Spontaneous symmetry breaking in cosmos: The hybrid symmetron as a dark energy switching device

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We consider symmetron model in a generalized background with a hope to make it compatible with dark energy. We observe a “no go” theorem at least in case of a conformal coupling. Being convinced of symmetron incapability to be dark energy, we try to retain its role for spontaneous symmetry breaking and assign the role of dark energy either to standard quintessence or \( F(R) \) theory which are switched on by symmetron field in the symmetry broken phase. The scenario reduces to standard Einstein gravity in the high density region. After the phase transition generated by symmetron field, either the \( F(R) \) gravity or the standard quintessence are induced in the low density region. We demonstrate that local gravity constraints and other requirements are satisfied although the model could generate the late-time acceleration of Universe.

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I. INTRODUCTION

One of the inconsistencies of hot big bang model is associated with the observed primordial density fluctuation necessary for large scale structure in Universe. The paradigm which successfully addresses this problem along with the resolution of other theoretical issues such as flatness and horizon problems is inflation. The standard model of Universe is also plagued with age crisis which is time lapse inconsistency. In the standard model, the only known way to address this problem is provided by late time acceleration of Universe. This serves as an indirect evidence of the presence of an exotic matter of repulsive nature in Universe dubbed dark energy in the framework of standard lore. The discovery of the accelerating expansion of Universe \[1, \, 2\] in 1998 supports this hypothesis. These observations tell us that the dark energy amounts to about 73\% of the total energy budget of the Universe at present \[3, \, 11\].

The simplest cosmological model of dark energy is based upon cosmological constant (\( \Lambda \)) but the framework faces the most difficult conceptual and theoretical issues associated with (\( \Lambda \)). With a hope to alleviate some of these problems, a variety of scalar field models such as quintessence, phantoms, k-essence and tachyons have been investigated in the literature. \[12-16\]. Naively, the mass of slowly rolling quintessence field should be of the order Hubble rate in the present Universe, \( H_0 \sim 10^{-33} \) eV. In case, it is much larger (smaller) than \( H_0 \), it would mimic stiff matter (\( \Lambda \)). The incredible small mass scale of dark energy gives rise to formidable problems. In case, the scalar field is coupled to matter, as it should be in general, the propagation of the scalar field could generate the large correction to the Newton law and could be excluded by the local physics constraints.

In order to address the problem related to local gravity constraints, three mechanisms of mass screening have been employed in cosmology. (1) Chameleon scenario: This mechanism operates with the field mass which becomes dependent on the local density of environment such that the latter gets large in high density regime thereby leading to effectively decoupling of field from matter or suppression of fifth force \[17, \, 18\]. (2) Vainshtein mass screening \[19\]: This mechanism is superior to chameleon and operates dynamically with non-linear derivative interactions. In the neighborhood of a massive body, the non-linear kinetic terms become strong leading to effectively decoupling of field from the source in a large region, around the massive body, specified by the so called Vainshtein radius.

Recently, a very interesting, third screening mechanism closely related to chameleon dubbed symmetron is proposed in Refs. \[20, \, 22\]. The idea of symmetron is related to the late time cosmic phase transition via spontaneous symmetry breaking. Similar to the chameleon model, the mass of the symmetron scalar field depends on the density of environment in a specific way due to its direct coupling to matter. For densities lower than some critical value, symmetron becomes tachyon and the symmetric vacuum state (\( \phi = 0 \)) is no longer a true vacuum. In this case, there are two vacua with \( Z_2 \) symmetry which breaks soon after we choose one of these. The mass around the true minimum is well behaved. It is clear that if symmetron is to be relevant to the dark energy, the phase transition should take place when the density of environ-
ment is low and the mass of the symmetron in the true vacuum is around $H_0$, which could be the first requirement. The second requirement is that the symmetron should be invisible locally, that is, the fifth force induced by the symmetron should be negligibly small as compared to the Newtonian force of gravity. The second requirement can be easily satisfied in the original symmetron model. As for the first, however, local physics imposes severe constraints on the symmetron mass which turns out to be quite heavy to derive late time acceleration. Although the symmetron might play some role during structure formation, the original idea of introduction of symmetron seems to be defeated.

No doubt that the symmetron presents a beautiful idea and we believe that beauty cannot go for waste. Though the original idea of the symmetron does not seem to work for dark energy without unnatural fine tunings but the role of symmetron in cosmic symmetry breaking could be retained. We propose a model where we use the symmetron to facilitate the cosmic phase transition and the role of dark energy is played either by a second quintessence field or $F(R)$ gravity which are switched on by the symmetron after the phase transition is over. In our proposal, the action reduces to that of the $F(R)$ gravity (for reviews, see [21, 22]) or standard quintessence field $\phi$ in the bulk, where the energy density is very small but action reduces to the usual Einstein-Hilbert one in the high density region, like in/on the earth, in the solar system, and in the galaxies, which makes the model consistent with the observations and local experiments.

The plan of the paper is as follows. We, first, very briefly revisit the standard symmetron scenario, specially focusing on its failure for dark energy. As an attempt to avoid the problem, we investigate a model where symmetron couples to curvature scalar but observe a no go theorem for late time acceleration. Hence we propose a model where $F(R)$ gravity couples with symmetron and show that there exists a parameter region which satisfies possible constraints. The last section is devoted to summary and discussion.

II. STANDARD SYMMETRON

In this section, we briefly review the standard symmetron model. This model is a generic attempt to implement the original Ginsburg-Landau idea of phase transitions in cosmology. The model is based upon Higgs type scalar field potential with $Z_2$ symmetry. In the high density regime, the system resides in the symmetric ground state, symmetry breaks spontaneously in low density region. The consistency of the model with local physics severely constraints the symmetron field to be relevant to dark energy [21, 22].

A. Failure of symmetron to be dark energy

Let us very briefly outline the basic features of the original symmetron scenario based upon the following Einstein frame [20, 22]

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right] + S_m \left[ A^2(\phi) g_{\mu\nu}, \Psi_m \right].$$

(1)

Here $S_m$ expresses the action of matter denoted by $\Psi_m$. We also choose $A(\phi) = 1 + \left[ \phi^2 / (2M^2) \right]$, where $M$ is a mass scale in the model. The effective potential then takes the following form $V_{eff} = (1/2) \left( \rho / M^2 - \mu^2 \right) \phi^2 + (1/4) \lambda \phi^4$. The mass of the field now depends upon the density of environment, naively, the field mass is given by, $m_{eff}^2 = \rho / M^2 - \mu^2$. In high density regime, the mass depends upon density linearly, $m_{eff}^2 \sim \rho / M^2 > 0$. In this case, the system resides in the symmetric vacuum $\phi = 0$. The requirement of local gravity constraints puts an upper bound on $M$ and there is no a priori reason for it to be consistent with dark energy. In case of chameleon, there is more flexibility, the mass depends on density non-linearly. As shown in [21, 22], $M \leq 10^{-4} M_{Pl}$ in case of symmetron. As the density redshifts with expansion and $\rho$ falls below $\mu^2 M^2$, tachyonic instability builds in the system and the symmetric state $\phi_0 = 0$ is no longer a true minimum; the true minima are then given by $\phi_0 = \pm \sqrt{\mu^2 - (\rho / M^2)} / \lambda$, and the mass of the symmetron about the true minimum is, $m_0 = \sqrt{2\mu}$ (at low density). Universe goes through a crucial transition when late time acceleration sets in around the redshift $z \sim 1$. One thus assumes that the phase transition or symmetry breaking takes place when $\rho$ is around $\rho_{cr}$ as $\rho_{cr} \sim M^2 \mu^2 \rightarrow \mu^2 \sim (H_0^2 M_{Pl}^2) / M^2 \rightarrow m_s \sim (H_0 M_{Pl}) / M$. This means that $m_s \geq 10^3 H_0$ which is larger than the required quintessence mass by four orders of magnitude. In this case, the field rolls too fast around the present epoch making itself untenable for cosmic acceleration. Invoking the more complicated potential with minimum with the required potential height does not solve the problem; field goes oscillating for a long time and does not settle in the minimum unless one arranges symmetry breaking very near to $z = 0$ by unnatural fine tuning of parameters [21, 22]. This also undermines the beauty of the underlying theory which is renormalizable and has an edge over chameleon theories which use potentials with complicated functional forms. In what follows we shall present a model which uses the $\lambda \phi^4$ theory in a modified background.

B. An effort to make symmetron compatible with dark energy that does not succeed

Since the complicated choice of original potential will not help. We thus modify the effective potential by virtue
of the modification of the background. We consider the following model,
\[ S = \int d^4x \sqrt{-g} \left[ F(\phi)R - \frac{1}{2}(\nabla \phi)^2 - V(\phi) \right] + S_m \left[ A^2(\phi)g_{\mu\nu}, \Psi_m \right], \quad (2) \]
where \( F(\phi) \) is an arbitrary function of the field which should satisfy the obvious requirements of \( Z_2 \) symmetry and the requirement that the model should not have instabilities. It is interesting to notice that if \( F = A^2 \), the model \( [2] \) is equivalent to a minimally coupled quintessence field.

Variation of action with respect to \( g_{\mu\nu} \) and \( \phi \) gives the following equations of motion
\[ F(\phi)G_{\mu\nu} = \frac{1}{2} \partial_{\mu} \phi \partial_{\nu} \phi - \frac{1}{2} g_{\mu\nu} \left[ \frac{1}{2}(\nabla \phi)^2 + V(\phi) \right] \]
\[ \quad + \nabla_{\mu\nu} F - g_{\mu\nu} \Box F + \frac{1}{2} T_{\mu\nu}, \quad (3) \]
\[ \Box \phi = V'(\phi) - R F' - \frac{A'}{A} T. \quad (4) \]

Taking the trace of Eq. \( [3] \), we can express \( R \) through \( T \) and combinations of \( \phi \) and substituting the same in the field equation gives
\[ (F + 3F'^2) \Box \phi + F' \left[ 3F'' + \frac{1}{2} (\nabla \phi)^2 \right] = \left( \frac{F'}{2} - \frac{A'}{A} F \right) T + F V' - 2F' V. \quad (5) \]

The second term in Eq. \( [5] \) is an extra kinetic term obtained due to field coupling with curvature. We should ensure that it does not give rise to instabilities. For simplicity, it is better to switch it off but keeping modification of gravity alive,
\[ F' \left( 3F'' + \frac{1}{2} \right) = 0. \quad (6) \]

In \( [6] \), an option \( F' = 0 \) corresponds to the general relativity by Einstein, which is not desirable for us. The second option is interesting,
\[ 3F'' + \frac{1}{2} = 0 \Rightarrow F(\phi) = \alpha + \beta \phi - \frac{\phi^2}{12}. \quad (7) \]

Since we like to have \( Z_2 \) symmetry to hold, we choose \( \beta = 0 \) and for \( \alpha \), the obvious choice is, \( \alpha = \frac{M_{Pl}^2}{2} \). We then have the expression for \( F(\phi) \),
\[ F(\phi) = \frac{M_{Pl}^2}{2} - \frac{\phi^2}{12}. \quad (8) \]

We notice that \( F(\phi) > 0 \) in generic range of \( \phi \) and corresponds to the conformal coupling. The field equation with this choice becomes
\[ \frac{M_{Pl}^2}{2} \Box \phi = \left( \frac{F'}{2} - \frac{A'}{A} F \right) T + F V' - 2F' V. \quad (9) \]

We now make use of generic functional forms of \( V(\phi) \) and \( A(\phi) \),
\[ V(\phi) = -\frac{1}{2} \mu^2 \phi^2 + \frac{1}{4} \lambda \phi^4, \quad A(\phi) = 1 + \frac{\phi^2}{2M^2}. \quad (10) \]

The field equation \( [9] \) then takes the following form \( \Box \phi = dV_{\text{eff}}/d\phi \) with
\[ V_{\text{eff}} = \left( -\frac{1}{2} \mu^2 + \frac{\rho}{2} \left( \epsilon M^2 + \frac{1}{6M_{Pl}^2} \right) \right) \phi^2 \]
\[ + \frac{1}{4} \left( \lambda - \frac{\mu^2}{6M_{Pl}^2} - \frac{\epsilon \rho}{12M^2 M_{Pl}^2} \right) \phi^4. \quad (11) \]

In order to carry out the consistency check, it is instructive to look at the expression of \( V_{\text{eff}} \) in low and high density regimes,

| Density regime          | \( V_{\text{eff}} \)                                                                 |
|-------------------------|-------------------------------------------------------------------------------------|
| High density regime     | \( V_{\text{eff}} \approx \frac{\rho}{2} \left[ \left( \frac{\epsilon}{M^2} + \frac{1}{6M_{Pl}^2} \right) \phi^2 - \frac{\epsilon}{24M^2 M_{Pl}^2} \phi^4 \right] \) |
| Low density regime      | \( V_{\text{eff}} \approx -\frac{1}{2} \mu^2 \phi^2 + \frac{1}{4} \left( \lambda - \frac{\mu^2}{6M_{Pl}^2} \right) \phi^4. \) |

\[ 1 - \frac{\mu^2}{6M_{Pl}^2} > 0, \quad M \geq \sqrt{6} M_{Pl}. \]

Let us now consider spontaneous symmetry breaking which takes place when the coefficient of \( \phi^2 \) in Eq. \( [11] \) turns negative which happens when
\[ \rho_{SB} = \mu^2 \left( \frac{1}{6M_{Pl}^2} - \frac{1}{M^2} \right)^{-1}. \quad (14) \]
Since we like to have the phase transition to take place at late times when Universe enters the regime of accelerated expansion, we shall identify $\rho_{SB}$ with $\rho_{cr}$

$$\rho_{SB} \simeq \rho_{cr} = H_0^2 M_{Pl}^2 \to \mu^2 \simeq \left( \frac{1}{6 M_{Pl}^2} - \frac{1}{M^2} \right) H_0^2 M_{Pl}^2. \tag{15}$$

Since the effective mass of the field in the low density regime is $m_e = \sqrt{2} \mu$ and we like to have it to be of the order of $H_0$, we define $\mu = a H_0$ where $a = O(1)$. Using the relation, $M > \sqrt{6} M_{Pl}$, we notice that

$$\alpha^2 H_0^2 \simeq \left( \frac{1}{6 M_{Pl}^2} - \frac{1}{M^2} \right) H_0^2 M_{Pl}^2 \to 0 < \alpha < 1/\sqrt{6}. \tag{16}$$

It is important to check whether or not the model is consistent with local physics or the local gravity constraint on symmetron mass are compatible with dark energy. In order to do that, we need to transform the action into the Einstein frame and obtain expressions for transformed potential and the coupling. It is then not difficult to demonstrate that the local gravity constraints in this case impose $\alpha > 10^4$ which do not match the cosmological condition derived previously.

Let us try to understand why it happens. Broadly, the modified background brings two changes from the point of view of Einstein frame. First, the effective potential, secondly, the coupling $A^2$ in the matter Lagrangian. Both terms are drastically modified but in the leading approximation, $\phi/M \ll 1$ $(\phi/M_{Pl} \ll 1)$, we recover the standard symmetron action. Thus roughly, the mass scale $M$ got replaced by $(-1/M^2 + 1/6 M_{Pl}^2)^{-1}$ which also defines the mass of symmetron in the broken phase, see Eq. (14), thereby leading to the same local gravity constraint as in the original symmetron model. Hence we see that the only difference would be a renormalization of the coefficients in the model, but the constraints will remain same, and therefore the symmetron cannot play the role of a dark energy fluid. It is important to say few words about this generic feature of modified gravity models.

Recently, a no go theorem related to the scope of modified theories based upon chameleon/symmetron was discussed in Ref. [32]. We have not tried to avoid the “no go” theorem, our proposal is rather in agreement with Ref. [32] that late time acceleration is not the result of gravity modification but is driven by some quintessence field or a cosmological constant. In the subsection to follow, we add some clarifications on the important findings of Ref. [32] in the background of our proposal.

### III. Scope of Chameleon/Symmetron Supported Modification of Gravity for Self Acceleration

There are two ways of gravity modification: (1) Apart from the massless spin-2 object, there is essentially at least an extra scalar degree of freedom which is exchanged apart from the graviton. In order to be relevant at large scales where modification is sought, the latter should gives rise to the effect of same strength as that gravitational interaction. This would then cause a havoc locally. The screening mechanism mentioned above address this problem by locally suppressing the exchange effects of the scalar degree of freedom. (2) The conformal coupling $A(\phi)$ also modifies the strength of gravitational interaction. To pass the local tests, $A(\phi)$ should be very closely equal to one in high density regime in theories based upon chameleon/symmetron mechanism. As mentioned above, Universe has undergone a phase transition during $0 < z < 1$ which is a large scale phenomenon and one might think that the screening which is a local phenomenon should not impose severe constraints on how $A(\phi)$ changes during the period acceleration sets in. It, however, turns out that the change the conformal coupling suffers as redshift changes from one to zero is negligibly small. Then a question arises, can such a conformal coupling be relevant to late time acceleration? It is well known that the de Sitter Universe is conformally equivalent to the Minkowski space-time. Has the conformal transformation changed physics? By ‘physics’, we mean the relationship between physical observables which is same in both the frames. In the Einstein frame we have the Minkowski space-time in which there is a scalar field sourced by the conformal coupling which is dynamical and couples to matter directly. The masses of all material particles are time dependent by virtue of $A(\phi)$. As a result, one would see the same relations between physical observables in both the frames. The acceleration dubbed self acceleration is generic which can be removed (caused) by virtue of conformal coupling [32]. Similar thing happens with Dvali-Gabadadze-Porrati (DGP) model [33, 34]. Late time cosmic acceleration which cannot be affected by conformal coupling is caused by the presence of slowly rolling (coupled) quintessence and is not a generic effect of modified theory of gravity.

Let us briefly check how it happens.

We have the following translation between the Einstein and Jordan frames,

$$a^J(t^J) = A(\phi) a^E(t^E), \quad dt^J = A(\phi) dt^E, \tag{17}$$

and conformal time which is same in both the frames, $dt = a(t) d\eta$. In (17), $a^J$ $(a^E)$ is a scale factor and $t^J$ $(t^E)$ is the cosmic time in the Jordan (Einstein) frame, respectively.

Following Ref. [32], it is easy to check that,

$$\ddot{a}^J a^J - \ddot{a}^E a^E = \left( \frac{A''}{A} - \frac{A'^2}{A^2} \right) = \left( \frac{A'}{A} \right)' \tag{18}.$$
Let notice that acceleration in the Einstein frame cannot be caused by conformal coupling,

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{Pl}^2} \left((\rho_\phi + 3P_\phi) + \beta_s \rho A(\phi)\right).$$  \hspace{1cm} (19)

It is clear that in case acceleration takes place in the Einstein frame, it can only be caused by slowly rolling quintessence ($\rho_\phi + 3P_\phi < 0$). This implies that acceleration in the Jordan frame and no acceleration in the Einstein frame is generic effect of conformal coupling or gravity modification. In this case, while passing from the Jordan to the Einstein frame, the acceleration is completely removed, its affects in the Einstein frame are contained in the coupling such that the relationship between physical observables are same in both the frames. The definition of self acceleration \[^{[32]}\], $\ddot{A} a^E < 0 \ (\ddot{a} a^J > 0)$, then implies

$$\left(\frac{A'}{A}\right) \geq \frac{\ddot{a}}{a}.$$

Since, $A' = \ddot{a} a^J \Delta A \ (\Delta A$ - change over one Hubble (Jordan) time), it follows that

$$\frac{a}{a} \frac{d}{dt} \left(\frac{\Delta A}{A}\right) \geq a \ddot{a}, \quad \Delta A = \left(\frac{1}{H J} \frac{dA}{dt}\right).$$  \hspace{1cm} (20)

Integrating left right the above relation, we get \[^{[32]}\]

$$\frac{\Delta A}{A} \geq 1.$$  \hspace{1cm} (22)

This quantity gives formally change of $A$ over one Hubble time in the Jordan frame. As mentioned earlier, $A$ is defined in the Einstein frame and naturally, the estimates of $A$ from screening are obtained in that frame

$$\left(\frac{\Delta A}{A}\right) = \frac{\left(\frac{1}{H E} \frac{dA}{dt}\right)}{1 + \left(\frac{1}{H J} \frac{dA}{dt}\right)}.$$  \hspace{1cm} (23)

In case, $\Delta A$ is small or is of the order of one in the Einstein frame, it will be so in the Jordan frame.

As demonstrated in Ref. \[^{[32]}\], screening imposes a severe constraint on the change of coupling during the last Hubble time, $\Delta A \ll 1$. Thus self acceleration cannot take place in this case. In most of the models supported by chameleon/symmetron screening, acceleration takes place in both frames such that $\ddot{a} a^J$ and $\ddot{a} E^a$ cancel each other with good accuracy or $\Delta A \ll 1$. In this case acceleration can only be caused by slowly rolling quintessence. In $F(R)$ theories, the scalar field and the coupling both are made of $F'(R)$ which imbibes the gravity modification. Since screening does not allow the conformal coupling to felicitate self acceleration, the problem simply reduces to coupled quintessence which one could deal with without really invoking $F(R)$ theories. It is in this sense, the chameleon/symmetron supported modified gravity models have limited scope for late time cosmic acceleration.

### IV. THE HYBRID SYMMETRON MODEL

The preceding discussion clearly shows that modifications of symmetron action (within the framework of conformal coupling at least) would not help to satisfy local gravity constraints and cosmological bounds suitable to late time evolution.

Late us note that the symmetron mass linearly depends upon the density of the environment, the only parameter which remains to be constrained by local gravity tests in this scenario is the mass scale appearing in the coupling function. On the contrary, the chameleon mass has a complicated non-linear dependence on density of environment depending on the specific chameleon potential which allows chameleon to satisfy the local physics constraints and also play the roll of dark energy. As for the symmetron modification, because the field is slowly varying in time, we have an approximate equivalence between a modified symmetron action and the standard symmetron. Thus within the framework of $\phi^4$ theory with conformal coupling, the bound imposed by local physics on symmetron mass cannot be improved. However, symmetron could still play the role of cosmic phase transition facilitator, the role of late time acceleration could be assigned to another field.

In what follows, we shall consider a model based upon the following action,

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{2\kappa^2} - \frac{1}{2}(\nabla \phi)^2 - V(\phi) \\
+ h(\phi)\mathcal{L}_{\text{dark energy}} \right\} + S_m [A^2(\phi)g_{\mu\nu}, \Psi_m].$$  \hspace{1cm} (24)

where $h(\phi)$ is a coupling function which satisfies the conditions

$$h(0) = 0, \quad h(\phi_0) = 1.$$  \hspace{1cm} (25)

for example, $h(\phi) = (\phi/\phi_0)^m$.

We may rewrite the original symmetron potential in the following convenient form:

$$V(\phi) = \lambda \left(\phi^2 - \phi_0^2\right)^2. \hspace{1cm} (26)$$

Here $\phi_0^2 = \mu^2/4\lambda$. Notice that we added an effective constant ($\lambda \phi_0^4$) compared to the potential previously studied. As we will see, this constant will be always sub-dominant in cosmology and hence do not contribute to the acceleration of Universe.

Let us note that in the high density regime such as the solar system or the galaxy, the action \[^{[24]}\] reduces to Einstein-Hilbert one with a sub-dominant symmetron field. On the other hand, in vacuum, the dark energy contribution to the action dominates.

#### A. $F(R)$ gravity switched by symmetron

In this section, we propose a model where we use the symmetron to generate the cosmic phase transition and
the role of dark energy is played by \( F(R) \) gravity which is switched on by the symmetron after the phase transition. This mechanism could alleviate some fundamental problems like the radiative corrections to the mass of the scalaron (or more generically, the mass of the quintessence field) \(^{28,29}\), and the Frolov singularity \(^{30}\).

The \( F(R) \) gravity is a famous model to explain the accelerating expansion of the present Universe. A problem in the \( F(R) \) gravity is the existence of the extra scalar mode which may give an observable correction to the Newton law. Such a correction could be observed by the solar system test, the experiments on the earth, etc. In the present model, however, since the action reduces to the general gravity. That is, the symmetron screens the scalar mode does not appear and could be consistent with any local test of the general gravity. Here, \( \phi \) with the EoS parameter of dark energy is played by \( F(R) \) gravity without introducing the real cold dark matter. In fact, the Einstein gravity in the region in solar systems, in galaxy, or on the earth, the scalar mode does not appear and could be consistent with any local test of the general gravity. We should also note that the de Sitter space-time is an exact solution of the wide class of the scalar tensor theories is more generic than that in those previous studied model when \( F(R) = R \) and \( m = 2 \). In the following part, we do not consider this case.

We know that \( F(R) \approx H_0^2M_0^{-2} \) which implies a negligible contribution in the Klein Gordon equation. In fact, when \( \phi \approx 0 \), the \( F(R) \) term is trivially negligible, in the case where \( \phi \approx \phi_0 \) we need to impose the condition \( F(R) \ll \alpha\phi_0^2\rho \). We will define the range of viability for the parameters in order to satisfy this condition. Assuming that the conditions are satisfied, the equation reduces locally to the standard form without the \( F(R) \)-term.

We consider the static spherically symmetric solution in the flat space-time, the equation has the following form:

\[
\frac{1}{r^2} d\left( r^2 \frac{d\phi}{dr} \right) = 4\lambda (\phi^2 - \phi_0^2) \phi + \alpha \rho \phi.
\]

We may assume

\[
\rho = \begin{cases} 
\rho_0 & \text{when } 0 \leq r < r_0 \\
0 & \text{when } r \geq r_0 
\end{cases}.
\]

We also assume \(-4\lambda\phi_0^2 + \alpha\rho_0 > 0\).

First, we consider the behavior of \( \phi \) in the region \( r \ll r_0 \) and \( \phi \) could be small. Then we may linearize \(^{30}\) as

\[
\frac{1}{r^2} d\left( r^2 \frac{d\phi}{dr} \right) = (-4\lambda\phi_0^2 + \alpha\rho_0) \phi.
\]

The solution is given by

\[
\phi = \phi_{in} = \frac{A \sinh \left( r \sqrt{-4\lambda\phi_0^2 + \alpha\rho_0} \right)}{r}.
\]

Here \( A \) is a constant and we have assumed \( \phi \) is finite at \( r = 0 \).

Second, we consider the behavior of \( \phi \) in the region \( r \gg r_0 \) and \( \phi \sim \phi_0 \). By writing

\[
\phi = \phi_0 + \delta \phi,
\]

we linearize the field equation \(^{30}\) with respect to \( \delta \phi \) as follows,

\[
\frac{1}{r^2} d\left( r^2 \frac{d\delta \phi}{dr} \right) = 8\lambda\phi_0^2\delta \phi.
\]

Then the solution is given by

\[
\phi = \phi_{out} = \phi_0 - \frac{B e^{-r \sqrt{8\lambda\phi_0^2}}}{r}.
\]

In the region \( r \ll r_0 \), the solution \(^{33}\) grows up very rapidly when \( r \) increases. Inversely if we fixed a value of...
\begin{align}
\phi & \text{ for finite } r < r_0, \; \phi \text{ decreases very rapidly and goes to vanish as } r \text{ decreases.} \\
\text{Different from the original symmetry model discussed in Ref. \textsuperscript{21} \textsuperscript{22}, we may assume the mass } \sqrt{8\lambda\phi_0^2} \text{ of the symmetron in the bulk (low density regime) can be large. Then even in the region } r \gtrsim r_0, \; \phi \text{ in the solution } [\textbf{30}] \text{ goes to } \phi_0 \text{ very rapidly when } r \text{ increases. The above behaviors tells us that } \phi \text{ is almost constant except the small region } r \sim r_0. \text{ Then in order to estimate the constants } A \text{ and } B, \text{ we match the solutions } \phi_{\text{in}} \text{ and } \phi_{\text{out}} \text{ by imposing the following boundary conditions}
\end{align}

\begin{equation}
\phi_{\text{in}}(r_0) = \phi_{\text{out}}(r_0), \quad \phi'_{\text{in}}(r_0) = \phi'_{\text{out}}(r_0).
\end{equation}

We find

\begin{align}
A &= \phi_0 \left(1 + r_0 \sqrt{8\lambda\phi_0^2}\right) - \phi_0 \left(1 + r_0 \sqrt{8\lambda\phi_0^2}\right) + \sqrt{-4\lambda\phi_0^2 + \alpha\rho_0} \cosh \left(r \sqrt{-4\lambda\phi_0^2 + \alpha\rho_0}\right), \\
B &= \phi_0 \sqrt{8\lambda\phi_0^2} \left(-\frac{1}{r_0} \sinh \left(r \sqrt{-4\lambda\phi_0^2 + \alpha\rho_0}\right) + \sqrt{-4\lambda\phi_0^2 + \alpha\rho_0} \cosh \left(r \sqrt{-4\lambda\phi_0^2 + \alpha\rho_0}\right)\right).
\end{align}

We may assume \( r_0 \) could be a radius of planet, star, or galaxy. Then \( \rho_0 \) should be much larger than the Compton lengths \( 1/\sqrt{8\lambda\phi_0^2} \) and \( 1/\sqrt{-4\lambda\phi_0^2 + \alpha\rho_0} \). Then since 
\[ \sinh \left(r \sqrt{-4\lambda\phi_0^2 + \alpha\rho_0}\right) \sim \cosh \left(r \sqrt{-4\lambda\phi_0^2 + \alpha\rho_0}\right) \sim e^{r\sqrt{-4\lambda\phi_0^2 + \alpha\rho_0}/2}, \]
we find

\begin{align}
A &\sim 2\phi_0 r_0 e^{-r_0 \sqrt{-4\lambda\phi_0^2 + \alpha\rho_0}} \frac{1 + \sqrt{\frac{\alpha\rho_0}{8\lambda\phi_0^2} - \frac{1}{2}}}{1 + \sqrt{\frac{\alpha\rho_0}{8\lambda\phi_0^2} - \frac{1}{2}}}, \\
B &\sim \phi_0 r_0 e^{r_0 \sqrt{8\lambda\phi_0^2}} \frac{\sqrt{\frac{\alpha\rho_0}{8\lambda\phi_0^2} - 1}}{1 + \sqrt{\frac{\alpha\rho_0}{8\lambda\phi_0^2} - 1}}.
\end{align}

We now consider the strength of the force \( F_\phi \) generated by the scalar field \( \phi \). Since the strength of the coupling is given by \( \alpha\phi \), we find

\begin{equation}
F_\phi(r) = m \alpha \phi \frac{d\phi}{dr}.
\end{equation}

Here \( m \) is the mass of the particle receiving the force. In the region \( r \gg r_0 \), by using \[ \textbf{30}, \] we find

\begin{align}
\frac{F_\phi(r)}{m} &= \alpha B \left( \phi_0 - \frac{B c^{-r\sqrt{8\lambda\phi_0^2}}}{r} \left( \frac{\sqrt{8\lambda\phi_0^2}}{r} + \frac{1}{r^2} \right) \right. \\
&\quad \times e^{-r\sqrt{8\lambda\phi_0^2}} \\
&\sim \alpha B \phi_0^2 \sqrt{8\lambda} \phi_0 \left( 1 - r \sqrt{8\lambda\phi_0^2} \right).
\end{align}

On the other hand, the Newtonian force of gravity \( F_g \) is given by \( F_g(r) = GMm/r^2 \), where \( M \equiv 4\pi\rho_0 r_0^3/3 \). We notice that in the region specified by \( r \gg r_0 \), the force \( F_\phi \) generated by the scalar field \( \phi \) can be neglected compared to the Newtonian force.

\section{Constraints on model parameters}

We now consider the constraints for the parameters. A constraint comes from the condition that the symmetry is restored when \( r < r_0 \) but broken in the vacuum:

\begin{equation}
\alpha\rho_0 > 2\lambda\phi_0^2 > \alpha\rho_\infty ~ \sim \alpha H_0^2 M_{Pl}^2.
\end{equation}

Here \( \rho_\infty \) is the energy density of the vacuum, \( H_0 \) is the Hubble constant in the present Universe, and \( M_{Pl} \) is the Planck mass. Another constraint may come from the condition that the Compton length of the scalar field even in the vacuum should be much smaller than the size of galaxies \( r_G \):

\begin{equation}
\frac{1}{\sqrt{8\lambda\phi_0^2}} \ll r_G.
\end{equation}

We also require that \( h(\phi)F(R) \) term does not affect the phase transition,

\begin{equation}
V(0) = \lambda\phi_0^4 \gg F(R_\infty) \sim H_0^2 M_{Pl}^2.
\end{equation}

In case of the galaxy \( \rho_0 \sim 10^5 H_0^2 M_{Pl}^2 \) and \( r_G \sim (10^{-7} - 10^{-5}) H_0^{-1} \).

Since \( H_0 \sim 10^{-61} M_{Pl} \) and \( G = 8\pi M_{Pl}^2 \), by introducing the new variables

\begin{align}
x &= \phi_0^2, \quad y = \lambda\phi_0^2, \quad z = \frac{1}{\alpha},
\end{align}

and putting \( r_0 = r_G \), the above constraints \[ \textbf{43}, \textbf{44}, \] and \[ \textbf{45} \] can be rewritten as

\begin{align}
yz &\ll 10^{-117} M_{Pl}^4, \quad yz \gg 10^{-122} M_{Pl}^4, \\
y &\gg 10^{-108} M_{Pl}^4, \quad xy \gg 10^{-122} M_{Pl}^4.
\end{align}
By using, \( z = 10^n M_{\text{Pl}}^2 \), we obtain
\[
y \ll 10^{-117-n} M_{\text{Pl}}^2, \quad y \gg 10^{-122-n} M_{\text{Pl}}^2, \quad (49)
y \gg 10^{-108} M_{\text{Pl}}^2, \quad xy \gg 10^{-122} M_{\text{Pl}}^4. \quad (50)
\]
We find \( xy \gg 10^{-122} M_{Pl}^4 \), and for \(-14 < n < -9\)
\[
10^{-117-n} M_{Pl}^2 \gg y \gg 10^{-108} M_{Pl}^2, \quad x \gg 10^{-5+n} M_{Pl}^2, \quad (51)
\]
and for \( n < -14 \)
\[
10^{-117-n} M_{Pl}^2 \gg y \gg 10^{-122-n} M_{Pl}^2, \quad x \gg 10^{-5+n} M_{Pl}^2. \quad (52)
\]
We therefore confirm that we have always a range of viability of the parameters, where all the constraints are satisfied.

**C. Phase transition in the early Universe**

As mentioned earlier, the phase transition should have occurred in rather early Universe, not in the late Universe. We now investigate how the transition could have occurred and if there could be any problem or not. The potential (20) tells that the critical density \( \rho_\text{cr} \), where the phase transition occurs, is given by
\[
\rho_\text{cr} = \frac{4 \lambda \phi^2}{\alpha} = 4yz. \quad (53)
\]
When the phase transition could have occurred, the Hubble rate \( H_\text{cr} \) is given by
\[
H^2_\text{cr} = \frac{\kappa^2}{3} \rho_\text{cr} \sim \frac{yz}{M_{Pl}^2}. \quad (54)
\]
Here we have used the notation defined in (40). As we are interested in the behavior when \( \phi \sim \phi_0 \), by using (51), we linearize the scalar field equation as
\[
\frac{d^2 \delta \phi}{dt^2} + 3H_\text{cr} \frac{d \delta \phi}{dt} + 8 \lambda \phi^2_0 \delta \phi = 0. \quad (55)
\]
Equation (55) has a form analogous to the equation of motion of the harmonic oscillator with drag (air resistance):
\[
\frac{d^2 x}{dt^2} + 2 \gamma \frac{dx}{dt} + \omega^2 x = 0. \quad (56)
\]
Here \( \gamma \) and \( \omega \) are positive constants and \( x \) expresses the position of the harmonic oscillator. As well-known, when \( \gamma^2 \geq \omega^2 \), the amplitude \( |x| \) decreases without oscillation and when \( \gamma^2 < \omega^2 \), the amplitude \( |x| \) decreases with oscillation by the angular velocity \( \sqrt{\omega^2 - \gamma^2} \). Hence the time scale relevant to the decrease of amplitude is given by \( T_{\text{dec}} = 1/\gamma \). Then Eq. (55) tells us that \( \delta \phi \) would vanish without oscillation if \( 9 H^2_\text{cr} = \frac{\gamma y z}{M_{Pl}^2} > 8 \lambda \phi^2_0 = 8y \), that is, \( \frac{\gamma y z}{M_{Pl}^2} \geq 1 \). On the other hand, \( \delta \phi \) vanishes with oscillation if \( \frac{\gamma y z}{M_{Pl}^2} < 1 \). By writing \( z = 10^n M_{Pl}^2 \) as before, and since \( n < -9 \), we find \( \delta \phi \) vanishes with oscillation. The time scale of decreasing the amplitude is given by
\[
T_{\text{dec}} = 1/\gamma \sim M_{Pl} \sqrt{yz}. \quad (57)
\]
Then Eq. (51), (52) tells when \(-14 < n < -9\)
\[
10^{58} M_{Pl}^{-1} \ll T_{\text{dec}} \ll 10^{54-n/2} M_{Pl}^{-1}, \quad (58)
\]
and for \( n \leq -14 \)
\[
10^{58} M_{Pl}^{-1} \ll T_{\text{dec}} \ll 10^{61} M_{Pl}^{-1}. \quad (59)
\]
Since \( M_{Pl}^{-1} \sim 10^{-44} \) sec \( \sim 10^{-51} \) years, we have for \(-14 < n < -9\)
\[
10^7 \text{ years} \ll T_{\text{dec}} \ll 10^{3-n/2} \text{ years}, \quad (60)
\]
and for \( n \leq -14 \)
\[
10^7 \text{ years} \ll T_{\text{dec}} \ll 10^{10} \text{ years}. \quad (61)
\]
We therefore conclude that it could take 10 million as minimum or 10 billion years as maximum for decreasing the oscillations. The period \( T_p \) of the oscillation is given by
\[
2\pi/\sqrt{8 \lambda \phi^2_0} \sim 1/\gamma. \quad (59)
\]
Therefore Eqs. (60), (61) gives
\[
10^{7+2\gamma} \text{ years} \ll T_p \ll 10^3 \text{ years}, \quad (62)
10^{7+2\gamma} \text{ years} \ll T_p \ll 10^{10+2\gamma} \text{ years}. \quad (63)
\]
for \(-14 < n < -9 \) and \( n \leq -14 \) respectively. For instance, if we choose \( n = -14 \), we have 1 year \( \ll T_p \ll 10^3 \) years. Therefore the oscillations are very slow and do not give rise to particle production.

Although the phase transition did not occur during the accelerating expansion, that is, when the redshift is \( z \sim 1 \), we may estimate when it could have occurred. Since we assume \( \rho_0 \sim 10^5 H_0^2 M_{Pl}^2 > \rho_{\text{cr}} > \rho_{\infty} \) and \( \rho \propto a^{-3} \), we may assume the phase transition could have occurred when the redshift \( z = a_0^{-1} - 1 \sim 10 \) (\( a_0 \) is the present value of the scale factor \( a \)), which corresponds to a few million years after the Big-Bang; the redshift of the cosmic microwave background radiation corresponds to \( z \sim 1000 \).

In the early universe, the \( Z_2 \) symmetry could be restored. Then Eq. (20) tells us that \( V(\phi) \) gives a contribution to the energy in the vacuum by \( \lambda \phi^4_0 \). The contribution is, however, not so large. Equation (10) tells \( \lambda \phi^4_0 = xy \). Using the previous results, we find \( \lambda \phi^4_0 = xy > 10^{-122} (M_{Pl})^4 \). On the other hand, for an example, the energy density of the matter in the present universe is \( 10^{-124} (M_{Pl})^4 \). When \( z > 10 \), since the matter density could be \( 10^{-124} \times (1+z)^3 (M_{Pl})^4 \sim 10^{-121} (M_{Pl})^4 \). Then the magnitude of \( V(0) = \lambda \phi^4_0 \) is comparable with the energy density of matter but could not be dominant. Therefore the contribution from the constant \( \lambda \phi^4_0 = xy \) can always be neglected.
V. SUMMARY AND DISCUSSION

In this paper, we examined the modified symmetron models with an aim to reconcile the latter with late time cosmic acceleration. We have investigated a symmetron type framework coupled to curvature scalar and to matter. In this case the effective symmetron potential drastically differs from the one in original scenario. We examined the consistency of the model with local physics and found constraints on the symmetron mass similar to the one in the standard model. The latter is related to the fact that local gravity tests are insensitive to the details of the effective symmetron potential and the new mass scale appearing in the over all coupling in the leading approximation \((\phi/M \ll 1, \phi/M_{Pl} \ll 1)\) in the model under consideration is same as the one that defines the symmetron mass. We therefore conclude that there is a “no go” theorem for dark energy symmetron as long as we confine to conformal coupling, situation might (might not) change in the disformal case.

Being inspired by the beauty of cosmic symmetry breaking, we have made a proposal in which symmetron facilitates the phase transition in low matter density regime; the symmetry is restored in high density ensuring the compliance of the model with local gravity constraints. The role of symmetron field ceases after phase transition thanks to its coupling to an \(F(R)\) gravity action or quintessence field.

In case of \(F(R)\) symmetron or equivalently a quintessence field, the model reduces to the standard Einstein gravity in the high density region whereas \(F(R)\) gravity is switched on in the low density region; the transition is generated by the symmetron field. Since in our proposal, symmetron field is responsible for phase transition only, its mass could be large even in the vacuum and therefore the local gravity constraint can easily be satisfied, which is different from the original symmetron model [21, 22]. We have also investigated other constraints and we have shown that there exists a parameter region which satisfies all constraints. In the model under consideration, the transition could have occurred around \(z \sim 10 – 10^2\) the dark age. After the transition, symmetron field oscillates around the true minimum but the time scale of decreasing of amplitude varies between 10 million to 10 billion years which is much smaller compared to the Hubble scale. On the other hand, the period of the oscillations varies from one to one thousand years, which could be large enough from thermodynamical point of view. Thus the oscillation would not conflict with the observed cosmology.

Last but not least, it might be interesting to enlarge the symmetron framework to disformal type of set up to check whether the disformal symmetron can reconcile with dark energy.

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\[ \phi/M \ll 1, \phi/M_{Pl} \ll 1 \]

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