Single and Multiple Volume Reflections of Ultra-Relativistic Electrons in a Bent Crystal as Tools for Intense Production of Electromagnetic Radiation

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Abstract. Electromagnetic radiation emitted by ultrarelativistic volume-reflected and multi-volume-reflected electrons in a bent crystal is very intense and takes place over a broad angular range of the incident beam. Such radiation results to be nearly independent from the particles trajectory and charge. This paper describes the possible applications these features allow, from the production of a gamma or a positron source to the crystal-assisted collimation of future linear \(e^+/e^−\) colliders and crystal-based electromagnetic calorimeters.

1. Introduction
In the middle 50s, the scientific researches of Ferretti, Ter-Mikaelian, Dyson and Uberall [1-3] on the influence of the crystalline lattice on the motion and on the radiation production by charged particles crossing a crystal led to the discovery of the so-called \textit{coherent bremsstrahlung} (CB) by Diambrini-Palazzi et al. [4]. In 1964, Jens Lindhard introduced the concept of channeling [5] that fostered the interest to the possibility to generate intense radiation by channeled \(e^+\) inside a straight crystal, thus called \textit{channeling radiation} (CR). Channeling consists in the trapping of charged particles with energy \(E\) and with an angle of incidence with the crystal planes (axes) smaller than \(\theta_{\text{critical}} = \sqrt{2U_0/E}\) in the planar (axial) potential well with depth \(U_0\). Radiation generation by electrons/positrons in straight crystals has been investigated for decades [6], mainly in view of its possibility to be used as a powerful hard X-ray and gamma-radiation source. However, the generation of radiation in straight crystals takes place in a narrow angular region of orientation between crystal and beam direction [7], which becomes still narrower with the increasing of beam energy, thus limiting the possible applications. An opportunity to overcome such limitation is offered by the volume reflection (VR) effect in a bent crystal. In 1976, Tsyganov proposed the idea that in a bent crystal channeled particles could deviate from their initial direction just following the crystal channel [8]. Important applications
of bent-crystal-assisted manipulation of charged particle trajectories are beam extraction and halo cleaning [9]. VR consists in the reversal of transverse momentum of over-barrier particles by the interaction with the planar potential barrier at the tangency point of the trajectory with the curved crystalline planes [10]. As a result, volume-reflected particles are deflected by an angle of the order of the Lindhard angle (i.e., the critical angle for channeling) to a direction opposite to that of crystal bending. VR has a smaller deflecting power than channeling in a bent crystal, but it has the great advantage of larger deflection efficiency and wider angular acceptance, the latter being equal to the bending angle of the crystal. In order to increase the deflecting power of VR while maintaining its advantages, one can profit from the effect of multiple volume reflection in one crystal (MVROC) [11]. MVROC occurs in a bent crystal when the particles move at few channeling angles with respect to a crystal axis, when correlated string of strings (SOS) scattering assures the possibility of repeated VR from the planes sharing the same axis, leading to a multi-reflection process. VR and MVROC have been experimentally proven to work for either positively or negatively charged particles [12-16]. The e.m. radiation emitted by charged particles under VR and MVROC conditions has been also studied both theoretically and experimentally [17-26]. The large angular acceptance makes radiation accompanying VR and MVROC interesting for applications such as a gamma or a positron source or as an innovative scheme for crystal-assisted collimation in future electron-positron linear colliders or for crystal-based e.m. calorimeters.

In this paper we present a study of features and possible applications for radiation accompanying VR and MVROC.

2. Features of radiation accompanying VR

It is known that the type of e.m. radiation generated by charged particles in an external field depends on the peculiarities of particles motion in such a field. In general [6, 7, 27, 28] the main features of the radiation process is determined by the ratio of the deflection angle $\theta$ and the typical radiation angle $1/\gamma$, $\gamma$ being the Lorentz factor. One limit is at $\theta << 1/\gamma$, when mainly one harmonic is emitted (dipole radiation) and the other limit appears when $\theta >> 1/\gamma$, which corresponds to the case of synchrotron-like radiation. The same approximations are valid for quasi-oscillatory particles motion in straight crystals. Following the formalism of [6, 7], if the angle between the particles trajectory and the crystal planes (axes) is larger than $U_0/m$, the radiation has a dipole nature; in the opposite case the radiation is synchrotron-like. $U_0/m$ is independent on the energy, while the critical angle decreases with energy as $1/E$. As energy increases, CR becomes more and more synchrotron-like. Under the VR condition, the type of radiation changes during the particle motion inside the crystal. In fact, the average misalignment angle between the particle trajectory and the crystalline planes decreases as the particle approaches the reflection point [19]. Therefore, either the amplitude or the period of the particle motion between planes increases and the radiation regime turns from dipole- to synchrotron-like. As a result of multiple reflections on several planes, the planar radiation is more synchrotron-like for MVROC conditions than for a single VR. By a combination of several planar reflections occurring for MVROC with the contribution of atomic strings, the spectral intensity is very strong as compared to that of an individual VR. Furthermore, since the axial electric field is stronger than that for planes, the SOS contribution considerably increases the probability of hard-photon emission [29].

An example of the experimental results on VR and on MVROC for 120 GeV/c electrons is shown in figure 1 and 2. Figure 1 shows the experimental distribution of reflected electrons inside a bent Si crystal 2mm long, with a curvature radius $R= (2.71 \pm 0.07) m$. The crystal is bent along the [111] direction and the bent (110) planes are used to steer particles. The VR deflection angle is quite small, being equal to $\theta_{vr} = (-11.4 \pm 0.7) \mu rad$, while the MVROC deflection angle is larger, being $\theta_{mvr} = (-43.1 \pm 2.2) \mu rad$. On the contrary, the deflection efficiency is higher for single than for multiple VR, being $\varepsilon_{vr} = (95.5 \pm 0.8)\%$ and $\varepsilon_{mvr} = (85 \pm 3)\%$, respectively. Nevertheless, both the deflection efficiency and the angular acceptance for VR/MVROC are larger than that for channeling. More details on the experiment can be found in [21, 26].
Figure 1 Experimental distribution of the horizontal deflection angle of particles interacting with the 2 mm bent Si crystal aligned on VR (dot-dashed line) [21] and MVROC (solid line) [26].

Figure 2 displays the experimental energy-loss distributions under VR (circles with bars) and MVROC (squares with bars) for the 120 GeV/c electrons in the 2 mm long crystal. The details about the procedure carried out to determine the distribution, (dn/dE)E vs. the energy lost by electrons, E, can be found in [20, 21]. The energy loss spectral intensity for either VR or MVROC is much larger than the Bethe-Heitler (BH) value (see figure 2 dashed line), which is the typical one for bremsstrahlung in amorphous materials [30]. The greater intensity for MVROC than for VR in the soft-medium region of the spectra is simultaneously determined by both the SOS scattering and the planar field contribution. Harder photons are emitted due to the SOS CB-like process in the field of axis [25, 26, 29], explaining the much stronger intensity of radiation accompanying MVROC in the hard region of the spectrum. The mean number of photons, with Eγ >1 GeV, emitted by each particles (multiplicity factor) is about 1.35 and 2.2 for VR and MVROC cases, respectively [25].

The high intensity of radiation accompanying VR and MVROC, combined with their large angular acceptance that exceeds those for CB and CR in a straight crystal [19–26], can be exploited for many applications. As described above, the radiation mechanism changes during the particle motion depending on the direction of the particle trajectory with respect to the crystal planes and axes. The main point is that the dynamics of particles under VR and MVROC conditions is similar to each other for different incidence angles within the total angular acceptance. In other words, particles trajectories under VR or MVROC conditions remain more or less in the same interval of directions of motion with respect to the crystal planes over the whole angular acceptance. Similar particle dynamics means similar radiation generation. Another advantage of such type of radiation is its weak dependence on particle charge [19, 25].

Figure 2. Energy loss spectra for 120 GeV/c electrons in the bent Si crystal [26]; VR (circles with bars); MVROC (squares with bars); amorphous estimation (dashed line).
3. Possible applications

3.1. Gamma and positron source

All the features presented in Sec. 2, such as the high-intensity and the wide angular acceptance, make the radiation accompanying VR and MVROC very attractive for several applications at once.

Additional advantage of radiation generation relying on VR is its robustness vs. crystalline imperfections as compared to channeling-based effects [31]. In contrast to channeling, the presence of crystal defects is not crucial in the case of VR and the deflection efficiency is practically the same as for a perfect bent crystal. Moreover, the local nature of the reflection effect will also assure the weak sensitivity to crystal mosaic structure.

Such possibility envisages also the usage of higher-Z materials are better than Si for e.m. generation because of their higher atomic potential [32], but they cannot be produced with the same perfection as Si. For instance, since the average electric field strength in W is about an order of magnitude stronger than in silicon, we expect that the advantages of the radiation accompanying VR and MVROC measured in Si at 120 GeV will be maintained in W at lower energies (about 10 GeV), available at many worldwide electron accelerators. In order to maintain the advantages of a broad angular acceptance at lower energies, one has simply to adjust the crystal curvature.

For all these reasons, e.m. radiation under VR and MVROC in a tungsten crystal could be exploited to convert high-emittance electron beams to high-intensity γ-beam. In order to obtain an high-intensity gamma source, MVROC could be a preferable choice than VR due to the contribution of both crystal axis and multiple reflecting planes. A further opportunity for exploitation of such type of radiation is as a positron source starting from the idea of Ref. [33], where the authors proposed to use axial channeling to produce a high-intensity gamma beam to be then converted in positron-electron pairs through the BH mechanism.

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Channeling application in positron sources is quite limited by several factors, like the W crystal mosaicity, the primary electron beam divergence and crystal-to-beam alignment precision. Moreover, the electron channeling role in positron sources is essentially limited by dechanneling in W crystals with thicknesses necessary exceeding the radiation length [33–35]. Because of these reasons, the main mechanism of γ emission in e+ sources is not really axial channeling radiation, but axial radiation emitted by e- moving at angles considerably exceeding the axial channeling angle. Due to the high-multiplicity contribution of multiple reflecting planes, the MVROC effect could be exploited for an additional amplification of γ-emission, in experimental conditions similar to those already used in positron sources based on axial channeling [33–35].

3.2. Crystal-assisted collimation

One of the most attractive applications that would take advantage of the features of VR and MVROC from both the points of view of radiation and deflecting power could be a crystal-assisted collimation device for future linear electron-positron colliders, e.g., the International Linear Collider (ILC) [36]. ILC is a proposed electron-positron collider with a planned collision energy of 500 GeV and a possible upgrade to 1000 GeV. The beam will be delivered in pulsed bunches; these bunches are not uniformly distributed and have ‘halo’ particles which need to be removed to avoid unacceptable background in the detectors. To remove the halo particles, the beam will be collimated in its delivery system. The baseline design requires two-part collimators made up of small aperture spoilers with 0.5-1 radiation length (X₀) thickness close to the beam, with downstream absorbers 30 X₀ thick [37]. Since the spoilers are placed very close to the beam, the beam-spoiler interactions result in wakefield perturbations [38]. The insertion of a short (1-2 mm ~ 0.02 X₀) Si crystal instead of a Ti spoiler of some cm of length would diminish the wakefield perturbations. A proposal in this sense has already been put forward by Seryi [39]. Such collimation scheme consists in replacing one or more spoilers with bent crystals oriented with the beam under VR. The choice of VR instead of channeling is connected to its larger angular acceptance and larger deflection efficiency with respect to channeling. Particles deflection by VR would increase halo-cleaning efficiency per unit of length as compared to the case of an amorphous spoiler. Furthermore, the larger energy loss distribution in VR than in
amorphous media would improve the discrimination of halo particles, which will be deflected by forward magnets. The larger deflection and the greater energy loss in MVROC condition than the case of an individual VR, makes MVROC radiation even more suitable for crystal-assisted collimation of future linear colliders [26]. The choice between VR and MVROC will depend only on the required deflection efficiency and on the possibility of crystal-beam alignment in one or two directions. Fig. 3 shows a possible scheme for crystal assisted collimation through VR-MVROC and briefly summarizes the various steps described above. Finally, it should be reminded that the energy loss distribution under VR-MVROC conditions is nearly independent on the particle charge [25, 26] and so it is almost the same for either electrons or positrons, which makes these two coherent effects even more attractive for collimation in e± colliders.

Figure 3. Concept of linear collider collimation system based on VR-MVROC radiation phenomenon [39]. In the figure a. is the beam, having a core and a halo; b. is the bent crystal oriented in VR-MVROC conditions; c. solid lines are the particles from the beam halo that have been deflected and have lost energy due to VR-MVROC; d. dashed-line represents the photons produced during VR-MVROC; e. is the bending magnet, separating halo particles that have lost a significant part of their energy during VR-MVROC and f. is the absorber of halo particles and photons.

3.3. Crystal-based electromagnetic calorimeter
In high energy physics and γ-astronomy, e± and γ energies are measured by electromagnetic calorimeters. Typically, they are composed of scintillating crystals, which collect the light emitted by electromagnetic showers inside the calorimeter material itself [40, 41]. Since the full length of the showers depth can exceed 20 X₀ for ultrahigh energies (>100 GeV), large-Z element scintillators are used to reduce the X₀ and, so, the calorimeter size. Usually the scintillator crystals are used just for their properties of scintillation and shower development, while the orientational effects are not taken into account. However, it has been demonstrated that an enhancement of the e.m. shower due to coherent interactions, namely coherent radiation and coherent e± pair production, with respect to random incidence occurs if the beam impinges on the crystal along a main axis direction [6, 7, 42]. This fact has as a consequence a much faster development of the e.m. shower as compared to what happens in amorphous material. Thereby, one could still reduce the calorimeter thickness through e.m. shower development acceleration. The limitation also in this case is the narrow angular region in which all coherent effects manifest themselves, which imply that one should align precisely each scintillator crystal axis with the incoming e± and γ direction from the interaction region and take into consideration the details of the orientation dependence of the radiation and pair production processes probabilities when data are processed.

The radiation and pair production under VR-MVROC conditions allow one to overcome these difficulties, permitting the acceleration of e.m. shower development in a greater angular range of orientation between the particle direction and the crystal. In order to achieve this goal, all the scintillator crystals that compose the calorimeter have to be bent by angles somewhat larger than the local divergences of e± and γ arriving from the interaction point and to align the crystals to ensure VR of all the incident particles. In fact, it could not be necessary to bent the crystal over its whole length,
which is normally quite large (tens of cm), to obtain an enhancement of the e.m. shower, but just the first layer of some mm, which ensures VR condition to be valid. Alternatively, one could bend some mm thick crystals via the well-established technique of superficial grooving to obtain self-bent crystals [43]. Some of these crystals can be stuck together [44] and then stuck to the entrance surface of a larger straight scintillator crystal.

5. Conclusions
We have described the radiation accompanying volume reflection in a bent crystal and some possible applications of this type of radiation. The most peculiar characteristics of VR-MVROC radiation are the high intensity, the wide angular acceptance and the weak dependence on the particle charge and trajectory. The combination of these features makes these types of radiation suitable for the conversion of high-emittance electron beams into hard gamma quanta or for a positron source in high-Z crystals (like tungsten). By combining their high deflecting capability with their high radiative power, VR and MVROC can be envisaged as good candidates for crystal-assisted collimation in future e± linear collider. Finally, VR or MVROC could be exploited to drastically reduce the thickness of crystalline electromagnetic calorimeters in High-Energy-Physics, by reducing the longitudinal length of the electromagnetic shower.

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