High-Q resonances supported by a single dielectric ridge on the surface of a slab waveguide

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Abstract. Diffraction of slab waveguide modes on a dielectric ridge located on the waveguide surface is studied. Calculations, based on aperiodic rigorous coupled-wave analysis, demonstrate the existence of sharp resonant features in the reflectance and the transmittance spectra occurring at oblique incidence of TE-polarized waveguide mode on the ridge. Angular widths of the resonant peaks and dips vary from units to less than a thousandth of a degree. Using the effective index method, we explain the resonances by the excitation of cross-polarized modes of the ridge. The existence of high-Q resonances makes this structure promising for filtering, sensing and all-optical signal processing.

1. Introduction
In a wide class of planar (integrated) optoelectronic systems, optical processing is performed in a slab waveguide [1–6]. Such geometry corresponds to the “insulator-on-insulator” platform and is suitable for the creation of fully integrated optical devices. In this case, the processed signal corresponds to a superposition of slab waveguide modes with different propagation directions (in the case of spatial filtering) or with different frequencies (in the case of spectral filtering).

In this work, we show that a very simple structure consisting of a single subwavelength ridge on the surface of a slab waveguide has interesting resonant properties resulting from the excitation of a cross-polarized mode of the ridge. The discovered high-Q resonances supported by the presented planar structure make it promising for various applications including spatial (angular) and spectral filtering, transformation of optical signals and chemical or biosensing.

2. Diffraction of slab waveguide modes on a ridge
Let us study the diffraction of slab waveguide modes on the ridge located on the surface of a slab waveguide (figure 1). For the analysis, the following parameters were chosen: refractive index of the waveguide core layer and of the ridge $n_c = 3.3212$ (GaP at the wavelength $\lambda = 630\,\text{nm}$), refractive indices of the substrate and superstrate $n_{\text{sub}} = 1.45$ (fused silica) and $n_{\text{sup}} = 1$, respectively, waveguide thickness $h_c = 80 \,\text{nm}$, waveguide thickness at the ridge $h_r = 110 \,\text{nm}$. In this case, at $\lambda = 630 \,\text{nm}$ the
waveguide is single-mode both at TE- and TM-polarizations, and the effective refractive indices of the modes amount to $n_{wg,TE} = 2.5913$ and $n_{wg,TM} = 1.6327$, respectively. Effective refractive indices of the modes at $h = 110$ nm equal $n_{r,TE} = 2.8192$ and $n_{r,TM} = 2.1867$.

![Figure 1. Geometry of the problem of diffraction of a waveguide mode on a dielectric ridge. One of the potential applications (spatial differentiation of the profile of an optical beam propagating in the waveguide) is schematically shown.](image)

Figure 2 shows the reflectance $|R_{TE}(\theta,l)|^2$ and the transmittance $|T_{TE}(\theta,l)|^2$ of the ridge vs. the angle of incidence $\theta$ and the ridge length $l$ in the case of an incident TE-polarized mode, where $R_{TE}(\theta,l)$ and $T_{TE}(\theta,l)$ are the complex reflection and transmission coefficients, respectively. The plots were calculated using an in-house implementation of the aperiodic rigorous coupled-wave analysis (aRCWA) technique [7]. RCWA, also called the Fourier modal method, is an established numerical technique for solving Maxwell’s equations.

It is evident that several resonances (sharp reflectance maxima and transmittance minima) are present in the spectra of figure 2. Resonant reflectance peaks (transmittance dips) are located in the angle of incidence range $\theta = 39.05^\circ - 57.55^\circ$, which is marked with dashed lines in figure 2. Let us explain these effects. In a general case, at oblique incidence of a TE-polarized guided mode on a ridge located on the waveguide surface, reflected and transmitted TE- and TM-polarized modes are generated, as well as a continuum of plane waves arising from “parasitic” scattering out of the guiding layer to the superstrate and substrate. However, if the angle of incidence exceeds a certain value $\theta_{cr,1}$, the reflected and transmitted fields contain only the TE-polarized mode, and no out-of-plane scattering and polarization conversion occur [4–6]. Indeed, let us denote by $k_{x,inc} = k_0 n_{wg,TE} \sin \theta$, where $k_0 = 2\pi/\lambda$ is the wavenumber, the x-component of the wavevector of the incident TE-mode, which is parallel to the ridge boundaries. According to the boundary conditions for the Maxwell’s equations, the $k_{x,inc}$ component has to be conserved, i.e. it is the same for all the waves constituting the reflected and the transmitted fields, including the radiation scattered out of the waveguide. Therefore, at angles of incidence $\theta > \theta_{cr,1} = \arcsin\left(n_{wg,TM}/n_{wg,TE}\right) = 39.05^\circ$, reflected and transmitted TM-polarized modes become evanescent. At $\theta > \theta_{cr,1}$, $k_{x,inc}$ also exceeds the wave vector magnitudes of the propagating plane waves over and under the waveguide (in the regions with the refractive indices $n_{sup}$ and $n_{sub}$, respectively) and
therefore the incident TE-mode is not scattered out of the waveguide layer. Thus, at $\theta > \theta_{c,1}$ the reflection and transmission coefficients corresponding to the TE-polarized mode satisfy the equality $|R_{\text{TE}}|^2 + |T_{\text{TE}}|^2 = 1$, since the materials of the structure are assumed to be lossless. The angle $\theta_{c,1}$ is shown with a dashed line in figure 2, which corresponds to the upper boundary of the “resonance region”. The lower boundary of this region corresponds to the cutoff angle of the TM-polarized mode in the ridge region, which amounts to $\theta_{c,2} = \arcsin\left(n_{r,TM}/n_{r,\text{TE}}\right) = 57.55^\circ$.

The presented analysis suggests that the resonances in figure 2 are associated with the excitation of TM-like eigenmodes of the ridge, which in this case acts as a leaky rib waveguide. Within the framework of the effective index method (EIM) [8], the TM-like modes of a rib waveguide can be approximately described by the dispersion relation of TE-polarized modes of a dielectric slab waveguide with the thickness $l$, in which the values $n_{r,TM}$ and $n_{\text{wg,TM}}$ are used as the refractive indices of the core layer and claddings, respectively.

The dispersion relation of a symmetric slab waveguide can be written in the form [9]

$$l = \pi m + \text{arg} \left(r(\theta)\right)/\left(k_{n_{r,TM}} \cos \theta\right),$$

where the integer $m$ is the mode order, $r(\theta)$ is the complex reflection coefficient of a TE-polarized plane wave from the interface between the media with refractive indices $n_{r,TM}$ and $n_{\text{wg,TM}}$. A straightforward refinement of this EIM-based dispersion relation consists in the use of the reflection coefficient of the TM-polarized mode $r_{\text{mode,TM}}(\theta)$ from the interface between two waveguides with the thicknesses $h_1 = 110 \text{ nm}$ and $h_2 = 80 \text{ nm}$ instead of the plane-wave reflection coefficient $r(\theta)$. The dispersion curves $l = l(\theta)$ corresponding to this refinement are shown in figure 2 with dotted curves and are in an excellent agreement with the resonance locations. This confirms the
hypothesis that the resonant features in the spectra of figure 2 are indeed due to the excitation of the cross-polarized modes of the rib waveguide.

Since in the resonant regime (at $\theta_{cr 1} < \theta < \theta_{cr 2}$) there is no out-of-plane scattering and polarization conversion in the reflected and transmitted radiation, the transmission coefficient strictly vanishes at the resonances [10, 11]. Moreover, at different ridge lengths the resonant peaks (dips) have different angular widths (different quality factor), which vary from units to less than a thousandth of a degree. This change in the Q-factor is caused by the interaction between the resonances of two types: Fabry-Perot resonance of the TE-mode and the guided-mode resonance of the TM-mode. When two types of resonances (two modes) interact, the so-called matrix Fabry-Perot resonances occur, which can have a very high Q-factor [12]. In fact, the Q-factor in the considered structure can reach infinity, i.e. the structure supports the so-called bound states in the continuum (BIC). A detailed theoretical study confirming the formation of robust BICs and high-Q resonances in the investigated structure will be the subject of a separate work.

3. Conclusion
In this work, we demonstrated that in the case of oblique incidence of TE-polarized modes of a slab waveguide on a dielectric ridge located on the waveguide surface, resonant changes in the reflectance and the transmittance spectra occur. The resonant peaks (dips) have different angular widths (different Q-factor), which can reach a thousandth of a degree or even less. Using a refinement of the effective index method, we explained these effects by the excitation of a cross-polarized mode of the ridge.

In our opinion, the presented planar structure can be used as an integrated optical spatial (angular) or spectral filter. Moreover, since the transmission coefficient of the ridge strictly vanishes at the resonances, the structure enables performing spatial optical differentiation in transmission (as it is schematically shown in figure 1) as well as spatial optical integration in reflection [13]. Finally, the existence of high Q-resonances and bound states in the continuum in the structure also makes it promising for chemical or biosensing. The authors believe that the presented configuration can also be extended to other platforms, e.g. Bloch surface waves propagating at the interfaces of photonic crystals.

Acknowledgments
This work was supported by Federal Agency of Scientific Organizations under Agreement 007-GZ/Ch3363/26 (theoretical analysis of the resonances of the ridge using the effective index method), and by Russian Science Foundation under Grant 14-19-00796 (implementation of the software tools for the rigorous simulation of the studied integrated structures).

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