General Static Spherically Symmetric Black Holes of Heterotic String on a Six Torus

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Abstract

We present the most general static, spherically symmetric solutions of heterotic string compactified on a six-torus that conforms to the conjectured “no-hair theorem”, by performing a subset of $O(8, 24)$ transformations, i.e., symmetry transformations of the effective three-dimensional action for stationary solutions, on the Schwarzschild solution. The explicit form of the generating solution is determined by six $SO(1, 1) \subset O(8, 24)$ boosts, with the zero Taub-NUT charge constraint imposing one constraint among two boost parameters. The non-trivial scalar fields are the axion-dilaton field and the moduli of the two-torus. The general solution, parameterized by unconstrained 28 magnetic and 28 electric charges and the ADM mass compatible with the Bogomol’nyi bound, is obtained by imposing on the generating solution $[SO(6) \times SO(22)]/[SO(4) \times SO(20)] \subset O(6, 22)$ (T-duality) transformation and $SO(2) \subset SL(2, R)$ (S-duality) transformation, which do not affect the four-dimensional space-time. Depending on the range of boost parameters, the non-extreme solutions have the space-time of either Schwarzschild or Reissner-Nordström black hole, while extreme ones have either null (or naked) singularity, or the space-time of extreme Reissner-Nordström black hole.

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I. INTRODUCTION

The black holes of the effective four-dimensional, heterotic string theory compactified on a six-torus have been the subject of active research, since they may shed light on quantum aspects of black holes as well as on the role such non-perturbative configurations may play in the full string dynamics.

In the past a class of such black holes with special charge configurations were considered and, in particular, all the electrically charged stationary black holes were constructed. Recently, progress has been made in constructing a more general class of BPS-saturated and non-extreme dyonic static, spherically symmetric black holes, which contain the previously constructed static spherically symmetric configurations as special cases.

Such dyonic black holes correspond to configurations with 28 electric and 28 magnetic charges of the $U(1)^{28}$ gauge symmetry, which are subject to one constraint. The 'generating solution' for this class of solutions is parameterized by two magnetic and two electric charges of $U(1)_{1,M} \times U(1)_{2,E} \times U(1)_{1,M} \times U(1)_{2,E}$ gauge group. Here the superscripts $(1, 2)$ denote the Kaluza-Klein and two-form gauge group factors, respectively, and the subscripts $(1, M)$ and $(2, E)$ denote the magnetic and electric charges associated with the gauge factors of the first and second compactified coordinates, respectively. Namely, the electric and magnetic charges arise from gauge factors associated with different compactified coordinates. The explicit form of this generating solution for the BPS-saturated and non-extreme configurations was obtained by solving directly the corresponding Killing spinor equations and the second order Euler-Lagrange equations, respectively.

The general configurations in this class are then obtained by imposing fifty $[SO(6) \times SO(22)]/[SO(4) \times SO(20)] \subset O(6, 22)$ symmetry transformations and one $SO(2) \subset SL(2, R)$ transformation on the generating solution. Here, $O(6, 22)$ and $SL(2, R)$ correspond to the T-duality and S-duality symmetries of the four-dimensional effective action, which do not affect the four-dimensional space-time metric. Thus, along with the four charge parameters of the generating solution, the subsequent T- and S-duality transformations introduce 51 new charge degrees of freedom. General solutions in this class are then specified by the ADM mass (compatible with the Bogomol’nyi bound) and by 28 electric and 28 magnetic charges which are subject to one charge constraint, i.e., there is one charge degree of freedom missing. On the other hand the most general, static, spherically symmetric configuration, consistent with the conjectured “no-hair theorem”, should be parameterized by the ADM mass and 56 unconstrained charges. In order to obtain such configurations one has

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1 For an overview see, e.g., Ref. and references therein.

2 In Ref. it was shown that the BPS-saturated generating solution is an exact string solution by proving the conformal invariance of the corresponding $\sigma$-model.

3 The content of the no-hair theorem would ensure that each black hole type solution would be uniquely specified by its mass, angular momentum and a set of conserved (electric and magnetic) charges. Such a theorem has not been proven in general and was studied in Ref. only within special theories like pure Einstein gravity or Einstein-Maxwell gravity and that only for configu-
to start with a generating solution, whose charge configuration is parameterized by five parameters.

The purpose of this paper is to complete the program by obtaining the explicit form of the generating solution for all the static, spherically symmetric configurations of the effective heterotic string theory compactified on a six-torus, compatible with the conjectured no-hair theorem. We shall employ a solution generating technique by which new solutions are obtained by performing symmetry transformations on a known solution. The symmetry transformations employed are those of the effective three-dimensional action for the stationary solutions of the toroidally compactified string. Related solution generating techniques were used to obtain the explicit form of a general class of stationary solutions in dilaton-Maxwell-Einstein gravity \[14,15\], all the static spherically symmetric black hole solutions of Abelian Kaluza-Klein theory \[7\] as well as all the electrically charged rotating black hole solutions of the heterotic string compactified on a six-torus \[7\]. The solution generating technique to obtain the most general class of static, spherically symmetric solutions of heterotic string compactified on a six-torus was already anticipated in Ref. \[7\]. In this paper we provide an explicit realization of the symmetry transformations and an explicit form of the generating solution.

The effective three-dimensional action for heterotic string compactified on a seven-torus configurations with regular horizons, \textit{i.e.}, it does not apply to configurations with naked singularities or singular horizons. Nevertheless, we assume that the no-hair theorem exists and can be applied also to the black hole type solutions of the low energy effective action of toroidally compactified heterotic string. See, for example, Ref. \[18\], where anti-gravitating black holes with scalar hairs were studied within \(N = 8\) supergravity, which contains scalar fields as well as gravity and vector fields. Since such scalar hairs are, in general, functions of \(U(1)\) charges and the ADM mass, our position is that in the broad sense the no-hair theorem is not violated in this and other related cases. Note also that some of the extreme solutions presented here have naked singularities or singular horizons, and thus general considerations of the no-hair theorem do not apply to such cases.

\[4\] The most general four-dimensional static spherically symmetric solutions of the \((4 + n)\)-dimensional Abelian Kaluza-Klein theories, as anticipated in Ref. \[16\], was obtained \[17\] by applying a specific subset of \(SO(2, n)\) symmetry transformations of the three-dimensional effective action on Schwarzschild solution. The explicit form of the generating solution, parameterized by four dyonic charges of \(U(1)_1 \times U(1)_2\) gauge factors of the first and second compactified directions, was obtained by applying four \(SO(1, 1) \subset SO(2, n)\) boosts to the Schwarzschild solution with zero Taub-NUT constraint imposing one constraint on two boost parameters, or equivalently, the four charges of the generating solution subjected to one constraint. (It was observed in Ref. \[18\] that such a boost transformation induces charges of the Kaluza-Klein \(U(1)\) gauge field within the context of the five-dimensional Kaluza-Klein theory.) The most general solution, specified by the ADM mass and \textit{unconstrained} \(n\)-magnetic and \(n\)-electric charges of \(U(1)^n\) gauge symmetry, is then obtained by imposing \(2n - 3\) \(SO(n)/SO(n - 2) \subset SO(n)\) transformations, \textit{i.e.}, target space symmetry transformations of the four-dimensional action, on the generating solution.
was given in Ref. [19], where it was shown to have a symmetry larger than the direct product of the three-dimensional target space $T$-duality $O(7, 23, R)$ and the four-dimensional $S$-duality $SL(2, R)$. Namely, since $SL(2, R)$ symmetry in four-dimensions does not commute with $O(7, 23)$ symmetry in three-dimensions, these two symmetries generate a larger symmetry group $O(8, 24)$ in three dimensions.

In order to obtain the explicit form of the general generating solution, we shall impose six $SO(1, 1) \subset O(8, 24)$ boost transformations on the Schwarzschild (neutral) black hole solution. The three boosts $SO(1, 1) \subset O(7, 23)$ on the Schwarzschild solution generate only electric charges. In fact, the three additional boosts $SO(1, 1) \subset O(8, 24) - O(7, 23)$ induce the necessary magnetic charges. The two ‘magnetic’ and two ‘electric’ boosts yield the previous generating solution with the charge configuration of $U(1)^{(1)}_{1,M} \times U(1)^{(1)}_{2,E} \times U(1)^{(2)}_{1,M} \times U(1)^{(2)}_{2,E}$ gauge group. The zero Taub-NUT constraint imposes one constraint among the two left-over boosts. Thus, the generating solution is specified by a five-parameter charge configuration (associated with the first two compactified dimensions), and the non-extremality parameter (related to the Schwarzschild black hole mass), which parameterizes a deviation of the ADM mass from the supersymmetric limit. The most general solution is then obtained by imposing fifty $[SO(6) \times SO(22)]/[SO(4) \times SO(20)] \subset O(6, 22)$ (four-dimensional $T$-duality) and one $SO(2) \subset SL(2, R)$ ($S$-duality) transformations, which do not affect the four-dimensional space-time metric.

In chapter II, we summarize the four-dimensional (chapter IIa) and three-dimensional (chapter IIb) effective actions of heterotic strings on a six-torus and a seven-torus, respectively, and give the relationship (chapter IIc) between fields of the three-dimensional and four-dimensional actions for the case of the static, spherically symmetric configuration. The solution generating technique with the explicit set of symmetry transformations is discussed in chapter IIIa, and the explicit form of the generating solution for all the static, spherically symmetric black holes (compatible with conjectured no-hair theorem) of the heterotic string compactified on a six-torus is given in chapter IIIb. The global space-time structures and thermal properties of such solutions are discussed in chapters IVa and IVb, respectively. Conclusions are given in chapter V.

II. ACTION OF HETEROYT STRING ON A SIX-TORUS AND A SEVEN-TORUS

For the purpose of fixing the notations, we first summarize (see Refs. [19,20] and references therein) structure of the effective three-dimensional and four-dimensional field theories of heterotic string compactified on a seven-torus and a six-torus, respectively. In addition we shall give the relationship between the fields of the effective three-dimensional and four-dimensional actions for the four-dimensional static, spherically symmetric configurations.

The starting point is the effective field theory of heterotic string in ten-dimensions, which is described by the $N = 1$ supergravity theory coupled to $N = 1$ super Yang-Mills theory in ten-dimensions. At generic points in the moduli space of heterotic string, the massless bosonic fields are given by $\hat{G}_{MN}, \hat{B}_{MN}, \hat{A}_M$ and $\Phi$ ($0 \leq M, N \leq 9, 1 \leq I \leq 16$), which correspond to the graviton, two-form field, $U(1)^{16}$ part of the ten-dimensional gauge group, and the dilaton field, respectively. The ten-dimensional action of these massless bosonic
modes is given by

$$\mathcal{L} = \sqrt{-G} \left[ \mathcal{R}_G + \hat{G}^{MN} \partial_M \Phi \partial_N \Phi - \frac{1}{12} \hat{H}_{MNP} \hat{H}^{MNP} - \frac{1}{4} \hat{F}^I_{MN} \hat{F}^I{}^{MN} \right],$$  \hspace{1cm} (1)

where $\hat{G} \equiv \det \hat{G}_{MN}$, $\mathcal{R}_G$ is the Ricci scalar of the metric $\hat{G}_{MN}$, $\hat{F}^I_{MN}$ is the field strength of $A^I_M$, and $\hat{H}_{MNP}$ = $\partial_M \hat{A}^N - \partial_N \hat{A}^M + \text{cyc. perms.}$ are the field strengths of $\hat{A}^I_M$ and $\hat{B}_{MN}$, respectively. We choose the mostly positive signature convention ($-++\cdots+$) for the metric $\hat{G}_{MN}$.

### A. Effective Four-Dimensional Action

We parameterize the solutions in terms of fields in effective four-dimensional heterotic string on a six-torus $[20,7]$, whose action is obtained by dimensionally reducing (1) on a six-torus down to four-dimensions. One uses the following Kaluza-Klein Ansatz for the string on a six-torus $[20,7]$, whose action is obtained by dimensionally reducing (1) on a six-torus down to four-dimensions. One uses the following Kaluza-Klein Ansatz for the metric:

$$\hat{G}_{MN} = \left( \begin{array}{cc} e^{2\varphi} g_{\mu\nu} + G_{m} A^{(1)}_{\mu} A^{(1)}_{\nu} & A^{(1)}_{\mu} G_{mn} \\ A^{(1)}_{\nu} G_{mn} & G_{mn} \end{array} \right),$$  \hspace{1cm} (2)

where $A^{(1)}_{\mu} (\mu = t, r, \theta, \phi; m = 1, \ldots , 6)$ are four-dimensional Kaluza-Klein $U(1)$ gauge fields and $\varphi \equiv \Phi - \frac{1}{2} \ln \det G_{mn}$ is the four-dimensional dilaton field. The effective four-dimensional action is specified by the following massless (four-dimensional) bosonic fields: the (Einstein-frame) graviton $g_{\mu\nu}$, the complex scalar field (dilaton-axion field) $\bar{S} = \Psi + ie^{-\varphi}$, 28 $U(1)$ gauge fields $A^I_{\mu} \equiv (A^{(1)}_{\mu}, A^{(2)}_{\mu}, A^{(3)}_{\mu})$ defined as $A^{(2)}_{\mu} \equiv \hat{B}_{\mu m} + \hat{B}_{mn} A^{(1)}_{n} + \frac{1}{2} \hat{A}^I_{\mu} A^{(3)}_{I}$, $A^{(3)}_{I} \equiv \hat{A}^I_{\mu} - \hat{A}^I_{m} A^{(1)}_{\mu}$, and the following symmetric $O(6,22)$ matrix of the scalar fields (moduli):

$$M = \left( \begin{array}{cccc} G^{-1} & -G^{-1} C & -G^{-1} a^T \\ -C^T G^{-1} & G + C^T G^{-1} C + a^T a & C^T G^{-1} a^T + a^T \\ -a G^{-1} C + a & a G^{-1} C + a & I + a G^{-1} a^T \end{array} \right),$$  \hspace{1cm} (3)

where $G \equiv [\hat{G}_{mn}]$, $C \equiv [\frac{1}{2} \hat{A}^I_{m} \hat{A}^I_{n} + \hat{B}_{mn}]$ and $a \equiv [\hat{A}^I_{m}]$ are specified in terms of the internal parts of ten-dimensional fields. The effective four-dimensional Lagrangian takes the form $[21,20]$:

$$\mathcal{L} = \sqrt{-g} \left[ \mathcal{R}_g + \frac{1}{2} \partial_{\mu} \varphi \partial^{\mu} \varphi - \frac{1}{2} e^{2\varphi} \partial_{\mu} \Psi \partial^{\mu} \Psi - \frac{1}{4} e^{-\varphi} f^{\mu \nu}_{i j} (LML)_{i j} f^{i j \mu \nu} + \frac{1}{8} \text{Tr}(\partial_{\mu} M L \partial^{\mu} M L) \right],$$  \hspace{1cm} (4)

5$\Psi$ is the axion which is equivalent to the two-form field $B_{\mu \nu}$ through the duality transformation $H^{\mu \nu \rho} = -\frac{e^{2\varphi}}{\sqrt{-g}} e^{\mu \nu \rho} \partial_\sigma \Psi$.

6We set $\alpha' = 2$, the four-dimensional Newton’s constant $G_N = \frac{1}{8} \alpha' = \frac{1}{4}$ and the compactification radii $R_m = \sqrt{\alpha'} = \sqrt{2}$.
where \( g \equiv \det g_{\mu\nu} \), \( R_g \) is the Ricci scalar of \( g_{\mu\nu} \), and \( F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu \) are the \( U(1)^8 \) gauge field strengths. The four-dimensional effective action (4) is invariant under the \( O(6,22) \) transformations (T-duality) \([21,20]\):

\[
M \to \Omega M \Omega^T, \quad A^i_\mu \to \Omega_{ij} A^j_\mu, \quad g_{\mu\nu} \to g_{\mu\nu}, \quad S \to S,
\]

where \( \Omega \) is an \( O(6,22) \) invariant matrix, i.e., with the following property:

\[
\Omega^T L \Omega = L, \quad L = \begin{pmatrix}
0 & I_6 & 0 \\
I_6 & 0 & 0 \\
0 & 0 & I_{16}
\end{pmatrix},
\]

where \( I_n \) denotes the \( n \times n \) identity matrix.

In addition, the corresponding equations of motion and Bianchi identities are invariant under the \( SL(2, R) \) transformations (S-duality) \([21]\):

\[
S \to \frac{aS + b}{cS + d}, \quad M \to M, \quad g_{\mu\nu} \to g_{\mu\nu}, \quad F^i_{\mu\nu} \to (c\Psi + d)F^i_{\mu\nu} + ce^{-2\varphi}(ML)_{ij}\tilde{F}^j_{\mu\nu},
\]

where \( \tilde{F}^i_{\mu\nu} = \frac{1}{2\sqrt{-g}}\varepsilon^{\mu\nu\sigma}F^i_{\sigma} \) and \( a, b, c, d \in R \) satisfy \( ad - bc = 1 \). At the quantum level, the parameters of both \( T- \) and \( S \)-duality transformations become integer-valued, corresponding to the exact symmetry of the perturbative string theory and the conjectured non-perturbative symmetry of string theory, respectively.

**B. Effective Three-Dimensional Action**

The dimensional reduction of the ten-dimensional action (1) on a seven-torus down to three dimensions \([19]\) can be achieved by the following choice of the Kaluza-Klein Ansatz for the metric:

\[
\hat{G}_{MN} = \begin{pmatrix}
\hat{e}^{2\hat{\varphi}}h_{\hat{\mu}\hat{\nu}} + \hat{G}_{\hat{m}\hat{n}}\hat{A}^\hat{m}_{\hat{\mu}}\hat{A}^\hat{n}_{\hat{\nu}} & \hat{A}^\hat{m}_{\hat{\mu}}\hat{G}_{\hat{m}\hat{n}} \\
\hat{A}^\hat{n}_{\hat{\nu}}\hat{G}_{\hat{m}\hat{n}} & \hat{G}_{\hat{m}\hat{n}}
\end{pmatrix},
\]

where \( \tilde{\varphi} \equiv \Phi - \frac{1}{2}\ln \det \hat{G}_{\hat{m}\hat{n}} \) is the three-dimensional dilaton and \( \hat{A}^\hat{m}_{\hat{\mu}} \) are the three-dimensional Kaluza-Klein \( U(1) \) gauge fields. The three-dimensional action for massless (three-dimensional) bosonic fields contains the following fields: the graviton \( h_{\hat{\mu}\hat{\nu}} \), the dilaton \( \varphi \), 30 \( U(1) \) gauge fields \( \hat{A}^\hat{i}_{\hat{\mu}} \equiv (\hat{A}^\hat{m}_{\hat{\mu}}, \hat{A}^{14+\hat{m}}_{\hat{\mu}}, \hat{A}^{14+\hat{i}}_{\hat{\mu}}) \) defined as \( \hat{A}^{14+\hat{m}}_{\hat{\mu}} \equiv \hat{B}_{\hat{\mu}\hat{m}} + \hat{B}_{\hat{m}\hat{\mu}} + \hat{A}^\hat{m}_{\hat{\mu}}\hat{A}^\hat{n}_{\hat{\nu}} + \frac{1}{2}\hat{A}^\hat{m}_{\hat{\mu}}\hat{A}^{14+\hat{i}}_{\hat{\nu}} \) and \( \hat{A}^{14+\hat{i}}_{\hat{\mu}} \equiv \hat{A}^\hat{i}_{\hat{\mu}} - \hat{A}^\hat{i}_{\hat{\nu}}\hat{A}^\hat{m}_{\hat{\mu}} \) with the field strengths \( \tilde{F}^\hat{i}_{\hat{\mu}\hat{\nu}} \equiv \partial_{\hat{\mu}}\hat{A}^\hat{i}_{\hat{\nu}} - \partial_{\hat{\nu}}\hat{A}^\hat{i}_{\hat{\mu}} \), the two-form field \( \tilde{B}_{\hat{\mu}\hat{\nu}} \) \([1]\), and the following symmetric \( O(7,23) \) matrix of scalar fields:

\[
\tilde{M} = \begin{pmatrix}
\hat{G}^{-1} & -\hat{G}^{-1}\hat{C} & -\hat{G}^{-1}\hat{a}^T \\
-\hat{C}^T\hat{G}^{-1} & \hat{G} + \hat{C}^T\hat{G}^{-1}\hat{C} + \hat{a}^T\hat{a} & \hat{C}^T\hat{G}^{-1}\hat{a}^T + \hat{a}^T \\
-\hat{a}\hat{G}^{-1} & \hat{a}\hat{G}^{-1}\hat{C} + \hat{a} & \hat{a}\hat{G}^{-1}\hat{a}^T + \hat{a}^T
\end{pmatrix}
\]

Since in three-dimensions \( \tilde{B}_{\hat{\mu}\hat{\nu}} \) has no physical degrees of freedom, its field strength \( \tilde{H}_{\hat{\mu}\hat{\nu}\hat{\rho}} \) can be set to zero.
defined in terms of internal parts \( \bar{G} \equiv [\hat{G}_{mn}] \), \( \bar{C} \equiv [\frac{1}{2} \hat{A}_{m}^{I} \hat{A}_{n}^{I} + \hat{B}_{mn}] \) and \( \bar{a} \equiv [\hat{A}_{m}^{I}] \) of the ten-dimensional fields.

The resulting three-dimensional action is invariant under the \( O(7, 23) \) transformations:

\[
\bar{M} \rightarrow \bar{\Omega} \bar{M} \bar{\Omega}^{T}, \quad \bar{A}_{\mu}^{I} \rightarrow \bar{\Omega}_{i j} \bar{A}_{\mu}^{j}, \quad \bar{h}_{\mu \nu} \rightarrow \bar{h}_{\mu \nu}, \quad \bar{\hat{B}}_{\mu \nu} \rightarrow \bar{\hat{B}}_{\mu \nu}, \quad \bar{\varphi} \rightarrow \bar{\varphi},
\]

where \( \bar{\Omega} \in O(7, 23) \), \( i.e. \), it has the following property:

\[
\bar{\Omega} \bar{L} \bar{\Omega}^{T} = \bar{L}, \quad \bar{L} = \begin{pmatrix} 0 & I_{7} & 0 \\ I_{7} & 0 & 0 \\ 0 & 0 & I_{16} \end{pmatrix}.
\] (11)

At the quantum level \( O(7, 23) \) becomes integer valued, and is the \( T \)-duality symmetry group of the perturbative heterotic string theory on a seven-torus.

Since in three-dimensions vector fields are dual to scalar fields, one can make the following duality transformations to trade the three-dimensional \( U(1) \) fields \( \bar{A}_{\mu}^{I} \) with a set of scalar fields \( \psi \equiv [\psi^{I}] \) [19]:

\[
\sqrt{-h} e^{-2\bar{\varphi}} h^{\bar{\mu} \bar{\nu}} (\bar{M} \bar{L})_{\bar{\mu} \bar{\nu}} \bar{F}_{\bar{\mu} \bar{\nu}} = \frac{1}{2} \bar{e}^{\bar{\mu} \bar{\nu}} \bar{M} \bar{\varphi} \psi^{\bar{I}}.
\] (12)

Then the three-dimensional action reduces to the following form [19]:

\[
\mathcal{L} = \frac{1}{4} \sqrt{-h} [\mathcal{R} + \frac{1}{8} h^{\bar{\mu} \bar{\nu}} \text{Tr}(\partial_{\bar{\mu}} \bar{M} \bar{L} \partial_{\bar{\nu}} \bar{M} \bar{L})],
\] (13)

where \( h = \text{det} \bar{h}_{\mu \nu} \), \( \mathcal{R} \) is the Ricci scalar of the three-dimensional metric \( h_{\mu \nu} \). \( \mathcal{M} \) is a symmetric \( O(8, 24) \) matrix of three-dimensional scalar fields defined as

\[
\mathcal{M} = \begin{pmatrix}
\bar{M} - \bar{e}^{2\bar{\varphi}} \psi^{I} \psi^{T} & \bar{e}^{2\bar{\varphi}} \psi & \bar{M} \bar{L} \psi - \frac{1}{2} \bar{e}^{2\bar{\varphi}} \psi (\psi^{T} \bar{L} \psi) \\
\bar{e}^{2\bar{\varphi}} \psi^{T} & -\bar{e}^{2\bar{\varphi}} & \frac{1}{4} \bar{e}^{2\bar{\varphi}} \psi^{T} \bar{L} \psi \\
(\psi^{T} \bar{M} - \frac{1}{2} \bar{e}^{2\bar{\varphi}} \psi^{T} (\psi^{T} \bar{L} \psi)) & \bar{L} \bar{M} \psi - \frac{1}{2} \bar{e}^{2\bar{\varphi}} (\psi^{T} \bar{L} \psi)^{2} & \frac{1}{4} \bar{e}^{2\bar{\varphi}} (\psi^{T} \bar{L} \psi)^{2}
\end{pmatrix}.
\] (14)

The action is manifestly invariant under the \( O(8, 24) \) transformations:

\[
\mathcal{M} \rightarrow \bar{\Omega} \mathcal{M} \bar{\Omega}^{T}, \quad h_{\mu \nu} \rightarrow h_{\bar{\mu} \bar{\nu}},
\] (15)

where \( \bar{\Omega} \in O(8, 24) \), \( i.e. \),

\[
\bar{\Omega} \bar{L} \bar{\Omega}^{T} = \bar{L}, \quad \bar{L} = \begin{pmatrix} \bar{L} & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.
\] (16)

Within the effective field theories described above, we shall be interested in four-dimensional static, spherically symmetric configurations. The relationship between such fields of the three-dimensional action (13) and the four-dimensional action (4) are given in the following subsection.
C. Relationship Between the Fields of the Three- and Four-Dimensional Actions

For the static, spherically symmetric configurations the four-dimensional (Einstein frame) space-time metric has the following form:

\[ g_{\mu\nu} dx^\mu dx^\nu = -\lambda(r) dt^2 + \lambda(r)^{-1} dr^2 + R(r)(d\theta^2 + \sin^2 \theta d\phi^2), \]  

and the four-dimensional scalar fields in \( M \) (cf. \( \ref{3} \)), \( i.e., \) moduli, and the dilaton-axion field \( S = \Psi + ie^{-\varphi} \) depend on the radial coordinate \( r \), only.

The spherical symmetry implies that 28 \( \mathcal{A}_i^I \) vector potentials have non-zero \( t \) and \( \phi \) components, whose asymptotic values \( (r \to \infty) \) specify the electric and magnetic charges, respectively. The Maxwell’s equations and Bianchi identities determine the \( U(1) \) field strengths to be

\[ \mathcal{F}_{\theta\phi}^i = L_{ij} \beta_j \sin \theta, \quad \mathcal{F}_{tr}^i = \frac{e^\varphi \lambda(r)}{r^2} M_{ij}(\alpha_j + \Psi_\beta_j), \]  

where \( \vec{\alpha} \) and \( \vec{\beta} \) correspond to the conserved (quantized) 28 electric and 28 magnetic charge vectors, and \( L \) is defined in \( \ref{3} \).

The physical magnetic and electric charges

\[ \vec{P} \equiv (P_1^{(1)}; P_2^{(1)}; P_3^{(3)}), \quad \vec{Q} \equiv (Q_1^{(1)}; Q_2^{(1)}; Q_3^{(3)}), \]  

are related to the conserved (quantized) charge vectors \( \vec{\alpha} \) and \( \vec{\beta} \) in the following way:

\[ P_i = L_{ij} \beta_j, \quad Q_i = e^{\varphi \infty} M_{ij \infty}(\alpha_j + \Psi_\infty \beta_j), \]  

where the subscript \( \infty \) refers to the asymptotic \( (r \to \infty) \) values of the corresponding fields.

The corresponding three-dimensional space metric takes the form:

\[ h_{\mu\nu} dx^\mu dx^\nu = dr^2 + R(r)(d\theta^2 + \sin^2 \theta d\phi^2), \]  

where the four-dimensional metric components are related to the three-dimensional ones as \( \tilde{R} = \lambda R \). The three-dimensional scalar fields in \( \mathcal{M} \) (cf. \( \ref{14} \)) depend on the radial coordinate \( r \), only.

For static, spherically symmetric configuration described above, the four-dimensional fields are related to the three-dimensional fields in the following way:

\[
\begin{align*}
G_{mn} &= \tilde{G}_{1+1,1+n}, & B_{mn} &= \tilde{B}_{1+1,1+n}, & a_{m}^I &= \tilde{a}_{1+m}^I, \\
\varphi &= \varphi + \frac{1}{2} \ln \det \tilde{G}_{m\bar{n}} - \ln \det G_{mn}, & \lambda &= e^{-\varphi}[\det \tilde{G}_{m\bar{n}}/\det G_{mn}]^{\frac{1}{2}}, & R &= \tilde{R}/\lambda, \\
A_{t}^{(1)} &= G^{mn} \tilde{A}_{1+1,n}, & A_{T}^{(1)} &= G^{mn} \tilde{A}_{m,1+n}, & A_{T}^{(3)} &= \tilde{A}_{1+m} - a_{m}^I \tilde{A}_{1+1,m}, \\
A_{t}^{(3)} &= \tilde{A}_{1+t} + a_{m}^I \tilde{A}_{m,1+n} - a_{m}^I \tilde{A}_{1+1,m} & A_{t}^{(3)} &= \tilde{B}_{1+1,n} + B_{mn} \tilde{A}_{m,1+n} + \frac{1}{2} a_{m}^I A_{t}^{(3)} I, \\
A_{\phi}^{(2)} &= \tilde{A}_{\phi}^{(8)} + \tilde{B}_{1+1,n} \tilde{A}_{\phi}^{(8)} + \frac{1}{2} a_{m}^I \tilde{A}_{1+t}^{(14+I)} + B_{mn} \tilde{A}_{\phi}^{(1)} + \frac{1}{2} a_{m}^I A_{\phi}^{(3)} I, \\
B_{\phi t} &= \tilde{B}_{\phi t} + \tilde{B}_{1+1,n} \tilde{A}_{\phi t} - \frac{1}{2} a_{m}^I \tilde{A}_{1+t}^{(14+I)} + \frac{1}{2} (A_{\phi}^{(1)} a_{m}^I A_{m,1+n} - a_{m}^I \tilde{B}_{1+1,n} - A_{\phi}^{(1)} B_{mn} \tilde{A}_{m,1+n}).
\end{align*}
\]
In this chapter we first spell out the solution generating technique, and the explicit sequence of symmetry transformations imposed on the Schwarzschild solution. Then we give the explicit form of the generating solution. We further spell out the $T$- and $S$-duality transformations on the generating solution, thus yielding the most general solution specified by 28 electric and 28 magnetic charges and the ADM mass. Finally, we compare the special cases of this general solution with existing examples in the literature.

A. Solution Generating Technique

An arbitrary asymptotic value of the scalar field matrix $\mathcal{M}$ can be transformed into the form

$$\mathcal{M}_\infty = \text{diag}(-1, 1, \ldots, 1, -1, \ldots, 1, -1, -1) \equiv I_{4,28}$$

by imposing an $O(8,24)$ transformation, i.e., $\mathcal{M}_\infty \to \Omega \mathcal{M}_\infty \Omega^T = I_{4,28}$ ($\Omega \in O(8,24)$). Thus, without loss of generality we can confine the analysis by choosing the asymptotic value of the form $\mathcal{M}_\infty = I_{4,28}$ and, then, obtain the solutions with arbitrary value of $\mathcal{M}_\infty$ by undoing the above constant $O(8,24)$ transformation. Then, the subset of $O(8,24)$ transformations that preserve this asymptotic value for $\mathcal{M}$ is $SO(8) \times SO(24)$.

The starting point of generating the static, spherically symmetric solutions with the most general charge configurations is the following three-dimensional form of the Schwarzschild black hole solution:

$$\mathcal{M} = \text{diag}( -\frac{r}{r-m}, 1, \ldots, 1, -\frac{r}{r-m}, 1, \ldots, 1, -\frac{r}{r-m}, -\frac{r}{r-m}) ,$$

and the three dimensional space metric (21) is specified by $\bar{R} = r(r-m)$. Here, $m$ is the ADM mass of the Schwarzschild black hole.

The subset of $O(8,24)$ transformations that generate new solutions from (24) is the quotient space $[O(22,2) \times O(6,2)]/[O(22) \times O(6) \times SO(2)]$, where $SO(2)$ is the subset of $SL(2,R)$ transformation in four-dimensions that preserves the asymptotic value $S_\infty = i$ as discussed in the previous footnote. The quotient space is parameterized by 57 parameters.

Similarly, the arbitrary asymptotic values of the four-dimensional moduli matrix $M$ and the dilaton-axion $S$ can be brought into the form $M_\infty = I_{28}$ and $S_\infty = i$ by using constant $O(6,22)$ and $SL(2,R)$ transformations, respectively. The subsets of four-dimensional symmetry transformations which preserve these asymptotic values are $SO(6) \times SO(22)$ and $SO(2)$, respectively.

To obtain the most general rotating charged black hole solution one imposes the same subsets of $O(8,24)$ transformations on the Kerr solution. For the sake of simplicity we confined ourselves to the static solutions, only.
One has to notice that in imposing a subset of $O(8, 24)$ transformations on the Schwarzschild solution, in general, one induces the unphysical Taub-NUT charge. To remove unphysical Taub-NUT charge one has to impose one constraint among the parameters of solution, in general, one induces the unphysical Taub-NUT charge. Therefore, these subsets of $O(8, 24)$ transformations, subject to zero Taub-NUT charge constraint, introduce 56 new charge degrees of freedom, which correspond to unconstrained 28 magnetic and 28 electric charges. Those are the necessary charge degrees of freedom specifying the most general static black hole solutions of heterotic string on a six-torus, compatible with the conjectured no-hair theorem.

We shall proceed with the following sequence of the symmetry transformations. In order to obtain the explicit form of the generating solution we shall apply six necessary $SO(1, 1) \subset O(8, 24)$ boost transformations on the Schwarzschild solution, which would yield, after imposing the zero Taub-NUT constraint, the generating solution specified by five parameter charge configurations. We then impose on the generating solution the $[SO(6) \times SO(22)]/[SO(4) \times SO(20)]$ and $SO(2)$ transformations, i.e., the subsets of $T$- and $S$-duality transformations, which do not affect the four-dimensional space-time. Thus, one in turn obtains the most general configuration specified by the mass and unconstrained 28 electric and 28 magnetic charges of $U(1)^{28}$ gauge group.

B. Explicit Form of the Generating Solution

For the purpose of obtaining the explicit form of the most general solution, it is convenient to first impose four successive $SO(1, 1)$ boosts $\Omega_{p1,p2,q1,q2}$ on the Schwarzschild solution (14) with boost parameters $\delta_{p1}, \delta_{p2}, \delta_{q1}, \delta_{q2}$, which generate non-extreme $U(1)_{1,M}^{(1)} \times U(1)_{2,E}^{(1)} \times U(1)_{M}^{(2)} \times U(1)_{E}^{(2)}$ solution (10). The boost transformation $\Omega_{p1}$ has the following form:

$$\Omega_{p1} \equiv \left( \begin{array}{ccccccc} 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cosh\delta_{p1} & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & I_6 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cosh\delta_{p1} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & I_{21} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cosh\delta_{p1} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{array} \right), \quad (25)$$

where the dots denote the corresponding zero entries and $\Omega_{p2}$ has the analogous form with the non-trivial entries (with positive signs in front of sinh’s) in the $\{9th, 32nd\}$ and (with negative signs in front of sinh’s) in the $\{2nd, 31st\}$, columns and rows. $\Omega_{q1}, [\Omega_{q2}]$ has the non-trivial entries (with positive signs in front of sinh’s) in the $\{8th, 10th\}$ $\{\{1st, 10th\}\}$ and (with negative signs in front of sinh’s) in the $\{1st, 3rd\}, \{3rd, 8th\}$ columns and rows.

This solution has among 28 magnetic $\vec{P}$ and 28 electric $\vec{Q}$ charges (cf. (13)) only four non-zero charges of the Kaluza-Klein and two-form gauge fields, which are associated with the first two compactified directions, only. The two magnetic and the two electric charges arise from different compactified directions. Namely:

$$P_{1}^{(1)} = m\sinh\delta_{p1}\cosh\delta_{p1} \equiv P_{1}, \quad Q_{1}^{(1)} = 0,$$
$$P_{2}^{(1)} = 0, \quad Q_{2}^{(1)} = m\cosh\delta_{q1}\sinh\delta_{q1} \equiv Q_{1},$$
\[ P_1^{(2)} = m \sinh \delta_{p2} \cosh \delta_{p2} \equiv P_2, \quad Q_1^{(2)} = 0, \]
\[ P_2^{(2)} = 0, \quad Q_2^{(2)} = m \cosh \delta_{q2} \sinh \delta_{q2} \equiv Q_2, \]

and the ADM mass given by
\[ M_{\text{ADM}} = \hat{P}_1 + \hat{P}_2 + \hat{Q}_1 + \hat{Q}_2, \]

where the hated quantities are defined as \( \hat{P}_1 \equiv \pm \sqrt{(P_1)^2 + \beta^2} \), etc. The signs \( \pm \) in the definitions of the hated quantities are determined \( \footnote{There are also hybrid solutions with the opposite relative signs for one type of charges, say, magnetic ones with \( |P_1| > |P_2| \), and the same relative signs for the other type of charges, i.e., electric ones. These solutions are singular, however, the non-extreme solutions satisfy \( M_{\text{ADM}} \geq M_{\text{BPS}} \) and, therefore, are in the spectrum, provided \( \sqrt{(P_1)^2 + \beta^2} \left( 1/\sqrt{(P_2)^2 + \beta^2} + 1/\sqrt{(Q_1)^2 + \beta^2} + 1/\sqrt{(Q_2)^2 + \beta^2} \right) \geq 1 \). This constraint can be satisfied only for a restricted range of \( \beta \) and charge parameters.} \] by the requirement that the above ADM mass formula approaches the Bogomol’nyi bound:
\[ M_{\text{BPS}} = |P_1 + P_2| + |Q_1 + Q_2|, \]

as \( \beta \to 0 \). Here \( \beta \equiv m/2 > 0 \) is the non-extremality parameter that measures a deviation from the supersymmetric limit. In the extreme (supersymmetric) limit (\( \beta \to 0 \)), one takes \( \delta_{p1,p2,q1,q2} \to \infty \) in a way that \( \beta e^{2\delta_{p1,p2,q1,q2}} \) become finite parameters, in order for charges \( \hat{P}_{1,2}, \hat{Q}_{1,2} \) to remain non-zero. Note, that when the relative signs of the \( P_{1,2} \) and \( Q_{1,2} \) parameters are opposite the corresponding non-extreme solutions are ultra-extreme \( \footnote{There are also hybrid solutions with the opposite relative signs for one type of charges, say, magnetic ones with \( |P_1| > |P_2| \), and the same relative signs for the other type of charges, i.e., electric ones. These solutions are singular, however, the non-extreme solutions satisfy \( M_{\text{ADM}} \geq M_{\text{BPS}} \) and, therefore, are in the spectrum, provided \( \sqrt{(P_1)^2 + \beta^2} \left( 1/\sqrt{(P_2)^2 + \beta^2} + 1/\sqrt{(Q_1)^2 + \beta^2} + 1/\sqrt{(Q_2)^2 + \beta^2} \right) \geq 1 \). This constraint can be satisfied only for a restricted range of \( \beta \) and charge parameters.} \], \( i.e., \) their ADM masses are not compatible with the Bogomol’nyi bound (their ADM masses are smaller than the BPS mass), and thus are not in the spectrum of states.

We shall concentrate on non-extreme (regular) solutions, \( i.e., \) those which reduce to the BPS solutions with the same relative signs for \( P_{1,2} \) and \( Q_{1,2} \) in the supersymmetric limit, and consequently \( \hat{P}_1 = +\sqrt{(P_1)^2 + \beta^2} \), etc. \( \footnote{There are also hybrid solutions with the opposite relative signs for one type of charges, say, magnetic ones with \( |P_1| > |P_2| \), and the same relative signs for the other type of charges, i.e., electric ones. These solutions are singular, however, the non-extreme solutions satisfy \( M_{\text{ADM}} \geq M_{\text{BPS}} \) and, therefore, are in the spectrum, provided \( \sqrt{(P_1)^2 + \beta^2} \left( 1/\sqrt{(P_2)^2 + \beta^2} + 1/\sqrt{(Q_1)^2 + \beta^2} + 1/\sqrt{(Q_2)^2 + \beta^2} \right) \geq 1 \). This constraint can be satisfied only for a restricted range of \( \beta \) and charge parameters.} \)

The explicit form of the above solutions was given in Ref. \( \footnote{There are also hybrid solutions with the opposite relative signs for one type of charges, say, magnetic ones with \( |P_1| > |P_2| \), and the same relative signs for the other type of charges, i.e., electric ones. These solutions are singular, however, the non-extreme solutions satisfy \( M_{\text{ADM}} \geq M_{\text{BPS}} \) and, therefore, are in the spectrum, provided \( \sqrt{(P_1)^2 + \beta^2} \left( 1/\sqrt{(P_2)^2 + \beta^2} + 1/\sqrt{(Q_1)^2 + \beta^2} + 1/\sqrt{(Q_2)^2 + \beta^2} \right) \geq 1 \). This constraint can be satisfied only for a restricted range of \( \beta \) and charge parameters.} \) by directly solving the Euler-Lagrange equations. Here, the same solution is obtained by the solution generating technique specified by the four boost parameters \( \delta_{p1,p2,q1,q2} \) and \( m \), or equivalently by four charge parameters \( P_1, P_2, Q_1, Q_2 \) and the non-extremality parameter \( \beta \). This solution now provides an intermediate step on the way to obtain the generating solution with the charge configuration specified by five parameters and the non-extremality parameter.

The missing one more charge degree of freedom can be obtained by performing on the above solution two more \( SO(1,1) \subset O(8,24) \) transformations, parameterized by two boost parameters \( \delta_{1,2} \). The corresponding \( \Omega_1 \left[ \Omega_2 \right] \) has the nontrivial entries (with positive signs in sinh’s) in the \( \{1st, 2nd\} \) \( \{\{3rd, 31st\} \) and (with negative signs in sinh’s) in the \( \{8th, 9th\} \) \( \{\{10th, 32nd\} \) columns and rows.

The above two boosts do introduce two new charges, \( i.e., \) \( P_2^{(2)} \) and \( Q_1^{(1)} \), as well as the
(unphysical) Taub-NUT charge \[\text{[72]}\]. The Taub-NUT charge can be eliminated by imposing the following constraint on the boost parameters \(\delta_{1,2}\):

\[
P_1 \tanh \delta_1 - Q_2 \tan \delta_2 = 0. \tag{29}\]

Without loss of generality, we assume that \(Q_2 \geq P_1\) and then \(\delta_2\) is expressed in terms of \(\delta_1\) as \[\text{[72]}\]

\[
cosh \delta_2 = Q_2 \cosh \delta_1 / \Delta, \quad \sinh \delta_2 = P_1 \sinh \delta_1 / \Delta,
\]

where \(\Delta \equiv \text{sign}(Q_2) \sqrt{(Q_2)^2 \cosh^2 \delta_1 - (P_1)^2 \sinh^2 \delta_1}\).

The final form of the generating solution (with zero Taub-NUT charge), expressed in terms of the four-dimensional fields, as specified in Section IIa and related to the fields of the three-dimensional action through (21) and (22), can be written in the following form \[\text{[72]}\]:

\[
\lambda = \frac{(r + \beta)(r - \beta)}{(XY - Z^2)^{1/2}}, \quad R = (XY - Z^2)^{1/2}, \quad e^{2\varphi} = \frac{W^2}{XY - Z^2},
\]

\[
\partial_r \Psi = \frac{1}{\Delta^3 W} \left[ \Delta^2 (P_1Q_1 + P_2Q_2) + P_1Q_2(P_1)^2(r + \hat{Q}_2) - (Q_2)^2(r + \hat{P}_1) \right]
\]

\[
\times \left[ \frac{P_1P_2(r - \hat{Q}_1)\sinh^2 \delta_1 + Q_1Q_2(r + \hat{P}_2)\cosh^2 \delta_1}{XY - Z^2} \right] \sinh \delta_1 \cosh \delta_1,
\]

\[
G_{11} = \frac{X}{(r + \hat{P}_1)(r + \hat{Q}_2)}, \quad G_{22} = \frac{Y}{(r + \hat{P}_1)(r + \hat{Q}_2)}, \quad G_{12} = -\frac{Z}{(r + \hat{P}_1)(r + \hat{Q}_2)},
\]

\[
B_{12} = -\frac{\Delta (r + \hat{P}_1)(r + \hat{Q}_2)}{\Delta (r + \hat{P}_1)(r + \hat{Q}_2)},
\]

\[
G_{ij} = \delta_{ij}, \quad B_{ij} = 0, \quad (i, j \neq 1, 2), \quad a_m^l = 0 \quad \tag{31}\]

with

\[\text{[72]}\]

\[\text{[72]}\]

Note that without loss of generality, one could have chosen other sets of two boosts, which would induce other pairs of U(1) charges, i.e., \((P_2^{(1)}, Q_1^{(2)})\), \((P_2^{(1)}, P_2^{(2)})\), etc. All such solutions are related to the above generating solution through \(SO(2) \times SO(2) \subset O(2, 2)\) and \(SO(2) \subset SL(2, R)\) transformations, i.e., subsets of the two-torus T-duality and S-duality transformations, which do not affect the four-dimensional space-time.

For the case \(Q_2 \leq P_1\), the role of the boost parameters \(\delta_1\) and \(\delta_2\) are interchanged.

The generating solution which saturates the corresponding Bogomol’nyi bound (BPS-saturated solution), i.e., the case with \(\beta = 0\) and \(\delta_1\) finite, was also found in Ref. \[\text{[72]}\] as (an exact string) solution of the conformal invariance constraints for the \(\sigma\)-model which corresponds to the chiral-null model with the non-trivial four-dimensional transverse part. There, the generating solution was parameterized by (five) non-zero charge parameters \(P_1^{(1,2)} \equiv P_{1,2}, Q_2^{(1,2)} \equiv Q_{1,2}\) and \(Q_1^{(1)} = -Q_2^{(2)} \equiv q\). These two (BPS-saturated) generating solutions are related to each another by subsets of \(SO(2) \times SO(2) \subset O(2, 2)\) (two-torus T-duality) and \(SO(2) \subset SL(2, R)\) (S-duality) transformations.
\[
X = r^2 + [(\dot{P}_1 + \dot{Q}_2)\cosh^2 \delta_1 + (\dot{Q}_1 - \dot{P}_1)\sinh^2 \delta_1]r + (\dot{P}_1 \dot{Q}_1\sinh^2 \delta_1 + \dot{Q}_1 \dot{P}_2\cosh^2 \delta_1), \\
Y = r^2 + \frac{1}{\Delta}[(P_1)^2(\dot{P}_2 - \dot{Q}_2)\sinh^2 \delta_1 + (Q_2)^2(\dot{P}_1 + \dot{Q}_1)\cosh^2 \delta_1]r \\
+ \frac{1}{\Delta}[(P_1)^2 \dot{Q}_2 \dot{P}_2 \sinh^2 \delta_1 + (Q_2)^2 \dot{Q}_1 \dot{P}_2 \cosh^2 \delta_1], \\
Z = \frac{1}{\Delta}[(P_1 P_2 + Q_1 Q_2) r + (\dot{P}_1 Q_2 + \dot{Q}_2 P_1)\cosh \delta_1 \sinh \delta_1], \\
W = r^2 + \frac{1}{\Delta}[(Q_2)^2(\dot{P}_1 + \dot{Q}_2)\cosh^2 \delta_1 + (P_1)^2(\dot{Q}_1 - \dot{Q}_2)\sinh^2 \delta_1]r \\
+ \frac{1}{\Delta}[(Q_2)^2 \dot{Q}_1 \dot{Q}_2 \cosh^2 \delta_1 + (P_1)^2 \dot{Q}_1 \dot{Q}_2 \sinh^2 \delta_1], \\
\] (32)

where again \( \Delta \equiv \text{sign}(Q_2)\sqrt{(Q_2)^2\cosh^2 \delta_1 - (P_1)^2\sinh^2 \delta_1}. \)

For the sake of simplicity of the above expressions, the radial coordinate \( r \) is chosen so that the outer horizon is at \( r = \beta \). This solution has the following non-zero charges:

\[
P_1^{(1)} = P_1 Q_2 / \Delta, \quad Q_1^{(1)} = (\dot{P}_1 - \dot{P}_2 - \dot{Q}_1 + \dot{Q}_2)\cosh \delta_1 \sinh \delta_1, \\
P_1^{(2)} = 0, \quad Q_2^{(1)} = (Q_1 Q_2 \cosh^2 \delta_1 + P_1 P_2 \sinh^2 \delta_1) / \Delta, \\
P_2^{(1)} = (Q_2 P_2 \cosh^2 \delta_1 + Q_1 P_1 \sinh^2 \delta_1) / \Delta, \quad Q_1^{(2)} = 0, \\
P_2^{(2)} = P_1 Q_2 (Q_2 - Q_1 - P_1 - P_2) \sinh \delta_1 \cosh \delta_1 / \Delta^2, \quad Q_2^{(2)} = \Delta, \\
\] (33)

and the ADM mass formula, compatible with the BPS bound [328], is given by:

\[
M_{\text{ADM}} = \frac{1}{\Delta}[(P_1)^2(\dot{P}_2 - \dot{Q}_2)\sinh^2 \delta_1 + (Q_2)^2(\dot{P}_1 + \dot{Q}_1)\cosh^2 \delta_1] \\
+ (P_2 + Q_2)\cosh^2 \delta_1 + (\dot{Q}_1 - \dot{P}_1)\sinh^2 \delta_1. \\
\] (34)

Note that the solution is specified by three electric and three magnetic charges (subject to zero Taub-Nut constraint) associated with the \( U(1)^{(1)}_1 \times U(1)^{(2)}_1 \times U(1)^{(1)}_2 \times U(1)^{(2)}_2 \) Kaluza-Klein and the two-form fields (the superscripts 1 and 2, respectively) of the first and the second compactified dimension (the subscripts 1 and 2, respectively). The solution is therefore parameterized by five parameters \( P_1, P_2, Q_1, Q_2 \) and \( \delta_1 \) as well as the non-extremality parameter \( \beta \). Alternatively, the configuration is specified by six charges (subject to one constraint) and the mass, compatible with the Bogomol’nyi bound. At this point, it seems to be difficult to express the generating solution in terms of physical parameters, such as physical charges and the ADM mass.

Among the scalar fields (cf. (31)), both the dilaton \( \varphi \) and the axion \( \Psi \) of the complex scalar field \( S \) vary with \( r \), and only the moduli of the two-torus \( (G_{11}, G_{22}, G_{12}, B_{12}) \) in the moduli \( M \) (cf. (3)) vary with \( r \). Note that in the limit \( \delta_1 \to 0 \) the solutions (33) reduce to the generating solution with four charge parameters (26), only. In this case, the axion and the off-diagonal toroidal moduli \( G_{12} \) and \( B_{12} \) are turned off.

C. S- and T-Duality Transformations

The additional 51 charge degrees of freedom necessary in parameterizing the most general static, spherically symmetric solutions are obtained through the subset of \( O(6, 22) \) (\( T \)-duality) and \( SL(2, R) \) (\( S \)-duality) transformations as stated before. The expressions for the charges after the transformations are given by:

\[
\tilde{Q}' = \frac{1}{\sqrt{2}} U'^T \begin{pmatrix} U_6 (e_u - e_d) \\ U_{22} (e_u + e_d) \\ 0_{16} \end{pmatrix}, \quad \tilde{P}' = \frac{1}{\sqrt{2}} U'^T \begin{pmatrix} U_6 (m_u - m_d) \\ U_{22} (m_u + m_d) \\ 0_{16} \end{pmatrix}, \\
\] (35)
$e_T^u \equiv (Q_1^{(1)} \cos \gamma + P_1^{(2)} \sin \gamma, Q_2^{(1)} \cos \gamma + P_2^{(2)} \sin \gamma, 0, ..., 0)$, $e_T^d \equiv (P_1^{(1)} \sin \gamma, Q_2^{(2)} \cos \gamma, 0, ..., 0)$,
$m_T^u \equiv (P_1^{(1)} \cos \gamma, Q_2^{(2)} \sin \gamma, 0, ..., 0)$, $m_T^d \equiv (P_1^{(2)} \cos \gamma + Q_1^{(1)} \sin \gamma, P_2^{(2)} \cos \gamma + Q_2^{(1)} \sin \gamma, 0, ..., 0)$, \(\gamma\) is the \(SO(2) \subset SL(2, R)\) rotational angle, \(U_6 \in SO(6), U_{22} \in SO(22), 0_{16}\) is a \((16 \times 1)\)-matrix with zero entries, and \(U \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} I_6 & -I_6 & 0 \\ I_6 & I_6 & 0 \\ 0 & 0 & I_{16} \end{pmatrix}\)

where \(\Psi, \epsilon^{-\varphi}\) and \(M\) are the axion, the dilaton and the moduli field of the generating solutions \((31)\). Then, the most general “string frame” metric \(g_{\mu\nu}^{\text{string}}\) is obtained through conformal transformation of the four-dimensional metric given by \(g_{\mu\nu}^{\text{string}} = g_{\mu\nu}/\Im(S)\), where \(g_{\mu\nu} = \text{diag}(-\lambda(r), \lambda(r)^{-1}, R(r), \sin^2 \theta R(r))\) is the “Einstein frame” metric for the generating solutions given in \((31)\).

Since these transformations do not affect the four-dimensional “Einstein-frame” spacetime metric, global space-time and thermal properties of the general solution are parameterized in terms of the six parameters of the generating solution. The study of the global space-time of such solutions is the topic of the next chapter.

In principle, one should be able to write the general solution, specified by 28 electric and 28 magnetic charges and the ADM mass, in terms of \(T\)- and \(S\)-duality invariant quantities. We shall not attempt to do that in this paper. \(^{14}\)

### D. Special Examples of the General Solution

The above general solution, \(i.e.,\) the generating solution accompanied with \(S\)- and \(T\)-duality transformations, contains as a proper subset the previously known spherically symmetric black holes in the heterotic string theory. Examples, known in the literature, correspond to the cases when \(\delta_1 = 0\) (Cf., Ref. \([10]\)), and special choices of the charges \(P_{1,2}\) and \(Q_{1,2}\). We conclude this section by illustrating these special cases.

- Spherically symmetric black hole solutions in heterotic string theory with different dilaton-gauge coupling \(a\) \([23, 25]\):
  \(P_1 = P_2 = Q_1 = Q_2 \neq 0\) case reduces to the Reissner-Nordström black hole solution, \(i.e., a = 0\); when three charges are non-zero and equal, the solution is that of \(a = 1/\sqrt{3}\) case \([26]\); when only two magnetic charges [or only two electric charges] are non-zero

\(^{14}\)For the BPS-saturated solutions, the ADM mass \([8, 26, 22]\) as well as the area of the horizon (for the regular solutions) \([22]\) was cast in the manifestly \(T\)- and \(S\)-duality invariant form.
and equal, the solution is that of \( a = 1 \) case \([3]\); when only one charge is non-zero, the solution reduces to that of \( a = \sqrt{3} \) case, which contains in the extreme limit \((\beta \to 0)\) as subsets \((i)\) \( P_1 \neq 0 \) case: the Kaluza-Klein monopole solution of Gross and Perry, and Sorkin \([27]\), and \((ii)\) \( P_2 \neq 0 \) case: H-monopole solution \([4]\).

- \( P_1 = P_2 \) and \( Q_1 = Q_2 \) solutions with imposed subsets of \( S \)- and \( T \)-duality transformations correspond to a general class of axion-dilaton black hole solutions found by Kallosh and Ortin \([3]\).

- The solutions with \( Q_1 \) and \( Q_2 \) non-zero, when supplemented by \( S \)- and \( T \)-duality transformations, correspond to a general class of electrically charged black holes in heterotic string constructed by Sen \([7]\). The \( S \)-duality counterpart of such general class of solutions is the purely magnetically charged solutions found by Behrndt and Kallosh \([28]\).

- The non-supersymmetric extreme solutions, i.e., those with \( \beta \to 0 \), \( P_2 = Q_1 = 0 \), \( |Q_2| - |P_1| \to 0 \) and \( \delta_1 \to \infty \), while keeping \( \beta e^{2|\delta_1|} \) and \( (|Q_2| - |P_1|) e^{2|\delta_1|} \) as finite parameters, reduce to the extreme dyonic five-dimensional Kaluza-Klein black hole studied in Ref. \([29]\), after \( S \)- and \( T \)-duality transformations are imposed.

IV. GLOBAL SPACE-TIME STRUCTURE AND THERMAL PROPERTIES OF THE SOLUTIONS

We shall now classify all the possible global space-time and thermal properties of the obtained solutions. Since the subsets of \( O(6, 22) \) and \( SL(2, R) \) transformations leave the four-dimensional (Einstein frame) space-time metric invariant, it is sufficient to consider, for that purpose, the generating solution specified by (31). Therefore, all the possible space-time properties of the solutions are determined by six parameters \( P_{1,2}, Q_{1,2}, \delta_1 \) and the non-extremality parameter \( \beta \). In the following we first discuss the global space-time properties, and then the thermal properties of the solutions. We separate the solutions into non-extreme, i.e., when \( \beta > 0 \), and extreme ones, i.e., when \( \beta = 0 \). Within each class we then analyze their properties according to the range of five parameters \( P_{1,2}, Q_{1,2} \) and \( \delta_1 \).

\[15^{1}\text{In the following analysis it is understood that } Q_2 \neq 0, \text{ always when } \delta_1 \neq 0. \text{ In the case when } Q_2 = 0, \text{ } P_1 \text{ has to be zero automatically due to our initial assumption that } |Q_2| \geq |P_1|. \text{ Then, the zero Taub-Nut constraint (24) does not restrict the values of } \delta_{1,2}. \text{ In this case, we have non-extreme four-parameter solution with non-zero charges given by } P_1^{(2)}, P_2^{(2)}, Q_1^{(1)} \text{ and } Q_2^{(1)}, \text{ equivalently parameterized by } Q_1, P_2, \text{ and } \delta_{1,2}. \text{ Such solutions can be related to the generating solutions through the subsets of } SO(2) \times SO(2) \subset O(2, 2) \subset O(6, 22) \text{ and } SO(2) \subset SL(2, R) \text{ transformations.}\]
A. Global Space-Time Structure

The singularity structure can be explored by studying the explicit form (31) of the four-dimensional metric (17). The Ricci scalar curvature blows up at the point \((r = r_{\text{sing}})\) where \(R = 0\), thus corresponding to the space-time singularity. The horizon(s) form at the point(s) \((r = r_{\text{hor}\pm})\) where \(\lambda = 0\), provided \(r_{\text{hor}\pm} > r_{\text{sing}}\). We shall now explore the global space-time properties for the two classes of solutions.

\(\text{(i) Non-extreme solutions (}\beta > 0)\)

Recall that we shall consider only the case where both pairs \((P_1, P_2)\) and \((Q_1, Q_2)\) have the same relative signs (and consequently \(\hat{P}_1 = +\sqrt{P_1^2 + \beta^2}\), etc. with plus signs in the hated quantities), since only for this case the ADM mass is compatible with the Bogomol’nyi bound for any values of \(\beta\).\(^{16}\) By analyzing the roots of \(XY - Z^2\) (cf. (31), (32)), one can see that the singularity always takes place at \(r_{\text{sing}} \leq -\beta\). On the other hand the zero(s) of \(\lambda\), corresponding in principle to the outer and/or inner horizons, are at \(r_{\text{hor}+} = \beta\) and/or \(r_{\text{hor}−} = -\beta\), respectively. Thus, the global space time of non-extreme solutions is that of non-extreme Reissner-Nordström black holes when \(r_{\text{sing}} < -\beta\), and that of Schwarzschild black hole when \(r_{\text{sing}} = -\beta\).

The single root of \(XY - Z^2\) at \(r_{\text{sing}} = -\beta\) takes place, in which case the singularity and the inner horizon coincide at \(r = -\beta\), when \(a\) \(\delta_1 \neq 0\) and \(P_1 = 0\), or \(b\) \(\delta_1 = 0\) and least one of \(P_{1,2}\) and \(Q_{1,2}\) is zero. On the other hand, the double root of \(XY - Z^2\) at \(r_{\text{sing}} = -\beta\) takes place, in which case the inner horizon disappears and the singularity forms at \(r = -\beta\), when \(a\) \(\delta_1 \neq 0\) and only \(Q_2\) is non-zero, or \(b\) \(\delta_1 = 0\) and at least two of \(P_{1,2}, Q_{1,2}\) are zero.

\(\text{(ii) Extreme solutions (}\beta \rightarrow 0)\)

When the boost parameter \(\delta_1\) is finite, these solutions saturate the Bogomol’nyi bound, \(\text{i.e.},\) they correspond to supersymmetric extreme solutions.

When both pairs of \((P_1, P_2)\) and \((Q_1, Q_2)\) have the same relative signs, the singularity always takes place at \(r_{\text{sing}} \leq 0\). The inner and outer horizons coincide at \(r_{\text{hor}+} = r_{\text{hor}−} = 0\). Global space-time of such supersymmetric extreme solutions is therefore that of the extreme Reissner-Nordström black holes when \(r_{\text{sing}} < 0\), or the singularity and the horizon coincide when \(r_{\text{sing}} = r_{\text{hor}±} = 0\). The latter case happens when at least one out of \(P_{1,2}, Q_1\) (and \(Q_2\)) parameters is zero with \(\delta_1 \neq 0\) (with \(\delta_1 = 0\)). The horizon at \(r_{\text{hor}} = 0\) disappears when only \(Q_2\) is non-zero with \(\delta_1 \neq 0\) (or when only one out of \(P_{1,2}, Q_{1,2}\) parameters is non-zero with \(\delta_1 = 0\)).

\(^{16}\)In the case when only one of the pairs has opposite sign, the ADM mass is compatible with the Bogomol’nyi bound for the restricted range of \(\beta\),\(^{16}\) only. (See also the footnote 9.) When both of the pairs have the opposite signs, the ADM mass is always less than the ADM mass of the corresponding supersymmetric solution for any \(\beta > 0\).
In the case of at least one of the pairs \((P_1, P_2)\) and \((Q_1, Q_2)\) having the opposite relative sign, the singularity at \(r_{\text{sing}} > 0\) is naked \(^{30,31}\).

There also exist non-supersymmetric extreme solutions, i.e., those with \(\beta \to 0\) but without saturating the Bogomol’nyi bound. To obtain such solutions, one sets \(P_2 = Q_1 = 0\), and takes the limits \(\beta \to 0\), \(|Q_2| - |P_1| \to 0\) and \(\delta_1 \to \infty\), while keeping \(\beta e^{2|\delta_1|}\) and \((|Q_2| - |P_1|) e^{2|\delta_1|}\) as finite parameters. In this case the time-like singularity is at \(r_{\text{sing}} < r_{\text{hor}} = 0\), i.e., the global space-time is that of the extreme Reissner-Nordström black holes.

\[S = \pi \Delta \left[ \left( \hat{Q}_2 + \beta \right) (\hat{P}_2 + \beta) \cosh^2 \delta_1 + (\hat{P}_1 + \beta)(\hat{Q}_1 - \beta) \sinh^2 \delta_1 \right] \times \left[ (Q_2)^2 (\hat{Q}_1 + \beta)(\hat{P}_1 + \beta) \cosh^2 \delta_1 + (P_1)^2 (\hat{Q}_2 + \beta)(\hat{P}_2 - \beta) \sinh^2 \delta_1 \right] - \left[ P_1 P_2 \hat{Q}_2 + Q_1 Q_2 (\hat{P}_1 + \beta) \right]^2 \cosh^2 \delta_1 \sinh^2 \delta_1 \right]^{1/2}, \tag{38}\]

where again \(\Delta \equiv \text{sign}(Q_2) \sqrt{(Q_2)^2 \cosh^2 \delta_1 - (P_1)^2 \sinh^2 \delta_1}\) and \(\hat{P}_1 = +\sqrt{P_1^2 + \beta^2}\), etc. The entropy increases with \(\delta_1\), approaching infinity [finite value] as \(\delta_1 \to \infty\) [non-supersymmetric extreme limit is reached]. For the supersymmetric extreme solutions, the entropy \(S\) is non-zero and finite, approaching infinity as \(\delta_1 \to \infty\), when \(P_{1,2}\) and \(Q_{1,2}\) are non-zero, and \((\delta_1 = 0)\) always zero when at least one of \(P_{1,2}, Q_1, (Q_2)\) is zero with \(\delta_1 \neq 0\) (with \(\delta_1 = 0\)).

The Hawking temperature \(T_H = |\partial_r \lambda(r = \beta)|/4\pi\) is given by

\[T_H = \frac{\beta}{\sqrt{2} S^{\frac{1}{2}}} . \tag{39}\]

As the boost parameter \(\delta_1\) increases, the temperature \(T_H\) decreases, approaching zero temperature. In the supersymmetric extreme limit with at least three of \(P_{1,2}, Q_{1,2}\) parameters non-zero, the temperature is always zero. With two of them non-zero, the temperature is non-zero and finite, approaching zero as \(\delta_1 \to \infty\). When only one of them is non-zero (only \(Q_2\) is non-zero for the case \(\delta_1 \neq 0\)), the temperature becomes infinite. In the non-supersymmetric extreme limit, the temperature is zero.

For the most general solution, it should be possible to write both the entropy \(^{38}\) and the temperature \(^{38}\) in the \(T\) - and \(S\)-duality invariant form, in terms of conserved (quantized) 28 electric charge \(\vec{\alpha}\) and 28 magnetic charge \(\vec{\beta}\) vectors, the asymptotic values of the moduli matrix \(M\) (cf. \(^{8}\)), and the dilaton-axion fields \(\varphi\) and \(\Psi\). In this case one would be able to address the dependence of the above thermal quantities on the asymptotic values of moduli and the couplings \(^{14}\).

\(^{17}\) Note that in the supersymmetric limit, the entropy \(^{38}\) has been cast \(^{22}\) in such a form and
V. CONCLUSIONS

In this paper, we obtained the most general static spherically symmetric solutions (compatible with no-hair theorem) of the effective four-dimensional heterotic string theory compactified on a six-torus, by performing a subset of $O(8, 24)$ transformations on the Schwarzschild solution. Here $O(8, 24)$ is the symmetry of the effective three-dimensional action of heterotic string, suitable for describing the stationary solutions of the four-dimensional heterotic string action. We provided an explicit sequence of symmetry transformations, which should be imposed on the Schwarzschild solution, in order to obtain the most general solution in this class.

We gave the explicit form of the generating solution. It was obtained by performing a sequence of six $SO(1, 1) \subset O(8, 24)$ boosts on the Schwarzschild solution, with the zero Taub-NUT charge constraint imposing one constraint among two boost parameters. The generating solution is therefore parameterized by six parameters: the mass of the Schwarzschild black hole and the six boost parameters, two of which subject to one constraint. Equivalently, these six parameters can be traded for the ADM mass of the generating solution and three electric and three magnetic charges (subject to zero Taub-NUT charge constraint), which are associated with the Kaluza-Klein and two-form gauge fields of the two compactified coordinates. The non-trivial scalar fields are the dilaton and the axion field, and the moduli of the two-torus.

The general solution, which are parameterized by unconstrained 28 magnetic and 28 electric charges and the ADM mass compatible with the Bogomol’nyi bound, is obtained by imposing on the generating solution fifty $[SO(6) \times SO(22)]/[SO(4) \times SO(20)] \subset O(6, 22)$ transformations and one $SO(2) \subset SL(2, R)$ transformation. Here $O(6, 22)$ and $SL(2, R)$ are the $T$- and $S$-duality symmetries of the effective four-dimensional heterotic string action. The above subset of $T$-duality transformations rotates the electric charges and like-wise magnetic charges as well as the moduli fields, while the above $S$-duality transformation rotates each electro-magnetic charge combination and the dilaton-axion field. Both subsets of transformations, however, do not affect the (Einstein frame) four-dimensional space-time, and thus the space-time properties of the whole class of solutions are determined by six parameters of the generating solution, which in turn enabled one to analyze explicitly their global space-time structure and thermal properties.

The work also sets a stage for addressing the moduli and coupling dependence of the thermal quantities for the general class of static spherically symmetric configurations, which may in turn shed light on quantum aspects of black holes in string theory.

is independent of the the moduli and couplings, as anticipated in Ref. [22]. We notice that for non-extreme solutions ($\beta > 0$) this is not the case anymore, and the entropy depends on the asymptotic values of the scalar fields.
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