Inverted Umbrella-type Hyperbolic Paraboloid Reinforced Concrete Shell Structures (Inverted Umbrella HP RC Shells)

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Abstract. The inverted umbrella HP RC shells became a predominant type of single column structure during the 1950s and 1960s. The paper provides a historical overview of architecturally most attractive inverted umbrella HP structures made out of reinforced concrete. It starts in the second quarter of the 20th century with the world’s oldest umbrella structures, designed by three pioneers: F. Aimond, A. Williams and K. Hruban. The most notable master in designing was F. Candela, as he constructed a number of this structures in Mexico between 1953-68. During the 1960s this form became widely used all over the Western world but suddenly disappear after 1975. The results of the paper are presented in three figures where the inverted umbrella HP RC shells are analysed according to several criteria (number of built elements, roof dimensions with shapes, use of the structures in relationship to year of completion). The similarities and differences between elements of the analysed buildings are compared and discussed. In the conclusion, the advantages and disadvantages are briefly exposed.

1. Introduction
This paper examines and investigates the inverted umbrella-type hyperbolic paraboloid shell structures made of reinforced concrete. Thirty-eight different umbrellas were studied and thirty-one are illustrated through the text with small sketches, made by the author of this paper. Inverted umbrella HP RC shells consist of three main elements: the highest element is a cantilever thin shell hyperbolic paraboloid roof; which is at its centre supported by the second element, a column, called also pillar or vertical beam; and the third element is a foundation, hidden in the ground. During the designing, constructing, describing and admiring these structures, many authors name them under several phrases. Usually they omit some words and call them inverted hyperbolic paraboloid shell structure, inverted hyperbolic paraboloid shell, hyperbolic paraboloid shell umbrella, hypar, HP, or just umbrella. But one should be careful because hyperbolic paraboloid shells could be supported by one, two or more supports. In this article they are called inverted umbrella HP RC shells or shorter umbrellas.

The idea of the inverted umbrella HP RC shells was especially popular in world architecture during the third quarter of the 20th century. A brief historical overview of tree-inspired dendriforms and fractal-like branching structures has been described by Rian and Sassone [1]. Furthermore, the thesis about Huddersfield's 1970 Market Hall written by Marsden [2] contains another important overview of the hyperbolic paraboloid shell architecture in the Western world and Britain especially. In recent years, numerous studies of roof structures, which are supported by only a single column, have been published in the field of construction history [3], [4]. Additionally, some case studies of Slovene mushroom and umbrella structures have been described in historical analysis of Slovene structuralism [5] and in a study on a distinction between mushroom and umbrella structures in Slovene architecture [6].
A growing interest on the work of structural engineers and architects of the first three quarters of the 20th century reveals new perspectives on the beginnings, development, and decline of inverted umbrella HP RC shells. But as each of the researchers usually focuses only on the oeuvre of one individual structural engineer, a wider comparison of these structures and their characteristics is needed. Thus, this study of different aspects of selected inverted umbrella HP RC shells is based on

- the number of built elements,
- the roof dimensions and their shapes,
- use of the structures.

2. The history of the inverted umbrella HP RC shells

The hyperbolic paraboloid shell structure is a doubly curved thin shell structure, made of reinforced concrete. Roofs, made as hyperbolic paraboloid shells, could be supported by a single column only, or by more columns. By joining four straight-edged hypar shell surfaces in concave shape and supporting it by only one column, the form of the inverted umbrella HP RC shells was introduced. The structures are also called hypar umbrellas or, even shorter, umbrellas.

2.1. Pioneers of the inverted umbrella HP RC shells.

Many structural engineers were searching for the ideal shape of single column roof structures during the 1930s and 1940s. Fernand Aimond, Amancio Williams and Konrad Hruban were among the first to use different inverted pyramid shape roofs as shelters for aircraft hangars, workshops, and factories.

2.1.1. Fernand Aimond. The first and also most productive in designing inverted umbrella HP RC shells during 1930s was French civil engineer Fernand Aimond (1902-84). He published two papers that introduced the notion of the concrete hypar [7], and gave precise calculations for hypar shells [8]. This was the first published suggestion and illustration of a hyperbolic paraboloid shell umbrella (figure 1), a structure where four thin hyperbolic paraboloids spring from a single column and cantilever out. He devised and constructed several different types of reinforced concrete hyperbolic paraboloid shell structures during this period. After Aimond's plans, three different types of single column structures were built [4]. The third one used four hyperbolic paraboloid shells that were supported by a single column: Workshops for the School of Naval Mechanics (figure 1) at Rochefort (1936) in France. Each structure is composed of an inclined rectangle roof, made out of four hyperbolic paraboloid shell surfaces, and is supported by a central column. This is the oldest inverted umbrella HP RC shell. After its shape, the structure is simply called parapluie, umbrella. Since Fernand Aimond made significant contributions in practice and theory of the thin reinforced concrete hyperbolic paraboloid shells, he could be considered the father of the inverted umbrella HP RC shells.

![Figure 1. Sketch of Aimond's concrete hyperbolic paraboloid shell umbrella (left) and of Aimond's Workshops for the School of Naval Mechanics structure (right) [9]](image-url)

2.1.2. Amancio Williams. Argentine architect Amancio Williams (1913-89) designed another shape of inverted umbrella reinforced concrete shell. Williams started to search for the ideal design of reinforced concrete shell of minimal thickness in 1939 [1]. The structure is based on three main ideas: it should rest only on a single column, be in balance by itself, and be capable of supporting extraordinary loads. The first result, called bóveda cascara, shell vault, was a reinforced concrete model of 5 cm thick inverted vault roof, supported by a central column. The shape of the roof was a concave square and the column had a gutter, where the rainwater could drain. Williams was the first to mention that "the thin shell umbrella roof of minimal thickness offers only little resistance to the wind" [10]. He developed the idea for shell vault roofs in 1948-53 [11], but as many of his ideas, these were also not built. His first...
built thin shell umbrella structure was the *Pabellón de exposición en Palermo*, Palermo Exhibition Pavilion (figure 2), sometimes named also *Pabellón Bunge y Born*, Bunge y Born Pavilion [12]. It was constructed in Buenos Aires in 1966, for the temporary exposition Exposición Rural. The Pavilion consisted of two diagonally placed thin shell umbrella roofs, each stood on hollow columns. As homage to Amancio Williams' work, two 15 m high shell vault umbrellas with hollow columns were built on the Buenos Aires coast (1999-2000) called the *Monumento del Fin de Milenio*, Monument to the End of the Millennium, in memory of the Bunge y Born Pavilion project.

![Figure 2. Sketch of Williams's Bunge y Born Pavilion structure [9]](image2)

### 2.1.3. Konrad Hruban

Just before the Second World War some engineers in Czechoslovakia also began to study and experiment with the thin shell structures. Among them was engineer Konrad Hruban (1893-1977) who, together with Otakar Štěpánek and Bedřich Hacar, designed *Továrny Jozef Černý a syn*, Jozef Černý and Son Factory (figure 3), in Nové Mesto nad Váhom (today in Slovakia). The factory was devised and built during the war years 1942-44 [13]. Each thin inverted hyperbolic paraboloid roof element is resting on a single column with exposed capital and four well seen beams. The whole reinforced concrete structure is monolithic and continuous, because the beams of the adjacent columns are interconnected. Between all three rows of the roof are two continuous strips of wide north light windows where natural light is reaching the factory floor.

![Figure 3. Sketch of Hruban's Jozef Černý and Son Factory structure [9]](image3)

### 2.2. Master of the inverted umbrella HP RC shells.

During the 1950s the idea of inverted umbrella HP RC shells became widely used. The master in calculating and designing them was Félix Candela. He studied the works of predecessors and upgraded them with the testing of prototypes. In collaboration with different architects Candela constructed inverted umbrella HP RC shells that covered together more than 28 hectares in only 10 years, from 1953 to 1963 [14].

#### 2.2.1. Félix Candela

Spanish-born architect Félix Candela (1910-97) immigrated to Mexico in 1939. He dedicated his work to shell structures among which he designed, calculated, constructed and tested many inverted umbrella HP RC shells and called them *hypar umbrellas*. Candela created the first hypar umbrella prototype structure (figure 4) in 1952 by joining four straight-edged hyperbolic paraboloid shell surfaces. Inverted roof has a form of a square with a side of 10 m, rose from the column’s capital 1 m, and was only 4 cm thick at the edge. He found out that the structure exhibited a tendency to flutter in the wind [14]. His second hypar umbrella prototype (figure 4) was built in 1953: this square side of inverted umbrella roof measured 8 m, rose from the column’s capital 60 cm, and was 8 cm thick at the edge. The foundations of both prototypes were also designed as the inverted umbrella structure. After these two experimental structures, Candela built Rio’s Warehouse (figure 4) in Mexico City in 1954 in a triangle plot. At almost the same time, he also constructed the High Life Textile Factory (figure 4) (1954-55), made with the perforated roof. In both cases, each roof is not horizontal, but inclined, and thus has a saw-tooth roof profile to provide sunlight into the interior. Candela also designed *Iglesia de la Medalla de la Virgen Milagrosa*, Our Lady of the Miraculous Medal Church (figure 4) (1953-55), in Mexico City, where the horizontal asymmetrical umbrella roof is transformed into a diagonal wall. The shapes of the hypar umbrellas remain the same through next years, but the roof surfaces become wider
and thinner, and the columns higher. Beside aforementioned buildings, Candela designed factories, markets, warehouses, slaughterhouse, low-cost dwellings, bank, and bungalows for hotel. Among his unbuilt projects, the plan for the Presidential palace in Havana, Cuba (1957) should be mentioned. His largest hypar umbrella was a square roof with 18 m side, another has a rectangular form of 8 m by 25 m. Some of the roofs have a shape of a triangular, rhomboidal or a polygonal form. As the roofs are designed as inverted pyramids, inside each hollow column is a gutter to drain the rainwater to the ground. His last built inverted umbrella HP RC shell was La Candelaria metro station roof (figure 4) (1967) in Mexico City. This building consists of 22 reinforced rectangular umbrellas built in two lines and with a continuous strip of windows between the both lines. Each umbrella is formed by 6 folded inverted hyperbolic paraboloids and measures 13 m by 6 m. Felix Candela became the master of inverted umbrella HP RC shells. He calculated and built many hypar umbrellas, as he called them, that varied in shape and size. Throughout the years Candela’s structures become ever more rational and economical but also graceful and reached perfection not only as engineering but also as art objects [14].

Figure 4. Sketches of Candela's structures: first prototype (far left), second prototype (left), Rio's Warehouse (centre), High Life Textile Factory (centre-right), Our Lady of the Miraculous Medal Church (right), La Candelaria metro station (far right) [9]

2.3. Inverted umbrella HP RC shells.
Through the 1960s, many architects together with structural engineers around the Western world started to design inverted umbrella HP RC shells. As a result, thousands of them were built. They were not used only as gas stations, pavilions, factories, warehouses or markets, but also for public buildings: churches, hotels, restaurants, casinos, banks, libraries, markets, museums, and even as low-cost housings. Some of the most notable inverted umbrella HP RC shells are mentioned and described in the text below.

2.3.1. John V. (Jack) Christiansen. Among the first to improve Candela's work was American structural engineer John V. (Jack) Christiansen (1927-2017), who calculated and designed many hyperbolic paraboloid structures in Washington State. Some of them were in a shape of inverted umbrella HP RC shells [15], [16]. One of his earliest is the project of 13 umbrellas that covered walkways in Pioneer Middle School (figure 5) (1957) in Wenatchee. In 1959 Jack Christiansen, together with Maury Proctor, designed the first moveable and reusable formwork system for prefabricated inverted umbrella HP RC shells. Formwork system was made in three different standardized square roof forms, side ranging from 7.3 m to 11 m, and the rectangle one. The system was adaptable to a variety of different building types and uses, and also allowed a variation in distance between columns. The first standardized umbrella structure was square roof with 7.3 m, built for gas station canopy in Olympia (1959). The second umbrella type was designed for the United Control Corporate Building (figure 5) in Redmond (1960) and has a square roof with 11 m. The third umbrella type, a square roof 9.1 m long, was developed for the Washington Correction Center in Shelton (1962). The rectangle form, with sides 8.5 m by 11 m, was first used for the Custom Manufacturing in Seattle (1969), Washington, but was not as popular as the previous three. Jack Christiansen used a unique design method for constructing 52 hexagonal umbrellas, used as International Pavilion (figure 5) in Seattle's World's fair (1962). He took the 9.1 m square formwork for inverted umbrella HP RC shell and cut off all four edges. The resulting hexagonal umbrella roof creates an interesting aesthetic: the deflections in all corners are not identical, and the roof has only two lines of symmetry and four warped surfaces. During his œuvre, Jack Christiansen chose to construct many thin concrete shell form roofs, including inverted umbrella-type hyperbolic paraboloid shell structures, not only for their economical construction using reusable formwork but also elegant design of structurally expressive forms.
2.3.2. Victor Lundy. Almost at the same time, American modernist architect Victor Lundy (1923) became nationally renowned for his Warm Mineral Springs Motel (figure 6) (1958) near Venice, Florida. The Motel consists of 75 precast reinforced concrete roof canopies, varying in two heights and thus providing light over clear glass panels and making the roofs appear to float [17]. The structural system is based on using inverted umbrella HP RC shells, each being a square side measuring 4.4 m only. The sign for the motel stands as a piece of sculpture in front of the building and imitates the precast canopies. It is formed out of three umbrella roofs, lifted up on higher columns of varying height. Today Warm Mineral Springs Motel is included among the National Register of Historic Places.

2.3.3. Kellam and Foley. One of the largest inverted umbrella HP RC shells was built in Columbus, Ohio. Scioto Downs grandstand roof (figure 7) (1959-60), designed by architectural firm Kellam and Foley, consists of 5 rectangle roofs with sides 36 by 18 m. As the structural system of each shell was unstable, additional stiffening was provided by two main ribs, one in each primary direction, and for stabilization a concrete post was added at the back. To control corner deflection, edge beams were extended around each shell. During the years, one more umbrella was built. Today, there are six shells, all well preserved [18].

2.3.4. Marcel Breuer. Hungarian-born American architect and designer Marcel Breuer (1902-81) built Hunter College Library (figure 8) (1957-60), today Lehman College Art Gallery, in New York, NY. The building consists of six inverted umbrella HP RC shells, each being a square measuring 18.3 m. Each umbrella is structurally independent but key-locked at the edges and tied together with dowels. The façade is made as a curtain wall of clear glass, which originally would have given views of the "grid of umbrellas" scheme inside of the library. In the interior, the formwork for the concrete roof, made of straight panels, is still seen today as the small gaps are left between the panels of undulating ceiling [19].

2.3.5. Claude Laurens. The progress in architecture and structures was well shown at the Brussels EXPO’58. Among new structures, built as an accompanying program, was Volkswagen showroom building, Le garage D’Ieteren (figure 9) (1957) just near Expo site in Heysel. Belgian architect Claude
Laurens (1908-2003) and engineer Hoite Cornelis Duyster constructed two inverted umbrella HP RC shells, each measured 31.4 by 23 m. Due to an only 5 cm thick roof, some additional tie beams had to be provided at the back façade to ensure balance of the structure, as well as at the side façade to ensure transverse stability of the roof. In front of the showroom, the gas station (figure 9) was built by the same team and the structure was again made as an umbrella. It was smaller and asymmetrical, in the form of a trapezoid that stood on a Minoan like column - wider at the top than at the base [20].

**Figure 9.** Sketches of Laurens's D'Ieteren showroom building (left) and gas station (right) [9]

2.3.6. Thomas (Tom) Hedley Bruce Burrough. British architect Thomas (Tom) Hedley Bruce Burrough (1910-2000), together with the engineer Derek Bond, devised the first inverted umbrella HP RC shell in Britain for the Mound Stand (figure 10) of the Gloucestershire County Cricket Club ground in Bristol (1959). One corner of the cricket ground was roofed with 8 asymmetric umbrellas that were tilted down to the back of the stand, eccentric to the column and without down column drainage. Each inverted roof has a form of a square with a side of approximately 9 m that stands on a circular column. The umbrellas are not designed in a straight line, they are curved-in-plan. The inverted umbrella HP RC shells become popular in Britain after these stands [2].

**Figure 10.** Sketch of Burrough's Mound Stand structure [9]

2.3.7. Yorke, Rosenberg and Mardall. Among other inverted umbrella HP RC shells built in Britain, at least one more structure should be mentioned: John Lewis Partnership’s warehouse (figure 11) in Stevenage (1962-3), designed by Yorke, Rosenberg and Mardall with help of Félix Candela. They used 75 rectangle tilted umbrellas and another 15 half shells that are joined in rows where north light is provided in a 1.5 m high interval. Each rectangle umbrella roof is 18.3 by 9.5 m large and supported by a 4 m square concrete column with a 10 cm diameter rainwater drainage.

**Figure 11.** Sketch of YMS's structure for John Lewis Partnership's warehouse [9]

2.3.8. Alfred (Al) Mansfeld. Russian-born Israeli architect Alfred (Al) Mansfeld (1912-2004) with Austrian-born Israeli engineer Eliyahu Traum (1924) devised the Israel Museum (figure 12) in Jerusalem in 1959-60, built in 1965. The Museum is designed as a series of modular roof units without any grid. Each 11.2 m square roof structure was constructed as an inverted hyperbolic paraboloid shell roof, supported by a single central column with a vertical hollow in which air conditioning, lightning and roof drainage are located [21]. The roof structure is enabling clerestory windows to separate the seemingly levitated roof from the freestanding walls of each modular unit.

**Figure 12.** Sketch of Mansfeld's Israel Museum structure [9]
2.3.9. Árpád Varga. Hungarian engineer Árpád Varga developed a series of hypar shells, supported at their centre only. The calculations, based on the membrane theory and considering symmetrical loading were provided first. Later, the full-scale prototype was built on site to study the overall stability of the structure, especially due to asymmetrical loading. The testing proved the initial design right. The adaptable system was used as roofing regional bus stations and built in many smaller Hungarian towns as Szolnok (figure 13) or Szombathely during the 1960s [22]. Varga’s roofs were appealing, ingenious and efficient structural designs, were neither particularly grand nor especially daring. Nevertheless, they have a touch of engineering romanticism, in terms of the idealistic unity of form and forces. His roofs were a perfect example of the general approach of Hungarian engineers of the era: modest, skilful designs that serve their purpose.

![Figure 13. Sketch of Varga's bus station structure [9]](image)

2.3.10. Warsaw Tigers. Another inverted umbrella HP RC shell was built in Eastern Bloc. In Poland, architects Wacław Kłyszewski (1910-2000), Jerzy Mokrzyński (1909-1997) and Eugeniusz Wierzbicki (1909-1991), known as Warsaw Tigers, devised the new Railway station building (figure 14) (1966-72) in Katowice. Together with structural engineer Wacław Zalewski they designed the main hall of the railway station building, consisting of 16 umbrella shells [23]. The distinctive part of the structure are two-floors high columns, carefully formed in an unusual hyperbolic geometry and show the raw texture of the timber formwork.

![Figure 14. Sketch of Warsaw Tiger's structure for Railway station building [9]](image)

2.3.11. Woldemar Baeckman. In Finland, architect Woldemar Baeckman (1911-94) built Sibelius Museum (figure 15) (1968) in Turku where the roof of the Sibelius Hall consists of four inverted umbrella HP RC shells. As each column is rotated, the structure has a unique impression and optimal acoustics. Again, the architect’s inspiration was the hypar umbrella developed by Félix Candela.

![Figure 15. Sketch of Baeckman’s Sibelius Museum structure [9]](image)

2.3.12. Edvard Ravnikar. Edvard Ravnikar (1907-93) designed and built a gas station (figure 16) in Ljubljana (1968-69), Slovenia. The building consists of three inverted umbrella HP RC shells in a row and a small, separate cashier’s building cubicle nearby. Each umbrella structure is a separate one, made up of a square roof with a side of 19 m and 2.8 m deep. Column in the shape of an octagon is 3 m high. A square foundation is measuring a 3 by 3 m [6]. Although the column of each modular unit is short, the shell roof is one of the widest.

![Figure 16. Sketch of Ravnikar's gas station structure [9]](image)
2.3.13. Jean Le Couteur. In France, architect Jean Le Couteur (1916-2010) and structural engineer Miroslav Kostanjevac (1926-2002) designed Criée aux Poissons, Fish Auction Hall (figure 17) (1966-69), in Sète. The structure should be mentioned as a juxtaposition of two shell types. The shape of the roofs was inspired by the flight of a seagull, emblematic bird of a fishing port [2]. It consists of 40 inverted umbrella HP RC shells, and of 4 saddle-type hyperbolic paraboloid shells measuring 11 m each side. In addition, columns differ through the height from the rhombus cross section at the highest point and twisted for 45 degrees to square shape at the base. While both structures were not a novelty, the incorporation of both hyperbolic paraboloid types, the apparent lightness and the transparency are appropriate.

Figure 17. Sketch of Le Couteur's Fish Auction Hall structure [9]

2.3.14. J Seymour Harris Partnership. Another structure made as inverted umbrella HP RC shells and built in the United Kingdom should be mentioned. The architectural firm J Seymour Harris Partnership, where Gwynfor (Gwyn) Roberts (1936-2004) was the project architect, designed Queensgate Market Hall (figure 18) in Huddersfield (1968-70). The interesting and economical freestanding structure is made up of 21 rectangle asymmetrical modular units. A rectangle roof, measuring 17 m by 9.5 m in plan and 3 m deep, stands asymmetrical on a column and cause that cantilever roof is 7 m to one side of it and 10 m to the other. The columns are also rectangular, with heights that vary from 3.3 m to 7.5 m. As the columns differ in height, each roof element is enabling clerestory windows to separate the seemingly floating roof from the freestanding façades of modular units. Thus, each unit functions independently and is not structurally connected to the other for bracing [2]. The developments in asymmetrical floating umbrella structures made the Queensgate Market Hall in Huddersfield unique not only in Britain.

Figure 18. Sketch of J Seymour Harris Partnership's Queensgate Market Hall structure [9]

2.4. Late inverted umbrella HP RC shells.
By the end of the 1960s some innovative inverted umbrella HP shells were also built out of reinforced concrete or even of prestressed concrete. Architects, together with structural engineers, were able to play a leading role in devising new architectural forms that were also structurally innovative. Thus, the new forms of single column roof structures were introduced. Inverted umbrella HP RC shells do not represent the mainstream in structural development after 1975.

2.4.1. Ildefonso Sánchez del Río y Písón. Spanish pioneer of single column roof structures Ildefonso Sánchez del Río y Písón (1898-1980) built his last umbrella structures in 1971-72. He used two different types of umbrella shells to cover the livestock market at Pola de Siero [3]. The first shell type is an octagon (figure 19), measuring 40 m in diameter and is only 3.5 cm thick at the end. By its dimensions, it was the largest umbrella constructed up until then. Today it is well preserved due to careful restoration. The second shell type were three equal square umbrellas (figure 19), built in one line, but unfortunately all three have been demolished. After more than 50 years of Sánchez del Rio career, he devised at least five different types of roof structures, supported by a single column. Thus, he should be recognized as one of the most important authors of single column roof structures.
2.4.2. Terminals A, B, C Newark Airport. One more structure from the United States should be described, the modernist complex of Terminal A (1973), Terminal B (1973), and Terminal C (1988) (figure 20) at Newark Liberty International Airport in New Jersey. Despite their phased construction all three terminals present a similar, consistent appearance that is defined by 87 inverted umbrella HP RC shells. The most significant part of each of the curved-in-plan terminals are the 26 umbrella roofs, that have almost a square shape with a 16 m side, while 3 further umbrellas are higher and have doubled length. Difference in height of both umbrella types allows the sunlight to shine into the interior of each terminal. The most remarkable part of each umbrella-type structure are four exposed beams at the bottom side of each roof that connect four hyperbolic paraboloid shells of which a separate roof is constructed [2].

2.4.3. Jean-François Zevaco. In Casablanca, Morocco, a French-Moroccan architect Jean-François Zevaco (1916-2003) designed the Rue D’Agadir Market (figure 21) (1973-75) with 13 square roofs covering a ground floor of 1200 m². Each roof is a freestanding inverted umbrella HP RC shell standing on a central column. The head of the column, at the point where the column meets the roof, is visibly narrower than the lower part of the column. Another distinction of this structure is that circular columns differ in six heights and the roofs have eight sizes in plan, measuring from 4 to 11.4 m [2].

3. Results and discussions
During the research of the inverted umbrella HP RC shells, 38 different structures were studied and sketched [9]. All the buildings are made as the hyperbolic paraboloid shell structures, supported by only a single column and made of reinforced concrete. The sketches are inserted between the text as small illustrations. Due to the limited pages of this paper, the main table with data about the structures mentioned in the text is omitted. Nevertheless, some of the results are shown in figures 22 to 24: the number of built elements of each structure, the roof dimensions with shapes, and use of the structures.

Inverted umbrella HP RC shells can be built as singular structural elements or as a grid of repeated structures. Even if the building consists of more than one structural system element, each structure is separated from another and thus acts as an independent structure. The number of built structures in relationship to the year of completion is shown in figure 22. Repetition of a bigger number of identical elements is preferable as it allows prefabrication which makes construction more economical. The highest number of the same structure was built by Lundy as a motel building and by Yorke, Rosenberg and Mardall as a factory. Furthermore, the structures with façade and without it are shown in different colours. Prototypes, single structures or structures with only a few elements, were usually built without façade.
Figure 22. Relationship between the year of completion of the structure (with or without façade) and the number of built elements of the structure.

Inverted umbrella HP RC shell consists of three main elements. The most visually exposed element is the cantilever roof. Due to the form of the shell, the stresses in the reinforced concrete roof is low and only a mesh of small reinforcing steel is necessary. Roofs differ in the shapes (square, rectangle, hexagon and octagon) and in side dimensions (from 4 m up to 40 m). Square and rectangle shapes were the most used forms for the roofs, only one type of hexagon and one octagon roof was constructed. Throughout those years, the dimensions of the roof surfaces become wider and thinner, while the shapes become more elegant. The dimensions and different shapes of roofs according to the year of completion of the buildings are shown in figure 23.

Figure 23. Relationship between the year of completion of the structure (with four different roof shapes) and the roof dimensions.

In the span of two decades, from 1955 to 1975, inverted umbrella HP RC shells became very suitable for roofing large areas economically. The amount of required concrete and reinforced steel is very low per area of floor. At the end of 1950s and during the 1960s, there was a major increase in constructing inverted umbrella HP RC shells which lasted almost 20 years until the beginning of 1970s. They were popular not only for factories, gas stations and transport buildings, but also for public buildings: pavilions, markets, sport stands, museums, and a church, library and motel. The use of the structures in relationship to the year of completion of the buildings are shown in figure 24. After 1975 they became unacceptable due to their weak resistance to horizontal forces in all earthquake areas and in areas with
strong wind. Only the Terminal C at Newark Airport was built after that year, but the structure is identical to Terminal A and B.

Figure 24. Relationship between the year of completion of the structure (with or without façade) and the use of the structure.

4. Conclusions

The paper overviews and investigates inverted umbrella-type hyperbolic paraboloid reinforced concrete shell structures, called inverted umbrella HP RC shells or shortly umbrellas. The 50 years long development is briefly explained and illustrated in this article. Due to the limited pages of this paper, the main table with data about the structures mentioned in the text (author, time, location, roof type, roof parameters, column type, column parameters, number of elements) is omitted. Nevertheless, all described structures are illustrated through the text with small sketches.

During the first 15 years some prototypes and factories were built, with a side of roof up to 15 m, but as many as 56 similar roof elements were built. Inverted umbrella HP RC shells enjoyed a golden age from 1954 to 1972, when they were used upon different purposes with the roof sides up to 40 m and 75 similar roofs were built to serve one building. After 1973 the decline was quick, only few structures were used for market or as buildings for transport with roof sides dimensions up to 32 m and 52 similar roofs were built.

The advancements and disadvantages of inverted umbrella HP RC shells should be briefly exposed. One of the benefits of inverted umbrella HP RC shell is that open space under the roof could be without supporting walls. They could be constructed as prefabricated elements: repetition of a greater number of identical elements is preferable as the prefabrication makes construction more economical and the same scaffolding could be used several times. Nevertheless, reinforced concrete thin roof structure, that stand on one column only, do not meet all safety standards. As some of the inverted umbrella HP RC shells become fragile and unstable, building codes become stricter through the years. Wind vibrations and unexpected live loads caused the structures to buckle and fail easily. Furthermore, earthquakes triggered the introduction of higher safety standards for earthquake-resistant design. Additionally, heat insulation and sustainability become important building requirements in building physics. Therefore, the designing of inverted umbrella HP RC shells is generally prohibited. Today, architects and engineers find new ways to design umbrella structures in several different materials, mostly steel or timber.

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