De retibus socialibus et legibus momenti

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Abstract – Online Social Networks (OSNs) are a cutting edge topic. Almost everybody —users, marketers, brands, companies, and researchers— is approaching OSNs to better understand them and take advantage of their benefits. Maybe one of the key concepts underlying OSNs is that of influence which is highly related, although not entirely identical, to those of popularity and centrality. Influence is, according to Merriam-Webster, “the capacity of causing an effect in indirect or intangible ways”. Hence, in the context of OSNs, it has been proposed to analyze the clicks received by promoted URLs in order to check for any positive correlation between the number of visits and different “influence” scores. That evaluation methodology is used in this letter to compare a number of those techniques with a new method firstly described here. That new method is a simple and rather elegant solution which tackles with influence in OSNs by applying a physical metaphor.

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Introduction. – This letter describes an eminently empirical study for which a number of experiments were conducted. Twitter was chosen for that purpose because it is relatively easy to obtain data from it in comparison to other services such as Facebook. For those experiments the dataset from [1] was used. That study completely describes the data assets but, still, a brief description follows the next introduction to Twitter topics.

Twitter is a microblogging service which allows users to publish text messages of up to 140 characters (tweets) which are shown to other users subscribed to the author feed (followers). Unlike other OSNs, relationships in Twitter are asymmetrical and it must be distinguished between people reading a given author messages (followers), and those persons that author reads (friends or followees).

In addition to such following behaviors, Twitter users get involved in tweeting behaviors: a tweet can be original content produced by the author or it can be non-original content; that is, the user is repeating a tweet by another user (retweeting, in Twitter parlance). Because retweeting is a form of citation, some syntax to provide attribution is needed: to do that, the name of the mentioned user is prepended with an at sign.

Let us suppose, for instance, that Alice had tweeted the message “Hello world!”. If Bob wanted to retweet it he just should have to post: “RT @alice: Hello world!”, where RT stands for retweet.

This mention syntax is not limited to retweets but can be used to address other users and engage into conversations similar to those within IRC (Internet Relay Chat).

Hence, users can tweet, retweet, mention, or combine all of them — e.g. retweeting something while addressing it to a third party: “RT @alice: Hello world! (cc @carol)”. For a deeper understanding of tweeting and retweeting behaviors the work by boyd et al. [2] is highly recommended.

Regarding the dataset used in this study, it consists of a collection of 27.9 million tweets and a user graph comprising 1.8 million users. Both were obtained using a number of methods of the Twitter API (Application Programming Interface). The tweets were collected from January 26 to August 31, 2009. Due to some network blackouts 4 days are missing and, thus, the dataset has got, on average, 130000 tweets per day. On 2009 Twitter received 2.5 million tweets per day [3], hence, the data

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corresponds to about 5.6% of the total amount of tweets published during the crawling period.

**Tweets** are associated with metadata such as the publishing author and, thus, a list of 4.98 million users was obtained from the previous dataset and used to crawl the user graph. At the moment of that second crawling many accounts had been suspended or changed their status from public to private. Additionally, users without links to other users in the list were considered isolated and removed, and there were also minor network blackouts. For these reasons the graph contains less users than those publishing tweets. Anyhow, it was checked that the crawling was uniform and, in fact, the graph corresponds to 4% of the Twitter’s worldwide user base of 44.5 million users as of mid-2009 [4].

**Computing influence in Twitter.** – The number of followers has been largely considered as equivalent to popularity because it seems rather obvious that the more popular a user is the more followers s/he has got —celebrities such as Lady Gaga or Britney Spears have got millions of followers, indeed.

Given this approach to popularity, some users have exploited a rule of etiquette to get massive audiences. In Twitter to follow back a new follower is considered good manners and, hence, abusive users (e.g. spammers) tend to follow thousands of people to get followers in return.

Because of this, the followers/followees ratio has been used as a “rule of thumb” for influence: users with a ratio greater than 1 are “influential” while those with a ratio lesser than 1 are “uninfluential”; besides, the larger the ratio the more “influential” the user.

A more complex way to estimate influence in Twitter exploits the fact that users and their relations constitute a directed graph and, thus, Twitter is amenable to analysis by means of eigenvector centrality algorithms. Such algorithms aim to compute the centrality of a node within a network starting from simple assumptions: 1) the centrality of a node depends on the centrality values of the nodes linking to it; 2) the more nodes linking to a given one, or the more central the few nodes linking to it, the more central that node will be; 3) centrality values for all of the nodes within the network are iteratively computed until convergence.

A number of algorithms exist to compute one or another “flavor” of these centrality scores. The “power iteration” method to compute the eigenvalues and eigenvectors of a matrix M is one of them. PageRank [5], HITS [6], TwitterRank [7], or TunkRank [8] compute different scores better adapted to the properties of the WWW or the Twitter graphs.

Another approach to estimate influence in social networks has been inspired by the work by Domingos and Richardson [9], and Kempe et al. [10]. Simply put, these researchers studied the way in which influence (e.g. related to purchase intention) virally spreads through a network, so a minimum set of influential users can be

**The implications of this are clear:** first, the number of followers and followees can be misleading if there are no interactions between users; second, centrality measures obtained from the “declared” user graph could be different from those obtained from the “hidden” interaction graph.

The second work is highly related to the first one; in it Romero et al. described the Influence-Passivity algorithm. This algorithm is closely related to PageRank, HITS, or TunkRank but, unlike them, the edges (their weights, indeed) and partial scores are inferred from user interactions, in concrete, retweets.

Its underlying assumptions are very appealing: 1) the influence of a user depends on the passivity of his followees and, conversely, the passivity depends on the influence of his followees. 2) For each pair of users, an acceptance and a rejection rate are computed for the follower user. The former is the amount of influence the follower accepts from his followee (i.e. the number of received messages s/he retweets) while the later is the amount of influence the follower rejects. 3) Thus, the passivity of a user is proportional to both his rejection rate and the influence of his followees while the influence of a user is proportional to both the acceptance rate and the passivity of his followers.

**A proposal for Twitter dynamics.** – Any of the above-mentioned methods to compute influence in Twitter requires the user graph. That graph alone is enough for eigenvector centrality methods but the rest of approaches also need the published tweets to find out

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1 That is, a user rejecting tweets from more influential users is more passive than a user rejecting the same amount but from less influential followees.
the *reposts*, *mentions*, diffusion cascades, and “hidden” relations between users. Both data assets (*tweets* and user graph) are distinct in nature and are also crawled in different ways.

*Tweets* are relatively easy to obtain as a data stream and most of the computation on them can be performed in near real time. The user graph, however, is a static snapshot or, at most, a series of snapshots. Nevertheless, *Twitter*, and *OSNs* in general, are highly dynamic systems, with users joining and quiting the network, and linking and unlinking among them continuously. Thus, *snapshots* are a pale approximation to the actual evolving network.

Arguably, a reasonable approximation is better than no approximation at all; still, in the light of the findings by Huberman *et al.* [15], it should be wondered: *Is the user graph really needed to get a picture of *Twitter*?* More concretely, *is there any way of inferring influence by just relying on the most basic actions of *Twitter* users?*

The method described here demonstrates that the user graph can be greatly disregarded, and *mention* counts alone are enough to provide an accurate and dynamic picture of *Twitter*. Given that *mentions* are citations this should be unsurprising; however, our approach is not based on bibliometrics but on physics, concretely on dynamic friction and uniformly accelerated linear motion.

To devise this new approach, concepts from dynamics such as *force*, *mass*, *acceleration* and *velocity* have been translated to an *OSN* scenario. Thus, a user’s *influence* is modeled as *velocity* and, thus, *acceleration* can be used to detect trending users in real time.

Newton’s second law (see eq. (1)) is the starting point; how does it translate to *Twitter*? First, the *mass* of a user is the number of *followers*. Second, the *force* applied to put a user in motion is the number of *mentions* received. This way, a user with a high number of *followers* needs more *mentions* to start “moving” while a “lighter” user (one with a lower number of *followers*) requires fewer *mentions*.

Equation (1), however, assumes instantaneous *forces* and *accelerations*, and continuous time. For implementation purposes it is much simpler to operate in discrete time intervals. Therefore, all of the experiments here described were performed using one-hour sampling intervals. This way, the *force* applied on a user is, indeed, the number of *mentions* addressing that user in a given hour.

Additionally, under real circumstances there are more forces at stake: mainly, the force of kinetic friction $F_f$ (see eq. (2)). Thus, *mentions* are actually the applied force, $F_m$, while $F$ is the resultant force of $F_f$ and $F_m$ which depends on the normal force $N$ and the coefficient of friction $\mu$. That way, acceleration would be defined by eq. (3).

Because eq. (3) is to be translated to a non-physical scenario it can be simplified by supposing that $\mu$, $g$, and $\cos(\Theta)$ are constant for every run of the method. Thus, *acceleration* in *Twitter* would be defined by eq. (4) where $\zeta$ is a damping constant responsible for the decay of users’ *acceleration* and *velocity* when they do not receive any *mention*.

Of course, the value for that constant must be empirically determined and should have the same dimensions as the quotient $F/m$, that is, *mentions* per hour per *follower*. Hence, $\zeta$ value would be the average number of *mentions* per hour per user, divided by the number of *followers* an average user has got in *Twitter*.

Finally, the *velocity* of a *Twitter* user would be computed according to eq. (5) taking into account that 1) time is discrete, using one-hour sampling; 2) $m$ is the number of *followers* of the user; 3) $F_m$ is the number of *mentions* addressing the user in the last hour; 4) $\zeta$ is a constant positive number; and 5) negative *velocities* are not allowed but they should be replaced by zero.

Experimental evaluation. – So far, another model to compute a score which may or many not relate to *influence* has been proposed. Thus, a way to correlate *velocity* with *influence* was also needed.

As has been said, *influence* should exert measurable effects and in this sense the evaluation approach by Romero *et al.* [16] is pretty clever: they argued that, in the context of *Twitter*, *influence* should correlate with attention and, therefore, *URLs* posted by influential *Twitter* users should receive more visits than those published by less influential ones.

Needless to say, the number of visits a given *URL* receives is just known to each website administrator. However, because of the length limit of the *tweets* virtually

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2That sampling interval was chosen after some preliminary experiments.

3In fact, smoother results can be obtained by applying natural logarithms to the number of *followers*.
every URL published in Twitter is shortened by means of one of several services⁴.

One of them, bit.ly, provides an API which allows anyone to check the number of clicks a given short URL has received. Hence, using that API, it was possible to associate to bit.ly URLs appearing in tweets from the aforementioned dataset the corresponding number of visits those URLs had received. Then, it was straightforward to check for any correlation between the influence of the users publishing the URLs and the visits for those URLs.

Some changes were made with respect to the methodology by Romero et al. They worked with URLs without taking into account for how long those URLs appeared in the Twitter stream. This is quite pertinent because some URLs can consistently appear for weeks or months, achieving a high number of visits with little or no relation at all with the influence of the users promoting them. Therefore, in addition to preparing a URL dataset in the fashion of Romero et al., a second one comprising URLs which appeared during one single week was compiled. An additional advantage of this second dataset is that it made possible to correlate URL visits with velocity values computed each week instead of comparing visits with one single final score for each user.

Finally, outliers were eliminated from both datasets using the common interquartile range method (k = 1.5). To that end, URL visits were considered and those URLs with exceedingly high numbers of visits were removed. In the second dataset, the outliers were computed for each different week and not for the whole dataset.

Hence, the first dataset was finally comprised of 22920 URLs while the second one contained 10120 URLs distributed in 29 weeks —from January 26 to August 16, 2009— with an average of 349 URLs and a standard deviation of 139.4.

Influence metrics compared. Romero et al compared the predictive power of their Influence-Passivity (I-P) score with both PageRank and the number of followers. For this study not only those metrics were compared but also the recently proposed TunkRank, and the new method described in this letter —i.e. velocity.

Therefore, the number of followers, I-P, PageRank, and TunkRank were determined for those users appearing in the Twitter dataset. Furthermore, velocities were computed and those reached at the end of each week were stored.

That way it was possible to associate every URL with both a number of visits, and a list of users who had “promoted” that URL in Twitter. Those users, in turn, had known “influence” scores. Therefore, it was just needed to look for any significant correlation between the number of clicks and each of the scores. To that end, the scores for those users promoting each URL were accumulated and, thus, for each URL a number of clicks and a single accumulated “influence” score were available.

Some caveats should be noted. Firstly, when correlating “influence” with clicks from the URL dataset which ignored week limitations, the velocities employed were those reached by users on August 16, 2009 no matter the date when the URL had been published. This is rather unrealistic but consistent with the way in which the rest of scores were obtained: after all, PageRank and TunkRank were computed from a graph crawled well after August 16, and the retweets required to applied the Influence-Passivity method were found across the whole dataset instead of using just the tweets predating the URLs.

Secondly, a single empirically found damping factor (0 < ζ < 1) was applied to compute velocity. Preliminary experiments showed that dynamic damping (i.e. a constant computed for each week or day based on the tweeting behavior of users during that period) did not provide better correlation. The same experiments revealed that although a frictionless scenario (ζ = 0) also shows a positive correlation between influence and clicks; that correlation is much weaker than when using a positive damping factor and, thus, such a frictionless model was disregarded.

Pearson’s r was employed to compare URL clicks with accumulated “influence”. Assuming a linear regression model between a given “influence” score and URL visits can be an oversimplification but, hopefully, it could shed some light on the relation between such scores and observable events and, besides, it would make the results of this study comparable to those obtained by Romero et al. who relied on similar assumptions.

Experimental findings. – Table 1 shows the results obtained when comparing the “influence” scores with the visits received by URLs in the dataset ignoring weekly limitations. Coefficients are not too high but, still, they are significant because of the sample size (22920 URLs). From those results, it seemed that all of the “influence” scores exhibit a positive correlation with URL visits.

Nevertheless, the attentive reader might have noted that a positive correlation between these “influence” scores and URL visits is not surprising but, instead, expected. Indeed,
Table 2: Correlation between the accumulated number of followers and the rest of “influence” scores using the information in the dataset ignoring weekly limitations. There exists a significant ($p \ll 0.001$) positive correlation between the number of followers and the “influence” scores. Influence-Passivity seems to be the method less sensitive to the number of followers and PageRank the most sensitive.

| “Influence” score | Pearson’s $r$ | Significance |
|-------------------|--------------|--------------|
| Influence (I-P)   | 0.27994      | $p \ll 0.001$|
| PageRank          | 0.37437      | $p \ll 0.001$|
| TunkRank          | 0.87284      | $p \ll 0.001$|
| velocity          | 0.37735      | $p \ll 0.001$|
| velocity          | 0.29921      | $p \ll 0.001$|
| velocity (on week)| 0.37735      | $p \ll 0.001$|
| velocity (prior week) | 0.37437  | $p \ll 0.001$|

Table 3: Correlation between different “influence” scores and clicks received by URLs in the dataset ignoring weekly limitations after correcting for the confounding variable audience (i.e., scores and clicks were divided by the accumulated number of followers of the users promoting the URLs). All of the scores, except for I-P, show significant positive correlations.

| “Influence” score | Pearson’s $r$ | Significance |
|-------------------|--------------|--------------|
| Influence (I-P)   | −0.01021     | Non-significant|
| PageRank          | 0.04399      | $p \ll 0.001$|
| TunkRank          | 0.13550      | $p \ll 0.001$|
| velocity          | 0.26532      | $p \ll 0.001$|

The high correlation between the number of followers and the clicks received provides a clue.

Algorithms such as PageRank, TunkRank, or Influence-Passivity are devised in such a way that users with few followers can still achieve rather high scores provided their few followers are “influential”. However, this is not the norm but the exception: most of the users with a high score also have a large number of followers. Hence, if users with a high PageRank, TunkRank, I-P, or velocity score have lots of followers, it is not strange that the URLs they promote receive more visits than those promoted by users with lower “influence” scores; after all, they have much larger audiences and, thus, more visits are expected.

Table 2 reveals that a highly significant positive correlation exist between the number of followers and the different “influence” scores. In other words, the accumulated number of followers for the URLs is a confounding variable and the data must be corrected for it.

To that end, both clicks and the different “influence” scores must be divided by the accumulated number of followers — i.e., the expected audience for each URL. This way, it would be checked if there exists any correlation between the probability for a member of a given audience to visit a URL and the fraction of the URL promoters’ influence that member is receiving.

Table 3 shows the results obtained after correcting the data for audience. The results for Influence-Passivity are inconclusive because there are no significant correlation while the rest of “influence” scores show significant positive correlations. Velocity seems to be the best predictor, followed by TunkRank and then, PageRank.

It should be remembered that all of these results were obtained using the first URL dataset which did not take into consideration weekly limitations and, because of that, velocity scores were those reached by users on August 16, 2009. Another set of results was obtained by using the second dataset, comprising URLs which appeared in one single week. For those experiments, three different velocity scores were employed: 1) velocities reached on August 16, 2009; 2) velocities computed at the end of each week; and 3) velocities computed at the end of the prior week. Clearly, the third “flavor” is the closest one to a real-time application.

The correlation coefficients reported in table 4 were obtained by averaging the coefficients found for each week (cf. [18]) while the significance was computed according to the average sample size (349 URLs per week). These results are consistent with those of table 3: the correlation between Influence-Passivity and clicks is again non-significant; the rest of scores exhibit a significant positive correlation with URL visits; and, again, velocity is the best predictor.

On a side note, velocities computed on the week when URLs were published are slightly better predictors than velocities computed the week before. This would be of course expected if velocity in Twitter was a valid proxy measure for influence.

Conclusions. — A new method to compute Twitter influence based on a physical metaphor has been proposed. It does not rely on the Twitter user graph which is costly to crawl, just provides static snapshots of a rapidly evolving network, and does not represent actual user interactions. Instead, the new method just requires the streamline of tweets to detect user mentions and, thus, it can be applied in near real time.

A number of experiments were conducted to check whether the new velocity score actually correlates with influence. Results from those experiments have been reported, revealing that most of the commonly applied scores such as the number of followers, or PageRank, and
recently proposed ones such as TunkRank, or Influence-Passivity, certainly exhibit a positive correlation with website visits. However, it has also been shown that the number of followers is a confounding variable which must be accounted for. Therefore, it is not the total number of visits and the different “influence” scores which have to be correlated but, instead, the probability of a user visiting a promoted URL and the fraction of the promoter’s influence a single user is receiving. After correcting the data for the audience, it was revealed that all of the “influence” scores except for Influence-Passivity exhibit positive correlation with user clicks and, thus, with influence in the sense of “attention gathering”. Velocity, the score inferred using the method proposed in this letter, was by a large margin the best predictor of user clicks.

Hence, this study makes a number of contributions. 1) It adds to the general understanding of the concept of influence in OSNs and its relation to “attention gathering”; 2) it has exposed the caveat due to the confounding nature of audience in this scenario; 3) it has shown how centrality measures can be used as rather good predictors of influence; and 4) it has described a new method that outperforms them with regards to influence scoring, and that can be applied in real time to rank users and to detect emerging “influentials”. In this sense, an interesting future line of work would be studying the feasibility of adapting this new model to tweets themselves to detect trending topics and compare its performance with Twitter’s own implementation.

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REFERENCES

[1] Gayo-Avello D., arXiv:1004.0816v1, 2010.
[2] Boyd D., Golden S. and Lottan G., HICSS, 2010 (IEEE Computer Society, Washington, DC, USA) 2010, pp. 1–10.
[3] Weil K., Measuring Tweets, blog post, 2010.
[4] Schonfeld E., Twitter Reaches 44.5 Million People Worldwide In June, blog post, 2009.
[5] Page L., Brin S., Motwani R. and Winograd T., The PageRank Citation Ranking: Bringing Order to the Web, Technical Report, 1998.
[6] Kleinberg J. M., Proceedings of the Ninth Annual ACM-SIAM Symposium on Discrete Algorithms, 1998 (Society for Industrial and Applied Mathematics, Philadelphia, Penn., USA) 1998, pp. 668–677.
[7] Weng J., Lim E. P., Jiang J. and He Q., WSDM 2010 (ACM, New York, NY, USA) 2010, pp. 261–270.
[8] Tunkelang D., A Twitter Analog to PageRank, informal publication, 2009.
[9] Domingos P. and Richardson M., Proceedings of the Seventh ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, 2001 (ACM, New York, NY, USA) 2001, pp. 57–66.
[10] Kempe D., Kleinberg J. and Tardos E., KDD ’03: Proceedings of the Ninth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, 2003 (ACM, New York, NY, USA) 2003, pp. 137–146.
[11] Bakshy E., Hofman J. M., Mason W. A. and Watts D. J., Identifying “Influencers” on Twitter, in Proceedings of the Fourth ACM International Conference on Web Search and Data Mining (ACM, New York, NY, USA) 2011.
[12] Galuba W., Aberer K., Chakraborty D., Despotovic Z. and Kellerer W., Proceedings of the 3rd Conference on Online Social Networks, 2010 (USENIX Association, Berkeley, Cal., USA) 2010, p. 3.
[13] Lee C., Kwak H., Park H. and Moon S., Proceedings of 19th World Wide Web (WWW) Conference, 2010, (ACM, New York, NY, USA) 2010, pp. 1137–1138.
[14] Java A., Kolari P., Finin T. and Oates T., WWW 2006 Workshop on Weblogging Ecosystem: Aggregation, Analysis and Dynamics, 2006, pp. 1–7.
[15] Huberman B. A., Romero D. M. and Wu F., First Monday, 14 (2009).
[16] Romero D. M., Galuba W., Asur S. and Huberman B. A., arXiv:1008.1253v1, 2010.
[17] West J. D., Bergstrom T. and Bergstrom C. T., arXiv:0911.1807v2, 2010.
[18] Cramer D. and Howitt D., The SAGE Dictionary of Statistics (SAGE Publications Ltd., London) 2004, p. 40.