The supply chain coordination model based on sales return and efforts

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Abstract. Taking a two-stage supply chain model of a single manufacturer and a single retailer as the research object, this paper initially proposes the random demand function of retailers’ effort to establish an integrated supply chain model based on dual factors of retailer effort and selling price. Then it discusses how to optimize the reward and punishment contracts to realize the win-win situation of supply chain members under decentralized decision-making by analyzing the coordination and optimization of supply chain under centralized and decentralized conditions. Finally, the conclusion is verified by numerical simulation experiment, effort and quantity will decrease along with the increase of the distribution coefficient and will increase with the increase of the buy-back price’s coefficient.

1. Introduction

The main aims of this article is to propose a supply chain hosted by a manufacturer and a retailer, where the manufacturer sells the products to the retailer[1][2]. The retailer allows customers to refund the purchased products at a discounted price within a limited period of time. With the efforts of the retailer, such as providing high-quality after-sales service guarantees, the demand for products will increase, and the amount of returns will decrease[3][4]; However high price will also affect the demand of the product to a certain extent and lead to the increase in the amount of returns[5]. Now, it is worthy to build a reasonable supply chain model about the efforts and sales price.

Many scholars have studied the supply chain coordination model with returns. Wen Jun [6] considered that the return of customers was only affected by the efforts of the retailer. Jing Chen et al.[7] demonstrated a single-inventory model in which customer return was related to sales volume and sales price. Frascatore and Mahmoodi[8] research showed that the implementation of the reward and punishment contract can effectively avoid the problem of insufficient production capacity of manufacturers. The reward and punishment contract means that the manufacturer sets a sales target for the retailer. If the retailer overfulfills the task, the excess will be rewarded; otherwise, the unfinished task will be punished. Hu Jun, Zhang Ga, etc. [9] demonstrated a supply chain coordination model considering quality control under the condition of linear demand, and discussed supply chain coordination in the context of revenue sharing contracts, reward and punishment contracts, franchise contracts, etc. It is an effort by a retailer or manufacturer to stimulate market demand for products, including early publicity, after-sales service, and product system upgrades. This article is based on Mo Jiangtao etc.[10], considering the supply chain coordination model that the return amount depends on the retailer's efforts and the sales price. The combination of reward and punishment contracts can achieve optimal supply chain in a decentralized situation.
2. Symbol descriptions

This paper will make the following assumptions, the cost price of the product is \( p_1 \), the sales price is \( p_2 \), and the retailer's wholesale price is \( p_3 \). The manufacturer will then take the products back if the retailer returns the unsold products to the manufacturer at the end of the sales period, the price is \( \alpha p_3 \) (\( 0 < \alpha < 1 \)). The demand \( F(t) \) for their products will increase if more efforts are made by retailer. \( t \) indicates the efforts of the retailer, and the cost of effort caused by the efforts is assumed \( E(t) \), \( E(t) = at, a > 1 \). The speed of demand growth will gradually slow down because of the limited potential customers, and \( \lim_{t \to \infty} F(t) = F_{\text{max}} \). The demand for the product during the actual sales period is determined by the retailer's efforts \( F(t) \) and a positive random variable with a mean of 1, and the demand form is as follows:

\[
G(x,t) = \frac{g(*)}{F(t)}
\]  

(1)

\( g(*) = g\left(\frac{x}{F(t)}\right) \) is a probability density function of a positive random variable with a mean of 1. The retailer's expected sales volume is \( H_1(Q,t) = \int_0^Q xG(x,t)dx + \int_0^{F_1(t)} QG(x,t)dx \), and the return volume is \( H_2(p_3,t) = (A + Bp_3 - \lambda t) \), \( A > 0, B \geq 0, \lambda > 0 \). The retailer deals with the unit price \( p_4 (p_3 > p_1 \geq p_4 > 0) \) of unsold products and the unsold volume is \( H_3(Q,t) = \int_0^Q (Q-x)G(x,t)dx \).

3. Centralization

In order to achieve the maximum profit of the supply chain and seek the optimal order quantity \( Q^* \) and efforts \( t^* \), an optimization model is established based on the previous assumptions, which the profit function of the system supply chain is \( \prod(Q,t) \).

\[
\prod(Q,t) = p_3H_1(Q,t) - p_1Q - \alpha p_3H_2(p_3,t) + p_4[H_3(Q,t) + H_2(p_3,t)] - E(t)
\]  

(2)

**Theorem 1.** In the case of centralized decision-making, a supply chain model can achieve system optimization where the demand has an efforts and the return amount depends on the sales price, and the optimal efforts \( t^* \) and order quantity \( Q^* \) are as follows:

\[
\left( p_3 - p_4 \right) F'(t^*) \int_0^{G^{-1}\left(\frac{p_3 - p_4}{p_3 - p_4}\right)} xg(x)dx = a - \lambda (\alpha p_3 - p_4)
\]  

(3)

\[
Q^* = F(t^*)G^{-1}\left(\frac{p_3 - p_4}{p_3 - p_4}\right)
\]  

(4)

\( G^{-1}(\cdot) \) is the inverse function of a positive random variable distribution function with a mean of 1.

4. Decentralization

In the case of no contract, if the retailer and the manufacturer unilaterally realize to optimize their profits, the retailer's expected profit function will be \( \prod_1(Q,t) \).

\[
\prod_1(Q,t) = p_3H_1(Q,t) - p_3Q - \alpha p_3H_2(p_3,t) + p_4[H_3(Q,t) + H_2(p_3,t)] - E(t)
\]  

(5)

The manufacturer's expected profit function is \( \prod_2(Q,t) \).
Comparing Eq. (2) with Eq. (5), we can get the unilateral profit function of the retailer.

**Theorem 2.** In the case of decentralized decision-making, the retailer can achieve the optimal profit in a supply chain model where the demand has efforts and the return amount depends on the sales price. At this time, the optimal order quantity and efforts are as follows:

\[
Q^* = F(t^*)G^{-1}\left[\frac{P_1 - P_2}{P_3 - P_4}\right]
\]

**Theorem 3.** The total profit of the system under centralized decision-making is greater than the sum of the profits of retailers and manufacturers under decentralized decision-making without contract coordination.

By comparing (3),(4) with (8),(9), the total profit of the system and the sum of the profits of retailers and manufacturers have the same form. Only considering functions \(\prod (Q', t')\), and \(p_2 > p_1\), then \(\frac{P_1 - P_2}{P_3 - P_4} < \frac{P_1 - P_2}{P_3 - P_4}\), So \(\prod (Q', t') > \prod (Q', t')\).

5. Coordination model of reward and punishment contract
The manufacturer set a sale target \(W\) for the retailer. \(H_1(Q,t) - H_2(p_1,t) - W \geq 0\), the retailer receives the reward \(\kappa[H_1(Q,t) - H_2(p_1,t) - W]\) (\(\kappa\) is a reward or penalty price per unit of product, and \(\kappa > 0\)), otherwise, he will receive penalty \(\kappa[W + H_2(p_1,t) - H_1(Q,t)]\).

The retailer's expected profit function is

\[
\prod s(Q,t,W) = p_1H_1(Q,t) + p_4[H_1(Q,t) + H_1(p_1,t)] - p_2Q - \alpha p_3H_2(p_1,t) - E(t) + \kappa[H_1(Q,t) - H_2(p_1,t) - W]
\]

\[
= (p_3 + \kappa)\left[\int_0^{F(t)} F(t)g(x)dx + \int_0^{\infty} Qg(x)dx\right] + p_4\left[\int_0^{F(t)} (Q - xF(t))g(x)dx + (A + Bp_3 - \lambda t)\right] - p_2Q - (\alpha p_3 - \kappa)(A + Bp_3 - \lambda t) - \alpha W
\]

It is the necessary first-order optimal conditions to make the retailer's optimal profit:

\[
\frac{\partial \prod s(Q,t,W)}{\partial Q} = 0
\]

Therefore,

\[
\int_0^{Q^*} g(x)dx = \frac{p_3 + \kappa - p_2}{p_3 + \kappa - p_4}
\]

\[
Q^* = F(t^*)G^{-1}\left[\frac{p_3 + \kappa - p_2}{p_3 + \kappa - p_4}\right]
\]

Then the optimal efforts \(t^*\) is obtained, and the following equation is satisfied,

\[
(p_3 + \kappa - p_4)F'(t^*)\int_0^{G^{-1}\left[\frac{p_3 + \kappa - p_2}{p_3 + \kappa - p_4}\right]} xg(x)dx = a - \lambda (\alpha p_3 - \kappa - p_4)
\]
If the supply chain is coordinated, the retailer's will order quantity $Q^*$ and efforts $t^*$ which must be equal to the optimal order quantity $Q^*$ and efforts $t^*$ under the system's decision. According to the analysis, it needs Eq. (12) and Eq. (4) to be equal. Therefore,

$$\frac{p_3 + \kappa - p_2}{p_3 + \kappa - p_4} = \frac{p_3 - p_1}{p_3 - p_4}$$

(13)

According to formula (13), then

$$p_2^* = \frac{(p_3 - p_4)(p_1 + (p_1 - p_4)\kappa)}{p_3 - p_4}$$

(14)

In the case of decentralized decision-making, the reward and punishment contract can realize the coordination of the supply chain, and the wholesale price $p_2^*$ should satisfy the formula (14) when the system reaches the optimal.

6. Numerical examples

The expected demand function is as follows. $F(t) = F_{\max} - (F_{\max} - F_0)e^{-\beta t}$, and $\beta > 0$. $F_0$ represents the market expected demand in the absence of the retailer’s efforts in advertising, after-sales service, etc. Assume that the random variable with a mean of 1 is a negative exponentially distributed random variable.

$$g(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & \text{other} \end{cases}$$

(15)

Substituting (15) into (3) and (4), we have

$$t^* = \frac{\ln[\beta(F_{\max} - F_0)/(p_1 - p_4)] + \ln\left[\frac{p_3 - p_1 + p_1 - p_4}{p_3 - p_1 + p_1 - p_4}\ln\left(\frac{p_3 - p_4}{p_3 - p_4}\right)\right]}{\beta} - \frac{\ln[a - \lambda(a p_1 - p_4)]}{\beta}$$

(16)

$$Q^* = F(t^*)[\ln(p_3 - p_4) - \ln(p_3 - p_4)]$$

(17)

A retailer buys a batch of products from a manufacturer, and the cost price $p_1$ is 35 yuan, the wholesale price $p_3$ is 45 yuan, and the sales price $p_3$ is 60 yuan. The residual value of the unsold product $p_4$ is 30 yuan, and the proportional coefficient of the product buy-back price $\alpha$ is 0.4. The effort cost factor $a$ is 1.50, and the factor of the return depends on the contribution of the efforts $\lambda$ is 0.8. $A$ is 400, $B$ is 10. The market demand for the product $F_0$ is 1500 units when the retailer does not invest the effort cost. And when the retailer invests the effort cost of 485.50 yuan, the market demand for the product is 4000 units; if the effort cost is 878.60 yuan, the market demand for the product is 5300 units.

$$\begin{cases} F_{\max} - (F_{\max} - F_0)e^{-485.50\beta} = 4000 \\ F_{\max} - (F_{\max} - F_0)e^{-878.60\beta} = 5300 \end{cases}$$

(18)

$F_{\max} = 8000, \beta = 0.001$.

Substitute the above data into equations (16) and (17), $t^* = 2806.40$ yuan, $Q^* = 13630$.

It is concluded that the optimal profit is $\prod(t^*, Q^*) = 110350.00$ yuan.

| Table 1 Optimal values of each parameter in the centralization and decentralization |
|---------------------------------|-------|------|------------------|------------------|
| $t^*$ (yuan) | $Q^*$ | $F(t^*)$ | $\prod(t^*, Q^*)$ (yuan) |
| centralization | 2806.40 | 13630 | 7607 | 110350.00 |
| decentralization | 1557.90 | 5273 | 6631 | 78937.00 |
It can be seen from Table 1 that the optimal profit under centralized is 1.4 times that of decentralized. If there is no contract coordination under decentralized, the supply chain will suffer a great loss. Therefore, it is of great significance to study how to achieve the optimal supply chain through contract coordination.

From the numerical simulation model of this example, we will discuss that the influence of the distribution coefficient $\lambda$ of return amount dependent effort and the proportion coefficient $\alpha$ of product buyback price on the optimal effort $t$ and order quantity $Q$ of the system, as shown in Figure 1,2 and Figure 3,4.

We find that effort and quantity will decrease along with the increase of the distribution coefficient from the figure 1 and 2. Namely the increase of share in return, most effort drops in the system, due to the return of the share account increase, the market demand will reduce accordingly, quantity is less, the retailer's investment will fall, thus effort will be less.

We find that effort and the quantity will increase with the increase of the buy-back price’s coefficient. from the figure 3 and 4. The buy-back price increases, and fluctuations in market risk is lesser. Retailer is taken on less risk, so he preferred to increase his input. As the demand of the market will increase, the quantity increase. However, buy-back price in the market will not keep going up , because manufacturers also need to consider the cost. It is of certain significance to determine a more appropriate quantity to make the supply chain reach the optimal value.
7. Conclusion
This article studies the supply chain coordination problem that depends on the efforts and sales price with a penalty contract. First, the market demand considered in the article is random, but what is given in the article is a fixed form. In reality, there are many factors affecting the demand, which cannot be determined. Second, the article considers fixed labor costs, however, in fact, manufacturers and retailers will make different efforts for their respective sales profits, so the cost of effort will also be different. Therefore, the supply chain model will be discussed to separate research in the future.

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