Baryonic acoustic oscillations in 21-cm emission: a probe of dark energy out to high redshifts

J. Stuart B. Wyithe,1⋆ Abraham Loeb2⋆ and Paul M. Geil1⋆

1School of Physics, University of Melbourne, Parkville, Victoria, Australia
2Harvard–Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA

Accepted 2007 October 23. Received 2007 October 21; in original form 2007 September 19

ABSTRACT

Low-frequency observatories are currently being constructed with the goal of detecting redshifted 21-cm emission from the epoch of reionization. These observatories will also be able to detect intensity fluctuations in the cumulative 21-cm emission after reionization, from hydrogen in unresolved damped Lyα absorbers (such as gas-rich galaxies) down to a redshift $z \sim 3.5$. The inferred power spectrum of 21-cm fluctuations at all redshifts will show acoustic oscillations, whose comoving scale can be used as a standard ruler to infer the evolution of the equation of state for the dark energy. We find that the first generation of low-frequency experiments (such as MWA or LOFAR) will be able to constrain the acoustic scale to within a few per cent in a redshift window just prior to the end of the reionization era, provided that foregrounds can be removed over frequency bandpasses of $\gtrsim 8$ MHz. This sensitivity to the acoustic scale is comparable to the best current measurements from galaxy redshift surveys, but at much higher redshifts. Future extensions of the first-generation experiments (involving an order of magnitude increase in the antennae number of the MWA) could reach sensitivities below 1 per cent in several redshift windows and could be used to study the dark energy in the unexplored redshift regime of $3.5 \lesssim z \lesssim 12$. Moreover, new experiments with antennae designed to operate at higher frequencies would allow precision measurements (≤1 per cent) of the acoustic peak to be made at more moderate redshifts ($1.5 \lesssim z \lesssim 3.5$), where they would be competitive with ambitious spectroscopic galaxy surveys covering more than 1000 deg$^2$. Together with other data sets, observations of 21-cm fluctuations will allow full coverage of the acoustic scale from the present time out to $z \sim 12$.

Key words: galaxies: high-redshift – intergalactic medium – cosmology: theory – diffuse radiation – large-scale structure of Universe.

1 INTRODUCTION

Measurement of the fluctuations in the intensity of redshifted 21-cm emission from neutral hydrogen promises to be a powerful probe of the reionization era (Furlanetto, Oh & Briggs 2006). The process of hydrogen reionization started with ionized (H II) regions around the first galaxies, which later grew to surround groups of galaxies. Reionization completed once these H II regions overlapped (defining the so-called overlap era) and filled up most of the volume between galaxies. Detection of the redshifted 21-cm signal will not only probe the astrophysics of reionization, but also the matter power spectrum during the epoch of reionization (McQuinn et al. 2006; Bowman, Morales & Hewitt 2007).

The conventional wisdom presumes that the 21-cm signal would disappear after the overlap epoch, because there is little neutral hydrogen left through most of intergalactic space. However, Wyithe & Loeb (2007) recently demonstrated that fluctuations in the 21-cm emission would remain substantial over a range of epochs following the end of the overlap era owing to the significant fraction by mass of neutral hydrogen that is locked up in the dense pockets that form the damped Lyα absorbers (DLAs) such as gas-rich galaxies. These systems trace the matter power spectrum on large scales. Hence observations of 21-cm fluctuations could in principle be used as a cosmological probe both during the reionization era and in the post-reionization intergalactic medium (IGM).

The sky temperature, which provides the limiting factor in the system noise at the low frequencies relevant to 21-cm studies, is proportional to $(1 + z)^2$, and so is a factor of $\sim 3.4(1 + z)/5)^2$ smaller at low redshifts than for observations at $z \sim 7$. As a result, detectability of fluctuations in 21-cm emission may not decline

⋆E-mail: swyithe@unimelb.edu.au (JSBW); loeb@cfa.harvard.edu (AL); pgeil@physics.unimelb.edu.au (PMG)
substantially following the overlap epoch. When combined with the large fluctuations in redshifted 21-cm emission during the reionization era, the detectability of a 21-cm power spectrum after the end of reionization will allow the measurement of cosmological parameters over a wide range of redshifts. Much of this constraining power originates with redshift-space distortions (McQuinn et al. 2006), which probe cosmology through the mapping between the vectors describing the wavenumber, and the observed spectral and angular scalelengths. The correct mapping produces an undistorted power spectrum. In this paper we consider baryonic acoustic oscillations (BAO). These provide constraints on cosmology that are related to redshift-space distortions, but which are particularly sensitive to the dark energy.

The BAO scale provides a cosmic yardstick that can be used to measure the dependence of both the angular diameter distance and Hubble parameter on redshift. The wavelength of the BAO is related to the size of the sound horizon at recombination. Its value depends on the Hubble constant, and on the matter and baryon densities. However, it does not depend on the amount or nature of the dark energy. Thus measurements of the angular diameter distance and Hubble parameter can in turn be used to constrain the possible evolution of the dark energy with cosmic time. This idea was originally proposed in relation to galaxy redshift surveys (Blake & Glazebrook 2003; Hu & Haiman 2003; Seo & Eisenstein 2003) and has since received significant theoretical attention (e.g. Glazebrook & Blake 2005; Seo & Eisenstein 2005; Angulo et al. 2007; Seo & Eisenstein 2007). Moreover, measurement of the BAO scale has been achieved within large surveys of galaxies at low redshift, illustrating its potential (Cole et al. 2005; Eisenstein et al. 2005). Galaxy redshift surveys are best suited to studies of the dark energy at relatively late times due to the difficulty of obtaining accurate redshifts for a large number of high-redshift galaxies. It has however been proposed that future power spectrum measurements based on the Lyα forest could be used probe the evolution of dark energy at higher redshifts through measurement of the BAO scale at $z < 4$ (McDonald & Eisenstein 2007). Similarly, high-resolution spectra may probe the evolution of dark energy through the Sandage–Loeb test (Corasaniti, Huterer & Melchiorri 2007). If the dark energy behaves like a cosmological constant, then its effect on the Hubble expansion is dominant only at $z \lesssim 1$ and becomes negligible at $z \gtrsim 2$. In this case studies of the BAO scale at low redshift would provide the most powerful measurement. However, the origin of the dark energy is not understood, and so it is not known a priori which redshift range should be studied in order to provide optimal constraints on possible theories for it.

At high redshifts, BAOs in the 21-cm power spectrum should be detectable during the reionization era using future low-frequency arrays (Bowman et al. 2007). Following the end of the reionization era, the detection of a large number of individual galaxies in redshifted 21-cm emission could be used to trace the matter power spectrum using the future generation of radio telescopes (Abdalla & Rawlings 2005), in a manner entirely analogous to optical galaxy redshift surveys. On the other hand, 21-cm observations using a low-frequency compact radio array could also detect fluctuations in the total neutral hydrogen content within volumes of IGM dictated by the telescope beam and frequency bandpass. Thus, in analogy to observations of the neutral IGM during the reionization era, one could construct the power spectrum of 21-cm intensity fluctuations in the cumulative 21-cm signal from all the unresolved pockets of neutral hydrogen in the IGM, regardless of their mass. An important advantage of studying the matter power spectrum through detection of fluctuations in the total emission of unresolved sources, rather than in the space density of individual sources, is that the requirement of object detection is removed which, as we show, allows the power spectrum to be determined at much higher redshifts where individual sources may not be resolved in sufficient numbers.

Our goal in this paper is to investigate the feasibility of using redshifted 21-cm observations to make precise measurements of the scale of the BAO in the matter power spectrum. We show that observations of the acoustic scale using 21-cm emission could be used to constrain the nature of the dark energy in the unexplored redshift range of $1.5 \lesssim z \lesssim 6$, as well as during the reionization era, and so would be complementary to galaxy redshift surveys. No other probes for precision cosmology are currently being applied to this cosmic epoch (Corasaniti et al. 2007). The outline is as follows. In Sections 2 and 3 we will describe our approach for calculating the 21-cm power spectrum, and its measurement uncertainties. The precision with which the scale of acoustic oscillations can be detected using upcoming 21-cm observatories will be analysed in Section 4, followed by a discussion of the possible influence of non-linear evolution of the power spectrum (Section 5). The potential constraints on the evolution of dark energy are presented in Section 6. Finally, we will summarize our main conclusions in Section 7.

2 THE POWER SPECTRUM OF 21-CM FLUCTUATIONS

A powerful statistical probe of the reionization era will be provided by the power spectrum of 21-cm emission which is naturally accessible to interferometric observations such as those to be carried out by the Mileura Wide-field Array (MWA) or the Low-Frequency Array (LOFAR2). We may write the following expression for the power spectrum of 21-cm fluctuations

$$P_21(k) = b_{21}(z, k)^2 P(k) D(z)^2,$$  

(1)

where $P(k)$ is the primordial power spectrum of the density field as a function of wavenumber $k$, extrapolated linearly to $z = 0$, and $D(z)$ is the growth factor for linear perturbations. We model the power spectrum, including baryonic oscillations using the transfer function from Eisenstein & Hu (1998), and normalized today to give an rms amplitude $\sigma_8 = 0.76$ for the mass density fluctuations within a sphere of radius $8\, h^{-1}\, \text{Mpc}$.

$$T(\delta, R) = 22.0\, \text{mK} \left(1 + \frac{z}{7.5}\right)^{1/2} [1 - Q_i(R, \delta)] \left(1 + \frac{4}{3} \delta\right),$$  

(2)

1. See http://www.haystack.mit.edu/ast/arrays/mwa/index.html.
2. See http://www.lofar.org/.

© 2007 The Authors. Journal compilation © 2007 RAS, MNRAS 383, 1195–1209
Figure 1. Upper panel: The bias ($b_{21}$) computed from our semi-analytic model at three different scales (dark solid, dashed and dotted lines). For comparison the figure also shows three curves for the bias corresponding to the 21-cm fluctuations given by spatially uniform mean neutral fractions, with fluctuations due to the density field. In the three cases the neutral fractions are equal to 1 (fully neutral, dashed grey line), 0.03 (the neutral fraction in DLAs at $z \lesssim 5$, dotted grey line) and the mean neutral fraction from our model (solid grey line). Lower panel: The bias ($b_{21}$) computed from our numerical model at four different wavenumbers $k$ (dark solid, dotted, short-dashed and long-dashed lines). The semi-analytic estimate at $R = 100$ Mpc is also shown for comparison.

where the pre-factor of $4/3$ on the overdensity refers to the spherically averaged enhancement of the brightness temperature due to peculiar velocities in overdense regions (Barkana & Loeb 2005a; Bharadwaj & Ali 2005). Given the distribution of $\delta$ from the primordial power spectrum of density fluctuations, we may find the probability distribution $dP/dT$ of brightness temperature $T$ in redshifted 21-cm intensity maps. The second moment of this distribution $\langle (T - \langle T \rangle)^2 \rangle$ corresponds to the autocorrelation function of brightness temperature smoothed on a scale $R$. The effective bias can be estimated directly from our calculation of the autocorrelation function

$$b_{21}(R)^2 = \frac{\langle (T - \langle T \rangle)^2 \rangle}{\sigma(R)^2},$$

where $\sigma(R)$ is the variance of the density field smoothed on a scale $R$.

The upper panel of Fig. 1 shows the bias ($b_{21}$) computed from our semi-analytic model. Curves are shown for model fluctuations computed at three different scales, each of which is larger than the typical bubble scale during the reionization era. For comparison, the figure also shows three curves for the bias corresponding to the 21-cm fluctuations given by spatially uniform mean neutral fractions, with fluctuations due to the density field. In the three cases, the neutral fractions are equal to 1 (fully neutral), 0.03 (the neutral fraction in DLAs at $z \lesssim 5$), and the mean neutral fraction from our model. We note that our analytic estimate of the bias is a weighted average of the bias on scales less than $R$, whereas the bias in equation (1) should be computed at a particular scale. The bias computed using our analytic model is quite insensitive to scale for $R \gtrsim 30$ Mpc, indicating that on large scales the bias is independent of wavenumber $k$.

Following the end of reionization (i.e. $z \lesssim 6$) there are no longer separate ionized bubbles as most of the IGM is ionized. Wyithe & Loeb (2007) have shown that the skewness of the 21-cm intensity fluctuation distribution will be small during the post-overlap epoch, which implies a linear relation between the matter and 21-cm power spectra. Moreover, since the mean free path of ionizing photons becomes very large in the post-overlap IGM, the ionization field is smooth. As a result the relation between the density and 21-cm emission can be reliably estimated using the semi-analytic model.

However, during the reionization epoch the relation between the power spectrum of 21-cm fluctuations and the underlying matter power spectrum is complex, and in the late stages is dominated by the formation of large ionized bubbles (Furlanetto et al. 2004). Thus prior to the end of reionization one must be careful about applying equation (3) to estimate the 21-cm power spectrum in equation (1). On very large scales such as those corresponding to the scale of BAOs, the fluctuations should average over many bubbles so that the 21-cm power spectrum is again expected to be linearly related to the matter power spectrum (McQuinn et al. 2006). Indeed, the largest discrete H II region that could be observed during the reionization era is much smaller than the BAO scale (Wyithe & Loeb 2004).

We have computed the matter and 21-cm power spectra from a hybrid simulation (e.g. Messinger & Furlanetto 2007) using both analytical and numerical techniques (Geil & Wyithe 2007). At each of redshifts $z = 12$, 8 and 6.5 we computed the three-dimensional ionization field within simulation boxes of side length 3000 co-moving Mpc. The simulations were computed with 256$^3$ resolution elements in each case. For these simulations we used an input power spectrum which includes the BAO signal. At $z = 12$, 8 and 6.5, the simulations have global neutral fractions of 98, 48 and 11 per cent, respectively. The hybrid scheme of Geil & Wyithe (2007) is able to compute the neutral fraction within pixels that are larger than the typical size of H II regions. This feature allows us to compute power spectra at very large scales even early in the reionization era when no H II regions are resolved.

The left-hand panels of Fig. 2 show the 21-cm emission from 12-Mpc slices through the simulation boxes at each redshift. The higher redshift example ($z = 12$) is early in the reionization era, and shows no H II regions forming at the resolution of the simulation (i.e. the IGM does not contain ionized bubbles with radii $\gtrsim 5$ co-moving Mpc). The fluctuations in the 21-cm emission are dominated by the density field at this time. The central redshift ($z \sim 8$) shows the IGM mid-way through the reionization process, and includes a few H II regions above the simulation resolution. The lower redshift example is just prior to overlap, when the IGM is dominated by large percolating H II regions, which are well resolved at the 5-Mpc resolution of our simulation.

The aim of this paper is to investigate the utility of 21-cm observations of the BAO scale as a probe of dark energy at high redshift. It is therefore important to demonstrate that the percolation process does not wash out the signature of BAOs in the 21-cm power spectrum towards the end of the reionization era. In the central left-hand panels of Fig. 2 we show the corresponding matter [$\Delta^2 = k^3 P/(2\pi^2)$] and 21-cm [$\Delta^2_{21} = k^3 P_{21}/(2\pi^2)$] power spectra. The $z = 6.5$ simulation exhibits a shoulder in the 21-cm power spectrum at a scale corresponding to the characteristic bubble size. This shoulder is also seen in analytic models of bubble growth (Furlanetto, Zaldarriaga & Hernquist 2004). In the two higher redshift cases this shoulder is located at a scale below the simulation resolution. We may use these simulations to estimate the bias $b_{21}(k)$ which is calculated from the
square root of the ratio between the simulated 21-cm power spectrum and the simulated matter power spectrum (using equation 3).

In the right-hand panels of Fig. 2 we show the resulting values of \( b_{21}(k) \) as a function of scale.

To illustrate the presence of BAOs in 21-cm power spectra computed using these simulations, we have also calculated the difference between the 21-cm power spectrum constructed from the simulation and a no wiggle reference 21-cm power spectrum [i.e. \( P_{21} - P_{21,\text{ref}}(k) \)]. This difference is plotted in the central right-hand panels of Fig. 2. To construct \( P_{21,\text{ref}}(k) \) we have multiplied a theoretical no wiggle reference matter power spectrum \( P_{\text{m,ref}} \) by the bias computed from the same simulation box [i.e. \( P_{21,\text{ref}}(k) = P_{\text{m,ref}}(k)b_{21}(k)D(z)^2 \)]. Note that since the boxes at each redshift were generated using the same realization of the matter power spectrum, the noise is correlated between the resulting 21-cm power spectra. For comparison we also plot the difference between the theoretical matter power spectrum and the theoretical no wiggle reference matter power spectrum, multiplied by the bias and growth factor squared [i.e. \( (P - P_{\text{m,ref}})D_z^2 \)], which is shown by the red line in Fig. 2]. The grey band around this line illustrates the level of statistical scatter in realizations of the power spectrum due to the finite size of the simulation volume (e.g. Peacock & West 1992). These simulations demonstrate that the 21-cm power spectrum will exhibit BAOs throughout the reionization epoch including the percolation phase of H II regions.

Our simulations show that \( b_{21} \) will be constant on scales much larger than the characteristic size of H II regions, but scale dependent at larger values of \( k \). The critical \( k \) scale below which \( b_{21} \) is nearly constant moves to smaller values of \( k \) as reionization proceeds and the ionized bubbles grow in size. Early in the reionization process our results show that there is only a very weak dependence of \( b_{21} \) on scale since the fluctuations are driven by the density field. Indeed, our modelling shows the scale dependence of \( b_{21} \) to be weak on the scales of interest for BAO at all times until the end of the reionization era. At this time, just prior to the full overlap between H II regions, the simulations show a strong dependence of \( b_{21} \) on scale for \( k \gtrsim 0.05-0.1 \text{ Mpc}^{-1} \) (comparable to the BAO scale) due to the formation of large bubbles.\(^3\) Our simulations suggest that the scale-dependent bias distorts the observed power spectrum of BAOs (see the panels

\(^3\) Note that the bias may never be observed to be scale dependent at values of \( k \) smaller than the BAO scale since the evolution of the power spectrum becomes more rapid than the light crossing time of the BAO scale (Wyithe & Loeb 2004).
in Fig. 2 corresponding to \(z = 6.5\). However, this distortion, which causes the ratio of peak heights to increase relative to a spectrum with constant bias, is not expected to affect the extraction of the BAO scale. This is because in practice, the matter power spectra could be fitted to the data using a scale-dependent bias (e.g. Seo & Eisenstein 2005). We will return to this point, as well as the effects of non-linear evolution of the power spectrum in Section 5.

The numerical values of \(b_{21}\) are plotted as a function of redshift in the lower panel of Fig. 1. Also shown is the semi-analytic estimate of \(b_{21}\) on a scale of \(R = 100\) Mpc, which has been replotted for comparison. Our analytic and numerical models predict similar values of \(b_{21}\) and similar behaviour with redshift. In particular, \(b_{21}\) has a value of a few tens of mK both at the very beginning and very end of reionization, despite the very different values of the global neutral fraction. In addition, on scales much greater than the typical bubble size there is a local minimum in the value of \(b_{21}\) mid-way through reionization, corresponding to the shift from fluctuations in 21-cm emission being dominated by fluctuations in the density to fluctuations in the ionization field. However, the semi-analytic and numerical models do not agree in detail. The semi-analytic model does not include a Poisson component of fluctuation due to the finite number of bubbles in a region of radius \(R\) and so underestimates the value of \(b_{21}\). On the other hand, our numerical scheme does not conserve photons [reflecting a limitation of seminumerical models of this type (Messinger & Furlanetto 2007)]. As a result, while our model predicts the topology of \(H\) at regions at a particular value of neutral fraction, it does not correctly predict the relation between the average neutral fraction and redshift. In the lower panel of Fig. 1 this manifests itself as an offset in the redshift where the local minimum of the bias is predicted to occur.

In light of the results described above, we have chosen to use the value for \(b_{21}\) computed at \(R = 100\) Mpc based on our semi-analytic model through the remainder of this paper. The semi-analytic model makes a conservatively low estimate of \(b_{21}\) at all redshifts and so will yield conservative estimates for the sensitivity of upcoming 21-cm facilities to the 21-cm power spectrum.

### 3 Measurement Uncertainties in the Power Spectrum of 21-cm Fluctuations

Calculations of the sensitivity to the 21-cm power spectrum for an interferometer have been presented by a number of authors. We follow the procedure outlined by McQuinn et al. (2006), drawing on results from Morales (2005) and Bowman, Morales & Hewitt (2006) for the dependence of the array antenna density on radius, \(\rho(r)\). The uncertainty in a measurement of the power spectrum has two separate components. The first, due to the thermal noise of the instrument is

\[
\Delta_{P_{21, N}}(k) = \left[ \frac{T_{\text{sys}}^2}{B_{\text{int}} n(k_{\perp})} \right]^{1/2} \frac{\lambda^2}{A_e \Delta f} \sqrt{\frac{1}{N_e}},
\]

where \(n(k_{\perp})\) is the density of baselines that observe the transverse component of the wavevector (Morales 2005; Bowman et al. 2006), \(T_{\text{sys}} \sim 250(1 + z)/7\) K is the system temperature of the telescope when observing the 21-cm line at redshift \(z\), \(B\) is the bandwidth over which the measurement of the power spectrum is made and \(f_{\text{int}}\) is the integration time. The quantities \(D\) and \(\Delta f\) are, respectively, the comoving distance to the survey volume, and the comoving depth of the survey volume (corresponding to the frequency bandpass within which the power spectrum is measured). The second component of uncertainty is due to sample variance within the finite volume of the survey, and equals

\[
\Delta_{P_{21, SV}}(k) = P_{21}(k) \sqrt{\frac{1}{N_e}}.
\]

The noise is evaluated within a \(k\)-space volume element \(d^3 k\). The total noise within a finite \(k\)-space bin may then be obtained by integration over the volume within the bin. In both equations (4) and (5) the quantity \(N_e = 2\pi k^2 \sin \theta d\theta dk \delta\left(V/(2\pi)^3\right)\) denotes the number of modes observed within a \(k\)-space volume element \(d^3 k = 2\pi k^2 \sin(\theta) dk d\theta\). Note that in computing \(N_e\), we have assumed symmetry about the polar angle and expressed the wavevector \(k\) in components of its modulus \(k\) and angle \(\theta\) relative to the line of sight. Because wavenumbers can only be observed if their line-of-sight component fits within the observer’s bandpass, we set \(N_e = 0\) if \(2\pi k \cos(\theta) > \Delta f\). The number of modes observed depends on the volume of the survey, \(V = \Delta f^2 \Delta k(2\pi / A_{\text{tile}})\), where \(A_{\text{tile}}\) is the total physical surface area of an antenna (this point is discussed further below).

The sensitivity to the 21-cm power spectrum is dependent on both the sensitivity of the telescope to a particular mode, and to the number of such modes in the survey. The former is set by the effective collecting area \((A_e)\) of each antenna element (as well as the total number of antennae), while the latter is sensitive to the total physical area covered by each antenna (which we refer to as \(A_{\text{tile}}\)). For a traditional interferometer consisting of a number of dishes in a phased array, these two areas are approximately equivalent \(A_e \sim A_{\text{tile}}\), since the solid angle of the primary beam and the sensitivity are both proportional to the physical collecting area of the dish. However, in constructing the above formalism for the sensitivity to the power spectrum, we have explicitly allowed \(A_e \neq A_{\text{tile}}\). This is because future interferometers being built to measure fluctuations from the epoch of reionization (like the MWA) will not comprise dishes, but rather a large number of ‘tiles’, each consisting of a phased array of \(N_{\text{dip}}\) dipoles distributed over an area \(A_{\text{tile}}\). Since the size of the dipole will be much lower than \(\lambda\) for observations of the 21-cm line at \(z \gtrsim 3.5\), the effective collecting area of each tile in this regime is \(A_e \sim N_{\text{dip}} \lambda^2 / 4\) (Bowman et al. 2005). Each tile forms an electronically steerable primary beam, with solid angle \(\Omega_{\text{beam}} \sim \lambda^2 / A_{\text{tile}}\). The MWA is designed to observe the 21-cm line from the epoch of reionization, and so has \(A_e \sim A_{\text{tile}}\) when observing at \(z \sim 8\), but \(A_e < A_{\text{tile}}\) at lower redshifts. In terms of measuring a power spectrum this reduces the efficiency of the MWA at higher frequencies. In this paper we consider the power spectrum at \(1.5 \lesssim z \lesssim 6\) as well as during the epoch of reionization at \(z \gtrsim 6\). When showing results at \(z > 3.5\) we use the specifications of the MWA since this observatory is already under construction and its design is not flexible. Observations of neutral hydrogen at \(z < 3.5\) are not accessible to the MWA, and so a new telescope would need to be constructed for probing this epoch. Thus, at \(z < 3.5\) we assume \(A_e \sim A_{\text{tile}}\), corresponding to a telescope with an optimal design (i.e. with dipoles spaced by \(\lambda / 2\)) for observations at these lower redshifts.

In the case of a spherically averaged power spectrum, \(P_{21}(k)\), each of the above noise components can be computed within \(k\)-space volumes of \(d^3 k = 4\pi k^2 d\Omega\), where \(d\Omega\) is a finite bin of values in \(k\). However, the power spectrum, \(P_{21}(k_1, k_2)\), can also be expressed in terms of the wavevector components that are parallel \((k_1)\) and

---

4 We assume \(T_{\text{sys}}\) to be dominated by the sky throughout this paper.

5 The relation between \(A_e\) and \(A_{\text{tile}}\) is often expressed in terms an aperture efficiency \(\epsilon = A_e / A_{\text{tile}}\) (e.g. Morales 2005).
perpendicular (\(k_\perp\)) to the line of sight, in which case the components \(\Delta P_{21,N}\) and \(\Delta P_{21,S,V}\) can be computed within \(k\)-space volumes of \(d^3k = 2\pi k_\perp dk_\parallel \Delta k_\parallel\).

The contamination by foregrounds provides an additional source of uncertainty in the estimate of the power spectrum. McQuinn et al. (2006) have shown that it should be possible to remove the power due to foregrounds to a level below the noise in the cosmological signal, provided that the region of frequency bandpass from which the power spectrum is estimated \((B)\) is substantially smaller than the total bandpass available \((B_{\text{total}})\). Following the approximation suggested in McQuinn et al. (2006), we combine the above components to yield the uncertainty in the estimate of the power spectrum. For the spherically averaged power spectrum we assume

\[
\Delta P_2(k) = \begin{cases} \frac{\Delta P_{21,N}(k) + \Delta P_{21,S,V}(k)}{N_{\text{est}} B_{\text{tot}} / B}, & \text{if } k > k_{\text{min}}, \\ \infty, & \text{otherwise}, \end{cases}
\]

while for noise in the power spectrum at \(k = (k_\perp, k_\parallel)\) we take

\[
\Delta P_2(k) = \begin{cases} \frac{\Delta P_{21,N}(k) + \Delta P_{21,S,V}(k)}{N_{\text{est}} B_{\text{tot}} / B}, & \text{if } k_\parallel > k_{\text{min}}, \\ \infty, & \text{otherwise}, \end{cases}
\]

where \(k_{\text{min}} = 2\pi / \Delta D\). In each of equations (6) and (7) the denominator represents the number of independent measurements made of the power spectrum within a bandwidth \(B\). The factor \(N_{\text{est}}\) is included because one independent measurement would be made per field imaged for time \(t_{\text{int}}\). As part of their analysis of foreground removal, McQuinn et al. (2006) have found that foreground power could be removed within a region \(B\) of the observed bandpass provided \(B\) is significantly smaller than \(B_{\text{tot}}\). However, in addition McQuinn et al. (2006) have also found that foreground removal is not sensitive to the location of \(B\) within the total processed bandpass. As a result, on scales where the power spectrum can be measured (i.e. at \(k < k_{\text{min}}\)) it can be determined within \(B_{\text{tot}} / B\) independent regions of the total processed bandpass \(B_{\text{tot}}\). Hence while foreground removal will limit the scale of fluctuations that can be observed, foregrounds should not affect the total sensitivity of the array to smaller scale modes. We note that, as with all studies of 21-cm fluctuations, the largest uncertainty in our analysis. In addition, we also note that the uncertainties assumed due to thermal noise and sample variance represent best case scenarios. It is possible that other experimental factors (such as calibration errors, radio frequency interference, and contamination due to polarized foregrounds) could further degrade the sensitivity to the BAO scale.

4 21-CM OBSERVATIONS OF BARYONIC ACOUSTIC OSCILLATIONS

Fig. 3 shows results for the power spectrum of 21-cm fluctuations, and sensitivity to the BAO scale at \(z = 3.5, 5, 6.5\) and 8. The spherically averaged model power spectra are marked on the left-hand panels as the thick dark lines (these are only valid on large scales as they do not capture the scale-dependent bias that is the signature of the H I bubbles on smaller scales). We also show (heavy grey lines) the component of the power spectrum due to the BAOs. (Note that we show the absolute value of the full power spectrum minus a representative no wiggle power spectrum.) Estimates of the sample variance (dotted lines) and thermal noise (dashed lines) components of the uncertainty for detection by the MWA are plotted in each of the panels. The MWA, which is currently under construction will comprise a phased array of 500 tiles. Each tile will contain 16 cross-dipoles to yield an effective collecting area of \(A_c = 16\lambda^2 / 4\) (the area is capped for \(\lambda > 2.1\) m). The tiles will be distributed over an area with diameter 1.5 km. The physical area of a tile is \(A_{\text{tile}} = 16\ m^2\).

In this paper we consider 1000 tiles for phase II of the MW A. We model the antenna distribution as having \(\rho(r) \propto r^{-2}\) with a maximum radius of 750 m and a finite density core of radius 18 m, and we assume a 1000-h integration on a single field, a bandpass over which foregrounds are removed of \(B = 8\ MHz\), and \(k\)-space bins of width \(\Delta k = k/10\). The total processed bandpass for the MWA is \(B_{\text{tot}} = 32\ MHz\). The combined uncertainty including the minimum \(k\) cut-off due to foreground subtraction is shown as the thin solid line.

The central panels of Fig. 3 show the power spectra at \(z = 3.5, 5, 6.5\) and 8 with the representative smooth power spectrum subtracted. The points with error bars show the accuracy attainable within a bin of width \(\Delta k/k = 0.1\). The vertical dotted line is the wavenumber corresponding to the bandpass, below which the error bars are very large. We have fitted the analytic approximation7 to the baryonic oscillation component of the spherically averaged power spectrum following Blake & Glazebrook (2003):

\[
P(k) = \frac{1}{A_k} \exp\left(-\frac{k}{0.07\ Mpc^{-1}}\right)^1.4 \sin(2\pi k_\parallel / k_\lambda). \tag{8}
\]

This function has two parameters \(A_\lambda\) and \(k_\lambda\). The value of \(A_\lambda\) is determined to high accuracy from observations of the cosmic microwave background. For the purposes of this analysis we therefore assume that \(A_\lambda\) is a known constant (namely \(A_\lambda = 2.5\)), and fit only for \(k_\lambda\) (providing the best-fitting value of \(k_\lambda = 0.0421\) in the absence of noise). We fit only to values of \(k < 0.25\ Mpc^{-1}\). The accuracy to which \(k_\lambda\) can be measured determines the constraints that 21-cm power spectra can place on the dark energy. The right-hand panels of Fig. 3 show the probability distributions for the recovered \(k_\lambda\) at each redshift considered. In a single field the MWA could detect the acoustic scale just prior to overlap, but could not make a precise measurement (less than a few per cent) at any redshift.

At values of \(k \sim 10^{-1}\ Mpc^{-1}\), the measurement of the power spectrum using the MWA will be limited by the thermal sensitivity of the array, and so the signal-to-noise ratio achievable in this regime will be greatly enhanced by a subsequent generation of instruments with a larger collecting area. As an example, we consider a hypothetical follow-up instrument to the MWA which would comprise five times the total collecting area. We refer to this follow-up telescope as the MWA5000. The design philosophy for the MWA5000 would be similar to the MWA, and we therefore assume antennae distributed as \(\rho(r) \propto r^{-2}\) with a diameter of 2 km and a flat density core of radius 80 m (see McQuinn et al. 2006). In Fig. 4 we repeat our analysis of the power spectrum and BAOs at \(z = 3.5, 5, 6.5\) and 8 for measurements using the MWA5000. The panels show the same results as described in Fig. 3. For the model overlapping at \(z = 6\), we find that the BAO scale could be detected at a range of redshifts

6 The signal-to-noise ratio is increased in proportion to \(\sqrt{\Delta k}\), and so will be substantially better per bin in measurements of the power spectrum at lower resolution in \(k\).

7 More recently a new technique has been proposed (Angulo et al. 2007; Percival et al. 2007) which replaces the analytic form of equation (8) with a scheme that uses a reference power spectrum derived from the observed power spectrum, a full linear perturbation theory power spectrum, plus a damping scale to account for non-linear evolution. This method improves the fit to BAO power spectra computed in numerical simulations, and provides a more general approach. However, we have chosen to employ the simpler approach of Blake & Glazebrook (2003) in this initial investigation.
and that very good measurements of $k_A$ ($\sim 1$ per cent) could be made at $z = 6.5$ with the MWA5000 in a single field.

Following the results of McQuinn et al. (2006) we do not make estimates for the SKA$^8$ in this paper. Current projections for the specifications of the SKA call for large antennae, with a small fraction of collecting area concentrated in a core. This design limits the field of view, as well as the fraction of the telescope that can be used to measure the large-scale modes which probe the BAOs. Thus, despite its increased collecting area, the SKA would be less powerful (with respect to measurement of the redshifted 21-cm power spectrum) than the MWA5000, whose design would be optimized for the measurement of the 21-cm power spectrum at high redshift. We note that at some redshifts the MWA5000 would be cosmic variance limited on scales relevant to BAO studies. As a result if an SKA were built with a design based on the MWA5000, but with 10 times the collecting area, no substantial gains could be made using observations of an individual field. Of course a telescope with a larger collecting area could reach the limit of cosmic variance in a shorter integration, allowing more fields to be observed.

To quantify the relative accuracy achievable on the measurement of $k_A$ we plot the variance of the recovered distribution divided by the best-fitting value in Fig. 5. In each panel of Fig. 5 we plot a vertical line at $z = 3.5$. The MWA and MWA5000 could observe to the right-hand side of this line, and we assume antennae with the specifications of the MWA in this region. We note that there

---

$^8$ See www.skatelescope.org/.

---

© 2007 The Authors. Journal compilation © 2007 RAS, MNRAS 383, 1195–1209

---
is non-zero probability for the recovered $k_A$ to lie at a harmonic of the true value (since the fitting function is quasi-periodic and the data is noisy). This may be seen in the distributions shown in Figs 3 and 4. For the results presented in Fig. 5 we assume a prior probability on $k_A$ which is constant for $k_A > 0.03$ Mpc$^{-1}$ but equal to zero otherwise. This effectively assumes that we know the accuracy of $k_A$ to $\sim 20$ per cent a priori. The upper left-hand panel of Fig. 5 shows results for the MW A, with $B = 6$, 8 and 12 MHz bandpasses, respectively, and $1000$ h of integration for a single field. If foreground subtraction could be achieved, the larger bandpasses would improve the accuracy significantly at low $z$ by giving access to the peak centred on $k \sim 0.05$ Mpc$^{-1}$. The MW A will not make a precision measurement of the BAO scale using only one field. The upper right-hand panel of Fig. 5 shows the corresponding results for MW A5000, again for a single field and $1000$ h of integration. In a single field the MW A5000 could make precise measurements $(\Delta k_A/k_A \sim 1 \text{ per cent})$ over an extended interval prior to overlap, but not at higher redshifts.

Since any one observing field can only be observed for a fraction of the time, measurement of the 21-cm power spectrum will be performed over several different fields. In addition, some phased arrays will have the capability to observe using several primary beams at once. In the lower panels of Fig. 5 we show results that assume an integration time of $1000$ h on each of three separate fields (note that this corresponds to $\sim 3$ h d$^{-1}$ per field for 1 yr). The noise on the power spectrum scales as the inverse square root of the number of fields (McQuinn et al. 2006), which results in an improved precision on measurements of $k_A$ over that achievable in a single field (see equations 6 and 7). The precision achieved at $z \sim 6.5$ would be as low as $\Delta k_A/k_A \sim 3$ per cent using the MW A (comparable with the best current measurements from galaxy surveys), while the MW A5000 could reach $\Delta k_A/k_A \sim 0.5$ per cent. In addition the MW A5000 could precisely $(\Delta k_A/k_A \lesssim 2 \text{ per cent})$ measure the acoustic scale at $z \sim 3.5$.

The antenna design of the MW A is optimized for the epoch of reionization measurements and only allows the 21-cm line to be observed at $z \gtrsim 3.5$. However, in Fig. 5 we show results down to a redshift of $z = 0$ because a future instrument could be constructed with antennae that are sensitive to a different frequency range. At $z \lesssim 3.5$ (to the left-hand side of the vertical grey lines in Fig. 5) we assume $A_e = A_{\text{sky}}$ and that the system temperature is dominated by the sky (an assumption implying that, unlike the MWA, such a telescope will need to have cooled receivers due to the lower sky temperature at shorter wavelengths). An instrument
constructed with the same number of antennae as the MWA but which operated at a higher frequency range could accurately measure the scale of acoustic oscillations at lower redshifts provided that foreground subtraction could be achieved over a sufficiently large bandpass. In constructing Fig. 5 we have computed the 21-cm power spectrum assuming a constant value of $b_{21}$ at $z < 2$. Our modelling of $b_{21}$ does not include quasars and assumes local absorption of ionizing photons. It therefore becomes unreliable at $z \lesssim 1$. However, the assumption of constant $b_{21}$ is reasonable given that the observed density parameter of neutral gas does not vary significantly with redshift (Prochaska, Herbert-Fort & Wolfe 2005). At low redshifts the precision for measurements of $k_A$ would be limited by the ability to remove foregrounds. The minimum value of $k$ which is probed by the 21-cm power spectrum becomes larger as the observing frequency is increased, and at $z \lesssim 1$ has moved to a value beyond the scale of the BAO peaks. Nevertheless, at $z \sim 1.5$ observations from three fields could yield precisions of $\sim 2$ per cent and $\sim 1$ per cent on measurements of $k_A$ using 1000 and 5000 antennae ($A_0 = A_{\text{tile}}$, respectively, with even higher precision ($\sim 0.7$ per cent and $\sim 0.5$ per cent, respectively) attainable at $z \sim 2.5$.

4.1 Sensitivity to the transverse and line-of-sight acoustic scales

In the previous section we have computed the sensitivity of 21-cm experiments to the angle-averaged value of $k_A$. However, observations of the three-dimensional power spectrum of 21-cm fluctuations provide constraints on both the radial and transverse measures of this scale. In Fig. 6 we present results for the sensitivity of 21-cm observations to the line-of-sight and transverse BAO scale at $z = 3.5, 5, 6.5$ and 8. In the left-hand panels we show the signal-to-noise ratio for observations of the full power spectrum $P_{21}(k_{1\perp}, k_{1})$. In the central panels we show the corresponding signal-to-noise ratio for observations of the difference between the full power spectrum $P_{21}(k_{1\perp}, k_{1})$ and the no wiggle reference power spectrum $P_{21,\text{ref}}(k_{1\perp}, k_{1})$. For this calculation we assume observation of a single field using the MWA5000, 1000 h of integration, and foreground removal within $B = 8$ MHz of bandpass. The signal-to-noise ratio has been computed in bins of volume $2\pi k_{1\perp} \Delta k_{1\perp} \Delta k_{1}$, where $\Delta k_{1\perp} = k_{1\perp}/10$ and $\Delta k_{1} = k_{1}/10$. We have used the analytical approximation from Glazebrook & Blake (2005):

$$\frac{P(k)}{P_{\text{ref}}(k)} = 1 + Ak \exp \left[ - \left( \frac{k}{0.07 \text{ Mpc}^{-1}} \right)^{1.4} \right] \times \sin \left( \frac{1}{2\pi} \sqrt{\frac{k_{1\perp}^2}{k_{A\perp}^2} + \frac{k_{1}^2}{k_{A\parallel}^2}} \right),$$

(9)

where $k^2 = k_{1\perp}^2 + k_{1}^2$, to estimate the corresponding constraints on the line-of-sight and transverse acoustic scales ($k_{A\perp}$ and $k_{A\parallel}$). On the right-hand panels of Fig. 6 we show contours of likelihood for the recovered values of $k_{A\perp}$ and $k_{A\parallel}$ around the true input value for our standard cosmology. Fig. 6 shows that redshifted 21-cm observations would be sensitive to both the transverse and line-of-sight components of the oscillation scale, with comparable uncertainty.

4.2 Comparison with galaxy surveys

Constraints on the BAO acoustic scale may be used to constrain parameters in models of dark energy, with the ability of a survey to discriminate among different models of dark energy governed by the accuracy achieved in measurements of the line-of-sight and transverse acoustic scales. Before proceeding we therefore pause to compare the accuracy of the 21-cm experiment with potential galaxy redshift surveys. Glazebrook & Blake (2005) present the simulated precision on measurements of $k_{A\perp}$ and $k_{A\parallel}$ from hypothetical galaxy redshift surveys. Spectroscopic surveys of $\sim 10^6$ galaxies within $\Delta z = 0.5$ redshift bins in the range $1 < z < 3.5$...
covering 1000 deg$^2$ would each measure the transverse and line-of-sight acoustic scales to accuracies of $\sim$1 per cent and $\sim$2 per cent, respectively. On the other hand a photometric redshift survey over 2000 deg$^2$ between 2.5 $< z < 3.5$ would measure the transverse scale to $\sim$1 per cent, but would not constrain the line-of-sight scale. Similar results were obtained at 1.5 $\lesssim z \lesssim 2.5$.

The SKA could also be used to do a galaxy survey of sufficient size to measure the BAO scale (Abdalla & Rawlings 2005), so long as it were designed to have a sufficiently large field of view. However, the full sensitivity of the SKA would be required to push the galaxy survey beyond $z \sim 1.5$, limiting the studies of BAO using galaxies to relatively low redshifts when compared with optical

---

**Figure 6.** Left-hand panels: The signal-to-noise ratio (S/N) for observations of the power spectrum $P_{21}(k_\perp, k_\parallel)$. Central panels: The S/N for observations of the difference between the full power spectrum $P_{21}(k_\perp, k_\parallel)$ and the no wiggle reference power spectrum $P_{21,\text{ref}}(k_\perp, k_\parallel)$. In each case the S/N has been computed in bins of volume $2\pi k_\perp \Delta k_\perp \Delta k_\parallel$, where $\Delta k_\perp = k_\perp/10$ and $\Delta k_\parallel = k_\parallel/10$. Right-hand panels: Contours of constant likelihood for the recovered $k_{A,\perp}$ and $k_{A,\parallel}$ around the true input value. Contours are shown at values of $\chi^2 - \chi^2_{\text{min}} = 1, 2.71$ and 4, where $\chi^2_{\text{min}}$ is the value corresponding to the best-fitting parameter set. When projected on to individual parameter axes ($k_{A,\perp}$ and $k_{A,\parallel}$), the extrema of these contours represent the 68, 90 and 95 per cent confidence intervals on values of the individual parameters. The four rows show results at $z = 3.5, 5, 6.5$ and 8. The observational parameters assume the layout and collecting area of the MWA5000 with 1000 h of integration on a single field, a bandpass of 8MHz and redshifts of $z = 3.5, 5, 6.5$ and 8.
spectroscopic surveys. By looking at fluctuations in the surface brightness of unresolved 21-cm emission, an array like the MWA could push measurement of the BAO to much higher redshift.

The very large areas of sky that must be surveyed in order to measure the BAO scale using galaxy redshift surveys arise because very large volumes must be sampled in order to beat down the statistical noise on the large-scale modes relevant to BAOs. For example, in units of the SDSS survey volume ($V_{\text{sdss}} \sim 5.8 \times 10^6 \text{Mpc}^3$), $V/V_{\text{sdss}} \sim 3$ and 1.8 are required to achieve 2 per cent accuracy on $k_0$ at $z$ $\sim$ 1 and 3, respectively (Blake & Glazebrook 2003). By comparison, in a single pointing within $B_{\text{total}} = 32 \text{MHz}$ of bandpass the MWA surveys $V/V_{\text{sdss}} \sim 2.4$, 4.8, 7.6 and 11 at $z$ $\sim$ 1.5, 2.5, 3.5 and 6.5 (where we have assumed $A_{\text{sdss}} = A_{\text{mwa}}$), respectively. These volumes must be multiplied by $N_{\text{field}}$, in order to get the total volume from which the power spectrum is to be constructed. Thus observations of the 21-cm power spectrum will probe very large volumes of the IGM, comparable to the most ambitious galaxy redshift surveys.

We find that observations using a low-frequency array the size of the MWA, but with an appropriate frequency range would achieve measurements of the acoustic scale at $z$ $\sim$ 1.5–2.5 that are comparable in precision ($\sim$ 1 per cent) to high-redshift galaxy surveys using the next generation of optical instruments. Moreover an instrument with the collecting area of the MWA5000 would extend this sensitivity out to $z$ $\sim$ 3.5. Observations of 21-cm fluctuations would also allow the acoustic scale to be measured with comparable precision at much earlier cosmic epochs ($z$ $\sim$ 6), using the MWA5000. Moreover, 21-cm observations will be comparatively sensitive to the line-of-sight and transverse acoustic scale. In contrast, spectroscopic galaxy redshift surveys are more sensitive to the transverse scale, while photometric redshift surveys will not be sensitive at all to the radial BAO (Glazebrook & Blake 2005).

5 POSSIBLE NON-LINEAR EFFECTS ON MEASUREMENT OF THE BAO SCALE

In estimating the precision with which the BAO scale can be measured we have assumed a constant relation between the linear matter power spectrum and the observed 21-cm power spectrum. However, there are several separate effects which could degrade the precision (as well as the accuracy) with which we can estimate the BAO scale (Seo & Eisenstein 2005). Two of these effects have been studied with respect to galaxy redshift surveys using numerical simulations.

The first effect is non-linear evolution of the mass power spectrum, which results in a change in its slope at small scales. Furthermore, the coupling of modes in the non-linear regime will tend to wash out the contrast of the BAO features. A second effect arises from the non-linear component of galaxy bias. On small scales, simulations show that galaxy bias deviates from the scale-independent bias that is predicted by linear theory. This small-scale bias is manifest in the power spectrum as anomalous power at large scales, and could result in a shift of the estimated BAO scale, as well as reduced contrast of BAO features in the observed power spectrum. Seo & Eisenstein (2005) have used a suite of $N$-body simulations to investigate the effects that these non-linear effects could have on the precision and accuracy of the recovered BAO scale in galaxy surveys. They find that estimation of the BAO scale is robust against non-linear effects in the linear and quasi-linear regimes.

In particular, Seo & Eisenstein (2005) show that the BAO features in the matter power spectrum may be recovered from an observed power spectrum through the subtraction of a smooth function which restores the matter power spectra overall shape. Thus, rather than use an analytic form for the BAO component of the power spectrum, Seo & Eisenstein (2005) relate the observed power spectrum $P_{\text{obs}}(k)$ to the linear matter power spectrum $P(k)$ through the expression

$$P_{\text{obs}}(k) = (b_0 + b_1 k)P(k/\alpha) + (a_0 + a_1 k + a_2 k^2),$$

(10)

and constrain the free parameters $b_0, b_1, a_0, a_1, a_2$ and $\alpha$. In this relation, the parameters $a_0, a_1$ and $a_2$ describe the smooth component of the power introduced through the combination of non-linear gravitational evolution and the anomalous power introduced through small-scale galaxy bias, while $b_0$ and $b_1$ describe the linear bias (with the parameter $b_1$ allowing for the possibility of scale-dependent bias in linear theory). The cosmological constraints are manifest in the parameter $\alpha$, which represents the amount of dilation in the BAO scale. The fluctuations in the recovered value of $\alpha$ are generated through the removal of non-linear effects as well as from the statistical uncertainties. The overall uncertainty in the recovered acoustic scale is provided by the size of these fluctuations. Seo & Eisenstein (2005) show that equation (10) is able to describe both the effects of non-linear evolution in the matter power spectrum and the anomalous power due to non-linear bias at a level which is sufficiently accurate so as not to degrade the precision of the recovered BAO scale.

In performing $\chi^2$ fits to their $N$-body simulated power spectra, Seo & Eisenstein (2005) found that inclusion of the parameter $b_1$ does not affect the statistical precision of the recovered BAO scale. However, setting $b_1 = 0$ resulted in an unbiased estimate of the BAO scale, while allowing $b_1$ to be an additional free parameter led to bias of the recovered scale (at a level of $<$ 1 per cent). The reason is the fitting procedure adjusts the parameter $b_1$ so as to describe the BAO features at large $k$. These features have been suppressed through non-linear evolution, which is not otherwise accounted for in the comparison between the linear power spectrum and the observed power spectrum (equation 10). Seo & Eisenstein (2005) suggest that this bias could be removed by including an appropriate recipe to account for the suppression of BAO features. This dependence on $b_1$ is important for the 21-cm analysis of the BAO scale. While galaxy bias is constant in linear theory ($b_1 = 0$), the effective bias in 21-cm observations ($b_{21}$) can be scale dependent in the region of interest. For example, Fig. 2 suggests that we would expect a scale-dependent bias at redshifts just prior to the end of the overlap era. Thus, while non-linear effects in the matter power spectrum and galaxy bias are well described by equation (10), careful account will need to be taken of the pre-factor to $P(k/\alpha)$ in measurements of the 21-cm power spectrum. We will return to this study in a future work, while in this paper we restrict our attention to a constant bias term. The finding of Seo & Eisenstein (2005) that introduction of scale dependence into the linear bias term (i.e. a non-zero $b_1$) does not increase the statistical uncertainty and also results in a systematic uncertainty of less than 1 per cent (prior to modifications which could reduce this effect) gives us confidence that our estimates will hold following a more detailed analysis.

Finally, we have neglected redshift-space distortions in this paper. In addition to non-linearities, Seo & Eisenstein (2005) have also considered the suppression of BAO features by redshift-space distortions. In particular they show that the non-linear power spectrum in redshift space can be related to the real space power spectrum via a fitting function, which allows the analysis described above to be performed in real space. However, Seo & Eisenstein (2005) find that the precision with which the line-of-sight BAO scale could be measured is degraded by redshift-space distortions, which will lower the contrast of BAO features. On the other hand Seo & Eisenstein (2005) also show that this effect becomes insignificant at $z \sim$ 3. Thus, we assume that redshift-space distortions will not limit the
of estimating the BAO scale from 21-cm observations at high redshifts.

6 CONSTRAINTS ON DARK ENERGY

The measurement of the angular scale of acoustic oscillations (i.e. oscillations transverse to the line of sight) provides a measure of the angular diameter distance at a redshift z where the oscillations are observed. The angular diameter distance can also be computed at a redshift z for a chosen cosmology

\[ D_A = (1 + z)^{-1} \int_0^z \frac{dz'}{H(z')} \]

where \( H(z) \) is the Hubble parameter at redshift \( z \), and we have set the speed of light equal to unity. Thus the measurement of \( D_A(z) \) probes the underlying geometry of the universe. On the other hand, measurement of the redshift scale of acoustic oscillations (i.e. oscillations along the line of sight) provides a direct measure of the Hubble parameter at a redshift \( z \), which may be written as

\[ H(z) = -H_0(1 + z)^{3/2} \times \left\{ \Omega_m + \Omega_Q \exp \left[ 3 \int_0^z \frac{dz'}{(1 + z')} w_Q(z') \right] \right\}^{1/2} \]

In writing the Hubble parameter we have parametrized the equation of state as

\[ p_i = w_i(z) \rho_i, \]

where the subscript \( i \) refers to either pressureless matter (with \( w = 0 \) and a subscript \( i = m \)) or dark energy (with a subscript \( i = Q \)). We have also used the energy conservation for the dark energy \( \rho_Q = -3H(z)[1 + w_Q(z)]\rho_Q \). The redshift dependence of the acoustic scale has been suggested as a powerful probe of the evolution of the dark energy (Blake & Glazebrook 2003; Hu & Haiman 2003; Seo & Eisenstein 2003; Glazebrook & Blake 2005; Angulo et al. 2007; Seo & Eisenstein 2007). If the size of the sound horizon is known, then the relative errors \( \Delta D_A/D_A \) in the angular diameter distance and \( \Delta H/H \) in the Hubble parameter are related to the relative errors \( \Delta k_{\perp}/k_{\perp} \) and \( \Delta k_{\parallel}/k_{\parallel} \) in the transverse and line-of-sight acoustic scales, respectively (Fig. 6).

To investigate the utility of the redshifted 21-cm emission as a probe of the dark energy equation of state we adopt a simple approach. In the remainder of this section we assume that the values of \( \Omega_m \), \( \Omega_Q \) and the present-day value of the Hubble parameter \( H_0 \) have been precisely determined a priori, so that we do not consider the uncertainty in the sound horizon. We also assume that the universe is flat. In the upper panels of Fig. 7 we plot the fractional change in the angular diameter distance and Hubble parameter, respectively, for several models with constant \( w_Q \), relative to a model with a cosmological constant (\( w_Q = -1 \)). The vertical bars illustrate the type of precision achievable by the MWA (grey bar) and MWA5000 (dark bars) in a 1000-h observation of three fields and a single field, respectively, at the redshifts where these observations will be most efficient. Based on the transverse measurement of the BAO scale, the MWA would be able to make a measurement of \( w_Q \) with \( \sim 20 \) per cent accuracy using a single measurement at \( z \sim 6.5 \), while the MWA5000 could make a measurement of \( w_Q \) at the \( \sim 7 \) per cent level from observations at each of \( z \sim 3.5 \) and 6.5, yielding a combined constraint of better than 5 per cent. Better precision could be achieved using observations of three fields. Since a cosmological constant does not affect the dynamics at high redshift the line-of-sight BAO scale is left unaffected by this change in the equation of state. Thus a cosmological constant would introduce an asymmetry in the BAO scale that would be easily detected by high-redshift observations.

We can obtain a more quantitative relation between the uncertainties in \( k_A \) and \( w_Q \). If we expand around \( w_Q = -1 \), then in the case of a constant \( w_Q \) model we can relate the uncertainty in \( w_Q \) to the observed uncertainty in the scale \( k_A \). We obtain

\[ \Delta w_Q = \frac{d w_Q}{d k_A} \Delta k_A, \]

where \( \Delta k_A \equiv \Delta k_{\perp}/k_{\perp} \) or \( \Delta k_A \equiv \Delta k_{\parallel}/k_{\parallel} \) for the transverse and line-of-sight components, respectively. In the lower left-hand panel

![Figure 7. Constraints on a constant \( w_Q \). Upper panels: The relative change in angular diameter distance (left-hand panel) and Hubble parameter (right-hand panel) as a function of redshift for different models of the evolution in \( w_Q \). The fractional changes are relative to a cosmological constant (\( w_Q = -1 \)). The thick black and grey bars illustrate an accuracy of 1 and 3 per cent on \( k_A \) which would be achievable in some redshift ranges in a 1000-h observation by the MWA5000 (one field) and MWA (three fields), respectively. Lower panels: The uncertainty in \( w_Q \) per unit relative uncertainty in the angular diameter distance \( D_A \) (left-hand panel) and Hubble parameter \( H \) (right-hand panel), as a function of redshift. The uncertainty is relative to a cosmological constant (\( w_Q = -1 \)).](https://academic.oup.com/mnras/article-abstract/383/3/1195/1037848/1195)
of Fig. 7 we plot the resulting uncertainty in $w_Q$ per unit fractional uncertainty in the transverse measurement of $k_A$. This figure shows that a 10 per cent uncertainty in $w_Q$ requires $k_A$ to be measured in the transverse direction with a precision of $\sim 1.5$ per cent. As mentioned above this will be achievable at each of redshifts $z \sim 3.5$ and 6 for the MWA5000. The small change in the radial BAO scale with $w_Q$ is reflected by the large value of the derivative in the lower right-hand panel of Fig. 7.

6.1 Constraints on parametrized models of evolving dark energy

In the remainder of this section we constrain two different parametrized dark energy models assuming observations with the MWA5000 at four different redshifts. Rather than consider the spherically averaged constraint on $k_A$, in this section we utilize independent constraints of the line-of-sight and transverse BAO scales (Glazebrook & Blake 2005). Fig. 6 illustrates that the MWA5000 will have a comparable sensitivity to measurements of $k_A$ along the line of sight ($k_{A,z}$) as compared to perpendicular to it ($k_{A,\perp}$). We therefore assume the transverse and line-of-sight sensitivities to be $\Delta k_{A,\perp}/k_{A,\perp} = \Delta k_{A,z}/k_{A,z} = \sqrt{2} \times \Delta k_A/k_A$, and constrain dark energy parameters using the joint constraints on $D_A$ and $H$ (i.e. we assume $\Delta k_{A,\perp}/k_{A,\perp} = \Delta D_A/D_A$ and $\Delta k_{A,z}/k_{A,z} = \Delta H/H$). Note that our relations between the transverse or line-of-sight sensitivities and the spherically averaged sensitivity to the BAO scale assume that the transverse and line-of-sight scales are independent, whereas Fig. 6 illustrates that there is some degeneracy. We therefore slightly underestimate the uncertainties on the transverse and line-of-sight BAO scales.

The equation of state of dark energy may not be constant. The possible evolution is often parametrized using the following simple form (Chevallier & Polarski 2001):

$$w_Q(z) = w_Q + w_1(1 - a),$$

where $w_1$ is the derivative of $w_Q(z)$ with respect to the scalefactor $a$. We have investigated the joint constraints that the 21-cm BAO observations might place on the parameters $w_Q$ and $w_1$, and present the results in Fig. 8. We construct the relative likelihoods for $w_Q$ and $w_1$ given a true model with $w_Q = -1$ and $w_1 = 0$,

$$\mathcal{L}(w_Q, w_1) = \exp \left\{ -\frac{1}{2} \left[ \left( \frac{\delta k_{A,z}}{\delta k_{A,z}} \right)^2 + \left( \frac{\delta k_{A,\perp}}{\delta k_{A,\perp}} \right)^2 \right] \right\},$$

where $\delta k_{A,z}(z)$ and $\delta k_{A,\perp}(z)$ are functions of $w_Q$ and $w_1$. We show results in four redshift bins and have assumed $\delta k_{A,z} = \delta k_{A,\perp} = \sqrt{2} \delta k_A/k_A$ with observed uncertainties of $\delta k_{A,z} = 0.007$ at $z = 1.5$, $\delta k_{A,\perp} = 0.007$ at $z = 2.5$, $\delta k_{A,z} = 0.02$ at $z = 3.5$ and $\delta k_{A,\perp} = 0.007$ at $z = 6.5$. The latter two cases are examples of the levels of precision in $\Delta k_A/k_A$ that may be achievable in 1000-h integrations over each of three fields with the MWA5000, while the cases at $z \sim 1.5$ and 2.5 correspond to an instrument with the antenna number of the MWA5000, but with a higher frequency range and $A_e = A_{\text{obs}}$ (again over three fields and 1000 h). In each case contours of the likelihood are shown at 64, 26 and 10 per cent of the maximum. The contours illustrate the degeneracy in $w_Q$ and $w_1$ from a single measurement of the acoustic scale at high redshift. This degeneracy arises if the cosmological constant is the true model, because the effect of dark energy is limited to the value observed for $D_A$. However, both $w_Q$ and $w_1$ could be constrained at $z \lesssim 2.5$. We note that models with $w_Q(z) < -1$ or $w_Q(z) > 1$ violate the weak energy condition, and shade these regions grey.

Figure 8. The joint likelihoods for $w_Q$ and $w_1$ relative to a model with a cosmological constant ($w_Q = -1, w_1 = 0$). The contours are at 64, 26 and 10 per cent of the maximum and were found from combining constraints on $D_A$ and $H$, assuming uncertainties $\Delta k_{A,z} = \Delta k_{A,\perp}$ and $\Delta k_A/k_A = \sqrt{2} \times 0.01$ (at $z = 1.5$, upper left-hand panel), $\Delta k_{A,z} = \Delta k_{A,\perp}$ and $\Delta k_A/k_A = \sqrt{2} \times 0.007$ (at $z = 2.5$, upper right-hand panel), $\Delta k_{A,z} = \Delta k_{A,\perp}$ and $\Delta k_A/k_A = \sqrt{2} \times 0.02$ (at $z = 3.5$, lower left-hand panel) and $\Delta k_{A,z} = \Delta k_{A,\perp}$ and $\Delta k_A/k_A = \sqrt{2} \times 0.007$ (at $z = 6.5$, lower right-hand panel). At $z \gtrsim 3.5$ these sensitivities correspond to three fields, each observed for 1000 h with the MWA5000. At $z \gtrsim 2.5$ these sensitivities correspond to three fields, each observed for 1000 h with an array having 5000 antennae (each with 16 dipoles) and $A_e = A_{\text{obs}}$. Models with $w_Q(z) < -1$ or $w_Q(z) > 1$ violate the weak energy condition, and we shade these regions grey.
occurred at $z \gtrsim 3.5$ where the BAO scale is accessible to the MWA5000. Nevertheless, the combination of observations at several redshifts at $z \gtrsim 3.5$ would provide tight limits on the value of either $w_0$ ($\approx 5\%$ per cent) if it is constant, or $w_1$ ($\pm 0.2$) if $w_0$ is known a priori. In constructing these limits we have utilized external information through our assumption that the cosmological parameters $\Omega_m$, $\Omega_b$, and $\Omega_{\Lambda}$ are known, but have not incorporated additional sources of constraint on the dark energy equation of state at lower redshift, which could include observations of Type Ia supernovae (e.g. Riess et al. 2007; Zhao et al. 2007), or the BAO scale from galaxy redshift surveys (Cole et al. 2005; Eisenstein et al. 2005). The constraints on the dark energy equation of state presented in Fig. 5 could of course be improved through a joint analysis of all available constraints. For example, while the BAO scale measured by Eisenstein et al. (2005) does not by itself constrain the parameters of dark energy, Wood-Vasey et al. (2007) have combined this measurement with observations of Type Ia supernovae from the ESSENCE supernova survey, and find (assuming a flat universe) that $w_0 = -1.05^{+0.11}_{-0.12}$ with a systematic uncertainty of $\pm 0.11$.

Equation (15) imposes a certain type of evolution on the dark energy. In particular, for all values of $w_1$ the parametrization in equation (15) assumes that 50 per cent of the evolution in $w_0(z)$ occurred at $z < 1$. Moreover, the constraints on model parameters are only meaningful should the model be a correct description of reality. Since 21-cm observations of BAO will be a powerful probe of dark energy at high redshifts, we require a parametrization that is more general than is provided by equation (15). Current observations indicate that the evolution of the cosmic expansion is consistent to within $\sim 10$ per cent with a pure cosmological constant at $z \lesssim 1$ (Riess et al. 2007; Zhao et al. 2007), while redshifted 21-cm observations might be the first working method of probing the dark energy at $z \gtrsim 3$. An alternative parametrization to equation (15) for the possible evolution of the vacuum energy density $[\rho_\Lambda(z)]$ at high redshifts is to adopt a constant value ($\rho_\Lambda$) in the redshift range $0 \leq z \leq z_{\text{min}}$ which is constrained by other observational probes (such as Type Ia supernovae or galaxy redshift surveys), and to represent the unknown evolution at higher redshifts $z > z_{\text{min}}$ by

$$\rho_\Lambda = \rho_{\Lambda,0} \left[ 1 + C_1(z - z_{\text{min}}) + C_2(z - z_{\text{min}})^2 + \cdots \right],$$

where we have kept only the two leading terms in the polynomial expansion of $\rho_\Lambda(z)$. We next require that the universe be flat, which yields the corresponding evolution for the Hubble parameter at $z > z_{\text{min}}$.

$$\left( \frac{H(z)}{H_0} \right)^2 = \Omega_{m,0}(1 + z)^3 + \Omega_{\Lambda,0} \left[ 1 + C_1(z - z_{\text{min}}) + C_2(z - z_{\text{min}})^2 + \cdots \right],$$

where the subscript 0 denotes present-day values. This expression can then be substituted directly into equation (11) to obtain the angular diameter distance, $D_A(z)$. If the vacuum energy density originates from a scalar field $\phi$ rolling down a potential $V(\phi)$, then measurements of the above expansion parameters can be used to infer the shape of $V(\phi)$ (see Linder 2007; Turner & Huterer 2007 for recent reviews).

We have investigated the joint constraints that the 21-cm BAO observations might place on the parameters $C_1$ and $C_2$, and present the results in Fig. 9. We do not constrain $\rho_{\Lambda,0}$ to be positive, but shade the regions where $\rho_{\Lambda,0} < 0$. For this figure we assume $z_{\text{min}} = 1$ and that the dark energy at $z < z_{\text{min}}$ is a cosmological constant. We construct the relative likelihoods for $C_1$ and $C_2$ given a true model with $C_1 = C_2 = 0$. As before we show results in four redshift bins and have assumed observed uncertainties of $\delta k_{\text{obs}}^c = \delta k_{\text{obs}}^l = \sqrt{2} \delta k_{\text{obs}}^c$ with $\delta k_{\text{obs}}^c = \delta k_{\text{obs}}^l = \delta k_{\text{obs}} = 0.007$ at $z = 2.5$, $\delta k_{\text{obs}}^c = \delta k_{\text{obs}}^l = \delta k_{\text{obs}} = 0.02$ at $z = 3.5$, $\delta k_{\text{obs}}^c = \delta k_{\text{obs}}^l = \delta k_{\text{obs}} = 0.007$ at $z = 6.5$ and $\delta k_{\text{obs}}^c = \delta k_{\text{obs}}^l = \delta k_{\text{obs}} = 0.04$ at $z = 12$. In each case contours of the likelihood are shown at 64, 26 and 10 per cent of the maximum. Since this model does not exclude a dark energy contribution to the expansion dynamics at high redshift, its

![Figure 9](https://academic.oup.com/mnras/article-abstract/383/3/1195/1037848/1037848)

Figure 9. The joint likelihoods for $C_1$ and $C_2$ relative to a model with a cosmological constant ($C_1 = C_2 = 0$). The contours are at 64, 26 and 10 per cent of the maximum and were found from combining constraints on $D_A$ and $H$, assuming uncertainties $\Delta k_{\text{obs}}^c/\Delta k_{\text{obs}}^l = \Delta k_{\text{obs}}/\Delta k_{\text{obs}} = \sqrt{2} \times 0.007$ (at $z = 2.5$, upper left-hand panel), $\Delta k_{\text{obs}}^c/\Delta k_{\text{obs}} = \Delta k_{\text{obs}}/\Delta k_{\text{obs}} = \sqrt{2} \times 0.02$ (at $z = 3.5$, upper right-hand panel), $\Delta k_{\text{obs}}^c/\Delta k_{\text{obs}}^l = \Delta k_{\text{obs}}/\Delta k_{\text{obs}} = \sqrt{2} \times 0.04$ (at $z = 6.5$, lower left-hand panel), and $\Delta k_{\text{obs}}^c/\Delta k_{\text{obs}}^l = \Delta k_{\text{obs}}/\Delta k_{\text{obs}} = \sqrt{2} \times 0.04$ (at $z = 12$, lower right-hand panel). At $z \geq 3.5$ these sensitivities correspond to three fields, each observed for 1000 h with the MWA5000. At $z = 2.5$ these sensitivities correspond to three fields, each observed for 1000 h with an array having 5000 antennae (each with 16 dipoles) and $A_{\text{e}} = A_{\text{dip}}$. Models with $\rho_{\Lambda,0} = 0$ are shaded grey.
parameters are well constrained by high-redshift observations of the BAO scale. The contours illustrate the level of degeneracy in $C_1$ and $C_2$ from a single measurement of the acoustic scale at high redshift. The combination of constraints at different redshifts would reduce this degeneracy. As with our previous examples, the constraints presented in Fig. 9 assume that there is no observational uncertainty in the size of the sound horizon.

7 DISCUSSION

In this paper we have shown that measurements of the power spectrum of fluctuations in the intensity of 21-cm emission could provide a precise determination of the scale of the acoustic peak in the matter power spectrum. By using the acoustic scale to measure the angular diameter distance and Hubble parameter, it would therefore be possible to use redshifted 21-cm studies to constrain the dark energy in the unexplored redshift range of $1.5 \lesssim z \lesssim 15$.

The main advantage of 21-cm observations is that in addition to studies in the redshift range where a cosmological constant is expected to dominate the expansion dynamics, measurement of the BAO scale would also be possible at much earlier cosmic epochs than are accessible by other methods. Indeed, observations of the redshifted 21-cm power spectrum at $3.5 \lesssim z \lesssim 12$ using a facility with 10 times the collecting area of the MWA would make a detailed study of the presence of dark energy at high redshift that differs from that measured at $z < 1$. Moreover, 21-cm observations will be comparably sensitive to the line-of-sight and transverse acoustic scales. In contrast, spectroscopic galaxy redshift surveys are more sensitive to the radial BAO at all scales (Glazebrook & Blake 2005). Moreover, since observations of the BAO scale in 21-cm fluctuations can be extended down to $z \sim 1.5$ there will be an opportunity to demonstrate that the results match those from other well studied techniques. However, at $z \gtrsim 3.5$, where the acoustic peak scale is not accessible through standard techniques, the 21-cm power spectrum would provide unique constraints on the dark energy.

ACKNOWLEDGMENTS

The authors thank Chris Blake, Matt McQuinn and Brian Schmidt for comments which have greatly contributed to this work. The research was supported in part by the Australian Research Council (JSBW) and Harvard University grants (AL). PMG acknowledges the support of an Australian Postgraduate Award.

REFERENCES

Abdalla F. B., Rawlings S., 2005, MNRAS, 360, 27
Alcock C., Paczynski B., 1979, Nat, 281, 358
Angulo R., Baugh C. M., Frenk C. S., Lacey C. G., 2007, preprint (astro-ph/0702543)
Barkana R., 2006, MNRAS, 372, 259
Barkana R., Loeb A., 2005a, ApJ, 624, L65
Barkana R., Loeb A., 2005b, MNRAS, 363, L36
Bharadwaj S., Ali S. S., 2005, MNRAS, 356, 1519
Blake C., Glazebrook K., 2003, ApJ, 594, 665
Bowman J. D., Morales M. F., Hewitt J. N., 2006, ApJ, 638, 20
Bowman J. D., Morales M. F., Hewitt J. N., 2007, ApJ, 661, 1
Chevalier M., Polarski D., 2001, Int. J. Mod. Phys. D, 10, 213
Cole S. et al., 2005, MNRAS, 362, 505
Corasaniti P.-S., Huterer D., Meichiotti A., 2007, Phys. Rev. D, 75, 062001
Eisenstein D. J. et al., 2005, ApJ, 633, 560
Eisenstein D. J., Hu W., 1998, ApJ, 496, 605
Furlanetto S. R., Zaldarriaga M., Hernquist L., 2004, ApJ, 613, 16
Furlanetto S. R., Oh S. P., Briggs F. H., 2006, Phys. Rep., 433, 181
Geil P. M., Wyithe J. S. B., 2007, preprint (arXiv:0708.3716)
Glazebrook K., Blake C., 2005, ApJ, 631, 1
Hu W., Haiman Z., 2003, Phys. Rev. D, 68, 063004
Linder E. V., 2007, Gen. Relativ. Gravit., preprint (arXiv:0704.2064)
McDonald P., Eisenstein D., 2007, Phys. Rev. D, 76, 063009
McQuinn M., Zahn O., Zaldarriaga M., Hernquist L., Furlanetto S. R., 2006, ApJ, 653, 815
Messinger A., Furlanetto S., 2007, ApJ, 669, 663
Morales M. F., 2005, ApJ, 619, 678
Nusser A., 2005, MNRAS, 364, 743
Peacock J. A., West M. J., 1992, MNRAS, 259, 494
Percival W. J., Cole S., Eisenstein D. J., Nichol R. C., Peacock J. A., Pope A. C., Szalay A. S., 2007, MNRAS, 381, 1053
Prochaska J. X., Herbert-Fort S., Wolfe A. M., 2005, ApJ, 635, 123
Riess A. G. et al., 2007, ApJ, 659, 98
Seo H.-J., Eisenstein D. J., 2003, ApJ, 598, 720
Seo H.-J., Eisenstein D. J., 2005, ApJ, 633, 575
Seo H.-J., Eisenstein D. J., 2007, ApJ, 665, 14
Spergel D. N. et al., 2007, ApJS, 170, 377
Turner M. S., Huterer D., 2007, preprint (arXiv:0706.2186)
Wood-Vasey W. M. et al., 2007, ApJ, 666, 694
Wyithe J. S. B., Loeb A., 2004, Nat, 432, 194
Wyithe S., Loeb A., 2007, preprint (arXiv:0708.3392)
Zhao G.-B., Xia J.-Q., Li H., Tao C., Virey J.-M., Zhu Z.-H., Zhang X., 2007, Phys. Lett. B, 648, 8

This paper has been typeset from a TeX file prepared by the author.