Unification and Model Building, 
Astroparticle Physics and Neutrinos 

WHEPP4 Working Group Report 

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Abstract 

This report summarises the work done during the Workshop on High Energy Physics Phenomenology 4 (S.N. Bose National Centre for Basic Sciences, Calcutta, India, Jan 2-14, 1996) in Working Groups IV (Unification and Model Building) and V (Astroparticle Physics and Neutrinos).

April 1996
1 Introduction

The original plan was to have two separate Working Groups called ‘Unification and Model Building’ and ‘Astroparticle Physics and Neutrinos’. However, at the beginning of the Workshop, it emerged that the members of these two working groups had a wide overlap of interest. Hence a decision was taken to merge them into one working group. In this working group several problems were initially identified for investigation. Smaller subgroups were formed to focus on each of these problems. The progress till the end of the workshop is summarised in this report.

2 Problems undertaken

1. CP odd $WW\gamma$ and $WWZ$ form-factors in the MSSM

Possible nonstandard couplings in the $WW\gamma$ and $WWZ$ vertices and how to probe them at the Next Linear Collider (NLC) have been well-studied in the CP-conserving sector. In principle, however, nonstandard interactions could lead to anomalously large CP-odd couplings too. These had not been studied so far. The CP-odd part of the $W-W-V$ vertex ($V = \gamma, Z$) can be expressed in terms of the two couplings $\tilde{\kappa}_V$ and $\tilde{\lambda}_V$ via

$$L_{eff}^{WWV} = \frac{i}{2} \left[ \tilde{\kappa}_V W^\mu_\nu W^-_\nu \epsilon^{\mu\nu\rho\sigma} V_{\rho\sigma} + \frac{\tilde{\lambda}_V}{m_W^2} W^+_{\alpha\mu} W^-_{\nu-\alpha} \epsilon^{\mu\nu\rho\sigma} V_{\rho\sigma} \right].$$ (1)

At the NLC $\tilde{\kappa}_V$ and $\tilde{\lambda}_V$ may be probed at the level of $10^{-3}$. Within the Standard Model (SM) the one loop contributions to these couplings vanish. Such is not the case for the Minimal Supersymmetric Standard Model (MSSM). It is found that

- Triangle diagrams with scalars in the loop do not contribute anything proportional to $\epsilon^{\mu\nu\rho\sigma}$.
- Diagrams with charginos and neutralinos in the loop yield non-vanishing effects. CP non-conservation is introduced through phases in the gaugino masses and the SUSY breaking parameter $\mu$.
- For $V \equiv \gamma$ there is no CP-violating contribution.
- For $V \equiv Z$ the analytical part of the calculation has been completed. The numerical evaluation remains to be done.
- It appears that, barring unexpected cancellations, contributions of $O(10^{-3})$ to $\tilde{\kappa}_Z$ and $\tilde{\lambda}_Z$ are possible for light charginos (masses $\sim 100 - 300$ GeV) and allowably large CP-violating phases.
2. Perturbative top Yukawa coupling in SUSY for $\tan \beta < 1.6$

The top quark Yukawa coupling in the MSSM is given in standard notation by

$$h_t(m_t) = m_t(m_t) \frac{\sqrt{2} \sqrt{1 + \tan^2 \beta}}{v \tan \beta}. \quad (2)$$

Here $v = (1/G_F \sqrt{2})^{1/2}$ and $\tan \beta$ is the ratio of two Higgs VEVs – the one yielding masses for up-type fermion over that for down-types. It is readily seen that $h_t$ increases as $\tan \beta$ decreases. Further, $h_t$ grows with the renormalisation scale $\mu$. Requiring the perturbative limit $h_t(\mu) < \sqrt{4 \pi}$ to be satisfied all the way up to $M_{\text{GUT}}$, one finds the bound $\tan \beta > 1.6$. On the other hand, the $\tan \beta < 1.6$ regime could be phenomenologically interesting [1]. Is it possible to evade the $\tan \beta$ bound in models with intermediate mass scales? The situation examined, as a part of this Working Group activity, is based on the symmetry sequence $SU(4)_c \times SU(2)_L \times SU(2)_R \to SU(3)_c \times SU(2)_L \times U(1)_Y$. By varying $M_R$ it is possible to find which choice will ensure $h_t(\mu) < \sqrt{4 \pi}$ at all scales and yield Yukawa couplings consistent with the known fermion masses.

3. $\theta_W$, fermion masses and horizontal $U(1)$ symmetry

The quark and charged lepton masses satisfy the empirical relations:

$$\frac{m_u}{m_t} = O(\lambda^8); \quad \frac{m_c}{m_t} = O(\lambda^4); \quad \frac{m_d}{m_b} = O(\lambda^4);$$

$$\frac{m_s}{m_b} = O(\lambda^2); \quad \frac{m_e}{m_t} = O(\lambda^4); \quad \frac{m_{\mu}}{m_{\tau}} = O(\lambda^2).$$

In addition, the following radiations are believed to hold at $M_{\text{GUT}}$.

$$m_b = m_{\tau}, \quad \frac{m_d m_e m_b}{m_e m_{\mu} m_{\tau}} = O(1)$$

It has been shown that these relations can be reproduced in the MSSM by introducing an additional $U(1)_H$ horizontal symmetry [2]. The $U(1)_H$ charges – in other words, the Yukawa couplings – are appropriately chosen to reproduce the required mass matrix textures. The procedure also entails the introduction of an electroweak gauge singlet $U(1)_H$ nonsinglet field. It turns out that the charge assignments are such that the $U(1)_H$ has anomalies. If the anomalies are cancelled by the Green-Schwarz mechanism (i.e., if the model originates from superstring theory), then that sets constraints on the mixed gauge anomalies. This, in turn, relates the gauge coupling constants, yielding $\sin^2 \theta_W = \frac{3}{8}$, a result that also emerges from GUT models.

As a part of the activities in this working group, a similar situation was investigated in the context of R-parity violating SUSY. It is of interest to examine whether this method might relate the $R$-couplings $\lambda$, $\lambda'$ and $\lambda''$ and might even forbid one or more of them. This work is in progress.
4. Can gauge coupling unification and gaugino mass unification be decoupled?

The equality of the three gaugino masses, corresponding to the $SU(3)$, $SU(2)$ and $U(1)$ gauge groups of the SM, is usually assumed at the GUT scale as one of the boundary conditions on the Renormalization Group evolution in the MSSM. Can one construct a consistent SUSY GUT in which this assumption is violated? This question has been addressed in the context of $SU(5)$ and a model was constructed with nonunified gaugino masses following the mechanism used in [3]. However, the model has a problem in that all fermionic partners of scalars in an adjoint representation of $SU(5)$ are left massless at the GUT scale. The issue of making a more consistent model is to be explored further.

5. Should squarks be degenerate?

It is well-known that Flavour Changing Neutral Current (FCNC) constraints following from the $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$ mass differences, $b \to s\gamma$ etc., set strong restrictions [4] in SUSY models. SUSY contributions to FCNC via sparticle exchange are usually tamed by choosing the squarks and sleptons to be almost mass degenerate. A comparison of the modes of suppressing FCNC in the supersymmetric extension of the SM can be summarised as:

\[
\begin{array}{cc}
\text{SM} & \text{SUSY} \\
\text{Quarks are degenerate} & \& \text{Squarks are degenerate} \\
\text{or} & \\
\text{CKM matrix is diagonal} & \& \text{SCKM matrix is diagonal}
\end{array}
\]

A special case of the latter is when the CKM (SCKM) matrix is proportional to the identity matrix. Such a situation obtains if in the SM the $u$-quark and $d$-quark mass matrices are matched while in SUSY it requires a matching of the quark and squark mass matrices [4]. The latter mode of satisfying the FCNC constraints in SUSY was examined and it was concluded that (a) a more careful analysis is called for to constrain the mass matrix structures and (b) the restrictions from CP-violation could turn out to be significant.

6. Non-SUSY resolution of $R_b$ anomaly and the $Z - t - \bar{t}$ vertex

Technicolour models may provide a solution to the discrepancy between the observed value of $R_b$ at LEP and its SM prediction. Such models also alter the $Z - t - \bar{t}$ vertex and may be probed at the NLC. How big are these effects?
One of the models examined involves the symmetry breaking $SU(N + 2)_{ETC} \times SU(2)_L \times U(1)_Y \rightarrow SU(N)_{TC} \times SU(2)_H \times SU(2)_L \times U(1)_Y \rightarrow SU(N)_{TC} \times SU(2)_L \times U(1)_Y \rightarrow SU(N)_{TC} \times U(1)_{EM}$.

The relevant Lagrangian can be written as:

$$L_{Zt\bar{t}} \sim \bar{t}(g_V \gamma^\mu F_V + g_A \gamma^\mu \gamma_5 F_A) t Z_\mu$$

where $g_{V,A}$ are the SM couplings and $F_V = 1 - 5.15\left(\frac{\xi^2 m_t}{4 \pi v} + \frac{s^4}{2x}\right)$, $F_A = 1 - 2.0\left(\frac{\xi^2 m_t}{4 \pi v} + \frac{s^4}{2x}\right)$. (4)

Here $x = u^2/v^2 \gg 1$ is a measure of the relative magnitudes of the heavy and light scales in the model and $\xi$ is a model-dependent Clebsch Gordan factor. Also, $s \equiv \sin \phi$ where $\phi$ is the neutral heavy boson – light boson mixing angle.

Two cases were considered:

- **Light case**: Choosing typical values of $x = 20$ and $\xi = 1.4$ and fitting the $R_b$ data implies $\delta F_V \simeq -0.25$, $\delta F_A \simeq -0.10$ and $\delta R_c/R_c \simeq -0.002$.
- **Heavy case**: Using $x = 380$ and $\xi = 0.8$ and fitting the $R_b$ data implies $\delta F_V \simeq -0.05$, $\delta F_A \simeq -0.02$ and $\delta R_c/R_c \simeq -0.002$.

Another model [7], that was also examined, has an additional $U(1)$ symmetry which couples to the third generation only and that too in the following manner:

(a) vectorially to the $\tau$, (b) axial vectorially to the $t$ and (c) in a left-handed way to the $\nu_\tau$. All the couplings have equal strengths. In this model $R_b$ receives a contribution through the mixing of the extra $U(1)$ gauge boson with the Z boson. The parameters of the model can be chosen to fit the experimental value of $R_b$. This is found to imply $\delta F_V = 0$, $\delta F_A \simeq -0.0275$, $\delta R_c/R_c = -0.008$. Recall that the $U(1)$ couples axial vectorially to the $t$-quark.

7. Maximal mixing and three degenerate neutrinos

The maximal mixing between three generations of neutrinos may be parametrised by the mixing matrix:

$$U = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega^2 & \omega & 1 \\ \omega & 1 & 1 \end{pmatrix}. \quad (5)$$

This choice automatically satisfies the neutrinoless double beta decay constraint since $< m > = m_0 \Sigma U^2_{ei} \sim 0$. In this model $P_{\nu_e\nu_e} = P_{\nu_e\nu_\mu} = P_{\nu_e\nu_\tau} = 1/3$. This
scenario was compared with the solar neutrino data and the following results were obtained.

| Theory          | Cl       | Ga          |
|-----------------|----------|-------------|
| $\phi(^{8}B)$ from Kamioka | $4.5 \pm 0.5$ | $123^{+8}_{-6}$ |
| Experiment      | $2.78 \pm 0.35$ | $75 \pm 9$   |
| 2 flavour maximal mixing | $3.64 \pm 0.4$ | $65^{+4}_{-4}$ |
| 3 flavour maximal mixing | $3.35 \pm 0.17$ | $45.7^{+7}_{-3}$ |

Notice that in the three-flavour case, the situation gets much worse for $Ga$, though there is a small change towards the observed value for the $Cl$ experiment.

### 8. Energy independent neutrino depletion and three generations

Another alternative - in some sense complementary to the previous one - is the situation where the neutrino mass differences are such that the neutrino oscillation probabilities are independent of the energy. With three generations, the probabilities relevant to the solar neutrino problem can then be written as:

$$P_{\nu e\nu e} = 1 - \frac{1}{2} c_{13}^4 \sin^2 2\theta_{12} - \frac{1}{2} \sin^2 2\theta_{13}$$

and

$$P_{\nu e\nu \mu} + P_{\nu e\nu \tau} = \frac{1}{2} c_{13}^4 \sin^2 2\theta_{12} + \frac{1}{2} \sin^2 2\theta_{13}$$

- It was found that the Kamiokande, $Cl$ and $Ga$ data cannot be simultaneously explained within this scenario at a 95% CL. A small allowed region in the parameter space is found at the $3\sigma$ level.
- If the flux of the $^{8}B$ neutrinos is taken from the Kamiokande data, then a much better fit is obtained.

### 9. Field theoretic formulation of neutrino and charged lepton oscillations

As a part of this project, several ideas in the recent literature pertaining to oscillations were critically examined. The following observations can be made.

- A careful field theoretic analysis of neutrino mixing leads to the realization that the vacuum corresponding to flavour basis states is a coherent state obtained from a condensation of mass eigenstates. The two are unitarily nonequivalent. This changes the momentum dependence of the oscillation probability $P_{\nu e\nu e}$, but the usual form is recovered in the ultra-relativistic limit (which is expected to be valid for neutrinos). Whether this idea can be tested was investigated.
- If the $\nu_\mu$ is a superposition of three mass eigenstates, then in pion decay ($\pi \rightarrow \mu + \nu_\mu$) one should obtain three different momenta of the muon in consistency with energy momentum conservation. This will affect the time...
evolution of the muon [10]. This idea was critically examined and it was concluded that, for the neutrino masses usually considered, the idea cannot be tested in experiments. Questions were also raised about the rigourousness of the result itself.

10. **Violation of Equivalence Principle, Gravitational effects in neutrino physics**

In this project there were several talks and group discussions. Problems, that were identified for further pursuit, include:

- Constraints on the Violation of Equivalence Principle (VEP) from existing data on neutrinos.
- Analysis of the parameter space for three neutrino generations including VEP and matter effects.
- Gravitational oscillations of Ultra-High-Energy (UHE) neutrinos.
- The issues of principle related to VEP.

11. **Singlet neutrino mixing and primordial nucleosynthesis**

Sterile neutrinos $\nu_s$ are of much current interest and have been invoked to explain neutrino oscillation data. If such a neutrino has a large mixing with either $\nu_e$ or $\nu_\mu$ then sterile neutrinos will equilibrate rather late in the early universe. This may contribute as much as 0.8 to $N_\nu$ – the effective number of neutrinos. Such a large contribution would be in conflict with the bounds from nucleosynthesis. Several possible loopholes to this line of argument were considered in this working group.

- Most recent observational data on $^4He$ and $^2H$ abundances allow $N_\nu$ up to 4.5 [11].
- Lepton and/or baryon number asymmetries are not usually taken into consideration when bounds on $N_\nu$ are set from nucleosynthesis.
- Nonlinear $\nu_e - \nu_s$ feedback is important.

3 **Acknowledgements:**

We thank all the participants in these two groups for their all-round cooperation. The work of AR has been supported by grants from the Department of Science and Technology and the Council of Scientific and Industrial Research, India.
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