Proposal for non-local electron-hole turnstile in the Quantum Hall regime.

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We present a theory for a mesoscopic turnstile that produces spatially separated streams of electrons and holes along edge states in the quantum Hall regime. For a broad range of frequencies in the non-adiabatic regime the turnstile operation is found to be ideal, producing one electron and one hole per cycle. The accuracy of the turnstile operation is characterized by the fluctuations of the transferred charge per cycle. The fluctuations are found to be negligibly small in the ideal regime.

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Transport along edge states in the integer quantum Hall regime has recently attracted large interest. The unidirectional transport properties of the edge states together with the possibility of using quantum point contacts as beam splitters has motivated a number of experiments on electronic analogues of optical interferometers, such as single particle Mach Zehnder and two particle Hanbury Brown Twiss interferometers. In the experiment by Altimiras et al. the electronic-optic analogue was supported by probing the non-equilibrium electronic distribution along the edge. Moreover, the prospect of entanglement generation in electronic two-particle interferometers has provided a connection between quantum information processing and edge state transport.

Another important aspect of edge state transport is the high-frequency properties. The experiment of Gabelli et al. confirmed the quantization of the charge relaxation resistance, predicted in Ref. [6]. In a pioneering experiment Fève et al. demonstrated that a mesoscopic capacitor coupled to an edge state can serve as an on-demand source for electrons and holes, operating at gigahertz frequencies. The experiment was followed by a number of theoretical works investigating the accuracy of the on-demand source and e.g. particle collisions with two synchronized sources. The successful realization of the electronic on-demand source also motivated new work on entanglement generation on-demand in the quantum Hall regime. A key feature of is that the on-demand source produces a single stream with alternating electrons and holes; the current has no dc-component, only ac-components. For quantum information tasks it would be desirable to have an on-demand source that produces two separate streams, one with electrons and one with holes. Such a source implemented in edge states and operating at gigahertz frequencies would also be of interest for metrological applications.

In this work we propose such an on-demand source. It comes as a non-local electron hole turnstile consisting of a double barrier (DB) formed by two quantum point contacts modulated periodically in time. A bias voltage is applied between the two sides of the turnstile, to have one resonant level of the DB in the bias window. An ideal operation cycle of the turnstile is shown in Fig. 1. (i) contact A is opened and one electron is transmitted into the region inside the DB, leaving a hole behind in the filled stream of electrons continuing towards terminal 3. (ii) Contact A closes and subsequently (iii) B opens and the electron trapped inside the DB is transmitted out through B and (iv) continues to terminal 2. Thus, during the cycle exactly one hole and one electron are emitted into spatially separate terminals.

Since the early turnstile experiments there has been large progress in operation speed and accuracy. Recent turnstiles or single electron pumps have demonstrated operation at gigahertz frequencies and single parameter pumping. The observed trend with increasing accuracy at large magnetic fields provides additional motivation for our quantum Hall turnstile.

Our proposal has a number of key features which have not been addressed together in earlier theoretical or
is found similarly, with $I^{bias}_2 = -I^{bias}_2(t + T/2)$ and the transferred charge per cycle is $Q_2 = -Q_3 = \int_0^T I_2(t) dt$.

The point contact scattering amplitudes $t_{A/B}, r_{A/B}$ are taken energy independent on the scale $\max\{kT, eV, \hbar \omega\}$, with $T$ the temperature. Motivated by the successful modelling in [3], we describe the contacts $A, B$ with saddle point potentials [21]. The time dependent scattering amplitudes are $t_{A/B}(t) = i\sqrt{r_{A/B}(t)}$ and $r_{A/B}(t) = \sqrt{1 - t_{A/B}(t)}$, where $T_{A/B}(t) = (1 + \exp((V_{A/B}(t) - V_{A/B}^0)/V_{A/B}^0)^{-1} - V_{A/B}^0)$ properties of the potential. Throughout the paper it is assumed that the product $T_{A}(t)T_{B}(t) \ll 1$, a typical driving scheme is shown in Fig. 2a. The top-gate suppresses charging effects [3, 7], supporting our non-interacting approximation.

In the rest of the paper we consider the case with $eV = \Delta$ giving one DB-level inside the bias window, optimal for the ideal turnstile operation shown in Fig. 1. We can then perform the energy integral in Eq. (1) giving

$$I_2(t) = (\Delta e/h) T_B(t) F(t - \tau),$$
$$F(t) = T_A(t) + R_A(t) R_B(t - \tau) F(t - 2\tau).$$ (2)

Quite remarkably, the current $I_2(t)$ depends only on the scattering probabilities $T_{A/B}(t) = 1 - R_{A/B}(t)$ of the contacts $A/B$ at times earlier than $t$. The result is independent on temperature and holds for arbitrary driving frequency. The recursively defined $0 \leq F(t) \leq 1$ is the probability that an electron injected in the bias window from terminal 1 at a time $t - 2n\tau$ ($n \geq 0$ integer) is propagating away from A towards B at time $t$.

In the adiabatic transport regime, the dwell time of the particles in the DB is much shorter than the drive period $T$. The maximum dwell time for particles injected in the bias window is $\sim \hbar/(\Delta \min[T_A(t) + T_B(t)])$, the inverse of the minimum resonant level width (taken over one period). Thus, at frequencies $\omega \ll \Delta \min[T_A(t) + T_B(t)]/\hbar$ the transport is adiabatic. The current is found by taking $\tau \to 0$ in Eq. (2), giving

$$I^{ad}_2(t) = (e\Delta/h) T_B(t) T_A(t)/(1 - R_A(t) R_B(t)).$$ (3)

This is simply the instantaneous DB current. Importantly, the corresponding transferred charge per period $Q^{ad}_2 \gg e$ (see Fig. 2a), i.e. many particles traverse the DB during one period. From Eq. (3) and Fig. 2a it is clear $Q^{ad}_2 \propto 1/\omega$ and that $I^{ad}_2(t)$ flows around times when $T_A(t)T_B(t)$ is maximal. Consequently, for a driving where contacts A and B are never both open at the same time there is no adiabatic current flow, or equivalently the adiabatic frequency limit $\Delta \min[T_A(t) + T_B(t)]/\hbar \to 0$.

From this reasoning it follows equally that for frequencies in the non-adiabatic regime, $\omega \gg \Delta \min[T_A(t) + T_B(t)]/\hbar$, we can neglect the current flow during times when both contacts are open. This leads to the standard physical picture in terms of charging and discharging of the DB-region: for the cycle $0 < t < T$ (mod $T$), i) at
times $0 < t < T/2$ contact B closed and charge is flowing into the DB-region through A. ii) at times $T/2 < t < T$ contact A is closed and charge is flowing out through B.

Focusing first on non-adiabatic frequencies much smaller than the level spacing, $\hbar \omega \ll \Delta$, the charge density inside the DB-region is uniform and a calculation of the charge in the DB-region, injected in the bias window, gives $Q(t) = eF(t)$. Thus, $F(t)$ is just the probability to find an electron inside the DB. The time development of the charge is found from Eq. (2),

$$Q(t) = \left\{ \begin{array}{ll}
  \bar{p}_A(t) + (1 - \bar{p}_A(t))Q(0) & \text{charging} \\
  (1 - \bar{p}_B(t))Q(T/2) & \text{discharging}
\end{array} \right. \quad (4)$$

where e.g. $\bar{p}_B(t) = 1 - \prod_{\tau=0}^{T/2} R_B(t-\tau\hbar)$ is the probability that an electron inside the DB at time $T/2$ has been transmitted out through contact B at time $t$, with $P_B = \int_{-\infty}^{\infty}[dt/R_B(t)]$. $\bar{p}_A(t)$ and $P_A$ are given analogously.

The charge at the opening/closing is $Q(T) = Q(0) = \bar{p}_A(1 - \bar{p}_B)/\bar{p}_A + \bar{p}_B - \bar{p}_A\bar{p}_B$ and $Q(T/2) = (1 - \bar{p}_B)$ where $\bar{p}_A = p_A(T/2), \bar{p}_B = P_B(T)$. The current $I_2(t) = (\Delta/\hbar)T_B(t)Q(t)$ is shown in Fig. 2.

For times $t$ not close to the opening times of $A$ and $B$, i.e. $P_A, P_B \gg 1$, we can write e.g. $1 - \bar{p}_B(t) = e^{\sum_{\tau=0}^{T/2}\ln[R_B(t-\tau\hbar)]} \approx e^{(1/2\omega)\int_{0}^{\omega} \ln[R_B(t')]dt'}$. It is instructive to compare this with the charging and discharging of a classical RC-circuit with a capacitance $C$ and a slowly time-varying resistance $R(t)$, for which $e^{-\int_{0}^{T/2}CR(t')^{-1}dt'}$ corresponds to $1 - p_B(t)$. This gives a capacitance $C = e^2/\Delta$ and a resistance $R(t) = (\hbar/e^2/\ln[1/R_B(t)])$, providing a turnstile analogy of the models for the on-demand source discussed in [7, 8].

The transferred charge per cycle, $Q(T/2) = Q(T)$, is

$$Q_2 = -Q_3 = e\bar{p}_A\bar{p}_B/[(\bar{p}_A + \bar{p}_B - \bar{p}_A\bar{p}_B)] \quad (5)$$

This gives that for $\omega \ll \omega_{\max}^A, \omega_{\max}^B$ with $\hbar\omega_{\max}^A, \omega_{\max}^B = \Delta\min\{1, \int_{0}^{T}(dt/T)\ln[1/R_A(t/B)]\}$, we have $\bar{p}_A, \bar{p}_B = 1$ and $Q_2 = -Q_3 = e_i.e.$ exactly one electron and one hole are transferred. Taken together, this yields a frequency interval $\Delta\min[T_A(t) + T_B(t)]/\hbar < \omega \ll \omega_{\max}^A, \omega_{\max}^B$ for the ideal turnstile cycle shown in Fig. 1. For higher frequencies electrons do not have time to completely charge or discharge the DB-region and $Q_2 < e$.

Importantly, for tunnelling contacts $T_A(t), T_B(t) \ll 1$ and $\omega \ll \Delta/\hbar$ we can directly expand $F(t - 2\tau) = F(t) - 2\tau dF(t)/dt$ in Eq. (2) and arrive at

$$dP_1/dt = -\Gamma_A(t)P_1(t) + \Gamma_B(t)P_0(t) \quad (6)$$

where $P_1(t) = F(t) = 1 - P_0(t)$ and $\Gamma_{A/B}(t) = T_{A/B}(t)/\Delta/\hbar$. This is a master equation with time dependent tunnelling rates, investigated in e.g. [16, 22, 23].

At frequencies $\omega \sim \Delta/\hbar$ the expression in Eqs. (4) and (6) break down and transport through higher/lower lying resonances become visible, manifested as sharp dips in the transferred charge as a function of frequency, see Fig. 2. The most pronounced set of dips, at frequencies

$$\hbar\omega = \Delta(2n + 1 \pm 1/m) \quad (7)$$

results from electrons, which after being injected at $A$ at maximal $T_A(t)$, circulate around the DB-region $m$ times during $(2n + 1) \pm 1$ periods before escaping back out at $A$ at maximal $T_A(t)$, not transferring any charge. For a long measurement time $t_0 = N\sqrt{T}, N \gg 1$, to characterize the accuracy of the turnstile it is important to investigate not only the average charge transferred per cycle, $Q_2 = (1/N)\int_{0}^{t_0} dtI_2(t)$, but also the fluctuations [19, 24, 25] experimentally accessible via current correlations [25]. To this end we first write the current $I_2(t) = \sum_{q,n} \langle i_{2,q}\rangle \exp(i \omega t)$, with $i_{2,q} = (e/\hbar)\int dEj_{2,q}(E)$ and $j_{2,q}(E) = j_{2,q}^{\text{bias}}(E) + j_{2,q}^{\text{pump}}(E)$ where

$$j_{2,q}^{\text{bias}}(E) = \sum_{n} [T_{21,q}^{n}(E) + T_{21,q}^{n}(E)] [f_0(E_n) - f_0(E)]$$

$$j_{2,q}^{\text{pump}}(E) = \sum_{n} [T_{21,q}^{n}(E) + T_{21,q}^{n}(E)] [f_V(E_n) - f_0(E_n)]. \quad (8)$$

Here $E_0 = E + n\hbar\omega, T_{21,q}^{n}(E) = f_2(E, E_n)t_{20}(E_0 - E_n), \alpha = 1, 4$ and $t_{20}(E, E_n) = \int_{0}^{T}(dt/T)e^{i\omega t}t_{20}(t, E)$ with $t_{20}(t, E)$ given above and $t_{24}(t, E) = \rho_B(t) + t_B(t)\sum_{q=0}^{\infty}e^{i2(q+1)\phi(E)}L_q(t)a_{2}(t-2q + 1)\gamma(t)B(t-2q + 1)\gamma$. The current at terminal 3 is found similarly.
The auto-correlations of transferred charge at terminal 2 is $S_{22} = (1/N) \int_0^{\infty} dt \int_0^{\infty} dt' \langle \Delta I_2(t) \Delta I_2(t') \rangle$ where $\Delta I_2(t)$ is the current fluctuations. Calculations following Ref. 19 give $S_{22} = g_{\text{eq}}^2 + S_{\text{th}}$ with

$$S_{\text{eq}} = \frac{T e^2}{h} \int dE \left[ j_{2,0} - 2 f_0(E) - \sum_q |j_{2,q}|^2 \right]^2 \quad (9)$$

and $S_{\text{th}} = 2T(e^2/h)kT$ the thermal noise in the absence of both drive and bias. The auto correlator $S_{33}$ and the cross correlators $S_{22} = S_{23}$ are found similarly.

We first consider the correlations at $\omega, kT < \Delta$. In this regime the fluctuations are minimized for DB-levels of both drive and bias. The auto correlator $S_{33}$ and the cross correlators $S_{22} = S_{23}$ are found similarly.

For large frequencies $\omega \sim \Delta$ both the components $j_{2,q}^{\text{pump}}$ and $j_{2,0}^{\text{bias}}$ contribute to the correlations. The correlations are evaluated numerically, shown in Fig. 2.

In conclusion we have analysed a mesoscopic turnstile implemented in a double barrier system in the quantum Hall regime. At ideal operation the turnstile produces one electron and one hole at different locations per driving cycle. The noise due to the driving is found to be negligibly small at frequencies for ideal operation.

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