BOSE - EINSTEIN EFFECT
IN MONTE CARLO GENERATORS

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Abstract

We briefly review various methods of implementing the Bose - Einstein effect into Monte Carlo generators. The weight methods are discussed in more detail; in particular, our method employing a clustering algorithm is applied for the process \( e^+e^- \rightarrow W^+W^- \). New results for the multiplicity distributions are presented.

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1 Introduction

Analyzing Bose - Einstein interference effects in multiparticle production may provide important information on the source geometry. However, more detailed investigations of the space-time development of this process are difficult. To compare data with models we should be able to calculate how the assumptions influence the interference effects. Only models using Monte Carlo generators allow us to produce arbitrary distributions and to investigate the effects of event selections or kinematical cuts reflecting the real experimental conditions. On the other side, however, these models use probabilities and not amplitudes. Thus incorporating quantum interference effects in them is not an easy task.

In this talk first we list shortly the existing approaches to the implementation of Bose - Einstein interference effects into Monte Carlo models. Then we discuss in more details the weight methods, and in particular a new method based on a clustering algorithm. We apply it to the process $e^+ e^- \rightarrow W^+ W^-$ (which was recently a subject of a hot debate) and present new results concerning the multiplicity in this process. We conclude with the outlook for further investigations in this field.

2 Implementation methods

The standard discussion of the BE effect in multiparticle production starts from the classical space-time source emitting identical bosons with known momenta. Thus the most natural procedure is to treat the original Monte Carlo generator as the model for the source and to symmetrize the final state wave function. This may be done in a more proper way using the formalism of Wigner functions. In any case, however, the Monte Carlo generator should yield both the momenta of produced particles and the space-time coordinates of their creation (or last interaction) points. Even if we avoid troubles with the uncertainty principle by using the Wigner function approach, such a generator seems reliable only for heavy ion collisions. It has been constructed also for the $e^+ e^-$ collisions, but localizing the hadron creation point in the parton-based Monte Carlo program for lepton and/or hadron collisions is a rather arbitrary procedure, and it is hard to say what does one really test comparing such a model with data.

It seems to be the best procedure to take into account the interference effects before generating events. Unfortunately, this was done till now only for the JETSET generator for a single Lund string, and a generalization for multi-string processes is not obvious. No similar modifications were yet proposed for other generators.

The most popular approach, applied since years to the description of BE effect in various processes, is to shift the final state momenta of events generated by the PYTHIA/JETSET generators. The prescription for a shift is such as to reproduce the experimentally observed enhancement in the ratio

$$c_2(Q) = \frac{\langle n >^2}{\langle n(n-1) >} \int d^3p_1 d^3p_2 \rho_2(p_1, p_2) \delta(Q - \sqrt{-\left(p_1 - p_2\right)^2})$$

which is a function of a single invariant variable $Q$. The value of this function is close to 1.
one for the default JETSET/PYTHIA generator. One parametrizes often this ratio by

$$c_2(Q) = 1 + \lambda \exp(-R^2 Q^2),$$  \hspace{1cm} (2)

where $R$ and $\lambda$ are parameters interpreted as the source radius and "incoherence strength", respectively.

After performing the shifts, all the CM 3-momenta of final state particles are rescaled to restore the original energy. In more recent versions of the procedure \cite{13} "local rescaling" is used instead of the global one. In any case, each event is modified and the resulting generated sample exhibits now the "BE enhancement": the ratio \cite{13} is no longer close to one, and may be parametrized as in \cite{2}.

There is no theoretical justification for this procedure, so it should be regarded as an imitation rather than implementation of the BE effect. Its success or failure in describing data is the only relevant feature. Unfortunately, whereas the method is very useful for the description of two-particle inclusive spectra, it fails to reproduce (with the same fit parameters $R$ and $\lambda$) the three-particle spectra \cite{14} and the semi-inclusive data \cite{15}. This could be certainly cured, e.g., by modifying the shifting procedure and fitting the parameters separately for each semi-inclusive sample of data. However, the fitted values of parameters needed in the input factor \cite{2} used to calculate shifts are quite different from the values one would get fitting the resulting ratio \cite{13} to the same form \cite{13}. This was shown recently in a much more detailed study \cite{17}. Thus it seems to be very difficult to learn something reliable on the space-time structure of the source from the values of fit parameters in this procedure.

All this has led to the revival of weight methods, known for quite a long time \cite{18}, but plagued with many practical problems. The method is clearly justified within the formalism of the Wigner functions, which allows to represent (after some simplifying assumptions) any distribution with the BE effect built in as a product of the original distribution (without the BE effect) and the weight factor, depending on the final state momenta \cite{19}. With an extra assumption of factorization in momentum space, we may write the weight factor for the final state with $n$ identical bosons as

$$W(p_1, \ldots p_n) = \sum \prod_{i=1}^n w_2(p_i, pp_{(i)}),$$  \hspace{1cm} (3)

where the sum extends over all permutations $P_n(i)$ of $n$ elements, and $w_2(p_i, p_k)$ is a two-particle weight factor reflecting the effective source size. A commonly used simple parametrization of this factor for a Lorentz symmetric source is

$$w_2(p, q) = \exp[-(p - q)^2R^2/2],$$  \hspace{1cm} (4)

The only free parameter is now $R$, representing the effective source size. In fact, the full weight given to each event should be a product of factors \cite{3} calculated for all kinds of bosons; in practice, pions of all signs should be taken into account. Only direct pions and the decay products of $\rho, K^*$ and $\Delta$ should be taken into account, since for other pairs much bigger $R$ should be used, resulting in negligible contributions.
3 Weight methods: problems and solutions

The main problem of the weight methods is that weights do change not only the Bose-Einstein ratio \( \frac{1}{1+e^{-x}} \), but also many other distributions. Thus with the default values of free parameters (fitted to the data without weights) we find inevitably some discrepancies with data after introducing weights.

We want to make clear that this cannot be taken as a flaw of the weight method. There is no measurable world “without the BE effect”, and it makes not much sense to ask, if this effect changes e.g. the multiplicity distributions. If any model is compared to the data without taking the BE effect into account, the fitted values of its free parameters are simply not correct. They should be refitted with weights, and then the weights recalculated in an iterative procedure. This, however, may be a rather tedious task.

Therefore we use a simple rescaling method proposed by Jadach and Zalewski \[21\]. Instead of refitting the free parameters of the MC generator, we rescale the BE weights (calculated according to the procedure outlined above) with a simple factor \( cV^n \), where \( n \) is the global multiplicity of ”direct” pions, and \( c \) and \( V \) are fit parameters. Their values are fitted to minimize

\[
\chi^2 = \sum_n \frac{[cV^n N^w(n) - N^0(n)]^2}{N^0(n)} \quad (5)
\]

where \( N^0(n) \) is the number of events for the multiplicity \( n \) without weights, and \( N^w(n) \) is the weighted number of events. This rescaling restores the original multiplicity distribution \[25\]. In addition, the single longitudinal and transverse momentum spectra are also restored by this rescaling \[25\].

Obviously, for a more detailed analysis of the final states, single rescaling may be not enough. E.g., since different parameters govern the average number of jets and the average multiplicity of a single jet, both should be rescaled separately to avoid discrepancy with data. Let us stress once again that such problems arise due to the use of generators with improperly fitted free parameters, and do not suggest any flaw of the weight method. Another problem is that our formula for weights \( (3) \) is derived using some approximations, which are rather difficult to control \[19\]. We can justify them only \textit{a posteriori} from the phenomenological successes of the weight method.

An obvious simplification introduced above is the Gaussian form of the two particle weight factor \( (4) \). The detailed form of this factor should reflect the space time characteristics of the source. Thus one may be forced to introduce asymmetry between various space-time components of momenta and/or more complicated functions (e.g., separate terms for direct particles and decay products of various resonances).

Last but not least, the main practical difficulty with formula \( (3) \) is the factorial increase of the number of terms in the sum with increasing multiplicity of identical pions \( n \). For high energies, when \( n \) often exceeds 20, a straightforward application of formula \( (3) \) is impractical \[20\], and some authors \[21, 22\] replaced it with simpler expressions, motivated by some models. It is, however, rather difficult to estimate their reliability.

We have recently proposed two ways of dealing with this problem. One method consists of a truncation of the sum \( (3) \) up to terms, for which the permutation \( P(i) \) moves no more than 5 particles from their places \[23\]. However, it is difficult to claim a priori that such a truncation does not change the results which would be obtained using the full series \( (2) \).
Therefore a second way of an approximate calculation of the sum (2) was proposed [24]. Since this sum, called a permanent of a matrix built from weight factors $w_{i,k}$, is quite familiar in field theory, one may use a known integral representation and approximate the integral by the saddle point method. However, this method is reliable only if in each row (and column) of the matrix there is at least one non-diagonal element significantly different from zero. Thus the prescription should not be applied to the full events, but to the clusters, in which each momentum is not far from at least one other momentum. The full weight is then a product of weights calculated for clusters, in which the full event is divided.

The considerations presented above suggested the necessity of combining these two methods. After dividing the final state momenta of identical particles into clusters, we used for small clusters exact formulae presented in [16, 23]. For large clusters (with more than five particles) we compared two approximations (truncated series and the integral representation) to estimate their reliability and the sensitivity of the final results to the method. Obviously, the results depend also on the clustering algorithm: if we restrict each cluster to particles very close in momentum space, the neglected contributions to the sum (3) from permutations exchanging pions from different clusters may be non-negligible, and if the cluster definition is very loose, the saddle point approximation may be unreliable. This was then also checked to optimize the algorithm used. We found that the truncated series method was sufficient in all cases [25].

### 4 Example: $e^+e^- \rightarrow W^+W^-$

As already noted, the main problem of the weight methods is the lack of a reliable reference model for "the world without the Bose - Einstein effect". Therefore it is particularly interesting to use the method for two processes, which are influenced by this effect in different ways. An important example is the comparison of two channels of the process named in the title: the decay of both $W - s$ into 2-j hadronic states, and the channel where one $W$ decays into leptons and the other one decays into hadrons.

With the advent of LEPII data one started to discuss in detail possible effects which could influence the final state obtained from double hadronic decay and break the simple factorization picture. The original motivation was the concern about the possibility of using these data for the precision measurements of $W$ mass, crucial for the tests of the standard model. Although the original suggestions of possible large $W$ mass shifts due to the BE interference and the colour reconnection (CR) effects [3] seem to be rather exaggerated (see refs. [4, 29], and references quoted therein), there are other possible observables which may discriminate between the existing models of space - time development of the hadronization process.

The simplest observable of this type is just the average multiplicity. Its value for the double hadronic $WW$ decay $\pi_{WW}$ may be not just twice the average multiplicity from single $W$ hadronic decay $\pi_W$. Moreover, one may predict which one is bigger.

For the $WW$ production at the energies near the threshold the decay products of both $W$ - s are formed in the same space - time region. Therefore the transition amplitudes to the
hadronic final states are not just products of two decay amplitudes. The symmetrization enhances the probabilities to obtain final states where the momenta of identical pions are close in momentum space. This is more likely for higher multiplicities, where many pions are slow in the CM frame. Thus one may expect that $\Delta n \equiv n_{WW} - 2n_{W} > 0$. This is to be contrasted with the CR effects, which reduce the multiplicity [27]. These two effects may well cancel to large extent (the so called BE conspiracy [13]). Thus it is particularly interesting to form quantitative predictions for each effect, which is quite simple for the BE effect implemented by our weight method [25].

For the MC without the BE effect the final state of the hadronic $WW$ decay is a simple superposition of two $W$ decay product systems. This means that the generating function of the multiplicity distribution

$$G(z) = \sum P(n)z^n. \quad (6)$$

should be just the square of the generating function for a single decay

$$G_{WW}(z) = [G_{W}(z)]^2. \quad (7)$$

Rescaling the distribution $P(n)$ by $cV^n$ factors rescales the argument of $G$ by $V$ (the normalization factor $c$ is irrelevant since any $G(z)$ has to fulfill the equation $G(1) = 1$). This does not spoil the relation (4). Rescaling may be interpreted as refitting the parameter which controls the density of particles from a single string. Thus the same value of the rescaling parameter $V$ should be used for single- and double $W$ decay.

In the single decay this value is fitted to restore exactly the average multiplicity obtained without weights (and compatible with the data). However, for the double decay such a reduction is insufficient: the BE weight for a final state from the double decay is always bigger than the product of weights for two independent decays. Since weights enhance the probability of states with high multiplicity, an excess of multiplicity appears. This should be contrasted with the momentum shifting method, in which by definition the multiplicity distributions are unchanged by the BE effect.

To be more precise, the rescaling should be performed separately for each channel of the process under consideration in order to avoid changes, e.g., of the $W$ branching ratios into $u\bar{d}$ and $c\bar{s}$ channels. However, we have checked that for our purposes a single rescaling is sufficient, since the difference of multiplicities obtained with the two rescaling procedures is much smaller than the estimated uncertainty of the final result.

We performed a quantitative analysis [28] with the value of only free parameter $R$ from the formula (4) determined by the fit to the Bose-Einstein ratio in the $Z^0$ decay [28]. We found the excess of the multiplicity $\Delta n = 2.1 \pm 0.9$ in a good agreement with the average preliminary data from all the LEPII experiments at 172 $GeV$ [30], which give $\Delta n = 2.4 \pm 1.8$. These results seemed to be confirmed by the first data from 183 $GeV$, where much bigger statistics is collected [31]. However, the average of all LEPII experiments at this energy shows no significant excess of multiplicity: the value of $\Delta n$ is $0.2 \pm 0.5$ [32]. The values from four experiments fluctuate rather widely around this number, as shown in the table below.
The estimates of the (negative) shift of the multiplicity due to the colour reconnection (CR) effect are only around $-0.5$ [27], and cannot explain the disagreement of our prediction with data. There is, however, a good reason why our prediction could overestimate seriously the multiplicity excess.

As already noted, we have used a single value of the parameter $R$ (reflecting the average distance between pions) for all pairs of ”direct” pions. For the pairs coming from different $W - s$ this is not quite correct already at the threshold of $WW$ production, since the strings formed in two $W \rightarrow q\bar{q}$ decays are not aligned, and only slow pions may be expected to originate really from the same volume. Moreover, using a single value of $R$ is an approximation which becomes worse with increasing energy.

Both BE and CR effects mentioned above are expected to decrease at higher energy. For the BE effect a simple approximate method to estimate this decrease would be to define an effective average distance for pairs of pions from different $W$-s as given by

$$R_{eff}^2 = R_W^2 + 4\beta^2 \gamma^2 c^2 \tau^2$$

where $\beta$ and $\gamma$ are the velocity and the Lorentz factor of each $W$ in the CM frame at given energy. $R_W$ defines the effective size of $W$, as measured by the BE effect in the $W$ (or $Z$) decay, and should be used for pairs coming from the same $W$. The proper time $\tau$ should represent the combined effect of the $W$’s lifetime and the hadronization time for slow $W$ decay products, which dominate the BE effect. We take for $\tau$ the value of 0.9fm/c, which gives for the second term in (8) the value of 1$fm^2$ at 183 $GeV$ and 0.5$fm^2$ at 172 $GeV$.

To apply the corrected algorithm we should be able to discriminate between pairs of pions coming from a single $W$ and from two $W - s$. Therefore we have modified our program. In our previous programs we were calling the procedures from the ”inside” of JETSET (at the same place, where the original LUBOEI procedure was called). Now we take the final state produced by JETSET, identify the ancestors of pions and select only ”direct” pions labelling them by the sign of $W$, from which they originate.

We found a strong reduction of the excess multiplicity due to the extra term in (8). Whereas without this term we reproduce the previous value of $\Delta\pi$, for the values quoted above we obtain $\Delta\pi = 1.1$ at 172 $GeV$, and only 0.7 at 183 $GeV$. Since we neglect the (negative) CR effect, the net excess of multiplicity may well be negligible for 183 $GeV$ and higher energies, as the data seem to indicate.

It is rather difficult to estimate the uncertainty of our predictions. Even without the second term in (8) we found that changes of $R_W$ at the level of 10% (experimentally allowed) lead to about 40% change of $\Delta\pi$ [8]. The value of $\tau$ is even less known. Allowing for the uncertainty of about 20% around the standard value of 1fm/c we find a 40% error of the second term in (8) and the corresponding uncertainty of $\Delta\pi$ between 60 and 80%.

In Fig.1 we show the resulting error band of our predictions for $\Delta\pi$ as a function of energy together with the two points representing the data averages at 172 $GeV$ [30] and 183 $GeV$.

|       | $\pi_{WW}$   | $\pi_{W}$    | $\Delta\pi$ |
|-------|-------------|-------------|-------------|
| ALEPH | 35.3±0.4±0.6| 17.0±0.3±0.2| +1.3±0.8    |
| DELPHI| 37.4±0.5±0.9| 19.5±0.4±0.6| -1.6±1.5    |
| L3    | 36.3±0.4±0.8| 18.6±0.3±0.4| -1.0±0.9    |
| OPAL  | 39.4±0.5±0.7| 19.3±0.3±0.3| +0.7±1.0    |
Obviously, neither our predictions nor the data are accurate enough to draw any strong conclusions, but there is a hope of confirming or disproving the effect in future investigations.

Fig. 1. The excess of multiplicity $\Delta \bar{n}$ as a function of CM energy $E_{CM}$. Shaded area represents our predictions from the BE effect, and the black points with errors are the averages of the LEP-II data at two energies.

A more direct way to investigate the BE effect for the joint $WW$ decay is to subtract the distributions for the joint $WW$ decay and for the single $W$ decay to get the separated BE effect for pairs from two different $W$-s. However, this has not given yet conclusive results [29, 32]. Some preliminary data suggest that at 172 GeV the value of $R_{eff}$ for such pairs should be similar to that of $R_W$, whereas other data show a suppression instead of BE enhancement at low $Q$ (!). Certainly better data are needed to clarify the situation. In any case the conclusions drawn from the multiplicity excess/deficit and from the low $Q$ enhancement/suppression should be compatible.

5 Conclusions and outlook

We have seen that our weight method of implementing the Bose - Einstein effect into Monte Carlo generators seems to work quite well. Moreover, for the process $e^+e^- \rightarrow W^+W^-$ it gives distinctly different predictions than the momentum shifting method [11, 12, 13]. Thus in principle it is possible to decide on the basis of data which method is better.

There are obvious directions to extend the applications of our method. The range of very small $Q^2$, for which the "intermittency signals" were observed, should be analysed in more detail, possibly using non-gaussian weight factors (instead of (4)). The possibility of non-symmetric sources, represented by non-symmetric weight factors should be considered. Semi-inclusive data should be investigated, looking for the possible dependence of the size parameter $R$ on different variables. Finally, higher order effects should be analysed too.
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