Models and algorithms of optimal structure synthesis of infocommunication network with specified parameters

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Abstract. The article considers the issues of the synthesis of the infocommunication network on the basis of its existing segments by introducing new elements, the modernization or elimination of existing ones. In this regard, there is a task to build a network corresponding to the specified parameters. The solution of the synthesis problem is based on the development of models and algorithms for the generation of alternative networks, the numerical evaluation of their performance indicators and choosing the best option. Generation involves the combination of a priori given modernization options of individual communication channels, introducing new ones or refusing to use existing ones. The choice of the optimal variant is carried out using the models of the Boolean linear programming. The proposed algorithm takes into account the features of the task and its dimension. It is based on the use of the branch and bound scheme.

1. Introduction
At the present time, there are constant changes in the conditions of work of organizations that are associated with changes in volume of financing and (or) their information needs. In this connection, changes in the functioning of infocommunication networks used by them become necessary in order to maintain specified requirements for the efficiency of operation of these networks [1-3].

Thus, there is the task of building a network corresponding to the necessary, specify parameters, by introducing new elements into already existing network segments (network), eliminating or modernizing existing elements.

Organizations can apply different options of using infocommunication networks:
- lease of communication channels of infocommunication networks from the provider with an ability to change the amount of provided network traffic;
- use of infocommunication networks owned by the organization;
- combination of the options described above.

Thus, changes in the operation of information and communication networks can be carried out on the basis of the following activities:
- modernization of current infocommunication network channels, avoiding use of channels with low capacity or development of new communication channels;
- changes in terms of a communication channels lease in the segments of public infocommunication networks.

These procedures will be referred to as infocommunication network modification options.
In order, to solve the problem of synthesizing the optimal network structure, first it is necessary to define performance indicators and criteria by which optimization will be carried out [4].

The following parameters can be used as performance indicators:
- value of a network weighted channel capacity rate, capacity rate for selected priority data transmission directions, which are usually chosen by organizations;
- value of financial costs for the modernization of the infocommunication network and the cost of information transfer for leased communication channels in public infocommunication networks.

The following requirements can be used as criteria:
- requirements to maximize both: the weighted channel capacity rate and channel capacities of certain data transmission directions;
- requirements to minimize financial costs of network modification (lease).

Nowadays there is no common approach to choosing the optimal variant of the structure of the infocommunication network.

This paper is devoted to the development of models and algorithms for synthesizing the structure of an infocommunication network corresponding to the specified characteristics based on a combination of specified modernization options of existing communication channels and changing the lease conditions for information transmission channels, numerical evaluation of network performance indicators and choosing the best option.

2. Evaluation of performance indicators

Let us introduce the following notation for infocommunication network elements:
- \( V = \{v_1,v_2,\ldots,v_{V_v}\} \) – set of nodes, where data transmission, data storage and data processing devices are located;
- \( E = \{e_1,e_2,\ldots,e_{E_e}\} \) – set of communication channels, used for data transmission.

Each communication channel \( e_k \in E \) in turn, is characterized by the following:
- \( r_{ik} \) – channel capacity rate;
- \( c_{jk} \) – financial costs of usage and modernization.

In this case, the infocommunication network model is a double-weighted directed graph \( G = (V,E,R,C) \) with a set of vertices \( V \), edges \( E \), and weights of edges \( R = \{r_1,r_2,\ldots,r_{E_e}\} \) and \( C = \{c_1,c_2,\ldots,c_{E_e}\} \).

In general, infocommunication networks can contain communication channels with either two-way or one-way data transmission. In order to unify mathematical description of random infocommunication networks the G graph, described above, is considered to be oriented. This is because each communication channel with two-way transmission corresponds to two oppositely directed edges (arcs). Yet this assumption doesn’t make development of the further model less common.

Assume that while using infocommunication network (or a network segment) a set of priority data transmission directions have been selected \( N = \{n_1,n_2,\ldots,n_{n_n}\} \), which provide effective performance of organization employing this infocommunication network (a public network segment). For each selected data transmission direction \( n_i \in N \), in turn, let us consider the following:
- \( \nu_i^b \) – data sender node;
- \( \nu_i^r \) – data receiver node;
- \( \alpha_i \) – importance of certain data transmission direction for organization performance;
- \( p_i \) – network capacity rate of certain data transmission direction.
Based on the introduced notation now it is possible to describe performance indicators of infocommunication network modification.

The potential of effective usage of infocommunication network (network segment) for data transmission is defined by the following indicator:

$$ P = \sum_{i=1}^{N} \alpha_i p_i $$

(1)

- network weighted capacity rate [5];

$$ S = \sum_{k=1}^{E} c_k $$

(2)

- financial costs of the infocommunication network usage and (or) modernization.

Data transmission in the priority direction $i$ from a data sender node $b_i$ to a data receiver node $e_i$ is a message flow, transferred via network links. This message flow can be redistributed in the nodes. In this case, the capacity rate $p_i$ of infocommunication network (1), for this priority data transmission direction is defined by a value of a maximum flow in the weighed graph $G$ from $b_i$ to $e_i$. This can be determined according to Ford–Fulkerson algorithm [6,7].

The importance of direction $\alpha_i$ is either set by organization, which uses the infocommunication network, or defined by experts, based on e.g. analytic hierarchy process (AHP) [8].

Note that equation (1) does not consider conflict situations, which can take place in case of using one communication channel in different data transmission directions and a necessity to transfer different data via the same channel at the same time.

Therefore, in practice, the network weighted channel capacity rate will be slightly lower. For its evaluation, a simulation algorithm based on the principles of information transfer in infocommunication networks with datagram switching is proposed [9].

It is assumed that the number of packets in the data sending nodes $b_i$ is unlimited.

When a packet is in the selection state, it is sent on the least loaded communication channel. Loading is determined by the following expression:

$$ r_k = \frac{1}{\sum_{\tilde{m}} m_{\tilde{m}}} $$

(3)

$m_{k_i}$ - the number of packets of direction $n_i$, that are in the queue to be sent on the communication channel $e_k$.

Every time when packets of different directions arrive at the input of a communication channel, they are placed in a queue for sending in proportion to the importance of direction $\alpha_i$.

The channel capacity of data transmission directions should be considered as an increase in the number of packets of a given direction at the data recipient nodes $e_i$ after stabilization of the process of passing them through the network.

3. The numerical example of evaluation of the network capacity rate of data transmission direction

Let there be a network segment with specified channels capacity rate (figure 1).

There are two directions on this segment: $n_1$ (1-6), which has an importance of 0.25 and $n_2$ (2-6), which has an importance of 0.75 (packets transmitted on directions are designated respectively • and...
When using the Ford-Fulkerson algorithm to evaluate the capacity rate of data transmission direction, capacity rate of the first direction $p_1 = 4$ and capacity rate of the second direction $p_2 = 6$.

$\alpha_1 = 0.25 \quad \alpha_2 = 0.75$

In the first step, the communication channels that have the highest capacity rate are filled (figure 2), since they are not yet loaded.

In the second step and subsequent steps, when determining the communication channel for packet transmission, emerging queues and importance of directional are taken into account (in this example, only for node 4) (figure 3).

$\alpha_1 = 0.25 \quad \alpha_2 = 0.75$
The established data transmission process becomes obvious already at the third step (figure 4), when the number of incoming packets to the receiving node 6 becomes steadily equal to 2 for directional $n_1$ and 5 for directional $n_2$.

These values are an evaluation of the capacity rate of selected data transmission directions, that has become more accurate as a result of taking into account conflicts that occur when transmitting packets in a network.

![Figure 4. The third and fourth steps of the simulation algorithm.](image)

**4. Formalization of infocommunication network modification options**

In order to develop an optimization model of choosing network modification options it is necessary to describe separate options. For this purpose, let us introduce the following notation:

- $\delta_{ks}$ – $s$-number modification option for a communication channel $e_k$;
- $\Delta_k = \{\delta_{k1}, \delta_{k2}, \ldots, \delta_{kl}\}$ – set of all network modification options for a communication channel $e_k$;
- $c(\delta_{ks})$ – financial costs of communication channel $e_k$ modification based on option $\delta_{ks}$;
- $\Theta = \{\theta_1, \theta_2, \ldots, \theta_{|[\Theta]|}\}$ – all infocommunication network modification options.

Each option can be, in turn, characterized by the following:

$$\theta_j = \{\delta_{l_{j1}}, \ldots, \delta_{l_{j[|\Theta|]}}\}. \quad (4)$$

For each network modification option $\theta_j$ it is possible to define:

$$S_j = \sum_{k=1}^{[\Theta]} c(\delta_{k{l_{j1}}}) \quad (5)$$

– financial costs of its implementation;
\[ p_i(\theta_j) = p_i(\delta_{i1},...,\delta_{iM}) \]  
(6)

– network capacity of \( i \) data transmission direction after implementation;

\[ P_j = \sum_{i=1}^{N} \alpha_i p_i(\theta_j) \]  
(7)

– network weighted channel capacity rate of all directions.

5. Optimization models of choosing network modification options

In order to describe the models of choosing an optimal modification option let us introduce the following variable

\[ x_j = \begin{cases} 1, & \text{if modification is made based on } \theta_j; \\ 0, & \text{otherwise.} \end{cases} \]  
(8)

Then for every network modification option \( \theta_j \) there defined:

\[ \sum_{j=1}^{M} S_j x_j = \sum_{j=1}^{M} x_j \sum_{i=1}^{N} c(\delta_{ix_j}) \]  
(9)

– financial costs of its implementation;

\[ \sum_{j=1}^{M} p_i(\theta_j) x_j \]  
(10)

– channel capacity rate of \( i \) priority data transmission direction after implementation;

\[ \sum_{j=1}^{M} P_j x_j = \sum_{i=1}^{N} \alpha_i \sum_{j=1}^{M} p_i(\theta_j) x_j \]  
(11)

– network weighted channel capacity rate.

The objectives of network modification may be the following:

1) maximization of a weighted capacity rate under set restrictions on achieving the desired capacity rate for selected priority data transmission directions and modification costs restrictions;

2) maximization of a capacity rate for selected data transmission directions under set capacity restrictions for selected priority data transmission directions, a network (network segment) weighted capacity rate and modification costs restrictions;

3) minimization of modification costs under set capacity restrictions for selected priority data transmission directions and a network (network segment) weighted capacity rate.

Some of restrictions in objectives 1), 2) and 3) may be missing.

Assume that in order to find an optimal infocommunication network modification option it is possible to set following restrictions:

- \( \hat{S} \) – maximum allowed modification costs;
- \( \hat{N} \) – minimum allowed value of a network weighted capacity rate;
- \( \hat{p}_1, \hat{p}_2, ..., \hat{p}_{|M|} \) – minimum allowed network capacity rates for priority data transmission directions.

The following optimization models of the variable \( X^* = (x_1, ..., x_{|M|}) \) provide achievement of the given objectives:

1) find
\[ X^* = \text{Arg} \max \sum_{i=1}^{[k]} \alpha \sum_{j=1}^{[n]} p_i(\theta_j) x_j \]  

(12)

under the following restrictions:

\[ \sum_{j=1}^{[k]} x \sum_{k=1}^{[n]} c(\delta_{k,x_j}) \leq \hat{S}; \]  

(13)

\[ \sum_{j=1}^{[k]} p_i(\theta_j) x_j \geq \hat{p}_i, i = 1, 2, \ldots, [N]; \]  

(14)

\[ \sum_{j=1}^{[k]} x_j = 1, x_j \in \{0, 1\}; \]  

(15)

2) find

\[ X^* = \text{Arg} \max \sum_{j=1}^{[k]} p_{\theta_j} x_j \]  

(16)

under the following restrictions:

\[ \sum_{j=1}^{[k]} x \sum_{k=1}^{[n]} c(\delta_{k,x_j}) \leq \hat{S}; \]  

(17)

\[ \sum_{j=1}^{[k]} x \sum_{i=1}^{[n]} \alpha_i p_i(\theta_j) \geq \hat{P}; \]  

(18)

\[ \sum_{j=1}^{[k]} p_i(\theta_j) x_j \geq \hat{p}_i, i = 1, 2, \ldots, [N]; \]  

(19)

\[ \sum_{j=1}^{[k]} x_j = 1, x_j \in \{0, 1\}; \]  

(20)

3) find

\[ X^* = \text{Arg} \min \sum_{j=1}^{[k]} x_j \sum_{k=1}^{[n]} c(\delta_{k,x_j}) \]  

(21)

under the following restrictions:

\[ \sum_{j=1}^{[k]} x \sum_{i=1}^{[n]} \alpha_i p_i(\theta_j) \geq \hat{P}; \]  

(22)

\[ \sum_{j=1}^{[k]} p_i(\theta_j) x_j \geq \hat{p}_i, i = 1, 2, \ldots, [N]; \]  

(23)

\[ \sum_{j=1}^{[k]} x_j = 1, x_j \in \{0, 1\}. \]  

(24)

The models (12)-(15), (16)-(20) and (21)-(24) are the models of the Boolean linear programming [10]. If the obtained models have a higher dimension, then it is advisable to find the optimal solution using the branch and bound scheme [11]. An advantage of methods, which use the given scheme, is a
possibility to receive quick approximate solution, which can be improved later. If there is not enough
time to solve the task, then it is possible to confine to the current approximate solution.

The options of using the branch and bound scheme differ in methods for building a partial solutions
tree, estimating partial solutions and traversing a partial solution tree.

Let us describe it's in more detail.

6. Method for building a partial solution tree
Partial solutions in this case are a set of infocommunication network modification options. A tree of
partial solutions is built as follows:
- a tree vertex $\Theta$ is a set of modification options;
- descendants of a vertex $\Theta_{i_1 \rightarrow \cdots \rightarrow i_s}$ are the vertices $\Theta_{i_1 \rightarrow \cdots \rightarrow i_{s+k}}$, built for all the possible sets $\delta_{1+k} = \theta_j$.

Note that the external vertices $\Theta_{i_1 \rightarrow \cdots \rightarrow i_s} = \theta_j$ have only one modification option.

7. Method for evaluating partial solutions
According to the general principle of using the branch and bound scheme:
- if minimization of parameter is carried out then the evaluations of partial solutions must be
  low and increase monotonically while descending a tree of partial solutions;
- if maximization of parameter is carried out then the evaluations of partial solutions must be
  high and decrease monotonically while descending a tree of partial solutions.

Let us define the following:
- $c_{\min}(\Delta_k) = \min\{c(\delta_{i_k})\}$ – minimum modification costs for data channel $e_k$;
- $p_{i, \max}(\Delta_k) = \max\{p_i(\delta_{i_k})\}$ – maximum network capacity rate of $i$ data transmission direction
  after data channel modification $e_k$;
- $\delta_{k, \max}$ – modification option provided maximum capacity after data channel modification $e_k$;
- $\Theta_{i_1 \rightarrow \cdots \rightarrow i_s}$ – set of modification options, which contain selected modifications options of certain
data channels $\delta_{1+k} = \theta_j$.

Note that if $k = |E|$, then $\Theta_{i_1 \rightarrow \cdots \rightarrow i_s} = \theta_j$, i.e. this set contains only one infocommunication network
modernization option.

Since in the task (21)-(24) when choosing an infocommunication network modernization option the
minimization of its costs is carried out, then the following function is used for evaluating modification
costs of partial solutions:

\[ S_{i_1 \rightarrow \cdots \rightarrow i_k} = \sum_{j=1}^{k} c(\delta_{i_j}) + \sum_{j=k+1}^{|E|} c_{\max}(\Delta_j). \]  \hfill (25)

Monotonicity of the corresponding function changes which appears while descending a tree of partial solutions can be seen directly from its form. The higher capacity of a communication channel, the higher network channel capacity of any data transmission direction.

Considering that while choosing infocommunication network modernization option in the task (12)-(15) the maximization of weight channel capacity is carried out, we will use the following function as evaluation of weighted capacity rate:

\[ p_{i_1 \rightarrow \cdots \rightarrow i_k} = \sum_{j=1}^{k} \alpha_j p_i(\delta_{i_1}, \ldots, \delta_{i_k}, \delta_{k+1, \max}, \ldots, \delta_{|E|, \max}), \]  \hfill (26)

and for selected $i_0$ direction –

\[ p_{i_0}(\delta_{i_1}, \ldots, \delta_{i_k}, \delta_{k+1, \max}, \ldots, \delta_{|E|, \max}). \]  \hfill (27)
8. Method for evaluating partial solutions

A process of traversing a partial solution tree consists of two steps:
- finding the first solution by descending down a tree of partial solutions;
- optimizing the first solution by limited traversing the vertices of partial solutions tree.

Let us describe these steps in more detail.

The first solution can be found based on algorithm, which uses method of local search (i.e. so called greedy algorithm [10,12]). We will describe it to a certain step.

Let us consider the vertex $\Theta_{n_1,\ldots,n_k}$. We determine all the possible options $\delta_{k+1,\ldots,k+|\mathcal{F}_{k+1}|}$ of modification $e_{k+1}$. Among them we chose a set $\Delta'_{k+1}$, which meets the following conditions:

\begin{align}
S(\delta_{k,n_1},\ldots,\delta_{k,n_s},\delta_{k+1,n_{s+1}}) & \leq \hat{S}, \\
P(\delta_{k,n_1},\ldots,\delta_{k,n_s},\delta_{k+1,n_{s+1}}) & \geq \hat{P}, \\
p(\delta_{k,n_1},\ldots,\delta_{k,n_s}) & \leq \hat{p}.
\end{align}

If costs minimization is carried out, then in $\Delta'_{k+1}$ here is such option $\delta_{k+1,s}$, that

\begin{equation}
t = \text{Arg min } S(\delta_{k+1,s}) \\
\quad s = 1,\ldots,|\Delta'_{k+1}|.
\end{equation}

The corresponding vertex is taken as follow-up branching of a tree of partial solutions.

If maximization of weighted channel capacity is carried out, then in $\Delta'_{k+1}$ there is such modification option $\delta_{k+1,s}$, that

\begin{equation}
t = \text{Arg max } P(\delta_{k+1,s}) \\
\quad s = 1,\ldots,|\Delta'_{k+1}|.
\end{equation}

and for a selected $i_0$ direction –

\begin{equation}
t = \text{Arg max } p_i(\delta_{k+1,s}) \\
\quad s = 1,\ldots,|\Delta'_{k+1}|.
\end{equation}

Similarly, the corresponding to this option vertex is taken as follow-up branching of a tree of partial solutions. Unlike classic branch and bound scheme, the optimizing indicator is also checked according to set restrictions. It reduces number of vertices searches in a tree of partial solutions. But at the same time it may lead to the situation when a solution will not be found. This means that the formulation of the optimization problem was not performed correctly.

Let us proceed to the second step of traversing a partial solution tree. The search within this step excludes considered earlier options.

If a new solution (which is better than previous) is found within any step of the method, it is called a record one. And all the following comparisons are made with it. If within any step there are no more options of descending down the tree of partial solutions, then the rise on one level up the tree of partial solutions is carried out.

9. Conclusion

Thus, the article presents models and an algorithm of synthesizing the optimal structure of an infocommunication network corresponding to the specified parameters, based on a priori given modifications options of infocommunication network of an organization.

One of the parameters of the models is the capacity rate of selected data transmission directions, which can use the same communication channels, that leads to conflicts in the transmission of
information in the network. Therefore, in the article proposed a simulation algorithm of evaluation of the capacity rate of selected data transmission directions, in which emerging conflicts are modeled based on the importance of directions.

The models of optimization of choice of the option of network modernization and its operating conditions are the models of the Boolean linear programming. The proposed algorithm for finding the optimal choice is based on the use of the branch and bound scheme, with the realization of which it is possible to quickly get the initial solution. If the size of the problem is large and there is not enough time to solve the problem, then we can limit ourselves to finding the current approximate solution. The solution can be later improved by traversing the partial solution tree.

Choice of optimization method for choosing infocommunication network modification options depends greatly on the task dimension. Performance indicators of an infocommunication network for which it is advisable to use the given method can be determined during a computing experiment.

The obtained models make it possible to increase the efficiency of choosing a solution how to modify or synthesize the structure of the infocommunication network of organization.

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