Bragg scattering of Cooper pairs in an ultra-cold Fermi gas

K. J. Challis, R. J. Ballagh, and C. W. Gardiner
Jack Dodd Centre for Photonics and Ultra-Cold Atoms
Department of Physics, University of Otago, P.O. Box 56, Dunedin, New Zealand
(Dated: October 10, 2018)

We present a theoretical treatment of Bragg scattering of a degenerate Fermi gas in the weakly interacting BCS regime. Our numerical calculations predict correlated scattering of Cooper pairs into a spherical shell in momentum space. The scattered shell of correlated atoms is centered at half the usual Bragg momentum transfer, and can be clearly distinguished from atoms scattered by the usual single-particle Bragg mechanism. We develop an analytic model that explains key features of the correlated-pair Bragg scattering, and determine the dependence of this scattering on the initial pair correlations in the gas.

PACS numbers: 03.75.Ss, 32.80.Cy

Bragg scattering provides a high precision spectroscopic technique that has been adapted from materials science to probe Bose-Einstein condensates [1, 2]. In condensed systems, signatures of soliton evolution [3], phase fluctuations [4], centre-of-mass motion [5], and vortex structure [6], are accessible due to the velocity selectivity of Bragg spectroscopy. It has been proposed [7, 8, 9, 10, 11] that Bragg spectroscopy of an ultra-cold Fermi gas can provide insight into the Cooper paired cold Fermi gas can provide insight into the Cooper paired cold Fermi gas can provide insight into the Cooper paired cold Fermi gas can provide insight into the Cooper paired cold Fermi gas can provide insight into the Cooper paired cold Fermi gas can provide insight into the Cooper paired cold Fermi gas can provide insight into the Cooper paired cold Fermi gas can provide insight into the Cooper paired cold Fermi gas can provide insight into the Cooper paired cold Fermi gas can provide insight into the Cooper paired cold Fermi gas can provide insight into the Cooper paired cold Fermi gas can provide insight into the Cooper paired cold Fermi gas can provide insight into the Cooper paired cold Fermi ga...
C is a constant chosen such that preserving Bragg field.

are scattered identically because of our choice of a spin $L$ defined by the number density at momentum $\hbar k$. Parameters are zero temperature. The momentum distribution of the initial cloud, centered at $k_0$, is given by the modified Fermi wave vector

\[
\mathcal{F}_{n/2}(k, t) = \langle \hat{\phi}_n(k, t) \hat{\phi}_n(k, t) \rangle = \int \frac{d^3r}{(L)^3/2} \langle \hat{\phi}_n(k, t) \hat{\phi}_n(k, t) \rangle,
\]

where $\phi_n(k, t) = \int \frac{d^3r}{L^3} \exp(-i k \cdot r) \psi_n(r, t)$ and $L$ is the computational volume. The two spin states are scattered identically because of our choice of a spin preserving Bragg field.

The initial cloud, centered at $k_0$, has an approximate width given by the modified Fermi wave vector $k_F'$, defined by $h^2 k_F'^2 / 2M = E_F - U(0)$ (e.g., 18). In the presence of the Bragg field the atoms scatter by two different mechanisms: (i) single-particle scattering, and (ii) correlated-pair scattering.

In single-particle Bragg scattering an atom receives a momentum kick of $n \hbar q$ and energy $n \hbar \omega$, where $n$ is an integer. A resonance condition selects primarily one value of $n$, and we consider $\omega$ in the range of first order Bragg scattering (e.g., 19), where $n = 1$ is dominant. This results in a cloud of atoms centered at $k = q$, as observed in Fig. 2(a)-(d). A faintly visible cloud at $k = -q$ is also observed in Fig. 2(a)-(d) due to off resonant scattering into the $n = -1$ order.

Figure 2(a)-(d) also shows scattering of atoms into a spherical shell centered at $k = q/2$. We refer to the mechanism responsible for the atom shell as correlated-pair Bragg scattering. Correlated-pair scattering has a frequency threshold denoted $\omega_{\text{thres}}$, at which atoms are scattered to $k = q/2$ [see Fig. 2(a)]. Above threshold [see Fig. 2(b)-(d)], the atoms are scattered into a spherical shell and the shell radius increases with Bragg frequency. Atoms scattered into the shell come primarily from the Fermi surface, and are correlated about the Bragg momentum.

We determine the Bragg spectrum of the degenerate Fermi gas by calculating the momentum transfer per atom along the Bragg axis, $P(t) = \int [\hbar \mathbf{k} \cdot \mathbf{q}] n(k, t) d^3k$, for a range of Bragg frequencies. The Bragg spectrum at zero temperature is given in Fig. 2(a), and is dominated by the broad single-particle resonance familiar from Bragg scattering of a Bose-Einstein condensate (e.g., 19). The single-particle Bragg resonance is due to two-photon scattering events that scatter atoms by the Bragg momentum $\hbar q$. The resonance condition is well approximated using energy conservation arguments for non-interacting particles, and by considering an atom scattered from momentum $\hbar k_R$ to $\hbar (k_R + q)$ we obtain the resonance condition

\[
\omega_{\text{sp}} = \frac{\hbar}{2M} \left( q^2 + 2 \mathbf{k}_R \cdot \mathbf{q} \right).
\]

The single-particle resonance [see Fig. 2(a)] is centered at $\omega = \hbar q^2 / 2M$, and its width is $\delta \omega \approx 2 \hbar q k_F' / M + A/\hbar$, where the first term accounts for the momentum width of the initial cloud, and the second term is due to power broadening (e.g., 19).

Correlated-pair Bragg scattering occurs on the red-detuned side of the single-particle resonance, leading to a slight asymmetry in the Bragg spectrum [see Fig. 2(a)]. The correlated scattering, which gives rise to the distinctive spherical shell of atoms in momentum space [see Fig. 2(a)-(d)], depends critically on the presence of Cooper pairs, and can not be understood by the usual single-particle scattering mechanism. Correlated-pair scattering is associated with the formation of a mov-
ing grating in the pair potential, which is well approximated by $\Delta(r, t) \approx \Delta_0(t) + \Delta_1(t) \exp[i(q \cdot r - \omega t)]$. The pair potential coefficients $\Delta_0$ and $\Delta_1$, after the Bragg pulse, are shown in Fig. 2(b), where we observe that the homogeneous term $\Delta_0$ is depleted in the region of the single-particle Bragg resonance, while the moving grating amplitude $\Delta_1$, is created over a slightly more extended region. Cooper pairs (of zero centre-of-mass momentum) scatter from the moving grating in the pair potential to centre-of-mass momentum $h\mathbf{q}$. At threshold, [see Fig. 2(a)] the pair potential grating provides just sufficient energy to scatter each atom of a pair to a final momentum $h\mathbf{q}/2$. Above threshold [see Fig. 2(b)-(d)], excess energy provided by the pair potential grating is distributed equally between the two atoms of a pair, and the individual momenta of the atoms become $h(q/2 \pm k_{rel})$. A resonance condition for correlated-pair Bragg scattering can be obtained from energy conservation arguments to be

$$\omega_{\text{pair}} = \frac{\hbar}{M} \left( \frac{q^2}{4} + k_{\text{rel}}^2 \right) - \frac{\hbar k_{\text{P}}^2}{2M},$$

which for $k_{\text{rel}} = 0$ gives the threshold frequency $\omega_{\text{thres}} = h\mathbf{q}^2/4M - \hbar k_{\text{P}}^2/2M$. Above threshold, the additional kinetic energy $\hbar^2 k_{\text{rel}}^2/2M$ carried by each atom of a scattered pair is given by $\omega - \omega_{\text{thres}} = \hbar k_{\text{P}}^2/M$, as confirmed by our numerical calculations.

We can understand some important features of Bragg scattering of correlated pairs with an analytic treatment. Due to the periodicity of the Bragg field, the solutions of Eq. 2 have the Bloch form, and can be expanded as

$$u_k(r, t) = e^{ik \cdot r} \sum_n a_n^k(t) e^{i(nq \cdot r - \omega t)}$$

$$v_k(r, t) = e^{ik \cdot r} \sum_n b_n^k(t) e^{i(nq \cdot r - \omega t)},$$

where $n$ is an integer. The self-consistent potentials are periodic, with the translational symmetry of the Bragg field, i.e.,

$$U(r, t) = \sum_n U_n(t) e^{i(nq \cdot r - \omega t)}$$

$$\Delta(r, t) = \sum_n \Delta_n(t) e^{i(nq \cdot r - \omega t)}.$$

Evolution equations for the coefficients $a_n^k(t)$ and $b_n^k(t)$ can be derived from Eq. 2 to be

$$i\hbar \frac{da_n^k(t)}{dt} = \hbar \omega_n^k(k) a_n^k(t) + \frac{A}{4} \left[ a_n^k(t) + a_{-n}^k(t) \right]$$

$$+ \sum_m U_m(t) a_n^k(t) + \sum_m \Delta_m(t) b_n^k(t),$$

and

$$i\hbar \frac{db_n^k(t)}{dt} = \hbar \omega_n^k(k) b_n^k(t) - \frac{A}{4} \left[ b_{n+1}^k(t) + b_{n-1}^k(t) \right]$$

$$- \sum_m U_m(t) b_n^k(t) + \sum_m \Delta_m(t) a_n^k(t),$$

where $\hbar \omega_n^k(k) = \pm \left[ \hbar^2 (k^2 + nq)^2/2M - E_P \right] - n\hbar \omega$. Initially the only non-zero coefficients are $a_0^k$ (for $|k| \gtrsim k_{\text{P}}$) and $b_0^k$ (for $|k| \lesssim k_{\text{P}}$). The mean-field coefficients $U_m$ and $\Delta_m$ must be obtained self-consistently, in particular

$$\Delta_m(t) = -\bar{V} \sum_k a_n^k(t) b_{n-m}^k,$$

with $\bar{V} = T/(1-\alpha T)$, where $T$ is the low energy $T$-matrix and $\alpha = M k_{\text{F}}/2\pi^2 \hbar^2$.

First order correlated-pair Bragg scattering is mediated by a moving grating in the pair potential, arising due to terms $m = 1, n = 0, 1$ in Eq. 10 becoming nonzero. Those terms are seeded by single-particle Bragg scattering events in which an atom on the Fermi surface is scattered by momentum $h\mathbf{q}$. It can be shown, from Eqs. 3 and 4, that single-particle transitions generate coefficients $a_0^k$ (for $|k| \gtrsim k_{\text{P}}$) and $b_0^k$ (for $|k| \lesssim k_{\text{P}}$). In the region of the Fermi surface, $|k| \approx k_{\text{F}}$. $\Delta_1$ is generated due to the formation of contributions $a_0^k b_{0}^k$ and $a_0^k b_{-1}^k$. Physically this corresponds to seeding pair correlations about $k = q/2$. When an atom with momentum...
\( \hbar \mathbf{R} \approx \hbar \mathbf{k}_R \) is scattered by a single-particle transition to \( \hbar (\mathbf{k}_R + \mathbf{q}) \), it remains correlated with its unscattered pair at momentum \(-\hbar \mathbf{k}_R\). This reduces the system pairing about \( \mathbf{k} = 0 \) (\( \Delta_0 \)), and generates a pair correlation about \( \mathbf{k} = \mathbf{q}/2 \) (\( \Delta_1 \)) [see Fig. 4(b)]. This correlation seeding can also be observed in \( \phi_{a,b}(\mathbf{k},t) \) [see Fig. 4(e)-(h)], where a point on the left most circle represents correlation between an atom on the Fermi surface of the initial cloud and its pair which has been scattered by \( \mathbf{q} \).

Following the initiation of the pair potential grating, its subsequent development can be understood in terms of a truncated version of Eqs. 3 and 4, i.e.,

\[
i \hbar \frac{d}{dt} \begin{bmatrix} a^k_n(t) \\ b^{-k}_{-1}(t) \end{bmatrix} = \begin{bmatrix} \epsilon^k_0(\mathbf{k}) & \Delta_1(t) \\ \Delta_1(t) & \epsilon^k_{-1}(\mathbf{k}) \end{bmatrix} \begin{bmatrix} a^k(t) \\ b^{-k}_{-1}(t) \end{bmatrix},
\]

(11)

which is appropriate for describing the scattered pairs. In Eq. (11), \( \epsilon^k_a(\mathbf{k}) = \hbar \omega^a(\mathbf{k}) \pm U_0 \), and \( \Delta_1(t) = -V \sum_k a^k_0(t) b^{-k}_{-1}(t) \). The term \( a^k_0 b^{-k}_{-1} \) only becomes significant if \( \epsilon = \epsilon^k_0(\mathbf{k}) - \epsilon^k_{-1}(\mathbf{k}) \approx 0 \), i.e., when correlated scattering transitions conserve energy (\( \omega \geq \omega_{\text{bragg}} \)). At threshold, the summands in \( \Delta_1 \) have a stationary phase, leading to enhancement of the grating amplitude \( \Delta_1 \) [see Fig. 4(b)]. The thickness \( \delta k \) of the spherical shell of scattered pairs can be estimated by assuming a frequency width \( \Gamma \), determined by the Bragg pulse length (\( \Gamma \approx \pi/\rho \)), and setting \( \delta k = \hbar \Gamma \), to find that

\[
\delta k \approx \sqrt{\frac{\pi M}{\hbar \rho} + k^2_{\text{rel}} - k_{\text{rel}}}. 
\]

(12)

We have investigated the dependence of correlated-pair Bragg scattering on a range of system parameters. In Fig. 1(b), \( \sim 0.2\% \) of the atoms are scattered by correlated-pair Bragg scattering, and the number of scattered pairs grows linearly with the length of the Bragg pulse (until \( t \sim 70/\omega \)). Over the range \( -0.18 \geq k_F a \geq -0.69 \), the number of pairs scattered increases quadratically with \( \Delta(0) \) indicating the coherent nature of the scattering process. For \( k_F a = -0.689 \) [all other parameters as per Fig. 1(b)] there \( \sim 6\% \) scattered pairs. However, we emphasize that for \( k_F a \gtrsim 1 \) the mean-field approach may not be quantitatively accurate (e.g., 11 and 12). The number of correlated pairs scattered can be further increased by enhancing the single-particle scattering processes that seed the pair potential grating, either by increasing the Bragg field strength \( A \), or by reducing the Bragg wave vector \( \mathbf{q} \) (to make the seeding more resonant).

In conclusion, we have calculated solutions of the time-dependent Bogoliubov de Gennes equations for a zero temperature homogeneous three-dimensional Bragg scattered Fermi gas, in the regime where the momentum transfer is well outside the Fermi surface. We predict Bragg scattering of correlated atom pairs, which has a distinctive signature in momentum space, namely a spherical shell of atoms centered at half the usual Bragg momentum transfer. Correlated-pair Bragg scattering occurs via a Bragg grating formed in the pair potential, and has a well defined frequency threshold on the red-detuned side of the familiar single-particle Bragg resonance. We have developed an analytic model that explains the mechanism by which the pair potential grating is generated, and observe that the number of scattered pairs is proportional to the square of the initial pairing field.

This work was supported by Marsden Fund UOO0509 and the Tertiary Education Commision (TAD 884).

* Electronic address: kchallis@physics.otago.ac.nz

[1] M. Kozuma, L. Deng, E. W. Hagley, J. Wen, R. Lutwak, K. Helmerson, S. L. Rolston, and W. D. Phillips, Phys. Rev. Lett. 82, 871 (1999).
[2] D. M. Stamper-Kurn, A. P. Chikkatur, A. Görlitz, S. Inouye, S. Gupta, D. E. Pritchard, and W. Ketterle, Phys. Rev. Lett. 83, 2876 (1999).
[3] K. Bongs, S. Burger, D. Hellweg, M. Kotthe, S. Dettmer, T. Rinkleff, L. Cacciapuoti, J. Arlt, K. Sengstock, and W. Ertmer, J. Opt. B 5, S124 (2003).
[4] S. Richard, F. Gerbier, J. H. Thywissen, M. Hugbart, P. Bouyer, and A. Aspect, Phys. Rev. Lett. 91, 010405 (2003).
[5] R. Geursen, N. R. Thomas, and A. C. Wilson, Phys. Rev. A 68, 043611 (2003).
[6] S. R. Muniz, D. S. Naik, and C. Raman, Phys. Rev. A 73, 041605(R) (2006).
[7] J. Ruostekoski, Phys. Rev. A 61, 033605 (2000).
[8] M. Rodriguez and P. Törmä, Phys. Rev. A 66, 033601 (2002).
[9] H. P. Büchler, P. Zoller, and W. Zwerger, Phys. Rev. Lett. 93, 080401 (2004).
[10] Bimalendu Deb, J. Phys. B 39, 529 (2006).
[11] R. Combescot, S. Giorgini, and S. Stringari, Europhys. Lett. 75, 695 (2006).
[12] P. B. Blakie, R. J. Ballagh, and C. W. Gardiner, Phys. Rev. A 65, 033602 (2002).
[13] P. G. de Gennes, Superconductivity of metals and alloys (W. A. Benjamin, Inc., New York, 1966).
[14] J. B. Ketterson and S. N. Song, Superconductivity (Cambridge University Press, Cambridge, 1999).
[15] K. J. Challis, R. J. Ballagh, and C. W. Gardiner, preparation.
[16] M. L. Chiofalo, S. J. M. F. Kokkelmans, J. N. Milstein, and M. J. Holland, Phys. Rev. Lett. 88, 090402 (2002).
[17] R. J. Ballagh, Computational methods for nonlinear partial differential equations, http://www.physics.otago.ac.nz (2000).
[18] N. Nygaard, G. M. Bruun, C. W. Clark, and D. L. Feder, Phys. Rev. Lett. 90, 210402 (2003).
[19] P. B. Blakie and R. J. Ballagh, J. Phys. B 33, 3961 (2000).
[20] Jan R. Engelbrecht, Mohit Randeria, and C. A. R. Sá de Melo, Phys. Rev. B 55, 15153 (1997).
[21] The maximum possible correlation for a pair of atoms with momenta \( \pm \mathbf{k} \) is \( \phi_{0}(\mathbf{k},t) = 1/2 \).