Effect of Color Screening on Heavy Quarkonia Regge Trajectories

M.M. Brisudová,* L. Burakovsky† and T. Goldman‡

Theoretical Division, MS B283
Los Alamos National Laboratory
Los Alamos, NM 87545, USA

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Abstract

Using an unquenched lattice potential to calculate the spectrum of a bottomonium-like system, we demonstrate numerically that the effect of pair creation is to produce termination of the real part of hadronic Regge trajectories, in contrast to the Veneziano model and the vast majority of phenomenological generalizations. Termination of the real part of Regge trajectories may have significant experimental consequences.

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It is well known that hadrons populate near-linear Regge trajectories; i.e., the orbital momentum ℓ of the state is proportional to the square of its mass: ℓ = α'M²(ℓ) + α(0), where the slope α' depends weakly (for light (u, d, s) quarks) on the flavor content of the states lying on the corresponding trajectory. This is based on the knowledge of the lowest
lying states. What happens, however, for highly excited states remains model-dependent. In the Veneziano model for scattering amplitudes [1] there are infinitely many excitations populating linear Regge trajectories. The same picture of Regge trajectories arises from a linear confining potential [2], and asymptotically for any potential where the dominant long distance behavior is linear.

In this paper we argue that in QCD, due to the pair creation that screens the potential at large distances, Regge trajectories become nonlinear and their real parts terminate. To illustrate this point, we consider a potential obtained in unquenched lattice QCD in ref. [3], and use it in the Schrödinger equation to calculate spectra of heavy quarkonia, in particular, for a bottomonium-like system, for which the use of the potential, as well as a nonrelativistic calculation, is best justified.

One can argue that an exact treatment of the problem would consist in the analysis of a multichannel potential scattering model which is characterized by the simultaneous presence of, and communication between Hamiltonians for two types of channels: one is the Hamiltonian of ordinary two-particle scattering channels (in our case, the open two-meson decay channels (e.g., $B\bar{B}$),) and the other is the Hamiltonian of permanently confined channels (in our case, the bare $b\bar{b}$ system). The scattering Hamiltonian has an absolutely continuous spectrum on the positive real axis (and perhaps a finite number of negative energy bound states corresponding to meson-meson molecules), and the confining Hamiltonian has only a point spectrum with an accumulation point at infinity. These two types of channels are connected in the full Hamiltonian for the multichannel system by off-diagonal local potentials.

The problem was studied in general, nonrelativistically, by Dashen et al. [4] for non-interacting scattering states. All of their results carry over unchanged for a class of scattering channel potentials [4] which includes a Yukawa-type interaction. They found that the spectrum of the full Hamiltonian consists of 3 parts: a finite number of negative energy eigenvalues, a discrete set of positive energy eigenvalues, and an “absolutely continuous spectrum” on the remainder of the positive real axis. Some of the bound states may be embedded in the continuum. Interestingly enough, in the examples studied, the number of positive energy bound states of the full Hamiltonian is finite, notwithstanding the fact that the number of such states of the unperturbed confining Hamiltonian is infinite, and Dashen et al. expected this to hold in general. Note that adding just one open channel is sufficient for this qualitative behaviour to develop.

With this established, let us concentrate on the potential energy as it varies with separation for the lowest lying channel in the case of a coupling to a single open channel. Proceeding out from zero separation, we expect to find that this energy corresponds to the value of the potential of the confining system at short distances, since the system cannot contain the additional (anti)quarks needed to produce the meson states without a significant energy cost. Conversely, as we proceed inward from infinite separation, we expect an attractive (except for particular quantum numbers) Yukawa potential between the meson-antimeson pair due to the exchange of a light meson between them in the t-channel. As we extend both considerations towards a matching point, we expect to find that the quark potential exceeds the meson one, due to the confining nature of the former. The quark potential dominates up to a separation at which the probability of light quark-
antiquark pair creation becomes significant. Quantum effects smooth the match, but we may still expect the potential to be non-monotonic in the matching region – overshooting the (two meson) threshold energy and then falling back below it again to reach the Yukawa value, as one moves out from the origin. A schematic picture of the 2-channel potential is shown in Figure 1. Details will depend on the particular channel considered.

In fact, data from recent lattice calculations are consistent with this form of the effective potential, although it may be that this only appears to be so due to statistical fluctuations \[5\]. Note that lattice studies include, in principle, a number of open channels.

We will show à posteriori that using a two-body potential which follows the energy of the lowest channel (i.e. a static potential found in an unquenched lattice QCD) in a two-body calculation leads to qualitatively similar results to those of ref. \[4\], and argue that the two approaches should be expected to differ only near the threshold. Note that the coupled-channel calculation would require more parameters than our two-body model where the coupling to open channels is included dynamically (through the lattice calculation).

Our purpose in this calculation is not to reproduce spectra; a better agreement with data could be achieved by adjusting the quark mass and/or fine tuning the parameters of the potential. Rather, we concentrate on general features of the Regge trajectories that arise for heavy quarkonia with a lattice QCD static potential once pair production of light quarks is allowed.

For this purpose, it is sufficient for us to consider only the leading, spin-independent part of the non-relativistic Hamiltonian, and argue that since spin-dependent interactions are at least \(O(1/M^2)\) suppressed for heavy quarks, the general features of the trajectories discussed below will persist in any more careful QCD bound-state calculation.

The bottomonium system is well described in leading order by a nonrelativistic, spin-independent Hamiltonian, viz.

\[
H = -\frac{1}{2m} \nabla^2 + V(r),
\]

where \(m = M/2\) is the reduced mass, with \(M = 5.2\) GeV, and \(V(r)\) denotes a potential. Relativistic corrections in this case can be expected to be on the order of 10\% \[13\]. To make our calculation represent QCD as closely as possible, we choose a potential that has been obtained in unquenched lattice QCD calculations for infinitely heavy sources \[3\]. One can expect that corrections to the potential due to the bottom mass being finite are small, of order \(O(\Lambda_{QCD}/M)\) (see \[14\] and references therein).

The screened static potential fitted to results of lattice calculations is \[3\]:

\[
V(r) = \left(-\frac{\alpha}{r} + \sigma r\right) \frac{1 - e^{-\mu r}}{\mu r},
\]

where \(\mu^{-1} = (0.9 \pm 0.2)\) fm = \(4.56 \pm 1.01\) GeV\(^{-1}\), \(\sqrt{\sigma} = 400\) MeV and \(\alpha = 0.21 \pm 0.01\). We obtain bottomonium spectra by diagonalizing the Hamiltonian (1) with the potential (2).

Before we proceed with the presentation of our results, a few comments regarding the lattice potential (2) are called for. First, the lattice screened potential in the analytic form
(2) does not have the asymptotic Yukawa approach to the screened constant that should occur as we argued above. Nevertheless, analytic results for massless quarks \[13\], which bracket Yukawa behavior, strongly suggest that results qualitatively \textit{and} quantitatively similar to those presented below hold also in that more physical case. Second, the potential (2) is monotonic, unlike the qualitative picture shown schematically in Fig. 1. The non-monotonic potential produces states which mix strongly with scattering states, thus giving rise to resonances. This does not occur in the Hamiltonian with a monotonic potential (in the lowest order bound state perturbation theory). However, the drop of the potential value in the matched region can be expected to be at most tens of MeV (based on a typical strength of Yukawa potentials, and also consistent with the latest lattice results \[5\]). This means that the only states (not likely more than one or two) which could be significantly affected would be those that happen to lie in the tens-of-MeV-wide band around the 2 B threshold. We conclude that the potential (2) is a sufficiently good approximation to the full coupled-channel problem for our purposes.

Our results are presented in Figs. 2-4. In Fig. 2 we show the parent trajectory for the minimum, average and maximum screening \(\mu\) extracted from the lattice study \[3\]. All three trajectories clearly indicate a discontinuity in slope when the mass of the bound state reaches the ionization level \((2M_b+\sigma/\mu)\). We identify the states above the discontinuity as scattering states. Note that the maximum \(\ell\) rapidly decreases with increasing screening \(\mu\). Daughter trajectories exhibit the same behavior.

In view of the comments regarding the potential (2), we argue that while we may not have precisely identified the trajectory termination point, it certainly occurs within a few states of the one below our model threshold. For example, in Fig. 4, the fourth daughter trajectory \(\ell = 0\) state (corresponding crudely to the \(\Upsilon(4S)\) state) may or may not be above threshold. Alternatively, there may be one or two states on succeeding daughter trajectories (not shown) which should also be identified as resonances with limited widths. In either case, the detail is not significant to the point we wish to make here: That the calculated trajectories (parent and daughter) have nonzero curvature, and that the trajectories have a limiting value of \(\ell\) (as well as \(M^2\)) beyond which no reasonably definable states exist.

The maximum \(\ell\) (see also Fig. 2 and Fig. 3) is consistent with

\[
L_{\text{max}} \simeq \frac{2}{\varepsilon} \sqrt{\frac{\sigma M_b}{\mu^3} - 1 - n} \geq 0,
\]

(3)

(where \(n = 0\) for the parent and \(1 \leq n \leq L_{\text{max}}(0)\) for the daughter trajectories), which has been derived in \[15\] for a potential similar (but not identical) to (2), viz.

\[
V(r) = a + \frac{\sigma}{\mu} (1 - e^{-\mu r}).
\]

Elsewhere, “square-root” Regge trajectories \[1\] have been studied as solutions to phenomenologically supported requirements of additivity of inverse slopes and intercepts \[11\].

\[1\] In practice, since for each hadronic Regge trajectories at most only three lowest lying states are known \[16\], the observed Regge trajectories can be well approximated by the well-known linear form. Nevertheless, some experiments indicate a nonzero curvature, for example, the nucleon trajectory \[15\].
which are also consistent with the heavy quark limit. In view of this success, it is interesting to see how much the QCD model trajectories found here deviate from the phenomenologically plausible form. In Fig. 4 we plot the parent trajectory for $\mu = 0.18$ GeV, compared to a “square-root” trajectory that has the same slope at the termination point.

$$\alpha(t) = \alpha(0) + \gamma \left[ \sqrt{T} - \sqrt{T - t} \right].$$  \hspace{1cm} (4)

where $\alpha(t) = \ell$ is the Regge trajectory, $\alpha(0)$ is the intercept of the trajectory, $T$ is its termination point (note that we allow $T$ to differ from the ionization threshold of the potential (2)), and $\gamma$ is the universal asymptotic slope (i.e. $\alpha(t) \sim \gamma(-t)^\nu$, $|t| \to \infty$).

Since the masses of the bound states with the lattice potential (2) lie very close to (i.e. in our case, within 1% of) the “square-root” trajectory, all of the phenomenology of “square-root” trajectories, such as additivity of inverse slopes and intercepts, is applicable to these QCD model bound state trajectories.

Note that the parameters of the “square-root” form are free parameters, a priori unknown, and must be fitted to the results of any bound-state calculation. Therefore, even though the calculation presented here should be expected to have uncertainties of up to 30%, due to the nonrelativistic approximation to the kinetic energy and due to the use of a static potential, it remains highly plausible that agreement of the exact QCD result with the phenomenologically supported “square-root” form can also be achieved (with similar parameter values).

As suggested by Eq. (3), $L_{\text{max}}$ should decrease by one unit for each consecutive daughter trajectory. Fig. 3 shows the parent and daughter trajectories for $\mu = 0.28$ GeV, and $L_{\text{max}}(n)$ given by Eq. (3) for each of the trajectories. In this case, according to (3), there should be only three daughter trajectories, the last of which consists of just one, $\ell = 0$, state.

Our numerical results are in agreement with Eq. (3). We observe that $L_{\text{max}}$ decreases by roughly one unit; there are two daughter trajectories with at least two bound states, and there is indeed one more $\ell = 0$ state near the threshold.

To summarize our findings: Using an unquenched lattice potential (2), we observe that there are a finite number of bound states occupying Regge trajectories of a near-square-root form which terminate in $\ell$, and a finite number of daughter trajectories. Quantitative results (i.e. the value of $L_{\text{max}}$ and consequently, the number of daughter trajectories) are sensitive to the exact value of the screening parameter $\mu$, but the qualitative behavior is the same over the entire range of $\mu$ allowed by [3].

We close these theoretical considerations with a few additional remarks.

First, we would like to emphasize that the existence of the ionization level (which is, in quark terms, an undesirable, but unavoidable feature of the potential under consideration)

Pomeron trajectory [8] and $a_2$ [4]. Also, the straight line connecting $\rho$ and $\rho_3$ gives an intercept smaller than the actual physical value [10] (0.48 vs 0.55). Note in Figs. 2, 3 that the trajectories are indeed approximately linear for $\ell \leq 3$. This can be also seen analytically by the expansion of Eq. (4) for $t \ll T$.

2This corresponds to matching (approximately) the size of the state, in addition to its mass and quantum numbers [12]. The parameters of the “square-root” trajectory matched here are: $\alpha(0) = -22.42$, $T = 128.05$ GeV$^2$, and $\gamma = 2.87$ GeV$^{-1}$.

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does not guarantee termination of Regge trajectories. For example, in QED, even though there exists an ionization level, there are infinitely many daughter trajectories, each with, in principle, infinitely many bound states with integer $\ell$, in contrast to finite $\ell$ in QCD. Since we do not know the solution to the potential (2) in an analytic form, we cannot conclude how many daughter trajectories it produces, but it seems unlikely that the numerical methods would have entirely missed evidence of some singular behavior or accumulation of trajectories in a small region. Hence, it is likely that the number of daughter trajectories is finite also (see [15]).

Second, our results are supported by recent lattice calculations which observe flux tube breaking [16], and the fact that there have been many lattice simulations showing flattening of the potential due to pair production with much smaller errors than the original lattice potential (used in our bound state calculation, see [17] and references therein).

We therefore conclude that, in QCD, the flattening of the potential due to pair creation produces termination of real part of Regge trajectories. This is in agreement with [18] where it is also suggested on different grounds that Regge trajectories must terminate.

Finally, we address some of experimental consequences of this effect, in particular, those regarding production processes, spectroscopy and the proposed quark-gluon plasma.

Our finding that the bottomonium bound states lie within 1% of the “square-root” trajectory may be useful for calculating cross sections for production processes, such as photoproduction, where an explicit form of the Regge trajectory exchanged is required. Moreover, evidence from an analytical calculation for massless quarks, and model-dependent studies [12], strongly suggest that light quarkonia populate near-square-root trajectories as well.

With regard to glueball spectroscopy: If one assumes that the same mechanism which produces termination of Regge trajectories applies to both gluonic bound states and quarkonia, and that they have different thresholds (e.g. due to different color factors), then it is conceivable that there is a range of masses where all $I = 0$ states are pure glue-balls, or, perhaps, with small admixtures of the lowest–lying heavy quarkonia. (A possible exception is the case of mesonic molecules which, however, would be naively expected to have much larger widths than ordinary resonances.)

The finite number of bound states that we find also affects understanding of the QCD phase transition from hadrons to the quark-gluon plasma. In order to discuss the QCD phase transition, both the hadron and the quark-gluon phases must be described by the corresponding equations of state, each of which depends on the corresponding degrees of freedom as functions of temperature. The critical temperature is found as the point where the two curves intersect. However, if the density of states in the hadron phase grows without bound (which does occur for the case of linearly rising trajectories [19]), then at high enough temperatures the hadron phase will be thermodynamically favored over the quark-gluon plasma phase because the effective number of degrees of freedom is constant for the plasma. This implies that the existence of the quark-gluon plasma would be restricted to a limited range of temperatures (if it existed at all), which is clearly unphysical. To be consistent with the concept of the QCD phase transition, therefore, the effective number of degrees of freedom in the hadron phase cannot grow indefinitely.
with temperature. (Of course, QED also has an infinite number of bound states for hydrogen atom, which are truncated by medium effects. This kind of physics could also resolve the QCD problem. What we have presented here is an alternative solution.)

Obviously, our results that show the termination of the real part of trajectories at a certain energy threshold (in the calculation presented here $E_{th} \simeq 2M b + \frac{\sigma}{\mu}$), lead to a finite number of effective degrees of freedom\(^3\) that freeze out at $E = E_{th}$. In this respect, the color screening discussed in this paper may be a manifestation of existence of the free phase of the hadron constituents – quarks and gluons. The idea that color screening may in fact be the only mechanism responsible for deconfinement has been suggested in ref. [20].

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\(^3\)The scattering states, of course, do not contribute to this number.
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Figure 1: A schematic representation of a two-coupled channel potential using parameters similar to those in text. Yukawa and linear extensions are shown.
Figure 2: The parent trajectory for bottomonium with the lattice unquenched potential (2) for various allowed values of $\mu$. Solid vertical lines represent the ionization level, $(2M_b + \sigma/\mu)^2$. 
Figure 3: The parent trajectory for bottomonium ($M_b = 5.2$ GeV) with the lattice unquenched potential (2) for $\mu = 0.18$ GeV, compared to a “square-root” trajectory with approximately the same slope at the threshold. Horizontal lines represent $L_{\text{max}}$ according to Eq. (2), and according to a numerical extrapolation based on the data points shown.
Figure 4: The parent and daughter trajectories for the bottomonium ($M_b = 5.2$ GeV) with the lattice unquenched potential (2) for $\mu = 0.28$ GeV, together with $L_{\text{max}}(n)$ according to Eq. (3). The line connecting the data points is to guide the eye.