Limits on temporal variation of quark masses and strong interaction from atomic clock experiments

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We perform calculations of the dependence of nuclear magnetic moments on quark masses and obtain limits on the variation of \(m_q/\Lambda_{QCD}\) from recent atomic clock experiments with hyperfine transitions in H, Rb, Cs, Hg\(^+\) and optical transition in Hg\(^+\).

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I. INTRODUCTION

Interest in the temporal and spatial variation of major constants of physics has been recently revived by astronomical data which seem to suggest a variation of the electromagnetic constant \(\alpha = e^2/hc\) at the \(10^{-5}\) level for the time scale 10 billion years, see \(\cite{1}\) (a discussion of other limits can be found in the review \(\cite{2}\) and references therein). However, an independent experimental confirmation is needed.

The hypothetical unification of all interactions implies that variation of the electromagnetic interaction constant \(\alpha\) should be accompanied by the variation of masses and the strong interaction constant. Specific predictions need a model. For example, the grand unification model discussed in \(\cite{3}\) predicts that the quantum chromodynamic (QCD) scale \(\Lambda_{QCD}\) (defined as the position of the Landau pole in the logarithm for the running strong coupling constant) is modified as follows

\[
\frac{\delta \Lambda_{QCD}}{\Lambda_{QCD}} \approx 34 \frac{\delta \alpha}{\alpha}
\]

(1)

The variation of quark and electron masses in this model is given by

\[
\frac{\delta m}{m} \sim 70 \frac{\delta \alpha}{\alpha}
\]

(2)

This gives an estimate for the variation of the dimensionless ratio

\[
\frac{\delta (m/\Lambda_{QCD})}{(m/\Lambda_{QCD})} \sim 35 \frac{\delta \alpha}{\alpha}
\]

(3)

The large coefficients in these expressions are generic for grand unification models, in which modifications come from high energy scales: they appear because the running strong coupling constant and Higgs constants (related to mass) run faster than \(\alpha\). This means that if these models are correct the variation of masses and strong interaction may be easier to detect than the variation of \(\alpha\).

Unlike for the electroweak forces, for the strong interaction there is generally no direct relation between the coupling constants and observable quantities. Since one can measure only variation of the dimensionless quantities, we want to extract from the measurements variation of the dimensionless ratio \(m_q/\Lambda_{QCD}\) where \(m_q\) is the quark mass (with the dependence on the normalization point removed). A number of limits on variation of \(m_q/\Lambda_{QCD}\) have been obtained recently from consideration of Big Bang Nucleosynthesis, quasar absorption spectra and Oklo natural nuclear reactor which was active about 1.8 billion years ago \(\cite{4, 5, 6, 7}\) (see also \(\cite{8, 9, 11, 12}\)). Below we consider the limits which follow from laboratory atomic clock comparison. Laboratory limits with a time base about a year are especially sensitive to oscillatory variation of fundamental constants. A number of relevant measurements have been performed already and even larger number have been started or planned. The increase in precision is very fast.

It has been pointed out by Karshenboim \(\cite{13}\) that measurements of ratio of hyperfine structure intervals in different atoms are sensitive to variation of nuclear magnetic moments. First rough estimates of nuclear magnetic moments dependence on \(m_q/\Lambda_{QCD}\) and limits on time variation of this ratio have been obtained in our paper \(\cite{10}\). Using H, Cs and Hg\(^+\) measurements \(\cite{14, 15}\), we obtained the limit on variation of \(m_q/\Lambda_{QCD}\) about \(5 \times 10^{-13}\) per year. Below we calculate the dependence of nuclear magnetic moments on \(m_q/\Lambda_{QCD}\) and obtain the limits from recent atomic clock experiments with hyperfine transitions in H, Rb, Cs, Hg\(^+\) and optical transition in Hg\(^+\). It is convenient to assume that the strong interaction scale \(\Lambda_{QCD}\) does not vary, so we will speak about variation of masses.

The hyperfine structure constant can be presented in the following form

\[
A = \text{const} \times \left[ \frac{m_e e^4}{\hbar^2} \right] [\alpha^2 F_{rel}(Z\alpha)][\mu_e m_e/M_p]
\]

(4)

The factor in the first bracket is an atomic unit of energy. The second “electromagnetic” bracket determines the dependence on \(\alpha\). An approximate expression for the relativistic correction factor (Casimir factor) for s-wave electron is the following

\[
F_{rel} = \frac{3}{\gamma(4\gamma^2 - 1)}
\]

(5)

where \(\gamma = \sqrt{1 - (Z\alpha)^2}\), \(Z\) is the nuclear charge. Vari-
tion of $\alpha$ leads to the following variation of $F_{rel}$ \[14\]:

$$\frac{\delta F_{rel}}{F_{rel}} = K \frac{\delta \alpha}{\alpha}$$

(6)

More accurate numerical many-body calculations \[16\] of the dependence of the hyperfine structure on $\alpha$ have shown that the coefficient $K$ is slightly larger than that given by this formula. For Cs ($Z=55$) $K=0.83$ (instead of 0.74), for Rb $K=0.34$ (instead of 0.29), for Hg\(^+\) $K=2.28$ (instead of 2.18).

The last bracket in eq. \[4\] contains the dimensionless nuclear magnetic moment $\mu$ in nuclear magnetons (the nuclear magnetic moment $M = \mu \bar{m}_e$), electron mass $m_e$ and proton mass $M_p$. We may also include a small correction due to the finite nuclear size. However, its contribution is insignificant.

Recent experiments measured time dependence of the ratios of hyperfine structure intervals of $^{199}\text{Hg}^+$ and H $^{1\text{H}}$, $^{133}\text{Cs}$ and $^{87}\text{Rb}$ \[17\] and ratio of optical frequency in Hg\(^+\) and $^{133}\text{Cs}$ hyperfine frequency \[18\]. In the ratio of two hyperfine structure constants for different atoms time dependence may appear from the ratio of the factors $F_{rel}$ (depending on $\alpha$) and ratio of nuclear magnetic moments (depending on $m_q/\Lambda_{QCD}$). Magnetic moments in a single-particle approximation (one unpaired nucleon) are:

$$\mu = (g_s + (2j-1)g_l)/2$$

(8)

for $j = l + 1/2$,

$$\mu = \frac{j}{2(j+1)}(-g_s + (2j+3)g_l)$$

(9)

for $j = l - 1/2$. Here the orbital g-factors are $g_l = 1$ for valence proton and $g_l = 0$ for valence neutron. The present values of spin g-factors $g_s$ are $g_p = 4.586$ for proton and $g_n = -3.826$ for neutron. They depend on $m_q/\Lambda_{QCD}$. The light quark masses are only about 1% of the nucleon mass ($m_q = (m_u + m_d)/2 \approx 5$ MeV). The nucleon magnetic moment remains finite in the chiral limit of $m_u = m_d = 0$. Therefore, one may think that the corrections to $g_s$ due to the finite quark masses are very small. However, there is a mechanism which enhances quark mass contribution: $\pi$-meson loop corrections to the nucleon magnetic moments which are proportional to $\pi$-meson mass $m_\pi \sim \sqrt{m_q/\Lambda_{QCD}}$, $m_\pi=140$ MeV is not so small.

According to calculation in Ref. \[19\] dependence of the nucleon g-factors on $\pi$-meson mass $m_\pi$ can be approximated by the following equation

$$g(m_\pi) = \frac{g(0)}{1 + am_\pi + bm_\pi^2}$$

(10)

where $a=1.37/\text{GeV}$, $b=0.452/\text{GeV}^2$ for proton and $a=1.85/\text{GeV}$, $b=0.271/\text{GeV}^2$ for neutron. This leads to the following estimate:

$$\frac{\delta g_p}{g_p} = -0.174 \frac{\delta m_\pi}{m_\pi} = -0.087 \frac{\delta m_q}{m_q}$$

(11)

$$\frac{\delta g_n}{g_n} = -0.213 \frac{\delta m_\pi}{m_\pi} = -0.107 \frac{\delta m_q}{m_q}$$

(12)

Eqs. \[8,9,11,12\] give variation of nuclear magnetic moments. For hydrogen nucleus (proton)

$$\frac{\delta \mu}{\mu} = \frac{\delta g_p}{g_p} = -0.087 \frac{\delta m_q}{m_q}.$$  \[13\]

For $^{199}\text{Hg}$ we have valence neutron (no orbital contribution), therefore the result is

$$\frac{\delta \mu}{\mu} = \frac{\delta g_n}{g_n} = -0.107 \frac{\delta m_q}{m_q}.$$  \[14\]

For $^{133}\text{Cs}$ we have valence proton with $j=7/2$, $l=4$ and

$$\frac{\delta \mu}{\mu} = 0.22 \frac{\delta m_\pi}{m_\pi} = 0.11 \frac{\delta m_q}{m_q}.$$  \[15\]

For $^{87}\text{Rb}$ we have valence proton with $j=3/2$, $l=1$ and

$$\frac{\delta \mu}{\mu} = -0.128 \frac{\delta m_\pi}{m_\pi} = -0.064 \frac{\delta m_q}{m_q}.$$  \[16\]

Deviation of the single-particle values of nuclear magnetic moments from the measured values is about 30%. Therefore, we tried to refine the single-particle estimates. If we neglect spin-orbit interaction the total spin of nucleons is conserved. The magnetic moment of nucleus changes due to the spin-spin interaction because valence proton transfers a part of its spin $<s_z>$ to core neutrons (transfer of spin from the valence proton to core protons does not change the magnetic moment). In this approximation $g_s = (1 - b)g_p + bg_n$ for valence proton (or $g_s = (1 - b)g_n + bg_p$ for valence neutron). We can use coefficient $b$ as a fitting parameter to reproduce nuclear magnetic moments exactly. The sign of $g_p$ and $g_n$ are opposite, therefore a small mixing $b \sim 0.1$ is enough to eliminate the deviation of the theoretical value from the experimental one. Note also that it follows from eqs. \[11,12\] that $\frac{g_p}{g_n} \approx \frac{g_p}{g_n}$. This produces an additional suppression of the effect of the mixing. This indicates that the actual accuracy of the single-particle approximation for the effect of the spin g-factor variation may be as good as 10%. Note, however, that here we neglected variation of the mixing parameter $b$ which is hard to estimate.

Now we can estimate sensitivity of the ratio of the hyperfine transition frequencies to variation of $m_q/\Lambda_{QCD}$. For $^{199}\text{Hg}$ and hydrogen we have

$$\frac{\delta[A(H)/A(H)]}{[A(H)/A(H)]} = 2.3 \frac{\delta \alpha}{\alpha} - 0.02 \frac{\delta m_q/\Lambda_{QCD}}{m_q/\Lambda_{QCD}}.$$  \[17\]
Therefore, the measurement of the ratio of Hg and hydrogen hyperfine frequencies is practically insensitive to the variation of masses and strong interaction. The result of measurement \[\frac{1}{\alpha} \frac{d\alpha}{dt} < 3.6 \times 10^{-14}/\text{year}\] (18)

Other ratios of hyperfine frequencies are more sensitive to \( m_q/\Lambda_{QCD} \). For \( ^{133}\text{Cs}/^{87}\text{Rb} \) we have

\[
\frac{\delta[A(Cs)/A(Rb)]}{[A(Cs)/A(Rb)]} = 0.49 \frac{\delta\alpha}{\alpha} + 0.17 \frac{\delta[m_q/\Lambda_{QCD}]}{[m_q/\Lambda_{QCD}]} \]

Therefore, the result of the measurement \[14\] may be presented as a limit on variation of the parameter \( X = \alpha^{0.49}[m_q/\Lambda_{QCD}]^{0.17} \):

\[
\frac{1}{X} \frac{dX}{dt} = (0.2 \pm 7) \times 10^{-16}/\text{year} \quad (20)
\]

Note that if the relation \[13\] is correct, variation of \( X \) would be dominated by variation of \( [m_q/\Lambda_{QCD}] \). The relation \[13\] would give \( X \propto \alpha^2 \) and limit on \( \alpha \) variation \( \frac{\delta\alpha}{\alpha} = (0.03 \pm 1) \times 10^{-16}/\text{year} \).

For \( ^{133}\text{Cs}/\text{H} \) we have

\[
\frac{\delta[A(Cs)/A(H)]}{[A(Cs)/A(H)]]} = 0.83 \frac{\delta\alpha}{\alpha} + 0.2 \frac{\delta[m_q/\Lambda_{QCD}]}{[m_q/\Lambda_{QCD}]} \quad (21)
\]

Therefore, the result of the measurements \[15\] may be presented as a limit on variation of the parameter \( X_H = \alpha^{0.83}[m_q/\Lambda_{QCD}]^{1.2} \):

\[
\frac{1}{X_H} \frac{dX_H}{dt} < 5.5 \times 10^{-14}/\text{year} \quad (22)
\]

If we assume the relation \[16\], we would have \( X_H \propto \alpha^8 \),

\[
\frac{1}{\alpha} \frac{d\alpha}{dt} < 0.7 \times 10^{-14}/\text{year}.
\]

The optical clock transition energy \( E(Hg) \) \((\lambda=282 \text{ nm})\) in Hg\(^+\) ion can be presented in the following form:

\[
E(Hg) = \text{const} \times \left[ \frac{mc^4}{\hbar^2} \right] F_{rel}(Z\alpha) \quad (23)
\]

Note that the atomic unit of energy (first bracket) is canceled out in ratios, therefore, we should not consider its variation. Numerical calculation of the relative variation of \( E(Hg) \) has given \[16\]:

\[
\frac{\delta E(Hg)}{E(Hg)} = -3.2 \frac{\delta \alpha}{\alpha} \quad (24)
\]

Variation of the ratio of the Cs hyperfine splitting \( A(Cs) \) to this optical transition energy is equal to

\[
\frac{\delta[A(Cs)/E(Hg)]}{[A(Cs)/E(Hg)]} = 6.0 \frac{\delta\alpha}{\alpha} + \frac{\delta[m_e/\Lambda_{QCD}]}{[m_e/\Lambda_{QCD}]} + 0.11 \frac{\delta[m_q/\Lambda_{QCD}]}{[m_q/\Lambda_{QCD}]} \quad (25)
\]

Here we have taken into account that the proton mass \( M_p \propto \Lambda_{QCD} \). The factor 6.0 before \( \delta\alpha \) appeared from \( \alpha^2 F_{rel} \) in the Cs hyperfine constant \( (2+0.83) \) and \( \alpha \)-dependence of \( E(Hg) \) (3.2). Therefore, the work \[15\] gives the limit on variation of the parameter \( U = \alpha^6[m_e/\Lambda_{QCD}][m_q/\Lambda_{QCD}]^{0.1} \):

\[
\frac{1}{U} \frac{dU}{dt} < 7 \times 10^{-15}/\text{year} \quad (26)
\]

If we assume the relation \[16\], we would have \( U \propto \alpha^{45} \),

\[
\frac{1}{\alpha} \frac{d\alpha}{dt} < 1.5 \times 10^{-16}/\text{year}. \quad \text{Note that we presented such limits on } \frac{1}{\alpha} \frac{d\alpha}{dt} \text{ as an illustration only since they are strongly model-dependent.}
\]

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