FAST ROTATION OF NEUTRON STARS

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ABSTRACT

We show that for realistic equations of state of dense matter, the universal proportionality factor relating the maximum rotation rate of neutron stars due to mass-shedding limit to the mass and radius of maximum allowable mass configuration of non-rotating models results from a universal proportionality between masses and radii of static maximum-mass neutron stars and those of maximally rotating configurations. These empirical relations cannot be obtained in the slow rotation approximation.

Subject headings: pulsars–stars: neutron–stars: rotation

1. INTRODUCTION

The question of how fast a pulsar can spin has recently been discussed by several authors (see e.g. Friedman & Ipser 1992, Cook et al. 1994a, b). Independently of the possible mechanisms of pulsar spin-up, for any particular equation of state there is always an upper limit for the final allowed rotational speed, $\Omega_{\text{max}}$. The limit is determined either

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by the mass shedding condition, \textit{i.e.} that the angular velocity of the configuration, stable with respect to axisymmetric perturbations, equals the Keplerian velocity at the surface, or by the condition of the onset of non-axisymmetric (e.g. gravitational radiation reaction) instabilities. In this paper we discuss the limit given by the mass shedding condition.

Haensel and Zdunik (1989; hereafter HZ) noticed that for realistic equations of state of dense matter, the numerically calculated values of the shedding limit $\Omega_{\text{max}}$ can be fitted, with an accuracy better than 5%, by an empirical formula

$$\Omega_{\text{max}} = C_{\Omega} \left( \frac{GM_{\text{max}}}{R_{\text{max}}^3} \right)^{1/2},$$  \hspace{1cm} (1.1)

where $M_{\text{max}}$ is the maximal mass of the non-rotating neutron stars with the same equation of state, $R_{\text{max}}$ is the radius corresponding to $M_{\text{max}}$, and $C_{\Omega}$ is a dimensionless phenomenological constant, independent of the equation of state. $\Omega_{\text{max}}$ is an angular velocity of rigid rotation as measured by a stationary observer at infinity. HZ determined that the best fit is for $C_{\Omega} = 0.67$.

The calculations of Friedman, Ipser and Parker (1989), performed for a very broad set of equations of state, yielded $C_{\Omega} = 0.66$, quoted in Friedman (1989), Friedman and Ipser (1992). It should be mentioned, that the value of $C_{\Omega} = 0.62$, quoted in the original paper of Friedman, Ipser and Parker (1989), was actually a very rough estimate of $C_{\Omega}$ (J.L. Friedman, private communication). Calculations based on recent very accurate numerical methods are in good agreement with the original HZ choice of $C_{\Omega}$. For example, values of $\Omega_{\text{max}}$ calculated for several equations of state by Lattimer et al. (1990) differ from the HZ version of the empirical formula by less than 4%. Most recent calculations in the full framework of General Relativity by Salgado et al. (1994a,b; hereafter SBGH) based on the spectral methods (Bonazzola et al. 1993) show that configurations with causal equations of state (EOS) satisfy (1.1) with $C_{\Omega} = 0.67$ with an accuracy of better than 5% (Haensel, Salgado and Bonazzola 1995, hereafter HSB). Results obtained for causal EOS by Cook et al. (1994b) lead to very similar “best fit” value of $C_{\Omega}$. HSB found that the empirical relation with $C_{\Omega} = 0.67$ fails for configurations constructed with non-causal equations of state, where the sound speed, $(\partial P/\partial \rho)_{\text{S}}^{1/2}$, may be greater than the speed of light within the neutron star models.

Empirical relations of universal character, valid for a broad range of realistic equations of state of dense matter, can obtained also for other bulk parameters of neutron star models. For example, a simple and surprisingly good universal relation, connecting the maximum moment of inertia for slow, rigid rotation, $I_{\text{max}}$, to the mass and radius of a static configuration with maximum allowable mass (which is different from that with $I_{\text{max}}$ !) was pointed out by Haensel (1992). The existence of such universal empirical relations is of
practical importance: for any realistic EOS, empirical formulae enable rapid and still quite precise calculation of the upper bounds on global parameters of neutron stars from the easily calculated parameters of the static configuration with maximum allowable mass. A different type of a universal formula was obtained by Ravenhall and Pethick (1994). Their formula, valid for a broad range of realistic equations of state of dense matter, and useful for all except lightest neutron stars, expresses the moment of inertia in terms of stellar mass and radius.

The few attempts to explain the empirical relation, Eq. (1.1), were not concluded with satisfactory results. The most elaborate discussion was presented by Weber and Glendenning (1991; 1992). They used numerical models of rotating neutron stars calculated in the slow rotation approximation of Hartle and Thorne (1968) to show that these also obey the empirical formula, albeit with $C_\Omega \approx 0.75$. Although this is obviously an important result, still lacking is a clear physical understanding of the problem.

In a recent work Glendenning and Weber (1994) derived a formula which relates $\Omega_{\text{max}}$ to $M_{\text{max}}/R_{\text{max}}^3$, in the slow rotation approximation, where $M_{\text{max}}$ and $R_{\text{max}}$ correspond to rotating configurations, but they do not discuss the connection between this relation and the formula (1.1). As we shall see below the slow rotation approximation cannot be applied consistently to maximally rotating neutron stars, as pointed out already by Hartle and Thorne (1968).

2. THE EMPIRICAL RELATIONS BETWEEN NON-ROTATING AND MAXIMALLY ROTATING CONFIGURATIONS

One can see that, to a good approximation, the “constant” $C_\Omega$ is in fact a slowly varying function of one variable, characterizing the EOS of dense matter, $C_\Omega(\text{EOS})$ (Table 1 and HSB). Indeed Fig. 1 shows the values of $C_\Omega$ as given by eq. (1.1) for 12 maximally rotating models calculated by SBGH, as a function of the parameter

$$x_s = \frac{2GM_{\text{max}}}{R_{\text{max}}c^2}$$

(2.1)

for static maximum mass configurations. It is clear that $C_\Omega(\text{EOS})$ is slowly varying, and for the range of parameters of interest, can be well approximated by a monotonic function of $x_s$. If one restricts to realistic EOS, which are both causal and stiff enough to support observed masses of pulsars, the range of relevant $x_s$ becomes rather narrow and the approximation of $C_\Omega(\text{EOS})$ by a constant is quite satisfactory (see HSB).
It is well known that for realistic EOS, rotation increases the value of maximum mass of a neutron star by about 20%. We find however that, with a very good approximation, there exists a *universal* relation between the maximum mass of non-rotating neutron star configuration and the mass of a configuration rotating with an angular speed $\Omega_{\text{max}}$. The relation is:

$$M_{\text{max}}(\text{rot}) = C M_{\text{max}}(\text{stat}),$$

(2.2)

where $M_{\text{max}}(\text{rot})$ and $M_{\text{max}}(\text{stat})$ are respectively the mass of the configuration in maximum rotation and the maximum mass of the static neutron star for the same equation of state. Of course, a similar relation holds trivially for any specific EOS, and yields a specific value of the proportionality constant, $C_M(\text{EOS})$. The values of $C_M(\text{EOS})$, calculated for a broad set of EOS using the results of SBGH, are given in Table 1. The best fit of relation (2.2) to numerical results of SBGH is obtained for $C_M = 1.18$. Then, relation (2.2) reproduces exact results within better than 3% (Fig.2).

A similar relation is found for the radii of the corresponding configurations:

$$R_{\text{max}}(\text{rot}) = C_R R_{\text{max}}(\text{stat}),$$

(2.3)

where $R_{\text{max}}(\text{rot})$ and $R_{\text{max}}(\text{stat})$ are the radii of the maximally rotating and the static configurations respectively. The best fit value of a constant $C_R$ in relation (2.3), based on numerical results of SBGH for causal EOS, is $C_R = 1.34$. Relation (2.3) reproduces then exact results for causal EOS within better than 4% (Fig. 3).

As in the case of relation (2.2), one can introduce the factor $C_R(\text{EOS})$ (Table 1). The dependence of $C_R$ on the EOS can be well approximated by a monotonically decreasing function of $x_s$ - this is visualized in Fig. 4. No such a trend was found for $C_M(\text{EOS})$, Fig. 5.

Our analysis have been based on a set of numerical results, obtained for twelve realistic EOS in SBGH. We have restricted to the SBGH set in order to keep the homogeneity of the sample of numerical results. Numerical results obtained by other authors (Friedman, Ipser & Parker 1989, Cook et al. 1994b) are found to be slightly different from those of SBGH. This is due to the fact, that the precise determination of the maximally rotating configuration – which requires very high precision of numerical procedure – turns out to be sensitive to such details as, e.g., the interpolation method used for the determination of the maximum frequency model (Stergioulas & Friedman 1995, E. Gourgoulhon, private communication). However, uncertainties resulting from this dependence on the specific sample of numerical results obtained for realistic EOS, are consistent with the precision of relations (2.2), (2.3), discussed above.
One can easily see that the empirical $C_\Omega$ constant may be obtained, within a very good approximation, from the formula:

$$C_\Omega \simeq C \equiv \left(\frac{C_M}{C_R^3}\right)^{1/2}.$$  \hfill (2.4)

As can be seen from Table 1, for causal EOS Eq. (2.4) reproduces the actual values of $C_\Omega$(EOS) within better than 5%. In the case when one considers also non-causal EOS, the precision of approximation (2.4) worsens to 7%.

Relation (2.4) implies, that the angular velocity for mass shedding is approximated by the formula

$$\Omega_{\max} \simeq \left(\frac{GM_{\max(\text{rot})}}{R_{\max(\text{rot})}^3}\right)^{1/2}.$$  \hfill (2.5)

The value of $\Omega_{\max}$ can be thus well approximated by the frequency of a particle in stable circular orbit at the equator of a fictitious non-rotating star of mass equal to $M_{\max(\text{rot})}$ and of the radius equal to the equatorial radius of the maximally rotating configuration, $R_{\max(\text{rot})}$ (in the case of non-rotating star the general relativistic formula for the particle frequency is identical to the newtonian one, see e.g. Misner, Thorne & Wheeler 1973).

Up to now, we considered only realistic EOS of dense matter. It is instructive to study also the case of polytropic configurations with an equation of state in the form $P = Kn_b^\Gamma$, where $n_b$ is baryon density of matter. Extensive calculations of rotating polytropic models were presented in Cook et al. (1992, 1994a). For a fixed value of $\Gamma$, static and rotating models exhibit useful scaling properties with respect to change of $K$ (Cook et al. 1992, 1994a). Consequently, the relativistic parameter $x_s$, and $C_M$(EOS), $C_R$(EOS) turn out to be functions of $\Gamma$ only; they are given in Table 2. For a fixed value of $\Gamma$, relations (1.1), (2.2), and (2.3) are thus exact, with $\Gamma$-dependent values of the numerical coefficients.

The realistic EOS are not polytropes, and their local adiabatic index, $\Gamma = (n_b/P)dP/dn_b$, depends on the density, $\Gamma = \Gamma(n_b)$. Two specific examples of the density dependence of $\Gamma$ are shown in Fig. 6. In both cases, the value of $\Gamma$ varies within the relevant interval of $n_b$ by more than 30% of its maximum value. Clearly, such EOS cannot be represented by single polytropes. This explains, e.g., the lack of monotonic trend in the dependence of $C_M$(EOS) on $x_s$(EOS) for realistic EOS, displayed in Fig. 5. Such a non-monotonic, irregular behavior, characteristic of realistic EOS, is to be contrasted with a monotonic dependence $C_M(x_s)$ for polytropes, Table 2. In spite of this irregular behavior of $C_M$(EOS) for realistic EOS, the values of $C$(EOS) for realistic EOS, calculated from (2.4), show a clear trend for a monotonic increase with $x_s$. This results from a clear trend in
\( C_R(\text{EOS}) \) to decrease with \( x_s \), which – magnified by the third power within the bracket of formula (2.4) – dominates the \( C - x_s \) relation.

The trend for a monotonic increase of \( C_\Omega \) with \( x_s \), Fig. 1, can be well reproduced by a linear function \( C_\Omega^{\text{lin}}(x_s) = 0.468 + 0.378x_s \). This function, obtained by a least squares fit to the points shown in Fig. 1, reproduces \( C_\Omega(\text{EOS}) \) (and \( \Omega_{\text{max}}(\text{EOS}) \), if inserted in Eq. (1.1)) with precision better than 1.5\%, with typical relative error being less than 1\%. It should be stressed, however, that such a linear approximation \( C_\Omega^{\text{lin}}(x_s) \) is valid only within a rather narrow interval of \( x_s \), characteristic of realistic EOS, \( 0.45 < x_s < 0.7 \). In contrast to the one-parameter empirical formula with \( C_\Omega = 0.67 \), \( C_\Omega^{\text{lin}}(x_s) \) does not work for the EOS of the free neutron gas (see HSB).

3. THE SLOW ROTATION APPROXIMATION

Weber and Glendenning (1991, 1992) tried to derive the empirical formula, equation (1.1), by using the slow rotation approximation of the Einstein equations. One should notice however that a neutron star rotating with \( \Omega_{\text{max}} \) may not be really considered to be a slow rotator (Hartle & Thorne 1978). Indeed, let us define a dimensionless angular velocity \( \Omega_* \),

\[
\Omega_*^2 \equiv \frac{\Omega^2}{\left(\frac{GM}{R^3}\right)},
\]

where \( M \) is the mass of the static star, \( R \) its radius and \( \Omega \) is the angular velocity measured by observers at infinity. The assumption of the slow rotation means that (Hartle 1967),

\[
\Omega_*^2 \ll 1.
\]

From equation (1.1) with \( C_\Omega = 0.67 \) it follows that

\[
\Omega_*^2 < 0.45,
\]

so that the use of slow-rotation approximation for maximally rotating configurations requires some additional justification. It is unlikely that this approximation will give accurate results considering the fact that at the stellar surface the linear speed of rotation is a significant fraction of the speed of light (Hartle & Thorne 1968):

\[
\frac{v_S}{c} = \frac{1}{\sqrt{2}} \left(\frac{x_s}{1 - x_s}\right)^{1/2}
\]
For the maximally rotating models of SBGH one gets typically $v_S/c \approx 0.7$. In the slow rotation approximation, one can obtain for $\Omega_{\text{max}}$ an expression of the form, which seems to be similar to that of the empirical formula (1.1) (see also Glendenning and Weber 1994):

$$\Omega_{\text{max}} = C_{sr} \left( \frac{GM_{\text{max}}(\text{rot})}{R_{\text{max}}^3(\text{rot})} \right)^{1/2} + \mathcal{O}(\Omega_*^3),$$

(3.5)

with

$$C_{sr} \approx \left[ 1 + \frac{I}{MR^2} \frac{R_G}{R} \left( 1 - 2.5 \frac{I}{MR^2} \left( \frac{R}{R_G} \right)^4 \left( 1 - \frac{R_G}{R} \right) Q_2^2(u) \cdot \left( 1 - \frac{QMc^2}{J^2} \right) \right) \right]^{-\frac{1}{2}}.$$

(3.6)

Here $R_G$ is the gravitational radius,

$$R_G = \frac{2GM}{c^2},$$

(3.7)

$J$ is the angular momentum, $I$ the moment of inertia, $Q$ is the quadrupole moment of the rotating configuration, and $Q_2^2(u)$ is the derivative, with respect to $u = 1 - R/R_G$, of the associated Legendre function of the second kind. The quantities $M$ and $R$ are those for the maximally rotating configuration, but within our approximation — we can as well replace them by $M_{\text{max}}(\text{stat}), R_{\text{max}}(\text{stat})$.

Equations (3.5) and (3.6) give $\Omega_{\text{max}}$ in terms of the mass, equatorial radius, moment of inertia, angular momentum and the quadrupole moment of the maximally rotating configuration. At first glance (and neglecting terms $\sim \Omega_*^3$ and higher), expression (3.5 – 3.6) may seem to be similar to empirical formula (1.1). It has been shown by Abramowicz and Wagoner (1978) and recently confirmed by Ravehall and Pethick (1994) that moments of inertia, expressed in the units of $MR^2$, are - to a good approximation - functions of only $x = R/R_G$. Moreover, results of SBGH show that $QM c^2/J^2$ is a decreasing function of $x_s$. Expression (3.5) seems thus to possess similar properties as the empirical formula. Unfortunately it is not the empirical formula since it involves $M_{\text{max}}$ and $R_{\text{max}}$ of rotating configurations, and therefore it corresponds to some approximation of eq. (2.5) and not to eq. (1.1). Further expansions of $M_{\text{max}}(\text{rot})$ and $R_{\text{max}}(\text{rot})$ in $\Omega_*$ are not justified - in view of the large value of this parameter (notice that rotation increases the value of $R_{\text{max}}$ by some 30%). So, only some external “empirical input” (such as assumption of a “typical” effect of rotation on $M_{\text{max}}, R_{\text{max}},$ and on eccentricity of rotating star, made in Weber & Glendenning (1992)) can lead to an expression of type (1.1). A consistent application of the slow rotation approximation cannot reproduce the empirical formula for $\Omega_{\text{max}}$. This is not very surprising if one considers that one should get to the 8-th order in $\Omega_*$ to get a precision of 4%, characteristic of empirical formula.
4. DISCUSSION AND CONCLUSIONS

The proportionality constant appearing in the empirical formula for $\Omega_{\text{max}}$ for realistic EOS of dense matter, is in fact a function of the relativistic parameter $x_s$. In the range of parameters describing maximally rotating configurations with causal equations of state $C_\Omega$ is a rather slowly varying function of $x_s$, which results in a high precision of empirical formula with an appropriate choice of a universal proportionality constant. We found universal relations connecting maximal masses and corresponding radii of static neutron stars, calculated for realistic EOS of dense matter, with those of maximally rotating configurations. The empirical formula follows from those relations.

Although the slow rotation approximation allows one to reproduce some of the properties of the empirical formula this approximation is not appropriate for maximally rotating configurations.

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FIGURE CAPTIONS

Fig. 1. Proportionality constant $C_{\Omega}(\text{EOS})$ for twelve realistic EOS versus relativistic parameter $x_s$. Filled circles for causal EOS, open circles for EOS which are non-causal within massive neutron star models.

Fig. 2. Mass of maximally rotating neutron stars versus maximum mass of non-rotating models for realistic EOS. Filled circles for causal EOS, open circles for EOS which are non-causal within massive neutron star models. Straight line gives the best fit to exact results obtained for causal EOS.

Fig. 3. Equatorial radius of maximally rotating neutron stars versus the radius of the static maximum mass configurations for realistic EOS. Filled circles for causal EOS, open circles for EOS which are non-causal within massive neutron star models. Straight line gives the best fit to exact results obtained for causal EOS.

Fig. 4. Proportionality constant $C_R(\text{EOS})$ for twelve realistic EOS versus relativistic parameter $x_s$. Filled circles for causal EOS, open circles for EOS which are non-causal within massive neutron star models. Dashed horizontal line corresponds to the best fit to exact results for causal EOS.

Fig. 5. Proportionality constant $C_M(\text{EOS})$ for twelve realistic EOS versus relativistic parameter $x_s$. Notation as in Fig. 4.

Fig. 6. Adiabatic index, $\Gamma$, versus baryon density, $n_b$, for AV14 + UVII (solid line) and UV14 + TNI (dashed line) EOS. The calculation of $\Gamma$ has been based on the polynomial fit of Kutschera & Kotlorz (1993) to the tabulated EOS of Wiringa et al. (1988).
### Table 1

**Realistic Equations of State**

| Equation of state                      | $x_s$ | $C_{\Omega}$ | $C_M$  | $C_R$  | $C$  |
|----------------------------------------|-------|--------------|--------|--------|------|
| Glendenning 1985 “case 2”             | 0.467 | 0.648        | 1.1741 | 1.3960 | 0.657|
| Glendenning 1985 “case 1”             | 0.480 | 0.659        | 1.1783 | 1.3794 | 0.670|
| Glendenning 1985 “case 3”             | 0.516 | 0.658        | 1.1746 | 1.3740 | 0.673|
| Diaz Alonso 1985 model II             | 0.524 | 0.666        | 1.1704 | 1.3575 | 0.684|
| Weber et al. 1991 $A_0^{16}$ Bonn + HV| 0.533 | 0.661        | 1.1983 | 1.3786 | 0.676|
| Bethe & Johnson 1974 model IH          | 0.554 | 0.672        | 1.1597 | 1.3409 | 0.693|
| Wiringa et al. 1988 model UV14 + TNI   | 0.569 | 0.687        | 1.1826 | 1.3277 | 0.711|
| Pandharipande 1970                    | 0.577 | 0.684        | 1.1633 | 1.3226 | 0.709|
| Haensel et al. 1980 “model 0.17”      | 0.614 | 0.704        | 1.2138 | 1.3103 | 0.7345|
| Friedman & Pandharipande 1981\(^a\)  | 0.619 | 0.701        | 1.1807 | 1.3145 | 0.721|
| Wiringa et al. 1988 model UV14 + UVII \(^a\) | 0.656 | 0.719        | 1.1824 | 1.2690 | 0.761|
| Wiringa et al. 1988 model AV14 + UVII \(^a\) | 0.667 | 0.722        | 1.1904 | 1.2671 | 0.765|

\(^a\) Non-causal within central cores of massive neutron stars
**TABLE 2**

POLYTROPIC EQUATIONS OF STATE

| $\Gamma$ | $x_s$  | $C_\Omega$ | $C_M$  | $C_R$  | $C$   |
|---------|-------|------------|-------|-------|------|
| 2.00    | 0.438 | 0.624      | 1.1504| 1.4242| 0.631 |
| 2.25    | 0.509 | 0.640      | 1.1766| 1.4032| 0.652 |
| 2.50    | 0.563 | 0.660      | 1.1961| 1.3745| 0.679 |
| 2.75    | 0.605 | 0.6745     | 1.2111| 1.3540| 0.6985|
| 3.00    | 0.637 | 0.6923     | 1.2225| 1.3288| 0.723 |