The combination of detections of anisotropy in the Cosmic Microwave Background radiation and observations of the large-scale distribution of galaxies probes the primordial density fluctuations of the universe on spatial scales varying by three orders of magnitude. These data are found to be inconsistent with the predictions of several popular cosmological models. Agreement between the data and the Cold + Hot Dark Matter model, however, suggests that a significant fraction of the matter in the universe may consist of massive neutrinos.

1. Introduction

Shortly after the Big Bang, the universe was smooth to a precision of one part in $10^5$. We measure this smoothness in the cosmic microwave background (CMB) radiation, the photons which provide us with a record of conditions in the early universe because they were last scattered about 300,000 years after the Big Bang. To remarkable precision, the early universe was characterized by isotropic, homogeneous expansion. However, temperature fluctuations are measured in the CMB ($1$), and complex structure surrounds us. There is a simple connection; the seeds of large-scale structure were infinitesimal density perturbations that grew via gravitational instability into massive structures such as galaxies and galaxy clusters.

One can search for the primordial seeds of large-scale structure by two complementary techniques. The cosmic microwave background fluctuations probe the density fluctuations in the early universe on comoving scales greater than $\sim 100$ Mpc. The gravity field of these density fluctuations also generates fluctuations in the luminous galaxy distribution, as well as deviations from the Hubble flow of universal expansion known as peculiar velocities. Optical redshift surveys of galaxies now examine a range of scales out to $\sim 100$ Mpc that overlaps with the range probed by fluctuations in the cosmic microwave background.
The expected rate of growth of density fluctuations depends on the precise cosmology that is adopted (2). One can therefore use the comparison between microwave background anisotropy and fluctuations in the galaxy distribution to discriminate among rival cosmological models. Scott, Silk, and White (3) illustrated this comparison. Several similar analyses (4-7) have been presented but have used only a portion of the compilation of observations that we present.

2. Structure Formation Models

We examined ten models of structure formation (Table 1), which represent the range of cosmological parameters currently considered viable (8). Each model gives transfer functions that predict how a primordial power spectrum of infinitesimal density perturbations in the early universe develops into CMB anisotropies and inhomogeneities in the galaxy distribution. A cosmological model whose predictions agree with both types of observations provides a consistent picture of structure formation on scales ranging from galaxy clusters to the present horizon size. The cosmological parameter $\Omega = \Omega_m + \Omega_\Lambda$ gives the ratio of the energy density of the universe to the critical density necessary to stop its expansion. Critical density is $\rho_c = 3H_0^2/8\pi G$ for a Hubble constant of $H_0 = 100h$ km/s/Mpc. The portion of this critical energy density contained in matter is $\Omega_m = \Omega_c + \Omega_\nu + \Omega_b$, the sum of the contributions from Cold Dark Matter (CDM), Hot Dark Matter (HDM) in the form of massive neutrinos, and baryonic matter. $\Omega_\Lambda = \Lambda/3H_0^2$ is the fraction of the critical energy density contained in a smoothly distributed vacuum energy referred to as a cosmological constant, $\Lambda$. The age of the universe in each model is determined by the values of $h, \Omega_m$, and $\Omega_\Lambda$; a critical matter density universe has an age of $6.5h^{-1}$Gyr (9).

Each model has a primordial power spectrum of density perturbations given by $P_p(k) = Ak^n$ where $A$ is the square of a free normalization parameter and $n$ is the scalar spectral index (10). Scale-invariance (11) corresponds to $n = 1$ for adiabatic (constant entropy) and $n = -3$ for isocurvature (constant potential) initial density perturbations. Instead of normalizing to the COBE result alone (12), we found the best-fit normalization of each model (Table 2) using the entire data compilation. Our rationale is that COBE is just one subset of the available data, albeit with small error bars, and is in fact the data most likely to be affected by a possible contribution of gravitational waves to microwave background anisotropies. These gravitational waves from inflation would have a significant impact only on large angular scales and are not traced by the large-scale structure observations. Normalizing to all of the data made our results less sensitive to the possible contribution of gravitational waves.

The first seven models (Table 1) are based on the Standard Cold Dark Matter (SCDM) model (13) and assume that the initial density perturbations in the universe were adiabatic, as is predicted by the inflationary universe paradigm. The Tilted CDM (TCDM)
and Cold + Hot Dark Matter (CHDM) models are both motivated by changing the shape of the SCDM matter power spectrum to eliminate its problem of excess power on small scales relative to large scales (14). The CHDM model has one family of massive neutrinos which contributes 20% of the critical density (15). For the cosmological constant (ΛCDM) and open universe (OCDM) models, Ω_m = 0.5, h = 0.6 guarantees roughly the right shape of the matter power spectrum (5). We have optimized some parameters of these models: n and Ω_b for TCDM, Ω_ν, Ω_b, n, and the number of massive neutrino families for CHDM, and Ω_m, Ω_b, h and n for OCDM and ΛCDM (16). The φCDM model (17) contains a vacuum energy contribution from a late-time scalar field with Ω_φ = 0.08. This energy behaves like matter today, but during matter-radiation equality and recombination it alters the shape of the matter and radiation power spectra from the otherwise similar SCDM model. The Baryonic + Cold Dark Matter (BCDM) model (18) contains nearly equal amounts of baryonic matter (Ω_b = 0.04) and CDM (Ω_c = 0.08). Its parameters have been tuned to produce a peak due to baryonic acoustic oscillations in the matter power spectrum at k = 0.05h Mpc^{-1}, where a similar peak is seen in the 3-dimensional power spectrum of rich Abell clusters (19) and the 2-dimensional power spectrum of the Las Campanas Redshift Survey (20).

The Isocurvature Cold Dark Matter (ICDM) model (21, 22) has a non-Gaussian (χ^2) distribution of isocurvature density perturbations produced by a massive scalar field frozen during inflation. This causes early structure formation, in agreement with observations of galaxies at high redshift and the Lyman α forest (23). The Primordial Black Hole Baryonic Dark Matter (PBH BDM) model (24) has isocurvature perturbations but no CDM. The primordial black holes form from baryons at high density regions in the early universe and thereafter behave like CDM. Only a tenth of the critical energy density remains outside the black holes to participate in nucleosynthesis. These black holes have the appropriate mass (M ∼ 1M_⊙) to be the Massive Compact Halo Objects (MACHOS) which have been detected in our Galaxy (25). Albrecht, Battye and Robinson found that critical matter density topological defect models fail to agree with structure formation observations (26). In the Strings+Λ model (27) that we examined, the nonzero cosmological constant causes a deviation from scaling and makes cosmic strings a viable model.

We used the CMBfast code (28) to calculate the predicted radiation and matter power spectra for the SCDM, TCDM, CHDM, OCDM, ΛCDM, and BCDM models.

### 3. Constraints on Cosmological Parameters

The models we consider are all consistent with the constraints on the baryon density from Big Bang nucleosynthesis, 0.012 < Ω_b h^2 < 0.026, allowed by recent observations of primordial deuterium abundance (29). A Hubble constant of 65 ± 15 encompasses the range of systematic variations between different observational approaches (30). The age “crisis” has abated with recent recalibration by Hipparcos of the distance to the oldest
Galactic globular clusters leading to a new estimate of their age of 11.5 ± 1.3 Gyr (31). All of our models have an age of at least 13 Gyrs except OCDM (12 Gyrs). Other constraints, however, appear to limit the viability of our models. Observations of high-redshift damped Lyman α systems are a concern for the CHDM and TCDM models, which have little power at small scales (32). Bartelmann et al. (33) used numerical simulations to compare the observed abundance of arcs from strong lensing by galaxy clusters with the predictions of various models and conclude that only OCDM works, and they found that critical density models seriously underpredict the number of arcs. Further support for low-Ωm models comes from the cluster baryon fraction of $\frac{\Omega_m}{\Omega_b} \leq 23h^{3/2}$ (34). This favors the ratio of total matter to baryons in the low matter density models considered here and is inconsistent with SCDM and φCDM. Observations of Type Ia supernovae at high redshift are progressing rapidly, and preliminary results argue in favor of a positive cosmological constant and strongly disfavor $\Omega_m = 1$ (35). The amount of vacuum energy is constrained to be $\Omega_\Lambda \leq 0.7$ by QSO lensing surveys (36). It is interesting that direct observations of cosmological parameters favor the low $\Omega_m$ models, but we found that the current discriminatory power of observations of structure formation outweighs that of direct parameter observations.

4. Comparison with Observations

Since the COBE DMR detection of CMB anisotropy (1), there have been over twenty-five additional measurements of anisotropy on angular scales ranging from 7° to 0°3. The models predict that the spherical harmonic decomposition of the pattern of CMB temperature fluctuations on the sky will have Gaussian distributed coefficients $a_{\ell m}$ with zero mean and variance $C_\ell$. Each observation has a window function $W_\ell$ which makes the total power measured sensitive to a range of angular scales given by $\theta \simeq 180^\circ/\ell$:

$$\left(\frac{\Delta T}{T}\right)_{\text{rms}}^2 = \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell W_\ell = \frac{1}{2} \frac{(dT/T_{\text{CMB}})^2}{\ell \ell + 1} \sum_\ell \frac{2\ell + 1}{\ell \ell + 1} W_\ell,$$

where COBE found $dT = 27.9 \pm 2.5\mu K$ and $T_{\text{CMB}} = 2.73K$ (37). This allows the observations of broad-band power to be reported as observations of $dT$, and knowing the window function of an instrument one can turn the predicted $C_\ell$ spectrum of a model into the corresponding prediction for $dT$ at that angular scale (Fig. 1).

We translated these observations of the radiation power spectrum into estimates of the matter power spectrum on the same scales (38). The matter power spectrum is determined by the matter transfer function $T(k)$ and primordial power spectrum $P_p(k)$ of each model, with $P(k) = T^2(k)P_p(k)$. The matter transfer function describes the processing of initial density perturbations from the Big Bang during the era of radiation domination; the earlier a spatial scale came within the horizon, the more its power was dissipated by radiation (and in the CHDM model, by relativistic neutrinos as well). If the baryon fraction is
large, the same acoustic oscillations of the photon-baryon fluid that give rise to peaks in the radiation power spectrum are visible in the matter power spectrum; otherwise, the baryons fall into the potential wells of the dark matter. Once matter domination and recombination arrive, $P(k)$ maintains its shape and grows as $(1 + z)^{-2}$. Thus, determining $P(k)$ today allows us to extract the power spectrum of primordial density fluctuations that existed when the universe was over a thousand times smaller.

Our compilation of observations of fluctuations in the large-scale distribution of galaxies and galaxy clusters (Fig. 2A) includes the determination of $\sigma_8$, the rms density variation in spheres of radius $8h^{-1}$ Mpc, based on the abundance of rich galaxy clusters (39). Another measurement of $\sigma_8$ is based upon the evolution of the abundance of rich clusters from redshift 0.5 until now (40). The predicted value of $\sigma_8$ is given by an integral over the matter power spectrum using a spherical top-hat window function (41)

$$\sigma_R^2 = \frac{1}{2\pi^2} \int \frac{d\epsilon}{\epsilon} \frac{k^2 P(k)}{(k\epsilon)^6} \left(\sin k\epsilon - k\epsilon \cos k\epsilon\right)^2, \quad (2)$$

which allows observations of $\sigma_8$ to determine the amplitude of $P(k)$ on scales $k \approx 0.2h$Mpc$^{-1}$. Another measurement of the amplitude of the power spectrum comes from observations of galaxy peculiar velocities (42).

Our data compilation includes power spectra from four redshift surveys, the Las Campanas Redshift Survey (LCRS), the combined IRAS 1.2 Jy and QDOT samples, the combined SSRS2+CfA2 survey, and a cluster sample selected from the APM Galaxy Survey (43). We also use the power spectrum resulting from the Lucy inversion of the angular correlation function of the APM galaxy catalog (44, 45). The APM galaxy power spectrum is measured in real space, whereas the others are given in redshift space. Each of these power spectra can be scaled by the square of an adjustable bias parameter, which is expected to be near unity for the galaxy surveys (46).

Following the methods of Peacock and Dodds (41), we performed model-dependent corrections for redshift distortions for each galaxy power spectrum (47, 48) and divided by the square of a trial value of the bias factor. We then corrected for non-linear evolution (49) to produce estimates of the unbiased linear power spectrum from these galaxy surveys. Comparison with the predicted linear $P(k)$ determined the best-fit bias parameter of each survey for each model (Table 2). We compared the corrected large-scale structure data, the CMB anisotropy observations and the predicted matter and radiation power spectra and calculated the $\chi^2$ value for each model (Table 3). Only points observed at $k \leq 0.2h$Mpc$^{-1}$ were used in selecting best-fit bias factors and normalizations and in calculating $\chi^2$ (50). On smaller scales, the linearization process yielded qualitative information despite systematic uncertainties.
5. Discussion

The current large-scale structure observations agree well with each other in terms of the shape of the uncorrected matter power spectrum (Fig. 2A). The APM clusters are biased compared to galaxies by about a factor of 3 and their power spectrum has a narrower peak and a possible small-scale feature. There is no clear evidence, however, for scale-dependence in the bias of the various galaxy surveys on linear scales. The observed galaxy power spectra are smooth, showing no statistically significant oscillations. A peak in the matter power spectrum appears near $k = 0.03h\text{Mpc}^{-1}$, which constrains $\Omega_m h$ by identifying the epoch of matter-radiation equality (44). The large-scale structure observations contain too much information to be summarized by a single shape parameter; no value of the traditional CDM shape parameter (51) can simultaneously match the location of this peak and its width.

We find a poor fit for SCDM (Fig. 2B) due to the difference in shape between the theory curve and the data. The best-fit normalization is only 0.91 that of COBE, as the model would otherwise overpredict the $\sigma_8$ measurements by an even greater amount. The fit to the CMB is poor, because the Saskatoon (SK) observations (52) would prefer more power. The fit of the data to the TCDM model (Fig. 2C) is better, although the peak of the matter power spectrum is still broader than that found in the data. Agreement with the CMB is harmed by the high normalization versus COBE and the tilt on medium scales.

The best-fit model is CHDM (Fig. 2D). The agreement with the location and shape of the peak of the matter power spectrum is remarkable, with the exception of the APM cluster power spectrum. The agreement with CMB anisotropy detections is excellent. The matter power spectrum of CHDM matches the linearized APM galaxy power spectrum down to non-linear scales, making this model a good explanation of structure formation far beyond the scales used for our statistical analysis (59).

For the OCDM model (Fig. 3A), $\Omega_m = 0.5$ is favored by the shape of $P(k)$ and the SK and CAT (54) CMB anisotropy detections and generates agreement between the two observations of $\sigma_8$. However, the location of the peak of $P(k)$ appears wrong. This model is our second-best fit but is statistically much worse than CHDM. The $\Lambda$CDM model (Fig. 3B) is nearly as successful as OCDM. It is a slightly better fit to the CMB but is worse in comparison to large-scale structure. The observations of $\sigma_8$ are again in agreement, but the shape of the matter power spectrum does not compare well with that of the APM galaxy survey.

The $\phi$CDM model is too broad at the peak and misses a number of APM galaxy datapoints (Fig. 3C), although its agreement with the other datasets is rather good. It remains to be seen whether other variations of scalar field models can match the observations better. The BCDM model (Fig. 3D) does not fit the data. Choosing parameters to place an acoustic oscillation peak near $k = 0.05h\text{Mpc}^{-1}$ has generated the wrong shape for $P(k)$, even though the APM galaxies and clusters seem to fit the
first and second oscillations, respectively (55). The main peak of $P(k)$ is in the wrong place; no model with similar oscillations and a baryon content consistent with Big Bang Nucleosynthesis can fix that problem (18).

For the Isocurvature CDM model (Fig. 4A), the fit to the CMB is poor, due to the rise of $C_\ell$ on COBE scales, too much power in the first peak near $\ell = 100$ and too little power compared to SK. The fit to large-scale structure is mediocre. The PBH BDM model has similar problems compared to the CMB, but the peak location and shape of the matter power spectrum are better (Fig. 4B). The Strings+$\Lambda$ model (Fig. 4C) underestimates the amplitude of the bias-independent measurements and therefore requires a large bias for all types of galaxies, which is difficult to justify.

6. Conclusions

The rough agreement of the Cosmic Microwave Background anisotropy and Large-Scale Structure observations over a wide range of models suggests that the gravitational instability paradigm of cosmological structure formation is correct. The current set of CMB anisotropy detections may be a poor discriminator among adiabatic models, but it strongly prefers them to non-adiabatic models. Several models (SCDM, TCDM, BCDM, and Strings+$\Lambda$) have a best-fit normalization significantly different from their COBE normalization and would have been unfairly penalized if normalized to COBE alone. The Strings+$\Lambda$ model already includes a tensor contribution, but SCDM and BCDM would benefit from adding a gravitational wave component that brought them into better agreement with COBE without changing the amplitude of their scalar perturbations. Adding gravitational waves is not, however, a panacea for those models. In general, the models which are the best fits to the shape of the matter power spectrum prefer to be close to their COBE normalization, which argues against there being a significant tensor contribution to large-angle CMB anisotropies.

Large-scale structure data have more discriminatory power at present than do the CMB anisotropy detections. The average ratio of best-fit biases (Table 2) is $b_{clus} : b_{cfa} : b_{lcrs} : b_{apm} : b_{iras} = 3.2 : 1.3 : 1.2 : 1.3 : 1$ (56). Most models allow optical galaxies to be nearly unbiased tracers of the dark matter distribution. The large-scale structure data are smooth enough to set a limit on the baryon fraction, $\Omega_b/\Omega_m$; when that fraction gets higher than 0.1 the fit worsens (57).

By restricting our analysis to the linear regime and correcting for the mildly scale-dependent effects of redshift distortions and non-linear evolution on those scales, we made it possible to test models quantitatively. The most likely cosmology is Cold + Hot Dark Matter, which is the only model allowed at the 95% confidence level. The disagreement between the data and the predictions of the other models is sufficient to rule out all of them at above 99% confidence unless there are severe systematic problems in the data (58). CHDM itself is not statistically very likely because of the APM cluster
survey $P(k)$, which no model fits much better, and which disagrees somewhat with the galaxy power spectra. Dropping the APM cluster $P(k)$ would give CHDM a $\chi^2$ of 66/62, which is within the 68% confidence interval. It is worth investigating whether the APM cluster power spectrum contains a scale-dependent bias or if its errors have somehow been underestimated.

We have extracted the spectrum of primordial density fluctuations from the data and found that it agrees well with that of the Cold + Hot Dark Matter model. This does not provide direct evidence for the existence of HDM, which requires experimental confirmation of neutrino mass. The CHDM model has other observational hurdles to overcome, including evidence for early galaxy formation on small scales where this model has little power, although it is impressive that CHDM agrees with the linearized APM data out to $k = 1h\text{Mpc}^{-1}$. If the rapidly improving Type Ia Supernovae observations follow current trends there may be enough statistical power in the direct observations of cosmological parameters to make OCDM and $\Lambda$CDM preferred to CHDM, although in that case none of these models would be a satisfactory fit to both the supernovae and structure formation observations.

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42. T. Kolatt and A. Dekel, Astrophys. J. 479, 592 (1997). We used only the measurement at $k = 0.1h$Mpc$^{-1}$ which is confirmed by the likelihood analysis of S. Zaroubi, I. Zehavi, A. Dekel, Y. Hoffman, T Kolatt, Astrophys. J. 486, 21 (1997). A full discussion of peculiar velocity data is given by M. Gramann, Astrophys. J. 493, 28 (1998).

43. Respectively, H. Lin et al., Astrophys. J. 471, 617 (1996); H. Tadros and G. Efstathiou, Mon. Not. R. Astron. Soc. 276, L45 (1995); L. N. Da Costa, M. S. Vogeley, M. J. Geller, J. P. Huchra, C. Park, Astrophys. J. 437, L1 (1994); H. Tadros, G. Efstathiou, G. Dalton, Mon. Not. R. Astron. Soc., submitted, (1997). We used the APM cluster $P(k)$ only for $k \leq 0.12h$Mpc$^{-1}$ to avoid possible artifacts of the survey window function at higher $k$. The APM cluster $P(k)$ was analyzed for several background cosmologies, and we used the version most appropriate to each model. We chose the $10h^{-1}$Mpc version of the SSRS2+CfA2 survey to avoid luminosity bias present in the deeper sample noted by C. Park, M. S. Vogeley, M. J. Geller, J. P. Huchra, Astrophys. J. 431, 569 (1994).

44. E. Gaztañaga and C. M. Baugh, Mon. Not. R. Astron. Soc. 294, 229 (1998). We dropped the first four reported APM points because they appear to be artifacts of the data analysis.

45. C. M. Baugh and G. Efstathiou, Mon. Not. R. Astron. Soc. 265, 145 (1993).

46. Galaxies may be more or less common than $1\sigma$ peaks of the dark matter distribution, so they are biased tracers of the mass. Each morphological type is expected to have a slightly different scale-independent bias. Clusters have a large bias because they trace high density peaks of the primordial density distribution and such peaks are themselves highly clustered (see N. Kaiser, Astrophys. J. 284, L9 (1984)). Our assumption that bias is scale-independent is supported by G. Kauffmann, A. Nusser, M. Steinmetz, Mon. Not. R. Astron. Soc. 286, 795 (1997); R. G. Mann, J. A. Peacock, A. F. Heavens, Mon. Not. R. Astron. Soc. 293, 209 (1998); and R. J. Scherrer and D. H. Weinberg, (1997), astro-ph/9712192. Peculiar velocities arise due to the gravity of the underlying dark matter, so they produce a bias-independent measurement of the matter power spectrum. Both observations of $\sigma_8$ are also bias-independent.

47. The power spectrum observed in redshift space is related to that in real space by $P_z(k) = (1 + \beta \mu^2)^2 D(k\mu \sigma_p)P_{\text{real}}(k)$, where the first term gives the Kaiser distortion (N. Kaiser, Mon. Not. R. Astron. Soc. 227, 1 (1987)) from coherent infall of galaxies with bias $b$ as a function of $\beta = \Omega_m^{0.6}/b$ and the second term is the damping of such distortions by the rms pairwise galaxy velocity dispersion ($\sigma_p$) measured in units of $H_0$. This velocity dispersion leads to the so-called fingers-of-God effect in redshift surveys. For an exponential velocity distribution, $D(k\mu \sigma_p) = (1 + (k\mu \sigma_p)^2/2)^{-1}$. We averaged over $\mu$, the cosine of the angle between the line of sight and a given wave vector $k$, to produce an estimate of the real-space power spectrum $P_{\text{real}}(k) = P_z(k)/f(k, b)$. Defining $K = k\sigma_p/\sqrt{2}$, this gave (W. E. Ballinger, Thesis, University of Edinburgh, 1997):
\[ f(k,b) = \frac{1}{K} \left[ \tan^{-1}(K) \left( 1 - \frac{2\beta}{K^2 + \beta^2} \right) + \frac{2\beta}{K} + \frac{\beta^2}{3K} - \frac{\beta^2}{K^3} \right]. \] (3)

For the pairwise velocity dispersion, we used the observation from S. D. Landy, A. S. Szalay, T. J. Broadhurst, *Astrophys. J.* 494, L133 (1998) of \( \sigma_p = 3.63h^{-1}\text{Mpc} \). Their determination that the velocity distribution is exponential motivated using that form for the damping term. We tried using the higher value of \( \sigma_p = 5.70h^{-1}\text{Mpc} \) (Y. P. Jing, H. J. Mo, G. Borner, *Astrophys. J.* 494, 1 (1998)), and it made only a small difference on quasi-linear scales; at \( k = 0.2h\text{Mpc}^{-1} \) the scale-dependence of the redshift distortions is a 15% effect for the value of 3.63h\(^{-1}\)Mpc and twice that for the higher one. This systematic uncertainty in the pairwise velocity dispersion makes the corrected real-space power spectrum somewhat unreliable on smaller scales. For the cluster power spectrum, we used only the scale-independent Kaiser distortion term to correct for redshift distortions as the clusters are engaged in coherent infall onto superclusters. The APM galaxy power spectrum was corrected for bias only, as it does not have redshift distortions.

48. C. Smith, A. Klypin, M. Gross, J. Primack, J. Holtzman, *Mon. Not. R. Astron. Soc.*, submitted (1997), astro-ph/9702099 give a full discussion of the effects of varying \( \sigma_p \) and propagating this systematic uncertainty through the linearization procedure; their results confirm that the systematic uncertainty is small up to \( k = 0.2h\text{Mpc}^{-1} \).

49. Because collapsing structure leads to a change of physical scale, the observed \( k_{nl} \) can be corrected to their linear values, given by \( k_l = (1 + \Delta_{nl}^2)^{-1/3}k_{nl} \), where \( \Delta^2 = k^3P(k)/2\pi^2 \). The non-linear evolution is given by \( \Delta_{nl}^2 = f(\Delta_l^2) \). A semi-analytic fit for this function with 10% accuracy compared to numerical simulations is given by J. A. Peacock and S. J. Dodds, *Mon. Not. R. Astron. Soc.* 280, L19 (1996). The accuracy of this formula is confirmed in (48). This correction is model-dependent, as it assumes a local slope for the original linear power spectrum based on the model being tested. By inverting the formula numerically, we linearized the unbiased real-space \( P(k) \). Non-linear evolution is significant only at \( k \geq 0.2h\text{Mpc}^{-1} \). At smaller \( k \), the linearization preserves the shape of the observed non-linear \( P(k) \) while sliding the data points to smaller \( k \); its main effect is to shrink the error bars slightly.

50. We have rebinned some of the galaxy survey data to make the points independent. Our conclusions are unaffected by varying the non-linear cutoff between \( k = 0.15h\text{Mpc}^{-1} \) and \( k = 0.25h\text{Mpc}^{-1} \).

51. Defined by G. Efstathiou, J. R. Bond, S. D. M. White, *Mon. Not. R. Astron. Soc.* 258, 1P (1992).

52. C. B. Netterfield et al., *Astrophys. J.* 474, 47 (1997). We used the recalibration of SK from E. Leitch, Thesis, Caltech, (1997).

53. J. A. Peacock, *Mon. Not. R. Astr. Soc.* 284, 885 (1997) and (48) also found good agreement between the linearized observations and linear theory under the CHDM model.

54. P. F. S. Scott et al., *Astrophys. J.* 461, L1 (1996); J. C. Baker, in *Proc. Particle Physics and the Early Universe Conference*, (1997), http://www.mrao.cam.ac.uk/ppeuc/proceedings

55. We averaged the predictions of the matter power spectrum over the window function of the observations to take into account the possible smoothing of these oscillations during observation. To make the linearization procedure work smoothly, we fixed the local slope of the linear power spectrum.

56. This is roughly consistent with the bias ratios found by (41).
57. See D. M. Goldberg and M. A. Strauss, Astrophys. J. 495, 29 (1998) for the future prospects of this constraint.

58. If the APM Galaxy $P(k)$ were ignored, OCDM, ΛCDM, and φCDM would become much better fits but would still be ruled out at 95% confidence.

59. The φCDM model has a scalar field energy density of $Ω_φ = 0.08$. The PBH BDM model has 30% of critical density in primordial black holes which act like CDM but actually contain baryonic matter.

60. The $χ^2$ category includes the contribution from peculiar velocity measurements. The degree of freedom used by normalizing is counted under $χ^2_{CMB}$, and each galaxy survey loses one degree of freedom in choosing a best-fit bias. The ICDM model has one less degree of freedom in the $χ^2_σ$ column and a total of 69.

61. The error bars include uncertainties due to instrument noise, calibration uncertainty, sample variance from observing only part of the sky, and cosmic variance from observing at only one location within the universe. The calibration errors were added in quadrature. Although calibration errors are correlated for multiple observations by the same instrument, they have been treated as independent, which is a good approximation after the recalibration of SK by Leitch (52).

62. CMB anisotropy observations are compiled in G. F. Smoot and D. Scott, in Review of Particle Properties, (1998), astro-ph/9711069 and at http://www.sns.ias.edu/~max/cmb/experiments.html. Shown here are COBE (M. Tegmark and A. Hamilton, (1997) astro-ph/9702019), FIRS (K. Ganga, L. Page, E. Cheng., S. Meyers, Astrophys. J. 432, L15 (1993)), Tenerife (C. M. Gutierrez, et al., Astrophys. J. 480, L83 (1997)), South Pole (J. O. Gundersen et al., Astrophys. J. 443, L57 (1994)), BAM (G. S. Tucker et al., Astrophys. J. 475, L74 (1997)), ARGO (S. Masi et al., Astrophys. J. 463, L47 (1996)), Python (S. R. Platt et al., Astrophys. J. 475, L1 (1997)), MAX (M. Lim et al., Astrophys. J. 469, L69 (1996); S. T. Tanaka et al., Astrophys. J. 468, L81 (1996)), MSAM (E. S. Cheng et al., Astrophys. J. 488, L59 (1997)), SK (52), and CAT (54).

63. The width of the box represents the range of spatial scales to which $σ_8$ is sensitive and the height shows the 68% confidence interval. The half-max window for $σ_8$ is from $k = 0.05$ to $k = 0.3$ but it has been narrowed for clarity. The width of the window function of the peculiar velocity observation is shown by the ends of its error bars, which include cosmic variance. This observation scales as $Ω_m^{-1.2}$ (the square of the growing mode). The determination of $P(k)$ from the value of $σ_8$ implied by the $z = 0$ cluster abundance scales roughly as $Ω_m^{-1}$ due to the relationship between the observed mass and the pre-collapse radius of rich clusters. The observation of $σ_8$ from the evolution of the cluster abundance is nearly independent of $Ω_m$.

64. The high-$k$ end of the LCRS data shows that the combination of deconvolving the fingers-of-God and linearizing the data has kept the shape the same but moved the points along that curve and reduced the error bars. The linearization of the APM dataset has removed the inflection at $k = 0.2h$Mpc$^{-1}$.

65. Each model has a particular value of $η_0$; varying $η_0$ moves the entire set of boxes horizontally. Comparison of the CMB anisotropy predictions of each model with observations gives the vertical placement of each box, showing the inferred amplitude of matter density fluctuations at that scale. The boxes for CMB anisotropy detections and $σ_8$ follow the local shape of...
each model’s $P(k)$ to indicate that they are a model-dependent averaging of the power over a range of $k$.

66. The strong rise of the matter power spectrum is caused by the sharp tilt of the model away from scale-invariance. The linearization procedure used was calibrated for Gaussian models, but non-linear evolution is expected to be similar under the $\chi^2$ distribution; see A. Stirling, Thesis, University of Edinburgh, 1998.

67. It is possible that the linearization procedure needs to be adjusted to account for non-Gaussianity in the matter distribution, but the reduced power of this model weakens the effects of non-linear evolution.

68. MAP and Planck parameters taken from J. R. Bond, G. Efstathiou, M. Tegmark, *Mon. Not. R. Astr. Soc.* 291, L33 (1997). SDSS is from M. Vogeley, personal communication (1997), and 2DF is from S. Hatton, personal communication (1997). No attempt has been made in this figure to account for redshift distortions or non-linear evolution. The overlap in scale between CMB anisotropy detections and large-scale structure observations will increase tremendously in the next several years, and the errors in these measurements will decrease significantly. W. Hu, D. J. Eisenstein, M. Tegmark, submitted to *Phys. Rev. Lett.*, (1998), astro-ph/9712057 examine how well $\Omega_\nu$ can be determined by SDSS observations, and Y. Wang, D. N. Spergel, M. A. Strauss, submitted to *Astrophys. J.*, (1998), astro-ph/9802231 discuss the ability of combined MAP and SDSS observations to constrain cosmological parameters.

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Our data compilation and full-size figures are available at http://cfpa.berkeley.edu/cmbserve/fluctuations/figures.html

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Table 1: Cosmological parameters of our models. Parameters marked with a * were optimized (59).

| Model  | $\Omega$ | $\Omega_\Lambda$ | $\Omega_m$ | $\Omega_\nu$ | $\Omega_b$ | h    | n    | Age (Gyr) |
|--------|----------|-------------------|------------|-------------|-----------|------|------|-----------|
| SCDM   | 1.0      | 0.0               | 1.0        | 0.95        | 0.05      | 0.5  | 1.0  | 13        |
| TCDM   | 1.0      | 0.0               | 1.0        | 0.90        | 0.10*     | 0.5  | 0.8* | 13        |
| CHDM   | 1.0      | 0.0               | 1.0        | 0.70        | 0.2*      | 0.10*| 0.5  | 1.0*      |
| OCDM   | 0.5      | 0.0               | 0.5*       | 0.45        | 0.05*     | 0.6* | 1.0* | 12        |
| ACDM   | 1.0      | 0.5               | 0.5*       | 0.45        | 0.05*     | 0.6* | 1.0* | 14        |
| $\phi$CDM | 1.0     | 0.0               | 0.92       | 0.87        | 0.05      | 0.5  | 1.0  | 13        |
| BCDM   | 1.0      | 0.8               | 0.12       | 0.08        | 0.04      | 0.8  | 1.6  | 15        |
| ICDM   | 1.0      | 0.8               | 0.17       | 0.03        | 0.7       | -1.8 | 15   |           |
| PBH BDM| 1.0      | 0.6               | 0.4        | 0.10        | 0.7       | -2.0 | 13   |           |
| Strings+Λ | 1.0     | 0.7               | 0.3        | 0.25        | 0.05      | 0.5  | $\sim$1 | 19        |

Table 2: Best-fit normalizations and biases. The normalization of each model is given by $\sigma_8$ or the value of $dT_\ell$ at $\ell = 10$, which can be compared to the COBE normalization of $dT_\ell = 27.9 \mu K$.

| Model  | $dT_{10} (\mu K)$ | $\sigma_8$ | $b_{clus}$ | $b_{cfa}$ | $b_{lcrs}$ | $b_{apm}$ | $b_{iras}$ |
|--------|-------------------|------------|------------|-----------|------------|-----------|------------|
| SCDM   | 25.4              | 1.08       | 2.12       | 0.83      | 0.72       | 0.89      | 0.57       |
| TCDM   | 31.2              | 0.79       | 2.73       | 1.13      | 1.01       | 1.18      | 0.83       |
| CHDM   | 27.1              | 0.75       | 2.52       | 1.11      | 1.01       | 1.13      | 0.78       |
| OCDM   | 29.0              | 0.77       | 2.67       | 1.25      | 1.11       | 1.10      | 0.93       |
| ACDM   | 26.8              | 1.00       | 2.14       | 0.91      | 0.82       | 0.87      | 0.68       |
| $\phi$CDM | 27.6         | 0.74       | 3.12       | 1.35      | 1.20       | 1.31      | 0.98       |
| BCDM   | 24.8              | 1.76       | 1.30       | 0.48      | 0.40       | 0.41      | 0.37       |
| ICDM   | 28.2              | 0.83       | 2.95       | 1.25      | 1.12       | 1.02      | 0.97       |
| PBH BDM| 29.9              | 0.78       | 2.74       | 1.21      | 1.09       | 1.10      | 0.92       |
| Strings+Λ | 21.2         | 0.32       | 6.95       | 3.10      | 2.86       | 2.62      | 2.48       |

Table 3: Chi-squared values for our models, computed from data at $k \leq 0.2 h$ Mpc$^{-1}$ (60). P is the probability of getting $\chi^2$ greater than or equal to the observed value if a model is correct.

| Model  | $\chi^2_{CMB}$ | $\chi^2_{\sigma_8}$ | $\chi^2_{clus}$ | $\chi^2_{cfa}$ | $\chi^2_{lcrs}$ | $\chi^2_{apm}$ | $\chi^2_{iras}$ | $\chi^2_{total}$ | $\chi^2$/d.o.f. | P |
|--------|----------------|---------------------|-----------------|----------------|----------------|----------------|----------------|----------------|---------------|---|
| SCDM   | 46             | 36                  | 37              | 0.2            | 8              | 121            | 18             | 266            | 3.8           | $< 10^{-7}$ |
| TCDM   | 51             | 5                   | 27              | 0.4            | 6              | 49             | 11             | 148            | 2.1           | $1.8 \times 10^{-7}$ |
| CHDM   | 30             | 4                   | 20              | 3              | 9              | 10             | 11             | 86             | 1.2           | 0.09         |
| OCDM   | 36             | 2                   | 24              | 2              | 11             | 42             | 12             | 128            | 1.8           | $2.9 \times 10^{-5}$ |
| ACDM   | 30             | 3                   | 26              | 2              | 12             | 46             | 13             | 132            | 1.9           | $1.1 \times 10^{-5}$ |
| $\phi$CDM | 32            | 4                   | 30              | 0.1            | 5              | 71             | 12             | 155            | 2.2           | $< 10^{-7}$ |
| BCDM   | 32             | 38                  | 33              | 1              | 125            | 225            | 56             | 511            | 7.3           | $< 10^{-7}$ |
| ICDM   | 61             | 3                   | 17              | 2              | 21             | 50             | 16             | 170            | 2.5           | $< 10^{-7}$ |
| PBH BDM| 65             | 4                   | 22              | 2              | 9              | 30             | 11             | 142            | 2.0           | $8.3 \times 10^{-7}$ |
| Strings+Λ | 64           | 37                  | 20              | 0.3            | 8              | 43             | 10             | 182            | 2.6           | $< 10^{-7}$ |
Fig. 1.— Compilation of CMB anisotropy results with horizontal error bars showing the full width at half maximum of each instrument’s window function and vertical error bars showing the 68% confidence interval (61). The detections shown here are COBE, FIRS, Tenerife, South Pole, BAM, ARGO, Python, MAX, MSAM, SK, and CAT (62). Predictions for the models with their best-fit normalizations are plotted as 

\[ dT_\ell = (\ell(\ell + 1)C_\ell/2\pi)^{1/2}T_{CMB} \]

for SCDM (solid black), TCDM (dashed black), CHDM (solid red), OCDM (dashed blue), ΛCDM (solid blue), φCDM (dotted black), BCDM (dotted blue), ICDM (dashed magenta), PBH BDM (solid magenta), and Strings+Λ (dotted magenta). The ICDM, PBH BDM, and Strings+Λ models disagree with the slope implied by COBE, SP, and BAM, which prefers the adiabatic models. SK favors a high acoustic peak near \( \ell = 250 \) and has small error bars, making it a challenge for most models.

Fig. 2.— A. Compilation of large-scale structure observations with \( P(k) \) for SCDM (solid curve) shown for reference. No corrections for bias, redshift distortions, or non-linear evolution have been made. \( k \) is the wave number in comoving units of \( h\text{Mpc}^{-1} \). The black and blue boxes are measurements of \( \sigma_8 \) from the present-day number abundance of rich clusters and its evolution, respectively (39, 40), and the black point with error bars is from peculiar velocities (42). \( \Omega_m = 1 \) is assumed (63). Uncorrected power spectra are shown for the APM galaxy survey (blue triangles), Las Campanas (red squares), IRAS (filled pink circles), APM clusters (orange circles), and SSRS2+CfA2 (green crosses) (44, 43). B. The SCDM model with its best-fit normalization compared to the large-scale structure data with its best-fit biases after model-dependent corrections for redshift distortions and non-linear evolution (64). Beyond \( k = 0.2h\text{Mpc}^{-1} \), the predicted matter power spectrum curve is dotted to indicate uncertainty in the data corrections. We plot each CMB anisotropy detection as a box, where the width of the box represents the range of \( k \) to which that experiment is most sensitive, and the height of the box shows the 68% confidence interval (65). C. The TCDM model. D. CHDM, our best-fit model. Note agreement even on non-linear scales.

Fig. 3.— A. OCDM, with scale-invariance of potential perturbations causing an increase in the matter power spectrum beyond the curvature scale. B. The ΛCDM model. C. The φCDM model. D. The BCDM model.

Fig. 4.— A. The ICDM model (66). B. The PBH BDM model. C. The Strings+Λ model (67). D. A simulation of high-precision future observations of CMB anisotropy by the MAP (red boxes) and Planck Surveyor (blue boxes) satellites. Green error bars show accuracy of the Sloan Digital Sky Survey and magenta are for the 2 Degree Field Survey. The simulated data are indistinguishable from the underlying model (CHDM) for a wide range of \( k \) (68).
