Electron-acoustic solitary structures in two-electron-temperature plasma with superthermal electrons

H. Chen • S. Q. Liu

Abstract The propagation of nonlinear electron-acoustic waves (EAWs) in an unmagnetized collisionless plasma system consisting of a cold electron fluid, superthermal hot electrons and stationary ions is investigated. A reductive perturbation method is employed to obtain a modified Korteweg–de Vries (mKdV) equation for the first-order potential. The small amplitude electron-acoustic solitary wave, e.g., soliton and double layer (DL) solutions are presented, and the effects of superthermal electrons on the nature of the solitons are also discussed. But the results shows that the weak stationary EA DLs cannot be supported by the present model.

Keywords superthermal electrons; Electron-acoustic solitary waves; Double layers; mKdV equation

1 Introduction

The EAWs can either exist in a two temperature (cold and hot) electron plasma (Watanabe and Taniuti, 1977; Yu and Shukla, 1983) or in an electron-ion plasma with ions hotter than electrons (Fried and Gould, 1961). EAWs are typically high-frequency (by comparison with the ion plasma frequency), dispersive plasma waves where the relatively cold inertial electrons oscillate against a thermalized background of inertialless hot electrons which provide the necessary restoring force.

In the long-wavelength approximation, the dispersion relation of EAWs is given as \( \omega = k \frac{n_{c0}}{n_{h0}} \frac{1}{2} v_{th} \), where \( v_{th} = (\kappa_B T_h/m_e)^{1/2} \) is the hot electron thermal speed, \( \kappa_B \) is the Boltzmann constant, and \( n_{c0}(n_{h0}) \) are the cold (hot) equilibrium electrons densities. The phase speed \( c_s = (n_{c0}/n_{h0})^{1/2} v_{th} \) of EAWs must be intermediate between cold and hot electron thermal velocities so that the Landau damping is avoided. In the Maxwellian plasmas, Gary and Tokar (1985) performed a parameter survey and found that the hot electron component constitutes a non-negligible fraction of the total electron density (more than \( \sim 20\% \)) for the existence of the EAWs. And for the Lorentzian plasmas, Mace and Hellberg (1990) showed that the EAWs were usually strongly Landau damped by the hot electrons, unless \( n_{c0}/n_{h0} \ll 1 \).

In the nonlinear wave studies, the propagation of solitary waves is important as it describes the characteristics of interaction between waves and plasmas. Solitary waves are localized nonlinear wave phenomena which arise due to a delicate balance between nonlinearity and dispersion. Among the best known paradigms used to investigate small-amplitude nonlinear wave behavior are different versions of KdV equation (Washimi and Taniuti, 1966), or nonlinear Schrödinger equation (NLSE) (Hasegawa, 1975). Some form of reductive perturbation technique is used to derive such equations. The KdV or mKdV describes the evolution of unmodulated wave, while the NLSE governs the dynamics of a modulated wave packet. In addition, for the arbitrary amplitude solitary waves, the Sagdeev pseudopotential method (Sagdeev, 1966) is used too. Electrostatic solitary structures are often observed in the space and laboratory plasma environment. EA soliton has been considered as one of the possible candidates for some of the observed solitary structures. Recently, the propagation of the linear as well as nonlinear EAWs has received a great deal of renewed interest not only...
because the two electron temperature plasma is very common in laboratory experiments (Derfler and Simonen, 1969; Henry and Treguier, 1972) and in space (Dubouloz et al., 1991, 1993; Pottelette et al., 1999; Berthomier et al., 2000; Singh and Lakhina, 2001), but also because of the potential importance in interpreting electrostatic component of the broadband electrostatic noise (BEN) as being solitary EA structures observed in the cusp of the terrestrial magnetosphere (Tokar and Gary, 1984; Singh and Lakhina, 2001), in the geomagnetic tail (Schriver and Ashour-Abdalla, 1989), in auroral region (Dubouloz et al., 1991, 1993; Pottelette et al., 1999), in the numerical simulation (Lu, Wang and Dou, 2005; Lu, Wang and Wang, 2005), and in laboratory experiment (Lefebvre, et al., 2011), etc.

On the other side, since Alfvén and Carlqvist (1967) had suggested the current disruption theory for solar flare, the subject of DL (sometimes also called shock or kinks) has attracted great attention (Li, 1984; Liu, 2010; reference therein). DLs occur naturally in a variety of space plasma environments. It turns out that DLs have the electrostatic potential and other relevant parameters monotonically changing from one value at one extreme to another at the other end, hence “kinks”. This is associated with adjacent positive and negative charge regions, which give rise to the name “double layers”. Such DLs are more difficult to generate and require a fine tuning of the plasma parameters, hence a more complicated plasma compositions with enough leeway to obey the necessary constraints (Verheest, 2006; Hellberg, 1992; Moslem, 2007).

The study of nonlinear EAWs has been focused by many authors with different particle distribution, i.e., Cairns distribution (Pakzad and Tribeche, 2010), Vortex-like distribution (Mamun and Shukla, 2002), q-nonextensive distribution (Gougam and Tribeche, 2011), quantum plasmas (Masood and Mushfaq, 2008), et al. And it had been found that the particles distributions play a crucial role in characterizing the physics of nonlinear waves.

Recently, the plasma with superthermal particles has gained much attention. Superthermal electrons are often observed in laboratory, space, and astrophysical plasma environments, viz., the ionosphere, auroral zones, mesosphere, lower thermosphere, etc (Pierrard, 2010, and reference therein). The Kappa functions (Vasyliunas, 1968) characterized by the spectral index $\kappa$ are found to represent more suitably the particle’s velocity distributions observed in number of space and astrophysical environment. The Kappa function may recover the Maxwellian distribution in the limit of $\kappa \rightarrow \infty$ and its mathematical characteristics and physical origin have recently been addressed by Hau and Fu (2007, the references within). It is worth noting that some theoretical work focused on the effects of superthermal particles on different types of linear and nonlinear collective processes in plasmas. For instance, the linear properties of plasmas in the presence of a Kappa distribution with excess superthermal particles have been investigated rather extensively (Summers and Thorne, 1991; Mace and Hellberg, 1995).

More recently, employing a Sagdeev pseudopotential method, the nonlinear arbitrary amplitude EAWs were studied, the weak stationary solitons and DLs were also given by expanding the Sagdeev potential in small amplitude limit (Sahu, 2010; Younsi and Tribeche, 2010). In their model, the plasma systems were assumed consisting of cold fluid electrons, superthermal hot electrons and stationary ions. The same procedure also was founded for the $q$-nonextensive distributed hot electrons plasma system (Gougam and Tribeche, 2011). Baboolal et al. (1991) have showed that, when ion-acoustic DLs in a plasma with negative ions are considered, one must be especially careful to ensure that one’s solutions meet the criteria for convergence of the original expansions. Verheest (1993) has arrived at a similar conclusion considering DLs in dusty plasmas. Mace and Hellberg (1993) showed that EA DLs can not be supported in an infinite, homogeneous, unmagnetized and collisionless plasma system consisting of cool fluid ions, cold fluid electrons and hot Boltzmann distributed electrons based on the mKdV model. Some same conclusions were also can be found (Hellberg, et al., 1992).

Thus, with these ideas in mind, we re-investigate here the small but finite amplitude solitary structures in such two-electron-temperature plasma with superthermal electron based on the mKdV model. One of our objective here is to study the nonlinear effects of superthermal distribution of hot electrons on the nature of the small amplitude solitary waves. Another one is to show whether the DL solution exist or not. This paper is organized as follows: in Section 2, the basic set of equations is introduced. In Section 3, we derive the mKdV equation, and the solutions of both solitons and DLs are given. Finally, some conclusions and discussions are given in Section 4.

2 Basic equation

We consider a homogeneous system of an unmagnetized collisionless plasma consisting of a cold electron fluid, and superthermal hot electrons obeying a Kappa distribution, and ions. It is well known to us that the linear spectrum of EAWs extends only up to the plasma frequency of the cold electron population, $\omega_{pe} =$
\[ \sqrt{4\pi n_0 e^2/m_e}, \] so at this high frequency, the ion population plays the role of a neutralizing background. For a small—but finite amplitude waves propagate one dimensionally, such system is governed by the following normalized equations:

\[ \frac{\partial}{\partial t} n_c + \frac{\partial}{\partial x} (n_c u_c) = 0, \]
\[ \frac{\partial u_c}{\partial t} + u_c \frac{\partial u_c}{\partial x} = \frac{\partial \varphi}{\partial x}, \]
\[ \frac{\partial^2 \varphi}{\partial x^2} = 1 \frac{\partial n_c}{\partial x} + n_h - \left( \frac{1 + 1}{\alpha} \right), \]
and the super-thermal hot electrons density \( n_h \) is given by (Saini, 2009; Rios, 2010):

\[ n_h = \left( 1 - \frac{\varphi}{\kappa - 3/2} \right)^{-(\kappa - 1/2)}, \]

where \( n_c(n_h) \) are cold (hot) electrons normalized densities to the total unperturbed density \( n_0 = n_{c0} + n_{h0}, u_c \) is the velocity of cold electrons normalized to the EA velocity \( c_s, \varphi \) is the electrostatic potential to \( \kappa B T_h/e \), respectively. Time and space variables are normalized, respectively, to the inverse of cold electron plasma frequency \( \omega_{pe}^{-1} \) and the hot electron Debye length \( \lambda_{De} = \sqrt{\kappa B T_h/4\pi n_{h0} e^2} \). The parameter \( \alpha = n_{h0}/n_{c0} \). It should be noted that the superthermal hot electron component constitutes a non-negligible fraction of the total electron density, as mentioned in the introduction.

### 3 The mKdV equation and the solution

Nonlinear electron-acoustic waves are governed by the full set of Eqs. [1] and [2]. To derive the mKdV equation describing the behavior of the system for longer times and small but finite amplitude EAWs, we employ the familiar reductive perturbation technique (Washimi and Taniuti, 1966; Mace and Hellberg, 1993). From the usual considerations of the small-wavenumber dispersion relation for electron-acoustic waves, we introduce the slow stretched coordinates: \( \xi = \varepsilon (x - \lambda t), \tau = \varepsilon^3 t, \) where \( \varepsilon \) is a small dimensionless expansion parameter and \( \lambda \) is the wave speed normalized by \( c_s \). All physical quantities appearing in Eq. [1] are expanded as a power series in \( \varepsilon \) about their equilibrium values as:

\[
\begin{pmatrix}
  n_c \\
  u_c \\
  \varphi
\end{pmatrix}
= \begin{pmatrix}
  1 \\
  0 \\
  0
\end{pmatrix}
+ \varepsilon \begin{pmatrix}
  n_c^{(1)} \\
  u_c^{(1)} \\
  \varphi^{(1)}
\end{pmatrix}
+ \varepsilon^2 \begin{pmatrix}
  n_c^{(2)} \\
  u_c^{(2)} \\
  \varphi^{(2)}
\end{pmatrix}
+ \cdots.
\]

We impose the boundary conditions that as \( |\xi| \to \infty, n_c = 1, u_c = 0, \varphi = 0 \). Substituting Eq. [3] into the system of Eq. [1], and equating coefficients of different powers of \( \varepsilon \), from the lowest-order equations in \( \varepsilon \), the following results are obtained:

\[
n_c^{(1)} = 0, \quad u_c^{(1)} = -\frac{\alpha \varphi^{(1)}}{\lambda^2}, \quad u_c^{(2)} = 0, \quad \varphi^{(2)} = 0.
\]

Eqs. [2] and [4] as well as Poisson equation give the linear dispersion relation as

\[
\lambda = \sqrt{\frac{2\kappa - 3}{2\kappa - 1}}.
\]

From the next order of \( \varepsilon \), with the aid of equation [10], we obtain the following equations:

\[
n_c^{(2)} = \frac{3\alpha^2}{2\lambda^3} \left( \varphi^{(1)} \right)^2 - \frac{\alpha}{\lambda^2} \varphi^{(2)},
\]

\[
u_{c}^{(2)} = \frac{\alpha^2}{2\lambda^3} \left( \varphi^{(1)} \right)^2 - \frac{\alpha}{\lambda} \varphi^{(2)},
\]

Poisson equation with the help of Eq. [2] at \( O(\varepsilon^2) \) yields

\[
A \left( \varphi^{(1)} \right)^2 = 0; \quad A = \frac{3\alpha}{2\lambda^3} + \frac{(2\kappa - 1)(2\kappa + 1)}{2(2\kappa - 3)^2}.
\]

Since we assume that \( \varphi^{(1)} \neq 0 \), it follows that at least \( |A| \sim O(\varepsilon) \). Hence it should be included in the next higher order, i.e., \( O(\varepsilon^3) \) of Poisson equation. Collecting the third order of \( \varepsilon \), that in \( O(\varepsilon^3) \), from equations [1] and [2] using [3]-[7], we obtain

\[
\frac{\partial}{\partial \xi} n_c^{(3)} = \frac{\partial}{\partial \tau} \left( -\frac{2\alpha}{\lambda^3} \varphi^{(1)} \right) + \frac{5\alpha^3}{2\lambda^5} \left( \varphi^{(1)} \right)^3 + \frac{3\alpha^2}{\lambda^3} \varphi^{(2)} \varphi^{(1)} - \frac{1}{\lambda^2} \alpha \varphi^{(3)},
\]

\[
n_{c}^{(3)} = \frac{2\kappa - 1}{2\kappa - 3} \varphi^{(3)} + \frac{(2\kappa - 1)(2\kappa + 1)}{(2\kappa - 3)^2} \varphi^{(2)} \varphi^{(1)} + \frac{(2\kappa - 1)(2\kappa + 1)(2\kappa + 3)}{6(2\kappa - 3)^3} \left( \varphi^{(1)} \right)^3.
\]

Eliminate the third-order perturbed quantities \( n_c^{(3)} \), \( u_c^{(3)} \), and \( \varphi^{(2)} \), we obtain the following mKdV equation for the first-order perturbed potential:

\[
\frac{2}{\lambda^4} \frac{\partial}{\partial \tau} \varphi + 3A_{2} \frac{\partial}{\partial \xi} \varphi = 4A_{3} \frac{\partial}{\partial \xi} \varphi^3 + \frac{\partial^3}{\partial \xi^3} \varphi = 0,
\]

with the coefficients read as

\[
A_2 = -\left( \frac{\alpha}{2\lambda^4} + \frac{(2\kappa - 1)(2\kappa + 1)}{6(2\kappa - 3)^2} \right).
\]
In Eq. (11), \( \varphi \) is used in place of \( \varphi^{(1)} \) for brevity. Let us introduce the variable, \( \eta = \xi - M_0 \tau \), where \( \eta \) is the transformed coordinates with respect to a frame moving with velocity \( M_0 \), for the steady-state solution of the mKdV equation (11). By using the boundary conditions \( \varphi \rightarrow 0 \) and \( d\varphi/d\eta \rightarrow 0 \) at \( |\eta| \rightarrow \infty \), we obtain

\[
\frac{1}{2} \left( \frac{d\varphi}{d\eta} \right)^2 + V(\varphi) = 0, \quad (14)
\]

where \( V(\varphi) \) is the Sagdeev pseudopotential (Sagdeev, 1966), reads as

\[
V(\varphi) = A_1 \varphi^2 + A_2 \varphi^3 + A_3 \varphi^4, \quad (15)
\]

with \( A_1 = -M_0/\lambda^3 \). It should be noted that when \( M_0 \ll \lambda \), the result of Sahu (2010), i.e., in which Eq.(18) is recovered for the small amplitude solitary waves by Sagdeev pseudopotential approach.

### 3.1 Solitons solution

If one neglects the term corresponding to \( \varphi^4 \) in the Sagdeev pseudopotential (15), the solution of Eq.(14) is

\[
\varphi = \varphi_m \sec h^2 \left( \frac{\eta}{\Delta} \right). \quad (16)
\]

where the soliton peak amplitude \( \varphi_m \) and width \( \Delta \) are given by \( \varphi_m = -A_1/A_2 \), \( \Delta = 2/\sqrt{2A_1} \). Solution (16) represents a small-amplitude stationary EAWs provided \( A_1 < 0 \) or \( M_0 > 0 \), which means that the solitary waves are supersonic, in agreement with the large amplitude case. It is clear that the nature of the solitary waves, i.e., whether the system will support compressive or rarefactive solitary waves, depends on the sign of \( A_2 \). If \( A_2 \) is positive (negative) a compressive (rarefactive) solitary wave exists. In our present case, \( A_2 < 0 \) since \( \kappa > 3/2 \) for a physically realistic thermal speed (Summers and Thorne, 1991; Mace and Hellberg, 1995), so that here we would have a rarefactive soliton.

### 3.2 Double layer solution

For the DL solution, the Sagdeev potential must satisfy the conditions (Mace and Hellberg, 1993), (a): \( V(\varphi) = 0 \) at \( \varphi = 0 \) and \( \varphi = \varphi_m \); (b): \( V'(\varphi) = 0 \) at \( \varphi = 0 \) and \( \varphi = \varphi_m \); and (c): \( V''(\varphi) < 0 \) at \( \varphi = 0 \) and \( \varphi = \varphi_m \). Applying the boundary conditions (a) and (b) into Eq.(15), we obtain

\[
\varphi_m = -\frac{A_2}{2A_3}, \quad M_0 = -\frac{(A_2)^2}{4A_3} \lambda^3, \quad (17)
\]

then Sagdeev pseudopotential (15) can be written as

\[
V(\varphi) = A_3 \varphi^2 \left( \varphi_m - \varphi^2 \right). \quad (18)
\]

The DL solution is then given by (if it exists)

\[
\varphi = \frac{\varphi_m}{2} \left[ 1 - \tanh \left( \frac{2\eta}{\Delta} \right) \right], \quad (19)
\]

with \( \Delta = \sqrt{-8/A_3/|\varphi_m|} \) represents the width of the DL provided \( A_3 < 0 \), i.e.,

\[
\frac{5\alpha^2}{8\lambda^6} - \frac{(2\kappa - 1) (2\kappa + 1) (2\kappa + 3)}{24 (2\kappa - 3)^3} < 0. \quad (20)
\]

For a DL exists, the condition (20) must be fulfilled, then we have that the hot electron concentration \( \alpha \) must satisfy \( \alpha < \sqrt{\frac{(2\kappa + 1)(2\kappa + 3)}{15(2\kappa - 1)^2}} \). It is indicates that \( \alpha < 1 \) in the both small and large value of superthermal index \( \kappa \). At such large cold electron density ratio \( (n_{e0}/n_{i0}) > 1 \), linear Landau damping by the cold electrons would become appreciable (Gary and Tokar, 1985; Mace and Hellberg, 1990). Furthermore, we note that for this case the mKdV model leads to a value of \( |\varphi_m| \sim 1 \) in the small superthermal index case and \( |\varphi_m| > 1 \) in the large superthermal index case, which is shown in the Fig.1. It clearly does not satisfy the expanding requirement \( |\varphi_m| \sim \varepsilon \). From the aforementioned considerations, we are let to conclude that Eq.(19) is not a valid solution of the mKdV equation for electron-acoustic waves in the present model. So we can draw the conclusion that the present model may not sustain a double layer structure. Such a point has also been studied extensively by Mace and Hellberg (1993) for the two-temperature electrons Maxwellian plasmas. Indeed, a systematic investigation should be be undertook to verify this conclusion, but it beyond the scope of present paper.

### 4 Remarks and Conclusion

To investigate the existence regions and nature of the EA solitary structures, we have done numerical calculations for different set of parameters.

Figs.2 and 3 present the effects of the superthermal index \( \kappa \) and hot electron concentration \( \alpha \) on the amplitude and width of the solitons in the slightly supersonic point. It can be seen from Figs.2 and 3 that the decreasing the superthermal index \( \kappa \) and increasing in
hot electron concentration ($\alpha$) will decrease the amplitude of soliton.

In summary, we have studied the nonlinear EAWs in an unmagnetized, collisionless plasma consisting of cold electrons, superthermal hot electrons, and stationary ions. A reductive perturbation method has been used to get the mKdV equation which describes the dynamics of solitons and DLs. The effects of super-thermal index $\kappa$ and concentration $\alpha$ of the hot electrons to the cold electrons on the nature of the solitons are also discussed. It is seen that the effects of the superthermal hot electrons have a very significant role on the amplitude and width of the weak amplitude EA solitons. It is also found that the small amplitude DLs cease to exist in our present model. Considering the wide relevance of nonlinear oscillations, we stress that the results of the present investigation should be useful in understanding the nonlinear features of localized electrostatic acoustic structures in different regions of the laboratory experiments and space environments.

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Fig. 1 Plot of EA DLs amplitude $|\varphi_m|$ vs $\kappa$, where $\alpha = 0.3, 0.25, 0.2$ for solid, dashed, dotted line, respectively

Fig. 2 Small amplitude soliton for different $\kappa$, i.e, $\kappa = 3$ for solid line, $\kappa = 6$ for dotted line, and $\kappa = 15$ for dashed line, where the other parameters are $\alpha = 0.5$ and $M_0 = 0.1\lambda$.

Fig. 3 Small amplitude soliton for different $\alpha$, i.e, $\alpha = 0.3$ for solid line, $\alpha = 0.5$ for dotted line, and $\alpha = 1.0$ for dashed line, where the other parameters are $\kappa = 3$ and $M_0 = 0.1\lambda$. 
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