On the number of Nflation fields

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Abstract. In this paper, we study the Nflation model, in which a collection of massive scalar fields drive the inflation, simultaneously. We observe that when the number of fields is larger than the square of the ratio of the Planck scale $M_p$ to the average value $\bar{m}$ of field masses, the slow roll inflation region will disappear. This suggests that in order to have Nflation responsible for our observable universe, the number of fields driving the Nflation must be bounded by the above ratio. This result is also consistent with recent arguments from black hole physics.

Keywords: inflation, physics of the early universe

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1. Introduction

The multiple-field inflation implemented by the assisted inflation mechanism proposed by Liddle et al [1] relaxes many limits for the single-field inflation models, and is beginning to provide a promising class of inflation models. There have been many studies on this [2,3]. Recently, Dimopoulos et al [4] showed that the many-axion fields predicted from the string vacuum can be combined and lead to a radiatively stable inflation, called Nflation, which may be an interesting embedding of inflation in string theory. Then a detailed study was made by Easther and McAllister [5] for quite specific choices of initial conditions for the fields. In the Nflation model, the spectral index of scalar perturbation is always redder than that of its corresponding single field, which is given numerically in [6,7] and is shown analytically in [8]; the ratio of tensors to scalars always has the same value as in the single-field case [10], and the non-Gaussianity is quite small [11,12]. There were some further studies [13].

In the single-field inflation model, the occurrence of inflation requires that the value of the field must be beyond the Planck scale, which can be obtained by imposing a slow roll condition upon the field. However, when the number of fields increases, this value will decrease rapidly, and can be far below the Planck scale, especially when the number of fields is quite large. This is a remarkable and interesting point as regards the Nflation model.

In addition, in the single-field inflation model, when the value of the field increases up to some value, the quantum fluctuation of the field will inevitably overwhelm its classical evolution along the potential. In this case, the inflaton field will undergo a kind of random walk, which will lead to the production of many new regions with different energy densities. In some regions, the field will wander down along its potential, so the classical variance dominates the evolution again and then inflation is able to cease when the field reaches its bottom. However, in other regions the field will fluctuate up and inflation will keep on endlessly. This so called slow roll eternal inflation [14,15] has been studied by using the stochastic approach [16–19].

1 See also different results for Nflation with small field potentials [9].
The critical value of field separating the field space into the slow roll inflation region and eternal inflation region can be obtained by requiring the change of classical rolling of the field in units of Hubble time to be equal to its quantum fluctuation. In the case of a single field, this value is far larger than the Planck scale, and so the end value of slow roll inflation. However, in the $N$flation model, it seems that when a number of fields are added, the total classical roll of fields is weakened, while the total quantum fluctuation is strengthened, which will lead to this critical value moving faster to some smaller value, which may bring in a bound for the number of fields participating in inflation. Here the ‘value’ for the multiple field means that what we take is the root of the square sum of changes of all fields, because here all fields contribute inflation, and thus the trajectory is given by the radial motion in field space. Thus it is interesting to check this possibility. This will be done in this paper. In section 2, we will study the case of $N$flation with massive fields. Firstly we show a simple estimate for the bound of the field number by taking $N$flation with equal mass fields as an example. Then we study a general case with mass distribution following the Marčenko–Pastur law proposed by Easther and McAllister [5], which further validates our result. In section 3, we discuss the case of $N$flation with $\phi^4$ fields. A summary and discussion are given in section 4.

2. A bound for $N$ for $N$flation

In the $N$flation model, the fields are uncoupled and the potential of each field is $V_i = \frac{1}{2} m_i^2 \phi_i^2$. The total change of all fields is determined by the radial motion in field space. In the slow roll approximation, we have

$$
\Delta \phi = \sqrt{\sum_i (\Delta \phi_i)^2} = \frac{\sqrt{\sum_i (\phi_i)^2}}{H} 
\simeq M_p^2 \sqrt{\sum_i (m_i^2 \phi_i)^2} \sum_i m_i^2 \phi_i^2, \tag{1}
$$

where $\Delta \phi_i \simeq |\dot{\phi}_i|/H$ and $\phi_i \simeq V_i/(3H)$ have been used, and the factor with order 1 has been neglected. In the meantime, the total quantum fluctuation of fields is

$$
\delta \phi \simeq \sqrt{\sum_i (\delta \phi_i)^2} \simeq \sqrt{\sum_i \left(\frac{H}{2\pi}\right)}^2 
\simeq \frac{\sqrt{N}}{M_p} \sqrt{\sum_i m_i^2 \phi_i^2}, \tag{2}
$$

where $N$ is the number of fields, $\delta \phi_i \simeq H/(2\pi)$ has been used and the factor with order 1 has been neglected. By requiring $\Delta \phi = \delta \phi$, we will obtain the critical point separating the slow roll inflation region and eternal inflation region, which is given by

$$
\left(\sum_i m_i^2 \phi_i^2\right)^{3/2} \simeq \frac{M_p^6}{N} \sum_i (m_i^2 \phi_i)^2. \tag{3}
$$
In slow roll inflation region, the end of slow roll inflation requires $\dot{H}/H^2 \simeq 1$, which may be reduced to

$$M_p^2 \sum_i (m_i^2 \phi_i)^2 \simeq \left( \sum_i m_i^2 \phi_i^2 \right)^2.$$  \hspace{1cm} (4)

### 2.1. The case with equal masses

Firstly, when the masses of all fields are equal, i.e. $m_i = m$, also for simplicity we take the values of all fields equal, i.e. $\phi_i = \phi$. From equations (3) and (4), we have

$$\phi \simeq \frac{1}{N^{3/4} \sqrt{M_p^3/m}},$$  \hspace{1cm} (5)

$$\phi \simeq \frac{M_p}{\sqrt{N}},$$  \hspace{1cm} (6)

respectively. Thus we see that the end point moves with $1/\sqrt{N}$, which is slower than the movement of the critical point separating the slow roll inflation region and eternal inflation region, since the latter changes with $1/N^{3/4}$. This suggests that when we plot the lines of the end point and the critical point moving with respect to $N$, there must be a value at which these two lines cross. Beyond this value, the slow roll inflation region disappears as shown in figure 1. This value can be obtained by taking equations (5) and (6) equal, which gives $N \simeq M_p^2/m^2$. Thus to have Nflation responsible for our observable universe, the number $N$ of fields in the Nflation model must satisfy

$$N \lesssim \frac{M_p^2}{m^2},$$  \hspace{1cm} (7)

since the existence of such a slow roll region is significant for solving the problems of standard cosmology and generating the primordial perturbation seeding large scale structures of our universe. It should be noted that in the case where the masses of all fields are equal, their field values being equal is not real, because even if initially the values of all fields are equal, they will be unequal after several e-folds due to the random walk of each field. However, this simplified analysis actually provides a simple estimate for the bound for $N$. In the next subsection, we will validate this result in a general case.

### 2.2. The case with mass distribution following the Marčenko–Pastur law

Now we will study a general case with mass distribution following the Marčenko–Pastur law proposed by Easther and McAllister [5], which appears for axions in string theory. The shape of the mass distribution of axions depends on the basic structure of the mass matrix, which is specified by the supergravity potential. In the simplest assumption, the mass matrix is a random matrix. When one diagonalizes this matrix, the fields will be uncoupled with the mass spectrum given by the distribution of eigenvalues. This distribution of the eigenvalues can be characterized by the Marčenko–Pastur law when the matrices are large. The mass distribution taken as the Marčenko–Pastur law is a function with respect to $\bar{m}$ and $\beta$, where $\bar{m}$ is the average value of the mass, i.e. $\langle m^2 \rangle = \bar{m}^2$, and $\beta$ is determined by the ratio of the number of axions to the dimension of the moduli space.
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Figure 1. The log change of the end point of slow roll inflation and the critical point separating the slow roll inflation region and eternal inflation region with respect to the number \( N \) of fields in the Nflation model with massive fields, in which \( m = 10^{-6} M_p \) is taken. Three regions separated by the two lines have been pointed out in the figure. We can see that when \( N \sim 10^{12} \), the slow roll inflation region disappears.

and a model dependent parameter, whose favoured value is expected to be about 0.5; see [5, 7]. In this case, the smallest and largest mass are given by \( m_1^2 = a \equiv \bar{m}^2(1 - \sqrt{\beta})^2 \) and \( m_N^2 = b \equiv \bar{m}^2(1 + \sqrt{\beta})^2 \), respectively.

In the slow roll approximation, the field value can be given by
\[
\phi_i(t) \simeq \phi_i(t_0)[\tau(t)]^{m_i^2/b},
\]
where \( \tau(t) \) is the ratio of the value of the heaviest field at time \( t \) to its initial value \( t_0 \), \( \tau(t) \equiv \phi_N(t)/(\phi_N(t_0)) \). Then defining \( c \equiv 2 \ln[\tau(t)]/b \), the parts including \( \phi_i^2 \) in the summation terms of equations (3) and (4) can be replaced with \( \phi_i^2(t_0) \exp[cm_i^2] \). When we ignore correlations between the mass distribution and the initial field distribution, we can immediately calculate their respective average values. Using power series expansions, the average value of the exponential term \( \exp[cm_i^2] \) can be written as
\[
\langle \exp[cm_i^2] \rangle = \sum_i \langle m_i^{2j} \rangle \frac{c^j}{j!}.
\]
The expectation value inside of summation in the left-hand side of equation (9) can be expressed with Narayana numbers \( T(i, j) \) (see equation (6.14) in [5]):
\[
\langle m_i^{2j} \rangle = \bar{m}^{2i} \sum_{j=1}^{i} T(i, j) \beta^{j-1}
= \bar{m}^{2i} F_1(1 - i, -i, 2, \beta),
\]
whose dependence on \( \beta \) will have the value \( \alpha \equiv \phi^2(t_0) \).

This result further validates the argument in the previous subsection, only replacing \( \phi \) with \( m \).

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where \( 2F_1 \) is a hypergeometric function. Then equation (9) can be rewritten as

\[
\langle \exp[cm_i^2] \rangle = \sum_{i=0}^{\infty} \bar{m}^{2i} 2F_1(1-i,-i,2,\beta) \frac{c^i}{i!}.
\] (11)

Therefore the summation terms in equation (3) with the expectation values of the initial conditions and of the distribution of the mass spectrum of the fields are

\[
\sum_i m_i^2 \phi_i^2 = N\alpha \bar{m}^2 \sum_{i=0}^{\infty} \bar{m}^{2i} 2F_1(-i,-i-1,2,\beta) \frac{c^i}{i!},
\] (12)

\[
\sum_i m_i^4 \phi_i^2 = N\alpha \bar{m}^4 \sum_{i=0}^{\infty} \bar{m}^{2i} 2F_1(-i-1,-i-2,2,\beta) \frac{c^i}{i!},
\] (13)

where \( \alpha \equiv \langle \phi_i^2(t_0) \rangle \).

Thus equation (3), with the help of equations (12) and (13), becomes

\[
\alpha \simeq \frac{M_p^2}{m N^{3/2} f_1(t, \beta)},
\] (14)

where

\[
f_1(t, \beta) = \frac{\left[ \sum_{i=0}^{\infty} \bar{m}^{2i} 2F_1(-i-1,-i-2,2,\beta)(c^i/i!) \right]^{1/2}}{\left[ \sum_{j=0}^{\infty} \bar{m}^{2j} 2F_1(-j,-j-1,2,\beta)(c^j/j!) \right]^{3/2}},
\] (15)

whose dependence on \( c \) is plotted in figure 2. We see that \( f_1(t, \beta) \) is approximately a constant with order 1 for a wide range of \( c \), i.e. different initial conditions and values of fields. Further, we can note that when all fields have equal values and masses, \( f_1(t, \beta) \) will have the value \( f_1(t, \beta) \simeq 1 \) with \( \alpha \equiv \phi^2 \) and \( \bar{m} = m \). In this case equation (14) will be exactly same as equation (5).

Equation (4), with the help of equations (12) and (13), becomes

\[
\alpha \simeq \frac{M_p^2}{N f_2(t, \beta)},
\] (16)

where

\[
f_2(t, \beta) = \frac{\left[ \sum_{i=0}^{\infty} \bar{m}^{2i} 2F_1(-i-1,-i-2,2,\beta)(c^i/i!) \right]}{\left[ \sum_{j=0}^{\infty} \bar{m}^{2j} 2F_1(-j,-j-1,2,\beta)(c^j/j!) \right]^{2}},
\] (17)

whose dependence on \( c \) is also plotted in figure 2. We see that, like \( f_1(t, \beta) \), \( f_2(t, \beta) \) is also approximately a constant with order 1 for a wide range of \( c \). When all fields have equal values and masses, \( f_2(t, \beta) \simeq 1 \) with \( \alpha \equiv \phi^2 \). In this case equation (16) will be exactly the same as equation (6).

Thus combining equations (14) and (16) to cancel \( \alpha \), we have \( N \simeq M_p^2/\bar{m}^2 \), where the ratio of \( f_1(t, \beta) \) to \( f_2(t, \beta) \) has been taken as roughly 1, which can be seen in figure 2. This is a point at which the slow roll inflation region will disappear. Thus to have a period of slow roll Nflation, the number of fields must be bounded by

\[
N \lesssim \frac{M_p^2}{\bar{m}^2}.
\] (18)

This result further validates the argument in the previous subsection, only replacing \( m \) with \( \bar{m} \).
3. The case of Nflation with $\phi^4$ fields

It is interesting to further check whether there is similar bound for the field number of Nflation with $\phi^4$ fields. Following the same steps as in the previous section, the critical point separating the slow roll inflation region and eternal inflation region and the end point of inflation can be given by

$$\left( \sum_i \lambda_i \phi_i^4 \right)^3 \simeq \frac{M_p^6}{N} \sum_i (\lambda_i \phi_i^2)^2,$$

$$M_p^2 \sum_i (\lambda_i \phi_i^3)^2 \simeq \left( \sum_i \lambda_i \phi_i^4 \right)^2,$$  \hspace{1cm} (19, 20)
Figure 3. The log change of the end point of slow roll inflation and the critical point separating the slow roll inflation region and eternal inflation region with respect to the number \( N \) of fields in the Nflation model with \( \phi^4 \) fields, in which \( \lambda = 10^{-12} \) is taken. Three regions separated by the two lines have been pointed out in the figure. We can see that the lines are parallel; thus there is no bound for \( N \).

respectively, where \( \lambda_i \) is the coupling constant of the corresponding field and the factors with order 1 have been neglected. For simplicity, we take all \( \phi_i = \phi \) and \( \lambda_i = \lambda \), and thus have

\[
\phi \simeq M_p/\lambda^{1/6}\sqrt{N},
\]

(21)

\[
\phi \simeq M_p/\sqrt{N}.
\]

(22)

Thus in this case the critical point and the end point approximately obey the same evolution with \( N \) increasing, which is plotted in figure 3. This suggests that there is not a bound for the number of fields imposed by the occurrence of slow roll inflation.

This result seems unexpected. The reason leading to it may be that for the \( \phi^4 \) field, its effective mass is \( \sim \lambda \phi^2 \), which is changed with \( \phi \), and its change in some sense sets off the fast moving of the critical point. When writing \( \lambda \phi^2 = m^2 \) in equation (21), one can find that the resulting equation will be the same as equation (5). Thus combining it with equation (22), we will have the same result as equation (7), which in turn suggests

\[
\lambda \phi^2 \lesssim \frac{M_p^2}{N}.
\]

(23)

Thus for the \( \phi^4 \) field, the bound relation between the mass \( m \) and \( N \) in equation (7) for massive field is transferred to that between the field value \( \phi \) and \( N \). The study for the
4. Summary and discussion

In this paper, we study the Nflation model, in which a collection of massive scalar fields drive inflation, simultaneously. We observe that with the increase in field number, both the critical point separating the slow roll inflation region and eternal inflation region and the end point of slow roll inflation will move towards smaller average values of fields—however, at different rates. In general, the critical point moves faster than the end point, which leads to the slow roll inflation region being eaten off by the eternal inflation region inch by inch. When the number of fields is enough large, i.e. $N \approx M_p^2/\bar{m}^2$, the points overlap, which means that the slow roll inflation region completely disappears. In this sense, in order to have Nflation responsible for our observable universe, the field number driving the Nflation must be bounded by $N \lesssim M_p^2/\bar{m}^2$.

Recently, it was shown that in theories with a large number $N$ of fields with a mass scale $m$, black hole physics imposes a bound relating $M_p$ and $N$ [20, 21], i.e. $Nm^2 \lesssim M_p^2$, which is actually the same as the result shown here. This can be explained as follows. In general, each field can contribute the factor $\sim m^2$ to the renormalization of the Planck mass; thus after the accidental cancellations are neglected, the net contribution of $N$ fields will be $\sim Nm^2 \approx M_p^2$. This indicates that with $N$ massive fields there exists a gravitational cut-off, beyond which the quantum gravity effect will become important [22]. When we focus on the inflation driven by $N$ massive fields, we observe that the same cut-off will also appear in a similar sense, i.e. beyond this cut-off the quantum effect will be dominant; it is this which leads to the disappearance of the slow roll inflation region. Thus in this sense our result also justifies the bound of [20] from a different point of view. In this case exactly the Planck mass should be replaced by the renormalized one, i.e. $\tilde{M}_p^2$, which includes the contributions of all massive fields for $M_p^2$. However, it can be noted that $\tilde{M}_p^2$ is actually of the same order as $M_p^2$. Thus when the renormalization of $M_p^2$ is considered, our result is not qualitatively altered.

However, this bound cannot be applied to a nearly massless scalar field. The reason is that when the masses of fields are negligible, they will not appear in the summation for fields in both sides of equations (3) and (4), which is actually also a reflection of the massless fields not affecting the motion of massive fields dominating the evolution of the universe, while the perturbations summed in equation (2) are those along the trajectory in field space, since the massless fields only provide the entropy perturbations orthogonal to the trajectory, which thus are not considered. The same case can also be seen in the argument of [20], since the contribution of the field to the renormalization of the Planck mass is proportional to $m^2$; thus the net contribution provided by all massless fields to the Planck mass may actually be neglected. Thus if there are some nearly massless fields and some massive fields with nearly same order, it should be the case that there is a bound $N \lesssim M_p^2/\bar{m}^2$ in which only massive fields are included in the definition of $\bar{m}$ and $N$. In our example, when the mass distribution is characterized by the Marčenko–Pastur law, as has been used here, in which the smallest and largest mass are given by $m^2_1 = \bar{m}^2(1 - \sqrt{\beta})^2$ and $m^2_N = \bar{m}^2(1 + \sqrt{\beta})^2$, respectively, the masses of all fields are approximately of the same order for $\beta \simeq 0.5$. In this case it is natural that all fields need to be considered.
The field $\lambda \phi^4$ in essence is different from the massive field $m^2 \phi^2$. The former corresponds to having a running mass $\sim \lambda \phi^2$, which is dependent on $\phi$. In this case, following [20], the net contribution of $N$ fields to the renormalization of the Planck mass will be $\sim N \lambda \phi^2 \simeq M_P^2$. Thus the same bound relation as in equation (23) can be obtained, which is seemingly one between the field value $\phi$ and $N$. Equation (23) can be written as

$$\phi \lesssim \frac{M_P}{\sqrt{\lambda N}}. \quad (24)$$

Note that for general $\lambda < 1$, when there is a slow roll inflation region, equation (24) is always satisfied, since the inequality given by equation (19) is actually included in equation (24). Thus it seems that there is not a bound for $N$ in the Nflation with $\lambda \phi^4$.

In principle, the bound shown here seems be only valid for massive scalar fields. Further, whether there are similar bounds for other fields with various potentials remains open, and needs to be studied. Be that as it may, however, the result observed, that there may be a large $N$ transition provided by the quantum effect in inflation, may be interesting, and might have deep relations with other large $N$ phenomena discussed, and thus merits further exploration.

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