UNIFICATION BEYOND GUTS–TOP MASS PREDICTIONS†,*

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ABSTRACT

Unification of Gauge and Yukawa sectors of GUTs is obtained by searching for renormalization group invariant relations among the couplings of the both sectors which hold beyond the unification scale. This procedure singled out two supersymmetric GUTs, the finite and the minimal $SU(5)$ models which, among others, were predicting successfully the top quark mass. The same procedure is currently extended in the soft supersymmetry-breaking sector of the theories, providing us with interesting, though preliminary, predictions on the Higgs and superpartner masses.

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1 Introduction

Unification of all interactions has always been a very attractive theoretical dream, and has proved in certain cases to be a powerful tool in our endeavor to reduce the plethora of free parameters of the Standard Model (SM). For instance, unification of the SM forces based on the SU(5) GUT was predicting one of the gauge couplings as well as the mass of the bottom quark. Now it seems that LEP data is suggesting that the symmetry of the unified theory should be further enlarged and become $N = 1$ globally supersymmetric.

Relations among gauge and Yukawa couplings, which are missing in ordinary GUTs, could be a consequence of a further unification provided by a more fundamental theory at the Planck scale. Moreover, it might be possible that some of these relations are renormalization group invariant (RGI) below the Planck scale so that they are exactly preserved down to the GUT scale. In our recent studies, we have been searching for RGI relations among gauge and Yukawa couplings in various GUTs. Two of our models are of particular interest. One based on the minimal supersymmetric SU(5) is very attractive due to its simplicity. The other, which is a finite SU(5) is exciting, since it is a realization of another old theoretical dream, namely that a truly unified theory should not have infinities. We should stress that the latter is a common view shared by many theoreticians including those working on strings, non-commutative geometry and quantum groups. Since it is widely expected that it is necessary to include gravity in the unified picture in order to achieve finiteness, we hope it is interesting for many to know that models which are finite to all orders in perturbation theory can be constructed, without introducing gravitational interactions, by searching for RGI relations beyond unification scale among gauge and Yukawa couplings.

Moreover both models among other successful postdictions provided us with predictions of the top quark mass, which have passed successfully so far all tests of progressively more accurate measurements.

2 Gauge-Yukawa Unification

A RGI relation among couplings, $\Phi(g_1, \ldots, g_N) = 0$, has to satisfy the partial differential equation (PDE) $\mu \frac{d\Phi}{d\mu} = \sum_{i=1}^{N} \beta_i \frac{\partial \Phi}{\partial g_i} = 0$, where $\beta_i$ is the $\beta$-function of $g_i$. There exist $(N-1)$ independent $\Phi$’s, and finding the complete set of these solutions is equivalent to solve the so-called reduction equations $\frac{dg_i}{dg} = \frac{\beta_i}{\beta_g}$, where $g$ and $\beta_g$ are the primary coupling and its $\beta$-function. Using all the $(N-1)$ $\Phi$’s to impose RGI relations, one can in principle express all the couplings in terms of a single coupling $g$. The complete reduction, which formally preserve perturbative renormalizability, can be achieved by demanding a power series solution, where the uniqueness of such a power series solution can be investigated at the one-loop level. An indication has been recently found in the soft supersymmetry-breaking sector as we will mention later.
Table 1: Representative predictions of the FUT $SU(5)$

| $m_{SUSY}$ [GeV] | $\alpha_3(M_Z)$ | $\tan\beta$ | $M_{GUT}$ [GeV] | $m_b$ [GeV] | $m_t$ [GeV] |
|------------------|----------------|-------------|----------------|-------------|-------------|
| 500              | 0.118          | 54.2        | $1.45 \times 10^{16}$ | 5.1          | 184.4       |

theory contains only one independent coupling with the corresponding $\beta$-function. In supersymmetric Yang-Mills theories with a simple gauge group, something more drastic can happen: the vanishing of the $\beta$-function to all orders in perturbation theory, if all the one-loop anomalous dimensions of the matter fields in the completely and uniquely reduced theory vanish identically.

This possibility of coupling unification is attractive, but it can be too restrictive and hence unrealistic. To overcome this problem, one may use fewer $\Phi$’s as RGI constraints.

2.1 The $SU(5)$ Finite Unified Theory [3]

This is a $N = 1$ supersymmetric Yang-Mills theory based on $SU(5)$ which contains one $24$, four pairs of $(\mathbf{5} + \bar{\mathbf{5}})$-Higgses and three $(\mathbf{5} + \mathbf{10})$’s for three fermion generations. It has been done a complete reduction of the dimensionless parameters of the theory in favour of the gauge coupling $g$ and the unique power series solution $\mathbb{E} \mathbb{E}$ corresponds to the Yukawa matrices without intergenerational mixing, and yields in the one-loop approximation $g_i^2 = g_c^2 = g_u^2 = (8/5)g^2$, $g_b^2 = g_d^2 = g_t^2 = g_s^2 = g_e^2 = (6/5)g^2$, where $g_i$’s stand for the Yukawa couplings. At first sight, this GYU seems to lead to unacceptable predictions of the fermion masses. But this is not the case, because each generation has an own pair of $(\bar{\mathbf{5}} + \mathbf{5})$-Higgses so that one may assume that after the diagonalization of the Higgs fields the effective theory is exactly MSSM, where the pair of its Higgs supermultiplets mainly stems from the $(\mathbf{5} + \bar{\mathbf{5}})$ which couples to the third fermion generation. (The Yukawa couplings of the first two generations can be regarded as free parameters.)

2.2 The minimal supersymmetric $SU(5)$ model [4]

The field content is minimal. Neglecting the CKM mixing, one starts with six Yukawa and two Higgs couplings. We then require GYU to occur among the Yukawa couplings of the third generation and the gauge coupling. We also require the theory to be completely asymptotically free. In the one-loop approximation, the GYU yields $g_{t,b}^2 = \sum_{m,n=1}^{\infty} \kappa_{i,m}^{(m,n)} h^m f^n g^2$ ($h$ and $f$ are related to the Higgs couplings). Where $h$ is allowed to vary from 0 to 15/7, while $f$ may vary from 0 to a maximum which depends on $h$ and vanishes at $h = 15/7$. As a result, we obtain $[\mathbb{E}, \mathbb{E}]: 0.97 g^2 \lesssim g_t^2 \lesssim 1.37 g^2$, $0.57 g^2 \lesssim g_b^2 = g_e^2 \lesssim 0.97 g^2$. We found $[\mathbb{E}, \mathbb{E}]$ that consistency with proton decay requires $g_t^2$, $g_b^2$ to be very close to the left hand side values in the inequalities.

3
Table 2: Representative predictions of the minimal SUSY $SU(5)$

| $m_{\text{SUSY}}$ [GeV] | $g_t^2/g_2^2$ | $g_b^2/g_2^2$ | $\alpha_3(M_Z)$ | $\tan \beta$ | $M_{\text{GUT}}$ [GeV] | $m_b$ [GeV] | $m_t$ [GeV] |
|-------------------------|----------------|----------------|----------------|-------------|----------------|-------------|-------------|
| 500                     | 0.97           | 0.57           | 0.118          | 47.7        | $1.39 \times 10^{16}$ | 5.3         | 178.9       |

In all of the analyses above, we have assumed that it is possible to arrange the susy mass parameters along with the soft breaking terms in such a way that the desired symmetry breaking pattern really occurs, all the superpartners are unobservable at present energies, and so forth. To simplify our numerical analysis we have also assumed a unique threshold $m_{\text{SUSY}}$ for all the superpartners. Using the updated experimental data on the SM parameters, we have re-examined the $m_t$ prediction of the two GYU $SU(5)$ models described above [3, 1]. They predict $m_t = (183 + \delta_{m_t}^{\text{MSSM}} \pm 5)$ GeV (FUT) and $m_t = (181 + \delta_{m_t}^{\text{MSSM}} \pm 3)$ GeV (Min. SUSY $SU(5)$), where $\delta_{m_t}^{\text{MSSM}}$ stands for the MSSM threshold corrections of few %.

3 Discussion and Conclusions

The gauge-Yukawa unification we have been proposing is based on the RGI relations among gauge and Yukawa couplings in GUTs beyond the unification scale. They correspond to a “technically natural fine tuning” and could point to a unified scheme at the Planck scale. We have not assumed the precise nature of the unified scheme, but these relations could be obtained at the Planck scale and being RGI survive down to the GUT scale. We have found [1, 3, 4, 5] that two supersymmetric $SU(5)$ GUTs with GYU can predict the bottom and top quark masses in accordance with the recent experimental data. This means that the top-bottom hierarchy could be explained in these models, in a similar way as the hierarchy of the gauge couplings of the SM can be explained if one assumes the existence of a unifying gauge symmetry at $M_{\text{GUT}}$. We would like to emphasize that the GYU scenario is the most predictive scheme as far as the mass of the top quark is concerned. We also have found [1, 3] that the lowest value of the infrared quasi-fixed-point value of $m_t$ for large $\tan \beta$ is $\sim 188$ GeV. Comparing this with the experimental value $m_t = (175.6 \pm 5.5)$ GeV [3] we may conclude that the present data on $m_t$ cannot be explained from the infrared quasi-fixed-point behaviour alone if $\tan \beta \gtrsim 2$.

Further, we have extended our approach [7] and have been searching for RGI relations among the soft supersymmetry-breaking (SSB) parameters of the models. Preliminary results in this case of the model discussed in 2.2 show that the low energy SSB sector of the theory contains a single arbitrary parameter, the unified gaugino mass $M_g$, which characterizes the scale of the supersymmetry breaking. In turn the lightest Higgs mass is predicted to be $\sim 120$ GeV, while the lightest supersymmetric particle is found to be a neutralino of $\sim 220$ GeV for $M_g(M_{\text{GUT}}) \sim 0.5$ TeV [7]. Moreover, it has been recently
found that RGI SSB scalar masses in gauge-Yukawa unified models satisfy a universal sum rule, which coincides with those obtained in a certain class of superstring models in which the origin of the sum rule has a symmetry interpretation, target-space duality.

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