Radiative Corrections to the Single Spin Asymmetry in Heavy Quark Photoproduction

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We analyze in the framework of pQCD the properties of the single spin asymmetry in heavy flavor production by linearly polarized photons. At leading order, the parallel-perpendicular asymmetry in azimuthal distributions of both charm and bottom quark is predicted to be about 20% in a wide region of initial energy. Using the soft gluon resummation formalism, we have calculated the next-to-leading order corrections to the asymmetry to next-to-leading logarithmic accuracy. It is shown that radiative corrections practically do not affect the Born predictions for the azimuthal asymmetry at energies of the fixed target experiments. Both leading and next-to-leading order predictions for the asymmetry are insensitive to within few percent to theoretical uncertainties in the QCD input parameters: \( m_Q, \mu_R, \mu_F, \Lambda_{QCD} \) and in the gluon distribution function. Our analysis shows that nonperturbative corrections to a \( B \)-meson azimuthal asymmetry due to the gluon transverse motion in the target are negligible. We conclude that measurements of the single spin asymmetry would provide a good test of pQCD applicability to heavy flavor production at fixed target energies.

I. INTRODUCTION

Presently, the basic spin-averaged characteristics of heavy flavor hadro-, photo- and electroproduction are known exactly up to the next-to-leading order (NLO) \[ \text{(1.1)} \]. Two main results of the exact pQCD calculations can be formulated as follows. First, the NLO corrections are large; they increase the leading order (LO) predictions for both charm and bottom production cross sections approximately by a factor of 2. For this reason, one could expect that the higher order corrections as well as the nonperturbative contributions can be essential in these processes, especially for the c-quark case. Second, the fixed order predictions are very sensitive to standard uncertainties in the input QCD parameters. In fact, the total uncertainties associated with the unknown values of the heavy quark mass, \( m_Q \), the factorization and renormalization scales, \( \mu_F \) and \( \mu_R \), \( \Lambda_{QCD} \) and the parton distribution functions are so large that one can only estimate the order of magnitude of the NLO predictions for total cross sections \[ \text{(1.2)} \]. For this reason, it is very difficult to compare directly, without additional assumptions, the fixed order predictions for spin-averaged cross sections with experimental data and thereby to test the pQCD applicability to the heavy quark production.

Since the spin-averaged characteristics of heavy flavor production are not well defined quantitatively in pQCD it is of special interest to study those spin-dependent observables which are stable under variations of input parameters of the theory \[ \text{[10]} \]. In this report we analyze the charm and bottom production by linearly polarized photons, namely the reactions

\[ \gamma + N \rightarrow Q(\overline{Q}) + X. \]

We consider the single spin asymmetry parameter, \( A(s) \), which measures the parallel-perpendicular asymmetry in the quark azimuthal distribution:

\[ A(s) = \frac{1}{\mathcal{P}_\gamma} \frac{d\sigma(s, \varphi = 0) - d\sigma(s, \varphi = \pi/2)}{d\sigma(s, \varphi = 0) + d\sigma(s, \varphi = \pi/2)}. \]

Here \( d\sigma(s, \varphi) \equiv \frac{d\sigma}{ds} (s, \varphi) \), \( \mathcal{P}_\gamma \) is the degree of linear polarization of the incident photon beam, \( \sqrt{s} \) is the centre of mass energy of the process \[ \text{[11]} \] and \( \varphi \) is the angle between the beam polarization direction and the observed quark transverse momentum.

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The properties of the single spin asymmetry at Born level as well as the contributions of nonperturbative effects (such as the gluon transverse motion in the target and the heavy quark fragmentation) have been considered in [10]. Using the methods of Refs. [11–14] for the threshold resummation of soft gluons, we have also calculated the next-to-leading order corrections to \( A(s) \) at next-to-leading logarithmic (NLL) level [13]. The main results of our analysis can be formulated as follows:

- At fixed target energies, the LO predictions for azimuthal asymmetry [1,2] are not small and can be tested experimentally. For instance,
  \[
  A(s = 400\text{GeV}^2) |^{\text{LO}}_{\text{Charm}} \approx A(s = 400\text{GeV}^2) |^{\text{LO}}_{\text{Bottom}} \approx 0.18. \tag{1.3}
  \]

- Radiative corrections practically do not affect the Born predictions for \( A(s) \) at fixed target energies.

- At energies sufficiently above the production threshold, both leading and next-to-leading order predictions for \( A(s) \) are insensitive (to within few percent) to uncertainties in the QCD parameters: \( m_Q, \mu_R, \mu_F, \Lambda_{QCD} \) and in the gluon distribution function. This implies that theoretical uncertainties in the spin-dependent and spin-averaged cross sections (the numerator and denominator of the fraction (1.2), respectively) cancel each other with a good accuracy.

- Nonperturbative corrections to the \( b \)-quark azimuthal asymmetry \( A(s) \) due to the gluon transverse motion in the target are negligible. Because of the smallness of the \( c \)-quark mass, the \( k_T \)-kick corrections to \( A(s) \) in the charm case are larger; they are of order of 20%.

We conclude that the single spin asymmetry is an observable quantitatively well defined, rapidly convergent in pQCD and insensitive to nonperturbative contributions. Measurements of asymmetry parameters would provide a good test of the fixed order QCD applicability to heavy flavor production.

II. SINGLE SPIN ASYMMETRY AT LEADING ORDER

At leading order, \( \mathcal{O}(\alpha_{em} \alpha_S) \), the only partonic subprocess which is responsible for heavy quark photoproduction is the two-body photon-gluon fusion:

\[
\gamma(k_\gamma) + g(k_g) \rightarrow Q(p_Q) + \bar{Q}(p_{\bar{Q}}). \tag{2.1}
\]

The cross section corresponding to the Born diagrams is [10]:

\[
\frac{d^2 \sigma^{\text{Born}}}{d\hat{s}d\varphi} = C e_q^2 \alpha_{em} \alpha_S \left( \mu_R^2 \right) \frac{\hat{s}}{s} \left( 1 + \hat{x}^2 \right) + \frac{2(1 - \hat{\beta}^2)(\hat{\beta}^2 - \hat{x}^2)}{(1 - \hat{x}^2)^2} \left( 1 + \mathcal{P}_\gamma \cos 2\varphi \right), \tag{2.2}
\]

where \( \mathcal{P}_\gamma \) is the degree of the photon beam polarization; \( \varphi \) is the angle between the observed quark transverse momentum, \( \vec{p}_{Q,T} \), and the beam polarization direction. In [2,2] \( C \) is the color factor, \( C = T_F = \text{Tr}(T^at^a)/(N_c^2 - 1) = 1/2 \), and \( c_Q \) is the quark charge in units of electromagnetic coupling constant. We use the following definition of partonic kinematical variables:

\[
\begin{align*}
\hat{s} &= (k_\gamma + k_g)^2; & \hat{t} &= (k_\gamma - p_Q)^2; \\
\hat{u} &= (k_g - p_Q)^2; & \hat{x} &= 1 + 2 \frac{\hat{t} - m^2}{\hat{s}}; \\
\hat{\beta} &= \sqrt{1 - \frac{4m^2}{\hat{s}}}; & \vec{p}_{Q,T}^2 &= \frac{\hat{s}}{4} (\hat{\beta}^2 - \hat{x}^2);
\end{align*} \tag{2.3}
\]

where \( m \) is the heavy quark mass.

Unless otherwise stated, the CTEQ5 [41] parametrization of the gluon distribution function is used. The default values of the charm and bottom mass are \( m_c = 1.5 \text{ GeV} \) and \( m_b = 4.75 \text{ GeV} \); \( \Lambda_4 = 300 \text{ MeV} \) and \( \Lambda_5 = 200 \text{ MeV} \). The default values of the factorization scale \( \mu_F \) chosen for the \( A(s) \) asymmetry calculation are \( \mu_F |_{\text{Charm}} = 2m_c \) for
the case of charm production and $\mu_F \big|_\text{Bottom} = m_b$ for the bottom case [8,17]. For the renormalization scale, $\mu_R$, we use $\mu_R = \mu_F$.

Let us discuss the pQCD predictions for the asymmetry parameter defined by (1.2). Our calculations of $A(s)$ at LO for the $c$- and $b$-quark are presented by solid lines in Fig.1. One can see that at energies sufficiently above the production threshold the single spin asymmetry $A(E_\gamma)$ depends weekly on $E_\gamma$, $E_\gamma = (s - m_N^2)/2m_N$.

![Single spin asymmetry comparison](image)

FIG. 1. Single spin asymmetry, $A(E_\gamma)$, in $c$- and $b$- quark production as a function of beam energy $E_\gamma = (s - m_N^2)/2m_N$; the QCD LO predictions with and without the inclusion of $k_T$-kick effect.

The most interesting feature of LO predictions for $A(E_\gamma)$ is that they are practically insensitive to uncertainties in QCD parameters. In particular, changes of the charm quark mass in the interval $1.2 < m_c < 1.8$ GeV affect the quantity $A(E_\gamma)$ by less than 6% at energies $40 < E_\gamma < 1000$ GeV. Remember that analogous changes of $m_c$ lead to variations of total cross sections from a factor of 10 at $E_\gamma = 40$ GeV to a factor of 3 at $E_\gamma = 1$ TeV. The extreme choices $m_c = 4.5$ and $m_b = 5$ GeV lead to 3% variations of the parameter $A(E_\gamma)$ in the case of bottom production at energies $250 < E_\gamma < 1000$ GeV. The total cross sections in this case vary from a factor of 3 at $E_\gamma = 250$ GeV to a factor of 1.5 at $E_\gamma = 1$ TeV. The changes of $A(E_\gamma)$ are less than 3% for choices of $\mu_F$ in the range $0.5 < \mu_F < 2m_b$. For the total cross sections, such changes of $\mu_F$ lead to a factor of 2.7 at $E_\gamma = 250$ GeV and of 1.7 at $E_\gamma = 1$ TeV. We have verified also that all the CTEQ3-CTEQ5 versions of the gluon distribution function [16] as well as the CMKT parametrization [18] lead to asymmetry predictions which coincide with each other with accuracy better than 1.5%.

III. NONPERTURBATIVE CONTRIBUTIONS

Let us discuss how the pQCD predictions for single spin asymmetry are affected by nonperturbative contributions due to the intrinsic transverse motion of the gluon in the target. In our analysis, we use the MNR model [17] parametrization of the gluon transverse momentum distribution,

$$\vec{k}_g = z\vec{k}_N + \vec{k}_{g,T},$$

(3.1)

where $\vec{k}_N$ is the target momentum in the $\gamma N$ centre of mass system. According to [17], the primordial transverse momentum, $\vec{k}_{g,T}$, has a random Gaussian distribution:

$$\frac{1}{N}\frac{d^2N}{d^2k_T} = \frac{1}{\pi \langle k_T^2 \rangle} \exp \left(-\frac{k_T^2}{\langle k_T^2 \rangle}\right),$$

(3.2)

where $k_T^2 = \vec{k}_{g,T}^2$. It is evident that the inclusion of this effect results in a dilution of azimuthal asymmetry. In [8,17], the parametrization (3.2) (so-called $k_T$-kick) have been used to describe the single inclusive spectra and the $Q\bar{Q}$ correlations. It was found that in charm photoproduction $0.5 < \langle k_T^2 \rangle < 2$ GeV$^2$. 

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Our calculations of the parameter $A(s)$ at LO with the $k_T$-kick contributions are presented in Fig.1 by dashed $(\langle k_T^2 \rangle = 0.5 \text{ GeV}^2)$ and dotted $(\langle k_T^2 \rangle = 1 \text{ GeV}^2)$ curves.

So, we can conclude that nonperturbative corrections to the $b$-quark asymmetry parameter $[1, 2]$ due to the $k_T$-kick effect practically do not affect predictions of the underlying perturbative mechanism: photon-gluon fusion.

Our calculations of the $p_T$- and $x_F$-distributions of the azimuthal asymmetry in heavy quark photoproduction are given in [10].

IV. NEXT-TO-LEADING ORDER CORRECTIONS

The perturbative expansion for the $\gamma g$ cross section, $\hat{\sigma}(\rho, \mu^2)$, is usually written in terms of scaling functions:

$$\hat{\sigma}(\rho, \mu^2) = \frac{\alpha_{em} \alpha_S(\mu^2) e_Q^2}{m^2} \sum_{k=0}^{\infty} (4\pi \alpha_S(\mu^2))^{k} \sum_{l=0}^{k} c^{(k,l)}(\rho) \ln^l \frac{\mu^2}{m^2}, \quad (4.1)$$

where $\rho = 4m^2/\hat{s}$, $\hat{s} = (k_T + k_T)^2$; $c^{(0,0)}(\rho)$ corresponds for the Born contribution [2,2], $c^{(1,1)}(\rho)$ and $c^{(1,0)}(\rho)$ describe the order-$\alpha_S^2$ corrections, factorization scale ($\mu_F = \mu_R = \mu$) dependent and independent, respectively.

To calculate the NLO corrections to the single spin asymmetry $A(s, \rho)$, we need to take into account the virtual $\mathcal{O}(\alpha_{em} \alpha_S^2)$ corrections to the Born process [2,3] and the real gluon emission in the photon-gluon fusion:

$$\gamma(\hat{k}_T) + g(\hat{k}_T) \rightarrow Q(p_Q) + \overline{Q}(p_{\overline{Q}}) + g(p_g). \quad (4.2)$$

The scale dependent term $c^{(1,1)}(\rho)$ which is the coefficient of $\ln \frac{\mu^2}{m^2}$ can be expressed explicitly in terms of the Born cross section using the renormalization group arguments [3]. Contribution of the scale independent cross section, $c^{(1,0)}(\rho)$, near the threshold can be obtained with help of the soft gluon resummation method [12, 14]. To the next-to-leading logarithmic accuracy, the soft gluon contribution to the photon-gluon fusion can be written in a factorized form as:

$$\delta(\hat{s} \hat{t}_1 \hat{u}_1) \approx B^{\text{Born}}(\hat{s} \hat{t}_1 \hat{u}_1) \bigg\{ \delta(\hat{s} + \hat{t}_1 + \hat{u}_1) + \sum_{n=1}^{\infty} \left( \frac{\alpha_S(\mu)}{\pi} \right)^n K^{(n)}(\hat{s} \hat{t}_1 \hat{u}_1) \bigg\}, \quad (4.3)$$

where $\hat{t}_1 = \hat{t} - m^2$, $\hat{u}_1 = \hat{u} - m^2$ and $B^{\text{Born}}(\hat{s}, \hat{t}_1, \hat{u}_1)$ describes the Born level $\gamma g$ cross section:

$$B^{\text{Born}}(\hat{s}, \hat{t}_1, \hat{u}_1) = e_Q^2 \alpha_{em} \alpha_S \left[ \frac{\hat{t}_1}{\hat{u}_1} + \frac{\hat{u}_1}{\hat{t}_1} + \frac{4\hat{m}^2\hat{s}}{\hat{t}_1 \hat{u}_1} \left( 1 - \frac{m^2 \hat{s}}{\hat{t}_1 \hat{u}_1} \right) \right] \quad (4.4)$$

At NLO, $\mathcal{O}(\alpha_{em} \alpha_S^2)$, the soft gluon corrections to NLL accuracy in the $\overline{\text{MS}}$ factorization scheme are (cf. Ref. [14]):

$$K^{(1)}(\hat{s} \hat{t}_1 \hat{u}_1) = 2C_A \left[ \ln \left( \frac{\hat{s}_4/m^2}{\hat{s}_4} \right) \right] + \left[ \frac{1}{\hat{s}_4} \right] + \left\{ C_A \left( \ln \left( \frac{\hat{t}_1}{\hat{u}_1} \right) + \text{Re} L_{\beta} - \ln \left( \frac{\mu^2}{m^2} \right) \right) - 2C_F (\text{Re} L_{\beta} + 1) \right\} + \delta(\hat{s}_4) C_A \ln \left( \frac{\mu^2}{m^2} \right) \ln \left( \frac{\mu^2}{m^2} \right), \quad (4.5)$$

where $C_A = N_c = 3$, $C_F = N_c^2 - 1 = 4/3$, $\hat{s}_4 = \hat{s} + \hat{t}_1 + \hat{u}_1$ and $\beta = \sqrt{1 - \rho}$. The function $L_{\beta}$ and the plus-distribution in (4.7) are defined as:

$$L_{\beta} = \frac{1 - 2m^2/\hat{s}}{\beta} \left[ \ln \left( \frac{1 - \beta}{1 + \beta} \right) + i\pi \right], \quad (4.6)$$

$$\left[ \ln \left( \frac{\hat{s}_4/m^2}{\hat{s}_4} \right) \right]_+ = \lim_{\Delta \rightarrow 0} \left\{ \ln \left( \frac{\hat{s}_4/m^2}{\hat{s}_4} \right) \theta(\hat{s}_4 - \Delta) + \frac{1}{l+1} \ln^{l+1} \left( \hat{t} \hat{u} \hat{s}_4 \right) \right\}. \quad (4.7)$$

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Eq. (4.5) describes very well the threshold behavior of the exact partonic cross sections \( c^{(1,0)}(\rho) \) and \( c^{(1,1)}(\rho) \) and gives, at NLO, practically whole contribution to the unpolarized bottom production in photon-hadron reactions at fixed target energies, \( E_\gamma \lesssim 1 \) TeV.

All terms in the r.h.s. of Eq. (4.3) originate from the so-called collinear \( \vec{p}_{g,T} \to 0 \), and soft, \( \vec{p}_g \to 0 \), components of cross section. Since the azimuthal angle \( \varphi \) is the same for both \( \gamma g \) and \( Q \bar{Q} \) centre of mass systems in the collinear and soft limits, Eq. (4.3) can be generalized to the spin-dependent case substituting the spin-averaged Born cross section by the \( \varphi \)-dependent one: \( B^{\text{Born}}(\hat{s}, \hat{t}_1, \hat{u}_1) \to B^{\text{Born}}(\hat{s}, \hat{t}_1, \hat{u}_1, \varphi) \).

The results of our calculations of the single spin asymmetry \( A(s) \) at NLO to NLL accuracy in \( c \)- and \( b \)-quark production are presented by dashed line in Fig. 2. The details of calculations as well as the higher order predictions for \( A(s) \) will be given in [15].

![FIG. 2. Single spin asymmetry, \( A(E_\gamma) \), in \( c \)- and \( b \)-quark production at LO (solid curve) and at NLO to NLL accuracy (dashed curve).](image)

One can see from Fig. 2 that the NLO NLL and Born predictions for \( A(s) \) coincide with each other with accuracy better than 2%. We have verified that the azimuthal asymmetry is independent (to within few percent) of theoretical uncertainties in the QCD input parameters \((m_Q, \mu_R, \mu_F \text{ and } \Lambda_{QCD})\) at NLO too.

V. CONCLUSION

Our analysis shows that the NLO corrections practically do not affect the Born predictions for the single spin asymmetry in heavy flavor production by linearly polarized photons at fixed target energies. So, the quantity \( A(s) \) is an observable quantitatively well defined, rapidly convergent and insensitive to nonperturbative contributions. Measurements of the azimuthal asymmetry in bottom production would be a good test of the conventional parton model based on pQCD.

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