Column Weight Two and Three LDPC Codes with High Rates and Large Girths

Abstract— In this paper, the concept of the broken diagonal pair in the chess-like square board is used to define some well-structured block designs whose incidence matrices can be considered as the parity-check matrices of some high rate cycle codes with girth 12. Interestingly, the constructed regular cycle codes with row-weights \( t, 3 \leq t \leq 20, t \neq 7, 15, 16 \), have the best lengths among the known regular girth-12 cycle codes. In addition, the proposed cycle codes can be easily extended to some high rate column weight-3 LDPC codes with girth 6. Simulation results show that the constructed column weight 3 QC LDPC codes remarkably outperform QC LDPC codes based on Steiner triple systems and integer lattices.

Keywords: LDPC Code, Tanner Graph, Girth.

I. INTRODUCTION

Low-density parity-check (LDPC) codes \(^1\) are the most promising class of linear codes due to their ease of implementation and excellent performance over noisy channels when decoded with message-passing algorithms \(^3\). Based on methods of construction, LDPC codes can be divided into two categories: random codes \(^2\) and structured codes \(^10\)\textendash\(^14\). Although randomly constructed LDPC codes of large length give excellent bit-error rate (BER) performance \(^2\), the memory required to specify the nonzero elements of such a random matrix can be a major challenge for hardware implementation. Structured LDPC codes can lead to much simpler implementations, particularly for encoding.

To each parity-check matrix \( H \) of an LDPC code, the Tanner graph \( TG(H) \) is assigned and the girth of the code, denoted by \( g(H) \), is defined as the length of the shortest cycle in \( TG(H) \). Cycles, especially short cycles, in \( TG(H) \) degrade the performance of LDPC decoders, because they affect the independence of the extrinsic information exchanged in the iterative decoding \(^2\). Accordingly, the design of LDPC codes with large girth is of great interest.

Cycle codes are a class of LDPC codes with parity-check matrices having fixed column weight-2, have shown potential in some applications such as partial response channels \(^9\). Also, it has been shown \(^5\) that designing cycle codes with large girth \(^6\), \(^15\), especially in the non-binary setting \(^7\), \(^8\), is highly beneficial for the error-floor performance.

Constructing cycle codes with large girth has been investigated by several authors. In \(^17\), cage graphs were used to construct cycle codes over a wide range of girths and rates. However, the problem of constructing cage graphs is very challenging and there is no deterministic approach to constructing arbitrary cages. In addition, in \(^16\), the authors used singer perfect difference sets to construct some non-binary cycle codes with girth 12 and regularity \( t = q + 1 \), \( q \) prime power, which achieved the Gallager bound. In \(^5\), the authors constructed a particular class of cycle codes with girth 8, \( e \geq 2 \), and rate \( 1/e \). Subsequently, in \(^13\), some girth-8 cycle codes were constructed whose parity-check matrices used as the mother matrices of some quasi cyclic (QC) cycle codes with girth 24.

In this paper, some girth-12 cycle codes are presented such that the constructed regular cycle codes with row-weights \( t, 3 \leq t \leq 20, t \neq 7, 15, 16 \), have the best known lengths among known regular girth-12 cycle codes \(^17\). Specially, for \( t = q + 1 \), \( q \) prime power, the Gallager bound has been achieved for the minimum lengths of the constructed codes. Our construction cause to obtain the memory efficiency in storing the parity-check matrices of the constructed codes in the decoder. In addition, the parity-check matrices of the proposed girth-12 cycle codes can be extended to some parity-check matrices of column weight three corresponding to some high rate column weight three LDPC codes with girth 6. Simulation results show that the constructed column-weight three QC LDPC codes remarkably outperform integer lattice and STS based LDPC codes over the additive white Gaussian noise channel.

II. PRELIMINARIES AND CONSTRUCTIONS

Let \( V = \{0, 1, \ldots, m - 1\} \) and \( B = \{B_1, B_2, \ldots, B_k\} \) be a collection containing subsets \( B_i \subseteq V, 1 \leq i \leq b \). The incidence matrix of \( B \) is an \( m \times b \) binary matrix \( H = (h_{ij})_{0 \leq i < m, 1 \leq j \leq b} \), in which \( h_{ij} = 1 \) iff \( i \in B_j \).

For a given integer \( m \), let \( L_m \) denote the \( m \times m \) square board whose columns (resp. rows) are indexed by \( 0, 1, \ldots, m - 1 \) from left to right (resp. up to down) starting from the most upper-left corner. So, by the square \((i, j)\), \( 0 \leq i, j \leq m - 1 \), we mean the square with row and column indices \( i \) and \( j \), respectively. The main diagonal of \( L_m \) is defined as the \( \{(i,i), 0 \leq i \leq m - 1\} \). By a coloring of \( L_m \), we mean a white-black coloring of the squares so that the main diagonal squares are white and the black squares are symmetric with respect to the main diagonal of \( L_m \), i.e. if the square \((i, j)\) is black, then the square \((j, i)\) is also black. For example, a random coloring of \( L_{10} \) is given in Figure \(\ref{fig:1}\) part (a). Since in an arbitrary coloring of \( L_m \), black and white squares are symmetric, thus we use \((i, j)\) to denote the squares \((i, j)\) or \((j, i)\). For an arbitrary coloring of \( L_m \) with black squares \( B_k = \{i_k, j_k\}, 1 \leq k \leq b \), let \( B = \{B_1, B_2, \ldots, B_k\} \) and \( H = (h_{p,q}) \) be the \( m \times b \) incidence matrix of \( B \).

Clearly \( H \) can be considered as the parity-check matrix of a cycle code with design rate \( R = 1 - m/b \) and block length \( b \). For example, the incidence matrix \( H \) corresponding to the random coloring of \( L_{10} \), shown in part (a) of Figure \(\ref{fig:1}\) is as follows, which can be considered as the parity-check matrix of a cycle code with girth 12 and design rate 0.25.

\[
H = \begin{pmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{pmatrix}
\]
g(H) is fully dependant on the coloring of \( \mathcal{L}_m \). Although increasing the number of black squares in \( \mathcal{L}_m \) increases the rate, in most cases this increment is accompanied by a reduction of the girth. Therefore for a fixed \( g \), the existence of a high rate cycle code with girth \( g \) will be guaranteed by an appropriate coloring of \( \mathcal{L}_m \). For example, an appropriate coloring of \( \mathcal{L}_m \) has been presented in (13) to construct some high rate girth-8 cycle codes with minimum lengths. In the sequel, some appropriate colorings of \( \mathcal{L}_m \) are presented to construct some high rate cycle codes with girth 12.

We begin with some notations and definitions. The pandiagonals of \( \mathcal{L}_m \) are those diagonal segments that are parallel to the main diagonal. Two pandiagonals that together contain \( m \) squares are called a broken diagonal pair. Now, let \( m \geq 14 \) and \( p \) be an odd positive integer less than \( m \). For even \( m \), we use \( L_m(p) \) to denote the broken diagonal pair in \( \mathcal{L}_m \) containing alternative white and black squares starting from \((0,0)\) such that the square \((0,0)\) is black. For odd \( m \), \( L_m(p) \) is obtained from \( L_{m+1}(p) \) by removing the last row and column of \( \mathcal{L}_{m+1} \). It can be easily seen that, for even \( m \), \( L_m(p) \) contains black squares \( \{ (2j, p + 2j \mod m) : 0 \leq j < \frac{m}{2} \} \) and for odd \( m \), \( L_m(p) = L_{m+1}(p) \setminus \{ (m-p, m) \} \). For example, \( L_{14}(5) = \{ (0, 5), (2, 7), (4, 9), (6, 11), (8, 13), (10, 2), (12, 4) \} \) and \( L_{13}(5) \) is the parity-check matrix of a regular cycle code with regularity \( t \). Thus, for odd \( m \), \( H_m(v) \) has \( m-1 \) rows of weight 1 and a row (indexed by \( m-v_i \)) of weight 0. Since for each \( i \neq j, m-v_i \neq m-v_j \), \( H_m(v) \) contains \( t \) rows of weight \( t-1 \) corresponding to the rows indexed by \( m-v_1, m-v_2, \ldots, m-v_t \), and \( m-t \) rows of weight 1.

It is noticed that for a given \((m, t)\)-vector \( v = (v_1, \ldots, v_t) \), we may assume that \( v_1 = 1 \), because the girth is invariant under any permutation on the rows (columns) of \( H_m(v) \). In addition, the design rate of the cycle code with the parity-check matrix \( H_m(v) \) is \( 1 - m/t \), which tends to 1, when \( t \) increases. In fact, the actual rate is \( r = 1 - (m-1)/t \), because the rank of \( H_m(v) \) is \( m-1 \).

Since for odd \( m \), \( H_m(v) \) is obtained from \( H_{m+1}(v) \), thus here in after we just consider the case that \( m \) is even. The Tanner graph \( \text{TG}(H_m(v)) \) consists check nodes \{0, \ldots, m-1\} and variable nodes \( c_{i,j} = [2j, v_i + 2j \mod m] \), \( 0 \leq j < \frac{m}{2}, 1 \leq i \leq t \), corresponding to the elements of \( L_m(v) \), where each \( c_{i,j} \) connects even check node \( 2j \) to odd check node \( v_i + 2j \mod m \). As an example, \( \text{TG}(H_{14}(1,5,13)) \) is shown in Figure 1 part (d), in which variable nodes and check nodes are denoted by white and black circles, respectively.

Since the components \( v_i, 1 \leq i \leq t \), of a \((m, t)\)-vector \( v = (v_1, \ldots, v_t) \) are distinct, thus \( \text{TG}(H_m(v)) \) is free of 4-cycles. On the other hand, variable nodes in \( \text{TG}(H_m(v)) \) connects even check nodes to odd check nodes and so cycles of lengths \( 6 \) and \( 10 \) are avoidable in \( \text{TG}(H_m(v)) \). In addition, the Tanner graph \( \text{TG}(H_m(v)) \) consists the 12-cycle containing check nodes \{0, v_2, v_2 - 1, v_2 + v_3 - 1, v_3 - 1, v_3 \} and variable nodes \{c_{2,0}, c_{1, (v_2-1)/2}, c_{3, (v_2-1)/2}, c_{2, (v_3-1)/2}, c_{1, (v_3-1)/2}, c_{3, 0}\}, as shown in Figure 2 part (a). Thus \( g(H_m(v)) \leq 12 \). Now, in the following theorem, necessary and sufficient conditions are given which guarantees that \( g(H_m(v)) = 12 \).

Fig. 1. A random coloring of \( \mathcal{L}_{10} \), lines \( L_{13}(5) \) and \( L_{14}(5) \) in \( \mathcal{L}_{14} \), \( L_{14}(1, 5, 13) \) and its Tanner graph, resp. from left to right.
be a TG two distinct even check nodes because each variable node connects an odd check node to $2 = (v_1, v_2, \ldots, v_t)$ with row weight $k = \left\lfloor \frac{m}{2} \right\rfloor$.

**Theorem 1:** Let $m \geq 14$ be even and $v = (v_1, v_2, \ldots, v_t)$ be a $(m, t)$-vector. Then $g(H_m(v)) = 12$ if and only if for every $k_1, k_2, k_3, k_4$ with $\{k_1, k_2\} \cap \{k_3, k_4\} = \emptyset$, we have

$$(v_{k_1} + v_{k_2}) - (v_{k_3} + v_{k_4}) \not\in \{0, \pm m\}.$$ 

**Proof.** To show $g(H_m(v)) = 12$, it is sufficient to prove that $\text{TG}(H_m(v))$ is free of 8-cycles. By the definition, for each $i$, $0 \leq i < \frac{m}{2}$, any check node $z = 2i \in \text{TG}(H_m(v))$ is only connected to check nodes $2i + v_i \pmod{m}$, $1 \leq k \leq t$, through variable node $c_{k,i}$, $1 \leq k \leq t$. It is easy to see that any 8-cycle in $\text{TG}(H_m(v))$ must contains exactly two distinct even check nodes $2i$ and $2j$, for some $i \neq j$, because each variable node connects an odd check node to an even check node. Now, set $A_1 = \{2i + v_i, k = 1, \ldots, t\}$ and $A_2 = \{2j + v_j, k = 1, \ldots, t\}$. As shown in Figure 2 part (b), an 8-cycle containing check nodes $2i$ and $2j$ exists if and only if $|A_1 \cap A_2| \geq 2$. Thus, let $|A_1 \cap A_2| \geq 2$ and $2i + v_i, 2i + v_j \in A_1 \cap A_2$, $k_i \neq k_j$, which implies that $2i + v_{k_i} \equiv 2j + v_{k_j}$ and $2i + v_{k_j} \equiv 2j + v_{k_i}$, for some $1 \leq k_i, k_j \leq t$ with $\{k_1, k_2\} \cap \{k_3, k_4\} = \emptyset$, where congruent relations are considered modulo $m$.

This means that $v_{k_1} - v_{k_2} \equiv v_{k_3} - v_{k_4} \pmod{m}$ or equivalently $m\{v_{k_1} + v_{k_2}\} - (v_{k_3} + v_{k_4}) \leq 2m - 8$. But $-2m - 8 < (v_{k_1} + v_{k_2}) - (v_{k_3} + v_{k_4}) \leq 2m - 8$, which implies that $\{v_{k_1} + v_{k_2}\} = \{0, \pm m\}$. This observation shows that 8-cycles are avoidable in $\text{TG}(H_m(v))$ and this completes the proof.

Now, in the following algorithm we generate $(m, t)$-vectors $v = (v_1, v_2, \ldots, v_t)$ such that $H_m(v)$ has girth 12. In fact, the following algorithm finds the smallest $m$ such that a $(m, t)$-vector exists. Using this algorithm, Table I presents such $(m, t)$-vectors $v$ corresponding to some girth-12 cycle codes with row-weight $t$, $3 \leq t \leq 20$ and rate $r$. Interestingly, the minimum length $n = mt/2$, of a regular cycle code with row weight $t \equiv q + 1, q$ prime power, and girth 12 determined by the Gallager bound [11] has been achieved by the codes given in Table I marked by an star. Moreover, for other values of $t$, $t \not\equiv 7, 15, 16$ the proposed cycle codes have minimum lengths among the known cycle codes [17].

**Algorithm.** Generating $(m, t)$-vectors $v$ with $g(H_m(v)) = 12$.

1. Let $m \geq 14$ be even and $t \geq 3$.
2. Let $k = 2$, $A_1 = \{1\}$, $A_2 = \{3, 5, \ldots, m - 1\}$ and $(v_1, v_2) \in A_1 \times A_2$ are chosen arbitrary.
3. If $k = 1$ then $m \rightarrow m + 2$ and go to step 2.
4. Choose $v_k \in A_k$. Define $A_{k+1}$ as the set of all elements

$$v \in \{v_1 + 2, v_k + 4, \ldots, m - 1\}$$

such that for all $i_1, i_2, i_3 \in \{1, 2, \ldots, k\}$ we have $v \not\equiv v_{i_1} + v_{i_2} - v_{i_3} \pmod{m}$ and $2v \equiv v_{i_1} + v_{i_2} \pmod{m}$.

5. If $A_k = \emptyset$, then set $k \rightarrow k - 1$, $A_k \rightarrow A_k - \{v_k\}$, and go to step 3.
6. If $k = t$, then go to step 8.
7. $k \rightarrow k + 1$ and go to step 4.
8. Print $v = (v_1, v_2, \ldots, v_t)$ as a solution.

**III. COLUMN WEIGHT 3 LDPC CODES**

One important invariant affecting the performance of an LDPC code is the column weight of the parity check matrix. For maximum-likelihood decoding, LDPC codes with larger column weight will give better decoding performances. Therefore, the smallest number of rows that can be added to the parity-check matrix of a cycle code with girth at least 6 to construct a girth-6 column-weight three LDPC code is an interesting problem. In the sequel, we give an approach to construct some girth-6 column-weight three LDPC codes from the proposed cycle codes such that number of added rows is small as possible. We begin with the following simple, but floristic lemma.

**Lemma 3.1:** Let $m \geq 14$ and $v = (v_1, v_2, \ldots, v_t)$ be an arbitrary $(m, t)$-vector. The minimum number of rows that must be added to $H_m(v)$ to construct girth-6 column-weight three LDPC code is at least $t$.

**Proof.** Consider an arbitrary row of $H_m(v) = (h_{ij})$ with regularity $t$ and non zero elements $h_{r,c_1} = h_{r,c_2} = \ldots = h_{r,c_t} = 1$. To avoid 4-cycles in any extension of $H_m(v)$ to a column-weight three parity-check matrix, we need at least $t$ new rows correspond to the column indexed by $c_1, c_2, \ldots, c_t$.

Now, we go through the details of the construction. Let $m \geq 14$ and $v = (v_1, v_2, \ldots, v_t)$ be an arbitrary $(m, t)$-vector. For each $i$, $1 \leq i \leq t$, set $B_i = \{e, o, m + i - 1 : \{e, o\} \in L_m(v_i)\}$ and $B_m(v) = \bigcup B_i$. For example $B_{14}(15, 13) = \{\{0, 1, 14\}, \{2, 3, 14\}, \{4, 5, 14\}, \{6, 7, 14\}, \{8, 9, 14\}, \{10, 11, 14\}, \{12, 13, 14\}, \{0, 5, 15\}, \{2, 7, 15\}, \{4, 9, 15\}, \{6, 11, 15\}, \{8, 13, 15\}, \{1, 10, 15\}, \{3, 12, 15\}, \{0, 13, 16\}, \{2, 1, 16\}, \{4, 3, 16\}, \{6, 5, 16\}, \{8, 7, 16\}, \{10, 9, 16\}, \{12, 11, 16\}\}$.

Let $M_m(v)$ denote the incidence matrix of $B_m(v)$. It is easy to see that $M_m(v)$ can be considered as the parity-check matrix of a column weight 3 LDPC code with girth 6, because for each $i$, $v_i \not\equiv j$, $L_m(v_i) \cap L_m(v_j) = \emptyset$ and the new points added to $L_m(v_i)$ and $L_m(v_j)$ are distinct. Clearly the first $m$ rows of $M_m(v)$ have the same regularity as $H_m(v)$ and the new $t$ added rows have regularity $\left\lfloor \frac{m}{t} \right\rfloor$. Therefore $M_m(v)$ is always irregular unless $m$ is even, $t = \frac{m}{2}$ and $v = (1, 3, \ldots, m - 1)$. The later case, was discussed in [12], which can be easily derived from our construction. Moreover, the rate of the constructed column weight 3 LDPC codes is $r' = 1 - \frac{\frac{m}{2} - 1}{\frac{m}{2} + 1}$, which tends to one when $m$ increases. As shown in Table I, the rate of the constructed column weight three LDPC codes derived from the proposed cycle codes, denoted by $r'$, is close to the rate of the proposed cycle codes.

Kim et al. [11] have shown that if the base matrix $H$ has girth 2g, then the maximum achievable girth of quasi cyclic LDPC codes having base matrix $H$ is at least 6g. Using this fact, the constructed column weights 2 and 3 LDPC codes
can be considered as the base matrices of some quasi cyclic LDPC codes with girth at least 36 and 18, respectively. In fact, using a similar approach posed in [12], the maximum achievable girth of the constructed quasi cyclic LDPC codes with column weight 3 is 20.

IV. SIMULATION RESULTS

In this section, we examine a performance comparison between the constructed column weight three LDPC codes with large girth, employing the proposed algorithm in [12], on one hand, and LDPC codes with different girths constructed in [13] based on Steiner triple system $STS(9)$ and the 15-points $3 \times 5$ integer lattice $L(3 \times 5)$, on the other hand.

In Figure 5 $STS(9)(N; gb)$ and $L(3 \times 5; N; gb)$ are used to denote $STS(9)$ and $L(3 \times 5)$-based LDPC codes with block-size $N$ and girth $b$, respectively. Moreover, $C3(m, N; gb)$ is used to denote the column-weight three QC LDPC code with girth $b$, block size $N$ which is lifted from the base matrix $M_{bn}(v)$. As shown Figure 5 the constructed column-weight three codes with different girths significantly outperform the codes based on $STS(9)$ and $L(3 \times 5)$ with the same girth.

REFERENCES

[1] R. G. Gallager, Low-Density parity-check Codes, Cambridge, MA: MIT Press, 1963.
[2] D. J. C. MacKay, Good error-correcting codes based on very sparse matrices, IEEE Trans. Inform. Theory, vol. 45, no. 2, pp. 399-431, March 1999.
[3] F. R. Kschischang, B. J. Frey, and H. A. Loeliger, Factor graphs and the sum-product algorithm, IEEE. Trans. Inform. Theory, vol. 47, pp. 498–519, 2001.
[4] R. M. Tanner, A recursive approach to low complexity codes, IEEE Trans. Inform. Theory, vol. IT-27, pp. 533-547, Sept. 1981.
[5] M. Gholami and M. Esmaeili, Maximum-girth Cylinder-type Block-circulant LDPC Codes, IEEE Trans. Commun., vol. 60, no. 4, pp. 952–962, April 2012.
[6] M. Gholami and M. Samadieh, Design of binary and nonbinary codes from lifting of girth-8 cycle codes with minimum lengths, IEEE Commun. Lett., vol. 17, no. 4, pp. 777-780, Apr. 2013.
[7] R. H. Peng and R. R. Chen, Design of nonbinary quasi-cyclic LDPC cycle codes, ITW, Lake Tahoe, California, Sept. 2-6, 2007.
[8] W. Chen, C. Poulliat, D. Declercq et al., Structured high-girth nonbinary cycle codes, in Proc. Asia Pacific Conference on Communications (APCC), October 2009.
[9] H. Song, J. Liu, and B. V. K. V. Kumar, Large girth cycle codes for partial response channels, IEEE Trans. Magn., vol. 40, no. 4, part 2, pp. 2084-2086, 2004.
[10] J. Thorpe, Low-density parity-check (LDPC) codes constructed from protograph, IPN Progr. JPL. Rep. 42-154, pp. 1-7, Aug. 2003.

Table I. Some $(m, t)$-vectors $v = (v_1, \ldots, v_t)$ associated to $H_m(v)$ with girth 12.

| m | t | $r$ | $r'$ | $n$ | Gallager bound |
|---|---|---|---|---|---|
| 13 | 3 | 0.38 | 0.24 | 27 | 23 |
| 20 | 4 | 0.52 | 0.44 | 52 | 52 |
| 25 | 5 | 0.61 | 0.56 | 105 | 105 |
| 62 | 6 | 0.67 | 0.64 | 186 | 186 |
| 90 | 7 | 0.71 | 0.7 | 336 | 301 |
| 114 | 8 | 0.75 | 0.71 | 449 | 426 |
| 146 | 9 | 0.78 | 0.77 | 567 | 657 |
| 182 | 10 | 0.8 | 0.79 | 710 | 910 |
| 240 | 11 | 0.82 | 0.81 | 1320 | 1221 |
| 260 | 12 | 0.83 | 1.008 | 1506 | 1506 |
| 330 | 13 | 0.85 | 2.379 | 2041 | 2041 |
| 360 | 14 | 0.86 | 2.562 | 2562 | 2562 |
| 510 | 15 | 0.87 | 3.825 | 3165 | 3165 |
| 510 | 16 | 0.85 | 4.080 | 3856 | 3856 |
| 540 | 17 | 0.88 | 4.631 | 4641 | 4641 |
| 614 | 18 | 0.89 | 5.256 | 5526 | 5526 |
| 720 | 19 | 0.90 | 6.840 | 6517 | 6517 |
| 762 | 20 | 0.9 | 0.89 | 7620 | 7620 |

---

Fig. 3. Comparisons between the proposed column-weight 3 LDPC codes with different girths and some well-known codes

[11] S. Kim, J. S. No, H. Chung, and D. J. Shin, Quasi-cyclic low-density parity-check codes with girth larger than 12, IEEE Trans. Inform. Theory, vol. 53, pp. 2885-2891, Aug. 2007.
[12] M. Gholami, M. Samadieh and G. Raeisi, Column-Weight Three QC LDPC Codes with Girth 20, IEEE Commun. Lett., vol. 17, pp. 1439-1442, May 2013.
[13] M. Esmaeili and M. Gholami, Structured QC LDPC codes with girth 18 and column-weight $J \geq 5$, IEEE Commun. Letter, Int. J. Electron. Commun. (AEUE), vol. 64, pp. 202-217, Mar. 2010.
[14] I. E. Bocharova, F. Hug, R. Johannesson, B. D. Kudryashov, and R. V. Satyukov, Searching for Voltage Graph-Based LDPC Tailbiting Codes with Large Girth, IEEE Trans. Inf. Theory, vol. 58, pp. 2265–2279, April 2012.
[15] G. Malena and M. Liebelt, High Girth Column-Weight-Two LDPC Codes Based on Distance Graphs, EURASIP Journal on Wireless Communications and Networking, vol. 2007, Article ID 48158, 5 pages, 2007.
[16] C. Chen, B. Bai and X. Wang, Construction of nonbinary quasi-cyclic LDPC code cycles based on singer perfect difference set, IEEE Commun. Lett., vol. 14, pp. 181–183, Feb. 2010.
[17] G. Esco, R. Jajcay, Dynamic Cage Survey, Electron. J. Combin., #DS16, pp. 1–55, Jul. 2013.
