Branes and Vector-like Supersymmetry Breaking Theories with Gauged Global Symmetry

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Abstract

We show that the brane configuration describing the Izawa-Yanagida-Intriligator-Thomas (IYIT) model with gauged $U(1)$ subgroup of the global symmetry contains inconsistent geometry, implying that there exists a stable vacuum where supersymmetry is dynamically broken.
1 Introduction

Dynamical supersymmetry breaking (DSB) is a longstanding problem in supersymmetric gauge theories. It provides a natural solution to the hierarchy problem that there is a large difference in magnitude between the electroweak scale and the Planck scale, by generating dynamically strong coupling scales which are far below the Planck scale (or the GUT scale) because of a logarithmic running of gauge couplings. There are many known examples of the theories which realize DSB as consequences of strong coupling dynamics, such as non-pertubatively generated superpotential, quantum deformation of moduli space, confinement, and so on. It is worth noting that most of the DSB models discovered recently depend on a series of the exact results in supersymmetric gauge theories, which are obtained by holomorphy and symmetry arguments. This tells us that if a new tool for investigating supersymmetric gauge theories is introduced, it will be a valuable attempt to apply it to the problem of determining if DSB occurs or not.

In recent developments in string theories and M-theory, the worldvolume dynamics on various branes has been proved to be a powerful alternative to analyzing supersymmetric gauge theories. Up to now, it has been found that many of the field theories with different number of supersymmetries can be realized on branes, and the properties of the theories which were understood using purely field theoretical methods so far can be derived from the brane analyses. For instance, we can construct $N = 4$ super Yang-Mills theories on parallel D3-branes in type IIB string theory, with Montonen-Olive duality originating from type IIB $SL(2,\mathbb{Z})$ duality. $N = 2$ supersymmetric QCD is realized by connecting suitably D4-branes and NS5-branes in type IIA string theory, or by reinterpretting them as describing a single 5-brane in M-theory to take quantum effects into account. One of the most remarkable consequences is that the Riemann surface which determines exactly the metric on Coulomb branch is a two dimensional part of the M5-brane, with the remaining four dimensional worldvolume being the flat space-time.

Brane construction of the theories with $N = 1$ supersymmetry will be most interesting for phenomenological applications. Non-perturbative aspects of field theories, such as runaway behaviour due to Affleck-Dine-Seiberg type superpotential, quantum modifi-
cation of moduli spaces, etc. are all rephrased in M5-brane language [18, 19, 14, 20, 21]. Furthermore, we can argue Seiberg’s $N = 1$ duality to be nothing but the exchange of two NS5-branes in type IIA configuration, or the smooth deformation of an M5-brane [22, 23, 24, 12, 25, 26, 27, 28]. Given that the correspondence between field theories and branes was established in this way, we are naturally led to a question as mentioned in the first paragraph of this section: Can brane configuration encode the information about DSB?

The authors of [20] gave an answer to this question, by considering the brane configuration describing the vector-like gauge theory which breaks supersymmetry dynamically, proposed by Izawa, Yanagida and Intriligator, Thomas [3] (we refer to this theory as the IYIT model in the following). In [29] it was argued that “t-configuration”, the transverse intersection of an M5-brane with an orbifold fixed plane, is incompatible with Dirac’s quantization condition for 4-form field strength in M-theory, and the M5-brane configuration for the IYIT model unavoidably contains this t-configuration, triggering DSB. Therefore, if we admit the interpretation of t-configuration in M5-brane as DSB in field theory, we may expect that certain field theories can be shown to have non-supersymmetric stable vacua by proving that t-configuration appears inevitably in the corresponding M5-brane configuration.

In this paper we adopt this brane approach to the problem of DSB in order to investigate the IYIT model with gauged subgroup of the flavour symmetry. Our motivation to consider this generalization of the original IYIT model comes from gauge-mediated supersymmetry breaking [30, 31, 32]. Most of the gauge-mediated supersymmetry breaking models consist of three sectors, the first one to break supersymmetry dynamically (hidden sector), the second one to mediate the effects of the broken supersymmetry (messenger sector), and the last one which governs the dynamics of the ordinary particles (observable sector). Generally, we gauge the $U(1)$ subgroup of the global symmetry possessed by the model exhibiting DSB which serves as the hidden sector, so that DSB effects should be transmitted to the messenger sector through the gauged $U(1)$ interaction. We conclude making use of the brane technique that the supersymmetry breaking stable vacuum still remains even if $U(1)$ subgroup of the global symmetry is gauged. This agrees with the naive expectation from the field theory that gauging $U(1)$ symmetry does not alter the
asymptotic behaviour of the potential in the original model.

The organization of this paper is as follows. In section 2, we briefly review the original IYIT model in both of field theory and brane context. In section 3, we discuss electric-magnetic duality in $Sp$ gauge theories with gauged global symmetry. In section 4, we demonstrate explicitly that the M5-brane configuration for the case of gauged $U(1)$ subgroup of the global symmetry involves t-configuration, which is a signal of the non-supersymmetric stable vacuum. The last section is devoted to conclusions.

2 Review of the Izawa-Yanagida-Intriligator-Thomas Model

2.1 The Original Model

The theory is based on $N = 1$ $Sp(\tilde{N}_c)$ SQCD with $2N_f$ quarks $\tilde{Q}_i^a$ and $N_f(2N_f - 1)$ gauge singlets $S^{ij} = -S^{ji}$ ($\tilde{a}, \tilde{b} = 1, 2, \cdots, 2\tilde{N}_c; i, j = 1, 2, \cdots, 2N_f$), with tree-level superpotential

$$\tilde{W}_{\text{tree}} = \frac{1}{2\mu} S^{ij} \tilde{Q}_i \tilde{Q}_j,$$

where $\tilde{Q}_i \tilde{Q}_j$ denotes $J_{\tilde{a} \tilde{b}} \tilde{Q}_i^\tilde{a} \tilde{Q}_j^\tilde{b}$, and $J_{\tilde{a} \tilde{b}} = (1_{\tilde{N}_c} \otimes i\sigma_2)_{\tilde{a} \tilde{b}}$ is the antisymmetric invariant tensor of $Sp(\tilde{N}_c)$. The representation of the quarks and the singlets under the gauge and the anomaly free global symmetry $SU(2N_f) \times U(1)_R$ is

$$\begin{array}{ccc}
Q & Sp(\tilde{N}_c) & SU(2N_f) \\
& & U(1)_R \\
S & 1 & 1 - \frac{\tilde{N}_c + 1}{N_f} \\
& & 2\tilde{N}_c + 1
\end{array}$$

The IYIT model corresponds to the case $N_f = \tilde{N}_c + 1$. Under this circumstance no non-perturbative superpotential is generated, because the tree-level superpotential \footnote{Since it is convenient to consider these theories as magnetic dual descriptions of electric theories in the next subsection, we interpret the singlets $S^{ij}$ as mesons of dimension two, and introduce mass scale $\mu$ in (2.1).} itself is the only possible one we can write down taking into account holomorphy and the symmetries of the theory. Instead of generating non-perturbative superpotential, however, strong coupling effects modify the classical constraint $\text{Pf}(\tilde{Q}_i \tilde{Q}_j) = 0$ to the quantum one
Pf(\tilde{Q}_i \tilde{Q}_j) = \tilde{\Lambda}^{2(\tilde{N}_c+1)}, \quad (2.3)

where \tilde{\Lambda} is the scale at which the gauge coupling blows up. Since the F term condition for \( S^{ij} \), \( \partial W/\partial S^{ij} = 0 \), clearly contradicts the constraint (2.3), supersymmetry is spontaneously broken.

We can ascertain that supersymmetry is indeed broken from another point of view, by considering large values for singlets, \( \mu^{-1} S^{ij} \gg \tilde{\Lambda} \). Since all the quarks acquire large masses under this condition, the low energy effective theory is described by pure \( Sp(\tilde{N}_c) \) SYM with the scale \( \tilde{\Lambda}_L \) which is related to \( \tilde{\Lambda} \) by the coupling matching, \( \tilde{\Lambda}_L^{3(\tilde{N}_c+1)} = Pf(\mu^{-1} S^{ij}) \tilde{\Lambda}^{2(\tilde{N}_c+1)} \). Then, gaugino condensation generates non-pertubative superpotential

\[ \tilde{W}_{\text{eff}} = (\tilde{N}_c + 1) \tilde{\Lambda}_L^3 = (\tilde{N}_c + 1)[Pf(\mu^{-1} S^{ij})]^{1/(\tilde{N}_c+1)} \tilde{\Lambda}^2. \quad (2.4) \]

Since the superpotential (2.4) is linear in \( S \) (recall \( N_f = \tilde{N}_c + 1 \)), F term condition \( \partial W_{\text{eff}}/\partial S = 0 \) cannot be satisfied, leading to DSB. Computation of the one-loop correction to the Kähler potential for \( S \) reveals that the scalar potential for \( S \) increases in the region \( \mu^{-1} S^{ij} \gg \tilde{\Lambda} \) where perturbative calculation is reliable. Therefore, we can assert that supersymmetry breaking stable vacuum exists, although its location in field space cannot be determined because strong coupling effects on the Kähler potential for small \( S \) is uncontrollable.

### 2.2 Brane Analysis

We can regard \( N = 1 \) \( Sp(\tilde{N}_c) \) gauge theory with matter representation (2.2) and tree-level superpotential (2.1) as a magnetic dual description of \( Sp(N_c(= N_f - \tilde{N}_c - 2)) \) SQCD with \( 2N_f \) massless quarks, provided that the number of flavours satisfies \( N_f > \tilde{N}_c + 2 \). If we put \( N_f = \tilde{N}_c + 1 \) ignoring this inequality, the IYIT model can be obtained as electric \( Sp(N_c(= -1)) \) SQCD. In [20], the IYIT model was analyzed by constructing the M5-brane configuration for \( Sp(-1) \) gauge theory, as we review in this subsection.

In order to realize \( Sp \) gauge theory with \( N = 1 \) supersymmetry on the worldvolume of the branes in type IIA string theory, we require the following branes and orientifold
where the numbers indicate the directions in the ten dimensional flat space-time along which the branes and the O4-plane are extending. It needs mentioning the peculiar nature of the O4-plane in the presence of the other branes in (2.5). If the O4-plane exists, we must identify the two points which are related to each other by the $\mathbb{Z}_2$ transformation $x^{4,5,7,8,9} \rightarrow -x^{4,5,7,8,9}$. There are two kinds of O4-plane distinguished by their Ramond-Ramond (RR) charges, namely the O4-planes with RR charge $+1$ and $-1$ which realize $Sp$ and $SO$ gauge theory respectively on the worldvolume of D4-branes which are set parallel to the O4-plane so as to obey the $\mathbb{Z}_2$ symmetry \([23]\). When an NS5-brane intersects with an O4-plane so that they share a real codimension one subspace of the O4-plane, the two regions into which the O4-plane is divided by the NS5-brane have opposite RR charges, as if the NS5-brane behaves as a domain wall \([33]\).

The type IIA brane configuration for $N = 1$ $Sp(N_c)$ SQCD with $2N_f$ massless quarks can be constructed as follows. We introduce the complex coordinates $v$ and $w$,

\[
\begin{align*}
  v &= x^4 + ix^5, \\
  w &= x^8 + ix^9.
\end{align*}
\]  

We first put an O4-plane at $v = 0, w = 0$ and place an NS5-brane at $w = 0$ and an NS'5-brane at $v = 0$, with their positions in $x^6$ direction different. Here, we choose the RR charge carried by the O4-plane between the NS5- and the NS'5-brane to be $+1$, in order to guarantee the gauge group to be $Sp(N_c)$. Then, we connect the two NS5-branes by $2N_c$ D4-branes, and locate $2N_f$ D6-branes at $v = 0$ and arbitrary $x^6$ positions on the interval between the two NS5-branes. The brane configuration made this way is shown in Figure 1. The $Sp(N_c)$ vector multiplet corresponds to the string both ends of which are stuck on the D4-branes, and $2N_f$ quarks are identified with the strings linking the D4-branes and the D6-branes.

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2 We use the terminology “NS5-brane” in two meanings, the one refers to general NS5-branes with their worldvolume extending in arbitrary directions including NS'5-brane in (2.5), and the other to the NS5-branes of the type specified in (2.5).

3 Throughout this paper, all the branes except the D6-branes lie on the hyperplane $x^7 = 0$. 
Figure 1: Type IIA brane configuration for the electric $Sp(N_c)$ SQCD.

Before lifting the above electric type IIA configuration up to a single M5-brane, we present here the type IIA brane configuration for the magnetic theory which expresses the IYIT model under the condition $N_f = \tilde{N}_c + 1$. The duality transformation in field theory is equivalent in the brane picture to moving the NS5-brane to the right of the NS'5-brane\[23\]. First, the NS5-brane passes through the $2N_f$ D6-branes, with a D4-brane created between the NS5-brane and a D6-brane just at the moment when the NS5-brane and the D6-brane coincide in the $x_6$ direction. At this stage we have $2N_f$ D4-branes, each of which is suspended between the NS5-brane and one of the $2N_f$ D6-branes. Then, we connect $2N_c$ D4-branes between the NS5-brane and the NS'5-brane to $2N_c$ of the $2N_f$ D4-branes ending on the $2N_f$ D6-branes, so that $2N_f - 2N_c$ D4-branes should connect the NS5-brane and $2N_f - 2N_c$ D6-branes. Finally we let the NS5-brane pass through the NS'5-brane. When the NS5-brane and the right NS'5-brane collide, two of the D4-branes and their mirror partners between the NS5-brane and D6-branes disappear \[23\], leading to $2\tilde{N}_c = 2(N_f - N_c - 2)$ D4-branes suspended between the NS'5-brane and the NS5-brane. We reach eventually the brane configuration shown in Figure 2.

We now discuss how the type IIA brane configurations given above are embedded in M-theory, by introducing an extra dimension with coordinate $x^{10}$, which is a circle of radius $R$. The O4-plane with RR charge $-1$ originates from the five dimensional orbifold fixed plane whose worldvolume is extending in the direction $x^{0,1,2,3,6,10}$, while the M-theory counterpart of the O4-plane with RR charge $+1$ is the fixed plane screened by
two M5-branes on top of it.

The D6-branes are interpreted as Kaluza-Klein monopoles, the four dimensional space transverse to which is described by the Taub-NUT geometry. Although the Taub-NUT space possesses hyperkähler structure, it suffices to choose one complex structure of that, for the purpose of embedding an M5-brane. In the present paper we express the Taub-NUT space by a two complex dimensional hypersurface in $C^3$ with complex coordinates $(x, y, v)$,

$$xy = v^{2N_f}.$$  \hspace{1cm} (2.7)

It is reflected on the $A_{2N_f-1}$ type singularity at the origin $x = y = v = 0$ that the D6-branes are located at the same position in the $v$ direction, $v = 0$. However, the singularity must be resolved in a generic situation where the $x^6$ positions of the D6-branes are different, to describe correctly the geometry around the D6-branes \[\square\]. The resolved surface is covered by $2N_f$ patches $U_i$ ($i = 1, 2, \cdots, 2N_f$) with coordinates $(x_i, y_i)$, which are glued by the transformation $x_i y_{i+1} = 1, x_i y_i = x_{i+1} y_{i+1}$. The three coordinates $(x, y, v)$ are related to the $U_i$ coordinates by

$$x = x_i^{2N_f-i+1} y_i^{2N_f-i}, \quad y = x_i^{-1} y_i, \quad v = x_i y_i.$$ \hspace{1cm} (2.8)

Upon resolving the singularity there appear $2N_f - 1$ exceptional $\mathbb{P}^1$ cycles $C_i$ ($i = 1, 2, \cdots, 2N_f - 1$) which are the loci $y_i = 0$ in $U_i$ or $x_{i+1} = 0$ in $U_{i+1}$, with two adjacent components intersecting transversely with one another at the position of a D6-brane.
Under the $\mathbb{Z}_2$ symmetry the coordinates on $U_i$ transform as $(x_i, y_i) \rightarrow ((-1)^i x_i, (-1)^{i+1} y_i)$, implying that $C_i$ with even $i$ and the two infinite planes $C_y$ and $C_x$ defined by $x_1 = 0$ and $y_{2N_f} = 0$ respectively are $\mathbb{Z}_2$ invariant, while $C_i$ with odd $i$ are $\mathbb{Z}_2$ reversed. The orbifold fixed plane is thus divided into the two infinite planes $C_y$, $C_x$ and the $N_f - 1$ $\mathbb{P}^1$'s, i.e. $C_i$ with even $i$. We depict the Taub-NUT space schematically in Figure 3.

To summarize, the structure of the eleven dimensional space-time is $\mathbb{R}^4 \times (M^7/\mathbb{Z}_2)$, where $\mathbb{R}^4$ is the ordinary four dimensional space-time, $M^7$ the product of the Taub-NUT space and the $\mathbb{R}^3$ parametrized by $x_7, 8, 9$, and the $\mathbb{Z}_2$ acts on $M^7$ as $(x, y, v, w, x_7) \rightarrow (x, y, -v, -w, -x_7)$.

On this manifold we embed a single M5-brane into which the D4-branes and the NS5-branes combine. The M5-brane worldvolume is of the form $\mathbb{R}^4 \times \Sigma$, where $\Sigma$ is a Riemann surface in $M^7/\mathbb{Z}_2$. Our general procedure to extract the information on the vacuum structure of a supersymmetric field theory from the Riemann surface $\Sigma$ is as follows [20]. We impose an appropriate boundary condition on the M5-brane which is holomorphic with respect to the complex structure of the manifold (non-holomorphic boundary conditions are imposed instead, when we consider the theories with supersymmetry explicitly broken). The M5-brane configurations which are holomorphic everywhere and satisfy the boundary condition ensure supersymmetry preserving vacua to exist. Generically, the holomorphic configurations are parametrized by a number of parameters, which correspond to the moduli space in the field theory. On the other hand, it is possible that no M5-brane can
fulfill the holomorphy and the boundary condition simultaneously. This situation indicates that supersymmetry is \textit{spontaneously} broken, even though we need more information to determine whether the field theory exhibits runaway behaviour or the potential has local minima where supersymmetry is dynamically broken.

As a result of applying the procedure explained above to the $Sp(-1)$ gauge theory, we obtain the unique \textit{holomorphic} M5-brane configuration which consists of two components,

\[
C_L \begin{cases} 
    y = 1, \\
    w = 0,
\end{cases} \quad C_R \begin{cases} 
    x = (\mu^{-1} \tilde{\Lambda}^2)^{2N_f}, \\
    v = 0,
\end{cases}
\]

where $C_L$ and $C_R$ reduce to the NS5-brane and the NS$'5$-brane in Figure 1 in a suitable type IIA limit. One might wonder how these everywhere holomorphic curves are consistent with DSB in the IYIT model. In order to solve this discrepancy, note that the component $C_L$ transversely intersects with the infinite fixed plane $C_y$, sharing $\mathbb{R}^4$ part of their worldvolumes (\(C_R\) is intersecting with $C_x$ in the same manner). The rules of possible intersections between M5-branes and orbifold fixed planes are obtained in [29] by examining the flux quantization condition for the 4-form field strength in M-theory. According to the rules, the transverse intersection between a single M5-brane and an infinite fixed plane is forbidden, while there is no obstruction to the same type of intersection if the infinite fixed plane is screened by a pair of M5-brane. Therefore, supersymmetry is spontaneously broken because the unique holomorphic configuration (2.9) is not allowed. The non-holomorphic M5-brane configuration corresponding to the supersymmetry breaking stable vacuum is expected to exist, although we cannot make any definite statements about its exact form or the vacuum energy, due to the breakdown of the supergravity approximation of M-theory in the vicinity of the orbifold fixed plane.

3 Electric-Magnetic Duality in SQCD with Gauged Global Symmetry

In this section we study the electric-magnetic duality in $N = 1$ $Sp(N_c)$ Supersymmetric QCD in the case where $SO(2N'_c)$ subgroup of the global symmetry is gauged. This duality

\[\text{In the following, the "transverse" intersection between an NS5-brane and the orbifold fixed plane always means this type of intersection.}\]
is useful when we discuss dynamically broken supersymmetry using the technique of M5-brane in the next section, since the M5-brane configuration is more easily obtained from the type IIA brane configuration of the electric theory than from that of the magnetic one, which describes the IYIT model with gauged global symmetry if we put formally $N_c = -1$ as in the previous section. Dualities in supersymmetric QCD with product gauge groups are also discussed in [34] from the ordinary field theory point of view, and in [12, 35, 36] by means of brane configurations.

The matter contents in the electric theory are $2N_f$ quarks $Q^i_a$ in the fundamental representation under $Sp(N_c)$ and a quark $q^a_s$ in the bifundamental representation under $Sp(N_c) \times SO(2N'_c)$ ($a, b = 1, 2, \cdots, 2N_c; s, t = 1, 2, \cdots, 2N'_c$). The anomaly free global symmetry is $SU(2N_f) \times U(1)_R$, and the representation of the quarks under the gauge and the global symmetries are summarized as follows.

$$
\begin{array}{c|cc}
  & Sp(N_c) & SO(2N'_c) \\
\hline
Q & \Box & 1 \\
q & \Box & \Box \\
\end{array}
\begin{array}{cc|c}
  & SU(2N_f) & U(1)_R \\
\hline
1 & 1 + \frac{N'_c(N'_c-1)}{N_fN_c} - \frac{N_c+1}{N_f} & \frac{1}{1 - \frac{N'_c-1}{N_c}} \\
\end{array}
$$

(3.1)

For $N_f + N'_c > N_c + 2$, we suspect that this electric theory has a dual magnetic description with gauge group $Sp(\tilde{N}_c) \times SO(2N'_c)$ where $\tilde{N}_c = N_f + N'_c - N_c - 2$, regarding $SO(2N'_c)$ gauge group as a spectator. The transformation properties of magnetic fields under the gauge and the global symmetries are

$$
\begin{array}{c|cc}
  & Sp(\tilde{N}_c) & SO(2N'_c) \\
\hline
\tilde{Q} & \Box & 1 \\
\tilde{q} & \Box & \Box \\
\Phi & 1 & \Box \\
N & 1 & \Box \\
S & 1 & 1 \\
\end{array}
\begin{array}{cc|c}
  & SU(2N_f) & U(1)_R \\
\hline
1 & 2 + \frac{N'_c(N'_c-1)}{N_fN_c} - \frac{N_c+1}{N_f} - \frac{N'_c-1}{N_c} & \frac{2 + 2N'_c(N'_c-1)}{N_fN_c} - \frac{2N_c+1}{N_f} \\
\end{array}
$$

(3.2)

where $\tilde{Q}^a_i$ and $\tilde{q}^a_s$ are magnetic quarks, and $\Phi^{st}$, $N^{is}$ and $S^{ij}$ are “mesons” described in terms of the electric fields as $\Phi^{st} = q^a q^t$, $N^{is} = Q^i q^s$ and $S^{ij} = Q^i Q^j$. In order for the two theories to flow into the same fixed point in the IR limit, we must also incorporate the following tree-level superpotential:

$$
\tilde{W}_{tree} = \frac{1}{2\mu} S^{ij} \tilde{Q}_i \tilde{Q}_j + \frac{1}{2\mu} \Phi^{st} \tilde{q}_s \tilde{q}_t + \frac{1}{\mu} N^{is} \tilde{Q}_i \tilde{q}_s.
$$

(3.3)
The dimensionful parameter $\mu$ in (3.3) relates the two scales $\Lambda, \Lambda'$ for $Sp(N_c), SO(2N'_c)$ in the electric theory to the scales $\tilde{\Lambda}, \tilde{\Lambda}'$ for $Sp(\tilde{N}_c), SO(2N'_c)$ in the magnetic theory as

$$
\Lambda^{3(N_c+1)-(N_f+N'_c)} \tilde{\Lambda}^{3(\tilde{N}_c+1)-(N_f+N'_c)} = (-1)^{N_f+N'_c-N_c-1} \mu^{N_f+N'_c},
$$

(3.4)

$$
\Lambda^{6(N_c+1)-2(N_f+N'_c)} \tilde{\Lambda}^{6(\tilde{N}_c'-1)-2N_c} = \mu^{2(N_f+N_c+N'_c)} \tilde{\Lambda}^{4(N'_c-1)-2(N_f+N_c)}.
$$

(3.5)

A non-trivial consistency check of the duality is provided by 't Hooft anomaly matching condition. For appropriate choices of $N_f, N_c$, and $N'_c$, we expect that the dual theories have a common moduli space with the origin where the full global symmetry is left unbroken. At least in such a situation, we can verify that the global anomalies match at the origin, from the charge assignments to the fermions in the two theories:

$$
SU(2N_f)^3, \quad SU(2N_f)^2U(1)_R, \quad U(1)^3_R \quad N_c(2N_c + 1) + N'_c(2N'_c - 1) + 4\frac{N'_c(N'_c-1)-N_c(N_c+1)^3}{N_f^2N_c^2} - 4\frac{N'_c(N'_c-1)^3}{N_c^3},
$$

(3.6)

In the remainder of this section, we consider various decoupling limits of the two theories to give further evidence for the duality, and in particular to confirm the coupling matching relations (3.4) and (3.5).

**Mass Deformations for $Q$**

Let us start with deforming the electric theory by adding a mass term for $Q^{2N_f-1}$ and $Q^{2N'_f}$, $W_{tree} = mQ^{2N_f-1}Q^{2N'_f}$. At an energy scale below the quark mass $m$, $Sp(N_c) \times SO(2N'_c)$ gauge group is unbroken and the matter spectrum consists of $2N_f-2$ quarks $Q^i$ and one bifundamental $q$, where $i$ runs over $N_f-1$ light flavours, $i = 1, 2, \cdots, 2N_f-2$. The dynamical scale for $Sp(N_c)$ in the low energy effective theory $\Lambda_L$ is thus determined by the relation

$$
\Lambda_L^{3(N_c+1)-(N_f-1+N'_c)} = m\Lambda^{3(N_c+1)-(N_f+N'_c)},
$$

(3.7)

while $\Lambda'$ remains unchanged. The tree-level superpotential in the magnetic theory takes the form

$$
\tilde{W}_{tree} = \frac{1}{2\mu} S^{ij} \tilde{Q}_i \tilde{Q}_j + mS^{2N_f-1,2N'_f} + \frac{1}{2\mu} \Phi^{st} \tilde{q}_s \tilde{q}_t + \frac{1}{\mu} N^{is} \tilde{Q}_i \tilde{q}_s.
$$

(3.8)

\(^5\) In this section, we omit the irrelevant numerical factors from the scale matching relations.
In the magnetic theory, \( \langle \tilde{Q}_{2N_f-1} \tilde{Q}_{2N_f} \rangle = -\mu m \), which means that by an appropriate gauge ans global rotation we can put \( \langle \tilde{Q}_{2N_f-1}^2 \rangle = (-\mu m)^{1/2} \delta_{2N_c-1}^2 \), \( \langle \tilde{Q}_{2N_f}^2 \rangle = (-\mu m)^{1/2} \delta_{2N_c}^2 \) with the vev’s of the other quarks vanishing. These vev’s break \( Sp(\tilde{N}_c) \) to \( Sp(\tilde{N}_c - 1) \) and give rise to the mass squared \( -\mu m \) of \( S^{2N_f-1}, S^{2N_f}, S^{2N_f-1,2N_f} \) and \( \tilde{Q}_{2N_f-1}, \tilde{Q}_{2N_f} \). Matching \( Sp(\tilde{N}_c) \) coupling at the scale \( -\mu m \), we obtain \( \tilde{\Lambda}_L \), the scale for \( Sp(\tilde{N}_c) \) in the low energy theory, as

\[
\tilde{\Lambda}_L^{3(\tilde{N}_c-1)+1 - (N_f-1+N_c')} = -(\mu m)^{-1} \tilde{\Lambda}_L^{3(N_c+1) - (N_f+N_c')}.
\]  

From eqs. (3.7) and (3.9) we reproduce eq. (3.4) with \( N_f \) and \( \tilde{N}_c \) replaced by \( N_f - 1 \) and \( \tilde{N}_c - 1 \). Due to the quark vev’s, \( SO(2N_f') \) fundamentals \( q^{2\tilde{N}_c-1}, q^{2\tilde{N}_c} \) and \( N^{2N_f-1}, N^{2N_f} \) also acquire the mass squared \( -\mu m \), leading to the relation between \( \tilde{\Lambda}' \) and the scale \( \tilde{\Lambda}_L' \) in the low energy theory

\[
\tilde{\Lambda}_L'^{(N_f'-1)-2(N_f-1+\tilde{N}_c-1)} = (\mu m)^2 \tilde{\Lambda}_L'^{(N_f'-1)-2(N_f+\tilde{N}_c)}.
\]  

Using eqs. (3.7) and (3.10), we find that the scales in the low energy effective theory satisfy eq. (3.6).

**Flat Directions for Q**

We can analyze another decoupling limit by the vev’s of the electric quarks \( Q \) along their \( D \)-flat directions. First examine the case where \( \langle Q^{2N_f-1} Q^{2N_f} \rangle = a^2 \) and all the other meson vev’s vanish. Below the scale \( a \) the electric theory flows to \( Sp(N_c - 1) \times SO(2N_f') \) SQCD with \( 2N_f - 2 \) quarks \( Q_{\tilde{a}} \), one bifundamental \( q_{\tilde{a}}^s \) (here, \( \tilde{a} \) refers to \( N_c - 1 \) colors, \( \tilde{a} = 1, 2, \cdots, 2N_c - 2 \)) and two \( SO(2N_f') \) fundamental quarks \( q_{2N_c-1}, q_{2N_c} \) with the dynamical scale \( \Lambda_L \) for \( Sp(N_c - 1) \) given by

\[
\Lambda_L^{3(N_c-1)+1 - (N_f-1+N_c')} = a^{-2} \Lambda_L^{3(N_c+1) - (N_f+N_c')}.
\]  

In the magnetic theory, \( \langle S^{2N_f-1,2N_f} \rangle = a^2 \) induces the mass \( \mu^{-1}a^2 \) of \( \tilde{Q}_{2N_f-1} \) and \( \tilde{Q}_{2N_f} \). Therefore, the dynamical scale \( \tilde{\Lambda}_L \) for \( Sp(\tilde{N}_c) \) in the low energy magnetic theory is related to \( \tilde{\Lambda} \) by

\[
\tilde{\Lambda}_L^{3(\tilde{N}_c-1)+1 - (N_f-1+N_c')} = \mu^{-1} a^2 \tilde{\Lambda}_L^{3(\tilde{N}_c+1) - (N_f+N_c')}.
\]  

which, combined with (3.11), shows that the relation (3.4) is preserved.
On the other hand, eq. (3.5) is not recovered in the low energy effective theory, since all the spectra with non-trivial $SO(2N'_c)$ representation do not decouple in both of the dual theories. In other words, the effective dual theories in a low energy scale are not described by just replacing $N_f$ and $N_c$ by $N_f - 1$ and $N_c - 1$ in the high energy dual theories. This fact is, for instance, reflected on the electric quarks $q_{2N_c-1}, q_{2N_c}$ which are coupled to the other fields via gauged $SO(2N'_c)$ interactions.

As a further comparison of the dual theories, we take one particular choice of the number of flavours, $N_f = N_c$, and let the electric quarks $Q$ develop such vev’s as to satisfy $\text{Pf}(Q^iQ^j) \neq 0$. Then, in the low energy theory $Sp(N_c)$ gauge group is completely broken, and remain $2N_c$ quarks $q_a$ in fundamental representation under $SO(2N'_c)$ ($a$ is now flavour index) as light matter. If $N_c \leq N'_c - 2$, strong coupling effects of $SO(2N'_c)$ gauge dynamics generate non-perturbative superpotential

$$W_{eff} = (N'_c - N_c - 1) \left[ \frac{\Lambda^{6(N'_c-1)-2N_c}}{\det(q_aq_b)} \right]^{\frac{1}{2(N'_c-1)-2N_c}}. \quad (3.13)$$

In the magnetic description, all the $2N_f$ quarks $\tilde{Q}$ acquire mass due to $\text{Pf}S^{ij} \neq 0$. Consequently, the number of $Sp(N_c)$ fundamental quarks $\tilde{q}_s$ in the low energy theory is $2N'_c = 2(N_c+2)$, which indicates that s-confinement of $Sp(N_c)$ charge occurs [7]. Using the notation $\tilde{\Phi}_{st} = \tilde{q}_a\tilde{q}_t$ as “gauge-invariant” composite states of the fundamental quarks $\tilde{q}$, s-confining effective superpotential (plus the tree level one with decoupled fields eliminated) is given by

$$\tilde{W}_{eff} = -\frac{\text{Pf}\tilde{\Phi}}{\Lambda^2_{L2^{N_c+1}}} + \frac{1}{2\mu}\Phi\tilde{\Phi}, \quad (3.14)$$

where $\Lambda^2_{L2^{N_c+1}} = \text{Pf}(\mu^{-1}S)\Lambda^{2^{N_c+1}-N_f}$. At the scale $\tilde{\Lambda}_L$, the elementary quarks $\tilde{q}$ combine into mesons $\tilde{\Phi}$, and decouple immediately together with the $SO(2N'_c)$ adjoint $\Phi$ because of the mass term in the r.h.s. of eq. (3.14) (note that $\Phi/\mu$ and $\tilde{\Phi}/\tilde{\Lambda}_L$ are canonically normalized fields). Eventually, at an energy scale lower than $\tilde{\Lambda}_L$ we are left with $N_f = N_c \leq N'_c - 2$ flavours of $SO(2N'_c)$ fundamentals $N^f$ which give rise to the non-perturbative \footnote{For $N_c = N'_c - 2$, there is another phase with no superpotential generated even non-perturbatively [8].}
superpotential

\[ \tilde{W}_{\text{eff}} = (N_c' - N_f - 1) \left[ \frac{\tilde{\Lambda}^{6(N_c' - 1) - 2N_f}}{\det(\mu^{-2} N^i N^j)} \right]^{\frac{1}{2(N_c' - 1) - 2N_f}}, \]  

(3.15)

where \( \tilde{\Lambda}^{6(N_c' - 1) - 2N_f} = \tilde{\Lambda}_L^{4N_c + 2}\Lambda^{4(N_c' - 1) - 2(N_f + N_c)} \). Applying the expression \( N^i s = J^a Q^i_a q^a \) and the scale matching relations (3.4) and (3.5) to (3.15), the effective superpotential in the electric theory (3.13) is correctly reproduced.

**Flat Directions for \( q \)**

Here, we consider the flat directions for the bifundamental quark \( q \), focusing our attention on one specific example where \( N_c > N_c' \) and \( \langle q^a_s \rangle = a\delta^a_s \) hold. In the electric theory, the vev’s for \( q \) break the gauge group \( Sp(N_c) \times SO(2N_c') \) to \( Sp(N_c - N_c') \times U(N_c') \), leaving in the low energy effective theory the field contents

\[
\begin{array}{c|cc|c}
R & Sp(N_c - N_c') & U(N_c') & SU(2N_f) \\
\hline
R & 1 & 1 & 0 \\
r & 1 & 1 & 0 \\
\bar{r} & 1 & 1 & 0 \\
\Phi_U & 1 & adj & 1 \\
\end{array}
\]  

(3.16)

where \( Q \) is decomposed into \( R, r, \) and \( \bar{r} \), and \( \Phi_U \) comes from \( q \). Note that the effective theory splits into the two systems which do not interact with one another, namely the fields with \( Sp(N_c - N_c') \) quantum number and the fields with \( U(N_c') \) quantum number. The coupling matching at the scale \( a \) determines the scale \( \Lambda_L \) for \( Sp(N_c - N_c') \) and the scale \( \Lambda_{SU} \) for \( SU(N_c') \) factor of \( U(N_c') \),

\[
\Lambda_L^{3(N_c - N_c' + 1) - 2N_f} = a^{-2N_c'} \Lambda^{3(N_c + 1) - (N_f + N_c)},
\]  

(3.17)

\[
\Lambda_{SU}^{2N_c' - 2N_f} = a^{-4N_c - 2N_c'} \Lambda^{6(N_c + 1) - 2(N_f + N_c')} \Lambda^{6(N_c' - 1) - 2N_c'}.
\]  

(3.18)

Turning on \( \langle q^a_s \rangle = a\delta^a_s \) induces \( \langle \Phi^{sf} \rangle = a^2(1_{N_f'} \otimes i\sigma_2)^{st} \), and the gauged global symmetry \( SO(2N_c') \) is broken to \( U(N_c') \) in the magnetic theory. The low energy magnetic theory is described by the following light fields

\[
\begin{array}{c|cc|c}
R & \tilde{\Phi} & \tilde{U} & \tilde{S} \\
\hline
\tilde{R} & 1 & 1 & 1 \\
\tilde{r} & 1 & 1 & 1 \\
\tilde{\bar{r}} & 1 & 1 & 1 \\
\tilde{\Phi}_U & 1 & adj & 1 \\
S & 1 & 1 & 1 \\
\end{array}
\]  

(3.19)
and the tree-level superpotential

\[ W = \frac{1}{2\mu} S^{ij} \tilde{R}_i \tilde{R}_j. \]  

(3.20)

Similarly to the electric theory, the effective theory of the magnetic description consists of two parts with irrelevant interaction between them. \( \tilde{R} \) and \( S \) with the Yukawa coupling (3.20) define \( Sp(\tilde{N}_c) \) gauge theory, which is dual to the \( Sp(N_c - N'_c) \) effective theory in the electric description. The other part is the \( U(N'_c) \) gauge theory with the matter contents in the same representation as those in the electric theory, the correspondence between the electric and the magnetic fields being \( \tilde{r} = (\mu^{-1}a)r, \tilde{\bar{r}} = (\mu^{-1}a)\bar{r}, \) and \( \tilde{\Phi}_U = (\mu^{-1}a)\Phi_U. \)

The scales \( \tilde{\Lambda}_L \) and \( \tilde{\Lambda}_{SU} \) for the two factors in \( Sp(\tilde{N}_c) \times U(N'_c) \) are related to the ones in the high energy theory as

\[ \tilde{\Lambda}_3^{3(N_c+1)-N_f} = (\mu^{-1}a^2)^{N'_c} \tilde{\Lambda}_3^{3(N_c+1)-(N_f+N'_c)}, \]  

(3.21)

\[ \tilde{\Lambda}_{SU}^{2N'_c-2N_f} = (\mu^{-1}a^2)^{2N_f-2N_c} \tilde{\Lambda}_{SU}^{4(N_c-1)-2(N_f+N_c)}. \]  

(3.22)

From (3.17) and (3.21), we can verify that the relation (3.4) is indeed preserved in the low energy dual theories. However, the two scales for \( SU(N'_c) \) in the dual theories must be identical only up to a constant in order for the relation (3.5) to be valid:

\[ \tilde{\Lambda}_{SU}^{2N'_c-2N_f} = (\mu^{-1}a)^{4N_f+2N'_c} \tilde{\Lambda}_{SU}^{2N'_c-2N_f}. \]  

(3.23)

The origin of the multiplicative factor in (3.23) is the difference in the normalization factors of the matter fields in the electric and the magnetic theories [37]. Suppose that we have canonically normalized chiral superfields \( \phi_i \) in the representation \( R_i \) of a gauge group \( G \). If we rescale \( \phi_i = Z_i^{-1/2} \tilde{\phi}_i \) and vary the cutoff to renormalize \( D \)-terms so that \( \tilde{\phi}_i \) should have the canonical kinetic term, the original scale for the gauge group \( G, \Lambda_G, \) is changed to the new one \( \tilde{\Lambda}_G \) due to the anomalous Jacobian \( |D(Z_i^{-1/2} \tilde{\phi}_i)/D\tilde{\phi}_i|^2 \neq 1 \), as

\[ \tilde{\Lambda}_G^{-b} = \Lambda_G^{-b} \prod_i Z_i^{T(R_i)}, \]  

(3.24)

where \( b \) is the coefficient of the one-loop \( \beta \) function and \( T(R) \) the Dynkin index of the representation \( R \). In the present case, \( G = SU(N'_c), b = -2N'_c + 2N_f, T(\square) = T(\square') = \frac{1}{2}, T(adj) = N'_c \) and \( Z_i = (\mu^{-1}a)^2 \) for all the matter. Substituting these in (3.24) yields (3.23).
4 Brane Realization of Dynamical Supersymmetry Breaking

In this section we gauge $SO(2N'_c)$ ($1 \leq N'_c \leq \tilde{N}_c$) subgroup of the flavour group $SU(2\tilde{N}_c + 2)$ in the IYIT model, and investigate the resulting theories by the brane method. In [20] was analyzed the case in which an maximal subgroup $SO(2\tilde{N}_c + 2) \subset SU(2\tilde{N}_c + 2)$ is gauged, the brane prediction agreeing with the field theory that the model exhibits runaway behaviour.

We can obtain the type IIA brane configuration realizing the IYIT model with gauged $SO(2N'_c)$ subgroup of the global symmetry (the magnetic description of the theory in the previous section), by replacing the leftmost $2N'_c$ of the $2N_f$ D6-branes in Figure 2 with an NS'5-brane. Then we take $N_f - N'_c \to N_f$ just as in the previous section, so that there should be $2N_f$ D6-branes, namely we should have $SU(2N_f)$ global symmetry. We can identify the fields (3.2) and the tree-level superpotential (3.3) in the magnetic theory from the resulting brane configuration depicted in Figure 4.

In the brane language, the duality transformation to the electric theory with respect to $Sp(\tilde{N}_c)$ gauge group corresponds to translating the NS5-brane into between the left NS'5-brane and the leftmost D6-brane. Performing the procedure explained in section 2 in reverse order, we have the type IIA brane configuration for the electric theory as shown
Figure 5: Type IIA brane configuration for the electric $Sp(N_c) \times SO(2N'_c)$ SQCD.

in Figure 5.

Let us now construct the corresponding M5-brane configuration by rotating the $N = 2$ configuration which is given by replacing both of the two NS'5-branes with two NS5-branes. For a finite mass $\mu_a$ for the adjoint chiral superfields in $N = 2 Sp(N_c)$ and $SO(2N'_c)$ vector multiplets, the M5-brane’s boundary conditions are given by

\begin{align}
(I) & \quad v \to \infty, \ w \sim \mu_a v, \ y \sim \Lambda_{N=2}^{N_c-4N'_c}, \ v^2 \sim v^{2N_c+2N'_c}, \\
(II) & \quad v \to \infty, \ w \sim 0, \ y \sim v^{2N_c+2N'_c}, \\
(III) & \quad v \to \infty, \ w \sim \mu_a v, \ x \sim \Lambda_{N=2}^{N_c+1+2N'_c}, \\
(IV) & \quad y \to \infty, \ w^2 \sim 0, \ v^2 \sim 0, \ (4.1)
\end{align}

where $\Lambda_{N=2}$ and $\Lambda'_{N=2}$ are the dynamical scales for $Sp(N_c)$ and $SO(2N'_c)$ gauge groups in the presence of $N = 2$ supersymmetry. From the type IIA point of view, the conditions (I), (II) and (III) characterize the bending pattern of the three NS5-branes due to D4-branes’ tension, and the condition (IV) indicates that the orbifold fixed plane is screened by an M5-brane and its mirror image in the region $y \sim \infty$, ensuring that the O4-plane has RR charge +1 on the left of the leftmost NS5-brane. Generally, the rotated curve is parametrized by two holomorphic equations: $F(y, v) = 0$ which characterizes the $N = 2$ configuration before the rotation, and $w = G(y, v)$ which implies that $w$ is a meromorphic function over the Riemann surface. If the M5-brane which has the asymptotic behaviour (4.1) consists of several components, we must determine the two functions $f$ and $g$ for each of the components. We consider the following five classes distinguished by the number
of M5-brane components which satisfy the first three boundary conditions, (I), (II) and (III). In the analysis, we regard the curves as compactified Riemann surfaces by adding a point to each of the asymptotic regions (I), (II) and (III).

**one component case**

Let us first examine if it is possible to enable only one component of M5-brane to fulfill the three conditions (I)∼(III) simultaneously. We introduce the complex parameter $z = w - \mu_a v$ which is, similarly to $w$, a meromorphic function over the surface. Then, the conditions (I)∼(III) inform us that $z$ has a simple pole at (II) and is finite at all the other points. The existence of such a meromorphic function forces the surface to be $\mathbb{P}^1$ and $z$ to be a global coordinate parametrizing the entire surface, $\mathbb{P}^1$. However, this is incompatible with the boundary conditions and $\mathbb{Z}_2$ symmetry which require $z$ to have the same value $z = 0$ at the two distinct points (I) and (III) on the surface. Therefore, we conclude that there is no holomorphically embedded single M5-brane with the boundary behaviour (I)∼(III).

**two components case 1**

We consider here the case in which there are two components, $C_I$ satisfying (I) and $C_{II-III}$ satisfying (II) and (III) simultaneously. Let us begin with $C_I$. $C_I$ is parametrized by two equations of the forms $y = f(v), w = g(v)$, where $f$ and $g$ are some meromorphic functions. If $f$ has a zero at a finite value of $v = v_0 \neq 0$, $x$ goes to infinity for $v \to v_0$ as can be seen from (2.7). This means that a semi-infinite D4-brane is stuck on the rightmost NS5-brane from the right, which is not contained in the type IIA brane configuration we started with. Thus, $f$ can have zeroes only at $v = 0$ or $v = \infty$. Besides, $f$ can have poles only at $v = 0$ or $v = \infty$ so that there should be no D4-brane extending semi-infinitely towards $y \to \infty$. $f$ is therefore a (non-negative or negative) power of $v$. $g$ must also be a power of $v$ to avoid unexpected boundary behaviour of the M5-brane. These restrictions uniquely determine $C_I$,

$$C_I \left\{ \begin{array}{l} y = \Lambda_{N=2}^{(N_e'-1)+2N_e} v^{2(N_e'-1)} \\ w = \mu_a v. \end{array} \right. \quad (4.2)$$

---

7 In the following of this paper, the $C$'s with roman subscripts refer to the M5-brane components which satisfy the boundary conditions specified by the subscripts.
We now turn to $C_{\text{II-III}}$. The same reasoning as explained in the previous case applied to the boundary conditions (II) and (III) shows that $C_{\text{II-III}}$ is a $\mathbb{P}^1$ with well-defined coordinate $w$. Hence, $v$ and $x$ are expressed as meromorphic functions of $w$, which are determined by the boundary conditions and the $\mathbb{Z}_2$ symmetry as

\begin{equation}
C_{\text{II-III}} \left\{ \begin{array}{l}
v = \mu_a^{-1}w^{-1}(w^2 - w_0^2), \\
x = \mu_a^{-2(N_c+1)}\Lambda_{N=2}^{-4(N_f+1)+2(N_f+N_c')} w^{2(N_c+1) - 2(N_f+N_c')}(w^2 - w_0^2)^{N_f+N_c'},
\end{array} \right. (4.3)
\end{equation}

where $(-w_0^2)^{N_f+N_c'-2(N_c+1)} = \mu_a^{2(N_f+N_c')-4(N_c+1)}\Lambda_{N=2}^{-4(N_c+1)+2(N_f+N_c')}$. Since both of the components $C_1$ and $C_{\text{II-III}}$ do not satisfy the boundary condition (IV), we require an additional component $C_{\text{IV}}$,

\begin{equation}
C_{\text{IV}} \left\{ \begin{array}{l}
x_1^2 = 0, \\
w = 0.
\end{array} \right. (4.4)
\end{equation}

Although $C_1$ intersects with the infinite orbifold fixed plane $C_y$ transversely for $N_c' = 1$, supersymmetry is not broken because the component $C_{\text{IV}}$ is a pair of M5-brane screening $C_y$. Since all the three components $C_1$, $C_{\text{II-III}}$ and $C_{\text{IV}}$ are holomorphic, they define supersymmetric vacua for a finite $\mu_a$.

In order to obtain the M5-brane configuration corresponding to the electric theory with the adjoint fields completely decoupled, we take $\mu_a \to \infty$ limit fixing the $N = 1$ scales $\Lambda$ and $\Lambda'$ which are related to the $N = 2$ scales by

\begin{equation}
\Lambda^{3(N_c+1)-(N_f+N_c')} = \mu_a^{N_c+1}\Lambda_{N=2}^{2(N_c+1)-(N_f+N_c')}, (4.5)
\end{equation}

\begin{equation}
\Lambda^{6(N_c'-1)-2N_c} = \mu_a^{2(N_c'-1)}\Lambda_{N=2}^{4(N_c'-1)-2N_c}. (4.6)
\end{equation}

For $N_c = -1$ which realizes the IYIT model with gauged global symmetry, $(-w_0^2)^{N_f+N_c'} = (\mu_a^2\Lambda^2)^{N_f+N_c'}$ diverges as $\mu_a \to \infty$. Therefore, the curve $C_{\text{II-III}}$ is infinitely elongated, implying that the supersymmetric vacua which exist for a finite adjoint mass $\mu_a$ run away towards far infinity in the limit $\mu_a \to \infty$.

**two components case 2**

Let us consider the M5-brane configuration consisting of two components $C_{\text{II}}$ and $C_{\text{I-III}}$. Following the argument on $C_1$ given in the previous case, $C_{\text{II}}$ can be determined as

\begin{equation}
C_{\text{II}} \left\{ \begin{array}{l}
y = \nu^{2N_c+2-2N_c'}, \\
w = 0.
\end{array} \right. (4.7)
\end{equation}
The holomorphic function $F(y, v)$ for $C_{\text{I-III}}$ is a second order polynomial of $y$ with its two roots behave like (I) and (III) for $v \to \infty$. We thus obtain

$$C_{\text{I-III}} \left\{ \begin{array}{l}
\Lambda_{N=2}^{4(N_c'-1)-2N_c} y^2 - P_{N_c'-1} (v^2) y + \Lambda_{N=2}^{4(N_c+1)-2(N_f+N_c')} v^2(N_f+N_c'-N_c-2) = 0, \\
w = \mu_a v,
\end{array} \right. \tag{4.8}$$

where $P_{N_c'-1} (v^2) = v^{2(N_c'-1)} + \sum_{n=0}^{N_c'-2} a_n v^{2n}$ is a polynomial of the $(N_c'-1)$-th order.

Let us put $N_c = -1$ in (4.7). Then, $C_{\text{II}}$ satisfies the condition (IV) beside (II) for $N_c' = 1$, while $C_{\text{II}}$ screens $C_y$ by more than two M5-branes for $N_c' \geq 2$, being incompatible with the condition (IV). Putting $N_c = -1$, $N_c' = 1$ and taking the limit $\mu_a \to \infty$ with the $N = 1$ scales given in (4.5) and (4.6) fixed, we find that $C_{\text{I-III}}$ breaks up into the two components $C_{\text{I}}$ and $C_{\text{III}}$.

The resultant configuration for $N_c' = 1$ is thus

$$C_{\text{I}} \left\{ \begin{array}{l}
y = \Lambda_{c}^{-2}, \\
v = 0,
\end{array} \right. \quad C_{\text{II}} \left\{ \begin{array}{l}
y = v^{-2}, \\
w = 0,
\end{array} \right. \quad C_{\text{III}} \left\{ \begin{array}{l}
x = \Lambda_{c}^{2(N_f+1)}, \\
v = 0,
\end{array} \right. \tag{4.9}$$

and there is no holomorphic configuration for $N_c' \geq 2$.

**two components case 3**

The last possibility with two M5-brane components is that $C_{\text{III}}$ and $C_{\text{I-II}}$ constitute the entire configuration. We can determine the forms of $C_{\text{III}}$ and $C_{\text{I-II}}$ in the same manner as in the previous cases,

$$C_{\text{III}} \left\{ \begin{array}{l}
x = \Lambda_{N=2}^{-4(N_c+1)+2(N_f+N_c')} v^{2(N_c+1)}, \\
w = \mu_a v,
\end{array} \right. \tag{4.10}$$

$$C_{\text{I-II}} \left\{ \begin{array}{l}
v = \mu_a^{-1} w^{-1} (w^2 - w_0^2), \\
y = \mu_a^{-2(N_c'-1)} \Lambda_{N=2}^{-4(N_c'-1)+2N_c'} w^{2(N_c'-1)-2N_c} (w^2 - w_0^2)^{N_c},
\end{array} \right. \tag{4.11}$$

where $(-w_0^2)^{N_c-2(N_c'-1)} = \mu_a^{2N_c-4(N_c'-1)} \Lambda_{N=2}^{4(N_c'-1)-2N_c}$. However, putting $N_c = -1$ in (4.11) indicates that $y \to \infty$ for $w^2 \to w_0^2$. Therefore, this possibility is excluded by the requirement that no semi-infinite D4-brane is present.

**three components case**

It is also possible that the M5-brane configuration consists of three components $C_{\text{I}}$, $C_{\text{II}}$, and $C_{\text{III}}$. As far as $N_c = -1$, this configuration is consistent with the boundary condition
Figure 6: M5-brane configuration for the electric $Sp(-1) \times U(1)$ gauge theory.

(IV) only for $N'_c = 1$. For $N_c = -1, N'_c = 1$, the three components reduce to (1.9) in the limit $\mu_a \to \infty$.

To summarize, there exists no holomorphic M5-brane configuration for the IYIT model with gauged $SO(2N'_c)$ ($N'_c \geq 2$) subgroup of the global symmetry. On the other hand, if the smallest subgroup $U(1) \simeq SO(2)$ is gauged, we can construct uniquely the holomorphic configuration (1.9) which we depict in Figure 6. Since $C_I$ and $C_{III}$ is intersecting transversely with $C_y$ and $C_x$ respectively, the theory possesses a stable vacuum with dynamically broken supersymmetry.

Finally, we check the validity of constructing M5-brane configuration from the electric $Sp(-1)$ theory, by demonstrating that the configuration (1.9) can be smoothly deformed to the one which resembles the magnetic type IIA configuration shown in Figure 4. We expect that the magnetic description becomes manifest for a small $Sp(\tilde{N}_c)$ scale $\tilde{\Lambda}$. For the purpose of taking the limit $\tilde{\Lambda} \to 0$, it is convenient to reparametrize the variables $x$ and $y$ in the way

\[
\begin{align*}
\tilde{x} &= \Lambda^{6(N_c+1)-2(N_f+N'_c)}x, \\
\tilde{y} &= \Lambda^{-6(N_c+1)+2(N_f+N'_c)}y.
\end{align*}
\] (4.12)

Under the condition $N_c = -1$ and $N'_c = 1$ we are now interested in, $C_I$, $C_{II}$ and $C_{III}$ in
where we have used the relation between the electric and the magnetic scales, (3.4) and (3.5). We take the magnetic limit by pushing $\tilde{\Lambda} \to 0$ while the magnetic $SO(2N_f')$ scale $\Lambda'$ is fixed, that is, we deform only the component $C_{II}$. The first equation in $C_{II}$ is written in terms of the coordinates $(\tilde{x}_i, \tilde{y}_i)$ of the patch $U_i$ in the Taub-NUT space as

$$\tilde{x}_i^{i+1}\tilde{y}_i^{i+2} = (\mu^{-1}\tilde{\Lambda}^2)^{2(\tilde{N}_c+1)}. \quad (4.14)$$

From this, we learn that $C_{II}$ is approximately wrapped on $C_y, C_1, C_2, \cdots, C_{2N_f-1}, C_x$ with multiplicities $2, 3, 4, \cdots, 2N_f + 1, 2N_f + 2$, for a sufficiently small $\tilde{\Lambda}$. As shown in Figure 7, the M5-brane configuration in the magnetic limit correctly reproduces the magnetic type IIA configuration with $N_f + N'_c = \tilde{N}_c + 1$ and $N'_c = 1$.

5 Conclusions

We have investigated $Sp(N_c) \times SO(2N'_c)$ SQCD which is obtained by gauging $SO(2N'_c)$ subgroup of the flavour symmetry in the IYIT model. For non-Abelian gauge group

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Figure 7: M5-brane configuration for the magnetic $Sp(\tilde{N}_c) \times U(1)$ gauge theory.

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\[ \text{Figure 7: M5-brane configuration for the magnetic } Sp(\tilde{N}_c) \times U(1) \text{ gauge theory.} \]
SO(2N'c) (N'c ≥ 2), the M5-brane configuration is necessarily non-holomorphic. Nevertheless, it is difficult in the brane framework to verify the presence of supersymmetry breaking local minima as stressed in section 2. On the contrary, in the case of gauged $U(1)$ symmetry we have reached the conclusion that the supersymmetry breaking stable vacuum exists, because the corresponding M5-brane configuration cannot avoid inconsistent configurations which we have encountered also in the original IYIT model. This observation will support the idea to construct the gauge-mediated models based on the IYIT model which plays the role of the hidden sector.

So far, the analyses of supersymmetric gauge theories via branes in string theories or M-theory have contributed to the understanding of brane dynamics for the most part, not of field theories themselves. That is, while we have successfully derived various properties of branes such as s-rule [38, 39], the inconsistency of t-configuration, etc. to account for the field theory results, there are few known examples of new consequences on field theories which are extracted from brane configurations. As a counterexample to this, we have proposed in this paper that branes can provide us with a new method for searching the models exhibiting DSB. However, t-configuration to which we have attributed DSB in field theories, is presumably one example of the inconsistent brane configurations which cause supersymmetry to be broken spontaneously. Studying the other models which are known to realize DSB in field theories, we may find other kinds of inconsistent configuration which allow us to discover new DSB models as we have achieved in this work.

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