SO(10) unification with horizontal symmetry

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Abstract

We extend the non-supersymmetric SO(10) grand unification theories by adding a horizontal symmetry, which connects the three generations of fermions. Without committing to any specific symmetry group, we re-investigate the 1-loop renormalization group evolutions of the gauge couplings with one and two intermediate breaking scales. We find that depending on the SO(10) breaking chains, gauge coupling unification is compatible with only a handful of choices of representations of the Higgs bosons under the horizontal symmetry.

1 Introduction

The quest for the unification of fundamental forces remains to be a major motivation over the last few decades for investigating theories beyond the Standard Model. Although the smallest possible group, SU(5) \cite{1}, has been ruled out, the idea of unification is very much alive, resting on supersymmetric versions, and/or bigger unification groups like SO(10) \cite{2–14}. In fact, SO(10) is the smallest unification group that contains all fermion fields of a single generation in one irreducible multiplet (irrep).

However, even a group like SO(10) cannot explain why there are three generations of fermions. Three copies of the irrep are obviously needed for three generations, though the number is theoretically arbitrary. Therefore, there is a lot of speculation whether an enhanced symmetry might shed some extra light on the number of fermion generations.

In this paper, we consider the possibility that the three fermion generations form a multiplet under some horizontal symmetry group $H$ that appears as a direct product with SO(10). In other words, the symmetry group of our model is $SO(10) \times H$. We will examine the unification of the gauge coupling constants of the Standard Model (SM) into SO(10). In this pursuit, we do not need to assume anything specific about the nature of $H$. For example, we do not need to know whether $H$ is a global symmetry or a gauge symmetry. Even more, we do not

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even need to specify whether it is a discrete group or a continuous group, or whether the three fermion generations comprise an irrep under this group. Multiplets of $H$ will only contribute in our analysis by providing multiple copies of the Higgs boson representations, which affect the evolution of the couplings of SM. Notably, several earlier attempts have been made to use horizontal symmetries in explaining the flavor hierarchy in the fermion sector of the SM [15–31]. However, in most of those cases, the horizontal symmetry is imposed on the fermion mass matrix to generate the required texture. Our approach in unifying the three generations, on the contrary is based on writing $H$-invariant renormalizable operators involving only SO(10) Higgs fields. The motivation of this analysis is to see, without delving into the flavor issues, what kinds of horizontal symmetries are compatible with the requirement of unification and can provide impetus for addressing the fermion masses and mixing.

The paper is organized as follows. In Sec. 2, we outline the general features of the model, with a short description of SO(10) unification, followed by how the horizontal symmetry group works. In Sec. 3, we introduce the various chains of symmetry breaking, indicating the Higgs boson representation whose vacuum expectation value (VEV) is responsible for each breaking, and the masses of the scalar bosons. In Sec. 4, we present all of our results for the symmetry breaking scales. These results show which chains are compatible with unification, and what would be the various scales of symmetry breaking in these chains. We end with some comments and outlook in Sec. 5.

2 SO(10) with horizontal symmetry

In SO(10) grand unified theory (GUT), all left-chiral fermion and antifermion fields belonging to the same generation constitute a 16-dimensional irrep. All fermions obtain mass at the level of electroweak symmetry breaking. The only exception is the left-chiral antineutrino field (conjugate of the right-chiral neutrino field), which is a singlet under the SM gauge group, and can therefore obtain a Majorana mass at a higher level of symmetry breaking.

The model that we explore in this paper has a symmetry group SO(10) × $H$, where $H$ acts between generations. As stated in the Introduction, we do not specify what this symmetry is, or whether it also breaks along with the gauge symmetry at different stages of symmetry breaking. The main relevance of the horizontal symmetry, in our calculation, is the multiplicity of various Higgs boson representations demanded by the symmetry, which affect the renormalization group (RG) evolution of the gauge coupling constants of SO(10).

To explain this point, let us consider the couplings of the fermion multiplets with the scalars, which can be generically written as $\Psi \Psi \Phi$. Since the fermions of a single generation transform like a 16 representation of SO(10), the Higgs bosons coupling to the fermions must belong to the direct product $16 \times 16$ in order that the coupling can be a singlet of SO(10). In SO(10),

$$16 \times 16 = 10 + 120 + 126,$$

(2.1)

of which the first and the last irreps shown on the right side constitute symmetric combinations, while the 120 is antisymmetric. We will consider only the symmetric combinations, since the 120 is not necessary for conducting the gauge symmetry breaking of SO(10) down to the electroweak symmetry breaking level.

The three generations of fermions must transform like a 3-dimensional representation of the horizontal symmetry group. As already stated, it is not relevant for our analysis whether it is reducible or irreducible under $H$. 


In writing the Yukawa couplings $\Psi \Psi \Phi$, we have omitted all indices. There are the Lorentz indices, the SO(10) indices, as well as the generation indices. The combination $\Psi \Psi$ should more explicitly be written as $\Psi^\top C \Psi$ where $C$ is a matrix, defined in such a way that $\Psi^\top C$ transforms like $\Psi$ under Lorentz transformation. The combination $\Psi^\top C \Psi$ must be antisymmetric in the two $\Psi$'s because the matrix $C$ is always antisymmetric, irrespective of the representation used for the Dirac matrices [32]. We have chosen to work with symmetric SO(10) combinations. Thus, according to the Pauli principle which dictates the overall exchange property to be antisymmetric with respect to the exchange of the two $\Psi$'s, the combination $\Psi \Psi$ must be symmetric under the horizontal group.

This requirement does not uniquely determine the dimensionality of the scalar representations under the horizontal group. As we said earlier, we make no assumption regarding whether the three generations of fermions constitute an irrep of $H$. All we know, without specifying the horizontal group, is that the generations form a 3-dimensional representation, which can be irreducible or reducible. Specification of the horizontal group then dictates whether that 6-dimensional representation, obtained by taking the symmetric product of two 3-dimensional representations, is irreducible or reducible. Accordingly, the representation of $\Phi$ might be irreducible, or might be one of the irreps appearing in the product $\Psi \Psi$.

It is easy to see that different choices of the group $H$ can lead to all sorts of dimensions for the irreps. For example, if the horizontal symmetry group is SU(3), the fermion generations form a 3-dimensional irrep, and the symmetric product is a 6-dimensional irrep. If the symmetry group is SO(3) and the fermion generations form an irrep, the symmetric product contains a 5-dimensional and a 1-dimensional irrep. However, in this case there is also the possibility that the fermion generations do not form an irrep. If they transform as a $2 + 1$ dimensional representation, i.e., one 2-dimensional irrep and a 1-dimensional irrep, then the symmetric product includes 3, 2 and 1 dimensional irreps. In the discrete group $A_4$, the symmetric product of $3 \times 3$ contains a 3-dimensional irrep, plus three different 1-dimensional irreps. If we choose $S_3$ as the horizontal group, which does not have a 3-dimensional irrep, the representation of the fermions can at best be $2 + 1$ dimensional. The symmetric product of such a reducible representation with itself contains both 2 and 1. In short, from symmetry consideration alone, we see anything from a 1-dimensional up to a 6-dimensional representation is allowed for $\Phi$.

If we use a $n$-dimensional representation for $\Phi$, with $1 \leq n \leq 6$, then there are $n$ copies of the SO(10) Higgs multiplets of that representation. These particles participate in the RG evolution above their mass thresholds, and affect gauge coupling unification. The demand of unification imposes what kind of representations are allowed for $\Phi$.

So far, we have been talking about Higgs multiplets which participate in Yukawa interaction. Symmetry breaking of the grand unified group SO(10) require other Higgs multiplets as well, as we will see in Section 3. All other Higgs bosons, which do not couple to fermions, are assumed to be singlets of the horizontal group in our discussion.

### 3 SO(10) breaking chains

SO(10) might break to the SM gauge group through only one intermediate group, i.e., the symmetry breaking chain may be of the form

$$
\text{SO}(10) \xrightarrow{M_c} G_1 \xrightarrow{M_1} \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \xrightarrow{M_2} \text{SU}(3)_c \times \text{U}(1)_{\text{em}},
$$

(3.1)
Table 1: Possible chains of SO(10) breaking with one intermediate scale. $G_U$ is the unification group. In the column SSB1, the number above the arrow sign gives the irrep of SO(10) whose VEV is responsible for the spontaneous symmetry breaking. The symbols below the arrows represent the multiplets of $G_1$ which contribute to the RG evolution as long as $G_1$ is unbroken. Breaking of $G_1$ is given in Eq. (3.3).

| Chain | $G_U$ | SSB1 | $G_1$ |
|-------|-------|------|-------|
| 1     | SO(10) | (210) | $\{2L2R4C\}$ |
|       |        | (2,2,1)_{10} (1,3,10)_126/(1,2,4)_{16} | |
| 2     | SO(10) | (54) | $\{2L2R4C\}$ |
|       |        | (2,2,1)_{10} (1,3,10)_126/(1,2,4)_{16} | |
| 3     | SO(10) | (45) | $\{2L2R1X3_{c}\}$ |
|       |        | (2,2,0,1)_{10} (1,3,1,1)_126/(1,2,1/2,1)_{16} | |
| 4     | SO(10) | (210) | $\{2L2R1X3_{c}\}$ |
|       |        | (2,2,0,1)_{10} (1,3,1,1)_126/(1,2,1/2,1)_{16} | |
| 5     | SO(10) | (45) | $\{2L1R4C\}$ |
|       |        | (2,1/2,1)_{10} (1,1,1,0)_126/(1,−1/2,4)_{16} | |
| 6     | SO(10) | (210) | $\{2L1R1X3_{c}\}$ |
|       |        | (2,1/2,0,1)_{10} (1,1,−1,1)_126/(1,−1/2,4)_{16} | |

where the symbols above the arrows represent the symmetry breaking scales. One possibility is that $G_1 = SU(5)$. We ignore this possibility, since then gauge coupling unification must happen at the SU(5) level, something that has been ruled out by precision data in non-supersymmetric models. The other possible choices of $G_1$ have been shown in Table 1.

From SO(10), the symmetry breaking can also go through two intermediate scales down to the SM gauge group, i.e., one can consider symmetry breaking chains like

$$SO(10) \xrightarrow{M_U} G_2 \xrightarrow{M_2} G_1 \xrightarrow{M_1} SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{M_2} SU(3)_c \times U(1)_{em},$$

with various choices for the intermediate groups $G_1$ and $G_2$. These possibilities have been shown in Table 2. Breaking chains with more than two intermediate scales have been considered in the literature [33] as well, but we do not discuss them.

In writing Table 1 and Table 2 as well as in subsequent discussions, we have adopted some shorthand notation used in many earlier articles on SO(10) breaking [7, 8, 14]. For the sake of completeness, we briefly explain the notations here.

- For example, let us look at the first chain given in Table 1. It shows that the intermediate gauge group is $2L2R4C$. This stands for $SU(2)_L \times SU(2)_R \times SU(4)_C$, where the SU(4) factor stands for the Pati-Salam group [34] which treats leptons as a fourth “color”, $SU(2)_L$ belongs to the SM electroweak gauge group, and the other one is a right-handed SU(2)
Table 2: Possible chains of SO(10) breaking with two intermediate scales.

| Chain | $G_U$ | SSB2 | $G_2$ | SSB1 | $G_1$ |
|-------|-------|------|-------|------|------|
| I     | SO(10) | $\begin{pmatrix} 210 \end{pmatrix}$ | $\{2L^2R^4C\}$ | $\begin{pmatrix} 45 \end{pmatrix}$ | $\{2L^2R^1_X3c\}$ |
|   |       | $\begin{pmatrix} (2,1,1)_{10} \\ (1,3,10)_{126}/(1,2,4)_{16} \end{pmatrix}$ |       | $\begin{pmatrix} (2,2,0,1)_{10} \\ (1,3,1,1)_{126}/(1,2,1,2)_{16} \end{pmatrix}$ |       |
| II    | SO(10) | $\begin{pmatrix} 54 \end{pmatrix}$ | $\{2L^2R^4CP\}$ | $\begin{pmatrix} 210 \end{pmatrix}$ | $\{2L^2R^1_X3P\}$ |
|   |       | $\begin{pmatrix} (2,1,1)_{10} \\ (1,3,10)_{126}/(1,2,4)_{16} \end{pmatrix}$ |       | $\begin{pmatrix} (2,2,0,1)_{10} \\ (1,3,1,1)_{126}/(1,2,1,2)_{16} \end{pmatrix}$ |       |
| III   | SO(10) | $\begin{pmatrix} 54 \end{pmatrix}$ | $\{2L^2R^4CP\}$ | $\begin{pmatrix} 45 \end{pmatrix}$ | $\{2L^2R^1_X3c\}$ |
|   |       | $\begin{pmatrix} (2,1,1)_{10} \\ (1,3,10)_{126}/(1,2,4)_{16} \end{pmatrix}$ |       | $\begin{pmatrix} (2,2,0,1)_{10} \\ (1,3,1,1)_{126}/(1,2,1,2)_{16} \end{pmatrix}$ |       |
| IV    | SO(10) | $\begin{pmatrix} 210 \end{pmatrix}$ | $\{2L^2R^1_X3P\}$ | $\begin{pmatrix} 45 \end{pmatrix}$ | $\{2L^2R^1_X3c\}$ |
|   |       | $\begin{pmatrix} (2,2,0,1)_{10} \\ (1,3,1,1)_{126}/(1,2,1,2)_{16} \end{pmatrix}$ |       | $\begin{pmatrix} (2,2,0,1)_{10} \\ (1,3,1,1)_{126}/(1,2,1,2)_{16} \end{pmatrix}$ |       |
| V     | SO(10) | $\begin{pmatrix} 54 \end{pmatrix}$ | $\{2L^2R^4C\}$ | $\begin{pmatrix} 45 \end{pmatrix}$ | $\{2L^1R^4C\}$ |
|   |       | $\begin{pmatrix} (2,1,1)_{10} \\ (1,3,10)_{126}/(1,2,4)_{16} \end{pmatrix}$ |       | $\begin{pmatrix} (2,2,1)_{10} \\ (1,3,1,1)_{126}/(1,2,1,2)_{16} \end{pmatrix}$ |       |
| VI    | SO(10) | $\begin{pmatrix} 54 \end{pmatrix}$ | $\{2L^2R^4CP\}$ | $\begin{pmatrix} 45 \end{pmatrix}$ | $\{2L^1R^4C\}$ |
|   |       | $\begin{pmatrix} (2,1,1)_{10} \\ (1,3,10)_{126}/(1,2,4)_{16} \end{pmatrix}$ |       | $\begin{pmatrix} (2,2,1)_{10} \\ (1,3,1,1)_{126}/(1,2,1,2)_{16} \end{pmatrix}$ |       |
| VII   | SO(10) | $\begin{pmatrix} 54 \end{pmatrix}$ | $\{2L^2R^4CP\}$ | $\begin{pmatrix} 210 \end{pmatrix}$ | $\{2L^2R^4C\}$ |
|   |       | $\begin{pmatrix} (2,1,1)_{10} \\ (1,3,10)_{126}/(1,2,4)_{16} \end{pmatrix}$ |       | $\begin{pmatrix} (2,2,1)_{10} \\ (1,3,1,1)_{126}/(1,2,1,2)_{16} \end{pmatrix}$ |       |
| VIII  | SO(10) | $\begin{pmatrix} 45 \end{pmatrix}$ | $\{2L^2R^1_X3c\}$ | $\begin{pmatrix} 45 \end{pmatrix}$ | $\{2L^1R^1_X3c\}$ |
|   |       | $\begin{pmatrix} (2,2,0,1)_{10} \\ (1,3,1,0)_{126}/(1,2,4)_{16} \end{pmatrix}$ |       | $\begin{pmatrix} (2,2,0,1)_{10} \\ (1,3,1,0)_{126}/(1,2,4)_{16} \end{pmatrix}$ |       |
| IX    | SO(10) | $\begin{pmatrix} 210 \end{pmatrix}$ | $\{2L^2R^1_X3P\}$ | $\begin{pmatrix} 45 \end{pmatrix}$ | $\{2L^1R^1_X3c\}$ |
|   |       | $\begin{pmatrix} (2,2,0,1)_{10} \\ (1,3,1,1)_{126}/(1,2,1,2)_{16} \end{pmatrix}$ |       | $\begin{pmatrix} (2,2,0,1)_{10} \\ (1,3,1,1)_{126}/(1,2,1,2)_{16} \end{pmatrix}$ |       |
| X     | SO(10) | $\begin{pmatrix} 210 \end{pmatrix}$ | $\{2L^2R^4C\}$ | $\begin{pmatrix} 210 \end{pmatrix}$ | $\{2L^1R^1_X3c\}$ |
|   |       | $\begin{pmatrix} (2,2,0,1)_{10} \\ (1,3,10)_{126}/(1,2,4)_{16} \end{pmatrix}$ |       | $\begin{pmatrix} (2,2,0,1)_{10} \\ (1,3,1,0)_{126}/(1,2,4)_{16} \end{pmatrix}$ |       |
| XI    | SO(10) | $\begin{pmatrix} 54 \end{pmatrix}$ | $\{2L^2R^4CP\}$ | $\begin{pmatrix} 210 \end{pmatrix}$ | $\{2L^1R^1_X3c\}$ |
|   |       | $\begin{pmatrix} (2,1,1)_{10} \\ (1,3,10)_{126}/(1,2,4)_{16} \end{pmatrix}$ |       | $\begin{pmatrix} (2,2,0,1)_{10} \\ (1,3,1,0)_{126}/(1,2,4)_{16} \end{pmatrix}$ |       |
| XII   | SO(10) | $\begin{pmatrix} 45 \end{pmatrix}$ | $\{2L^1R^4C\}$ | $\begin{pmatrix} 45 \end{pmatrix}$ | $\{2L^1R^1_X3c\}$ |
|   |       | $\begin{pmatrix} (2,1,1)_{10} \\ (1,1,10)_{126}/(1,2,4)_{16} \end{pmatrix}$ |       | $\begin{pmatrix} (2,2,0,1)_{10} \\ (1,1,10)_{126}/(1,2,4)_{16} \end{pmatrix}$ |       |
under which the right-chiral quark and lepton fields transform separately as doublets. Similarly, presence of the symbol $3_c$ would imply the SU(3) gauge group of QCD. The direct product factor $P$, which appears in some chains, is a discrete symmetry between the two SU(2) factors, which ensures that the coupling constants of the two SU(2)’s are equal.

- As for $1_X$ which is a U(1) subgroup, the quantum numbers shown in Table 1 and Table 2 equal to $(B - L)/2$. With this definition, the normalization of $X$ does not agree with that of the non-Abelian factors, so the quantum numbers will have to be multiplied by a factor of $\sqrt{3/2}$ in order to be used in RG equations. For the $1_R$ subgroup however, we put the eigenvalues of the corresponding generator of the $2_R$ subgroup, which have the proper normalization.

- In Table 1, the column bearing the heading ‘SSB1’ contains information about symmetry breaking and masses of the Higgs bosons. The number above the arrow gives the SO(10) representation which has a neutral component whose VEV can perform the desired symmetry breaking. Of course these multiplets have VEVs at the unification scale, and therefore all their components are expected to have masses at the unifications scale, meaning that they do not affect the RG equations. But, in the regime below the SO(10) breaking scale where the group $G_1$ is the unbroken gauge group, the RG equations contain contributions from some Higgs boson submultiplets which contain VEVs that affect one of the lower stages of symmetry breaking. They have been shown below the arrow, indicating their transformations under the unbroken group at that stage, and marked with a subscript that tells us which SO(10) representation contains them. The rationale for choosing the masses of the Higgs bosons will be described in Section 4.

- We have not shown in Table 1 which multiplet of SO(10) breaks the intermediated symmetry down to the SM symmetry group, and further trigger the electroweak symmetry breaking. This is because this part is the same for all chains, and is given by

\[
G_1 \xrightarrow{(126/16)} 2L1Y3c \xrightarrow{(10)} 1Q3c.
\]  

As seen here, we have considered two alternatives for breaking $G_1$ to the SM gauge group, separated by a slash in Eq. (3.3). One is by the 126-dimensional representation of SO(10), which has Yukawa coupling with fermions. The other is by using the 16-dimensional irrep, which does not have Yukawa coupling at the tree-level, but can contribute to fermion masses through loops [35]. In subsequent tables and discussions, for the first alternative, the chain is specified by adding the letter ‘a’, whereas the letter ‘b’ is added for the second alternative.

- The same notations are used for chains with two intermediate scales. The only difference is that, in this case, one needs to specify the Higgs boson multiplets which perform the intermediate scale symmetry breaking, $G_2 \rightarrow G_1$. Thus, the column with the heading ‘SSB2’ contains information about the Higgs multiplet that performs the breaking SO(10) $\rightarrow G_2$, and the submultiplets of $G_2$ which we consider for the RG equations above the $G_2$-breaking scale. Similarly, the column marked ‘SSB1’ contains information about the multiplet of SO(10) that is responsible for the breaking $G_2 \rightarrow G_1$, and the submultiplets of $G_1$ which are assumed to be light above the $G_1$-breaking scale.
4 Results

4.1 Outline of the strategy

The 1-loop RG evolution of the gauge coupling $g$ for an SU($N$) factor above the weak scale is governed by the equation

$$\frac{d\omega}{d \ln \mu} = \frac{1}{2\pi} \left( \frac{11}{3} N - 4 - \frac{T(S)}{6} \right),$$

(4.1)

where

$$\omega = \frac{4\pi}{g^2},$$

(4.2)

and $T(S)$ is the scalar contribution. For a U(1) factor in the symmetry group, one should take $N = 0$ in Eq. (4.1). Note that, in writing the fermion contribution in Eq. (4.1), we assumed for simplicity that symmetry breaking scales are heavier than the masses of all fermions which obtain masses at that stage of breaking. In fact, we consider the right-handed neutrinos to have Majorana masses which are smaller than $M_{1}$, the scale at which all symmetries specific to the right-handed fermions break down in the chain of symmetry breaking from SO(10), i.e. below which the SM group appears.

For the scalar contribution, we need to have some idea of the Higgs boson masses. As is usual practice in this kind of analysis, we use the extended survival hypothesis [36,37] to estimate the masses. This means that, at any stage of symmetry breaking, the entire submultiplet containing the VEV obtains mass at that scale, and any particle not controlled by this rule obtains mass at the unification scale. We have already listed, in Table 1 and Table 2, the Higgs boson submultiplets which contribute to $T(S)$ in all different regimes. The overall contributions to $T(S)$ for all intermediate symmetry groups have been listed in earlier literature [7,8] where only one 10-dimensional and one 126-dimensional SO(10) multiplets of Higgs boson were considered. For the present purpose, all we need is to multiply the contributions of 10 and 126 irreps by the appropriate number of multiplets.

So we start with the values of the gauge coupling constants of the SM gauge group at the Z-scale [38]:

$$\omega_{1Y}(M_Z) = 59.042 \pm 0.003,$$

(4.3a)

$$\omega_{2L}(M_Z) = 29.596 \pm 0.005,$$

(4.3b)

$$\omega_{3c}(M_Z) = 8.47 \pm 0.02.$$  

(4.3c)

As has already been said, the evolution depends on the chain of symmetry breaking, for which we use the nomenclature used by earlier authors and repeated here in Table 1 and Table 2. In addition, the evolution depends on the numbers of 10 and 126 irreps of Higgs bosons used, which we will denote by $r_{10}$ and $r_{126}$ respectively. For each chain, and each choice of $r_{10}$ and $r_{126}$, we employ the following checks to determine whether a given chain is allowed.

- There must be a solution for all gauge couplings meeting at a scale.
- When a mathematical solution is obtained, it must be physically meaningful. For example, in Eq. (3.2), if SO(10) breaks at $M_U$, whereas $G_2$ and $G_1$ break at the scales $M_2$ and $M_1$
respectively, one must have
\[ n_U > n_2 > n_1 > n_Z, \]  
where the \( n_i \)'s represent a logarithmic notation that we use for the various energy scales:
\[ n_i = \log_{10}\left( \frac{M_i}{1 \text{ GeV}} \right). \]  

\[ (4.4) \]

Further, one must have \( M_U \lesssim 10^{18} \text{ GeV} \) because gravity effects become strong at higher scales, and the analysis that ignores gravity makes no sense.

The unification scale must be consistent with proton decay bound. The lifetime of proton in terms of the GUT scale and couplings can be written as
\[ \tau_p \simeq \frac{\omega^2 M_U^4}{m_p^5}, \]  
where \( m_p \) denotes proton mass. Present limit on the proton lifetime is [38]
\[ \tau_p(p \rightarrow e^+ \pi^0) > 1.6 \times 10^{34} \text{ yr}. \]  
\[ (4.7) \]

However, taking cue from [14], we have made an order of magnitude estimate to accommodate the higher-loop effects in the proton decay bound. Our estimate shows that proton lifetime as calculated using 1-loop results for the GUT parameters can be enhanced by two orders of magnitude if 2-loop effects are incorporated. In view of this conservative estimation, we also allow those chains which satisfy
\[ \tau_p(p \rightarrow e^+ \pi^0) > 1.6 \times 10^{32} \text{ yr}. \]  
\[ (4.8) \]

We clearly mark which chains satisfy Eq. (4.7) and which ones satisfy only this relaxed bound, Eq. (4.8).

All couplings are consistent with the perturbative limit
\[ \omega > \frac{1}{4\pi}. \]  
\[ (4.9) \]

If any one of these conditions is not satisfied for a chain, the chain is ruled out, and its details are not given in the tables.

In our calculations, whenever it is relevant, we take kinetic mixing of U(1) gauge factors into account, something that was not done in the early papers on the subject but was incorporated in the later papers [14,39–41]. Our results for the allowed chains, and the allowed values of various scales of symmetry breaking, are presented in the rest of this section. For the sake of convenience, we divide the discussion into two parts: one in which there is only one intermediate scale, and the other in which there are two such scales.
Table 3: Results for chains with a-type symmetry breaking with one intermediate scale: intermediate scale ($n_1$), unification scale ($n_U$) and unification coupling ($\omega_U$) have been shown for each case where we find an acceptable solution. Nomenclatures for the chains can be read off from Table 1. A downarrow to the right of the value of $n_U$ indicates that the given combination of unification scale and coupling does not satisfy the proton lifetime bound in Eq. (4.7), but allowed by the conservative bound in Eq. (4.8).

| ($r_{10}$, $r_{126}$) | Quantity | Breaking chains |
|----------------------|----------|----------------|
| (1, 1)               | $n_1$    | 11.75 13.71 9.02 10.11 |
|                      | $n_U$    | 16.06 15.22↓ 16.66 15.77 |
|                      | $\omega_U$ | 45.70 41.20 46.19 43.89 |
| (1, 2)               | $n_1$    | 6.16 |
|                      | $n_U$    | 16.54 |
|                      | $\omega_U$ | 45.89 |
| (2, 1)               | $n_1$    | 12.19 10.24 10.95 |
|                      | $n_U$    | 15.53↓ 15.90 15.29↓ |
|                      | $\omega_U$ | 44.11 44.23 42.66 |
| (2, 2)               | $n_1$    | 8.28 |
|                      | $n_U$    | 15.75 |
|                      | $\omega_U$ | 43.84 |
| (2, 3)               | $n_1$    | 4.49 |
|                      | $n_U$    | 15.46↓ |
|                      | $\omega_U$ | 43.09 |
| (3, 1)               | $n_1$    | 4.49 |
|                      | $n_U$    | 15.30↓ |
|                      | $\omega_U$ | 42.69 |

Table 4: Results for chains with b-type symmetry breaking with one intermediate scale. Notations are same as in Table 3.

| ($r_{10}$) Quantity | Breaking chains |
|---------------------|----------------|
| (1)                 | $n_1$ 13.63 13.71 10.41 10.67 |
|                      | $n_U$ 15.39↓ 15.35↓ 16.72 16.49 |
|                      | $\omega_U$ | 45.07 44.64 46.34 45.75 |
| (2)                 | $n_1$ 11.23 11.40 |
|                      | $n_U$ 15.97 15.82 |
|                      | $\omega_U$ | 44.42 44.02 |
| (3)                 | $n_1$ 11.89 12.00 |
|                      | $n_U$ 15.37↓ 15.27↓ |
|                      | $\omega_U$ | 42.89 42.62 |
4.2 One intermediate scale

If there is only one intermediate scale, the possibilities of the intermediate gauge group have been shown in Table 1. The results obtained from the 1-loop RG equations have been summarized in two tables, where we show the intermediate scale, the unification scale and the unification coupling for all chains which admit an acceptable solution. In Table 3, we consider the SM symmetry group to appear through the breaking using 126 multiplet of Higgs bosons, whereas in Table 4, we consider the possibility that the said intermediate symmetry breaking occurs through a 16 multiplet of Higgs bosons. If any chain does not appear in these tables, or there is no entry for the scales corresponding to any chain, it means that there is no solution consistent with the criteria outlined earlier.

For all chains, the values $r_{10} = r_{126} = 1$ correspond to the model with no horizontal symmetry. The differences of our results with that obtained earlier [7] using 126-irrep are because of two reasons. First, we use updated inputs for the couplings at the weak scale, given in Eq. (4.3) and for the constraint on proton lifetime. Second, while we have performed only a 1-loop calculation here, 2-loop results are available in the literature [7]. However, note that we keep some room for the 2-loop effects using some simple estimates as mentioned earlier.

We now highlight the new results found in the present paper. First, in a-type chains, we analyze the effects of the values of $r_{10}$ and $r_{126}$ larger than 1, which are necessary for discussing horizontal symmetry. Second, cases for b-type symmetry breaking [42, 43], which use a 16 instead of a 126 for breaking down to the level of the SM symmetry group, have been analyzed here with $r_{10} > 1$.

Note that we obtain no solution at all for chains 5 and 6 in Table 1. This means that, if there is only one intermediate stage between the GUT group and the SM group, then that intermediate stage must contain the full SU(2)$^R$ symmetry.

If we look at models where at least one of $r_{10}$ and $r_{126}$ is bigger than 1, we see that chain 2 is also ruled out, and chain 1a is allowed only when $r_{10} = 2$. This means that the full SU(4)$^C$ symmetry at the intermediate stage is disfavored, or equivalently that the SU(3)$^C$ group of QCD should appear right at the grand unified symmetry breaking.

It is also seen that there is no solution in any symmetry breaking chain when either $r_{10}$ and $r_{126}$ is bigger than 3. The intermediate group $2_L 2_R 1_X 3_c$ seems to be most suitable, in the sense that it is most flexible with the numbers of the Higgs boson multiplets. Its companion version with the extra parity symmetry does equally well for b-type chains, but not so well with the a-type chains.

4.3 Two intermediate scales

We now turn to the symmetry breaking chains which involve two intermediate stages. As is well known, it is not possible to uniquely solve the values of the various breaking scales in this case. Since there are only three boundary conditions corresponding to the experimentally measured values of the SM gauge couplings at the weak scale, while there are four unknown variables, namely $n_1$, $n_2$, $n_U$ and $\omega_U$, one can at most find some regions allowed by the selection criteria for each of them. All solutions which pass these tests have been listed. Table 5 contain results for a-type breaking chains, whereas Table 6 contain results for b-type breaking chains. Since the number of possible chains is huge, we had to break each of these tables into two parts for
Table 5: The range of allowed values for a-type symmetry breaking with two intermediate scales. Nomenclatures for the chains can be read off from Table 2. In case of $n_U$, an arrow on either side of the separator points to the combination of unification scale and coupling that does not satisfy the proton lifetime bound in Eq. (4.7), but allowed by the conservative bound in Eq. (4.8).

| Chains Ia to Va | Quantity | Ia          | IIa         | IIIa        | IVa         | Va          |
|-----------------|----------|-------------|-------------|-------------|-------------|-------------|
| **(r_{10}, r_{126})** |          |             |             |             |             |             |
| **(1, 1)**      |          |             |             |             |             |             |
| $n_1$           | 9.02 – 11.17 | 10.12 – 13.70 | 9.02 – 13.70 | 9.02 – 10.11 | 11.36 – 11.58 |
| $n_2$           | 16.66 – 11.19 | 15.77 – 13.71 | 16.66 – 13.72 | 16.65 – 10.15 | 13.63 – 11.66 |
| $n_U$           | 16.66 – 16.75 | 15.77 – 15.35 | 16.66 – 15.36 | 16.66 – 15.77 | 15.10 – 16.00 |
| $\omega_U$      | 46.19 – 46.42 | 43.89 – 41.03 | 46.19 – 41.03 | 46.19 – 43.91 | 44.75 – 45.62 |
| **(1, 2)**      |          |             |             |             |             |             |
| $n_1$           | 2.00 – 6.15 | 12.66 – 13.71 | 6.16 – 13.71 | 6.16 – 9.17  |             |
| $n_2$           | 15.94 – 16.54 | 14.02 – 13.71 | 16.54 – 13.71 | 16.54 – 10.49 |             |
| $n_U$           | 16.66 – 16.75 | 15.77 – 15.32 | 16.66 – 15.22 | 16.54 – 15.13 |             |
| $\omega_U$      | 46.87 – 45.89 | 38.59 – 37.51 | 45.88 – 37.51 | 45.88 – 42.25 |             |
| **(1, 3)**      |          |             |             |             |             |             |
| $n_1$           | 10.24 – 11.79 | 10.95 – 12.72 | 10.24 – 13.34 | 10.24 – 10.95 | 11.91 – 12.07 |
| $n_2$           | 15.89 – 11.79 | 15.29 – 14.28 | 15.90 – 13.94 | 15.87 – 10.98 | 12.99 – 12.09 |
| $n_U$           | 15.90 – 16.02 | 15.29 – 15.13 | 15.90 – 15.13 | 15.89 – 15.29 | 15.12 – 15.50 |
| $\omega_U$      | 44.23 – 44.53 | 42.66 – 41.32 | 44.22 – 40.96 | 44.22 – 42.67 | 43.80 – 44.07 |
| **(2, 1)**      |          |             |             |             |             |             |
| $n_1$           | 2.00 – 8.28 | 2.00 – 8.28  | 2.00 – 8.28  | 8.28 – 9.55  |             |
| $n_2$           | 14.34 – 15.75 | 14.34 – 15.75 | 14.34 – 15.75 | 15.74 – 12.78 |             |
| $n_U$           | 17.35 – 15.75 | 17.35 – 15.75 | 17.35 – 15.75 | 15.75 – 15.12 |             |
| $\omega_U$      | 44.64 – 43.84 | 44.64 – 43.84 | 44.64 – 43.84 | 43.84 – 42.24 |             |
| **(2, 2)**      |          |             |             |             |             |             |
| $n_1$           | 2.00 – 4.49 | 2.00 – 4.49  | 2.00 – 4.49  | 4.49 – 5.84  |             |
| $n_2$           | 15.39 – 15.46 | 15.45 – 14.55 | 15.45 – 14.55 | 15.45 – 14.18 |             |
| $n_U$           | 15.72 – 15.46 | 15.45 – 15.14 | 15.45 – 15.14 | 15.45 – 15.12 |             |
| $\omega_U$      | 43.05 – 43.09 | 43.09 – 39.04 | 43.09 – 42.24 |             |             |
| **(2, 3)**      |          |             |             |             |             |             |
| $n_1$           | 11.19 – 12.29 | 11.19 – 11.99 | 11.19 – 11.99 | 11.19 – 11.38 |             |
| $n_2$           | 15.30 – 12.30 | 15.30 – 14.80 | 15.30 – 13.75 |             |             |
| $n_U$           | 15.30 – 15.42 | 15.30 – 15.13 | 15.30 – 15.13 |             |             |
| $\omega_U$      | 42.69 – 43.00 | 42.69 – 41.88 | 42.69 – 42.25 |             |             |
| **(3, 1)**      |          |             |             |             |             |             |
| $n_1$           | 2.00 – 9.85 | 2.00 – 9.85  | 2.00 – 9.85  | 9.86 – 9.92  |             |
| $n_2$           | 12.80 – 15.16 | 15.16 – 15.06 | 15.16 – 14.99 |             |             |
| $n_U$           | 17.04 – 15.16 | 15.16 – 15.12 | 15.16 – 15.12 |             |             |
| $\omega_U$      | 42.50 – 42.33 | 42.32 – 42.04 | 42.32 – 42.25 |             |             |
| **(3, 2)**      |          |             |             |             |             |             |
| $n_1$           | 2.00 – 4.95 | 2.00 – 4.95  | 2.00 – 4.95  | 4.95 – 4.95  |             |
| $n_2$           | 14.56 – 14.74 | 14.56 – 14.74 | 14.56 – 14.74 | 14.74 – 14.74 |             |
| $n_U$           | 15.40 – 15.13 | 15.40 – 15.13 | 15.40 – 15.13 |             |             |
| $\omega_U$      | 41.10 – 41.40 |             |             |             |             |
| **(3, 3)**      |          |             |             |             |             |             |
| $n_1$           | 2.00 – 9.17 | 2.00 – 9.17  | 2.00 – 9.17  | 9.17 – 9.17  |             |
| $n_2$           | 11.32 – 13.99 | 11.32 – 13.99 | 11.32 – 13.99 |             |             |
| $n_U$           | 16.75 – 15.13 | 16.75 – 15.13 | 16.75 – 15.13 |             |             |
| $\omega_U$      | 40.44 – 41.01 |             |             |             |             |

(Table continued to page 12)
### Table 5: continued from page 11

#### Chains Ia to Va

| $(r_{10}, r_{126})$ | Quantity | Ia     | IIa     | IIIa    | IVa     | Va     |
|---------------------|----------|--------|----------|---------|---------|--------|
| $(5, 2)$            | $n_1$    | 2.00 – 8.26 |
|                     | $n_2$    | 9.90 – 12.67 |
|                     | $n_U$    | 16.46 – 15.14 |
|                     | $\omega_U$ | 38.47 – 39.57 |
| $(6, 2)$            | $n_1$    | 2.00 – 7.26 |
|                     | $n_2$    | 8.54 – 11.22 |
|                     | $n_U$    | 16.19 – 15.15 |
|                     | $\omega_U$ | 36.56 – 37.98 |

#### Chains VIa to Xa

| $(r_{10}, r_{126})$ | Quantity | VIa    | VIIa    | VIIIa   | IXa     | Xa     |
|---------------------|----------|--------|---------|---------|---------|--------|
| $(1, 1)$            | $n_1$    | 13.55 – 13.71 |
|                     | $n_2$    | 13.77 – 13.71 |
|                     | $n_U$    | 15.13 – 15.16 |
|                     | $\omega_U$ | 41.31 – 41.11 |
| $(1, 2)$            | $n_1$    | 11.75 – 13.71 |
|                     | $n_2$    | 16.06 – 13.71 |
|                     | $n_U$    | 16.06 – 15.22 |
|                     | $\omega_U$ | 45.69 – 41.21 |
| $(1, 3)$            | $n_1$    | 17.36 – 15.18 |
|                     | $n_2$    | 18.88 – 15.15 |
|                     | $n_U$    | 18.88 – 15.15 |
|                     | $\omega_U$ | 28.65 – 37.84 |
| $(1, 4)$            | $n_1$    | 2.00 – 13.59 |
|                     | $n_2$    | 14.88 – 13.72 |
|                     | $n_U$    | 14.88 – 15.15 |
|                     | $\omega_U$ | 45.69 – 41.21 |
| $(2, 1)$            | $n_1$    | 12.19 – 13.23 |
|                     | $n_2$    | 15.53 – 14.29 |
|                     | $n_U$    | 15.53 – 15.13 |
|                     | $\omega_U$ | 44.11 – 41.80 |
| $(2, 2)$            | $n_1$    | 12.19 – 13.23 |
|                     | $n_2$    | 15.53 – 14.29 |
|                     | $n_U$    | 15.53 – 15.13 |
|                     | $\omega_U$ | 44.11 – 41.80 |
| $(2, 3)$            | $n_1$    | 12.19 – 13.23 |
|                     | $n_2$    | 15.53 – 14.29 |
|                     | $n_U$    | 15.53 – 15.13 |
|                     | $\omega_U$ | 44.11 – 41.80 |
(Table 5: continued from page 12)

| (r_{10}, r_{126}) | Quantity | VIa | VIIa | VIIIa | IXa | Xa |
|-------------------|----------|-----|------|-------|-----|-----|
| (3, 1)            | n_1      | 2.00 – 10.97 |
|                   | n_2      | 10.12 – 10.97 |
|                   | \omega_U | 15.18–15.28 |
|                   | \omega_U | 42.40 – 42.63 |
| (3, 2)            | n_1      | 2.00 – 11.65 |
|                   | n_2      | 14.88 – 13.92 |
|                   | \omega_U | 17.72–15.16 |
|                   | \omega_U | 29.22 – 36.60 |
| (3, 3)            | n_1      | 2.81 – 9.91 |
|                   | n_2      | 10.80 – 12.70 |
|                   | \omega_U | 16.55–15.25 |
|                   | \omega_U | 0.08 – 23.70 |
| (4, 2)            | n_1      | 2.00 – 10.47 |
|                   | n_2      | 14.88 – 14.03 |
|                   | \omega_U | 17.19–15.16 |
|                   | \omega_U | 29.48 – 35.85 |
| (4, 3)            | n_1      | 2.53 – 7.69 |
|                   | n_2      | 10.73 – 12.11 |
|                   | \omega_U | 16.16–15.32 |
|                   | \omega_U | 0.36 – 17.23 |
| (5, 1)            | n_1      | 2.00 – 4.71 |
|                   | n_2      | 3.89 – 4.71 |
|                   | \omega_U | 16.13–16.03 |
|                   | \omega_U | 38.84–39.03 |
| (5, 2)            | n_1      | 2.00 – 9.11 |
|                   | n_2      | 14.88 – 14.17 |
|                   | \omega_U | 16.70–15.17 |
|                   | \omega_U | 29.72 – 34.99 |
| (6, 2)            | n_1      | 2.00 – 7.53 |
|                   | n_2      | 14.88 – 14.33 |
|                   | \omega_U | 16.24–15.17 |
|                   | \omega_U | 29.94 – 33.98 |
Table 6: Results for b-type symmetry breaking with two intermediate scales. Notation for the separators have been explained in the caption of Table 5.

### Chains Ib to Vb

| $(r_{10})$ | Quantity | Ib   | IIb  | IIIb | IVb   | Vb   |
|-----------|----------|------|------|------|-------|------|
| **(1)**   | $n_1$    | 10.41 – 13.62 | 10.67 – 13.71 | 10.41 – 13.71 | 10.41 – 13.71 | 13.05 – 13.59 |
|           | $n_2$    | 16.72 – 13.62 | 16.49 – 13.71 | 16.72 – 13.71 | 16.72 – 13.71 | 14.20 – 13.59 |
|           | $n_U$    | 16.72 – 15.56 | 16.49 – 15.51 | 16.72 – 15.51 | 16.72 – 15.51 | 15.11 ↔ 15.37 |
|           | $\omega_U$ | 46.34 – 45.26 | 45.74 – 44.77 | 46.34 – 44.77 | 46.34 – 44.77 | 44.80 – 45.05 |
| **(2)**   | $n_1$    | 11.23 – 13.64 | 11.41 – 13.71 | 11.23 – 13.71 | 11.23 – 13.71 | 15.11 – 13.40 |
|           | $n_2$    | 15.97 – 13.64 | 15.81 – 13.71 | 15.97 – 13.71 | 15.97 – 13.71 | 15.93 – 11.42 |
|           | $n_U$    | 15.97 – 15.20 | 15.82 – 15.16 | 15.97 – 15.16 | 15.97 – 15.16 | 15.97 – 15.82 |
|           | $\omega_U$ | 44.42 – 43.96 | 44.02 – 43.58 | 44.42 – 43.58 | 44.42 – 44.02 |               |
| **(3)**   | $n_1$    | 11.89 – 12.77 | 12.01 – 12.59 | 11.89 – 12.75 | 11.89 – 12.00 |               |
|           | $n_2$    | 15.37 – 14.51 | 15.27 – 14.73 | 15.37 – 14.59 | 15.37 – 12.03 |               |
|           | $n_U$    | 15.37 ↔ 15.12 | 15.27 ↔ 15.12 | 15.37 ↔ 15.12 | 15.37 ↔ 15.27 |               |
|           | $\omega_U$ | 42.89 – 42.83 | 42.62 – 42.57 | 42.89 – 42.70 | 42.88 – 42.62 |               |

### Chains VIb to Xb

| $(r_{10})$ | Quantity | VIb  | VIIb | VIIIb | IXb   | Xb   |
|-----------|----------|------|------|-------|-------|------|
| **(1)**   | $n_1$    | 13.24 – 13.71 | 13.63 – 13.71 | 13.00 – 11.11 | 2.00 – 11.07 | 2.00 – 12.45 |
|           | $n_2$    | 14.14 – 13.71 | 15.32 – 13.78 | 9.92 – 10.11 | 10.89 – 11.07 | 12.29 – 12.45 |
|           | $n_U$    | 15.11 ↓ 15.28 | 15.39 ↓ 15.35 | 16.70 – 16.71 | 16.09 – 16.12 | 15.67 – 15.72 |
|           | $\omega_U$ | 44.45 – 44.39 | 45.05 – 44.65 | 46.29 – 46.31 | 44.72 – 44.79 | 45.30 – 45.36 |
| **(2)**   | $n_1$    | 13.24 – 13.71 | 13.63 – 13.71 | 13.00 – 11.11 | 2.00 – 11.07 | 2.00 – 12.45 |
|           | $n_2$    | 14.14 – 13.71 | 15.32 – 13.78 | 9.92 – 10.11 | 10.89 – 11.07 | 12.29 – 12.45 |
|           | $n_U$    | 15.11 ↓ 15.28 | 15.39 ↓ 15.35 | 16.70 – 16.71 | 16.09 – 16.12 | 15.67 – 15.72 |
|           | $\omega_U$ | 44.45 – 44.39 | 45.05 – 44.65 | 46.29 – 46.31 | 44.72 – 44.79 | 45.30 – 45.36 |
| **(3)**   | $n_1$    | 13.24 – 13.71 | 13.63 – 13.71 | 13.00 – 11.11 | 2.00 – 11.07 | 2.00 – 12.45 |
|           | $n_2$    | 14.14 – 13.71 | 15.32 – 13.78 | 9.92 – 10.11 | 10.89 – 11.07 | 12.29 – 12.45 |
|           | $n_U$    | 15.11 ↓ 15.28 | 15.39 ↓ 15.35 | 16.70 – 16.71 | 16.09 – 16.12 | 15.67 – 15.72 |
|           | $\omega_U$ | 44.45 – 44.39 | 45.05 – 44.65 | 46.29 – 46.31 | 44.72 – 44.79 | 45.30 – 45.36 |
convenience of display, first giving results for chains I to V, and next for chains VI to X. For chains XI and XII, we find no solution, so they do not appear in the tables.

In the results presented in Table 5 and Table 6, different quantities given on the left sides of the dashes as a set, correspond to one solution, whereas the those of the right sides correspond to another solution. Thus, for example, with chain Ia and \((r_{10}, r_{126}) = (1, 1)\), one extreme of the solution range lies at \(n_1 = 9.02\) and \(n_2 = n_U = 16.66\), whereas the other extreme solution is at \(n_1 = 11.17, n_2 = 11.19\) and \(n_U = 16.75\). Of course, all intermediate values are allowed. Since the RG equations are linear in the scales and also in \(\omega_U\), the values of \(n_2, n_U\) and \(\omega_U\) corresponding to any intermediate value of \(n_1\) can be obtained by linear interpolation.

As for the cases with single intermediate stage, the solutions with \(r_{10} = r_{126} = 1\) merely reproduce the solutions with single-generation unification. Our results agree roughly with the results in earlier 1-loop calculations [8]. Slight deviations are due to reasons mentioned in the context of symmetry breaking with one intermediate scale. Also, in chains VIII to XII where there are two U(1) factors, we take kinetic mixing into account.

5 Discussions

Here we briefly outline some important aspects and limitations of this work.

- It has to be understood that our results are based on 1-loop calculations only. It is known that higher-loop calculation and threshold corrections may sometimes produce large changes in the RG solutions [14, 44]. To take into account the effect of 2-loop corrections we somewhat relaxed the proton decay bound using an order of magnitude estimation. We emphasize that 2-loop estimation and 1-loop threshold corrections carry additional baggage of uncertainties due to the involvement of more parameters arising from specific details of model spectra. All these effectively relax the proton decay bound, which constitutes the basis of our strategy.

- From our 1-loop results, one trend is quite clear: too many copies of the Higgs multiplets is not good for a model. For a-type symmetry breaking using the 126-plets which contribute to renormalizable Yukawa interactions, more than three copies of the 10-plet or the 126-plet works only for very few chains. For b-type symmetry breaking which utilizes 16-plets of Higgs bosons instead of 126, we obtain results for at most three copies of the 10-plet. We said earlier that we consider only Yukawa couplings which are symmetric in the generation indices. With three generations, there are six symmetric combinations. Thus, if there is a symmetric 6-dimensional irrep of the horizontal symmetry group \(H\), that will provide the only source of couplings of scalars and fermions. A horizontal symmetry group of SU(3) is therefore ruled out by our criteria, since it would require both 10-plets and 126-plets to transform like a 6-plet under \(H\), and our tables show that there is no acceptable solution.

- SO(3) also disqualifies as the horizontal symmetry \(H\) for the following reason. For SO(3), the symmetric combination of \(3 \times 3\) contains a 5-plet and a singlet. However, from the Table 5, \(r_{10} = 5\) necessarily requires \(r_{126} = 2\). The only escape route is to consider two copies of 126-dimensional Higgs which are singlet under SO(3).
• It is of course not necessary to think in terms of irreps only. If, for example, we need 4 copies of the 10, they need not form a 4-dimensional irrep of $H$. One can have a triplet and a singlet, or maybe two doublets. All we need is that they are part of the symmetric product of two triplets of $H$, since we are dealing with three generations.

In fact, reducible representations of $H$ might be preferable, or even essential, for another reason. If all the 10-plets of SO(10) belong to one irrep of $H$, there will be only one Yukawa coupling with the fermions. That may not suffice for producing the variety of masses and mixings for quarks and leptons. On the other hand, if we have 10-plets belonging to more than one irreps of $H$, there will be different couplings for different irreps, which will help produce acceptable masses and mixings. Alternatively, the fermions can also live in reducible representations under $H$. For example, if the horizontal symmetry is $S_3$, the SM quarks and leptons can transform as $2 + I$, where the third generation fermions form the singlet. This might also be useful in order to fulfill the phenomenological requirement of reproducing the correct flavor hierarchy. Models of quark masses and mixings with multiple irreps have been explored to some extent in the literature [45–48] though not in the context of SO(10). Reproducing the structure of CKM matrix in the quark sector as well as the PMNS matrix in the neutrino sector in this context might be a challenging future direction.

• Additional SO(10) Higgs multiplets might also be required to meet some phenomenological criteria in the context of flavor physics, which can affect the unification of the gauge couplings. Here, we consider the minimal possibility from the perspective of grand unification only.

• Values of $r_{10}$ larger than 1 imply multiple Higgs doublets with masses around the weak scale. Such scenarios may receive severe constraints from flavor changing neutral current processes, from the direct searches of charged Higgs bosons, etc. Thus our selection criteria can, in principle, confront the presence of multi-Higgs models at the weak scale.

• In this paper we work with three generations of chiral fermion, though our analysis can be easily extended to accommodate more generations. We have observed that for four chiral generations, charged under $H$, even fewer chains of symmetry breaking would be allowed.

• As a final comment, we mention that our analysis can also be extended to GUT models based on $E_6$ group as well, which might constitute an interesting exercise.

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