Non-linear Behavior of Low Strength RC Beams Strengthened with CFRP Sheets

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Abstract
There are many modern technologies for strengthening concrete. One of them is Carbon Fiber Reinforcement Polymer (CFRP) sheets. There are many cases that require concrete strengthening, including the weakness of concrete from the design strength due to the exposure of the concrete elements to fires or less control of concrete quality. This research provides a theoretical study to analyze the behavior of low strength RC beams strengthened with CFRP sheets using ANSYS software. The research contained (75) RC beam model specimens. Six of them for verifying results with experimental tests. Forty five specimens were used for studying flexural behavior and twenty four beam models used for studying shear behavior. The study conducted with variable parameters includes CFRP thickness, concrete strength, the yield stress of steel, and the application of CFRP plies. The theoretical results were very similar to experimental test results. The results proved that strengthening RC beams is increasing load capacity and it is very effective in case of low yield stress of steel.

Keywords: Shear Behavior; Flexural Behavior; Low Strength Concrete; CFRP Sheets; Finite Element Method by ANSYS.

1. Introduction
There are several reasons leading to strengthen reinforced concrete RC structural elements. One of the most familiar reason is the low strength of concrete which may occur due to poor quality control during construction [1, 2]. Structure elements exposed to accidental fire may need to be strengthened by using CFRP due to the significant reduction in the concrete strength [3, 4].

Using CFRP for strengthening RC elements could be considered as one of the recent effective maintenance techniques [5, 6]. It can be considered as one of the optimum solutions for strengthening RC elements due to its light weight and high tensile resistance compared to steel and other fabric polymers [7]. Many researchers have studied the different CFRP effect of using CFRP sheets on RC element behavior [8, 9]. They found that CFRP can enhance shear and flexural capacities of RC elements according to method of application by wrapping (strips or sheets) on one side, two sides, three sides (U jacket) or complete wrapping [10, 11]. The theoretical studies were conducted to give area for exploring the behavior of beams in flexural and shear by using many advanced software programs [12, 13].

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The non-linear behavior of low strength RC beams strengthened with CFRP sheets was numerically investigated using 3D FEM program (ANSYS) v14.5. The analytical results were successfully verified by experimental results. In this research two experiments were conducted by Balamuralikrishnan and Jeyasehar (2009) and Alagusundaramoorthy et al. (2002) [14, 15] are simulated analytically to investigate flexural and shear behavior of beams strengthened with CFRP respectively. Several parameters were considered for investigating behavior of beams with low strength concrete strengthened with CFRP sheets. These include compressive strength of concrete, yield stress of steel reinforcement [16] and the application of CFRP plies. The flexural and shear capacities of beams with varied parameters were found closer to those estimated according to EGYPTIAN CODE NO. ECP 208 -2005 [17] for the proposed values of concrete compressive strength. In This research, author concentrated on the concrete with low strength in many cases and when the strengthening is more effective. The research results confirmed the previous studies around CFRP effect on RC beams [18, 19]. The additional results in this research were obtained about beams with low strength concrete and low yield stress of steel reinforcement which were strengthened with CFRP sheets.

2. Constitutive Modeling

For simulating the non-linear behavior of RC beams in flexural and shear, perfect bond between steel reinforcement and concrete is assumed. In addition, perfect bond between Concrete and CFRP is also assumed to avoid complications in beams modeling. This assumption has no significant effect on load-deflection response according to Isenburg (1993) [20]. The finite element analysis in this research was conducted as load control. The specimens considered for flexural and shear performance of RC beams strengthened with CFRP are illustrated in Figures 2 and 3 respectively.

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F_y(\text{ten&comp.steel}) = 512 \text{ MPa}, F_y(\text{stirrup}) = 280 \text{ MPa}, F_{cu} = 27.54 \text{ MPa}
\]

Figure 2. Set up of RC beam flexural test [14]
The material model used for concrete is defined as linear isotropic and multilinear isotropic material. Steel reinforcement is defined as linear isotropic and bilinear isotropic. The steel plates used for loading and supporting are defined as linear isotropic. The CFRP material is defined as linear orthotropic transversally isotropic, which means that the mechanical properties are varied from the fiber axis to the other perpendicular axes.

3. Experimental Validation of FE Model

The results of FE model are compared to those from experimental tests, flexural test and shear test [14, 15].

3.1. Flexural Behavior of Control RC Beam

A simply supported control beam under two-point loading was analytically simulated using a mesh size of 25×25×25 mm Figure 4(a). The cracks propagations and location were fairly elaborated in Figures 4(b, c and d). The flexural behavior is shown in Figure 5 where the beam failure occurred due to yielding of steel reinforcement and crashing of concrete. Load-displacement relationships of analysis and experiment were found table in good agreement as shown in Figure 6.

![Figure 4. Control beam (Analysis)](image_url)
Figure 5. Failure cracking pattern in control beam (Experiment) [14]

\[ F_{y(\text{ten.}\&\text{comp.steel})} = 512 \text{ MPa}, \quad F_{y(\text{stirrup})} = 280 \text{ MPa}, \quad F_{cu} = 27.54 \text{ MPa} \]

Figure 6. Comparison between experiment and FE analysis (Flexural control beam)

Strengthening RC beams with one layer of CFRP soffit increases the flexural capacity and reduces the cracks propagation as shown in Figures 7 and 8. The beam failure occurred due to yielding of steel reinforcement and complete crushing of concrete.

(a) Cracks at 10 kN
(b) Cracks at 30 kN
(c) Crushing at 52.5 kN

Figure 7. Cracks in beam strengthened with one layer CFRP (Flexural analysis)

Figure 8. Failure pattern in beam strengthened with one layer CFRP (Experiment) [14]

The analytical results of RC beam strengthened with one layer of CFRP are close to those from experimental test, Figure 9.
3.2. Shear Behavior of RC Control Beam

A simply supported control beam subjected to one point loading was analytically simulated using a mesh size of 25×25×25 mm, Figure 9(a). The cracks propagation pattern is elaborated in Figures 9(b, c, d, and e). Load-displacement relationships of analysis and experiment are compared in Figure 11. For the beam strengthened with CFRP sheets with inclination angle 90°, the cracks propagation pattern is in Figure 12. Load-displacement relationships of analysis and experiment are compared in Figure 13. Good agreement can be seen between the experimental and analytical results.
Figure 12. Cracks in beam strengthened with CFRP 90° (shear analysis)

| Load (kN) | Displacement (mm) |
|-----------|-------------------|
| 0         | 0                 |
| 50        | 5                 |
| 100       | 10                |
| 150       | 15                |
| 200       | 20                |
| 250       | 25                |
| 300       | 30                |
| 350       | 35                |
| 400       | 40                |
| 450       | 45                |
| 500       | 50                |

$F_{y(\text{ten} \& \text{comp.steel})} = 414 \text{ MPa}, F_{y(\text{stirrup})} = 414 \text{ MPa}, F_{cu} = 31 \text{ MPa}$

Figure 13. Comparison between experiment and FE analysis (Beam with one layer CFRP 90° in shear)

4. Parametric Study

In the flexural analysis, the main parameters include compression strength of concrete (10, 15, 20, 25, and 30 MPa), steel yielding stress [main reinforcement (512, 360, 270 MPa) and web reinforcement (280, 200 MPa)] and number of CFRP layers. In the Egyptian code, the minimum compression strength is considered as 20 MPa for design. In the current research, concrete strength was considered lower than the minimum allowable value by ECP to investigate the effect of strengthening extremely poor quality concrete with CFRP.

In shear analysis, the main parameters include compression strength of concrete (10, 15, 20, and 25 MPa), steel yielding stress (414, 250 MPa) and inclination of CFRP layer on the longitudinal axis of beam.

4.1. Flexural Analysis Results

The results obtained from flexural FE analysis show a significant increase in the ultimate load capacity with the increase in concrete compressive strength. Figure 14(a) shows the control beams behavior as the compressive strength increased from 30 MPa to 10 MPa by changing rate of 5 MPa. The reduction rate in flexural load capacity was found to be as 11, 0.0, 13.3 and 10.1%, respectively. The reduction rate of 0.0% was due to reduction of compressive strength from 25 MPa to 20 MPa where the two beams gave almost identical maximum load. As seen in Figure 14(a), the deflection of the beam having compressive strength of 20 MPa was higher than that of 25 MPa during all the load increments up to failure. The average reduction rate in flexural load capacity of control beams can be calculated as 11.47% for the case of using steel yield stress for main reinforcement as 512 MPa, and for web reinforcement as 280 MPa.

In a similar way, the rates of reduction in flexural capacity of control beams for the cases of using yielding stress for main reinforcement as 360 MPa, and for web reinforcement as 280 MPa and for main reinforcement as 270 MPa, and for web reinforcement as 200 MPa, were (3.2%, 2.1%, 7.7%, 16.6%) and (1.6%, 6.7%, 9.1%, 10%), respectively. The average reduction rates in flexural load capacity of control beams can be calculated as 7.4 and 6.85% for the cases of using steel yield stress for main reinforcement as 360 MPa, and for web reinforcement as 280 MPa and for main reinforcement as 270 MPa, and for web reinforcement as 200 MPa, respectively.
The overall average reduction rate in flexural load capacity of the pre-mention twelve cases be calculated as 8.1% w.r.t 5 MPa reduction rate in compressive strength. Considering the same sequence of calculation for control beams, it was found that using one layer of CFRP, Figure 14(b), (for yield stress of main reinforcement as 512 MPa, and for web reinforcement as 280 MPa leads to an overall average reduction rate in flexural load capacity of 13.3% w.r.t 5 MPa reduction rate in compressive strength. Generally, the reduction rate in flexural capacity increases with the increase of the number of CFRP layers as shown in Figure 15.
Figure 15. Comparison of flexural capacities of RC beams w.r.t change in concrete strength and layers of CFRP. 
(a) $F_{y(ten.comp.steel)}$ =512 MPa, $F_{y(stirrup)}$ =280 MPa; (b) $F_{y(ten.comp.steel)}$ =360 MPa, $F_{y(stirrup)}$ =280 MPa; 
(c) $F_{y(ten.comp.steel)}$=270 MPa, $F_{y(stirrup)}$=200 MPa
According to the Figures 14 and 15, it is obvious that:

- In case of high yield stress of steel and poor compressive strength of concrete (poor quality control), the strengthening of RC beams is very effective. This efficiency is increased by recasting the concrete with high compressive strength.

- In case of low yield stress of steel and poor compressive strength of concrete (the structure exposed to accidental fire), the strengthening of RC beams is more effective than the previous case.

4.2. Shear Analysis Results

The results obtained from shear FE analysis show a significant increase in the ultimate load capacities with the increase in concrete compressive strength. Figure 16 (a) shows the control beams behavior of the compressive strength is reduced from 25MPa to 10MPa by changing rate of 5MPa. The reduction rate in shear load capacity was found to be as 20.7, 2.9 and 13.5%, respectively. The average reduction rate in shear load capacity of control beams can be calculated as 12.37% for the case of using steel yield stress as 414 MPa.

In a similar way, the rates of reduction in shear capacity of control beams for the case of using yielding stress 250 MPa were 5.9, 10.5 and 22.6%. The average reduction rates in shear load capacity of control beams can be calculated as 13% for the case of using steel yield stress as 250 MPa.

The overall average reduction rate in shear load capacity of the pre-mention six cases can be calculated as 12.68 % w.r.t 5 MPa reduction rate in compressive strength.

![Graphs and Figures]

Figure 16. Shear capacities of RC beams
Considering the same sequence of calculation for control beams, it was found that using one layer of CFRP with inclination angle 90° as shown in Figure 16(b) for steel yield stress as 414 MPa leads to an overall average reduction rate in shear load capacity of 11.9% w.r.t 5 MPa reduction rate in compressive strength.

Considering the same sequence of calculation for control beams and beams strengthened with one layer CFRP with inclination angle of 90°, it was found that using one layer of CFRP with inclination angle of 45° as shown in Figure 16(c) for steel yield stress as 414 MPa leads to an overall average reduction rate in shear load capacity of 13.11% w.r.t 5 MPa reduction rate in compressive strength. Generally, the reduction rate in shear capacity increases with the increase of the number of CFRP as shown in Figure 17.

![Comparison of shear capacities of RC beams w.r.t change in concrete strength and inclination angle of CFRP fibers.](image)

According to the Figures 14 and 15, it is obvious that:

- In case of high yield stress of steel and poor compressive strength of concrete (poor quality control), the strengthening of RC beams is very effective. Recasting the concrete with higher compressive strength is increasing the ultimate load capacity with the same efficiency.

- In case of low yield stress of steel and poor compressive strength of concrete (The structure exposed to accidental fire), the strengthening of RC beams is less effective than the previous case. Recasting the concrete with higher compressive strength increase the ultimate load capacity till concrete compressive strength 20MPa then the increase in concrete grade has no effect on ultimate load capacity.
5. Results Comparison with Design Codes

The load capacities of beams with varied parameters were found closer to those estimated according to EGYPTIAN CODE NO. ECP 208 - 2005 [16] for proposed values of concrete compressive strength. In reality; debonding of CFRP sheets occurs, while the reinforced concrete compressive strength is lower than 20 MPa.

Tables 2 to 5 are showing a comparison between load capacities of FE analysis and those calculated according to the Egyptian design code [17] for strengthened beams the results show that the current Egyptian code can be used safely to estimate ultimate capacity of RC beams strengthened with CFRP.

Table 2. Comparison between flexural load capacities of FE analysis and ECP NO. 208-2005 [17] for beams strengthened with one layer CFRP

| #       | Fc, = 512 MPa | Fc, = 360 MPa | Fc, = 270 MPa |
|---------|---------------|---------------|---------------|
| MODEL   | FE (kN)       | ECP (kN)      | FE (kN)       | ECP (kN)      | FE (kN) | ECP (kN) |
| Fc, = 10 MPa | 35.3          | 31.1          | 28.7          | 29.5          | 25      | 29       |
| Fc, = 15 MPa | 38.7          | 41.8          | 33.8          | 38.5          | 30      | 35.7     |
| Fc, = 20 MPa | 45            | 48.7          | 42.5          | 43.1          | 37.5    | 39       |
| Fc, = 25 MPa | 50            | 52.9          | 47.5          | 45.8          | 37.5    | 41       |
| Fc, = 30 MPa | 60            | 55.6          | 55            | 47.6          | 45      | 42.4     |

Table 3. Comparison between flexural load capacities of FE analysis and ECP NO. 208-2005 [17] for beams strengthened with two layers CFRP

| #       | Fc, = 512 MPa | Fc, = 360 MPa | Fc, = 270 MPa |
|---------|---------------|---------------|---------------|
| MODEL   | FE (kN)       | ECP (kN)      | FE (kN)       | ECP (kN)      | FE (kN) | ECP (kN) |
| Fc, = 10 MPa | 37.5          | 32.2          | 30            | 30.5          | 25      | 30       |
| Fc, = 15 MPa | 42            | 45            | 35            | 44.9          | 32.5    | 43       |
| Fc, = 20 MPa | 57.5          | 56            | 57.5          | 53.3          | 40      | 51       |
| Fc, = 25 MPa | 60            | 63.5          | 59.5          | 58.4          | 43      | 54.7     |
| Fc, = 30 MPa | 65            | 68            | 61.5          | 61.7          | 59.5    | 57.4     |

Table 4. Comparison between shear load capacities of FE analysis and ECP NO. 208-2005 [17] for beams strengthened with one layer of CFRP 90°

| #       | Fy= 414 MPa | Fy= 250 MPa |
|---------|-------------|-------------|
| MODEL   | FE (kN)     | ECP (kN)    | FE (kN)     | ECP (kN)    |
| Fcu= 10 MPa | 220         | 173         | 149         | 145         |
| Fcu= 15 MPa | 240         | 200         | 185         | 173         |
| Fcu= 20 MPa | 275         | 217         | 195         | 190         |
| Fcu= 25 MPa | 330         | 230         | 215         | 201         |

Table 5. Comparison between shear load capacities of FE analysis and ECP NO. 208-2005 [17] for beams strengthened with one layer of CFRP 45°

| #       | Fy= 414 MPa | Fy= 250 MPa |
|---------|-------------|-------------|
| MODEL   | FE (kN)     | ECP (kN)    | FE (kN)     | ECP (kN)    |
| Fc, = 10 MPa | 210         | 187         | 149         | 160         |
| Fc, = 15 MPa | 275         | 217         | 195         | 190         |
| Fc, = 20 MPa | 295         | 235         | 215         | 209         |
| Fc, = 25 MPa | 340         | 247.5       | 220         | 221         |

6. Conclusions

Non-linear behavior of RC beams with low strength concrete strengthened with CFRP sheets is analytically investigated considering a number of parameters such as concrete compressive strength, yielding stress of steel reinforcement and configuration of CFRP plies.
6.1. For Flexural Case Study

- Adding CFRP layer bonded to the control beam soffit increases ultimate load capacity by 11 to 46% considering different values of concrete compressive strength and yielding stress of steel reinforcement.
- Doubling the number of CFRP layers bonded to the beam soffit increases ultimate load capacity by 11 to 78% considering different values of concrete compressive strength and yielding stress of steel reinforcement.
- The overall average reduction rate in flexural load capacity of (control beams, beams strengthened with one layer of CFRP sheet and beams strengthened with two layers of CFRP sheet) were 8.1, 13.3 and 16%, respectively for each 5 MPa reduction rate in compressive strength.
- The overall average reduction rate in flexural load capacity of (control beams, beams strengthened with one layer of CFRP sheet and beams strengthened with two layers of CFRP sheet) were 13.6, 11 and 12.7% respectively w.r.t 100 MPa reduction rate in yielding stress of steel reinforcement.

6.2. For Shear Case Study

- Adding CFRP layer warped as U jacket with inclination angle of 90° to the main axis of beam increases the stiffness of the beam, and ultimate load capacity at failure with a value between 2.76 and 14.58%.
- Adding CFRP layer warped as U jacket with inclination angle of 45° to the main axis of beam increases the stiffness of the beam, and increases ultimate load capacity at failure with a value between 9.7 and 23%.
- The overall average reduction rate in flexural load capacity of (control beams, beams strengthened with one layer CFRP with inclination angle of 90° sheet and beams strengthened with one layer CFRP with inclination angle of 45° sheet) were 12.6, 11.67 and 13 % respectively w.r.t 5 MPa reduction rate in compressive strength.
- The overall average reduction rate in flexural load capacity of (control beams, beams strengthened with one layer of CFRP with inclination angle of 90° sheet and beams strengthened with one layer of CFRP with inclination angle of 45° sheet) were 26.5, 27.2 and 27.5%, respectively w.r.t 150 MPa reduction rate in yielding stress of steel.

7. Declarations

7.1. Author Contributions

The basic theme of the research was discussed and decided by A.M.Y.M., A.R.M.A. and H.A.A.; The manuscript was written by A.M.Y.M. and A.R.M.A.; The finite element analyses were conducted by A.R.M.A.; The main revision and correction by A.M.Y.M. and H.A.A.; The results and discussions and conclusion section were completed by A.M.Y.M., A.R.M.A. and H.A.A. All authors have read and agreed to the published version of the manuscript.

7.2. Data Availability Statement

The data presented in this study are available in article.

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7.4. Conflicts of Interest

The authors declare no conflict of interest.

8. References

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