Investigation for Suitable Target-Projectile Combination for Fusion from the Isotopes of Ti and Nd using Intrinsic Fusion and Fission Barriers Analysis

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ABSTRACT

Background: A configuration is most suitable for the fusion if it corresponds to a minimum intrinsic fusion barrier and maximum fission barrier.

Purpose: To find a suitable target-projectile combination from the isotopes of Ti and Nd by analyzing the intrinsic fusion and fission barriers theoretically by including the deformations up to hexadecapole order.

Methods: The fragmentation theory has been used for the calculations.

Results: The intrinsic fusion barrier is minimum and fission barrier is maximum for the target-projectile combination: \(^{44}\text{Ti}+^{142}\text{Nd}\) in belly-belly configuration, and the inclusion of deformation of higher order leads to the decrease of fission barrier for the prolate shaped cases and compactness for most of the cases.

Conclusions: The most suitable target-projectile combination from the isotopes of Ti and Nd for the fusion is \(^{44}\text{Ti}+^{142}\text{Nd}\).

1. Introduction

In deformed heavy-ion collisions the final-state observables as well as properties of the dynamics of the fusion or fusion-fission process is expected to be different from the spherical cases. This is due to the distinct differences in the overlap region of aligned deformed nuclei in the process of fusion and hence an investigation of the shape effects during collisions is important to understand the mechanism and dynamics of the process. The interaction potential is not only the function of the deformations of the interacting nuclei but it also depends on their orientations. The orientation which corresponds to the maximum height of fission barrier \(V_\beta\) and minimum interaction radius \(R_\beta\) is known as hot compact and which gives minimum \(V_\beta\) and maximum \(R_\beta\) is cold elongated [1]. In a recent work of ref. [2] hot compact and cold elongated configurations have been obtained for the isotopes of Ti and Nd, with nuclei oriented at belly-belly (b-b) and tip-tip (t-t) configurations, respectively, with quadrupole deformation only. As the nuclei are not quadrupole deformed only so investigations with respect to the higher deformations will be interesting.

In this paper, we have obtained a suitable target-projectile for the fusion from the various combinations of the deformed and oriented isotopes of Ti and Nd: \(^{44}\text{Ti}\) (\(\beta_1=0\), \(\beta_2=0\)), \(^{46}\text{Ti}\) (\(\beta_1=-0.042, \beta_2=0.012\)), \(^{48}\text{Ti}\) (\(\beta_1=0.011\), \(\beta_2=0\)), \(^{143}\text{Nd}\) (\(\beta_1=-0.125\), \(\beta_2=-0.006\)), and \(^{149}\text{Nd}\) (\(\beta_1=0.237\), \(\beta_2=-0.110\)) by investigating the \(V_\beta\) and \(R_\beta\) for the b-b and t-t configurations, and intrinsic fusion barrier \(B_{fu}\) in mass asymmetric coordinates, for b-b configuration only due to lesser values of \(V_\beta\) for t-t configuration. It may be noted that \(\beta_1=0\) for these nuclei and the deformation values are of ref. [3]. These barriers play an important role in understanding the competition between quasi-fission and complete fusion. Following the definition of \(B_{fu}\) in charge asymmetry coordinate of ref. [4], \(B_{fu}\) in mass asymmetry coordinate is defined as the height of the saddle point (the maximum fragmentation potential) from the potential which corresponds to the incoming channel mass asymmetry or mass number of the projectile. The smaller value of \(B_{fu}\) favours complete fusion while the larger value of it is a hindrance to the fusion process.

In the following we discussed the methodology, calculations and results, and conclusion of the study.
2. Methodology

The fragmentation theory [5, 6] used to obtain the fragmentation potentials is worked out in terms of mass asymmetry coordinate \( \eta = (A_i - A_j)/(A_i + A_j) \) or charge asymmetry coordinate \( \eta_z = (Z_i - Z_j)/(Z_i + Z_j) \), relative separation \( R \), the neck-length parameter \( \ell \) and the deformations of the interacting nuclei \( \beta_{\lambda i} \) (\( i = 1, 2 \) and \( \lambda = 2, 3, 4 \) for the quadrupole, octupole, and hexadecapole deformations). According to the fragmentation theory the fragmentation potential between two deformed and oriented nuclei colliding in a plane \((\phi = 0)\) at fixed inter-nuclear separation \( R_s \) is

\[
V(\eta, \eta_z, R) = \sum_{i=1}^{3} V_{LDM}(A_i, Z_i) + \sum_{i=1}^{2} \delta U_i + V_c(R, A_i, \beta_{\lambda i}, \theta_i, \varphi) + V_L(R, A_i, \beta_{\lambda i}, \theta_i, \varphi)
\]

where \( V_{LDM}(A_i, Z_i) \), \( \delta U_i \), \( V_c \), and \( V_L \) respectively are the liquid drop energies, shell corrections, proximity potential, Coulomb potential and centrifugal potential between the fragments. For a given nucleus \( V_{LDM}(A, Z) \) [7] is

\[
V_{LDM}(A, Z) = \alpha A + \beta A^{2/3} + \left( \frac{4 \gamma - 4 \delta}{A} \right) \left( 1 + 0.7636 \frac{Z^2}{A^{2/3}} - 2.29 \left( \frac{R_0 A^{1/3}}{Z^{2/3}} \right)^2 + \frac{6}{\lambda} \frac{f(Z, A)}{A^{3/4}} \right)
\]

where \( \alpha, \beta, \gamma, \delta \) and \( a_\perp \) are Seeger’s constants [8], \( t_\perp = a_\perp (Z - N) \) is the asymmetry term and \( a_\parallel \) is the asymmetry constant. The shell correction \( \delta U \) for a given nucleus is taken from [9]. The bulk \( \alpha \) and asymmetry \( a \) constants are obtained by equating the ground state mass excess of AME2016 [10] or of FRDM2012 [3] by with \( V_{LDM}(A, Z) + \delta U \) [11]. The Coulomb and proximity potentials for deformed and oriented nuclei for \( \phi = 0 \) are given as:

\[
V_c(R, Z_i, \beta_{\lambda i}, \theta_i) = \frac{Z_i Z_j e^2}{R} + \sum_{\lambda=1,2} \frac{3Z_i Z_j e^2 R^\lambda (\alpha_\lambda)}{2\lambda + 1} \frac{Y^{(0)}_\lambda(\theta_i)}{R^{\lambda+1}} \left[ \beta_{\lambda i} + \frac{4}{7} \beta_{\lambda i}^2 Y^{(0)}_\lambda(\theta_i) \right]
\]

\[
V_p(R, A_i, \beta_{\lambda i}, \theta_i) = 4\pi R^2 b \Phi(s_b)
\]

where \( \gamma = 0.9517[1 - 1.7826((N-Z)/A)^2] \) MeV fm\(^2\) is the nuclear surface energy constant, \( b = 0.99 \) fm is the nuclear surface thickness and \( \Phi(s_b) \) is the universal function, given as

\[
\Phi(s_b) = \begin{cases} 0.5(s_b - 2.54)^2 - 0.0852(s_b - 2.54)^3 & \text{for } s_b \leq 1.2511 \\ -3.437 \exp(-s_b/0.75) & \text{for } s_b \geq 1.2511 \end{cases}
\]

Figure 1: Interaction potential for tip-tip and belly-belly configurations of the oblate shaped P-T.

3. Calculations and Results

Fig. 1 shows the interaction potential for the oblate shaped projectile-target combination \(^{48}\)Ti\(^{148}\)Nd oriented at \((0°, 0°)\).
for hot and (90°, 90°) for cold configurations called belly-belly and tip-tip configurations, respectively. The barrier height (V_b) is maximum and interaction radius (R_b) is minimum for the belly-belly configuration while for the tip-tip configuration corresponds to the minimum of V_b and maximum of R_b, whatever may be the sign of the deformation.

This has been explored for various combinations of Ti and Nd: oblate-oblate, oblate-prolate, oblate-spherical, prolate-oblate, prolate-prolate, prolate-spherical, spherical-oblate and spherical-prolate, as tabulated in Table 1 and 2 below along with the inclusion of deformation of higher order (±).

Table 1: P-T oriented for belly-belly (b-b) configuration (Hot fusion).

| S. N. | P-T (b-b configurations) | P-T Orientation (θ, 0) | P-T with β_2, β_4, (i=1,2) | V_b (MeV) | R_b (fm) | P-T with β_2+β_4+β_6, (i=1,2) | V_b (MeV) | R_b (fm) |
|------|---------------------------|-------------------------|-----------------------------|--------|--------|-------------------------------|--------|--------|
| 1.   | 48Ti+183Nd                | (0°, 0°)                | o – o                        | 148.79 | 11.80  | o’ – o’                        | 148.79 | 12.50  |
| 2.   | 48Ti+183Nd                | (0°, 90°)               | o – p                        | 154.19 | 11.36  | o’ – p’                        | 152.96 | 11.46  |
| 3.   | 48Ti+183Nd                | (0°, 0°)                | o – s                        | 152.60 | 11.60  | o’ – s                         | 152.60 | 11.70  |
| 4.   | 48Ti+183Nd                | (90°, 0°)               | p – o                        | 146.77 | 11.97  | p – o’                         | 146.77 | 11.97  |
| 5.   | 48Ti+183Nd                | (90°, 90°)              | p – p                        | 151.46 | 11.58  | p – p’                         | 151.00 | 11.60  |
| 6.   | 48Ti+183Nd                | (90°, 0°)               | p – s                        | 149.69 | 11.86  | p – s’                         | 149.69 | 11.91  |
| 7.   | 48Ti+183Nd                | (0°, 0°)                | s – o                        | 147.99 | 11.90  | s – o’                         | 147.99 | 12.60  |
| 8.   | 48Ti+183Nd                | (0°, 90°)               | s – p                        | 153.01 | 11.48  | s – p’                         | 151.80 | 11.58  |

Table 2: P-T oriented for tip-tip (t-t) configuration (Cold fusion).

| S. N. | P-T (t-t configurations) | P-T Orientation (θ, 0) | P-T with β_2, β_4, (i=1,2) | V_b (MeV) | R_b (fm) | P-T with β_2+β_4+β_6, (i=1,2) | V_b (MeV) | R_b (fm) |
|------|---------------------------|-------------------------|-----------------------------|--------|--------|-------------------------------|--------|--------|
| 1.   | 48Ti+184Nd                | (90°, 90°)              | o – o                        | 142.41 | 12.50  | o’ – o’                        | 142.41 | 12.50  |
| 2.   | 48Ti+184Nd                | (90°, 0°)               | o – p                        | 142.07 | 12.80  | o’ – p’                        | 136.86 | 13.30  |
| 3.   | 48Ti+184Nd                | (90°, 0°)               | o – s                        | 151.28 | 11.70  | o’ – s                         | 151.28 | 11.70  |
| 4.   | 48Ti+184Nd                | (90°, 0°)               | p – o                        | 149.35 | 11.91  | p – s’                         | 149.35 | 11.91  |
| 5.   | 48Ti+184Nd                | (0°, 90°)               | p – s                        | 142.24 | 12.60  | s – o’                         | 142.24 | 12.60  |
| 6.   | 48Ti+184Nd                | (0°, 90°)               | s – p                        | 142.14 | 12.80  | s – p’                         | 136.99 | 13.30  |

It can be seen from Table 1 and 2 that the V_b is maximum and R_b is minimum for the reaction 48Ti+150Nd in belly-belly configuration, and hence is predicted to be the best target-projectile for fusion. The addition of higher order deformation leads to the decrease of height of fission barrier (V_b) in p’ (prolate with β_2=+ve) cases and increase of the interaction radius in most of the cases. This means that the systems become less compact with the inclusion of deformation of higher order and compact configuration is expected to be at some other orientation, as can be seen in ref. [14].

Thus a configuration of compactness is most suitable for the fusion process. The suitability is explored further in terms of B_m. It may be noted that the inclusion of the deformations of higher order (±) does not change the optimal orientation, but leads to a decrease in the compactness.

Fig. 2 illustrate the intrinsic fusion barrier B_m for 48Ti+183Nd reaction when target and projectile considered are: (i) spherical, (ii) quadrupole deformed (β_2), and (iii) with deformation of higher order such as octupole (β_3) and hexadecapole (β_6) i.e., p^2, o^4. The intrinsic fusion barrier B_m increases with the inclusion of the deformation of higher order, i.e., B_m(β_2) < B_m(β_2+β_4+β_6), which seems to be due to the decrease of the compactness with the higher deformations [14].
Table 3 clearly shows that for $^{43}$Ti+$^{150}$Nd$\rightarrow^{193}$Pb$^*$ the intrinsic fusion barrier $B_{\text{ fus}}$ is zero when target and projectiles are considered quadrupole deformed ($\beta_2$) and is minimum ($\approx0.84$ MeV) when the deformations of higher orders ($\beta_3$ and $\beta_4$) are included. Thus $^{43}$Ti+$^{150}$Nd reaction or target-projectile combination seems to be the best for the fusion when oriented at belly-belly configuration.

**Table 3: Intrinsic fusion barriers for hot optimum or belly-belly Configuration.**

| S. N. | P-T (b-b configurations) | P-T with $\beta_2$ (i=1,2) | $B_{\text{ fus}}$ (MeV) | P-T with $\beta_2+\beta_3+\beta_4$ (i=1,2) | $B_{\text{ fus}}$ (MeV) |
|-------|---------------------------|-----------------------------|--------------------------|---------------------------------|--------------------------|
| 1.    | $^{43}$Ti$^{181}$Nd$\rightarrow^{224}$Pb* | $o$ − $o$ | 22.39 | $o$ − $o$ | 29.94 |
| 2.    | $^{43}$Ti$^{190}$Nd$\rightarrow^{209}$Pb* | $o$ − $p$ | 0 | $o$ − $p$ | 0.84 |
| 3.    | $^{43}$Ti$^{142}$Nd$\rightarrow^{185}$Pb* | $o$ − $s$ | 0.75 | $o$ − $s$ | 6.39 |
| 4.    | $^{43}$Ti$^{181}$Nd$\rightarrow^{229}$Pb* | $p$ − $o$ | 43.44 | $p$ − $o$ | 152.51 |
| 5.    | $^{43}$Ti$^{190}$Nd$\rightarrow^{198}$Pb* | $p$ − $p$ | 5.71 | $p$ − $p$ | 47.70 |
| 6.    | $^{43}$Ti$^{142}$Nd$\rightarrow^{190}$Pb* | $p$ − $s$ | 8.64 | $p$ − $s$ | 148.15 |
| 7.    | $^{43}$Ti$^{181}$Nd$\rightarrow^{235}$Pb* | $s$ − $o$ | 18.83 | $s$ − $o$ | 48.42 |
| 8.    | $^{43}$Ti$^{190}$Nd$\rightarrow^{194}$Pb* | $s$ − $p$ | 4.95 | $s$ − $p$ | 26.63 |

**Figure 2:** Fragmentation potential illustrating intrinsic fusion barrier $B_{\text{ fus}}$ for (i) spherical and deformed with (ii) $\beta_2$ and (iii) $\beta_2+\beta_3+\beta_4$.

**Conclusions**

An analysis of intrinsic fusion barrier, fission barrier height and position reveals that the most compact orientation (belly-belly) of the colliding nuclei seems to be the best for the fusion of heavy-ions and can be obtained using the barrier analysis, here predicting $^{43}$Ti+$^{150}$Nd as the best target-projectile combination for the synthesis of Pb.

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**Authorship Contribution**

Conception of the idea, interpretation of the data/results and manuscript preparation by Dalip Singh Verma, running codes, extracting the data for figures by Kushmakshi, and manuscript preparation and interpretation of the results by Monika Manhas. All authors discussed the results and contributed to the final manuscript.

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**Conflict of Interest**

There is no conflict of interests.

**Declaration**

It is declared that the results presented in this paper are original and has neither been sent elsewhere, nor published anywhere.

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