Computing the split domination number of grid graphs

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Abstract

A set $D \subseteq V$ is a dominating set of $G$ if every vertex in $V - D$ is adjacent to some vertex in $D$. The dominating number $\gamma(G)$ of $G$ is the minimum cardinality of a dominating set $D$. A dominating set $D$ of a graph $G = (V, E)$ is a split dominating set if the induced graph $(V - D)$ is disconnected. The split domination number $\gamma_s(G)$ is the minimum cardinality of a split domination set. In this paper we have introduced a new method to obtain the split domination number of grid graphs by partitioning the vertex set in terms of star graphs and also we have obtained the exact values of $\gamma_s(G_{m,n})$, $m \leq n$, $m, n \leq 24$.

Keywords: separate, domination number, split domination, grid graph, star graph

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1. Introduction

The graphs considered here are finite, connected, undirected without loops or multiple edges and without isolated vertices. As usual $n$ and $q$ denote the number of vertices and edges of a graph $G$. For any undefined term or notation in this paper can be found in Harary [2].

A set $D \subseteq V$ is a dominating set of $G$ if every vertex in $V - D$ is adjacent to some vertex in $D$. The dominating number $\gamma(G)$ of $G$ is the minimum cardinality of a dominating set $D$ [5]. V.R.
Kulli and B. Janakiram had introduced a concept of split domination [3]. A dominating set $D$ of a graph $G = (V, E)$ is a split dominating set, if the induced graph $(V - D)$ is disconnected. The split domination number $\gamma_s(G)$ is the minimum cardinality of a split domination set.

A two dimensional grid graph $G_{m,n}$ is the graph Cartesian product $P_m \times P_n$ of paths on $m$ and $n$ vertices. The Cartesian graph product of $G_1 \times G_2$ with disjoint vertex sets and edge sets in $G_1, G_2$ is the graph with the vertex set $V_1 \times V_2$ and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent in $G_1 \times G_2$ whenever $[u_1 = v_1$ and $u_2 \neq v_2]$ or $[u_2 = v_2$ and $u_1 \neq v_1]$. A star graph is a complete bipartite graph of the form $K_{1,n-1}$ with $n$ vertices. The neighborhood of a vertex in the graph $G$ is the set of vertices adjacent to $v$ and is denoted by $N(v)$.

Computing of domination of grid graphs has been studied in [1, 4]. In this paper we have introduced a new method to obtain the split domination number of a grid graphs by partitioning the vertex set in terms of $K_{1,3}, K_{1,2}, K_2$ and $K_1$ and also we have obtained the exact values of $\gamma_s(G_{m,n}), m \leq n, m, n \leq 24$.

2. Preliminaries

To simplify the description of the algorithm, we first define an order of the vertices of an $mn$ grid graph with vertices $v_{i,j}, 1 \leq i \leq m, 1 \leq j \leq n$. Every minimum dominating set can be constructed by an exhaustive search where in each step any undominated vertex is picked, after which all possible ways of dominating this vertex are considered in turn.

The procedure to construct the minimum split dominating set is as follows:

Pick the vertex of degree 3 say $v_i, E = v_i \cup N(v_i)$. Next choose the vertex in $V(G) - E$, that has the vertex of degree 3, if not choose the vertex of degree 2 otherwise choose the vertex of degree 1 say $v_j$, if not choose $v_k \in E$. Let $D = v_i \cup v_j \cup v_k$ such that the number of vertices in $D$ is minimum and $< V(G) - D >$ is disconnected, this procedure is continued unless $V(G) - E$ is an empty set. Suppose if $< V(G) - D >$ is connected, then we need one more vertex to make the graph disconnected.

3. Algorithm to Find the Split Domination Number of grid graph by partitioning the vertex set.

Step 1: Divide the grid graph $G_{m,n}$ by partitioning the vertex set in terms $K_{1,3}, K_{1,2}, K_{1,1}$ and $K_1$ such that

(i) Partitioning $A = \{P_1, P_2, P_3 \ldots P_n\}$ is minimum.

(ii) $< V(P_1) > \cap < V(P_2) > \cap \ldots \ldots \cap < V(P_n) > = \phi$ and $v_1 \in < V(P_1) >$,$v_2 \in < V(P_2) >$ if deg($v_1$) is maximum in $< V(P_1) >$ and $v_2 \notin N(v_1)$ in $G_{m,n}$.

(iii) $E(P_1) \cap E(P_2) = \phi, E(P_1)$ is the edges in $< V(P_1) >$.

Step 2: Suppose $A$ contains atleast two partition set say $P_1, P_2$ such that $< V(P_1) > = K_1$ and $< V(P_2) > = K_1$ and $H = \{v_c \in N(V(P_1) \cap V(P_2))$ in $G_{m,n}$, $A = \{A - \{P_1, P_2\}\}$. 

Step 3: Let $D = \{v_i \in < V(P_1) >, P_i \in A/v_i$ is of max deg of $< V(P_i) >$ and $< V(P_i) > \neq K_{1,1}\}$
Step 4: IF $A$ contains atleast one partition $P_j$, such that $< V(P_j) > = K_{1,1}$ with $< V(P_j) > = \{v_m, v_r\}$.
   GOTO STEP 5.
   ELSE IF $H = \phi$
   $C = D$
   ELSE
   $C = D \cup H$
   GOTO STEP 7.

Step 5: For each partition $P_j$.
   IF atleast one vertex say $v_s, v_s \in N(v_m), v_s \notin < V(P_j) >$ and $N(v_s) - \{v_m\} \in D$.
   $\{v_m\} \in S$.
   ELSE
   $\{v_m\}$ or $\{v_r\} \in S$.
   GOTO STEP 5.
   Let $S$ be the set of all such vertices.

Step 6: IF $H = \phi$ and $S = \phi$
   $C = D$
   ELSE IF $H = \phi$ and $S \neq \phi$
   $C = D \cup S$.
   ELSE
   $C = D \cup S \cup H$.

Step 7: Find the split adjacency matrix
   \[
   a_{ij} = \begin{cases} 
   v_{ij} = 1 & \text{if } i \text{ is adjacent to } j \text{ and } v_i \notin C \\
   v_{ij} = 0 & \text{otherwise.}
   \end{cases}
   \]

Step 8: IF $a_{ij}$ contains atleast one zero row then, $\gamma_s = |C|$
   GOTO STEP 11.
   ELSE
   GOTO STEP 9.

Step 9: Let $\{v_k\}$ is the row in $a_{ij}$ in which sum of all the elements in $\{v_k\} = 1$ and 1 is present in $v_p$ column.

Step 10: $\gamma_s = |C| + |\{v_p\}| = |C| + 1$.

Step 11: END

4. Examples

   $A = \{P_1, P_2, P_3, P_4, P_6\}$
   $D = \{v_2, v_{11}, v_9, v_{18}, v_5\}$
For partition $P_5, v_s = v_{10}, v_m = 15, S = \{v_{15}\}$

$C = D \cup S = \{v_2, v_{11}, v_9, v_{18}, v_5, v_{15}\}$

The Split adjacent matrix $a_{ij}$ is:

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Since there exists a zero row $v_{10}$ in $a_{ij}$, therefore the split domination number $\gamma_s = |C| = 6$. 
Figure 2. A grid graph $G_{3,6}$

\[ A = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\} \]

Since $< V(P_5) >$ and $< V(P_6) >= K_1$ and $H = \{v_c = v_{12}\}$

\[ A = \{P_1, P_2, P_3, P_4, P_7\} \]

\[ D = \{v_3, v_6, v_7, v_{14}, v_{16}\} \]

\[ C = D \cup H = \{v_3, v_6, v_7, v_{14}, v_{16}, v_{12}\} \]

The Split adjacent matrix $a_{ij}$ is:

\[
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]
Since there exists a zero row $v_{13}$ in $a_{ij}$, $\gamma_s = |C| = 6$

![Figure 3. A grid graph $G_{4,4}$](image)

$A = \{P_1, P_2, P_3, P_4\}$
$D = \{v_3, v_5, v_{12}, v_{14}\}$

$H = \phi$

$C = D = \{v_3, v_5, v_{12}, v_{14}\}$

The Split adjacent matrix $a_{ij}$ is:

$$
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}
$$

Since there exists a non-zero row in $a_{ij}$ and $v_{16}$ contains 1 in $v_{16}^{th}$ column, $\gamma_s = |C| + 1 = 5$
**Theorem 4.1.** For any grid graph $G_{m,n}$,

$$
\gamma_s(G_{m,n}) = \begin{cases} 
\gamma(G_{m,n}) & \text{if } \langle V(G_{m,n}) - \gamma(G_{m,n}) \rangle \text{ is disconnected} \\
\gamma(G_{m,n}) + 1 & \text{otherwise.}
\end{cases}
$$

**Proof.** Let $D$ be the $\gamma$-set of $G$.

**Case 1:** if $\langle V(G) - D \rangle$ is disconnected, the result follows from the definition of split dominating set.

**Case 2:** if $\langle V(G) - D \rangle$ is connected, then there exists atleast one vertex say $v_i$ in $D$ which is of degree 3 and $v_i \in N(v_j)$, $v_j$ is of degree 2. Then $D \cup v_k$, $v_k \in N(v_j) \neq v_i$. Since $v_j$ is of degree 2, $D \cup v_k$ is disconnected. Hence $\gamma_s(G_{m,n}) = \gamma(G_{m,n}) + 1$.

\[\square\]

5. **Exact values of $\gamma_s(G_{m,n})$, $m \leq n$.**

The exact values of $\gamma_s(G_{m,n})$ are obtained by using Theorem 4.1 and [1, 4]

$$
\gamma_s(G_{1,n}) = \left\lfloor \frac{n+2}{2} \right\rfloor
$$

$$
\gamma_s(G_{2,n}) = \begin{cases} 
\left\lfloor \frac{n+2}{2} \right\rfloor & n \text{ is even or } n = 3 \\
\left\lfloor \frac{n+2}{2} \right\rfloor + 1 & \text{otherwise.}
\end{cases}
$$

$$
\gamma_s(G_{3,n}) = \begin{cases} 
\left\lfloor \frac{3n+4}{4} \right\rfloor & n \equiv 0 (mod4) \text{ or } n = 3 \\
\left\lfloor \frac{3n+4}{4} \right\rfloor + 1 & \text{otherwise.}
\end{cases}
$$

$$
\gamma_s(G_{4,n}) = n + 1.
$$

$$
\gamma_s(G_{5,n}) = \begin{cases} 
\frac{6n+13}{5} & n = 5p + 2, p \geq 2 \\
\left\lfloor \frac{6n+8}{5} \right\rfloor & \text{otherwise.}
\end{cases}
$$

$$
\gamma_s(G_{6,n}) = \begin{cases} 
\left\lfloor \frac{10n+10}{7} \right\rfloor & n \equiv 1 (mod7) \\
\left\lfloor \frac{10n+19}{7} \right\rfloor & n = 7p + 3, p \geq 1 \\
\left\lfloor \frac{10n+12}{7} \right\rfloor & \text{otherwise}
\end{cases}
$$

$$
\gamma_s(G_{7,n}) = \begin{cases} 
\left\lfloor \frac{5n+6}{3} \right\rfloor & n = 3p, p \geq 3 \\
\left\lfloor \frac{5n+3}{3} \right\rfloor & \text{otherwise.}
\end{cases}
$$

$$
\gamma_s(G_{8,n}) = \begin{cases} 
\left\lfloor \frac{15n+22}{8} \right\rfloor & n = 8p + 6, p \geq 1 \\
\left\lfloor \frac{15n+14}{8} \right\rfloor & \text{otherwise.}
\end{cases}
$$

$$
\gamma_s(G_{9,n}) = \begin{cases} 
\left\lfloor \frac{23n+41}{11} \right\rfloor & n = 11p + 2, p \geq 1 \\
\left\lfloor \frac{23n+40}{11} \right\rfloor & \text{otherwise.}
\end{cases}
$$

$$
\gamma_s(G_{10,n}) = \begin{cases} 
\left\lfloor \frac{30n+37}{13} \right\rfloor & n \neq 13, 16, n \equiv 0, 3 (mod13), n = 13p + 7, p \geq 1 \\
\left\lfloor \frac{30n+24}{13} \right\rfloor & \text{otherwise.}
\end{cases}
$$
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\[\gamma_s(G_{11,n}) = \begin{cases} \frac{38n+21}{15} & n = 11, 20, 22 \\ \frac{38n+36}{15} & n = 15p + 3, p \geq 3 \\ \text{otherwise.} & \end{cases} \]

\[\gamma_s(G_{12,n}) = \begin{cases} \frac{80n+95}{29} & n = 29p - 3, p \geq 1 \\ \frac{80n+66}{29} & \text{otherwise.} \end{cases} \]

\[\gamma_s(G_{13,n}) = \begin{cases} \frac{98n+111}{33} & n \equiv 14, 15, 17, 20 \,(mod\,33), n = 33p + 12, p \geq 1 \\ \frac{98n+78}{33} & \text{otherwise.} \end{cases} \]

\[\gamma_s(G_{14,n}) = \begin{cases} \frac{35n+40}{11} & n \equiv 18 \,(mod\,22), n = 11p + 13, p \geq 1 \\ \frac{35n+29}{11} & \text{otherwise.} \end{cases} \]

\[\gamma_s(G_{15,n}) = \begin{cases} \frac{44n+27}{13} & n \equiv 5 \,(mod\,26), \\ \frac{44n+53}{13} & n = 13p + 5, p \geq 1 \\ \frac{44n+40}{13} & \text{otherwise.} \end{cases} \]

\[\gamma_s(G_{16,n}) = \begin{cases} \frac{18n+21}{5} & n = 5p + 13, p \geq 1 \\ \frac{18n+36}{5} & \text{otherwise.} \end{cases} \]

\[\gamma_s(G_{17,n}) = \begin{cases} \frac{19n+23}{5} & n = 5p + 13, p \geq 1 \\ \frac{19n+38}{5} & \text{otherwise.} \end{cases} \]

\[\gamma_s(G_{18,n}) = 4n + 5. \]

\[\gamma_s(G_{19,n}) = \begin{cases} \frac{21n+27}{5} & n = 5p + 13, p \geq 2 \\ \frac{21n+42}{5} & \text{otherwise.} \end{cases} \]

\[\gamma_s(G_{20,n}) = \begin{cases} \frac{22n+29}{5} & n = 5p + 13, p \geq 2 \\ \frac{22n+44}{5} & \text{otherwise.} \end{cases} \]

\[\gamma_s(G_{21,n}) = \begin{cases} \frac{23n+31}{5} & n = 5p + 13, p \geq 2 \\ \frac{23n+46}{5} & \text{otherwise.} \end{cases} \]

\[\gamma_s(G_{22,n}) = \begin{cases} \frac{24n+33}{5} & n = 5p + 13, p \geq 2 \\ \frac{24n+48}{5} & \text{otherwise.} \end{cases} \]

\[\gamma_s(G_{23,n}) = 5n + 7. \]

\[\gamma_s(G_{24,n}) = \begin{cases} \frac{26n+37}{5} & n = 5p + 13, p \geq 3 \\ \frac{26n+52}{5} & \text{otherwise.} \end{cases} \]
Table 1. Split domination numbers $\gamma_s(G_{m,n})$, $m, n \leq 25$

| m \div n | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|-----------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1         | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  | -  |
| 2         | -  | 2  |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 3         | 1  | 2  | 3  |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 4         | 2  | 3  | 4  | 5  |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 5         | 2  | 4  | 5  | 6  | 8  |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 6         | 2  | 4  | 6  | 7  | 9  | 11  |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 7         | 3  | 5  | 7  | 8  | 10 | 12 | 13 | 15 | 17 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 8         | 3  | 5  | 7  | 9  | 12 | 13 | 15 | 17 | 21 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 9         | 3  | 6  | 8  | 10 | 13 | 15 | 17 | 19 | 21 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 10        | 4  | 6  | 9  | 11 | 14 | 17 | 18 | 21 | 23 | 25 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 11        | 4  | 7  | 10 | 12 | 15 | 18 | 20 | 23 | 25 | 28 | 30 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 12        | 4  | 7  | 10 | 13 | 17 | 19 | 22 | 25 | 27 | 30 | 33 | 36 |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 13        | 5  | 8  | 11 | 14 | 18 | 21 | 23 | 27 | 30 | 32 | 36 | 39 | 41 |    |    |    |    |    |    |    |    |    |    |    |    |
| 14        | 5  | 8  | 12 | 15 | 19 | 22 | 25 | 29 | 32 | 35 | 38 | 41 | 45 | 48 |    |    |    |    |    |    |    |    |    |    |    |
| 15        | 5  | 9  | 13 | 16 | 20 | 23 | 27 | 30 | 34 | 37 | 41 | 44 | 48 | 51 | 54 |    |    |    |    |    |    |    |    |    |    |
| 16        | 6  | 9  | 13 | 17 | 21 | 25 | 28 | 32 | 36 | 39 | 43 | 47 | 50 | 54 | 58 | 61 |    |    |    |    |    |    |    |    |    |
| 17        | 6  | 10 | 14 | 18 | 23 | 27 | 30 | 34 | 38 | 42 | 46 | 50 | 54 | 57 | 61 | 65 | 69 |    |    |    |    |    |    |    |    |
| 18        | 6  | 10 | 15 | 19 | 24 | 28 | 32 | 36 | 40 | 44 | 48 | 52 | 56 | 61 | 65 | 69 | 73 | 77 |    |    |    |    |    |    |    |
| 19        | 7  | 11 | 16 | 20 | 25 | 29 | 33 | 38 | 42 | 46 | 51 | 55 | 59 | 64 | 68 | 72 | 76 | 81 | 85 |    |    |    |    |    |
| 20        | 7  | 11 | 16 | 21 | 26 | 31 | 35 | 40 | 44 | 49 | 53 | 58 | 63 | 67 | 71 | 76 | 80 | 85 | 89 | 93 |    |    |    |    |
| 21        | 7  | 12 | 17 | 22 | 27 | 32 | 37 | 42 | 46 | 51 | 56 | 61 | 65 | 70 | 75 | 79 | 84 | 89 | 93 | 98 | 102 |    |    |    |
| 22        | 8  | 12 | 18 | 23 | 29 | 33 | 38 | 44 | 48 | 53 | 58 | 63 | 68 | 73 | 78 | 83 | 88 | 93 | 97 | 102 | 107 | 112 |    |    |
| 23        | 8  | 13 | 19 | 24 | 30 | 35 | 40 | 45 | 50 | 54 | 60 | 65 | 70 | 76 | 81 | 87 | 92 | 97 | 102 | 107 | 112 | 117 | 122 |    |
| 24        | 8  | 13 | 19 | 25 | 31 | 37 | 42 | 47 | 53 | 58 | 64 | 69 | 74 | 80 | 85 | 90 | 95 | 101 | 106 | 111 | 116 | 121 | 127 | 132 |    |
| 25        | 9  | 14 | 20 | 26 | 32 | 38 | 43 | 49 | 55 | 60 | 66 | 72 | 77 | 83 | 88 | 94 | 99 | 105 | 110 | 115 | 121 | 126 | 132 | 137 | 142 |
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