A Note on the Majorana-Oppenheimer Quantum Electrodynamics

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Abstract

A group theoretical description of the Majorana-Oppenheimer quantum electrodynamics is considered. Different spinor realizations of the Maxwell and Dirac fields are discussed. A representation of the Majorana-Oppenheimer wave equations in terms of the Gel’fand-Yaglom formalism is given.

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In 1950, Gupta [14] and Bleuler [3] proposed to quantize the electromagnetic field via the four-potential $A_\mu$. As is known, such a quantization gives rise to the following difficulties: non-physical degrees of freedom, indefinite metric, null helicity, peculiar opposition between electromagnetic field and other physical fields. More recently, Bogoliubov and Shirkov said: “Among all the physical fields the electromagnetic field is quantized with the most difficulty” [4]. For that reason at present time many physicists considered a situation with the quantization of the electromagnetic field as unsatisfactory. It is clear that the Gupta-Bleuler phenomenology should be replaced by a more rigorous alternative.

Beyond all shadow of doubt, one of the main candidates on the role of such an alternative is the Majorana-Oppenheimer quantum electrodynamics. At the beginning of thirties of the last century Majorana [17] and Oppenheimer [21] proposed to consider the Maxwell theory of electromagnetism as the wave mechanics of the photon. They introduced a wave function of the form

$$\psi = E - iB,$$  \hspace{1cm} (1)
where \( \mathbf{E} \) and \( \mathbf{B} \) are electric and magnetic fields. In virtue of this the standard Maxwell equations can be rewritten

\[
\text{div} \, \psi = \rho, \quad i \text{ rot} \, \psi = j + \frac{\partial \psi}{\partial t},
\]

where \( \psi = (\psi_1, \psi_2, \psi_3), \psi_k = E_k - i B_k \) \((k = 1, 2, 3)\). In accordance with correspondence principle \((-i \partial/\partial x_i \to p_i; +i \partial/\partial t \to W)\) and in absence of electric charges and currents the equations (2) take a Dirac-like form

\[
(W - \mathbf{a} \cdot \mathbf{p}) \psi = 0
\]

with transversality condition

\[
\mathbf{p} \cdot \psi = 0.
\]

At this point, three matrices

\[
\alpha^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \quad \alpha^2 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \alpha^3 = \begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\]

satisfy the angular-momentum commutation rules

\[
[\alpha_i, \alpha_k] = -i \varepsilon_{ilk} \alpha_l \quad (i, k, l = 1, 2, 3).
\]

Further, for the complex conjugate wave function

\[
\psi^* = \mathbf{E} + i \mathbf{B}
\]

there are the analogous Dirac-like equations

\[
(W + \mathbf{a} \cdot \mathbf{p}) \psi^* = 0
\]

From the equations (3) and (6) it follows that photons coincide with antiphotons (truly neutral particles). In such a way, \( \psi \) \((\psi^*)\) may be considered as a wave function of the photon satisfying the massless Dirac-like equations. In contrast to the Gupta-Bleuler phenomenology, where the non-observable four-potential \( A_\mu \) is quantized, the main advantage of the Majorana-Oppenheimer formulation of electrodynamics lies in the fact that
it deals directly with observable quantities, such as the electric and magnetic fields.

Let us consider now a relationship between the Majorana-Oppenheimer formulation of electrodynamics and a group theoretical framework of quantum field theory. It is widely accepted that the Lorentz group (a rotation group of the 4-dimensional space-time continuum) is a kernel of relativistic physics. Fields and particles are completely formulated within irreducible representations of the Lorentz or Poincaré group (see Wigner and Weinberg works [40, 39], where physical fields are considered in terms of induced representations of the Poincaré group). According to Wigner [40], a quantum system, described by an irreducible representation of the Poincaré group, is called an elementary particle. The group theoretical formulation of quantum field theory allows one to describe all the physical fields in equal footing, without any division on ‘gauge’ and ‘matter’ fields as it accepted in modern gauge phenomenologies, such as Standard model and so on. On the other hand, there is a close relationship between linear representations of the Lorentz group and Clifford algebras (for more details see [31, 33] and references therein). As is known, a double covering of the proper orthochronous Lorentz group \( \mathfrak{G}_+ \), the group \( SL(2, \mathbb{C}) \), is isomorphic to the Clifford–Lipschitz group \( \text{Spin}_+(1, 3) \), which, in its turn, is fully defined within a biquaternion algebra \( \mathbb{C}_2 \), since

\[
\text{Spin}_+(1, 3) \simeq \left\{ \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \in \mathbb{C}_2 : \quad \det \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = 1 \right\} = SL(2, \mathbb{C}).
\]

Thus, a fundamental representation of the group \( \mathfrak{G}_+ \) is realized in a spin-space \( S_2 \). The spin-space \( S_2 \) is a complexification of the minimal left ideal of the algebra \( \mathbb{C}_2 \): \( S_2 = \mathbb{C} \otimes I_{2,0} = \mathbb{C} \otimes \mathcal{A}_{2,0}e_{20} \) or \( S_2 = \mathbb{C} \otimes I_{1,1} = \mathbb{C} \otimes \mathcal{A}_{1,1}e_{11} \) \( (\mathbb{C} \otimes I_{0,2} = \mathbb{C} \otimes \mathcal{A}_{0,2}e_{02}) \), where \( \mathcal{A}_{p,q} \ (p+q = 2) \) is a real subalgebra of \( \mathbb{C}_2 \), \( I_{p,q} \) is the minimal left ideal of the algebra \( \mathcal{A}_{p,q} \), \( e_{pq} \) is a primitive idempotent.

Further, let \( \mathbb{C}_2 \) be the biquaternion algebra, in which all the coefficients are complex conjugate to the coefficients of the algebra \( \mathbb{C}_2 \). The algebra \( \mathbb{C}_2 \) is obtained from \( \mathbb{C}_2 \) under action of the automorphism \( \mathcal{A} \to \mathcal{A}^* \) (involution), or the antiautomorphism \( \mathcal{A} \to \tilde{\mathcal{A}} \) (reversal), where \( \mathcal{A} \in \mathbb{C}_2 \) (see [27, 28, 29]). Let us compose a tensor product of \( k \) algebras \( \mathbb{C}_2 \) and \( r \) algebras \( \mathbb{C}_2 \):

\[
\mathbb{C}_2 \otimes \mathbb{C}_2 \otimes \cdots \otimes \mathbb{C}_2 \otimes \mathbb{C}_2 \otimes \cdots \otimes \mathbb{C}_2 \simeq \mathbb{C}_{2k} \otimes \mathbb{C}_{2r},
\]
The tensor product (7) induces a spinspace

\[ S_2 \otimes S_2 \otimes \cdots \otimes S_2 \otimes \hat{S}_2 \otimes \cdots \otimes \hat{S}_2 = S_{2k+r} \]  

(8)

with ‘vectors’ (spintensors) of the form

\[ \xi_{\alpha_1} \otimes \xi_{\alpha_2} \otimes \cdots \otimes \xi_{\alpha_k} \otimes \hat{\xi}_{\hat{\alpha}_1} \otimes \hat{\xi}_{\hat{\alpha}_2} \otimes \cdots \otimes \hat{\xi}_{\hat{\alpha}_r}. \]  

(9)

The full representation space \( S_{2k+r} \) contains both symmetric and antisymmetric spintensors \( \text{Sym}(k, r) \). Usually, at the definition of irreducible finite-dimensional representations of the Lorentz group physicists confined to a subspace of symmetric spintensors \( \text{Sym}(k, r) \subset S_{2k+r} \). Dimension of \( \text{Sym}(k, r) \) is equal to \( (k+1)(r+1) \) or \( (2l+1)(2l'+1) \) at \( k = \frac{l}{2}, \ l' = \frac{r}{2} \). The space \( \text{Sym}(k, r) \) can be considered as a space of polynomials

\[ p(z_0, z_1, \bar{z}_0, \bar{z}_1) = \sum_{(\alpha_1, \ldots, \alpha_k) \atop (\hat{\alpha}_1, \ldots, \hat{\alpha}_r)} \frac{1}{k! r!} a^{\alpha_1 \ldots \alpha_k \hat{\alpha}_1 \ldots \hat{\alpha}_r} z_{\alpha_1} \cdots z_{\alpha_k} \bar{z}_{\hat{\alpha}_1} \cdots \bar{z}_{\hat{\alpha}_r} \]  

(10)

where the numbers \( a^{\alpha_1 \ldots \alpha_k \hat{\alpha}_1 \ldots \hat{\alpha}_r} \) are unaffected at the permutations of indices. Some applications of the functions (10) contained in [35, 13].

In accordance with the Wigner-Weinberg scheme all the physical fields are described within \( \tau_{ii} \)-representations of the proper Lorentz group \( \mathfrak{g}_+ = SL(2, \mathbb{C})/\mathbb{Z}_2 \) or within the proper Poincaré group \( SL(2, \mathbb{C}) \otimes T_4/\mathbb{Z}_2 \), where \( T_4 \) is a subgroup of four-dimensional translations (since \( T_4 \) is an Abelian group, then all its representations are one-dimensional). At this point, any representation \( \tau_{ii} \) can be written as \( \tau_{i_0} \otimes \tau_{i_0} \), where \( \tau_{i_0} \) (\( \tau_{i_0} \)) are representations of the group \( SU(2) \). In a sense, it allows us to represent the group \( SL(2, \mathbb{C}) \) by a product \( SU(2) \otimes SU(2) \) as it done by Ryder in his textbook [25]. Moreover, in the works [1, 6] the Lorentz group is represented by a product \( SU_R(2) \otimes SU_L(2) \), where the spinors \( \psi(p^\mu) = \begin{pmatrix} \phi_R(p^\mu) \\ \phi_L(p^\mu) \end{pmatrix} \) (\( \phi_R(p^\mu) \) and \( \phi_L(p^\mu) \) are the right- and left-handed spinors) are transformed within \( (j, 0) \oplus (0, j) \) representation space, in our case \( j = l = \frac{l}{2} \). All these representations for \( SL(2, \mathbb{C}) \) follow from the Van der Waerden representation of the Lorentz group which was firstly given in the brilliant book [38] and further widely accepted by many authors [2, 26, 24, 25]. In [38] the
group $SL(2, \mathbb{C})$ is understood as a complexification of the group $SU(2)$, $SL(2, \mathbb{C}) \sim \text{complex}(SU(2))$ (see also [34]). Further, in accordance with the Wigner-Weinberg scheme a helicity $\lambda$ of the particle, described by the representations $\tau_{ll}$, is defined by an expression $l - \dot{l} = \lambda$ (Weinberg Theorem [39]). However, the Wigner-Mackey-Weinberg scheme does not incorporate the Clifford algebraic framework of the Lorentz group. On the other hand, there is a more powerful method based on a generalized regular representation [36, 37] (GGR-theory) which naturally incorporates Clifford algebraic description [34] and includes the Wigner-Weinberg approach as a particular case.

As is known, basic equations of quantum electrodynamics in any formulation are the Maxwell equations

\[ \begin{align*}
\text{div } & E = 4\pi \rho, \\
\text{div } & B = 0, \\
\text{rot } & B = \frac{4\pi}{c} j + \frac{1}{c} \frac{\partial E}{\partial t}, \\
\text{rot } & E = -\frac{1}{c} \frac{\partial B}{\partial t},
\end{align*} \tag{11} \]

and the Dirac equations

\[ \left( i \gamma_{\mu} \frac{\partial}{\partial x_{\mu}} - m \right) \psi(x) = 0, \tag{12} \]

where $\gamma$-matrices in a canonical basis have the form

\[ \gamma_0 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & \sigma_0 \end{pmatrix}, \quad \gamma_1 = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}, \quad \gamma_3 = \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix}. \]

It is well known that Maxwell and Dirac equations can be rewritten in a spinor form (see [16, 23, 24]). In this form these equations look very similar. Indeed, the spinor form of the equations (11) and (12) is

\[ \begin{align*}
\partial_{\lambda}^\rho f^\lambda_{\rho} & = 0, \\
\partial_{\lambda} f^\lambda_{\rho} & = s^\mu_{\rho}; \\
\partial_{\lambda}^\rho \eta_{\mu} + im \xi^\lambda & = 0, \\
\partial_{\lambda} \xi^\lambda + im \eta_{\mu} & = 0.
\end{align*} \]

Moreover, in the vacuum ($s^\mu_{\rho} = 0$) and for the massless field ($m = 0$) these
equations take the form
\[
\begin{align*}
\partial_{11} f_{11} + \partial_{12} f_{12} &= 0, \\
\partial_{21} f_{11} + \partial_{22} f_{12} &= 0, \\
\partial_{11} f^{11} + \partial_{21} f^{12} &= 0, \\
\partial_{12} f^{11} + \partial_{21} f^{12} &= 0.
\end{align*}
\]

The latter equations formally coincide if we suppose \( \xi^\lambda = (f_{11}, f_{12})^T \), \( \eta_\mu = (f^{11}, f^{12})^T \) as it done by Rumer in [23]. In spite of this similarity there is a deep difference between Dirac and Maxwell equations. Namely, spinors \( \xi^\lambda \) and \( \eta_\mu \) are transformed within \( \tau_{1,0} \) and \( \tau_{0,1} \) representations, whereas the spintensors \( f^{\lambda\mu} \) and \( f^\lambda_{\mu} \) are transformed within \( \tau_{1,0} \) and \( \tau_{0,1} \) representations of the Lorentz group. Indeed, the Dirac electron-positron field \( (1/2, 0) \oplus (0, 1/2) \) corresponds to the algebra \( \mathbb{C}_2 \oplus \mathbb{C}_2 \). It should be noted that the Dirac algebra \( \mathbb{C}_4 \), considered as a tensor product \( \mathbb{C}_2 \otimes \mathbb{C}_2 \) (or \( \mathbb{C}_2 \otimes \mathbb{C}_2^* \)), leads to spintensors \( \xi^{\alpha\alpha_2} \) (or \( \xi^{\alpha_1\alpha_1} \)), but it contradicts with the usual definition of the Dirac bispinor as a pair \((\xi^1, \xi^2)\). Therefore, the Clifford algebra associated with the Dirac field is \( \mathbb{C}_2 \oplus \mathbb{C}_2 \), and a spinspace of this sum in virtue of unique decomposition \( S_2 \oplus \bar{S}_2 = S_4 \) (where \( S_4 \) is a spinspace of \( \mathbb{C}_4 \)) allows one to define \( \gamma \)-matrices in the Weyl basis.

In contrast to the Dirac field \((1/2, 0) \oplus (0, 1/2)\), the Maxwell field is defined in terms of spintensors of the second rank. According to (7), the Clifford algebras, corresponded to the Maxwell fields, are \( \mathbb{C}_2 \otimes \mathbb{C}_2 \) and \( \mathbb{C}_2 \otimes \mathbb{C}_2^* \). The algebras \( \mathbb{C}_2 \otimes \mathbb{C}_2 \) and \( \mathbb{C}_2 \otimes \mathbb{C}_2^* \) induce spinspaces \( S_2 \otimes \bar{S}_2 \) and \( S_2 \oplus \bar{S}_2 \).

1On the other hand, these algebras can be obtained by means of an homomorphic mapping \( \epsilon : \mathbb{C}_5 \rightarrow \mathbb{C}_4 \) (see [32]). Indeed, for the algebra \( \mathbb{C}_5 \) we have a decomposition

\[
\lambda_+ \cup \lambda_- \quad \text{with} \quad \lambda_+ = \frac{1}{2}(e_1 e_2 e_3 e_4 e_5), \quad \lambda_- = \frac{1}{2}(e_1 e_2 e_3 e_4 e_5),
\]

where the central idempotents \( \lambda_+ = \frac{1}{2}e_1 e_2 e_3 e_4 e_5 \), \( \lambda_- = \frac{1}{2}e_1 e_2 e_3 e_4 e_5 \) correspond to the helicity projection operators of the Maxwell field. As is known, for the photon there are two helicity states: left and right handed polarizations. Hence it follows that in
\( \hat{S}_2 \otimes \hat{S}_2 \). These spinspaces are full representation spaces for tensor products \( \tau_{\frac{1}{2},0} \otimes \tau_{\frac{1}{2},0} \) and \( \tau_{0,\frac{1}{2}} \otimes \tau_{0,\frac{1}{2}} \). In accordance with (9) the basis ‘vectors’ (spintensors) of the spinspaces \( S_2 \otimes S_2 \) and \( \hat{S}_2 \otimes \hat{S}_2 \) have the form

\[
\begin{align*}
\xi_{11} &= \xi^1 \otimes \xi^1, \\
\xi_{12} &= \xi^1 \otimes \xi^2, \\
\xi_{21} &= \xi^2 \otimes \xi^1, \\
\xi_{22} &= \xi^2 \otimes \xi^2,
\end{align*}
\]

(13)

Further, the representations \( \tau_{\frac{1}{2},0} \otimes \tau_{\frac{1}{2},0} \) and \( \tau_{0,\frac{1}{2}} \otimes \tau_{0,\frac{1}{2}} \) are reducible. In virtue of the Clebsh–Gordan formula

\[
\tau_{\frac{1}{2},1} \otimes \tau_{\frac{1}{2},\hat{1}} = \sum_{|l_1-l_2| \leq k \leq l_1+l_2} \tau_{l_1,l_2}
\]

we have

\[
\begin{align*}
\tau_{\frac{1}{2},0} \otimes \tau_{\frac{1}{2},0} &= \tau_{0,0} \oplus \tau_{1,0}, \\
\tau_{0,\frac{1}{2}} \otimes \tau_{0,\frac{1}{2}} &= \tau_{0,0} \oplus \tau_{0,1}. 
\end{align*}
\]

At this point, the spinspaces \( S_2 \otimes S_2 \) and \( \hat{S}_2 \otimes \hat{S}_2 \) decompose into direct sums of the symmetric representation spaces:

\[
\begin{align*}
S_2 \otimes S_2 &= \text{Sym}_{(2,0)} \oplus \text{Sym}_{(0,2)}, \\
\hat{S}_2 \otimes \hat{S}_2 &= \text{Sym}_{(0,0)} \oplus \text{Sym}_{(2,2)}. 
\end{align*}
\]

The spintensors \( \xi_{11}, \xi_{12} = \xi_{21}, \xi_{22} \) and \( \xi_{11}, \xi_{12} = \xi_{21}, \xi_{22} \), obtained after symmetrization from (13) compose the bases of three-dimensional complex spaces \( \text{Sym}_{(2,0)} \) and \( \text{Sym}_{(0,2)} \), respectively. Let us introduce independent complex coordinates \( F_1, F_2, F_3 \) and \( \hat{F}_1, \hat{F}_2, \hat{F}_3 \) for the spintensors \( f^{\lambda\mu} \) and \( f^{\lambda\hat{\mu}} \) (spiner representations of the electromagnetic tensor), where \( F_i = E_i - iB_i \) and \( \hat{F}_i = E_i + iB_i \). Explicit expressions of the spinor representations

common with other massless fields (such as the neutrino field \( (1/2,0) \cup (0,1/2) \)) the Maxwell electromagnetic field is also described within the quotient representations of the Lorentz group \( \mathfrak{so}(1,3) \). In accordance with Theorem 4 in \( \mathfrak{so}(1,3) \) the photon field can be described by a quotient representation of the class \( \chi^0 \mathfrak{so}_4 \cup \chi^0 \mathfrak{so}_4 \). This representation admits time reversal \( T \) and an identical charge conjugation \( C \sim I \) that corresponds to truly neutral particles (see Theorem 3 in \( \mathfrak{so}(1,3) \)).
of the electromagnetic tensor are

\[
\tau_{1,0} : \quad \begin{cases} 
  f_{11} & \sim 4(F_1 + iF_2), \\
  f_{12} & \sim 4F_3, \\
  f_{22} & \sim 4(F_1 - iF_2); 
\end{cases} \\
\tau_{0,1} : \quad \begin{cases} 
  f_{11} & \sim 4(F_1^* + iF_2^*), \\
  f_{12} & \sim 4F_3^*, \\
  f_{22} & \sim 4(F_1^* - iF_2^*). 
\end{cases}
\]

In such a way, we see that complex linear combinations \( F = E - iB \) and \( \dot{F} = E + iB \), transformed within \( \tau_{1,0} \) and \( \tau_{0,1} \) representations, are coincide with the Majorana-Oppenheimer wave functions (11) and (15). Therefore, the Majorana-Oppenheimer formulation of quantum electrodynamics is a direct consequence of the group theoretical framework of quantum field theory. In virtue of the Weinberg Theorem \[39\] for the Maxwell field \((1,0) \oplus (0,1)\) (or \((1,0) \cup (0,1)\)) we have two helicity states \( \lambda = 1 \) and \( \lambda = -1 \) (left- and right-handed polarizations). In contrast to this, the electromagnetic four-potential \( A_\mu \) is described within \( \tau_{\frac{3}{2}, \frac{1}{2}} \)-representation of the Lorentz group. Therefore, the Gupta–Bleuler quantum electrodynamics, based on the quantization of the non–observable quantity \( A_\mu \) has led to the null helicity, what, as is known, contradicts with experience. For that reason many authors (see, for example, \[24\]) considered the Gupta-Bleuler quantization of the electromagnetic field in terms of \( A_\mu \) as a phenomenological description. Moreover, such a description is not incorporated properly into a group theoretical scheme of quantized fields.

In 1932, Majorana proposed the first construction of a relativistically invariant theory of arbitrary half integer or integer spin particles \[18\]. As is known \[22\], the photon case is an initial point of this construction. There is a close relationship between Majorana’s construction for the photon and other higher spin formalisms proposed well after. For example, Weinberg considered the following \((1,0) \oplus (0,1)\)-field equations \[39\]:

\[
\nabla \times [E - iB] + i(\partial / \partial t)[E - iB] = 0, \\
\nabla \times [E + iB] - i(\partial / \partial t)[E + iB] = 0,
\]

\[\text{It should be noted here that at all times in electrodynamics the four-potential } A_\mu \text{ is understood as an auxiliary mathematical tool.}\]
which, as it easy to see, are equivalent to the equations \(13\) and \(16\). At present time the Majorana-Oppenheimer formulation of quantum electrodynamics is studied in terms of the Joos-Weinberg formalism (with respect to Hammer-Tucker and Proca equations) and the Bargmann-Wigner theory \[8\].

One of the most powerful higher spin formalisms is a Gel’fand-Yaglom approach \[10\] based primarily on the representation theory of the Lorentz group. In contrast to the Bargmann-Wigner and Joos-Weinberg formalisms, the main advantage of the Gel’fand-Yaglom formalism lies in the fact that it admits naturally a Lagrangian formulation. Indeed, an initial point of this theory is the following lagrangian \[10, 11, 2\]:

\[
L = - \frac{1}{2} \left( \bar{\psi} \Gamma_\mu \frac{\partial \psi}{\partial x_\mu} - \frac{\partial \bar{\psi}}{\partial x_\mu} \Gamma_\mu \psi \right) - \kappa \bar{\psi} \psi, \tag{14}
\]

where \(\Gamma_\mu\) are \(n\)-dimensional matrices, \(n\) equals to the number of components of the wave function \(\psi\). Varying independently the functions \(\psi\) and \(\bar{\psi}\), one gets general Dirac-like (Gel’fand-Yaglom \[10\]) equations

\[
\Gamma_\mu \frac{\partial \psi}{\partial x_\mu} + \kappa \psi = 0, \quad \Gamma^\top_\mu \frac{\partial \bar{\psi}}{\partial x_\mu} - \kappa \bar{\psi} = 0.
\]

Let us consider a massless case of \(14\):

\[
L_M = - \frac{1}{2} \left( \psi^\ast \alpha_\mu \frac{\partial \psi}{\partial x_\mu} - \frac{\partial \psi^\ast}{\partial x_\mu} \alpha_\mu \psi \right), \tag{15}
\]

where \(\psi = E - iB, \psi^\ast = E + iB\), and \(\alpha_\mu (\mu = 0, 1, 2, 3)\) are the matrices \(4\), at this point \(\alpha_0 = W\) is a unit matrix. Varying independently the functions \(\psi\) and \(\psi^\ast\) in \(15\), we obtain

\[
\frac{\partial L}{\partial \psi^\ast} = - \frac{1}{2} \alpha_\mu \frac{\partial \psi}{\partial x_\mu}, \quad \frac{\partial L}{\partial \psi^\ast_{\mu}} = \frac{1}{2} \alpha_\mu \psi; \tag{16}
\]

\[
\frac{\partial L}{\partial \psi} = \frac{1}{2} \alpha_\mu \frac{\partial \psi^\ast}{\partial x_\mu}, \quad \frac{\partial L}{\partial \psi_{\mu}} = - \frac{1}{2} \psi \alpha_\mu. \tag{17}
\]
Substituting the relations (16) and (17) into the Euler equation
\[
\frac{\partial L}{\partial u_i} - \frac{\partial}{\partial x} \frac{\partial L}{\partial u_i} = 0,
\]
we come to equations
\[
\alpha_\mu \frac{\partial \psi}{\partial x_\mu} = 0, \\
\alpha_T \frac{\partial \psi^*}{\partial x_\mu} = 0
\]
which, obviously, are equivalent to the equations (3) and (6). At this point, we have \(j_\mu \neq 0\) (so-called neutral current), and the ‘charge’ \(Q = \int d^3x j_0(x) = \int d^3x \psi^* \alpha_0 \psi\) is proportional to the energy \(E^2 + B^2\) of electromagnetic field.

In conclusion it should be noted that Majorana-Oppenheimer formulation is an antithesis to the widely accepted gauge paradigm based on the so-called Standard Model, in which all the physical fields are divided into the ‘gauge’ and ‘matter’ fields. One of the main preferences of the Majorana-Oppenheimer quantum electrodynamics lies in the fact that it allows one to avoid this opposition and to consider all the fields in equal footing. Moreover, the Majorana-Oppenheimer formulation based naturally on the ground of group theoretical methods which include a wide variety of powerful mathematical tools.

References

[1] D. V. Ahluwalia, D. J. Ernst, Int. J. Mod. Phys. E 2, 397, (1993).

[2] A. I. Akhiezer, V. B. Berestetskiï, Quantum Electrodynamics, New York, John Wiley & Sons, 1965.

[3] K. Bleuler, Helv. Phys. Acta 23, 567, (1950).

[4] N. N. Bogoliubov, D. V. Shirkov, Quantum Fields, Moskow, Nauka, 1993.

[5] L. de Broglie, Compt. Rend. 195, 862, (1932).

[6] V. V. Dvoeglazov, Nuovo Cimento B 111, 483, (1996).
[7] V. V. Dvoeglazov, *The Weinberg Formalism and a New Look at the Electromagnetic Theory*, in The Enigmatic Photon. Vol. IV (Eds.: M. Evans, J.-P. Vigier, S. Roy and G. Hunter) p. 305–353, Series Fundamental Theories of Physics. Vol. 90 (Ed. A. van der Merwe) Dordrecht, Kluwer Academic Publishers 1997.

[8] V. V. Dvoeglazov, Hadronic J. Suppl. **12**, 241, (1997).

[9] S. Esposito, Found. Phys. **28**, 231, (1998).

[10] I. M. Gel’fand, A. M. Yaglom, Zh. Ehksp. Teor. Fiz. **18**, 703, (1948).

[11] I. M. Gel’fand, R. A. Minlos, Z. Ya. Shapiro, *Representations of the Rotation and Lorentz Groups and their Applications*, Oxford, Pergamon Press, 1963.

[12] E. Giannetto, Lettere al Nuovo Cimento **44**, 140, (1985).

[13] D. M. Gitman, A. L. Shelepin, Int. J. Theor. Phys. **40**, 603, (2001).

[14] S. N. Gupta, Proc. Phys. Soc. A **63**, 681, (1950).

[15] P. Jordan, Z. Phys. **93**, 434, (1935).

[16] O. Laport, G. E. Uhlenbeck, Phys. Rev. **37**, 1380, (1931).

[17] E. Majorana, *Scientific Papers*, unpublished, deposited at the “Domus Galileana”, Pisa, quaderno 2, p.101/1; 3, p.11, 160; 15, p.16; 17, p.83, 159.

[18] E. Majorana, Nuovo Cimento, **9**, 335, (1932).

[19] R. Mignani, E. Recami, M. Baldo, Lettere al Nuovo Cimento **11**, 568, (1974).

[20] M. A. Naimark, *Linear Representations of the Lorentz Group*, London, Pergamon, 1964.

[21] J. R. Oppenheimer, Phys. Rev. **38**, 725, (1931).

[22] E. Recami, *Possible Physical Meaning of the Photon Wave-Function, According to Ettore Majorana*, in Hadronic Mechanics and Non-Potential Interactions (Nova Sc. Pub., New York, 1990), pp. 231–238.

[23] Yu. B. Rumer, *Spinorial Analysis*, Moscow, 1936, [in Russian].

[24] Yu. B. Rumer, A. I. Fet, *Group Theory and Quantized Fields*, Moscow, Nauka, 1977, [in Russian].
[25] L. Ryder, *Quantum Field Theory*, Cambridge, Cambridge University Press, 1985.

[26] S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory*, New York, Harper & Row, 1961.

[27] V. V. Varlamov, Hadronic J. **22**, 497, (1999).

[28] V. V. Varlamov, Int. J. Theor. Phys. **40**, 769, (2001).

[29] V. V. Varlamov, *Clifford Algebras and Discrete Transformations of Spacetime*, Proceedings of Third Siberian Conference on Mathematical Problems of Spacetime Physics, Novosibirsk, 20–22 June 2000. (Institute of Mathematics Publ., Novosibirsk, pp. 97–135, 2001).

[30] V. V. Varlamov, Mathematical Structures and Modelling **7**, 114, (2001).

[31] V. V. Varlamov, *Clifford Algebras and Lorentz Group*, arXiv:math-ph/0108022 (2001).

[32] V. V. Varlamov, Annales de la Fondation de Louis de Broglie **27**, 273, (2002).

[33] V. V. Varlamov, *Group Theoretical Description of Space Inversion, Time Reversal and Charge Conjugation*, arXiv:math-ph/0203059 (2002).

[34] V. V. Varlamov, Hadronic J. **25**, 481, (2002).

[35] M. A. Vasiliev, Int. J. Mod. Phys. D**5**, 763, (1996).

[36] N. Ya. Vilenkin, *Special Functions and the Theory of Group Representations*, Providence, AMS, 1968.

[37] N. Ya. Vilenkin, A. U. Klimyk, *Representations of Lie Groups and Special Functions*, vols. 1–3, Dordrecht, Kluwer Acad. Publ., 1991–1993.

[38] B. L. van der Waerden, *Die Gruppentheoretische Methode in der Quantenmechanik*, Berlin, Springer, 1932.

[39] S. Weinberg, Phys. Rev. **133B**, 1318–1332 & **134B**, 882–896 (1964) & **181B**, 1893–1899 (1969).

[40] E. P. Wigner, Ann. Math. **40**, 149, (1939).