that the differentiation enhances the impact of high-frequency pixels which directly includes noises (Anoraganingrum, 1999; Loupas, McDicken, & Allan, 1989).

Median Filter is a popular noise reduction method in medical image processing while keeping edges. This filter is a nonlinear, low pass filter with the low speed with a computation time of \( n \times \log(n) \), where \( n \) is the width of the window. An important advantage of the median filter in using medical images is that effectively removes impulsive spikes from signals. It also decreases noise without generating a smooth ramp at the border between a dark and a light area.

Median filter employs \( m \times n \) neighbourhood and arranges the neighbourhoods in ascending order and select the middle element of the ordered numbers and replace the central pixel.

In this study, \( 5 \times 5 \) median filters has been used to eliminate the noises from the medical images. By increasing the size of the mask, the noise reduction can be performed better, but it loses the edges. Once the median filter is performed on a grayscale image, the median value of the pixels below the mask is located in each pixel. Figure 3 shows an example of the noise reduction on a breast cancer based on a median filter.

### 4. Image segmentation based on Kapur thresholding

Image thresholding is a pre-processing step for most works of digital images. It is one of the most widely used methods of image segmentation. In image processing, a
5. Interval based image edge detection

5.1. Interval analysis

Consider an interval integer. The definition for this integer can be determined as follows:

\[ \mathbb{IR} = \{X|X = [x, \bar{x}], X = \{x|x \in \mathbb{R} \cup (-\infty, \infty), x \leq \bar{x}\}\} \]

where, \( X \) is an interval integer over \( \mathbb{IR} \) and \( \bar{x} \) and \( x \) are the upper and the lower intervals, respectively (Razmjooy & Ramezani, 2018a, 2018b).

If \( x = \bar{x} \), interval integer will be called degenerate.

In the following, some important definitions for interval integers including mean value, the width, and the radius of an interval integer \( X \) are given (Hamian et al., 2018; Razmjooy & Ramezani, 2018b; Razmjooy & Ramezani, 2019).

\[ x_c = \frac{1}{2}(x + \bar{x}) \]

(8)

\[ x_w = \bar{x} - x \]

(9)

\[ x_r = \frac{1}{2}x_w \]

(10)

Based on the above definitions, an interval integer can be defined as follows:

\[ X = [x] = x_c + \delta \]

\[ \delta = \begin{cases} -x_r, & x_r \end{cases} \]

(11)

where, \( \delta \) represents the symmetric interval of \([x]\).

5.2. Basic mathematical operations based on interval analysis

By considering two interval integers including \([x] = [x, \bar{x}]\) and \([y] = [y, \bar{y}]\) and also by assuming that \( \diamond = \{+,-,\times, /\} \), the primary algebraic operations including summation (+), subtraction (-), multiplication (\(\times\)) and division (/) are as follows:

\[ [x] \odot [y] = \{x \odot y \in \mathbb{IR}|x \in [x], y \in [y]\} \]

(12)

where, \( 0 \notin [y] \) and \( [x] + [y] = [x + y, \bar{x} + \bar{y}] \)

(13)

\[ [x] - [y] = [x - y, \bar{x} - \bar{y}] \]

(14)

\[ [x] \times [y] = [\min\{xy, \bar{x}y, \bar{y}x, \bar{x}\bar{y}\}, \max\{xy, \bar{x}y, \bar{y}x, \bar{x}\bar{y}\}] \]

(15)

By assuming \( \frac{1}{[y]} = \left[ \frac{1}{y}, \frac{1}{\bar{y}} \right] \), division is defined as follows:

\[ [x]/[y] = [x] \times \frac{1}{[y]} \]

(16)

More information about interval operations can be found in (Moore, Kearfott, & Cloud, 2009).
5.3. Hukuhara difference method

Ordinary interval difference (Minkowski) has a big drawback for providing the correct difference between two interval integers; in other words, here, [x] + (−[x]) ≠ [0] where [0] determines the degenerate zero. This causes the difference between two equal interval integers not to reach the degenerate zero.

Several works have been developed to resolve this drawback. In the meantime, Hukuhara introduced a simple and efficient difference to resolve this problem. Hukuhara difference (H-difference) as a set [z] where, [x] ⊕ [y] = [z] ⇔ [x] = [y] + [z] which results [x] ⊕ [y] = [0] (Akbari, Ghiasi, Pourkheranjan, Alipour, & Ghadimi, 2019; Hukuhara, 1967; Lupulescu, 2013; Malinowski, 2012).

H-difference condition is satisfied if and only if for [x] ⊕ [y] = [z], [x] contains a translate ([z]) + [y] of [y]. A generalized version of the Hukuhara difference (gH-difference) (Leng et al., 2018) for all conditions is introduced by Stefania et al.

By considering gH-difference between two interval integers ([x], [y]) where [x] = [x̅, ̅x] and [y] = [y̅, ̅y] (Stefanini & Bede, 2009),

\[ [x] ⊕ [y] = [z] ⇔ \begin{cases} (I) : [x] = [y] + [z] \\ (II) : [y] = [x] + (-1)[z] \end{cases} \] (17)

Therefore, gH-difference between two interval integers [x] = [x̅, ̅x] and [y] = [y̅, ̅y] will exist, if:

\[ [x̅, ̅x] ⊕ [y̅, ̅y] = [z̅, ̅z] \] (18)

Such that:

\[ \bar{z} = \min(|\phi|) \]
\[ \bar{z} = \max(|\phi|) \] (19)

where,

\[ \emptyset = [x - y̅, x̅ - y] \] (20)

More information can be found in (Bede & Stefanini, 2013; Hüllermeier, 1997; Leng et al., 2018; Stefanini & Bede, 2009).

Because most of image edge detection methods use differentiation for detection the object area. Interval image edge detection should be determined by implementing the combination of interval differentiation and image edge detection on the input image.

In the following, a new method for improving the interval derivation is determined and utilized for image edge detection.

5.4. Interval derivation

The first order derivation of an interval function \( f \) in the interval \([x]\) is formulated as follows (Leng et al., 2018; Morsali, Mohammadi, Maleksaeedi, & Ghadimi, 2014):

\[ f'(x) = \frac{f_w([x])}{x_w} \] (21)

By considering \( x_0 \in [a, b] \) and \( h \) whereas \( x_0 + h \in ]a, b[ \), the derivative equivalently satisfies the property along with the generalized Hukuhara (gH) derivative of the function \( ]a, b[ \to \mathbb{R} \) in \( x_0 \) as follows:

\[ f'(x_0) = \lim_{h \to 0} \frac{1}{h} [f(x_0 + h) \Theta_g f(x_0)] \] (22)

In the above equation, \( f'(x_0) \) satisfies the equation if \( f'(x_0) \in \mathbb{R} \) and \( f \) in \( x_0 \) satisfies the gH derivative condition.

The continuity of the interval function can be proved as follows:

\[ \lim_{x \to x_0} f(x) = f(x_0) \Leftrightarrow \lim_{x \to x_0} [f(x) \Theta g f(x_0)] = [0] \] (23)

gH derivation could be satisfied once the function is continuous and the right derivative \( (f'_r([x_0])) \) and the left derivative \( (f'_l([x_0])) \) of the function are equal, i.e.

\[ f'_r([x_0]) = \lim_{h \searrow 0} \frac{1}{h} [f(x_0 + h) \Theta_g f(x_0)] \]
\[ = \lim_{h \searrow 0} \frac{1}{h} [f(x_0 + h) \Theta_g f(x_0)] = f'_l([x_0]) \] (24)

Assuming the central definition from (Razmjooy & Ramezani, 2018), by considering \([x] = x_c + x_l c,\)

\[ f'(x_0) = \lim_{h \to 0} \frac{1}{h} [f(x_0 + h) \Theta_g f(x_0)] \]
\[ = \lim_{h \to 0} \frac{1}{h} [f_c(x_0 + h) - f_c(x_0)] \]
\[ + |f_l(x_0 + h) - f_l(x_0)| c \] (25)

The partial derivative in this condition can be achieved by extending the above equation,

\[ \frac{\partial f(x_0, ..., x_i, ..., x_n)}{\partial x_i} = \lim_{h \to 0} \frac{1}{h} [f(x_0, ..., x_i, ..., x_n + h) \Theta_g f(x_0, ..., x_i, ..., x_n)] \]
\[ = \lim_{h \to 0} \frac{1}{h} [f_c(x_0, ..., x_i, ..., x_n + h) - f_c(x_0, ..., x_i, ..., x_n)] \]
\[ + \lim_{h \to 0} \frac{1}{h} [|f_l(x_0, ..., x_i, ..., x_n + h) - f_l(x_0, ..., x_i, ..., x_n)| c] \] (26)
5.5. Interval Taylor method

Interval Taylor method can be derived by implementing the interval centred definition into the higher derivatives. A second-order improvement of the definition is given in the following (Nedialkov, 1999; Rihm, 1994; Stauning & Madsen, 1997):

\[
F_t([x]) = f(x_c) + g^T(x_c)([x] - x_c) + \frac{1}{2}([x] - x_c)^T[H]([x] - x_c) \tag{27}
\]

where, \(g\) represents the gradient of \(f\) related to \(x\) and \([H]\) is the Hessian matrix.

5.6. Interval edge detection

Image edge detection is a technique for determining the boundaries of objects in the image. In the edge detection, the Sobel method adopts approximations to extract the edges. To do so, this function search for edges where the image gradient is maximized. The Sobel operator uses a pair of vertical and horizontal gradient metrics with a dimension of \(3 \times 3\).

Indeed, a Sobel operator is a first-order derivative of the image. Generally, performing derivative operator on an image highlights its edges with high frequencies; therefore, the Sobel filter can be used for extracting the edges on an image.

The Sobel filter, \(L(i,j)\) of an image can be determined as a pixel intensity values \(\varphi(i,j)\) by:

\[
L(i,j) = \sqrt{\left(\frac{\partial \varphi}{\partial i}\right)^2 + \left(\frac{\partial \varphi}{\partial j}\right)^2} \tag{28}
\]

where, the equation above can be obtained by convolution operator.

In images processing, the first operation is image acquisition. This process is performed based on discretizing the kernel of the approximating of the Sobel filter. The vertical and the horizontal kernels for the Sobel filter are illustrated in Figures 4 and 5.

A significant problem in Sobel filtering is that doesn’t perform correctly at the places where the intensity level is changing and uncertain. Therefore, by considering uncertainties, the performance of the method is improved. In the following, the interval extension of the Sobel operator is determined.

\[
\varphi_{i,j} = \varphi_{i,j} + \frac{\partial \varphi_{i,j}}{\partial x_c} \frac{[h]}{i} + \frac{\partial^2 \varphi_{i,j}}{\partial x_c^2} \frac{[h]^2}{2i} + \ldots
\]

where,

\[
\left[ \frac{\partial \varphi(x)}{\partial x} \right] = \frac{\varphi_{i+1}(x) - \varphi_{i}(x)}{h} \tag{32}
\]

Therefore,

\[
\left[ \frac{\partial \varphi(x)}{\partial x} \right] = \frac{\varphi_{i+1}(x) - \varphi_{i}(x)}{h} + \left[ \frac{\varphi_{i+1}(x) - \varphi_{i}(x)}{h} \right]_{i} \tag{33}
\]

**Figure 4.** small kernel for the horizontal derivative of Sobel filter.

|   | -1 | 0 | +1 |
|---|----|---|----|
| -2 | 0 | +2 |
| -1 | 0 | +1 |

**Figure 5.** small kernel for the vertical derivative of Sobel filter.

|   | -1 | -2 | -1 |
|---|----|----|----|
| 0 | 0 | 0 |
| +1 | +2 | +1 |

\[
\begin{align*}
\varphi_{i,j} &= \frac{\partial \varphi_{i,j}}{\partial x_c} \frac{[h]}{i} + \frac{\partial^2 \varphi_{i,j}}{\partial x_c^2} \frac{[h]^2}{2i} + \ldots \\
&+ \frac{\partial^{(n-1)} \varphi_{i,j}}{\partial x_c^{(n-1)}} \frac{[h]^{(n-1)}}{(n-1)!} + \frac{\partial^{(n)} \varphi_{i,j}}{\partial x_c^{(n)}} \frac{[h]^{(n)}}{(n)!} \tag{30}
\end{align*}
\]
Figure 6. Interval-based Sobel.

Figure 7. Interval-based Sobel filter.

Figure 8. Results of some examples of the processed images based on the proposed method: (A) original image, (B) noisy image, (C) filtered image, (D) thresholding process, and (E) interval based edge detection method.
Hence, for an image matrix $i \times j$, the gradient matrix will be achieved as follows:

$$\begin{align*}
\left[ \frac{\partial \psi_i(x)}{\partial x} \right] & = \frac{\psi_{i+1,j}(x_c) - \psi_{i,j}(x_c)}{h} + \frac{\psi_{i,j+1}(x_c) - \psi_{i,j}(x_c)}{h} \\
& + \frac{\psi_{i+1,j}(x_r) - \psi_{i,j}(x_r)}{h} + \frac{\psi_{i,j+1}(x_r) - \psi_{i,j}(x_r)}{h}
\end{align*}$$

(34)

The interval extension of the interval Sobel as a kernel is shown in Figures 6 and 7.

### 6. Implemented results

In this section, the performance of the proposed method is studied and compared with some different state of the art methods on a standard breast mammography database. Simulation results are implemented by Matlab R2017® platform on a 64-bit laptop with 32 GB RAM and an Intel® core™ i7 CPU.

In this paper, Mammographic Image Analysis Society Digital Mammogram Database (MIAS) is utilized for the analysis of the proposed system and the other compared methods (Suckling et al., 1994). Figure 8 shows some examples of the processed images based on the proposed method.

Figure 8 shows that by considering the system uncertainties like intensity variations, the efficiency of the presented edge detection gives good results for medical imaging purposes, especially for breast cancer detection. To the better analysis of the proposed system, it is compared with Laplacian of Gaussian filter (Zhitao, Chengming, Ming, & Qiang, 2002), Prewitt method (Yang, Wu, Zhao, Li, & Zhai, 2011), Canny filter (Bao, Zhang, & Wu, 2005), and Confidence intervals (Buenestado & Acho, 2018). In this study, the signal to noise ratio (PSNR) has been employed. The PSNR is employed on images with two different noises including salt & pepper and Gaussian.

Table 1 illustrates different variations of the PSNR for the analyzed database as for salt & pepper and Gaussian noise with varying variance ($\sigma$) and for the compared methods (from 0.2 to 1).

From the table, it can be concluded that by considering uncertainties in the system, the quality of the PSNR is increased. It is also clear that increasing the value of $\sigma$ makes the results of the methods to get worst while the

| Variance | Proposed method | Laplacian of Gaussian filter Zhitao et al. (2002) | Prewitt method Yang et al. (2011) | Canny filter Bao et al. (2005) | Confidence intervals Buenestado & Acho, 2018 |
|----------|-----------------|---------------------------------|---------------------------------|-------------------------------|---------------------------------|
| $\sigma = 0.2$ | 8.69 | 7.53 | 6.37 | 8.81 | 7.36 |
| $\sigma = 0.4$ | 5.24 | 4.67 | 3.14 | 5.10 | 5.28 |
| $\sigma = 0.6$ | 3.61 | 2.96 | 1.52 | 3.48 | 3.11 |
| $\sigma = 0.8$ | 3.78 | 1.50 | 2.43 | 2.67 | |
| $\sigma = 1$ | -1.06 | -9.07 | -10.19 | -8.26 | -3.54 |

Table 2. Analysis of the PSNR variation for MIAS dataset with Gaussian noise for mean value ($\mu = 0.1$) and varying variance ($\sigma$).

| Variance | Proposed method | Laplacian of Gaussian filter Zhitao et al. (2002) | Prewitt method Yang et al. (2011) | Canny filter Bao et al. (2005) | Confidence intervals Buenestado and Acho (2018) |
|----------|-----------------|---------------------------------|---------------------------------|-------------------------------|---------------------------------|
| $\sigma = 0.2$ | 1.87 | -3.43 | -4.32 | -2.81 | -1.38 |
| $\sigma = 0.4$ | 0.27 | -5.12 | -8.45 | -4.83 | -2.16 |
| $\sigma = 0.6$ | 0.79 | -6.21 | -9.31 | -5.17 | -3.64 |
| $\sigma = 0.8$ | 1.05 | -6.44 | -10.43 | -5.6080 | -0.72 |
| $\sigma = 1$ | 0.94 | -7.57 | -10.53 | -5.41 | 0.17 |

Table 3. Analysis of the PSNR Variation for MIAS dataset with Gaussian noise for mean value ($\mu = 0.5$) and varying variance ($\sigma$).

| Variance | Proposed method | Laplacian of Gaussian filter Zhitao et al. (2002) | Prewitt method Yang et al. (2011) | Canny filter Bao et al. (2005) | Confidence intervals Buenestado and Acho (2018) |
|----------|-----------------|---------------------------------|---------------------------------|-------------------------------|---------------------------------|
| $\sigma = 0.2$ | -1.48 | -8.92 | -11.28 | -6.19 | -2.12 |
| $\sigma = 0.4$ | 0.02 | -8.37 | -11.76 | -7.52 | -5.43 |
| $\sigma = 0.6$ | 0.25 | -8.22 | -11.37 | -7.41 | -3.42 |
| $\sigma = 0.8$ | 1.3 | -8.71 | -12.71 | -7.19 | -3.71 |
| $\sigma = 1$ | 0.16 | -8.67 | -12.41 | -7.74 | -1.45 |
presented method has robust results against the uncertainties.

In the following, the impact of Gaussian noise variations on the analyzed methods is studied. The results of different variations of the mean value for the Gaussian noise are illustrated in Table 2 ($\mu = 0.1$) and Table 3 ($\mu = 0.5$).

The results show that the canny filter gives better results than the other compared methods for the salt and pepper noise variations with low variance. Instead, by increasing the variance value, especially in Gaussian noise, the efficiency of the proposed method is increased.

7. Conclusions

A significant application of image processing is to use it in medical image analysis. This method is a nondestructive testing method for primary analysis and early detection of the illness. The main purpose of this paper is to use medical imaging to the early detection of breast cancer. To do so, the first step is to sample and quantizing the acquired image. Generally, this process is performed for using them as digital images in the computers by eliminating some uncertain information from the image. In addition, in the process of image acquisition, there may be occurred different types of noises. Since, the utilization of the ordinary methods for image processing is not very efficient. To deal with this problem, several techniques have been introduced. The main purpose of this paper is to propose a new approach for edge detection of breast cancer tumour based on Sobel filtering, interval analysis, and Hukuhara difference. Simulation results showed that due to the considering of the intensity variations uncertainties in the proposed method, it gives better results than the classic methods. The simulations are compared with some different state of the art methods on MIAS database and by implementing Gaussian and salt and pepper noises for analyzing the performance of the proposed method. Final results showed that the presented method has a satisfying efficiency in the presence of uncertainties.

Disclosure statement

No potential conflict of interest was reported by the authors.

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