Oscillations in solar–type stars tidally induced by orbiting planets

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0.1 Introduction

We examine the effect of dynamical tides raised by a companion on a solar–type star. In these binaries, gravity or g mode oscillations are excited by the companion in the radiative region beneath the convective envelope of the star. They become evanescent in the convection zone.

This is of particular interest in connection with the newly discovered planets, some of which are found to orbit around solar–type stars with a period comparable to that of the high order g modes of the star. One such example is 51 Pegasi (8; 7).

Here, we determine the magnitude of the perturbed velocity induced by the tides at the stellar surface. We show that, in the case of 51 Pegasi, this velocity is too small to be observed. This result is insensitive to the magnitude of the stellar turbulent viscosity assumed and is not affected by the possibility of resonance, which occurs when the frequency of the tidal disturbance is close to that of some normal mode of the star. We also discuss the orbital evolution and synchronization timescales associated with the tidal interaction, a detailed calculation of which will be presented elsewhere.

0.2 Tidal response to a companion in circular orbit

In the case of 51 Pegasi, observations indicate that the rotational period of the star is between 30 and 40 days (8). Since this is much larger than the orbital period (4.23 days), we neglect the rotational angular velocity of the primary compared to the orbital frequency. Quadrupolar tidal forcing thus occurs through potential perturbations with periods which are half the orbital period.

The tides raised on the star are damped by turbulent friction (10) in the convective envelope, and by non–adiabaticity arising from heat transport in the radiative interior.
For the periods of interest, these dissipative mechanisms are predominant away from and close to resonance respectively. We then calculate the tidal response in resonance taking radiative damping alone into account. Away from resonance, we should in principle include turbulent dissipation to calculate the tide. However, dissipation in the convection zone of solar-type stars is weak enough so that the tidal response away from resonance is well approximated by assuming adiabaticity.

The linearized momentum and mass equations governing the response of the non-rotating star to the perturbing potential $\Psi_T$ may be written (\[9\])

\[
\frac{\partial^2 \xi}{\partial t^2} = -\frac{1}{\rho} \nabla P' + \frac{\rho'}{\rho^2} \nabla P - \nabla \Psi_T,
\]

\[
\rho' = -\nabla \cdot (\rho \xi),
\]

where $P$ is the pressure, $\rho$ is the density, $\xi$ is the Lagrangian displacement vector and the primed quantities are Eulerian perturbations.

In the adiabatic approximation, the energy equation is $\Delta S = 0$, where $S$ is the entropy per unit mass and $\Delta$ denotes the Lagrangian perturbation. When non-adiabaticity in the radiative core is taken into account, this equation takes on the form:

\[
\frac{\partial}{\partial t} \frac{\Delta P}{\rho} = -\nabla \cdot F',
\]

where $T$ is the temperature and $F'$ is the perturbed radiative flux. We suppose that close to resonance, the response behaves exactly like a free g mode with very large radial wavenumber so that WKB theory can be used together with the local dispersion relation to estimate $\nabla \cdot F'$ in this last equation. In addition, $\Delta S$ is related to $\Delta P$ and $\Delta \rho$ through a standard thermodynamic relation.

Here, only the companion’s dominant tidal term is considered in the perturbing potential (\[3\]). For a system with a circular orbit, this is given in spherical polar coordinates \((r, \theta, \phi)\) by the real part of

\[
\Psi_T (r, \theta, \phi, t) = fr^2 P_n^m |\cos \theta| e^{im(\phi-\omega t)}
\]

with $n = m = 2$, $P_n^m$ being the associated Legendre polynomial. Here $\omega$ is the orbital angular velocity, $D$ is the orbital separation, and $f = -GM_p/4D^3$, with $M_p$ being the mass of the companion. The Lagrangian displacement can then be sought in the form:

\[
\xi = \left[ \xi_r (r), \xi_h (r) \frac{\partial}{\partial \theta} \xi_h (r) \frac{\partial}{\sin \theta \partial \phi} \right] P_n^m |\cos \theta| e^{im(\phi-\omega t)}.
\]

We can eliminate $P'$ and $\rho'$ using the non-radial momentum and energy equations. The radial momentum and mass equations then reduce to a pair of ordinary differential equations for $\xi_r$ and $\xi_h$:

\[
\frac{d\xi_r}{dr} = \left( -\frac{2}{r} + \bar{A} - \frac{\Delta \ln \rho}{dr} \right) \xi_r + \left[ -\frac{m^2 \omega^2 \rho}{\Gamma_1 P} + \frac{n(n+1)}{r} \right] \xi_h + \frac{fr^2 \rho}{\Gamma_1 P},
\]

\[
\frac{d\xi_h}{dr} = \frac{1}{r} \left( 1 - \frac{\bar{A} \Delta P}{m^2 \omega^2 \rho} \frac{\Delta \ln P}{dr} \right) \xi_r - \left( A + \frac{1}{r} \right) \xi_h + \frac{Af r}{m^2 \omega^2},
\]

where $A = \Delta \ln \rho/dr - (\Delta \ln P/dr)/\Gamma_1$, $\Gamma_1$ being the adiabatic exponent. We have defined $\bar{A}$ such that $\bar{A} = A$ in the adiabatic approximation, and, when radiative damping is taken
into account, \( \tilde{A} = A/(1 + i\epsilon) \) with \( \epsilon = 16acT^4N^2/(5m^3\omega^3\kappa P r^2) \) in the radiative core and \( \epsilon = 0 \) elsewhere. Here \( a \) is the Stefan–Boltzmann radiation constant, \( c \) is the velocity of light, \( \kappa \) is the opacity and \( N^2 = -Ag \) is the square of the Brunt-Väisälä frequency, \( g \) being the acceleration due to gravity.

The solution of this system requires two boundary conditions. At the surface of the star we take a free boundary. At \( r = 0 \), where equations (5) and (6) have a regular singularity, the boundary condition is that the solutions be regular.

0.3 Numerical results: tidal response and velocity at the surface of the star

The calculations presented in the previous section are applied to a standard solar model (2). We solve numerically the differential equations (3) and (4) using a shooting method to an intermediate fitting point. We define \( x \equiv r/R_c \), where \( R_c \) is the outer radius of the convective envelope. With this notation, the equations are integrated from \( x_{in} = 10^{-6} \) to \( x_{out} = 1.00071256 \). The radiative core extends from \( x = 0 \) to \( x \approx 0.7 \).

In Figure 1, we plot the spatial distribution of the real parts of \( m\omega \xi \) and \( m\omega \xi_h \) for an orbital period of \( P_o = 4.23 \) d away from resonance and in the adiabatic approximation. Away from resonance, the magnitude of the imaginary parts of these quantities is much smaller than that of their real parts (which is why the adiabatic approximation can be used). Therefore, they represent typical values of the radial and horizontal velocities, the maximum values being three and six times larger respectively. Since these quantities depend on the perturbing mass through the ratio \( M_p/(M_p + M_\odot) \), they have been represented in units of this factor.

We see from Figure 1 that a companion orbiting around the star with a period \( P_o = 4.23 \) d induces a radial velocity at the stellar surface the maximum of which is between \( 10^{-2} \) and \( 6 \) m/s for \( M_p \) between \( 10^{-3} \) and \( 1 \) M_\odot. This is at least one order of magnitude smaller than the observed velocity. The period of the tidal oscillation corresponding to this orbital period is \( 2.115 \) d. For the oscillation to have a period of \( 4.23 \) d, the orbital period would have to be \( 8.46 \) d. The maximum perturbed radial velocity at the surface of the star induced by the companion would then be between \( 2 \times 10^{-3} \) and \( 1 \) m/s for a perturbing mass between \( 10^{-3} \) and \( 1 \) M_\odot. These velocities are at least about 50 times smaller than the observed ones. These numbers do not depend on the magnitude of the turbulent viscosity assumed. We have also checked that, because of evanescence in the convective envelope, they are not affected by the possibility of resonance.

0.4 Discussion and Conclusion

The planetary companion interpretation has been questioned recently by the reported \( 4.23 \) d modulation in the line profile of 51 Pegasi (3), and the possibility that this modulation may be due to g mode oscillations has been considered (3). We note that, according to our results, such a modulation could not be due to g mode oscillations tidally driven by a companion.

From the calculations presented above, it is also possible to compute the various timescales associated with the tidal interaction. For \( P_o = 4.23 \) d and \( M_p = 10^{-3} \) M_\odot, it is found that if the turbulent viscosity of the star is calibrated such that the calculations
Figure 1: Real part of $m\omega \xi_r$ (solid lines) and $m\omega \xi_h$ (dotted lines) in units $M_p/(M_p + M_\odot)$ m/s versus $x$ for $x_{in} \leq x \leq 0.01$ (top panel), $0.01 \leq x \leq 0.1$ (middle panel) and $0.1 \leq x \leq x_{out}$ (bottom panel), and for $P_\odot = 4.23 \, d$. 
can account for the observed circularization rates of main sequence solar-type binaries, the tidal orbital evolution, circularization, stellar spin up and convective envelope spin up timescales are respectively 143, 24, 131 and 18 Gyr. All of these timescales are long compared with the inferred age of 51 Pegasi ([4]). If the companion is a low-mass star of 0.1\(M_\odot\), as has been recently suggested, these numbers drop to 1.7, 0.25, 0.016 and 0.0022 Gyr respectively. We then expect the primary star to be synchronized with the orbit, in which case exchange of angular momentum is no longer taking place. Synchronization is actually expected if the mass of the companion is larger than about 10 Jupiter masses. We note that since the orbital decay timescale is larger than the synchronization one, tidal interaction stops before the companion has plunged into the central star.

If a simple estimate of the turbulent viscosity based on the usual mixing length theory is used, all these timescales have to multiplied by \(\sim 50\). In that case, synchronization is expected if the mass of the companion is larger than about 70 Jupiter masses.

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