The Ameyalli-Rule: Logical Universality in a 2D Cellular Automaton

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Dedicated to the memory of John Horton Conway
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Abstract
We present a new spontaneously emergent glider-gun in a 2D Cellular Automaton and build the logical gates NOT, AND and OR required for logical universality. The Ameyalli-rule is not based on survival/birth logic but depends on 102 isotropic neighborhood groups making an iso-rule, which can drive an interactive input-frequency histogram for visualising iso-group activity and dependent functions for filtering and mutation. Neutral inputs relative to logical gates are identified which provide an idealized striped-down form of the iso-rule.

keywords: cellular automata, iso-rule, glider-gun, logical gates, universality

1 Introduction

The Ameyalli-rule[1] is a new result that continues our search for 2D Cellular Automata (CA) with glider-guns and eaters capable of logical universality. Since the publication of Conway’s Game-of-Life[5] with its survival/birth s23/b3 logic, many other “Life-Like” survival/birth combinations have been examined[3] but none seem to have come close[11] to achieving the complexity of behaviour of the Game-of-Life itself.

To study the basic principles of CA universal computation in a more general context, glider-guns and logical gates have been demonstrated outside survival/birth “Life-Like” constraints, but still within isotropic rule-space where all possible flips/spins of a neighborhood pattern give the same input. Isotropic
rules are preferable to enact logical universality because their dynamics have no directional bias and computational machinery operates in any orientation. Examples include the 3-value 7-neighbour hexagonal Spiral-rule[1], and for a binary 2d Moore neighborhood, the Sapin-rule[13], and four rules in our own published results demonstrating logical universality — the first was the anisotropic X-Rule[7], followed by the isotropic Precursor-rule[8], the Sayab-rule[9] and the Variant-rule[10].

These rules were found from a short-list[7,8] within an input-entropy scatter-plot[17,18] sample of 93000+ isotropic rules, which classify rule-space between order, chaos and complexity. The input-entropy criteria in this sample follow “Life-Like” constraints to the extent that the rules are binary, isotropic, with a $3 \times 3$ Moore neighborhood — $\square$ — and with the $\lambda$ parameter, the density of $1$s in the full 512 rule-table, similar to the Game-of-Life where $\lambda = 0.273$. The short-list consists of rules that feature emergent gliders within the ordered zone of the plot. The Ameyalli-rule was found from the same short-list. Its 2-phase orthogonal emergent glider — one type found to date — is shown in figure 2. Its glider-gun (figures 3 and 4) also emerges spontaneously from a random initial state in a similar way to the Sapin-rule[13], the Sayab-rule[9], and the Spiral-rule[1], but with a lower probability, whereas the glider-guns for the other rules listed above, including the Game-of-Life, require careful construction.

The Ameyalli-rule is most efficiently defined and mutated as a 102-bit iso-rule[19], where any mutation conserves isotropy. An in-depth definition of isotropic rules, iso-rules based on iso-groups, including alternative notations — the full rule-table, symmetry classes, iso-rule prototypes, and the iso-rule in hexadecimal — are presented in section 5. These notations relate to DDLab[20] and can be redefined for Golly[6]. Figure 1 shows the Ameyalli iso-rule, and the mutations that were made to create the eaters A and B.

![Figure 1: The Ameyalli 102-bit iso-rule shown as a DDLab graphic, indexed from 101-0 (left-right) — the positions of mutants from the rule originally found are indicated below the graphic in blue. For eater-A, indices 43 and 36 were flipped from 1 to 0. For eater-B, indices 78 and 67 were flipped from 0 to 1. The iso-rule expressed in hexadecimal is 26 42 a3 a9 08 8a 44 90 00 0b 54 50 90.](image)

It turn out that a large proportion, 60/102, of Ameyalli iso-rule inputs are neutral with respect its glider-gun/eater system (section 6.1) so this system can be equivalently generated by a vast family of mutant iso-rules. The identities of the Ameyalli and other logically universal rules are centered on their glider-gun/eater systems, which is best represented by a stripped-down “idealized” version of the iso-rule with all neutral inputs set to zero.

The paper is organised into the following sections: (2) describes the glider-gun, gliders, eaters, and collisions, (3) outlines logical universality, (4) demonstrates the logical gates, (5) defines the iso-rule, (6) gives methods for iso-rule activity by the input-frequency histogram (IFH), for filtering and mutation, and for idealising the iso-rule, and (7) is a summary and discussion of the issues.
2 The glider-gun, gliders, eaters, and collisions

The essential ingredients for a recipe to create CA logical universality are gliders, glider-guns, eaters, and appropriate collisions.

A glider is a special kind of oscillator, a mobile pattern that recovers its
Figure 4: The Ameyalli glider-gun. Left: a snapshot with green trails indicating motion. Right: 274 time-step isometric with IFH colors (section 5). 2-phase orthogonal gliders are shot in 4 directions North/South/East/West with period 22 and speed of $c/2$, where $c$ is the speed of light. The gliders spacing in the glider-stream is 11 cells. In this example North and West glider streams are stopped by the stationary eater B, East and South by the 2-period eater A.

form but in a displaced position, thus moving at a given velocity. A rule with the ability to support a glider, together with a stable eater, and a diversity of interactions between gliders and eaters, provides the first hint of potential universality.

Although many rules can be found with these properties, the really essential and most elusive ingredient is a glider-gun, a dynamic structure that ejects gliders periodically into space. A glider-gun can also be seen as an oscillator that adds to its form periodically to shed gliders. It can be regarded as a periodic attractor[17], especially if confined by eaters as in figure 3.

In the case of the Ameyalli-rule, as already noted, a glider-gun may emerge spontaneously from a random pattern — only one type has been observed so far. In other cases a glider-gun can sometimes be constructed from smaller components, as for the very first glider-gun created by Gosper for the Game-of-Life. These glider-guns are such elaborate structures that the probability of their spontaneous emergence is negligible.

Another essential requirement is that gliders colliding sideways can self-destruct leaving no residue, and that the resulting gap in the glider-gun-stream is sufficiently wide to allow a following glider to pass through. The Ameyalli-rule satisfies all these requirements as illustrated in figures 4 to 9.
Figure 5: There are two types of eater, type-A oscillates with period 2, type-B is stationary. We illustrate above how a glider self-destructs when colliding with an eater. If the collision dynamics are exactly coordinated in time and space, the eater will re-establish its original configuration, and will be able to destroy the next glider in a glider-gun-stream. Green trails indicate motion.

Figure 6: A sideways glider collisions between two gliders approaching at 90° can be arranged so that the gliders self-destruct leaving no residue. The resulting gap in the glider-gun-stream is wide enough for the following glider to pass through. Green trails indicate motion.

3 Logical Universality

Traditionally the proof for universality in CA is based on the Turing Machine or an equivalent mechanism, but in another approach by Conway, a CA is universal in the full sense if it is capable of the following,

1. Data storage or memory.
2. Data transmission requiring wires and an internal clock.
3. Data processing requiring a universal set of logic gates NOT, AND, and OR, to satisfy negation, conjunction and disjunction.

This paper is confined to proving condition 3 only — for universality in the logical sense. To demonstrate universality in the full sense as for the Game-of-Life, it would be necessary to also prove conditions 1 and 2, or to prove universality in terms of the Turing Machine, as was done by Randall for the Game-of-Life.
4 Logical Gates

Logical universality in the Ameyalli-rule, as in the Game-of-Life, is based on Post’s Functional Completeness Theorem (FCT)\[4\]. This theorem guarantees that it is possible to construct a conjunctive (or disjunctive) normal form formula using only the logical gates NOT, AND and OR.

Using a specific right-angle collision, two gliders can self-destruct leaving no residue as shown in figure\[6\]. Applying this property between glider-gun streams and a glider/gap sequence with the correct spacing and phases representing a “string” of information, its possible to build the logical gates NOT, AND and OR, illustrated in figures\[7\],\[8\] and\[9\]. Gaps in a string are indicated by grey circles, dynamic trails are included, and B-type eaters are positioned to eventually stop gliders. Note that the leading bit of a moving glider/gap information sequence appears on the left in its string representation.

4.1 NOT gate

Figure 7: An example of a NOT gate: \((\neg 1, 1 \rightarrow 0\) and \(\neg 0, 0 \rightarrow 1)\) or inverter, which transforms a stream of data to its complement, represented by gliders and gaps (grey discs). \textit{Left:} The 5-bit input string A (10001) moving East is about to interact with a glider-stream moving North. \textit{Right:} The outcome is NOT-A (01110) moving North, shown after about 133 time-steps.
4.2 AND gate

Figure 8: An example of the AND gate \((1 \land 1 \rightarrow 1, \text{else} \rightarrow 0)\) making a conjunction between two streams of data, represented by gliders and gaps (grey discs). 

*Left:* The 5-bit input strings A (10001) and B (10100) both moving East are about to interact with a glider-stream moving North. *Right:* The outcome is A-AND-B (10000) moving East shown after about 197 time-steps.

The dynamics making this AND gate first makes an intermediate NOT-A (North 01110 – figure 7) which interacts with input B to simultaneously produce both A-AND-B (East 10000), and the A-NOR-B (North 01010) which will be required to make the OR gate in figure 9.
4.3 OR gate

Figure 9: An example of the OR gate (1 ∨ 1 → 1, else → 0) making a disjunction between two stream of data represented by two streams of gliders and gaps (grey discs). Top: The 5-bit input strings A (10001) and B (10100) both moving East are about to interact with two glider-streams, the lower shooting North, and the upper shooting East. Below: The outcome is A-OR-B (10101) moving East shown after 220 time-steps.

The dynamics making this OR gate first makes an intermediate NOT-A (North 01110 – figure 7) which interacts with input B to make A-NOR-B (North 01010 – figure 8) which interacts with the upper glider-stream shooting East to make A-OR-B (East 10101). A residual bi-product is A-AND-B (East 10000 – figure 8).
5 The Ameyalli-Rule definition

An isotropic CA rule based on a 2d binary $3 \times 3$ Moore neighborhood — — can be defined by a series of methods that become ever simpler, clearer, and more concise, illustrated in figures 10, 11, and 12. In all these methods, a descending order (from left to right) of the neighborhood’s decimal equivalent is employed, in line with Wolfram’s classic convention — rule-tables can then be expressed in decimal or hexadecimal.

The decimal equivalent of a $3 \times 3$ pattern is taken as a string in the order — for example, the pattern is the binary string $001110111$ (119 in decimal), representing the full rule-table index 119 (figure 10), the symmetry class 119 (figure 11), and iso-rule index 66 (figure 12).

5.1 full lookup-table

- Neighborhood patterns in descending decimal equivalent order from to . (511 to 0) if each row from the upper matrix is continued in the lower matrix. Ameyalli-rule neighborhoods that input 1 are colored red.

- The Ameyalli-rule showing just inputs follows the upper and lower matrices above, set alongside each other giving a continuous descending decimal equivalent order 511 to 0. This is the full rule-table (rcode). 154 inputs=1 are colored black. $\lambda=154/512=0.3$.

Figure 10: (a) The Ameyalli-rule showing details of all neighborhood patterns, and (b) the full rule-table, rcode in DDLab. To restrict the table to isotropic rules only, algorithms ensure that all related neighborhoods by spins/flips give the same input. In these presentations the diagonal symmetry of each $8 \times 8$ block is a necessary but insufficient indication of isotropy.
5.2 symmetry class table — iso-groups

Figure 11: The symmetry class table (equivalent to iso-groups in figure 12) lists all 102 prototype neighborhoods together with their group of spins/flips listed in sequence — here in 3 columns. The prototype is the smallest decimal equivalent and serves as the symmetry class index which is discontinuous between 511 and 0. Patterns within each symmetry class are ordered from lowest (left) to highest decimal equivalent — opposite to the usual order. This convention was employed in [7, 8, 9, 10] where only symmetry classes with inputs 1 are usually listed, but here we show all 102. Input 1 neighborhoods are colored red. The Ameyalli-Rule is defined by the 31 red symmetry classes which input 1, the rest input 0. Numbered labels (101:511 to 0:0) show the iso-rule index 101 to 0, followed by the symmetry class index.
5.3 iso-rule

The iso-rule\cite{19,18} depends on just the 102 prototype neighborhoods in figure 12(a) with a rule-table index 101-0, following the symmetry class table in figure 11 but with the rest of the group taken as read and omitted. The iso-rule shown in figure 12(b) is then just a 102 length bit-string (indexed 101-0) listing prototype inputs, 1 or 0.

Besides simplicity and brevity, an advantage of the iso-rule is that any mutation conserves isotropy. The iso-rule has many other advantages\cite{19,18} when studying isotropic CA as compared to a full or symmetry class rule-table. The iso-rule provides a more direct entropy scatter plot for automatically classifying isotropic rule-space, and as shown in section 6, it provides a concise input-frequency histogram (IFH) with its functions of mutation and filtering.

![Figure 12: (a) The 102 iso-group prototypes indexed 101 to 0. The prototype order is from highest decimal equivalent (511 top-left) to lowest (0 bottom-right), which is also the left to right order of an iso-rule, indexed 101 to 0, the simplest expression of an isotropic rule. The Ameyalli is defined by the 31 red prototypes which input 1, the rest input 0.](image)

(b) The Ameyalli iso-rule compacts the information in (b) into a simple bit-string, here shown as a DDLab graphic with 1/0 inputs shown as black/green. The iso-rule is expressed in hexadecimal (iso-hex)\textasciitilde 2642a3a9088a4490000b545090 by breaking the bit-string into 4-bit segments (with a complete segment on the right). For example, the hexadecimals of the 7 segments on the right \ldots b545090 are given by \ldots 1011\textunderscore b, 0101\textunderscore 5, 0100\textunderscore 4, 0101\textunderscore 5, 0000\textunderscore 0, 1001\textunderscore 9, 0000\textunderscore 0.

The Hensel string translated for Golly\cite{6}, generated automatically in DDLab\cite{20,18} is B2ci3ar4krtz5cq6c7ce/S01e2ek3qj4kt5ceayq6cki7c8.

\footnote{The iso-rule method applies in general to a variety of CA neighborhood templates, in 1d, hex/square-2d, and 3d, including multi-value as well as binary\cite{19,18}.}
6  iso-rule input-frequency-histogram (IFH)

The input frequency histogram (IFH)\textsuperscript{[17,19]}, a method in DDLab\textsuperscript{[20,18]}, allows a dynamic view of the activity of a rule-table in relation to the current iterating space-time pattern, with options to interactively filter and mutate the current rule. Any isolated periodic pattern can be investigated to ascertain which rule-table inputs\textsuperscript{3} are responsible for maintaining the pattern and to what extent.

Four Ameyalli gliders moving East. \textit{far left}: colors by value with green dynamic trails. \textit{near left}: colors corresponding to IFH colors.

Figure 13: The IFH for the Ameyalli glider, with 15 active bars.

For example, in figure 13 the space-time pattern consists of four non-stop Ameyalli gliders on a lattice with periodic boundaries. The IFH accommodates 102 columns to indicate the level of activity\textsuperscript{4} of each iso-group — the frequency of iso-rule inputs, indexed from 101 to 0 (left to right) as in figure 12. Only 15 bars appear showing which inputs are active and to what extent, noting that the bar height is log\textsubscript{2} of the actual frequency to amplify infrequent hits. Mutating the iso-rule at any of these bar indices disrupts the gliders, but mutating at any other index makes no difference to the gliders themselves, but would very probably disrupt any other Ameyalli pattern. We call such inactive inputs “neutral” relative to the particular space-time pattern.

In figure 14 the (periodic) space-time pattern is the Ameyalli glider-gun system contained by eaters-B, similar to figure 3. In this case 32 iso-groups are active, the rest are neutral indicated by missing columns. In DDLab, on-the-fly key-hits will progressively filter active columns from high to low, marked by black blocks. Filtering excludes drawing these cells in the space-time pattern, or alternatively just marks the neutral bars. Other key-hits progressively mutate the iso-rule at column positions from low to high, marked by black blocks.

\textsuperscript{3}The IFH applies to any type of rule-table, full and totalistic as well as iso-rules.

\textsuperscript{4}In figures 13,14,15 the measures are averaged over a moving window of 100 time-steps to stabilise the IFH. The number of trailing hits \(h_i\) (lookups at index \(i\) of the iso-rule-table) at each time-step is recorded, and converted to \(f_i\), the fraction (0 to 1) of all possible hits \(h_i/(n \times w \times S)\), where \(n\) is lattice size, \(w\) (100) is the size of the moving window of time-steps, and \(S\) (102) is the iso-rule size. \(f_i\) is converted to log\textsubscript{2} to amplify infrequent hits, which sets the column height.
The Ameyalli-rule glider-gun contained by type-B eaters as in figure 3.

far left: colors by value with green dynamic trails.
near left: colors corresponding to IFH colors.

Figure 14: The log$_2$ iso-rule IFH of the Ameyalli glider-gun/eater-B system. All 32 active bars were firstly filtered (black blocks), then all 70 neutral inputs were mutated (red blocks) — the dynamics were not affected. Below the IFH the corresponding iso-rule tables are shown, firstly the original Ameyalli-rule, and below that the mutated rule. The position of entries for eater A and B are indicated, noting that there is no activity for the absent eater A.

The iso-rules in hexadecimal hex are compared below:

- Ameyalli-rule
  26 42 a3 a9 08 8a 44 90 00 0b 54 50 90
- after all 70 inactive mutations
  19 bd 5d 56 af 65 bb 7c 1a 37 fd 54 90

so neutral inputs are preferentially mutated, either by flipping the input, or setting zero. In this way its possible to mutate any or all neutral inputs, and as expected, experiment confirms there is no effect on the given glider-gun system.

6.1 The idealized iso-rule

Here we construct the IFH compatible with the logical gates in section 4 in order to derive a stripped down “idealized” form of the logically universal Ameyalli iso-rule. Employing the logical gates themselves is impractical because their dynamics is non-periodic. Instead, in figure 15 we construct an “intensive” periodic space-time pattern that includes all the dynamical structures that make the logical gates, with two interacting glider-guns to make mutually destructive 90° glider collisions, as well as gliders destroyed by collisions with both eaters A and B. This results in 47 active bars, but the majority are absent representing neutral inputs. Any or all mutations to these neutral inputs has no affect on the “intensive” space-time pattern, and when tested, the logical gates in section 4 are preserved. We suggest that if all these neutral inputs are set to zero, the resulting idealized iso-rule will form an appropriate starting point for further Ameyalli studies.
An intensive Ameyalli periodic space-time pattern

*far left*: colors by value with green dynamic trails.

*near left*: colors corresponding to IFH colors.

Figure 15: The log₂ iso-rule IFH of the Ameyalli-rule driven by an “intensive” periodic space-time pattern which includes all structures making logical gates. All 47 active bars were firstly filtered (black blocks), then all 53 neutral inputs were mutated to zero (red blocks), turning the Ameyalli-rule into its idealized form. The IFH and the intensive space-time pattern, and logical gates, are preserved, though other dynamics would be drastically altered. Below the IFH the corresponding iso-rule tables are shown, firstly the original Ameyalli with 31 inputs of 1, and below that the idealized Ameyalli with only 12 inputs of 1.

The iso-rules in hexadecimal hex are compared below:

26 42 a3 a9 08 8a 44 90 00 0b 54 50 90 — Ameyalli iso-rule
00 00 01 00 08 00 00 10 00 03 54 50 90 — idealized Ameyalli iso-rule

The idealized Hensel string translated for Golly[6] is B2ci3ar5q/S01e2ek3qj4t5y

7 Summary and Discussion

The Ameyalli iso-rule is another example of the search for glider-guns, then building logical gates. Its spontaneously emergent glider-gun was found from the input-entropy scatter-plot samples that favoured both order and emergent gliders. Minor mutations created two types of eater to stop the glider stream. With these ingredients, and by adjusting collision dynamics according to precise timing and points of impact, we were able to build the logical gates NOT, AND and OR required for logical universality.

Further mutations would possibly uncover other artefacts of interest, but for a better appreciation of the causal links between these significant dynamical patterns and the responsible iso-groups inputs, we applied the input-frequency histogram (IFH) method, which also revealed neutral inputs where mutations have no effect. As we have shown for the Ameyalli, setting neutral inputs to zero relative to an intensive glider-gun/eater system will reduce logically universal
iso-rules to their stripped down or “idealized” form where gliders, eaters, glider-guns, and logical gates continue to be supported. The essential identity of Ameyalli, and other logically universal CA, are these dynamical objects together with the significant part of the iso-rule table that drives them. However, the possibility is there to configure neutral inputs to create other unpredicted but relevant structures.

An idealized logically universal iso-rule is the primitive of its huge family of mutants that perform the same basic functions. The dynamics of idealized versions of all the logically universal iso-rules mentioned in this paper, including the game-of-Life, are worth investigating in further work.

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