Variable methods to estimate the ionospheric horizontal gradient

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Abstract. DGPS or differential Global Positioning System is a system where the range error at a reference station (after eliminating the error due to its’ clock, hardware delay and multipath) will be eliminated from the range measurement at the user, which view the same satellite, presuming that the satellites path to both the reference station and the user experience common errors due to the ionosphere, clock errors etc. In this assumption, the error due to the ionospheric refraction is assumed to be the same for the two closely spaced paths (such as a baseline length between reference station and the user of 10km as used in simulations throughout this paper, unless otherwise stated) and thus the presence of ionospheric horizontal gradient is ignored. If a user’s path is exposed to a drastically large ionosphere gradient, the large difference of ionosphere delays between the reference station and the user can result in significant position error for the user. Several examples of extremely large ionosphere gradients that could cause the significant user errors have been observed. The ionospheric horizontal gradient could be obtained instead from the gradient of the Total Electron Content, TEC observed from a number of received GPS satellites at one or more reference stations or based on empirical models updated with real time data. To investigate the former, in this work, the dual frequency method has been used to obtain both South-North and East-West gradients by using four different receiving stations separated in those directions. In addition, observation data from Navy Ionospheric Monitoring System (NIMS) receivers and the TEC contour map from Rutherford Appleton Laboratory (RAL) UK have also been used in order to define the magnitude and direction of the gradient.

1. Introduction
DGPS is a system where the range error at a reference station (after eliminating the error due to its’ clock, hardware delay and multipath) will be eliminated from the range measurement at the user, which view the same satellite, presuming that the satellites path to both the reference station and the user experience common errors due to the ionosphere, clock errors etc. In this assumption, the error due to the ionospheric refraction is assumed to be the same for the two closely spaced paths and thus the presence of ionospheric horizontal gradient is ignored. If a user’s path is exposed to a drastically large ionosphere gradient, the large difference of ionosphere delays between the reference station and the user can result in significant position error for the user. Several examples of extremely large ionosphere gradients that could cause the significant user errors have been observed. As an example, Datta-Barua et al. [1] had found an ionosphere gradient as large as 316mm/km which is more than 60 times the one-sigma nominal gradient (one-sigma nominal gradient is at 3~5mm/km) [2-3].

The ionospheric horizontal gradient could be obtained instead from the gradient of the TEC observed from a number of received GPS satellites at one or more reference stations or based on empirical models updated with real time data. To investigate the former, as an example, the dual frequency method has been used to obtain both South-North and East-West gradients by using four different
receiving stations separated in those directions. In addition, observation data from NIMS receivers and the TEC contour map from RAL UK have also been used in order to define the magnitude and direction of the gradient.

2. Determining the magnitude of the gradient from GPS data

There are a number of ways in which an approximate value for the gradient could be determined to input into the empirical formula using either a 3D ionosphere model such as IRI or real-time measurements or a model updated with real-time data.

2.1 Gradient from four GPS observation stations

The dual frequency model has been used to obtain the slant TEC ($sTEC$) from four different receiving stations separated in both South-North and West-East directions. Then, by using appropriate simultaneous equations, both the South-North and West-East gradients in units $\delta TEC/°lat$ and $\delta TEC/°longi$ respectively can be obtained. The latitudinal gradient ($g_1$) is considered positive in the South-North direction and the longitudinal gradient ($g_2$) is considered positive in the West-East direction.

Figure 1 below shows the four GPS stations that have been used in this gradient determination method. Those stations are being operated by the UK Ordnance Survey. They are Church Lawford Met at $(52.36°N, 1.33°W)$ as station A, Leeds at $(53.8°N, 1.66°W)$ as station B, Lincoln $(53.25°N, 0.52°W)$ as station C and Nottingham $(52.96°N, 1.2°W)$ as station D.

![Figure 1. Approximate location of four GPS receiver stations in UK used to determine the magnitude and direction of the ionospheric horizontal gradient.](image)

The simultaneous equations that were used to obtain $g_1$ and $g_2$ are as follows;

\begin{align*}
TEC_{sb} &= TEC_{sa} + g_1(x_B - x_A) + g_2(y_B - y_A) \quad (1) \\
TEC_{sc} &= TEC_{sa} + g_1(x_C - x_A) + g_2(y_C - y_A) \quad (2) \\
TEC_{sd} &= TEC_{sa} + g_1(x_D - x_A) + g_2(y_D - y_A) \quad (3)
\end{align*}

A, B, C and D stand for the respective reference stations. $x$ and $y$ represent the latitude and longitude respectively and $g_1$ and $g_2$ represent the gradient of TEC in latitude and longitude respectively.

To obtain $sTEC$ from every station for a period of time, the dual frequency model was used;

\[
TEC_{slant} = \frac{1}{40.3} \left( \frac{f_2^2 \cdot f_2^2}{f_2^2 - f_2^2} \right) (P_2 - P_1) \quad (4)
\]
where \( f_1 \) is 1557.42MHz (\( L_1 \)), \( f_2 \) is 1227.6MHz (\( L_2 \)), \( P_1 \) and \( P_2 \) are GPS pseudorange measurements on \( L_1 \) and \( L_2 \) respectively.

2.2 Gradient from NIMS observation data

Another method that has been tried to obtain the magnitude of the gradient was to use the NIMS observation data. Since NIMS satellites are polar orbiting satellites, their observation data should be able to give the magnitude of the horizontal gradient in the latitudinal direction.

The Navy Ionospheric Monitoring System, briefly known as NIMS satellites (formerly known as NNSS), are polar orbiting satellites at an altitude of 1100km above the Earth [4]. It is a constellation of 6 satellites. These navigation satellites were used by the US Navy before GPS became available. NIMS satellites transmit in the form of dual frequencies at 150 and 400 MHz. The sTEC along each path of the ray was obtained by a chain of ground-based NIMS receiver. Since the satellites are in low-Earth polar orbit (LEO), they move latitudinally across the sky to provide the scan of the ionosphere. Although NIMS could measure only the ionisation below the altitude of the satellite’s orbit of about 1100km, compare with GPS which can measure up to 20200km altitude, at least during the day and for solar maximum, the TEC derived from NIMS well represents the total TEC to higher altitude.

Pryse et al. [5] obtained the TEC data from 3 NIMS Earth receiver stations in UK. These stations were Aberystwyth (52.42°N, 4.06°W), Hawick (55.42°N, 2.80°W) and Reay (58.57°N, 3.76°W). To obtain the sTEC, they processed the observations from individual satellites that pass over these stations by using standard, well documented techniques to produce the images of electron density in the height versus latitude plane. Then, vertical integration over the electron density was done to obtain the vertical TEC (vTEC), as a function of latitude for each satellite pass. A set of parameters was defined in the plot of vTEC versus latitude in order to represent the position and shape of electron density troughs. The parameters were chosen so that both the location of the trough and the form of the latitudinal gradients could be characterized. A schematic diagram showing these parameters is shown in figure 2.

![Figure 2. Schematic diagram of TEC versus latitude, illustrating the parameters defined to characterize the location and shape of the trough [5].](image-url)

D_E and D_P are the limits of the usable data range, where E and P refer to the equatorward and poleward extremes respectively. The average vTEC was calculated for the entire range of the data between the extreme limits and was used to define the locations of two ‘breakpoints’ on the equatorward, B_E and poleward, B_P sides of the trough minimum. H_E and H_P are the halfway points between the minimum and equivalent breakpoint. Data on the poleward maximum, L_P, were used to define the poleward gradient of the trough wall.
The $vTEC$ in between all these points have been compared with the $vTEC$ from IRI at the same time, day and location. The comparison was done in order to see if there is any correlation in the value of $vTEC$ from NIMS and IRI. If there is correlation, it will be helpful to obtain the magnitude of the ionospheric gradient and eventually will be helpful to improve the accuracy of location in DGPS. The work than furthered to define the magnitude of the gradient in DGPS by using the tomographic images of the ionosphere from RAL UK’s TEC map.

2.3 Gradient from RAL UK’s TEC Map
The Rutherford Appleton Laboratory, RAL UK provides the real-time $vTEC$ profile (ionospheric tomographic images or contour of TEC) for the whole of Europe. The contour maps are updated every 10 minutes in order to see if there is any change in the structure of the TEC [6]. The contour was obtained by using the daily GPS Observation data provided by the International GPS Service for Geodynamics (IGS). The $vTEC$ was computed by using the method developed by [7]. The ionospheric pierce point (IPP) was fixed at 400km. However, for this work, the TEC contour was taken only for the region of the UK.

Tomography is a technique that has been developed from a medical diagnostics tool to become an imaging technique for many applications. Tomographic imaging of the ionised atmosphere is one of the most recent advances in the use of TEC measurements. By using this method, the measured $sTEC$ from a number of satellites can be inverted in a reconstruction algorithm to produce the two-dimensional spatial distribution of the electron concentration throughout the region of the ray-path intersections at a fixed IPP of 400km.

Several authors have reviewed the application of computerised reconstruction of radio tomographic imaging of the ionosphere. Leitinger [8] has explained the basis and the limitations of the ionospheric application. Kunitsyn et al. [9] have also derived a number of theoretical formulations regarding ionospheric tomography. Pryse [10] has analyzed the experimental results from the application of radio tomography to ionospheric imaging. All the development work carried out has demonstrated that radio tomography is really reliable in providing images of large scale spatial structures in a wide area of the ionosphere from a limited number of ground stations. Yizengaw et al. [11] has also expressed the view that tomographic imaging technique has the potential to provide a more accurate description of the average electron density of the ionosphere. Structures such as troughs, traveling ionospheric disturbances (TIDs), polar cap patches and horizontal gradients are some of the ionospheric features suitable for study by this technique.

3. Results and analysis

3.1 GPS observation stations
By using the simultaneous Equations 1, 2 and 3 and the $sTEC$ values from Equation 4, the value of $g_1$ and $g_2$ were obtained for the same period of time. The $sTEC$ can be seen in figure 3, whereas the $g_1$ and $g_2$ can be seen in figure 4.

It can be seen that the gradients $g_1$ and $g_2$ change quite rapidly (~every minute) and some temporal averaging seems to be needed. Though the temporal averaging can be done, the drastic variation of $g_1$ and $g_2$ does not seems to be helpful to give better gradients ($g_1$ and $g_2$) solution. Without a better $g_1$ and $g_2$, the idea of improving the position of a user in the vicinity of the four stations (figure 1) by inputting the effect of latitudinal and longitudinal gradient ($g_1$ and $g_2$ respectively), cannot be done.
3.2 NIMS observation stations

In order to determine this correlation, the difference between vTEC from NIMS and IRI was obtained on a few days in the months of September 2002, January 2003 and June 2003. Some of the findings are shown in figures 5, 6, 7 and 8 below.

**Figure 3.** The sTEC at four different stations for a 75 minute daytime period.

**Figure 4.** The value of $g_1$ and $g_2$ for the same duration of time as for figure 2.

**Figure 5.** Difference in vTEC for NIMS and IRI at early morning in September, 2002.
From all those figures above, it can be seen that there is no correlation at all between $vTEC$ from NIMS and IRI. Other than that, the $vTEC$ was also not available from NIMS for each and every day.
for the same time and same location (same range of latitude). This is the reason the plot of difference between \(vTEC\) from NIMS and IRI on different days varies at different locations of latitude. Further, NIMS \(vTEC\) data could only show the variation of TEC in latitude and not in longitude.

To further the analysis, comparison was also done between the variation of \(vTEC\) derived from NIMS and IRI with the \(K_p\) index. The \(K_p\) index is derived from the standardized K index from 13 magnetic observatories. It was found out that the correlation between the TEC from real observation data and IRI is good for higher \(K\) indices [12-13].

\(K_p\) indices were obtained from NGDC. Figures 9(a), 9(b) and 9(c) below show the \(K_p\) indices on the days the comparison was done between the \(vTEC\) from NIMS and IRI.

Figure 9(a). \(K_p\) indices at every 3 hours for a 24 hour period on some particular days of the month in September 2002.

Figure 9(b). \(K_p\) indices at every 3 hours for a 24 hour period on some particular days of the month in February 2003.

Analysis has been carried out to see if the \(K_p\) index does give any indication of the variation between the \(vTEC\) from NIMS and IRI. As an example, from figure 5 above, the variation of the \(vTEC\) from NIMS and IRI from 6 to 9am on 14 September 2002 is increasing as latitude increases. When we see the \(K_p\) index from Figure 9(a) for that duration of time on the same day, the index decreases. It shows that the variation of \(vTEC\) between NIMS and IRI does not correlate with the variation of the \(K_p\) index. The same goes for the \(K_p\) indices for other NIMS and IRI differences. However, another analysis has
been done to investigate the variation of $K_p$ indices itself for 24 hours a day for seven consecutive days.

\[ \text{Figure 9(c). } K_p \text{ index indices at every 3 hours for a 24 hour period on some particular days of the month in June 2003.} \]

\[ \text{Figure 10(a). } K_p \text{ indices for seven consecutive days on September 2002.} \]

\[ \text{Figure 10(b). } K_p \text{ indices for seven consecutive days on February 2003.} \]
Figure 10(c). $K_p$ indices for seven consecutive days on June 2003.

As was mentioned above, the plots of the $K_p$ indices for seven consecutive days have been done in order to see the coherence between the indices from day 1 and 2, day 2 and 3 and so on and so forth. Unfortunately, from all those figures above, 10(a), 10(b) and 10(c), the correlation of the $K_p$ index on consecutive days is very poor. This poor correlation of the $K_p$ index cannot be used to explain the variation of $\nu$TEC between NIMS and IRI.

3.3 RAL’s UK TEC

Figure 11. Contours of the TEC map on 29 March 2004 at 06:05UT (RAL, UK).
Figure 11 shows the contour map of TEC from RAL UK that was obtained on 29 March 2004 at 06:05UT. In order to know the $vTEC$ at every degree of latitude, the contours of TEC will be interpolated linearly at every 1° of latitude at the longitude of 358°E. By doing so, the $vTEC$ that is obtained at each degree of latitude can be compared with the $vTEC$ from IRI at 400km. Contours of TEC were taken at dawn because the correlation of this $vTEC$ with IRI is normally good at this time.

![Figure 12](image)

**Figure 12.** $vTEC$ from RAL contour map (after linear interpolation) and IRI at dawn.

As expected, the $vTEC$ from the contours correlates very well with the $vTEC$ from IRI at dawn. This has shown that we can obtain the magnitude of gradient from IRI for this particular time and day. The numerical differentiation then was done to obtain the magnitude of the gradient (in unit $dTEC/dLat$) as shown in figure 13.

![Figure 13](image)

**Figure 13.** Gradient of TEC from RAL’s contour map and IRI at dawn.

The differentiation ($dTEC/dLat$) also shows that the magnitude of the gradient does agree well between the $vTEC$ from contour map and IRI values. Another contour map of TEC was also chosen to do the same analysis, but at dusk rather than dawn.
Figure 14. Contours of the TEC map on 29/3/2004 at 19:05UT (RAL, UK).

Figure 14 shows the variation of $v\text{TEC}$ is much larger than the variation of $v\text{TEC}$ at dawn (figure 13). So, in order to get the better $v\text{TEC}$ observation data, the interpolation was done at every 0.5° of latitude. Using the same method as before, the $v\text{TEC}$ from the contour map at every 0.5° is compared with the $v\text{TEC}$ from IRI, as shown in figure 15 below.

Figure 15. $v\text{TEC}$ from RAL contour map (after linear interpolation) and IRI at dusk.

Again, it shows that the $v\text{TEC}$ from the RAL UK’s TEC map and IRI correlate very well. This will make us easier to obtain the magnitude of the gradient. The plot of $d\text{TEC}/d\text{Lat}$ is shown below.
Figure 16. Gradient of TEC from RAL’s contour map and IRI at dusk.

It shows that IRI really correlates well with the contour of TEC from RAL for the UK region, both at dawn and dusk. Both are giving about the same magnitude of gradient. However, figures 13 and 16 show that the magnitude of the gradient appears to be the same for a range of 1° of latitude, which is about 111km in baseline length. This is because, in region like UK, which lies in geographical mid-latitude of Northern Hemisphere, the horizontal variation of electron density latitudinally is generally not as large as in the equatorial region, where a significant latitudinal variation of electron density can occur for a short a distance as 10km of baseline length. Nevertheless, since the magnitude of the gradient can be obtained from RAL UK’s TEC map, this can be used to show the improvement in the final position of a user station in DGPS.

4. Conclusion
In order to make a correction in practice, the horizontal electron density gradient needs to be known. It can be estimated from the gradient in $sTEC$ observed from the received GPS satellites (since it has also been shown that the ratio of $sTEC$ gives the ratio of electron density at the IPP) or based on an empirical model updated with real-time data, or both. As described above, a number of methods have been used to obtain the magnitude and direction of the gradient from real data. However, all of these methods have their own advantageous and disadvantages.

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