IMPACT AND IMPLICATION OF BI-LARGE NEUTRINO MIXINGS ON GUTS∗

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Under the assumptions that 1) the quark/lepton mass matrices take Froggatt-Nielsen’s factorized power form $\lambda^{\psi_i + \psi_j}$ with anomalous $U(1)$ charges $\psi_i$, and 2) the $U(1)$ charges $\psi_i$ respect the $SU(5)$ GUT structure, we show that the quark mass data necessarily implies the large 2-3 mixing in the MNS mixing matrix $U_{\text{MNS}}$. If we further add the data of the mass squared difference ratio of solar and atmospheric neutrinos, then, it implies that the 1-2 mixing in $U_{\text{MNS}}$ is also large, so explaining the bi-large mixing. This analysis also gives a prediction that $U_{e3} \equiv (U_{\text{MNS}})_{13}$ should be of order $\lambda \sim (0.1 - 0.5)$.

1. Introduction

Existence of a certain grand unified theory (GUT) beyond the standard model is guaranteed by i) the anomaly cancellation between quarks and leptons and ii) the unification of the gauge coupling constants at energy scale around $\mu \sim 10^{15-16}$ GeV. The strongest candidate for the unified gauge group is $E_6$, which is not only suggested by string theory but also unique in the property that it is the maximal safe simple group allowing complex representations in the $E$-series; $E_3 = SU(3) \times SU(2)$, $E_4 = SU(5)$, $E_5 = SO(10)$, $E_6$, $E_7$, $E_8$.

The purpose of this talk is to analyze the implications of the neutrino data on the possible GUTs. This is based on a work2 in collaboration with Masako Bando. The particular facts of the neutrino data are3,4,5,6

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(1) Bi-large mixing
\[ \sin^2 2\theta_{12} \sim (0.86 - 1.0), \quad \sin^2 2\theta_{23} \sim 1. \]  

(2) Mass-squared difference ratio
\[ \frac{\Delta m^2_{\odot}}{\Delta m^2_{\text{atm}}} \sim \frac{7 \times 10^{-5} \text{eV}^2}{3 \times 10^{-3} \text{eV}^2} \sim \lambda^2 . \]  

where \( \lambda \) defined below is a quantity of magnitude \( \lambda \sim 0.22 \).

These show a sharp contrast to the quark sector, in which the mixings are very small and the mass spectrum is hierarchical. The mutual relations of masses and mixing angles between quarks and leptons/neutrinos will be great clues for the GUTs.

As an working hypothesis we here assume an supersymmetric SU(5) GUT and the Froggatt-Nielsen mechanism\(^7\) to generate effective Yukawa coupling matrices of the form
\[ y_{\Psi_i \Psi_j H} \left( \frac{\Theta}{M_{\text{Pl}}} \right)^{\psi_i + \psi_j + h}, \]  

where the Yukawa couplings \( y \) can in principle depend on the generation label \( i, j \) but are assumed to be all order 1 and so are denoted by \( y \) collectively. \( \Theta \) is the Froggatt-Nielsen field carrying the \( U(1)_X \) charge \(-1\) and the \( U(1)_X \) charges of the other Higgs field \( H \) and matter fields \( \Psi_i \) \( (i = 1, 2, 3) \) are denoted by the corresponding lower-case letters:
\[ X(\Theta) = -1, \quad X(H) = h, \quad X(\Psi_i) = \psi_i (\geq 0). \]

After the Froggatt-Nielsen field \( \Theta \) develops a vacuum expectation value (VEV) \( \langle \Theta \rangle \), which is assumed to be smaller than the Planck scale by a factor of Cabibbo angle \( \theta_C \)
\[ \frac{\langle \Theta \rangle}{M_{\text{Pl}}} \equiv \lambda \sim 0.22 \simeq \sin \theta_C, \]  

the effective Yukawa couplings induced from Eq. (3) are given by
\[ y_{ij}^{\text{eff}} = y \times \lambda^{\psi_i + \psi_j + h} = O(1) \times \lambda^{\psi_i + \psi_j + h}. \]

That is, the mass matrix \( M \) takes the form
\[ M = yv\lambda^h \times \begin{pmatrix} i \hfill \sum_j \end{pmatrix} \begin{pmatrix} \lambda^{\psi_i + \psi_j} \end{pmatrix} \]
with $\langle H \rangle = v$. $\psi_R^i$ and $\psi_L^j$ are the $U(1)_X$ charges of the right-handed and left-handed matter fields $\Psi_R^i$ and $\Psi_L^j$, respectively. Thus, in this Froggatt-Nielsen mechanism, the hierarchical mass structure can be explained by the difference of the $U(1)_X$ charges $\psi_R^i$, $\psi_L^j$ of the matter fields. Note that this type of ‘factorized’ mass matrix can be diagonalized as

$$VMU^\dagger = M^{\text{diag}}.$$  

by unitary matrices $V$ and $U$ taking also a similar power forms:

$$U \sim i \left( \lambda |\psi_{R}^i - \psi_{L}^j| \right), \quad V \sim i \left( \lambda |\psi_{R}^i - \psi_{L}^j| \right).$$

2. $U(1)_X$ charge assignment

I assume $SU(5)$ structure at least for the $U(1)_X$ charge assignment. Then, first, we consider the Yukawa coupling responsible for the up-quark sector masses. In order for the effective Yukawa coupling

$$y_u \Psi_i^{(10)} \Psi_j^{(10)} H_u^{(5)} \left( \frac{\Theta}{M_{Pl}} \right)^{\psi_i^{(10)} + \psi_j^{(10)} + h_u} \rightarrow y_{u_{ij}}^{\text{eff}} = y_u \times \lambda^{\psi_i^{(10)} + \psi_j^{(10)} + h_u}$$

(10)

to reproduce the observed up-type quark mass hierarchy structure

$$m_t : m_c : m_u = 1 : \lambda^4 : \lambda^7,$$

(11)

we are led to choose the following values for the $U(1)_X$ charges of three generation $\Psi_i^{(10)}$ fermions taking $h_u = 0$ for simplicity: 

$$(\psi_1^{(10)}, \psi_2^{(10)}, \psi_3^{(10)}) = (3, 2, 0)$$

(12)

Next we consider the mass matrices of down-type quarks and charged leptons which come from the couplings

$$y_d \Psi_i^{(10)} \Psi_j^{(5^*)} H_d^{(5^*)} \left( \frac{\Theta}{M_{Pl}} \right)^{\psi_i^{(10)} + \psi_j^{(5^*)} + h_d} \rightarrow y_{d_{ij}}^{\text{eff}} = y_d \times \lambda^{\psi_i^{(10)} + \psi_j^{(5^*)} + h_d}.$$  

(13)

Note that this yields the transposed relation between the down-type quark mass matrix $M_d$ and the charged lepton one $M_l$: $M_d^T \sim M_l$. This is because the $\Psi_i^{(5^*)}$ multiplets contain the right-handed component $d^c$ for the
down-type quarks while the left-handed component \( l \) for the charged leptons. Therefore the unitary matrices for diagonalizing those mass matrices, satisfy the relations

\[
\begin{align*}
V_d M_d U_d^\dagger &= M_d^{\text{diag}}. \\
V_l M_l U_l^\dagger &= M_l^{\text{diag}}.
\end{align*}
\]

so that we have

\[
U_d^*(M_l \sim M_d^T)U_l^\dagger = \text{diag.}
\]

That is, the mass matrix takes the form

\[
M_d^T \sim M_l \sim y v \lambda h_d \times \begin{pmatrix}
\begin{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
\end{bmatrix}
\end{bmatrix}
\end{bmatrix}
\end{pmatrix}
\end{aligned}
\]

In order for this \( M_d \) to reproduce the mass ratio of the top and bottom quarks

\[
\frac{m_b}{m_t} \sim \lambda^{2-3}
\]

we take \( \psi_3(5^*) = 2 - h_d \). Further, to reproduce the down-type quark mass hierarchy

\[
m_b : m_s : m_d = 1 : \lambda^2 : \lambda^4,
\]

we take \( \psi_2(5^*) = 2 - h_d \) and \( \psi_1(5^*) = 3 - h_d \); thus, we have

\[
(\psi_1(5^*), \psi_2(5^*), \psi_3(5^*)) = (3 - h_d, 2 - h_d, 2 - h_d),
\]

and the mass matrix (16) now reduces to

\[
M_d^T \sim M_l \sim y v \lambda^2 \times \begin{pmatrix}
\begin{bmatrix}
\begin{bmatrix}
\end{bmatrix}
\end{bmatrix}
\end{pmatrix}
\end{aligned}
\]

This form of mass matrix is called lopsided.
3. Mixing matrices

Mixing matrices in the quark sector and lepton sector are called Cabibbo-Kobayashi-Maskawa (CKM) and Maki-Nakagawa-Sakata (MNS) matrices and they are given by

\[ U_{\text{CKM}} = U_u U_d^\dagger, \quad U_{\text{MNS}} = U_l U_\nu^\dagger. \]  

(21)

In our case both \( U_u \) and \( U_d \) takes the form

\[ U_u \sim U_d \sim (\lambda |\psi_i(10)\rangle - |\psi_j(10)\rangle), \]

(10)

so that the CKM matrix, generally, also has the same form

\[ U_{\text{CKM}} = U_u U_d^\dagger \sim (\lambda |\psi_i(10)\rangle - |\psi_j(10)\rangle) \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \]  

(22)

This is all right. For the charged lepton sector we have

\[ U_l \sim (\lambda |\psi_i(5^*)\rangle - |\psi_j(5^*)\rangle) \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}. \]  

(23)

If the mixing matrix \( U_\nu \) in neutrino sector is \( \sim 1 \), this beautifully explains the observed large 2-3 neutrino mixing! However, this alone fails in explaining the large 1-2 mixing. We thus have to discuss the neutrino mixing matrix \( U_\nu \) now.

4. Neutrino mass matrix and mixing

Generally in GUTs, there appear some right-handed neutrinos \( \Psi_I(1) = \nu_R I \) \((I = 1, \cdots, n)\); for instance, \( n = 3 \) in \( SO(10) \) and \( n = 6 \) in \( E_6 \). They will generally get superheavy Majorana masses denoted by an \( n \times n \) mass matrix \( (M_R)_{IJ} \), and also possesses the Dirac masses (R-L transition mass terms)

\[ (M_D^T)_{II} \sim y_\nu v \lambda h_u \times (\lambda |\psi_i(5^*)\rangle + |\psi_i^R\rangle) \]  

(24)

induced from

\[ y_\nu \Psi_i(5^*) \Psi_I(1) H_u(5) \left( \frac{\Theta}{M_{pl}} \right) \psi_i(5^*) + |\psi_i^R\rangle + h_u \quad \rightarrow \quad y_{\nu ij}^{\text{eff}} = y_\nu \times \lambda |\psi_i(5^*)\rangle + |\psi_j^R\rangle + h_u \]  

(25)

Here \( |\psi_i^R\rangle \) denotes the \( U(1)_X \) charges of the right-handed neutrinos \( \Psi_I(1) \).

The Majorana mass matrix \( M_\nu \) of (left-handed) neutrino is induced from these masses \( M_R \) and \( M_D \) by the see-saw mechanism \(^{10}\) and evaluated as

\[ (M_\nu)_{IJ} \sim (M_D^T)_{II} (M_R^{-1})_{IJ} (M_D)_{JJ} \sim \lambda |\psi_i(5^*)\rangle \left( \lambda |\psi_j^R(5^*)\rangle + \lambda |\psi_j^R(5^*)\rangle \right) \]  

(26)
Note here that the dependence on the $U(1)_X$ charges $\psi^R_I$ of the right-handed neutrinos has completely dropped off.\footnote{We should however take it account that this occurs only for a generic case and may be broken in particular cases in which $(M_D^T)_{II}$ brings about correlation between the left-handed neutrino index $i$ and right-handed one $I$.\footnote{8}} Plaguing the values (19) for $\psi(5^*)$, we thus have

$$M_\nu \propto \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}. \quad (27)$$

This neutrino mass matrix happens to take the same form as one of the models that have been proposed by Ling and Ramond.\footnote{11} This form is very interesting.

First, this matrix implies the large 2-3 mixing in the diagonalization matrix $U_\nu$. The 2-3 mixing is also large in the charged lepton mixing matrix $U_l$ as we have seen above, and so is it generally in the MNS matrix $U_{\text{MNS}} = U_l U_\nu^T$ unless a cancellation occurs between $U_l$ and $U_\nu$.

Second, it is natural to assume that three neutrino masses are not so degenerate accidentally. Then, the mass squared difference ratio (2) of solar and atmospheric neutrinos implies the mass ratio of the second and third neutrinos:

$$\frac{m_{\nu_2}}{m_{\nu_3}} \sim \lambda. \quad (28)$$

In order for the $M_{\nu}$ to reproduce this mass ratio, the $2 \times 2$ bottom-right submatrix of this $M_\nu$ should not be naturally-expected order 1, but should be $O(\lambda)$; that is, it is diagonalized by an $2 \times 2$ unitary matrix $u_\nu$ as

$$\det \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \sim O(\lambda^1) \quad \rightarrow \quad u_\nu^* \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} u_\nu \sim \begin{pmatrix} \lambda & 0 \\ 0 & 1 \end{pmatrix}. \quad (29)$$

If this is the case, the mass matrix $M_\nu$ takes the following form after the diagonalization of this $2 \times 2$ bottom-right submatrix:

$$M_\nu \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & u_\nu^* \end{pmatrix} M_\nu \begin{pmatrix} 1 & 0 \\ 0 & u_\nu \end{pmatrix} \sim \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & \lambda & 0 \\ \lambda & 0 & 1 \end{pmatrix}. \quad (30)$$

If we note the $2 \times 2$ top-left submatrix of this matrix

$$\begin{pmatrix} \lambda^2 & \lambda \\ \lambda & \lambda \end{pmatrix}, \quad (31)$$
we see that this also gives the large mixing in the 1-2 sector so that it explains the bi-large mixing.

Therefore, the experimental fact

$$\frac{\Delta m^2_{20}}{\Delta m^2_{atm}} \sim \lambda^{2-3} \quad \Leftrightarrow \quad \frac{m_{\nu 2}}{m_{\nu 3}} \sim \lambda$$

(32)

necessarily implies the bi-large mixing!

We note that a very similar neutrino mass matrix $M_{\nu}$ to ours (27) was also proposed by Maekawa:12

$$M_{\nu} \propto \begin{pmatrix}
\lambda^2 & \lambda^{1.5} & \lambda^1 \\
\lambda^{1.5} & \lambda^1 & \lambda^{0.5} \\
\lambda^1 & \lambda^{0.5} & 1
\end{pmatrix}.$$  

(33)

5. Prediction on $U_{e3}$

We should note that there is one more prediction in our framework, that is, the magnitude of the $U_{e3} \equiv (U_{MNS})_{13}$:

$$U_{e3} \sim O(\lambda^1) \sim \left( \frac{0.5}{\lambda^{0.5}} - \frac{0.1}{\lambda^{1.5}} \right)$$

(34)

This is seen as follows. First, we have

$$(U_l)_{11} \sim O(1), \quad (U_l)_{12} \text{ and } (U_l)_{13} \sim \lambda^{\psi_1(5^*) - \psi_2(5^*)} = \lambda^1,$$

(35)

which have resulted from down-type quark masses and an $SU(5)$ relation. Second, we have for the matrix elements of $U_\nu$,

$$(U_\nu)_{31} \sim \lambda^{\psi_1(5^*) - \psi_3(5^*)} = \lambda^1, \quad (U_\nu)_{32} \text{ and } (U_\nu)_{33} \sim O(1).$$

(36)

These clearly give rise to $U_{e3} \equiv (U_{MNS})_{13} = (U_lU_\nu^\dagger)_{13} \sim O(\lambda)$.

This prediction gives a crucial test for the idea of Froggatt-Nielsen mechanism.

6. Conclusion

I have shown the following points in this paper:

1. If we assume Froggatt-Nielsen’s factorized form for the quark/lepton mass matrices and the $SU(5)$ structure for the $U(1)_X$ charges, an input of up- and down-type quark masses necessarily implies that the 2-3 mixing is large in the MNS matrix $U_{MNS}$.
(2) If we further add the data $\sqrt{\Delta m^2_{\odot}/\Delta m^2_{\text{atm}}} \sim \lambda$, then, it implies that the 1-2 mixing in $U_{\text{MNS}}$ is also large, so leading to bi-large mixing.

(3) The measurement of $U_{e3}$ will confirm or kill the basic idea of Froggatt-Nielsen mechanism for explaining the hierarchical mass structures of quarks and leptons.

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