One-Loop Electroweak Corrections to the Muon Anomalous Magnetic Moment Using the Pinch Technique

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Abstract

The definition of the physical properties of particles in perturbative gauge theories must satisfy gauge invariance as a requisite. The Pinch Technique provides a framework to define the electromagnetic form factors and the electromagnetic static properties of fundamental particles in a consistent and gauge-invariant form. We apply a simple prescription derived in this formalism to check the calculation of the gauge-invariant one-loop bosonic electroweak corrections to the muon anomalous magnetic moment.

A definition of the neutrino charge radius that satisfies good physical requirements, i.e. it is a physical observable, has been provided recently \cite{1} in the framework of the Pinch Technique (PT) formalism \cite{2}. In the PT formalism, the construction of a gauge-independent and gauge-invariant one-loop vertex and, in particular, of an effective electromagnetic form
factor for the neutrino amounts to compute the one-loop vertex corrections using a simple prescription in the linear $R^L_\xi$ gauge, where gauge-boson propagators

$$P^V_{\mu\nu}(q) = \frac{-i}{q^2 - M^2_\nu} \left[ g_{\mu\nu} + \left(1 - \xi\right) \frac{q_\mu q_\nu}{q^2 - M^2_\nu} \right],$$  \hspace{1cm} (1)

are taken in the 't Hooft-Feynman gauge $\xi = 1$, and the usual three-boson vertex

$$\Gamma_{\alpha\mu\nu}(q, k, -q - k) = (q - k)_\nu g_{\alpha\mu} + (2k + q)_\alpha g_{\mu\nu} - (2q + k)_\mu g_{\alpha\nu},$$  \hspace{1cm} (2)

is replaced by the truncated vertex [3]:

$$\Gamma^F_{\alpha\mu\nu} = (2k + q)_\alpha g_{\mu\nu} + 2q_\nu g_{\alpha\mu} - 2q_\mu g_{\alpha\nu},$$  \hspace{1cm} (3)

which satisfies a simple Ward identity:

$$q^\alpha \Gamma^F_{\alpha\mu\nu} = (k + q)^2 g_{\mu\nu} - k^2 g_{\mu\nu}.$$

We emphasize that this prescription should be applicable not only for the case of the neutrino but also for the electromagnetic form factors of quarks and leptons [4]. In particular, it looks very appealing to compute in a simple form the electroweak contributions to the static properties of fermions. In this note we apply the PT prescription to give an alternative evaluation of the one-loop $W$-boson contribution to the anomalous magnetic moment of the muon, $a_\mu \equiv (g - 2)/2$.

The complete one-loop electroweak corrections to $a_\mu$ were computed long time ago in refs. (the Higgs boson contribution and subleading muon mass terms are neglected):

$$a_\mu^{\text{weak}} = \frac{G_F m^2_\mu}{8\pi^2\sqrt{2}} \left\{ \frac{10}{3} + \frac{1}{3} \left[ (1 - 4 \sin^2 \theta_W)^2 - 5 \right] \right\}. \hspace{1cm} (4)$$

The first term in Eq. (4) accounts for the $W$-boson (plus unphysical scalars) contributions and the second term for the $Z^0$-boson correction to the vertex. Each one of these contributions is independent of the $\xi$-boson correction parameters (in the linear $R^L_\xi$ gauges) [5]. In the following, we are concerned first with the derivation of the first term in Eq. (4) using the PT prescription mentioned above. It is worth to mention that, in contradistinction with the Pinch Technique, the evaluation of the muon anomalous magnetic form factor (for a non-vanishing $q^2$ value) is gauge-dependent with the methods used in refs. [5].

Instead of performing an explicit evaluation of the $W$-boson corrections to the vertex, we can take advantage of a result derived, in another context, by Brodsky and Sullivan and
Figure 1: $W$-boson contributions to $a_\mu$: in the BFM formalism only diagrams (a-b) are required, while the four diagrams are necessary in the $R_\xi^L$ gauge calculation.

independently by Burnett and Levine in the late sixties. Using the $W$-boson propagator of Eq. (1) and the electromagnetic vertex of the $W$-boson as proposed by Lee and Yang (all particles are incoming, namely $k_1 + k_2 + k_3 = 0$):

$$V_{\mu\alpha\beta} = i e \{ g_{\alpha\beta}(k_1 - k_2)_{\mu} - g_{\alpha\mu}(k_1 + \kappa_W k_1 + \xi k_2 + \kappa_W k_2)_\beta + g_{\beta\mu}(k_2 + \kappa_W k_2 + \xi k_1 + \kappa_W k_1)_\alpha \} , \quad (5)$$

it can be shown that the prescription of the PT formalism for the $W$-boson propagator and electromagnetic vertex (see eqs. (1) and (3)) is obtained by choosing $\xi = 1$ and $\kappa_W = 1$.

The $W$-boson contribution (Fig. 1a) to $a_{\mu}^{\text{weak}}$ obtained in refs. using the Feynman rules of Eqs. (1) and (5) is:

$$a_{\mu}^{WW} = \frac{G_F m_\mu^2}{8\pi^2 \sqrt{2}} \left\{ 2 (1 - \kappa_W) \ln \xi + \frac{10}{3} \right\} . \quad (7)$$

As it can be easily checked by inserting the values given in eq. (6), the PT prescription for this correction gives the correct result for the $W$-boson contributions to $a_\mu$ (first term in Eq. (4)). The contribution from the $Z^0$-boson corresponding to the PT prescription ($\xi = 1$) computed in must be added to Eq. (7). Therefore, we recover, in the leading muon mass approximation, the usual result for the electroweak corrections to $a_\mu$ at the one-loop level.

As a further check of the results obtained using the PT formalism, we have computed the one-loop electroweak corrections to $a_\mu$ using the Background Field Method (BFM). It has

\footnote{The usual electromagnetic vertex for the $W$-boson in gauge theories is recovered for the special choice $\xi = 0$ and $\kappa_W = 1$ in eq. (5).}
been shown that when we fix the gauge parameter in the electroweak BFM to a particular value, namely $\xi_Q = 1$, one recovers the results of the PT formalism at the one-loop level \[9\]. This equivalence of both formalisms has been shown to hold at the two-loop level in Ref. \[10\] and very recently it has been proved at all orders in \[11\]. Since the different diagrams that contribute to the S-matrix amplitude in the framework of the PT have been rearranged in a gauge-invariant form to produce an effective electromagnetic vertex, this becomes a useful test of the calculation. Using the BFM formalism, the calculation of the $W$-boson and two-scalar contribution (The BFM only requires the contribution of Figs. 1a and 1b) to the anomalous magnetic moment of the muon becomes (we keep some terms of higher order in the muon mass):

$$a_W^{\mu \mid BFM}_{\text{Fig.1a}} = \frac{G_F m_\mu^2}{8\pi^2\sqrt{2}} \left\{ \frac{10}{3} + \frac{5}{6} \left( \frac{m_\mu}{m_W} \right)^2 + \frac{7}{6} \left( \frac{m_\mu}{m_W} \right)^4 + \ldots \right\} , \quad (8)$$

$$a_{\phi\phi}^{\mu \mid BFM}_{\text{Fig.1b}} = \frac{G_F m_\mu^2}{8\pi^2\sqrt{2}} \left\{ \frac{1}{3} \left( \frac{m_\mu}{m_W} \right)^2 - \frac{1}{4} \left( \frac{m_\mu}{m_W} \right)^4 + \ldots \right\} . \quad (9)$$

The sum of Eqs. (8) and (9) give :

$$a_\mu^{\mu \mid BFM}_{\text{Fig.1a+b}} = \frac{G_F m_\mu^2}{8\pi^2\sqrt{2}} \left\{ \frac{10}{3} + \frac{1}{2} \left( \frac{m_\mu}{m_W} \right)^2 + \frac{11}{12} \left( \frac{m_\mu}{m_W} \right)^4 + \ldots \right\} , \quad (10)$$

in good agreement with the results obtained using the PT recipe and with previous results \[5\].

As a further check of the subleading terms in $m_\mu$, one can compute the different contributions from Figs. (1.a-d) in the linear $R_\xi^L$ gauge. Choosing the gauge $\xi = 1$ we obtain:

$$a_W^{\mu \mid R_\xi^L}_{\text{Fig.1a}} = \frac{G_F m_\mu^2}{8\pi^2\sqrt{2}} \left\{ \frac{7}{3} + \frac{1}{2} \left( \frac{m_\mu}{m_W} \right)^2 + \frac{5}{6} \left( \frac{m_\mu}{m_W} \right)^4 + \ldots \right\} , \quad (11)$$

$$a_{\phi\phi}^{\mu \mid R_\xi^L}_{\text{Fig.1b}} = \frac{G_F m_\mu^2}{8\pi^2\sqrt{2}} \left\{ -\frac{1}{3} \left( \frac{m_\mu}{m_W} \right)^2 - \frac{1}{4} \left( \frac{m_\mu}{m_W} \right)^4 + \ldots \right\} , \quad (12)$$

$$a_W^{\mu \mid R_\xi^L}_{\text{Fig.1c}} = a_W^{\mu \mid R_\xi^L}_{\text{Fig.1d}} = \frac{G_F m_\mu^2}{8\pi^2\sqrt{2}} \left\{ \frac{1}{2} + \frac{1}{6} \left( \frac{m_\mu}{m_W} \right)^2 + \frac{1}{6} \left( \frac{m_\mu}{m_W} \right)^4 + \ldots \right\} . \quad (13)$$

When we add Eqs. (11–13), we obtain the same result as in Eq. (10). Let us note that to obtain the above results we have made extensive use of the expressions for the two- ($B_0$) and three-point ($C_0$) Passarino-Veltman functions derived in Ref. \[12\].

In summary, the application of the prescription given in Eqs. (1) (with $\xi = 1$) and (3), shows the robustness and simplicity of the PT formalism. In particular, the PT could be useful to verify the independence of the result with respect to the gauge-parameter in a given
gauge structure and to clarify the evaluation of the complete contributions to the two-loop electroweak corrections to $a_\mu$, since it has been proved that gauge invariance is satisfied to all orders \cite{1, 11} in this method. Note that the two-loop electroweak contributions to $a_\mu$ were computed in ref. \cite{13}. These corrections were computed using the linear $R_\xi$ gauge in the 't Hooft-Feynman gauge and also a nonlinear gauge structure, and neglecting the contributions that involve two or more scalar couplings \cite{13} since they are suppressed by additional powers of $m_\mu^2/m_W^2$. The two-loop electroweak corrections amount to a reduction of $-22.6\%$ with respect to the one-loop electroweak result and it is at the level of the sensitivities expected in current experiments. The PT formalism can therefore provide an additional check of these results in a consistent, gauge-invariant and gauge-parameter independent way.

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