Probabilistic assessment of permanent displacement of soil slopes: how many ground motions are needed?

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Abstract: As the occurrence of earthquakes is highly uncertain, the permanent displacement of a slope during a given exposure time is also uncertain. Probabilistic analysis can be used to assess the effect of uncertainty in the ground motions on the permanent displacement. However, the number of ground motions to be considered in a probabilistic assessment of permanent displacement of a slope is still not clearly defined. In this paper, a numerical study is conducted to investigate the effect of number of ground motions on the variability of fragility curves and the hazard curve of the permanent displacement. It is found that for the slope study in this paper, the median fragility curve and hazard curve are not sensitive to the number of ground motions. The width of the confidence interval of the fragility curve can be reduced by half when the number of ground motions is increased by four times. It can be learned from the COV of the hazard curve of permanent displacement that the determination of the number of ground motion records should be based on a concerned critical permanent displacement.

1. Introduction

To overcome the drawbacks of conventional pseudo-static analysis [1], there is an increasing interest on assessing the performance of slopes through permanent displacement, e.g., [2-4]. To consider the uncertainty in the ground motions, probabilistic methods have been suggested for permanent displacement analysis, e.g., [2]. As the required computational burden generally increases with the number of ground motions [5], how to determine the number of ground motions used in probabilistic permanent displacement analysis is an important issue.

In structural engineering, studies have been conducted on the number of ground motions to be
considered in performance-based design of structures. Shome et al. [6] and Hancock et al. [7] found that 20 or 25 ground motions are adequate to accurately estimate the median seismic response of a structure when the ground motion records were selected and scaled according to a target spectra. Eads et al. [8] found that when 50 ground motion records is used for a given specific intensity level in the incremental dynamic analysis, the uncertainty in the fragility curve can be considerable. Gehl et al [9] indicated that to reduce the coefficient of variation (COV) of the parameters of the fragility curve to be less than 8% in the incremental dynamic analysis, the number of ground motions to be used for a given intensity measure level should not be less than 40. Jalayer et al. [10] observed that the COV of the statistics of the fragility curve is greater than 20% when the number of ground motions is 20, when the cloud method is used for developing the fragility curve. Based on incremental dynamic analysis, Baltzopoulous et al. [11] found that to ensure a COV of 10% for the failure rate of a structure, the number of ground motions that should be used for a given intensity measure level may varies between 40 and 100, depending on the efficiency of the adopted intensity measure and the characteristics of the seismic hazard of the site. For slope stability problems, however, very few studies have been made to examine the effect of number of ground motions on the fragility curve and the failure rate of the slope.

The objective of this paper is to investigate the impact of the number of ground motions on probabilistic assessment of permanent displacement of slopes. The structure of this paper is as follows. Firstly, the slope under study is briefly described. Then, the method for probabilistic assessment of slopes is introduced. Finally, the influences of the number of ground motions on the probabilistic permanent displacement assessment are discussed.

![Figure 1. Geometry of the study slope and mean annual exceedance rate of PGA at the site.](image)

2. Study slope
A simple slope at a hypothetical site is studied here, as shown in Figure 1. The cohesion and the friction angle of the soil are 10.0 kPa and 20.0°, respectively. The average shear-wave velocity from the surface to a depth of 30 m of the base rock is \( V_{s30} = 760 \text{ m/s} \). Assume the slope is potentially affected by a point-source rupture with a Joyner-Boore distance of \( R_{JB} = 10 \text{ km} \). Suppose the occurrence of the earthquake follows the bounded Gutenberg and Richter reoccurrence law [12] as
follows

\[ f_M(m) = \frac{b \ln(10) \cdot 10^{-b(m-m_{\text{min}})}}{1-10^{-b(m_{\text{max}}-m)}} \]

where \( m \) denotes the earthquake magnitude and \( f_M(m) \) is the probability density function of earthquake magnitude, \( b \) is a constant, \( m_{\text{max}} \) is the maximum earthquake that a given source can produce and \( m_{\text{min}} \) is the minimum magnitude and the earthquake with a magnitude smaller than \( m_{\text{min}} \) will be ignored. In this paper, \( b \) is determined as 1.0, \( m_{\text{max}} \) and \( m_{\text{min}} \) are set as 8.0 and 5.0, respectively. In this paper, the ground motion prediction equations proposed by Boore and Atkinson [13] are used to develop the intensity measure hazard curve of the hypothetical site. As an example, Figure 1 shows the hazard curve of \( PGA \) at this site.

3. Methods for slope probabilistic permanent displacement assessment

3.1. Permanent displacement analysis method

In this paper, the coupled method [14] is used to calculate the permanent displacement of the slope. When the acceleration of the potential sliding mass is below the critical acceleration of the slope, no sliding will occur. The governing equation in such a case is as follows [14]

\[ A\ddot{u} + C\dot{u} + Ku = -A\cdot 1 \cdot \ddot{u}_g \]

(2)

where \( A \) is mass matrix; \( C \) is damping matrix; \( K \) is stiffness matrix; \( \ddot{u} \) is the vector of nodal relative acceleration; \( \dot{u} \) is the vector of nodal relative velocities; \( u \) is vector of nodal relative displacements; \( \ddot{u}_g \) is the acceleration-time history at the base; \( 1 \) is a vector of order \( R \) (\( R \) is the number of degrees of freedom) composed of 1. When the acceleration of the potential sliding mass exceeds the critical acceleration of the slope, sliding will be initiated. The dynamic response of the sliding mass and the relative movement between the sliding mass and the base are evaluated via governing motion equations as follows [14]:

\[ A\ddot{u} + C\dot{u} + Ku = -A\cdot 1 \cdot (\ddot{s} + \ddot{u}_g) \]

(3)

\[-A_T \left( \ddot{s} + \ddot{u}_g \right) - 1^T A \cdot \ddot{\bar{u}} = \mu A_T g \]

(4)

where \( \ddot{s} \) is the sliding acceleration at the base of the sliding mass; \( A_T \) is the total mass above the sliding interface; and \( \mu \) is the coefficient of friction. The permanent displacement can be calculated by solving Eqs. (3) and (4). In this paper, SLAMMER [15] will be used to solve the above equations to obtain the permanent displacement of a slope.

3.2. Ground motion selection method

The algorithm as suggested in Bradley [16] is used in this paper to select ground motions. The essence of this algorithm is to select as-recorded ground motions and render the empirical distribution function of the concerned intensity measure of the selected ground motion records to approximate the corresponding cumulative distribution function of the concerned intensity measure. For the permanent displacement of slopes, Bray and Travasarou [17] and Saygili and Rathje [18] suggested that the peak ground acceleration (\( PGA \)) and peak ground velocity (\( PGV \)) are optimal intensity measures. Thus,
PGV and PGA are adopted in this paper and PGA is regarded as the conditioning intensity measure. One may refer to Bradley [16] for more details of the ground motion selection algorithm.

3.3. Fragility curve and hazard curve

When the number of ground motions is limited, the developed fragility curve and hazard curve will be subjected to statistical uncertainty. In this study, the effect of the number of ground motions records on the probabilistic permanent displacement assessment will be assessed through the variabilities of the slope fragility curves and permanent displacement hazard curves.

For a given PGA, the slope may experience permanent displacement when subjected to one ground motion record but may not experience permanent displacement under another ground motion record. To consider the above uncertainty associated with the occurrence of the permanent displacement, the occurrence probability of the permanent displacement can be predicted through an equation as follows [19]:

$$P(S \leq S_d) = 1 - \Phi(c_1 + c_2 \ln PGA)$$  \hfill (5)

where $S_d$ is a threshold permanent displacement below which the permanent displacement is considered as negligible, and $c_1$ and $c_2$ are two parameters to be calibrated. In this study, $S_d = 0.01$ m is adopted [19]. To calibrate these parameters, one can evaluate the permanent displacements of the slope at $m$ PGA levels. At each PGA level, suppose $N$ ground motions are used. As such, $m \cdot N$ values of the permanent displacements can be obtained, based on which the values of $c_1$ and $c_2$ can then be calibrated based on the maximum likelihood method [19].

If the permanent displacement occurs at a given PGA, its magnitude may also vary with the ground motions. The fragility function can be used to consider such an uncertainty. Assume that $S$ is a linear function of PGA in the logarithmic space [19], the relationship between $S$ and PGA can be written as follows

$$\ln S = \ln q + k \cdot \ln PGA + \varepsilon \sim N(0, \beta^2)$$  \hfill (6)

where $\ln q$, $k$ and $\beta$ are parameters to be calibrated. When calibrating the parameters of Eq. (2), $m \cdot N$ values of the permanent displacements have been obtained. Out of these $m \cdot N$ permanent displacements, the displacements with values greater than $S_d$ can then be used to calibrate the above linear function based on the method of least-squares regression, e.g., [9].

After Eq. (6) is established, the probability that the permanent displacement $S$ is greater than a critical threshold of a damage stage, i.e., the fragility function, can be written as follows [9]

$$P(S > S_d \mid PGA, S > 0.01 \text{ m}) = \Phi\left[k \ln PGA - (\ln S_d - \ln q)\beta\right]$$  \hfill (7)

After the fragility function is obtained, the annual rate of exceedance of the permanent displacement, i.e., the hazard curve, can be calculated using the following equation, e.g., [20]:

$$\lambda_S(S > S_d) = \left[1 - P(S \leq 0.01 \text{ m})\right] \sum_{i=1}^{20} P(S > S_d \mid PGA = pga_i, S > 0.01 \text{ m}) \Delta \lambda_{PGA}(PGA = pga_i)$$  \hfill (8)

where $\Delta \lambda_{PGA}(PGA = pga)$ is the annual occurrence rate of an earthquake with $PGA = pga$, which can be calculated by performing the forward difference technique on the annual exceedance rate of $PGA$.
In this paper, the PGA is discretized into ten discrete values, i.e., 0.1 g, 0.2 g, ..., and 1.0 g. As such, \( m = 10 \).

### 3.4. Variability of the fragility curves and hazard curves

When a limited number of ground motions are used, the derived fragility curves and hazard curve will be subjected to statistical uncertainty. For example, the fragility curve derived based on one set of \( N = 25 \) ground motions is not exactly the same as the fragility curve derived based on another set of \( N = 25 \) ground motions. As mentioned previously, the variability of the fragility curves will be used to measure the impact of the number of ground motions. In this study, the bootstrap method [21] will be used to assess the variability of the fragility curves and hazard curves. To use such a method, the samples of permanent displacements will be first generated for the cases of \( m = 10 \) and a given \( N \), resulting \( 10\cdot N \) permanent displacement values. The bootstrap method will then be used to generate samples of the permanent displacement. By calibrating Eq. (6) using samples generated from the bootstrap method repeatedly, one can obtain different values of \( \ln q \), \( k \) and \( \beta \), and hence different fragility curves and hazard curves are evaluated without selecting additional ground motion records.

One may refer to Efron and Tibshirani [21] for more details on the bootstrap technique.

![Figure 2](image-url)

*Figure 2.* The median and 95% confidence interval of the fragility curves evaluated using different size of ground motions as \( S_d \) varies: (a) \( S_d = 0.05 \) m; (b) \( S_d = 0.10 \) m; (c) \( S_d = 0.20 \) m; (d) \( S_d = 0.40 \) m.

### 4. Effect of number of ground motions
4.1. Effect of the number of ground motions on fragility curves

Figure 2(a) shows the median as well as the 95% confidence interval of the fragility curve for the case of $S_d = 0.05$ m when the number of ground motions, i.e., $N$, is 6, 25, and 100, respectively. It can be seen from this figure that the median fragility curves evaluated using different sizes of ground motion sets are very similar, indicating that the number of ground motion records has a limited impact of the median slope fragility curves. Figure 2(a) also shows that the confidence intervals of fragility curves with different $N$ values overlap with each other, indicating that the fragility curves calculated based on different numbers of ground motions are consistent with each other. On the other hand, Figure 2(a) also reveals that as the number of ground motion records increases, the width of the confidence interval decreases, which is reasonable.

Using the same procedure, Figure 2(b)-(d) show the fragility curves of the slope calculated based on different numbers of ground motions for the cases of $S_d = 0.10$ m, 0.20 m, and 0.40 m, respectively. In these figures, the median fragility curves calculated based on different numbers of ground motions are also close to each other. Comparing Figure 2(a)-(d), it seems that the width of the confidence interval increases as the value of $S_d$ increases. This is because, as the value of $S_d$ increases, the number of samples of the permanent displacement with values greater than $S_d$ decreases, which resulting greater statistical error.

Figure 2 shows the maximum width of the 95% confidence interval of the fragility curves when the $PGA$ is fixed with different numbers of ground motions and different values of $S_d$ being adopted. As can be seen from this table, when the value of $S_d$ is the same, the width of the confidence interval obtained based on $N = 6$ is approximately 2 and 4 times of that obtained based on $N = 25$ and 100, respectively. It seems that the variability of the fragility curves as characterized by the maximum width of the confidence interval can be roughly reduced by a half when the number of ground motions used is increased by four times. Such a phenomena is also observed in Shome et al. [6], Hancock et al. [7] and Baltzopoulos et al. [11].

### Table 1. The maximum width of the 95% confidence interval of the fragility curves.

| $N$ | $S_d$ (m) | 0.05 | 0.10 | 0.20 | 0.40 |
|-----|-----------|------|------|------|------|
| 6   |           | 0.279| 0.318| 0.441| 0.579|
| 25  |           | 0.162| 0.136| 0.218| 0.305|
| 100 |           | 0.067| 0.076| 0.108| 0.162|

4.2. Effect of the number of ground motion records on hazard curves

Figure 3 presents the median and the 95% confidence intervals of the annual exceedance rate of the permanent displacement of the slope, i.e., hazard curve, when the number of ground motion records is $N = 6$, 25, and 100, respectively. The median hazard curves are also quite close to each other, indicating that the median hazard curve can be estimated using a small number of ground motions. Figure 3 also shows that the confidence interval generally shrinks as the number of ground motion records used increases.

Based on Figure 3, Table 2 shows the COVs of hazard curves when different number of ground motions are used as the value of $S_d$ varies. As can be seen from this table, when the $S_d$ is fixed, the
COV of hazard curve generally decreases as $N$ increases, which is reasonable. When $N$ is fixed, the COV of hazard curve increases along with the growth of $S_d$. This is because, as the value of $S_d$ increases, the number of ground motions that results in a permanent value greater than $S_d$ tends to decrease, which then may lead to greater statistical uncertainty in the fragility curve. Therefore, the accuracy of the hazard curve does not depend solely on the number of ground motions, and it is also affected by the value of $S_d$.

Based on Table 2, one then can determine the required number of ground motions based on the desired accuracy of the hazard curve and the target hazard level. For example, if the target COV of the hazard curve is 20% (e.g., [11]) and the target hazard level is $S_d = 0.20$ m, then $N \geq 25$ should be used for the slope studied in this paper.

**Table 2.** The COVs of hazard curves with $N = 6$, 5 and 100.

| $N$  | $S_d$ (m) | 0.05 | 0.10 | 0.20 | 0.40 |
|------|-----------|------|------|------|------|
| 6    | 0.189     | 0.194| 0.251| 0.411|
| 25   | 0.111     | 0.138| 0.185| 0.256|
| 100  | 0.056     | 0.066| 0.087| 0.135|

**Figure 3.** The median and the 95% confidence interval of the permanent displacement hazard curves with different size of ground motion sets.

5. Conclusions
In this paper, the effect of the number of ground motion records on the probabilistic assessment of permanent displacement of a slope is studied. The conclusions obtained from this paper are summarized as follows:

- For the slope studied in this paper, the medium fragility curve and hazard curve are not sensitive to the number of ground motions.
• The width of confidence interval of the fragility curve can be reduced by half when the number of ground motions is increased by four times.
• The number of ground motion records required depends on the permanent displacement of slopes and the demanded COV of the hazard curve.

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