Reliability Assessment Method of Complex Electronic Equipment with Multivariate Competing Failure

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Abstract. This paper presents a generalized reliability assessment model for complex electronic equipment with multivariate competing failure. The key of multivariate competing failure model is separated into two parts, multi-degradation measure probability density function (MDM-PDF) and traumatic failure reliability function (TFRF) related to degradation level. Principal component analysis is used to solve the calculation problem of MDM-PDF; Logistic function and least square method are introduced to solve the problem of TFRF. Finally, a case study of sine-wave oscillation circuit is given to validate the effectiveness of the model.

Keywords: Reliability estimation; Multivariate competing failure; Degradation failure; Traumatic failure; Principal component analysis; Logistic function.

1. Introduction
Based on the failure mechanisms, product failure can be classified into degradation failure and traumatic failure, in which degradation means that the products will gradually lose their function and performance until the characteristic measurements reach a certain level, and traumatic failure means that products will suddenly failure during the working process. For most products, the two main failure modes exist simultaneously.

In Ref [1], the traumatic failure and the degradation failure were supposed to be independent of each other and they both fitted Weibull distribution. Zhao et al [2] considered the interaction between degradation and traumatic failure and proposed a competing failure model based on the known distribution of degradation. LIU Hongdou researched one reliability evaluation model based on multiple critical performance parameters of accelerated degradation testing Ref [3].

The above references discussed the competing failure problem from different views, and solved some practical problem. However, most of research has only considered single degradation measure for degradation failure. In practice, electronic system has multiple degradation variables for degradation failure, so it is necessary to simultaneously consider multiple degradation variables for competing failures.

2. Basic Assumptions and Multivariate Competing Failure Modeling

2.1. Basic Assumptions
In a reliability testing experiment, degradation variables can be detected for each sample unit at \( t_1, t_2, \ldots, t_p \). The objective is to estimate the reliability from the observed testing data. The general assumptions are following:
(1) Assume the vector \( \mathbf{y}(t) \) contains \( k \) degradation variables \( \{y_1(t), y_2(t), \ldots, y_k(t)\} \), and the standard definition of a degradation failure event is \( \bigcup_{j=1}^{k} \{y_j(t) > D_j\} \), where \( D_j \) is the critical level for the \( j \)-th degradation variable;

(2) The fault of electronics is also related to traumatic failure, and assume the product have \( l \) traumatic failure modes;

(3) Probability of traumatic failure is not only related to time, but also related to the degradation performance \( \mathbf{y} \) at time \( t \). For given \( \mathbf{y}(t) \), each traumatic failure is independent.

### 2.2. Traumatic Failure Modeling Corresponding to Degradation Variables

During the degradation testing, we can use a logistic function and composite exponential model to define the probability of a failure event.

(1) Logistic function

\[
P\{\text{failure} | \mathbf{y}\} = \frac{e^{\beta(\mathbf{y})}}{1 + e^{\beta(\mathbf{y})}} = \frac{\exp\left(\sum_{i=1}^{m} \beta_i y_i\right)}{1 + \exp\left(\sum_{i=1}^{m} \beta_i y_i\right)}
\]

Logistic function in Eq.1 is a standard statistical technique to predict the distribution of a binary variable given predictors of \( \mathbf{y} \). When given the sample data with pairs of the observed degradation variables and response value of whether a hard failure occurs, the coefficients \( \beta \) can be estimated in Eq.1 by using least square method (LSM). Logistic function is suitable for traumatic failure modeling when each variable \( y_i \) is independent, and composite negative exponential model is introduced to solve this problem.

(2) Composite Negative Exponential Model

In order to reflect the correlation of each degradation measure, polynomial is the simplest method to describe \( \beta(y) \). But it is more difficult relatively for parameter estimation of polynomial. In elementary function, exponential function can be used to describe polynomial \( \beta(y) \). So we introduce the negative exponential model to express \( \beta(y) \), that is

\[
\beta(y) = -e^{a(y)}
\]

Where, \( a(y) = a_0 + a_1 y_1 + a_2 y_2 + \ldots + a_q y_q \). The reason to choose negative exponential model in Eq.2 is that \( \beta(y) \in (-\infty, 0) \) and \( e^{a(y)} \in (0, 1) \), which is very suitable the requirement of reliability function of product. Based on these, this paper choose composite negative exponential model (Eq.3) to describe the relationship between multiple dependent degradation measure and traumatic failure event.

\[
P\{\text{failure} | \mathbf{y}\} = \exp\{-e^{a(\mathbf{y})}\}
\]

### 2.3. Multivariate Competing Failure Modeling

Suppose vector \( \mathbf{z}(t) \) contains \( q \) degradation variables \( \{z_1(t), z_2(t), \ldots, z_q(t)\} \), whose cumulative distribution function is \( G_s(\mathbf{z}, t) \) respectively and corresponding probability density function is \( g_s(\mathbf{z}, t) \). The relationship of \( G_s(\mathbf{z}, t) \) and \( g_s(\mathbf{z}, t) \) can be defined in Eq.4.

\[
g_s(\mathbf{z}, t) = \frac{\partial G_s(\mathbf{z}, t)}{\partial \mathbf{z}}
\]

Assume the \( \mathbf{z}(t) \) is monotonic increasing, \( T_s \) is the failure time for degradation failure mode and \( D_j \) is the critical failure value, so the failure probability at \( t \) can be defined in Eq.5.

\[
F_s(z, t) = P(T_s \leq t) = P(z(t) \geq D_j) = 1 - G_s(z, t)
\]
From the above analysis, the occurrence probability of traumatic failure is not only related to time, but also related to degradation measure $z(t)$. In this paper, $T_{hj}$ is defined as the failure time resulted from the $j$th ($j=1,2,...,N$) traumatic failure mode, which is also a random variable. And $\lambda_{hj}(t,z)$ is defined as the hazard rate function of $T_{hj}$. Then the reliability function of the $j$th traumatic failure mode can be obtained by Eq.6.

$$R_{hj}(t|z) = P(T_{hj} > t|z) = \exp\left\{-\int_0^t \lambda_{hj}(s,z) \, ds\right\}$$

The conditional probability density function of $T_{hj}$ can be expressed by Eq.7.

$$f_{hj}(t|z) = \lambda_{hj}(t,z) \exp\left\{-\int_0^t \lambda_{hj}(s,z) \, ds\right\}$$

So the conditional probability of no occurring traumatic failure is

$$R_s(t|z) = P(T_{hj} > t, j = 1,2,...,N|z) = \prod_{j=1}^N R_{hj}(t|z) = \exp\left\{-\sum_{j=1}^N \int_0^t \lambda_{hj}(s,z) \, ds\right\}$$

From above discuss, reliability function at $t$ can be obtained by

$$R(t) = P(T > t) = P(T_{1} > t, T_{h1} > t, T_{h2} > t, ..., T_{hN} > t) = \int_{-\infty}^{D_1} \sum_{j=1}^{N} R_{hj}(t|z) dG_s(z,t)$$

$$= \int_{-\infty}^{D_1} R_s(t|z) dG_s(z,t) = \int_{-\infty}^{D_1} e^{-\Lambda(t,z)} g_s(z,t) \, dz$$

Where, $\Lambda(t,z) = \sum_{j=1}^N \int_0^t \lambda_{hj}(s,z) \, ds$.

3. Reliability Assessment with Multivariate Competing Failure Based on Principal Component Analysis

3.1. Theory of Principal Component Analysis

Suppose that a product has $k$ performance degradation variables $y$, and $y$ is a random variable, and define $y=(y_1, y_2, ..., y_k)^T$, so its orthogonal transformation equation is described as follows.

$$Z = P^T y$$

Where $P$ is orthogonal matrix and each component of $Z$ is unrelated, dispersion of the first component of $Z$ is the greatest, the second is greater, and the others follow by analogy. PCA compress variable space and reduce variable dimension by retaining components with greater dispersion and ignoring the components with less dispersion. Generally, cumulative percent of variance (CPV) is used to ascertain the number of principal components, $q$. CPV can be expressed as follows:

$$CPV = \frac{\sum_{i=1}^q \lambda_i}{\sum_{j=1}^k \lambda_j}, \quad q=1,2,...,k$$

Where, $\lambda_i$ is the eigenvalue of covariance matrix $\Sigma$ for data matrix $y$, and $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_k \geq 0$. Generally, when CPV of the forward $q$ component is larger than 85%, we can recognize the $q$ principal components could represent the initial data, and the number of principal components is $q$.

3.2. Basic Flow of PCA Algorithm for Reliability Assessment

Suppose that, $k$ degradation variables can be got for each sample unit in a random sample of size $n$ at $t_1$, $t_2$, ..., $t_p$. for sample unit $i$ ($i=1,2,...,n$), the observed value at $t_s (s=1,2,...,p)$ is \( (y_{i1}(t_s), y_{i2}(t_s), ..., y_{ik}(t_s))^T \), which is a vector with $k$ dimensions and is configured as follows:
Define the critical failure level of \( m \) characteristic degradation variables as \( D_f=(D_{f1}, D_{f2}, \ldots, D_{fk}) \).

(1) Standardization processing of performance degradation data. In practice, the observed performance parameters have different dimensionality and format, so we need to standardize the multiple performance degradation variables. The general method of standardization processing is expressed as follows:

\[
y^*_j = \frac{y_j - \bar{y}_j}{\sqrt{\text{var}(y_j)}}, \quad i=1, 2, \ldots, n; \quad j=1, 2, \ldots, k
\]

(13)

Where \( \bar{y}_j \) and \( \text{var}(y_j) \) are respectively mean value and dispersion of \( j^{th} \) characteristic measure. The standardized data can eliminate the effect of dimensionality by using this method, but it makes the mean value equates 0 and dispersion equates 1 for each standardized parameters. In order to solve the problem, the initial data can be processed by Eq.14.

\[
y^*_j = \frac{y_j}{\bar{y}_j}, \quad i=1, 2, \ldots, n; \quad j=1, 2, \ldots, k
\]

(14)

Where \( \bar{y}_j = \frac{1}{n} \sum_{i=1}^{n} y_j \), \( j=1, 2, \ldots, k \). Ref [4] proved that the covariance matrix of standardized data not only eliminate the dimensionality problem of initial data, but also reflect the difference and effect level of each variable fully. Correspondingly, we can get the expression of standardization critical failure level by Eq.15.

\[
D_{\beta} = \frac{D_f}{\bar{y}_j}, \quad j=1, 2, \ldots, k
\]

(15)

(2) Calculation of correlation matrix. PCA involves determining the eigenvalues and eigenvectors of a covariance or correlation matrix of \( k \) characteristics variables. Since the degradation variables of a product are in widely different units, PCA is performed based on correlation matrix, say \( r \) observed in Ref [5] and is expressed as follows:

\[
r(t_s) = \frac{1}{k-1} Y^T(t_s) Y(t_s)
\]

(16)

(3) The decomposition formula of \( r \) is expressed as follows:

\[
r(t_s) = \frac{1}{k-1} Y^T(t_s) Y(t_s) = U \Lambda U^T
\]

(17)

Where \( \Lambda = \text{diag}(\lambda_1(t_s), \lambda_2(t_s), \ldots, \lambda_k(t_s)) \) is the characteristic matrix of correlation matrix \( r \), eigenvalues \( \lambda_1(t_s) \geq \lambda_2(t_s) \geq \ldots \geq \lambda_k(t_s) \); \( U=[u_1, u_2, \ldots, u_k] \) is the identity matrix, which are consist of orthonormal vector corresponding to \( \Lambda \).

(4) Calculation of dispersion contribution rate and CPV. Using the eigenvalues \( \lambda_1(t_s) \geq \lambda_2(t_s) \geq \ldots \geq \lambda_k(t_s) \), find \( q \) and make \( \frac{\sum_{i=1}^{q} \lambda_i(t_s)}{\sum_{i=1}^{k} \lambda_i(t_s)} \geq 0.85 \). Then, we can ascertain \( q \) principal component, and the dispersion contribution rate of \( i^{th} \) principal component is get form Eq.18.
\[
\alpha_i = \frac{\lambda_i(t_i)}{\sum_{i=1}^{k} \lambda_i(t_i)}, \quad i=1, 2, \ldots, k
\]

(18)

(5) Calculation of principal component values and its critical failure level. The formula of principal component can be expressed by Eq.19

\[
z_i(t_i) = Xu_i, \quad i=1, 2, \ldots, k
\]

(19)

And the critical failure level of principal component is

\[
D_{iq}(t_i) = D_iu_i, \quad i=1, 2, \ldots, k
\]

(20)

The above steps should be repeated at each observing time. When calculating principal component values and its critical failure levels at all observing time, we can get reliability function \(R_{zl}(l=1,2,\ldots,q)\) of each principal component by Eq.9. Based on these, we can get the formula of comprehensive reliability function \(R\) as follows:

\[
R = \sum_{l=1}^{q} \alpha_l R_{zl}
\]

(21)

3.3. Parameters Estimation of Traumatic Failure

Since each principal component degradation measure is independent, we choose logistic function (Eq.1) to describe the relationship between degradation measure and reliability.

\[
R\{t\mid z_i\} = 1 - P\{\text{failure}\mid z_i\} = 1 - \frac{e^{\beta z_i(1)}}{1 + e^{\beta z_i(1)}} = \frac{1}{1 + \exp(\beta_{0l} + \beta_{1l}z_i)}
\]

(22)

In this paper, distribution function method is used to calculate the failure probability. The sample size is \(n\), and failure time is arranged by the order \(t_1\leq t_2\leq \ldots \leq t_n\). Median of distribution function \(F_\alpha(t_i)\) for Each sample statistics \(t_i\) is called middle rank, which is the fractile quantile when \(F_\alpha(t_i)\) is 0.5. According to beta distribution function, we can get

\[
I_\rho(i, n-i+1) = 0.5
\]

(23)

Where \(I_\rho(i, n-i+1)\) is partial beta distribution. For given \(n\), fractile quantile \(p_i\) can be obtained by Eq.23. \(1-p_i\) is the reliability \(R_i\) related to degradation measure, then we can get the values of \((R_i, z_i)\). By Eq.22, we can get

\[
\frac{1}{R\{t\mid z_i\}} - 1 = \exp(\beta_{0l} + \beta_{1l}z_i)
\]

(24)

Transform Eq.24 by logarithmic computation, its linearization model can be obtained as follows:

\[
\log\left(\frac{1}{R\{t\mid z_i\}} - 1\right) = \beta_{0l} + \beta_{1l}z_i
\]

(25)

Using the values of \((R_i, z_i)\) and least square estimation method(LSEM), the value of equation coefficient \(\beta_{0l}\) and \(\beta_{1l}\) can be obtained.

4. Reliability Assessment with Multivariate Competing Failure Example

Fig.1 is the circuit schematic diagram of 20 kHz sine-wave oscillation circuit. Its standard signal model is \(y = A\sin(2\pi ft), \) where \(A \leq 8V, f \leq 21kHz\).The degradation failure and traumatic failure data were measured every 250 hours, until 5000 hours. Fig.2 shows the amplitude data and frequency data respectively, broken line indicates the traumatic failure time.
4.1. Principal Component Analysis

a) PCA of degradation data

Process the degradation data of amplitude and frequency by using PCA method introduced in section 3. Fig.3 respectively shows the principal component value $Z$ and critical failure level $D_f$ at each observed time; standardization characteristic vector $U$ and eigenvalue $\lambda$ are showed in Table 1 and Table 2.
Table 1. Standardization characteristic vector $U$ at each observed time

| time/h | $U$ | time/h | $U$ | time/h | $U$ |
|-------|-----|-------|-----|-------|-----|
| 0     | -0.7073 0.7069 | 1750   | -0.7073 0.7069 | 3500   | -0.7073 0.7069 |
| 250   | -0.7073 0.7069 | 2000   | -0.7073 0.7069 | 3750   | -0.7073 0.7069 |
| 500   | -0.7073 0.7069 | 2250   | -0.7073 0.7069 | 4000   | -0.7073 0.7069 |
| 750   | -0.7073 0.7069 | 2500   | -0.7073 0.7069 | 4250   | -0.7073 0.7069 |
| 1000  | -0.7073 0.7069 | 2750   | -0.7073 0.7069 | 4500   | -0.7073 0.7069 |
| 1250  | -0.7073 0.7069 | 3000   | -0.7073 0.7069 | 4750   | -0.7073 0.7069 |
| 1500  | -0.7073 0.7069 | 3250   | -0.7073 0.7069 | 5000   | -0.7073 0.7069 |

Table 2. Eigenvalue $\lambda$ of at each observed time

| time/h | $\lambda_1/10^{-3}$ | $\lambda_2/10^{-3}$ | time/h | $\lambda_1/10^{-3}$ | $\lambda_2/10^{-3}$ | time/h | $\lambda_1/10^{-3}$ | $\lambda_2/10^{-3}$ |
|--------|----------------------|----------------------|--------|----------------------|----------------------|--------|----------------------|----------------------|
| 0      | 2.2867 0.3172        | 1750 2.2867 0.2856   | 3500   | 2.2868 0.3156        |                      |
| 250    | 2.2867 0.3103        | 2000 2.2867 0.2824   | 3750   | 2.2868 0.2959        |                      |
| 500    | 2.2867 0.3018        | 2250 2.2867 0.2813   | 4000   | 2.2868 0.2851        |                      |
| 750    | 2.2867 0.3115        | 2500 2.2867 0.3095   | 4250   | 2.2868 0.2839        |                      |
| 1000   | 2.2866 0.2688        | 2750 2.2867 0.2882   | 4500   | 2.2868 0.2764        |                      |
| 1250   | 2.2866 0.2574        | 3000 2.2867 0.3089   | 4750   | 2.2868 0.2797        |                      |
| 1500   | 2.2866 0.2744        | 3250 2.2867 0.3103   | 5000   | 2.2868 0.2738        |                      |

From Fig.3, we can see that the variation of principal value at each observed time is not obvious; the changing of principal component critical failure value is monotonic. Besides, from Table 2, we know that CPV at each observed time is greater than 99.98%, so we choose the first principal component as the degradation measure of sine-wave oscillation circuit.

b) PCA of traumatic failure

The PCA results of traumatic failure time are shown in Table 3 and CPV at traumatic failure time is greater than enough 85%, so the first principal component is chosen as the principal component measure at traumatic failure time.

Table 3. PCA results at traumatic failure time

| failure time/h | $Z_1$ | $Z_2$ | $\lambda_1/10^{-3}$ | $\lambda_2/10^{-3}$ |
|----------------|-------|-------|----------------------|----------------------|
| 2875           | -1.4219 | -0.0104  | 2.2509                 | 0.2758               |
| 3125           | -1.4189 | -0.0131  | 2.2510                 | 0.2949               |
| 3625           | -1.4122 | -0.0181  | 2.2510                 | 0.3132               |
| 3875           | -1.4092 | -0.0194  | 2.2509                 | 0.3061               |

4.2. Reliability Analysis

Firstly, adopt the LSEM to get the parameters estimation values of Logistic model, the values are as follows:

$$\hat{\beta}_0 = 139.1823, \quad \hat{\beta}_1 = 195.3176$$

Further the Logistic model of traumatic failure can be obtained as follows

$$R(z) = \frac{1}{1 + \exp(139.1823 + 195.3176z)}$$

Based on these, using Eq.9, we can get the competing failure model of sine-wave oscillation circuit, that is
Using the competing failure model expressed as Eq. 26, we can get reliability at each observed time shown in Table 4.

Table 4. Reliability $R_{z_1}$ at each monitoring time

| time/h | $R_{z_1}$ | time/h | $R_{z_1}$ | time/h | $R_{z_1}$ |
|--------|-----------|--------|-----------|--------|-----------|
| 0      | 1.0000    | 1750   | 0.9986    | 3500   | 0.8192    |
| 250    | 1.0000    | 2000   | 0.9963    | 3750   | 0.7431    |
| 500    | 1.0000    | 2250   | 0.9911    | 4000   | 0.6527    |
| 750    | 1.0000    | 2500   | 0.9788    | 4250   | 0.5569    |
| 1000   | 1.0000    | 2750   | 0.9596    | 4500   | 0.4645    |
| 1250   | 0.9999    | 3000   | 0.9226    | 4750   | 0.4064    |
| 1500   | 0.9996    | 3250   | 0.8775    | 5000   | 0.3854    |

We know that Weibull distribution (Eq. 27) is extensively applied to describe product’s reliability function, so this paper uses Weibull distribution with two parameters to describe the reliability function of sine-wave oscillation circuit by the data in Table 4.

$$ R(t) = e^{(-\frac{t}{\eta})^m} $$  

(27)

Transform it to linear model by logarithmic computation, that is

$$ \log \left[ \log \left( \frac{1}{R(t)} \right) \right] = m \log(t) - m \log(\eta) $$  

(28)

Then use LSEM and the data in Table 4, the estimation value $m$ and $\eta$ are as follows:

$$ m = 6.8556, \eta = 4.5769 \times 10^3 $$

So the reliability function is

$$ R(t) = \exp \left[ -\left( \frac{t}{4.5769 \times 10^3} \right)^{6.8556} \right] $$  

(29)

Based on the above discussion, the plot of reliability under multivariate competing failure model is shown in Fig. 4. In which, the dash curve is the reliability diagram of multivariate degradation failure and the solid curve is the reliability diagram of multivariate competing failure.

![Figure 4. Reliability curves](image-url)
From Fig.4 we can see that there are great differences between the curves considering multivariate competing failure or not. If we only consider the degradation failure, the estimated results will comparably optimistic, which will lead to draw wrong conclusion and take wrong actions.

5. Conclusion
This paper introduces principal component analysis and logistic function to model and analyze complex electronic equipment reliability assessment with multivariate competing failure. We first give the general model of multivariate competing failure, and then by exploring the model we separate the model into two parts, that is, probability density function with multi-degradation measure and traumatic failure probability function related to degradation level. Based on these analysis results, we introduce PCA to change the multivariate probability density function problem to unilabiate problem. Logistic function and LSEM are adopted to calculate the reliability function. Finally, an example is used to validate the method proposed in this paper.

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