Non-unified gaugino masses in supersymmetric missing partner models with hypercolor

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Abstract

The gaugino mass relations $m_3/g_3^2 = m_2/g_2^2 = m_1/g_1^2$ are considered to be robust signals for supersymmetric grand unification. In this letter, we point out that these relations may be significantly modified in an interesting class of models which solve the doublet-triplet splitting problem using a missing partner mechanism together with a strong hypercolor gauge group. The observation of non-unified gaugino masses, together with unified sfermion masses, provides a distinctive signature for these models.

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Supersymmetric grand unified theory (SUSY-GUT) \([\[1\]]\) is one of the most attractive candidates of new physics beyond the standard model. It is supported by the precision measurements of the gauge coupling constants of \(SU(3)_C \times SU(2)_L \times U(1)_Y\) \([2]\). However, it may be that the unification of gauge couplings is an accident. What further evidence can support the existence of a SUSY-GUT? If supersymmetric particles are discovered, we may hope to see signatures for unification in the superpartner spectrum. In particular, the pattern of sfermion masses can probe the unification of quark and lepton multiplets, while the gaugino masses can signal the existence of a unified gauge group at the GUT scale. Between the two, the weak scale scalar mass relations are more sensitive to the physics between the weak and GUT scales, whereas the gaugino mass relations\([1]\)

\[
\frac{m_3}{g_3^2} : \frac{m_2}{g_2^2} : \frac{m_1}{g_1^2} = \frac{m_{\text{GUT}}}{g_{\text{GUT}}^2} : \frac{m_{\text{GUT}}}{g_{\text{GUT}}^2} : \frac{m_{\text{GUT}}}{g_{\text{GUT}}^2} = 1 : 1 : 1, \tag{1}
\]

where \(g_3, g_2, g_1\) and \(g_{\text{GUT}}\) (\(m_3, m_2, m_1\) and \(m_{\text{GUT}}\)) represent the gauge coupling constants (gaugino masses) for \(SU(3)_C\), \(SU(2)_L\), \(U(1)_Y\), and \(G_{\text{GUT}}\), respectively\([2]\). They are a more robust prediction of gauge unification. However, these simple relations \([1]\) may also arise in models with no unified gauge group at high scales, as in some string theories with dilaton-dominated SUSY breaking \([3]\), or theories with low energy dynamical SUSY breaking \([4]\). Thus, we conclude that if the sfermion masses do not satisfy the GUT mass relations, grand unification is by no means ruled out. On the other hand, verifying the gaugino mass relations, while extremely exciting, would not suffice as a proof for grand unification.

However, what if the opposite happened? Then there would be little question as to quark-lepton unification, but what of gauge unification? In this letter, we point out that non-unified gaugino masses can in fact arise naturally in a very interesting class of grand unified theories, providing a unique signature for these type of models and a window into physics above the GUT scale.

Grand unified models suffer from a serious problem, the “doublet-triplet splitting problem”. With a unified gauge group like \(SU(5)\) or \(SO(10)\), Higgs doublets are accompanied by color-triplet Higgses. Higgs doublets are responsible for the electro-weak symmetry breaking, and hence their masses are of the order of electro-weak scale. On the other hand, the

\[1\] We neglect the higher loop effects of \(SU(3)_C \times SU(2)_L \times U(1)_Y\) gauge coupling constants, as they are quite small.

\[2\] We choose the GUT normalization for \(g_1\), i.e., \(g_1 = \sqrt{\frac{5}{3}} g_Y\).

\[3\] In the case of low energy dynamical SUSY breaking, the usual gaugino mass relations only follow if the vector-like fields transmitting SUSY breaking to the ordinary sector form complete representations of \(SU(5)\), which can perhaps be taken as indirect evidence for unification.
stability of the nucleon and/or successful unification of the gauge coupling constants require
the colored Higgs masses to be of the order of the GUT scale, \(M_{\text{GUT}} \sim 10^{16}\) GeV, which is
much larger than the doublet Higgs masses \[5\]. In the minimal SUSY SU(5) model, this mass
hierarchy is obtained by an extreme fine tune among several parameters in the superpotential.
Many attempts have been made to solve this problem \[5, 6, 7\].

Recently, an interesting mechanism has been proposed to solve the doublet-triplet splitting
problem \[9, 10, 11, 12\]. It is based on an enlarged gauge group, \(G_{\text{GUT}} \times G_{\text{H}}\), like SU(5)_{\text{GUT}} \times
SU(3)_{\text{H}} (\times U(1)_H) or SO(10)_{\text{GUT}} \times SO(6)_{\text{H}}\) (where the subscript ”H” stands for hypercolor).
A characteristic feature of these models is that the SU(3)_{\text{C}} group is a diagonal subgroup of
SU(3)_{\text{GUT}} and SU(3)_{\text{H}}, where SU(3)_{\text{GUT}} \in G_{\text{GUT}} and SU(3)_{\text{H}} \in G_{\text{H}}, while SU(2)_{\text{L}} is embed-
ded only in \(G_{\text{GUT}}\). The doublet-triplet splitting problem is solved by the missing partner
mechanism \[6\], which in this case can work with smaller matter multiplet representations,
guaranteeing a perturbative picture of \(G_{\text{GUT}}\) up to the Planck scale.

Even though the unified gauge group is not simple, unification of the gauge coupling
constants of SU(3)_{\text{C}} \times SU(2)_{\text{L}} \times U(1)_Y is not spoiled if the gauge couplings of \(G_{\text{H}}\) are large
enough. In this case, the corrections to the gauge couplings of the low energy gauge group can
be smaller than what can be distinguished by precision tests of the gauge coupling constants.
However, we will show that the gaugino masses may deviate from the usual GUT relation
(1), while scalar masses should still be unified at the GUT scale if they belong to the same
multiplet of the GUT group.

Let us first review the \(G_{\text{GUT}} \times G_{\text{H}}\) model. To make our points clear, we will concentrate
on the model based on the gauge group SU(5)_{\text{GUT}} \times SU(3)_{\text{H}} \times U(1)_{1\text{H}} given in Ref. \[10\]. The
generalization to other models is straightforward.

In the SU(5)_{\text{GUT}} \times SU(3)_{\text{H}} \times U(1)_{1\text{H}} model, the particle content which is responsible for
the breaking of \(G_{\text{GUT}} \times G_{\text{H}}\) group consists of the following chiral supermultiplets:
\(Q(5^*, 3, 1), \bar{Q}(5, 3^*, -1), q(1, 3, 1), \bar{q}(1, 3^*, -1), \Sigma(24, 1, 0), H(5, 1, 0),\) and \(\bar{H}(5^*, 1, 0),\) where the num-
bers in brackets denote the transformation properties under SU(5)_{\text{GUT}}, SU(3)_{\text{H}} and U(1)_{1\text{H}},
respectively. The superpotential of the model is given by

\[
W = \bar{Q}_A^\alpha (m_Q q_B^A \lambda \Sigma^A_B) Q^B_{\alpha} + \frac{1}{2} m_{\Sigma} \text{tr}(\Sigma^2) + h H_A Q^A_{\alpha} q^{\alpha} + h' \bar{H}^A \bar{Q}^A_{\alpha} q_{\alpha},
\]

where \(\lambda, h\) and \(h'\) are dimensionless coupling constants, while \(m_Q\) and \(m_{\Sigma}\) are mass parameters
of the order of the GUT scale \(M_{\text{GUT}} \sim 10^{16}\) GeV. Minimizing the potential, we find that there
is a vacuum in which the scalar components of \(Q, \bar{Q}, \) and \(\Sigma\) have the following vacuum
expectation values (VEVs),

\[
\langle Q \rangle = \langle \bar{Q}^T \rangle = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix},
\]

\[
\langle \Sigma \rangle = \frac{m_Q}{2\lambda} \text{diag}(3, 3, -2, -2, -2),
\]

with \( v^2 = 5m_Qm_\Sigma/\lambda^2 \), while other fields do not acquire a VEV.

In the vacuum given in eqs. (3) and (4), the gauge symmetry SU(5)_{GUT} \times SU(3)_H \times U(1)_H is broken down to the standard model gauge group SU(3)_C \times SU(2)_L \times U(1)_Y at the GUT scale. As we can see from the last two terms in eq. (2), the colored Higgses, \( H_I \) and \( \bar{H}^I \) \((I = 3 - 5)\), get large masses from the \( Q, \bar{Q} \) VEVs by marrying with \( q^\alpha \) and \( \bar{q}^\alpha \), while doublet Higgses, \( H_i \) and \( \bar{H}_i \) \((i = 1, 2)\), remain massless. Thus, the doublet-triplet splitting is naturally achieved in this model due to the missing partner mechanism.

An important feature of the G_{GUT} \times G_H model is that the low energy SU(3)_C symmetry is the diagonal subgroup of SU(3)_{GUT} \times SU(3)_H, where SU(3)_{GUT} is embedded in SU(5)_{GUT}. The gauge field of SU(3)_C, \( G_\mu \), is given by a linear combination of the gauge field of SU(5)_{GUT}, \( A_{GUT,\mu} \), and of SU(3)_H, \( A_{H3,\mu} \), as

\[
G_\mu = \frac{1}{\sqrt{g^2_{H3} + g^2_{GUT}}} (g^2_{H3} A_{GUT,\mu} + g^2_{GUT} A_{H3,\mu}),
\]

where \( g_{GUT} \) and \( g_{H3} \) are the gauge coupling constants for SU(5)_{GUT} and SU(3)_H at the GUT scale, and the gauge coupling \( g_3 \) of SU(3)_C is given by, (in the following equations (6) – (11), all relations are understood to hold at the GUT scale)

\[
g_3^2 = \frac{g^2_{H3} g^2_{GUT}}{g^2_{H3} + g^2_{GUT}},
\]

or in terms of \( \alpha(= g^2/4\pi) \),

\[
\frac{1}{\alpha_3} = \frac{1}{\alpha_{GUT}} + \frac{1}{\alpha_{H3}}.
\]

Similarly, U(1)_Y is a subgroup of U(1)_{GUT} \times U(1)_H, and its gauge field \( B_\mu \) and gauge coupling constant \( g_1 \) are given by

\[
B_\mu = \frac{1}{\sqrt{15g^2_{H1} + g^2_{GUT}}} (\sqrt{15} g^2_{H1} A_{GUT,\mu} + g^2_{GUT} A_{H1,\mu}),
\]

\[
g_1^2 = \frac{15g^2_{H1} g^2_{GUT}}{15g^2_{H1} + g^2_{GUT}}.
\]
On the other hand, $SU(2)_L$ is embedded only in $SU(5)_{GUT}$, and hence its gauge field $W_\mu$ and gauge coupling constant $g_2$ are given by

\begin{align}
W_\mu &= A_{GUT,\mu}, \\
g_2^2 &= g_{GUT}^2.
\end{align}

The important point is that $g_1(M_{GUT}) \simeq g_2(M_{GUT}) \simeq g_3(M_{GUT})$ if $g_{H3}(M_{GUT}), g_{H1}(M_{GUT}) \gg g_{GUT}(M_{GUT})$. Thus, the gauge coupling unification is not spoiled if the gauge coupling constants of $G_H$ are large enough. As a result, it is very difficult to distinguish $G_{GUT} \times G_H$ model with the ordinary GUTs by the precise measurements of the gauge coupling constants.

In fact, recent analyses show that the predicted strong coupling constant from SUSY-GUT without including threshold corrections, $\alpha_s(M_Z) = 0.130$ \cite{13, 14}, is a little bit higher than the world averaged experimental value, $\alpha_s(M_Z) = 0.118 \pm 0.003$ \cite{15}. We can see from eqs.\((\ref{eq:10})\) and \((\ref{eq:11})\) that the correction from the hypercolor gauge coupling reduces the $SU(3)_C$ gauge coupling, and hence shifts the prediction in the right direction to be consistent with the experimental value. The correction from the $U(1)_H$ coupling also moves the prediction in the right direction by changing the unification scale. Let the shift of the inverse couplings of $SU(3)_C$ and $U(1)_Y$ at the GUT scale due to $G_H$ couplings be $\delta_3$ and $\delta_1$,

\begin{align}
\delta_3 &= \frac{1}{\alpha_3(M_G)} - \frac{1}{\alpha_{GUT}(M_G)} = \frac{1}{\alpha_{3H}} - \frac{1}{\alpha_{GUT}(M_G)} = \frac{1}{15\alpha_{1H}}.
\end{align}

A simple calculation using one-loop renormalization group equations (RGEs) gives the shift of the inverse of the predicted strong coupling constant due to $\delta_3$ and $\delta_1$ as

\begin{align}
\Delta \left(\frac{1}{\alpha_s}\right) = \delta_3 + \frac{5}{7} \delta_1.
\end{align}

So, the prediction of $\alpha_s(M_Z)$ in this model can be written as

\begin{align}
\alpha_s(M_Z) &\simeq 0.130 - \frac{0.014}{\alpha_{3H}} - \frac{0.010}{15\alpha_{1H}} + \Delta_{\alpha_s},
\end{align}

where the first term is the ordinary SUSY-GUT prediction, and the last term represents the extra threshold corrections. Typically it is found that $|\Delta_{\alpha_s}| < 0.01$ \cite{13}. Then in order to be consistent with the experimental value, we require $\alpha_{3H} > 0.6$ (for $\delta_1 = 0$) and $\alpha_{1H} > 0.03$ (for $\delta_3 = 0$).

Now, we are in a position to discuss the gaugino masses in the $G_{GUT} \times G_H$ model. The gaugino masses originate in the soft SUSY breaking terms, whose origin is related to the mechanism of the SUSY breaking. In the main part of this letter, we assume a hidden-sector
SUSY breaking scenario, in which SUSY breaking is mediated by supergravity. The gauginos have the following mass terms above the GUT scale,

\[ \mathcal{L} = -\frac{1}{2} m_{\text{GUT}} \lambda_{\text{GUT}} \lambda_{\text{GUT}} - \frac{1}{2} m_{H3} \lambda_{H3} \lambda_{H3} - \frac{1}{2} m_{H1} \lambda_{H1} \lambda_{H1} + h.c., \]  

(15)

where \( \lambda_{\text{GUT}}, \lambda_{H3} \) and \( \lambda_{H1} \) (\( m_{\text{GUT}}, m_{H3} \) and \( m_{H1} \)) are gauginos (gaugino masses) for SU(5)\(_{\text{GUT}}\), SU(3)\(_{H}\) and U(1)\(_H\) gauge groups, respectively. Heavy particles decouple at the GUT scale, and below the GUT scale, we have the SUSY standard model as a low energy effective theory.

In particular, gauginos for the standard model gauge group SU(3)\(_C\), SU(2)\(_L\) and U(1)\(_Y\) (which we denote \( \tilde{G}, \tilde{W} \) and \( \tilde{B} \)), are given by, (eqs.(16) – (22) are understood to hold at the GUT scale)

\[ \tilde{G} = \frac{1}{\sqrt{g_{H3}^2 + g_{\text{GUT}}^2}} (g_{H3} \lambda_{\text{GUT}} + g_{\text{GUT}} \lambda_{H3}), \]  

(16)

\[ \tilde{W} = \lambda_{\text{GUT}}, \]  

(17)

\[ \tilde{B} = \frac{1}{\sqrt{15 g_{H1}^2 + g_{\text{GUT}}^2}} (\sqrt{15} g_{H1} \lambda_{\text{GUT}} + g_{\text{GUT}} \lambda_{H1}). \]  

(18)

Substituting eqs.(16) – (18) into eq.(15), we obtain masses for \( \tilde{G}, \tilde{W} \) and \( \tilde{B} \) as

\[ m_3 = g_3^2 \left( \frac{m_{H3}}{g_{H3}^2} + \frac{m_{\text{GUT}}}{g_{\text{GUT}}^2} \right), \]  

(19)

\[ m_2 = m_{\text{GUT}}, \]  

(20)

\[ m_1 = g_1^2 \left( \frac{m_{H1}}{15 g_{H1}^2} + \frac{m_{\text{GUT}}}{g_{\text{GUT}}^2} \right). \]  

(21)

We can see that the GUT relation on the gaugino masses (1) is modified

\[ \frac{m_3}{g_3^2} : \frac{m_2}{g_2^2} : \frac{m_1}{g_1^2} = \left( \frac{m_{\text{GUT}}}{g_{\text{GUT}}^2} + \frac{m_{H3}}{g_{H3}^2} \right) : \frac{m_{\text{GUT}}}{g_{\text{GUT}}^2} : \left( \frac{m_{\text{GUT}}}{g_{\text{GUT}}^2} + \frac{m_{H1}}{15 g_{H1}^2} \right). \]  

(22)

The above relations receive negligible modification in running from the GUT to the weak scale. Thus, if the ratio \( m_{H3}/g_{H3}^2 \) or \( m_{H1}/g_{H1}^2 \) at the GUT scale is comparable to \( m_{\text{GUT}}/g_{\text{GUT}}^2 \), significant deviations from the usual GUT relations (1) can be observed when gaugino masses are measured. Notice that the combinations \( m_{H3}/g_{H3}^2 \) and \( m_{H1}/g_{H1}^2 \) are renormalization group invariants at the one loop level. So, in contrast with the gauge coupling, at one loop the corrections to the gaugino mass relations do not diminish as the hyper gauge couplings become large. On the other hand, in this model, squarks and sleptons are contained in \((5^*, 1, 0)\) or \((10, 1, 0)\) representation of the unified gauge group, as in the ordinary GUT. Thus, sfermion
mass unification is still expected. These facts suggest that the mass spectroscopy of the superparticles can give us a signal for these kind of models.

Now, let us discuss the magnitude of $m_H/g_H^2$. In the hidden sector SUSY breaking scenario, gaugino masses are usually given in the form

$$L = \sum_G \int d^2 \theta \frac{k_G}{M_{PL}} SW^GW^G + h.c.,$$

(23)

where $S$ denotes the chiral multiplet which is responsible for the SUSY breaking, $M_{PL} \approx 2.4 \times 10^{18}$ GeV is the reduced Planck scale, $k_G$ denote the coupling constants, and $W^G$ is the superfield for the gauge multiplet. (Here, G indicates gauge group.) Then, when SUSY is broken ($F_S \equiv \langle f d^2 \theta S \rangle \neq 0$), we will get gaugino masses of order $k_GF_S/M_{PL}$. In general, we do not expect that $k_{H3}$ and $k_{H1}$ are much smaller than $k_{GUT}$, and hence the deviation from the GUT relation is expected to be non-negligible. To make a more definite statement we have to make assumptions about how SUSY is broken. For example, in the superstring inspired model with dilaton-dominated SUSY breaking, the combination $m_G/g_G^2$ is universal for all the gauge groups $G$ at the string scale. Then, large corrections to the GUT gaugino mass relations are expected. In particular, the gaugino masses for SU(3)$_C$ and SU(2)$_L$ obey $m_3/g_3^2 : m_2/g_2^2 = 2 : 1$, neglecting the higher order corrections.

In general, the ratio,

$$R_{3/2} = \frac{m_3/g_3^2}{m_2/g_2^2},$$

(24)

is related to the boundary condition,

$$R_{H/G} \equiv \left. \frac{m_{H3}/g_{H3}^2}{m_{GUT}/g_{GUT}^2} \right|_{\mu = M_{PL}},$$

(25)

as $R_{3/2} = R_{H/G} + 1$ in the one loop approximation (with $\mu$ being the renormalization point).

So far, we considered only one loop RGEs. However, as we discussed before, gauge coupling constants for $G_H$ have to be large at the GUT scale, and hence the results based on the one loop RGEs may not be a good approximation. In fact, the ratio of the gaugino mass to the gauge coupling constant squared receives higher order corrections and does not remain constant. Therefore, $R_{3/2}$ will depend explicitly on $g_{H3}(M_G)$ as well as on $R_{H/G}$ if we take into account the higher loop effects due to the large coupling $g_{H3}$.

In order to demonstrate the effect of the higher order terms, we use two loop RGEs to evolve the ratios between the GUT scale and the Planck scale. The two loop RGEs for gauge

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For U(1)$_H$, we do not know the normalization of the charges of the chiral multiplets, and hence we cannot give definite prediction on U(1)$_Y$. 

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couplings and gaugino masses are given in [16]. We fix \( \alpha_{\text{GUT}} = g_{\text{GUT}}^2 / 4\pi = 1/25 \) at the GUT scale, and numerically evaluate \( R_{3/2} \) as a function of \( \alpha_{\text{H3}}(M_{\text{GUT}}) \) for different initial values of \( R_{\text{H/G}} \) at \( M_{\text{PL}} \). The result is shown in Fig. 1 for \( 0.5 < \alpha_{\text{H3}}(M_{\text{GUT}}) < 2 \). We can see that \( R_{3/2} \) is close to the one loop value, \( R_{\text{H/G}} + 1 \), for smaller \( \alpha_{\text{H3}} \) and \( R_{\text{H/G}} \), but deviates significantly from the one loop result for larger \( \alpha_{\text{H3}} \) and \( R_{\text{H/G}} \). Notice that the apparent blow up of \( R_{3/2} \) in the case with \( R_{\text{H/G}} = 2 \) is due to \( m_{\text{GUT}} \) being scaled to zero in the course of running. Below \( M_{\text{GUT}} \), \( R_{3/2} \) stays approximately constant since there is no large coupling to make higher loop contributions important. For very large \( \alpha_{\text{H3}} \), the perturbative calculation should break down and the results based on the two loop calculation are not reliable. In that case, we have no control on \( m_{\text{H3}}/g_{\text{H3}}^2 \) near the GUT scale. However, based on our results for moderate large \( \alpha_{\text{H3}} \), we do not expect that \( m_{\text{H3}}/g_{\text{H3}}^2 \) quickly goes to zero for finite values of \( \alpha_{\text{H3}} \).

Some comments are in order. First of all, we would like to discuss the models based on the gauge group other than \( \text{SU}(5)_{\text{GUT}} \times \text{SU}(3)_\text{H} \times \text{U}(1)_\text{H} \), i.e. models based on \( \text{SU}(5)_{\text{GUT}} \times \text{SU}(3)_\text{H} \) [11] or \( \text{SO}(10)_{\text{GUT}} \times \text{SO}(6)_\text{H} \) [12]. In those cases, \( \text{SU}(3)_C \) is a diagonal subgroup of \( \text{SU}(3)_{\text{GUT}} \) and \( \text{SU}(3)_\text{H} \) as in the previous case, while \( \text{SU}(2)_L \) and \( \text{U}(1)_Y \) are embedded only in \( \text{G}_{\text{GUT}} \). Then, the gaugino masses obey the relation (22) with \( m_{\text{H1}} = 0 \), i.e., the gaugino masses for \( \text{SU}(2)_L \) and \( \text{U}(1)_Y \) obey the usual GUT relation, while that for \( \text{SU}(3)_C \) does not. This kind of signal, together with the unifications of the sfermion masses, will give us an information on the structure of the \( \text{G}_{\text{GUT}} \times \text{G}_\text{H} \) model.

So far, we have concentrated on the hidden sector SUSY breaking scenario. There is another interesting scenario where supersymmetry is broken dynamically at a low energy scale, then mediated to the observable sector by a messenger sector [4]. To preserve gauge coupling unification, the messenger sector should fill out complete multiplets under ordinary \( \text{SU}(5) \). In this case, the GUT relation (1) is not affected even in the \( \text{G}_{\text{GUT}} \times \text{G}_\text{H} \) model.

In summary, we have investigated the gaugino masses in supersymmetric unified models based on an enlarged gauge group like \( \text{SU}(5)_{\text{GUT}} \times \text{SU}(3)_\text{H} \times \text{U}(1)_\text{H} \) or \( \text{SO}(10)_{\text{GUT}} \times \text{SO}(6)_\text{H} \), in which the doublet-triplet splitting problem can be solved naturally. In these models, the GUT relations on the gaugino masses can be broken completely or partially, while we can still hope that the unification of the sfermion masses is unaffected. Therefore, by the accurate spectroscopy of the superparticles, we may have a window into physics at and beyond the GUT scale.

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Figure 1: The ratio $R_{3/2}$ as a function of $\alpha_{H3}(M_{\text{GUT}})$ for different values of the ratio $R_{H/G}=2$ (solid), 1 (dotted), 0 (short dashed), $-1$ (long dashed) and $-2$ (dot-dashed).