ISOSPIN OF NEW PHYSICS IN $|\Delta S| = 1$ CHARMLESS $B$ DECAYS

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ABSTRACT

New physics (NP) in charmless strangeness-changing $B$ and $B_s$ decays, which are dominated by the $b \to s$ penguin amplitudes, can either preserve isospin or change it by one unit. A general formalism is presented studying pairs of processes related to each other by isospin reflection. We discuss information on $\Delta I$ in NP amplitudes, provided by time-integrated CP-violating rate asymmetries in $B^+$ and $B^0$ decays (or in $B_s$ decays), differences between rates for isospin-reflected processes, and coefficients $S$ of $\sin \Delta m t$ in time-dependent CP asymmetries. These four asymmetries in $B^+$ and $B^0$ decays (or five asymmetries in $B_s$ decays) are shown to determine the magnitude and CP-violating phase of a potential isovector NP amplitude, and the imaginary part of an isoscalar amplitude, assuming that strong phases in NP amplitudes are negligible. This information may be compared with predictions of specific models, for which we discuss a few examples.

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I. INTRODUCTION

The $b \to s$ penguin amplitude is a promising place to look for effects of new physics in charmless strangeness-changing ($|\Delta S| = 1$) $B$ meson decays [1–5]. Observables which can shed light on its nature include decay rates, direct CP-violating rate asymmetries, time-dependent CP asymmetries, differences between processes related by isospin, and polarization measurements in decays to two vector mesons. Using these, one can obtain information on new physics (NP) entering into the $b \to s$ amplitude.

The most general $b \to sq\bar{q}$ operator, where $q$ is a light quark $u$, $d$, $s$ or a charmed quark $c$, is a combination of operators with isospin $\Delta I = 0$ and $\Delta I = 1$, $\Delta I_3 = 0$. In the Standard Model (SM), the penguin operator involving a virtual $b \to s$ transition accompanied by a flavor-symmetric $q\bar{q}$ pair has $\Delta I = 0$ to the extent that it is dominated by intermediate charm or top quarks, while intermediate on-shell $u$ quarks induce a small $\Delta I = 1$ component. Transitions treating $b \to su\bar{u}$ and $b \to sd\bar{d}$
differently, including tree and electroweak penguin amplitudes, contain both $\Delta I = 0$ and $\Delta I = 1$ components. Four-quark operators associated with NP in $b \to sq\bar{q}$ transitions can change isospin by either 0 or 1 unit.

The present paper is devoted to the question of how one may use all possible observables in charmless strangeness-changing $B$ meson decays to diagnose the value of $\Delta I$ in NP operators, regardless of the chiral or color structure of these operators. This question has been addressed in the past by studying a small sample of a few processes including $B \to \phi K$ and $B \to \pi K$. Fleischer and Mannel [6] have shown how to separate from one another $\Delta I = 0$ and $\Delta I = 1$ NP effects in $B \to \phi K$ by comparing CP asymmetries in charged and neutral $B$ decays. Sum rules for decay rates and CP asymmetries in $B \to K\pi$ have been proposed [7–13], whose violation tests NP in $\Delta I = 1$ transitions.

One of the quantities to be studied together with decay rates and direct CP asymmetries is the coefficient $S$ of the sin $\Delta mt$ term in time-dependent CP asymmetries in $B^0$ decays to CP-eigenstates. A canonical value, $S = -\eta_{CP} \sin 2\beta$ for final states with CP-eigenvalue $\eta_{CP}$, is expected approximately for $b \to s$ penguin amplitudes dominated by top or charm quarks [2, 4, 14]. Small process-dependent deviations from this value are expected within the SM [15]. Measurements of this quantity, averaged over a large number of processes including (quasi) two-body and three-body decays, yield $\sin 2\beta_{\text{eff}} = 0.53 \pm 0.05$, to be compared with the value of $\sin 2\beta = 0.678 \pm 0.025$ obtained from $B^0 \to J/\psi K_{S,L}$ [16]. Thus, the question arises as to what kind of NP is probed by a deviation of $S$ from its nominal value. Is it sensitive to new-physics contributions with $\Delta I = 0$? Does it reflect only new $\Delta I = 0$ contributions?

In this paper we shall address the above questions. Our purpose is to develop a general formalism for studying the isospin structure of potential NP effects in a broad class of (quasi) two-body and multi-body $B$ and $B_s$ decays mediated by $b \to sq\bar{q}$ transitions. We shall see that this information can be obtained almost completely by combining measurements of $S$ in $B^0$ (or $B_s$) decays with isospin asymmetry measurements and direct CP asymmetry measurements in $B^0$ and $B^+$ (or in $B_s$) decays. It will be shown that the magnitude and CP-violating phase of the $\Delta I = 1$ NP term, and the imaginary part of the $\Delta I = 0$ NP term, can be determined, assuming negligible strong phases in NP amplitudes. Individual models may be tested against this information when it becomes available.

In Section II we prove a general relation between charmless $|\Delta S| = 1$ decay amplitudes for pairs of $B$ and $B_s$ decay processes, obtained from each other under isospin reflection. This relation permits introducing a general language suitable for decomposing processes mediated by $b \to sq\bar{q}$ into $\Delta I = 0$ and $\Delta I = 1$ contributions. In Section III we discuss evidence for $\Delta I = 0$ penguin dominance provided by asymmetries measured in these processes. Section IV presents an expansion of four asymmetries in $B^+$ and $B^0$ decays, and five asymmetries in $B_s$ decays, in terms of small ratios involving non-penguin $\Delta I = 0$ and $\Delta I = 1$ contributions, showing the amount of information which can be learned from these observables about NP $\Delta I = 0$ and $\Delta I$ amplitudes. Section V demonstrates the extraction of NP amplitudes and CP-violating phases in $B \to K\phi, B \to K\pi$ and $B_s \to K\bar{K}$ decays, assuming that strong phases in these amplitudes are negligible. Section VI deals with some aspects...
of specific models, including isospin properties of new $b \to sq\bar{q}$ effective operators, while Section VII concludes.

II. AMPLITUDES FOR ISOSPIN-REFLECTED DECAYS

We start by pointing out a general relation between amplitudes for pairs of $B$ or $B_s$ decay processes mediated by $\bar{b} \to \bar{q}q$, which are obtained from each other under isospin reflection,

$$R_I : u \leftrightarrow d , \bar{u} \leftrightarrow -\bar{d} .$$

(1)

The $\Delta I = 0$ and $\Delta I = 1$ operators, $\bar{b}s(\bar{d}d + \bar{u}u)$ and $\bar{b}s(\bar{d}d - \bar{u}u)$, obtain a relative minus sign under this transformation. This suggests that the $\Delta I = 0$ and $\Delta I = 1$ amplitudes behave the opposite way under isospin reflection. Denoting the $\Delta I = 0$ and $\Delta I = 1$ contributions in $B^+ \to f$ by $B$ and $A$, the corresponding $\Delta I = 0$ and $\Delta I = 1$ contributions in $B^0 \to R_I f$ are either $B$ and $-A$, or $-B$ and $A$, depending (as we will show next) on the isospin structure of the final state,

$$A(B^+ \to f) = B + A , \quad A(B^0 \to R_I f) = \pm (B - A) .$$

(2)

Note that while $B$ and $A$ correspond to $\Delta I = 0$ and $\Delta I = 1$ operators, respectively, these amplitudes do not necessarily involve final states with a well-defined isospin. The state $f$ in (2) may be any (quasi) two-body or multi-body state. In our discussion below we will focus our attention on final states in $B^0$ decays, for which a somewhat low value of $\sin 2\beta$ was measured [16], and on isospin-related charged $B$ decays.

Our analysis relies largely on Eqs. (2). Before proving this general structure, fixing the signs in the second equation, we will treat several cases involving specific final states.

The simplest example is the pair of processes $B^+ \to K^+\phi$ and $B^0 \to K^0\phi$, where the final states are pure $I = 1/2$, obtaining contributions from a single $\Delta I = 0$ amplitude, $B$, and a single $\Delta I = 1$ amplitude, $A$. The corresponding Clebsch-Gordan coefficients imply [6]

$$A(B^+ \to K^+\phi) = B + A , \quad A(B^0 \to K^0\phi) = B - A .$$

(3)

Here and below we absorb Clebsch-Gordan coefficients in isospin amplitudes. Similar decompositions into amplitudes $B \pm A$ apply to such decays as $B \to K\eta', K\omega$, and $Kf_0(980)$, where the final kaon is accompanied by an $I = S = 0$ meson.

The frequently-discussed decays $B \to K\pi$, where final states are admixtures of $I = 1/2$ and $I = 3/2$, obtain contributions from a single $\Delta I = 0$ amplitude and two $\Delta I = 1$ amplitudes. Denoting the final state isospin by a subscript on $B$ and $A$, one has [17, 18]

$$-\sqrt{2}A(B^+ \to K^+\pi^0) = B_{1/2} + A_{1/2} - 2A_{3/2} ,$$

$$\sqrt{2}A(B^0 \to K^0\pi^0) = B_{1/2} - A_{1/2} + 2A_{3/2} ,$$

$$A(B^+ \to K^0\pi^+) = B_{1/2} + A_{1/2} + A_{3/2} ,$$

$$-A(B^0 \to K^+\pi^-) = B_{1/2} - A_{1/2} - A_{3/2} .$$

(4)
The four processes may thus be grouped into two pairs, with each member of a pair related to the other by isospin reflection, as in Eq. (2):

\[-\sqrt{2}A(B^+ \to K^+\pi^0) = B + A, \quad -\sqrt{2}A(B^0 \to K^0\pi^0) = -B + A,\]  
\[A(B^+ \to K^0\pi^+) = B + A', \quad A(B^0 \to K^+\pi^-) = -B + A'.\]  

Here \(B \equiv B_{1/2}, A \equiv A_{1/2} - 2A_{3/2}, A' \equiv A_{1/2} + A_{3/2}.\) A similar structure applies to \(B \to K\rho\) and \(B \to K^*\pi\) decays. The processes \(B^0 \to K^0\pi^0\) and \(B^0 \to K^0\rho^0\) have been used to obtain the somewhat low averaged value for \(\sin 2\beta_{\text{eff}}[16]\). In all these cases, the coefficient of the \(\Delta I = 0\) amplitude changes sign under isospin reflection, while the coefficients of the \(\Delta I = 1\) amplitudes do not.

Final states in \(B^0\) decays involving three neutral kaons are interesting as they are dominated by \(b \to s\bar{s}s\). Hence in the SM deviations of \(S\) from \(-\sin 2\beta\) are expected to be very small in \(B^0 \to K_SK_SK_S\). The final state \(|f\rangle \equiv |KKK\rangle\) is a superposition of three isospin states, \(|I_{KK} = 0, I_{tot} = \frac{1}{2}\rangle, |I_{KK} = 1, I_{tot} = \frac{1}{2}\rangle\) and \(|I_{KK} = 1, I_{tot} = \frac{3}{2}\rangle\). Consequently, these decay processes are described by five independent isospin amplitudes which are functions of the three kaon momenta. Suppressing the momentum dependence of the physical amplitudes and of the five isospin amplitudes, \(A_{\Delta I, I_{tot}}^{KKK}\), one finds expressions for the four physical decay amplitudes [19],

\[
A(B^+ \to K^+K^+K^-) = 2A_{1\frac{1}{2}}^0 - 2A_{1\frac{1}{2}}^1 + A_{1\frac{3}{2}}^1, \\
A(B^0 \to K^0K^0K^0) = -2A_{1\frac{1}{2}}^0 - 2A_{1\frac{1}{2}}^1 + A_{1\frac{3}{2}}^1, \\
A(B^+ \to K^+K^0\overline{K}^0) = A_{0\frac{1}{2}}^0 - A_{1\frac{1}{2}}^1 - A_{1\frac{3}{2}}^1 + A_{1\frac{3}{2}}^1, \\
A(B^0 \to K^0K^+K^-) = -A_{0\frac{1}{2}}^0 + A_{0\frac{1}{2}}^1 - A_{1\frac{1}{2}}^1 + A_{1\frac{3}{2}}^1 + A_{1\frac{3}{2}}^1.
\]

Amplitudes on the left-hand side correspond to given momenta of the three particles in the final state. Denoting \(B \equiv 2A_{1\frac{1}{2}}^0, A \equiv -2A_{1\frac{1}{2}}^1 + A_{1\frac{3}{2}}^1, B' \equiv A_{0\frac{1}{2}}^0 - A_{1\frac{3}{2}}^1, A' \equiv -A_{1\frac{3}{2}}^1 + A_{1\frac{1}{2}}^1 + A_{1\frac{3}{2}}^1\), one has a situation similar to \(B \to K\pi\) for the two pairs consisting of isospin-reflected processes,

\[
A(B^+ \to K^+K^+K^-) = B + A, \quad A(B^0 \to K^0K^0\overline{K}^0) = -B + A, \\
A(B^+ \to K^+K^0\overline{K}^0) = B' + A', \quad A(B^0 \to K^0K^+K^-) = -B' + A'.
\]

Finally, we consider decays of the form \(B \to K\pi\pi\) of which \(B^0 \to K_S\pi^0\pi^0\) has been used recently to measure \(\sin 2\beta_{\text{eff}}[20]\). Here three pairs of processes are related to one another by isospin reflection [21]:

\[
B^+ \to K^+\pi^+\pi^- \iff B^0 \to K^0\pi^-\pi^+, \quad (12) \\
B^+ \to K^+\pi^0\pi^0 \iff B^0 \to K^0\pi^0\pi^0, \quad (13) \\
B^+ \to K^0\pi^+\pi^0 \iff B^0 \to K^+\pi^-\pi^0. \quad (14)
\]

The amplitudes of these processes may be expanded in terms of invariant isospin amplitudes \(A_{\Delta I, I_{tot}}^{KKK}\) as follows, where we have absorbed common factors into the amplitudes as in previous examples:

\[
A(B^+ \to K^+\pi^+\pi^-) = A_{0\frac{1}{2}}^{1/2} - A_{1\frac{1}{2}}^{1/2} - A_{1\frac{3}{2}}^{1/2} + A_{1\frac{1}{2}}^{1/2} + A_{1\frac{3}{2}}^{3/2} - A_{2\frac{3}{2}}^{1/2},
\]
Table I: Behavior under isospin reflection of coefficients of invariant isospin amplitudes describing decays $B \rightarrow KX$.

| $X$ | $\Delta I = 0$ | $\Delta I = 1$ |
|-----|---------------|---------------|
| $\phi$ | + | − |
| $\pi$ | − | + |
| $K\bar{K}$ | − | + |
| $\pi\pi$ | + | − |

\[
A(B^0 \rightarrow K^0 \pi^- \pi^+) = A_0^{0.1/2} - A_0^{1.1/2} + A_1^{0.1/2} - A_1^{1.1/2} - A_1^{1.3/2} + A_1^{2.3/2}, \quad (15)
\]
\[
A(B^+ \rightarrow K^+ \pi^0 \pi^0) = -A_0^{0.1/2} + A_1^{1.1/2} - 2A_1^{2.3/2},
A(B^0 \rightarrow K^0 \pi^0 \pi^0) = -A_0^{0.1/2} - A_1^{1.1/2} + 2A_1^{2.3/2}, \quad (16)
\]
\[
\sqrt{2}A(B^+ \rightarrow K^0 \pi^+ \pi^0) = 2A_0^{1.1/2} - 2A_1^{1.1/2} + A_1^{1.3/2} + 3A_1^{2.3/2},
\]
\[
\sqrt{2}A(B^0 \rightarrow K^+ \pi^- \pi^0) = 2A_0^{1.1/2} + 2A_1^{1.1/2} - A_1^{1.3/2} - 3A_1^{2.3/2}. \quad (17)
\]

Defining $B \equiv A_0^{0.1/2} - A_0^{1.1/2}, A \equiv -A_1^{0.1/2} + A_1^{1.1/2} + A_1^{1.3/2} - A_1^{2.3/2}, B' \equiv -A_0^{0.1/2}, A' \equiv A_1^{0.1/2} - 2A_1^{2.3/2}, B'' \equiv 2A_0^{1.1/2}, A'' \equiv -2A_1^{1.1/2} + A_1^{1.3/2} + 3A_1^{2.3/2},$ the three pairs of physical amplitudes can be expressed again as in $B \rightarrow K\phi$,

\[
A(B^+ \rightarrow K^+ \pi^+ \pi^-) = B + A, \quad A(B^0 \rightarrow K^0 \pi^- \pi^+) = B - A, \quad (18)
\]
\[
A(B^+ \rightarrow K^+ \pi^0 \pi^0) = B' + A', \quad A(B^0 \rightarrow K^0 \pi^0 \pi^0) = B' - A', \quad (19)
\]
\[
\sqrt{2}A(B^+ \rightarrow K^0 \pi^+ \pi^0) = B'' + A'', \quad \sqrt{2}A(B^0 \rightarrow K^+ \pi^- \pi^0) = B'' - A''. \quad (20)
\]

As in $B \rightarrow K\phi$, under isospin reflection, the coefficients of all $\Delta I = 1$ amplitudes change sign, while those of the $\Delta I = 0$ amplitudes do not.

We summarize in Table I the behavior of the coefficients of the $\Delta I = 0$ and $\Delta I = 1$ amplitudes for processes $B \rightarrow KX$ under isospin reflection. We shall now present a general proof for Eq. (2), showing that the $\Delta I = 0$ and $\Delta I = 1$ coefficients always behave in opposite fashion under this reflection.

The proof makes use of the property

\[
(j_1j_2 - m_1 - m_2|j_1j_2j - m) = (-1)^{j-j_1-j_2}(j_1j_2m_1m_2|j_1j_2jm) \quad (21)
\]

of Clebsch-Gordan coefficients. The total isospin $I_{tot} = 1/2$ or $3/2$ is formed by the coupling of $\Delta I = 0, 1$ to $I_B = 1/2$, giving rise to a phase $(-1)^{I_{tot} - \Delta I - 1/2}$ under isospin reflection. In turn, $I_{tot}$ couples to the product of $I_X$ and $I_K = 1/2$, giving rise to a phase $(-1)^{I_{tot} - I_X - 1/2}$. Finally, there is a phase $\eta_X(-1)^{I_X}$, where $\eta_X$ depends on the specific particles in $X$. For example, $\eta_X = +1$ for $X = \phi, \pi\pi$ and $\eta_X = -1$ for $X = \pi, K\bar{K}$. We use the fact that $2I_{tot} - 1$ is always even, so that all phases except those due to $\Delta I$ and $\eta_X$ cancel, and the coefficients of amplitudes describing transitions with $\Delta I = 0, 1$ acquire phases $\eta_X(-1)^{\Delta I}$ under isospin reflection.

A similar formulation in terms of $\Delta I = 0$ and $\Delta I = 1$ amplitudes $B$ and $A$ applies also to $B_s$ decays, where the initial state has $I = 0$ and final states in $b \rightarrow sq\bar{q}$
transitions are admixtures of $I = 0$ and $I = 1$. Thus, $B_s$ decay amplitudes for pairs of isospin-reflected final states can always be expressed in a form similar to Eq. \[2\]. For instance, one has

$$A(B_s \to K^+ K^-) = B + A , \quad A(B_s \to K^0 \bar{K}^0) = -B + A , \quad \text{(22)}$$

and

$$A(B_s \to K^+ \bar{K}^0 \pi^-) = \bar{B} + \bar{A} , \quad A(B_s \to K^0 K^- \pi^+) = \bar{B} - \bar{A} . \quad \text{(23)}$$

### III. TESTS FOR PENGUIN DOMINANCE

In the SM, strangeness-changing decays of $B$ and $B_s$ mesons to final states consisting of $u, d$ and $s$ quarks are expected to be dominated by $\Delta I = 0$ penguin amplitudes involving a CKM-favored factor $V_{cb}^* V_{cs}$. NP could, in principle, change this behavior by introducing comparable $\Delta I = 0$ or $\Delta I = 1$ contributions involving new CP-violating phases. In this section we discuss circumstantial evidence showing that this is not the case. Namely, potential $\Delta I = 0$ and $\Delta I = 1$ contributions beyond the Standard Model are most likely much smaller than the Standard Model penguin amplitudes.

Consider a pair of $B^+$ and $B^0$ decay processes as discussed in Section II, $B^+ \to K^X$ and $B^0 \to K_{R_1} X_{R_1}$, related to each other under isospin reflection. Here $K$ may be a $K^+$ or a $K^0$, and $X$ is an arbitrary charmless and nonstrange hadronic state with a corresponding total charge. As we have shown, the amplitudes for this pair of processes can be expressed in a model-independent way as

$$A(B^+ \to KX) = B + A , \quad A(B^0 \to K_{R_1} X_{R_1}) = \pm (B - A) , \quad \text{(24)}$$

where $B$ and $A$ are $\Delta I = 0$ and $\Delta I = 1$ amplitudes depending on $X$. The corresponding decay rates are

$$\Gamma_+ \equiv \Gamma(B^+ \to KX) = |B + A|^2 , \quad \Gamma_0 \equiv \Gamma(B^0 \to K_{R_1} X_{R_1}) = |B - A|^2 , \quad \text{(25)}$$

where inessential kinematic factors have been omitted including phase space integration in the case of three-body or other multi-body decays. The rates for $B^-$ and $\bar{B}^0$ decays into charge-conjugate final states are

$$\Gamma_- \equiv \Gamma(B^- \to \bar{K}X) = |\bar{B} + \bar{A}|^2 , \quad \Gamma_0 \equiv \Gamma(\bar{B}^0 \to \bar{K}_{R_1} \bar{X}_{R_1}) = |\bar{B} - \bar{A}|^2 . \quad \text{(26)}$$

The charge-conjugated amplitudes $\bar{B}$ and $\bar{A}$ are related to $B$ and $A$ by a change in sign of all weak phases, whereas strong phases are left unchanged.

In the Standard Model, the isoscalar amplitude $B$ contains a dominant penguin contribution, $B_P$, with a CKM factor $V_{cb}^* V_{cs}$. The residual isoscalar amplitude,

$$\Delta B \equiv B - B_P , \quad \text{(27)}$$

and the amplitude $A$, consist each of smaller contributions. These include terms with a much smaller CKM factor $V_{ub}^* V_{us}$, and a higher order electroweak penguin amplitude
Table II: Isospin-dependent asymmetries $A_I$ defined in (29), CP-asymmetries $A_{CP}^{+0}$ defined in (30), and mixing-induced CP asymmetries $S$ for $B \rightarrow KX$ [16].

| $X$     | $A_I$       | $A_{CP}^+$ | $A_{CP}^0$ | $-\eta_{CP}S$ |
|---------|-------------|------------|------------|----------------|
| $\phi$  | $-0.037 \pm 0.077$ | $0.034 \pm 0.044$ | $-0.01 \pm 0.13$ | $0.39 \pm 0.18$ |
| $\eta'$ | $-0.001 \pm 0.033$ | $0.031 \pm 0.026$ | $0.09 \pm 0.06$ | $0.61 \pm 0.07$ |
| $\omega$| $0.14 \pm 0.07$   | $0.05 \pm 0.06$ | $0.21 \pm 0.19$ | $0.48 \pm 0.24$ |
| $f_0(980)$ | $0.18 \pm 0.08$  | $-0.026^{+0.068}_{-0.064}$ | $0.02 \pm 0.13$ | $0.42 \pm 0.17$ |
| $\pi^0$ | $0.087 \pm 0.038$ | $0.047 \pm 0.026$ | $-0.12 \pm 0.11$ | $0.33 \pm 0.21$ |
| $\rho^+$ | $0.051 \pm 0.026$ | $0.009 \pm 0.025$ | $-0.097 \pm 0.012$ |$-$|
| $\rho^0$ | $-0.16 \pm 0.10$ | $0.31^{+0.11}_{-0.10}$ | $-0.64 \pm 0.46$ | $0.20 \pm 0.57$ |
| $\rho^+$ | $-0.14 \pm 0.12$ | $-0.12 \pm 0.17$ | $0.17^{+0.15}_{-0.16}$ |$-$|
| $K^+K^-$ | $0.25 \pm 0.07$ | $-0.02 \pm 0.04$ | $-0.15 \pm 0.09$ | $0.58^{+0.18}_{-0.13}$ |
| $K^0\bar{K}^0$ | $0.018 \pm 0.080$ | $-0.04 \pm 0.11$ | $0.14 \pm 0.15$ | $0.58 \pm 0.20$ |
| $\pi^+\pi^-$ | $0.064 \pm 0.039$ | $0.023 \pm 0.025$ |$-$ |$-$|
| $\pi^0\pi^0$ | $-0.23 \pm 0.54$ | $-0.72 \pm 0.71$ |$-$ |$-$|

$a$ Values of branching ratio and asymmetry for $B^+ \rightarrow K^0 \rho^+$ taken from Ref. [27].

$b$ Values obtained by subtracting $B \rightarrow K\phi$ contributions, assuming that the subtracted amplitudes are symmetric with respect to interchanging $K^+$ and $K^-$ momenta and $K^0$ and $\bar{K}^0$ momenta [21].

c Values include measurements reported in Ref. [28].

d Values from Ref. [20].

with CKM factor $V_{tb}^*V_{ts}$. In $B \rightarrow K\pi$ decays, for instance, the largest term of the first kind is a color-favored tree amplitude. This amplitude and a comparable electroweak penguin term are suppressed by about an order of magnitude relative to $B_P$ [22–26]. In $B \rightarrow K\phi$, the first amplitude is even smaller. Thus, in general one expects

$$|\Delta B| \ll |B_P|, \quad |A| \ll |B_P|.$$  \hspace{1cm} (28)

Tests for the hierarchy (28) are provided by an isospin-dependent asymmetry,

$$A_I \equiv \frac{\Gamma_+ + \Gamma_- - \Gamma_0 - \Gamma_0}{\Gamma_+ + \Gamma_- + \Gamma_0 + \Gamma_0}, \hspace{1cm} (29)$$

and by two CP-violating asymmetries, in charged and neutral $B$ decays,

$$A_{CP}^+ \equiv \frac{\Gamma_- - \Gamma_+}{\Gamma_- + \Gamma_+}, \quad A_{CP}^0 \equiv \frac{\Gamma_0 - \Gamma_0}{\Gamma_0 + \Gamma_0}. \hspace{1cm} (30)$$

The asymmetries $A_I$ and $A_{CP}^{+0}$ are expected to be small in the CKM framework, of order $2|A||B|$ and $2|\Delta B||B_P|$. In contrast, potentially large contributions to $\Delta B$ and $A$ from NP, comparable to $B_P$, would most likely lead to large asymmetries of order one. An unlikely exception is the case when both $\Delta B/B_P$ and $A/B_P$ are purely
imaginary, or almost purely imaginary. This would require very special circumstances such as fine-tuning in specific models.

Current values for the asymmetries \(A_I, A^+_{CP}, A^0_{CP}\), and the mixing-induced asymmetry \(S\) multiplied by minus the CP eigenvalue \(-\eta_{CP}\) of the final state in corresponding \(B^0\) decays are quoted in Table [11] [16]. We have used the ratio of \(B^+\) and \(B^0\) lifetimes, \(\tau_{+}/\tau_{0} = 1.076 \pm 0.008\), for translating ratios of \(B^+\) and \(B^0\) branching ratios into ratios of corresponding decay rates. The asymmetries \(A_I\) for \(X = K^+K^-\) and \(X = K^0\bar{K}^0\) were obtained after subtracting \(B \to K\phi\) contributions in \(B^+\) and \(B^0\) decays, assuming that the subtracted amplitudes are symmetric with respect to interchanging \(K^+\) and \(K^-\) momenta and \(K^0\) and \(\bar{K}^0\) momenta. This symmetry assumption, which is required for obtaining \((B^+ \to K^+K^0\bar{K}^0)\) from the measured \((B^+ \to K^+K_SK_S)\) and \((B^0 \to K^0\bar{K}^0)\) from \((B^0 \to K_SK_SK_S)\) [21], is motivated by data, but may hold only approximately [29,30]. This may explain the somewhat large value of \(A_I\) for \(X = K^+K^-\).

In general, CP asymmetries in charged \(B\) decays involve smaller experimental errors than corresponding asymmetries in neutral \(B\) decays, which require flavor tagging and time-dependent measurements. An exception is the self-tagged mode \(B^0 \to K^+\pi^-\), where a nonzero asymmetry has been measured with a small error. We will focus attention on the two asymmetries \(A^+_{CP}\) and \(A_I\) which involve the smallest errors.

Central values measured for the asymmetries \(A^+_{CP}\) are in most cases at a level of several percent with comparable errors, implying asymmetries of no more than ten or fifteen percent. Larger asymmetries, at a level of twenty or thirty percent, are possible in \(B^+ \to \rho^0K^+, \rho^+K^0, K^+K^0\bar{K}^0\). A similar situation occurs in isospin asymmetries \(A_I\), which are fairly small for several final states involving \(X = \phi, \eta', \pi^0, \pi^+, K^0\bar{K}^0, \pi^+\pi^-\), while larger asymmetries at a level of twenty or thirty percent are possible for other final states. (Note that the asymmetries \(A_I\) for \(X = \omega, f_0, \rho^0\) depend on modeling the Dalitz plot for \(B^{+,0} \to K^{+,0}\pi^+\pi^-\) [31–33].) Thus, since \(A^+_{CP}\) and \(A_I\) are expected to be of order \(2|\Delta B|/|B_P|\) and \(2|A|/|B|\), this confirms the hierarchy (28), excluding NP contributions to \(\Delta B\) and \(A\) comparable to \(B_P\).

A test for the smallness of the magnitude of \(A/B\) regardless of its phase is provided by a sum rule suggested several years ago for the four \(B \to K\pi\) decay rates [7,8]. Using Eqs. (6) and (7), one has

\[
2\Gamma(K^+\pi^0) + 2\Gamma(K^0\pi^0) = 2(|B|^2 + |A|^2) ,
\]

\[
\Gamma(K^0\pi^+) + \Gamma(K^+\pi^-) = 2(|B|^2 + |A'|^2) .
\]

This implies a sum rule also for charge-averaged decay rates, \(\bar{\Gamma}(B \to f) \equiv [\Gamma(B \to f) + \Gamma(B \to \bar{f})]/2\),

\[
2\bar{\Gamma}(K^+\pi^0) + 2\bar{\Gamma}(K^0\pi^0) = \bar{\Gamma}(K^0\pi^+) + \bar{\Gamma}(K^+\pi^-) .
\]

A quadratic multiplicative correction to the sum rule, \((|A'|^2 - |A|^2)/|B|^2,\) is a few percent in the Standard Model [26,34,35]. Since \(A\) and \(A'\) involve two independent \(\Delta I = 1\) amplitudes, the fact that this sum rule holds experimentally within 5% [36]
seems to rule out large $\Delta I = 1$ NP contributions of order $B_P$, which would have to cancel within a few percent.

IV. LINEAR EXPANSION IN $\Delta B$ and $A$

As we have shown, the hierarchy (28) expected in the SM is confirmed by experiments, and can therefore be assumed to hold also in the presence of NP. Thus, we will expand the four asymmetries in Table II to leading order in $\Delta B/B_P$ or $A/B_P$. We will take by convention the dominant penguin amplitude $B_P$ to have a zero weak phase and a zero strong phase, referring all other strong phases to it. Writing

$$B = B_P + \Delta B , \quad \bar{B} = B_P + \Delta \bar{B} ,$$

and defining for $B^0$ decays to CP-eigenstates

$$S = \frac{2 \text{Im} \lambda}{1 + |\lambda|^2} , \quad \lambda \equiv \eta_{CP} \frac{\bar{B} - \bar{A}}{\bar{B} - A} e^{-2i\beta} ,$$

we find

$$\Delta S \equiv -\eta_{CP} S - \sin 2\beta \cos 2\beta \left[ \frac{\text{Im}(\bar{A} - A)}{B_P} - \frac{\text{Im}(\Delta \bar{B} - \Delta B)}{B_P} \right] ,$$

$$A_I = \frac{\text{Re}(\bar{A} + A)}{B_P} ,$$

$$A^+_{CP} = \frac{\text{Re}(\bar{A} - A)}{B_P} + \frac{\text{Re}(\Delta \bar{B} - \Delta B)}{B_P} ,$$

$$A^0_{CP} = -\frac{\text{Re}(\bar{A} - A)}{B_P} + \frac{\text{Re}(\Delta \bar{B} - \Delta B)}{B_P} .$$

Eq. (35) is familiar from its implication in the SM, where one considers the effect on $S$ of a small amplitude with weak phase $\gamma$ and a strong phase $\delta$ relative to the dominant penguin amplitude $B_P$, $\Delta B - A = |\Delta B - A| e^{i\delta} e^{i\gamma}$. When inserted into (35) this implies

$$\Delta S = 2 \frac{|\Delta B - A|}{B_P} \cos 2\beta \sin \gamma \cos \delta ,$$

which is a well-known result [37]. This result is used to calculate deviations of $-\eta_{CP} S$ from $\sin 2\beta$, and to argue that $\Delta S > 0$ when one expects $|\delta| < \pi/2$ [15].

The relations (35)–(38) tell us several things:

1. The $\Delta I = 0$ and $\Delta I = 1$ contributions in $A_{CP}$ may be separated from one another by taking sums and differences [6]:

$$A^\Delta I=0 \equiv \frac{1}{2} (A^+_{CP} + A^0_{CP}) = \frac{\text{Re}(\Delta \bar{B} - \Delta B)}{B_P} ,$$

$$A^\Delta I=1 \equiv \frac{1}{2} (A^+_{CP} - A^0_{CP}) = \frac{\text{Re}(\bar{A} - A)}{B_P} .$$
(2) One can separate the $\Delta I = 1$ terms $\text{Re} A/B_P$ and $\text{Re} \bar{A}/B_P$ from one another using information from $A_{CP}^{\Delta I=1}$ and $A_I$.

(3) The deviation of $S$ from its nominal value of $-\eta_{CP} \sin(2\beta)$ is governed by an imaginary part of a combination of $\Delta I = 0$ and $\Delta I = 1$ terms. Thus, without the help of a specific model, it is impossible to determine whether any deviation of $S$ from its nominal value is due to $\Delta I = 0$ or $\Delta I = 1$ or a combination. One may, however, test the predictions of specific models for the four asymmetries $(35) - (38)$.

A similar expansion in terms of small ratios of amplitudes can be performed for asymmetries in $B_s$ decays. Consider, for instance, the pair of isospin-reflected decays $B_s \to K^+ K^-$ and $B_s \to K^0 \bar{K}^0$, whose amplitudes are given in Eq. (22), where the observed final state in the second process is $K_S K_S$. In this case, one may measure in principle five asymmetries (instead of four in $B^+$ and $B^0$ decays): an isospin asymmetry, a pair of direct asymmetries $A_{CP}$ in decays to charged and neutral kaons, and a pair of asymmetries $S$ in these decays. The first three asymmetries are given by expressions as in Eqs. $(36) - (38)$. Denoting the small phase of $B_s - \bar{B}_s$ mixing by $\chi$, where in the SM $(39)$

$$\sin \chi = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{\sin \beta \sin \gamma}{\sin(\beta + \gamma)},$$

the other two asymmetries are given by

$$S_{K^+ K^-} - \sin 2\chi = \cos 2\chi \left[ \frac{\text{Im}(\bar{A} - A)}{B_P} + \frac{\text{Im}(\Delta \bar{B} - \Delta B)}{B_P} \right],$$

$$S_{K_S K_S} - \sin 2\chi = \cos 2\chi \left[ -\frac{\text{Im}(\bar{A} - A)}{B_P} + \frac{\text{Im}(\Delta \bar{B} - \Delta B)}{B_P} \right].$$

Thus, a fourth lesson is the following:

(4) In contrast to $B^0$ and $B^+$ decays, these two deviations of $S$ from $\sin 2\chi$ enable separating imaginary parts of contributions from $\Delta I = 0$ and $\Delta I = 1$ amplitudes. The difficult part is measuring time-dependence in $B_s \to K_S K_S$. Alternatively, one may measure time-dependent CP asymmetries $S$ in $B_s \to K^{*+} K^{*-}$ and $B_s \to K^{*0} \bar{K}^{*0}$, where the decay time is obtained by reconstructing $K^{*} \to K \pi$. Separating between CP-even and CP-odd final states requires analyzing the distributions of transversity angles in $K^*$ decays.

V. EXTRACTING NEW PHYSICS AMPLITUDES

Under some circumstances one can carry the above discussion further without referring to a specific model, by assuming that the strong phases associated with NP amplitudes are small relative to those of the SM and can be neglected $(40-42)$. This assumption is reasonable since rescattering from a leading $b \to sc\bar{c}$ amplitude is likely the main source of strong phases, while rescattering from a smaller $b \to sq\bar{q}$ NP amplitude is then a second-order effect. Under this assumption, all magnitudes and
weak phases of NP contributions in $A$ and $\Delta B$ can be combined into one effective magnitude and weak phase which changes sign under CP conjugation. We note that while this assumption is intuitive and plausible, it is an approximation which must be confronted by data. (See the discussion at the end of Section V.B.)

The amount of SM contributions in $A$ and $\Delta B$, involving a CKM phase different than in $B_P$, is process-dependent. These contributions are very small in $B \to K\phi$, implying a tiny value for $\Delta S$ within the SM. They are larger in $B \to K\pi$, leading on the one hand to a potentially larger value of $\Delta S$ in the SM, but on the other hand to a more subtle treatment of NP effects in these processes. We will now demonstrate the extraction of NP amplitudes in these two cases, where the current measurement of $\Delta S$ is 1.6$\sigma$ away from zero for each process (see Table II), and in $B_s \to K\bar{K}$.

A. $B \to K\phi$

Consider first the decays $B^+ \to K^+\phi$ and $B^0 \to K^0\phi$, where both amplitudes are dominated by the same penguin term $B_P$, with weak phase $\simeq \text{Arg}(V_{cb}V_{cs}^*) \simeq 0$. [There will be a negligible contamination in $B_P$ of order 2% (the ratio of CKM elements) from $V_{ub}V_{us}^*$.] The SM terms in $\Delta B$ and $A$ include subleading electroweak penguin (EWP) contributions [43], with a vanishing weak phase similar to $B_P$. (We neglect a small annihilation amplitude in $B \to K^+\phi$.) The SM $\Delta I = 0$ EWP amplitude, which can acquire a strong phase by rescattering from $B_P$, will be included in the definition of $B_P$. In contrast, the SM $\Delta I = 1$ EWP term can be assumed to get no strong phase, since it involves no rescattering from the dominant $\Delta I = 0$ amplitude $B_P$. Thus, in our definition the amplitude $\Delta B$ is purely NP, while $A$ involves a SM EWP amplitude and NP contributions, both of which are assumed to involve no strong phases.

We will now study the amount of information which can be learned about magnitudes and weak phases of isoscalar and isovector NP amplitudes, using the four observables (35) (36) (40) and (41). Since these four asymmetries are first order in small ratios of amplitude, we can take $B_P$ as given by the square root of $\Gamma(B \to K\phi)$, thereby neglecting second order terms. We will assume that the EWP contribution to the $\Delta I = 1$ amplitude $A$ is calculable within the SM [35,44], or can be obtained independently by fitting within flavor SU(3) other $B$ decay rates and asymmetries [45].

In our convention (33), where the strong phase of $B_P$ is set equal to zero, $\Delta B$ and $A$ have the same strong phase $\delta$, and involve weak phases $\phi_B$ and $\phi_A$, respectively,

$$\Delta B = |\Delta B|e^{i\delta}e^{i\phi_B} , \quad A = |A|e^{i\delta}e^{i\phi_A} .$$

(46)

In the charge-conjugated amplitudes $\Delta \bar{B}$ and $\bar{A}$ the phase $\delta$ is left unchanged, while $\phi_B$ and $\phi_A$ change signs. This leads to

$$\frac{\text{Re}(\bar{A} - A)}{\text{Re}(\Delta \bar{B} - \Delta B)} = \frac{\text{Im}(\bar{A} - A)}{\text{Im}(\Delta \bar{B} - \Delta B)} = \frac{|A|\sin \phi_A}{|\Delta B|\sin \phi_B} \equiv r ,$$

(47)

and

$$\frac{\text{Re}(\Delta \bar{B} - \Delta B)}{\text{Im}(\Delta \bar{B} - \Delta B)} = -\tan \delta ,$$

(48)

$$\text{Im}(\Delta \bar{B} - \Delta B) = -2|\Delta B|\sin \phi_B \cos \delta ,$$

(49)

and

$$\text{Re}(\bar{A} + A) = 2|A|\cos \phi_A \cos \delta .$$

(50)
The parameter $r$ is seen to be determined by the ratio of $\Delta I = 1$ and $\Delta I = 0$ CP asymmetries given in Eqs. (41) and (40),

$$r = \frac{A_{CP}^{\Delta I=1}}{A_{CP}^{\Delta I=0}}.$$  (51)

The second ratio in (47) implies

$$\Delta S = \frac{\text{Im}(\Delta \bar{B} - \Delta B)}{B_P}(r - 1) \cos 2\beta,$$  (52)

leading by (48) to a determination of $\tan \delta$,

$$\tan \delta = \frac{A_{CP}^{\Delta I=0}}{\Delta S}(1 - r) \cos 2\beta.$$  (53)

This fixes $\tan \phi_A$ from the ratio of the other two asymmetries,

$$\tan \phi_A \tan \delta = \frac{A_{CP}^{\Delta I=1}}{A_I}.$$  (54)

Once $\delta$ and $\phi_A$ are given (mod $\pi$), knowledge of $B_P$ and a measurement of $A_I$ or $A_{CP}^{\Delta I=1}$ gives $|A|$ through (56) and (50) or (41) and (47). (Note that $A$ includes a potential NP contribution and a SM EWP amplitude, both of which were assumed to have equal strong phases.) Similarly, a measurement of $A_{CP}^{\Delta I=0}$ and $\Delta S$ permit one to obtain $\text{Re}(\Delta \bar{B} - \Delta B)$ and $\text{Im}(\Delta \bar{B} - \Delta B)$, each of which yields $|\Delta B| \sin \phi_B$ from Eqs. (48) and (49). The combination $|\Delta B| \cos \phi_B$ adds coherently to $B_P$ and cannot be fixed independently.

B. $B \to K\pi$

In $B \to K\pi$, an isolation of NP contributions in $\Delta B$ and $A$ requires taking into account two subleading SM amplitudes involving a weak phase $\arg(V_{ub}^* V_{us}) = \gamma$ [22]. These are a color-favored tree amplitude $T$ contributing to $B^0 \to K^+\pi^-$ and $B^+ \to K^+\pi^0$, and a color-suppressed tree amplitude $C$ contributing to $B^+ \to K^+\pi^0$ and $B^0 \to K^0\pi^0$. We consider the two pairs of isospin-reflected processes ($B^+ \to K^+\pi^0, B^0 \to K^0\pi^0$) and ($B^+ \to K^0\pi^+, B^0 \to K^+\pi^-$). The four measured asymmetries in the first pair and the three asymmetries in the second pair (see Table III) combine an interference of the dominant penguin amplitude $B_P$ with $T$ and $C$ and with potential NP contributions.

Using the notations of Eqs. (6) and (7) and expressing NP contributions explicitly, one has [22, 34]

$$\Delta B = \frac{1}{2} T + \Delta B_{NP},$$  \hspace{1cm} (55)

$$A = \frac{1}{2} T + C + P_{EW} + \frac{1}{2} P_{EW}^c + A_{NP},$$  \hspace{1cm} (56)

$$A' = -\frac{1}{2} T - \frac{1}{2} P_{EW}^c + A_{NP}'$$  \hspace{1cm} (57)
where $P_{EW}$ and $P^c_{EW}$ are color-favored and color-suppressed EWP amplitudes. The dominant amplitude $B_P$ with weak phase $\simeq \text{Arg}(V_{cb}V^\ast_{cb}) \simeq 0$ includes a small EWP contribution, $\frac{1}{6}P^c_{EW}$. An annihilation amplitude with weak phase $\gamma$, expected to be smaller than $T$ and $C$ [22, 26], is included in the definition of $T$ in $B$ and $A$, but is neglected in $A'$. Proceeding as before under the assumption that subleading NP amplitudes involve negligible strong phases, we apply the same assumption to all subleading contributions including $T, C, P_{EW}$ and $P^c_{EW}$, which involve no rescattering from the dominant amplitude $B_P$. In our convention, where the strong phase of $B_P$ vanishes, expression [46] for $\Delta B$ and $A$ hold also in $B^+ \to K^+\pi^0$ and $B^0 \to K^0\pi^0$. Consequently, the entire analysis following these expressions holds too, where $B_P$ is now given by the square root of $\Gamma(B^+ \to K^0\pi^+)$ (which involves tiny second order corrections). Thus, measurements of the four asymmetries, $A_I, A^+_CP, A^0_CP$ and $\Delta S$, in the pair $(B^+ \to K^+\pi^0, B^0 \to K^0\pi^0)$ leads to a determination of the magnitude $|A|$ and weak phase $\phi_A$ of the amplitude $A$ and of the quantity $|\Delta B|\sin\phi_B$. Assuming, as in the case of $B \to K\phi$, that the magnitudes of all subleading SM amplitudes are calculable [24–26] (or can be fitted independently within flavor SU(3) to other decays into two pseudoscalars [46, 47]), and that the weak phase $\gamma$ is given [48, 49], this provides information on the magnitude and CP-violating phase of $A_{NP}$ and partial information on $\Delta B_{NP}$.

The pair of decays, $B^+ \to K^0\pi^+$ and $B^0 \to K^+\pi^-$, provides three observables, $A_I$ and $A^0_CP$, which are insufficient for determining the four parameters $|\Delta B|, \phi_B, |A'|$ and $\phi_{A'}$.

A recent study [50], comparing different values measured for $A^+_CP(B^+ \to K^+\pi^0)$ and $A^0_CP(B^0 \to K^+\pi^-)$, two processes unrelated by isospin-reflection, concluded that a sizable negative strong phase difference between $C$ and $T$ amplitudes of comparable magnitude is required to account for the asymmetries within the SM. (In Ref. [51, 52] the different asymmetries have been argued to provide a clue for NP.) A similar conclusion about Arg($C/T$) was reached in Ref. [46, 47, 56] studying $B \to K\pi$ and $B \to \pi\pi$ decays in flavor SU(3). This seems to stand in contrast to QCD calculations using a factorization theorem, which obtain small values for this strong phase difference [25, 35, 53, 54]. In our arguments above we have assumed that Arg($C/T$) is negligible, in accordance with QCD calculations. The nonzero strong phase difference measured in $A^+_CP(B^+ \to K^+\pi^0)/A^0_CP(B^0 \to K^+\pi^-)$ may be due to rescattering from $T$ to $C$ (if $C$ were smaller than $T$ without rescattering), or the effect of an annihilation amplitude which we included in $T$. In either case, the nonzero and negative value of Arg($C/T$) must be included in the extraction of NP parameters.

C. $B_s \to K\bar{K}$

The structure of the amplitudes $\Delta B$ and $A$ in the pair $(B_s \to K^+K^-, B_s \to K^0\bar{K}^0)$ is similar to Eqs. (55) and (56), however the term $C + P_{EW}$ is absent in $A$ [22]. As noted, these processes (or $B_s \to K^{*+}K^{*-}$, $B_s \to K^{*0}\bar{K}^{*0}$) provide in addition to an isospin-dependent asymmetry and two direct CP asymmetries also the two time-dependent asymmetries $S_{K^+K^-}$ and $S_{K^0\bar{K}^0}$ (or $S_{K^{*+}K^{*-}}$ and $S_{K^{*0}\bar{K}^{*0}}$) given
in (44) and (45). While $S_{K_S K_S}$ is difficult to measure, the first four asymmetries suffice for determining the magnitude and weak phase of a potential NP $\Delta I = 1$ amplitude and the imaginary part of the $\Delta I = 0$ amplitude. A theoretical advantage of the pair $(B_s \rightarrow K^+ K^-, B_s \rightarrow K_S K_S)$ over the pair $(B^+ \rightarrow K^+ \pi^0, B^0 \rightarrow K^0 \pi^0)$ is the absence of SM color-suppressed tree ($C$) and color-favored EWP ($P_{EW}$) amplitudes in the first pair of processes. Thus, in these decays NP amplitudes cannot masquerade as unusual EWP amplitudes, as may happen in the second pair of processes [34, 35, 55, 56].

VI. ASPECTS OF SOME SPECIFIC MODELS

In models with extra isosinglet quarks, such as considered in Refs. [57–61], mixing of the isosinglet quark with ordinary quarks can induce flavor-changing neutral currents (FCNC). Thus, for example, effective four-fermion operators of the flavor structure $b \rightarrow sq \bar{q}$ ($q = u, d, s, c$) can arise, with arbitrary couplings $U_{sb}$ at the $bsZ$ vertex and couplings to $u, d, s, c$ as in the standard electroweak theory. These differ for left-handed and right-handed quarks, being given by the interaction Lagrangian

$$\mathcal{L}_{Zff} = -i\sqrt{g^2 + g'^2} \bar{f} \gamma^\mu (I_{3L} - Q_{EM} x) Z_\mu f.$$  \hfill (58)

Here $g$ and $g'$ are the SU(2) and U(1) couplings of the SU(2) × U(1) electroweak theory, $I_{3L}$ is the left-handed isospin and $Q_{EM}$ the charge of fermion $f$, and $x \equiv \sin^2 \theta_W$ is the electroweak SU(2)–U(1) mixing parameter. At values of $Q \leq 1$ GeV, relevant for color-favored couplings of the neutral current to a light meson, one expects $x \approx 0.238$ [62], running to $x = 0.23152 \pm 0.00014$ at $Q = M_Z$ [63]. (Here we are taking the “effective” $x$ as measured in leptonic asymmetries at $M_Z$.) The low-energy expectation has been confirmed by a recent measurement of $x = 0.2397 \pm 0.0010 \pm 0.0008$ at $Q^2 = 0.026$ GeV$^2$ [64].

Many extended grand unified theories can have gauge bosons beyond those of the SM at relatively low (TeV) masses. The implications of such bosons for FCNC processes contributing to $B$ decays have been examined, for example, in Refs. [65]. These authors have considered the effects of operators of various chiralities. As in the case of FCNC couplings of the $Z$, a variety of couplings of the $Z'$ to the $q \bar{q}$ pair in $b \rightarrow sq \bar{q}$ is possible. The $Z_\chi$, associated with the U(1) in the symmetry-breaking chain $SO(10) \rightarrow SU(5) \times U(1)_\chi$, couples differently to different SU(5) representations. As the SU(5) assignments of up-type and down-type quarks are different, such a boson will have both isoscalar and isovector couplings. On the other hand, the $Z_\psi$, associated with the U(1) in the chain $E_6 \rightarrow SO(10) \times U(1)_\psi$, couples in the same way to up- and down-type quarks, and so will have a purely $\Delta I = 0$ nature. Couplings of mixtures of $Z_\chi$ and $Z_\psi$, such as arise in certain versions of superstring compactifications, were discussed in Ref. [66].

The relative $Z_\chi$ charges $Q_\chi$ of SU(5) representations in a (left-handed) 16*-plet of SO(10) are (3, $-1$, $-5$) for the 5* = ($\bar{d}, e^-, \nu_e$), 10 = ($u, d, \bar{u}, e^+$), and 1 = $\bar{N}_e$, respectively. Here $\bar{N}_e$ denotes the (presumably heavy) left-handed antineutrino. To calculate the $Z_\chi$ charges of right-handed quarks and leptons, we bear in mind that CP-conjugation reverses the sign of all charges.
Table III: Values of charges $I_{3L} - Q_{EM}x$, $Q_{X}$, and $Q_{ψ}$, governing $Z(ℓ)f f$ couplings. Normalizations are arbitrary and independent for $Z$, $Z_{X}$, and $Z_{ψ}$ couplings.

|       | $u$    | $d$    | $u + d$ | $u - d$ | $e$   | $ν$  |
|-------|--------|--------|---------|---------|-------|------|
| $Z :$ | $L$    | $\frac{1}{2} - \frac{2}{3}x$ | $-\frac{1}{2} + \frac{1}{3}x$ | $-\frac{1}{3}x$ | $1 - x$ | $-\frac{1}{2} + x$ | $\frac{1}{2}$ |
|       | $R$    | $-\frac{2}{3}x$ | $\frac{1}{3}x$ | $-\frac{1}{3}x$ | $-x$ | $x$ | 0 |
| $R + L$ | $\frac{1}{2} - \frac{4}{3}x$ | $-\frac{1}{2} + \frac{2}{3}x$ | $-\frac{2}{3}x$ | $1 - 2x$ | $-\frac{1}{2} + 2x$ | $\frac{1}{2}$ |
| $R - L$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | 0 | $-1$ | $\frac{1}{2}$ | $-\frac{1}{2}$ |
| $Z_{X} :$ | $L$ | $-1$ | $-1$ | $-2$ | 0 | 3 | 3 |
|       | $R$ | 1 | $-3$ | $-2$ | 4 | 1 | 5 |
| $R + L$ | 0 | $-4$ | $-4$ | 4 | 4 | 8 |
| $R - L$ | 2 | $-2$ | 0 | 4 | $-2$ | 2 |
| $Z_{ψ} :$ | $L$ | 1 | 1 | 2 | 0 | 1 | 1 |
|       | $R$ | $-1$ | $-1$ | $-2$ | 0 | $-1$ | $-1$ |
| $R + L$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $R - L$ | $-2$ | $-2$ | $-4$ | 0 | $-2$ | $-2$ |

A simpler rule applies to the $Z_{ψ}$ charges $Q_{ψ}$. In the 27-plet of $E_{6}$ these charges are $(1, -2, 4)$ for the 16-, 10-, and 1-dimensional $SO(10)$ representations. All the ordinary left-handed fermions belong to the 16 of $SO(10)$. Thus all left-handed $u, d, e$, and $ν$ have charge 1 while right-handed $u, d, e$, and $ν$ have charge $-1$.

In models with FCNC $b \rightarrow s$ transitions, whether due to standard $Z$ or $Z'$ exchange, the $Z(0)f f$ couplings may be decomposed into $ΔI = 0$ and $ΔI = 1$ contributions for each chirality (left or right) of fermion. Now, matrix elements will depend on fermion chiralities, so the interpretation of specific ratios of $ΔI$ amplitudes will depend on calculations of these matrix elements as in Refs. [58, 67–69]. In the last three references, NP operators are expressed in terms of SM operators $O_{3,7,9}$ and evolved down to low $Q^{2}$, where additional operators arise. The isospin content of flavor-changing $Z$ couplings in $B \rightarrow Kπ$ is studied in Ref. [68].

The isospins and chiralities of the $Zf f$ couplings are directly relevant in the case of contributions to the color-favored electroweak penguin amplitudes. In this case the $Z$ couples directly to a meson, such as $π^{0}$, $η$, $η'$, $ρ^{0}$, $ω$, or $ϕ$. Neglecting QCD corrections to factorization, we may calculate the relative couplings of a $Z$, $Z_{X}$, or $Z_{ψ}$ to various pseudoscalar or various vector mesons using the values of $I_{3L} - Q_{EM}x$, $Q_{X}$, and $Q_{ψ}$ shown in Table III.

The isoscalar ($u + d$) couplings of $Z$ and $Z_{X}$ are purely vector ($L = EM$) while that of $Z_{ψ}$ is purely axial ($L = -R$). The isovector couplings of $Z$ and $Z_{X}$ contain both vector and axial-vector contributions, while those of $Z_{ψ}$ vanish identically. A
Table IV: Relative couplings $g_M$ and their squares for transitions between $Z$, $Z_\chi$, or $Z_\psi$ and pseudoscalar or vector mesons. Normalizations for $Z$, $Z_\chi$, and $Z_\psi$ are arbitrary and independent of one another; no relation is implied between pseudoscalar and vector mesons. For $Z$ couplings to vector mesons we have taken $x = 0.238$.

|     | $Z$ | $Z_\chi$ | $Z_\psi$ |
|-----|-----|----------|----------|
| $M = \pi^0$ | $g_M = \frac{1}{\sqrt{2}}$ | $g_M = \frac{1}{2}$ | $g_M = -\frac{4}{\sqrt{2}}$ | $g_M = 8$ | $g_M = 0$ | $g_M = 0$ |
| $M = \eta$  | $g_M = \frac{1}{2\sqrt{3}}$ | $g_M = \frac{1}{4\sqrt{3}}$ | $g_M = -\frac{2}{\sqrt{3}}$ | $g_M = \frac{4}{3}$ | $g_M = \frac{2}{3}$ | $g_M = \frac{4}{3}$ |
| $M = \eta'$ | $g_M = \frac{1}{\sqrt{6}}$ | $g_M = \frac{1}{6}$ | $g_M = -\frac{4}{\sqrt{6}}$ | $g_M = \frac{8}{3}$ | $g_M = -\frac{8}{3}$ | $g_M = \frac{3}{3}$ |
| $M = \rho^0$ | $g_M = \frac{1}{\sqrt{2}}(2x - 1)$ | $g_M = 0.137$ | $g_M = -\frac{4}{\sqrt{2}}$ | $g_M = 8$ | $g_M = 0$ | $g_M = 0$ |
| $M = \omega$ | $g_M = -\frac{\sqrt{2}}{3}x$ | $g_M = 0.0126$ | $g_M = -\frac{4}{\sqrt{2}}$ | $g_M = 8$ | $g_M = 0$ | $g_M = 0$ |
| $M = \phi$  | $g_M = \frac{2}{3}x - \frac{1}{2}$ | $g_M = 0.117$ | $g_M = -4$ | $g_M = 16$ | $g_M = 0$ | $g_M = 0$ |

Pseudoscalar meson couples to the axial current, while a vector meson couples to the vector current. We use the quark content $\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\eta \simeq (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}$, $\eta' \simeq (2s\bar{s} + u\bar{u} + d\bar{d})/\sqrt{6}$, $\rho^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\omega = (d\bar{d} + u\bar{u})/\sqrt{2}$, $\phi = s\bar{s}$. The results are shown in Table IV.

The relative couplings in Table IV have some interesting properties. The standard $Z$ couples more strongly to $\pi^0$ than to $\eta$ or $\eta'$, and much more strongly to $\rho^0$ and $\phi$ than to $\omega$. The $Z_\chi$ also couples more strongly to $\pi^0$ than to $\eta$ or $\eta'$, but its coupling to $\phi$ is somewhat stronger than to $\rho^0$ or $\omega$. The $Z_\psi$ couples to $\eta$ and overwhelmingly to $\eta'$ but not to $\pi^0$. By virtue of its purely axial coupling to ordinary matter (members of the SO(10) 16-plet), it does not couple at all to vector mesons. As mentioned, QCD corrections could modify this conclusion. In fact, QCD corrections have been calculated for models involving flavor-changing $Z$ exchange and were shown to change the above pattern [70].

FCNC operators which emulate SM electroweak penguins are expected to contribute also to $B \to X_s\ell^+\ell^-$ and $B \to X_s\nu\bar{\nu}$ processes. The fact that no anomalous behavior of these processes has yet been seen provides constraints on such operators. The current agreement between SM calculations of $(B \to X_s\ell^+\ell^-)$ [71] and experiment [16, 72], $(B \to X_s\ell^+\ell^-) = (4.50^{+1.03}_{-1.01}) \times 10^{-6}$ for $M_{\ell^+\ell^-} > 0.2$ GeV/c$^2$, is somewhat above the level of $1 \times 10^{-6}$. For a NP contribution of no more than this magnitude, this leads to upper bounds on flavor-changing couplings, such as $|U_{sb}/V_{cb}| < 6 \times 10^{-3}$ for the $bsZ$ coupling in models with an extra isosinglet quark [1]. A coupling at this upper limit would contribute less than 1% of the measured $B_s$-$\bar{B}_s$ mixing [73], while its EWP-like contributions to $B \to K\phi$ and $B \to K\pi^0$ would be at a level of several percent of the dominant penguin amplitude [1].
VII. CONCLUSIONS

We have studied the question of how to determine the isospin structure of potential NP operators occurring in the effective Hamiltonian describing $b \to s q \bar{q}$. We have shown that this question may be answered by studying four asymmetries in pairs of isospin-reflected decay processes: an isospin asymmetry, direct CP asymmetries in $B^+$ and $B^0$ decays, which can be translated into $\Delta I = 0$ and $\Delta I = 1$ asymmetries, and a deviation $\Delta S$ from $\pm \sin 2\beta$ of the coefficient $S$ of the $\sin \Delta m t$ term in time-dependent CP asymmetry. These four observables permit, in principle, determining the magnitude and CP-violating phase of a $\Delta I = 1$ NP amplitude and the imaginary part of a corresponding $\Delta I = 0$ amplitude. Similar considerations apply to $B_s$ decays, where one may use five asymmetries instead of four for certain pairs of isospin-reflected decays.

The current precision in asymmetry measurements is insufficient for carrying out this program at this time. So far, a single nonzero direct CP asymmetry, $A_{CP}(B^0 \to K^+\pi^-) = -0.097 \pm 0.012$, has been clearly observed. The isospin-reflected pair of processes, $B^+ \to K^0\pi^+$ and $B^0 \to K^+\pi^-$, where $A^I=0_{CP} = -0.044 \pm 0.014, A^I=1_{CP} = 0.053 \pm 0.014$ (see Table III), may soon provide first direct evidence for separate nonzero $\Delta I = 0$ and $\Delta I = 1$ asymmetries. A sum rule among the four $B \to K\pi$ CP asymmetries [9, 12], valid within first order $\Delta I = 1$ corrections, predicts [36] $A_{CP}(B^0 \to K^0\pi^0) = -0.140 \pm 0.043$, using the values of the other three measured asymmetries. This implies a sizable $\Delta I = 1$ asymmetry, $A^I=1_{CP} = 0.094 \pm 0.025$, and $A^I=0_{CP} = -0.047 \pm 0.025$ in the pair $B^+ \to K^+\pi^0, B^0 \to K^0\pi^0$. Measurements of the corresponding isospin asymmetry, $A_I = 0.087 \pm 0.038$, and $\Delta S = -0.35 \pm 0.21$ must be improved for a useful implementation of the proposed method.

Certain models of New Physics, such as those involving flavor-changing neutral currents mediated either by the standard $Z$ or by new neutral gauge bosons, have distinctive isospin patterns in the operators giving rise to deviations from Standard Model observables. We have given examples of some of these patterns for models with additional singlet quarks or with the gauge bosons $Z_X, Z_\psi$ of SO(10) and $E_6$ theories.

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