A new online static voltage stability monitoring method for power systems is proposed by using phasor measurement unit (PMU) data in this paper. This approach uses the real-time power, voltage, and phase angle data collected by the PMU to estimate the power flow Jacobian matrix of the system, and then the static voltage stability is monitored via the minimum singular values (MSVs) of the power flow Jacobian matrix. The novelty of the approach lies in the fact that it only utilizes PMU data for implementing online monitoring of the power system static voltage stability, independent of the physical model and its parameters. The application results on the IEEE 57-bus test system verify the effectiveness of the proposed approach.

1. Introduction

Voltage stability assessment (VSA) has been recognized as an important task to ensure the secure and economical operation of power systems [1, 2]. Problems arising from the growing integration of intermittent renewable power generation in a variety of forms such as active distribution networks [3] and microgrids [4, 5] and integrated energy systems [6] are nudging power systems toward potential dynamic instability scenarios due to the inherent uncertainties of renewable generation [7]. As a new generation of DC transmission technique, voltage source converter-based high voltage direct current (VSC-HVDC) has become a popular opinion of power transmission due to its significant advantages such as independent adjustments of active and reactive powers [8], asynchronous interconnection between islands [9], and black-start capability [10]. In addition, the growing integration of energy storage [11], electric vehicles [12], cyber attacks [13–15], and increasingly diversified demands [16, 17] affect the stable operation of the system to a certain extent. All these shifts pose new challenges in maintaining the system working reliably and securely. In recent years, the wide-area measurement system (WAMS) using time-stamped phasor measurement units (PMUs) has been receiving ever increasing attention from both academia and industry, which makes it possible to explore wide-area protection and control (WAPaC) schemes to avoid the system collapse [18–21]. Therefore, the voltage stability monitoring and assessment of power systems based on PMU data are of great significance in the new context [22–25].

Since the first dQ/dU criterion proposed by the Soviet scholar N.M in the 1940s, a large number of voltage stability analysis approaches of power systems have been developed, such as the sensitivity method [26], continuous power flow method [27], the singular value decomposition method [28], and so on. The sensitivity method is only suitable for simple power systems, and sometimes there are discriminant errors when it is used in multimachine systems. In theory, the feasible solution domain method can calculate the voltage stability margin of a given operation mode, but the calculation of critical injection vectors involves complex nonlinear problems, and the computation is heavy. The multivalue method of power flow needs to track and calculate the multisolution value of the injection quantity repeatedly, and the static stability margin of the system voltage is approximate. References [29–31] illustrate the feasibility of the singular value decomposition theory of the trend Jacobian matrix, but the above methods need to know the
2. Establishment of the Jacobian Matrix Model for Power Flow

The power flow equation of the power system in polar form can be expressed as follows:

\[
\begin{align*}
P_i &= V_i \sum_{j \in \mathcal{I}} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}), \\
Q_i &= V_i \sum_{j \in \mathcal{I}} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}),
\end{align*}
\]

where \(j \in \mathcal{I}\) denotes that buses \(i\) and \(j\) are connected; \(U_{ij}\) and \(U_{ji}\) represent the voltage amplitude of buses \(i\) and \(j\), respectively; \(G_{ij}\) and \(B_{ij}\) represent the values of the real and imaginary parts of the admittance matrix between buses \(i\) and \(j\), respectively; \(\theta_{ij}\) represents the phase angle difference between buses \(i\) and \(j\); and \(P_i\) and \(Q_i\) represent the active and reactive power of bus \(i\), respectively.

Excluding the equilibrium bus of the system, according to the Newton–Raphson method, formula (1) can establish the following linearization modified equations:

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} =
\begin{bmatrix}
H_{ij} & N_{ij} \\
I_{ij} & L_{ij}
\end{bmatrix}
\begin{bmatrix}
\Delta \theta \\
\Delta V
\end{bmatrix},
\]

where \(\Delta P\) and \(\Delta Q\) represent the microincrement of active power and reactive power of the injection bus; \(\Delta \theta\) and \(\Delta V\) represent the microincrement of the voltage phase angle \(\theta\) and the voltage amplitude \(V\), respectively; \(H_{ij}\) and \(N_{ij}\) represent the partial derivative of the active power \(P\) to the voltage phase angle \(\theta\) and the voltage amplitude \(V\), respectively. In equation (2),

\[
J = \begin{bmatrix}
H_{ij} & N_{ij} \\
I_{ij} & L_{ij}
\end{bmatrix},
\]

where \(J\) is the Jacobian matrix of the linearized power flow equation.

3. Evaluation of Voltage Stability by Singular Value Decomposition

Suppose the system has \(n\) buses in addition to the slack bus, of which there are \(m\) PV buses, and the singular value decomposition (SVD) \([47, 48]\) is performed on equation (3). Then, the following formula can be obtained:

\[
J = E \delta U^T = \sum_{i=1}^{(2n-m)} e_i \delta_i u_i^T,
\]

where \(e_i\) and \(u_i\) represent the \(i\)-th column elements of \(E\) and \(U\), respectively, and \(\delta_i\) is the diagonal element of the diagonal matrix. If the Jacobian matrix is nonsingular, the effect of the increment \(\Delta P\) and \(\Delta Q\) of the injection powers \(P\) and \(Q\) on \(\theta\) and \(V\) can be obtained by equations (2) and (4):

\[
\begin{bmatrix}
\Delta \theta \\
\Delta V
\end{bmatrix} = J^{-1} \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = \sum_{i=1}^{2n-m} \delta_i^{-1} u_i e_i^T \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}.
\]

It can be seen from equation (5) that when a singular value \(\delta_i\) is very small (close to zero), the small change of injection power \(P\) and \(Q\) will cause great fluctuation of \(\theta\) and \(V\). The response of the system is completely determined by the minimum singular value (MSV) \(\delta_{\text{min}}\) and its corresponding left and right singular vectors \(e_{\text{min}}\) and \(u_{\text{min}}\), that is:

\[
\begin{bmatrix}
\Delta \theta \\
\Delta V
\end{bmatrix} = \sum_{i=1}^{2n-m} \delta_i^{-1} u_{\text{min}} e_{\text{min}}^T \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}.
\]

Here, \(\epsilon_{\text{min}}\) and \(u_{\text{min}}\) are normalized to
\[
\sum_{i=1}^{n} \theta_i^2 + \sum_{i=1}^{n-m} \nu_i^2 = 1, \\
\sum_{i=1}^{n} p_i^2 + \sum_{i=1}^{n-m} q_i^2 = 1, \tag{7}
\]

and then
\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = \epsilon_{\min}, \tag{8}
\]

and thus
\[
\begin{bmatrix}
\Delta \theta \\
\Delta V
\end{bmatrix} = \frac{\epsilon_{\min}}{\delta_{\min}} \tag{9}
\]

From equations (8) and (9), it can be concluded that since the MSV is sufficiently small, a small change in the injection power can cause a large change in voltage. The MSV of the Jacobian matrix can be used as a good indicator of static voltage stability [49, 50].

4. Evaluation of Static Voltage Stability by PMU Data

The aforementioned static voltage stability method by using the MSV of Jacobian matrix can give the static voltage stability quantitatively, but it depends on the fixed physical model and cannot judge the static voltage stability online.

4.1. Estimation of Power Flow Jacobian Matrix by PMU Data

In polar coordinates, the terms of the Jacobian matrix of power flow can be represented by the partial derivative of \( \theta_j \) and \( V_j \) by \( P_i \) and \( Q_i \), respectively, and then formula (3) can be expressed as

\[
\begin{bmatrix}
H_{ij} = \frac{\partial P_i}{\partial \theta_j} \\
N_{ij} = \frac{\partial P_i}{\partial V_j} \\
I_{ij} = \frac{\partial Q_i}{\partial \theta_j} \\
L_{ij} = \frac{\partial Q_i}{\partial V_j}
\end{bmatrix} \tag{10}
\]

Suppose both \( \theta_j \) and \( V_j \) have a very small change, represented by \( \Delta \theta_j \) and \( \Delta V_j \), respectively. Then, \( P_i \) and \( Q_i \) will also change with the slight changes above, denoted by \( \Delta P_{i}^{\Delta \theta_j} \), \( \Delta P_{i}^{\Delta V_j} \) and \( \Delta Q_{i}^{\Delta \theta_j} \), \( \Delta Q_{i}^{\Delta V_j} \), respectively; then, equation (10) can be expressed as [51]

\[
\begin{bmatrix}
H_{ij} = \frac{\partial P_i}{\partial \theta_j} \approx \frac{\Delta P_{i}^{\Delta \theta_j}}{\Delta \theta_j}, \\
N_{ij} = \frac{\partial P_i}{\partial V_j} \approx \frac{\Delta P_{i}^{\Delta V_j}}{\Delta V_j}, \\
I_{ij} = \frac{\partial Q_i}{\partial \theta_j} \approx \frac{\Delta Q_{i}^{\Delta \theta_j}}{\Delta \theta_j}, \\
L_{ij} = \frac{\partial Q_i}{\partial V_j} \approx \frac{\Delta Q_{i}^{\Delta V_j}}{\Delta V_j}.
\end{bmatrix} \tag{11}
\]

At time \( t \):

\[
\begin{bmatrix}
\Delta P_i(t) \\
\Delta Q_i(t)
\end{bmatrix} \approx \sum_{j \in N_g \cup N_l} \Delta \theta_j(t) H_{ij} + \sum_{j \in N_l} \Delta V_j(t) N_{ij}, \tag{12}
\]

where \( N_g \) represents the number of PV buses and \( N_l \) represents the number of PQ buses, and substituting equation (11) into (12) yields

\[
\begin{bmatrix}
\Delta P_i(t) \\
\Delta Q_i(t)
\end{bmatrix} \approx \sum_{j \in N_g \cup N_l} \Delta \theta_j(t) I_{ij} + \sum_{j \in N_l} \Delta V_j(t) L_{ij}. \tag{13}
\]

Further consolidation of equation (13) is available:

\[
\Delta P_i \approx \left[ \left( \Delta \theta_j \right)_{j \in N_g \cup N_l} \left( \Delta V_j \right)_{j \in N_l} \right] \left[ \begin{array}{c}
H_i \\
N_i
\end{array} \right], \tag{14}
\]

\[
\Delta Q_i \approx \left[ \left( \Delta \theta_j \right)_{j \in N_g \cup N_l} \left( \Delta V_j \right)_{j \in N_l} \right] \left[ \begin{array}{c}
I_i \\
L_i
\end{array} \right], \tag{15}
\]

where \( H_i = [(H_{ij}), j \in N_g \cup N_l], N_i = [(N_{ij}), j \in N_l], I_i = [(I_{ij}), j \in N_g \cup N_l], L_i = [(L_{ij}), j \in N_l]. \)

4.2. Model Solution. In the assumption of equations (14) and (15), both the regression matrix and the measurement vector have measurement errors. For the convenience of expression, here we make \( A = [(\Delta \theta_j)_{j \in N_g \cup N_l} (\Delta V_j)_{j \in N_l}], b_i = P_i \); then, equation (14) can be expressed as

\[
b_i \approx A \left[ H_i^T \ N_i^T \right]^T. \tag{16}
\]

Since equation (16) is an overdetermined equation, in this paper, \( [H_i^T \ N_i^T]^T \) is calculated by using the total least square (TLS) [52, 53]. This method is also suitable for equation (15).

In the ordinary least squares estimate (LSQ), since the regression matrix is assumed to be error-free, the principle of this method is to correct \( b_i \) as little as possible under the
Euclidean norm \([51]\), which forms the following optimization problem \([54]\):

\[
\begin{align*}
\min_{\tilde{b}_i \in \mathbb{R}^M} & \| \Delta b_i \|_2, \\
\tilde{b}_i = A [ H_i^T \, N_i^T ]^T,
\end{align*}
\]

where \( M \) represents sets of synchronized measurements, \( \Delta b_i = b_i - \tilde{b}_i \) (\( \tilde{b}_i \) is a very small value), and any \( [ H_i \, N_i ]^T \) that satisfies \( \tilde{b}_i = A [ H_i \, N_i ]^T \) is the solution of LSE of equation (16).

Assuming that \( A \) is full rank, the special solution of the closed form of equation (17) is

\[
[ H_i \, N_i ]^T = ( A^T A )^{-1} A^T b_i.
\]

Unlike LSE, TLS also takes into account the measurement error in \( A \); similar to the Euclidean norm, the problem is to find a minimized F-norm, as follows:

\[
\begin{align*}
\min_{[ \tilde{A} \, \tilde{b}_i ] \in \mathbb{R}^{M+(N+1)}} & \| \Delta A \, \Delta b_i \|_F, \\
\tilde{b}_i = \tilde{A} [ H_i^T \, N_i^T ]^T,
\end{align*}
\]

where \( N = N_g + 2 N_i \) and \( \Delta A = A - \tilde{A}, \Delta b_i = b_i - \tilde{b}_i \).

The TLS solution of equation (19) depends on the SVD. We describe this process and write equation (16) as follows:

\[
[ A \, b_i ] [ H_i^T \, N_i^T ]^T - 1 = 0.
\]

By using the SVD, the above formula can be rewritten as

\[
[ A \, b_i ] [ H_i^T \, N_i^T ]^T = E \delta U^T,
\]

where \( E = [e_1, e_2, \ldots, e_m] \), \( U = [u_1, u_2, \ldots, u_{N+1}] \) is a symmetric matrix whose diagonal element \( \delta_i \) is the singular value of \( [ A \, b_i ] \). If \( \delta_m \neq 0 \); then, the rank of \( [ A \, b_i ] \) is \( (N + 1) \), and the only solution of equation (20) is zero vector. In order to obtain the nonzero solution of equation (20), \( [ A \, b_i ] \) must be reduced to the rank \( N \). The matrix approximation theorem shows that

\[
[ \tilde{A} \, \tilde{b}_i ] = E \tilde{\delta} U^T.
\]

Since the rank of the approximate matrix \( [ \tilde{A} \, \tilde{b}_i ] \) is equal to \( N \), equation (20) has a nonzero solution. According to the nature of the SVD, \( u_{N+1} \) is the unique vector belonging to the zero vector space \( [ \tilde{A} \, \tilde{b}_i ] \), and the TLS solution is obtained by scaling the vector \( u_{N+1} \) until the last element is \(-1\):

\[
[ H_i^T \, N_i^T ]^T - 1 = \frac{1}{u_{N+1}^T} u_{N+1}.
\]

where \( u_{N+1}^T \) represents the \( (N + 1) \)th element of \( u_{N+1} \); then, the unique TLS solution of equation (16) is

\[
[ \tilde{H}_i^T \, \tilde{N}_i^T ]^T = \frac{1}{u_{N+1}^T} u_{N+1}.
\]

\[
(24)
\]

5. Algorithm Flow and Basic Steps

The flowchart of the proposed method is shown in Figure 1. The specific steps are as follows:

1. Collect multiple sets of PMU data from different buses at the \( (t + \Delta t) \) time, and the number of collected groups is greater than the order of the Jacobian matrix.
2. Obtain \( \Delta P_i, \Delta Q_i, \Delta \theta_i, \Delta V_i \) by difference.
3. Taking the data obtained in step 2 into equation (14), the solution of the overdetermined equations is obtained by the TLS method, and the Jacobian matrix of power flow is obtained.
4. Calculate the MSV of the Jacobian matrix obtained in step 3 and compare it with the threshold \( \tau \), if the difference exceeds the margin \( \xi \).
5. Repeat steps 1 to 4 to realize real-time monitoring of the power grid.

By collecting the electrical quantities of different buses at time \( (t + \Delta t) \), including \( P_i, Q_i, \theta_i \), and \( V_i \), the power flow Jacobian matrix can be obtained according to equations (14) and (15). This process is shown in Figure 2.

Note that the number of samples is greater than the order of the Jacobian matrix. The results of the MSV of the power flow Jacobian matrix estimated by the Newton–Raphson method are shown in Table 1.

Comparing the data in Table 1, it can be seen that the difference is small in terms of the MSV obtained by the Newton–Raphson method and the PMU data. This shows that the proposed method can be used to evaluate the static voltage stability of the system.

6. Case Study

In this study, it is assumed that all buses of the system are equipped with PMUs. In order to simulate the process of voltage collapse in the power system, the active power \( P \) is increased by 0.01 steps in the fifth node of the IEEE 57-bus system, and the MSV of the Jacobian matrix calculated by the proposed method is tracked in real time. The results are shown in Figure 3:

\[
P_{8/V8} - \delta_{\min}, \quad P_{13/V13} - \delta_{\min}, \quad P_{31/V31} - \delta_{\min}, \quad \text{and } P_{V33} - \min_{\delta_{\min}}
\]

above represent the process of the voltage \( V \) changing and the MSV \( \delta_{\min} \) of the buses 8, 13, 31, and 33 changing with the change of power \( P \), respectively. \( P/\delta_{\min} \) represents the process in which the MSV of the system varies with power \( P \).

It can be seen from the \( P/\delta_{\min} \) curve that as the power increases for the different buses, the MSV \( \delta_{\min} \) of the system decreases continuously. When \( \delta_{\min} \) is close to zero, the system collapses. Comparing the \( P/\delta_{\min} \) and \( P/V \) (pu) curves,
it can be seen that as $\delta_{\text{min}}$ decreases ($P$ increases continuously), the voltage value also decreases and the rate of decrease is different. When $\delta_{\text{min}}$ is close to zero, a small change of $P$ causes a large fluctuation in voltage, which is consistent with the context in equations (8) and (9).

During normal operation of the two systems, the sampling frequency of PMU is taken as 50 Hz/s, and the MSV values of the power flow Jacobian matrix are calculated every 4 seconds according to equations (14) and (15). In this way, the evolution curves of the MSV values are demonstrated in Figure 4.

$\delta_{\text{min}}$, $\delta_{1\text{min}}$ and $\delta_{2\text{min}}$ represent the smallest singular value obtained by the Newton–Raphson method and PMU data, respectively. By comparing the matching degree of the curves in Figure 4, it can be seen that the curves obtained by the two methods are basically consistent, indicating that the method proposed in this paper can monitor the static voltage stability of the system online.

The active power $P$ is increased by the step of 0.005 at bus 6 of the IEEE 57-bus system, and the static voltage stability of the power system is estimated from every 200 PMU measurements, as shown in Figure 5.

By comparing the matching degree of the curves in Figure 5, we can see that the curves obtained by the two
methods are basically consistent, which shows that the method proposed in this paper can realize online monitoring in the process of system instability.

7. Conclusion

A method for real-time evaluation of static voltage stability of a power system based on PUM data is proposed. The effectiveness of the proposed method is verified by the IEEE 57-bus system. The conclusions are as follows:

(1) The proposed method is capable of realizing online monitoring of static voltage stability of power systems.

(2) Compared with the traditional method, the proposed method does not need any model and parameter information of the power grid and only needs PMU data for the implementation.

(3) The proposed method can be used in an online stability detection system of power systems for situational awareness improvement.

Our future work will focus on extending the proposed approach to estimate an accurate voltage stability boundary and address stability constraints in voltage stability-constrained optimal power flow [55–58]. It is interesting to investigate the voltage stability of power systems with new elements such as distributed generations [59–61] and combined heat and power plants [62]. Besides, the optimal placement of PMU will be further studied to realize the monitoring of the whole network with fewer PMUs. Another potential topic in future research is to develop voltage stability assessment model using new machine learning techniques such as the lasso algorithm [63], least squares support vector machines [64], and deep learning [65].

Data Availability

The IEEE 57-bus system data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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