The Intrinsic Gamma-Ray Spectrum of TXS 0506+056: Intergalactic Propagation Effects

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ABSTRACT

The recent observation of high-energy neutrinos from the 2017 flare of the blazar TXS 0506+056, together with counterparts across the whole electromagnetic spectrum, opens up new possibilities for investigating the properties of this class of objects as well as the traversed medium. Propagation effects such as the attenuation of the very-high-energy gamma-ray component by the extragalactic background light are well known, and usually taken into account when fitting spectral energy distributions of objects. Other effects such as those of intergalactic magnetic fields are, however, often neglected. In this work we present a comprehensive study of the influence of these fields and the extragalactic background light on the determination of the intrinsic gamma-ray spectrum of this blazar.

Key words: BL Lacertae objects: individual: TXS 0506+056 – cosmic background radiation – gamma-rays: galaxies – magnetic fields – neutrinos – relativistic processes

1 INTRODUCTION

Active Galactic Nuclei (AGNs) are sources of electromagnetic radiation across the whole spectrum, from radio to gamma rays (for a review see, e.g., Padovani et al. 2017). Blazars are a type of AGN with relativistic jets pointing approximately towards Earth, making them interesting objects to study the extreme processes taking place near accreting supermassive black holes at their centres. They have been known to be sources of energetic gamma rays for decades (Punch et al. 1992).

The high-energy (HE; \( \gtrsim 1 \) GeV) and very-high-energy (VHE; \( \gtrsim 100 \) GeV) electromagnetic component of the spectral energy distribution (SED) of AGNs is sometimes attributed to leptonic processes (Dermer et al. 1992; Schlickeiser 1996; Mastichiadis & Kirk 1997; Ghisellini et al. 1998; Böttcher et al. 2013; Diltz & Böttcher 2014), involving interactions of electrons with photon and magnetic fields pervading the environment. The emission by these objects can also be due to hadronic processes (see e.g. Mannheim & Biermann 1992; Mannheim 1995; Mastichiadis 1996; Halzen & Zas 1997; Tavecchio & Ghisellini 2015; Cerruti et al. 2015; Khiali & de Gouveia Dal Pino 2016; Murase et al. 2016; Zech et al. 2017), where high-energy cosmic rays interact with the environment producing pions, whose decays create very energetic gamma rays and neutrinos. Lepto-hadronic models also exist (see, e.g., Mücke & Protheroe 2001; Cerruti et al. 2019; Rodrigues et al. 2019). They are commonly fits to the observed multi-wavelength SED based on single or multiple zones. A typical SED for this class of objects features two humps. The first one, in the X-ray band, is mostly due to synchrotron emission by relativistic electrons and positrons within the blazar’s environment. The high-energy peak can be explained by inverse Compton scattering of low-energy background photons by electrons, by synchrotron emission due to highly relativistic protons, or by photon-pion production of HE photons; see Böttcher et al. (2013) for more details. Naturally, combinations of these processes are also plausible explanations.

It is difficult to ascribe a purely hadronic/leptonic interpretation to the VHE emission by blazars based solely on gamma rays. For this reason, in addition to multi-wavelength observations, neutrino measurements are important, since they allow us to distinguish among the aforementioned scenarios.

The observation of the high-energy neutrino event IC 170922A (\( E_\nu \approx 290 \) TeV) correlating with the position of the blazar TXS 0506+056 (IceCube et al. 2018), in combination with an electromagnetic counterpart (IceCube et al. 2018), was the cornerstone of multi-messenger astronomy. TXS 0506+056 is a laboratory for testing models of non-thermal emission by blazars. High-energy emission from this object had already been observed by EGRET (Dingus &

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Bertsch 2001). Located at \( z = 0.3365 \) (Paiano et al. 2018), TXS 0506+056 also had another (even stronger) episode of neutrino emission during 2014-2015. A total of 13 ± 5 neutrinos were detected within a time window of 110 days (IceCube Collaboration 2018). Archival analyses of Fermi-LAT data for this period reveal that TXS 0506+056 was at a low state during this period (Garrappa et al. 2019). The scarcity of multi-wavelength observations make it difficult to draw a picture of this object capable of accommodating both the 2014-2015 and the 2017 flares.

Hadronic and lepto-hadronic models were put forth to interpret the observations of TXS 0506+056 (Ansoldi et al. 2018; Sahakyan 2018; Keivani et al. 2018; Liu et al. 2019; Gao et al. 2019; Cerruti et al. 2019; Rodrigues et al. 2019; Samui et al. 2018). They infer a maximum cosmic-ray energy of \( E_{\text{CR}} \sim 10^{15} - 10^{18} \) eV. Following the common procedure of SED-fitting, the intrinsic parameters of the object were constrained. One step in this procedure is to include the opacity of the Universe to pair production \( (\tau_{\gamma\gamma}) \), whose dominant contribution corresponds to infrared photons from the extragalactic background light (EBL). This photon field effectively leads to an exponential attenuation of the flux, of the form \( \exp(-\tau_{\gamma\gamma}) \). An issue that arises naturally is: if VHE gamma rays create electron-positron pairs in the intergalactic medium, what happens to these charged particles in the presence of intergalactic magnetic fields?

The question posited before relates to a yet unanswered problem in cosmology: the origin of the magnetic fields in the Universe. Currently, two general classes of magnetogenesis scenarios are considered in the literature: cosmological and astrophysical. For the former, one assumes that strong seed magnetic fields have been created in the very early Universe (for example, during the electroweak or QCD phase transition, or during inflation – see e.g. Durrer & Neronov 2013 and Subramanian 2016) and then evolve up to the present day, which may be simulated in full or (semi-)analytical magnetic-hydrodynamic simulations (Saveliev et al. 2012, 2013; Campanelli 2014; Kahniashvili et al. 2013, 2016, 2017; Brandenburg et al. 2020). On the other hand, the basic idea of the astrophysical scenario is that weak magnetic field seeds were created during later stages of the evolution of the Universe (and then amplified by a battery mechanism), e.g. during Reionization (Langer & Durrive 2018), from cosmic-ray currents (Miniati & Bell 2011; Ohira 2020) or galactic (nuclei) outflows (Furlanetto & Loeb 2001; Beck et al. 2013; Samui et al. 2018).

Lower limits on the strength of IGMFs were obtained by several authors (Neronov & Vovk 2010; Tavecchio et al. 2010, 2011; Dolag et al. 2011; Essey et al. 2011; Finke et al. 2015; Ackermann et al. 2018) using the deflection of electron-positron pairs produced in electromagnetic cascades in IGMFs. An immediate consequence of the existence of such fields is an angular spread of the arrival directions of secondary gamma rays produced via inverse Compton scattering (see e.g Dolag et al. 2009; Neronov et al. 2010; Chen et al. 2015; Yang & Dai 2015). Evidently, temporal profiles are also expected to depend on properties of the fields (Plaga 1995; Murase et al. 2008; Olkononou et al. 2014; Yang & Dai 2015). For this effect, however, the duration of the emission has to be comparable to the other time scales involved (observation time). Another observable that is directly influenced by IGMFs is the flux of gamma rays observed at Earth (d’Avezac et al. 2007; Neronov & Vovk 2010; Vovk et al. 2012). For longer time delays, part of the HE band of the spectrum cannot be detected, as not all photons can arrive within a fixed time window. Moreover, the more the electrons (and positrons) in the cascade are deflected, the more diluted the arrival distribution of photons from a single source is, and the less likely it is that they will be contained within a fixed angular window corresponding to the object. Consequently, IGMFs induce a flux suppression of secondary photons; but they are not accounted for in SED fits. It should be noted here that there is another explanation for this suppression: electrons/positrons present in the cascade might interact with the intergalactic medium and generate plasma instabilities, leading to energy losses and consequently to a lower flux of high-energy photons arriving at the observer (Broderick et al. 2012; Miniati & Elyiv 2013; Schlickeiser et al. 2012; Schlickeiser et al. 2013; Sironi & Giannios 2014; Vafin et al. 2018; Broderick et al. 2018; Yan et al. 2019; Alves Batista et al. 2019b).

In light of the preceding discussion, in this work we address how the presence of intervening IGMFs may interfere with the modelling of the intrinsic spectral parameters of blazars. We focus on the particular case of the blazar TXS 0506+056. We start off by describing the three-dimensional simulation setup adopted, in section 2, and then we explain the procedure to post-process these simulations in section 3. In 4 we present the results of a first intuition-builder study required for the interpretation of the results. The procedural details concerning the fitting of the observations is extensively addressed in section 5, followed by the results in 6. Finally, in section 7, we discuss our results, and draw our conclusions in 8.

### 2 SIMULATION SETUP

We simulate the propagation of gamma rays in the intergalactic space using the CRPropa code (Alves Batista et al. 2016). It is a modular code designed for the propagation of high-energy particles in the Universe. We consider pair production \( (\gamma_{\text{HE}} + \gamma_{bg} \rightarrow e^+ + e^-) \) and inverse Compton scattering \( (e^\pm + \gamma_{bg} \rightarrow e^\pm + \gamma_{\text{HE}}) \) both in the EBL and CMB; here the subscript ‘HE’ denotes a high-energy particle, and ‘bg’ a background photon. Adiabatic energy losses due to the expansion of the Universe are considered as well. In this case, a particle with initial energy \( E_0 \) would be observed with an energy \( E(z) = E_0/(1+z) \), where \( z \) is the redshift of emission. We also consider synchrotron radiation by charged particles; this is, however, small, and produce photons below the energy range of interest (\( \lesssim 1 \) GeV).

The source is assumed to be located at the centre of a sphere of radius \( D \), which corresponds to the (co-moving) distance of the Earth to TXS 0506+056. Events are emitted isotropically until one of the following conditions are met: (i) they hit the sphere; (ii) their energy drops below a minimum energy threshold, assumed here to be 1 GeV; (iii) the total trajectory length described by the particle exceeds a maximum threshold of 4000 Mpc.

We use the built-in CRPropa integrator, which solves the equations of motion with a 5th-order Runge-Kutta method with adaptive steps. The minimum and maximum
step sizes are, respectively, $10^{13}$ m and 10 Mpc. This choice yields a time resolution of ~1 day.

The magnetic field is assumed to be a turbulent zero-mean Gaussian random field with a Kolmogorov spectrum, and redshift evolution given by $B(z) = B_0(1 + z)^2$. It is sampled in the Fourier space, transformed into real space, and projected onto a uniformly spaced cubic grid with $N = 500^3$ cells. The minimum scale that can be resolved is $\ell_{\text{min}} \equiv 2L_{\text{c}}/N^{1/3}$ following the Nyquist criterion. The maximum scale is chosen to achieve the desired coherence length. The grid is periodically repeated to cover the whole volume between TXS 0506+056 and Earth. At each run, we change the seed used to generate the magnetic-field grids to prevent spurious features inherent to one realisation of the field from affecting the results.

The magnetic fields considered range from $10^{-19}$ G up to $10^{-14}$ G, in logarithmic steps of 1. The coherence length ($L_c$) lies in the range 10 kpc–1 Gpc, also in logarithmic steps of 1. In addition, we consider the case $B = 0$. The range of $B$ we adopt covers the typical lower bounds derived using electromagnetic cascades. We could not simulate stronger magnetic fields because the Larmor radii of electrons start to become excessively small, rendering it impractical to track single particles due to the high computational load—especially if they can get trapped within a small region of space.

The values of coherence length were chosen to encompass the most common values of $L_c$ according to various constraints (see Durrer & Neronov 2013 for a review).

### 3 POST-PROCESSING OF THE SIMULATIONS

We reject all events whose time delays exceed $\Delta t$ to compute the gamma-ray fluxes at Earth. For the enhanced emission state, we set $\Delta t = \Delta t_{\text{E}} = 6$ months. The duration of gamma-ray activity of TXS 0506+056 during its quiescent state is unknown, so we use $\Delta t = \Delta t_{\text{AGN}} = 10^4, 10^5$ years.

We apply a posteriori cuts to the simulations in order to mimic the desired injection spectrum. This is done by applying weights to each simulated event according to the desired injection spectrum. Note that in our analysis we are not concerned with the gamma-ray spectrum within the source environment. Instead, we consider only the gamma rays that escape the source; these two quantities are not necessarily the same. For a spectrum $dN/dE \propto E^{-\alpha} \exp(-E/E_{\text{max}})$, we consider the following parameter range: $0 \leq \alpha \leq 4$ and cut-off energy $10^{10,+6} \leq E_{\text{max}}/\text{eV} \leq 10^{14}$, in steps of 0.2 for $\alpha$ and log($E_{\text{max}}/\text{eV}$).

The actual spectrum that is observed is a combination of those for the low state (subscript ‘l’) and the enhanced emission (‘h’) periods, and is given by

$$
\frac{dN}{dE} \propto E^{-\alpha} \exp\left(-\frac{E}{E_{\text{max,l}}}\right) + \eta E^{-\alpha_h} \exp\left(-\frac{E}{E_{\text{max,h}}}\right),
$$

where $\eta$ denotes the flux enhancement in the high state with respect to the low state.

We consider TXS 0506+056 to have a perfectly collimated jet pointing directly to Earth. While the viewing and jet opening angles could impact our predictions quantitatively, our arguments still hold.

TXS 0506+056 is a point-like source. Therefore, before confronting our simulations with the data, we have to fold in the point spread function of the detectors. These are introduced as energy-dependent windows around the position of the blazar. For energies below 300 GeV, in the range of Fermi-LAT observations, we consider a 68% containment radius and use the values from the instrument response function P8R2_V6. For $E > 300$ GeV, we assume an angular resolution of $0.1^\circ$ up to $E = 1$ TeV and $0.06^\circ$ above this energy; these are similar to MAGIC’s angular resolution (Aleksić et al. 2016). In the simulations there are no background events contributing to the flux within the containment radius. However, simulated events whose angular distances to the source position exceed the size of the window associated with the containment radius have to be rejected to ensure consistency with the observations (IceCube Collaboration 2018; IceCube et al. 2018; Ansoldi et al. 2018). This is important to guarantee that the significance of the correlation between the gamma-ray signals and the electromagnetic/neutrino counterparts is preserved.

### 4 A FIRST STUDY

Before we proceed and fit the observations, it is in order to provide further grounds for our forthcoming results. The kinematic energy threshold for the production of pairs by high-energy gamma rays is $E_{\text{thr}} \sim m_e^2/c^2$, where $e$ denotes the energy of the target photon. For the EBL, $e \sim 0.001 - 10$ eV. Thus, from purely kinematic arguments, we expect that at least a fraction of the gamma rays from TXS 0506+056 be comprised of secondaries produced in the electromagnetic cascade process.

The EBL attenuates the high-energy component of the very-high-energy emission, leading to a spectral suppression. This effect could be easily mistaken by a cut-off due to the maximum energy ($E_{\text{max}}$) of gamma rays leaving the object. A natural question that then arises is: how can we distinguish between these two mechanisms? If $E_{\text{max}} \lesssim 100$ GeV, most of the observed flux is due to prompt emission. However, if $E_{\text{max}} \gtrsim 400$ GeV, cascade photons could significantly contribute to the gamma-ray flux above 1 GeV.

In this section, to better understand the problem and
Figure 2. Cumulative distribution of time delays of arrival times (upper panels) and average time delays as a function of the observed energy (lower panels). The grey shaded region indicates the period of enhanced activity of the object (Δtflar = 180 days). This particular figure assumes a spectral index α = 2.0 and Emax = 1 TeV. These plots are for the whole energy range considered in the simulation (E ≥ 1 GeV).

by showing the cumulative time-delay distribution for one specific scenario (left panel). An energy-dependent version of the average time delays for different (observed) energies is shown in the right-hand-side panel.

Figure 2 suggests that for strong enough magnetic fields, the total flux is diluted over a large period of time. In other words, if the cumulative distributions reach ~ 1 within the shaded region corresponding to the neutrino flare, then the effects of IGMFs are small and so is the flux suppression due to the field. This general behaviour holds qualitatively for all EBL models studied. Moreover, the dependence of this effect on ΔtAGN is small.

5 FITTING THE SIMULATIONS

In order to perform the analysis described in this work, a stringent multi-stage data analysis is necessary. To this end, we carried out the procedure described below. First, we determined the best fit for the low state. The only parameter in this fit is the overall normalisation factor (ν), which scales the simulated spectrum (Jsim(E) = EdNsim/dE) to fit the observations (Jobs(E) = EdNobs/dE) resulting from a simulation, the fitted spectrum is given by J(E) = νJ0(E). To calculate the optimal ν, we calculate the normalized log-likelihood ln L of the fit using the data points (Ei, Ji) with the formula

\[ \ln L = -\sum_{i=1}^{n} \frac{(J_{\text{sim}}(E_i) - J_{\text{obs}}(E_i))^2}{2\sigma(E_i)} \]  

where σ(Ei) is given by

\[ \sigma(E_i) = \begin{cases} \sigma^+ & \text{if } J_{\text{obs}}(E_i) \geq J_i, \\ \sigma^- & \text{if } J_{\text{obs}}(E_i) < J_i \end{cases} \]

for which σ+ and σ− are, respectively, the lower and upper standard deviations for the i-th point. In other words, we consider the split normal distribution as the asymmetric generalisation of the normal distribution. Now, in order to find the value of ν that fits the data, ln L has to be maximised, which is done numerically for each simulation.

Next, we reduce the size of the parameter space to be
were found to be tiny ($\ll$) the spectral parameters to the magnetic field. Moreover, the impact of $\Delta$ negligible.

$L_{\text{coherence length}}$ for in general, time delays of electromagnetic cascades are pre-

EBL models the change of $\Delta$ case, whereas for the upper limit model by Stecker et al. (2016), point increases with respect to the $\Delta$ medium.

The change in the value of the maximal energy due to magnetic fields can be assessed by estimating the change in $E_{\text{max},h}$ with respect to the case without magnetic fields. This quantity will be henceforth called $\Delta E_{\text{max},h}$. Its average value, marginalised over all other quantities except $B$ and $L_c$, is shown in figure 3. From this figure it is clear that the actual value of $E_{\text{max},h}$ may change considerably, depending on the EBL model, if magnetic-field effects are considered. Interestingly, the two bottom panels, corresponding to the upper and lower limit EBL model by Stecker et al. (2016), point to an interesting trend: in the presence of IGMFs, $E_{\text{max},h}$ increases with respect to the $B = 0$ case for the lower limit case, whereas for the upper limit model $E_{\text{max},h}$ decreases.

In addition, one can see that for three out of the four EBL models the change of $\Delta E_{\text{max},h}$ has a stronger dependence on the magnetic field strength ($B$) than on the coherence length ($L_c$). This may be explained by the fact that, in general, time delays of electromagnetic cascades are predicted (Neronov & Semikoz 2009) to be independent of the coherence length for $L_c \gg D_e$ (where $D_e$ is the electron energy loss length due to inverse Compton scattering) which, in turn, results in a smaller sensitivity of the estimates of the spectral parameters to the magnetic field.

The effects of the magnetic field on the spectral index were found to be tiny ($\lesssim 10^{-3}$) and are, thus, not shown. Moreover, the impact of $\Delta \text{MAGN}$ on the results is virtually negligible.

### 7 GENERAL REMARKS

A key aspect of the present work concerns the injection of VHE gamma rays in the intergalactic medium by TXS 0506+056. The fact that the combined observations by IceCube, MAGIC, and VERITAS (IceCube et al. 2018; Ansoldi et al. 2018; Abeysekara et al. 2018) show a flux suppression above $E \gtrsim 400$ GeV calls for an explanation. The first hypothesis is that this is due to EBL attenuation. The optical depth for a $\approx 100$ GeV photon for a source at $z = 0.34$ is $\sim 0.1$, meaning that this hypothesis alone does not account for the data. Another possibility is that the absorption is due to the presence of a weak broad line region (Padovani et al. 2019). If this is true, then the emitting region of TXS 0506+056 would be located in the outer regions of the object, thus explaining why the spectrum extends up to $\sim 400$ GeV; otherwise, these VHE photons would be completely absorbed if they were to traverse a much larger region before escaping into intergalactic space. A third hypothesis attributes the cut-off to intrinsic absorption (Keivani et al. 2018; Petropoulou et al. 2020). We suggest that the observed time dependence of gamma-ray fluxes from TXS 0506+056 is affected by intervening IGMFs. This, however, does not exclude any of the aforementioned possibilities.

We assumed that VHE gamma rays can indeed escape the blazar, which, as we argue below, is a valid approach, even though the details of the underlying mechanism are currently an active field of research. VHE gamma rays and neutrinos are associated to each other if they are created via processes that lead to the production of mesons (nucleus-nucleus and nucleus-photon interactions), whose decays are responsible for generating the observed particles. Murase et al. (2016) estimated the intrinsic optical depth ($\tau_{\gamma\gamma}$) for photons in the source to be $\tau_{\gamma\gamma}(E_0) \approx 1000\tau_{\gamma\gamma}(E_{\text{CR}})$, wherein $\tau_{\gamma\gamma}$ is the corresponding optical depth for photons production by protons, the prime indicates that the quantities should be taken in the co-moving frame, and $E_0 = 500(E_{\text{CR}}/100 \text{ PeV}) \text{ GeV}$ (see also Mannheim et al. 2001). Here, the factor 1000 stems from the ratio between the $\gamma\gamma$ and $\gamma p$ cross sections (cf. also fig. 5 in Murase et al. 2016 for an illustration, including some limiting cases). Thus, it is reasonable to expect the gamma-ray spectrum to extend up to $\beta$-energies — although energies much higher than 1 TeV do represent a theoretical challenge.

Reimer et al. (2019) claim that the photon fields within TXS 0506+056 required for the production of high-energy neutrinos would imply $\tau_{\gamma\gamma} \gg 1$ at $E_{\gamma} \sim 1 \text{ GeV}$ energies and, as a consequence, neutrinos and $\beta$-decays would not have the same origin. While this is indeed a plausible possibility, there is some freedom to change $\tau_{\gamma\gamma}$ and $E_{\text{CR}}$ at the source. Moreover, we could easily evade this particular constraint by invoking cosmic-ray nuclei instead of nucleons. It suffices to demonstrate the plausibility of scenarios with $E_{\text{max}} \gtrsim 1 \text{ TeV}$ during the flaring state, and point out the limitations inherent to simple one-dimensional single- or few-zone models that neglect IGMFs, such that any conclusions relying on these simplified models ought to be extrapolated with caution.

The picture changes if we consider cosmic-ray nuclei instead (see e.g. Rodrigues et al. 2018). In this case, $\beta$-decays during nuclear cascades triggered by photodisintegration may provide a significant contribution to the neutrino...
flux and the main gamma-ray production channel (photomeson production) may become subdominant. One could also consider a neutral beam, as done by Zhang et al. (2020). In this case, however, the gamma-ray flux would be much lower compared to the neutrino one, given that pairs generated via Bethe-Heitler process would respond for a large fraction of the total gamma-ray flux.

Padovani et al. (2018) found that gamma rays from the object PKS 0502+049 contaminate the signal at energies $\lesssim$ a few GeV. If a part of the flux measured by Fermi-LAT could, indeed, be attributed to this source, our results would change quantitatively. Nevertheless, this effect would be small, since only a few data points would be affected.

The determination of the maximal energy attainable by cosmic rays in blazars such as TXS 0506+056 is an important issue, given its intrinsic connection with ultra-high-energy cosmic rays (UHECRs; $E \gtrsim 1$ EeV), whose origins elude us (see Alves Batista et al. 2019a for a review). This connection, in the context of blazars, has been discussed by Resconi et al. (2017); Rodrigues et al. (2018); Murase et al. (2014); Padovani et al. (2015). As pointed out by Keivani et al. (2018), the X-ray emission in the 0.1–100 keV band, combined with the HE and VHE data, seems to dis-favour the possibility that this object would be an UHECR source, with $E_{\text{CR}} \gtrsim 1$ EeV. This connects with another issue regarding the classification of TXS 0506+056 as a BL Lac or FSRQ (see Padovani et al. 2019). Neutrino production is more efficient in the latter, due to the rapid photodisintegration of nuclei (Palladino et al. 2019), whereas the former may, indeed, accelerate cosmic rays to ultra-high energies (Rodrigues et al. 2018; Yoshida & Murase 2020).

In light of our results, claims of correlations between HE neutrinos and HE/VHE gamma rays must be carefully made if the objects in question are not steady gamma-ray sources. For instance, Kadler et al. (2016) found a temporal correlation between the blazar PKS 1424+240 and neutrino events. This object is located at $z = 0.60 \pm 1.20$ (Rovero et al. 2016), so the VHE part of the gamma-ray flux is attenuated by the EBL and reprocessed to lower energies, potentially arriving many years after the emission, depending on the properties of the IGMFs (Neronov & Semikoz 2009). Similar associations were reported by ANTARES Collaboration (2012), whose results suggest coincidences between neutrino events and Fermi-LAT flaring blazars. Nevertheless, more recent analyses using a larger data set do not confirm the previous findings (ANTARES Collaboration 2015; Ayala So-
las et al. 2019). In these cases, like in the one we studied (TXS 0506+056), the actual contribution of cascade photons to the gamma-ray flux at the \( \sim \) GeV–TeV band depends on the maximal energy and spectral index of the gamma rays escaping the source during the flaring activity, but if \( E_{\text{max},h} \gtrsim 1 \) TeV, this contribution may be dominant at 1-100 GeV.

In our simulations we included energy losses due to synchrotron emission by electrons interacting with IGMFs, but we did not compute the associated spectrum. One could argue that this contribution could affect the X-ray part of the spectral energy distribution of TXS 0506+056. While this is a valid concern, the total irradiated synchrotron power is very small, namely tens of orders of magnitude below measurements by Swift/NuSTAR (IceCube et al. 2018; Keivani et al. 2018).

Our simulations were restricted to the case in which the blazar jet points exactly towards Earth, with no misalignment. Multi-wavelength fits of the SED of TXS 0506+056 suggest a small misalignment angle of \( \theta_{\text{los}} \simeq 0.8^\circ \) (Ansoldi et al. 2018). Moreover, to reduce the computational load we have neglected the jet opening angle of the object. In general, this angle is \( \theta_{\text{jet}} \sim 1^{-1} \), where \( \Gamma \) is the bulk Lorentz factor of the jet. For TXS 0506+056, this angle is estimated to be \( \theta_{\text{jet}} \simeq 2.5^\circ \) (Ansoldi et al. 2018; Sahakyan 2018; Keivani et al. 2018; Gao et al. 2019).

Recent work by Halzen et al. (2019) presents a phenomenological model similar to ours (see eq. 1). The authors claim a successful description of the observations for \( E_{\text{max},l} \simeq 100 \) GeV and \( E_{\text{max},h} \simeq 1 \) TeV, for \( B \approx 10^{-19} \) G and \( L_{\text{c}} \approx 1 \) Mpc, analysing the 2014/2015 flare of TXS 0506+056 in their studies, together with the 10-year Fermi-LAT flux that we also considered. These results are order-of-magnitude compatible with ours. However, we found stronger magnetic fields \( B \simeq 10^{-16} \) G for \( L_{\text{c}} \sim 0.1 \)–10 Mpc to provide better description of the data. This is not surprising, given that our analysis considers the 2017 flare and, in addition to Fermi-LAT data, takes into account the observations by MAGIC (Ansoldi et al. 2018). Furthermore, we have scanned a much broader range of magnetic-field parameters, as shown in figure 3, using detailed three-dimensional simulations.

Comprehensive multi-messenger studies of blazar flares such as that of TXS 0506+056, but with much larger samples, will enable us to properly infer the value of the cut-off energy \( E_{\text{max}} \). Current observatories such as HAWC (HAWC Collaboration 2017), as well as the upcoming Cherenkov Telescope Array (Cherenkov Telescope Array Consortium et al. 2019) will have enough sensitivity in the 0.1–100 TeV region to provide temporal information at these energies. In particular, electromagnetic follow-ups of high-energy neutrino events from flaring blazars with real-time networks such as AMON (Ayala Solares et al. 2020) will play a key role in understanding VHE emission by blazars.

8 SUMMARY AND OUTLOOK
We have here made the case for high-energy gamma-ray emission by TXS 0506+056 during the 2017 flare. We argued that the 1-100 GeV region of the measured flux may contain a significant contribution of cascade photons produced in electromagnetic cascades in the intergalactic medium. Some of these secondary photons may suffer from very large time delays and angular spreading due to the intergalactic magnetic field.

Our central thesis that the determination of the maximal intrinsic gamma-ray energy \( E_{\text{max},h} \) during the flaring state is influenced by IGMFs holds regardless of the actual mechanism responsible for the apparent flux suppression above \( \gtrsim 400 \) GeV. Nevertheless, there are theoretical arguments that make high-energy gamma-ray emission by blazars inefficient if neutrinos are efficiently produced. If this is the case, then \( E_{\text{max},h} \) may be relatively low, and so can be the contribution of secondary cascade photons. Either way, it is worth considering that IGMFs may influence the inferred intrinsic spectral properties of TXS 0506+056 and other objects.

In the future we plan to carry out a more thorough analysis, similar to this one, for other sources. Moreover, we intend to perform a more general combined analysis performing the usual SED fits using, for instance, one-zone models, in addition to propagation effects due to IGMFs.

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DATA AVAILABILITY
The data underlying this article will be shared on reasonable request to the corresponding author.

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