Orthogonal iteration process of determining K value on estimator of Jackknife ridge regression parameter

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Abstract. Jackknife is re-sampling bias estimation method, in predict standard deviations. The principle of the Jackknife method in regression parameters estimation is to eliminate one data and repeat it as much as the number of existing data samples. The step in estimating regression parameter of Jackknife Ridge (JR) was the first to transform data through cantering and scaling process and orthogonalization on independent variables. The second determines initial value $k_0$ and perform the iteration process. The third by transform initial estimator JR and finally by testing the feasibility of the resulting model of Jackknife ridge regression. Results obtained, iteration process is conducted until obtained a value of $\left| \left( \tilde{\alpha}_{GR} \tilde{\beta}_{GR} \right)^1 - \left( \tilde{\alpha}_{GR} \tilde{\beta}_{GR} \right)^{1}\right| \leq 0.0001$ and iteration stops at 5th iteration, so obtain estimator value of generalized ridge coefficient of orthogonal independent variable ($\alpha^*_GR$) value of generalized ridge coefficient of orthogonal independent variable ($\tilde{\alpha}_{GR}$).

1. Introduction

Regression analysis is a useful statistical analysis to determine the relationship between two or more variables so that one variable can be estimated from other variables. In this regression analysis can be known form or pattern of relationships that occur and can be done predictions based on the value of variables that have been known. If regression analysis is performed for one dependent variable (y) with more than one independent variable (x) then this regression is called multiple linear regression [1]. One requirement in multiple regression analysis other than normality is multicollinearity. Multicollinearity is the absence of a linear relationship between independent variables. If there is a linear relationship between the independent variables then it can be said that the model is exposed to multicollinear, when two or more independent variables are highly correlated. If there is a relationship between independent variables then this variable is not orthogonal. The orthogonal variable is an independent variable which correlation value inter-independent equal to zero [2]. Some characteristics of model that exposed to multicollinear or strong relationship among independent.

Some alternatives to solve multicollinear problems include merges cross-sectional and time-series data removes one or more independent variables that have high correlations from the
regression model and identifying other independent variables to help prediction, transforming variables [3]. Transformation can be Natural Logarithm (ln), using models with independent variables that have high correlation solely for prediction, and not interpreting regression coefficients, using better analytical methods such as Bayesian regression or in certain cases with Ridge Regression (RR). For RR methods often produce biased estimators, to solve them introduced a technique for reducing bias of RR estimators is called estimation techniques such as Jackknife ridge has smaller bias and lower MSE (Mean Square of Error) than RR estimators [4]. Conducting the process of Orthogonalization of the Independent Variables that will be orthogonalized are the free variables resulting from centering and scaling. First find the eigenvalue of the independent variable, through the iteration process. The iteration process is terminated when the generalized ridge coefficient estimator values are ≤ 0.0001.

2. Literature Review

In general, multiple linear regression equation with some independent variable c is expressed by:

\[ y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_c x_{ic} + \epsilon_i, \quad i = 1, 2, 3, \ldots, n, \]  

(1)

The multiple linear regression equation can be elaborated into matrix notation, as a form of extension of the general linear regression model expressed as follows:

\[ y = X\beta + \epsilon, \]  

(2)

where:

\[ y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad X = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1c} \\ 1 & x_{21} & x_{22} & \cdots & x_{2c} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nc} \end{bmatrix}, \quad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_c \end{bmatrix}, \quad \epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix} \]

Assumption of Multicollinearity

A term of multicollinearity was first discovered by [5] it means that there is a linear relationship between some or all independent variables (X) in the regression model. Multicollinearity is an ill condition where there is a strong correlation between independent variables \( (x_j) \) involved in the formation of a linear regression model, so \( X'X \) does not meet the classical assumptions. According to Montgomery and Peck [6], that a very high correlation will produce biased, unstable estimator and probably far from true estimates so that the resulting residual becomes large and the variance of parameters becomes indefinite. According to [7] there are several ways to detect the presence of multicollinearity or not, as follows:

a. Variation Inflation Factor (VIF)

Variation Inflation Factor (VIF) is one way of detecting the multicollinearity [8]. This is based on the fact that the increase of variance depends on \( \sigma^2 \) and VIF itself. VIF is expressed by the formula:

\[ \text{VIF} = \frac{1}{1-R_j^2} \]  

(3)

where \( R_j^2 \) is the coefficient of determination of \( x_j \) and \( x_j' \) of other independent variables. The value of \( R_j^2 \) has an effect on the resulting VIF value, where the greater of \( R_j^2 \) then the greater of VIF value. If the value of VIF > 10 then it can be concluded that there is multicollinearity in the variable [9].

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b. Tolerance Value Method

According to [10] that to detect multicollinearity, in addition to using coefficient correlation and VIF can also use the Tolerance Value Method or TOL. TOL is an indication of percent variance in predictors that cannot be quantified by predictor variables. The formula of TOL is as follows:

\[ \text{TOL} = \frac{1}{VIF} \]

X is said to have a high collinearity with another X if has TOL < 0.1.

Regression Jackknife Ridge

Jackknife is a re-sampling method introduced by Tukey [11] for bias estimation and Iskandar et al [12] introduces Jackknife to predict standard deviations. The principle of Jackknife method in estimating regression parameters is to eliminate one data and repeat as many existing data samples [12]. To avoid a bias on RR estimator, Hinkley [13] suggests using Jackknife technique as follows:

\[ y_{(-i)} = Z_{(-i)} \alpha + \varepsilon^* \] (4)

where

- \( y_{(-i)} \) is vector y with \( i^{th} \) value eliminated
- \( Z_{(-i)} \) is matrix Z with each of \( i^{th} \) rows eliminated
- \( \varepsilon^* \) is residual vector with \( i^{th} \) coordinate eliminated

JR solution is given as: [14] and [15].

\[ \hat{\alpha}_{JR(-i)} = (Z_{(-i)}'Z_{(-i)} + K)^{-1}Z_{(-i)}y_{(-i)} \] (5)

From equation (5) we get \( \hat{\alpha}_{JR} \) estimator as follows:

\[ \hat{\alpha}_{JR} = [I + B^{-1}K]\hat{\alpha}_{BB} \] (6)

Jackknife ridge coefficients can be formulated as follows:

\[ \hat{\beta}_{JR} = \gamma \hat{\alpha}_{JR} \] (7)

Centering and Scaling Methods

Data centering and scaling is part of standardizing variables. A simple modification of standardization of these variables is correlation transformation. The centering measure is any measure indicating the center of a series of data, which has been ordered from the smallest to the largest or vice versa from the largest to the smallest while the scaling includes the observational on the standard deviation unit of observation for the variable [16].

Correlation transformation is a simple function of variable standardization, so through transformation is obtained:

\[ Y_i^* = \frac{Y_i - \bar{Y}}{\sqrt{n-1} S_Y} \text{ dengan } S_Y = \sqrt{\frac{\sum_{i=1}^{n} (Y_i - \bar{Y})^2}{n-1}} \] (8)

\[ X_{ij}^* = \frac{X_{ij} - \bar{X}_j}{\sqrt{n-1} S_{Xj}} \text{ dengan } S_{Xj} = \sqrt{\frac{\sum_{i=1}^{n} (X_{ij} - \bar{X}_j)^2}{n-1}}, j = 1,2,\ldots,c \] (9)

where:

- \( \bar{Y} \) = Mean of Y
- \( \bar{X}_j \) = Mean of observations \( X_j \)
- \( S_Y \) = standard deviation of Y
- \( S_{Xj} \) = standard deviation of \( X_j \)
Based on the transformation of variables $Y_i^*$ and $X_{ij}^*$ defined by the transformation of correlation on models (6) and (7) above and obtained by the regression model as follows:

$$Y_i^* = \beta_1^* X_{i1}^* + \beta_2^* X_{i2}^* + \cdots + \beta_c^* X_{ic}^* + \varepsilon_i^*$$  \hspace{1cm} (10)

The above model (8) is called the standardized regression model. Among the parameters $\beta_1^*, \beta_2^*, \ldots, \beta_k^*$ on the standardized regression model with original parameters $\beta_1, \beta_2, \ldots, \beta_k$ on the multiple linear regression model which usually in a linear relationship [17].

**Significance Test of Parameter**

Significance test of a parameter is used to determine the effect of the independent variables on the dependent variable either simultaneously or partially [18].

**Simultaneous Testing**

Simultaneous parameter test is done to test the significance of the relationship between all independent and dependent variables, and to determine whether all parameters can describe the data well. Testing simultaneously using F-test based on the hypothesis:

$H_0$: $\beta_1 = \beta_2 = \beta_3 = \cdots = \beta_c = 0$.

$H_1$: at least there are one $\beta_j \neq 0, j = 1, 2, \ldots, c$

Then, F-test statistic is:

$$F_{\text{count}} = \frac{\text{MSR}}{\text{MSE}} = \frac{\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2}{c} \div \frac{\sum_{i=1}^{n}(y_i - \hat{Y})^2}{n - c - 1}.$$  \hspace{1cm} (11)

Criterion of decision-making:

$$\begin{cases} 
\text{Accept } H_0, \text{if } F_{\text{count}} \leq F_{\text{table}} \\
\text{Reject } H_0, \text{if } F_{\text{count}} > F_{\text{table}}
\end{cases}$$

where $F_{\text{table}} = F_{(c; n-c-1)}$, and if reject $H_0$ it can be concluded that there is a significant relationship simultaneously between independent and dependent variables in a model, and vice versa.

**Partial Testing**

Partial parameter test is done to determine the significance of regression parameter estimator in arranging model based on the following hypothesis:

$H_0$: $\beta_j = 0$ (no significant relationship), $j = 1, 2, \ldots, c$

$H_1$: $\beta_j \neq 0$ (at least one significant relationship)

Then, t-test statistic is:

$$t_{\text{hitung}} = \frac{\hat{\beta}_{JR}}{\text{SE}(\hat{\beta}_{JR})}$$

where $\text{SE}(\hat{\beta}_{JR}) = \hat{\sigma} \sqrt{c_{jj}}$, $c_{jj}$ is the main diagonal of matrix $(X'X)^{-1}$ and $\sigma$ is the root of the MSE.

Criterion of decision-making:

$$\begin{cases} 
\text{Accept } H_0, \text{if } t_{\text{count}} \leq t_{\text{table}} \\
\text{Reject } H_0, \text{if } t_{\text{count}} > t_{\text{table}}
\end{cases}$$
where the value of t-table is \( t_{(\alpha/2, n-c-1)} \). [19].

3. Analysis Method

The steps undertaken based on the objectives of the research are as follows:

1. Generating simulation data
2. Testing classical assumptions and detecting multicollinearity by looking at correlations between independent variables and VIF values.
3. Estimating the regression parameter jackknife ridge:
   a. Transform data through centering and scaling process
   b. Orthogonalization on independent variables
   c. Determine the initial value of \( k^0 \) to determine the initial estimator of the generalized ridge for orthogonal independent variables \( \hat{\alpha}_{GRj}^0 = (Z'Z + kI)^{-1}Z'y \). Then, estimator \( \hat{\alpha}_{GRj}^0 \) are used to calculate the of \( k^1 \). The value of \( k^1 \) is used to calculate the estimator of \( \hat{\alpha}_{GRj}^1 \) and so on. The iteration process is stopped when \( \left| (\hat{\alpha}_{GRj}^1 - (\hat{\alpha}_{GRj}^0)^{i-1}) \right| \leq 0.0001 \).
   d. After the stopped iteration, obtained estimator value of generalized ridge coefficient from an orthogonal free variable (\( \hat{\alpha}_{GRj} \)).
   e. Specifies initial estimator jackknife ridge:
   f. Transforming initial estimator JR is \( \hat{\beta}_{JR} = \hat{y}\hat{a}_{GR} \).
   g. Examine the feasibility of resulting model of jackknife ridge regression.

4. Result and Discussion

Estimator of Generalized Ridge Regression Parameters

The estimator of generalized ridge regression is obtained by minimizing the sum of the residual squares for the regression model in (8)

\[
SSE = \epsilon'\epsilon = (y - Za_{GR})'(y - Za_{GR})
\] (12)

By using Lagrange multiplier, where \( a_{GR} \) is a value that minimizes the objective function with a condition \( (a_{GR}'a_{GR}) \leq c^2 \):

\[
F \equiv (y - Za_{GR})'(y - Za_{GR}) + k(a_{GR}'a_{GR} - c^2)
\]

\[
F \equiv (y' - a_{GR}'Z')(y - Za_{GR}) + k(a_{GR}'a_{GR} - c^2)
\]

\[
F \equiv y'y - y'Za_{GR} - a_{GR}'Z'y + a_{GR}'Z'Za_{GR} + k(a_{GR}'a_{GR} - c^2)
\] (13)

Since \( a_{GR}'Z'y \) is a scalar, then by using transpose \( (a_{GR}'Z'y)' = y'Za_{GR} \), so equation (12) becomes:

\[
F \equiv y'y - 2a_{GR}'Z'y + a_{GR}'Z'Za_{GR} + k(a_{GR}'a_{GR} - c^2)
\] (14)

The value of F minimum if \( \frac{\partial F}{\partial a_{GR} |_{a_{GR}=\tilde{a}_{GR}}} = 0 \), then:

\[
\frac{\partial}{\partial a_{GR}} (y'y - 2a_{GR}'Z'y + a_{GR}'Z'Za_{GR} + k(a_{GR}'a_{GR} - c^2)) = 0
\]

\[
-2Z'y + 2\tilde{a}_{GR}'Z'Z + 2k\tilde{a}_{GR} = 0
\]

\[
2\tilde{a}_{GR}'Z'Z + 2k\tilde{a}_{GR} = 2Z'y
\]

\[
\tilde{a}_{GR} (Z'Z + kI) = Z'y
\]

\[
(Z'Z + kI) = Z'y\tilde{a}_{GR}
\]
\[ \hat{\alpha}_{GR} = (Z'Z + kI)^{-1}Z'y \]  

**Estimator of Jackknife Ridge Regression Parameter**

Jackknife ridge regression is the development of generalized ridge regression where in the estimator of parameter regression is to eliminate one data and repeat as many as the number of data samples. The general equations of the Jackknife ridge regression model are

\[ y_{(-i)} = Z_{(-i)} \alpha + \varepsilon^* \]  

where \( y_{(-i)} \) is a vector \( y \) with the \( i \)th value is eliminated, \( Z_{(-i)} \) is matrix \( Z \) with each \( i \)th row are eliminated, \( \varepsilon^* \) is a residual vector with \( i \)th coordinate is eliminated.

Matrix \( Z_{(-i)} \) does not always have full column rank. JR solution is given:

\[ \hat{\alpha}_{GR(-i)} = (Z'_{(-i)}Z_{(-i)} + kI)^{-1}Z'_{(-i)}y_{(-i)} \]  

Suppose \( z_i \) and \( y_i \) are \( i \)th column vectors of \( Z \) and \( i \)th coordinates of \( y \), then:

\[ \hat{\alpha}_{GR(-i)} = (Z' - z_iz_i' + kI)^{-1}(Z' - z_iy_i) \]  

If replaced by \( B = (Z'Z + kI) \) and \( c = z_i \), then \( (B - cc')^{-1} = B^{-1} + \frac{B^{-1}cc'B^{-1}}{1 - c'B^{-1}c} \), obtained:

\[ (Z'Z - z_iy_i' + kI)^{-1} = B^{-1} + \frac{B^{-1}z_iz_i'B^{-1}}{1 - z_i'B^{-1}z_i} \]  

Inserting equation (17) in equation (18), obtained:

\[ \hat{\alpha}_{GR(-i)} = \left[ B^{-1} + \frac{B^{-1}z_iz_i'B^{-1}}{1 - h_i} \right] (Z' - z_iy_i) \]  

where \( h_i = z_i'B^{-1}z_i \). Equation (18) can be simplified as follows:

\[ \hat{\alpha}_{GR(-i)} = \hat{\alpha}_{GR} = \frac{B^{-1}z_iy_i}{1 - h_i} \]  

where \( e_i = y_i - z_i'\hat{\alpha} \).

Hinkley (1977) proposed the value of weighted pseudo as follows:

\[ Q_i = \hat{\alpha}_{GR} + n(1 - h_i)(\hat{\alpha}_{GR} - \hat{\alpha}_{GR(-i)}) = \hat{\alpha}_{GR} + nB^{-1}z_i e_i \]  

Estimator of JR was given as follows:

\[ \hat{\alpha}_{JR} = \frac{1}{n} \sum Q_i = \hat{\alpha}_{GR} + B^{-1} \sum z_i e_i = \hat{\alpha}_{GR} + B^{-1}Z'\varepsilon \]

\[ = \hat{\alpha}_{GR} + B^{-1}(y - \hat{\alpha}_{GR}) = \hat{\alpha}_{GR} + \hat{\alpha}_{GR} - B^{-1}Z\hat{\alpha}_{GR} \]

\[ = \hat{\alpha}_{GR} + [1 - B^{-1}Z'Z]\hat{\alpha}_{GR} = \hat{\alpha}_{GR} + (B^{-1}kI)\hat{\alpha}_{GR} \]

\[ = [1 + B^{-1}kI]\hat{\alpha}_{GR} \]

From equation (21) obtained estimator \( \hat{\beta}_{JR} \) as follows:

\[ \hat{\beta}_{JR} = \gamma \hat{\alpha}_{JR} \]
Classical Assumption Testing
Classical assumption testing is necessary for estimating the parameters of linear regression to be tested on this research are:

1. Normality test
Normality test of a residual can be done by the Kolmogorov-Smirnov test. The hypothesis used as follows:
   \[ H_0 : \text{normal distributed residual} \]
   \[ H_1 : \text{not normal distributed residual} \]
   With test criteria, accept \( H_0 \) if p-value > (\( \alpha = 0.05 \))
   Also, a normality test can be done by looking for a residual plot
   It is seen that diagonal line data indicate normally distributed data. In addition, obtained p-value = 0.599 > (\( \alpha = 0.05 \)), it causing \( H_0 \) accepted. It can be concluded that normal distributed residual (normality assumption met).

2. Heteroscedasticity Test
The Linear regression model is assumed not occur in heteroscedasticity. A way to detect heteroscedasticity by forming a plot of standardized predicted value with studentized residual. This assumption is met if the plot has not a certain pattern. For scatterplot output seems that scatter point and not forming a clear pattern so that it can be concluded that it does not occur heteroscedasticity.

3. Multicollinearity Test
To testing the multicolinearity assumption of a variable, it can be done by looking for VIF value for each independent variable as table 1 as follows:

| Predictor | Coef  | SE Coef | T   | P    | VIF  |
|-----------|-------|---------|-----|------|------|
| \( X_1 \) | 0.0122 | 0.011   | 6.02| 0.000| 1.051|
| \( X_2 \) | 3.223  | 1.901   | 1.89| 0.056| 12.980|
| \( X_3 \) | -0.07  | 0.171   | -0.29| 0.699| 3.011|
| \( X_4 \) | -0.601 | 0.400   | -0.59| 0.120| 18.032|
| \( X_5 \) | -0.029 | 0.201   | -0.20| 0.901| 2.969|

Source: Simulation data processing

Based on Table 1 it seems that VIF value in which its value is larger of 10 by \( X_2 \) and \( X_4 \) variables so that it can be concluded that occur multicollinearity for these independent variables.

Transformation using Centering and Scaling Methods
It has been explained that the centering and rescaling method is a part of variable standardized. Its transformation is:

\[
Y_i^* = \frac{Y_i - \bar{Y}}{\sqrt{n - 1} S_Y}
\]

\[
X_{ij}^* = \frac{X_{ij} - \bar{X}_j}{\sqrt{n - 1} S_{x_j}}
\]

Where:
\[ \bar{Y} : \text{Mean of } Y \quad ; \quad \bar{X}_j : \text{Mean of observation } X_j \]

\[ S_Y : \text{Standard deviation of } Y \quad ; \quad S_{X_j} : \text{Standard deviation of } X_j.\]

In calculating transformation, we first were looking for mean and standard deviation. These results can be written in Table 2 as follows:

**Table 2. Mean and standard deviation of variables**

| Variable | Mean \((\bar{Y})\) | Standard Deviation |
|----------|------------------|-------------------|
| \(X_1\)  | 190.11           | 89.99             |
| \(X_2\)  | 3,030            | 0.4001            |
| \(X_3\)  | 50.1             | 1.889             |
| \(X_4\)  | 29.71            | 1.998             |
| \(X_5\)  | 29.98            | 2.011             |

Source: Simulation data processing

**Determining K Value and Estimator of GR Regression Parameter**

The determination of initial value k is obtained by \(k^0 = \frac{\hat{\sigma}^2}{\hat{\alpha}_j^2}\), where \(\sigma^2\) is MSE (Mean Square Error) of OLS (Ordinary Least Square) and \(\hat{\alpha}_j\) is estimator of parameter OLS of the transformed data. The value of \(k^0\) will be used to obtain estimator parameter \(\hat{\alpha}_{GR,j}^0\). Parameters \(\hat{\alpha}_{GR,j}^0\) are used to find the value of \(k^1\), and \(k^1\) is used to obtain estimator value of parameter \(\hat{\alpha}_{GR,j}^0\) and so on. This iteration will stop when \(|\hat{\alpha}_{GR} - \hat{\alpha}_{GR,j}^0| - (\hat{\alpha}_{GR}^0 \hat{\alpha}_{GR,j}^0)^{-1}| \leq 0.0001\).

By using Matlab R2018b software, we obtain estimator value of the initial parameter of GR \((\hat{\alpha}_{GR,j}^0)\) and it can be seen in the following Table 3.

**Table 3. Estimator result of initial iteration generalized ridge parameter**

| Parameter | \(\hat{\alpha}_{GR,j}^0\) |
|-----------|-------------------------|
| \(\hat{\alpha}_1\) | -0.0943                |
| \(\hat{\alpha}_2\) | -0.7195                |
| \(\hat{\alpha}_3\) | -0.0940                |
| \(\hat{\alpha}_4\) | -0.1250                |
| \(\hat{\alpha}_5\) | -0.8647                |

Source: Simulation data processing

Iteration process continues until we obtain value \(|\hat{\alpha}_{GR} - \hat{\alpha}_{GR,j}^0| - (\hat{\alpha}_{GR}^0 \hat{\alpha}_{GR,j}^0)^{-1}| \leq 0.0001\). Iteration process for the estimator of the generalized ridge parameter will stop at 5th iteration. Estimator value of generalized ridge parameter can be seen in the following Table.
Table 4. Estimator result of generalized ridge parameter

| Parameter | $\hat{\alpha}_{GR}$ |
|-----------|---------------------|
| $\hat{\alpha}_1$ | -0.0943 |
| $\hat{\alpha}_2$ | -0.7190 |
| $\hat{\alpha}_3$ | -0.0938 |
| $\hat{\alpha}_4$ | -0.1245 |
| $\hat{\alpha}_5$ | -0.8475 |

Source: Simulation data processing

Based on Table 4 it can be seen that the regression model that can be formed for the estimator of generalized ridge regression are:

$$\hat{y}_{GR} = -0.0943X_1 - 0.7190X_2 - 0.0938X_3 - 0.1245X_4 - 0.8475X_5$$

The Estimator of Jackknife Ridge Regression Parameter

The estimator of jackknife ridge regression parameters was calculated and the estimators were obtained as follows:

Table 5. Estimation result of Jackknife Ridge regression parameters

| Parameter | $\alpha_{JRR}$ |
|-----------|----------------|
| $\hat{\alpha}_1$ | -0.0946 |
| $\hat{\alpha}_2$ | -0.725 |
| $\hat{\alpha}_3$ | -0.0959 |
| $\hat{\alpha}_4$ | -0.1303 |
| $\hat{\alpha}_5$ | -1.0469 |

Source: Simulation data processing

Estimator result on Table 5 are transformed and obtained estimator $\beta_{JRR}$ as follows:

Table 6. Transformation result of Jackknife Ridge regression parameter estmation

| Parameter | $\beta_{JRR}$ |
|-----------|---------------|
| $\hat{\beta}_1$ | 0.8001 |
| $\hat{\beta}_2$ | 0.6998 |
| $\hat{\beta}_3$ | -0.0600 |
| $\hat{\beta}_4$ | -0.6991 |
| $\hat{\beta}_5$ | -0.0392 |

Source: Simulation data processing

Based on Table 6 we obtain jackknife ridge regression equation as follows:

$$Y^* = 0.8001X_1 + 0.6998X_2 - 0.0600X_3 - 0.6991X_4 - 0.0392X_5$$

(24)
Transformation of JRR Equations into Initial Form 1

The process of forming $X^*$ to $X$ can be performed on the estimator (21) by using a linear relationship:

$$\beta_j = \left( \frac{s_y}{s_{xj}} \right) \hat{\beta_j}^*, \ j = 1, 2, \ldots, k$$  \hspace{1cm} (25)

$$\beta_0 = \bar{Y} - \beta_1 \bar{X}_1 - \beta_2 \bar{X}_2 - \ldots - \beta_k \bar{X}_k$$  \hspace{1cm} (26)

So, we obtain

$$\beta_1 = \left( \frac{s_y}{s_{x1}} \right) \beta_1^* = 0.0140; \quad \beta_2 = \left( \frac{s_y}{s_{x2}} \right) \beta_2^* = 2.9769$$

$$\beta_3 = \left( \frac{s_y}{s_{x3}} \right) \beta_3^* = -0.0601; \quad \beta_4 = \left( \frac{s_y}{s_{x4}} \right) \beta_4^* = -0.5917$$

$$\beta_5 = \left( \frac{s_y}{s_{x5}} \right) \beta_5^* = -0.0403$$

$$\beta_0 = \bar{Y} - \beta_1 \bar{X}_1 - \beta_2 \bar{X}_2 - \beta_3 \bar{X}_3 - \beta_4 \bar{X}_4 - \beta_5 = 15.8973$$

Thus, regression equation obtained by using jackknife ridge assumption is:

$$\bar{Y} = 15.8973 + 0.0140X_1 + 2.9769X_2 - 0.0601X_3 + 0.5917X_4 - 0.0403X_5$$

To see the goodness of a regression model we can use the value of $R^2$ (the coefficient of determination), where a good regression model is a model with the largest $R^2$ value. The value of $R^2$ of MKT model is 29.77%, while $R^2$ value of jackknife ridge model is 65.21%. So, it can be concluded that the regression model with Jackknife ridge method is a good model used if occur violation of multicollinearity assumption on a data.

5. Conclusion

As a result of research that has been done and based on the explanations given, it can be taken some conclusions as follows:
- The iteration process is conducted until obtained a value of

$$\left| \left( \hat{\beta}_{{GR}} - \hat{\beta}_{{GR}}^{(-1)} \right) \right| \leq 0.0001$$

and iteration stops at 5th iteration, so we obtain estimator value of generalized ridge coefficient of the orthogonal independent variable ($\hat{\beta}_{{GR}}$).
- Obtained estimator of Jackknife Ridge regression coefficient parameter based on simulation data $\bar{Y} = 15.8973 + 0.0140X_1 + 2.9769X_2 - 0.0601X_3 + 0.5917X_4 - 0.0403X_5$.

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