Physical and Mathematical Modeling of Heat Transfer in Intumescent Thermal Protective Coatings Under Radiative Heating

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Abstract. Radiative heating of a metal plate protected by an intumescent coating layer has been studied experimentally and theoretically. Special aspects of physical modeling of intumescent coating heating on a test bench for radiative heating are considered. Heat resistance testing of metal structures is justified. A conjugate mathematical model of radiative-conductive heat transfer in an intumesced material layer is proposed. The experimental and calculation data on the metal plate temperature beneath the coating are shown to agree well.

Introduction

Different-purpose structures operating under extreme conditions require an improvement of their heat resistance to fires of different origin, intensity and duration. The notion of heat resistance implies the ability of structures to operate properly during fire [1]. The heat resistance of metal structures is defined by the time of achieving the ultimate temperature $T_{fr}$ at which material strength decreases. For example, $T_{fr}$ for steel is equal to 500 °C and $T_{fr}$ for aluminum is 250 °C.

A solution to this problem is the application of thermal protective materials [1]. Intumescent coatings have a range of advantages over conventional materials and are currently one of the most efficient means for passive thermal protection of structures.

Intumescent coatings operate through an increase of the protective layer thickness during heating and formation of char that has low porosity and thermal conductivity. The char serves as a barrier for heat transfer and reduces the heat flux to the substrate [2]. The development and testing of intumescent thermal protective coatings on structures requires that tests reproduce the full-scale fire conditions and theoretical investigation methods be developed to determine optimal parameters of fire protection systems.

In view of the fact that it is very difficult and expensive to perform full-scale experiments, a large part of investigations can be carried out using test benches. In this case objects for investigation are materials and structural elements, and modeled thermal loading conditions are equivalent to those in real fires. This approach has certain advantages as it allows reproducing results, reduces material and time costs, and provides more information. It is widely applied in industry [3] to improve the heat resistance of structures [3] and hence it can be naturally used to study, test and optimize the application of fire protection materials.
This paper is aimed to study experimentally and theoretically heat transfer in intumescent coatings under radiative heat load, to model fire resistance tests of metal structures using a test bench for radiative heating, and to describe mathematically the heating of intumescent coatings with regard to radiation transfer in the char layer.

1. Analysis of thermal loading on a test bench for radiative heating.

Thermal radiation is one of the hazardous factors in fire [1]. To analyze the heat resistance of materials and structures, it is important to model thermal loading conditions of fire. Modeling is usually performed with the use of various setups operating on the principle of fuel combustion, electric or sun heating. Most of them model a narrow heat load range or do not allow repeated reproduction of real fire parameters.

Test benches for radiative heating offer more opportunities for creating the space-time laws of thermal loading [3]. They use tubular incandescent halogen lamps as the radiator. The lamps are simple to use and can be easily mounted into heating panels of various shape and size. Figure 1a shows the principal scheme of a test bench for radiative heating [3].

The variation of the spatial field of radiative heat load can be estimated in calculations and measurements using heat flux sensors. Figure 1b schematically illustrates the calculation of the radiative heat flux field in the approximation of a continuously emitting panel $A$ of dimension $(a \times b)$, where $A_1$-$A_4$ are the parts of panel $A$, $A'$ is the plane $(a' \times b')$, $d\sigma$ is the unit surface area, and $x_*$ is the fixed cross section along $x$.

The unit surface area $d\sigma(x, y, z)$ is under radiation flux of density

$$q_r(x, y, z) = q_{r0} \Phi_{d\sigma \to A}(x, y, z),$$

where $T_r$, $e_r$ is the radiation temperature, emissivity factor of the radiator, and $\sigma$ is the Stefan–Boltzmann constant. The angular coefficient $\Phi_{d\sigma \to A}$ between the unit surface area $d\sigma$ and parallel plane $A'$ spaced distance $x$ is equal to [4]:

$$\Phi_{d\sigma \to A} = \varphi(x, a', b') = \frac{1}{2\pi} \left\{ \frac{a'}{\sqrt{a'^2 + x^2}} \arctg \frac{b'}{\sqrt{a'^2 + x^2}} + \frac{b'}{\sqrt{b'^2 + x^2}} \arctg \frac{a'}{\sqrt{b'^2 + x^2}} \right\}$$

For the unit surface area $d\sigma(x, y, z)$ we have:

$$\Phi_{d\sigma \to A} = \Phi_{d\sigma \to A_1} + \Phi_{d\sigma \to A_2} + \Phi_{d\sigma \to A_3} + \Phi_{d\sigma \to A_4} =$$

$$= \varphi(x, a + y, b + z) + \varphi(x, a + y, b - z) + \varphi(x, a - y, b - z) + \varphi(x, a - y, b + z).$$
Dependences (3) for the points lying on the x axis (y = 0, z = 0) and y axis (z = 0, x = x*) for a laboratory emitting panel with dimensions a = b = 0.25, 0.5 m and industrial panel with dimensions a = b = 1 m are given in Fig. 2.

Analysis of curve 1 in Fig. 2a shows that radiative heat load decreases significantly with distance from the panel along the x axis. For example, for distances 0.1 < x < 0.2 m for a laboratory panel it comprises 65% and 33% of the initial value. An increase in the panel dimensions up to a = b = 0.5 m (curve 1') reduces this dependence down to 88% and 66%, and at a = b = 1 m (curve 1'') down to 97% and 88%, respectively. This imposes certain requirements on the performance of tests on intumescent coatings which are related to the coating thickness growth during heating.

For example, the surface of the coating of initial thickness 2 mm located at the distance x* = 0.15 m from the emitting panel would have the coordinate x* ≈ 0.11 m at a 20-fold expansion of the material. This increases the heat load up to 32% in the first case (a = b = 0.25 m) and up to 12% in the second (a = b = 0.5 m).

Thus, to preserve a given load level on the coating surface, larger panels or a special device for moving the sample for small laboratory panels should be used [1].

The radiative flux in each section along x (Fig. 2b) decreases with y increase. The decrease is less pronounced for large x (curves 2, 3), and at y < 0.1 m all curves almost coincide. According to calculations, the irregularity degree of the radiative flux field within the range -50 ≤ y ≤ 50 mm is less than 5%, which corresponds to the data from [3]. This means that the required sample dimensions must not exceed 100×100 mm if we model uniform radiative loading of the surface for a laboratory panel of dimensions a = b = 0.25 m.

2. Modeling of fire resistance test of metal structures.

Let us consider in detail the performance of fire resistance tests of metal structures. Metal structures have different cross section shapes. Owing to high thermal conductivity of metal there is no temperature gradient in the bulk of metal structure, and the equation for its heating in fire has the form:

$$\rho_m c_m V_m \frac{dT}{dt} = \{\alpha_f (T_f (t) - T) + \varepsilon_{fs} \sigma (T_f^4 (t) - T^4)\} S_f , \quad T(0) = T_0 ,$$  (4)

\(T_f (t) - T_0 = 345 \cdot 1\mathrm{g}(0.133t + 1)\)
where \( T, T_f(t) \) is the temperature of metal structure and “standard fire” \([1]\), \( t \) is the time, \( \rho_m, c_m, V_m \) is the density, specific heat capacity and metal structure volume, \( S_f \) is the heated surface area, \( \varepsilon_{fw}=(1/\varepsilon_f +1/\varepsilon_w -1)^{-1} \) is the reduced emissivity factor of the “flame – metal structure surface” system, and \( \alpha_f \) is the convective heat transfer coefficient. The subscript \( m \) is the metal, \( f \) is the flame, and \( w \) is the metal structure surface.

Let us denote the reduced metal thickness through \( \delta_{red} = V_m/S_f \), for elongated metal structures \( \delta_{red} \approx S_{sec}/P_f \), where \( S_{sec} \) is the cross section area, \( P_f \) is the heated area of the metal structure perimeter. Then, Eq. (4) takes the form:

\[
\rho_m c_m \delta_{red} \frac{dT}{dt} = \left[\alpha_f (T_f(t) - T) + \varepsilon_{fw} \sigma(T^4_f(t) - T^4)\right],
\]

which coincides with the equation for heating a plane metal plate of thickness \( \delta_{red} \) with thermally insulated back surface. As a result, the full-scale fire resistance testing of metal structures on a test bench for radiative heating can be reduced to testing of a plane plate with reduced thickness \( \delta_{red} \).

Heat resistance tests can also be conducted directly on metal structure fragments. Let us consider it using an I-beam as an example (Fig. 3). Since thermal loading is single-sided, only a part of the surface is subjected to radiative heating. The reduced metal thickness under test bench conditions is \( \delta_{stand} = V_m/S_{inc} \), where \( S_{inc} \) is the irradiated surface. Thus, \( \delta_{stand} \) exceeds \( (S_{sec}/S_{inc}) \) times the \( \delta_{red} \) value determined by the cross section geometry. Consequently, the bench test results correspond to full-scale test results on fire resistance of a similar section with other dimensions, for which \( \delta_{red} = \delta_{stand} \).

3. Heat transfer in intumescent coatings under radiative heating.

The mathematical models and approaches used to study heat transfer processes in intumescent coatings are reviewed elsewhere \([5]\). Of practical interest are theoretical models closed by integral characteristics of intumescent coatings which can easily be obtained within a simplest laboratory experiment. These characteristics can be the mass loss and volume change determined under isothermal heating conditions. The model of this type is proposed in \([6]\). In order for mathematical modeling to be more accurate, it is necessary to account for additional heat transfer mechanisms among which is the radiation transfer in the coating layer.

The intumescent coating will be considered as a porous two-phase reacting medium that contains a solid (char) and gaseous phase \([6]\). Heating causes a mass loss in the material. We assume that its expansion is irreversible and one-dimensional. In view of high porosity we will consider radiation transfer in the char layer, neglecting its heterogeneous burnout and shrinkage. Without going into details of the intumescence and thermal destruction mechanisms, we determine the local mass fraction and expansion ratio of an elementary intumescent coating volume through \( m = m_0/\tilde{m} \) and \( \tilde{V} = V/V_0 \).
In the Lagrangian coordinate system \( s \) related to solid material the laws of conservation of mass and energy for the intumescent coating read [6]:

\[
\rho_0 \frac{\partial \hat{m}}{\partial t} + \frac{\partial G_g}{\partial s} = 0, \quad s \in (0,h_0), \quad x = h_b + \int_0^s \hat{u} \, ds,
\]

\[
(\rho c_p)_{ef} \frac{\partial T}{\partial t} + (Gc_p)_{g} \frac{\partial T}{\partial s} = \frac{\partial}{\partial s} \left( \frac{\lambda_{ef} \partial T}{\theta} - q_r \right) + Q \rho_0 \frac{\partial \hat{m}}{\partial t},
\]

\[
\frac{\partial}{\partial s} \left( \frac{1}{3(k + \beta) \theta} \frac{\partial U_r}{\partial s} \right) + k \bar{\theta}(4\sigma T^4 - U_r) = 0, \quad q_r = -\frac{1}{3(k + \beta) \theta} \frac{\partial U_r}{\partial s},
\]

\[
\frac{\partial \hat{m}}{\partial t} = -R_s(T,\bar{\theta}),
\]

\[
(\rho c_p)_{ef} = (\rho c_p)_{g} + (\rho c_p)_{s}, \quad \lambda_{ef} = (\lambda \phi)_{g} + (\lambda \phi)_{s}, \quad \phi_g + \phi_s = 1, \quad \rho_g / \rho_g0 = T_0 / T.
\]

The thermal state of an opaque substrate is described by the heat conduction equation

\[
(\rho c)_{b} \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda_b \frac{\partial T}{\partial x} \right), \quad x \in (0,h_b).
\]

The boundary conditions for Eqs. (6), (7), (10) have the form:

\begin{align*}
 & x = 0: \quad \lambda_b \frac{\partial T}{\partial x} = 0, \\
 & x = h_b, \quad s = 0: \quad T|_{h_b} = T|_{s=0}, \quad \lambda_b \frac{\partial T}{\partial x}|_{h_b} = \frac{\lambda_{ef} \partial T}{\theta}|_{s=0} - q_r, \quad G = 0, \\
 & -\varepsilon_b U_r - 2(2 - \varepsilon_b) q_r + 4\varepsilon_b \sigma T^4 = 0, \\
 & s = h_0: \quad \frac{\lambda_{ef} \partial T}{\theta} = \alpha_f(T_f(t) - T), \\
 & \varepsilon_f U_r - 2(2 - \varepsilon_f) q_r - 4\varepsilon_f \sigma T_f^4 = 0.
\end{align*}

The initial conditions correspond to the thermal equilibrium between the substrate – coating system and environment:

\[
t = 0: \quad T = T_0, \quad m = 1, \quad \bar{\theta} = 1, \quad s = x - h_b.
\]

Here, \( \rho, c_p, \lambda \) are the density, heat capacity and thermal conductivity, \( G_g \) is the filtration gas mass flux, \( s, x \) are the Lagrangian (\( s = 0 \) for substrate – coating) and Euler (\( x = 0 \) for substrate back surface) coordinates, \( Q, R_r \) are the thermal effect and rate of thermal coating destruction, \( \phi \) is the volume fraction of phase, \( \varepsilon, \sigma \) are the emissivity factor and Stefan–Boltzmann constant, \( U_r \) is the quantity proportional to the bulk radiation density, \( q_r \) is the radiative heat flux, \( k, \beta \) are the absorption and scattering coefficients, \( \alpha \) is the heat transfer coefficient, and \( h_b, h_0 \) are the substrate and coating thickness (initial). The subscripts \( s \) is the solid material, \( g \) is the gaseous phase, \( f \) is the environment, \( r \) is the radiation, \( b \) is the substrate, \( ef \) is the effective quantities, and \( 0 \) are the initial values.

Equation (5) is the mass conservation law for intumescent coating, Eq. (6) is the energy equation, Eq. (7) is the radiation transfer equation in the diffusion approximation [4, 7], Eq. (8) is the equation of thermal destruction of material, Eq. (9) are the effective thermophysical characteristics of porous medium of the coating [1]. Condition (11) corresponds to perfect heat insulation of the substrate, Eq. (12) corresponds to the thermal conjugation of the substrate – coating interface and to radiation reflection, Eq. (13) to convective heating of the coating surface and to specifying the radiative heat load. The model is closed by determining the parameters \( m, \bar{\theta} \) that integrally characterize material expansion [6].

The system of equations was solved numerically using Petukhov’s difference method of the 4th order of accuracy with respect to coordinate and 1st order with respect to time [8]. The comparison of calculation results and experimental data for temperature is illustrated in Fig. 4a for a two-phase
system heated on the test bench: steel – 2 mm, heat insulation (glass fiber cloth) – 40 mm; and in Fig. 4b for a three-layer system with a 2-mm thick layer of coating SGK-1 [6, 9]. The incident radiative heat flux was 59 kW/m². As one can see, there is a good agreement with experiment. Calculations revealed a strong dependence of the obtained results on the coefficients $k, \beta$ affecting the radiation energy transfer mechanism within the volume.

![Graph](image)

Fig. 4. Heating of a metal plate unprotected (a) and protected (b) by coating SGK-1. Calculation: 1 – metal, 2 – heat insulation (back surface), 3 – coating surface, $\Delta$ – experiment. $h_0 = 2$ mm, $h_b = 2$ mm

**Conclusion**

1. The possibility of fire resistance testing of metal structures with intumescent coatings on a test bench for radiative heating has been shown experimentally and theoretically. The spatial distribution of radiative heat load has been studied, and optimal dimensions of the studied samples have been determined.

2. A conjugate mathematical model of intumescent coating heating with regard to radiation transfer in the char layer has been developed.

3. The thermal regime of a metal plate with coating SGK-1 has been studied on the test bench for radiative heating. Calculation results are shown to be in a good agreement with experimental data, which bears witness to the adequacy of the proposed model for describing heat transfer processes in the char layer.

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