Heavy Quark Phenomenology from Lattice QCD

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Recent results relevant for the $B$-physics phenomenology, obtained from lattice QCD simulations by the APE Collaboration, are reviewed. This includes the $B^0 - B^0$ mixing amplitude, $B \to \pi$ semileptonic decay and the relative width difference of $B_s^0$ mesons, ($\Delta \Gamma/\Gamma)_{B_s}$.

The main theoretical obstacles in determining the amount of CP-violation that comes from the Standard Model (SM) are related to the uncertainties in computation of various hadronic quantities. In this talk, I focus on several such quantities/processes involving heavy-light mesons, for which APE group provided new lattice results. Technical details about the simulations are given in the references which will be quoted with each quantity I discuss in this paper. Here I only stress that all our results are obtained at $\beta = 6.2$, in the quenched approximation, and by using the (fully propagating) $\mathcal{O}(a)$ improved, Wilson fermions.

1. Decay Constants ($f_B$ and others) \cite{1,2,3}

One of the essential hadronic quantities entering the $B^0_s - B^0_s$ mixing amplitude is the $B$-meson decay constant $f_{B_a}$ ($a = d, s$), defined as

$$\langle 0 | \bar{b} \gamma_\mu \gamma_5 q | B_a(p) \rangle = i p_\mu f_{B_a} \ .$$

(1)

The central results from our two simulations (see Tab. \ref{table}) are obtained by: (i) linearly extrapolating (interpolating) $f_{H_a}/\sqrt{m_{H_a}}$ in $1/m_{H_a}$, to the $B$ ($D$) meson mass; (ii) including the KLM factor in a way discussed in \cite{3}; (iii) converting to the physical units by using $a^{-1}(m_{K^*}) = 2.71$ (GeV). To estimate the systematic errors, we combine in quadrature the following differences between our central values and the ones obtained when: (a) extrapolating in $1/m_{H_a}$ quadratically; (b) omitting the KLM factor, (c) using the ratio $f_H/f_\pi$ to extrapolate to $m_B$. Since the extrapolation from the directly accessed heavy meson masses ($2$ GeV $\leq m_H \leq 3$ GeV) to $m_B$ is rather long, the source (a) largely dominates the systematics. This error has not been included in the results \cite{2}, where only 3 heavy quarks were considered. Note, however, that this error completely cancels in the SU(3) breaking ratio, $f_B/f_{B_s}$. For comparison with results of other lattice groups, see \cite{3}. In \cite{3} we have also computed the vector meson decay constants, which I now update. In addition we compute the coupling of the vector meson to the tensor current, \textit{i.e.}

$$\langle 0 | b_\mu | p, \lambda \rangle | B^*_q(p, \mu) \rangle = i e^{(\lambda)} m_{B^*_q} f_{B_q^*} \ ,$$

$$\langle 0 | b_{\mu \nu} | p, \lambda \rangle | B^*_q(p, \mu) \rangle = i \left( e^{(\lambda)} p_\mu - p_\nu e^{(\lambda)} \right) f_{B_q^*}(\mu) \ ,$$

where $\mu$ is the scale at which the tensor density is non-perturbatively renormalized (in the RI-MOM scheme). These decay constants are particularly important in testing the validity of the factorization in non-leptonic decays of heavy-light mesons. Our new results are

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
 & Ref. \cite{1} & Ref. \cite{2} \\
\hline
$f_B$ & 173(13)\textsuperscript{+26}_{-8} & 175(22)\textsuperscript{+8}_{-6} \\
$f_B/f_B$ & 1.14(2)(1) & 1.17(4) \\
$f_D$ & 216(11)(5) & 207(11)\textsuperscript{+4}_{-0} \\
$f_D/f_D$ & 1.11(1)(1) & 1.13(3)(1) \\
\hline
\end{tabular}
\end{table}

Our new results are

\begin{align*}
& f_{B^*} = 199(14)\textsuperscript{+34}_{-4} \text{ MeV} : f_{D^*} = 258(14)(6) \text{ MeV} ; \\
& f_{B^*}/f_{B^*} = 1.14(2)(1) : f_{D^*}^2/(m_b)_{f_{B^*}} = 0.88(2)(2) ; \\
& f_{D^*}/f_{D^*} = 1.10(2) : f_{D^*}^2/(2 \text{ GeV})_{f_{D^*}} = 0.90(2) .
\end{align*}
2. $B^0_q$–$\bar{B}^0_q$ Mixing and $(\Delta \Gamma/\Gamma)_{B_s}$ \[2, 10\]

To access any bare continuum $\Delta E = 2$ operator from the lattice, by using Wilson fermions, one first has to subtract the effect of mixing with other dimension-six 4-fermion operators which is due to the explicitly broken chiral symmetry in the Wilson quark action. A bare (continuum) operator should then be appropriately renormalized. The whole procedure can be shortened as

$$\langle B^0_q(Q_\mu) | B^0_q \rangle =$$

$$\langle \bar{B}^0_q(\sum_j Z_{ij}(g_0^2, \mu) \left(Q_{j}}^{\text{latt.}} + \sum_{k \neq j} A_{k}(g_0^2)Q_k^{\text{latt.}} \right) | B^0_q \rangle,$$

where $i, j, k$ run over the basis of parity conserving operators ($Q_i$), $\Delta_j(g_0^2)$ and $Z_{ij}(g_0^2, \mu)$ are the subtraction and renormalization (RC) constants, respectively. A technique to compute the constants $\Delta_j$ and $Z_{ij}$ non-perturbatively, in the RI-MOM renormalization scheme, has been developed in ref. \[10\]. We work in Landau gauge, apply the technique \[3\] at three different values of the scale $\mu$, and verify that the scale dependence of $Z_{ij}(\mu)$, for the operators discussed below, is indeed well described by the perturbative NLO anomalous dimension matrix \[3\]. This allows us to express our matrix elements in the renormalization group invariant (RGI) form.

$B^0_q$–$\bar{B}^0_q$ Mixing: The needed parameter, $B_{B_s}$, ($q = d, s$), is defined as

$$\langle \bar{B}^0_q(Q_L(\mu)) | B^0_q \rangle = \frac{8}{3} m^2_B f B_{B_s}(\mu),$$

where $Q_L = \bar{b} \gamma_5 (1 - \gamma_5) q \bar{b} \gamma_5 (1 - \gamma_5) q$, and $i, j$ are the color indices. From the definitions \[4\] and \[2\], it is clear that the $B$-parameter can be directly accessed if (for each heavy light-meson $H_q$) we compute the ratio of correlation functions

$$\frac{3}{2} \cdot \sum_{\vec{y}, \vec{y}'} \langle P(x)Q_L(0; \mu) P'(y) \rangle \rightarrow B_{H_q}(\mu).$$

The last limit is valid when the operator $Q_L$ and the pseudoscalar sources $P$ are sufficiently separated on the temporal axis. To reach the physically relevant $B_{H_q}$ from the extracted $B_{H_q}$ (which

\[2\] The procedure sketched above can be highly simplified if one uses the Ward identity to relate the parity conserving to the parity violating operators (for which $\Delta_j = 0$) \[3\]. This idea is yet to be implemented in practice.

should scale with heavy meson mass as a constant), we make the linear $1/m_H$ fit and extrapolate to $m_B$. Our final results are

$$\hat{B}_{B_d} = 1.38(11)^{+0.09}_{-0.06}, \quad \hat{B}_{B_s} = 1.35(5)^{+0.09}_{-0.08},$$

$$\hat{B}_{B_d}/\hat{B}_{B_d} = 0.98(5),$$

where we converted (to NLO) the directly computed $B_{B_s}^{\text{RI-MOM}}(\mu)$ to the RGI form $\hat{B}_{B_s}$. We also obtained, $\hat{B}_D = 1.24(4)^{+0.09}_{-0.06}$, which may be useful in the non-SM phenomenology. Since we clearly see $1/m^2_H$ corrections from our data, one can try to constrain the extrapolation by using the static result for $\hat{B}_B$ \[8\]. Such an exercise leads to a $\sim 5\%$ lower value of $\hat{B}_B$, which is well within our error bars (a similar conclusion is reached in \[8\]).

After combining the above results with the ones for $f_{B_h}$, also computed in \[2\], we get

$$f_{B_d} \sqrt{\frac{\hat{B}_{B_d}}{\hat{B}_{B_d}}} = 206(28)(7) \text{ MeV};$$

$$\xi = \frac{f_{B_s}}{f_{B_d}} = 1.69(7).$$

In the last result, most of the systematic uncertainties cancel in the ratio. To exemplify the phenomenological benefit of this result, I combine our value for $\xi$ with the updated experimental value for the frequency of $B^0_d$ mesons oscillations $(\Delta m^2_{d})_{\text{exp}} = 0.486(15) \text{ ps}^{-1}$ \[3\], to get

$$\Delta m_{s} = \frac{\left| V_{ts}^2 \right|^2}{\left| V_{td} \right|^2} \xi^2 \left( \frac{m_B}{m_{B_d}} \Delta m_{d} \right)_{\text{exp}} = 16.2 \pm 2.1 \pm 3.4 \text{ ps}^{-1},$$

where $\left| V_{ts} \right|^2 / \left| V_{td} \right|^2 = 24.4 \pm 5.0$ is assumed. Experimental lower bound is $\Delta m^2_{s} > 14.9 \text{ ps}^{-1}$ \[8\].

$(\Delta \Gamma/\Gamma)_B$: In the framework of the heavy quark expansion, the leading contribution in the expression for the width difference of $B^0_d$ mesons, comes from $\Delta B = 2$ operator, $Q_S = \bar{b} (1 - \gamma_5) s \bar{b} (1 - \gamma_5) s$, the matrix element of which is parameterized as

$$\langle \bar{B}^0_q(Q_S(\mu)) | B^0_q \rangle = -\frac{5}{3} \left( \frac{m^2_B f B_{B_s}}{m_B(\mu) + m_s(\mu)} \right)^2 B_S(\mu),$$

where $B_S(\mu)$ is the wanted bag parameter. An important observation made in ref. \[10\] is that if we only replace $Q_L/8 \rightarrow -O_5/5$ in eq. \[4\], we see a very large dependence on the inverse heavy meson...
mass $1/m_{H_2}$. On the contrary, if in denominator of eq. (3) we also replace the axial current by the pseudoscalar density, $A_0 \to P$, the $1/m_{H_2}$ dependence becomes much weaker and the extrapolation to $m_B$ is more under control (which is why our central results are those obtained using the latter procedure). Obviously, the large $1/m_{H_2}$ dependence comes from the ratio of the heavy meson/heavy quark mass, $[m_{H_2}/(m_Q + m_s)]^2$. It will be interesting to see whether the inconsistency of the two procedures disappears with the simulations performed closer to $m_{B^0}$. In this calculation, we also needed to compute the matrix element of the operator $Q_5$ which is the one obtained by reversing the color indices in $Q_5$, and which mix with $Q_8$ in the continuum. Once we extract the values for $B_S(\mu)$ and $B_S(\mu)$, we converted RI-MOM to $\overline{MS}$ since the Wilson coefficients, $G(z)$ and $G_5(z) (z = m_2^2/m_6^2)$, were computed in the $\overline{MS}$ scheme [11]. After a linear extrapolation to $1/m_{B^0}$, we get

$$B_S^{\overline{MS}}(m_b) = 0.86(2)(3); \quad \overline{B}_S^{\overline{MS}}(m_b) = 1.25(3)^{+0.02}_{-0.05}.$$  

For the physical prediction of $(\Delta \Gamma/\Gamma)_{B_s}$, one needs the ratio of the matrix elements (3) and (6). We obtain

$$\frac{R^{\overline{MS}}}{R}(m_b) = -0.93(3)(1)$$  

which is in a good agreement with results obtained by using the effective theories [8, 12]. We proposed in [13] a safer way to predict

$$(\frac{\Delta \Gamma}{\Gamma})_{B_s} = K \left(\tau_{B_s} \Delta m_{B_d} \frac{m_{B_d}}{m_{B_s}}\right)^{(exp.)} \times \left[\frac{G(z) - G_5(z)R(m_b) + \delta_{1/m}}{\xi^2}\right]$$  

where $K$ is the known constant and $\delta_{1/m}$ encodes $1/m_b$ corrections (which are estimated by using the vacuum saturation approximation). The advantage of using this formula is that it contains experimentally well determined quantities, and $\xi$ and $R(m_b)$ ratios, in which (again) most of the systematic uncertainties cancel. Finally, we obtained

$$(\frac{\Delta \Gamma}{\Gamma})_{B_s} = [(0.5 \pm 0.1) - (13.8 \pm 2.8)R(m_b)] + (15.7 \pm 2.8)\delta_{1/m}] \cdot 10^{-2}$$  

$$= (4.7 \pm 1.5 \pm 1.6) \cdot 10^{-2}, \quad (10)$$

where we show how the explicit cancellation occurs between the leading $R(m_b)$ and the $1/m_b$ correcting terms. Therefore, to improve the above result it is necessary to gain a better control over the dimension-seven operators which appear in $\delta_{1/m}$.

3. $|V_{ab}|$ from $B(B \to \pi l \nu)$ [13]

Compared to “Lattice 99” [4], we now have the final results for the $D$ decay modes (see [13]), and also for the most challenging one, $B \to \pi$. The relevant form factors, $F_{+0}/(q^2)$ ($q = p_H - p$),

$$\langle \pi(p) | q \gamma_{\mu} Q | H(p_H) \rangle = \frac{m_H^2 - m_\pi^2}{q^2} q_\mu F_{0}(q^2) + \left(p_H - p - \frac{m_H^2 - m_\pi^2}{q^2}\right)_{\mu} F_{+}(q^2), \quad (11)$$

are extracted for 3 different light ($q$) and 4 heavy ($Q$) quark masses and for 13 inequivalent kinematical configurations $(p_H, q)$. The mass extrapolations of form factors are known to be trickier because of the interplay between $m_B$ and $q^2$ dependences. A parameterization for the $q^2$-dependence, which contains most of the theoretically available constraints, has been proposed in [14]:

$$F_{+}(q^2) = \frac{c_H(1 - \alpha_H)}{(1 - \alpha_H)^2} \left(1 - \frac{1}{q^2}ight)$$  

$$F_{0}(q^2) = \frac{c_H(1 - \alpha_H)}{(1 - \alpha_H^2)^2} \left(1 - \frac{1}{q^2}ight), \quad (12)$$

where $q^2 = q^2/m_B^2$. The parameters $\phi \in \{c_H\sqrt{m_H}, (1 - \alpha_H)m_H, (\beta - 1)m_H\}$ should scale as a constant (plus corrections $\propto 1/m_B$). To reach $B \to \pi$ we first fit the form factors to the parameterization (12) for each combination of the heavy and light quarks and then adopted the following two methods:

**Method I** We smoothly extrapolate to the final pion (kaon) state, for every heavy quark, and then use the scaling laws for all three parameters to extrapolate to $B$, namely

$$\phi = a_0 + \frac{a_1}{m_H} + \frac{a_2}{m_H^2}$$  

**Method II** Following the proposal of UKQCD group [14], we first extrapolate to the final pion at fixed $v \cdot p = \frac{M_H^2 + m_0^2 - q^2}{2M_H}$, namely

$$v \cdot p = \frac{M_H^2 + m_0^2 - q^2}{2M_H}.$$  

**Method II** Following the proposal of UKQCD group [14], we first extrapolate to the final pion at fixed
Table 2
APE lattice results for the $B \to \pi$ form factors \cite{15} are compared to the predictions obtained by using LCSR \cite{16}.

|                | Method I    | Method II   | LCSR \cite{16} |
|----------------|-------------|-------------|----------------|
| $c_B$          | 0.42(13)(4) | 0.51(8)(1)  | 0.41(12)       |
| $\alpha_B$     | 0.40(15)(9) | 0.45(17)+0.06−0.13 | 0.32+0.21−0.07 |
| $\beta_B$      | 1.22(14)+0.12−0.03 | 1.20(13)+0.15−0.00 | —              |
| $F(0)$         | 0.26(5)(4)  | 0.28(6)(5)  | 0.28(5)        |
| $F_{B^+\to K}^{B^+\to \pi}(0)/F_{B^+\to \pi}(0)$ | 1.21(9)+0.00−0.09 | 1.19(11)+0.03−0.11 | 1.28(11)+0.18−0.10 |
| $F_0(m_B^2)$   | 1.3(6)+0.4−0.4 | 1.5(5)+0.4−0.4 | —              |
| $g_{B\to B\pi}$ | 20±7        | 24±6        | 22±7           |

$|V_{ub}|^{-2}\Gamma(B^0 \to \pi^-\ell^+\nu) [\text{ps}^{-1}]$ & 6.3±2.4±1.6 & 8.5±3.8±2.2 & 7.3±2.5

and then extrapolate to $B$ by using the HQET scaling laws. The $q^2$-behavior is then fit by using eq. (12).

The results obtained by using both methods are shown in Tab. 3, where we also make a comparison with the light cone QCD sum rules (LCSR) predictions \cite{16}. Besides a very good consistency of the results of the two (different) methods, a pleasant feature of this analysis is also the agreement with lattice results of \cite{15} as well as with the LCSR results \cite{16}. For a further comparison with presently available lattice results, see \cite{15}.

The central values in Tab. 3 are obtained through a quadratic extrapolation to $m_B$, which provides a better consistency with the Callan-Treiman relation, $F_0(m_B^2) = f_{B}/f_{\pi}$ \cite{16}. The complete account of the systematic uncertainties is detailed in \cite{15}. I emphasize that $F_{B^+\to \pi}(0)$ in Tab. 3 as obtained after extrapolating $F_{H^+\to \pi}(m_H^{3/2})$ to $m_B$, is indistinguishable from the one obtained by combining separately extrapolated $c_B$ and $\alpha_B$ into $F_{B^+\to \pi}(0) = c_B(1-\alpha_B)$.

To read the content of the parameterization \cite{15} I recall that the parameter $c_B$ measures the residue of $F_{+}(q^2)$ at $m_B^2$, thus allowing determination of the $g_{B^+\to \pi}$. By using the standard definitions, one has

$$g_{B\to B\pi} = 2m_Bc_B/f_B$$

and the other singularities contributing to $F_{+}(q^2)$ can be mimicked by an effective pole corresponding to $m_{1-} \simeq 8 \pm 1$ GeV, whereas the contributions to the scalar form factor $F_0(q^2)$ are represented by an effective pole with the mass $m_{0\tau} \simeq 6 \pm 3$ GeV.

Finally, by comparing our results for the decay width, with the experimental branching ratio $B(B^0 \to \pi^+\ell^+\nu)$, we obtain $|V_{ub}| = (4.1 \pm 1.1)10^{-3}$.

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\footnote{Research on verification of this relation on the lattice has been discussed at this conference \cite{17}.}