Constructions of $f(R,G,T)$ Gravity from Some Expansions of the Universe

Ujjal Debnath*

Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah-711 103, India.

Here we propose the extended modified gravity theory named as $f(R,G,T)$ gravity where $R$ is the Ricci scalar, $G$ is the Gauss-Bonnet invariant and $T$ is the trace of the stress-energy tensor. We derive the gravitational field equations in $f(R,G,T)$ gravity by taking least action principle. Next we construct the $f(R,G,T)$ in terms of $R$, $G$ and $T$ in de Sitter as well as power law expansion. We also construct $f(R,G,T)$ if the expansion follows the finite time future singulary (big rip singularity). We investigate the energy conditions in this modified theory of gravity and examine the validities of all energy conditions. Finally, we analyze the stability of the constructed modified gravity.

Keywords: $f(R,G,T)$ gravity, energy conditions, stability.

I. INTRODUCTION

Recent observational data suggests that our universe is experiencing an accelerated expansion [1–4]. This acceleration is caused by some unknown matter known as dark energy which has the property that positive energy density ($\rho$) and negative pressure ($p$) satisfying $\rho + 3p < 0$. It is believed that our present Universe is made up of about 4% ordinary matter, about 74% dark energy and about 22% dark matter. A simple candidate for dark energy is the cosmological constant [5, 6]. Also several models play the roles of dark energy such as quintessence [7], phantom [8], quintom [9, 10], tachyon [11], k-essence [12], dilaton [13], hessence [14], DBI-essence [15, 16] etc. There are other alternative of dark energy is modified gravity theory [17, 18] which represents a classical generalization of general relativity. In theoretical model, the standard Einstein-Hilbert action is replaced by different functions of the Ricci scalar $R$ [19, 20] (i.e. $f(R)$ gravity) or Gauss-Bonnet invariant $G$ [21, 22] (i.e., $f(G)$ gravity). These modifications should consistently describe the early-time inflation and late-time acceleration, without introduce any other dark component and consistent with the solar system constraints [23]. A generalization of $f(R)$ modified gravity theory was proposed by Bertolami

* ujjaldebnath@gmail.com
et al \cite{24} by including an explicit coupling of arbitrary function of the Ricci scalar \( R \) with the matter Lagrangian \( L_m \). Nojiri et al \cite{25} studied the non-minimally coupling of \( f(R) \) and \( f(G) \) gravity theories with \( L_m \) and found that this coupling unifies the inflationary era with recent cosmic accelerated expansion. The geodesic deviation of \( f(G) \) gravity (for small curvature) is weaker than the non-minimal \( f(R) \) gravity.

Harko et al \cite{26} proposed another extensions of standard general relativity, the \( f(R, T) \) and \( f(R, T^\phi) \) modified theories of gravity, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar \( R \) and of the trace of the stress-energy tensor \( T \) and \( T^\phi \) is the stress-energy tensor of scalar field. The implications in \( f(R, T) \) gravity have been extensively studied in several works \cite{27-32}. Another extension of modified gravity is \( f(R, G) \) gravity \cite{33}. In \( f(R, G) \) gravity, the energy conditions, future finite time singularities and other cosmological implications have been extensively studied by several authors \cite{34-40}. Recently, Sharif et al \cite{41} have introduced another kind of extension of modified gravity theory like \( f(G, T) \) gravity theory. They have reconstructed the \( f(G, T) \) gravity through power law, de Sitter expansions of the Universe and also investigated the validities of all the energy conditions. Also Sharif et al \cite{42} have analyzed the stability of some reconstructed cosmological models in \( f(G, T) \) gravity. Shamir et al \cite{43} have studied the Noether symmetry approach of some cosmologically viable \( f(G, T) \) gravity models. Motivated by these works, here we propose another extension of modified theories of gravity like \( f(R, G, T) \) gravity and our main aim is to find the forms of the function \( f(R, G, T) \) from de Sitter, power law and future singularity models.

In section 2, we derive the gravitational field equations in \( f(R, G, T) \) gravity. Next we construct the \( f(R, G, T) \) in terms of \( R, G \) and \( T \) in de Sitter expansion in section 3. We construct the \( f(R, G, T) \) due to the power law expansion model in section 4. We also construct \( f(R, G, T) \) in section 5 if the expansion follows the finite time future singularity. In section 6, we study the energy conditions in modified theory of gravity. In section 7, we analyze the stability of \( f(R, G, T) \) gravity model. Finally we draw some concluding remarks in section 8.

II. GRAVITATIONAL FIELD EQUATIONS IN \( f(R, G, T) \) GRAVITY

Here we formulate the Einstein’s field equations for \( f(R, G, T) \) modified gravity theory. For this purpose, we consider the action for \( f(R, G, T) \) gravity theory in the form

\[
S = \frac{1}{2\kappa^2} \int f(R, G, T) \sqrt{-g} \, d^4x + \int L_m \sqrt{-g} \, d^4x
\]  

(1)
where $f(R, G, \mathcal{T})$ is the arbitrary function of Ricci scalar $R$, Gauss-Bonnet invariant $G$ and of the trace $\mathcal{T}$ of stress-energy tensor of the matter. Also $L_m$ is the matter Lagrangian, $g = |g_{\mu\nu}|$ and $\kappa^2 = 8\pi G$ (choosing $c = 1$). The stress-energy tensor of the matter is defined as

$$ T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^{\mu\nu}} \quad (2) $$

The trace of the stress-energy tensor is $\mathcal{T} = g^{\mu\nu} T_{\mu\nu}$. Here we assume that the matter Lagrangian $L_m$ depends only on the metric tensor $g_{\mu\nu}$, so we obtain

$$ T_{\mu\nu} = g_{\mu\nu} L_m - 2 \frac{\partial L_m}{\partial g^{\mu\nu}} \quad (3) $$

Now the variation of action (1), we obtain the following integral:

$$ \delta S = \frac{1}{2\kappa^2} \int \left[ f_R \delta R + f_G \delta G + f_T \delta \mathcal{T} + f(\sqrt{-g}) \delta(\sqrt{-g}) + 2\kappa^2 \frac{\delta L_m}{\delta g^{\mu\nu}} \right] \sqrt{-g} d^4 x \quad (4) $$

where $f_R = \frac{\partial f}{\partial R}$, $f_G = \frac{\partial f}{\partial G}$ and $f_T = \frac{\partial f}{\partial T}$. The Ricci scalar $R$ and Gauss-Bonnet invariant $G$ are as follows:

$$ R = g^{\mu\nu} R_{\mu\nu}, \quad G = R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\xi\eta} R^{\mu\nu\xi\eta} \quad (5) $$

The variations of $\sqrt{-g}$, $R$, $R_{\mu\nu}$, $G$, $\mathcal{T}$ are as follows:

$$ \delta(\sqrt{-g}) = -\frac{1}{2} \sqrt{-g} g_{\mu\nu} \delta g^{\mu\nu}. \quad (6) $$

$$ \delta R = (R_{\mu\nu} + g_{\mu\nu} \nabla^2 - \nabla_\mu \nabla_\nu) \delta g^{\mu\nu}. \quad (7) $$

$$ \delta R_{\mu\nu} = \nabla_\lambda \delta \Gamma^\lambda_{\mu\nu} - \nabla_\nu \delta \Gamma^\lambda_{\mu\lambda}, \quad (8) $$

$$ \delta \Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha} (\nabla_\nu \delta g_{\alpha\mu} + \nabla_\mu \delta g_{\alpha\nu} - \nabla_\alpha \delta g_{\mu\nu}). \quad (9) $$

$$ \delta G = 2 R \delta R - 4 \delta (R_{\mu\nu} R^{\mu\nu}) + \delta (R_{\mu\nu\xi\eta} R^{\mu\nu\xi\eta}), \quad (10) $$

$$ \delta \mathcal{T} = (T_{\mu\nu} + \Theta_{\mu\nu}) \delta g^{\mu\nu} \quad (11) $$

with

$$ \Theta_{\mu\nu} = g^{\alpha\beta} \frac{\partial T_{\alpha\beta}}{\partial g^{\mu\nu}} \quad (12) $$
Now putting $\delta S = 0$ in equation (4) and using (6) - (12), we obtain the field equations of $f(R, G, T)$ gravity as

\[
(R_{\mu\nu} + g_{\mu\nu}\nabla^2 - \nabla_\mu \nabla_\nu)f_R - \frac{1}{2} f g_{\mu\nu} + (2RR_{\mu\nu} - 4R^\xi_\mu R_{\xi\nu} - 4R_{\mu\xi\nu\eta} R^{\xi\eta} + 2R^\xi_{\mu\lambda} R_{\nu\xi\lambda}) f_G
\]

\[
+ (2g_{\mu\nu}\nabla^2 - 2R\nabla_\mu \nabla_\nu - 4g_{\mu\nu} R^{\xi\eta} \nabla_\xi \nabla_\eta - 4R_{\mu\nu}\nabla^2 + 4R^\xi_\mu \nabla_\nu \nabla_\xi + 4R^\xi_\nu \nabla_\mu \nabla_\xi + 4R_{\mu\xi\nu\eta} \nabla^{\xi\eta}) f_G
\]

\[
= \kappa^2 T_{\mu\nu} - (T_{\mu\nu} + \Theta_{\mu\nu}) f_T
\]

where $\nabla^2 = \nabla_\mu \nabla^\mu$ is the d’Alembert operator. If we put $f(R, G, T) = f(R, T)$ ($G$ independent), we can recover the field equations in $f(R, T)$ gravity which was proposed in Ref [26]. If we put $f(R, G, T) = f(G, T)$ ($R$ independent), we can recover the field equations in $f(G, T)$ gravity which was proposed in Ref [41] and if we put $f(R, G, T) = f(R, G)$ ($T$ independent), we can recover the field equations in $f(R, G)$ gravity [33]. Taking the trace of the above field equation (13) (multiplying both sides by $g^{\mu\nu}$) we have

\[
(R + 3\nabla^2) f_R - 2f - (2G - 2R\nabla^2 + 4R^{\mu\nu} \nabla_\mu \nabla_\nu)f_G = \kappa^2 T - (T + \Theta) f_T
\]

where $\Theta = \Theta_{\mu\nu} g^{\mu\nu}$. Taking covariant divergence of equation (13), we obtain [26, 41]

\[
\nabla^\mu T_{\mu\nu} = \frac{f_T}{\kappa^2 - f_T} \left[ (T_{\mu\nu} + \Theta_{\mu\nu}) \nabla^\mu \ln f_T + \nabla^\mu \Theta_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \nabla^\mu T \right]
\]

We see that the above expression is independent of $f_R$ and $f_G$. Also we may obtain [26, 41]

\[
\Theta_{\mu\nu} = -2T_{\mu\nu} + g_{\mu\nu} L_m - 2g^{\alpha\beta} \frac{\partial^2 L_m}{\partial g^{\mu\nu} \partial g^{\alpha\beta}}
\]

If $L_m$ is known then we can find $\Theta_{\mu\nu}$. The energy momentum tensor for perfect fluid is assumed as

\[
T_{\mu\nu} = (\rho + p) u_\mu u_\nu + pg_{\mu\nu}
\]

where $\rho$ and $p$ are respectively energy density and pressure of perfect fluid. The four velocity $u_\mu$ satisfies $u_\mu u^\mu = -1$ and $u^\mu \nabla_\nu u_\mu = 0$. Now assume that the matter Lagrangian is $L_m = p$. So the equation (16) reduces to

\[
\Theta_{\mu\nu} = -2T_{\mu\nu} + pg_{\mu\nu}
\]

Here we assume the flat FRW model of the universe described by the line element

\[
ds^2 = -dt^2 + a^2(t) \left[ dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]
\]
where $a(t)$ is the scale factor. From equation (5), we can obtain

$$R = 6(\dot{H} + 2H^2), \quad G = 24H^2(\dot{H} + H^2)$$

(20)

where $H = \dot{a}/a$ is the Hubble parameter and dot means the derivative w.r.t. cosmic time $t$. For the above metric, we obtain the trace $T = 3p - \rho$ and $\Theta = 2(\rho - p)$. So $T + \Theta = 2(\rho + p)$. From equation (15), we obtain the non-conservation equation

$$\dot{\rho} + 3H(\rho + p) = \left(\frac{1}{2} \dot{T} - \dot{p}\right) f_T - (\rho + p) \dot{f}_T$$

(21)

Now, the standard conservation equation $\nabla^\mu T_{\mu\nu} = 0$ for perfect fluid gives

$$\dot{\rho} + 3H(\rho + p) = 0$$

(22)

So from equation (21), we obtain

$$(\dot{\rho} - \dot{p}) f_T + 2(\rho + p) \dot{f}_T = 0$$

(23)

From equation (13) we obtain the field equations for $f(R, G, T)$ gravity as

$$3H^2 = \frac{1}{f_R} \left[ \kappa^2 \rho + (\rho + p)f_T + \frac{1}{2}(R f_R - f) - 3H \dot{f}_R + 12H^2(\dot{H} + H^2)f_G - 12H^3 \dot{f}_G \right]$$

(24)

and

$$(2\dot{H} + 3H^2) = -\frac{1}{f_R} \left[ \kappa^2 p - \frac{1}{2}(R f_R - f) + 2H \dot{f}_R + \ddot{f}_R - 12H^2(\dot{H} + H^2)f_G + 8H(\dot{H} + H^2)f_G + 4H^2 \ddot{f}_G \right]$$

(25)

The above two field equations can be written in the standard Einstein’s field equations as

$$3H^2 = \kappa^2 \rho_{\text{eff}} \quad \text{and} \quad (2\dot{H} + 3H^2) = -\kappa^2 p_{\text{eff}}$$

(26)

where

$$\rho_{\text{eff}} = \frac{1}{\kappa^2 f_R} \left[ \kappa^2 \rho + (\rho + p)f_T + \frac{1}{2}(R f_R - f) - 3H \dot{f}_R + 12H^2(\dot{H} + H^2)f_G - 12H^3 \dot{f}_G \right]$$

(27)

and

$$p_{\text{eff}} = \frac{1}{\kappa^2 f_R} \left[ \kappa^2 p - \frac{1}{2}(R f_R - f) + 2H \dot{f}_R + \ddot{f}_R - 12H^2(\dot{H} + H^2)f_G + 8H(\dot{H} + H^2)f_G + 4H^2 \ddot{f}_G \right]$$

(28)

Now the first and 2nd derivatives of $f_R$ and $f_G$ w.r.t. $t$ are given below:

$$\dot{f}_R = \dot{R} f_{RR} + \dot{G} f_{RG} + \dot{T} f_{RT}, \quad \dot{f}_G = \dot{R} f_{RG} + \dot{G} f_{GG} + \dot{T} f_{GT},$$

(29)
\[ f_R = \ddot{R}f_{RR} + \ddot{G}f_{RG} + \ddot{T}f_{RT} + \dddot{R}f_{RRR} + 2\ddot{R}\dot{G}f_{RGG} + 2\dot{R}\dddot{T}f_{RRT} + \dddot{G}f_{RGG} + 2\dddot{G}f_{RGT} + \dddot{T}^2f_{RTT}, \quad (30) \]

\[ f_G = \ddot{R}f_{RG} + \ddot{G}f_{GG} + \dddot{T}f_{GT} + \dddot{R}f_{RRG} + 2\ddot{R}\dot{G}f_{RGG} + 2\dot{R}\dddot{T}f_{RGT} + \dddot{G}f_{GGG} + 2\dddot{G}f_{GGT} + \dddot{T}^2f_{GTT} \quad (31) \]

Using (29)-(31), the field equations (24) and (25) reduce to the following equations

\[
\kappa^2 \rho - \frac{1}{2} f + \frac{1}{2} (R - 6H^2) f_R + 12H^2 (\dot{H} + H^2) f_G + (\rho + p) f_T - 3H\dot{R}f_{RR} \\
- 3H(4H^2 \dddot{R} + \dddot{G}) f_{RG} - 3H\dddot{T}f_{RT} - 12H^2 \dddot{G}f_{GG} - 12H^3 \dddot{T}f_{GT} = 0, \quad (32) \]

and

\[
\kappa^2 p + \frac{1}{2} f + \frac{1}{2} (4\dot{H} + 6H^2 - R) f_R - 12H^2 (\dot{H} + H^2) f_G + (2\dot{R} + \dddot{R}) f_{RR} \\
+ [8H(\dot{H} + H^2) \dddot{R} + 4H^2 \dddot{R} + 2\dot{H}\dddot{G} + \dddot{G}] f_{RG} + (2\dddot{T} + \dddot{T}) f_{RT} + [8H(\dddot{H} + H^2) \dddot{G} + 4H^2 \dddot{G}] f_{GG} \\
[8H(\dddot{H} + H^2) \dddot{T} + 4H^2 \dddot{T}] f_{GT} + \dddot{R}^2 f_{RRR} + (2\dddot{R}\dddot{G} + 4H^2 \dddot{R}) f_{RGG} + 2\dddot{T} f_{RRT} + (\dddot{G}^2 + 8H^2 \dddot{R}\dddot{G}) f_{RGG} \\
+ (2\dddot{G}\dddot{T} + 8H^2 \dddot{R}\dddot{T}) f_{RGT} + \dddot{T}^2 f_{RTT} + 4H^2 \dddot{G} f_{GGG} + 8H^2 \dddot{G}\dddot{T} f_{GGT} + 4H^2 \dddot{T}^2 f_{GTT} = 0 \quad (33) \]

In the next sections, we’ll construct the function \( f(R, G, T) \) in terms of \( R, G \) and \( T \) for de Sitter, power law and future singularity expansion models.

### III. CONSTRUCTION OF \( f(R, G, T) \) IN DE SITTER MODEL

Now we want to construct the function \( f(R, G, T) \) in terms of \( R, G, T \) for de-Sitter universe. For de Sitter model of the universe, the scale factor can be written as [41]

\[ a(t) = a_0 e^{H_0 t} \quad (34) \]

where \( a_0 \) and \( H_0 \) are positive constants. For this model, we have

\[ H = H_0, \quad R = 12H_0^2, \quad G = 24H_0^4, \quad \dot{R} = \dddot{G} = \dddot{R} = \dddot{T} = 0, \quad (35) \]

Also we assume the fluid of the Universe obeys barotropic equation of state \( p = w\rho \), where \( w \) is constant. So from the conservation equation [22], we obtain the energy density \( \rho = \rho_0 a^{-3(1+w)} \), where \( \rho_0 \) is positive constant. Now we obtain

\[ \rho = \frac{T}{3w - 1}, \quad \dot{T} = -3H_0(1 + w)T, \quad T = 9H_0^2(1 + w)^2T \quad (36) \]
Using these values, the field equations (32) and (33) reduce to
\[
\frac{\kappa^2 T}{3w - 1} \left( -\frac{1}{2} f + 3H_0^2 f_R + \frac{1 + w}{3w - 1} T f_T + 12H_0^2 f_G + 9H_0^2 (1 + w) T f_{RT} + 36H_0^4 (1 + w) T f_{GT} \right) = 0
\] (37)
and
\[
\frac{\kappa^2 w T}{3w - 1} + \frac{1}{2} f - 3H_0^2 f_R - 12H_0^2 f_G + 3H_0^2 (1 + w) (1 + 3w) T f_{RT} + 12H_0^4 (1 + w) (1 + 3w) T f_{GT} + 9H_0^2 (1 + w)^2 T^2 f_{RTT} + 36H_0^4 (1 + w)^2 T^2 f_{GTT} = 0
\] (38)
The solution of the above equations is obtained as
\[
f(R, G, T) = c_1 e^{a_1 R + a_2 G} T^{b_1} + c_2 e^{a_3 R} T^{b_2} + c_3 e^{a_4 G} T^{b_3} + c_4 e^{a_5 R} + c_5 e^{a_6 G} + c_6 T^{b_4} + c_7 T
\] (39)
where \(c_i (i = 1, ..., 7)\), \(a_i (i = 1, ..., 6)\) and \(b_i (i = 1, ..., 4)\) are constants satisfying (i) \(a_1 + 4H_0^2 a_2 = -\frac{1}{42H_0^2}\), \(a_3 = -\frac{1}{42H_0^2}\), \(a_4 = -\frac{1}{108H_0^4}\), \(a_5 = \frac{1}{24H_0^4}\), \(a_6 = \frac{1}{24H_0^4}\), \(b_1 = b_2 = b_3 = 1\), \(c_0 = 0\), \(c_7 = -\kappa^2\) for \(w = 1\); (ii) \(a_1 + 4H_0^2 a_2 = \frac{1}{6H_0^2}\), \(a_3 = \frac{1}{6H_0^2}\), \(a_4 = \frac{1}{12H_0^2}\), \(a_5 = \frac{1}{24H_0^2}\), \(a_6 = \frac{1}{24H_0^2}\), \(c_0 = 0\), \(c_7 = -\frac{\kappa^2}{2}\) for \(w = -1\).

So the above solution can be written as
\[
f(R, G, T) = \begin{cases} 
e^{-\frac{R}{42H_0^2}} \left( c_1 e^{-4a_2 H_0^2 R + a_2 G} + c_2 \right) T + c_3 T e^{-\frac{G}{168H_0^2}} + c_4 e^{\frac{R}{6H_0^2}} + c_5 e^{\frac{G}{24H_0^2}} - \kappa T & \text{for } w = 1 \\
e^{\frac{R}{6H_0^2}} \left( c_1 e^{-4a_2 H_0^2 R + a_2 G} T^{b_1} + c_2 T^{b_2} + c_3 T^{b_3} + c_4 \right) + e^{\frac{G}{24H_0^2}} (c_3 T^{b_3} + c_5) - \frac{\kappa^2 T}{2} & \text{for } w = -1 \end{cases}
\] (40)
We see that \(f(R, G, T)\) is the combinations of exponential and power forms of \(R, G\) and \(T\). Now we draw the function \(f(R, G, T)\) against \(t\) in figure 1. From the figure we observe that \(f(R, G, T)\) increases as \(t\) increases. From the figure we observe that \(f(R, G, T)\) sharply increases as \(t\) increases (upto \(\approx 2\)) and then it takes the value 5.4365 which is nearly parallel to \(t\) axis (i.e., slope of the curve \(\approx 0\)) throughout the evolution.

**IV. CONSTRUCTION OF \(f(R, G, T)\) IN POWER LAW MODEL**

Now we consider the universe expands in the power law form of the scale factor \(41\):
\[
a(t) = a_0 t^n
\] (41)
where \(a_0\) and \(n\) are positive constants. It should be noted that for acceleration phase of the Universe \(\ddot{a} > 0\), we must have \(n > 1\). So from (41) we obtain
\[
H = \frac{n}{t}, \quad \dot{H} = -\frac{n}{t^2}, \quad R = \frac{6n(2n - 1)}{t^2}, \quad \frac{\dot{R}}{R} = -\frac{2}{t}, \quad \frac{\ddot{R}}{\dot{R}} = \frac{6}{t^2}, \quad G = \frac{24n^3(n - 1)}{t^4}, \quad \frac{\dot{G}}{G} = -\frac{4}{t}, \quad \frac{\ddot{G}}{G} = \frac{20}{t^2}
\] (42)
Using equations (42) and (43), the field equation (32) simplifies to the form

\[ c = \frac{T}{3w - 1}, \quad \dot{T} = \frac{3n(1 + w)}{t}, \quad \ddot{T} = \frac{3n(1 + w)[3n(1 + w) + 1]}{t^2} \] (43)

Using equations (42) and (43), the field equation (32) simplifies to the form

\[ \frac{\kappa^2 T}{3w - 1} - \frac{1}{2} f + \frac{n - 1}{2(2n - 1)} Rf_R + \frac{1}{2} Gf_G + \frac{1 + w}{3w - 1} Tf_T + \frac{1}{2n - 1} R^2 f_{RR} \]

\[ + \frac{4n - 3}{(n - 1)(2n - 1)} RGf_{RG} + \frac{3n(1 + w)}{2(2n - 1)} RTf_{RT} + \frac{2}{n - 1} G^2 f_{GG} + \frac{3n(1 + w)}{2(n - 1)} GTf_{GT} = 0 \] (44)

From the above equation (44), we get the solution as in the following form:

\[ f(R, G, T) = c_1 R^{a_1} G^{a_2} T^{a_3} + c_2 R^{a_4} G^{a_5} + c_3 R^{a_6} T^{a_7} + c_4 G^{a_8} T^{a_9} + c_5 R^{b_1} + c_6 G^{b_2} + c_7 T^{b_3} + c_8 + c_9 \] (45)

where \( c_i \) (\( i = 1, \ldots, 9 \)), \( a_i \) (\( i = 1, \ldots, 9 \)), \( b_i \) (\( i = 1, 2, 3 \)) are constants satisfying

\[ c_8 = \frac{2\kappa^2}{w - 3}, \quad b_3 = \frac{3w - 1}{2(1 + w)}, \quad b_2 = -\frac{n - 1}{4}, \quad 2b_1^2 + (n - 3)b_1 = 2n - 1, \] (46)

\[ a_9 = \frac{(3w - 1)(1 - a_8)(n - 1 + 4a_8)}{(1 + w)[2(n - 1) + 3n(3w - 1)a_8]}, \quad a_7 = \frac{(3w - 1)[2n - 1 - a_6(n - 1 - 2a_6)]}{(1 + w)[2(2n - 1) + 3n(3w - 1)a_6]}, \] (47)

\[ [4(2n - 1)a_5 + 2(4n - 3)a_4 + (n - 5)(2n - 1)]a_5 = (n - 1)[2n - 1 - a_4(n - 3 + 2a_4)], \] (48)

\[ a_3 = \frac{(3w - 1)[(n + 1)(2n + 1)(1 - a_2) - a_1(n + 3 - 2a_1)] + 2(4n + 3)a_1a_2 + 4(2n + 1)a_2(a_2 - 1)}{(1 + w)[2(n + 1)(2n + 1) + 3n(3w - 1)][(n + 1)a_1 + (2n + 1)a_2]} \] (49)

We see that \( f(R, G, T) \) is the power forms of \( R, G \) and \( T \). Now we draw the function \( f(R, G, T) \) against \( t \) in figure 2. From the figure we observe that \( f(R, G, T) \) decreases as \( t \) increases. From the figure we observe that \( f(R, G, T) \) sharply decreases as \( t \) increases (upto \( \approx 2 \)) and then it is nearly parallel to \( t \) axis (i.e., slope of the curve \( \approx 0 \)) throughout the evolution.
V. CONSTRUCTION OF $f(R, G, T)$ IN FUTURE SINGULARITY MODEL

If there is a finite time future singularity occurs at $t = t_s$ in the evolution of the Universe, we may consider the universe expands as in the following form of the scale factor [41]

$$a(t) = \frac{a_0}{(t_s - t)^n} \tag{50}$$

where $a_0$ and $n$ are positive constants and $t < t_s$. Now in this case we obtain $H = n/(t_s - t)$. Since $H$ becomes singular in the limit $t \rightarrow t_s$ so $t_s$ is the future time when a singularity appears. This singularity is known as type I or big rip singularity [45] because $a(t) \rightarrow \infty$, $\rho \rightarrow \infty$ and $|p| \rightarrow \infty$ in the limit $t \rightarrow t_s$. Similar to the power law form, the field equation (32) reduces to

$$\frac{\kappa^2 T}{3w - 1} - \frac{1}{2} f + \frac{n + 1}{2(2n + 1)} Rf_R + \frac{1}{2} Gf_G + \frac{1 + w}{3w - 1} T f_T - \frac{1}{2n + 1} R^2 f_{RR}$$

$$- \frac{4n + 3}{(n + 1)(2n + 1)} R G f_{RG} + \frac{3n(1 + w)}{2(2n + 1)} R T f_{RT} - \frac{2}{n + 1} G^2 f_{GG} + \frac{3n(1 + w)}{2(n + 1)} G T f_{GT} = 0 \tag{51}$$
From the above equation, we get the solution in the following form:

$$f(R, G, \mathcal{T}) = d_1 R^{x_1} G^{x_2} \mathcal{T}^{x_3} + d_2 R^{x_4} G^{x_5} + d_3 R^{x_6} \mathcal{T}^{x_7} + d_4 G^{x_8} \mathcal{T}^{x_9} + d_5 R^{y_1} + d_6 G^{y_2} + d_7 \mathcal{T}^{y_3} + d_8 \mathcal{T} + d_9 G$$  \( (52) \)

where \( d_i \) (i = 1, ..., 9), \( x_i \) (i = 1, ..., 9), \( y_i \) (i = 1, 2, 3) are constants satisfying

$$d_8 = \frac{2n^2}{w - 3}, \quad y_3 = \frac{3w - 1}{2(1 + w)}, \quad y_2 = \frac{n + 1}{4}, \quad 2y_1^2 - (n + 3)y_1 + (2n + 1) = 0,$$  \( (53) \)

$$x_9 = \frac{(3w - 1)(1 - x_8)(n + 1 - 4x_8)}{(1 + w)(2n + 1) + 3n(3w - 1)x_8}, \quad x_7 = \frac{(3w - 1)[2n + 1 - x_6(n + 1 - 2x_6)]}{(1 + w)[2(2n + 1) + 3n(3w - 1)x_6]},$$  \( (54) \)

$$x_5 = (n + 1)[x_4(n + 3 - 2x_4) - 2n + 1],$$  \( (55) \)

$$x_3 = \frac{(3w - 1)\{(2n - 1)(1 - x_2) - x_1(n - 3 + 2x_1)\} - 2(4n - 3)x_1x_2 - 4(2n - 1)x_2(x_2 - 1)}{(1 + w)[2(n - 1)[2n + 1 + 3n(3w - 1)]\{(n - 1)x_1 + (2n - 1)x_2\}]}$$  \( (56) \)

When the universe expands with big rip singularity, the solution \( f(R, G, \mathcal{T}) \) is in the combinations of power forms of \( R, G \) and \( \mathcal{T} \). We see that \( f(R, G, \mathcal{T}) \) is the power forms of \( R, G \) and \( \mathcal{T} \). Now we draw the function \( f(R, G, \mathcal{T}) \) against \( t \) in figure 3. From the figure we observe that \( f(R, G, \mathcal{T}) \) nearly parallel to \( t \) axis (i.e., slope of the curve \( \approx 0 \)) up to a certain period of time \( t \approx 5 \) then sharply increases as \( t \) increases near future singularity (\( t \approx 6 \)).

VI. ENERGY CONDITIONS

Here we study the energy conditions for \( f(R, G, \mathcal{T}) \) gravity in FRW Universe. The concept of energy conditions came from the Raychaudhuri equation \([40, 41]\)

$$\frac{d\theta}{d\tau} = -\frac{1}{2} \theta^2 - \sigma_{\mu\nu} \sigma^{\mu\nu} + \omega_{\mu\nu} \omega^{\mu\nu} - R_{\mu\nu} k^{\mu} k^{\nu}$$  \( (57) \)

where \( \theta, \sigma_{\mu\nu} \) and \( \omega_{\mu\nu} \) are expansion scalar, shear tensor and rotation tensor respectively associated to congruence defined by the null vector field \( k^{\mu} \). From the Raychaudhuri equation, we observe that \( \sigma^2 = \sigma_{\mu\nu} \sigma^{\mu\nu} \geq 0 \) and for any hypersurface orthogonal congruences, \( \omega_{\mu\nu} = 0 \) and for attractive gravity, \( \frac{d\theta}{d\tau} \leq 0 \), so above equation reduces to \( R_{\mu\nu} k^{\mu} k^{\nu} \geq 0 \). In Einstein’s gravity, the above condition becomes \( T_{\mu\nu} k^{\mu} k^{\nu} \geq 0 \), which is the null energy condition (NEC). For timelike vector field \( v^{\mu} \), the above Raychaudhuri equation can be written as

$$\frac{d\theta}{d\tau} = -\frac{1}{3} \theta^2 - \sigma_{\mu\nu} \sigma^{\mu\nu} + \omega_{\mu\nu} \omega^{\mu\nu} - R_{\mu\nu} v^{\mu} v^{\nu}$$  \( (58) \)
From this equation we get $R_{\mu \nu}v^\mu v^\nu \geq 0$ and hence $T_{\mu \nu}v^\mu v^\nu \geq 0$, which is the weak energy condition (WEC). Similarly the strong energy condition (SEC) is $(T_{\mu \nu} - \frac{1}{2} T g_{\mu \nu})v^\mu v^\nu \geq 0$ and the dominant energy condition (DEC) is $T_{\mu \nu}v^\mu v^\nu \geq 0$ and $T_{\mu \nu}v^\nu$ is not space-like imply locally measured energy density to be always positive and the energy flux is time-like or null. Now we write the NEC, WEC, SEC and DEC for modified $f(R,G,T)$ gravity theory in general values of $f(R,G,T)$.

- Null energy condition (NEC):

$$\rho_{eff} + p_{eff} = \frac{1}{\kappa^2 f_R} \left[ \kappa^2(\rho + p) + (\rho + p)f_T - H \dot{f}_R + \ddot{f}_R + 4H(2\dot{H} - H^2)\dot{f}_G + 4H^2 \ddot{f}_G \right] \geq 0,$$  

(59)
Figs. 8 and 9 show the plots of $\rho_{\text{eff}} + p_{\text{eff}}$ and $\rho_{\text{eff}}$ against time $t$ for power law expansion model respectively.

Figs. 10 and 11 show the plots of $\rho_{\text{eff}} + 3p_{\text{eff}}$ and $\rho_{\text{eff}} - p_{\text{eff}}$ against time $t$ for power law expansion model respectively.

- **Weak energy condition (WEC):**
  \[
  \rho_{\text{eff}} = \frac{1}{\kappa^2 f_R} \left[ \kappa^2 \rho + (\rho + p) f_T + \frac{1}{2} (R f_R - f) - 3 H f_R + 12 H^2 (\dot{H} + H^2) f_G - 12 H^3 \dot{f}_G \right] \geq 0, \quad (60)
  \]
  \[
  \rho_{\text{eff}} + p_{\text{eff}} = \frac{1}{\kappa^2 f_R} \left[ \kappa^2 (\rho + p) + (\rho + p) f_T - H \dot{f}_R + \ddot{f}_R + 4 H (2 \dot{H} + H^2) \dot{f}_G + 4 H^2 \ddot{f}_G \right] \geq 0, \quad (61)
  \]

- **Strong energy condition (SEC):**
  \[
  \rho_{\text{eff}} + 3p_{\text{eff}} = \frac{1}{\kappa^2 f_R} \left[ \kappa^2 (\rho + 3p) + (\rho + p) f_T - (R f_R - f) + 3 H f_R + 3 \dot{f}_R \\
  - 24 H^2 (\dot{H} + H^2) f_G + 12 H (2 \dot{H} + H^2) \dot{f}_G + 12 H^3 \ddot{f}_G \right] \geq 0, \quad (62)
  \]
Figs. 12 and 13 show the plots of \( \rho_{\text{eff}} + p_{\text{eff}} \) and \( \rho_{\text{eff}} \) against time \( t \) for future singularity model respectively.

\[
\rho_{\text{eff}} + p_{\text{eff}} = \frac{1}{\kappa^2 f_R} \left[ \kappa^2 (\rho + p) + (\rho + p) f_T - H \dot{f}_R + \ddot{f}_R + 4H (2 \dot{H} - H^2) \dot{f}_G + 4H^2 \ddot{f}_G \right] \geq 0 \quad (63)
\]

- Dominant energy condition (DEC):

\[
\rho_{\text{eff}} = \frac{1}{\kappa^2 f_R} \left[ \kappa^2 \rho + (\rho + p) f_T + \frac{1}{2} (R f_R - f) - 3H \dot{f}_R + 12H^2 (\dot{H} + H^2) f_G - 12H^3 \dot{f}_G \right] \geq 0, \quad (64)
\]

\[
\rho_{\text{eff}} + p_{\text{eff}} = \frac{1}{\kappa^2 f_R} \left[ \kappa^2 (\rho + p) + (\rho + p) f_T - H \dot{f}_R + \ddot{f}_R + 4H (2 \dot{H} - H^2) \dot{f}_G + 4H^2 \ddot{f}_G \right] \geq 0, \quad (65)
\]

\[
\rho_{\text{eff}} - p_{\text{eff}} = \frac{1}{\kappa^2 f_R} \left[ \kappa^2 (\rho - p) + (\rho + p) f_T + (R f_R - f) - 5H \dot{f}_R - \ddot{f}_R \right]
\]
\[ +24H^2(\dot{H} + H^2)T_G - 4H(2\dot{H} + 5H^2)\ddot{T}_G - 4H^2\dddot{T}_G \geq 0 \] (66)

Since the above expressions for NEC, WEC, SEC and DEC are complicated form which contain partial derivatives of \( f(R, G, T) \) w.r.t. \( R, G \) and \( T \) as well as time derivatives. So to examine the validities of NEC, WEC, SEC and DEC, we need graphical representations. For de Sitter model, we plot \( \rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}} \) and \( \rho_{\text{eff}} - p_{\text{eff}} \) against \( t \) in figures 4, 5, 6 and 7 respectively. We have taken the parameters \( H_0 = 72, a_0 = 1, \rho_0 = 1, k = 1, a_2 = 1, b_1 = 2, b_2 = 3, b_3 = 1, b_4 = 2, c_1 = 2, c_2 = 1, c_3 = 4, c_5 = 3, c_6 = 0, \) 0. For these choices of the parameters, from figures we observe that \( \rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}} \) and \( \rho_{\text{eff}} - p_{\text{eff}} \) are all positive during evolution of the Universe. So NEC, WEC, SEC and DEC are satisfied for our constructed \( f(R, G, T) \) gravity for de Sitter expansion model. Also, we see that in this model, \( \rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}} \) and \( \rho_{\text{eff}} - p_{\text{eff}} \) are all decreasing as time increases. Initially, these quantities have sharp decrease behaviour (about \( t \approx 2 \)) and after that these are nearly parallel to \( t \) axis and tending to zero but keeps positive sign.

For power law model, we plot \( \rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}} \) and \( \rho_{\text{eff}} - p_{\text{eff}} \) against \( t \) in figures 8, 9, 10 and 11 respectively. We have taken the parameters \( n = 3, a_0 = 1, \rho_0 = 1, k = 1, a_2 = 1, a_4 = 3, a_5 = 2, a_6 = 3, a_8 = 2, b_1 = 2, c_1 = 1, c_2 = 3, c_3 = 2, c_5 = 4, c_7 = 2, c_9 = 2, \) 0. For these choices of the parameters, from figures we observe that \( \rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}} \) and \( \rho_{\text{eff}} - p_{\text{eff}} \) are all positive during evolution of the Universe. So NEC, WEC, SEC and DEC are satisfied for our constructed \( f(R, G, T) \) gravity for power law expansion model. Also, we see that in this model, \( \rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}} \) and \( \rho_{\text{eff}} - p_{\text{eff}} \) are all decreasing from high values to nearly zero as time increases.

For future singularity model, we plot \( \rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}} \) and \( \rho_{\text{eff}} - p_{\text{eff}} \) against \( t \) in figures 12, 13, 14 and 15 respectively. We have taken the parameters \( n = 2, a_0 = 1, \rho_0 = 1, k = 1, t_1 = 6, x_1 = 1, x_2 = 1, x_4 = 3, x_5 = 2, x_6 = 3, x_8 = 2, y_1 = 2, d_1 = 1, d_2 = 3, d_3 = 2, d_5 = 4, d_5 = 1, d_6 = 1, d_7 = 3, d_9 = 2. \) For these choices of the parameters, from figures we observe that \( \rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}} \) and \( \rho_{\text{eff}} - p_{\text{eff}} \) are all positive during evolution of the Universe. So NEC, WEC, SEC and DEC are satisfied for our constructed \( f(R, G, T) \) gravity for future singularity model. Also, we see that in this model, \( \rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}} \) and \( \rho_{\text{eff}} - p_{\text{eff}} \) are all increasing from lower values to higher values as time increases. At future singularity, all the quantities are blows up.
VII. STABILITY ANALYSIS

In this section, we test the viability of $f(R, G, \mathcal{T})$ gravity model by exploring its stability against perturbation. The square speed of sound is the key quantity for investigation of the stability. So for stability analysis of the model against small perturbation we derive the squared speed of sound defined by

$$v_s^2 = \frac{dp_{\text{eff}}}{d\rho_{\text{eff}}} = \frac{\dot{p}_{\text{eff}}}{\dot{\rho}_{\text{eff}}}$$

(67)

The sign of $v_s^2$ plays a crucial role in determining the classical stability or instability of the background evolution. If $0 < v_s^2 < 1$, the model is classically stable while $v_s^2 < 0$ or $v_s^2 > 1$ represent a classically unstable model against the perturbation respectively. The square speed of light $v_s^2$ vs time $t$ for de Sitter, power law and future singularity models have been drawn in figures 16, 17 and 18 respectively. From these figures, we observe that $v_s^2$ for all the models are lying in $(0, 1)$. So we may conclude that $f(R, G, \mathcal{T})$ gravity models are classically stable for de Sitter, power law and future singularity expansions.

VIII. DISCUSSIONS AND CONCLUDING REMARKS

Here we have introduced the extended modified gravity theory named as $f(R, G, \mathcal{T})$ gravity after the modifications of gravities like $f(R, \mathcal{T})$, $F(R, G)$, $f(G, \mathcal{T})$ gravities where $R$ is the Ricci scalar, $G$ is the Gauss-Bonnet invariant and $\mathcal{T}$ is the trace of the stress-energy tensor. We have obtained the gravitational lagrangian by adding $f(R, G, \mathcal{T})$ with matter Lagrangian $L_m$ in the Einstein-Hilbert action. We have derived the gravitational field equations for $f(R, G, \mathcal{T})$ gravity by least action principle.
If we put $f(R, G, T) = f(R, T)$ ($G$ independent), we can recover the field equations in $f(R, T)$ gravity which was proposed in Ref [26]. If we put $f(R, G, T) = f(G, T)$ ($R$ independent), we can recover the field equations in $f(G, T)$ gravity which was proposed in Ref [41] and if we put $f(R, G, T) = f(R, G)$ ($T$ independent), we can recover the field equations in $f(R, G)$ gravity [33]. Next we have constructed the $f(R, G, T)$ in terms of $R, G$ and $T$ in de Sitter as well as power law expansion of the universe. We have also constructed $f(R, G, T)$ where the expansion follows the finite time future singularity (big rip singularity). It has been observed that for de Sitter expansion, the form of $f(R, G, T)$ contains the combinations of exponential and power forms of $R, G$ and $T$ but for power law and big rip singularity expansion models, the forms of $f(R, G, T)$ contain only the power forms of $R, G$ and $T$.

We have drawn the function $f(R, G, T)$ against $t$ in figure 1, 2 and 3 for de Sitter, power law and future singularity models respectively. From figure 1, we have observed that $f(R, G, T)$ sharply increases as $t$ increases (upto $\approx 2$) and then it takes the value 5.4365 which is nearly parallel to $t$ axis.
(i.e., slope of the curve \(\approx 0\)) throughout the evolution of the Universe for de Sitter expansion. From figure 2, we have seen that \(f(R, G, T)\) sharply decreases as \(t\) increases (upto \(t \approx 2\)) and then it is nearly parallel to \(t\) axis (i.e., slope of the curve \(\approx 0\)) throughout the evolution of the Universe for power law expansion. On the other hand, from figure 3, we have observed that \(f(R, G, T)\) nearly parallel to \(t\) axis (i.e., slope of the curve \(\approx 0\)) upto certain period of time \(t \approx 5\) then sharply increases as \(t\) increases near future singularity \((t \approx 6)\).

We have investigated all the energy conditions (NEC, WEC, SEC, DEC) in \(f(R, G, T)\) modified theory of gravity. If \(\rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}}\) and \(\rho_{\text{eff}} - p_{\text{eff}}\) are all non-negative, then all the energy conditions are satisfied. For this purpose, in de Sitter model, we have plotted \(\rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}}\) and \(\rho_{\text{eff}} - p_{\text{eff}}\) against \(t\) in figures 4, 5, 6 and 7 respectively. From these figures, we have seen that \(\rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}}\) and \(\rho_{\text{eff}} - p_{\text{eff}}\) are all positive during evolution of the Universe. So NEC, WEC, SEC and DEC are satisfied for our constructed \(f(R, G, T)\) gravity for de Sitter expansion model. Also, we have seen that in this model, \(\rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}}\) and \(\rho_{\text{eff}} - p_{\text{eff}}\) are all decreasing as time increases. Initially, these quantities have sharp decrease behaviour (about \(t \approx 2\)) and after that these are nearly parallel to \(t\) axis and tending to zero but keeps positive sign. For power law model, we have plotted \(\rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}}\) and \(\rho_{\text{eff}} - p_{\text{eff}}\) against \(t\) in figures 8, 9, 10 and 11 respectively. From these figures, we have observed that \(\rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}}\) and \(\rho_{\text{eff}} - p_{\text{eff}}\) are all positive during evolution of the Universe. So NEC, WEC, SEC and DEC are satisfied for our constructed \(f(R, G, T)\) gravity for power law expansion model. Also, we have seen that in this model, \(\rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}}\) and \(\rho_{\text{eff}} - p_{\text{eff}}\) are all decreasing from high values to nearly zero as time increases. Also for future singularity model, we have plotted \(\rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}}\) and \(\rho_{\text{eff}} - p_{\text{eff}}\) against \(t\) in figures 12, 13, 14 and 15 respectively. From figures we have seen that \(\rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}}\) and \(\rho_{\text{eff}} - p_{\text{eff}}\) are all positive during evolution of the Universe. So NEC, WEC, SEC and DEC are satisfied for our constructed \(f(R, G, T)\) gravity for future singularity model. Also, we have seen that in this model, \(\rho_{\text{eff}} + p_{\text{eff}}, \rho_{\text{eff}}, \rho_{\text{eff}} + 3p_{\text{eff}}\) and \(\rho_{\text{eff}} - p_{\text{eff}}\) are all increasing from lower values to higher values as time increases. At future singularity, all the quantities are blows up.

Finally, we have examined the stability of our constructed \(f(R, G, T)\) gravity for de Sitter, power law and future singularity models. For this purpose, the sign of \(v^2_s\) plays a crucial role to determine the classical stability or instability of the background evolution. The square speed of light \(v^2_s\) vs time \(t\) for de Sitter, power law and future singularity models have been drawn in figures 16, 17 and 18 respectively. From these figures, we have seen that \(v^2_s\) for all the models are lying in \((0, 1)\). So we have concluded that \(f(R, G, T)\) gravity models are classically stable for de Sitter, power law and
future singularity expansions.

Acknowledgement: The author is thankful to SERB DST (MATRICS Scheme), Govt. of India for providing research project grant (No. MTR/2019/000751/MS).

[1] A. G. Riess et al (Supernova Search Team Collaboration), Astron. J. 116, 1009 (1998);
[2] S. Perlmutter et al (Supernova Cosmology Project Collaboration), Astrophys. J. 517, 565 (1999).
[3] D. N. Spergel et al (WMAP Collaboration), Astrophys. J. Suppl. Ser. 148, 175 (2003).
[4] D. N. Spergel et al, Astrophys. J. Suppl. 170, 377 (2007).
[5] T. Padmanabhan, Phys. Rep. 380, 235 (2003).
[6] V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D 9, 373 (2000).
[7] B. Ratra, P. J. E. Peebles, Phys. Rev. D 37, 3406 (1988).
[8] R. R. Caldwell, Phys. Lett. B 545 23 (2002).
[9] B. Feng, X. L. Wang and X. M. Zhang, Phys. Lett. B 607, 35 (2005).
[10] Z. K. Guo, Y. S. Piao, X. M. Zhang and Y. Z. Zhang, Phys. Lett. B 608 177 (2005).
[11] A. Sen, JHEP 0207 065 (2002).
[12] C. Armendariz - Picon, V. F. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. 85, 4438 (2000).
[13] M. Gasperini et al, Phys. Rev. D 65, 023508 (2002).
[14] H. Wei, R.G. Cai and D.F. Zeng, Class. Quantum Grav. 22 3189 (2005).
[15] B. Gumjudpai and J. Ward, Phys. Rev. D 80 023528 (2009).
[16] J. Martin and M. Yamaguchi, Phys. Rev. D 77 123508 (2008).
[17] V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D 15, 2105 (2006).
[18] S. Nojiri and S. D. Odintsov, Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007).
[19] K. Bamba, S. Nojiri and S. D. Odintsov, JCAP 0810 045 (2008).
[20] S. Capozziello and M. Francaviglia, Gen. Rel. Grav. 40, 357 (2008).
[21] M. Sami, A. Toporensky, P. V. Tretjakov and S. Tsujikawa, Phys. Lett. B 619, 193 (2005).
[22] S. Nojiri and S. D. Odintsov, Phys. Lett. B 631 1 (2005).
[23] A. De Felice, S. Tsujikawa, Phys. Rev. D 80, 063516 (2009).
[24] O. Bertolami, C. G. Boehmer, T. Harko and F. S. N. Lobo, Phys. Rev. D 75, 104016 (2007).
[25] S. Nojiri, S. D. Odintsov and P. V. Tretjakov, Prog. Theor. Phys. Suppl. 172, 81 (2008).
[26] T. Harko, F. S. N. Lobo, S. Nojiri and S. D. Odintsov, Phys. Rev. D 84, 024020 (2011).
[27] M. Jamil, D. Momeni, M. Raza, Eur. Phys. J. C 72, 1999 (2012).
[28] M. J. S. Houndjo, C. E. M. Batista, J. P. Campos, O. F. Piattella, Can. J. Phys. 91, 548 (2013).
[29] M. Sharif, M. Zubair, JCAP 1203, 028 (2012).
[30] F. G. Alvarenga, M. J. S. Houndjo, A. V. Monwanou, J. B. C. Orou, J. Mod. Phys. 4, 130 (2013).
[31] R. Myrzakulov, Eur. Phys. J. C72, 2203 (2012).
[32] S. Chakraborty, Gen. Rel. Grav. 45, 2039 (2013).
[33] K. Bamba, S. D. Odintsov, L. Sebastiani and S. Zerbini, Eur. Phys. J. C 67, 295 (2010).
[34] A. De Felice, J. -M. Gerard, T. Suyama, Phys. Rev. D 82, 063526 (2010).
[35] A. De Felice, T. Tanaka, Prog. Theor. Phys. 124, 503 (2010).
[36] A. De Felice, T. Suyama, Prog. Theor. Phys. 125, 603 (2011).
[37] A. De Felice, T. Suyama, T. Tanaka, Phys. Rev. D 83, 104035 (2011).
[38] A. de la Cruz-Dombriz, D. Saez-Gomez, Class. Quant. Grav. 29, 245014 (2012).
[39] A. N. Makarenko, V. V. Obukhov, I. V. Kirnos, Astrophys. Space Sci. 343, 481 (2013).
[40] K. Atazadeh and F. Darabi, Gen. Relat. Grav. 46, 1664 (2014).
[41] M. Sharif and A. Ikram, Eur. Phys. J. C 76, 640 (2016).
[42] M. Sharif, A. Ikram, Phys. Dark Univ. 17, 1 (2017).
[43] M. F. Shamir and M. Ahmad, Eur. Phys. J. C 77, 55 (2017).
[44] L. D. Landau and E. M. Lifshitz, “The Classical Theory of Fields”, Butterworth-Heinemann, Oxford (1998).
[45] S. Nojiri, S. D. Odintsov and S. Tsujikawa, Phys. Rev. D 71, 063004 (2005).