Orthogonal-state-based protocols of quantum key agreement

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Abstract

Two orthogonal-state-based protocols of quantum key agreement (QKA) are proposed. The first protocol of QKA proposed here is designed for two-party QKA, whereas the second protocol is designed for multi-party QKA. Security of these orthogonal-state-based protocols arise from monogamy of entanglement. This is in contrast to the existing protocols of QKA where security arises from the use of non-orthogonal state (non-commutativity principle). Further, it is shown that all the quantum systems that are useful for implementation of quantum dialogue and most of the protocols of secure direct quantum communication can be modified to implement protocols of QKA.

Keywords: Quantum key agreement, multi-party key agreement, quantum cryptography, orthogonal-state-based quantum key agreement.

1 Introduction

Since Bennett and Brassard [1] proposed the first protocol of unconditionally secure quantum key distribution (QKD), several aspects of secure quantum communication have been explored [2,3,4,5,6,7]. One such idea is quantum key agreement (QKA) [8,9,10,11]. There are two notions of QKA. In the weaker notion of QKA that was followed in [12] a key is generated by two or more parties through the negotiation which happens in public. Under this weaker notion of QKA many of the existing protocols of QKD can be viewed as protocols of QKA. For example, well-known BB84 [1], Ekert [2] and B92 [3] protocols of QKD qualify as protocols of QKA if we follow the weaker notion of QKA introduced in [12]. However, we are interested in a stronger notion of QKA that was introduced in Ref. [8] and is subsequently followed in all the recent works on QKA [9,10,13,14,15,16,17,18,19]. In this notion of QKA all the parties involved in the key generation process contribute equally to construct the key. This is in contrast to QKD where a single party can control the entire key. Before we introduce new protocols of QKA it is important to understand the differences between key distribution (KD) and key agreement (KA) in further detail. In a KD protocol, a trusted authority (TA) chooses a secret key that will be used in future for communication, and transmits (distributes) it to other parties who want to communicate. In contrast, in a KA scheme (KAS): two or more parties establish a secret key on their own. Thus in two-party scenario we may say that in protocols of KD, a key is created by Alice and the same is securely transmitted to Bob, while in the protocols of KA, both Alice and Bob contribute information that is subsequently used to derive the shared secret key. Further, in a good KAS each party contributes equally to the shared key and a dishonest party or a group of dishonest parties cannot control or completely decide the final key. The last point shows why all the traditional protocols of quantum cryptography e.g., BB84 [1], B92 [3], ping-pong (PP) [3], LM05 [3] etc. are not protocols of QKA in their original forms.

Several protocols of classical key agreement are studied since the well known Diffie-Hellman (DH) key agreement protocol or the exponential key agreement protocol was introduced by Diffie and Hellman in 1976 [20]. A large number of the classical key agreement protocols are actually variant of the DH protocol as they are based on intractability of the DH problem [21] and references therein. To be precise, security of these protocols depends on the intractability of discrete logarithm (DL) problem which may be stated as follows: given a generator $g$ of a cyclic group $G$ and an element $g^x$ in $G$, determine $x$. Quite similarly, the DH problem is stated as: given $g^x$ and $g^y$, determine $g^{xy}$ [22]. Clearly if we can solve DL problem in polynomial time then we will be able to solve DH problem in polynomial time. As there is no efficient classical algorithm for DL problem, modified and improved DH protocols have been considered to be secure for long. Interestingly...
in 1997, Shor introduced polynomial-time quantum algorithms for prime factorization and discrete logarithms [23]. These two quantum algorithms clearly established that neither the RSA protocol nor the DH based KA protocols would remain secure if a scalable quantum computer is built. This fact along with the already established unconditional security of QKD enhanced the interest on QKD and QKA.

First protocol of QKA was introduced by Zhou et al. in 2004 [8] using quantum teleportation. Almost simultaneously Hsueh and Chen [24] proposed another protocol of QKA. However, in 2009, Tsai and Hwang [13] showed that quantum teleportation based Zhou et al. protocol was not a true protocol of QKA as a particular user can completely determine the final (shared) key without being detected. Next year Tsai et al. [14] showed that even protocol of Hsueh and Chen does not qualify as a protocol of QKA. In 2010, Chong and Hwang [9] developed a protocol of QKA using mutually unbiased bases (MUBs). Apparently Chong Hwang (CH) protocol was the first successful protocol of QKA. They claimed that their protocol is based on BB84. However, a deeper analysis would show that their protocol is closer to LM05 protocol [6]. Of course the security of both LM05 and BB84 protocols arises from the non-commutativity and cloning principles. In 2011, Chong, Tsai and Hwang [15] proposed a modified version of Hsueh and Chen protocol that is free from the limitations of the original protocol mentioned in Ref. [14]. All the successful and unsuccessful efforts of designing protocols of QKA until recent past were limited to two party case. Recently an enhanced interest on multi-party QKA schemes has been observed and several protocols have been reported [10, 16, 17, 18, 19]. A systematic review of all these existing works leads us to the following observations.

1. The amount of works reported to date on QKA is much less compared to the amount of works reported on other aspects of quantum cryptography, such as QKD, deterministic secure quantum communication (DSQC), quantum secure direct communication (QSDC) and quantum dialogue (QD). Thus we may conclude that QKA is not yet studied rigorously and probably many more combinations of quantum states and protocols of QKA can be found. Keeping this in mind we show that majority of the existing protocols of QSDC, DSQC and QD can be turned into protocol of QKA by introducing a delayed measurement technique.

2. Security of all the protocols of two-party and multi-party QKA reported to date is based on conjugate coding, i.e., the security is obtained using two or more MUBs and thus the protocols are essentially of BB84 type. This lead to a question: Is it essential to use non-orthogonal states (2 or more MUBs) for designing of protocols of QKA? The question is not yet answered, but the expected answer is “no” as QKA is related to QKD and a few orthogonal-states-based protocols of QKD (e.g., Goldenberg-Vaidman (GV) protocol [14] and N09 or counter-factual protocol [25]) are known since a few years. Further, some of the present authors have recently shown that protocols of QSDC and DSQC can be designed using orthogonal states [26, 27]. In addition several exciting experiments on orthogonal-state-based QKD are reported in recent past [28, 29, 30, 31]. These recent experimental observations and the recently proposed orthogonal-state-based protocols are very interesting as they are fundamentally different from the traditional conjugate coding based protocols where two or more MUBs (set of non-orthogonal states) are used to provide security. Keeping these in mind present paper aims to provide orthogonal-state-based protocols of 2-party and multi-party QKA.

Remaining part of the paper is organized as follows. In the next section we present a protocol of QKA for 2-party scenario. In Section 3 we provide a protocol of 3-party QKA and discuss the possibilities of extending it to n-party (n > 3) scenario. Specifically, we have shown that the proposed 3-party protocol can be extended to a 5-party protocol of QKA that uses 4-qubit $|\Omega\rangle$ state or 4-qubit cluster state. In Section 4 security and efficiency of the proposed protocols are discussed and are compared with that of existing protocols of QKA. In Section 5 we investigate the possibilities of transforming the existing protocols of QSDC, DSQC and QD to protocols of QKA. Finally the paper is concluded in Section 6.

2 Protocol 1: A 2-party orthogonal-state-based protocol of QKA

**Step 1:** Alice prepares $|\psi^+\rangle^\otimes n$ where $|\psi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$. She uses first qubits of each Bell state to form an ordered sequences $p_A = \{p_A^1, p_A^2, p_A^3, \cdots, p_A^n\}$. Similarly she forms an ordered sequence $q_A = \{q_A^1, q_A^2, q_A^3, \cdots, q_A^n\}$ with all the second qubits. Here $p_A^i$, $q_A^i$ denote the first and second particles of $i^{th}$ copy of the Bell state $|\psi^+\rangle$, for $1 \leq i \leq n$. She also prepares a random sequence $K_A = \{K_A^1, K_A^2, K_A^3, \cdots, K_A^n\}$, where $K_A^i$ denotes the $i^{th}$ bit of sequence $K_A$ and $K_A^i$ is randomly chosen from $\{0, 1\}$. $K_A$ may be considered as Alice’s key.

**Step 2:** Alice prepares a sequence of $\frac{4}{n}$ Bell states ($|\psi^+\rangle^\otimes \frac{4}{n}$) as decoy qubits and concatenates the sequence with $q_A$ to form an extended sequence $q_A'$. She applies a permutation operator $\Pi_{2n}$ on $q_A'$ to create a new sequence $\Pi_{2n}q_A' = q_A''$ and sends that to Bob.

**Step 3:** After receiving the authentic acknowledgment of the receipt of the entire sequence $q_A''$ from Bob, Alice announces the coordinates of the qubits ($\Pi_{2n}$) sent by her. Using the information Bob rearranges the qubits and performs Bell measurements on the decoy qubits and computes the error rate. Ideally in absence of Eve all the decoy Bell states
are to be found in \(|\psi^+\rangle\). If the error rate is found to be within the tolerable limit, they continue to the next step, otherwise they discard the protocol and go back to Step 1.

**Step 4**: Bob drops the decoy qubits to obtain \(q_A\). Now he prepares a new random sequence \(K_B = \{K_{B_1}^1, K_{B_2}^2, K_{B_3}^3, \cdots, K_{B_n}^n\}\), where \(K_{B_i}^j\) denote the \(i\)th bit of sequence \(K_B\), for \(1 \leq i \leq n\) and \(K_{B_i}^j\) is randomly chosen from \{0, 1\}. \(K_B\) may be considered as Bob’s key. He applies a unitary operation on each qubit of sequence \(q_A\) to encode \(K_B\). The encoding scheme is as follows: to encode \(K_B^i = 0\) and \(K_B^i = 1\) he applies \(I\) and \(X\) respectively on \(q_A^i\). This forms a new sequence \(q_B\). After encoding operation, Bob concatenates \(q_B\) with a sequence of \(\frac{2}{3}\) Bell states (\(|\psi^+\rangle \otimes \frac{2}{3}\)) that is prepared as decoy qubits and subsequently applies the permutation operator \(I_{2n}\) to obtain an extended and randomized sequence \(q_B'\) which he sends to Alice.

**Step 5**: After receiving the authenticated acknowledgment of the receipt of the entire sequence \(q_B'\) from Alice, Bob announces the position of the decoy qubits (note that he does not disclose the actual order of the message qubits) i.e., \(\Pi_n \in I_{2n}\). Alice checks the possibility of eavesdropping by following the same procedure as in Step 3. If the error rate is found to be within the tolerable limit, they continue to the next step, otherwise they discard the protocol and go back to Step 1.

**Step 6**: Alice publicly announces her key \(K_A\) and Bob uses that and his own key (sequence) \(K_B\) to form the shared key: \(K = K_A \oplus K_B\).

**Step 7**: Bob announces the actual order of the message qubits (\(\Pi_n \in I_{2n}\)) and Alice uses that information to obtain \(q_B\). Now she combines \(p_A\) and \(q_B\) and performs Bell measurements on \(p_A'q_B'\). This would reveal \(K_B\) as she knows the initial state and the encoding scheme used by Bob.

**Step 8**: Using \(K_A\) and \(K_B\) Alice prepares her copy of the shared key i.e., \(K = K_A \oplus K_B\).

The protocol discussed above is an orthogonal-state-based 2-party protocol of QKA. However, several multi-party protocols of QKA are introduced in recent past [10, 16, 17, 18, 19]. Of course none of these recently introduced multi-party QKA protocols are based on orthogonal state. Keeping these in mind we aim to provide a completely orthogonal-state-based three-party protocol into \(n\)-party case with \(n > 3\) is also discussed in the following section.

### 3 Protocol 2: A multi-party protocol of QKA

In analogy to the previous protocol Alice, Bob and Charlie produce their secret keys:

\[
K_A = \{K_A^1, K_A^2, K_A^3, \cdots, K_A^n\}, \quad K_B = \{K_B^1, K_B^2, K_B^3, \cdots, K_B^n\}, \quad K_C = \{K_C^1, K_C^2, K_C^3, \cdots, K_C^n\},
\]

where \(K_A^i, K_B^i, K_C^i\) denote \(i\)th bit of key of Alice, Bob and Charlie respectively and \(i = 1, 2, \cdots, n\). We describe a protocol of multi-party QKA in the following steps.

**Step 1**: Alice, Bob and Charlie separately prepare \(|\psi^+\rangle_{A}^\otimes n, |\psi^+\rangle_{B}^\otimes n\) and \(|\psi^+\rangle_{C}^\otimes n\), respectively. As in Step 1 of the previous protocol Alice prepares two ordered sequences \(p_A = \{p_A^1, p_A^2, p_A^3, \cdots, p_A^n\}\) and \(q_A = \{q_A^1, q_A^2, q_A^3, \cdots, q_A^n\}\) composed of all the first and the second qubits of the Bell states that she has prepared. Similarly, Bob and Charlie prepare \(p_B = \{p_B^1, p_B^2, p_B^3, \cdots, p_B^n\}, q_B = \{q_B^1, q_B^2, q_B^3, \cdots, q_B^n\}\) and \(p_C = \{p_C^1, p_C^2, p_C^3, \cdots, p_C^n\}, q_C = \{q_C^1, q_C^2, q_C^3, \cdots, q_C^n\}\) from \(|\psi^+\rangle_{B}^\otimes n\) and \(|\psi^+\rangle_{C}^\otimes n\), respectively.

**Step 2**: Each of Alice, Bob and Charlie separately prepares sequence of \(\frac{2}{3}\) Bell states (\(|\psi^+\rangle \otimes \frac{2}{3}\)) with \(j \in \{A, B, C\}\) as decoy qubits and concatenates the sequence with \(q_j\) to form extended sequences \(q_j'\). Subsequently user \(j\) applies permutation operator \((\Pi_{2n})_j\) on \(q_j'\) to create a new sequence \((\Pi_{n})_jq_j' = q_j''\) and sends that to user \(j + 1\).

Here we follow a notation in which \(j \in \{A, B, C\}\) and \(A, B, C\) follows a modulo 3 algebra that gives us the relations: \(A + 3 = B + 2 = C + 1 = A\), \(A = C + 1\), \(B = A + 1\), \(C = B + 1\) and so on.

**Step 3**: After receiving the authentic acknowledgment of receipt from the receiver (user \(j + 1\)) corresponding sender (user \(j\)) announces the coordinates of the qubits \((\Pi_{2n})_j\) sent by him/her. Each receiver computes error rate as in Step 3 of the previous protocol. If the computed error rates are found to be within the tolerable limit, they continue to the next step, otherwise they discard the protocol and go back to Step 1.

\(^1\)Here subscripts A, B, C denote Alice, Bob and Charlie, respectively.
Step 4: After discarding the decoy qubits each user $j$ encodes his/her secret bits by applying the unitary operation on each qubit of the sequence received by him (i.e., on $q_{j-1}$) in accordance with his/her key $K_j$. The encoding scheme is as follows: If $K_j = 0 (1)$ then user $j$ applies $I (X)$ on $q_{j-1}$. As a result of encoding operations, user $j$ obtains a new sequence $r_j$. After the encoding operation user $j$ concatenates $r_j$ with a sequence of $\frac{n}{2}$ Bell states ($\langle \psi^+ \rangle \otimes \frac{1}{2}$) $j$ that is prepared as decoy qubits and subsequently applies the permutation operator $(\Pi_{2m})_j$ to obtain an extended and randomized sequence $r_j'$ which he/she sends to the user $j + 1$.

Step 5: After receiving the authentic acknowledgment of the receipt of the sequence $r_j'$ from the receiver $j + 1$, the sender $j$ announces the coordinates of the decoy qubits i.e., $(\Pi_m)_j \in (\Pi_{2m})_j$. User $j + 1$ uses the information for computing the error rate as before and if it is below the threshold value then they go on to the next step, otherwise they discard the communication. In absence of eavesdropping user $j$ announces the coordinates of the message qubits i.e., $(\Pi_m)_j \in (\Pi_{2m})_j$.

Step 6: Same as Step 4 with only difference that if $K_j = 0$ and $K_j' = 1$ then user $j$ applies $I$ and $Z$ respectively on $r_j$. As a result of encoding operations user $j$ obtains a new sequence $s_j$ and after insertion of decoy qubits and applying permutation operator he/she obtains a randomized sequence $s_j'$ which he/she sends to the user $j + 1$.

Step 7: Same as Step 5.

Step 8: After discarding the decoy qubits each user rearranges the sequence received by him/her. Now each user $j$ has two ordered sequences $p_j$ and $s_j$. Each of the users $j$ performs Bell measurements on $p_j s_j'$ after following the strategy for eavesdropping checking. Subsequently, user $j$ announces the coordinates of the decoy qubits $i.e.$, $(\Pi_m)_j \in (\Pi_{2m})_j$.

| Initial state prepared by user $j$ | First operator applied by user $j + 1$ | Second operator applied by user $j + 2$ | Final State |
|-----------------------------------|---------------------------------|---------------------------------|-------------|
| $\langle \psi^+ \rangle$          | $I \otimes I$                   | $I \otimes Z$                  | $\langle \psi^+ \rangle$ |
| $I \otimes I$                     | $I \otimes Z$                  | $\langle \psi^+ \rangle$       |
| $I \otimes X$                     | $I \otimes I$                   | $\langle \phi^- \rangle$       |
| $I \otimes X$                     | $I \otimes Z$                  | $\langle \phi^- \rangle$       |

Table 1: Transformation of $\langle \psi^+ \rangle$ based on two operations. Here $+$ refers to modulo 3 operations. $j \in \{A, B, C\}$ where $A, B, C$ stands for Alice, Bob and Charlie, respectively. Thus $A + 2 = C = A - 1$ and so on. Further, to denote the Bell states, we have used the following conventions: $\langle \psi^+ \rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and $\langle \phi^- \rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$.

Here we note that the $\{I, X, iY, Z\}$ is a modified Pauli group under multiplication and $\{I, X\}, \{I, Z\}$ are its disjoint subgroups. Here disjoint subgroups refer to two subgroups $g_i$ and $g_j$ of a group $G$ that satisfy $g_i \cap g_j = \{I\}$, where $I$ is the identity element. Thus except identity element $g_i$ and $g_j$ do not contain any other common element. Now we assume that $G$ is a group of order $M$ under multiplication and elements of $G$ are $x$-qubit unitary operators. Further, we assume that there exist $n$ mutually disjoint subgroups $g_i$ with $i = 1, \ldots, n$ of the group $G$ such that $g_i$’s are of equal size (say each of the $g_i$’s has $2^y$ elements) and $\Pi_m^e g_i = g_1 \otimes g_2 \otimes g_3 \otimes \cdots \otimes g_m = \{U_1, U_2, \ldots, U_{(2^y)^m}\}$ where $(2^y)^m \leq M$; $U_i \in G$ and $U_i \neq U_i \forall i, l \in \{1, 2, \ldots, (2^y)^m\}$. Now if we have $T^{e(N-x)}U_i|\phi_0\rangle = |\phi_i\rangle$ and $\langle \phi_i|\phi_i\rangle = \delta_{i,l}$ where $|\phi_i\rangle$ is an $N$-qubit quantum state with $N > x$, then we can have an $(m + 1)$-party version of Protocol 2 of QKA. In this $(m + 1)$-party protocol of QKA all the $(m + 1)$ parties create quantum state $|\phi_0\rangle$ in the beginning. Each user keeps the first $N - x$ qubits of $|\phi_0\rangle$ with himself/herself and sends the remaining qubits to the user $j + 1$ after following the strategy for eavesdropping checking. Subsequently, user $j$ encodes his/her $y$-bit secret key ($N > x \geq y$) by applying unitary operators from $g_1$ on the $x$ qubits that he/she has received from the user $j - 1$ in the previous step and sends the key encoded state to user $j + 1$. After $m$ rounds of such encoding (in $k^{th}$ round of encoding each of the users encodes their keys using elements of $g_k$) and communication operations user $j$ measures the $N$ qubits of his/her possession using $\{|\phi_i\rangle\}$ basis. From the input state $|\phi_0\rangle$ and output state (say, $|\phi_{\text{final}}\rangle = |\phi_k\rangle$) he/she would know the unitary operator $U_k$ that has converted the initial state into the final state. Now the condition $\Pi_m^e g_1 = \{U_1, U_2, \ldots, U_{(2^y)^m}\}$ where $U_i \in G$ and $U_i \neq U_i$ ensures that every sequence of encoding operations will lead to different $U_k$ and this is how user $j$ can know the key encoded by the other users and he/she can use that to create the shared key $K_1 \oplus K_2 \oplus \cdots \oplus K_m$, where the secret key of the user $j$ is $K_j$.

\[1\] In the stabilizer formalism of quantum error correction Pauli group is frequently used (see Section 10.5.1 of [32]). It is usually defined as $G_1 = \{ \pm I, \pm X, \pm Y, \pm Z, \pm \sigma_x, \pm \sigma_y, \pm \sigma_z, \pm i\sigma_z \}$, where $\sigma_i$ is a Pauli matrix. The inclusion of $\pm 1$ and $\pm i$ ensures that $G_1$ is closed under standard matrix multiplication, but the effect of $\sigma_x, -\sigma_x, i\sigma_x, -i\sigma_x$ on a quantum state is the same. So in [32] we redefined the multiplication operation for two elements of the group in such a way that global phase is ignored from the product of matrices. This is consistent with the quantum mechanics and it gives us a modified Pauli group $G_1 = \{ I, \sigma_x, i\sigma_y, \sigma_z \} = \{ I, X, iY, Z \}$. 
In Protocol 2 we have used modified Pauli group \( G = G_1 = \{I, X, iY, Z\} \). It has three disjoint subgroups: \( g_1 = \{I, X\}, g_2 = \{I, Z\}, g_3 = \{I, iY\} \) which satisfy \( g_1 \otimes g_2 = g_2 \otimes g_3 = g_3 \otimes g_1 = G_1 \). Further, \( |\phi_0\rangle = |\psi^+\rangle \) and as \( G_1 \) is the set of elements used for dense coding using Bell states so it naturally implies \( U_i|\phi_0\rangle = |\phi_i\rangle \forall U_i \in G : \langle \phi_i|\phi_i\rangle = \delta_{i,1} \). Thus Protocol 2 is a special case of a more general scenario described here. Many more examples can be obtained from the properties of Pauli groups discussed in Ref. [33]. Just to provide specific examples we may note that for the modified Pauli group

\[
G_2 = G_1 \otimes G_1 = \{I, X, iY, Z\} \otimes \{I, X, iY, Z\} = \{I \otimes I, I \otimes X, I \otimes iY, I \otimes Z, X \otimes I, X \otimes X, X \otimes iY, X \otimes Z, iY \otimes I, iY \otimes X, iY \otimes iY, iY \otimes Z, Z \otimes I, Z \otimes X, Z \otimes iY, Z \otimes Z\}
\]

we have following disjoint subgroups of order 2: \( g_1 = \{I \otimes I, I \otimes X\}, g_2 = \{I \otimes I, X \otimes I\}, g_3 = \{I \otimes I, I \otimes Z\}, g_4 = \{I \otimes I, Z \otimes I\}, g_5 = \{I \otimes I, iY \otimes iY\} \) and \( g_6 = \{I \otimes I, iY \otimes I\} \). Further, these disjoint subgroups satisfy

\[
g_1 \otimes g_2 \otimes g_3 \otimes g_4 = g_1 \otimes g_2 \otimes g_5 \otimes g_6 = g_3 \otimes g_4 \otimes g_5 \otimes g_6 = G_2
\]

and the elements of \( G_2 \) can be used for dense coding using 4-qubit maximally entangled \( |\Omega\rangle \) state and cluster \( (|C\rangle) \) state if the elements of \( G_2 \) operate on 1\(^{st}\) and 3\(^{rd}\) qubits of these states. Here

\[
|\Omega\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |0110\rangle + |1001\rangle - |1111\rangle),
\]

\[
|C\rangle = \frac{1}{\sqrt{2}}(|0001\rangle + |0110\rangle - |1001\rangle - |1111\rangle).
\]

The table of dense coding for these states using elements of \( G_2 \) is explicitly shown in our earlier work (see Table 1 of Ref. [33]). As the elements of \( G_2 \) can be used for dense coding using \( |\Omega\rangle \) and \( |C\rangle \) states, output states obtained on application of the elements of \( G_2 \) on \( |\Omega\rangle \) or \( |C\rangle \) are mutually orthogonal. This clearly implies that we can construct a 5-party protocol of QKA using \( |\Omega\rangle \) or \( |C\rangle \) state where each user prepares a large number copies of one of these two states and keeps 2\(^{nd}\) and 4\(^{th}\) qubit with himself/herself and sends the remaining qubits to next user and later encodes his/her secret key using \( g_i \)'s. In a 5-party protocol, encoding operation take place in 4 rounds and the users would use either \( g_1, g_2, g_3 \) or \( g_1, g_2, g_5, g_6 \) or \( g_3, g_4, g_5, g_6 \). We can generate many more examples of multi-party protocols of QKA using similar strategy and properties of modified Pauli group.

## 4 Security and efficiency analysis

Protocol 2 is designed along the line of existing protocol of Yin, Ma and Liu [10] with a modified strategy of eavesdropping checking that converts the non-orthogonal-state-based protocol of Yin, Ma and Liu into an orthogonal-state-based protocol. Unconditional security of the eavesdropping checking using this technique is already shown in our earlier works [26, 27] where we have also established that security of this orthogonal-state-based technique of eavesdropping checking originates from the monogamy of entanglement [24]. Thus the protocol is secure against external attacks (eavesdropping). Remaining part of the protocol is technically equivalent to Yin Ma Liu (YML) protocol and consequently the security of YML protocol against the internal attacks (i.e., the attempts of malicious Alice, Bob and Charlie to completely control the key either individually or by mutual cooperation of any two users) is applicable here, too. Thus Protocol 2 is a secure protocol of QKA and it does not need any separate elaborate discussion. Keeping this in mind in the remaining part of the present section we have explicitly analyzed the security of Protocol 1.

### 4.1 Security against eavesdropping

Our Protocol 1 and also the protocol of Chong and Hwang [11] may be viewed as protocols of secure direct communication of \( K_B \) from Bob to Alice added with a classical communication of \( K_A \) from Alice to Bob. Specifically, instead of sending a meaningful message Alice and Bob send random keys to each other. While Bob sends his key \( K_B \) by using a DSQC or QSDC scheme, Alice announces her key \( K_A \) publicly. Security proofs of the existing protocols of DSQC and QSDC ensures that the key communicated by Bob (i.e., \( K_B \)) using DSQC or QSDC scheme is unconditionally secure. Thus Eve has no information about \( K_B \). On the other hand the key communicated by Alice (i.e., \( K_A \)) is a public knowledge. However, it does not affect the secrecy of the shared key as the final shared key to be produced and used is \( K_A \oplus K_B \), knowledge of \( K_A \) alone does not provide any information about \( K_A \oplus K_B \). Thus the shared key produced in this manner is secure from external attacks of Eve. However, there may exist insider attacks in which Alice or Bob tries to completely control the shared key. Security of Protocol 1 against such attacks is described below.

#### 4.1.1 Security against dishonest Alice

To communicate \( K_B \) if Alice and Bob use a standard protocol of DSQC or QSDC (say they use PP protocol), then it would be possible for Alice to know Bob’s secret key before she announces \( K_A \). In that case she will be able to completely
control the shared key by manipulating $K_A$ as per her wish. To circumvent this attack we have modified the protocol in such a way that Bob does not announce the coordinates of the message qubits sent by him till he receives $K_A$. This strategy introduces a delay in measurement of Alice and this delayed measurement strategy ensures that Alice cannot control the key by knowing $K_B$ prior to her announcement of $K_A$.

### 4.1.2 Security against dishonest Bob

Alice announces her key only after receiving the message qubits (without their actual order) from Bob. This ensures that Bob cannot control the key by knowing Alice’s key. Only thing that Bob can do after knowing $K_A$ is to change/modify the coordinates of $q'_B$, but any modification in that would lead to entanglement swapping in our case and that would lead to probabilistic outcomes without any control of Bob. Further, Bob will be completely unaware of $K_B$ to be generated by Alice in that case and as a consequence any such effort of Bob would lead to different keys at ends of Alice and Bob. Thus the protocol ensures that Bob cannot control the key. Here we may note that similar strategy was used in Chong and Hwang protocol. In their protocol modified QSDC scheme that was used for Bob to Alice communication was equivalent to LM05 protocol. In contrast here we have used a modified orthogonal version of PP-type protocol which may be referred as PP$^{GV}$ protocol.

### 5 Turning existing protocols of quantum communication to protocols of QKA

In the previous sections we have seen that there exist a strong link between protocols of DSQC/QSDC and those of QKA. For example, PP and LM05 protocols of QSDC have already been employed to design protocols of QKA (Protocol 1 presented here and CH protocol). This observation leads to an important question: Is it possible to convert all protocols of secure direct quantum communication into protocols of QKA? In what follows we aim to answer this question. We also aim to study the possibilities of transforming other protocols of quantum communication to protocols of QKA.

#### 5.1 Turning a protocol of QSDC/DSQC to a protocol of QKA

Recently we have shown that maximally efficient protocols for secure direct quantum communications can be constructed using any arbitrary orthogonal basis. However, all of them will not lead to protocol of QKA. To be precise, eavesdropping can be avoided in all protocols of DSQC and QSDC and by randomizing the sequence of key encoded bits sent by Bob (i.e., by delaying the measurement to be performed by Alice) we can circumvent the attacks of dishonest Alice, but it is not sufficient to build a protocol of QKA. We also need to avoid the attacks of dishonest Bob. To do so we need to restrict the information available to Bob. Specifically, Bob must not have complete information of the basis that is used to prepare the qubits on which he has encoded his key. In our Protocol 1 and in all orthogonal-state-based two-way DSQC/QSDC protocols this can be achieved if Alice keeps some of the qubits of each entangled state with her as that would restrict Bob from changing $K_B$ after receiving $K_A$. The same can be achieved in a non-orthogonal-state-based protocol by using more than one MUBs. If Alice prepares the state randomly using one of the basis sets and don’t disclose the basis set used by her till Bob discloses the sequence then Bob will not have complete access of the basis used for preparation of the message qubits. As a consequence he will not be able to control the key. This is shown in a particular case in Ref.

The above discussion shows that the DSQC/QSDC protocol to be used to implement a QKA protocol cannot be one-way as in that case Bob will have complete access to the basis in which the quantum state used for encoding of his key is prepared (since in a one-way protocol Bob himself will prepare the quantum state). Thus none of the one-way protocol of DSQC or QSDC would lead to QKA. However, most of the two-way protocols of secure quantum communication would lead to QKA. As example, we may note both Deng Long Liu (DLL) protocol and Cai Li (CL) protocol can be viewed as variant of PP protocol, but DLL being a one-way protocol would not give us a QKA protocol, but two-way CL protocol would lead to a QKA protocol.

#### 5.2 Turning a protocol of QD to a protocol of QKA

A very interesting two-way quantum communication scheme is QD and references therein. Since in the above we have already seen that two-way secure direct communication is useful for QKA and since a large number of alternatives for implementing quantum dialogue are recently proposed by us (see Table 4 of Ref.), it would be worthy to investigate the relation between QKA and QD. In a Ba An type QD protocol, Alice keeps part of an entangled state with herself and encodes her secret on the remaining qubits by applying unitary operation $U_A$ and subsequently sends the message encoded qubits to Bob who applies $U_B$ on them and returns the qubits to Alice with appropriate strategy of eavesdropping checking. Now Alice measures the final state and announces the outcome. As the states and
operators are chosen in such a way that $|\phi\rangle_1$ and $|\phi\rangle_f$ are mutually orthogonal, from the announcement of Alice we know $U_A U_B$. As Alice (Bob) knows $U_A (U_B)$ she (he) can easily obtain $U_B (U_A)$ using $U_A U_B$ obtained from the announcement of Alice. For a detailed discussion see Ref. [33] where it is explicitly shown that if we have a set of mutually orthogonal $n$-qubit states \{$(|\phi_0\rangle, |\phi_1\rangle, \ldots, |\phi_i\rangle, \ldots, |\phi_{2^n-1}\rangle)$\} and a set of $m$-qubit unitary operators \{$(U_0, U_1, U_2, \ldots, U_{2^m-1})$\} such that $U_i(|\phi_i\rangle) = |\phi_i\rangle$ and \{$U_0, U_1, U_2, \ldots, U_{2^m-1}$\} forms a group under multiplication then it would be sufficient to construct a quantum dialogue protocol of Ba An type. Now assume that $n > m$ and Alice encodes nothing (i.e., she always choose $U_A = I_m$) and keeps $(n - m)$-qubits with herself and sends the remaining $m$-qubits to Bob who encodes his key by applying an $m$-qubit unitary operation $U_B$ and sends that back to Alice, but only after changing the order so that Alice cannot measure the final state immediately. Alice announces her key after receiving the key encoded qubits from Bob as in Protocol 1 and subsequently Bob announces the sequence of the message qubits sent by him. In QKA Alice does not need to disclose her measurement outcome. This modified QD protocol is equivalent to our Protocol 1. This clearly shows that all protocols of QD with $n > m$ would lead to protocols of QKA. It is interesting because in [33] we have shown that a large number of alternative combinations of quantum states and unitary operators can be used to implement QD. All of them (if $n > m$) will be useful for QKA, too.

5.3 Efficiency analysis

A well-known measure of efficiency of secure quantum communication is known as qubit efficiency [33] which is given as

$$\eta = \frac{c}{q + b}.$$  (3)

where $c$ denotes the total number of transmitted classical bits (message bits), $q$ denotes the total number of qubits used and $b$ is the number of classical bits exchanged for decoding of the message (classical communication used for checking of eavesdropping is not counted). This measure was introduced by Cabello in 2000 and it has been frequently used since then to compare protocols of secure direct communication. As we are not interested in communicating a message here, so we may modify the meaning of $c$ in $\eta_2$ to make it suitable for comparison of protocols of QKA. In the modified notion $c$ is the length of the shared key generated by the protocol. Thus in case of our first protocol if we generate an $n$-bit shared key then $c = n$. Further, in the entire protocol we have used $2n$ Bell states i.e., $4n$ qubits (of which $n$-Bell states were used as decoy qubits). Thus $q = 4n$. Now Alice and Bob announces the coordinates of the message qubits and Alice announces $K_A$, each of these three steps require communication of $n$ classical bits. Thus $b = 3n$. All other classical communications incurred in the process are related to the checking of eavesdropping and classical bits exchanged for eavesdropping checking are not counted in $b$. Thus $b = 3n$. This makes $\eta = \frac{n}{4n+3n} = \frac{1}{7} = 14.29\%$. In the similar manner if an $n$-bit shared key is prepared through Protocol 2 then $c = n$ and $q = 3(2n + 3n)$ as each party creates $n$ Bell states for key encryption and $\frac{3n}{2}$ Bell states for eavesdropping checking. Further, each party uses $3n$ bits of classical information for the disclosure of coordinates of the message qubits. Thus $b = 3 \times 3n = 9n$ and consequently $\eta = \frac{n}{15n+9n} = \frac{1}{24} = 4.17\%$. As YML protocol is similar to the Protocol 2 with only difference in the strategy adopted for eavesdropping checking, for YML protocol also we obtain $\eta = 4.17\%$. Clearly, Protocol 1 is more efficient than Protocol 2 and YML protocol, but Protocol 1 is less efficient than its QSDC counterpart (PP$^G$V protocol) whose qubit efficiency as per the unmodified definition is $\eta = \frac{n}{3n+2n} = \frac{1}{5} = 16.67\%$. This is expected as with the increase on number of parties contributing to the key, $q$ and $b$ required to generate the key of same size should also increase. This point can be further established by noting that $\eta$ for the 5-party protocol described above will be $\frac{1}{10} = 1.43\%$ as $q = 4n \times 5 = 20n$, $b = 2n \times 5 \times 5 = 50n$ and $c = n$.

6 Conclusions

In the present work we have proposed two orthogonal-state-based protocols of QKA. The first one works for 2-party case and the second one works for multi-party case. These are first set of orthogonal-state-based protocols of QKA as all the existing protocols of QKA are based on conjugate coding. Thus the proposed protocols are fundamentally different from all the existing protocols of QKA. orthogonal-state-based protocols show that the use of conjugate coding or in other words use of non-commutativity principle is not essentially required for unconditional security. Thus it requires lesser quantum resources in a sense. To be precise, monogamy of entanglement is sufficient to protect these protocols [27]. We have also shown that most of the existing protocols of QSDC and DSQC and all the protocols of QD can be turned into protocols of QKA. Thus the present work leads to several new options for implementation of QKA. Further, as the orthogonal-state-based protocols of QSDC and QKD are experimentally implemented in recent past, the protocols proposed here seem to be experimentally realizable.

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[33] In PP$^G$V Alice does not need to disclose her key $K_A$. Everything else is the same and as a consequence $b = 2n$, $q = 4n$ and $c = n$ with $c$ being the number of bits in the message or key that is transmitted.
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