Intermittency is commonly observed by practicing power supply engineers in their design workbenches. It shows to be a symmetrical period-doubling bifurcation in time domain with fixed long intermittent period. Sometimes it is called “breathing”. The article studies on the intermittency in the parallel-connected buck converter under master–slave operation, and opens out the reason of the intermittency, then gives a scheme called phase-shifting to realize or control the intermittency. The experimental results provide useful information for the design of the system.

© 2011 Published by Elsevier Ltd. Open access under [CC BY-NC-ND license](https://creativecommons.org/licenses/by-nc-nd/). Selection and/or peer-review under responsibility of ICAE2011.

**Keywords:** bifurcation; chaos; intermittency; phase-shifting; poincaré section

1. Introduction

Recently, paralleling converters has become a popular technique in power-supply design for improving power processing capability, reliability and practicability[11]. Nonlinear dynamics and bifurcation behaviour are important topics of investigation in power electronics. The paper attempts to probe into some nonlinear phenomena of a system of parallel-connected buck converters controlled under a master–slave current-sharing scheme.

2. Master-slave controlled parallel-connected BUCK converter

The system under study consists of two DC-DC converters which are connected in parallel feeding a common load. The current drawn by the load is shared properly between the two buck converters by the action of a master–slave control scheme, as mentioned briefly in the preceding section. Fig. 1 shows the block diagram of this master–slave configuration. Denoting the two converters as Converter 1 and

* Corresponding author. Tel.: +8613965075061.
E-mail address: lily_wang_502@yahoo.com.cn.
Converter 2 as shown in Fig. 1, the operation of the system can be described as follows. Both converters are controlled via a simple pulse-width modulation (PWM) scheme, in which a control voltage $V_{\text{con}}$ is compared with a saw tooth signal to generate a pulse-width modulated signal that drives the switch, as shown in Fig. 2. The saw tooth signal of the PWM generator is given by

$$v_{\text{con1}} = V_{\text{offset}} - K_{v1}(v - V_{\text{ref}})$$  \hspace{1cm} (1)

where $V_L$ and $V_U$ are the lower and upper voltage limits of the ramp, and $T$ is the switching period. The PWM output is “high” when the control voltage is greater than $V_{\text{ramp}}$, and is “low” otherwise. For Converter 1, the control voltage is derived from a voltage feedback loop, i.e.

$$v_{\text{con2}} = V_{\text{offset}} - K_{v2}(v - V_{\text{ref}}) - K_i(i_s - m_i)$$  \hspace{1cm} (2)

Where $V_{\text{offset}}$ is dc offset voltage that gives the steady-state duty cycle, $V_{\text{ref}}$ is reference voltage and $k_{v1}$ is voltage feedback gain for Converter 1.

For Converter 2, an additional current error signal, which is proportional to the weighted difference of the output currents of the two converters, determines the control voltage. Specifically we write the control voltage for Converter 2 as

$$v_{\text{ramp}} = V_L + (V_U - V_L)u (t \mod T)$$  \hspace{1cm} (3)

where $k_{v2}$ is voltage feedback gain of Converter 2, $k_i$ is current feedback gain and $m$ is current weighting factor.

Suppose as $V_{\text{con}} > V_{\text{ramp}}$, $u = 0$ and as $V_{\text{con}} < V_{\text{ramp}}$, $u = 1$. Fig. 3 shows two buck converters connected in parallel. When the converters are operating in continuous conduction mode, diode $D_i$ is always in complementary state to switch $S_i$, for $i = 1,2$. The state equations corresponding to these switch states can be written as

$$\dot{x} = A_1 x + B_1 E \text{ (for } S_1 \text{ and } S_2 \text{ on)}$$

$$\dot{x} = A_2 x + B_2 E \text{ (for } S_1 \text{ on and } S_2 \text{ off)}$$

$$\dot{x} = A_3 x + B_3 E \text{ (for } S_1 \text{ off and } S_2 \text{ on)}$$

$$\dot{x} = A_4 x + B_4 E \text{ (for } S_1 \text{ and } S_2 \text{ off)}$$  \hspace{1cm} (4)
3. Intermittency in the parallel-connected buck converter

We now begin our investigation with computer simulations. Our investigation is based on the exact state equations (5). By the equation, we can get the model of Simulink. The circuit parameters used in simulations are shown in Table 1.

The simulation divided into two steps. One is the simulation of the condition when two frequencies of saw-tooth are same; the other is the simulation of the condition when two frequencies of saw-tooth are different. In the following, a large number of trajectories diagrams are shown, which serve to exemplify the main findings concerning the bifurcation behaviour of a system of parallel buck converters under a master–slave sharing scheme.

Table 1. Values used in simulation

| Circuit Components   | Values       | Circuit Components | Values   |
|----------------------|--------------|--------------------|----------|
| Switching period $T$ | 400 $\mu s$  | Inductance $L_1$   | 0.02 $H$ |
| Input voltage $E$    | 30 $V$       | Inductance $L_2$   | 0.04 $H$ |
| Offset voltage $V_{offset}$ | 5 $V$ | Load resistance $R$ | 12.5 $\Omega$ |
| Reference voltage $V_{ref}$ | 24 $V$ | Current feedback gain $k_1$ | 5 |
| Capacitance $C$     | 47 $Mf$     | Current weighting factor $m$ | 1       |

3.1 Same frequencies of two saw-teeth

We vary $k_{v1}$ and $k_{v2}$ simultaneously, and the corresponding phase diagram[21] is shown in Fig. 4. The diagram shows that the converter experiences a typical period-doubling bifurcation and eventually enters chaos.

(a) Period-1 (kv1 = kv2 = 4.5)  (b) Period-2 (kv1 = kv2 = 5)  (c) Period-4 (kv1 = kv2 = 5.8)  (d) Chaos (kv1 = kv2 = 6.3)

Fig. 4. Trajectory diagram
3.2 Different frequencies of two saw-teeth

Let $f_1 = 2500 \text{Hz}$, $f_2 = 2498 \text{Hz}$, $k_{12} = 6$, the time bifurcation diagram is shown in Fig. 5. The diagram shows that the system changes from period 1 to period-doubling bifurcation, and to chaos, then change in opposite direction to period 1, which is called intermittency or breathing [3], and the intermittent period $T_{in} = 1/|f_1 - f_2|$ is 0.5 seconds.

![Fig. 5 Bifurcation diagram with time as bifurcation parameter](image)

![Fig. 6 Bifurcation diagram with $\theta$ as bifurcation parameter](image)

4. Analysis of intermittency

When the two frequencies of the two saw-teeth are different, we define the voltage of the first saw tooth as

$$v_{\text{ramp1}} = V_L + (V_U - V_L) \frac{t}{T_1} \mod 1 = V_L + (V_U - V_L) t \cdot f_1 \mod 1$$  \hspace{2cm} (6)

Then, the second one is defined as

$$v_{\text{ramp2}} = V_L + (V_U - V_L) \frac{t}{T_2} \mod 1 = V_L + (V_U - V_L) t \cdot f_2 \mod 1$$

$$= V_L + (V_U - V_L) t \cdot (f_1 + (f_2 - f_1)) \mod 1 = V_L + (V_U - V_L) t \cdot \Delta f \mod 1 = V_L + (V_U - V_L) \frac{\theta}{2\pi} \mod 1$$ \hspace{2cm} (7)

Now, two frequencies are same, while there is a delay phase $\theta$, and $\theta = 2\pi \Delta f \cdot t = 2\pi |f_1 - f_2| \cdot t$, so, we can get the bifurcation of $\theta$ as shown as Fig. 6. when it change among $[0,2\pi]$ correspond the intermittent period $T_{in} = 0.5s$. From the Fig. 6, we can know when $\theta = 0$, that to say, there is no phase delay between two saw tooth, the system is in the chaos state, and when $\theta = 2.7 \sim 4$ the system is steady, that is to say the chaos is controlled. The method is called phase-shifting. We can prove the conclusion by phase diagram as shown as Fig. 7. When $\theta = 0$, there are countless trajectories and the poincaré section is made of countless points, for it is in chaos state. While when $\theta = 26\pi / 25$, there is only one trajectory and poincaré section [4] is only one point, for it is in stable state.

![Fig. 7. comparisons between phase diagram and poincaré section](image)

5. Conclusion

Despite the popularity of parallel converter systems in power electronics applications, their bifurcation phenomena are rarely studied. This paper reports some selected bifurcation phenomena in a parallel
system of two buck converters which share current under a master–slave control scheme, especially describes the processing of intermittent chaos. And the paper gives the method to control the nonlinear action, it will give the guidance in the engineering.

Acknowledgements

The authors would like to thank Zhou Yufei and Chen Junning of Anhui University for their instructive advice and useful suggestions. Project supported by the Natural Science Foundation of the Higher Education Institutions of Anhui Province, China (Grant No. KJ2010B046).

References

[1] H. H. C. Iu and C. K. Tse “Bifurcation Behavior in Parallel-Connected Buck Converters,” IEEE Transactions on circuits and systems—I: fundamental theory and applications applications. vol.48, NO. 2, february 2001.

[2] Y. Zhou, C. K. Tse, S. S. Qiu and F. C. M. Lau, “Applying resonant parametric perturbation to control chaos in the buck dc/dc converter with phase shift and frequency mismatch considerations,” Int. J. of Bifurcation and Chaos, vol. 13, no. 11, p. 3459–3472, 2003.

[3] C.K. Tse, "Chaos from a buck switching regulator operating in discontinous mode," Int. J. Circ. Theory Appl., vol. 22, pp. 263–278, 1994.

[4] Zhou Y F, Tse C K, Qiu S S, et al. Applying resonant parametric perturbation to control chaos in the buck dc/dc converter with phase shift and frequency mismatch considerations[J]. Int. J. Bifur. Chaos, 2003, 13(11): 3459–3471.