Experimental and numerical studies on the mixing at the intersection of millimetric channels

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Abstract.
In this work, experimental and numerical results on the effect of diffusion and geometrical dispersion on the mixing of confluent flows are presented. Two channels with an internal diameter \( D_h = 4 \text{ mm} \) intersect with an angle \( \alpha = 30, 60, 90, 120, 150, 180^{°} \). The experimental setup allows to accurately control the flow rate and assures a constant flow at both inlets. The mixing properties are studied by injecting pure water in one inlet and colored water at the other. The effects of the inlet flow and the intersection angle on the diffusion of ink is analyzed. We observed that the mixing by convection is only important for \( \alpha = 180^{°} \). For other angles, diffusion is the main mechanism for mixing.

1. Introduction
In both theoretical and experimental research, porous media are modeled in different ways such as obstacles distribution, network of channels of different sizes, granular media and artificial fractures just to mention a few [1–8]. An additional approximation made is to study the flow using Hele-Shaw cells, that is, to employ two-dimensional configuration in order to simplify the theoretical analysis or, in experiments, to optimize the flow visualization [9, 10].

A subject, closely related to the presented here, which has recently received a great interest deals with the mixing at the micrometer scale. Its study is essential to improve our understanding of the transport phenomena in channels as well as in network of channels. When the Reynolds numbers are small, as usual in the flows in porous media, the main mechanism of interaction is through molecular diffusion [1, 15]. Due to this fact, the diffusion time is a key parameter to be taken into account when designing micromixers. Nevertheless, the importance of the geometry has been recognized and, recently, many geometrical configurations have been explored [11–14], all of them trying to reduce the mixing length and mixing time.

Despite the large number of works on this field, a systematic study on the impact of the geometry parameters on the mixing process has been seldom studied. Here, we show results in the investigation of the effect of the crossing angle \( \alpha \) on the mixing of the species. This has been achieved by analyzing the behavior of the flow and its dispersion in the intersection of two channels in the stationary regime.
2. Experimental setup

A schematic view of the experimental setup is shown in figure 1. The liquids are injected into two crossing channels at the same flow rate by using a double-syringe pump (Harvard Apparatus [16]). The millimetric pipes cross with angles, defined as the angle between the injection channels, of $\alpha = 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ$ and $\alpha = 180^\circ$. The inner diameter of the channels is $D_h = 4mm$, constructed by drilling a solid block of transparent plexiglass, as shown in figure 2. The external and internal surfaces of the channels were carefully polished in order to obtain an optimal visualization of the flow. The drainage pipes are identical in length and diameter and to ensure the same impedance at both outlets, they are submerged in a vessel at the same depth.

The injected liquids are (a) pure distilled water and (b) a mixture of distilled water with blue ink (water blue). The density and viscosity of both liquids are the same, given the low concentration of ink needed to obtain a reasonable intensity of color. The diffusion constant for the ink in water is $D = 10^{-11} m^2/s$. The blue color of the ink allows to analyze only the red channel of the RGB images, where a darker level indicates a higher concentration of blue tonality. The channels were illuminated by transmission using a uniform light source, the illumination being controlled and kept constant during all the experiences. The analysis of the images is carried out after subtracting each image with ink from the image without ink.

The study presented here corresponds with stationary state regime, so the images are taken after an injection of about 10 times the volume of the mixing zone. We do not quantify the amount of ink in the experiments but only qualitatively compare the images with those obtained from the simulations.

![Figure 1.](image1.jpg) 
**Figure 1.** Experimental setup for the study of the flow in the X mixer.

![Figure 2.](image2.jpg) 
**Figure 2.** Top view of the X millichannel.
3. Formulation and numerical issues
The flow is described by the Navier-Stokes and continuity equations:

\[
\rho \frac{\partial u}{\partial t} - \eta \nabla \cdot (\nabla u + (\nabla u)^T) + \rho (u \cdot \nabla) u + \nabla p = 0
\]  

(1)

\[
\nabla \cdot u = 0
\]  

(2)

where \(\rho\) is the density, \(u\) is the velocity, \(\eta\) is the viscosity and \(p\) the pressure. The considered fluid is water with \(\eta = 1 \times 10^{-3} \text{N.s/m}^2\) and \(\rho = 1 \times 10^{3} \text{kg/m}^3\). The convection-diffusion process is governed by the following equation:

\[
\frac{\partial c}{\partial t} = -\nabla \cdot (-D \nabla c) - u \cdot (\nabla c) = 0,
\]  

(3)

where \(D = 10^{-11} \text{m}^2/\text{s}\) represents the diffusion coefficient and \(c\) is the ink concentration. The boundary conditions at the inlets are \(c = 0\) and \(c = 1 \text{mol/m}^3\), similar to the experiments. Considering this low concentration value, it is licit to assume that the solute molecules only interact with water molecules (without interacting with themselves) and thus, it is possible to use Fick’s law to describe the diffusion process. We also assume that density and viscosity of the water is not modified by the presence of the ink. Under these assumptions, it is legitimate to first solve the Navier-Stokes equations at steady state and then use its solution as a base flow to solve the steady convection-diffusion equation. Nevertheless, in order to check the validity of the numerical solutions and explore some experimental issues not shown in this work, we solved the complete partial differential equation given in (3) until the steady state is reached. The convergence criteria employed to assume a stationary solution are that the concentration at the outlets and the velocity at several control points vary less than 0.5% with time.

In order to simplify the simulation and to exploit the symmetry of the problem, we have studied only one half of the volume, as shown in figure 3(a). The geometry is meshed using tetrahedral elements, as schematized in Figure 3(b). The maximum elements size are \(5 \times 10^{-4} \text{m}\) at the intersection volume and \(8 \times 10^{-4} \text{m}\) at the outlets channels. In total, we have a mesh of about 10600 finite elements, which corresponds to approximately 68100 degrees of freedom. As solver we have chosen the direct PARDISO. For the diffusion equations, the model includes an isotropic artificial diffusion as stabilization procedure. Briefly, the method increases the effective value of \(D\). This increment, proportional to the cell size and local velocity, is kept as small as possible to obtain meaningful and realistic solutions. The artificial diffusivity is iteratively reduced down to a value that assures a stationary solution without spurious oscillations. The artificial isotropic diffusion is necessary in order to keep the local Peclet number near 1 (\(\text{Pe} = \nu h / 2D\), where \(\nu\) is the convective velocity, \(h\) is the mesh element size and \(D\) is the diffusion coefficient). In our case, due to the computer memory limitations it is not possible to reduce \(h\). Instead, we need to adjust the coefficient diffusion as explained above.

4. Results
The working regime of mixers as well as micromixers is usually characterized in terms of the Reynolds number, \(Re = UD_h/\nu\), and the Peclet number, \(Pe = D_h V / D\), where, \(V\) is a typical velocity, \(D_h\) is the hydraulic diameter and \(\nu\) the kinematic viscosity. Nevertheless, both dimensionless numbers are related to each other through the Schmidt number, \(Sc = Pe/Re = \nu / D\). For a given fluid, \(Sc\) is a constant, in our problem \(Sc = 10^5\). Thus, in what follows, both numbers are indistinctly employed to describe the flow.

Figure 4(a) shows the streamlines for a case with \(Re = 40\). The flow is symmetric with respect to a bisecting plane that is not crossed by the streamlines. The velocity at the central
Figure 3. (a) Sketch of the numerical domain and position of the inlets and outlets. (b) Size distribution of the tetrahedral elements.

Figure 4. Case with $Q = 6\text{ml/min}$ (Re=40), $\alpha = 90^\circ$. (a) Streamlines are symmetric with respect to a bisecting plane. (b) Velocity: the highest value occurs at the mixing zone. In both figures, inlets are at right and top channels.

zone, Figure 4(b), is increased from $0.01m/s$ (velocity at the inlets) to $0.0139m/s$, an increment that exactly corresponds to a section narrowing that occurs in the central zone.

Figure 5(a) shows the ink distribution for an experiment with $\alpha = 90^\circ$ and an inflow $Q = 6\text{ml/min}$. The numerical results are in good agreement with the experiments, as shown in Figure 5(b). In this experiment, ink is not detected at the outlets while in the simulation the ink concentrations at the outlets are $0.93\text{mol/m}^3$ and $0.06\text{mol/m}^3$, respectively. This slight difference between the experiment and the simulation can be explained in terms of the isotropic artificial diffusion employed to solve the convection-diffusion equation, which induces diffusion of a small amount of ink towards the opposite outlet.
Figure 5. Concentration of blue ink for case with $Q = 6\text{ml/min}$ and $\alpha = 90^\circ$. (a) Experiment. The image was rotated for an easier comparison with numerical result. (b) Simulation. Here, the red color represents the maximum of ink concentration, the blue color is pure water. In both figures, inlets are at right and top channels.

In general, experiments show that diffusion is almost negligible. This observation is in agreement with numerical results and can be explained in terms of the large values for the Peclet numbers ($Pe > 10^3$). Effectively, $Pe$ is the ratio between the diffusion time, $\sim D^2/h$ to the characteristic transit time in the mixing zone, $\sim Dh/V$, $V$ being a characteristic velocity. Figure 6(a) shows that for an inlet flow of $Q = 1\text{ml/min}$, there is a presence of ink at the water outlet (upper right corner of the figure). When $Q$ is increased, diffusion is less important (see figure 6(b)) and, for example for the large inflow $Q = 20\text{ml/min}$, ink can not be detected at the water outlet.

We analyzed the effect of the crossing angle $\alpha$ on the diffusion of the ink. Although several angles were studied, only two typical values (60° and 90°), for the sake of simplicity are presented. For the range of $5 < Re < 150$, given $Q$, we did not appreciate significant differences in the amount of ink in the water outlet when $\alpha$ is changed. The exception is the case with $\alpha = 180^\circ$, for which the ink equally convects to both outlet pipes, as shown in figure 7. Nevertheless, it is worth to point out that the geometry for $\alpha = 180^\circ$ differs from other cases in that the outlets
channels are not aligned to the inlets and, thus, the ink is equally splitted to the outlets.

Figure 7. Flow for the case with $Q = 10\text{ml/min}$ and $\alpha = 180^\circ$. On the contrary of what happens for other values of $\alpha$, here the ink convects toward both outlet pipes. Here, the inlets are on the left, outlets on the right.

5. Conclusions
The convection-diffusion behavior of a colored liquid injected in a X millichannel is characterized. Numerical results show that the flow is symmetric with respect to a bisection plane that is not crossed by the streamlines. The velocity is increased in the confluence zone due to a reduction of the channel volume. The effect of the crossing angle on the diffusion of the ink towards the pure water outlet is negligible except for the case $\alpha = 180^\circ$, for which the ink equally convects to both outlet channels.

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