Thermal Model Description of Strangeness Enhancement 

at Mid-Rapidity in 

Pb–Pb collisions at 158 GeV A/c.

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The results of the WA97 collaboration for strange particle production at mid-rapidity in Pb–Pb collisions at 158 GeV A/c at CERN display a strong strangeness enhancement with system size at mid-rapidity which is dependent on the strangeness of the particle concerned, and saturates at values of participating nucleons greater than 120. These results are phenomenologically described by the mixed canonical ensemble, with canonical (exact) strangeness conservation involving all strange resonances, and grand canonical conservation of charge and baryon number. A detailed quantitative analysis shows that the data are well described by an equilibrium ($\gamma S \equiv 1$) hadron gas.

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I. INTRODUCTION

The WA97 collaboration at CERN has shown that strange particle yields per wounded nucleon reach a saturation level for the most central Pb–Pb collisions (the Pb–Pb data is presented in 4 centrality bins) and show a pronounced increase when compared to a p–Be system. A mixed canonical description of these data has been performed. The dependence of the parameters on the size of the system consistent with the WA97 observations. In addition Hamieh et al. have shown that if reasonable values are used for the strangeness partition function for a hadron gas conserving strangeness exactly is described by the mixed canonical ensemble, with canonical (exact) strangeness conservation involving all strange resonances, and grand canonical conservation of charge and baryon number. A detailed quantitative analysis shows that the data are well described by an equilibrium ($\gamma S \equiv 1$) hadron gas.

II. MIXED CANONICAL FORMALISM

The mixed canonical partition function for a hadron gas conserving strangeness exactly is described by the following partition function:

$$Z^C_{S=0} = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi \exp \left( \sum_{n=-3}^{3} S_n e^{i n \phi} \right),$$

where $S_n = V \sum_k Z_k^n$, $V$ is the volume, and the sum is over all particles and resonances carrying strangeness $n$. For a particle of mass $m_k$, with spin-isospin degeneracy factor $g_k$, carrying baryon number $B_k$ and charge $Q_k$ with baryon chemical potential $\mu_B$ and charge chemical potential $\mu_Q$, the one-particle partition function is expressed in the Boltzmann approximation as:

$$Z_k^1 \equiv \frac{g_k}{2\pi^2} m_k^2 T K_2 \left( \frac{m_k}{T} \right) \exp(B_k \mu_B + Q_k \mu_Q).$$

As described in Ref. 4, the partition function for this ensemble may be rewritten in the following form which is well suited for numerical calculation:

$$Z^C_{S=0} = e^{S_0} \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} a_0^p a_2^n x_1^{-2n-3p} \times I_n(x_2) I_p(x_3) I_{-2n-3p}(x_1),$$

where

$$a_i = \sqrt{S_i/S_{-i}},$$

$$x_i = 2\sqrt{S_i S_{-i}},$$

and $I_i$ are modified Bessel functions.

The expression for the particle density, $n_i$, may be obtained from the partition function Eq. (3) by following the standard method 3. For a particle $i$ having strangeness $s$ the result is

$$n_i = \frac{Z_k^1}{Z^C_{S=0}} \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} a_0^p a_2^n x_1^{-2n-3p-s} \times I_n(x_2) I_p(x_3) I_{-2n-3p-s}(x_1).$$
III. RESULTS

In this section we explore whether canonical strangeness suppression at small volumes (compared to grand canonical equilibrium particle yields) is able to explain the enhancement of strange particle yields per wounded nucleon from small to large systems, as measured by the WA97 collaboration at CERN\[7\]. The thermal model is well suited for to \(4\pi\) integrated data and to the central rapidity region if a boost invariant plateau exists around at mid-rapidity\[8\]. We assume the validity of the latter in order to analyze the particle yields. A thorough comparison of \(4\pi\) particle numbers with the thermal model has been made in Ref. \[1\]. As has been the case in other calculations\[8,9\], we find our findings at mid-rapidity to be consistent with full strangeness equilibration (i.e. \(\gamma_s = 1\)). This is in contrast to \(4\pi\) integrated particle yields which are incompatible with full strangeness chemical equilibrium and deviate from it by several standard deviations. Experimentally, ratios of particle to anti-particle yields\[10\] at mid-rapidity\[10\] have been compared to \(4\pi\) integrated yields and shown to be in agreement for S+S collisions at 200 A GeV by the CERN NA35 collaboration\[10\]. In addition, and perhaps of greater interest, the \(4\pi\) integrated ratio \(\Xi/\Xi\) measured by CERN NA49 for Pb+Pb collisions at 158 A GeV has been shown to agree with the corresponding CERN WA97 mid-rapidity ratio\[11\].

As the mixed canonical formalism has been derived here for the case of Boltzmann statistics, it is worth noting that the second term in a series expansion of the correct quantum statistical distribution function for kaons gives a correction to the kaon numbers of the order of 3% at a temperature of 150 MeV. The corrections to the Boltzmann distribution functions due to quantum mechanics are expected to affect the kaons more than any other strange particle, as they are the lightest of the strange particles. With errors of 3% and less, the use of Boltzmann statistics to describe the strange particles is justified. For all mixed canonical analyses the parameter \(\mu_Q\) fit to zero, and has been subsequently removed as a free parameter. For a \(4\pi\) integrated Pb+Pb system, \(\mu_Q\) is expected to be small and negative.

A. Analysis A

The main results of analysis A are based on fitting the ratios of the strange particle yields to the yield of negatives for each case in Table I. The negatives yield is used for each ratio due to the good statistics. In addition it provides a measure of the entropy of the collision. For negative pions, the quantum mechanical correction to the Boltzmann distribution function at 150 MeV is of the order of 30% . It is unacceptable to ignore this correction when attempting to describe the data. Considering that the non-strange particle yields predicted by the model are independent of the exact conservation of strangeness, the full quantum mechanical grand canonical particle number expression is evaluated for all non-strange particles. The parameters obtained in this way are shown in Table II as analysis A\[17\]. The \(\chi^2\) for each system is good, and there are three (two) degrees of freedom for the p+Pb, and Pb+Pb systems (p+Be system). The large uncertainties in the radius parameter for the Pb+Pb systems shows that we have hit the grand canonical limit, and have no volume dependence, this is discussed briefly in a later section.

B. Analysis B

Analysis B has only three data points (\(\Lambda/\Xi\) and \(\bar{\Xi}/\Xi\)) for each fit. This analysis has been motivated by the reasons mentioned previously in the text. This analysis has no free parameters and shows reasonable agreement with analysis A. The large uncertainties of the fitted parameters are unavoidable. Interestingly, fitting these ratios proves impossible for the p+Pb system. The hypothesis of \(\Lambda\) of a special ‘interaction volume’ for strange particles in this system cannot be tested, as the particle ratios would depend only on this interaction volume, as the regular volume cancels in the ratio of the particle multiplicities.

C. Analysis C

Analysis C determines the fits to the data using the grand canonical ensemble with quantum statistics. In this case, the volume dependence cancels out in the particle ratios and we have included \(\mu_S\) and \(\mu_Q\). This procedure fails for the p+Be system. As may be expected for a system of this size, a canonical treatment\[12\] is required. In this analysis, the parameters fitted for Bin 4 immediately catch the eye – with high uncertainty values even though \(\chi^2 < 1\). This is especially puzzling when one notices that this is not the case in analyses A or B. The \(T\) and \(\mu_B\) parameters agree with those obtained by Baccetti et al.\[13\], for a grand canonical description of the \(4\pi\)-integrated data for central symmetric Pb\(^{208}\) collisions at 158 A GeV by the CERN NA49 collaboration. The parameters in analyses A, B, and C agree within th each other one standard deviation.

D. Thermal Model Parameters

In order to fully reproduce the data obtained by the WA97 collaboration , it is necessary to determine a relationship between the parameters \(\mu_B\), the radius \(R\) and the temperature \(T\) obtained from the fits of analysis A and the number of wounded nucleons. The fits used are shown in Figs. I (temperature), and \(\mu_B\) as well as the parameters obtained from a full Boltzmann treatment\[13\]. The volume parameterisation is described
TABLE I: Parameters obtained for various analyses as described in the text. The temperature \((T)\) and chemical potential \((\mu_B)\) are in MeV. The radii \((R)\) are given in fm. If \(\chi^2\) is greater than 50, the fit parameters are not shown.

|        | p+Be | p+Pb | Pb+Pb |
|--------|------|------|--------|
|        | Bin1 | Bin2 | Bin3   | Bin4   |
| \(\chi^2\) | 0.756 | 1.85 | 2.07   | 0.427  | 1.58  | 1.45   |
| \(T\)  | 162±4 | 172±2 | 161±4  | 164±4  | 166±4 | 159±5  |
| \(\mu_B\) | 111±7 | 157±39 | 201±25 | 230±38 | 228±28 | 204±30 |
| \(R\)  | 1.39±0.14 | 1.16±0.39 | 6.8±8.73 | 9.88±10.9 | 6.78±9.39 | 10.4±6.8 |

### Analysis A

#### Ratios of Strange Particle to Negatives Multiplicities

|        | Bin1 | Bin2 | Bin3   | Bin4   |
|--------|------|------|--------|--------|
| \(\chi^2\) | \(\sim 10^{-6}\) | 60287 | \(\sim 10^{-3}\) | \(\sim 10^{-3}\) | \(\sim 10^{-2}\) |
| \(T\)  | 157±25 | - | 171±17 | 161±15 | 153±35 | 163±45 |
| \(\mu_B\) | 106±31 | - | 229±55 | 222±46 | 190±61 | 220±30 |
| \(R\)  | 1.50±0.83 | - | 2.17±2.03 | 6.06±8.92 | 9.38±7.12 | 8.08±6.83 |

### Analysis B

#### Ratios of Particle Yields to that of their Anti-Particles

### Analysis C

#### Grand Canonical Fit

|        | Bin1 | Bin2 | Bin3   | Bin4   |
|--------|------|------|--------|--------|
| \(\chi^2\) | 78 | 17.5 | 1.53   | 0.15   | 0.70  | 0.97   |
| \(T\)  | - | 143±8 | 173±20 | 165±4  | 166±5 | 171±20 |
| \(\mu_B\) | - | 205±106 | 282±96 | 227±39 | 203±45 | 265±120 |
| \(\mu_Q\) | - | -109±52 | -54±68 | 0±50  | 0±60 | -62±86 |
| \(\mu_S\) | - | 105±63 | 110±73 | 54±25  | 39±28 | 94±97 |

by equation \(3\). The parameters obtained from the p+Pb data have not been considered because of their prediction of a volume smaller than that of the p+Be system.

1. **Temperature**

As may be seen in Fig. 1, it appears initially that the temperature increases with centrality. However, the last data point does not fit this hypothesis, and the variation of chemical freeze-out temperature with number of participants has been assumed to be constant \((T = 163\text{ MeV})\). This is in agreement with the fit to the NA49 data by Becattini et al. \([7]\) to central Pb+Pb data. This temperature value is slightly less than the one obtained by Hamieh et al. \([2]\).

2. **Radius**

In order to reproduce the WA97 data, knowledge of the variation of system size with average number of wounded nucleons is required. We have chosen a function of the form:

\[
<N_{\text{wound}}> = R^3 + b
\]  

with \(b < 0.5\) adjusted to reproduce the data of the WA97 collaboration (Figs. 3 and 4). This differs from the dependence of the radius on \(A_{\text{part}}\) assumed by Hamieh et al. \([2]\) where \(A_{\text{part}} \sim 1.3 - 1.7R^3\).

The large uncertainty in the radius for the larger systems is not surprising, as the ratios being fitted are expected to show a small volume dependence, due only to canonical strangeness suppression. The effects of canonical strangeness suppression decrease with system size as one approaches the grand canonical limit where ratios of particle multiplicities have no volume dependence.

3. **Baryon Chemical Potential**

Figure 2 shows the variation of the baryon chemical potential with system size. This function increases rapidly before saturating. All the Pb+Pb bins are described by the saturation value of approximately 210 MeV. The obtained values of \(\mu_B\) in the central Pb+Pb bin are in agreement with that of Becattini et al. \([7]\). The \(\mu\) used...
for the Pb+Pb bins by Hamieh et al. is just outside one standard deviation of the values obtained during this analysis. For the p+Be system the value of $\mu_B$ used in Ref. (150 MeV) is much larger than that obtained in this analysis ($111 \pm 7$ MeV).

The exact variation of $\mu_B$, $T$ and $R$ in the range between the p–Be and Pb–Pb may differ from what is shown here and new data from CERN NA57 in this range are eagerly awaited. The figures mentioned above, and also show the variation of the parameters $T$, and $\mu_B$ with centrality in the case where all particle multiplicities are assumed to be Boltzmann. These parameters are seen to be in agreement with the parameters from Analysis A. The value of the $\chi^2$ parameter obtained for the purely Boltzmann fits were of order 1 for all systems considered.

E. Predicted Evolution of Particle Yields

Figs. 3 and 4 show the ability of the model to reproduce the data using the functions in Figs 1 and 2, and equation 7 to predict the variation in chemical freeze-out temperature $T$, baryon chemical potential $\mu_B$, and radius $R$ with average number of wounded nucleons. As is clear, the agreement of the model with experimental data is good. The exact shape of the thermal model predictions shown in Figure 3 and 4 show a large dependence on the exact relationship between number of wounded nucleons and system size, and a weaker dependence on the functional fit to the other two parameters. This is rather unfortunate as the thermal model $R$ values obtained have uncertainties equal to their magnitude, so this relationship is not constrained by the model.

IV. CONCLUSIONS

A thermal model conserving baryon number and charge on average, and strangeness exactly has been used to describe particle yields from heavy ion collisions. The formalism includes all strange particles. The model has been applied to the CERN WA97 data, and is shown to be able to reproduce the data for all centrality classes (Figs. 3 and 4). The accuracy of the model parameters (temperature $T$, radius $R$, and baryon chemical potential $\mu_B$) is restricted by the lack of $4\pi$ integrated particle yields for multi-strange particles. In order to reproduce WA97 graphical presentation of their data accurately, some knowledge of the variation of $T$, $R$, and $\mu_B$ was required. A functional form of the variation of these parameters with the number of wounded nucleons has been determined and is presented in Figs. 3 and 4, and equation 7.

The parameters obtained in this model for the central Pb+Pb bins are in agreement with the $4\pi$ thermal model application of Becattini et al. This lends some weight to the accuracy of the model when fitting particle ratios in a limited kinematic region. This grand canonical model includes the factor $\gamma_S \neq 1$ to predict the yields of strange particles. To differentiate between canonical strangeness suppression, and suppression of strange particles by anomalous phase space occupancy, the yield of the $\phi$ particle could be used. Yields of this particle are not sensitive to canonical strangeness suppression, but as it contains an s and an $\bar{s}$ quark, these yields will be sensitive to $\gamma_S \neq 1$.

The enhancement of strange particles measured by CERN WA97 has been considered a signal for QGP formation. At first glance, the ability of a full equilibrium thermal model to reproduce the data suggests the existence of a deconfined state, since equilibrium strange particle yields have been proposed as a possible signal for deconfinement. A deconfined phase is not, however, expected to be formed in the p+Be system, as it is only expected in large dense systems. The ability of the model to reproduce the p+Be data suggests that it is possible for strange particles to reach equilibrium yields by hadronic interactions alone.

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FIG. 3: Comparison of the hadron gas model with exact strangeness conservation and CERN WA97 data for negatives and strange particles.

FIG. 4: Comparison of the hadron gas model with exact strangeness conservation and CERN WA97 data for the Ω and strange anti-particles.

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[15] The reason for checking particle to anti-particle ratios is that the masses of the particles in the ratio are equal - this leads to these particles being equally affected by flow. This in turn minimizes the errors introduced by considering a limited kinematic region [14].
[16] In symmetric collisions the majority of new particles are produced at mid-rapidity.
[17] The table also includes the $\chi^2$ and fit parameters for two other analyses described in detail below.
[18] Using Boltzmann statistics for strange, and non-strange particles.
[19] parameterised by $\gamma s \neq 1$ in thermal models.