Stochastic cellular automata model for wildland fire spread dynamics

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Abstract. A stochastic cellular automata model for wildland fire spread under flat terrain and no-wind conditions is proposed and its dynamics is characterized and analyzed. One of three possible states characterizes each cell: vegetation cell, burning cell and burnt cell. The dynamics of fire spread is modeled as a stochastic event with an effective fire spread probability \( S \) which is a function of three probabilities that characterize: the proportion of vegetation cells across the lattice, the probability of a burning cell becomes burnt, and the probability of the fire spread from a burning cell to a neighboring vegetation cell. A set of simulation experiments is performed to analyze the effects of different values of the three probabilities in the fire pattern. Monte-Carlo simulations indicate that there is a critical line in the model parameter space that separates the set of parameters which a fire can propagate from those for which it cannot propagate. Finally, the relevance of the model is discussed under the light of computational experiments that illustrate the capability of the model catches both the dynamical and static qualitative properties of fire propagation.

1. Introduction
The wildland fire spread is a combustion reaction where the necessary ingredients for its occurrence are: the vegetation, which provides the combustible source for the reaction; the oxygen in the air, which acts as an oxidizing agent; and a heat source responsible by the initiation and the self-sustainability of the reaction [5]. The fire spreads across the landscape consuming the vegetation and this process can be decomposed into four combustion phases, the so called: pre-heating, ignition, combustion and extinction [5]. The fire front is the region of intense flaming combustion where a large quantity of heat is released. Part of this heat is transmitted to the vegetation not yet burning, and heating it up to the ignition temperature. When this happens, the flames rise and the fire front occupies a new position ahead. The flames remain as the vegetation is burnt out.

In this work a simple model for wildland fire dynamics under flat terrain and no-wind conditions is proposed and its dynamics is analyzed. The model formulation is based on...
stochastic cellular automata and its dynamics is analyzed qualitatively and quantitatively. Cellular automata are models that assume space, state and time discrete [1, 6]. A square lattice represents the space and each element that constitutes the lattice is called cell. Each cell has a neighborhood, set of internal states variables, and a local rule that describes the transition between the states variables and define the future state as a function of the cell present state and the neighborhood present states. In stochastic cellular automata the rules for cell states transition are performed by means of probabilities. Stochasticity is used to represent the vegetation heterogeneity and to include random component in the dynamics of the vegetation combustion and ignition process during the fire spread.

The paper is structured as follow. In the Section 2 the modeling approach and the model parameters are described. In the section 3 the model dynamics is characterized and analyzed. Finally, in the last section, the model relevance is discussed.

2. Model description

The model is based on the spatially explicit representation and the landscape is depicted as a square and two-dimensional lattice \( L \) of dimensions \( L_x \times L_y \). Each cell is defined by:

- its discrete position \((i, j) \in \mathbb{Z}^2\) in the lattice, where \( i = 1, \ldots, L_x \) is the column and \( j = 1, \ldots, L_y \) is the row;
- the finite set of internal states variables that describes the possible behavior of the cells in a given time step \( t \) which are \( S_{t(i,j)} \in \{E, V, F, O\} \) where: \( E \) is an empty cell, which denotes unburnable cells or without vegetation; \( V \) is a vegetation cell, with denote cells with potential to burn; \( F \) is burning cell, which denotes a cell whose the vegetation in its inside is burning; and \( O \) is a burnt cell, which denote vegetation cell that was burned by the fire;
- the finite set of neighborhood cells \( N(i,j) \), where the Moore neighborhood, as illustrated in the Figure 1(a), illustrate the neighborhood relations in the model and comprises the eight cells surrounding \((i^*, j^*)\) of a central cell \((i, j)\) according with the definition \( N(i, j) = \{(i^*, j^*) : |i - i^*| \leq 1, |j - j^*| \leq 1\} \);
- the local rule that determines the future cell state as a function of the present cell state and present neighborhood cell states \( f : S_{t(i,j)}^t \times S_{N(i,j)}^t \rightarrow S_{(i,j)}^{t+1} \), where the time \( t \) is also represented by discrete values or time steps. Thus, the time evolution of the model is driven by the interaction between the cell states and the cell neighborhood states. Starting from a given configuration of cells initial states, the cellular automaton self-replicates the sequent cell states. The cellular automata model is stochastic because the state transition function is performed according to probabilities values.

The heat transfer from burning regions to non-burning regions governs the fire spread. Thus, the fire spread is modeled as a set of ignitions of vegetation cells as the burning cells persist. Stochasticity is used to include the heterogeneity of spatial conditions present in real vegetation patterns and to include a random component to the combustion dynamics and ignition process [3, 4, 2]. The dynamics of fire spread is modeled as a stochastic event with an effective fire spread probability \( S \) which is as a function three probabilities, which are:

- the probability \( D \), that represents the heterogeneity of fuel conditions along the lattice and determines the proportion of cells with vegetation across the lattice. Thus, for each cell in \( t = 0 \), there is a probability \( D \) for its state to be vegetation cell and the probability \( 1 - D \) to it is empty cell;
- the probability \( B \), that models the combustion process, where, in each time step, a burning cell has a probability \( B \) to change its state to burnt cell:
• the probability $I$, that models the ignition process, where, there is a probability $I$ for the
fire spreads from a burning cell to a neighboring vegetation cell.

The local rule for perform the transition between the states are assessed according to these
probabilities values. The cell state transition diagram is showed in the Figure 1(b). An empty
cell is unchangeable and always remains in this state. The fire spread is considered a diffusion
contagious process and the fire can spreads only from a burning cell to a neighbor vegetation
cell. Thus, the transition $V \Rightarrow F$ is conditioned for a vegetation cell that has at least one
burning cell in this neighborhood. Given two neighbors cells, one burning cell and the other a
vegetation cell, in each time step, there is a probability $I$ for the burning cell ignites the neighbor
vegetation cell. Once ignited, in each time step, there is a probability $1 - B$ for a burning cell
remain burning, otherwise its state changes to burnt cell, which is the transition $F \rightarrow O$.

The model input parameters are the probabilities $D$, $B$ and $I$, the lattice size, and the
maximum time step $N_{STEPS}$. A complete visit to all cells of the lattice is called a sweep. A
simple simulation is performed in the two stages: initialization and fire spreading algorithm. A
complete visit to all cells of the lattice is called a sweep. The initialization stage includes: (1)
define the model input parameters; (2) execute a sweep and for each cell and change its state
to vegetation cell with probability $D$ or empty cell with probability $1 - D$; and (3) select one or
more vegetation cells and change its state to burning cell. In the fire spreading algorithm, for
each time step $t = 1, \ldots, N_{STEPS}$ execute a sweep and: (1) for each burning cell, evaluate the
transition $B \rightarrow O$; (2) for each neighbor of a burning cell evaluate the transition $V \Rightarrow F$.
Time simulation stop if $t = N_{STEPS}$ or if there are none burning cell in a given time step.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{(a) The Moore neighborhood comprises eight cells (yellow cells) which surround
the central cell (black cells). (b) In the cell state transition diagram, arrows indicate the state
transitions paths. The double arrow indicates that the transition depends on the neighbor
cell state. The round dashed arrows indicate that the state transitions are conditioned by the values
of other probabilities.}
\end{figure}

3. Simulations and results
3.1. Qualitative analysis of the fire patterns
The effective fire spread probability $S$ describes the fire behavior across the lattice as a function
of the probabilities $D$, $B$ and $I$. Different fire patterns, with different size and shape, can be
obtained varying the values of these probabilities. The Figure 2 characterizes some fire patterns
using different values of $D$, $B$ and $I$, for a lattice of size $201 \times 201$ and the fire starting from a
cell at the middle of the lattice $S^0_{(100,100)} = F$. Each cell state along the lattice is represented by
colors that which are, empty cell (black), vegetation cell (green), burning cell (red) and burnt
cell (gray).

The proportion of vegetation cells across the lattice determines the spatial distribution of
available fuel along the lattice. Higher values of $D$ implies in more quantity of available fuel
along the landscape and therefore the fire propagates with more facility. This effect can be
observed comparing the Figures 2(d) and 2(f), which can be observed that in the Figure 2(f)
the burned area is larger than in the Figure 2(d).
Figure 2. Different fire patterns for $t = 100$ using a lattice with size $201 \times 201$ and the fire starting from the middle cell positioned at $(i, j) = (100, 100)$. The parameters values are showed immediately bellow the figures.

The probability $B$ asserts the combustion latency for a burning cell ($B^{-1}$ is a conception for the mean reaction time). A burning cell with a high value of $B$ burns most quickly (i.e., in less time steps) than those that have a low value. This behavior can be observed comparing the
Figures 2(e) and 2(f). Furthermore, how smaller is the values of $B$, more high is the possibility of the fire spread from the burning cell to neighbors vegetation cells, because the cells remain burning in more time steps. This effect can be observed comparing the Figure 2(c) and 2(d).

The ignition probability $I$ determines how lightly the fire spreads along the lattice. The effects of different fire rates of spread can be observed by the different burned areas comparing the Figures 2(a), 2(b) and 2(c). Higher values of $I$ are related with fire fronts which spread most quickly.

3.2. Monte-Carlo simulations
Because the model stochasticity, a same set of parameters values can generate fire patterns that are slightly different, according showed in the Figure 3(a), 3(b) and 3(c). Thus, is necessary to obtain the mean behavior computed during $N$ simulations based on different sequence of generated random numbers. This is the objective of the Monte-Carlo simulations (MCS). For a given set of model input parameters, a large quantity of simulations are carried out and for each cell is computed the number of times that it burns. The number of time that a cell burn divided by the total number of MCS is the estimative of the cell burning risk. The Figure 3(d) shows the cell burning risk computed by a MCS for $D = 0.6$, $B = 0.5$, $I = 0.5$ and $N = 100$.

3.3. Evaluating the value of $S$
The fire spreads along the lattice following a pathway of interconnected cells which varies as a function of the effective probability $S$. Studies in percolation theory [7] corroborate that there is a critical value $S^*$, called percolation threshold, so that when $S > S^*$ always there is the pathway for the fire spreads from any starting cell to some other point inside the lattice. The main question here is how to characterize the probability $S$ for the model. The existence of the percolation threshold and the consequent description of the critical line as a function of the probabilities $D$, $B$ and $I$ are investigated using MCS.

A set of $N$ MCS are performed using identical lattices and different values of $D$, $B$ and $I$. The fire starts at the left border of the lattices and during the $N$ simulations is computed the number of times that the fire reaches the right border of the lattice. If the fire propagates from one side to the other then the fire percolate the lattice. Thus, the approximation of $S$, denoted by $\langle S \rangle$, is calculated as:

$$\langle S \rangle = \frac{1}{N} \sum_{j=1}^{N} C_j,$$

where $C_j = 1$ if the fire percolate the lattice and $C_j = 0$ otherwise.

The Figure 4 characterizes the values of $\langle S \rangle$ for different values of $D$ varying the values of $B$ and $I$. For each set of parameters values a number of $N = 1000$ MCS are carried out using one lattice of size $52 \times 52$. The color map displays values varying from $\langle S \rangle = 0$ (black) to $\langle S \rangle = 1$ (white). If $\langle S \rangle = 0$ the fire not percolates the lattice, in other words, the fire extinction regime predominates. Otherwise, if $\langle S \rangle = 1$, the fire propagation regime predominates and the fire spreads incessantly across the lattice. The Figs. 4(a)–4(f) indicate the existence of a critical line defining a division on the model parameter space that separates the set of parameters for which a fire can propagate from those for which it cannot.

3.4. The fire rate of spread
The fire rate of spread $R$ is a common measure of the fire behavior. To compute its value and its relation with the model parameters, a MCS is used to compute the mean fire front edge position, computed in the central cells in the direction of spread, for fires initiated at the left edge of the lattice. The time evolution of the fire front edge position is compatible with a linear
Figure 3. (a)–(c) Fire patterns slightly different for a lattice with size $201 \times 201$, $D = 0.6$, $B = 0.5$, $I = 0.5$, $t = 100$ and the fire starting from the middle cell positioned at $(i, j) = (100, 100)$. (d) Burning risk for $N = 10000$ Monte-Carlo simulation. The color map in the figure varies from 0 (black) to 1 (white), and the cells that not burn (green cells).

fit, and thus a rate of spread can be defined as the angular coefficient of the linear fit. The rate of spread unit is calculated in cell dimensions per time steps, assuming that the cells have the same dimensions.

The fire pattern generated by the model is resultant of two coupled processes: fire diffusion and fire persistence. The fire diffusion process is the dynamics of fire transmission between the cells, and it is governed by the ignition probability $I$. The fire persistence process supports the fire remaining in a burning cell and is responsible by the dynamics of fire pattern formation during the time and drives the fire diffusion process.

The combination of these two processes can generate a large quantity of different fire patterns and the values of $I$ and $B$ are the weights of each one. A high fire persistence and one low fire diffusion provides fire fronts that propagate slowly during the time. Keeping constant the persistence and increasing the diffusion, the fire rate of spread increases. This effect can be observed comparing the dependence of rate of spread on $I$ in the Figs. 5(a) – 5(d).

The probability $D$ is also important to the fire pattern formation. De facto, the diffusive fire spread require a continuous pathway or clusters of vegetation cells and the spatial distribution of vegetation cells across the lattice depends on $D$. Lower values of $D$ generate a fuel distribution
Figure 4. Values of $\langle S \rangle$ for different values of $D$ varying the values of $B$ (x-axis) and $I$ (y-axis). A total of $N=1000$ Monte-Carlo simulations are carried out for each set of values $D$, $B$ and $I$ using a lattice of size $52 \times 52$.

that is sparse enough to the fire not propagate across it, and that can be observed in the Fig. 4(a). As $D$ is increased, the vegetation cells cluster take place and there is fuel enough to support the
fire spread. Thus, there is a critical value $D^*$, such that for $D \geq D^*$ there is the consequent formation of a percolation cluster of neighbor vegetation cells. This percolation cluster not indicate that the fire can propagate uninterruptedly because the fire spread probability is also conditioned to the values of $B$ and $I$ according the critical line characterized in the Figure 4. In the Figs. 4(a)–4(f) can be observed that when $D$ increases, also increases the proportion of the set of model parameters for which the fire can propagate.

![Graphs showing dependence of the rate of spread on $I$ for different values of $D$ and $B$.]

**Figure 5.** Dependence of the rate of spread on $I$ for different values of $D$ and $B$.

4. Final considerations

Although the model formulation include only fire spread dynamics under flat terrain and no-wind conditions, the qualitative and quantititative analysis performed in this paper indicate that this model constitutes a qualitative framework for wildland fire spread dynamics simulation. However, for further ecological applications of this model, the relation of the model parameters with meteorological, vegetation and topographical factors remain to be quantitatively established.
The effects of wind and slope on ignition process can be represented according to the incorporation of a directional bias that proportions an anisotropic diffusive process across the lattice [3]. Different values of vegetation density $D$ can be used to represent several phytophysiognomy vegetation clusters. The probability $B$ can be adjusted to include different values of fuel load over the surface and different fuel moisture conditions. This model parameterization consists in finding an explicit expression between the model parameters and the environmental conditions of historical and documented forest fires.

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