Point-Form Hamiltonian Dynamics and Applications

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Abstract This short review summarizes recent developments and results in connection with point-form dynamics of relativistic quantum systems. We discuss a Poincaré invariant multichannel formalism which describes particle production and annihilation via vertex interactions that are derived from field theoretical interaction densities. We sketch how this rather general formalism can be used to derive electromagnetic form factors of confined quark-antiquark systems. As a further application it is explained how the chiral constituent quark model leads to hadronic states that can be considered as bare hadrons dressed by meson loops. Within this approach hadron resonances acquire a finite (non-perturbative) decay width. We will also discuss the point-form dynamics of quantum fields. After recalling basic facts of the free-field case we will address some quantum field theoretical problems for which canonical quantization on a space-time hyperboloid could be advantageous.

Keywords Point-form dynamics · Relativistic quantum mechanics · Hadron structure · Quantum field theory

1 Introduction

The formal problem of constructing a relativistically invariant quantum theory amounts to finding a representation of the Poincaré generators in terms of self-adjoint operators that act on an appropriate Hilbert space and satisfy the Poincaré algebra. If one is interested in interacting theories this is a non-trivial task. The commutation relation

\[ [\hat{P}^0, \hat{K}^j] = i\hat{P}^j \]  

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between the generator for time translations $\hat{P}^0$ and the boost generators $\hat{K}^j$ already indicates that interaction terms must appear in more than one of the Poincaré generators. If interaction terms show up in $\hat{P}^0$, then $\hat{K}^j$ or $\hat{P}^l$ (or both) must be interaction dependent too. A particular form of relativistic dynamics is then characterized by the set of Poincaré generators that contain interactions. In Dirac’s seminal paper on the forms of classical relativistic dynamics the interaction dependent (dynamical) Poincaré generators were called “Hamiltonians” [1]. The most prominent forms are the “instant form” (IF), the “front form” (FF), and the “point form” (PF) with the corresponding sets of Hamiltonians being given by \{ $\hat{P}^0, \hat{K}^1, \hat{K}^2, \hat{K}^3$, \}$, \{ $\hat{P}^0 - \hat{P}^3, \hat{P}^1 - J^2, F^2 = K^2 + J^1$, \}$, and \{ $\hat{P}^0, \hat{P}^1, \hat{P}^2, \hat{P}^3$, \}$, respectively. Among these forms the point form is the least known and, although it has definite virtues, the least utilized. An obvious advantage is the clear separation of Poincaré generators that are interaction dependent from those which are free of interactions. The former are components of a 4-vector $\hat{P}^\mu$ whereas the latter can be combined into an antisymmetric Lorentz tensor $\hat{j}^{\mu\nu}$. This makes it possible to express equations in point form in a manifestly Lorentz covariant way and makes for simple behavior under rotations and boosts. The conditions for Poincaré invariance can, e.g., be phrased in terms of the “point-form equations”

\[
[\hat{P}^\mu, \hat{P}^\nu] = 0, \\
\hat{U}_A \hat{P}^\mu \hat{U}_A^\dagger = (A^{-1})^\mu_\nu \hat{P}^\nu,
\]

where $\hat{U}_A$ is the unitary operator representing the Lorentz transformation $A$ on the Hilbert space. The Lorentz part of the Poincaré commutation relations has been integrated out, which is possible due to the kinematic nature of the rotation and boost generators.

The usual procedure for constructing a representation of the Poincaré algebra on a Hilbert space is canonical quantization of a local field theory. The interaction dependence of the Poincaré generators is then determined by the quantization surface. If the surface is chosen as the forward hyperboloid $x_\mu x^\mu = \tau^2$ all interactions go into $\hat{P}^\mu$ and one ends up with point-form quantum field theory (PFQFT) [2]. A quantized field theory is intrinsically a many-body theory with infinitely many degrees of freedom with the representation space being a Fock space. Relativistic invariance follows automatically if one starts from a Poincaré-scalar Lagrangean density.

The problem of constructing a relativistically invariant quantum theory for a restricted number of particles is more intricate. Interaction terms that show up in the dynamical Poincaré generators are, in general, subject to non-linear constraints which are imposed by the Poincaré algebra. A particular solution to the problem of finding a consistent set of 10 Poincaré generators for an interacting quantum mechanical system with finitely many degrees of freedom was discovered by Bakamjian and Thomas [3]. If their construction is carried out in point form the resulting (interacting) 4-momentum operator $\hat{P}^\mu$ is seen to separate into an interacting mass operator $\hat{M}$ and a free velocity operator $\hat{V}^\mu_{\text{free}}$,

\[
\hat{P}^\mu = \hat{P}^\mu_{\text{free}} + \hat{P}^\mu_{\text{int}} = (\hat{M}_{\text{free}} + \hat{M}_{\text{int}}) \hat{V}^\mu_{\text{free}}.
\]

The PF Eqs. (2) imply that the interacting part of the mass operator $\hat{M}_{\text{int}}$ must be a Lorentz scalar that has to commute with the free velocity operator, i.e. $[\hat{M}_{\text{int}}, \hat{V}^\mu_{\text{free}}] = 0$, to ensure Poincaré invariance. Within the Bakamjian-Thomas framework the operator
for the total spin of the (interacting) system is not affected by interactions. However, different forms of dynamics are, in general, associated with different definitions of the spin (canonical spin, helicity, front-form spin, ...), which are related via the unitary “Melosh transformation” [4]. Remarkably, in the Bakamjian-Thomas approach it is possible to construct Poincaré invariant models with instantaneous interactions (action-at-a-distance). What one loses with instantaneous interactions as compared to a local quantum field theory is, however, microcausality. In relativistic quantum mechanics this property is replaced by the weaker condition of cluster separability (or macrocausality) [5]. Macrocausality roughly means that disjunct subsystems of a quantum mechanical system should behave independently of each other, if they are separated by sufficiently large space-like distances.

In Sec. 2 we will show that retardation effects in particle-exchange interactions can be accommodated within the Bakamjian-Thomas framework by means of a multichannel formulation. This multichannel framework will then be used to derive electroweak current matrix elements and form factors of bound few-body systems. In this context it will be necessary to discuss the question of cluster separability, as it plays a role in the extraction of form factors. A further example shows that such a coupled-channel approach could also be used to formulate constituent quark models that lead to unstable excited states, i.e. resonances with finite (non-perturbative) decay widths. Section 3 is devoted to point-form quantum field theory. We will give a few applications for which field quantization on a space-time hyperboloid could be advantageous as compared to usual equal-time quantization. Section 4 finally contains our conclusions and an outlook.

2 Point-form quantum mechanics

2.1 Relativistic multichannel framework

Our first aim will be to describe particle-exchange interactions within the Bakamjian-Thomas approach in such a way that retardation effects are fully taken into account. This can be done within a multichannel framework that allows for the creation and annihilation of a finite number of additional particles. From Eq. (3) it is clear that it suffices to study the eigenvalue problem for the mass operator. Let us start with a 2-channel system in which an \( N \)-particle channel is coupled to an \( (N+1) \)-particle channel. The mass-eigenvalue problem for such a system can be written as

\[
\begin{pmatrix}
\hat{M}_N & \hat{K} \\
\hat{K}^\dagger & \hat{M}_{N+1}
\end{pmatrix}
\begin{pmatrix}
|\Psi_N\rangle \\
|\Psi_{N+1}\rangle
\end{pmatrix} = m
\begin{pmatrix}
|\Psi_N\rangle \\
|\Psi_{N+1}\rangle
\end{pmatrix},
\]

with a matrix mass operator that acts on the direct sum of \( N \)- and \( (N+1) \)-particle Hilbert spaces. \( |\Psi_N\rangle \) and \( |\Psi_{N+1}\rangle \) are the \( N \)- and \( (N+1) \)-particle components of the mass eigenstate with mass eigenvalue \( m \). The diagonal elements of the mass operator \( \hat{M}_n, n = N, N+1 \) represent the invariant masses of the uncoupled \( N \)- and \( (N+1) \)-particle systems. In addition to the relativistic kinetic energies of the particles these operators may contain instantaneous interactions between the particles. In the forthcoming applications to constituent-quark models an instantaneous confinement between the (anti)quarks is, e.g., put into \( \hat{M}_n \). \( \hat{K} \) is the vertex operator that accounts for the creation of the additional particle.
For our further purposes it is useful to apply a Feshbach reduction to Eq. (1) and eliminate the \((N+1)\)-particle channel in favor of the \(N\)-particle channel:

\[ m |\Psi_N\rangle = \left( \hat{M}_N + \hat{K}^\dagger (m - \hat{M}_{N+1})^{-1} \hat{K} \right) |\Psi_N\rangle = : \left( \hat{M}_N + \hat{V}_{\text{opt}}(m) \right) |\Psi_N\rangle. \tag{5} \]

The optical potential \(\hat{V}_{\text{opt}}(m)\) introduced in this way is a function of the mass eigenvalue \(m\) and consists of all possibilities to exchange the \((N+1)\)st particle between the remaining particles. It also includes loop contributions, i.e. absorption by the emitting particle. \((m - \hat{M}_{N+1})^{-1}\) describes the propagation of the \((N+1)\)-particle intermediate state and is thus responsible for retardation effects.

In order to analyze Eq. (3) in some more detail it is appropriate to introduce a set of basis states that is tailored to the point-form version of the Bakamjian-Thomas construction:

\[ |v; k_1, \mu_1; \ldots; k_n, \mu_n\rangle = \hat{U}_{B_\nu(v)} |k_1, \mu_1; \ldots; k_n, \mu_n\rangle \quad \text{with} \quad \sum_{i=1}^n k_i = 0. \tag{6} \]

Such states are called “velocity states”. A velocity state is a usual multiparticle momentum state in the rest frame \(\sum_{j=1}^n k_j = 0\) that is boosted to overall 4-velocity \(v\) \((v_\nu v^\mu = 1)\) by means of a canonical spin boost \(B_\nu(v)\). The \(\mu_i\)'s denote the spin projections of the individual particles. A velocity state is a simultaneous eigenstate of the free \(n\)-particle mass operator \(\hat{M}_{\text{free},n}\) (eigenvalue \(M_n = \sum_{i=1}^n \sqrt{m_i^2 + k_i^2}\)) and the 4-velocity operator \(\hat{V}_\nu\) \(\text{free},n\) (eigenvalue \(v^\mu\)). In a velocity-state representation of a Bakamjian-Thomas type model the overall velocity of the system can always be factored out as a velocity-conserving delta function, leaving an equation for the pure internal motion. A further advantage of velocity states is that angular momenta can be coupled together as in non-relativistic quantum mechanics.

The velocity-state representation can now be used to specify the vertex operator \(\hat{K}\) by relating it to an appropriate field theoretical interaction density \(\hat{\mathcal{L}}_{\text{int}}(x)\). Since the total velocity of a Bakamjian-Thomas type system should be conserved, one is led to define matrix elements of \(\hat{K}\) via the relation \(\text{[7]}\)

\[ \langle v'; k'_1, \mu'_1; \ldots; k'_{n+1}, \mu'_{n+1} |\hat{K}| v; k_1, \mu_1; \ldots; k_n, \mu_n\rangle = \mathcal{N}_{n+1,n} v^0 \delta^4(v - v') \langle k_1, \mu_1; \ldots; k'_n, \mu'_n |\hat{\mathcal{L}}_{\text{int}}(0) f(\Delta m) |k_1, \mu_1; \ldots; k_n, \mu_n\rangle, \tag{7} \]

with \(\mathcal{N}_{n+1,n} = (2\pi)^3 \sqrt{m_{n+1} m_n} f(\Delta m = |M_{n+1} - M_n|)\) denotes a vertex form factor that one may introduce to compensate for missing off-diagonal velocity contributions, as well as regulate integrals. The prescription, Eq. (4), preserves the Lorentz structure of field theoretical vertex interactions, but it violates locality of the vertex due to the \textit{overall} velocity conserving delta function \(\delta(v - v')\) and the kinematical factor \(\mathcal{N}_{n+1,n}\). These quantities do not only depend on the kinematics at the vertex, but also on the kinematics of the spectator particles. Later on we will comment on the consequences for macroscopic causality.

This relativistic coupled-channel framework with field-theory motivated vertex interactions has already been applied to several problems. In Ref. [3] the mass operator for \(\pi\)-\(N\) and \(N\)-\(N\) scattering has been derived using chiral perturbation theory. In the following we are going to show how electromagnetic hadron form factors can be extracted from the optical one-photon-exchange potential for electron scattering off confined (anti)quarks [9]. A most recent application, which we are also going to discuss, is the construction of constituent quark models that lead to resonance-like behavior for hadron excitations.
2.2 Electromagnetic structure of hadrons

Electromagnetic hadron form factors are observables that encode the electromagnetic structure of hadrons. The theoretical analysis of hadron form factors amounts to the question of how the electromagnetic current $J^\mu$ of a hadron may be expressed in terms of the electromagnetic currents of its constituents. It has long been recognized that binding effects must appear in $J^\mu$ and it cannot be a simple sum of constituent currents \[10\]. The 3 basic constraints that a model for the electromagnetic hadron current has to satisfy are Poincaré covariance, current conservation and the requirement that the hadron charge should not be renormalized by binding effects (i.e. correct normalization of form factors at vanishing momentum transfer) \[11\]. General procedures for the construction of model currents of bound few-body systems that satisfy these constraints are given in Refs. \[11,12\]. Both papers use the point form of relativistic quantum mechanics and the Breit frame, in which the constraints on $J^\mu$ are most easily satisfied, for their analysis. Due to the kinematic nature of Lorentz boosts in point form the model current constructed in the Breit frame is then readily transformed into an arbitrary frame. Given the bound-state wave function of a few-body system, this construction generates a whole class of currents that satisfy the 3 basic constraints. One of the admissible currents is the spectator current, in which only one of the constituents is active, whereas the other ones are just spectators.

In the following we will present a different strategy and rather try to derive a microscopic model for a hadron current that is consistent with the binding forces than to construct it (cf. Ref. \[9\]). To do this we will use the Poincaré invariant coupled channel framework just introduced and study the full physical electron-hadron scattering process and not just the emission and absorption of a virtual photon by the hadron. To be more specific, let us consider a pion within a constituent-quark model. Electron-pion scattering is then treated as a 2-channel problem. The first channel is the $eq$ channel, the second the $eq\gamma$ one. In this way the dynamics of the exchange photon is fully taken into account. The diagonal terms of the mass operator in Eq. (4) are given by $M_3 = M_{\text{free},eq} + V_{\text{conf}}$ and $M_4 = M_{\text{free},eq\gamma} + V_{\text{conf}}$, respectively. $V_{\text{conf}}$ is an instantaneous confinement potential that acts only between the quark and antiquark. The vertex operator $\hat{K}$ is defined via velocity-state matrix elements as in Eq. (7) with $\hat{L}_{\text{int}}(\alpha)$ being the Lagrangean density for the coupling of a photon to a spin-1/2 (anti)quark.

Hadron form factors can be extracted from the invariant 1-photon-exchange amplitude for electron-hadron scattering. In our case this amplitude is given by on-shell matrix elements of (the 1-photon-exchange part of) the optical potential between velocity eigenstates of $M_3$ with the $q\bar{q}$ cluster possessing the quantum numbers of the pion. As one would expect, these on-shell matrix elements of the optical potential can be written as a contraction of the usual point-like electron current with a hadronic current times the (covariant) photon propagator:

$$\langle v'; k'_{e}, \mu'_{e}; k'_{M}, \alpha_{M} | \hat{V}_{\text{opt}}(m) | v; k_{e}, \mu_{e}; k_{M}, \alpha_{M} \rangle_{\text{on-shell}} \propto \frac{1}{(k'_{e} - k_{e})^2} \int_0^\infty d\beta \langle k'_{e}, \mu'_{e}; k_{e}, \mu_{e} | J^\mu (k'_{M}; k_{M}) \rangle | J^\mu (k'_{M}; k_{M}) \rangle_{\text{on-shell}}.$$  

(8)

"On-shell" means that $m = k'^0_\alpha + k'^0_{M} = k^0_\alpha + k^0_{M}$ and $k^0_\alpha = k^0_{M} = k^0_\alpha$, $k^0_{M} = k^0_\alpha$, $\alpha_{M}$ denotes the discrete quantum numbers of $q\bar{q}$ bound state. The detailed derivation of Eq. (8) and the explicit expression for the pion current $J^\mu (k'_{M}; k_{M})$ can be...
Dependence of the physical pion form factor $f_1$ and of the spurious pion form factor $f_2$ (cf. Eq. (10)) on the meson center-of-mass momentum $k = |k_M|$ for different values of the squared momentum transfer $Q^2$.

found in Ref. [9]. Since $k_M$ and $k'_M$ are momenta defined in the center of mass of the electron-meson system $J^\mu(k'_M; k_M)$ does not behave like a 4-vector under a Lorentz transformation $A$. It rather transforms by the Wigner rotation $R_W(v, A)$. The current with the correct transformation properties is the one involving the physical meson momenta $p_M^{(l)} = B_c(v) k_M^{(l)}$:

$$J^\mu(p'_M; p_M) := [B_c(v)]^\mu_\nu J^\nu(k'_M; k_M).$$

(9)

In addition one can show that $(p'_M - p_M)_\mu J^\mu(p'_M; p_M) = 0$, i.e. $J^\mu$ is a conserved current. If $J^\mu(p'_M; p_M)$ were a correct model for the pion current it would be possible to write it in the form $J^\mu(p'_M; p_M) = (p_M + p'_M)^\mu F(Q^2)$ (with $Q^2 = (p'_M - p_M)^2$) which should hold for arbitrary values of the meson momenta $p_M$ and $p'_M$. This is, however, not possible in our case. The reason is that our derivation of the current is based on the Bakamjian-Thomas construction which is known to provide wrong cluster properties [4]. Therefore we cannot be sure that the hadronic current we get does not also depend on the electron momenta. What we find is indeed that $J^\mu(p'_M; p_M)$ cannot be fully expressed in terms of hadronic covariants, but one needs 1 additional (current conserving) covariant, which is the sum of the 2 electron momenta:

$$J^\mu(p'_M; p_M) = (p_M + p'_M)^\mu f_1(Q^2, s) + (p_e + p'_e)^\mu f_2(Q^2, s).$$

(10)

This decomposition holds for arbitrary values of the meson momenta $p_M$ and $p'_M$ with only one exception, the Breit frame (i.e. backward scattering in the electron-meson center of mass system). In this frame the 2 covariants become proportional and it is not possible to separate the 2 form factors. Wrong cluster properties may also influence the form factors $f_1$ such that they not only depend on the squared 4-momentum transfer at the photon-pion vertex, but also on Mandelstam $s$, i.e. the square of the invariant mass of the electron-meson system.

The influence of cluster-separability violating effects can be studied numerically. To this end we have taken the simple harmonic oscillator wave function for the $q\bar{q}$ bound state that has already been used in Ref. [9] to calculate the microscopic current $J^\mu(k'_M; k_M)$ and separate the form factors by means of Eq. (10). The result is shown in Fig. 1. One first observes that both, the physical form factor $f_1$ and the spurious form
factor $f_2$ depend on $|k_M|$ (or Mandelstam $s$). This $s$ dependence of $f_1$ vanishes rather quickly with increasing invariant mass. At the same time the spurious form factor $f_2$ is seen to vanish. It is thus suggestive to take the limit $s \to \infty$ to get a sensible result for the pion form factor. In this limit the microscopic pion current factorizes explicitly into a point-like pion current times a $Q^2$ dependent integral which can be identified as pion form factor:

$$J^\mu(k'_M; k_M) \xrightarrow{s \to \infty} (k_M + k'_M)\mu F(Q^2).$$

The final result for the form factor has a simple analytical form and can be proved to be equivalent to the standard front form expression for a spectator current in the $q^+ = 0$ frame \[9\]. The fact that the $s \to \infty$ limit removes effects due to violation of cluster separability seems to have a plausible explanation: In order to separate subsystems by space-like distances in point form it is natural to use boosts. In this way one stays on the quantization hypersurface. Separation by boosts, however, increases the invariant mass of the whole system. Or, reversing the argument, by taking $s$ large we separate the electron from the meson such that it does not affect the meson structure.

This procedure for the calculation of form factors can immediately be generalized to electroweak form factors of arbitrary bound few-body systems. Very recently it has been applied to electroweak form factors of heavy-light mesons \[13\]. We have been able to derive a simple analytical expression for the Isgur-Wise function and to prove that the form-factor relations due to heavy-quark symmetry are recovered in the heavy-quark limit $m_Q \to \infty$. The electromagnetic form factors of spin-1 bound states, like the deuteron, are presently under investigation. Here it turns out that one cannot completely get rid of the problems associated with cluster separability by taking the $s \to \infty$ limit. Covariant structures in which $(k_e + k'_e)$ is contracted with the in- and outgoing polarization vectors of the spin-1 particle do not vanish in this limit. But our findings resemble those of Ref. \[14\] that have been obtained within a covariant light-front approach. In this paper the authors also encounter spurious contributions to the current which are associated with a 4-vector $\omega^\mu$ that characterizes the orientation of the light front. These spurious contributions correspond exactly to our spurious contributions which have their origin in the violation of cluster separability. Their presence guarantees that the angular condition for the (spin) matrix elements of the physical part of the current is satisfied \[14\]. This means that there is also a connection between the angular condition and cluster separability.

2.3 Dressing of hadrons and decays of resonances

The resonance character of hadron excitations is usually ignored when calculating hadron spectra within constituent quark models. Hadron excitations rather come out as stable bound states and their bound-state wave function is subsequently used to calculate partial decay widths perturbatively by assuming a particular model for the decay vertex on the constituent level. The observation that the predicted strong decay widths are notoriously too small \[15\]-\[17\] is an indication that a physical hadron resonance is not just a simple bound state of valence (anti)quarks, but it should also contain (anti)quark-meson components. A natural starting point for including such components is the chiral constituent-quark model. The physical picture behind this model is that the effective degrees of freedom emerging from chiral symmetry breaking of QCD are constituent quarks and Goldstone bosons which couple directly to the constituent quarks. The Goldstone bosons can be identified with the lightest pseudoscalar
meson octet and singlet. With a linear confinement and the hyperfine interaction being given by the instantaneous approximation to Goldstone boson exchange the chiral constituent-quark model has been very successful in reproducing mass spectra [1] and electroweak form factors of light baryons [10]. These calculations (and also those in Refs. [15,16]) were already done within the framework of relativistic point-form quantum mechanics.

We will now go one step further and take the dynamics of the Goldstone-boson exchange fully into account by employing the relativistic coupled channel framework introduced at the beginning of this section. If we want to study baryon resonances, we have in the first channel 3 quarks and in the second channel 3 quarks plus a Goldstone boson. A physical mass eigenstate is then a superposition of a 3-quark component \(|\Psi_3\rangle\) and a 3-quark plus Goldstone-boson component \(|\Psi_4\rangle\). These components are solutions of the mass eigenvalue equation (4) with the diagonal terms of the mass operator being given by \(M_3 = M_{\text{free},3q} + V_{\text{conf}}\) and \(M_4 = M_{\text{free},3q+GB} + V_{\text{conf}}\), respectively. \(V_{\text{conf}}\) is an instantaneous confinement potential that acts only between the quarks. The vertex operator \(K\) is defined via velocity-state matrix elements as in Eq. (7) with \(\mathcal{L}_{\text{int}}(x)\) being the Lagrangean density for the coupling of a pseudoscalar particle to a spin-1/2 particle. The physical picture that emerges from this model becomes most obvious if we study, instead of Eq. (4), the equivalent (non-linear) eigenvalue equation (5). In order to convert it into an algebraic equation we expand \(|\Psi_3\rangle\) in terms of (velocity) eigenstates \(|v;\alpha\rangle\) of \(M_3\), i.e. eigenstates of the pure confinement problem:

\[
|\Psi_3\rangle = \sum_\alpha \int \frac{d^3v}{(2\pi)^3} m_\alpha^2 \delta^3(v - \mathbf{V}) A_\alpha |v;\alpha\rangle.
\]  

Here, \(\mathbf{V}\) denotes the overall velocity of the coupled 3q-GB system and \(\alpha\) the whole set of discrete quantum numbers that are necessary to specify a particular eigenstate of the pure confinement problem uniquely. \(v\) is the velocity of such an eigenstate and \(m_\alpha\) its mass. For reasons which will become clear very soon we will call \(|v;\alpha\rangle\) a “bare” baryon state and \(|\Psi_3\rangle\) the (3-quark component of the) “physical” baryon state. By projecting Eq. (6) onto bare baryon states \(|v;\alpha\rangle\) and inserting the completeness relation for such states between the operators and \(|\Psi_3\rangle\) we end up with an infinite set of algebraic equations for the expansion coefficients \(A_\alpha\):

\[
2v^0 \delta^3(v - \mathbf{V})(m_\alpha - m) A_\alpha = \sum_\alpha' \int \frac{d^3v}{(2\pi)^3} m_{\alpha'}^2 \langle v;\alpha|\tilde{V}_{\text{opt}}(m)|v';\alpha'\rangle \delta^3(v' - \mathbf{V}) A_{\alpha'}.
\]  

This equation is now a mass-eigenvalue equation for hadrons rather than for quarks. It describes how a physical hadron of mass \(m\) is composed of bare hadrons with masses \(m_\alpha\). The bare hadrons are mixed via the optical potential \(\tilde{V}_{\text{opt}}(m)\) which we will now analyze in some more detail. By inserting completeness relations for eigenstates of \(M_3\), \(M_4\), \(M_{\text{free},3q}\), and \(M_{\text{free},3q+GB}\) at appropriate places one infers that the optical potential has the following structure:

\[
\langle v;\alpha|\tilde{V}_{\text{opt}}(m)|v';\alpha'\rangle = g^2 \sum_{\alpha''} v^0 \delta^3(v - v') \int \frac{d^3\kappa}{2\sqrt{m_{GB}^2 + \kappa^2}} \frac{1}{2\sqrt{m_{\alpha''}^2 + \kappa^2}} \frac{1}{\sqrt{m_{\alpha}^2 m_{\alpha'}} m - \sqrt{m_{\alpha''}^2 + \kappa^2} - \sqrt{m_{GB}^2 + \kappa^2} + i\epsilon} \times \frac{f_{\alpha\alpha''}(|\kappa|) f_{\alpha'\alpha''}(|\kappa|)}{f_{\alpha\alpha'}(|\kappa|)}.
\]  

(14)
This means that the optical potential couples bare baryon states |v′; α′⟩ and |v; α⟩ via a Goldstone boson loop such that any bare baryon state |v''; α'')⟩ (that is allowed by conservation laws) can be excited in an intermediate step (cf. Fig. 2). g is the quark-Goldstone boson coupling, $f_{\alpha\alpha'}(|\kappa|)$ are (strong) transition form factors that show up at the (bare) baryon Goldstone-boson vertices. With this expression for the matrix elements of the optical potential the velocity dependence in Eq. (13) is seen to cancel out completely. The eigenvalue problem that one ends up with describes thus bare baryons, i.e. eigenstates of the pure confinement problem, that are mixed and dressed via Goldstone-boson loops. The only places where the quark substructure enters, are the vertex form factors. Here we want to emphasize that due to the instantaneous nature of the confinement potential the dressing happens on the baryonic level and not on the quark level, i.e. emission and absorption of the Goldstone boson by the same quark must not be interpreted as mass renormalization of the quark. This was done, e.g., in Ref. [20] where this kind of coupled channel approach has been applied for the first time to calculate the mass spectrum of vector mesons within the chiral constituent quark model.

Note that the optical potential, Eq. (13), becomes complex if the mass eigenvalue $m$ is larger than the lowest threshold $m_{th} = m_0 + m_{GB}$, i.e. the mass of the lightest bare baryon plus the Goldstone-boson mass. As a consequence also the physical mass eigenvalues $m$ will become complex as soon as their real part is larger than $m_{th}$ and we will get unstable baryon excitations. The mass of such an excitation can then be identified with Re($m$), its width $\Gamma$ with 2 Im($m$). The mass eigenvalue problem, Eq. (13), can be solved by an iterative procedure. One first has to restrict the number of bare states, that are taken into account, to a certain number $\alpha_{max}$. The first step is to insert a start value for $m$ into $\hat{V}_{opt}(m)$ and solve the resulting linear eigenvalue equation. This leads to $\alpha_{max}$ (possibly complex) eigenvalues. From these one has to pick out the right one, reinsert it into $\hat{V}_{opt}(m)$, solve again, etc. Appropriate start values are, e.g., the eigenvalues of the pure confinement problem. In order to pick out the correct eigenvalue in each step it is helpful to perform this procedure by gradually increasing the strength of the optical potential (until it has reached its full strength) and observe the path of the eigenvalues in the complex plane.

A first numerical test of the ideas just presented has been performed for a toy model in Ref. [21]. The toy model can be understood as a simplified constituent-quark model for mesons with its degrees of freedom being a quark, an antiquark and a real scalar meson which couples to the (anti)quark. Both, quark and antiquark are treated as spin- and flavorless particles that are confined by an instantaneous harmonic-oscillator potential. To simplify things further, only bare hadron states that correspond to radial
(and not orbital) excitations of the pure confinement problem are taken into account when solving Eq. (13). Reasonable convergence for the ground state and the first excited state is already achieved with $\alpha_{\text{max}} = 4$ after 5 iterations. With a meson-quark coupling compatible with the Goldberger-Treiman relation and a reasonable parameterization of the confinement potential the 2 lowest lying states are found to have masses of about 0.8 and 1.45 GeV, respectively. The first excited state has a width of 0.026 GeV. These are promising results in view of the simplicity of the model and it will be interesting to see whether typical decay widths of 0.1 GeV or more can be achieved in the case of baryon resonances with the full chiral constituent-quark model.

The picture that emerges from the chiral constituent-quark model with instantaneous confinement and dynamical Goldstone-boson exchange corresponds to the kind of hadronic resonance model that has been developed and refined by Sato and Lee [22] and that is now heavily applied at the Excited Baryon Analysis Center (EBAC) at JLab to extract $N^*$ properties from the world data on $\pi N$, $\gamma N$, and $N(e,e')$ reactions. Admittedly, our attempts are still far away from the degree of sophistication of the Sato-Lee model, but unlike this pure hadronic model we could also predict the meson-baryon couplings and vertex form factors from the underlying constituent-quark model. Our approach is in a certain sense inverse to the Sato-Lee approach. Whereas they want to “undress” physical resonances to end up with “bare” quantities that can, e.g., be compared with results from naive constituent quark models, we rather want to “dress” constituent-quark models to end up with physical quantities that can be directly compared with experiment.

3 Point-Form Quantum Field Theory

We now turn to infinite degree-of-freedom systems satisfying the point-form equations. Irreducible representations of the Poincaré group generate one-particle Hilbert spaces for massive and massless particles. Infinite tensor products of these one-particle Hilbert spaces then generate many-body Fock spaces. Alternatively creation and annihilation operators with the correct Poincaré transformation properties form an algebra of operators, in which the Fock space is generated by the action of products of creation operators acting on the Fock vacuum. Further both the free and interacting four-momentum operators are polynomials in the creation and annihilation operators.

3.1 Free quantum fields

Consider first massive particles of spin $j$. The single-particle Hilbert space is then $L^2(SO(1,3)/SO(3)) \times V^j$, the space of square-integrable functions over the forward hyperboloid times a 2$j$+1 dimensional spin space. On this space the natural variable is the four-velocity, $v := p/m$, the four-momentum divided by mass. Under a Poincaré transformation, a one-particle state of four-velocity $v$ and spin projection $\sigma$ transforms as

$$\hat{U}_{a} |v, \sigma\rangle = e^{i p \cdot a} |v, \sigma\rangle,$$

$$\hat{U}_{\Lambda} |v, \sigma\rangle = \sum |\Lambda v, \sigma'\rangle D^{j}_{\sigma'\sigma}(R_{W}(v, \Lambda)),$$
where $a$ is a four-translation and $D^j_{\sigma\alpha}(\ldots)$ an $SU(2)$ matrix element for spin $j$. $R_W(v, A)$ is a Wigner rotation, an element of the rotation group $SO(3)$ defined by

$$R_W(v, A) = B_v^{-1}(Av)AB_v(v).$$  \hspace{1cm} (17)

$B_v(v)$ is a boost, a Lorentz transformation satisfying $v = B_v(v)v^{\text{rest}}$, with $v^{\text{rest}} = (1, 0, 0, 0)$. Here boosts are canonical spin boosts.

Classical irreducible fields over Minkowski space-time, with a differentiation inner product, are then naturally related to the one-particle wave functions over the four-velocity. As an example, consider a scalar particle with wave function $\phi(v) \in L^2(R^3)$:

$$||\phi||^2 = \int \text{d}v |\phi(v)|^2; \quad \text{d}v = \frac{\text{d}^3v}{(2\pi)^3/2v_0}$$  \hspace{1cm} (18)

$$\psi(x) = \int \text{d}v e^{-imv\cdot x}\phi(v)$$  \hspace{1cm} (19)

$$||\psi||^2 = \int \text{d}x^\mu \left(\frac{\partial\phi^*}{\partial x^\mu}\psi - \psi^* \frac{\partial\phi}{\partial x^\mu}\right) = ||\phi||^2.$$  \hspace{1cm} (20)

Note that the integration on space-time $\text{d}x^\mu := \text{d}^4x \delta(x \cdot x - \tau^2)\theta(x_0)x^\mu$ is over the forward hyperboloid, typical of the point form.

To generate a many-particle theory introduce creation and annihilation operators with the usual commutation relations for fermions ($\hat{a}(v, \sigma, \alpha)$), antifermions ($\hat{b}(v, \sigma, \alpha)$) and bosons ($\hat{c}(v, \lambda, i)$). Products of bilinears $\hat{a}^\dagger\hat{a}, \hat{a}^\dagger\hat{b}, \hat{b}\hat{b}^\dagger$ along with products of boson creation and annihilation operators form the algebra of operators $\mathcal{A}$, which, acting on the Fock vacuum, generates the usual Fock space.

The free four-momentum operator is made from this algebra of operators:

$$\hat{P}_\text{free}^\mu := m \sum \int \text{d}v v^\mu \left(\hat{a}^\dagger(v, \sigma, \alpha)\hat{a}(v, \sigma, \alpha) - \hat{b}(v, \sigma, \alpha)\hat{b}^\dagger(v, \sigma, \alpha)\right) + \kappa \left(\hat{c}^\dagger(v, \lambda, i)\hat{c}(v, \lambda, i)\right),$$  \hspace{1cm} (21)

where $\kappa$ is a dimensionless relative bare boson mass parameter and $m$ is a constant with the dimensions of mass. $\alpha$ and $i$ are internal symmetry variables. It is easily checked that the free four-momentum operator satisfies the point-form equations.

To generate quantum fields for massive particles, it is straightforward to use the mapping between irreps of the Poincaré group and classical irreducible fields to define (for example spin 1/2 and 0)

$$\hat{\Psi}_\alpha(x) = \sum \int \text{d}v (\hat{a}(v, \sigma, \alpha)u_\sigma(v)e^{-imv\cdot x} + \hat{b}^\dagger(v, \sigma, \alpha)v_\sigma(v)e^{imv\cdot x}),$$  \hspace{1cm} (22)

$$\hat{\phi}_i(x) = \int \text{d}v \left(\hat{c}(v, i)e^{-imv\cdot x} + \hat{c}^\dagger(v, i)e^{imv\cdot x}\right).$$  \hspace{1cm} (23)

It is then possible to define the usual free Lagrangean and get $\hat{P}_\text{free}^\mu$ from the stress energy tensor,

$$\hat{P}_\text{free}^\mu = \int \text{d}^4x \delta(x \cdot x - \tau^2)\theta(x_0)x^\mu \hat{T}^{\text{free}}_{\mu\nu} = \int \text{d}x_\nu T^{\text{free}}_{\nu\mu},$$  \hspace{1cm} (24)
As an example, consider the free charged scalar field,
\[ \hat{L} = \partial^\mu \hat{\phi}^\dagger (x) \partial_\mu \hat{\phi}(x) - m^2 \hat{\phi}^\dagger (x) \hat{\phi}(x); \]  
\[ \hat{T}^{\mu\nu} = \partial^\mu \hat{\phi}^\dagger (x) \partial^\nu \hat{\phi}(x) + \partial^\nu \hat{\phi}^\dagger (x) \partial^\mu \hat{\phi}(x) - \eta^{\mu\nu} \hat{L}. \]  

As shown in Ref. [2] the free four-momentum operator obtained from such a Lagrangean is the same as the one obtained from irreps of the Poincaré group.

The same sort of analysis can be carried out for massless particles. However, now the one-particle Hilbert space is given by square integrable functions over the forward light cone times a spin space, where the light cone is given by \( SO(1,3)/E(2) \), the homogeneous space defined by the ratio of the Lorentz group divided by the two dimensional Euclidean group. In the following we will consider only gluon representations, as that is what is relevant for point-form QCD. In that case the spin space can be chosen to be the four dimensional nonunitary irrep of \( E(2) \) to get four polarization degrees of freedom (labeled \( \rho \)). The standard four-vector \( k^a = (1,0,0,1) \) leaves \( E(2) \) invariant; a helicity boost, \( B(k) := R(\hat{k}) A_z(|k|) \), to a four-momentum \( k \) then generates a gluon state with transformation properties
\[ \hat{U}_e|k^a, \rho, a\rangle = \sum |k^{a'}, \rho', a', e\rangle \Lambda_{\rho' \rho} (e_2), \]  
\[ |k, \rho, a\rangle := \hat{U}_{B(k)} |k^a, \rho, a\rangle, \]  
\[ \hat{U}_{A}|k, \rho, a\rangle = \hat{U}_{A} \hat{U}_{B(k)} |k^a, \rho, a\rangle \]  
\[ = \sum |Ak, \rho', a, A_{\rho' \rho} (e_W)\rangle, \]  
\[ \hat{U}_{c}|k, \rho, a\rangle = \sum |k, \rho, a', D_{a' a}(c)\rangle, \]  
where \( A(e_W) = B^{-1}(Ak)AB(k) \) is a Euclidean Wigner "rotation", \( c \) is an element of the internal symmetry (color) group and \( a, a' \) are color indices.

A Lorentz invariant gluon inner product is given by
\[ ||\phi||^2 = -\int dk \eta_{\rho\rho} |\phi(k, \rho, a)|^2, \]  
and shows the origin of gauge degrees of freedom, for this inner product is not positive definite. By imposing a Lorentz invariant auxiliary condition \( \phi(k, 0, a) = \phi(k, 3, a) \), the inner product becomes positive definite, and then only two polarizations are physical, as required for massless particles.

A classical gluon field is defined by
\[ G^\mu_a(x) = \int dk B^{\mu a}(k) \phi(k, \rho, a) e^{-ik\cdot x}, \]  
with a norm given by a differentiation inner product over the forward hyperboloid. It should be noted that the polarization matrix is a Lorentz boost. Further it can be shown that \( ||G||^2 = ||\phi||^2 \).

As with massive particles, many-gluon states are generated by gluon creation and annihilation operators such that
\[ |k, \rho, a\rangle = \hat{g}^\dagger (k, \rho, a)|0\rangle, \]  
\[ \hat{g}(k, \rho, a)|0\rangle = 0, \quad \forall k, \rho, a. \]
\[
[\hat{g}(k, \rho, a), \hat{g}^\dagger(k', \rho', a')] = -\eta_{\rho \rho'} k_0 \delta^3(\mathbf{k} - \mathbf{k}') \delta_{aa'}, \tag{34}
\]

\[
\hat{U}_A \hat{g}(k, \rho, a) \hat{U}_A^{-1} = \sum \hat{g}(A, k, \rho', a) A_{\rho \rho'}(\epsilon_W), 
\tag{35}
\]

\[
\hat{U}_c \hat{g}(k, \rho, a) \hat{U}_c^{-1} = \hat{g}(k, \rho, a') D_{a'a}(c), \tag{36}
\]

\[
\hat{P}^\mu_{\text{free}} = -\sum \int \frac{d^3 k}{k_0} k^\mu \hat{g}^\dagger(k, \rho, a) \eta^{\rho \rho} \hat{g}(k, \rho, a). \tag{37}
\]

The auxiliary condition eliminating the 0 and 3 components on the physical many-body Fock space is the annihilation operator condition,

\[
\sum k^\mu \eta^{\rho \rho} \hat{g}(k, \rho, a) |\phi\rangle = 0, \tag{38}
\]

which is Lorentz invariant.

Gauge transformations, which change the 0 and 3 components only, are given by

\[
\begin{align*}
\hat{g}(k, \rho, a) &\to \hat{g}'(k, \rho, a) = \hat{g}(k, \rho, a) + k^\mu f(k, a) I, \\
\hat{g}^\dagger(k, \rho, a) &\to \hat{g}'^\dagger(k, \rho, a) = \hat{g}^\dagger(k, \rho, a),
\end{align*}
\tag{39}
\]

and leave the commutation relations and auxiliary condition unchanged.

The free gluon field is again defined through the gluon creation and annihilation operators:

\[
\hat{G}^\mu_a(x) = \int dk B^{\mu \rho}(k) \left( e^{-ik \cdot x} \hat{g}(k, \rho, a) + e^{ik \cdot x} \hat{g}^\dagger(k, \rho, a) \right),
\tag{41}
\]

\[
\frac{\partial}{\partial x^\mu} \hat{G}^\mu_a = 0. \tag{42}
\]

Under a gauge transformation this gives

\[
\begin{align*}
\hat{G}^\mu_a(x) &\to \hat{G}'^\mu_a(x) = \hat{G}^\mu_a(x) + \frac{\partial f(x, a)}{\partial x^\mu} I, \\
\hat{f}(x, a) &\to \hat{f}'(x, a) = \int \frac{d^3 k}{2k_0} e^{ik \cdot x} f(k, a),
\end{align*}
\tag{43}
\]

which preserves the Lorentz gauge on the physical Fock space:

\[
\partial \hat{G}_{\mu a}(x) / \partial x^\mu |\phi\rangle = i \int dk \frac{k^\mu B^{\mu \rho}(k) \eta_{\rho \rho} e^{-ik \cdot x} \hat{g}(k, \rho, a) |\phi\rangle }{k_0} = i \sum \int dk \frac{e^{-ik \cdot x} k^\mu \eta_{\rho \rho} \hat{g}(k, \rho, a) |\phi\rangle }{k_0} = 0. \tag{44}
\]

The Lagrangean for free gluon fields generates a stress energy tensor and a free gluon four-momentum operator, which agrees with the four-momentum operator given by gluon creation and annihilation operators. Rather than writing this out, we next turn to interacting quantum fields and give as an example the self-coupling interaction of gluon fields.
3.2 Interacting quantum fields

Interactions in PFQFT are generated by integrating vertices over the forward hyperboloid. A vertex  $\hat{L}_\text{int}(x)$ is a local space-time scalar density operator,

$$e^{-iP_\text{free}^\mu a} \hat{L}_\text{int}(x) e^{iP_\text{free}^\mu a} = \hat{L}_\text{int}(x + a),$$  \hspace{1cm} (46)

$$\hat{U}_A \hat{L}_\text{int}(x) \hat{U}_A^{-1} = \hat{L}_\text{int}(Ax),$$  \hspace{1cm} (47)

$$[\hat{L}_\text{int}(x), \hat{L}_\text{int}(y)] = 0, \quad (x - y)^2 \text{ spacelike.}$$  \hspace{1cm} (48)

Then the interacting four-momentum operator is defined by

$$\hat{P}_\mu^\text{int} = \int dx^\mu \hat{L}_\text{int}(x)$$

and satisfies the point-form equations. Further, the total four-momentum operator, $\hat{P}_\mu := \hat{P}_\mu^\text{free} + \hat{P}_\mu^\text{int}$ also satisfies the point-form equations.

As an example consider the gluon self-coupling interaction:

$$\hat{F}_{\mu\nu}^\alpha(x) = \frac{\partial \hat{G}_\nu^\alpha(x)}{\partial x_\mu} - \frac{\partial \hat{G}_\mu^\alpha(x)}{\partial x_\nu} - \alpha c_{abc} \hat{G}_\mu^b(x) \hat{G}_\nu^c(x),$$  \hspace{1cm} (50)

$$\hat{T}^{\mu\nu}(x) = \hat{F}_{\alpha\beta}^\mu(x) \eta_{\alpha\beta'} \eta_{\mu\nu} + \eta_{\mu\beta'} \eta_{\nu\alpha} - \frac{1}{2} \eta_{\mu\nu} \eta_{\alpha\beta'} \hat{F}_{\alpha\beta'} \eta_{\mu\nu},$$  \hspace{1cm} (51)

$$\hat{P}_\mu^\text{gluon} = \int \text{hyper} dx_\mu \hat{T}^{\mu\nu};$$  \hspace{1cm} (52)

here $\hat{F}_{\mu\nu}^\alpha(x), \hat{T}^{\mu\nu}(x)$ are the field tensor and stress energy tensor respectively, and the four-momentum operator is integrated over the forward hyperboloid. Also, $c_{abc}$ are the color structure constants and $\alpha$ is the strong bare coupling constant.

Given the total four-momentum operator as a sum of free and interacting operators, the goal is to find the vacuum, one-particle and scattering states. The vacuum problem is to find a state $|\Omega\rangle$ such that it carries a one-dimensional representation of the Poincaré group and is invariant under internal symmetries:

$$\hat{P}_\mu |\Omega\rangle = 0$$  \hspace{1cm} (53)

$$\hat{U}_A |\Omega\rangle = |\Omega\rangle$$  \hspace{1cm} (54)

$$\hat{U}_c |\Omega\rangle = |\Omega\rangle,$$  \hspace{1cm} (55)

where $c$ is an element of the internal symmetry group.

Several points can be made about the vacuum: First, it is not possible to add a multiple of the identity operator to $\hat{P}_\mu$ without violating point-form equations. This means the vacuum state cannot be shifted to give zero energy and momentum. Second, it suffices to calculate $\hat{P}_\mu^\text{int}|\Omega\rangle = 0$, for invariance under Lorentz transformations implies $\hat{P}_\mu|\Omega\rangle = 0$. Finally, writing $|\Omega\rangle = e^S |0\rangle$, generates a set of vacuum equations. For simple one-dimensional models these vacuum equations can be used to transform away the gluon self-coupling terms; it is not known whether the same procedure also works for the full space-time vacuum equations [23].
One of the main advantages of the point form is its explicit Lorentz structure. Candidates for the vacuum state are greatly restricted by the condition that they be Lorentz scalars. Similarly one-particle states must transform properly under Lorentz transformations. There are simple models where the four-momentum eigenstates can be solved exactly because of this transparent Lorentz structure.

To conclude this section we describe the point-form interaction picture and its accompanying covariant perturbation theory. Starting with the relativistic Schrödinger equation, write:

\[ i\partial_{\mu}\Psi(x) = (\hat{P}_{\text{free}}^\mu + \hat{P}_{\text{int}}^\mu)\Psi(x) ; \quad (56) \]

\[ \hat{P}_{\text{int}}^\mu(x) := e^{i\hat{P}_{\text{free}}\cdot x} \hat{P}_{\text{int}}^\mu e^{-i\hat{P}_{\text{free}}\cdot x} ; \quad (57) \]

\[ \Psi(x) = \hat{U}(x, x_0)\Psi(x_0), \quad (58) \]

\[ i\frac{\partial \hat{U}(x, x_0)}{\partial x_\mu} = \hat{P}_{\text{int}}^\mu(x)\hat{U}(x, x_0), \quad (59) \]

\[ \hat{U}(x, x_0) = \hat{I} - i \int_{C(x, x_0)} dy_\mu \hat{P}_{\text{int}}^\mu(y)\hat{U}(y, x_0), \quad (60) \]

\[ \hat{U}(x, x_0) = \hat{P} e^{-i \int_{C} dy_\mu \hat{P}_{\text{int}}^\mu(y)}, \quad (61) \]

which is the starting point for covariant perturbation theory \((C\) is a contour in space-time). Note that the four-momentum is not conserved for intermediate states; nor is the four-velocity conserved for intermediate states. This is to be contrasted with point-form relativistic quantum mechanics, where the four-velocity is conserved.

4 Concluding Remarks

Though point-form relativistic quantum theory is the least explored of the three common forms of relativistic dynamics, it has several properties that make it well suited for applications to hadronic physics. The most important properties are the kinematic nature of Lorentz transformations and the fact that those operators that have interactions (the energy and momentum operators) commute with one another. The kinematic nature of Lorentz transformations is particularly important for calculating hadronic form factors. Using a coupled channel approach we have constructed hadronic current matrix elements that satisfy Lorentz covariance and current conservation. However, in using a Bakamjian-Thomas approach, problems involving cluster separability arise; surprisingly, by taking appropriate limits the point form inspired form factors turn out to be closely related to front form inspired form factors. Further the coupled channel approach has been used to provide more realistic models of hadronic decay processes.

One of the goals of point-form quantum field theory is to analyze point-form QCD. To that end we have constructed massless gluon fields from irreps of the Poincaré group and begun investigating model solutions for the gluon self-interactions. In both finite and infinite degree of freedom systems the goal is to exploit some of the unique features of the point form.

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References

1. Dirac, P. A. M.: Forms of relativistic dynamics, Rev. Mod. Phys. **21**, 392 (1949)
2. Biernat, E. P., Klink, W. H., Schweiger, W., and Zelzer, S.: Point-form quantum field theory, Annals Phys. **323**, 1361 (2008)
3. Bakamjian, B. and Thomas, L. H.: Relativistic particle dynamics. 2, Phys. Rev. **92**, 1300 (1953)
4. Keister, B. D. and Polyzou, W. N.: Relativistic Hamiltonian dynamics in nuclear and particle physics, Adv. Nucl. Phys. **20**, 225 (1991)
5. Foldy L. L.: Relativistic particle systems with interactions, Phys. Rev. **122**, 275 (1961)
6. Klink, W. H.: Relativistic simultaneously coupled multiparticle states, Phys. Rev. C **58**, 3617 (1998)
7. Klink, W. H.: Constructing point form mass operators from vertex interactions, Nucl. Phys. A **716**, 123 (2003)
8. Girlanda, L., Viviani M., and Klink W. H.: Bakamjian-Thomas mass operator for the few-nucleon system from chiral dynamics, Phys. Rev. C **76**, 044002 (2007)
9. Biernat, E. P., Schweiger W., Fuchsberger, K., and Klink, W. H., Electromagnetic meson form factor from a relativistic coupled-channel approach, Phys. Rev. C **79**, 055203 (2009)
10. Siegert A. J. F.: Note on the interaction between nuclei and electromagnetic radiation, Phys. Rev. **52**, 787 (1937)
11. Lev, F. M.: Exact Construction of the electromagnetic current operator for relativistic composite systems, Annals Phys. **237**, 355 (1995)
12. Klink, W. H.: Point form relativistic quantum mechanics and electromagnetic form factors, Phys. Rev. C **58**, 3587 (1998).
13. Gómez Rocha, M. and Schweiger, W.: Paper in preparation
14. Carbonell, J., Desplanques, B., Karmanov, V. A., and Mathiot, J. F.: Explicitly covariant light-front dynamics and relativistic few-body systems, Phys. Rept. **300**, 215 (1998)
15. Melde, T., Plassas, W., and Sengl, B.: Covariant calculation of nonstrange decays of strange baryon resonances, Phys. Rev. C **76**, 025204 (2007)
16. Sengl, B., Melde, T., and Plassas, W.: Covariant calculation of strange decays of baryon resonances, Phys. Rev. D **76**, 054008 (2007)
17. Metsch, B.: Quark models, Eur. Phys. J. A **35**, 275 (2008)
18. Glozman, L. Y., Plassas, W., Varga, K., and Wagenbrunn, R. F., Unified description of light- and strange-baryon spectra, Phys. Rev. D **58**, 094030 (1998)
19. Boffi, S., Glozman, L. Y., Klink W. H., Plassas, W., Radici, M., and Wagenbrunn, R. F.: Covariant electroweak nucleon form factors in a chiral constituent quark model, Eur. Phys. J. A **14**, 17 (2002)
20. Krassnigg, A., Schweiger, W., and Klink, W. H.: Vector mesons in a relativistic point-form approach, Phys. Rev. C **67**, 064003 (2003)
21. Kleinhappel, R.: Resonances and decay widths within relativistic point-form quantum mechanics, Master’s thesis, Karl-Franzens-Universität Graz (2010)
22. Sato, T. and Lee, T. S.: Dynamical Models of the excitations of nucleon resonances, J. Phys. G **36**, 073001 (2009)
23. Murphy, K. C.: The structure of gluons in point form quantum chromodynamics, PhD thesis, University of Iowa (2009)