Digital holographic 3D imaging spectrometry (a review)

Kyu Yoshimori
Faculty of Science and Engineering, Iwate University, 4-3-5 Ueda Morioka, Iwate 0208551 Japan
yosimori@cis.iwate-u.ac.jp

Abstract. This paper reviews recent progress in the digital holographic 3D imaging spectrometry. The principle of this method is a marriage of incoherent holography and Fourier transform spectroscopy. Review includes principle, procedure of signal processing and experimental results to obtain a multispectral set of 3D images for spatially incoherent, polychromatic objects.

1. Introduction
We have considered possibility to obtain both three-dimensional (3D) spatial information and spectral information of spatially incoherent, polychromatic objects by a fully interferometric technique [1]. We have proposed a method which uses a two-wavefront folding interferometer to obtain a suitably arranged 3D spatial coherence function. This spatial coherence function is computed from originally measured 5D interference data-set [2]. This paper reviews recent progress in interferometric 3D imaging spectrometry. Review includes principle of method, procedure of signal processing and experimental results to obtain a multispectral set of 3D images for spatially incoherent, polychromatic objects.

2. Principle and experimental method
Figure 1 shows schematic of the two-wavefront folding interferometer that is used in our method. It is one of interferometers to measure 5D spatial coherence function. A propagating light wave, which is emitted from a polychromatic light source, placed on the $x$-$y$ stage, is incident upon the interferometer. Each wave front, split by the beam splitter BS, is reversed left to right or up to down by the right-angle prism, P or $P'$, and superposed again on the BS. A revealed interference pattern, called the elementally interference pattern, is detected by CCD and, then, recorded. This measurement process is repeated while moving $x$-$y$ stage and $z$ stage (PZT) stepwise. The 5D interferogram measured with this interferometer contains the 5D spatial coherence function of the form: $\Gamma(-\xi + \hat{x}, \eta + \hat{y}, z_0; \xi + \hat{x}, -\eta + \hat{y}, z_0 + Z)$. Here,
Z denotes the path difference along the optical axis that is introduced by translation of z stage, \((\hat{x}, \hat{y})\) indicates the position of the cross point of apexes of the right-angle prisms P and P’ that are set orthogonally, \((\xi, \eta)\) stands for the coordinate system taken over the observation plane. The relationship between the coordinate system \((\xi, \eta)\) and \((x, y)\) components of the original Cartesian coordinate system that is fixed on the x-y stage are written as \(\xi = x - \hat{x}\) and \(\eta = y - \hat{y}\). Since each elementally interference pattern is parameterized by \(\hat{x}, \hat{y}\), and \(Z\), one may imagine the whole 5D interferogram as a set of 2D elementally interference patterns, namely \((\xi, \eta)\), arranged in the 3D \((\hat{x}, \hat{y}, Z)\) space.

To retrieve a set of spectral components of the object images, we begin by applying the synthetic aperture technique to the 5D interferogram to compute a volume interferogram that is suitable for our purpose [3]. This procedure is performed by choosing one pixel value in each elementally interference pattern and, then, rearranging in a new 3D space. For synthesis an in-line interferogram, the selection rule is written as \(\xi = \hat{x}, \eta = -\hat{y}\) and the rearrangement rule as \(X = 2\hat{x}, Y = 2\hat{y}\). The resultant volume interferogram contains the 3D spatial coherence function of the form \(\Gamma(R_0, R_0 + \rho)\), with \(R_0 = (0, 0, z_0)\) indicating the phase tracking center on the observation plane \(z = z_0\), and \(\rho = (X, Y, Z)\) denoting 3D shear. For paraxial regime, this spatial coherence function is related to the cross-spectral density \(W\) by the following relationship:

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\Gamma(R_0, R_0 + \rho) = \int_0^\infty W(R_0, R_0 + \rho_\perp; \omega)e^{i\omega z}d\omega,
\]

where \(\rho_\perp = (X, Y)\), \(\omega = ck\) is the angular frequency and \(c\) is the speed of light. Then, similar to the Fourier transform spectroscopy, one may obtains a set of the cross-spectral densities \(W(R_0, R_0 + \rho_\perp; \omega)\) from the volume interferogram by taking Fourier transform with respect to \(Z\) and simply extracting positive frequency components. Since each cross-spectral density is proportional to a spectral component of the complex amplitude \(U(R_0 + \rho_\perp, \omega)\), it is regarded as an incoherent complex hologram at frequency \(\omega\). It is then possible to retrieve a spectral component of 3D image of the light source by a usual inverse propagation technique.

### 3. Results and discussion

We now demonstrate the performance of the digital holographic 3-D imaging spectrometry, in which the measured object is composed of three planar light sources of different shapes, each having different continuous spectrum, located at different depth. We used three different color LEDs as primary light sources. These light sources are attached to the ends of acrylic rods. The cross sections on the other ends of the three rods are shaped as a rectangle and two different triangles. These planar light sources are set up on the x-y stage of the two-wavefront folding interferometer. The x-y stage is moved along the x and y axes at 12.9 \(\mu\)m in every step. The prism P, placed on the piezo translator (PZT), is also moved 0.08 \(\mu\)m in every step as well. The total steps of experiment, i.e., the number of acquired elementally interference pattern is 64x64x64.

The object under measurement is composed of these planar light sources. The peak wavelengths of the light sources are 458 nm, 504 nm and 630 nm. The optical depths of these light sources from the observation plane are 70 mm, 61 mm and 51 mm, respectively. Figure 2 shows the acquired continuous spectral profile of the object

![Figure 2 Continuous spectral profile of each light source: S1, S2 and S3.](image-url)
measured over the observation plane. Three spectral peaks around 458 nm, 504 nm and 630 nm are clearly seen. The phase distributions of the cross-spectral density at the three spectral peaks are shown in Fig. 3. Each phase distribution is essentially equivalent to that of incoherent hologram of the corresponding spectral component.

Finally, Fig. 4 partly shows a whole set of the retrieved in-focus images for many spectral bands. Variation of intensity across the images agree well with the retrieved spectral profile shown in Fig 2. We also find that the size and depth of these images agree well with the actual ones of the original light sources. As a result, we find that a set of spectral components of 3D images of three planar light sources with different shapes of cross section are successfully retrieved by our method.

4. Concluding remarks
We have shown that both 3-D spatial information and spectral information of spatially incoherent polychromatic light sources with different continuous spectra, located at different positions, can be obtained by the digital holographic 3-D imaging spectrometry. This enables a fully interferometric hyper-multispectral 3D imaging for usual polychromatic objects.

References
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