A Survey on Autonomous Vehicle Control in the Era of Mixed-Autonomy: From Physics-Based to AI-Guided Driving Policy Learning

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Abstract

This paper serves as an introduction and overview of the potentially useful models and methodologies from artificial intelligence (AI) into the field of transportation engineering for autonomous vehicle (AV) control in the era of mixed autonomy. It is the first-of-its-kind survey paper to comprehensively review literature in both transportation engineering and AI for mixed traffic modeling. We will discuss state-of-the-art applications of AI-guided methods, identify opportunities and obstacles, raise open questions, and help suggest the building blocks and areas where AI could play a role in mixed autonomy. We divide the stage of autonomous vehicle (AV) deployment into four phases: the pure HVs, the HV-dominated, the AV-dominated, and the pure AVs. This paper is primarily focused on the latter three phases. Models used for each phase are summarized, encompassing game theory, deep (reinforcement) learning, and imitation learning. While reviewing the methodologies, we primarily focus on the following research questions: (1) What scalable driving policies are to control a large number of AVs in mixed traffic comprised of human drivers and uncontrollable AVs? (2) How do we estimate human driver behaviors? (3) How should the driving behavior of uncontrollable AVs be modeled in the environment? (4) How are the interactions between human drivers and autonomous vehicles characterized? Hopefully this paper will not only inspire our transportation community to rethink the conventional models that are developed in the data-shortage era, but also reach out to other disciplines, in particular robotics and machine learning, to join forces towards creating a safe and efficient mixed traffic ecosystem.

Keywords: Artificial intelligence (AI), Autonomous vehicle (AV) control, Mixed autonomy

1. Introduction

We are transitioning into a big data era from a data-shortage era, thanks to the popularity of ubiquitous sensors, such as GPS [Hecker et al., 2018a,b; Hammit et al., 2018; Flores et al., 2018; Zhang et al., 2018a], blue tooth [Allström et al., 2014a], and smart phones [Herrera et al., 2010]. Autonomous vehicles (AV), mounted with sensors like camera and LiDAR, will potentially provide exploding volumes of transportation data [SAS, 2015]. While moving from a data-sparse to a data-rich era, we, the transportation community, urgently need a methodological paradigm shift from physics-based models to artificial intelligence (AI)-guided methods, which can project future traffic dynamics comprised of AVs driving alongside human-driven vehicles (HV) and assist in socially optimal policy-making. **Physics-based** (or rule-based [Zhou and Laval, 2019]) models refer to all the scientific hypotheses about the movement of cars or traffic flow, including traffic models on micro-, meso-, and macro-scale; while **AI-guided** methods refer to cutting-edge models that mimic human intelligence, leveraging deep neural networks, reinforcement learning, imitation learning, and other advanced machine learning methods.

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This paper serves as an introduction and overview of the potentially useful models and methodologies from AI into the field of transportation engineering in the era of mixed autonomy. We will discuss the state-of-the-art applications of AI-guided methods to AV controls, identify opportunities and obstacles, raise open questions, and help suggest the building blocks and areas where AI could play a role in mixed autonomy. It is the first-of-its-kind survey paper to comprehensively review literature in both transportation engineering and AI for mixed traffic modeling. Hopefully this paper will not only inspire our transportation community to rethink the conventional models that are developed in the data-shortage era, but also reach out to other disciplines, in particular robotics and machine learning, to join forces towards creating a safe and efficient mixed traffic ecosystem.

Vehicles’ driving choices contain three levels: operational level (including pedal and brake control, turn signal), tactical level (including lane-changing, lane-keeping), and strategic level (including routing). This paper is mainly focused on the operational and tactical controls of AVs. The operational and tactical controls can be further categorized into longitudinal control (i.e., car-following, lane-keeping) and lateral control (i.e., lane-change). Longitudinal control has been studied in various scenarios, including: platooning (Gong et al., 2016; Zhou et al., 2017b; Wei et al., 2017; Li et al., 2018d), speed harmonization (Ma et al., 2016; Malikopoulos et al., 2018; Arefizadeh and Talebpour, 2018), longitudinal trajectory optimization (Wei et al., 2017; Li et al., 2018b), and eco-approach and departure at signalized intersections (Altan et al., 2017; Hao et al., 2018; Yao et al., 2018). Most of the existing studies are limited to a single AV navigating along a highway or dense with human drivers, or all AVs dominate the road with negligible interactions with HVs (Katrakazas et al., 2015).

1.1. Modeling complexity

To date, the vast majority of existing research has focused - perhaps unsurprisingly - on two polar scenarios, where either a single AV navigates in an ecosystem dense with human drivers, or a platoon of AVs move along a highway, with negligible interaction with human-controlled counterparts. Much less attention has been accorded to the far more realistic, yet challenging transition path between these two scenarios. However, it is precisely this hybrid human-machine space that deserves our concerted attention now, so-called “mixed autonomy” (Wu et al., 2017b).

We divide the stage of AV deployment into four phases: the pure HVs, the HV-dominated, the AV-dominated, and the pure AVs. This paper focuses on the latter three phases. Figure 1 demonstrates the modeling complexity for each phase. It is most challenging to model the HV-dominated and AV-dominated phases, in other words, mixed autonomy. This is an understudied phase due to the unknown and complex interactions among different types of vehicles. We further divide mixed autonomy by the relative proportion of AVs and HVs using the following notions (indicated in red boxes in Figure 1):

- 1 AV + 1 HV: one AV interacts with one HV;
- 1 AV + m HVs: one AV navigates the HV-dominated traffic environment;
- n AVs + m HVs: multiple AVs navigate the HV-dominated traffic environment;
  - n AVs + 1 HV: multiple AVs interact with one HV in the AV-dominated traffic environment; It is one special case of n AVs + m HVs.
- n AVs: a pure AV market where all vehicles are replaced by AVs. Accordingly, AVs interact with one another.

Remark 1.1. In the process of preparing this paper, we discover another survey paper (Zhou and Laval, 2019) on longitudinal control of AVs and their impact on traffic congestion. The main difference is that Zhou and Laval (2019) focus on training a single AV on an empty highway or with a few HVs surrounded (corresponding to 1 AV + m HVs), while ours reviews a broader literature in mixed autonomy.
Figure 1: Modeling complexity at each stage (Four research questions will be answered through models of each phase. White boxes indicate the model types for each phase. Research communities are enclosed by dashed green boxes.)

In Figure 1, the community associated with each phase is enclosed by a green dotted box. The transportation community has primarily focused on modeling the pure HVs, the AV-dominated, and the pure AVs, while AI and control communities are more focused the HV-dominated phase where a single AV or a finite number of AVs navigate the traffic environment. Below we will further elaborate how these communities diverge in autonomous control modeling.

1.2. Divergence in the communities

Researchers from the transportation community and the robotics community refer to same quantities with different terminologies (Fernandez Fisac, 2019). Here we present all the relevant terminologies across the communities in Table 1.

| Transportation | Control | AI | Game Theory |
|----------------|---------|----|-------------|
| Traffic        | System  | Environment | Game |
| Traffic evolution | Dynamics | Transition | Dynamics |
| Traffic state | State space | State | State |
| CAV            | CACC | AV | AV |
| Car-driver unit | Controller | Agent | Player |
| Vehicle control | Control | Action | Play |
| Vehicle control | Control | Action | Play |
| Vehicle control law | Control law | Policy | Strategy |
| Vehicle control objective | Cost | Reward | Payoff |
| Car-following | Longitudinal control | - | - |
| Lane-change | Lateral control | - | - |
| Driving behavior | Driver model | Driver intent | Rationality |
| Traffic outcome | Optimal control | Optimal policy | Equilibrium |

When it comes to the autonomous driving controller design, these two communities take two different paths. First of all, these two communities share different goals. Transportation researchers aim to understand the influence of AVs on the transportation system performance (from the SYSTEM perspective),
such as traffic congestion (Zhou and Laval, 2019), while robotics researchers are primarily focused on the development of optimal driving policies for AVs to learn and adapt in a stochastic environment (from the VEHICLE perspective). As a consequence, these communities investigate problems on different scales.

The robotics community aims to design AV controllers with human-in-the-loop. In particular, researchers on human-cyber-physical systems design AVs that actively influence human drivers through mutual interactions, in order to achieve efficient driving. Their impact on system level performance remains unknown though. The transportation systems community aims to understand the influence of AVs on the transportation system performance, including travel time, traffic delay, traffic safety, and emissions. Because multi-class microscopic models are not scalable for the varying topology of mixed vehicle types, researchers are more focused on modeling mixed traffic using the multi-class approach on a macro scale (Talebpour and Mahmassani 2016; Levin and Boyles 2016; Melson et al. 2018; Chen et al. 2016, 2017; Kockelman 2017). For example multi-class Lighthill-Whitham-Richards (LWR) model (Lighthill and Whitham 1955) has been used to capture the evolution of hybrid traffic dynamics, assuming AVs are powered by stable controllers (Levin and Boyles 2016; Patel et al. 2016). On road networks, static (Chen et al. 2016, 2017) or dynamic (Dresner and Stone 2007; Levin and Boyles 2015, 2016; Patel et al. 2016; Melson et al. 2018) traffic assignment models are developed to capture AVs’ intersection coordination and routing behavior. Those models on a macroscopic scale may lack detailed interpretation of how different types of vehicles interact at the micro scale.

Second, with different goals, different AV decision-making frameworks have been employed. Academic researchers have to make various assumptions to implement AV components in their models or simulations, because real-world AVs are primarily developed and tested by private companies which are not willing to reveal how the existing AV test fleets on public roads are actually programmed to drive and interact with other road users. Accordingly, different driving models lead to different driving behavior and traffic patterns.

The transportation and the control communities assume AVs are particles or fluids following the physics-based models, including both the micro- and macroscopic traffic models that were originally developed for human drivers, and tailors AV behavior on that of HVs in which AVs are essentially human drivers but react faster, “see” farther, and “know” the road environment better. For instance, a majority of studies equate AVs to advanced driver-assistance system, Connected Adaptive Cruise Control (CACC), or commercial semi-autonomous functionality (e.g., Tesla’s Autopilot). Accordingly, models of the dynamic response of these systems are used as AV driving models (Naus et al. 2010; Qin and Orosz 2013; Shladover et al. 2015; Delis et al. 2016; Zhou et al. 2020). Otherwise AVs are treated like humans but with modified parameters (Schakel et al. 2010; Naus et al. 2010; Ploeg et al. 2011; Milanés et al. 2014; Milanés and Shladover 2014; Jin and Orosz 2014). Also, because these automated driving systems are only enabled in designated traffic scenarios, such as platooning, control models are thus constrained to these scenarios, not applicable to a generic traffic environment.

The robotics community, on the other hand, treats AVs like AI robots or agents who can continuously explore environments and exploit optimal actions (Sadigh et al. 2016b; Liu and Tomizuka 2016). When the environment is observable, AVs select optimal strategies based on predefined reward functions in cooperative or non-cooperative games. Reinforcement learning, a cutting-edge learning paradigm initially developed for optimal control of robotics, has been naturally deployed for AVs. In this framework, human drivers are modeled as part of the environment where AVs move and explore, using either an Markov decision process (Mukadam et al. 2017) or a simulated model-free environment (Wu et al. 2017b; Jia 2018a; Kreidieh et al. 2018b).

The aforementioned modeling difference arises from the fundamentally different assumptions of vehicle automation levels. The transportation and control community is focused on Level-2 or 3 automated vehicles (International Standard J3016 2010) enabled with vehicle-to-vehicle (V2V) or vehicle-to-infrastructure (V2I) communication. The most studied scenario is collaborative platooning, in which connected and automated vehicles (CAVs) drive in a platoon to optimize certain systematic performance measures. On the other hand, the robotics and AI community is focused on Level-5 automated vehicles, which are autonomous vehicles. A variety of generic traffic scenarios are studied such as lane-keeping, lane-change, merging, and crossing.

Third, behavioral modeling of HVs and AVs is different. The transportation community differentiates HVs and AVs with different models: HVs tend to exhibit unstable, stochastic behavior, while AVs can...
overcome traffic instability with stable controller design. In contrast, because the robotics community does not account for collective traffic patterns, they believe human drivers are intelligent for AVs to emulate. Thus those studies do not usually distinguish between HVs and AVs. Instead, both HVs and AVs are modeled as AI agents.

Fourth, there is a discrepancy in how the interactions between AVs and HVs are modeled. The transportation community does not formalize how AVs interact with HVs in driving processes. On the microscopic level, car-following models (CFM) are applied that implicitly encode how one follows its immediate or far upstream leaders. On the macroscopic level, usually multi-class traffic models are adopted, which do not define micro level interactions in detail. The robotics community tries to explicitly design microscopic interactions between one or a few AVs and one or multiple HVs.

1.3. Driving Policy Mapping: Overview

To further demonstrate the divergence of two communities, we provide a generic mathematical form of driving policy mappings. Usually, a driving behavior model, or a driving control or policy is a mathematical mapping from states, i.e., observations of a traffic environment, to actions, i.e., acceleration and steering angle. The mapping can be a mathematical formula or a neural network (NN).

\[
\phi : s \rightarrow a
\] (1.1)

We summarize a variety of mapping forms for longitudinal control policies in Table 2. The driving policy mapping is categorized into physics-based and AI-based. Physics-based mapping can be characterized by mathematical formulas, while AI-based mapping is usually represented by a variety of machine learning models. Within each mapping type, we also compare how HVs and AVs are modeled differently. As pointed out before, when both HVs and AVs are modeled by physics-based mappings, the main difference between HVs and AVs lie in the parameters, reflecting that AVs “sense” better, “see” farther, and “react” faster. The AI-based mappings assume there exists a complex, highly nonlinear mapping from driving perception to machine activation. In these mappings, the difference of HVs and AVs may not be so notable because the goal is to train AVs to exhibit human-like performance. In the last column, we list communities along with sample references for each mapping category. Most of the listed references may be revisited in the rest of the paper.
Table 2: Mapping categories in the car-following scenario by communities

| Models    | Goal                                                                 | Input features                                                                 | Behavioral difference                                                                 | Community (Sample references) |
|-----------|----------------------------------------------------------------------|-------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|--------------------------------|
| Physics-Based |                                                                      |                                                                               |                                                                                       |                                |
| HV        | CFM                                                                | $v_{i-1} - v_i, h_i$                                                         | less, local continuously changing                                                     | transportation (Newell 1961)                |
|           |                                                                     |                                                                               | long, long ($\sim 1.5$ sec)                                                          | Gipps et al., 1981 Treiber et al., 2000 Kesting et al., 2010 |
| AV        | Linear or non-linear controllers                                    | $h_i, s_i - s_i^*, v_{i-1} - v_i, h_i, others' acceleration$                   | rich, set from different speed options, heterogeneous acceleration rate                | transportation & control (Schakel et al., 2010) Nans et al., 2010 Ploeg et al., 2011 Milanes et al., 2014 Milanes and Shladover 2014 Orosz et al., 2010 Jin and Orosz, 2014 Qin and Orosz, 2017 Chen et al., 2019a) |
| AI-Based  | (Recurrent) NN                                                      | (time series of) $v_{i-1} - v_i, h_i$, reaction delay                          | local, unstable behavior, including asymmetric driving behavior, traffic oscillation   | transportation (Panwai and Dia 2007 Khodayari et al., 2012 Zhu et al., 2018b) |
| AV        | (Deep) RL                                                          | images                                                                        | local, end-to-end nonlinear, uninterpretable controllers                              | robotics (Lillicrap et al., 2015 Zhang et al., 2016 Sallab et al., 2017 Perot et al., 2017 Jaritz et al., 2018) |
Physics-based driving models, widely used by the transportation and control communities, assume each unit behaves like an automated particle or automaton, within which human cognitive process and the machine’s mechanical dynamics are highly simplified. Car-following is the most studied driving behavior, which is categorized in terms of spatial resolution: microscopic driving models and macroscopic traffic flow models. In microscopic models, cars are assumed to select their driving velocity and acceleration dynamically based on the following distance from their immediate leader, speed difference, and other features. The mathematical tool is ordinary differential equation. Some of the widely used microscopic car-following models include Newell (Newell, 1961), Gipps’ model (Gipps, 1981), IDM (Treiber et al., 2000; Kesting et al., 2010), and OVM (Orosz et al., 2010; Jin and Orosz, 2014; Qin and Orosz, 2017). In macroscopic traffic flow models, cars are assumed to follow hydrodynamics. The evolution of aggregate traffic density and velocity are determined using partial differential equation. Popular traffic flow models include LWR (Lighthill and Whitham, 1955; Richards, 1956), PW (Payne, 1971), and ARZ models (Aw and Rascle, 2000). MOBIL (Kesting et al., 2007) introduces the “politeness” parameter and corresponding utility function to capture intelligent driving behavior in steering and acceleration.

These physics-based models, however, usually suffer from two major weaknesses (Chen et al., 2019c). First, different models have been developed for different driving behaviors. For example, car-following and lane-change behaviors are usually modeled separately. The model developed for one behavior has to be redesigned manually for different scenarios and tasks. Second, the predefined motion heuristics usually make strong assumptions about driving behaviors with a small set of parameters, which may not be able to capture human’s strategic planning behaviors and may not generalize well to diverse driving scenarios in a highly interactive environment.

1.4. AI for decision-making of AVs

“It is not the strongest … that survives, nor the most intelligent …. It is the one that is most adaptable to change.” Instead of hypothesizing explicitly how AVs would drive, we believe the futuristic AVs should be designed to act as rational, utility-optimizing agents that play best strategies at each level of driving choices. By doing so, it would allow AVs to react according to the impending traffic situations and closely mimic human drivers’ intelligence. However, the major advantage AVs will have over human driver is its ability to access the situation promptly with a better set of information, and thereby enable AVs to react in an optimal way compared to a human driver. Natural traffic experiments are, however, costly and highly risky to perform. We thus seek an innovative AI-guided methodological framework for complex multi-agent learning and adaptation.

Despite a significant amount of machine learning efforts given to computer vision, the intelligence of AVs lies in its optimal decision-making at the stage of motion planning. We believe the key to empowering AVs’ driving intelligence is AI or even a broader area “Artificial general intelligence” (AGI) (Ramamoorthy and Yampolskiy, 2018). We are seeing a growing number of studies that have employed AI methods to discover humans’ driving behaviors, including deep learning (Tanaka, 2013; Colombo and Fusco, 2014; Zhou et al., 2017a; Wang et al., 2018; Zhu et al., 2018a), reinforcement learning (VanderWerf et al., 2001), and imitation learning (Kuefler et al., 2017; Bhattacharyya et al., 2018a). More recently, many attempts also focus on human behavior prediction, such as lane changing (Kumar et al., 2013; Woo et al., 2017; Wei et al., 2019; Shou et al., 2020), merging (Rios-Torres and Malikopoulos, 2016; Bevly et al., 2016), and stop behavior (Kumagai and Akamatsu, 2006), to predict with high confidence when a human would change lanes. However, applications of AI to the decision-making processes of AVs are still emerging and remain understudied.

Game theory, a mature field for modeling strategic interactions of rational players, has empowered intelligence of multiple interacting machines and is revolutionizing the field of AI (Tennenholtz, 2002). Fortunately, we have seen a gradual convergence in the control, transportation, and AI communities that have employed game-theoretic models to design algorithmic decision-making processes for AVs (Yoo and Langari, 2012, 2013; Kim and Langari, 2014; Talebpour et al., 2015; Yu et al., 2018; Huang et al., 2019, 2020a). We also believe gaming traffic would be a key feature of future AVs to strategically interact with and navigate through a complex traffic environment.
1.5. Organization of the paper

The remainder of the paper is organized as follows: In Section 2, we will provide a general problem statement for AV control in mixed traffic, along with the existing knowledge gaps. We will then examine the existing models and methods for AV control in Sections 3-6. Section 7 presents the methods and models of human and autonomous driving policy learning, respectively. Section 8 summarizes all the models that have been reviewed. In Section 9, we present the challenges and insights into modeling the mixed traffic with AI methods and provide potential research areas.

2. AI-guided driving policy learning for AVs

2.1. Multi-vehicle systems (MVS) in mixed autonomy

A mixed traffic system is comprised of a large number of intelligent agents, which are AVs and human drivers. They dynamically select driving actions while interacting with the traffic environment. Their actions are interdependent in the sense that one’s driving action depends on others’, via either coupled reward functions, the common traffic environment state, or the action constraints. Due to this coupling among agents, the mixed transportation system is a multi-agent system (MAS) - a widely used term in the control and robotics community. Specifically, we call it a “multi-vehicle system (MVS).”

Definition 2.1. (AV control problem statement in mixed-autonomy.) In a mixed traffic system, there are $N$ controllable AVs indexed by $n \in \{1, 2, \ldots, N\}$ driving along a stretch of a road, with initial states $s_0^{(1)}, \ldots, s_0^{(N)}$. Each car aims to select a sequence of optimal driving controls (e.g., acceleration or steering angle) in discrete (i.e., $a_1^{(n)}, \ldots, a_T^{(n)}$) or continuous time steps (i.e., $a^{(n)}(t), t \in [0, T]$) over a predefined planning horizon $[0, T]$. The selected controls are solved by minimizing a common or an individual cost functional. Human drivers and other uncontrollable AVs serve as the background traffic, which evolves according to some dynamics. What is a scalable distributed control strategy for these AVs?

2.2. Research questions

While reviewing the methodologies, we primarily focus on the following research questions:

1. What scalable driving policies are to control a large number of AVs in mixed traffic comprised of human drivers and uncontrollable AVs? (Sections 3-6)
2. How do we estimate human driver behaviors? (Section 7.1)
3. How should the driving behavior of uncontrollable AVs be modeled in the environment? (Section 7.2)
4. How are the interactions between human drivers and autonomous vehicles characterized? (Section 7.3)

Below we will present the knowledge gaps that exist in the literature with respect to each research question.

2.3. Roadmap to navigate literature: Control dimensions

In the subsequent sections, we will give an overview of how the existing literature addresses these three gaps. We categorize these studies based on how many AVs and HVs are involved. They include: one AV interacts with one HV (1 AV + 1 HV), one AV navigates in the HV-dominated traffic environment, i.e., one AV interacts with multiple HVs (1 AV + m HVs), multiple AVs interact with many HVs (n AVs + m HVs, $n \ll m$), multiple AVs interact with one HV (n AVs + 1 HV), and a pure AV market (n AVs).

Other than the number of AVs and HVs, we further categorize the multi-AV control problem based on two dimensions: the first dimension is whether these controllable AVs are cooperative or not, while the second dimension is whether the AV control takes into account uncertainty arising from the external environment. Fig (2) provides a roadmap to navigate readers to literature on AV controls in mixed traffic.
The colored rectangles represent the classification criteria and the red round-corner rectangles (as the end nodes) represent the methodologies used to control AVs.

Figure 2: Roadmap to mixed traffic models

3. 1 AV + 1 HV, 1 AV + 1 AV: General-sum Game-Based Control

Game theory is a natural approach to model the non-cooperative strategic interactions among AVs or between one AV and one HV, who are usually taken as intelligent agents aiming to optimize an individual objective function. In the game theoretic framework, cars are referred to as “agents” or “players”.

3.1. One-shot game

The one-shot two-person game is applied to model two cars’ strategic actions at one step. Driving (Yoo and Langari, 2012), merging (Liu et al., 2007; Yoo and Langari, 2013), lane-changing (Talebpour et al., 2015; Yu et al., 2018; Zhang et al., 2019b; Yoo and Langari, 2020), and unprotected left-turning behavior (Rahmati and Talebpour, 2017) is modeled as either a two-person non-zero-sum non-cooperative game (Liu et al., 2007; Talebpour et al., 2015), a Stackelberg game (Yoo and Langari, 2012, 2013; Yu et al., 2018; Zhang et al., 2019b; Yoo and Langari, 2020) or a mixed-motive game (Kim and Langari, 2014). The outcome of these games can be a pure or mixed Nash equilibrium, based on the payoff bimatrix. The payoff of a given strategy accounts for traffic safety and efficiency, depending on current driving speed, relative positions, reaction and perception time, aggressiveness, and collision avoidance. When human driving behavior is modeled, human driver data is collected to estimate parameters of payoff functions, using bi-level optimization (Liu et al., 2007), simulated moments (Talebpour et al., 2015), and maximum likelihood (Rahmati and Talebpour, 2017). When one of the game player is an AV, utility or reward needs to be designed while accounting for aggressiveness of surrounding drivers (Yoo and Langari, 2020; Zhang et al., 2019b). Further develops a game theoretic model predictive controller that solves a Stackelberg equilibrium with multiple interacting vehicles continuously.

3.2. Dynamic/Continuous game: perfect information, full observability

The one-shot game cannot model vehicles’ dynamic driving actions. To solve for time-varying controls, dynamic optimal control (Wang et al., 2015), model predictive control (MPC) (Wang et al., 2016) Gong
A simultaneous differential game between one AV and one HV is formulated below:

\[ \dot{s} = f\left( s, a^{(AV)}, a^{(HV)} \right) \tag{3.1a} \]

\[ a^{(AV)}(t) = \arg \min_{s^{(AV)}=f^{(AV)}(s,a^{AV})} \int_{t}^{t_f} r^{(AV)}(s, a^{(AV)}, a^{(HV)}) dt \tag{3.1b} \]

\[ a^{(HV)}(t) = \arg \min_{s^{(HV)}=f^{(HV)}(s,a^{HV})} \int_{t}^{t_f} r^{(HV)}(s, a^{(AV)}, a^{(HV)}) dt \tag{3.1c} \]

where,

- \( t_f \): a predefined planning horizon;
- \( s \): the system state including the states of both the AV and the HV;
- \( f \): the state dynamic function for continuous-time, or the state transition function for discrete-time;
- \( a^{(AV)}, a^{(HV)} \): the dynamic driving actions or policies of the AV and the HV, respectively;
- \( r^{(AV)}, r^{(HV)} \): the reward function of the AV and the HV, respectively.

If the AV were to be able to predict the HV’s strategy in the entire planning horizon, it then optimize its own objective function that depends on both its own current and future strategies as well as the HV’s current and future strategies to generate a continuous sequence of control strategies along this horizon and implement it. The same process holds for the HV. Due to the dynamic coupling, it is challenging to solve this equilibrium.

To simplify, several techniques have been applied. Sadigh et al. (2016b); Lazar et al. (2018a) have simplified the original two-player differential game to a leader-follower game (or Stackelberg game) played at discretized time steps. In this game, the AV takes actions first. Then the HV observes actions taken by the AV and predicts the AV’s future action based on the AV’s historical actions, maximizes its own objective and calculates its own future actions for a short period of time. Then the AV maximizes its own objective using the HV’s future actions and replans repeatedly using MPC at each iteration. In other words, in a leader-follower scheme, the AV directly solves an optimization based upon its prediction of human driver actions rather than human’s actual strategies. The advantage of the Stackelberg game is that the AV can be designed beforehand to influence uncontrolled HVs via a carefully selected reward function (Sadigh et al., 2016b). The reward function contains two parts: one controls the AV’s driving efficiency and safety, while the other determines the influence the AV would like to impose to neighboring HVs. Lazar et al. (2018a) extends this framework to a Stackelberg game between one AV and multiple HVs, but assumes that one AV only influences one HV and the actions of others HVs are fixed.

Fisac et al. (2019) further develops a hierarchical game-theoretic planning scheme, where the strategic planner solves a closed-loop dynamic game with approximate dynamics in a relatively long planning horizon (e.g., 5 second), while the tactical planner solves an open-loop trajectory optimization with high-fidelity vehicle dynamics over a shorter planning horizon (e.g., 0.5 second). On the strategic planner level, the AV and the HV still play a feedback Stackelberg dynamic game in which their driving actions are recursively solved through successive application of dynamic programming. The solved optimal Q-value obtained from the strategic level is then introduced to the objective function of the tactical level as a guiding terminal reward representing an optimal reward-to-go. On the tactical planner level, the trajectory of the AV is iteratively optimized using a nested optimization problem that estimates the human’s best trajectory response to each candidate plan in the short-term planning horizon. The hierarchical game-theoretic model is tested on two scenarios with merging and overtaking maneuvers: one on a straight empty multi-lane highway with only
two-vehicle interaction and one with the presence of a third vehicle (i.e., a truck with a slower moving speed). Li et al. (2018a); Tian et al. (2018, 2019) assume two different game structures for HVs and AVs, respectively. Human drivers play a game based on hierarchical reasoning. A level-0 player ignores the interaction of other players, while a level-1 player assumes all other players are level-0 players. Similarly, a level-\(k\) player assumes that all other players act according to level-\((k-1)\) models. In other words, in a two-driver scenario, when the ego driver is level-1, the action of her opposite driver is assumed a level-0 driver whose actions are solved without accounting for the vehicle interaction. Then the ego driver selects her actions based on the fixed actions of the opposite driver. The simultaneous game is reduced to solving two optimal control problems sequentially. When the ego driver is level-2, the action of this ego driver depends on that of the opposite vehicle, which in turn depends on that of ego vehicle. This becomes an embedded game like proposed in Sadigh et al. (2016b). Accordingly, the AV plays an adaptive game against HVs in which the AV predicts the opponent vehicle’s actions based on the opponent vehicle’s driver type, and updates its belief of the driver type using the difference between the actual action and the predicted action and then update its own actions. This game method is tested on multi-lane highways and unsignalized intersections (including four-way, T-shape, and roundabout).

4. 1 AV + m HV

In this section, we will first briefly mention stability-oriented controls, and then introduce two types of AI-based modeling approaches: game based and reinforcement learning based. There exist only a few studies using game-theory based control, partly due to high dimensionality of the coupled game system. Accordingly, a majority of studies employ reinforcement learning based AV control, which will occupy the most space in this section.

4.1. Deterministic stability-oriented control

When the environment is deterministic, the traffic control community aims to understand how one AV can stabilize a HV platoon using linear (Cui et al., 2017; Wang, 2018) or nonlinear controllers (Jin and Orosz, 2014, 2018), based on the concept of head-to-tail stability (i.e., stability from the first vehicle to the last vehicle in a platoon (Jin and Orosz, 2014)). Field experiments have also demonstrated the feasibility of using one AV to stabilize HVs (Stern et al., 2018; Jin and Orosz, 2018). Because the physics-based models are not the focus of this paper, interested readers can refer to Li et al. (2014) for a comprehensive survey of stability-based controls.

4.2. Differential game based control

Assume the HV-dominated environment is deterministic and every vehicle interacts among one another in a game-theoretic framework, we can formulate a simultaneous differential game between one AV and multiple HVs below:

\[
\dot{s} = f(s, a^{(AV)}(s), a^{(HV)}_1, \ldots, a^{(HV)}_M),
\]

\[
a^{(AV)}(t) = \arg \min_{s^{(AV)} = f^{(AV)}(s, a^{(AV)})} \int_t^{t_f} r^{(AV)}(s, a^{(AV)}, a^{(HV)}_1, \ldots, a^{(HV)}_M) \, dt,
\]

\[
a^{(HV)}_m(t) = \arg \min_{s^{(HV)}_m = f^{(HV)}(s, a^{(HV)}_m)} \int_t^{t_f} r^{(HV)}(s, a^{(AV)}, a^{(HV)}_m) \, dt, m = 1, \ldots, M.
\]

where \(a^{(HV)}_m\) is the dynamic driving actions or policies for HV \(m, m = 1, \ldots, M\). Other notations carry the same meaning as before.

Schwarting et al. (2019) develops an autonomous control policy by solving an iterative best-response, with embedded levels of tacit negotiation. In a two-agent case, an iterative best-response can be written as \(a^{(AV)}(a^{(HV)}(a^{(AV)}(\cdot)))\) where one’s strategy is solved using Equ (3.1). In an MAS, a system of interdependent optimization is reduced to a single-level optimization using KKT conditions. The resulting
Nash equilibrium not only offers a control law for the AV but also predicted actions for other HVs. The innovation of this study is to include a term “Social Value Orientation (SVO)” into the reward function of HVs, representing HVs’ driving aggressiveness. One can adjust its SVO value while interacting with another vehicle. The control law is validated in highway merging and unprotected left turn. Social preference learning can improve the AV’s performance by 25%.

Liu and Tomizuka (2015, 2016) combine multiple HVs as one effective human and assume a sequential game in which HVs lead and the AV play reactive strategies. By mapping a baseline control law to a set of safe control, an online algorithm is developed for the AV controller to incorporate human intentions as safety constraints.

4.3. Reinforcement learning based control

In the HV-dominated traffic, a single AV’s driving policy selection can be treated as a sequential decision-making process in a partially or fully observable random environment. Learning driving policies are needed to predict vehicles’ acceleration and steering angle using their environmental information as input.

Reinforcement learning, which enables the intelligent agents to learn optimal policies driven by a reward, has made breakthroughs to achieve super-human-level performance in game playing, such as Atari (Mnih et al., 2015), Go game (Silver et al., 2016), Poker (Brown and Sandholm, 2018, 2019), Dota 2 (OpenAI, 2018), and StarCraft II (Vinyals et al., 2019). Its application to autonomous driving has become a promising direction. The dynamic motion planning of a single AV is usually modeled using a Markov decision process (MDP) (Puterman, 1994) or reinforcement learning (RL) (Sutton and Barto, 1998).

When RL is used to control AVs in a stochastic environment, the basic idea is demonstrated in Fig 3. Let us first discuss the single-agent RL setting. One controllable AV (the host vehicle whose controller needs to be designed) perceives the state of the mixed traffic environment, comprised of HVs, other AVs that are not controllable, and road users. Based on some predefined reward function, it executes an action (such as acceleration or steering angle), which in turn transforms the state of the traffic environment. In return, the environment provides a reward to the AV. Based on the received reward and the new environment state, the AV further solves an optimal policy based on the reward function and selects an action. This process iterates till the AV finishes its entire control process. When there are multiple AVs that are all required to make decisions simultaneously, we need a multi-agent RL (MARL) framework, which will be discussed in Section 6.2.

MDP implicitly assumes that the agent can fully observe the state dynamics. In other words, after the agent applies an action, he knows the probability of the next state the system will move to. A majority of transportation studies assume connectivity among vehicles via V2V or V2I. Thanks to these communication technologies, every driver obtains full information of other drivers and the system state. Control and robotics researchers, on the other hand, make various assumptions on observability (Liu and Tomizuka, 2015, 2016; Bouton et al., 2017, 2018). One may observe others’ positions, headings, and sometimes longitudinal and lateral velocity, but not accelerations. Even a driver observes the entire state of other drivers, she may not know the intention of those drivers. Accordingly, the AV has to maintain a belief state space over all possible states based on its observations. Therefore a partially observable Markov decision
process (POMDP) model is widely used in modeling a single AV’s motion planning. Assuming each AV follows a (partially observable) Markov decision process, the single-AV control problem in the HV-dominated traffic can be formulated as an (PO)MDP, whose components are described below.

- **S**: The state space of the HV-dominated traffic environment. A state $s \in S$ contains the information of the ego car and all surrounding vehicles. For instance, the physical state of the ego car at time $t$ can be represented by $s^e_t = (p^e_t, p^y_t, \theta_t, v_t, a_t)$, where $p^e_t, p^y_t$ are its longitudinal and lateral positions, $\theta_t$ is the axle angle, $v_t, a_t$ are speed and acceleration. The physical state of the $n$ other cars at time $t$ is $s^n_t = (p^n_t, p^n_y, \theta_t, v_t, a_t)$.

- **A**: The action space. The action of the AV, denoted as $a \in A$, is a two-dimensional vector, including acceleration and steering angle.

- **O**: The observation space. The observation of the AV is $o \in O$. For instance, the observation space for the $i^{th}$ car at time $t$ is $O_i = (p^i_t, p^y_t, \theta_i, v_i)$. The $i^{th}$ car’s acceleration $a_i$ is usually not observable.

- **G**: The observation function. For the AV, $s$ may not be fully observable. Instead, it draws an observation $o \in O$ that is correlated with $s$ according to an observation function $G : S \times O \rightarrow [0, 1]$, i.e., $o \sim \mathcal{F}(o|s)$.

- **P**: The state transition function, i.e., $S \times A \times S \rightarrow [0, 1]$. The action $a$ triggers a state transition $s \rightarrow s'$ according to function $P(s'|s, a)$. This state transition can rely on a specific form of $P$ or provided by a traffic simulator (i.e., model-free).

- **R**: The reward space. Along with the state transition, the AV receives an immediate reward, i.e., $r \in R : S \times A \times S \rightarrow \mathbb{R}$. The reward $r$ may include traffic safety (e.g., off-road/collision avoidance), efficiency (e.g., fast speed), and emissions.

The AV aims to derive an optimal policy $\pi^{*(AV)}$ by maximizing its expected cumulative reward.

In contrast to theory-driven AV controllers, such as game-theory driving model, RL-based AV controllers are model-free and End-to-End (End2End), which directly maps sensory inputs to control commands. Behavioral cloning (BC) simplifies the AV policy learning as a supervised-learning problem, which usually performs well when driving data are sufficient or the driving task is for limited regions. [Pomerleau, 1989] introduce a multi-layer network learned from simulated road images to control a vehicle to follow real roads. The next milestone of AVs is to employ convolutional neural networks (CNNs) to efficiently process raw camera images, which helps AVs drive through an obstacle-filled road after training on similar scenarios [Muller et al., 2006]. Later on, CNN-based AV controllers are widely studied, and recently work includes NVIDIA’s PilotNet [Bojarski et al., 2016, 2017] to control AVs in real traffic situations, Rausch’s deep CNN policy [Rausch et al., 2017] and DeepPicar [Bechtel et al., 2018] for steering angle control, Agile driving [Pan et al., 2018] for steering angle and velocity controlling in aggressive scenarios. Temporal dependencies of the driving data have been considered to improve the performance of AV control, and recently, long-short-term memory (LSTM) and its variants have been leveraged for End2End AV policy learning. [Xu et al., 2017] propose FCN-LSTM, a combination of a fully-convolutional network (FCN) and LSTM, which can predict a distribution of future vehicle egomotion data. [Eraqi et al., 2017] develop a convolutional LSTM (C-LSTM) for learning both visual and dynamic temporal dependencies of driving. [Hecker et al., 2018a] introduce Drive360, which combines CNN, fully connected layers and LSTM to integrate information from multiple sensors to predict the driving maneuvers. [Bansal et al., 2019] from Waymo presents ChauffeurNet, a mid-to-mid driving policy learning framework, in which inputs are prepossessed before receiving by an RNN to generate low-level controls. BC has several shortcomings: (1) BC requires the collection of huge amount of expert driving data, which is time-consuming and expensive; (2) It can only learn the driving skills that are covered in the data, and may not generalize to diver real-world driving scenarios; and (3) Since BC is based on supervised learning using human actions as target, it can never exceed the human-level performance of experts.

Different from BC, general deep reinforcement learning (DRL) driving systems are mainly developed in simulation, which provides consequential information, such as reward, for AVs to learn from. [Wu et al., 2019].
train an autonomous driving controller using (deep) reinforcement learning with a designated reward function that avoids crashes into other agents, and applied trust region policy optimization (TRPO) method to train a Gaussian Multilayer perceptron (MLP) policy in SUMO simulator for improving traffic efficiency. Lillicrap et al. (2015) apply a deep deterministic policy gradient (DDPG) RL algorithm to control a car in a simulation environment. Their work designed a reward, which provides a positive reward at each step for the velocity of the car projected along the track direction and a penalty of -1 for collisions. Sallab et al. (2017) develop a integrated deep Q-network (DQN), which integrates attention models to make use of glimpse and action networks to direct the CNN kernels for steering command in TORCS simulator, which provides a positive/negative reward for on/off-lane situations. Perot et al. (2017) propose an asynchronous advantage Actor-Critic (A3C) method for training a policy network for realistic games, such as World Rally Championship 6 and TORCS simulator. This work has been enhanced by Jaritz et al. (2018) with an improved convergence and generalization. Both of these studies have designed the reward as a function of the distance to the road center and angle between the road’s and cars heading.

Monte Carlo tree search (MCTS) is one of the most effective methods for solving decision making problems online (Browne et al., 2012) and has begun to be applied to AVs in recent years. Paxton et al. (2017) integrate MCTS with hierarchical neural net control policies trained on Linear Temporal Logic (LTL) constraints for motion and path planning in complex road environments. They designed the reward function as a combination of cost terms upon current continuous states (e.g., location or speed), and a bonus terms based on completing immediate goals (e.g., stopping at the sign or existing a region), and a penalty term for constraint violations. Sunberg et al. (2017) use MCTS to infer the internal state of traffic participants for operating safe lane changes on a highway. Their reward design penalized the average time taken for the ego to reach the target lane and the number of hard braking maneuvers that any vehicle undertakes during the time for the ego vehicle to reach the target lane. Hoel et al. (2020) combine MCTS with DRL to achieve tactical highway driving, and in their work, a deep neural network is trained to guide MCTS to the relevant regions of the search tree, while MCTS is used to improve the training process of the neural network at the same time. The associated reward is a combination of cost terms concerning the number of lane changes and difference from the desired speeds, and a bonus terms for highway exit (i.e., goal achieved).

There is another direction of DRL-based AV control, using prior knowledge or classical theory-driven controllers to constrain the learning and behaving of neural network-based driving models. Bouton et al. (2017) impose a computational safety factor as a penalty in the reward function rather than a hard constraint (Bouton et al. 2017), and as a result, the driving policy solved from MDP cannot avoid accidents. Bouton et al. (2018) add a model checking step to enforce probabilistic guarantees of the trained driving policy on an RL agent, and they used simplified reward function to penalize the number of action steps and award goal accomplishment. Zhang et al. (2016) propose to combine the traditional MPC method with RL in the framework of guided policy search for controlling autonomous aerial vehicles, where a deep neural network policy is trained on data generated by MPC for training robustness and generalizable control. The RL training is based on a cost function that measures the distribution difference between the action generated from the policy model and the data generated from MPC. Chen et al. (2019b) develops a hierarchical control framework, where the higher-level controller employs MDP to solve a reference driving policy and the lower-level controller implements it accounting for safety concerns.

For more examples of using single-agent RL for AV controlling, we refer readers to recent surveys such as Zhou and Laval (2019; Grigorescu et al., 2020; Kiran et al., 2020).

5. n AVs: A Driverless World

Control of a single AV is far from sufficient to exploit the potentials of AVs in the era of mixed autonomy, when an increasing number of AVs are introduced to public roads. Systems with multiple AVs have attracted increasing interest in recent years. In this and the next sections, we go against the AV deployment timeline by first discussing the pure driverless world, denoted as “n AVs”, followed by the mixed market (of n AVs + m HVs). The pure AV market precedes the mixed one, because the former can be generalized to the latter by adding a traffic background comprised of HVs.
5.1. Classification of multi-AV control models

Based on whether all the vehicles solve for a common or an individual objective function, the multi-AV control models can be divided into two classes: cooperative control and non-cooperative game.

The multi-AV control problem can also be categorized based on whether the traffic environment that AVs navigate is deterministic or stochastic. The transportation community generally assumes that every CAV has global connectivity with the leader and/or with other vehicles in a platoon via V2V or V2I. Accordingly, the traffic environment is known and deterministic. These studies are primarily focused on optimization of a centralized or a distributed control, with a few exceptions that consider measurement errors or communication delays. The robotics community, on the other hand, assumes each AV can only observe local information using its own sensors, such as camera, LiDAR. Accordingly, the traffic environment is full of uncertainty, including but not limited to stochasticity induced by other neighboring vehicles’ dynamic driving actions and exogenous randomness.

In this and the next sections, we will categorize the literature using vehicle cooperation as the primary category and the environmental stochasticity as the secondary category. In the pure AV market, a majority of studies assume the environment is deterministic because all the AVs are controllable and fully observable, while stochasticity could originate from measurement errors or communication delay. In the mixed market, the environment is highly random due to stochasticity of human driving behavior, but there are also studies by the traffic control community that assume a deterministic environment.

5.2. Cooperative control

5.2.1. A deterministic environment

In a deterministic environment, assuming that the central planner knows the state of every vehicle and aims to optimize a total system performance, from the system perspective, the multi-AV control problem is formulated as an optimization problem:

\[
\min_{a} J^N(s_1, a_1; \ldots; s_N, a_N), \quad (5.1a)
\]

\[
\text{s.t.} \quad \dot{s}_i(t) = f(s_i, a_i), \quad (5.1b)
\]

\[
s_i \in S_i(s_{-i}, a_{-i}), \quad (5.1c)
\]

\[
a_i \in A_i(s_{-i}, a_{-i}), \quad (5.1d)
\]

\[
a(t) \in A. \quad (5.1e)
\]

where,

\(J^N(\cdot)\): the common objective function shared by a total \(N\) vehicles;

\(S_i(s_{-i}, a_{-i}), A_i(s_{-i}, a_{-i})\): the vehicle \(i\)'s state and control constraints. Vehicle \(i\)'s state and control are constrained by other vehicles. Other notations remain.

When AVs are programmed to optimize its own objective and not cooperate with other AVs, the multi-AV control becomes a non-cooperative game. In a non-cooperative system, vehicles select their own controls to achieve individual goals, which may likely conflict with others’ goals. Compared to the cooperative control, the non-cooperative interactions among AVs are relatively understudied. A non-cooperative framework for a simultaneous game is formulated as:

\[
\min_{a_i} J_i^N(s_i, a_i; s_{-i}, a_{-i}), i = 1, \ldots, N \quad (5.2a)
\]

\[
\text{s.t.} \quad \dot{s}_i(t) = f(s_i, a_i), \quad (5.2b)
\]

\[
s_i \in S_i(s_{-i}, a_{-i}), \quad (5.2c)
\]

\[
a_i \in A_i(s_{-i}, a_{-i}), \quad (5.2d)
\]

\[
a(t) \in A. \quad (5.2e)
\]
where $J^N_i$ is vehicle $i(i=1,\cdots,N)$’s individual objective function.

In the next two subsections, we will review the existing models of each category that materialize the above two control schemes.

A majority of research on control of multiple AVs falls within the category of cooperative coordination. In other words, AVs are assumed to communicate with one another for global traffic information and optimize a common goal of traffic flow improvement. Cooperative control has been widely studied in multi-robotic systems. Swarm intelligence (Bogue 2008; Venayagamoorthy and Doctor 2004), formation control (Chen and Wang 2005), and consensus control (Zegers et al. 2017; Li et al. 2018c) have been widely used for a group of robots with a centralized goal to accomplish a task collaboratively, so is in multi-AV control (Wu et al. 2018a; Lazar et al. 2018b).

In a cooperative MVS, the movement of vehicles is coordinated by a central controller or planner to achieve a common goal, such as to collectively stabilize traffic flow and smoothen traffic jam (Wang et al., 2016; Gong et al., 2016; Gong and Du, 2018), to optimize driving comfort (Wang et al., 2014b; Zhou et al., 2017b), or to improve fuel efficiency (Wang et al., 2014a; Yao et al., 2018). To achieve coordination, full observability and full controllability is required, meaning that all vehicles’ states and controls are known to the central controller and every vehicle can be controlled in a centralized or distributed manner. The communication topology in a platoon of vehicles determines the degree of cooperation among CAVs (Li et al., 2014). For example, control protocols can be designed for a platoon of vehicles to reach an equilibrium state using a consensus based approach. Accordingly, the car-following coupling among vehicles are modeled as a consensus problem and a distributed nonlinear delay-dependent control algorithm is used to solve a safe velocity (Li et al., 2018d).

Assuming connectivity between predecessors and followers as well as between platoon leaders and followers, CACC contains two control policies: constant spacing (Swaroop and Hedrick 1996; Darbha and Rajagopal 1999; Swaroop et al. 2001) and constant time headway (Ioannou and Chien 1993; Rajamani and Shladover 2001; Van Arem et al. 2006; Naus et al. 2010; VanderWerf et al. 2001; Zhou et al., 2017b; Arefizadeh and Talebpour 2018; Stern et al., 2018). AVs longitudinal acceleration control can also be modeled using nonlinear CFMs, which is discussed in Sec. 1.3. All the aforementioned studies aim to develop a string stable car-following controller in order to smoothen traffic flow and prevent stop-and-go waves. But none of them considers control and physical safety constraints (Gong and Du, 2018). In other words, interactions among vehicles are not explicitly modeled (Li et al., 2018d). To explicitly model the physical interaction between vehicles, a growing body of literature formulates a platoon of AV longitudinal control as optimal control problems. The control policies based on linear spacing policies or non-linear CFMs are special cases of optimal control problems (Wang et al., 2014b).

Define $c_{run}, c_{ter}$ as running cost and terminal cost of a platoon, respectively, and $t_f$ as the planning horizon.

\[
\begin{align*}
\min_a J^N = & \min_a \int_0^{t_f} c_{run}(s(\tau), a(\tau)) \, d\tau + c_{ter}(s(t_f), a(t_f)) , \\
\text{s.t.} & \quad \dot{s} = f(s(t), a(t)) , \\
& \quad s(t) \in S , \\
& \quad a(t) \in A .
\end{align*}
\] (5.3a)

Centralized control requires the central controller to solve for an optimal control for each car at each time step. It is challenging to solve a centralized control of this type, because: (1) all vehicles’ states and controls are coupled through objective functions and constraints; (2) A longer planning horizon requires prediction of future traffic dynamics, which may suffer from both curse of dimensionality and disturbances.

To resolve the first issue of state coupling, a distributed algorithm is usually designed and implemented on each vehicle (Wang et al., 2016; Gong et al., 2016; Gong and Du, 2018; Li et al., 2018d). Consensus based approaches are also employed to design a control protocol for a platoon of vehicles to reach a consensus and a distributed nonlinear delay-dependent control algorithm is designed to solve a safe velocity (Li et al., 2018d). To resolve the second issue of prediction horizons, the original optimal control problem can be approximated as a one-step MPC and a distributed algorithm is developed. The MPC control is close to optimal control
strategies if the planning horizon is short, but may deviate when the planning horizon is long. MPC is also employed as a higher level control model to compute reference planning trajectories (Wang et al., 2014a,b; Gong et al., 2016; Gong and Du, 2018; Zhou et al., 2017b).

5.2.2. A stochastic environment

The optimal control framework can be extended in several ways. When there are measurement errors or when there is only partial observability, a measurement equation is introduced into the state-space model (Wang et al., 2014a,b; Zhou et al., 2017b). Considering stochastic communication delay arising from packet drops, decomposition is proposed for the stability analysis of a large system of CAVs (Qin and Orosz, 2017; Jin and Orosz, 2018; Jin et al., 2018).

5.3. Noncooperative control: a deterministic environment

The noncooperative control of multiple AVs is modeled as N-player game-theoretic models. To the best of our knowledge, all the work on multi-AV competitive control assumes a deterministic environment. The first group of studies assume there is a small number of AVs to control in specific scenarios such as platooning, in other words, \( n \) is a finite number. Wang et al. (2015) formulates AVs discrete lane change and continuous acceleration selections as a differential game, where agents’ optimal strategies are obtained from solving optimal control problems. The outcome of a differential game is a dynamic equilibrium. Computation of such a dynamic equilibrium involving \( N \) players is mathematically intractable when the number of coupled agents becomes large. To get around, Wang et al. (2015) decomposes the problem into a finite number of sub-problems and applies MPC to each vehicle. Dreves and Gerdts (2018) solves a generalized Nash equilibrium by summing up all vehicles objective functions, which is essentially a cooperative control. Because the game-based control suffer from scalability issues, all the aforementioned studies had to constrain their applications to a limited number of AVs. As a growing number of AVs are put on public roads, a scalable and computational efficient algorithm is needed for a large number of AV controllers.

Another school of research assumes a more generic traffic scenario, which is a large number of AVs interacting with one another on a transportation system, in other words, \( n \) goes to infinity. Mean field game (MFG) has shown to be a scalable model for the \( N \)-car differential game, as the AV population grows (Huang et al., 2019, 2020a,b). MFG is a game-theoretic framework to model complex multi-agent dynamics arising from the interactions of a large population of rational utility-optimizing agents whose dynamical behaviors are characterized by optimal control problems (Lasry and Lions, 2007; Huang et al., 2006). By exploiting the “smoothing” effect of a large number of interacting individuals, MFG assumes that each agent only responds to and contributes to the density distribution of the whole population. It has become increasingly popular in finance (Guéant et al., 2011; Lachapelle et al., 2010), engineering (Djehiche et al., 2016), and pedestrian crowds (Lachapelle and Wolfram, 2011). In the longitudinal control of AVs, each car solves its optimal velocity backward in time, the aggregate effect of which is formulated by a Hamilton-Jacobi-Bellman (HJB) equation; while the mean field approximation derives the evolution of traffic density solved by a transport equation (with many other names like continuity equation, flow conservation equation) forward in time. To solve the mean field equilibrium, The distributed velocity controller derived from the MFE is shown to be an \( \epsilon \)-equilibrium of the \( N \)-car differential game.

Huang et al. (2020a) has also established a connection between an MFG-based macroscopic continuum model and the existing traffic flow theory. The LWR model, which implicitly assumes that cars move according to hydrodynamics without modeling driving intent, is proved to be a myopic MFG with a specially designed objective function. In conclusion, MFG embodies classical traffic flow models with behavioral interpretation, thereby providing a flexible behavioral foundation and a promising direction to accommodate new traffic entities like AVs. Under the more intelligent objective function of AVs, the LWR velocity does not represent a socially optimal driving strategy as demonstrated by larger deviations from the actual equilibrium in Fig. 4(d). Fig. 4(a-b) illustrate that the MFG mitigates traffic oscillation faster than LWR. Fig. 4(c) reveals the rationale at one time instant. Around a jam area with symmetric traffic density, vehicles driven by MFG controllers tend to slow down farther upstream before joining the jam and immediately speed up after leaving the jam; in contrast to those driven by LWR controllers whose speed remains symmetric.
before and after the jam area. This is because LWR’s velocity is determined only through traffic density at that location, while that of the MFG depends on traffic density of the entire horizon.

![LWR and MFG density graphs](image)

(a) LWR  
(b) MFG

(c) Velocity  
(d) Solution gap

Figure 4: LWR (HV) v.s. MFG (AV)

6. n AV + m HV: controllable AVs navigating the HV-dominated traffic

As mentioned in the previous section, in this section, we add HVs into the traffic environment where multiple AVs navigate. Likewise, we will use vehicle cooperation as the primary category and the environment stochasticity as the secondary category.

6.1. Cooperative control

When multi-AV control is cooperative, the existing literature covers both a deterministic and a stochastic environment.

6.1.1. A deterministic environment: Mixed Vehicle platooning

In a mixed traffic platoon comprised of multiple AVs and multiple HVs, how to design a AV controller to stabilize a mixed traffic platoon remains largely unsolved, due to the scalability issue, in other words, the topology of AVs and HVs in a mixed platoon.

To avoid enumerating various topology of a mixed platoon, a majority of studies use a general concept of head-to-tail stability in which the stability of a platoon only depends on the total numbers of AVs and HVs, not their topology. Using simulations, Talebpour and Mahmassani (2016); Yao et al. (2019) implement CACC on CAVs and investigated the string stability of the mixed traffic system. Different controller parameters and the CAV’s penetration rates are tested to illustrate their relations to the stability.

While accounting for the topology of a mixed platoon, by decomposing the entire platoon into small subsystems, Zhou et al. (2020) introduce a more practical head-to-tail stability criterion for subsystems and analyzes the mixed traffic system with multiple CAVs and multiple HVs under the new stability criterion.
Gong and Du (2018) solve a p-step MPC instead of a one-step MPC to mitigate the uncertainty of human driver trajectories.

One alternative approach to address the scalability issues is the PDE approximation (Barooah et al., 2009; Zheng et al., 2016). This approach suggests to study the stability of continuum traffic flow models which are the limits of microscopic models. Traffic stability is then defined by whether the deviations on the density and velocity profile from uniform flows are controlled as time increases. (Darbha and Rajagopal, 1999). Building on MFG, Huang et al. (2019, 2020b) analyze traffic stability for mixed traffic, assuming that HVs are modeled by ARZ and AVs are modeled by an MFG. Linear stability analysis demonstrates that the MFG traffic flow model behaves differently from traditional traffic flow models. The impact of AV’s penetration rate and controller design on traffic stability are quantified on ring roads.

6.1.2. A stochastic environment

Wu et al. (2017a,b); Vinitsky et al. (2018) assume a fully observable system where the goal of multi-AV control is to optimize total system performances, such as velocity, energy consumption. A model-free MARL is employed. In other words, there is no need to define a state transition matrix explicitly. Instead, the state transition is computed from the simulation platform. A traffic simulator has been developed in SUMO to simulate HVs and uncontrolled AVs using IDM models. Given actions selected by controlled AVs, the simulator updates every car’s position based on selected actions of controllable AVs and IDM models of uncontrollable vehicles. Then the centralized training and execution with trust region policy optimization (TRPO) policy gradient is implemented to solve for an optimal policy. Kreidieh et al. (2018a) trains AVs on a multi-lane ring road and implements transfer learning to execute the AV control on an open multi-lane highway.

6.2. Noncooperative control

In a multi-AV system where human drivers exist and dominate the traffic environment, uncertainty arises from human driving behavior. Controllable AVs have to learn the environment while selecting optimal driving policies with a maximum reward. Built upon single-agent RL, multi-agent reinforcement learning (MARL) extends the control of single robot to multiple ones. In a multi-agent system with stochasticity and uncertainty, MARL becomes a natural tool for control of multiple AVs. MARL tasks can be broadly grouped into three categories, namely, fully cooperative, fully competitive, and a mix of the two, depending on different applications (Zhang et al., 2019a): (1) In the fully cooperative setting, agents collaborate with each other to optimize a common goal; (2) In the fully competitive setting, agents have competing goals, and the return of agents sums up to zero; (3) The mixed setting is more like a general-sum game where each agent cooperates with some agents while competes with others. For instance, in the video game Pong, an agent is expected to be either fully competitive if its goal is to beat its opponent or fully cooperative if its goal is to keep the ball in the game as long as possible (Tampuu et al., 2017). A progression from fully competitive to fully cooperative behavior of agents was also presented in Tampuu et al. (2017) by simply adjusting the reward. Fig. [3] illustrates the classification of MARL based on if agents are collaborative (to optimize a common goal) or competitive (with competing goals) and if they share information with others (i.e., being independent or coordinate). There exists a void in which multi-AV control in a competitive environment is modeled.
The future AVs will be manufactured by different companies with different technical specifications. It will thus be challenging for AVs to collaborate with a common goal. We believe it is reasonable to picture that each AV is an independent and fully decentralized agent with its own goal (e.g., to send its occupant to her destination on a shortest path). Below we will lay out a feasible MVS framework where MARL algorithms can potentially be applied to this context. Without cooperation and with the presence of human drivers, it is also challenging for AVs to sense and perceive the environment precisely. Therefore we assume a stochastic environment. Fig. [3] illustrates the multi-AV control framework. The main different from the single-AV control is the interaction among multiple AVs when they simultaneously explore the mixed traffic environment. A Markov game is defined by a tuple $(S, O_1, O_2, \ldots, O_N, A_1, A_2, \ldots, A_N, P, R_1, R_2, \ldots, R_N, N, \gamma)$, where $N$ is the number of agents and $S$ is the environment state space. Environment state $s \in S$ is not fully observable. Instead, agent $i$ draws a private observation $o_i \in O_i$ which is correlated with $s$. $O_i$ is the observation space of agent $i$, yielding a joint observation space $O = O_1 \times O_2 \times \cdots \times O_N$. $A_i$ is the action space of agent $i \in \{1, 2, \ldots, N \}$, yielding a joint action space $A = A_1 \times A_2 \times \cdots \times A_N$. $P : S \times A \times S \rightarrow [0, 1]$ is the state transition probability, $R_i : S \times A \times S \rightarrow \mathbb{R}$ is the reward function for agent $i$, and $\gamma$ is the discount factor. The MARL components are:

- $S$. The state space of the mixed traffic environment. A state $s \in S$ contains the information of all controllable and uncontrollable AVs and HVs.
- $A$. The joint action space, i.e., $A_1 \times A_2 \times \cdots \times A_N$. The action of the $i^{th}$ AV, denoted as $a_i \in A_i$, is a two-dimensional vector, including acceleration and steering angle. The joint action of AVs is $a = (a_1, \ldots, a_N)$.
- $O_i$. The joint observation space for the $i^{th}$ AV. $O_i$ is the observation space for the $i^{th}$ AV, yielding a joint observation space $O = O_1 \times O_2 \times \cdots \times O_N$.
- $\mathcal{G}_i$. The set of observation functions, i.e., $\{\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_N\}$. For the $i^{th}$ AV, $s$ may not be fully observable. Instead, it draws an observation $o_i \in O_i$ that is correlated with $s$ according to an observation function $\mathcal{G}_i : S \times O_i \rightarrow [0, 1]$, i.e., $o_i \sim \mathcal{G}_i(o_i|s)$.
- $P$. The state transition function, i.e., $S \times A \times S \rightarrow [0, 1]$. This state transition can be computed from specific form (model-based) or from a mixed traffic simulator (model-free).
- $R_i$. The joint reward function, i.e., $R_1 \times R_2 \times \cdots \times R_N$. Along with the state transition, car $i$ receives an immediate reward, i.e., $r_i \in R_i : S \times A \times S \rightarrow \mathbb{R}$. The reward $r_i$ may include traffic safety (e.g., off-road/collision avoidance), efficiency (e.g., fast speed), and emissions. One aims to maximize its discounted expected cumulative reward by deriving an optimal policy, which is the best response to other AVs’ policies.

![Figure 5: Literature on MARL](image-url)
A controllable AV indexed by $i$ aims to derive an optimal policy $\pi^*(AV)_i: \Omega_i \times A_i \rightarrow [0, 1]$ by maximizing its expected cumulative reward. It samples an action from the policy after drawing observation $o_t$. After all AVs take actions, the joint action $a$ triggers a state transition $s \rightarrow s'$ based on the state transition probability $P(s'|s,a)$. Agent $i$ draws a private observation $o'_i$ corresponding to $s'$ and receives a reward $r_i(s,a,s')$. Agent $i$ aims to maximize its discounted expected cumulative reward by deriving an optimal policy $\pi^*_i$ which is the best response to other agents’ policies. This process repeats until agents reach their own terminal state. Due to the existence of other agents, the Q-value function for agent $i$, i.e., $Q_i$, is now dependent on the environment state $s \in S$ and the joint action $a \in A$ of all agents, i.e, $Q_i = Q_i(s,a)$. Similarly, the value function of agent $i$, i.e., $V_i = V_i(s)$, is dependent on the environment state $s$.

**MARL algorithms**

Depending on if AVs can exchange information and learn the environment information, driving policy learning can be categorized into joint or independent learners. Local observation leads to independent learners, while information sharing can change AVs’ learning behavior to joint learners.

**Independent Learners.** If AVs only sense neighboring vehicles’ information, each AV learns the environment and policies independently. From the vehicle perspective, vehicle $i$’s objective functional $J^N_i(s_i,u_i,s_{-i},a_{-i})$ depends on all other vehicles’ state and controls. The optimal driving strategy for vehicle $i$ $(i = 1, \cdots, N)$ is computed as (Liu et al., 2018):

$$
\begin{align}
\min_{a_i} J^N_i(s_i,a_i) \\
\text{s.t.} \quad \dot{s}_i(t) &= f(s_i,a_i), \\
& \quad a(t) \in A_i, \\
& \quad s_i \in S_i(\hat{s}_{-i}, \hat{a}_{-i}), \forall -i \in \mathcal{N}_i.
\end{align}
$$

where,

- $\mathcal{N}_i$: the neighboring vehicles of vehicle $i$;
- $\hat{s}_{-i}(t)$: the state of vehicles other than $i$ estimated by vehicle $i$;
- $\hat{a}_{-i}$: the action of vehicles other than $i$ estimated by vehicle $i$;
- $S_i(\hat{s}_{-i}, \hat{a}_{-i})$: the state space of vehicle $i$ given others’ states and actions.

Conceptually, most single-agent RL techniques can be directly applied to multi-agent scenarios for independent learners. Popular examples include Deep Q Network (DQN) (Mnih et al., 2015), deep deterministic policy gradient (Lillicrap et al., 2015) and soft actor-critic (Haarnoja et al., 2018). However, difficulties also arise (Omidshafiei et al., 2017; Nguyen et al., 2018): (1) non-stationarity of Q-value estimation due to co-existence of other adaptive AVs; (2) invalid theoretical convergence in multi-AV scenarios because the Markovian property may not apply; (3) confusing domain stochasticity from both environments and other AVs; and more importantly, (4) the curse of dimensionality, i.e., the search space in state and action is too large, making the learning intractable. Advanced MARL algorithms are developed to mitigate some of the aforementioned challenges. Decentralized hysteretic deep recurrent Q-networks (Dec-HDRQNs) utilizes different learning rates for different partially-observable domains (Omidshafiei et al., 2017). This approach exploits the robustness of hysteresis to non-stationarity and alter-exploration, in addition to the representational power and memory-based decision making of DRQNs. More recently, lenient DQN (Palmer et al., 2018) is proposed, with which lenient agents map state-action pairs to decaying temperature values that control the amount of leniency applied towards negative policy updates that are sampled from the experience replay. This introduces optimism in the value function update, and can facilitate cooperation in tabular fully-cooperative MARL problems.

A key challenge arises in MARL when independent agents have no knowledge of other agents, that is, the theoretical convergence guarantee is no longer applicable since the environment is no longer Markovian and stationary (Matignon et al., 2012). To tackle this issue, one way is to exchange or share information among agents.
Joint Learners. If AVs receive global information regarding all others’ state and action (e.g., via V2V/V2I), they can learn their optimal policies jointly. The performance of the agents could be better off through coordination.

To the best of our knowledge, there only exist a small amount of studies (Wu et al., 2017b,d,a, 2018a) on multi-AV control using MARL. They assume that all controllable AVs share a common objective, which constitutes a fully observable cooperative MVS taking into account uncontrollable vehicles. The policy network is trained with a TRPO policy gradient method, and transfer learning (Kreidieh et al., 2018b) is applied to transfer the policy from multi-lane ring roads to highway merging scenarios.

A joint learning framework suffers from the curse of dimensionality, as the agent size grows. Thus the centralized learning (i.e., based on global information) and decentralized execution (i.e., based on local observation) paradigm has become an increasingly popular paradigm for independent learners (Foerster et al., 2016; Lowe et al., 2017; Lin et al., 2018; Li et al., 2019). While training is stabilized conditioning on the information of other agents, scalability becomes a critical issue in MARL because the joint state space and joint action space grow exponentially with the number of agents. To mitigate the curse of dimensionality, mean field reinforcement learning (Yang et al., 2018) has become a popular technique, where the interactions within the population of agents are approximated by those between a single agent and the average effect from the overall population or neighboring agents. In this way, learning of individual agent’s optimal policies depends on the population dynamics, which makes possible a scalable policy learning for achieving Nash equilibrium in multi-agent environments. Its potential to multi-AV control could be one direction to explore.

6.3. $n$ AVs + $m$ HV ($n >> m$): A special case

There is little research on the AV-dominated world, partly because that it is highly likely that human drivers will adapt their driving behavior when surrounded with AVs. But it remains unclear how such behavior evolves. Different hypotheses could drive the evolution of human driving behavior toward opposite directions.

One hypothesis is that humans may gradually adapt their driving behavior in the presence of AVs and consequently develop moral hazards (Pedersen, 2001, 2003; Chatterjee and Davis, 2013; Chatterjee, 2016; Millard-Ball, 2016; Di et al., 2020). These speculations cannot be validated in the existing market with a too low penetration rate of AVs. Laboratory driving simulator using driving simulators could serve as a safe and effective alternative (Creech et al., 2019; Tilbury et al., 2020). In spite of the fact that participants could possibly exhibit unrealistic behaviors on a driving simulator, the value of these simulators should not be ignored for advancement of people’s behavioral adaptation for a future scenario.

7. Data-driven policy learning

A major challenge in the study of AVs, different from other autonomous systems, is the highly dynamic, uncertain, complex environment in which it navigates. Unlike training robots in a controlled laboratory environment, training intelligent AVs requires them to interact continuously with the traffic environment to learn optimal driving policies. Such a traffic environment, primarily comprised of intelligent actors including human drivers and other uncontrollable AVs, needs to be learned from real data.

7.1. Human driving policy learning

Human movement trajectories are treated as hard safety constraints or boundaries for robots motion planning. To this end, accurate and precise models of human behavior are required to ensure safety-critical applications. Driving is a complex task. It is a sequential decision-making process with a complex mapping from the perception of neighboring traffic or the prediction of global traffic environment onto driver actions. Human driving behavior has long been studied in the transportation community. It has recently gained growing attentions from the control and robotics communities for its importance in designs of AVs that will drive alongside human drivers.
7.1.1. Dataset

There are aggregate traffic data and individual trajectory based data. Aggregate traffic data are collected from various sensors, including loop detectors (Rakha et al., 2010; Rakha and Crowther, 2002, 2003), surveillance cameras (Mao et al., 2018; Tang et al., 2017; Lu and Skabardonis, 2007), Bluetooth detection (Singer et al., 2013; Allström et al., 2014b), roadside radar/LiDAR (Zhang et al., 2018b).

Emerging traffic sensors, including connected vehicles, smart phones, on-board cameras, and LiDARs, are expected to generate terabytes of streaming data daily (SAS, 2015). These new datasets would offer new opportunities to understand human driving behavior. Collecting real-time vehicle trajectory data, however, is costly and may infringe privacy, as it involves placing sensors inside individual vehicles (e.g., naturalistic driving devices continuously collecting vehicle movement information in the real traffic environment (Hecker et al., 2018a,b; Hammit et al. 2018; Flores et al., 2018; Zhang et al., 2018a; Zhu et al., 2018a)). Albeit lower cost, laboratory driving simulators (Sadigh et al., 2016b; Sadigh et al., Abbeel and Ng, 2011; Ziebart et al., 2008) allow only one driver to test at a time, unable to offer realistic experience of interacting with other vehicles on roads. To understand the emergent dynamics arising from human drivers requires information of all the vehicles dynamically moving in a traffic stream. By far there are only a few such public datasets.

Next Generation Simulation (NGSIM) is the mostly widely used human driver trajectory dataset. It provides all vehicle trajectories across a time span along some multi-lane highways. The shortcoming is that no camera images are recorded for each vehicle, which may limit the usage of image features for human driving policy learning.

Naturalistic data, collected while driving in the real traffic environment, provide an non-intrusive approach of personal driving data collection. The largest naturalistic dataset has been collected via the Strategic Highway Research Program (SHRP2) (NDS, 2018; McLaughlin and Hankey, 2015; Hankey et al., 2016). There were 3,400 participating vehicles instrumented with a data acquisition system recording speed, acceleration, latitude and longitude. Forward radar detects distance and speed relative to other vehicles. Four video views are also available. Such a dataset can train a driving policy using camera sensing information.

7.1.2. Physics-based model parameter calibration

Human driving behavior includes driving intent identification and prediction of internal states. Without communication among one another or via turning on signals, neither the intent nor internal states of neighboring vehicles are unknown and has to be estimated. We will first present the estimation of internal states in the car-following behavior, which is extensively studied in the transportation community, and then the prediction of driving intent.

CFMs have been extensively calibrated using a maximum likelihood approach (Hoogendoorn and Hoogendoorn, 2010), Bayesian estimation (van Hinsbergen et al., 2009; Kasai et al., 2013; Rahman et al., 2015; Lee and Ozbay, 2009a; Davis, 2017), fundamental diagram regression (Qu et al., 2015; Phegley et al., 2014), or heuristics (Lee and Ozbay, 2009b; Ma and Abdulhai, 2002). Most of them are calibrated using a pair of leading and following vehicle trajectories. It loses the information of how perturbation in one vehicle may propagate to those far behind in the platoon, thus may not capture instability of traffic.

With a rising volume of data generated by vehicles and their sensors, the conventional traffic models cannot predict generalizable driving behaviors. Leveraging big data, researchers are able to leverage data-hungry machine learning methods to learn the policies underlying the diverse human driving behaviors. In the context of driving, states are observations of a driver’s environment and actions are acceleration and steering angle. We have seen a growing body of literature characterizing driving behaviors using (deep) artificial neural networks (Khodayari et al., 2012; Panwai and Dia, 2007; Zhou et al., 2017a; Huang et al., 2018) and reinforcement learning (Zhu et al., 2018b). These models aim to capture various phenomena arising from human drivers, including asymmetric behaviors, traffic oscillations.

Compared to the car-following behavior, lane-change is more challenging to estimate, partly because of intent identification. Driving intents, which are intended actions, can be represented by discrete categories, including driving straight with a constant speed or acceleration or deceleration (or lane-keeping), turning (or preparing to change lanes), and changing to its left or right lane (or lane-changing). The driving intent estimation problem is commonly modeled as a classification problem, which will be discussed in the next
subsection. However, one school of researchers argue that human’s unpredictability, randomness, and non-Markovian property makes it infeasible to learn true dynamics (Driggs-Campbell et al., 2017). Instead, task-specific Bayesian optimization (Bansal et al., 2017), stochastic reachable set (Driggs-Campbell et al., 2018), non-parametric driver model (Driggs-Campbell et al., 2017), and probabilistic approaches (Bouton et al., 2017) have been developed. Humans usually convey intent through motion, which plays a crucial role in social interactions (Becchio et al., 2012). Built upon such understanding, Driggs-Campbell and Bajcsy (2016) assume drivers tend to follow some nominal trajectory, given by the spatial empirical distributions on a cost map. Accordingly, the lane-change intent can be formulated as an optimal control problem (and can be reduced to an MPC control). The parameters of the control objective function are estimated using 10 subjects’ 200 lane-change trajectories. Bansal et al. (2017) learns human dynamics via Bayesian optimization. The learned dynamic model is the one that achieves the best control performance for the task at hand but could be different from the true dynamic. Driggs-Campbell et al. (2017, 2018) solves a mixed integer linear program to estimate a stochastic reachable set that encapsulates the likely trajectories of human drivers intent and this model can generate trajectories that are similar to those performed by humans.

Physics-based models simplify the complex decision-making processes of human beings and may lack predictive powers due to its open-loop procedure of parameter estimation.

7.1.3. AI-based methods

Estimation of discrete human intent can be essentially formulated as a classification problem. Support vector machine (SVM) (Aoude et al., 2012), hidden Markov model (HMM) (Li et al., 2016), dynamic Bayesian Networks (Kasper et al., 2012), and Bayesian filtering (BF) (Li et al., 2016) are commonly used for online classification of human intent. Features used for classification include longitudinal acceleration, deceleration light, turn signal, speed relative to traffic flow (Liu and Tomizuka, 2015, 2016), steering angle, lateral acceleration, yaw rate (Li et al., 2016), and lane occupancy (Kasper et al., 2012).

Once human intentions are known, the internal state of a vehicle, i.e., its future trajectory, is estimated using Gaussian mixture models (Wiest et al., 2012), dynamic Bayesian Networks (Gindele et al., 2010), Kalman filter with parameter adaptation algorithm (Liu and Tomizuka, 2015, 2016).

Vehicle behavior estimation and prediction is built upon vehicle detection and tracking that happens within one’s perception system (Sivaraman and Trivedi, 2013). Vision-based or feature-based tracking is widely used to detect the presence of moving objects and associate vehicles between frames (Darms et al., 2008). These vehicle tracking techniques provide a foundation for end-to-end (or perception-to-control) training of autonomous driving policies (Amini et al., 2020).

The mainstream research on human’s driving policy learning is imitation learning, which will be primarily discussed subsequently.

Imitation learning

Imitation learning (IL) approaches learn the policy directly from expert demonstration data in order to behave similarly to an expert. Popular IL approaches include BC (Pomerleau, 1989; Bojarski et al., 2016; Syed and Schapire, 2008), inverse reinforcement learning (IRL) (Abbeel and Ng, 2004a; Gonzalez et al., 2016; Sadigh et al., 2016a), and generative adversarial imitation learning (GAIL) (Ho and Ermon, 2016).

In early attempts to model human driving behavior, BC formulates IL as a supervised learning problem and directly learns a mapping from states to actions using available datasets (Pomerleau, 1989). Compared to rule-based models, the advantage of these systems is that no assumptions are made about road conditions or driver behaviors. While BC approaches are conceptually sound (Syed and Schapire, 2008), they may fail in practice when there are states and conditions unrepresented in the dataset. As a result, even the post-trained policy model performs well on the observed states, small inaccuracies will compound resulting in cascading errors (Ross and Bagnell, 2010). In the case of driving behavior, for example, when the vehicle drifts from the center of the lane, a human driver should correct itself and move back to the centre. However, since this condition does not happen very often for human drivers, data on the correcting action is scarce, resulting in the cascading error problem, and the learned policy will continue to deviate from the center.

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and drive off-road. Dataset aggregation (DAGGER) is one popular technique to mitigate propagation errors of BC by augmenting original training data with expert demonstration for missing states (Ross et al., 2011). Assuming human drivers follow hierarchical reasoning decision-making, Tian et al. (2019) employs DAGGER to establish a mapping from the ego car’s state, all others’ state, and the ego car’s reasoning level \( k \) to the ego car’s level \( k \) action. To accommodate heterogeneity in human drivers, different HVs are assumed to follow different reasoning levels.

Instead of directly learning actions from observed states, IRL estimates one’s underlying reward function that drives observed actions, thus avoiding the issue of missing states. Assuming that the expert follows an optimal policy with respect to an unknown reward function, IRL (Ng and Russell, 2000; Abbeel and Ng, 2004b, 2011) and its variants (Ziebart et al., 2008) have become increasingly popular to learn optimal sequential policies from expert demonstration. In general, IRL attempts to recover the reward function prior to finding the policy that behaves identically to the expert. Because the recovered reward function extends to unseen states, the corresponding policy can generalize much more efficiently and mitigate the cascading errors from which BC approaches suffer. For example, when driving on the highway, the vehicle knows to return to the centre of the lane when it is close to the side, because the reward function gives a high penalty in this situation. As to BC, due to scarce learning samples of driving at rare situations, such as driving at the side of the road, this would be a problem for BC to handle these situations. IRL has been used for modeling human driving behavior (Gonzalez et al., 2016; Sadigh et al., 2016a). In particular, the reward function is specified as a linear combination of features (or a DNN) (Sadigh et al., 2018; Buyik and Sadigh, 2018; Abbeel and Ng, 2011). Sadigh et al. (2016b) employs a continuous-time version of IRL, which is the continuous inverse optimal control with locally optimal examples (Levine and Koltun, 2012). Schwarting et al. (2019) models heterogeneity in human drivers by introducing a social preference value into one’s reward function. An online IRL learning algorithm is developed for the AV to learn such value while interacting with HVs. Despite the increasing potential in imitation learning, IRL approaches are typically computationally expensive toward recovery of the expert reward function (or cost function).

Instead of learning the expert cost function directly and learning the policy based on it, recent work has attempted to learn the expert behavior through direct policy optimization and skip the step of cost function recovery. These methods have been successfully applied to modeling human driving behavior (Ho et al., 2016). With the advent of the generative adversarial network (GAN) (Goodfellow et al., 2014) and generative adversarial imitation learning (GAIL) (Ho and Ermon, 2016), new policy learning methods have become available, performing well on certain benchmarking tasks. GANs are based on a two-layer minimax game where one network acts as a discriminator to learn the difference between real and generated samples. The second network, i.e. the generator, is to generate fake samples to fool the discriminator. The goal is to find a Nash-equilibrium of the racing game between the generator and discriminator. More recently, Wasserstain GAN (Arjovsky et al., 2017) is proposed to replace the standard KL divergence objective with Wasserstein distance, which solves the mode collapse issue in standard GAN. GANs can be extended to imitation learning domain by replacing the generator with the policy network, i.e., the action generator given states. The generator generates actions based on a learned policy, which is derived via the the objective of fooling the discriminator. The discriminator distinguishes between the generated actions and expert actions given states. GAIL uses the GAN technique in combination with TRPO. TRPO updates the policy within a properly bounded region, and based on which, a monotonic improvement in policy over iterations is guaranteed (Schulman et al., 2015a). For more stable training, generalized advantage estimation (GAE) (Schulman et al., 2015b) is used to adjust variance-bias trade-off and reduce the variance in learning. TRPO combined with GAE is able to learn complicated high-dimensional control tasks (Schulman et al., 2015b). GAIL combined with recurrent policy learning (Wierstra et al., 2010; Heess et al., 2015), in particular, has been used for modeling human driving behavior, achieving advanced results (Kuefler et al., 2017). Later on, other algorithms combining GAIL with Wasserstein GAN (WGAIL), and gradient penalty (WGAIL-GP) (Gulrajani et al., 2017) are explored and show improved performances in some conditions compared to standard GAIL (Greveling, 2018). GAIL has been used to imitate human driving behaviors (Kuefler et al., 2017; Bhattacharyya et al., 2018a, 2019). It however may not reflect realistic human driving behaviors. For instance, humans on a straight road would drive without maneuvering steering wheels, but the trained driving polices could
alternate between small left and right wheel-turning actions, indicating instability of policies. Moreover, the newest algorithms, such as WGAIL-GP proposed in Greveling (2018), have not been thoroughly evaluated to form a well-built conclusion about the performance for modelling driving behavior, and are open to further practice in future study.

In summary, when only a small portion of states are visited in training datasets, BC suffers from cascading errors in prediction over that unseen states. IRL mitigates the cascading error issue by learning an expert’s unknown reward function, because the inferred reward would provide a feedback to the learned actions generated from unseen states. However, IRL approaches are typically computationally expensive. Instead of learning reward functions, GAIL learns expert behavior through direct policy optimization (Ho et al., 2016). GAIL can extract a generalizable policy from limited driving scenarios compared to BC, and has a relatively faster learning speed compared to IRL. We believe GAIL could be one promising tool for human driving policy learning.

7.2. Autonomous Driving Models for Uncontrollable AVs

Uncontrollable AVs refer to those AVs that interact with controllable AVs in the traffic environment but cannot be controlled, probably because they are manufactured from different companies and their driving algorithms are unknown to the host AV. Their driving behavior also needs to be learned by controllable AVs. Unfortunately, researchers’ inaccessibility of AV data has greatly hindered such understanding. Due to manufacturers’ proprietary protection, however, no documentation has revealed how the existing AVs are actually programmed to drive and interact with other road users on public roads. In this subsection, we strive to provide some insights into how researchers may leverage some public datasets collected for computer vision to model the driving behavior of existing AV fleets on public roads.

7.2.1. Dataset

Researchers should be very careful when they claim an AV dataset or when they need to seek some AV related data, because most public AV datasets are actually collected by HVs. We summarize a non-exhaustive list of AV datasets in Table 3. These data are collected by vehicles equipped with a variety of sensors, such as radar, LiDAR, GPS, cameras, and inertial measurement units (IMU). These sensor data, if collected from HVs, are solely used to train computer vision algorithms for object detection, segmentation, 3D tracking, pedestrian detection, and Simultaneous Localization and Mapping (SLAM). Once trained, these computer vision algorithms are mounted to AVs for testing. Fortunately, there exist several public datasets collected directly from AVs, that were pre-trained by academic institutes or AV technology companies. It would be a good strategy for academic researchers to make use of these public AV-collected datasets to learn uncontrollable AV models for simulation, which might behave similarly to existing AV fleets.
To train driving behavioral models of AVs (i.e., end-to-end driving policies), we need not only data from sensors mounted for computer vision, but also driving data directly collected from AVs’ motion sensors. Thus, some of AV-collected datasets in Table 3 might not be ideal for AV policy training in their raw format. For example, A*3D and Argoverse did not provide acceleration, which requires inference using other sensor

| Data collection vehicle | Dataset   | Purpose                                         | Sensor setup                                                                 | Location         | Institute                                      |
|------------------------|-----------|-------------------------------------------------|------------------------------------------------------------------------------|------------------|-----------------------------------------------|
| HV                     | KITTI     | 3D object detection tracking                    | grayscale/ color cameras, a rotating 3D laser scanner, GPS, IMU               | Karlsruhe        | Karlsruhe Institute of Technology            |
|                        | KAIST     | object detection, drivable region detection, depth estimation | 2 RGB & 1 thermal camera, 1 integrated GPS/IMU device                       | Seoul            | Korea Advanced Institute of Science and Technology |
|                        | H3D       | 3D detection, 3D multi-object tracking          | GPS/IMU device, a LiDAR, 3 cameras                                          | San Francisco    | Honda Research Institute                     |
|                        | A2D2      | 3D semantic segmentation, object detection      | 5 LiDARs, 5 surround cameras                                                | Germany          | Audi AG                                       |
|                        | ApolloCar3D | 3D car instance understanding                   | GPS, 2 laser scanners, 6 video cameras, a combined IMU/GNSS system, LiDARs | Various cities in China | Baidu                                         |
|                        | nuScenes  | 3D detection, tracking                          | 6 cameras, 5 radars and 1 LiDAR, IMU, GPS                                   | Boston, Singapore| Aptiv Autonomous Mobility (Aptiv)             |
| AV                     | A*3D      | 3D object detection                             | 2 Chameleon3 USB3 cameras, 1 Velodyne 64-beam 3D-LiDAR                       | Singapore        | Agency for Science, Technology And Research (A*STAR) |
|                        | Argoverse | 3D tracking and motion forecasting              | 2 long-range LiDARs, 9 cameras for 360° coverage, GPS and other localization sensors | Pittsburgh, PA: Miami, PT | Argo AI and Carnegie Mellon Univ.           |
|                        | Lyft L5   | perception systems, motion prediction           | 2 40-beam and 1 64-beam LiDARs, 360° cameras built in-house, a long-focal camera points upward | Palo Alto, CA    | Lyft level 5 self-driving system             |
|                        | Waymo Open| 2/3D object detection, 2/3D tracking            | 1 mid-range LiDAR, 4 short-range LiDARs, 5 cameras (front and sides), IMUs  | Various places in USA | Waymo self-driving cars                     |
information, such as GPS. Fortunately, Waymo and Lyft datasets provide complete acceleration records, based on which AV policy training can be made.

To the best of our knowledge, Waymo/Lyft data (Waymo, 2019; Kesten et al., 2019) are the only two public datasets on how Level-5 automated vehicles drive and interact with other road users on public roads. Both sets are composed of sensor data collected from accelerometer (i.e., IUM), camera, and LiDAR. They were originally released for the purpose of object detection and tracking algorithm development. These datasets also offer valuable insights into the AV driver models and the interaction between AVs and the environment. Such datasets are however distinct from conventional traffic data that our transportation community is used to handle and thus novel methods are required.

7.2.2. AV driving model for Waymo cars

Leveraging Waymo’s sensor data, Gu et al. (2020) apply BC to learn generalizable autonomous driving polices for two reasons: First, AVs are assumed to follow the same driving policies in the same traffic environment, which is different from human drivers who behave highly heterogeneously. Second, Waymo datasets cover a wide range of traffic scenarios, including on highways or urban streets, at intersections with traffic lights or stop signs, Car-following scenarios were selected from a vast amount of Waymo video data to validate the algorithm performance. An LSTM-based learning model is trained, which takes sensor inputs from accelerometer and camera of the past ten frames and predicts acceleration for the next frame. Figure (7) illustrates three scenarios in one video: the ego car follows a truck, the truck leaves, and another leading car decelerates. This model could be a basis to build a mixed traffic environment that captures the interactions between AVs and their sounding environment.
(a) In the first frame of the video, this is a typical "car-following" scenario: the AV follows the truck steadily, and the initial value of the acceleration is around zero.

(b) In the 108th frame of the video, the truck turns left and at this moment, there is no detected front car and the AV accelerates. The ground-truth and the prediction acceleration curves climb simultaneously.

(c) In the last frame of the video, the declining acceleration curves shows that the AV brakes accordingly as the front car stops due to the traffic ahead.

Figure 7: LSTM prediction on longitudinal and lateral accelerations (left) and video snapshot captured from one AV camera (right) (Three key frames are extracted from segment-10289507859301986274 from tar validation_0001.)

7.2.3. AV Simulators in Mixed Traffic

It is crucial to validate efficiency and safety of designed AV controllers. High-risk and high-cost of real AV test urgently requires the development of a mixed traffic simulation environment for virtual testing of AVs. We want to stress that there are tons of “AV simulators” out there but their settings and purposes differ significantly. Researchers should be aware of the types of traffic simulators in the market and select the one tailored to their own purposes.

We divide the existing AV simulators into two primary types based on their purposes: one for traffic performance assessment implemented with preprogrammed physics-based CAV driving models, and the other for AV driving policy training and evaluation by creating a static or interactive environment. The former encode the already calibrated driving models (such as IDM) of each agent to simulate outcomes without further updating these driving models, while the latter require continuous interactions of the trained AV system with the environment. The transportation community is focused on the first category aiming to evaluate the impact of AVs on traffic congestion or emission, while the robotic community has widely used
the second category for RL based AV policy training.

The first category of simulators include VISSIM (Group) and AIMSUN (AIMSUN), which have accommodated AV driving modules. VISSIM, compatible to vehicle dynamics simulators including CarMaker, is able to simulate the full spectrum of vehicle automation from Level 1 to 5. Aimsun Auto can be integrated with sensor testing tools and vehicle dynamics simulation tools, such as Simcenter PreScan. These simulators are good to answer systematic questions like the impact of various AV market penetration rates on traffic performances and tipping points, which would help operators and policy makers better assess the impact of AVs on safety, mobility, and sustainability. But they suffer from two issues: First, vehicle motion is simulated using physics-based traffic models, which limits their potential to include AI-based controls trained with high-dimensional features. Second, these simulators cannot update driving models online. Driving parameters have to be calibrated offline before running online simulation. This may prevent CAVs from adapting to the imminent traffic environment.

The second category of simulators can be further divided into two types: game-playing and customizable (see Table 4). Game-playing AV simulators are task-oriented with specified tasks for AV players to accomplish. It is not allowed to change the environment and surrounding vehicles because they are pre-programmed and fixed. TORCS ([Wymann et al., 2000]) is a racing simulator, which provides real-time observations like speed, position on roads, distance to proceeding car, and image. This simulator has been used for AV training in the lane-keeping and racing scenarios ([Chen et al., 2015] [Sallab et al., 2017] [Yang et al., 2017]). Another popular racing simulator currently used for learning a racing AV policy is World Rally Championship 6 (WRC 6) ([Perot et al., 2017] [Jaritz et al., 2018]). WRC 6 provide front view image and speed information for players to control steering, brake, and gas. Compared to TORCS, WRC 6 has a more realistic physics engine. In addition to racing simulator, AV communities recently extend their work to action-adventure games, such as Grand Theft Auto V (GTA V), in which multiple vehicle-related missions need to be completed ([Richter et al., 2016] [Johnson-Roberson et al., 2017] [Richter et al., 2017]).

Customizable AV simulators provide APIs for users to design their environments, surrounding vehicles, and sensor suites at will. Once setting up the initial scenarios, the built-in engine will simulate the involved vehicles and traffic scenarios of interest. Various AV training tasks, such as object detection, lane keeping and collision avoidance, can be made on the platforms provided by customizable simulators. SUMO ([DLR Institute of Transportation Systems]) is a typical example, with which users can design road networks, surrounding vehicles’ policies and sensor systems. SUMO can provide state information, such as speed, location on a road, 2D top-down image of the road, and the state of other designated vehicles in the simulation. CARLA ([Chen et al., 2019b] [Codevilla et al., 2018]) has a 3D engine, and can provide more realistic traffic simulation, though the computation is heavier. CARLA also has a flexible setup of sensor suites and signals, including GPS, collision, LiDAR, 3D images and etc. The 3D road traffic can be configured in the simulator. FLOW ([Wu et al., 2017c]) is another customizable simulator, which utilizes SUMO based engine. FLOW incorporates RL libraries, such as rilab and RLib, and thus, it has convenient interface for RL developers to train and evaluate their AV policies in the simulation provided by FLOW. FLOW has recently been used to train muti-AV policies in a mixed traffic environment ([Wu et al., 2017c] [Jang et al., 2019]).
Table 4: Simulators for training AV policies

| Type of AV Sim. | Simulator | Features | Reference |
|-----------------|-----------|----------|-----------|
| Game-playing    | TORCS     | speed, position on the road, distance to proceeding car, 3D image | Wymann et al. (2000) |
| (vehicle sensor, environment and surrounding vehicles are fixed) | World Rally Championship 6 (WRC 6) | 84x84 front view image, speed | Perot et al. (2017); Jaritz et al. (2018) |
|                 | Grand Theft Auto V (GTA V) | images from various viewpoints of vehicle | Richter et al. (2016); Johnson-Roberson et al. (2017); Richter et al. (2017) |
| Customizable (can configure vehicle sensor systems, environment specification, controlling of surrounding vehicles) | SUMO | 2D top-down view of road, speed, location on roads, state info. of designated surrounding veh., and etc. | DLR Institute of Transportation Systems |
|                 | CARLA     | GPS, speed, acceleration, collision sensor, LiDAR, 3D images, and other sensor suites | Chen et al. (2019b); Codevilla et al. (2018) |
|                 | FLOW      | Same to SUMO’s features | Wu et al. (2017c); Jang et al. (2019) |

7.3. AV-HV Interaction

Ideally how one AV interacts with HVs and other AVs should be learned from data. But the lack of such data along with an extremely low penetration rate of AVs makes this task infeasible. All the existing studies resort to theoretical modeling. We will first point out what needs to be learned if we have data and then turn our attention to modeling approaches.

1. HV-HV: How human drivers interact with one another has been extensively studied in the existing literature. Its characterization uses both physics-based and AI-based methods (detailed in Section 7.1).

2. HV-AV: How human drivers react to the presence of AVs has two directions: (1) Most studies assume that HVs drive the same way as they do in the pure HV traffic environment. In other words, even when they encounter AVs, they cannot identify AVs and interact as if AVs were HVs. (2) If HVs have the capability of identifying AVs, AVs are essentially another vehicle class and HVs may likely interact differently. On one hand, the interaction of heterogeneous vehicle classes can provide comparative studies (Ossen and Hoogendoorn, 2011). On the other hand, AVs are fundamentally different from other vehicle types propelled by human drivers and may transform humans’ car-following behavior significantly. Unfortunately, due to lack of behavioral data for HV-AV scenarios, this arena is understudied.

A limited number of studies have all pointed out that humans’ behavioral reaction to AVs highly depends on their trust to the technology. Frber (2016) studies the importance of communications among road users, such as eye contact, gestures, or anticipatory behavior, on the safety of human drivers. Thus, the challenge arises when humans attempt to communicate but cannot get feedback from AVs. As a result, humans may not predict what will happen and behave cautiously. Dekker (2019) raises a concern that lack of local traffic culture, such as when to honk rather than light signals or yield modestly, may cause people’s distrust, and increase the risk of conflicts between human drivers and AVs. Unfortunately, most current AV training processes mainly focus on learning a general and culture-blind AV policy instead of learning how to drive like a local driver. Zhao et al. (2020) conducts a sequence of field experiments with ten recruited drivers for HV-AV scenarios. Using headways, gaps,
and speed deviation, the participants are grouped into three types: AV-believers, AV-skeptics, and AV-insensitives. In other words, one’s car-following reaction to AVs highly depends on her trusts on AV technologies.

3. **AV-HV**: For those uncontrollable AVs, how they interact with the HV-dominated traffic depends on the autonomous driving algorithms programmed into these AVs. The existing AV data, such as Waymo open data, contains rich information of how one AV interacts with its surrounding environment. Our work (Gu et al., 2020) proposes an LSTM model to understand how an AV follows HVs.

   For those AVs that are controllable, effective mutual interaction would help AVs to effectively communicate and exchange messages with HVs and other road users. To design AVs’ interaction interface is an emerging field in marriage of human factors and human-machine interaction (Vinkhuyzen and Celkin, 2016; Müller et al., 2016; Woff, 2016; Zhang et al., 2017).

4. **AV-AV**: For those uncontrollable AVs, with a low market penetration rate, it is less likely for an AV to encounter another AV. Thus, at this point it is difficulty to model how two AVs interact in the HV-dominated traffic environment using data-driven approaches.

   For those AVs that are controllable, AVs should be capable of communicating with one another via V2V/V2I or cloud-based technologies.

In the modeling aspect, there does not exist any formal definition of what constitutes “interactions” between AVs and HVs at a microscopic level. Here we provide an abstract definition of vehicular interactions.

**Definition 7.1. Interactions**: how the presence of other vehicles influences the driving strategies of AVs and vice versa. The vehicular interaction can be modeled through joint states, physical constraints, coupled rewards or objective functions. It can be categorized into local and global interactions, relying on information technologies.

In a platoon of CAVs, vehicles interact either locally (e.g., the immediate leader and the follower) or globally. The local pairwise interaction between the immediate leader and the follower is captured in all CFMs, where the speed difference and headway with the immediate leader influences one’s acceleration (Talebpour and Mahmassani, 2016; Cui et al., 2017; Wu et al., 2018a). When V2V communication links are introduced in a platoon, additional interaction terms that reflect the speed and headway influence of far upstream leading vehicles are accounted for in CFMs by adding accelerations of k-ahead-vehicle (Jin and Orosz, 2014; Qin and Orosz, 2017; Jin and Orosz, 2018; Jin et al., 2018) or interaction links (Li et al., 2017a, 2018a, 2018c). These models do not consider physical collisions. To fix it, physical distance constraints must be imposed. Accordingly, (Wang et al., 2014a,b, Gong et al., 2016; Gong and Du, 2018; Zhou et al., 2017b) encode these hard constraints into optimal control problems.

Researchers from control and robotics communities primarily focus on local interaction and formalize it using different tools. Sadigh et al. (2016b) assumes that AVs actions can influence HVs immediately through carefully selected reward functions. Lazar et al. (2018a) further illustrates that an “interaction-aware” AV can maximize road capacity leveraging such interaction. Furthermore, Sadigh et al. employs the concept of adversarial game for a game-theoretic interaction: the HV serves as the AV’s adversarial and play an adversarial game with the AV. The goal is to find a sequence of human driving actions that could lead to the AV’s unsafe behavior. In this game, the AV takes actions to maximize its cumulative reward, while the HV tries to falsify the AV’s action by selecting driving actions that minimize the AV’s reward function. The driving actions of the AV solved from this game ensures a robust controller design that accommodates measurement or prediction errors of the HV behavior.

Transportation researchers are more interested in the impact of microscopic AV-HV interactions on macroscopic traffic flow patterns (Chen et al., 2019a) and its implication for traffic controls in the presence of AVs (Levin and Boyles, 2016). A majority of studies use simulations, due to the complex project from micro to macro scales. On the macroscopic level, a multi-class traffic modeling approach is commonly adopted. Among a large amount of studies on the multiclass LWR for the interaction between multiple types of traffic flows, (Levin and Boyles, 2016; Patel et al., 2016; Kockelman, 2017; Melson et al., 2018).
have applied it to AV-HV mixed traffic and proposed networked traffic controls. To capture the effect of communication and information sharing on traffic flow, Ngoduy et al. (2009); Ngoduy (2013b,a) propose a multiclass non-equilibrium gas-kinetic theory based model to characterize traffic flow dynamics for connected and automated vehicles and analyzed stability of the developed controllers. These models, however, may lack detailed interpretations of how two types of vehicles interact on a microscopic level. We urgently need a micro-macro analytical framework to offer insights into how microscopic interactions are designed for desirable traffic flow patterns.

We summarize the above vehicular interaction types in Table 5.

Table 5: Vehicle interaction types

| Interaction Type | Reference | Community |
|------------------|-----------|-----------|
| **Micro**        |           |           |
| Local pairwise   | Talebpour and Mahmassani (2016); Cui et al. (2017); Wu et al. (2018a) | Transportation |
| Local influence by design | Sadigh et al. (2016b); Lazar et al. (2018a) | Robotics |
| Local adversarial game | Sadigh et al. | Robotics |
| **Global**       |           |           |
| Global k-ahead-vehicle term | Jin and Orosz (2014); Qin and Orosz (2017); Jin and Orosz (2018); Jin et al. (2018) | Control |
| Global interaction link | Li et al. (2014); 2017a,b 2018d | Transportation \\ \\ & control |
| Global hard constraints | Wang et al. (2014a,b); Gong et al. (2016); Gong and Du (2018); Zhou et al. (2017b) | Transportation |
| **Macro**        |           |           |
| Macro multiclass | Levin and Boyles (2016); Patel et al. (2016); Kockelman (2017); Melson et al. (2018) | Transportation |
| Macro gas-kinetic | Ngoduy et al. (2009); Ngoduy (2013b,a) | Transportation |

8. Model Summary

In this section, we summarize all the mixed traffic models based on physics-based and AI-based categories.
| Interaction scenario | Coop. or comp. | Model | AV controller | Goal | HV driving model | HV data & estimation | Traffic scenario | Simulation | Algorithm | Reference |
|----------------------|----------------|-------|---------------|------|------------------|---------------------|-----------------|------------|-----------|-----------|
| n AVs                | Coop.          | Linear controller | string stability | CACC | - | CF | numerical, field experiments | - | Schakel et al. (2010) | Naus et al. (2010); Ploeg et al. (2011); Milanés et al. (2014); Milanès and Shladover (2014); Cui et al. (2017) |
| Coop.               | linearly con- | General longitudinal vehicle dynamics, (serial distributed) MPC | robust local and string stability (with uncertainties) | - | - | CF | - | Sequential distributed algorithm | Zhou et al. (2017c); Zhou and Ahn (2019); Zhou et al. (2019) |
|                     | strained linear quadratic Gaussian, optimal control |                |               |      |      |      |      |                        |
| n AVs + 1 HV        | Coop.          | CCC   | n-car-ahead OVM | optimal velocity, close to uniform flow | spacing and speed feedback | sweeping least square | CF | numerical, field experiments | recursive method for LQ with distributed delay | Qin and Oroz (2013); Jin and Oroz (2014); Qin and Oroz (2017); Jin and Oroz (2018); Jin et al. (2018) |
| Coop.               | Optimal control | fully observable one- or p-step MPC | transient traffic smoothness, asymptotic stability | Newell | Online curve matching NGSIM data | CF | - | Dual-based distributed algorithms | Gong et al. (2016); Gong and Du (2018) |
| n AVs + m HV        | Coop.          | Linear quadratic regulator optimization | closed form | Equilibrium spacing, speed difference, acceleration rates | (Chained) asymmetric behavior model | - | CF | A 10-vehicle platoon | - | Chen et al. (2019a) |
| Coop. | distributed frequency-domain-based | hierarchical control | String stability for mixed platoons | Linearized general form | CF | NGSIM | Two mixed vehicular platoons led by two HVs sampled from NGSIM in MATLAB | H-infinity control | Zhou et al. (2020) |

(Abbreviation: coop. – cooperative, comp. – competitive. CF – Car-following; LC – Lane-change.)
| Scenario | Reward | AV controller | HV model | HV data & estimation | Traffic scenario | Solution algorithm | Control solution algorithm | Evaluation | Reference |
|----------|--------|---------------|----------|----------------------|----------------|-------------------|---------------------------|------------|-----------|
| 1 AV + 1 HV | spacing, safety | mixed-motive game | NGSIM | CF, NHTSA | CF, L.C. | PD controllers | Simulated driving trajectories | Continuous inverse optimal control | Li et al. (2018a); Li et al. (2018b) |
| 1 AV + m HV | collision, on-road, distance-to-object, safe separation, lane position, speed, effort | Reactive game | NCSIM or simulated | BO | DAgger BO (hierarchical reasoning) | Simulated driving trajectories | Stackelberg, decision tree policies | Feedback Stackelberg dynamic program | Sadigh et al. (2016b); Sadigh et al. (2016a); Fisac et al. (2019) |
| 1 AV + m HVs | Similarity to experts behavior in data | End-to-End controller based on BC | - | - | - | - | - | CNN, LSTM, and their combinations | Pomerleau (1989); Muller et al. (2016); Bojarski et al. (2017); Baechle et al. (2017); Xu et al. (2017); Enas et al. (2017); Hackbarth et al. (2018a); Bansal et al. (2019) |

Table 7: AI-based mixed traffic model summary
| - | MDP | End-to-end controller based on DRL | cost related to collisions, location on the road, angle between vehicle and road headings, difference from desired speed, et al. | IDM, PD controllers or other pre-programmed HV in the gaming simulator | simulated CF, LC and racing scenarios | Deep policy networks, deep Q networks using training methods, such as TRPO, DDPG, advantage Artor-critic, A3C | Lillicrap et al. (2015); Zhang et al. (2016); Sallab et al. (2017); Perot et al. (2017); Jaritz et al. (2018) |
| - | (PO)MDP | MCTS + DRL | collision avoidance, keep on the road, task completion bonus and etc. | IDM, PD controllers or pre-programmed HV in the simulator | simulated tactical LC, path planning | self-developed simulators, such as POMDPs.jl SIMULATE function | Paxton et al. (2017); Sunberg et al. (2017); Hoel et al. (2020) |
| - | MDP with probabilistic guarantees | probabilistic specification expressed with linear temporal logic, modified $\epsilon$-greedy exploration policy | make a left turn safely and efficiently | IDM, pedestrians move at a constant speed | Unsignalized intersections | - | - |
| $n$ AVs | Comp., Coop. | differential game | (Multi-anticipative) ACC, rolling horizon control | safety, equilibrium, control, travel efficiency, route, lane preference, lane switch | IDM, Helly CFM | CF, LC | one-lane highway | Pontryagins Minimum Principle | Wang et al. (2015, 2016) |
| $n$ AVs + $m$ HVs | Comp. | Multi-population differential game | MFG | efficiency, safety, kinetic energy | ARZ | CF | Numerical | Multigrid preconditioned Newton’s finite difference | Huang et al. (2019, 2020a,b) |
| Coop. | Model-free fully observable DRL | GRU NN | close to desirable system-level velocity, collision penalty | IDM | - | CF, LC, merge | FLOW (based on SUMO) | centralized training and execution with TRPO policy gradient in state equivalence class representation, transfer learning |
|-------|---------------------------------|--------|----------------------------------------------------------|-----|----|----------------|-------------------|-------------------------------------------------------------------------------------------------------------------------------------|

(Abbreviation: coop. – cooperative, comp. – competitive. CF – Car-following; LC – Lane-change.)
9. Conclusions and Open Questions

We will discuss open questions that are unanswered in the existing literature and provide several promising research directions.

9.1. Scalable Multi-AV Controls for Social Optima

There are few successful applications for MARL in autonomous driving, especially in complex multi-AV driving scenarios. Most previous research focuses on either using centralized but computationally-heavy MARL approaches for cooperative policy to achieve long-term traffic efficiency (Vinitsky et al., 2018), or applying decentralized parameter-sharing but non-cooperative techniques for collision-free driving for multiple AVs (Bhattacharyya et al., 2018b). In addition, MARL is a fast evolving research area, but its application to multi-autonomous driving has lagged behind. Most researchers are still using basic deep RL algorithms such as deep Q network, which is not able to solve some complex problems with more than one AV. As having been discussed above, much more powerful MARL algorithms were developed in recent years but few of them have been applied to multi-AV tasks and traffic domain. In the sense, research is highly in demand on extending existing MARL algorithms or developing brand-new MARL for multi-AV and mixed AV-HV scenarios.

Nevertheless, AV algorithmic designers tend to program AVs for individual welfare, such as protecting occupants or selecting a fastest route selfishly, with no incentive for improved traffic performance. City planners have to regulate the behavior of AVs or their designers for social good. Such competing goals pose difficulty in upper level control imposed by planners, which has not been explored in the existing literature. A socially optimal control scheme needs to be devised for city planners to guide the autonomous driving technology toward social optima.

9.2. Human Driving Policy Learning

Only when we begin to study AVs, have we learned that as humans, we know little about our own driving behavior.

9.2.1. Physics-Informed AI Models

Both physics-based and AI-guided methods have limitations: the former highly relies on existing physical traffic models, which may only capture limited dynamics of real-world traffic, resulting in low-quality estimation; While the latter requires massive data in order to perform accurate and generalizable estimation. Nevertheless, AI-based models may not capture all traffic phenomena observed in the real-world (Zhou and Laval, 2019). To mitigate the limitations, a physics-informed deep learning (PIDL) framework could potentially predict one’s driving behavior with small amounts of observed data more efficiently. PIDL contains both model-driven and data-driven components, making possible to leverage the advantages of both scientific models and deep learning techniques while overcoming the shortcomings of either.

The integration of PDE-based physics and deep learning has recently become gradually popular as an effective alternative PDE solver (Raissi, 2018; Raissi and Karniadakis, 2018). Increasing attentions have been paid to the application of PIDL in scientific and engineering areas, to name a few, the discovery of constitutive laws for flow through porous media (Yang and Perdikaris, 2019), the prediction of vortex-induced vibrations (Raissi et al., 2019), 3D surface reconstruction (Fang and Zhan, 2020), and the inference of hemodynamics in intracranial aneurysm (Raissi et al., 2020). We have seen a few studies that adopt the similar framework in the transportation community. Hofleitner et al. (2012) and Wang et al. (2019) embed a probabilistic models into a dynamic Bayesian network to estimate the evolution of travel time on links. Wu and Work (2018) use neural networks with different structures to capture the general behavior of a car-following model and predict the acceleration of a vehicle using velocity and distance information.

In conclusion, available human driver data is usually sparse, likely leading to sample bias issues. PIDL has shown its predictive robustness with smaller datasets and could become a promising direction in human driving behavior learning. Existing traffic models would provide some prior knowledge and help constrain the admissible solutions of AI approaches.
9.2.2. Experimental Design

Little research discusses optimal data or experiment selection for a robust traffic model calibration. The interactions of HVs on roads could generate emergent dynamics, i.e., traffic jam in the form of stop-and-go wave or oscillation. A driving model calibrated by one dataset may not be capable of predicting the emergent dynamics arising from another dataset. In other words, the predictive power of a driving model heavily depends on its training and test datasets. Field experiments are, of course, not just costly but highly risky to perform. Thus optimal experimental design can help collect representative training and validation datasets. What experiments to perform, what data to collect, and what data to use for behavioral learning and policy training are all unresolved questions.

9.2.3. Heterogeneity

The mixed traffic model need to account for heterogeneity of human drivers (e.g., different capabilities and risk profiles, human driving errors) and AVs (e.g. acceleration and braking capacity, as well as manufacturer’s choice of risk tolerance). In particular, humans are highly heterogeneous, due to personal taste and preference, randomness or aggressiveness, and driving experience. With the same environment and information, different drivers may maneuver their cars differently. A robust model has to be able to accommodate these deviations and predict a distribution of actions that is consistent with real-world observations.

9.3. Multi-Scale Human-Machine Ecosystem Modeling

The overarching goal of researchers is to understand the new traffic pattern comprised of large numbers of AVs and HVs and the systematic impact of AVs on traffic safety and efficiency. Such an understanding of the macroscopic traffic behavior should be rooted in both the microscopic behavior of AVs (Chen et al., 2019a) and evolution of the driving behavior of HVs over time (Di et al., 2020). This topic can be positioned to a broader context which is the collective behavior of hybrid human-machine (Rahwan et al., 2019). Thus, bridging traffic models on both the micro- and micro-scale using a multi-scale scheme needs to be understood.

A majority of existing studies on AVs are primarily focused on highways. An urban traffic environment consists traffic entities including cars, traffic lights, pedestrians, (motor)cyclists, scooters, and other road users. This multimodal mixed traffic environment will further complicate the control of AVs driving alongside various road users.

9.4. Accountable, Fair, and Ethical AVs

When automated systems can make life or death decisions for humans, questions arise related to AI-based algorithmic decision-making:

1. Accountability: We need to determine and assign responsibility for damages or injuries caused by AVs.
2. Fairness: AVs should make unbiased decisions.
3. Ethics: Ethical collection and use of data should be guaranteed for privacy-preserving. AVs should also make ethical decisions.

There are qualitative studies on above topics, but how to integrate these aspects into engineering decision remains unanswered. Interdisciplinary collaboration with legal experts and social scientists is the key to success.

9.5. A Pathway to Artificial General Intelligence (AGI)

There is still a long way to reach AGI, which is the ultimate intelligence of machines, AVs need to reach human-level AIG, with the capabilities of reasoning, knowledge representation, planning, learning, communicating in natural language, and integration of all these skills towards common goals (Hodson, 2020). This not only requires bridging gaps with AI tools, but also a convergence of engineering, cognitive science, and social science. If achieved, it is not only a breakthrough to AV controls, but also to humanity.

In summary, with a rapid growing AV fleet on public roads, it is crucial to develop analytical tools for mixed traffic, which will help traffic engineers better understand the impact of AVs on transportation system performances, for the AV industry to develop a scalable autonomous driving control algorithm, and ultimately, for city planners, policymakers, and lawmakers to manage AVs for social good.
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