Ramsey-Bordé interferometer for electrons

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A scheme to realize an electron interferometer using low-intensity, bi-chromatic laser pulses as beam splitter is proposed. The splitting process is based on a modification of the Kapitza-Dirac effect, which produces a momentum kick for electrons with a specific initial momentum. A full interferometric setup in Ramsey-Bordé configuration is theoretically analyzed.

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Introduction. — Interferometry with matter has a long history that started with experiments on electrons \([1]\) shortly after de Broglie hypothesized the existence of matter waves. Much later the interference between matter waves was also demonstrated for neutrons \([2]\), atoms \([3, 4]\), molecules \([5]\) and ions \([6]\). In most of these experiments the particle beam is split by mechanical means such as gratings or crystal lattices. In atom interferometry, however, one can also use laser light to split the beam \([7, 8]\) by transferring the momentum of absorbed photons to the atoms. This technique enables temporal control of the splitting process, makes the set-up of an interferometer very flexible, and has had an enormous influence on atom optics, metrology, and quantum information.

Atom interferometers are excellently suited for many purposes, but some phenomena, such as the Aharonov-Bohm effect \([9]\), require the use of charged particles. It would be desirable to use fields as beam splitters for electrons and thus acquire the same flexibility in setting up an interferometer. For neutral particles in magnetic fields the Stern-Gerlach effect may be used, but for electrons the Lorentz force makes it very difficult to design a beam splitter \([10, 11]\). For electrons in electric fields, the Kapitza-Dirac effect \([12, 13]\) may be suitable, which has been observed experimentally in the high light intensity \([14]\) and low light intensity regime \([15]\). Current efforts to design temporal lenses for electrons require very high light intensities \([16, 17]\).

In this paper, I propose to realize beam splitters for electrons by using two counter-propagating laser pulses with a specific frequency difference. This modification of the Kapitza-Dirac effect is related to diffraction without grating \([18]\) but uses a resonance condition to achieve a specific momentum transfer from the light field to the electrons. I will then show how such beam splitters may be used to set up a complete interferometer of Ramsey-Bordé type \([7, 19–21]\).

Kapitza-Dirac beam splitter. — The principle behind the current proposal is conservation of energy and momentum in the interaction between an electron and two counter-propagating laser fields of frequency \(\omega_i\) \((i = 1, 2)\) and wavenumber \(k_i = \omega_i/c\). If an electron absorbs a photon from laser 1 and subsequently emits a photon stimulated by laser 2, its momentum will change by \(2\hbar k_L\) and its energy by \(-\hbar \Delta \omega\), where \(\Delta \omega \equiv \omega_2 - \omega_1\) and \(k_L \equiv (k_1 + k_2)/2\). If for a given initial momentum the change in kinetic energy is equal to \(\hbar \Delta \omega\), this process will be resonant and can be accomplished with light intensities as used for Bragg scattering \([22]\).

To describe this effect we consider the Hamiltonian

\[
\hat{H} = (-i\hbar \nabla - q\hat{A})^2/(2m) \quad \text{with vector potential} \quad \hat{A} = \hat{e}(A^+ + A^-),
\]

is the positive-frequency part of the field and \(A^\mp = (A^\pm)^\ast\). \(E_i\) is the electric field amplitude of laser \(i\). The unit vector \(\hat{e}\) describes the polarization direction in the \(x\)-\(y\)-plane and \(\theta_i\) are phase factors. By expanding the Hamiltonian we obtain terms that are linear or quadratic in the vector potential. The optical frequencies \(\omega_i\) are much larger than any other frequency scale in our system, so that linear terms and terms of the form \((A^\pm)^2\) are rapidly oscillating. After time averaging \([23]\), only terms of the form \(A^+ A^-\) survive and we obtain the averaged Hamiltonian

\[
\hat{H}_{\text{avg}} = \frac{\hbar^2}{2m} \Delta + \frac{q^2}{8m} \left( \frac{E_1^2}{2\omega_1^2} + \frac{E_2^2}{2\omega_2^2} \right)
\]

with \(\Delta \theta = \theta_2 - \theta_1\). Restricting our considerations to the \(z\)-direction and performing a spatial Fourier transformation yields

\[
\imath \partial_t \psi(t, k) = \left( \frac{\hbar k^2}{2m} + g_1^2 + g_2^2 \right) \psi(t, k) + g_1 g_2 \left( e^{i\Delta \omega t - i\Delta \theta} \psi(t, k - 2k_L) + e^{-i\Delta \omega t + i\Delta \theta} \psi(t, k + 2k_L) \right),
\]

with \(g_i \equiv qE_i/(4\omega_i\sqrt{m\hbar})\). The coupling between discrete momentum components in this equation is similar to the problem of calculating band gaps for particles in periodic potentials. Following standard methods we introduce the quasi wavenumber \(\vec{k} \in [-k_L, k_L]\) and express
the electron wavenumbers as \( k = \tilde{k} + 2nk_L \) for \( n \in \mathbb{Z} \). Setting \( \psi_n(t, \tilde{k}) \equiv \psi(t, \tilde{k} + 2nk_L) \) we then obtain

\[
i\partial_t \psi_n(t, \tilde{k}) = \left( \frac{\hbar(\tilde{k} + 2nk_L)^2}{2m} + g_1^2 + g_2^2 \right) \psi_n(t, \tilde{k}) + g_1g_2 \\
\times \left( e^{i\Delta\omega t - i\Delta\theta} \psi_{n-1}(t, \tilde{k}) + e^{-i\Delta\omega t + i\Delta\theta} \psi_{n+1}(t, \tilde{k}) \right).
\]

(4)

A unitary transformation

\[
\psi_n(t, \tilde{k}) = e^{-it(n\Delta\omega + g_1^2 + g_2^2 + \frac{\hbar^2}{2m}k^2) + in\Delta\theta} \tilde{\psi}_n(t, \tilde{k})
\]

yields

\[
i\partial_t \tilde{\psi}_n(t, \tilde{k}) = n \left( \frac{2\hbar k^2}{m} + \frac{2\hbar k_L}{m} + \Delta\omega \right) \tilde{\psi}_n(t, \tilde{k}) \\
+ g_1g_2 \left( \tilde{\psi}_{n-1}(t, \tilde{k}) + \tilde{\psi}_{n+1}(t, \tilde{k}) \right).
\]

(6)

For \( \Delta\omega = 0 \) this equation describes Bragg scattering [22], but we can use \( \Delta\omega \) to make the interaction resonant for a specific initial momentum. For concreteness we consider an initial electron wavepacket with mean momentum zero and spatial width \( w \),

\[
\tilde{\psi}_0(0, \tilde{k}) = e^{-i^2w^22^\frac{1}{2}}w^{\frac{3}{2}}\pi^{-\frac{1}{2}},
\]

(7)

such that \( wk_L \ll 1 \), and \( \tilde{\psi}_n(0, \tilde{k}) = 0 \) for \( n \neq 0 \). The energy difference between the state \( n = 0 \) and the states \( n = \pm 1 \) is then given by

\[
\Delta E_{\pm 1,0} = \omega_{\text{rec}} \pm \frac{2\hbar k_L}{m} \Delta\omega \approx \omega_{\text{rec}} \pm \Delta\omega,
\]

(8)

with \( \omega_{\text{rec}} = 2\hbar k_L^2/m \) the recoil shift. For \( \Delta\omega = \mp\omega_{\text{rec}} \) the \( n = 0 \) state is resonant with the state \( n = 1 \) or \( n = -1 \), respectively. In the limit of weak coupling, \( g_1g_2 \ll \omega_{\text{rec}} \), all other components \( \tilde{\psi}_n(t, \tilde{k}) \) can be neglected, so that, e.g., for \( \Delta\omega = -\omega_{\text{rec}} \) the Schrödinger equation reduces to

\[
i\partial_t \begin{pmatrix} \tilde{\psi}_0(t) \\ \tilde{\psi}_1(t) \end{pmatrix} = \begin{pmatrix} 0 & g_1g_2 \\ g_1g_2 & \frac{2\hbar k_L}{\hbar} \end{pmatrix} \begin{pmatrix} \tilde{\psi}_0(t) \\ \tilde{\psi}_1(t) \end{pmatrix}.
\]

(9)

For \( \tilde{k} = 0 \) and \( t = \pi/(4g_1g_2) \) the solution is given by

\[
\begin{pmatrix} \tilde{\psi}_0(t) \\ \tilde{\psi}_1(t) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \begin{pmatrix} \tilde{\psi}_0(0) \\ \tilde{\psi}_1(0) \end{pmatrix},
\]

(10)

which corresponds to a perfectly balanced beam splitter. For \( \tilde{k} \neq 0 \) the beam splitter will not be perfectly balanced, but if \( g_1g_2 \gg 2\hbar k_L \Delta k/m \), where \( \Delta k = 1/(2w) \) is the width of the wavepacket in momentum space, Eq. (10) still provides an excellent approximation.

To estimate the feasibility of this proposal we consider a laser wavelength of 1064 nm, so that \( \omega_{\text{rec}} = 2\pi \times 1.3 \) GHz. The generation of two phase-locked laser pulses with such a detuning is well within the range of current experimental techniques [24]. For lasers of equal intensity, \( g_1 = g_2 \), the weak coupling condition then results in the constraint \( q^2I/(32\omega^2m\epsilon_0^2) \ll \omega_{\text{rec}} \), where \( I = 2\epsilon_0E^2 \) denotes the light intensity. In our case this implies \( I \ll 8W/\mu m^2 \), which is considerably less than the intensity used in Ref. [15]. For an intensity of \( I = 0.5W/\mu m^2 \), i.e., \( g_1g_2 = 2\pi \times 80 \) MHz, the pulse duration is \( \pi/(4g_1g_2) \approx 1.5 \) ns. The spatial width of the electron wavepacket needed to obtain a balanced beam splitter is then \( w \approx 1 \mu m \).

**Ramsey-Bordé interferometer.**—Kapitza-Dirac beam splitters could be utilized to implement a Ramsey-Bordé interferometer for electrons, with a setup as shown in Fig. 1. The electron beam is split and recombined by four bichromatic laser pulses. The first two pulses are detuned by \( \Delta\omega = -\omega_{\text{rec}} \), thus coupling electrons with an initial momentum \( p \approx 0 \) to \( p + 2\hbar k_L \). During the time \( T \) between these pulses the electron beam is spatially split into two beams, which after the second pulse are separated by a distance \( \Delta z = T2v = 2\hbar k_L T/m \). The third and fourth pulse are detuned by \( \Delta\omega = \omega_{\text{rec}} \), so that \( p \approx 0 \) is coupled to \( p - 2\hbar k_L \). This enables us to recombine (parts of) the electron beam, leading to a closed interferometer that corresponds to output I (and V) in Fig. 1.

To understand the details of this interferometer we describe the electrons by the time-dependent Schrödinger equation. For simplicity we will only consider the motion along the \( z \)-axis (perpendicular to the initial beam direction). During the free evolution for a time \( T \) between two pulses the dynamics is governed by Hamiltonian \( \hat{H}_0 = -\hbar^2\partial_z^2/(2m) \). During the time \( T' \) between the beam splitting and recombination processes we admit a constant electric field \( E \) to study its effect on the phase difference between the beams. The corresponding Hamiltonian is \( \hat{H}_a = \hat{H}_0 - \hat{m}az \), with acceleration \( a = eE/m \). Inhomogeneous electric fields could be studied with the methods of Ref. [20].
To describe the interference experiment, we start with an initial state of the form \( \psi_{\text{init}}(z) = \int dp \, \tilde{\psi}(p) \phi_p(z) \), with momentum eigenstates \( \phi_p(z) = e^{ipz/\hbar} \) and \( \int dp \, |\tilde{\psi}(p)|^2 = 1 \). The evolution between the laser pulses can then be described by the unitary transformations \( \exp(-iTH_0/\hbar)\phi_p(z) = \exp(-iTE(p)/\hbar)\phi_p(z) \), where \( E(p) = p^2/(2m) \), and
\[
e^{-\frac{i}{\hbar} T \hat{H}} \phi_p(z) = e^{-i\tau(p)} \phi_{p + maT'}(z), \tag{11}
\]
\[
\tau(p) = \frac{1}{\hbar} \int_0^{T'} dt' E(p + maT'). \tag{12}
\]
A simplified description of Eq. (10) for the first two pulses, with \( \Delta \omega = -\omega_{\text{rec}} \), can be given by the unitary transformation
\[
\hat{U} \phi_p(z) = \frac{1}{\sqrt{2}} \left( \phi_p(z) + \phi_{p + 2h\omega L}(z) \right) \tag{13}
\]
\[
\hat{U} \phi_{p + 2h\omega L}(z) = \frac{1}{\sqrt{2}} \left( -\phi_p(z) + \phi_{p - 2h\omega L}(z) \right). \tag{14}
\]
For the second pair of pulses we chose \( \Delta \omega = +\omega_{\text{rec}} \), which results in a similar unitary transformation \( \hat{U}_+ \), which is equal to \( \hat{U}_- \) with \( k_L \) replaced by \( -k_L \). These expressions are only valid for momenta \( |p| \ll \hbar k_L \). Eq. (14) is not accurate but captures the essential physics of each pulse. A more complete treatment will be given below.

The final state of the electrons after passing through the interferometer can be found by concatenating all unitary transformations. This splitting process, corresponding to the pair of pulses labeled by \( \Delta \omega \) in Fig. 1, can be described by a unitary operator \( \hat{U}_{\text{split}} = \hat{U}_+ \exp(-iT\hat{H}_0/\hbar)\hat{U}_- \). Analogously, the recombination process can be described through the unitary operator \( \hat{U}_{\text{recomb}} = \hat{U}_+ \exp(-iT\hat{H}_0/\hbar)\hat{U}_+ \), which results in the final state
\[
\hat{U}_{\text{recomb}} e^{-\frac{i}{\hbar} T \hat{H}} \hat{U}_{\text{split}} \psi_{\text{init}}(z) = \frac{1}{4} \int dp \, \tilde{\psi}(p) \phi_{p + maT'}(z)
\times e^{-i\tau(p,T')} \left( e^{-\frac{iE}{2} T(p + 2h\omega L)} e^{-\frac{iE}{2} T(p - 2h\omega L + maT')} + \text{rest.} \tag{15}\right.
\]
In this expression, “rest” refers to seven terms that are similar to the ones displayed and correspond to partial beams represented by dashed arrows in Fig. 1. The two terms that are displayed correspond to the solid black lines, which realize a spatially closed interferometer geometry. In an atomic Ramsey-Bordé interferometer, this geometry is useful because the phase difference between the two beams is not influenced by the Doppler effect [25]. This is also the case for electrons, for which the phase difference is given by
\[
\Delta \varphi = \frac{\hbar^2}{2m} T' - 2ak_LTT'. \tag{16}
\]
This differs from previous results (e.g., Eq. (45) of Ref. [21]) because the momentum transfer \( 2h k_L \) is twice as big as in atom interferometers and because we assumed that the acceleration is only present during \( T' \).

With the simplified description (14) of Kapitza-Dirac beam splitters and initial state (7) one can evaluate the final state Eq. (15) in a closed form. The full solution is rather involved and will not be displayed here. However, it can be shown that the final spatial wavefunction of partial beam I corresponds to a (dispersing) Gaussian wavepacket that is centered at \( z = \frac{1}{2} aT'^2 + aTT' \). As is to be expected because of Ehrenfest’s theorem, its mean position follows a classical trajectory that is determined by momentum kicks due to the laser pulses combined with free or accelerated motion between the pulses. All other partial beams do also correspond to Gaussian wavepackets that follow classical trajectories.

To provide a more accurate description of the beam splitting process we also solved the Schrödinger equation numerically, including the Kapitza-Dirac effect as described by Eq. (6). The spatial profile of the electron probability density after passing through the interferometer is shown in Fig. 2, where we have used an initial wavepacket of the form (7) with a mean width of \( w = 3 \mu \text{m} \), a laser wavenumber of \( k_L = 2\pi/1064 \text{ nm} \) and an acceleration of \( a = 10^{10} \text{m/s}^2 \). The time \( T \) between the two pulses of the beam splitter and recombiner is \( T = 12 \text{ ns} \) and the electrons are accelerated for \( T' = 10 \text{ ns} \) so that their final velocity transverse to the direction of the beam is \( 100 \text{ m/s} \) [27]. To resolve all partial beams (for clarity of presentation) we have added another free evolution for a time \( T'' = 40 \text{ ns} \) after the last laser pulse. The two partial beams that form a closed geometry produce peak I in Fig. 2. The offset of this peak from the origin corresponds to the distance travelled due to the acceleration. The position of all other peaks is mainly determined by the sequence of momentum transfers that they obtain. For instance, peak VIII obtains a momen-
tum of $2\hbar k_L$ at the first bichromatic pulse. Because all other partial beams either obtain this momentum transfer later, or obtain no or negative momentum transfer, it is pulse VIII that will travel to the rightmost position. If we had set $T'' = 0$, i.e., if the interference pattern was observed right after the last laser pulse, wavepackets IV, V, and VI would be overlapping with wavepackets II, I, and III, respectively. These partial beams only differ by a momentum kick generated by the last laser pulse.

The position of the partial beams in Fig. 2 is not in perfect agreement with the classical trajectories discussed above. The reason is that the latter ignores the finite duration of the interaction between the laser pulses and the electrons. If we assume that during the interaction time each partial beam moves with the average of its momentum before and after the pulse, the agreement between analytical and numerical solution is excellent.

Ramsey-Bordé interferometers for atoms and electrons have a similar geometry but differ significantly in some details. First, the atom interferometer has more partial beams than the electron interferometer and even includes a second pair of beams with a closed geometry (see Fig. 1 of Ref. [21], for instance). The reason is that in an atom interferometer the resonance condition is determined by atomic energy levels. Hence, light transfers momentum to an atom regardless of the value of its center-of-mass momentum. On the other hand, in a Kapitza-Dirac beam splitter the kinetic energy determines the resonance condition, so that only electrons with a specific initial momentum will resonantly interact with the bichromatic laser pulses. For this reason, partial beams VII and VIII will not be affected by the second pair of laser pulses, resulting in a reduced number of partial beams.

Second, it is worth to remark that there are no spatial interference fringes in Fig. 2 because for our choice of $T, T', T''$ all partial beams are separated. However, the phase difference (16) has a strong influence on the relative intensities of the partial beams. This situation is similar in atom interferometers, where the probability for the atoms to be in the excited state instead of a spatial interference pattern is observed [7]. This probability is a function of the partial beam intensities and thus is sensitive to a phase difference.

**Conclusion.**— In this paper I have proposed to use a modification of the Kapitza-Dirac effect to devise beam splitters and a Ramsey-Bordé interferometer for electrons. Numerical simulations suggest that such an experiment may be realized using current technology. Field-based electron beam splitters would allow for a much more flexible setup that could lead to novel applications of electron interferometers. For instance, “figure 8” interferometer geometries could be used to test the phase shift induced by a spatial variation of an electric field, rather than its amplitude [25, 26]. Many more applications that involve electric and magnetic fields are possible and will be explored in future publications.

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