Research Article

On the $q$-Extension of Apostol-Euler Numbers and Polynomials

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Recently, Choi et al. (2008) have studied the $q$-extensions of the Apostol-Bernoulli and the Apostol-Euler polynomials of order $n$ and multiple Hurwitz zeta function. In this paper, we define Apostol’s type $q$-Euler numbers $E_{n,q,ξ}$ and $q$-Euler polynomials $E_{n,q,ξ}(x)$. We obtain the generating functions of $E_{n,q,ξ}$ and $E_{n,q,ξ}(x)$, respectively. We also have the distribution relation for Apostol’s type $q$-Euler polynomials. Finally, we obtain $q$-zeta function associated with Apostol’s type $q$-Euler numbers and Hurwitz’s type $q$-zeta function associated with Apostol’s type $q$-Euler polynomials for negative integers.

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1. Introduction

Let $p$ be a fixed odd prime. Throughout this paper, $\mathbb{Z}_p$, $\mathbb{Q}_p$ and $\mathbb{C}$ and $\mathbb{C}_p$ will, respectively, denote the ring of $p$-adic rational integers, the field of $p$-adic rational numbers, the complex number field, and the completion of algebraic closure of $\mathbb{Q}_p$. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{Z}_+ = \mathbb{N} \cup \{0\}$. Let $\nu_p$ be the normalized exponential valuation of $\mathbb{C}_p$ with $|p|_p = p^{-\nu_p(p)} = p^{-1}$. When one talks of $q$-extension, $q$ is variously considered as an indeterminate, a complex number $q \in \mathbb{C}$, or a $p$-adic number $q \in \mathbb{C}_p$. If $q \in \mathbb{C}$, one normally assumes $|q| < 1$. If $q \in \mathbb{C}_p$, then one assumes $|q - 1|_p < 1$. We also use the notations

$$[x]_q = \frac{1 - q^x}{1 - q}, \quad [x]_{-q} = \frac{1 - (-q)^x}{1 + q} \quad \forall x \in \mathbb{Z}_p$$ (1.1)

For a fixed odd positive integer $d$ with $(p, d) = 1$, let

$$X = X_d = \lim_{N \to \infty} \frac{\mathbb{Z}}{dp^N \mathbb{Z}}, \quad X_1 = \mathbb{Z}_p,$$
\[ X^* = \bigcup_{0 \leq a < dp} (a + dp\mathbb{Z}_p), \]
\[ a + dp^N\mathbb{Z}_p = \{ x \in X \mid x \equiv a \pmod{dp^N} \}, \tag{1.2} \]

where \( a \in \mathbb{Z} \) lies in \( 0 \leq a < dp^N \). The distribution is defined by
\[ \mu_q(a + dp^N\mathbb{Z}_p) = \left[ \frac{q^a}{[dp^N]_q} \right]. \tag{1.3} \]

We say that \( f \) is a uniformly differentiable function at a point \( a \in \mathbb{Z}_p \) and denote this property by \( f \in UD(\mathbb{Z}_p) \), if the difference quotients \( F_f(x, y) = (f(x) - f(y))/(x - y) \) have a limit \( l = f'(a) \) as \( (x, y) \to (a, a) \). For \( f \in UD(\mathbb{Z}_p) \), the \( p \)-adic invariant \( q \)-integral is defined as
\[ I_q(f) = \int_{\mathbb{Z}_p} f(x)d\mu_q(x) = \lim_{N \to \infty} \frac{1}{[pN]_q} \sum_{x=0}^{pN-1} f(x)q^x. \tag{1.4} \]

The fermionic \( p \)-adic \( q \)-measures on \( \mathbb{Z}_p \) are defined as
\[ \mu_{-q}(a + dp^N\mathbb{Z}_p) = \left[ \frac{(-q)^a}{[dp^N]_{-q}} \right], \tag{1.5} \]

and the fermionic \( p \)-adic invariant \( q \)-integral on \( \mathbb{Z}_p \) is defined as
\[ I_{-q}(f) = \int_{\mathbb{Z}_p} f(x)d\mu_{-q}(x) = \lim_{N \to \infty} \frac{1}{[pN]_{-q}} \sum_{x=0}^{pN-1} f(x)(-q)^x \tag{1.6} \]

for \( f \in UD(\mathbb{Z}_p) \). For details see [1–10].

Classical Euler numbers are defined by the generating function
\[ \frac{2}{e^t + 1} = \sum_{n=0}^{\infty} E_n \frac{t^n}{n!}, \tag{1.7} \]

and these numbers are interpolated by the Euler zeta function which is defined as
\[ \zeta_E(s) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^s}, \quad s \in \mathbb{C}. \tag{1.8} \]

After Carlitz [11] gave \( q \)-extensions of the classical Bernoulli numbers and polynomials, the \( q \)-extensions of Bernoulli and Euler numbers and polynomials have been studied by several authors (cf. [1–16, 18–26, 34–39]).
By using $p$-adic $q$-integral, the $q$-Euler numbers $E_{n,q}$ are defined as

$$E_{n,q} = \int_{\mathbb{Z}_p} [t]^n d\mu_{-q}(t), \quad \text{for } n \in \mathbb{N}. \quad (1.9)$$

The $q$-Euler numbers $E_{n,q}$ are defined by means of the generating function

$$F_q(t) = [2]_q \sum_{n=0}^{\infty} (-1)^n q^n e^{[n]_q t}$$

(cf. [8, 26]). Kim [22] gave a new construction of the $q$-Euler numbers $E_{n,q}$ which can be uniquely determined by

$$E_{0,q} = [2]_q^q,$$

$$E_{n,q} = \begin{cases} [2]_q^n & \text{if } n = 0, \\ 0 & \text{if } n \neq 0, \end{cases} \quad (1.11)$$

with the usual convention of replacing $E^n$ by $E_{n,q}$.

The twisted $q$-Euler numbers and $q$-Euler polynomials are very important in several fields of mathematics and physics, and so they have been studied by many authors. Simsek [37, 38] constructed generating functions of $q$-generalized Euler numbers and polynomials and twisted $q$-generalized Euler numbers and polynomials. Recently, Y. H. Kim et al. [27] gave the twisted $q$-Euler zeta function associated with twisted $q$-Euler numbers and obtained $q$-Euler’s identity. They also have a $q$-extension of the Euler zeta function for negative integers and the $q$-analogue of twisted Euler zeta function. Kim [24] defined twisted $q$-Euler numbers and polynomials of higher order and studied multiple twisted $q$-Euler zeta functions.

The Apostol-Bernoulli and the Apostol-Euler polynomials and numbers have been studied by several authors (cf. [15, 17, 32, 33, 40, 41]). Recently, $q$-extensions of the Apostol-Bernoulli and the Apostol-Euler polynomials and numbers have been studied by many authors with great interest. In [15], Cenkci and Can introduced and investigated $q$-extensions of the Bernoulli polynomials. Choi et al. [16] have studied some $q$-extensions of the Apostol-Bernoulli and the Apostol-Euler polynomials of order $n$ and multiple Hurwitz zeta function.

In this paper, we define Apostol’s type $q$-Euler numbers and $q$-Euler polynomials. Then, we have the generating functions of Apostol’s type $q$-Euler numbers and $q$-Euler polynomials and the distribution relation for Apostol’s type $q$-Euler polynomials. In Section 2, we define Apostol’s type $q$-Euler numbers $E_{n,q,\xi}$ and $q$-Euler polynomials $E_{n,q,\xi}(x)$. Then, we obtain the generating functions of $E_{n,q,\xi}$ and $E_{n,q,\xi}(x)$, respectively. We also have the distribution relation for Apostol’s type $q$-Euler polynomials. In Section 3, we obtain $q$-zeta function associated with Apostol’s type $q$-Euler numbers and Hurwitz’s type $q$-zeta function associated with Apostol’s type $q$-Euler polynomials for negative integers.
2. On the $q$-extensions of the Apostol-Euler numbers and polynomials

In this section, we will assume $q \in \mathbb{C}_p$ with $|q - 1|_p < 1$. For $n \in \mathbb{Z}_+$, let $C_{p^n} = \{ \xi \mid \xi^{p^n} = 1 \}$ be the cyclic group of order $p^n$, and let $T_p$ be the space of locally constant space, that is,

$$T_p = \lim_{n \to \infty} C_{p^n} = \bigcup_{n \geq 0} C_{p^n}. \quad (2.1)$$

Let $\xi \in T_p$. We define Apostol’s type $q$-Euler numbers by

$$E_{n,q,\xi} = \int_{\mathbb{Z}_p} q^{-x} \xi^x [x]_q^n d\mu_q(x). \quad (2.2)$$

Then, we have

$$E_{n,q,\xi} = \frac{[2]_q}{(1 - q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l \frac{1}{1 + q^l \xi}. \quad (2.3)$$

where $\binom{n}{l}$ are the binomial coefficients.

Apostol’s type $q$-Euler polynomials are defined as

$$E_{n,q,\xi}(x) = \int_{\mathbb{Z}_p} q^{-y} \xi^y [x + y]_q^n d\mu_q(y). \quad (2.4)$$

Since

$$[x + y]_q^n = ([x]_q + q^x [y]_q)_q^n = \sum_{l=0}^{n} \binom{n}{l} [x]_q^{n-l} q^l [y]_q^l, \quad (2.5)$$

we have from (2.4) that

$$E_{n,q,\xi}(x) = \sum_{l=0}^{n} \binom{n}{l} [x]_q^{n-l} q^l \int_{\mathbb{Z}_p} q^{-y} \xi^y [y]_q^l d\mu_q(y). \quad (2.6)$$

By (2.2) and (2.6), we have

$$E_{n,q,\xi}(x) = \sum_{l=0}^{n} \binom{n}{l} [x]_q^{n-l} q^l E_{l,q,\xi}. \quad (2.7)$$

Since

$$[x + y]_q^n = \frac{1}{(1 - q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{(x+y)^l} = \frac{1}{(1 - q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^l q^l, \quad (2.8)$$
we have

\[
\int_{x,y} q^{-y} x^n dμ_{y}(x) \cdot y^n dμ_{y}(y) = \frac{1}{(1-q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{lx} \int_{x,y} \int \frac{q^{(l-1)y} \xi-y dμ_{y}(y)}{x^n \plusmath 1 + q^{lx}.}
\]

Therefore, we also have

\[
E_{n,q,ξ}(x) = [2]_q \frac{1}{(1-q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l \frac{1}{1 + q^{lx}}.
\]

Note that (2.7) and (2.10) are two representations for \( E_{n,q,ξ}(x) \). Hence, we have the following result.

**Theorem 2.1.** For \( n \in \mathbb{Z} \) and \( ξ \in T_p \), one has

\[
E_{n,q,ξ} = \frac{[2]_q}{(1-q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l \frac{1}{1 + q^{lx}}.
\]

\[
E_{n,q,ξ}(x) = \frac{[2]_q}{(1-q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l \frac{1}{1 + q^{lx}}
\]

\[
= \sum_{l=0}^{n} \binom{n}{l} [x]^{n-1} q^{lx} E_{n,q,ξ}.
\]

Now, we will find the generating function of \( E_{n,q,ξ} \) and \( E_{n,q,ξ}(x) \), respectively. Let \( F(t) \) be the generating function of \( E_{n,q,ξ} \). Then, we have

\[
F(t) = \sum_{n=0}^{∞} E_{n,q,ξ} \frac{t^n}{n!}
\]

\[
= \sum_{n=0}^{∞} \left[ \frac{2}{(1-q)^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l \frac{1}{1 + q^{lx}} \right] \frac{t^n}{n!}
\]

\[
= \frac{[2]_q}{(1-q)^n} \sum_{n=0}^{∞} \left( \frac{1}{1 - q} \right)^n \sum_{l=0}^{n} \binom{n}{l} (-1)^l \left( \sum_{m=0}^{∞} q^{lm} q^m (-1)^m \right) \frac{t^n}{n!}
\]

\[
= \frac{[2]_q}{(1-q)^n} \sum_{m=0}^{∞} \frac{1}{1 - q} \sum_{n=0}^{∞} \frac{1}{1 - q^n} \sum_{l=0}^{n} \binom{n}{l} (-1)^l \frac{t^n}{n!}
\]

\[
= \frac{[2]_q}{(1-q)^n} \sum_{m=0}^{∞} \frac{1}{1 - q^m} \frac{t^n}{n!}
\]
\[ F(t) = \sum_{n=0}^{\infty} E_{n,q}(x) \frac{t^n}{n!} = [2]q \sum_{m=0}^{\infty} (-1)^m q^m e^{m|x|_q}. \]  
(2.13)

Note that
\[ \int_{\mathbb{Z}_p} q^{-x} \xi^x e^{[x]_q t} \, d\mu_{-q}(x) = \sum_{n=0}^{\infty} \int_{\mathbb{Z}_p} q^{-x} \xi^x [x]_q^n \, d\mu_{-q}(x) \frac{t^n}{n!} = \sum_{n=0}^{\infty} E_{n,q}(x) \frac{t^n}{n!} = F(t). \]  
(2.14)

For the generating function of \( E_{n,q}(x) \), we have
\[ \int_{\mathbb{Z}_p} q^{-y} \xi^y e^{[y]_q t} \, d\mu_{-q}(y) = [2]q \sum_{m=0}^{\infty} (-1)^m q^m e^{m|y|_q}. \]  
(2.15)

Hence, we obtain the following theorem.

**Theorem 2.2.** For \( \xi \in T_p \), one has
\[ \int_{\mathbb{Z}_p} q^{-x} \xi^x e^{[x]_q t} \, d\mu_{-q}(x) = [2]q \sum_{m=0}^{\infty} (-1)^m q^m e^{m|x|_q}, \]  
(2.16)
\[ \int_{\mathbb{Z}_p} q^{-y} \xi^y e^{[y]_q t} \, d\mu_{-q}(y) = [2]q \sum_{m=0}^{\infty} (-1)^m q^m e^{m|y|_q}. \]  
(2.17)

Since (2.16) equals to the generating functions (2.17) equals to the generating functions \( \sum_{n=0}^{\infty} E_{n,q}(x) \left( \frac{t^n}{n!} \right) \), we have the following result.

**Corollary 2.3.** For \( n \in \mathbb{Z}_+ \) and \( \xi \in T_p \), one has
\[ E_{n,q}(x) = [2]q \sum_{m=0}^{\infty} (-1)^m q^m \left[ m \right]_q^n, \]  
(2.18)
\[ E_{n,q}(x) = [2]q \sum_{m=0}^{\infty} (-1)^m q^m \left[ m + x \right]_q^n. \]
Now, we will find the distribution relation for \(E_{n,q,\xi}(x)\). By (2.4), we have

\[
E_{n,q,\xi}(x) = \int_X q^{-y} \zeta_q^n [x + y]_q^n d\mu_q(y)
\]

\[
= \lim_{N \to \infty} \frac{1}{[dp^N]_{-q}} \sum_{y=0}^{p^N-1} \zeta_q^n (-1)^y [x + y]_q^n
\]

\[
= \lim_{N \to \infty} \frac{1}{[dp^N]_{-q}} \sum_{a=0}^{d-1} \sum_{y=0}^{p^N-1} \zeta_q^n (x + a + dy)q^n
\]

Note that for odd numbers \(d\) and \(p\),

\[
[dp^N]_{-q} = [d]_{-q} [p^N]_{-q^d},
\]

\[
[x + a + dy]_q = [d]_q \left[ \frac{x + a}{d} + y \right]_{q^d}.
\]

By (2.19), we have

\[
E_{n,q,\xi}(x) = \frac{1}{[d]_{-q}} \sum_{a=0}^{d-1} \zeta_a (-1)^a \lim_{N \to \infty} \frac{1}{[p^N]_{-q^d}} \sum_{y=0}^{p^N-1} \zeta_q^n (x + a + dy)^n [x + a + y]_{q^d}^n
\]

\[
= \frac{[d]_q}{[d]_{-q}} \sum_{a=0}^{d-1} (-1)^a \int_{\mathbb{Z}_p} \zeta_q^a y (q^d - y) \left[ \frac{x + a}{d} + y \right]_{q^d}^n d\mu_{-q^d}(y).
\]

Therefore, we obtain the distribution relation for \(E_{n,q,\xi}(x)\) as follows.

**Theorem 2.4.** For \(n \in \mathbb{Z}_+\), \(\xi \in T_p\), and \(d \in \mathbb{Z}_+\) with \(d \equiv 1 \pmod{2}\), one has

\[
E_{n,q,\xi}(x) = \frac{[d]_q}{[d]_{-q}} \sum_{a=0}^{d-1} \zeta^a (-1)^a E_{n,q^d,\xi^a} \left( \frac{x + a}{d} \right).
\]

3. **Further remark on the basic \(q\)-zeta functions associated with Apostol’s type \(q\)-Euler numbers and polynomials**

In this section, we assume that \(q \in \mathbb{C}\) with \(|q| < 1\). Let \(\xi \in T_p\). For \(s \in \mathbb{C}\), \(q\)-zeta function associated with Apostol’s type \(q\)-Euler numbers is defined as

\[
\zeta_{q,\xi}(s) = \frac{1}{[2]_q} \sum_{n=1}^{\infty} \frac{\zeta^n (-1)^n}{[n]_q}. \quad (3.1)
\]
which is analytic in whole complex $s$-plane. Substituting $s = -k$ with $k \in \mathbb{Z}_+$ into $\zeta_{q,\xi}(s)$ and using Corollary 2.3, then we arrive at

$$\zeta_{q,\xi}(-k) = \left[2\right]_q \sum_{n=1}^{\infty} \xi^n (-1)^n [n]_q^k = E_{k,q,\xi}. \quad (3.2)$$

Now, we also consider Hurwitz’s type $q$-zeta function associated with the Apostol’s type $q$-Euler polynomials as follows:

$$\zeta_{q,\xi}(s, x) = \left[2\right]_q \sum_{n=0}^{\infty} \frac{\xi^n (-1)^n}{[n+x]_q}. \quad (3.3)$$

Substituting $s = -k$ with $k \in \mathbb{Z}_+$ into $\zeta_{q,\xi}(s, x)$ and using Corollary 2.3, then we arrive at

$$\zeta_{q,\xi}(-k, x) = \left[2\right]_q \sum_{n=0}^{\infty} \xi^n (-1)^n [n+x]_q^k = E_{k,q,\xi}(x). \quad (3.4)$$

Hence, we obtain $q$-zeta function associated with Apostol’s type $q$-Euler numbers and Hurwitz’s type $q$-zeta function associated with Apostol’s type $q$-Euler polynomials for negative integers.

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