Modeling interaction of relativistic and nonrelativistic winds in binary system PSR 1259-63/SS2883. I. Hydrodynamical limit

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ABSTRACT
In this paper, we present a detailed hydrodynamical study of the properties of the flow produced by the collision of a pulsar wind with the surrounding in a binary system. This work is the first attempt to simulate interaction of the ultrarelativistic flow (pulsar wind) with the nonrelativistic stellar wind. Obtained results show that the wind collision could result in the formation of an "unclosed" (at spatial scales comparable to the binary system size) pulsar wind termination shock even when the stellar wind ram pressure exceeds significantly the pulsar wind kinetical pressure. Moreover, the post-shock flow propagates in a rather narrow region, with very high bulk Lorentz factor (\(\gamma \sim 100\)). This flow acceleration is related to adiabatical losses, which are purely hydrodynamical effects. Interestingly, in this particular case, no magnetic field is required for formation of the ultrarelativistic bulk outflow. The obtained results provide a new interpretation for the orbital variability of radio, X-ray and gamma-ray signals detected from binary pulsar system PSR 1259-63/SS2883.

Key words: HD – shock waves - pulsars: binaries.

1 INTRODUCTION

Pulsars lose their rotation energy through relativistic winds, the collision of which with the Interstellar Medium results in the formation of the Pulsar Wind Nebulae (regions of nonthermal synchrotron radiation of ultrarelativistic electrons accelerated at the termination of pulsar winds (Rees & Gunn\(^1\) 1974 [Kennel & Coroniti\(^2\) 1994]). The Crab Nebula is the most famous example of such an object. The recent X-ray and TeV gamma-ray observations (Gansler & Slane\(^1\) 2006) show that this is a common phenomenon.

A very interesting situation arises when a pulsar is located in a binary system. In this case the pulsar wind interacts with the wind from the companion star. This case, in particular, is realized in the binary system PSR1259-63/SS2883 which consists of a \(\sim 48\)ms pulsar in an elliptic orbit around a massive B2e optical companion (Johnston\(^1\) 1994). The density and velocity of the stellar wind depend on the separation distance between two stars. Thus, the processes related to the interaction of two winds, in particular particle acceleration and radiation proceed under essentially different physical conditions depending on the orbital phase. This makes this object a unique laboratory for the study of nonthermal processes in "on-line" regime, due to the short acceleration and cooling time-scales characterizing this system, especially close to the periastron (Khangulyan et al.\(^2\) 2007). Observations show that this system is indeed a strong source of nonthermal time-dependent emission extending from radio to TeV gamma-rays (see e.g. Neronov & Chernyakova\(^1\) 2007a).

Two variable TeV galactic gamma-ray sources, LS 5039 and LSI 61 303 (see e.g. Paredes\(^1\) 2006), are discussed as possible, although less evident, candidates representing “binary pulsar” source population (Dubus\(^1\) 2006 [Mirabel\(^1\) 2006] Neronov & Chernyakova\(^1\) 2007b). LS 5039 consists of O6.5V star and an unidentified compact object in a 3.9 day orbit. This object has been detected as the source of gamma rays by EGRET (source 3EG J1824-1514) (Paredes et al.\(^2\) 2000) and by HESS (Aharonian et al.\(^1\) 2005). LSI 61 303 is a binary system with a B0Ve star in a 26.5 day orbit. This source presumably associates with a low energy gamma-ray source 2CG 135+01/3EG J0241+6103 (Maraschi & Treves\(^1\) 1981 [Tavani et al.\(^1\) 1998]). Recently, it has been detected in TeV gamma-rays (Albert et al.\(^1\) 2006). The nature of the compact objects (black hole or a neutron star) in both sources is not yet firmly established.

The collision of supersonic winds from two stars located in a binary system results in the formation of two terminat-
ing shock fronts and a tangential discontinuity separating relativistic and nonrelativistic parts of the shocked flow. It is believed that the shocked flow should propagate into a limited solid angle with rather high velocity. However the properties of the flow downstream the shock is unknown.

The collision of the pulsar wind with the supersonic flow of the nonrelativistic plasma has been recently numerically modeled by several groups (Bucciantini 2002; Swaluw 2003; Vigelius et al. 2006; Romero et al. 2007) using different nonrelativistic versions of hydrodynamical codes. However, the dynamics of the relativistic plasma is rather specific, and the use of nonrelativistic codes cannot be a priori justified. The modeling of collision of a relativistic pulsar wind with a nonrelativistic one demands an adequate treatment of distinct features of relativistic outflows. Moreover, in many previous studies the stellar wind has been approximated as plane parallel, while both winds initially expand radially. However, this purely geometric difference in the formulation of the problem, leads, in fact, to significantly different results and conclusions.

In this paper we present the results of our studies conducted in the hydrodynamical limit, i.e. the role of the magnetic field in dynamics has been ignored. The impact of the magnetic field will be published elsewhere.

2 PROPERTIES OF THE PULSAR AND STELLAR WINDS OF PSR 1259-63/SS2883

Below we describe the properties of PSR 1259-63/SS2883 in which the interaction of the pulsar and stellar winds leads to the nonthermal emission observed in radio, X-ray and TeV gamma-ray energy bands.

2.1 Pulsar wind

The rotational losses of pulsars are released in the form of electromagnetic and kinetic energy fluxes of the wind. Interpretation of observations usually results in a conclusion that the energy flux in the wind at large distances from the pulsar is concentrated in the particle kinetic energy. The ratio of the electromagnetic energy flux over the kinetic energy flux is described using $\sigma$-parameter. Typically $\sigma$ is much less than 1. Therefore, it is natural in the first approximation to consider the pulsar wind as purely hydrodynamical. Below we assume that the wind is ejected isotropically with the total energy flux equal to the total rotational losses of the pulsar $E_{\text{rot}}$. Thus the momentum flux density of the wind varies with the distance $r$ to the pulsar as

$$L = \frac{E_{\text{rot}}}{4\pi c r^2}. \quad (1)$$

For a given Lorentz factor of the wind $\Gamma_0$, the proper density of the plasma in the pre-shock region is equal to

$$n = \frac{E_{\text{rot}}}{4\pi mc^3 \Gamma_0 r^2}. \quad (2)$$

2.2 Parameters of the stellar wind

In the system PSR 1259-63/SS2883 the 47.8 ms radio pulsar rotates around the Be star in an elliptic orbit with a period 3.4 yr. The eccentricity of the orbit is 0.87. The distance between stars in apastron is $\sim 10^{13}$ cm (Johnston et al. 1994). The Be star has a mass $\sim 10M_\odot$ and radius $R_* = 6R_\odot$. Due to the very fast rotation of the star, the mass outflow is strongly anisotropic. The mass flux is concentrated along the equatorial plane forming a disk-like flow. In addition, there is also an isotropic component of the wind (so-called polar wind) with smaller mass flux, but higher velocity.

The disks in Be stars are dense and expand rather slowly. Typical velocities of the plasma in the disk are $150 - 300$ km/s (Waters et al. 1988). The velocity of the plasma in the polar wind is much higher achieving $1500 - 2000$ km/s (Snow 1993). The ratio of the mass flux density in the disk over the polar wind is estimated (Waters et al. 1988) as

$$\frac{n_d n_p}{n_p n_p} = 30 - 100. \quad (3)$$

The mass flux of the disk-like wind from Be SS2883 has been estimated by several authors. The estimates were made basically using eclipses by the disk-like flow of the pulsed radio emission from the pulsar. Following Johnston (1999) the mass loss rate $\dot{M} = 5 \times 10^{-6}M_\odot$ yr$^{-1}$ and the opening angle of the disk $\theta \sim 15^\circ$. Then the ratio of the momentum flux-densities from the Be star and the pulsar, $\eta$, in the case of pulsar–disk interaction, is

$$\eta = \frac{E_{\text{rot}} \sin(\theta/2)}{cMv}. \quad (4)$$

For the given parameters characterizing PSR 1259-63/SS2883, $E_{\text{rot}} = 8 \times 10^{35}$ ergs$^{-1}$, $\theta = 15^\circ$, and the velocity of the equatorial outflow $v = 200$ km/s, one obtains $\eta \geq 5.5 \times 10^{-2}$ with an uncertainty by a factor of 2-3. In the polar wind the velocity is much larger, $v \sim 2000$ km/s, but, at the same time the mass-flux is reduced by a factor of 30-100. Therefore, in the polar wind of the Be star, $\eta$ appears in the range 0.2 - 0.6. Thus, given the uncertainties in the mass-flux from the Be star and the variation of the momentum flux with distance due to the acceleration of the stellar wind by radiation pressure, the parameter $\eta$ along the pulsar orbit may vary between $10^{-2}$ and 1.

3 COMPUTATIONAL METHODS

The scheme of the plasma flow formed at the collision of the pulsar wind with the wind from the Be star is shown in Fig. 1. Both winds are terminated with formation of two shock waves. The post shock flow of the nonrelativistic plasma is separated from the shocked relativistic plasma flow by a contact discontinuity. Within the hydrodynamical classification, the contact discontinuity is treated as tangential discontinuity (Landau & Lifshits 1987).

A remarkable feature of the post-shock flow arising from the collision of the two supersonic winds is that the flow is subsonic only in the region close to the line connecting the stars (i.e. the symmetry line). Propagating downstream the flow crosses sound line (shown in Fig. 1) by the thick black lines) and becomes supersonic. We note that the locations of the sound lines in the nonrelativistic and relativistic flows are different. While the flow in the subsonic region is described by elliptic-type equations, in the supersonic region the flow is hyperbolic. Thus the plasma flow in the
post shock region appears to be of mixed-type. The critical lines or separatrix characteristics play a crucial role in the mixed-type hydrodynamical [Landau & Lifshitz 1987; Hayes & Probstein 1959] and magnetohydrodynamical [Bogovalov 1994; Tsinganos et al. 1996] flows. The separatrix characteristics are located in the hyperbolic region, and in Fig. 1 they are shown by thin red lines. The separatrix characteristics and sound lines may coincide only in a particular case when they are orthogonal to the flow lines. The separatrix characteristics separate the flow regions with different causal connection. In the “subsonic” region limited by the separatrix characteristics every couple of points are causally connected through hydrodynamical or MHD signals. Downstream the separatrix characteristics any two points are causally disconnected unless they are located in the cone formed by two different characteristics. Moreover, in a case of steady state mixed type flow, the solution setting on the separatrix characteristics specifies a unique solution in the hyperbolic region.

The most general numerical method of mixed-type problem solution is the relaxation method. In this method, the time dependent problem of the plasma flow is solved starting from some initial state. The method was used for the mixed-type equation solution by many groups in the context of the problem of astrophysical jet formation (Bogovalov & Tsinganos 1999; Romanova et al. 1993; Ouyed & Pudritz 1997; Krasnopolsky, Li & Blandford 2000). However, the applicability of this method is rather limited. Usually transient equations are solved in a rather small computational domain. Typically, the outer boundaries of the domain are located in the region of the hyperbolic flow. Theoretically they can be located at arbitrarily large distances downstream of the critical surfaces. However, in practice they are located not very far away due to the limitations imposed by memory and power of computers. At the same time, one is often interested in the plasma flow rather far downstream from the critical lines. Therefore, it was proposed by Bogovalov & Tsinganos (1999) to extend the solution obtained by the relaxation method to arbitrarily large distances downstream of the separatrix characteristics. This extension is based on the fact that in the hyperbolic region a Cauchy problem can be formulated with the initial data specified at some point located slightly downstream of the separatrix characteristic. Thus, the solution of the problem of the mixed-type flow can be reduced to a two-step treatment. At the first step (nearest zone solution) the solution can be found in the limited computational domain using the relaxation method. At the second step (far zone solution) the solution for large distances downstream of the separatrix characteristics can be obtained using the approach of Bogovalov & Tsinganos (1999).

3.1 Method for the solution in the nearest zone

The computational method used in this work was developed for the solution for the problem of the interaction of the relativistic pulsar wind with the interstellar medium [Koldoba, Kuznetsov & Ustyugova 2002] (Tsinganos et al. 2005). This method has certain advantages compared to other approaches published in the literature. Namely:

(i) The equations which describe the dynamics of the plasma are solved only in the shocked region. There is no need to compute the plasma flow in the pre-shock region as it is done in previous studies (e.g. Bucciantini 2002; Vigelius et al. 2006; Swaluw 2003) because the flow in the pre-shock region is a priori known;

(ii) All discontinuities are considered as having zero thickness mathematical breaks. This allows us to avoid numerical diffusion of discontinuities. This property is especially important for calculations of radiation from the shocked material;

(iii) The method has no limitations on the Lorentz factor of the wind (e.g. in the approach suggested by Komissarov & Lyubarsky 2003) the Lorentz factor of the pulsar wind cannot exceed 10).

We assume that the winds from the pulsar and Be star are isotropic. In this case flow formed after the interaction is axially isotropic. In this paper we assume that both winds are cold. Strictly speaking, the stellar wind is hot. However, the sound speed in the stellar wind is significantly below the bulk motion speed, making this approximation well justified.

The relativistic hydrodynamical (RHD) equations describing the post-shock flow are

\[
\frac{\partial n\gamma}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r n\gamma v_r) + \frac{\partial}{\partial z} (n\gamma v_z) = 0
\]

\[
\frac{\partial \gamma v_r}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \gamma v_r^2 + p) + \frac{\partial}{\partial z} ( \gamma v_r v_z ) = \frac{p}{r}
\]

\[
\frac{\partial \gamma v_z}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \gamma v_z^2 + p) + \frac{\partial}{\partial z} ( \gamma v_z v_r ) = 0
\]

\[
\frac{\partial (\gamma w - p)}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \gamma w^2 + p) + \frac{\partial}{\partial z} ( \gamma w^2 v_z ) = 0.
\]

Here \( p, \nu, n \) and \( v \) are the pressure, enthalpy, particle density, and flow velocity, respectively. At the termination shock front the bulk motion energy is transformed into the energy

\[\frac{1}{2}u^2 \left( \frac{p}{\gamma - 1} \right)^{\gamma / (\gamma - 1)} \]

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Here all thermodynamic quantities have their proper values. \( p \) and \( u \) are taken per unit volume in the local rest frame.
of the plasma’s chaotic motion. The efficiency of this transformation depends on the incident angle with which plasma crosses the shock wave.

The plasma flow in the nonrelativistic region is described by the following equations:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial \rho v_r}{\partial r} + \frac{\partial \rho v_z}{\partial z} &= 0 \\
\frac{\partial \rho v_r}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( \rho v_r^2 + p \right) + \frac{\partial}{\partial z} \rho v_r v_z &= \frac{p}{r} \\
\frac{\partial \rho v_z}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( \rho v_z^2 + p \right) &= 0 \\
\frac{\partial}{\partial t} \left( \frac{\rho v_r^2}{2} + \epsilon \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{\rho v_r^2}{2} + \epsilon + p \right) + \frac{\partial}{\partial z} v_z \left( \frac{\rho v_z^2}{2} + \epsilon + p \right) &= 0
\end{align*}
\]

Here \( \rho \) and \( \epsilon \) are the densities of mass and thermal energy, respectively, and \( p \) \((= 2/3\epsilon)\) is the pressure.

Generally, one faces certain difficulties while solving numerically the equations which describe interaction of relativistic and nonrelativistic winds. The post shock flow is described by the systems of equations which are essentially different for the relativistic and nonrelativistic flow domains. The locations of discontinuities (shock waves and the contact discontinuity) are unknown \( \text{a priori} \). In addition, there are other difficulties related to large differences in velocities of the relativistic and nonrelativistic flows. The velocity of relativistic plasma is equal to \( c \) in the pre-shock region and larger than \( c/3 = 10^{10} \text{ cm/s} \) in the post shock region. The velocities of the nonrelativistic stellar wind in PSR 1259/SS2883 are \( \sim 3 \cdot 10^8 \text{ cm/s} \) and \( \sim 10^8 \text{ cm/s} \) in the pre-shock and post-shock regions, respectively. The spacial scales in the nonrelativistic and relativistic flows are similar and have to be meshed with cells of similar space resolution. This means that at integration of the full system of equations with explicit numerical schemes, the time step will be limited by the Courante condition in the region of the relativistic flow. In the region of the nonrelativistic plasma flow this time-step will be less than the Courant limited time-step by two orders of magnitude.

To overcome all these difficulties, a special numerical method has been developed for integration of transient HD and RHD equations. The integration of the HD equations was performed in the region limited by the nonrelativistic termination shock wave and by the contact discontinuity. The RHD equations have been integrated in the region located between the relativistic termination shock wave and the contact discontinuity. The plasma flow outside these regions is known.

An adaptive computational mesh has been used. The mesh boundaries were located on the shock waves and the contact discontinuity. The boundary node location varied with change of the discontinuity locations. Usually the nodes were located on the system of beams which were fixed. The nodes moved only along these beams. The beams were directed along an axis of symmetry of the problem or spread radially from a fixed point (pulsar position for example) as it is shown in Fig. 2. The nodes on the beams located on the discontinuities were specified. The internal nodes were distributed between them uniformly.

![Figure 2. Typical mesh in the computational domain of the nearest zone region. Only every tenth cell is shown.](image-url)
cold (p = 0) relativistic plasma inflow (state "0") and hot relativistic plasma outflow (state "1") is shown in Fig. 3. At the calculation of the decay problem it was assumed that:
(i) the termination shock is close to the steady state and the variation of all parameters can be described by linearized (in relation to the steady state) Gugonio relationships.
(ii) the variation of the parameters in the post shock region behind any rarefication or shock wave can be described by the linearized equations of RHD.

Amplitudes of the waves propagating in the states "0" and the state "1" are taken to satisfy the conditions at the contact (tangential) discontinuity: pressure and the normal component of the velocity are constant. The diagram of the RHD discontinuity decay in variables (p, v) is shown in Fig. 3. The scheme of the RHD discontinuity decay. The discontinuity can decay into the shock terminating the infalling cold plasma, tangential discontinuity and a shock or rarefication wave propagating in the shocked material. The case when the shock propagates in the shocked material is shown. In the linearized case there is no difference between the shock and the rarefication wave in the analytical representation.

Figure 3. The scheme of the RHD discontinuity decay. The discontinuity can decay into the shock terminating the infalling cold plasma, tangential discontinuity and a shock or rarefication wave propagating in the shocked material. The case when the shock propagates in the shocked material is shown. In the linearized case there is no difference between the shock and the rarefication wave in the analytical representation.

3.2 Parameterization of the solution

The steady state equation systems which describe both the nonrelativistic and relativistic outflows can be scaled through the value of the momentum flux of the nonrelativistic wind, pν^2, at the distance D between the star and the pulsar. Also, it is natural to normalize all geometrical variables to D. In all figures below all the distances are given in this normalization unless stated otherwise. With such a normalization pν^2 = 1 at the dimensionless distance D = 1. At the same time the momentum flux of the relativistic wind at D = 1 becomes equal to the parameter η given by Eq. (4).

In this way the interaction of the two flows is described by two parameters: η and the initial Lorentz factor of the pulsar wind Γ_0. In the limit Γ_0 ≫ 1, the enthalpy per particle ψ becomes equal to the parameter η given by Eq. (4). In the limit Γ_0 ≫ 1, the enthalpy per particle ψ becomes equal to the parameter η given by Eq. (4).

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The flow in the far zone is supersonic. The system of relativistic HD equations allow a simple scaling of variables: t → t/α, v → αv, p → p/α^2, ρ → ρ. Moreover, this scaling does not change the conditions |p| = 0, v_n = 0 (which are viable in the steady state regime) at the contact discontinuity. Thus, the difficulties related to large difference in velocities of the relativistic and nonrelativistic flows may be overcome by rescaling of initial conditions for Eqs. (9)-(12) in such a way that ν → c in the nonrelativistic wind. This approach allows us also to stabilize the flow at the contact discontinuity which in fact is a tangential discontinuity.

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3.3 Method for the solution in the far zone

For magnetohydrodynamical nonrelativistic and relativistic flows, a method (hereafter BT method) has been suggested by Bogovalov & Tsinganos [1999]. Below we discuss a pure hydrodynamical version of this method which can be applied to the far zone.

The flow in the far zone is supersonic. The system of equations describing the steady state flow is hyperbolic. Therefore, an initial-value Cauchy problem can be formulated for this flow.

It is convenient to introduce the flux function ψ as follows

\[ n\mu_p = \frac{\nabla \psi \times \hat{\phi}}{r} \quad \text{(14)} \]

where \( \hat{\phi} \) is the azimuthal unit vector.
The stationary HD equations accommodate the energy conservation law
\[(e + \pi)\gamma = W(\psi),\] (15)
where \(e\) and \(\pi\) are the thermal energy and pressure per particle. According to this law, the average energy of particles remains constant along the flow line. This equation should be supplemented by the relativistic relationship between the components of the four-velocity \(u\)
\[\gamma^2 = 1 + u^2.\] (16)

For analysing the behavior of the plasma at large distances, it is convenient to deal with this equation in an orthogonal curvilinear coordinate system \((\psi, \alpha)\) formed by the flow lines and by the lines perpendicular to them. While \(\psi\) varies with the motion along the flow lines, the coordinate \(\alpha\) varies with the motion across the flow lines. The geometrical interval in these coordinates can be expressed as
\[(d\mathbf{r})^2 = g_\psi^2 \, d\psi^2 + g_\alpha^2 \, d\alpha^2 + r^2 \, d\phi^2,\] (17)
where \(g_\psi^2, g_\alpha^2\) are the corresponding line elements (components of the metric tensor).

According to \textbf{Landau & Lifshits} (1994) the equation \(T^\psi_\psi = 0\) (where \(T^\psi_\psi\) is the energy-momentum tensor and \(^\text{";}\)) represents covariant differentiation in these coordinates) can be written in the form
\[\frac{\partial p}{\partial \psi} - u^2 w \frac{1}{\alpha} \, \frac{\partial g_\alpha}{\partial \psi} = 0.\] (18)

The unknown variables here are \(z(\alpha, \psi)\) and \(r(\alpha, \psi)\). The metric coefficient \(g_\alpha\) can be obtained from Eq. (18).
\[g_\alpha = \exp \left( \int_0^\psi G(\alpha, \psi) \, d\psi \right),\] (19)
where
\[G(\alpha, \psi) = \frac{1}{u^2 w} \, \frac{\partial p}{\partial \psi}.\] (20)

The lower limit of the integration in Eq. (19) is chosen to be 0 such that the coordinate \(\alpha\) is uniquely defined. In this way \(\alpha\) coincides with the coordinate \(z\) where the surface of constant \(\alpha\) crosses the axis of rotation.

The metric coefficient \(g_\psi\) can be obtained from Eq. (14) in terms of the magnitude of the poloidal velocity as follows
\[g_\psi = \frac{1}{r \, u_\psi}.\] (21)

The orthogonality condition
\[r_\alpha r_\psi + z_\alpha z_\psi = 0,\] (22)
and the relationships
\[g_\psi^2 = r_\alpha^2 + z_\alpha^2 \quad \text{and} \quad g_\psi^2 = r_\psi^2 + z_\psi^2\] (23)
result in
\[r_\alpha = -\frac{z_\psi g_\alpha}{g_\psi} \quad \text{and} \quad z_\alpha = \frac{r_\psi g_\alpha}{g_\psi},\] (24)
with \(g_\alpha\) given by Eq. (19). Here \(r_\alpha = \partial r/\partial \alpha, z_\alpha = \partial z/\partial \alpha, r_\psi = \partial r/\partial \psi, z_\psi = \partial z/\partial \psi\).

Eqs. (24) should be supplemented by appropriate boundary conditions and initial values on some initial surface of constant \(\alpha\) located in the nearest zone, but downstream from all the critical surfaces. The form of the initial surface of constant \(\alpha\) was obtained numerically via integration of the following equations
\[\frac{\partial r}{\partial \psi} = \frac{n u_z}{r (n u_\rho)^2}, \quad \frac{\partial z}{\partial \psi} = -\frac{n u_r}{r (n u_\rho)^2}.\] (25)
where \(n u_z\) and \(n u_r\) as well as the integral \(W(\psi)\) are given by solution in the nearest zone.

The boundary conditions on the axis of rotation and the equatorial plane are the same as the conditions in the nearest zone. No conditions at infinity are specified.

Fig. 5 demonstrates a typical structure of the solution and the coordinate system obtained with the BT method. The terminating shocks are well reproduced. The nonrelativistic and relativistic flow are perfectly separated. The distribution of the selected flow lines and correspondingly of the coordinate system lines was chosen to reproduce accurately the structure of the shocks.

4 RESULTS

The modeling of hydrodynamical collision of two winds from a massive star and pulsar has been performed for a wide range of \(\eta\) parameter: from \(\eta = 10^{-4}\), to \(\eta = 20\), which covers both cases when momentum flux is dominated by the wind of pulsar \((\eta > 1)\) or by optical star \((\eta < 1)\).

Two computational methods have been applied for solutions in the near and far zones. The comparison of two
methods in the far zone is shown in Fig. 6. Although the two solutions are in good agreement, being practically identical, nevertheless there is a difference between these solutions, namely the shock structures predicted by two methods are somewhat different at the backside point of the pulsar wind termination shock. In the backside point the post-shock supersonic relativistic flow converges with the axis, thus a reflecting shock is formed. The structure of the reflecting shock depends on the angle to the symmetry axis. This behavior of the flow is analogous to the reflection of the shock wave from a solid wall [Landau & Lifshits, 1987]. Two different structures are possible at the reflection: (i) perfect reflection and (ii) Mach reflection. In the last case a triple configuration of the shock front with a tangential discontinuity is formed (see Fig. 7). In this structure a region of subsonic flow is formed. While the BT method for the problem solution in the far zone is applicable only for pure supersonic flow, the subsonic region is not reproduced properly. However, the subsonic domain occupies a small region on plots shown in Fig. 6 otherwise the BT solution correctly reproduces all features relevant to the interpretation of observations. The interactions of the pulsar wind with a plane parallel nonrelativistic flow always result in the formation of a closed termination shock wave as shown in Fig. 8. Similar structure has been revealed in calculations by Bucciantini (2002); Vigelius et al. (2006); Swaluw (2003). However, this assumption cannot be applied to the radially expanding winds. The shock wave associated with the termination of the pulsar wind becomes closed only in the case when the momentum flux in the nonrelativistic wind significantly exceeds the momentum flux in the pulsar wind, namely for \( \eta < 1.25 \times 10^{-2} \). For larger \( \eta \) the shock is open as shown in Fig. 9.

The properties of the post shock flow can be characterized by the location of the termination shocks at the symmetry line and by the asymptotical properties of the shocks. Dependence of the location of the shocks and the contact discontinuity on \( \eta \) at the symmetry line (at the bow shock) is shown in Fig. 11. At small \( \eta \), the position of the discontinuity relative to the pulsar \( r_d \approx \sqrt{\eta} \). The distance to the backward point of the relativistic shock front does not follow this law; it diverges at \( \eta \approx 1.25 \times 10^{-2} \).

An important parameter characterizing the post-shock flow is the asymptotic opening angle \( \theta \). Dependence of \( \theta \) on \( \eta \) is shown in Fig. 12. The function \( \theta(\eta) \) can be interpolated as

\[
\theta = \exp(4.7516)\eta^{0.217} = 115.7693\eta^{0.217}
\]

(26)

for the nonrelativistic shock wave,

\[
\theta = 41.068 \log \eta + 71.693
\]

(27)

for the relativistic shock wave, and

\[
\theta = 28.64(2 - \eta^{2/5})\eta^{1/3}
\]

(28)

for the contact discontinuity.

Interestingly, Eq. (28) agrees quite well with the analytical interpolation of the asymptotic opening angle of the contact discontinuity proposed by Eichler & Usov (1993), despite the fact that the interpolation of Eichler & Usov (1993) is based on the results of simulations of Girard & Willson (1987) performed for collisions of nonrelativistic winds.

It follows from Fig. 9 that the post-shock flow is accelerated to rather large Lorentz factors. Even in the nearest zone (limited by the radial distance equal to the distance between the two stars from the symmetry axis) the bulk motion Lorentz factor increases up to \( \gamma = 2 \). In the far zone the Lorentz factor becomes very large, e.g. in the zone limited by \( r = 50 \) (the size of the computational domain) the bulk Lorentz factor can be as large as 100. The results of calculations shown in Fig. 9 are performed for \( \eta = 1 \) which for the system PSR 1250/SS2893 approximately corresponds to

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**Figure 6.** Comparison of the results obtained by different methods in the general computational domain for \( \eta = 1.110^{-3} \). The color represents the distribution of temperature in the post shock region. The left panel shows the solution obtained by the relaxation method (nearest zone solution). The middle panel shows the solution obtained in the far zone and the right panel shows the comparison of the solutions obtained by these methods. The solid lines here show the position of the flow lines and shock fronts obtained by the relaxation method. Color represents the solution obtained by BT method. In the region of the relativistic flow the temperature is expressed in the units \( mc^2/20 \). In the nonrelativistic region the temperature is expressed in the units \( mc^2/20 \), where \( r_0 \) is the initial velocity.

**Figure 7.** The possible structure of the shock waves at the reflection from the symmetry axis. The thick solid line - shock fronts. The thin solid line - flow line. Dashed line - the tangential discontinuity. Two variants are possible. On the left panel the perfect reflection. On the right panel - Mach reflection.
the case when the pulsar interacts with the polar wind of the Be star. In Fig. 10 we show also the results for $\eta = 0.05$ which corresponds to the location of the pulsar in the equatorial wind of the Be star. This case is shown in Fig. 10. Even in this case, the post-shock flow can be accelerated in the downstream region to very large Lorentz factors.

This interesting effect obviously is related to adiabatic losses. According to (15) the full energy per particle, which is the product of the bulk motion Lorentz factor $\gamma$ and enthalpy $(e + \pi)$, is conserved along the flow line. Thus the acceleration of the flow is reduced to the transfer of the thermal energy to the bulk motion. Initially the energy of particles in the cold pulsar wind is domined by the kinetic energy of the motion. At the shock the total energy per particle remains unchanged, but a part of the energy is transformed to the thermal energy of particles.

The relationship between bulk motion Lorentz factor and post shock enthalpy depends on the incident angle between the shock front and the flow line. At the symmetry line, practically all bulk motion energy is transformed to the internal energy (enthalpy) of particles. Downstream of the shock the pressure reduces because of the flow expansion, and thus the bulk motion Lorentz factor increases. Correspondingly the temperature falls down, in other words we deal with adiabatic losses. Formally, the bulk motion Lorentz

**Figure 8.** The structure of the post shock flow in the nearest zone solution at the interaction of the relativistic wind with the plane parallel flow of the nonrelativistic plasma. The color represents the Lorentz factor. Red thick lines show the sound lines. Geometry is normalized on the distance between the pulsar and the relativistic shock front at the symmetry axis.

**Figure 9.** The post shock flow for $\eta = 1$. The color represents the bulk Lorentz factor.

**Figure 10.** The post shock flow for $\eta = 0.05$. The color represents the bulk Lorentz factor.
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Figure 11. $\eta$ dependence of the location of the shock waves at the symmetry line. The solid line corresponds to the analytical approximation defined by Eq. (13).

Figure 12. $\eta$ dependence of the asymptotical opening angle of the shocks and discontinuities in the post shock flow together with interpolations defined by Eqs. (26)–(28).

Figure 13. The post shocked flow in the far zone for $\eta = 1.2 \cdot 10^{-4}$. The color represents the Lorentz factor.

factor can achieve the initial value provided that the post-shock is not terminated by the interstellar medium. The bulk motion acceleration to very large Lorentz factors is possible not only at the collision of the pulsar and stellar winds of comparable power as shown in Fig. 9. Even in the case when the momentum flux in the Be wind strongly exceeds the momentum flux in the pulsar wind, the post-shock flow can be reaccelerated to $\gamma \gg 1$ as it shown in fig. 13.

5 DISCUSSION: IMPLICATIONS AND LIMITATIONS

In this paper, we have conducted a detailed numerical study of the interaction of the relativistic and nonrelativistic winds assuming isotropic, radially expanding winds and ignoring the role of the magnetic field. In fact, the cold ultrarelativistic pulsar winds are generally believed to be highly anisotropic (Bogovalov 1999; Lyubarsky 2002). The anisotropy of the energy flux in the wind can result in a nonaxisymmetric form of the termination shock wave at the
vicinity of the symmetry axis, as it follows from the studies of [Vigiliius et al. 2006]. Whether the anisotropy has a noticeable impact on the result and conclusions of this paper is a subject of further investigations and will be discussed elsewhere. This concerns also the role of the magnetic field. Generally, as it follows from calculations of the interaction of the pulsar wind with the interstellar medium [Bogovalov et al. 2005; Komissarov & Lyubarsky 2003; Bucciantini 2002], the role of the magnetic field is rather important in the post shock region almost independent of the strength of the field. This is explained by fast amplification of the magnetic field in the post shock region due to deceleration of the flow (Khangulyan & Bogovalov 2004). However, as shown in this paper, the flow is not decelerated. Just the opposite - it can be accelerated to large bulk motion Lorentz factor. Thus although one should expect an impact of the magnetic field on the post shocked flow, this affect most likely will not be so strong as in the case of the interaction of the pulsar wind with interstellar medium. Detailed MHD calculations are needed to to clarify the role of the magnetic field.

The effect of reacceleration of the post shock flow to relativistic bulk motion Lorentz factors has direct implication to the interpretation of observations of high energy γ- and X-rays from binary pulsar systems like PSR 1259-63/SS2883. This effect strongly modifies the relationship between the synchrotron X-ray and inverse Compton gamma-ray fluxes produced by the same population of relativistic electrons. It is well known that in the pulsar wind nebulae (plerions) which are formed, by the interaction of pulsar winds with the interstellar medium, the ratio between the X-ray and VHE γ-ray fluxes are defined by the ratio of the energy density of the magnetic field to the energy density of soft radiation field, provided that Compton scattering takes place in the Thompson regime (Aharonian, Atoyan & Kifune 1997). In binary systems inverse Compton scattering proceeds in the Klein-Nishina regime which changes the relationship between the X-ray and VHE gamma-ray fluxes (Khangulyan & Aharonian 2005). Due to our calculations it becomes clear that a significant deviation from the standard relations should be expected also from hydrodynamics.

Let us consider these processes in more detail, assuming that the magnetic field is present in the wind. Relativistic particles moving in the magnetic field usually produce synchrotron radiation. However, if the wind is cold (in this case the velocity of particles coincides with the bulk motion velocity) these particles do not produce synchrotron radiation. However, they can produce gamma-radiation with a specific sharp spectral feature through the IC scattering (Bogovalov & Aharonian 2000). An example of such a system is a cold pulsar wind which does not produce synchrotron radiation in the pre-shock region. In such a system the “standard” relation between synchrotron and IC radiation components is violated. Remarkably, the possibility for the formation of relativistic flows in the post-shock region, as revealed in this paper, shows that in binary pulsar systems we should expect a “non-standard” relation between synchrotron and IC radiation components in post-shock region as well.

Moreover, due to the large bulk motion Lorentz factor we should expect strong modulation of the observed nonthermal radiation of electrons. Indeed in the case of binary-pulsar systems the direction of the post shock flow varies with the motion of the pulsar along the orbit around the star. This implies significant changes of the Doppler factor, δ, given the large value of the Lorentz factor. Namely, δ ≪ 1 for large viewing angles φ (e.g. close to 90°) and δ ≳ 1 for small viewing angles. Correspondingly this will have a strong impact on the lightcurve of nonthermal radiation of electrons, Fγ ∝ δ^n where typically n ≳ 3. This effect, in particular, can naturally explain the interesting feature of the nonthermal emission of PSR 1259/SS2883, the both synchrotron and inverse Compton components of which disappear during the periastron passage of the pulsar (see e.g. Neronov & Chernyakova 2007a). This interesting issue will be discussed elsewhere.

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