Adaptive Control of Robot Manipulators With Uncertain Kinematics and Dynamics

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Abstract

In this paper, we investigate the adaptive control problem for robotic systems with both the uncertain kinematics and dynamics. By a new formulation of the unknown kinematic system, we propose an adaptive control scheme that includes a new kinematic parameter adaptation law to realize the objective of task-space trajectory tracking irrespective of the uncertain kinematics and dynamics. Unlike most existing results that rely on the approximate transpose Jacobian feedback, the proposed controller employs the inverse Jacobian feedback. The new kinematic parameter adaptation law and the inverse Jacobian feedback supplies the proposed control scheme with the desirable decomposition property and the convenient accommodation of the performance issues. The performance of the proposed control is shown by numerical simulations.

Index Terms

Adaptive control; Kinematic uncertainty; Robot manipulators; Performance.

I. INTRODUCTION

The study on the adaptive control of robot manipulators with dynamic parameter uncertainty has a long and rich history (see, e.g., the early results in [1], [2], [3]), and the employment of adaptive control provides robot manipulators with the ability of performing tasks in the unknown environment. The recent advances in adaptive robot control occur in [4], [5], [6], [7], [8] aiming at handling the kinematic parameter uncertainty. Kinematic uncertainty is frequently encountered as the robots perform various work in the task space (e.g., Cartesian space or image space) (see,
e.g., [4], [9]), among which is the now actively studied visual servoing problem (see, e.g., [9], [10], [11]). These control schemes (e.g., [4], [5], [6], [7], [8], [10], [11]) are characterized by the use of an approximate Jacobian matrix (due to the kinematics uncertainty), and the prominent part of the control scheme may be the approximate transpose Jacobian control with/without a kinematic parameter adaptation law. In particular, the transpose Jacobian based adaptive tracking control (e.g., [5], [6], [7], [10], [11]) inherits the advantage of the Slotine and Li adaptive scheme [2], i.e., neither measurement of joint acceleration nor inversion of the estimated inertia matrix is required.

At the present stage, one may say that the stability properties of the adaptive Jacobian control system under both the uncertain kinematics and dynamics are fully addressed, as can be seen in the above mentioned results, yet, it remains unclear about the performance of the system in the sense that some performance issues regarding, e.g., tracking accuracy and transient response, are not adequately studied. In fact, the performance of the now commonly adopted transpose Jacobian feedback (e.g., [5], [6], [7], [10], [11]), as stated in [12], is not desirable especially when the manipulator moves in a large range although the transpose Jacobian feedback for robot task-space control problem shows excellent stability property (refer to the pioneering work in [13] on the regulation problem and to [5], [6] on the tracking problem). Another commonly adopted task-space control approach is inverse Jacobian feedback (see, e.g., [12]), and the stability analysis of the inverse Jacobian feedback for regulation problem is given in [14], which seems much more involved than that of its counterpart (i.e., transpose Jacobian feedback).

It is well known that the performance of a linear time-invariant system is ensured by appropriately designating the poles of the closed-loop system. For the nonlinear robotic system, this is almost not achievable except for the known parameter case (e.g., the standard computed torque control can result in a linear error dynamics with guaranteed performance). Let us now contemplate the standard control problem for a frictionless mass that is governed by $\ddot{y} = u$, where $y \in R$ denotes the position of the mass, $m$ the mass and $u \in R$ the control input. From the standard linear system theory, if the control $u$ takes a PD action, the design of the gains must take into account the mass property of the system and one advisable design is to choose mass-dependent gains. This standard idea has already appeared in the robot control problem with or without dynamic uncertainties, e.g., the computed torque control actually takes inertia-matrix-dependent PD control plus certain feedforward terms, and the adaptive control in [15]...
chooses the feedback gain based on the estimated inertia matrix (see [15, Sec. 3.2]). However, it is unclear how to ensure the performance of the robot system under both the kinematic and dynamic uncertainties.

On the other hand, we know that kinematic uncertainty is, in certain sense, different from dynamic uncertainty since kinematic uncertainty occurs at the kinematic system (see, e.g., [4]). In our opinion, decomposition of the handling of the kinematic and dynamic uncertainties is highly preferred, and due to the nature of the kinematic uncertainty, the best position for handling the kinematic uncertainty is within the kinematic system. Furthermore, it is well known that the control objective at the kinematic level is to design a suitable joint velocity so that the task-space position converges to the desired one, and the available information for the design of the joint velocity is the joint-space/task-space position (without involving joint velocity at best). However, the kinematic parameter adaptation in the existing schemes (e.g., [5], [6], [7], [16]) are all based on a kinematic regressor that depends on the joint velocity. This motivates us to wonder whether it is possible to avoid the use of joint velocity in the kinematic parameter adaptation.

In this paper, we propose a new solution to the adaptive control problem for robots with both the uncertain kinematics and dynamics, and this new solution is derived by a new formulation of the unknown kinematic system (different from the results in, e.g., [5], [6], [7], [16]). The proposed control law employs the inverse Jacobian control rather than the approximate transpose Jacobian control, and can ensure the performance of the closed-loop system with essentially the same modification as in [15]. The proposed kinematic parameter adaptation law, unlike those in [5], [6], [7], [16], adopts a new kinematic regressor matrix that depends on the joint reference velocity rather than the joint velocity (which means that this regressor depends on the kinematic parameter estimate and thus is adaptive). The superior/desirable properties of the proposed scheme are summarized as follows.

1) It realizes decomposition of the designs of the kinematic and dynamic loops (i.e., the kinematic loop gives a joint reference velocity, and this reference velocity then acts as the joint velocity command for the dynamic loop) thanks to the employment of a new kinematic regressor matrix and of a control law (with the same structure as the Slotine and Li adaptive scheme in the task space [2, Sec. 3]) that does not use the approximate transpose Jacobian matrix, while the two designs are mixed in existing results due to the adoption of approximate transpose Jacobian feedback (e.g., [5], [6], [7]) and of the
joint-velocity-dependent kinematic regressor (e.g., \cite{5, 6, 7, 16});

2) the proposed control with appropriate modifications that follow the result in \cite{15} ensures
the performance of the closed-loop system, extending the performance guaranteed scheme
in \cite{15} to be capable of handling both the kinematic and dynamic uncertainties, and it is
also shown that even under constant-gain feedback, the proposed control shall give better
performance than the approximate transpose Jacobian feedback.

Here, we would like to emphasize that the feature given in 1) becomes more prominent in
industrial robotic applications in that joint velocity control mode is very common in most
industrial manipulators. Under the joint velocity control mode, we cannot modify the joint
servoing module and what we can design is the joint velocity command. The decomposition
property of the proposed controller makes one reduced case of our main result serve well for
this application scenario [i.e., taking the joint reference velocity as the joint velocity command
of the joint servoing module (see Remark 2)], while the adaptive transpose Jacobian control does
not fit this circumstance due to the transpose Jacobian feedback in the torque input.

II. Robot Kinematics and Dynamics

Let $x \in \mathbb{R}^n$ be the position of the end-effector in the task space (e.g., Cartesian space or
image space), and it is relevant to the joint position via the nonlinear mapping \cite{12, 17}

$$ x = f(q) $$  \hspace{1cm} (1)

where $q \in \mathbb{R}^n$ denotes the joint position, and $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is the mapping from joint space to
task space.

Differentiating (1) with respect to time gives the relation between the task-space velocity and
joint-space velocity \cite{12, 17}

$$ \dot{x} = J(q)\dot{q} $$  \hspace{1cm} (2)

where $J(q) \in \mathbb{R}^{n \times n}$ is the Jacobian matrix. In the case that the kinematic parameters are
unknown, we cannot obtain the task-space position/velocity by the direct kinematics given above.
Instead, we assume that certain task-space sensors (e.g., a camera) are employed to give the task-
space position/velocity information.

The kinematics (2) has the following linearity-in-parameters property \cite{5}.
Property 1: The kinematics \( J(q)\xi = Y_k(q, \xi)a_k \) depends linearly on a constant parameter vector \( a_k \), which gives rise to

\[ J(q)\xi = Y_k(q, \xi)a_k \]  

where \( Y_k(q, \xi) \) is the kinematic regressor matrix, and \( \xi \in \mathbb{R}^n \) is a vector.

The equations of motion of the manipulator can be written as \cite{18}, \cite{17}

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau \]  

where \( M(q) \in \mathbb{R}^{n \times n} \) is the inertia matrix, \( C(q, \dot{q}) \in \mathbb{R}^{n \times n} \) is the Coriolis and centrifugal matrix, \( g(q) \in \mathbb{R}^n \) is the gravitational torque, and \( \tau \in \mathbb{R}^n \) is the joint control torque. For the convenience of later reference, three well-understood properties associated with the dynamics \( 4 \) are listed as follows (see, e.g., \cite{18}, \cite{17}).

Property 2: The inertia matrix \( M(q) \) is symmetric and uniformly positive definite.

Property 3: The Coriolis and centrifugal matrix \( C(q, \dot{q}) \) can be appropriately determined such that \( \dot{M}(q) - 2C(q, \dot{q}) \) is skew-symmetric.

Property 4: The dynamics \( 4 \) depends linearly on a constant parameter vector \( a_d \), which leads to

\[ M(q)\dot{\zeta} + C(q, \dot{q})\zeta + g(q) = Y_d(q, \dot{q}, \zeta, \dot{\zeta})a_d \]  

where \( Y_d \) is the dynamic regressor matrix, \( \zeta \in \mathbb{R}^n \) is a differentiable vector, and \( \dot{\zeta} \) is the time derivative of \( \zeta \).

III. Adaptive Control Scheme

In this section, we will investigate the adaptive controller design for robot manipulators with both the uncertain kinematics and dynamics, and the control objective is to drive the robot end-effector to asymptotically track the desired trajectory in the task space, i.e., to ensure that \( x - x_d \to 0 \) as \( t \to \infty \), where \( x_d \) denotes the desired task-space trajectory and it is assumed that \( x_d, \dot{x}_d, \ddot{x}_d \) are all bounded.

Following \cite{5}, \cite{6}, we define a joint reference velocity using the estimated Jacobian matrix as

\[ \dot{q}_r = \hat{J}^{-1}(q)\dot{x}_r \]  

where \( \hat{J}(q) \) is the estimated Jacobian matrix.
where $\dot{x}_r = \dot{x}_d - \alpha \Delta x$, $\Delta x = x - x_d$ denotes the task-space position tracking error, $\alpha > 0$ is a positive design constant, and $\hat{J}(q)$ is the estimated Jacobian matrix which is obtained by replacing $a_k$ in $J(q)$ with its estimate $\hat{a}_k$.

Differentiating equation (6) with respect to time gives the joint reference acceleration

$$\ddot{q}_r = \hat{J}^{-1}(q) \left[ \ddot{x}_r - \dot{\hat{J}}(q) \dot{q}_r \right].$$

(7)

Let us now define a sliding vector

$$s = \dot{q} - \dot{q}_r$$

(8)

and using (2) and (6), we can rewrite equation (8) as

$$s = J^{-1}(q) \left[ \dot{x} - J(q) \dot{q}_r \right] = J^{-1}(q) \left[ \dot{x} - \dot{\hat{J}}(q) \dot{q}_r + Y_k(q, \dot{q}_r) \Delta a_k \right]$$

(9)

which can further be written as

$$\Delta \dot{x} = -\alpha \Delta x - Y_k(q, \dot{q}_r) \Delta a_k + J(q) s$$

(10)

where $\Delta a_k = \hat{a}_k - a_k$ is the kinematic parameter estimation error, and $Y_k(q, \dot{q}_r)$ is the kinematic regressor matrix. It is noticed that the regressor $Y_k(q, \dot{q}_r)$ generated by the new formulation (9) is different from the one in, e.g., [5], [6], [7], [16] since it does not depend on the joint velocity.

Now we propose the following control law

$$\tau = -Ks + Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \hat{a}_d$$

(11)

where $K \in \mathbb{R}^{n \times n}$ is a symmetric positive definite matrix, and the estimated dynamic parameter $\hat{a}_d$ (i.e., the estimate of $a_d$) and the estimated kinematic parameter $\hat{a}_k$ are, respectively, updated by

$$\dot{\hat{a}}_d = -\Gamma_d Y_d^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) s$$

(12)

$$\dot{\hat{a}}_k = \Gamma_k Y_k^T(q, \dot{q}_r) \Delta x$$

(13)

where $\Gamma_d$ and $\Gamma_k$ are both symmetric positive definite matrices.

**Remark 1:** The differences between the proposed adaptive controller and the one in [5], [6] are that 1) the feedback part in (11) can be rewritten as $-K J^{-1}(q) \left[ \Delta \dot{x} + \alpha \Delta x + Y_k(q, \dot{q}_r) \Delta a_k \right]$, which can thus be intuitively interpreted as *inverse Jacobian feedback of both the task-space...*
tracking error and the kinematic parameter estimation error rather than the approximate trans-
pose Jacobian feedback and 2) the kinematic parameter adaptation law (13) is based on a new 
regressor that depends on the joint reference velocity \( \dot{q}_r \) rather than \( \dot{q} \). The dynamic parameter 
 adaptation law (12) is actually the same as the one in [6]. The control law (11) expands the inverse 
Jacobian based task-space adaptive scheme in [2, Sec. 3] to additionally include the inverse 
Jacobian feedback of the kinematic parameter estimation error, which supplies the proposed 
control with the ability of handling the kinematic uncertainties.

Substituting the control law (11) into the dynamics (4) yields
\[
M(q) \ddot{s} + C(q, \dot{s})s = -Ks + Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\Delta a_d
\]
(14)
where \( \Delta a_d = \hat{a}_d - a_d \) is the dynamic parameter estimation error.

The closed-loop robotic system can be described by
\[
\begin{cases}
\Delta \dot{x} = -\alpha \Delta x - Y_k(q, \dot{q}_r)\Delta a_k + J(q)s, \\
M(q) \dot{s} + C(q, \dot{s})s = -Ks + Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\Delta a_d,
\end{cases}
\]
(15)
and the adaptation laws (12) and (13). System (15) is obviously cascaded with \( s \) acting as the 
cascade variable. Due to the new formulation of the unknown kinematic system, the system \( \Psi \) in 
the first subsystem of (15) depends on a new regressor \( Y_k(q, \dot{q}_r) \) and is driven by \( J(q)s \), which 
renders it possible to employ the inverse Jacobian feedback \(-Ks\) in the control torque [thanks 
to the fact that the boundedness of \( J(q) \) is independent of \( \hat{a}_k \)], while the kinematic system in 
[5], [6], [7], [16] relies on the regressor \( Y_k(q, \dot{q}) \) and is driven by \( \hat{J}(q)s \).

We are presently ready to formulate the following theorem.

**Theorem 1:** The control law (11), the dynamic parameter adaptation law (12), and the kinematic 
parameter adaptation law (13) ensure that the task-space tracking errors converge to zero, i.e., 
\( \Delta x \to 0 \) and \( \Delta \dot{x} \to 0 \) as \( t \to \infty \).

**Proof:** Following [2], [19], we take into account the Lyapunov-like function candidate
\[
V_1 = \frac{1}{2} s^T M(q)s + \frac{1}{2} \Delta a_d^T \Gamma_\Delta^{-1} \Delta a_d,
\]
differentiating \( V_1 \) with respect to time along the trajectories of the second subsystem in (15) and of the adaptation law (12) and using Property 3, we obtain
\[
\dot{V}_1 = -s^T Ks \leq 0,
\]
which implies that \( s \in L_2 \cap L_\infty \) and \( \dot{a}_d \in L_\infty \).

Using the fact that the Jacobian matrix \( J(q) \) is bounded, we have that \( J(q)s \in L_2 \), and thus, 
there exists a constant \( l_M > 0 \) such that 
\[
\int_0^t s^T J^T(q)J(q)sdr \leq l_M \text{ for all } t \geq 0.
\]
Considering
the following nonnegative function

\[
V_2 = \frac{1}{2} \Delta x^T \Delta x + \frac{1}{2\alpha} \left[ l_M - \int_0^t s^T J^T(q)J(q)sd\tau \right] + \frac{1}{2} \Delta a_k^T \Gamma_k^{-1} \Delta a_k
\]

(16)

where the term \(\Pi^*\) in \(V_2\) follows the result in [20] p. 118, and taking the derivative of \(V_2\) along the first subsystem in (15) gives

\[
\dot{V}_2 = -\alpha \Delta x^T \Delta x - \Delta a_k^T Y_k^T(q, \dot{q}_r) \Delta x + \Delta a_k^T \Gamma_k^{-1} \dot{a}_k + \Delta x^T J(q)s - \frac{1}{2\alpha} s^T J^T(q)J(q)s.
\]

(17)

Using the standard inequality \(\Delta x^T J(q)s \leq (\alpha/2) \Delta x^T \Delta x + [1/(2\alpha)] s^T J^T(q)J(q)s\) and substituting the adaptation law (13) into equation (17) yields \(\dot{V}_2 \leq - (\alpha/2) \Delta x^T \Delta x \leq 0\), which directly gives the result that \(\Delta x \in L_2 \cap L_\infty\) and \(\dot{a}_k \in L_\infty\). From equation (6), if the estimated Jacobian matrix \(\hat{J}(q)\) is nonsingular, we have that \(\dot{q}_r \in L_\infty\) since \(\dot{x}_r \in L_\infty\). Then, we obtain that \(\dot{q} \in L_\infty\) since \(s \in L_\infty\), and that \(\dot{x} \in L_\infty\) based on the kinematics (2). Therefore, \(\Delta x\) must be uniformly continuous, and from the properties of uniformly continuous functions belonging to the space \(L_p\) [20] p. 117, we obtain \(\Delta x \to 0\) as \(t \to \infty\). From (13), we have that \(\dot{a}_k \in L_\infty\) since \(Y_k(q, \dot{q}_r)\) and \(\Delta x\) are both bounded, which then implies the boundedness of \(\hat{J}(q)\). Thus, from (7), we obtain that \(\ddot{q}_r \in L_\infty\).

From (14), we obtain that \(\dot{s} \in L_\infty\) by using Property 2. This leads us to obtain that \(\ddot{q} = \ddot{q}_r + \ddot{s} \in L_\infty\), and that \(\ddot{x} \in L_\infty\) from the differentiation of the kinematic equation (2), i.e.,

\[
\ddot{x} = J(q)\ddot{q} + \ddot{J}(q)\dot{q}.
\]

Therefore, \(\Delta \ddot{x} \in L_\infty\), and then \(\Delta \dot{x}\) must be uniformly continuous. Due to the result that \(\Delta x \to 0\) as \(t \to \infty\), we obtain from Barbalat’s Lemma [18] that \(\Delta \dot{x} \to 0\) as \(t \to \infty\).

Remark 2: The formulation (10) and the associated analysis given in the proof of Theorem 1 demonstrates that the joint reference velocity (6) and the kinematic parameter adaptation law (13) serve well for industrial robotic applications. In fact, the joint velocity servoing module ensures that the joint velocity tends sufficiently fast to the joint reference velocity, i.e., \(s \equiv 0\).

Now consider the Lyapunov function candidate \(V'_2 = (1/2) \Delta x^T \Delta x + (1/2) \Delta a_k^T \Gamma_k^{-1} \Delta a_k\) and the derivative of \(V'_2\) along (10) (with \(s \equiv 0\)) and (13) can be written as \(\dot{V}'_2 = -\alpha \Delta x^T \Delta x \leq 0\), which implies the convergence of the task-space tracking errors regardless of the kinematic uncertainty.

IV. Performance Guaranteed Adaptive Control

In this section, we show how the previous adaptive controller ensures the performance of the robotic system under both the uncertain kinematics and dynamics by a suitable modification. This
modification follows the fundamental work in [15], yet extends it to the case of task-space robot control with kinematic uncertainties. The extension turns out to be direct thanks to the proposed new formulation in the previous section, yet, here, our emphasis is on demonstrating why this modification implies good performance. Moreover, it will be shown that even under constant-gain feedback, inverse Jacobian feedback yields better performance than transpose Jacobian feedback.

Following [15], we specify the feedback gain \( K \) in the control law (11) as
\[
K = \lambda_C \hat{M}(q)
\]
and at the same time modify the dynamic parameter adaptation law (12) as
\[
\dot{\hat{a}}_d = -\Gamma_d Y^T_d (q, \dot{q}, \ddot{q}_r, \ddot{q}_r^*) s
\]
where \( \lambda_C > 0 \) is a design constant, \( \hat{M}(q) \) is the estimated inertia matrix which is obtained by replacing \( a_d \) in \( M(q) \) with \( \hat{a}_d \), and \( \ddot{q}_r^* = \ddot{q}_r - \lambda_C s \). The gain selection (18) and the modification (19) yields (the same as the case in [15])
\[
\dot{V}_1 = -\lambda_C s^T M(q) s \leq 0.
\]
Then, the same result as in Theorem 1 follows. The choice of the feedback gain like (18), as is interpreted in [15], follows the idea that specifying large feedback gain for the joint with relatively large inertia.

Next, let us focus on interpreting the performance issues. Based on (9), we can rewrite the definition of \( \dot{q}_r \) in (6) as
\[
\dot{q}_r = J^{-1}(q) [\dot{x}_r - Y_k(q, \dot{q}_r) \Delta \dot{a}_k]
\]
whose differentiation with respect to time gives a new formulation of the joint reference acceleration [different from (7)] as
\[
\ddot{q}_r = J^{-1}(q) \left[ \ddot{x}_r - \dot{Y}_k(q, \dot{q}_r) \Delta \dot{a}_k - Y_k(q, \dot{q}_r) \dot{\hat{a}}_k \right] - J^{-1}(q) \dot{J}(q) \dot{q}_r.
\]

With the gain selection (18) and the above new formulation of \( \dot{q}_r \) and \( \ddot{q}_r \), the control law (11) can now be written as
\[
\tau = \hat{M}(q) J^{-1}(q) \left[ \ddot{x}_d - (\alpha + \lambda_C) \Delta \ddot{x} - \alpha \lambda_C \Delta x - \left( \dot{Y}_k(q, \dot{q}_r) + \lambda_C Y_k(q, \dot{q}_r) \right) \Delta \dot{a}_k - Y_k(q, \dot{q}_r) \dot{\hat{a}}_k \right]
+
\left[ \dot{C}(q, \dot{q}) J^{-1}(q) - \hat{M}(q) J^{-1}(q) \dot{J}(q) J^{-1}(q) \right] [\dot{x}_r - Y_k(q, \dot{q}_r) \Delta \dot{a}_k] + \dot{g}(q).
\]

The control (22) is quite similar to the certainty-equivalence form of the standard task-space inverse dynamics (see, e.g., [21], [12], [17]) and thus ensures the performance, where \( \alpha \) and \( \lambda_C \) can be tuned to fulfill the performance requirements (e.g., robustness and speed of response).
Remark 3: With the new formulation of the kinematic system, the adaptive control scheme given by (22), (19), and (13) yields

\[
\begin{align*}
\Delta \dot{x} &= -\alpha \Delta x - Y_k(q, \dot{q}_r) \Delta a_k + J(q)s, \\
M(q)\dot{s} + C(q, \dot{q})s &= -\lambda C M(q)s + Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r^*) \Delta a_d.
\end{align*}
\]

(23)

From equation (23), it can be observed that the feedback gains in both the systems of (23) are indeed inertia-dependent (note that the apparent inertia of the first subsystem in (23) can be considered as \(I_n\)). This gives the additional demonstration on why the adaptive scheme given by (22), (19), and (13) implies guaranteed task-space tracking performance. The approximate transpose Jacobian feedback adopted in [5] [i.e., \(-\hat{J}^T(q)K \hat{J}(q)s\)] with the gain selection (18) and appropriate modification of the dynamic parameter adaptation law would render the feedback gain (with respect to \(s\)) in the closed-loop dynamics as \(-\lambda C \hat{J}^T(q)M(q)\hat{J}(q)\), which, in most cases, cannot guarantee the tracking performance due to the (possibly large) discrepancy between \(\hat{J}^T(q)M(q)\hat{J}(q)\) and \(M(q)\). This also suggests that for the task-space tracking problem, transpose Jacobian control may not be preferred although it can successfully ensure the stability of the closed-loop system (refer to, e.g., [5], [6]).

Remark 4: The statement in Remark 3 holds even for the case of constant-gain feedback (i.e., \(K\) is chosen to be constant). It is well known that the task-space inertia \(J^{-T}(q)M(q)J^{-1}(q)\) involves the inversion of the Jacobian matrix. In the case of using transpose Jacobian feedback as is the case in [5], [6], the inversion of the transpose of the approximate Jacobian matrix would cancel the transpose of the approximate Jacobian matrix and render the feedback gain to be \(K\), which implies that we have to rely on the constant gain \(K\) to compensate for the task-space inertia \(J^{-T}(q)M(q)J^{-1}(q)\). In the case of using inverse Jacobian feedback without involving the transpose of the Jacobian matrix as in our result, the task-space formulation renders the inverse Jacobian feedback premultiplied by \(J^{-T}(q)\), i.e., using the Jacobian-dependent varying gain \(J^{-T}(q)KJ^{-1}(q)\) to compensate for the varying task-space inertia \(J^{-T}(q)M(q)J^{-1}(q)\) (tending to be easier). The performance superiority of inverse Jacobian feedback control is thus obvious.

Remark 5: In the visual tracking problem for robots with uncertainties in the camera model and/or manipulator kinematics, most results, e.g., [22], [23], [10], [24], [25], [11], are fully/partly based on the approximate transpose Jacobian feedback. The use of constant-gain feedback in the joint space (i.e., in the form \(-Ks\)) occurs in [16], [26] [which employ the indirect kinematic
parameter adaptation law yet, and additionally require the persistent excitation of the joint-velocity-dependent kinematic regressor $Y_k(q, \dot{q})$ for avoiding the use of task-space velocity in the kinematic parameter adaptation], and also appears in [7], [23], [10], [24], [25] as part of the overall feedback action (the use of $-Ks$ alone in this case, yet, cannot ensure stability), which is the same as the one proposed in our result and may also be interpreted as inverse Jacobian feedback, yet the rationality/interpretation of doing so and the performance issues associated with the closed-loop dynamics are not adequately addressed. Moreover, all these results rely on the joint-velocity-dependent kinematic regressor.

V. Simulation Results

Let us consider a standard 2-DOF (degree-of-freedom) planar manipulator that grasps an unknown tool. The physical parameters of the 2-DOF manipulator are not listed for saving space. The sampling period is chosen as 5 ms.

The desired trajectory of the manipulator end-effector is chosen as $x_d = [1.8 + 0.4 \cos \pi t, 3.8 + 0.4 \sin \pi t]^T$. The controller parameters $K$, $\alpha$, $\Gamma_d$, and $\Gamma_k$ are determined as $K = 40I_2$, $\alpha = 10$, $\Gamma_d = 160I_4$, and $\Gamma_k = 40I_3$, respectively. The initial parameter estimates are chosen as $\hat{a}_d(0) = [0, 0, 0, 0]^T$ and $\hat{a}_k(0) = [4.0, 5.0, 2.0]^T$, while their actual values are $a_d = [7.9628, -0.9600, 19.2828, 10.1495]^T$ and $a_k = [2.0000, 3.3856, 0.8000]^T$. The initial joint position and velocity of the manipulator are set as $q(0) = \left[\pi/12, \pi/3\right]^T$ and $\dot{q}(0) = 0$, respectively. Simulation results of the proposed adaptive controller which employs the inverse Jacobian feedback and the new kinematic parameter adaptation law are shown in Fig. 1 to Fig. 4.

Under the same context, we also conduct the simulation when the controller given in [5], [6] is adopted. The kinematic parameter adaptation law in this case depends on the regressor $Y_k(q, \dot{q})$, and the control law employs the approximate transpose Jacobian feedback $-\hat{J}^T(q)K\hat{J}(q)s$. The controller parameters are chosen to be the same as in the proposed controller. Simulation results in this context are shown in Fig. 5 to Fig. 8.

One obvious difference between the simulation results under the proposed controller and those under the one in [5], [6] is that the proposed controller results in faster parameter convergence which are reflected in both the kinematic and dynamic parameter estimation process (see Fig. 3 and Fig. 4 as compared with Fig. 7 and Fig. 8), and consequently better tracking performance and more adequate utilization of the joint torques (see Fig. 1 and Fig. 2 as compared with
Fig. 5 and Fig. 6). This is possibly due to the employment of the new joint-reference-velocity-dependent kinematic regressor and of the inverse Jacobian feedback. As can be clearly seen from the definition of the joint reference velocity [i.e., equation (6)], it contains the information concerning the task-space tracking error. The rationality of using inverse Jacobian feedback has already been discussed in Remark 4, and is now reflected in the simulation results.

The performance of the proposed controller under an inertia-dependent feedback action $-\lambda C\hat{M}(q)s$ is shown in Fig. 9 and Fig. 10, where we choose $\lambda C = \alpha = 5$ so that the closed-loop dynamics is approximate to a critically damped linear dynamics, and the other parameters are chosen to be the same as above. The performance is comparable to the case of constant-gain feedback (see Fig. 1 and Fig. 2) although the parameter $\alpha$ is now decreased by half.

VI. CONCLUSION AND DISCUSSION

In this paper, we consider the adaptive tracking problem for robot manipulators subjected to both the kinematic and dynamic uncertainties. We propose a new formulation of the unknown kinematic system, which generates a new kinematic regressor matrix that contains the kinematic parameter estimate and the information concerning the task-space tracking error. With this new formulation, we propose an adaptive control scheme that enjoys the desirable decomposition property. The feedback control adopted can be intuitively interpreted as inverse Jacobian feedback, and the kinematic parameter adaptation is based on a new kinematic regressor that depends on the joint reference velocity. Due to the proposed new control framework, the performance
can be conveniently ensured with essentially the same modification of the control law and of the dynamic parameter adaptation law as in [15]. Our study also suggests that, to guarantee task-space tracking performance, inverse Jacobian feedback seems preferable than the now commonly adopted transpose Jacobian feedback. The proposed adaptive control can thus be considered as a qualified (or perhaps superior) alternative to the existing results (e.g., [5], [6]). The performance of the proposed adaptive control scheme is shown by numerical simulations.

One desirable feature of the proposed control scheme that we would like to emphasize is that the decomposition of the handling of the kinematic and dynamic uncertainties (both within their respective domains) makes one reduced version of our control scheme rather suitable
for industrial robotic applications. This originates from the fact that the kinematic control law (represented by the joint reference velocity $\dot{q}_r$) plus the associated kinematic parameter adaptation law will ensure the task-space tracking error convergence so long as the low-level joint servoing loop (which is now commonly embedded in most industrial robots) can ensure that the joint velocity tends sufficiently fast to the joint reference velocity.

REFERENCES

[1] J. J. Craig, P. Hsu, and S. S. Sastry, “Adaptive control of mechanical manipulators,” *The International Journal of Robotics Research*, vol. 6, no. 2, pp. 16–28, Jun. 1987.
[2] J.-J. E. Slotine and W. Li, “On the adaptive control of robot manipulators,” *The International Journal of Robotics Research*, vol. 6, no. 3, pp. 49–59, Sep. 1987.

[3] R. H. Middleton and G. C. Goodwin, “Adaptive computed torque control for rigid link manipulators,” *Systems & Control Letters*, vol. 10, no. 1, pp. 9–16, Jan. 1988.

[4] C. C. Cheah, M. Hirano, S. Kawamura, and S. Arimoto, “Approximate Jacobian control for robots with uncertain kinematics and dynamics,” *IEEE Transactions on Robotics and Automation*, vol. 19, no. 4, pp. 692–702, Aug. 2003.

[5] C. C. Cheah, C. Liu, and J.-J. E. Slotine, “Adaptive tracking control for robots with unknown kinematic and dynamic properties,” *The International Journal of Robotics Research*, vol. 25, no. 3, pp. 283–296, Mar. 2006.

[6] ——, “Adaptive Jacobian tracking control of robots with uncertainties in kinematic, dynamic and actuator models,” *IEEE Transactions on Automatic Control*, vol. 51, no. 6, pp. 1024–1029, Jun. 2006.

[7] D. Braganza, W. E. Dixon, D. M. Dawson, and B. Xian, “Tracking control for robot manipulators with kinematic and dynamic uncertainty,” in *Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference 2005*, Seville, Spain, 2005, pp. 5293–5297.

[8] W. E. Dixon, “Adaptive regulation of amplitude limited robot manipulators with uncertain kinematics and dynamics,” *IEEE Transactions on Automatic Control*, vol. 52, no. 3, pp. 488–493, Mar. 2007.

[9] Y.-H. Liu, H. Wang, C. Wang, and K. K. Lam, “Uncalibrated visual servoing of robots using a depth-independent interaction matrix,” *IEEE Transactions on Robotics*, vol. 22, no. 4, pp. 804–817, Aug. 2006.

[10] H. Wang, Y.-H. Liu, and W. Chen, “Uncalibrated visual tracking control without visual velocity,” *IEEE Transactions on Control Systems Technology*, vol. 18, no. 6, pp. 1359–1370, Nov. 2010.

[11] H. Wang, “Adaptive visual tracking for robotic systems without visual velocity measurement,” *arXiv preprint arXiv:1401.6904*, 2014.

[12] J. J. Craig, *Introduction to Robotics: Mechanics and Control*, 3rd ed. Upper Saddle River, NJ: Prentice-Hall, 2005.

[13] M. Takegaki and S. Arimoto, “A new feedback method for dynamic control of manipulators,” *Journal of Dynamic Systems, Measurement, and Control*, vol. 103, no. 2, pp. 119–125, Jun. 1981.

[14] C. C. Cheah and H. C. Liaw, “Inverse Jacobian regulator with gravity compensation: Stability and experiment,” *IEEE Transactions on Robotics and Automation*, vol. 21, no. 4, pp. 741–747, Aug. 2005.

[15] J.-J. E. Slotine and W. Li, “Composite adaptive control of robot manipulators,” *Automatica*, vol. 25, no. 4, pp. 509–519, Jul. 1989.

[16] A. C. Leite, A. R. L. Zachi, F. Lizzaralde, and L. Hsu, “Adaptive 3D visual servoing without image velocity measurement for uncertain manipulators,” in *18th IFAC World Congress*, Milano, Italy, 2011, pp. 14 584–14 589.

[17] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*. New York: John Wiley & Sons, 2006.

[18] J.-J. E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, NJ: Prentice-Hall, 1991.

[19] R. Ortega and M. W. Spong, “Adaptive motion control of rigid robots: A tutorial,” *Automatica*, vol. 25, no. 6, pp. 877–888, Nov. 1989.

[20] R. Lozano, B. Brogliato, O. Egeland, and B. Maschke, *Dissipative Systems Analysis and Control: Theory and Applications*. London: Spinger-Verlag, 2000.

[21] J. Y. S. Luh, M. W. Walker, and R. P. C. Paul, “Resolved-acceleration control of mechanical manipulators,” *IEEE Transactions on Automatic Control*, vol. AC-25, no. 3, pp. 468–474, Jun. 1980.

[22] C. C. Cheah, C. Liu, and J.-J. E. Slotine, “Adaptive vision based tracking control of robots with uncertainty in depth...
information,” in Proceedings of the IEEE International Conference on Robotics and Automation, Roma, Italy, 2007, pp. 2817–2822.

[23] H. Wang, Y.-H. Liu, and D. Zhou, “Dynamic visual tracking for manipulators using an uncalibrated fixed camera,” IEEE Transactions on Robotics, vol. 23, no. 3, pp. 610–617, Jun. 2007.

[24] H. Wang, Y.-H. Liu, and W. Chen, “Visual tracking of robots in uncalibrated environments,” Mechatronics, vol. 22, no. 4, pp. 390–397, Jun. 2012.

[25] X. Li and C. C. Cheah, “Adaptive regional feedback control of robotic manipulator with uncertain kinematics and depth information,” in Proceedings of the American Control Conference, Montréal, Canada, 2012, pp. 5472–5477.

[26] F. Lizarralde, A. C. Leite, L. Hsu, and R. R. Costa, “Adaptive visual servoing scheme free of image velocity measurement for uncertain robot manipulators,” Automatica, vol. 49, no. 5, pp. 1304–1309, May 2013.