Abstract

Metastatic tumor blood perfusion and interstitial fluid transport based on 3D microvasculature response to inhibitory effect of angiostatin are investigated. 3D blood flow, interstitial fluid transport, and transvascular flow are described by the extended Poiseuille’s, Darcy’s, and Starling’s law, respectively. The simulation results demonstrate that angiostatin has the capacity to regulate and inhibit the formation of new blood vessels and has an obvious impact on the morphology, growth rate, and the branches of microvascular network inside and outside the metastatic tumor. Heterogeneous blood perfusion, wide-spread interstitial hypertension, and low convection within the metastatic tumor have obviously improved under the inhibitory effect of angiostatin, which suits well with the experimental observations. They can also result in more efficient drug delivery and penetration into the metastatic tumor. The simulation results may provide beneficial information and theoretical models for clinical research of antiangiogenic therapy strategies.

Keywords: blood perfusion, interstitial transport, metastatic tumor, angiostatin, 3D simulation

1. Introduction

Cancer is the second leading cause of mortality worldwide [1], right behind cardiovascular disease. Metastatic tumors, the ultimate causes of death for the majority of cancer patients, are the important biological characteristics of malignant tumors. Metastasis occurs when cancer
cells spread from a primary tumor to distant and vital organs (secondary sites) in human body. Angiogenesis is necessary for tumor growth, invasion, and metastasis [2], since it supplies the nutrients and oxygen for continued tumor growth. The neovascularization accelerates the growth of tumor while simultaneously offering an initial route by which cancer cells can escape from a primary tumor to form metastatic tumor. Cancer cells migrate into the blood stream and surrounding tissues via microcirculation, then continue to grow giving rise to metastases [3]. The blood perfusion and interstitial fluid flow have been recognized as critical elements in metastatic tumor growth and vascularization [4]. However, tumor vessels are dilated, saccular, tortuous, and heterogeneous in their spatial distribution. These abnormalities result in heterogeneity of blood flow and elevated interstitial fluid pressure (IFP), which forms a physiological barrier to the delivery of therapeutic agents to tumors [5]. Abnormal microvasculature and microenvironment further lowers the effectiveness of therapeutic agents.

Experimental research showed that the primary tumor in the Lewis lung model system was capable of generating a factor which was named angiostatin later suppressing the neovascularization and expansion of tumor metastases [6]. Angiostatin is a 38-kD internal peptide of plasminogen, which is a potent inhibitor of angiogenesis in vivo, and selectively inhibits endothelial cell (EC) proliferation and migration in vitro. Tumor cells express enzymatic activity which is capable of hydrolyzing plasminogen to generate angiostatin [7]. Angiostatin is then transported and accumulated in the blood circulation in excess of the stimulators and thus inhibiting angiogenesis of a metastatic tumor. Angiostatin, by virtue of its longer half-life in the circulation [8], reaches the vascular bed of metastatic tumor. As a result, growth of a metastasis is restricted by preventing and inhibiting angiogenesis within the vascular bed of the metastasis itself. A schematic diagram of this process is given in Figure 1. Indeed, anti-angiogenic treatments directly targeting angiogenic signaling pathways as well as indirectly modulating angiogenesis show normalization of tumor microvasculature and microenvironment at least transiently in both preclinical and clinical settings.

In spite of several mathematical models of metastatic tumors, there appears to be little in the literature by way of mathematical modeling of the mechanisms of antiangiogenic activity of angiostatin on blood flow and interstitial fluid pressure in a metastatic tumor. Liotta et al. [9] first

Figure 1. Schematic representation of angiostatin transported from a fully vascularized primary tumor to its relation to a distant secondary tumor.
developed an experimental model to quantify some of the major processes initiated by tumor transplantation and culminating in pulmonary metastases. The study suggested that “dynamics of hematogenously initiated metastases depended strongly on the entry rate of tumor cell clumps into the circulation, which in turn was intimately linked to tumor vascularization.” Later in the study, Liotta et al. [9] confirmed their former observation and raised the idea that “larger clumps produce significantly more metastatic foci than do smaller clumps matched for the number of cells.” Saidel et al. [10] proposed a lumped-parameter, deterministic model of the hematogenous metastatic process from a solid tumor, which provided a general theoretical framework for analysis and simulation. Numerical solutions of the model were in good agreement with their experimental results [9]. The possibilities of anti-invasion and antimetastatic strategies in cancer treatment have bestowed an added preponderance with the keen interest in the mathematical modeling in the areas of tumor invasion and metastasis. Orme and Chaplain [11] presented a simple mathematical model of the vascularization and subsequent growth of a solid spherical tumor and gave a possible explanation for tumor metastasis, whereby tumor cells entered the blood system and secondary tumor may rise with the transportation function of blood. Sleeman and Nimmo [12] modified the model of fluid transport in vascularized tumors by Baxter and Jain [13] to take tumor invasion and metastasis into consideration. Although these models did provide some features of tumor metastasis and interstitial fluid transportation such as perturbation analysis, they lacked in providing more detail information of metastatic tumor and as such were of limited predicted value. More realistic models of metastasis and interstitial fluid transportation were developed to better understand its mechanism. Anderson et al. [14] presented a discrete model from the partial differential equations of the continuum models which implied that haptotaxis was important for tumor metastasis. Iwata et al. [15] proposed a partial differential equation (PDE) that described the metastatic evolution of an untreated tumor, and its predicted results agreed well with successive data of a clinically observed metastatic tumor. Benzekry et al. [16] proposed an organism-scale model for the development of a population of secondary tumors that takes into account systemic inhibiting interactions among tumors due to the release of a circulating angiogenesis inhibitor. Baratchart et al. [17] derived a mathematical model of spatial tumor growth compared with experimental data and suggested that the dynamics of metastasis relied on spatial interactions between metastatic lesions. Stéphanou et al. [18] investigated chemotherapy treatment efficiency by performing a Newtonian fluid flow simulation based on a study of vascular networks generated from a mathematical model of tumor angiogenesis. Wu et al. [19] extended the mathematical model into a 3D case to investigate tumor blood perfusion and interstitial fluid movements originating from tumor-induced angiogenesis. Soltani and Chen [20] first studied the fluid flow in a tumor-induced capillary network and the interstitial fluid flow in normal and tumor tissues. The model provided a more realistic prediction of interstitial fluid flow pattern in solid tumor than the previous models. Some related works have been done on tumor-induced angiogenesis, blood perfusion, and interstitial fluid flow in the tumor microenvironment by using 2D mathematical methods [5, 21–23]. In spite of the valuable body of work performed in simulation of blood perfusion, interstitial fluid flow, and metastasis, previous studies have not examined blood perfusion and interstitial fluid pressure in the metastatic tumor microcirculation based on the 3D microvascular network response to the inhibitory effect of angiostatin which plays a significant role in suppressing tumor growth and metastasis.
Metastatic tumor blood perfusion and interstitial fluid transport based on 3D microvasculature response to inhibitory effect of angiostatin are investigated for exploring the suppression of metastatic tumor growth by the primary tumor. The abnormal geometric and morphological features of 3D microvasculature network inside and outside the metastatic tumor, and relative complex and heterogeneous hemodynamic characteristics in the presence and absence of angiostatin can be studied in the 3D case. The simulation results may provide beneficial information and theoretical basis for clinical research on antiangiogenic therapy.

2. 3D mathematical models

2.1. Metastatic tumor angiogenesis

3D mathematical model we present in this section originates from the previous 2D tumor antiangiogenesis mathematical model [5, 21] describing how capillary networks form in a metastatic tumor in response to angiostatin released by a primary tumor. The conservation equation of endothelial cells (EC) indicates the migration of EC influenced mainly by four factors: random motility, inhibitory effect of angiostatin, chemotaxis, and haptotaxis. Subsequently, from a discretized form of the partial differential equations governing endothelial-cell motion, a discrete biased random-walk model will be derived enabling the paths of individual endothelial cells located at the sprout tips, and hence the individual capillary sprouts, to be followed. Hence, realistic capillary network structures were generated by incorporating rules for sprout branching and anastomosis. The generated microvascular network inside and outside the metastatic tumor in the presence of angiostatin and in the absence of angiostatin is shown in Figure 2. General morphological features of the network such as growth speed, capillary number, vessel branching order, and anastomosis density in/outside the metastatic tumor are consistent with the physiologically observed results, which indicate that angiostatin secreted by the primary tumor dose has an inhibitory effect on metastatic tumor [5, 11].

2.2. Blood perfusion

To calculate blood flow through a given 3D microvascular network of interconnected capillary elements to the metastatic tumor, assuming flux conservation and incompressible flow at each junction where the capillary elements meet

$$\sum_{n=1}^{6} Q_{(l,m,j)}^{n} \cdot B_{(l,m,j)}^{n} = 0$$

(1)

where $B_{(n)}^{n}$ takes the integer 1 or 0, representing the connectivity between node $(l, m, j)$ and its adjacent node $n$. $Q_{(l,m,j)}^{n}$ is the flow rate from node $(l, m, j)$ to node $n$ and is given by $Q_{(l,m,j)}^{n} = Q_{v,(l,m,j)}^{n} - Q_{t,(l,m,j)}^{n}$, where $Q_{v,(l,m,j)}^{n}$ is the vascular flow rate without fluid leakage, described locally by Poiseuille’s law

$$Q_{v,(l,m,j)}^{n} = \frac{\pi R_{n}^{4} (p_{v,(l,m,j)}^{n} - p_{v,(n)})}{8 \mu_{n} \Delta L_{n}}$$

(2)

and $Q_{t,(l,m,j)}^{n}$ is the transvascular flow rate, following Starling’s law.
\[ Q^n_{\alpha,(l,m,j)} = 2\pi R_n \Delta L_n \cdot L_{pv} \left[ (\bar{p}^v_{\alpha,(l,m,j)} - \bar{p}^v_{\beta,(l,m,j)}) - \sigma_t (\pi_v - \pi_i) \right] \] (3)

where \( p^v_{\alpha,(l,m,j)} \) and \( p^v_{\alpha,(l,m,j)} \) are the intravascular pressure of node \((l,m,j)\) and node \(n\); \( \bar{p}^v_{\alpha,(l,m,j)} \) is the mean pressure in vascular element \((l,m,j)\); \( \bar{p}^v_{\beta,(l,m,j)} \) is the mean interstitial pressure outside of vascular element \((l,m,j)\). \( \mu_n, R_n, \) and \( \Delta L_n \) are the blood viscosity, radius, and length of the vessel element \(n\), respectively; \( L_{pv} \) is the hydraulic permeability of vascular wall; \( \sigma_t \) is the average osmotic reflection coefficient for plasma proteins; \( \pi_v \) and \( \pi_i \) are the colloid osmotic pressure of plasma and interstitial fluid.

2.3. Interstitial flow in metastatic tumor

Considering the metastatic tumor tissue as an isotropic porous medium, its interstitial flow is modeled by Darcy’s law [24]:

\[ u_i = -\kappa \nabla p_i \] (4)

where \( u_i \) is the interstitial fluid velocity; \( \kappa \) is the hydraulic conductivity coefficient of the interstitium; \( p_i \) is the interstitial pressure.

The continuity equation is given by:
\[ \nabla \cdot \mathbf{u}_i = \phi_e - \phi_L \]  

(5)

where \( \phi_e = \frac{L_v S_v}{V} (p_e - p_i - \sigma (\pi_e - \pi_i)) \) is the fluid source term leaking from blood vessels. \( \phi_L = \frac{L_v S_v}{V} (p_v - p_i) \) is the lymphatic drainage term, which is proportional to the pressure difference between the interstitium and the lymphatics.

Mass conservation at each junction where the interstitial fluid pressure satisfies equation:

\[ \nabla^2 p_i = \frac{\alpha^2}{\kappa^2} (p_i - \bar{p}_c) \cdot B \]  

(6)

where \( p_c = (L_v S_v (p_e - \sigma (\pi_e - \pi_i)) + L_v S_p) / (L_v S_v + L_v S_p) \) is the effective pressure and \( \alpha = R L \sqrt{(L_v S_v + L_v S_p) / \kappa V} \) is the ratio of interstitial to vascular resistances to fluid flow. \( L_v \) is the hydraulic permeability of lymphatic vessel wall. \( S_v / V \) and \( S_p / V \) are the surface areas of blood vessel wall and lymphatic vessel wall per unit volume of tissue. In the model, \( L_v S_v / V \) is assumed zero for tumor tissue, and given a uniform value for normal tissue referring to Baxter and Jain [13]. The continuity of pressure and flux on the interconnected boundary between the tumor and normal tissue \( \Gamma: p_i |_\Gamma = p_j |_\gamma \), \(-\kappa \nabla p_i |_\Gamma = -\kappa \nabla p_j |_\gamma \) and \( \kappa_v \) and \( \kappa_h \) are the hydraulic conductivity coefficients of normal tissue and tumor tissue, respectively.

Table 1 shows the values of the parameters used in the microcirculation simulations.

| Parameter | Name | Value | Parameter | Name | Value |
|-----------|------|-------|-----------|------|-------|
| \( \sigma_v \) | Average osmotic reflection coefficient for plasma proteins | \( 8.7 \times 10^{-3} \) | \( \kappa_v \) | Hydraulic conductivity coefficient of interstitium | \( 2.5 \times 10^{-7} \) cm²/mmHg s |
| \( \pi_v \) | Colloid osmotic pressure of plasma | \( 198 \) mmHg | \( \kappa_h \) | Surface area per unit volume for transport in interstitium | \( 50 \) cm⁻¹ |
| \( p_v \) | Lymphatic pressure | \( 0.5 \) mmHg | | | |
| \( \pi_i \) | Colloid osmotic pressure of interstitium | \( 173 \) mmHg | \( \kappa_h \) | Absorption capacity of lymphatic system | \( 0.1 \) 1/mmHg s |
| \( \mu \) | Blood viscosity | \( 1.0 \) cP | | | |

*Jain et al. [5].
*Stephanou et al. [18].
*Zhao et al. [25].

Subscript “\( \text{N} \)” and “\( \text{T} \)” represents the values in normal and tumor tissues, respectively.

Table 1. Baseline parameter values used in the simulations.

### 3. Simulation results

#### 3.1. 3D blood perfusion of metastatic tumor

We simulated the evolution of blood flow pressure in the presence/absence of angioptatin for 14 days representing the typical timescale for tumor vasculature to grow. Figure 3 shows...
the snapshots of the pressure profiles of blood flow through each vessel segment in a three-dimensional microvascular networks. We keep the inlet pressure and outlet pressure across parent vessel fixed at 25 and 16 mmHg [25] in the simulation, in accordance with physiological values at the capillary scale. Figure 3 highlights a direct comparison of blood pressure distributions (Figure 3a–c shows the blood pressure distribution in the presence of angiostatin, Figure 3d–f shows the blood pressure distribution in the absence of angiostatin). We observe that the overall blood pressure is higher in the presence of angiostatin than that in the absence of angiostatin over the same growth duration. The blood flow distribution is complex and chaotic which makes the variety of blood pressure small in the interior of the metastatic tumor compared to its exterior, contributing to the difficulties of efficient drug delivery in metastatic tumor. In the presence of angiostatin, the pressure-flows within some of the daughter vessels are elevated from the branching points to the metastatic tumor surface which provides effective blood perfusion and thus efficient therapeutic agents to the tumor. The simulation results indicate that blood perfusion varies significantly with the complex and chaotic three-dimensional microvascular networks inside and outside the metastatic tumor. The poor blood perfusion can be improved through the increased intravascular pressure with the presence of angiostatin. These results suggest that the inhibitory effect of angiostatin can affect the distribution of blood flow pressure and improve drug delivery to tumor.
3.2. 3D interstitial fluid flow of metastatic tumor

Figure 4 shows the distribution of interstitial fluid pressure (IFP) within the metastatic tumor under the two mentioned situations. From the simulation results, we obtain that maximum IFP near the tumor center significantly dropped from 3.3, 11.48, and 11.53 to 0, 4.7, and 10.3 mmHg.

Figure 4. Interstitial fluid pressure distributions within the metastatic tumor: (a–c) in the presence of angiostatin; (d–f) in the absence of angiostatin on the same other conditions.
with the presence of angiostatin at $t = 3, 7$ and 14 days, respectively, which indicated the IFP plateau is well relieved. As the growth days increase, IFP gradually elevates throughout the 3D metastatic tumor and the high pressure zone is at the center of the tumor and diminishes to the periphery and later becomes flatter. Comparing Figure 4a–c to Figure 4d–f, we come to conclude that angiostatin decreases the high IFP in the tumor, thus with the lower transvascular pressure in the 3D heterogeneous capillary networks, leading to an significantly improved situation for interstitial convection which plays a significant role in nonuniform distribution of drug delivery to the metastatic tumor. These results provide important references for cancer prevention and treatment. Furthermore, antiangiogenic therapies can normalize tumor vasculature and microenvironment, at least transiently in both preclinical and clinical settings [5].

4. Conclusion

The inhibitory effect of angiostatin on the growth of metastatic tumor has been observed in some clinical and experimental malignancies. In this chapter, we develop three-dimensional mathematical models describing the metastatic tumor microvasculature and microenvironment to investigate the inhibitory effect of antiangiogenic factor angiostatin secreted by the primary tumor on metastatic tumor angiogenesis, blood perfusion, and interstitial fluid flow. Simulation results demonstrate that angiostatin has an obvious impact on the morphology, expansion speed, capillary number, and vessel branching order inside and outside the metastatic tumor. 2D and 3D mathematical models of tumor antiangiogenesis predict similar morphological behavior, such as vessels’ length, branching patterns, anastomosis density, or geometric distribution, for metastatic tumor angiogenesis under the inhibitory efficiency of angiostatin. However, capillary number and microvascular density due to space growth of vessel networks are increased in the 3D model. Furthermore, the simulations reflect the influences of heterogeneous blood perfusion, widespread interstitial hypertension, and low convection within the 3D metastatic tumor by carrying out a comparative study relating to the inhibitory effect of angiostatin. We find that 2D antiangiogenesis model may be well suited to studying morphological behavior of vessel networks in the metastatic tumor, but 3D antiangiogenesis model can better analyze blood perfusion, interstitial fluid flow, or oxygen and nutrient transport within the metastatic tumor microenvironment based on its more realistic 3D microvascular networks. Although 3D simulation results are consistent with the experimental observed facts and can provide more detailed space information, however, angiogenesis and hemodynamics in the metastatic tumor by the antiangiogenic therapy are very complex. To further research tumor angiogenic mechanisms and help to improve antiangiogenic cancer therapy, more realistic features and complex biology factors need to be incorporated within the 3D model, such as the anatomy and physiology of the metastatic tumor, drug delivery of antiangiogenic therapy, behaviors of cells adhesion and interaction and coupled with the various factors or other therapy strategies.

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Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this chapter.

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