Determination of $V_{ub}$ at the $Z$–boson resonance

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Abstract

We show that the observation of energetic charged leptons from semileptonic $b$ decays at the LEP and SLC $Z$–boson factories offers a unique opportunity to measure the quark mixing matrix element $V_{ub}$. We present various distributions of the $b$ decay products to show that the $b \to u$ decays can be cleanly separated from the $b \to c$ decays, with a signal of $O(100)$ events per million $Z$ bosons produced.

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Our knowledge of the CKM or quark mixing matrix elements $V_{cb}$ and $V_{ub}$ comes from studies of the semileptonic decays of $B$ mesons. The larger element, $V_{cb}$, is well determined both from inclusive $B \to X_c l \nu$ decays and from the exclusive $B \to D^* l \nu$ decay. The measurements have been obtained from studies of $B \bar{B}$ production in $e^+e^-$ collisions at $\Upsilon(4S)$ [1,2] and at the $Z$ boson resonance [3]. The results have been reviewed recently by Patterson [4]. Using the inclusive decays it is found

$$|V_{cb}| = 0.039 \pm 0.001 \pm 0.005 \quad [\text{at } \Upsilon(4S)]$$

$$|V_{cb}| = 0.042 \pm 0.002 \pm 0.005 \quad [\text{at } Z]$$

where the first error is experimental and the second is due to the dependence on theoretical models. The observation of the exclusive $B \to D^* l \nu$ decay gives

$$|V_{cb}| = 0.040 \pm 0.002 \pm 0.002$$

where here the theoretical error is smaller since the form factors for the $b \to c$ (‘heavy-quark $\to$ heavy-quark’) transition are well studied and under control [5]. Model dependence only enters at the level of power corrections, which are suppressed by at least a factor of $(\Lambda_{QCD}/m_c)^2$, see ref.[6] for a recent review.

The situation for $V_{ub}$ is very different. The present information comes from $\Upsilon(4S) \to B \bar{B}$. The extraction of $V_{ub}$ is based on the fact that only $b \to u l \nu$ is kinematically able to populate the high end of the inclusive momentum spectrum of the charged lepton $l$. The much more numerous $b \to c l \nu$ decays have $p(l) < 2.5$ GeV. By determining the small excess of events over the off resonance continuum in the short interval $2.5 < p(l) < 3$ GeV it is found that [4]

$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.09 \pm 0.03$$

where the error is almost entirely due to the sensitivity of the analysis to the various theoretical models. Recently it has been proposed that measurements of the
hadronic invariant mass [7,8] or hadronic energy spectrum [9] of inclusive semileptonic $B$ decays may be useful in extracting $V_{ub}$ in a less model dependent manner. In future asymmetric $B$–factories, with vertex detection, these offer alternative ways of identifying $b \to u$ transitions.

Turning now to the exclusive decays, we note that the rates for $B \to \rho l \nu$, $\omega l \nu$ are so low that only the upper limit [1,2,4]

$$|\frac{V_{ub}}{V_{cb}}| < 0.10 - 0.13$$

has been obtained, where the range indicates the model dependence. Even if appreciable statistics for the exclusive channels become available, the theoretical description will be more problematic than $b \to c$ since $b \to u$ is a ‘heavy-quark $\to$ light-quark’ transition.

In this letter we investigate the possibility of measuring $|V_{ub}|$ at $Z$–factories (LEP, SLC) by searching for particular distinctive kinematic characteristics of inclusive $B \to X_u l \nu$ decays. We have

$$\Gamma(Z \to b \bar{b}) \simeq 0.38 \text{ GeV}$$

with a branching fraction of about 15% of $Z$ decays leading to $b \bar{b}$ final states [10]. For the purposes of illustration we will assume that the ‘$b$-quark $\to B$-meson’ fragmentation is described by the Peterson et al. function [11]

$$f(x) = N \frac{x(1-x)^2}{[(1-x)^2 + \epsilon_p x^{2}]^{2}}$$

with $x = |p(B)/p(b)|$. We allow the parameter $\epsilon_p$ to span the range $\epsilon_p = 0.006 \pm 0.003$. We use the ACCMM model [12] to relate the inclusive $B \to X_q l \nu$ decays (with $q = u$ or $c$) to the $b \to q l \nu$ transition. In this model the Fermi motion of the $b$ quark inside $B$ meson is assumed to correspond to a Gaussian momentum
distribution

\[ \phi(p) = \frac{4}{\sqrt{\pi} p_F^3} \exp\left(-\frac{p^2}{p_F^2}\right) \]

where \( p_F \simeq 0.3 \) GeV is widely used in analyses. To investigate the sensitivity to \( p_F \) we also take \( p_F = 0.5 \) GeV, a value found in a relativistic quark model calculation\[13\]. In the ACCMM model we have

\[ d\Gamma_B(p_F, m_{sp}, m_q) = \int_0^{p_{\text{max}}} dp \, p^2 \phi(p) d\Gamma_b(m_b) \]

where \( p_{\text{max}} \) is the maximum kinetically allowed value of \( p = |p| \). We take \( m_c = 1.6 \) GeV for \( b \to c \) decay and \( m_u = 0.15 \) GeV for \( b \to u \), and we ensure the correct hadronic end-point by assuming that the spectator quark mass to be \( m_{sp} = m_d - m_c \) for \( b \to c \) and \( m_{sp} = m_{\pi} - m_u \approx 0 \) GeV for \( b \to u \). To investigate the sensitivity of the results to the choice of \( m_u \) we repeated the calculation with \( m_u = 0.3 \) GeV (keeping \( m_{sp} = 0 \)).

The results are given in Figs. 1–3, where for each kinematic variable (\( x \equiv E(l), E(X) \) and \( p_{\text{TX}}(l) \)) we show the normalized distribution

\[ \frac{1}{\Gamma(Z \to bb)} \frac{d\Gamma}{dx}(Z \to bb [\to (B \to X_q l\nu)]) \]

in units of \( \text{BR}(B \to X_q l\nu)/\text{GeV} \) with \( q = c \) or \( u \) and \( l = e^\pm \). In other words the total integral over each distribution is normalized to unity when no cuts are applied (and GeV units are used). To the right-hand-side of each figure we show the event numbers/GeV for \( 10^6 \) \( Z \) bosons produced. We take \( \text{BR}(B \to X_c e^- \bar{\nu}) = 10.4\% \) \[10\] and we assume \( |V_{ub}/V_{cb}| = 0.1 \). If we include the \( e^+ \) and \( \mu^\pm \) final states then the rate shown is multiplied by a factor of 4. The variables that are most relevant for the extraction of \( V_{ub} \) from \( B \to Xl\nu \) decays are the charged lepton energy \( E(l) \), the energy of the hadronic system \( E(X) \) and the perpendicular momentum \( p_{\text{TX}}(l) \) of the charged lepton with respect to the hadronic jet \( X \). We use the \( Z \) rest frame.
Fig. 1 shows the energy spectrum of the charged lepton, $E(l)$. In Fig. 1(a) no cuts are applied and we see that events with $E(l) \gtrsim 35$ GeV originate dominantly from $b \to u$ transitions. Indeed for $E(l) > 35$ GeV we have a very clean sample $B \to X_u l \nu$ decays corresponding to $\sim 40\ b \to u$ events per $10^6$ $Z$ bosons produced, taking $l = e^\pm, \mu^\pm$.

We can increase the $b \to u$ sample by observing the hadronic jet $X$. In Fig. 1(b) we again show the charged lepton spectrum but now with the hadronic energy cut $E(X) > 5$ GeV and the perpendicular momentum cut $p_T^X(l) > 5$ GeV imposed. Indeed these hadronic cuts have the effect of eliminating $b \to c$ events completely. Even if we conservatively consider only events with $E(l) > 25$ GeV in order to eliminate any cascade events from $b \to c \to ul\nu$, then Fig. 1(b) shows we should have a clean $b \to u$ sample of $\sim 110$ events per $10^6$ $Z$ bosons produced. With these leptonic and hadronic cuts ($E(l) > 25$ GeV and $E(X), p_T^X(l) > 5$ GeV) the momentum flow to the unobservable neutrino is kinematically restricted, so we can use the direction of the other side $B$ (or $\bar{B}$) to determine $p_T^X(l)$ more correctly even with the loss of neutrals.

The effects of the hadronic cuts are shown in Figs. 2 and 3. Fig. 2 illustrates the discriminatory power of the $p_T^X(l)$ distribution in selecting $b \to u$ events. However, this variable can only be employed if the hadronic jet $X$ is identified, as explained above, and we show in Fig. 2 the effect of imposing cuts of $E(l) > 25$ GeV and $E(X) > 5$ GeV. For completeness we show in Fig. 3 the $E(X)$ spectrum with $p_T^X(l) > 5$ GeV and $E(l) > 25$ GeV.

A few remarks are in order. (a) Although the invariant mass of the decay hadrons is the best observable [7] to separate the $b \to u$ from the $b \to c$ events, its precise measurement at $Z$ factories is difficult (even with a vertex detector to identify the decay hadrons) since it requires knowledge of all the momentum components of the decay hadrons, and there could well be neutral particles which escape detection. (b) We explored the possible model dependence by varying the input parameters in the ranges: $\epsilon_p = 0.006 \pm 0.003$, $m_u = 0.15 - 0.3$ GeV and $p_F = \ldots$
0.3 – 0.5 GeV. We also adopted normalized distributions to eliminate theoretical uncertainty from total event rates of $Z \to b\bar{b}$. We found very little difference in the shape of the spectrum of the $b \to u$ decays, as shown in figures. For the $b \to c$ transition, we can see a pronounced difference in shape as we vary the input parameters over the ranges. (c) Recently, Bigi et. al. [14] proposed a model independent decay spectrum using heavy quark effective theory by including nonperturbative $1/m_Q^2$ corrections. They found that their spectrum can be well reproduced by the ACCMM model with $p_F \approx 0.3$ GeV. This model [14] cannot describe properly the end-point region, $\delta E(l) \approx 300$ MeV, of $E(l)$ spectrum, which is the important region for measuring $|V_{ub}|$ at the $\Upsilon(4S)$. At the $Z$ resonance, since we can use at least events with $35 \lesssim E(l) \lesssim 45$ GeV, we can avoid any difficulty in applying this model. (d) There could also be some model dependence from ‘$b$-quark $\to B$-meson’ fragmentation. With the large $b$ production rate at the $Z$ resonance, this is the best place to study the fragmentation effect of the heavy quark.

In conclusion we note that the observation of very energetic charged leptons ($E(l) > 35$ GeV) in $b\bar{b}$ production at $Z$ factories offers a sizeable clean sample of $b \to u$ events, with little or no contamination from $b \to c$ processes. Moreover statistics can be significantly increased by the observation and measurement of the hadronic system accompanying the decay. Particular event topologies are shown which eliminate the $b \to c$ transitions regardless, in principle, of the lepton energy $E(l)$. However, in practice, it will be best to use Monte Carlo studies to optimize both the hadronic and leptonic cuts so as to extract the cleanest possible $b \to u$ data sample.
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Figure Captions

Fig.1 (a) The normalized energy distribution of electron, $1/\Gamma(Z \rightarrow b\bar{b})d\Gamma(Z \rightarrow b\bar{b}[\rightarrow (B \rightarrow X_qe^-\bar{\nu})])/dE(e^-)$ in units of BR($B \rightarrow X_qe^-\bar{\nu}$)/GeV with $q = c$ or $u$. On the right-hand-side of the figure we show the event numbers/GeV for $10^6$ $Z$ bosons produced. We take BR($B \rightarrow X_ce^-\bar{\nu}$) = 10.4% [10] and we assume that $|V_{ub}/V_{cb}| = 0.1$. If we include the $e^+$ and $\mu^\pm$ final states then the rate shown is multiplied by a factor of 4. The narrow band, visible only for $B \rightarrow X_c$, results from varying the input parameters over the ranges $\epsilon_p = 0.006 \pm 0.003$, $m_u = 0.15 - 0.3$ GeV and $p_F = 0.3 - 0.5$ GeV. We take $m_c = 1.6$ GeV. (b) The normalized $E(e^-)$ distribution after the imposition of the cut, $E(X) > 5$ GeV, on the energy of hadronic system and the cut, $p_T^X(e^-) > 5$ GeV, on the perpendicular momentum of electron with respect to the hadronic jet.

Fig.2 The normalized $p_T^X(e^-)$ distribution with cuts $E(e^-) > 25$ GeV and $E(X) > 5$ GeV imposed. The narrow band arises as in Fig.1.

Fig.3 The normalized $E(X)$ distribution with cuts $E(e^-) > 25$ GeV and $p_T^X(e^-) > 5$ GeV imposed.
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