Impact of RGE-induced $\mu - \tau$ Reflection Symmetry Breaking on the Effective Majorana Neutrino Mass in $0\nu\beta\beta$ Decay

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Abstract
We make an attempt to study the impact of renormalization-group equations (RGE) induced $\mu - \tau$ reflection symmetry breaking on the effective Majorana neutrino mass $|\langle m_{ee} \rangle|$ in neutrinoless double beta ($0\nu\beta\beta$) decay. At present, the $0\nu\beta\beta$ decay serves as a unique process to address the Majorana nature of massive neutrinos. The rate of such decay process depends on $|\langle m_{ee} \rangle|$. On the other hand, $\mu - \tau$ reflection symmetry predicts $\theta_{23} = 45^\circ$ and $\delta = \pm 90^\circ$ together with trivial values of the Majorana CP-phases ($\rho, \sigma$). Moreover, based on the recent global best-fit values which prefer higher octant of $\theta_{23}$ and third quadrant of $\delta$, it is hard to believe the exactness of such symmetry. Also, any non-trivial values of $\rho, \sigma$ may have some significant impact on $|\langle m_{ee} \rangle|$. In this context, we study the spontaneous breaking of the symmetry via one-loop RGE-running from a superhigh energy scale ($\Lambda_{\mu\tau}$) down to electroweak scale ($\Lambda_{EW}$). Given the broken symmetry, we perform some systematic analysis for $|\langle m_{ee} \rangle|$ in substantial details. Further, we also extend this analysis for other lepton-number violating effective Majorana masses.

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I. INTRODUCTION

Among a number of open questions in neutrino physics, the nature of neutrinos whether they are Majorana or Dirac fermions is yet unanswered. At the current juncture, the only appreciable process which can uncover the Majorana nature of neutrinos is the neutrinoless double beta ($0\nu\beta\beta$) decay \cite{2,3}. In this process, the following lepton-number violating decay takes place,

\[(Z, A) \longrightarrow (Z + 2, A) + 2e^-,\]

which offers a unique opportunity to test the violation of lepton-numbers by two-units. In the standard three neutrino framework, the decay rate of such process is controlled by the effective Majorana neutrino mass \(\langle m_{ee} \rangle = \sum_i |m_i U_{ei}^2|\) where \(U_{ei}\)’s (for \(i = 1, 2, 3\)) represent the elements of the first row of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix \cite{5}. Furthermore, the observation of $0\nu\beta\beta$ decay process can also provide some information about the absolute neutrino masses and constrain the Majorana CP-phases. At present, the most stringent upper bound on \(\langle m_{ee} \rangle\) arises from KamLAND-Zen collaboration \cite{6} and their latest data reports \(\langle m_{ee} \rangle < (0.061 - 0.165) \text{ eV}\), whereas Planck Collaboration \cite{7} gives bounds on absolute neutrino mass scale as \(\sum m_\nu < 0.12 \text{ eV} (95\%, \text{Planck TT, TE, EE + lowE + lensing + BAO})\).

On the other hand, theory behind the dynamical origin of neutrino mass and leptonic flavor mixing is yet need to be answered. Among a large varieties of theoretical models which address neutrino masses, the type-I seesaw mechanism \cite{8,11,12}, has been considered as the most simplest and elegant one. Furthermore, flavor symmetry has been very successful to explain the observed leptonic mixing pattern \cite{13,17}. In this context, the $\mu - \tau$ reflection symmetry which was originally proposed in Ref. \cite{18} (for latest review see Ref. \cite{19} and the references therein) leads to \(|U_{\mu i}| = |U_{\tau i}|\), (for \(i = 1, 2, 3\)) where \(U\) is the PMNS flavor mixing matrix receives numerous attention. This symmetry predicts the maximal values of the atmospheric mixing angle \(\theta_{23} = 45^\circ\) as well as the Dirac CP-phase \(\delta = \pm 90^\circ\) along with non-zero \(\theta_{13}\). Moreover, it also predicts the trivial Majorana CP-phases \(\rho, \sigma = 0^\circ, 90^\circ\). Considering the latest global best-fit results which favors non-maximal \(\theta_{23}, \delta\) \cite{20,22}, it is hard to strict on the exactness of the symmetry. On the other hand, as the flavor symmetries

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1 In 1937, E. Majorana first hypothesized that a fermion can be its own antiparticle in Ref. \cite{1}.
2 The testing of Majorana’s theory was first suggested in Ref. \cite{4}.
are generally imposed at a superhigh energy scale to address tiny neutrino masses and their flavor mixing at low energies, the renormalization group equations (RGE) running effect may lead possible corrections and naturally break the exact symmetry. Indeed, in recent times breaking of the $\mu - \tau$ reflection symmetry via quantum corrections achieve lots of attention [23–30]. Recently, the impact of such symmetry for the upcoming long baseline neutrino oscillation experiment has been addressed in Ref.[31].

In this work, in best of our knowledge for the first time, we make an attempt to study the impact of RGE-induced $\mu - \tau$ reflection symmetry breaking on $0\nu\beta\beta$ decay. We implement the concerned symmetry at the superhigh energy scale $\Lambda_{\mu\tau}$ and analyze the RGE-triggered quantum corrections all the way from $\Lambda_{\mu\tau}$ down to the electroweak (EW) scale $\Lambda_{EW}$. Keeping the current global best-fit result in mind which prefers $\theta_{23} > 45^\circ$ and $\delta < 270^\circ$, we study correlation among this two parameters due to RGE corrections at low energies. Further, the Majorana nature of massive neutrinos which also suggests the existence of Majorana CP-phases have some significant impact on $0\nu\beta\beta$ decay. Note that as the Majorana CP-phases $(\rho, \sigma)$ take fixed values (i.e., $0^\circ$ or $90^\circ$) at the energy scale $\Lambda_{\mu\tau}$, depending on their initial choices at $\Lambda_{\mu\tau}$ one would expect their distinct behavior at low energies due to quantum corrections. Therefore, we also perform correlation study between the Majorana CP-phases $(\rho, \sigma)$ at low energies. Further, we systematically study the impact of RGE-induced symmetry breaking on $0\nu\beta\beta$ decay for the different boundary values of $\rho, \sigma$ in substantial details. We show our result in the two dimensional conventional Vissani graph in $(m_{\text{light}}, |\langle m_{ee} \rangle|)$ plane [32]. We also extend our analysis for the remaining effective Majorana neutrino masses which may appear in different lepton-number violating (LNV) processes.

We outline this work as follows: in next Sec. (II), we give a description of the $\mu - \tau$ reflection symmetry and describe its breaking considering one-loop renormalization-group equations. In Sec. (III), we present a detailed set-up of our numerical procedure. Sec. (IV) is devoted to our results where in first sub-Sec. (IVA), we perform correlation studies considering the different parameters which are fixed by the symmetry at $\Lambda_{\mu\tau}$. Later, in sub-Sec. (IVB), our results corresponding to $0\nu\beta\beta$ decay has been addressed. We discuss various LNV processes other than $|\langle m_{ee} \rangle|$ in sub-Sec. (IVC). Finally, we summarize our conclusion in Sec. (V).
II. SPONTANEOUS BREAKING OF $\mu - \tau$ REFLECTION SYMMETRY

Let us consider the following transformations of the neutrino fields:

$$
\nu_{L,e} \leftrightarrow \nu_{R,e}^c, \quad \nu_{L,\mu} \leftrightarrow \nu_{R,\tau}^c, \quad \nu_{L,\tau} \leftrightarrow \nu_{R,\mu}^c,
$$

(2)

where $\nu_{L,\alpha}$'s (for $\alpha = e, \mu, \tau$) are the left-handed neutrino fields in the flavor basis, and $\nu_{R,\alpha}^c$'s are the right-handed neutrino charge-conjugated fields of $\nu_{R,\alpha}$'s. These transformations lead to the effective Majorana neutrino mass matrix of the form,

$$
M_\nu = \begin{pmatrix}
\langle m \rangle_{ee} & \langle m \rangle_{e\mu} & \langle m \rangle_{e\mu}^* \\
* & \langle m \rangle_{\mu\mu} & \langle m \rangle_{\mu\tau} \\
* & * & \langle m \rangle_{\mu\mu}^*
\end{pmatrix}.
$$

(3)

Note that the different entries of the most general Majorana neutrino mass matrix obey following equalities,

$$
\langle m \rangle_{ee} = \langle m \rangle_{ee}^*, \quad \langle m \rangle_{e\mu} = \langle m \rangle_{e\tau}^*, \quad \langle m \rangle_{\mu\tau} = \langle m \rangle_{\mu\tau}^*, \quad \langle m \rangle_{\mu\mu} = \langle m \rangle_{\tau\tau}^*.
$$

(4)

The Majorana neutrino mass matrix $M_\nu$ can be diagonalized as $U^\dagger M_\nu U^* = m_\nu^d = \text{diag}\{m_1, m_2, m_3\}$. In the standard PDG parameterization, the unitary mixing matrix $U = P_l V P_\nu$ can be decomposed as

$$
U = P_l \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\
 s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23}
\end{pmatrix} P_\nu,
$$

(5)

where $c_{ij}(s_{ij})$ (for $i < j = 1, 2, 3$) stands for $\cos \theta_{ij}(\sin \theta_{ij})$, $P_l = \text{diag}\{e^{i\phi_e}, e^{i\phi_\mu}, e^{i\phi_\tau}\}$ contains three unphysical phases which can be absorbed by the rephasing of charged lepton fields, and $P_\nu = \text{diag}\{e^{i\rho}, e^{i\sigma}, 1\}$ is the diagonal Majorana phase matrix. Given the form of $M_\nu$ in eq. (3), we find six predictions for the elements of the unitary matrix $U$, namely,

$$
\theta_{23} = 45^\circ, \quad \delta = \pm 90^\circ, \quad \rho, \quad \sigma = 0 \text{ or } 90^\circ, \phi_e = 90^\circ, \quad \phi_\mu = -\phi_\tau.
$$

(6)

A detailed description of $\mu - \tau$ reflection symmetry with the proper phase convention has been discussed in Ref. [28].

Having discussed the framework of $\mu - \tau$ reflection symmetry, we now proceed to describe the breaking of $\mu$-$\tau$ reflection symmetry due to RGE-running in the context of the minimal
supersymmetric standard model (MSSM) \(^3\). In this study, we introduce the concerned flavor symmetry at superhigh energy scale \(\Lambda_{\mu\tau} \equiv 10^{14}\) GeV which is much higher compared to electroweak (EW) scale \(\Lambda_{EW} \sim 10^2\) GeV. Therefore, one must consider the effect of RGE-running while addressing the neutrino oscillation phenomenology at low energies. It is relevant to scrutinize such effect because during the RGE-running process the Yukawa coupling corresponding to \(\mu\) and \(\tau\) may gain significant difference and later this can severely impact the breaking of \(\mu - \tau\) reflection symmetry.

The energy dependence of neutrino mass matrix \(M_\nu\) is expressed by its RGE-running equation, which at the one-loop level can be written as \[^{[33-36]}\]

\[
16\pi^2 \frac{dM_\nu}{dt} = C \left(Y_l^\dagger Y_l\right)^T M_\nu + CM_\nu \left(Y_l^\dagger Y_l\right) + \alpha M_\nu .
\]

(7)

Note here that \(t\) stands for \(\ln(\mu/\mu_0)\) while \(\mu\) signifies the renormalization scale, whereas \(C\) and \(\alpha\) in the MSSM can be read as

\[
C = 1 , \alpha \simeq -\frac{6}{3} g_1^2 - 6 g_2^2 + 6 y_t^2 .
\]

(8)

In the diagonal basis of the charged lepton Yukawa coupling matrix, we have \(Y_l = \text{diag}\{y_e, y_\mu, y_\tau\}\). In the limit \(y_e \ll y_\mu \ll y_\tau\), one can rule out the contributions of \(y_e\) and \(y_\mu\) compared to \(y_\tau\). The evolution of \(M_\nu\) due to the one-loop RGE-running from high energy scale \(\Lambda_{\mu\tau}\) down to \(\Lambda_{EW}\) can be expressed as \(^{[37-39]}\)

\[
M_\nu(\Lambda_{EW}) = I_\alpha I_\tau^\dagger M_\nu(\Lambda_{\mu\tau}) I_\tau^* ,
\]

(9)

where one defines \(I_\tau \simeq \text{diag}\{1, 1, 1 - \Delta_\tau\}\) along with

\[
I_\alpha = \exp \left( \frac{1}{16\pi^2} \int_{\ln \Lambda_{\mu\tau}}^{\ln \Lambda_{EW}} \alpha \ dt \right) , \quad \Delta_\tau = \frac{C}{16\pi^2} \int_{\ln \Lambda_{\mu\tau}}^{\ln \Lambda_{EW}} y_\tau^2 \ dt .
\]

(10)

Here, \(y_\tau^2 = (1 + \tan^2 \beta) m_\tau^2 / v^2\) in the MSSM with \(v \approx 174\) GeV being the Higgs vacuum expectation value. We notice that \(\Delta_\tau\) depends on \(\tan \beta\) which eventually determines the strength of symmetry breaking pattern. To observe a reasonable amount of deviation from the symmetry, we fix \(\tan \beta = 30\) throughout this work. Further, diagonalization of \(M_\nu\) leads

\[^{3}\text{Note that we consider MSSM as our theoretical framework at high energies which can serve as a possible ultraviolet extension of the Standard Model.}\]
to the approximate expressions of light neutrino masses at low energies as

\[ m_1(\Lambda_{EW}) \simeq m_1(\Lambda_{\mu\tau})[1 - \Delta_\tau(1 - c_{12}^2 c_{13}^2)] I_\alpha, \]

\[ m_2(\Lambda_{EW}) \simeq m_2(\Lambda_{\mu\tau})[1 - \Delta_\tau(1 - s_{12}^2 c_{13}^2)] I_\alpha, \]

\[ m_3(\Lambda_{EW}) \simeq m_3(\Lambda_{\mu\tau})[1 - \Delta_\tau c_{13}^2] I_\alpha. \] (11)

Moreover, the different flavor mixing angles at low energies can be given by

\[ \theta_{12}(\Lambda_{EW}) \simeq \theta_{12}(\Lambda_{\mu\tau}) + \frac{\Delta_\tau}{2} s_{12}^2 (\zeta_{31} - \zeta_{32}) + c_{12}^4 \zeta_{31} - c_{12}^2 \zeta_{32} s_{12}^2, \]

\[ \theta_{13}(\Lambda_{EW}) \simeq \theta_{13}(\Lambda_{\mu\tau}) + \frac{\Delta_\tau}{2} [c_{12} s_{31}^2 + s_{12} c_{32}^2] \zeta_{13} s_{13}, \]

\[ \theta_{23}(\Lambda_{EW}) \simeq \theta_{23}(\Lambda_{\mu\tau}) + \frac{\Delta_\tau}{2} [s_{12}^2 \zeta_{31} - c_{12}^2 \zeta_{32}^2], \] (12)

whereas one can find three CP-phases at low energies as

\[ \delta(\Lambda_{EW}) \simeq \delta(\Lambda_{\mu\tau}) + \frac{\Delta_\tau}{2} \frac{s_{12} c_{12} s_{13}^2}{s_{13}} (\zeta_{31} - \zeta_{32}^2) - \frac{s_{13}}{c_{12} s_{12}} (c_{12}^4 \zeta_{31} - s_{12}^2 \zeta_{31}^2 + \zeta_{21}^2), \]

\[ \rho(\Lambda_{EW}) \simeq \rho(\Lambda_{\mu\tau}) + \frac{\Delta_\tau}{2} s_{12} c_{12} s_{13}^2 [s_{12}^2 (\zeta_{31} - \zeta_{32}) + \frac{1}{2} (\zeta_{31} - \zeta_{21}^2)], \]

\[ \sigma(\Lambda_{EW}) \simeq \sigma(\Lambda_{\mu\tau}) + \frac{\Delta_\tau}{2} [s_{12} c_{12} s_{13}^2 (\zeta_{31} - \zeta_{32}) - c_{12}^2 (2\zeta_{32} - \zeta_{31} - \zeta_{21}^2)]. \] (13)

Here, \( \eta_\rho = \cos 2\rho = \pm 1 \) and \( \eta_\sigma = \cos 2\sigma = \pm 1 \) represents the choices of \( \rho, \sigma \) at high energies whereas \( \zeta_{ij} \) with \( i, j = 1, 2, 3 \) are defined at low energies as \( \zeta_{ij} = (m_i - m_j)/(m_i + m_j) \).

## III. NUMERICAL SET-UP

We illustrate our numerical procedure that has been carried out throughout this work in this section. By adopting the framework of MSSM, we study the RGE-running effect from \( \Lambda_{\mu\tau} \) down to \( \Lambda_{EW} \). In our numerical analysis, we set \( \tan \beta = 30 \), and the high and low energy boundary scales are fixed to be \( \Lambda_{\mu\tau} = 10^{14} \) GeV and \( \Lambda_{EW} = 91 \) GeV, respectively. As the \( \mu-\tau \) reflection symmetry predicts maximal value of the atmospheric mixing angle, \( \theta_{23} \), and the Dirac CP-phase, \( \delta \), along with the trivial values of Majorana CP-phases, \( \rho, \sigma \) (0°, 90°), we consider here four different scenarios, namely, (i) **S1**, \( \rho = 0^\circ, \sigma = 0^\circ \), (ii) **S2**, \( \rho = 90^\circ, \sigma = 90^\circ \), (iii) **S3**, \( \rho = 0^\circ, \sigma = 90^\circ \) and (iv) **S4**, \( \rho = 90^\circ, \sigma = 0^\circ \). Note that for all these four scenarios, we fix \( \theta_{23} = 45^\circ \) and \( \delta = 270^\circ \) whereas remaining oscillation parameters (namely, \( \sin^2 \theta_{12}, \sin^2 \theta_{13}, \Delta m_{21}^2, \Delta m_{31}^2 \)) are scanned over wide ranges with the help of the
nested sampling package MultiNest program \cite{40} \cite{42} at $\Lambda_{\mu\tau}$ \footnote{Note that the $\mu - \tau$ reflection symmetry also allows $\delta = 90^\circ$, however current global-fit seems to favor $\delta = 270^\circ$ \cite{20} \cite{22}, hence we perform this study for the latter scenario.}. We define the Gaussian-$\chi^2$ function that has been considered in numerical scan as,

$$\chi^2 = \sum_i \frac{[\xi_i - \bar{\xi}_i]^2}{\sigma_i^2},$$

(14)

where $\xi_i$ represents the neutrino oscillation parameters i.e., $\xi_i = \{\theta_{12}, \theta_{13}, \theta_{23}, \delta, \Delta m_{21}^2, \Delta m_{31}^2\}$ at $\Lambda_{EW}$. Also, $\bar{\xi}_i$ stands for the best-fit values from the recent global-fit results \cite{43}, while $\sigma_i$ signifies the symmetrized 1σ errors. The best-fit values and the corresponding 1σ errors that we have considering in our numerical simulations \cite{43} are $\sin^2 \theta_{12} = 0.304^{+0.014}_{-0.013}, \sin^2 \theta_{13} = 0.0214^{+0.0009}_{-0.0007}, \sin^2 \theta_{23} = 0.551^{+0.019}_{-0.070}, \delta = 1.32^{+0.23}_{-0.18}\pi, \Delta m_{21}^2 = 7.34^{+0.17}_{-0.14} \times 10^{-5}\text{eV}^2, \Delta m_{31}^2 = 2.455^{+0.035}_{-0.032} \times 10^{-3}\text{eV}^2$. As the current neutrino oscillation data favors normal neutrino mass ordering (i.e., $m_1 < m_2 < m_3$) with more than 3σ C.L. \cite{20} \cite{22} over inverted neutrino mass ordering (i.e., $m_3 < m_1 \sim m_2$), we concentrate this study considering the former mass ordering. Also, in this study the smallest neutrino mass $m_1$ is allowed to vary in the range $[0, 0.2]\text{eV}$. Based on our numerical analysis, in next section we describe our results considering correlation between $\theta_{23}, \delta$ as well as $\rho, \sigma$ at low energies which arises due to RGE-triggered $\mu - \tau$ symmetry breaking. Further, we proceed to discuss their impacts on the different Majorana neutrino masses in $0\nu\beta\beta$ decay and LNV processes.

IV. RESULTS

A. Correlation Study

By investigating global-fit of neutrino oscillation data as well as the predictions of the $\mu - \tau$ reflection symmetry, we choose initial value of $\theta_{23} = 45^\circ, \delta = 270^\circ$ at superhigh energy scale. On the other hand, there are no such preference for the Majorana phases as there are no experimental results. However, concerned symmetry predicts that both the Majorana phases can either be $0^\circ$ or $90^\circ$. Thus, we perform this analysis for both the choices. The effective Majorana neutrino mass $|\langle m \rangle_{ee}|$ which can be tested at $0\nu\beta\beta$ experiments can behave very differently depending on the values of different Majorana CP-phases. In this context, $\theta_{13} \rightarrow 0$ limit is a good approximation to have analytical understanding of $|\langle m \rangle_{ee}|$
(which we later mention in eq.\([17]\)). We notice that in this limit \(|\langle m_{ee}\rangle|\) depends on CP-phases \(\rho\) and \(\sigma\), whereas dependency on \(\delta\) can be neglected\(^5\). However, RGE-running effect of \(\delta\) also plays some significant role for the remaining effective Majorana masses. Therefore, we study its behavior considering two less known parameters \(\theta_{23}, \delta\) in \((\theta_{23}, \delta)\) plane.

![Correlation plots between \(\theta_{23}, \delta\) at \(\Lambda_{EW}\). Here, we show four different cases with possible initial values of \((\rho, \sigma)\), as shown by the plots label at \(\Lambda_{\mu\tau}\), whereas \(\theta_{23} = 45^\circ, \delta = 270^\circ\) are adopted at \(\Lambda_{\mu\tau}\) for all cases.](image)

\(^5\) Note that throughout this work we have numerically analyzed RGE-running behavior of all the oscillation parameters, however cancellation among different components of the effective Majorana masses only arise from relative phase factors. Hence, we mainly concentrate on the behavior of different CP-phases.
gray-scatter points of the figure have $\chi^2 < 30$. The results describe in fig. 1 are in well agreement with the analytical expressions as mentioned in the third line of eq. (12) and the first line of eq. (13) for $\theta_{23}$ and $\delta$, respectively. One can notice from these equations that because of the factor $\Delta_\tau$ which eventually depends on $\tan \beta$, the difference of the parameters (i.e., $x(\Lambda_{\text{EW}}) - x(\Lambda_{\mu\tau}) \neq 0$ for $x = \theta_{23}, \delta$) can never be zero. Thus, from all the panels of fig. 1 we observe that in none of the scenarios scatter curve touches $(\theta_{23}, \delta) = (45^\circ, 270^\circ)$ which are fixed by the symmetry at high energies. Further, by inspecting all scenarios, we notice that after the breaking of the symmetry, $\theta_{23}$ tends to lie always in the higher octant which is in well agreement with the latest global-fit data. However, the amount of deviation from its initial value are different for different initial boundary values of $\rho, \sigma$ at $\Lambda_{\mu\tau}$. This can also be understood from the approximate formula of $\theta_{23}$ (see the third line of eq. (12)).

As one can notice that $O(\Delta_\tau)$ term depends on different CP-phase factors (i.e., $\zeta_{31}^{-\eta_\nu}, \zeta_{32}^{-\eta_\nu}$) which adds very distinct contribution to $\theta_{23}$ depending on the initial options of $\rho, \sigma$. We notice that the current best-fit value of $\theta_{23} \simeq 48^\circ$ can be reached by the scenarios $S_1, S_4$ as shown by the first and the last panels, respectively. Also in both this scenarios $\theta_{23}$ can reach value as large as $52^\circ$.

Considering the RGE-running effect on $\delta$, we also observe distinct running behavior of $\delta$ for four initial options of $\rho, \sigma$ at high energies. The behavior for distinct deviations remains same as explained for $\theta_{23}$ and which is apparent from the first line of eq. (13). We find $O(1^\circ)$ deviations of $\delta$ from its maximal value for both the scenarios of the top row, whereas deviations of $O(90^\circ)$ have been noted for both the cases of the bottom row. We also notice that deviations of $\delta$ can reach its current best-fit value of $\delta \simeq 238^\circ$ in both the panels of the bottom row. Furthermore, comparing the top and the bottom row, we observe that the current best-fit value of $(\theta_{23}, \delta) \simeq (48^\circ, 238^\circ)$ is achievable only for the scenario $S_4$.

Similarly, in fig. 2, we describe the correlation between $\rho, \sigma$ at low energies for four different initial choices of $\rho, \sigma$ at $\Lambda_{\mu\tau}$ (see figure label for details). By inspecting both the scenarios of the top row where initial boundary values of $\rho, \sigma$ are chosen to be same, such as $0^\circ$ (left panel), $90^\circ$ (right panel), we observe a maximum deviation of $O(2^\circ)$ for $\rho$, whereas a deviation of less than $O(1^\circ)$ has been identified for $\sigma$. However, a noticeable deviation of $\rho, \sigma$ from their initial values have been observed from the bottom row where both the Majorana phases take distinct boundary values. We find deviations of $O(90^\circ)$ for $\rho$, and $O(15^\circ)$ for $\sigma$ from both the scenarios, $S_3$ and $S_4$. Similar to the radiative correction of $\theta_{23}, \delta,$
FIG. 2: Correlation plots between $\rho, \sigma$ at $\Lambda_{\text{EW}}$. We show four different cases with possible initial values of $(\rho, \sigma)$, as shown by the plots label at $\Lambda_{\mu\tau}$, whereas $\theta_{23} = 45^\circ, \delta = 270^\circ$ are adopted at $\Lambda_{\mu\tau}$ for all cases.

The numerical pattern of $\rho, \sigma$ due to RGE-running can also be understood from their analytical results as given by eq.(13). We notice very modest correction to $\rho, \sigma$ from the upper row of fig.(2). To explain this, we observe $\zeta_{31}^{-\eta_{\rho}}, \zeta_{32}^{-\eta_{\eta}}$ phase dependency in $\mathcal{O}(\Delta r)$ term of eq.(13) for both $\rho, \sigma$. One finds a large cancellation between the terms $\zeta_{31}^{-\eta_{\rho}}, \zeta_{32}^{-\eta_{\eta}}$ when CP-phases take same values at high energies. Therefore, we notice a very small correction from the upper row, whereas a significant amount of radiative correction has been observed from the bottom row where both $\rho, \sigma$ take different initial boundary values as this leads very small cancellation among the different phase factors. From this figure, we also notice that due to correction term $\mathcal{O}(\Delta r)$ the initial value of $\rho, \sigma$ at $\Lambda_{\mu\tau}$ will never be same at $\Lambda_{\text{EW}}$, in other words $x(\Lambda_{\text{EW}}) - x(\Lambda_{\mu\tau}) \neq 0$ for $x = \rho, \sigma$. This is apparent from the scenarios, S1 and S2.
as shown by the top row. However, due to large deviation this behavior is not apparent for S3 and S4 as described by the bottom row but our careful zooming-in analysis shows similar non-zero deviation for both the panels of the bottom row. From the above discussion it is now quite apparent that depending on the initial values of the Majorana phases their behavior at low energies are very distinct. This may lead some significant impact on the effective Majorana neutrino mass matrix elements. In next subsections, we first concentrate our study on \(|\langle m \rangle_{ee}|\) and proceed further to discuss other \(|\langle m \rangle_{\alpha\beta}|\) with \(\alpha, \beta = e, \mu, \tau\) which may appear in some other lepton-number violating processes.

**B. Impact on 0\(\nu\beta\beta\) Decay**

In this subsection, we scrutinize the impact of RGE-triggered symmetry breaking effect on neutrinoless double beta (0\(\nu\beta\beta\)) decay experiments. At the current juncture, 0\(\nu\beta\beta\) decay is the only feasible process which can address the issue that whether the massive neutrinos are the Majorana fermions. This kind of decay process violate lepton number by two-units and the half-life of such decay process can be written as \[44, 45\],

\[
(T_{1/2}^{0\nu})^{-1} = G_{0\nu}|M_{0\nu}(A, Z)|^2 |\langle m \rangle_{ee}|^2 ,
\]  

(15)

where \(G_{0\nu}\) stands for two-body phase-space factor, \(M_{0\nu}\) is the nuclear matrix element (NME), whereas \(|\langle m \rangle_{ee}|\) represents the effective Majorana neutrino mass \(^6\). The expression of \(|\langle m \rangle_{ee}|\) is given by,

\[
|\langle m \rangle_{ee}| = |m_{ee}| = \left| \sum_{i=1}^{3} m_i U_{ei}^2 \right| ,
\]  

(16)

where \(U\) stands for PMNS mixing matrix as mentioned in eq. (5). In the standard PDG parameterization one can read \(|m_{ee}|\) as,

\[
|m_{ee}| = |m_1 c_{12}^2 c_{13}^2 e^{2i\rho} + m_2 s_{12}^2 c_{13}^2 e^{2i\sigma} + m_3 s_{13}^2 e^{-2i\delta}| \]
\[
= |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i(\sigma-\rho)} + m_3 s_{13}^2 e^{-2i(\delta+\rho)}| .
\]  

(17)

Here, \(c_{ij}(s_{ij})\) represents \(\cos \theta_{ij}(\sin \theta_{ij})\), which are the leptonic mixing angles while \(\delta\) stands for the Dirac CP-phase and, \(\rho, \sigma\) signify Majorana CP-phases. In principle, we can see from the last line of eq. (17) that \(|m_{ee}|\) depends on two effective phases. As the neutrino

\(^6\) Note that now onwards, we adopt the notation \(|\langle m \rangle_{\alpha\beta}| = |m_{\alpha\beta}|\) for \(\alpha, \beta = e, \mu, \tau\) throughout this work.
oscillation data provides information about the mass-squared differences $\Delta m^2_{21}$, $\Delta m^2_{31}$ but not about the absolute neutrino masses, we define masses $m_2$, $m_3$ in terms of the lightest neutrino mass $m_1$ as $m_2 = \sqrt{m_1^2 + \Delta m^2_{21}}$ and $m_3 = \sqrt{m_1^2 + 0.5\Delta m^2_{21} + \Delta m^2_{31}}$ for the normal mass ordering $^7$.

FIG. 3: Prediction for the effective Majorana neutrino mass $m_{ee}$ which is involved in $0\nu\beta\beta$ decay where four panels represent different initial choices of $\rho, \sigma$. The most stringent upper bound on $|m_{ee}|$ from KamLAND-Zen collaboration are shown by gray-horizontal band. The latest result on lightest neutrino mass is shown by gray-vertical band from Planck Collaboration which gives $\sum m_\nu < 0.12$ eV at the 95% C.L.

The different experiments which are looking for the signature of neutrinoless double beta ($0\nu\beta\beta$) decay are GERDA Phase II $^{16}$, CUORE $^{17}$, SuperNEMO $^{18}$, KamLAND-Zen

$^7$ In this study, we define $\Delta m^2_{21} = m_2^2 - m_1^2$ and $\Delta m^2_{31} = m_3^2 - 0.5(m_1^2 + m_2^2)$. 
At present, the most stringent upper bound on the effective Majorana neutrino mass $|m_{ee}|$ arises from KamLAND-Zen experiment [6]. Their latest results have reported the bound $|m_{ee}| < (0.061 - 0.165) \text{ eV}$ by taking into account the uncertainty in the estimation of the nuclear matrix elements.

In fig.(3), we describe our results of $0\nu\beta\beta$ decay for the four boundary values of Majorana phases in $(m_1, |m_{ee}|)$ plane. This results show the pattern of the effective Majorana neutrino mass $|m_{ee}|$ due to RGE-triggered breaking of the $\mu - \tau$ symmetry at low energies. The latest results on $|m_{ee}|$ from KamLAND-Zen collaboration are shown by gray-horizontal band in each scenario. On the other hand, the current results reported by the Planck Collaboration [7] gives $\sum m_{\nu} < 0.12 \text{ eV} (95\%, \text{Planck TT, TE, EE} + \text{lowE} + \text{lensing} + \text{BAO})$. Thus, an upper bound on lightest neutrino mass has been established as shown by gray-vertical band. Clearly, we notice that depending on the initial values of Majorana phases one obtains different results of $|m_{ee}|$ at low energies.

The expression of $|m_{ee}|$ at $\Lambda_{\mu\tau}$ for four different initial values of $\rho, \sigma$ is given by

$$|m_{ee}|(\Lambda_{\mu\tau}) = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13} - m_3 s_{13}^2|; \quad \text{for } \rho = 0^\circ, \sigma = 0^\circ,$$

$$= |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13} + m_3 s_{13}^2|; \quad \text{for } \rho = 90^\circ, \sigma = 90^\circ,$$

$$= |m_1 c_{12}^2 c_{13}^2 - m_2 s_{12}^2 c_{13} - m_3 s_{13}^2|; \quad \text{for } \rho = 0^\circ, \sigma = 90^\circ,$$

$$= |m_1 c_{12}^2 c_{13}^2 - m_2 s_{12}^2 c_{13} + m_3 s_{13}^2|; \quad \text{for } \rho = 90^\circ, \sigma = 0^\circ. \quad (18)$$

Here, we kept $\delta = 270^\circ$ for all cases.

As the cancellation among the various terms of $|m_{ee}|$ depends on CP-phases, their behavior at low energies play very vital role to understand the numerical results of fig.(3). In the limit $\theta_{13} \to 0$ which implies $s_{13} \to 0, c_{13} \to 1$, in addition if one finds the deviation of $\rho, \sigma$ less than $O(1^\circ)$ (as shown by the top row of fig.2) then there will not have any significant cancellation among the different components of $|m_{ee}|$. This is apparent from the first two lines of the analytical expressions of eq.(18). Note that in numerical analysis, we do not adopt any approximation. Thus, from both the panels of the top row of fig.(3), we notice that minimum of $|m_{ee}|$ never approaches to zero. We find that $|m_{ee}|$ can reach around $\sim 1.5 \text{ meV, } \sim 6 \text{ meV}$ for $m_1 \to 0.1 \text{ meV}$ in the left and right panel, respectively. On the other hand, experimentally constraint upper limits of $|m_{ee}|$ can achieve value $\sim 40 \text{ meV}$.

We now proceed to elaborate our results for the second row of fig.(3). Together with $\theta_{13} \to 0$ limit, if one consider quasi-degenerate scenario (i.e., $m_1 \approx m_2 \approx m_3$) then there
could be possible cancellation among the different terms of \(|m_{ee}|\) as it is apparent from the last two expressions of eq.\((18)\). This is exactly what we notice from the bottom row where \(|m_{ee}| \to 0\) for higher values of \(m_1\). In other words, one can notice a two dimensional “well” in Vissani graph for the normal neutrino mass ordering in both the scenarios \(S3, S4\). As we notice from the bottom row of fig.\((2)\) where \(\rho, \sigma\) take distinct boundary values, the RGE-running makes significant contribution on CP-phases which allows some major cancellation among various components of \(|m_{ee}|\) and leads \(|m_{ee}| \to 0\) for some specific range of lightest neutrino mass. One can make a noteworthy outcome that as the latest neutrino oscillation data favors normal neutrino mass ordering and if this turns out to be a true mass spectrum then this result will play an important role to rule out or verify the result of 0\(\nu\beta\beta\) decay experiments in the standard three-neutrino paradigm. On the other hand, this results will also help us to constrain or determine both the Majorana phases \(\rho, \sigma\).

C. The effective Majorana Neutrino Masses

We extend this study for the remaining effective Majorana neutrino masses \(|m_{\alpha\beta}|\) for \(\alpha, \beta = e, \mu, \tau\) with \(\alpha = \beta \neq e\). It’s important to study \(|m_{\alpha\beta}|\) as the number of unknowns involving \(|m_{ee}|\) cannot simply be addressed in 0\(\nu\beta\beta\) decay\(^8\). Further, if 0\(\nu\beta\beta\) decay experiments observe null results then one needs to extend their search for other lepton-number violating (LNV) processes to address the Majorana nature of neutrinos. Moreover, the LNV processes involving the effective Majorana neutrino masses can play an important role in the decay rates of \(H^{++} \to l^{+}_{\alpha}l^{+}_{\beta}\) and \(H^{+} \to l^{+}_{\alpha}\bar{\nu} [44, 50, 51]\), in the neutrino-antineutrino oscillation probabilities [50, 52], and in rare mesons decay involving \(B\) and \(D\) mesons [44, 53].

We list the mathematical form of the effective Majorana neutrino mass matrix elements considering the concerned form of PMNS mixing matrix as given by eq.\((5)\) at high energies as

\[
\sqrt{2}|m_{e\mu}| = |\bar{m}_1 c_{12} c_{13} (s_{12} - i c_{12} s_{13}) - \bar{m}_2 s_{12} c_{13} (c_{12} + i s_{12} s_{13}) - i m_3 c_{13} s_{13}|, \\
2|m_{\mu\mu}| = |\bar{m}_1 (s_{12} - i c_{12} s_{13})^2 + \bar{m}_2 (c_{12} + i s_{12} s_{13})^2 + m_3 c_{13}^2|, \\
2|m_{\mu\tau}| = |\bar{m}_1 (s_{12}^2 + c_{12} s_{13}^2) + \bar{m}_2 (c_{12}^2 + s_{12} s_{13}^2) - m_3 c_{13}^2|, \\
\tag{19}
\]

\(^8\) Note that the unknowns in \(|m_{ee}|\) involves the absolute neutrino mass scale, correct mass ordering and the Majorana CP-phases.
FIG. 4: Prediction for the effective Majorana masses $m_{\alpha\beta}$. The gray, black, light-blue, dark-blue patterns show the behavior of $m_{\alpha\beta}$ for different initial choices of $\rho, \sigma$ (see figure label for details). Here, the latest result on lightest neutrino mass is shown by gray-vertical band from Planck Collaboration which gives $\sum m_\nu < 0.12$ eV at the 95% C.L.

where $m_1 = \pm m_1$ and $m_2 = \pm m_2$ with ‘±’ stands for $\rho, \sigma = 0^\circ$ or $90^\circ$. Note that the remaining elements of $M_\nu$ satisfy the symmetry equations as mentioned in eq.(4). Fig.(4) illustrates our results where the impact of spontaneous breaking of the concerned symmetry are introduced on $|m_{\alpha\beta}|$. We show the latest Planck [7] bounds on lightest neutrino mass by gray-vertical band at the 95% C.L. Note that the current upper bounds on this five $|m_{\alpha\beta}|$ can reach multi-MeV to TeV [44, 53]. One expects, the experimental sensitivity of some of the effective Majorana masses would improve to sub-eV level in near future.

In fig.(4), the different color patterns like gray, black, light-blue, and dark-blue describe different boundary values of $\rho, \sigma$ i.e., $(0^\circ, 0^\circ)$, $(90^\circ, 90^\circ)$, $(0^\circ, 90^\circ)$ and $(90^\circ, 0^\circ)$, at high energies, respectively. We notice that these scenarios are indistinguishable for $m_1 < 0.01$
eV, in other words the impact of RGE remains insignificant when \( m_1 \) lies below 0.01 eV. However, one can distinguish them when \( m_1 \) lies in \( 0.01 \text{ eV} \leq m_1 \leq 0.04 \text{ eV} \) which also falls in the latest experimentally allowed regime. By inspecting all the different cases, we notice that in none of the scenarios significant cancellation among different terms of \( |m_{\alpha\beta}| \) occur which may lead \( |m_{\alpha\beta}| \rightarrow 0 \) except for \( |m_{\tau\tau}| \) with some specific values of \( \rho, \sigma \) (see gray curve) but the latest constrain on \( m_1 \) rule out that possibility. However, we find the effective Majorana masses like, \( |m_{\mu\mu}|, |m_{\mu\tau}|, |m_{\tau\tau}| \) take values as large as \( \sim 80 \text{ meV} \) for \( m_1 \rightarrow 0 \), on the other hand channels like, \( |m_{e\mu}|, |m_{e\tau}| \) constrains themselves around \( \sim 8 \text{ meV} \) in the limit \( m_1 \rightarrow 0 \). Further, we also notice very modest impact of RGE-induced symmetry breaking on \( |m_{e\mu}|, |m_{e\tau}| \) as they show almost same patterns.

V. CONCLUSION

Numerous neutrino oscillation as well as non-oscillation experiments have opened a new window to probe some intriguing fundamental properties of neutrinos. At the same time, it is also interesting to look for various models which may strengthen our theoretical understanding. However, in near future with more and more data from both oscillation and non-oscillation sectors will help us to verify some definite predictions of different models or rule out some specific models.

In this work, we mainly concentrate on the impact of RGE-induced breaking of the \( \mu - \tau \) reflection symmetry in \( 0\nu\beta\beta \) decay. The experimental result of \( 0\nu\beta\beta \) decay would allow us to establish the Majorana nature of massive neutrinos. Thus, in current time searching for the signal of \( 0\nu\beta\beta \) decay becomes one of the most important task for the non-oscillation experiments in neutrino physics. The decay rate of such process depends on the effective Majorana neutrino mass \( |m_{ee}| \) involving Majorana phases \( (\rho, \sigma) \). Therefore, it is very significant to study \( |m_{ee}| \) which can put bounds on \( \rho, \sigma \). Besides this, the \( \mu - \tau \) reflection symmetry predicts \( \theta_{23} = 45^\circ \) and \( \delta = \pm 90^\circ \) which are in good agreement with global-fit data along with the trivial values of \( \rho, \sigma \) (i.e., \( 0^\circ, 90^\circ \)). However, in order to explain the low-energy neutrino oscillation data, one imposes such symmetry at a superhigh energy scale. In this scenario, it is necessary to consider the RGE-induced quantum corrections which naturally breaks the exact symmetry and leads some interesting aspects in neutrino oscillation data. This also brings some noteworthy correction to the Majorana phases which
has very significant impact on $|m_{ee}|$. In this prospect, we make an attempt to explore some salient features of the effective Majorana neutrino mass $|m_{ee}|$.

In this study, considering the broken scenario, we perform various correlation study among the parameters (i.e., $\theta_{23}, \delta, \rho, \sigma$) which are fixed by the symmetry at high energies for the normal neutrino mass ordering. We adopt four different scenarios depending on the initial options of $\rho, \sigma$ at high energies (i.e., (i) $S_1, \rho = 0^\circ, \sigma = 0^\circ$, (ii) $S_2, \rho = 90^\circ, \sigma = 90^\circ$, (iii) $S_3, \rho = 0^\circ, \sigma = 90^\circ$ and (iv) $S_4, \rho = 90^\circ, \sigma = 0^\circ$). We observe very distinct corrections to both $\theta_{23}, \delta$ at low energies depending on the initial values of $\rho, \sigma$. A deviation of $\sim O(6^\circ)$ from the maximal value of $\theta_{23}$ has been noticed for the scenarios $S_1, S_4$, whereas a minute correction of less than $O(1^\circ)$ has been registered for the scenarios $S_2, S_3$. Similarly, for the Dirac CP-phase $\delta$ a less than $O(1^\circ)$ has been observed for the scenarios $S_1, S_2$. However, a large deviation of $\sim O(90^\circ)$ has been found for the scenarios $S_3, S_4$. Further, we also notice that only the scenarios $S_4$ is able to reach latest best-fit values of both $\theta_{23}, \delta$. Now, inspecting the correlation between both the Majorana phases ($\rho, \sigma$), we draw following conclusions. We observe a correction of less than $O(2^\circ)$ for both $\rho, \sigma$ when they take same initial values at high energies which are explained by the scenarios $S_1, S_2$. Further, we find a deviation of $O(90^\circ), O(15^\circ)$ for $\rho, \sigma$, respectively from both the scenarios $S_3, S_4$.

Furthermore, by adopting the conventional Vissani graph, we illustrate our result for the effective Majorana neutrino mass in $(m_1, |m_{ee}|)$ plane. Depending on the initial choices of $\rho, \sigma$, we observe two distinct behavior of $|m_{ee}|$. Our numerical analysis show that for the scenarios $S_3, S_4$, the effective Majorana neutrino mass falls into the “well” where $|m_{ee}| \rightarrow 0$. This scenario arises because of major cancellation among the different components of $|m_{ee}|$ and becomes unobservable for $0\nu\beta\beta$ decay. However, for the scenarios where the initial options of $\rho, \sigma$ take same values, we find no possible cancellation among the terms of $|m_{ee}|$ as shown by the scenarios $S_1, S_2$. We observe that the magnitude of $|m_{ee}|$ varies from 1.5 meV to 40 meV when the lightest neutrino mass lies in $0.1 \text{ meV} < m_1 < 0.04 \text{ eV}$ for the scenario $S_1$, whereas $|m_{ee}|$ takes values from 6 meV to 40 meV for the same mass range of $m_1$ in the scenario $S_2$. Note that, we fix the upper limit on the effective Majorana neutrino mass as well as the lightest neutrino mass from the latest experimental bound. This analysis will also help us to constrain or determine both the Majorana phases $\rho, \sigma$. At the end, we proceed to discuss the remaining effective Majorana masses $|m_{\alpha\beta}|$ with $\alpha, \beta = e, \mu, \tau$ which may have some interesting consequences for different lepton-number
violating channels. In our careful analysis, we find that in none of the scenarios of $|m_{\alpha\beta}|$ with $\alpha, \beta \neq e$ significant cancellation among different terms of $|m_{\alpha\beta}|$ occurs. We observe the effective Majorana masses like, $|m_{\mu\mu}|, |m_{\mu\tau}|, |m_{\tau\tau}|$ take values $\sim 80$ meV for $m_1 \to 0$, whereas channels, $|m_{e\mu}|, |m_{e\tau}|$ constrains themselves $\sim 8$ meV in the limit $m_1 \to 0$.

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