SUSY GUT model building

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Abstract The evolution of SUSY GUT model building is discussed, starting with the construction of 4D GUTs, to orbifold GUTs and finally to orbifold GUTs, within the heterotic string model. This evolution is an attempt to obtain realistic string models, perhaps relevant for the LHC. This review is in memory of the sudden loss of Julius Wess, a leader in the field, who will be sorely missed.

1 Motivation

Before beginning this review on SUSY GUTs, it is probably worthwhile spending a very brief moment motivating the topic. What are the virtues of SUSY GUTs? The following is a list of all the issues that SUSY GUTs either address directly or provide a framework for addressing.

1. \( M_Z \ll M_{\text{GUT}} \) “natural”
2. Explains charge quantization and family structure
3. Predicts gauge coupling unification
4. Predicts Yukawa coupling unification
5. + Family symmetry \( \Rightarrow \) hierarchy of fermion masses
6. Neutrino masses via see-saw scale \( \sim 10^{-3} - 10^{-2} M_G \)
7. LSP—dark matter candidate
8. Baryogenesis via leptogenesis
9. SUSY desert \( \Rightarrow \) LHC experiments probe physics \( O(M_{\text{Planck}}) \) scale
10. SUSY GUTs are natural extension of the standard model.

In the following review we shall discuss some of these issues in great detail. Let us start by defining our notation for the standard model.

2 Standard model

Let us define the generators for the gauge group \( SU(3) \otimes SU(2) \otimes U(1)_Y \) as \( T_A, A = 1, \ldots, 8 \) for \( SU(3) \), \( T_a, a = 1, 2, 3 \) for \( SU(2) \) and the hypercharge operator \( Y \) for \( U(1)_Y \).

The gauge interactions of the quarks and leptons of the standard model are then completely defined in terms of their gauge quantum numbers. The quark and lepton fields for one family are given in terms of the left-handed Weyl spinors:

\[
q = \begin{pmatrix} u \\ d \end{pmatrix}, \quad u^c, \quad d^c, \quad l = \begin{pmatrix} \nu \\ e \end{pmatrix}, \quad e^c, \quad \nu^c,
\]

with SM charges given by

\[
T_A q = \frac{1}{2} \lambda_A q, \quad T_A u^c = -\frac{1}{2} \lambda_A^a T_a u^c,
\]

\[
T_A d^c = -\frac{1}{2} \lambda_A^a T_a d^c, \quad T_A l = T_A e^c = T_A \nu^c = 0
\]

(where \( \lambda_A \) are the \( 3 \times 3 \) Gell-Mann matrices);

\[
T_a q = \frac{1}{2} \tau_a q, \quad T_a l = \frac{1}{2} \tau_a l,
\]

\[
T_a u^c = T_a d^c = T_a e^c = T_a \nu^c = 0
\]

(where \( \tau_a \) are the \( 2 \times 2 \) Pauli matrices);

\[
Y q = \frac{1}{3} q, \quad Y u^c = -\frac{4}{3} u^c, \quad Y d^c = \frac{2}{3} d^c,
\]

\[
Y l = -l, \quad Y e^c = +2 e^c, \quad Y \nu^c = 0.
\]

With this notation, the gauge covariant derivative is given by

\[
D_\mu = \left( \partial_\mu + ig_s T_A G_{\mu A} + ig T_a W_{\mu a} + ig \frac{Y}{2} B_\mu \right),
\]

and the electric charge operator is given by \( Q = T_3 + \frac{Y}{2} \).
The gauge–fermion Lagrangian is given by

$$L_{\text{gauge–fermion}} = \left[ i \bar{\psi}_a \gamma^\mu \partial_\mu \psi_a + \cdots \right] + \left[ -\frac{1}{2} \mathrm{Tr}(G_{\mu\nu} G^{\mu\nu}) + \cdots \right].$$

(6)

\(^1\)In order to make contact with phenomenology it is sometimes useful to use Dirac four component notation. For example, the Dirac four component electron field, in terms of the two component Weyl spinors, is given by

$$\psi_e = \begin{pmatrix} e \\ i \sigma_2 (e^\ast) \end{pmatrix}.$$  

(7)

In addition, we must add the Higgs bosons. We shall introduce the minimal set of Higgs doublets consistent with supersymmetry:

$$H_u = \begin{pmatrix} h^+_1 \\ h^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} \tilde{h}^0 \end{pmatrix},$$

satisfying

$$T_A H_u = T_A H_d = 0,$$

(8)

$$T_a H_u = \frac{1}{2} \tau_a H_u, \quad T_a H_d = \frac{1}{2} \tau_a H_d,$$

(9)

$$Y H_u = + H_u, \quad Y H_d = - H_d;$$

$$-\mathcal{L}_{\text{Yukawa}} = \lambda^{ij}_d l_i e^c_j H_d + \lambda^{ij}_u q^c_i d^c_j H_d + \lambda^{ij}_u q^c_i u^c_j H_u - \frac{1}{2} M_{ij} v^c_i v^c_j.$$  

(10)

The generalization to the minimal supersymmetric standard model is then quite simple. One defines the left-handed chiral superfields. For example, the electron left-handed Weyl field, \( e \), is contained in the left-handed chiral superfield, \( E \), with

$$E(y, \theta) = \bar{e}(y) + \sqrt{2} (\theta e(y)) + (\theta \theta) F_\varepsilon(y),$$

where the product \((\theta e(y)) = \theta^a e(y)_a\) and \(y^\mu = \chi^\mu = \chi^\mu - i \sigma^\mu \sigma^\ast\).

Then the supersymmetric Lagrangian includes the gauge–matter terms

$$L_{\text{gauge–matter}} = \int d^4 \theta \bar{L}_i \exp \left( -2 \left[ g_s V_\varepsilon + g V_W + \frac{1}{2} Y V_B \right] L_i + \cdots \right) + \frac{1}{8 g_s^2} \int d^2 \theta \mathrm{Tr} \left( W_\varepsilon^{\mu\nu} W_\varepsilon_{\mu\nu} + h.c. \right) + \cdots.$$  

(12)

\(^1\)We gave \( G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + i [\partial_\mu, G_\nu] \) with \( G_\mu = T^A_B G_{\mu A} \) and \( T^A_B \) is the generator in the fundamental representation.

\[ V_\varepsilon = V_A T_A, V_W = V_\varepsilon T_\varepsilon \] and the flavor index is \( i = 1, 2, 3 \).

We have \( \mathcal{L}_{\text{Yukawa}} = \int d^2 \theta W \), with the superpotential given by

$$W = \lambda^{ij}_d l_i E^c_j H_d + \lambda^{ij}_u Q^c_i D^c_j H_d + \lambda^{ij}_u Q^c_i U^c_j H_u + \lambda^{ij}_u L_i N^c_j H_u - \frac{1}{2} M_{ij} N^c_i N^c_j - \mu H_u H_d.$$  

(13)

3 Two roads to grand unification

One can first unify quarks and leptons into two irreducible representations of the group \( SU(4)_C \otimes SU(2)_L \otimes SU(2)_R \), i.e. the so-called Pati–Salam group [1], where lepton number is the fourth color.

Then the PS fields

$$Q = (q l), \quad Q^c = (q^c l^c),$$

(14)

where

$$q^c = \begin{pmatrix} u^c \\ d^c \end{pmatrix}, \quad l^c = \begin{pmatrix} \nu^c \\ e^c \end{pmatrix},$$

(15)

transform as \((4, 2, 1) \oplus (\bar{4}, 1, \bar{2})\) under PS. One can check that baryon number minus lepton number acting on a 4 of \( SU(4) \) is given by

$$B - L = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}.$$  

(16)

and similarly electric charge is given by

$$Q = T_{3L} + T_{3R} + \frac{1}{2} (B - L).$$

(17)

Note that charge is quantized since it is embedded in a non-abelian gauge group. One family is contained in two irreducible representations. Finally, if we require parity \((L \leftrightarrow R)\) then there are two independent gauge couplings.

What about the Higgs? The two Higgs doublets \( H_u, H_d \) are combined into one irreducible PS Higgs multiplet

$$\mathcal{H} = (H_d H_u)$$

(18)

transforming as a \((1, 2, \bar{2})\) under PS. Thus, for one family there is a unique renormalizable Yukawa coupling given by

$$\lambda \mathcal{Q}^c \mathcal{H} \mathcal{Q},$$

(19)

giving the GUT relation

$$\lambda_t = \lambda_b = \lambda_e = \lambda_{V_\varepsilon} \equiv \lambda.$$  

(20)