Quantum Rotor Atoms in Light Beams with Orbital Angular Momentum: Highly Accurate Rotation Sensor

Igor Kuzmenko$^{1,2,4}$, Tetyana Kuzmenko$^{1,4}$, Y. B. Band$^{1,2,3,4}$

$^1$Department of Physics, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel
$^2$Department of Chemistry, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel
$^3$Department of Electro-Optics, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel
$^4$The Ilse Katz Center for Nano-Science, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel

Atoms trapped in a red detuned retro-reflected Laguerre-Gaussian beam undergo orbital motion within rings whose centers are on the axis of the laser beam. We determine the wave functions, energies and degeneracies of such quantum rotors (QRs), and the microwave transitions between the energy levels are elucidated. We then show how such QR atoms can be used as high-accuracy rotation sensors when the rings are singly-occupied.

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Introduction: We show that quantum rotor (QR) atoms (atoms whose motion is constrained to a circular ring) $^{1}$ can be formed in light beams having orbital angular momentum and that they can be used as an extremely high accuracy rotation sensor. QR atoms are trapped in a red-detuned linearly polarized retro-reflected Laguerre-Gaussian (LG) beam. A line of singly-occupied rings filled with QR atoms can be easily formed (see Fig. 1). Single-occupation and negligible tunneling between rings are important to suppress deleterious spin exchange collisions between QR atoms for sensor applications. The accuracy obtained here suggests that this can be the highest precision rotation sensor proposed so far in the literature.

A LG beam propagating along the z-axis with orbital angular momentum $l$ and polarization $e_\alpha$ can be written in terms of a slowly varying envelope $u_{l,p}(r,\phi,z)$ of the electric field as

$$E_{\alpha,l,p}(r,t) = u_{l,p}(r,\phi,z) e^{i(kz-k_0\omega t)} e_\alpha + c.c., \quad (1)$$

with field amplitude mode LG$_p^l(r,t) = u_{l,p}(r,\phi,z) e_\alpha$.

$$u_{l,p}(r,\phi,z) = \sqrt{\frac{2^l p!}{\pi (p+|l|)!}} \frac{\sqrt{P_0/\epsilon}}{w(z)} \frac{r \sqrt{2}}{w(z)} \exp \left( - \frac{r^2}{w^2(z)} \right) \times L_p^{|l|}(\frac{2r^2}{w^2(z)}) \exp \left( - \frac{i k r^2 z}{2(z^2+z_R^2)} \right) \exp (-i l \phi) \times \exp \left( i (2p + |l| + 1) \tan^{-1} \left( \frac{z}{z_R} \right) \right), \quad (2)$$

Here $z$ is the longitudinal distance from the beam waist located at $z = 0$, $P_0$ is the laser beam power, $w_0$ is the beam waist at $z = 0$, $R(z) = z(1+(z_R/z)^2)$ is the radius of curvature of the beam wavefront, $w(z) = w_0(1+(z_R/z))^l/2$ is the radius at which the beam intensity falls to $1/e$ of its axis value at $z$, $z_R = \pi w_0^2/\lambda$ is the Rayleigh range for the laser with wavelength $\lambda = 2\pi/k$ where $k = \omega/c$ is the wavenumber, $0 < z_0 < \lambda/2$ is a phase parameter, $L_p^{|l|}(\phi)$ is the associated Laguerre polynomial, $\phi$ is the azimuthal angle, and $\tan^{-1}(z/z_R)$ is the Gouy phase. Figure 1(a) is a schematic diagram of a retro-reflected LG beam propagating along the z-axis, and Fig. 1(b) shows superposition of two counter-propagating beams that form a standing wave along the z-axis. The slowly varying envelope $u_{l,p}(r,\phi,z)$ of the counter-propagating (cp) standing wave has the form

$$E_{\alpha,l,p}^{cp}(r,t) = u_{l,p}(r,\phi,z) e_\alpha (e^{i(kz-k_{0z}\omega t)} + e^{-i(kz+k_{0z}\omega t)}). \quad (3)$$

This standing wave configuration results in a series of ring shaped optical potentials [see the orange rings in Fig. 1(b)] stacked perpendicular to the axis of the beams. Since our interest is in trapping atoms in the light beam, the light is red-detuned from atomic resonance, and atoms will be trapped in the ring shaped optical potentials that are singly occupied so that spin relaxation collisions are suppressed.

QR Bound States in LG Rings: The QR Hamiltonian operator in cylindrical coordinates is

$$H = \frac{\hbar^2}{2M} \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right) + V(r,\phi,z), \quad (4)$$

where the first term is the atom kinetic energy in polar coordinates and $V(r,\phi,z)$ is the optical potential resulting from the LG beams, which is calculated as a second-order ac Stark effect.
FIG. 2: Energies $\epsilon(n_z, n_r, m_t)$ of the QR (relative to the QR ground state energy) trapped in an LG beam with $w_0 = 10 \mu m$, $l = 5$, $p = 0$, and $V_0 = 10 \mathcal{E}_0$, where $\mathcal{E}_0$ is the recoil energy. This gives $r_i \equiv r_i(z_j)$ with $j = 0$ equal to 15.81 $\mu m$. Energies with $n_z = 0$ are shown as purple lines, $n_z = 1$ as blue lines and $n_z = 2$ as sky-blue lines. Quantum states with $n_z \geq 1$ have very high energies and fall out of the scale of the figure, hence only $n_z = 0$ are shown.

shift \[11\] and is given in terms of the ac polarizability $\alpha(\omega)$ by $V(r) = -\alpha(\omega) \mathcal{E}_{\text{ac}}^2 (r,t)^2$. In the standing wave configuration in the nearly constant beam waist region, the optical potential can be taken to be two-dimensional since the width of the rings are very small. For a LG$_0^l$ mode ($p = 0$ and $l \neq 0$), the potential is independent of $\phi$.

$$V(r, z) = -V_0 \cos^2 \left( k(z-z_0) \right) \frac{\rho^2(z)}{w^2(z)} e^{-|\rho^2(z) - 1|}, \quad (5)$$

where $\rho(z) = r/r(z)$ and $w(z) = w(z)/w_0$. Potential \[5\] has a minimum at

$$z = z_j \equiv \frac{\pi}{k} j + z_0, \quad r = r_j(z) \equiv w(z_j) \sqrt{|l|/2}, \quad (6)$$

where $j$ is integer. The trapped atoms execute circular motion around the $z$ axis, i.e., they are QRs. $V(r_j(z_j), z_j)$ is given by $V(r_j(z_j), z_j) = -V_0 \left( w_0/w(z_j) \right)$ for $z$ close to $z_j$,

$$V(r, z) \approx V(r) + W_j(z), \quad (7)$$

where

$$V(r) = V(r, z_j), \quad W_j(z) = \frac{V_0 k^2}{w^2(z_j)} (z - z_j)^2. \quad (8)$$

$W_j(z)$ is a harmonic potential in $z - z_j$. The corresponding harmonic frequency and length are

$$\omega_z(z_j) = \frac{2}{w(z_j)} \frac{\sqrt{\mathcal{E}_0 V_0}}{\hbar}, \quad b_z(z_j) = \frac{\sqrt{\omega_z(z_j)}}{k} \left( \frac{E_0}{V_0} \right)^{1/4}, \quad (9)$$

where $\mathcal{E}_0 = \hbar^2 k^2/(2M)$ is the recoil energy.

The optical potential \[5\] is invariant with respect to rotations about the $z$ axis, therefore the quantum states of the QR in the harmonic approximation \[7\] are parametrized by radial and vertical quantum numbers $n_r$ and $n_z$ describing radial and $z$ motion ($n_r, n_z = 0, 1, 2, \ldots$), the orbital momentum quantum number $m_t$ and projection $m_F$ of the hyperfine orbital momentum $\mathbf{F}$ on the $z$ axis. The ground state has $n_z = n_r = m_t = 0$, and is $2F + 1$ fold degenerate. Orbitally excited states with $m_t \neq 0$ are $(2F + 1)$ fold degenerate, and have angular momentum $\pm m_t$. Radial and vertical excitations have $n_r \neq 0$ and $n_z \neq 0$ respectively. For simplicity, in this paragraph, let us consider an atom trapped at the $z_0$ site (i.e., near $z_j$ with $j = 0$). The QR wave functions and eigen-energies satisfy the Schrödinger equation,

$$\left[ -\frac{\hbar^2}{2M} \nabla^2 + V(r, z) - \epsilon_n \right] \Psi_n(r) = 0, \quad (10)$$

where $\mathbf{n} = (n_z, n_r, m_t)$. The wave function can be written in cylindrical coordinates $r = (r, \phi, z)$ as

$$\Psi_n(r) = \frac{1}{\sqrt{2\pi}} \eta_n(z) \phi_{n,m}(r)e^{im\phi}, \quad (11)$$

where $\eta_n(z)$ and $\phi_{n,m}(r)$ satisfy the equations,

$$\left[ -\frac{\hbar^2}{M} \frac{d^2}{dr^2} + W_0(z) - \epsilon_n(z) \right] \phi_{n,m}(z) = 0, \quad (12)$$

$$\left[ -\frac{\hbar^2}{2Mr} \left( \frac{d}{dr} \right)^2 + V_l(r) + m_t^2 \right] \Psi_{n,m}(r) = 0. \quad (13)$$

$V_l(r)$ and $W_j(z)$ are given by Eq. \[8\], and $C(r) = \frac{\epsilon^2(r)}{M \omega_z}$ is the rotational energy of the QR around the $z$ axis. The eigenenergy of the trapped atom is

$$\epsilon(n) = \epsilon_z(n_z) + \epsilon_r(n_r, m_t), \quad (14)$$

where $r_i \equiv r_i(z_j)$ with $j = 0$. The QR vertical, radial and orbital excitation energies are

$$\epsilon_z = \epsilon_z(1) - \epsilon_z(0) \approx \hbar \omega_z, \quad \epsilon_r = \epsilon_r(1, 0) - \epsilon_r(0, 0), \quad \epsilon_t = \epsilon_t(1, 0) - \epsilon_t(0, 0) \approx C(r).$$

We assume they satisfy the inequalities,

$$\epsilon_z \gg \epsilon_r \gg \epsilon_t, \quad (15)$$

i.e., the orbital excitations are the lowest-energy excitations, whereas the radial and longitudinal excitations have relatively high energies. The energies $\epsilon(n_z = 0, n_r, m_t)$, Eq. \[14\], are shown in Fig.\[2\] For $l = 5$, $w_0 = 10 \mu m$, $r_i = 15.81 \mu m$ and $V_0 = 10 \mathcal{E}_0 = k_B \times 35.36 \mu K$, the excitation energies are $C(r) = k_B \times 0.1613 \mu K$, $\hbar \omega_z = k_B \times 0.4776 \mu K$ and $\hbar \omega_z = k_B \times 22.36 \mu K$, so the inequalities \[15\] are valid. Quantum states with $n_z \geq 1$ have high energies and are not shown in Fig.\[2\].

Rabi Oscillation Method with Raman Pulses: In order to measure the excitation energies of the QR atoms with quantum numbers $n_r$, $m_t$ and $m_F$, we propose to subject the QRs
fore we use a LG pulse that allows quantum transitions between the quantum states \(|n_e, m_e⟩\) and \(|n'_e, m'_e⟩\) and we can approximate \(B_0(r, t)\) by \(B_0(0, t)\). Hence, the dipole magnetic interaction between the QR and the radio waves, \(H_R = -g_F \mu_B F \cdot B(t)\), does not depend on \(r\) (the position of the atom), and therefore \((n_e, m_e|H_R|n'_e, m'_e⟩ \propto \delta_{n_e, n'_e} \delta_{m_e, m'_e}^\prime\). For Raman transitions \(|n_e, m_e⟩ \rightarrow |n_e, m'_e⟩\) with \(m_e \neq m'_e\), an optical square pulse which breaks the cylindrical symmetry of the QR is required. The electric field of the optical pulse is \(E_r(r, t) = \frac{1}{\hbar} (\mu_{\text{e}(r)} + n_{-L, 0}(r)) \cos(k_2 \cdot \Theta)(\Theta - \tau) e^{-i\omega t} e_+ c.c.,\) where \(k_2 = \omega c / L\) is the wavenumber of the optical pulse and \(n_{-L, 0}(r)\) is given by Eq. (2). We choose the waist radii of the LG pulse \(w_e\) and the LG beam waist \(w_0\), and their orbital angular momenta \(L\) and \(l\) in such a way that \(w_e \sqrt{|L|^2} = w_0 \sqrt{|l|^2}\).

The stimulated Raman process corresponding to absorption of a pump photon and stimulated emission of a Stokes photon gives rise to excitation of the QR. This scattering is described by the time-dependent Hamiltonian with matrix elements given by

\[
\mathcal{H}_{n_e, m_e; n'_e, m'_e}(t) = -2V \cos(\omega_p t) \Theta(t)\Theta(\tau - t) \delta_{n_e, n'_e} \sum_{m = 2L} \delta_{m; m'_e + m},
\]

where \(\omega_p = \omega_p - \omega_s\) is the difference of frequencies of the pump and Stokes pulses, and

\[
\mathcal{V} = \frac{V_c V_b}{\hbar \Delta_{\text{hf}}} \quad \mathcal{V}_b = \frac{g_s^2 \mu_B^2 B_p^{(0)} B_s^{(0)}}{3 \hbar \Delta_{\text{hf}}} \quad \mathcal{V}_c = \frac{4\alpha(\omega_e)}{\pi L!} \frac{P_e L^2 e^{-L}}{w_c^2 c}
\]

(17)

\(P_e\) and \(w_c\) are the power and the beam waist of the optical pulse, \(B_p^{(0)}\) and \(B_s^{(0)}\) are the magnetic field strengths of the pump and Stokes radio-frequency pulses. The subscript \(e\) symbolizes the electric dipole interaction of the atom with the optical pulse, and the subscript \(b\) symbolizes the magnetic dipole interaction of the atoms with the pump and Stokes radio-frequency pulses. The detuning \(\Delta_{\text{hf}}\) of the pump pulse frequency from the \(2S_{1/2}(F = 1/2) \rightarrow 2S_{1/2}(F = 3/2)\) quantum transition frequency greatly exceeds the frequency \(\omega_{2L, 0}\) of the \(|0⟩ \rightarrow |2L⟩\) quantum transition,

\[
\Delta_{\text{hf}} \gg \omega_{2L, 0},
\]

(18)

hence we can assume that \(\omega_p\) and \(\omega_s\) have the same detuning \(\Delta_{\text{hf}}\) from resonance. Details of the Raman scattering of the pump and Stokes light, the Rabi oscillations with Raman pulses used to measure the energy differences of the QR states are presented Ref. [1] and also in the SM [14].

Consider the QR initially in the ground state \(|0⟩\) subjected to Raman pulses of duration \(\tau\) and generalized Rabi frequency \(\Omega_R = 2\sqrt{2} V / h\), such that \(\Omega_R \tau \approx \pi\). In the 3-level rotating wave approximation [17,], the temporal evolution of a single QR wave function is given by \(|\Psi(\tau)⟩ = e^{-iH_R\tau / h}|0⟩\), where \(\mathcal{H}_R\) is the Hamiltonian (see SM [14]),

\[
\mathcal{H}_R = \hbar \begin{pmatrix}
0 & \Omega_R \sqrt{2}/4 & \Omega_R \sqrt{2}/4 \\
\Omega_R \sqrt{2}/4 & -\delta & 0 \\
\Omega_R \sqrt{2}/4 & 0 & -\delta
\end{pmatrix},
\]

(19)
\[ \delta = \omega_{pl} - \omega_{2L,0} \], and the basis vectors are,

\[ |0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad |2L\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad |-2L\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}. \]

The probability of finding the quantum rotor in the final state \(|f\rangle = 2^{-1/2}(|2L\rangle + |-2L\rangle)\) is

\[ P_0(\delta, \Omega_R) = \frac{\Omega_R^2}{\Omega_R^2 + \delta^2} \sin^2 \left( \frac{\tau}{2} \sqrt{\Omega_R^2 + \delta^2} \right). \tag{20} \]

\( P_0(\delta, \Omega_R) \) has a peak, \( P_0(0, \Omega_R) = 1 \) at \( \delta = 0 \). The peak width at half maximum is \( 1.597 \Omega_R \).

When we have \( N = \frac{1}{2}j_{max} + 1 \) atoms singly occupying the sites with \( |j| \leq j_{max} \), the detuning of \( \omega_{pl} \) from the resonant frequency of the \( j \)-th quantum rotor is \( \delta_j = \delta + 4L^2 \beta^2 (\omega_0(r_j(0)) - \omega_0(r_j(z_j))) \), where \( \delta \) is the detuning of \( \omega_{pl} \) from the resonant frequency of the \( j = 0 \) QR, \( \omega_0(r_j(z_j)) = \hbar/(2Mr_j^2(z_j)) \) and \( r_j(z_j) \) is the radius of the classical circular trajectory. As a result, the peak in the probability to find a QR in the final state is shifted and broadened. The probability to find a QR in the final state is

\[ P(\delta, \Omega_R) = \frac{1}{2j_{max} + 1} \sum_{j=-j_{max}}^{j_{max}} P_0(\delta_j, \Omega_R). \tag{21} \]

![Figure 4](image)

**Figure 4:** The probability \( P \) in Eq. (21) for absorption of a pump photon and stimulated reemission of a Stokes photon with QRs quantum transition from state \(|m_r = 0\rangle\) to \(|m'_r = 50\rangle\) as a function of the difference \( \omega_{pl} = \omega_p - \omega_0 \) of the pump and Stokes frequencies \( \omega_p \) and \( \omega_0 \) (solid blue curve). The dashed red curve shows the fit of \( P(\delta, \Omega_R) \) obtained as explained in the text.

Figure 4 shows \( P(\delta, \Omega_R) \) of Eq. (21) plotted as a function of \( \delta \), for \( \omega_0 = 21.13 \text{ s}^{-1} \), which corresponds to a LG beam with \( w_0 = 10 \mu m \) and \( \ell = 25 \), Rabi frequency \( \Omega_R = 3.142 \text{ s}^{-1} \), pulse duration \( \tau \approx \pi/\Omega_R = 1 \text{ s} \), \( j_{max} = 80 \), \( m_r = 0 \) and \( m'_r = 50 \). The solid blue curve shows that \( P(\delta, \Omega_R) \) has a peak, \( P_{\max} = P(\delta_{max}, \Omega_R) = 0.6989 \), at \( \delta_{max} = -0.5374 \Omega_R \). The dashed red curve shows the fitting of \( P(\delta, \Omega_R) \) by the function \( P(\delta, \Omega_R) \approx A P_0(\delta - \delta_0, \Omega_R) \) with \( A = 0.67987 \) and \( P_0 \) is given by Eq. (20) with \( \delta_0 = -0.64002 \Omega_R \), \( \Omega_R = 1.4865 \Omega_R \).

Note that \( \delta_{max} \neq \delta_0 \). This is partly because the function \( P(\delta, \Omega_R) \) is not symmetric with respect to the inversion \( \delta \rightarrow -\delta \rightarrow \delta_{max} \), whereas the function \( P_0(\delta, \Omega_R) = P_0(-\delta, \Omega_R) \) is symmetric.

We now show that ground-state QRs in an LG beam can serve as extremely accurate rotation sensors.

**Rotation Sensor:** Consider a QR in a non-inertial frame rotating with angular velocity \( \Omega = \Omega \mathbf{e} \) (see Fig. 2 in the SM [14]). The additional term needed in the Hamiltonian is

\[ H_{\Omega} = \hbar \Omega \ell_z, \tag{22} \]

where \( \ell_z = -i\hbar \ell \) is the orbital momentum operator. The Hamiltonian (22) lifts the symmetry under the transformation \((x, y, z) \rightarrow (-x, y, z) \) but not the rotational symmetry about the \( z \)-axis. As a result, \( m_r \), the eigenvalue of \( \ell_z \), is a good quantum number, and the energy levels \( \epsilon_{m_r} \) of the rotational motion become \( \epsilon_{m_r}(\Omega) = \epsilon_{m_r} + \hbar \Omega m_r \). Hereafter we use the inequalities (15) and restrict ourselves by considering the quantum states with \( n_L = n_r = 0 \). Moreover, using the inequalities (15), we approximate the rotational energies as \( \epsilon_{m_r} = m_r^2 C(r_l) \), where \( r_l \) is given by Eq. (6).

Let us consider the frequencies of the quantum transitions between the quantum states \(|m_r\rangle\) and \(|m'_r\rangle\) with \( m_r = 0, \pm 1 \) and \( m'_r = m_r \pm 2L \). We have six spectral lines with frequencies \( \omega_{m_r,m'_r+2L}(\Omega) \) which are convenient to order as follows: \( \omega_{0,2L}(\Omega), \omega_{\pm 1, (1,1,2L)}(\Omega), \) and \( \omega_{\pm 1, (1,1,2L)}(\Omega) \). The frequencies of the quantum transitions between \( \zeta(m_r) \) and \( \zeta(m_r + 2\zeta L) \) are

\[ \omega_{m_r,m_r+2\zeta L}(\Omega) = 4L(L + m_r)\omega_0 + 2\zeta L\Omega. \tag{23} \]

where \( \zeta = \pm 1 \), and the rotational frequency is

\[ \omega_0 = C(r_l)/\hbar. \tag{24} \]

Hence, when \( \Omega = 0 \), \( \omega_{m_r,m_r+2L}(0) = \omega_{-m_r,-m_r+2L}(0) \). One can measure the three spectral lines \( \omega_{0,2L}(\Omega), \omega_{1,1,2L}(\Omega) \), and \( \omega_{-1,1,2L}(\Omega) \) due to \( \Omega \) distinguishes between clockwise and counter-clockwise rotations. All the spectral lines have the same splittings. Hence, the frequencies satisfy the periodic condition,

\[ \Delta \omega_{m_r,m_r+2L} = \omega_{m_r,m_r+2L}(\Omega) - \omega_{m_r,-m_r-2L}(\Omega) = 4L\Omega. \tag{25} \]

Eq. (25) shows that measuring the splitting of the spectral lines (25), can be used to determine \( \Omega \). Figure 5 shows the transition frequencies of the \(|m_r\rangle \rightarrow |m_r \pm 50\rangle \) transitions as functions of \( \Omega \). The frequency splitting (25) due to \( \Omega \) distinguishes between clockwise and counter-clockwise rotations. All the spectral lines have the same splittings. Hence, the frequencies satisfy the periodic condition,

\[ \omega_{m_r,m_r+2L}(\Omega - 2m_0\omega_0) = \omega_{m_r,m_r+2L}(\Omega), \]

where \( m_0 \) is integer.

**Rotation measurement accuracy estimate:** Note that Eq. (25) does not contain any information regarding the optical potential, the laser frequency or the intensity. Therefore
FIG. 5: The frequencies of the quantum transitions \(|m_r| \rightarrow |m_r \pm 50|\) versus the magnitude of the rotational velocity \(\Omega\). \(\omega_0\) is given by Eq. (24).

the uncertainty of \(\Omega\), \(\delta \Omega\), is determined solely by uncertainty \(\delta \omega\) of the pump and Stokes frequencies,

\[
\delta \Omega = \frac{1}{\sqrt{4L}} \frac{\delta \omega}{\omega_0}.
\] (26)

Here \(N = 2j_{\text{max}} + 1\) is the number of atoms singly occupying the sites with \(|j| \leq j_{\text{max}}\). \(\delta \omega = \delta \omega_p + \delta \omega_s\), and \(\delta \omega_p\) and \(\delta \omega_s\) are the uncertainties of the pump and Stokes frequencies. For \(^6\text{Li}\) atoms, we take \(\omega_p \approx 1.43 \times 10^9\) s\(^{-1}\). \(\delta \omega\) can be estimated as \(\delta \omega = 2 \times 10^{-18} \omega_p \approx 2.86 \times 10^{-9}\) s\(^{-1}\). From Eq. (26) we see that the larger \(L\), the smaller is \(\delta \Omega\); when \(L = 25\) and \(j_{\text{max}} = 80\), \(\delta \Omega = 2.25 \times 10^{-12}\) s\(^{-1}\).

In order to measure angular velocity when gravity \(g\) is present, place the QRs in the plane perpendicular to \(g\) (let us call this the \(x\)-\(y\) plane), in order to measure the component of the angular velocity in the \(z\) direction. If an additional acceleration \(a\) in the \(x\)-\(y\) plane is present, there is an additional splitting of the QR ground state degeneracy due to the acceleration, and the frequency splitting in Eq. (25) becomes dependent on \(m_r\). Hence, turn the plane of the QRs to be perpendicular to \(g' = g - a\) so that the frequency splitting (25) is independent on \(m_r\), and obtain the energy splitting of the levels caused just by \(\Omega' = \Omega \cdot e_z^x\), where \(e_z^x\) is the unit vector along \(-g'\). For details see the SM [14].

Summary and Conclusion: Cold atoms trapped in a Laguerre-Gauss optical potential \(5\) are confined to circular rings (donuts) of radius \(r_l\) with centers on the axis of the Laguerre-Gauss beam. Rings with one atom per site (to suppress spin-exchange collisions) can be used as high-accuracy rotation sensors. When \(r_l = 15.81\) \(\mu\)m, the accuracy obtained with \(N = 161\) atoms singly occupying the sites with \(|j| \leq 80\) is \(\delta \Omega = 2.25 \times 10^{-12}\) s\(^{-1}\). This is better than the accuracy \(\delta \Omega = 6.4 \times 10^{-10}\) s\(^{-1}\) reported in Ref. [11]. Moreover, the rotation sensor accuracy is much better than \(\delta \Omega_{\text{NIST}} = 3 \times 10^{-8}\) s\(^{-1}\) reported in Ref. [13].

The rings between the lenses in Fig. 1 have slightly different radii [see Eq. (6)]. As a result, different transition frequencies are obtained from of the \(m_r \rightarrow m'_r\) transition for different \(z_j\): \(\omega_{m_r,m'_r}(z_j) = \omega_{m_r,m'_r}(0)C(r_j(z_j))/C(r_j(0))\). Hence, the width of the Rabi oscillation is broadened. For example, when \(\omega_0 = 21.13\) s\(^{-1}\), \(\Omega_e = 3.142\) s\(^{-1}\), and there are \(N = 161\) singly occupied sites with \(|j| \leq 80\), the resulting effective width of the Rabi oscillation peak is \(\tilde{\Omega}_e = 1.4865 \Omega_e\) instead of \(\Omega_e\) for a single QR (see Fig. 4).

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I. RABI OSCILLATIONS OF QUANTUM ROTOR STATES IN LG BEAMS

Here we expand the discussion of the main text \[1\] as follows. Section \[2\] contains details regarding the Rabi oscillation method used to probe the energy levels of quantum rotors (QR) trapped in Laguerre-Gaussian (LG) beams to induce stimulated Raman transitions with radio-frequency pulses and optical transitions between the ground state with \( m \) states, with \( \ell \) trapping optical potential, Eq. (5) in \[1\], and allows quantum transitions with \( \Delta \), delta-function, from the hyperfine splitting, and a laser beam far detuned from the excitation frequency of the \( ^2P_{3/2} \) state by a detuning \( \Delta_e \). Diagrams contributing to the quantum transitions \( |0\rangle \to |\pm 2L\rangle \) in fourth-order perturbation theory are illustrated in Figs. \[1(a)\] and \[1(b)\]. The frequencies of the pump and Stokes pulses are \( \omega_p \) and \( \omega_s \), and the frequency of the laser pulse is \( \omega_L \). The detuning frequencies from the resonances are given by

\[
\Delta_e = \omega_e - \omega(2P_{3/2} - ^2S_{1/2}), \quad \Delta_{hf} = \omega_p - \omega_{hf} \geq \omega_s - \omega_{hf},
\]

where \( \omega(2P_{3/2} - ^2S_{1/2}) \) is a resonant frequency of the \( ^2S_{1/2} \to ^2P_{3/2} \) quantum transition, and \( \omega_{hf} \) is the hyperfine splitting.

We assume that

\[
|\Delta_e| \gg |\Delta_{hf}| \gg \omega_{2L,0},
\]

where \( \omega_{2L,0} \) is the frequency of the quantum transition \( |0\rangle \to \langle 2L| \). Hence, we can neglect \( \omega_{2L,0} \) compared with \( \Delta_e \), and take the same detuning \( \Delta_e \) of \( \omega_e \) from resonance with the \( ^2S_{1/2} \to ^2P_{3/2} \) transition [as shown in Figs. \[1(a)\] and \[1(b)\]]. Moreover, we assume that the detuning \( \omega_p - \omega_{hf} \) of the pump pulse is approximately equal to the detuning \( \omega_s - \omega_{hf} \) of the Stokes pulse.

Interactions of the trapped atom with the pump, Stokes and optical pulses are described by the Hamiltonians,

\[
H_p = -g \mu_B s \cdot B_p(t), \quad H_e = -p \cdot E_e(r, t),
\]

where \( \mu = p, s \) for the pump and Stokes pulses, \( s \) is the spin-1/2 vector operator, and \( p \) is the atomic electric dipole operator. The subscript \( e \) here and in Eq. (17) of \[1\] symbolizes the electric dipole interaction of the atom with the optical pulse, and the subscript \( b \) symbolizes the magnetic dipole interaction of the atoms with the pump and Stokes radio-frequency pulses. The electric field \( E_e(r, t) \) and the magnetic field \( B_p(t) \) are given by

\[
B_p(t) = B_p^{(0)} \cos(\omega_p t),
\]

\[
E_e(r, t) = \frac{1}{2} \left[ u_{L,0}(r) + u_{-L,0}(r) \right] \cos(k_z z) e^{-i\omega_k t} e_x + \text{c.c.},
\]

where \( u_{L,0}(r) \) is given by Eq. (2) in the main text \[1\].

When a trapped atom absorbs and reemits an optical pulse photon, it gets a rotational-kicking quantum transition with \( \Delta m = \pm 2L \), where \( m \) is the orbital quantum number of orbital motion of the atom around the \( z \) axis, and \( L \) is the orbital
trapped atom are $|^2 S_{1/2}(1/2), m_r, m_f \rangle$ states with the electronic configuration $^2 S_{1/2}(F = 1/2)$, magnetic quantum number $m_f = \pm 1/2$ and the motion of the center of mass parametrized by $m_r = 0, \pm 2L$. High-energy states are $|^2 S_{1/2}(3/2), m_r, m_f \rangle$ with electronic configuration $^2 S_{1/2}(F = 3/2)$, $m_r = 0, \pm 2L$ and $m_f = \pm 1/2$, and $|^2 P_{3/2}, m_r, m_f \rangle$ with electronic configuration $^2 P_{3/2}, m_r = \pm L$ and $m_f = \pm 1/2$, as illustrated in Fig. 1.

In this section, we use the following basis,

$$
\begin{align*}
|0\rangle &= |^2 S_{1/2}(1/2), 0, m_f \rangle, \\
|1\rangle &= \frac{1}{\sqrt{2}} \left( |^2 S_{1/2}(1/2), 2L, m_f \rangle + |^2 S_{1/2}(1/2), -2L, m_f \rangle \right), \\
|2\rangle &= e^{i \omega_p t} |^2 S_{1/2}(3/2), 0, m_f \rangle, \\
|3\rangle &= \frac{e^{i \omega_p t}}{\sqrt{2}} \left( |^2 S_{1/2}(3/2), 2L, m_f \rangle + |^2 S_{1/2}(3/2), -2L, m_f \rangle \right), \\
|4\rangle &= \frac{e^{i \omega_p t}}{\sqrt{2}} \left( |^2 P_{3/2}, L, m_f \rangle + |^2 P_{3/2}, -L, m_f \rangle \right).
\end{align*}
$$

The “non-perturbed” Hamiltonian of the trapped atom without the pump, Stokes and optical pulses is given by the matrix elements,

$$
\langle \psi | H_{0} | \psi' \rangle = \epsilon_r \delta_{\psi \psi'},
$$

where

$$
\epsilon_0 = 0, \quad \epsilon_1 = \epsilon_2L, \quad \epsilon_2 = h \Delta_{st}, \quad \epsilon_3 = h \Delta_{st} + \epsilon_2L, \quad \epsilon_4 = h \Delta_c.
$$

The energy of the rotational motion of the atom is $\epsilon_{r} = m_r^2 C(r_1)$, where $C(r_1) = h^2/(2M r_1^2)$ and $r_1$ is the radius of the classical circular trajectory. $\Delta_{st} = \omega_p - \omega_{2L}/h$ and $\Delta_c = \omega_r - \epsilon_r/h$ are the detuning of the pump and optical frequencies from the resonant frequencies of the quantum transitions $|0\rangle \rightarrow |3\rangle$ and $|0\rangle \rightarrow |4\rangle$, respectively.

Nontrivial matrix elements of $H_p, H_s$ and $H_r$, Eq. (5), are

$$
\langle 1 | H_p | 3 \rangle = \langle 2 | H_s | 4 \rangle = g \mu_B B_p (1/2, m_f) \left| S_{1/2}(3/2, m_f) \right|
$$

$$
\left( \begin{array}{c}
\frac{\sqrt{3}}{3} g \mu_B B_p e^{i (\omega_p - \omega_{2L}) t} m_f, \\
\epsilon_r \delta_{\psi \psi'}
\end{array} \right),
$$

$$
\langle 1 | H_c | 5 \rangle = \langle 2 | H_c | 5 \rangle = \frac{1}{2} \left[ u_{L,0}(r, 0, 0) + u_{-L,0}(r, 0, 0) \right] \\
x \langle ^2 S_{1/2}, m_f | p \rangle | ^2 P_{3/2}, m_f \rangle,
$$

where $u_{L,0}(r, \phi, z)$ are given by Eq. (2) in the main text [1].

Here we chose the x axis as a quantization axis.

A. Adiabatic elimination of the $^2 S_{1/2}(F = 3/2)$ hyperfine state

As a first step, we apply the following unitary transformations,

$$
|\psi_0\rangle = u_b |0\rangle - v_b |2\rangle, \quad |\psi_1\rangle = u_b |1\rangle - v_b |3\rangle,
$$

$$
|\psi_2\rangle = v_b^* |0\rangle + u_b |2\rangle, \quad |\psi_3\rangle = v_b^* |1\rangle + v_b |3\rangle.
$$

moment of the optical Laguerre-Gaussian (LG) pulse, see the main text [1] for details. From the other side, absorbing a pump photon and reemitting a Stokes photon, the atom gets the energy needed for quantum transitions. In this section, we apply fourth order perturbation theory and derive an effective Hamiltonian (16) in the main text [1]. This derivation is rather standard, but cumbersome.

We derive here an effective Hamiltonian describing Rabi oscillations for the QR in the $m_r = 0$ and $\pm 2L$ states. For this purpose, we assume that the low-energy states of the

FIG. 1: Far-off resonance stimulated Raman scattering due to pump and Stokes radiofrequency waves (blue arrows) and optical rotation-kicking optical pulses (red arrows). The two diagrams of processes that contribute are shown. $\Delta_c$ is a detuning of $\omega_r$ from resonance with the $^2 S_{1/2} \rightarrow ^2 P_{3/2}$ quantum transition, and $\Delta_{st}$ is a detuning of $\omega_p$ from resonance of the $^2 S_{1/2}(F = 1/2) \rightarrow ^2 S_{1/2}(F = 3/2)$ quantum transition. The detuning $\Delta_c$ and $\Delta_{st}$, and the frequency of the quantum transition $\omega_{2L} = (\epsilon_{2L} - \epsilon_0)/h$ satisfy the inequalities (4), therefore the optical pulses are taken to have the same detuning $\Delta_c$ in both diagrams (a) and (b).
and $|\psi_4\rangle = |4\rangle$, where
\[
\nu_b = \frac{\sqrt{2}}{3} \frac{g \mu_B (B_p + B_s e^{-i \omega_{pt}})}{\hbar \Delta_{hf}}, \quad u_b = \sqrt{1 - |\nu_b|^2}, \tag{9}
\]
and $\omega_{pt} = \omega_p - \omega_s$. We assume here that $|\nu_b| \ll 1$ and keep terms up to $v_e^2$ and neglect $v_e^3$ with $n \geq 3$. This transformation makes $H_0 + H_p + H_s$ diagonal,
\[
\langle \psi_n | H_0 + H_p + H_s | \psi_{n'} \rangle = \tilde{e}_n \delta_{n,n'},
\]
where
\[
\tilde{e}_0 = -\Delta_{hf} |\nu_b|^2, \quad \tilde{e}_1 = \epsilon_{2L} - \Delta_{hf} |\nu_b|^2, \\
\tilde{e}_2 = \Delta_{hf} (1 + |\nu_b|^2), \quad \tilde{e}_3 = \epsilon_{2L} + \Delta_{hf} (1 + |\nu_b|^2), \\
\tilde{e}_4 = \tilde{e}_4.
\]
Matrix elements of $H_e$ are
\[
h_e = \langle \psi_0 | H_e | \psi_4 \rangle = \langle \psi_1 | H_e | \psi_4 \rangle = \frac{1}{2} \left[ u_{L,0}(r, 0, 0) + u_{-L,0}(r, 0, 0) \right] \times \\
\langle \psi_0 | H_e | \psi_4 \rangle = \epsilon_{2L} |\nu_b|^4 \left( 1 - \frac{|\nu_b|^2}{2} \right), \tag{10}
\]
where $u_{L,0}(r, \phi, z)$ is given by Eq. (2) in the main text [1].

B. Adiabatic elimination of the $2^P_{3/2}$ excited state

In a second step, we apply the following unitary transformations,
\[
|\tilde{\psi}_0\rangle = u_4 |\psi_0\rangle + \frac{\nu_e^2}{2} |\psi_1\rangle - v_e |\psi_4\rangle, \\
|\tilde{\psi}_1\rangle = -\frac{\nu_e^2}{2} |\psi_0\rangle + u_e |\psi_1\rangle - v_e |\psi_4\rangle, \\
|\tilde{\psi}_4\rangle = v_e |\psi_0\rangle + v_e |\psi_1\rangle + u_e |\psi_4\rangle,
\]
where
\[
v_e = \frac{h_e}{\Delta_e}, \quad u_e = \sqrt{1 - |v_e|^2}, \quad u_e = \sqrt{1 - |v_e|^2}.
\]
We assume here that $|v_e| \ll 1$ and keep terms up to $v_e^2$ and neglect terms proportional to $v_e^3$ with $n \geq 3$. Then the transformed Hamiltonian $H = H_0 + H_p + H_s + H_e$ is given by the matrix elements,
\[
\langle \tilde{\psi}_n | H | \tilde{\psi}_{n'} \rangle = \tilde{e}_n \delta_{n,n'} - \frac{2 |\nu_e|^2}{\Delta_e}, \\
\langle \tilde{\psi}_4 | H | \tilde{\psi}_4 \rangle = \Delta_e + \frac{4 |\nu_e|^2}{\Delta_e}, \\
\langle \tilde{\psi}_n | H | \tilde{\psi}_4 \rangle = 0,
\]
where $n, n' = 0, 1$.

Omitting the high-energy state $|\tilde{\psi}_4\rangle$, we get the two-level effective Hamiltonian describing Rabi oscillations,
\[
H = \begin{pmatrix} 0 & -2 |\nu_e|^2 / \Delta_e - \epsilon_{2L} / \Delta_e \\ -2 |\nu_e|^2 / \Delta_e - \epsilon_{2L} / \Delta_e & 1 - \frac{2 |\nu_e|^2}{\Delta_e} \end{pmatrix}.	ag{11}
\]
where we shift the chemical potential by adding the term
\[
\left( \Delta_{hf} |\nu_b|^2 + \frac{2 |\nu_e|^2}{\Delta_e} \right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.
\]
Taking into account Eqs. (10) and (9), we get
\[
\frac{2 |\nu_e|^2}{\Delta_e} = \frac{\alpha(\omega_e)}{2} \left| u_{L,0}(r, 0, 0) + u_{-L,0}(r, 0, 0) \right|^2 \times \\
\left( 1 - \frac{1}{3} \frac{g_B^2}{\hbar^2 \Delta_{hf}^2} B_p^2 + B_s^2 + 2 B_p B_s \cos(\omega_{pt} t) \right). \tag{12}
\]
On the right hand side of Eq. (12) there are time-independent and time-dependent terms. The former can be considered as weak perturbation which does not contribute to the Rabi oscillations, whereas the second term gives rise to Rabi oscillations. Off-diagonal part of the Hamiltonian (11) after omitting the time-independent terms gives us the Rabi oscillation Hamiltonian (16) in the main text [1].

II. ROTATION SENSOR ACCURACY

When QRs are placed in a non-inertial frame rotating with angular velocity $\Omega = \Omega e$ (as described in the main text [1] and as illustrated in Fig. 2), they can be used as a high accuracy rotation sensor to determine $\Omega$. The accuracy of the rotation sensor due to uncertainty of the pump and Stokes frequencies is analyzed in the main text. Here we derive the uncertainty in the angular velocity due to Rabi frequency fluctuations and due to shot noise in the pump and Stokes pulses.
A. Uncertainty due to Rabi frequency fluctuation

The energies $\epsilon_{m_1} = m_1^2 C(r_I)$ do not depend on the laser frequency and intensity. Therefore, the frequencies of the quantum transitions $|m_1| \rightarrow |m'_1|$ are independent of the laser frequency and intensity. Hence, the only source of uncertainty is fluctuations in frequencies of the Stokes, pump and “kick” pulses which results in fluctuations in the Rabi oscillations. Indeed, fluctuations of frequencies result in fluctuations in the Rabi frequency (2),

$$\phi_R = \Omega_R \tau_\pi = \pi \pm \delta\phi_\omega,$$

where

$$\delta\phi_\omega = \sqrt{\sum_{k=p,s} \left( \frac{\partial\phi_R}{\partial\omega_k} \delta\omega_k \right)^2} = \phi_R \sqrt{\left( \frac{\delta\omega_p}{\Delta\omega_B} \right)^2 + \left( \frac{\delta\omega_s}{\Delta\omega_B} \right)^2}.$$  

Taking $\Delta\omega_p \approx \Delta\omega_s = 2.86 \times 10^{-9} \text{ s}^{-1}$ (see the main text [1]) and $\Delta\omega_B = 1.26 \times 10^8 \text{ s}^{-1}$, we get

$$\delta\phi_\omega = \frac{3.21 \times 10^{-17}}{\phi_R}. \quad (15)$$

When the energy levels $\epsilon_{2L}$ are not degenerate, but $\Delta\epsilon = \epsilon_{2L} - \epsilon_0$, the difference $\Delta\phi_\omega = \phi_{2L} - \phi_{-2L}$ is

$$\Delta\phi_\omega = \frac{\Delta\epsilon}{4\hbar \Omega_R}, \quad (16)$$

where $\Omega_R$ is the Rabi frequency (2). The levels $\epsilon_{2L}$ can be distinguished when $\Delta\phi_\omega > \delta\phi_\omega$, or

$$\frac{\Delta\epsilon}{C(r_I)} > \frac{\delta\phi_\omega}{C(r_I)} = \frac{4\hbar \Omega_R}{C(r_I)} \delta\phi_\omega = 5.998 \times 10^{-17}, \quad (17)$$

where we take $\Omega_R = 3.142 \text{ s}^{-1}$. Then, taking $C(r_I) = \xi_B \times 0.1613 \text{ nK}$ (which corresponds to the Laguerre-Gaussian beam with $l = 5$, $p = 0$ and $v_0 = 10 \mu\text{m}$, see Eq. (2) in the main text [1]), we get the uncertainty of the energy splitting $\delta\epsilon_\omega$ as

$$\delta\epsilon_\omega = \hbar \times 1.267 \times 10^{-15} \text{ s}^{-1}. \quad (18)$$

Knowing the uncertainty $\delta\epsilon_\omega$ of the energy splitting, we are able to estimate the uncertainty $\delta\Omega_\omega$ of the rotation sensor,

$$\delta\Omega_\omega = \frac{1}{\sqrt{N}} \frac{\delta\epsilon_\omega}{4L\hbar}, \quad (19)$$

where $N = 2j_{max} + 1$ is the number of singly-occupied rings filled with QR atoms. When $L = 25$ and $N = 161$, then

$$\delta\Omega_\omega = 9.985 \times 10^{-19} \text{ s}^{-1}. \quad (19)$$

Comparing this result with $\delta\Omega$ in Eq. (23) in the main text [1], one can see that uncertainty in $\Omega$ due to fluctuations of the Rabi frequency is very small with respect to the uncertainty $\delta\Omega$ due to the fluctuations of the pump and Stokes frequencies.

B. Uncertainty due to shot noise in the pump and Stokes pulses

Another source of uncertainty arises from shot noise in the Stokes, pump and kick pulses. Shot noise results in fluctuations in the position and amplitude of the population oscillations of the Ramsey fringes because of fluctuation of the Rabi frequencies,

$$\phi_R = \Omega_R \tau_\pi = \pi \pm \delta\phi_\ell,$$

where

$$\delta\phi_\ell \approx \pi \left( \frac{1}{\sqrt{N_p}} + \frac{1}{\sqrt{N_s}} \right). \quad (21)$$

where $N_p$ and $N_s$ are the number of photons in the pump and Stokes pulses during the time $\tau_\pi = \pi/\Omega_R$. When $N_p \sim N_s \sim 10^{20}$, then

$$\delta\phi_\ell = 1.987 \times 10^{-14}. \quad (22)$$

Uncertainty in the energy splitting can be estimated in a fashion similar to Eq. (17),

$$\frac{\Delta\epsilon}{4\hbar \Omega_R} > \frac{\delta\phi_\ell}{4\hbar \Omega_R} = \frac{\delta\phi_\ell}{\phi_R} = 6.325 \times 10^{-15}. \quad (22)$$

For $\Omega_R = 3.142 \text{ s}^{-1}$, we get $\delta\phi_\ell = \hbar \times 7.949 \times 10^{-14} \text{ s}^{-1}$.

Knowing the uncertainty $\delta\epsilon_\ell$ of the energy splitting, we are able to estimate the uncertainty $\delta\Omega_\ell$ of the rotation sensor,

$$\delta\Omega_\ell = \frac{1}{\sqrt{N}} \frac{\delta\epsilon_\ell}{4L\hbar}, \quad (23)$$

where $N = 2j_{max} + 1$ is the number of singly-occupied rings filled with QR atoms. When $L = 25$ and $N = 161$, the uncertainty is

$$\delta\Omega_\ell = 6.265 \times 10^{-17} \text{ s}^{-1}. \quad (23)$$

Comparing these results with Eq. (23) in the main text [1], one can see that the uncertainty in $\Omega$ due to shot noise in the pump, Stokes and kick pulses is of the same order of magnitude as the uncertainties due to fluctuations in the pump and Stokes frequencies.

III. DISCRIMINATING AGAINST IN-PLANE ACCELERATION

An additional term $H_{gy} = -M g \cdot r$ must be added into the QR Hamiltonian to model the affects of a gravitational field $g$ (2). This preserves the rotational symmetry in the plane perpendicular to $g$ (the $x$-$y$ plane in Fig. 3), but lifts the rotational symmetry in the $x$-$z$ and $y$-$z$ planes. As a result, the QRs rotating in the $x$-$y$ plane clockwise and counterclockwise with the same quantum number $|m_1|$ have the same energy. When the QRs are placed in the $x$-$z$ or $y$-$z$ plane (such that $g$ is in the plane of the QRs), the degeneracy of the quantum states $|m_1|$ and $|-m_1|$ are split and the splitting depends on $m_1$. Hence,
FIG. 3: Elimination of the in-plane acceleration $a$ by inclining the QRs. Here $g = -g_e$ is the acceleration due to gravity, $\Omega$ is the angular velocity, and $\mathbf{g}' = \mathbf{g} - \mathbf{a}$ is the total acceleration, where $e_x$, $e_y$, and $e_z$ are unit vectors parallel to the $x$-, $y$-, and $z$-axes. $\Omega' = \Omega' e'_z$, where $\Omega' = (\Omega \cdot e'_z)$ and the unit vector $e'_z$ is antiparallel to $\mathbf{g}'$. The angle between $\mathbf{g}$ and $\mathbf{g}'$ is $\theta_a$. Blue ellipse is the QR placed in the plane perpendicular to $\mathbf{g}'$.

Now consider QRs placed in a non-inertial frame moving with acceleration $\mathbf{a}$. The effective gravity $\mathbf{g}'$ in the non-inertial frame is given by $\mathbf{g}' = \mathbf{g} - \mathbf{a}$. Therefore, when we turn the QRs to the plane perpendicular to $\mathbf{g}'$, we obtain the splitting of the energy levels caused just by $\Omega' = \Omega \cdot e'_z$, as illustrated in Fig. 3, where $e'_z$ is the unit vector along $-\mathbf{g}'$.

[1] I. Kuzmenko, T. Kuzmenko, Y. B. Band, “Quantum Rotor Atoms in Light Beams with Orbital Angular Momentum: Highly Accurate Rotation and Acceleration Sensing”, to be published.

[2] I. Kuzmenko, T. Kuzmenko, Y. Avishai and Y. B. Band, Phys. Rev. A 100, 033415 (2019).