Dynamics in the turbulent wake of a curved circular cylinder

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Abstract. Understanding the physics of turbulent wakes is an essential, yet complex task in the study of turbulent flows. In the present paper we investigate the flow past a curved body of circular cross-section. The inflow velocity is aligned with the curvature of the cylinder and directed towards its convex face. We conduct direct numerical simulations at a Reynolds number of 3900 in order to obtain a fully turbulent wake. The instantaneous vortical structures reveal that the primary vortices are roughly aligned with the curved axis. Despite the presence of isolated splitting events in the frequency of the vortex shedding, there is one single shedding frequency that dominates this process. Velocity time-traces confirm that the shear layers exhibit intermittency, which is manifested as large amplitude fluctuations. The intensity of these instabilities is increased by the secondary flow along the recirculation region, thereby influencing the dynamics of the near wake. Several spots of zero mean velocity reside next to the baseline within the recirculation region, displacing the back-flow region further downstream. It is suggested that this displacement is induced by the secondary flow, in combination with the symmetry boundary condition imposed at the top plane.

1. Introduction

The study of turbulent wakes is of great interest from an academic and industrial point of view. This type of flow, commonly encountered in nature and engineering applications, is associated with the flow over and obstacle or the movement of a body through fluid, and poses significant challenges due to its complex three-dimensional nature. Of special interest in this context is the direct numerical simulation (DNS) of turbulent wakes, which gives detailed information about the flow physics that is sometimes difficult, or even impossible, to obtain by experiments.

Inspired by the flow around catenary risers used to convey fluids from the sea-bed, the flow around a stationary body of circular cross-section with axial curvature was investigated by Miliou et al. (2007). These authors conducted simulations at Reynolds numbers of 100 and 500 (laminar and transitional wakes), reporting vortex shedding occurring at one single frequency along the whole span of the cylinder when the flow was directed towards its convex face, and suppression of the vortex shedding when the flow was directed in the opposite direction. Although transition to turbulence was observed at a Reynolds number of 500 for the convex flow configuration, additional features of vortex formation and shedding for this particular bluff body may arise.
when the wake is fully turbulent. Interestingly, a certain degree of similarity was found between the flow past this curved body and the flow past a straight circular cylinder (Williamson, 1996).

In the present paper, we present the results obtained from the DNS of the flow past a curved circular cylinder at a Reynolds number of 3900 (i.e. a fully turbulent wake). We have used the same geometry as that used by Miliou et al. (2007), with the inflow directed towards the convex face of the curved cylinder. We have focused this work on the instantaneous vortical structures in the near wake, the instability of the shear layers, the recirculation region and the dominant shedding frequencies. To the knowledge of the authors, there is no experimental or numerical data available for this flow configuration at a Reynolds number of 3900. For this reason and the similarity with the flow past a straight circular cylinder, many of the results presented here are compared with previous studies conducted on straight circular cylinders at the same Reynolds number.

2. Numerical methodology

2.1. Numerical solution of the Navier–Stokes equations

The flow field is governed by the incompressible Navier–Stokes equations for a fluid with constant density and viscosity. The governing equations are directly solved using the code MGLET (Manhart, 2004). This is a second-order finite volume code that has been used for the DNS of a variety of flow configurations, including channel flows with obstructions and different types of bluff-bodies. The flow field is defined on a staggered Cartesian grid with non-equidistant spacing. Linear interpolation is used to approximate the velocities at the faces of the control volumes, and a central-difference formulation is used for the approximation of the first-order derivatives arising from the diffusive term in the momentum equation. Time integration is performed with an explicit low-storage third-order Runge–Kutta scheme (Williamson, 1980); this method provides high accuracy and good conservation properties. The Poisson equation for pressure correction is solved iteratively by the Stone’s strongly implicit procedure (SIP) (Stone, 1980).

Inclusion of the curved geometry inside the computational domain is taken into account by an immersed boundary method (IBM). This method relies on direct forcing to specify the non-slip and non-impermeability boundary conditions at the walls. The basic procedure is to transform the cells at the fluid-solid interface into internal boundary conditions on the corresponding computational domain by using high order interpolation from the fluid cells in the vicinity of the body. A detailed description and discussion on the stability of this IBM is given by Peller et al. (2006).

2.2. Flow parameters

A schematic diagram of the flow model used for the numerical simulations is presented in figure 1. The streamwise, cross-stream and vertical directions are denoted by $x$, $y$ and $z$, respectively. The geometry of the bluff body consists of a quarter segment of a ring which is connected to a horizontal extension of length $L_e$ on its lower end; both segments have circular cross-section with diameter $D$. The non-dimensional radius of curvature $R/D$ is defined as the ratio of the radius of curvature of the quarter-ring $R$ to its cross-sectional diameter $D$. Throughout the present paper, $\theta$ denotes the angle with vertex at the center of the quarter-ring measured from the top plane in the $(z,x)$-plane of symmetry. The incoming flow is parallel to the $x$-direction and directed towards the convex face of the cylinder with a uniform velocity $U_0$, as shown in figure 1.

A uniform velocity profile $U_0$ was specified at the inlet, and a fully developed zero-stress condition is imposed at the outlet. Symmetry boundary conditions were used at the sides, as
The size of the computational domain is $L_x$, $L_y$ and $L_z$ in the $x$-, $y$- and $z$-directions, respectively. The gray-shaded plane corresponds to the symmetry $(z,x)$-plane. The Reynolds number $Re = U_0 D/\nu$ was set at 3900, yielding a turbulent wake and shear-layer transition. The smallest spatial scales of the turbulence were considered to define the minimum grid size by comparing the grid sizes with the Kolmogorov length scale $\eta$ defined as $(\nu^3/\epsilon)^{1/4}$. Here, the pseudo-dissipation rate $\bar{\epsilon}$ is estimated as $\nu \partial u_i/\partial x_j \partial u_i/\partial x_j$. The resolutions achieved at different locations in the wake were always lower than $4\eta$.

Taking into account the resolution requirements, a $N_x \times N_y \times N_z = 1200 \times 440 \times 450$ Cartesian grid was used, yielding a total of $238 \times 10^6$ grid points. The computations were run in parallel on an IBM p575+ machine. A total of 64 large-memory processors were used, the average time required to compute one time-step was 35 seconds approximately. A first simulation over a time-interval of $T U_0/D = 100$ was run in order to reach a quasi-periodic state, after which statistics were collected over a time interval of $T U_0/D = 136$, corresponding to approximately 30 vortex shedding cycles.

3. Results

3.1. Instantaneous vortical structures in the near wake

The evolution of the instantaneous vortical structures in time and space offers a clear picture of the structure of the flow. Figure 2 shows a time sequence of the wake topology viewed on the $(z,x)$-plane spanning approximately 1.25 times a vortex shedding period. Here, the vortex identification method proposed by Jeong & Hussain (1995) is used to regions of minimum pressure in the planes perpendicular to the vortex axis, i.e. vortex cores. In order obtain a better view of the wake evolution, only half of the wake (considering the wake sliced by the symmetry plane shown in figure 1) is shown. Despite the random distribution of small-scale vortical structures, the alternating pattern of the vortex shedding which dominates the large scales dynamics of the flow is clear. Close to the solid body, primary vortices arise due to the rolling-up of the shear layer; in addition to this, there are secondary vortices due to the
Figure 2. Time sequence of instantaneous vortical structures visualized as iso-surfaces of $\lambda_2$ (Jeong & Hussain, 1995); samples start from $202D/U_0$ time units. The view is on the $(x, z)$-plane, with an extent of $10 \leq x/D \leq 23$ and $7 \leq x/D \leq 16$.

instability of the shear layers. These vortices are more or less aligned with the axial curvature of the body, producing a certain degree of tilting with respect to the vertical direction as they evolve downstream. In the first two rows of vortices seen in the time sequence the small structures coexist with larger structures, but as the wake evolves downstream the structures increase in size due to merging and destruction of the small-scale structures.

Since at this Reynolds number instabilities in the shear layer are expected, it is also of interest to visualize the instantaneous enstrophy, which in addition to vortices enables visualization of shear layers. Figure 3 shows a plot of instantaneous enstrophy sliced at $z/D = 8, 12$ and 16. The sampling points used to conduct the time analysis presented in the next section are also shown in this figure; the points are taken at a distance $d_a$ of 0.7D, 1.0D and 1.3D from the axis, in accordance with previous studies for straight cylinders (Prasad & Williamson, 1997; Rai, 2010). Looking downstream, the points are denoted as $A$, $B$ and $C$ for the right shear layer, and $D$, $E$ and $F$ for the left shear layer.

At $z/D = 8$, there are vortices within the recirculation region and the shear layers exhibit a wavy shape. These vortices are produced by the secondary flow that arises due to a pressure gradient along the baseline. An interesting finding by Rai (2010), who investigated the instabilities in the shear layers on a straight circular cylinder, was that the intermittent appearance of instabilities is a result of the interaction between entrained vortices within the recirculation region, and the shear layers at the sides of the cylinder. This interaction produces deflection and further tilting of the shear layers. The higher degree of instability in the shear layer at $z/D = 8$ may therefore be attributed to the secondary flow that induces the presence of vortices in the vicinity of the shear layers. In addition to the secondary flow induced by the pressure gradient, the flow dynamics are affected by the cross-sectional shape of the body; for instance, at $z/D = 8$ the cross-sectional shape corresponds to that of an oval, as figure 3 depicts. The deflection of the shear layers seen at $z/D = 12$ is rather small compared to those at $z/D = 8$. There is a stronger back flow in this case, and the cross-sectional shape is closer to that of a circle. At the upper location ($z/D = 16$) we observe that the shear layers are longer and more stable than those at $z/D = 8$ and 12. This may be due to the symmetry boundary condition imposed at the top $(x, y)$-plane (i.e. the normal velocity component set to zero), which gives a two-dimensional character to the flow at the upper part. The study of the effects of this
3.2. Instability of the detached shear layers

Previous studies in straight circular cylinders have shown that the instability on the free shear layers occurs intermittently (Prasad & Williamson, 1997; Rajagopalan & Antonia, 2005). This means that the time-traces of the velocities will exhibit regions of high fluctuations interspersed with a relatively flat signal. Prasad & Williamson (1997) noticed that as the Reynolds number increases, these high-fluctuation regions will occur more frequently while the region of instability will move upstream. In order to investigate these phenomena for the present case, time-traces of the fluctuating component of the streamwise and cross-stream velocities at \( z/D = 8, 12, \) and \( 16 \) were taken close to the edges of the shear layer (see figure 3). The plot of these time-traces is shown in figure 4 for a period of \( 60U_0/D \). It is evident that intermittency at \( z/D = 8 \) and \( 12 \) manifest close to the axis of the cylinder, while the signal at \( z/D = 16 \) exhibits the large amplitude fluctuations further downstream. As discussed in the previous section, this apparent delaying at \( z/D = 16 \) is probably associated to the symmetry boundary condition imposed at the top \((x,y)\)-plane.

Prasad & Williamson (1997) suggested that the intermittency in the shear layer was caused by the random streamwise motion of the transition point, which is influenced by temporal changes in near-wake three-dimensional structures. However, as pointed out in the previous section, the recent study by Rai (2010) showed that ingested vortices arising within the recirculation region interact with the shear layer, thereby producing the intermittent fluctuations in the signal. For the straight cylinder case, this phenomenon was found to occur locally along the span of a straight cylinder. In the present case, visual inspection of the time signals in figure 4 reveals that the spots of large amplitude fluctuations are not synchronized.

The velocity spectra in the vicinity of the shear layer are characterized by a sharp peak at the Strouhal frequency \( (f_{St}) \), together with a broadband peak at the shear layer frequency \( (f_{SL}) \). The following power-law fit \( (f_{SL}/f_{St} \text{ versus } Re) \) was proposed by Prasad & Williamson (1997):

\[
\frac{f_{SL}}{f_{St}} = 0.0235Re^{0.67}.
\]

Based on the time-series for the velocity fluctuations, velocity spectra corresponding to the right shear layer are shown in figure 5 for the present case together with the estimate given by equation (1). It can be noticed that the estimate proposed by Prasad & Williamson (1997) gives a fairly good estimate of \( f_{SL}/f_{St} \) at \( 1.3D \) from the axis of the cylinder. Close to the axis, at \( 0.7D \), the signal at \( z/D = 12 \) and \( 16 \) does not exhibit high amplitude fluctuations, and the corresponding spectra only exhibit the \( f_{St} \) peak. Since the shear layer at \( z/D = 16 \) is longer and more stable at the locations sampled, a signal taken further downstream would perhaps have yielded a better agreement with the estimate given by equation (1). The length of the time-series for the frequency analysis was \( 136U_0/D \); due to intermittency, however, long-time-averaged spectra tend to diminish the peak at the shear layer frequency.
Figure 4. Time traces of the fluctuating component of the streamwise (black line) and cross-stream (light-gray line) velocities in the sampled points of the shear layers.

Figure 5. One-dimensional streamwise and cross-stream velocity spectra in the right shear layer at $z/D = 8$, 12 and 16. The vertical dashed line corresponds to the estimate given by equation (1). Here, —— is taken at $0.7D$ from the axis of the cylinder; ·····, at $1.0D$ from the axis; and ——, at $1.3D$ from the axis.
Additional statistical parameters are shown in table 1 for the right and left shear layers. A general trend at the different vertical positions is the increase in the r.m.s. values of the velocities with the distance from the axis \(d_a\); also the r.m.s. values are higher for at \(z/D = 8\). At \(z/D = 8\) and \(12\), the correlation coefficients between the streamwise and cross-stream velocities fluctuate between 0.4 and 0.5, being the only exception the right shear layer at \(z/D = 12\) at \(d_a/D = 0.7\) which exhibits relatively flat signals. At \(z/D = 16\), on the other hand, high correlation coefficients are obtained due to the low degree of fluctuation that characterize the signals. Additionally, the sign of these correlation coefficients is in accordance with the directions in which the instability occurs (positive at the right shear layer and negative at the left). Previous investigations have shown that, in general, the intermittency does not occur at the same time at the lower and upper shear layers. Correlation coefficients between the velocities at the upper and lower shear layers support this observation; the coefficients are relatively low except at \(z/D = 16\), where the signals exhibit a lower degree of fluctuation.

The instabilities seen in figure 4 are related to transfer of energy from regions of large intensity towards low intensity regions; skewness of the distributions is associated to this process. All the streamwise fluctuations in table 1 have negative skewness factors, confirming the tendency of this velocity component towards negative fluctuations. The skewness of the cross-stream velocities, on the other hand, show negative values for the right shear layer and positive values for the left shear layer, in accordance with the direction of the fluctuations. The large peaks in the velocity fluctuations make the flatness factor exceed its normal value of three, indicating that the intensity of the fluctuations is spotty.

3.3. Recirculation region and dominant shedding frequency

Figure 6 shows the values of the mean streamwise velocity taken at the symmetry \((z,x)\)-plane, where the recirculation region defined by the iso-contours of \(\tau = 0\) is highlighted. The recirculation region extends up to \(\theta_r \approx 50^\circ\), and is characterized by a pointed-end shape and spots of zero velocity. The larger spot is located close to the top of the recirculation region, and is induced by the secondary flow that arises due to the symmetry boundary condition that sets the flow through the top \((x,y)\)-plane equal to zero. There are, however, other spots of zero velocity along the baseline that may be induced by the secondary flow; the largest spot is close to the end of the recirculation region.

This apparent displacement of the mean negative velocities induces a longer recirculation length compared to that of a straight cylinder at the same Reynolds number. Here, the recirculation length \(L_r\) as the distance between the base of the cylinder and the location of the change of sign of the mean streamwise velocity. At \(z/D = 16\), we obtained \(L_r = 1.86\), while at \(z/D = 12\), the recirculation length is 1.57. The study by Parnaudeau et al. (2008) give an estimation of \(L_r \approx 1.67\) for a straight circular cylinder at \(Re = 3900\), and points out that the uncertainty of this estimation decreases for longer time-series. They collected statistics over 52 vortex shedding cycles, which lead to an uncertainty of \(\pm 12\%\) in the estimation of \(L_r\). Although the length of the time series considered for the statistics in the present case may not be ideal, the instantaneous shear layers seen in figure 3 are longer at the upper part of the cylinder, producing thus longer recirculation lengths.

Time traces of the cross-stream velocity at the centreline of the wake were taken in order to study the frequency of the vortex shedding at different vertical positions. Figure 7(a) shows the spectrum at \(x/D = 26\) and \(z/D = 16\), which clearly depicts the peak corresponding to the dominant Strouhal frequency and the inertial range. In figure 7(b), the Strouhal frequency at different vertical positions is plotted together with values of \(St\) for a straight circular cylinder obtained in different studies. In the present case, there is a single vortex shedding frequency that prevails along the vertical direction. This was also observed at \(Re = 100\) and 500 for this same flow configuration; Miliou et al. (2007) commented that the vortex shedding at the upper
Table 1. Statistical parameters for the velocity fluctuations on the right and left shear layers (points shown in figure 3). Here \( r \) denotes right shear layer; and \( l \), left shear layer. The correlation coefficient between variables \( i \) and \( j \) is denoted as \( \rho_{ij} \), and \( S_i \) and \( K_i \) denotes the skewness and flatness factors for the variable \( i \), respectively.

| \( z/D \) | \( d_a/D \) | Point | \( \overline{u'_{\text{rms}}}/U_0 \) | \( \overline{v'_{\text{rms}}}/U_0 \) | \( \rho_{u'u'_{ij}} \) | \( \rho_{v'v'_{ij}} \) | \( S_{u'} \) | \( S_{v'} \) | \( K_{u'} \) | \( K_{v'} \) |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.7 | A | 0.031 | 0.047 | 0.489 | -0.322 | 0.258 | -0.345 | -0.143 | 3.2 | 3.12 |
| | D | 0.065 | 0.054 | -0.442 | 0.079 | 0.161 | -0.451 | -0.897 | 33.5 | 5.9 |
| 8 | B | 0.076 | 0.067 | 0.457 | -0.174 | 0.161 | -3.74 | 1.12 | 17.8 | 8.35 |
| | E | 0.138 | 0.076 | -0.5 | -0.174 | 0.161 | -3.74 | 1.12 | 17.8 | 8.35 |
| 1.3 | C | 0.131 | 0.093 | 0.404 | -0.19 | 0.133 | -2.9 | 1.23 | 13.4 | 6.83 |
| | F | 0.181 | 0.102 | -0.488 | -0.19 | 0.133 | -3.02 | 1.33 | 12.8 | 8.61 |
| 0.7 | A | 0.019 | 0.028 | 0.604 | -0.079 | -0.278 | -0.968 | -0.504 | 10.4 | 5.98 |
| | D | 0.023 | 0.028 | -0.501 | -0.079 | -0.278 | -1.04 | 0.404 | 5.68 | 3.65 |
| 12 | B | 0.045 | 0.043 | 0.428 | -0.118 | -0.139 | -4.65 | -1.34 | 38.7 | 21.3 |
| | E | 0.064 | 0.064 | -0.433 | -0.118 | -0.139 | -2.03 | 2.25 | 11.7 | 23.5 |
| 1.3 | C | 0.09 | 0.078 | 0.403 | -0.179 | 0.075 | -4.15 | -1.12 | 29.5 | 8.9 |
| | F | 0.094 | 0.102 | -0.475 | -0.179 | 0.075 | -2.96 | 1.81 | 21 | 16.3 |
| 0.7 | A | 0.015 | 0.014 | 0.905 | 0.582 | -0.717 | -1.21 | -0.274 | 4.96 | 3.13 |
| | D | 0.016 | 0.013 | -0.889 | 0.582 | -0.717 | -0.814 | -0.041 | 3.98 | 2.93 |
| 16 | B | 0.047 | 0.019 | 0.803 | 0.686 | -0.686 | -2.14 | -0.156 | 7.93 | 3.23 |
| | E | 0.051 | 0.02 | -0.74 | 0.686 | -0.686 | -1.45 | -0.077 | 5.17 | 4.11 |
| 1.3 | C | 0.08 | 0.039 | 0.586 | 0.567 | -0.279 | -2.34 | -0.294 | 9.5 | 5.62 |
| | F | 0.083 | 0.047 | -0.543 | 0.567 | -0.279 | -1.94 | -0.153 | 8.68 | 13.8 |

Figure 6. Mean streamwise velocity contours (normalized by \( U_0 \)) taken at the symmetry \((z,x)\)-plane. The thick blue line defines the recirculation region.
Figure 7. (a) One-dimensional cross-stream velocity spectra in the wake centreline at \( x/D = 29 \) taken at \( z/D = 8 \) (——), 12 (· · · ·), and 16 (——). (b) Dominant shedding frequency (St) along the vertical direction with reference values for St in a straight circular cylinder: •, Parnaudeau et al. (2008); ◊, Kravchenko & Moin (2000); ■, Cardell (1993); and △, Ma et al. (2000).

plane drives the vortex shedding on the lower part. There are, however, isolated events in which there is splitting of the vortex shedding frequency at some locations below \( z/D \approx 12 \), but a long enough time series smoothes these differences.

4. Concluding remarks

We have performed direct simulation of turbulent flow past a curved circular cylinder, where the inflow velocity was parallel to the plane of curvature and directed towards the convex face of the body. Although the vortex shedding exhibits a periodic pattern similar to that observed in previous studies (Miliou et al., 2007) at lower \( Re \), a certain degree of tilting of the primary vortices with respect to the vertical direction was noticed in the instantaneous vortical structures.

A characteristic feature of the flow past straight circular cylinders at this \( Re \) is the instability of the shear layers separating from the bluff body. For the present case, the time-series analysis shows that these instabilities exhibit different degrees of intensity depending on their spanwise location (i.e. along the curved axis). In particular, at \( z/D = 8 \) the presence of intermittency in the fluctuating velocity signal is enhanced by the secondary flow within the recirculation region. This flow is driven downwards and generates vortices that interact with the shear layer, increasing thus the degree of instability. Furthermore, the shear layers at the upper position (\( z/D = 16 \)) are more stable and longer than those at lower positions. This is likely an effect of the symmetry boundary condition imposed at the top plane of the computational domain.

A surprising result was the presence of spots of zero velocity within the mean recirculation region, and the downstream displacement of the back flow region (\( \overline{v} < 0 \)). The larger spot was located in the vicinity of the top \( (x, y)-\)plane, and is clearly related to the symmetry boundary condition which induces a recirculating flow. Additional regions of \( \overline{v} = 0 \) arise next to the base line until the recirculation region is suppressed with a pointed end-shape. One possible explanation for this regions of zero mean streamwise velocity is that higher \( Re \) creates a stronger secondary flow. Such flow interacts with the recirculation bubble and displaces the back flow region further downstream. We intend to perform systematic simulations varying the Reynolds
number and suppressing the effect of the symmetry boundary condition in order to elucidate this behavior.

Finally, the spectral analysis revealed that one single shedding frequency dominates along the vertical direction ($St = 0.2197$). Nevertheless, we noticed local splitting of the vortex shedding frequencies between $z/D = 10$ and $12$. Fourier analysis with a long time-series, however, tends to smooth these isolated events. Perhaps a different approach, for instance wavelet analysis, may be used to extract information about these phase differences.

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