Cancellation of the
Chiral Anomaly
in a Model with Spontaneous Symmetry Breaking\textsuperscript{1}

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Abstract

A perturbatively renormalized Abelian Higgs-Kibble model with a chirally coupled fermion is considered. The Slavnov identity is fulfilled to all orders of perturbation theory, which is crucial for renormalizability in models with vector bosons. BRS invariance, i.e. the validity of the identity, forces the chiral anomaly to be cancelled by Wess-Zumino counterterms. This procedure preserves the renormalizability in the one-loop approximation but it violates the Froissart bounds for partial wave amplitudes above some energy and destroys renormalizability from the second order in $\hbar$ onwards due to the counterterms.

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0. Introduction

An approach to quantizing an anomalous gauge theory in the sense of Jackiw [1] is presented. The model under study is the simplest one involving a chiral or $\gamma_5$-anomaly in which all particles are massive, i.e. an Abelian Higgs-Kibble model with a chirally coupled Dirac fermion.

The spontaneous symmetry breaking (SSB) which is responsible for the masses possesses three remarkable features. First of all, the mass of the vector boson renders the model consistent although generally the anomaly violates the conservation of the $U(1)$ charge beyond the tree approximation. However, because of this one faces non-renormalizability due to the high-energy behaviour of the massive vector boson propagator. Instead, in this work it is suggested to write down Wess-Zumino counterterms by means of the inhomogeneous gauge transformation of the Goldstone boson due to SSB and to cancel the anomaly by adding these terms to the Lagrangian. This preserves the gauge invariance of the model, keeps renormalizability at least in the one-loop approximation, and therefore provides a method of quantizing anomalous gauge theories. Finally, the introduction of masses avoids problems related to infrared singularities.

The same model has already been discussed by Gross and Jackiw in section III stage ii of ref. [2] with the result that it is non-renormalizable. In addition, they considered the possibility of anomaly cancellation by those counterterms and did not even then reach renormalizability. This paper confirms that result to become relevant in the second order in perturbation. Those linear combinations of counterterms are given which additionally become necessary to define that order.

In contrast to ref. [2] which is based on the concepts and techniques provided by Lee (for the model without fermion) and Adler [3] later approaches to the renormalization of gauge models do not start any more from the gauge invariance of the regularization procedure used. In the presence of an anomaly this is in any case not possible in a consequent way. Becchi, Rouet, and Stora [5] carried out the renormalization of the model under investigation without fermion based on the use of the scheme introduced by Bogoliubov, Parasiuk, Hepp, and Zimmermann (BPHZ) [8] which is not gauge invariant. By this they succeeded for the first time in giving that model an interpretation in the framework of a Fock space operator theory by means of the finite mass renormalization of both the physical and the unphysical degrees of freedom. Such an interpretation is very convenient for a discussion of asymptotic problems, e.g. the unitarity of the $S$ matrix. This way they have shown the independence of physical observables on parameters of the gauge fixing sector. The condition for the application of this concept which can easily be generalized to other (e.g. non-Abelian) gauge models is the invariance under a generalized gauge transformation, the BRS transformation. This invariance is accomplished by the Slavnov identity in higher orders in perturbation, which governs the renormalization procedure.

Putting emphasis to the consistency of the model under study and facing the proofs of the unitarity and the fact that physical observables do not depend on parameters of the gauge fixing sector this work is therefore based on these later achievements as summarized in refs. [6].

Another difference between ref. [2] and this paper is the following. The spontaneous symmetry breaking makes one degree of freedom of the Higgs field unphysical and massless due to the Goldstone theorem. In order to give it a mass the ’t Hooft gauge is
used. This choice makes the introduction of Faddeev-Popov ghosts necessary because the Goldstone boson is not free.

The subsequent paper is organized as follows. In section 1 the most general invariant and power-counting renormalizable Lagrangian is constructed giving the field content and the BRS transformations. CP invariance is imposed which plays an analogous role as C invariance in the Higgs-Kibble model [5]. Section 2 is devoted to the Slavnov identity and the anomaly cancellation. However, it is not carried through how to fulfill the Slavnov identity because of lack of space. This standard procedure is reviewed for more general cases in refs. [6] or [7]. The consequences of introducing the Wess-Zumino counterterms are illustrated in a certain example for a physical scattering amplitude. Section 3 presents a closer look at the non-renormalizability.

1. Tree Approximation

The Fields and their BRS Transformations

The field content of the model is given by a Dirac particle $\psi$, a complex scalar Higgs field $\Phi$ which breaks the symmetry spontaneously, a gauge field $A_\mu$, and the ghost fields $c$ and $\bar{c}$. An auxiliary (Lagrange multiplier) field $b$ is useful to obtain the full BRS invariance off-shell. The fields have the following power-counting dimensions, graduations, and ghost numbers

|        | dim | grad | gh |
|--------|-----|------|----|
| $\psi$, $\bar{\psi}$ | 3/2 | 1    | 0  |
| $\Phi$ | 1   | 0    | 0  |
| $A_\mu$ | 1   | 0    | 0  |
| $c$    | 1   | 1    | 1  |
| $\bar{c}$ | 1   | 1    | -1 |
| $b$    | 2   | 0    | 0  |
| $s$    | 1   | 1    | 1  |

(1)

The BRS transformations are

$$s \psi_R = iq_R c\psi_R,$$
$$s \psi_L = iq_L c\psi_L,$$
$$s \Phi = iq c\Phi,$$
$$s A_\mu = -\partial_\mu c,$$
$$s c = 0,$$
$$s \bar{c} = i b,$$
$$s b = 0,$$

(2.i) (ii) (iii) (iv) (v) (vi) (vii)

where $q_R$, $q_L$, and $q$ are charge numbers.
Discrete Symmetries

The anomaly reads

$$A \propto c \epsilon^\mu{}^\nu{}^\rho{}^\sigma \partial_\mu A_\nu \partial_\rho A_\sigma,$$

thus being even under charge conjugation $C$ (where the ghost $c$ is arbitrarily assumed to be even). Now, the simultaneous charge conjugation invariance of the action and the anomaly implies a contradiction due to the renormalized action principle. Hence charge conjugation invariance must be relinquished if one wants to investigate an anomaly. Therefore the two chirality parts of the Dirac field (eq. (2.i and ii)) should transform differently whereby the gauge field couples to the axial fermion charge, too.

But one can insist upon invariance under time reversal $T$ (or $CP$ respectively). This discrete symmetry plays a role analogous to $C$ in the Higgs-Kibble model. Therefore the result is the same non-fermionic part of the Lagrangian as in ref. [5]2.

The phase of the complex field $\Phi$ is chosen such as to give the real part the quantum numbers of the vacuum which is assumed to be time reversal invariant. Consequently, the imaginary part is $T$ odd. Under BRS transformations the two parts of the Higgs field transform as

$$\phi = \frac{1}{\sqrt{2}} ((v + \varphi) + i \chi);$$

$$s \varphi = -q c \chi,$$

$$s \chi = q c (v + \varphi)$$

with the vacuum expectation value $\frac{1}{\sqrt{2}} v \equiv \langle 0 | \Phi | 0 \rangle$.

The following Hermitian Lorentz tensors that can be constructed from the fields are even (+) or odd (−) under $CP$ and $T$; space-time arguments and indices transform respectively.

|               | $CP$ | $T$ |
|---------------|------|-----|
| $\varphi, \bar{\psi} \psi, i \bar{\psi} \slashed{\partial} \psi, i \bar{\psi} \slashed{\gamma}_5 \psi, g^{\mu\nu}$ | +   | +   |
| $\chi, i \bar{\psi} \gamma_5 \psi, \bar{\psi} \gamma^{\mu} \psi, \epsilon_{\mu\nu\rho\sigma}, \partial_\mu A_\nu$ etc., $b$ | −   | −   |
| $\partial_\mu \varphi$ etc. | +   | −   |
| $\partial_\mu \chi$ etc., $\bar{\psi} \gamma^{\mu} \psi, \bar{\psi} \gamma^{\mu} \gamma_5 \psi, A^\mu$ | −   | +   |
| $s$ | −   | −   |
| $c, \bar{c}$ | +   | +   |

Lagrangian

In the framework of BPHZ renormalization [8] the definition of a quantized model is based on Zimmermann’s effective Lagrangian

$$L_{\text{eff}} := \sum_{i=1}^{n} \lambda_i M_i,$$

2 except for an additional BRS invariant in ref. [5] which is excluded here by introduction of the auxiliary field $b$ in the BRS transformations
where $\mathcal{M}_i$ are Hermitian, Lorentz invariant, homogeneous polynomials of the model fields, called in the following “monomials” by abuse of language. The parameters $\lambda_i$ have a perturbative expansion

$$\lambda_i = \sum_{l=0}^{\infty} \lambda_i^{(l)}.$$

(7)

To lowest order they are the renormalized parameters of the model and they are given by the classical Lagrangian. The parameters of higher order ($l > 0$) correspond to local, finite counterterms which compensate for the loop corrections at the points where the corresponding vertex functions are renormalized.

The task is to define all parameters $\lambda_i^{(l)}$ by imposing BRS invariance and the normalization conditions. In a power-counting renormalizable theory the number $n$ of the monomials is fixed, finite, and defined by imposing the power-counting dimensions of the monomials not to exceed the space-time dimension. The BPHZ regularization besides respects discrete symmetries. Thus that set of normalization conditions which defines the model in the tree approximation is necessary and sufficient to define all orders. The most general ansatz for eq. (6) is the one that has the right quantum numbers and works at the classical level.

A Lagrangian $\mathcal{L}$ always consists of a gauge invariant physical part $\mathcal{L}_{\text{inv}}$ which depends neither on the ghosts nor on the auxiliary field, and a gauge fixing and ghost part $i s \mathcal{Q}$ which is unphysical [6] [9]:

$$\mathcal{L} = \mathcal{L}_{\text{inv}}(\phi) + i s \mathcal{Q}(\phi, c, \bar{c}, b).$$

(8)

The latter part is separately BRS invariant because the BRS operator is nilpotent

$$s^2 = 0$$

(9)

where $s$ is Hermitian.

In addition to the four gauge invariants of the Higgs-Kibble model there are three invariants containing the Dirac field in this model: two kinetic terms, $i \bar{\psi} \slashed{D} \psi$ and $i \bar{\psi} \slashed{D} \gamma_5 \psi$, and a Yukawa coupling, $\bar{\psi}_R \psi_L \Phi^* + \bar{\psi}_L \psi_R \Phi$. The covariant derivative $D$ is given by

$$D_\mu \psi_R := (\partial_\mu + iq_R A_\mu) \psi_R,$$

$$D_\mu \Phi := (\partial_\mu + iq A_\mu) \Phi,$$

(10)

where $q = q_L - q_R$ because of the Yukawa coupling.

The non-vanishing of the normalization of the gauge field dependent part of $\mathcal{Q}$ is necessary in order to implement a gauge fixing and to make the propagation of the gauge field and the Goldstone field well-defined. Attention is drawn to the interesting fact that one is led to this in a natural way by the generality of the BRS invariant ansatz for $\mathcal{L}$. The function $\mathcal{Q}$ is odd under $CP$, has dimension 3, ghost number $-1$, and therefore the following form

$$\mathcal{Q} = \bar{c} \left(-\frac{\alpha}{2} b + \partial \cdot A + \zeta \chi + \vartheta \varphi \chi \right)$$

(11)
where \( \alpha, \zeta, \) and \( \vartheta \) are additional free parameters. Thus there are four unphysical BRS invariants. One may now choose a special class of gauge fixing without changing the physical content. First of all, one is only interested in a linear gauge function, i.e. \( \vartheta = 0. \) (12)

Secondly, the mixing propagator \( \langle 0 | T \partial \cdot A \chi | 0 \rangle \) vanishes by choice of \( \zeta = \alpha v. \) (13)

This is the so-called “restricted ’t Hooft gauge”. The \( b \)-propagator is skillfully “diagonalized” by the substitution \( b' := \alpha b - \partial \cdot A - \zeta \chi. \) (14)

After rescaling \( A_\mu \to e A_\mu, \) the Lagrangian gets the form

\[
\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - e A^\mu \varphi \overleftrightarrow{\partial_\mu} \chi \\
- \frac{1}{4} \kappa (\varphi^2 + \chi^2) - \kappa v \varphi (\varphi^2 + \chi^2) - \kappa v^2 \varphi^2 \\
+ \bar{\psi} \left( i \frac{\gamma_5}{2} \right) e A \frac{1 + \gamma_5 \varepsilon}{2} - m \left( 1 + \frac{\varphi + i \gamma_5 \chi}{v} \right) \psi \\
- \frac{1}{4} F^2 + \frac{1}{2} e^2 v^2 A \cdot A \left( 1 + 2 \frac{\varphi}{v} + \frac{\varphi^2 + \chi^2}{v^2} \right) \\
+ \frac{\lambda}{2 e^2} v^4 - \frac{\lambda}{2} (\partial \cdot A)^2 - \frac{1}{2} \frac{e^2 v^2}{\lambda} \chi^2 - \kappa v \partial \cdot (A \chi) \\
+ i \bar{c} \left( \square + \frac{e^2 v^2}{\lambda} \left( 1 + \frac{\varphi}{v} \right) \right) c
\]

with \( \lambda := e^2 / \alpha. \) The choice of the charge numbers is \( q_R = 1, q_L = 0. \)

At this stage one may impose a necessary and sufficient number of normalization conditions in view of higher orders of perturbation theory, namely in order to fix the wave functions, the vacuum expectation value of the physical Higgs field \( \langle 0 | \varphi | 0 \rangle = 0, \) the free parameters \( v, \kappa, m, e, \lambda, \zeta, \vartheta, \) and the ratio of the charge numbers in such a way, that at the classical level one gets the above Lagrangian.

The resulting propagator of the gauge field

\[
\int d^4x \, e^{ip \cdot x} \langle 0 | T A_\mu(0) A_\nu(x) | 0 \rangle = i \left( \frac{-g_{\mu \nu} + p_\mu p_\nu / e^2 v^2}{p^2 - e^2 v^2 + i \varepsilon} - i \frac{p_\mu p_\nu / e^2 v^2}{p^2 - e^2 v^2 / \lambda + i \varepsilon} \right)
\]

behaves like \( p^{-2} \) for large momentum \( p \)—it “scales” at high energy and the (non-) renormalizability is read off the vertices alone. In turn there is a propagating scalar ghost which should however be cancelled by the Goldstone boson with the propagator

\[
\int d^4x \, e^{ip \cdot x} \langle 0 | T \chi(0) \chi(x) | 0 \rangle = \frac{i}{p^2 - e^2 v^2 / \lambda + i \varepsilon}
\]

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3. This is identical with the gauge function from ref. [5].

4. Here and in the following \( \langle 0 | T \cdot \cdot | 0 \rangle \) denotes the free two-point function.
because of BRS invariance. This is quite similar to section III stage ii in ref. [2], which would be equivalent to \( \zeta = 0 \) here and also results in a scaling propagator. The advantage of the restricted 't Hooft gauge is that the scalar part of the gauge field and the Goldstone boson simultaneously become massive and do not cause infrared singularities. Thereby one is on the safe side, for the Goldstone boson (\( \chi \)) - ghost (\( \partial \cdot A \)) cancellation is not yet achieved in the non-invariant regularization procedure. It is established by renormalization by means of the Slavnov identity. Besides Faddeev-Popov fields have to be introduced for the compensation of the effect of the Goldstone mass.

2. Perturbation Theory of Higher Orders

Slavnov identity

Now the task is to define all parameters \( \lambda_{l}^{(j)} \) with \( l > 0 \). For that a generalization of the BRS transformation to higher orders in \( \hbar \) is needed. Because it is non-linear in the quantized and interacting fields (i.e. composite operators) due to the non-trivial Faddeev-Popov sector one has to add sources to the Lagrangian which generate the field transformations:

\[
\mathcal{L}_{\text{new}} = \mathcal{L}_{\text{old}} + q_{\alpha} s \phi^{\alpha},
\]

\[
q_{\alpha} s \phi^{\alpha} = \sigma c\chi - \tau c(v + \varphi) + i \bar{\rho} c \psi_{R} - i c \bar{\psi}_{R} \rho - \frac{1}{e} v^{\mu} \partial_{\mu} c.
\]

Dimensions and charges of the anti-Hermitian sources \( q_{\alpha} \) are given by

| dim  | gh |
|------|----|
| \( \sigma, \tau, v^{\mu} \) | 2   | 1  | -1 |
| \( \bar{\rho}, \rho \)       | 3/2 | 0  | -1 |

\( \mathcal{L}_{\text{new}} \) is BRS invariant if one requires \( s q_{\alpha} = 0 \). One has to introduce further normalization conditions for the new terms.

The generalized BRS transformation of the quantum vertex functional \( \Gamma[\phi; q] \)

\[
S(\Gamma) = \int d^{4}x \left( \frac{\delta \Gamma}{\delta \sigma} \frac{\delta \Gamma}{\delta \varphi} + \frac{\delta \Gamma}{\delta \tau} \frac{\delta \Gamma}{\delta \chi} + \frac{\delta \Gamma}{\delta \bar{\rho}_{\alpha}} \frac{\delta \Gamma}{\delta \psi_{\alpha}} + \frac{\delta \Gamma}{\delta \rho_{\alpha}} \frac{\delta \Gamma}{\delta \bar{\psi}_{\alpha}} + \frac{\delta \Gamma}{\delta v^{\mu}} \frac{\delta \Gamma}{\delta A_{\mu}} + i \frac{\delta \Gamma}{\delta c} \right)
\]

is the Slavnov transformation. The Slavnov identity which generalizes the BRS invariance reads

\[
S(\Gamma) = 0.
\]

The renormalized action principle for the Slavnov transformation

\[
S(\Gamma) = \Sigma^{(l)} + \mathcal{O}(\hbar^{l+1})
\]

is valid; \( \Sigma[\phi; q] \) is a local functional of ghost number one. There is now a local differential operator \( \mathcal{B} \) with

\[
\mathcal{B} \Sigma^{(l)} = 0
\]
which is nilpotent and identical with the classical BRS operator \( s \) if applied to the fields, \( \mathcal{B} \phi^\alpha = s \phi^\alpha \). Eq. (23) is the Wess-Zumino consistency condition [4].

Its solution has the form [9]

\[
\Sigma^{(l)} = A + \mathcal{B} \gamma, \quad A = \int d^4x \, A = \mathcal{B} \gamma',
\]

where \( A \) and \( \gamma \) are of formal order \( h^l \). This is quite analogous to the form of the Lagrangian (8). \( A \) is a “genuine” anomaly. The terms \( \mathcal{B} \gamma \) are not actually anomalous, i.e. in the sense of an inconsistency, because it is possible to absorb them recursively to all orders \( l \) in the course of the renormalization procedure. In this way the Slavnov identity can be fulfilled if there is no term \( A \). The question of the existence of a “genuine” anomaly \( A \) is a local cohomology problem and just answered by algebraic methods [9]. It is a property of the model and does not depend on the renormalization procedure.

**Cancellation of the Anomaly**

In this model (3) is no “genuine” anomaly—and there is no “genuine” anomaly at all because there is a function of the Higgs boson which transforms into const. \( \cdot c \), i.e. its phase relative to the vacuum

\[
\omega := v \arctan \frac{\chi}{v + \varphi}.
\]

It is asymptotically identical with the Goldstone field which has the inhomogeneous BRS transformation. In the case \( v = 0 \) this expansion does not exist. But with \( v > 0 \) the model contains a field \( \omega \) which transforms as

\[
s \omega = -v \, c,
\]

so the anomaly is \( \mathcal{B} \) exact:

\[
cF\bar{F} = \mathcal{B} \left( -\frac{\omega}{v} F\bar{F} \right).
\]

The BRS invariance violating vertex insertion \( \Sigma^{(1)}_{\text{chiral}} \) contributed by the diagrams shown in fig. 1 is computed to be

\[
\Sigma^{(1)}_{\text{chiral}} = \int d^4x \, \frac{e^2}{48\pi^2} cF\bar{F} = \mathcal{B} \int d^4x \, \frac{-e^2}{48\pi^2} \omega F\bar{F} = \mathcal{B} \gamma^{(1)}_{\text{chiral}}.
\]

(The upper vertices of the diagrams are terms of the insertion.)

The counterterm \(-\gamma_{\text{chiral}}\) consists of infinitely many monomials:

\[
\mathcal{M}_{\ell'} = \varphi^{m_{\ell'}} \chi^{2n_{\ell'} + 1} F\bar{F}, \quad m_{\ell'}, n_{\ell'} \in \{0, 1, 2, \ldots\}
\]

\((n \neq \infty \text{ any more in eq. (6)}\)). They are all power-counting non-renormalizable.
fig. I: The diagrams contributing to the chiral anomaly.

The triangles of solid lines denote the sum over the two fermion loops with resp. opposite direction.

fig. II: The diagrams with the unphysical poles and the anomaly which contribute to the fermion pair annihilation.

The dashed line denotes the Goldstone boson propagator and the full circle is the Wess-Zumino term.

Discussion

Before analyzing the renormalizability a physical scattering amplitude involving the unphysical poles and the anomaly shall be considered in order to see their cancellation on the one hand and the high-energy behaviour which is the origin of the non-renormalizability on the other hand.

The corresponding contributions of leading order to the fermion pair annihilation (fig. II) have also been considered by Gross and Jackiw [2]. As they already pointed out, the residue of the unphysical pole (here at $\left(p + q\right)^2 = \frac{e^2 v^2}{\lambda}$ in contrast to ref. [2]) is proportional to

$$i (p + q) \lambda \Gamma_{\lambda(0)}^{(1)} A_{\lambda(p)} \tilde{A}_{\sigma(q)} + ev \Gamma_{\chi(0)}^{(1)} A_{\lambda(p)} \tilde{A}_{\sigma(q)}.$$

This sum vanishes because the Slavnov identity in the one-loop approximation (or the
Ward identity resp.) is valid and thus the Goldstone boson ($\chi$) - ghost ($\partial \cdot A$) cancellation occurs. Diagrammatically this is so because the well-known anomalous violation of PCAC is cancelled by the Wess-Zumino counterterm $\frac{e^2}{48\pi^2}\chi F\tilde{F}$ which contributes locally to $G^{(1)}_{\chi\tilde{A}_\nu\tilde{A}_\sigma}$ with $-\frac{e^2}{12\pi^2}\epsilon^{\mu\nu\rho\sigma}p_\mu q_\rho$.

Whereas the high-energy behaviour of the complete pair annihilation amplitude without Wess-Zumino counterterm is as expected, the counterterm causes trouble however. In order to see this one can apply the criterion of “tree-unitarity” introduced by Cornwall, Levin, and Tiktopoulos [10]. It says that any physical $T$ matrix element in the tree approximation of a Lagrangian model with $N$ initial and final particles must not grow faster than $E^{4-N}$ in the limit of high center-of-momentum energy $E$ and fixed, non-exceptional angles in order not to exceed the Froissart bounds.

Because in the tree approximation the model is power-counting renormalizable it is formally “tree-unitary”. However, the considered amplitude which belongs to the first order in $\hbar$ does not fulfill the criterion. In the high-energy limit the counterterm contributes a factor proportional to $\epsilon^{0\nu\rho\sigma}\hat{p}_\rho E^2$ with $\hat{p}^\mu = (0, p/|p|)\mu$ and $\lim_{E \to \infty} p_\mu = \frac{E}{2} (1, p/|p|)\mu$. Therefore the corresponding contribution to the $T$ amplitude grows like $E^1$ in contrast to all other contributions which tend at most to constants (apart from logarithmical factors; note that spinors grow like $\sqrt{E}$).

Thus the requirement of BRS invariance of this anomalous gauge model violates the Froissart bounds in the one-loop approximation. One may view this as indication for the fact that perturbation theory fails above some energy [11] because above that energy the Wess-Zumino contribution which is of first order in $\hbar$ dominates for instance the pair annihilation.

3. Non-Renormalizability

As claimed in ref. [10] the on-shell property “tree-unitarity” is a necessary condition for renormalizability, thus the model cannot be renormalizable.

An essential input of the BPHZ regularization is the power-counting formula

$$d(\Gamma) = 4 - B - \frac{3}{2} F - \sum_a (4 - \delta_a)$$

with $\delta_a = b_a + \frac{3}{2} f_a + D_a + c_a$ (30)

($B$, $F$: external boson, fermion line; $b_a$, $f_a$ count the bosons or fermions resp. at the vertex, $D_a$ is the power of momenta belonging to the vertex, $c_a$ is a non-negative degree of oversubtraction, and the index $a$ numbers the vertices) which specifies the degree of superficial divergence of a Feynman diagram or sub-diagram $\Gamma$. Conventionally, the degree $c_a$ of oversubtraction is chosen such as to equate $\delta_a = 4$ and to determine the superficial divergence of a whole vertex function by the external lines only. This is possible in power-counting renormalizable theories, i.e. if all monomials $M_i$ have field dimension $b_a + \frac{3}{2} f_a + D_a \leq 4$.

5 who derived gauge invariance from high-energy unitarity bounds
In this model one has to consider the full formula (30). Vertex functions which are convergent up to first order in $\hbar$ may consequently become undetermined with growing order of perturbation theory because they contain the vertices of eq. (29) which make some contributing (sub-)diagrams divergent. One must therefore add further counte-
terms of field dimension $> 4$ to the effective Lagrangian with respect to these vertex functions.

The result is that already in second order perturbation theory new normalization con-
ditions become necessary. In order to see this it is sufficient to search for new BRS invariants of dimension $> 4$ which are linear combinations of (both old and new) counterterms of the corresponding order. In order to find out new counterterms one looks for diagrams which become divergent just because of the vertices (29) and which have external fields corresponding to really new terms. It was found by inspection that the unique divergent functions of second order whose counterterms are not determined by invariants of dimension $\leq 4$ have the structure shown in fig. III. The upper vertices originate from the monomials of (29) and are of formal order $\hbar^{1}$, i.e. just determined by eq. (28) without further corrections. The BRS invariants which have to be renormalized read

$$\bar{\psi} \sigma^{\mu \nu} (v + \varphi + i \gamma_{5} \chi) \psi F_{\mu \nu} f (|\Phi|^{2}).$$

(31)

Because of the arbitrariness of the function $f$ they are even infinitely many. The model is therefore non-renormalizable from the second order onwards in the following sense: The first order is still well-defined by the symmetry and the classical normalization conditions only but the higher orders cannot be defined by a finite set of normalization conditions at all.

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fig. III: Diagrams of second order
which make further normalization conditions necessary

One may ask if this simple and superficial inspection based on power-counting in the framework of the BPHZ renormalization scheme is actually satisfactory since one could attempt to reach renormalizability by pushing all “non-renormalizabilities” onto one field by formulating the model in terms of the Higgs phase $\omega$ and the radial part $\varrho$ (with $\langle 0 | \varrho | 0 \rangle = 0$) instead of Cartesian variables as fundamental fields. The kinetical Lagrangian term for the Higgs field would read

$$\frac{1}{2} \partial_{\mu} \varrho \partial^{\mu} \varrho + \frac{1}{2} \left(1 + \frac{\varrho}{v}\right)^{2} \left(\partial_{\mu} \omega \partial^{\mu} \omega - 2e v A^{\mu} \partial_{\mu} \omega + e^{2} v^{2} A^{2}\right)$$
and the Yukawa coupling
\[-m \left( 1 + \frac{\theta}{\nu} \right) \bar{\psi} \exp \left( i\gamma_5 \frac{\omega}{\nu} \right) \psi.\]

Because of loss of power-counting renormalizability the first order already needs additional normalization conditions in this case, and therefore this alternative is non-renormalizable from the first order onwards. However, the number of Wess-Zumino counterterms shrinks to one, cf. eqs. (27) and (28). Consequently there is only a finite number of normalization conditions which are additionally necessary for each order in \(\hbar\).

One could now try to give \(\omega\) power-counting dimension 0 by modifying the bilinear part of the Lagrangian with a view of proving the renormalizability of the complete model together with the Wess-Zumino term by power counting. However, exactly this modification is not possible without destroying the consistency because it would necessarily mean a propagator for \(\omega\)
\[
\int d^4 x \, e^{i p \cdot x} \langle 0 | T \omega(0) \omega(x) | 0 \rangle_0 \propto \frac{i}{p^2 - m_1^2 + i\epsilon} - \frac{i}{p^2 - m_2^2 + i\epsilon}
\]
which behaves like \(p^{-4}\) for large \(p\). Thus the model would be afflicted with an additional ghost field due to the “wrong” sign of the one part of the propagator. This ghost must be physical since an unphysical modification cannot alter the high-energy behaviour of a physical scattering amplitude like the discussed fermion pair annihilation.

4. Conclusion and Discussion

The Abelian Higgs-Kibble model with a chirally coupled fermion is an anomalous gauge model. It has been perturbatively quantized preserving the Slavnov identity in arbitrary orders in \(\hbar\). Since there is a Wess-Zumino counterterm this is possible because of the inhomogeneous gauge transformation of the Goldstone boson due to the SSB. Since BRS invariance is established one can easily transcribe the proof of the unitarity from ref. [5] or [6] to this model. Thus a consistent, unitary, Lorentz covariant, four-dimensional, massive gauge model is obtained.

It has been shown that the model is not renormalizable in the sense that it cannot be defined by a finite number of normalization conditions. The net result of imposing BRS invariance is only that one has pushed the non-renormalizability which at first appeared as bad-behaved vector boson propagator to second order in \(\hbar\). According to this it is expected that perturbation theory breaks down above a certain energy.

The shown cancellation of the anomaly is suggested as a method of quantizing anomalous gauge models in the sense of ref. [1]. The role of the “chiral field” is played here by the Higgs phase \(\omega\). In ref. [1] the method of decoupling additional anomaly-cancelling fermions is mentioned. That method has already been explicitly applied to the model under study in ref. [2]. The decoupling of the fermion with the other axial-vector charge by sending the corresponding Yukawa coupling to infinity leaves behind the same Wess-Zumino term as in this work. However, it is stressed that the decoupling method differs from the demonstrated one by the fact that the resulting effective action contains other non-renormalizable terms in addition to the Wess-Zumino term (see also ref. [12]). Even
if one is only interested in the perturbation series up to the first order which is still meaningful\(^6\) the two methods give different results. The decoupling method results in an effective low-energy theory per definition. Apparently, the shown quantization method which is a “minimal” anomaly cancellation in the sense that it manages without additional effective terms has no corresponding interpretation for high energies.

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5. References

[1] Jackiw R: Update on Anomalous Theories, MIT preprint CTP 1436 (1986)  
[2] Gross D J, Jackiw R: Phys. Rev. D6 (1972) 477  
[3] Lee B W: Phys. Rev. D5 (1972) 823  
   Adler S L: Lectures on Elementary Particles and Quantum Field Theory (Eds. Deser S, Grisaru M, Pendelton H; MIT Press 1970)  
[4] Wess J, Zumino B: Phys. Lett. 49B (1974) 52  
[5] Becchi C, Rouet A, Stora R: Comm. math. Phys. 42 (1975) 127  
[6] Piguet O, Rouet A: Phys. Rep. 76C (1981) 1  
   Becchi C: Relativity, Groups and Topology II (Les Houches 1983 Session XL, Eds. DeWitt B S, Stora R; North-Holland 1984) 789  
[7] Sibold K: Lecture Notes in Physics 303 (Eds. Breitenlohner P, Maison D, Sibold K; 1987) 32  
[8] Lowenstein J H: Renormalization Theory (Eds. Velo G, Wightman A S; 1975) 94  
   Lowenstein J H: Seminars on Renormalization Theory (given in Maryland 1972)  
[9] Brandt F, Dragon N, Kreuzer M: Nucl. Phys. B332 (1990) 224  
[10] Cornwall J M, Levin D N, Tiktopoulos G: Phys. Rev. D10 (1974) 1145  
[11] Llewellyn Smith C H: Phys. Lett. 46B (1973) 233  
[12] D’Hoker E, Farhi E: Nucl. Phys. B248 (1984) 59

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\(^6\) i.e. completely defined by the normalization conditions determining the tree approximation