On Holographic Non-relativistic Schwinger Effect

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Abstract

Using the AdS/CFT correspondence, we study the Schwinger effect in strongly coupled theories with an anisotropic scaling symmetry in time and spatial direction. We consider Lifshitz and hyperscaling violation theories and use their gravity duals. It is shown that the shape of the potential barrier depends on the parameters of theory. One concludes that the production rate for the pair creation of particle and anti-particle will not be the same as the relativistic case.

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Introduction

One of the interesting effects in QED is pair production in a constant electric field which is known as Schwinger effect \[1\]. This is a non-perturbative effect in quantum field theories. The pair production of charged particles like particle \( q \) and anti-particle \( \bar{q} \) with mass \( m \) is exponentially suppressed for homogeneous background electric field \( E \) as

\[ P \propto e^{-\frac{m^2}{|eE|}} \]  

(1)

where \( e \) is the charge of the particle. It is clear that there is a tunneling process with a classical Euclidean action. Increasing the electric field leads to increasing the production rate so that there is not a phase transition. There is a smooth behavior even for larger values of \(|eE|\) where the saddle-point approximation ceases to be applicable. This is not the case in string theory, where there exists a critical value for the electric field \( E_c \) \[2, 3\]. This critical value is proportional to the string tension \(|eE_c| \sim \frac{1}{2\pi\alpha'}\). As a result, at \( E_c \) phase transition occurs and one finds an instability. It means that the pair production drops to zero and potential barrier disappears. One should notice that this instability happens even for a neutral string with opposite charges at the ends.

Based on AdS/CFT correspondence \[4\], one may study this instability in some field theories. This study has been done for \( \mathcal{N} = 4 \) super Yang-Mills (SYM) in four dimensions in \[5\]. They consider SYM theory in the Coulomb branch and spontaneously break the gauge group \( SU(N+1) \) to \( SU(N) \times U(1) \). Then turn on an electric field of the \( U(1) \) gauge theory and pair creation of the massive W-bosons is similar to the Schwinger effect. It was shown that the critical value of the electric field is expressed as

\[ E_c = \frac{2\pi m^2}{\sqrt{\lambda}} \]  

(2)

where \( m \) and \( \lambda \) are W-boson mass and 't Hooft coupling, respectively. The \( \frac{1}{\sqrt{\lambda}} \) correction to this value has been studied in \[6\] \[3\].

The Schwinger effect in the context of AdS/CFT correspondence has been studied in different papers. The universal aspects of this effect in the general backgrounds are studied in \[7\]. The pair production in confining geometries is investigated in \[8\] and \[9\]. They find two kinds of critical electric field in this case. The potential barrier for the pair creation is analyzed in \[10\]. The effect of pair production in the conductivity of a system of flavor and color branes is discussed in \[11\]. In de Sitter space time also the Schwinger effect has been studied in \[12\]. This effect is also studied in the world-line formalism of quantum field theory in \[13\] and in \[14\] as a WKB exact path integral. The holographic Schwinger effect with constant electric and magnetic fields has been studied in \[15\] and \[16\]. The vacuum instability in the presence of a constant electric field in \( \mathcal{N} = 2 \) supersymmetric QCD has been studied in \[17\]. For study of this effect in the Sakai-Sugimoto model, see \[18\]. They compute the creation rate of the \( q\bar{q} \) pair by evaluating the imaginary part of the Dirac-Born-Infeld (DBI) action including a constant electromagnetic field. On the other hand, the authors of \[19\] evaluate the Nambu-Goto action on the probe D3-brane.  

\[^3\text{From the AdS/CFT correspondence, the 't Hoof coupling \( \lambda \) is related to the curvature radius \( L \) and the string tension (\( \frac{1}{2\pi\alpha'} \)) by \( \sqrt{\lambda} = \frac{l^2}{\alpha'} \).}\]
It is known that methods based on the AdS/CFT correspondence relate gravity in AdS space to the conformal field theory on the boundary. These conformal field theories are invariant under the following scaling transformation
\[(t, \vec{x}) \rightarrow (w t, w \vec{x}),\]  
where \(t\) is time and \(\vec{x}\) is spatial directions of the space time. The scaling factor \(w\) is a constant. However, in field theories near a critical phenomena there is an anisotropic scaling symmetry as follows:
\[(t, \vec{x}) \rightarrow (w^z t, w \vec{x}),\]  
where \(z\) is called dynamical exponent. In the case of \(z = 1\), theory shows relativistic scale invariance. The non-relativistic geometries have been considered in [19] and [20]. The holographic description with the Lifshitz fixed point has been studied in [21]. The gravity in 5-dimensions is described by the following metric
\[ds^2 = L^2 \left( -r^{2z}dt^2 + \frac{dr^2}{r^2} + r^2d\vec{x}^2 \right),\]  
where \(L\) is the radius of curvature. This geometry has a genuine null singularity which may be resolved by considering stringy effects. We ignore these issues and point out that our results may only be valid in certain range of energies. One should notice that in the pure cosmological Einstein gravity the space time is isotropic and to produce an anisotropic space time one should consider other fields like a massive gauge field. Including both a scalar field with nontrivial potential and a gauge field lead to the following metric
\[ds^2 = \frac{L^2}{r^{2\theta/d}} \left( -r^{2z}dt^2 + \frac{dr^2}{r^2} + r^2d\vec{x}^2 \right),\]  
where \(d\) is the spatial dimension of the boundary and \(\theta\) is hyperscaling violation exponent. To satisfy the null energy condition, one should assume the following relations between \(\theta\), \(z\) and \(d\)
\[(z - 1)(d + z - \theta) \geq 0, \quad (d - \theta)(d z - \theta - d) \geq 0.\]  
Actually, based on the AdS/CFT correspondence non-zero \(\theta\) means hyperscaling violation in the dual field theory. An anomalous scaling dimension is also introduced in [22, 24] to describe the anomalous temperature scaling of strange metals by studying the frequency and temperature dependence of the conductivity. Two classes of non-zero anomalous scaling dimension, are Einstein-Maxwell-Dilaton systems or probe branes in backgrounds with non-zero \(\theta\) [23].

Now we extend the previous studies of the Schwinger effect in strongly coupled theories with an anisotropic scaling symmetry in time and spatial direction. We consider Lifshitz and hyperscaling violation theories and use their gravity duals in eqs. [5] and [10]. We investigate if the potential barrier for creation of \(q\bar{q}\) depends on the dynamical exponent \(z\) and hyperscaling violation \(\theta\). Our purpose is to study the shape of the barrier potential versus parameters of \(z\) and \(\theta\) to find how the rate of \(q\bar{q}\) changes, qualitatively. We find analytic equations for the barrier potential and the distance between \(q\) and \(\bar{q}\) in these theories. One may even ask whether the barrier potential disappears by changing \(\theta\) and

\footnote{We would like to thank A. Karch for discussion on this scaling parameter.}
We will show that shape of the potential changes and the production rate for the pair creation will not be the same as the relativistic case. Interestingly, we find that increasing hyperscaling parameter $\theta$ and dynamical exponent $z$ have different effects on the shape of the barrier potential, i.e., by increasing $\theta$ the height and the width of the barrier increase while by increasing $z$ they decrease, significantly.

This paper is organized as follows. In sections one and two, we will present the potential analysis in the Lifshitz geometry and hyperscaling violation, respectively. We study behavior of barrier potential as a function of distance between $q$ and $\bar{q}$. We consider the case with $\theta < d$. We also discuss the spacetimes with anisotropic in spatial direction in section three. In the last section we summarize our results.

## 2 Potential analysis in Lifshitz geometry

To study the holographic Schwinger effect, one may estimate the barrier potential, holographically. The general form of this tunneling barrier depends on the rest masses $2m$ and the energy that $q$ and $\bar{q}$ at distance $x$ gained from an external homogeneous electric field $E$. By adding Coulomb interactions, the barrier potential has this profile

$$V = 2m - E x - \frac{\alpha}{x}$$

where $\alpha$ depends on the electric charge. Simple holographic arguments in [5] lead to an estimate for the critical electric field which exhibits deviation from the DBI result. This deviation has been studied by potential analysis in [10]. We will follow the same setup and consider a probe D-brane at finite position in the bulk.

The Schwinger effect in general backgrounds with an external electric field has been studied in [7]. Using holographic potential analysis, they find some universal aspects for this effect. Although, we have considered an anisotropic scaling symmetry in time and spatial direction but our anisotropic geometries belong to their class and one expects a critical $E_c$ in the theory. As it was pointed out, we are going to investigate the shape of the barrier potential versus parameters of $z$ and $\theta$. Our purpose is to find how the creation rate of $q\bar{q}$ changes, qualitatively.

Using the $AdS/CFT$ correspondence, we consider $q\bar{q}$ pair on the D-brane which is located at finite position in the bulk. They can be realized as the endpoints of the U-shaped string hanging from the D-brane to the IR-region. The brane covers the boundary directions and locates at $r_0$ [10]. We use the usual orthogonal Wilson lines and assume the $q\bar{q}$ is aligned in the $x$ direction

$$t = \tau, \quad x = \sigma, \quad r = r(\sigma),$$

one finds the generic formula for the distance between $q$ and $\bar{q}$ in [7]. Considering the metric in [5], one obtains the lagrangian from the Euclidean Nambu-Goto action as

$$\mathcal{L} = L^2 \sqrt{\tau^{2s+2} + r^{2s-2}(\partial_\sigma r)^2}.$$
\[ x(z) = \frac{2L^2}{r_c} \int_{r_0/r_c}^{r_0} \frac{dy}{y^2 \sqrt{y^{2z+2} - 1}} \]

\[ = \frac{2L^2}{r_0} \left\{ \frac{\sqrt{\pi} \Gamma \left( \frac{2+z}{2+2z} \right)}{a\Gamma \left( \frac{1}{2+2z} \right)} \right\} - \left( \frac{a^{1+z}}{2+z} \right) _2 F _1 \left( \frac{1}{2}, \frac{2+2z}{2+2z} a^{2+2z} \right) \].  \hspace{1cm} (11)

where \( y = \frac{r}{r_c} \) is a new coordinate and \( a = \frac{r_c}{r_0} \) measures the tip of the U-shaped string with respect to the location of the D-brane in the bulk. For the relativistic case of \( z = 1 \), this formula reduces to the case of an isotropic strongly coupled theory \[10\].

Using the standard calculations of Wilson lines, one finds the sum of the \( q\bar{q} \) static energy and Coulomb potential as follows:

\[ V_1(z) = \frac{r_0^2}{\pi \alpha'} \int_{r_0/r_c}^{r_0} \frac{y^{2z}}{\sqrt{y^{2z+2} - 1}} dy \]

\[ = \frac{r_0^2}{\pi \alpha'} \left\{ \frac{a^{z} \sqrt{\pi} \Gamma \left( \frac{2+z}{2+2z} \right)}{(1+2z)\Gamma \left( -1 + \frac{1}{2+2z} \right)} + \left( \frac{1}{z} \right) _2 F _1 \left( \frac{1}{2}, \frac{-z}{2+2z}, \frac{2+z}{2+2z}, a^{2+2z} \right) \right\}. \hspace{1cm} (12)

In this potential the UV part of the the string solution is absent, because we put the D-brane at finite radius in the bulk not in the boundary. As a result the short distance behavior of \( q\bar{q} \) is modified in this approach. By putting off the D-brane to the boundary at fixed \( r_c \) which means \( a \to 0 \), the Hypergeometric function goes to 1 and the second term in (13) reduces to the static energy of infinitely massive \( q \) and \( \bar{q} \). The first term will also coincide with the Coulomb potential in the presence of dynamical \( z \) exponent which is proportional to \( 1/x^z \). The Wilson loop calculations in the Lifshitz spacetime which shows the same results for \( q\bar{q} \) potential was performed in \[26, 27\]. In the limit \( a \to 1 \), Hypergeometric function in the second term of (11) goes to \( \sqrt{\pi (2+z)} \Gamma \left( \frac{2+z}{2+2z} \right) / \Gamma \left( \frac{1}{2+2z} \right) \). Then \( x \) and \( V_1 \) vanish. This is the same as the case of \( z = 1 \) in \[10\].

In the presence of external electric field \( E \) along the \( x \) direction, the total potential is given by

\[ V(z) = V_1(z) - E x(z) \]

\[ = \frac{r_0^2}{\pi \alpha'} \left\{ \frac{a^{z} \sqrt{\pi} \Gamma \left( \frac{2+z}{2+2z} \right)}{z\Gamma \left( \frac{1}{2+2z} \right)} + \left( \frac{1}{z} \right) _2 F _1 \left( \frac{1}{2}, \frac{-z}{2+2z}, \frac{2+z}{2+2z}, a^{2+2z} \right) \right\} - \frac{b}{a} \left( \frac{\sqrt{\pi} \Gamma \left( \frac{2+z}{2+2z} \right)}{\Gamma \left( \frac{1}{2+2z} \right)} - \left( \frac{a^{2+z}}{2+z} \right) _2 F _1 \left( \frac{1}{2}, \frac{2+z}{2+2z}, \frac{4+3z}{2+2z}, a^{2+2z} \right) \right\}. \hspace{1cm} (13)

where \( b = \frac{E}{E_c} \) and \( E_c = \frac{r_0^{2z+1}}{2\pi \alpha' L^2} \).

\(^5\)Higher derivative correction to the \( q\bar{q} \) potential has been studied in \[25\]
In Fig. 1, we assume $E < E_c$ and call $V_{tot}$ as barrier potential in the non-relativistic Schwinger effect. This barrier potential disappears when $E > E_c$ and vacuum becomes unstable catastrophically. We plot in this figure the barrier potential versus the inter quark distance $x$ in the presence of new Lifshitz scaling symmetry $z$. In the left and right plots, we consider $b = 0.8$ and $b = 0.2$, respectively. In all of these plots from top to bottom the dynamical exponent $z$ increases as $z = 1, 2, 3, 4, 5$. It is clear that the height and the width of the barrier depend on the dynamical exponent $z$. As $z$ increases, the shape of barrier changes significantly i.e. the height and the width of barrier decrease. As a result, one finds that the rate of producing the $q\bar{q}$ pairs should be changed in the presence of an anisotropic scaling symmetry in time and spatial direction. We checked that increasing the electric field, leads to decreasing the height and the width of the barrier.

Why should one expects that by turning on the dynamical exponent $z$, the barrier potential also significantly changes? We use some simple field theory arguments to show that it is expectable. One should consider that the pair production is a tunneling process through a barrier of height which is proportional to $2m$. The width of the barrier is also given in terms of $x \sim \frac{2m}{E}$. The production rate has exponential suppression which is approximately $(2m)\left(\frac{2m}{E}\right) \sim \frac{m^2}{E}$. Then the final results depends strongly on the mass of the quarks. We see that the mass is given by

$$m = \frac{\sqrt{\lambda r_0^2}}{2\pi} \quad (14)$$

It depends on the dynamical exponent $z$. Then the shape of the barrier changes by changing $z$.

When the electric field is larger than $E_c$, the barrier potential disappears. We show the total potential in (13) versus the distance $x$ in the Fig. 2. As it is clear in this figure, there is no barrier potential and $q\bar{q}$ pairs create freely and the vacuum decays catastrophically. By changing $z$, this phenomena does not change. Although the shape of the potential depends on $z$.

### 3 Potential analysis in theories with hyperscaling violation

Next, we consider barrier potential analysis in hyperscaling violation geometry in (6). We call again the distance between $q\bar{q}$ pair on the boundary and the $q\bar{q}$ pair potential energy
as \( x(\theta, z) \) and \( V_{\text{tot}}(\theta, z) \), respectively. One finds the analytic solutions as follows:

\[
x(\theta, z) = \frac{1}{a} \Gamma\left(4 + 2z - \frac{4\theta}{d}\right) \times 
\left(\frac{2\sqrt{\pi}}{\Gamma\left(\frac{1}{2} + 2z - \frac{4\theta}{d}\right)} - \frac{a^{2+z-\frac{2\theta}{d}}}{1 + z - 2\theta/d}\right) F_1\left(\frac{1}{2}, \frac{4 + 2z - \frac{4\theta}{d}}{4 + 4z - \frac{8\theta}{d}}; \frac{8 + 6z - \frac{12\theta}{d}}{4 + 4z - \frac{8\theta}{d}}; a^{2+2z-\frac{4\theta}{d}}\right),
\]

and

\[
V_1(\theta, z) = 2a^{-z-\frac{2\theta}{d}} \times 
\left(\frac{d\sqrt{\pi} \Gamma\left(\frac{-d + z + 2\theta}{2(d + dz - 2\theta)}\right)}{(d + 2dz - 4\theta)\Gamma\left(\frac{-d - 2dz + 4\theta}{2(d + 2dz - 4\theta)}\right)} + \frac{a^{-z+\frac{2\theta}{d}}}{2} F_1\left(\frac{1}{2}, \frac{-d + z + 2\theta}{2(d + dz - 2\theta)}; \frac{d(2z) - 2\theta}{2(d + dz - 2\theta)}; \frac{a^{2+z-\frac{4\theta}{d}}}{z - 2\theta/d}\right)\right),
\]

where \( V_{\text{tot}}(\theta, z) = V_1(\theta, z) - E x(\theta, z). \) In Fig. 4 we show the effect of the hyperscaling parameter \( \theta \) on the barrier potential. In the left and right plots, the dynamical exponent is \( z = 1 \) and \( z = 2 \), respectively. It is clearly seen that by increasing \( z \) the height and width of the barrier decreases. One finds also that increasing \( \theta \) leads to increasing the height and the width of the barrier. It is interesting that increasing hyperscaling parameter \( \theta \) and dynamical exponent \( z \) have different effects on the shape of the barrier potential. Then one can change the shape of the barrier potential by changing the values of these parameters. As a result, the rate of producing the \( q\bar{q} \) pairs depends on these parameters. It means that pair production is easier when \( z \) increases.

4 Anisotropic scaling in spatial direction

We have considered an anisotropic scaling symmetry in time and spatial direction. It would be interesting to discuss the case of anisotropic scaling in one of the spatial directions. Then one may follow \cite{28, 29} and consider anisotropic scaling in one of the spatial directions. Using a double Wick rotation in (6) as \( t \to iy \) and \( x_d \to it \) one finds the corresponding background as

\[
ds^2 = \frac{L^2}{r^{2\theta/d}} \left( -r^2 dt^2 + \frac{dy^2}{r^{2z}} + r^2 dx^2 + \frac{dr^2}{r^2}\right).
\]
Figure 3: The barrier potential versus the inter quark distance in hyperscaling violation geometry. Left: $z = 1$. Right: $z = 2$. In all of the plots $d = 3$, $b = 0.6$ and from top to bottom $\theta = 2, 1$.

We turn on the external electric field in $x$ and $y$ directions, respectively. It is straightforward to find the related equations in this geometry. If we assume $q\bar{q}$ pair in the $x$ direction and turn on the external electric field, one finds the same behavior in the Fig. [1] It means that by increasing $\theta$ the barrier potential becomes stronger. Next, we consider the $q\bar{q}$ pair in the $y$ direction and turn on the electric field. In this case both of parameters i.e. $\theta$ and $z$ appear in the total potential equation. Then by changing them the shape of the barrier also changes, significantly.

The type IIB solutions dual to Lifshitz theories with anisotropic scaling symmetry have been discussed in [30]. They consider the following anisotropic spacetime

$$ds^2 = -r^{2z}dt^2 + r^{2z}dx^2 + r^2dy^2 + \frac{dr^2}{r^2}.$$  \hspace{1cm} (18)

These solutions describe anisotropic theory in the IR regime and the $AdS_5$ solutions in the UV regime. They considered geometries with intersecting D3 and D7 branes. The energy loss of heavy quarks in this background has been studied in [31]. The computation of the total potential of the Schwinger effect in this background is straightforward and one finds the same results as before. Simply, one should turn on the electric field in the different directions and study whether the barrier potential gets stronger or weaker.

5 Conclusion

In this paper, we have studied the holographic Schwinger effect in the non-relativistic setting. We considered Lifshitz and hyperscaling violation theories. They are strongly coupled theories with an anisotropic scaling symmetry in time and spatial direction. An understanding of how this effect changes by these theories may be essential for theoretical predictions. We discussed how the barrier potential changes in the presence of non-relativistic parameters $z$ and $\theta$. One motivation for studying the behavior of the barrier potential is that the tunneling trajectory which is a circle in the relativistic case, may not be so simple any more for $z \neq 1$. We found the analytical solutions for distance $d$ and total potential. It was shown that the shape of the potential barrier depends on these parameters. As a result, we found qualitatively how the production rate of $q\bar{q}$ pairs change. In this way, one concludes that the creation rate for the pair creation will not be the same as the relativistic case. By increasing $z$, $q\bar{q}$ pairs produce easier while increasing $\theta$ leads to harder production of $q\bar{q}$
pairs. It will be very interesting to investigate the creation rate of the $q\bar{q}$ pair by evaluating the imaginary part of the DBI action in the case of the Lifshitz geometry. Also, the analysis of the barrier potential could be generalized to the finite temperature.

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