PARAMETRISED TOPOLOGICAL COMPLEXITY
OF GROUP EPIMORPHISMS

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ABSTRACT. We show that the parametrised topological complexity of Cohen, Farber and Weinberger gives an invariant of group epimorphisms. We extend various bounds for the topological complexity of groups to obtain bounds for the parametrised topological complexity of epimorphisms. Several applications are given, including an alternative computation of the parametrised topological complexity of the planar Fadell–Neuwirth fibrations which avoids calculations involving cup products. We also prove a homotopy invariance result for parametrised topological complexity of fibrations over different bases.

1. Introduction

The topological complexity TC(X) of a space X is defined to be the Schwarz genus, or sectional category, of the endpoint fibration \( \pi: X^I \to X \times X \) which sends a free path to its pair of endpoints. This notion due to Michael Farber [10] is of potential applicability in the field of robotics, since sections of \( \pi \) correspond to motion planning algorithms for mechanical systems with X as their space of configurations.

One potential obstacle to applying TC(X) to robotics problems is that it assumes that the configuration space is known in advance. In order to address this, Cohen, Farber and Weinberger [4], [5] have introduced a parametrised version of topological complexity, which models motion planning problems for which

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