A Linear Narrowband Filtering Method Based on Migration-Enhanced Singular Value Decomposition and Its Application to Reduce Noise

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ABSTRACT With the development of the aviation industry, the limit of aeroengine noise is becoming increasingly stringent. Vibration mode extraction is the most commonly used method for reducing the noise of compressor, which is the main source of aeroengine noise. In vibration mode extraction, inherent errors occur due to the inherent defects in the measurement and analysis. To solve the problem, the acoustic pressure signal, which is among the most common and accurately measured signals and contains much useful information, is selected for analysis to eliminate the inherent errors. In addition, to compensate for the lack of analysis methods, based on the characteristics of the singular value decomposition (SVD) and acoustic pressure signals, a linear narrowband filtering method based on migration-enhanced singular value decomposition (LNF-MSVD) is proposed to accurately extract the acoustic mode from acoustic pressure signal to reduce the noise of a high-pressure-ratio axial flow compressor. The steps of LNF-MSVD are listed after determining the method’s feasibility and effectiveness, as well as the amplitude and frequency standards of the excitation signal. Using this method, the acoustic mode was extracted accurately and clearly shown using an acoustic modal diagram, which was used to acquire experimental acoustic modal order. The superiority of LNF-MSVD for extracting acoustic mode was confirmed by a comparison with SVD and wavelet analyses. By comparing experimental acoustic modal order with theoretical acoustic modal order calculated through an empirical formula, the likely source of noise of the compressor was successfully located.

INDEX TERMS Acoustic mode, high-pressure-ratio axial flow compressor, migration-enhanced singular value decomposition, noise source location.

I. INTRODUCTION With the development of technology and society in recent decades, noise limitation standards for aircraft engines have become increasingly stringent [1]. According to the core idea, methods for the noise reduction of aeroengines can be divided into four categories, which are: spectrum analysis, power spectrum analysis, coherence analysis, and acoustic holography [2]–[6]. Among the four categories of methods, three kinds of characteristic signals are used for noise reduction analysis: mechanical vibration signals, acoustic pressure signals, and acoustic power signals [7]–[9]. Subdivision methods for noise reduction based on vibration signals include time – frequency characteristic analysis, finite element analysis, dynamic modeling, and modal analysis [10]–[15]. These methods are widely used to reduce the noise of aeroengine blades, combustion chambers, intake and exhaust systems, drive systems, and casing components [16]–[21]. In the process of reducing the noise of a high-pressure-ratio axial flow compressor, structural vibration analysis forms a complete theoretical system that provides efficient and feasible guidance for the optimization and
improvement of the compressor [22]. However, most compressor vibration systems are complex, their vibration signals are disturbed considerably, and the installation of measurement points is restricted at the intake casing, the intermediate casing, and the turbine’s rear casing. In addition, methods for vibration analysis are complex and difficult to be used to analyze the vibration signals accurately. These problems pose significant obstacles for the accurate measurement of the vibration signals and hinder the accurate analysis of the vibration signals. There are always some systematic errors in the results of structural vibration analysis [23], [24]. To solve the problem, it is necessary to improve the accuracy and analysis method of signals. On the one hand, to improve the accuracy of the signals needed for analysis, acoustic pressure signals that are easy to accurately measure and contain much useful information have attracted considerable attention [25]. On the other hand, to improve the signal analysis method, LNF-MSVD is proposed by combining the traditional SVD method, signal resonance, and the characteristics of acoustic pressure signals.

SVD has a wide range of applications, on the basis of which many data-driven mode composition techniques have been developed. SVD has difficulty in isolating specific harmonic components of the resulting modes. To solve this problem, Nobach proposed proper orthogonal decomposition (POD) based on SVD [26]. Then, Mendez proposed multi-scale proper orthogonal decomposition (mPOD) by combining multi-resolution analysis (MRA) with standard POD. mPOD can split the correlation matrix into the contributions of different scales, retaining non-overlapping portions of the correlation spectra [27]. However, POD is not suitable when the relevant coherent structures occur at low energies or at multiple frequencies. To improve the analysis effect of POD, Sieber presented spectral proper orthogonal decomposition (SPOD) by improving POD [28]. The origin of dynamic mode decomposition (DMD) is similar to the origin of POD. SVD has difficulty in isolating modes of a single frequency. Schmid presented dynamic mode decomposition (DMD) by improving SVD, and the aim of isolating modes of a single frequency was accomplished [29]. Broatch successfully solved the problem that the decomposition effect of the acoustic pressure signal of a centrifugal compressor was not ideal using DMD [30]. Semlitsch successfully separated the flow characteristic signal of a centrifugal compressor during surging using DMD [31]. Because DMD is incapable of producing an input-output model, Proctor proposed a DMD method with control (DMDc) by adding control conditions in DMD. An input-output model was successfully produced [32].

SVD can decompose complex signals into a series of component signals, where each signal corresponds to a singular value. The singular values are arranged from large to small according to the magnitude of component signals. The larger the amplitude of component signal, the larger the corresponding singular value. Some major component signals with larger amplitudes are easily obtained. However, determining the corresponding singular value of the weak target signal is a difficult problem. Inspired by the development process of POD, DMD, sPOD, mPOD, and DMDc, LNF-MSVD is proposed in this paper to solve this problem. LNF-MSVD originated from a combination of the traditional SVD method, signal resonance, and the characteristics of acoustic pressure signals. By calculating example signals and comparing the results, the feasibility and effect of this method were verified, and criteria for the amplitude and frequency of the excitation signal were proposed. The detailed implementation of this method is also shown.

In the field of aeroacoustics, which is closely related to compressor noise, many pioneers have done a significant amount of research, which lays a solid theoretical and application foundation for aeroacoustics. The most core breakthrough achievements follow. In 1962, Tyler and Sofrin first systematically studied the generation mechanism and influencing factors of the aeroengine gas path acoustic mode through the analysis of the turbine noise and proposed the rotating stator interference theory [33]. To further explore the main aerodynamic noise sources, Holste and Neise studied the noise generation mechanism of the interference between two rows of rotors and further improved the theoretical basis of the acoustic mode of the aeroengine [34]. In the measurement of the actual acoustic pressure signal, Wang achieved the accurate measurement of the acoustic pressure signal in a fan duct by installing a microphone array with a uniform circumferential distribution on the wall of the axial fan duct [25]. Liang proposed a measurement and analysis method for an acoustic pressure signal with a non-uniform distribution of sensors and solved the problem of the analysis of the acoustic pressure signal with a non-uniform distribution of measuring points [35]. Based on the above research results, the acoustic pressure signal has been applied in engineering practice. However, because of the restriction of signal processing method, the application of the acoustic pressure signal is limited to the frequency solution or modal parameter calculation of the acoustic mode, and the acoustic mode cannot be displayed to extract the acoustic mode characteristics. In addition, there are some restrictions of extraction method on the effective extraction of the target acoustic pressure signal [36], [37]. If the target acoustic pressure signals can be extracted efficiently and the corresponding acoustic modes can be displayed simply with diagrams, the characteristic information of compressor noise source can be easily obtained from the diagrams. With the characteristic information, the effect of the noise reduction of the compressor can be significantly improved using a more convenient method, which would further improve the compressor performance. Now, LNF-MSVD, as proposed in this paper, can extract the target acoustic pressure signals efficiently and the acoustic mode can be displayed with the proposed acoustic modal diagram. This goal of improving the effect of the noise reduction of the compressor can be effectively achieved.
In the process of noise reduction, LNF-MSVD was used to extract the target acoustic pressure signal. The acoustic modes are comprehensively and concisely presented using acoustic modal diagrams. After constructing the acoustic modal diagrams, the superiority of this method in extracting acoustic modes was confirmed. Based on the theory of acoustic modal analysis proposed by Tyler and Sofrin [33], the theoretical order of the compressor’s stator and rotor interference acoustic mode was calculated using an empirically derived formula. The experimental acoustic modal order can be obtained from the acoustic modal diagram and compared with the theoretical order to reduce the noise due to the interactions of the compressor’s rotor, stator, and intake support plate.

The rest of this paper is organized as follows. In Section II, a linear narrowband filtering method based on migration-enhanced singular value decomposition (LNF-MSVD) is proposed. The feasibility and effectiveness of the method are examined, and the amplitude and frequency standards of the excitation signals are determined. The extraction and display of the acoustic modes are provided in Section III. The superiority of LNF-MSVD in extracting acoustic modes is confirmed in Section IV. In Section V, analysis of the noise reduction of a compressor is presented. Finally, the conclusions are given in Section VI. The flow of proposing LNF-MSVD and using it to improve the noise reduction effect is expressed in Fig. 1.

II. LINEAR NARROWBAND FILTERING METHOD BASED ON MIGRATION-ENHANCED SINGULAR VALUE DECOMPOSITION (LNF-MSVD)

SVD, as a mathematical method, has been introduced in signal processing [38], [39]. It has been combined with other processing methods, such as wavelet analysis and empirical mode decomposition (EMD), to improve the effect of signal processing [40], [41].

A. MIGRATION-ENHANCED SINGULAR VALUE DECOMPOSITION (MSVD)

SVD has three distinct characteristics: linear decomposition, frequency-domain disorder of the reconstructed components, and band-pass filtering [42]. The results of the decomposition appear in the order of the amplitude of the component signal, which is independent of its own frequency. If the amplitude of the target component signal is similar to or less than those of the other component signals, traditional SVD cannot extract it effectively because the corresponding singular value is difficult to find. By using the linear decomposition characteristic of SVD, an excitation signal of the same or similar frequency as that of the target signal can be added to the original signal to implement the migration enhancement of the target signal to maximize its amplitude. In this way, the target signal after migration enhancement can be extracted by the first singular value reconstruction, and the target signal that preserves the
amplitude and phase can then be obtained by subtracting the excitation signal.

1) FEASIBILITY OF METHOD AND EVALUATION OF ITS EFFECT

Is the proposed target signal migration enhancement extraction method feasible? If it is, what is the effect of the amplitude of the signal on extraction? What is the effect when the frequency of the excitation signal deviates from that of the target signal? To answer these questions, the signal $S_1(t) = \sin(2\pi f_1 t) + \sin(160\pi t)$ was selected, and the $\sin(160\pi t)$ component in $S_1(t)$ was extracted by the excitation signal $S_2(t) = A_2 \times \sin(2\pi f_2 t)$. The difference between the extracted result and the target signal was defined as $S$ (reconstructive signal difference) and was used to evaluate the effect of the extraction. $S$ is expressed as follows:

$$S = \frac{1}{n} \sqrt{\frac{\sum_{i=1}^{n} (X_i - X_i^1)^2}{\sum_{i=1}^{n} X_i^2}} \times 100\% \quad (1)$$

where $n$ is the number of sampled data items, $X_i$ is the original signal, and $X_i^1$ is the signal extracted by the proposed method.

While maintaining the frequency $f_2 = 80$, the amplitude of the excitation signal ($A_2$) changes constantly, and $S$ is calculated as shown in Fig. 2(a). To consider all conditions (the amplitude of the excitation signal is not restricted to a fixed level), we kept $A_2$ constant at a certain value and varied $f_2$. We then kept $A_2$ constant at another value and varied $f_2$. $S$ was subsequently calculated, and this process was repeated for many cycles, as shown in Fig. 2(b).

The purpose of normalization is to facilitate the comparison of the numerical size relationship between multi-dimensional data and to narrow the range of the data representation [43]. In Fig. 2(b), only the numerical value of reconstructive signal difference should be compared. The reconstructive signal difference has dimensionless value, so its numerical value is easy to compare. In addition, the data range of the reconstructive signal difference was not large. For these two reasons, the reconstructive signal difference does not require a normalized representation. If the reconstructive signal difference is normalized, the data will be distorted. The frequency change only represents the change of the experimental condition. There is no need to compare the numerical value of each frequency, and the selected frequency range was not very large. Thus, the frequency did not need to be normalized. In addition, if the frequency is normalized, the corresponding points of each frequency will be unevenly distributed on the horizontal axis, which will affect the description of the reconstruction signal difference trend (the reasons the frequency is not normalized in Figs. 6 and 7 are the same).

As shown in Figs. 2(a) and (b), as long as the frequency of the excitation signal approached that of the target signal, the effect of extraction could be improved with a low excitation energy, which shows that the excitation signal was relatively easy to construct and that this method is feasible.

The advantage of MSVD over SVD is that it can improve the extraction accuracy of the target signal from other signals with small amplitude differences. If the amplitude of the target signal is different from those of the other signals, pure SVD can extract the target signal with high accuracy. Therefore, the comparison of MSVD and SVD is limited to cases where the amplitude of the target signal is close to those of the other signals.

For the signal $S_1(t)$, traditional SVD and MSVD were used to extract the component signal $\sin(160\pi t)$. The extracted signal was compared with the original component signal. The results are shown in Fig. 3.

By comparing Figs. 3(a) and (b), it is clear that MSVD can reduce the distortion rate of the extracted signal to extract a high-fidelity target signal, which shows that it can greatly improve the extraction of the signal.

2) AMPLITUDE STANDARD OF EXCITATION SIGNAL

For MSVD, is there a clear criterion for the amplitude of the excitation signal? To answer this question, the original signal $S_3(t) = \text{wgn}(m, n, p) + \sin(6t)$ was used, and the $\sin(6t)$ component of $S_3(t)$ was extracted by using the excitation signal $S_4(t) = A_4 \times \sin(6t)$. By constantly changing the amplitude $p$ of the noise signal $\text{wgn}(m, n, p)$, how the signal-to-noise...
ratio (SNR) of the extracted signal changes with the change in the amplitude of the excitation signal under different SNRs is examined in this section, and the ideal range of the amplitude of the added excitation signal is determined.

Signals with SNRs of $-20$, $-10$, and 0 were analyzed. The spectra of the original signals in the three cases are as follows:

In the above three cases, the target signal was extracted using excitation signals of different amplitudes (the amplitude of $S_4(t)$ was constantly changing), and the SNR of the extracted signal was calculated. The relationship between the amplitude of the excitation signal and the SNR of the extracted signal was thus obtained.

Fig. 5 shows that with a continuous increase in the excitation signal amplitude, the SNR of the extracted signal significantly increased and tended to be finally stable. Thus, MSVD could effectively improve the extraction effect. The weaker the target signal energy was (the smaller the SNR was), the larger the amplitude of the excitation signal was that was needed to extract a target signal with high fidelity (the SNR of the extracted signal became finally stable).

To ensure that the amplitude of the target signal reaches its maximum value after migration enhancement, the amplitude of excitation signal should be constructed to be equal to the component of the maximum amplitude from the results of the spectrum analysis, so that the target component has a maximum relative energy and corresponds to the first singular value in the singular value sequence.

3) FREQUENCY STANDARD OF EXCITATION SIGNAL

In engineering practice, due to errors or signal fluctuations, the excitation and target signals cannot maintain the same frequency at all times. Setting a reasonable frequency standard for the excitation signal can maintain the quality of the extracted target signal in a real engineering environment.

To determine the appropriate frequency standard, the signal $S_5(t) = \sin(160\pi t) + 3\sin(60\pi t)$ was selected, and the excitation signal $S_6(t) = A_6\sin(2\pi f_6 t)$ was constructed to extract the target signal $\sin(160\pi t)$ in $S_5(t)$. The sampling frequency was 2048 Hz, the number of samples was 2048, the frequency resolution was 1 Hz, and the frequency of the target signal was 80 Hz. According to the result of Section IIB 2), the amplitude of the excitation signal ($A_6$) should be 3. For the case of $A_6 = 3$, the values of $f_6$ were constantly changing around 80 Hz. In this case, $S$ was calculated. The result is as follows:
Fig. 6 shows that when the amplitude of the excitation signal met the requirement \( A_6 = 3 \), the excitation frequency that could exhibit a good extraction effect was in the 79.5–80.5 Hz range. Beyond this frequency band, the larger the difference was between the excitation frequency and the frequency of the target signal, the larger the reconstructive signal difference.

The above results can be explained using the Fourier transform resolution of signals. When the frequency resolution was 1 Hz, the scale of the abscissa axis of the Fourier transform spectrum was 1 Hz, as shown in Fig. 7(a). The energy of a signal component whose frequency is between the two spectral lines will distribute to the two nearest spectral lines after the transformation. The closer the frequency of a spectral line and the signal frequency are, the more energy the spectral line shares. As shown in Figs. 7(b) and (c), when \( A_6 = 3 \), the target signal could obtain enough migration enhancement to ensure its amplitude was a maximum only when \( f_6 \) was between 79.5 and 80.5 Hz. When \( f_6 \) was in the range 79–79.5 or 80.5–81 Hz, although the target signal still underwent migration enhancement, its amplitude was smaller than that of the signal whose frequency was 79 or 81 Hz.

Therefore, the first singular value reconstruction could not be used to extract the target signal.

In summary, when the signal resolution is \( \Delta f \) and the target signal frequency is \( f_B \), the excitation frequency should be selected as \( f_B \pm 0.5\Delta f \).

B. APPLICATION FLOW OF LINEAR NARROWBAND FILTERING METHOD BASED ON MSVD

By fully utilizing the signal extraction feature at the specified frequency of MSVD, an improved signal processing method, called LNF-MSVD, is proposed. The steps are as follows.
(1) Define the center frequency of the band-pass filter and add the excitation signal that meets the amplitude and frequency standard of MSVD to the original signal.

\[ X = X_1 + X_2 = [x(1), x(2), \ldots, x(N)] \]  

(2) Construct a Hankel matrix \( H^1 \) with an equal number of rows and columns (\( q = N/2 \)) [42].

\[
H^1 = \begin{bmatrix}
    x(1) & x(2) & \cdots & x(q)
g(2) & x(3) & \cdots & x(q+1)
    \vdots & \vdots & \ddots & \vdots
g(p) & x(p+1) & \cdots & x(N)
\end{bmatrix} \]  

(3) Perform SVD to obtain this formula.

\[
H^1 = U \begin{bmatrix}
    \sigma_1 & 0 & \cdots & 0 \\
    0 & \sigma_2 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & \sigma_q
\end{bmatrix} V^T \]  

(4) Reconstruct the component signal by the first singular value:

\[
X^1 = \sigma_1 u_1 v_1^T \]  

where \( \sigma_1 \) is the first and largest singular value, \( u_1 \) is the first column of matrix \( U \), and \( v_1^T \) is the first row of matrix \( V^T \).

(5) Subtract the excitation signal from the reconstructed component signal.

\[
X_{\text{ideal}} = X^1 - X_2 \]  

The target signal sequence \( X_{\text{ideal}} \) is thus obtained.

Based on the signal analysis requirements, the above five steps can be repeated until all the target signals are represented.

III. EXTRACTION AND DISPLAY OF ACOUSTIC MODE OF HIGH-PRESSURE-RATIO AXIAL COMPRESSOR

A. MEASUREMENT OF ACOUSTIC PRESSURE SIGNAL AND FORMATION MECHANISM OF ACOUSTIC MODE

1) STRUCTURE OF HIGH-PRESSURE-RATIO AXIAL COMPRESSOR AND MEASUREMENT OF ACOUSTIC PRESSURE SIGNAL

The structural parameters of a two-stage transonic axial compressor with a high pressure ratio (hereinafter referred to as “compressor”) are shown in Table 1. The structure and positions of the sensor installation are shown in Fig. 9.

The acoustic pressure signal measurement system is composed of pressure sensors, connecting cable, data acquisition instrument, and computer. The connecting cable between the sensors and the data acquisition instrument is twisted pair-shielded wire, which can effectively reduce information leakage, can also block the interference of external electromagnetic environment, and it has a higher transmission rate. This advantage can play an important role in the measurement of acoustic pressure signal, reduce the time error of speed transmission, and lay a good foundation for subsequent analysis. The data acquisition instrument (DEWETRON Co., Austria) has 32 channels and adopts 24 V constant voltage DC power to ensure its working stability. The sampling mode is the interrupt sampling mode, and the maximum sampling frequency can reach 10 kHz, which can meet the requirements of collecting the acoustic pressure information of the aeroengine channel.
To collect the acoustic pressure signal, 24 sensors were arranged uniformly on the wall of the air passage 10 mm in front of the intake support plate of the compressor. After the sensors were installed, the measurement system was turned on, and the motor was started to drive the compressor, so that the compressor could complete a complete working process including start, acceleration, stability, and deceleration. With the compressor running, the acoustic pressure signals of the compressor in various states were collected. The rotating direction of the rotor was clockwise, and the sampling frequency was 25,000 Hz.

2) FORMATION MECHANISM OF ACOUSTIC MODE

By analogy with the vibration modes of solid structures, air in a space can be treated as an elastic body. When it is stimulated by different acoustic pressure signals, different morphological changes occur. These changes are called acoustic modes [44], [45].

The flow field of the compressor can be simplified as a pipe. Assuming the pipe has a circular cross section and the flow is uniform and inviscid, the equations of conservation of momentum and continuity govern the flow, and a mode for the theoretical analysis of sound propagation, called the convective wave equation, can be obtained [46]–[48]:

\[
\nabla^2 p - \frac{1}{c_0^2} \frac{D^2 p}{Dt^2} = 0
\]

(7)

where \(c_0\) is the speed of sound, \(p\) is the acoustic pressure, and \(\nabla^2\) in the cylindrical coordinate system is given as follows:

\[
\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}
\]

(8)

Using equation (8), a special solution of (7) can be obtained:

\[
p_{m,n}(\theta, t) = A_{m,n}(t)e^{-ih\theta + i\omega t}e^{i\phi}(9)
\]

where \(\theta\) is the azimuthal angle, \(t\) is the time, \(A_{m,n}(t)\) is the amplitude of the \((m, n)\)th-order wave acoustic pressure at \(t\), \(h\) is the harmonic order, \(z\) is the number of rotor blades, \(\omega\) is the phase velocity of the component, and \(\phi\) is its phase angle.

For each numerical combination \((m, n)\), equation (9) has a corresponding deterministic algebraic formula corresponding to the acoustic pressure of \((m, n)\)th-order waves. Therefore, the total acoustic pressure in the tube can be expressed as follows:

\[
P_{m,n}(\theta, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} p_{m,n}(\theta, t)
\]

(10)

Under the action of the acoustic pressure, the air begins to deform and vibrate like an elastomer, and acoustic modes are generated. The acoustic modes consist of downstream and upstream acoustic modes, which are called forward and backward acoustic modes, respectively [49].

B. METHOD OF DISPLAYING ACOUSTIC MODE

To further extract the acoustic modal information after LNF-MSVD is used to extract the real-time acoustic pressure signal, it is necessary to plot the real-time acoustic pressure signal to display the acoustic mode. The method for displaying acoustic modes is proposed as follows.

The acoustic modal diagram can be obtained by plotting the extracted acoustic pressure signal. By using the coordinate plane to represent the measurement section of the sensor and the horizontal axis to represent the time, the actual acoustic mode cylinder diagram in the compressor can be drawn. Different colors (the warm colors represent high values, and the cold colors represent low values) represent different amplitudes of the measured acoustic pressure (unit: Pa) (as shown in Fig. 10). Similar to the acquisition of the world map, the plane acoustic modal diagram can be obtained by expanding the acoustic modal cylinder (as shown in Fig. 11). In each time section, if the diagram of the acoustic mode has only \(n\) peaks and \(n\) troughs, the order of the acoustic mode \(m = n\). At any rotational speed and at any passing frequency, the acoustic mode may be forward or backward, but its order is not affected by the propagation direction. Thus, the application effect of the two modes is the same when only considering the order of the acoustic mode [46].
The selected technique is widely known in the field of turbomachinery acoustics, and it is a classical method to display the acoustic pressure signal of an axial-flow compressor effectively. However, it is limited by the constraints of the geometric transformation of a single surface [46]. If the proposed method is applied to process 3D data, such as data coming from a CFD simulation, 3D display techniques using iso-surfaces should be considered as well. Because the research object of this paper is an axial-flow compressor with a high pressure ratio, this method is suitable for obtaining the acoustic modal diagram.

C. PROCEDURE TO EXTRACT AND DISPLAY ACOUSTIC MODE

1. An example acoustic pressure signal (acoustic pressure signal at a compressor speed of 3348 r/min) was intercepted, and the results of the spectral analysis are shown in Fig. 12.

2. The target signal was determined. For example, the compressor’s rotor blades in the first stage consisted of 17 pieces. For a rotation speed (55.8 t/s), the 948.6-Hz signal in the spectrum shown in Fig. 12 should be the signal at the tip’s passage and the target signal to be extracted below.

3. The target signal was extracted by LNF-MSVD.

4. The diagram of the acoustic mode was drawn using the extracted target signal, as shown in Fig. 13.

The procedure used to extract and display acoustic mode is expressed concisely in Fig. 14.

Based on the requirements of the acoustic modal analysis, the above four steps can be repeated until all the target acoustic modes are extracted and displayed.

IV. COMPARATIVE ANALYSIS OF THE SUPERIORITY OF LNF-MSVD IN EXTRACTING ACOUSTIC MODE

To confirm the superiority of LNF-MSVD in extracting acoustic modes, the data at a speed of 4050 r/min was selected for comparative analysis with SVD and wavelet analyses (representative of traditional signal extraction methods).

Fig. 15 shows that, the amplitudes of many other spectral lines near 1131 Hz were larger than the amplitude of the signal at 2262 Hz. When the SVD method was used to extract the acoustic mode at 2262 Hz, it was difficult to determine which singular value corresponded to the target signal. Thus, SVD is not conducive to the extraction of the target acoustic mode.
As shown in Fig. 16, when LNF-MSVD was used for extraction, the target signal and corresponding acoustic modal diagram could be obtained from the reconstruction result of first-order singular value by using an excitation signal with a frequency of 2262 Hz and amplitude of $6.407 \times 10^5$. However, the target signal and corresponding acoustic modal diagram could not be obtained with SVD. This shows that LNF-MSVD has the ability to extract weak energy target signals that SVD does not have. Therefore, the extraction effect of LNF-MSVD for the target acoustic mode was greatly improved compared with SVD.

As shown in Fig. 17, when wavelet analysis was used to extract the acoustic mode, the obtained acoustic mode diagram was irregular, and the relevant information could not be obtained from the acoustic mode diagram, which indicated that wavelet analysis could not extract the acoustic mode effectively. However, LNF-MSVD could effectively extract the acoustic mode. This proved that LNF-MSVD had a better effect over some traditional signal extraction methods represented by wavelet analysis in extracting the acoustic mode.

V. ANALYSIS FOR REDUCING NOISE OF COMPRESSOR

For a compressor, locating the likely source of the noise will be helpful for reducing the noise of the compressor, at least by reducing the search range of the noise source [50], [51]. The generation of compressor noise is connected to its work, and the acoustic modal characteristics of the compressor change with changes in its operating state [52]. Therefore, the characteristics of the extracted acoustic mode can be associated with the noise generation of the compressor.

A. RELATION BETWEEN THEORETICAL ACOUSTIC MODAL ORDER AND COMPRESSOR STRUCTURE

The theoretical orders of the interference acoustic modes of the compressor can be calculated based on the acoustic modal analysis theory proposed by Tyler and Sofrin [33]. The formula for calculating the theoretical orders of the acoustic modes is as follows:

$$m = nB \pm Kv$$

where $B$ represents the number of rotor blades, $V$ represents the number of stator blades, $n$ represents the number of harmonics, and $K$ is a variable integer [53].

Table 2 shows the corresponding theoretical orders of the acoustic modes under 1BPF (which denotes the first-order bypassing frequency; a similar notation is used to refer to other orders), 2BPF, and 3BPF for the interference between the first-stage rotor and the intake support plate of the compressor. Table 3 shows the corresponding theoretical orders of the acoustic modes under 1BPF, 2BPF, and 3BPF for the interference between the first-stage rotor and the first-stage stator of the compressor. Because the distance between the second level of the compressor and the acoustic pressure sensor was too long, the noise generated by the interference between the second-stage rotor and the second-stage stator had little influence on the measured acoustic pressure signal, and thus, this influence could be neglected.

For a common acoustic pressure signal $X = A \sin 2\pi ft$, its derivative $\dot{X} = v = 2\pi f A \cos 2\pi ft$ indicates the corresponding vibration speed, and the required excitation energy $E = \frac{1}{2}mv^2$. When the frequency ($f$) of the signal is too high, because the excitation energy cannot be excessively concentrated, the excitation energy is not increased significantly. Consequently, the enlargement of the vibration speed ($v$) is not evident, so the vibration amplitude corresponding to the signal is significantly reduced. If the frequency of the acoustic pressure signal is too high, the frequency of the excitation signal will be very high, and the multiple of this frequency with respect to the natural frequency of the air in the compressor will also be very large, i.e., the order of
the acoustic mode is too high. Thus, in actual engineering conditions, acoustic modes with orders that are too high will not be considered generally. As \( k \) deviates further from the \( k \) value corresponding to the minimum order in each
case, the calculated theoretical order will be larger and fur-
ther beyond the scope of consideration. Therefore, only the
calculated minimum order and its adjacent order are listed
in Tables 2 and 3.

Based on practical engineering experience, the orders of
the compressor’s acoustic modes that can be measured are
shown in Tables 2 and 3: $m = -1$ at 1BPF, $m = -2$ at 2BPF,
and $m = -3$ at 3BPF when the first rotor interfered with the
intake support plate; $m = -8$ at 2BPF and $m = 9$ at 3BPF
when first rotor interfered with the first stator [46].

B. ANALYSIS OF NOISE REDUCTION OF COMPRESSOR

If the experimental acoustic modal order $m$ can be obtained
from the acoustic diagram, the likely location of the noise
source can be determined by comparing it with the theoret-
cal acoustic modal order, which is useful for reducing the
compressor noise. The example is as follows.

The acoustic modal characteristics at 5883 r/min were
analyzed. Fig. 18(a) shows that in the spectrum, there was
only one signal at 1BPF whose amplitude was far greater than
the amplitudes of the other signals, which indicated that there
was only one main noise source in the compressor casing.

Generally, the only acoustic mode at this frequency was
extracted for the analysis of noise reduction. To test the effect
of this method at other simple signals, the signal at 2BPF was
selected to conduct analysis of the signal at 1BPF. As shown
in Fig. 18(b), the experimental order of the acoustic mode was
$m = 1$. Table 2 shows that in the state of 1 BPF (1,667 Hz), the
theoretical order of the acoustic mode $m = 1$ was generated
by the interference between the first-stage rotor and the intake
support plate. For this order relationship, the interference
between the first-stage rotor and the intake support plate
could be the source of this noise, and this should be taken

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**TABLE 2. Theoretical acoustic modal orders for the interference between the first-stage rotor and intake support plate.**

| Harmonic number | BPF | $k$ | Number of rotor and support plate blades | Order number |
|-----------------|-----|-----|------------------------------------------|--------------|
| 1               | 1   | -2  | $B = 17$                                 | -19          |
|                 |     | -1  | $V = 18$                                 | -1           |
|                 |     | 0   |                                          | 17           |
| 2               | 2   | -3  |                                          | -20          |
|                 |     | -2  |                                          | -2           |
|                 |     | -1  |                                          | 16           |
| 3               | 3   | -3  |                                          | -21          |
|                 |     | -2  |                                          | -3           |
|                 |     | -1  |                                          | 15           |

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**TABLE 3. Theoretical acoustic modal orders for the interference between the first-stage rotor and stator.**

| Harmonic number | BPF | $k$ | Number of rotor and stator blades | Order number |
|-----------------|-----|-----|----------------------------------|--------------|
| 1               | 1   | -2  | $B = 17$                         | -65          |
|                 |     | -1  | $V = 42$                         | -25          |
|                 |     | 0   |                                  | 17           |
| 2               | 2   | -3  |                                  | -92          |
|                 |     | -2  |                                  | -50          |
|                 |     | -1  |                                  | 8            |
| 3               | 3   | -3  |                                  | -75          |
|                 |     | -2  |                                  | -33          |
|                 |     | -1  |                                  | 9            |

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FIGURE 18. Acoustic modal extraction results at 5883 r/min.
(a) Spectrum. (b) Plane acoustic modal diagram at 1BPF (1667 Hz).
(c) Plane acoustic modal diagram at 2BPF (3334 Hz).
into account to reduce the noise of the compressor. Because the signs of the experimental and calculated results were opposite, the acoustic mode at this condition was a backward acoustic mode [46]. Fig. 18(c) shows the acoustic mode at 2 BPF (3,334 Hz). At this time, the experimental order of the acoustic mode was m = 2. Similarly, according to the theoretical order listed in Table 2 at 2BPF, the interference between the first-stage rotor and the intake support plate may have also been the likely source of the noise. The acoustic mode at this condition was a backward acoustic mode because the experimental result was opposite to the calculated result [46]. Because this speed is in the range that a compressor operates in for long periods, the first-stage rotor and intake support plate were the characteristic structures that must be considered in the design of the compressor to reduce noise.

VI. CONCLUSION

Inherent errors are present in signal measurement and vibration modal analysis of classical noise reduction method. To improve the noise reduction, the first step is to re-select the acoustic pressure signal, which is easily accurately measured for analysis. In addition, an improved method (LNF-MSVD) is proposed to improve the accuracy of the analysis. The basis of proposing LNF-MSVD is the limitations of traditional SVD and the characteristics of acoustic pressure signals, and the process and calculation parameters for the implementation of LNF-MSVD were given after a rigorous case study. The data measurement process and formation mechanism of the acoustic mode were briefly described. Acoustic modal diagram was first obtained by extracting the target signal from the measured pressure signal of compressor using LNF-MSVD and displaying target signal. The experimental order of the acoustic mode was obtained from the acoustic modal diagram. These diagrams were compared with the acoustic modal images extracted by SVD and wavelet analyses, and the advantages of LNF-MSVD in extracting the acoustic mode were proven. Finally, the experimental acoustic modal order was compared with the theoretical acoustic modal order to find the likely location of the noise source, which could be helpful for reducing the noise of the compressor. The main achievements of the study are as follows:

1. The target signal could be accurately extracted by LNF-MSVD after changing the relationship between the amplitude of the target signal and the amplitudes of the surrounding frequency components through the application of migration enhancement. This method solves the problem of disorder in the frequency domain (the difficulty in extracting signals whose amplitude characteristics are not evident) of traditional SVD, implements linear narrow-band filtering near any given frequency, and integrally extracts the amplitude, frequency, and phase characteristics of the target signal. In addition, this method also provides a new solution for weak signal extraction.

2. The acoustic mode of the compressor was successfully extracted with LNF-MSVD and display method by combining the structure of the compressor and the measurement of the acoustic pressure signal. Through a comparison, LNF-MSVD was shown to have apparent advantages for the precise extraction of acoustic mode, which could ensure the effectiveness for the subsequent use of acoustic mode diagram for compressor noise reduction.

3. The likely source of noise in the compressor could be found by analyzing the relationship between the experimental acoustic modal order and theoretical acoustic modal order after acquiring the experimental acoustic modal order from the acoustic mode diagram. The result could be helpful to effectively reduce the compressor noise and the workload.

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