High-precision Flux Linkage Observation of Induction Motor based on Spectral Decomposition Theory

Yuedou Pan 1, Weifeng Zhang 2

1 University of Science and Technology Beijing, USTB, BeiJing, 100000, China
2 University of Science and Technology Beijing, USTB, BeiJing, 100000, China

Abstract. In the vector control system of induction motor, there is a delay in the switching device, which causes current distortion and motor performance degradation. Especially for high-power induction motors, the switching frequency of the inverter system is low, and the time delay effect is obvious. The mathematical model of magnetic flux observation of induction motors generally ignores the delay of the switching device. Therefore, the mathematical model is not accurate and the accuracy of the magnetic flux observation is not high. To address this problem, the mathematical model of the induction motor with delay is established, and the spectral decomposition flux linkage observer is designed to observe the rotor flux linkage in this paper. The designed spectral decomposition magnetic flux observer has the advantages of high magnetic flux observation accuracy. Simulation results prove the feasibility and effectiveness of the designed spectral decomposition flux observer.

1. Introduction

With the development of power electronics technology and transmission technology, induction motors are widely used in rail transit, aviation applications, electric locomotives and other industrial fields [1]. To achieve high-performance vector control of induction motors, the magnetic field of the induction motor must be accurately oriented [2]. Under the reference coordinate system, vector control according to the rotor magnetic field orientation is the key to achieving flux linkage and torque decoupling control. In order to decouple the magnetic flux generated by current and the component generated by torque accurately, the rotor flux linkage must be accurately estimated or observed [3]. Scholars at home and abroad have conducted a lot of research and proposed a variety of methods to obtain accurate flux linkage information. Literature [4] pointed out that the voltage model method has a simple structure and only needs to know the stator structure parameters. However, due to its characteristic of pure integration, it relies too much on the initial value of integration, which tends to cause integral drift, large errors in potential calculation, and poor system robustness and low-speed performance. In literature [5], the current model method is adopted to solve the problem that the voltage model is prone to integral drift and the initial flux linkage cannot be established, but the observation accuracy is related to the speed and is easily affected by the change of the motor speed. Literature [6] proposes a stator flux linkage observation method based on the second-order sliding mode suboptimal algorithm. The controller has a simple structural design and strong robustness, but its chattering problem cannot always be completely eliminated. Literature [7] proposed a discretization model flux linkage observation method for induction motors at low sampling frequency, which solved the instability problem of the first-order Euler discretization model of the full-order flux linkage.
observer for induction motors, but the algorithm requires a large amount of calculation, which greatly limits its application.

As mentioned above, there are many methods of flux linkage observation for induction motors, which improve the accuracy of flux linkage observation to a certain extent, but they do not take into account the delay of switching devices at low switching frequencies, leading to an inaccurate mathematical model of flux linkage observation and low accuracy. In response to these problems, this paper proposes a rotor flux linkage observation method based on spectral decomposition theory.

2. Considering the mathematical model of induction motor with delay
   In order to facilitate the establishment of the spectral decomposition observer model, in this paper, the \( M - T \) coordinate system is oriented according to the rotor magnetic field, and the \( M \) axis is oriented on the full flux linkage axis \( \psi_r \) to establish a mathematical model of the induction motor [8]:

\[
\begin{bmatrix}
\psi_{t} \\
i_{sm}
\end{bmatrix} = \begin{bmatrix}
k_1 & k_2 \\
k_3 & k_4
\end{bmatrix} \begin{bmatrix}
\psi_r \\
i_{st}
\end{bmatrix} + \begin{bmatrix}
0 & 0 \\
k_5 & k_6
\end{bmatrix} \begin{bmatrix}
i_s \\
u_{sm}
\end{bmatrix}
\]

(1)

Therein, \( k_1 = \frac{1}{T_r} \), \( k_2 = \frac{L_m}{T_r} \), \( k_3 = \frac{L_m}{\delta L_s L_r T_r} \), \( k_4 = \frac{R_s L_s^2 + R_m L_m^2}{\delta L_r^2} \), \( k_5 = \omega_s \), \( k_6 = \frac{1}{\delta L_r} \). \( u_{sm} \) is the component of the stator voltage on the \( M \) axis; \( i_{sm} \) and \( i_s \) are the components of the stator current on the \( M \) axis and the \( T \) axis; \( L_m \) is the mutual inductance of the stator and rotor coaxial equivalent windings; \( L_s \) and \( L_r \) are the self-inductance of the stator and rotor equivalent two-phase windings; \( \psi_r \) is the total flux linkage of the rotor Vector; \( \delta \) is the magnetic flux leakage coefficient of the motor, \( \delta = 1 - L_m / (L_s L_r) \); \( T_r \) is the rotor electromagnetic time constant; \( R_s \) and \( R_m \) are the stator resistance and rotor resistance; \( \omega_s \) is the synchronous angular velocity, \( \omega_s = \omega_s + \omega ; \omega_s \) is the slip angular velocity; \( \omega \) is the electrical angular velocity.

Transforming (1) into a matrix form:

\[
\begin{bmatrix}
\psi_{t} \\
i_{sm}
\end{bmatrix} = \begin{bmatrix}
k_1 & k_2 & 0 & 0 \\
k_3 & k_4 & 0 & 0 \\
k_5 & k_6
\end{bmatrix} \begin{bmatrix}
\psi_r \\
i_{st} \\
i_s \\
u_{sm}
\end{bmatrix}
\]

(2)

Considering the influence of the sampling period of the system and the input filter parameters in the actual application process, especially when applied to medium and high voltage and high power applications [9], the rotor electromagnetic time constant changes and the assumptions in the previous modeling process are no longer to be true, and the state delay term needs to be added in (2). Within a certain range, the effect of time lag on the system depends on the size of the corresponding coefficient matrix. Thus, the mathematical model of the induction motor with delay can be obtained:

\[
\begin{bmatrix}
\psi_{t} \\
i_{sm}
\end{bmatrix} = (1 - \lambda(\vartheta)) \begin{bmatrix}
k_1 & k_2 & 0 \\
k_3 & k_4 & 0 \\
k_5 & k_6
\end{bmatrix} \begin{bmatrix}
\psi_r \\
i_{st} \\
i_s \\
u_{sm}
\end{bmatrix} + \lambda(\vartheta) \begin{bmatrix}
0 & 0 & k_2 & k_3 \\
k_1 & k_4 & 0 & 0 \\
k_5 & k_6
\end{bmatrix} \begin{bmatrix}
\psi_r(t-\tau) \\
i_{sm}(t-\tau) \\
i_s(t-\tau) \\
u_{sm}(t-\tau)
\end{bmatrix}
\]

(3)

Among (3), \( \lambda(\vartheta) \) is the impact factor, which represents the degree of influence of the delay on the system, and \( \vartheta \) is the corresponding matrix; \( \tau \) is the total delay time. And the impact factor \( \lambda(\vartheta) \) is related to \( \vartheta \) as follow: \( \lambda = 1 / \| \vartheta \|_\infty \).

3. Spectral decomposition system theory and observer design

3.1. Spectral decomposition system theory
Spectral decomposition theory adopts the idea of equivalent transformation to transform a system that originally contained delay into a system that does not contain delay. This article adopts this idea to
design a spectral decomposition observer, configure the poles of the system with delays, eliminate the
effects of the system poles caused by delays deviating from the expected position, and then equate
the delay system as delay-free system. Consider the following delay system:

\[
\begin{align*}
S_\tau: & \quad \dot{x}(t) = A_\tau x(t) + A_1 x(t - \tau) + Bu(t) \\
y(t) &= C x(t) 
\end{align*}
\]

(4)

\(\tau\) is the delay time, which is a constant greater than zero; \(x(t) \in R^n, u(t) \in R^m, A_\tau \in R^{n \times n}, A_1 \in R^{n \times n}\)
\(, B \in R^{m \times n}, C \in R^n\) are the corresponding dimensional matrix. Defining: \(x_\tau(\theta) = x(t+\theta)\), where
\(\theta \in [-\tau, 0]\). The initial function satisfies: \(x_0 \in C([-\tau, 0]; R^n)\).

For the system \(S_\tau\), make the following linear transformation:

\[
z(t) = (T_\tau (x, u))(t) = x(t) + \int_{-\tau}^{t} e^{(t-\tau-\theta)} A_\tau x(\theta) d\theta
\]

(5)

From (6), we can get: \(z(t) \in AC([0, \tau]; R^n)\). Therefore, \(z(t)\) can be differentiated for any variable \(t\).

From (7), the following differential equations can be obtained:

\[
S_0: \dot{z}(t) = Az(t) + Bu(t)
\]

(7)

Comparing the delay system \(S_\tau\) and the non-delay system \(S_0\), if there is a state matrix \(A\) that
satisfies: \(A = A_\tau + e^{-\tau} A_1\), then the delay system can be equivalently converted into a non-delay
system.

In order to further clarify the relationship between the delay system \(S_\tau\) and the non-delay system
\(S_0\), the system \(S_\tau\) and \(S_0\) Laplace transform are performed respectively.

\[
X(s) = (s I - A_\tau - e^{-\tau} A_1)^{-1} BU(s)
\]

(8)

\[
Z(s) = (s I - A)^{-1} BU(s)
\]

(9)

Assuming \(\Delta(s) = [s I - A_\tau - e^{-\tau} A_1]\), the relationship between the delay system \(S_\tau\) and the non-delay
system \(S_0\) is shown in Figure 1.

From figure 1 we can easily get: After equivalent conversion of the delay system \(S_\tau\), the system
\(S_\tau\) can be transformed into a delay-free system \(S_0\). And the essence of the equivalent transformation
idea is configuring unstable poles in the delayed system \(S_\tau\) to eliminate the influence of unstable
poles on the system. In this paper, we design a spectral decomposition observer for the time-delay
system and configure unstable poles in the system to make the time-delay system and the time-delay-
free system equivalent.
3.2. Induction motor spectral decomposition observer design
Considering a time-delay system $S_d$:

$$\begin{align*}
\dot{x}(t) &= A_0 x(t) + A_1 x(t - \tau) + B u(t) \\
y(t) &= C x(t)
\end{align*}$$

Designing the following state observer [10]:

$$\begin{align*}
\dot{x}(t) &= A_0 \hat{x}(t) + A_1 \hat{x}(t - \tau) + B u(t) + P L_p w(t) + \int_{-\tau}^{0} A P [\exp(J \tau)] L_p w(t - \tau) d\tau \\
\dot{\hat{y}}(t) &= C \hat{x}(t)
\end{align*}$$

In (12), $'$ represents the transpose of the matrix; $\wedge$ represents the estimate of the term; $P$ is the matrix composed of the right eigenvector of the system $S_d$; $L_p$ is the state feedback matrix of the system. And after configuring the poles, it can be obtained by the method Bass – Gura [20-22]; $J$ is the Jordan standard type corresponding to the pole that the system $S_d$ needs to be configured; $w(t)$ is the auxiliary variable on $[-\tau, 0]$, satisfying the following equation:

$$w(t) = y(t) - \hat{y}(t)$$

In conjunction with (3), (10), (11), an induction motor spectrum decomposition flux linkage observer can be established:

$$\begin{align*}
\dot{\hat{x}}(t) &= A_0 \hat{x}(t) + A_1 \hat{x}(t - \tau) + B u(t) + P L_p w(t) + \int_{-\tau}^{0} A P [\exp(J \tau)] L_p w(t - \tau) d\tau \\
\dot{\hat{y}}(t) &= C \hat{x}(t)
\end{align*}$$

In (13), $\hat{x}(t) = \begin{bmatrix} \hat{\psi}_r & \hat{\psi}_q \end{bmatrix}$, $u(t) = \begin{bmatrix} i_{sl} \\ i_{sm} \end{bmatrix}$, $A_0 = (1 - \hat{\lambda}(\vartheta)) \begin{bmatrix} -k_1 & k_2 \\ k_3 & -k_4 \end{bmatrix}$, $A_1 = \hat{\lambda}(\vartheta) \begin{bmatrix} -k_1 & k_2 \\ k_3 & -k_4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ k_3 & k_6 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $w(t) = \hat{\psi}_r(t) - \hat{\psi}_r(t)$ is the rotor flux error.

4. Proof of the stability of the observer
For system $S_d$, by numerical calculation, we can get all the characteristic root $\Lambda$ sets of the system in the $S$ plane. Assume that the set of characteristic roots is:

$$\Lambda = \{s_1, s_2, \ldots, s_r, s_{r+1}, \ldots, s_s | s_s \in \sigma(S_d)\}$$
In 14, $\sigma(S_d)$ is the spectrum of system $S_d$; $S_i (k = 1, ..., r)$ is the unstable characteristic root of the system or is the unstable spectrum of the system; $S_i (k = 1 + r, ..., n)$ is the stable characteristic root of the system or is the stable spectrum of the system; Defining the set of finite-dimensional subspaces corresponding to the unstable spectrum set as $C_r (C_r = \oplus_{i=1}^{r} S_i )$ and $\oplus$ represents the direct sum of space. According to the principle of space division, there is a complementary space $C_\perp$, satisfying: $C_r \oplus C_\perp = C_\perp$.

Assumption $\Delta(s) = [sI - A_0 - e^{-rt}A_1 ]$ is the characteristic matrix of the system $S_d$, $C_p \in \mathbb{R}^{p \times p}$, and $C_p = \left\{ C \in \mathbb{R}^{p \times p} \mid \Delta(s) \neq 0 \right\}$. According to the principle of space division, there is a complementary space $C_\perp$, satisfying:

$$\Delta(s) = \left[ \begin{array}{cc} sI - A_0 & -e^{-rt}A_1 \end{array} \right]$$

Lemma 4 [10]: In system $S_d$, for any unstable characteristic root $S_i (S_i \in C_r )$, satisfying: $\text{Rank}[sI - A_0 - e^{-rt}A_1 | B] = n$, then there is a pair $(C_p, J)$ which is completely observable.

Defining the observer system error as: $\epsilon(t) = x(t) - \hat{x}(t)$. Sorting out formulas (10) and (13), we can get:

$$\dot{\epsilon}(t) = A_\epsilon \epsilon(t) + A_\epsilon(t - \tau) - PL_{\epsilon} \omega(t) - \int_0^\tau A_p \left[ \exp(J' \tau) \right] L_p \omega(t - \tau)d\tau$$

Applying Laplace transform to (12) and (15):

$$\left[ \begin{array}{c} \Delta(s) \\ C(s) \end{array} \right] \left[ \begin{array}{c} \Delta(s) P(sI - J')^{-1} L_p \\ -\Gamma, (s) \end{array} \right] \left[ \begin{array}{c} E(s) \\ W(s) \end{array} \right] = [I, C]$$

$$E(s) = \varnothing \epsilon(t), W(s) = \varnothing \omega(t) \cdot \varnothing \text{ represents the inverse Laplace transform of the term, and } [I, C] \text{ is the initial error function of the observer. Therefore, the observer feature matrix is:}$$

$$\rho(s) = \det \left[ \begin{array}{cc} \Delta(s) & \Delta(s) P(sI - J')^{-1} L_p \\ C(s) & -\Gamma, (s) \end{array} \right] = \det \left[ \begin{array}{cc} \Delta(s) & \Delta(s) P(sI - J')^{-1} L_p \\ C(s) & -\Gamma, (s) \end{array} \right] \left[ \begin{array}{cc} I_n & -P(sI - J')^{-1} \end{array} \right]$$

$$= \det \left[ \begin{array}{cc} \Delta(s) & 0 \\ C(s) & -[I_p + C_p(sI - J')^{-1} L_p] \end{array} \right] = (-1)^p \left\{ \frac{\det \Delta(s)}{\prod_{i=1}^{p} \det(sI - D_i)} \right\}$$

$$\left\{ \frac{\det \Delta(s)}{\prod_{i=1}^{p} \det(sI - D_i)} \right\} \text{ is a complete function. According to (17), it can be deduced that if } (C_p, J') \text{ is observable, there must be a feedback coefficient matrix } L_p \text{ such that } \sigma(J' - L_p C_p) \subset C_\perp, \text{ which means the observer (16) is stable.}$$

5. Simulation
The simulation parameters of the motor are shown in Table 1.

| Parameter       | Rate            | Power/ {{{Kw}}}/ | Leakage inductance/ {{{mH}}} | Rotor Resistance/ Ω | Friction/ N.m.s | Number of poles |
|-----------------|-----------------|------------------|-----------------------------|---------------------|-----------------|-----------------|
| Value           | 4               | 178              | 1.39                        | 0.0323              | 2               |                 |
When the switching frequency is 500 Hz, the total delay time of the system [13] is generally 1.5 / f = 0.003 seconds, which means τ = 0.003. We can use the parameters in Table 1 to calculate (14), and configure the unstable pole according to the method of Basset-Gura [10-12]. Then the following spectral decomposition flux observer can be got:

\[
\begin{align*}
\dot{x}(t) &= \begin{bmatrix} -7.8253 & 1.3475 \\ 663.4223 & -237.1830 \end{bmatrix} x(t) + \begin{bmatrix} -0.0118 \\ 0.9985 \end{bmatrix} \dot{x}(t-0.003) + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \omega(t) + \begin{bmatrix} 0.002 \\ -0.3570 \end{bmatrix} w(t) + \\
0 &= \begin{bmatrix} -241.7656 & 0 \\ 0 & -399.7883 \end{bmatrix} \begin{bmatrix} x(t-0.003) \\ w(t) \end{bmatrix} + \begin{bmatrix} 4.2364 \\ -554.4815 \end{bmatrix} \omega(t) + \\
0 &= \begin{bmatrix} -3.2207 \\ 1.0158 \end{bmatrix} w(t-0.003) d \tau
\end{align*}
\]

The simulation time is 2 seconds. In addition, the motor speed is set to 400 r/min, and the given flux linkage is 1 Wb.

Figure 2. Rotation Speed Flux Error and Observed by Spectral Decomposition Observation Method

Figure 3. Flux Error Observed by Voltage Model Method and Second-Order Sliding Film Method
From (a) of figure 2, we can get that the motor reaches the given speed in 0.14 seconds, which means the speed response is fast, and when the motor runs stably, the speed overshoot is kept within 0.1 revolution and the stability is good. It can be seen from Figure 2 and Figure 3 that when the induction motor is running stably, the flux linkage errors observed by the spectral decomposition observation method, the voltage model method, and the second-order sliding film method are: 0.06 \( Wb \), 0.10 \( Wb \), and 0.07 \( Wb \). Meanwhile, in the response process of the speed, the error fluctuation of the spectral decomposition flux linkage observation method is relatively small, which is kept within 0.05 \( Wb \) and the error fluctuation of the voltage model method and the second-order sliding film method are all over 0.10 \( Wb \). It can be seen from Figure 4 that compared with the second-order sliding mode flux linkage observation method, the spectral decomposition flux linkage observation method has a smoother observation waveform and a more concentrated curve distribution. In summary, the spectral decomposition flux linkage observation method has higher observation accuracy.

6. Conclusion
Based on the theory of spectral decomposition, this paper proposes a motor flux linkage observation method and applies this method to the vector control system of the motor. The simulation results show that the high-precision flux linkage observation of the induction motor based on the spectral decomposition theory proposed in this paper can effectively improve the precision of flux linkage observation.

Acknowledgments
This work was supported by the National Key R&D Program of China, under Grant 2019YFB1309900.

Reference
[1] Wang, X. Application of high-performance induction motor drive in industry[J]. Private Science and Technology, 2015(11):48.
[2] Zhang, Z., Zhao, Y., Qiao, W. A space-vector-modulated sensor less direct-torque control for direct-drive PMSG wind turbines[J]. IEEE Transactions on Industry Applications, 2014, 50(4): 2331-2341.
[3] Tripathi, A., Hari V., Narayanan, G. Closed-loop rotor flux estimation in vector controlled induction motor drives operated at low switching frequencies[C] //2016 IEEE International Conference on Power Electronics, Drives and Energy Systems (PEDES). IEEE, 2016: 1-6.

[4] Wei, H., Zhu, B. P., Wei, H. F., Zhang, Y. The rotor flux linkage of a new induction motor considering maximum efficiency[J]. Motors and Control Applications, 2018, 45(02): 81-85.

[5] Ying, K., Jiang, L., Li, H. B., Yang J. Simulation study of MRAS system based on stator current model[J]. Electrical Technology, 2012(09): 19-23.

[6] Pan, Y. D., Chen, Z. P., Guo, Y. W. Design of stator flux linkage observer for second-order sliding mode suboptimal algorithm of induction motor[J]. Control Theory and Applications, 2015, 32(05): 641-645.

[7] Li, J., Zhan, R., Song, W. X. Discretization model of flux linkage observer with low sampling frequency for induction motor[J]. Journal of Electrical Engineering and Technology, 2019, 34(15): 3136-3146.

[8] Ma, H. X. Improved flux linkage observation method for induction motor based on rotor magnetic field oriented vector control[J]. Motors and Control Applications, 2017, 44(08): 65-68.

[9] Kan, J. B. Research on high-performance control technology of asynchronous motor at low switching frequency[D]. Huazhong University of Science and Technology, 2017.

[10] Fiagbedzi, Y., Pearson, A. Output feedback stabilization of delay systems via generalization of the transformation method[J]. International Journal of Control, 1990, 51(4): 801-822.

[11] Rong, P. X., Lü, N., Lu, H. L. The Unified Proof of Bass-Gura and Ackerman and Controllable Standard Form Formula[J]. Journal of Harbin University of Science and Technology, 1999(01): 22-24.

[12] Lun, G., Mi. H. The estimation of an aircraft motions by using the Bass-Gura full-order observer. 2012 International Conference on Applied and Theoretical Electricity (ICATE), 2012, 10: 25-27.

[13] Qi, L. Y., Wang, C. C., Zhou, M. L., Wang, J. A current loop decoupling control method for asynchronous motors[J]. Journal of Electrical Engineering and Technology, 2014, 29(05): 174-180.