Disturbing action of a cubical obstacle on the turbulent vertical-plate free convection boundary layer: RANS-based simulation

A M Levchenya, E M Smirnov, A A Smirnovsky, and M A Zasimova

Department of Fluid Dynamics, Combustion and Heat Transfer, Peter the Great St.Petersburg Polytechnic University, Polytechnicheskaya str. 29, St.Petersburg, 195251 Russia

E-mail: levchenya_am@spbstu.ru

Abstract. Steady-state RANS simulations were performed to establish disturbing action of a cubical obstacle on the turbulent free convection boundary layer in air along a vertical isothermally heated plate. In the case considered, the size of the cube mounted on the plate is about one-third the full thickness of the incoming boundary layer. Using the $k$-$\omega$ SST turbulence model, refined-grid numerical solutions have been obtained assuming the obstacle surfaces either adiabatic or isothermal. Characteristic three-dimensional features of the free convection flow past the cubical obstacle, such as the horseshoe-like vortex structures and the near-wake recirculation, are analyzed in comparison with the previously studied case dealing with the disturbing action of a circular cylinder of unit aspect ratio. Data on heat transfer augmentation near the obstacle are presented and discussed.

1. Introduction

Extended knowledge of dynamic and heat transfer phenomena peculiar to turbulent free convection boundary layers is of great importance for improvement of performance of various moderate- and large-scale devices, as well as for building heat transfer optimization. The generic case for studying these phenomena is the free convection layer on a vertical heated flat plate. In a more general case, the plate can have a series of protuberances (or a single protuberance), which are structural elements of a industrial device or obstacles specially introduced for sake of heat transfer intensification (see, for instance, [1]).

A great effort has been made for last decades to study the related problem of forced-convection turbulent heat transfer augmentation by mean of inserting wall-mounted obstacles into the boundary layer. Among the obstacles used, the most common are protuberances in the form of a circular cylinder or in the shape of a rectangular parallelepiped [2-5]. Despite general similarity in disturbing effects produced by 3D wall-mounted obstacles in the forced- and free-convection flows, there are also considerably distinctions caused by the wall-jet like profile of the free convection boundary layer. It concerns both the separation region in front of the obstacle, where horseshoe-like vortices occur, and the back-side recirculation flow.

Recently we have reported results of an extended RANS-based numerical study of 3D turbulent flow and heat transfer near a finite-height circular cylinder disturbing the turbulent free convection vertical-plate boundary layer [6]. In particular, the effects of the cylinder surface thermal conditions and the height-to-diameter ratio on the wall shear stress and heat transfer patterns in the front and in
the rear of the obstacle were analyzed in detail. The present paper covers results of RANS simulations of disturbing action of a wall-mounted cubical obstacle on the turbulent free convection boundary layer developing along a vertical heated plate. A comparison with the results obtained previously for a corresponding case of a cylindrical obstacle [6] is given as well. Computations were carried out using ANSYS Fluent in version 16.2.

2. Problem formulation and computational methodology

2.1. Flow domain and determining parameters

Figure 1 illustrates the flow configuration under consideration. A cube of size \( a \) mounted on a vertical heated plate disturbs the incoming free convection boundary layer that has reached the fully turbulent statistically 2D state far away upstream of the obstacle. The plate is kept at constant temperature \( T_w \). It is assumed that the ambient temperature \( T_a \) is less than \( T_w \), and \( (T_w- T_a)/T_a \ll 1 \). The \( y \)-axis of the Cartesian system used is directed vertically upward; \( xy \)-planes are parallel to the plate.

![Figure 1](image.png)

The local state of the incoming turbulent layer can be characterized by the Grashof number, \( Gr_\delta = g \beta_r (T_w-T_a) \delta^3/\nu^2 \), based on a characteristic local thickness of the layer, \( \delta \). The gravity acceleration, \( g \), the thermal expansion coefficient, \( \beta_r \), and the fluid kinematic viscosity, \( \nu \), are assumed constant. As in [6], the thickness \( \delta \) is defined as integral of the normalized streamwise velocity profile, \( \frac{v}{v_{max}} \), from the plate surface (\( z=0 \)) to \( z=\delta_T \), where \( \delta_T \) is the thermal boundary layer thickness evaluated as a distance from the plate where the fluid temperature differs from \( T_a \) by 1\% of \( (T_w-T_a) \).

It is assumed that the cubical obstacle is placed at a position where \( \delta=\delta^* \) in case of no obstacle. Correspondingly, the flow simulated is determined by three dimensionless parameters. These are the Grashof number \( Gr_\delta^* \) evaluated with \( \delta^* \), the ratio of the layer thickness to the cube size, \( \beta^*=\delta^*/a \), and the Prandtl number, \( Pr \). The present simulation covers the case of \( Pr=0.7 \), \( \beta^*=3/2 \), and \( Gr_\delta^* \approx 1.1 \times 10^6 \).

With the ANSYS Fluent package, the calculations were carried out in fact using a dimensional problem formulation. Thermal boundary conditions were set according to the experimental study of air convection along a vertical flat plate, as reported by Tsuji and Nagano [7]: \( T_w=60^\circ C \), \( T_a=16^\circ C \). Following [7], physical properties of media were taken at the mean temperature, evaluated as 38\(^\circ\)C, except that the thermal expansion coefficient was taken at \( T=T_a \).

The computational domain used (Figure 1a) has a form of a parallelepiped with the inserted cube of size \( a=40 \) mm. The cube centre is positioned in the domain middle (vertical) plane, at a distance of 10.5\( a \) from the inlet section. The computational domain size is \( 21a \times 21a \times 5\delta^* \) m in \( x \)-, \( y \)- and \( z \)-direction correspondingly.

2.2. Mathematical model and computational aspects

Under the above assumption, the flow dynamics and heat transfer processes are described by the Reynolds-averaged Navier-Stokes and energy equations, with the Boussinesq approach for incorporation of the buoyancy effect. Following [6], the \( k-\omega \) SST turbulence model, as implemented in
ANSYS Fluent, is employed for turbulent viscosity calculations. The turbulent Prandtl number, \( Pr_t \), is set to 0.85.

The no-slip condition is imposed at the plate (1 in Figure 1a) and at the cube surface (2). Two variants of cube-surface thermal conditions are considered. In the first case, the cube is treated as adiabatic. In the second case, its surface is assumed isothermal, with temperature \( T_w \). In order to get inlet profiles of velocity components, temperature, \( k \) and \( \omega \), prescribed at the inlet section (3), two-dimensional RANS simulation of the incoming free convection boundary layer is carried out with the same turbulence model (see [6] for more detail). A generalized inflow/outflow (“pressure-outlet”) condition is used for the outlet section (4). The “pressure-inlet” condition is applied at the boundary counterpart (5) to the heated plate. Symmetry conditions are prescribed at both boundaries limiting the computational domain in the spanwise direction (6, 7).

The 3D quasi-structured grid used for computations was created via translating a 2D \( xy \)-plane grid along the \( z \)-direction. The translated 2D grid consisted of several hexa-blocks. The points of translation were clustered near the plate. The average normalized distance from the centre of the first computational cell to the wall, \( Y^+ \), was about 0.2. A special attention was paid to achieve a good grid resolution near edges of the cube (Figure 1b). Overall, the computational grid includes about 4 million cells.

The numerical results presented below were obtained using the second-order scheme for convective flux evaluation in the momentum and energy equations. The second-order scheme was used also for integration of the \( k-\omega \) SST model transport equations. The flow fields computed were steady-state and symmetrical with respect to the mid vertical plane.

3. Results and discussion

For the adiabatic cube case, Figure 2a illustrates the predicted pattern of limiting streamlines on the plate and the obstacle surface, together with the normalized wall shear stress distribution. Hereinafter, the superscript \(^{(0)}\) denotes a value calculated for the undisturbed 2D boundary layer at the obstacle center position. Note also, that a practically same plot was obtained in the isothermal cube case (not shown).

Figure 2b presents a corresponding plot for the adiabatic cylindrical obstacle, which diameter, \( d \), and height are equal to \( a \). Among others, this case was simulated in [6] at \( \beta = \delta^*/d = 3/2 \). One can see that the separation region forming in front of the cube is considerably wider than in the cylindrical obstacle case. On the plate surface, there is a footprint of a secondary horseshoe-like vortex, which isn’t observed upstream of the cylinder. Contrary to that, structure of the near-plate flow in the rear of the cylinder is more complicated, as compared with the cubical obstacle case. A comparison of 3D flow patterns visualized by volume streamlines for different shape obstacles is given in Figure 3. In flow past the cube, the separation zone developing above the obstacle top is much larger in size than that above the cylinder. Separation zones occur also at both side faces of the cube.

![Figure 2](image-url)  

**Figure 2.** Surface streamlines superimposed on the wall shear stress distributions: (a) cubical obstacle (present) and (b) cylindrical obstacle [6].
Figure 3. 3D streamline patterns in flow past (a) the cubical (present) and (b) cylindrical obstacle [6].

For the symmetry plane, Figure 4 shows streamline patterns superimposed on distributions of the vertical velocity normalized by the buoyancy velocity $u_b = \left[ g \beta \left( T_w - T_a \right) \right]^{1/3} = 0.0292 \, \text{m/s}$. Hereinafter the coordinates are scaled with the cube size or the cylinder diameter. In case of the cubical obstacle, the vortex structure of the upstream separation zone is more complex: the near-wall secondary vortex is well pronounced, and the primary and the tertiary vortices are more intensive, as compared with the cylinder case. The low velocity zone in the rear of the cube, as expected, is considerably larger than that formed behind the cylinder.

Figure 4. Midplane velocity distributions in cases of (a) the cubical and (b) cylindrical obstacle [6].

Figure 5 illustrates influence of the cube-surface thermal conditions on the temperature field around the obstacle: here maps of the reduced temperature, defined as $\theta = (T - T_a)/(T_w - T_a)$, are given. This influence covers mostly the separation zone in the rear of the cube, where the fluid becomes much more heated in the isothermal cube case. Besides, relatively thin temperature layers develop over the obstacle perimeter.

Influence of the cube-surface thermal conditions on distributions of the normalized wall shear stress and heat flux over the plate surface and the cube top is demonstrated in Figure 6. As seen, the wall shear stress distributions are quite close for two variants of the thermal conditions on the cube surface. Slight discrepancies are seen only at the cube top and far away downstream of the obstacle. Contrary to that, effect of the cube-surface conditions on the plate heat transfer is well pronounced. In particular, in the heated cube case, a belt of high heat fluxes caused by transport action of the primary horseshoe vortex is less expressive. Besides, heat transfer augmentation in the rear of the obstacle is notably reduced as compared with the adiabatic cube case. All this compensates considerably the positive effect of increasing the heat-release surface in case of the heat-conducting obstacle.
Figure 5. Temperature distributions at the midplane for (a) adiabatic and (b) isothermal cube.

Figure 6. Wall shear stress and heat flux distributions (alternate) for (a) adiabatic and (b) isothermal cubical obstacle.

Figure 7 gives a general comparison of the plate-surface heat flux distributions over the symmetry line predicted for the cubical and cylindrical obstacles. In the upstream region plot (given on the left) one can see again that the disturbed part of the boundary layer is notably larger in the cube case. The influence of the cube-surface thermal conditions penetrates upstream at a relatively small distance, to one third the cube size. Data for the obstacle rear (right) points that in all cases the region of significant heat transfer increase extends to a distance that is one order higher than the obstacle size. The influence of the cube-surface conditions is not strong in the obstacle rear. In particular, peak values are reduced by about 15% only after changing the adiabatic surface to isothermal. A more significant distinction is seen between the cases of cubical and cylindrical obstacles: it achieves 25%.

Figure 7. Plate-surface heat flux distributions along the symmetry line.
Some general flow and heat transfer characteristics are summarized in Table 1 for all the cases considered. This table includes: maximal normalized values of skin friction and heat flux on the plate surface, the coordinate of flow separation in front of the obstacle, $y_S$ (point S in Figure 2), the position of the main horseshoe vortex center, $y_{c1}$ (shown in Figure 4), the position of the tertiary horseshoe vortex center, $y_{c3}$ (shown also in Figure 4), and the position of the flow reattachment point downstream of an obstacle, $y_R$ (point R in Figure 2).

**Table 1. Flow and heat transfer characteristics for different cases.**

| Obstacle            | $\max(\tau_w/\tau_w^{(0)})$ | $\max(q_w/q_w^{(0)})$ | $y_S$  | $y_{c1}$ | $y_{c3}$ | $y_R$  |
|---------------------|-----------------------------|------------------------|--------|----------|----------|--------|
| Adiabatic cube      | 9.39                        | 6.70                   | -1.423 | -0.822   | -1.128   | 1.764  |
| Isothermal cube     | 9.74                        | 5.70                   | -1.415 | -0.822   | -1.124   | 1.635  |
| Adiabatic cylinder  | 4.40                        | 5.50                   | -1.125 | -0.734   | -0.882   | 1.747  |

The table quantitative data confirm all the above considerations. A peak value of the plate-surface heat flux is about 15% higher in case of the adiabatic cube. A similar result was reported in [6] for the case of a circular cylinder being adiabatic or isothermal. Characteristic dimensions of the horseshoe-shaped vortex structures are practically independent of the thermal boundary condition on the obstacle surface. The area of disturbing action of a cubical obstacle is sufficiently larger, as compared with the case of a unit-aspect-ratio cylinder of same height.

4. Conclusions

Steady-state RANS/$k-\omega$ SST numerical simulation has been carried out to analyse disturbing action of a cubical obstacle on the turbulent free convection boundary layer in air along a vertical isothermally heated plate. The analysis covers the case of a plate-mounted cube, which size is about one-third the full thickness of the incoming boundary layer. Because of interaction of the turbulent free convection boundary layer with the cube, a complicated 3D flow field develops both upstream of the obstacle-plate junction, where a system of horseshoe-shaped vortex structures is formed, and in the rear of the obstacle. The plate-surface heat transfer augmentation caused by disturbing action of the heat-conducting (isothermal) cube is reduced as compared with the adiabatic cube case. It compensates the positive effect of increasing the heat-release surface in case of the heat-conducting obstacle. A detailed comparison with the previously studied case of the unit-aspect-ratio cylindrical obstacle has been made. It has been revealed, in particular, that the area of disturbing action of a cubical obstacle is sufficiently larger than in the circular cylinder case.

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References

[1] Tsuji T, Kajitani T and Nishino T 2007 *Int. J. of Heat and Fluid Flow* 28 pp 1472–83
[2] Sahin B, Ozturk N, Akilli H 2007 *Flow Meas. Instrum.* 18 pp 57–68
[3] Borello D and Hanjalić K 2011 *J. Phys. Conf. Ser.* 318 042046
[4] Martinuzzi R, Tropea C 1993 *J. Fluids Eng.* 115 pp 85-92
[5] Yakhot A, Liu H, Nikitin N 2006 *J. Heat Fluid Flow* 27 pp 994-1009
[6] Levchenya A M, Smirnov E M and Zhukovskaya V D 2019 *Int. J. Heat Mass Transfer* 144 118573
[7] Tsuji T and Nagano Y 1988 *Int. J. Heat Mass Transfer* 31 pp 1723-34