Light-Front Quark Model Analysis of Exclusive $0^{-} \rightarrow 0^{-}$ Semileptonic Heavy Meson Decays

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Abstract

We present the analysis of exclusive $0^{-} \rightarrow 0^{-}$ semileptonic heavy meson decays using the constituent quark model based on the light-front quantization. Our model is constrained by the variational principle for the well-known linear plus Coulomb interaction motivated by QCD. Our method of analytic continuation to obtain the weak form factors avoids the difficulty associated with the contribution from the nonvalence quark-antiquark pair creation. Our numerical results for the heavy-to-heavy and heavy-to-light meson decays are in a good agreement with the available experimental data and the lattice QCD results. In addition, our model predicts the two unmeasured mass spectra of $^1S_0(b\bar{b})$ and $^3S_1(b\bar{s})$ systems as $M_{b\bar{b}}=9657$ MeV and $M_{b\bar{s}}=5424$ MeV.

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In recent years, the exclusive semileptonic decay processes generated a great excitement not only in measuring the most accurate values of the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements but also in testing diverse theoretical approaches to describe the internal structure of hadrons. Especially, due to the anticipated abundance of accurate experimental data from the $B$-factories (e.g., HERA-B at HERA, BaBar at SLAC and Belle at KEK), the heavy-to-heavy and heavy-to-light meson decays such as $B \to D$, $B \to \pi$, $D \to \pi (K)$ etc. become invaluable processes deserving thorough analysis. While the available experimental data of heavy meson branching ratios have still rather large uncertainties\cite{1}, various theoretical methods have been applied to calculate the weak decay processes, e.g., lattice QCD\cite{2, 3, 4}, QCD sum rules\cite{5}, Heavy quark effective theory\cite{6}, and quark models\cite{7, 8, 9, 10, 11, 12, 13}. In particular, the weak transition form factors determined by the lattice QCD\cite{4} provided a useful guidance for the model building of hadrons, making definitive tests on existing models, even though the current error bars in the lattice data are yet too large to pin down the best phenomenological model of hadrons. These weak form factors, however, are the essential informations of the strongly interacting quark/gluon structure inside hadrons and thus it is very important to analyze these processes with the viable model that has been very successful in analyzing other processes.

In this letter, we report the analysis of exclusive semileptonic decays of a heavy pseudoscalar meson into another heavy or light pseudoscalar meson using the light-front quark model (LFQM) which has been quite successful in the analysis of electromagnetic form factors, radiative decays and $K \to \pi$ transition form factors\cite{7, 8}. The LFQM takes advantage of the equal LF time ($\tau = t + z/c$) quantization\cite{14} and includes important relativistic effects in the hadronic wave functions. The distinguished feature of the LF equal-$\tau$ quantization compared to the ordinary equal-$t$ quantization is the rational energy-momentum dispersion relation\cite{15} which leads to the suppression of vacuum fluctuations with the decoupling of complicated zero modes\cite{16, 17} and the conversion
of the dynamical problem from boost to rotation\cite{15}. The recent lattice QCD results\cite{19} indicated that the mass difference between $\eta'$ and pseudoscalar octet mesons due to the complicated nontrivial vacuum effect increases (or decreases) as the quark mass $m_q$ decreases (or increases), i.e., the effect of the topological charge contribution should be small as $m_q$ increases. This supported in building the constituent quark model\cite{5} in the LF quantization approach because the complicated nontrivial vacuum effect in QCD can be traded off by the rather large constituent quark masses. We have also circumvented the problem of assigning the dynamical quantum numbers $J^{PC}$ to hadrons by using the Melosh transformation of each constituent from equal $t$ to equal $\tau$\cite{7}.

Moreover, one of the most distinctive advantages in the LFQM has been the utility of the well-established Drell-Yan-West ($q^+ = q^0 + q^3 = 0$) frame for the calculation of various form factors\cite{20}. By taking the “good” components of the current ($j^+$ and $j_\perp$), one can get rid of the zero mode\cite{16} problem and compute the full theoretical prediction for the spacelike form factors in $q^+ = 0$ frame. The weak transition form factors that we are considering, however, are the timelike $q^2 > 0$ observables. Our method is to rely on the analytic continuation from the spacelike region to the timelike region calculating the “good” components of the current in the $q^+ = 0$ frame\cite{8}. If we were to take the $q^+ \neq 0$ frame, then we must take into account the higher Fock-state (nonvalence) contributions arising from quark-antiquark pair creation (so called “Z-graph”) as well as the valence configurations. In fact, we notice that a few previous analyses\cite{11} were performed in the $q^+ \neq 0$ frame without taking into account the nonvalence contributions. We find that such omission leads to a large deviation from the full results \cite{8}. Our method is to rely on the analytic continuation from the spacelike region to the timelike region calculating the “good” components of the current in the $q^+ = 0$ frame.

The key idea in our LFQM\cite{6} for mesons is to treat the radial wave function as a trial function for the variational principle to the QCD-motivated Hamiltonian saturating
the Fock state expansion by the constituent quark and antiquark. The spin-orbit wave function is uniquely determined by the Melosh transformation. We take the QCD-motivated effective Hamiltonian as the well-known linear plus Coulomb interaction given by

\[ H_{qq} = H_0 + V_{qq} = \sqrt{m_q^2 + k^2} + \sqrt{m_{\bar{q}}^2 + k^2} + V_{qq}, \]  

(1)

where

\[ V_{qq} = V_0 + V_{\text{hyp}} = a + br - \frac{4\kappa}{3r} + \frac{2\vec{S}_q \cdot \vec{S}_{\bar{q}}}{3m_q m_{\bar{q}}} \nabla^2 V_{\text{Coul}}. \]  

(2)

We take the Gaussian radial wave function \( \phi(k^2) = N \exp(-k^2/2\beta^2) \) as our trial wave function\(^1\) to minimize the central Hamiltonian\(^7\). Among the light-quark mass and the potential parameters \( \{m_u(=m_d), \beta_{ud}(=\beta_{u\bar{d}}), a, b, \kappa\} \), only 4 parameters are independent because of the constraint from the variational principle. Furthermore, the string tension \( b=0.18 \text{ GeV}^2 \) and the constituent \( u \) and \( d \) quark masses \( m_u=m_d=0.22 \) GeV are rather well known from other quark model analyses commensurate with Regge phenomenology\(^9\). Thus, using the experimental values of \( \rho \) and \( \pi \) masses and the variational constraint, we can fix the remaining parameters \( a, \kappa, \) and \( \beta_{ud} \) as \( a = -0.724 \) GeV, \( \beta_{ud} = 0.3695 \) GeV, and \( \kappa = 0.313 \), respectively\(^4\). More detailed procedure of determining the model parameters of light-quark sector \((u \text{ and } s)\) can be found in \(^7\). It is very important to note that all other model parameters such as \( m_c, m_b, \beta_{uc}, \beta_{ub}, \) etc. are then uniquely determined by the same procedure as the light-quark analysis \(^6\). The procedure of determining model parameters constrained by the variational principle\(^8\) is shown in Fig. 1, where the lines of \( qq \) and \( qc \) \((q=u \text{ and } d)\) etc. represent the sets of \( \{m_q, m_{\bar{q}}, \beta_{qq}\} \) and \( \{m_q, m_c, \beta_{qc}\} \), respectively, etc. Because all the lines in

\(^1\) Even though one can in principle expand the radial function \( \phi_{n,l=0}(k^2) \) with a truncated set of HO basis states\(^8\), our choice of radial wave function turns out to be sufficient for the analysis of the ground state \( 0^{-+} \) and \( 1^{-+} \) ground state meson properties\(^6\).
Fig. 1 should go through the same point of \((b=0.18 \text{ GeV}^2, \kappa = 0.313)\), the parameters of \(m_c, m_b, \beta_{uc}, \beta_{ud}\), etc. are all automatically determined without any adjustment. Our model parameters obtained by the variational principle are summarized in Table 1.

Our predictions of the ground state meson mass spectra and the decay constants of various heavy pseudoscalar mesons are summarized in Tables 2 and 3, respectively, and compared with the available experimental data\[^1\] and the lattice QCD results\[^2\]. Our predictions of ground state meson mass spectra agree with the experimental data\[^1\] within 6% error. Furthermore, our model predicts the two unmeasured mass spectra of \(^1S_0(b\bar{b})\) and \(^3S_1(b\bar{s})\) systems as \(M_{b\bar{b}}=9657\text{ MeV}\) and \(M_{b\bar{s}}=5424\text{ MeV}\), respectively. Our values of the decay constants are also in a good agreement with the results of lattice QCD\[^2\] anticipating future accurate experimental data.

The matrix element of the current \(j^\mu = \bar{q}_2 \gamma^\mu Q_1\) for \(0^- (Q_1 \bar{q}) \to 0^- (q_2 \bar{q})\) decay is given by two weak form factors \(f_+\) and \(f_-\), viz.,

\[
\langle P_2 | \bar{q}_2 \gamma^\mu Q_1 | P_1 \rangle = f_+(q^2)(P_1 + P_2)^\mu + f_-(q^2)q^\mu, \tag{3}
\]

where \(q^\mu = (P_1 - P_2)^\mu\) is the four-momentum transfer to the lepton and \(m_i^2 \leq q^2 \leq (M_1 - M_2)^2\). In the heavy quark limit \(M_{1(2)} \to \infty\), the form factor \(f_+(q^2)\) is reduced to the universal Isgur-Wise (IW) function, \(\xi(v_1 \cdot v_2) = \left[2\sqrt{M_1 M_2}/(M_1 + M_2)\right]f_+(q^2)\), where \(v_{1(2)} = P_{1(2)}/M_{1(2)}\). In LFQM, the matrix element of the weak vector current can be obtained by the convolution of initial and final LF meson wave functions in \(q^+=0\) frame:

\[
\langle P_2 | \bar{q}_2 \gamma^\mu Q_1 | P_1 \rangle = -\int_0^1 dx \int d^2k_\perp \frac{\phi_2(x, k'_\perp)|\phi_1(x, k_\perp)}{2(1-x) \prod_{i=1}^2 \sqrt{M_{i0}^2 - (m_i - m_q)^2}} \\
\times \text{Tr}\left[\gamma_5 (\not{p}_2 + m_2) \gamma^\mu (\not{p}_1 + m_1) \gamma_5 (\not{p}_q - m_q)\right], \tag{4}
\]
where $M_0^2 = (k_+^2 + m_i^2)/(1 - x) + (k_+^2 + m_q^2)/x$. In Eq. (4), the form factors $f_+(q^2)$ and $f_-(q^2)$ are obtained by taking the “good” components of the current ($j^+$ and $j_+$). The detailed derivation of $f_\pm(q^2)$ can be found in [8]. Our analytic continuation to the timelike region has verified [8] the equivalence to the dispersion method [12].

Our numerical results of the decay rates for $D \to \pi(K)$, $D_s \to \eta(\eta')$, and $B \to \pi(D)$ processes are consistent with the experimental data as summarized in Table 4. It is interesting to note that our value of $\eta-\eta'$ mixing angle, $\theta_{SU(3)} = -19^\circ$ presented in [7], are also in agreement with the data for $D_s \to \eta(\eta')$ decays. One should note that the number of events for the $D \to \pi$ data is currently very small compared to other processes [1].

In Figs. 2(a) and 2(b), we present the form factors $f_{DK}^+(q^2)$ and $f_{DK}^0(q^2)$, respectively, with the definition of $f_0(q^2)$ as $f_0(q^2) = f_+(q^2) + q^2 f_-(q^2)/(M_1^2 - M_2^2)$, and compare with the experimental data as well as the lattice QCD results [4]. Note that our value of $f_{DK}^+(0) = 0.736$ is within the error bar of the measured value [1], $f_{DK}^{\text{Expt.}}(0) = 0.7 \pm 0.1$. In Fig. 3, we present the form factor $f_{B\pi}^+(q^2)$ and compare with the results from lattice QCD [3]. Our result is very close to the UKQCD [3] results for a wide range of momentum transfer. In Fig. 4, our prediction of the IW function for $B \to D$ transition are compared with the experimental data [21, 22]. Our prediction of the slope $\rho^2 = 0.8$ of the IW function at the zero-recoil point defined as $\xi(v_1 \cdot v_2) = 1 - \rho^2(v_1 \cdot v_2 - 1)$ is quite comparable with the current world average $\rho_{\text{avg}} = 0.66 \pm 0.19$ [1] extracted from exclusive semileptonic $\bar{B} \to D\ell\bar{\nu}$ decay.

In conclusion, in this paper, we analyzed the exclusive $0^- \to 0^-$ semileptonic heavy meson decays using the LFQM constrained by the variational principle for the QCD-motivated effective Hamiltonian with the well-known linear plus Coulomb interaction. Our model not only provided overall a good agreement with the available experimental data and the lattice QCD results for the weak transition form factors and branching ratios of the heavy-to-heavy and heavy-to-light meson decays but also rendered a large number of predictions to the heavy meson mass spectra and decay constants. Our
model in fact predicted the masses of heavy mesons, i.e., $M_{b\bar{b}}(^1S_0) = 9657$ MeV and $M_{b\bar{s}}(^3S_1) = 5424$ MeV. We have overcome the difficulty associated with the nonvalence $Z$-graph contribution in timelike region by the analytic continuation of weak form factors from the spacelike region. Our numerical computation confirmed the equivalence of our analytic continuation method and the dispersion relation method\textsuperscript{[12]}. We think that the success of our model hinges on the advantage of light-front quantization realized by the rational energy-momentum dispersion relation. It is crucial to calculate the “good” components of the current in the reference frame which deletes the complication from the nonvalence $Z$-graph contribution. We anticipate further stringent tests of our model with more accurate data from future experiments and lattice QCD calculations.

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\textsuperscript{2}If we were to use the model parameters given in \textsuperscript{[12]}, we obtain the values of $f_+(0) - f_+(q_{\text{max}}^2)$ as follows: 0.783 (0.781) - 1.2 (1.2) for $D \rightarrow K$, 0.682 (0.681) - 1.61 (1.63) for $D \rightarrow \pi$, 0.682 (0.684) - 1.12 (1.12) for $B \rightarrow D$, and 0.293 (0.293) - 2.7 (2.3) for $B \rightarrow \pi$, where the values in parentheses are the results obtained by the author in \textsuperscript{[12]}. 
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Table 1: The constituent quark masses $m$[GeV] and the Gaussian parameters $\beta$[GeV] for the linear potential obtained by the variational principle. $q=u$ and $d$.

| $m_q$ | $m_s$ | $m_c$ | $m_b$ | $\beta_{q\bar{q}}$ | $\beta_{s\bar{s}}$ | $\beta_{q\bar{c}}$ | $\beta_{s\bar{c}}$ | $\beta_{q\bar{s}}$ | $\beta_{s\bar{b}}$ | $\beta_{q\bar{b}}$ | $\beta_{c\bar{c}}$ | $\beta_{s\bar{b}}$ | $\beta_{c\bar{b}}$ |
|-------|-------|-------|-------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| 0.22  | 0.45  | 1.8   | 5.2   | 0.3659               | 0.4128               | 0.3886               | 0.4679               | 0.5016               | 0.6509               | 0.5266               | 0.5712               | 1.1452               |

Table 2: Fit of the ground state meson masses [MeV] with the parameters given in Table I. Underline masses are input data. The masses of $(\omega - \phi)$ and $(\eta - \eta')$ were used to determine the mixing angles of $\omega - \phi$ and $\eta - \eta'$ [7], respectively.

| $^1S_0$ | Expt. [1] | Prediction | $^3S_1$ | Expt. [1] | Prediction |
|---------|-----------|------------|---------|-----------|------------|
| $\pi$   | 135 ±0.0006 | 135        | $\rho$  | 770 ± 0.8 | 770        |
| $K$     | 498 ± 0.016 | 478        | $K^*$   | 892 ± 0.26 | 850        |
| $\eta$  | 547 ± 0.12  | 547        | $\omega$| 782 ± 0.12 | 782        |
| $\eta'$ | 958 ± 0.14  | 958        | $\phi$  | 1020 ± 0.008 | 1020     |
| $D$     | 1865±0.5    | 1836       | $D^*$   | 2007 ± 0.5 | 1998       |
| $D_s$   | 1969±0.6    | 2011       | $D_s^*$ | 2112±0.7   | 2109       |
| $\eta_c$| 2980±2.1    | 3171       | $J/\psi$| 3097±0.04  | 3225       |
| $B$     | 5279±1.8    | 5235       | $B^*$   | 5325±1.8   | 5315       |
| $B_s$   | 5369±2.0    | 5375       | $(bs)$  | –          | 5424       |
| $(b\bar{b})$ | –        | 9657       | $\Upsilon$ | 9460±0.21 | 9691       |
Table 3: Decay constants [MeV] for various heavy pseudoscalar mesons.

| References | \( f_D \) | \( f_{Ds} \) | \( f_B \) | \( f_{Bs} \) |
|------------|----------|-----------|----------|----------|
| Ours       | 139.2    | 164.8     | 121.2    | 144.2    |
| Lattice[2] | 141.4±21.2 | 155.6±21.2 | 120.2±24.7 | 137.9±24.7 |
| Expt.[1]   | < 219    | 137 – 304 | –        | –        |

Table 4: Form factors \( f_+ (0) \) and branching ratios (Br.) for various heavy meson semileptonic decays for \( 0^- \rightarrow 0^- \). We use \( \theta_{\eta' \eta}^{SU(3)} = -19^\circ \) for \( D_s \rightarrow \eta (\eta') \) decays and the following CKM matrix element: \( |V_{cs}| = 1.04 \pm 0.16 \), \( |V_{cd}| = 0.224 \pm 0.016 \), \( |V_{ub}| = (3.3 \pm 0.4 \pm 0.7) \times 10^{-3} \), and \( |V_{bc}| = 0.0395 \pm 0.003 \) [1].

| Processes | \( f_+ (0) \) | Br. | Expt.[1] |
|-----------|-------------|-----|---------|
| \( D \rightarrow K \) | 0.736 | (3.75 ± 1.16)% | (3.66 ± 0.18)% |
| \( D \rightarrow \pi \) | 0.618 | (2.36 ± 0.34) \times 10^{-3} | (3.9^{+2.4}_{-1.4} ± 0.4) \times 10^{-3} |
| \( D_s \rightarrow \eta \) | 0.421 | (1.8 ± 0.6)% | (2.5 ± 0.7)% |
| \( D_s \rightarrow \eta' \) | 0.585 | (9.3 ± 2.9) \times 10^{-3} | (8.8 ± 3.4) \times 10^{-3} |
| \( B \rightarrow \pi \) | 0.273 | (1.40 ± 0.34) \times 10^{-4} | (1.8 ± 0.6) \times 10^{-4} |
| \( B \rightarrow D \) | 0.709 | (2.28 ± 0.20)% | (2.00 ± 0.25)% |
Figure 1: The parameters $m_s, m_c, m_b, \beta_{qs}, \beta_{qc}$, etc. satisfying variational principle. The $qq$ and $qc$ etc. represents the sets of $(m_q, m_q, \beta_{qq})$ and $(m_q, m_c, \beta_{qc})$ etc., respectively.
Figure 2.a: The form factor $f_+(q^2)$ for $D\to K$ transition compared with the experimental data [1](full dot) as well as the lattice QCD results [4].
Figure 2.b: The form factor $f_0(q^2)$ for $D \rightarrow K$ transition compared with the lattice QCD results [4].
Figure 3: The form factor $f_+(q^2)$ for $B \rightarrow \pi$ transition compared with the lattice QCD results [3].
Figure 4: The IW function $\xi(v_1 \cdot v_2)$ for $B \rightarrow D$ transition compared with the experimental data of ARGUS [21] (square) and CLEO [22] (circle).