Supporting information

Adsorption-based membranes for air separation using metal oxide membranes

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1. Molecular Dynamics simulations

1.1 Lennard-Jones potential

In this work, all the interactions were modeled using the Lennard-Jones (LJ) potential with LAMMPS. A description of this potential is found in [2]. The form of the LJ 12-6 potential used is given by

$$ E(r) = \begin{cases} 4\varepsilon \left[ \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^{6} \right] & (r < r_{\text{cut}}) \\ 0 & (r \geq r_{\text{cut}}) \end{cases} \quad (S1) $$

A cutoff radius $r_{\text{cut}}$ of 10 Å was used. The values of $\sigma$ and $\varepsilon$ used in the simulations are listed in table S0.

Table S0: LJ potential parameters used in the simulation

|          | $\varepsilon$ (kcal/mol) | $\sigma$ (Å) |
|----------|--------------------------|--------------|
| C-N      | 0.102                    | 3.516        |
| C-O      | 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 1.0 | 3.401        |
| N-N      | 0.077                    | 3.263        |
| N-O      | 0.086                    | 3.148        |
| O-O      | 0.096                    | 3.033        |

1.1.2 Bond description

Along with the bonds in the nanoporous graphene (NPG) membrane, the bonds in the diatomic gases were also fixed in length and not allowed to vibrate.
1.2 The pore in NPG membrane

Since the selectivity isn’t based on molecular sieving, we employed an NPG membrane with a pore large enough to allow unhindered motion of both gases. In the energy profile, this would ensure that the translocation step in the adsorbed-phase pathway wouldn’t have the highest energy barrier.

Fig S1: The bare nanoporous graphene (NPG) membrane.

1.3 Obtaining adsorption energies from LJ parameters and resulting permeability and selectivity

A setup as shown in Fig S2a was used where a dimer of O₂ and N₂ were separately placed vertically at various distances from the membrane surface and the potential energy was tabulated. The potential energy plots are depicted in Fig S2b and the minimum energies obtained for each of the LJ parameter are indicated in Table S1. The selectivity as a function of the LJ parameters is plotted in Fig S3.
Permeability was calculated using equation S2

\[ P = \frac{Jl}{AN_0 \Delta p} \]  

(S2)

Here, \( J \) is the flow rate, \( P \) is the partial pressure, \( A \) is the surface area, \( l \) is the thickness, \( N_0 \) is the Avogadro constant, and \( P \) is the permeability. The following values were used to compute permeability: \( A = 3.6 \times 3.15 \times 10^{-18} \text{ m}^2 \), \( N_0 = 6.023 \times 10^{23} \), \( l = 3.4 \times 10^{-10} \text{ m} \). \( \Delta p \) values were obtained from the slope of the curves depicted in Figure 7.

Table S1: Adsorption energies corresponding to the LJ parameters used and the calculated selectivity

| \( \varepsilon \) (kcal/mol) | \( E_{\text{ads}} \) (eV) | \( E_{\text{ads}}(O_2) - E_{\text{ads}}(N_2) \) (eV) | Permeability (Barrer) | Selectivity \((O_2/N_2)\) |
|---|---|---|---|---|
| \( O_2 \) | \( N_2 \) | \( O_2 \) | \( N_2 \) | \( O_2 \) | \( N_2 \) | \( O_2 \) | \( N_2 \) |
| 0.11 | 0.12 | -0.12 | -0.13 | 0.01 | 1094±80 | 1335±77 | 0.8±0.1 |
| 0.2 | 0.12 | -0.21 | -0.13 | -0.08 | 3695±250 | 1106±71 | 3.3±0.4 |
| 0.3 | 0.12 | -0.31 | -0.13 | -0.18 | 4594±281 | 610±36 | 7.5±0.9 |
| 0.4 | 0.12 | -0.41 | -0.13 | -0.28 | 4976±286 | 174±8 | 29±3 |
| 0.5 | 0.12 | -0.51 | -0.13 | -0.38 | 4746±258 | 34±2 | 138±16 |
| 0.6 | 0.12 | -0.62 | -0.13 | -0.49 | 3973±187 | 4.9±0.2 | 840±81 |
Fig S3: A plot of selectivity vs C-O LJ interaction parameter. A similar trend is observed when plotted against difference in adsorption energies.

1.4 Theoretical framework for permeability and selectivity

The theoretical framework used to obtain selectivity is by adopting the methodology used by Sun et al.\textsuperscript{19} Flow rate expressed as a function of permeability is given by Equation S3

\[ J = \frac{dN}{dt} = PA\Delta p \]  

(S3)

Where \( J \) is flow rate, \( A \) is the surface area, \( \Delta p \) is the difference of partial pressure and \( P \) is the permeability. \( \Delta p \), in turn, can be expressed as a function of the initial pressure \( p_{in} \), number of molecules in the permeate region \( N \), number of molecules adsorbed \( N_{ads} \), and total number of molecules \( N_{tot} \) and is given by Equation S4.

\[ \Delta p = \frac{N_{tot} - N_{ads} - 2N}{N_{tot}} p_{in} \]  

(S4)

Combining equation S3 and S4 gives us equation S5,

\[ J = \frac{dN}{dt} = PA(N_{tot} - N_{ads} - 2N) \frac{p_{in}}{N_{tot}} \]  

(S5)

Integrating equation S5, we get the following:
Equation S6 is in the following form and all our calculations are fitted to the equation S7.

\[ N = a (1 - e^{-bt}) \]  

1.5 Pressure calculation

Pressure is calculated using the ideal gas law as shown in equation S8.

\[ p = \frac{Nk_B T}{V} \]  

Where \( p, N, k_B, T, \) and \( V \) represent the pressure, number of molecules, the Boltzmann constant, temperature, and volume respectively. For obtaining the initial pressure in the feed side, we use \( N = 200 \) molecules (100 O\(_2\) and 100 N\(_2\)), \( T = 500 \) K, and \( V = 31.5 \times 36 \times 175 \times 10^{-30} \) m\(^3\). This results in an initial pressure of around 68.6 atm in the feed side.

2. Density Functional theory calculations

Density Functional Theory (DFT) was employed using Vienna Ab initio Simulation Package (VASP)\(^{20-22} \) to obtain the adsorption energies of oxygen and nitrogen molecules on two transition metal oxides: \( \alpha\)-Fe\(_2\)O\(_3\) and Co\(_3\)O\(_4\). The projector augmented wave (PAW) method\(^{21} \) was used and the exchange-correlation effects are described by the generalized gradient approximation (GGA) as developed by Perdew, Burke and Ernzerhof (PBE).\(^{24} \) Since these materials are magnetic, spin polarized calculations were performed. In order to capture the physisorption of N\(_2\) molecules, van der Waals (vdW) forces were incorporated using the D3 correction method of Grimme et al.\(^{25} \) We used an energy cutoff of 650 eV for both systems. Monkhorst-Pack \( k \)-points meshes of 4x4x2 (4x4x1) and 2x2x2 (2x3x1) were used for bulk (surface) Fe\(_2\)O\(_3\) and Co\(_3\)O\(_4\) systems, respectively. To account for correlations in the 3d orbitals in Fe\(_2\)O\(_3\) and Co\(_3\)O\(_4\), we used Hubbard U-J=4\(^{26} \) and U-J=3\(^{27} \) parameters respectively in the Dudarev approach.\(^{28} \)

All the geometry relaxation was converged to within 1x10\(^{-5} \) eV of the total energy. For surface relaxations, it was found to be advantageous to relax the structure in stages. A rough relaxation to 1x10\(^{-4} \) eV of the total electronic energy and 0.03 eV/Å of total force was followed by a refined relaxation to 1x10\(^{-6} \) eV of the total electronic energy and 0.001 eV/Å of total force. For asymmetric surfaces, a dipole correction\(^{29,30} \) was added at the end.

2.1 Fe\(_2\)O\(_3\)

A lot of theoretical study have been done for the (0001) surface.\(^6,7 \) The bulk and surface properties as well as the parameters used in this study have been listed in Table S2. The lattice
parameters match closely with other theoretical studies as well as experimental ones. The surface energy obtained for the Fe-O$_3$-Fe termination is within the range of previous DFT studies (1.01-1.70 J/m$^2$)$^5,7,8$

**Table S2: Lattice parameters, magnetic moment and surface energy of the Fe$_2$O$_3$ system**

|                  | Present work | Berger et al. | Gattinoni et al.$^5$ | Tang and Liu$^4$ | Dzade et al.$^{11}$ | Wang et al.$^7$ | Exp. |
|------------------|--------------|---------------|-----------------------|------------------|--------------------|----------------|------|
| **Functional**   | PBE+U+D3     | PBE           | opt86b-vdW+U          | PBE+U            | PW91+U+D2         | FP-LAPW        | -    |
| **Energy cutoff (eV)** | 650          | 400           | 550                   | 400              | 400                | 18 Ry          | -    |
| **K-points mesh (bulk)** | 4x4x2        | 4x4x1         | 4x4x2                 | 5x5x2            | 11x11x7           | -              | -    |
| **K-points mesh (surface)** | 4x4x1        | 4x4x1         | 4x4x1                 | 5x5x1            | 5x5x1              | -              | -    |
| **U, J**         | 5, 1         | -             | U-J=4                 | 5, 1             | 5, 1               | -              | -    |
| **a (Å)**        | 5.052        | 4.995         | 5.035                 | 5.027            | 5.024              | 5.025          | 5.035$^7$ |
| **c (Å)**        | 13.823       | 13.858        | 13.763                | 13.728           | 13.658             | 13.671         | 13.747$^7$ |
| **Bulk magnetic moment ($\mu_B$)** | 4.159        | 3.5           | 4.24                  | 4.15             | 4.23               | 3.39           | 4.6-4.9$^{9,10}$ |
| **Surface energy (J/m$^2$)** | 1.34         | -             | 1.54                  | 1.28             | 1.66               | 1.52           |      |

**2.2 Co$_3$O$_4$**

The bulk and surface properties as well as the parameters used in this study have been listed in Table S3. The lattice parameters match closely with other theoretical studies as well as experimental ones.

**Table S3: Lattice parameters, magnetic moment and formation energy of the Co$_3$O$_4$ system**

|                  | Present work | Xu et al.$^{18}$ | Wang et al.$^{17}$ | Beatty et al.$^{15}$ | Ren et al.$^{14}$ | Dong et al.$^{16}$ | Exp. |
|------------------|--------------|-----------------|-------------------|----------------------|------------------|--------------------|------|
| **Functional**   | PBE+U+D3     | PBE+U           | PBE+U             | PBE+U+D3             | PBE+U+D3         | PBE+U+D3          | -    |
| **Energy cutoff (eV)** | 650          | 4.5 Å           | 380               | 500                  | 500              | 400                | -    |
| **K-points mesh (bulk)** | 2x2x2        | 5x5x5           | 4x4x4             | 6x6x6                | 8x8x8            | -                  | -    |
| K-points mesh (surface) | 2x3x1 | 2x3x1 | 3x2x1 | 2x2x1 | 2x2x1 | - |
|-------------------------|-------|-------|-------|-------|-------|---|
| U, J                    | 4, 1  | 3, 1  | 4, 1  | U =5.9 | U-J=3 | U-J=3 |
| a (Å)                   | 8.096 | 8.084 | 8.243 | 8.113 | 8.072 | 7.99 |
| Bulk magnetic moment (µB) | 2.608 | 2.631 |       |       |       | 2.26 |
| Formation energy (J/m²) | -9.394 | -10.435 |       |       |       |     |

3. References

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