The Toroid Moment of Majorana Neutrino

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Abstract

If neutrino is the Majorana particle it can possess only one electromagnetic characteristic, the toroid dipole moment (anapole) in the static limit and nothing else. We have calculated the diagonal toroid moment (form factor) of the Majorana neutrino by the dispersion method in the one-loop approximation of the Standard Model and found it to be different from zero in the case of massive as well as massless neutrinos. All external particles are on the mass shells and there are no problems with the physical interpretation of the final result. Some manifestations of the toroid interactions of Majorana neutrinos, induced by their toroid moments, are also remarked.

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I. INTRODUCTION

The electromagnetic properties of Dirac and Majorana neutrinos are the subject of great interest at present [1]. The difference between massive Dirac and massive Majorana neutrinos is clearly exhibited by their electromagnetic properties. As early as 1939, Pauli remarked that the Majorana neutrinos have neither a magnetic dipole moment nor an electric dipole moment in vacuum [2]. Nevertheless the electromagnetic properties of Dirac andMajorana neutrinos can manifest themselves via the anapole moment also [3].

The anapole moment of $\frac{1}{2}$-spin Dirac particle was introduced by Zel’dovich [4] for a T-invariant interaction which does not conserve P-parity and C-parity individually. Subsequently, a more convenient characteristic was pointed out to describe of this kind of interaction, the toroid dipole moment (TDM) [5]. As was shown, the TDM is a general case of the anapole, it coincides with an anapole on the mass-shell of the particle under consideration and has a simple classical analog. Similar to an electric dipole and a magnetic dipole moments the TDM is the first term of third multipole family, the toroid moments. This type of static multipole moments does not produce any external electromagnetic fields in vacuum but generates a free-field (gauge-invariant) transverse-longitudinal potential [6] which is responsible for topological effects such as the Aharonov–Bohm effect. As was pointed out in [7] the Majorana particles of any value of the spin are characterized by toroid moments and nothing else.

A calculation of the vacuum anapole moment (TDM) of Dirac particle was started in [8]. Then, a number of articles about the problems of renormalizability, gauge non-invariance and, consequently, observability of the neutrino anapole moment and neutrino charge radius (NCR) was published. (See [8] [11] and references therein.) However, as was pointed out in [12], these quantities are finite and well-defined in the Standard Model (SM) as being the axial-vector (TDM) and the vector (NCR) contact interactions with an external electromagnetic field, respectively.

In this paper we calculate the vacuum diagonal TDM (form factor) of the Majorana neutrino in the framework of the SM. In Sec. 2, we start by clarifying the reason why the Majorana neutrinos can possess only one electromagnetic characteristic, the TDM. We also discuss the distinctions and similarities of the anapole and TDM of $\frac{1}{2}$-spin particle and correctly define the TDM of the Majorana neutrino. Using some examples in Sec. 3, we illustrate the toroid interactions of the Majorana neutrinos. Further in Sec. 4, using the dispersion method, we calculate the TDM of the Majorana neutrino in the one-loop approximation of the SM. All external particles are on the mass shells and there are no problems with the physical interpretation of the final result. We summarized our results in Sec. 5.

II. THE TOROID DIPOLE MOMENT OF THE MAJORANA NEUTRINO

In the case of Majorana neutrinos, the amplitude of the interaction with an external electromagnetic field $A^\mu$, $M \propto e J^{EM}_\mu(q) A^\mu(q)$ is defined by the particle and antiparticle contributions

$$ J^{EM}_\mu(q) = \left[ \overline{u}_f(p') \Gamma_\mu(q) u_i(p) + \overline{v}_i(p) \Gamma_\mu(q) v_f(p') \right] $$

$$ \equiv \overline{u}_f(p') \left[ \Gamma_\mu(q) - \left( C^{-1} \Gamma_\mu(q) C \right)^T \right] u_i(p), $$

(1)
where $e$ is the charge of an electron, $q_{\mu} = p'_{\mu} - p_{\mu}$ is the transferred 4-momentum, $u(p)$ and $v(p)$ are bispinors and $C$ is the charge conjugation matrix (we will use the chiral representation of the gamma matrices with $C = i\gamma_0\gamma_2$ and the normalization $\pi(p)u(p) = 1$; here $\Gamma_{\mu}(q)$ is the electromagnetic vertex, which is characterized by a set of electromagnetic form factors [3,5]. One popular way of defining the Lorentz structure of $\Gamma_{\mu}(q)$ is as follows:

$$\Gamma_{\mu}(q) = F(q^2)\gamma_{\mu} + M(q^2)\sigma_{\mu\nu}q^\nu + E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + A(q^2)[q^2\gamma_{\mu} - \tilde{q}q_{\mu}]\gamma_5,$$

(2)

where $F$, $M$, $E$ and $A$ are called the normal magnetic, anomalous magnetic, electric, and anapole dipole form factors, respectively. These form factors are physically observable quantities as $q^2 \to 0$ and their combinations define the well-known magnetic ($\mu$), electric ($d$) and anapole ($a$) dipole moments. In the non-relativistic limit, the energy of interaction with an external electromagnetic field has the following form:

$$\mathcal{H}_{\text{int}} \propto -\mu(\boldsymbol{\sigma} \cdot \mathbf{B}) - d(\boldsymbol{\sigma} \cdot \mathbf{E}) - a(\boldsymbol{\sigma} \cdot \text{curl} \mathbf{B}),$$

where $\mathbf{B}$ and $\mathbf{E}$ are the strengths of the magnetic and electric fields. In the case of Majorana neutrinos, imposing the restriction of the CPT-invariance and using the C-, P-, T-properties of $\Gamma_{\mu}(q)$ and $\mathcal{H}_{\text{int}}$, which we have combined in Table I, we see that the magnetic and electric dipole moments are absent in the static limit when the masses of the initial $m_i$ and final $m_f$ neutrino eigenstates are equal to each other. This means that Majorana neutrinos possess only one electromagnetic characteristic, the anapole moment [3,5]. But as was pointed out in Ref. [5], the anapole moment does not have a simple classical analog – $A(q^2)$ does not correspond to certain multipole distribution and, therefore, a more convenient characteristic, the TDM, was proposed to describe the T-invariant interaction with non-conservation of P and C symmetries. For clarity, we rewrite the axial part of the electromagnetic vertex (2) in the multipole parametrization [13]

$$\Gamma_{\mu}(q) \propto \left\{ i\varepsilon_{\mu\nu\lambda\sigma} P^\nu q^\lambda q^\sigma\gamma_5 T(q^2) + \sigma_{\mu\nu} q^\nu D(\Delta m^2) \right. - \frac{q^2 P_{\mu} - (q \cdot P) q_{\mu}}{q^2 - \Delta m^2} \left[D(q^2) - D(\Delta m^2)\right]\gamma_5,$$

(3)

where $\varepsilon_{\mu\nu\lambda\sigma}$ is the Levi-Civita unit antisymmetric tensor, $P_{\nu} = p'_{\nu} + p_{\nu}$, $\Delta m = m_i - m_f$ and $D(\Delta m^2)$, $D(q^2)$ and $T(q^2)$ are the charge dipole moment, charge dipole, and toroid dipole form factors, respectively. In this parametrization, there is a one-to-one correspondence in the definition of multipole moments by their form factors: the electric dipole moment by $D(\Delta m^2)$, the TDM by $T(\Delta m^2)$, etc. That is not a case in (2), where, for instance, the electric dipole moment

$$d \propto ie \left[ E(\Delta m^2) - \Delta m A(\Delta m^2) \right]$$

is defined in terms of the electric and anapole dipole form factors. Using the following identities:

$$\mathcal{U}_f(p') \left\{ q^2 \sigma_{\mu\nu} q^\nu + \Delta m \left[q^2 \gamma_{\mu} - \tilde{q} q_{\mu}\right] + \left[q^2 P_{\mu} - (q \cdot P) q_{\mu}\right] \right\} \gamma_5 u_i(p) = 0,$$

$$\mathcal{V}_f(p') \left\{ \Delta m \sigma_{\mu\nu} q^\nu + \left[q^2 \gamma_{\mu} - \tilde{q} q_{\mu}\right] - i\varepsilon_{\mu\nu\lambda\sigma} P^\nu q^\lambda q^\sigma\gamma_5 \right\} \gamma_5 u_i(p) = 0,$$

(4)
we obtain the connection between the anapole and TDM

\[ A(q^2) = T(q^2) + \frac{m_i^2 - m_f^2}{q^2 - \Delta m^2} \left[ D(q^2) - D(\Delta m^2) \right]. \]  

(5)

As can be seen, they coincide only on the mass-shell of the particle under consideration. The definition of the anapole by two independent form factors leads to confusion in the classical limit and gives no way for an analytical continuation of this form factor on the mass-shell. Hence, the TDM is a more convenient electromagnetic characteristic of the particle than the anapole.

Using the standard definitions of dipole moments, we define the TDM of the Majorana neutrino as a pseudovector \( T \) that is directed along the spin of the particle, the only vector characteristic in its own rest frame,

\[ T_\mu = eT(0)\overline{\sigma}_\mu \sigma u(0), \quad \mathbf{T} = eT(0)^\dagger \sigma \varphi, \]

where \( \varphi \) is the Pauli spinor. In the coordinate representation, \( \mathbf{T} \) is the total moment of the following density distribution:

\[ g(r) = \frac{1}{10} \left[ r(Jr) - 2r^2J \right], \]

where \( J \) is the current density that produces the magnetic field. The interaction of TDM with an external electromagnetic field has the following form:

\[ \mathcal{H}_{\text{int}} = eT(q^2)\overline{N}(x) \left[ q^2 \gamma_\mu A^\mu(x) - \gamma_\mu q^\nu q_\nu A^\mu(x) \right] \gamma_5 N(x) \]

\[ = eT(q^2)\overline{N}(x) \gamma_\mu \gamma_5 N(x) \frac{\partial^2 A^\mu(x)}{\partial x^\nu \partial x_\mu} - \frac{\partial^2 A^\nu(x)}{\partial x^\mu \partial x_\mu} \]

\[ = eT(q^2)\overline{N}(x) \gamma_\mu \gamma_5 N(x) \frac{\partial F^{\mu\nu}(x)}{\partial x^\nu}, \]  

(6)

where \( N(x) \) is the Majorana neutrino field, which has a usual plane wave expansion, and satisfies the following condition

\[ N^c(x) = C N(x) C^{-1} = \overline{N}^T(x) = N(x). \]

\( F^{\mu\nu}(x) \) is the tensor of the electromagnetic field, which, in turn, produces an external current

\[ \partial_\nu F^{\mu\nu}(x) = -JEM^\mu(x). \]

It is easy to see that in the nonrelativistic limit, when the particle is in its own rest system of reference,

\[ q^2 \to 0, \quad \overline{N}\gamma_0\gamma_5 N \to 0, \quad \overline{N}\gamma_5 N \to \varphi^\dagger \sigma \varphi, \]

the corresponding interaction energy is

\[ \mathcal{H}_{\text{int}} = -\mathbf{T} \cdot \mathbf{J} = -eT(0)^\dagger \sigma \varphi \left( \text{curl} \, B - \dot{E} \right). \]  

(7)

It has the moment of force

\[ \mathbf{M} = \mathbf{T} \left[ \sigma \times \mathbf{J} \right], \]

and represents a T-invariant toroid (anapole) interaction of the particle which does not conserve P-parity and C-parity individually, and defines the axial-vector contact interaction with an external electromagnetic field.
III. REMARKS ON SOME PROPERTIES OF THE TOROID INTERACTIONS OF MAJORANA NEUTRINOS

The TDM has many different applications, both in classical electrodynamics and in quantum theories \[5,6,11\]. The simplest model of TDM (anapole) was given by Zel’dovich \[4\] as a conventional solenoid folded into a torus and having only a poloidal current. For such a stationary solenoid, having neither an azimuthal (toroidal) component of the current nor electric fields around the torus, there is only a nonzero azimuthal magnetic field inside the torus. Therefore, fields outside the permanent toroid dipole are zero in vacuum, as well. However, as was pointed out by Ginzburg and Tsytovich \[16\], when the toroid dipole moves in a medium, the latter might be regarded as permitting the dipole itself and the fields outside the dipole appear to produce, for instance, Vavilov-Cherenkov radiation. Also, the toroid dipole is responsible for the transition radiation when it goes through the interface between two media whose indices of refraction are \( n_1 \) and \( n_2 \) (\( n_1 \gg n_2 \)). However, we should stress here that the result of \[16\] was derived in the framework of classical electrodynamics and it might change in quantum theory with the consideration of the Vavilov-Cherenkov and transition radiations of neutrinos induced by their toroid moments moving in the medium.

The toroid interactions of Dirac or Majorana neutrinos manifest themselves in collisions of the neutrinos with charged particles where the TDM conserves the helicity of the neutrino and gives an extra contribution, as a part of the radiative corrections, to the total cross section of the scattering of neutrinos by electrons, quarks and nuclei. In this regard, the toroid moment is similar to NCR. Both conserve the helicity in coherent neutrino scattering, but have different natures. They define the axial-vector (TDM) and the vector (NCR) contact interactions with an external electromagnetic field. Such interactions are the subject of interest in low-energy scattering processes and give one way to probe the NCR and TDM. For further implications and references in the question of the observability of NCR, see Refs. \[10,17\].

The toroid interactions of neutrinos may have a very interesting consequences in different media. As was pointed out in \[14\], the electromagnetic properties of Dirac and Majorana neutrinos in the medium are similar to the vacuum case. For instance, the Majorana neutrinos do not have the induced electric and magnetic dipole moments but have an induced anapole (toroid) dipole moment. However, it has a nonzero value in an anisotropic medium such as ferromagnetic material and absent in an isotropic medium. Nevertheless the vacuum TDM may play a very important role itself, in particular, for neutrino oscillations. For instance, let us consider the evolution equation for three neutrino flavors \( \vec{\nu} = (\nu_e, \nu_\mu, \nu_\tau)^T \) in the presence of toroid interactions

\[
i \frac{d\vec{\nu}}{d\tau} = K \left[ \frac{1}{2E} \text{diag} (m_1^2, m_2^2, m_3^2) + T(\tau) \right] K^\dagger \vec{\nu},
\]

where \( K \) is the mixing matrix connecting the flavor basis, \( \nu_\ell \) (\( \ell = e, \mu, \tau \)), and the mass basis, \( N_i \) (\( i = 1, \ldots, k \)), of Majorana neutrinos as \( \nu_\ell = \sum_i K_{i\ell} N_i \), and has the \( 3(k+1) \) mixing angles and \( 3(k+1) \) CP-violating phases (for details see \[19\].) The matrix \( T \) is, in general, a \( 3 \times 3 \) matrix whose elements

\[
T_{if}(\tau) \propto T_{if} \vec{\sigma} \cdot \text{curl} \mathbf{B}(\tau),
\]

(8)
are functions of time $\tau$ different from zero in the presence of the inhomogeneous vortex magnetic field which, in a concrete experimental situation, may be realized according to Maxwell’s equations as a displacement current or the current of the particle colliding with the considered neutrino at the space point where the interaction (3) is determined.

In this sense, this problem is an analog of the well-known Wolfenstein equation for the propagation of neutrinos through a medium [20], but resonance conversion of neutrinos can occur even in vacuum, due to the toroid interactions of neutrinos (if $T_{ii} \neq T_{jj}$ [21]). The off-diagonal matrix elements $T_{ij}$, induced by the transition toroid moments, are nontrivial factors in the Wolfenstein equation and had no previous analogs in the SM (beyond the scope of SM this role was played by the so-called flavor changing neutral currents). Since the Hamiltonian of evolution of the three neutrino flavors contains at least one time-dependent varying external parameter $\mathbf{\sigma} \cdot \text{curl} \mathbf{B}(\tau)$, we should take into account the topological phases in the evolution operator [22], which may be very important for neutrino oscillations [23]. If, in addition, a neutrino beam intersects some fluctuations of density and element compositions in the background of matter, a new phenomenon, geometric resonance, in neutrino oscillations occurs [24]. The role of the two time-dependent parameters of the Hamiltonian (varying independently from one another), which are need for the geometric resonance, can be played by the external electromagnetic field (in the medium, it can be the electron current and/or intrinsic sources) and the medium itself ($n_\nu \neq 1$). For example, if the curl $\mathbf{B}(\tau)$ and particle number density $\rho(\tau)$ vary cyclically when a neutrino beam propagates through the medium, i.e., curl $\mathbf{B}(\tau) = \text{curl} \mathbf{B}(0)$ and $\rho(\tau) = \rho(0)$ for some time $\tau$, they form a closed contour on the plane $(\mathbf{\sigma} \cdot \text{curl} \mathbf{B}, \rho)$, and for some neutrino momentum the geometric resonance takes place (for details, see [24].)

The transitions in a system of Majorana neutrinos with anapole and transition magnetic moments propagating in matter with a twisting non-potential magnetic field have recently been investigated within the asymmetric left-right model by Boyarkin and Rein [25]. It has been shown that the resonance conversion of neutrinos appears not only in response to the influence of matter, but also due to the availability of electromagnetic moments.

These effects can manifest themselves in numerous astrophysical and cosmological situations. Among them are neutrino propagation through the solar interior and young supernova envelopes, neutrino radiation of accreting neutron stars and black holes, where inhomogeneous vortex magnetic fields can have large values, etc., and the toroid interactions of neutrinos should be taken into account in each of them. But, the conclusions about the magnitude of these effects have been the subject of separate investigations, which is beyond the scope of our present work.

Since there is great interest in neutrino properties at the moment, we present here the calculation of the diagonal TDM, $T_i(0)$, of the mass eigenstate, $N_i$, of the Majorana neutrino in the framework of the SM.

**IV. ONE-LOOP RESULT**

The TDM of the Majorana neutrino can be defined in the one-loop approximation of the SM of electroweak interactions from the Feynman graphs shown in Figs. 1, 2. As one can see from (3) and (4), the transition TDM is equal to the diagonal one plus the part proportional to the neutrino mass difference. Therefore, to calculate the diagonal TDM and to estimate the transition one of the Majorana neutrino, only the anapole parametrization is sufficient.
To illustrate, we shall give some details of our calculations for the two graphs with $\ell\ell W$ states, see Fig. 3. The amplitudes and contributions to the imaginary part of the toroid form factor for the other diagrams are given in Appendix B. It is easy to verify that contributions of the particle and antiparticle currents are equal to each other, and we will consider one of them, and thus multiply the amplitude by a factor of 2. Using the Feynman rules and notation summarized in Appendix A, we can write the amplitude in the following form:

$$
\mathcal{M} = 2 \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} (2\pi)^4 \delta^4 \left[ p_1 - p_2 - (k_1 - k_2) \right] \\
\times \bar{u}(p_1) \left[ i\Gamma_{\chi}^{(\ell_N)} \right] i\Delta_F(k_1)(-ie\gamma_\mu) i\Delta_F(k_2) \\
\times \left[ i\Gamma_{\nu}^{(\ell_N)} \right] u(p_2) \left[ i\Delta_{W}^{\mu}(p_1 - k_1) \right] \mathcal{A}^\mu(k_1 - k_2). \quad (9)
$$

For convenience in our calculations, we pass to the $t$-channel where the momenta of particles transform as

$$
p_1 \rightarrow p_-, \quad p_2 \rightarrow -p_+, \\
k_1 \rightarrow k_- \equiv k_1, \quad k_2 \rightarrow -k_+ \equiv -k_2, \\
t = q^2 = (k_1 + k_2)^2 = (p_- + p_+)^2,
$$

and using the transformation

$$
\frac{1}{k^2 - m^2} \rightarrow (-2\pi i) \delta \left( k^2 - m^2 \right) \Theta(k_0), \quad (10)
$$

which is valid when we take into account the unitary condition for the S-matrix $\cite{26,27}$, we can write the imaginary part of the amplitude as

$$
\text{Im} \mathcal{M} = -e \int d\tau \frac{\mathcal{A}^\mu(q)}{(p_- - k_1)^2 - m_W^2} \\
\times \bar{u}(p_-)\Gamma_{\chi}^{(\ell_N)} (\tilde{k}_1 + m_\ell) \gamma_\mu \\
\times (\tilde{k}_2 - m_\ell) \Gamma_{\nu}^{(\ell_N)} g^{\lambda\nu} v(p_+). \quad (11)
$$

Here, we have denoted the two-body phase-space factor as

$$
d\tau = \frac{1}{(2\pi)^2} d^4k_1 d^4k_2 \delta^4(p_- + p_+ - k_1 - k_2) \\
\times \delta \left( k_1^2 - m_\ell^2 \right) \Theta(k_{10}) \delta \left( k_2^2 - m_\ell^2 \right) \Theta(k_{20}).
$$

Now, keeping only the terms with $\gamma_\mu \gamma_5$, and using the identity

$$
\text{Im} \mathcal{M} = e\pi(p_-) \text{Im} \left[ T_i(t)(t\gamma_\mu - \tilde{q}\gamma_\mu)\gamma_5 \right] v(p_+) \mathcal{A}^\mu(q),
$$
and the gauge $q_{\mu}A^\mu = 0$, we perform the two-particle phase space integration, following Refs. [8], and obtain the contribution to the imaginary part of the diagonal toroid form factor for the $\ell\ell W$ diagrams:

$$\text{Im} \, tT_i(t) = \frac{|A_L^{(EN)}|^2 - |A_R^{(EN)}|^2}{16\pi} (L_t - I_t - J_t),$$

(12)

where

$$L_k = \frac{1}{\lambda} \ln \left| \frac{1 + b_k}{1 - b_k} \right|, \quad J_k = \frac{\sqrt{a_k}}{\lambda^2} \left( 2 - \lambda b_k L_k \right),$$

$$I_k = a_k \left[ \frac{b_k}{\lambda} + \frac{1}{2} (1 - b_k^2) L_k \right], \quad \lambda = \sqrt{1 - \frac{4m_i^2}{t}},$$

$$a_k = \left( 1 - \frac{4m_i^2}{t} \right), \quad b_k = \frac{a_k + 2(m_W^2 + m_i^2 - m_i^2)/t}{\lambda \sqrt{a_k}}.$$ (13)

The real part of the toroid form factor can be derived by using the dispersion relation with one subtraction [26],

$$tT_i(t) - \phi = \frac{t}{\pi} \int_{4m_i^2}^{\infty} \frac{\text{Im} \, t'T_i(t')}{t'(t' - t - i0)} dt',$$

(14)

where $\phi$ is some constant, which we put zero in agreement with (2). Since $T_i(t) \rightarrow \text{Const} \neq \infty$ as $t \rightarrow 0$, we calculate the real part of the toroid form factor for $t \leq 0$, where $T_i(t) = \text{Re} \, T_i(t)$. Introducing the new variable $x = \frac{t'}{2m_W^2}$ and putting $m_i = 0$, for simplicity, we obtain

$$T_i(t) = \frac{|A_L^{(EN)}|^2 - |A_R^{(EN)}|^2}{32\pi^2 m_W^2} \int_{2\beta}^{\infty} F(x, \beta) dx$$

(15)

where $\alpha = \frac{-t}{2m_W^2} > 0$ and the integrand reads

$$F = \left( \frac{\beta - 1}{x} - 3 \right) \sqrt{1 - \frac{2\beta}{x}}$$

$$+ \left[ 2 \left( 1 + \frac{1}{2x} \right)^2 - \frac{\beta}{x^2} \left( 1 + x - \frac{\beta}{2} \right) \right]$$

$$\times \ln \left[ 1 + x - \beta + \sqrt{x(x - 2\beta)} \right]$$

$$\times \ln \left[ 1 + x - \beta - \sqrt{x(x - 2\beta)} \right],$$

with $\beta = \frac{m_i^2}{m_W^2}$. Finally, using the definition of the matrices $A_{L,R}^{(x)}$ and $B_{L,R}^{(x)}$, see eq. (21), and performing elementary integrations for these two graphs and others (making appropriate expansions in $\frac{m_i^2}{m_W^2}$ and denoting $f = e, \mu, \tau, u, d, s, c, b, t$), we obtain for $|t| = 0$: 8
where

\[ T_i(0) = \frac{\sqrt{2}G_F}{12\pi^2} \left[ C^{i}_{WWZ} + C^{i}_{W\phi Z} + C^{i}_{\phi\phi Z} + C^{i}_{f_f Z} \right. \]

\[ + \sum_{\ell=e,\mu,\tau} \left( C^{\ell}_{WW} + C^{\ell}_{W\phi} + C^{\ell}_{WW\phi} + C^{\ell}_{W\phi\ell} + C^{\ell}_{\phi\phi\ell} + C^{\ell}_{\phi\phi\ell} \right) \left(1 + \ln \frac{m^2_W}{4m^2_{s_{\ell}}} \right) - P_i \]

\[ C^{\ell}_{WW} = |K_{\ell\ell}|^2 \left[ \frac{11}{6} - \ln 4\beta + \beta \left( \frac{7}{4} + \frac{\ln 2}{2} - \frac{9}{8} \ln 3 \right) + O(\beta^2) \right] , \]

\[ C^{\ell}_{W\phi} = |K_{\ell\ell}|^2 \beta \left[ \frac{1}{6} - \frac{1}{2} \ln 4\beta + O(\beta) \right] , \]

\[ C^{\ell}_{WW\phi} = |K_{\ell\ell}|^2 \left[ -\frac{5}{6} + \frac{7}{6}\beta + O(\beta^2) \right] , \]

\[ C^{\ell}_{W\phi\ell} = |K_{\ell\ell}|^2 \left[ \frac{1}{8} + O(\beta^2) \right] , \]

\[ C^{\ell}_{\phi\phi\ell} = |K_{\ell\ell}|^2 \left[ -\frac{1}{12} + O(\beta^2) \right] , \]

\[ C^{\ell}_{f_f Z} = - \Omega_{\ell\ell} \sum_{j \neq f} (g^f_L + g^f_R) \left[ \frac{8}{3} - c_f - \frac{3}{2} \sqrt{c_f} \left( 1 - \frac{c_f}{3} \right) \ln \left| \frac{1 + \sqrt{c_f}}{1 - \sqrt{c_f}} \right| \right. \]

\[ - \Omega_{\ell\ell} (g^f_L + g^f_R) \left[ \frac{8}{3} + c_f - 3\sqrt{c_f} \left( 1 - \frac{c_f}{3} \right) \arctan \left( \frac{1}{\sqrt{c_f}} \right) \right] , \]

\[ C^{\ell}_{WWZ} = - \Omega_{\ell\ell} \left[ \frac{77}{6} + 2c_W - \frac{\sqrt{c_W}}{2} (27 + 4c_W) \arctan \frac{1}{\sqrt{c_W}} \right] , \]

\[ C^{\ell}_{W\phi Z} = - \frac{3}{4} \Omega_{\ell\ell} \sin^2 \theta_W \left[ \frac{2}{3} + c_W - \sqrt{c_W} (1 + c_W) \arctan \frac{1}{\sqrt{c_W}} \right] , \]

\[ C^{\ell}_{\phi\phi Z} = - \frac{1}{2} \Omega_{\ell\ell} (1 - 2\sin^2 \theta_W) \left[ \frac{1}{3} - c_W - \sqrt{c_W} (1 - \sqrt{c_W}) \right] \]

where

\[ c_k = \left| 1 - \frac{4m^2_k}{m^2_Z} \right| , \quad k = f, W, \]

\[ g^U_L = -\frac{1}{2} + \sin^2 \theta_W , \quad g^U_R = \sin^2 \theta_W , \]

\[ g^U_L = \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W , \quad g^U_R = -\frac{2}{3} \sin^2 \theta_W , \quad U = u, c, t , \]

\[ g^D_L = -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W , \quad g^D_R = \frac{1}{3} \sin^2 \theta_W , \quad D = d, s, b , \]

and \( K_{\ell\ell}, \Omega_{\ell\ell} \) are elements of the mixing matrices \( K \) and \( \Omega \) (see Appendix A.) We have also considered the \( f f \phi, f f \phi^0, c c Z^0, W W \phi^0, \phi \phi \phi^0 \) and \( c c c \phi^0 \) graphs, since they appear in the 't Hooft-Feynman gauge, but their amplitudes do not contain terms with the \( \gamma_\mu \gamma_5 \) structure and, therefore, they do not contribute to the TDM of the Majorana neutrino.

Summing over all contributions, we get

\[ T_i(0) \approx \frac{\sqrt{2}G_F}{12\pi^2} \sum_{\ell=e,\mu,\tau} |K_{\ell\ell}|^2 \left( 1 + \ln \frac{m^2_W}{4m^2_{s_{\ell}}} \right) - P_i \]
where $P_i = -\left(C_{WWZ}^i + C_{\phi Z}^i + C_{\phi\phi Z}^i + C_{fZ}^i\right)$ is the polarization-type contribution to the TDM. As one can see from (16), the $C_{\ell\ell W}^i$, $C_{W W \ell}^i$ and $P_i$ contributions are the leading terms that define the TDM. Since the quark mass values have various values in different hadronic models, the $C_{fZ}^i$ contribution contains large uncertainties. Taking into account the masses of leptons, $W, Z$-bosons and limits on the “current-quark masses” [18], we obtain [28]

$$T_i(0) \approx 1.277 \times 10^{-33} \left( |K_{ei}|^2 + 0.547 |K_{\mu i}|^2 
+ 0.307 |K_{\tau i}|^2 - 0.425 \times 10^{-2} P_i \right) \quad (\text{cm}^2),$$

$$P_i \in [8.585 \text{ to } 10.870] \Omega_{ii}. \quad (17)$$

Our result shows some very interesting peculiarities of TDM. In particular, as one can see from (16) and (17): (i) the value of the diagonal TDM depends on the masses of leptons, quarks, $W, Z$-bosons, and matrix elements of the mixing matrices $K$ and $\Omega$; (ii) the diagonal TDM of the massive as well as massless neutrino has a finite value and depends only very slightly on the mass of neutrino, $\mathcal{O}(m_i^2/m_W^2)$; (iii) the strong dependence of the TDM from the fermion masses shows it is a good tool to test the SM, as well as the new generation of fermions and bosons.

V. SUMMARY

While a Dirac neutrino has three dipole moments, a Majorana neutrino possesses only one electromagnetic characteristic, in the static limit, the toroid (anapole) dipole moment. Nevertheless, the Majorana neutrinos can have nonzero transition magnetic, electric and toroid dipole moments. We have calculated the diagonal toroid dipole moment of the mass eigenstate of the Majorana neutrino in the one-loop approximation of the SM. It determines by the matrix elements of the mixing matrices $K$ and $\Omega$, leptons, quarks and $W, Z$ bosons masses and has the absolute value of the order of $10^{-33} - 10^{-34}$ cm$^2$. This value is very sensitive to uncertainties of quark masses which define the up and down limits of TDM of the mass eigenstate of the Majorana neutrino, see eq. (17). We also found that TDM has a finite value in the case of massive as well as massless neutrinos. If there is no mixing in the lepton sector, $K = \Omega = 1$, we can define the singular electromagnetic characteristics, the toroid moments, of the three weakly interacting massless neutrinos as:

$$T_{\nu_e}(0) \approx [+ 6.873 \text{ to } + 8.112] \times 10^{-34} \quad (\text{cm}^2),$$

$$T_{\nu_\mu}(0) \approx [+ 1.090 \text{ to } + 2.329] \times 10^{-34} \quad (\text{cm}^2),$$

$$T_{\nu_\tau}(0) \approx [-1.971 \text{ to } -0.732] \times 10^{-34} \quad (\text{cm}^2). \quad (18)$$

Toroid interactions are a part of the radiative corrections to the scattering of neutrinos on electrons, quarks and nuclei both in vacuum and in medium. Therefore it gives a way to extract the experimental information about the magnitude of neutrino toroid moments in the low-energy scattering processes. Since the toroid interactions manifest themselves in the presence of an external electromagnetic field, they should be taken into account in various astrophysical and cosmological situations. As an example, in this paper we are touched the problem of neutrino oscillations. We are noticed that the toroid interactions can mediate the
resonance conversion of neutrinos even in vacuum in the presence of time-dependent external electromagnetic field. Also, they may have a very interesting consequences on neutrino oscillations in the medium. In fact, when the neutrino flux travels in the medium with a time-dependent non-potential vortex magnetic field at some conditions the geometrical resonance in neutrino oscillations occurs. However the conclusions about the magnitude of these effects requires a separate investigation.

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APPENDIX A

Here we give a short list of the Feynman rules used in our calculations. Weak interactions of Majorana neutrinos $N$ and charged leptons $\ell$ with gauge bosons $W^\pm$, $Z^0$, non-physical scalars $\phi^\pm$, $\phi$ and Higgs particles $\phi^0$ may be described by five Lagrangians [29]:

\[
\mathcal{L}_{\text{int}}^{NW^\pm} = \overline{N} \Gamma^{(\ell N)}_\mu W^{\pm\mu} + \overline{\ell} \Gamma^{(N)\ell} W^{-\mu},
\]
\[
\mathcal{L}_{\text{int}}^{NZ^0} = \overline{N} \Gamma^{(N)\ell} Z^{\mu} + \overline{\ell} \Gamma^{(\ell N)} \ell Z^{\mu},
\]
\[
\mathcal{L}_{\text{int}}^{N\phi^\pm} = \overline{N} \Gamma^{(\ell N)} \ell \phi^{\pm} + \overline{\ell} \Gamma^{(N)\ell} N\phi^{-},
\]
\[
\mathcal{L}_{\text{int}}^{N\phi^\pm} = \overline{N} \Gamma^{(N)\ell} \phi^{\pm} + \overline{\ell} \Gamma^{(\ell N)} N\phi^{-},
\]
\[
\mathcal{L}_{\text{int}}^{N\phi^0} = \overline{N} \Gamma^{(N)\ell} \phi^{-} + \overline{\ell} \Gamma^{(\ell N)} N\phi^{-}.
\]

The relevant Feynman rules are:

- $i\Gamma^{(\ell N)}_\mu$ for outgoing $W^-$ or incoming $W^+$,
- $i\Gamma^{(\ell N)}_\mu$ for outgoing $W^+$ or incoming $W^-$,
- $i\Gamma^{(\ell N)}_\mu$ for outgoing $\phi^-$ or incoming $\phi^+$,
- $i\Gamma^{(\ell N)}_\mu$ for outgoing $\phi^+$ or incoming $\phi^-$,
- $i\Gamma^{(N)\ell} + i\Gamma^{(NC)\ell}$ for $Z^0$,
- $i\Gamma^{(N)} + i\Gamma^{(NC)}$ for $\phi^0$,
- $i\Gamma^{(N\phi)} + i\Gamma^{(NC\phi)}$ for $\phi$.

Here

\[
\Gamma^{(NC)}_\mu \equiv C \left[ \Gamma^{(N)}_\mu \right]^T C^{-1}, \quad \Gamma^{(NC)} \equiv C \left[ \Gamma^{(N)} \right]^T C^{-1},
\]

and similarly, for $\Gamma^{(NC\phi)}$.

As was pointed out in Ref. [29], we should use the following rule for the Dirac-Majorana transition in a Feynman graph in real calculations: for an incoming (outgoing) Dirac particle, the outgoing (incoming) Majorana neutrino must be treated as a particle, and vice versa, for antiparticles.

Introducing the notation $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$, we can write the general forms for all vertices

\[
\Gamma^{(x)}_\mu = \gamma_\mu \left[ P_L A_{L(x)} + P_R A_{R(x)} \right], \quad x = \ell, N, \ell N,
\]
\[
\Gamma^{(\ell N)}_\mu \equiv \gamma_0 \left[ \Gamma^{(\ell N)}_\mu \right]^\dagger \gamma_0 = \gamma_\mu \left[ P_L A_{L(\ell N)^*} + P_R A_{R(\ell N)^*} \right],
\]

and

\[
\Gamma^{(x)} = P_L B_{L(x)} + P_R B_{R(x)}^\dagger, \quad x = N, N\phi, \ell N,
\]
\[
\Gamma^{(\ell N)} \equiv \gamma_0 \left[ \Gamma^{(\ell N)} \right]^\dagger \gamma_0 = P_R B_{R(\ell N)^*}^\dagger + P_L B_{L(\ell N)^*}^\dagger.
\]
The other Feynman rules used in our calculations are well known and are taken from \cite{30}. Equations (19–20) have a general form and must be specified for a given gauge group. Below, we present these matrices in the SM.

We will use the following definitions of charged and neutral currents:

\[ J^-_\mu = \frac{1}{2} \bar{\ell} \gamma_\mu (1 - \gamma_5) K N, \quad J^0_\mu = \frac{1}{4} \bar{\bar{\ell}} \gamma_\mu (1 - \gamma_5) \Omega N, \]

where \( K \), in general, is a rectangular matrix, an analog of the Kobayashi-Maskawa matrix in the quark sector, such that \( KK^\dagger = 1 \) and \( \Omega = K^\dagger K \neq 1 \). In the SM with 3 neutrino flavors from SU(2) doublets and \( k \)-singlets, the matrices \( K \) and \( \Omega \) have \( 3 \times (3 + k) \)- and \( (3 + k) \times (3 + k) \)-dimensions, respectively. In this manner, we define the required matrices \( A^{(x)}_{L,R} \) and \( B^{(x)}_{L,R} \) as:

\[
A^{(\ell N)}_L = \frac{g}{\sqrt{2}} K, \quad A^{(\ell N)}_R = 0, \\
A^{(f)}_L = \frac{g g_f}{\cos \theta_W}, \quad A^{(f)}_R = \frac{g g_f}{\cos \theta_W}, \\
A^{(N)}_L = \frac{g \Omega}{2 \cos \theta_W}, \quad A^{(N)}_R = 0, \\
B^{(\ell N)}_L = -\frac{g m_i K}{\sqrt{2} m_W}, \quad B^{(\ell N)}_R = \frac{g m_i K}{\sqrt{2} m_W},
\]

with

\[ G_F = \frac{\sqrt{2} g^2}{8 m_W^2}, \]

and use the propagators in the ‘t Hooft-Feynman gauge

\[
i \Delta_F(k) = \frac{i}{k - m_\ell + i\epsilon}, \\
i \Delta^{\mu\nu}_{W,Z}(k) = \frac{-ig^{\mu\nu}}{k^2 - M^2_{W,Z} + i\epsilon}, \\
i \Delta_{\phi^\pm}(k) = \frac{i}{k^2 - m^2_{W,Z} + i\epsilon}, \\
i \Delta_{\phi}(k) = \frac{i}{k^2 - m^2_{Z} + i\epsilon},
\]
APPENDIX B

In this appendix, we present the full list of amplitudes, in the $t$-channel, and the contributions to the imaginary parts of the diagonal toroid form factor for triangular- and polarization-type diagrams:

**$\ell\ell\phi$ triangular diagrams**

$$
\mathcal{M} = 2 \int d\theta \, \bar{u}(p_-) \left[ i\Gamma^{(\ell N)}_\lambda \right] i\Delta_F(k_1) (-ie\gamma_\mu) \times i\Delta_F(-k_2) \left[ i\Gamma^{(\ell N)}_\nu \right] v(p_+) \left[ i\Delta_\phi(p_- - k_1) \right] A^\mu(q),
$$

$$
\text{Im} \, tT_i(t) = \frac{1}{32\pi} \left( |B_L^{(\ell N)}|^2 - |B_R^{(\ell N)}|^2 \right) \left( I_\ell - J_\ell - L_\ell \right),
$$

where

$$
dq = \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \delta^4(p_- + p_+ - k_1 - k_2).
$$

**$WW\ell$ triangular diagrams**

$$
\mathcal{M} = 2 \int d\theta \, \bar{u}(p_-) \left[ i\Gamma^{(\ell N)}_\lambda \right] i\Delta_W^{\alpha}(k_1) \left[ -ieV_{\alpha\beta}(q, -k_1, -k_2) \right] \times i\Delta_W^{\beta}(k_2) \left[ i\Gamma^{(\ell N)}_\nu \right] v(p_+) \left[ i\Delta_F(p_- - k_1) \right] A^\mu(q),
$$

$$
\text{Im} \, tT_i(t) = \frac{1}{16\pi} \left( |A_L^{(\ell N)}|^2 - |A_R^{(\ell N)}|^2 \right) \left( I_W - J_W + \frac{1}{2} a_W L_W - \frac{3}{2} L_W \right).
$$

Here $V_{\alpha\beta}(r_1, r_2, r_3)$ is the usual vertex function:

$$
V_{\alpha\beta}(r_1, r_2, r_3) = (r_1 - r_2)\beta g_{\mu\alpha} + (r_2 - r_3)\mu g_{\alpha\beta} + (r_3 - r_1)\alpha g_{\beta\mu}, \quad r_1 + r_2 + r_3 = 0.
$$

**$W\phi\ell$ triangular diagrams**

$$
\mathcal{M} = 2 \int d\theta \, \bar{u}(p_-) \left[ i\Gamma^{(\ell N)}_\lambda \right] i\Delta_W^{\alpha}(k_1) \left( iem_W g_{\alpha\mu} \right) \times i\Delta_\phi(-k_2) \left[ i\Gamma^{(\ell N)}_\nu \right] v(p_+) \left[ i\Delta_F(p_- - k_1) \right] A^\mu(q),
$$

$$
\text{Im} \, tT_i(t) = \frac{m_W}{16\pi t} \left[ \left( A_R^{(\ell N)} B_L^{(\ell N)*} - A_L^{(\ell N)} B_R^{(\ell N)*} \right) L_W m_\ell \right.
$$

$$
+ \left. \left( A_L^{(\ell N)} B_L^{(\ell N)*} - A_R^{(\ell N)} B_R^{(\ell N)*} \right) (L_W + J_W) m_\ell \right],
$$

**$\phi W\ell$ triangular diagrams**
\[
\mathcal{M} = 2 \int d\phi \ \bar{\pi}(p_-) \left[i \Gamma^{(\text{EN})} \right] i \Delta_{\phi^+}(k_1) (i e m_W g_{\mu\beta})
\times i \Delta^{\beta'}_{\nu'}(-k_2) \left[i \bar{T}^{(\text{EN})}_{\nu'} \right] v(p_+) \left[i \Delta_{F}(p_- - k_1) \right] \mathcal{A}^\mu(q),
\]

\[
\text{Im } tT_i(t) = \frac{m_W}{16\pi t} \left[ \left( B_L^{(\text{EN})} A_R^{(\text{EN})} - B_R^{(\text{EN})} A_L^{(\text{EN})} \right) L_W m_e \right.
\]
\[
+ \left( B_L^{(\text{EN})} A_R^{(\text{EN})} - B_R^{(\text{EN})} A_L^{(\text{EN})} \right) (L_W + J_W) m_i \right].
\]

\textbf{ϕϕℓ triangular diagrams}

\[
\mathcal{M} = 2 \int d\phi \ \bar{\pi}(p_-) \left[i \Gamma^{(\text{EN})} \right] i \Delta_{F}(p_- - k_1) [-i e (k_2 - k_1)_{\mu}]
\times \left[i \bar{T}^{(\text{EN})}_{\nu} \right] v(p_+) i \Delta_{\phi^+}(k_1) i \Delta_{\phi^+}(-k_2) \mathcal{A}^\mu(q),
\]

\[
\text{Im } tT_i(t) = \frac{I_W}{32\pi} \left( |B_L^{(\text{EN})}|^2 - |B_R^{(\text{EN})}|^2 \right).
\]

\textbf{ffZ polarization diagram}

\[
\mathcal{M} = - \int d\phi \ \bar{\pi}(p_-) \left[i (\Gamma^{(\text{N})} + \Gamma^{(\text{NC})}) \right] i \Delta_{\nu_{\mu}}^\nu (q) \left[i \Gamma^{(f)}_{\nu} \right]
\times i \Delta_{F}(-k_2) \left[-i e Q_f \gamma_{\mu} \right] i \Delta_{F}(k_1) v(p_+) \mathcal{A}^\mu(q),
\]

\[
\text{Im } tT_i(t) = \frac{1}{16\pi} \left[ A_R^{(\text{N})} - A_L^{(\text{N})} \right] \left[ A_R^{(f)} + A_L^{(f)} \right] \sqrt{a_f} \left( 1 - \frac{a_f}{3} \right) \left( 1 - \frac{m_{Z}^{2}}{t} \right)^{-1}.
\]

\textbf{WWZ polarization diagram}

\[
\mathcal{M} = \int d\phi \ \bar{\pi}(p_-) \left[i (\Gamma^{(\text{N})} + \Gamma^{(\text{NC})}) \right] i \Delta_{\nu_{\mu}}^\nu (q) \left[-i g \cos \theta_W V_{\rho_{\alpha}}(-q, k_2, k_1) \right] i \Delta_{W}^{\alpha\alpha'}(k_1)
\times i \Delta_{W}^{\beta\beta'}(k_2) \left[-i e V_{\mu_{\alpha'}}(q, -k_1, -k_2) \right] v(p_+) \mathcal{A}^\mu(q),
\]

\[
\text{Im } tT_i(t) = \frac{g \cos \theta_W}{96\pi} \left[ A_R^{(\text{N})} - A_L^{(\text{N})} \right] \frac{(23t + 16m_{W}^{2})\sqrt{a_W}}{t - m_{Z}^{2}}.
\]

\textbf{WϕZ polarization diagram}

\[
\mathcal{M} = \int d\phi \ \bar{\pi}(p_-) \left[i (\Gamma^{(\text{N})} + \Gamma^{(\text{NC})}) \right] i \Delta_{Z}^{\nu_{\mu}}(q)
\times \left(-i g m_Z \sin^2 \theta_W g_{\rho_{\alpha}} \right) i \Delta_{W}^{\alpha\alpha'}(k_1) i \Delta_{\phi^+}(-k_2) (i e m_W g_{\alpha'\mu}) v(p_+) \mathcal{A}^\mu(q),
\]

\[
\text{Im } tT_i(t) = \frac{g \sin^2 \theta_W}{16\pi} \left[ A_R^{(\text{N})} - A_L^{(\text{N})} \right] \frac{m_W m_Z \sqrt{a_W}}{t - m_{Z}^{2}}.
\]

\textbf{φϕZ polarization diagram}
\[ M = \int d\varphi \, \pi(p_{-}) \left[ i(\Gamma^{(N)}_{\nu} + \Gamma^{(NC)}_{\nu}) \right] i\Delta^\nu_{\rho}(q) \times \left[ -ig \frac{1 - 2\sin^2\theta_{W}}{2\cos\theta_{W}}(k_{1} - k_{2})_{\rho} \right] i\Delta_{\phi}^{+}(-k_{2})i\Delta_{\phi}^{-}(k_{1})\left[ ie(k_{1} - k_{2})_{\mu} \right] v(p_{+})A^\mu(q), \]

\[ \text{Im } t_{i}(t) = \frac{g(1 - 2\sin^2\theta_{W})}{96\pi \cos\theta_{W}} \left[ A_{R}^{(N)} - A_{L}^{(N)} \right] \frac{a_{W}^{3/2}}{1 - m_{Z}^{2}/t}. \]
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For resonance conversion of neutrinos in the medium, the MSW effect, the role of the matrix elements $\mathcal{T}_{ii}$ plays the combinations $(n_{\nu_\beta} - n_{\nu_\gamma}) p_\nu$, where $n_{\nu_\ell}$ is the neutrino index of refraction and $p_\nu$ is the neutrino momentum.

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## TABLES

TABLE I. C-, P-, T-properties of the spin, the electromagnetic field, and their interactions.

|         | C | P | T |
|---------|---|---|---|
| $\sigma$ | + | + | - |
| B       | - | + | - |
| E       | - | - | + |
| $\text{curl } B$, $\dot{E}$ | - | - | - |
| $\sigma \cdot B$ | - | + | + |
| $\sigma \cdot E$ | - | - | - |
| $\sigma \cdot \text{curl } B$ | - | - | + |
| $\sigma \cdot \dot{E}$ | - | - | + |
FIG. 1. Triangle diagrams responsible for the toroid moment of the Majorana neutrino.
FIG. 2. Polarization-type diagrams responsible for the toroid moment of the Majorana neutrino.
FIG. 3. Feynman graphs with $\ell \ell W$ intermediate states.