FLUORESCENT Lyα EMISSION FROM THE HIGH-REDSHIFT INTERGALACTIC MEDIUM
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ABSTRACT

We combine a high-resolution hydro simulation of the ΩCDM cosmology with two radiative transfer schemes (for continuum and line radiation) to predict the properties, spectra, and spatial distribution of fluorescent Lyα emission at z ∼ 3. We focus on line radiation produced by recombinations in the dense intergalactic medium ionized by UV photons. In particular, we consider both a uniform background and the case in which gas clouds are illuminated by a nearby quasar. We find that the emission from optically thick regions is substantially less than predicted from the widely used static, plane-parallel model. The effects induced by a realistic velocity field and by the complex geometric structure of the emitting regions are discussed in detail. We make predictions for the expected brightness and size distributions of the fluorescent sources. Our results account for recent null detections and can be used to plan new observational campaigns both in the field (to measure the intensity of the diffuse UV background) and in the proximity of bright quasars (to understand the origin of high column density absorbers).

Subject headings: cosmology: theory — intergalactic medium — large-scale structure of universe — line: formation — quasars: absorption lines — radiative transfer

Online material: color figure

1. INTRODUCTION

Hydrogen absorption-line systems observed shortward of Lyα emission in quasar spectra constitute an important probe of the physical state of the intergalactic medium (IGM) at high redshift. These spectral features are shaped by the combined action of gravity, hydrodynamics, and photoionization processes, which determine the local density and the velocity field of neutral hydrogen within the absorbers. Numerical simulations suggest that the so-called Lyα forest is generated by diffuse, sheetlike, and filamentary structures with a mean density that is between 1 and 10 times higher than the cosmic average (Cen et al. 1994; Zhang et al. 1995; Hernquist et al. 1996; Miralda-Escude et al. 1996). These low column density systems are highly ionized by the extragalactic background of Lyman continuum photons generated by young stellar populations and quasars. At the opposite extreme, Lyman-limit systems (LLSs; \( N_{\text{H}_1} > 10^{17.2} \text{ cm}^{-2} \)) and damped Lyα systems (DLAs; \( N_{\text{H}_1} > 10^{20.3} \text{ cm}^{-2} \)) correspond to concentrations of atomic hydrogen that are optically thick to the cosmic ionizing background. Numerical simulations suggest that they arise in dense gas clouds with a meatball topology. On cosmological scales, they appear to form a collection of isolated clouds that trace the cosmic web.

Optically thick clouds are expected to emit fluorescent Lyα photons produced in hydrogen recombinations (Hogan & Weymann 1987; Gould & Weinberg 1996). This emission is concentrated in the outer parts of the clouds where hydrogen is significantly ionized by the external UV background (τLL ∼ 1). However, Lyα photons cannot directly escape the clouds because of the large optical depth in the center of the line (τLyα ∼ 10−3 τLL). Each photon thus suffers a large number of resonant scatterings (more precisely, absorptions and recombinations) by neutral hydrogen atoms in the ground state. Each scattering adds a small Doppler shift to the frequencies of the photons due to the thermal (and turbulent) motions of the atoms. Therefore, photons execute a random walk both in frequency and in physical space until their frequencies are shifted sufficiently away from the line center and they are able to escape the medium in a single flight (Zanstra 1949).

Monte Carlo simulation (e.g., Ahn et al. 2001; Zheng & Miralda-Escudé 2002b, and references therein) is the most popular method for addressing the radiative transfer problem. Analytical solutions only exist for highly symmetric systems. For instance, the emerging spectrum from a plane-parallel and static homogeneous slab is characterized by two sharp peaks in the Doppler wings of the line (Neufeld 1990 and references therein). The plane-parallel solution approximately holds also for self-shielded systems in which the ionized layer that surrounds the neutral region is thin with respect to the characteristic radius of the cloud. In this ideal case, optically thick systems act as efficient mirrors, which convert nearly 60% of the impinging ionizing flux into Lyα photons (Gould & Weinberg 1996).

Direct imaging of fluorescent sources would lead to a major advance in our understanding of galaxy formation. Determining the size distribution of LLSs at \( z \gtrsim 3 \) would be crucial to distinguish whether they arise from photoionized clouds in galactic halos (Steidel et al. 1995; Mo & Miralda-Escudé 1996) or in minihalos formed prior to reionization (Abel & Mo 1998). At the same time, the intensity of the cosmic UV background could be inferred from the observed brightness of the fluorescent emission.

With present-day technology, the detection of fluorescent emission from high-redshift gas condensations is challenging but not impossible. At \( z \sim 3 \), the intensity of the diffuse ionizing background (e.g., Haardt & Madau 1996) corresponds to a Lyα surface brightness on the order of \( 10^{−20} \text{ ergs cm}^{-2} \text{ s}^{-1} \) arcsec^{-2}. It is, then, not surprising that blind searches have only produced a number of null results (Lowenthal et al. 1990; Martínez-Gonzalez et al. 1995; Bunker et al. 1998). Positive fluctuations in the ionizing background can be used to increase the signal. For instance, clouds lying close to a bright quasar are exposed to a stronger UV flux (with respect to an “average” cloud) and are then expected to be brighter in fluorescent Lyα.
Very recently, Francis & Bland-Hawthorn (2004) presented a deep narrowband search for Ly$\alpha$ emission in a field that lies next to the quasar PKS 0424–131. Based on quasar absorption-line statistics and on simple models for fluorescent emission (Gould & Weinberg 1996), they expected to detect more than six clouds, but none were seen. These null results highlight the need for a more sophisticated analysis of fluorescent Ly$\alpha$ emission in realistic environments.

In this paper, we present accurate models of the fluorescent Ly$\alpha$ emission from LLSs at redshift $z \sim 3$. Our study proceeds in three steps. First, we perform a hydrodynamical simulation of structure formation to compute the cosmological distribution of the baryons at $z = 3$. A simple radiative transfer scheme is then used to propagate the ionizing radiation through the computational box and to compute the distribution of neutral hydrogen and of recombinations. Finally, a three-dimensional Monte Carlo code is used to follow the transfer of Ly$\alpha$ photons. As ionizing radiation, we first consider the diffuse background generated by the UV emission of galaxies and quasars (Haardt & Madau 1996). We then discuss an inhomogeneous case in which the ionizing flux from a quasar (which lies in the foreground of the gas clouds) is superimposed on the uniform background. Our detailed numerical analysis shows that simplified models (e.g., Gould & Weinberg 1996) tend to overpredict the Ly$\alpha$ flux emitted from optically thick regions.

The structure of the paper is as follows. We describe our numerical techniques in §2 and present our results in §3, where we also discuss the implications of our analysis for present and future observations. Finally, we discuss the limitations of our approach in §4, and we conclude in §5.

2. METHOD

2.1. Cosmological Simulation

The formation and evolution of the large-scale structure in a “concordance” ΛCDM cosmological model is followed by means of an Eulerian, grid-based total-variation–diminishing hydro+$\alpha$N-body code (Ryu et al. 1993). We assume that the mass density parameter $\Omega_m = 0.3$ (with a baryonic contribution $\Omega_b = 0.04$), the vacuum energy density parameter $\Omega_k = 1 - \Omega_m = 0.7$, and the present-day value of the Hubble constant $H_0 = 100$ km s$^{-1}$ Mpc$^{-1}$ with $h = 0.67$. The simulation is started at redshift $z = 60$ and follows the evolution of Gaussian density fluctuations characterized by a primordial spectral index $n = 1$ and “cluster normalization,” $\sigma_8 = 0.9$ (with $\sigma_8$ the rms linear density fluctuation within a sphere with a comoving radius of $8$ h$^{-1}$ Mpc). This is consistent with the most recent joint analyses of temperature anisotropies in the cosmic microwave background and galaxy clustering (e.g., Tegmark et al. 2004 and references therein). We use a comoving computational box size of $10^3$ h$^{-1}$ Mpc in which the dark matter distribution is traced by 256$^3$ particles and the gas component is evolved on a comoving grid with 512$^3$ zones. The nominal spatial resolution for the gas (the mesh size) is $\sim 20$ h$^{-1}$ kpc (comoving) with the mean baryonic mass in a cell being $\sim 10^5$ h$^{-1}$ M$_\odot$. On the other hand, each dark matter particle has a mass of $5 \times 10^6$ h$^{-1}$ M$_\odot$. All the results presented in this work are derived from the $z = 3$ output of a simulation that does not include radiative cooling of the gas. The limitations of this assumption are briefly discussed in §4. We defer a detailed analysis of the radiative case to future work.

2.2. Radiative Transfer of UV Radiation

In order to compute the distribution of neutral hydrogen within a snapshot of the computational box, we need to simultaneously solve the radiative transfer problem for UV radiation and the rate equations describing the balance between the ionization and recombination rates.

For simplicity, we assume that hydrogen is in ionization equilibrium and use the on-the-spot approximation (Baker & Menzel 1962):

$$ (1 - x)n_H \int_{\nu_0}^{\nu_{\text{opt}}} \frac{d\nu}{\hbar \nu} \int_{4\pi} d\Omega \, J_{\nu}(\Omega) = x n_H n_e \alpha_B(T), $$

where $h_p$, $n_e$, $x$, $n_H$, $\sigma$, $T$, and $\alpha_B$, respectively, denote the Planck constant, the electron number density and the hydrogen ionized fraction, volume number density, ionization cross section, temperature, and case B recombination coefficient (for which we use the fit by Hui & Gnedin 1997). The intensity of ionizing radiation per unit frequency and solid angle is given by $J_\nu$ (in ergs cm$^{-2}$ s$^{-1}$ sr$^{-1}$ Hz$^{-1}$). The frequency integral in equation (1) extends from the hydrogen ionization threshold, $h_p \nu_0 = 13.6$ eV to a maximum frequency $\nu_{\text{opt}}$ (which is formally infinite). A good approximation for our purposes is to assume $\nu_{\text{opt}} = 4\nu_0$ (i.e., set the intensity of radiation to zero at frequencies above the ionization threshold for He i). The motivation is twofold. First, nearly all the photons with $\nu > 4\nu_0$ (which anyway contribute only a few percent of the energy available for H ionization in the UV background) will be absorbed by He atoms (Haardt & Madau 1996). Second, He i recombinations faster than H i, and the intensity of radiation at the He i Lyman limit is typically lower than at $\nu_0$. Therefore, He i is more easily shielded from the ionizing background with respect to H i (Miralda-Escudé & Ostriker 1990). This implies that He i ionizing photons are absorbed in the outer regions of the gas concentrations where H is nearly fully ionized. In order to describe the hydrogen shielding layers we thus neglect He ii and assume that the neutral fraction of He coincides with $1 - x$ (Zheng & Miralda-Escudé 2002a). For a helium abundance of $Y = 0.24$, this corresponds to assuming $n_e = 0.3 n_H$ with $\beta \simeq 1.08$ (see also §2.3). Other than this, the presence of He atoms is neglected in equation (1). Given that He ii recombinations produce H i–ionizing photons and the relatively small number density of helium atoms and ions, this approximation should be reasonably accurate. Note that recombination radiation from He iii can also ionize H i. However, considering the different spatial distribution of He iii and H i discussed above and the characteristic He iii–recombination timescales, we neglect the small local corrections to the H i–ionizing background deriving from this effect.

In each cell of the simulation, the diffuse ionizing background is approximately described by following the radiative transfer along six “light rays” that propagate parallel (and anti-parallel) to the main axes of the computational box. With this numerical trick we can treat anisotropic backgrounds (created, for instance, by shadowing effects) with a minimal request of CPU time (see the Appendix for a test of this approximation). Let us denote by $\tau_i(\nu)$ the optical depth of a given cell along the $i$th ray. This quantity is computed by integrating the product $(1 - x)n_H \sigma_\nu$ from a given starting location (a light source) in the box (see below) up to the first point of the cell crossed by the ray. The closest face of the cell is then exposed to a radiation field with intensity $J_\nu^{\text{in}} e^{-\tau_i(\nu)}$, where $J_\nu^{\text{in}}$ denotes the input ionizing radiation before it is filtered by the gas distribution in the box. Let us also indicate with $\Delta \tau(\nu) = (1 - x)n_H \sigma_\nu L$ (with $L$ the cell size in physical units) the optical-depth variation within the cell measured along one of its principal axes. In order to
implement a photon-conserving scheme, we replace the left-hand side in equation (1) with the quantity
\[
\frac{4\pi}{6} \sum_{i=1}^{6} \int_{v_0}^{v_{ei}} dv \frac{J_{\nu}^{in}}{\hbar \nu v} e^{-\sigma_\nu(v)} \frac{1 - e^{-\Delta \tau_\nu(v)}}{L},
\]
where the sum is taken over the six rays (labeled by the index \(i\)). This corresponds to the number of ionizing photons (per unit volume and time) that are deposited in a given cell by the six rays. To describe the diffuse UV background, we assume that \(J_{\nu}^{in} = J^{HM}_{\nu}\), with \(J^{HM}\) the intensity of radiation derived by F. Haardt & P. Madau (2005). In preparation, the “HM” superscript refers to this work) considering the emission from observed quasars and galaxies after it is filtered through the Ly\(\alpha\) forest.\(^1\) We assume that underdense cells are exposed to the full, isotropic background. On the other hand, overdense cells see an anisotropic radiation field that is computed by using equation (2) to propagate the input background starting from the surface \(\rho = \rho_0\). The intensity of radiation (and thus \(x\)) in each overdense cell depends on the ionized fraction of the surrounding region. To solve the nonlocal equations, we start our calculations by assuming that the whole simulation box is optically thin (i.e., it is exposed to the input radiation field), and we iterate the radiative transfer and ionization-equilibrium calculations until convergence (within 1%) is reached in each overdense cell.

We use a similar approach to discuss the anisotropic radiation field generated by a quasar lying in the foreground of the simulation box along the observer’s line of sight. For simplicity, we assume that the quasar lies distant enough from the simulated region that its emission can be modeled as a train of plane waves impinging onto a face of the simulation box. We also assume that the quasar input spectrum is identical to that of the cosmic background. Given that \(J^{HM}\) is well described by a power law of index \(-1.25\) between \(v_0\) and \(3v_0\), this is a sufficiently good approximation for our purposes (see also the extensive discussion in § 4.4). We then write the quasar ionizing flux (in ergs cm\(^{-2}\) s\(^{-1}\) Hz\(^{-1}\)) as \(F_\nu = \pi b JM_{\nu} G_\nu\), with \(G_\nu\) the Kronecker symbol and \(b\) a dimensionless constant. This is equivalent to using \(J_{\nu}^{in} = 1.5 b J^{HM} G_\nu\) in equation (2). In this case, we compute the optical depth starting from the face of the simulation box that is first reached by quasar light (i.e., along the direction \(i = 1\)).

A self-consistent calculation of the gas temperature requires a joint treatment of radiative transfer and hydrodynamics, which is still beyond present-day computing capabilities. Assuming that the photoionized gas is in thermal equilibrium, we find that \(T \approx (1-3) \times 10^4\) K for the typical densities in the shielding layers (\(100 \leq \rho / \rho_0 \approx 300\)). However, shock heating can easily drive the gas temperature to \(10^6 - 10^7\) K. This is particularly important for the low-density regions (\(\rho \lesssim 100\rho_0\)) where cooling processes are inefficient and the shocked material remains hot (Theuns et al. 1998). In our analysis, we assume that \(T = 2 \times 10^4\) K everywhere. This is an excellent approximation for highly overdense regions (\(\rho \approx 100\rho_0\)) where the cooling time is shorter than the Hubble time and the gas temperature rapidly approaches the equilibrium solution (Theuns et al. 1998).

\(^{\text{1}}\) This is obtained using the most recent results regarding the quasar luminosity function and cosmic evolution within a concordance cosmological model. It assumes that the galaxy escape fraction of ionizing radiation is \(f_{\text{esc}} = 0.1\) and that the energy spectral index for quasar radiation is \(\alpha = 1.8\). The resulting hydrogen ionization rate is a factor of 1.16 smaller than in the models by Haardt & Madau (1996) used by Gould & Weinberg (1996). The spectrum is available at http://pitto.mib.infn.it/~haardt/refmodel.html.

\(^{\text{2}}\) Note that the spectral resolution of the observational data roughly corresponds to our box size. Therefore, we can safely compute the hydrogen column density by integrating \(n_{HI}\) along the entire box.
fraction $\epsilon_{\text{thick}} = \alpha_{2p}^\text{eff}/\alpha_A \sim 0.36$ (where $\alpha_{2p}^\text{eff}$ and $\alpha_A$ denote the effective recombination coefficient to the $2P$ level and the case A total recombination coefficient, respectively) of the recombinations yield a Ly$\alpha$ photon. However, in the optically thick case, continuum photons produced by recombinations to the ground level can be captured by neutral atoms and produce additional Ly$\alpha$ radiation. The asymptotic yield in the extremely thick case (case B approximation, where no continuum photon can leave the cloud) is $\epsilon_{\text{thick}} = \alpha_{2p}^\text{eff}/\alpha_B \sim 0.65$. We use this value to compute the emission rate of fluorescent Ly$\alpha$ photons in the simulation box.

2.5. Resolving the Optical Depth

When we apply the method described above to our simulation, we find that the shielding layers (where the transition between optically thin and optically thick regions occurs) are poorly resolved (see Fig. 1). Typically, they consist of very few cells, which each correspond to an H I optical depth variation (at the Lyman limit) of $\Delta \tau_{\text{cell}} \equiv \Delta \tau(i_0) \gtrsim 1$. However, for a proper treatment of the radiative transfer problem, more stringent requirements on the grid spacing must be met. In particular, the Ly$\alpha$-emitting regions must be resolved with $\Delta \tau_{\text{cell}} \leq 1$. If not, both the spatial distribution of recombinations and the escape probabilities of Ly$\alpha$ photons along different directions (see § 2.6) are spuriously altered.

To solve this problem, we adaptively refine the Ly$\alpha$-emitting regions by interpolating the original density and velocity fields of the input simulation. We use the solution of the radiative-transfer problem for the original (unrefined) grid to select the regions to interpolate and the factor of refinement. Given the memory limitations of the available machines, we use a $10^3$ cell subbox (which is particularly rich of structures) of the original simulation, and we interpolate every cell with a significant recombination rate (>0.1% of the maximum) and $\Delta \tau_{\text{cell}} > 1$. The level of refinement is scaled proportionally to $\Delta \tau_{\text{cell}}$ (up to a factor of 32 in each dimension) in order to have a subgrid of cells with $\Delta \tau_{\text{cell}} \leq 1$. Eventually, we recompute the radiative transfer for the adaptively refined grid. Figure 1 shows that the fraction of recombinations originated in cells with $\Delta \tau_{\text{cell}} > 1$ decreases from 30% to 7% as a result of this refinement. Moreover, in the finer grid, only a negligibly small number of recombinations takes place in extremely thick cells ($\Delta \tau_{\text{cell}} > 10$) compared with 12% of the original grid.

As discussed in § 2.3, we account for unresolved substructure in our simulation box by using a nonvanishing clumping factor in the equation of ionization equilibrium. Density variations within a parent cell of the original simulation due to the refinement procedure described above could, in principal, significantly contribute to the clumping factor. If this is the case, we should then adopt a value $C < 6$ for the refined simulation to reproduce the observed abundance of LLSs. We find that the clumping associated with the refinement is severe in the densest zones of the simulation (which typically lie in the self-shielded regions and do not contribute to the Ly$\alpha$ flux) but amounts to only a few percent in the most rapidly recombing cells. For these, we can then safely adopt $C = 6$ also for the refined box.

Increasing the spatial resolution of the simulation complicates the radiative transfer of ionizing radiation generated by recombinations. Equation (1) assumes that every ionizing photon generated by an H ii recombination is absorbed in the same cell in which it is generated. However, this is no longer a good approximation for the adaptively refined cells that are optically thin to UV radiation. In this case, ionizing photons generated by recombinations can be absorbed in a different cell with respect to where they are created. This process is too complicated to follow without an accurate radiative transfer scheme and we use equation (1) also for the refined cells. How does this affect our results for the distribution of H i? First, the propagation of recombinations can slightly extend (of a few cells) the thickness of the shielding layer of a gas cloud with respect to our results. The effect is probably more pronounced in the outer shells where the gas density is lower. This should only redistribute the birth point of a small fraction of line photons. On the other hand, in the central part of the shielding layer (which contributes most recombinations) we expect that the flows of incoming and outcoming recombinations should nearly balance given that the hydrogen density shows little variations. In summary, our approximated treatment of recombinations should only slightly modify the spectral energy distribution of the emerging Ly$\alpha$ line.

2.6. Ly$\alpha$ Radiative Transfer

We now combine the results of the previous sections (namely, a set of arrays containing the Ly$\alpha$ emission rate, the H i density, and the gas velocity field as a function of spatial position) to compute the spectra and the projected image on the plane of sky of the fluorescent sources. The radiative transfer of resonant Ly$\alpha$ photons is modeled using a three-dimensional Monte Carlo scheme analogous to that employed by Zheng & Miralda-Escudé (2002b; see also Ahn et al. 2001). The method follows a large number of photon trajectories as they are scattered within the H i density and velocity distribution of the hydro simulation.

2.6.1. Emission of Ly$\alpha$ Photons

We assume that Ly$\alpha$ photons are isotropically emitted with frequency $i_0$ in the frame of the recombing atoms (the natural
line width is negligibly small for our purposes). In the cosmic frame (e.g., for an observer who is lying at the center of the simulation box and participates in the free expansion of the universe), the frequencies of the resonant photons appear Doppler shifted by the projected velocities of the atoms along the photon trajectories. The velocity of a hydrogen atom with respect to the cosmic frame is given by the superposition of the Hubble flow with the bulk motion of the gas (i.e., the peculiar velocity of the fluid in the corresponding cell of the simulation) and a random thermal velocity:

\[ \mathbf{v} = H(z) \mathbf{r} + v_{\text{gas}} + v_{\text{th}}, \]

with \( \mathbf{r} \) the atom position with respect to the center of the simulation box. The component of \( v_{\text{th}} \) along the direction of the emitted photon is generated by extracting a Gaussian deviate out of a distribution with zero mean and dispersion \( \sigma_{\text{th}} = (k_B T/m_H)^{1/2} = 12.8(T/2 \times 10^4 \, \text{K})^{1/2} \) km s\(^{-1} \) (with \( k_B \) the Boltzmann constant and \( m_H \) the atomic mass).

2.6.2. Absorption

The photon frequency can be conveniently expressed in terms of the variable

\[ x = \frac{\nu - \nu_0}{\Delta}, \]

which measures the frequency shift from the Ly\( \alpha \) line center in units of the Doppler width, \( \Delta = \sqrt{2} \nu_0 \sigma_{\text{th}} / c \), where \( c \) denotes the speed of light. The mean scattering cross section of Ly\( \alpha \) photons in the fluid frame is

\[ \sigma_{\text{Ly}}(x) = \sqrt{\pi} f_{\text{Ly}} \frac{c r_e}{\Delta} H(a, x) \]

where \( f_{\text{Ly}} = 0.416 \) is the Ly\( \alpha \) oscillator strength, \( r_e = 2.82 \times 10^{-15} \) m is the classical electron radius, and

\[ H(a, x) = \frac{a}{\pi} \int_{-\infty}^{+\infty} \frac{e^{-y^2}}{(x-y)^2 + a^2} \, dy \]

is the Hjerting-Voigt function. For the relatively low densities we are interested in, atomic collisions are not important, and the damping coefficient \( a \) can be expressed in terms of the spontaneous decay rate \( \Gamma \) as \( a = \Gamma/(4\pi \Delta) = 3.3 \times 10^{-4} (T/2 \times 10^4 \, \text{K})^{-1/2} \).

We use equation (7) to determine the distance covered by each photon before it is scattered by an atom. We first extract a random deviate, \( R \), from an exponential distribution function, and then we integrate the product \( n_{\text{H}} \sigma_{\text{Ly}}(x) \) along the photon direction of motion until the resulting optical depth equals \( R \). If the photon still lies within the computational volume, we select the velocity of the scatterer. Note that in order to be able to absorb line radiation, an atom must have a velocity component along the trajectory of the incoming photon, \( \mathbf{v}_{||} \), which closely matches the Doppler shift. From equation (8), it follows that in the fluid frame, \( x_i = v_{||} / (\sqrt{2} \sigma_{\text{th}}) \) is characterized by the following probability distribution

\[ P(x_i) = \frac{a}{\pi H(a, x_i)} \frac{e^{-x_i^2}}{(x-x_i)^2 + a^2}. \]

We use the method presented by Zheng & Miralda-Escudé (2002b) to generate deviates that follow this statistic. The perpendicular component of the thermal velocity in the scattering plane, \( x_{\perp} \), is then extracted from a Gaussian distribution with a temperature-dependent dispersion, as described above.

2.6.3. Reemission

A new direction for the photon is then randomly selected according to a phase function, \( P(\cos \theta) \) (with \( \theta \) the scattering angle), determined by atomic physics. Resonant scattering has an isotropic angular distribution, \( P = 1 \), while wing scattering is characterized by the Rayleigh phase function, \( P = 3(1 + \cos^2 \theta)/4 \) (Stenflo 1980). We find that the two angular distributions give consistent outputs. All the results presented in this work are obtained assuming isotropic reemission.

To determine the new photon frequency, we assume that the scattering process is coherent in the reference frame of the scatterer (partially coherent scattering). This is appropriate when the excited atom undergoes no collisions before reemission and the radiative damping coefficient is small (Avery & House 1968). Both conditions apply to Ly\( \alpha \) radiation emitted by gas in the typical conditions of the shielding regions in the IGM. Once the scattering angle and the photon velocity of the scatterer are specified, it is straightforward to compute the frequency shift of the reemitted photon in the fluid frame:

\[ x = (x_i - x_i) + x_i \cos \psi + x_{\perp} \sin \psi, \]

where \( x_i \) is the frequency shift of the incoming photon and \( \psi \) is the angle between the direction of the incident photon and the direction of the scattering atom. A Lorentz transformation is finally used to compute the frequency shift in the cosmic frame.

The set of calculations described above is iterated until the photon escapes the computational box.

2.6.4. Ly\( \alpha \) Spectra

To produce spectra (and broadband images) of the fluorescent emitters, we compute the surface-brightness of the computational box along the observer’s line of sight (hereafter the \( x \)-axis). At each scattering, the probability that a photon will be remitted along this direction is

\[ \frac{1}{4\pi} P(\cos \theta_i) e^{-x_i}, \]

where \( \theta_i \) is the angle between the incoming photon and the \( x \)-axis and \( x_i \) denotes the Ly\( \alpha \) optical depth of the scattering site along the observer’s line of sight.\(^3\) For each photon and for each scattering, we sum this quantity to a counter in correspondence of the projected position of the scattering site and of the photon frequency. We thus obtain a three-dimensional array containing the surface brightness of fluorescent Ly\( \alpha \) photons as a function of two spatial coordinates plus frequency. Note that a simulated photon tends to remain for many scatterings in a rather small region before it eventually escapes. This means that photons contribute only to a few pixels surrounding their emission site.

Following Zheng & Miralda-Escudé (2002b), we test our implementation of the Monte Carlo scheme against the analytical approximation by Neufeld (1990) for the optically thick, plane-parallel case. Figure 2 shows that our code accurately

\(^3\) This optical depth includes the effects of neutral hydrogen lying in the foreground of the computational box.
reproduces the analytical solution, which becomes exact in the limit of extremely large optical depths.

3. RESULTS

In order to have an acceptable compromise between spectral resolution and CPU time, we only apply the Monte Carlo radiative transfer to the adaptively refined grid corresponding to a $100^3$ region of the original simulation box. To achieve a good signal-to-noise ratio, we generate $10^6$ photon trajectories for every simulation. We thus obtain high-resolution spectra for each pixel of the resulting image, which can be combined to simulate slit, line-emission integral field, or broadband observations.

3.1. Diffuse Background and Static Gas

We first discuss the ideal case of a static gas distribution illuminated with a uniform and isotropic background of ionizing radiation. This is obtained by artificially setting to zero the velocity field of the gas within our refined box.

In the left panels of Figures 3 and 4, we show, respectively, the HI column density distribution and the broadband images [$\lambda \approx 90 \, \text{Å}$ in the observed frame, centered at $\lambda = (1 + z)1216 = 4864 \, \text{Å}$] of the selected region illuminated with the diffuse UV background. The color code in Figure 4 gives the fluorescent Ly$\alpha$ emission rate (photons per unit time, surface, and solid angle) in units of the impinging rate of ionizing photons times $\epsilon$ (i.e., the fraction of the recombinations yielding a Ly$\alpha$ photon):

$$R_{HI} = \epsilon_{\text{thick}} \int_{\nu_0}^{4\nu_0} \frac{\nu H_{\text{HI}}}{h \nu^2} \, d\nu = 2.44 \times 10^4 \, \text{cm}^{-2} \, \text{s}^{-1} \, \text{sr}^{-1}$$

with $\epsilon_{\text{thick}} \simeq 0.65$. For an observer at redshift $z = 0$, this corresponds to a Ly$\alpha$ surface brightness of

$$SB_{HI} = 3.67 \times 10^{-20} \, \text{ergs cm}^{-2} \, \text{s}^{-1} \, \text{arcsec}^{-2}.$$  

There is some observational evidence that the UV background at $z = 3$ is dominated by quasar emission with a negligible contribution from star-forming galaxies (e.g., Scott et al. 2000). In this case, the models by F. Haardt & P. Madau (2005, in preparation) give $R_{HI} = 1.88 \times 10^4 \, \text{cm}^{-2} \, \text{s}^{-1} \, \text{sr}^{-1} - 2$. The spectral shape of the UV background between $\nu_0$ and $4\nu_0$ is nearly identical to the general case discussed in the main text. Therefore, our predictions for the surface brightness of fluorescent sources can be simply scaled down by 30% if future observations prove that galaxies do not significantly contribute to the ionizing background at $z = 3$.  

![Image of LYα spectrum](image-url)  

Fig. 2.—LYα spectrum emitted by a uniform slab with a midplane source with optical depth $\tau_0$. The results of our Monte Carlo code (solid histograms) are compared with an analytical approximation (Neufeld 1990), which becomes exact in the limit $\tau_0 \rightarrow \infty$ (dotted lines). A temperature of $T = 10 \, \text{K}$ is assumed.

![Image of Column-density distribution](image-url)  

Fig. 3.—Column-density distribution of neutral hydrogen at $z = 3$. In the left panel, the gas is exposed to a diffuse UV background generated by the population of galaxies and quasars. In the right panel, the ionizing flux from a foreground quasar, located a short distance in front of the region and corresponding to a boost factor $b = 6$ (see eq. [14]) is superimposed on the diffuse background.
The brightest fluorescent sources correspond to compact gas clouds with a meatball topology. This is because the diffuse UV background is bright enough to fully ionize gas concentrations with $N_{H_1} > 10^{18}$ cm$^{-2}$. In general, the shielding regions either lie within virialized structures or correspond to dense gas shells that are accreting onto collapsed objects. As we see below, the velocity field of the infalling gas produces specific signatures in the Ly$\alpha$ spectra.

The compact fluorescent sources lie along the filaments and sheets that characterize the distribution of neutral hydrogen on cosmological scales. For ease of reading, we label the three largest structures (which each has a diameter of $\sim 0.4$ comoving Mpc) with the letters A, B, and C (see Fig. 3). Cloud C is composed of two subunits and is a part of an elongated structure that extends toward cloud B. Similarly, a filamentary plume of gas bridges clouds A and B.

Simple reasoning based on the plane-parallel model for line transfer suggests that in the absence of photon sinks (e.g., dust), self-shielded (isotropically illuminated) objects should shine with a surface brightness of $SB_{BM}$ (Hogan & Weymann 1987; Gould & Weinberg 1996). In our static simulation (Fig. 4, left), the SB of self-shielded objects closely matches the predictions of this simple plane-parallel model. The SB distribution in the simulation (Fig. 5, dotted histogram) shows a narrow peak at this expected value. In general, the SB scales proportionally to $N_{H_1}^{1/2}$ in the optically thin regions and asymptotically approaches its maximum value for self-shielded objects (Fig. 6, top left). The brightest lines of sight in fluorescent Ly$\alpha$ correspond to optically thick systems with column densities $N_{H_1} > 10^{18}$ cm$^{-2}$, which are thus associated with LLSs and DLAs. All the photons of the ionizing background are converted into Ly$\alpha$ radiation within the shielding layers of these optically thick systems. In the absence of other sources of ionizing radiation, it is impossible to produce a stronger Ly$\alpha$ flux. This explains why the brightest objects in the left panel of Figure 4 have a uniform SB and sharp boundaries that correspond to the regions with $N_{H_1} \simeq 10^{18}$ cm$^{-2}$ in the left panel of Figure 3.

3.2. Diffuse Background and Realistic Gas Velocities

We are now ready to discuss the more realistic case in which we include the gas velocity field of the hydro simulation. The corresponding Ly$\alpha$ emission rate is shown in the right panel of...
Figure 4. The overall pattern is similar to the static case, but a number of striking differences are noticeable, as follows: (1) the SB of self-shielded objects is no longer uniform (e.g., the right-hand side of cloud A is nearly a factor of 2 fainter than the left-hand side); (2) the boundaries of the emitting regions are less sharp, and self-shielded objects are surrounded by large, low-SB halos; (3) self-shielded objects can be significantly fainter (or, very rarely, brighter) than in the static case.

The top right panel in Figure 6 shows that the gas velocity field introduces additional scatter into the SB-$N_{\text{HI}}$ relation with respect to the static case. The brightest lines of sight still correspond to $N_{\text{HI}} > 10^{18}$ cm$^{-2}$, but now two regions with the same column density can be associated with brightnesses that differ up to a factor of 5. In consequence, the SB distribution of optically thick regions is broader, and it is slightly shifted to fainter fluxes with respect to the static case (see the peak of the solid histogram in Fig. 5). We find that the median SB of the self-shielded objects amounts to nearly 75% of the value predicted by Gould & Weinberg (1996). At the same time, a larger fraction of the sky has SB < SB_{HM} compared to the static case (the power-law part of the solid histogram in Fig. 5). As we show below, this excess is caused by foreground scattering of the Ly$\alpha$ photons and is related to the presence of extended Ly$\alpha$ halos around self-shielded objects.

A better understanding of the “velocity-field effect” can be achieved by comparing the spectra of the fluorescent emission in the static and in the general case. In the left and central panels of Figure 7, we show the corresponding spectral energy distributions of the Ly$\alpha$ photons. These have been obtained by positioning four slit spectrographs (width $\approx 0.9''$ and variable length) on top of the three brightest sources, as shown in the right panel of Figure 4. In a static gas distribution, spectra have a characteristic double humped shape and are symmetric with respect to the line center. On the other hand, in the general case the energy distribution is no longer symmetric. In fact, particular configurations of the velocity and density fields are able to strongly suppress one of the wings of the Ly$\alpha$ line and significantly lower the observed SB of the self-shielded objects. In the particular case of cloud A, a low-density concentration of neutral hydrogen is infalling onto the Ly$\alpha$-emitting region. The relative velocity (along the line of sight) corresponds to $\sim 4\sigma_p$ and thus to a very high optical depth. Therefore, most of the photons that in the static case, leave the shielding layers along the line of sight in the red Doppler wing will scatter within the infalling cloud and escape in other directions lowering the observed surface brightness. These photons will then form the extended Ly$\alpha$ halos that surround the brightest objects in Figure 4. The phase-space distribution of neutral gas in the vicinity of the emitting regions thus plays a fundamental role in reshaping the Ly$\alpha$ spectral energy distribution. In broad terms, infalling material diminishes the red wing of the spectrum, while gas that is receding from the emitting region (which could also mean that the shielding layer is infalling onto a central object more rapidly than the surrounding gas) damps out the blue peak of the spectrum. We find that the velocity dispersion of the gas within the regions crossed by the Ly$\alpha$ photons broadens the red and blue peaks of the spectrum by up to 10 $\AA$. On the other hand, when both peaks are detectable, their separation is nearly independent from the detailed properties of the emitting regions and is set by the thermal velocity dispersion of the original cloud. In fact, in analogy with the plane-parallel case, the escape probability of a Ly$\alpha$ photon peaks at a frequency that only depends on the optical depth of its emission site and on the temperature of the medium. For a typical self-shielded cloud ($\tau_{\text{LL}} \gtrsim 1$), the spectrum peaks at $\sim \pm 4x$, which for the assumed temperature, corresponds to a separation of $\sim 8$ $\AA$.

The two-dimensional spectra in the central panel of Figure 7 clearly show that the gas velocity field within and in the vicinity of the shielding layers has a complicated structure that does not show the characteristic pattern of ordered rotation or symmetric infall considered by Zheng & Miralda-Escudé (2002b).

3.3. Quasar Plus Diffuse Background

We now discuss a case of anisotropic illumination obtained by superimposing the ionizing flux from a quasar on the diffuse UV background. The quasar is imagined to lie a short distance in front of the computational box as seen by us. Therefore, gas clouds exposed to quasar light are subject to enhanced UV illumination. Note that the “boost” factor $b$ (defined in §2.2) is determined by the intrinsic luminosity of the quasar and by its actual separation from the simulated region. The definition above can be generalized to any given quasar spectrum using the emitted rate of ionizing photons. At a physical distance $r$ from a quasar with monochromatic luminosity $L_\nu = L_{\text{LL}} (\nu/\nu_{\text{LL}})^{-\alpha}$, we find

$$b = 15.2 \frac{L_{\text{LL}}}{10^{30} \text{ ergs s}^{-1} \text{ Hz}^{-1}} \cdot \frac{0.7 (\frac{r}{1 \text{ Mpc}})^{-2}}{\alpha},$$

The resulting $N_{\text{HI}}$ distribution (assuming a boost factor $b = 6$) is shown in the right panel of Figure 3. The corresponding broadband image (obtained accounting for gas velocities) is presented in the left panel of Figure 8. As expected, the self-shielded regions (and thus the fluorescent sources) are smaller with respect to the isotropic background case due to the extra ionizing radiation produced by the quasar. This also makes the fluorescent sources brighter (Fig. 5, dot-dashed histogram).
since more recombinations will be produced to balance a stronger ionization rate. Based on the (plane-parallel) slab model, where \( \text{Ly}\alpha \) photons are emitted following a cosine law (Gould & Weinberg 1996), one would have naively expected an increase in the \( \text{Ly}\alpha \) surface brightness toward the observer by a factor \( 1 + b = 7 \) with respect to the diffuse background case. However, Figures 5 and 6 clearly indicate that the slab model overestimates the SB of the self-shielded objects. This is not due to shadowing effects. In fact, the attenuation of the quasar flux by diffuse gas lying in front of the fluorescent clouds is generally negligible. Comparing with a static simulation, we also find that gas motions can only explain a small part of this discrepancy. In fact, in the presence of a quasar, foreground scattering is reduced due to the lower neutral fraction present in low-density gas, and broadband images tend to be more uniform than in the case of isotropic illumination. On the other hand, the slab approximation no longer applies when the size of the shielding layers is comparable to the radius of a cloud. In this case, \( \text{Ly}\alpha \) photons produced at a particular point leave the cloud with a different angular distribution with respect to the plane-parallel case. For approximately spherical clouds and in the presence of uniform illumination, this effect is suppressed for symmetry reasons. However, when the ionizing flux is anisotropic, the \( \text{Ly}\alpha \) SB does depend quite strongly on the geometry of the shielding layers.

To study how the SB of self-shielded objects along the quasar direction, \( \text{SB}_{||} \), depends on the impinging flux, we performed a series of simulations with increasing \( b \). Our results are summarized in Figure 9, in which we express \( \text{SB}_{||} \) in terms of an “effective boost factor” defined by

\[
\text{SB}_{||} = (1 + b_{\text{eff}})\text{SB}_{\text{HM}}. \tag{15}
\]

Points with error bars mark the 25th, 50th, and 75th percentiles of \( 1 + b_{\text{eff}} \) among the DLAs. The solid line represents the best-fitting relation,

\[
1 + b_{\text{eff}} = 0.74 + 0.50b^{0.89}, \tag{16}
\]
while the dashed line shows the predictions of the slab model. Note that the geometric effect becomes more and more important as $b$ is increased.

Where do the "missing" Lyα photons go? In the right panel of Figure 8, we show the fluorescent emission along a line of sight perpendicular to the direction of quasar illumination (assuming $b = 6$, as in the left panel). In this case, the plane-parallel model predicts that the self-shielded objects should emit at SB $\text{HM}$. In our simulations, however, the shielding layer deeply penetrates in the clouds along the quasar direction, and the slab model does not apply. In consequence, self-shielded objects are much brighter than a slab along this line of sight. Typically, $\text{SB}_\perp \approx 0.5\text{SB}_\parallel$ for $b \gg 1$, while $\text{SB}_\perp \approx \text{SB}_\parallel$ for $b \ll 1$. In other words, Lyα photons generated by the quasar ionizing flux are emitted within a wide solid angle. As a consequence of this partial isotropization, self-shielded clouds are fainter than expected (based on the slab approximation) along the quasar direction and brighter in the perpendicular directions.

Finally, in the bottom panels of Figure 6, we show the SB-$N_{\text{HI}}$, scatter plot for anisotropic illumination (when observer, quasar, and the simulation box are aligned). It is worth noting that, while the SB keeps nearly constant for LLSs, it steadily increases with $N_{\text{HI}}$ for DLAs. This phenomenon can be explained by as follows. Let us assume that self-shielded objects are nearly spherically symmetric. Then (1) the ionizing flux from the quasar depends on the incident angle with respect to the local density gradient in the clouds; (2) this cosine approaches unity for the central projected regions of self-shielded objects; (3) the column density reaches the highest values along these lines of sight.

#### 3.4. Size Distribution of Lyα Sources

Knowing the size distribution of fluorescent Lyα sources is fundamental to planning an observational campaign for their detection. Regrettably, our refined box is too small (its size being $\sim 2 \, h^{-1} \text{Mpc}$) to provide a statistically representative sample of optically thick sources. On the other hand, performing the line transfer on the $10 \, h^{-1} \text{Mpc}$ box would require an excessive amount of computer time. For these reasons,
we decided to propagate only the ionizing radiation through the 10 h⁻¹ Mpc box and to use the scatterplots in Figure 6 to convert the neutral-hydrogen column densities into Lyα fluxes. In fact, independently of the value of $b$, all lines of sight with $N_{H_1} > 10^{18}$ cm⁻² are approximately associated with a constant Lyα surface brightness (within a factor of 2 uncertainty caused by the gas motion and cosine effects discussed above). We then adopt this threshold value to derive the size distribution of fluorescent objects. In Figure 10, we present our results for an isotropic ionizing background ($b = 0$). Solid and dashed histograms, respectively, refer to objects with $N_{H_1} > 10^{18}$ cm⁻² and to DLAs. It is worth remembering that we fixed the value of the clumping factor in our simulation so as to reproduce the observed sky covering factor of LLSs. In consequence, if a significant fraction of the real systems have a characteristic size that is smaller than our numerical resolution, our simulation will overpredict the number of large systems in order to preserve the required normalization.

In Figure 11, we plot the number density of self-shielded objects as a function of $b$. We use three different thresholds for the source size: 3 (which corresponds to barely resolved objects), 20, and 80 arcsec². In all cases, the number of sources rapidly drops with increasing $b$. In fact, higher values of $b$ characterize regions that are closer to a given quasar (see eq. [14]), and they obviously correspond to a lower number density of self-shielded objects.

From this figure, it is also possible to determine the number density of sources that are brighter than a certain threshold value $(1 + b_{\text{th}})SB_{\text{HIM}}$. Let us consider a Lyα source that is optically thick to ionizing radiation at a given distance from a quasar. Let us also imagine that we can move the cloud toward the quasar, thus increasing the $b$ factor. As long as the cloud remains optically thick, $SB_1$ monotonically increases. However, there exists a particular value of the boost factor, $b_{\text{th}}$, at which the cloud is no longer able to self-shield. Therefore, for $b > b_{\text{th}}$, $SB_1$ keeps roughly constant.⁶ Thus, the number of self-shielded objects at a given $b$ coincides with the number of sources (which are not necessarily optically thick) with $SB > (1 + b_{\text{th}})SB_{\text{HIM}}$. In other words, the number of sources that are brighter than a given threshold can be computed with the following procedure. First, convert the threshold SB into an effective boost factor, $b_{\text{eff}}$. Second, invert equation (16) to find the value of $b_{\text{th}}$ such that $b_{\text{eff}}(b_{\text{th}}) = b_{\text{eff}}$. Third, use $b = b_{\text{th}}$ in Figure 11 to determine the number density of the sources. Fourth, use equation (14) to find the volume within which it is possible to have $b > b_{\text{th}}$.

The variation of the number density of fluorescent sources as a function of $b$ is somehow related to the proximity effect. Hydrogen clouds in the vicinity of a quasar are strongly ionized and emit fluorescent radiation. In other words, the missing absorption systems that determine the proximity effect can be detected in emission through their recombination radiation. Therefore, in analogy with studies of the proximity effect (e.g., Bajtlik et al. 1988; Scott et al. 2000), the number density variation of fluorescent sources around a quasar can be used to infer the intensity of the UV background. In this case, reliable models of fluorescent emission are fundamental to convert the observed counts into a background amplitude.

3.5. Comparison with Recent Observational Data

We can use the above to compare the predictions of our models with the recent observational results by Francis & Bland-Hawthorn (2004, hereafter FBH04). These authors performed a deep narrowband search for fluorescent Lyα emission in the vicinity of the $z = 2.168$ quasar PKS 0424−131.

⁶ The fraction of recombinations yielding a Lyα photon decreases from $\sim 0.65$ to $\sim 0.36$ when a cloud becomes optically thin. Therefore, we expect a fully ionized cloud to be a factor of $\sim 2$ fainter in Lyα with respect to the optically thick case.
\(L_{\text{LL}} = 1.67 \times 10^{30} \text{ ergs s}^{-1} \text{ Hz}^{-1}, \alpha \simeq 0.7\). At the 5 \(\sigma\) confidence level (corresponding to a surface brightness of \(4.7 \times 10^{-19} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ arcsec}^{-2}\) for sources larger than 100 arcsec\(^2\) and to \(9.6 \times 10^{-18} \text{ ergs cm}^{-2}\) for unresolved sources) no source was been detected. Based on the observed abundance of LLSs, FBH04 expected to find about six fluorescent clouds with a size of 100 arcsec\(^2\). This estimate, however, does not take into account the ionizing radiation from the quasar.

Assuming that our results at \(z = 3\) are approximately valid at the quasar redshift,\(^7\) we find that the sensitivity limits of FBH04 correspond to \(b_{\text{min}} \sim 11.2\) for sources that are larger than 100 arcsec\(^2\). Assuming that the ionizing background keeps roughly constant in the redshift interval \(2 < z < 3\), from equation (14) we find that this corresponds to a distance from the quasar of \(r_{\text{max}} \sim 1.5\) (physical) Mpc. This is the maximum distance from the quasar at which an optically thick cloud could have been detected. Based on our simulations, we expect to find, on average, one to two objects within a sphere of radius \(r_{\text{max}}\) around the quasar. However, FBH04 limited their search to distances smaller than 1 Mpc, thus reducing the sampled volume by a factor 3.4 with respect to the theoretical limit. In this case, the expected number of sources ranges between 0.3 and 0.6. Therefore, the probability of detecting (at least) one source with one single observational run is \(0.25 < P < 0.45\) (assuming Poisson statistics). Our simulations clearly show that the results by FBH04 are thus perfectly consistent with our understanding of the IGM at high \(z\).

There are caveats to the simple calculations described above. For instance, we have assumed that all the Ly\(\alpha\) sources lying within a distance \(r_{\text{min}}\) from the quasar are brighter than \(b_{\text{min}}\). This assumption tends to overestimate the number of detectable objects. In fact, (1) self-shielded clouds lying in front of the quasar are much fainter and hardly detectable (their SB actually depends on the angle between the line of sight and the quasar direction); (2) fully ionized clouds in the foreground of the quasar tend to be a factor of \(\epsilon_{\text{thick}}/\epsilon_{\text{thin}} \simeq 1.8\) fainter than assumed above; (3) if the age of the quasar is shorter than the hydrogen recombination timescale, no fluorescent sources will be detectable (see the discussion at the end of §2.2). On the other hand, we have assumed that our simulation box is representative of the gas distribution surrounding a quasar. Since optically selected quasars at high \(z\) tend to sit within the most massive dark-matter halos formed at that epoch (Porciani et al. 2004), it is reasonable to expect that matter clusters (and moves with larger peculiar velocities) are attracted by them. Therefore, we could have underestimated the number densities of fluorescent sources lying close to a quasar.

Despite of the approximations listed above, we believe that our results provide the most reliable estimates for the abundance of fluorescent Ly\(\alpha\) sources at high \(z\) carried out to date. Optimized sampling strategies are certainly required to observe these objects. Our simulations then represent a fundamental tool to plan observations around a given quasar.

4. UNCERTAINTIES

4.1. Radiative Cooling

While numerical simulations are a useful tool to guide our understanding, they cannot be considered a perfect model of reality. A potential limit of our simulation is the lack of radiative cooling. While this is not a concern for the diffuse IGM, it becomes worrisome for highly overdense regions. Fluorescent Ly\(\alpha\) sources have intermediate overdensities (\(\rho/\rho_0 \sim 200\)) and are likely to be in equilibrium with the UV ambient radiation. We thus expect them to be mildly affected by cooling processes. In any case, most of the results discussed in this paper are nearly independent of the details of the gas distribution. Both the velocity and the geometric effects will be present anyway. On the other hand, the size distribution of the sources might be more affected by the cooling processes. It is also worth stressing that there is no way of accounting for the effects of cooling and heating in a self-consistent way. In fact, simulations in which radiative transfer is fully coupled with hydrodynamics are still not viable with current supercomputers. Moreover, other poorly understood processes (such as energy and momentum feedback) play an important role. Therefore, it is hard to judge the level of approximation that current simulations including gas cooling may reach.

4.2. Resolution and Substructure

Is the finite resolution of our simulation affecting our results? Multiple metal lines are often associated with single DLAs (e.g., Prochaska & Wolfe 1997), thus suggesting the presence of a clumpy medium. Unresolved substructure in our simulation might reduce the velocity effect and modify the outcome of spectra. The adopted value of \(C\) implies that at least \(\epsilon/\zeta\) of the volume is in dense clumps. If these substructures have a diameter that is comparable to the cell size, we only expect a minor modification of our results. In fact, the gas velocity in the simulation should closely approximate the motion of these clumps. The only effect is then a slight Doppler shift of the entire Ly\(\alpha\) spectrum of each cell. The opposite case, in which each cell contains a large number of small substructures (Abel & Mo 1998), can be approximately discussed by considering an additional contribution to the thermal velocity dispersion. Assuming a value of \(c_{\text{vth}} \sim 50 \text{ km s}^{-1}\) (corresponding to roughly half the virial velocity of the host halos; Haehnelt et al. 1998), we find that the velocity effect is strongly suppressed and the spectra are in better agreement with the slab model. Note, however, that the existence of a sea of small subclumps is disfavored by observations. In fact, this scenario would produce broad absorption features instead of the multiple metal systems associated with single DLAs (see Haehnelt et al. 1998; McDonald & Miralda-Escudé 1999). In any case, the velocity dispersion within our simulated DLAs is of the order of 100 km s\(^{-1}\), in good agreement with observational data.

4.3. Additional Sources and Dust

Beyond fluorescent emission, additional Ly\(\alpha\) radiation might be produced in the inner regions of the clouds. Within gravitationally collapsed objects, the gas tends to dissipate its internal energy by emitting line photons (e.g., Haiman et al. 2000; Fardal et al. 2001; Furlanetto et al. 2005). Similarly, internal star formation could act as a copious source of Ly\(\alpha\) photons, but whereas fluorescent emission is expected to extend over several tens of kiloparsecs, the Ly\(\alpha\) emission from star-forming regions should be more concentrated near the centers of galaxies (Furlanetto et al. 2005). We have focused here on the fluorescent emission generated by recombinations and these extra sources of line photons are not considered in our analysis. We will present a comprehensive model of Ly\(\alpha\) emitters in a future paper.

\(^7\) We simply assume that the Ly\(\alpha\) surface brightness scales as \((1 + z)^{-4}\); i.e., SB\(\alpha\) = 2.168g \(\times 2.54\beta\) (\(\epsilon = 3\)).

\(^8\) The detection limit for unresolved sources is less interesting. It corresponds to \(b_{\text{min}} \sim 388\) (assuming that eq. [16] can be extrapolated to such high values of \(b\) and \(r_{\text{max}} \sim 250\) kpc. The number of expected sources in the associated volume of \(\sim 0.06\) Mpc\(^3\) is therefore negligibly small.
At the same time we did not consider the destruction of Ly$\alpha$ photons by dust grains. Little is known about the dust properties within the IGM at $z \sim 3$, even though there is some evidence for the presence of dust in DLAs (Fall & Pei 1993). Nevertheless, the associated absorption of fluorescent photons is likely to be minimal due to the relatively low $N_{\text{HI}}$, of the shielding layer (e.g., Gould & Weinberg 1996). On the other hand, absorption is expected to be more severe for Ly$\alpha$ photons produced close to and within the star-forming regions, where dust is likely to be more abundant and the Ly$\alpha$ escape probability is lower.

4.4. The UV Background

Our results are based on the Haardt-Madau model for the UV background. At $z = 3$, the adopted model has an amplitude at the Lyman limit of $J(\nu_0) = 4.04 \times 10^{-22}$ ergs cm$^{-2}$ s$^{-1}$ sr$^{-1}$ Hz$^{-1}$ and corresponds to a hydrogen ionization rate $\Gamma = 1.15 \times 10^{-12}$ s$^{-1}$. Even though this is on the low side, it is consistent with recent observational studies of the proximity effect, which give $J(\nu_0) = 7.0^{+4.4}_{-3.4} \times 10^{-22}$ ergs cm$^{-2}$ s$^{-1}$ sr$^{-1}$ Hz$^{-1}$ and $\Gamma = 1.9^{+1.3}_{-1.6} \times 10^{-12}$ s$^{-1}$ (Scott et al. 2000). Our results should then be considered conservative, because they account for a UV background intensity that is a factor of 1.75 lower than current observational estimates. Note, however, that a different normalization of the UV background cannot change our predictions for the number density of fluorescent sources. In fact, we determined the clumping factor of the gas by imposing that the mean number of LLFs in the simulation matches the observational data. Therefore, a stronger ionizing radiation field would require a larger clumping factor to fit the observed counts. If both $J_{\text{UV}}$ and $C$ are multiplied by the same multiplicative factor, $A_J$, the solution of the ionization equilibrium does not change. In fact, this is equivalent to multiplying both sides of equation (1) by a constant factor. Therefore, our results for the number density and size distribution of fluorescent sources are robust with respect to the amplitude of the UV background. Note, however, that the Ly$\alpha$ surface brightness of optically thick cloud would be higher by a factor $A_J$ with respect to equation (13). Consistently, the boost factor in equation (14) would decrease by a factor $A_J$, while equation (16) would remain unaltered.

Some (most likely minor) modification of our number counts is instead expected if the spectral energy distribution of the UV background significantly differs from the assumed one. This crucially depends on the relative contribution of galaxies and quasars to the cosmic ionizing background. Similarly, the spectral energy distribution of quasar radiation plays an important role. For simplicity, we assumed that the quasar spectrum is identical to that of the cosmic background. This corresponds to a power index $\alpha \approx 1.25$, i.e., to a much flatter spectrum than inferred from quasar observations ($1.5 \lesssim \alpha \lesssim 1.8$). What could be the effect of this assumption? The average mean free path of ionizing photons from a spectrum with $\alpha = 1.8$ is only a factor of 20% smaller than for $\alpha = 1.25$. Therefore, radiation from a steep quasar spectrum should penetrate a bit less within hydrogen clouds than it does in our models. This might slightly modify the emerging Ly$\alpha$ spectra and reduce the importance of the geometric effect. Note, however, that quasar radiation will be partially filtered by the IGM before reaching the fluorescent clouds. Thus, a spectrum with $\alpha = 1.8$ at emission will be transformed into a flatter distribution in the shielding layer of a cloud.

5. SUMMARY

We have presented a new method to produce realistic simulations of fluorescent Ly$\alpha$ sources at high redshift. We started by simulating the formation of baryonic large-scale structure in the $\Lambda$CDM cosmology. A simple radiative transfer scheme was then used to propagate ionizing radiation through the computational box and to derive the distribution of neutral hydrogen. Finally, the transport of Ly$\alpha$ photons generated by hydrogen recombinations was followed using a three-dimensional Monte Carlo code. As ionizing radiation, we first considered the smooth background generated by galaxies and quasars. Then, as a second case, we superimposed on the background the UV flux produced by a quasar lying in the vicinity of the simulation box.

Our detailed numerical treatment improves on previous work, which was either based on rather crude approximations for the transfer of resonantly scattered radiation (Hogan & Weymann 1987; Gould & Weinberg 1996) or on highly symmetric semi-analytical models for the gas distribution (Zheng & Miralda-Escude 2002b). Our results show that simple models (e.g., Gould & Weinberg 1996) tend to overpredict the Ly$\alpha$ flux emitted from optically thick clouds. In fact, we identified two effects that reduce the fluorescent Ly$\alpha$ flux (and modify the spectral energy distribution) with respect to the widespread static and plane-parallel model.

Velocity effect.—The velocity field inside and around the shielding layers of a gas cloud influences the emerging line profile. The symmetry of the double-humped spectrum is lost, and in most cases, one of the two peaks is severely suppressed. On average, the SB of a cloud is reduced by 25% with respect to the static situation.

Geometric effect.—For anisotropic illumination and in the presence of a strong ionizing flux, the thickness of the shielding layer is comparable to the size of the gas cloud. In this case, the angular distribution of the emerging radiation is very different than in the plane-parallel approximation. For instance, close to a quasar, a cloud emits much less than predicted by the slab model in the direction of the quasar and much more in the other directions.

The importance of these effects (in particular of the angular redistribution of Ly$\alpha$ photons) depends on the intensity of the impinging radiation. In equation (16), we provided a fitting function for the maximum Ly$\alpha$ brightness of optically thick sources as a function of the incident ionizing flux. In Figure 11, we presented our predictions for the number density of fluorescent sources with different sizes.

These results are consistent with the recent null detection by Francis & Bland-Hawthorn (2004) and represent a fundamental tool for planning future observations.

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approximation in the optically thin (dashed line) and optically thick (dotted line) cases. [See the electronic edition of the Journal for a color version of this figure.]

APPENDIX

RADIATIVE TRANSFER FOR A DIFFUSE BACKGROUND

The fluorescent Lyα flux from a source is proportional to the number of hydrogen ionizations taking place in the corresponding gas cloud. In order to simplify the radiative transfer problem for continuum radiation, in equation (2) we only followed the propagation of ionizing photons along the six principal directions of the simulation box. In this appendix, we first provide justification for the multiplicative factor $2\pi/3$, which appears in equation (2), and then test the six-direction approximation.

The factor $2\pi/3$ in equation (2) is obtained by discretizing the left-hand side in equation (1) along six preferred directions. Note, however, that equation (1) applies to an infinitesimal volume element, while our discretized version is used to describe finite cubic cells. It is clear that the approximation is correct for optically thin cells, but where does it break down? Let us consider a homogeneous cube of side $L$ (corresponding to an optical depth $\Delta_\text{cell}$) illuminated by a uniform and monochromatic background with intensity $I$ (photons per unit time, surface, and solid angle). The ionization rate within the cube generated by the radiation background impinging on one face can be written as $\alpha \pi IL^2 e^{-\Delta_\text{cell}}$, with $\alpha$ a numerical coefficient of order unity. We use a Monte Carlo method to compute $\alpha$ as a function of $\Delta_\text{cell}$. Our results are plotted in Figure 12. When the cell is optically thin $\alpha \approx 2/3$, while $\alpha \rightarrow 1$ when $\Delta_\text{cell} \rightarrow \infty$. A rather sharp transition between the two asymptotic regimes is observed for $\Delta_\text{cell} \sim 10$. Note that in the presence of a uniform background, the optically thin approximation adopted in the main text is accurate even for $\Delta_\text{cell} = 1$, where $\alpha = 0.70$. Given that in our adaptively refined cosmological simulation, $\sim 93\%$ of recombinations take place in cells with $\Delta_\text{cell} < 1$, we used $\alpha = \frac{2}{3}$ in our calculations.

All this, however, applies to an isolated cell exposed to the ionizing background. We now want to test how accurately equation (2) works in the shielding layer of an optically thick cloud. We thus consider a spherically symmetric cloud that is optically thin at his external boundary and thick at the center [$\log \Delta_\text{cell} = 2 - 4(r/R)$], with $r$ the radial coordinate, which vanishes at the cloud center, and $R$ the characteristic size of the cloud, which roughly mimics one of the clouds in our refined simulation). We then illuminate the outer boundaries of the cloud with a uniform (over the external $2\pi r \, \text{sr}$) and monochromatic background. This is injected from the surface of a cube of side $2R$ centered on the cloud and divided into $151^3$ mesh points. In Figure 12, we compare the outcome of a detailed radiative transfer code (C. Porciani & P. Madau 2005, in preparation) with our approximated method. Note that in our six-direction approximation the ionizing flux penetrates slightly deeper into the cloud with respect to the exact solution. This happens because our method ignores photon trajectories that are oblique to the principal axes of the box. On the other hand, we find that our approximation produces $92\%$ of the ionizations that take place in the cloud. Given the simplicity of our algorithm, this is a remarkable achievement. Note that had we adopted the optically thick approximation ($\alpha = 1$), we would have overestimated the number of recombinations by $38\%$. These figures are rather stable and, for realistic cases, do not depend on the details of the optical depth distribution.

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