The Rate Difference between the Weak Decays of $t$ and $\bar{t}$ in Supersymmetry

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**Abstract**

We find that the $CP$-violating asymmetry

$$\frac{\Gamma (t \to bW^+) - \Gamma (\bar{t} \to \bar{b}W^-)}{\Gamma (t \to bW^+) + \Gamma (\bar{t} \to \bar{b}W^-)}$$

at the one-loop order within the minimal supersymmetric extension of the standard model is of the order of few per cent for maximal $CP$ violation. It could be measured by considering the rate difference in the one-lepton events.
1. The weak decay of the $t$ quark has been increasingly advocated as a promising process for testing violations of $CP$ invariance that could arise in extending the standard model [1].

At the supercolliders, as well as at the Next Linear Collider (NLC), both $t$ and $\bar{t}$ will be copiously produced and their decay modes studied. Because of its large mass [2], the $t$ decays before forming any hadronic bound state. Events in which three or more jets are hard, there is missing transverse energy, and a lepton is identified in the final states can in principle be used to study the difference in the rate between the weak decays of the $t$ and $\bar{t}$ quarks. At the NLC, a sensitivity of $10^{-4}$ in branching ratios is not unconceivable [3].

$CP$ invariance can be violated in the minimal, supersymmetric extension of the standard model [4] to a larger degree than in the standard model. It is therefore of some interest to estimate the size of the $CP$-violating asymmetry

$$\xi_{CP} \equiv \frac{\Gamma(t \to bW^+) - \Gamma(\bar{t} \to \bar{b}W^-)}{\Gamma(t \to bW^+) + \Gamma(\bar{t} \to \bar{b}W^-)},$$

which can be induced at the one-loop level by supersymmetry.

Notice that the asymmetry (1) within the standard model implies $CPT$ violation if we neglect generation mixing, the effect of which cannot be larger than $10^{-4}$, the square of the largest off-diagonal element of the quark mixing matrix.

The corresponding supersymmetry-induced asymmetry in the decay of the $W$ is already ruled out by the present bounds on the supersymmetrical masses [6].

2. Let us then consider the minimal supersymmetric extension of the standard model. We neglect generation mixing. Hence, only three terms in the supersymmetric Lagrangian can give rise to $CP$-violating phases that cannot be rotated away [7]: The superpotential contains a complex coefficient $\mu$ in the term bilinear in the Higgs superfields. The soft supersymmetry-breaking operators introduce two further complex terms, the gaugino masses $\tilde{m}$ and the left- and right-handed squark mixing term.

The possible supersymmetric one-loop diagrams are depicted in Figs. 1 and 2.

Since only the imaginary part of the supersymmetric loop amplitude enters the asymmetry (1), the main contribution to it comes from the diagrams in which one
of the two on-shell internal particles is the lightest supersymmetric particle, that is the lightest neutralino. These are the diagrams of Fig. 1. The contribution of the other diagrams (see Fig. 2) is either strongly suppressed (for the diagram with a charged Higgs, which is proportional to the mass of the $b$ quark) or closed by the current experimental bounds on the corresponding supersymmetric particles [3]. We also neglect squark mixing, which would only make the calculation more involved without changing the order of magnitude of the final result.

To build the relevant diagrams, we use the Lagrangian

$$\mathcal{L} = L_{\tilde{q}\tilde{q}W} + L_{\tilde{q}\tilde{q}x^+} + L_{W^{-}\chi^0} + L_{\tilde{q}\tilde{q}x^0},$$  \hspace{1cm} (2)

where

$$L_{\tilde{q}\tilde{q}W} = -\frac{ig}{\sqrt{2}} W^a_\alpha \left( \bar{\tilde{b}}_L \gamma^\alpha \tilde{t}_L \right) + H.c.,$$  \hspace{1cm} (3)

$$L_{\tilde{q}\tilde{q}x^+} = \frac{g}{2} \sum_i \left\{ \bar{\tilde{t}}_i \left[ U_{i1}(1 + \gamma_5) - \frac{m_t}{\sqrt{2}m_W \sin \beta} V_{i2}^*(1 - \gamma_5) \right] \chi^+_i \bar{\tilde{b}}_L ight. \\
+ \left[ V_{i1}(1 + \gamma_5) - \frac{m_b}{\sqrt{2}m_W \cos \beta} U_{i2}^*(1 - \gamma_5) \right] \chi^+_{i'} \tilde{t}_L \\
- \frac{m_b}{\sqrt{2}m_W \cos \beta} U_{i2} \bar{\tilde{t}}_i (1 + \gamma_5) \chi^+_{i'} \bar{\tilde{t}}_L \\
- \frac{m_t}{\sqrt{2}m_W \sin \beta} V_{i2} \bar{\tilde{b}}_i (1 + \gamma_5) \chi^+_{i'} \tilde{t}_L \left\} \right\},$$  \hspace{1cm} (4)

$$L_{W^{-}\chi^0} = \frac{g}{2} W^a_\alpha \sum_{k,i} \tilde{\chi}^0_k \gamma^\alpha \left[ O^L_{ki}(1 - \gamma_5) + O^R_{ki}(1 + \gamma_5) \right] \chi^+_i + H.c.,$$  \hspace{1cm} (5)

$$L_{\tilde{q}\tilde{q}x^0} = \frac{g}{2} \sum_{k,f} \bar{\tilde{q}}_f \left[ f^f_k(1 + \gamma_5) - \frac{\sqrt{2}m_f}{2m_W B_f} N_{k,5-f}\gamma_5 N^*_k(1 - \gamma_5) \right] \chi^0_k \tilde{q}_f L \\
+ \frac{g}{2} \sum_{k,f} \bar{\tilde{q}}_f \left[ g^f_k(1 - \gamma_5) - \frac{\sqrt{2}m_f}{2m_W B_f} N_{k,5-f}^* \gamma_5 N^*_k(1 + \gamma_5) \right] \chi^0_k \tilde{q}_f R + H.c.,$$  \hspace{1cm} (6)

and

$$f^f_k \equiv -\sqrt{2} \left[ T_{3f} N_{k2} - \tan \theta_W (T_{3f} - e_f) N_{k1} \right]; \hspace{1cm} g^f_k \equiv \sqrt{2} \tan \theta_W e_f N^*_{k1},$$

$$O^L_{ki} \equiv -\frac{1}{\sqrt{2}} N_{k4} V^*_i + N_{k2} V^*_i,$$

$$O^R_{ki} \equiv \frac{1}{\sqrt{2}} N^*_{k3} U_{i2} + N^*_{k2} U^*_i,$$  \hspace{1cm} (7)

In (3) and below, $\chi^0$ and $\chi^+$ are the four-component spinors of the neutralino and chargino physical fields, $\chi^+_{i'}$ are the chargino charge-conjugate states, $t$ and $b$ are
the quark fields and \( \tilde{t}, \tilde{b} \) their scalar partners. The index \( f \) stands for the flavor of, respectively, the \( t \) and \( b \) quark; therefore, \( T_3^f, e_f \) and \( m_f \) are, respectively, the third component of the weak isospin, the charge and the mass of the corresponding quark. The mass mixing matrices \( N \) for the neutralinos, \( U \) and \( V \) for the charginos, contain the \( CP \)-violating phases of \( \mu \) and \( \tilde{m} \).

3. By neglecting the effect of the mass of the \( b \) quark, we can write the amplitude for the decay as

\[
\mathcal{M} = \frac{g}{2\sqrt{2}} \bar{u}(p') \left[ \gamma_\alpha (1 - \gamma_5) + A\gamma_\alpha (1 - \gamma_5) + B\gamma_\alpha (1 + \gamma_5) \right] u(p)\epsilon^\alpha(q),
\]

where the coefficients \( A \) and \( B \) contain the radiative corrections; \( p \) is the momentum of the decaying \( t \) quark, \( p' \) of the \( b \) and \( P \equiv p + p' \). The width is therefore proportional to

\[
|\mathcal{M}|^2 = \frac{g^2}{8} p \cdot p' \left[ \left(-2 + \frac{m_t^2}{m_w^2}\right) (1 + 2 \text{Re} A) + 2 \text{Re} B m_t \left(-1 + \frac{m_t^2}{m_w^2}\right) \right].
\]

The supersymmetric one-loop contribution, arising from the diagrams in Fig. 1, can thus be denoted as two sums

\[
A = A_a + A_b + A_c \quad \text{and} \quad B = B_a + B_b + B_c,
\]

the subscripts following the labelling of the diagrams in Fig. 1. A straightforward computation by means of (2) gives

\[
A_a = \sqrt{2}g^2 V_{i1} \left\{ \tilde{m}_i^+ \tilde{m}_k^0 O^l_{ki} f^{l*}_{k} I - O^R_{ki} f^{l*}_{k} \right\} \left[ m_i^2 (a_1 + a_2 + c_4 + 2c_1 + c_2 - 4c_3) - 2p \cdot p' (c_1 - c_2) \right] - \frac{\sqrt{2} m_t^2}{2 \sin \beta} N^*_{k4} \left[ \tilde{m}_i^+ m_w^2 O^L_{ki}(a_1 + a_2 + I) - \tilde{m}_k^0 m_w^2 O^R_{ki}(a_1 + a_2) \right]
\]

\[
A_b = -2g^2 m_t^2 f^{l*}_{j} f^{l*}_{k} c_3'
\]

\[
A_c = g^2 m_t^2 \sin \beta V_{i2} \left\{ \tilde{m}_i^+ \tilde{m}_k^0 O^l_{ki}(a_1 + a_2 + I) - \tilde{m}_k^0 m_w^2 O^L_{ki}(a_1 + a_2) \right\}
\]
and

\[ B_a = \sqrt{2}g^2V_1m_t \left\{ O^R_{ki}f_k^* (a_1 + a_2 + 2c_1 + 2c_2) \\
+ \sqrt{2} \frac{2}{2 \sin \beta} m^+_{k4} O^L_{ki}(a_1 - a_2) - \frac{m^0_{k4}}{m_W} O^R_{ki}(a_1 + a_2) \right\} \]

\[ B_b = -g^2m_t f^b_k \left[ f^*_k(2c'_1 + 2c'_2 + a'_1 + a'_2) - \frac{\sqrt{2}m^0_{k4}}{2m_W \sin \beta} N^*_{k4}(2a'_2 + I') \right] \]

\[ B_c = -g^2m_t \left\{ \frac{g^*_{k2}}{m_W} \left[ \frac{m^0_{k4}}{m_W} O^L_{ki}(a_1 + a_2) - \frac{m^0_{k4}}{m_W} O^R_{ki}(a_1 - a_2) \right] \\
- \sqrt{2} \frac{m^2_t}{m^2_W} \right\} N^*_{k4} \left[ O^L_{ki}(a_1 + a_2 + 2c_1 + c_2) \right] \}

(12)

The coefficients \( I, a_i \) and \( c_i \), as well as the primed ones, are defined by the loop momentum integrals as follows:

\[ a_1 = \frac{A}{q^2} - B \frac{P \cdot q}{P^2 q^2} \]

\[ a_2 = \frac{B}{P^2} \]

\[ c_1 = \frac{E}{q^2 P^2} - 3 \frac{P \cdot q}{2 q^2 P^2} \left[ \frac{C}{P^2} - \frac{1}{3} \left( \frac{F - D}{q^2} \right) \right] \]

\[ c_2 = \frac{3}{2 P^2} \left[ \frac{C}{P^2} - \frac{1}{3} \left( \frac{F - D}{q^2} \right) \right] \]

\[ c_3 = -\frac{1}{2m^2_t} \left[ \frac{C}{P^2} - \frac{F}{q^2} + \frac{D}{q^2} \right] \]

\[ c_4 = -\frac{1}{2q^2} \left[ -2 \frac{D}{q^2} + 4 \frac{P \cdot q}{q^2 P^2} E - \frac{C}{P^2} \left( 1 + 3 \frac{(P \cdot q)^2}{q^2 P^2} \right) \\
+ \left( 1 + \frac{(P \cdot q)^2}{q^2 P^2} \right) \left( F - \frac{D}{q^2} \right) \right] \]

(13)

where

\[ \{ I, A, B, C, D, E, F \} \equiv \]

\[ 8\pi \int \frac{d^4k}{(2\pi)^4} \left\{ 1, (k \cdot q), (k \cdot \hat{P}), (k \cdot \hat{P})^2, (k \cdot q)^2, (k \cdot q)(k \cdot \hat{P}), k^2 \right\} \frac{k^2 - m_1^2}{[(k + p')^2 - m_2^2][(k + p)^2 - m_3^2]} \]

(14)

each coefficient being defined by the corresponding term inside the curly brackets.

In the integrals above

\[ m_1 = \tilde{m}_{\bar{q}} \quad m_2 = \tilde{m}_i^+ \quad m_3 = \tilde{m}_k^0 \]

(15)
for the diagrams of Figs. 1(a) and 1(c), and
\[ m_1 = \tilde{m}_k^0 \quad m_2 = \tilde{m}_q \quad m_3 = \tilde{m}_\tilde{q} \]  \hfill (16)
for the diagrams of Fig. 1(b), thus giving rise to the primed coefficients; the two orthogonal momenta are defined as follows:
\[ q = p' - p \quad \text{and} \quad \tilde{P} = P - \frac{P \cdot q}{q^2} q. \]  \hfill (17)

The asymmetry (1) is now readily obtained by using (9) and is
\[ \xi_{CP} = 2 \left| \text{Re} \ A + \frac{-1 + m_t^2/m_W^2}{-2 + m_t^2/m_W^2} m_t \text{Re} \ B \right|. \]  \hfill (18)

Equation (18) shows that, in order to have a \( \xi_{CP} \) different from zero, we need, at the same time, a non-vanishing absorptive part of the loop integrals and a \( CP \)-violating imaginary coupling in the Lagrangian.

4. The contribution of any of the possible supersymmetric \( CP \)-violating phases can thus be computed. However, we would like to have a reliable estimate of the effect without having to commit ourselves to a definite model of supersymmetry breaking. A possible way is the following. We consider the case in which there is one common phase \( \delta_{CP} \) for both the chargino and the neutralino mixing matrices. Accordingly, we can factorize out the phase and take the matrix elements to be all equal. Because of the threshold in the absorptive part of the loop integral, in the sum over the neutralino states we keep only the lightest neutralino; we sum over all chargino states, which we take to be degenerate in mass (and therefore giving no contribution to \( \delta_{CP} \)).

We still have many terms. To obtain a reliable estimate, we use Table 1, where we have computed the coefficients \( I_i, a_i, c_i \) and the primed ones numerically and listed the values for a possible choice of the supersymmetric masses and different values of \( m_t \).

For maximal \( CP \) violation (\( \sin \delta_{CP} = 1 \)), we take an averaged value \(|V_{i1,2}O^{L,R}f_k| = |V_{i2}O^{R,L}g_k| = |V_{i1,2}N_{k3}O^{L,R}| = 1/4 \) and \(|N_{k4}f_k| = \|f_k\| = 1/2 \) for the mixing matrix elements. For \( \sin^2 \beta = 1/2, \ m_t \simeq 130 \ \text{GeV}, \ m_{\tilde{q}} = m_{\chi^0} = 100 \ \text{GeV} \) and \( m_{\chi^0} = 18 \ \text{GeV} \) (at the experimental bound), we obtain
\[ \xi_{CP} = 1.80 \times \alpha_w \simeq 0.05, \]  \hfill (19)
that is, an asymmetry of five per cent.

The asymmetry does not vary by much within the present possible experimental range of $m_t$; while the two terms $A$ and $B$ independently grow for larger values of $m_t$, they enter in the asymmetry with the opposite sign, so that, for example at $m_t = 150$ GeV, the asymmetry is slightly smaller, being of three per cent. It becomes smaller and eventually vanishes as we approach the threshold, where the masses of the on-shell supersymmetric particles are taken close to the value of the mass of the $t$ quark.

The study of such an asymmetry can be useful in providing indirect evidence for the existence of supersymmetry and in setting new bounds on the size of supersymmetric phases [8].

E.C.’s work has been partially supported by the Bulgarian National Science Foundation, Grant Ph-16.
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Table 1: $\tilde{m}^0 = 18$ GeV and $\tilde{m}^+ = m_{\tilde{q}} = 100$ GeV

| $m_t$ | $I$       | $a_1$  | $a_2$  | $c_1$  | $c_2$  | $c_3$  | $c_4$  |
|-------|-----------|--------|--------|--------|--------|--------|--------|
| 120   | $-2.0 \times 10^{-5}$ | $-8.2 \times 10^{-6}$ | $-8.4 \times 10^{-6}$ | $-3.3 \times 10^{-6}$ | $-3.5 \times 10^{-6}$ | $4.4 \times 10^{-8}$ | $1.2 \times 10^{-5}$ |
| 130   | $-4.0 \times 10^{-5}$ | $-1.5 \times 10^{-5}$ | $-1.6 \times 10^{-5}$ | $-4.5 \times 10^{-6}$ | $-6.7 \times 10^{-6}$ | $5.3 \times 10^{-7}$ | $3.1 \times 10^{-5}$ |
| 150   | $-4.7 \times 10^{-5}$ | $-1.4 \times 10^{-5}$ | $-1.8 \times 10^{-5}$ | $-8.3 \times 10^{-7}$ | $-7.3 \times 10^{-6}$ | $1.6 \times 10^{-6}$ | $3.6 \times 10^{-5}$ |

| $m_t$ | $I'$      | $a_1'$ | $a_2'$ | $c_1'$ | $c_2'$ | $c_3'$ | $c_4'$ |
|-------|-----------|--------|--------|--------|--------|--------|--------|
| 120   | $1.6 \times 10^{-5}$ | $1.4 \times 10^{-6}$ | $1.3 \times 10^{-6}$ | $1.3 \times 10^{-7}$ | $1.1 \times 10^{-7}$ | $-3.5 \times 10^{-8}$ | $-7.5 \times 10^{-7}$ |
| 130   | $4.5 \times 10^{-5}$ | $6.0 \times 10^{-6}$ | $4.1 \times 10^{-6}$ | $1.2 \times 10^{-6}$ | $4.8 \times 10^{-7}$ | $-5.9 \times 10^{-7}$ | $-8.9 \times 10^{-6}$ |
| 150   | $1.4 \times 10^{-4}$ | $4.5 \times 10^{-5}$ | $8.6 \times 10^{-6}$ | $2.3 \times 10^{-5}$ | $5.9 \times 10^{-7}$ | $-4.8 \times 10^{-6}$ | $-1.6 \times 10^{-4}$ |
Figure captions:

Fig. 1: The diagrams relevant in computing $\xi_{CP}$.

Fig 2: Other possible diagrams, which are closed by threshold; $\tilde{\lambda}$ is the gluino, $H$ the charged Higgs.