Counting pairs in $^{44}$Ti with various interactions

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We count the number of pairs in the single $j$–shell model of $^{44}$Ti for various interactions. For a state of total angular momentum $I$, the wave function can be written as\[\Psi = \sum_{J_P J_N} D(J_P J_N) [ (j_P^2)^J_P (j_N^2)^J_N ]^I, \]
where $D(J_P J_N)$ is the probability amplitude that the protons couple to angular momentum $J_P$ and the neutrons to $J_N$. For $I = 0$ there are three states with $(I = 0, T = 0)$ and one with $(I = 0, T = 2)$. The latter is the double analog of $^{44}$Ca. In that case $(T = 2)$, the magnitude of $D(JJ)$ is the same as that of a corresponding two particle fractional parentage coefficient. In counting the number of pairs with an even angular momentum $J$ we find a new relationship obtained by diagonalizing a unitary nine–j symbol. We are also able to get results for the ‘no-interaction’ case for $T = 0$ states, for which it is found that there are less $(J = 1, T = 0)$ pairs than on the average. Relative to this ‘no-interaction case’ we find for the most realistic interaction used that there is an enhancement of pairs with angular momentum $J_A = 0, 2, 1$ and 7, and a depletion for the others. Also considered are interactions in which only the $(I = 0, T = 1)$ pair state is lowered, only the $(J = 1, T = 0)$ pair state is lowered and where both are equally lowered.

I. INTRODUCTION

In this work we expand upon a note in a previous work on charge operator sum rules and proton-neutron $T = 0$ and $T = 1$ pairing interactions [1]. The note in question has to do with counting the number of pairs of particles with a given angular momentum in $^{44}$Ti in a single $j$–shell approximation.

In the single $j$–shell model $^{44}$Ti consists of two valence protons and two valence neutrons in the $f_{7/2}$ shell. The allowed states for two identical particles have angular momenta $J = 0, 2, 4$ and 6 and isospin $T = 1$. For a neutron-proton pair we can have these and also states of isospin $T = 0$ with angular momenta $J = 1, 3, 5$ and 7. In other words, for even $J$ the isospin is one and for odd $J$ the isospin is zero.

The wave function of a given state of total angular momentum $I$ can be written as\[\Psi = \sum_{J_P J_N} D^I(J_P, J_N) [ (j_P^2)^J_P (j_N^2)^J_N ]^J. \]

In the above $D(J_P, J_N)$ is the probability amplitude that the protons couple to angular momentum $J_P$ and the neutrons to $J_N$. The normalization condition is\[\sum_{J_P J_N} [ D^I(J_P, J_N) ]^2 = 1, \]
and the orthonormality condition is\[\sum_{\alpha} D^\alpha(J_P, J_N) D^\alpha(J_P', J_N') = \delta_{J_P, J_P'} \delta_{J_N, J_N'}. \]

For states of angular momentum $I = 0$, with which we will here be concerned, $J_P$ must be equal to $J_N$ $(J = J_P = J_N)$
\[\Psi(I = 0) = \sum_J D(JJ) [ (j_P^2)^J (j_N^2)^J ]^0. \]
In the single \( j \)-shell configuration of \( ^{44}\text{Ti} \) there are three \( I = 0 \) states of isospin \( T = 0 \) and one of isospin \( T = 2 \). The latter is the double analog of a state in \( ^{44}\text{Ca} \), i.e., of a state of four neutrons in the \( f_{7/2} \) shell. For the unique \((I = 0, T = 2)\) state in \( ^{44}\text{Ti} \), the magnitudes of the \( D(JJ) \)'s are the same as those of two particle coefficients of fractional parentage

\[
D(JJ)_{(I=0, T=2)} = \left( (j^2)_f (j^2)_f \right) j^4 0. \tag{1.5}
\]

We thus have for \((I = 0, T = 2)\)

\[
D(00) = -0.5, \quad D(2,2) = 0.3727, \quad D(4,4) = 0.5, \quad D(6,6) = 0.600 \tag{1.6}
\]

For the \((I = 0, T = 0)\) states however the \( D \)'s do depend on the interaction. We show in Table 1 the values of the \( D(JJ) \)'s for the lowest energy state for the following interactions

- A : \((J = 0, T = 1)\) pairing (all two particle states are degenerate except \((J = 0, T = 1)\), which is lowered relative to the others.
- B : \((J = 1, T = 0)\) pairing. Only \((J = 1, T = 0)\) is lowered.
- C : Equal \( J = 0 \) and \( J = 1 \) pairing. Both \((J = 0, T = 1)\) and \((J = 1, T = 0)\) are lowered by the same amount.
- D : MBZ interaction used by McCullen, Bayman and Zamick [2].
- E : Spectrum of \( ^{42}\text{Sc} \). This is the same as the MBZ calculation except that the correct spectrum of \( ^{42}\text{Sc} \) is used (some of the \( T = 0 \) states were not known in 1964). We equate the matrix elements

\[
\left\langle (f_{7/2})^2 | V | (f_{7/2})^2 \right\rangle \tag{1.7}
\]

with \( E(J) \), the excitation energy of the lowest state of angular momentum \( J \) in \( ^{42}\text{Sc} \). The experimental values for \( J = 0 \) to \( J = 7 \) are (in MeV) 0.0, 0.6111, 1.5863, 1.4904, 2.8153, 1.5101, 3.2420, and 0.6163, respectively. Note that the three lowest states have angular momenta \( J = 0, 1, 7 \).

One can add to all those numbers a constant equal to the pairing energy \( E(2^{\text{Sc}}) + E(40\text{Ca}) - E(41\text{Sc}) - E(41\text{Ca}) \). The value is \(-3.182\) MeV. However, adding this constant will not affect the spectrum or wave functions of \( ^{44}\text{Ti} \). Note that for the even-\( J \) states of \( ^{42}\text{Sc} \) the isospin is one, while for the odd-\( J \) states the isospin is zero.

The eigenvalues and eigenfunctions of interaction E are given in Table 2.

II. THE NUMBER OF PAIRS. A NEW RELATIONSHIP

As previously noted [1] for a system of nucleons with total isospin \( T \), we have the following result for the number of pairs:

- Total number of pair states is \( n(n - 1)/2 \)
- Number with isospin \( T = 0 \) is \( n^2/8 + n/4 - T(T + 1)/2 \)
- Number with isospin \( T = 1 \) is \( 3n^2/8 - 3n/4 + T(T + 1)/2 \)

Hence, for the \( T = 0 \) state of \( ^{44}\text{Ti} \) \((n = 4)\) we have three \( T_0 = 0 \) pairs and three \( T_0 = 1 \) pairs. For the \( T = 2 \) state however we have six \( T_0 = 1 \) pairs. The important thing to note is that the number of pairs does not depend on the two-body interaction, except for the fact that it conserves isospin.

In \( ^{44}\text{Ti} \) the number of pairs with total angular momentum \( J_{12} \) \((J_{12} = 0, 1, 2, 3, 4, 5, 6, 7)\) is given by

\[
2 |D(J_{12}J_{12})|^2 + |f(J_{12})|^2, \tag{2.1}
\]

where

\[
f(J_{12}) = 2 \sum_{J_F} U_{0j}(J_F, J_{12}) D(J_FJ_F), \tag{2.2}
\]
where we introduce the abbreviated symbol $U_{9j}$ to represent the unitary $9j$ symbol,

$$U_{9j}(J_P J_{12}) = \langle (j^2)J_P(j^2)J_P|(j^2)J_{12}(j^2)J_{12} \rangle^0$$

$$= (2J_P + 1)(2J_{12} + 1) \left\{ \begin{array}{ccc}
  j & j & J_P \\
  j & j & J_P \\
  J_{12} & J_{12} & 0
\end{array} \right\}$$

A derivation of the results up to now in this section is given in Appendix A. Since the last publication [1] we have found a new relationship which in some cases simplifies the expression. The new relationship pertains to even $J_{12}$

$$\sum_{J_P} U_{9j}(J_P J_{12}) D(J_P J_P) = D(J_{12} J_{12})/2 \quad \text{for} \quad T = 0$$

$$= -D(J_{12} J_{12}) \quad \text{for} \quad T = 2$$

This remarkable relationship does not depend upon which isospin conserving interaction is used. Using this result we find that the number of pairs for even $J_{12}$ is equal to $3|D(J_{12}, J_{12})|^2$. We do not have a corresponding simple expression for odd $J_{12}$.

We can prove the relationship by regarding the unitary $9j$ symbol as a four by four matrix where $J_P$ and $J_{12}$ assume only even values (0,2,4,6). The eigenvalues of this matrix are $-1$ (singly degenerate) and 0.5 (triply degenerate). The eigenvalue $-1$ corresponds to the $(J = 0, T = 2)$ state of $^{44}$Ti and indeed the values of $D(JJ)$ are identical to those obtained with a charge independent Hamiltonian $D(00) = -0.5, D(22) = 0.3727, D(44) = 0.5, D(66) = 0.6009$. As previously mentioned these are the two particle coefficients of fractional parentage.

The triple degeneracy with eigenvalue 0.5 corresponds to the three $T = 0$ states being degenerate with this unitary $9j$ hamiltonian. This means that any linear combination of the three $T = 0$ states is an eigenvector. This then proves the relationship.

A. Results for the $(I=0, T=2)$ state

Since $(I = 0, T = 2)$ state is unique, we will give the results for this case first. Since the $^{44}$Ti $T = 2$ state is the double analog of $^{44}$Ca, a system of four identical particles, each pair must have even $J_{12}$. The number of pairs is $6 |< (j^2)J_{12}(j^2)J_{12}|j^40)|^2$, i.e., proportional to the square of the two particle coefficient fractional parentage. The number of pairs is:

- 1.5 for $J_{12} = 0$; 0.8333 for $J_{12} = 2$; 1.5 for $J_{12} = 4$; and 2.16667 for $J_{12} = 6$.

This is also the result for $^{44}$Ca. Hence, even though the $I = 0$ ground state of $^{44}$Ca has angular momentum zero and seniority zero, there are more $J_{12} = 6$ pairs in $^{44}$Ca than there are $J_{12} = 0$ pairs. This should not be surprising. As noted by Talmi [3] for the simpler case of a closed neutron shell i.e. $^{48}$Ca the number of pairs with angular momentum $J$ is equal to $2J + 1$. There is only one $J = 0$ pair in $^{48}$Ca.

B. Number of pairs for all states

We can count the number of pairs for all the four states, three of isospin $T_{12} = 0$ and one of isospin $T_{12} = 1$.

Using the relation

$$\sum_{\alpha} D_{\alpha} (J_{12} J_{12}) D_{\alpha} (J'_{12} J'_{12}) = \delta_{J_{12} J'_{12}}$$

we eliminate the $D$’s and find

$$(\text{Number of pairs})/4 = \frac{1}{2} \delta_{J_{12}, \text{even}} + \frac{1}{2} [1 - U_{9j}(J_{12} J_{12})]$$

The values for $T_{12} = 0$ are

- 0.9375 for $J_{12} = 0$; 0.8542 for $J_{12} = 2$; 0.9375 for $J_{12} = 4$; and 1.0208 for $J_{12} = 6$. The total sum is 3.75.

The values for $T_{12} = 1$ are

- 0.3244 for $J_{12} = 1$; 0.6761 for $J_{12} = 3$; 0.7494 for $J_{12} = 5$; and 0.5001 for $J_{12} = 7$. The total sum is 2.25.
C. Results for the T=0 ground state of $^{44}$Ti including the no-interaction case

In Table 3 we give results for the number of pairs for the four interactions defined above. We also consider the 'no-interaction' case. This is obtained by getting the total number of pairs for all three $T = 0$ states and dividing by three.

The results are given in Table 3. We start with the 'no-interaction' result in the last column. Since there are six pairs and eight $J_{12}$'s if there were an equal distribution, then we could assign 0.75 pairs to each angular momentum. This serves us as a good basis for comparison. We find that even in the 'no-interaction' case the results do depend on $J_{12}$. The minimum number of pairs comes with the $(J_{12} = 1, T_{12} = 0)$ case, only 0.432. This is of interest because there has been a lot of discussion in recent times about $(J_{12} = 1, T_{12} = 0)$ pairing. We start out at least with a bias against it. The maximum number of pairs in the 'no-interaction' case is for $(J_{12} = 5, T_{12} = 0)$, a mode that has largely been ignored.

However, of greater relevance is what happens to the ground state wave function when the interaction is turned on. Therefore we compare the no-interaction case with the Spectrum of $^{42}$Sc interaction case E. We see striking differences. Relative to the no-interaction case there is an increase in the following number of pairs: a) $J_{12} = 0$ from 0.75 to 1.8617; b) $J_{12} = 2$ from 0.861 to 0.9458; c) $J_{12} = 1$ from 0.432 to 0.6752 and d) $J_{12} = 7$ from 0.667 to 1.8945. Since the sum of all pairs in both cases is six, there must be a decrease in the number of pairs with the other angular momenta and there is. For example the number of $J_{12} = 6$ pairs decreases from 0.639 to 0.0457 and the number of $J_{12} = 5$ pairs from 1.00 to 0.1587. There are also large decreases in the number of $J_{12} = 4$ and $J_{12} = 3$ pairs. The results with MBZ are qualitatively similar to the correct spectrum of $^{42}$Sc.

We next look at the schematic interactions in the first three columns. For the $(J = 0, T = 1)$ pairing interaction, there are a lot of $J_{12} = 0$ pairs (2.25) but very few $J_{12} = 1$ pairs (0.250). The number of $J_{12} = 7$ pairs is fairly large (1.250).

For the $(J = 1, T = 0)$ pairing interaction there are a lot of $J_{12} = 1$ pairs (1.297) and relatively few $J_{12} = 0$ pairs (0.433). But still there are substantial number of $J_{12} = 7$ pairs (1.311). However, if we examine the wave function for this case it is very different from that of MBZ and this case represents a rather unrealistic ground state wave function.

For the column equal $(J = 0, T = 1)$ and $(J = 1, T = 0)$ pairing we get much better agreement in the wave function as compared with MBZ. The number of $J_{12} = 0$ pairs is 2.043 as compared with 1.736 from MBZ. For $J_{12} = 1$ the values are 0.618 and 0.746, and for $J_{12} = 7$ are 1.654 and 1.948. Amusingly when we lower the $J = 0$ and $J = 1$ matrix elements together we get more $J_{12} = 7$ pairs than we do for them separate (1.250 and 1.311). There is one main deficiency in the $(J = 0 + J = 1)$ case. The number of $(J = 2, T = 0)$ pairs is only 0.497 as compared with 0.9458 for the Spectrum of $^{42}$Sc case. The enhancement is undoubtedly due to the quadrupole correlations in the nucleus, an important ingredient which is sometimes forgotten when all the emphasis is on $(J = 0, T = 1)$ and $(J = 1, T = 0)$ pairing. However if one restricts oneself to $(J = 0, T = 1)$ and $(J = 1, T = 0)$, then equal admixtures in the interaction yield much more realistic results than do either one of them.

III. CLOSING REMARKS

In this work we have studied the effects of the nucleon-nucleon interaction on the number of pairs of a given angular momentum in $^{44}$Ti. We have found that the more attractive the nucleon-nucleon interaction is in a state with angular momentum $J$, the more pairs of that given $J$ will be found in $^{44}$Ti. As a basis of comparison we have defined the no-interaction case in which we average over all three $T = 0$ states (in the single $j$–shell approximation) in $^{44}$Ti, even if the number of $J$ pairs is not independent of $J$ and there are for example less $J = 1$ pairs than the average (0.432 vs. 6/8=0.75). When the realistic interaction is turned on one gets, relative to this no-interaction case, an increase in the number of $J = 0, 1, 2$ and 7 pairs and a decrease in the others. This is in accord with the fact that in $^{42}$Sc the states with angular momentum $J = 0, 1, 2$ and 7 are lower than the others.

For the $T = 2$ state of $^{44}$Ti, the double analog of $^{44}$Ca (a system of particles of one kind), the number of $J = 6$ pairs is the largest. This suggests that the more deformed the state, and the $T = 0$ ground state of $^{44}$Ti is certainly more deformed than the ground state of $^{44}$Ca), the less the number of high angular momentum pairs with the exception of $J$(maximum)=7. Or to put it in another way, the more spherical the state the higher the number of high angular momentum pairs.

The number of pairs obviously is of relevance to two nucleon transfer experiments and we plan to address this more explicitly in the near future. For example, the pickup of an np pair in $^{44}$Ti to the $J = 1$ state in $^{44}$Sc will be enhanced, relative to the no-interaction case by a factor of (0.675/0.432)$^2$. Of course $^{44}$Ti is not stable so this experiment cannot be performed, so we will address other cases.
In the course of this work we also found a new relationship which simplifies the expression for the number of even \( J \) pairs

\[
\sum_{J_B} D(J_B J_B) < (jj) J_B(jj) J_B|0 |(jj) J_A(jj) J_A|0 > = D(J_A J_A)/2
\]

for \( T=0 \), and is equal to \(-D(J_A J_A)\) for \( T = 2 \), where in the above we have a unitary 9j symbol.

One way of looking at this is to say that we can also write the wave function as

\[
\sum_{J, \text{even}} D(J J) [p(1)n(2)|p(3)n(4)]^{I=0}
\]

**IV. APPENDIX A. THE NUMBER OF PAIRS OF A GIVEN ANGULAR MOMENTUM IN THE SINGLE \( J \)-SHELL IN \( ^{44}\text{Ti} \)**

The wave function of a given state in \( ^{44}\text{Ti} \) is

\[
\psi = \sum_{J_P, J_N} D^J(J_P J_N) \left[ (j^2)^J_P (j^2)^J_N \right]^I
\]

where \( I \) is the total angular momentum.

In the single \( j^- \) shell there are 8 two-body interaction matrix elements

\[
E(J) = \left\langle \left( f_{T/2} \right)_J^2 | V | \left( f_{T/2} \right)_J^2 \right\rangle
\]

\( J = 0, 1, \ldots, 7 \). For even \( J \) the isospin is \( T = 1 \), for odd \( J \) is \( T = 0 \). The energy of a \( ^{44}\text{Ti} \) state can be written as \( < \psi H \psi > \). This can be written as a linear combination of the eight two-body matrix elements \( E(J) \)

\[
E(\text{^{44}Ti}) = \sum_{J=0}^{7} C_J E(J)
\]

We can identify \( C(J) \) as the number of pairs in \( ^{44}\text{Ti} \) with a given angular momentum \( J \)

\[
< \psi H \psi > = \sum D(J_P' J_N') D(J_P J_N) \left\langle [J_P' J_N']^I H [J_P J_N]^I \right\rangle
\]

\[
\left\langle [J_P' J_N']^I H [J_P J_N]^I \right\rangle = [E(J_P) + E(J_N)] \delta_{J_P' J_P} \delta_{J_N' J_N}
+ 4 \sum_{J_A J_B} \left\langle (j^2)^J P (j^2)^J N | (j^2)^J A (j^2)^J B \right\rangle^I
\times \langle (j^2)^J P (j^2)^J N | (j^2)^J A (j^2)^J B \rangle^I E(J_B)
\]

In the above, the first two terms are the pp and nn interactions and the last one is the pn interaction. The factor of 4 is due to the fact that there are 4 np pairs. The unitary 9j symbol recombines a proton and a neutron. Note that \( J_P \) and \( J_N \) are even but \( J_A \) and \( J_B \) can be even or odd.

By identifying the number of pairs with angular momentum \( J_B \) we get the expression for \( I = 0 \) (for which \( J_P = J_N \))

**Number of \( J_B \) pairs = \( 2D^2(J_P J_P)_{(J_P \text{ even})} \)**

\[
+ 4 \sum_{J_P J_N} D(J_P J_N) \left\langle (j^2)^J P (j^2)^J N | (j^2)^J A (j^2)^J B \right\rangle^0
\times \sum_{J_P' J_N'} D(J_P' J_N') \left\langle (j^2)^J P' (j^2)^J N' | (j^2)^J A (j^2)^J B \right\rangle^0
\]

The first term contributes only for \( J_B \) even.
We can rewrite this as

\[
\text{Number of } J_B \text{ pairs} = 2D^a(J_P^2)(J_P \text{ even}) + |f(J_B)|^2 \tag{A.7}
\]

with

\[
f(J_B) = 2 \sum_{J_P J_N} D(J_P J_N) \langle (j^2) J_P (j^2) J_N | (j^2) J_B (j^2) J_B \rangle^0 \tag{A.8}
\]

\[
\langle j_1 j_2 J_P (j_3 j_4 J_N) J_A (j_2 j_4 J_B) \rangle^I = \sqrt{(2J_P + 1)(2J_N + 1)(2J_A + 1)(2J_B + 1)} \left\{ \begin{array}{ccc} j_1 & j_2 & J_P \\ j_3 & j_4 & J_N \\ J_A & J_B & I \end{array} \right\} \tag{A.9}
\]

We can get the number of pairs with a given isospin \( T \) by using a simple interaction \( a + bt(1) \cdot t(2) \), where \( a \) and \( b \) are constants. The value of this interaction for two particles is

\[a - 3b/4 \text{ (for } T_0 = 0) \text{ and } a + b/4 \text{ (for } T_0 = 1)\] \tag{A.10}

For \( n \) nucleons we have

\[
\sum_{i<j} (a + bt(i) \cdot t(j)) = \frac{a}{2} n(n-1) + \frac{b}{2} \sum_{i,j} t(i) \cdot t(j) - \frac{1}{2} \sum_{i} t(i)^2 = \frac{a}{2} n(n-1) + \frac{b}{2} T(T + 1) - \frac{3}{8} nb \tag{A.11}
\]

We can write this as

\[C_0(a - 3b/4) + C_1(a + b/4)\] \tag{A.12}

and identify \( C_T \) as the number of pairs with isospin \( T \). We then get the result of section 2.

\[
C_0 = \frac{n^2}{8} + \frac{n}{4} - \frac{T(T + 1)}{2} \tag{A.13}
\]

\[
C_1 = \frac{3n^2}{8} - \frac{3n}{4} + \frac{T(T + 1)}{2} \tag{A.14}
\]

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Table 1. Wave functions of $^{44}$Ti for various interactions: A ($J = 0$, $T = 1$) pairing; B ($J = 1$, $T = 0$) pairing; C equal $J = 0$, $J = 1$ pairing; D (MBZ); E spectrum of $^{42}$Sc.

| $D(JJ)$ | A   | B   | C   | D   | E   | $T = 2$ any interaction |
|---------|-----|-----|-----|-----|-----|------------------------|
| $J = 0$ | 0.866 | 0.380 | 0.826 | 0.7608 | 0.7878 | -0.5 |
| $J = 2$ | 0.213 | 0.688 | 0.405 | 0.6090 | 0.5617 | 0.3727 |
| $J = 4$ | 0.289 | 0.416 | 0.373 | 0.2093 | 0.2208 | 0.5 |
| $J = 6$ | 0.347 | -0.457 | 0.126 | 0.0812 | 0.1234 | 0.6009 |

Table 2. Excitation energies [MeV] and eigenvectors of the Spectrum of $^{42}$Sc interaction.

| excitation energies | eigenvectors | $D(00)$ | $D(22)$ | $D(44)$ | $D(66)$ |
|---------------------|--------------|---------|---------|---------|---------|
| 0                   |              | 0.78776 | 0.56165 | 0.22082 | 0.12341 |
| 5.5861              |              | -0.35240 | 0.73700 | -0.37028 | -0.44219 |
| 8.2840              |              | -0.50000 | 0.37268 | 0.50000 | 0.60093 |
| 8.7875              |              | -0.07248 | -0.04988 | 0.75109 | -0.65432 |

Table 3. Number of pairs for various interactions: A ($J = 0$, $T = 1$) pairing; B ($J = 1$, $T = 0$) pairing; C equal $J = 0$, $J = 1$ pairing; D (MBZ); E spectrum of $^{42}$Sc; F no interaction.

| $J_{12}$ | A   | B   | C   | D   | E   | F   |
|----------|-----|-----|-----|-----|-----|-----|
| $J_{12} = 0$ | 2.250 | 0.433 | 2.045 | 1.736 | 1.862 | 0.750 |
| $J_{12} = 2$ | 0.139 | 1.420 | 0.492 | 1.126 | 0.946 | 0.861 |
| $J_{12} = 4$ | 0.250 | 0.320 | 0.416 | 0.114 | 0.146 | 0.750 |
| $J_{12} = 6$ | 0.361 | 0.626 | 0.048 | 0.020 | 0.046 | 0.639 |
| $J_{12} = 1$ | 0.250 | 1.297 | 0.618 | 0.746 | 0.675 | 0.432 |
| $J_{12} = 3$ | 0.583 | 0.388 | 0.165 | 0.216 | 0.271 | 0.902 |
| $J_{12} = 5$ | 0.916 | 0.003 | 0.564 | 0.091 | 0.159 | 1.000 |
| $J_{12} = 7$ | 1.250 | 1.311 | 1.654 | 1.948 | 1.895 | 0.667 |