Bounds on Graviton mass using weak lensing and SZ effect in Galaxy Clusters

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Abstract

In General Relativity (GR), the graviton is massless. However, a common feature in several theoretical alternatives of GR is a non-zero mass for the graviton. These theories can be described as massive gravity theories. Despite many theoretical complexities in these theories, on phenomenological grounds the implications of massive gravity have been widely used to put bounds on graviton mass. One of the generic implications of giving a mass to the graviton is that the gravitational potential will follow a Yukawa-like fall off. We use this feature of massive gravity theories to probe the mass of graviton by using the largest gravitationally bound objects, namely galaxy clusters. In this work, we use the mass estimates of galaxy clusters measured at various cosmologically defined radial distances measured via weak lensing (WL) and Sunyaev-Zeldovich (SZ) effect. We also uses the model independent values of Hubble parameter $H(z)$ smoothed by a non-parametric method, Gaussian process. Within 1σ confidence region, we obtain the mass of graviton $m_g < 5.9 \times 10^{-30}$ eV with the corresponding Compton length scale $\lambda_g > 6.82$ Mpc from weak lensing and $m_g < 8.31 \times 10^{-30}$ eV with $\lambda_g > 5.012$ Mpc from SZ effect. This analysis improves the upper bound on graviton mass obtained earlier from galaxy clusters.

1 Introduction

General theory of Relativity (GR) is one of the most elegant theories of gravity. It had been introduced only on the basis of theoretical principles before being tested and confirmed by observations \cite{1}. Till date all of its predictions have been tested and verified in different limits \cite{2}. In the weak field approximation, observations like the precise measurement of the perihelion advance of Mercury \cite{3}, deflection of light by the Sun \cite{4}, gravitational time delay \cite{5}, equivalence principle \cite{6}, the Nordtvedt effect in lunar motion \cite{7}, frame-dragging \cite{8} etc., show an impeccable agreement with the observations at solar system length scales \cite{9}. The recent detection of gravitational waves \cite{10} as well as time lag measurements on binary radio pulsar \cite{11} verified the consistency of GR even in the strong field limit \cite{12}. However, there is still a lack of direct observations at large length scales ( $\sim$ Mpc) that can establish the consistency of GR \cite{13}. Further, there is a need to introduce a dark component in the energy budget of the Universe to make it compatible with cosmological observations \cite{14}. This dark sector of the Universe remains unobserved which in turn could be taken as an opportunity to look for alternatives to GR at large (cosmological) length scales. The study of any deviation and modification of GR remains an exciting topic of research and has immense theoretical importance. Many alternative theories like $f(R)$ gravity \cite{15}, Chameleon theory \cite{16}, Galileon models \cite{17} etc., have been proposed. For a detailed review of alternative theories of gravity, see ref \cite{18, 20}.

Massive gravity theories are a class of alternative theories to GR that can explain the cosmic acceleration without invoking a dark component in the Universe \cite{21, 22}. In 1939, Fierz and Pauli (F&P) proposed a very elegant theory of massive spin 2 gravitons, in which they added a mass term to the Einstein-Hilbert action in such a way that GR is recovered when the mass term tends to zero \cite{23}. However, this theory of massive gravity failed to reproduce the results of GR at small scales (specially at solar system scales). This incompatibility is known as van Dam, Veltman, and Zakharov (vDVZ) discontinuity (1970) in literature \cite{24}. But Vainshtein (1972) presented a mechanism where he showed that the vDVZ discontinuity can be cured by taking into account non-linearities \cite{25}. Soon thereafter, Boulware and Deser (1972) found that a ghost-like negative kinetic term appears in the non-linear massive gravity theory which destabilizes the background. This instability is known as the BD instability \cite{26}. However, in the last decade de Rham, Gabadadze, Tolley (dRGT 2011) provided a nonlinear completion to the Fierz-Pauli massive gravity theory that evades the BD ghost instability \cite{27}. Recently, LIGO has reported a gravitational wave event GW170718 which rules out many scalar-tensor theories though many massive gravity models survive this test \cite{28, 29}. Presently, the DGP model \cite{30} as well as Bigravity models \cite{31} which support modification of GR at large scales have emerged to address some fundamental issues in cosmology such as Dark matter, Dark energy, inflation etc.

Due to many conceptual and theoretical difficulties, massive gravity theories are yet to emerge as
strong contenders for replacing GR. However, various generic phenomenological features of the motivation of these theories can be used to probe graviton mass. Three traditionally used motivations for this purpose are as follows: [for details of all methods see ref. [32–34]].

a) In the case of a massive graviton, the left hand side of the basic Poisson equation (i.e. $\nabla^2 \phi$) which governs the gravitational potential in a linear regime, gets modified to $(\nabla^2 - m^2)\phi$ and the corresponding form of gravitational potential gets modified from the Newtonian potential to a Yukawa-line potential. Giving a small mass to the graviton, at small length scales (solar system scales) the departure of the Yukawa potential from the Newtonian potential would be very small but at large distances (galactic and extragalactic scales) it would become significant.

b) A massive graviton would not travel at the speed of light. This would modify the corresponding dispersion relation. One way to test this is by comparing the arrival times of a gravitational wave and the electromagnetic counterpart [35,36]. The most recent and reliable bound on the graviton mass from this approach is obtained from GW170104 which is $m_\gamma < 7.7 \times 10^{-23}$ eV [37].

c) The above mentioned approaches are straightforward and can easily be inferred from the linear theory. However, many massive gravity theory also give rise to a fifth-force kind of interaction due to non-linear effects. Several bounds on graviton mass from models in this category have been obtained by studying the sensitivity of the fifth force effect on the precession of the Earth-Moon by using Lunar Laser Ranging ($m_\gamma < 10^{-38}$ eV) [38], radiated power from binary pulsar systems ($m_\gamma < 10^{-27}$ eV) [39] and structure formation ($m_\gamma < 10^{-32}$ eV) [40]. However, all these bounds are restricted to the DGP and dRGT model within their decoupling limit approximations and sensitive to the details of the model. [32].

In this work, we use the fact that in the case of a massive graviton the gravitational potential due to a static massive object of mass $M$ changes from the Newtonian to the Yukawa type fall-off and can be parametrized as

$$V = \frac{GM}{r} e^{-r/\lambda_g}$$

where $\lambda_g$ is a length scale that represents the range of interaction due to exchange of gravitons of mass $m_\gamma = \frac{\hbar}{\lambda_g^2}$.

Hare [1973] first proposed this phenomenological approach of the probing graviton mass [41]. Goldhaber and Nieto (hearafter GN74) used Galaxy Clusters for the first time to limit the graviton mass. Using the Holmberg galaxy cluster they found the bound on the graviton mass of the order of $m_\gamma < 1.1 \times 10^{-29}$ eV or $\lambda_g > 10^{20}$ km (3.24 Mpc). Choudhury et. al. (2004) [42] derived the convergence power spectra of weak lensing under Newtonian as well as Yukawa gravity. To obtain the bound on the graviton mass they compared the corresponding cosmic shear with observations of weak lensing from a cluster. Within a 1σ confidence region they constrained the graviton mass to $m_\gamma < 6 \times 10^{-32}$ eV or $\lambda_g > 3 \times 10^{21}$ km ($\sim$ 97 Mpc).

Recently, S. Desai (2017) used the dynamical features of the Abell 1689 galaxy cluster to probe the graviton mass [45] and obtained the upper limit on the graviton mass of $m_\gamma < 1.37 \times 10^{-29}$ eV or $\lambda_g > 9.1 \times 10^{19}$ km (2.95 Mpc) within 90% confidence level. Zakharov et. al. [2016] also use a similar phenomenological consequence of massive gravity and show that an analysis of bright star trajectories near the Galactic Center could bound the graviton mass. They found the upper bound for graviton mass from their work to be $m_\gamma < 2.9 \times 10^{-21}$ eV within 90% confidence level [46,47].

In this era of precision cosmology with access to detailed observations and improved knowledge, it is desirable to check and explore these limits with extended datasets. Here we further extend this approach to limit the graviton mass by analysing the acceleration profile of Newtonian and Yukawa gravity. For this we use the full catalog of galaxy clusters, obtained by using weak lensing and SZ effect. In the past, a single galaxy cluster has been used to constrain the graviton mass. However, we have not come across any work in literature where the presently available full catalog of galaxy clusters has been used for this purpose. The structure of the paper is as follows: In Section 2, we discuss the datasets and the method of mass estimation of galaxy cluster using weak lensing and SZ effect. The method and the result are discussed in the Section 3 and 4, respectively. Conclusions are summarized in Section 5.

2 Dataset

Galaxy clusters are the most massive gravitationally bound structures that emerge in the large scale structure (LSS) web. Several characteristic features of galaxy clusters like the emission of X-Ray radiation from intercluster medium through thermal bremsstrahlung phenomenon and the thermal shift in the black-body spectrum of CMB photons through inverse Compton scattering (SZ effect) play a crucial role in probing the Universe [48]. Weak gravitational lensing of background objects by clusters also provides crucial information about the evolution of large scale structure and composition of the universe [49]. We use two different mass measurements of galaxy clusters from the weak lensing and SZ effect to probe the graviton mass.
2.1 Mass estimation of Galaxy cluster using Weak lensing

Weak lensing (WL) is the cleanest method to estimate the mass of large scale structures because gravitational lensing is sensitive to the total matter distribution and is not affected by the physical and dynamical state of clusters. The observable quantity measured in weak lensing surveys is the small change in the ellipticity or the tidal distortion of a galaxy’s image known as shear. If this shear is caused by large scale structures like clusters, then it is known as cosmic shear. It is directly related to the projected foreground mass of lensing objects enclosed within the cluster radius \( R_{200} \[50\].

In this work, we use the weak lensing mass measurement of 50 most massive galaxy clusters (redshift range \( 0.15 < z < 0.3 \) ) analysed by Okaba et. al (2015) in the Local Cluster Substructure Survey (LoCuSS) \[51,52\]. To compute the mass of a galaxy cluster, a model of the shear profile of individual cluster has been fitted to the observational data by adopting the universally accepted Navarro, Frank & White (NFW) mass density profile \[54\] of dark matter halo. The effect of systematic bias in measurement, contamination of background galaxies and intrinsic asphericity of galaxy clusters have also been taken care of in the dataset.

In this analysis, we use mass estimates of galaxy clusters calculated by using the same approach at radius \( R_{200}, R_{500}, R_{1000} & R_{2500} \) and defined as \( M_{200}^{WL}, M_{500}^{WL}, M_{1000}^{WL} \) & \( M_{2500}^{WL} \) (For data see Table B1 in ref. \[51\]).

2.2 Mass estimation of Galaxy cluster from SZ effect

The SZ effect accounts for the distortion in the black body spectrum of the Cosmic Microwave Background (CMB) radiation generated via inverse Compton scattering of CMB photons by the hot and energetic free electrons in the inter-cluster medium (ICM). The magnitude of SZ distortion in the CMB spectrum is measured by a parameter called the Compton parameter \( y \), which is a measure of gas pressure integrated along the line of sight. The gas pressure is directly related to the gravitational potential of clusters. Hence, the mass of a cluster can be calculated from the SZ signal through the pressure profile of the galaxy cluster \[55\]. Arnaud et. al. (2010) proposed a cluster electron pressure profile as a function of radius \( r \) of cluster, modeled by using a generalized NFW density profile and named it as the Universal Pressure Profile (UPP) \[56\]. Hasselfield et. al (2017) use this pressure profile to develop a scaling relation between the SZ observable and cluster mass and hence provide an estimate of the mass of galaxy cluster observed in the Atacama Cosmology Telescope (ACT) \[57\].

Recently, Hilton et. al (2017) present a catalog of 182 optically confirmed galaxy clusters detected via the SZ effect at the Atacama Cosmology Telescope (ACT) survey in the redshift range \( 0.1 < z < 1.4 \[58\]. The clusters’ mass \( M_{500} \) has been calculated by using the SZ signal under the assumption of the Universal Pressure Profile of cluster electron pressure within a radial distance \( R_{500} \). In this work, we used these mass estimates of 182 galaxy clusters and represented them as \( M_{SZ}^{500} \). For details of method see ref. \[57,58\] and for the dataset see Table A.3 in \[58\] with index \( M_{UPP}^{500} \).
3 Method

Given the mass of a galaxy cluster at any particular radial distance, the gravitational acceleration, $a_n$, in Newtonian gravity can be written as

$$a_n = \frac{GM_\Delta}{R_\Delta^2}$$

(2)

where $M_\Delta$ represents the mass of the galaxy cluster within a radius $R_\Delta$, which is defined as a distance from the core of the cluster at which the density of galaxy cluster becomes $\Delta$ times the critical density $\rho_c$ of the Universe at that epoch. The mass of the galaxy cluster can be defined as;

$$M_\Delta = \Delta \times \rho_c \times \frac{4\pi}{3} R_\Delta^3$$

(3)

The critical density of the Universe is given by $\rho_c = \frac{3H^2(z)}{8\pi G}$, where $H(z)$ represents the Hubble parameter. On the other hand, if we assume a modified theory with massive gravitons, the corresponding gravitational acceleration at any particular radial distance would take the Yukawa form and we get,

$$a_y = \frac{GM_\Delta}{R_\Delta} \exp(-R_\Delta/\lambda_y) \left( \frac{1}{R_\Delta} + \frac{1}{\lambda_y} \right)$$

(4)

By using Eq. 3, one can rewrite the above mentioned acceleration expressions $a_n$ and $a_y$ as,

$$a_n(z, M_\Delta) = (GM_\Delta)^{1/3} \left( \frac{H^2(z)\Delta}{2} \right)^{2/3}$$

(5)

and

$$a_y(z, M_\Delta, \lambda_y) = (GM_\Delta)^{2/3} \left( \frac{H^2(z)\Delta}{2} \right)^{1/3} \times \exp \left[ -\frac{1}{\lambda_y} \left( \frac{2M_\Delta G}{H^2(z)\Delta} \right)^{1/3} \right] \left[ \frac{1}{\lambda_y} + \left( \frac{H^2(z)\Delta}{2M_\Delta G} \right)^{1/3} \right]$$

(6)

In order to put any limit on graviton mass by using these acceleration profiles, we require independent information about $M_\Delta$ and Hubble parameter $H(z)$. For $M_\Delta$, we use the masses of galaxy clusters estimated by using the SZ effect and weak lensing properties, as mentioned in the previous section.

To estimate the value of the Hubble parameter $H(z)$ at the corresponding redshift, we use the 38 observed Hubble parameter values of $H(z)$ in the redshift range $0.07 < z < 2.34$ calculated by using the differential ages of galaxies, Baryonic Acoustic Oscillation (BAO) and Lyman $\alpha$ measurement [59]. As seen in Fig. 3, we apply a non-parametric technique (Gaussian process) to smooth it which enables us to find the model independent value of $H(z)$ at all desired redshifts of the galaxy clusters [60]. [For more details of Gaussian process see [60]].

Once the acceleration corresponding to the Newtonian potential and Yukawa potential are known, we define a chi-square $\chi^2$ as:

$$\chi^2 = \sum_i \left[ \frac{a_{n,i}(z, M_\Delta) - a_{y,i}(z, M_\Delta, \lambda_y)}{\sigma_{a,i}} \right]^2$$

(7)

where $\sigma_a$ gives the error in acceleration obtained by adding the errors of mass estimate, $\sigma_{M_\Delta}$ and Hubble parameter $\sigma_H$ in quadrature, given by,

$$\sigma_a = \frac{a_n}{3} \left( \frac{\sigma_{M_\Delta}}{M_\Delta} \right)^2 + 16 \left( \frac{\sigma_H}{H(z)} \right)^2$$

(8)
In this analysis, we have only one model parameter, \( \lambda_g \), related to the graviton mass by the expression, \( \lambda_g = h/m_g c \) and the \( \chi^2 \) has been summed over all the datapoints present in catalog. It can be seen easily that as \( \lambda_g \to \infty \) or \( m_g \to 0 \) then \( a_g(z, M_\Delta, \lambda_g) \) reduces to \( a_n(z, M_\Delta) \) and \( \chi^2 \to 0 \). Hence it is obvious that the best fit value of \( m_g \) for which \( \chi^2 \) would minimize (i.e. \( \chi^2_{min} \)) is zero. To get a bound on graviton mass with 68.3% or 1σ confidence we put a threshold limit \( \Delta \chi^2 < 1.0 \), where \( \Delta \chi^2 = \chi^2 - \chi^2_{min} \). Similarly for 90.0% (1.64σ), 95.5% (2σ) and 99.7% (3σ) confidence limits we have threshold limits at \( \Delta \chi^2 = 2.714 \) and 9 respectively (see Fig. 1).

| Data   | Parameter | 1 σ (68.3%) | 1.64 σ (90%) | 2 σ (95.5%) | 3 σ (99.7 %) |
|--------|-----------|-------------|--------------|-------------|-------------|
| \( M_{WL}^{500} \) | \( m_g < (\text{in eV}) \) | \( 5.902 \times 10^{-30} \) | \( 7.849 \times 10^{-30} \) | \( 8.715 \times 10^{-30} \) | 1.106 \times 10^{-29} |
|        | \( \lambda_g > (\text{Mpc}) \) | 6.822 | 5.132 | 4.622 | 3.643 |
| \( M_{WL}^{1000} \) | \( m_g < (\text{in eV}) \) | \( 8.003 \times 10^{-30} \) | \( 1.053 \times 10^{-29} \) | 1.175 \times 10^{-29} | 1.48 \times 10^{-29} |
|        | \( \lambda_g > (\text{Mpc}) \) | 5.033 | 3.824 | 3.427 | 2.713 |
| \( M_{WL}^{2500} \) | \( m_g < (\text{in eV}) \) | \( 1.088 \times 10^{-29} \) | \( 1.421 \times 10^{-29} \) | \( 1.598 \times 10^{-29} \) | \( 2.017 \times 10^{-29} \) |
|        | \( \lambda_g > (\text{Mpc}) \) | 3.700 | 2.821 | 2.520 | 1.997 |
| \( M_{WL}^{5000} \) | \( m_g < (\text{in eV}) \) | \( 1.952 \times 10^{-28} \) | \( 2.583 \times 10^{-28} \) | \( 2.894 \times 10^{-28} \) | \( 3.641 \times 10^{-28} \) |
|        | \( \lambda_g > (\text{Mpc}) \) | 2.060 | 1.560 | 1.390 | 1.100 |
| \( M_{SZ}^{500} \) | \( m_g < (\text{in eV}) \) | \( 3.807 \times 10^{-30} \) | 1.051 \times 10^{-29} | \( 1.169 \times 10^{-29} \) | 1.461 \times 10^{-29} |
|        | \( \lambda_g > (\text{Mpc}) \) | 5.012 | 3.831 | 3.443 | 2.747 |

Table 1: Bounds on the graviton mass \( m_g \) and corresponding Compton length scale \( \lambda_g \) within 1σ, 1.64σ, 2σ and 3σ confidence limits estimated by using \( M_{WL}^{200} \), \( M_{WL}^{500} \), \( M_{WL}^{1000} \), \( M_{WL}^{2500} \) and \( M_{SZ}^{500} \).

Figure 3: Plot of Hubble Parameter \( H(z) \) vs redshift \( z \). The black points with error bars are the 38 measurements of \( H(z) \) and the solid line with 1σ and 2σ confidence limits are the corresponding smoothed values of the \( H(z) \) plot by using a model independent non-parametric method, "Gaussian Process".

4 Result

The deviation of the Newtonian potential at large scales depends on how the total mass of galaxy cluster changes with respect to the radial distance. Presently, the weak lensing and SZ effect related surveys enable us to find out the mass of these large celestial objects, i.e. galaxy clusters with great precision at a radial distance where all the clusters have the same density. In this work, we use two such catalogs, in which first one contains the mass estimates of 50 galaxy clusters calculated by using the weak lensing properties and defined as \( M_{WL}^{200} \), \( M_{WL}^{500} \), \( M_{WL}^{1000} \) and \( M_{WL}^{2500} \). The second catalog contains the mass estimate of 182 galaxy cluster derived by using the SZ property of clusters and defined as \( M_{SZ}^{500} \) [Note: “Universal Pressure Profile” (UPP) of pressure distribution within the cluster has been used to estimate mass ]. In the notation of mass, the subscript contains a number (\( \Delta = 200, 500, 1000 \) etc.), which represents the radial extent of cluster upto a region where the density of cluster is \( \Delta \) times of the critical density of the universe.

The mass estimates of galaxy clusters indirectly depend upon the form of the potential. It requires input about the mass profiles for dark matter halos. In both datasets used in this analysis, dark mass distribution have been assumed to follow the NFW profile. This is an empirical mass profile identified in N-body simulations of structure formation and widely accepted in the literature [63, 64]. Since these simulations implicitly assume a Newtonian potential, it would not be quite correct to claim that this analysis is completely independent of any cosmological assumption.

As deduced from Fig. 1, Table 1 presents the bounds on the graviton mass obtained through different mass estimates of galaxy clusters at different confidence limits (1σ, 1.64σ, 2σ & 3σ). One obvious point that can be directly inferred from Fig. 2 as well as from Table. 1 is that as the length scale increases, the fractional change between the Newtonian & Yukawa acceleration profile \( a_n - a_y \) becomes significant. In the left panel of Fig. 2 we explore this fractional change by using WL upto 2.3 Mpc length scale, where it becomes quite significant (approximately 15%). Similarly, in the right panel of Fig. 2 this difference is approximately 7% at a radial distance 1.3 Mpc which is calculated by using the mass
measurements of 182 galaxy clusters studied through the 
SZ effect. As expected this behaviour of increasing dif-
ference between the two potentials with increasing dis-
tance has been shown by all clusters irrespective of their 
redshift.

In this work, the strongest bound on the graviton mass 
obtained is $m_g < 5.9 \times 10^{-30}$ eV or $\lambda_g > 6.822$ Mpc by 
using $M^{WL}_{200}$ estimate of 50 galaxy clusters within a 1$\sigma$
confidence limit. The catalog having 182 independent 
mass measurements $M_{500}^{SZ}$ of galaxy clusters, derived by 
SZ effects constrains the graviton mass to $m_g < 8.307 \times$
$10^{-30}$ eV or $\lambda_g > 5.012$ Mpc. We also notice that 
the graviton mass is seen to be sensitive to the length scale. 
It is clear from the bounds given in Table 1 that the 
graviton mass bounds improve as one moves out in the 
radial distance (i.e. from $R_{2500}$ to $R_{200}$).

5 Conclusion

Galaxy clusters are the largest gravitationally bound 
objects in the Universe and occupy a special place in 
the hierarchy of cosmic structures. The observational 
characteristics of galaxy clusters can be extensively used 
to study the properties of the cluster galaxy population 
and those of the hot diffuse intracluster medium (ICM).
In recent times many dedicated surveys, like Chandra, 
Newton, ACT, LoCuSS etc. have studied the X-ray 
properties, SZ effect and weak lensing properties of 
galaxy clusters. There has also been extensive work to 
probe different features of cosmology and alternative 
theories. However, there has not been much work on 
using galaxy clusters to test gravity and probe the 
graviton mass. The very first proposal of using 
galaxy cluster for probing graviton mass was given by 
Goldhaber and Nieto (GN74) [42]. They gave a rough 
estimate of graviton mass by assuming that the orbits of 
galaxies within galaxy clusters are gravitationally bound 
& closed and that the distance of outermost galaxies 
from the core of the Holmberg galaxy cluster is 580 kpc. 
GN74 further assumed that only Newtonian potential 
could give rise to such closed and bound orbits and 
obtained an upper limit on $m_g < 1.1 \times 10^{-29}$ eV. We 
call it a rough estimate because neither any statistically 
significant confidence limit had been defined on the 
upper limit nor any details about the complicated 
dynamics of galaxy cluster had been taken into account.
The assumption that only a Newtonian potential can 
give bound and close orbits has been invalidated in 
several recent works [65, 67]. Desai (2017) has made 
an effort to overcome these limitations of GN74 and 
 improve the bounds on graviton mass from a galaxy 
cluster [45]. He used a single galaxy cluster Abell 1689 in order to compare the acceleration profile of 
the galaxy cluster calculated under both Newtonian as 
well as Yukawa framework. To find the mass enclosed 
within the galaxy cluster at any particular radius, 
different mass distribution profiles of Dark matter, 
baryonic matter and galaxy distribution have been 
used [68, 70, 74]. However both of these analysis use 
only a single galaxy cluster which also include many 
features of the models. We have made an effort to use 
the complete catalog of galaxy clusters and to obtain 
significantly stronger bounds on the mass of graviton.

In this work we also extended the same approach fur-
ther and use the difference between the Newtonian and 
Yukawa acceleration profile to put bounds on graviton 
mass by using the mass estimates of galaxy clusters 
obtained by using weak lensing and SZ effect. However, 
it is important to emphasize that this analysis is not 
completely independent of any cosmological assumption. 
This is because these mass estimates are obtained by 
using the NFW and generalized NFW density profiles 
of dark matter halos. These density profiles are the 
outcome of N-body simulations of structure formation 
which are performed under a Newtonian framework. 
The ideal way out to overcome this limitation would 
be to run the N-body simulations in Yukawa gravity 
and refit all scaling relations and obtain separate mass 
estimates of galaxy clusters in the Yukawa gravity 
framework. But the N-body simulations in Yukawa 
gravity need the graviton mass as a prior input or 
else one would get the output as a function of the 
graviton mass. Hence it may not be much of a help 
in constraining the graviton mass in a self consistent 
way. Running such simulations at cluster scales is 
complicated since to use the Yukawa potential, we have 
to deal with multiple model parameters and density 
profiles which will further make the process highly 
sensitive to multiple parameters. We would like to 
add that in this work we have followed previous work 
where alternative gravity models (modified gravity 
models [71, 73], chameleon gravity [75, 76], Galileon 
gravity [77] etc.) have been tested using galaxy cluster 
observations with a similar dependence on dark matter 
halo density profiles obtained from N-body simulations.

The main conclusion of our work is as follows:

• Instead of using a single galaxy cluster, (as used by 
Desai (2017) and GN74), we use the presently available 
observational catalogs of mass measurements of galaxy 
clusters at different cosmologically defined radii. The use 
of the complete catalogue including hundreds of indepen-
dent observations averages out any possible random error 
contribution, which is not possible when using a single 
cluster.

• We write down the acceleration profile under New-
tonian and Yukawa gravity, in terms of the cluster mass 
and Hubble parameter. The mass measurements of clus-
ters are obtained through weak lensing and SZ effect 
scaling relations of galaxy clusters. These mass estimates are 
more reliable because the weak lensing and SZ effect of
Various bounds on graviton mass

| Hypothesis          | Method                                                                 | \( m_g \) (in eV) |
|---------------------|------------------------------------------------------------------------|-------------------|
| Yukawa potential    | 1σ bound from weak lensing power spectrum at \( z=1.2 \) [44]          | \( 6.00 \times 10^{-23} \) |
|                     | Using Holmberg galaxy cluster by assuming scale size around 580 kpc [42] | \( 1.10 \times 10^{-29} \) |
|                     | 1.64σ (90%) bound from galaxy cluster Abell 1689 [45]                  | \( 1.37 \times 10^{-29} \) |
|                     | 2σ bound from the precession of Mercury [78]                          | \( 7.20 \times 10^{-23} \) |
|                     | 1.64σ (90%) bound using trajectories of S2 stars near the galactic center [46] | \( 2.91 \times 10^{-21} \) |
|                     | 1σ bound from \( M_{WL}^{500} \) mass estimate of 50 galaxy cluster (This work) | \( 5.90 \times 10^{-30} \) |
|                     | 1σ bound from \( M_{SZ}^{500} \) mass estimate of 182 galaxy cluster (This work) | \( 8.31 \times 10^{-30} \) |
| Dispersion Relation | 90% upper limit from GW150914 [35]                                     | \( 1.20 \times 10^{-22} \) |
|                     | 90% upper limit from binary pulsar observations [36]                  | \( 7.60 \times 10^{-20} \) |
|                     | 90% upper limit from GW170104 [37]                                    | \( 7.70 \times 10^{-23} \) |
|                     | By studying the impacts of a \( m_g \) on the B-mode polarization of CMB [79] | \( \sim 9.7 \times 10^{-33} \) |

Table 2: Some robust bounds on graviton mass \( m_g \) in eV obtained by using the phenomenological implications of massive gravity theories. For some more interesting works and detailed review see [32, 38, 46, 80, 81].

galaxy clusters provide direct observational results with least input assumptions about cluster dynamics. Moreover, no modelling or assumptions are required about the baryonic mass profile and galaxy distribution within the galaxy cluster in weak lensing studies of clusters, as required for instance in Desai’s work. However, it would not be quite correct to claim that this analysis is completely model independent because the NFW mass profile for dark matter halo has been used to derive the scaling relations.

- We also need the model independent values of the Hubble parameter \( H(z) \) corresponding to each cluster. To find out \( H(z) \) at the redshift of each cluster, we smoothen the observational dataset of \( H(z) \) by using a model independent non-parametric smoothing technique, Gaussian process [see Fig. 3]. One point to be noted here is that the Hubble parameter enters the expression for the critical density \( \rho_{cr} \) of the universe, which is an outcome of a flat FLRW universe. But in order to calculate the value of \( H(z) \), we don’t use \( \Lambda CDM \) or any other cosmological model.

- The Yukawa potential decreases rather quickly in comparison to the Newtonian potential. For a given mass of the graviton of the order \( m_g \sim 10^{-29} \) eV (i.e. \( \lambda_g \sim 4 \) Mpc), the fractional difference between the Yukawa and Newtonian acceleration is approximately \( 10^{-14} \) at a length scale of 1 pc, \( \sim 10^{-10} \) at a length scale of 100 pc, \( \sim 10^{-6} \) at a length scale of 10 kpc and of the order of \( 10^{-2} \) at a length scale of 1 Mpc. Hence, it becomes important to extend this analysis up to very large (Mpc) length scales to observe a significant difference between these potentials. In this work, the extended catalogs of galaxy clusters gives us an opportunity to probe the graviton mass by using the length scale beyond 2 Mpc, where the fractional difference between both potentials widens up to 15% and beyond. This is a significant improvement as compared to earlier studies.

In Table 1, we mention all possible bounds obtained in this analysis with different confidence levels (i.e. 68.3%, 90.0% 95.5% and 99.7% corresponding to 1σ, 1.64σ, 2σ and 3σ). In Table 2, we summarize the bounds obtained on \( m_g \) using phenomenological approach. We finally conclude that:

- In this work, the strongest bound on graviton mass is obtained by using the catalog containing 50 independent mass measurement of galaxy cluster \( M_{WL}^{200} \) from weak lensing at a radial distance \( R_{500} \). It is \( m_g < 5.9 \times 10^{-30} \) eV with the corresponding Compton length \( \lambda_g > 6.822 \) Mpc within 1σ confidence interval. The bound on \( m_g \) from the catalog having 182 mass measurements of galaxy clusters \( M_{500} \) is \( m_g < 8.307 \times 10^{-30} \) eV or \( \lambda_g > 5.012 \) Mpc.

- The bound on graviton mass estimated by using the \( M_{WL}^{500} \) and \( M_{SZ}^{500} \) are almost similar which seems to indicate some degree of consistency in the mass measurement methods (WL & SZ) of galaxy clusters.

With the ongoing and future surveys, our understanding of mass distribution in large scale structures like galaxies, clusters, super-clusters and filaments will improve and more reliable and precise bounds can be obtained using a similar method.

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