A more general treatment of the philosophy of physics and the existence of universes

Jonathan M. M. Hall

University of Adelaide, Adelaide, South Australia 5005, Australia

Abstract

Natural philosophy necessarily combines the process of scientific observation with an abstract (and usually symbolic) framework, which provides a logical structure to the development of a scientific theory. The metaphysical underpinning of science includes statements about the process of science itself, and the nature of both the philosophical and material objects involved in a scientific investigation. By developing a formalism for an abstract mathematical description of inherently non-mathematical, physical objects, an attempt is made to clarify the mechanisms and implications of the philosophical tool of Ansatz. Outcomes of the analysis include a possible explanation for the philosophical issue of the ‘unreasonable effectiveness’ of mathematics as raised by Wigner, and an investigation into formal definitions of the terms: principles, evidence, existence and universes that are consistent with the conventions used in physics. It is found that the formalism places restrictions on the mathematical properties of objects that represent the tools and terms mentioned above. This allows one to make testable predictions regarding physics itself (where the nature of the tools of investigation is now entirely abstract) just as scientific theories make predictions about the universe at hand. That is, the mathematical structure of objects defined within the new formalism has philosophical consequences (via logical arguments) that lead to profound insights into the nature of the universe, which may serve to guide the course of future investigations in science and philosophy, and precipitate inspiring new avenues of research.

*jonathan.hall@adelaide.edu.au
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1 Introduction

The study of physics requires both scientific observation and philosophy. The tenants of science and its axioms of operation are not themselves scientific statements, but philosophical statements. The profound philosophical insight precipitating the birth of physics was that scientific observations and philosophical constructs, such as logic and reasoning, could be married together in a way that allowed one to make predictions of observations (in science) based on theorems and proofs (in philosophy). This natural philosophy requires a philosophical ‘leap’, in which one makes an assumption or guess about what abstract framework applies most correctly. Such a leap, called Ansatz, is usually arrived at through inspiration and an integrated usage of faculties of the mind, rather than a programmatic application of certain axioms. Nevertheless, a programmatic approach allows enumeration of the details of a mathematical system. It seems prudent to apply a programmatic approach to the notion of Ansatz itself and to clarify its process metaphysically, in order to gain a deeper understanding of how it is used in practice in science; but first of all, let us begin with the inspiration.

2 A metaphysical approach

In this work, a programme is laid out for addressing the philosophical mechanism of Ansatz. In physics, a scientific prediction is made firstly by arriving at a principle, usually at least partly mathematical in nature. The mathematical formulation is then guessed to hold in particular physical situations. The key philosophical process involved is exactly this ‘projecting’ or ‘matching’ of the self-contained mathematical formulation with the underlying principles of the universe. No proof is deemed possible outside the mathematical framework, for proof, as an abstract entity, is an inherent feature of a mathematical (and philosophical) viewpoint. Indeed, it is difficult to imagine what tools a proof-like verification in a non-mathematical context may use or require.

It may be that the current lack of clarity in the philosophical mechanism involved in applying mathematical principles to the universe has implications for further research in physics. For example, in fine-tuning problems of the Standard Model of particle interactions (such as for the mass of the Higgs boson\[1\] and the magnitude of the cosmological constant\[3\]) it has been spec-
ulated that the existence of multiple universes may alleviate the mystery surrounding them, in that a mechanism for obtaining the seemingly finely-tuned value of the quantity would no longer be required- it simply arises statistically. However, if such universes are causally disconnected, e.g. in disjoint ‘bubbles’ in Linde’s chaotic inflation framework, there is a great challenge in even demonstrating such universes’ existence, and therefore draws into question the rather elaborate programme of postulating them. Setting aside for the time being the use of approaches that constitute novel applications of known theories, such as the exploitation of quantum entanglement to obtain information about the existence of other universes, a more abstract and philosophical approach is postulated in this paper.

2.1 The universality of mathematics

Outside our universe, one is at a loss to intuit exactly which physical principles continue to hold. For example, could one assume a Minkowski geometry, and a causality akin to our current understanding, to hold for other universes and the ‘spaces between’, if indeed the universes are connected by some sort of spacetime? Indeed, such questions are perhaps too speculative to lead to any real progress; however, if one takes the view of Mathematical Realism, which often underpins the practice of physics, as argued in Section and the tool of Ansatz, one can at least identify mathematical principles as principles that should hold in any physical situation- our universe, or any other. This viewpoint is more closely reminiscent of Level IV in Tegmark’s taxonomy of universes. One may imagine that mathematical theorems and logical reasoning hold in all situations, and that all ‘universes’ (a term in need of a careful definition to match closely with the sense it is meant in the practice of physics) are subject to mathematical inquiry. In that case, mathematics (and indeed, our own reasoning) may act as a ‘telescope to beyond the universe’ in exactly the situation where all other senses and tools are drawn into question.

To achieve the goal of examining the process of the Ansatz- of matching a mathematical idea to a non-mathematical entity (or phenomenon), one needs to be able to define a non-mathematical object abstractly, or mathematically. Of course, such an entity that can be written down and manipulated is indeed not ‘non-mathematical’. This is so in the same way that, in daily speech, an object can be referred to only by making an abstraction (c.f. ‘this object’, ‘what is meant by this object’ ‘what is meant by the phrase ‘what is meant by this object’
This nesting feature is no real stumbling block, as one can simply identify it as an attribute of a particular class of abstractions - those representing non-mathematical objects. Thus a rudimentary but accurate formulation of non-mathematical objects in a mathematical way will form the skeleton outline for a new and fairly general formalism.

After developing a mathematics of non-mathematical objects, one could then apply it to a simple test case. Using the formalism, one could derive a process by which an object is connected or related somehow to its description, using only the theorems and properties known to hold in the new framework. The formalism could then be applied to the search for other universes, and the development of a procedure to identify properties of such universes. In doing so, one could make a real discovery so long as the phenomenological properties are not introduced ‘by hand’. This follows the ethos of physics, whereby an inspired principle (or principles) is followed, sometimes superficially remote from a phenomenon being studied, but which has profound implications not always perceived contemporaneously (and not introduced artificially), which ultimately guide the course of an inquiry or experiment.

There is an additional motivation behind this programme beyond addressing the mechanism of the Ansatz, which is to attempt to clarify philosophically Wigner’s ‘unreasonable effectiveness’ of mathematics itself. It is the hope of this paper to identify this kind of ‘effectiveness’ as a kind of fine-tuning problem, i.e. that it is simply a feature that naturally arises from the structure of the new formalism.

### 2.2 Evidences

In the special situation where one uses mathematical constructs exclusively, the type of evidence required for a new discovery would also need to be mathematical in nature, and testing that it satisfies the necessary requirements to count as evidence in the usual scientific sense could be achieved by using mathematical tools within the new framework. To explain how this might be done, consider that evidence is usually taken to mean an observation (or collection of observations) about the universe that supports the implications of a mathematical formulation prescribed by a particular theory. Therefore, it is necessary to have a strict separation between objects that are considered ‘real/existing in the universe’, and those that are true mathematical statements that may be applied or projected (correctly or otherwise) onto the universe.

Note that, for evidence in the usual sense, any observations experienced by the scientist
are indeed abstractions also. For example, in examining an object, photons reflected from its surface can interact in the eye to produce a signal in the brain, and the interpretation of such a signal is an object of an entirely different nature to that of the actual photons themselves—much observational data is, in fact, discarded, and most crucially, the observation is then fitted into an abstract framework constructed in the mind. In a very proper sense, the more abstract is the more tangible to experience, and the more material is the more alien to experience. Therefore, it seems reasonable to suggest that a definition of evidence in familiar scientific settings is already equivalent to evidence in an entirely mathematical framework; in fact, the distinction between the two is purely convention.

3 Mathematical Realism

3.1 Historical Background

In the ‘world of ideals’ (as developed from the notion of Plato’s ‘universals’[14] rather than Berkeley’s Idealism[15]) there are certain abstract objects (‘labels’ or ‘pointers’), which refer to material objects. The Ansatz arises by guessing and then assuming a particular connection between those pointers and other abstract objects. This allows material objects to be entirely objective, (to avoid solipsism), but also entirely subservient in some sense, to abstract objects. An observer can only indirectly interact via interpretation. Thus, the abstract affects the abstract, and abstract the physical.

One might argue, contrarily, that entities existing in the abstract mind are altered via natural or material means[16] Certain mental states are invoked upon interpretation of the empirical, regardless of the existence of patterns, which are abstractions (and that this would be true even if the universe were ‘unreasonable’—not in general amenable to rational inquiry). It is the point of view expounded in this treatise, however, that it is not true that material objects can interact so directly with other material objects, but only indirectly, since the interactions themselves would otherwise need to be materials. Yet an interaction is necessarily an abstract link (i.e. it has to follow some pattern, rule or law) even when at rest. That is, the notion of ‘interaction’ is necessarily abstract. It is the feature of positing only indirect relation among physical objects that takes the form of a view opposite to that of epiphenomenalism[16][17] The
difference in viewpoint may appear to hinge on the semantics of the terms ‘interaction’ and ‘abstract’, but the goal is simply to characterise the oft-proved successful methods of physics *as already applied* in practice, through a particular choice in philosophy. Our goal is, by simply identifying (and thus labelling) the salient features of the philosophy, an investigation into the more general (and philosophical) aspects of the practice of physics can be conducted. On the other hand, the discussion of the soundness of such a philosophy on other grounds constitutes a tangent topic, and the presentation of a complete enumeration of various emendations or contrary viewpoints on the choice of philosophy will be left for further investigation.

### 3.2 Contemporary developments

The evolution of metaphysics from rudimentary Cartesian Dualism to that proposed by Bohm demonstrates the usefulness of a mathematical viewpoint in clarifying an enriching philosophical ideas as they pertain to physics, and the universe as a whole. Abstract relationships are centralised, and underlying principles of matter, rather than a catalogue of the immediate properties, are interpreted to have the greater influence in accessing the fundamental nature of the physical world. The shift in perspective is that the simple and elegant descriptions of a physical system are those which incorporate its seemingly disparate features into an integrated whole. One then goes on to postulate a relationship between the physical object in question and the machinery of its abstract description.

Similar philosophical views have found success in the field of neuroscience, such as the work of Damásio in characterising consciousness. Instead of treating the mind and body as separate entities, one postulates an integrated system, whereby mechanisms in the body, such as internal and external stimuli, result in neurological expressions such as emotion. Emotion and reason are thus brought together on equal footing, since both actions are the result of comparing and evaluating a variety of stimuli, including other emotions, to arrive at a response. That is, from a modern perspective, just as relationships between physical objects are fundamental in characterising the intangible properties of their whole, it is the abstract relationships between faculties in the body and brain, such as interactions and stimuli, that characterise consciousness.

The identification of phenomena with abstract descriptions, such as behaviour and interactions, was formalised in Putnam and Fodor’s Functionalism. Functionalism provides the
ability to consider indirect or ‘second order’ explanations for the nature of objects. Unlike Physicalism, which identifies the nature of objects with the instances themselves that occur in the real-world, Functionalism entails the generalisation of the objects in terms of their functional behaviour. These more general classes of object are identified by the features that all relevant instances of the object have in common, and so the nature of the object becomes more ubiquitous, even if more abstract. This may be nothing more than a semantic shift, in cases where one is at liberty to allow the definitions of certain abstractions more scope as needed (such as pain or consciousness), so that they more closely match their use in daily human endeavours. Further abstractions, such as ‘causes’, an integral part of many areas of science, follow naturally.

Taken on its own, Functionalism represents a deprecated metaphysic, insufficient for a complete account of internal states of a physical system, and is therefore commonly employed simultaneously with another metaphysic (such as Physicalism). By not providing for a ‘real’ or material existence independent of an object’s functional behaviour, a bare Functional philosophy is not wholly suitable for describing the process of physics, which involves identifying material objects whilst projected upon them an Ansatz from some (abstract) theory.

As an example of the shift in perspective that Functionalism brings, consider the following scenario of an abstract entity based on real-world observations, such as an emotion/state of affairs, etc., whose cause is in want of identifying. Let this object be denoted a ‘feature’. Instead of the cause simply being identified specific phenomenon, or a mechanism based on the real world, the cause is characterised by an abstract object, $G$, which represents a collection of pointers to the relevant parts of the mechanism. If we posit, for the moment, an abstract interpreting function, $i : R \to A$, relating the real-world, $R$, to the world of ideals, $A$, and similarly, a pointer function, $r : A \to R$, one can establish a relationship between the cause and the feature. For a mechanism, $M$, a set of features, $F$, and an element $g_1 \in G$, define $M = r(g_1)$, which resides in $r(G)$, and then $i(r(g_1)) = i(M) = F$. In this (fairly loose) symbolic description, the features and the cause are related in the statement $i(r(G)) = F$. The ‘reverse-epiphenomenal’ philosophy, akin to Interactionism, is to realise that the relation itself between the two entities is an abstract one, whose attributes it will benefit us to characterise.
4 Formalism

In this section, a new formalism is outlined in order to capture the essential features of the philosophical problem at hand. A set of very general abstraction operators are defined, such that they may act upon each other in composition. By introducing another special kind of operation, the projection $\mathcal{P}$, general objects may be constructed such that they, at the outset, obey the basic principles expected to hold for objects and attributes used in a recognisable context, such as in language. To avoid semantic trouble, when one is free to assume or assign a property in a given context, the choice made is that which is most closely aligned with ‘what is commonly understood’ by a term. Note that other definitions are (unless logically non-viable) completely acceptable also- it is simply a choice of convenience to try to align the concepts chosen to be investigated with those of a language (such as a spoken or written language). In fact, it is judicious to do so, given that any philosophical problems one may wish to address are usually cast in such a language.

4.1 Projective algebra and abstraction classes

Though Cantor’s Theorem\textsuperscript{25} prohibits a consistent scheme classifying the space of all such abstract entities, (as echoed by Schmidhuber\textsuperscript{5}), the abstractions considered here are limited to a set, $W$, of ‘world objects’, representing a set of a specific type of object with certain (very general) properties. Very little mandatory structure for the objects, $w$, inhabiting $W$ is assumed, and they may be represented by a set, a group or other more specific mathematical objects. Thus, one may define $W$ in a consistent fashion using an appropriate axiomization, such as that of ZFC\textsuperscript{26} so long as none of the properties of the formalism is contravened.

In Section\textsuperscript{4.2.3} the properties of the real-world objects $w$ are clarified. Some basic rules of composition are assumed, but the spaces mapped-into in doing so are simply definitions rather than theorems; the tone of this work is not to impose any more specific details on the framework than is required in order to fulfill the aforementioned goals, namely, the construction of a mathematical-like theory in order to address the mechanism of Ansätze. Other mathematical formulations for obtaining general information about a system, such as Deutsch’s Constructor Theory\textsuperscript{27}, take a similar approach in determining suitable definitions for objects required for certain tasks in an inquiry.
4.1.1 The labelling principle

Firstly, the projection operator, \( \mathcal{P} \), obeys what shall be known as the *labelling principle*:

\[
\mathcal{P} \circ \mathcal{P} = \mathcal{P}.
\] (1)

A direct consequence of the labelling principle is that \( \mathcal{P} \) has no inverse, \( \mathcal{P}^{-1} \).

*Proof:* Assume \( \mathcal{P}^{-1} \) exists. Then:

\[
\mathcal{P} \circ \mathcal{P}^{-1} = 1
\]

\[
\neq 1. \text{ Therefore, } \mathcal{P}^{-1} \text{ DNE.}
\]

where 1 is the identity operator. An equivalent argument follows for an operator \( \mathcal{P}^{-1} \) acting on the left of \( \mathcal{P} \).

The projection operator may be applied to a world object \( w \), and the resultant form, \( \mathcal{P}(w) \), constitutes a new object, inhabiting a different space from that of \( w \). Firstly, the consequences of the lack of inverse of \( \mathcal{P} \) directly affect the projected space \( \mathcal{P}W \), which will be interpreted philosophically in the next section. Suffice to say, the judicious design of \( \mathcal{P}W \) lends itself to a particular view of abstractions, whereby very little information can be gained from an object in the real world *directly*, as expostulated regarding the definition of evidence, in Section 2.2.

4.1.2 Abstractions

The notion of ‘abstraction’ is codified by postulating a certain operator \( \mathcal{A} \), which may act on objects residing in a space \( W \), much like the projection operator. It will have, however, different properties to those of the projection operator. Using the abstraction operator, one is able to go ‘up a level’, \( (\mathcal{A} \circ \mathcal{A}(w) \neq \mathcal{A}(w)) \), and establish new features of the object \( w \). The sequential application of the abstraction operator creates a chain, in a reminiscent fashion to that of (co-)homologies, however, the properties of the abstraction operators are more general.
One may define the abstraction classes, $\Omega^i$, as

\begin{align*}
1 & \in \Omega^0, \quad \text{(2)} \\
\mathcal{A} & \in \Omega^1, \quad \text{(3)} \\
\mathcal{A} \circ \mathcal{A} & \in \Omega^2, \quad \text{(4)} \\
& \vdots \quad \text{(5)}
\end{align*}

For the moment, the properties of the classes are no more extensive than, say, a collection of elements (the operators). The range of $\mathcal{A}$, namely $\Omega^1 \equiv \{ \mathcal{A}_i \}$, is a class of any type of $\mathcal{A}$. (What is meant by the set symbols $\{ \}$ will be discussed in Section 4.2.2) It follows that, for $w \in W$, $\Omega^0(w) = w \in W$.

The sequential actions of the projection and the abstraction operators do not cancel each other, and it can be shown that $\mathcal{A} \circ \mathcal{P}(w) \neq w$:

**Theorem 1.** $\mathcal{A} \circ \mathcal{P}(w) \neq w$.

**Proof:** Assume $\mathcal{A} \circ \mathcal{P}(w) = w$. Then:

\[
\mathcal{A} \circ \mathcal{P} \in \Omega^0 \\
\Rightarrow \mathcal{A} \circ \mathcal{P} = 1 \\
\Rightarrow \mathcal{A} = \mathcal{P}^{-1}, \text{ DNE.}
\]

Thus, $\mathcal{A} \circ \mathcal{P}(w) \in \Omega^1(\mathcal{P}(w))$, for some $w \in \Omega^0(w)$.

Consider the complex of maps:

\[
\Omega^0 \rightarrow \Omega^1 \rightarrow \Omega^2 \cdots \\
\downarrow \mathcal{X} \downarrow \rightarrow \\
\mathcal{P} \Omega^0 \rightarrow \mathcal{P} \Omega^1 \rightarrow \mathcal{P} \Omega^2 \cdots
\]

It is possible to design a function, $\chi \equiv \mathcal{P} \circ \mathcal{A}$, which exists, and will be utilised in Section 4.2.3. However, it is important to note that our constraints on $\mathcal{P}$ do not allow the construction of a function $\varphi : \mathcal{P} \Omega^0 \rightarrow \mathcal{P} \Omega^1$, or any other mapping between projections of abstraction classes. Mathematically, $\varphi$ would have the form $\mathcal{P} \circ \mathcal{A} \circ \mathcal{P}^{-1}$, which does not exist; but philosophically,
it is supposed of $\mathcal{P}(w)$ that it encode the behaviour of the actual world object \textit{meant} by $w$. One interprets the ‘non-mathematical’ object, $\mathcal{P}(w) \in \mathcal{PW}$, knowing that the fact it is necessarily an abstraction is already encoded in the behaviour of $\mathcal{P}$ by construction. Note that the failure to construct a function $\varphi$ as a composite of abstraction operators and their inverses does not mean that such a mapping does not exist. However, the principal motivation for supposing the non-existence of such a function is to encode the features one expects in an abstract modelling of non-abstract objects.

This view of the general structure of abstraction is an opposite view to the metaphysic of epiphenomenalism\textsuperscript{16,17} in that, colloquially speaking, changes to ‘real-world’ objects can only occur via some abstract state, and it does not make sense to set up a relationship between non-mathematical entities and insist that such a relationship must be non-abstract.

Different instances of $w$ cannot be combined in general, but their abstractions can be compared by composition. The objects $\mathcal{A}(w_1)$ and $\mathcal{A}(w_2)$ can also be defined to be comparable, via use of the commutators, in Section\textsuperscript{4.1.3}

In considering the properties of $\Omega^0(W)$, one finds that

$$\Omega^0(\Omega^0(W)) = W$$  \hspace{1cm} (6)

$$\Rightarrow \Omega^0 \circ \Omega^0 = \Omega^0.$$  \hspace{1cm} (7)

Generalising to higher abstraction classes, we find the level addition property:

$$\Omega^i \circ \Omega^j = \Omega^{i+j}.$$  \hspace{1cm} (8)

The non-uniqueness of $\mathcal{A}$ means that many abstract objects can describe an element of $W$. In general, $\mathcal{A}_i \circ \mathcal{A}_j(w) \neq \mathcal{A}_j \circ \mathcal{A}_i(w)$, so $\Omega^1(\mathcal{A}_i(w)) \neq \mathcal{A}_i(\Omega^1(w))$, though both $\Omega^1(\mathcal{A}_i(w))$ and $\mathcal{A}_i(\Omega^1(w))$ are in $\Omega^2(w)$. The set $\Omega^0$ includes the identity operator $1$, but also contains elements constructed from abstractions and other inverses, e.g. $\mathcal{A}_{L,i}^{-1} \circ \mathcal{A}_j(w)$, to be discussed in Section\textsuperscript{4.1.4}.
4.1.3 Commutators

Define the commutator as an operator that takes the elements of the $i$th order abstraction space, acting on a world object $w_1$, to the same abstraction space acting on another world object $w_2$, $\Phi^i_{W=\Omega^i(W)} : \Omega^i(w_1) \rightarrow \Omega^i(w_2)$. The subscripts on the commutator symbol indicate the space inhabited by the objects whose abstractions are to be commuted, and the labels of the discarded and added objects, respectively. The superscript denotes the order of abstraction (plus one) at which the commutation takes place. As a simple example, $\Phi^1_{W,1,2}A(w_1) = A(w_2)$. In general, let

$$\Phi^1_{W,i,j}A(w_i) = A(w_j), \quad (9)$$

$$\Phi^2_{\Omega^i(w),i,j}A_k \circ A_i(w) = A_k \circ A_j(w), \quad (10)$$

$$\text{and } \Phi^{b+1}_{\Omega^b(w),i,j}A \circ \cdots \circ A_i \circ \cdots \circ A(w) = \underbrace{A \circ \cdots \circ A_j}_a \circ \cdots \circ A(w). \quad (11)$$

In order to construct the new object from the old object, one must successively apply ‘inverse’ operations of the relevant abstractions to the left of the old object (as discussed in the next section), and rebuild the new object by re-applying the abstractions. This is not possible in general, where objects may include operators that have no inverse, such as the projection operator.

4.1.4 The left inverse of the abstraction

Define the left inverse

$$A^{-1}_L \circ A = 1, \quad (12)$$

or more generally, $A^{-1}_L \circ A(w) = w$. A right inverse is not assumed to exist in general, which will be important in establish certain kinds of properties in Section 4.3.

As a generalization, one can define a chain of negatively indexed abstraction classes $\Omega^{-|i|}$. The level addition property can accommodate this scenario. The elements of $\Omega^0$ are populated by objects of the form $A^{-1}_{L_2} \circ A_j$, or $A_i \circ A^{-1}_{L,j} \circ A_k \circ A^{-1}_{L,n}$, etc. That is, successive abstractions and inverses in any combination such that the resulting abstraction space is order zero. This includes the identity operator.
By using the left inverse, it can be shown that the following theorem holds (which comple-
ments Theorem 1), as a consequence of the choice of the philosophical properties of \( \mathcal{P} \):

**Theorem 2.** \( \mathcal{P} \circ \mathcal{A}(w) \neq w \).

**Proof:** Assume \( \Omega^{-2} \neq \Omega^{-1} \), and \( \mathcal{P} = \mathcal{A}_L^{-1} \). Then:

\[
\mathcal{P} \circ \mathcal{P}(w) = \mathcal{A}_L^{-1} \circ \mathcal{A}_L^{-1}(w) = \mathcal{P}(w) \quad (\text{Eq. (1)})
\]

\[
= \mathcal{A}_L^{-1}(w)
\]

\[
\Rightarrow \mathcal{A}_L^{-1} \circ \mathcal{A}_L^{-1} = \mathcal{A}_L^{-1} \Rightarrow \]

As a corollary, it also follows that

\[
\mathcal{P} \circ \mathcal{A}(w) \neq \mathcal{A} \circ \mathcal{P}(w).
\]

In summary, this non-commutativity property of the abstraction operators in Eq. (13) is an important consequence of the reverse-epiphenomenal philosophical motivation behind the labelling principle, and it will be the starting point for the construction of the generalised objects in Section 4.2.3.

4.1.5 Auxiliary maps

In order for a successful description of the relationships among different objects in general, a definition of the mapping between objects of the form \( \mathcal{A}\mathcal{P}(w_1) \) and \( \mathcal{A}\mathcal{P}(w_2) \) is sought.

Up until now, maps of the following types have been considered:

- \( w \rightarrow \mathcal{A}(w) \), where the notion of the map itself is an abstraction of \( \mathcal{A} \) of the form:
  \( \mathcal{A}_{\text{map}} \circ \mathcal{A} \in \Omega^2(w) \);

- \( \mathcal{A} \circ \mathcal{A}(w) \rightarrow w \), where the map is now of the form \( \mathcal{A}_{\text{map}} \circ \mathcal{A} \circ \mathcal{A} \in \Omega^3(w) \), and

- \( \mathcal{A}(w) \rightarrow w \), identical to the first case, except that the direction is reversed. By convention, let the definition of map be chosen such that the direction of mapping is not important, but simply the relationship between the two objects. Therefore, the map is always taken such that it exists in a space \( \Omega^{\geq 1} \), that is, we define it as the modulus of the map. Note that each of these maps is constructed as a composite of other abstractions, and shall be denoted literal maps.
It cannot, in general, be attested that there is no such mapping between some $w_1$ and $w_2$ in $W$. However, one is free to define to exist a non-composite type of map, denoted auxiliary maps, which relate objects of the form $AP(w_1)$ and $AP(w_2)$, or $APA_1(w)$ and $APA_2(w)$, etc. The map itself exists in the space $\Omega^1(w)$, that is, the application of what is meant by the map does not change the order of the objects to which it is applied.

Using the concept of auxiliary maps, a generalized version of the commutator, denoted $\hat{\Phi}$, may be defined in a way not possible for the naïve construction of the commutator

$$\hat{\Phi}_{\Omega^1(w),i,j}^2 A PA_i(w) = A PA_j(w).$$

The auxiliary commutator will be important for the neat formulation of the general condition for a specific kind of existence, in Sec 4.3.

4.1.6 The total set

One may define the total set of an object $w$ as

$$S(w) = \bigcup_{i=0}^{\infty} \Omega^i(w),$$

$$S(P(w)) = \bigcup_{i=0}^{\infty} \Omega^i \circ P(w).$$

The $\bigcup$ symbol denotes the range of the multiple objects, indexed by an integer $i, j$, etc. over which the set should be specified. Here, one postulates a certain supposition of physics, that $PS(w)$ spans at least $W$.

**Supposition 1.** $P$ is surjective, i.e. $\forall w_i \in W, \exists \sigma_i \in S(w_i)$ such that $P(\sigma_i) = w_i$.

This condition represents the ‘working ethos’ of the practice of physics. It is expected that there is some abstract description, however elaborate or verbose, to describe every real-world object.

4.2 Relationships

In this section, the tools introduced in the preceding section will be used to define more general relationships between objects. In addition, a generalised object notation will be defined, and the nature of the real-world objects $w$ will be clarified.
4.2.1 Ansätze

An Ansatz is formed by adding a structure, or additional layer of abstraction, and imposing it on what one considers ‘the real world’. Cast in the new framework, this is simply the successive application of an abstraction and a projection operator upon some object

\[ Z_i(w) = \mathcal{P} \circ \mathcal{A}_i(w). \] (17)

As a consequence of the labelling principle in Eq. (1), the Ansatz of an object, \( w \), and what is meant by the real-world object corresponding to \( w \), cannot be related directly, recalling that \( \varphi : \mathcal{P} \circ \mathcal{A}(w) \rightarrow \mathcal{P}(w) \) does not exist. This simply means that there is no way of generating the label of an object directly from the object itself; it is a free choice. The Ansatz is akin to the statement: ‘let this new label (with possible additional information) be linked to the real object’. The notion of Ansatz, particularly the special examples considered in Sec. 4.3, will be useful in understanding the formal structure of existence as it pertains to real-world objects.

4.2.2 Collections and relationships

A collection or set of objects, \( \{w_i\} \) (indexed by integer \( i \)), in the formalism, is simply treated as an abstraction, \( \mathcal{A}_{\text{set}} \), used in conjunction with a commutator:

\[ \{w_1, \ldots, w_N\} = \bigcup_{j=1}^{N} \mathcal{A}_{\text{set}} \circ \Phi_{W,i,j}^1 w_i. \] (18)

By further imposing that there should be a relationship (other than the collection itself) among the objects \( w_i \), the addition information is simply added by another abstraction, say, \( \mathcal{A}' \), and what is meant be this particular relationship is simply:

\[ r(w_i) = \mathcal{P} \mathcal{A}' \circ \bigcup_{j=1}^{N} \mathcal{A}_{\text{set}} \circ \Phi_{W,i,j}^1 w_i. \] (19)

A relationship in general, \( r = \mathcal{A}' \circ \Phi_{W,i}^1 \mathcal{A} \in \Omega^2 \), does not have to specify that there be a particular relationship among objects \( w_i \).
Example: In identifying a ‘type’ of object, such as all objects that satisfy a particular function or requisite, one means something slightly more abstract than a particular instance of an object itself. In order to produce a notion similar to the examples: ‘all chairs’ or ‘all electrons’, one must construct a relationship among a set of $w_i$’s, each of which is a set of, say, $n$ observations: $w_A, w_B, \ldots \in W' \subset W$. Let

$$w_A = \bigcup_{j=1}^{N_A} \mathcal{A}_{\text{set}} \circ \Phi_W^{1}(w^{(A)}_i), \quad (20)$$

$$w_B = \bigcup_{j=1}^{N_B} \mathcal{A}_{\text{set}} \circ \Phi_W^{1}(w^{(B)}_i), \quad \text{etc.} \quad (21)$$

$$w^{(A)}_i, w^{(B)}_i, \ldots \in W. \quad (22)$$

Then, \(\{w_A, w_B, \ldots\} = \bigcup_{j'=1}^{N_{j'}} \bigcup_{j=1}^{n} \mathcal{A}_{\text{set}} \circ \Phi_W^{1}(w^{(i')}_i) \mathcal{A}_{\text{set}} \circ \Phi_W^{1}(w^{(i')}_i) \subset \Omega^3(W) (= \Omega^2(W')), \quad (23)$$

The relationship itself that constitutes the ‘type’ thus takes the form:

$$r_{\text{type}} = \mathcal{A}' \circ \bigcup_{j'=1}^{n} \bigcup_{j=1}^{N_{j'}} \mathcal{A}_{\text{set}} \circ \Phi_W^{1}(w^{(i')}_i) \mathcal{A}_{\text{set}} \circ \Phi_W^{1}(w^{(i')}_i) \in \Omega^3(W) (= \Omega^2(W')), \quad (24)$$

for some $\mathcal{A}'$, and $W' = \Omega^1(W)$. This formula represents the notion of ‘types’ of object in a fairly general fashion, in order to resemble as closely as possible the way in which objects are typically characterised and subsequently handled in the frameworks of language and thought.

4.2.3 Generalised objects

Up until now, discussion of the nature of the real-world objects $\{w_i\} \in W$ has been avoided. However, in order to incorporate them in the most general way into the framework of the abstraction algebra, one may posit that the real-world objects are simply a chain of successive projection or abstraction operators. In general, one can construct ‘sandwiches’, such as:

$$\mathcal{A}_1 \circ \cdots \circ \mathcal{A}_{i_1} \circ P \circ \mathcal{A}_2 \circ \cdots. \quad (25)$$

\(^{\dagger}\)The form, $W' = \Omega^1(W)$, makes sense, in that the notion of ‘being a subset’ is a single-level abstraction, residing in $\Omega^1$.\]
All objects considered in the framework thus far can be expressed in this form, noting that \( \mathcal{P}(\text{anything}) \in \Omega^0 \mathcal{P}(\text{anything}) \). Due to the corollary in Eq. (13), the projection operators cannot be ‘swapped’ with any of the abstraction operators, and so the structure of the object is nontrivial. Let \( c \) denote a generalised object, living in the space:

\[
c \in \Omega^{i_1} \mathcal{P} \Omega^{i_2} \mathcal{P} \cdots \mathcal{P} \Omega^{i_n}(W) \equiv C_{W}^{i_1 \cdots i_n}
\]  

(26)

The space \( W \) here could stand for any other general space constructed in this manner, not necessarily the space inhabited by \( c \) itself; thus Eq. (26) is not recursive as it may initially appear. Because the internal structure, so-to-speak, of \( c \) contains a collection of a possible many abstractions, it may be expressed in terms of type. Here are two examples:

Let \( c^{(1)} = \mathcal{P}\{\mathcal{P} \circ A_1(w), \ldots, \mathcal{P} \circ A_n(w)\} \in \Omega^0 \mathcal{P} \Omega^1 \mathcal{P} \Omega^1(w) = C_{W}^{011} \)  

(27)

\[
= \mathcal{P} \bigcup_{j=1}^{n} A_{\text{set}} \circ \Phi_{\Omega^j, i,j}^2 \mathcal{P} A_i(w).
\]  

(28)

Or \( c^{(2)} = \mathcal{P}\{\mathcal{P}(w_1), \ldots, \mathcal{P}(w_n)\} \in \Omega^0 \mathcal{P} \mathcal{P} \mathcal{P} \mathcal{P}(W) = C_{W}^{010} \)  

(29)

\[
= \mathcal{P} \bigcup_{j=1}^{n} A_{\text{set}} \circ \Phi_{\mathcal{P}^j, i,j}^1 \mathcal{P}(w_i),
\]  

(30)

where, in the first case, \( \Omega^0(w) \in \Omega^0(W) \).

Consider the behaviour of an Ansatz \( \mathcal{Z} = \mathcal{P} \circ A_Z \) acting on a generalised object \( c \in C_{W}^{i_1 \cdots i_n} \):

\[
\mathcal{Z} \circ c = \mathcal{P} \circ A_Z \circ A_1 \circ \cdots \circ A_i \circ \mathcal{P} \circ \cdots.
\]  

(31)

\( \mathcal{Z} \) maps \( c \) into a space \( C_{W}^{0i_1+1 \cdots i_n} \). If we define \( \text{rank}(c) = n \), then \( \text{rank}(\mathcal{Z} \circ c) = n + 1 \). Note that the rank of \( \mathcal{Z} \in C_{c}^{01} \) can also be read off easily: \( \text{rank}(\mathcal{Z}) = 2 \). Objects of the form of \( \mathcal{Z} \) are the principal rank 2 Ansätze. Note that other rank 2 objects besides \( \mathcal{Z} \) exits, such as objects of the form \( \mathcal{P} \circ A_1 \circ A_2 \).

A more general description of Ansätze also exist, analogously to the generalised objects. By constructing an object of the form: \( \chi = \underbrace{\mathcal{A} \circ \cdots \circ \mathcal{A} \circ \mathcal{P} \circ \mathcal{A} \circ \cdots \circ \mathcal{A}}_{j_1} \circ \mathcal{P} \circ \cdots \), that is, for an object residing in a space \( C_{c}^{j_1 \cdots j_m} \), the composition \( \chi \circ c \) lies in \( C_{W}^{j_1 \cdots (j_m+i_1) \cdots i_n} \), which is of rank \( n + m - 1 \).
By convention, an Ansatz must contain a projection operator. Therefore, there is no rank 1 Ansatz, and we arrive at our general definition of Ansatz:

Any object acting on $c$, with a rank $> 1$, is an Ansatz. (32)

In addition, there are no objects with rank $\leq 0$.

**Proof:** Let $\xi$ exist such that $\text{rank}(\xi) \leq 0$, and $c \in C_W^{i_1...i_n}$. Then:

$$\text{rank}(\xi \circ c) = \text{rank}(\xi) + \text{rank}(c) - 1 < \text{rank}(c) = n$$

$\Rightarrow \xi \circ c \in C_W^{i_1...i_{n-1}}$

$\Rightarrow \xi$ is of the form $X \circ \bigcup_{j=1}^{k} \mathcal{P}^{-1} \circ \bigcup_{i=1}^{ij} \mathcal{A}^{-1}_{i,L}$, where $\text{rank}(X) \leq k$

$\Rightarrow \xi$ DNE, for any $X$. \hfill $\Box$

For example, in the case $k = 1$, $X$ is a rank 1 Ansatz, and $\xi = X \circ \mathcal{P}^{-1} \circ \bigcup_{i=1}^{ij} \mathcal{A}^{-1}_{i,L}$, such that $\xi \circ c \in C_W^{i_2...i_n} \cong C_W^{i_1...i_{n-1}}$. The rank of $\xi \circ c$ is $n - 1$, and the rank of $\xi$ is therefore 0. Because of the usage of the operation $\mathcal{P}^{-1}$, such an object is inadmissible.

We would like to use the Ansätze to investigate the properties of generalised objects. However, there are a variety of properties in particular, discussion of which shall occupy the next section. The notion of ‘existence’ is a key example that urgently requires clarification, and it will be found that such a property (and those similar to it), when treated as an abstraction, must have additional constraints.

### 4.3 $I$-extantness

Firstly, one must make a careful distinction between what is meant by ‘existence’ in the sense of mathematical objects, and in the sense of the ‘real world’. In the former case, one may assume that an object exists if it can be defined in a logically consistent manner. In the latter case, it is a nontrivial property of an object, which must be investigated on a case-by-case basis, and the alternative word ‘extantness’ will be used in order to avoid confusion. The goal of the formalism is to relate the two terms- that an object’s extantness can be tested by appealing to the existence (in the mathematical sense) of some construction.
We begin by assuming that extantness is an inferred property of an object, and thus added by an Ansatz. Define its abstraction, $A_E$, such that an object $c = P \circ A_E(w)$ is extant if such a construction exists; i.e. $c$ is extant if it can be written in this form (for any $w$). For $c = P \circ A_E \circ A_1 \circ \cdots \in C_W^{i_1 \cdots i_n}$; the operator $A_E$ must occur in the left-most position of all the abstractions in $c$. Clearly then, it must not necessarily be the case that $P \circ A_E(w)$ exists, if this abstraction is to be equivalent to how extantness (or existence in the conventional sense) is understood.

**Example:** Consider the object $P \circ A_E(1)$, where 1 is the abstraction identity. $A_E(1) = A_E$ is the extantness itself, and $P \circ A_E$ is ‘what is meant’ by extantness, which is itself extant- it is the trivial extant object.

This leads us to the first property of $A_E$, that its right inverse, $A_{E,R}^{-1}$, does not exist, as anticipated in Section 4.1.4.

**Proof:** Consider $c = P \circ A \circ \cdots (w)$ such that it is not extant. Assume $A_{E,R}^{-1}$ exists also. Then:

$$c = P \circ A_E \circ A_{E,R}^{-1} \circ A \circ \cdots (w)$$

$$= P \circ A_E(w'), \text{ where } w' \equiv A_{E,R}^{-1}(w)$$

$\Rightarrow c$ is extant. $\Rightarrow \Leftarrow$

It is not necessary at this stage to suppose that the left inverse of $A_E$ does not exist either; however, if that were the case, then $A_E$ would share a property with $P$, in lacking an inverse. The two are unlike, however, in that $A_E \circ A_E \neq A_E$. Since $A_E$ lives in a restricted class $\tilde{\Omega}^1 \subset \Omega^1$, indicating the additional constraint of lacking an inverse, then the level addition property of Eq. (8) means $A_E \circ A_E \notin \tilde{\Omega}^2 \neq \tilde{\Omega}^1$. A further consequence of the non-existence of $A_{E,L}^{-1}$ is that the statement $A_E(a) = A_E(b)$ does not mean that $a = b$. One may interpret this as the fact that two abstractions may simply be labels for the same extant object. Note that the definition of the literal commutator requires the existence of an inverse of each abstraction operator that occurs in sequence to the left of the object being commuted, but that is not the case for auxiliary commutators.

Recalling the supposition of physics, that $P S(w)$ spans at least $W$, a further clarification may now be added:
Supposition 2. All extants have Ansätze, but not all elements of $\mathcal{P}(W)$ or $\mathcal{P}S(w)$ are extants.

From the point of view of Mathematical Realism, one would argue that projected quantities, $\mathcal{P} \circ \cdots$, are those which are ‘real’ (and not dependent on their extantness), since such a definition of ‘real’ would then encompass a larger variety of objects, regardless of their particular realisation in our universe. Such a semantic choice for the word ‘real’ seems to align best with the philosophy of Mathematical Realism. Nevertheless, it is still important to have a mechanism in the formalism to determine the extantness of an object.

Although extantness has been singled out as a key property, a similar argument may be made for the truth of a statement, whose abstraction can be denoted as $\mathcal{A}_T$. Like extantness, the object $\mathcal{P} \circ \mathcal{A}_T(w)$ may not exist for every $w$, and the trivially true object is $\mathcal{P} \circ \mathcal{A}_T(1)$. Let us label all properties of this sort, ‘$I$-extantness’, since their enumeration in terms of common words is not of interest here. For any $I$-extant abstraction $\mathcal{A}_I$, we call $\tilde{c}^{i_1, \ldots, i_n}_W$ the restricted class of generalised objects, $c_I$.

A formula is now derived, which is able to distinguish between objects that are $I$-extant and those that are not, by virtue of their mathematical existence. Consider the case that $\mathcal{P} \circ \mathcal{A}_I(w_1)$ exists, but $\mathcal{P} \circ \mathcal{A}_I(w_2)$ does not. $w_1$ must contain an addition property, $\mathcal{A}_I'$, that is not present in $w_2$. Unlike $\mathcal{A}_I$, it is not required that $\mathcal{A}_I'$ occur in a particular spot in the list of abstractions that comprise $w_1$. Nor is there a restriction in the construction of an inverse, which would prevent a commutator notation being employed. Let $w_1$ be represented by a collection of objects defined by: $w_1 = \{ \mathcal{A}_I \circ \mathcal{A} \circ \cdots, \mathcal{A} \circ \mathcal{A}_I \circ \cdots, \text{etc.}\}$. That is, $w_1$ takes the form of a set of generalised objects, $c$, but for the replacement of an operator, $\mathcal{A}$, with $\mathcal{A}_I'$. It is important to note that the $\mathcal{A}_I'$ that distinguishes $w_1$ from a non-extant object, such as $w_2$, is particular to $w_1$. For an object $c$ to be extant, it would have to include an abstraction $\mathcal{A}_I^c$, specific to $c$; otherwise, any object related to $c$ in any way would also be extant, which doesn’t reflect the behaviour expected of extant objects in the universe.

In commutator notation, one would need to write out a geometric composition of the form

$$w_1 = \mathcal{A}_\text{set} \bigcup_{m=0}^{i_1-1} \mathcal{P} \mathcal{I}_{\Omega_l}^{i_1-m+1}(w), m+1, p \mathcal{H}_1 \circ \mathcal{P} \circ \bigcup_{m'=0}^{i_2-1} \mathcal{P} \mathcal{I}_{\Omega_2}^{i_2-m'+1}(w), m'+1, p' \mathcal{H}_2 \circ \cdots, \quad (33)$$

$$= \mathcal{A}_\text{set} \circ \mathcal{H}_1 \circ \bigcup_{p=2}^{n} \mathcal{P} \bigcup_{m=0}^{i_p-1} \mathcal{P} \mathcal{I}_{\Omega_p}^{i_p-m+1}(w), m+1, p \mathcal{H}_p, \quad (34)$$
for $c = H_1 \circ \mathcal{P} \circ H_2 \circ \cdots$. A more elegant formula may be defined simply in terms of $c$ itself, without the need of introducing new symbols, $H_1, \ldots, H_n$. One can achieve this using auxiliary commutators

$$w_1 = \left\{ \Phi_{c_1^{i_1 \cdots i_n}} (\sum_{j=1}^{n} i_j) + 1, P, c, \Phi_{c_1^{i_1 \cdots i_n}} (\sum_{j=1}^{n-1} i_j) + 1, P, c, \ldots, \Phi_{c_1^{i_1 \cdots i_n}} (\sum_{j=1}^{n-2} i_j) + 1, P, c, \ldots \right\}$$

$$= \bigcup_{m=0}^{i_1-1} A_{set} \circ \Phi_{c_1^{i_1 \cdots i_n}} (\sum_{j=1}^{n} i_j) + 1 - m \bigcup_{m'=0}^{i_2-1} A_{set} \circ \Phi_{c_1^{i_1 \cdots i_n}} (\sum_{j=1}^{n-2} i_j) + 1 - m'$$

$$= \bigcup_{p=1}^{n} \bigcup_{m=0}^{i_p-1} A_{set} \circ \Phi_{c_1^{i_1 \cdots i_p \cdots i_n}} (\sum_{j=1}^{n} i_j) + 1 - m$$

It follows then, that a generalised object that is $I$-extant takes the form

$$c_I = \mathcal{P} \circ A_I \circ \bigcup_{p=1}^{n} \bigcup_{m=0}^{i_p-1} A_{set} \circ \Phi_{c_1^{i_1 \cdots i_p \cdots i_n}} (\sum_{j=1}^{n} i_j) + 1 - m$$

where $c_I$ is of the form $\mathcal{P} \circ A_I (w_I)$. This is a powerful formula, as it represents the condition for $I$-extantness for a generalised object, $c$. Note that it would be just as correct to define $c_I$ as an element of a set characterised by the righthand side (i.e. using ‘$\in$’ instead of ‘$=$’), but because the notion of a ‘set’, $A_{set}$, is simply an element of $\Omega^1$, it can be incorporated into the general form of $c_1^{i_1 \cdots i_n}$.

One might wonder how to relate the properties of a proof (i.e. verifying the truth of a statement) with the existence of an abstraction, $A_T$. In an example, consider the object representing the existence of truth, $w_T$. The validity of the ‘excluded middle’\[29\] in this situation means that the proof is very simple:

**Proof:**

\[w_T \Rightarrow w_T\]
\[\neg w_T \Rightarrow (\neg \neg w_T) = w_T.\]

Since $w_T$ is the statement of truth itself, i.e. $w_T = \mathcal{P} \circ A_T$, the inconsistency of $\mathcal{P} \circ A_T (w_T)$ means the inconsistency of $\mathcal{P} \circ A_T$. Such a statement is not true by construction. One can now identify the abstraction, $A_T^{w_T}$, as being $A_T$ itself. Thus, this exercise demonstrates that the
proof of a statement has consequences for the abstract form of the statement, allowing one to identify more specific properties. Note that this does not, at this stage, provide extra proof methods, since there is no procedure, as yet, for acquiring knowledge of the form of an object’s relevant $A_T$ in advance. The content of the proof must rely on standard means.

4.4 Cardinality

In the derivation of the general condition for an object $c$ to be $I$-extant (Eq. (38)), one arrives at a set of elements. In this notation, the set is not intended to specify all the possibilities that each abstraction operator, $A$, can take. Rather, the set can be thought of as being ‘the set of alterations from a general $c$’ that encompass the required condition.

If one seeks an absolute measure of the ‘size’ of the object, in terms of the overall possibilities, one may define a type of cardinality, $|c|$, in terms of the total possible number of abstractions. Recalling Cantor’s Theorem, there is no consistent description of such a universal class. However, since the formalism accommodates the imposition of restrictions on the kind of objects that can be represented, let the number of possibilities for $A$ be assumed consistently definable, and denote as $L$. $L$ need not be finite, nor even countable, however, it can be used to obtain formulae for the cardinality of an object.

Define the number of abstractions, $A$, in $c \in C_{W}^{i_1\ldots i_n}$ as $\bar{n} \equiv \sum_{j=1}^{n} i_j$. Thus one finds that

$$|c| = L^\bar{n}, \text{ and}$$

$$|c_I| = \bar{n} L^{\bar{n}-1}. \quad (39)$$

The latter formula is simply a consequence of there being $\bar{n}$ possibilities for restricting one abstraction operator to be $A_I$. If one enforces $N$ restrictions on the set of $A$’s, then it follows that

$$|c_N| = \left( \frac{1}{2}(\bar{n}^2 - N^2) + 1 \right) L^{\bar{n}-N}. \quad (40)$$

This formula will become relevant in the next section.


5 Unreasonable effectiveness

The goal is to use the general framework, described in Section 4, to encapsulate the essence of describing phenomena using a theory, in the sense used in physics. Thus, the issue of Wigner’s ‘unreasonable effectiveness’ of mathematics to describe the universe may be addressed by transporting the problem to a metaphysical context. There, the tools from philosophy, such as logic and proof theory, can be directed at the question that involves not so much the behaviour of the universe, as the behaviour of descriptions of the universe (i.e. physics itself). It is important to be able to transport certain features of physics into a context where an analysis may take place, and such a context is, by definition, metaphysics.

The notion of ‘effectiveness’ is that, given a consistent set of phenomena, \( v_i \in V \), One can extend \( V \) to include more phenomena such that (general) Ansätze able to explain the phenomena satisfactorily can still be found. In this general context, what is meant by an ‘explanation’ will be taken to be a relationship among the phenomena, \( v_i \), in the form of abstractions. The essence of the mystery of the effectiveness of mathematics is not whether one can always ‘draw a box’ around an arbitrary collection of objects, or that laws and principles (of any kind) are obeyed, but the identification of particular principle(s) such that phenomena \( v_1, v_2, \ldots \) are consequences of them; and that via the principles, the whole of \( V \) may be obtained, indicating a more full explanation of the phenomena. That is, the phenomena are extant because of the truth of the underlying principles, rather than being identified ‘by hand’ (which would hold no predictive power in the scientific sense). Note that the set \( V \) may, in fact, only include a subset of the possible phenomena to discover in the universe, and so would represent a subset of the set, \( W \), as discussed in Section 4.

Let \( v_1, v_2, \ldots \) have descriptions \( A_{v_1}, A_{v_2}, \ldots \in V \subset W \), which are extant. Let there be some principle, (or even collection of principles with complicated inter-dependencies), described by the general object, \( c_{\text{princ}} \), such that each element of \( V \) may be enumerated. It is our goal to investigate under what conditions the following statement holds:

\[
c_{\text{princ}} \text{ is true } \Rightarrow v_1, v_2, \ldots \text{ are extant},
\]

i.e.

\[
[ c_{\text{princ}} = \mathcal{P} \circ A_T(w_{\text{princ}}) ] \Rightarrow [ A_{v_i} = \mathcal{P} \circ A_E(A_{y_i}) \in V ].
\]

If there is a principle that implies such a statement, we wish to identify it, and investigate whether or not it is true.
The circumstances of the truth of Eq. (43) depends on how \( w_{\text{princ}} \) is related to the phenomena, \( v_i \). \( w_{\text{princ}} \) itself represents principle(s) whose truth is not added by hand in Ansatz form. This does not mean that it is not true, since the form of \( w_{\text{princ}} \) is as yet unspecified. The most general way of relating \( w_{\text{princ}} \) and all \( v_i \)'s is to use the method of substituting abstractions into the formula for a generalised object, such as that used to derive the general condition of \( I \)-extantness in Eq. (38). In the same way that the set of all possible locations of \( A_{v_i} \) in \( c \) was considered, here, all possible combinations of locations of abstractions describing \( v_i \) in \( c \) must be considered, such that each \( A_{v_i} \) occurs at least once. This formula can be developed inductively.

Consider only two phenomena, \( v_1 \) and \( v_2 \), with corresponding abstract descriptions defined as \( A_{v_1} \) and \( A_{v_2} \). For a generalised object, \( c \in C_{W}^{i_1...i_n} \), one finds

\[
\begin{align*}
\mathcal{w}_{\text{princ}}^{(N=2)} &= \bigcup_{p=1}^{n} \bigcup_{m'=0}^{i_p-1} \bigcup_{m=0}^{i_p-1} \mathcal{A}_{\text{set}} \circ \tilde{\Phi}_{C_{W}^{i_1...i_p...i_{m'}...i_n},m'+1,v_2} \circ \tilde{\Phi}_{C_{W}^{i_1...i_p...i_{m'}...i_n},m+1,v_1} \\
&= \bigcup_{m=(N-1)=0}^{i_p-1} \bigcup_{m'=(N-1)=0}^{i_p-1} \bigcup_{m=(1)=0}^{i_p-1} \bigcup_{k=0}^{N-1} \bigcup_{m=(i_p)_k \in [0,i_p-1] \setminus \bigcup_{\mu=0}^{1} \{m(\mu)\}}.
\end{align*}
\]

In the case of \( N \) phenomena, it is assumed that \( N \leq i_p \): the number of abstractions available in the general formula for \( c \) may be made arbitrary large to accommodate the number of phenomena. One may make use of the following formula

\[
\begin{align*}
\mathcal{w}_{\text{princ}}^{G} &= \bigcup_{m=(N-1)=0}^{i_p-1} \bigcup_{m'=(N-1)=0}^{i_p-1} \bigcup_{m=(1)=0}^{i_p-1} \bigcup_{k=0}^{N-1} \bigcup_{m=(i_p)_k \in [0,i_p-1] \setminus \bigcup_{\mu=0}^{1} \{m(\mu)\}} \mathcal{A}_{\text{set}} \circ \tilde{\Phi}_{C_{W}^{i_1...i_p...i_{m'}...i_n},m'+1,v_2} \circ \tilde{\Phi}_{C_{W}^{i_1...i_p...i_{m'}...i_n},m+1,v_1} \circ \tilde{\Phi}_{C_{W}^{i_1...i_p...i_{m'}...i_n},m+1,v_k}. \tag{46}
\end{align*}
\]

In order for \( c_{\text{princ}} \) to be true, \( w_{\text{princ}} \) must contain information about the objects \( y_i \), such that \( A_{v_i} = \mathcal{P} \circ A_{E}(A_{y_i}) \). Therefore, we seek only those elements of Eq. (46) such that the phenomena \( v_i \) take this form. This is a more restrictive set, as each abstraction of \( y_i \) must be applied to the right of \( A_{E} \), which must be applied to the right of \( \mathcal{P} \). There are only \( n-1 \) such
occurrences of $P$ in $c$, so in making this restriction, we are free to choose

\begin{itemize}
  \item $N \leq n - 1,$ \hspace{1cm} (47)
  \item $N \leq i_p.$ \hspace{1cm} (48)
\end{itemize}

The form of the more restricted version of $w_{princ}$ is thus

$$w_{princ}^R = \bigcup_{k=0}^{N-1} \bigcup_{p \in [2,n] \setminus \bigcup_{\pi=0}^{k} \{p^{(0)}\}} \mathcal{A}_{set} \circ \Phi_{c_{w}^{i_1 \ldots i_p \ldots i_n}} \circ \Phi_{c_{w}^{i_1 \ldots i_p \ldots i_n} + 1, E} \circ \Phi_{c_{w}^{i_1 \ldots i_p \ldots i_n} + 2, y_k+1, c, 1}.$$ \hspace{1cm} (49)

where \{p^{(0)}\} = \emptyset.

If $c_{princ}$ can be constructed consistently, i.e. if it exists, then the form of $w_{princ}$ must be restricted to include an abstraction, $\mathcal{A}_{w_{princ}T}$, that ensures the existence of $c_{princ}$. This uses the same argument as in deriving Eq. (58), with $c_T = P \circ A_T(w_T)$, and involves the union of Eq. (49) with the object $w_T$. Thus, the condition under which Eq. (43) is true can now be written.

**Theorem 3.** The condition under which the principles of a theory describe certain phenomena takes the form

$$w_{princ} \subseteq w_{princ}^{R,T'} = \bigcup_{k=0}^{N-1} \bigcup_{p \in [2,n] \setminus \bigcup_{\pi=0}^{k} \{p^{(0)}\}} \mathcal{A}_{set} \circ \Phi_{c_{w}^{i_1 \ldots i_p \ldots i_n}} \circ \Phi_{c_{w}^{i_1 \ldots i_p \ldots i_n} + 1, E} \circ \Phi_{c_{w}^{i_1 \ldots i_p \ldots i_n} + 2, y_k+1, c, 1} \cup w_T.$$ \hspace{1cm} (50)

**Proof:** The statement of the theorem, that ‘$w_{princ} \subseteq w_{princ}^{R,T'}$ constitutes the condition for which Eq. (43) is true’, is only fulfilled if the general form of $w_{princ}$ (in Eq. (46)) includes a description of extant phenomena explicitly, which takes the form shown in Eq. (49). That is, one must show that $w_{princ}^{R,T'} \subseteq w_{princ}^{G,T'}$. This entails that the elements of $w_{princ}^{R,T'}$ and $w_{princ}^{G,T'}$ are of the same form, differing only by use of a restriction. Therefore, in this case, the abstraction $\mathcal{A}_{w_{princ}T}$ is sufficient to ensure the truth of the elements in both sets. Note that the inclusion of $\mathcal{A}_{w_{princ}T}$ takes the same form for both $w_{princ}^{R,T'}$ and $w_{princ}^{G,T'}$. Therefore, it is sufficient to show that $w_{princ}^{R,T'} \subseteq w_{princ}^{G,T'}$. 

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Express Eq. (49) in terms of abstractions, $A_i$, recalling that $P \circ A_E(A_{y_i}) = P \circ P \circ A_E(A_{y_i}) = P \circ A_{v_i}$:

$$w^R_{\text{princ}} = \bigcup_{k=0}^{N-1} \bigcup_{p \in [2, n]} \bigcup_{\pi=0}^{k} \Phi_{\bar{w}_{1}^{i} \cdots i \cdots i_{n}^{1}, 1, v_{k+1}}^{(\sum_{j=1}^{n-p+2} i_j)+1} A_{\text{set}}.$$

Choosing the value $m = 0$ in $w^G_{\text{princ}}$ yields

$$w^G_{\text{princ}, m=0} = \bigcup_{p=1}^{n} \bigcup_{k=0}^{N-1} \bigcup_{\pi=0}^{k} \Phi_{\bar{w}_{1}^{i} \cdots i \cdots i_{n}^{1}, 1, v_{k+1}}^{(\sum_{j=1}^{n-p+1} i_j)+1} A_{\text{set}}.$$

The only difference between Eqs. (51) and (52) is the choice of values of the iterator $p$. To obtain $w^R_{\text{princ}} \subseteq w^G_{\text{princ}}$, it is sufficient to show that

$$[2, n] \setminus \bigcup_{\pi=0}^{k} \{p(\pi)\} \subseteq [2, n] \quad \forall k \in [0, N - 1].$$

Recalling the restrictions of Eqs. (47) and (48), take $N \ll n$. Now, $\bigcup_{\pi=1}^{k} \{p(\pi)\}$ is a finite set of integers that is a subset of $[2, n]$

$$\bigcup_{\pi=1}^{k} \{p(\pi)\} \subseteq [2, n],$$

where $\{p(0)\} = \emptyset$.

$$\therefore \bigcup_{\pi=0}^{k} \{p(\pi)\} \subseteq [2, n]$$

$$\Rightarrow w^R_{\text{princ}, T'} \subseteq w^G_{\text{princ}, T'}\qed$$

Note that the fact that $w^R_{\text{princ}}$ is a more restrictive set than $w^G_{\text{princ}}$ does not mean that it is ‘smaller’ in the sense of cardinality. Assuming a sufficiently large $n$ value to accommodate all $N + 1$ restrictions, one finds that

$$|w^G_{\text{princ}, T'}| = \left(\frac{1}{2}(\bar{n}^2 - (N + 1)^2) + 1\right) L^{\bar{n}-(N+1)} = |w^R_{\text{princ}, T'}|$$

$$\Rightarrow w^R_{\text{princ}, T'} \equiv w^G_{\text{princ}, T'}.$$
The above observation provides a possible explanation for the appearance of the ‘unreasonable effectiveness’ of mathematics. The set of relationships among extant phenomena, \( w_{princ}^{R,T'} \), is not smaller, in any strict sense, than the general set of relationships among abstractions, \( w_{princ}^{G,T'} \). The countability of sets of phenomena filtering into a more restrictive and still countable form, \( w_{princ}^{R,T'} \), combined with the formalism for describing non-mathematical objects in a mathematical way, constitutes the metaphysical explanation for the ‘unreasonable effectiveness’ of mathematics. In other words, there is no unreasonableness at all, but it is simply a mathematical consequence of the countability of phenomena, and the abstract description of objects that are not innately abstract.

This demonstrates the power of metaphysical tools, in the form of principles and proofs, to address key philosophical issues in physics. That is, the process employed here was not physics itself, but philosophical argumentation applied to the abstractions of objects used in the practice of physics.

6 Evidence for universes

6.1 Defining evidence

The definition of evidence relies on the connection between a set of phenomena (called evidence), and the principles of a theory that the evidence supports. It is assumed here that the sense in which the phenomena support or demonstrate an abstraction, such as the theory, is the same sense in which a theory can be said to entail the extantness of the phenomena. The symmetry between the two arguments has not been proved, however, since it relies on the precise details of often-imprecisely defined linguistic devices.

The description of evidence, using the formalism of Section 4, takes a similar form to that of the description of Ansatz for phenomena in Eq. (43), except that the direction of the correspondence is reversed

\[
A_{v_1}, A_{v_2}, \ldots \text{ are extant } \Rightarrow c_{princ} \text{ is true,} \quad (56)
\]

i.e.

\[
\left[ A_{v_i} = \mathcal{P} \circ \mathcal{A}_E(\mathcal{A}_{v_i}) \in V \right] \Rightarrow \left[ c_{princ} = \mathcal{P} \circ \mathcal{A}_T(w_{princ}) \right]. \quad (57)
\]
The lefthand side of Eq. (57) restricts the form of the object, $w_{\text{princ}}$ (representing the set of principles) through $w_{\text{princ}} \subseteq w_{\text{R,princ}}^R$. For the form of $w_{\text{princ}}$ to entail the righthand side of Eq. (57), it must also include the abstraction, $A_{T'}$. Thus, the condition under which Eq. (57) is true, where phenomena constitute evidence for a set of principles, is

$$\left[w_{\text{princ}} \subseteq w_{\text{princ}}^{R,T'}\right] \quad (= c_{\text{cond}}).$$

(58)

This is the same condition obtained for the examination of principles entailing extant phenomena, in Eq. (43).

There is a duality between the two scenarios, which can be expressed in the following manner. If the condition of Eqs. (50) and (58) is true, then

$$\left[c_{\text{princ}} = \mathcal{P} \circ A_T(w_{\text{princ}})\right] \iff \left[A_{v_i} = \mathcal{P} \circ A_E(A_{y_i})\right].$$

(59)

That is, relationship between principles and evidence is symmetrical in a sense. The sort of phenomena entailed by a theory is of exactly the same nature as the sort of phenomena that constitutes evidence for such a theory. This leads one to postulate an object, $c_D = \mathcal{P} \circ A_T(w_D)$, which represents this duality, and one may identify a Duality Theorem, which takes the form

$$w_D = \left\{\left[\mathcal{P} \circ A_T(w_{\text{cond}}) \Rightarrow \left[c_{\text{princ}} = \mathcal{P} \circ A_T(w_{\text{princ}})\right] \iff \left(A_{v_i} = \mathcal{P} \circ A_E(A_{y_i})\right)\right]\right\}.$$

(60)

The theorem is a consequence of the fact that the application of the restrictions acting upon $w_{\text{princ}}$ commute with each other in the formalism.

Note that, in attempting to clarify a term ill-defined in colloquial usage, we have arrived at quite a strict definition of evidence: if $w_{\text{princ}}$ is to constitute a set of principles describing the elements, $v_i$, it must at least take the form of a description based explicitly on all $v_i$ elements. Any part of $w_{\text{princ}}$ that does not lie in $w_{\text{princ}}^{R,T'}$ is not relevant for consideration as being supported by the evidence.
6.2 The relating theorem and the fundamental object

Since the $I$-extantness of some $c_I$ has been related to the mathematical existence of an object $A_I(w)$, a primary question to investigate would be the $I$-extantness of the statement of this relation itself. The statement of ‘the tying-in of the mathematical and non-mathematical objects’ has certain properties that should deem the investigation of its own $I$-extantness a nontrivial exercise.

Denote the above statement, which is an Ansatz, as $Z_I(c_I)$, and let $c_I$ be $I$-extant. That is, for $c_I \equiv P \circ A_I(w)$, $A_I(w)$ exists; and let $Z_I(c_I) \equiv P \circ A_Z \circ c_I$, for some $A_Z$. Recall that the assumed existence (in the mathematical sense) of the statement, $Z_I(c_I)$, does not trivially entail $I$-extantness, under Supposition 2. To show that $Z_I$ is $I$-extant, it is required that it can be put in the form

$$Z_I(c_I) = P \circ A_I(w_Z),$$

which implies that

$$P \circ A_Z \circ c_I = P \circ A_I(w_Z).$$

We would like to attempt to understand under what conditions this holds.

Consider the scenario in which the $I$-extant form of $Z_I$ does not exist. In this case, it is not possible to say that $Z_I$ is not $I$-extant, since the statement relating existence and $I$-extantness has not been proved, and no information about $I$-extantness can be gained using this method. If, however, the $I$-extant form of $Z_I$ does exist, then it is indeed certain that $Z_I$ is $I$-extant. In other words, there is a logical subtlety that entails an ‘asymmetry’: the demonstration of the existence of an object is enough to prove it, but the equivalent demonstration of its non-existence is not enough to disprove it, since the relied-upon postulate would then be undermined. Therefore, in this particular situation, unless further a logical restriction is found to be necessary to add in later versions of the formalism, it is sufficient to show that the $I$-extant can exist, for $Z_I$ to be $I$-extant. This is not true in general, due to Supposition 2.

**Theorem 4.** $Z_I(c_I)$ is $I$-extant.

The above theorem, denoted the relating theorem, may be verified in proving Eq. (62). It is enough to show that $A_Z \circ c_I = A_I \circ w_Z$ for any $c_I$, where there exists an $A_Z$ such that $A_I$ obeys the property: $A_{I,R}^{-1}$ DNE, which is, in our general framework, the only distinguishing
feature of $A_I$ at this point. The demonstration is as follows:

**Proof:** Let $Z_I$ exist, such that

$$Z_I = A_Z \circ P \circ A_I \circ \bigcup_{p=1}^{n} \bigcup_{m=0}^{i_p-1} A_{\text{set}} \circ \hat{\Phi}_{C_{W}^{n-p+1,1-m}}^{(\sum_{j=1}^{n-p+1} i_j)+1-m} \bigcup_{p=1}^{n} \bigcup_{m=0}^{i_p-1} A_{\text{set}} \circ \hat{\Phi}_{C_{W}^{n-p+1,1-m}}^{(\sum_{j=1}^{n-p+1} i_j)+1-m} \bigcup_{p=1}^{n} \bigcup_{m=0}^{i_p-1} A_{\text{set}} \circ \hat{\Phi}_{C_{W}^{n-p+1,1-m}}^{(\sum_{j=1}^{n-p+1} i_j)+1-m}$$

$$= A_I \circ w_Z,$$

for any $w_Z$. Due to the labelling principle, this can only be true if $w_Z \equiv c_I$ and $A_I \equiv A_Z$; that is, the abstraction of $c_I$ (above) is an $I$-abstraction: $A_Z \in \hat{\Omega}(W)$. This is a valid choice, since the existence of $A_{Z,R}^{-1}$ was not assumed.

Therefore, the form of $Z_I$ is now known:

$$Z_I(c_I) = P \circ A_I(c_I)$$

$$= P A_I \circ P A_I(w). \quad (64)$$

In words, what has been discovered is that the Ansatz of $I$-extantness is equivalent to the Ansatz in the statement ‘the $I$-extantness of $c_I$ is related to existence’. That is, the operation associated with, say, ‘it is $I$-extant’ ($P \circ A_I(w)$), when applied twice, forms the statement ‘its $I$-extantness is related to existence’ ($P A_I \circ P A_I(w)$); and it is the same operation. This needn’t be the case in general, and so it is a nontrivial result that

$$Z_I = P \circ A_I. \quad (65)$$

Note that Eq. (65), in this case, is not a definition, but a *theorem*, to be known as the *correspondence corollary* to the relating theorem.

In a sense, $Z_I$ is the fundamental $I$-extant object, in that it is the most obvious starting point for the analysis of the existence of $I$-extant objects in general. It also constitutes the first example of an object demonstrated to exist in a universe (though, a clarification of distinguishing different universes is still required, and investigated in Section 6.3).

Recall, in construction of ‘types’ in Eq. (24), that familiar notions such as ‘chair’, or other such objects, are brought into a recognisable shape using this formula. Though the types may
not appear more recognizable at face value, the properties of such a construction align more closely with what is meant phenomenologically but such objects. In a similar fashion, the type of $Z_I$ can be established, to create a more full, complete, or ‘dressed’ version of the object.

$Z_I$ is an example of a $c_I$. Since projected objects cannot be related directly, a type will be constructed from instances of $A_I$ (of which $A_E$ is one), and the dressed fundamental object will be a projection of the type. Using the same argument used in deriving Eq. (24), the set of $n$ observed instances of $A_I$ takes the following form (acting on some set $W'$)

$$
\bigcup_{j'=1}^{n} A_{\text{set}} \circ \hat{\Phi}_{\Omega^1(W'),i',j'}^2 A_I^j(W').
$$

(66)

If each observed instance may be identified as the set of a certain $N$ characteristics residing in $W$, then our intermediate set $W'$ can be dropped, and we find

$$
A_I \circ \bigcup_{j=1}^{N_{i'}} A_{\text{set}} \circ \hat{\Phi}_{W,i,j}^1 \mathcal{U}_i(W_{i'}) \in A_I^{i'}(\Omega^1(W)).
$$

(67)

In this case, an instance of $A_I^{i'}$ contains more information than just a set of characteristics, since it is also known that it is $I$-extant. Therefore, it contains an additional abstraction operator.

What is meant by the fundamental type therefore takes the form

$$
R_Z \equiv \mathcal{P}_Z \mathcal{A} \equiv \mathcal{P} \cdot A' \circ \bigcup_{j'=1}^{N_{i'}} \bigcup_{j=1}^{N_{j'}} A_{\text{set}} \circ \hat{\Phi}_{W,i',j'}^1 \mathcal{A}_I \circ A_{\text{set}} \circ \hat{\Phi}_{W,i,j}^1 \mathcal{U}_i(W_{i'}) \in \mathcal{P} \Omega^2 \circ \tilde{\Omega}^1 \circ \Omega^1(W),
$$

(68)

where $A'$ is the abstraction of relationship. Note that $R_Z$ is, in general, an element of $\mathcal{P} \Omega^4(W)$.

### 6.3 Distinguishing universes

In this section, we address the issue of classifying universes by their properties in a general fashion. An attempt can then be made to identify features that distinguish universes from one another, and thus clarify the definition of our own universe in a way that is convenient in the practice of physics.

Suppose the definition of a universe, $\mathcal{U}$, to be the ‘maximal’ list of objects that have the same character, that is, obeying the same list of basic properties. The list need not necessarily
be finite, as each collection of properties could, in principle, represent a collection of infinitely many objects themselves. In the language of the formalism developed so far, a formula may be constructed from a generalised object \( c \) by ensuring that each element of \( \mathcal{U} \) is related to the content of the underlying principles, \( w_{\text{princ}} \). This formula will be analogous to Eq. (46).

**Supposition 3.** \( \exists \mathcal{U} \), such that a list of underlying principles may be configured to be enumerable as a countable set, \( w_{\text{princ}} = u_1 \cup \cdots \cup u_N \), for \( N \) elements. (It is not required that \( u_1 \cup \cdots \cup u_N \in \mathcal{U} \)). A universe based on these principles is the object represented by the largest possible set of the form:

\[
\mathcal{U} = \mathcal{P} \circ \bigcup_{p=1}^{n} \bigcup_{k=0}^{N-1} \bigcup_{m\in[0,i_p-1]} \bigcup_{j=0}^{k} \mathcal{A}_{\text{set}} \circ \hat{\Phi}_{\mathcal{A}_{W}}^{(\sum_{j=1}^{n-p+1} i_j)+1-m, u_k+1, c_i}, \tag{69}
\]

with respect to a generalised object, \( c \).

Note that extantness is a universal property, in that it can be defined in the formalism regardless of the universe in which it is extant. It may, but it is not required that it constitute one of the \( N \) underlying principles of a universe.

The distinction between different universes is largely convention, based on the most convenient definition in practising physics. One such convenience is the ability to arrive at a consistent definition of the universe. This is not the case for a naively defined universe required to contain all possible objects, due to Cantor’s Theorem\(^{25} \). In order to establish two universes as distinct, the following convention is adopted:

**Supposition 4.** Consider two consistently definable universes \( \mathcal{U} \) and \( \mathcal{U}' \). If the universe defined as the union, \( X \equiv \mathcal{U} \cup \mathcal{U}' \) is inconsistent, then the universes \( \mathcal{U} \) and \( \mathcal{U}' \) are distinct.

Introducing a square-bracket notation, where \( \mathcal{U}[[\ldots]] \) indicates that the underlying principles to be used in defining \( \mathcal{U} \) are listed in \( [\ldots] \), one may write \( \mathcal{U} = \mathcal{U}[w_{\text{princ}}] \) and \( \mathcal{U}' = \mathcal{U}[w'_{\text{princ}}] \). Define the following lists of principles to be consistent:

\[
w_{\text{princ}} = u_1 \cup \cdots \cup u_{N-1}, \tag{70}
\]
\[
w'_{\text{princ}} = u_N \cup u'_1 \cup \cdots u'_{N'}, \tag{71}
\]
but suppose the inclusion of both principles \( u_{N-1} \) and \( u_N \) to lead to inconsistency. It then follows that

\[
X \equiv U[u_1 \cup \cdots u_{N-1}] \cup U'[u_N \cup u'_1 \cup \cdots u'_{N'}]
\]
is inconsistent. (72)

**Example:** The inconsistency of a set of principles can emerge in the combination of negation and recursion, as clearly demonstrated by Gödel\(^{30}\) and Tarski\(^{31}\). If \( u_{N-1} \) were to express a negation, such as ‘only contains elements that don’t contain themselves’, and \( u_N \) were to enforce a recursion, such as ‘contains all elements’, then Russell’s paradox would result\(^{32}\).

### 6.4 Evidence for other universes

The final denouement is to demonstrate that the fundamental type constitutes evidence for a universe distinct from our own. Consider \( N \) abstract objects, \( A_{v_k} \), each of which represents an element of the fundamental type in Eq. \((68)\). They may be expressed analogously to the operator \( A_I' \) defined in Eq. \((67)\), for \( N \) sub-characteristics:

\[
A_{v_k} \equiv \mathcal{P} \circ A_I \circ \bigcup_{j=1}^{N} A_{\text{set}} \circ \Phi^{1}_{W,i,j} u_{(k)}^{(j)}.
\]  

(73)

Though the sub-characteristics themselves are not vital in this investigation, one can simply see that the objects are \( I \)-extant (by construction), by expressing them in the form:

\[
A_{v_k} = \mathcal{P} \circ A_I(A_{y_k}) \in V',
\]  

(74)

where \( V' \) denotes a set that contains at least all \( N \) elements, \( A_{v_k} \). By determining the underlying principles describing \( V' \), one may denote its maximal set as \( U' \). Since \( V' \) is an abstract object existing as a subset of the objects that comprise our formalism, \( F \), it follows that \( U' \) is the set of all abstracts, and cannot be consistently defined\(^{25}\). That is, by taking the maximal set of objects obeying this restriction, one arrives at a set containing itself, and all possible abstract objects, which is Cantor’s universal set. This does not mean, however, that \( V' \) itself is inconsistent; since \( V' \subset F \subset U' \), we are free to assume that \( V' \) is constructed in such a way as to render it consistent, just as was assumed for \( F \).
Now consider our universe, $\mathcal{U}$, which takes the form of Eq. (69), with the restriction that one of its underlying principles must be that all elements are extant. That is, let $\mathcal{U}$ only contain elements of the form $w = \mathcal{P} \circ \mathcal{A}_E(c) \in \mathcal{U}$. $\mathcal{U}$ may be still be consistently defined. Since $\mathcal{U}'$ is inconsistent, the definition of a composite universe $X \equiv \mathcal{U} \cup \mathcal{U}'$ is also inconsistent. Therefore, by the convention established in Supposition 4, $\mathcal{U}$ and $\mathcal{U}'$ are distinct. It also follows that $V' \subset \mathcal{U}'$ is also distinct from $\mathcal{U}$.

The pertinent underlying principle for $V'$ is simply that it be consistent; or more specifically, that an extant description of it be consistent:

$$w'_{\text{princ}} = Y', \quad \text{such that} \quad V' = \mathcal{P} \circ \mathcal{A}_E(Y'),$$

(75)

for some consistently definable $Y'$. It is reasonable to expect that the elements that comprise $V'$ constitute evidence for the extantness of $V' \in \mathcal{U}'$. This can be checked by determining that the condition for evidence is satisfied:

$$w'_{\text{princ}} \subseteq w^{R,I'}_{\text{princ}}[v_k].$$

(76)

This condition, however, is not satisfied in general, due to the fact that $Y'$ must contain an abstraction, $\mathcal{A}_{E'}$, which is not present in the general formulation of $w^{R,I'}_{\text{princ}}$. This makes sense that the truth of a statement does not necessarily entail an extantness. In the specific case above, though, we have considered the fundamental type of the general $I$-extantness, encompassing all abstractions of a specific form, as described in Section 4.3. In this special case, the extantness required by both sides of Eq. (76) is the same, and we are required to demonstrate that

$$w'_{\text{princ}} = Y' \subseteq w^{R,I'}_{\text{princ}}[v_k],$$

(77)

for $V' = \mathcal{P} \circ \mathcal{A}_I(Y')$.

(78)

**Proof:** $Y'$ is a set containing $N + 1$ abstractions, $\mathcal{A}_{v_k}$, with $\mathcal{A}_{N+1} \equiv \mathcal{A}_I$:

$$Y' = \{ \mathcal{A}_{v_1}, \ldots, \mathcal{A}_{v_N}, \mathcal{A}_I \} = \bigcup_{k=0}^{N} \mathcal{A}_{\text{set}} \circ \mathcal{Q}_{V',i,k}^{-1} \mathcal{A}_{v_k}.$$  

(79)
The righthand side of Eq. (77) takes the form:

\[ w_{\text{princ}}^{R,I'} = \bigcup_{k=0}^{N-1} \bigcup_{p \in [2,n] \setminus \bigcup_{k=0}^{k} \{p(r)\}} A_{\text{set}} \circ \hat{\Phi}_{C_{W}^{i_{1} \ldots i_{p} \ldots i_{n},1,v_{k+1}+1} \bigcup w_{I}}. \] (80)

The left term can be pared down by choosing \( n = 2, i_{1} = 0 \) and \( i_{2} = 1 \), and the right term, \( w_{I} \), by choosing \( n = 1, i_{1} = 1 \):

\[ \Rightarrow \left( \bigcup_{k=1}^{N} A_{\text{set}} \circ \hat{\Phi}_{P \Omega_{1}(W),1,v_{k+1}+1} \bigcup \left( A_{\text{set}} \circ \hat{\Phi}_{\Omega_{1}(W),m+1,I'} \right) \right) \]
\[ = \{ P_{A} v_{1}(w), \ldots , P_{A} v_{N}(w) \} \bigcup \{ A_{I'}(w) \} \]
\[ = \bigcup_{k=0}^{N} A_{\text{set}} \circ \hat{\Phi}_{V',i,k} A_{v_{k}}, \quad \text{for} \quad A_{N+1} \equiv A_{I'}. \quad \square \]

The final line follows from the labelling principle, \( P \circ P = P \), and the fact that each \( A_{v_{k}} \) is in extant form, \( P \circ A_{I}(A_{y_{k}}) \).

### 7 Conclusion

This manuscript has attempted to address key issues in physics from the point of view of philosophy. By adopting a metaphysical framework closely aligned with that used in the practice of physics, the philosophical tool of Ansatz was examined, and the process of its use was clarified. In this context, a mathematical formalism for describing intrinsically non-mathematical objects was expounded. In examining the consistency of such a framework, a careful distinction between existence (in the mathematical sense) and ‘extantness’ (in the sense of phenomena existing in the universe) was made. Using the formalism, a general condition for extantness was derived in terms of a generalised object, which incorporates the salient features of abstraction and projection to the non-mathematical world in a way easily manipulated. In principle, the formalism may make verifiable predictions, since properties (and the consequences of combinations of properties in the form of theorems) can be arranged to make strict statements about the behaviour or nature of a system or other general objects.
As an example, a possible explanation of Wigner’s ‘unreasonable effectiveness’ of mathematics was derived. This demonstrates the ability of a metaphysical framework to address important mysteries inaccessible from within the physics itself being described.

Lastly, an attempt was made to classify other universes in a general fashion, and to clarify the characteristics and role of evidence for theories that provides at least a partial description of a universe. The connection between phenomena that constitute evidence and the theory itself was established in a Duality Theorem. Instead of focusing on attempting an ad-hoc identification of extra-universal phenomena from experiment, the formalism was used to derive basic properties of objects that do not align with our universe. As a first example toward such a goal, a fundamental object was identified, which satisfies the necessary properties of evidence, and whose extantness does not coincide with our universe. This paves the way for future investigations into more precise details of the properties (perhaps initially bizarre and unexpected) that objects may possess outside our universe.

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