Calculating Millimeter-Wave Modes of Copper Twisted-Pair Cables Using Transformation Optics

ALI MOHAJER HEJAZI, (Graduate Student Member, IEEE), GERT-JAN STOCKMAN, YANNICK LEFEVRE, VINCENT GINIS, AND WERNER COOMANS

1Data Laboratory, Applied Physics Research Group, Vrije Universiteit Brussel, 1050 Brussel, Belgium
2Nokia Bell Labs, 2018 Antwerp, Belgium

ABSTRACT Recent research has indicated considerable potential for millimeter-waves in copper access, with data-rate estimations up to 1 Terabit per second for a reach of 100 m. This line of research exploits millimeter-waves and their corresponding higher-order propagation modes inside the twisted pair cable binder. Unlike the conventionally used transmission-line mode (currents through copper wires), the approach relies on the copper and plastics present in these cables to form a low-loss waveguide. Here, we take a closer look at the potential of millimeter-wave propagation in twisted pair cables by refining the idealized assumptions, used by Cioffi et al., and identifying the limiting factors. To this end, we introduce the concept of transformation optics as an efficient method of calculating the propagating modes on a twisted pair. Leveraging this technique allows us to calculate modal propagation using realistic material parameters, exposing an important trade-off between loss and confinement. Our modeling results yield achievable data rates that are orders of magnitude lower than those achieved under idealized assumptions. According to our results, 1 Terabit per second can be achieved up to a distance of about 10 m over a twisted-pair with a plastic sheath.

INDEX TERMS Copper access, millimeter wave propagation, TDSL, twisted pair cables, waveguides.

I. INTRODUCTION
Over the last three decades, telecommunication providers have largely succeeded in delivering broadband services in line with the increasing demand for bandwidth by leveraging a hybrid fiber-copper network. One of the deployed technologies in the copper part of such a network is Digital Subscriber Line (DSL) technology (for twisted-pairs) [1]. Today it is capable of delivering 1 Gbps using the G.fast specification. The next generation successor of G.fast, called G.mgfast, will be able to deliver downstream data rates close to 5 Gbps on short copper pairs [2]. The conversion from a hybrid-fiber-copper network to a fully-fledged fiber network has proven difficult, mainly due to economic reasons. Today, as the operational cost of replacing the last copper drop with fiber is large and unpredictable for the telecommunication provider, FTTH solutions are mostly available in densely populated areas. In such cases, those costs can be divided over many subscribers reducing the average deployment cost per subscriber. But even there where fiber is readily available, exploiting the last copper drop is still an attractive solution for telecommunication providers. In cities for example, connecting each apartment in a multi-dwelling unit to the access network still largely relies on copper solutions.

Recently, in an effort to push the technological boundaries of a DSL network even further, Cioffi et al. have proposed the concept of Terabit DSL (TDSL) [3]. TDSL aims to use the waveguide transmission modes of a twisted-pair cable by using the cable bundle as a guiding medium in the 100 – 300 GHz range. At these frequencies, the conventional transmission line modes – with currents flowing to and from the other end of the cable – are extremely lossy. Nonetheless, at high frequencies, another type of (wireless) mode can exist that supports propagation of electromagnetic waves traveling through the plastic insulators of the copper wires, internal air gaps, the plastic surrounding sheath, and the surrounding air. According to the initial assessment in [3] (using certain

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idealized modeling assumptions), the loss at these frequencies would be sufficiently low to allow data-rates up to 1 Tbps at a distance of 100 m. Based on measurements of a waveguide consisting of two parallel wires and a metal sheath, a more recent assessment found that a reach of about 10 m for 1 Tbps is feasible [4].

In this paper, we make a self-consistent calculation of the data-rates by investigating the dispersion relations of the propagating modes. To this end, we take into account effects that were not considered in the modeling of [3], such as conductor loss, dielectric loss of the plastic and the effect of twisting the wires. The modeling in [5] addresses a similar question for Ethernet cabling (CAT5e) with a metallic shield, and in which losses are calculated using conventional three-dimensional electromagnetic solvers. While [5] focused on a shielded cable, we consider unshielded twisted pairs that are used in fixed access networks (over which xDSL technologies run today). This naturally reduces mode confinement and thereby inflates simulation complexity. We therefore also propose another simulation methodology leveraging transformation optics to deal with the twisting of the wires and reducing the three-dimensional simulations to two-dimensional simulations. This approach reduces the computational complexity and allows more accurate simulations, thus enabling more detailed investigations such as for instance simulating the influence of the twist rate of the wires. Furthermore, it can be used to simulate geometries under conditions that would be difficult for a three-dimensional simulator, such as a very strong twist rate close to the waveguide port. Here, the result from the three-dimensional simulator can be inaccurate as it assumes the injected mode at the waveguide port to be that of a waveguide with constant cross-section over the direction of propagation.

In Section II, we start by explaining how the twisted wires can be simulated using the geometrical technique known as transformation optics. In Section III, we will investigate the propagating modes on different types of twisted-pair waveguides based on the methodology discussed in Section II. Consequently, in Section IV, we leverage the results from these previous sections to refine the assumptions made by [3] in order to obtain a more realistic rate-reach (capacity versus loop length) for millimeter-wave propagation in the DSL network. We conclude in Section V.

II. HOW TO MODEL ELECTROMAGNETIC MODES ON TWISTED-PAIRS

In copper-access networks, two wires are typically twisted around one another to form a twisted-pair. In this section, we will investigate how we can apply transformation optics to mitigate the computational complexity inherent in simulating millimeter-wave propagation over twisted-pairs.

Generally, a valid simulation approach would be to use conventional full-wave three-dimensional electromagnetic solvers. These solvers discretize Maxwell’s equations and can generally tackle any kind of problem geometry. However, for high-accuracy meshes or electromagnetically large domains, these simulations quickly induce large computational complexity and long simulation times. This is especially true for waveguide systems with a poor confinement. In those systems, the field profiles can extend relatively far away from the waveguide structure, which means that the simulation domain has to be chosen sufficiently large to capture all relevant effects. This is why three-dimensional problems are often solved using two-dimensional simulations, by exploiting the symmetry of the problem, e.g., by solving the modes on a cross-section of a waveguide. Due to the twisting of the wires, however, a twisted-pair is not translationally invariant and would thus normally have to rely on a three-dimensional full-wave solver. In addition, adjusting the twist rate requires a completely new full-wave simulation which makes it difficult and time-consuming to investigate the effect of increasing/decreasing twist rate on the propagating modes. Nonetheless, the translational invariance can be restored by taking advantage of the ideas underlying the technique of transformation optics [6], [7]. A transformation of the coordinate system can be leveraged to perform low-complexity simulations in an auxiliary space (where there is translational invariance) and transform those results back to be applicable for a twisted-pair. The transformation allows for the simulations to be performed in two dimensions rather than in three dimensions, allowing calculations with lower computational complexity or higher mesh-densities for better accuracy.

Maxwell’s equations are form-invariant under coordinate transformations meaning the mathematical form of those equations is preserved when an electromagnetic problem is reformulated in another coordinate system. However, the change of the coordinate system needs to be compensated for by changing the material properties such as the permittivity, permeability and conductivity tensors $\varepsilon$, $\mu$, and $\sigma$, respectively. In other words, to change from one coordinate system to another, one needs to replace the initial material parameters with new material parameters corresponding to the new coordinate system.

Technically this boils down to exploiting the equivalence between two ‘spaces’. On the one hand, there is the so-called ‘physical space’, which is expressed in a Helical coordinate system (since the twisted waveguide’s shape follows Helical coordinate system) and which contains isotropic and homogeneous materials parameters $\varepsilon$, $\mu$, and $\sigma$. In this space, the wires are twisted. On the other hand, we identify the ‘electromagnetic space’. In this space, the wires are straight, and the system is expressed in Cartesian coordinates. Actively changing the coordinate system – and the geometry – generally changes the physics of the system. However, the effects of the new coordinate system can be absorbed by changing the material properties of the materials. By defining new material parameters $\varepsilon'$, $\mu'$, and $\sigma'$ in the electromagnetic space, it is possible to reproduce the exact same Maxwell’s equations in both systems. This allows simulating the complex geometry of twisted wires with isotropic and homogeneous material properties using an equivalent system with a much simpler
geometry (straight wires) and anisotropic, inhomogeneous materials properties.

For a general coordinate transformation, the correspondence is given by the following equivalence relations:

\[ \epsilon' = \frac{J \cdot \epsilon \cdot J^T}{\det(J)}, \]

\[ \mu' = \frac{J \cdot \mu \cdot J^T}{\det(J)}, \]

\[ \sigma' = \frac{J \cdot \sigma \cdot J^T}{\det(J)}, \]

where \( \epsilon, \mu \) and \( \sigma \) are the material parameters in the physical space and \( \epsilon', \mu' \) and \( \sigma' \) are the material parameters in the electromagnetic space. In these equations, \( J \) is the Jacobian matrix that relates the elements of the two corresponding coordinate systems. The superscript ‘T’ refers to the transpose of the Jacobian.

The geometry of our twisted waveguide is extremely elegant when expressed in a Helical coordinate system. The equations relating the Cartesian and the helical coordinate system are given by [8]:

\[ x = \zeta_1 \cos \alpha \zeta_3 + \zeta_2 \sin \alpha \zeta_3 \]

\[ y = -\zeta_1 \sin \alpha \zeta_3 + \zeta_2 \cos \alpha \zeta_3 \]

\[ z = \zeta_3. \]

In these equations, \( \zeta_1, \zeta_2, \zeta_3 \) are coordinates of the helical system and the parameter \( \alpha \) specifies the twist rate in the waveguide (expressed in rad/cm). The Jacobian matrix consists of the partial derivatives of the new coordinate system with respect to the initial one. In the case of a transformation from the Helical to the (new) Cartesian system, it is given by:

\[ J_{\text{hel}} = \begin{bmatrix}
\cos \alpha \zeta_3 & \sin \alpha \zeta_3 & \alpha \zeta_2 \cos \alpha \zeta_3 - \alpha \zeta_1 \sin \alpha \zeta_3 \\
-\sin \alpha \zeta_3 & \cos \alpha \zeta_3 & \alpha \zeta_2 \sin \alpha \zeta_3 - \alpha \zeta_1 \cos \alpha \zeta_3 \\
0 & 0 & 1
\end{bmatrix} . \]

Let us now consider our twisted-pair, expressed in the helical coordinate system. All the material parameters are isotropic and homogeneous. We use (1)-(3) to retrieve the new material parameters in the transformed coordinate system (electromagnetic space). The transformation matrix that transforms the material properties from a Helical to a Cartesian coordinate system is given by:

\[ J_{\text{hel}}^{-T} = \frac{J_{\text{hel}}^T \cdot J_{\text{hel}}}{\det(J_{\text{hel}})} = \begin{bmatrix} 1 & 0 & \alpha \zeta_2 \\
0 & 1 & -\alpha \zeta_1 \alpha \zeta_2 \\
\alpha \zeta_2 & -\alpha \zeta_1 & 1 + \alpha^2(\zeta_1^2 + \zeta_2^2) \end{bmatrix} . \]

Fig. 1 illustrates the concept of using the transformation-optics approach to tackle the twisting problem. In Fig. 1 (bottom), the physically twisted waveguide is shown, following the Helical coordinate system. The material properties in this system are homogeneous and isotropic. In Fig. 1 (top), by means of coordinate transformation, we obtain a geometrically simpler waveguide. However, the material properties of the waveguide become anisotropic and inhomogeneous, in agreement with the equivalence relations, shown in (1)-(3).

Using transformation formula (8), we can set the material properties to their transformed values. The new material parameters can then be simulated by a mode solver to calculate modes of the waveguide and the dispersion relation for each mode. Thus, obtained field distributions of each mode are specified in the coordinates \((\zeta_1, \zeta_2, \zeta_3)\). The fields can be recalculated in the coordinates \((x,y,z)\) based on the following formula:

\[ \begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix} = J_{\text{hel}}^{-T} \begin{bmatrix}
E_{\zeta_1} \\
E_{\zeta_2} \\
E_{\zeta_3}
\end{bmatrix} , \]

where \( J_{\text{hel}}^{-T} \) is the transpose of the Jacobian’s inverse.

It is interesting to note that also more complex twisted wire bundles could be simulated efficiently using this technique. For instance, larger cables, where twisted wire pairs are combined and twisted around each other, could be simulated using a super-transformation that combines several local transformations into one [9]. This is outside the scope of this paper.

As a validation example, we consider two dielectric rods that are twisted together. We generate the field modes using a full-wave three-dimensional simulation (COMSOL) and compare the result with the modes obtained through the transformation optics approach. Here, we consider a twist rate of \( \alpha = 2\pi/15.5 \text{ rad/cm} \) and a permittivity of the dielectric rods of 2.25. We choose a large twist length as for higher twist rates (smaller twist lengths), the three-dimensional simulation quickly becomes too complex. For simplicity, we consider the dielectric to be lossless. The radius of each dielectric rod is equal to 0.5 mm. The simulation domain length is exactly equal to one twist length (15.5 cm). The frequency at which we validate the approach equals 100 GHz.

For the full-wave three-dimensional simulation, the simulation domain around the waveguide is chosen as a cylinder
after Sommerfeld’s initial publication, Georg Goubau was the first to investigate the electromagnetic modes propagating along a metal wire covered by a plastic insulator [13]. As a result, a single insulated metal wire is now often referred to as a Goubau line. Another more accurate investigation of the propagating electromagnetic modes on an insulated wire was conducted by King and Wiltse [14]. They generalized the previous study of Goubau and extended the analysis to millimeter-waves above 100 GHz. Their analysis only considered the fundamental mode propagating on the wire and did not include any higher-order modes. Their derivation relied on a low-frequency approximation to obtain analytical dispersion relations.

Based on the theory elaborated in Section II, we now study the propagating modes of a realistic waveguide made of twisted insulated copper wires.

The twisted-pair waveguide under consideration has a wire diameter of 0.5 mm and a cladding radius of 0.5 mm (added thickness of the plastic cladding equals 0.25 mm), which is typical for twisted pair cables in copper access networks as well as Ethernet cables. The copper conductivity is considered 5.813 \cdot 10^{7} \text{ S/m} and the relative permittivity of the dielectric’s cladding is chosen equal to 2.25. This is the value for a typical material used for plastic insulation in twisted-pair cables, namely PE (polyethylene, in accordance with EN 50290-2-23). For PEs loss-tangent, we assume \tan\delta = 0.0004. These dielectric parameters are tabulated at 10 GHz in [15]. A similar material to PE, namely PTFE (polytetrafluoroethylene, also known as Teflon), has no dipolar mechanisms such that its permittivity (and as such its loss-tangent, which is the ratio of the permittivity’s imaginary and real part) is remarkably constant well into the millimeter-wave region [16]. Based on this consideration, we make the assumption that PE’s permittivity and loss-tangent remain constant over our frequency range of interest. The amount of twist, or twist rate, applied to the waveguide, represented by \alpha in Section II, is equal to 2\pi / 2.5 \text{ rad/cm} which amounts to 2.5 cm for a complete twist. This is also in line with typical values found in the fixed access network.

We use a finite elements method solver (COMSOL Multiphysics) to numerically retrieve the dispersion relation of the waveguide modes. Based on the techniques discussed in the previous section, the dispersion relation (and corresponding mode profiles) were found using a two-dimensional simulation of the cross-section of the twisted waveguide. The dispersion relation can be visualized in different ways: by plotting the longitudinal component of the wave vector \kappa, or by showing the effective mode index \n_{\text{eff}} versus frequency. In this work, we focus on the latter, where \n_{\text{eff}} is defined as \kappa / \omega. The effective mode index is an intrinsically complex number. The real part of \n_{\text{eff}} expresses the phase velocity of the confined modes, whereas the imaginary part yields the damping experienced by these modes (in dB/m).

In Fig. 3 the real part of the longitudinal component of the electric field for the first six modes is shown.

![Figure 2](image-url)

**FIGURE 2.** The real part of the z-component of the electric field at 100 GHz as obtained by (a) simulating the physically twisted waveguide in three dimensions and (b) simulating the waveguide using transformation optics in two dimensions. The twist length is equal to 15.5 cm.
In a conventional waveguide, modes are often classified as transverse electric (TE) or transverse magnetic (TM) depending on their characteristics with respect to a mirror symmetry plane of the waveguide’s geometry [17]. Due to the waveguide’s twisted geometry, no such symmetry planes are present here, which makes this typical classification not possible.

The mode profiles can change their forms drastically as frequency increases. Here, we plot the different modes at a frequency in which they are in their first form. From this figure we can discern some spatial properties of the modes on a twisted pair. First of all, the modal distributions are slightly distorted due to the twisting of the wires. This is especially noticeable for some of the lower-order modes, for instance mode 2. This distortion appears particularly for lower-order modes since the energy in those modes is located close to the waveguide structure, strongly feeling the effect of twist. This also explains why for higher frequencies the impact is stronger as well. For the higher-order modes, a majority of the field is positioned in vacuum, and the twisting of the wires therefore has less impact on these modes. Secondly, it is obvious from Fig. 3 that for higher-order modes, the energy is concentrated predominantly in the surrounding (lossless) vacuum. We expect these modes to suffer less from conductor/dielectric loss. Note that particular attention needs to be paid to the meshing of the point of contact between the two wires as the point where two cylinders touch can induce a singularity in the modal spectrum [18], [19].

We can classify the different modes in these kinds of waveguides by counting the number of nodes in the azimuthal plane of the real part of the electric field \( \Re(E_z) \), using

\[
N = 2 \times (m - 1),
\]

where \( N \) is the number of nulls in the pattern, and \( m \) is the (azimuthal) mode number. Take Fig. 3 (c), for example. Here, the number of zeros in the azimuthal plane is \( N = 4 \), which translates into the unique mode number \( m = 3 \). For Fig. 3 (e) and (f) on the other hand, we obtain \( m = 5 \) for both modes. This is the reason that a subdivision into mode 5a and 5b has been made. Note that there is no true null in the profile of the first mode (Fig. 3 (a)), as the zero-crossing does not appear for all radii and the transition between the maximum and the minimum occurs asymptotically close to the point of contact.

From the COMSOL simulation, we also retrieve the dispersion relations. For each mode, Fig. 4 shows as a function of frequency (top) the real part of the effective index \( \Re(n_{\text{eff}}) \), which quantifies the phase variation experienced by the mode as it propagates, and (bottom) the damping, corresponding to the loss experienced by the mode as it propagates. From the increasing \( \Re(n_{\text{eff}}) \) we can conclude that the mode profiles change and that they tend to become more confined to the waveguide for increasing frequency (\( \Re(n_{\text{eff}}) = 1 \) would indicate free space propagation or no confinement at all).
From the damping-curves, it is especially evident that the higher-order modes suffer less from attenuation in the lower frequency range. Here, the higher-order modes are poorly confined and are mostly propagating in lossless free space. However, as frequency increases, the modes profiles become more closely concentrated to the metal and dielectric, leading to stronger interaction between these lossy materials. Consequently, this leads to higher losses, as seen on the bottom curves of Fig. 4. To illustrate the increased confinement for increasing frequency, Fig. 5 shows the mode profiles of the first two modes at two different frequencies. The plot shows that for higher frequencies, the mode profiles become more concentrated around the twisted pair. This leads to an important conclusion of this work that there is a fundamental trade-off between the confinement of the modes and the losses they experience during propagation. In the two extremes, the waves either experience low loss at the expense of poor confinement, or good confinement at the expense of high loss.

We now extend the analysis to include a dielectric sheath (typical for access), as well as a metallic sheath (not typical for fixed access networks, but only for high-quality Ethernet cabling). We use the same material properties for the dielectric (PE) and the conductor (copper) as described in Section III. In the first case, the twisted pair is surrounded by a lossy dielectric sheath (thickness = 2 mm). The second case is a twisted pair with metal shield (thickness = 0.5 mm). The plastic sheath, as well as the metal shield, could in theory improve confinement and perhaps reduce the overall propagation losses. The permittivity of the dielectric sheath, as well as the dielectric loss tangent are chosen equal to the insulator around the wires. The material used for the metal sheath is copper. The twist rate used is \( \alpha = \frac{2\pi}{2.5} \text{ rad/cm} \). The resulting damping is shown in Fig. 6 and their accompanying effective refractive indices in Fig. 7. In the case of the dielectric sheath (see Figs. 7(top) and 6(top)), several sudden variations in the slopes of the mode characteristics are present. We believe this behavior is caused by mode hybridization, which is known to occur in asymmetric multi-mode waveguides [20].

The results reveal that for the twisted-pair in a plastic sheath, at low frequencies, the higher-order modes are propagating mainly in vacuum around the plastic sheath. Around 50 GHz the modes start to propagate closer to the cable and become more and more confined. This is also why we see lower damping of those modes in the frequency range smaller than 50 GHz (see Fig. 6(top)). At higher frequencies, where larger damping is observed, the modes are mainly propagating inside the dielectric sheath. Consequently, the interaction between the modes and the dielectric sheath introduces significant losses. Typically, twisted pairs are encapsulated in a PVC sheath, which induces even higher losses than the PE sheath used in the simulations here. In the case of twisted-wires covered by a metal shield, the modes seem to be better confined. However, there are also stronger interactions with the metal and the lossy dielectric parts of the waveguide. Consequently, the damping is also large for all modes (Fig. 6 bottom). From these simulations, it is clear that, although the plastic sheath and metal shield improve the confinement of the waves, overall, the total loss in the system has worsened (especially below 200 GHz) due to the increased exposure to the lossy dielectric and metal materials. The use of PVC instead of PE is expected to make this effect even worse.
As mentioned in Section II, one of the advantages of exploiting the techniques of transformation optics is that it allows to study effects such as a varying twist rate. In Fig. 8, the damping of the first four modes of a twisted-pair in free space are depicted for varying twist length. The simulation frequency is fixed at 100 GHz. The horizontal axis represents the twist length which has a reciprocal relation with the twist rate $\alpha$ (as twist length equals $2\pi/\alpha$). In Fig. 8, we see that the first mode is the most resistant and the damping does not change significantly with varying twist rate (this mode is the most confined one). For higher order modes, for an increase in the twist length (less twist), we generally observe less interaction of the fields with the lossy dielectric and the conductive parts of the waveguide in the mode profile. This explains the slightly decreasing trend in damping for these modes.

IV. CAPACITY CALCULATIONS

To calculate rate-reach curves, different assumptions need to be made. Here, we mainly follow the assumptions made in [3], but use our more realistic channel model. The COMSOL simulations above are used to construct the direct channel of the different modes propagating in twisted-pair cables. We will focus on the twisted pair in free space (see Fig. 4) and the twisted pair surrounded by a plastic sheath (see Fig. 6 and 7 top) as these examples are most relevant in an access network context. In accordance with [3], to make a comparison of both modeling attempts possible, we also assume four modes per home (the lowest-order modes). The channel capacity can be calculated from Shannon’s theorem [21]:

$$C = \Delta f \sum_k \log_2 \left(1 + \frac{\text{SNR}_k}{\Gamma}\right),$$

for which the different parameters are in accordance with [3]. SNR$_k$ is the signal-to-noise ratio for tone $k$ (with a total of 4096 tones) and $\Delta f$ denotes the frequency resolution for a bandwidth reaching up to 300 GHz. Furthermore, $\Gamma$ is the SNR gap defined in dB as $\Gamma = \text{Gap to capacity} - \text{Coding Gain} + \text{SNR Margin} = 9.75 \text{ dB} - 7 \text{ dB} + 6 \text{ dB} = 8.75 \text{ dB}$. Here, the gap to capacity originates from using a sub-optimal modulation scheme, compared to what is theoretically achievable. Coding gain can reduce this gap, depending on the type of code that has been implemented, and SNR margin is a buffer that is built in to increase robustness. A total transmit power of 20 dBm is assumed, to be divided over the four modes, and the PSDs of the four modes are jointly optimized using a discrete waterfilling algorithm [22]. The noise floor used in [3] was $-160 \text{ dBm/Hz}$ (as a reference, the thermal noise floor is $-174 \text{ dBm/Hz}$). The bit loading is capped at 12 bit, and 10% physical-layer overhead was assumed.

The modes simulated in Section III were obtained under the assumption of an ideal waveguide structure, in which the waveguide geometry and twist ratio remains constant along the longitudinal direction. For such ideal waveguides, all the modes propagate independently without any crosstalk between them. However, practical twisted-pairs are characterized by non-idealities, such as variations in the waveguide geometry or twisting, which cause a scattering of the modes, leading to further losses and a coupling or crosstalk between the modes. This crosstalk can be compensated and even exploited using multiple-input multiple-output signal processing. A detailed analysis of the scattering in practi-
when no requirement on confinement is initiated (\(\Re(\eta_{\text{eff}}) > 1.0\). Data-rates than the twisted pair in a plastic sheath (red) and a twisted pair in free space (blue), the curves for the confinement of the propagating modes. Note that for the twisted pair in a plastic sheath (red) with added conditions on rate-reach curves for a twisted pair in free space (blue) and a twisted pair in a plastic sheath (red) with added conditions on the confinement of the propagating modes. Note that for the twisted pair in free space (blue), the curves for \(\Re(\eta_{\text{eff}}) > 1.1\) and \(\Im(\eta_{\text{eff}}) > 1.2\) coincide.

The twisted pair in free space (blue) shows higher data-rates than the twisted pair in a plastic sheath (red) when no requirement on confinement is initiated (\(\Re(\eta_{\text{eff}}) > 1.0\)). This is in accordance with the damping curves shown in Figs. 4 and 6, where the twisted pair in free space is seen to have some very low-loss modes at the lower part of the band compared to the modes of the twisted pair in a plastic sheath. However, it is clear that the very low-loss modes of the twisted pair in free space originate from modes propagating mainly in the free space environment, with poor confinement to the cable. For the twisted pair in a plastic sheath, the confinement is higher (as is apparent from the higher real part of the effective mode index in Fig. 6 compared to Fig. 4), but due to this increased confinement, there is more interaction with the dielectrics in the system, and consequently higher losses are observed. Nonetheless, due to increased confinement, the desired confinement requirements do not influence the capacity of the cable as much as is the case for the twisted pair in free space. For the free space case, the capacity rapidly drops, as the modes cannot satisfy the condition and can thus not be loaded. The twisted pair in a plastic sheath is less influenced as most of the modes are in accordance with the confinement condition and can be leveraged. It is clear that the combination of both loss and confinement is important in an access network system. In the remainder of the work, we will impose the effective index \(\eta_{\text{eff}}\) to at least equal 1.35, to impose sufficient confinement.

Applying this condition to the twisted pair in free space and the twisted pair in a plastic sheath leads to Fig. 10, where a comparison is also made with G.mgfast 424 MHz [2]. Due to the new imposed condition of \(\eta_{\text{eff}} > 1.35\), the rates of the system with a twisted pair in free space drop to zero. The twisted pair in free space only meets this condition at frequencies where losses are already too high to contribute to the system capacity. The twisted pair in a plastic sheath, on the other hand, shows better results. Due to its improved confinement, more modes satisfy the condition and can be leveraged to yield higher data-rates. It is clear that an improved confinement makes the system more practical for use in a real access network scenario.

For this simulation (twisted pair in plastic sheath), we obtain rates at least an order of magnitude lower than those predicted by the initial model in [3]. According to the projections, Tbps rates are only possible up to about 10 m, which is in-line with assessments based on measurements of a two-wire waveguide [4].

Other aspects that are to be expected but that were not included in our analysis are typical DSL impairments (bridge tap, bad connection, cable bends and kinks) leading to additional signal loss, the close presence of other cables or the effects of external surroundings to the higher order modes (the modes used in this analysis are assumed to propagate partly in lossless free space surrounding the cable), and the actual properties of the plastic (e.g., color pigments of different color wires in a twisted-pair could introduce additional losses in the dielectric and therefore increase its loss tan-
gent). In real twisted pair cables, PVC (as opposed to PE, as assumed in Fig. 10) is also often used as cable insulation. In that case, the loss tangent would be at least an order of magnitude larger and also increasing with frequency [23]. This would inflate the dielectric losses and significantly reduce the achievable data-rates.

V. CONCLUSION
The prospect of Terabit data-rates in legacy copper access technologies makes investigating the TDSL potential worthwhile. In this work we demonstrated that transformation optics is an efficient method to calculate propagation properties for twisted pairs. The approach allows for much shorter simulation times at a lower complexity and a higher accuracy. This was validated by comparing to full-wave three-dimensional simulations. This method was leveraged to calculate the modes on a twisted pair with realistic loss assumptions and we observed a clear trade-off between confinement and loss. Further improving the modeling efforts from the idealized modeling in [3] led to a drastic reduction in achievable data-rate (e.g., 10 Gbps at 53.5 m for our modeling compared to 3.75 Tbps at that same length in [3]). Modeling improvements that we introduced consist of including dielectric loss (which is substantial), simulating the modes on an actual twisted pair cable using corresponding material parameters and including the effects of twisting the cables by utilizing transformation optics. Our projections indicate that the presence of a plastic sheath is essential for achieving Terabit data rates. For free-space twisted-pairs, the confinement of the millimeter-wave modes is insufficient to even achieve Gigabit data rates. In case of a plastic sheath, Terabit data rates are projected up to about 10 m. For longer reaches, the potential of millimeter-wave communication over twisted pairs is limited even in case of a plastic sheath, as G.mgfast, the latest generation of DSL, is projected to achieve higher data rates from about 60 m onwards.

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ALI MOHAJER HEJAZI (Graduate Student Member, IEEE) received the master’s degree in photonics from the Chalmers University of Technology, Gothenburg, Sweden, in 2017. He is currently pursuing the Ph.D. degree with Vrije Universiteit Brussel (VUB). His research interests include electrodynamic properties of photonic complex structures and devices, metamaterials, plasmonics, nonlinear optics, and optical communications.

GERT-JAN STOCKMAN received the M.Sc. and Ph.D. degrees in electrical engineering from Ghent University. From 2016 to 2020, he was active as an Access Technology Research Engineer with Nokia Bell Labs. He is currently active as the Network Partnership Manager with Proximus. His research interest includes next-generation fixed access technologies that leverage legacy copper infrastructure.
YANNICK LEFEVRE received the master’s degree in engineering sciences from Vrije Universiteit Brussel (VUB), Brussels, Belgium, the second master’s degree in engineering sciences from Universiteit Gent, Ghent, Belgium, in 2010, and the Ph.D. degree in applied sciences and engineering from VUB, in 2014. He joined as a Research Engineer with Nokia Bell Labs, Antwerp, Belgium, in 2015, where he works on next-generation copper and optical access technologies, and is involved with the standardization of digital subscriber line (DSL) and passive optical network (PON) technologies. He has authored or coauthored more than 70 articles, patent applications and standard contributions. His research interests include digital signal processing, equalization, forward error correction, constellation shaping, and modulation. He was a recipient of an Aspirant Grant from the Research Foundation-Flanders (FWO).

VINCENT GINIS is currently an Assistant Professor of mathematics and physics with Vrije Universiteit Brussel. He works as a Visiting Professor with the Group of Prof. Federico Capasso, Harvard University. Working in collaboration with international scientists, his research is published in prominent scientific journals, including Science, Nature Photonics, and PRL. His research interests include applied physics, data analytics, and computer science with a topical focus on electromagnetism and neural networks. He is also the Co-President of the Young Academy of Belgium. He regularly gives invited presentations at international conferences, and his scientific work has been recognized by IEEE, SPIE, Solvay, the Royal Academy of Belgium, and the Research Foundation Flanders (FWO).

WERNER COOMANS received the M.Sc. degree in electrical engineering and the Ph.D. degree in engineering sciences from the Vrije Universiteit Brussel, in 2009 and 2013, respectively, with a focus on semiconductor lasers. He joined Bell Labs, in 2013. He currently heads the Fixed Networks Department, Nokia Bell Labs, focusing on next-generation gigabit access technologies leveraging legacy infrastructure. He actively contributes to several standardization bodies, such as ITU-T and CableLabs, has filed more than 15 patents. He was a recipient of the Ph.D. Fellowship of the Research Foundation Flanders (FWO), in 2009, the Bell Labs Outstanding Results Award, in 2018, and the Edison Patent Award for Telecommunications, in 2019.

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