Many-body localization with synthetic gauge fields in disordered Hubbard chains

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We analyze the localization properties of the disordered Hubbard model in the presence of a synthetic magnetic field. An analysis of level spacing ratio shows a clear transition from ergodic to many-body localized phase. The transition shifts to larger disorder strengths with increasing magnetic flux. Study of dynamics of local correlations and entanglement entropy indicates that charge excitations remain localized whereas spin degree of freedom gets delocalized in the presence of the synthetic flux. This residual ergodicity is enhanced by the presence of the magnetic field with dynamical observables suggesting incomplete localization at large disorder strengths. Furthermore, we examine the effect of quantum statistics on the local correlations and show that the long-time spin oscillations of a hard-core boson system are destroyed as opposed to the fermionic case.

I. INTRODUCTION

The phenomenon of many-body localization (MBL) has attracted a significant interest in condensed matter physics over past several years, both theoretically [1–4] and experimentally [5–9]. The MBL is an extension of Anderson localization (which describes localization of single-particle eigenstates in the presence of disorder potential) to highly excited eigenstates of interacting many-body systems. The characteristic properties of MBL phase are Poisson eigenvalue statistics [10–16], an absence of thermalization [17–20], a vanishing transport [21–23], and the logarithmic spreading of the entanglement entropy [24–26]. Starting from early works, many aspects of the topic have been examined to date. The ultracold atomic systems are ideal platform to explore the localization phenomena due to their ability to tune dimensionality, nature of applied disorder, atomic interactions, lattice geometry and synthetic gauge fields. The existence of MBL phase has been confirmed in recent quantum gas experiments using quantum simulators such as optical lattices [5, 6, 27, 28] and trapped ions [8]. Moreover, the flux dependent mobility edge of disordered chain and the significance of entropy on the localization properties in the presence of synthetic gauge field have been explored in experiments [29, 30].

The nonergodic behaviour of disorder Hubbard chain at strong disorder and temporal evolution of its correlation functions have been examined theoretically in [26, 31]. The SU(2) symmetry of the model limits the full MBL as the charge degrees of freedom are localized but spins remain delocalized and reveal subdiffusive dynamics [32–34]. The partial MBL is due to the separation of time scale between charge and spin sectors in the presence of correlated disorder [35] and the decay rate of the transport strongly depends on the density of singly occupied sites in the initial state [36]. The implementation of synthetic gauge field in ultracold atoms and recently on disorder systems allows one to study the effect of time-reversal symmetry breaking on the localization properties of the disordered fermions [29, 37]. Furthermore, it is possible to couple internal atomic spin states which allows for an interpretation in terms of an additional synthetic lattice dimension [38, 39]. The gauge field in synthetic dimension leads to flux ladders where the hopping and atomic interactions between different states can be controlled in experiments [40]. The presence of the random gauge fields delocalizes the interacting system [41] and many-body states become extended into a single Landau level [42]. Despite numerous studies on the topic of MBL, the effects of time-reversal symmetry breaking on localization phenomena have been sparsely studied [43] and this is the subject of the present study.

In particular, we examine the spectral and dynamical properties of disordered Hubbard chains in the presence of the synthetic gauge fields. At lower disorder strengths, in ergodic regime, the breaking of time-reversal invariance (TRI) by gauge field results in spectral statistics well described by Gaussian Orthogonal Ensemble (GOE) of random matrices instead of Gaussian Unitary Ensemble (GUE) as expected for broken TRI. This is due to a residual discrete symmetry. Only when the residual reflection symmetry is broken by local field or asymmetric tunneling rate of spin-up and down fermions, the level statistics is characterized by GUE. The time dynamics of charge and spin correlations for random initial states reveal the localization of charges and a subdiffusive decay of spin correlations. The introduction of a synthetic flux damps the spin oscillations and finally delocalizes them. Furthermore, the entanglement entropy confirms the delocalization of spins in the presence of the synthetic gauge fields.

The paper is organized as follows. In Sec. II we introduce the Hubbard model with synthetic gauge field and disorder. In Sec. III we analyze the effect of symmetry breaking and synthetic flux on the spectral properties of the system. We further study the local charge and spin dynamics of fermions and hard-core bosons in Sec. IV. In Sec. V we examine the bipartite entanglement entropy. The local correlations in the presence of spin-dependent disorder are discussed in Sec. VI. Finally, we conclude in Sec. VII.
FIG. 1. Schematic visualization of the system studied (1) in which up (down)-spin corresponds to upper (lower) rung of the ladder.

II. THE MODEL

We consider interacting spin-1/2 fermions in a quasi-one dimensional lattice. The two spin components can be interpreted as the realization of a two-leg ladder geometry, with two legs corresponding to two spin states. Two components may be realized as in the recent experiment [5] for $^{40}$K atoms with spin up state being $|F,m_F| = |\frac{9}{2}, -\frac{7}{2}\rangle \equiv |\uparrow\rangle$ and the spin-down state $|\frac{9}{2}, -\frac{5}{2}\rangle \equiv |\downarrow\rangle$. The two spin components may be coupled by e.g. Raman coupling realizing the Hamiltonian [39, 44] $\hat{H} = \hat{H}_0 + \hat{H}_{sh}$, wherein

$$\hat{H}_0 = -\sum_{j,\sigma} \left( J \hat{c}^\dagger_{j,\sigma} \hat{c}_{j+1,\sigma} + K e^{-\gamma j} \hat{c}^\dagger_{j,\uparrow} \hat{c}_{j,\downarrow} + \text{H.c.} \right) + U \sum_j \hat{n}_{j,\uparrow} \hat{n}_{j,\downarrow} + \sum_{j,\sigma} \epsilon_j \hat{n}_{j,\sigma}. \tag{1}$$

Here, $j$ and $\sigma = \uparrow, \downarrow$ are the spatial and spin indices, $J$ is the hopping amplitude between neighbouring lattice sites on the same leg and $K$ is the strength of complex hopping in the synthetic dimension, $\hat{c}^\dagger_{j,\sigma} (\hat{c}_{j,\sigma})$ creates (annihilates) fermions with spin $\sigma$ at site $j$, and the occupation number operator $\hat{n}_{j,\sigma} = \hat{c}^\dagger_{j,\sigma} \hat{c}_{j,\sigma}$. The local “charge” density is $n_j = n_{j,\uparrow} + n_{j,\downarrow}$ while the spin magnetization is $m_j = n_{j,\uparrow} - n_{j,\downarrow}$. The on-site interaction strength of two spins is assumed to be repulsive i.e. $U > 0$ and $\epsilon_j$ is the uniform distributions of a random spin-independent on-site potential, $\epsilon_j \in [-W/2, W/2]$ with $W$ being the disorder amplitude. The parameter $\gamma = 2k_B a$ determines the synthetic magnetic flux with the flux per plaquette of the lattice being $\Phi = \gamma/2\pi$. Here $k_B$ is the recoil wave vector of the Raman beams and $a$ is the lattice spacing [44]. The finite value of $\gamma$ leads to complex hoppings along the rungs of the ladder (corresponding to flipping the spins). The system length along the synthetic dimension is two, although it may be increased by trapping few Fermi mixtures together. The gauge is chosen in such a way that the Peierls phase is along the rung and not on the legs of the ladder - compare Fig. 1. The above model Hamiltonian without a synthetic gauge field has been realized in quantum gas experiments in optical lattices [5] similarly physics with the synthetic dimension in clean, disorder-free experiments has also been studied [40].

The hopping amplitude sets the unit of energy scale, $J = 1$ and we use periodic boundary conditions throughout this work. We study the system at unit filling (the total number of fermions $N = N_\uparrow + N_\downarrow = 1$ is conserved). Note that such a situation is sometimes called a half-filling in condensed matter community. In the absence of the gauge field ($K = 0$), the model Hamiltonian preserves purity, time-reversal and SU(2) spin symmetry [45], however, pseudospin SU(2) [46] and particle-hole symmetries are broken due to the on-site disorder potential. The parity and SU(2) spin symmetry can be removed by adding a local weak magnetic field ($h_0$) at the edge of the chain [26]. The symmetry breaking Hamiltonian is

$$\hat{H}_{sh} = h_0 (\hat{n}_{1,\uparrow} - \hat{n}_{1,\downarrow}). \tag{2}$$

Additionally, the presence of gauge field in the synthetic dimension breaks the time-reversal symmetry of the system.

We use an exact diagonalization for different system size and flux quanta obtaining both spectral and time evolution properties of systems studied. While using more sophisticated techniques one could extend the study to slightly larger systems, we restrict ourselves in this exploratory approach to small $L \ll 8$ system amenable to exact diagonalization. We estimate influence of finite size effects on our results by comparison with smaller system sizes. While the question of a possible instability of MBL phase in thermodynamic limit is debated [47–50], we believe that our results are robust on experimentally relevant time scales.
Hamiltonian (1) is preserved. Without the symmetry breaking term, Hamiltonian (2), the individual spectra from diagonalizations consist of independent subsets of eigenvalues due to conserved quantities. Hence, the value of the average gap ratio \( \langle r \rangle \) is close to the Poisson value for arbitrary disorder strength \( W \) vs. \( |h_0| \). To observe the transition between ergodic and MBL phases, we use the explicit forms of generators of SU(2) symmetry

\[
S^z = \frac{1}{2} \sum_j (\hat{n}^+_{j,\uparrow} - \hat{n}^+_{j,\downarrow}), \quad S^+ = (S^-)^\dagger = \sum_j \hat{c}^+_j \hat{c}_{j,\downarrow} \quad (3)
\]

\[
S^z = \frac{1}{2} (S^+ S^- + S^- S^+) + (S^z)^2. \quad (4)
\]

For \( K = 1 \), the total number of up/down fermions, \( N_\uparrow/N_\downarrow \) is not conserved and \( \{H_0, S^z\} \neq 0 \). However, the Hamiltonian still commutes with \( S^x = (S^+ + S^-)/2 \) and \( S^y \) operators as can be checked by a direct calculation. Calculating the Hamiltonian matrix in a basis composed of eigenstates of \( S^x \) and \( S^z \) and performing exact diagonalization within a single block of the matrix, we explicitly observe the crossover between ergodic and MBL regimes as demonstrated in the inset of Fig. 2(a).

Alternatively, the crossover can be observed by applying the symmetry breaking local Hamiltonian (2) [26] indeed, as Fig. 2(b) demonstrates at weak \( W \), \( \langle r \rangle \) reaches the value \( \langle r_{\text{Poisson}} \rangle \), while at strong disorder it approaches \( \langle r_{\text{GOE}} \rangle \). This indicates the existence of two phases at \( \gamma = 0 \).

In order to extract the critical disorder strength \( W_c \) and exponent \( \nu \) of transition we use the finite-size scaling technique. We collapse the curves of different system sizes \( L \) as a function of \( (W - W_c)/L^{1/\nu} \) as shown in the inset of the Fig. 2(b). The critical disorder strength is the value of \( W \) at which the curves cross or merge and indicates the ergodic to MBL transition. We estimate \( W_c \sim 13.2 \) and \( \nu \sim 0.8 \). Observe that as in the spinless fermions case (equivalent to the spin Heisenberg chain) [11] the critical exponent breaks the Harris bound which is 2 for one dimensional system [55]. In the latter case it has been suggested that the possible explanation may be related to the character of the transition resembling a Kosterlitz-Thouless transition [56] with possible important logarithmic corrections.

The remaining symmetries of Hamiltonian (1) for TRI case \( (\gamma = 0) \) may be alternatively removed by making the tunneling \( J \) spin dependent, i.e. \( J_{\uparrow} = J_{\downarrow} + \Delta J \). Both cases are visualized in Fig. 3 - top row. At \( W = 3 \) the system is characterized by almost Poissonian mean gap ratio in the presence of symmetries. The finite value of \( h_0 \) couples different symmetry classes and reveals the true extended character of the system with \( \langle r \rangle \) corresponding to GOE (top left panel). A very similar behavior is observed when spin dependent tunneling is present (top right panel). In both cases the system the smaller value of the symmetry breaking parameter is required to fully break symmetry constrains.

Let us now consider the case of nonvanishing phase \( \gamma \) in tunnelings - compare Hamiltonian (1). Non-zero \( \gamma \) breaks

![FIG. 3. Mean gap ratio for different system sizes in the delocalized regime (\( W = 3 \)) as a function of the strength of the symmetry breaking local magnetic field (left) or the assumed difference between tunneling rates for spin up and down fermions (right) which also breaks the symmetry. The top row represents TRI case, \( \gamma = 0 \) where presence of symmetries results in a superposition of independent spectra yielding Poisson-like statistics. For complex fluxes, the residual reflection symmetry between spin up and down fermions (for unit filling) combined with TRI leads to generalized time-reversal symmetry (see discussion in the text) leading to an apparent GOE-like behaviour. Only breaking this symmetry by a local field or difference in tunneling rates, the GUE-like statistics corresponding to broken TRI is observed.](image-url)
simple TRI - one might naively expect in that case the GUE-like behavior in delocalized regime. Yet, as shown in the bottom row in Fig. 3 without $h_b$ (or spin dependent tunneling) the mean gap ratio $\langle r \rangle \approx 0.53$ points towards GOE statistics. This is explained by the fact that while standard TRI is broken, there exists a generalized TRI in our system (1), namely TRI combined with reflection in $x-y$ plane (i.e. change of the spin direction). That generalized TRI leads to GOE statistics (for an excellent discussion of symmetries in different universality classes see [57]).

With the introduction of $h_b$ or spin dependent tunneling this generalized TRI symmetry is broken and, once the symmetry is fully broken, the GUE statistics is fully recovered in the delocalized regime as shown in the bottom row of Fig. 3.

Consider now the crossover from the delocalized to localized regime for nonzero $\gamma$, i.e., in the presence of a synthetic field. Let us discuss first the case with $h_b = 0$ and a generalized TRI - compare Fig. 4. Please note that the observed GOE – Poisson transition in mean gap ratio values seems not to depend on $\gamma$ (once it is sufficiently big to mix different symmetries). This could in principle be quantified by comparison of finite size scaling parameters obtained - we refrain from such a procedure as the obtained values of $W_c$ must be regarded as only rough estimates due to small system sizes. Thus we rely for this observation on the crossing points of curves in Fig. 4 for different system sizes.

On the contrary, in the presence of additional local field [i.e. adding the term (2) to the Hamiltonian (1)] the transition between GUE and Poisson-like behaviour becomes dependent on $\gamma$ as apparent from data presented in Fig. 5. One might think that this shift of the transition is related to the way in which the symmetry is broken, namely by a local magnetic field. Such a local perturbation might differently affect the transition to the localized phase for different value of $\gamma$. However, we have checked, by adding random fields on all of the lattice sites that this is not the case.

IV. DYNAMICS AND DENSITY CORRELATIONS

To study the dynamical properties, we choose random Fock state $|\psi(0)\rangle$ as an initial state for the temporal evolution and examine the dynamics of the local correlations and the entanglement entropy for $L = 8$. The local charge and spin correlations are $C(t) = A \sum_j \langle \rho_j(t) \rho_j(0) \rangle$ and $S(t) = B \sum_j \langle m_j(t) m_j(0) \rangle$, where $\rho_j = n_j - \bar{n}$ with the average density $\bar{n} = \sum_j n_j / L$, and $A$ and $B$ are normalization constants such that $C(0) = S(0) = 1$. Recall that $n_j$ ($m_j$) are the sum (difference) site occupations of up and down polarized fermions, respectively. We assume a unit filling, $\bar{n} = 1$. The memory of the initial state is lost for ergodic system and this leads to decay of the charge and spin correlations. Interesting situation occurs for the localized phase where (in the absence of symmetry breaking local magnetic field or other effects destroying the system symmetries) the charge sector appears localized but spin sector does not show localization [32, 35]. The absence of fully localized phase is due to the SU(2) symmetry of the model. The choice of a different random potential for up and down polarized fermions, a random magnetic field or a weak spin asymmetry which breaks the SU(2) spin symmetry recovers the full MBL phase [34, 58, 59]. Moreover, the subdiffusive transport of spins is due to a singular random distribution of effective spin exchange interactions [33] and at long time evolution the transport is strongly suppressed [35]. The particle transport rate is exponentially small in $J/W$, and the rate depends strongly on the initial state of the system. The states occupied with only doublons or holons exhibit full MBL
Consider now the evolution of the spin correlation $S(t)$ which is shown in Fig. 6(b). In the absence of the flux, the decay of $S(t)$ at long time is suppressed and eventually spins are localized [26, 35]. For $K = 0$ the intermediate time decay of $S(t)$ is monotonic (modulo fluctuations resulting from relatively small number of disorder realizations equal to 300) while the presence of non-zero $K$ leads to oscillations in $S(t)$. The frequency of oscillations is simply $K$ as may be verified in the inset of Fig. 6(b). Interestingly the oscillations have the envelope given by the $S(t)$ curve for $K = 0$. Denoting by $H_K$ the term of $H_0$ proportional to $K$ and by $H_H = H_0 - H_K$, we verify that $[H_H, H_K] = 0$ as long as $\gamma = 0$. Thus, the evolution operator factorizes: $\exp(-iH_0t) = \exp(-iH_Ht) \exp(-iH_Kt)$ and average number of fermions with spin $\sigma$ at site $j$ is given by
\[ n_{j\sigma}(t) = \langle \psi(0) | e^{iH_Ht} e^{iH_Kt} \hat{n}_{j\sigma} e^{-iH_Kt} e^{-iH_Ht} | \psi(0) \rangle, \]
so that the time evolution of spin correlation function is a superposition of dynamics determined by the $H_H$ Hamiltonian and oscillations driven by the $H_K$ term. The long-time average of the oscillations is zero, which is in agreement with previous study on the localization of coupled chains with correlated disorder [62]. In the presence of a finite flux, $[H_K, H_H] \neq 0$, the oscillations are damped and the damping rate is growing with $\gamma$. Thus, the spin sector of the system gets delocalized by the introduction of gauge field.

To examine the effect of quantum statistics, we further analyze the time dependence of local correlations when instead of fermions we consider hard-core bosons in the Hamiltonian (1). The correlations of hard-core bosons for different values of $K$ and $\gamma$ are shown in Fig. 7. The behaviour of charge correlation, $C(t)$, is similar to the fermionic case and its finite saturation value represents the localization of particles in
the presence of the synthetic flux. The saturation value of $C(t)$ decreases with $0 \leq \gamma \leq 1$ at $K = 1$. The spin sector is localized as $S(t)$ saturates to a finite value when the two hyperfine spin states are decoupled. The coupling of these two states, even for $\gamma = 0$, leads to oscillations, which in contrast to fermionic $S(t)$, are damped to zero. A finite flux $\gamma$ only weakly affects the damping and spin sector becomes fully delocalized. The stark contrast of $S(t)$ between hardcore bosons and fermions for $K = 1$ and $\gamma = 0$ is a very nice demonstration of effects induced by quantum statistics and different commutation properties of fermions and hardcore bosons (for ground state properties this observation goes back to [63]).

Finally, we return to the fermionic system and calculate time evolution of charge and spin correlation functions as shown in Fig. 8 for non-zero value of symmetry breaking field $h_0$. In this case, even at $\gamma = 0$ the commutator $[H_H + H_{sb}, H_K] \neq 0$ and the oscillations of $S(t)$ are damped. The effects of broken generalized TRI symmetry, while altering significantly spectral properties of the system, have relatively small effect on time dynamics of correlation functions $C(t)$ and $S(t)$ for $\gamma > 0$.

V. ENTANGLEMENT ENTROPY GROWTH

To corroborate our study of spin delocalization in the presence of the synthetic gauge field we investigate the entanglement entropy growth in the system. The two-leg ladder system can be partitioned in several ways with two prime options: the first version is to cut the ladder perpendicular to the direction of the spatial dimension in two equal (left-right) parts and the second approach is to cut the ladder perpendicular to the direction of synthetic dimension decoupling the hyperfine states. This allows us to study both the entanglement between two left and right ladder subsystems but also between an ensemble of up and down polarized fermions.

Regardless of the splitting the von Neumann entanglement entropy is defined in a standard way as

$$S_E(t) = -\text{Tr}\rho_A(t) \ln \rho_A(t),$$

where the two subsystems are denoted as $A$ and $B$ with $\rho_A(t) = \text{Tr}_B |\psi(t)\rangle \langle \psi(t)|$ being the reduced density matrix of subsystem $A$. Here $\text{Tr}_B$ is the trace over degrees of freedom of subsystem $B$.

In the MBL regime the entropy of entanglement of initially separable state should grow logarithmically in time [24, 25] after an initial transient eventually saturating for a finite system size [25]. Such a behavior is indeed observed in our model for decoupled spin chains ($K = \gamma = 0$) both for the standard left-right splitting [Fig. 9(a)] and for the up-down splitting [Fig. 9(b)].

The coupling between up and down polarized fermions introduces strong oscillations in the entanglement entropy, with the same period as revealed by the spin correlation function - compare Fig. 9(a). Similarly, the entropy curve for $K = 0$ forms an envelope (here low lying envelope) to the oscillations. Again this fact is related to vanishing commutators between different kinetic energy terms in the Hamiltonian (1), as described in the previous Section. The presence of complex flux makes these commutators non-zero and introduces the damping of the oscillations. The resulting entropy growth is significantly faster than in the $K = \gamma = 0$ case reflecting the delocalization of spin sector.

Similar, even more spectacular oscillations of the entanglement entropy are observed when the partition devides up and down polarized fermions (i.e. the rungs in the ladder are cut). The coupling between up and down fermions ($K = 1$) makes

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig8}
\caption{Same as Fig. 6 but in the additional presence of the local magnetic field term, [Eq.(2)] breaking the generalized TRI symmetry. The influence of coupling $K$ and synthetic flux becomes similar to that of hard-core bosons. The strength of local field $h_0 = 0.5$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig9}
\caption{(a) Left-right bipartite entanglement entropy in the localized regime. With no coupling of spin up and spin down polarized fermions a typical logarithmic growth of $S_E$ is observed after initial transients. Coupling $K$ induces oscillations of the entanglement entropy. Complex flux $\gamma$ leads to damping of these oscillations with also faster entropy growth. (b) Bipartite entanglement entropy dynamics when rungs of the ladder are cut, i.e. a partition devided up and down polarized fermions.}
\end{figure}
the entropy oscillate maximally, with lower envelope given by $K = 0$ curve as before while the upper envelope is system size and disorder strength dependent. Again the non-zero flux introduces damping of these oscillations - on the time scale discussed the entropy growth practically saturates at upper allowed value [Fig. 9(b)].

VI. SPIN-DEPENDENT DISORDER

Up till now we considered the disorder identical for both spin components. Such a situation (albeit for quasiperiodic disorder) is realized in experiments [5]. However, it is possible (and feasible) experimentally to consider a situation when disorder is different (and uncorrelated) for both spin components with the last term in the Hamiltonian (1) changed to $\sum_{j,\sigma} \epsilon_{j,\sigma} \hat{n}_{j,\sigma}$, where now $\epsilon_{j,\sigma}$ are independent random variables drawn from uniform distribution in the interval $[-W/2, W/2]$. The resulting charge and spin correlation functions are shown in Fig. 10. We observe a clear localization of both charge and spin sectors.

FIG. 10. (a) Charge $C(t)$ and (b) spin $S(t)$ correlation functions for uncorrelated disorder $\epsilon_{j,\sigma} \in [-W/2, W/2]$, the chemical potential is site and spin dependent. The system size is $L = 8$, disorder strength $W = 32$, various values of coupling $K$ and flux $\gamma$ are considered. Both $C(t)$ and $S(t)$ saturate signalling localization of both charge and spin sectors.

We observe that the spin sector is localized as $S(t)$ quickly saturates whereas the charge correlations decay suggesting delocalization of charge degrees of freedom.

Note that when disorder becomes spin-dependent the time evolution of both charge and spin correlation function is no longer dependent on the phase of the coupling $\gamma$ between spin components. While for individual realizations of disorder the evolution may differ substantially, the differences average out when taking the disorder average. Once the spin degree of freedom becomes localized the information transfer between the rungs of the ladder stops regardless of $\gamma$ value. It is worth mentioning that similar independence on the phase of flux is also observed in the entanglement entropy for uncorrelated and “spin disorder”.

One may make this observation even stronger. Consider the

FIG. 11. (a) Charge $C(t)$ and (b) spin $S(t)$ correlation functions for spin disorder $\epsilon_{j,\sigma} = -\epsilon_{j,\bar{\sigma}}$ (where $\bar{\sigma}$ denotes spin opposite to $\sigma$). The system size is $L = 8$, disorder strength $W = 32$, various values of coupling $K$ and flux $\gamma$ are considered. The saturation of $S(t)$ signals localization of the spin sector whereas the charge sector remains delocalized.

FIG. 12. Averaged over disorder realizations inverse participation ratio (a) for Hamiltonian (1) and (b) for disorder uncorrelated between spin components - leading to full MBL of both charge and spin.
inverse participation ratio of \( n \)-th eigenstate \(|n\):

\[
I_n = 1/\sum_i |(n|b_i)|^4,
\]

where \(|b_i\rangle\) are basis functions (we chose a natural basis of Fock states on lattice sites). We define the averaged over disorder inverse participation ratio as \(\text{IPR}(n) = \overline{I_n}\), with the overbar denoting disorder average. For identical random disorder for both spin components [Hamiltonian (1)] IPR defined in this way depends on \(K\) and \(\gamma\) - compare Fig. 12(a). Significant values of IPR indicate that localization is not complete. For the disorder uncorrelated between spin components with the same amplitude \(W\) we observe significantly smaller IPR (indicating strong localization), moreover, IPR becomes independent of \(\gamma\) (recall that IPR\((n)\) are obtained after disorder averaging). Thus not only \(C(t)\) or \(S(t)\) but also other observables may be expected to be \(\gamma\)-independent once the spin component is localized.

In this respect we observe a clear asymmetry between a real dimension (along the chain) and the synthetic dimension (represented by spin components). The localization of the latter is essential for \(\gamma\)-independence of disorder averaged observables while the charge localization plays little role. Observe that for pure “spin disorder”, as shown in Fig. 11 the correlations for \(K = 1\) are flux independent while charge degrees of freedom are delocalized.

VII. CONCLUSIONS

We have analyzed properties of the disordered chain of spin-1/2 fermions in the presence of the synthetic magnetic field. While the spectral properties such as the average gap ratio indicate the transition to many-body localized phase, the time dynamics suggest that MBL is realized in charge (density) sector only. The presence of the synthetic magnetic field delocalizes the spin sector as revealed by the decay of spin time correlation function. Similarly, the entropy of entanglement in the system grows much faster in the presence of the magnetic field flux.

Interestingly the spectral properties of the system strongly depend on the realized symmetries providing a nice example of the effects due to a generalized TRI (i.e. a TRI combined with a discrete symmetry). In effect, despite the standard time reversal symmetry being broken the spectral properties in the delocalized regime resemble that of GOE unless additional symmetry breaking terms are introduced into the model.

A comparison of the dynamics of correlation functions for fermions and hard-core bosons in the absence of the flux \((\gamma = 0)\) but when the driving term \(K\) couples up and down polarized particles shows sensitivity of the observed phenomena to quantum statistics. For fermions, due to their commutation relations, a kinetic energy part corresponding to the transition between spin up and down fermions commutes with the rest of the Hamiltonian. In effect, the exact quantum dynamics is given by a rapidly oscillating solution whose one envelope is given by \(K = 0\) (i.e. no Rabi coupling) solution. This behavior is absent for hard-core bosons.

Last but not least, we considered different types of random disorder, in particular spin dependent disorder that leads to strong localization in the spin sector. In such a case time-dependent observables become, after disorder average, independent of flux \(\gamma\). We believe that the system sizes examined in present work are sufficient to grasp robust features of considered systems on time scales relevant for experiments with ultracold atoms.

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