Dynamics and Synchrony from Oscillatory Data via Dimension Reduction

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Complex, oscillatory data arises from a large variety of biological, physical, and social systems. However, the inherent oscillation and ubiquitous noise pose great challenges to current methodology such as linear and nonlinear time series analysis. We exploit the state of the art technology in pattern recognition and specifically, dimensionality reduction techniques, and propose to rebuild the dynamics accurately on the “cycle” scale. This is achieved by deriving a compact representation of the cycles through global optimization, which effectively preserves the topology of the cycles that are embedded in a high dimensional Euclidian space. Our approach demonstrates a clear success in capturing the intrinsic dynamics and the subtle synchrony pattern from uni/bivariate oscillatory data over traditional methods. Application to the human locomotion data reveals important dynamical information which allows for a clinically promising discrimination between healthy subjects and those with neural pathology. Our results also provide fundamental implications for understanding the neuromuscular control of human walking.

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Time series exhibiting complex oscillatory dynamics are widely observed in the real world: from finance to biomedical systems \cite{1}. In this Letter we focus on an important class of data with a stable oscillation frequency but irregular waveform fluctuations, also called pseudoperiodic time series. Such data arise from broad application domains and have gained particular interest in recent years, with examples ranging from human ECG and gait data\cite{2,3}, white blood-cell count and tremor data in patients\cite{4,5}, epidemic dynamics\cite{6}, light intensity of laser\cite{7}, sun spot numbers and global temperature variation\cite{8,9}. Accurate extraction and characterization of the dynamics of these time series is a general problem, which holds the key to understanding the inner workings of many important physical and biological systems. To this end, traditional methods rely primarily on linear spectral analysis or computing nonlinear characteristics, and recent attempts include producing pseudoperiodic surrogate data\cite{10}, or to establish a nonlinear transform from cycles in time domain to nodes in a dual complex network domain\cite{11}.

However, there are still no generic, systematic and robust approaches to handle such oscillatory time series. The Fourier transform, though widely applied to oscillatory data, is inherently a linear method and cannot account for the nonlinear nature of the data, if any. For nonlinear approaches, the cyclic trend overlying the signal, together with noise from unknown sources, can often mask the intrinsic dynamics\cite{12,13} and pose great challenges to the nonlinear time series analysis/modeling techniques. For example, the most popular Poincare section method, which reduces the flow data to intersections of the trajectories with a lower-dimensional subspace, may produce highly corrupted results under noisy environment, in that the intersection points can no longer preserve the original dynamics in the presence of noise. Things get even worse when the cycles possess complex noncoherent waveforms. In this paper we propose a novel, generic approach that can effectively capture the dynamics of oscillatory time series. By mapping the cycles in the time series to a multi-dimensional Euclidian space, we seek a low-dimensional representation which topologically preserves the important proximity relation among cycles, through efficient global optimization. It is the first time that advances in dimension reduction are explored in reconstructing the dynamics of complex oscillatory data. Our approach utilizes the richer information of pairwise cycle correlation, therefore it not only excavates the inherent dynamics obscured by the cyclic trend, but also offers an extra robustness to noise due to the global nature of the method. Exploration of our approach to probing the synchrony between bivariate oscillatory time series is shown to be able to reveal the subtle synchrony for which traditional approaches will fail. We applied the proposed method to the human locomotion data, and are capable of discriminating between the healthy subjects and those with neuropathology reliably. Based on the results we are able to make the important biological inference that the human walking is not critically dependent on the peripheral neural feedback.

\textbf{THEORY}

\textbf{Reconstructing Dynamics Underlying Cyclic Trend by Spectral Clustering}

We demonstrate how intrinsic dynamics of pseudoperiodic data can be extracted using Laplacian Eigenmaps\cite{14}, a nonlinear dimension reduction method based...
on spectral graph theory. We illustrate with the X-component of the chaotic Rössler system. Motivated by the fact that such data usually exhibit a highly redundant patterns in the form of repeated cycles, we first partition the time series into individual cycles $C_i$'s ($i = 1, ..., k$) by local minima as shown in Fig. 1 (a). Each cycle is then mapped to a high dimensional vector $y_i$ whose dimension equals the number of points in that cycle. Our goal is to compute a set of new, low-dimensional (in the simplest case, $1D$) representation, or embedding $y_i$'s, such that the proximity relations among $x_i$'s are maximally preserved in their low-dimensional counterparts $y_i$'s and that the redundancy in the cycles are removed.

To achieve this, a weighted graph $\xi$ is constructed, where each node corresponds to a cycle $x_i$ and edges are assigned between all pairs of nodes. The graph can either be fully connected or only bind those vertices within the $k$-nearest-neighbors and we aims at extracting the dynamics from the topology of the graph. We use $W_{ij}$ to denote the similarity between cycle $x_i$ and $x_j$, which can be chosen conveniently as the correlation coefficient $\rho_{ij} = \text{Cov}(x_i, x_j) / (\sigma_x \sigma_x)$ (in case $x_i$ and $x_j$ differ in length, shift the shorter vector on the longer one until $\rho_{ij}$ maximizes). Then, the low-dimensional representation $y_i$'s can be cast as the solution of the following optimization problem, $\min \sum_{i,j \in \xi} W_{ij} ||y_i - y_j||^2$, which penalizes those mappings where nearby points $x_i$'s are relocated far apart in the space of $y_i$'s. In case of univariate $y_i$'s, the objective can be written as $y^T Ly$, where $y = [y_1, y_2, ..., y_k]^T$, $L = D - W$ is the graph Laplacian, and $D$ is the diagonal degree matrix such that $D_{ii} = \sum_j W_{ji}$. Hence the name Laplacian Eigenmap. To prevent arbitrary scaling in the solution, one can enforce the constraint $y^T Dy = 1$.

The above constrained minimization is solved by the generalized eigenvalue problem $Ly_i = \lambda_i Dy_i$, where $\lambda_i$'s ($i = 1, 2, ..., k$) are eigenvalues sorted in an ascending order, and $y_i$'s are the corresponding eigenvectors. It can be shown that the minimum eigenvalue $\lambda_1$ is zero, corresponding to an eigenvector ($y_1$) of all 1's. Therefore it is degenerate and the optimal solution is actually provided by $y_2$, the eigenvector of the second smallest eigenvalue $\lambda_2$. As is shown in Fig. 1, the eigenvector $y_2(i)$ provides a unique, reduced representation of the original time series by capturing the dynamics of the oscillatory data on the cycle scale. We use a general notation $c(i)$ ($c$ for cycle) for such reduced series representation (i.e., other dimension reduction schemes can also be applied, and we denote the reduced series all as $c(i)$). More generally, to obtain an $m$-dimensional solution for $y_i$'s, one can simply extract the $m$ trailing eigenvectors $y_i$'s ($i = 2, 3, ..., m + 1$).

The Significance of Extracting Dynamics on Cycle Scale

The comparison between Laplacian embedding $c(i)$ and the Poincare section points $P(i)$ (obtained by collecting the local minima $X_{min}$) indicates that they are dynamically identical by sharing the same chaotic invariants, i.e., Lyapunov exponent and correlation dimension. However, in case of significant noise (see Fig. 2), our approach can capture more of the deterministic structure than does $P(i)$ and is therefore more robust. This is not surprising, since acquisition of $c(i)$ is based on an optimization process that preserves the proximity relation between all

FIG. 1: (a) Time series form X-component of the chaotic Rössler system, which is divided into cycles by local minima (circles). (b) Laplacian embedding $c(i)$ extracted on the cycle scale for the time series shown in (a), and (c) the return plots of $c(i)$ derived by two dimension reduction methods, i.e., the Laplacian Eigenmaps and Multidimensional scaling.

FIG. 2: Return plots for the time series from (a) Poincare section points $P(i)$, which is obtained by collecting the local minima $X_{min}$, and (b) the extracted $c(i)$. The Rössler system here is corrupted by 5% dynamical noise and 30% measurement noise. It is obvious that the return plots from $c(i)$ takes on a clearer form of quadratic function, indicating that the original nonlinear dynamics is sufficiently kept.
pairs of cycles simultaneously, while \( P(i) \) is obtained by treating each cycle independent of each other. It is reasonable to expect the former to excavate more useful, global patterns than the latter.

Many other forms of dimension reduction can also be applied here, which produce similar results. For example, the *Multidimensional Scaling* method that preserves the pairwise distance yields almost the same outputs as the *Laplacian Eigenmaps*, see Fig. 1(c). Actually, the applicability of dimension reduction techniques in extracting the dynamics shall be generally justifiable, considering the intrinsically low correlation dimensions of most real world pseudoperiodic data. The corresponding trajectories in phase space tend to have similar orientations for nearby cycles (see, e.g., Fig. 3). Such redundancy can be effectively removed through dimension reduction, leaving only the useful degrees of freedom for manifestation of the dynamics of interest.

Reconstructing the dynamics on the cycle scale not only enhances the robustness to noise, but bring new vitality to a number of nonlinear methods which are otherwise not suitable for oscillatory data due to the inherent periodicity, such as detrended fluctuation analysis, surrogate data method, recurrence plot, causality, entropy and so on. For example, the detrended fluctuation analysis, which computes the variance of the detrended data at different scales, will produce similar variance curves (i.e., uprising at small scale and saturate at the periodicity of the signal) for all pseudoperiodic data despite their distinct dynamics. This is because the periodic trend dominating in the data will conceal the underlying, subtle dynamics. Thus extracting the dynamics on the cycle scale proves to be crucial for subsequent analysis.

**Detecting Synchrony from Oscillatory Data**

A topic which is of great interest to oscillatory data analysis in recent years is to detect the degree of synchronization between self-sustained oscillators. For example, the peakness of the phase difference distribution and the consistency of mutual nearest neighbors are used to characterize the phase synchrony \( [16, 17] \) and the dynamical interdependence \( [18] \), respectively. Here we are especially interested in the case where the phase relation between two oscillator is evident, but is hard to estimate due to non-phase-coherence and noise, or is not sensitive to degree of changes of synchrony. For example, the blood pressure interacted with heartbeat, where each cycle of blood pressure variation correspond to one heartbeat. The phase index in this case may not be quite informative of the synchrony strength. On the other hand, the presence of noise in most real world data will inevitably degrade the dependency measures.

Unlike traditional methods, we propose to quantify the synchrony the noisy, noncoherent pseudoperiodic data *on their cycle scale* through their Laplacian embedding \( c(i) \). We first test with the \( X \) and \( Y \) components of the noisy Rössler system. Although these two components are non-separable and cannot be strictly defined as being synchronized, they are actually “in phase” and therefore suitable in our context. The two time series are first segmented into cycles by the local maximums of one of the data (i.e., the \( i \)th cycle in both time series share the same time span). We find that the Poincare section points picked up from the two time series can hardly capture the interdependency due to noise, in that the scatter plot between the Poincare section points of \( X \) and \( Y \) leads to a group of random points (Fig. 3(a)). Comparatively, the extracted Laplacian embedding \( c(i) \)'s extracted from \( X \) and \( Y \) by our approach successfully reveal the synchrony pattern by demonstrating an uprising trend in the corresponding scatter plot (Fig. 3(b)).

**APPLICATION TO HUMAN LOCOMOTION DATA**

**Characterizing the Locomotion Dynamics**

Now we apply our approaches to human gait data collected from electrogoniometers at the knee and ankle joints that measure their sagittal plane kinematics during overground walking. Human locomotion is a highly complex, rhythmic process that involves multilevel control of central nerve system and feedback from various peripheral sensors. Here we consider two groups of subjects from healthy controls (CO) and diabetic patients (NP, with significant peripheral neuropathy), each with 10 subjects \( [3] \).

Typically, the human gait time series (Fig. 3(a) and
long range correlation (i.e., the strides separated by a large time span are still statistically correlated). In comparison, the spectrum of the diabetic patients are mostly flat resembling white noise processes ($\beta = 0.37 \pm 0.16$), which means that the strides at different times are mostly uncorrelated. This difference, however, has not been found with either the stride interval (SI) series (Fig. 5 (c)-(d)) or the raw data (Fig. 5 (e)-(f)). This is because, the periodicity in the raw data can obscure the underlying fluctuations and that a linear Fourier transform is not capable of characterizing nor distinguishing the nonlinear dynamics in the data; on the other hand, although SI removes the cyclic trend and has been widely used to quantify pathological locomotion [15], it loses much of the dynamical information by only recording the duration of cycles. In comparison, the transform $c(i)$ successfully removes the periodic trend while preserving the dynamical fluctuation within each cycle, in particular, the specific patterns of the four phases within a pace. Therefore it keeps more valuable information about the neuromuscular control of human walking.

Synchrony Detection between Knee and Ankle Movement

The human walking involves the coordination of two major joints, i.e., the knee and the ankle. So we are also interested in examining the synchrony between the knee and ankle data, which may provide further insights in understanding the neural control of walking. The knee and ankle movements during continuous walking are obviously in phase due to the physical connection of the two joints. The strength of coupling, however, can hardly be recognized from the phase due to the noncoherence [19] (see Fig. 4). Also, noise tends to destroy the local structure in phase space and thus hampers the dynamical dependency measures [20]. To circumvent these difficulties, we compare the dynamics of the two time series by using their Laplacian Embedding $c(i)$’s. Note that each time series can be segmented by either its own local maxima, or those of its partner series (markers in Fig. 4). Therefore we will segment each time series twice and compute the averaged correlation coefficients $\rho_{ij}$’s between $c_{\text{ankle}}$ and $c_{\text{knee}}$.

Then we examine the interrelation between the Laplacian embeddings from the ankle and the knee data. For healthy subjects, the scatter plots demonstrate a significant uprising trend (Fig. 6 (a), $\rho = 0.68 \pm 0.19$), indicating that the knee and ankle movements are highly synchronized; while for diabetics patients, the synchrony almost vanishes (Fig. 6 (b), $\rho = 0.26 \pm 0.18$). Again, the discrimination between CO and NP groups is unavailable by stride interval series, which always exhibits a strong correlation between the joints (Fig. 6 (c)-(d)), corresponding to high degree of phase synchronization. Note that our approach avoids the difficulty of defining...
the phase. On the other hand, it is superior to directly computing the correlation between pairs of equal-time cycles picked respectively from the two time series, which is sensitive to noise and nonstationarity. Finally, we point out that a more comprehensive description of synchrony can be achieved by examining more Laplacian eigenvectors. In the current case the single eigenvector $y_k$ already encodes the primary variability and is thus sufficient for the discriminative task.

Note that we have observed a lack of long range correlation in the ankle kinematics of the NP group (Fig. 5(b)). This suggests the alteration of the locomotor pattern in the lower limbs, due to the nerve deterioration in feet and toes typical of the diabetics. Despite the impaired peripheral feedback caused by the dying nerves, the knee kinematics of the NP group are still found to demonstrate a stable long range correlation indistinguishable from the CO group. This “mismatch” between the ankle/knee dynamics for diabetics can be reasonably explained by the weak synchrony between these two joints, as is shown in Fig. 5(b). From these findings, we can see that the impaired neural feedback from the feet influences only the lower limb locomotion while not that of the knees. We therefore conclude that the human walking is not critically dependent on the feedback from peripheral nervous system. It should be noted that although Gates et al. [21] checked the same data, they do not consider the interaction among the knee and angle locomotion which may be important in characterizing understanding neural control of human walking. Furthermore the authors relied solely on the extracted stride interval series, which have been shown to contain limited dynamical information and therefore may be insufficient for the analysis and evaluation of the human walking.

CONCLUSION

To conclude, we have for the first time circumvented the problem of rebuilding the complex dynamics in oscillatory data from the viewpoint of dimension reduction. We use a global optimization procedure to compute the inherent, low-dimensional representation that maximally preserves the topology of the cycles when embedded in the multi-dimensional Euclidian space. Our approach has been shown to be very robust to noise, and is capable of extracting the underlying dynamics and identifying the subtle synchrony pattern which usually degrade traditional linear or nonlinear methods. Application to human gait data provides clinically promising information in discriminating the healthy from the neuropathological patients, and further enables us to make fundamental inference on the neuromuscular control mechanism of human walking.

Our approach may be of great relevance, and is expected to provide more accurate and powerful diagnostics, to the general biological and biomedical fields where complex oscillatory data abound. For example, we have applied the proposed method to the phonation data from the Parkinson patients, and find a significant lack of long range correlation among the cycles in the signal compared to the healthy people. Our approach can therefore serve as an alternative tool in early Parkinson disease (PD) detection, where there are no blood or laboratory tests currently that can help in diagnosing PD. Other potential applications which will be involved in the future works include the discrimination between Parkinson and essential tremor, and the evaluation of the interaction between the blood pressure variation with the heart beat.

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