Energy optimal trajectories for electro-mechanical systems

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Abstract. Electro-mechanical systems (EMS) efficiency improvement can result not only in significant energy savings but also can have positive impact on our environment. Progressive applications of EMS control as e.g. dynamical system of multi-parts robots are described by system of non-linear mutually coupled differential equations considering gravity, acceleration, Coriolis and centrifugal couplings together with parameter changes as a function of load. Using Euler-Lagrange optimization we discuss some results in decreasing energy demands for given electro-mechanical systems exploiting electric drives, which are typical for industrial and transportation applications e.g. robotic arm control, train movement control etc.

1. Introduction

Efficiency improvement of electro-mechanical systems (EMS) can result in significant energy savings and can have positive impact on environment what can be significant contribution as stated in [1]. Simple consideration can show that by halving the speed of the drive with viscous friction 50 % reduction of the energy expenditures can be achieved and for friction depending quadratically it can reach 75 %. Up to now the individual approaches to energy saving control of EMS differ mainly by formulation of energy optimized task what reflects in energy optimization which minimizes the only mechanical losses [2] or in energy optimization taking into account overall losses (mechanical and electrical) [3] or in energy savings by partially completing of energy optimal control based on sub-optimal solution [4] as is shown further. The difference also appears in respects of energy optimal control demands resulting in sub-optimal solutions. The three steps velocity function in the shape of symmetrical trapeze was proposed by Kim at al [5] for minimization of energy consumption of battery powered wheeled mobile robot. To complete this task a new objective function considering practical energy consumption was developed together with an efficient iterative search algorithm finding its numerical solution. The same speed function was used by Dodds, for sliding mode control of the drive position [6]. Both papers report nearly 30 % less energy expenditures if compared with conventional concept of control. Sheta at al developed servosystem for the drive with dc motor respecting principles of optimal control theory [7]. Similar approach was applied also by Dodds et al, for the drives with PMSM [8]. Both proposed control systems achieve minimization of input energy by decreasing of the losses via exploitation of the drive’s kinetic energy during decelerating phase of motion with respect to the speed profile and the field current. Drawbacks of these strategies are assumptions of constant friction components, which depend on environmental conditions. Position control system with the minimization of the total dissipated energy taking into account Coulomb and viscous friction has been proposed by Zhu at al [9]. The optimal drive velocity and current functions are obtained as a function of the optimal zero crossing time. Comparison of the dc motor drive consumptions for optimized algorithm and trapezoidal velocity function has revealed substantial energy savings of optimized algorithm which were proportional to a moment of inertia. Optimized trajectory of servo-drives was developed by Wang et al
[10] considering Hamiltonian approach and energy optimal trajectory respecting also electrical losses of the drive was developed by Izumi at all in [11]. Energy optimal and near-optimal control of trains is described in detail [12]. This paper states that optimal and energy efficient control techniques can substantially decrease service-expenses and increase sustainability of railway transportation. Complex models and control algorithms based on Pontryagin maximum principle capable to consider route profiles, speed limits and recuperative braking into traction net used in the last decade are capable to compute model of effective control between two stops. The task of work [13] is to optimize train speed profile with respect of train timetable while satisfying the main goal to decrease energy demands, when density of trains and transport limitations are taking into account. Two models are used to minimize energy consumption. The first one minimizes energy consumption and the second one considers possibilities of energy recuperation. Possibility to exploit symmetrical trapezoidal speed profile for position manoeuvre was investigated also at our department for robotic arm control and control of trains [14].

2. Theoretical background

Many fundamental problems in applied mathematics and mechanics are posed in terms of finding the extremum (minima or maxima) and they can be formulated as variational statements. The mathematical theory of optimization methods and variational principles may be found in [15].

Variational principles form also a powerful basis for obtaining approximate solutions to practical problems. We consider a variational problem of finding extremum with control function \( u(t) \), that is:

\[
\Gamma = \int_{t_A}^{t_B} L_0(x, u, t) dt \to \min.
\]

The function \( L_0 \) describes a minimization problem with the constraints (equations of motion for some physical problem, e.g. Newton’s equations of motion \( F = ma \) and dynamics equations for EMS):

\[
\frac{dx}{dt} = f(x, u, t).
\]

The complete cost function is defined by the relation:

\[
I = L_0(x, u, t) + \lambda^T \left[ \frac{dx}{dt} - f(x, u, t) \right],
\]

where \( \lambda^T \) is a vector of Lagrange multipliers. As it’s well known, the solution of the minimizing problem requires to find a corresponding solution of Euler-Lagrange equations:

\[
\frac{\partial I}{\partial x} = \frac{d}{dt} \left( \frac{\partial I}{\partial \dot{x}} \right) \quad \text{and} \quad \frac{\partial I}{\partial u} = 0
\]

with some boundary conditions:

\[
x(t_A) = A \quad \text{and} \quad x(t_B) = B.
\]

Dynamical system of electric drive we assume in the form:

\[
\frac{d}{dt} (j \omega) = M_e - M_l,
\]

\[
\dot{\varphi} = \frac{d \varphi}{dt} = \omega,
\]

where \( j \) is reduced moment of inertia to the shaft of electric motor, \( \varphi \) is rotor position, \( \omega \) is rotor speed and \( M_e \) is electric torque. Remember that the moment of inertia \( j \) for many applications may not be constant. Load torque \( M_l \) is expressed as a function of rotor speed with constant \( A \), linear \( B \) and quadratic components \( C \), that is:

\[
M_l = A + B \omega + C \omega^2.
\]

The constants \( A \), \( B \) and \( C \) represent Coulomb, viscous and aerodynamic friction terms.
The functional criteria for consumed energy minimization covering electrical-copper losses during manoeuvre time $T_m$ is defined as:

$$Q \int_{0}^{T_m} M_e^2 \, dt \rightarrow \text{min},$$

where $Q$ is a constant characteristic of electric drive and unknown electric torque plays a role of control function. Our problem is now strictly defined: find optimal trajectories of electric drive, that is, $\varphi$ and $\omega$ as functions of time.

Overall cost function $I_p$ for drive’s losses minimization consists of minimization functional (9) and two other terms with Lagrange multipliers (6), (7):

$$I_p = Q M_e^2 + \lambda_1 [k \omega + J \dot{\omega} - M_e + A + B \omega + C \omega^2] + \lambda_2 [\dot{\varphi} - \omega].$$

In corresponding Lagrange equations we state: $x = (\varphi, \omega)$ and $u = M_e$.

3. Simple kinematic structure

Now we will use the basic idea from the previous section, and we will analyze simple kinematic structure (figure 1) with variable moment of inertia:

$$J = J_0 + J_1 \sin(\alpha),$$

where the angle depends on time linearly, $\alpha = k t$ and $k$ is a constant.

For simplicity we assume that $\alpha(T_m) = \frac{\pi}{2}$, where $T_m = 1\text{s}$ is a maneuver time. From this condition we find $k = \frac{\pi}{2}$.

![Figure 1. Simple kinematic structure consisting of two parts, where part 1 (the black one) is rotating around his own axis and part 2 (the blue one) is changing moment of inertia of whole system due moving along the angle $\alpha$ and the segment $D$ represents the drive of the system.](image)

The equations of motion and the cost function have forms:

$$\frac{d}{dt}(J \omega) = k J_1 \cos(kt) \omega + [J_0 + J_1 \sin(kt)] \dot{\omega} = M_e - A - B \omega - C \omega^2,$$

$$\frac{d \varphi}{dt} = \omega,$$

and

$$I_p = Q M_e^2 + \lambda_1 [k \cos(kt) \omega + [J_0 + J_1 \sin(kt)] \dot{\omega} - M_e + A + B \omega + C \omega^2] + \lambda_2 [\dot{\varphi} - \omega].$$
To solve the minimization problem, we derive corresponding nonlinear Lagrange equations (4) with the equations of the motions (11), (12):

\[
\frac{d\lambda_1}{dt}[J_0 + J_1 \sin(kt)] = B\lambda_1 - \lambda_2 + 2C\omega \lambda_1, \tag{15}
\]

\[
\frac{d\lambda_2}{dt} = 0, \tag{16}
\]

\[
\frac{d}{dt}(J_0 + J_1 \sin(kt))\omega + [J_0 + J_1 \sin(kt)]\dot{\omega} = M_e - A - B\omega - C\omega^2, \tag{17}
\]

\[
\frac{d\phi}{dt} = \omega. \tag{18}
\]

Now we get four equations with (for simplicity) four boundary conditions:

\[
\omega(0) = 0, \tag{19}
\]

\[
\omega(T_m) = 0, \tag{20}
\]

\[
\phi(0) = 0, \tag{21}
\]

\[
\phi(T_m) = 2\pi. \tag{22}
\]

The relation between control function $M_e$ (electric torque) and Lagrange multiplier is $M_e = \frac{\lambda_1}{2\omega}$ and it results from the condition $\frac{\partial I_p}{\partial M_e} = 0$.

4. Reached results

Given system’s equations (15), (16), (17) and (18) and corresponding boundary conditions (19), (20), (21) and (22), was solved by MatLab script function bvp5c which is boundary value problem solver of fifth order. The computed results can be seen on figures 2 and 3.

![Figure 2](image_url)

**Figure 2.** Position and angular speed of the system for Coulomb $A = 0.2$, viscous $B = 0.02$ and aerodynamic $C = 0.01$ friction terms. (The $x$-axis represents the time in seconds and the $y$-axis represents the position in radians, respectively angular velocity in radians per second).
Figure 3. Position and angular speed of the system for Coulomb $A = 1$, viscous $B = 0.5$ and aerodynamic $C = 0.2$ friction terms. (The $x$-axis represents the time in seconds and the $y$-axis represents the position in radians, respectively angular velocity in radians per second).

From these two examples it can be seen, that if the system has lower friction values, the increase of the angular speed is much greater in opposite to the case when the system has higher friction values. However, the deceleration at the end of the maneuver is more remarkable in the second case with higher friction due to the less efficient energy recovery of the system.

5. Conclusion
Areas of applications of energy optimal and sub-optimal control are assumed control of position of industrial machines, elevators, robotic arms and traction drives control. Theoretical base of energy optimal and sub-optimal control is Euler-Lagrange optimization method, which was applied to simple electro-mechanical systems with variable moment of inertia.

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