Uncertainty Quantification of Bifurcations in Random Ordinary Differential Equations

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ABSTRACT

Many real-world phenomena can be described by nonlinear ordinary differential equations. Key characteristics of the model appear in terms of parameters. Oftentimes, these parameters need to be estimated using data assimilation techniques [1]. Parameter-related uncertainty can have a major impact on the system’s dynamics. It is well-known in the theory of dynamical systems that parameter variation can induce bifurcations [2]. This means that a qualitative change in the flow of the vector field corresponding to the ODE happens.

In this talk, I will address the effect of parameter uncertainty on the occurrence of different bifurcation types. While some of them cause rather smooth qualitative changes, others can cause the system to suddenly jump to a new equilibrium far away from previous dynamics. Concerns have been raised that subsystems of the earth might undergo such critical transitions under sustained global warming [3] with severe impacts on various ecosystems and human habitat. This phenomenon is not restricted to climate science but appears also in ecology and epidemiology [4]. Therefore, we need estimates for the bifurcation type probabilities to contribute to a risk assessment of the exposure to critical transitions. In this talk, I will combine known statistical and probabilistic concepts with classical analysis and bifurcation theory. This will be done in a two-step procedure. The first step consists in performing reduction and transformation steps leading to a normal form coefficient. Its sign is known to determine the bifurcation type [2]. The probabilistic analysis of the normal form coefficient is challenging as it generically comes as a nonlinear transformation of the uncertain input parameters. This problem is addressed in a second step by using probabilistic techniques from Uncertainty Quantification and classical analysis. In a numerical case study, I will illustrate the performance of the estimation procedure.

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