Anikushin, Mikhail
Frequency theorem and inertial manifolds for neutral delay equations. (English)
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Summary: We study the infinite-horizon quadratic regulator problem for linear control systems in Hilbert spaces, where the cost functional is in some sense unbounded. Our motivation comes from delay equations with the feedback part containing discrete delays or, in other words, measurements given by δ-functionals, which are unbounded in $L_2$. Working in an abstract context in which such (and many others, including parabolic boundary control problems) equations can be treated, we obtain a version of the Frequency Theorem. It guarantees the existence of a unique optimal process and shows that the optimal cost is given by a quadratic Lyapunov-like functional. In our adjacent works it is shown that such functionals can be used to construct inertial manifolds and allow to treat and extend many works in the field in a unified manner. Here we concentrate on applications to delay equations and especially mention the works of R.A. Smith on developments of convergence theorems and the Poincaré-Bendixson theory; and also the works of Yu.A. Ryabov, R.D. Driver and C. Chicone on inertial manifolds for equations with small delays and their recent generalization for equations of neutral type given by S. Chen and J. Shen.

MSC:
35B42 Inertial manifolds
34K35 Control problems for functional-differential equations
34K40 Neutral functional-differential equations
37L45 Hyperbolicity, Lyapunov functions for infinite-dimensional dissipative dynamical systems
37L15 Stability problems for infinite-dimensional dissipative dynamical systems
47D06 One-parameter semigroups and linear evolution equations

Keywords:
frequency theorem; delay equations; inertial manifolds; Lyapunov functionals

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