Jet Quenching and Azimuthal Anisotropy of Large $p_T$ Spectra in Non-central High-energy Heavy-ion Collisions

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Parton energy loss inside a dense medium leads to the suppression of large $p_T$ hadrons and can also cause azimuthal anisotropy of hadron spectra at large transverse momentum in non-central high-energy heavy-ion collisions. Such azimuthal anisotropy is studied qualitatively in a parton model for heavy-ion collisions at the RHIC energies. The coefficient $v_2(p_T)$ of the elliptic anisotropy at large $p_T$ is found to be very sensitive to parton energy loss. It decreases slowly with $p_T$ contrary to its low $p_T$ behavior where $v_2$ increases very rapidly with $p_T$. The turning point signals the onset of contributions of hard processes and the magnitude of parton energy loss. The centrality dependence of $v_2(p_T)$ is shown to be sensitive to both size and density dependence of the parton energy loss and the later can also be studied via variation of the colliding energy. The anisotropy coefficient $v_2/\varepsilon$ normalized by the spatial ellipticity $\varepsilon$ is found to decrease significantly toward semi-peripheral collisions, differing from the hydrodynamical results for low $p_T$ hadrons. Constrained by the existing WA98 experimental data at the SPS energy on parton energy loss, both hadron spectra suppression and azimuthal anisotropy at high $p_T$ are predicted to vanish for $b > 7.8$ fm in Au + Au collisions at $\sqrt{s} = 130$-200 GeV when the hadron rapidity density per unit area of the initial overlapped region is less than what is achieved in the central Pb + Pb collisions at the SPS energy.

I. INTRODUCTION

In the study of the dense matter that is created in high-energy heavy-ion collisions, one crucial issue is the degree of thermalization through secondary scatterings. If thermalization has been, at least partially, achieved, there should be collective effects developed through the evolution of the system. Azimuthal anisotropy in hadron spectra or elliptic flow had been proposed as a good signature of collective transverse flow in relativistic heavy-ion collisions [1]. Such an elliptic flow had been observed in experiments at the BNL/AGS [2], CERN/SPS [3] and most recently BNL/RHIC by the STAR experiment [4]. The strength of the elliptic flow, $v_2$, experimentally defined as the second coefficient in the Fourier decomposition of the particle azimuthal distribution [5] with respect to the reaction plane, was found [2,5] to be between the limits of low-density rescattering [6] and hydrodynamic expansion [7,8,9,10], indicating approach to a higher degree of thermalization with increasing colliding energies. At RHIC energies, one expects to see $v_2$ becoming closer to the hydrodynamic limit as the initial energy density increases and the lifetime of the initial dense matter is getting longer. Because system evolution will eliminate the geometrical anisotropy which generates the anisotropy in momentum space, the elliptic flow was argued [11,12] to be sensitive to the early dynamics of the system.

In hydrodynamic models, the strength of the differential elliptic flow, $v_2(p_T)$, increases almost linearly with $p_T$ [13,14] (at $p_T \sim 0$, however, $v_2$ increases quadratically with $p_T$) because of collective expansion. At large transverse momentum, the hydrodynamic model will likely cease to be a valid mechanism for particle production in high-energy nuclear collisions. Instead, particle production at $p_T > 2$ GeV/$c$ will be dominated by hard or semihard processes. The hadron spectra in this region of phase space typically exhibit a power-law behavior and depend on how an energetic parton propagates through the dense medium created in the heavy-ion collisions. The corresponding azimuthal anisotropy of hadron spectra at high $p_T$ would also have completely different behavior from hydrodynamic model results.

Recent theoretical studies of fast partons propagating inside a dense medium all suggest a large energy loss caused by multiple scattering and induced gluon radiation [15,16]. Of particular interest is the quadratic dependence of the total energy loss on the distance of propagation [17] due to the non-Abelian nature of gluon radiation in QCD. Such a parton energy loss will cause the suppression of large $p_T$ hadron spectra [18,19] in heavy-ion collisions if the lifetime of the dense medium is long enough to influence the propagation of fast partons. The suppression will depend on the average distance that partons propagate inside the medium during the lifetime of the dense matter. Since the transverse distance depends on the azimuthal direction of the parton propagation in non-central heavy-ion collisions, one should expect the hadron suppression caused by parton energy loss to depend on the azimuthal angle with respect to the reaction plane, thus leading to azimuthal anisotropy in high $p_T$ hadron spectra.

When the momentum transfer $\mu$ for each of the few scatterings suffered by an energetic parton in a finite system is small so that $\mu/p_T \ll 1$, the effect of the elastic
scattering on azimuthal anisotropy in large $p_T$ hadron spectra is negligible. Only parton energy loss can cause sizable azimuthal anisotropy for large $p_T$ hadrons. Therefore, azimuthal anisotropy and hadron spectra suppression at large $p_T$ should accompany each other in high energy heavy-ion collisions. For a parton energy loss $dE/dx$ that has a weak dependence on the parton energy, the hadron spectra suppression and the accompanying azimuthal anisotropy should all decrease with $p_T$ as we shall show in this paper. This is in sharp contrast with the effect of hadronic interactions whose cross sections, especially for inelastic processes, increase with energy and thus lead to increased hadron suppression and azimuthal anisotropy at large $p_T$. Therefore, the $p_T$ dependence of hadron spectra suppression and azimuthal anisotropy is a unique signal of parton scattering and energy loss in the early stage of heavy-ion collisions.

Azimuthal anisotropy due to jet quenching has been studied before with the HIJING Monte Carlo model. In this paper, we will study the azimuthal anisotropy in hadron spectra at large $p_T$ using a parton model in which one incorporates the parton energy loss via modified parton fragmentation functions. We will study the sensitivity of the azimuthal anisotropy, its $p_T$ and centrality dependence to the parton energy loss $dE/dx$ and in particular the distance and density dependence of the parton energy loss.

We should emphasize here that our study in this paper is only qualitative. First of all, the hard sphere geometry we use does not give an accurate description of the spatial anisotropy $\varepsilon$ (or ellipticity). However, the ratio $v_2/\varepsilon$ will reduce the sensitivity to models of geometry. Secondly, our theoretical understanding of parton energy loss is only qualitative so far. There are many uncertainties in the estimate of $dE/dx$ in QCD. The purpose of our study is to demonstrate the effect of jet quenching in hadron spectra and the azimuthal anisotropy for a given averaged $dE/dx$. Furthermore, in our model we will neglect both longitudinal and transverse expansion. The longitudinal expansion will change the particle density and thus the effective total parton energy loss in the medium while the transverse expansion will influence the spatial ellipticity of the medium as the system evolves with time. Since the parton energy loss in a QGP is estimated to be much larger than that in a hadronic gas, we assume that the parton energy loss happens mostly in the partonic medium. Then an energetic parton is likely to travel to the outside of the partonic matter before the transverse expansion changes the spatial ellipticity of the partonic medium significantly. In this case, the expansion will just change the overall average parton energy loss. For a qualitative study in this paper, we are only concerned with the average parton energy loss and we will just consider the dependence of $dE/dx$ on the average particle density in the initial overlapped region.

II. HADRON SPECTRA AT LARGE $p_T$

Hadron spectra in $pp$, $pA$ and $AA$ collisions have been systematically studied in a parton model. This model extends the collinear factorized parton model to include intrinsic transverse momentum and its broadening due to multiple scattering in nuclear matter. The value of the intrinsic transverse momentum and its nuclear broadening are adjusted once and the model can reproduce most of the experimental data in $pp$ and $pA$ collisions. In $AA$ collisions, we model the effect of parton energy loss by the modification of parton fragmentation functions in which the distribution of leading hadrons from parton fragmentation is suppressed due to parton energy loss. In non-central collisions, the single inclusive hadron spectra in this parton model are given by

$$\frac{d\sigma_{AB}}{dy dp_T d\phi} = K \sum_{abcd} \int_{b_{min}}^{b_{max}} d^2 b d^2 r A(r) T(b-r)$$

$$\int dx_a dx_b d^2 k_{aT} d^2 k_{bT} g_A(k_{aT}, Q^2, r) g_B(k_{bT}, Q^2, |b-r|)$$

$$f_{a/A}(x_a, Q^2, r) f_{b/B}(x_b, Q^2, |b-r|)$$

$$\frac{D_{h/c}(z_c, Q^2, L(\phi))}{\pi z_c} \frac{d\sigma}{dt}(ab \rightarrow cd).$$

The factor $K = 1.5 - 2.0$ is used to account for higher order corrections. The nuclear thickness function $t_A(b)$ is normalized to $\int d^2 T_A(b) = A$ using the Woods-Saxon form of nuclear distributions. To take into account of the intrinsic transverse momentum and its nuclear broadening, $g_A(k_T, Q^2, r)$ is assumed to have a Gaussian form and its width is parametrized to fit the $pp$ and $pA$ data. The impact-parameter dependence of $g_A(k_T, Q^2, r)$ comes from the nuclear broadening of the intrinsic transverse momentum. The parton distributions in nuclei $f_{a/A}(x_a, Q^2, r)$ are assumed to be factorized into the parton distributions in a nucleon and the impact-parameter dependent nuclear modification factor,

$$f_{a/A}(x, Q^2, r) = S_{a/A}(x, r) \left[ \frac{Z}{A} f_{a/p}(x, Q^2) + (1 - \frac{Z}{A}) f_{a/n}(x, Q^2) \right].$$

We will use the parameterization of the nuclear modification in HIJING. Constrained by the existing $pA$ data, nuclear modification of parton distributions and $p_T$ broadening produce about 10-30% increase in the $p_T$ spectra in central $Au + Au$ collisions relative to $pp$. However, they will not give any azimuthal anisotropy in the hadron spectra.

Eq. (1) also applies to $pp(\bar{p})$ collisions in which one simply sets $A = 1$ without intrinsic $p_T$ broadening and nuclear modification of the parton distributions. The parton fragmentation functions $D_{h/c}(z_c, Q^2)$ are then given by the parameterization of $e^+e^-$ data. As an example, we show in Fig. [1] the comparison of the parton...
model calculation (solid line) with the experimental data \cite{22} for \( p\bar{p} \) collisions at \( \sqrt{s} = 200 \text{ GeV} \). We used a \( K = 1.5 \) factor in the calculation. The model describes the data very well down to \( p_T \sim 1.5 \text{ GeV/c} \). Below this scale we expect such a parton model calculation to fail and non-perturbative physics to dominate. As an illustration, we fit the experimental data below \( p_T = 1 \text{ GeV/c} \) with an exponential distribution (thin solid line),

\[
\frac{d\sigma_{\text{soft}}}{dydp_T} = \frac{\sigma_{\text{in}}}{2\pi T_0^2} \frac{dn_{\text{ch}}}{d\eta} e^{-p_T/T_0}, \quad (3)
\]

with \( T_0 = 0.2 \text{ GeV} \). The normalization of the fit is determined by \( dn_{\text{ch}}/d\eta = 2.3 \) in the central rapidity region and \( \sigma_{\text{in}} = 42 \text{ mb} \) at \( \sqrt{s} = 200 \text{ GeV} \). One can notice that the data differ significantly from the exponential fit already at around \( p_T = 1.5 \text{ GeV/c} \). Beyond this scale, the spectra have a power-law behavior characteristic of hard parton scattering. We can therefore use the parton model to describe the hadron spectra for \( p_T > 2 \text{ GeV/c} \). In Fig. 1 we also plot the hadron spectra for \( pp \) collisions at \( \sqrt{s} = 130 \text{ GeV} \). The spectrum differs very little from that at \( \sqrt{s} = 200 \text{ GeV} \) at intermediate \( p_T \) and is only about a factor 2 lower at \( p_T = 7 \text{ GeV/c} \).

![FIG. 1. Transverse momentum spectra of charged hadrons in \( pp \) collisions at \( \sqrt{s} = 200 \text{ (solid) and 130 GeV (dot-dashed) as compared to UA1 data \cite{22} on \( pp \) collisions at \( \sqrt{s} = 200 \text{ GeV} \). The thin solid line is a parameterization of the data below \( p_T < 1 \text{ GeV/c} \).](image)

To take into account the effect of parton energy loss in \( AA \) collisions, we use modified effective fragmentation functions, \( D_h/c(z_c, Q^2, L(\phi)) \) for a produced parton \( c \) which has to travel in the medium a distance \( L(\phi) \) that depends on the azimuthal angle in non-central \( AA \) collisions. This will be the dominant source of anisotropy in hadron spectra in this parton model. We will use a phenomenological model \cite{20} for the modified fragmentation functions. The modification in this model depends on two parameters: the energy loss per scattering \( \epsilon \), and the mean free path \( \lambda_c \) for a propagating parton \( c \). The energy loss per unit length of distance is then \( dE_c/dx = \epsilon_c/\lambda_c \). We also assume that a gluon’s mean free path is half that of a quark and then the energy loss \( dE/dx \) is twice that of a quark. According to a pQCD study of the parton energy loss \cite{13},

\[
\frac{dE}{dx} \approx \frac{\alpha_s N_c L}{4} \frac{1}{\lambda_c^2}, \quad (4)
\]

where \( \alpha_s \) is the strong coupling constant, \( N_c = 3 \) and \( \mu \) is the average transverse momentum kick the propagating parton suffers in the medium per scattering. So the energy loss per scattering \( \epsilon \) in our model is now related to \( \mu \) by

\[
\epsilon = \frac{\alpha_s N_c L}{4} \frac{1}{\lambda_c^2}. \quad (5)
\]

![FIG. 2. Transverse momentum spectra of charged hadrons (solid) and \( \pi^0 \) (dot-dashed) in central \( Au + Au \) collisions at \( \sqrt{s} = 200 \text{ and 130 GeV without energy loss and with } dE/dx = 0.5 \text{ GeV/fm (dashed lines for charged hadrons and dotted lines for } \pi^0 \text{.)} \)](image)

Shown in Fig. 2 are the calculated spectra for charged hadrons, averaged over azimuthal angle, in central \( Au + Au \) collisions at \( \sqrt{s} = 130 \text{ and 200 GeV with (dashed lines) and without (solid lines) parton energy loss. In the spectra we also add the soft component in Eq. (3) with } dn_{\text{ch}}/d\eta = 520 \text{ (700) for } \sqrt{s} = 130 \text{ (200) GeV from HIJING calculations \cite{24} and } \sigma_{\text{in}} = \pi b_{\text{max}}^2. \text{ We used a constant } dE/dx = 0.5 \text{ GeV/fm and } \lambda = 1 \text{ fm in the calculation to demonstrate the effect of parton energy loss. The contribution from the soft component is negligible at } p_T > 2 \text{ GeV/c as compared to the hard contribution when there is no parton energy loss. However, when parton energy loss is assumed, the hard contribution is strongly suppressed so that the soft component} \]
become important at some smaller $p_T$. This is why the suppression of the large $p_T$ spectra due to parton energy loss as compared to the spectra without energy loss becomes smaller at smaller $p_T$. However, with $dE/dx = 0.5$ GeV/fm, the contribution from the soft component again is negligible at $p_T > 3$ GeV/c. The suppression is proportional to parton energy loss. The detailed dependence of the suppression on the form and values of parton energy loss can be found in Ref. [16]. Also shown in Fig. 2 are the $\pi^0$ spectra with (dotted lines) and without (dot-dashed lines) parton energy loss. Notice that $\pi^0$ spectra at high $p_T$ are about 40-30% lower than $(h^-+h^+)/2$. This means that $\pi^\pm$ only contribute to about 60-70% of the total charged hadron spectra. Detailed study of the flavor dependence of the parton energy loss effects can also be found in Ref. [16].

### III. AZIMUTHAL ANISOTROPY

For non-central $A + A$ collisions, the averaged distance a parton travels through the medium varies with the azimuthal angle and so does the averaged total parton energy loss. This will give azimuthal anisotropy in the hadron spectra at large transverse momentum with respect to the reaction plane. The reaction plane is defined by the beam direction and the impact parameter $b$ of heavy-ion collisions. Experimentally, the reaction plane should be determined by low $p_T$ particles that constitute the bulk of matter produced in heavy-ion collisions. Large momentum hadrons will not affect the determination of this reaction plane.

![Diagram of reaction plane](image)

**FIG. 3.** Illustration of the geometry of the overlapped region of two colliding nuclei $A$ and $B$ in the transverse plane.

In this model calculation, we will use a hard-sphere nuclear distribution to describe the initial geometry of the medium and to estimate the distance of parton propagation inside the dense medium. As illustrated by Fig. 3 at a given impact parameter $b$, a parton produced at point $r$ has to travel a distance $L(\phi, r, b)$ inside the overlapped region in the azimuthal direction $\phi$. With $L(\phi, r, b)$, one can then calculate the modified fragmentation function $D_{n/f}(z, Q^2, L(\phi, r, b))$ and the hadron spectra. The cross section is weighted with the overlap function of two colliding nuclei according to Eq. (4). To help us to understand the azimuthal distribution of hadron spectra, we show in Fig. 3 the averaged distance,

$$\langle L(\phi) \rangle = \frac{1}{T_{AB}(b)} \int d^2 r t_A(r) t_B(|b - r|) L(\phi, r, b),$$

as a function of $\phi$, that the parton has to travel across the overlapped region for different impact parameters, where $T_{AB}(b) = \int d^2 r t_A(r) t_B(|b - r|)$. As we can see, $\langle L(\phi) \rangle$ on the average decreases with impact parameter. However, the anisotropy increases toward large impact parameters. Since jet quenching is directly proportional to $\langle L(\phi) \rangle$, one should expect similar azimuthal distribution of the hadron suppression at large $p_T$. The azimuthal anisotropy of the hadron spectra, however, should depend on both the averaged value of $\langle L(\phi) \rangle$ and its anisotropy.

![Average distance graph](image)

**FIG. 4.** The averaged distance a fast parton has to travel across the overlapped region of two colliding nuclei as a function of the azimuthal angle $\phi$ for different impact parameters.

The azimuthal angle dependence or azimuthal anisotropy of the distance $\langle L(\phi) \rangle$ is directly related to the spatial deformation of the dense medium which can be characterized by the spatial ellipticity $\varepsilon$. For a hard-sphere distribution,

$$\varepsilon(b) \equiv \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle} = \frac{b}{2R_A}$$

is assumed to be given by the initial overlap of two colliding nuclei with a radius $R_A \approx 1.12 A^{1/3}$ at impact parameter $b$. Such a simple geometry is far from realistic. But it is sufficient to illustrate the qualitative feature of azimuthal anisotropy caused by parton energy loss.
elliptic flow $v_2(p_T)$ for low $p_T$ hadrons increases with $p_T$, the turning point where $v_2(p_T)$ starts to saturate and decrease with $p_T$ will provide information on the interplay between hard and soft components of particle production. Combined with the magnitude of the azimuthal anisotropy at higher $p_T$, it also allows one to extract the average parton energy loss in the medium. Since one cannot make quantitative estimate of parton energy loss, we can only give qualitative predictions of $v_2$ and its dependence on $p_T$.

**IV. CENTRALITY DEPENDENCE**

In non-central heavy-ion collisions, the average distance of parton propagation depends on the impact parameter of collisions. The suppression of hadron spectra at large $p_T$ and the elliptic anisotropy will then also depend on the centrality. Shown in the upper panel of Fig. 6 is the impact parameter dependence of the calculated $v_2$ at fixed $p_T=5$ GeV/c for two different forms of parton energy loss, one constant $dE/dx=0.5$ GeV/fm and another $dE/dx=0.1(L/\text{fm})$ GeV/fm with a linear distance dependence. In both cases, $v_2$ increases with the impact parameter, reflecting the increased spatial ellipticity at large impact parameters. In central collisions, the average parton energy loss $dE/dx$ in the second case is larger than the constant $dE/dx=0.5$ GeV/fm, leading to a larger $v_2$. However, as the impact parameter increases further, the average parton energy loss per unit length with a linear distance dependence becomes smaller than the constant $dE/dx=0.5$ GeV/fm. This leads to smaller $v_2$. Eventually, in more peripheral collisions, $v_2$ decreases with the impact parameter in the second case. Therefore, as compared to the case of a constant $dE/dx$, a linear distance dependence of $dE/dx$ gives a slower impact parameter dependence of $v_2$.

Since the elliptic anisotropy in hadron spectra is directly related to the spatial ellipticity, it is useful to study...
the impact parameter dependence of the ratio $v_2/\varepsilon$ as shown in the lower panel of Fig. 6. Such a ratio will also reduce the sensitivity to the modeling of the initial density distribution of the overlapped region, e.g., hard sphere vs. Woods-Saxon nuclear distributions. For both forms of parton energy loss, the normalized elliptic anisotropy coefficient increases with impact parameter very rapidly in central collisions and then starts to decrease at relatively small impact parameters. The decrease is much faster for the case of linear distance dependence of $dE/dx$. This feature is markedly different from the hydrodynamic models [8], in which $v_2/\varepsilon$ remains constant for a very large range of impact parameters. So far for a constant $dE/dx$, the hydrodynamic models [8], in which $\sigma_{\rho}$ remains constant for a very large range of impact parameters. So far experimental measurements of the impact parameter dependence of $v_2$ [4] for low $p_T$ hadrons show a behavior roughly similar to the hydrodynamic results. A different behavior of $v_2/\varepsilon$ at large $p_T$ that decreases with the impact parameter will be a clear indication of the effect of parton energy loss.

For a qualitative study in this paper, we will neglect the effect of expansion and we are only concerned with the average parton energy loss. In this case, the parton energy loss $dE/dx$ should depend on the averaged particle density in the overlapped region of heavy-ion collisions, which in turn depends on the centrality of the collisions.

According to Eq. (4), $dE/dx$ is proportional to $\mu^2/\lambda = \mu^2\sigma\rho$ with $\mu$ being the momentum transfer per scattering. For a parton scattering cross section $\sigma$ that is screened by $\mu^2$, $\mu^2\sigma$ is roughly a constant. Then $dE/dx$ will be proportional to the parton density $\rho$. We assume that the initial parton density is proportional to the final hadron multiplicity per unit transverse area, $\rho \sim dN/dy/(\tau_0S(b))$, with $S(b)$ the transverse area of the initial overlapped region and $\tau_0$ the initial time. Using the two-component (soft and hard) model of particle production in HIJING [18], $dN/dy$ has one term (soft) proportional to the number of participant nucleons $N_{\text{part}}$ and another (hard) proportional to the number of binary collisions $N_{\text{binary}}$. In a Glauber model of nuclear collisions, $S(b) \sim N_{\text{part}}^{2/3}(b)$ and $N_{\text{binary}}(b) \sim N_{\text{part}}^{4/3}(b)$. Parameterizing the centrality dependence of $dN/dy$ in the HIJING calculation [18] for $Au + Au$ collisions at $\sqrt{s} = 130$ GeV, we have

$$\frac{dE}{dx} = \left( \frac{dE}{dx} \right)_0 \rho(b);$$

$$\rho(b) = 0.072N_{\text{part}}^{1/3}(b) \left[ 1 + 0.13N_{\text{part}}^{1/3}(b) \right], \quad (9)$$

where the impact parameter dependence of $N_{\text{part}}(b)$ can be calculated in a Glauber model of nuclear collisions with Woods-Saxon nuclear distributions. The above centrality dependence of the average $dE/dx$ is normalized to central collisions, $dE/dx(b = 0) = (dE/dx)_0$, or $\rho(b = 0) = 1$. Such an impact parameter dependence of $dE/dx$ is plotted in Fig. 8 as the solid line for $(dE/dx)_0 = 0.1(L/fm)$ GeV/fm.

Theoretical studies have predicted a large parton energy loss in a hot partonic medium as compared to cold nuclear matter [3]. However, analysis of the large $p_T$ $\pi^0$ spectra in central $Pb + Pb$ collisions from WA98 experiment [24] at the SPS energy $\sqrt{s} = 17$ GeV shows no indication of parton energy loss [24]. Since the high $p_T$ spectra at SPS are very sensitive to any change of the jet cross section, the constraint on parton energy loss by the WA98 data is very stringent and gives a limit that is much smaller than the most conservative estimate of parton energy loss in a hadronic matter [14]. This casts serious doubts on the accuracy of the theoretical esti-

FIG. 7. Upper panel: The impact parameter dependence of azimuthal anisotropy coefficient $v_2$ for charged hadrons with $p_T = 5$ GeV/c in $Au + Au$ collisions at $\sqrt{s} = 130$ GeV for a constant $dE/dx = 0.5$ GeV/fm (open circle) and distance-dependent $dE/dx = 0.1(L/fm)$ GeV/fm (solid circle) parton energy loss. Lower panel: The impact parameter dependence of $v_2/\varepsilon$. Here, $\varepsilon = b/2R_A$ is the spatial ellipticity of the initial overlapped region of collisions with a hard sphere nuclear distribution.

FIG. 8. Three scenarios of impact parameter dependence of parton energy loss $dE/dx = 0.1(L/fm)\rho(b)$ GeV/fm
mates which are based on a scenario of a static and infinitely large \((L/\lambda \gg 1)\) dense partonic gas. Though recent studies \([23, 29]\) have considered the finite number of scatterings in a medium with finite size, it is still not clear whether the estimates are numerically consistent with the WA98 data. The experimental data of WA98 could also imply that the lifetime of the dense partonic system is too short to induce any parton energy loss. If we take the implication of the WA98 data at its face value, we can assume that the effective parton energy loss vanishes in non-central collisions in which the average particle density is equal to or less than that of central \(Pb+Pb\) collisions at the SPS energy. Using HIJING simulations we find that the particle rapidity density per unit transverse area in \(|y| < 1\) for \(Au+Au\) collisions becomes less than that of central \(Pb+Pb\) collisions at the SPS energy for impact parameters larger than 7 fm at \(\sqrt{s} = 130\) GeV, or 8 fm at \(\sqrt{s} = 200\) GeV. For smaller impact parameters, we will assume the normal centrality dependence of the parton energy loss in Eq. \((9)\). Such a scenario is denoted as density dependence \(\rho_1(b)\) and is shown in Fig. \(8\) as the dashed line. The sudden drop of \(dE/\text{d}x\) with impact parameter might be extreme and is only to illustrate the effect of a possible critical behavior in jet quenching.

Shown in Fig. \(8\) are the calculated coefficients of the azimuthal anisotropy \(v_2\) and \(v_2/\varepsilon\) for charged hadrons at \(p_T = 5\) GeV/c with the above two scenarios of density dependence of parton energy loss (solid and dashed lines). As a comparison, we also show the results of a density-independent \(dE/\text{d}x\) (dot-dashed lines). Comparing the results of scenario I of density dependence (solid lines) to a density-independent \(dE/\text{d}x\) (dot-dashed lines), it is obvious that the density dependence of the parton energy loss \(dE/\text{d}x\) reduces the azimuthal anisotropy at large impact parameters because of the reduced parton energy loss in peripheral collisions as compared to the central ones. The ratio \(v_2/\varepsilon\) therefore decreases faster toward the peripheral collisions as compared to the case of a density-independent \(dE/\text{d}x\). Because of the complication of such a density dependence, it is difficult to single out the quadratic distance dependence of the parton energy loss by studying the centrality dependence of the azimuthal anisotropy alone. One has to combine it with the study of density dependence by measuring the anisotropy for a fixed system at different colliding energies. In the second scenario of the density dependence of parton energy loss, \(v_2\) will vanish above impact parameter \(b > 7\) fm, where \(dE/\text{d}x\) is assumed to vanish when the particle density is below what was reached in the central \(Pb+Pb\) collisions at the SPS energy. The drop should occur at larger impact parameters for higher colliding energies because of the increased particle density in the collision region. The behavior of \(v_2\) at small or intermediate impact parameters is dictated by the geometry of the medium and is not very sensitive to small changes of the density dependence of the parton energy loss. It is more sensitive to the density dependence at large impact parameters in more peripheral collisions. The vanishing \(dE/\text{d}x\) at large impact parameter in the second scenario has very dramatic effect on \(v_2\). Such a scenario will be easy to verify from experimental data.

At RHIC energies the initial energy density is expected to be higher than at the SPS with longer lifetime. Indeed the first experimental measurement of \(dN_{ch}/dy\) by PHOBOS \([34]\) shows that the particle production for central \(Au+Au\) collision at \(\sqrt{s} = 130\) GeV is about 50% higher than in central \(Pb+Pb\) collisions at the SPS energy, consistent with the default value of HIJING simulations. If we take the implication of the WA98 data at its face value, we can assume that the effective parton energy loss vanishes in non-central collisions in which the average particle density is equal to or less than that of central \(Pb+Pb\) collisions at the SPS energy. Using HIJING simulations we find that the particle rapidity density per unit transverse area in \(|y| < 1\) for \(Au+Au\) collisions becomes less than that of central \(Pb+Pb\) collisions at the SPS energy for impact parameters larger than 7 fm at \(\sqrt{s} = 130\) GeV, or 8 fm at \(\sqrt{s} = 200\) GeV. For smaller impact parameters, we will assume the normal centrality dependence of the parton energy loss in Eq. \((9)\). Such a scenario is denoted as density dependence \(\rho_1(b)\) and is shown in Fig. \(8\) as the dashed line. The sudden drop of \(dE/\text{d}x\) with impact parameter might be extreme and is only to illustrate the effect of a possible critical behavior in jet quenching.

Shown in Fig. \(8\) are the calculated coefficients of the azimuthal anisotropy \(v_2\) and \(v_2/\varepsilon\) for charged hadrons at \(p_T = 5\) GeV/c with the above two scenarios of density dependence of parton energy loss (solid and dashed lines). As a comparison, we also show the results of a density-independent \(dE/\text{d}x\) (dot-dashed lines). Comparing the results of scenario I of density dependence (solid lines) to a density-independent \(dE/\text{d}x\) (dot-dashed lines), it is obvious that the density dependence of the parton energy loss \(dE/\text{d}x\) reduces the azimuthal anisotropy at large impact parameters because of the reduced parton energy loss in peripheral collisions as compared to the central ones. The ratio \(v_2/\varepsilon\) therefore decreases faster toward the peripheral collisions as compared to the case of a density-independent \(dE/\text{d}x\). Because of the complication of such a density dependence, it is difficult to single out the quadratic distance dependence of the parton energy loss by studying the centrality dependence of the azimuthal anisotropy alone. One has to combine it with the study of density dependence by measuring the anisotropy for a fixed system at different colliding energies. In the second scenario of the density dependence of parton energy loss, \(v_2\) will vanish above impact parameter \(b > 7\) fm, where \(dE/\text{d}x\) is assumed to vanish when the particle density is below what was reached in the central \(Pb+Pb\) collisions at the SPS energy. The drop should occur at larger impact parameters for higher colliding energies because of the increased particle density in the collision region. The behavior of \(v_2\) at small or intermediate impact parameters is dictated by the geometry of the medium and is not very sensitive to small changes of the density dependence of the parton energy loss. It is more sensitive to the density dependence at large impact parameters in more peripheral collisions. The vanishing \(dE/\text{d}x\) at large impact parameter in the second scenario has very dramatic effect on \(v_2\). Such a scenario will be easy to verify from experimental data.

Since the azimuthal anisotropy in large \(p_T\) hadron spectra in a parton model is caused predominantly by jet quenching or parton energy loss, such a study should be complemented with direct measurements of the suppression of large \(p_T\) hadron spectra relative to \(pp\) or \(pA\) collisions at the same energy. To facilitate such direct measurements, one needs to scale the spectra in \(AA\) collisions by that of \(pp\) and geometric factors. According to the parton model in Eq. \((9)\), the ratio

\[
R_{AA}(b, p_T) \equiv \frac{dN_{AA}/dyd^2p_T}{\langle N_{\text{binary}} \rangle (b) dN_{pp}/dyd^2p_T} \tag{10}
\]

will be 1, independent of \(p_T\) and impact parameter \(b\) if there are no nuclear effects in the large \(p_T\) hadron production. Jet quenching will suppress the large \(p_T\) hadron spectra and reduce the above ratio for non-vanishing parton energy loss \([10]\). Here \(\langle N_{\text{binary}} \rangle (b)\), defined as

\[
\langle N_{\text{binary}} \rangle (b) = \frac{\sigma_{\text{in}}^{pp}}{\sigma_{\text{in}}^{AA}(b_{\text{min}}, b_{\text{max}})} \int_{b_{\text{min}}}^{b_{\text{max}}} d^2b T_{AA}(b), \tag{11}
\]
is the averaged number of binary collisions that can be calculated in a Glauber model of nuclear collisions with given $\langle b \rangle = (b_{\text{min}} + b_{\text{max}})/2$.

![Figure 10](image.png)

**FIG. 10.** Impact parameter dependence of the ratio $R_{AA}(p_T, b)$ of the charged hadron spectra at $p_T=5$ GeV/$c$ in $Au+Au$ and $pp$ collisions at $\sqrt{s} = 130$ GeV for three different scenarios of density dependence of parton energy loss.

**V. CONCLUSIONS AND DISCUSSIONS**

In this paper we have studied in a parton model the azimuthal anisotropy of large $p_T$ hadron spectra in noncentral high-energy heavy-ion collisions due to parton energy loss. We demonstrated that the anisotropy is very sensitive to the parton energy loss $dE/dx$ and its dependence on the size and density of the medium. We predict that the coefficient of the anisotropy $v_2$ decreases slowly with $p_T$. The parton model also predicts an early saturation of $v_2$ as a function of the impact parameter $b$. Once divided by the spatial anisotropy $\varepsilon(b)$, the ratio $v_2/\varepsilon$ will increase with $b$ for very central collisions and then will decrease toward peripheral collisions. This is in sharp contrast to the centrality dependence of the elliptic flow $v_2$ for low $p_T$ hadrons. Hydrodynamic models predict almost a constant $v_2/\varepsilon$ in a very large range of centralities. We found that the centrality dependence of $v_2$ is sensitive to the size and density dependence of parton energy loss $dE/dx$ in the medium.

Since the azimuthal anisotropy is directly related to parton energy loss, one should combine the study of azimuthal anisotropy with the direct measurements of hadron spectra suppression at large $p_T$. We propose to study the density dependence of parton energy loss by measuring the hadron suppression and the anisotropy at different colliding energies for the same system. The centrality dependence of the hadron suppression and anisotropy can then provide information on the size dependence of the parton energy loss. Perturbative QCD studies predict a nonlinear dependence of the total energy loss due to non-Abelian gluon radiation.

Based on constraints by WA98 experimental data on high $p_T$ pion spectra, we also proposed a density dependence of the parton energy loss that will vanish in peripheral collisions, where the particle density is equal to or smaller than what has been achieved in the central $Pb+Pb$ collisions at the SPS energy. Then both hadron spectra suppression and the azimuthal anisotropy will disappear in these peripheral collisions. Such a strong centrality dependence would indicate an onset of parton energy loss in a dense medium.

Parton energy loss is always associated with momentum broadening perpendicular to the parton propagation direction. Such transverse momentum broadening can also lead to azimuthal anisotropy in the final hadron spectra. One can characterize the broadening by the momentum transfer $\mu$ in each parton-medium scattering. The effect of the broadening on the azimuthal anisotropy can then be characterized by the diffraction angle of each scattering $\phi_0 = \mu/p_T$. For $dE/dx = 0.1(L/fm)$ GeV/fm and $\lambda \sim 1$ fm, which we have used to demonstrate the effect of parton energy loss in this paper, $\mu \sim 0.3$ GeV/$c$ according to Eq. (4) and $\phi_0 \sim 0.06$ for $p_T = 5$ GeV/$c$. In a medium with a size of only a few mean free paths, the contribution to $v_2$ due to such a small momentum transfer is almost negligible. Therefore, the dominant cause of elliptic anisotropy at large $p_T$ will be the radiative parton energy loss in the medium. If any suppression of large $p_T$ hadron is observed in experiments, there should be non-vanishing azimuthal anisotropy, and vice versa.

We have considered a parton energy loss that has only a weak (logarithmic) dependence on the initial parton energy. Recent studies [27], however, show a stronger energy dependence as a result of the kinetic limits in the estimate of energy loss for a parton with finite initial energy. Such a strong energy dependence would give a different $p_T$ dependence of hadron spectra suppression and the accompanying azimuthal anisotropy.

For a qualitative study in this paper, we did not consider the expansion of the system. The longitudinal ex-
pansion will change the particle density in the medium, while the transverse expansion will decrease the spatial anisotropy as the system evolves with time. The decrease of particle density will reduce the average parton energy loss over the course of the evolution. The change of spatial anisotropy will reduce the resultant anisotropy in hadron spectra. It could also affect the centrality dependence of the jet quenching effect. Detailed study of the jet quenching in a dynamically evolving system will be the subject of future studies and is needed for a more quantitative analysis of any experimental data.

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