ON THE DOUBLE NATURED SOLUTIONS OF THE TWO-TEMPERATURE EXTERNAL SOFT PHOTON COMPTONIZED ACCRETION DISKS

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ABSTRACT

We have analyzed pair production in the innermost region of a two-temperature external soft photon Comptonized accretion disk. We have shown that, if the viscosity parameter is greater than a critical value $\alpha_c$, the accretion rate has to satisfy $\dot{m}_1 \leq \dot{m} \leq \dot{m}_c$, in order to have two steady-state solutions. It is shown that these critical parameters $\dot{m}_1$, $\dot{m}_c$ are functions of $r$, $\alpha$, and $\theta_e$, and $\alpha_c$ is a function of $r$ and $\theta_e$. Depending on the combination of the parameters, the advection-dominated solution may not be physically consistent. It is also shown that the electronic temperature is maximum at the onset of the thermal instability, from which results this inner region. These solutions are stable against perturbations in the electron temperature and in the density of pairs.

Key words: accretion, accretion disks – black hole physics – elementary particles – radiation mechanisms: general

1. INTRODUCTION

The idea that pair production could occur in astrophysical plasmas is not an old one. It dates from a pioneering paper by Bisnovatyi-Kogan et al. (1971). In their study, these authors have considered the production of pairs through particle–particle interactions and have discovered that, under these conditions, there is a maximum temperature above which production overwhelms annihilation, equilibrium is no longer possible, and positron density approaches infinity. Soon, after Bisnovatyi-Kogan et al. (1971) conclusions on pairs on relativistic plasmas, lot of works followed (Stoeger 1977; Pozdnyakov et al. 1977; Liang 1979) which failed to confirm the existence of this maximum temperature. However, though considering different processes for pair production, these works have focused on a rather limited range of the parameter space. A full understanding of Bisnovatyi-Kogan et al. (1971) results was only possible after the work of Lightman (1982), who showed the limitations of Bisnovatyi-Kogan et al. (1971) results. Accordingly, these results only hold for very small photon densities, or, equivalently, very small scattering depths. Much before attaining the maximum temperature, photon–photon processes will dominate pair production. In that regime, the relation between pair production and annihilation is quite different, not allowing for a critical temperature as is the case of particle–particle interaction. As a matter of fact, Lightman (1982) has shown the existence of two branches of solution for a given $(\tau_p, T_e)$: one, the low $n_+$, with particle–particle dominating pair production, and the other, the high $n_+$, with photon–photon interactions dominance in pair production. In the upper branch, the high $n_+$, the specific heat is negative, and as the heating rate and the luminosity increases, the temperature decreases; in the lower branch, the low $n_+$, as the heating rate increases, the temperature increases and eventually reaches a $T_c$, where both branches merge. For a further increase in the heating rate, the behavior follows that of the high $n_+$ branch, with temperature decreasing till the plasma becomes effectively thick. These findings of Lightman (1982) have been confirmed independently by Svensson (1982) and Svensson (1984). When these results are applied to flows in accretion disks, the role of the temperature in the existence of equilibrium production–annihilation is now played by the accretion rate. If flows in accretion disks have conditions such that the dynamical time is shorter than the characteristic time for the ions to transfer energy to the electrons, radiative cooling will be inefficient and most of the energy will be advected with the ions. A two-temperature disk will follow, with both $T_i$ and $T_e$ close to their virial values, with $T_i \gg T_e \approx m_e c^2$, close to the inner radius. One should then expect a significant pair production in these flows. Kusunose & Takahara (1988) have shown that, for accretion rates below a critical one, there exist two branches of solution under equilibrium production–annihilation: one of them, the high pair, has $z > 1$, and the other, the low pair solution, has $z < 1$, where $z$ is the ratio pair density over proton density. For accretion rates greater than the critical one, no equilibrium solution is possible. An interesting result from these works is that the existence of a critical accretion rate is independent of pairs being created by particle–particle, particle–photon, or photon–photon processes, as long as the photons are internally produced. When photons are externally produced, the rate of pair creation is no longer nonlinear in the particle density. As far as important issues, such as the number of solutions accessible to the disk for given physical conditions, the role of the accretion rate and viscosity in determining steady-state solutions for accretion disks with pairs produced by internal photons, are concerned, reasonable agreement is achieved by several authors. However, if one is concerned with pair production by external photons, the situation is less clear, and the results are even discrepant. Tritz & Tsuruta (1989), White and Lightman (1989), and Lightman (1982) obtain rather similar results for disks with pairs produced by external soft photons. According to them, pairs produced in that way do not substantially modify the disk, the density of pairs being negligibly small. Since they failed to find a critical accretion rate, pair production–annihilation equilibrium is always possible. Their solutions are not self-consistent in the sense that the ion temperature exceeds its virial value and the disk scale height is larger than the radial distance. Kusunose & Takahara (1990), on the other hand, relaxing the imposition of setting the $y$ Kompaneetz parameter equal to 1, have studied disks with pair production by external soft photons, with
different soft photons input. They found a critical accretion rate and pair densities smaller than the proton density. The critical accretion rate is proportional to the viscosity parameter and depends on the soft photon energy. They also have found a unique solution to the disk equations. However, their result is only numerical and they have exploited only a small range for the soft photon energy, and they have not included advection in their energy equation. The effect of advection was later included by Kusunose & Mineshige (1996). They, however, considered the soft photon energy, and they have not included advection in only numerical and they have exploited only a small range for depends on the soft photon energy. They also have found a
cal accretion rate is proportional to the viscosity parameter and pair densities smaller than the proton density. The criti-

No. 2, 2009 DOUBLE NATURED SOLUTIONS OF COMPTONIZED ACCRETION DISKS 1087

1991; Niedzwiecki et al. 1995). This, besides being suggestive the interstellar medium, close to some sources, there are ev-

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tual soft photons and internal bremsstrahlung photons. They have included a term of energy cooling due to photon escape, but they have not included advection. In recent years, a large amount of observational data on compact X-ray and γ-ray sources are highly indicative of electron temperature close to the virial value and, in the interstellar medium, close to some sources, there are evidences of pair annihilation features (Bjornsson & Svensson 1991; Niedzwiecki et al. 1995). This, besides being suggestive of the presence of pairs, implies not only the importance of advective cooling, due to the increase of the scale height as compared to the radial distance, but as well that the inner region, where the disk has thickened, will be subject to a large amount of soft photons, irradiated from the outer disk. If the picture we have about these sources, surrounded by accretion disks, is correct, the irradiation of the inner region is unavoidable. Moreover, it is interesting to remind that pair processes can lead to nonstationary behavior of the accretion flow, giving rise to strong X-ray flare activity (Moskalik & Sikora 1980; Misra & Melia 1995; Ghisellini & Celotti 1999). Therefore, it will be very useful to revisit disks with pairs produced by external soft photons. We, then, propose, in this paper, to undertake this task. Our special emphasis will be the search for

1. multiple solutions for soft photon Comptonized two-
temperature accretion disks, with photons supplied by an external source;
2. critical values for both the accretion rate and the viscosity parameter;
3. conditions under which the solutions are physically consist-
tent;
4. the maximum electron temperature in the disk;
5. conditions for the instability against perturbations in the
electron temperature and in the density of pairs.

The outline of this paper is as follows. In Section 2, we present our model of accretion disk, together with some assumptions. In Section 3, we explain the units and obtain the disk equations. In Section 4, we obtain the solution to the disk equations. In Section 5, we obtain the critical parameters. In Section 6, we obtain the maximum electron temperature in the disk. In Section 7, we analyze stability of the disk against perturbations in the electron temperature and in the density of pairs. In Section 8, the conclusions are presented.

2. ABOUT THE MODEL

The model of accretion disk we shall be considering is a soft photon Comptonized two-temperature accretion disk, with pair production by photon–photon interaction. The photons are externally produced and, after impinging the disk, they are upscattered in energy. The region we are most interested is the innermost one, where the disk has thinned. Most of the standard assumptions will be kept: Keplerian velocity, stress tensor \( \approx \alpha P \), with \( \alpha \) being the viscosity parameter, and \( P \) the pressure, hydrostatic equilibrium in \( z \)-direction, particles obey a Maxwell–Boltzmann distribution. The energy transport will be radiative in \( z \)-direction, perpendicular to the plane of the disk, and advective in the radial direction. The advection of energy will be treated locally. The coupling between protons and electrons will be only through Coulomb collisional energy exchange. For the radiation field, we shall follow the treatment of Svensson (1984) and Zdziarski (1985), and assume it may be represented by an inverse power law with an exponential cutoff at \( KT_e \) plus a Wien bump. For the lepton energy equation we shall follow the Comptonization treatment as given by Zdziarski (1985), which is equivalent to obtain a relation between the spectral index \( \gamma \), the electron temperature, and the probability of a power-law photon into the region \( x = \frac{h}{KT_e} \simeq 1 \), where the Wien spectrum is formed. As usual, it is assumed that heating of the disk by viscous processes locally balances cooling.

3. DISK EQUATIONS

Throughout this article we shall adopt the following set of units: radial distance and the disk scale height are expressed in units of \( R_S = \frac{2GM}{c^2} \), where \( R_S \) is the Schwarzschild radius, electron temperature \( \theta_e \), and proton temperature \( \theta_p \) in units of \( m_e c^2 \), \( M \) is the mass of the central object in units of 10 solar masses, \( \dot{m} \) is the accretion rate in units of \( \frac{L_E}{c^2} \), where \( L_E \) is the Eddington luminosity, \( z \), the pair density, will be expressed in units of the electron number density \( N_e \). In the following, we shall adopt the same procedure of Bjornsson & Svensson (1992) and Bjornsson et al. (1996), and employ, as much as possible, as variables, the pressure \( P \) in units of rest-
mass energy density, the compactness parameter \( \ell \), or radiative flux in units of \( \frac{4\pi}{3} \frac{m_e c^2}{\sigma_T n_e H} \), and the Thomson scattering depth, \( \tau_p = \sigma_T N_e H \), where \( \sigma_T \) is the Thomson cross section for electron scattering, and \( H \) is the disk scale height. Using these variables, the disk structure equations, hydrostatic equilibrium, conservation of angular momentum, and energy balance are written, respectively, as

\[
P = \frac{1}{2} \frac{H^2}{r^5}, \tag{1}
\]

\[
P = \frac{1}{2^{1.5}} \alpha \tau_p \dot{m} r^{-1.5} S(r), \tag{2}
\]

\[
\dot{\ell} = \frac{\pi m_p}{2 m_e} A r^{-2} \left( \frac{H}{r} \right) S(r), \tag{3}
\]

where

\[
S(r) = 1 - 1.732 r^{-0.5}, \tag{4}
\]

under the assumption that \( R_1 = 3R_S \), the inner radius being \( R_1 \). Now, defining \( \eta \) in the same way as Bjornsson et al. (1996), i.e.,

\[
\eta = \dot{m} r^{1.5} S(r), \tag{5}
\]

and substituting the scale height by the pressure, we obtain

\[
P \tau_p = 2^{-1.5} \frac{\eta}{\alpha} \tag{6}
\]

and

\[
\dot{\ell} = \frac{\pi m_p}{\sqrt{2} m_e} A \eta r^{0.5}, \tag{7}
\]
where

\[ A = 1 - \frac{Q_{\text{adv}}}{Q_+}, \]  

(8)

\[ Q_+, \text{ and } Q_{\text{adv}} \text{ being, respectively, the flux of heat generated by} \]

viscous processes and the advective flux. For the advective cooling, we shall use an approximate local expression (Abramowicz et al. 1995),

\[ Q_{\text{adv}} = Q_+ \left( \frac{H}{r} \right)^2. \]  

(9)

Therefore,

\[ A = 1 - 2 P r. \]  

(10)

It should be remarked that Equation (7) is equivalent to

\[ F_r = Q_+ - Q_{\text{adv}}, \]  

(11)

where \( F_r \) is the radiative cooling. In steady state, the leptons emit exactly the energy they receive from the protons through Coulomb collisional energy exchange, which may be written as given by Stepney & Guilbert (1983), approximated to reproduce the correct behavior in the limit of low and high temperature,

\[ F_r = 4.54 \times 10^{26} \frac{\tau_p^2}{H \theta_e^{1.5}} (\theta_p - \theta_e)(1 + 2z)(1 + \theta_e^{1/2}). \]  

(12)

The energy equation for electrons and pairs comes from the treatment of Comptonization, as given Zdziarski (1985), and it reads

\[ \tau = (1 + 2z) \tau_p = \left( \frac{1 + \theta_e^{0.5}}{\theta_e^{0.5}} \right) \left( 1 + 12 \theta_e^2 \right). \]  

(13)

In writing Equation (13), we have assumed spectral index \( = 1 \), which is not very far from observed spectral indexes in a large number of X-ray and \( \gamma \)-ray sources. However, it should be remarked that the above expression only reproduces the correct spectral index, given by \( s = -\frac{2}{3} \), where the amplification factor is \( a = 1 + 4 \theta_e + 16 \theta_e^2 \), for \( \theta_e \gg 1 \). Substituting Equation (12) into Equation (11), we obtain from the definition of the compactness parameter

\[ \ell = \frac{4 \pi}{3} \frac{\sigma_T H}{m_e c^3} F_r \]  

(14)

or

\[ \ell = 0.0504 \tau_p (\theta_p - \theta_e) \frac{(1 + \theta_e^{0.5})^2}{\theta_e^2 (1 + 12 \theta_e^2)}. \]  

(15)

Now, writing the equation of state for a gas composed of protons, electrons, pairs, and radiation, i.e.,

\[ P = \frac{m_e}{m_p} \left( \theta_p + (1 + 2z) \theta_e + \ell \frac{4 \pi}{(1 + \tau_W)} \right), \]  

(16)

where \( m_e, m_p \) are, respectively, electron and proton masses. \( \tau_W \) is the Wien-averaged scattering optical depth (Svensson 1984), and it reads

\[ \tau_W = \frac{G(\theta_e)}{\theta_e^{0.5} (1 + 12 \theta_e^2)}. \]  

(17)

where

\[ G(\theta_e) = (1 + \theta_e^{0.5})(1 + 5 \theta_e + 0.4 \theta_e^2), \quad 1 \geq \theta_e \]

\[ = \frac{3}{16} \frac{(\ln(1.12 \theta_e) + 0.75)}{\theta_e^2 (1 + \theta_e)}, \quad 1 \leq \theta_e, \]

Before we write the equation for pair production–annihilation equilibrium, we must talk about photon balance, from which we can have the radiation field to obtain the rate of pair production by photon–photon interactions. Following Svensson (1984), we treat the radiation field as

\[ N(x) = \frac{e^{-x}}{2} \left( N_p x^2 + N_W x^2 \right), \]  

(18)

where we have specialized for an spectral index of 1. The ratio of intensities is given by Sunyaev & Titarchuk (1980), i.e.,

\[ \frac{N_W}{N_p} = \frac{\Gamma(1)}{\Gamma(5)} P_{\text{sct}}, \]  

(19)

where \( \Gamma \) is the Gamma function, \( P_{\text{sct}} \) is the probability of scattering, and \( N_p \) is related to the soft input \( N_{\text{soft}} \) by conservation of the total number of photons. If \( N_{\text{soft}} \) is the rate of soft photon creation, and we have equilibrium between input of soft photons and output of photons from the disk,

\[ \dot{N}_\gamma = \dot{N}_{\text{soft}} \]  

(20)

and

\[ N_{\gamma} = \frac{F_r}{m_e c^3 \theta_e} (1 + \tau). \]  

(21)

Since the photons escape from the disk with an energy \( m_e c^2 \theta_e \),

\[ N_{\gamma} = \frac{F_r}{m_e c^3 \theta_e} (1 + \tau). \]  

(22)

However,

\[ N_{\gamma} = \int_{x_0}^{\infty} N(x) dx, \]  

(23)

where \( x_0 \) is the soft photon energy in units of \( k T_e \). Therefore,

\[ \frac{N_p}{2} \left( \int_{x_0}^{\infty} x^{-2} e^{-x} dx + 2 \frac{\Gamma(1)}{\Gamma(5)} \tau \right) = \frac{F_r}{m_e c^3 \theta_e} (1 + \tau), \]  

(24)

where we have used (Zdziarski 1985),

\[ P_{\text{sct}} = \tau. \]  

(25)

Now, using Equations (13), (14), (21), and (24), we may write

\[ N_p = \frac{3}{2 \pi \sigma_T H} \left( \int_{x_0}^{\infty} x^{-2} e^{-x} dx + 0.258 \tau \right) \left( \frac{4 \pi}{1 + \tau_W} \right)^{-1} \ell (1 + \tau), \]  

(26)

which may be used to write the rate of pair production (Lightman 1982; Svensson 1982),

\[ \dot{n}_+ = \frac{3}{8} c \sigma_T \theta_e e^2 N_{\gamma}^2 \left( x_1^{-1} - x_2^{-1} \ln \left( 4 \theta_e^4 x_1 x_2 \right) \right), \]  

(27)

where

\[ \left( x_1^{-1} - x_2^{-1} \ln \left( 4 \theta_e^4 x_1 x_2 \right) \right) \]

\[ = \int_{x_0}^{\infty} x_1^{-1} N(x_1) dx_1 \int_{x_0}^{\infty} x_2^{-1} N(x_2) \ln \left( 4 \theta_e^4 x_1 x_2 \right) dx_2, \]  

(28)

with \( x_0 = \frac{1}{x_1 x_2} \). If we now write the pair annihilation rate as (Lightman 1982; Svensson 1982),

\[ \dot{n}_A = \frac{3}{16} \frac{c \sigma_T \theta_e e^2 N_{\gamma}^2 e^2 (1 + z) \left( \ln(1.12 \theta_e) + 1.3 \right)}{2 \theta_e + (\ln(1.12 \theta_e) + 1.3)}, \]  

(29)
and assume equilibrium between production and annihilation, we obtain, taking only leading terms in the double integral in Equation (28),

\[ \frac{9}{4 \pi^2} (1 + \tau)^2 \left( \frac{\ell}{\theta_e} \right)^2 f(x_0, \theta_e) = B(\theta_e) \left( \tau^2 - \tau_p^2 \right), \tag{30} \]

where

\[ B(\theta_e) = \frac{(\ln(1.12 \theta_e + 1.3))}{2 \theta_e + (\ln(1.12 \theta_e + 1.3))}, \tag{31} \]

and

\[ f(x_0, \theta_e) = 0.693 \left( \int_{x_0}^{\infty} X^{-2} e^{-x} dx + 0.258 \tau \right)^{-2} e^{-\frac{\tau}{\theta_e}} \times (\theta_e^5 + (0.128 \tau^2) \theta_e^{-1} + 0.128 \tau (\theta_e + \theta_e^3)). \tag{32} \]

To obtain Equation (30), we have assumed only photon–photon pair production. Particle–particle production of pairs, despite being a second order process in the fine structure constant, dominates production whenever \( \tau_{\text{opt}} \ll 1 \). This is due to the fact that the rate of pair production is quadratic in the density of particles, and photon–photon pair production is quadratic in the density of photons, Equation (27), which is proportional to the radiation field, or to the compactness parameter. As the scattering depth diminishes, the radiation field gets too diluted, which explains the dominance of particle–particle processes when the photons are internally produced. We have assumed that the external soft photon source is so copious, being the main responsible for the radiation field, producing a density of photons much greater than the density of particles. This assumption allows us to neglect particle–particle interactions.

4. THE NUMBER OF CONSISTENT SOLUTIONS TO THE DISK EQUATIONS

Using Equations (5), (6), (14), and (15), we finally reduce our system of equations to

\[
\frac{m_p}{\sqrt{32} m_e} A \eta P^{0.5} \left( (1 + \tau_W)(1 + \theta_e^{0.5})^2 + 249.33 \theta_e^2 (1 + 12 \theta_e^2) \right) \]

\[
- \left( \frac{(1 + \theta_e^{0.5})^2}{(1 + 12 \theta_e^2)} \right) \left( 1819.2 P \tau_p - \tau_p \theta_e \right) (1 + 12 \theta_e^2) \]

\[
- \theta_e^{0.5} (1 + \theta_e^{0.5}) = 0 \tag{33} \]

and

\[
\frac{9}{8} \left( \frac{m_p}{m_e} \right)^2 (1 + \tau)^2 \frac{A^2 \eta^2 P}{\theta_e^2} f(x_0, \theta_e) \]

\[ = B(\theta_e) \left( \tau^2 - \frac{1}{8 P^2} \left( \frac{\eta}{\alpha} \right)^2 \right), \tag{34} \]

where \( A, \eta, \tau, f(x_0, \theta_e), B(\theta_e), \tau_p \) are given, respectively, by Equations (5), (6), (10), (13), (31), and (32). For given \( \eta, \alpha, x_0, \theta_e, \) this is a system of two-coupled equations on the variables \( P \) and \( \theta_e \). We now solve this system of equations specializing for Cygnus X-1, for which we assume canonical values of \( M = 10 M_\odot \) and \( m = 6.52 \times 10^{-2} \) (Liang & Nolan 1984).

The solutions to Equations (33) and (34) are displayed in Figures 1–4. The first thing to be remarked is the existence of two solutions. In Figure 1, we have \( \theta_e \), the electron temperature, along the radial distance, for three different set of parameters. It is not hard to see that, for these parameters, the lower solution decreases as we go close to the hole. The upper solutions have a little decrease as we come from the outer region, and then start to increase at about \( r = 10 \), reaching their highest value at \( r = 5 \). Inspecting the lower solution, we see that the greater \( \alpha \), the lower \( \theta_e \). Concerning the accretion rate, the greater \( m \), the higher \( \theta_e \), for \( r \geq 15 \); for \( r \leq 15 \), this tendency is inverted. As far as the upper solutions are concerned, we see the larger \( \alpha \), the greater \( \theta_e \), and the larger \( m \), the smaller \( \theta_e \). In Figure 2, the Thomson scattering depth \( \tau_p \), for the lower branches, increases a little bit as we come from the outer radius. Then, it starts to decrease at about \( r = 15 \). The upper solutions increase monotonically as we approach the inner radius. The lower branch solution decreases with increasing \( \alpha \), and increases with increasing \( m \). The upper branch solution increases with increasing \( \alpha \). As a general trend, it decreases as \( m \) increases. If \( \alpha \) is kept constant, it increases with \( m \), for \( r \approx 10–15 \). In Figure 3, the proton temperature shows a monotonically increasing behavior, for both branches, as we come from the outer region. It also shows, in the lower branches, \( \theta_p \), increasing with \( m \), and decreasing with \( \alpha \). This behavior is inverted in the upper branches. In Figure 4, the pair density exhibits a monotonically increasing behavior as we come from the outer region. It also shows, in the lower branches, a lack of sensitivity with \( \alpha \), and an increasing behavior with \( m \). In the upper branch, \( \gamma \) grows with both \( \alpha \) and \( m \). In Figure 12, the stability \( y \) parameter indicates that, for the lower branch, the disk is more stable for increasing \( \alpha \) and decreasing \( m \).
Figure 2. \(\tau_p\), the Thomson scattering depth, along the radial distance. The upper and the lower solutions are represented by similar lines. For the continuous line (—), we have adopted \(\alpha = 0.1, x_0 = 0.01, \dot{\tau} = 0.065217\); for the dotted line (...), we have adopted \(\alpha = 0.1, x_0 = 0.01, \dot{\tau} = 0.13034\); for the dashed line (- - -), we have adopted \(\alpha = 0.2, x_0 = 0.01, \dot{\tau} = 0.065217\).

Figure 3. \(\theta_p\), the proton temperature, along the radial distance. The upper and the lower solutions are represented by similar lines. For the continuous line (—), we have adopted \(\alpha = 0.1, x_0 = 0.01, \dot{\tau} = 0.065217\); for the dotted line (...), we have adopted \(\alpha = 0.1, x_0 = 0.01, \dot{\tau} = 0.13034\); for the dashed line (- - -), we have adopted \(\alpha = 0.2, x_0 = 0.01, \dot{\tau} = 0.065217\). The continuous and dashed upper lines are coincident.

Figure 4. \(z\), the pair density, along the radial distance. The upper and the lower solutions are represented by similar lines. For the continuous line (—), we have adopted \(\alpha = 0.1, x_0 = 0.01, \dot{\tau} = 0.065217\); for the dotted line (...), we have adopted \(\alpha = 0.1, x_0 = 0.01, \dot{\tau} = 0.13034\); for the dashed line (- - -), we have adopted \(\alpha = 0.2, x_0 = 0.01, \dot{\tau} = 0.065217\). The continuous and dashed lower lines are coincident.

In the upper branch, the disk is more stable for decreasing \(\alpha\). As far as \(x_0\) is concerned, for the values used for \(\alpha\) and \(\dot{\tau}\), we have not found a noticeable dependence. Figures 5 and 6 show the thermal curves, exhibiting the relation between the accretion rate \(\dot{\tau}\) and the scattering depth \(\tau_p\) for \(\alpha = 0.1\) and \(\alpha = 0.3, x_0 = 0.01, \) at \(r = 5\), and \(r = 10\). The existence of a maximum and a minimum accretion rate is shown, such that, between them, the solution is double. One can easily see that the gap between them increases with \(\alpha\), and decreases with \(r\), which is essentially due to the increase of \(\dot{\tau}\), with \(\alpha\) and with decreasing \(r\). Concerning the nature of the solutions, i.e., the kind of disk, we see radiative cooling dominating in the lower
branch ($A \to 1$), and advective cooling dominating in the upper branch ($A \to 0$). It is interesting to stress that, as we go farther out, the solutions become more mixed, i.e., radiative efficiency decreases in the lower branch and increases in the upper branch. Advective efficiency is subject to opposite behavior.

In Figures 7 and 8, we have plotted $\dot{m}$ as a function of $A$, for $\alpha = 0.1$ and $\alpha = 0.3$, for $r = 5$ and $r = 10$, respectively. Clearly, it is shown that the range in the accretion rate increases with $\alpha$ and decreases with $r$. No matter, $\alpha$ or $r$, this gap is larger for $A \leq 0.5$, meaning that $\dot{m}_c$ lies in the region where advection is more important, independent of $\alpha$ or $r$. However, it seems that, for very small $\dot{m}$, we obtain both $A \to 0$, and $A \to 1$. So, it seems we cannot classify the flow as advective or radiative, in terms of the accretion rate.

5. CRITICAL CONDITIONS

We now look for critical conditions for the disk, i.e., parameters values which define the transition from a steady state to a time dependent solution. In order to do so, we start with Equation (33) and treat it as a function of $P$, assuming all other variables as parameters. Besides, a cursory analysis reveals that
the term $\tau_r \theta_e (1 + 12 \theta_e^2)$ is negligible in most situations. We, then, write Equation (33) as

$$3.22 \times 10^2 \eta A P^{0.5} = g_0,$$

where

$$g_0 = \frac{(643.18 \frac{\eta}{\alpha} (1 + 12 \theta_e^2) - \theta_e^{0.5} (1 + \theta_e^{0.5})}{(1 + \tau_W)(1 + \theta_e^{0.5})^2 + 249.33 \theta_e^2 (1 + 12 \theta_e^2)}.$$

The left-hand side of Equation (31) has a maximum at $P = \frac{1}{\beta r}$, which equals $87.8 \eta r^{-0.5}$. Then, in order to have solution, this maximum should be greater or equal to the right-hand side of Equation (31), i.e.,

$$87.8 \eta r^{-0.5} \geq g_0.$$  

(37)

Since, physically, the left-hand side of Equation (37) is positive defined, we must have $g_0 \geq 0$, which implies

$$\eta \geq \frac{\theta_e^{0.5} (1 + \theta_e^{0.5}) \alpha}{(1 + 12 \theta_e^2) (643.18)},$$

(38)

defining the least value $\eta$ can assume in the disk. Now, if we write Equation (37) in a more detailed manner, we will have

$$(87.5 r^{-0.5} \mu_0 \alpha - (1 + 12 \theta_e^2) (643.18 - 12.12 r^{0.5} \theta_e)) \eta \geq -\theta_e^{0.5} (1 + \theta_e^{0.5}),$$

(39)

where

$$\mu_0 = \frac{(1 + 12 \theta_e^2)}{(1 + \theta_e^{0.5})^2 (1 + \tau_W)(1 + \theta_e^{0.5})^2 + 249.33 \theta_e^2 (1 + 12 \theta_e^2)}.$$  

(40)

Then, criticality is determined by the value of $\alpha$: if $\alpha$ is greater than the value of $\alpha_c$, defined below,

$$\alpha_c = \frac{(1 + 12 \theta_e^2) (643.18 - 2.12 r^{0.5} \theta_e)}{87.5 r^{-0.5} \mu_0},$$

(41)

there is no critical condition, and the system of equations for the disk will admit two steady solutions, as long as $\eta$ satisfies Equation (38). If $\alpha \leq \alpha_c$, we must have

$$\eta \leq \frac{\theta_e^{0.5} (1 + \theta_e^{0.5}) \alpha}{(1 + 12 \theta_e^2) (643.18 - 2.12 r^{0.5} \theta_e) - 87.5 r^{-0.5} \mu_0 \alpha}.$$

(42)

which defines the critical value of $\eta$, i.e., the largest value $\eta$ can attain, resulting in just one steady-state solution for the disk equations. If $\alpha < \alpha_c$, and

$$\eta_1 < \eta < \eta_c,$$

(43)

where $\eta_1$ is defined by Equation (38), and $\eta_c$ is defined by the equality sign in Equation (42), the system of equations for the disk admits two steady-state solutions. Clearly, in terms of $\dot{m},$ Equation (43) may be written as

$$\dot{m}_1 \leq \dot{m} \leq \dot{m}_c,$$

(44)

where

$$\dot{m}_1 = \frac{\theta_e^{0.5} (1 + \theta_e^{0.5}) \alpha r^{1.5}}{S(r)(1 + 12 \theta_e^2) (643.18 - 2.12 r^{0.5} \theta_e)}$$

(45)

and

$$\dot{m}_c = \frac{\theta_e^{0.5} (1 + \theta_e^{0.5}) \alpha r^{1.5}}{S(r)((1 + 12 \theta_e^2) (643.18 - 2.12 r^{0.5} \theta_e) - 87.5 r^{-0.5} \mu_0 \alpha)}.$$  

(46)

We now, assuming $\alpha < \alpha_c$, and using the critical conditions, i.e., $P = \frac{1}{\beta r}, \ A = \frac{r}{2},$ Equation (37), $\eta = \eta_c$ into Equation (34), obtain

$$m_p^2 \eta_c^2 f(x_0, \theta_e) (1 + r)^2 = 12 m_r^2 r \theta_e^2 B(\theta_e) \times \left( r^2 - 4.5 \left( \frac{\eta_c}{\alpha_c} \right)^2 \right),$$

(47)

with $r$ given by Equation (13). Clearly, Equation (47) defines a critical local $\theta_e$, for given $(r, \alpha, x_0)$.

Figure 9 depicts the critical local temperature, solution to Equation (47), for several values of $\alpha$, for $r = 5$ and $x_0 = 0.1$. Figure 10 shows $\alpha_c$, the critical viscosity parameter, as a function of the temperature, for $r = 5$ and $x_0 = 0.1$. Figure 11 shows $\eta_1$ and $\eta_c$, for $r = 5$ and $x_0 = 0.1$, as a function of $\theta_e$. Let us clear things a little bit. Let us take our solution for $\theta_e$, Figure 1, for $\alpha = 0.1$, $x_0 = 0.01$, and $\dot{m} = 0.065217$. For $5 \leq r \leq 20$, $\alpha \geq \alpha_c$, $\theta \geq \theta_c$, where $\theta_c$ is the critical local temperature. Therefore, in that region, there is no criticality, and we find solutions, no matter what the value of $\eta$ is. For $r \geq 20$, $\alpha \leq \alpha_c,$
\[ \theta_e \leq \theta_c. \] However, we found \( \eta \leq \eta_c \), and we were able to find two steady-state solutions. For \( r \geq 20, \alpha \leq \alpha_c \), and \( \eta \) decreases faster than \( \eta_c \). Our results differ from others in the literature in the sense that \( \alpha_c \) depends on \( r, \theta_e \), and \( \dot{m}_c \) depends on \( r, \theta_e \), and \( \alpha \). This implies that for any reasonable \( \alpha \) we may find a critical temperature such that \( \alpha = \alpha_c(r, \theta_e) \). For \( \theta_e > \theta_c, \alpha > \alpha_c \), and for \( \theta_e < \theta_c, \alpha < \alpha_c \). This may be easily seen in Figures 9 and 10.

Let us consider \( r = 7 \) and \( \alpha = 0.2 \), then \( \alpha = 0.2 \) becomes critical for \( \theta_e = 0.62 \). Therefore, at \( r = 7 \), for \( \alpha = 0.2 \) and \( \theta_e > 0.62 \), the system of equations for the disk admits two steady solutions. For \( \alpha = 0.2 \) and \( \theta_e \leq 0.62 \), the system of equations for the disk admits solutions if and only if

\[ 7.72 \times 10^{-5} \leq \eta \leq 1.57 \times 10^{-2} \quad (48) \]

or equivalently,

\[ 4.14 \times 10^{-3} \leq \dot{m} \leq 0.842. \quad (49) \]

Looking at Figures 5 and 6, we obtain

\[ \dot{m}_c \approx 0.340, \dot{m}_1 \approx 2.38 \times 10^{-3} \rightarrow \alpha = 0.1, r = 5, \]
\[ \dot{m}_e \approx 1.939, \dot{m}_1 \approx 9.14 \times 10^{-3} \rightarrow \alpha = 0.3, r = 5, \]
\[ \dot{m}_e \approx 0.190, \dot{m}_1 \approx 3.31 \times 10^{-3} \rightarrow \alpha = 0.1, r = 10, \]
\[ \dot{m}_e \approx 1.1, \dot{m}_1 \approx 1.3 \times 10^{-2} \rightarrow \alpha = 0.3, r = 10. \]

Now, use of Equations (38), (42), and (47) yields
\[ \dot{m}_e \approx 0.200, \dot{m}_1 \approx 1.97 \times 10^{-3} \rightarrow \alpha = 0.1, r = 5, \]
\[ \dot{m}_e \approx 1.140, \dot{m}_1 \approx 7.87 \times 10^{-3} \rightarrow \alpha = 0.3, r = 5, \]
\[ \dot{m}_e \approx 0.114, \dot{m}_1 \approx 2.53 \times 10^{-3} \rightarrow \alpha = 0.1, r = 10, \]
\[ \dot{m}_e \approx 0.647, \dot{m}_1 \approx 1.03 \times 10^{-2} \rightarrow \alpha = 0.3, r = 10. \]

These results are about 41% lesser for \( \dot{m}_e \), and at the most 30% for \( \dot{m}_1 \) as compared to the previous values, obtained directly from the thermal curves.

It should be remarked that production of pairs is not rampant increased when the disk is fed with a critical accretion rate, as it happens for disks with pairs created by internally produced photons.

6. ON THE MAXIMUM ELECTRON TEMPERATURE

The more efficiently the disk emits, the less is its temperature. Conversely, the higher is the temperature, the less is the radiative flux. In our formulation, the radiative efficiency is given by \( \alpha \), which, of course, satisfies \( 0 \leq A \leq 1 \). If we take the least radiative efficiency, i.e., \( A = 0 \), we are obviously taking the maximum electron temperature. Setting \( A = 0 \) in Equations (29) and (30), we obtain, respectively,
\[ \tau_p (1819.2 P - \theta_e) (1 + 12 \theta_e^2) - \theta_e^{0.5} (1 + \theta_e^{0.5}) = 0 \] (50)

and
\[ \tau^2 = \frac{1}{8} \frac{\eta}{\alpha} = 0. \] (51)

Using Equations (5), (9), and (12), we can recast Equations (50) and (51) into a more suitable form, i.e.,
\[ \theta_e = \frac{1819.2}{4 \tau} \] (52)

and
\[ \frac{1}{\theta_e^{0.5}} \left( \frac{1 + \theta_e^{0.5}}{1 + 12 \theta_e^2} \right) = r \frac{\eta}{\alpha}. \] (53)

Solving for \( r \) in Equation (52) and substituting into Equation (53), we obtain, using the definition of \( \eta \), Equation (4),
\[ \frac{\alpha}{m} (1 + \theta_e^{0.5}) - (1 + 12 \theta_e^2) (0.03316 \theta_e - 0.00259 \theta_e^{1.5}) = 0. \] (54)

For \( \alpha = 0.1 \) and \( m = 6.52 \times 10^{-2} \), yields \( \theta_e = 2.246 \) and \( \theta_e = 148.86 \) which correspond, respectively, to \( r = 202.46 \) and \( r = 2.77 \). Clearly, \( \theta_e = 148.86 \) is not acceptable, because it would imply that instability only occurs inside the inner radius, \( r_1 = 3 \), in the assumption of a Schwarzschild black hole. We, then, interpret \( r = 202.46 \approx 67.5 r_1 \) as the onset of a thermal instability which drives the disk to a two-temperature regime. When the instability starts, the disk is suddenly heated to a high temperature, with protons and electrons in thermal equilibrium. Soon after, there is an electron and proton temperature decoupling, which develops all the way down to the hole. The electron temperature found that this way is very close to the electron temperature for the upper branch, and looking at the behavior of \( \theta_e \) along \( r \), Figure 1, it seems that the maximum electron temperature is obtained at the onset of the instability.

7. STABILITY

We shall examine the stability of the two-temperature external soft photon Comptonized accretion disk against perturbations in the electron temperature and pair density. Before we write the equations to study stability, it will be very profitable if we recall some aspects related to Comptonization. If \( F_0(v) \) is the soft photons spectral flux, then \( F(v) \), the resulting spectral flux of upscattered photons, will be
\[ F(v) = \int_0^\infty n(v, v_1) F_0(v_1) dv_1, \] (55)

where \( n(v, v_1) \) is the energy amplification factor for a photon with energy \( v_1 \) that is upscattered to an energy \( v \). \( n(v, v_1) = n(x, \theta_e) \) is given by Dermer et al. (1991). \( x \) and \( \theta_e \) are, respectively, the initial and final energies of the photon. According to Dermer et al. (1991),
\[ n(x, \theta_e) = 1 + \frac{p(a - 1)}{1 - pa} \left[ 1 - \left( \frac{x}{3 \theta_e} \right)^{-1} e^{\frac{\eta \theta}{\alpha \theta_e}} \right]. \] (56)

where
\[ p = 1 - \exp -\tau \] (57)
is the scattering probability, and
\[ a = 1 + 4 \theta + 16 \theta^2 \] (58)
is the fractional energy change of the photon after one scattering. Our main concern will be the limiting form of \( n(x, \theta_e) \), when \( \tau \ll 1, a \gg 1 \), with \( a \gg 1 \). Under these conditions,
\[ n(x, \theta_e) \approx \frac{3 \theta}{x}. \] (59)

Therefore, the cooling due to the Comptonization will be
\[ F_c = \frac{8 \pi}{c^5} \left( 3 \theta_e \right) \int_0^\infty m e c^2 v^3 \left( \exp \frac{h v}{K T_0} - 1 \right)^{-1} d v \] (60)
or
\[ F_c = \sigma T_0^4 \left( \frac{T_e}{T_0} \right) \]. (61)

where we have associated a temperature \( T_0 \) for the soft photons, and \( \sigma \) is the Stefan–Boltzmann constant. We now write \( Q^* = M \Omega^2 S(r) \), in terms of \( \eta \), i.e.,
\[ Q^* = 7.782 \times 10^{35} \frac{\eta}{r_1^{1.5}}, \] (62)

and express the cooling due to Comptonization, normalized by the locally generated heat, i.e.,
\[ f_c = 7.573 \times 10^9 \frac{r_1^{1.5}}{\eta} \theta_0^3 \theta_e. \] (63)

Now, using Equations (11)–(15), we may write the heating of the leptons, through Coulomb collisional energy exchange, in units of \( Q^* \), as
\[ f_{eh} = \frac{1.485 (1 + \theta_e^{0.5})^2}{\alpha P^{1.5} \theta_e^2 (1 + 12 \theta_e^2)} I(P, \alpha, \eta, \theta_e). \] (64)
where

\[ I(P, \alpha, \eta, \theta_e) = P \left( \frac{m_p}{m_e} - 2^{1.5} \frac{\alpha (1 + \theta_e^{0.5}) \theta_e^{0.5}}{\eta (1 + 12 \theta_e^2)} \right) - \theta_e. \] (65)

We are, now, in a position to define \( F_{eb} \), a function that describes energy balance, i.e.,

\[ F_{eb} = f_{eb} - f_c, \] (66)

and, \( F_{pb} \), a function that describes pair production and annihilation balance. From Equation (30),

\[ F_{pb} = \frac{9}{4 \pi^2} (1 + \tau)^2 \frac{\ell^2}{\theta_e^2} f(x_0, \theta_e) - B(\theta_e) \left( \tau^2 - \tau_p^2 \right). \] (67)

Following a similar treatment as that given by Sikora & Zbyszewska (1986), we obtain

\[ \delta \theta_e = - \left( \frac{\partial F_{eb}}{\partial \tau} \right) \left( \frac{\partial F_{eb}}{\partial \theta_e} \right)^{-1} \delta \tau, \] (68)

where the subscript implies that the expression in parenthesis should be calculated at equilibrium parameters. Finally, perturbing \( F_{pb} \), given by Equation (67), and using Equation (68), we obtain

\[ \delta F_{pb} = \left[ - \frac{\partial F_{eb}}{\partial \tau} \frac{\partial \theta_e}{\partial \tau} \left( \frac{\partial F_{eb}}{\partial \theta_e} \right)^{-1} + \frac{\partial F_{pb}}{\partial \tau} \right] \delta \tau. \] (69)

Then, instability condition is given by

\[ \delta \tau \delta F_{pb} > 0, \] (70)

which is satisfied if the term in brackets is positive. Defining \( y \), a kind of stability parameter as

\[ y = \frac{\delta F_{pb}}{B(\theta_e)(\tau^2 - \tau_p^2)}, \] (71)

we obtain

\[ y = -2 \theta_e^{1.5} (\theta_p - \theta_e) (1 + \theta_e^{0.5}) \frac{X_0}{\tau X_1} + X_2, \] (72)

where

\[ X_0 = \frac{\partial}{\partial \theta_e} \ln \left( \frac{f(x_0, \tau, \theta_e)}{\theta_e^2 B(\theta_e)} \right), \] (73)

\[ X_1 = (\theta_p - \theta_e) (\theta_e - 3 \theta_e^{0.5} - 2 \frac{f_c}{f_{ep}} \theta_e^{0.5} (1 + \theta_e^{0.5})) \]
\[ - 2 \theta_e^{1.5} (1 + \theta_e^{0.5}), \] (74)

\[ X_2 = \frac{\partial}{\partial \tau} \ln \left( \frac{(1 + \tau^2) f(x_0, \tau, \theta_e)}{\tau^2 - \tau_p^2} \right). \] (75)

Before we talk about results for the stability, we should stress that Equation (59) was obtained under the assumptions \( p \approx \tau \), \( a \gg 1 \), \( p a \gg 1 \), and \( \ln \frac{\eta}{\alpha} \approx 0 \), i.e., a flat spectrum which is not consistent with our assumption \( s = -\frac{a}{\alpha} = 1 \). However, a cursory estimate leads to \( ap \approx 4 \) in the region of interest. This, together with \( s = 1 \), gives, keeping only the leading term,

\[ n(x, \theta_e) \simeq \left( \frac{3 \theta_e}{x} \right)^2. \] (76)

The main change implied by Equation (76) is an increase in \( f_c \) given by Equation (63). If we call the new one \( f_{nc} \), we will have

\[ f_{nc} \approx f_c \frac{3 \theta_e}{\theta_0}. \] (77)

This increase in the cooling leads to a decrease in the amplitude of temperature perturbations, as it may be seen in Equation (68), reinforcing tendency to stability.

Figure 12 depicts \( y \) along the disk, for various sets of parameters \((\alpha, x_0, \eta)\). It is shown that, for the chosen sets of parameters, the disk is stable against perturbations in \( \theta_e \) and \( z \).

8. CONCLUSIONS

The results we have obtained differ from those of White & Lightman (1989), Kusunose & Takahara (1990), and Kusunose & Mineshige (1996), mainly by the number of solutions to the disk equations and by the conditions under which the system reaches criticality. The critical viscosity parameter \( \alpha_c \), below which the system may develop time behavior, is a function of \( r \) and \( \theta_e \). For a given \( \alpha \), we may find a region in \( \theta_e \), where \( \alpha = \alpha_c \).
Even under this condition, the system may have two steady-state solutions, as long as the accretion rate satisfies $\dot{m}_1 \leq \dot{m} \leq \dot{m}_c$, where $\dot{m}_1$ and $\dot{m}_c$ are functions of $\alpha$, $r$, and $\theta_e$. To make these points more clear, let us take $\alpha = 0.2$ and $r = 7$. We, then, obtain $\alpha_c = 0.2$ for $\theta_e = 0.62$. For $\theta_e > 0.62$, $\alpha > \alpha_c$ and the system of equations admits two solutions no matter what the value of $\dot{m}$ is. For $\theta_e \leq 0.62$, $\alpha < \alpha_c$ and the system of equations admits solutions only if

$$4.14 \times 10^{-3} \leq \dot{m} \leq 0.842.$$  

In other words, for a given $\alpha$, there is a maximum accretion rate, above which there is no solution. The range of $\dot{m}$ for which there are solutions is $\alpha$ dependent. The solutions we have found yield quite different values for the physical variables in the upper and lower branches, except for the density of pairs which are, practically, the same in both branches. These solutions practically do not depend on the soft photon energy, and the maximum electron temperature in the disk is achieved for the lower branch. Close to the onset of the instability from which results the inner region. Besides, the density of pairs we have found is always much smaller than the proton density. Advection cooling tends to be more important in the upper branch, radiative cooling going in the opposite way, being more important in the lower branch. Exception made for $\theta_e$, the variation of the physical variables is rather small along the disk. Also, we have not found any abruptly increased pair production as we approach critical conditions.

As compared to White & Lightman (1989), our results reflect the fact that we have included advection in the energy equation and have adopted a different approach to the treatment of Comptonization. It seems that the inclusion of advection in the pair equation, as White & Lightman (1989) did, is a minor effect as compared to the inclusion of this term in the energy equation. We have included advection in a rather local manner (Abramowicz et al. 1995), explicitly dependent on $r$, and this has introduced an explicit $r$ dependence in our equations, and not only an implicit dependence through $\eta$. This somehow helps to explain our different result as compared to Kusunose & Mineshige (1996) results, besides the fact they have used different radiative cooling, and used another local approach to the advection treatment. Though Kusunose & Takahara (1990) have not included advection in their disk with pairs produced by soft external Comptonized photons, they obtain a rather similar result to the maximum accretion rate. Just for comparison, for $\alpha = 0.1$, and $r = 10$, Kusunose & Takahara (1990) and White & Lightman (1989) obtain, respectively, for $\dot{m}_c$, 0.29 and 0.55. Though these results are about three or five times bigger than ours, they are similar to Abramowicz et al. (1995) result, which is about 20% of ours. Their results are, however, very sensitive to the soft photon energy. Our results, qualitatively, are in agreement with those of Chen et al. (1995), who have analyzed solutions to the disk equations in terms of $\dot{m}$ and $\alpha$ and have found a critical $\alpha_c$, above which there is always at least one optically thin advection-dominated solution for a given accretion rate. According to them, for $\alpha = \alpha_c$, there is a maximum accretion rate, above which there are no steady-state solutions. It should be stressed that Chen et al. (1995) work predicts a local minimum to $\alpha_c$, which is $\alpha_c \approx 0.21$ at $r = 14$ (in their units, $r = 7R_g$), which does not exist in our formulation. Finally, concerning the behavior of the disk against perturbations in the electron temperature and in the density of pairs, we have found that the disk is always stable.

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