Quantitative description of the $^{20}\text{Ne}(p,\alpha^{16}\text{O})$ cross section for probing the surface $\alpha$ amplitude

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The proton-induced $\alpha$ knockout reaction has been utilized for decades to investigate the $\alpha$ cluster states of nuclei, of the ground state in particular. However, even in recent years, it is reported that the deduced $\alpha$ spectroscopic factors from $\alpha$ knockout experiments and reaction analyses with a phenomenological $\alpha$ cluster wave function diverge depending on the kinematical condition of the reaction. In the present study we examine the theoretical description of the $^{20}\text{Ne}(p,\alpha^{16}\text{O})$ cross section based on the antisymmetrized molecular dynamics and the distorted wave impulse approximation by comparing with existing experimental data. We also investigate the correspondence between the $\alpha$ cluster wave function and the $\alpha$ knockout cross section. The existing $^{20}\text{Ne}(p,\alpha^{16}\text{O})$ data at 101.5 MeV is well reproduced by the present framework. Due to the peripherality of the reaction, the surface region of the cluster wave function is selectively reflected to the knockout cross section. A quantitatively reliable $\alpha$ cluster wave function, $p-\alpha$ cross section, and distorting potentials between scattering particles, $\alpha^{16}\text{O}$ in particular, are crucial for the quantitative description of the $^{20}\text{Ne}(p,\alpha^{16}\text{O})$ cross section. Due to the peripherality of the reaction, the $(p,\alpha^{16}\text{O})$ cross section is a good probe for the surface $\alpha$ amplitude.

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As it is schematically illustrated in the Ikeda diagram [1], the $\alpha$ particle is expected to emerge as a subunit in nuclear systems, in the light mass region in particular, reflecting the fact that the $\alpha$ particle is an enormously tight-binding system of four nucleons. Various cluster theories have been developed so far, and in recent decades microscopic cluster theories based on the nucleon degrees of freedom with fermionic quantum statistics are available, as reviewed in a recently published article [2]. Among them, the antisymmetrized molecular dynamics (AMD) has been applied to many systems and succeeded in describing cluster structures [3–12]. One of the questions remaining today is how much $\alpha$ cluster states exist or not in the ground state of nuclei, far below its $\alpha$ threshold.

From a nuclear reaction point of view, the proton-induced $\alpha$ knockout reaction, $(p,\alpha)$, is a good probe for the $\alpha$ cluster state in the ground state of a target nucleus. Much effort has been made on the $(p,\alpha)$ reaction studies [13–23], but even today quantitative understanding of the $(p,\alpha)$ cross section and its relation with the $\alpha$ cluster wave function in the ground state of the target nucleus have not yet fully established. In Ref. 20, it is reported that the deduced $\alpha$ spectroscopic factor ($S_\alpha$) of $^{12}\text{C}$ from the $^{12}\text{C}(p,\alpha)^{16}\text{Be}$ reaction at 100 MeV has large uncertainty of $S_\alpha = 0.19$–1.68 depending on the kinematics of the reaction, while $S_\alpha$ should be determined purely from the structure of $^{12}\text{C}$ and should not depend on the kinematical condition of the reaction. This uncertainty may be arising from the ambiguity in the reaction theory, particularly in the $\alpha^{8}\text{Be}$ optical potential and its energy dependence, since $^{8}\text{Be}$ is unstable and hence the $\alpha^{8}\text{Be}$ elastic scattering is not well understood. Another important ingredient in describing the $(p,\alpha)$ reaction is the appropriate $p-\alpha$ effective interaction within the required range of the $p-\alpha$ scattering energy and angle in the $(p,\alpha)$ reaction. The $\alpha$ cluster wave function adopted to the reaction analysis is also important.

Considering the above-mentioned situation, in the present study $^{20}\text{Ne}(p,\alpha)^{16}\text{O}$ reaction is considered within the distorted wave impulse approximation (DWIA) framework [24, 25]. Since the target and the residue of this reaction are typical stable nuclei and the distorting potentials of them are well known, the description of the reaction will be free from ambiguities of the optical potential. In addition, in the existing data of the $^{20}\text{Ne}(p,\alpha)^{16}\text{O}$ experiment [15], the emitting angles of the $p$ and $\alpha$ are fixed and the corresponding $p-\alpha$ binary scattering angle is very limited. This may also help to reduce the ambiguity arising from the angular dependence of the $p-\alpha$ cross section required in the DWIA calculation.

As for the $\alpha$ cluster wave function, the $\alpha^{16}\text{O}$ reduced width amplitude (RWA) of the $^{20}\text{Ne}$ ground state obtained by the AMD framework [12] is adopted as an input of the DWIA calculation. The RWA is the probability amplitude to find the clusters at inter-cluster distance $R$, and is defined as the overlap between the ground state wave function of $^{20}\text{Ne}$ and

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the reference wave function for the $\alpha + ^{16}\text{O}$ clustering,

$$y(R) = \sqrt{\frac{20!}{16!4!4\pi}} \left( \frac{\delta(r-R)}{r^2} \Phi_\alpha \Phi_{^{16}\text{O}} \right) \Psi_{^{20}\text{Ne}}. \tag{1}$$

The integral of the RWA called $\alpha$ spectroscopic factor $S_\alpha$ is often used as a measure of the clustering,

$$S_\alpha = \int_0^\infty R^2 dR |y(R)|^2. \tag{2}$$

In the definition of the RWA given by Eq. (1), the bra vector represents the reference cluster state, in which the $\alpha$ and $^{16}\text{O}$ clusters are coupled to angular momentum zero with the inter-cluster distance $R$. The ground state wave functions of $\alpha$ and $^{16}\text{O}$ clusters ($\Phi_\alpha$ and $\Phi_{^{16}\text{O}}$) are the harmonic oscillator wave functions having the double closed shell configurations. The ket state $\Psi_{^{20}\text{Ne}}$ is the ground state wave function of $^{20}\text{Ne}$ calculated by AMD. The AMD wave function is a superposition of the parity and angular momentum projected Slater determinants, and it was shown that the known experimental properties of the $^{20}\text{Ne}$ ground and excited states such as the radius, energies, electromagnetic transitions, and $\alpha$ decay widths are reasonably described. Once the ground state wave function is obtained, Eq. (1) is calculated by using the Laplace expansion method [12]. For more details of the calculation, readers are directed to the references [6, 9]. It should be noted that other cluster models yield the similar RWA and reasonably reproduce the observed decay widths of the excited states [6, 26–29]. The use of AMD in the present study is aimed to extend our framework to the investigation of the $\alpha$ clustering in unstable nuclei in the future.

We employ the DWIA framework in the present study. Details of DWIA for the description of the knockout reaction can be found in a recent review paper [25]. The incident and emitted protons are labeled as particles 0 and 1, respectively. The wave number and its solid angle, the total and kinetic energies of particle $i$ ($i = 0, 1, \alpha$) are represented by $K_i$, $\Omega_i$, $E_i$, and $T_i$, respectively. Quantities with (without) the superscript L are evaluated in the laboratory (center-of-mass) frame.

The triple differential cross section (TDX) of the $A(p, p\alpha)B$ reaction within the so-called factorized form of the DWIA framework is given by

$$\frac{d^3\sigma}{dE_1^L d\Omega_1^L d\Omega_2^L} = F_{\text{kin}} C_0 \left| \frac{d\sigma_{p\alpha}}{d\Omega_{p\alpha}}(\theta_{p\alpha}, T_{p\alpha}) \right|^2 T_{K_i}. \tag{3}$$

It is essentially a product of the absolute square of the reduced transition matrix $T_{K_i}$ and the $p-\alpha$ two-body differential cross section $d\sigma_{p\alpha}/d\Omega_{p\alpha}$ at a given scattering angle (energy) $\theta_{p\alpha}$ ($T_{p\alpha}$). Since a $p-\alpha$ binary collision in the $(p, p\alpha)$ three body kinematics is in principle off-the-energy-shell scattering, the final-state prescription of the on-shell approximation is adopted; $T_{p\alpha}$ is determined by the $p-\alpha$ relative momentum in the final state. The kinematical factor (or also referred to as the phase space factor) $F_{\text{kin}}$ and a constant $C_0$ are defined by

$$F_{\text{kin}} = \int J_L \left[ \frac{K_1 K_\alpha}{(hc)^2} E_1 E_\alpha \right. \left. \left[ 1 + \frac{E_\alpha}{E_B} \frac{K_1 \cdot K_\alpha}{K_\alpha^2} \right] \right] \tag{4}$$

$$C_0 = \frac{E_0}{(hc)^2 K_0 (2\pi)^2 \mu_{p\alpha}}. \tag{5}$$

where $\mu_{p\alpha}$ is the reduced mass of the $p-\alpha$ binary system and $J_L$ is the Jacobian from the center-of-mass frame to the L frame. The reduced transition matrix is given by

$$T_{K_i} \equiv \int dR F_{K_i}(R) y(R) Y_{00}(\hat{R}), \tag{6}$$

$$F_{K_i}(R) \equiv \chi_i^{\alpha(-)}(R) \chi_\alpha^{\alpha(-)}(R) \chi_0^{00}(R) e^{-iK_i R} A_\alpha / A. \tag{7}$$

where $A_\alpha = 4, A = 20$, and $\chi_i^{\alpha(-)}$ is a distorted wave between particle $i$ and A when $i = 0$, otherwise between $i$ and B. The outgoing and incoming boundary conditions of the scattering waves are specified by the superscript (+) and (−), respectively. Note that the $S_\alpha$ does not appear explicitly in Eqs. (3)–(7) because it is already taken into account in the RWA.

The RWA is calculated by AMD and the Laplace expansion method [12]. The Hamiltonian used in the AMD calculation is given by

$$\hat{H} = \sum_i \hat{t}_i - \hat{c}_\text{em} + \sum_{i<j} \hat{v}_{\text{NN}} + \sum_{i<j} \hat{v}_{\text{Coul}}, \tag{8}$$

where $\hat{t}_i$, $\hat{c}_\text{em}$, $\hat{v}_{\text{NN}}$, and $\hat{v}_{\text{Coul}}$ being the nucleon and the center-of-mass kinetic energies, effective nucleon-nucleon interaction, and the Coulomb interaction, respectively. As for $\hat{v}_{\text{NN}}$, the Gogny D1S interaction [30] is adopted. The obtained RWA [12] (AMD-RWA) is shown in Fig. 1, and its $\alpha$ spectroscopic factor is $S_\alpha = 0.26$.

![FIG. 1. $\alpha + ^{16}\text{O}$ RWA of the $0^+_1$ state taken from Fig. 2 of Ref. 12.](image)

The kinematical condition of the $^{20}\text{Ne}(p, p\alpha)^{16}\text{O}$ reaction is shown in Fig. 2. In the experiment [15] the so-called energy sharing distribution was measured; the emitted angle of p ($\alpha$) is fixed at $-70^\circ$ (46.3°), and the proton emission energy $T_p$ is varied in the range of 40–75 MeV. By the energy conservation, $T_\alpha$ is ranging from 55 to 21 MeV accordingly. The incident energy is set to 101.5 MeV and all the scattering particles are kept in a coplanar. With such kinematical setup the recoil-less
FIG. 2. Kinematical condition of the $^{20}$Ne($p,p\alpha$)$^{16}$O reaction [15].

condition, i.e., the reaction residue is at rest in the final state, is achieved at around $T_p = 67$ MeV ($T_\alpha = 30$ MeV). The TDX has a peak at this condition reflecting the fact that the struck $\alpha$ is bound in the $s$-wave. For the optical potential of the incident and emitted protons, the EDAD1 optical potential of the Dirac phenomenology [31, 32] is adopted. The optical model parametrization by F. Michel et al. [33] is adopted for the $\alpha$-$^{16}$O scattering in the final state. As shown in Fig. 1 of Ref. 33, this parameterization of the $\alpha$-$^{16}$O optical potential provides excellent agreement with the $\alpha$-$^{16}$O elastic scattering data of 30–150 MeV $\alpha$ incident energies.

Since $\theta_p$ and $\theta_\alpha$ are fixed in the present ($p,p\alpha$) kinematics, the required $p-\alpha$ differential cross section lies in the very limited range of $\theta_{pa} = 84^\circ$–$86^\circ$ in the $p-\alpha$ center-of-mass frame and $E_{pa} = 75$–$100$ MeV in the $p-\alpha$ laboratory frame. Its energy and angular dependence are obtained by the microscopic single-folding model [34] with a phenomenological $\alpha$ density and the Melbourne nucleon-nucleon $g$-matrix interaction [35]. As shown in Fig. 3, the calculated $p-\alpha$ differential cross section at 85 MeV (dashed line) deviates from the experimental data [36] by about a factor of 2 at around $\theta_{pa} = 84^\circ$–$86^\circ$. Therefore the cross section scaled by a factor of 2.0 (dotted line) is adopted to the knockout calculation to guarantee a quantitative agreement. It should be noted that this correction is valid thanks to the limited range of required $\theta_{pa}$, which is the outcome of the kinematical condition in which $\theta_p$ and $\theta_\alpha$ are fixed.

In Fig. 4 the energy sharing distribution calculated with the AMD-RWA is shown. The present framework well reproduces both the height and distribution of the experimental data [15] without any additional adjustment. It should be stressed that the quantitative reproduction of the cross section data is guaranteed by the combination of the sophisticated RWA in $^{20}$Ne, $p-\alpha$ cross section, and $\alpha$-$^{16}$O optical potential. In particular, $\alpha$-$^{16}$O optical potential suggested in Ref. 33 which reproduces the low-energy $\alpha$-$^{16}$O differential cross section up to very backward angle was essential for this success of quantitative description. This result indicates that once proper ingredients mentioned above are adopted, the ($p,p\alpha$) reaction can be a quantitative probe for the $\alpha$ amplitude in the ground state of target nuclei.

In order to investigate how the peak height of the energy sharing distribution is contributed by the RWA, three different types of cluster wave functions shown in Fig. 5 are considered as artificial input data. They are constructed by superposing two Brink-Bloch (BB) wave functions [37] of the $\alpha$-$^{16}$O cluster. See text for details.

FIG. 3. $p-\alpha$ differential cross section at 85 MeV obtained by the folding model [34] with the Melbourne $g$-matrix interaction [35] (dashed line). The scaled result at around $\theta_{pa} = 85^\circ$ is shown in dotted line. The experimental data are taken from Ref. 36.

FIG. 4. The comparison between the calculated energy sharing distribution with the AMD-RWA (solid line) and the experimental data taken from Ref. 15.
and the resulting $S_\alpha$ of the three RWAs. As shown in Fig. 5,

### Table I. Coefficients for the BB wave functions. The resulting $S_\alpha$ are also shown.

|       | original | smaller peak | shorter tail |
|-------|----------|--------------|--------------|
| $C_1$ | 0.67     | 0.55         | 0.69         |
| $C_2$ | 0.17     | 0.17         | 0.09         |
| $S_\alpha$ | 0.24     | 0.18        | 0.21         |

$C_i$ of one labeled “original” (dashed) is determined so as to reproduce the largest peak and the tail behavior of the AMD-RWA as much as possible, while for “smaller peak” (dotted) and “shorter tail” (dot-dashed), they are tuned to reduce the peak and tail region of the original RWA, respectively.

To investigate how these RWAs contribute to the TDX at the peak of the energy sharing distribution, it is useful to consider the transition matrix density (TMD) defined by [25]

$$\delta^{\text{Tr}}(R) \equiv \tilde{T}_{K_i} \int d\Omega R^2 F_{K_i}(R)\gamma(R)\gamma_0(R).$$

(9)

From Eqs. (6) and (9) it has the following properties:

$$\int dR \text{Re}[\delta^{\text{Tr}}(R)] = |\tilde{T}_{K_i}|^2,$$

(10)

$$\int dR \text{Im}[\delta^{\text{Tr}}(R)] = 0.$$

(11)

Thus, one may regard $\text{Re}[\delta^{\text{Tr}}(R)]$ as a radial distribution of the cross section and its integrated value is proportional to the TDX. In Fig. 6 $\text{Re}[\delta^{\text{Tr}}(R)]$ at the recoil-less condition ($T_p = 67$ MeV) is shown. It is clearly seen that the internal region of the RWAs are suppressed by the absorption effect and the surface region contributes to the $(p,pa)$ cross section. In Fig. 7 the energy sharing distributions with the RWAs of Fig. 5 are shown. Due to the peripherality of the reaction the peak height of the energy sharing distribution is reduced significantly in the shorter tail case (dot-dashed), while it is similar to the original one in the smaller peak case (dotted). Another response of the difference in the RWAs is the width of the energy sharing distribution. Since the RWA with smaller peak (shorter tail) has narrower (wider) momentum distribution, its energy sharing distribution has a narrower (wider) width accordingly. From these results it is shown that the $(p,pa)$ cross section is a quantitative probe for the $\alpha$ cluster state, putting emphasis on the $\alpha$ amplitude around the nuclear surface.

In summary, the $^{20}\text{Ne}(p,\alpha)^{16}\text{O}$ reaction at 101.5 MeV was investigated within the DWIA framework. AMD was adopted to describe the $\alpha^{16}\text{O}$ cluster state of $^{20}\text{Ne}$ and its RWA was obtained by the Laplace expansion method. The existing experimental data were compared with the theoretical calculation of the present study, and it was found that once quantitatively reliable $p-\alpha$ differential cross section and distorting potentials are employed together with the AMD-RWA, the DWIA calculation well reproduces the experimental data without any additional correction or scaling in describing the $(p,pa)$ cross section.

Through the analyses using the BB wave functions with the different spatial distribution and corresponding $\alpha$ spectroscopic factors, it was shown that the surface region of the target is selectively probed by the $^{20}\text{Ne}(p,\alpha)^{16}\text{O}$ reaction. Therefore the $(p,pa)$ cross section is a good probe for the surface amplitude of the cluster wave function, which should be directly related to the $\alpha$ clustering of interest. One may also deduce the momentum distribution of the $\alpha$ cluster wave function from the width of the energy sharing distribution. It would be beneficial if the $(p,pa)$ cross section is measured very precisely down to the tail region, which will make it possible to settle or put constraints on the momentum distribution of $\alpha$ cluster wave function.

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