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SVM Classification and Kalman Filter Based Estimation of the Tire-Road Friction Curve

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Abstract: An estimation method is proposed for the maximum tire-road friction coefficient $\mu_{\text{MAX}}$ and its corresponding wheel-slip $\lambda_{\text{peak}}$. After a preliminary analysis of Burckhardt’s model analytic properties, a machine learning approach involving Support Vector Machines is used to classify the curves representing different road surfaces. A metric to evaluate the misclassification risk is used to determine boundaries for the curve coefficients. These boundaries are then used to form linear inequality constraints for two recursive parameter estimation algorithms based on the Kalman Filter, in its Extended and Unscented forms. Finally, the proposed method is evaluated via simulations in MATLAB environment.

Keywords: Automotive control, Nonlinear parameter identification, Machine learning, Extended Kalman filters

1. INTRODUCTION

In dynamic vehicle control, active safety applications such as traction control and yaw rate control often require the knowledge of the longitudinal tire-road friction coefficient. As both the road surface properties and the slip dynamics change during the ECU intervention, it is extremely valuable to have a complete set of data for the tire-road friction. There are many methods for the estimation of this coefficient, such as the least squares algorithm, which implies a direct relation between the $\mu$ curve and $\lambda_{\text{MAX}}$. However, given the difficulties in applying these methods to real-time control systems, the possibility of obtaining such an estimation in real-time, for each wheel, creates new possibilities for optimized, multi-actuated control system based on the slip-control of each individual wheel.

Several static and dynamic models have been investigated in the past (see Svendenius (2007)), including those which combine longitudinal and lateral dynamics. Among the static models, Paceja’s “Magic Formula”, Burckhardt’s and Dugoff’s models are generally adopted in slip-control applications.

In various research works, it is assumed that the estimation of the maximum friction coefficient $\mu_{\text{MAX}}$ can be inferred from an analysis of the relation between $\lambda$, $\mu$ at low slip levels, which implies a direct relation between the $\mu$ curve at micro-slip and $\mu_{\text{MAX}}$ (Müller et al. (2003), Rajamani et al. (2012)). Nevertheless, given the difficulties in applying the same control laws for wheel-slip control in off-road conditions, such as gravel or cobblestone (Ivanov et al. (2015)), we assume that a different approach, which includes multiple parameters estimation at the same time, might be beneficial. A similar idea stands behind the work presented by Tanelli et al. (2009), where a recursive least squares algorithm for online estimation of the tire-road friction curve is adopted and validated by on vehicle testing. In de Castro et al. (2010) the same approach is used on several tire-road friction models and verified via simulation.

Several other examples of parameter estimation can be found in the literature, in which the $\mu(\lambda)$ curve (or a section of it) is derived.

In Umeno (2002) the gradient of $\mu(\lambda)$ at the operational working point is estimated in different road conditions. Starting from a Lu-Gre dynamic friction model, Alvarez et al. (2005) estimate its internal state from the wheel angular speed and longitudinal vehicle acceleration. The unknown parameters of the dynamic friction model are estimated through a parameter adaptation law. The Kalman Filter (KF), in its Unscented version, is employed in Nakatsuji et al. (2007) to estimate $\mu_{\text{MAX}}$ of winter road, in the case of active lateral forces.

A different approach, for the acquisition of information concerning the road conditions, is the one involving machine learning techniques. Support Vector Machines (SVMs) have been applied to road surface identification by Ward and Iagnemma (2009), although using an approach which involves the modelling of the vertical vehicle dynamics. From a measured suspension acceleration signal, the terrain profile is estimated, through a dynamic vehicle model. Then a supervised SVM is employed to classify profile segments as members of pre-defined classes (such as asphalt, brick, gravel, etc.).

In this work, we propose an approach to the estimation of the entire $\mu(\lambda)$ characteristic, which differs from previous works, as it focuses on both coordinates $\lambda_{\text{peak}}$, $\mu_{\text{MAX}}$ with a combination of techniques which, to the best of our knowledge, was not applied before. We start with a preliminary study on the parameter sensitivity properties of Burckhardt’s model, which are compared to those of a generic rational function model. Then, we use SVMs for the classification of the different road type surfaces: in particular we analyze in which circumstances there is a high risk of selecting the wrong class. Finally, by using the class information to build linear constraints, we perform an online parameter estimation using nonlinear variants of the Kalman Filter.

The paper is organized as follows. Section 2 illustrates the tire-road friction model selection criteria. In Section 3 the road type classification with the use of SVM is presented. In Section 4 we apply different KF techniques to the problem of online parameter identification for the tire-road friction model, while in Section 5 simulation results obtained with MATLAB are illustrated and analyzed. Some conclusions are gathered in Section 6. The workflow followed during the road type classification and the iden-
The approximation $\theta_2 \approx \text{const}$ was made, based on values reported in Tanelli et al. (2009). In both models the behavior of $\lambda_{\text{peak}}$ appears to be influenced by a unique parameter: due to the shape of functions (4)-(5), we expect, during parameter identification, more accurate convergence for low values of these parameters. For $\theta_2$ this means good accuracy for $\theta_2<30$ and worse performances for $\theta_2>40$.

We are interested in maximizing the sensitivity of the candidate model equations to the variation of parameters $\theta_2$, $p_3$, in the slip region where the samples $(\lambda_i, \mu_i)$ are most likely to be found. The parameter sensitivity functions are defined as follows:

$$\left\| \frac{\partial}{\partial \theta_i} \mu(p, \lambda) \right\|, i = 1, \ldots, m$$

which yields

$$\frac{\partial}{\partial \theta_2} \mu_{\text{exp}}(\theta, \lambda) = \theta_1 \lambda e^{-\theta_2 \lambda}$$

$$\frac{\partial}{\partial p_3} \mu_{\text{rat.fun}}(p, \lambda) = \frac{p_1 \lambda}{1 + p_2 |\lambda| + p_3 \lambda^2}$$

As it can be seen in Figure 2, the normalized sensitivity of the parameter $\theta_1$ in model (2) is preferable to the one of parameter $p_3$ in (3) for $\lambda \leq \lambda_{\text{peak}}$, which makes the selection of model (2) the natural choice. Different $\mu(\lambda)$ curves for various surfaces, obtained based on the values for model (2) reported in Tanelli et al. (2009), are shown in Figure 3. It is evident how, compared to the dry asphalt condition, the nonlinear parameter $\theta_2$ affects the shape of the curves in case of cobblestone and gravel, whereas wet asphalt only sees a reduction of maximum grip, but no shifting of the maximum grip slip-ratio. In case of extreme low-grip surfaces such as ice and snow, the slope in the micro-slip region is lower, mainly due to the drop in $\theta_1$.

3. ROAD SURFACE IDENTIFICATION VIA SVM

The purpose of using SVMs is to classify the different regions of the $\lambda, \mu$ space based on a score assigned to each surface type. In the next subsections we provide a brief introduction to how the scores for each road type are calculated (Subsection 3.1), and an explanation of the SVMs application to the road type classification (Subsection 3.2). A more exhaustive explanation of the SVM principle can be found in Gunn (1998).

3.1 SVM Scores Calculation

In the linear case, training a SVM for binary classification is equivalent to finding a hyperplane $<\mathbf{w}, x> + b = 0$, with $<\cdot, \cdot>$ scalar product in $\mathbb{R}^n$, such that the training data $D = \{\mathbf{x}_i, y_i\}_{i=1}^n$ where $\mathbf{x}_i \in \mathbb{R}^n$ and $y_i \in \{-1, 1\}$ are pairs of training features and their corresponding labels. The SVM optimization problem is formulated as:

$$\min_{\mathbf{w}, b} \frac{1}{2} \mathbf{w}^T \mathbf{w} + C \sum_{i=1}^n \max(0, 1 - y_i (\mathbf{w}^T \mathbf{x}_i + b))$$

where $C$ is a regularization parameter, and the optimal separating hyperplane is given by $\mathbf{w}^* \mathbf{x} + b^* = 0$. The SVM scores are then defined as the signed distance of a point $\mathbf{x}$ to the hyperplane:

$$s(\mathbf{x}) = \mathbf{w}^* \mathbf{x} + b^*$$
which weighs the score function \( \alpha \) under the Karush-Kuhn-Tucker (KKT) conditions.

\[
y_i \left( \langle w, x_i \rangle + b \right) \geq 1, \quad i = 1, \ldots, l
\]

The criteria for the selection of the parameters \( w, b \) is to maximize, for both classes, the distance between the hyperplane and the nearest element of the class. Defining the distance as the euclidean norm \( || \cdot || \), it can be proven that maximizing such distance is equivalent to finding the minimum of \( \Phi = \frac{1}{2} ||w||^2 \) (Gunn (1998)). This is a constrained optimization problem which, following a standard procedure, can be expressed as

\[
\Phi = \frac{1}{2} ||w||^2 - \sum_{i=1}^{l} \alpha_i (y_i \langle w, x_i \rangle + b) - 1)
\]

under the Karush-Kuhn-Tucker (KKT) conditions

\[
\alpha_i (y_i \langle w, x_i \rangle + b) - 1 = 0, \quad i = 1, \ldots, l
\]

This concept can be extended to nonlinear hyperplanes, by re-defining the internal product \( \langle w, x \rangle \) as a function performing a nonlinear mapping from the space of input vectors \( x \) into a high dimensional feature space. In this case, \( w \) cannot be explicitly calculated: defining \( K(\cdot, \cdot) \) as the kernel function performing the nonlinear mapping, the internal product is:

\[
\langle w, x \rangle = \sum_{i=1}^{l} \alpha_i y_i K(x_i, x)
\]

Note that the generalization to non-separable sets, not covered here, is obtained by relaxing constraints (9) with proper tolerances.

The KKT conditions (11) impose that the only samples \( (x_i, y_i) \) associated with positive Lagrange multipliers \( \alpha_i > 0 \) are those satisfying

\[
y_i \left( \langle w, x_i \rangle + b \right) = 1
\]

These vectors are called Support Vectors (SVs). The SVM classification obtained is therefore fully described by the SVs and their respective coefficients \( \alpha_i > 0 \).

In binary classification problems, once the SVM is trained and the coefficients \( \alpha_i, i = 1, \ldots, l \) are assigned, the evaluated samples \( x_i, i = 1, \ldots, m \) are assigned to the classes by means of a classifier which weighs the score function

\[
f(x) = \langle w, x \rangle + b = \sum_{i \in SV} \alpha_i y_i K(x_i, x) + b
\]

Relying on the definition of score function (14), the classifier can be defined as a suitable function of \( f(x) \). In case of binary classification, an example of a hard classifier is \( c_{hard}(x) = sign(f(x)) \), while a soft classifier is \( c_{soft}(x) = sat(f(x)) \). In the present paper, where classification among multiple classes is desired, the number of hyperplanes to be found is equal to the number of classes defined, and the classifier is the rule which allows one to identify the appropriate class index, i.e.

\[
c_1(x) = \arg \max_{c \in C} f_c(x)
\]

where \( C \) is the set containing all classes defined, and \( f_c(x) \) is the score function of class \( c \). Note that the use of the name \( c_1 \) in (15) is motivated by the fact that the selected class is the first ranked in terms of score.

### 3.2 Application to Road Surface Identification

Now, the previously illustrated approach is applied to the road surface identification. MATLAB functions used for the classification of the road types are `fitcsvm` and `predict`, both included in the Statistics and Machine Learning Toolbox. They perform respectively: a) the training of the SVMs, i.e. the calculation of the \( \alpha_i \) coefficients for each class; b) the calculation of the score for each trained SVM over the entire \( \lambda, \mu \) plane.

3D maps are calculated offline from the score functions \( f_c(\lambda, \mu), c = 1, \ldots, m \) (\( m \) being the number of different road types considered), and then used as simple look-up tables (LUTs) during the online identification of the road type: this procedure avoids the burden of performing an optimization at each iteration, thus going in the direction of an online real-time implementation.

After applying equations (14)-(15) to our problem, we can get a visual rendering of the maximum score, for all road types defined on the \( \lambda, \mu \) plane, reported in Figure 4. Each “wave” of contiguous local maximums corresponds to the area of maximum probability for a specific road type. Problems arise when multiple maximums overlap. Several situations in which this occur can be identified:

1. at micro-slip for all road types;
2. at low slip (\( \lambda < 0.03 \)) for high grip surfaces: below \( \mu \approx 0.6 \) the behavior of all 3 high grip surfaces is similar, which would make it necessary to obtain more information in the proximity of the nonlinear area for a more accurate classification;
3. at \( \lambda \approx 0.2 \) and \( \lambda > 0.45 \) in case of Dry Cobblestone, due to intersections with high grip surface curves.

In the latter case, the correct identification of the surface type could be obtained by considering the history of the classification within the algorithm: at \( \lambda \approx 0.2 \) the conflicting curves have different derivatives, at \( \lambda \approx 0.5 \), if a slip-control system is active, the slip cannot have such high values.

In case of the former two situations, discerning the correct surface from the score is not trivial, but some considerations can be made, which are valid in the frame of the present work. First, the percentage of samples gathered in the micro-slip region drops in situations where TC or ABS control are active, so these samples can be ignored during the classification process. Secondly, for what concerns the high-grip surfaces, it is in general acceptable to identify just the group of possible curves, as in many application it might be sufficient to estimate the slip-ratio corresponding to
maximum slip, rather than knowing the exact shape of the whole $\mu(\lambda)$ curve.

4. CURVE COEFFICIENTS IDENTIFICATION VIA KALMAN FILTERING

The problem of recursively identifying the set of parameters $\theta$ of the nonlinear model (2), can be solved by using a Kalman Filter (KF) in one of its nonlinear versions, while imposing the state dynamics function to be the identity matrix. The following process observation model is therefore considered:

$$\begin{align*}
\theta_{k+1} &= \hat{\theta}_k + w_k \\
\mu_k &= g(\hat{\theta}_k, \lambda_k) + v_k
\end{align*}$$

(16)

where the function $g(\cdot)$ represents the nonlinear model (2), and the process and measurement noise are

$$\begin{align*}
w_k &\sim \mathcal{N}(0, Q_k) \\
v_k &\sim \mathcal{N}(0, R_k)
\end{align*}$$

(17)

The noise affecting the estimation of the wheel-slip $\lambda_k$ is not considered in this model, although it has a great impact on the estimation performance (Tanelli et al. (2012)) and will be added during the simulations.

In order to make the convergence of the coefficients faster and more robust, we use the information obtained with the SVM classification to restrict the accepted values for $\hat{\theta}_i$, $i = 1, 2, 3$. In this way, it is possible to reduce the gain of the KF without deteriorating dramatically the response time, while at the same time reducing the impact of the uncertainty of the variances $Q_k, R_k$, which directly affect the update of the estimation error variance $P_{\theta_k}$.

When the road surface is classified as the j-th, with nominal coefficients $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3$, the KF parameter identification starts with constraints on the parameters around a certain interval of the nominal values, e.g. $\pm 20\%$. In case the road type identification via SVM is found to be uncertain between 2 different road types, the accepted interval is the union of the respective intervals. The resulting accepted interval is fed to the KF in the form of a linear inequality constraint

$$\mathbb{D} \theta \leq \mathbb{d}$$

(18)

In our work, we have used 2 different KF variants for the estimation of the parameters $\hat{\theta}_i$. Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF), both featuring the additional linear inequality constraint (18). The first variant is reported in Algorithm 1. It is based on the modified version of EKF proposed in Alessandri et al. (2007).

Algorithm 1 EKF Parameter Estimation

$$H_k = \left[ \frac{\partial g(\theta, \lambda)}{\partial \theta_i} \right]_{\theta = \hat{\theta}_k, \lambda = \lambda_k}$$

$$= [1 - e^{-\lambda_k \theta_2, k-1}, -\lambda_k \theta_1, k-1, -e^{-\lambda_k \theta_2, k-1}, -\lambda_k]$$

(19)

$$K_k = P_{\theta_{k-1}} H_k^T (H_k P_{\theta_{k-1}} H_k^T + R_k)^{-1}$$

(20)

$$\hat{\theta}_k = \hat{\theta}_{k-1} + K_k (\mu_k - g(\hat{\theta}_{k-1}, \lambda_k))$$

(21)

$$P_{\theta_k} = (\varepsilon_{EKF} + 1)(P_{\theta_{k-1}} - K_k H_k P_{\theta_{k-1}} + Q_k)$$

(22)

In Algorithm 1, $\varepsilon_{EKF} > 0$, $P_0, R_0$ are symmetric positive definite matrices, and (21), (22) are initialized with proper values $P_0, \theta_0$. The difference with a conventional EKF algorithm lays in the error variance update equation (22), which in the conventional EKF algorithm is

$$P_{\theta_k} = (P_{\theta_{k-1}} - K_k H_k P_{\theta_{k-1}} + Q_k)$$

(23)

In the new formulation, the process variance term $Q_k$ is neglected, whereas a tunable factor $\varepsilon_{EKF}$ is used to adjust the convergence rate of the error variance $P_{\theta_k}$. In fact, estimating the process variance is in general a challenging task, due to its volatile nature and its dependence on the surface type.

The UKF algorithm for parameter identification (see Wan and van der Merwe (2000) for a detailed explanation of the algorithm) is an alternative for the estimation of the $\theta$ parameters. Three tunable parameters, $\alpha_{UKF}$, $\beta_{UKF}$ and $\kappa_{UKF}$, determine the distribution of the sigma samples. They affect the higher order terms of the nonlinear estimation, but have little relation with the estimation accuracy or stability of UKF (Han et al. (2009)). As a matter of fact, the convergence properties of the algorithm depend mainly on the variance matrices $Q_k, R_k$. The latter, in particular, can be tuned to higher values, in order to decrease the Kalman gain and thus oscillations.

State or parameter estimation by means of KF techniques can be adapted to include linear equality constraints on the estimated variable $\mathbb{D} \theta = \mathbb{d}$. The case of inequality constraints can be reductively solved by imposing at each iteration which components of $\theta$ do not satisfy condition (18). The so-called gain projection method for the application of linear equality constraints to KF is applied (Simon (2010)). If the unconstrained $a$ posteriori estimate $\hat{\theta}_{unc}$ does not satisfy the constraints, then the estimation * at time $\hat{\theta}_{k-1}$ can be projected in the direction of $\hat{\theta}_{unc}$ until it reaches the constraint boundary. This effectively gives a modified Kalman gain $K_{unc} = \beta K_k$, with $\beta \in (0, 1)$ and $K_k$ is the standard unconstrained Kalman gain. Having defined

$$e_k = \mu_k - g(\hat{\theta}_{k-1}, \lambda_k)$$

(24)

eq \mathbb{D} \theta_{unc} - \mathbb{d}$$

(25)

5. SIMULATIONS

Several simulations were conducted for the validation of the proposed algorithms. In the next subsections we illustrate how the SVMs were trained (Subsection 5.1), then show the results of road surface classification on a sequence of rapidly changing road surfaces (Subsection 5.2), and eventually verify the effectiveness of the KF estimation techniques for the tire-road friction curve coefficients (Subsection 5.3). Note that the procedure is the one anticipated in Figure 1.

5.1 SVM Training

In the training phase of high grip surfaces, regardless of the Kernel function used, it is only possible to obtain good placement of the SVs by splitting the classes into low and high slip regions. The sets of $N$ samples $\lambda_i \in \{\lambda_{i,j} \in \{\lambda_{j,1}, ..., \lambda_{j, M_j} \} : j = 1, ..., \kappa_{max}, \kappa_{max} \geq m \}$ (m number of classes), used for the training are generated differently for low and high slip. For low slip ratios, they are generated according to a Gamma distribution with shape parameter $\theta_1 = 1.5$ and scale parameter $k_l = 2$, normalized for each surface type, such that at least 90% of the $\lambda_{i,j}$ samples satisfy $\lambda_{i,j} \leq \lambda_{peak,j}$ (see Figure 5, first plot). For high slip ratios, samples $\Lambda_j$, generated from a uniform distribution on the interval $[\lambda_{peak,j}, 1]$, are used.

The j-th SVM kernel (chosen gaussian) is trained with samples $(\lambda_{i,j}, H_{i,j})$, with $\mu_{i,j}$ samples calculated from the set $\Lambda_j$ through equation (2). Additional noise of variance:

$$R_{\text{train}} = 5 \cdot 10^{-4}, \quad R_{\text{train}} = 2 \cdot 10^{-4}$$

(26)

is added on $\mu$ and $\lambda$ respectively.

The SVMs, resulting from sets of $N = 2000$ training samples, are used for the classification of the SVs such as the ones in the second plot of Figure 5. In case of the high grip surface, dry asphalt, performing the training for low slip samples alone allows to obtain an area of the plane clearly delimitated by the SVs for slips $\lambda < \lambda_{peak}$. To
avoid misclassification of the less frequent samples acquired at high slip, an additional training set, specific for these, is used. In case of lower grip surfaces, such as gravel, samples diffusion is more spread, thus making a single training set sufficient.

5.2 SVM Classification Results

In all simulations illustrated in this section, process and observation noise variances defined in (17) are:
\[ Q_r = Q = 10^{-4} \cdot \text{diag}(5, 100, 1) \]  
\[ R_r = R = 2 \cdot 10^{-3} \]  
Additionally, we define the variance observation noise of the wheel slip-ratio \( \lambda \) as
\[ R_\lambda^2 = R_\lambda^2 = 10^{-3} \]  
Samples \( \lambda_t \) are generated with a normal distribution centered around the lambda value corresponding to \( \approx 40\% \) of \( \lambda_{peak} \), with variance \( var_{\lambda} = 0.04 \). For the SVM classification test, the road type is changed every 2 seconds according to the sequence in Table 1. For all simulations in this paper, the sampling time is 0.01 s.

Table 1. SVM Classification

| Step Nr. | Time (s) | Road Type | Surface Idx | Avg. Norm. Radf | Avg. Norm. Idx | Radf |
|---------|----------|-----------|-------------|-----------------|---------------|-------|
| 1       | 0-2      | Dry Asphalt | 1           | 0.857           | 0.555         |
| 2       | 2-4      | Gravel    | 6           | 0.846           | 0.628         |
| 3       | 4-6      | Dry Cobblestone | 8 | 0.665           | 0.048         |
| 4       | 6-8      | Wet Cobblestone | 7 | 0.866           | 0.732         |
| 5       | 8-10     | Wet Asphalt | 2           | 1               | 1             |
| 6       | 10-12    | Snow      | 4           | 0.623           | 0.273         |
| 7       | 12-14    | Ice       | 5           | 0.539           | 0.202         |
| 8       | 14-16    | Dry Concrete | 3           | 0.752           | 0.261         |

Due to noise, scores appear noisy and overlapping. A clearer result (first plot of Figure 6) is obtained by filtering the scores with a first order digital filter
\[ LPF(z^{-1}) = \frac{1}{1 - az^{-1}} \]  
Let \( f_s^{filt}(x) \) be the score obtained by filtering (14) with (30). By using a constant \( a = 0.9 \), it is possible to obtain in several cases a clear highest score, without significantly compromising the quickness in recognizing the new road type. This is confirmed by several simulations, in which the maximum delay in the identification of the new grip condition is generally less than 10 samples for all surfaces.

In the first plot of Figure 6, an unclear situation (wet asphalt) and a clear one (snow) are presented after filtering the scores. In the first case, several classes present comparable scores \( f_s^{filt}(\lambda, \mu) \), while in the second there is a clear separation. Starting from these considerations, based on the definition of score function (14), and having defined \( c_1 \) as the output of (15) for the i-th sample (highest scoring class) and \( c_2 \) as the class with second highest score, we define the High Score Ratio
\[ \text{Rat}_{HS} = \frac{f_{c_2}(\lambda, \mu)}{f_{c_1}(\lambda, \mu)} \leq 1 \]  
It is used as a measure of the classification risk connected to the selection of the highest scoring class in that instant. In the first interval \((8 - 10s)\) \( \text{Rat}_{HS} \) is higher than in the following one \((10 - 12s)\). In particular, there are instances where the highest score does not correspond to the right surface. The dry cobblestone road condition poses a problem, as it is conflicting with several other road types. It can be seen in Figure 3 that its ideal characteristic overlaps with other curves.

To reduce misclassifications, it is possible to manually re-weight the score functions \( f_s(\lambda, \mu) \), as well as to introduce a logic in the classification algorithm, which recognizes potential conflicts considering the behavior of \( \text{Rat}_{HS} \). The effectiveness of such logic (Algorithm A) can be appreciated in the second plot of Figure 6, when compared to the basic one (Algorithm B).

To assess how likely it is for a certain terrain to be correctly identified, we define a Risk Index, which penalizes High Score Ratio values above a defined limit value, namely \( \text{RAT}_{lim} \):
\[ \text{Id}_{risk} = \frac{1}{N_{seq}} \sum_{i=1}^{N_{seq}} \left( \frac{\text{Rat}_{HS}(i) - \text{RAT}_{lim}}{1 - \text{RAT}_{lim}} \right)^+ \]  
where \( N_{seq} \) is the number of samples for each step of the sequence, and the operator \( [\cdot]^+ \) returns the maximum between the argument and 0. Averages of Risk Indices, calculated for \( \text{RAT}_{lim} = 0.7 \), and High Score Ratio mean values obtained from 10 different simulations are reported in Table 1, and their normalized value is plotted in Figure 7.

From these data, it appears that a high risk of misclassification for certain surfaces (wet asphalt, dry cobblestone, etc) is in line with the preliminary considerations formulated in Subsection 3.2.

5.3 Parameter Estimation Results

In the simulation presented here, where the same variance noises (27)-(29) of the classification test are used, the two different KF techniques are compared.

In the EKF case, the gain \( \alpha_{EKF} \) is set to be small, in order to reduce oscillations, as convergence rate is already helped by the KF constraints. In the other case, the UKF Algorithm necessitates...
The identified road surface determines the constraints on the wheel slip and friction coefficient. A preliminary classification via SVM is performed, followed by a recursive parameter identification through a constrained nonlinear KF. Convergence of the coefficients is helped considerably by the pre-classification via SVMs. Convergence is a direct effect of the pre-classification via SVMs. In order to reduce oscillations in the estimated parameters, $R_k$ is therefore increased.

Convergence of the coefficients is helped considerably by the preliminary road type classification. Threshold $RAT_{lim}$ is used to determine when the SVM classification is considered unreliable, and therefore the constraints on the parameters estimation (18) are relaxed to include classes with lower scores.

In Figure 8 the estimation of $\hat{\lambda}_{\text{peak}}$ and $\hat{\mu}_{\text{MAX}}$ is shown, which reflect directly the estimation performance of $\hat{\theta}_1$ and $\hat{\theta}_2$ based on equations (2) and (4). One can observe that the EKF provides a better estimation than UKF. All estimations fall within 20% of actual value for $\hat{\lambda}_{\text{peak}}$ and $\hat{\mu}_{\text{MAX}}$ in less than 10 samples, which is a direct effect of the pre-classification via SVMs. Convergence to ±5% error margin is obtained in less than 50 samples. As a term of comparison, we can use the simulation performed with noisy data in Tanelli et al. (2009), where, for the same model (2), 10% $\hat{\mu}_{\text{MAX}}$ estimation error is achieved after $\approx 40$ samples, and 5% error after $\approx 70$ samples.

6. CONCLUSIONS

In this paper a novel approach to the identification of the tire-road friction curve $\mu(\lambda)$ is presented. A preliminary classification of the road surface via SVM is performed, followed by a recursive parameter identification through a constrained nonlinear KF. The identified road surface determines the constraints on the estimation.

The proposed algorithm was evaluated in MATLAB with noisy wheel slip $\lambda$ and friction coefficient $\mu$ samples, showing promising results. Further work will be devoted to the experimental assessment of the proposed procedure, by acquiring real data during acceleration and braking sessions from an experimental set-up, and producing the corresponding $\lambda$, $\mu$ estimations. This will allow us to validate the algorithm in more realistic conditions. Additional benefits to the surface classification could be introduced by improved modelling and reference road surfaces selection.

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