Magnetic Polarizability of Virtual \((s\bar{s})\) and \((c\bar{c})\) Pairs in the Nucleon

Peter Filip
Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, Bratislava 845 11, SK
E-mail: peter.filip@savba.sk

Abstract. We suggest \(3P_0\) quantum state of virtual \((s\bar{s})\) pairs in the nucleon can be polarised by the internal fields permeating the volume of the nucleon (proton or neutron). Due to the quadratic Zeeman interaction, \(3P_0\) wavefunction of virtual \((q\bar{q})\) pairs acquires the admixture of \(1P_0\) quantum state in the magnetic field, which generates the antiparallel polarization of \(s\) and \(\bar{s}\) quarks (in the nucleon). Considering the internal magnetic fields of neutron and proton (originating from their measured magnetic dipole moments), we suggest the induced \(s\)-quark polarization in the neutron to be of the opposite direction compared to the proton case. We mention the influence of the internal chromo-magnetic fields on the quantum state of \((q\bar{q})\) pairs in the nucleon and we discuss also the expected behaviour of virtual \((c\bar{c})\) pairs.

1. Introduction
Internal structure of the nucleon is an important and challenging question of modern physics. Due to remaining uncertainties in the spin (angular momentum) structure of Proton (only 1/3 of the expected part of the proton spin was found [1] to be directly related to the spin orientations of quarks), the intensive experimental efforts continue with the goal to determine the spin contribution of gluons (\(\Delta G\)), and quark orbital angular momentum \(L_q\). The spin of virtual \((ss)\) pairs may also contribute to the total angular momentum \((J = h/2)\) of proton.

The presence of virtual \((s\bar{s})\) pairs in the nucleon is naturally expected within Parton model, and the experimental data from lepton-proton scattering as well as the observed OZI rule violations in \(\bar{p}p\) annihilation [2] suggest the polarization of virtual \((s\bar{s})\) pairs [3] in the nucleon.

In this contribution we consider a polarization of \(3P_0\) \((s\bar{s})\) quantum state, which may occur due to the interaction of quark magnetic (or chromo-magnetic) moments with the oriented internal electromagnetic or gluonic fields in the nucleon.

2. \(3P_0\) quantum state in external fields
Quantum state of virtual \((s\bar{s})\) component in the proton is not fully fixed by theoretical arguments, and several possibilities (e.g. \(1S_0, 3S_1, 3P_0\) or \(1P_1\) type of wavefunctions) exist [3]. Experimental data indicate that \(1S_0\) and \(3S_1\) configurations are unlikely [3], while \(3P_0\) state of \((s\bar{s})\) pair with vacuum quantum numbers \(J^{PC} = 0^{++}\) appears to be a reasonable assumption.

Bound state of real \((c\bar{c})\) quark-antiquark pair in \(3P_0\) configuration \((0^{++})\) is observed as \(\chi_{c0}\) meson [4], and the properties of \(3P_0\) state in Positronium \((e^+e^-)\) are well understood [5]. If we presume the response of \((e^+e^-)\) and \((q\bar{q})\) bound systems (in \(3P_0\) state) to the external field to
be similar, the existing knowledge about the Positronium $^3P_0$ state behavior in external fields [5] can be used to make the predictions regarding the state of virtual $(s\bar{s})$ pairs in the nucleon.

2.1. Internal magnetic fields in the nucleon

Neutron and proton have anomalous magnetic moments ($\mu_n = -1.91\mu_N$ and $\mu_p = 2.79\mu_N$) due to their nontrivial internal structure. Using SU(6) symmetric constituent quark $[qqq]$ wavefunction, the nucleon magnetic moment is [4]: $\mu_N = (4\mu_u - \mu_d)/3$. Constituent quark magnetic moments then are: $\mu_u = 1.85\mu_N$ and $\mu_d = -0.97\mu_N$. Hyperons $\Sigma^+$ and $\Xi^0$ also agree with such a simple ansatz (within 20% precision) and $\mu_s \approx -0.61\mu_N$ value of $s$-quark magnetic moment agrees well also with the magnetic moments of $\Lambda^0$ and $\Omega^-$ ($3\mu_s$) baryons [4].

Magnetic moments ($\mu_{\bar{p}}, \mu_n$) of proton and neutron are the source of dipole magnetic field $B_p$, which is generated within the interior (MIT bag volume) of the nucleon. Compton length of the proton ($\lambda_c = h/m_pc = 1.3$ fm) is comparable to the proton radius (size), and one can imply a dipole field $B_p$ does penetrate the interior (partonic matter) of the nucleon. Direction of internal magnetic field $B_p$ within the proton (or neutron) volume is parallel (or antiparallel) to the spin orientation, and its magnitude is approximately $|B_p| \approx 10^{13}$ T = $10^7$ Gauss. This estimate can be obtained at semi-classical level, requiring the electric current $I$ inside the coil of radius $R_p \approx 0.82$ fm to generate the magnetic dipole of size $\mu = \pi R_p^2 I \approx 2\mu_N$. Magnetic field inside such a theoretically considered coil [6] then is: $B = \mu_0 I/2R_p \approx 10^{13}$ T (here $\mu_0 = 4\pi \times 10^{-7} N/A^2$).

2.2. Magnetic polarizability of $^3P_0$ state

Let us consider the wavefunction of bound fermion-antifermion $(\bar{f}f)$ pair in $^3P_0$ quantum state with $J^{PC} = 0^{++}$ quantum numbers. Using the Positronium notation [7] we have

$$^3P_{00} = \frac{1}{\sqrt{3}} (Y_{11}x_{1-1} + Y_{1-1}x_{11} - Y_{10}x_{10})$$

(1)

where $Y_{lm}$ are orbital (spherical harmonics) functions, and $\chi_{s,s'}$ denote the spin part of $^3P_{00}$ wave function. Now we shall assume that quantum state of the virtual $(s\bar{s})$ pairs in the nucleon has a similar (the same) quantum structure as given by Eq.(1). Using the uncoupled spin eigenstates $\chi_{1-1} = |\downarrow\downarrow\rangle$, and $\chi_{1+1} = |\uparrow\uparrow\rangle$, and $\chi_{10} = (|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)/\sqrt{2}$, one has

$$^3P_{00}[s\bar{s}] = \frac{1}{\sqrt{3}} \left[ Y_{11}|\downarrow\downarrow\rangle + Y_{1-1}|\uparrow\uparrow\rangle - Y_{10}|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle \right]$$

(2)

The probability of antiparallel $s, \bar{s}$ spins ($\uparrow\downarrow$ or $\downarrow\uparrow$) is 1/3, and parallel orientations ($\uparrow\uparrow$) and ($\downarrow\downarrow$) occur with the same probability (1/3). For the net helicities of $s$ and $\bar{s}$ quarks in such $^3P_{00}$ state we have $\Delta s = s^z - \bar{s}^z = 0$ and $\Delta \bar{s} = \bar{s}^z - s^z = 0$, and $L_s + L_{\bar{s}} = 0$ also applies.

It has been suggested [8] that component $Y_{11}|\downarrow\downarrow\rangle$ in $^3P_{00}[s\bar{s}]$ wave function (2) is enhanced in the proton, due to strong attraction of the valence $u$ and virtual $\bar{s}$ quarks in the pseudoscalar channel, which results in the polarization of strange quarks in the nucleon. Magnetic fields can also modify the spin structure of $^3P_{00}$ state. In the Positronium [7] case, $^3P_{00}$ wavefunction acquires the admixture of $^1P_{10}$ state. Eigenstates $\tilde{P}_{00}, \tilde{P}_{10}, \tilde{P}_{20}$ in small magnetic fields are [7]:

$$^3P_{00}(B) \approx ^3P_{00} + \varepsilon_B 3P_{10}/\sqrt{3}$$

$$^1P_{10}(B) \approx ^1P_{10} - \varepsilon_B^2 3P_{00}/\sqrt{3}$$

$$^3P_{20}(B) \approx ^3P_{20} + \varepsilon_B^2 3P_{10}/\sqrt{3}$$

(3)

(4)

(5)

where admixture parameters $\varepsilon_B = \frac{2\mu_B}{3\Delta E_{10}}$ and $\varepsilon_B' = \frac{2\mu_B}{3\Delta E_{21}}$ depend on the magnetic field $B$, and on eigenenergy differences $\Delta E_{10}, \Delta E_{21}$. Strange quarks also have their magnetic moment $\mu_s$, and therefore, a similar behavior of $^3P_{00}[s\bar{s}]$ wavefunction in external $B$ field can be anticipated.
Figure 1. Probability of the spin components in $\tilde{3P}_{00}$ state in the magnetic field, as a function of the mixing angles (value $\phi = 45^\circ$ corresponds to Paschen-Back limit of very large fields: $B \to \infty$).

Using $^1P_{10} = Y_{10}(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle)/\sqrt{2}$, the virtual $\tilde{3P}_{00}$ state of $(s\bar{s})$ pair in small magnetic fields is:

$$3\tilde{P}_{00}[s\bar{s}](B) \approx \frac{1}{\sqrt{3}} \left[ Y_{11}|\downarrow\downarrow\rangle + Y_{1-1}|\uparrow\uparrow\rangle \right] - \frac{Y_{10}}{\sqrt{3}} \left( |\downarrow\uparrow\rangle \frac{1 + \varepsilon_B}{\sqrt{2}} + |\uparrow\downarrow\rangle \frac{1 - \varepsilon_B}{\sqrt{2}} \right)$$  \hspace{1cm} (6)

Relative imbalance of $\uparrow\downarrow$ and $\downarrow\uparrow$ spin configurations in the wavefunction (6) generates the induced magnetic moment of $3\tilde{P}_{00}$ state. Such behavior corresponds to magnetic polarizability [9].

Energy of the Positronium $3\tilde{P}_{00}$ state decreases in the magnetic field [5], which corresponds to $\uparrow\mu_e$ and $\downarrow\mu_e$ orientation of $e^-$, $e^+$ magnetic moments. In Paschen-Back limit (see Fig. 1), component $|\downarrow\uparrow\rangle$ becomes dominant in $3\tilde{P}_{00}$ state. Therefore, one can expect enhanced probability of $\uparrow\mu_e\uparrow\mu_e$ configuration $|\uparrow\uparrow\rangle$ for the virtual $(s\bar{s})$ pairs in the polarized proton as well. The behavior of $3\tilde{P}_{00}$ Positronium wavefunction in the electric field is well known [7], and our observation (analysis) gives: No polarization of $s, \bar{s}$ spins occurs, if $3\tilde{P}_{00}$ state is exposed to the electric fields.

3. Virtual $s\bar{s}$ and $c\bar{c}$ pairs in the proton and neutron

If other effects (e.g. pseudoscalar QCD interaction [8]) do not determine the polarization of $(q\bar{q})$ pairs in the nucleon, and the polarizability of virtual $(s\bar{s})$ pairs by the internal magnetic field is indeed significant, the following qualitative predictions can be inferred from Eq.(6):

- Net spin polarization of $s$ quarks in proton is antiparallel to the proton spin.
- Net spin polarization of $s$ quarks in the neutron is parallel to the spin of the neutron.
- Polarization of $\bar{s}$ quarks in the nucleon favors to be opposite to the $s$ quark polarization.
- The polarization of $c$ quarks in the nucleon is opposite to the polarization of $s$ quarks.
- The polarization of $\bar{c}$ (and $\bar{u}$) quarks is parallel to the polarization of $s$ quarks.

The first two statements originate from the internal magnetic dipole field $B_\mu$ orientation relative to the spin of proton and neutron, and the last two statements come from the fact, that $c$ quarks have positive charge $(2/3)e$, and thus their magnetic moment is parallel to their spin (contrary to the case of negatively charged $s$ quarks). The third statement applies only if $\uparrow\uparrow$ and $\downarrow\downarrow$ components of the function (6) have the same probability (QCD interaction [8] is inactive).

4. Magnetic moments of virtual quarks and magnitude of the effect

Magnetic polarizability of $(q\bar{q})$ pair depends on the quark magnetic moment $|\mu_q|$. The observed magnetic moments of baryons [4] require $\mu_u=1.85$, $\mu_d=-0.97$, $\mu_s=-0.61$ (in $\mu_N$ units), for
the constituent quarks \( u, d, s \). This agrees with Dirac equation (demanding \( \mu_q = hQ/2m^*_q \)), if constituent quark masses are: \( m^*_u \approx 338, m^*_d \approx 322, m^*_s \approx 510 \) (in MeV units). One can estimate the magnetic moment of \( c \) quark to be \( \mu_c = +0.4\mu_N \) (a significant value) using \( m^*_c \approx 1.5 \) MeV.

However, for virtual \((q\bar{q})\) quarks, the constituent masses are unjustifiably large and one should use smaller (e.g. current quark [4]) masses: \( m_u = 2.3, m_d = 4.8, m_s = 95 \) and \( m_c = 1.27 \) (MeV). In such a case, magnetic moments \( \mu_q \) of light virtual quarks are scaled up according to \( (m^*_q/m_q) \) ratios, which gives factors 135\( \times \), 67\( \times \), and 5\( \times \) for \( u, d, s \) quarks. The current quark magnetic moments thus are: \( \mu_u = +250\mu_N \), \( \mu_d = -65\mu_N \), \( \mu_s = -3\mu_N \) (and \( \mu_c = +0.47\mu_N \)).

Using \( \mu_s = 3\mu_N \) for current quark \( \bar{s} \), one has \( |\mu_s B| \approx 1 \) MeV for \( B \approx 10^{13} \) T in the nucleon (\( \mu_N = 3.15 \times 10^{-14} \) MeV/T). Assuming hyperfine splitting \( \Delta E_1^{ss} \approx 100 \) MeV, for \( ^3P_0 \) and \( ^1P_0 \) states of virtual \((s\bar{s})\) pair, mixing parameter \( \varepsilon_B = 2\mu_s B/\Delta E_1^{ss} = 0.019 \). From Eq.(6) we obtain

\[
\Delta s = s^\uparrow - s^\downarrow = [(1 - \varepsilon_B)^2 - (1 + \varepsilon_B)^2]/6 = -0.0126 \quad \Delta \bar{s} = \bar{s}^\uparrow - \bar{s}^\downarrow = +0.0126
\]

(7)

for proton, and \((\Delta s + \Delta \bar{s}) = 0\). For virtual \( c \) quarks we have \( \Delta c = +0.002 \), using \( |\mu_{s/c}| \approx 5 \), \( \Delta E_1^{ss} \approx 100 \) MeV. One can also expect \( \Delta \bar{d} > \Delta \bar{s} > 0 \) and \( \Delta \bar{u} < 0 \), for the virtual anti-quarks.

Parallel orientation of magnetic moments \( |\mu_{s/c}| \) is more probable than the opposite \((|\mu_{s/c}|)\) one in the polarized \(^3P_0\) state. Induced magnetic moment \( \langle \mu \rangle_{ss} \) of virtual \((s\bar{s})\) pairs can thus contribute [10] to the magnetic moment of proton, while the overall spin as well as total (and orbital) angular momentum of virtual \((s\bar{s})\) pairs remain to be zero.

5. Summary and conclusions

We have considered a simple model of \(^3P_0[qq]\) quantum state polarization of virtual \((s\bar{s})\) and \((c\bar{c})\) pairs in the nucleon, occurring due to the internal magnetic field \( B \approx 10^{13} \) T, which originates from the nucleon magnetic moment. Our estimates are based on the analogy with the behavior of \(^3P_0\) state of Positronium \((e^+e^-)\) in the magnetic field [7]. The angular momentum of virtual \((s\bar{s})\) pairs remains zero, while \( s \) quarks are polarized oppositely (and \( \bar{s} \) parallel) to the proton spin. The estimated polarizations are: \(-\Delta \bar{s} = \Delta s \approx -0.013\), and \( \Delta c = -\Delta \bar{c} \approx +0.002\).

However, quarks interact also via strong interactions, and QCD effects [8] can be larger than the electromagnetic interactions we have considered here. Moreover, internal volume of the polarized nucleon presumably contains an oriented chromo-magnetic fields \( B^a \) [11]. The interaction \( H^\text{int}_{2CD} = -\mu^a q B^a \) of such gluonic fields \( B^a \) with chromo-magnetic moments \( \mu^a_q \) of virtual quarks may induce a stronger polarization of \(^3P_0[qq]\) state. Our inferences about the \((s\bar{s})\) polarization magnitude, the polarization orientation of \((c\bar{c})\) pairs, and the opposite polarization of \( s, d, \) and \( \bar{u} \) quarks in neutron (relative to proton case) should thus be taken with caution.

Acknowledgments

Author is indebted to the organizers of DSPIN 2017 conference for the invitation to participate in this valuable scientific meeting, and to JINR, for the financial support and kind hospitality.

References

[1] Adams D, et al. (SMC Collaboration) 1997 Physical Review D 56 5330
[2] Ansler C 1998 Review of Modern Physics 70 1293
[3] Ellis J, Karliner M, Kharzeev D E and Sapochnikov M G 2000 Nuclear Physics A 673 256
[4] Patrignani C, et al. (Particle Data Group) 2016 Chinese Physics C 40 100001
[5] Curry S M 1973 Physical Review A 7 447
[6] Filip P 2014 Proc. of DSPIN-2013 Conference, October 8-12, 2013, Dubna, 53-56
[7] Dermer C D and Weisheit J C 1989 Physical Review A 40 5526
[8] Alberg M, Ellis J and Kharzeev D E 1995 Physics Letters B 356 113
[9] Petrukhin V A 1981 Soviet Journal of Particles and Nuclei 12 278
[10] Young R D 2017 Nature 544 419
[11] Barone V, Calarco T and Drago A 1998 Physics Letters B 431 405