Evaluation of the Spatial Consistency Feature in the 3GPP GSCM Channel Model

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Abstract—Since the development of 4G networks, Multiple-Input Multiple-Output (MIMO) and later multiple-user MIMO became a mature part to increase the spectral efficiency of mobile communication networks. An essential part of simultaneous multiple-user communication is the grouping of users with complementing channel properties. With the introduction of Base Station (BS) with large amount of antenna ports, i.e. transceiver units, the focus in spatial precoding is moved from uniform to heterogeneous cell coverage with changing traffic demands throughout the cell and 3D beamforming. In order to deal with the increasing feedback requirement for Frequency-Division Duplex (FDD) systems, concepts for user clustering on second order statistics are suggested in both the scientific and standardization literature. Former 3rd Generation Partnership Project (3GPP) Geometry-based Stochastic Channel Model (GSCM) channel models lack the required spatial correlation of small-scale fading. Since the latest release of 3GPP Geometry-based Stochastic Channel Model this issue is claimed to be solved and hence our contribution is an evaluation of this spatial consistency feature.

Index Terms—GSCM, MIMO, Channel, Model, Spatial, Consistency

I. INTRODUCTION

In mobile communications, the abstraction of the underlying wireless channel is an essential part. Parallel to increased computation and storage capacities, also the complexity of channel models has increased over the years [1], capturing more and more effects of real environments. Channel models can be categorized into statistical and deterministic channel models [1]. A combination of the statistical and deterministic modeling components can be found in the so-called Geometry-based Stochastic Channel Models (GSCMs). This type of model has been used as a basis for the standardization of the fourth generation (4G) of the mobile network. One important requirement for this work were standardized channel models that can be used to evaluate and compare the different proposals against each other. This was provided with the 3rd Generation Partnership Project (3GPP)-Spatial Channel Model (SCM) in 2003 [2]. Since then, the family of GSCMs is under constant development, evolving to enable performance evaluation of new techniques such as “massive” Multiple-Input Multiple-Output (MIMO) [3], which led to the three dimensional (3D) extension of GSCM [4] or the extension to frequencies above 6 GHz [5].

Another recently added feature is “spatial consistency” [5], because one major drawback of the GSCMs channel has been the lack of realistic correlation in the small-scale fading (SSF) [6]. While Large Scale Parameters (LSPs), such as Angle of Arrival (AoA) spread, Angle of Departure (AoD) spread, delay spread, K-factor, and Shadow Fading (SF), are correlated both in space and with each other to ensure consistent channel properties of closely located mobile users, SSF has been uncorrelated [2]. As a consequence, the signal paths of users located next to each other had a correlated angular spread (AoA), but their individual directions were independently generated. These uncorrelated Multi Path Component (MPC) directions are caused by “random” generation of positions of the scattering clusters. This behavior intuitively contradicts experienced causality in reality and observations in channel measurements [7].

Uncorrelated SSF was also the reason why massive MIMO schemes that rely on spatial consistency could not be reliably investigated with GSCMs before. One promising massive MIMO schemes is Joint Spatial Division and Multiplexing (JSDM) [8] which is based on the assumption that users can be clustered into groups with “similar” covariance matrices [9], [10]. What “similar” covariance matrices exactly means and how it is measured is described in Section IV. However, in publicly available evaluations of the spatial consistency feature, metrics that measure the similarity of SSF are missing, e.g. in [5] (further reference to R1-1700990) the coupling loss, wideband Signal to Interference and Noise Ratio (SINR), cross-correlation of delays, AoAs, and Line of Sight (LoS)/Non Line Of Sight (NLoS) status are investigated, in [11] the impact on the signal to interference ratio is studied, and in [12] the path power, delay, AoA and AoD of a single user is provided.

In contrast to previous work, this paper evaluates spatial consistency in terms of AoA difference and covariance matrix similarity between users. The investigated two dimensional parameter space is the distance between the two users and the spatial decorrelation distance denoted by $d_\lambda$, a parameter that adjusts the degree of “correlation” between the positions of the scattering clusters observed from distinct user positions. For evaluation we use the open source available Quasi Deterministic Radio Channel Generator (QuaDRiGa) channel model [12], where the spatial consistency feature is integrated since version 2.0 and which is calibrated against the 3GPP New Radio (NR) channel model specification [5]. The details of the implementation are available in the source code at [12].
The remainder of this paper is organized as follows. In Section II the principles of the 3GPP GSCM channel model are introduced. Section III describes the positioning of the scattering clusters, with and without the spatial consistency feature. Following, Section IV defines the performance metric used for numerical evaluation in Section V. Finally, Section VI concludes this paper.

II. CHANNEL MODEL

The wireless channel between a transmitter and a receiver in the frequency domain is denoted as $H \in \mathbb{C}^{n_r \times n_t}$, where $n_r$ and $n_t$ are the number of antennas at the receiver and the transmitter, respectively. $H_n$, where $n$ is the sample frequency, can be decomposed as

$$H_n = \sum_{l=1}^{L} G_l \cdot e^{-2\pi j \cdot f_n \cdot \tau_l},$$

where the index $l = [1, \ldots, L]$ denotes the path number and $G_l$ is the MIMO coefficient matrix. $f_n$ is the $n$-th sample frequency in [Hz] relative to the beginning of the used bandwidth and $\tau_l$ is the delay of the $l$-th path in seconds. The coefficient matrix $G_l$ has $n_r$ rows and $n_t$ columns. It contains one complex-valued channel coefficient for each antenna pair.

One major advantage of GSCMs is that they allow the separations of propagation and antenna effects. Therefore, it is essential to have a description of the antenna that captures all relevant effects that are needed to accurately calculate the channel coefficients in the model. Antennas do not radiate equally in all directions. Hence, the radiated power is a function of the angle. Consequently, the individual values of the coefficients are a result of the attenuation of a path, the weighting by the antenna radiation patterns, and the polarization.

Thus, the complex-valued amplitude $g_{r,t,l}$ of the $l$-th path between the $t$-th transmit and $r$-th receive antenna is given by

$$g_{r,t,l} = \sqrt{P_l} \cdot F_r(\phi^r_l, \theta^r_l)^T \cdot M \cdot F_t(\phi^t_l, \theta^t_l) \cdot e^{-j \frac{2\pi}{\lambda} d},$$

where $F_r$ and $F_t$ describe the polarimetric antenna response at the receiver and the transmitter, respectively, $\tau_l$ is the transmit antenna index and $t = [1, \ldots, n_t]$ is the receive antenna index and $t = [1, \ldots, n_t]$ is the transmit antenna index. $P_l$ is the power of the $l$-th path, $\lambda$ is the wavelength, $d$ is the length of the path, $(\phi^r_l, \theta^r_l)$ are the arrival and $(\phi^t_l, \theta^t_l)$ the departure angles that are defined by the position of the receiver, transmitter, and scattering clusters. $M$ is the $2 \times 2$ polarization coupling matrix. This matrix describes how the polarization changes on the way from the transmitter to the receiver. The Cross Polarization Ratio (XPR) quantifies the separation between two polarized channels due to different polarization orientations. $M$ is then often modeled by using random coefficients $(Z_{\theta\theta}, Z_{\theta\phi}, Z_{\phi\theta}, Z_{\phi\phi})$ as

$$M = \begin{pmatrix} Z_{\theta\theta} & \sqrt{1/\text{XPR}} \cdot Z_{\theta\phi} \\ \sqrt{1/\text{XPR}} \cdot Z_{\phi\theta} & Z_{\phi\phi} \end{pmatrix},$$

where $Z \sim \exp \{j \cdot U(-\pi, \pi)\}$ introduces a random phase.

Of all the different ways to describe polarization [13], the polar spherical polarization basis is the most practical for GSCMs. In the polar spherical basis, the antenna coordinate system has two angles and two poles. The elevation angle $\theta$ is measured relative to the pole axis. A complete circle will go through each of the two poles, similar to the longitude coordinate in the World Geodetic System (WGS). The azimuth angle $\phi$ moves around the pole, similar to the latitude in WGS. Thus, the antenna is defined in geographic coordinates, the same coordinate system that is used in the channel model. Hence, deriving the antenna response from the previously calculated departure and arrival angles is straightforward. The electric field is resolved onto three vectors which are aligned to each of the three spherical unit vectors $\hat{e}_\theta$, $\hat{e}_\phi$, and $\hat{e}_r$ of the coordinate system. In this representation, $\hat{e}_r$ is aligned with the propagation direction of a path. In the far-field of an antenna, there is no field in this direction. Thus, the radiation pattern consists of two components, one is aligned with $\hat{e}_\theta$ and another is aligned with $\hat{e}_\phi$. The polarimetric antenna responses $F_r$ and $F_t$ are each described by by a 2-element vector

$$F(\theta, \phi) = \begin{pmatrix} F_r(\theta, \phi) \\ F_t(\theta, \phi) \end{pmatrix}.$$
would result in 2232 random variables. Compared to the 7 variables needed for the LSF model, the filtering approach would require prohibitively large amounts of memory.

In order to reduce the memory requirements, the distance-dependent correlation of the SSF parameters can be modeled by means of the sum-of-sinusoids (SOS) method. The method for generating these correlations is adopted from [16]. The main advantage of using the SOS method lies in the low memory requirements since it only requires a relatively small number of sinusoids coefficients to generate spatially correlated parameters for a large volume of space. Spatially correlated random variables \( k(x, y, z) \) generated by the SOS method are Normal-distributed.

\[
k(x, y, z) = \mathcal{N}(\mu, \sigma^2)
\]

where \( \mathcal{N}(\mu, \sigma^2) \) denotes a Normal distribution with mean \( \mu \) and standard deviation \( \sigma \). Other distributions can be generated by a mapping operation. The parameter value \( k \) is a function of the MT location in 3D Cartesian coordinates \((x, y, z)\). The one dimensional (1D) spatial Auto-Correlation function (ACF) describes how fast the local mean of \( k(x, y, z) \) evolves as the MT moves. The ACF is usually modeled as an exponential decay function

\[
\rho(d) = \exp \left( -\frac{d}{d_{\lambda}} \right),
\]

with \( d \) as the distance between two positions and \( d_{\lambda} \) as the so-called decorrelation distance, i.e. the distance at which the correlation drops to \( e^{-1} \approx 0.37 \) [17]. The SOS method now approximates \( k(x, y, z) \) as

\[
\hat{k}(x, y, z) = \sum_{n=1}^{N} a_n \cos\{2\pi(f_n x + f_{y,n} y + f_{z,n} z + \psi_n)\}
\]

with \( N \) sinusoids. The variables \( a_n, f_n, \) and \( \psi_n \) denote the amplitude, the frequency, and the phase of a sinusoid, respectively. The amplitudes \( a_n \) and the frequencies \( f_n \) are determined in a way that \( \hat{k}(x, y, z) \) has the same approximate ACF as \( k(x, y, z) \) and the Cumulative Distribution Function (CDF) is close to Gaussian density if \( N \) is sufficiently large. The phases \( \psi_n \) are random variables distributed in the range from \(-\pi\) to \(\pi\). Hence, exchanging the \( \psi_n \) while keeping \( a_n \) and \( f_n \) fixed creates a new set of spatially correlated random variables at minimal computational cost. The main challenge of using the SOS method is to find the frequencies \( f_{x,n}, f_{y,n}, \) and \( f_{z,n} \) for a given ACF. Suitable methods for doing this have been introduced by [18] which have been adopted for the implementation of the spatial consistency feature.

IV. PERFORMANCE METRICS

In this section, we introduce three performance metrics that are considered representative for evaluating the spatial consistency feature. These metrics are based on either angular distance or the comparison between the covariance matrices of the users.

A. Angular Distance

In wireless channels, spatial consistency is shown in three aspects: 1) Large-scale parameters, 2) LoS/NLoS state of a link and 3) Distribution of scattering clusters according to user positions. Since the large-scale parameters are always spatially consistent, we focus only on the SSF parameters in this paper. For example, two closely spaced receivers will not only experience similar SF and angular spread, they should also be able to see the same clusters. In the GSCM, the exact positions of each scattering cluster are calculated. Moreover, the Azimuth Angle of Arrival (AAoA) \( \phi \) and Elevation Angle of Arrival (EAOA) \( \theta \) of each path are known at each user. Therefore, the arrival angles are the direct representations of each MPC from the user’s perspective. Here, in order to examine the similarity of the SSF parameters, we consider the absolute angular difference between two receivers as the angular distance. Given the AAoAs and EAOAs of receiver \( i \) and \( j \), the angular distance for MPC \( l \) is defined by

\[
\Delta \phi_l = \left\{ \begin{array}{ll}
|\phi_{i,l} - \phi_{j,l}|, & \text{if } |\phi_{i,l} - \phi_{j,l}| < \pi \\
2\pi - |\phi_{i,l} - \phi_{j,l}|, & \text{otherwise}
\end{array} \right.
\]

\[
\Delta \theta_l = |\theta_{i,l} - \theta_{j,l}|,
\]

where Eq. (8a) always selects the value smaller than \( \pi \) as the angular distance due to the circular nature of AAoA. This nature is not observed in EAOA since the EAOA value is restricted to \([-\pi/2, \pi/2]\). It is evident to see that the angular distance is lower bounded by 0 and a small value of angular distance indicates high similarity in the SSF parameters and therefore a high spatial correlation. A more general way to study the spatial consistency regardless of the number of MPCs involved is to average all angular distances over existing MPCs. Section V shows this average angular distance as a function of distance between users. The average angular distance is therefore given by

\[
\Delta \phi = \frac{1}{L} \sum_l \Delta \phi_l \quad \text{and} \quad \Delta \theta = \frac{1}{L} \sum_l \Delta \theta_l,
\]

where \( L \) is the total number of MPCs.

B. Covariance Matrix

The authors in [8] proposed JSDM, a novel way to enable massive MIMO gains in Frequency-Division Duplex (FDD) systems. With this approach the training overhead by partitioning users into groups is significantly decreased. All users within the same group share similar second-order statistics (e.g. in the form of covariance matrices). Therefore the channel covariance has been extensively researched to study the spatial correlation between users. With the given channel coefficients from Eq. (1), we are able to obtain the covariance matrix \( \mathbf{R} \in \mathbb{C}^{n_t \times n_t} \) by the following equation

\[
\mathbf{R} = \mathbb{E}[\mathbf{H}^H \mathbf{H}],
\]

The covariance matrix is assumed to be slow-varying in time and depends on user mobility. The idea of grouping receivers with sufficiently similar covariance matrices inspired several
similarity measures, some of which are also suitable for measuring the spatial consistency feature. Motivated by this, we investigate the two performance metrics based on covariance matrices in this subsection. The first makes use of the chordal distance criterion, the second adapts the Correlation Matrix Distance (CMD) measure introduced in [19].

1) Chordal distance: The chordal distance has been introduced by [20] and is widely used as a distance metric between subspaces. The authors in [21] used chordal distance as a metric for packings in Grassmannian spaces. Moreover, [9], [10] came up with different user grouping algorithms for JSDM, with chordal distance being the distance criterion. The idea of grouping closely placed users by studying the similarity between channel covariance also suits our intention well. Hence we utilize chordal distance as our second evaluation metric for spatial consistency. Given the covariance matrices of two users \( \mathbf{R}_1, \mathbf{R}_2 \), the chordal distance is expressed by

\[
d_C(\mathbf{R}_1, \mathbf{R}_2) = \| \mathbf{R}_1^H \mathbf{R}_2 - \mathbf{R}_2^H \mathbf{R}_1 \|_F^2,
\]

where \( \| \cdot \|_F \) denotes the Frobenius norm. When studying the spatial correlation of two users, we should expect the chordal distance to decrease as the distance between the users drops. In fact, we consider two users are strongly spatially correlated if the chordal distance is smaller than a selected threshold \( c_C \).

2) Correlation Matrix Distance: The CMD proposed in [22] was a novel measure to track the changes in spatial structure of non-stationary MIMO channels. The same metric was used in [19] as a covariance matrix measure for user clustering for JSDM. In this way, it is possible to lower the complexity of predicting the threshold involved in the chordal distance based clustering algorithms. According to [19], the similarity measure based on CMD is expressed as

\[
d_{CMD}(\mathbf{R}_1, \mathbf{R}_2) = 1 - CMD(\mathbf{R}_1, \mathbf{R}_2) = \frac{\text{Tr}(\mathbf{R}_1^H \mathbf{R}_2)}{\| \mathbf{R}_1 \|_F \| \mathbf{R}_2 \|_F},
\]

where \( \text{Tr} \) denotes the “trace” operator as the sum of the diagonal elements. This similarity measure is 1 in the case of \( \mathbf{R}_1 \) and \( \mathbf{R}_2 \) being collinear, 0 in the case of \( \mathbf{R}_1 \) and \( \mathbf{R}_2 \) being orthogonal. Therefore, with the spatial consistency feature, \( d_{CMD} \) between two spatially correlated users should have a value close to 1. With a properly defined threshold value \( c_{CMD} \), we are able to find the distance within which the users are considered strongly spatially correlated.

V. NUMERICAL RESULTS

For the evaluation of the spatial consistency feature we used QuaDRiGa version 2.0 which implements the 3GPP new radio model [5] together with the SOS-based spatial consistency model. The center frequency was set to 2 GHz in an urban macro NLoS scenario, where the number of modeled MPCs is \( L = 5 \). A summary of the simulation parameters is listed in Table I.

![Table I: Simulation Assumptions](image)

A scenario with two users separated by a distance of 20 m is considered, see Fig. 1. Therein, user two moves straight towards user one, labeled as “Track” in Fig. 1. The distance between user one and user two is denoted as \( d^{(1,2)} \). We assume downlink transmission from the Base Station (BS) to the users such that the AoA means angles observed by the users.

First, the AoA differences \( \Delta \phi \) and \( \Delta \theta \) according to Eq. (9) are shown in Fig. 2, the AAoA on the left hand side and the EAOA on the right hand side. It can be observed that depending on the on the spatial decorrelation distance \( d_{\lambda} \) the slope of angular distance decreases at a certain point, e.g. for \( d_{\lambda} = 5 \) m at \( \approx 7 \) m. Note, that in Fig. 2 the mean over the \( L = 5 \) MPCs is given in [radian]. The difference in the value range of the AAoA and EAOA in case of uncorrelated scattering positions \( d_{\lambda} = 0 \) m is caused by the “urban macro” scenario assumption, where in measurements BSs are on the roof top of buildings limiting the elevation angular spread which is smaller than the azimuth angular spread.

Fig. 3 shows the Chordal distance \( d_C(\mathbf{R}_1, \mathbf{R}_2) \) according to Eq. (11) between user one and two. Therein, two observation are worthwhile to mention. First, the chordal distance is not a normalized measure and the value range, which is \( < 6 \times 10^{-15} \) shown in Fig. 3, depends on the power of the channel coefficients. Thus, any threshold \( c_C \) for similarity between two users depends on the LSPs and path-loss of the
two users. Second, the chordal distance $d_C$ decreases to zero as the distance drops, superposed by a $d_A$ depened threshold, where the slope of the decrease changes significantly, e.g. for $d_A = 20$ m at approximately 5 m.

Finally, Fig. 4 shows the $d_{CMD}$ according to Eq. (12) over the distance between user one and two. In contrast to the chordal distance, the $d_{CMD}$ is a normalized metric between zero and one, where one is reached for collinear covariance matrices, see Section IV-B2. Thus, large scale parameter independent thresholds can be defined, e.g. in [19] a similarity threshold of $\epsilon_{CMD} = 0.95$ is set as a condition for users to be in the same cluster. In our numerical evaluation such a value is approximately achieved at 1 m distance for $d_A \geq 15$ m. On the other hand, even for $d_A = 0$ m (e.g. spatial consistency feature disabled), a decreasing $d_{CMD}$ is observed in Fig. 4 with a saturation at $\lim_{d(1,2) \to 0} d_{CMD}$ ($d_A = 0$ m) $\approx 0.64$. This is caused by the correlation of the LSP in space, see Section III. The difference of the AoA and AoD spreads, delays, coupling loss, K-factor, and SF also decreases with the distance and thus increases the $d_{CMD}$ even for $d_A = 0$ m.

VI. CONCLUSION

In this work, the recently proposed spatial consistency feature is evaluated. Thereby, several performance metrics are considered, showing that the spatial consistency feature works as expected. In a scenario where one user drifts to another, the angle distance of observed scattering clusters and chordal distance of the covariance matrices converge to zero, whereas the $d_{CMD}$ approaches one. Results also show the impact of the “decorrelation distance” parameter $d_A$, which tunes the degree of correlation between scattering cluster positions. The larger the $d_A$ the stronger the correlation.

The results in the paper enable researchers to parametrize the QuaDRIGa channel model for simulations that require spatial consistent SSF. Furthermore, this work confirms that the 3GPP GSCM can be used for reliable research and performance evaluations of JSDM or similar user clustering based schemes.

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