Isometry-preserving boundary conditions in the Kerr/CFT correspondence

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Abstract

The near-horizon geometries of the extremal Kerr black hole and certain generalizations thereof are considered. Their isometry groups are all given by $SL(2,\mathbb{R}) \times U(1)$. The usual boundary conditions of the Kerr/CFT correspondence enhance the $U(1)$ isometry to a Virasoro algebra. Various alternatives to these boundary conditions are explored. Partial classifications are provided of the boundary conditions enhancing the $SL(2,\mathbb{R})$ isometries or separately the $U(1)$ isometry to a Virasoro algebra. In the case of $SL(2,\mathbb{R})$-enhancing boundary conditions of a near-horizon geometry of the type considered, the conserved charges associated to the generators of the asymptotic Virasoro symmetry form a centreless Virasoro algebra.
1 Introduction

Quantum gravity on three-dimensional anti-de Sitter (AdS) space was found in [1] to be holographically dual to a two-dimensional conformal field theory (CFT). In the spirit of this work, it was recently argued [2] that the extremal four-dimensional Kerr black hole [3], for which the angular momentum $J$ saturates the regularity bound $J \leq GM^2$, is holographically dual to a two-dimensional chiral CFT in two dimensions. This Kerr/CFT correspondence was subsequently [4] generalized to a similar correspondence for the extremal Kerr-Newman black hole as well as for its AdS and dS generalizations. We refer to [5] for earlier work on a dual description of the Kerr black hole, and to [6, 7, 8, 9, 10, 11, 12] for further progress in the wake of [2].

Excitations around the near-horizon extremal black holes can be controlled by imposing appropriate boundary conditions. To every consistent set of boundary conditions, there is an associated asymptotic symmetry group generated by the diffeomorphisms obeying the conditions. The conserved charge of an asymptotic symmetry is constructed as a surface integral and can be analyzed using the formalism of [13, 14] based on [1, 15] and discussed extensively in [16]. An asymptotic symmetry transformation with vanishing conserved charge is rendered trivial.

The near-horizon metrics of the extremal black holes of our interest all have an $SL(2, \mathbb{R}) \times U(1)$ isometry group. In the studies [2, 4] of the Kerr/CFT correspondence and its generalizations, the $SL(2, \mathbb{R})$ becomes trivial while the $U(1)$ is enhanced to a Virasoro algebra. This is in contrast to the situation in studies of the Gödel black hole [17] and warped $AdS_3$ [18] in which an $SL(2, \mathbb{R})$ isometry is enhanced to a Virasoro algebra.

The boundary conditions imposed in [2, 4] are relevant for describing the ground-state entropy of the extremal Kerr black hole or its generalization. A discussion of the microscopic origin of the Bekenstein-Hawking entropy [19] for a class of black holes may be found in [20] and references therein.

Once constructed, boundary conditions enhancing the $SL(2, \mathbb{R})$ isometry of the extremal Kerr black hole, on the other hand, are speculated [2] to be relevant for the understanding of the entropy of near-extremal fluctuations. It is an objective of the present work to devise such boundary conditions and to study the resulting asymptotic symmetry group. The proposed boundary conditions are isometry-preserving in the sense that the original exact $SL(2, \mathbb{R})$ isometries correspond to the global conformal transformations (generated by the Virasoro modes $\ell_n, n = -1, 0, 1$) of the dual two-dimensional CFT. Following the standard approach [13, 14], the conserved charges associated to these Virasoro-generating asymptotic Killing vectors are well-defined and non-vanishing, but yield a centreless Virasoro algebra. Challenges from so-called back-reaction effects are briefly indicated.

After reviewing the construction in [2], we argue that there are infinitely many choices of boundary conditions yielding the same centrally-extended Virasoro algebra as the $U(1)$-enhanced one obtained in [2]. We also show that there is a related class of boundary conditions giving rise to a centreless $U(1)$-enhanced Virasoro algebra. We then present a class of boundary conditions enhancing the $SL(2, \mathbb{R})$ isometries to a centreless Virasoro algebra. The corresponding asymptotic symmetry is generated by an unusual differential-operator realization of the Virasoro algebra. A well-defined central extension of the algebra generated by the associated conserved charges is not permitted in this context – not even when ignoring back-reaction effects. Nor does it seem possible, within our approach, to construct boundary conditions resulting in two copies of the Virasoro algebra – one enhancing the $U(1)$ isometry; the other enhancing the $SL(2, \mathbb{R})$ isometries. It certainly is possible, though, at the level of asymptotic Killing vectors, but the charges associated to at least
one of the two Virasoro copies are ill-defined or simply vanish.

While this work was being completed, the paper [21] on the Kerr/CFT correspondence appeared. It shares some of our objectives and has an overlap in approach, but is based on a particular choice of boundary conditions not considered here. As in our similar cases, the $SL(2, \mathbb{R})$ isometries of the extremal Kerr black hole are enhanced to a Virasoro algebra generated by a set of asymptotic Killing vectors. Contrary to our cases, one verifies that the associated conserved charges all vanish, thus rendering the corresponding asymptotic symmetries trivial. The subsequent analysis of finite-temperature effects and entropy in [21] is based on quasi-local charges [22]. Analyses of that kind are not carried out here. A continuation of the work [21] can be found in [23].

2 Near-horizon extremal geometry

We are interested in the class [24] of extremal, stationary and rotationally symmetric four-dimensional black holes whose near-horizon metric and gauge field are of the form

$$d\bar{s}^2 = \Gamma(\theta)\left(-r^2 dt^2 + \frac{dr^2}{r^2} + \alpha(\theta)d\theta^2\right) + \gamma(\theta)(d\phi + krdt)^2$$

$$\bar{A} = f(\theta)(d\phi + krdt)$$

(2.1)

where $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi)$. Among these, is the extremal Kerr-Newman black hole as well as its $AdS$ and $dS$ generalizations. The corresponding isometry group $SL(2, \mathbb{R}) \times U(1)$ is generated by the Killing vectors

$$\left\{ \partial_t, t\partial_t - r\partial_r, \left(t^2 + \frac{1}{r^2}\right)\partial_t - 2tr\partial_r - \frac{2k}{r}\partial_\phi \right\} \cup \left\{ \partial_\phi \right\}$$

(2.2)

In this work, we only consider the gravitational part but hope to discuss the gauge transformations elsewhere.

The near-horizon extremal Kerr metric, in particular, is obtained by setting

$$\Gamma(\theta) = a^2(1 + \cos^2 \theta), \quad \gamma(\theta) = \frac{4a^2 \sin^2 \theta}{1 + \cos^2 \theta}, \quad \alpha(\theta) = k = 1$$

(2.3)

and the ADM mass and angular momentum of the extremal Kerr black hole are given by

$$M = \frac{a}{G}, \quad J = \frac{a^2}{G}$$

(2.4)

The Kerr metric itself [25] describes a rotating black hole as a solution to the four-dimensional vacuum Einstein equations.

3 Asymptotic symmetry group

We are interested in fluctuations of the near-horizon geometry of the extremal black hole whose background metric $\bar{g}_{\mu\nu}$ is defined in (2.1). We denote the corresponding perturbation of this metric by $h_{\mu\nu}$. Asymptotic symmetries are generated by the diffeomorphisms whose action on the metric generates metric fluctuations compatible with the chosen boundary conditions. We are thus looking for contravariant vector fields $\eta$ along which the Lie derivative of the metric is of the form

$$\mathcal{L}_\eta \bar{g}_{\mu\nu} \sim h_{\mu\nu}$$

(3.1)
The asymptotic symmetry group is generated by the set of these transformations modulo those whose charges, to be defined below, vanish. The boundary conditions should therefore be strong enough to ensure well-defined charges, yet weak enough to keep the charges non-zero.

It is of interest to determine the window of suitable boundary conditions. Once the asymptotic group of a consistent set of boundary conditions has been determined, one may scan, by weakening or strengthening the conditions, for other boundary conditions yielding the same or a closely related asymptotic group. Generally speaking, a strengthening of the boundary conditions will disallow certain asymptotic symmetries but not allow new ones. A weakening of the boundary conditions, on the other hand, will typically allow new asymptotic symmetries but may render some charges ill-defined. A change in boundary conditions strengthening some parts of the metric fluctuations but weakening others, may result in a different asymptotic symmetry group or in an equivalent group obtained as an enhancement of different exact isometries.

To the asymptotic symmetry generator \( \eta \) satisfying (3.1), one associates \([13, 14]\) the conserved charge

\[
Q_\eta = \frac{1}{8\pi G} \int_{\partial \Sigma} \sqrt{\bar{g}} k_\eta[h; \bar{g}] = \frac{1}{8\pi G} \int_{\partial \Sigma} \sqrt{-g} \frac{1}{4} \epsilon_{\alpha\beta\mu\nu} d^\mu_\eta[h; \bar{g}] d\alpha \wedge d\beta
\]

(3.2)

where

\[
d^\mu_\eta[h; \bar{g}] = \eta^\nu \bar{D}^\mu h - \eta^\nu \bar{D}_\sigma h^{\mu\sigma} + \eta_\sigma \bar{D}^\nu h^{\mu\sigma} - h^{\nu\sigma} \bar{D}_\sigma \eta^\mu + \frac{1}{2} h \bar{D}^\nu \eta^\mu + \frac{1}{2} h^{\sigma\nu} (\bar{D}^\mu \eta_\sigma + \bar{D}_\sigma \eta^\mu)
\]

(3.3)

and where \( \partial \Sigma \) is the boundary of a three-dimensional spatial volume, ultimately near spatial infinity. Here, indices are lowered and raised using the background metric \( \bar{g}_{\mu\nu} \) and its inverse, \( \bar{D}_\mu \) denotes a background covariant derivative, while \( h \) is defined as \( h = \bar{g}^{\mu\nu} h_{\mu\nu} \). To be a well-defined charge in the asymptotic limit, the underlying integral must be finite as \( r \to \infty \). If the charge vanishes, the asymptotic symmetry is rendered trivial. The algebra generated by the set of well-defined charges is governed by the Dirac brackets computed \([13, 14]\) as

\[
\{Q_\eta, Q_\hat{\eta}\} = Q_{[\eta, \hat{\eta}]} + \frac{1}{8\pi G} \int_{\partial \Sigma} \sqrt{\bar{g}} k_\eta[L_{\hat{\eta}} \bar{g}; \bar{g}]
\]

(3.4)

where the integral yields the eventual central extension.

### 4 Kerr/CFT correspondence

For ease of comparison, we here use the global coordinates of the near-horizon extremal Kerr black hole used in \([2]\) in which the metric reads

\[
ds^2 = 2GJ \Omega^2 \left( - (1 + r^2) d\tau^2 + \frac{dr^2}{1 + r^2} + d\theta^2 + \Lambda^2 (d\varphi + r d\tau)^2 \right)
\]

(4.1)

where

\[
\Omega^2 = \Omega^2(\theta) = \frac{1 + \cos^2 \theta}{2}, \quad \Lambda = \Lambda(\theta) = \frac{2 \sin \theta}{1 + \cos^2 \theta}
\]

(4.2)
In these coordinates, the rotational $U(1)$ isometry is generated by the Killing vector $\partial_\phi$, while the $SL(2, \mathbb{R})$ isometries do not concern us here. Written in the ordered basis $\{\tau, r, \varphi, \theta\}$, the imposed boundary conditions read

$$h_{\mu\nu} = \mathcal{O} \begin{pmatrix} r^2 & -r^{-2} & 1 & r^{-1} \\ r^{-2} & r^{-3} & r^{-1} & r^{-2} \\ 1 & r^{-1} & r^{-1} & r^{-1} \end{pmatrix}, \quad h_{\mu\nu} = h_{\nu\mu}$$ (4.3)

To preserve extremality of the Kerr black hole, one imposes the additional condition $Q \partial_\tau = 0$. The asymptotic Killing vectors are given by

$$K_\epsilon = \mathcal{O}(r^{-3}) \partial_\tau + (-r \epsilon'(\varphi) + \mathcal{O}(1)) \partial_r + (\epsilon(\varphi) + \mathcal{O}(r^{-2})) \partial_\varphi + \mathcal{O}(r^{-1}) \partial_\theta$$ (4.4)

where $\epsilon(\varphi)$ is a smooth function, in addition to the ‘trivialized’ $\partial_\tau$. The generators of the corresponding asymptotic symmetry read

$$\xi = -r \epsilon'(\varphi) \partial_r + \epsilon(\varphi) \partial_\varphi$$ (4.5)

and form the centreless Virasoro algebra

$$[\xi, \xi'] = \xi \epsilon'' - \epsilon' \xi$$ (4.6)

The usual form of the Virasoro algebra is obtained by choosing an appropriate basis for the functions $\epsilon(\varphi)$ and $\hat{\epsilon}(\varphi)$. This symmetry is an enhancement of the exact $U(1)$ isometry generated by the Killing vector $\partial_\phi$ of (4.1) as the latter is recovered by setting $\epsilon(\varphi) = 1$. The associated charges are computed using

$$\sqrt{-g} k_{\xi}[h; \bar{g}] = \left( \frac{1}{2} \epsilon' \Lambda r h_{r\varphi} - \frac{1}{4} \epsilon \Lambda \left( \Lambda^2 \frac{h_{r\tau}}{r^2} + 2 r \partial_\varphi h_{r\varphi} + (\Lambda^2 + 1) h_{\varphi\varphi} \right) \right) d\varphi \wedge d\theta + \ldots$$ (4.7)

where we have introduced the shorthand notation $\epsilon = \epsilon(\varphi)$. The dots indicate that terms not contributing to the charge (3.2) have been omitted. With respect to the basis $\xi_n(\varphi)$, where $\epsilon_n(\varphi) = -e^{-im\varphi}$, one introduces the dimensionless quantum versions of the conserved charges

$$L_n = \frac{1}{\hbar} \left( Q \xi_n + \frac{3J}{2} \delta_{n,0} \right)$$ (4.8)

After the usual substitution $\{\ldots\} \rightarrow -\frac{i}{\hbar} [\ldots]$ of Dirac brackets by quantum commutators, the quantum charge algebra is recognized as the centrally-extended Virasoro algebra \cite{2}

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c}{12} n(n^2 - 1) \delta_{n+m,0}, \quad c = \frac{12J}{\hbar}$$ (4.9)

4.1 Partial classification of $U(1)$-enhancing boundary conditions

The Lie derivative along $\xi$ of the background metric $\bar{g}_{\mu\nu}$ is given by

$$\mathcal{L}_\xi \bar{g}_{\mu\nu} = -2GJ \Omega^2 \begin{pmatrix} 2(\Lambda^2 - 1) r^2 \epsilon' & 0 & 0 & 0 \\ 0 & \frac{2}{(1+r^2)} r^2 \epsilon' & \frac{r}{1+r^2} \epsilon'' & 0 \\ 0 & \frac{r}{1+r^2} \epsilon'' & \frac{r^2}{1+r^2} \epsilon' & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$ (4.10)
and has only four (independent) non-trivial entries. It is thus natural to ask if the boundary conditions (4.3) can be strengthened while maintaining the Virasoro algebra (with its non-trivial central charge (4.9)) as an enhancement of the $U(1)$ isometry generated by the Killing vector $\partial_\varphi$. The strongest such boundary conditions are

$$h_{\mu\nu} = \begin{pmatrix} O(r^2) & 0 & 0 & 0 \\ 0 & O(r^{-4}) & 0 & 0 \\ 0 & 0 & O(r^{-1}) & 0 \\ 0 & 0 & 0 & O(1) \end{pmatrix}$$

and have asymptotic symmetries generated by $\xi$. The expression (4.7) remains and the Virasoro algebra generated by the associated charges $Q_\xi$ has the same central charge as above. The additional condition $Q_\partial_\tau = 0$ is still imposed.

Determining the weakest boundary conditions yielding a $U(1)$-enhanced Virasoro algebra is more delicate. As already indicated, new symmetries may arise. Even if the charges of the Virasoro algebra in question remain well-defined, the new symmetries may be ill-defined and thus render the boundary conditions inconsistent. It is, a priori, unclear if such ill-defined charges can be dealt with by imposing additional boundary conditions setting them equal to zero, as in the case $Q_\partial_\tau = 0$ above. It is beyond the scope of the present work to classify such possibilities. Instead, we merely point out that the weakest boundary conditions keeping the Virasoro-generating charges themselves well-defined are of the form

$$h_{\mu\nu} = \begin{pmatrix} r^2 & r^{p_{\tau\tau}} & r & r^{p_{\tau\varphi}} \\ r^{-2} & r^{-1} & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}, \quad h_{\mu\nu} = h_{\nu\mu}$$

Here, $p_{\tau\tau}$ and $p_{\tau\varphi}$ are real parameters bounded from above by the applicability, at infinity, of the linear theory assumed in the charge formula (3.2) [14]. Explicit values for the bounds are not discussed here, though. The general form of (4.12) follows from a simple inspection of the $r$-powers in

$$\sqrt{-g}k_\xi[h; \bar{g}]_{d\varphi^\perp, d\theta} = - \frac{\Lambda^3}{4(r^2 + 1)} h_{\tau\tau} + \frac{r\epsilon'' + 2r\Lambda^4 \epsilon - 2(r^2 + 1)\Lambda^2 \epsilon \partial_r - 2r\epsilon' \partial_\varphi}{4\Lambda(r^2 + 1)} h_{r\varphi}$$

$$- \frac{r\epsilon'}{2(r^2 + 1)} \partial_\theta(\Lambda h_{\tau\theta}) + \frac{\Lambda^3}{4(r^2 + 1)} \partial_r(\Lambda h_{r\tau}) + \frac{\Lambda}{2} (r\epsilon' - r\epsilon \partial_\varphi + \epsilon \partial_t) h_{r\varphi}$$

$$- \frac{\Lambda^2 ((\Lambda^2 + 1)r^2 + 1) \epsilon - 2\Lambda^2 r(r^2 + 1)\epsilon \partial_r - 2r\epsilon' \partial_\varphi}{4\Lambda(r^2 + 1)} h_{\varphi \varphi} + \frac{r^2 \epsilon'}{2(r^2 + 1)} \partial_\theta(\Lambda h_{\varphi \theta})$$

$$- \frac{\Lambda}{4(r^2 + 1)} ((\Lambda^2 + 1)\epsilon - r^3 \epsilon'' + 2r^2 \epsilon' \partial_\varphi - 2r\epsilon' \partial_t) h_{\theta \theta}$$

where subleading terms have been included to show that $p_{\tau\tau}$ and $p_{\tau\varphi}$ are unaffected by this particular evaluation. Compared with (4.10), we see that the contributions from $L_\xi g_{\tau\tau}$, $L_\xi g_{r\varphi}$ and $L_\xi g_{\varphi \varphi}$ to the central charge are independent of the particular choice of boundary conditions considered here, while $L_\xi g_{\tau \theta}$ does not contribute to any of them. The central charge (4.9) is thus the same for all these choices.
4.1.1 Centreless Virasoro algebra

As we are about to demonstrate, one obtains a centreless Virasoro algebra as an enhancement of the $U(1)$ isometry by imposing boundary conditions $h_{\mu\nu}$ in one of the three ‘ranges’

\[
\begin{pmatrix}
0 & 0 & O(r) & 0 \\
0 & 0 & 0 & 0 \\
O(r) & 0 & O(1) & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} < \begin{pmatrix}
h_{\tau\tau} & O(r^{\rho r}) & O(1) & O(r^{\rho \varphi}) \\
h_{r\tau} & h_{r\varphi} & O(r^{\rho \varphi}) & O(r^{\rho \theta}) \\
h_{\rho \varphi} & h_{\rho \varphi} & O(1) & O(1) \\
0 & 0 & O(1) & O(1)
\end{pmatrix}, \quad h_{\mu\nu} = h_{\nu\mu} \tag{4.14}
\]

where

\[h_{\tau\tau}, h_{rr}, h_{r\varphi} = O(r), O(r^{-2}), O(r^{-1})\]

or

\[h_{\tau\tau}, h_{rr}, h_{r\varphi} = O(r^2), O(r^{-5}), O(r^{-1})\]

or

\[h_{\tau\tau}, h_{rr}, h_{r\varphi} = O(r^2), O(r^{-2}), O(r^{-2})\] \tag{4.15}

in addition to $Q_{\partial r} = 0$. As in the discussion following (4.12), the real parameters $p_{rr}$, $p_{r\theta}$, $p_{r\varphi}$ and $p_{\varphi \theta}$ are bounded from above. Either of the new conditions $h_{\tau\tau} = O(r)$, $h_{r\varphi} = O(r^{-2})$ or $h_{rr} = O(r^{-5})$ reduces the symmetry generator $\xi$ from $\xi = -r \epsilon'(\varphi) \partial r + \epsilon(\varphi) \partial \varphi$ to $\xi = \epsilon(\varphi) \partial \varphi$. Further conditions may have to be imposed to ensure finiteness of eventual charges different from the ones generated by the new asymptotic symmetry $\hat{\xi} = \epsilon(\varphi) \partial \varphi$, but this question is not addressed here. Along $\xi = \epsilon(\varphi) \partial \varphi$, the Lie derivative of the background metric reads

\[
\mathcal{L}_\xi g_{\mu\nu} = 2GJ\Omega^2 \Lambda^2 \epsilon' \begin{pmatrix}
0 & 0 & r & 0 \\
0 & 0 & 0 & 0 \\
r & 0 & 2 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} \tag{4.16}
\]

The associated conserved and central charges follow from

\[
\sqrt{-g} k_\xi [h; g] = -\frac{\Lambda^3 \epsilon}{4(r^2 + 1)} h_{\tau\tau} + \frac{\Lambda \epsilon}{2} \left(\frac{\Lambda^2 r^2}{r^2 + 1} - \partial_r \right) h_{r\varphi} + \frac{\Lambda^3}{4} (r^2 + 1) \epsilon h_{rr}
\]

\[
+ \frac{\Lambda}{4} \left( r \epsilon' - 2r \epsilon \partial \varphi + 2 \epsilon \partial \theta \right) h_{r\varphi} - \frac{\Lambda \epsilon}{4} \left( \frac{\Lambda^2 r^2}{r^2 + 1} + 1 - 2r \partial_r \right) h_{\varphi \varphi} - \frac{\Lambda^3}{4} \epsilon \theta \theta \tag{4.17}
\]

and

\[
\frac{1}{8\pi G} \int_{\partial \Sigma} \sqrt{-g} k_\xi [\mathcal{L}_\xi g; g] = -\frac{J}{\pi} \int_0^{2\pi} \epsilon(\varphi) \epsilon'(\varphi) d\varphi \tag{4.18}
\]

where $\xi = \epsilon(\varphi) \partial \varphi$ and $\hat{\xi} = \hat{\epsilon}(\varphi) \partial \varphi$. Using the same basis $\xi_n(\varphi)$ as above, but with

\[
L_n = \frac{1}{\hbar} (Q\xi_n + J\delta_{n,0}) \tag{4.19}
\]

the quantum charge algebra is recognized as the centreless Virasoro algebra

\[
[L_n, L_m] = (n - m) L_{n+m} \tag{4.20}
\]
5 Enhancing the $SL(2, \mathbb{R})$ isometries

Returning to the general near-horizon geometry of an extremal black hole whose background metric $\bar{g}_{\mu\nu}$ is defined in (2.1), we are now looking for fluctuations $h_{\mu\nu}$ of the background metric enhancing the $SL(2, \mathbb{R})$ isometries to a Virasoro algebra. That is, some of the generators of the asymptotic symmetry group should correspond to the generators of the $SL(2, \mathbb{R})$ isometries. Also, since these fluctuations are expected to be relevant for the description of near-extremal perturbations, we refrain from imposing $Q_{\partial_t} = 0$. Aside from this weakening of the boundary conditions, it is natural to expect otherwise stronger boundary conditions than the ones enhancing the $U(1)$ isometry. To select such boundary conditions, we reverse-engineer the problem.

First, though, we note that supplementing the boundary conditions (4.3) with the condition $Q_{\partial_t} = 0$ corresponds to restricting to solutions with vanishing energy. This zero-energy condition was imposed in [2] not only to preserve extremality and study the ground states of the Kerr black hole, but also to ensure finiteness of the associated conserved charges. It was subsequently argued in [11] that this additional condition actually follows from the boundary conditions (4.3). It was also argued that so-called “back-reaction effects” at orders higher than linear could impose vanishing conditions, known as “linearization-stability constraints”, on seemingly well-defined charges. In particular, boundary conditions admitting near-extremal perturbations of the near-horizon extremal Kerr geometry preserving any of the $SL(2, \mathbb{R})$ isometries are believed to back-react so strongly that the Kerr asymptotics would break down. Despite these assertions, we find it worthwhile to continue our ‘linear’ analysis of $SL(2, \mathbb{R})$-enhancing boundary conditions of the near-horizon geometry (2.1).

Thus, we now consider one of the simplest possible sets of asymptotic Killing vectors generating a Virasoro algebra whose ‘global’ subalgebra (generated by $\ell_n$, $n = -1, 0, 1$) corresponds to the $SL(2, \mathbb{R})$ isometries, namely

$$K_\epsilon = \left[ \epsilon(t) + \frac{\epsilon''(t)}{2r^2} + O(r^{-4}) \right] \partial_t + \left[ - r \epsilon'(t) + O(r^{-1}) \right] \partial_r + \left[ - \frac{kr''(t)}{r} + O(r^{-3}) \right] \partial_\phi + \left[ O(r^{-2}) \right] \partial_\theta$$

(5.1)

Here, $\epsilon(t)$ is a smooth function and it follows that

$$[K_\epsilon, K_{\epsilon'}] = K_{\epsilon''} - \epsilon' \epsilon_\epsilon$$

(5.2)

The associated asymptotic symmetry generator is given by the contravariant vector field

$$\kappa_\epsilon = \left( \epsilon(t) + \frac{\epsilon''(t)}{2r^2} \right) \partial_t - r \epsilon'(t) \partial_r - \frac{kr''(t)}{r} \partial_\phi$$

(5.3)

and the three $SL(2, \mathbb{R})$ generators in (2.2) follow by setting $\epsilon(t) = t^{n+1}$, $n = -1, 0, 1$. The Lie derivative along $\kappa_\epsilon$ of the background metric is given by

$$\mathcal{L}_{\kappa_\epsilon} \bar{g}_{\mu\nu} = -\epsilon''' \begin{pmatrix} \Gamma + k^2 \gamma & 0 & k\gamma \\ 0 & 0 & 0 \\ \frac{k\gamma}{2\pi} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(5.4)

here written in the ordered basis \{t, r, \phi, \theta\} and in terms of the shorthands

$$\Gamma = \Gamma(\theta), \quad \gamma = \gamma(\theta), \quad \alpha = \alpha(\theta)$$

(5.5)
A set of boundary conditions compatible with these diffeomorphisms are

\[
\begin{pmatrix}
1 & r^{-3} & r^{-1} & r^{-2} \\
r^{-4} & 1 & r^{-3} & r^{-2} \\
r^{-3} & r^{-3} & 1 & r^{-2} \\
r^{-2} & r^{-2} & r^{-2} & 1
\end{pmatrix}, \quad h_{\mu \nu} = h_{\nu \mu}
\]  \tag{5.6}

There are several problems with this construction. First, the associated conserved charges \( Q_{\kappa_{\ell}} \) vanish, thereby rendering the asymptotic symmetries trivial. Second, the asymptotic symmetry generators \( (5.3) \) do not quite form a Virasoro algebra as we have

\[
[\kappa_{\ell}, \kappa_{\ell'}] = \kappa_{\ell} \epsilon_{\ell'} - \epsilon_{\ell} \kappa_{\ell'} + \frac{\epsilon''(t) \epsilon'''(t) - \epsilon'''(t) \epsilon''(t)}{4r^4} (\partial_t - 2kr \partial_\phi)
\]  \tag{5.7}

A proper differential-operator realization of the Virasoro algebra containing the \( SL(2, \mathbb{R}) \) isometries \( (2.2) \) does exist, though, and is discussed in the following. The issue with the charges is subsequently addressed and resolved, and we are ultimately left with a well-defined and non-vanishing set of conserved charges realizing the Virasoro algebra. As we will see, the symmetry generators \( \kappa_{\ell} \) \( (5.3) \) differ from the proper Virasoro generators by diffeomorphisms rendered trivial by their vanishing charges.

### 5.1 Asymptotic Virasoro symmetry

For every smooth function \( \epsilon(t) \), we introduce the contravariant vector field

\[
\xi = \xi^\mu \partial_\mu = \cosh \epsilon(t) \partial_t - r^2 \sinh \epsilon(t) \partial_r - k \sinh \epsilon'(t) \partial_\phi
\]  \tag{5.8}

where

\[
\cosh \epsilon(t) := \cosh \left( \frac{\partial_t}{r} \right) \epsilon(t) = \sum_{n=0}^{\infty} \frac{1}{(2n)! r^{2n}} \epsilon^{(2n)}(t)
\]

\[
\sinh \epsilon(t) := \sinh \left( \frac{\partial_t}{r} \right) \epsilon(t) = \sum_{n=0}^{\infty} \frac{1}{(2n+1)! r^{2n+1}} \epsilon^{(2n+1)}(t)
\]  \tag{5.9}

with the \( n \)’th derivative of \( \epsilon(t) \) denoted by \( \epsilon^{(n)}(t) \). We note that

\[
\partial_t \xi^r = r^4 \partial_t \xi^t, \quad \partial_t \xi^t = r^2 \partial_t \left( \frac{\xi^t}{r^2} \right), \quad \xi^\phi = \frac{k}{r^2} \partial_t \xi^r
\]  \tag{5.10}

After a bit of algebra, one verifies that the vectors \( (5.8) \) satisfy \( (4.6) \), thus providing a somewhat unusual differential-operator realization of the centreless Virasoro algebra. With \( \xi \) as the candidate for the generator of the corresponding asymptotic symmetry, we now continue the reverse-engineered selection of suitable \( SL(2, \mathbb{R}) \)-enhancing boundary conditions.

Along \( \xi \), the Lie derivative of the background metric is worked out to be

\[
\mathcal{L}_\xi \bar{g}_{\mu \nu} =
\begin{pmatrix}
2r \left( k \gamma (\partial_t \xi^\phi - (\Gamma - k^2 \gamma) (\xi^r + r \partial_t \xi^t)) \right) & k \gamma r \left( k r \partial_t \xi^t + \partial_t \xi^\phi \right) & \gamma \left( k (\xi^r + r \partial_t \xi^t) + \partial_t \xi^\phi \right) & 0 \\
k \gamma r \left( k r \partial_t \xi^t + \partial_t \xi^\phi \right) & \frac{2rt}{r^2} \partial_r \left( \frac{\xi^t}{r^2} \right) & \gamma \left( k r \partial_t \xi^t + \partial_t \xi^\phi \right) & 0 \\
\gamma \left( k (\xi^r + r \partial_t \xi^t) + \partial_t \xi^\phi \right) & \gamma \left( k r \partial_t \xi^t + \partial_t \xi^\phi \right) & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]  \tag{5.11}
Its non-vanishing components can be written as

\[
\mathcal{L}_{\xi} g_{tt} = -4 \sum_{n=0}^{\infty} r^{-2n} \frac{n+1}{(2n+3)!} (\Gamma + 2(n+1)k^2\gamma) \epsilon^{(2n+3)}(t)
\]

\[
\mathcal{L}_{\xi} g_{tr} = \mathcal{L}_{\xi} g_{rt} = 2k^2\gamma \sum_{n=0}^{\infty} r^{-3-2n} \frac{n+1}{(2n+3)!} \epsilon^{(2n+4)}(t)
\]

\[
\mathcal{L}_{\xi} g_{t\phi} = \mathcal{L}_{\xi} g_{\phi t} = -4k\gamma \sum_{n=0}^{\infty} r^{-1-2n} \frac{(n+1)^2}{(2n+3)!} \epsilon^{(2n+3)}(t)
\]

\[
\mathcal{L}_{\xi} g_{rr} = 4\Gamma \sum_{n=0}^{\infty} r^{-4-2n} \frac{n+1}{(2n+3)!} \epsilon^{(2n+3)}(t)
\]

\[
\mathcal{L}_{\xi} g_{r\phi} = \mathcal{L}_{\xi} g_{\phi r} = 2k\gamma \sum_{n=0}^{\infty} r^{-4-2n} \frac{n+1}{(2n+3)!} \epsilon^{(2n+4)}(t)
\]

and it is observed that

\[
\mathcal{L}_{\xi} g_{tt} + r^4 \mathcal{L}_{\xi} g_{rr} = 2kr \mathcal{L}_{\xi} g_{t\phi}, \quad \mathcal{L}_{\xi} g_{tr} = kr \mathcal{L}_{\xi} g_{r\phi}
\]

(5.13)

It is straightforward, albeit rather tedious, to compute the relevant part of the integrand in the surface integral \( \tilde{\Phi} \) defining \( Q_{\xi} \), and we find

\[
\sqrt{-g} k_{\xi}[h; \bar{g}]|_{\phi=\bar{\phi}, \theta=\bar{\theta}} = \sqrt{\frac{\alpha}{4\gamma \Gamma}} \left( \frac{k^2\gamma r^2}{2} \partial_r(r\xi^t) (r^3 h_{tt} - \frac{h_{tt}}{r}) + H_{t\phi} + H_{r\phi} + H_{\phi\phi} + H_{\theta\theta} \right)
\]

\[
+ \frac{1}{2r^2} \partial_\theta \left( \sqrt{\frac{\gamma}{\alpha \Gamma}} (r^4 \xi^t h_{t\phi} + \xi^r (h_{t\theta} - kr h_{\phi\theta})) \right) + \partial_\phi \Phi
\]

(5.14)

where

\[
H_{t\phi} = \left( \frac{k^2\gamma}{2} \partial_r(r\xi^t) + \frac{\gamma}{2r} \partial_r(r\xi^\phi) \right) h_{t\phi} - k\gamma r \partial_r(r\xi^t) \partial_r h_{t\phi}
\]

\[
H_{r\phi} = -\frac{\gamma}{2} \left( kr^2 \partial_r(\frac{\xi^t}{r}) + \partial_t \xi^\phi \right) h_{r\phi} + k\gamma r \partial_r(r\xi^t) \partial_t h_{r\phi}
\]

\[
H_{\phi\phi} = \left( (1 + \frac{k^2\gamma}{2\Gamma}) (\Gamma - k^2\gamma) r \partial_r(r\xi^t) - \frac{k\gamma}{2} \partial_r(r\xi^\phi) \right) h_{\phi\phi} - \frac{\Gamma \xi^t}{r^2} \partial_t h_{\phi\phi}
\]

\[
- \Gamma \xi^t - k^2\gamma \partial_r(r\xi^t)) r^2 \partial_r h_{\phi\phi}
\]

\[
H_{\theta\theta} = \frac{\gamma}{\alpha} \left( (1 - \frac{k^2\gamma}{2\Gamma}) r \partial_r(r\xi^t) h_{\theta\theta} - \xi^r \partial_t h_{\theta\theta} - r^2 \xi^t \partial_r h_{\theta\theta} \right)
\]

(5.15)

and

\[
\Phi = \sqrt{\frac{\alpha}{4\gamma \Gamma}} \left( \frac{\Gamma}{r^2} \xi^t h_{t\phi} + (\Gamma \xi^t - k^2\gamma \partial_r(r\xi^t)) r^2 h_{r\phi} + \frac{k\gamma \xi^t}{\alpha r} h_{\phi\phi} \right)
\]

(5.16)

It is emphasized that these expressions are valid for all \( r \), and we note that \( (5.14) \) is independent of \( h_{t\tau} = h_{\tau t} \). The total \( \phi \)-derivative \( \partial_\phi \Phi \) can be ignored in the surface integral \( \tilde{\Phi} \).
5.2 Boundary conditions

We initially require the boundary conditions to be of the form

\[ h_{\mu\nu} = O(r^{p_{\mu\nu}}), \quad h_{\mu\nu} = h_{\nu\mu} \quad (5.17) \]

where we allow \( p_{\mu\nu} = -\infty \) for some coordinates, as for several entries in (4.11), for example. The strongest such conditions compatible with the Virasoro symmetry generator \( \xi \) (5.8) are given by

\[ h_{\mu\nu} = O\left( \begin{array}{cccc}
1 & r^{-3} & r^{-1} & 0 \\
r^{-4} & 1 & r^{-2} & 0 \\
r^{-1} & 0 & 1 & 0 \\
0 & 0 & 0 & 0
\end{array} \right), \quad h_{\mu\nu} = h_{\nu\mu} \quad (5.18) \]

but one verifies that the associated conserved charges vanish.

The weakest boundary conditions (5.17), compatible with \( \xi \) and yielding well-defined charges \( Q_\xi \), are given by

\[ h_{\mu\nu} = O\left( \begin{array}{cccc}
r & p_{tr} & 1 & r \\
r^{-3} & 1 & r^{-2} & 0 \\
r^{-1} & 0 & 1 & 0 \\
0 & 0 & 0 & 0
\end{array} \right), \quad p_{tr} \geq -3, \quad h_{\mu\nu} = h_{\nu\mu} \quad (5.19) \]

These bounds on the asymptotic boundary conditions follow from analyzing the leading terms in (5.14) given by

\[
\sqrt{-g} \kappa [h; \bar{g}]_{\phi\theta} = \epsilon \sqrt{\frac{\alpha}{4\Gamma}} \left( k^2 \frac{\gamma^2}{2\Gamma} (r^3 h_{rr} - h_{rt}) + \frac{k^3 \gamma^2}{\Gamma} h_{t\phi} + k\gamma r (\partial_t h_{r\phi} - \partial_r h_{t\phi}) \right) \\
+ (\Gamma - k^2 \gamma) \left( 1 + \frac{k^2 \gamma}{2\Gamma} \right) r h_{\theta\phi} + r^2 \partial_t h_{r\phi} + \frac{\gamma}{\alpha} \left( 1 - \frac{k^2 \gamma}{2\Gamma} \right) r h_{t\theta} - \frac{\gamma r^2}{\alpha} \partial_t h_{t\theta} \right) \\
+ \frac{1}{2} \partial_\phi \left( \sqrt{\frac{\gamma}{\alpha \Gamma}} (r^2 h_{r\theta} - \epsilon (\frac{h_{t\theta}}{r}) - kh_{\phi\theta}) \right) + \ldots
\]

(5.20)

where the total \( \phi \)-derivatives have been ignored. Including these derivatives might prompt one to strengthen the allowed fluctuation \( h_{r\phi} \) unnecessarily from \( O(r^{-1}) \) to \( O(r^{-2}) \). As in the discussion following (4.12), the real parameter \( p_{tr} \) is bounded from above by the linear theory underlying (3.2).

The conserved charges \( Q_\xi \) corresponding to boundary conditions in the range from (5.18) to (5.19) are non-zero if at least one of the bounds in (5.19), \( h_{tr} \) excluded, is saturated. In all such cases, the charges generate a centreless Virasoro algebra since a simple \( r \)-power counting asymptotically gives \( \mathcal{L}_\xi \bar{g}_{\mu\nu} < h_{\mu\nu} \) for all \( \mu, \nu \).

With reference to the comments following (5.7), we note that \( Q_{\kappa - \kappa\epsilon} = 0 \) for all boundary conditions in the range from (5.18) to (5.19). This implies the announced triviality of the difference between the naive symmetry generators \( \kappa \) and the proper Virasoro generators \( \xi \) for every smooth function \( \epsilon(t) \).

Many alternatives exist to boundary conditions of the simple type (5.17). Imposing the separate condition \( h_{t\theta} = kr h_{\phi\theta} \), for example, renders (5.14) independent of \( h_{t\theta} \) and \( h_{\phi\theta} \) thus weakening the conditions on the real parameter \( p_{\phi\theta} \) in \( h_{\phi\theta} = O(r^{p_{\phi\theta}}) \), and subsequently in \( h_{t\theta} = O(r^{p_{\theta\phi}+1}) \). Imposing conditions resembling the relations (5.13) is another interesting possibility.
We emphasize that there are infinitely many sets of consistent boundary conditions simultaneously admitting two copies of Virasoro-generating asymptotic Killing vectors enhancing the SL(2,\mathbb{R}) and U(1) isometries separately. Within the realm of boundary conditions considered here, however, at least one of these copies gives rise to vanishing or ill-defined charges, and we are left with at most one quantum charge Virasoro algebra.

5.3 Spurious asymptotic symmetries

There may be many more asymptotic Killing vectors and asymptotic symmetry generators than conserved charges since the surface integrals (3.2) producing the charges from the generators may vanish. The corresponding contravariant vector fields thus generate spurious asymptotic symmetries. Let us illustrate this by considering the diffeomorphisms whose action on the background metric generate fluctuations compatible with

\[
\begin{pmatrix}
O(2^{\ell_0}) & 0 & O(2^{\ell_0} - 2) & 0 \\
0 & O(2^{\ell_0} - 2) & 0 & 0 \\
O(2^{\ell_0} - 2) & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\] (5.21)

for \(\ell_0\) a non-negative integer. Since these conditions are stronger than (5.18), the diffeomorphisms to be discussed are compatible with all boundary conditions in the range from (5.18) to (5.19). For every integer \(\ell \geq \ell_0 + 2\), we find the contravariant vector field

\[
\eta(2\ell) = \frac{\rho(2\ell)(t)}{2\ell r^{2\ell}} \partial_t - \frac{\rho(2\ell)(t)}{r^{2\ell - 3}} \partial_r - \frac{k\rho'(2\ell)(t)}{(2\ell - 1)r^{2\ell - 1}} \partial_\phi
\] (5.22)

The Lie derivative along \(\eta(2\ell)\) of the background metric is found to have the following non-trivial components

\[
\mathcal{L}_{\eta(2\ell)} \bar{g}_{tt} = \frac{2\ell(2\ell - 1)(\Gamma - k^2\gamma)r^2\rho(2\ell)(t) - ((2\ell - 1)\Gamma + k^2\gamma)\rho''(2\ell)(t)}{\ell(2\ell - 1)r^{2\ell - 2}}
\]
\[
\mathcal{L}_{\eta(2\ell)} \bar{g}_{t\phi} = \mathcal{L}_{\eta(2\ell)} \bar{g}_{\phi t} = -\frac{k\gamma(2\ell(2\ell - 1)r^2\rho(2\ell)(t) + \rho''(2\ell)(t))}{2\ell(2\ell - 1)r^{2\ell - 1}}
\]
\[
\mathcal{L}_{\eta(2\ell)} \bar{g}_{rr} = \frac{(4\ell - 4)\Gamma \rho(2\ell)(t)}{r^{2\ell}}
\] (5.23)

It follows that the corresponding charges \(Q_{\eta(2\ell)}\) all vanish.

Our final example concerns the fate of the original U(1) isometry in conjunction with the SL(2,\mathbb{R})-enhanced Virasoro algebra. Since \(\mathcal{L}_{\partial_\phi} \bar{g}_{\mu\nu} = 0\), it survives all consistent sets of boundary conditions. We wish to demonstrate that it can be enhanced to a U(1) current, though this current generates a spurious asymptotic symmetry. To see this, let us weaken the boundary conditions (5.21) and consider

\[
\begin{pmatrix}
O(r) & 0 & O(1) & 0 \\
0 & O(r^{-3} - 2\ell_0) & 0 & 0 \\
O(1) & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\] (5.24)
For every non-negative integer \( \ell \) of the corresponding Lie derivatives are
and for every \( \ell \geq \ell_0 \), we find the additional vector field
\[
\zeta_{(2\ell)} = -\frac{\psi_{(2\ell)}'(t)}{(2\ell + \delta_{\ell,0})(2\ell + 3)r^{2\ell+3}} \partial_t + \frac{\psi_{(2\ell)}(t)}{(2\ell + \delta_{\ell,0})r^{2\ell}} \partial_r + \frac{k\psi_{(2\ell)}'(t)}{(2\ell + \delta_{\ell,0})(2\ell + 2)r^{2\ell+2}} \partial_\phi.
\]
We note that the exact \( U(1) \) isometry follows from setting \( \psi(t) = 1 \) in \( \zeta \). The non-trivial components of the corresponding Lie derivatives are
\[
\mathcal{L}_\zeta \bar{g}_{tt} = 2k\gamma r \psi'(t), \quad \mathcal{L}_\zeta \bar{g}_{t\phi} = \mathcal{L}_\zeta \bar{g}_{\phi t} = \gamma \psi'(t)
\]
and
\[
\mathcal{L}_{\zeta_{(2\ell)}} \bar{g}_{tt} = -\frac{(2\ell + 2)(2\ell + 3)(\Gamma - k^2 \gamma)r^2 \psi_{(2\ell)}(t) - ((2\ell + 2)\Gamma + k^2 \gamma)\psi_{(2\ell)}''(t)}{(2\ell + \delta_{\ell,0})(\ell + 1)(2\ell + 3)r^{2\ell+1}},
\]
\[
\mathcal{L}_{\zeta_{(2\ell)}} \bar{g}_{t\phi} = \mathcal{L}_{\zeta_{(2\ell)}} \bar{g}_{\phi t} = \frac{k\gamma \left((2\ell + 2)(2\ell + 3)r^2 \psi_{(2\ell)}(t) + \psi_{(2\ell)}''(t)\right)}{(2\ell + \delta_{\ell,0})(2\ell + 2)(2\ell + 3)r^{2\ell+2}},
\]
\[
\mathcal{L}_{\zeta_{(2\ell)}} \bar{g}_{rr} = -\frac{2(2\ell + 1)\Gamma \psi_{(2\ell)}(t)}{(2\ell + \delta_{\ell,0})r^{2\ell+3}}
\]
Now, weakening the boundary conditions (5.24) to (5.19), compatible with the Virasoro generators \( \xi \) (5.8), we find
\[
\sqrt{-g}k_\xi [h; \bar{g}]_{d\phi\wedge d\theta} = -\sqrt{\frac{\alpha}{4\gamma}} \frac{k\gamma r \psi(t) \partial_\phi h_{r\phi}}{\Gamma} + \mathcal{O}(r^{-1})
\]
where \( h_{r\phi} = \mathcal{O}(r^{-1}) \). However, we can ignore total \( \phi \)-derivatives, implying that \( Q_\xi = 0 \). One could, perhaps, still wonder if the formalism would allow a central extension when combining \( \xi \) and \( \zeta \). This is not the case, though, since we have
\[
\sqrt{-g}k_\xi [\mathcal{L}_\zeta \bar{g}; \bar{g}]_{d\phi\wedge d\theta} = \sqrt{\frac{\alpha\gamma^5}{\Gamma}} \frac{\partial_r (r\xi^\phi) \psi'(t)}{4r} = \mathcal{O}(r^{-4})
\]
while \( \int \sqrt{-g}k_\xi [\mathcal{L}_\xi \bar{g}; \bar{g}] \) = 0 since \( Q_\xi = 0 \) and \( \xi \) is compatible with all the boundary conditions considered here. We also find that \( Q_{\zeta_{(2\ell)}} = 0 \).

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