The Vector Manifestation and Effective Degrees of Freedom At Chiral Restoration

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Abstract

The role of effective degrees of freedom on the vector and axial-vector susceptibilities and the pion velocity at chiral restoration is analyzed. We consider two possible scenarios, one in which pions are considered to be the only low-lying degrees of freedom – that we shall refer to as “standard” – and the other in which pions, vector mesons and constituent quarks (or quasiquarks in short) are the relevant low-lying degrees of freedom – that we shall refer to as “vector manifestation (VM).” We show at one-loop order in chiral perturbation theory with hidden local symmetry Lagrangian that while in the standard scenario, the pion velocity vanishes at the chiral transition, it instead approaches unity in the VM scenario. If the VM is realized in nature, the chiral phase structure of hadronic matter can be much richer than that in the standard one and the phase transition will be a smooth crossover: Sharp vector and scalar excitations are expected in the vicinity of the critical point. Some indirect indications that lend support to the VM scenario, and in consequence to BR scaling, are discussed.

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1 Introduction

In describing the chiral restoration transition at the critical temperature $T_C$ and/or critical density $n_C$, one of the essential ingredients is the relevant degrees of freedom that enter in the vicinity of the critical point. Depending upon what enters there, certain aspects of the phase transition scenario can be drastically different. These different scenarios will eventually be sorted out by experiments or by QCD simulations on lattice.

The standard way of looking at the problem at high temperature and low density currently accepted by the majority of the community as the “standard picture” is to assume that near the critical point, the only relevant low-lying degrees of freedom are the (pseudo-)Goldstone pions and a scalar meson that in the $SU(2)$ flavor case, makes up the fourth component of the chiral four-vector of $SU(2)_L \times SU(2)_R$. For the two-flavor case, one then maps QCD to an $O(4)$ universality class etc. Here lattice measurements will eventually map out the phase structure. On the contrary, at zero temperature and high density, the situation is totally unclear. In fact as density increases, the possibility is that one may not be able to talk about quasiparticles of any statistics at all: The concept of a hadron may even break down. Unfortunately lattice cannot help here, at least for now, because of the notorious sign problem.

The situation is markedly different if the vector manifestation à la Harada and Yamawaki [1, 2] scenario is viable. In this picture, certain hadrons other than pions can play a crucial role in both high temperature and high density with a drastically different phase structure. In particular, light-quark vector mesons and constituent quarks (called quasiquarks from here on) can become the relevant degrees of freedom near the phase transition, becoming “sharper” quasiparticles even near the critical density, thereby increasing the number of degrees of freedom that enter from the hadronic sector, with – in sharp contrast to the standard picture – narrow-width excitations near the critical point. This can then lead to a form of phase change that is a lot smoother than that of the standard scenario.

The objective of my talk is to describe how this can happen.

2 The vector manifestation

To approach the chiral symmetry restored phase “bottom up” from the broken phase, we need an effective field theory (EFT) that represents as closely as possible the fundamental theory of strong interactions, QCD. In fact, according to Weinberg’s unproven theorem [3], QCD at low energy/momentum can be encapsulated in an effective field theory with a suitable set of colorless fields subject to the symmetries and invariance required by QCD. So the question is how to construct an EFT that captures the essence of QCD in describing the relevant physics at the phase transition. In order to do this, we first need to identify the scale at which we want to define our EFT and the relevant degrees of freedom and symmetries that we want to implement. I will consider two possibilities. One is the standard scenario based on linear sigma model that assumes that the only low-excitation degrees of freedom relevant to chiral restoration, apart from the nucleons, are the pions and possibly a scalar (denoted $\sigma$ in linear sigma model) with all other degrees of freedom integrated out. The other is the vector manifestation scenario based on hidden local symmetry (HLS) in which light-quark vector mesons figure crucially.

In this talk, we are principally interested in what new physics can be learned in the second scenario which seems to be currently ignored by the community in the field. We consider
for definiteness the three-flavor case, i.e., $SU(3)_L \times SU(3)_R$. In going to nuclear matter and beyond in this scenario, we must keep vector-meson degrees of freedom explicit. This is because of the vector manifestation (VM) \cite{1, 2} for which vector degrees of freedom are indispensable.

To understand the VM, we consider the HLS Lagrangian \cite{4} in which the pion and vector mesons are the effective degrees of freedom. For the moment, we ignore fermions and heavier excitations as e.g., $a_1$, glueballs etc. To make the discussion transparent, we consider three massless flavors, that is, in the chiral limit \cite{3}. The relevant fields are the (L,R)-handed chiral fields $\xi^{L,R} = e^{i\sigma/F_\sigma}e^{\mp i\pi/F_\pi}$ where $\pi$ is the pseudoscalar Goldstone boson field and $\sigma$ the Goldstone scalar field absorbed into the HLS vector field $\rho_\mu$, coupled gauge invariantly with the gauge coupling constant $g$. If one matches this theory to QCD at a scale $\Lambda_M$ below the mass of the heavy mesons that are integrated out but above the vector ($\rho$) meson mass, it comes out – when the quark condensate $\langle \bar{q}q \rangle$ vanishes as in the case of chiral restoration in the chiral limit – that

$$g(\bar{\Lambda}) \to 0, \quad a(\bar{\Lambda}) \equiv F_\sigma/F_\pi \to 1.$$  

(1)

Now the renormalization group analysis shows that $g = 0$ and $a = 1$ is the fixed point of the HLS theory and hence at the chiral transition, one approaches what is called the “vector manifestation” fixed point. The important point to note here is that this fixed point is approached regardless of whether the chiral restoration is driven by temperature $T$ \cite{5} or density $n$ \cite{6} or a large number of flavors \cite{7}. At the VM, the vector meson mass must go to zero in proportion to $g$, the transverse vectors decouple and the longitudinal components of the vectors join in a degenerate multiplet with the pions.

### 3 Vector and Axial-Vector Susceptibilities

As an illustration of what new features are encoded in the HLS/VM scenario, we first consider approaching the critical point in heat bath. We will come to the density problem later. Specifically consider the vector and axial vector susceptibilities defined in terms of Euclidean QCD current correlators as

$$\delta^{ab}\chi_V = \int_0^{1/T} d\tau \int d^3\vec{x} \langle V^a_0(\tau, \vec{x})V^b_0(0, \vec{0}) \rangle_\beta,$$  

(2)

$$\delta^{ab}\chi_A = \int_0^{1/T} d\tau \int d^3\vec{x} \langle A^a_0(\tau, \vec{x})A^b_0(0, \vec{0}) \rangle_\beta$$  

(3)

where $\langle \rangle_\beta$ denotes thermal average and

$$V^a_0 \equiv \bar{\psi} \gamma^0 \frac{\tau^a}{2} \psi, \quad A^a_0 \equiv \bar{\psi} \gamma^0 \gamma^5 \frac{\tau^a}{2} \psi$$  

(4)

with the quark field $\psi$ and the $\tau^a$ Pauli matrix the generator of the flavor $SU(2)$.

\footnote{The two-flavor case is a bit more subtle and has not yet been fully worked out. I will leave that issue for later.}

\footnote{HLS is essential for the VM since local gauge symmetry is required for doing a consistent chiral perturbation calculation in the presence of vector mesons. Other theories where local gauge symmetry is absent are moot on this issue. See \cite{2} for a clear discussion on this point.}

\footnote{Masses and symmetry breaking can be introduced with attendant complications. Up to date, the effects of quark masses have not been investigated in detail in this formalism. It is possible that the detail structure of the phase diagram can be substantially different from the chiral limit picture we are addressing here.}
3.1 The standard (linear sigma model) scenario

The standard picture with pions figuring as the only degrees of freedom in heat bath has been worked out by Son and Stephanov [8]. The reasoning and the result are both very simple and elegant. They go as follows.

If the pions are the only relevant degrees of freedom near the chiral transition, then the axial susceptibility (ASUS) for the system is encoded in the chiral Lagrangian of the form

$$L_{\text{eff}} = \frac{f_{\pi}^2}{4} \left( \text{Tr} \nabla_0 U \nabla_0 U^\dagger - v_{\pi}^2 \text{Tr} \partial_i U \partial_i U^\dagger \right) - \frac{1}{2} \langle \bar{\psi} \psi \rangle \text{Re} M^\dagger U + \cdots \quad (5)$$

where $v_{\pi}$ is the pion velocity, $M$ is the mass matrix introduced as an external field, $U$ is the chiral field and the covariant derivative $\nabla_0 U$ is given by $\nabla_0 U = \partial_0 U - \frac{i}{2} \mu_A (\tau_3 U + U \tau_3)$ with $\mu_A$ the axial isospin chemical potential. The ellipsis stands for higher order terms in spatial derivatives and covariant derivatives. Now given (5) as the full effective Lagrangian which would be valid if it could be given in a local form as is, then the ASUS would take the simple form

$$\chi_A = -\frac{\partial^2}{\partial \mu_A^2} L_{\text{eff}}|_{\mu_A=0} = f_{\pi}^2. \quad (6)$$

Within the scheme, this is the entire story: There are no other terms that contribute. That the ASUS is given solely by the square of the temporal component of the pion decay constant follows from the fact that the Goldstone bosons are the only relevant degrees of freedom in the system, with those degrees of freedom integrated out being totally unimportant. The effective theory of course cannot tell us what $f_{\pi}^t$ is. However one can get it from lattice QCD. To do this, we exploit that at chiral restoration, the vector correlator and the axial correlator must be equal to each other, which means that

$$\chi_A|_{T=T_c} = \chi_V|_{T=T_c}. \quad (7)$$

Now from the lattice data of Gottlieb et al [9], we learn that

$$\chi_V|_{T=T_c} \neq 0 \quad (8)$$

and hence from (6) that

$$f_{\pi}^t \neq 0. \quad (9)$$

Next, we know that the space component of the pion decay constant $f_{\pi}^s$ must go to zero at chiral restoration. This is because it should be related directly to the quark condensate $\langle \bar{q} q \rangle$, i.e., the order parameter of the chiral symmetry of QCD. Thus one is led to the conclusion that at $T = T_c$ the velocity of the pion must be zero,

$$v_{\pi} \propto f_{\pi}^s/f_{\pi}^t \rightarrow 0 \quad \text{as} \quad T \rightarrow T_c. \quad (10)$$

That the pion velocity is zero at the critical point is both elegant and reasonable. It is analogous to the sound velocity in condensed matter physics which is known to go to zero on the critical surface. But the trouble is that there is a caveat here which throws doubt on the simple result. One might naively think that the vector susceptibility (VSUS) could also be described by the same local effective Lagrangian but with the covariant derivative now defined.
with the vector isospin chemical potential $\mu_V$ as $\nabla_0 U = \partial_0 U - \frac{1}{2} \mu_V (\tau_3 U - U \tau_3)$. If the local form of the effective Lagrangian (5) is valid as well for the VSUS, one can do the same calculation as for $\chi_A$, i.e.,

$$\chi_V = - \frac{\partial^2}{\partial \mu_V^2} L_{\text{eff}} \big|_{\mu_V = 0}. \quad (11)$$

Now a simple calculation shows that $\chi_V = 0$ for all $T$. This is at variance with the lattice result. It is also unacceptable on general grounds. Thus either the local effective Lagrangian is grossly inadequate for the VSUS or else the assumption that the pions are the only relevant degrees of freedom is incorrect. Indeed, Son and Stephanov suggest that diffusive modes in hydrodynamic language that are not describable by a local Lagrangian can be responsible for the non-vanishing VSUS.

### 3.2 The HLS/VM scenario

The situation is dramatically different in the VM scenario [1, 2]. Both the hidden gauge fields and quasiquark fields enter importantly at the phase transition [10]. The reason for this is that the masses of both tend to zero near the VM fixed point and hence they must enter on the same footing as the Goldstone pions.

In the HLS/VM scheme, the parameters of the effective Lagrangian are defined at the matching scale $\Lambda_M$ in terms of the QCD parameters that encode the vacuum change in heat bath and/or dense medium. In computing physical observables like the current correlators, one takes into account both quantum loop effects that represent how the parameters run as the scale is changed from the matching scale to the physical (on-shell) scale and thermal and/or dense loop effects induced in the renormalization-group flow. Now the former implies an intrinsic temperature and/or density dependence in the parameters — called “parametric dependence.” The one-loop calculations in [10] show that both effects are governed by the VM fixed point at $T \to T_c$,

$$g \to 0, \quad a \to 1. \quad (12)$$

The results [10] that follow from this consideration are

$$f^s_\pi|_{T = T_c} = f^s_\pi|_{T = T_c} = 0, \quad v_\pi|_{T = T_c} = 1 \quad (13)$$

and

$$\chi_A|_{T = T_c} = \chi_V|_{T = T_c} = 2 N_f \left[ \frac{N_f}{12} T_c^2 + \frac{N_c}{6} T_c^2 \right]. \quad (14)$$

One can understand these results as follows. As $T \to T_c$, both $f^s_\pi$ approach $\bar{f}_\pi \propto \langle \bar{q}q \rangle$ which approaches zero [5]. At one loop, they approach the latter in such a way that the ratio goes to 1, thereby making the pion velocity approach the velocity of light. Both $\chi_{V,A}$ get contributions from the flavor gauge vector mesons and the quasiquarks whose masses approach zero and chiral symmetry forces them to become equal to each other. Since both the space and time

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4This dependence is missing in most of the effective field theory calculations that are based on effective Lagrangians determined in the matter-free and zero temperature vacuum. Most of the treatments found in the literature nowadays belong to this category.
components of the pion decay constant are vanishing, they do not figure in the formulas for the susceptibilities (14).

If realized, the VM scenario will present an interesting phase structure. It will give a phase diagram drastically different from the standard sigma model one. For instance, it would imply that there are a lot more degrees of freedom than the standard picture just below the critical temperature. We do not know how fast the masses actually drop as one approaches, bottom-up, the critical point but if the presently available lattice results are taken at their face value, then they do not seem to drop appreciably up to near \( T_c \). But it is still possible that they drop to zero in a narrow window near the critical point in a way consistent with the VM and account for the rapid increase of energy density observed in the lattice calculations. In any event, that would provide a natural explanation of a smooth transition with a possible coexistence of excitations of various quantum numbers below and above \( T_c \) as seems to be indicated by the MEM analysis of [11].

I must mention that there is a caveat here also. The results (13) and (14) are one-loop results and one may wonder whether two-loop or higher orders would not change the qualitative features. The space component of the pion decay constant is undoubtedly connected to the chiral order parameter, so remains zero to all orders but there is no general argument to suggest that the time part cannot receive non-vanishing contributions at higher orders. If it did, then we would fall back to the Son-Stephanov result of a vanishing pion velocity.

4 Multiplet structure

The multiplet structure of hadrons implied by the VM at the phase transition at \( T = T_c \) or \( n = n_c \) or \( N_f = N_f^c \) is basically different from that of the standard one based on linear sigma model. Continuing with three flavors in the chiral limit, at the phase transition where the VM is realized, the longitudinal components of the vector mesons \( \rho_\parallel \) join the \((1,8) \oplus (8,1) \) multiplet of the Goldstone pions and the transverse vectors, massless, decouple from the system as the gauge coupling vanishes. This contrasts with the linear sigma model picture where a scalar joins the Goldstone pions in \((3,3^*) \otimes (3^*,3) \) multiplets. If one were to explicitly incorporate the \( a_1 \) vector mesons and the scalar meson in the scheme (so far integrated out), then the \( a_1 \) would be in the same multiplet \((3,3^*) \otimes (3^*,3) \) with the scalar in the VM while in the standard picture, the \( a_1 \) would be in the multiplet \((1,8) \oplus (8,1) \) with the vectors \( \rho \).

As stressed by Harada and Yamawaki, physically there is no sense to be on the VM point. It makes sense only to approach it from below. One can however ask what happens precisely at the VM. Since the VM is in the Wigner mode, one may wonder whether the decoupled vector mesons should not have chiral partners. If they should, then the HLS/VM would be in difficulty since the theory does not contain such chiral partners in the same multiplet. The only way that I see to avoid this obstruction is that the transverse vectors become “singlet” under chiral transformation. This can be made possible if the chiral transition is considered as the flavor vector mesons getting de-Higgsed to the color gauge bosons, i.e., gluons, as proposed by Wetterich [12]. Specifically one can think of the flavor vector mesons as the vectors excited in the color-flavor locking (CFL) transition

\[
SU(3)_L \times SU(3)_R \times SU(3)_c \rightarrow SU(3)_{L+R+c}
\]  

(15)

in analogy to the CFL in color superconductivity in QCD at asymptotic density. In this case, the flavor vectors un-lock the color and flavor and turn in a sort of relay [13] into the color
gauge vector mesons. This phenomenon can be summarized by writing

\[ \xi^\alpha_{L(R)i} = [\xi_{L(R)}v]^\alpha_i, \quad U = \xi^\dagger_L \xi_R \]  

in terms of a color-singlet \( \xi \) field and a \( v \in SU(3)_C \), both of which are unitary. One can then relate [12, 13] the vector meson fields \( \rho_\mu \) and the baryon fields \( B \), to the gluon fields \( A_\mu \) and the quark fields \( \psi \) as

\[
B = Z_\psi^{1/2} \xi^\dagger \psi v^\dagger, \\
\rho_\mu = v(A_\mu + i \frac{g}{2} \partial_\mu) v^\dagger 
\]

where \( Z_\psi \) is the quark wave function renormalization constant. This CF unlocking scenario appears to be highly appealing and fascinating. Up to date, however, this idea has not been fully worked out -- the mechanism for CFL and CF-unlocking is not known -- and although quite plausible, it is not proven yet that it is not inconsistent with the known structure of QCD. It remains to be investigated.

## 5 Evidences for the VM?

The VM prediction is clean and unambiguous for \( SU(3)_L \times SU(3)_R \) chiral symmetry in the chiral limit at the chiral restoration point. At present, there are no lattice measurements that would validate or invalidate this picture. Are there experimental indications that Nature exploits this scheme?

So far, there are no indications from relativistic heavy-ion processes as to whether the VM is realized in the vicinity of the critical temperature or density. So to answer this question, one would have to work out what happens at temperatures and/or densities away from the critical point. One would also have to consider two-flavor cases and quark mass terms to make contact with Nature. To do all these is a difficult task and no theoretical work has been done up to date on this matter. What is available up to date are some indications in nuclear systems at low temperature. Nuclei involve many nucleons in the vicinity of nuclear matter density and the density regime involved here is rather far from the density relevant to the VM. Thus we are compelled to invoke a certain number of extrapolations toward the VM point. Near nuclear matter density, however, we have a many-body fixed point known as “Fermi-liquid fixed point” [14, 15, 13] which involves quantum critical phenomenon and this makes the connection to the VM tenuous even when one is at a much higher density.

To make progress in this circumstance, we have to make a rather drastic simplification of the phase structure. Here we will adopt what is called “double-decimation approximation” [16] which consists of (1) extrapolating downwards from the VM to the Fermi-liquid fixed point and (2) extrapolating upwards from the zero-density regime where low-energy theorems apply to the Fermi-liquid fixed point at nuclear matter density. The spirit here is close to BR scaling proposed in 91 [17].

Close to the VM, the vector meson mass must go to zero in proportion to \( g \). Specifically

\[ m^*_\rho / m_\rho \approx g^*/g \approx \langle \bar{q}q \rangle^*/\langle \bar{q}q \rangle \rightarrow 0 \]  

as the transition point \( n = n_c \) is reached. One can understand this as follows. Near the critical point the “intrinsic term” \( \sim g^*F^*_\pi \) in the vector mass formula drops to zero faster than the dense
loop term that goes as \( \sim g^* H(n) \) where \( H \) is a slowly (i.e., logarithmically) varying function of density. So the dense loop term controls the scaling. Now it seems to be a reasonable thing to assume that near the VM fixed point, we have the scaling

\[
\frac{m^*_\rho}{m_\rho} \approx \frac{g^*}{g} \approx \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle}.
\]

(19)

Our conjecture [16] is that this holds down to near nuclear matter density.

Let us now turn to the low-density regime, that is, a density below nuclear matter density. At near zero density, one can apply chiral perturbation theory with a zero-density HLS Lagrangian matched to QCD at a scale \( \Lambda_M \sim \Lambda_\chi \). We expect to have [18]

\[
\frac{m^*_\rho}{m_\rho} \approx \frac{f^*_\pi}{f_\pi} \approx \sqrt{\langle \bar{q}q \rangle^* / \langle \bar{q}q \rangle}.
\]

(20)

This result follows from an in-medium GMOR relation for the pion if one assumes that at low density the pion mass does not scale (as indicated experimentally [19]), that the vector meson mass is dominantly given by the “intrinsic term” \( \sqrt{aF_\pi g} \) with small loop corrections that can be ignored and that the gauge coupling constant does not get modified at low density (as indicated by chiral models and also empirically). The double-decimation approximation is to simply assume that this relation holds from zero density up to nuclear matter density [16]. Note that we are essentially summarizing the phase structure up to chiral restoration by two fixed points, namely, the Fermi-liquid fixed point and the vector-manifestation fixed point. Here we are ignoring the possibility that there can be other phase changes, such as kaon condensation (or hyperon matter), color superconductivity etc. which can destroy the Fermi liquid structure before chiral symmetry is restored.

The one important feature that distinguishes the HLS/VM theory from other EFTs is the parametric dependence on the background of the “vacuum” – density and/or temperature – which intricately controls the fixed point structure of the VM. At low density, this dependence is relatively weak, so hard to pinpoint. But in precision experiments, it should be visible. One such case is the recent experiment of deeply bound pionic atoms. For this, we can consider a chiral Lagrangian in which only the nucleon and pion fields are kept explicit with the vectors and other heavy hadron degrees of freedom integrated out from the HLS/VM Lagrangian. The relevant parameters of the Lagrangian are the “bare” nucleon mass, the “bare” pion mass, the “bare” pion decay constant, the “bare” axial-vector coupling and so on which depend non-trivially on the scale \( \Lambda_M \) and density \( n \). This Lagrangian takes the same form as the familiar one apart from the intrinsic dependence of the parameters on \( n \). (In the usual approach, the scale \( \Lambda_M \) is fixed at the chiral scale and the dependence on \( n \) is absent). As shown by Harada and Yamawaki [20, 2], the local gauge symmetry of HLS Lagrangian enables one to do a systematic chiral perturbation theory even when massive vectors are present. Since the vectors are integrated out, the power counting will be the same as in the conventional approach. Now if the density involved in the system is low enough, say, no greater than nuclear matter density, then one could work to leading order in chiral expansion. Suppose that one does this to the (generalized) tree order. To this order, the parameters of the Lagrangian can be identified with physical quantities. For instance, the bare pion decay constant \( F_\pi \) can be identified with the physical constant \( f_\pi \), the parametric pion mass with the physical pion mass \( m_\pi \) etc. Now in the framework at hand, the only dependence of the constants on density will then be the intrinsic one determined by the matching to QCD immersed in the background of density \( n \).

If we apply the above argument to the recent measurement by Suzuki et al [19] of deeply bound pionic atom systems, we will find that the measurement supplies information on the
ratio \( f_\pi^*/f_\pi \) at a density \( n \lesssim n_0 \). There is a simple prediction for this quantity [14, 21]. We have from (20)

\[
\Phi(n) \equiv \frac{f_\pi^*/f_\pi}{\sqrt{\langle \bar{q}q \rangle^*/\langle \bar{q}q \rangle}}. \tag{21}
\]

Instead of calculating the quark condensate in medium which is a theoretical construct, we can extract the in-medium pion decay constant by extracting \( \Phi \) from experiments. Indeed, the scaling \( \Phi \) has been obtained from nuclear gyromagnetic ratio in [14, 21]. At nuclear matter density, it comes out to be

\[
\Phi(n_0) = 0.78 \tag{22}
\]

with an uncertainty of \( \sim 10\% \). Thus it is predicted that

\[
(f_\pi^*(n_0)/f_\pi)_{\text{th}} \approx 0.61. \tag{23}
\]

This agrees with the value extracted from the pionic atom data of [19],

\[
(f_\pi^*(n_0)/f_\pi)_{\text{exp}}^2 = 0.65 \pm 0.05. \tag{24}
\]

It is perhaps important to stress that this “agreement” cannot be taken as an evidence for “partial chiral restoration” as one often sees stated in the literature. If one were to go to higher orders in chiral expansion, the parametric pion decay constant cannot be directly identified with the physical pion decay constant since the latter should contain two important corrections, i.e., quantum corrections governed by the renormalization group equation as the scale is lowered from \( \Lambda_M \) to the physical scale and dense loop corrections generated by the flow. At chiral restoration, it is this latter that signals the phase transition: The parametric pion decay constant with the scale fixed at the matching scale does not go to zero even at the chiral restoration point [2]. Thus when one does a higher-order chiral perturbation calculation of the same quantity, one has to be careful which quantity one is dealing with.

What one can say with some confidence is that (23) goes in the right direction in the context of BR scaling.

A variety of other evidences that lend, albeit indirect, support to the scaling [17] and in consequence to the notion of the VM are discussed in [21, 16]. If the VM were verified by going near the chiral transition point, it would constitute a nice illustration of how the mass of the hadrons making up the bulk of ordinary matter around us is made to “disappear,” a deep issue in physics [22].

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