Strong Dynamics and Inflation: a review

Phongpichit Channuie

School of Science, Walailak University, Nakhon Si Thammarat, 80160 Thailand

Abstract

In this article, we review how strong dynamics can be efficiently employed as a viable alternative to study the mechanism of cosmic inflation. We examine single-field inflation in which the inflaton emerges as a bound state stemming from various strongly interacting field theories. We constrain the number of e-foldings for composite models of inflation in order to obtain a successful inflation. We study a set of cosmological parameters, e.g., the primordial spectral index $n_s$ and tensor-to-scalar ratio $r$, and confront the predicted results with the joint Planck data, and with the recent BICEP2 data.

Keywords: Strongly interacting field theories, Composite Inflaton, Non-minimal coupling, PLANCK and BICEP2 experiments

*channuie@gmail.com
1 Introduction

The underlying theory of inflation constitutes a cornerstone of the standard model of modern cosmology. By definition, it is the mechanism responsible for an early rapid expansion of our universe which is supposed to take place in the very early time. So far, new scalar fields are traditionally used to describe two prominent physics problems, i.e., the origin of mass of all particle in the standard model and cosmic inflation [1–6]. However, the elementary scalar field in field theories is plagued by the so-called hierarchy problem. Commonly, this means that quantum corrections generate unprotected quadratic divergences which must be fine-tuned away if the models must be true till the Planck energy scale. Similarly the inflaton, the field needed to initiate a period of rapid expansion of our Universe, suffers from the same kind of untamed quantum corrections. Therefore, finding its graceful exit is one of the great campaigns.

Recently, the claimed detection of the BICEP2 experiment on the primordial B-mode of cosmic microwave background polarization suggests that cosmic inflation possibly takes place at the energy around the grand unified theory scale given a constraint on the tensor-to-scalar ratio, i.e., $r \approx 0.20$. Since then, a series of papers on model updates has been reviving by this new results. Theses recent efforts include the Higgs-related inflationary scenarios [7–11], several paradigms of chaotic inflation [12, 13], some interesting analyses related to supersymmetry [14,15], and other compelling scenarios [16–36].

Nevertheless, the situation is still controversial since some serious criticisms to the BICEP2 results appeared in the literature, e.g., [39]. Furthermore, the Planck collaboration has very recently released the data concerning the polarized dust emission [38], while some attempts making a joint analysis of Planck and BICEP2 data have been publicised (see, for example, [39,40]). However, the recent improvement yields the value of $r$ lower than the one initially claimed by Ref. [59].

In this work, we anticipate to solve the cosmological “hierarchy problem” in the scalar sector of the inflation. In doing so, we have posted the compelling assumption that the
inflaton needs not be an elementary degree of freedom called the “composite inflaton” [45, 48,51] and remarkably showed that the energy scale of inflation driven by composite inflaton is around the GUT energy scale. Moreover, there has been shown that the composite models of inflation nicely respect tree-level unitarity for the scattering of the inflaton field all the way to the Planck energy scale [48,51] and some efforts have already implemented to study on their phenemenology [54–56].

Here we show how strong dynamics can be efficiently used as a viable alternative to study the mechanism of cosmic inflation. In Sec. 2 we derive equations of motion to figure out background evolutions. This will allow us to lay out the setup for a generic model of inflation. In Sec. 3 we derive $n_s$, $r$ and $A_s$ for composite models. In Sec. 4, we compute the power spectra for the curvature perturbations by using the usual slow-roll approximations and constrain the model parameters of various composite inflationary models using the observational data from Planck and recent BICEP2 observations. Finally, we conclude in the last section 5.

2 Composite Setup and background evolutions

In this section, we will start by laying out the setup for a generic models of composite inflation. We aim to derive equations of motion to figure out background evolutions and to obtain inflationary expressions. In so doing, we introduce the action for composite models in the Jordan frame (J) in which the inflaton non-minimally couples to gravity taking the form for scalar-tensor theory of gravity as [54]

$$S_J = \int d^4x \sqrt{-g} \left\{ \frac{M_P^2}{2} F(\Phi) R - \frac{1}{2} G(\Phi) g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - V(\Phi) \right\}.$$  \hspace{1cm} (1)

Here $F(\Phi)$ and $G(\Phi)$ in this action are functions of the field $\Phi$ and can be written as

$$F(\Phi) = 1 + \frac{\xi}{M_P^2} \Phi^{2/D} \text{ and } G(\Phi) = \frac{1}{D^2} G_0 \Phi^{(2-2D)/D},$$  \hspace{1cm} (2)

where the composite field $\Phi$ has mass dimension $D$. In the following calculations, we will set $M_P^2 = 1$. The non-minimal coupling to gravity is controlled by the dimensionless coupling $\xi$. Here we introduce a constant $G_0$ and $1/D^2$ for later convenience. However, the action under our consideration can practically written in the standard form of the scalar-tensor theory of gravity. To this end, we just redefine the field and write the potential in the form

$$V(\Phi) = \Phi^{4/D} f(\Phi) \quad \text{with} \quad \Phi \equiv \varphi^D,$$  \hspace{1cm} (3)

where the field $\varphi$ possesses a unity canonical dimension and $f(\Phi)$ can be in general a function of the field $\Phi$. At first glance, the non-minimal term $\xi \Phi^{2/D} R/M_P^2$ has purely phenomenological origin. However, we can read between the lines that one can revoke the unacceptable large amplitude of the primordial power spectrum if one takes $\xi = 0$ or smaller than $O(10^4)$. According to the above action, the Friedmann equation and the evolution equations for the background field are respectively given by

$$3FH^2 + 3\dot{H} = 3H^2 F (1 + 2\mathcal{F}_i) = \frac{1}{2} \dot{\Phi}^2 + V(\Phi),$$  \hspace{1cm} (4)
\[ 3FH^2 + 2\dot{F}H + 2F\ddot{H} + \ddot{F} = -\frac{1}{2}G\dot{\Phi}^2 + V(\Phi), \]  

(5)

\[ G\ddot{\Phi} + 3HG\dot{\Phi} + \frac{1}{2}G_{\Phi} \dot{\Phi}^2 + V_{\Phi} = 3F_{\phi} \left( \dot{H} + 2H^2 \right), \]  

(6)

where \( \mathcal{F}_t = \dot{F}/(2HF) \), \( H \) is the Hubble parameter, subscripts “\( \Phi \)” denote a derivative with respect to \( \Phi \), and the dot represents derivative with respect to time, \( t \). In order to derive the observables, it is common to apply the standard slow-roll approximations such that

\[ |\ddot{\Phi}/\dot{\Phi}| \ll H, \quad |\dot{\Phi}/\Phi| \ll H \quad \text{and} \quad |G\dot{\Phi}^2/2| \ll V(\Phi). \]  

(7)

It is more fashionable to work in the Einstein frame (E) instead of the Jordan one. However, the Einstein and Jordan frame are equivalent and related by a conformal transformation of the metric, which amounts to rescaling all length scales. In our presentation below, we will first derive some inflationary parameters in the Einstein frame and then transform to the Jordan one in order to figure out the relation between two frames.

### 3 Inflationary Observables

The non-minimal coupling between a scalar field and the Ricci scalar may be diagonalized to the minimally coupled system in which the system can basically transformed to the GR form of the action. This approach is well-known as the Einstein frame and is equivalent to the Jordan frame analysis at the classical level. It is often more convenient to perform calculations in Einstein frame and (under the assumption, that the Jordan frame is the physical one) to express results in terms of physical variables. For instance, there has been an interesting discussion about the Jordan and Einstein frames of Brans-Dicke gravity [49]. By performing a conformal transformation, we take the following replacement:

\[ g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = F(\Phi) g_{\mu\nu}. \]  

(8)

With the above rescaling replacement, we obtain the action in Eq. (1) transformed into the new frame – the Einstein frame – as

\[ S_E = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{M_p^2}{2} \tilde{R} - \frac{1}{2} \partial^\mu \chi \partial^\nu \chi - U(\chi) \right], \]  

(9)

where \( \tilde{g} \) and \( \tilde{R} \) are basically computed from \( \tilde{g}_{\mu\nu} \); “tildes” represent the quantities in the Einstein frame, and

\[ \frac{\partial \Phi}{\partial \chi} = \sqrt{GF + 3F_{\Phi}^2/2} \quad \text{and} \quad U(\chi) = \frac{V(\Phi)}{F^2(\Phi)} \bigg|_{\Phi=\Phi(\chi)}, \]  

(10)

where the subscript denotes a derivative with respect to \( \Phi \). We can reexpress inflationary parameters and all relevant quantities in terms of the field \( \chi \) if we solve

\[ \chi \equiv \int \frac{\sqrt{GF + 3F_{\Phi}^2/2}}{F} d\Phi. \]  

(11)
Using the expression for the slow-roll parameter in the Einstein frame, \( \tilde{\epsilon} \), such that

\[
\tilde{\epsilon} = \frac{1}{2} \left( \frac{1}{U} \frac{\partial U}{\partial \chi} \right)^2 ,
\]

we can simply obtain the relation between that of two frames, and we see that

\[
\tilde{\epsilon} = \frac{1}{2} \left( \frac{F^2 \partial \Phi}{V} \frac{\partial}{\partial \chi} \left( \frac{V}{F^2} \right) \right)^2 = \epsilon + \mathcal{F}_t ,
\]

where \( \mathcal{F}_t \equiv \dot{\tilde{F}}/2HF \); \( \epsilon \) is the slow-roll parameter in the Jordan frame given by \( \epsilon \equiv \mathcal{F}_t - (V_\Phi/V)(F/F_\Phi)\mathcal{F}_t \); and the dot denotes a derivative with respect to time, \( t \). It is well known that the power spectrum for the scalar perturbation generated from inflaton field \( \chi \) in the Einstein frame is given by

\[
P_\zeta \simeq \frac{U}{24 \pi^2 \tilde{\epsilon}} \bigg|_{k|\tau|=1} ,
\]

where the above expression is evaluated at the conformal time \( \tau \) when the perturbation with wave number \( k \) exits the horizon and the tensor-to-scalar ratio is

\[
r \simeq 16 \epsilon .
\]

Since the power spectra are frame independent, we can use Eq.(13) to write the power spectrum in Eq.(14) and the tensor-to-scalar ratio in Eq.(15) in terms of the Jordan frame parameters as

\[
P_\zeta \simeq \frac{V}{24 \pi^2 F (\epsilon + \mathcal{F}_t)} \bigg|_{k|\tau|=1} ,
\]

\[
r \simeq 16 (\epsilon + \mathcal{F}_t) .
\]

Here, it is convenient (although tricky) to use the results in the Einstein frame, and then we transform the quantities in the Einstein frame into the Jordan one. It is noticed that one obtains the relation between two frames: \( \tilde{\epsilon} \leftrightarrow \epsilon + \mathcal{F}_t \). Having computed the field \( \Phi \) at the end of inflation \( \Phi_e \) by using the condition \( \epsilon(\Phi_e) = 1 \), one can determine the number of e-foldings via

\[
N(\Phi) = \int_{\Phi}^{\Phi_e} \frac{H}{\Phi'} d\Phi = \int_{\Phi}^{\Phi_e} \frac{1}{\Phi'} d\Phi' ,
\]

where the subscript “e” denotes the evaluation at the end of inflation and \( \Phi' \) is given by

\[
\Phi' = \frac{1}{\left(1 + \frac{3F_\Phi^2}{2F^2\mathcal{G}}\right)} \left( \frac{2F_\Phi}{G} - \frac{V_\Phi}{V} F \right) .
\]

Here, we have used the Friedmann equation and the evolution equations for the background field and apply the standard slow-roll approximations. Determining the value of \( \Phi \) and \( \Phi' \) when the perturbations exit the horizon allows us to compute the spectral index and the
amplitude of the power spectrum in terms of the number of e-foldings. The spectral index for this power spectrum can be computed via

$$n_s = \frac{d \ln P_\zeta}{d \ln k} + 1 \simeq 1 - 2\epsilon - 2\mathcal{F}_t - \Phi' \frac{d \ln (\epsilon + \mathcal{F}_t)}{d \Phi}.$$  \hspace{1cm} (20)

The amplitude of the curvature perturbation can be directly read from the power spectrum and we find

$$A_s \equiv \log \left[ |\zeta|^2 \times 10^{10} \right] \simeq \log \left[ \frac{V \times 10^{10}}{24\pi^2 F^2 (\epsilon + \mathcal{F}_t)} \right]_{c_s k|\tau|=1}.$$ \hspace{1cm} (21)

It is noticed from Eq. (20) and (21) that the spectral index and the amplitude of the curvature perturbation in the Einstein frame respectively reads

$$n_s = 1 - 6\bar{\epsilon} + \ldots \quad \text{and} \quad A_s \equiv \log \left[ |\zeta|^2 \times 10^{10} \right] \simeq \log \left[ \frac{U \times 10^{10}}{24\pi^2 \bar{\epsilon}} \right]_{c_s k|\tau|=1}.$$ \hspace{1cm} (22)

where an ellipsis represents the contributions from other inflationary parameters, e.g. the second slow-roll parameter $\eta$. In the next section, we will examine single-field inflationary models in which the inflaton is a composite state stemming from various four-dimensional strongly coupled theories.

4 Theoretical predictions & observational constraints

In this section, we compute the power spectra for the curvature perturbations by using the usual slow-roll approximations. We will constrain the model parameters of various composite inflationary models using the observational bound for $n_s$ and $r$ from Planck and recent BICEP2 observations, and use $A_s$ from Planck data.

4.1 Composite Inflation from Technicolor

The underlying gauge theory for the technicolor-inspired inflation is the SU(N) gauge group with $N_f = 2$ Dirac massless fermions. The two technifermions transform according to the adjoint representation of SU(2) technicolor (TC) gauge group, called SU(2)$_{TC}$. Here we engaged the simplest models of technicolor known as the minimal walking technicolor (MWT) theory [41–44] with the standard (slow-roll) inflationary paradigm as a template for composite inflation and name it, in short, the MCI model. In order to examine the symmetry properties of the theory, we arrange them by using the Weyl basis in the following vector transformation according to the fundamental representation of SU(4) group. The field contents in the Wely basis are

$$Q^a = \begin{pmatrix} U_L^a \\ D_L^a \\ -i\sigma^2 U_R^a \\ -i\sigma^2 D_R^a \end{pmatrix},$$ \hspace{1cm} (23)

where $U_L$ and $D_L$ are the left-handed techniup and technidown respectively, and $U_R$ and $D_R$ are the corresponding right-handed particles and the upper index $a = 1, 2, 3$ is the TC index.
indicating the three dimensional adjoint representation. With the standard breaking to the maximal diagonal subgroup, the SU(4) global symmetry spontaneously breaks to SO(4). Such a breaking is driven by the formation of the following condensate:

$$\left\langle Q_i^\alpha Q_j^\beta \epsilon_{\alpha\beta} \mathcal{E}^{ij} \right\rangle = -2 \left\langle \bar{U}_R U_L + \bar{D}_R D_L \right\rangle,$$

where $i, j = 1, \ldots, 4$ denote the components of the tetraplet of $Q$, and $\alpha, \beta$ indicate the ordinary spin. The $4 \times 4$ matrix $\mathcal{E}^{ij}$ is defined in terms of the 2-dimensional identical matrix, 1, as

$$\mathcal{E} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

with, for example, $\epsilon_{\alpha\beta} = -i \sigma^2_{\alpha\beta}$ and $\left\langle U_L^\alpha U_R^\beta \epsilon_{\alpha\beta} \right\rangle = -\left\langle \bar{U}_R U_L \right\rangle$. The connection between the composite scalar fields and the underlying technifermions can be obtained from the transformation properties of SU(4). To this end, we observe that the elements of the matrix $\mathcal{M}$ transform like technifermion bilinears such that

$$\mathcal{M}_{ij} \sim Q_i^\alpha Q_j^\beta \epsilon_{\alpha\beta} \quad \text{with} \quad i, j = 1, \ldots, 4.$$  

The composite action can be built in terms of the matrix $\mathcal{M}$ in the Jordan frame as

$$S_{\text{MCI},J} = \int d^4x \sqrt{-g} \left[ \frac{M^2}{2} R + \frac{1}{2} \xi \text{Tr} \left[ \mathcal{M} \mathcal{M}^\dagger \right] R + \mathcal{L}_{\text{MWT}} \right],$$

where $\mathcal{L}_{\text{MWT}}$ is the Lagrangian density of the MWT sector, see [45] for more details. The details of this sector are not relevant for the present discussion. From the above action, the non-minimally coupled term corresponds at the fundamental level to a four-fermion interaction term coupled to the Ricci scalar in the following way:

$$\frac{1}{2} \xi \text{Tr} \left[ \mathcal{M} \mathcal{M}^\dagger \right] R = \frac{1}{2} \xi \frac{\langle QQ \rangle_{\Lambda_{\text{Ex}}}}{\Lambda_{\text{Ex}}^2} R,$$

where $\Lambda_{\text{Ex}}$ is a new high energy scale in which this operator generates. Here the non-minimal coupling is added at the fundamental level showing that the non-minimal coupling is well motivated at the level of the fundamental description. Moreover, at the level of the fundamental theory, the transition from the Jordan frame to the Einstein frame can be also implemented by redefining the fundamental fields investigated in [46]. However, an instructive analysis of the generated coupling of a composite scalar field to gravity has been initiated in the Nambu-Jona-Lasinio (NJL) model [47]. With this regard, the non-minimal coupling apparently seems rather natural. Using the renormalization group equation for the chiral condensate, we find

$$\langle QQ \rangle_{\Lambda_{\text{Ex}}} \sim \left( \frac{\Lambda_{\text{Ex}}}{\Lambda_{\text{MCI}}} \right)^\gamma \langle QQ \rangle_{\Lambda_{\text{MCI}}},$$

where the subscripts indicate the energy scale at which the corresponding operators are evaluated, and basically $\Lambda_{\text{Ex}} \gg \Lambda_{\text{MCI}}$. If we assume the fixed value of $\gamma$ is around two the explicit dependence on the higher energy $\Lambda_{\text{Ex}}$ disappears. This is since we have $\mathcal{M} \sim \langle QQ \rangle_{\Lambda_{\text{MCI}}} / \Lambda_{\text{MCI}}^2$. According to this model at the effective description, the relevant effective
Figure 1: The plot shows the relation between the amplitude of the power spectrum $A_s$ and the non-minimal coupling $\xi$ within a range of $10^{-3} \lesssim \xi \lesssim 10^6$ for $N = 50, 60$ predicted by the MCI model. The horizontal bands represent the 1\(\sigma\) (yellow) and 2\(\sigma\) (purple) CL for $A_s$ obtained from Planck.

theory consisting of a composite inflaton ($\varphi$) and its pseudo scalar partner ($\Theta$), as well as nine pseudo scalar Goldstone bosons ($\Pi^A$) and their scalar partners ($\tilde{\Pi}^A$) can be assembled in the matrix form such that

$$
\mathcal{M} = \left[ \frac{\varphi + i\Theta}{2} + \sqrt{2} \left( i\Pi^A + \tilde{\Pi}^A \right) X^A \right] \mathcal{E},
$$

(30)

where $X^A$'s, $A = 1, ..., 9$, are the generators of the SU(4) gauge group which do not leave the vacuum expectation value (vev) of $\mathcal{M}$ invariant, i.e. $\langle \mathcal{M} \rangle = v\mathcal{E}/2$, $v \equiv \langle \varphi \rangle$. In this model, the composite inflaton is the lightest state $\varphi$, and the remaining composite fields are massive. This provides a sensible possibility to consider the $\varphi$ dynamics first. In terms of the component fields, the resulting action in the Jordan frame is given by [45]:

$$
S_{\text{MCI}} = \int d^4x \sqrt{-g} \left[ \frac{1 + \xi \varphi^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \frac{m^2}{2} \varphi^2 - \frac{\kappa}{4} \varphi^4 \right],
$$

(31)

in which $\kappa$ is a self coupling and the inflaton mass is $m_{\text{T1}}^2 = 2m^2$. Since $m_{\text{T1}}$ is order of the GeV energy scale, $\kappa$ should be order of unity and $\varphi$ during inflation is order of Planck mass, we therefore neglect $m_{\text{T1}}^2$ term in our calculation. In the large $\xi$ limit, we obtain $n_s$, $r$ and $|\zeta|^2$ in terms of $N$ as

$$
n_s \simeq 1 - \frac{8}{3\varphi^2\xi} + \mathcal{O}(1/\xi^2) \simeq 1 - \frac{2}{N},
$$

(32)

$$
r \simeq \frac{64}{3\varphi^4\xi^2} - \frac{32}{9\varphi^4\xi^3} + \mathcal{O}(1/\xi^4) \simeq \frac{12}{N^2},
$$

(33)

$$
|\zeta|^2 \simeq \frac{\kappa \varphi^4}{128\pi^2} + \frac{\kappa (-12\varphi^2 + \varphi^4)}{768\pi^2 \xi} + \mathcal{O}(1/\xi^2) \simeq \frac{\kappa N^2}{12\pi^2 \xi^2}.
$$

(34)

Notice that the above relations lead to the consistency relation, allowing us to write

$$
r \simeq \frac{6}{N}(1 - n_s).
$$

(35)
Figure 2: The contours show the resulting 68% and 95% confidence regions for the tensor-to-scalar ratio $r$ and the scalar spectral index $n_s$. Left: The red contours are for the Planck+WP+highL data combination, while the blue ones display the BICEP2 constraints on $r$ [59]. Right: The figure shows the results from Planck plus various ancillary sets of data [58]. The plots also show the analytical and numerical predictions given by the MCI model.

In this model with $\kappa \sim \mathcal{O}(1)$, the amplitude $A_s$ is well consistent with the Planck data up to $2\sigma$ CL for $\mathcal{N} = 60$ and $4.7 \times 10^4 \lesssim \xi \lesssim 5.0 \times 10^4$, for instance, illustrated in Fig (1). However, $A_s$ does strongly depend on $\mathcal{N}$, and thus the coupling can be lowered (or raised) if $\mathcal{N}$ changes. We also find for $\xi \gg 1$ that the predictions lie well inside the joint 68% CL for the Planck+WP+highL data for $\mathcal{N} = [40, 60]$, whilst for $\mathcal{N} = 60$ this model lies on the boundary of 1σ region of the Planck+WP+highL data (the left-hand side of Figure (2)). However, with $\xi \gg 1$, the model predictions is in tension with the recent BICEP2 contours (the right-hand side of Figure (2)). This is so since the model predictions yield quite small values of $r$. Concretely, the model predicts $\epsilon \sim 1/N^2$ which no longer holds in light of the BICEPS results for $r = 16\epsilon$ such that $r = 0.2_{-0.05}^{+0.07}$. Nevertheless, this tension can be relaxed if $\xi$ is very small, i.e. $\xi \sim 10^{-3}$. If this is the case, $A_s$ cannot satisfy the Planck data unless $\kappa$ gets extremely small. Unfortunately, the prediction with very small $\kappa$ is opposed to the underlying theory. This model predicts $n_s \simeq 0.960$ and $r \simeq 0.0048$ for $\mathcal{N} = 50$ with $\xi \gg 1$. Likewise, the Higgs inflation is also in tension with the recent BICEP2 data.

4.2 Composite Inflation from pure Yang-Mills Theory

The underlying gauge theory for glueball inflation is the pure SU(N) Yang-Mills gauge theory. The inflaton in this case is the interpolating field describing the lightest glueball. In the same manner with the preceding section, the connection between the composite field and the underlying fundamental description can be also obtained. In this case, the inflaton field is

$$\Phi = \frac{\beta}{g} \text{Tr} \left[ G_{\mu\nu} G^{\mu\nu} \right], \tag{36}$$

where $G_{\mu\nu}$ is the standard non-Abelian field strength, $\beta$ is the full beta function of the theory in any renormalisation scheme, and $g$ is the gauge coupling. We can also demonstrate that
the fundamental degrees of freedom are naturally non-minimally coupled to gravity, and features the description at the fundamental level. In doing so, we introduce the non-minimal coupling term as follows:

$$
\xi \left( \frac{\beta}{g} \text{Tr} \left[ G^{\mu\nu} G_{\mu\nu} \right] \right)^{1/2} R \equiv \xi \Phi^{1/2} R, \quad (37)
$$

where the $\xi$ is a dimensionless quantity. Here, together with the preceding section, we have explicitly explained how the introduction of the non-minimal coupling is motivated in a natural way with the underlying fundamental descriptions. In this case, the inflaton emerges as the interpolating field describing the lightest glueball associated to a pure Yang–Mills theory. It is worthy to note here that the theory we are using describes the ground state of pure Yang–Mills theory, and of course is not the simple $\phi^4$ theory. For this model, we have

$$
f(\varphi) = 2 \ln(\varphi/\Lambda), \quad (38)
$$

so that the effective Lagrangian for the lightest glueball state, constrained by the Yang–Mills trace anomaly, nonminimally coupled to gravity in the Jordan frame reads [48]

$$
S_{GB} = \int d^4 x \sqrt{-g} \left[ \frac{F(\varphi)}{2} R - 16 g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - 2 \varphi^4 \ln (\varphi/\Lambda) \right]. \quad (39)
$$

Here we call it, in brief, the GB model. Also the modified version of this model has been considered in [57]. In this work, we consider only the large $\xi$ limit and find for this case

$$
N \simeq 3 \left( \ln^2 (\varphi/\Lambda) - \ln^2 (\varphi_e/\Lambda) \right) + O(1/\xi). \quad (40)
$$
Here, we can write \( \varphi \) in terms of \( \mathcal{N} \) and use Eqs. (20), (17), and (21) to write \( n_s \), \( r \), and \( |\zeta|^2 \) in terms of \( \mathcal{N} \). Finally, we obtain for a large \( \xi \) limit [54]

\[
\begin{align*}
    n_s &\simeq 1 - \frac{3}{2\mathcal{N}} + \mathcal{O}(\xi), \\
r &\simeq \frac{4}{\mathcal{N}} + \mathcal{O}(\xi), \\
|\zeta|^2 &\simeq \frac{\mathcal{N}^{3/2}}{3\sqrt{3\pi^2\xi^2}} + \mathcal{O}(1/\xi^3).
\end{align*}
\]

Notice that the above relations lead to the consistency relation, allowing us to write

\[
r \simeq \frac{8}{3}(1 - n_s).
\]

We discover that \( \mathcal{A}_s \) is well consistent with the Planck data up to 2\( \sigma \) CL for \( \mathcal{N} = 60 \) with \( 7.3 \times 10^4 \lesssim \xi \lesssim 7.5 \times 10^4 \), illustrated in Fig (3). However, \( \mathcal{A}_s \) does strongly depend on the number of e-foldings implying that the coupling can be lowered (or raised) with changing \( \mathcal{N} \). This model provides \( n_s \simeq 0.967 \) and \( r \simeq 0.089 \) for \( \mathcal{N} = 45 \) with \( \xi \gg 1 \).

From the above estimations, we see that when \( \xi \gg 1 \), \( n_s \), \( r \), and \( |\zeta|^2 \) can satisfy the 95\% C.L. observational bound from Planck data for \( 50 < \mathcal{N} < 60 \) and \( \xi \sim 10^4 \); see Fig. (3) and (4). Nevertheless, for such range of \( \mathcal{N} \), \( r \) lies outside the 2\( \sigma \) C.L. with BICEP2 results shown in Fig. (4). The value of \( r \) will increase and then satisfy the bound from BICEP2 results when \( \mathcal{N} \lesssim 45 \). However, it is obvious that \( \mathcal{N} \) is a model-dependent quantity. However, it is quite subtle if we have \( \mathcal{N} \lesssim 45 \) for models of inflation to be viable. This is so since, in order to solve the horizon problem, in the common formulation one frequently uses at least \( \mathcal{N} \subset [50, 60] \). We anticipate this can be further verified by studying the reheating effect. The compatibility between our analytical and numerical results of this model is illustrated in Fig. (4).

### 4.3 Composite Inflation from super Yang-Mills Theory

The underlying gauge theory of this model is initiated in [50] based on the following considerations. Let us consider the pure \( N = 1 \) supersymmetric Yang-Mills (SYM) gauge theory proposed by suitably modifying that of the ordinary QCD. The theory we are considering is the \( \text{SU}(N_c) \) gauge group featuring a one flavor \( (N_f = 1) \) gauge group with Weyl fermions in the adjoint representation. The Lagrangian can be written as

\[
\mathcal{L}_{\text{SYM}} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \frac{i}{2} \bar{\lambda}^{a,\alpha} P_{ab} \lambda_{b}^{\alpha} + \ldots,
\]

where \( \alpha \) is an ordinary spin, \( a = 1, \ldots, N_c^2 - 1 \), \( \lambda^a \) is the spinor field and \( G^a_{\mu\nu}, P_{ab} \) are the usual Yang-Mills strength tensor and a covariant derivative, respectively. The dots in principle represent “gauge fixing, ghost terms and auxiliary fields” of those are not relevant for our current discussion. This theory is supersymmetric of an arbitrary \( N_c \). If a strongly interacting regime takes place, the spinor fields (gluon fields) do condensate into a composite field, called super-glueball, which will be identified as the inflaton \( \Phi \) in this case. The precise form of the inflaton field is prior given in [51] such that \( \Phi = -3\lambda^{a,\alpha}\lambda_{a}^{\alpha}/64\pi^2 N_c \). As examined in the previous two examples, the fundamental degrees of freedom are naturally
Figure 4: The contours show the resulting 68% and 95% confidence regions for the tensor-to-scalar ratio $r$ and the scalar spectral index $n_s$. Left: The red contours are for the Planck+WP+highL data combination, while the blue ones display the BICEP2 constraints on $r$ [59]. Right: The figure shows the results from Planck plus various ancillary sets of data [58]. The plots also show the analytical and numerical predictions given by the GB model.

non-minimally coupled to gravity, and features the description at the fundamental level. We start with the introduction of the non-minimal coupling term as follows:

$$\frac{N_c^2}{2} \xi \left( \frac{-3\lambda^{\alpha\beta} \lambda^{\alpha\beta}}{64\pi^2 N_c} \right)^{2/3} R \equiv \frac{N_c^2 \xi \Phi^{2/3} R}{2}.$$  \hspace{1cm} (46)

Again, the $\xi$ is the dimensionless coupling. We have just demonstrated how the introduction of the non-minimal coupling is motivated in a natural way with the underlying fundamental descriptions. According to this model, the inflaton is designed to be the gluino-ball state in the super–Yang–Mills theory. For this model, we have

$$f(\phi) = 4\alpha N_c^2 \ln^2(\phi/\Lambda).$$  \hspace{1cm} (47)

As it is always investigated in standard fashion, we take the scalar component part of the superglueball action and coupled it nonminimally to gravity. Focusing only on the modulus of the inflaton field and taking the next step in order to write the non-minimally coupled scalar component part of the superglueball action to gravity, the resulting action in the Jordan frame reads [54]

$$S_{sGB} = \int d^4x \sqrt{-g} \left[ \frac{F(\phi)}{2} R - \frac{9 N_c^2}{2\alpha} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 4\alpha N_c^2 \phi^4 (\ln[\phi/\Lambda])^2 \right],$$  \hspace{1cm} (48)

with $N_c$ a number of colors, and $\alpha$ a $N_c$-independent quantity. Here we call it, in brief, the sGB model. Using the similar approximations to those of the above consideration, the number of e-foldings for this inflation model in the large $\xi$ limit is approximately given by

$$N \approx \frac{3}{2} \left( \ln^2 \left( \frac{\phi}{\Lambda} \right) - \ln^2 \left( \frac{\phi_0}{\Lambda} \right) \right) + \mathcal{O}(1/\xi).$$  \hspace{1cm} (49)
Figure 5: The plot shows the relation between the amplitude of the power spectrum $A_s$ and the nonminimal coupling $\xi$ with $10^{-3} \lesssim \xi \lesssim 10^6$ for $N = 50, 60$ predicted by the sGB model. The horizontal bands represent the $1\sigma$ (yellow) and $2\sigma$ (purple) C.L. for $A_s$ obtained from Planck.

Regarding to the above relations between the number of e-foldings and $\varphi$, we can write $n_s$, $r$ and $|\zeta|^2$ in terms of $N$ for a large $\xi$ limit to yield

$$ n_s \simeq 1 - \frac{2}{N} + \mathcal{O}(1/\xi), \quad (50) $$
$$ r \simeq \frac{8}{N} + \mathcal{O}(1/\xi), \quad (51) $$
$$ |\zeta|^2 \simeq \frac{2\alpha N^2}{81N_c^2\pi^2\xi^3} + \mathcal{O}(1/\xi^3). \quad (52) $$

The consistency relation of the above relations reads

$$ r \simeq 4(1 - n_s). \quad (53) $$

We discover that the predictions of this model are fully consistent with BICEP2 constraints for $N \subseteq [50, 60]$. Moreover, the model can also be consistent with the Planck contours at $1\sigma$ CL. We discover that $A_s$ is well consistent with the Planck data up to $2\sigma$ CL for $N = 50$ and $N_c = 3$ with $9.2 \times 10^4 \lesssim \xi \lesssim 9.5 \times 10^4$, illustrated in Fig (5). This model provides $n_s \simeq 0.960$ and $r \simeq 0.16$ for $N = 50$ with $\xi \gg 1$, see Fig. (6). Here we can use the BICEP2 results to constrain $\Lambda_{sGB}$ since the data provides us the lower bound on $r$. According to the recent BICEP2 data, we roughly opt $r \simeq 0.12$ and use $N_c = 1(3)$ predicting $\Lambda_{sGB} > 10^{-3}(10^{-4})$ which corresponds to, at least, the GUT energy scale in this investigation, in order to satisfy the BICEP2 data at $1\sigma$ CL. We hope that the future observations will provide significant examination for this model.

### 4.4 Composite Inflation from Orientifold Theory

The authors of [51] examined the supersymmetric low-energy effective action to study inflation driven by the gauge dynamics of SU(N) gauge theories adding one Dirac fermion in
Figure 6: The contours show the resulting 68% and 95% confidence regions for the tensor-to-scalar ratio $r$, the scalar spectral index $n_s$ and $N_c=1$. Left: The red contours are for the Planck+WP+highL data combination, while the blue ones display the BICEP2 constraints on $r$ [59]. Right: The figure shows the results from Planck plus various ancillary sets of data [58]. The plots also show the analytical and numerical predictions given by the sGB model.

either the two-index antisymmetric or symmetric representation of the gauge group. Such theories are known as orientifold theories [52]. Here the gluino field of supersymmetric gluodynamics is replaced by two Weyl fields which can be formed as one Dirac spinor. The background framework of this model is to slightly deform an effective Lagrangian for the pure $N=1$ supersymmetric Yang-Mills theory derived in [53]. For investigating the inflationary scenario, we write the action by using the real part of the field $\phi$ in which the orientifold sector non-minimally coupled to gravity in the Jordan frame

$$S_{OI,J} \supset \int d^4x \sqrt{-g} \left[ -\frac{F(\phi)}{2} R - \frac{9F(N_c)}{\alpha} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 4\alpha F(N_c)\phi^4 \left( \ln(\phi/\Lambda)^2 - \gamma \right) \right] , \quad (54)$$

where $F(\phi) = 1 + N_c^2 \xi \phi^2$, $F(N_c) = N_c^2(1 + O(1/N_c))$, $\gamma = 1/9 N_c + O(1/N_c^2)$ and hereafter we will keep only leading order in $1/N_c$. However, we can impose the conformal transformation and then find the resulting action in the Einstein frame

$$S_{OI,E} \supset \int d^4x \sqrt{-g} \left[ -\frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{4\alpha F(N_c) M_P^4}{N_c^4} \frac{M_P^4}{\xi^2} \left( \ln \left( \phi/\Lambda \right)^2 - \gamma \right) \right] , \quad (55)$$

with $N_c$ being a number of colours. Note that at large-$N_c$ the theory features certain super Yang-Mills properties, i.e. $F(N_c) \to N_c^2$. With this limit, the transformed potential reduces to that of Section (4.3). With the large field limit, we can derive the following slow-roll parameter in terms of the number of $N_c$-foldings as

$$n_s = 1 - 6\epsilon + 2\eta \simeq 1 - \frac{2}{N_c} \left( 1 + \frac{9\gamma}{2N_c} \right) , \quad r = 16\epsilon \simeq \frac{8}{N_c} \left( 1 + \frac{3\gamma}{N_c} \right) . \quad (56)$$

Notice that for large $N_c$ the observables given above features the Super Yang-Mills inflation since $\gamma \to 0$. 

13
5 Conclusions

We revisit single-field (slow-roll) inflation in which the inflaton is a composite field stemming from various strongly interacting field theories. With regard to our framework, the cosmological “hierarchy problem” in the scalar sector of the inflation can effectively be solved, which not be solved by Higgs or another elementary scalar field paradigms. We constrain the number of e-foldings for composite models of inflation in order to obtain a successful inflation. We study a set of cosmological parameters, e.g., the primordial spectral index $n_s$ and tensor-to-scalar ratio $r$, and confront the predicted results with the joint Planck data, and with the recent BICEP2 data. Last but not the least, we anticipate that the composite paradigms and their verifiable consequences, e.g., reheating mechanism, can possibly receive considerable attention for inflationary model buildings.

Acknowledgments. P.C. is financially supported by the Thailand Research Fund (TRF) under the project of the “TRF Grant for New Researcher” with Grant No. TRG5780143.

References

[1] A. A. Starobinsky, JETP Lett. 30, 682 (1979) [Pisma Zh. Eksp. Teor. Fiz. 30, 719 (1979)].

[2] A. A. Starobinsky, Phys. Lett. B 91, 99 (1980).

[3] V. F. Mukhanov and G. V. Chibisov, JETP Lett. 33, 532 (1981) [Pisma Zh. Eksp. Teor. Fiz. 33, 549 (1981)].

[4] A. H. Guth, Phys. Rev. D 23, 347 (1981).

[5] A. D. Linde, Phys. Lett. B 108, 389 (1982).

[6] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).

[7] K. Nakayama and F. Takahashi, Phys. Lett. B 734, 96 (2014)

[8] J. L. Cook, L. M. Krauss, A. J. Long and S. Sabharwal, Phys. Rev. D 89, 103525 (2014)

[9] Y. Hamada, H. Kawai, K. y. Oda and S. C. Park, Phys. Rev. Lett. 112, 241301 (2014)

[10] C. Germani, Y. Watanabe and N. Wintergerst, arXiv:1403.5766 [hep-ph].

[11] I. Oda and T. Tomoyose, arXiv:1404.1538 [hep-ph].

[12] K. Harigaya, M. Ibe, K. Schmitz and T. T. Yanagida, Phys. Lett. B 733, 283 (2014)

[13] H. M. Lee, arXiv:1403.5602 [hep-ph].

[14] K. Harigaya and T. T. Yanagida, arXiv:1403.4729 [hep-ph].

[15] M. Czerny, T. Higaki and F. Takahashi, Physics Letters B (2014) 167-172

[16] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Lett. B 732, 380 (2014)
[17] S. Viaggiu, arXiv:1403.2868 [astro-ph.CO].
[18] A. Kehagias and A. Riotto, Phys. Rev. D 89, 101301 (2014)
[19] T. Kobayashi and O. Seto, Phys. Rev. D 89, 103524 (2014)
[20] M. P. Hertzberg, arXiv:1403.5253 [hep-th].
[21] N. Okada, V. N. ?eno?uz and Q. Shafi, arXiv:1403.6403 [hep-ph].
[22] S. Ferrara, A. Kehagias and A. Riotto, Fortsch. Phys. 62, 573 (2014)
[23] Y. Gong and Y. Gong, Phys. Lett. B 734, 41 (2014)
[24] K. Bamba, R. Myrzakulov, S. D. Odintsov and L. Sebastiani, arXiv:1403.6649 [hep-th].
[25] P. Di Bari, S. F. King, C. Luhn, A. Merle and A. Schmidt-May, arXiv:1404.0009 [hep-ph].
[26] E. Palti and T. Weigand, JHEP 1404, 155 (2014)
[27] K. S. Kumar, J. Marto, N. J. Nunes and P. V. Moniz, JCAP 1406, 064 (2014)
[28] T. Fujita, M. Kawasaki and S. Yokoyama, arXiv:1404.0951 [astro-ph.CO].
[29] Y. C. Chung and C. Lin, JCAP 1407, 020 (2014)
[30] S. Antusch and D. Nolde, JCAP 1405, 035 (2014)
[31] M. Bastero-Gil, A. Berera, R. O. Ramos and J. G. Rosa, arXiv:1404.4976 [astro-ph.CO].
[32] S. Kawai and N. Okada, arXiv:1404.1450 [hep-ph].
[33] M. . W. Hossain, R. Myrzakulov, M. Sami and E. N. Saridakis, Phys. Rev. D 89, 123513 (2014) [arXiv:1404.1445 [gr-qc]].
[34] K. Kannike, A. Racioppi and M. Raidal, JHEP 1406, 154 (2014)
[35] C. M. Ho and S. D. H. Hsu, JHEP 1407, 060 (2014)
[36] J. Joergensen, F. Sannino and O. Svendsen, Phys. Rev. D 90 (2014) 043509 [arXiv:1403.3289 [hep-ph]].
[37] M. J. Mortonson and U. Seljak, JCAP 1410 (2014) 10, 035 [arXiv:1405.5857 [astro-ph.CO]].
[38] R. Adam et al. [Planck Collaboration], arXiv:1409.5738 [astro-ph.CO].
[39] M. J. Mortonson and U. Seljak, JCAP 1410 (2014) 10, 035 [arXiv:1405.5857 [astro-ph.CO]].
[40] C. Cheng, Q. G. Huang and S. Wang, arXiv:1409.7025 [astro-ph.CO].
[41] F. Sannino and K. Tuominen, Phys. Rev. D 71, 051901 (2005) [hep-ph/0405209].
[42] D. K. Hong, S. D. H. Hsu and F. Sannino, Phys. Lett. B 597, 89 (2004) [hep-ph/0406200].

[43] D. D. Dietrich, F. Sannino and K. Tuominen, Phys. Rev. D 73, 037701 (2006) [hep-ph/0510217].

[44] D. D. Dietrich, F. Sannino and K. Tuominen, Phys. Rev. D 72, 055001 (2005) [hep-ph/0505059].

[45] P. Channuie, J. J. Joergensen and F. Sannino, JCAP 1105, 007 (2011) [arXiv:1102.2898 [hep-ph]].

[46] J. Garcia-Bellido, D. G. Figueroa and J. Rubio, Phys. Rev. D 79, 063531 (2009) [arXiv:0812.4624 [hep-ph]].

[47] C. T. Hill and D. S. Salopek, Annals Phys. 213, 21 (1992).

[48] F. Bezrukov, P. Channuie, J. J. Joergensen and F. Sannino, Phys. Rev. D 86 (2012) 063513 [arXiv:1112.4054 [hep-ph]].

[49] M. Artymowski, Y. Ma and X. Zhang, Phys. Rev. D 88 (2013) 10, 104010 [arXiv:1309.3045 [gr-qc]].

[50] G. Veneziano and S. Yankielowicz, Phys. Lett. B 113, 3 (1982)

[51] P. Channuie, J. J. Jorgensen and F. Sannino, Phys. Rev. D 86, 125035 (2012) [arXiv:1209.6362 [hep-ph]].

[52] F. Sannino and M. Shifman, Phys. Rev. D 69 (2004) 125004 [hep-th/0309252].

[53] G. Veneziano and S. Yankielowicz, Phys. Lett. B 113 (1982) 231.

[54] K. Karwan and P. Channuie, JCAP 1406 (2014) 045 [arXiv:1307.2880 [hep-ph]].

[55] P. Channuie, Int. J. Mod. Phys. D 23 (2014) 1450070 [arXiv:1312.7122 [gr-qc]].

[56] P. Channuie and K. Karwan, Phys. Rev. D 90 (2014) 047303 [arXiv:1404.5879 [astro-ph.CO]].

[57] O. Svendse, “Natural Models of Inflation,”

[58] P. A. R. Ade et al. [Planck Collaboration], “Planck 2013 results. XXII. Constraints on inflation,” arXiv:1303.5082 [astro-ph.CO].

[59] P. A. R. Ade et al. [BICEP2 Collaboration], “BICEP2 I: Detection Of B-mode Polarization at Degree Angular Scales,” arXiv:1403.3985 [astro-ph.CO].