Revisiting supernova constraints on a light CP-even scalar

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Abstract. A light CP-even Standard Model (SM) gauge-singlet scalar $S$ can be produced abundantly in the supernova core, via the nucleon bremsstrahlung process $NN \to NNS$, due to its mixing with the SM Higgs boson. Including the effective $S$ coupling to both nucleons and the pion mediators, we evaluate the production amplitude for the $S$ particle and point out a key difference with the well-known light CP-odd scalar (axion) and vector boson (dark photon) cases. Taking the subsequent decay and re-absorption of $S$ into account, we present a complete calculation of the energy loss rate for the $S$ particle. We then use the SN1987A luminosity constraints to derive an updated limit on the mixing of the scalar $S$ with the SM Higgs boson. We find that the mixing angle $\sin \theta$ with the SM Higgs is excluded only in the narrow range of $3.5 \times 10^{-7}$ to $2.5 \times 10^{-5}$, depending on the scalar mass up to the two-pion threshold, beyond which the supernova limit disappears. This result has important consequences for the laboratory searches for light scalars.

Keywords: Supernova, Beyond Standard Model
1 Introduction

In the absence of any new TeV-scale particles appearing at the Large Hadron collider, the possibility that new physics beyond the Standard Model (SM) of particle physics could manifest itself in the low-energy regime, e.g. below GeV-scale has recently attracted considerable attention. Typical candidates include light CP-odd scalar particles, such as the widely discussed QCD axion (originally proposed to solve the strong CP problem [1–3]) or similar axion-like particles (ALPs), SM-singlet CP-even scalars, and light vector bosons such as dark photons, all of which can be searched for in the high-intensity experiments [4–8]. Such particles have the important property that if their masses are below a few hundred MeV or so, they can also manifest themselves in the compact astrophysical objects like supernovae or neutron stars and affect their properties, thereby making these astrophysical systems an ideal laboratory for beyond SM physics search [9]. For example, the light axion or ALPs \(a\) can be produced abundantly in the supernova core via the nuclear bremsstrahlung process \(NN\rightarrow NNa\) (where \(N=p,n\) collectively stands for protons and neutrons). If their mean free path is larger than the size of the supernova core, these light particles could carry away a significant fraction of the energy. Therefore, their couplings to matter can be constrained [10–16] from the inferred energy loss rate \(\mathcal{L}_\nu \sim (3-5) \times 10^{53} \text{ erg/sec}\), derived from the detection of late-time neutrino events from SN1987A by Kamiokande [17]. This is the so-called ‘Raffelt criterion’ [9] (see Ref. [18] for a critical assessment of the Raffelt criterion).

Following the same calculation as in the axion case, supernova limits on a light CP-even scalar \(S\) have been discussed in Refs. [19–22] (see also Ref. [23] for the limit on saxion which to some extent is similar to the \(S\) particle). In this paper, we revisit the calculation for \(S\) production and its mean free path in a supernova core, and point out some important differences with the pseudoscalar axion/ALP case, which lead to some modifications of the supernova constraints on \(S\), as compared to the existing bounds in the literature.

The light scalar \(S\) has been discussed as a natural dark matter candidate [24–28] or dark force mediator [29–34], or a particle which can assist in the generation of the baryon asymmetry of the Universe [35–38]. The singlet scalar \(S\) with mass around 100 MeV and...
mixing angle with the SM Higgs of $\theta \sim 3 \times 10^{-4}$ could also be responsible for the recent anomalous excess in the flavor-changing kaon decay $K_L \to \pi^0\nu\bar{\nu}$ observed in the KOTO experiment [39–43]. This makes a careful (re)examination of the supernova constraints on light scalar even more relevant and timely.

The couplings of light $S$ to the SM particles can be induced in two ways: (i) through mixing with the SM Higgs $h$, and (ii) through the introduction of new heavy particles such that $S$ can couple radiatively to photons or gluons (or other SM particles) via the heavy particle loops. A good example of type (ii) couplings naturally occur in the left-right symmetric model (LRSM) based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [44–46], where the light scalar originates from the $SU(2)_R$-breaking triplet scalar field, and the heavy $W_R$ boson and heavy scalars in the LRSM mediate the decay $S \to \gamma\gamma$ even if $S$ does not mix with the SM Higgs [47–49]. This restricts the applicability of the supernova bound to only a narrow mass range of $m_S = 20–30$ MeV [41]. Therefore, we will consider only the $S$ couplings of type (i), where all the couplings of $S$ to the SM particles are induced from the $h-S$ mixing, with a universal mixing angle $\sin \theta$. Then we can use the Raffelt criterion on energy loss from SN1987A to set limits on the scalar mass $m_S$ and the mixing angle $\sin \theta$.

Through mixing with the SM Higgs, the light scalar $S$ couples to nucleons, pions, electrons and photons. Similar to the axion/ALP case, $S$ can be produced in the supernova core via the following processes [11]:

- nucleon bremsstrahlung $NN \to NNS$;
- $e^+e^-$ annihilation $e^+e^- \to \gamma S$;
- $\gamma\gamma$ annihilation $\gamma\gamma \to S$;
- Compton-like scattering $e\gamma \to eS$;
- Primakoff-like process $\gamma(e, N) \to S(e, N)$;
- plasma ($\gamma_{pl}$) decay $\gamma_{pl} \to \gamma S$.

The coupling of $S$ to electrons is highly suppressed by the small Yukawa coupling $y_{e}$ of the SM Higgs field and the coupling to photons is loop suppressed. Thus the production of $S$ in the supernova core is dominated by the nucleon bremsstrahlung process.

After being produced in the supernova core, the scalar $S$ can decay into light SM particles such as $\gamma\gamma$, $e^+e^-$ and $\mu^+\mu^-$. If these decays happen inside the supernova core, then the energy loss constraints will not apply. The $S$ particle can also scatter with nucleons and be re-absorbed via the inverse nucleon bremsstrahlung process $NNS \to NN$. Here again we neglect the scattering of $S$ with electrons and photons; for instance, the processes $Se \to e\gamma$ and $S\gamma \to e^+e^-$ are suppressed by either the small couplings to electrons or by the loop factor from photon coupling. From the scattering rate of the $3 \to 2$ re-absorption process we can obtain the mean free path (MFP) of $S$, and compare it to the supernova core radius to see whether the $S$ particle can escape from the core for a given mixing angle. It turns out that for $m_S \lesssim 100$ MeV, the MFP of $S$ is of the order of the supernova core size (roughly 10 km) for a mixing angle of $\sin \theta \sim 10^{-6}$ (see Fig. 4). After taking into account both the decay and re-absorption of $S$, we find that the supernova luminosity limit excludes the scalar mass up to $m_S \simeq 2m_\pi \simeq 270$ MeV (with $m_\pi$ being the pion mass) for mixing angle $\sin \theta$ ranges from $3.5 \times 10^{-7}$ to $2.5 \times 10^{-5}$ (see Fig. 5). Beyond $m_S = 2m_\pi$, the channel $S \to \pi\pi$ is kinematically open and the scalar $S$ decays too fast inside the core itself. Our final result
As stated in the introduction, the dominant channel for the production of $S$ in the supernova core is through the nucleon bremsstrahlung process

$$N + N \rightarrow N + N + S.$$  \hfill (2.1)

At the leading-order, this process is mediated by one-pion exchange (OPE) between nucleons, as shown in Fig. 1. In the calculation below we include both types of diagrams with $S$ coupling to nucleons and pions [50, 51] and pions [52, 53]. These diagrams are labeled by $(a), (b), (c), (d), (a'), (b'), (c'), (d')$, and denoted respectively by the crosses and black blobs in Fig. 1. As shown below, at the leading order in $m_S^2/m_N E_S$ (where $m_N$ is the nucleon mass and $E_S$ the energy of $S$), the Feynman diagrams $(a), (b), (c)$ and $(d), (a'), (b'), (c'), (d')$ with $S$-nucleon couplings cancel out with each other, and we have to go to the next-to-leading order in $m_S^2/m_N E_S$ for these diagrams. As a result when the scalar mass $m_S \lesssim 10$ MeV, the contributions of $S$-nucleon coupling diagrams are suppressed by the scalar mass via \((m_S/E_S)^4\) [cf. Eq. (2.12)], and the energy loss due to the existence of $S$ will be dominated by the $S$-pion coupling diagrams $(e')$ and $(e')$, as shown in Fig. 2.

The $S$ couplings to both nucleons and pions are induced by the mixing of $S$ with the SM Higgs $h$, with the Lagrangian given by

$$\mathcal{L} = \sin \theta S \left[ y_{hNN} \nabla N + A_\pi (\pi^0 \bar{\pi}^0 + \pi^+ \pi^-) \right],$$  \hfill (2.2)

where $y_{hNN}$ is the effective coupling of SM Higgs to nucleons. Through the couplings to quarks inside nucleons, it turns out that $y_{hNN} \approx 10^{-3}$ [50, 51]. From chiral perturbation
theory, the effective coupling of light $S$ to pions in Eq. (2.2) can be written as \[52, 53\]
\[
A_\pi = \frac{2}{9v_{\text{EW}}} \left( m_S^2 + \frac{11}{2} m_\pi^2 \right),
\]
with $v_{\text{EW}} \simeq 246$ GeV is the electroweak vacuum expectation value. In order to discuss supernova limits, the relevant range of scalar mass is below the two-pion threshold, i.e. $m_S \lesssim 2m_\pi$, and it is clear in Eq. (2.3) that the coupling $A_\pi$ is highly suppressed by the ratio $m_\pi/v_{\text{EW}} \sim 10^{-3}$ which is of the same order as $y_{hNN}$.

Following the calculations in Refs. [54–58], the energy emission rate per unit volume in the supernova core is given by
\[
Q = \int d^5S \sum_{\text{spins}} |\mathcal{M}|^2/(2\pi)^4 \delta^4(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4 - \mathbf{k}_S) E_S \mathcal{P}_{12} P_{\text{decay}} P_{\text{abs}},
\]
where $d^5S$ is the $2 \to 3$ phase space factor, $S$ is the symmetry factor for identical particles, being 1 for $np$ scattering process and $1/4$ for $pp$ and $nn$ processes, $\mathcal{M}$ is the scattering amplitude for $N+N \to N+N+S$, and $f_{1,2}$ the non-relativistic Maxwell-Boltzmann distributions of the two incoming nucleons in the non-degenerate limit, defined by
\[
f(\mathbf{p}) = \frac{n_B}{2} \left( \frac{2\pi}{m_N T} \right)^{3/2} e^{-\mathbf{p}^2/2m_NT},
\]
with $T \simeq 30$ MeV the supernova core temperature and $n_B \simeq 1.2 \times 10^{38}$ cm$^{-3}$ the typical baryon number density in the supernova core. In Eq. (2.4),
\[
P_{\text{decay}} = \exp\{-R_c \Gamma_S\}
\]
is the decay probability with $R_c$ being the supernova core size (only the $S$ particles decaying outside the supernova core will contribute to energy loss), and
\[
P_{\text{abs}} = \exp\{-R_c/\lambda\}
\]
is the re-absorption probability, taking into account the fact that the light scalar $S$ can be re-absorbed inside the core via the $3 \to 2$ inverse nucleon bremsstrahlung process
\[
N + N + S \to N + N,
\]
and $\lambda$ in Eq. (2.7) is the MFP of $S$.

The calculation details of the squared amplitude $\sum |\mathcal{M}|^2$ and the emission rate $Q$ are collected respectively in Appendices A.1 and A.2. It is interesting to notice that at the leading order in $m_N E_S \gg m_S^2$ [cf. Eq. (A.2)], the four $t$ and $u$-channel diagrams cancel out among themselves:
\[
\mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d \simeq 0,
\]
\[
\mathcal{M}_{d'} + \mathcal{M}_{u'} + \mathcal{M}_{c'} + \mathcal{M}_{d'} \simeq 0.
\]
Incidentally, this cancellation occurs only for the scalar case we are considering. For a vector boson (dark photon) or CP-odd scalar (axion/ALP) case, the Lorentz structures of the matrix elements are very different and no such cancellation occurs (see e.g. Refs. [54, 55]). This is a
key difference between the CP-even scalar and other kinds of light particles discussed in the literature that makes the CP-even case somewhat special.

Thus, to obtain a non-vanishing result for the sum of amplitudes \( \sum |M|^2 \) in Eq. (2.4), we have to go to next-to-leading order in the small parameter \( m_S^2/m_N E_S \). After a long but straightforward calculation (cf. Appendix A), the emission rate in (2.4) can be simplified to

\[
Q = \left( \frac{P_\text{decay} P_\text{abs}}{\mathcal{I}_{\text{tot}}} \right) \frac{m^2_B \alpha_x^4 f_{pp}^4 T^{7/2} \sin^2 \theta}{8 \pi^3/2 m_N^{9/2}} \int du dv dz x \sqrt{u} \sqrt{v} \sqrt{z} - \sqrt{u} - \sqrt{v} \sqrt{z} \delta(u - v - z) \mathcal{I}_{\text{tot}},
\]

where \( \alpha_x \equiv (2m_N/m_\pi)^2/4\pi \lesssim 15 \), and the effective pion-nucleon coupling \( f_{pp} \approx 1 \), and \( u, v, z \) are dimensionless parameters defined in Eq. (A.17). The dimensionless function

\[
\mathcal{I}_{\text{tot}} = 2 y_{hNN} \left( \frac{q}{x} \right)^4 \mathcal{I}_A + \frac{1}{81} \left( \frac{m_N}{v_{\text{EW}}} \right)^2 \mathcal{I}_B + 2 \left( \frac{y_{hNN}}{y_{hNN}} \right) \left( \frac{m_N}{v_{\text{EW}}} \right) \mathcal{I}_C,
\]

where we have summed up the contributions from the \( pp, nn \) and \( np \) processes:

\[
\mathcal{I}_{A,B,C} = \mathcal{I}_{A,B,C}^{(pp)} + \mathcal{I}_{A,B,C}^{(nn)} + 4 \mathcal{I}_{A,B,C}^{(np)},
\]

with \( \mathcal{I}_{A,B,C} = \mathcal{I}_{A,B,C}^{(np)} \). For the \( pp \) and \( nn \) channels, there is a factor of \( 1/2 \times 1/2 = 1/4 \) for identical particles in both the initial and final states, thus we have an extra factor of 4 for the \( np \) term in Eq. (2.13). The dimensionless functions \( \mathcal{I}_{A,B,C} \) can be found in Eqs. (A.24) to (A.29) in Appendix A.2. In Eq. (2.12) the \( \mathcal{I}_A \) term denotes the contribution from the diagrams (a), (b), (c), (d) and (a'), (b'), (c'), (d'), with the scalar \( S \) coupling to the nucleons. Besides the small Yukawa coupling \( y_{hNN} \), the function \( \mathcal{I}_A \) is also suppressed by the scalar mass \( m_S \) via \( (q/x)^4 = (m_S/E_S)^4 \) when \( S \) is light (note that the scalar energy \( E_S \sim 3T \) when \( m_S \ll 3T \)). The \( \mathcal{I}_B \) function is the contribution from the diagrams (e) and (e') with \( S \) coupling to the pion mediator, and is highly suppressed by the ratio \( (m_N/9v_{\text{EW}})^2 \). The \( \mathcal{I}_C \) term stands for the interference contributions from the two sets of diagrams above, and is expected to be always in between \( \mathcal{I}_A \) and \( \mathcal{I}_B \). This is explicitly shown in Fig. 2 with a benchmark value of the mixing angle \( \sin \theta = 10^{-6} \).

In Fig. 2, the \( \mathcal{I}_{A,B,C} \) contributions to energy loss as functions of the scalar mass \( m_S \) are presented respectively by the blue, red and green curves, while the black curve is the total contribution

\[
Q = QV_c,
\]

where \( V_c = 4\pi R_c^3/3 \) is the supernova core volume with the core radius \( R_c \approx 10 \text{ km} \). As expected in Eq. (2.12) and as can be seen from Fig. 2, the \( \mathcal{I}_A \) term is suppressed by the scalar mass for \( m_S \lesssim 100 \text{ MeV} \), via \( (q/x)^4 = (m_S/E_S)^4 \). It originates from the cancellation in Eq. (2.9). The \( \mathcal{I}_B \) contribution is almost a constant in the limit of small scalar mass \( m_S \), and dominates over other terms for \( m_S \lesssim 10 \text{ MeV} \), although it is suppressed by \( (m_N/v_{\text{EW}})^2 \), as can be seen from Eq. (2.12). As expected, the contribution of the cross term \( \mathcal{I}_C \) is always in between \( \mathcal{I}_A \) and \( \mathcal{I}_B \). Also all the three contributions are Boltzmann-suppressed in the limit of large scalar mass \( m_S \gtrsim 300 \text{ MeV} \).
3 Decay of $S$

From mixing with the SM Higgs, the light scalar $S$ can decay into the SM particles. The supernova limits are only relevant if the scalar mass is $\lesssim 300$ MeV; otherwise the production rate will be highly Boltzmann-suppressed (see Fig. 2). Therefore only the following decay channels are relevant:

$$
\Gamma_0(S \to e^+e^-) = \frac{2m_SM_S^2\sin^2\theta}{8\pi v_{EW}^2} \left( 1 - \frac{4m_e^2}{m_S^2} \right) \frac{3}{2},
$$

$$
\Gamma_0(S \to \mu^+\mu^-) = \frac{2m_SM_S^2\sin^2\theta}{8\pi v_{EW}^2} \left( 1 - \frac{4m_\mu^2}{m_S^2} \right) \frac{3}{2},
$$

$$
\Gamma_0(S \to \gamma\gamma) = \frac{m^2_S^2\sin^2\theta}{256\pi^3v_{EW}^2} \left| \sum_f N_f Q_f^2 A_{1/2}(\tau_f) + A_{1}(\tau_W) \right|^2,
$$

$$
\Gamma_0(S \to \pi^0\pi^0) = \frac{\sin^2\theta}{648\pi m_S v_{EW}^2} \left( m^2_S + \frac{11}{2} m^2_{\pi^0} \right)^2 \left( 1 - \frac{4m^2_{\pi^0}}{m_S^2} \right)^{1/2},
$$

$$
\Gamma_0(S \to \pi^+\pi^-) = \frac{\sin^2\theta}{324\pi m_S v_{EW}^2} \left( m^2_S + \frac{11}{2} m^2_{\pi^\pm} \right)^2 \left( 1 - \frac{4m^2_{\pi^\pm}}{m_S^2} \right)^{1/2},
$$

with $m_{e,\mu,\pi^0,\pi^\pm}$ being respectively the masses of $e$, $\mu$, $\pi^0$ and $\pi^\pm$. In Eq. (3.3) we include only the SM fermions and $W$ bosons running in the loop for the diphoton channel, with $\alpha = e^2/4\pi$ the fine-structure constant, $N_C = 3$ (1) the color factor for quarks (charged
Figure 3. Decay widths of $S$ into $e^+e^-$ (red), $\mu^+\mu^-$ (green) and $\gamma\gamma$ (blue), with $\sin \theta = 10^{-6}$. We also show the corresponding proper decay length $c\tau_S$.

In addition to the two-body decays above, if kinematically allowed, the light scalar $S$ has also the following four-body decays via the SM $W$ and $Z$ bosons:

\begin{align}
S &\to W^+W^- \to \ell^+\ell^-\nu\bar{\nu} \quad \text{with} \quad \alpha, \beta = e, \mu, \\
S &\to Z^*Z^* \to \nu\bar{\nu}\nu\bar{\nu}, \quad e^+e^-e^-e^-, \quad e^+e^+e^-e^-.
\end{align}

The four-body decay widths can be calculated, e.g. following the procedure in the Appendix of Ref. [59]. Suppressed by the phase space, it turns out that the partial widths of the four-body decays are much smaller than the two-body decays above, and can be safely neglected. Then we can insert the following width into Eq. (2.6):

$$\Gamma_S = \frac{m_S}{E_S} \Gamma_{S,0},$$

where $\Gamma_{S,0}$ is the proper total width, i.e. the sum of all partial widths listed in Eqs. (3.1)-(3.5), and $m_S/E_S$ the inverse Lorentz boost factor.

The partial widths of $S$ into $e^+e^-$, $\mu^+\mu^-$ and $\gamma\gamma$ and the corresponding proper decay length $c\tau_S$ (where $\tau_S = 1/\Gamma_S$) are shown in Fig. 3 respectively by the red, green and blue curves. For concreteness we have set the mixing angle to be the benchmark value of $\sin \theta = 10^{-6}$. It is clear from Fig. 3 that the decay of $S$ is dominated by $e^+e^-$ channel when the scalar mass $m_S \lesssim 2m_\mu \simeq 210$ MeV and by the decay $\mu^+\mu^-$ above the $2m_\mu$ threshold. When the scalar $S$ is heavier than the two-pion threshold $2m_\pi \simeq 270$ MeV, it will decay mostly into pions. However, above the $2m_\pi$ threshold the scalar $S$ decays so fast that the range...
\(m_S \gtrsim 270\) MeV can no longer be excluded by supernova energy loss criteria (see Fig. 5). Therefore we do not show the partial decay width of \(S\) into pions in Fig. 3.

### 4 Re-absorption of \(S\)

As mentioned earlier, after being produced, the light scalar \(S\) may be re-absorbed via the inverse nucleon bremsstrahlung process in Eq. (2.8). As a result, the scalar \(S\) can only travel a finite distance in the supernova core, and we can define the corresponding inverse MFP as [54, 60]

\[
\lambda^{-1} \equiv \frac{1}{2E_S} \frac{dN_S(-k_S)}{d\Pi_S} = \frac{1}{2E_S} \int d\Pi_4 S \sum_{\text{spins}} |\mathcal{M}'|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4 + k_S) f_1 f_2, \tag{4.1}
\]

with \(N_S(-k_S)\) the total \(S\) number per unit volume produced in the supernova core with momentum \(-k_S\) (note that for the re-absorption process the scalar \(S\) is in the initial state, while in the production process \(S\) is in the final state), \(\Pi_S\) the phase space of \(S\), \(d\Pi_4\) the four-body phase space for the initial and final state nucleons, and \(\mathcal{M}'\) the scattering amplitude for the inverse nucleon bremsstrahlung process in Eq. (2.8). It is straightforward to prove that

\[
\sum_{\text{spins}} |\mathcal{M}(k_S)|^2 = \sum_{\text{spins}} |\mathcal{M}'(-k_S)|^2. \tag{4.2}
\]

Following the same procedure as above for the emission rate \(Q\), the twelve-dimensional integration can be greatly simplified:

\[
\lambda^{-1} = \frac{n_B^2}{64\pi^{3/2} m_N^{5/2} T^{1/2} x} \int du dv dz \sqrt{uve - u} \delta(u - v + x) S \sum_{\text{spins}} |\mathcal{M}|^2
\]

\[
= \frac{n_B^2 \pi^{1/2} T^{1/2} f_4^4}{m_N^{9/2} T^{1/2} x} \int du dv dz \sqrt{uve - u} \delta(u - v + x) I_{\text{tot}}, \tag{4.3}
\]

with the dimensionless variables \(u, v, z, x\) and the dimensionless function \(I_{\text{tot}}\) being the same as in Eq. (2.11). It is obvious from Eq. (4.3) that the MFP \(\lambda\) not only depends on the scalar mass \(m_S\) and mixing angle \(\sin \theta\), but also on the scalar energy \(E_S = xT\) (note that in Eq. (4.3) the variable \(x\) is not integrated out, unlike in Eq. (2.11)). To simplify the calculation of emission rate \(Q\) in Eq. (2.11), we average over the distributions of \(S\) and replace the MFP \(\lambda\) in the \(P_{\text{abs}}\) factor in Eq. (2.7) by the effective energy-independent MFP \(\langle \lambda \rangle\), with its inverse defined by [19]

\[
\langle \lambda^{-1} \rangle \equiv \int dE_S \frac{E_S^3}{\sigma_{\text{abs}} T^{-1}} \lambda^{-1}(E_S) \left( \int dE_S \frac{E_S^3}{\sigma_{\text{abs}} T^{-1}} \right) = \int dx \frac{x^3}{\sigma_{\text{abs}} T^{-1}} \lambda^{-1}(x). \tag{4.4}
\]

The dependence of the effective MFP on the scalar mass \(m_S\) and mixing angle \(\sin \theta\) is shown in Fig. 4, where the orange, green, blue and purple lines correspond respectively to
the mean free path values of 0.1 km, 1 km, 10 km and 100 km. From Fig. 4, one can make a good estimate of the order-of-magnitude MFP by the following empirical formula:

$$\langle \lambda \rangle \approx 10 \text{ km} \times \left( \frac{\sin \theta}{10^{-6}} \right)^{-2} \text{ for } m_S \lesssim 100 \text{ MeV}. \quad (4.5)$$

This can be estimated from the prefactor in Eq. (4.3) and the fact that $I_{\text{tot}} \sim 10^{-6}$ [cf. Eq. (2.12)]. In the limit of large scalar mass, $\lambda^{-1}$ is highly suppressed by the $f_1 f_2$ factor in Eq. (4.1); therefore in Fig. 4 the MFP $\langle \lambda \rangle$ is largely enhanced for $m_S \gtrsim 300$ MeV.

5 Luminosity limit

With the decay and re-absorption effects in Section 3 and 4, we are now ready to calculate the emission rate $Q$ in Eq. (2.11) in the supernova core due to the existence of the light scalar $S$. Based on the observed neutrinos from SN1987A [17], the energy loss $L_\nu$ due to neutrino emission is expected to be $(3 - 5) \times 10^{53}$ erg/sec [9] and can be used to set limits on the scalar mass $m_S$ and the mixing angle $\sin \theta$. Setting the total energy emission rate $Q$ to be $3 \times 10^{53}$ erg/sec and $5 \times 10^{53}$ erg/sec, we can exclude the orange and purple shaded regions in the $m_S$ and $\sin \theta$ parameter space in Fig. 5 respectively. Here we have taken a fixed supernova core temperature of $T = 30$ MeV and a constant nuclear core density of $n_B = 1.2 \times 10^{38}$ cm$^{-3}$ for simplicity, although there might be some uncertainty in these parameters due to the uncertainty in the progenitor proto-neutron star mass and neutrino flux estimations [57, 61, 62]. For comparison, the previous limit from Ref. [21] with the same supernova parameters is shown as the pink shaded region, which provides a good order-of-magnitude estimate (cf. Eq. (39) in Ref. [21]) but did not take into account the cancelation effect in Eq. (2.9) and the contributions from the diagrams with $S$ coupling to the pion mediators. So the updated limit presented here should supersede the old limit from Ref. [21].
As shown in Fig. 5, there is an upper “cut-off” of the scalar mass $m_S$ around $2m_\pi \approx 270$ MeV, as the scalar with mass $m_S \gtrsim 2m_\pi$ decays too fast such that it can not effectively take away energy from the supernova core. The excluded mixing angles range from $3.5 \times 10^{-7}$ to $2.5 \times 10^{-5}$ for the luminosity limit of $3 \times 10^{53}$ erg/sec and from $4.3 \times 10^{-7}$ to $2.5 \times 10^{-5}$ for $5 \times 10^{53}$ erg/sec, depending on the scalar mass. The shape of excluded regions in Fig. 5 is a combined effect of the superposition of the three contributions $I_{A,B,C}$, the decay of $S$, and the dependence of effective MFP $\langle \lambda \rangle$ on the scalar mass and mixing angle. One can expect that when large-scale neutrino detectors like IceCube-DeepCore [63, 64], Hyper-Kamiokande [65, 66] and DUNE [67, 68] collect more neutrinos from the explosion of a nearby supernovae, the luminosity limits on the scalar mass $m_S$ and mixing angle can be significantly improved.

The supernova luminosity limits on the scalar mass $m_S$ and mixing angle $\sin \theta$ obtained here are largely complementary to other laboratory, astrophysical and cosmological limits, as summarized in Fig. 6. Through mixing with the SM Higgs, the scalar $S$ could obtain flavor-changing neutral current (FCNC) couplings to the SM quarks at 1-loop level, such as $S\bar{s}d$. These couplings induce the FCNC decays $K \rightarrow \pi + X$, $B \rightarrow K + X$ and $B \rightarrow \pi + X$ with $X = e^+e^-, \mu^+\mu^-, \gamma\gamma$ or missing energy. As the corresponding branching fractions are all highly suppressed in the SM, these decays provide the most stringent laboratory limits on the light scalar $S$. Combining the limits from NA48/2 [69, 70], E949 [71], KOTO [72], NA62 [73, 74], KTeV [75–78], BaBar [79, 80], Belle [81], and LHCb [82], the meson decay limits are shown as the gray shaded regions in Fig. 6. The scalar $S$ might also be produced in the high-intensity beam-dump experiments. The current most stringent limits are from kaon decays in the CHARM experiment [83], which is presented as the blue shaded region in Fig. 6. With a higher luminosity, the DUNE experiment can improve current limits by roughly two orders of magnitude [84], as indicated by the dashed blue line. All the calculation details can
Figure 6. Complementarity of the supernova limits on $m_S$ and $\sin \theta$ (shaded purple and orange, cf. Fig. 5) and those from FCNC meson decays (shaded gray), CHARM (shaded blue) and BBN (shaded pink). The dashed blue curve shows the future prospect at DUNE.

be found e.g. in Refs. [41, 49].

For sufficiently small mixing angle $\sin \theta$, if the lifetime of light scalar $\tau_S \gtrsim 1$ sec, then $S$ will contribute an extra degree of freedom in the early universe and spoil the success of big bang nucleosynthesis (BBN). The corresponding limit on $m_S$ and $\sin \theta$ is shown as the pink shaded region in Fig. 6. More details can be found e.g. in Ref. [85]. A sufficiently light scalar $S$ could also contribute to the light degrees of freedom $N_{\text{eff}}$ in the early universe and thus constrained by the precision Planck data [86]. However, the limit from $\Delta N_{\text{eff}}$ is very weak, i.e. requiring that $\sin \theta \lesssim 0.01$ [41], hence it is not shown in Fig. 6.

Taking into consideration all the existing laboratory, astrophysical and cosmological limits on the light CP-even scale $S$ and its mixing angle $\sin \theta$ with the SM Higgs, it is clear in Fig. 6 that the luminosity limits from SN1987A are one or two orders of magnitude lower than the current laboratory constraints, depending on the scalar mass, and overlap sizably with the BBN limit. It is remarkable that the future DUNE experiment could reach the supernova excluded regions for $m_S \gtrsim 100$ MeV.

6 Conclusion

Due to their high nucleon density, supernova cores provide an ideal testing ground for light hypothetical particles, such as axion/ALP, dark photon or a light CP-even scalar, weakly coupling to the SM particles. Taking into consideration the core energy loss from the emission of these particles, stringent limits on their couplings to the SM can be derived. While there has been considerable literature on this for the dark photon and axion cases, the discussion of supernova limits on a light CP-even scalar is very limited. Motivated to fill this gap, we have presented a complete calculation of the supernova luminosity limit on a generic light CP-even scalar with mass $m_S$ and mixing angle $\sin \theta$ with the SM Higgs. We consider the scalar production via the nucleon bremsstrahlung process $N N \rightarrow N N S$ in the supernova
core and point out that, as a result of the distinct Lorentz structure of the couplings, the supernova limits on the CP-even scalar $S$ are very different from those on the dark photon and axion/ALP.

We have included the contributions from the diagrams with $S$ coupling to both nucleons and the pion mediators, as shown in Fig. 1. We find that to leading order in $m_S^2/m_N E_S$, the contributions from the $S - N - N$ diagrams cancel out with each other. As a result, the $S - N - N$ diagram contributions are suppressed by $(m_S/E_S)^4$. For light scalar with $m_S \lesssim 10$ MeV, the $S$-induced energy loss is therefore dominated by the diagrams with $S$ coupling to the pion mediators, as shown in Fig. 2. In order to get the final supernova limits, we have calculated the decay and re-absorption rates of $S$, and find that the re-absorption via inverse nucleon bremsstrahlung $NNS \rightarrow NN$ plays a crucial role in the energy loss mechanism. The resultant dependence of the average mean free path on the scalar mass and mixing angle is shown in Fig. 4. With the production, decay and re-absorption of $S$ properly taken into account, the supernova luminosity limit on $m_S$ and $\sin \theta$ is presented in Fig. 5. Complementarity with the existing and future laboratory constraints is demonstrated in Fig. 6.

As pointed out in Refs. [56, 58] for the case of dark photon, new limits on the properties of $S$ can arise from the decay of $S$ inside the mantle of stars which can blow away the outer layers of the stellar material and generate intense light emission. This is likely if the $S$ decay products are mainly $e^+e^-$, which then will annihilate and generate MeV $\gamma$-rays. We do not consider this here since this is beyond the main scope of this paper. Moreover, our calculations in this paper is at the leading order of OPE. The beyond OPE effects have been considered for neutrino [87] and axion [57, 88] production in the supernova core, and might also be important for the scalar $S$. In addition to supernova explosion, the light scalar $S$ might also significantly impact neutron star mergers, through either cooling the mergers or heat transportation inside the stars, similar to the axion case [89]. As can be seen from Fig. 6, there exists an allowed parameter space around $\sin \theta \simeq 10^{-5} - 10^{-4}$ (depending on the scalar mass) where the light scalar can be trapped in the supernova or neutron star core (see also Fig. 4), which could in principle affect the merger dynamics in a nontrivial way. A proper investigation of these issues, taking into account the complications arising due to the degenerate nuclear matter, nuclear equations of state and general relativity effects in a merger environment, is well beyond the scope of this work and will be pursued in a future communication.

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A Details of the emission rate calculation

A.1 The amplitude

The master formula of the emission rate $Q$ is given in Eq. (2.4), and the corresponding Feynman diagrams are shown in Fig. 1. For the sake of concreteness let us denote the momentum of the nucleon $N_i$ in Fig. 1 as $p_i$ ($i = 1, 2, 3, 4$), and the momentum of $S$ as $k_S$, the momenta exchanged in the $t$ and $u$-channels are then respectively $k \equiv p_2 - p_4$ and $l \equiv p_2 - p_3$.

Let us start with the proton bremsstrahlung process $p + p \rightarrow p + p + S$. The effective $p - p - \pi$ coupling is $(2m_N/m_\pi)f_{pp\pi^0}\bar{p}i\gamma_5p$ with $f_{pp} \simeq 1$. The amplitude for the diagram (a) is

$$
\mathcal{M}_a = \left[ \left( \frac{2m_N}{m_\pi} \right)^2 \frac{f_{pp}^2(\sin \theta g_{hNN})}{k^2 - m_\pi^2} \right] \frac{1}{(p_3 + k_S)^2 - m_N^2} \times \bar{u}(p_4)\gamma_5u(p_2)\bar{u}(p_3)(\bar{p}_3 + \bar{k}_S + m_N)\gamma_5u(p_1).
$$

(A.1)

The factors in the bracket are common to all the four diagrams $(a), (b), (c)$ and $(d)$ in the $t$-channel. As the nucleons are non-relativistic in the supernova core, we can use the approximation that $p_i \cdot k_S \simeq m_N E_S$ which is much larger than the other term in the propagator, i.e. $m_N E_S \gg m_N^2$. Then the second propagator in Eq. (A.1) can be simplified to be

$$
\frac{1}{(p_i \pm k_S)^2 - m_N^2} \simeq \frac{1}{\pm 2m_N E_S}.
$$

(A.2)

Here we have included also the corresponding propagators for the diagrams $(b), (c)$ and $(d)$, with the $+$ sign for the diagrams $(a)$ and $(b)$ with $i = 3, 4$ and the $-$ sign for $(c)$ and $(d)$ with $i = 1, 2$. It is easy to obtain the spinor parts for the diagrams $(b), (c)$ and $(d)$, which are respectively

$$
\bar{u}(p_3)\gamma_5u(p_1)\bar{u}(p_4)(\bar{p}_1 + \bar{k}_S + m_N)\gamma_5u(p_2),
$$

$$
\bar{u}(p_4)\gamma_5u(p_2)\bar{u}(p_3)\gamma_5(\bar{p}_1 - \bar{k}_S + m_N)u(p_1),
$$

$$
\bar{u}(p_3)\gamma_5u(p_1)\bar{u}(p_4)\gamma_5(\bar{p}_2 - \bar{k}_S + m_N)u(p_2).
$$

(A.3)

Using the kinematic relations [55]

$$
p_1 \cdot p_2 = m_N^2 - \frac{1}{2}m_S^2 - \frac{1}{2}k^2 - \frac{1}{2}l^2 - (k \cdot l) + (k \cdot k_S) + (l \cdot k_S),
$$

$$
p_1 \cdot p_3 = m_N^2 - \frac{1}{2}m_S^2 - \frac{1}{2}k^2 + (k \cdot k_S),
$$

$$
p_1 \cdot p_4 = m_N^2 - \frac{1}{2}m_S^2 - \frac{1}{2}l^2 + (l \cdot k_S),
$$

$$
p_2 \cdot p_3 = m_N^2 - \frac{1}{2}l^2, 
$$

$$
p_2 \cdot p_4 = m_N^2 - \frac{1}{2}k^2, 
$$

$$
p_3 \cdot p_4 = m_N^2 - \frac{1}{2}k^2 - \frac{1}{2}l^2 + (k \cdot l),
$$

(A.4)

it is easy to prove that with the approximation in Eq. (A.2) the four amplitudes $\mathcal{M}_{a,b,c,d}$ cancel out with each other [see Eq. (2.9)], as at the leading order in Eq. (A.3) $p_i \cdot p_j \simeq m_N^2$
with \(i, j = 1, 2, 3, 4\). To obtain a non-vanishing result, we expand the propagators to the next-to-leading order in \(m_S^2/m_N E_S\), in the form of

\[
\frac{1}{(p_i^2 - m_N^2)^2 - m_N^2} \simeq \frac{1}{\pm 2m_N E_S + m_N^2} \simeq \frac{1}{\pm 2m_N E_S} \left[ 1 + \frac{m_N^2}{2m_N E_S} \right], \tag{A.5}
\]

where the \(+\) and \(−\) signs are again respectively for \(i = 3, 4\) and \(1, 2\). As a result of the opposite sign in the bracket for the diagrams \((a), (b)\) and \((c), (d)\), the sum of amplitudes are non-vanishing only at the next-to-leading order in \(m_S^2/m_N E_S\). The contributions of the diagrams \((a), (b), (c)\) and \((d)\) to the production rate will be suppressed by the ratio of \((m_S/E_S)^4\) in the limit of small \(m_S\) [see Eq. (2.12)].

The amplitude for the \(t\)-channel diagram \((e)\) is

\[
\mathcal{M}_e = \left( \frac{2m_N}{m_\pi} \right)^2 \frac{f_{pp}^2}{k^2 - m_S^2} \frac{1}{(k - k_S)^2 - m_\pi^2} \bar{u}(p_4)\gamma_5 u(p_2) \bar{u}(p_3)\gamma_5 u(p_1). \tag{A.6}
\]

It is expected that \(|k_S| \sim T\), and \(|k|^2 \sim m_N T\). We can then neglect the momentum \(k_S\) in the second propagator. The subsequent calculation of \(\mathcal{M}_e\) is trivial.

Evaluations of the \(u\)-channel diagrams \((a'), (b'), (c'), (d')\) and \((e')\) and the \(t\) and \(u\)-channel cross terms are quite similar (note that there is an overall relative minus sign for the \(u\)-channel diagrams). With the approximations \(k^2 \simeq -|k|^2\), \(l^2 \simeq -|l|^2\), \((k \cdot l) \simeq -(k \cdot l)\) and \(k^2, l^2, (k \cdot l) \gg m_N^2, E_S^2\), the squared amplitude for the \(pp\) process is proportional to

\[
\frac{y_{NN}^2 m_N^4}{E_S^4} I_A^{(pp)} + \frac{m_N^2}{81 \sqrt{2} E_W} I_B^{(pp)} + \frac{2 \sqrt{3} y_{NN} m_N^2}{9 E_S^2} \frac{m_N}{v_{EW}} I_C^{(pp)}, \tag{A.7}
\]

where the dimensionless functions of the momenta are respectively

\[
I_A^{(pp)} = \frac{k^4}{(k^2 + m_\pi^2)^2} + \frac{l^4}{(l^2 + m_\pi^2)^2} - \frac{k^2 l^2 - 2(k \cdot l)^2}{(k^2 + m_\pi^2)^2(l^2 + m_\pi^2)^2}, \tag{A.8}
\]

\[
I_B^{(pp)} = \left( \frac{m_N^2 + 11 m_\pi^2}{2 m_\pi^2} \right)^2 \left[ \frac{k^4}{(k^2 + m_\pi^2)^4} + \frac{l^4}{(l^2 + m_\pi^2)^4} - \frac{k^2 l^2 - 2(k \cdot l)^2}{(k^2 + m_\pi^2)^2(l^2 + m_\pi^2)^2} \right], \tag{A.9}
\]

\[
I_C^{(pp)} = \left( \frac{m_N^2 + 11 m_\pi^2}{2 m_\pi^2} \right)^2 \left[ \frac{k^4}{(k^2 + m_\pi^2)^3} + \frac{l^4}{(l^2 + m_\pi^2)^3} - \frac{1}{2} \frac{k^2 l^2 - 2(k \cdot l)^2}{(k^2 + m_\pi^2)^2(l^2 + m_\pi^2)^2} \right] - \frac{1}{2} \frac{k^2 l^2 - 2(k \cdot l)^2}{(k^2 + m_\pi^2)^2(l^2 + m_\pi^2)^2} \tag{A.10}
\]

Here the \(I_A^{(pp)}\) terms are respectively the contributions from the diagrams with the scalar \(S\) coupling to the protons and pions, and \(I_C^{(pp)}\) are from the cross terms.

Let us now move on to the \(nn\) and \(np\)-channel production of \(S\). Isospin invariance requires that \(f_{nn} = -f_{pp}\) and \(f_{np} = -\sqrt{2} f_{pp}\). This implies that the \(nn\) contribution is the same as that for the \(pp\) process, i.e.

\[
I_{A,B,C}^{(nn)} = I_{A,B,C}^{(pp)}. \tag{A.11}
\]

For the \(np\) processes, there will be an extra factor of \((-\sqrt{2})^2 = 2\) for the amplitudes of \(u\)-channel diagrams. When amplitudes are squared, there will be an extra factor of 4 for
the \(u\)-channel diagrams, and a factor of \(1 \times (-1) \times (-\sqrt{2})^2 = -2\) for cross terms of \(t\) and \(u\)-channel diagrams. As a result, we get

\[
I_A^{(np)} = \frac{k^4}{(k^2 + m_{\pi}^2)^2} + \frac{4l^4}{(l^2 + m_{\pi}^2)^2} + \frac{2 [k^4 l^2 - 2(k \cdot l)^2]}{(k^2 + m_{\pi}^2)(l^2 + m_{\pi}^2)},
\]

(A.12)

\[
I_B^{(np)} = \left( \frac{m_{\pi}^2}{2} \right)^2 \left[ \frac{k^4}{(k^2 + m_{\pi}^2)^2} + \frac{4l^4}{(l^2 + m_{\pi}^2)^2} + \frac{2 [k^4 l^2 - 2(k \cdot l)^2]}{(k^2 + m_{\pi}^2)(l^2 + m_{\pi}^2)^2} \right],
\]

(A.13)

\[
I_C^{(np)} = \left( \frac{m_{\pi}^2}{2} \right)^2 \left[ \frac{k^4}{(k^2 + m_{\pi}^2)^2} + \frac{4l^4}{(l^2 + m_{\pi}^2)^2} + \frac{2 [k^4 l^2 - 2(k \cdot l)^2]}{(k^2 + m_{\pi}^2)(l^2 + m_{\pi}^2)^2} \right].
\]

(A.14)

Summing up the \(pp\), \(nn\) and \(np\) channels, we get the full squared amplitude expression:

\[
\sum_{\text{spins}} |\mathcal{M}|^2 \approx \frac{256\pi^2 n_{\pi}^2 f_{\pi}^4 \sin^2 \theta}{m_N^2} \left[ \frac{y_{hNN} m_S^4 I_A}{E_S} + \frac{m_N^2}{81 v_{EW}} I_B \right.
\]

\[
\left. + \frac{2 y_{hNN} m_S^2}{9} \frac{m_N}{E_S} v_{EW} I_C \right],
\]

(A.15)

where

\[
I_{A,B,C} = I_{A,B,C}^{(pp)} + I_{A,B,C}^{(nn)} + 4 I_{A,B,C}^{(np)}.
\]

(A.16)

As in Eq. (2.13), the factor of 4 for the \(np\) channel is for non-identical particles in the initial and final states.

### A.2 The emission rate

To simplify the fifteen-dimensional phase space integral in Eq. (2.4) for the emission rate, we define the following dimensionless variables \([54, 55]\):

\[
u \equiv \frac{p_i^2}{m_N T}, \quad v \equiv \frac{p_j^2}{m_N T}, \quad x \equiv \frac{E_S}{T}, \quad y \equiv \frac{m_{\pi}^2}{m_N T},
\]

\[
r \equiv \frac{T}{m_N}, \quad q \equiv \frac{m_S}{T}, \quad z \equiv \cos(\theta_{if}),
\]

(A.17)

with

\[
p_1 \equiv P + p_i, \quad p_2 \equiv P - p_i, \quad p_3 \equiv P + p_f, \quad p_4 \equiv P - p_f.
\]

(A.18)

We have taken the approximation that \(p_i \gg k_S\) such that \(p_3 + p_4 \approx p_1 + p_2 = P\). In Eq. (A.17) \(z\) is defined as the angle \(\theta_{if}\) between \(p_i\) and \(p_f\). Then it is trivial to obtain the following relations:

\[
k^2 = (u + v - 2z\sqrt{uv})m_N T \equiv c_k m_N T,
\]

(A.19)

\[
k^2 + m_{\pi}^2 = (u + v + y - 2z\sqrt{uv})m_N T \equiv c_{k\pi} m_N T,
\]

(A.20)

\[
l^2 = (u + v + 2z\sqrt{uv})m_N T \equiv c_l m_N T,
\]

(A.21)
\[ I^2 + m_N^2 = (u + v + y + 2z\sqrt{uv})m_NT \equiv c_{l\pi}m_NT, \]  
\[ k^2 l^2 - 2(k \cdot l)^2 = [u^2 + v^2 + 2uv(-3 + 2z^2)]m_N^2T^2 \equiv (c_{kl}m_NT)^2. \]  

It is now straightforward to arrive at the final expression for the emission rate \( Q \) in Eq. (2.11), with the dimensionless components for the \( pp \), \( nn \) and \( np \) processes respectively:

\[ I_A^{(pp)} = I_A^{(nn)} = \frac{c_k^2}{c_{k\pi}} + \frac{c_l^2}{c_{l\pi}} - \frac{c_{kl}^2}{2c_{k\pi}c_{l\pi}}, \]  
\[ I_B^{(pp)} = I_B^{(nn)} = \left( q^2 r + \frac{11}{2} y \right)^2 \left[ \frac{c_k^2}{c_{k\pi}} + \frac{c_l^2}{c_{l\pi}} - \frac{c_{kl}^2}{2c_{k\pi}c_{l\pi}} \right], \]  
\[ I_C^{(pp)} = I_C^{(nn)} = \left( q^2 r + \frac{11}{2} y \right) \left[ \frac{c_k^2}{c_{k\pi}} + \frac{c_l^2}{c_{l\pi}} - \frac{c_{kl}^2}{2c_{k\pi}c_{l\pi}} - \frac{c_{kl}^2}{2c_{k\pi}c_{l\pi}} \right], \]  
\[ I_A^{(np)} = \frac{c_k^2}{c_{k\pi}} + \frac{4c_l^2}{c_{l\pi}} + \frac{2c_{kl}^2}{c_{k\pi}c_{l\pi}}, \]  
\[ I_B^{(np)} = \left( q^2 r + \frac{11}{2} y \right)^2 \left[ \frac{c_k^2}{c_{k\pi}} + \frac{4c_l^2}{c_{l\pi}} + \frac{2c_{kl}^2}{c_{k\pi}c_{l\pi}} \right], \]  
\[ I_C^{(np)} = \left( q^2 r + \frac{11}{2} y \right) \left[ \frac{c_k^2}{c_{k\pi}} + \frac{4c_l^2}{c_{l\pi}} + \frac{2c_{kl}^2}{c_{k\pi}c_{l\pi}} \right]. \]  

Simplification of the twelve-dimensional integration in Eq. (4.1) for the inverse MFP \( \lambda^{-1} \) is quite similar, which leads to the formula in Eq. (4.3).

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