1. Introduction

The machining of titanium alloys has been a great challenge in the manufacturing community due to the large material flow stress and low heat conductivity, especially for the Ti-6Al-4V (Pan, Shih et al., Hong, Ding et al. 2001, Arrazola, Garay et al. 2009, Dandekar, Shin et al. 2010). The excessive tool wear significantly increases the machining cost and reduces the machining end-product dimensional accuracy (Barry, Byrne et al. 2001, Shivpuri, Hua et al. 2002). Also, the low thermal conductivity will result in the high temperature concentration near the tool/workpiece interface. Combined with large strain and high strain rate condition in the shear zone, considerable material microstructure evolution will take place. The material microstructure evolution in the shear zone includes phase transformation, augmented/elongated grain size and localized dislocation density change (Molinari, Musquar et al. 2002, Shivpuri, Hua, Mittal et al. 2002). Also, the plastic deformation localization in the shear zone could induce material ductile to brittle transition, which would result in the segmented chip formation (Xie, Bayoumi et al. 1995, Molinari, Musquar and Sutter 2002). The material mechanical response in the shear zone is the dominating factor that determines the machining force. The optimization of machining process parameters has been a long-term focus to reduce the
machining costs. Therefore, it is of great importance to understand the material deformation behavior in the shear zone in the machining process.

Tremendous efforts have been devoted for the material flow stress development for the application of machining (Rhim and Oh 2006, Liu, Melkote et al. 2013). Comprehensive review work has been done on the material constitutive modeling in machining. As one of the most popular empirical model, Johnson-Cook (JC) flow stress model takes the strain, strain rate and temperature as the input. Due to the severe plastic deformation and high temperature in the shear zone, significant material microstructure evolution has been reported. This microstructure change could also influence the material flow stress. Semi-empirical model models have been developed to include those effects. Calamaz et al. (Calamaz, Coupard et al. 2008) investigated the chip formation in the machining of Ti-6Al-4V and pointed out that the segmented chip formation was influenced by the strain softening and thermal softening. A modified chip formation model was developed by introducing the softening term in to the traditional JC model. In the shear zone, the material flow stress not only depends on the current states, the mechanical loading history also influence the material flow stress. A more physical based BCJ model was proposed by Guo et al. (Guo, Wen et al. 2006) to account for the kinematic hardening and dislocation effects in the adiabatic shear zone. However, the microstructure attributes are not directly calculated from the BCJ model. For a multiphase material, such as Ti-6Al-4V, the high temperature and large strain rate could promote the phase transformation effect. Zhang et al. (Zhang, Shivpuri et al. 2014) introduced a material microstructure based flow stress model to consider the material phase transformation effect in the machining of Ti-6Al-4V. The material phase composition is purely dependent on the current temperature. The temperature history effect is not fully considered. From a time temperature transformation diagram, Pan et al. (Pan, Tabei et al. , Pan, Liang et al. 2016) introduced an isothermal transformation model to predict the instantaneous phase composition of Ti-6Al-4V in the machining process. A mixture rule is used to correlate the material phase composition with the material flow stress. In addition to the phase transformation effect, the material dynamical recrystallization strongly affects the material mechanical behavior in the shear zone. Courbon et al. (Courbon, Mabrouki et al. 2013) applied a dynamic recrystallization based flow stress model to predict the machining forces. The material model categorizes the material deformation into three stages, work hardening, dynamic recovery and dynamic recrystallization. The recrystallized volume fraction or grain size evolution is not directly connected to the material flow stress. An explicit grain size calculation scheme was introduced by Arisoy et al. (Arisoy and Özel 2015) for the machining of Ti-6Al-4V. However, the strain size effect on the flow stress is not clearly indicated. A so-called microstructure sensitive flow stress model was proposed by Pan et al. (Pan, Shih, Garmestani et al.) to introduce a grain size term into the classic JC model for the force prediction in the machining of steel alloys. However, the explicit grain size are not provided. Additionally, the modified term is based on a homogenized average grain size value. The grain element distribution and texture information are missing.

Pure physics based model, such as viscoplasticity model has been developed to describe the material response by taking account the dislocation density, grain morphology. Due to the computational efficiency, the application of viscoplasticity model into the machining is still facing great challenges. Ti-6Al-4V, as a typical α + β alloy, has hexagonal-close-packed α phase and body-centered-cubic β phase (Murr, Quinones et al. 2009). The strain hardening, strain rate sensitivity and microstructure evolution are the dominating factors for the material flow stress. A physical based flow stress model is believed to yield a more accurate prediction. In the current study, a physical based mechanical threshold stress (MTS) model will be provided for the machining force prediction of Ti-6Al-4V, for the first time. Both an analytical force model and FEA force model will be developed. The MTS will be applied into machining forces prediction model. The orthogonal turning experimental data from previous literature are provided for the model validation.

2. MTS model

Traditional material constitutive model applied in the machining process mainly includes continuum level model scheme, such as JC model, Zerilli-Armstrong model (Zerilli and Armstrong 1987). The material flow stress parameters are purely from mathematic model fitting. The material microstructure evolution effect is not fully considered. The MTS model links the material flow stress with the material microstructural states through a structural parameter $s_j$ measured at the temperature $0\ K$. The MTS model takes the material deformation dislocation density and grain boundary into the flow stress consideration. For the Ti-6Al-4V, the MTS could be denoted as,
\[ \sigma = \sigma_a + \sum_j s_j(\dot{\varepsilon}, T)\sigma_i \frac{\mu}{\mu_0} \]  

(1)

where \( \sigma_a \) is the athermal stress, \( \sigma_i \) is the internal atom threshold stress, \( \sigma_s \) is the so-called stress from strain hardening, \( \sigma_d \) is the solution strengthening stress from dislocation interaction; \( \mu \) is the material shear modulus, \( \mu_0 \) is the reference shear modulus at 0 K. The shear modulus temperature dependence could be described as (Kotkunde, Deole et al. 2014),

\[ \frac{\mu}{\mu_0} = 1 - \frac{a}{\mu_0 \exp\left(\frac{T_0}{T} - 1\right)} \]

(2)

where \( a \) is material constant, selected to be 4.355 GPa, \( T_0 \) is the reference temperature, 198 K, \( \mu_0 \) is 49.02 GPa. The structure term is in the form of,

\[ s_j(\dot{\varepsilon}, T) = \begin{cases} 
1 & \left(1 - \frac{kT}{g_0 b^\eta} \ln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)\right)^{1/p_j} \\
0 & \text{otherwise}
\end{cases} \]

(3)

where \( k \) is the Boltzmann constant, \( g_0 \) is the normalized activation energy, \( b \) is the burgers vector, \( p_j \) and \( q_j \) are obstacle profile constants.

The athermal stress term \( \sigma_a \) is defined to be independent of the temperature. The microstructural states, such as grain size and dislocation density will affect the athermal stress term. By assuming a strong strain dependent relation, the athermal stress could be approximated by the power law as,

\[ \sigma_a = \sigma_a^0 \dot{\varepsilon}^n \]

(4)

where \( \sigma_a^0 \) and \( n \) are power law hardening term, determined to be 900 MPa, and 0.075 respectively. \( \dot{\varepsilon} \) is the plastic strain. The \( \sigma_i \) and \( \sigma_s \) are constant terms, which are selected to be 1050 MPa, and 873 MPa. The dislocation interaction stress could yield in the equation as,

\[ \sigma_d = \sigma_{d0} \left[1 - \left(1 - \frac{\sigma_{d0}}{\sigma_{dS}}\right) \exp\left(-\Theta_0 \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)\right] \]

(5)

where \( \sigma_{d0} \) is the initial dislocation interaction stress before any work hardening. The Ti-6Al-4V material parameters for the MTS model are summarized in Table 1 (Follansbee and Gray 1989).

| Obstacle type            | \( p \) | \( q \) | \( g^0 \) | \( \dot{\varepsilon}_0 \) | \( \sigma \) |
|--------------------------|---------|---------|-----------|--------------------------|-----------|
| Internal atom \( \sigma_i \) | 1       | 2       | 0.264     | 1.0 \times 10^{10}       | 1050 MPa  |
| Strain hardening \( \sigma_s \) | 1       | 2       | 0.8       | 5.8 \times 10^{6}        | 873 MPa   |
| Dislocation interaction \( \sigma_d \) | 2/3     | 1       | 1.6       | 1.0 \times 10^{10}       | Equ. (5)  |
| Other parameters         |         |         | \( \sigma_{d0}=538 \text{ MPa, } b = 0.296 \times 10^{-10}, \kappa =1, \Theta_0 = 2721 \text{ MPa} \) |

In order to see the difference between the traditional JC model and the MTS model, the material flow stresses are plotted as a function of strain at three different temperature levels, as shown in Fig. 1 (a). In a typical machining process of Ti-6Al-4V, the maximum temperature would not exceed 1000 °C. So the temperature range of 20 °C to 1000 °C is selected. The strain rate is fixed at 0.01 s^{-1}, the strain is in the range of 0 to 0.1. The biggest flow stress difference between the JC model and the MTS model is around 6.4 %. Also, the MTS flow stress has larger increase rate with the
increasing strain compared with the JC model which indicates the stronger strain sensitivity.

Fig. 1 The comparison of the flow stress between the JC model and MTS model at the strain rate of 0.01 s⁻¹, three different temperature conditions.

3. Analytical machining force model

The machining involves the friction between the cutting tool and the material plastic deformation in the shear zone, as shown in Fig. 2. The shear strain in the shear zone is estimated as,

\[ \varepsilon_{AB} = \frac{\cos \alpha}{2\sqrt{3} \sin \phi \cos(\phi - \alpha)} \]  

(6)

Where \( \phi \) is the shear angle, \( \alpha \) is the rake angle. Assuming a constant cutting speed \( V_C \), the strain rate in the shear zone could be determined to be,

\[ \dot{\varepsilon}_{AB} = \frac{C_{Oxley} V_C \sin^2 \phi}{\sqrt{3} \tau} \]  

(7)

where \( C_{Oxley} \) is the Oxley constant, \( \tau \) is the undeformed chip thickness. The material deformation in the shear zone would induce considerable heat generation, which could be calculated as,

\[ \Delta T = \frac{(1 - \beta) F_S V_S}{C_p \rho \dot{V} \tau w} \]  

(8)

where \( \beta \) is the heat dissipation coefficient, \( F_S \) is the shear force, \( V_S \) is the shear velocity in the shear plane, \( C_p \) is the specific heat capacity, \( \rho \) is density, \( w \) is the width of cut. With all the material states as input, the material flow stress in the shear zone could be calculated from the MTS model as,

\[ \sigma_{AB} = \sigma_a^0 \varepsilon_{AB}^n + \sum \int_{j}^{i} \left( \dot{\varepsilon}_{AB}, T_0 + \Delta T \right) \frac{\sigma_j}{\mu_0}. \]  

(9)
From the contact mechanics theory, the chip average flow stress could be alternatively calculated as $K_{int}$ from the interaction force between the tool and workpiece (Pan, Shih et al. 2017). An iterative scheme is used by matching the $\sigma_{AB}$ and $K_{int}$ for the $\phi$ calculation. The largest $\phi$ value is selected to be the shear angle (Pan, Lu et al. 2016). The machining forces in the cutting direction and thrusting direction could be determined as,

$$F_T = \frac{\sigma_{AB}tw\cos(\lambda - \alpha)}{\sin \phi \cos \theta}$$  \hspace{1cm} (10)

$$F_C = \frac{\sigma_{AB}tw\sin(\lambda - \alpha)}{\sin \phi \cos \theta}$$  \hspace{1cm} (11)

By implementing the above MTS based analytical model into a Matlab script, the machining forces in both cutting direction and thrusting direction could be obtained. The Ti-6Al-4V material thermomechanical properties are listed in Table 2 (Welsch, Boyer et al. 1993). At a tool rake angle of 8°, width of cut 3.8 mm, cutting speed 0.5 m/s, the machining forces are calculated as a function of different feed rate, as shown in Fig. 3. The experimental machining force data is obtained from the chapter four in Su’s thesis (Su 2006). The two-dimensional face turning experiments were conducted in the reference. For a more comprehensive comparison, both the experimental measurement data and classic JC flow stress prediction data are provided. Both the MTS model and JC model show good approximation for the machining forces $F_C$ and $F_t$ at all four different cutting conditions. $F_C$ and $F_t$ monotonically increase with the increasing feed rate. Also as noted from the MTS model is that, the predicted forces error would increase with the increasing feed rate. This could be explained by the strain sensitivity of the MTS model, as in Fig. 1. At a higher strain, the MTS flow stress tends to be larger than the JC model, which would result in larger forces at higher feed rate. Three more comparisons are provided at varying rake angle, as shown in Fig. 3. The cutting speed is 0.5 m/s, width of cut is 3.8 mm, feed rate is 0.153 mm/rev. The rake angle varies from 5° to 15°. As the rake angle increases, the force in the cutting direction $F_C$ dose not have obvious change, as indicated in Fig. 4 (a). The ploughing force $F_t$ monotonically decreases with the increasing rake angle, which results from the less ploughing effect, as shown in Fig. 4 (b). The largest prediction error is found to be in the feed rate of 101.6 μm, which is around 41.7 %.

### Table 2. The thermal mechanical model of Ti-6Al-4V

| Property                             | Value                  |
|--------------------------------------|------------------------|
| Density ($kg/m^3$)                   | 4420                   |
| Elastic modulus (MPa)                | 0.7412T+113375         |
| Poisson’s ratio                      | 0.31                   |
| Thermal conductivity ($W/mK$)        | $7.039 \times 10^{0.001T}$ |
| Heat capacity ($J/mm^3K$)            | $3.10 \times 10^{0.0007T}$ |
| Thermal expansion coefficient ($K/mm$) | $3 \times 10^{7T}+7 \times 10^{6}$ |
Fig. 3 The predicted cutting force as a function of feed rate, at the cutting speed 0.5 m/s, width of cut 3.8 mm, rake angle 8°.

Fig. 4 Predicted cutting forces as a function of rake angle, at the cutting speed of 0.5 m/s, width of cut 3.8 mm, depth of cut = 0.153 mm.

4. FEA machining force model

For further validation of the proposed MTS flow stress model in the application of machining force prediction, a finite element based machining force calculation is conducted. The MTS model is implemented into a commercial software DEFORM as a user subroutine. The Coulomb’s friction model (Dahl 1968) is used to describe the interaction between the tool and workpiece, as

$$\tau = \begin{cases} \mu \sigma_n & \tau < \sigma_y \\ \sigma_y & \tau \geq \sigma_y \end{cases}$$  \hspace{1cm} (12)

where $\tau$ is the shear stress in the tool/workpiece interface, $\mu$ is the friction coefficient, selected to be 0.7 from a previous study, $\sigma_n$ is the normal stress on the interface, $\sigma_y$ is the material flow stress. The tool is assumed to be rigid to improve the computational efficiency. The tool and workpiece are allowed to change heat at a heat transfer coefficient of 40 W/m²K. The workpiece and tool are exchanging the heat with the environment at a rate of 0.2 W/m²K. The FEA model of the machining process is illustrated in Fig. 5. The workpiece is in the dimension of 2 mm x 0.7 mm. A total...
number of 9000 elements are used for the workpiece. The boundary condition is applied by fixing the tool and moving the workpiece in the x-direction at a constant speed. The advanced remeshing technique is adopted which takes the strain and strain rate into the mesh size determination. Appropriate weighting factors over the material deformation state and geometrical shape are selected to make the solution convergent.

Given a tool rake angle of −4°, edge radius 20 \( \mu m \), the machining forces are predicted at a constant cutting speed 60 \( m/min \) and feed rate of 100 \( mm/rev \), as shown in Fig. 6 (a). The predicted forces in both direction are close to the experimental measurement data given from the work of Calamaz et al. (Calamaz, Coupard and Girot 2008). At a lower cutting speed of 60 \( m/min \), the static force dominates where the machining forces do not show a periodic fluctuation. By increasing the cutting speed to 180 \( m/min \), the cutting forces are plotted in Fig. 6 (b) against the experimental data. The forces show a periodic fluctuation with a frequency around 10 kHz. The chip segmentation has been reported to be a function of the cutting speed for the machining of Ti-6Al-4V material (Sun, Brandt et al. 2009). With a higher cutting speed, the fracture tends to occur in the chip which results in the segmented chip morphology.

Additional three machining force validation tests are conducted by varying the feed rate. To match the experimental condition in the work of Sima et al. (Sima and Özel 2010), the tool edge radius is 5 \( \mu m \). The cutting configuration gives a rake angle of 0°. The cutting speed is fixed at a constant value of 150 \( m/min \) to match the experimental condition.
provided in the reference. The feed rate ranges from 75 $\mu$m, 100 $\mu$m to 125 $\mu$m. The machining forces in cutting direction are thrust direction are plotted against the experimental data in Fig. 7. Similar result is found as in the analytical model part with the increasing feed rate. The forces in both direction increase with the increasing feed rate. Also, the predicted forces in both direction are consistently larger than the experimental data. The largest prediction error is 8.78% for $F_C$, and 29.2% for $F_t$. Compared with the experimental data provided, slight improvement on the force prediction is obtained. The inclusion of the material microstructure consideration from the MTS based flow stress model, makes it especially useful for the severe machining conditions.

The originality of work is the first forever, implementation of the MTS flow stress model for the machining process modeling. The reason for MTS model selection for the machining process modeling comes from the material microstructure consideration of the MTS flow stress model. Extensive experimental data has reported the material microstructure change in machining. However, traditional JC model provides only an empirical fitting. Easy modification could be made to include the grain size, phase composition, and the dislocation density to the MTS model for a purely physical based machining process modeling.

![Fig. 7](image)

**Fig. 7** The predicted machining force and experimental measurement in the cutting direction (a) and ploughing direction (b) at a cutting speed of 150 m/min, with varying feed rate.

5. Conclusion

A physical based material flow stress model MTS is introduced for the machining of Ti-6Al-4V. Based on the MTS model, an analytical turning force prediction model and FEA based model are developed for the 2D orthogonal turning force calculation. A comprehensive validation test are conducted by comparing the proposed model prediction with the experimental measurement for both analytical model and FEA model. The predicted machining forces show a good approximation to the experimental data as obtained from literature. The largest prediction error for the analytical model is found to be 41.7%, and 29.2% for the FEA model, which are all in the ploughing direction. In a general case, the model gives a more accurate prediction in the cutting direction than in the ploughing direction. The MTS model is proved to be an alternative flow stress model for he machining process modeling. For the first time, the MTS flow stress model is implemented for the machining process modeling. We are not trying to show the superiority of the MTS model for machining process application over the JC model in the current study. Intrinsically as a microstructure based model, the MTS model could be easily modified to include the material microstructure components, such as grain size, phase composition and dislocation, which makes it especially useful for the severe machining process model. Future work on the modified MTS model will be provided.

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