Search for time modulations in the Gallex/GNO solar neutrino data

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Abstract

The final data of the Gallex and GNO radiochemical experiments have been searched for possible time modulations of the solar neutrino capture rate using the Lomb-Scargle periodogram and the maximum likelihood methods. Both analyses, performed using the raw data, do not support the presence of a time variability with characteristic periods resembling those of rotation of the solar magnetic fields, as previously suggested in the literature by some authors. In this context, the potential sensitivity of the Lomb-Scargle method to radiochemical \textsuperscript{71}Ga data has also been explored with simulated data. This allowed to set an exclusion plot in the frequency/amplitude plane from the Gallex-GNO dataset.

Key words: Solar neutrinos, time series analysis, radiochemical experiments
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1 Introduction

In recent years it has been suggested \cite{1,2,3,4,5,6,7} that solar neutrino flux measured by Cl and Ga radiochemical experiments and by Super-Kamiokande shows a time variability with characteristic periods resembling those of rotation of the solar magnetic fields. Since this could be a very strong hint for sub-leading Resonant-Spin-Flavour-Precession (RSFP) of pp neutrinos occurring in the solar convection zone \cite{8} and for a non-negligible neutrino transition magnetic moment, it is of paramount importance to perform an extensive

\textsuperscript{1} As a member of the GNO Collaboration
search for possible time modulations in the $^{71}$Ga data using different methods and exploiting the total information of the available solar runs.

The Gallex [9] and GNO [10,11,12] radiochemical experiments at Laboratori Nazionali del Gran Sasso, Italy, monitored the low energy solar neutrino flux using a 30-ton gallium detector via the inverse $\beta$ reaction $^{71}$Ga($\nu_e,e^-)^{71}$Ge. GNO is the successor project of Gallex, which continuously took data between 1992 and 1997, and has been in operation from April 1998 to April 2003. The gallium detector at Gran Sasso consists of 30.3 tons of natural gallium in the form of 103 tons of gallium chloride ($\text{GaCl}_3$) acidic solution. The energy threshold of the $^{71}$Ga($\nu_e,e^-)^{71}$Ge reaction, 233 keV, is sufficiently low to make gallium detectors sensitive to the fundamental $pp$ neutrinos. Presently only radiochemical experiments based on $^{71}$Ga are sensitive to these neutrinos. $^{71}$Ge nuclei produced by $\nu_e$ interactions are unstable and transform back to $^{71}$Ga by electron capture, with a life time $\tau$ of 16.49 days. The gallium target is exposed to solar neutrinos for a time $t_{\text{exp}} = t_{\text{EOE}}-t_{\text{SOE}}$ (SOE = start of exposure, EOE = end of exposure) which is typically 3 or 4 weeks. After this, the accumulated $^{71}$Ge ($\sim$8 atoms for a 4-week-exposure) is extracted from the target solution and converted in germane gas $\text{GeH}_4$. The germane is added with low-activity xenon and is used as counting gas for a miniaturized proportional counter, which is able to detect the electron capture decays of $^{71}$Ge. The interaction rate of solar neutrinos with the target is measured, in the hypothesis it is constant in time, by counting the decay signals of $^{71}$Ge in the proportional counter. For details about the experimental procedure and data analysis see Ref. [10]. The final result from Gallex (65 solar runs, 1594 days of exposure time) is $77.5\pm6.2\pm4.5$ SNU. From April 1998 to April 2003, 58 solar runs, with exposure time of 3 or 4 weeks, were successfully performed within the GNO project, for a overall time of 1713 days [10,11,12]: the resulting solar neutrino interaction rate is $62.9^{+5.5}_{-5.3}\pm2.5$ SNU. The combined Gallex/GNO rate (123 solar runs, 3307 days of exposure time) is $69.3\pm4.1\pm3.6$ SNU. Results from the single solar runs are displayed in Fig. 1.

In this paper, possible significant time periodicities in the solar neutrino data are looked for using the Lomb-Scargle periodogram method (which has already been used in the literature for this kind of analysis); the potential sensitivity is also tested with simulated data sets. The search for time periodicities is then repeated by applying the maximum likelihood method to the candidate $^{71}$Ge decays in the single solar runs; the constant solar neutrino rate hypothesis is tested and a likelihood spectrum is derived.

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2 1 SNU = $10^{-36}$ interactions per second and per target nucleus.
Fig. 1. Measured solar neutrino capture rate (SNU) in the 123 solar runs of Gallex/GNO [12].

2 Lomb-Scargle periodogram method

2.1 Analysis of the solar runs

The search for periodicities in the Gallex/GNO solar neutrino data can be performed using the Lomb-Scargle periodogram method (see Ref. [2,6,13]). Given a set of $N$ data points $h_i \equiv h(t_i)$ at the times $\{t_i\}$, $i = 1 \ldots N$, with mean and variance $\bar{h}$ and $\sigma^2$ respectively, the Lomb-Scargle periodogram is defined as [14,15]

$$z(\nu) = \frac{1}{2\sigma^2} \left\{ \frac{\sum_j (h_j - \bar{h}) \cos(\omega(t_j - \theta))}{\sum_j \cos^2 \omega(t_j - \theta)} + \frac{\sum_j (h_j - \bar{h}) \sin(\omega(t_j - \theta))}{\sum_j \sin^2 \omega(t_j - \theta)} \right\}, \quad (1)$$

where $\omega = 2\pi\nu$ is the angular speed and $\nu$ is the frequency; the offset $\theta$ is defined by the relation

$$\tan(2\omega\theta) = \frac{\sum_j \sin(2\omega t_j)}{\sum_j \cos(2\omega t_j)} \quad (2)$$

and makes the periodogram $z(\nu)$, which is also called power, independent of shifting all the times $\{t_i\}$ by any constant [15].
Following the procedure used in Ref. [13], the periodogram for the Gallex/GNO data presented in Ref. [2] is first reproduced. Fig. 2 shows the Lomb-Scargle periodogram obtained from the analysis of the first 84 Gallex/GNO solar runs [9,10], that can be directly compared with Fig. 2 at the page 1369 of Ref. [2].

The most relevant features of the original periodogram are correctly reproduced: the peaks (and in particular the main one, at frequency $\sim 13.7 \, \text{y}^{-1}$) are approximately in the same position and the vertical scale of power, which fixes the statistical significance of the peaks themselves, is exactly equal. This level of agreement is completely satisfactory for the present testing purposes $^3$.

After this test of the Lomb-Scargle code, the same analysis has been extended to the whole set of Gallex and GNO data [9,10,11,12]. As in the preliminary test of Fig. 2, the results $\{h_i\}$ of the $N = 123$ solar runs are conventionally referred to the end-of-exposure $\{t_{\text{EOE}}\}$ times[2]. In order to test the region of major astrophysical interest, the frequency range taken into account spans from

$$
\nu_{\text{min}} = \frac{1}{2(t_N - t_1)} = 0.04 \, \text{y}^{-1} \quad \text{to} \quad \nu_{\text{max}} = 5 \cdot \nu_{\text{cr}} = 26 \, \text{y}^{-1},
$$

(3)

where $\nu_{\text{cr}} = \frac{N}{2(t_N - t_1)} = 5.19 \, \text{y}^{-1}$ is the average Nyquist frequency. In the periodogram analysis all the data points $\{h_j\}$ are used with equal statistical weights and are corrected to take into account the expected $1/d^2$ geometrical modulation of the neutrino flux, which is due to the variation of the Sun-Earth distance during the year. If $z_{\text{max}}$ is the highest peak found in the periodogram, one has to establish if this is compatible with statistical fluctuations, i.e. one

$^3$ Minor differences in the positions or in the powers of the single peaks, that are probably related to different used approximations, are not significant in this context.
has to test the null hypothesis $H_0$ of a constant neutrino signal with random noise. The probability $P(z_{\text{max}} > z_{\text{max}} | H_0)$ that statistical fluctuations give a peak in the periodogram larger than $z_{\text{max}}$ is the false-alarm probability of the null hypothesis and a small value of $P(z_{\text{max}} > z_{\text{max}} | H_0)$ indicates an highly significant periodic signal. Following the procedure of Ref. [16], the false-alarm probability has been estimated by generating 100,000 Monte Carlo data samples \{$h_i$\}, $i = 1 \ldots 123$, in the assumption of the hypothesis $H_0$. As the probability depends also on the detailed time spacing of the data, the \{$h_i$\} have been referred to the same times \{$t_i$\} as the real runs. The Lomb-Scargle method is then applied to each data sample, to get the spectrum $z(\nu)$ and the maximum power $z_{\text{max}}$. The resulting maximum power distribution and the false-alarm probability are displayed in Fig. 3; the $z_{\text{max}}$ power values corresponding to 50%, 5%, 1% and 0.1% false-alarm probabilities are respectively 6.69, 9.12, 10.57 and 12.55.\footnote{The 50% false-alarm level, $z_{\text{max}}=6.69$, corresponds to the average value of $z_{\text{max}}$ expected from a null hypothesis signal.}

The Lomb-Scargle periodogram obtained from the analysis of the 123 solar runs of Gallex and GNO is shown in Fig. 4. The highest peak in the interval [$\nu_{\text{min}}, \nu_{\text{max}}$] is found at $\nu_{\text{max}} = 6.37 \pm 0.04$ $\text{y}^{-1}$; the corresponding power is $z_{\text{max}} = 5.26$ and the false-alarm probability for the null hypothesis is $\sim$95%. The Lomb-Scargle periodogram for Gallex/GNO solar neutrino data is hence statistically consistent with the expectation of a constant interaction rate and it does not contain any hint of time modulation. If this same analysis is repeated for the 58 solar runs of GNO only, the highest peak is found at the frequency $\nu_{\text{max}} = 2.9 \pm 0.1$ $\text{y}^{-1}$, with false-alarm probability $\sim$55%.

Fig. 3. Distribution of maximum power $z_{\text{max}}$ (left) and false-alarm probability (right) for the null hypothesis (i.e. constant signal with random noise), assuming $N = 123$ data with times \{\(t_i\)\} as in the real solar runs and frequency in the range [$\nu_{\text{min}}, \nu_{\text{max}}$].
2.2 Test of the sensitivity with simulated data

In order to test the actual sensitivity of this kind of data to possible time modulations, Monte Carlo repetitions \( \{h_i\} \) of the Gallex/GNO experiments have been generated, in the hypothesis that the \(^{71}\)Ge neutrino-induced production rate is modulated in time as

\[
n(t) = n_0[1 + A \sin(2\pi \nu t + \varphi)],
\]

with \( 0 \leq A \leq 1 \). The mean production rate \( n_0 \) is assumed to be \( n_0 = 0.63 \) atoms/day = 70 SNU, close to the experimental value. The statistical fluctuations on the single runs can be approximated with a Gaussian having \( \sigma \sim 40 \) SNU.\(^5\) The search for periodicities in the simulated data has been then performed using the Lomb-Scargle periodogram method in the frequency range \( [\nu_{\text{min}}, \nu_{\text{max}}] \) defined above. Being the Lomb-Scargle periodogram invariant for time translations \([15]\), it can be assumed without loss of generality that \( \varphi = 0 \), since this is simply equivalent to shift the time origin. For each data set, the maximum power \( z_{\text{max}}(A, \nu) \) was evaluated. Fig. 5 shows the \( z_{\text{max}}(A, \nu) \) for frequencies ranging in 1-20 \( y^{-1} \) and for amplitudes \( A = 1, \frac{1}{2} \) and \( \frac{1}{4} \). It can be seen from Fig. 5 that the method clearly indentifies the time modulation if the frequency is low and the amplitude is large enough. However, even for \( A=100\% \), the sensitivity rapidly decreases for frequencies \( \nu \) higher than \( \sim 12 \)

\(^5\) The actual distribution of the results of single runs is not exactly Gaussian, because of the asymmetry introduced by Poissonian effects. However, it has been verified that the assumption of Gaussian fluctuations does not affect the final result in this particular application.
Fig. 5. Maximum power $z_{\text{max}}(A, \nu)$ vs. frequency obtained from the Lomb-Scargle analysis of simulated data, in the hypothesis time modulation with frequency $\nu$ and amplitude $A = 1, \frac{1}{2}$ and $\frac{1}{4}$. The statistical error on the single run is assumed to be 40 SNU. Dashed lines are the 50%, 5%, 1% and 0.1% false-alarm levels.

$y^{-1}$: in this case, the modulation signal cannot be statistically distinguished from a constant signal with Gaussian fluctuation, i.e. it is impossible to reject the null hypothesis using this kind of data and analysis.

### 2.3 Exclusion plot

Since no positive modulation signal has been found with the periodogram analysis of the Gallex/GNO dataset in the region of sensitivity derived in Sect. 2.2, it is possible to set an exclusion plot. In the assumption of time modulation of Eq. (4), the experimental data allow to rule out (see Fig. 6) variations of the solar neutrino flux with low frequency and large amplitude. As expected, there is a rapid loss of sensitivity for $\nu \gtrsim \frac{1}{t_{\text{exp}}}$.

### 3 Maximum likelihood analysis

An other option is to apply the maximum likelihood method to the time list of the $^{71}\text{Ge}$ candidate events in the solar runs, including (on the contrary of what happens in the standard Gallex/GNO analysis) a time-modulated term in the likelihood function. The general maximum likelihood method described
Fig. 6. 68% and 95% CL exclusion plots in the frequency/amplitude plane derived from the Lomb-Scargle analysis of GALLEX/GNO data in Ref. [17] can then be applied to the case of a production rate varying as in Eq. (4). In this condition the mean number of $^{71}$Ge atoms produced in the target depends on the times of start and end of exposure (and not only from the total exposure time) and can be calculated as

$$N_{71} = \int_{t_{SOE}}^{t_{EOE}} n(t; n_0, A, \nu, \varphi) e^{-\lambda_{71}(t_{exp}-t)} \, dt$$

(5)

(see Eq. 12 of Ref. [17]), where $\lambda_{71}$ is the decay rate of $^{71}$Ge. It is hence possible to estimate from the solar run data the best-fit values for the four free parameters $n_0$, $A$, $\nu$ and $\varphi$ of Eq. (4).

A known time modulation is actually expected in the solar neutrino data because of the seasonal variation of the Sun-Earth distance. Since the eccentricity of the Earth’s orbit is $\epsilon=0.0167$ and the perihelion is on January $4^{th}$, the expected parameters for the $1/d^2$ geometrical modulation in the solar neutrino flux are: $A \sim 2\epsilon = 0.033$, $\nu = 1 \, y^{-1}$ and $\varphi=3.81 \, rad^6$. Although, at

\footnote{The time origin is conventionally set to May $14^{th}$, 1991, so that $t_{SOE}^{(1)} = 0$.}
least at the present stage, the $^{71}$Ga data have not enough sensitivity to reveal such a low-amplitude modulation ($A \cdot n_0 \sim 2.3$ SNU, to be compared with the overall Gallex/GNO error, $\sim 5.5$ SNU), this is anyway explored using the maximum likelihood method: the frequency and phase are fixed to the known values reported above while the amplitude is left as a free parameter. The best-fit coming from the combined analysis of the 123 Gallex/GNO solar runs is $n_0 = 69.3 \pm 5.9$ SNU and $A = 0.01 \pm 0.11$. The amplitude is well consistent with what expected, but it is nevertheless consistent with zero, confirming that the presently reached sensitivity is not good enough to test this modulation.\footnote{The exposure needed to reduce the statistical error to $A \cdot n_0 \sim 2.3$ SNU would be of $\sim 700$ ton·y, to be compared with 270 ton·y of Gallex and GNO. Moreover the present systematic error, which is 2.5 SNU for GNO, should be further reduced in order to reach the required level of sensitivity.}

The data analysis was then repeated including two modulation terms in the likelihood: one, whose parameters are fixed, to account for the annual modulation and the other, whose parameters are left free, for other possible unknown periodicities. In this case the best-fit point for the 123 solar runs is (statistical errors only) $n_0 = 69.1 \pm 6.1$ SNU, $A = 0.57 \pm 0.38$, $\nu = 13.9 \pm 0.4$ y$^{-1}$ (in good agreement with the frequency $13.59$ y$^{-1}$ reported in Ref. [5]) and $\varphi = -0.3 \pm 1.0$ rad. The hypothesis of modulated solar neutrino flux can be compared with the usual null hypothesis of stationary flux (with the geometrical correction) using the likelihood ratio test [18]. If the constraint $A = 0$ is applied (i.e. one makes the hypothesis of a constant neutrino flux) the best-fit value is of course $n_0 = 69.3 \pm 4.1$ SNU (statistical error only). If

$$\lambda = \frac{L_{\text{max}}(A = 0)}{L_{\text{max}}} \quad (6)$$

is the likelihood ratio, the statistics $-2 \ln \lambda$ has the same distribution of $\chi^2(3)$ [18]; in the present case, $-2 \ln \lambda = 3.40$ and the corresponding $p$-value is 33.4%. The experimental data are hence statistically consistent with the hypothesis of a time-constant neutrino interaction rate, in agreement with the results of the Lomb-Scargle analysis.

In Fig. 7 it is shown the likelihood spectrum $-2 \ln \lambda$ obtained fixing the frequency and fitting all the other parameters from the data. It can be seen from the plot that the absolute maximum of the likelihood is not much larger than the other local maxima: therefore, no clear modulation frequencies can be derived from the data. Using the same likelihood ratio test, it is possible to compare the hypothesis of stationary rate with the hypothesis of a time-modulated rate with frequency $13.59$ y$^{-1}$ [5]: in this case $-2 \ln \lambda = 3.28$ (2 d.o.f.) and the corresponding $p$-value is 19.4%.
4 Conclusions

The experimental data of the Gallex and GNO solar neutrino experiments (123 solar runs, for a total exposure time of 3307 days) have been searched for possible time modulations in the capture rate using the Lomb-Scargle periodogram and the maximum likelihood methods. For the latter analysis, the time list of candidate $^{71}$Ge events in the single runs has been taken into account. In both cases no statistical evidence of time variations has been found: given the present sensitivity, the results are fully consistent with the expectation of a constant solar neutrino capture rate over the data taking period ($\gtrsim 10$ y). Though this fact does not automatically exclude other hypotheses that predict a time-dependent rate, modulations with low frequency and large amplitude can be ruled out.

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