Modelling of Simple Assembly Line Balancing Problem Type 1 (SALBP-1) with Machine and Worker Constraints

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Abstract—This paper presents a mathematical model for Simple Assembly Line Problem Type 1 (SALBP-1) with resource constraints; machine and worker. The existing model of SALBP-1 assumes that all the workstations have similar capability, while in reality the workstation has different capability because of limitation in term of machines and worker skills. The proposed model is aimed to mathematically represent the SALBP-1 with resource constraints. Besides that, three objective functions which to minimize number of workstation, machine used and number of worker are also presented. The machine considered to have different types of machine needed to produce a product in an assembly line while worker are considered to have different abilities and skills. The model is then illustrated and validated using some examples. The proposed model for SALBP-1 with machine and worker constraints is able to minimize the resources in assembly. Therefore, the assembly cost can be reduced.

1. Introduction
Assembly line balancing is one of important activities prior to the mass production of a product. It will determine the line efficiency, productivity and as well as the assembly cost. Assembly line consists of workstations and as well as one or more dedicated machines/tools together with workers. The process of balancing the assembly task to the workstations with respect to some objective function is known as the assembly line balancing problem (ALBP)[1].

ALBP can be divided into two groups; Simple Assembly Line Balancing Problem (SALBP) and Generalised Assembly Line Balancing Problem (GALBP). SALBP involves production of single product in serial line layout on one sided workstations and GALBP consider different objectives with different assembly line such as mixed model assembly line, parallel, U-shaped and two sided lines [2]. SALBP can be categorized into different categories based on the objective function as in Table 1 [3].

The SALBP addresses the assignment of the tasks to the stations with one or more optimization criteria while satisfying the occurrence constraint, precedence constraint and cycle time constraint and other constraint which may be put into consideration such as resource constraint [4]. The issue of line balancing with the minimum or limited number of resources (machines and workers) has always been a serious problem in industry. Equipment and workers should assign to task and workstations so that maximum efficiency; maximum usage of resources and minimum number of workstations of production line can be achieved.
Table 1. Classification of SALBP

| Type    | Given                     | Objective                           |
|---------|---------------------------|-------------------------------------|
| SALBP-1 | Cycle Time                | Minimise no. of workstation         |
| SALBP-2 | Number of workstation     | Minimise cycle time                 |
| SALBP-E | -                         | Maximise line efficiency            |
| SALBP-F | Cycle Time & Number of workstation | Obtain feasible balance              |

The study on assembly line with resource constraint has been analyzed and published by several numbers of researchers. In 2005, a study on resource constrained assembly line balancing (RCALB) problem with objective function to minimize number of resources used was introduced by K. Agpak with using 0-1 integer-programming models. Resources considered in this paper is a general resources either machine or workforce [5]. Meanwhile, in 2009 I. Moon study on worker skill constraint for integrated assembly line balancing using a mixed integer linear program with a genetic algorithm in order to minimize the annual cost and cost for workers [6]. O. Mutlu in 2013, discussed on worker assignment of assembly line problem type-II and using iterative genetic algorithm (IGA) to assign worker to workstation and at the same time optimize the assembly line [7]. Machine assignment for SALBP-2 have been discussed by H. Triki (2014) where Hybrid Multi Objective Genetic Algorithm (HMOGA) is used to obtained minimum cycle time and cost [8].

In 2014, S. Jayaswal conducted a research on assign tasks to workstations, and resources (equipments and assistants) to tasks with the objectives function is to minimized total cost of workstation and resource utilization. This research is modelled to a U-shaped assembly line balancing using Simulated Annealing [9]. M. Mura and G. Dini (2016) proposes a multi-objective optimization with aim to minimize number of workstation and also number of skilled worker and equipment used in the SALBP-1 by using a Genetic Algorithm (GA) approach [10].

This paper aims to propose a model for understanding and analysing of SALBP-1 with multi-resource constraints so as to minimize number of workstation, number of machines used and also number of workers.

2. Modelling Problem of SALBP-1

Simple assembly line balancing problem is simple assembly line with a number of task are carried in designated workstation. SALBP produce a single product in a serial workstation with a fixed cycle time as illustrated in Figure 1.
2.1 Precedence relation
The precedence graph of SALBP-1 with 9 tasks is presented in Figure 2. The arrows linked between task shows that a task can be performed only after its predecessor tasks are performed.

![Figure 2. Precedence graph of 9 task example problem](image)

2.2 Data presentation
Table 2 presents the precedence matrix for the above precedence diagram in a matrix form. In this matrix, when task $k$ have precedence relation with task $j$, ‘1’ will be put into the matrix, otherwise it will be ‘0’.

| $k$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|-----|----|----|----|----|----|----|----|----|----|
| 1   | 0  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  |
| 2   | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  |
| 3   | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  |
| 4   | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  |
| 5   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  |
| 6   | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  |
| 7   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  |
| 8   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  |
| 9   | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |

Table 3 meanwhile shows the data matrix for the assembly line with resource constraints which include the task time, types of machine required and also worker skill. The worker column with tick mark shows that the worker is able to conduct the task.

2.3 Problem description
In this paper, several assumption has been made in order to describe the problem and evaluate SALBP-1 with resource constraint. The assumptions are as follows:

a) The precedence relationships are known.

b) Task can be assigned to workstation without violating the precedence relation.
c) A worker can be assigned to a task depending upon skills.
d) A worker can only be assigned to one workstation.
e) Machine required to process one task may be more than one type.
f) Tasks using same type of machine can share the same machine.
g) The line is balanced for a single product.
h) The processing time is deterministic.

Table 3. Data Matrix

| Task | Task Time (min) | Machine | Worker |
|------|----------------|---------|--------|
|      |                |         | 1 2 3 4 5 6 7 8 9 |
| 1    | 6              | A, B    | / / / / / / / / |
| 2    | 2              | -       | / / / / / / / / |
| 3    | 5              | A       | / / / / / / / / |
| 4    | 7              | A, B, C | / / / / / / / / |
| 5    | 1              | A       | / / / / A / / |
| 6    | 2              | B       | / / / / / / / / |
| 7    | 3              | A       | / / / / / / / / |
| 8    | 6              | A, C    | / / / / / / / / |
| 9    | 5              | C       | / / / / / / / / |

2.4 Mathematical Equation

The notations considered in the mathematical model are clarified as follows:

\( s \) \quad \text{workstation,} \quad s = 1, 2, \ldots, n

\( m \) \quad \text{machine,} \quad m = 1, 2, \ldots, r

\( w \) \quad \text{worker,} \quad w = 1, 2, \ldots, h

\( k \) \quad \text{task,} \quad k = 1, 2, \ldots, j

\( C \) \quad \text{cycle time}

\( A_s \) \quad \text{number of workstation}

\( B_m \) \quad \text{number of machine}

\( x_{ks} \) \quad 1 \text{ if task } k \text{ is assigned to workstation } s \quad 0 \text{ otherwise}

\( y_{ms} \) \quad 1 \text{ if machine } m \text{ is allocated to workstation } s \quad 0 \text{ otherwise}

\( z_{ws} \) \quad 1 \text{ if worker } w \text{ is applied to workstation } s \quad 0 \text{ otherwise}

\( t_k \) \quad \text{time required by task } k

\( E_a \) \quad \text{earliest task in precedence relation}

\( L_a \) \quad \text{latest task in precedence relation}

\( x_{ias} \) \quad 1 \text{ if earliest task } i \text{ is assigned to workstation } s \quad 0 \text{ otherwise}
The mathematical equation of SALBP-1 with resource constraints is presented in below equations.

\[ f_1 = \sum_{k=1}^{j} t_k \]  
(1)

The first objective function (1) is to minimize the number of workstations for a given cycle time which subject to certain constraint as in equation (2) till equation (5) as follows:

\[ \sum_{s=E_k}^{l_k} x_{ks} = 1 \]  
(2)

\[ \sum_{k \in F_s} t_k x_{ks} \leq C \]  
(3)

\[ \sum_{s=E_a}^{l_a} s x_{as} \leq \sum_{s=E_b}^{l_b} s x_{bs} \quad \text{for } \forall \ (a, b) \in P \]  
(4)

\[ \sum_{k \in F_s} x_{ks} \leq \|F_s\| A_s \]  
(5)

Constraint (2) is an assignment constraint, which ensures that each task is assigned only once. Constraint (3) is a cycle time constraint, which ensure that the total times in each workstation does not exceed the given cycle time. Constraint (4) is a precedence relation constraint, which guarantees that precedence relation among tasks is not violated. Constraint (5) is a workstation constraint, which guarantees that a workstation is utilized if the task(s) is/are assigned to it.

The second objective function (6) is to minimize the number of machines used which subject to certain constraint as in equation (7).

\[ f_2 = \sum_{m=1}^{r} y_{ms} \]  
(6)

\[ \sum_{m=1}^{r} y_{ms} \leq B_m \]  
(7)

Constraint (7) is a resource availability constraint, which ensure that the total number of resources in workstation is not more than the number of available machines.
The third objective function (8) is to minimize the total number of workers used in an assembly line.

\[ f_3 = \sum_{w=1}^{n} z_{ws} \quad (8) \]

\[ \sum_{w=1}^{n} z_{ws} \leq 1 \quad (9) \]

Constraint (9) is to restrict only one worker to be assigned to exactly one workstation depending upon his/her skills.

2.5 Fitness Function

Optimal solutions of the first, second and third objectives are obtained by solving each objective function separately using the above mathematical equation. Then, a single objective is employed to minimize the summation of normalized differences between each objective and its optimal value by determining the fitness function.

\[ F = w_1 f_1 + w_2 f_2 + w_3 f_3 \quad (10) \]

Where;

\[ w_1 = w_2 = w_3 = 0.33 \]

\( w_i \) represent the weights of objectives and \( f_1, f_2 \) and \( f_3 \) represent respectively the normalized values derived from the equations.

\[ \frac{\bar{f}_i - \bar{f}_{i\text{ min}}}{\bar{f}_{i\text{ max}} - \bar{f}_{i\text{ min}}} = \frac{x - x_{\text{ min}}}{x_{\text{ max}} - x_{\text{ min}}} \quad (11) \]

3. Numerical Example

A numerical example is used to explain the objective functions in section 2. In this example, a feasible sequence, \( \text{seq} \ [1, 2, 6, 8, 3, 4, 5, 7, 9] \) are considered in distributing task to workstation. In evaluating the optimisation objective, the assembly tasks are assigned to workstations with the \( \text{ct}_{\text{ max}} \) constraint where the total processing time for tasks assigned to the workstation must not exceed the cycle time, set as \( \text{C}=10 \). The example of the assignment of assembly task for this sequence is presented in Figure 3.
The task distribution is considered by distributing each task to any workstation of the assembly line, as long as the machine and worker assigned to that station can perform the task and the precedence ratios are obeyed.

### 3.1 Problem Evaluation

Based on the precedence graph in Figure 2, a feasible assembly sequence is established to evaluate the optimization objective. As an example, a valid feasible assembly sequences ($seq_1$), $seq_1[1, 2, 6, 8, 3, 4, 7, 5, 9]$ as in Figure 3 and two other sequence $seq_2[1, 5, 3, 4, 7, 2, 6, 8, 9]$ and $seq_3[1, 3, 4, 7, 2, 6, 8, 5, 9]$ are considered. For given, cycle time maximum for this assembly line is to be 10. To assign task to workstation ($w$), the processing time ($p_t$) which is the sum of task time ($t_t$) for each workstation must not exceed 10. Resources; machines ($m$) and workers ($w$) are then assigned to the task base on the compatibility by following the data given in Table 2. The results of assembly tasks assignment for $seq_1$, $seq_2$ and $seq_3$ are presented in Table 4, 5 and 6.

#### Table 4. Assembly Task Assignment for $seq_1$

| $seq_1$ | 1  | 2  | 6  | 8  | 3  | 4  | 7  | 5  | 9  |
|---------|----|----|----|----|----|----|----|----|----|
| $t_t$   | 6  | 2  | 2  | 6  | 5  | 7  | 3  | 1  | 5  |
| $p_t$   | 10 | 6  | 5  | 10 | 3  | 1  | 5  |    |    |
| $s$     | 1  | 2  | 3  | 4  | 5  |    |    |    |    |
| $m$     | A, B | A, C | A | A, B, C | A, C |    |    |    |    |
| $w$     | 5, 2 | 4  | 3  | 8  | 1  |    |    |    |    |

#### Table 5. Assembly Task Assignment for $seq_2$

| $seq_2$ | 1  | 5  | 3  | 4  | 7  | 2  | 6  | 8  | 9  |
|---------|----|----|----|----|----|----|----|----|----|
| $t_t$   | 6  | 1  | 5  | 7  | 3  | 2  | 2  | 6  | 5  |
| $p_t$   | 7  | 5  | 10 | 10 | 2  | 2  | 10 | 5  |    |
| $s$     | 1  | 2  | 3  | 4  | 5  |    |    |    |    |
| $m$     | A, B | A  | A, B, C | A, B, C | C  |    |    |    |    |
| $w$     | 9  | 3  | 8  | 2  | 1  |    |    |    |    |
Table 6. Assembly Task Assignment for seq3

| seq3 | 1   | 3   | 4   | 7   | 2   | 6   | 8   | 5   | 9   |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( t_i \) | 6   | 5   | 7   | 3   | 2   | 2   | 6   | 1   | 5   |
| \( p_i \) | 6   | 5   | 10  | 6   | 10  | 6   |     |     |     |
| \( s \) | 1   | 2   | 3   | 4   | 2   | 4   | 5   |     |     |
| \( m \) | A, B| A   | A, B, C | A, B, C | A, B, C | A, C |     |     |     |
| \( w \) | 9   | 7   | 8   | 2   | 2   | 1   |     |     |     |

Number of workstation: The number of workstations required when assigning task to workstation for seq1, seq2 and seq3 are both 5 number of workstations.

Machines: Total 10 units of machine; 5A, 2B and 3C required for assembly task assignment seq1 and 4A, 3B and 3C for seq2 and 11 units of machine for seq3; 5A, 3B and 3C.

Workers: The number of workers required to perform the assembly task assignment seq1 is 6 while only 5 workers need to be allocate in seq2 and seq3.

3.2 Minimize number of workstation
The first objective function is to minimize number of workstation subjected to the equation (1).

\[
f_1 = \sum_{s=1}^{n} A_s
\]

\[
f_{1 \text{ min}} = \frac{\sum t_i}{C_{t_{\text{max}}}} = \frac{37}{10} = 3.7 \sim 4
\]

\[
f_{1 \text{ max}} = \frac{\sum t_i}{\text{max}(t_i)} = \frac{37}{7} = 5.3 \sim 6
\]

3.3 Minimize number of machine
The second objective function that subject to equation (6) are as follows.

\[
f_2 = \sum_{m=1}^{r} y_{ms}
\]

\[
f_{2 \text{ min}} = \text{total machine type} = 3
\]

\[
f_{2 \text{ max}} = f_{1 \text{ max}} (\text{total machine type}) = 6(3) = 18
\]
3.4 Minimize number of worker

\[ f_3 = \sum_{w=1}^{h} z_{ws} \]

\[ f_{3\text{ min}} = f_{1\text{ min}} = 4 \]

\[ f_{3\text{ max}} = \text{number of task} = 9 \]

3.5 Fitness value

Based on the above solution for sequence \( seq_1 \), \( seq_2 \) and \( seq_3 \), the fitness function for both is calculated by using equation 10 and 11 and the result obtained presented in Table 6.

| Sequence | Workstation | Machine | Worker | Fitness value |
|----------|-------------|---------|--------|---------------|
| \( seq_1 \) | 5           | 10      | 6      | 4.62          |
| \( seq_2 \) | 5           | 10      | 5      | 3.96          |
| \( seq_3 \) | 5           | 11      | 5      | 4.29          |

Based on the results in Table 6, all feasible sequences tested shows the same number of workstation. Feasible sequence \( seq_1 \) and \( seq_2 \) have the same number of machine used. However \( seq_2 \) have less number of worker assigned to workstation compare to \( seq_1 \). Meanwhile feasible sequence \( seq_3 \) have the highest number of machine used compare to \( seq_1 \) and \( seq_2 \) but less worker assigned to workstation compare to \( seq_1 \). \( seq_2 \) give the best fitness value followed by \( seq_3 \) and \( seq_1 \).

The model presented in this paper shows that the model is valid in presenting simple assembly line balancing type 1 (SALBP-1) with resource constraint; machine and worker. This paper considers different types of machine used in an assembly line as well as multi skilled worker which can be assigned to workstation. The number of machines and workers used in SALBP-1 can be optimize as suggested and tested in the proposed model.

4. Conclusion

In this paper, Simple Assembly Line Balancing Problem Type 1 (SALBP-1) with resource constraint was modelled and discussed. The model is design by considering three objective functions which is the minimization number of workstation, number of machines used and number of multi-skilled workers. The model is explained in details and evaluation on the model is briefly presented with a particular equation and calculation. The obtained results have described the SALBP-1 with resource constraint features. From the results of the calculation, it shows that this tested model for SALBP-1 with resource constraint could have better performance in optimization.
For future work, this proposed model of SALBP-1 with resource constraint might be extend for future research by tested using algorithm by referring to the solution procedure. The case study on real industrial problem on SALBP-1 with machine and worker constraint might be conducted to validate the propose model.

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