Protocol for Energy-Efficiency using Robust Control on WSN

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Abstract – The present work analyzes the feasibility of obtaining a single controller (robust), with theoretical guarantees of stability and performance, valid for a total set of network configurations in designed the controller for an uncertain success probability obtain the protocol for Energy-Efficiency in Networked Control System NCS. In particular, this work investigates the performance degradation, in terms of the $H_\infty$ guaranteed cost, between optimal controller design (precisely known probability) and the sub-optimal controller design (robust to probability uncertainties). The feasibility of the proposed methodology is validated by a numerical example.

keywords: uncertain MJLS, Energy-Efficiency, NCS

1 Introduction

A full-reliable communication, independent of the network topology, corresponds to the most expensive configuration at the resource level. However, a full-reliable communication is necessary to implement the classical control design for dynamic systems. To improve the energy-efficiency in Network Control Systems (NCS) [1], particularly in multi-hop networks used to transmit packet of measurements or control signals, there exists in the literature an approach called Trade-Off in dynamic systems [2, 3]. The Trade-Off approach, for a particular network configuration, provides resource savings in wireless networks, admitting the employment of semi-reliable networks, because it allows a certain level of performance degradation associated to the controller [4].

The protocol for energy-efficiency proposed in [5] extends the Trade-Off approach to deal with time-varying network configurations. Such protocol modifies the maximum number of transmissions based on of the modification to the maximum number of transmissions per packet (MNTP) per node, in function of the current energy level of each wireless unit (node). To confine the MNTP in a bounded interval generates an impact on the probability of successful transmission per package between the plant and controller. A probability of successful transmission between source and sink depends on the current power of the wireless units (battery level in the node) [5], the number of possible configurations in the network depends on; i) number of nodes that modify their MNTP. ii) set of values that the MNTP can assume, for node. The example shown in [5] corresponds to a network with 16 nodes that modify their MNTP and each node can assume two different values of MNTP, implying $2^{16}$ possible probability of successful transmission.

The closed-loop system for is composed by different operation modes (depending on the reception or not of the signals) which can be conveniently modeled by called Markov jump linear systems (MJLS). In the literature [6], there exist design conditions for this class of systems that provide theoretical guarantees of stability and performance. For precisely known systems subject to packet loss, it is possible to design the optimal controller using the MJLS theory. Nevertheless, such controllers are valid only for unique network configuration (probability), which implies that in the example given in [5] the node responsible for calculating the control signal must store $2^{16}$ controllers, one for each network configuration. In terms of computational cost, the storage of $2^{16}$ controllers in Wireless Sensor Networks (WSN) can be inviable.

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The present work analyzes the feasibility of obtaining a single controller (robust), with theoretical guarantees of stability and performance, valid for a total set of network configurations (distinct probabilities of successful transmissions). The designed controller is robust for an uncertain success probability [7]. This project is based on a Markov chain whose state transitions are ruled by a polytopic matrix probability, which includes all the possible network configurations. In particular, this work investigates the performance degradation, in terms of the $H_\infty$ guaranteed cost, between optimal controller design (precisely known probability) and the sub-optimal controller design (robust to probability uncertainties). The feasibility of the proposed methodology is validated by a numerical example where the $H_\infty$ guaranteed cost obtained for robust approach is numerically identical to the one computed with optimal controller (precisely known system). Therefore, for the study case, the employment of a single robust controller provides the same results than using the optimal solution (that requires a storage of $2^{16}$ controllers) when applied to applications using the energy-efficiency protocol.

2 Preliminaries

Consider the notation used in [7], and the system

$$
go = \begin{cases} x(k+1) = A(\theta_k)x(k) + B(\theta_k)u(k) + E(\theta_k)w(k) \\ z(k) = C_z(\theta_k)x(k) + D_z(\theta_k)u(k) + E_z(\theta_k)w(k) \end{cases}$$

(1)

where $x(k) \in \mathbb{R}^{n_x}$ is the state vector, $u(k) \in \mathbb{R}^{n_u}$ is the control input, $w(k) \in \mathbb{R}^{n_w}$ is the noisy input, and $z(k) \in \mathbb{R}^{n_z}$ is the controlled output. The state-feedback control law is $u(k) = Kx(k)$. The state-space matrices depend upon a discrete-time homogeneous Markov chain $\theta_k$ which is associated to a uncertain transition probability matrix given by $\Gamma(\alpha) = [p_{ij}(\alpha)], \forall i, j \in \mathcal{X}$, where $p_{ij}(\alpha) = \Pr(\theta_{k+1} = j \mid \theta_k = i), \forall k \geq 0$, satisfying $p_{ij}(\alpha) \geq 0$ and $\sum_{j=1}^{\sigma} p_{ij}(\alpha) = 1$. In the study case, the random variable $\theta_k$ has a generalized Bernoulli distribution [8], which means that $\Gamma(\alpha)$ has $\sigma$ identical rows or $p_{ij}(\alpha) = p_j(\alpha), \forall i, j \in \mathbb{K}$, more details in [7]. As performance criterion, this work uses the $H_\infty$ norm ($\|\go\|_\infty$). One of the possible definitions for this norm is a ratio between the expected values of the exogenous input $w(k)$ and the output $z(k)$ for the worst case scenario of the signal $w(k) \in \mathcal{L}_2$ in [9], and reproduced below,

$$
\|\go\|_\infty^2 = \sup_{0 \neq w \in \mathcal{L}_2, \theta_0 \in \mathcal{K}} \frac{\|z(k)\|^2_2}{\|w(k)\|^2_2}.
\tag{2}
$$

3 Main Results: robust vs optimal $H_\infty$ Controller

The performance is quantified by the $H_\infty$ guaranteed cost associated with the optimal controller ($H_\infty^{OP}$) and with robust controller ($H_\infty^{RO}$). The $H_\infty$ guaranteed cost for optimal control design corresponds to actual system norm. On the other hand, the robust control design provides an upper bound for the $H_\infty$ norm of the system ($H_\infty$ guaranteed cost), being of interest the distance of the robust with respect to the optimal. To illustrate this, it is used an example of level control plant composed of two coupled tanks as shown in Fig. 1 (borrowed from [3]). The controlled output ($z(k)$) is a level variation of tanks 1 and 2, the physical parameters and space-state matrix for the system used in this example are given in [3] Section 6.1.

The state-space representation (1) of the system given in Fig. 1a is obtained according to the steps shown in [10] Example 2] and [5] Section 4.2]. The system (1) models the loss of the control signal $u(k)$ by the approach Zero-Input [11], forming two operation modes: a nominal system and a system with loss in the control signal represented by a matrix $B(\theta_k)$ with null elements.

$H_\infty$ guaranteed cost

The Fig. 2 shows the values of: the $H_\infty$ guaranteed cost ($H_\infty^{OP}(q)$) of the closed-loop system with the optimal control (precisely known probability) obtained from [7] Theorem 1, Corollary 1 and $\zeta = 10^4$, in function of the probability q of successful transmission of the control signal $u(k)$. Fig. 2 also displays the $H_\infty$ guaranteed cost ($H_\infty^{RO}(q)$) of the closed-loop system with the robust controller (polytopic probability matrix $P_\alpha^\sigma(\alpha)$ obtained from [7] Theorem 1 and $\zeta = 10^4$) in function of $q$. In this case the probability of successful transmission of the control signal $u(k)$ is $P_\alpha^\sigma(\alpha) \in [1, q]$. Note that both the optimal and the robust cases are numerically equal, implying that, in terms of the $H_\infty$ guaranteed cost, the design of a robust controller does not add conservatism to the studied problem.
Figure 1: (a) Operating system diagram of Example; (b) Level control plant used in Example.

4 Conclusion

The example proposed in [5] that employs Energy-Efficiency protocol based on WSN yield $2^{16}$ possible network configurations, requiring the same number of optimal controllers to obtain a certain $H_\infty$ performance level. In the present work, for the study case, it is shown that a single robust controller (sub-optimal design) can provide the same $H_\infty$ guaranteed costs without incorporate any conservatism. For resource-constrained networks, as WSN, the robust design presents two advantages: it eliminates the necessity of storing a hard number of controllers and also avoids the real-time monitoring of the current network configuration required to switching the controller. In this case, even a worse performance is acceptable, but in the case study, the improvements were obtained without loss of performance.

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Figure 2: $H_\infty$ guaranteed cost for the robust and optimal controller in function of $q$.

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