Modeling of neutron-star mergers: 
a review while awaiting gravitational-wave detection

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Abstract.
I review some recent results on simulations of mergers of binary neutron stars, highlighting some of the several significant advances published in the literature in the years 2013 to 2015.

1. Introduction
We are living in an exciting time for research on binary neutron-star (BNS) mergers. In fact, we are in a very dynamical phase of research, in which many accomplishments have been achieved (especially in the years 2013-2015), while many more need to be achieved to describe such fascinating physical phenomena. The first detection of gravitational waves [1] adds excitement to the field. Such first detection concerned a binary black-hole coalescence, but measurements of BNS mergers are also expected in the next months.

The details of the genesis of BNS systems are still unclear, but the general picture seems well accepted [2]. The more massive star of the binary explodes as a supernova once it has evolved off the main sequence and through its giant phase. The remnant of such an explosion becomes the first neutron star of the binary, with a potential recoil. Subsequently, the secondary star evolves off the main sequence and explodes as a supernova becoming the second neutron star.

Many astronomical observations revealed that BNS indeed exist [3]. The inspiral and merger of two neutron stars in binary orbit is the inevitable fate of close-binary evolution, since their angular momentum is dissipated through the emission of gravitational radiation. Here is a brief description of their evolution. As the inspiral progressively speeds up and the two neutron stars approach each other, tidal waves produced by the tidal interaction become visible at their surface. At the merger, the two stars collide with a rather large impact parameter. A vortex sheet (or shear interface) develops, where the tangential component of the velocity exhibits a discontinuity. This condition is known to be unstable to very small perturbations and it can develop a Kelvin–Helmholtz instability, which curls the interface forming a series of vortices at all wavelengths [4, 5]. Even if this instability is purely hydrodynamical and it is likely to be important only for binaries with very similar masses, it can have strong consequences if the stars have magnetic fields. It has been shown in most simulations [6, 7, 8, 9, 10, 11] that, in the presence of an initially poloidal magnetic field, this instability may lead to an exponential growth of the toroidal component. Such a growth is the result of the exponentially rapid formation of vortices that curl magnetic field lines that were initially purely poloidal. The exponential growth
caused by the Kelvin–Helmholtz instability leads to an overall amplification of the magnetic field of up to a few orders of magnitude [10, 11].

After the merger, unless the progenitor stars have very small masses or for some particular equations of state (EoSs), the compact object newly formed is expected to collapse to a black hole, either promptly or after a certain amount of time. If the collapse is not prompt, the merged object is expected to be temporarily a hypermassive neutron star (HMNS), namely a neutron star whose rest mass exceeds the maximum rest mass of nonspinning or uniformly spinning neutron stars and is sustained against collapse by differential rotation. The merged object oscillates violently, with a dominant $m = 2$ non-axisymmetric character [12]. Dissipative effects like viscosity, magnetic fields or gravitational-wave emission bring then the star towards a configuration which is unstable to gravitational collapse. The time interval between merger and collapse can be roughly estimated to be of the order of $1 − 100$ s or more. The large uncertainty in determining this is due particularly to the difficulty of performing accurate numerical simulations over these very long time-scales. During this time, if the merged object has a sufficiently high ratio of rotational to gravitational binding energy, it could also become dynamically unstable to nonlinear instabilities leading to a bar-mode deformation [13, 14, 15]. Both magnetic fields [7, 10, 11] and radiative processes [16, 17, 18, 9, 19] can modify significantly the lifetime of the HMNS and hence change the properties of the black hole and of the surrounding torus. In most cases when a black hole is formed some amount of matter remains outside of it. This leads to the formation of an accretion torus that may be rather dense ($10^{12} − 10^{13}$ g cm$^{-3}$) and extended horizontally for tens of kilometers and vertically for a few tens of kilometers.

BNS mergers are expected (i) to be significant sources of gravitational radiation, not only during the inspiral, but also during and after the merger, (ii) to be possible progenitors for short-gamma-ray bursts (GRBs), whose short rise times suggest that their central sources have to be highly relativistic objects, and (iii) to be the possible sources of other electromagnetic (e.g. macronovae [20, 21]) and neutrino emission.

The typical scenario leading to GRBs assumes that a system composed of a rotating black hole and a surrounding massive torus is formed after the merger. If the disc had a mass $\gtrsim 0.01−0.1 M_\odot$, it could provide the large amount of energy observed in SGRBs, either through neutrino processes or by extracting the rotational energy of the black hole via magnetic fields [22, 23, 8]. Such a configuration is expected to form in most BNS mergers.

As far as gravitational-wave emission is concerned, BNS systems are expected to produce signals of amplitude large enough to be relevant for Earth-based gravitational-wave detectors such as LIGO [24], Virgo [25], or Kagra [26]. They are also expected to be sufficiently frequent sources: Advanced interferometric detectors are in fact expected to observe them at a rate of $\sim 0.4 − 400$ events per year, with a most realistic rate of $\sim 40$ per year [27]. Among other results, observations of gravitational waves, photons, and neutrinos emitted during and after BNS mergers will give strong indications about the EoS of matter at nuclear densities, which cannot be probed in laboratories on Earth and is not fully understood at the theoretical level [28].

During the inspiral phase, when the two stars are still well separated, sophisticated post-Newtonian expansions offer a good approximate description of the dynamics, but only full numerical simulations can describe the highly nonlinear regime of the merger and post-merger phases. The final goal of numerical computations may be a simulation that includes the solution of the Einstein equations, the relativistic hydrodynamic and (resistive) magnetohydrodynamics (MHD) equations, EoSs based on microphysical calculations, neutrino and photon radiation transport, nuclear-reaction networks. On top of all this, codes should possibly implement high-order, high-accuracy numerical methods and run fast enough to allow parameter-space exploration. This is a goal still very far in the future. But when we achieve all this, the reward will be great, namely an accurate interpretative and predictive description of the processes related to BNS systems and involving and causing short GRBs, jet formation and emission,
particle formation and emission, other electromagnetic emission like that of macronovae, heavy-element abundance. As far as the study and interpretation of gravitational-wave observations is concerned, we may be able to obtain satisfactory results even without the full machinery introduced above. In fact, the interesting gravitational radiation is probably emitted before neutrino, radiation transport, and nuclear reactions become important. The evolution of the merged object, instead, depends on a large number of details, such as the mass-ratio of the initial stars, the EoS, the neutrino physics and precise energy transport, the dynamical relevance of the magnetic field, etc.

Several groups are working on BNS simulations with their own independent codes. Most of the codes solve the full Einstein Equations without approximations (except for the truncation error and the errors associated to numerical methods), but there are two notable exceptions, namely the code of Rosswog [29, 30, 31, 32] and the code of Janka/Bauswein [33, 34, 35]. These two smoothed-particle-hydrodynamics (SPH) codes make theoretical approximations to the evolution of gravity / spacetime but are among the most advanced in treating microphysical processes.

The first successful fully general-relativistic simulations of BNS mergers were presented at the end of the past century in [36] and huge advances have been made since. Several groups have developed numerical codes for general-relativistic simulations. All of these codes share the following capabilities:

- Implementation and use of high-order finite-differencing techniques\(^1\) for the solution of the Einstein equations
- Implementation and use of high-resolution shock-capturing (HRSC) methods for the solution of the GRHD/GRMHD equations
- Implementation and use of adaptive mesh-refinement (AMR) techniques that provide higher resolution around the orbiting stars and the merged object
- Use of consistent initial data representing a system of BNS in quasi-circular orbits\(^2\)
- Accurate evolution of matter and spacetime (including long-term evolutions of the formed black holes and accretion discs), thanks to the numerical methods mentioned above
- Analysis of the properties of the black holes produced in the merger, e.g. through the calculation of trapped surfaces
- Extraction of gravitational waveforms (usually through computation of the Weyl tensor)

Intense work is ongoing and first satisfactory results are being obtained on linking future gravitational-wave observations to physical properties of the emitting system (e.g. relating the main frequency of postmerger oscillations to the neutron-star masses) [41, 42, 43, 44, 45, 46, 47, 48] and on heavy-element production and macronovae [49, 31, 32, 51, 52, 53, 54, 55, 56, 57, 58, 59, 19, 60]. There are then still issues that are very open, like the accurate dynamics of magnetic fields after the merger [8, 10, 11] (and before the merger for resistive MHD [61, 62, 63, 64, 65]), neutrino treatment [16, 17, 66, 18, 67, 68, 9, 69, 70, 71, 19] and photon radiation transport [70].

Most of the general-relativistic codes [36, 72, 73, 74, 10, 75, 76, 77, 78, 79, 61, 80, 81, 82, 83, 62, 84, 85, 86, 87] solve the field equations in the Baumgarte-Shapiro-Shibata-Nakamura formulation [88, 89]. The remainder [90, 91, 92, 93] use the generalized-harmonic formulation [94, 95]. Most groups use finite-difference methods for the metric evolution and

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\(^1\) One group is developing a code based on pseudo-spectral techniques, but no publication on BNS mergers has been produced yet, except the preliminary studies of [37, 38]. It is named the SpEC-hydro code and employs a dual-grid method in which the metric fields are solved using pseudospectral methods on a multidomain spectral grid, and the matter fields are solved using high-resolution shock-capturing finite volume schemes on a Cartesian finite-difference grid.

\(^2\) Some studies focus on purpose on different types of initial data [39, 40, 31, 32].
conservative, HRSC schemes for the hydrodynamics evolution. In all cases, these unigrid algorithms are then interfaced with some sort of AMR.

Some groups have implemented the MHD equations in full general relativity; since these codes all make use of conservative HRSC methods, their main differences are in how they meet the challenge of preserving the $\text{div} \cdot \mathbf{B} = 0$ constraint, especially in AMR. Besides MHD, the other major advances in the physical modelling for numerical relativity codes have been in the arena of microphysics. While the polytropic or ideal-fluid EoSs have been the community standard for quite some time, most codes now allow for a nuclear theory-based EoS and/or use various parameterized, piecewise polytropic EoSs inspired by the range of plausible nuclear EoSs. Some groups have also started to account for neutrino transport via simplified leakage schemes and are also studying formulations for more accurate treatments, which show much promise for numerical relativity simulations with neutrino physics beyond the leakage approximation.

2. Status of models, techniques, and results for general-relativistic simulations

General relativistic hydrodynamics simulations of BNS started being performed more than 15 years ago. Even if nowadays many state-of-the-art codes are able to solve more physical equations (for magnetic fields, neutrinos, radiation, etc.), simulations involving only general-relativistic hydrodynamics (pure GRHD) are still the benchmark for any new code and the necessary testbed for more advanced codes. Furthermore, in many cases, results obtained with pure GRHD - notably inspiral gravitational waveforms - are a sufficiently good description of BNS systems. During the merger and after a single object forms, possibly surrounded by a massive disc, it is necessary to consider additional physical components in order to have a satisfactory description.

One particularly important aspect of BNS simulations is the EoS. While detectable differences between simulations that employed different EoSs already appear during the inspiral, it is really the post-merger phase that is markedly different. Hence, an accurate description of the post-merger evolution is essential to extract information concerning the neutron-star interior structure. These particularly interesting parts of the waveforms that are more likely to yield fundamental clues to the physics beyond nuclear density are at very high frequencies and thus probably only marginally detectable by detectors like advanced LIGO. Third-generation detectors, such as ET, may provide the first realistic opportunity to use gravitational waves to decipher the stellar structure and EoS.

First attempts to single out the effects of the EoS on the inspiral and merger were done in Refs. [102, 103, 74, 104]. The authors there focus mostly on the dynamics of equal-mass binaries, as these are thought to be the most common and are easier and faster to compute, since symmetries of the configuration can be exploited to save computational resources. In Baiotti et al. [104], an investigation was made using two analytic EoSs: a cold, i.e. polytropic, EoS, or a hot, i.e. ideal-fluid (also called simple fluid or gamma-law), EoS. Although the polytropic EoS is isentropic and thus unrealistic for describing the post-merger evolution, it provides an adequate description of the inspiral phase, during which the neutron stars are expected to interact only gravitationally. The most marked differences between cold and hot EoSs are in the post-merger phase.

The study of the effect of realistic EoSs in GRHD simulations has been subsequently especially brought forward by the Shibata group, starting from Ref. [107], where they employ the Akmal-Pandharipande-Ravenhall (APR) EoS augmented, after the onset of merger, by a hot part through an ideal-fluid EoS. In another work of the same group, the authors studied the

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3 Straightforward finite-difference evolutions of the magnetic field would generically lead to magnetic monopoles and, hence, unphysical behavior.

4 Other groups have implemented the ability to use tabulated EoSs on their codes, see [68, 9]
dependence of the dynamical behavior of BNS mergers on the EoS of the supernuclear-density matter with piecewise-polytropic EoSs [110]. Table I in [109] usefully summarizes the piecewise polytropic parameters of several realistic EoSs.

In a work published soon after, the same group showed results of simulations performed by incorporating both nucleonic and hyperonic finite-temperature EoS [17, 18]. Reported results are of interest, even if the hyperonic EoS used here is limited by the fact that it only takes Λ hyperons into account and that it cannot produce a stable neutron star with realistic masses. It was found that also for the hyperonic EoS, a HMNS is first formed after the merger and subsequently collapses to a black hole. The radius of such a HMNS decreases in time because of the increase of the mass fraction of hyperons and the consequent decrease in supporting pressure. Such a shrinking is noticeably larger than the one simply due to angular-momentum loss through gravitational-wave emission that is present also in nucleonic EoSs. These dynamics are clearly visible in the gravitational-wave signal, whose characteristic peak frequency has an increase of 20%-30% during the HMNS evolution. By contrast, for the nucleonic EOS the peak gravitational-wave frequency in the HMNS phase is approximately constant. The authors stress that their results raise a warning about using the peak frequency of the gravitational-wave spectrum to extract information of the neutron-star matter, because it may evolve and so make the relation of the peak frequency with the HMNS structure ambiguous. Finally, the authors found that the torus mass for the hyperonic EoS is smaller than that for the nucleonic EoS. This makes hyperonic EoSs less favorite for the description of short gamma-ray bursts.

The most recent works of the Shibata group on EoSs are Ref. [44, 99], where they use their AMR SACRA code [74] to perform a large number of simulations with a variety of EoSs and mass ranges. On the basis of their results they derive a fitting formula for the quasiperiodic gravitational waveforms, which they say may be used for the analysis of future observed gravitational-wave signals. They choose two types of EoSs: piecewise polytropic EoSs [110] and the tabulated Shen EoS [114]. In both cases, they add approximate finite-temperature effects to the cold EoSs through an additional ideal-fluid term.

They put in evidence that the universal features of gravitational waves emitted by HMNSs include, in order: (i) a peak in frequency and amplitude soon after the merger starts, (ii) a decrease in amplitude during the merger and an new increase when the HMNS forms, (iii) a damped oscillation of the frequency during the HMNS phase lasting for several oscillation periods and eventually settling to an approximately constant value (although a long-term secular change associated with the change of the state of HMNSs is always present), (iv) a final decrease in the amplitude during the HMNS phase, either monotonical or with modulations. Based on this, they find an optimal 13-parameter fitting function. Using the fitted functions it may be possible to constrain the neutron-star radius with errors of about 1 km.

3. Outlook

In this article, because of limited space, I have been able to review only one topic about BNS merger simulations, namely the influence of the EoS. The phenomenology and physics of BNS mergers, however, are much richer. Simulations of BNS merger and post-merger phases will soon be connected to observations of gravitational waves and will allow us to interpret them and therefore to uncover the mysteries of the physical and astrophysical properties of relativistic stars.

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5 Hyperonic EoSs, which are generally soft, are currently disfavored by the observation of a 2M⊙ star [111, 112]. However, the existence of exotic phases in neutron stars remains unconstrained [113].
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