A method to define the energy threshold depending on noise level for rare event searches

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ABSTRACT: Solid state detectors and cryogenic detectors are widely employed in rare event searches, such as direct Dark Matter detection or Coherent Neutrino Nucleus Scattering experiments. The excellent sensitivity and, consequently, their low energy thresholds are among the most appealing features of such detectors. In this paper we discuss a method based on the optimum filtering technique to quantify the lowest trigger threshold achievable as a function of the acceptable amount of noise events, given that the entire raw data stream from the detector is available.

KEYWORDS: Trigger algorithms, cryogenic detectors, solid state detectors, rare event searches
1 Introduction

There is a widespread tendency, among rare event searches, to continuously lower the energy range of interest. Among those using cryogenic and solid state detectors it is certainly the case for light dark matter direct detection experiments (CRESST[1], Edelweiss[2], SuperCDMS[3]) and Coherent Neutrinos Nucleus Scattering (COHERENT[6], MINER[5], ν-cleus[4], RICOCHET[8], TEXONO[7]). The latter are relatively new since the necessary detector sensitivity to access the sub-keV energy scale was only recently achieved. Such impressive accomplishments come as a result of a worldwide intense effort dedicated to detector development. At low energies a thorough understanding of the noise contribution is essential.

In this work we investigate how the noise affects the measured energy spectrum in the vicinity of the trigger threshold. We quantify the noise trigger rate for a specific experimental set-up, in order to define an optimization procedure for assessing the optimal energy threshold.

2 Optimal data filtering

Signal filtering is a powerful tool to reduce the noise contribution in a signal, leading to an improved signal-to-noise ratio, and thus to a better resolution and lower energy threshold. In the case of a known detector response, matched filters usually provide the best result. The transfer function of a matched filters is obtained via maximization of the filter’s response with respect to a certain quantity of interest, such as signal amplitude, signal shape parameter or signal-to-noise ratio. Since their definition follows an optimization problem, matched filters are usually called optimum filters. We will consider the filter which maximizes the signal-to-noise ratio. A required input to build such a filter is the detector’s response function (i.e. the expected signal after a particle interaction). This is usually a pulse-like function which follows from the underlying physics in the detector. In the time domain it typically shows an exponential rise caused by phonon collection, electron-hole
pair creation or scintillation process (depending on the technology) and an exponential decay of the signal height corresponding to the time that the detector needs to recover to the equilibrium state. For illustration purposes we will use data from a particular cryogenic calorimeter but in general all the following considerations apply to any calorimetric detector, namely a detector that provides a measurement of the total energy deposit. In such a detector the signal amplitude is usually the parameter measured from which the energy spectrum is obtained. The signal shape of the detector used in sec. 5 is shown in fig. 1(red).

Once the measurement condition is established, a particle interaction in the detector generates a time dependent output signal $v(t)$, that can be modelled as follows:

$$v(t) = A s(t - t_0) + n(t)$$  \hspace{1cm} (2.1)

where $A$ is the amplitude, which contains the information on the deposited energy, $s(t)$ is the pulse shape function, $t_0$ is the signal onset (arrival time) of the event and $n(t)$ describes the noise.

The most effective way to reduce the noise contribution to the pulse resolution is to use a matched-filter. Given the linearity of the Fourier transform, does not affect the value of $A$.

We denote with $H(\omega)$ the transfer function of the filter in the frequency domain. The filtered signal $\tilde{v}(t)$ can be written as:

$$\tilde{v}(t) = \frac{A}{2\pi} \int_{-\infty}^{+\infty} H(\omega) V(\omega) e^{-j\omega t} d\omega$$  \hspace{1cm} (2.2)

where $V(\omega)$ is the Fourier transform of $v(t)$.

We chose a matched-filter where the criterion of optimization is the maximization of the signal-to-noise ratio $\rho$ at the measurement time $\tau_M$ after filtering:

$$\rho = \frac{\frac{A}{2\pi} \int_{-\infty}^{+\infty} H(\omega) V(\omega) e^{-j\omega \tau_M} d\omega}{\left( \frac{1}{2\pi} \int_{-\infty}^{+\infty} |H(\omega)|^2 N(\omega) d\omega \right)^{1/2}}$$  \hspace{1cm} (2.3)
In the equation $\rho$ is the ratio between the filtered pulse at time $\tau_M$ and the noise RMS after filtering. There is a unique solution for the transfer function $H(\omega)$ [9] which maximises $\rho$ and it is:

$$H(\omega) = K' \frac{S^*(\omega)}{N(\omega)} e^{-j\omega\tau_M}.$$ (2.4)

Where $S^*(\omega)$ is the complex conjugate of the Fourier transform of the pulse shape function. This transfer function $H(\omega)$ is solely built using the noise power spectrum $N(\omega)$ and a reference pulse $s(t)$. The transformed signal in the time domain is illustrated in fig. 1 (blue).

3 Trigger algorithm

The operation of a basic trigger consists in setting a threshold on the detector’s output: as soon as the signal exceeds a fixed value, the trigger flags it as an event to be processed. The threshold is chosen as a function of the noise level to avoid tagging noise fluctuations as physical events. As the signal amplitude approaches the noise level, the discrimination power of pulse shape analysis rapidly decreases. This effect is most pronounced close to the threshold energy. Random fluctuations in the noise become an irreducible background contribution, possibly misleading the interpretation of the results if not accounted for.

Many applications require a low energy threshold and it is therefore desirable to tag events as close as possible to the noise level. Compared to the raw data at the output of the detector, the output of the optimum filter shows a larger signal-to-noise ratio, allowing for a lower threshold. However, the numerical filtering of the entire stream of raw data is not straightforward. The numerical procedure used for this purpose is described in [10].

The optimization of the trigger requires a dedicated step in data analysis, as shown in fig. 2. The algorithm needs the transfer function of the filter as an input, implying that the expected shape of signal and noise power spectra have to be known. In order to obtain them, a short data processing (step 1 in fig. 2) is needed before the actual data triggering (step 2 in fig. 2). The expected signal shape (template pulse) is obtained by averaging a set of selected pulses. To derive a noise power spectrum a large set of baselines (waveforms of pure noise) has to be collected from the data and selected according to well defined criteria ensuring good data quality. To do so, a small set of baselines is selected by visual inspection and characterised in order to automatically select additional waveforms with the same parameters\(^1\). In practice, we compute the average RMS value of the small baseline set as a reference value and the additional baselines are selected such that their RMS values are within one standard deviation of the RMS distribution of the initial set. This procedure does not exclude the presence of small pulses hidden in the selected noise windows, nevertheless this is not a critical issue in the calculation of the noise power spectrum, which is the average power spectrum of all the selected baselines. Thus the contribution of the few\(^2\) hidden pulses becomes negligible in the final noise power spectrum.

Once the transfer function has been built, the data can be filtered to obtain the maximum signal-to-noise ratio. The pulse output of this particular filter is symmetric with respect to the position of the

\(^1\)The same set of baselines will be used in the following section to analyse the noise triggers

\(^2\)The amount of hidden pulses is limited by the total rate of the expected signal, which in general is low for rare event searches
maximum (fig. 1) and the transformation produces lobes in the tails of the signal in the presence of periodic noise such as e.g. 50Hz. Due to these effects an algorithm to trigger only the maximum of a pulse is required. Instead of firing a trigger as soon as the signal exceeds the threshold value, a sliding window is then adopted in order to trigger only the maximum of the pulse, preventing triggering on the local maxima in the tails. An optimized performance is achieved by defining the sliding time window large enough to contain the full filtered pulse. The sliding window advances forward in time, while looking for local maxima values inside the window. When a local maximum above threshold is found the search for the pulse maximum starts. The local maximum found can be a pulse maximum or a lobe in the tails. In order to resolve such cases, the algorithm searches for an absolute maximum inside a new window centred in the current maximum. If a new maximum is found the process is repeated again. In the end when the maximum in the centre of the window is the absolute maximum of the windows it is selected as a triggered event.

An appropriate adjustment of the dead time has to be applied for this triggering method. As a triggered event has to be separated by more than half window length from another trigger, the whole time window length has to be considered dead time for any energy smaller than the triggered pulse. The resulting energy dependent dead time can be easily obtained from the final spectrum and be included in the trigger efficiency which would depend then not only on rate but also on spectral shape. This simple dead time calculation is valid for data-sets where the rate is low enough to consider negligible the probability of three or more events that pile-up, certainly the case in rare event search.

**Figure 2:** Block diagram of the data processing chain. Step 1 represents the data preprocessing in order to build the filter and obtain the input for step 2 where the trigger and its optimization is defined. Step 3 contains the data processing and analysis.
4 Analytical description of the noise trigger rate

The threshold value for the trigger algorithm can be chosen freely, a common choice is 5 times the standard deviation of the baseline noise distribution. We propose instead to set it according to the total rate of accepted noise triggers, which then allows to evaluate the background contribution due to triggering in the noise.

In order to evaluate the number of noise triggers for the algorithm illustrated above, given a particular noise spectrum, we have to describe the probability that in a time window containing only noise, the maximum value of the samples\(^2\) exceeds the threshold value. This first requires to describe the distribution probability \(P(x)\) of filtered noise samples and then to define the length of the trigger window, \(d\). Once these two parameters are fixed it is possible to describe the distribution of the maxima for a set of filtered baseline windows using combinatorics. In other words, we look for the probability \(P_d(x_{\text{max}})\) that in a defined noise window of length \(d\), after filtering, the maximum sample value is equal to a certain value \(x_{\text{max}}\).

Assuming that each sample follows the distribution \(P(x)\) and that their are statistically independent, the joint probability of one sample being equal to \(x_{\text{max}}\) and all other being smaller can be expressed as:

\[
P_d(x_{\text{max}}) = \frac{d!}{1!(d-1)!} (P(x_{\text{max}})) \left( \int_{-\infty}^{x_{\text{max}}} P(x) dx \right)^{d-1}.
\] (4.1)

In case of pure white noise \(P(x)\) can be described by a Gaussian function. This is in fact the case of interest given that the particular filter used whitens the noise [9]. Using Gaussian error function \(erf(x)\) to describe the integral of a Gaussian \(P(x)\), the probability distribution \(P_d(x_{\text{max}})\) can be written as:

\[
P_d(x_{\text{max}}) = \frac{d}{\sqrt{2\cdot\pi\cdot\sigma}} \cdot \left( e^{-\left(\frac{x_{\text{max}}}{\sqrt{2}\sigma}\right)^2} \right) \cdot \left( \frac{1}{2} + \frac{erf\left(\frac{x_{\text{max}}}{(\sqrt{2}\sigma)}\right)}{2} \right)^{d-1}.
\] (4.2)

The total Noise Trigger Rate (NTR) above threshold is

\[
NTR = \frac{1}{t_{\text{win}} \cdot N_{\text{win}} \cdot m_{\text{det}}} \int_{x_{\text{th}}}^{\infty} P_d(x_{\text{max}}) dx_{\text{max}}
\] (4.3)

where \(t_{\text{win}}\) is the trigger time window, \(N_{\text{win}}\) is the number of noise windows used, \(x_{\text{th}}\) is the threshold value and \(m_{\text{det}}\) is the detector mass. Obviously the NTR does not scale with the detector mass, but the background rate often scales with time and mass therefore it is useful to divide the NTR by the detector mass in order to compare it with different expected signals.

From eq. 4.3 it is possible to determine the value of the threshold, \(x_{\text{th}}\), according to an accepted rate of noise triggers. This approach provides a tool to define the value of the energy threshold considering the acceptable rate of misidentified noise events according to the application under consideration. Furthermore, eq. 4.1 also describes the energy distribution of this background, which can then be accounted for in background models.

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\(^{2}\)We use ‘sample’ to define single digitized value of the detector output, and ‘window’ for a set of sequential samples that describe a waveform.
To validate the model we simulate a set of 40000 baseline windows, all with the same characteristics: average $= 0$ V, $\sigma = 3$ V and 25000 samples. Fig. 3 shows an example of a white noise baseline simulated numerically. The distribution of the samples is shown on the left panel. For each noise window we computed the maximum value of the samples. A fit of the spectrum of the baselines maxima with eq. 4.2, where the parameters $\sigma$ and $d$ are left free, is displayed in fig. 4. The result of the fit (tab. 1) is in agreement with the input parameters within the statistical error.

![Figure 3: Right: white noise simulated numerically. Left: the sample distribution fitted with a Gaussian function.](image)

![Figure 4: Distribution of the maxima of 40000 baseline windows of 25000 samples, the black curve is the result of the fit according eq.4.2.](image)

| Parameter | input value | fit result |
|-----------|-------------|------------|
| $\sigma$  | 3 V         | 3 ± 0.003  |
| $d$       | 25000       | 25010 ± 490|

**Table 1:** Fit result for the distribution of the baseline maxima.

5 **Experimental results**

We now show the result of the noise trigger effect applied to an interesting real case. We use the data acquired with a gram scale cryogenic detector prototype developed for rare event search [11]. This R&D project aims for a detection of low energy nuclear recoils for light dark matter direct detection [12] and coherent neutrinos-nucleus scattering [4], processes for which a lower energy threshold enhances the sensitivity [13]. This measurement provides a suitable test bench for the method proposed. The detector consists of a gram-scale calorimeter, an Al$_2$O$_3$ cube of $5 \times 5 \times 5$ mm$^3$. 

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with a mass of 0.5 g.

The detector was measured in a dilution cryostat at the Max-Planck-Institut (MPI) for Physics in Munich, Germany. The absorber was held on three Al₂O₃ spheres with a diameter of 1 mm resting on a copper plate in order to provide point-like contacts with the detector housing. The temperature readout is obtained by a tungsten TES (Transition Edge Sensor) evaporated on the top surface of the crystal. The TES design was adjusted to provide a calorimetric measurement [14]. Electrical and thermal connections are made with Ø 25 μm Al and Au bond wires, respectively. The cryostat reached a stable base temperature of 11 mK. The W-TES showed a normal-to-superconducting transition at ~22 mK and the detector was stabilized at this temperature via a gold heater sputtered on the Al₂O₃ absorber’s surface. A ⁵⁵Fe X-ray calibration source with an activity of 0.6 Bq was placed about 2 cm away from the detector. Commercial SQUID magnetometers were used for signal readout, combined with typical CRESST detector control system [15]. More detailed description of the measurement can be found in [11].

We apply the procedure described in sec. 3-4 on the data. The reference pulse used to construct the filter is the one shown in fig. 1, that is obtained by averaging a set of pulses normalized to 1. The data are continuously filtered in the frequency domain with the optimum filter described in sec. 2. The filtered waveforms are reconstructed in time domain. Baseline windows are selected as described in sec. 3. For consistency with the selection of baselines, a similar quality cut is applied on pulses. The same RMS cut used for the selection of the baselines is applied to the baseline samples before the onset of the pulse to ensure the same noise conditions.

A first simple trigger without optimization is used for calibration and performance studies. The baseline fluctuation after filtering corresponds to $\sigma = 3.45 \pm 0.37$ eV as can be seen in fig. 5 (left) and the resulting calibration factor obtained with the calibration source is 1.1 mV/eV. The resolution of the calibration peak is significantly larger than the baseline fluctuation, the degradation in energy resolution can be attributed in this case to the reconstruction of the pulse height for energies above saturation for which the truncated template fit technique was used [16].

The first data pre-processing provides the set of empty baselines suitable to evaluate a trigger threshold for the measurement considered. Noise time windows (empty baselines) of 400 ms, which is the length of the trigger sliding window, are selected from the data to build the distribution

![Baseline fluctuation and Calibration](image.png)

**Figure 5:** Left: baseline fluctuation of filtered baseline describing the irreducible noise contribution. Right: Histogram of the ⁵⁵Fe source
of the maxima. The fit of the distribution using eq. 4.2 is shown in fig. 6 (left). Like for the simulated case, we let the parameters $\sigma$ and $d$ free. The result of the fit can be used to compute the noise trigger rate (NTR) as a function of the trigger threshold (fig. 6 (right)). Despite the distribution is well fitted ($\chi^2=0.96$), the parameter $d = 392 \pm 6$ is far from the expected value of 8000 and $\sigma = 4.5 \pm 0.2$ eV differs significantly from baseline fluctuation $\sigma = 3.45$ eV.

As $d$ is the number of statistically independent samples in a trigger window, the result suggests that data after filtering still have large components of correlated noise unlike in the simulated data where only ergodic noise is considered. The presence of time-correlated noise jeopardizes the interpretation of the parameter $d$, nevertheless eq. 4.2 still holds describing the maxima distribution and, therefore, the spectral shape of random noise triggers. The detector baseline fluctuation of $\sigma = 3.45$ eV allows for an identification of ~20 eV signals with high efficiency [11] however with this noise condition the noise trigger rate results in ~$10^6$ counts/(kg day). In fig. 6 (right) different threshold values are highlighted corresponding to noise trigger rate from $10^5$ to $10^8$ counts/kg day. Compared to the expected value, a smaller value of $d$ extends the tail of the noise trigger distribution to higher energy (eq. 4.2), explaining why this detector, triggered at a threshold of ~20 eV ($> 5 \cdot \sigma$ of the baseline distribution), still has a high NTR. In fact pronounced noise components populating the same frequency of the signal can not be filtered completely. In this test case an important 50 Hz component is present, by reducing it the condition for eq. 4.1 of having statistically independent sample would be better fulfilled and therefore a further improvement of noise leakage above threshold would be expected.

In order to set the ideal threshold value for the measurement under consideration we compare the expected signal rate with the background rate due only to noise triggers. The background is $10^8$ counts/(kg day) in the energy region below 1 keV [12]. In this condition an energy threshold of 19.6 eV corresponding to a NTR of $10^6$ counts/(kg day) can be considered acceptable since it will contribute to the total rate with a negligible fraction of noise triggered events.

We can also consider the case where the detector is used for its application (i.e. CNNS measurements). It is interesting to evaluate the threshold in the hypothetical case that this detector is deployed with the same noise condition near a 4 GW nuclear power plant in order to detect the neutrinos spectrum. The expected neutrino rate in sapphire 15 m far from a such reactor is ~ 10 counts/(kg

![Figure 6: Left: blue: histogram of the maximum founded in ~ 1400 baselines available, red: fit result of eq.4.2. Right: blue line: Noise Trigger Rate as function of threshold, three representative values are displayed (see text).](image)

\[ \sigma = 3.45 \text{ eV} \]
day) in the energy interval between 20 and 40 eV [4]. Supposing that the background is significantly lower than the neutrino interaction rate, a reasonable choice can be a threshold which ensures a NTR five times lower than the process under investigation. Given the expected rate in this energy interval the suggested threshold would be 30.3 eV (fig. 6). With this threshold the misidentified triggers in the noise give a negligible contribution to the spectral shape.

6 Conclusion

For many applications the energy threshold is a crucial parameter for the success of the experiment. Therefore, the knowledge of the triggered noise background spectrum is an important information. We have shown an analytical description of the noise trigger rate using solid state calorimeter detectors. We have evaluated the irreducible background contribution due to triggering of baseline fluctuations with a general description applicable to many data sets. Eq. 4.1 describes the spectral shape of this noise-induced background and then can be included in future background models for a better data description. We believe this is an important step forward in rigorous data analysis at threshold energy.

We have succeeded to evaluate the NTR and noise induced spectrum on simulated data and real data using the method described in sec. 4 with a good agreement with the analytical description. The use of the data itself to derive the NTR makes the method general enough to be applied to a wide range of detectors, giving the possibility to set the best threshold value knowing the acceptable NTR according to the application.

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