Ambiguities of neutrino(antineutrino) scattering on the nucleon
due to the uncertainties of relevant strangeness form factors

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Strange quark contributions to neutrino(antineutrino) scattering are investigated on the nucleon level in the quasi-elastic region. The incident energy range between 500 MeV and 1.0 GeV is used for the scattering. All of the physical observable by the scattering are investigated within available experimental and theoretical results for the strangeness form factors of the nucleon. In specific, a newly combined data of parity violating electron scattering and neutrino scattering is exploited. Feasible quantities to be explored for the strangeness contents are discussed for the application to neutrino-nucleus scattering.

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Knowledge of neutrino-nucleus interactions plays vital roles in nuclear detectors for the neutrino($\nu$) physics, such as $\nu$ oscillations and $\nu$ masses. Therefore neutrino-nucleus ($\nu - A$) scattering has become to be widely interested in different fields of physics such as astrophysics, cosmology, particle, and nuclear physics. Not only the $\nu$ physics, but also the hadron physics is closely related to the $\nu - A$ scattering. In particular, the scattering of neutrino and antineutrino($\bar{\nu}$) on nuclei enables us to obtain some invaluable clues on the strangeness contents of the nucleon. Along this line, Brookhaven National Laboratory (BNL) reported that a value of a strange axial vector form factor of the nucleon, $G_A^s(Q^2 = 0)$, does not have zero through the experimental data of $\nu(\bar{\nu})$ scattering on the proton. But the extraction of the exact value $G_A^s(0)$ depends on other variables, such as the axial mass, \textit{i.e.}, the axial vector dipole mass, and the parameters on strange vector form factors.\textsuperscript{2,3}

Unfortunately, the BNL data is the only one for $\nu(\bar{\nu})$ scattering on the nucleon available until now, so that one uses complementary data from the parity violating (PV) polarized

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electron scattering by HAPPEX\cite{4} and $G^0$ data\cite{5} in searches for the strangeness on the nucleon\cite{6}. Of course, there are on going experiments, FINeSSE\cite{7}, or proposed experiments in J-PARC\cite{8}.

On the other hand, many calculations\cite{9, 10, 11, 12, 13, 14, 15} about $\nu - A$ scattering are carried out to disentangle effects of the strangeness from $\nu$ interactions with exotic nuclei matter, which is one of the key ingredients of understanding the supernova explosion. But there remained still some uncertainties from the undetermined strangeness form factors of the nucleon to be pinned down for further nuclear application.

Recently, however, Pete\cite{16} suggested a method to combine $\nu(\bar{\nu})$ - nucleon(N) data and PV electron scattering data, and displayed a data set composed of 11 data for the strangeness form factors\cite{17}. Therefore, by exploiting the new data set, it is necessary and meaningful to investigate again the ambiguities regarding the interpretation of $\nu - N$ scattering data, which stems from the uncertainties of strangeness form factors. This could constrain the uncertainties relevant to the strangeness on the nucleon and give more reasonable analysis for further study of the $\nu - A$ scattering.

In this paper, we consider the neutral current (NC) and charged current (CC) scattering on the nucleon within the relevant data in the quasi-elastic (QE) region, where inelastic processes like pion production and $\Delta$ resonance are excluded. Beyond the QE region, one has to include such inelastic processes. For example, ref.\cite{18} showed that the contribution of $\Delta$ excitation is comparable to that of the QE scattering at the neutrino energies above 1 GeV.

We start from a weak current on the nucleon level. The weak current, $W^\mu$, takes a $V^\nu - A^\nu$ current form by the standard electro-weak theory, which has isoscalar and isovector parts for NC interactions

$$W^\mu = V^\mu_3 - A^\mu_3 - 2\sin^2\theta_W J^\mu_{em} - \frac{1}{2}(V^\mu_s - A^\mu_s)$$

$$= (1 - 2\sin^2\theta_W) V^\mu_3 - A^\mu_3 - 2\sin^2\theta_W V^\mu_0 - \frac{1}{2}(V^\mu_s - A^\mu_s)$$ \quad (1)

with Weinberg angle $\theta_W$, where we used $J^\mu_{em} = V^\mu_3 + V^\mu_0$. Strangeness contributions, which are isoscalar parts, are considered in $-\frac{1}{2}V^\mu_s + \frac{1}{2}A^\mu_s$. For CC interactions, only $V^\mu_3 - A^\mu_3$ term is involved, while $J^\mu_{em} = V^\mu_3 + V^\mu_0$ is concerned with the meson electro-production. Therefore the CC scattering of $\nu(\bar{\nu})$ is independent of strangeness contents. For the elastic scattering of polarized electron on nucleon, $J^\mu = -2\sin^2\theta_W J^\mu_{em} - \frac{1}{2}V^\mu_s$ is exploited.
For a free nucleon, the current operator comprises the vector and the axial vector form factors, $F^V_i(Q^2)$ and $G_A(Q^2)$,

$$W^\mu = F^V_i(Q^2)\gamma^\mu + F^A_i(Q^2)\frac{i}{2M_N}\sigma^{\mu\nu}q_\nu + G_A(Q^2)\gamma^\mu\gamma^5 + \frac{G_F(Q^2)}{2M}q^\mu\gamma^5.$$  \hfill (2)

By the conservation of the vector current (CVC) hypothesis with the inclusion of an isoscalar strange quark contribution, $F^s_i$, the vector form factors for protons and neutrons, $F^V_{p,n}(Q^2)$, are expressed as \cite{12}

$$F^V_{p,n}(Q^2) = \left( \frac{1}{2} - 2\sin^2\theta_W \right)F^p_{i}(Q^2) - \frac{1}{2}F^n_{i}(Q^2) - \frac{1}{2}F^s_{i}(Q^2) \quad \text{for NC}$$  \hfill (3)

$$= \left( F^p_{i}(Q^2) - F^n_{i}(Q^2) \right) \quad \text{for CC}.$$  

Strange vector form factors are usually given as a dipole form \cite{2}, independently of the nucleon isospin,

$$F^s_i(Q^2) = \frac{F^s_i(Q^2)}{(1 + \tau)(1 + Q^2/M_V^2)^2}, \quad F^s_i(Q^2) = \frac{F^s_i(0)}{(1 + \tau)(1 + Q^2/M_V^2)^2},$$  \hfill (4)

where $\tau = Q^2/(4M_N^2)$, $M_V = 0.843$ GeV is the cut off mass parameter usually adopted for nucleon electro-magnetic form factors. If we assume the same $Q^2$ dependence as the non-strange Sachs form factors

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2),$$  \hfill (5)

one obtains electro and magnetic strangeness form factors

$$G^s_M(Q^2) = \frac{Q^2F^s_i + \mu_s}{(1 + \tau)(1 + Q^2/M_V^2)^2}, \quad G^s_E(Q^2) = \frac{Q^2F^s_i - \mu_s\tau}{(1 + \tau)(1 + Q^2/M_V^2)^2},$$  \hfill (6)

where $\mu_s = F^s_i(0) = G^s_M(0)$ is a strange magnetic moment. If we define the root means square (rms) value of strangeness, $<r^2_s> = -6\frac{dG^s_E(Q^2)}{dQ^2}|_{Q^2=0}$, it can be approximated as $<r^2_s> \sim -6F^s_i$ at small $Q^2$. Therefore, $F^s_i$ can be deduced from $G^s_E(Q^2)$ \cite{17}. There exists a bit different form from the above one \cite{12}.

The axial form factor is given by \cite{20}

$$G_A(Q^2) = \frac{1}{2}(\mp g_A + g^*_A)/(1 + Q^2/M_A^2)^2 \quad \text{for NC}$$  \hfill (7)

$$G^*_A(Q^2) = -g_A/(1 + Q^2/M_A^2)^2 \quad \text{for CC},$$

where $g_A$ and $M_A$ are the axial coupling constant, the axial cut off mass, respectively. $-$(+) coming from the isospin dependence denotes the knocked-out proton (neutron), respectively.
The $g_A^s$ represents the strange quark contents in the nucleon. Usually it is interpreted as the integral over the polarized parton distribution function by the strangeness quark $g_s^A$. Since we take $+$ sign for $G_A(Q^2)$ in Eq.(2), the axial form factor in Eq.(7) is just negative to the form factor elsewhere, for example, in ref.[12]. The induced pseudoscalar form factor is usually parameterized by the Goldberger-Treimann relation

$$ G_P(Q^2) = \frac{2M_N}{Q^2 + m_\pi^2} G_A(Q^2), $$

where $m_\pi$ is the pion mass. But the contribution of the pseudoscalar form factor vanishes for the NC reaction because of the negligible final lepton mass participating in this reaction.

In principle, relevant strangeness form factors, $G_{E,M}^s(Q^2)$ and $G_A^s(Q^2) = g_A^s/(1 + Q^2/M_A^2)^2$, can be deduced from experimental data, for instance, $G^0$ and HAPPEX data by PV electron scattering and/or BNL data by $\nu - N$ scattering. But the available experimental data are not enough to fix the strangeness form factors, although theoretical calculations, such as chiral soliton nucleon model or quark model [22, 23], show results consistent, but scattered with each other, with the experimental data. Therefore, values of $F_1^s$, $\mu_s$, and $g_A^s$ in the form factors have some ambiguities due to the uncertainties persisting in the strangeness form factors [6, 17].

But the newly combined data set [17] for those strangeness form factors (see figure 1 at ref.[17]) makes it possible to constrain the relevant parameters to some extent by adjusting the parameters to the data. In table 1, relevant coupling constants and parameters adopted in previous calculations are summarized with our values adjusted to the experimental data [17].

Model IV in ref. [2] deduced from the reanalysis of BNL data is the most common value in $\nu - N(A)$ scattering calculations, and difference between model III and IV is the axial mass used. Ref. [13] did not take the strangeness into account, but focus on the $\Delta$ region. We fixed $\sin^2\theta_W, g_A, M_A$ as the most common values. But other parameters are adjusted to satisfy newly combined data [17] and theoretical calculations, by presuming a dipole form. It is remarkable that $\mu^s$ value is constrained to be positive according to the newly combined data. This contradicts the previous values used in previous $\nu - A$ scattering calculations.

Our lower and upper limits of the parameters constrained to experimental data lead to some inescapable ambiguities of the physical observables. They are discussed in detail in the following. Since the first term in Eq.(3) rarely contributes due to the given Weinberg
angle, the elastic cross section on the proton, $\sigma(\nu p \rightarrow \nu p)$, is sensitive mainly on the $F_i$ and $g_A$ values. But measuring of the cross section itself is not easy experimental task, so that one usually resorts to the cross section ratio between the proton and the neutron, $R_{p/n} = \sigma(\nu p \rightarrow \nu p)/\sigma(\nu n \rightarrow \nu n)$. Measuring of this ratio has also some difficulties in the neutron detection.

Therefore, the ratio, $R_{NC/CC} = \sigma(\nu n \rightarrow \nu n)/\sigma(\nu n \rightarrow \mu^- p)$ and $R_{NC/CC} = \sigma(\bar{\nu} p \rightarrow \bar{\nu} p)/\sigma(\bar{\nu} p \rightarrow \mu^+ n)$, are suggested as plausible signals for the nucleon strangeness, for example, in FINeSSE experiments \cite{7}, because the charged current (CC) cross section is insensitive to the strangeness \cite{25}. In this paper, we investigate other possible observables, mainly asymmetries between $\nu$ and $\bar{\nu}$ scattering.

If we assume that the standard Sachs form factors also hold for vector form factors

$$ G_E^V(Q^2) = F_1^V(Q^2) - \frac{Q^2}{4M^2} F_2^V(Q^2), \quad G_M^V(Q^2) = F_1^V(Q^2) + F_2^V(Q^2), \quad (9) $$

we can define Sachs vector form factors, similarly to Eq.(3),

$$ G_{M,E}^{V,p(n)}(Q^2) = (\frac{1}{2} - 2 \sin^2 \theta_W) G_{M,E}^{V,p(n)}(Q^2) - \frac{1}{2} G_{M,E}^{n(p)}(Q^2) - \frac{1}{2} G_{M,E}^s(Q^2) \quad \text{for NC} \quad (10) $$

$$ = (G_{M,E}^{p(n)}(Q^2) - G_{M,E}^{n}(Q^2)) \quad \text{for CC}. $$

Then the NC $\nu(\bar{\nu}) - N$ cross section is expressed in terms of the Sachs vector and axial form factors \cite{3}

$$ \left( \frac{d\sigma}{dQ^2} \right)_{\nu(\bar{\nu})}^{NC} = \frac{G_E^2}{2\pi} \left[ \frac{1}{2} y^2 (G_M^V)^2 + (1 - y - \frac{M}{2E_{\nu}}) (G_E^V)^2 + \frac{E_{\nu}}{2M} y (G_M^V)^2 \right] \quad (11) $$

$$ + \left[ \frac{1}{2} y^2 + 1 - y + \frac{M}{2E_{\nu}} \right] y (G_A)^2 \mp \mp 2y(1 - \frac{1}{2} y) G_M^V G_A. $$

Here $E_{\nu}$ is the energy of incident $\nu(\bar{\nu})$ in the laboratory frame, and $y = p \cdot q/p \cdot k = Q^2/2p \cdot k$ with $k, p$ and $q$, initial 4 momenta of $\nu(\bar{\nu})$ and target nucleon, and 4 momentum transfer to the nucleon, respectively. $\mp$ corresponds to the cases of the $\nu$ and $\bar{\nu}$. Therefore the difference and the sum of the cross sections are simply summarized as

$$ \left( \frac{d\sigma}{dQ^2} \right)_{\nu}^{NC} - \left( \frac{d\sigma}{dQ^2} \right)_{\bar{\nu}}^{NC} = -\frac{G_E^2}{2\pi} 4y(1 - \frac{1}{2} y) G_M^V G_A, \quad (12) $$

$$ \left( \frac{d\sigma}{dQ^2} \right)_{\nu}^{NC} + \left( \frac{d\sigma}{dQ^2} \right)_{\bar{\nu}}^{NC} = \frac{G_E^2}{\pi} \left[ \frac{1}{2} y^2 (G_M^V)^2 + (1 - y - \frac{M}{2E_{\nu}}) (G_E^V)^2 + \frac{E_{\nu}}{2M} y (G_M^V)^2 \right] \quad (13) $$

$$ + \left[ \frac{1}{2} y^2 + 1 - y + \frac{M}{2E_{\nu}} \right] y (G_A)^2. $$
As shown in Eq.(12), the difference between $\nu$ and $\bar{\nu}$ scattering depends only on the product of $G_M^V$ and $G_A$. Consequently, the asymmetries, $A_{NC}^{\nu(p)}$, or the ratios, $R_{NC}^{\nu/p}$, could be good observables for the strangeness study

$$A_{NC}^{\nu(p)} = \frac{(\sigma_{NC}^{\nu} - \sigma_{NC}^{\bar{\nu}})^{p(n)}}{(\sigma_{NC}^{\nu} + \sigma_{NC}^{\bar{\nu}})^{p(n)}} = \frac{1 - R_{NC}^{\nu/p}}{1 + R_{NC}^{\nu/p}},$$

where $\sigma_{NC}^{\nu(p)}$ means differential cross sections, Eq.(11), by incident $\nu$ and $\bar{\nu}$ on proton(neutron).

Since the $y$ variable in Eq.(12) is given as $Q^2/2E_\nu M$ in the nucleon rest frame, $y$ is always positive, but less than 1 for the energy region, $E_\nu < 1$ GeV and $Q^2 < 1$ GeV$^2$, considered here. It means that the asymmetry $A_{NC}$ could be very sensitive on the $g_A$ value because $A_{NC}$ is approximated as $2y G_M^V G_A / (1 - y)(G_E^V G_A^2)$ if $O(y^2)$ and $\frac{E_\nu}{2M} O(y)$ terms are neglected. Moreover, the Eq.(12) has a positive sign irrespective of the proton and the neutron. Consequently, $\nu$ cross section is always larger than that of $\bar{\nu}$ on the nucleon level.

Detailed results on the nucleon level are shown in Figs. 1 and 2, where the results for $g_A^s = -0.19$ and 0.0 are presented on the proton and the neutron. In Fig. 1 the cross sections by the incident $\nu(\bar{\nu})$ on the proton are usually enhanced in the whole $Q^2$ region by the $g_A^s$, while they are reduced on the neutron. But, as shown in Fig.2, the asymmetry on the proton is maximally decreased in the $Q^2 \sim 0.6$ GeV$^2$ region about 15%, while on the neutron it is maximally increased in that region. The ratio, $R_{\nu/\bar{\nu}}$, on the contrary, shows reversed behaviors. Therefore, the $g_A^s$ effects can be detected in the asymmetry $A_{NC}$ around the $Q^2 \sim 0.6$ GeV$^2$ region, more clearly than the cross sections.

Since the neutrino energy was not known exactly at the BNL experiments, one usually defines the flux averaged cross section

$$< \sigma > = < \frac{d\sigma}{dQ^2} >^{NC}_{\nu(\bar{\nu})} = \frac{\int dE_{\nu(\bar{\nu})}(d\sigma/dQ^2)_\nu^{NC}(E_{\nu(\bar{\nu})})}{\int dE_{\nu(\bar{\nu})}(\Phi_{\nu(\bar{\nu})}(E_{\nu(\bar{\nu})}))},$$

where $\Phi_{\nu(\bar{\nu})}(E_{\nu(\bar{\nu})})$ is neutrino and antineutrino energy spectra. The experimental result at BNL, $R_{\nu/\bar{\nu}}^{BNL} = < \sigma(\bar{\nu}p \rightarrow \bar{\nu}p) > / < \sigma(\nu p \rightarrow \nu p) >$, turned out to be about 0.32 [1, 3]. Therefore, the flux averaged asymmetry $< A_{NC} > = ( < \sigma_{NC}^\nu > - < \sigma_{NC}^{\bar{\nu}} > )/ ( < \sigma_{NC}^\nu > + < \sigma_{NC}^{\bar{\nu}} > )$ is 0.5 on the nucleon level. This value is approximately consistent with our $A_{NC}$ values in Fig.2 if they are averaged by $Q^2$.

Dependence on the strangeness parameters on the vector form factors, $\mu_s$ and $F_1^s$, is presented in Fig.3. Results in left figure are obtained by fixing $\mu_s$ to 0.4, but varying $F_1^s$.
to the constrained interval, $0.0 \sim 0.53$. Those of right panel are the case of $F_1^s = 0.53$ and $\mu_s = 0.0 \sim 0.5$. One can see that only a few percent difference is found for the cross sections. It means that the most sensitive effect by the strangeness stems from the axial strangeness form factor, $g_A^s$, as shown in Figs.1 and 2.

The cross section for CC scattering is given with the following replacement to NC cross section

$$\frac{d\sigma}{dQ^2}_{CC} = \frac{d\sigma}{dQ^2}_{NC}(G_E \to G_E^{CC}, G_M \to G_M^{CC}, G_A \to G_A^{CC}),$$

(16)

where

$$G_E^{CC} = G_E^p(Q^2) - G_E^n(Q^2), \quad G_M^{CC} = G_M^p(Q^2) - G_M^n(Q^2).$$

(17)

One can see that form factors in CC scattering are not influenced by the strangeness in the vector and axial form factors, but depends only on the axial mass $M_A$, and axial coupling constant, $g_A$. Therefore CC scattering is usually used to extract the $M_A$ and $g_A$ values [6].

The left and right panel in Fig.4 corresponds to those of $\nu$ and $\bar{\nu}$ scattering, respectively. The CC cross sections are about $4 \sim 6$ times larger rather than those of the NC, and the cross section of $\sigma_{CC}^\nu = \sigma(\nu n \to \mu^- p)$ is larger than that of $\sigma_{CC}^\bar{\nu} = \sigma(\bar{\nu} p \to \mu^+ n)$. Our CC results are consistent with other calculations [6]. Effect of $M_A$ difference, i.e. cases of $M_A = 1.032$ and $1.026$ GeV, is also presented in Fig.4 but the effect is nearly indiscernible (see solid and long-dashed curves).

The difference and the sum of $\nu$ and $\bar{\nu}$ scattering for CC are also given as Eqs.(12) \sim (13) with the above replacement. If we denote $\sigma_{CC}^{\nu(\bar{\nu})}$ as differential cross sections by $\sigma(\nu n \to \mu^- p)$ and $\sigma(\bar{\nu} p \to \mu^+ n)$, respectively, the asymmetry in CC is given as

$$A_{CC} = \frac{\sigma_{CC}^\nu - \sigma_{CC}^\bar{\nu}}{\sigma_{CC}^\nu + \sigma_{CC}^\bar{\nu}} = \frac{1 - R_{CC}^{\bar{\nu}/\nu}}{1 + R_{CC}^{\bar{\nu}/\nu}}.$$  

(18)

Our results for CC scattering are given in Fig.5 where dashed and solid curves are the asymmetry and the ratio for CC, respectively.

Finally we discuss the ratios and the asymmetries between CC and NC cases. The ratios of NC and CC scattering are given as

$$R_{NC/CC} = \frac{\sigma_{NC}^{\nu n}}{\sigma_{CC}^\nu}, \quad \bar{R}_{NC/CC} = \frac{\sigma_{NC}^{\bar{\nu} p}}{\sigma_{CC}^{\bar{\nu}}}, \quad \tilde{R}_{NC/CC} = \frac{\sigma_{NC}^{\bar{\nu} p}}{\sigma_{CC}^{\nu}}.$$  

(19)

These ratios have been suggested for probing the strangeness on the nucleon or nuclei because the CC scattering is independent of the strangeness and any possible nuclear structure effects.
are expected to be cancelled out. Results for $R_{NC/CC}$ are shown in Fig. 6. One can see large strangeness effects due to the $g^s_A$ in the ratios (see solid and dashed curves). Here we $F^s_1 = 0.53$ and $\mu_s = -0.4$ are used.

It would be interesting which is the larger of the two strangeness effect, vector and axial strangeness parts. To distinguish each strangeness contribution, we consider asymmetries between NC and CC, because they are expressed only in terms of the ratios between strangeness and non-strangeness as follows

$$A^p_{NC/CC} = \frac{\sigma^p_{NC} - \sigma^p_{NC}}{(\sigma^p_{CC} - \sigma^p_{CC})} = \frac{G^V_M G_A}{G^3_M G^A_{CC}} = 0.12 - 0.12 \frac{g^s_A}{g_A} - 0.13 \frac{G^s_M}{G^3_M} ,$$

$$A^n_{NC/CC} = \frac{\sigma^n_{NC} - \sigma^n_{NC}}{(\sigma^n_{CC} - \sigma^n_{CC})} = \frac{G^V^n_M G_A}{G^3_M G^A_{CC}} = 0.16 + 0.16 \frac{g^s_A}{g_A} + 0.13 \frac{G^s_M}{G^3_M} ,$$

where $G^s_M(Q^2 = 0) = \mu_s$, $G^3_M = (G^p_M - G^n_M)/2$, $\mu_p = 2.79$, and $\mu_n = -1.91$ are used. Since $|G^s_M/G^3_M|$ and $|g^s_A/g_A|$ are approximately 0.2, strangeness effects from vector and axial parts are comparable, in principle.

Remarkable point is that one can decide the $\mu_s$ sign. If $g^s_A$ and $G^s_M$ have different $\pm$ sign, $A^p(n)_{NC/CC}$ values become constants 0.12, 0.16, respectively, because the last two terms in Eq.(20) are nearly cancelled. Any deviations from these constants by the same sign would imply the $Q^2$ dependence of form factors as shown in Fig. 6, in which we used $g^s_A = -0.19$ and $\mu_s = -0.4$.

Finally we consider a difference and a sum of asymmetries

$$DA^p(n)_{NC/CC} = A^p_{NC/CC} - A^n_{NC/CC} = (\frac{G^V_M}{G^3_M} G^A_{CC} G^V^n_M G^A_{CC}) \simeq -0.04 - 0.28(\frac{g^s_A}{g_A} + \frac{G^s_M}{G^3_M}) ,$$

$$SA^p(n)_{NC/CC} = A^p_{NC/CC} + A^n_{NC/CC} = (\frac{G^V_M}{G^3_M} G^A_{CC} G^V^n_M G^A_{CC}) = 0.28 + 0.04 \frac{g^s_A}{g_A} .$$

Sum of asymmetry, $SA^p(n)_{NC/CC}$, is given only in terms of axial part. But the 2nd term is very small by the factor 0.04, so that SA is nearly independent of the strangeness, but DA depends strongly on the axial strangeness as shown in Fig. 7. If $g^s_A$ and $\mu_s$ have different signs, DA would be constant, but it depends on $Q^2$ if they have same signs as in Fig. 7.

In the following, we make brief summaries and conclusions. A newly combined data by parity violating electron scattering and $\nu(\bar{\nu})$ scattering shed a valuable light on relevant strangeness form factors, $G^s_{E,M}(Q^2)$ and $G^s_A(Q^2)$. Using a conventional dipole form, we extracted lower and upper limits of the parameters related with the strangeness for factors
by adjusting them to the experimental data. We found that $G_M^s(Q^2 = 0) = \mu_s$ has a positive value contrary to the negative values exploited for $\nu - A$ scattering calculations.

Effects of the strangeness form factors are investigated for cross sections and their ratios by varying the parameters within the limits. We found ambiguities by $F_1^s, \mu_s$ due to the vectorial and $g_A^s$ by the axial form factors are within maximally 3 %, 5 %, and 15 %, respectively. Change of axial mass in these calculations affect results within only a 1 %.

In specific, effects of the main parameters, $g_A^s$, are detailed. It shows that cross sections, ratios and asymmetries for protons are increased, decreased, and increased, respectively, by the $g_A^s$, while those of neutrons show reversed behaviors of each quantity.

Finally, in order to search for more efficient observable for the strangeness, relevant asymmetries between $\nu$ and $\bar{\nu}$ scattering are studied. As expected, there appeared larger strangeness effects in the ratios and asymmetries rather than cross sections. In specific, asymmetry between NC and CC scattering could be meaningful methods to look for the $Q^2$ dependence of the strangeness form factors. It is also remarkable that sum of asymmetry is nearly independent of the strangeness. Therefore it could be a measure of nuclear effects independent of the strangeness. Nuclear application based on this work are under progress. One of our preliminary results show that $g_A^s$ effects are strongly cancelled in nuclei by the enhancement of proton knockout processes and the decrease of neutrons processes.

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TABLE I: Relevant parameters to the axial couplings and the strangeness form factors.

| $\sin^2 \theta_W$ | $g_A$ | $M_A$ | $g_A^s = \Delta s$ | $F_1^s$† | $\mu^s = F_2^s(0)$ | $\rho^{***}$ | Ref. |
|-------------------|-------|-------|---------------------|--------|---------------------|-----------|-----|
| 0.23143           | 1.26  | 1.026 ± 0.021 [21] | -0.10   | 0.4    | -0.50              | 2         | [12] (2006) |
| 0.2313            | 1.26  | 1.026 ± 0.021 [21] | -0.19   | 0.53   | -0.40              | x         | [12] (2004) |
| 0.2224            | 1.262 | 1.032   | x         | x      | x                  | x         | [13] |
| 0.2325            | 1.256 ± 0.003 | 1.012 ± 0.032* | -0.21 ± 0.10 | 0.53 ± 0.70 | -0.40 ± 0.72 | x         | [2] (Model IV) |
| 0.2325            | 1.256 ± 0.003 | 1.049 ± 0.023 | -0.13 ± 0.09 | 0.49 ± 0.70 | -0.39 ± 0.70 | x         | [2] (Model III) |
| x                 | x     | 1.026 ± 0.021 [21] | -0.04 ~ -0.09 | x      | 0.08 ~ 0.32      | x         | Theory [17, 22, 23] |
| 0.232             | 1.256 | 1.032   | -0.21 ~ 0.0  | 0 ~ 0.53 | 0.0 ~ 0.4 ††      | x         | Ours |

* a value cited from a table at ref. [2], but actually the world average value prior to the BNL data, $1.032 \pm 0.036$, seems to be used at the reference.

† $F_1^s = - < r_s^2 > / 6$ is the rms value of strangeness.

** comes from ref. [12] in which a bit different form for $F_i^s(Q^2)$ with a constant $\rho^s$ is used.

†† has a positive sign (see the data at ref. [17]) compared to previous values.
FIG. 1: Differential cross sections, Eq. (11), by the NC scattering on proton (left) and neutron (right) in an arbitrary unit as a function of $Q^2$ for the incident energy $E_{\nu(\bar{\nu})} = 500$ MeV. They are calculated for $g_A^s = -0.19$ and 0.0 cases, respectively. Lower parts are for the $\bar{\nu}$. 
FIG. 2: $R_{\nu/\mu}$ (upper) and asymmetries (lower), Eq.(14), by the NC scattering on proton (left) and neutron (right) as a function of $Q^2$ for the incident energy $E_\nu = 500$ MeV. They are calculated for $g_A^s = -0.19$ and 0.0 cases, respectively.
FIG. 3: $F_1^s$ (right) and $\mu_s$ (left) dependence for the incident $\nu(\bar{\nu})$ on proton target.
FIG. 4: Differential cross sections, Eq.(16), for the CC scattering on the nucleon in an arbitrary unit as a function of $Q^2$ for the incident energy $E_{\nu(\bar{\nu})} = 500$ MeV. They are calculated for $M_A = 1.032$ and 1.026 cases, respectively. Left panel is w.r.t. $\sigma(\nu n \rightarrow \mu^- p)$ and right is $\sigma(\bar{\nu} p \rightarrow \mu^+ n)$. Effects of axial mass difference turned out to be nearly indiscernible.

FIG. 5: Ratio and asymmetry of the CC cross sections in fig.4
FIG. 6: Ratio on the neutron, $R_{NC/CC}$ (Eq.(19)), and asymmetries, $A_{NC/CC}^{p}$ and $A_{NC/CC}^{n}$ (Eq.(20)), between for NC and CC. They are calculated for $g_{A}^{s} = -0.19$ and 0.0 cases, respectively.
FIG. 7: Difference and sum, Eq.(21), of asymmetries in Fig[6]