Shear viscosity, cavitation and hydrodynamics at LHC

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We study evolution of quark-gluon matter in the ultrarelativistic heavy-ion collisions within the frame work of relativistic second-order viscous hydrodynamics. In particular, by using the various prescriptions of a temperature-dependent shear viscosity to the entropy ratio, we show that the hydrodynamic description of the relativistic fluid becomes invalid due to the phenomenon of cavitation. For most of the initial conditions relevant for LHC, the cavitation sets in very early stages. The cavitation in this case is entirely driven by the large values of shear viscosity. Moreover we also demonstrate that the conformal terms used in equations of the relativistic dissipative hydrodynamic can influence the cavitation time.

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Presently the viscosity of the strongly-interacting matter produced in the heavy-ion collision experiments at LHC and RHIC is under extensive investigations. The measurements of the elliptical flow parameter \( v_2 \) show a strong collectivity in the fluid-flow implying the existence of a very low viscous stress due to shear viscosity [1]. According to the AdS/CFT conjecture, ratio of the shear viscosity to entropy density \( \eta/s \) may not be lower than \( 1/4\pi \) which is now known as KSS bound [2]. It has been argued that in order to explain the collective flow data \( \eta/s \) cannot be larger than twice the KSS-bound [3]. It must be noted that the applications of the viscous hydrodynamics discussed above regard \( \eta/s \) as independent of temperature. However, recently it has been argued that constant \( \eta/s \) is in sharp contrast with the observed fluid behavior in nature where it can depend on temperature [4, 5]. It has been demonstrated that the temperature-dependence of \( \eta/s \) can strongly influence the transverse momentum spectra and elliptical flow in the heavy-ion collision experiments at LHC [5, 6]. It should be emphasized here that the ratio of bulk viscosity to entropy density \( \zeta/s \) as a function of temperature was already considered by several authors and interesting consequences like cavitation were studied [7, 10]. A similar analysis with a temperature-dependent \( \eta/s \) has not been performed so far, which we intend to address here. Cavitation has also been studied recently with a holographic formulation of sQGP [11].

It is generally expected that \( \eta/s \) for QGP has a minimum at the critical temperature \( T_c \), while it increases with the temperature beyond \( T_c \) [5, 12, 13]. In this work we use \( \eta/s \) prescriptions arising from lattice QCD (lQCD) as in Ref. [5], virial theorem type of arguments [12] as well as the analytical expressions for \( \eta/s \) as given in Ref [6]. We show that the large values of \( \eta/s \), relevant for LHC energies, can make the effective pressure of the fluid very small in a time less than 2 fm/c. This would cause cavitation in the fluid which in turn would limit the applicability of hydrodynamics. It must be noted that the cavitation at RHIC energies studied in Refs. [8, 9, 14] earlier was driven by the high values of the bulk viscosity near the critical temperature. However, the bulk viscosity can play an insignificant role in the temperatures \( T >> T_c \). In the present study we demonstrate that for LHC energies cavitation is solely driven by the shear viscosity.

We use relativistic boost invariant causal viscous hydrodynamics equations in 1+1 dimensions [15, 16]. One may argue against the validity of applying (1+1)-dimensional flow in studying the relativistic heavy-ion collisions by ignoring the transverse flow. As will be shown later for a central collision at LHC energies the cavitation sets during the initial stage of the evolution in a time less than 2 fm/c. Since the transverse flow is negligible during the earlier stages of a heavy-ion collision, it will not have a significant effect on the cavitation time. We use the parametrization of the coordinates \( t = \tau \cosh \eta_p \) and \( z = \tau \sinh \eta_p \), with the proper time \( \tau = \sqrt{t^2 - z^2} \) and space-time rapidity \( \eta_p = \frac{1}{2} \ln \frac{t + \sqrt{t^2 - z^2}}{t - \sqrt{t^2 - z^2}} \). Now the 4-velocity can be written as \( u^\mu = (\cosh \eta_p, 0, 0, \sinh \eta_p) \). Within the second order theory (for more details on this theory and its application to relativistic heavy ion collisions we refer to Refs. [9, 17, 18]) the equations dictating the longitudinal ex-

![Fig. 1. Different prescriptions of \( \eta/s \) as function of temperature, with \( T_c = 0.2 \) GeV. The horizontal curve show \( \eta/s = 1/4\pi \) obtained from the AdS/CFT correspondence.](image)
pansion of the medium are given by [19] [22]:

\[
\frac{\partial \varepsilon}{\partial \tau} = -\frac{1}{\tau} (\varepsilon + P + \Pi - \Phi),
\]

(1)

\[
\frac{\partial \Phi}{\partial \tau} = -\frac{\Phi}{\tau} + \frac{2}{3} \beta_2 \tau - \frac{1}{\tau_\pi} \left[ \frac{4\pi}{3} \frac{\Phi + \lambda_1}{2\eta^2} \Phi^2 \right],
\]

(2)

\[
\frac{\partial \Pi}{\partial \tau} = \frac{\Pi}{\tau_\Pi} - \frac{1}{\beta_0 \tau},
\]

(3)

The effects due to shear and bulk viscosity are represented via \( \Phi \) and \( \Pi \) respectively and they can contribute to the effective pressure of the fluid. Equations (2,3) are evolution equations for \( \Phi \) and \( \Pi \) governed by their relaxation times \( \tau_\pi \) and \( \tau_\Pi \) respectively. Last term on the right-hand side of Eq. (2) is due the conformal symmetry [23]. In order to close the system of Equations (1,2,3), one needs to use equation of state (EoS). We have used recent IQCD results [24] for this purpose. At LHC energies the bulk viscosity is expected to be negligible as \( \varepsilon \approx 3P \) and one can ignore Eq. (3).

In the local rest frame the shear stress describes the deviation from the isotropy of the stress tensor. To quantify this anisotropy one can define the longitudinal pressure \( P_z \) [8] [25] in absence of the bulk stress as

\[
P_z = P - \Phi
\]

(4)

where \( P \) is the equilibrium hydrodynamic pressure.

We use recent IQCD estimate for \( \eta/s \) in QGP sector calculated by Nakamura et. al [13]. The resulting \( \eta/s \) from IQCD has the expected minimum near the critical temperature \( T_c \). It should be noted that recent lattice studies indicate a crossover rather than a phase transition [20]. However, for the present work this may not be an issue since we are interested in temperature dependence of \( \eta/s \) where \( T_c \) is a parameter. We use the parametrization of \( \eta/s \) given in Ref. [27], where the minimum value of \( \eta/s \) is \( 1/4\pi \). Another prescription for shear viscosity that we use is from Ref. [12], where using virial expansion techniques, the authors calculate \( \eta/s \) in QGP. Fig.1 shows the plots of various \( \eta/s \) prescriptions versus temperature with \( T_c =0.2 \) GeV. The top curve shows values of \( \eta/s \) obtained from the lattice results, while the middle curve corresponds to \( \eta/s \) values obtained from the virial expansion. The horizontal line corresponds to the KSS value. Finally, we consider the temperature-dependent forms of \( \eta/s \) as given in Ref. [6]: \( (\eta/s)_1 = 0.2 + 0.3 \frac{T-T_{chem}}{T_{chem}}, (\eta/s)_2 = 0.2 + 0.4 \left( \frac{T-T_{chem}}{T_{chem}} \right)^2 \) and \( (\eta/s)_3 = 0.2 + 0.3 \sqrt{\frac{T-T_{chem}}{T_{chem}}}, \)

with \( T_{chem} = 0.165 \) GeV.

Relaxation time \( \tau_\pi = 2\eta \beta_2 \) can be determined by an underlying theory other than the hydrodynamics. It ought to be mentioned that in the relativistic viscous hydrodynamic literature there is some ambiguity regarding the value of the relaxation times associated with shear and bulk viscosity. In this work we have taken the relaxation time for shear viscosity \( \tau_\pi = \frac{5\eta}{\tau}, \) which is motivated by kinetic theory [5] [27]. In addition we also solve

| \( \text{LHC} \) | \( IS (\tau_x = \frac{2\mu_2}{\tau}) \) | \( IS+C (\tau_x = \frac{2\mu_3}{\tau}) \) |
|----------------|-----------------|-----------------|
| \( \tau_f \) | \( \tau_{cav} \) | \( \tau_f \) | \( \tau_{cav} \) |
| \( T_0 = 0.3 \) fm/c | \( \eta/s \) IQCD | \( T_{cav} = 21.18 \) \( 0.57 \) | \( 0.40 \) | \( 14.21 \) \( 0.52 \) | \( 0.43 \) |
| \( T_0 = 0.506 \) GeV | \( \eta/s \) virial | \( 20.61 \) \( 0.63 \) | \( 0.410 \) | \( 13.93 \) | \( 0.93 \) | \( 0.374 \) |
| \( T_0 = 0.3 \) fm/c | \( \eta/s \) IQCD | \( 31.58 \) \( 0.57 \) | \( 0.465 \) | \( 20.31 \) | \( 0.52 \) | \( 0.479 \) |
| \( T_0 = 0.560 \) GeV | \( \eta/s \) virial | \( 25.06 \) \( 0.68 \) | \( 0.444 \) | \( 18.12 \) | 1.20 | \( 0.385 \) |
| \( T_0 = 0.6 \) fm/c | \( \eta/s \) IQCD | \( 12.40 \) \( 1.20 \) \( 0.333 \) | \( 10.85 \) \( 1.55 \) | \( 0.316 \) |
| \( T_0 = 0.405 \) GeV | \( \eta/s \) virial | \( 15.30 \) | \( 1.27 \) | \( 0.329 \) | \( 11.88 \) | - |
| \( T_0 = 0.6 \) fm/c | \( \eta/s \) IQCD | \( 18.36 \) \( 1.21 \) \( 0.369 \) | \( 15.63 \) | \( 1.29 \) | \( 0.365 \) |
| \( T_0 = 0.450 \) GeV | \( \eta/s \) virial | \( 19.84 \) | \( 1.43 \) | \( 0.353 \) | \( 16.07 \) | - |
| \( T_0 = 1.0 \) fm/c | \( \eta/s \) IQCD | \( 10.48 \) | - | - | 9.98 | - |
| \( T_0 = 0.350 \) GeV | \( \eta/s \) virial | \( 13.04 \) | \( 2.16 \) | \( 0.283 \) | \( 11.17 \) | - |

TABLE I. Column IS corresponds to the case when the conformal terms are neglected from the hydrodynamics equations. In this case the relaxation time \( \tau_\pi \) from the kinetic theory is taken in to account. The column IS+C corresponds to the case when the conformal terms and \( \tau_\pi \) obtained from the supersymmetric Yang-Mills theory are included in the equations of hydrodynamics. The cavitation time \( \tau_{cav} \) and \( \tau_f \) are measured in the unit of fm/c and the cavitation temperature \( T_{cav} \) is shown in the units of GeV. \( \tau_{cav} \) and \( T_{cav} \) are left blank when there is no cavitation.

Eqs. (1,2) by taking \( \tau_\pi = \frac{2\eta}{\tau} \approx \frac{2\eta}{\tau_\pi} \) with \( \lambda_1 = \frac{2\eta}{\tau_\pi} \), inspired by results from \( N=4 \) supersymmetric Yang-Mills theory [23] [28].

Next we present the numerical solutions for the equations of hydrodynamics. First we consider the case with temperature-dependent \( \eta/s \) taken from IQCD calculations. Fig.2 shows the plots of longitudinal pressure \( P_z \) versus the proper time for the cases of pure Israel-Stewart type (IS) hydroy by neglecting the conformal terms in Eq. (2) and with conformal terms (IS+C). In the case of IS we use \( \tau_\pi \) from the kinetic theory and from the supersymmetric Yang-Mills theory when we consider IS+C case. We plot \( P_z \) for these two cases with the initial temperatures 0.405 and 0.450 GeV. The starting time \( \tau_0 \) is chosen to be 0.6 fm/c. Let us first consider the case with \( T_0 = 0.405 \) GeV. From the figure it is clear that longitudinal pressure becomes negative in the IS case around cavitation time \( \tau_{cav} = 1.20 \) fm/c. The temperature \( T_{cav} \) at which the cavitation occurs is about 0.333 GeV which is much larger than the critical temperature \( T_c \). Thus the cavitation can take place very early during the evolution. This, we believe, provides a posteriori justification for neglecting the transverse flow; as the hydrodynamic treatment may not be valid for the time larger than \( \tau_{cav} \). Further, if we include the conformal terms in Eq. (2) together with the relaxation time obtained from supersymmetric Yang-Mills (IS+C), the cavitation time increases marginally and becomes \( \tau_{cav} = 1.53 \) fm/c. Similarly \( T_{cav} = 0.316 \) GeV is less than the cavitation temperature without the conformal terms. Next we consider a higher initial temperature \( T_0 = 0.450 \) GeV. Here also we observe cavitation for both IS and IS+C cases as in the previous case with \( T_0 = 0.405 \) GeV. For IS case cavitation happens at a time \( \tau_{cav} = 1.21 \)
3

FIG. 2. The longitudinal pressure $P_z$ as function of time for IS and IS+C hydrodynamics. Initial time is taken to be 0.6 fm/c with initial temperatures 0.405 and 0.450 GeV. $\eta/s(T)$ is obtained from the IQCD curve shown in Fig. 1.

FIG. 3. The longitudinal pressure $P_z$ as function of time for IS and IS+C hydrodynamics. Initial time is taken to be 0.6 fm/c with initial temperatures 0.405 and 0.450 GeV. $\eta/s(T)$ taken from the virial expansion techniques curve in Fig. 1.

fm/c which is only marginally greater than the corresponding $T_0 = 0.405$ GeV case considered previously. However, here the temperature at which cavitation occurs is higher ($T_{\text{cav}} = 0.369$ GeV) than the previous case. This difference is expected since the initial temperature for the latter case is also larger. IS+C case with $T_0 = 0.450$ GeV, cavitation sets in at $\tau_{\text{cav}} = 1.29$ fm/c with $T_{\text{cav}} = 0.365$ GeV. Again we note that there is not much difference between the cavitation times in IS and IS+C cases.

In Fig. 2, we show $P_z$ as function of time by taking $\eta/s$ values using the virial expansion techniques given in Ref. [12]. Values for $\tau_0$ and $T_0$ are same as in Fig. 2. Here in the IS case with $T_0 = 0.450$ GeV we can see that cavitation sets in around 1.43 fm/c when the system temperature is 0.353 GeV. However, as one can see from Fig. 3, when we include conformal terms (IS+C case) cavitation scenario is avoided. Next we lower the initial temperature to 0.405 GeV and consider the IS case. Here system reaches a negative longitudinal pressure stage at $\tau_{\text{cav}} = 1.27$ fm/c with $T_{\text{cav}} = 0.329$ GeV. But with conformal terms included, as one can see from the figure, the longitudinal pressure remains positive although it assumes a very small value by 2 fm/c. Since the values of $\eta/s$ for the virial expansion techniques are systematically smaller than $\eta/s$ for the IQCD results as shown in Fig. 1, the corresponding cavitation time is larger than that shown in Fig. 2. However, the cavitation temperature $T_{\text{cav}}$ is smaller than the corresponding cases discussed in Fig. 2.

Further, we have changed the values of the initial time by considering the case $\tau_0 = 0.3$ fm/c and $\tau_0 = 1.0$ fm/c. These results are summarized in Table I. For $\tau_0 = 0.3$ fm/c and $T_0 = 0.560$ GeV case, the cavitation occurs around $\tau_{\text{cav}} = 0.6$ fm/c for the IQCD $\eta/s$ while it occurs around $\tau_{\text{cav}} = 0.68$ fm/c for $\eta/s$ obtained from the virial expansion. For the case with $\tau_0 = 1.0$ fm/c and $T_0 = 0.350$ GeV, for $\eta/s$ from virial expansion, the cavitation occurs around $\tau_{\text{cav}} = 2.16$ fm/c. However, in this case when the $\eta/s$ values from IQCD are used there is no cavitation. We would like to note that the table shows no entries for $\tau_{\text{cav}}$ and $T_{\text{cav}}$ for certain cases. For such instances the longitudinal pressure remains positive and there is no cavitation. Table I indicates for the given initial conditions there are more number of no-cavitation instances when the conformal terms in the equations of the hydrodynamics are taken into account.

We also summarise the results for $\tau_f$, the total time taken by the system to reach $T_c$ by ignoring the cavitation in Table I. One can see that with $T_0 = 0.405$ GeV for IQCD (virial) case $\tau_f$ = 12.40 (15.30) fm/c without the conformal term and $\tau_f$ = 10.83 (11.88) fm/c if the term is included. Thus the inclusion of the conformal terms reduces $\tau_f$. We would like to emphasize that in this work we have taken a rather conservative initial value $\Phi(\tau_0) = 0$ so that the initial value of the longitudinal pressure is always positive [29]. Instead if one includes the first-order (Navier-Stokes) initial value $\Phi(\tau_0) = 4\eta(T_0)/(3\tau_0)$, then the cavitation can occur at even earlier time and higher temperature.

Next, we repeat our analysis using the temperature-dependent $\eta/s$ prescriptions given in Ref. [9]. With the same initial conditions as in Ref. [6] we find that the longitudinal pressure becomes negative very early $\sim 1$ fm/c for all the cases they have considered. Fig. 7 shows $P_z$ versus $\tau$ for initial temperature $T_0 = 0.419$ GeV and $\tau_0 = 0.6$ fm/c. In this case also cavitation sets in early in about $\tau_{\text{cav}} \sim 1$ fm/c.

Further, we also consider $\eta/s = 1/4\pi$ and $\zeta/s$ as function of $T$ as in Ref. [8] for LHC energies. It is found that the cavitation does not occur in this case unlike the results for RHIC energies [8, 9]. One may naively expect that when the system temperature reach $T \sim T_c$, the bulk viscosity become large enough to drive cavitation. However, the cavitation occurs when the viscous stress ($\Pi$ and/or $\Phi$) has a peak in its temporal profile.
The maximum value of $\Phi$ and longitudinal pressure $P_z$ is taken to be 0.450 GeV with initial time 0.6 fm/c. We have further considered the effect of anomalous viscosity ($\eta_A$), which may be important during the early time evolution in the hydrodynamics [30]. We use an effective shear viscosity $\eta^{-1} = \eta_A^{-1} + \eta_C^{-1}$, as discussed in Ref. [30]. Here, $\eta_C$ the collisional viscosity is taken from lQCD and for $\eta_A/s$ we use the expression from Ref. [30]. In this case, (with $\tau_0 = 0.6$ fm/c and $T_0 = 450$ MeV), cavitation sets in at a time 1.46 fm/c when the system is at a temperature 351 MeV. The initial value of anomalous viscosity to entropy density ratio is $\approx 0.23$. The results are presented in Fig. 5, where we plot the shear stress term $\Phi$ and longitudinal pressure $P_z$ as a function of proper time. As is clear from Fig. 5, the shear stress $\Phi$ increases sharply from its initial value. The maximum value of $\Phi$ and the time it takes to reach that value strongly depend upon $\tau_c$. This sharp rise of $\Phi$ result in a sharp reduction of $P_z$, which, finally becomes negative at $\tau_c$. 

Thus to summarise, we have shown by using various prescriptions for a temperature-dependent $\eta/s$ that at LHC energies the higher values of shear stresses can induce the cavitation. This will in turn make the hydrodynamical treatment invalid beyond cavitation time $\tau_c$. We have studied shear viscosity induced cavitation using one dimensional boost invariant causal dissipative hydrodynamics of Israel-Stewart. One would of course like to do an analysis using a (3+1)-dimensional viscous hydrodynamics like e.g. in Ref. [31]. Since cavitation occurs during the early stages of the collision, we believe that the inclusion of transverse flow will not alter the result qualitatively. However, as a caveat, we would like to mention that the difference between the initial conditions for the “cavitation” and “no-cavitation” cases is rather small, see Table I. It remains to be seen if the inclusion of transverse flow can alter the cavitation scenario in a qualitative way. It is worth noting here the negative pressure scenario may be circumvented by considering anisotropic corrections in the distribution functions [32]. It should be emphasized that there exist alternate formulations of dissipative relativistic fluid dynamics where the longitudinal pressure remains positive e.g. in Ref. [33]. It has been shown recently that the inclusion of the cavitation condition in boost invariant hydrodynamics can change the particle spectra from expanding QGP [31, 33]. Based on the various prescriptions of $\eta/s$ our results indicate that the hydrodynamical description is valid about $\tau_c \approx 2$ fm/c at LHC energies. Beyond $\tau_c$, the fluid might fragment [7] or form inhomogeneous clusters. Let us note that one of the assumptions of the statistical hadronisation models lies in creation of extended clusters of quark matter which hadronize statistically [34]. Alternately, as has been attempted recently one can possibly use a hybrid approach for the description of fire ball expansion applying viscous hydrodynamics for the QGP stage and then coupling it to a microscopic kinetic evolution for the hadronic stage [35]. Mere integration of the equations of hydrodynamics may not tell us about cavitation. We therefore believe that the conditions for cavitation may be required to be incorporated in the hydrodynamical codes.

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![Figure 4](image_url)  
**FIG. 4.** Cavitation with various $\eta/s$ prescriptions considered by Shen et al. in Ref. [3]. The initial temperature is taken to be 0.419 GeV with initial time 0.6 fm/c.

![Figure 5](image_url)  
**FIG. 5.** Cavitation along with anomalous viscosity. The longitudinal pressure $P_z$ and $\Phi$ as function of time. The initial temperature is taken to be 0.450 GeV with initial time 0.6 fm/c.
