Abstract

Answering Conjunctive Queries (CQs) and solving Constraint Satisfaction Problems (CSPs) are arguably among the most fundamental tasks in Computer Science. They are classical NP-complete problems. Consequently, the search for tractable fragments of these problems has received a lot of research interest over the decades. This research has traditionally progressed along three orthogonal threads. a) Reformulating queries into simpler, equivalent, queries (semantic optimization) b) Bounding answer sizes based on structural properties of the query c) Decomposing the query in such a way that global consistency follows from local consistency.

Much progress has been made by various works that connect two of these threads. Bounded answer sizes and decompositions have been shown to be tightly connected through the important notions of fractional hypertree width and, more recently, submodular width. Recent papers by Barceló et al. study decompositions up to generalized hypertree width under semantic optimization. In this work, we connect all three of these threads by introducing a general notion of \textit{semantic width} and investigating semantic versions of fractional hypertree width, adaptive width, submodular width and the fractional cover number.

1 Introduction

Answering conjunctive queries (CQs) is one of the central themes of database theory. As the problem is known to be NP-complete in general, identifying tractable classes of CQs has been the focus of much research (see e.g., [8] and many references therein). The study of bounded decompositions and answer sizes in particular has produced many remarkable results and has significantly advanced our understanding of what makes CQs hard in general.

A largely separate line of research has followed the question of minimizing the query itself. Given a query it seems enticing to only want the minimal (in the number of atoms) version that produces the same result, i.e., a semantically equivalent query. This is referred to as semantic optimization. In a classic result, Chandra and Merlin [8] show that queries are (semantically) equivalent iff they are homomorphically equivalent. The minimal equivalent CQ is called the core of a query. However, while semantic optimization simplifies the query...
in a sense, even finding the minimal equivalent query still provides little information on the complexity of its execution.

The equivalence classes induced by semantic equivalence can be viewed as the collection of all possible ways to formulate the same question to the database (in the language of CQs). It is then natural to ask if some formulations can be answered more efficiently than others and how to identify and derive these efficiently answerable formulations. Work by Dalmau et al. [8] and Barceló et al. [4, 5] investigates the treewidth, acyclicity and generalized hypertree width (ghw) under semantic equivalence. Specifically in the case of acyclicity the term semantic acyclicity is used. This theme motivates the introduction of semantic width (for any notion of width) as a measure for the complexity of the underlying question to the database rather than the complexity of a specific formulation.

Definition 1. The semantic width of a conjunctive query \( q \) is the minimal width over all conjunctive queries that are equivalent to \( q \).

In this sense, the main goal of this work is the investigation of semantic fractional hypertree width, semantic adaptive width and semantic submodular width. Section 2 provides preliminaries and formal definitions of the central concepts. In Section 3 we introduce a formal version of Definition 1 and our central machinery. We show that the semantic fractional cover number (analogous to semantic width) is determined by the fractional cover number of the core in Section 4. The same result is then also derived, for all the semantic widths stated above, in Section 5.

2 Preliminaries

The initial definitions here are adapted from [10]. A relation schema \( R \) consists of a name \( r \) and an ordered list of attributes. An attribute \( a \) has an associated countable domain \( \text{dom}(a) \). A relation instance of a schema with attributes \( (a_1, \ldots, a_k) \) is a finite subset of \( \text{dom}(a_1) \times \cdots \times \text{dom}(a_k) \). The elements of relation instances are called tuples. A database schema is a finite set of relation schemas. A database (instance) \( D \) over a database schema \( \{R_1, \ldots, R_m\} \) consists of relation instances for every schema \( R_1, \ldots, R_m \). The universe of a database \( D \) is the set of all values occurring for attributes of the relation instances of \( D \).

A (rule based) conjunctive query \( q \) on a database schema \( DS = \{R_1, \ldots, R_m\} \) is a rule of the form \( q: \text{ans}(\overline{x}) \leftarrow r_1(\overline{x_1}) \land \cdots \land r_n(\overline{x_n}) \) where \( r_1, \ldots, r_n \) are relation names of \( DS \), \( \text{ans} \) is a relation name not in \( DS \) and \( \overline{x}, \overline{x_1}, \ldots, \overline{x_n} \) are ordered lists of terms matching the number of attributes of the respective relation schema. We refer to the atoms in the body as \( \text{atoms}(q) \) and the variables occurring in an atom \( R \) as \( \text{var}(R) \).

The answer of \( Q \) on a database \( D \) with universe \( U \) is a relation \( \text{ans} \) with attributes \( \overline{x} \). The tuples of \( \text{ans} \) are all tuples \( \text{ans}(\overline{x})\sigma \) such that: \( \sigma: \text{var}(Q) \rightarrow U \) is a substitution and \( r_i(\overline{x_i})\sigma \) is in a relation instance of \( D \) for \( 1 \leq i \leq n \). A substitution \( \sigma \) is applied to an atom \( A \) by replacing each variable \( X \) in \( A \) with \( \sigma(X) \).

For conjunctive queries \( q_1, q_2 \), a homomorphism from \( q_1 \) to \( q_2 \) is a mapping \( f: \text{var}(q_1) \rightarrow \text{var}(q_2) \cup \text{constants} \) such that:
1. For every variable \( x \) in the head of \( q_1 \), \( f(x) = x \)
2. For every atom \( R(x_1, \ldots, x_k) \in \text{atoms}(q_1) \) there exists an atom \( R(f(x_1), \ldots, f(x_k)) \in \text{atoms}(q_2) \).

If there exists a homomorphism in both directions we say \( q_1 \) and \( q_2 \) are homomorphically equivalent, we write \( q_1 \simeq q_2 \).

We call CQs equivalent if they have the same answer over any database instance. It is known that two queries are equivalent if they are homomorphically equivalent [5]. For
a CQ $q$, a minimal (in the number of atoms) equivalent CQ is called a core. All cores of $q$ are isomorphic and it is therefore common to refer to the core of $q$ (we sometimes write $\text{Core}(q)$). Another observation by Chandra and Merlin is important in our context: The core can always be obtained by an endomorphism on the query or equivalently by deleting atoms.

A hypergraph $H = (V(H), E(H))$ is a pair, where $V(H)$ is a the set of vertices and the set of hyperedges $E(H)$ is a set of subsets of $V(H)$. In this work we assume hypergraphs are finite, and all vertices are contained in some hyperedge (there are no isolated vertices). We write $I_v$ for the set of all incident edges of a vertex $v$. A homomorphism $G \rightarrow H$ for hypergraphs is a mapping $f: V(G) \rightarrow V(H)$ s.t. if $e \in E(G)$, then $\{f(v) \mid v \in e\} \in E(H)$. Function application is extended to hyperedges and sets of hyperedges in the usual, element-wise, fashion: for instance, for $e \in E(G)$, we write $f(e)$ to denote $\{f(v) \mid v \in e\}$. Likewise, for $E' \subseteq E(G)$, we write $f(E)$ to denote $\{f(e) \mid e \in E'\}$. Note that if two CQs are homomorphic, then also their associated hypergraphs are homomorphic, while the converse is, in general, not true.

A fractional edge cover $x$ of a set $W \subseteq V(H)$ for a hypergraph $H = (V(H), E(H))$ is a (not necessarily optimal) solution of the linear program:

\[
\begin{align*}
\text{minimize} & \quad \sum_{e \in E} x_e \\
\text{subject to} & \quad \sum_{e \in \gamma(v)} x_e \geq 1 \quad \text{for all } v \in W \\
& \quad x_e \geq 0 \quad \text{for all } e \in E(H)
\end{align*}
\]

If no subset is specified the cover of all vertices is assumed. We call the result of the objective function the total weight of a cover. The minimal total weight of a fractional edge cover of $H$ is the fractional edge cover number $\rho^*(H)$. For a CQ $q$, $\rho^*(q)$ is the fractional cover number of the hypergraph associated with $q$.

### 3 Semantic Properties and Core Minimal Functions

As stated in the introduction, our main interest is the complexity of the underlying question posed by a query. To this end we move away from looking at specific queries that implement the question and instead consider the whole equivalence class of queries with the intended output. Instead of looking at the width of a query we now want to study the semantic width of a query, the width of the most efficient way to equivalently formulate the query.

- **Definition 2.** Let $Q$ be the class of all conjunctive queries and $w: Q \rightarrow \mathbb{R}^+$. We define the **semantic variant of $w$** as $\text{sem-}w(q) := \inf\{w(q') \mid q' \simeq q\}$.

  The previous definition also illustrates one of the main issues of semantic width, as there are infinitely many equivalent CQs, it is inherently unclear how it can be computed. The rest of this section provides a framework to determine these semantic variants in a more practical manner.

  Barceló et al. investigate what they call the reformulation problem for ghw in [3]. This is the problem whether, given a CQ $q$, there exists an equivalent query with a ghw less or equal to some specified threshold. We generalize their reformulation result to the following Lemma [4] for a more general class of functions.

- **Definition 3.** Let $Q$ be the class of all conjunctive queries. We call a function $w: Q \rightarrow \mathbb{R}^+$ **core minimal** if it is invariant under isomorphisms and for any $q \in Q$: $w(\text{Core}(q)) \leq w(q)$.
Lemma 4. Fix \( k \geq 1 \), and let \( w \) be a core minimal function. For each conjunctive query \( q \) the following are equivalent:

1. There exists a \( q' \) equivalent to \( q \) with \( w(q') \leq k \).
2. \( w(\text{Core}(q)) \leq k \).

Proof. The core of \( q \) is always equivalent to \( q \) and therefore the upward implication follows. For the downward implication \( w(\text{Core}(q')) \leq w(q') \) by definition. If \( q' \) is equivalent to \( q \), then their cores must be isomorphic, thus \( w(\text{Core}(q)) = w(\text{Core}(q')) \leq w(q') \leq k \). ▶

It is easy to see that \( w \) being core minimal is in fact also a necessary condition for Lemma 4 above. It also follows that for core minimal functions, the minimal value among equivalent queries is always found in the core. This leads us to the following convenient lemma.

Lemma 5. A function \( w \) is core minimal if and only if for all conjunctive queries \( q \) \( \text{sem}-w(q) = w(\text{Core}(q)) \).

Proof. The implication from left to right is immediate from Lemma 4. For the other direction we note that if \( q' \simeq q \), then \( \text{sem}-w(q') \leq w(q) \) by definition. Thus, from \( \text{Core}(q) \simeq q \) we see \( w(\text{Core}(q)) = \text{sem}-w(q) \leq w(q) \). ▶

## 4 Edge Covers for Homomorphic Hypergraphs

We prove that the fractional cover number is core minimal as a consequence of the fact that fractional edge covers of hypergraphs are preserved by homomorphisms. This fact is used again in the proof of our result for semantic fractional hypertree width. By showing that \( \rho^* \) is core minimal we also determine \( \text{sem}-\rho^* \).

Lemma 6. Let \( f \) be a homomorphism from \( G \) to \( H \). Given a fractional edge cover \( x \) of \( G \), define \( x' \) s.t.

\[
x'_h = \sum_{g \in f^{-1}(h)} x_g \quad h \in E(H).
\]

Then \( x' \) is a fractional edge cover of \( f(V(G)) \) with the same total weight as \( x \).

Proof. We first show that \( x' \) is fractional edge cover. To see this, choose an arbitrary \( w \in f(V(G)) \). For every \( v \in f^{-1}(w) \), we have that \( \sum_{g \in I_v} x_g \geq 1 \). For every \( E \subseteq E(G) \), \( E \subseteq f^{-1}(f(E)) \) and, therefore, we also have

\[
\sum_{h \in f(E)} x'_h = \sum_{h \in f(E)} \sum_{g \in f^{-1}(h)} x_g \geq \sum_{g \in f(E)} \sum_{g \in f^{-1}(f(E))} x_g \geq \sum_{g \in E} x_g.
\]

From this we conclude:

\[
\sum_{h \in I_w} x'_h \geq \sum_{h \in f(I_v)} x'_h \geq \sum_{g \in I_v} x_g \geq 1
\]

The leftmost inequality holds, because \( f(I_v) \subseteq I_w \). The rightmost inequality holds, because we are assuming that \( x \) is a fractional edge cover of \( G \). We have thus shown that \( x' \) covers \( w \). Since \( w \in f(V(G)) \) was arbitrarily chosen, we conclude that \( x' \) is a fractional edge cover of \( f(V(G)) \).

To see that the total weights of both covers are the same, observe:

\[
\sum_{h \in f(E(G))} x'_h = \sum_{h \in f(E(G))} \sum_{g \in f^{-1}(h)} x_g = \sum_{g \in E(G)} x_g
\]
The right equality follows from the fact that every edge of $G$ is present in exactly one set $f^{-1}(h)$, i.e., for $E = E(G)$, we actually have $E = f^{-1}(f(E))$.

\begin{lemma}
The fractional edge cover number $\rho^*$ of a conjunctive query is core minimal.
\end{lemma}

\begin{proof}
Let $G$ be the hypergraph of $q$ and $H$ be the hypergraph of $Core(q)$. Since there is a surjective homomorphism from $q$ to $Core(q)$, there exists a surjective homomorphism from $G$ to $H$. Then, by Lemma [6] for any fractional edge cover of $G$ there exists a cover of $H$ with equal weight.
\end{proof}

\begin{theorem}
For all conjunctive queries $q$:

$$\text{sem-\rho}^*(q) = \rho^*(Core(q)).$$
\end{theorem}

\section{Core Minimal Notions of Widths}

In this section we show that fractional hypertree width, adaptive width and submodular width are all core minimal. The proof is constructive by transforming tree decompositions of a query to tree decompositions of its core in a way that can only decrease the width of the decomposition.

We follow Marx [14] in our definitions of various notions of widths. A tuple $(T, (B_u)_{u \in V(T)})$ is a \textit{tree decomposition} of a hypergraph $H$ if $T$ is a tree, every $B_u$ (the bags) is a subset of $V(H)$, for every $e \in E(H)$ there is a node in the tree s.t. $e \subseteq B_u$, and for every vertex $v \in V(H)$, $\{u \in V(T) \mid v \in B_u\}$ is connected in $T$. For functions $f : 2^{V(H)} \rightarrow \mathbb{R}^+$, the \textit{f-width} of a tree decomposition is $\sup\{f(B_u) \mid u \in V(T)\}$ and the $f$-width of a hypergraph is the minimal $f$-width over all its tree decompositions. Let $\mathcal{F}$ be a class of functions from subsets of $V(H)$ to the non-negative reals, then the $\mathcal{F}$-width of $H$ is $\sup\{f\text{-width}(H) \mid f \in \mathcal{F}\}$. All such widths are implicitly extended to conjunctive queries by taking the width of the associated hypergraph.

The following properties of functions $b : 2^{V(H)} \rightarrow \mathbb{R}^+$ are important:

\begin{itemize}
  \item $b$ is called \textit{submodular} if $b(X) + b(Y) \geq b(X \cap Y) + b(X \cup Y)$ holds for every $X \subseteq V(H)$.
  \item $b$ is called \textit{modular} if $b(X) + b(Y) = b(X \cap Y) + b(X \cup Y)$ holds for every $X \subseteq V(H)$.
  \item $b$ is called \textit{edge-dominated} if $b(e) \leq 1$ for every $e \in E(H)$.
  \item Finally, $b$ is \textit{monotone} if $X \subseteq Y$ implies $b(X) \leq b(Y)$.
\end{itemize}

For $X \subseteq V(H)$, let $\rho_H(X)$ be the size of the smallest integral edge cover of $X$ by edges in $E(H)$ and $\rho_H^*(X)$ the size of the smallest fractional edge cover of $X$ by edges in $E(H)$. We are now ready to define the specific widths that are being investigated:

\begin{definition}
For a hypergraph $H$:

\begin{itemize}
  \item \textit{Generalized hypertree width of $H$} \cite{11}: $\text{ghw}(H) := \rho_H$-width.
  \item \textit{Fractional hypertree width of $H$} \cite{12}: $\text{fhw}(H) := \rho_H^*$-width.
  \item \textit{Adaptive width of $H$} \cite{13}: $\text{adw}(H) := \mathcal{F}$-width$(H)$, where $\mathcal{F}$ is the set of all monotone, edge-dominated, modular functions $b$ on $2^{V(H)}$ with $b(\emptyset) = 0$. (Equivalently, $\mathcal{F}$ can be defined as the set of all functions $b : 2^{V(H)} \rightarrow \mathbb{R}^+$ obtained as $b(X) = \sum_{x \in X} f(x)$, where $f$ is a fractional independent set of $H$.)
  \item \textit{Submodular width of $H$} \cite{14}: $\text{subw}(H) := \mathcal{F}$-width$(H)$, where $\mathcal{F}$ is the set of all monotone, edge-dominated, submodular functions $b$ on $2^{V(H)}$ with $b(\emptyset) = 0$.
\end{itemize}
\end{definition}
For \( ghw \) the following result was already shown in \cite{11}. However, because it comes for free with our proof that \( fhw \) is core minimal, we include \( ghw \) in the following Lemma \ref{lem:core_minimal} to illustrate how the proof applies to \( ghw \).

\begin{lemma}
The functions \( ghw, fhw, adw, \) and \( subw \) are core minimal.
\end{lemma}

\begin{proof}
Let \( q \) be a conjunctive query and \( f \) an endomorphism from \( q \) to \( Core(q) \). W.l.o.g., we may assume \( f(v) = v \) for all \( v \in f(q) \). This can be seen as follows: suppose that \( f(v) = v \) does not hold for all \( v \in f(q) \). Clearly, \( f \) restricted to \( Core(q) \) must be a variable renaming. Hence, there exists the inverse variable renaming \( f^{-1} : Core(q) \rightarrow Core(q) \). Now set \( f^* = f^{-1}(f(v)) \). Then \( f^* : q \rightarrow Core(q) \) is the desired endomorphism from \( q \) to \( Core(q) \) with \( f^*(v) = v \) for all \( v \in f^*(q) \).

Let \( H = (V(H), E(H)) \) denote the hypergraph of \( q \) and \( H' = (V(H'), E(H')) \) the hypergraph of \( Core(q) = f(q) \). Furthermore, let \( (T, (B_u)_{u \in V(T)}) \) be a tree decomposition of \( H \). Then we create \( (T, (B'_u)_{u \in V(T)}) \) with the same structure as the original decomposition and \( B'_u = B_u \cap V(H') \). This gives a tree decomposition of \( H' \): for every edge \( e \in E(H') \) with \( e \subseteq B_u \), also \( e \subseteq B_u \cap V(H') \) holds, because \( e \subseteq V(H') \). Removing vertices completely from a decomposition cannot violate the connectedness condition. Actually, some bags \( B'_u \) might become empty but this is not problematic: either we simply allow empty bags in the definition of the various notions of width; or we transform \( (T, (B'_u)_{u \in V(T)}) \) by deleting all nodes \( u \) with empty bag from \( T \) and append every node with a non-empty bag as a (further) child of the nearest ancestor node with non-empty bag.

\textbf{fhw and ghw:} We show that if \( (T, (B_u)_{u \in V(T)}) \) has \( \rho_H \)-width \( k \), then \( (T, (B'_u)_{u \in V(T)}) \) has \( \rho_{H'} \)-width \( \leq k \). By assumption, there is a fractional edge cover \( \gamma_u \) of every set \( B_u \) with weight \( \leq k \). By Lemma \ref{lem:part_submodular_width} there exists a cover \( \gamma'_u \) of \( f(B_u) \) with weight \( \leq k \) and because \( B'_u \subseteq f(B_u) \), \( \gamma'_u \) also covers \( B'_u \).

The proof for \( ghw \) is analogous (the cover created in Lemma \ref{lem:part_submodular_width} preserves integrality).

\textbf{subw and adw:} Let \( F \) and \( F' \) be the sets of monotone, edge-dominated, submodular functions on \( V(H) \) and \( V(H') \) respectively. We show that for every \( b' \in F' \) there exists \( b \in F \) such that \( b'-width(H') \leq b-width(H) \): Consider an arbitrary monotone edge-dominated submodular function \( b' : 2^{V(H')} \rightarrow \mathbb{R}^+ \). For each \( \emptyset \). This function can be extended to a monotone, edge-dominated, submodular function \( b : 2^{V(H)} \rightarrow \mathbb{R}^+ \) on \( V(H) \) by setting \( b(X) = b'(X \cap V(H')) \) for every \( X \subseteq V(H) \). Now, assume \( (T, (B_u)_{u \in V(T)}) \) has \( b'-width \leq k \) because \( b'(B_u) = b'(B_u \cap V(H')) = b(B_u) \) for every \( u \in V(T) \). Thus, the \( b'-width \) of \( H' \) is less or equal the \( b'-width \) of \( H \). As submodular width considers the supremum over all permitted functions we see that \( subw(H') \leq subw(H) \).

For \( adw \) observe that the definition of function \( b \) and the line of argumentation above still holds if we start off with a \textit{modular} function \( b' : 2^{V(H')} \rightarrow \mathbb{R}^+ \).

\end{proof}

\begin{theorem}
For every conjunctive query \( q \):
\begin{itemize}
  \item \textsc{sem-ghw}(q) = ghw(Core(q))
  \item \textsc{sem-fhw}(q) = fhw(Core(q))
  \item \textsc{sem-adw}(q) = adw(Core(q))
  \item \textsc{sem-subw}(q) = subw(Core(q))
\end{itemize}
\end{theorem}

To see that the semantic variants of functions from Theorems \cite{11} and \cite{8} bring non-trivial improvements consider grids of atoms with the same relation name. Under the right circumstances – some care is necessary regarding the output variables and the ordering of
variables in the atoms – their cores are often vastly simpler structures, in some cases even a single atom.

6 Conclusion & Future Work

In this work we introduce the concept of semantic width, a measure for the complexity of the underlying semantics of a query. We extend a result of Barceló by showing that the problem of determining the semantic width of a query, which seems inherently undecidable, can in fact be reduced to determining the width of the core for various common notions of width and for the fractional cover number. Therefore, we can compute the various presented semantic widths by first computing the core and then its width. However, finding the core of a CQ is an NP-complete problem [5]. Some properties of CQs are known to make the computation of generalized and fractional hypertree decompositions tractable [7]. It warrants study if these conditions could also be used to make computing the core tractable. If so, the respective semantic width also becomes tractable.

A natural next step would be to extend known complexity results to the semantic viewpoint. As an example, consider the classical result by Atserias et al. [2]. It says that for full conjunctive queries (CQs where every variable of the body is also in the head) if a class of CQs \( Q \) has bounded fractional cover number, computing the answers is in polynomial time for \( Q \). Using the semantic fractional edge cover number and Theorem 8 we can extend this result to general CQs whose cores are efficiently computable as stated in the following corollary. A similar result, generalizing the bounds on the answer size to general CQs was shown in [9].

\[ \textbf{Corollary 12.} \text{ Let} \ Q \text{ be a class of conjunctive queries whose core is computable in polynomial time. Then, if queries in} \ Q \text{ have bounded } \text{sem-}p^*, \text{ queries in} \ Q \text{ can be computed in polynomial time.} \]

The transformation of tree decompositions used in the proof of Lemma 10 suggests that the problem of finding a decomposition for \( \text{Core}(q) \) is already included in the problem of finding a decomposition for a CQ \( q \). There may be further ways to exploit this connection. We are particularly interested in the possibility of using tree decompositions of a query to speed up finding the core.

Preliminary results of ongoing work suggest that knowledge of the semantic generalized hypertree width of a query can be used to solve the query efficiently even without knowing the core. This opens up an exciting area of applications and we aim to expand on this topic soon.

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