A Blade Dynamic Strain Inversion Method for Rotating Blades Based on Blade Tip Timing

Haonan Guo\textsuperscript{1,a}, Yongmin Yang\textsuperscript{1,b*}, Fengjiao Guan\textsuperscript{1,c}, Haifeng Hu\textsuperscript{1,e}, Guoji Shen\textsuperscript{1,f}, Zifang Bian\textsuperscript{1,g}, Guangrong Teng\textsuperscript{2,d}

\textsuperscript{1} Laboratory of Science and Technology on Integrated Logistics Support
College of Intelligence Science and Technology
National University of Defense Technology, Changsha, China
\textsuperscript{2} Sichuan Gas Turbine Establishment Aero Engine Corporation of China, Chengdu, China
* Corresponding author: yangyongmin@163.com

Abstract—Based on the vibration response transfer ratio and Natural Neighbor Interpolation (NNI), the research on the noncontact inversion method of dynamic strain of high-speed rotating blades is carried out. The high-speed rotating blade is analyzed through the motion equation, and the correction method of the transfer ratio is proposed through NNI. The three-dimensional model of the rotating blade is established, and the Finite Element Analysis (FEA) considering stress stiffening is carried out to obtain the transfer ratio between the blade tip displacement and the blade strain at a certain speed. The vibration experiment of the high-speed rotating blade is carried out, and the blade tip displacement and the dynamic strain of the key points of the blade are obtained based on Blade Tip Timing (BTT) and strain gauges. The experimental data are processed to obtain the actual transfer ratio of the key points of the blade, and the theoretical transfer ratio is corrected based on NNI. The results show that when the \textsuperscript{1}\textsuperscript{st} order resonance of the rotating blade occurs at the rotating speed of 7960RPM, the strain inversion error of the test point can be reduced to less than 5\%, which verifies the proposed correction method. Therefore, the blade dynamic strain can be inverted more accurately based on BTT.

1. INTRODUCTION
The turbine can provide stable and sufficient power, which is widely used in various fields such as aviation. As the core components, blades compress air step by step to form high temperature and high pressure gas [1]. During operation, excitations such as aerodynamic loads and mechanical loads can easily lead to blade vibration and high-cycle fatigue [2]. Especially during resonance, the dynamic stress will reach the extreme value, which has a higher destructiveness. Therefore, real-time dynamic stress monitoring of blades is of great significance [3]. The traditional method is to measure through strain gauges attached to the blade, which is a contact measurement method. However, the number and location of strain gauges installed in turbines are greatly restricted, and the life of strain gauges is significantly reduced under high temperature and high pressure [4, 5]. Thus, noncontact measurement methods have received more and more attention.

For any mode of the blade, there is a stable mapping relationship defined as the transfer ratio between the displacement or strain of any two points of the blade [6]. After obtaining the transfer ratio between
the blade tip displacement and the blade strain based on vibration data of the blade, the strain can be obtained noncontact based on the blade tip displacement measured by Blade Tip Timing (BTT). MTU of German claims that it can make the inversion error less than 20% when the signal-to-noise ratio is within an acceptable range [7]. The "High Cycle Fatigue (HCF) Science and Technology Program" in the United States pointed out that the inversion error of the dynamic strain should be less than 25% [8]. Zielinski et al. obtained the transfer ratio between blade tip displacement and blade root strain through Finite Element Analysis (FEA) and experiment, and the results were basically the same [9]. Liu et al. obtained the maximum blade root strain based on the blade tip displacement measured by BTT, and performed crack identification and life prediction on the blade [10]. Duan et al. obtained the ratio between the maximum dynamic strain of the blade and the blade tip displacement during the 1st-order resonance of the blade based on FEA [11]. Ao et al. studied the transfer ratio in the frequency domain, and obtained the transfer ratio between blade tip displacement and blade root dynamic strain at any speed through FEA [6]. However, due to the complex structure of the blade, there are usually differences between the processed entity and the three-dimensional model. This leads to the transfer ratio error obtained by FEA, which limits the further development of blade condition monitoring technology based on BTT. Thus, if the transfer ratio can be corrected, the dynamic strain of the blade can be measured noncontact with higher accuracy, so as to better meet the requirements of blade condition monitoring.

In summary, there is error in the transfer ratio between blade tip displacement and blade strain obtained by FEA. Based on the blade motion equation and Natural Neighbor Interpolation (NNI), a correction method of the transfer ratio is proposed. In the experiment of high-speed rotating blades, the actual transfer ratio is obtained through BTT system and strain gauges. Based on the actual transfer ratio, the transfer ratio obtained based on FEA is corrected, and the transfer ratio before and after the correction of the test point is compared with the actual transfer ratio.

2. INTRODUCTION OF BLADE TIP TIMING

The structure of measurement system based on BTT is shown in Fig. 1 [12]. The rotation direction of the blisk is set to clockwise, and the rotation period is set to \( T_a \). The reference sensor is installed directly above the rotating shaft and generates a pulse signal in each rotation cycle of the blisk to determine the time reference. \( N_b \) blades are evenly installed on the blisk along the circumferential direction, and the installation angle of each blade relative to the reference sensor is \( \tau_i \). \( N_b \) BTT sensors are installed in the casing and the installation angle relative to the reference sensor is \( \tau_b \), which generates pulse signals when blades pass by. When the blade does not vibrate, the theoretical time for the \( N_b \)-th blade to reach the \( N_b \)-th BTT sensor after \( C \) cycles is calculated as follows:

\[
\tau_{n,c} = \frac{2\pi}{2\pi C + \tau_b - \tau_i} 
\] (1)

When the blade vibrates, the actual arrival time of the blade tip deviates from the theoretical arrival time. The continuous time of theoretical arrival and actual arrival are set to \( \{t_{\text{expected}}\} \) and \( \{t_{\text{actual}}\} \) respectively, and the time difference can be obtained by substituting the following equation:

\[
\{At\} = \{t_{\text{expected}} - t_{\text{actual}}\} 
\] (2)

Substitute the time difference into the following equation and solving it, the continuous vibration signal of the blade tip can be obtained:

\[
s(t) = \frac{2\pi R}{T_a} \{At\} 
\] (3)

Where \( R \) is the rotation radius of the blade. The blade vibration measurement based on BTT is to sample the continuous vibration signal, and a single blade can generate \( N_b \) pulse signals in one cycle. The Dirichlet sampling function is set to \( \delta(t) \), and the pulse signal sequence obtained by sampling the \( N_b \)-th blade is as follows:
The pulse signal sequence in (4) is the blade tip vibration displacement sequence, and the vibration parameters of the blade can be obtained by processing it.

\[ m_N(t) = s(t) \sum_{n=1}^{N} \sum_{c=1}^{C} \delta(t - (c - 1)T_n - \frac{r_n}{2\pi}T_n) \]  

(4)

The pulse signal sequence in (4) is the blade tip vibration displacement sequence, and the vibration parameters of the blade can be obtained by processing it.

3. CORRECTION METHOD OF TRANSFER RATIO

3.1. Introduction of Transfer Ratio

The rotation vector diagram of the discretized blade is shown in Fig. 2. OXYZ is a static global coordinate system, and the axis Z coincides with the direction of the rotation axis. OrXrYrZr is a nonstatic rotating coordinate system, Or is the intersection of the plane XY and the root of the blade, and the vertical distance from the rotation axis is \( R \). Initially, the Xr axis and the X axis coincide. A certain node \( A_r \) is set as the research node, which moves from \( A_{in}(x,y,z) \) to \( A_{i1}(x + u,y + v,z + w) \). The displacement of the node is \( d_i = (u,v,w) \), and the three components correspond to the radial stretching direction, the circumferential bending direction, and the axial bending direction of the blade respectively. The motion equation of the rotating blade is as follows:

\[ M\ddot{q} + C\dot{q} + Kq = F \]  

(5)

Where \( q \) is the generalized displacement vector, \( M \) is the mass matrix, \( C \) is the damping matrix, \( K \) is the stiffness matrix, and \( F \) is the external load. The generalized displacement of the blade node can be obtained by solving (5).

For a element with \( n \) nodes, the coordinate of a certain node is set to \( (X_j,Y_j,Z_j) \), and the coordinate of a certain internal point is set to \( (x_j,y_j,z_j) \). The displacement of any internal point can be calculated as follows:

\[ D = [u \ v \ w]^T = NQ \]  

(6)

Where \( N \) and \( Q \) are the shape function matrix and the displacement matrix of nodes respectively, which can be obtained as follows:
The strain of any internal point can be calculated as follows:

\[
\epsilon = \frac{\partial}{\partial x} \begin{bmatrix} N_1 & N_2 & \cdots & N_n \end{bmatrix} \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix}^T
\]

Where \( \epsilon \) can be obtained as follows:

\[
\epsilon = \frac{\partial}{\partial x} \begin{bmatrix} N_1 & N_2 & \cdots & N_n \end{bmatrix} \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix}^T \quad \text{(10)}
\]

Based on the chain rule, the following equation can be obtained:

\[
\frac{\partial N_i}{\partial x_j} = \frac{\partial}{\partial x_j} \begin{bmatrix} \frac{\partial N_1}{\partial x_j} & \frac{\partial N_2}{\partial x_j} & \cdots & \frac{\partial N_n}{\partial x_j} \end{bmatrix} = -J \begin{bmatrix} \frac{\partial N_1}{\partial x_j} & \frac{\partial N_2}{\partial x_j} & \cdots & \frac{\partial N_n}{\partial x_j} \end{bmatrix}
\]

Where \( J \) is the Jacobian matrix, which can be calculated as follows:

\[
J = \begin{bmatrix} \frac{\partial x_1}{\partial X_1} & \frac{\partial y_1}{\partial Y_1} & \frac{\partial z_1}{\partial Z_1} \\ \frac{\partial x_2}{\partial X_2} & \frac{\partial y_2}{\partial Y_2} & \frac{\partial z_2}{\partial Z_2} \\ \cdots & \cdots & \cdots \\ \frac{\partial x_n}{\partial X_n} & \frac{\partial y_n}{\partial Y_n} & \frac{\partial z_n}{\partial Z_n} \end{bmatrix}
\]

Based on the above equations, the strain of any internal point can be obtained. When the blade resonates, there is a stable correspondence between the displacement and strain of any two points of the blade. Researchers define the correspondence as the transfer ratio [6]. When the blade is excited, the displacement response \( Y(\omega) \) and strain response \( \epsilon(\omega) \) of any point in the frequency domain are as follows:

\[
Y(\omega) = H^d(\omega)F(\omega)
\]
\[
\epsilon(\omega) = H^e(\omega)F(\omega)
\]

Where \( \omega \) is the frequency of excitation, \( g \) is the freedom degree of the blade, \( F(\omega) = [f_1, f_2, \ldots, f_g]^T \) is the excitation vector, \( H^d(\omega) \) and \( H^e(\omega) \) are the response function matrix of displacement and frequency respectively. The displacement and strain responses of all points of the blade are as follows:

\[
\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_g \end{bmatrix} = \begin{bmatrix} H_{y1}^d & H_{y1}^e & \cdots & H_{y1}^g \\ H_{y2}^d & H_{y2}^e & \cdots & H_{y2}^g \\ \vdots & \vdots & \ddots & \vdots \\ H_{yg}^d & H_{yg}^e & \cdots & H_{yg}^g \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_g \end{bmatrix}
\]

\[
\begin{bmatrix} \frac{\partial y_1}{\partial x_1} \\ \frac{\partial y_2}{\partial x_2} \\ \vdots \\ \frac{\partial y_g}{\partial x_g} \end{bmatrix} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_g} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \cdots & \frac{\partial y_2}{\partial x_g} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial y_g}{\partial x_1} & \frac{\partial y_g}{\partial x_2} & \cdots & \frac{\partial y_g}{\partial x_g} \end{bmatrix} \begin{bmatrix} \frac{\partial x_1}{\partial x_1} & \frac{\partial y_1}{\partial y_1} & \frac{\partial z_1}{\partial z_1} \\ \frac{\partial x_2}{\partial x_2} & \frac{\partial y_2}{\partial y_2} & \frac{\partial z_2}{\partial z_2} \\ \cdots & \cdots & \cdots \\ \frac{\partial x_g}{\partial x_g} & \frac{\partial y_g}{\partial y_g} & \frac{\partial z_g}{\partial z_g} \end{bmatrix}
\]

\[
\begin{bmatrix} \frac{\partial y_1}{\partial x_1} \\ \frac{\partial y_2}{\partial x_2} \\ \vdots \\ \frac{\partial y_g}{\partial x_g} \end{bmatrix} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_g} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \cdots & \frac{\partial y_2}{\partial x_g} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial y_g}{\partial x_1} & \frac{\partial y_g}{\partial x_2} & \cdots & \frac{\partial y_g}{\partial x_g} \end{bmatrix} \begin{bmatrix} \frac{\partial x_1}{\partial x_1} & \frac{\partial y_1}{\partial y_1} & \frac{\partial z_1}{\partial z_1} \\ \frac{\partial x_2}{\partial x_2} & \frac{\partial y_2}{\partial y_2} & \frac{\partial z_2}{\partial z_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial x_g}{\partial x_g} & \frac{\partial y_g}{\partial y_g} & \frac{\partial z_g}{\partial z_g} \end{bmatrix}
\]
Where \( i \) is the response point and \( j \) is the excitation point. \( H^x_\phi \) and \( H^\gamma_\phi \) are the response functions of the displacement and strain of the point \( i \) respectively when only the point \( j \) is excited, which can be obtained as follows:

\[
\begin{bmatrix}
    e_1 \\
e_2 \\
\vdots \\
e_g
\end{bmatrix}
= \begin{bmatrix}
    H^1_{11} & H^1_{12} & \cdots & H^1_{1g} \\
    H^2_{21} & H^2_{22} & \cdots & H^2_{2g} \\
\vdots & \vdots & \ddots & \vdots \\
    H^g_{g1} & H^g_{g2} & \cdots & H^g_{gg}
\end{bmatrix}
\begin{bmatrix}
f_1 \\
f_2 \\
\vdots \\
f_g
\end{bmatrix}
\]

(16)

Where \( \phi_\phi \) is the \( r \) th-order displacement mode of point \( j \), \( \psi_\phi \) is the \( r \) th-order displacement mode of point \( i \). \( K_{pr} \) is the main stiffness of the structure of the \( r \) th-order mode, \( \lambda_r \) is the ratio of \( \omega_r \) to the natural frequency \( \omega_v \) of the \( r \) th-order mode, and \( \xi_s \) is the damping ratio of the \( r \) th-order mode. When the blade resonates in the \( s \) th-order mode, the ratio of the strain response of point \( i \) to the displacement response of point \( j \) can be obtained as follows:

\[
H^\gamma_\phi(\omega) = \sum_{r=1}^{n} \frac{\phi_{sr} \psi_{sj}}{K_{pr}(1 - \lambda_s^2 + i2\xi_s \lambda_s)}
\]

(17)

\[
H^x_\phi(\omega) = \sum_{r=1}^{n} \frac{\phi_{sr} \phi_{sj}}{K_{pr}(1 - \lambda_s^2 + i2\xi_s \lambda_s)}
\]

(18)

Where \( \phi_\phi \) is the \( r \) th-order displacement mode of point \( j \), \( \psi_\phi \) is the \( r \) th-order displacement mode of point \( i \). \( K_{pr} \) is the main stiffness of the structure of the \( r \) th-order mode, \( \lambda_r \) is the ratio of \( \omega_r \) to the natural frequency \( \omega_v \) of the \( r \) th-order mode, and \( \xi_s \) is the damping ratio of the \( r \) th-order mode. When the blade resonates in the \( s \) th-order mode, the ratio of the strain response of point \( i \) to the displacement response of point \( j \) can be obtained as follows:

\[
\frac{e^i_s}{y^j_s} = \frac{\sum_{k=1}^{n} \frac{\psi_{sk} \phi_{sj}}{K_{pr}(1 - \lambda_s^2 + i2\xi_s \lambda_s)} f_k(\omega)}{\sum_{k=1}^{n} \frac{\phi_{sk} \phi_{sj}}{K_{pr}(1 - \lambda_s^2 + i2\xi_s \lambda_s)} f_k(\omega)} = \frac{\psi_{si} \phi_{sj}}{\phi_{sj}}
\]

(19)

The displacement-strain transfer ratio can be obtained by solving the above equation. Where \( k \) is the \( k \) th-point of the blade, \( \psi_\phi \) is the \( s \) th-order strain mode of point \( i \), \( \phi_\phi \) is the \( s \) th-order displacement mode of point \( j \), \( K_{pr} \) is the \( s \) th-order displacement mode of point \( k \), \( \lambda_s \) is the main stiffness of the structure of the \( s \) th-order mode, \( \lambda_s \) is the ratio of \( \omega_s \) to the natural frequency \( \omega_v \) of the \( s \) th-order mode, and \( \xi_s \) is the damping ratio of the \( s \) th-order mode.

In summary, the transfer ratio depends on the mode shape of the structure, which is only related to the inherent characteristics of the system. When the blade tip displacement is obtained by BTT, the blade strain can be inverted through the transfer ratio. For the blade radial strain \( e_x \), the transfer ratio between it and the blade tip displacement can be obtained as follows:

\[
p_x = \frac{e_x}{y_0}
\]

(20)

Where \( y_0 \) is the blade tip displacement, and the transfer ratio obtained by each element can be gathered together to obtain the transfer ratio of all points of the blade. The diagram of blade tip displacement and radial strain during blade bending vibration is shown in Fig. 3. In this study, the displacement unit is set to \( \text{mm} \), the strain unit is set to \( 10^{-3} \), and the transfer ratio unit is \( \text{m}^{-1} \).

\[\begin{array}{c}
y_0 \\
\gamma_0 \\
e_x \\
\end{array}\]

Figure 3. Diagram of blade tip displacement and radial strain.
3.2. Transfer Ratio Correction Based on Natural Neighbor Interpolation

It is found that the shape function and strain of each element are determined by the displacement of the boundary nodes, so the transfer ratio of the blade is locally continuous, which lays the foundation for the transfer ratio correction based on local interpolation. The current spatial interpolation methods mainly include Kriging interpolation, Bessel interpolation, Minimum Curvature Splines, bilinear interpolation, NNI, etc. [13]. NNI is an interpolation method based on the Voronoi Diagram. The point to be interpolated is only affected by the neighboring points, and the first derivative of the interpolation function is continuous except for the origin. Therefore, NNI can fit the discretely distributed irregular nodes well, and is widely used in surface reconstruction, visual design, and geophysical modeling [14].

The Voronoi Diagram is an unstructured grid diagram as shown in Fig. 4, and each solid line element that composes it is defined as Voronoi Cell. The Voronoi Cell corresponding to each original node in the Voronoi Diagram is defined as Natural Neighbors (NN) that are adjacent to each other, and the NN of node \( p \) include \( p_1, p_2, p_3, p_4, p_5 \). \( \Omega \) is set as the convex space of any dimensional space \( R^n \), and \( \{ p_i \} \) is set as the node set \( \{ p_i \in \Omega, i = 1, \cdots, N_p \} \), which is arbitrarily distributed in \( \Omega \). For each node set \( p_i \), a subspace \( T_i \) that satisfies the following equation can be defined:

\[
T_i = \{ p \mid d(p, p_i) < d(p, p_j), p \in \Omega, \forall j \neq i \}
\]  

(21)

Where \( d \) is the Euclid distance, \( T_i \) is the Voronoi Cell corresponding to node \( p_i \). The size of Voronoi Cell depends only on the density of nodes, which is a dual relationship with Delaunay Triangles. Delaunay Triangles can be formed by connecting the nodes of the Voronoi Cell with a common boundary, which can minimize the number of long narrow triangles and maximize the balance. Correspondingly, the Voronoi Cell can be obtained by making vertical bisectors on each side of Delaunay Triangles. The above characteristics make Voronoi Cell can be used for interpolation [15-17].

Through NNI, interpolation can be performed according to the contribution rate of the NN to the point to be interpolated. The interpolation for \( p_x \) is as follows:

\[
p_x (p) = \sum_i \eta_i (p) p_{si}, \quad p \in \Omega, \quad i \in (1, 2, \cdots, N_p)
\]  

(22)

Where \( p_x (p) \) is the value of the point to be interpolated, \( i \) is the number of the original node, \( p_{si} \) is the value of node \( p_i \). And \( \eta_i (p) \) is the interpolation basis function corresponding to node \( p_i \), which is determined by Natural Neighbors Coordinates (NNC). The area of the Voronoi Cell where the point \( p \) is located is set as \( T_p \), it is divided into \( N_p \) parts by the Voronoi Cell where the NN are located, and the area of each part is \( T_{si} \). The ratio of the area of each part to the total area is the interpolation basis function \( \eta_i (p) = T_{si} / T_p \). The basis function has the orthogonality and isoparametric properties as the traditional finite element interpolation function, and there is no discontinuity on the unit boundary [14, 18].

4. Acquisition of the Transfer Ratio

4.1. Finite Element Analysis of the Blade

Based on Solidworks, the integral rotor model with 8 blades in total is established as shown in Fig. 5(a), and its parameters are shown in Table 1. FEA of the blade vibration mode is carried out by ANSYS. The
rotor is axisymmetric, and only a single blade is analyzed in order to improve calculation efficiency. The mesh element type is set to SOLID187, the number of elements and nodes after division are 251089 and 352652 respectively, and fixed constraints are added on both sides of the blade root to simulate the actual working condition. Finite Element Model (FEM) of the blade is generated, as shown in Fig. 5(b), where points A and B are set as key nodes.

![Figure 5. Blade model. (a) 3D model of the integral rotor. (b) FEM of the single blade.](image)

| TABLE 1 PARAMETERS OF THE BLADE. |
|-----------------------------------|
| **Properties** | **Value** |
| Length | 47.5mm |
| Width | 25.4mm |
| Thickness | 1.78mm |
| Radius of rotation | 371mm |
| Material | GH4169 |

The results of the first three modes are shown in Fig. 6 and Table 2.

![Figure 6. Mode shapes of the blade. (a) Mode 1st. (b) Mode 2nd. (c) Mode 3rd.](image)

| TABLE 2 NATURAL FREQUENCY AND MODE SHAPE CLASSIFICATION. |
|----------------------------------------------------------|
| **Mode order** | **Natural frequency (Hz)** | **Mode shape classification** |
| 1st | 665 | Bending vibration |
| 2nd | 2711 | Torsional vibration |
| 3rd | 4128 | Bending vibration |

The blade mode only depends on the inherent parameters such as size and material. However, the blade is affected by centrifugal force during rotation, and its rigidity changes to affect the inherent properties, which is defined as stress stiffening. The 1st order modal analysis is performed on the blade with different rotational speeds, and the Cambell diagram is obtained, as shown in Fig. 7. From the intersection of the 1st natural frequency line and the rotating speed doubling line, it can be found that in the range of 0 to 10000 RPM, the blade may resonate at about 7900 RPM, and the corresponding excitation order is 6Oe, where Oe is the frequency multiplier.
4.2. Plotting of the Transfer Ratio Cloud Diagram

The displacement-strain transfer ratio is affected by stress stiffening, which increases slightly with the increase in rotating speed [6]. The blade modal analysis is performed at the rotating speed of 7900RPM, the blade tip displacement and the radial strain of all nodes are obtained. The transfer ratio cloud diagram is plotted, as shown in Fig.8.

5. EXPERIMENT ANALYSIS

5.1. Structure of the Test Bench

The purpose of the experiment is to verify the transfer ratio correction method proposed in this study, which was carried out on the TDI high-rotating speed test bench shown in Fig. 9(a). The parameters of the rotor are the same as the FEM in section IV. The test bench is equipped with BTT vibration measurement system of Hood Company and strain gauges of EDAS Company. There are 8 BTT sensors, and their installation angles are respectively 57, 68, 100, 123, 184, 260, 271, 283 degrees. The position of the strain gauges shown in Fig. 9(b) correspond to points A and B in Fig. 5. The blade tip vibration displacement and blade dynamic strain can be measured based on the two measurement systems respectively.
5.2. Measurement Result

The speed-up experiment with the speed range of 0 to 8100 RPM was carried out, and the blade tip vibration signal of the No. 1 blade was obtained as shown in Fig. 10. It is easy to be found that the blade resonates when the experiment is carried out to about 315s. After processing, the resonance rotating speed is 7960 RPM and the resonance frequency is 796Hz, which is the same as the analysis result in section IV. The strain at the key points is shown in Fig. 11, which is the same as the trend of the blade tip vibration signal.

5.3. Correction Based on Actual Transfer Ratio

The transfer ratio of point A and point B are obtained as shown in Table 3 based on FEA and the experiment. It is easy to find that although the parameters of the FEM are the same as those of the real blade, there are still errors.

| Position | Theoretical transfer ratio ($m^{-1}$) | Actual transfer ratio ($m^{-1}$) | error |
|----------|--------------------------------------|----------------------------------|-------|
| Point A  | 0.84                                 | 0.94                             | 10.64%|
| Point B  | 1.03                                 | 1.11                             | 7.20% |

The transfer ratios of the blade are locally continuous, so the transfer ratios of adjacent nodes are correlated. However, the gradient is different in different directions, based on the actual transfer ratio of
point B and NNI, the transfer ratios of its neighboring nodes are corrected. Firstly, the nodes within the interpolation radius with point B as the center point are deleted, leaving space for the interpolation nodes, as shown in Fig. 12. Then, the transfer ratio at the location of the strain gage at point B is set to the actual value. Finally, interpolation is performed in the interpolation area to correct the transfer ratio based on NNI. The interpolation radius gradually increases from point B to the edge of the blade, and the interpolation nodes are spaced at intervals of 0.1mm along the length and width of the blade.

Figure 12. Schematic diagram of interpolation.

The results of the correction are shown in Table 4. It is easy to find that as the interpolation radius gradually increases, the error tends to decrease first and then increase. The initial interpolation radius is too small to include point A in the interpolation range. When the interpolation radius is increased to include point A in the interpolation range, the transfer ratio can be corrected. However, as the interpolation radius increases, the number of points to be interpolated is too large, resulting in poor interpolation effect. In the experiment, the errors of the corrected transfer ratio are smaller than the initial error, which verifies the effectiveness of the proposed method. After the transfer ratio is corrected, the blade tip displacement measured based on BTT can be used to invert the blade strain more accurately, so as to better monitor the blade state.

**TABLE 4 CORRECTION RESULT OF TRANSFER RATIO AT POINT A.**

| Interpolation radius (mm) | Actual transfer ratio (m⁻¹) | Theoretical transfer ratio (m⁻¹) | Correction transfer ratio (m⁻¹) | Error before correction | Error after correction |
|--------------------------|-----------------------------|---------------------------------|---------------------------------|------------------------|-----------------------|
| 5                        | 0.94                        | 0.84                            | 0.84                            | 10.64%                 | 10.64%                |
| 10                       | 0.94                        | 0.84                            | 0.92                            | 10.64%                 | 2.33%                 |
| 15                       | 0.94                        | 0.84                            | 0.93                            | 10.64%                 | 1.06%                 |
| 20                       | 0.94                        | 0.84                            | 0.96                            | 10.64%                 | 1.99%                 |
| 25                       | 0.94                        | 0.84                            | 0.97                            | 10.64%                 | 3.21%                 |
| 30                       | 0.94                        | 0.84                            | 0.98                            | 10.64%                 | 4.36%                 |
| 35                       | 0.94                        | 0.84                            | 0.99                            | 10.64%                 | 5.59%                 |
| 40                       | 0.94                        | 0.84                            | 1.01                            | 10.64%                 | 6.97%                 |

6. CONCLUSIONS

In this study, a correction method for the transfer ratio is proposed to perform the higher-precision inversion of blade strain. A three-dimensional model of the blade was established and the FEA was performed to obtain the theoretical transfer ratio. For the 1st-order resonance of the blade (7960RPM), the theoretical transfer ratio of point B can be corrected based on the actual transfer ratio to invert the strain of point A with higher accuracy, which verifies the effectiveness of the transfer ratio correction method proposed in this study. The correction effect is related to the distance between the key points and the interpolation radius. When the interpolation radius is set to 15mm, the error after correction can be
minimized. Through the method proposed in this paper, the transfer ratio can be corrected by the actual transfer ratio of a few points in the blade design and manufacturing stage. The dynamic strain of key points can be measured non-contact with higher accuracy, so as to better meet the indicators proposed in the “HCF Science and Technology Program”. In future work, a larger number of strain gauges will be installed on the high-speed rotating bench, and the transfer ratio will be corrected and analyzed based on the actual values to further verify the method and make improvements.

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