On neutrino oscillations and time–energy uncertainty relation

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Abstract
We consider neutrino oscillations as a non-stationary phenomenon. We show that the time–energy uncertainty relation plays a crucial role in neutrino oscillations.

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The recent accelerator K2K [1] and MINOS experiments [2] are important steps in the study of the problem of neutrino masses and mixing. We would like to draw attention to the fact that the K2K and MINOS experiments are neutrino oscillation experiments of a new type: in these experiments, the time of production and the time of detection of neutrinos were measured for the first time. From our point of view, these experiments are important not only for the confirmation of the existence of the phenomenon of neutrino oscillations, but also for the understanding of the physics of this new phenomenon.

It was established by the K2K and MINOS experiments that neutrino oscillations is a phenomenon with a finite time interval during which the neutrino state is significantly changed. For such phenomena, the time–energy uncertainty relation

\[ \Delta E \Delta t \geq 1 \] (1)

holds (see, for example, [3]). This means that neutrinos are described by non-stationary states.

A neutrino \( \nu_l \) produced in a weak CC process together with a lepton \( l^* \) is described by a mixed flavour state

\[ |\nu_l\rangle = \sum_i U_{li}^* |\nu_i\rangle, \] (2)

where \( U \) is a unitary mixing matrix and \( |\nu_i\rangle \) is the state of a neutrino (Majorana or Dirac) with mass \( m_i \) and momentum \( \vec{p} \).

The evolution equation in the quantum field theory is the Schrödinger equation

\[ i \frac{\partial}{\partial t} |\Psi(t)\rangle = H |\Psi(t)\rangle. \] (3)

If, at \( t = 0 \), a flavour neutrino \( \nu_l \) is produced, it follows from (3) that, at time \( t \), the neutrino is described by the non-stationary state

\[ |\nu_l\rangle = \sum_i e^{-iE_i t} U_{li}^* |\nu_i\rangle = \sum_i |\nu_f\rangle \sum_j U_{ji} e^{-iE_j t} U_{li}^*. \] (4)

From (4) it follows that the state of the neutrino is significantly changed at the time \( t \), which satisfies the inequality

\[ (\Delta E)_\nu t \geq \frac{\Delta m^2_{\nu} L}{2E} \geq 1. \] (5)

From the results of K2K and MINOS experiments, it follows that \( t \simeq L \). Taking into account this relation from (4), we come to the standard expression for transition probability

\[ P(\nu_l \rightarrow \nu_f) = |\delta_{lf} + \sum_{i \neq f} U_{li} (e^{-i\Delta m^2_{\nu}\frac{L}{2E}} - 1) U_{li}^*|^2. \] (6)

It is obvious that the time–energy uncertainty relation (5) coincides with the well-known necessary condition to observe neutrino oscillations

\[ \frac{\Delta m^2_{\nu} L}{2E} \geq 1. \] (7)

We assumed that the flavour neutrino state is determined by its momentum. In the approach based on the Schrödinger equation, this is the only possibility compatible with experiment. However, it was stated in the literature (see [4] and references therein) that in neutrino oscillations, time is not measured and only distance \( L \) is relevant. In this approach, the oscillation probabilities are averaged over time and energies of different mass components of flavour neutrino states are the same. In spite of the final expression for the transition probability coinciding with the standard one (equation (6)),
this approach to neutrino oscillations does not correspond to the results of the accelerator neutrino oscillation experiments.

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