I. INTRODUCTION

A large number of neutron stars (NS) has been discovered as close companions of low mass and high mass stars (so called low mass and high mass X-ray binaries, see [1, 2]). These binaries are characterized by a strong X-ray emission that is produced as a result of matter accretion from the companion star onto the neutron star. In some cases, the matter arrives up to the NS surface at the magnetic polar cap region which creates a very hot spot. This class of X-ray binaries shows pulsations correlated with the rotational period of the NS (so called X-ray pulsars, see [3]). X-ray binaries may contain a long period NS as well as a millisecond NS. The long period X-ray pulsars are usually found in isolated binary systems whereas the millisecond pulsars (MSPs) are mostly contained within globular clusters [4]. According to the standard scenario, the MSPs are old pulsars spun up by the angular momentum transferred to the NS from the accreting matter. Many of them have been captured by stars whose density inside the globular cluster is very large. On the other hand, long period X-ray pulsars are found inside binary systems in which NS is formed in a supernova explosion of a more massive companion.

The matter that is falling onto the NS as a result of the overflow through the inner Lagrangian point usually contains a large angular momentum. This matter forms an accretion disk that can be disrupted at the inner radius due to the pressure of the magnetic field of the NS. In the case of accretion from the wind of the massive star, the process occurs more spherically due to the isotropization of the accretion flow by the shock in the massive star wind. In some binaries the process of accretion can be additionally stimulated by the re-radiation of stellar surface of the companion star by the X-ray radiation from the NS. Accreting pulsars can suffer spin up and spin down periods depending on the conditions inside the binary system and the parameters of the MSP (i.e. the mass loss rate of the companion star, distance between the stars, neutron star velocity, surface magnetic field strength and the MSP rotational period). These basic parameters determine the specific accretion phase of the matter onto the NS, i.e. whether it is in the accretor, the propeller, or the ejector phase [5]. The ejector phase is characteristic for a fast rotating NS whose magnetized wind is able to prevent any accretion below the light cylinder radius. In the propeller phase, matter can enter the inner pulsar magnetosphere. However, it is blocked by the NS fast rotating magnetosphere at a certain distance from the NS surface due to the centrifugal force. Then, most of the matter is expelled from the vicinity of the pulsar and the accretion onto the neutron star surface may occur only episodically when the pressure of the accumulated matter overcomes the pressure of the rotating magnetosphere.

In this paper we are interested in the acceleration of hadrons during the accretion of matter from the companion star onto the magnetized neutron star. Hadrons interact with the thermal radiation from the NS surface and produce neutrinos which might be observable by the large scale neutrino telescopes (IceCube, KM3NET). Note that production of neutrinos inside the binary systems has been up to now considered in a few different scenarios. For example, hadrons accelerated in the jet can interact with the radiation of the accretion disk [6], the stellar wind [7] and/or the massive star [8] or hadrons accelerated in the pulsar inner magnetosphere or the pulsar wind shock can interact with the matter of the massive star [9] and/or the accretion disk [10]. More recently, production of neutrinos has been considered in the context of the massive binary systems detected in the TeV γ-rays, e.g. [11, 12].
...sphere onto the surface of the neutron star. It is characteristic for NSs which are relatively slow rotators with a weak surface magnetic field. The gravitational energy of the accreting matter is released on the neutron star surface, producing a hot spot around the magnetic pole. The amount of released energy is re-emitted as thermal X-ray emission whose the power can be estimated from:

\[ L_X = GM_{\text{acc}}M_{NS}/R_{NS} \approx 2 \times 10^{36} M_{16} \text{ erg s}^{-1}, \tag{1} \]

where \( M_{\text{acc}} = 10^{16} M_{16} \text{ g s}^{-1} \) is the accretion rate, and \( L_X \) is the X-ray thermal emission from the polar cap region. The radius and the mass of the NS is assumed to be \( R_{NS} = 10^6 \text{ cm} \) and \( M_{NS} = 1.4 M_{\odot} \).

The distance from the NS surface at which the magnetic field starts to dominate the dynamics of the infalling matter (the Alfvén radius) can be estimated by comparing the magnetic field energy density to the kinetic energy density of the wind, \( B_A^2/8\pi \rho = \rho v_t^2/2 \), where \( B_A \) is the magnetic field in the inner neutron star magnetosphere, \( \rho = M_{\text{acc}}/(\pi R_A^2 v_t) \) is the density of the accreting matter, \( v_t = (2GM_{NS}/R_A)^{1/2} \) is the free fall velocity of the accreting matter, \( R_A \) is the Alfvén radius, and \( G \) is the gravitational constant.

By applying Eq. (1) and assuming that the magnetic field in the neutron star magnetosphere is of the dipole type, i.e. \( B_A = B_{NS}(R_{NS}/R_A)^3 \), we estimate the location of \( R_A \) with respect to the NS surface:

\[ R_A = 7.8 \times 10^6 B_9^{4/7} M_{16}^{-2/7} \text{ cm}, \tag{2} \]

where the magnetic field at the neutron star surface is \( B_{NS} = 10^9 B_9 G \).

Then, we can estimate the magnetic field strength at the transition region,

\[ B_A = 3.3 \times 10^6 M_{16}^{6/7} B_9^{-5/7} \text{ G.} \tag{3} \]

The observed thermal luminosity in X-rays, re-radiated from the region of the polar cap, can be calculated from \( L_X = \pi R_{\text{cap}}^2 \sigma T_{\text{cap}}^4 \), where \( \sigma \) is the Stefan-Boltzmann constant. In fact, the emission from the polar cap region is well described by a black body spectrum (see e.g. [10]). The radius of the polar cap region on the NS surface, onto which the matter falls, and from which thermal X-ray radiation is emitted, can be estimated from (assuming a dipole structure of the magnetic field):

\[ R_{\text{cap}} = (R_{NS}/R_A)^{1/2} \approx 3.6 \times 10^5 B_9^{-2/7} M_{16}^{1/7} \text{ cm.} \tag{4} \]

Then, the surface temperature of the polar cap has to be,

\[ T_{\text{cap}} = (L_X/\pi R_{\text{cap}}^2 \sigma)^{1/4} \approx 1.8 \times 10^7 B_9^{1/7} M_{16}^{5/28} \text{ K.} \tag{5} \]

In general, the accretion of matter onto the NS can occur provided that the radius of the transition region (i.e. the Alfvén radius \( R_A \)) lies inside the light cylinder radius of the neutron star, i.e. \( R_A < R_{LC} = cP/2\pi \), where \( P \) is the rotational period of the neutron star in seconds, and \( c \) is the velocity of light. This condition is fulfilled for,

\[ P > P_p = 1.7 \times 10^{-3} B_9^{4/7} M_{16}^{-2/7}. \tag{6} \]

Moreover, the rotational velocity of the magnetosphere at \( R_A \) has to be larger than the keplerian velocity of the accreting matter. If the rotational velocity, given by

\[ v_{\text{rot}} = 2\pi R_A/P \approx 4.8 \times 10^7 B_9^{4/7} M_{16}^{-2/7}/P \text{ cm s}^{-1}, \tag{7} \]

is larger than the keplerian velocity,

\[ v_{\text{kep}} = (GM_{NS}/R_A)^{1/2} \approx 4.8 \times 10^8 B_9^{-2/7} M_{16}^{1/7} \text{ cm s}^{-1}. \tag{8} \]

then the matter cannot accrete directly onto the NS surface. It is partially accumulated close to the transition region and partially expelled by the centrifugal force. By comparing Eqs. (7) and (8) we get the limiting rotational period of the NS below which the matter can accrete onto the NS surface:

\[ P > P_a = 0.01 B_9^{6/7} M_{16}^{-3/7}. \tag{9} \]

NSs with periods within this range, \( P_p \) and \( P_a \), can only accrete matter in the propeller scenario. However, for periods longer than \( P_a \), the accretion process occurs in the accretor phase. This last stage is the most interesting for this paper.

### III. ACCELERATION OF HADRONS

As we discussed above, the transition region contains turbulent magnetized plasma which provides good conditions for the acceleration of particles. Both electrons and hadrons can be accelerated there. Electrons scatter on thermal radiation from the polar cap region and produce \( \gamma \)-rays by the IC \( e^\pm \) pair cascade process as discussed recently in [14]. However, also relativistic hadrons can suffer significant energy losses due to radiative processes in collisions with dense thermal radiation from the polar cap. These processes produce \( \gamma \)-rays (that undergo similar processes as the accelerated electrons) and additionally neutrinos that escape from the production region without any absorption. The observation of these neutrinos by large scale detectors will provide crucial information on the relative importance of the acceleration of hadrons and electrons in the strongly magnetized and turbulent plasma during the accretion process onto the NS.

The maximum power available for the acceleration of particles is limited by the extracted energy in the transition region. This energy can be supplied by two mechanisms. In the case of a quasi-spherical accretion from the stellar wind, the matter has to be accelerated to the velocity of the rotating magnetosphere at \( R_A \). The rotating NS decelerates, providing energy to the turbulent region. In the case of accretion through the Lagrangian...
point, the matter has a large angular momentum, which has to be partially lost in the transition region in order to guarantee the accretion process up to the NS surface. The rotational energy of the accreting matter is then supplied partially to the transition region and to the NS. As a result, the NS reaches the angular momentum and accelerates. In the first case, the power which has to be transferred from the rotating NS to the accreting matter can be estimated from:

$$L_w = M_{\text{acc}} v_{\text{rot}}^2/2 \approx 10^{31} B_9^{8/7} M_{16}^{3/7} P^{-2} \text{ erg s}^{-1}. \quad (10)$$

By Using Eq. (9), we can estimate the maximum power which can be extracted via accretion from the quasi-spherical wind,

$$L_{w}^{\text{max}} \approx 10^{35} B_9^{4/7} M_{16}^{9/7} \text{ erg s}^{-1}. \quad (11)$$

In the case of accretion through the accretion disk, the matter arrives to the transition region with the Keplerian velocity. This region is now closer to the NS surface than estimated above in $R_A$ (see Eq. 2) by a factor $\chi \sim 0.1 - 1$ (see 13). In order to accrete onto the NS surface, the matter from the disk has to be slowed down to the rotational velocity of the NS magnetosphere, i.e. from $v_{\text{kep}}$ to $v_{\text{rot}}$. Then, the maximum available power extracted in the transition region is

$$L_d \approx \frac{1}{2} M_{\text{acc}} (v_{\text{kep}}^2 - v_{\text{rot}}^2) = L_{w}^{\text{max}} \left(1 - \frac{\chi^2 P_9^2}{P_\text{rot}^2}\right) \text{ erg s}^{-1} \quad (12)$$

which gives $L_d \approx L_{w}^{\text{max}}/\chi$ for $P \gg P_a$.

In conclusion, in both considered here accretion scenarios, from the wind and through the accretion disk, the energy is transferred to the turbulent, magnetized plasma at $R_A$. However, in the first case the pulsar loses its rotational energy but in the second case the pulsar can gain rotational energy from the accreting matter and accelerate its rotation.

A part, $\eta$, of the power, $L_w$ or $L_d$, can be used for the acceleration of hadrons. Below we estimate the characteristic energies of accelerated hadrons. The acceleration rate of hadrons with energy $E$ (and Lorentz factor $\gamma_p$) can be parametrized by a simple scaling to the Larmor radius of particles within a medium with magnetic field $B$.

$$\dot{P}_{\gamma \rightarrow \pi} = \sigma_{p\gamma} n K E \approx 1.5 \times 10^6 B_9^{4/7} M_{16}^{33/28} E \frac{\text{erg s}^{-1}}{s}, \quad (15)$$

where $n = n_{\text{bb}} (R_{\text{cap}}/(R_A - R_{\text{NS}}) + R_{\text{cap}})^2$ (a) is the density of the black body photons coming from the polar cap at the distance of the transition region $R_A$, and $\sigma_{p\gamma} \approx 3 \times 10^{-28}$ cm$^{-2}$ and $K \approx 0.15 - 0.35$ are the cross section and the inelasticity for pion production due to collision of relativistic protons with thermal photons. In our estimations we apply the average value for $K = 0.25$. As above, we can estimate the maximum possible energies of accelerated hadrons due to their energy losses on pion production in collisions with thermal photons by comparing Eqs. (13) and (15). We obtain the limit on the Lorentz factor of hadrons.

$$\gamma_{p\gamma \rightarrow \pi} \approx 1.5 \times 10^6 \gamma_{-1} B_9^{4/7} M_{16}^{33/28}. \quad (16)$$

These maximum Lorentz factors of hadrons are typically above the maximum Lorentz factors of hadrons estimated above based on their escape from the acceleration region especially in the case of large accretion rates (note the dependence on the accretion rate in Eqs. (14) and (16)). Therefore, we conclude that hadrons are at first accelerated to maximum energies allowed by the escape mechanism and after that they interact with thermal photons during their fall onto the NS surface. Note that, density of thermal photons increases towards the NS surface with the distance as $\propto R^2$. Therefore, hadrons interact not very far from the acceleration region.

We consider hadrons which are accelerated with a power law or a mono-energetic spectra at the transition region. In the first case hadrons with energies below the threshold for pion production in collisions with thermal photons also exist. These hadrons can only interact with the matter inside the transition region and the matter accreting onto the NS surface. We estimate the density of matter at the transition region,
\[ \rho = \frac{\dot{M}_{\text{acc}}}{(\pi R_s^2 u_t)} \approx 2.2 \times 10^{15} B_9^{6/7} M_{16}^{10/7} \text{ cm}^{-3}, \]
and the interaction rate of hadrons for pion production \( N_{\text{pp-\pi}} = \sigma_{\text{pp}} \rho \pi (R_s/u_t) \approx 2 \times 10^{-3} M_{16} \). Therefore, hadron-hadron interactions are not efficient at the transition region. These lower energy hadrons are convected with the in-falling matter on the NS surface. They interact with much denser matter in the accretion column over the polar cap region. However, neutrinos produced in hadron-hadron collisions have on average lower energies to produce any observable signal in large scale neutrino telescopes.

IV. PRODUCTION OF NEUTRINOS

We calculate the spectra of muon neutrinos coming from the decay of pions produced in collisions of hadrons with the thermal radiation form the NS surface by applying a numerical code developed for the interaction of hadrons with photons within supernova envelope where the condition are quite similar (see [17]). Two types of hadron injection spectra are considered: (a) a power law differential spectrum with the spectral index 2.1 which cuts at energies estimated by Eq. (14) and (b) a mono-energetic spectrum with energies given by Eq. (14). The second case is considered as a limiting case of the acceleration process of hadrons occurring in the reconnection of magnetic field inside the transition region. The electric field induced in the reconnection region can in principle accelerate particles to a very flat spectrum (spectral index lower than 2) in which most of the energy is accumulated with particles at the highest possible energies.

Pions and muons (from their decay) are produced in a relatively strong magnetic field at \( R_s \). Therefore, they can suffer significant synchrotron energy losses. In order to check whether or not the synchrotron process can change the energies of the produced charged pions and muons before their decay, we calculate their synchrotron energy loss time scales and compare them with their life times. From this comparison, we put an upper limit on the Lorentz factor of pions,

\[ \gamma_{\pi}^s \approx 7 \times 10^7 B_9^{10/7} M_{16}^{-12/7}, \tag{17} \]

that are able to decay before losing a significant amount of energy. We take the effects of synchrotron energy losses of pions on the produced spectra of neutrinos in the case of their production by hadrons with the Lorentz factors above \( \gamma_{\pi}^s \) into account by simply replacing their \( \gamma_{\pi} \) by \( \gamma_{\pi}^s \). However, in the case of muons, the critical value of the Lorentz factors is about two orders of magnitude lower. Therefore, neutrinos from muons decays have rather low energies. They are not included in our calculations of the neutrino event rates.

For the example calculations, we fix the following parameters describing acceleration of hadrons: \( \xi = 0.1 \), and \( \zeta = 0.1 \). In table 1, we show the muon neutrino event rates that are expected in a \( \text{km}^2\text{sr} \) neutrino detector from millisecond pulsars at the distance of 5 kpc which accrete matter from the stellar wind. The typical parameters for the MSPs and the accretion rates are considered. The results are shown for a power law spectrum of hadrons (marked by power) and a mono-energetic spectrum (marked by mono). The number of neutrino events is estimated by integrating the muon neutrino spectra over the probability of their detection,

\[ N_{\mu} = \frac{S}{4\pi D^2} \int_{E_{\nu}^{\text{min}}}^{E_{\nu}^{\text{max}}} P_{\nu-\mu}(E_{\nu}) \frac{dN_{\nu}}{dE_{\nu} dt} dE_{\nu}, \tag{18} \]

where \( S = 1 \text{ km}^2 \) is the surface of the detector, \( D \) is the distance to the source, \( P_{\nu-\mu}(E_{\nu}) \) is the energy dependent detection probability of a muon neutrino (see Eqs. (14) and (17)). The maximum energy of neutrinos produced in this model is typically in the range \( E_{\nu}^{\text{max}} = 10^8 \) to \( 10^9 \text{ GeV} \). This maximum energy of neutrinos can not be simply expressed since it depends on the parameters of the model (see Eqs. (14) and (17)). Therefore, it is obtained numerically in our calculations. The expected neutrino event rates from millisecond pulsars is up to a few per \( \text{km}^2 \text{yr} \) (depending on the model). These event rates should be detected significantly by the IceCube neutrino detector.

| B [G] | M [g s\(^{-1}\)] | \( \gamma_{\pi}^\text{max} \) | \( P_{\nu-\mu} \) [ms] | \( N_{\mu} \) [km\(^{-2}\text{yr}^{-1}\)] |
|-------|----------------|----------------|----------------|----------------|
| \( 3 \times 10^8 \) | \( 10^{17} \) | \( 2.3 \times 10^5 \) | 13 | 1.0 | 5.1 |
| \( 10^9 \) | \( 3 \times 10^{17} \) | \( 4.3 \times 10^5 \) | 23 | 2.1 | 5.6 |
| \( 3 \times 10^9 \) | \( 10^{18} \) | \( 8.4 \times 10^5 \) | 36 | 5.8 | 5 |

We also show the neutrino event rates that are expected from the two nearby X-ray pulsars inside the binary systems GROJ 1744-28 and A 0535+262, which are characterized by long rotational periods, strong surface magnetic fields, and high accretion rates. Also from these sources a few neutrino events per year are expected in an IceCube size neutrino detector.
similar spectra and energies. However, these \( \gamma \)-rays with MeV-GeV energies can escape from the NS surface due to a change in the accretion rate caused e.g. by the eccentric orbit of the neutron star. Moreover, in the case of the accretion process through the accretion disk, the power available for the acceleration of particles can be enhanced by a factor of \( \chi^{-1} \) (\( \chi \) is typically in the range \( \sim 0.1 - 1 \), see Eq. 12) with respect to the case of the quasi-spherical accretion. Therefore, the \( \gamma \)-ray photons with MeV-GeV energies can escape from the radiation field of the hot NS surface. Therefore, we propose that the neutron stars accreting at high rates can be observable by the recently launched Fermi LAT detector in the GeV \( \gamma \)-rays but also by the large scale neutrino detectors such as IceCube and KM3NET.

As an example we estimated the neutrino event rates in the km\(^2\) detector for two relatively nearby binary systems which accrete at large rate, GRO 1744-2 and A 0535+262. Other considered sources are expected to emit lower neutrino fluxes due to the larger distances or lower accretion rates. We also estimate the neutrino event rates expected from the millisecond pulsars at the typical distance of globular clusters. It is clear (from Table 1) that some accreting millisecond pulsars could become detectable neutrino sources.

Different factors can enhance or reduce the neutrino event rates reported in Tables 1 and 2. The obvious ones are related to the distance to the source and the variable neutrino emission due to a change in the accretion rate caused e.g. by the eccentric orbit of the neutron star. Moreover, in the case of the accretion process through the accretion disk, the power available for the acceleration of particles can be enhanced by a factor of \( \chi^{-1} \) (\( \chi \) is typically in the range \( \sim 0.1 - 1 \), see Eq. 12) with respect to the case of the quasi-spherical accretion. Therefore, the \( \gamma \)-ray event rates reported in Table 1 should be scaled as well by that factor. From another site, most of the observed millisecond pulsars belong to globular clusters. It is expected that a massive globular cluster can contain up to \( \sim 100 \) MSPs[21]. In fact, inside the globular clusters Tuc 47 and Ter 5, already \( \sim 20 - 30 \) MSP have been discovered [4]. A significant amount of the MSP cannot be discovered directly by their pulsed radio emission since they are inside compact binary systems (so called hidden MSPs [22]). These hidden MSP can become neutrino sources via the production mechanisms, that have been discussed in our model. Therefore, the neutrino event rates expected from the whole population of MSPs in a specific globular cluster can be enhanced due to the cumulative contribution from many sources. On the other hand, non-observation of neutrinos from the globular clusters give important constraint on the number of hidden MSPs in these objects.

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