Optical unidirectional amplification in a three-mode optomechanical system

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We study the directional amplification of an optical probe field in a three-mode optomechanical system, where the mechanical resonator interacts with two linearly-coupled optical cavities and the cavities are driven by strong optical pump fields. The optical probe field is injected into one of the cavity modes, and at the same time, the mechanical resonator is subject to a mechanical drive with the driving frequency equal to the frequency difference between the optical probe and pump fields. We show that the transmission of the probe field can be amplified in one direction and demagnified in the opposite direction. This directional amplification or de-amplification results from the constructive or destructive interference between different transmission paths in this three-mode optomechanical system.

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I. INTRODUCTION

With the rapid development of microfabrication technology, cavity optomechanical system [1–4] is becoming an appealing candidate to connect a broad spectrum of photonic, electronic, and atomic devices, besides being studied for fundamental questions of macroscopic systems in the quantum limit [5]. Recently, enormous progresses have been achieved that aim at the applications of optomechanical systems in ultra-high precision measurement [6–12], quantum information processing [13], quantum illumination [14] to optomechanically induced transparency [15–21], absorption [22, 23], and amplification [24–28].

Among these applications, nonreciprocal transmission and amplification are of great interest in the study of the quantum analogue of photonic and electronic devices, such as diode, circulator, and transistor, which are crucial for scalable quantum information processing in integrated circuits [29]. In the past, nonreciprocal devices have been investigated broadly in optical systems [30–38]. In these devices, the occurrence of nonreciprocal lightpropagation is associated with the symmetry breaking induced by various mechanisms, such as magneto-optical Faraday effect [30], parametric modulation [31, 32], optical nonlinearity [33, 34], and chiral light-matter interaction [35].

In recent years, it has been shown that the optomechanical system can be utilized to realize nonreciprocal effects for propagating light fields [34–38]. The nonreciprocal optical diodes are achieved in multimode optomechanical systems with effective breaking of time-reversal symmetry generated by on-demand gauge-invariant phases [34, 36]. Nonreciprocal phenomena with directional amplification have been explored theoretically in general coupled-mode systems [39]. The phenomena of optical directional amplification have also been implemented experimentally very recently in multi-mode optomechanical systems [50–54].

In this paper, we study a scheme to achieve directional amplification of an optical probe field in a three-mode optomechanical system, where a mechanical resonator is coupled to two optical modes that directly interact with each other. In this system, controllable phase difference between the linearized optomechanical couplings, which breaks the time-reversal symmetry of this three-mode system, is generated by the strong pump fields on the optical cavities. Meanwhile, the probe field is applied to one of the cavities and the mechanical resonator is subject to a mechanical drive with the driving frequency equal to the frequency difference between the optical probe and pump fields. The constructive (destructive) interference between the transmission paths for the optical probe field and its mechanical counterpart via the optomechanical interaction results in the amplification of the probe field [24]. Strong directional amplification of the optical field with high amplification ratio can be achieved in this system. In comparison with the previous works [50–54] in multi-mode optomechanical systems, where the directional amplification results from the blue-detuned pump fields, here we use the red-detuned pump fields as well as the additional mechanical drive to achieve the optical directional amplification in a three-mode optomechanical system. Since the blue-detuned (red-detuned) pump field will heat (cool) the motion of mechanical resonator in an optomechanical system, our scheme avoiding pumping with blue-detuned light can improve the stability of the amplification scheme in optomechanical systems. As a tradeoff, the additional mechanical drive with the driving frequency equal to the frequency difference between the optical probe and pump fields is required to achieve the directional amplification in our scheme. Our work
provides an alternative method to achieve the optical directional amplification in optomechanical systems, which could stimulate future studies of optomechanical interfaces in the implementation of nonreciprocal and nonlinear photonic devices.

This paper is organized as follows. In Sec. II, we present the Hamiltonian of the three-mode optomechanical system for nonreciprocal amplification and our derivation of the transmission coefficients in this system. Details of the directional amplification and de-amplification of the optical probe field are studied in Sec. III. Conclusions are given in Sec. IV.

II. MODEL AND TRANSMISSION MATRIX

The optomechanical system under consideration consists of a mechanical oscillator with resonance frequency $\omega_m$ and two optical cavities with resonance frequencies $\omega_1$ and $\omega_2$, respectively, as illustrated in Fig. 1. We first focus on the case that the probe field is incident from the left side to the cavity 1. The total Hamiltonian of this system has the form

$$H = H_0 + H_I + H_d.$$  (1)

The first term describes the free Hamiltonian of the cavity modes and the mechanical one with $(\hbar = 1)$

$$H_0 = \omega_1 a_1^\dagger a_1 + \omega_2 a_2^\dagger a_2 + \omega_m b^\dagger b,$$  (2)

where $a_i^\dagger (a_i)$ for $i = 1, 2$ and $b^\dagger (b)$ are the creation (annihilation) operators for the cavity modes and the mechanical one. The second term

$$H_I = J \left( a_1^\dagger a_2 + a_1 a_2^\dagger \right) + \sum_i g_i a_i^\dagger a_i (b + b^\dagger)$$  (3)

characterizes the linear coupling between the cavity modes with coupling strength $J$ and the radiation-pressure force interaction between the cavities and the mechanical resonator with single-photon coupling strength $g_i$. The third term $H_d$ describes the mechanical drive, the optical pump fields on the cavities, and the probe field (incident from the left side to cavity 1, see the thin solid arrow in Fig. 1)

$$H_d = \sum_i \left( i \varepsilon_i a_i^\dagger e^{-i\omega_d t} e^{i\theta_i} + \text{h.c.} \right) + \left( i \varepsilon_p a_1^\dagger e^{-i\omega_p t} + i \varepsilon_p e^{i(\omega_m - \omega_p) t} + \text{h.c.} \right),$$  (4)

where $\omega_d$ is the frequency, $\varepsilon_i$ is the amplitude, and $\theta_i$ is the phase of the two pump fields, $\omega_p$ ($\omega_b$) is the frequency and $\varepsilon_p$ ($\varepsilon_b$) is the amplitude of the probe field on cavity 1 (the mechanical drive applied on the mechanical resonator). It is worth pointing out that the mechanical drive can be easily realized in experiments through an external electric drive [55–58]. Here without loss of generality, we have assumed that $J, g_{1,2}$, and $\varepsilon_{1,2}$ are real numbers.

In the rotating frame with respect to the frequency of the pump fields, the quantum Langevin equations (QLEs) for the operators in the system are given by

\begin{align*}
\dot{a}_1 &= \{-\gamma_1 - i \left[ \Delta_1 + g_1 (b + b^\dagger) \right]\} a_1 - iJa_2 + \varepsilon_1 e^{i\theta_1} + \varepsilon_p e^{i(\omega_m - \omega_p) t} + \xi_1, \quad (5) \\
\dot{a}_2 &= \{-\gamma_2 - i \left[ \Delta_2 + g_2 (b + b^\dagger) \right]\} a_2 - iJa_1 + \varepsilon_2 e^{i\theta_2} + \xi_2, \quad (6) \\
\dot{b} &= \{-\gamma_m - i\omega_m\} b - i \left( g_1 a_1^\dagger a_1 + g_2 a_2^\dagger a_2 \right) + \varepsilon_b e^{-i\omega_b t} + \xi_m. \quad (7)
\end{align*}

Here $\Delta_i = \omega_i - \omega_d$ $(i = 1, 2)$ are the optical detunings of the cavities, $\gamma_i$ ($\gamma_m$) are the decay rates of the two cavities (mechanical resonator), $\xi_i$ $(\xi_m)$ are the noise operators of the cavities (mechanical mode) with $\langle \xi_i \rangle = \langle \xi_m \rangle = 0$.

We first derive the steady-state solution of the three-mode system under strong pump fields. Neglecting the
effects of the optical probe and mechanical drive, we can obtain the steady-state solution as

\[
\langle a_1 \rangle = \frac{(\gamma_2 + i\Delta'_2) e^{i\theta_1} - iJ\xi e^{i\theta_2}}{\gamma_1 + i\Delta'_1} \frac{e^{i\theta_1} - iJ\xi e^{i\theta_2} + J^2}{e^{i\theta_1} + J^2},
\]

(8)

\[
\langle a_2 \rangle = \frac{(\gamma_1 + i\Delta'_1) e^{i\theta_2} - iJ\xi e^{i\theta_1}}{\gamma_2 + i\Delta'_2} \frac{e^{i\theta_2} - iJ\xi e^{i\theta_1} + J^2}{e^{i\theta_2} + J^2},
\]

(9)

\[
\langle b \rangle = \frac{-i(\gamma_1 |a_1|^2 + g_2 |a_2|^2)}{\gamma_m + i\omega_m},
\]

(10)

where \( \langle a_i \rangle \) (\( \langle b \rangle \)) are the steady-state averages of the cavities (mechanical mode), and \(\Delta'_i = \Delta_i + g_i(\langle b \rangle + \langle b \rangle^*) \) (\(i = 1, 2\)) are the cavity detunings shifted by the radiation-pressure force. These equations are coupled to each other and can be solved self-consistently.

Each operator of this system can be written as a sum of the steady-state solution and its fluctuation with \(a_i = \langle a_i \rangle + \delta a_i\) and \(b = \langle b \rangle + \delta b\), where \(\delta a_i\) are the fluctuations of the cavities and \(\delta b\) is that of the mechanical mode. Neglecting the nonlinear terms in the radiation-pressure interaction in Eqs. \(8\)-\(10\), we obtain a set of linear QLEs for the fluctuation operators:

\[
\delta \dot{a}_1 = (\gamma_1 - \gamma_2) \delta a_1 - iG_1 \delta b + \varepsilon e^{i(\omega_a - \omega_p)t} + \xi_1,
\]

(11)

\[
\delta \dot{a}_2 = (\gamma_1 - \gamma_2) \delta a_2 - iG_2 \delta b + \varepsilon e^{i(\omega_a - \omega_p)t} - i \delta a_1 + \xi_2,
\]

(12)

\[
\delta \dot{b} = (\gamma_m - i\omega_m) \delta b - i(G_1 \delta a_1 + G_2 \delta a_2) = \varepsilon_b e^{-i\omega_m t} + \xi_m,
\]

(13)

where \(G_i = g_i \langle a_i \rangle\) (\(i = 1, 2\)) represent the pump-enhanced linear optomechanical couplings.

In what follows, we fix \(\omega_b = \omega_p - \omega_d\) in our scheme, i.e., the frequency of the mechanical drive is always equal to the frequency difference between the optical probe and pump fields. To solve the above QLEs, we transform all the operators to another rotating frame with \(\delta a_i \rightarrow \delta a_i e^{-i(\omega_p - \omega_d)t}, \xi_i \rightarrow \xi_i e^{-i(\omega_p - \omega_d)t}, \delta b \rightarrow \delta b e^{-i\omega_m t}\), and \(\xi_m \rightarrow \xi_m e^{-i\omega_m t}\). In addition, we assume that the cavities are driven by the red-detuned pump fields and \(\Delta'_i \sim \omega_m\). In this case, by using the rotating-wave approximation, one can neglect the fast-oscillating counter-rotating terms and obtain the following linearized QLEs

\[
\delta \dot{a}_1 = -\Gamma_1 \delta a_1 - iG_1 \delta b - i \delta \dot{a}_2 + \varepsilon_p + \xi_1,
\]

(14)

\[
\delta \dot{a}_2 = -\Gamma_2 \delta a_2 - iG_2 \delta b - i \delta \dot{a}_1 + \xi_2,
\]

(15)

\[
\delta \dot{b} = -\Gamma_m \delta b - iG_1^* \delta a_1 - iG_2^* \delta a_2 + \varepsilon_b + \xi_m,
\]

(16)

The cavity output fields \(\langle \delta a_i^{out} \rangle\) (\(i = 1, 2\)) can be derived from the input-output theorem with

\[
\langle \delta a_i^{out} \rangle + \langle \delta a_i^{in} \rangle = \sqrt{2\gamma_i} \langle \delta a_i \rangle,
\]

(20)
where $\gamma_1^e$ represents the cavity loss related to coupling between the cavity and the input (output) modes, and is part of the total cavity loss rate $\gamma_i$ with $\gamma_i^e = \eta_i \gamma_i$ and $\eta_i \leq 1$. For simplicity of discussion, we focus on the case of over-coupled cavities with $\eta_i \approx 1$ and neglect cavity intrinsic dissipation [58–61]. With this assumption, \( \langle \delta a_1^{in} \rangle = \varepsilon_p / \sqrt{2\gamma_1^e} \), \( \langle \delta a_2^{in} \rangle = 0 \). The input field on the mechanical resonator can then be written in terms of the cavity input with \( \langle \delta b^{in} \rangle = \sqrt{\gamma_1^e / \gamma_m(y)e^{i\varphi}} \langle \delta a_1^{in} \rangle \). The transmission coefficient that describes the dependence of the output field of cavity 2 on the input field \( \langle \delta a_1^{in} \rangle \) can be defined as

\[
t_{21} \equiv \partial \langle \delta a_2^{out} \rangle / \partial \langle \delta a_1^{in} \rangle. \tag{21}
\]

With Eqs. (18) and (20), we derive

\[
t_{21} = -2 \sqrt{\gamma_1^e \gamma_2^e} \left[ \frac{(iJ \Gamma_m + G_1^e G_2^e) (\Gamma_m - iG_1 y e^{i\varphi}) + iG_2 y e^{i\varphi} (\Gamma_1 \Gamma_m + G_1^2)}{(\Gamma_1 \Gamma_m + |G_1|^2) (\Gamma_2 \Gamma_m + |G_2|^2) - (iJ \Gamma_m + G_1 G_2) (iJ \Gamma_m + G_1^2 G_2)} \right], \tag{22}
\]

where we have defined the amplitude of the mechanical drive through $\varepsilon_b / \varepsilon_p = ye^{i\varphi}$ ($y > 0$).

Similarly, we can derive the transmission coefficient for a probe field applied to cavity 2 from the right side (see the thin dashed arrow in Fig. 11). In this case, we have \( \langle \delta a_1^{in} \rangle = 0 \), \( \langle \delta a_2^{in} \rangle = \varepsilon_p / \sqrt{2\gamma_2^e} \), and still fix $\omega_b = \omega_p - \omega_d$. Here the transmission coefficient is defined as $t_{12} \equiv \partial \langle \delta a_2^{out} \rangle / \partial \langle \delta a_1^{in} \rangle$. We derive that

\[
t_{12} = -2 \sqrt{\gamma_1^e \gamma_2^e} \left[ \frac{(iJ \Gamma_m + G_2^e G_1^e) (\Gamma_m - iG_2 y e^{i\varphi}) + iG_1 y e^{i\varphi} (\Gamma_2 \Gamma_m + G_2^2)}{(\Gamma_2 \Gamma_m + |G_2|^2) (\Gamma_1 \Gamma_m + |G_1|^2) - (iJ \Gamma_m + G_2 G_1) (iJ \Gamma_m + G_2^2 G_1)} \right]. \tag{23}
\]

This equation shows that the propagation of the optical probe field in the three-mode optomechanical system depends strongly on the interference between various paths of the probe field via the optical cavity with amplitude $\varepsilon_p$ and the frequency-matched mechanical drive with amplitude $\varepsilon_b$ via the optomechanical interaction. And the transmission is not symmetric between cavities 1 and 2.

## III. DIRECTIONAL AMPLIFICATION OF OPTICAL PROBES

In this section, we will study the transmission of optical probe and the asymmetry in the transmission systematically. We will show that amplification of optical probe fields can be directional. Consider $G_1 = G > 0$ and $G_2 = Ge^{i\theta}$ for simplicity of discussion. The transmission coefficients can be rewritten as

\[
t_{21} = -2 \sqrt{\gamma_1^e \gamma_2^e} \left[ \frac{(iJ \Gamma_m + G_2 e^{i\theta}) (\Gamma_m - iG y e^{i\varphi}) + iG (\Gamma_1 \Gamma_m + G_2^2) ye^{i(\theta + \varphi)}}{(\Gamma_1 \Gamma_m + G^2) (\Gamma_2 \Gamma_m + G^2) - (iJ \Gamma_m + G_2 e^{i\theta}) (iJ \Gamma_m + G_2^2 e^{i\theta})} \right], \tag{24}
\]

and

\[
t_{12} = -2 \sqrt{\gamma_1^e \gamma_2^e} \left[ \frac{(iJ \Gamma_m + G_2 e^{-i\theta}) (\Gamma_m - iG y e^{i(\theta + \varphi)}) + iG (\Gamma_2 \Gamma_m + G^2) ye^{i\varphi}}{(\Gamma_1 \Gamma_m + G^2) (\Gamma_2 \Gamma_m + G^2) - (iJ \Gamma_m + G_2 e^{-i\theta}) (iJ \Gamma_m + G^2 e^{i\theta})} \right]. \tag{25}
\]

When $\varepsilon_b = 0$ ($y = 0$), the model reduces to that studied in [44], where the directional transmission of the
FIG. 2: The transmission probabilities $T_{21}$ and $T_{12}$ versus $\Delta_m = \omega_m - (\omega_p - \omega_d)$ for different values of $\theta$ and $\varphi$: (a) $\theta = 0$, $\varphi = \pi/2$; (b) $\theta = \pi/2$, $\varphi = 0$; (c) $\theta = \pi/2$, $\varphi = \pi/2$. Other parameters are $y = 20$, $\eta_{1,2} = 1$, $\gamma_1 = 1.1\gamma_m$, $\gamma_2 = 1.5\gamma_m$, $G = |G_{1,2}| = J = \gamma_m$, and $\Delta''_{1,2} = \Delta_m$.

probe field can be achieved under optimal parameters. In such a scheme, the introduction of the nontrivial phase $\theta$ breaks the time-reversal symmetry of this system and results in nonreciprocal propagation of the probe field. In contrast, in the presence of the frequency-matched mechanical drive with the optical probe field. In particular, at certain optimal values of $\theta$ and $\varphi$, e.g., $\theta = \pi/2$, $\varphi = \pi/2$, $T_{21} \to 0$ and $T_{12} \gg 1$, as shown in Figs. 2(c). The transmission from cavity 1 to cavity 2 is strongly amplified; whereas, the transmission on the opposite direction is suppressed. In this case, the amplification of the probe field results from phonon-photon parametric process due to the existence of the frequency-matched mechanical drive [20].

We plot the probability of the transmission $T_{21}$ and $T_{12}$ as functions of $\theta$ and $\varphi$ in Fig. 3. It is also shown that the directional propagation can be achieved with $\theta = \pi/2$ in Fig. 3(a) or $\varphi = \pi/2$ in Fig. 3(b). Note that, when $\theta = \pi/2$ with other parameters given in the caption of Fig. 3(b), the probability of transmission $T_{12}$ is independent of $\varphi$, which can be given through Eq. (25).

To further understand the effect of the frequency-matched mechanical drive on the transmission property of the probe field, we assume that the parameters $G = |G_{1,2}| = J = \gamma_m$, $\Delta_m = \Delta''_{1,2} = 0$, $\theta = \pi/2$, and $\varphi = \pi/2$. Then the corresponding transmission coefficients $T_{12}$ and $T_{21}$ are simplified to be

\begin{align*}
T_{21} &= 4\gamma_1\gamma_2 \left( \frac{2\gamma_m(y+1) - y(\gamma_1 + \gamma_m)}{(\gamma_1 + \gamma_m)(\gamma_2 + \gamma_m)} \right)^2, \\
T_{12} &= 4y^2 \frac{\gamma_1\gamma_2}{(\gamma_1 + \gamma_m)^2}.
\end{align*}

In the absence of the mechanical drive ($y \to 0$), the directional transmission of the probe field can occur with $T_{12} \to 0$ and $T_{21} > 0$ (particularly, $T_{21} = 1$ at $\gamma_{1,2} = \gamma_m$) as shown in [44]. On the contrary, in the presence of frequency-matched mechanical drive with $y = y_c \equiv 2\gamma_m/(\gamma_1 - \gamma_m)$ and $|y_c| \gg 1$, we have $T_{21} \to 0$ and $T_{12} \gg 1$. The directional amplification of the optical
FIG. 3: Plot of the probability of transmission $T_{21}$ and $T_{12}$ as functions of $\theta$ and $\varphi$, respectively. (a) $\varphi = \pi/2$. (b) $\theta = \pi/2$. Other parameters are $y = 20$, $\eta_1, 2 = 1$, $G = |G_{1,2}| = J = \gamma_m$, $\Delta_m = \Delta''_{1,2} = 0$, $\gamma_1 = 1.1\gamma_m$, and $\gamma_2 = 1.5\gamma_m$. One can see that at certain optimal values of $\theta$ and $\varphi$, e.g., $\theta = \pi/2$, $\varphi = \pi/2$, $T_{12} \to 0$ and $T_{21} \gg 1$.

FIG. 4: The transmission probabilities $T_{21}$ and $T_{12}$ versus $y$. Other parameters are $\theta = \pi/2$, $\varphi = \pi/2$, $G = |G_{1,2}| = J = \gamma_m$, $\Delta_m = \Delta''_{1,2} = 0$, $\gamma_1 = 1.1\gamma_m$, and $\gamma_2 = 1.5\gamma_m$.

probe field can be observed due to the presence of the mechanical drive frequency-matched to the probe field, and the direction of the amplification is opposite to that in the case of directional transmission in [44]. Strong amplification requires $|y_c| \gg 1$, i.e., the cavity damping rate $\gamma_1$ is approximately equal to the mechanical damping rate $\gamma_m$.

To study the role of the mechanical drive, we plot $T_{21}$ and $T_{12}$ as functions of $y$ in Fig. 4. This plot clearly demonstrates that the propagation of the optical field is strongly amplified with $T_{12} \sim 600$ when the mechanical drive becomes large ($|y| \gg 1$). Meanwhile, when $y \sim y_c = 20$ under the parameters given in the caption of Fig. 4 the transmission in the opposite direction quickly drops with $T_{21} \to 0$.

IV. CONCLUSIONS

To conclude, we investigate the transmission of an optical probe field in a three-mode optomechanical system, where the mechanical resonator is subject to a mechanical drive with the driving frequency being equal to the frequency difference between the optical probe and pump fields. Under appropriate parameters, the directional amplification of the probe field resulting from the interference between different optical path and phonon-photon parametric process can be achieved. Amplification far exceeding unity can be achieved when the mechanical drive becomes strong. Such optomechanical setups could be used to switch and amplify weak probe signals in quantum networks.

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