WORMHOLES SUPPORTED BY CHIRAL FIELDS

Kirill A. Bronnikov\textsuperscript{a,1}, Sergey V. Chervon\textsuperscript{b,2}, and Sergey V. Sushkov\textsuperscript{c,3}

\textsuperscript{a} Center of Gravitation and Fundamental Metrology, VNIIMS, Ozyornaya St. 46, Moscow 117361, Russia; Institute of Gravitation and Cosmology, PFUR, Miklukho-Maklaya St. 6, Moscow 117198, Russia

\textsuperscript{b} Department of Theoretical Physics, Ulyanov State University, Leo Tolstoy St. 42, 432000 Ulyanovsk, Russia; Department of General Physics, Ulyanov State Pedagogical University, Lenin’s 100 years Sq., 4, 432700 Ulyanovsk, Russia

\textsuperscript{c} Department of General Relativity and Gravitation, Kazan State University, Kremlyovskaya St. 18, Kazan 420008, Russia; Department of Mathematics, Tatar State University of Humanities and Education, Tatarstan St. 2, Kazan 420021, Russia

We consider static, spherically symmetric solutions of general relativity with a nonlinear sigma model (NSM) as a source, i.e., a set of scalar fields $\Phi = (\Phi^1, ..., \Phi^n)$ (so-called chiral fields) parametrizing a target space with a metric $h_{ab}(\Phi)$. For NSM with zero potential $V(\Phi)$, it is shown that the space-time geometry is the same as with a single scalar field but depends on $h_{ab}$. If the matrix $h_{ab}$ is positive-definite, we obtain the Fisher metric, originally found for a canonical scalar field with positive kinetic energy; otherwise we obtain metrics corresponding to a phantom scalar field, including singular and nonsingular horizons (of infinite area) and wormholes. In particular, the Schwarzschild metric can correspond to a nontrivial chiral field configuration, which in this case has zero stress-energy. Some explicit examples of chiral field configurations are considered. Some qualitative properties of NSM configurations with nonzero potentials are pointed out.

PACS: 04.20.-q, 04.20.Jb, 04.40.-b

1. Introduction

For many years scalar fields have been an object of great interest for at least two reasons. The first one is quite pragmatic: models with scalar fields are relatively simple, and therefore it appeared possible to study them in detail and then extrapolate the results to more realistic and complicated models. Another reason has a more physical basis. Though so far there in no direct observational evidence, it is generally supposed that there exist fundamental scalar fields of great importance for the structure of the Universe. As a bright example, one may mention numerous inflationary models in which inflation in the early Universe is typically driven by a fundamental scalar field called an inflaton.

Studies of scalar fields in general relativity trace back to the paper by Fisher [1] who first found a static spherically symmetric solution of the Einstein-scalar equations with a single scalar field $\phi$. Then this solution was repeatedly rediscovered (see a historical review in [2]) and discussed from various points of view along with its various generalizations [3–11].

An obvious generalization of models with a single scalar field is to invoke a set of such fields, $\Phi = (\Phi^1, ..., \Phi^n)$. Such multi-field models have been applied in inflationary Universe theory (see, e.g., [12] and references therein). A more significant generalization is represented by nonlinear sigma models (NSM) in which the fields of the multiplet take part in a geometric interaction by forming an inner space (the so-called target space) with a Riemannian metric $h_{ab}(\Phi)$ [13]. The action of the theory then must be invariant under coordinate transformations both in our space-time and in the target space.

The terms “chiral model” and “chiral fields” are used to stress that interaction is inserted in a purely geometric way, unlike scalar fields in multi-field models. In the latter, interaction is inserted by simply adding the interaction Lagrangian to the one of free fields [13]. In [13], the term “chiral model” is used as an equivalent to NSM. A historical comment on the term “chiral” can be found in [2].

Chiral NSM have been introduced by Schwinger [14] and Skyrme [15]. Gell-Mann and Levy [16] pointed out how to realize the chiral symmetry and partial conservation of the axial vector current. A two-dimensional version of the model was studied because of its analogy in many respects to non-Abelian gauge theories. The main results of these studies can be found in the review [13].

A bridge to four-dimensional NSM was built only with inclusion of a coupling to gravity in [17]. NSM as a source of gravity were also considered by G. Ivanov [18] (see also [19]).

Applications of chiral NSM in inflationary cosmology have been proposed in [20, 21]. Chiral NSM with self-interaction potentials, leading to the so-called chiral cosmological models, contain self-interacting scalar field
Field equations

2.1. General formalism

The action for a self-gravitating chiral model based on a nonlinear sigma mode with \( n \) scalar fields \( \Phi^a \), minimally coupled to gravity, and an interaction potential \( V(\Phi) \) has the form

\[
S = \int d^4 x \sqrt{-g} \left\{ R - g^{\mu \nu} h_{ab} \Phi_{\mu \nu}^{a} \Phi_{\mu \nu}^{b} - 2V(\Phi) \right\},
\]

where \( g_{\mu \nu}(x) \) is the spacetime metric\(^4\), \( R \) is the scalar curvature, \( h_{ab}(\Phi) \) is a metric in the target space, Latin indices run from 1 to \( n \), and \( \Phi = (\Phi^1, ..., \Phi^n) \); commas and semicolons in the indices stand for partial \( (\partial/\partial x^\mu) \) and covariant \( (\nabla_\mu) \) derivatives, respectively.

Varying the action \( S \) with respect to the metric \( g_{\mu \nu} \) gives the Einstein equations

\[
R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = -T_{\mu \nu},
\]

with the stress-energy tensor (SET) of the chiral fields

\[
T_{\mu \nu} = h_{ab} \Phi_{\mu \nu}^a \Phi_{\mu \nu}^b - g_{\mu \nu} \left[ \frac{1}{2} R_{\mu \nu} h_{ab} \Phi_{\mu \nu}^a \Phi_{\mu \nu}^b + V(\Phi) \right].
\]

Eqs. (2) can be easily transformed to

\[
R_{\mu \nu} = h_{ab} \Phi_{\mu \nu}^a \Phi_{\mu \nu}^b + g_{\mu \nu} V(\Phi).
\]

Varying the action \( S \) with respect to the chiral fields \( \Phi^c \) gives the chiral field equations of motion:

\[
h_{ab} \Phi^{b \mu \nu} + \left[ \frac{\partial h_{ab}}{\partial \Phi^c} - \frac{1}{2} \frac{\partial h_{bc}}{\partial \Phi^a} \right] \Phi^{a k \mu} \Phi^{c \nu} - \frac{\partial V}{\partial \Phi^a} = 0. \tag{5}
\]

As usual, one should check whether the SET obeys the usual energy conditions. In particular, the null energy condition (NEC) reads \( T_{\mu \nu} k^\mu k^\nu \geq 0 \), where \( k^\mu \) is an arbitrary null vector. For chiral fields with the stress-energy tensor (5), the NEC yields

\[
\Xi := h_{ab} \Phi^a \Phi^b \Phi^{k \mu} k^\nu \geq 0. \tag{6}
\]

This means that \( \Xi \) should be a positive-definite quadratic form with respect to the vectors \( \zeta^a = \Phi^a k^\mu \) in the target space. Evidently, the NEC is violated if \( \Xi < 0 \).

2.2. Two-component sigma model

A simple example of a nonlinear sigma model is that consisting of two scalar field components, i.e., \( \Phi = (\phi, \psi) \). A general quadratic form in the target space now is

\[
g^{\mu \nu} [h_{11} \phi_\mu \phi_\nu + 2h_{12} \phi_\mu \psi_\nu + h_{22} \psi_\mu \psi_\nu] \tag{7}
\]

where \( h_{ab} = h_{ab}(\phi, \psi) \). After appropriate transformations in the target space: \( \phi = \phi(\tilde{\phi}, \tilde{\psi}), \psi = \psi(\tilde{\phi}, \tilde{\psi}) \), the quadratic form \( (7) \) can, in principle, be reduced to its canonical form

\[
g^{\mu \nu} [\phi_\mu \phi_\nu + h \psi_\mu \psi_\nu], \tag{8}
\]

where \( h \) is, generally speaking, a function of \( \phi \) and \( \psi \), i.e., \( h = h(\phi, \psi) \). The especially simple case, which corresponds to rotational symmetry in the target space, is \( h = h(\phi) \). In this case the action \( S \) reduces to

\[
S = \int d^4 x \sqrt{-g} \left\{ R - g^{\mu \nu} [\phi_\mu \phi_\nu + h(\phi) \psi_\mu \psi_\nu] \right\} - 2V(\phi, \psi).
\]

Now the chiral field \( \phi \) can be interpreted as an ordinary scalar field and \( \psi \) as a scalar field coupled with \( \phi \) via the kinetic coupling function \( h(\phi) \).

Static, spherically symmetric solutions

3.1. Equations and geometry

In this section we will consider static, spherically symmetric solutions of the theory \( S \) without a potential, i.e., assuming \( V(\Phi) = 0 \). For a general static spherically symmetric configuration, \( \Phi^a = \Phi^a(u) \), where \( u \) is an arbitrary radial coordinate, and the spacetime metric can be written as

\[
d\sigma^2 = -e^{2\gamma(u)} dt^2 + e^{2\alpha(u)} du^2 + e^{2\beta(u)} d\Omega^2. \tag{10}
\]

where \( d\Omega^2 = (d\theta^2 + \sin^2 \theta d\varphi^2) \) is the linear element on a unit sphere.

The SET of the scalar fields has the form

\[
T_{\mu \nu} = h_{ab} \Phi^a \Phi^b \partial_a \partial_b \log(1 - 1, 1, 1), \tag{11}
\]

i.e., has the same structure as for a single massless scalar field (the prime denotes \( d/du \)). Therefore the metric has the same form as in this simple case and should be reduced to the Fisher metric [1] if the scalar fields behave as a canonical scalar with positive kinetic energy and to the metric of the corresponding solution for a phantom scalar, first found by Bergmann and Leipnik [3] (it may be called “anti-Fisher”), by analogy with anti-de Sitter, if the scalar fields behave in a phantom way. Let us reproduce this solution in the simplest joint form, suggested in [8].

Two combinations of the Einstein equations for the metric \( S \) and the SET \( T \) read \( R^0_0 = 0 \) and \( R^0_0 +
$R^2_s = 0$. Choosing the harmonic radial coordinate $u$, such that $\alpha(u) = 2\beta(u) + \gamma(u)$, we easily solve these equations. Indeed, the first of them reads simply $\gamma'' = 0$, while the second one is written as $\beta'' + \gamma'' = e^{2(\beta + \gamma)}$. Solving them, we have

$$
\gamma = -mu,
$$

$$
e^{-\beta - \gamma} = s(k, u) := \begin{cases} 
  k^{-1} \sinh ku, & k > 0, \\
  u, & k = 0, \\
  k^{-1} \sin ku, & k < 0.
\end{cases} \quad (12)
$$

where $k$ and $m$ are integration constants; two more integration constants have been suppressed by choosing the zero point of $u$ and the scale along the time axis. As a result, the metric has the form [8]

$$
ds^2 = -e^{-2mu} dt^2 + \frac{e^{2mu}}{s^2(k,u)} \left[ \frac{du^2}{s^2(k,u)} + d\Omega^2 \right]. \quad (13)
$$

In addition, with these metric functions, the $(1)$ component of the Einstein equations [2] leads to

$$
k^2 \sinh k u = m^2 + h_{ab} \Phi^a \Phi^b, \quad (14)
$$

whence it is clear that

$$
h_{ab} \Phi^a \Phi^b = N = \text{const}. \quad (15)
$$

The scalar field equations read

$$
2(h_{ab} \Phi^b) + \frac{\partial h_{bc}}{\partial \Phi^b} \Phi^b \Phi^c = 0 \quad (16)
$$

and obviously cannot be solved in a general form, but [15] is their first integral.

The metric (13) is defined (without loss of generality) for $u > 0$, it is flat at spatial infinity $u = 0$, and $m$ has the meaning of the Schwarzschild mass in proper units. The metric properties crucially depend on the sign of $k$, which in turn depends on $N$ and ultimately on the nature of the matrix $h_{ab}$.

If $h_{ab}$ is positive-definite, we have $N = k^2 - m^2 > 0$ for all nontrivial scalar field configurations and obtain the Fisher metric: then $k > 0$, and the substitution $e^{-2ku} = 1 - 2k/r$ converts (13) to

$$
ds^2 = -(1 - 2k/r)^{a} dt^2 + (1 - 2k/r)^{a} dr^2 \\
+ (1 - 2k/r)^{1-a} r^2 d\Omega^2, \quad (17)
$$

where $a = m/k = (\text{sign} m)(1 - N/k^2)^{1/2} < 1$. The solution is defined for $r > 2k$, $r \to \infty$ is a flat asymptotic, and $r = 2k$ is a central singularity. The Schwarzschild metric is restored for $N = 0$, $a = 1$. In full agreement with the well-known general results [23, 24], in this case, due to $\Xi > 0$, throats (i.e., minima of the spherical radius $R(r) = e^\beta$) are absent, and wormholes are impossible.

If $h_{ab}$ is not positive-definite, $N$ can have any sign. Thus, in particular, for some nontrivial scalar field configurations we may have $N = 0$, hence the Schwarzschild metric. It is an interesting new feature which is absent in the case of a single scalar field. More generally, there can be now $N < 0$, which corresponds to “anti-Fisher” phantom-field metrics. Let us briefly describe different cases of these metrics, following [25] and the book [26]. According to the three variants of the function $s(k, u)$ at different $k$ in [12], the solution with $N < 0$ splits into three branches with the following properties.

(a) $k > 0$: the metric has again the form (17), but now, due to $N < 0$, it holds $|a| > 1$. For $m < 0$, that is, $a < -1$, we have, just as in the Fisher solution, a repulsive central singularity at $r = 2k$.

The situation is, however, drastically different for $a > 1$. Indeed, the spherical radius $R$ then has a finite minimum at $r = r_{th} = (a + 1)k$, corresponding to a throat of the size

$$
R(r_{th}) = R_{th} = k(a + 1)^{(a+1)/2}(a - 1)^{(1-a)/2}, \quad (18)
$$

and tends to infinity as $r \to 2k$. Moreover, for $a = 2, 3, \ldots$ the metric exhibits a horizon of order $a$ at $r = 2k$ and admits a continuations to smaller $r$. A peculiarity of such horizons is their infinite area. Such asymptotically flat configurations with horizons of infinite size have been named cold black holes since all of them have zero Hawking temperature (see [25] and references therein for more detail).

Furthermore, one can verify that all nonzero components $R_{\mu\nu\rho\sigma}$ of the Riemann tensor behave as $P^{\mu\nu\rho\sigma}$ (where $P = 1 - 2k/r$) as $r \to 2k$ and $P \to 0$. An exception is the value $a = 1$, corresponding to $N = 0$, when the Schwarzschild solution is reproduced. Hence, at $r = 2k$ the metric has a curvature singularity if $a < 2$ (except for $a = 1$), there is finite curvature if $a = 1$ and $a = 2$ and zero curvature if $a > 2$.

For non-integer $a > 2$, the qualitative behavior of the metric as $r \to 2k$ is the same as near a horizon of infinite area, but a continuation beyond it is impossible due to non-analyticity of the function $P^{\mu\nu}(r)$ at $r = 2k$. Since geodesics terminate there at a finite value of the affine parameter, it is a space-time singularity (a singular horizon as it was termed in [25]) even though the curvature invariants tend there to zero.

(b) $k = 0$, $N = -m^2$: the metric is defined in the range $u \in \mathbb{R}_+$ and is rewritten in terms of the coordinate $r = 1/u$ as follows:

$$
ds^2 = -e^{-2m/r} dt^2 + e^{2m/r} (dr^2 + r^2 d\Omega^2), \quad (19)
$$

As before, $r = \infty$ is a flat infinity, while at the other extreme, $r \to 0$, the behavior is different for positive and negative mass. Thus, for $m < 0$, $r = 0$ is a singular center ($r = 0$, the curvature invariants are infinite). On the contrary, for $m > 0$, $r \to \infty$ and the curvature tensor tends to zero as $r \to 0$. This is again

\footnote{To our knowledge, this metric was for the first time found by Yilmaz [4].}
a singular horizon: despite the vanishing curvature, the non-analyticity of the metric in terms of $r$ makes its continuation impossible. The geometries (17) with $a > 1$ and (19) with $m > 0$ can be called wormholes, but they are asymptotically flat only as $r \to \infty$, whereas on the other side of the throat there is a singular or nonsingular horizon of infinite area.

(c) $k < 0$, $-N = m^2 + k^2$: the solution describes a wormhole with two flat asymptotics at $u = 0$ and $u = \pi/|k|$. The metric has the form

$$ds^2 = -e^{-2mu}dt^2 + \frac{k^2 \omega_{2mu}}{\sin^2(ku)} \left[ k^2 du^2 + d\Omega^2 \right],$$

where $u$ is expressed in terms of the coordinate $r$, defined on the whole real axis $\mathbb{R}$, by $|k|u = \cot^{-1}(r/|k|)$. If $m > 0$, the wormhole is attractive for ambient test matter at the first asymptotic ($r \to \infty$) and repulsive at the second one ($r \to -\infty$), and vice versa in case $m < 0$. For $m = 0$ one obtains the simplest possible wormhole solution, called the Ellis wormhole, although Ellis [7] actually discussed these wormhole solutions with any $m$.

The wormhole throat occurs at $r = m$ and has the size

$$R_{th} = (m^2 + k^2)^{1/2} \exp \left( \frac{m}{k} \cot^{-1} \frac{m}{k} \right).$$

### 3.2. Scalar field configurations

The metrics (17), (19), and (20) describe all possible kinds of static, spherically symmetric spacetime geometries for the model (1). The scalar field configuration can be determined from Eqs. (16) which, as has been noted, cannot be solved in a general form. Let us therefore consider some specific examples for the action (9) with two fields $\phi$ and $\psi$ and the kinetic coupling $h(\phi)$.

Then, in terms of the harmonic coordinate $u$, Eqs. (16) take the form

$$2\phi'' - \frac{dh}{d\phi}\phi' = 0,$$

(22)

$$\left(h\psi'\right)' = 0,$$

(23)

while their integral (15) reads

$$\phi'^2 + h(\phi)\psi'^2 = N.$$  

(24)

It is easy to verify that Eq. (22) follows from (23) and (24). Eq. (23) leads to

$$h\psi' = C = \text{const},$$

(25)

and both (25) and (24) with $\psi'$ substituted from (25) are easily integrated by quadratures in a general form: indeed, substituting $\psi'$ from (25) into (24), one obtains an integrable first-order equation with respect to $\phi(u)$. Moreover, it is clear that $N > 0$ as long as $h(\phi) \geq 0$ for all nontrivial scalar field configurations, and we have the Fisher geometry (17), $|a| < 1$. In other words, throats and wormholes can only be obtained with $h < 0$.

Let us present three examples of $h(\phi)$ for which the quadratures are found explicitly.

1. $h(\phi) = 1/\phi$. In this case, integrating (24) with (25), we find

$$\phi = \frac{N}{C^2} - \frac{C^2}{4}(u-u_0)^2,$$

(26)

and substituting it into (25), we finally get

$$\psi = \psi_0 + \frac{N}{C}u - \frac{C^3}{12}(u-u_0)^3,$$

(27)

where $u_0$ and $\psi_0$ are integration constants. The main observation is that the solution is defined for any $N$, hence all geometries described above are possible. In particular, in case $N = 0$ we have the Schwarzschild geometry despite the existence of two nontrivial scalar fields.

2. $h(\phi) = \phi^2$. Integration gives

$$\phi = \sqrt{N} \sqrt{C^2/N^2 + (u-u_0)^2},$$

(28)

$$\psi = \psi_0 + \arctan \left( \frac{N}{C}(u-u_0) \right).$$

(29)

3. $h(\phi) = \sin^2 \phi$, which corresponds to SO(3) symmetry of the target space. We get in a similar way

$$\phi = \sqrt{1-C^2/N} \sin \left( \sqrt{N}(u-u_0) \right),$$

(30)

$$\psi = \psi_0 + \arctan \left( \frac{C}{\sqrt{N}} \tan \left[ \sqrt{N}(u-u_0) \right] \right).$$

(31)

In the second and third cases, $h \geq 0$ and accordingly $N > 0$, i.e., only the Fisher metric (17) with $|a| < 1$ takes place.

In all three cases, expressions for $\phi$ and $\psi$ in terms of the radial coordinate $r$ used in the metrics (17), (19), (20) are easily obtained by substituting the appropriate expressions for $u(r)$.

### 4. Concluding remarks

We have considered static, spherically symmetric solutions of the theory (1) without a potential, i.e., assuming $V(\Phi) \equiv 0$. In this case, the stress-energy tensor of chiral fields has the same algebraic structure as for a single massless scalar field, namely, $T_{\mu}^\nu = h_{ab}(\phi^a\phi^b)\delta^{(4)}(1, -1, 1, 1)$. As a consequence, a geometry of static, spherically symmetric spacetime with chiral fields turns out to be the same as in a simple model with a single scalar field; its properties are determined by the quadratic form $h_{ab}$. All possible kinds of these geometries are described by the metrics (17), (19), and (20).

Thus, if it is positive-definite, we obtain the Fisher metric (17) [1] with a naked singularity, originally found
for a canonical scalar field with positive kinetic energy. In $h_{ab}$ is negative-definite, the metric corresponds to the so-called anti-Fisher solution for a phantom scalar, first found by Bergmann and Leipnik [3], including singular and nonsingular horizons of infinite area, with the metrics (17) and (19) as well as traversable wormholes with the metric (20).

If $h_{ab}$ is neither positive- nor negative-definite, all branches of the (anti-)Fisher solution are possible, depending on a particular scalar field configuration. A new interesting feature of NSM that appears in this case but is absent with a single scalar field is that the family of solutions includes the Schwarzschild metric corresponding to some nontrivial chiral field configurations, whose SET is in such cases equal to zero.

Some explicit examples of chiral field configurations have been obtained for the action (1) with two fields $\phi$ and $\psi$ and the kinetic couplings $h(\phi) = \phi^{-1}, \phi^2, \sin^2 \phi$.

For configurations with nonzero potentials $V(\Phi)$, it is hard to find exact solutions but some qualitative results are known, which, taken together, give rather a clear picture of what can and what cannot be expected from static, minimally coupled NSM in general relativity as well as some its extensions, including (multi-)scalar-tensor and multidimensional theories [28]. Let us enumerate some results valid for static, spherically symmetric configurations with the action (11), the metric (11) and $\Phi^a = \Phi^a(u)$.

1. The no-hair theorem generalizing that of Adler and Pearson [29] to NSM: it claims that if the matrix $h_{ab}(\Phi)$ is positive-definite and the potential $V(\Phi) \geq 0$, asymptotically flat black holes cannot have nontrivial external scalar fields.

2. Nonexistence of particlelike solutions (i.e., asymptotically flat solutions with a regular center) for NSM with positive-definite $h_{ab}(\Phi)$ and $V(\Phi) \geq 0$.

3. Nonexistence of regular solutions without a centre (wormholes, horns, flux tubes) for NSM with positive-definite $h_{ab}(\Phi)$ and any potentials $V(\Phi)$.

4. The causal structure theorem generalizing that of [30]: it asserts that, with any $h_{ab}(\Phi)$ and $V(\Phi)$, the list of possible types of global causal structures (and the corresponding Carter-Penrose diagrams) is the same as in the trivial case $\Phi^a = \text{const}$, namely: Minkowski (or AdS), Schwarzschild, de Sitter and Schwarzschild — de Sitter.

The fourth statement holds irrespective of any assumptions on the spatial asymptotics and admits many generalizations. It is therefore the most universal property of self-gravitating scalar fields in various theories of gravity [28].

Acknowledgments

This work was supported in part by the Russian Foundation for Basic Research grants No. 08-02-91307, 08-02-00325, 09-02-0677-a.

References

[1] I.Z. Fisher, Zh. Eksp. Teor. Fiz. 18, 636 (1948); gr-qc/9911008
[2] S.V. Chervon, Non-linear fields in theory of gravitation and cosmology (Middle-Volga Scientific Centre, Ulyanovsk State University, Ulyanovsk, 1997).
[3] O. Bergmann and R. Leipnik, Phys. Rev. 107, 1157 (1957).
[4] H. Yilmaz, Phys. Rev. 111, 1417 (1958).
[5] H.A. Buchdahl, Phys. Rev. 115, 1325 (1959).
[6] A.I. Janis, D.C. Robinson, and J. Winicour, Phys. Rev. 186, 1729 (1969).
[7] H. Ellis, J. Math. Phys. 14, 104 (1973).
[8] K. A. Bronnikov, Acta Phys. Pol. B 4, 251 (1973).
[9] M. Wyman, Phys. Rev. D 24, 839 (1981).
[10] C. Armendáriz-Picón, Phys. Rev. D 65, 104010 (2002).
[11] S.V. Sushkov and Y.-Z. Zhang, Phys. Rev. D 77, 024042 (2008).
[12] A.R. Liddle and D.H. Lyth, Cosmological Inflation and Large-Scale Structure, (Cambridge University Press, 2000)
[13] A.M. Perelomov, Phys. Rep. 146, No 3, 136 (1987).
[14] J. Schwinger, Ann. Phys. 2, 407 (1957).
[15] T.H.R. Skyrme, Proc. Roy. Soc. Lond. A247, No 1249, 260 (1958).
[16] M. Gell-Mann and M. Levy, Nuovo Cim. 26, No 4, 705 (1960).
[17] V. De Alfarro, S. Fubini, and G. Furlan, Nuovo Cim. A 50, 523 (1979).
[18] G.G. Ivanov, Teor. Mat. Fiz. 57, 45 (1983).
[19] S.V. Chervon, Izv. Vuzov, Fiz. (Russ. Phys. J., New York) 26, No 8, 89 (1983).
[20] S.V. Chervon, Grav. Cosmol. 1, 91 (1995).
[21] S.V. Chervon, Grav. Cosmol. 3, 145 (1997).
[22] V.D. Ivashchuk and V.N. Melnikov, Exact solutions in multidimensional gravity with antisymmetric forms, topical review. Class. Quantum Grav. 18 R87-R152 (2001); hep-th/0110274
[23] M.S. Morris and K.S. Thorne, Am. J. Phys. 56, 395 (1988).
[24] D. Hochberg and M. Visser, Phys. Rev. D 56, 4745 (1997).
[25] K.A. Bronnikov, M.S. Chernakova, J.C. Fabris, N. Pinto-Neto and M.E. Rodrigues, Int. J. Mod. Phys. D 17, 25–42 (2008); gr-qc/0609084
[26] K.A. Bronnikov and S.G. Rubin, Lectures on Gravitation and Cosmology (MIFI Press, Moscow, 2008, in Russian).
[27] S.V. Chervon, J. Astroph. Astron. 16, Suppl. 65 (1995).
[28] K.A. Bronnikov, S.B. Fadeev and A.V. Michtchenko, Gen. Rel. Grav. 35, 505 (2003); gr-qc/0212065.
[29] S. Adler and R.B. Pearson, Phys. Rev. D 18, 2798 (1978).
[30] K.A. Bronnikov, Phys. Rev. D 64, 064013 (2001); gr-qc/0104092