Dynamics of Relativistic Shock Waves Subject to a Strong Radiation Drag: Similarity Solutions and Numerical Simulations

Ilia Leitus\(^1\) and Amir Levinson\(^1\)\(^\text{a)}\)

\(^1\)Raymond and Beverly Sackler School of Physics & Astronomy, Tel Aviv University, Tel Aviv 69978, Israel

(Dated: 1 July 2018)

We examine the effect of Compton drag on the dynamics of a relativistic shock wave. Similarity solutions describing a radiation-supported shock are obtained for certain profiles of the external radiation intensity and the density of the unshocked ejecta, and are compared with 1D numerical simulations of a blast wave expanding into an ambient medium containing isotropic seed radiation. Both the analytic model and the simulations indicate that under realistic conditions the radiation drag should strongly affect the dynamics and structure of the shocked layer. In particular, our calculations show significant strengthening of the reverse shock and weakening of the forward shock over time of the order of the inverse Compton cooling time. We conclude that the effect of radiation drag on the evolution of the emitting plasma should affect the resultant light curves and, conceivably, spectra of the observed emission during strong blazar flares.

I. INTRODUCTION

High-energy emission is a defining property of compact relativistic astrophysical sources. There is ample evidence that the emission is produced in relativistic outflows launched by an accreting black hole or a magnetar. The interaction of these outflows with their environments results in formation of expanding cocoons and shocks, that decelerate the flow and lead to lower energy emissions over a large range of scales, e.g., afterglow emission in GRBs and radio lobes in AGNs and micro-quasars. In certain circumstances the relativistic jets interact also with ambient radiation fields emitted by the surrounding gas. This interaction can considerably alter the dynamics of dissipative fluid shells at relatively small scales, and is likely to be the origin of the variable gamma-energy emission observed in certain classes of high-energy sources.

Direct evidence for such interactions is found in blazars, where the properties of the ambient radiation field can be measured. This radiation is contributed primarily by the accretion disk around the black hole, by gaseous clouds in the broad line region (BLR) and, at larger scales, by a dusty molecular torus\(^\text{1,2}\). The conventional wisdom has been that the high-energy emission seen in powerful blazars results from inverse Compton scattering of ambient seed photons by non-thermal electrons accelerated in dissipative regions in the jet\(^\text{3–6}\). In most early works the dynamics of the radiating fronts is not computed in a self-consistent manner. More recent works incorporated realistic shock models to compute light curves of flared emission\(^\text{1,7}\). However, the effect of the radiation drag on the structure and dynamics of the shock has been neglected.

In this paper we construct a model for the dynamics of a relativistic shock wave in the presence of intense radiation field. In section II we derive the governing equations of a radiation-supported shock. In section III we present self-similar solutions, obtained under several simplifying assumptions regarding the properties of the ejecta and the intensity profile of the external radiation field, and study the dependency of the evolved structure on the

\(^\text{a)}\)Electronic mail: Levinson@wise.tau.ac.il

\[\text{arXiv:1707.08654v1 [astro-ph.HE]} \ 26 \text{ Jul 2017}\]
magnitude of the radiation drag. In section [IV] we present results of numerical simulations of a uniform, spherical ejecta propagating through an ambient medium containing isotropic seed radiation. The simulations enable analysis of the temporal evolution from the initial stage under more realistic conditions. While the analysis is motivated by the application to blazars, mainly because there we have sufficient information to assess the conditions in the source, it might also be relevant to other high-energy astrophysical systems.

II. RADIATION-SUPPORTED SHOCK MODEL

Quite generally, the collision of relativistic fluid shells leads to the formation of a double-shocked layer. Some examples include the interaction of expanding ejecta with an ambient medium and collision of fast and slow shells (internal shocks). In the absence of external forces, the dynamics of the double-shock system is dictated solely by the properties of the unshocked media. In the presence of external radiation field, the shocked layers are subject to radiation drag that can alter the dynamics significantly. When the drag is strong enough it should lead to a weakening of the forward shock and strengthening of the reverse shock. In essence, the compression of the shocked ejecta (or the shocked plasma of the fast shell in case of internal shocks) is supported by the radiation pressure imposed on it. Thus, as a simple approximation one can ignore the entire region beyond the contact discontinuity, and obtain solutions describing a radiation-supported shock. This is the treatment adopted in section [III] below. The validity of this approximation is checked in section [IV] using numerical simulations.

Below we adopt the following notation: The velocities of the reverse shock and the contact surface are denoted by $V_r$ and $V_c$, respectively, and the corresponding Lorentz factors by $\Gamma_r$ and $\Gamma_c$. Plasma quantities are denoted by lower case letters, and subscripts $u$ and $s$ refer to the unshocked and shocked ejecta, respectively.

A. Flow equations

The dynamics of a relativistic fluid is governed by the continuity equation,

$$\partial_\mu \left( \rho u^\mu \right) = 0,$$

and energy-momentum equations,

$$\frac{\partial}{\partial x^\alpha} T^{\alpha\mu} = S^\mu,$$

here expressed in terms of the energy-momentum tensor

$$T^{\mu\nu} = w u^\mu u^\nu + g^{\mu\nu} p,$$

where $\rho$, $p$ and $w$ are, respectively, the proper density, pressure and specific enthalpy, $u^\mu = \gamma(1, v_1, v_2, v_3)$ is the fluid 4-velocity, and $g^{\mu\nu}$ is the metric tensor, here taken to be the Minikowsky metric $g^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$. The source term on the right hand side of Eq. (2) accounts for energy and momentum exchange between the fluid and the ambient radiation field, and is derived explicitly below. The above set of equations must be augmented by an equation of state. We adopt the relation

$$w = \rho + \frac{\gamma}{\gamma - 1} p \equiv \rho + ap,$$

with $a = 4$ for a relativistically hot plasma and $a = 5/2$ for sub-relativistic temperatures.
We shall restrict our analysis to spherical (or conical) flows, and adopt a spherical coordinate system \((r, \theta, \phi)\). Equations (1) and (2) for the shocked fluid then reduce to:

\[
\frac{\partial_t (\rho \gamma)}{\gamma} + \frac{1}{r^2} \frac{\partial_r (r^2 \rho \gamma v)}{\gamma} = 0, \tag{5}
\]

\[
\frac{\partial_t (w \gamma^2 - p)}{\gamma} + \frac{1}{r^2} \frac{\partial_r (r^2 w \gamma^2 v)}{\gamma} = S^0, \tag{6}
\]

\[
w \frac{d}{dt} \ln \gamma + \frac{dp}{dt} - \gamma^{-2} \partial_t \rho = v(S^0 - vS^0), \tag{7}
\]

here \(d/dt = \partial_t + v \partial_r\) denotes the convective derivative.

A common approach is to seek solutions of Eqs. (5)-(7) inside the shocked layer, given the properties of the unshocked flows upstream of the forward and reverse shocks as input. The jump conditions at the forward and reverse shocks can, in principle, be derived in a similar manner, but are redundant under our radiation-supported shock approximation.

B. Jump conditions at the reverse shock

For the range of conditions envisaged here the shocked plasma is optically thin. The shock, in this case, is collisionless, i.e., mediated by collective plasma processes (as opposed to radiation mediated shocks), and its width, roughly the skin depth, is much smaller than the Thomson length. Thus, the shock transition layer can be treated as a discontinuity in the flow parameters. The local jump conditions at the reverse shock can be expressed in terms of the local shock velocity \(V_r = dR_r/dt\), where \(R_r(t)\) is the shock radius at time \(t\), as:

\[
\rho_u \gamma_u (v_u - V_r) = \rho_s \gamma_s (v_s - V_r), \tag{8}
\]

\[
w_u \gamma_u^2 v_u (v_u - V_r) + p_u = w_s \gamma_s^2 v_s (v_s - V_r) + p_s, \tag{9}
\]

\[
w_u \gamma_u^2 (v_u - V_r) + p_u V_r = w_s \gamma_s^2 (v_s - V_r) + p_s V_r. \tag{10}
\]

We shall henceforth assume that the unshocked medium is cold and set \(p_u = 0\). The jump conditions at the forward shock can, in principle, be derived in a similar manner, but are redundant under our radiation-supported shock approximation.

C. Compton drag terms

Behind the shock electrons are heated and accelerated to relativistic energies. We denote the energy distribution of electrons there by \(F_e(\gamma_e)\), where \(\gamma_e = \epsilon/m_ec^2\) is the dimensionless electron energy, as measured in the fluid frame. The total electron proper density is then given by \(n_e = \int F_e(\gamma_e) d\gamma_e\), and the average energy and second energy moment by

\[
< \gamma_e >= \frac{1}{n_e} \int \gamma_e F_e(\gamma_e) d\gamma_e, \quad < \gamma_e^2 >= \frac{1}{n_e} \int \gamma_e^2 F_e(\gamma_e) d\gamma_e. \tag{11}
\]

If only the thermal electrons are taken into account one expects \(m_e \gamma_e^2 n_e < \gamma_e^2 > \approx m_p \gamma_u'/2\) for plasma in rough equipartition, where \(\gamma_u' = \gamma_u \Gamma_s (1 - v_u V_r)\) is the Lorentz factor of the upstream flow, as measured in the shock frame, and it is assumed that \(\gamma_u' > 1\). If the contribution of non-thermal electrons is substantial, then \(\gamma_e^2 \gamma > \) may be larger. Since the shocked electron plasma is relativistic we have \(m_e c^2 n_e < \gamma_e^2 > \approx w < \gamma_e >\).
The derivation of the source terms is given in Ref [10]. In terms of the energy density of the radiation intercepted by the flow, \( u_{\text{rad}}(r) \), and the quantities defined above they can be expressed as

\[
S^0 = -\frac{8}{3} \gamma_s^3 < \gamma_e^2 > u_{\text{rad}} \sigma_T n_e, \tag{12}
\]

and

\[
S^e = v_s S^0 + S^0/3\gamma_s^2. \tag{13}
\]

From Eqs. (7) and (13) it is readily seen that the deceleration time of the shocked fluid is given by \( t_{\text{dec}} = -3u\gamma_s^2/v_s S^0 = 2\gamma_s t'_IC, \) where \( t'_IC \approx 3m_c c/(4\sigma_T u'_{\text{rad}} < \gamma_e >) \) is the inverse Compton cooling time of an electron having an energy \( m_v c^2 < \gamma_e >, \) as measured in the fluid rest frame, and \( u'_{\text{rad}} = 4\gamma_s^2 u_{\text{rad}}/3. \) Substantial deceleration occurs if \( t_{\text{dec}} \) is much shorter than the outflow time, \( t_f = r/c. \) For convenience, we define the drag coefficient as

\[
\alpha = 3t_f/t_{\text{dec}} = \frac{8}{3m_v c^2} \gamma_s < \gamma_e > \sigma_T u_{\text{rad}} r. \tag{14}
\]

As noted above, for thermal electrons \( \gamma_s < \gamma_e > \) \( \approx \) \( (m_p/2m_e) \gamma_u^2 \gamma_s = (m_p/4m_e) \gamma_u. \) If only the contribution of thermal electrons is accounted for, then

\[
\alpha = \frac{8m_p \sigma_T}{12m_v^2 c^2} \gamma_u u_{\text{rad}} r. \tag{15}
\]

The energy loss term can now be expressed as:

\[
S^0 = -\alpha \gamma_s^2 w r^{-1}. \tag{16}
\]

To get an estimate of the value of \( \alpha \) anticipated in blazars, suppose that a fraction \( \eta \) of the nuclear luminosity, \( L = 10^{45} L_{45} \) erg/s, is scattered across the jet. The corresponding energy density of the radiation intercepted by the jet is roughly \( u_{\text{rad}}(r) = \eta L/4\pi r^2 = 3 \times 10^{-4} \eta L_{45}/r_{pc}^2 \) ergs cm\(^{-3}\), where \( r_{pc} \) is the radius in parsecs. From Eq. (15) one obtains \( \alpha = 0.7 \gamma_u \eta L_{45} r_{pc}^{-1}. \) A more realistic estimate can be obtained using recent calculations of the ambient radiation intensity contributed by the various radiation sources surrounding a blazar jet. It is found that for a prototypical blazar the intensity profile of radiation intercepted by the jet is flat up to a distance of about one parsec, with \( u_{\text{rad}} \approx 10^{-3} \) erg cm\(^{-3}\), followed by a gradual decline. For an internal shock produced by colliding shells, Equation (15) yields for the drag coefficient \( \alpha \approx 3 \gamma_u r_{pc}. \) Typically \( \gamma_u > 10, \) implying significant drag on sub-parsec scales in blazars.

D. Global energy conservation

The change over time in the total energy of the shocked ejecta (or shocked fast shell) must equal the difference between the energy injected through the reverse shock and the energy radiated away through the contact. The energy accumulated behind the forward shock is neglected in our approximate treatment. The time change of the total energy per steradian of the shocked ejecta is

\[
\partial_t E = \partial_t \int_{R_c(t)}^{R_s(t)} T^{00} r^2 dr = \int_{R_c(t)}^{R_s(t)} \partial_t (T^{00} r^2) dr + R_c^2 T^{00}(R_s)V_c - R_c^2 T^{00}(R_c)V_s, \tag{17}
\]

where \( R_c(t) \) and \( R_s(t) \) are, respectively, the radii of the reverse shock and the contact surface, and \( V_c = dR_c/dt, \) \( V_s = dR_s/dt \) are the corresponding velocities. Integrating Eq. (17) from the shock, \( R_c(t) \), to the contact, \( R_c(t) \), one obtains:
The trajectory of the reverse shock is given by 

\[ O \text{ determined from global energy conservation, as will be shown below.} \]

Now, to order the contact discontinuity have a similar time evolution, viz., \( \Gamma \) and \( v \) of shocked ejecta and region 2 (shocked ambient medium) at the contact discontinuity 

\[ \text{(it can be readily seen that the continuity equation is satisfied for this choice of} \]

For an ambient density profile of the form 

\[ \rho \]

where 

\[ a \]

characterized by a velocity profile 

\[ v \]

subsequently analyzed 

Medium, in the absence of radiative losses, were derived in Ref 8, and their stability was 

III. SELF-SIMILAR SOLUTIONS

Similarity solutions describing the interaction of a relativistic shell with an ambient medium, in the absence of radiative losses, were derived in Ref 8, and their stability was 

obtained self-similar solutions provided the energy source term scales as 

\[ \rho \propto r^{-k} \]

For an ambient density profile of the form \( \rho \propto r^{-k} \), the index \( m \) is given by 

\[ m = \frac{6 - 2k}{n + 2} \]

In cases where the shocked plasma is subject to strong radiative losses it is still possible to obtain self-similar solutions provided the energy source term scales as 

\[ S^0 \propto \gamma^2 w r^{-1} \] (see Eq. 19 and 17), that is, \( \alpha \) in Eq. 16 is constant. We shall henceforth make this assumption even though it implies a somewhat artificial intensity profile of the external radiation field. We note, however, that as long as the deceleration length is smaller than the radius of the shock these details are unimportant. This is confirmed by numerical simulations, presented in the next section.

As explained above, in the presence of a strong radiation drag the dynamics of the system is dictated by the interaction of the external radiation field with the shocked plasma enclosed between the contact and the reverse shock. To a good approximation one can then ignore the contribution of the forward shock to the overall dynamics. The evolution of the radiation-supported shock still satisfies 

\[ \Gamma_2 = B t^{-m} \]

\[ \Gamma_3 = C t^{-m} \]

but the index \( m \) is now determined from global energy conservation, as will be shown below. Now, to order \( O(\Gamma^{-2}) \) the trajectory of the reverse shock is given by 

\[ R_\sigma(t) = \int_0^t \left( 1 - \frac{1}{2\Gamma_\sigma^2} \right) dt' = \frac{t}{2(m + 1)\Gamma_\sigma^2}, \]
from which we obtain for the velocity of the ejecta crossing the shock: \( v_u(R_t) = R_t/t = 1 - 1/[2(m + 1)\Gamma_r^2] \). The corresponding Lorentz factor is thus given, to the same order, by

\[
\gamma_u^2 = (m + 1)\Gamma_r^2,
\]

and the density profile by

\[
\rho_u = \frac{\alpha_u}{(m + 1)^{n/2}B^{3/m}r^{(6/m-n)}}.
\]

We adopt the similarity parameter introduced in Ref.\[12\]

\[
\chi = [1 + 2(m + 1)\Gamma_r^2](1 - r/t).
\]

With this choice the reverse shock is located at \( \chi = 1 \), and the contact at \( \chi_c = \Gamma_r^2/\Gamma_c^2 = B/C < 1 \). Following Ref.\[8\] we define the self-similar variables, \( G(\chi), H(\chi) \) and \( F(\chi) \), such that at the reverse shock they satisfy the boundary conditions \( G(1) = H(1) = F(1) = 1 \). Upon solving the jump conditions (8)-(10), the shocked fluid quantities can be expressed in terms of the self-similar variables as

\[
\gamma_s^2 = q\Gamma_c^2G(\chi),
\]

\[
\rho_s\gamma_s = \frac{m\rho_u\gamma_u}{(m + 1)(q - 1)}H(\chi),
\]

\[
p_s = \frac{m\rho_u}{a(q - 1) + 2(1 - \sqrt{q/(m + 1)})F(\chi)},
\]

\[
w_s = \rho_s + ap_s \equiv K(\chi)p_s,
\]

where the parameter \( \sqrt{q} \) is the only positive root of the polynomial equation

\[
\hat{\gamma}x^3 + (2 - \hat{\gamma})\sqrt{m + 1}x^2 - (2 - \hat{\gamma})x - \hat{\gamma}\sqrt{m + 1}x = 0.
\]

Upon substituting these relations into Eqs. (5)-(7), and using Eq. (16) with a constant drag coefficient, \( \alpha = \text{const.} \) we obtain the following set of equations for the self-similar variables:

\[
2(1 + qG\chi)\partial_\chi \ln F - (1 - qG\chi)K\partial_\chi \ln G = \frac{mn - 6 - (m - \frac{3}{2}\alpha)K}{(m + 1)}qG,
\]

\[
2(1 - qG\chi)\partial_\chi \ln F - \hat{\gamma}(1 + qG\chi)\partial_\chi \ln G = \frac{6 - mn + (m - 4)\hat{\gamma} - \frac{2}{3}(\hat{\gamma} - 1)\alpha K}{m + 1}qG,
\]

\[
2(1 - qG\chi)\partial_\chi \ln H - 2\partial_\chi \ln G = \frac{-(mn - m - 2)}{(m + 1)}qG,
\]

subject to the boundary conditions \( G(1) = F(1) = H(1) = 1 \). These equations reduce to those derived in Ref.\[8\] in the special case \( \alpha = 0 \). As can be seen, at the contact, \( \chi = \chi_c \), the Lorentz factor must satisfy \( qG\chi \chi_c = 1 \), where for short we denote \( G(\chi_c) = G_c \). This relation defines the limit of integration.

The self-similar equations involve two eigenvalues; the index \( m \) and the location of the contact \( \chi_c \). Thus, two conditions are needed to find them. The first one is the relation

\[
qG\chi \chi_c = 1
\]

mentioned above. The second one is global energy conservation, Eq. \[19\]. To order \( O(\gamma_r^{-2}) \) we have \( T_r^{90} = w_s\gamma_s^2 - p_s \approx w_s\gamma_s^2, r = t \) and \( dr = -td\chi/[2(m + 1)\Gamma_r^2] \), from which we obtain

\[
E(t) = \int_{R_t(t)}^{R_c(t)} T_r^{90}r^2dr = \frac{t^{mn/2}qma_u(1 - \sqrt{q/(m + 1)})}{2(m + 1)^{n/2+1}B^{n/2}[a(q - 1) + 2]}
\]

\[
\times \int_{\chi_c}^{1} K(\chi)F(\chi)G(\chi)d\chi,
\]

(35)
and \( \partial_t E = n m E/2t \). To the same order one has \( \int_{R_e} S^0 r^2 dr = -\alpha E/4t \). Substituting these results into Eq. (19) yields the constraint

\[
\int_{\chi_c}^1 K(\chi) F(\chi) G(\chi) d\chi = \frac{(m+1)}{q(\alpha + nm/2)} \times \left[ \frac{a(q-1) + 2}{1 - \sqrt{q/(m+1)}} - 2F_c \right],
\]

where \( F_c = F(\chi_c) \).

A. Results

For a given choice of the drag coefficient \( \alpha \) we guess the value of \( m \) and integrate Eqs. (31)-(33) from \( \chi = 1 \) to the point \( \chi_c \) at which Eq. (34) is satisfied. We then check if Eq. (36) is satisfied. If not, we change the value of \( m \) and repeat the process until a solution satisfying all constraints is found. Sample profiles are shown in Fig. 1 for \( n = 1 \) and different values of \( \alpha \). The radiation free case \((\alpha = 0)\) is shown for a comparison. It was computed using the full solution of the two-shock model described in Ref. [8]. As seen, the width of the shocked layer, \( \Delta \chi = 1 - \chi_c \), decreases with increasing drag coefficient \( \alpha \), as naively expected. Moreover, larger radiative losses lead to increased non-uniformity of the Lorentz factor and pressure in the shocked layer. The divergence of the density at the contact is a basic feature of the similarity solutions [9] and occurs even in the absence of radiative losses \((\alpha = 0)\), as seen in the upper panel of Fig. 1.

Figure 2 depicts the dependence of the index \( m \) on \( \alpha \). For a comparison, the value of \( m \) obtained in the case \( \alpha = 0 \) for a blast wave propagating in a uniform density medium \((k = 0)\) is \( m = 2 \) (see Eq. (21) with \( k = 0 \) and \( n = 1 \)). The 4-velocity of the unshocked fluid, as measured in the frame of the reverse shock, is given by

\[
u'_u = \gamma_u \Gamma_r (v_u - V_r) = \frac{m}{2\sqrt{m+1}},
\]

and it is seen that the shock becomes substantially stronger as \( \alpha \) increases. Note that the power dissipated behind the shock, \( \rho_u \gamma'_u u'_u \), increases roughly linearly with the index \( m \). At sufficiently large drag the entire power incident through the shock is radiated away, and the solution becomes independent of \( \alpha \), as seen in Fig. 2. For our choice of parameters, specifically \( n = 1 \), this occurs at \( \alpha \gtrsim 50 \), for which \( m \approx 19 \) and \( u'_u \approx 2.1 \). We emphasize that this limit can be approached provided the Lorentz factor \( \gamma_u \) of the unshocked shell is sufficiently large. To be more concrete, Eq. (23) implies that \( \gamma_u \approx 4.6 \Gamma_r \) as \( \alpha \to \infty \), while our analysis is valid only for \( \Gamma_r > 1 \). The Compton drag terms given in Eqs. (12) and (13) assume that the intensity of ambient radiation is highly beamed in the frame of the shocked fluid. Once \( \gamma_s \) decelerates to modest values, \( \gamma_s \gtrsim 1 \), the drag force is strongly reduced, ultimately becoming ineffective.

IV. NUMERICAL SIMULATIONS

Numerical simulations were performed using the PLUTO code [14], that was modified to incorporate energy and momentum losses of the shocked plasma through Compton scattering. To clarify the presentation we adopted an external intensity profile of the form \( u_s \propto r^{-1} \) for which \( \alpha \) in Eq. (16) is constant. Over the deceleration scale this profile is a reasonable approximation to the flat profile expected in blazars [1]. We start with the basic spherical blast wave problem, releasing an ejecta with a uniform Lorentz factor \( \gamma_e = 10 \) into a stationary ambient medium (henceforth, quantities in the unshocked ejecta are designated by subscript "e"). In our setup, both the ejecta and the ambient medium are taken to be uniform initially with a density ratio \( \rho_e/\rho_i = 100 \). The calculations are performed in
"simulation units" in which a fluid element traveling with a unit velocity (the speed of light) passes a unit of length per unit of time. The initial impact occurs at a radius $R_0 = 10^3$ and time $t = 0$. Due to the collision a double-shock structure forms and a shocked layer is created. As the simulation progresses the shocked layer widens and Compton drag is then applied on the shocked fluid contained between the forward and the reverse shocks. In order to suppress artificial transients created by abrupt changes we apply the radiation
drag gradually by increasing the drag coefficient over time as \( \alpha(t) = \alpha(1 - e^{-t/t_0}) \). As our model assumes that the drag only affects the hot shocked plasma, the in-simulation drag is applied only in the region where the shocked fluid velocity is in the range \([0.1u_e, 0.9u_e]\), where \(u_e\) is the 4-velocity of the unshocked ejecta. Preliminary runs have shown that for

\[ t/t_0 \]

of the external radiation in the rest frame of the shocked fluid, overestimates the actual evolution of the entire structure. The \(\alpha\) initially less than in the case and decelerates it (Fig. 5). The shocked layer gradually expands as time progresses, but rise to compression of the fluid near the contact which is communicated to the reverse shock. The radiation force gives rise to compression of the fluid near the contact which is communicated to the reverse shock and weakening of the forward shock is clearly seen. The radiation force gives rise to compression of the fluid near the contact which is communicated to the reverse shock and decelerates it (Fig. 5). The shocked layer gradually expands as time progresses, but initially less than in the case \(\alpha = 0\), the shocked layer decelerates and the reverse shock quickly becomes radiation-supported.

### A. Test case

As a check, we ran a test case with \(\alpha = 0\) for the same setup described above, and compared the results with an analytic solution obtained under the thin shell approximation, whereby the shocked layers are assumed to be uniform. Under this approximation the shocked ejecta and shocked ambient medium have the same Lorentz factor, \(\gamma_s = \Gamma_c\). In terms of the ratios \(q_e = (\gamma_e/\Gamma_c)^2\) and \(q = (\gamma_s/\Gamma_c)^2\), the jump conditions, Eqs. (8) - (10), yield

\[
2q_e \left(1 - \sqrt{\frac{q}{q_e}}\right) + (a(q - 1) + 2) \left(\frac{q - q_e}{q + 1}\right) = 0, \tag{38}
\]

and

\[
\frac{q}{q_e} = \frac{3}{4\gamma_e^2} \frac{\rho_e}{\rho_c} \frac{(q_e - 1)}{q(q - 1) + 2}, \tag{39}
\]

where \(\rho_e, q\) and \(q_e\) are functions of time. Solving these equations for the given initial conditions we obtain \(q_e = 3.765, q = 1.2\) at \(t = 0\). As the ejecta expands, its density just upstream of the reverse shock evolves as \(\rho_e(t) \propto [R_e(t)]^{-2}\). Solving the above equations at any given time \(t\) using \(\rho_e(t)\) in Eq. (39), one obtains the Lorentz factor of the contact discontinuity, \(\Gamma_c(t) = \gamma_c \sqrt{\frac{q(t)}{q_e(t)}}\). The contact 4-velocity, \(U_c(t) = \sqrt{\Gamma_c^2(t) - 1}\), is shown as a dashed line in Fig. 3 along with the 4-velocity, pressure and density obtained from the simulations of the radiation free case \((\alpha = 0)\), at different times. As seen from Fig. 3 the jump at the shock agrees well with the analytic result.

### B. Simulation Results

For numerical reasons we were only able to run cases with modest values of \(\alpha\), however these suffice to illustrate the main trends. Below we present results for \(\alpha = 3, 6, 10, 15\). The initial conditions in all cases were set as described above.

Fig 3 displays snapshots of the plasma 4-velocity at different times for \(\alpha = 10\), showing the evolution of the entire structure. The \(x\)-axis gives the distance from the reverse shock, \(x = R - R_e(t)\), so that the reverse shock is located at \(x = 0\) at all times. The vertical red and blue lines indicate the location of the contact discontinuity \(x_c\) and the forward shock \(x_f\), respectively. The deceleration of the shocked fluid, that leads to strengthening of the reverse shock and weakening of the forward shock is clearly seen. The radiation force gives rise to compression of the fluid near the contact which is communicated to the reverse shock and decelerates it (Fig. 5). The shocked layer gradually expands as time progresses, but initially less than in the case \(\alpha = 0\). The apparent sudden expansion of the shocked layer at late times (between \(t = 600\) and \(t = 900\)) commences when the shock becomes mildly relativistic, as can be observed from Fig. 5. This is mainly due to the fact that the width of the shocked layer evolves with time as \(t/\Gamma_c^2\). We note that at such small Lorentz factors (\(\Gamma_r < 2.5\) at \(t > 600\)) our choice of Compton drag terms, that assume perfect beaming of the external radiation in the rest frame of the shocked fluid, overestimates the actual
FIG. 3. Profiles of the 4-velocity (blue solid line), pressure (black solid line), and density (red solid line) obtained from the test run of a radiation free system ($\alpha = 0$), plotted as a functions of the distance from the reverse shock, $x = R - R_r(t)$. The value of the contact 4-velocity $U_c$, computed from the analytic model, is marked by the dashed line. As seen, the agreement is excellent.

FIG. 4. Profiles of the plasma 4-velocity (solid line) and pressure (dashed line) at different times, for the case $\alpha = 10$. For clarity of presentation, the pressure is scaled by a factor of 5. Time progresses from top to bottom panels, as indicated.

drag. In reality the drag is expected to be suppressed as the shocked fluid becomes mildly relativistic, so that the Lorentz factor of the reverse shock will eventually saturate once $\Gamma_r$ becomes sufficiently low.

Figs shows the profiles of the 4-velocity, pressure and density around the shock at the same simulation time, $t = 800$, for four values of $\alpha$ as indicated in the figure label. The radius of the reverse shock at this time is also indicated for convenience, and it ranges from $R_r \approx 1776$ (or $(R_r - R_0)/R_0 \approx 0.77$) for $\alpha = 3$ to $R_r \approx 1716$ for $\alpha = 15$. The result of the run with no radiation drag ($\alpha = 0$), presented in Fig. 3, is plotted as a dotted-dashed line and is exhibited for a comparison. The contact location is found from the density profile, noticing that, as in the radiation-free case, the density peaks towards the contact
FIG. 5. Time evolution of the reverse shock Lorentz factor. Up to \( t \approx 30 \) the shock structure is not sufficiently developed and the determination of \( \Gamma_r \) is uncertain.

and drops to a minimum right after it. In the resulting structure we see the transition from "material-supported" to "radiation-supported" shock; as \( \alpha \) is increased the forward shock becomes progressively weaker and ultimately negligible, while the reverse shock strengthens. This trend justifies the neglect of the shocked ambient pressure in the self-similar solution outlined in section [III].

Fig. 6 confirms that, as long as the shock is sufficiently relativistic, the width of the shocked layer shrinks and the shocked fluid velocity decreases with increasing radiation drag, in accord with the self-similar solution. The widening of the shocked layer for the \( \alpha = 10 \) and \( \alpha = 15 \) cases stems from the transition to the mildly relativistic regime, as explained above. The increase of the pressure with increasing \( \alpha \) further indicates strong compression by the radiation force. The formation of the cold dense shell near the contact, seen in the lower left panel of Fig. 6, results from the rapid cooling of the compressed plasma.

The left panel of figure 7 exhibits the evolution of the 4-velocity of the shocked ejecta. Gradual deceleration of the shocked flow over a timescale of the order of the cooling time is observed, as expected. The evolution of the 4-velocity of the unshocked ejecta with respect to the reverse shock is exhibited in the right panel of figure 7, indicating a substantial increase in shock efficiency with increasing \( \alpha \).

V. SUMMARY AND CONCLUSIONS

We studied the effect of radiation drag on the dynamics of shocks that form in relativistic outflows. Such situations are expected in cases where the outflow propagates through a quasi-isotropic, ambient radiation field, on scales at which the inverse Compton cooling time is significantly shorter than the outflow expansion time. The observations of the continuum emission in blazars suggest that these conditions may prevail on sub-parsec to parsec scales in those objects.

For certain profiles of the external radiation intensity and the density of the unshocked ejecta we were able to find self-similar solutions of the radiation hydrodynamics equations, describing a radiation-supported shock. We also performed 1D numerical simulations of a uniform, spherical shell interacting with an ambient medium that contains cold gas and seed radiation. For that purpose we used the PLUTO code, that we modified to incorporate energy and momentum losses of the shocked plasma through Compton scattering.

In both, the analytical model and the simulation results, we find significant alteration of the shock dynamics when the ratio of dynamical time and Compton cooling time exceeds a factor of a few. Quite generally, substantial radiation drag leads to a faster deceleration, strengthening of the reverse shock and weakening of the forward shock. In the self-similar model with \( n = 1 \) the 4-velocity of the upstream flow with respect to the shock increases from \( u'_u \approx 0.5 \) for a uniform ambient density profile and no radiation (\( \alpha = 0 \)), to about \( u'_u \approx 2.1 \) in the asymptotic limit of maximum efficiency (obtained for \( \alpha > 50 \)). The Lorentz factor in the asymptotic limit evolves roughly as \( \Gamma \propto t^{-10} \).

For numerical reasons the simulations were limited to modest values of \( \alpha \), but show similar trends. The numerical experiments enabled us to follow the evolution of the system from the
onset of fluid collision, and demonstrate the gradual strengthening of the reverse shock (and weakening of the forward shock) over time. For convenience we adopted intensity profile (of the seed radiation) that scales as $r^{-1}$, which is close to the flat profile obtained for blazars from detailed calculations. Since for a sufficiently large drag the deceleration scale is much smaller than the shock radius, the details of the intensity profile has little effect on the evolution. For our choice of constant drag coefficient, the deceleration continues.

FIG. 6. Profiles of the 4-velocity (upper panel), density (lower left panel) and pressure (lower right panel) at simulation time $t = 800$, plotted as a functions of the distance from the reverse shock, for different values of the drag coefficient $\alpha$. The location of the reverse shock is at $x = 0$. The locations of the contact and the forward shock are marked by red and blue vertical lines respectively. The thin dotted-dashed lines correspond to the case $\alpha = 0$, and are shown for comparison. As seen, the strengthening of the reverse shock and weakening of the forward shock become more substantial as $\alpha$ increases. Strong compression by the radiation force is also seen in the pressure and density plots. Note the logarithmic scale of the y-axis in the density plots.
FIG. 7. Time evolution of the 4-velocity of the shocked ejecta (left panel), and of the unshocked ejecta, as measured in the shock frame (right panel).

indefinitely. In reality the drag will be strongly suppressed once the Lorentz factor of the emitting (shocked) fluid drops to values at which our "beaming approximation" breaks down. In this regime strong cooling still ensues, but with little momentum losses. Our late time results for the cases $\alpha = 10$ and $\alpha = 15$ are therefore not reliable. More detailed calculations of the drag terms are needed to follow the evolution in the mildly relativistic regime.

The effect of radiation drag on the dynamics of the shock might have important implications for the resultant emission. On the one hand, the strengthening of the reverse shock leads to enhanced efficiency of internal shocks, in particular under conditions at which the reverse shock is sub-relativistic in the absence of external radiation. On the other hand, the deceleration of the shocked fluid, that emits the observed radiation, leads to a dramatic change in the beaming factor. This temporal change in beaming factor can significantly alter the light curves, particularly for observers viewing the source off-axis. Detailed calculations of variable emission from dragged shocks is beyond the scope of this paper, but our analysis suggests that detailed emission models, at least in blazars, should account for such effects.

ACKNOWLEDGMENTS

This research was supported by a grant from the Israel Science Foundation no. 1277/13.

1M. Joshi, A. P. Marscher, and M. Bottcher, “Seed photon fields of blazars in the internal shock scenario,” Astrophys. J. 785, 132 (2014).
2M. Sikora, L. Stawarz, R. Moderski, K. Nalewajko, and G. M. Madejski, “Constraining emission models of luminous blazar sources,” Astrophys. J. 704, 38–50 (2009).
3C. D. Dermer and R. Schlickeiser, “Model for the high-energy emission from blazars,” Astrophys. J. 416, 458 (1993).
4M. Sikora, M. C. Begelman, and M. J. Rees, “Comptonization of diffuse ambient radiation by a relativistic jet: The source of gamma rays from blazars?” Astrophys. J. 421, 153–162 (1994).
5R. D. Blandford and A. Levinson, “Pair cascades in extragalactic jets. 1: Gamma rays,” Astrophys. J. 441, 79–95 (1995).
6G. Ghisellini and P. Madau, “On the origin of the gamma-ray emission in blazars,” Mon. Not. Roy. Ast. Soc. 280, 67–76 (1996).
7M. Joshi and M. Bottcher, “Time-dependent radiation transfer in the internal shock model scenario for blazar jets,” Astrophys. J. 727, 21 (2011).
8K. Nakamura and T. Shigeyama, “Self-similar solutions for the interaction of relativistic ejecta with an ambient medium,” Astrophys. J. 645, 431–4325 (2006).
9A. Levinson, “Relativistic rayleigh-taylor instability of a decelerating shell and its implications for gamma-ray bursts,” GAPFD 104, 85–111 (2010).
10O. Golan and A. Levinson, “Blazar flares from compton dragged shells,” Astrophys. J. 809, 23 (2015).
11A. Levinson, “Convective instability of a relativistic ejecta decelerated by a surrounding medium: An origin of magnetic fields in gamma-ray bursts?” Astrophys. J. 705, L213–L216 (2009).
12R. D. Blandford and C. F. McKee, “Fluid dynamics of relativistic blast waves,” Phys. Fluids 19, 1130–1138 (1976).
More generally, $\alpha$ can be taken to be any function of the similarity coordinate $\chi$. In the presence of strong drag the width of the shocked layer is much smaller than its radius, hence we anticipate little variations in $\alpha$ across it.

A. Mignone, G. Bodo, S. Massaglia, and et al., “Pluto: A numerical code for computational astrophysics,” Astrophys. J. Supp. 170, 228 (2007).