Performing application of cooper-jacob method for identification of storativity

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Abstract. Transmissivity is estimated by fitting a straight line between time versus drawdown on semi-logarithmic paper. The test data of both confined and unconfined aquifers were analyzed by many practitioners that used cooper-Jacob method to estimate the drawdown equation, regardless of the differences between theoretical and practical conditions. The other parameter is storativity overestimated, so the aim of this research is to analyze ten pumping test data located in Makassar area by using cooper-Jacob’s (1946) time drawdown approximation of Theis method to estimate the aquifer parameters, also in order to determine the reasons which are affecting the reliability of the storativity value and obtain the important aspect behind that in practice. The pump well is installed to a depth of 20 m, the end is 2 meters into the hard consistency clay layer. The radius of the pump well used in a single well test to estimate the storativity value of the type of aquifer layer unconfined with the Cooper-Jacob method gives excessive value, but if the radius of the pump well is used larger by inserting the radius of the gravel filter layer is obtained a significant value that approaches the storativity value that has been estimated by Theis method uses one pump well and nine observation wells.

1. Introduction

The most important method for estimating aquifer parameter values is testing with many observation wells, but the cost is greater than the single well test [1]. Single well test with Cooper-Jacob method has been carried out to estimate aquifer parameter values, but the estimated storativity value is excessive. The purpose of this study was to analyze a single well test that had been carried out in the Makassar city by using Cooper-Jacob's drawdown time approach (1946) from Theis method to determine the factors that influence the value of storativity [2]. The soil profile around the investigation well has a similar soil profile [2,4]. This is obtained from visual interpretation during investigation well drilling and from reports of soil investigation at drill points BH1, BH2, BH3 and BH4, that at a depth of 0-15 meters is a relatively moderate density fine sand layer, depth of 15-18 meters is a consistency clay layer very stiff and a depth of 18-40 meters is a layer of hard consistency clay. Thus the observation pump well is installed to a depth of 20 m, the end is 2 meters into the hard consistency clay layer where the well is used as a pipe 6”. It is easier for water to enter the well if the aquifer material around the pipe is replaced with gravel material [3-6]. When the well is pumped, the gravel will store a lot of water that will enter the well. The thickness of the gravel material must be in the range of 8 to 20 cm. Gravel package material must be clean and round fine grains.
2. Metode Theis (1935)

Field data from pumping tests such as drawdown, flow rate as a function of time at constant pumping rate conditions and constant head in wells are needed to analyze pumping test data. Theis (1935) assumes that the pumping rate is constant in confined, homogeneous, isotropic areas, infinite areas of extent and transient flow without recharge are:

\[ \Delta^2 h = \frac{S}{Kb} \frac{\partial h}{\partial t} \]  

(1)

\[ \Delta^2 h = \frac{S}{Kb} \frac{\partial h}{\partial t} \]  

(2)

Using well boundary conditions:

\[ Q = q \cdot A \]  

(3)

\[ q = \frac{Q_{well}}{2 \pi r b} \]  

(4)

\[ h = h_{initial}, \quad @r = r_w \atop @r = r_\infty \]  

(5)

So the equation of Theis (1935)

\[ s = \frac{Q}{4\pi Kb} \int_0^\infty \int_0^\infty e^{-u} \ln u + u \cdot \frac{u}{2.2!} + \frac{u}{3.3!} + \ldots, \quad u = \frac{r^2 S}{4Tt} \]  

(6)

\[ W(u) = [\gamma - \ln u + u \cdot \frac{u}{2.2!} + \frac{u}{3.3!} + \ldots], \quad \gamma = \text{Euler number} = -0.5772 \]  

(7)

\[ u = \frac{r^2 S}{4Tt} \]  

(8)

Or

\[ s = \frac{Q}{4\pi T} W(u) \]  

(9)

Where \( s \) is drawdown (m); \( Q \) is constant rate pumping test (m³/min); \( T \) is transmissivity (m²/min); \( S \) is storativity (unit less); \( r \) is radial distance (m); \( u \) is well constant; \( W(u) \) is well function; \( t \) is time of pumping (min); \( \gamma \) is Euler number = -0.5772; \( h \) is aquifer thickness in water-table aquifer (m); \( b \) is the aquifer thickness in case confined (m) [6].

The above equation is known as Theis matching curve by plotting the field data obtained during pumping tests on curves using semi-logarithmic paper, then two curves superimposed on each other where unknown values are obtained to calculate aquifer parameters [7].

3. Metode Cooper-Jacob’s (1946)

Cooper and Jacob suggest that the Theis method (1935) can be simplified if the time is long and the value of \( r \) is small, so the value of \( u \) must be small or equal to 0.01, therefore only the first two terms are taken into account [7]. The equation can also be used for water-table type aquifer if \( 2s << b \), which \( b \) is the thickness of the aquifer.
Testing of single well aquifers can provide transmissivity values, where the test data is analyzed by
the simple straight-line method Cooper-Jacob's (1946). Transmissivity is estimated by placing a
straight line between time versus drawdown on semi-logarithmic paper. Theis drawdown equation:

\[ s = \frac{Q}{4\pi T} \left[ -0.5772 - \ln u + \frac{u}{2.2!} + \frac{u^2}{3.3!} + \ldots \right] \]  \hspace{1cm} (10)

Where

\[ u = \frac{r_w^2 S}{4Tt} \]  \hspace{1cm} (11)

According to Jacob the assumption of drawdown equations is simplified to:

\[ s = \frac{Q}{4\pi T} \left[ -0.5772 - \ln u \right] \]  \hspace{1cm} (12)

\[ s = \frac{Q}{4\pi T} \left[ -\ln 1.78 - \ln \frac{r_w^2 S}{4Tt} \right] \]  \hspace{1cm} (13)

\[ s = \frac{Q}{4\pi T} \left[ -\ln \frac{1.78r_w^2 S}{4Tt} \right] \]  \hspace{1cm} (14)

\[ s = \frac{Q}{4\pi T} \ln \frac{4Tt}{1.78r_w^2 S} \]  \hspace{1cm} (15)

\[ s = \frac{2.3Q}{4\pi T} \log \left[ \frac{2.25Tt}{r_w^2 S} \right] + \frac{2.3Q}{4\pi T} \log t \]  \hspace{1cm} (16)

\[ Y = B \text{ (intercept)} + A \text{ (slope)} x \]  \hspace{1cm} (18)

The plot \( s \) to \( \log t \) is a straight line, the extension of the straight line in the drawdown is zero, \( t = t_o \) so
that:

\[ \frac{2.25Tt_o}{r_w^2 S} = 1 \]  \hspace{1cm} (19)

\[ S = \frac{2.25Tt_o}{r_w^2} \]  \hspace{1cm} (20)

And

\[ \Delta s = \frac{2.3Q}{4\pi T} \]  \hspace{1cm} (21)

where \( r_w \) is well radius (m), \( \Delta s \) is slope of the line per one log cycle (m), \( t_o \) is the initial time of
pumping test at zero drawdown (min).

The solution may be used for the interpretation of pumping tests in unconfined aquifers through the
application of the following simple correction to drawdown data measured during a test:

\[ s' = s - s^2 / 2b \]
where \( b \) is saturated thickness (m); \( s \) is observed drawdown (m); \( s' \) is corrected drawdown (m).

### 4. Results of pumping tests

The well pumping test results which consist of pumping time to groundwater drawdown can be seen in figure 1. Pumping test results are plotted on semi-logarithmic paper and line slope with the pumping test start time, \( t_o \) is determined as presented in figure 2.

![Figure 1. Pump time (t) vs corrected drawdown (s').](image1)

![Figure 2. Time-drawdown straightline plot.](image2)

From the single well test, it can be estimated the transmissivity and storativity values as shown in table 1.

| Parameter                          | Unit            | Value  |
|------------------------------------|-----------------|--------|
| Pumping rate (Q)                   | m³/minute       | 0.1205 |
| Well radius (r_w)                  | m               | 0.0762 |
| Slope of the line per one log cycle (Δs) | m       | 0.60   |
| Initial time of pumping test at zero drawdown (t_o) | minute | 0.4 |
| Transmissivity (T)                 | m³/minute       | 0.0368 |
| Hydraulic Conductivity (K)         | m/minute        | 0.0031 |
| Storativity (S)                    |                | 5.7004 |
| Corrected Storativity (S') dengan r_w = 0.2762 m | -       | 0.4339 |

Based on the results of the analysis of the single well test data, it was found that the value of storativity was very large at 5.7004, this was due to the high drawdown found in the time-straight line graph against drawdown, and the radial distance representing the well radius (\( r_w \)), which is the filtered part of the well. The storativity values for unconfined aquifer types are in the range of 0.05 to 0.3. By using the value of \( r_w \) equal to 0.2762 m which is the value of the pipe radius which is added to the
radius of the gravel material, the calculation results with Cooper-Jacob Method obtained the value of storativity is 0.4339. Based on calculations the Theis method with 1 point pump is well and 9 points of observation well at the same location obtained value of storativity (S) ranges between 0.084 - 0.472 approaching the storativity value corrected value is 0.4339.

5. Conclusions
The radius of the pump well used in a single well test to estimate the storativity value of the type of aquifer layer unconfined with the Cooper-Jacob method gives excessive value, but if the radius of the pump well is used larger by inserting the radius of the gravel filter layer is obtained a significant value that approaches the storativity value that has been estimated by Theis method uses one pump well and nine observation wells.

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