Renormalizability of the Nuclear Many-Body Problem with the Skyrme Interaction Beyond Mean Field

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Phenomenological effective interactions like Skyrme and Gogny forces are currently used in mean-field calculations in nuclear physics. Mean-field models represent the first order in the perturbative many-body problem and effective interactions are adjusted and used at this order. In this work, we analyze the renormalizability of the nuclear many-body problem in the case where the effective Skyrme interaction is employed in its standard form and the perturbative problem is solved up to second order. For simplicity, we focus on symmetric nuclear matter and its equation of state, which can be calculated analytically at this order. Specific density dependences are found for which the problem is renormalizable, one of which generates a reasonable equation of state over a quite large range of densities around saturation. However, it is shown that the standard Skyrme interaction does not in general lead to a renormalizable theory. For renormalizability, other terms should be added to the interaction and employed perturbatively only at first order.

Bulk properties of medium-mass and heavy nuclei are very well described by phenomenological effective interactions treated in the mean-field picture. Despite this success, the necessity of increasing the accuracy of theoretical predictions and of extending them to the exotic regions of the nuclear chart has motivated several groups to formulate beyond-mean-field models, which explicitly include more correlations in their formal scheme. Several directions have been explored, for instance: projection techniques in the framework of the generator coordinate method; second random-phase approximation (SRPA) calculations with Skyrme and Gogny forces, as well as with an interaction derived from a realistic force; particle–vibration coupling (PVC) techniques with the Skyrme interaction and with a relativistic Lagrangian; multiparticle–multihole configuration mixing (mpmhCM) methods with both Skyrme and Gogny forces.

A challenge faced by all these models is how to overcome the overcounting of correlations when conventional forces or Lagrangians are used. Conventional forces and Lagrangians are actually designed for mean-field–based models, and the adjustment of their parameters is performed at this level. When the same interactions, with the same values of the parameters, are employed in calculations where different types of correlations are explicitly taken into account, a risk of double counting is present. In other words, when beyond-mean-field methods are used, the adjustment of the parameters should be done at the same level (at the same order) in the perturbative many-body problem. Apart from this general problem, several technical difficulties are encountered in many of these sophisticated models. Let us mention for instance the irregularities and the divergences that may be found in projection calculations and the ultraviolet (UV) divergences that are present when zero-range interactions are employed in SRPA, PVC or mpmhCM calculations.

The issue of UV divergences in second-order calculations with the zero-range Skyrme force has been addressed by two of us in the case of nuclear matter using cutoff- and dimensional-regularization techniques. New-generation Skyrme-type interactions have been designed to provide a reasonable equation of state for nuclear matter by including first- and second-order contributions in the evaluation of the energy. This approach produces well-defined results that avoid overcounting but depend on the arbitrary regularization procedure.

The specific problem of designing new interactions to be used in beyond-mean-field calculations can be viewed as a part of a more general issue: the formulation of an interaction that provides a renormalizable theory order by order in the perturbative many-body problem. Renormalizability means that the theory is independent of the arbitrary regularization procedure. High-energy physics eliminated from loops by the regulator is accounted for in the coefficients of the interactions, which are then cutoff dependent in such a way as to ensure that observables are not. Renormalizability is guaranteed once all interactions allowed by the symmetries of the underlying dynamics are included. The framework to accomplish this is that of effective field theories (EFTs), which has been successfully applied to the physics of light nuclei over the last two decades.

Ensuring renormalizability is, in turn, a step towards an even more general objective, that of searching for the correct power counting which indicates the proper hierarchy of allowed interactions. A consistent power counting generates at each order enough interactions so that any remaining regularization dependence can be eliminated with a sufficiently high value for the regulator parameter. Thus, imposing renormalizability is a guide for theory construction, the best-known example being the development of the electroweak theory known as the Standard Model. A nuclear example is provided by Pionless EFT, where the existence of a three-body force in leading order was discovered by demanding renormalizability of the theory’s description of the three-body system.
should be a controlled expansion around it, but the expansion parameter has remained mysterious. Renormalizability has not yet been extensively explored in the case of phenomenological effective interactions like Gogny and Skyrme forces. In this exploratory study, we focus on the zero–range Skyrme force, which bears formal similarities with the interactions in Pionless EFT. We are thus implicitly assuming that non–relativistic nucleons are the only degrees of freedom relevant for the low–energy dynamics of the nuclei of interest, which is far from obvious. The analysis is performed by including first– and second–order contributions in the EoS of symmetric nuclear matter. The objective is to reveal the implications of demanding renormalizability through a redefinition of the existing parameters at each order (at second order in the present work). A similar procedure can be followed for more complex forces, higher orders, different isospin composition, and finite nuclei.

We consider the standard Skyrme force \[25\], which contains central, density–dependent, and velocity–dependent terms of zero range. (The spin–orbit term does not contribute in infinite matter.) We denote by \( \vec{r}_n \) the coordinate of nucleon \( n \), and by \( \vec{\nabla}_n \) the gradient with respect to it. We also introduce \( \vec{\nabla}' = (\vec{\nabla}_1 - \vec{\nabla}_2)/2i \) and its complex conjugate \( \vec{\nabla}'^\dagger \), which acts to the left. The spin–exchange operator is written as \( P_\sigma = (1 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)/2 \) in terms of the spin \( \sigma_n/2 \) of nucleon \( n \). The interaction may then be written as

\[
V(\vec{r}_1, \vec{r}_2) = t_0(1 + x_0 P_\sigma) \delta(\vec{r}_1 - \vec{r}_2) + T_3(1 + x_3 P_\sigma) k_F^3 \delta(\vec{r}_1 - \vec{r}_2) \\
+ \frac{t_1}{2}(1 + x_1 P_\sigma)[\vec{\nabla}'^2 \delta(\vec{r}_1 - \vec{r}_2) + \delta(\vec{r}_1 - \vec{r}_2) \vec{\nabla}'^2] + t_2(1 + x_2 P_\sigma) \vec{\nabla}' \cdot \delta(\vec{r}_1 - \vec{r}_2) \vec{\nabla},
\]

where \( k_F \) is the Fermi momentum and \( \alpha \) a real number. The usual Skyrme parameters \( t_0,1,2 \) and \( x_0,1,2,3 \) are present, while the parameter \( T_3 \) is defined in terms of the Skyrme parameter \( t_3 \) as \( T_3 = (2/3\pi^2)^2 t_3/6 \).

The EoS for symmetric matter is given, up to second order, by the diagrams shown in Fig. 1. The upper (lower) line displays first– (second–) order diagrams, while direct (exchange) contributions are shown on the left (right) column. Straightforward evaluation gives for the energy per nucleon

\[
\frac{E}{A}(k_F, \Lambda) = \frac{3\hbar^2}{10m} k_F^2 + \frac{t_0}{4\pi^2} k_F^3 + \frac{T_3}{4\pi^2} k_F^{3+3\alpha} \\
+ \frac{\theta_s}{4\pi^2} k_F^5 + \frac{\Delta E^{(2)}}{A}(k_F, \Lambda).
\]

The first four terms are first–order, with \( m \) the medium nucleon mass and

\[
\theta_s = \frac{1}{10} \left[ 3t_1 + t_2(5 + 4x_2) \right].
\]

The last term collects the second–order contributions, which depend on the momentum cutoff \( \Lambda \). The expression for \( \Delta E^{(2)}(k_F, \Lambda)/A \) can be found in Ref. [18].

The asymptotic (\( \Lambda \gg k_F \)) behavior of \( \Delta E^{(2)}(k_F, \Lambda)/A \) can be split into three terms,

\[
\frac{\Delta E^{(2)}(k_F, \Lambda)}{A} = \frac{\Delta E^{(2)}_f(k_F)}{A} + \frac{\Delta E^{(2)}_a(k_F, \Lambda)}{A} \\
+ \frac{\Delta E^{(2)}_d(k_F, \Lambda)}{A},
\]

where the subscripts \( f, a, \) and \( d \) denote, respectively, the finite part, the contribution where the \( \Lambda \) dependence can be absorbed with a redefinition of the parameters, and the part that cannot in general be regrouped with mean–field terms and thus can diverge when \( \Lambda \to \infty \). These three contributions can be written as

\[
\frac{\Delta E^{(2)}_f(k_F)}{A} = \frac{3m}{2\pi^2 \hbar^2} k_F^4 \left[ A_0 + A_1 T_3 k_F^{3\alpha} + A_2 T_3^2 k_F^{6\alpha} + A_3 k_F^2 + A_4 T_3 k_F^{2+3\alpha} + A_5 k_F^4 \right],
\]

\[
\frac{\Delta E^{(2)}_a(k_F, \Lambda)}{A} = -\frac{m}{8\pi^4 \hbar^2} \Lambda k_F^3 \left[ B_0(\Lambda) + B_1(\Lambda) T_3 k_F^{3\alpha} + B_2(\Lambda) k_F^2 \right],
\]

\[
\frac{\Delta E^{(2)}_d(k_F, \Lambda)}{A} = -\frac{m}{8\pi^4 \hbar^2} \Lambda k_F^3 \left[ C_0 T_3^2 k_F^{6\alpha} + C_1 T_3 k_F^{2+3\alpha} + C_2 k_F^4 \right],
\]

where the constants \( A_i, B_i(\Lambda), \) and \( C_i \) are the combinations of Skyrme parameters and cutoff shown explicitly.
Figure 1. First–order (upper line) and second–order (lower line) contributions to the energy of nuclear matter. Direct (exchange) terms are shown on the left (right). Particles and holes are denoted by oriented solid lines and the interaction by a dashed line.

in the Appendix.

From the $k_F$ dependence in Eq. (9) one sees that $\Delta E_d^{(2)}(k_F, \Lambda)/A$ may be regrouped with mean–field terms in the EoS. Its cutoff dependence can be eliminated by a parameter redefinition,

$$
\begin{align*}
t_0^R &= t_0(\Lambda) - \frac{m\Lambda}{2\pi^2\hbar^2} B_0(\Lambda), \\
T_3^R &= T_3(\Lambda) \left[1 - \frac{m\Lambda}{2\pi^2\hbar^2} B_1(\Lambda)\right], \\
\theta_s^R &= \theta_s(\Lambda) - \frac{m\Lambda}{2\pi^2\hbar^2} B_2(\Lambda),
\end{align*}
$$

where we allow the “bare” parameters $t_0$, $T_3$, and $\theta_s$ to depend on the cutoff so that the “renormalized” parameters $t_0^R$, $T_3^R$, and $\theta_s^R$ are cutoff independent. Up to terms of higher order beyond the mean–field approximation, we can replace the bare parameters in Eqs. (3) and (7) by the renormalized ones. We indicate this by a superscript $R$ in $A^R$ and $C^R$.

However, Eq. (7) shows that the cutoff dependence in $\Delta E_d^{(2)}(k_F, \Lambda)/A$ cannot be similarly handled. The $k_F^7$ divergence cannot in general be absorbed in first–order terms unless $\alpha = 4/3$ (with $T_3^R \neq 0$), but in this case the $k_F^{3+6\alpha}$ and $k_F^{3+3\alpha}$ divergences are too severe.

One possibility is to have the $k_F^7$ divergence cancel out against other second–order divergences. In this special case the problem becomes renormalizable by imposing that $\Delta E_d^{(2)}(k_F, \Lambda)/A = 0$. To do this, the parameter $\alpha$ has to be constrained by $6\alpha = 2 + 3\alpha = 4$, that is, $\alpha = 2/3$. In addition, we must have

$$
C_0^R T_3^{R2} + C_1^R T_3^R + C_2^R = 0. \tag{11}
$$

As long as the discriminant $\Delta \geq 0$, the solutions for $T_3^R$ are real. Note that a solution is a function of all the other Skyrme parameters, except $t_0^R$.

The $k_F^7$ problem clearly arises from the very singular contributions generated by the two–derivative two–body force, and can be eliminated by setting $t_{1,2} = 0$. Then,

$$
C_1^R = 0, \quad C_2^R = 0, \tag{12}
$$

and the $k_F^{3+6\alpha}$ divergence can be absorbed in the mass $m$ for $\alpha = -1/6$. A less drastic alternative is to require Eq. (12) without demanding that $t_{1,2}^R = 0$. If $\alpha = 1/3$, the $k_F^{3+6\alpha}$ term can be absorbed in $\theta_s^R$; instead of Eq. (10),

$$
\theta_s^R = \theta_s(\Lambda) - \frac{m\Lambda}{2\pi^2\hbar^2} B_2(\Lambda) - \frac{m\Lambda}{2\pi^2\hbar^2} C_0 T_3^2(\Lambda). \tag{13}
$$

By solving $C_1^R = 0$ we get a first–order equation in $x_3$, and a quadratic equation in $x_1$ (or $x_2$) upon solving $C_2^R = 0$.

After the cutoff dependence has been eliminated, the remaining second–order terms given by the $A^R$s bring in new, cutoff–independent, non–analytic dependence on the density $\rho = (2/3\pi^2)^{1/3} k_F^3$, modifying the behavior of the EoS. Thus the problem can be renormalized for specific values of $\alpha$ and constraints over the Skyrme parameters. Typical values of the parameter $\alpha$ range from 1/6, for instance in the Saclay–Lyon forces [24], up to 1, for instance in SIII [27]. Two of the values found here are located inside this interval. The third value, $\alpha = -1/6$, is located outside. The two–derivative two–body force is usually thought to be required for good fits of data. Indeed, following the procedure described below we were unable to find a set of parameters that provides a reasonable EoS in this case, which we do not consider further in this paper. As examples of the effects of the extra constraints over the Skyrme parameters, we focus on the two cases $\alpha = 1/3$ and $\alpha = 2/3$.

If we impose the constraints that $\alpha = 2/3$ and $T_3$ is a solution of Eq. (11) with $\Delta \geq 0$, the renormalized EoS for symmetric nuclear matter evaluated up to second order is given by

$$
\begin{align*}
\frac{E}{A}(k_F) &= \frac{3\hbar^2}{16m} k_F^2 + \frac{1}{4\pi^2} \frac{A_R^R T_3^R + A_S^R}{2\pi^2\hbar^2} k_F^6 + \frac{1}{4\pi^2} \left(\theta_s^R + T_3^R\right) k_F^8 \\
&\quad + \frac{3m}{2\pi^2\hbar^2} \left(A_R^{R2} T_3^R + A_S^R\right) k_F^6 + \frac{3m}{2\pi^2\hbar^2} \left(A_R^{R3} T_3^R + A_S^R T_3^R + A_S^R\right) k_F^8.
\end{align*}
$$

Note that the density–dependent term plays a crucial role in ensuring renormalization, but it gives in first order a contribution that cannot be separated from that of the two–derivative terms with coefficients $t_{1,2}$; only at second order are $t_{1,2}$ and $T_3$ disentangled through their different contributions to loops. This situation is reminiscent of the different roles in vacuum between momentum– and energy–dependent interactions.
In contrast, for $\alpha = 1/3$, Eq. (12) has to be satisfied and the EoS is instead

$$\begin{align*}
\frac{E}{A}(k_F) &= \frac{3\hbar^2}{10m}k_F^2 + \frac{1}{4\pi^2}t_0^Rk_F^3 + \frac{1}{4\pi^2} \left( T_3^R + \frac{6m}{\pi^2\hbar^2}A_0^R \right) k_F^4 + \frac{1}{4\pi^2} \left( \theta_s^R + \frac{6m}{\pi^2\hbar^2}A_1^RT_3^R \right) k_F^5 \\
&+ \frac{3m}{2\pi^2\hbar^2} \left( A_2^R T_3^R + A_3 \right) k_F^6 + \frac{3m}{2\pi^2\hbar^2} A_4^R T_3^R k_F^7 + \frac{3m}{2\pi^2\hbar^2} A_5^R k_F^8,
\end{align*}$$

where $\theta^R_s$ is now given by Eq. (13). The density–dependent term has, as in more general cases, a different behavior in first order than the two–derivative terms. In second order, it generates a new $k_F^5$ term compared to Eq. (14).

We show in Fig. 2 the $\Lambda$–independent result for the EoSs in terms of the density obtained for $\alpha = 1/3, 2/3$, with $m = 938.91$ MeV and the parameters of the force SLy5 [20]. It can be observed that the renormalized problem does not immediately lead to a reasonable EoS. This is not surprising in view of the overcounting problem raised earlier: the parameters have to be readjusted when we go beyond the mean field.

To demonstrate the importance of refitting, we have performed $\chi^2$ fits of the SLy5 mean–field EoS, with the appropriate constraints, Eqs. (11) or (12). Following Ref. [15], we fit $N$ energies $E_i$ to reference points $E_{i,\text{ref}}$, with

$$\chi^2 = \frac{1}{N-1} \sum_{i=1}^N \left( \frac{E_i - E_{i,\text{ref}}}{0.01 \times E_{i,\text{ref}}} \right)^2. \quad (16)$$

We have added in the fitting procedure an additional constraint on the pressure and the incompressibility at the saturation density only. For $\alpha = 2/3$, we fit $N = 8$ energies in the density range from 0.10 to 0.24 fm$^{-3}$, while for $\alpha = 1/3$, we use $N = 12$ energies in the density range from 0.08 to 0.30 fm$^{-3}$. The resulting parameters, found in Table I, give fits with $\chi^2 = 1.08$ and $\chi^2 = 1.02$ for $\alpha = 2/3$ and $\alpha = 1/3$, respectively.

![Graph of EoS as a function of density](image)

Figure 2. (Color online) EoS as a function of the density $\rho$: the renormalized second–order EoS using SLy5 parameters with $\alpha = 1/3$ ($\alpha = 2/3$), represented by the dotted red (dashed green) line, is compared with the SLy5 mean–field EoS, given by the solid black line.

The corresponding curves are shown in Fig. 3. For $\alpha = 2/3$, the saturation point is slightly shifted with respect to the SLy5 mean–field value and is equal to 0.156 fm$^{-3}$, with an incompressibility $K_\infty = 275.42$ MeV at saturation. The fit has a good quality in the region around the saturation density, deteriorates slightly at densities below 0.08 fm$^{-3}$, and has a qualitatively incorrect behavior for densities larger than about 0.26 fm$^{-3}$. Rather large higher–order corrections would be needed to restore the successful first–order form of the EoS. In contrast, for $\alpha = 1/3$ the saturation point is not shifted with respect to the SLy5 mean–field value and is equal to 0.16 fm$^{-3}$. The value of the incompressibility at saturation density is $K_\infty = 229.13$ MeV. The fit has only slightly better $\chi^2$ but in a larger region around saturation, which translates into good overall reproduction of SLy5.

It is interesting that we were able to find a renormal-

| $\alpha$ | $t_0^R$ (MeV fm$^3$) | $t_1^R$ (MeV fm$^3$) | $x_0$ | $t_2^R$ (MeV fm$^3$) | $x_1$ | $t_3^R$ (MeV fm$^{3,4,5}$) | $x_2$ | $T_3^R$ (MeV fm$^{3,4,5}$) | $x_3$ |
|---|---|---|---|---|---|---|---|---|---|
| 2/3 | $-1.34 \times 10^3$ | $4.58 \times 10^{-4}$ | $1.34 \times 10^3$ | $6.47$ | $-1.87 \times 10^3$ | $4.33$ | $5.27 \times 10^2$ | $3.11$ |
| 1/3 | $-1.84 \times 10^3$ | $3.72 \times 10^{-2}$ | $8.82 \times 10^3$ | $2.25$ | $-8.21 \times 10^3$ | $1.65$ | $5.70 \times 10^2$ | $-1.25$ |

Table I. Parameter sets obtained in fits of the renormalized second–order EoS to the SLy5 mean–field EoS.
ized EoS in second order based on the Skyrme force for \( \alpha = 1/3 \), which could potentially serve for a description of finite nuclei. Even in the other case we considered explicitly, \( \alpha = 2/3 \), the EoS is not wholly unrealistic. Nevertheless, the renormalization requirements \( \text{(11)} \) or \( \text{(12)} \) are highly unusual. It is not a coincidence that they involve the singular two-derivative two-body terms and the term in Eq. \( \text{(11)} \) that depends explicitly on the density. At least one of these terms is required for saturation and, it is believed, both are needed for a good fit at the mean-field level. Both solutions found above include a delicate balance among these terms. It is unlikely that the requirement of renormalization at higher orders can be fulfilled with such a constrained set of interactions.

A more general renormalization would be achieved only if all the cutoff-dependent second-order terms could be regrouped with first-order terms without extra constraints. For this, additional terms have to be added to the interaction. For example, let us consider the special case where \( \alpha = 1 \), which at mean-field level is a proxy for a three-body force. In this case, the second-order contributions provide terms proportional to \( k_F^3 \), \( k_F^4 \), and \( k_F^5 \) in Eq. \( \text{(10)} \), and to \( k_F^7 \), \( k_F^8 \), and \( k_F^9 \) in Eq. \( \text{(11)} \). As already shown, the first three types of terms can be regrouped with leading-order contributions and lead to a redefinition of the parameters. The last three terms diverge. From their \( k_F \) dependence, we can recognize which terms should be added to the interaction to provide the same \( k_F \) dependence in the EoS: for the \( k_F^7 \) terms, one would need a two-body term of the type \( \nabla^4 \delta(r_1 - r_2) \); for the \( k_F^5 \) dependence a three-body term of the type \( \nabla^2 \delta(r_1 - r_2) \delta(r_2 - r_3) \); and, finally, for the \( k_F^3 \) dependence, a four-body term \( \delta(r_1 - r_2) \delta(r_2 - r_3) \delta(r_3 - r_4) \).

The inclusion of such additional terms would provide of course a much more complicated interaction and calculations would become more difficult to be performed in practice already at the mean-field level. More importantly, if these additional terms are treated on the same footing as the terms in Eq. \( \text{(11)} \), they will generate further cutoff dependence. The situation is familiar in field theory, where it is recognized that renormalization requires all possible interactions allowed by the symmetries. In this case, to have any predictive power one should be able to argue that some “sub-leading” terms should be included in first order only when “leading” terms are included in second order. For \( \alpha = 1 \), this could be the case for the four-derivative two-body, two-derivative three-body, and no-derivative four-body terms.

This issue should be addressed within a more general study where a systematic analysis of the correct power counting within the perturbative many-body problem with effective interactions is performed. To our knowledge, this aspect, which we reserve for future work, has not been addressed so far in the framework of the energy-density functional theories based on Skyrme interactions.

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**APPENDIX**

Defining the combinations of Skyrme parameters

\[
\begin{align*}
D_0 &= t_0^3 (1 + x_0^2), & D_1 &= 2t_0 (1 + x_0^2), & D_2 &= 1 + x_0^2, \\
F_{12} &= t_1 + t_2, & x_{12} &= t_1 + t_2/2, \\
e_i &= 3t_1 + 4t_2 + x_i (3x_{12} + 2t_2), \\
h &= t_{12}^2 + x_{12}^2 + t_{12}^2 x_{12}, & s &= t_{12}^2 t_{12}^2 + x_{12}^2 + 4t_1 x_{12}, \\
y_{12} &= t_{12} + x_{12} + 2 (x_{12} + x_{12} t_{12}), \\
&-j[t_{12} + x_{12} + 2 (x_{12} + x_{12} t_{12})],
\end{align*}
\]

we can write the constants \( A_i, B_i(\Lambda), \) and \( C_i \) appearing in Eqs. \( \text{(5)}, \text{(6)}, \text{and (7)} \), respectively, as

\[
\begin{align*}
A_0 &= d_0 I_1, & A_1 &= d_1 I_1, & A_2 &= d_2 I_1, & A_3 &= t_0 c_0 I_2, \\
A_4 &= e_3 I_2, & A_5 &= 8h I_3 - \frac{17}{18} s I_4, \\
B_0(\Lambda) &= d_0 + \frac{\Lambda^2}{9} t_0 y_{01} + \frac{4\Lambda^4}{15} (4h - s), \\
B_1(\Lambda) &= d_1 + \frac{\Lambda^2}{9} y_{31}, \\
B_2(\Lambda) &= \frac{2}{15} \left[ t_0 y_{02} + \frac{2\Lambda^2}{3} (2h - s) \right], \\
C_0 &= d_2, & C_1 &= \frac{2}{15} y_{32}, & C_2 &= \frac{16}{175} (6h - s),
\end{align*}
\]
where

\[ I_1 = \frac{11 - 2 \ln 2}{140}, \quad I_2 = \frac{167 - 24 \ln 2}{2835}, \]
\[ I_3 = \frac{83 - 10 \ln 2}{3465}, \quad I_4 = \frac{115 - 12 \ln 2}{3465}. \]

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