Automation engineering of adaptive industrial warehouse

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Abstract. The modern automatic warehouse of industrial production is equipped with sensors to determine the type, size, weight, expire date of each SKU. There are also robots for moving SKUs, control algorithms, and an interface for interacting with external services. When developing such a cyber physical system, it is necessary to optimize its main parameters. In engineering, only 2-3% of the whole time is spent on the processing of parts. The substantial time is spent on interoperational storage and transportation. The productivity of the automatic warehouse determines the economic performance of the enterprise as a whole. In addition, the assortment and emergent stock of products stored in the warehouse should ensure stable work under conditions of a change in the production program. This paper is devoted to the algorithm that underlies the digital interaction of an automatic warehouse as a cyber-physical system.

1. Introduction

The implementation of the interactive digital technologies provided the wide employment of the flexible manufacturing systems (FMS). It allows to generalize their component elements to shape their structure [1], in which the automatic warehouse (AW) is one of the most important elements greatly determining the indices of the entire production. According to the information furnished by the manufacturing companies, an average factory uses up to 40% of industrial areas for warehousing facilities at high cost, while their volumetric space is used very little [2]. Besides, the actual time of treatment of the articles makes up some 3% and less, whence it follows that the tempo and character of the AW operation set limits on the productivity of the FMS, output capacity and diversity of nomenclature, which influence the degree of flexibility of the whole production and the basic technical-and-economical characteristics.

While the problem of raising the flexibility of the basic equipment of the FMS is solved as a whole by both the development of the software and great diversity of the treating tools and fixtures, the automatic warehouses at present are busy mainly with perfecting algorithms of operation of the robot-stackers which has a certain limit conditioned by the rigidly determined structure of the great majority [3] of AW constructions.
Since all goods (SKU - stock keeping unit) arriving at AW are digitally labeled (barcode, QR codes, RFID), which allowed machine-to-machine interaction (M2M), information about dimensions and weight of goods is available to the warehouse management server.

In order to increase the degree of flexibility [4] of automatic warehouses and make it more relevant to other elements of FMS, it is necessary to ensure the possibility of flexible rearrangement of its design to suite the parameters of goods to be stored. The research and analysis of contemporary warehouses prove that at present, taking into consideration the technical capabilities, the most rational solution is the realization of automatic warehouses with cells of rearrangeable dimensions. For efficient development of such systems, it is necessary to elaborate methods of calculation of their basic indices [5] with due regard to main peculiarities of operation as a component of FMS [6,7].

2. The main formalisms
Automatic warehouses designed and operated at present represent a system of multi-cell racks served by a robot-stacker, the dimensions of the cells being determined during designing and not changed after construction of the warehouse. The volume $V$ of goods kept in the warehouse is variable and the limits of the range of its change are conditioned by the capacities of the equipment used in FMS and auxiliary facilities (transport, robots and the like), and are known beforehand, which can be expressed by the relation $V_{\text{min}} < V < V_{\text{max}}$ (figure 1a and b). Besides, the volumes of goods in the flow of supply to such warehouses are very stochastic values, which can be characterized by the density of the distribution which is also influenced [8] by such basic factors as the orientation of the production, the equipment used and also the production program, the control executed over the process of the production program selection being that particular lever you can use to influence the indices of the FMS as a whole and the operation of the automatic warehouse in particular.

The described model of a rigid warehouse will serve as a starting point for the analysis. It is evident that in such a warehouse the volume of cells [9] corresponds to the maximum warehousing of goods $V_{\text{max}}$, besides, due to elaboration of optimum control algorithms [10], several units of goods of smaller volume can be warehoused in one cell.

3. Calculation
Let $n$ - number of unitary goods in a cell, versus volume $V$ (characteristic $D_1$); another characteristic $D_2$ corresponding to the ideal variant of arrangement, featuring multiple volumes of goods in warehousing, while any characteristic of arrangement in a rigid warehouse will be always below $D_2$. For estimation of indices of an automatic warehouse the data on distribution of volumes in the goods flow were used.

![Figure 1. The relation $V_{\text{min}} < V < V_{\text{max}}$.](image)
4. Mathematical model.

Assuming, that \( V_1 = V_{\text{max}} \), \( V_2...V_p = V_{\text{min}} \) are steps on the characteristic curve of arrangement \( D_1 \), then \( C_N \), the number of cells required for arrangement of \( N \) units of goods will be calculated [11] from the formula: 
\[
C_N = \sum_{i=1}^{p-1} \text{enter} \left( \frac{N}{i} \int_{V_{i+1}}^{V_i} P_j(V)dV \right) .
\]
For optimum arrangement of goods there are the values:
\[
C_N = \text{enter} \left( \frac{N}{n} \int_{V_{\text{min}}}^{V_{\text{max}}} P_j(V)dV \right) + \sum_{i=1}^{p-1} \text{enter} \left( \frac{N}{i} \int_{V_{i+1}}^{V_i} P_j(V)dV \right). \]

It is evident that \( C_N \geq \overline{C}_N \). On the basis of the values obtained one can determine the most important parameter of the warehouse, that is the volume utilization factor, which is numerically equal to the value of relation of the busy volume of cells, where the goods [12] are warehoused, to their total volume, and designated as \( K_v \). Taking it into account, one has for the rigid warehousing (index “r”): 
\[
V_{\text{max}}K_v^{(r)} = V_N / C_N V_{\text{max}},
\]
where \( V_N = N \int_{V_{\text{min}}}^{V_{\text{max}}} P_j(V)dV \) is the natural value of \( N \) goods in the warehouse, while in estimating the value \( K_v^{(r)} \) it is necessary to use the relation: 
\[
K_v^{(r)} \leq \overline{K}_v^{(r)},
\]
where \( \overline{K}_v^{(r)} = V_N / \overline{C}_N V_{\text{max}} \). It should be noted that \( K_v \) depends on \( P_j(V), V_{\text{max}}, V_{\text{max}}, n^* \), besides, it depends to a certain extent on \( N \), however with great values of \( N \) its influence is so insignificant, that it can be neglected in most cases. Further, with the normalized distribution [13] commonly used, the influence is exerted only by the value of the relation \( q = V_{\text{max}} / V_{\text{min}} \).

For quantitative description of discreteness and the degree of rearrangement and hence the character of the construction solution of the warehouse, here goes the introduction of the value \( m \)-index of discreteness of a flexible rearrangeable automatic warehouse which is numerically equal to the number of possible rearrangements depending on the inner size of a single cell of a rigid warehouse, while to simplify the calculation, there takes a place an assumption that the rearrangement is effected in equal steps. For calculating the volume of the warehouse required for arrangement of \( N \) units of goods in a flexible warehouse, it is necessary to use the same initial data of the goods flow, as for the calculation of the arrangement in a rigid warehouse. Since the discreteness index fully characterizes the conversion [14] to a flexible warehouse, the result is the following calculation formula of \( V_N^{(m)} \):
\[
V_N^{(m)} = N \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{1}{m} \int_{V_{j-1}}^{V_j} P_j(V)dV .
\]
It should be noted here, that it always holds true:
\[
V_N^{(m)} \geq V_N^{(m)},
\]
with \( m_i < m_j \), and with the increase [15] of the discreteness index \( m \) there is the following limit relation (which is also evident from the logical consideration):
\[
V_N^{(\infty)} = \lim_{m \to \infty} V_N^{(m)} = N \int_{V_{\text{min}}}^{V_{\text{max}}} P_j(V)dV .
\]

That is by determining the volume utilization factor in the absolutely flexible warehouse \( K_v = 1 \), whence an important relation for numerical evaluation [16] is obtained: \( V_N^{(m)} \geq V_N^{(\infty)} \).

Further the volume utilization factor should be determined. Another basic parameter is the weight of goods. For calculation [17] of the said limitation an additional function \( P_j(\gamma, V) \) is introduced, describing the distribution of density of probability of the specific volume weight against the volume of goods. In this case, there can be written the following equation [18] for the amount of the goods in warehousing:
\[ N \leq N_{\text{max}} = M \int_{V_{\text{max}}}^{V_{\text{min}}} \int_{V_{\text{max}}}^{V_{\text{min}}} V P_\gamma(V)P_\gamma(V)d\gamma dV, \]

where \( N_{\text{max}} \) is the maximum possible amount of the goods in warehousing, \( \gamma_{\text{min}}, \gamma_{\text{max}} \) are change limits of the specific weight by volume of goods. In this way the limitation of the value of the utilization factor with full loading of an AW is determined by the following inequality: \( K_v \leq \min(1, \overline{K}_v) \), where \( \overline{K}_v = N_{\text{max}} \int_{V_{\text{max}}}^{V_{\text{min}}} V P_\gamma(V)dV / V_0 \); \( V_0 \) is the total volume of the warehouse. Now there is total data [19] required for the calculation of the value \( \Delta K_v \), that is the increment of the utilization factor of the AW volume, when changing over to the flexible organization of its structure [20] with rearrangeable dimensions [21] of the cells:

\[
\Delta K_v = \int_{V_{\text{max}}}^{V_{\text{min}}} V P_\gamma(V)dV \left[ N \sum_{i=1}^{n} \sum_{i=1}^{n'} \int_{V_{\text{max}}}^{V_{\text{min}}} V P_\gamma(V)dV \right] - \\
- V_{\text{max}} \text{entier} \left( N \int_{V_{\text{max}}}^{V_{\text{min}}} P_\gamma(V)dV \right) + \sum_{i=1}^{n} \text{entier} \left( N \int_{V_{\text{max}}}^{V_{\text{min}}} P_\gamma(V)dV \right) \right]^{11} \cdot 100\%. \]

That means, that the lower boundary is known, which even with incomplete information always allows for a preliminary evaluation of the benefit of the rearrangement. The value \( \Delta K_v \) is the function of the law of distribution of volumes, discreteness index and characteristics of the load carrying capacity. In spite of the apparent complexity of the proposed equation, the results of the calculation with this formula can be represented rather clearly.

5. Conclusion

Well, the above calculations allow to determine the change of the basic AW parameters after conversion to their organization with rearrangeable dimensions of the cells. The results obtained concern only the functioning of the warehousing zone proper and can be represented as algorithms combining concrete characteristics of goods flows (including quantitative and qualitative indices of the goods), structural characteristics of AW, as well as the data of the robot-stacker, but only those, which were necessary during the calculations and taken in the most generalized form. For development of a flexible automatic warehousing system, it is necessary to determine the interrelation between the said two groups of parameters, with addition of dynamical and structural characteristics of the robot-stacker, which will enable their tying up together as a unit. This is necessary for the reason that in a FMS interaction with an AW occurs practically always through the robot-stacker, which therefore is an important link in the production chain, and in our case its development should be performed together with development of the design of the warehouse with rearrangeable dimensions of the cells.

References

[1] Provotorov V V, Sergeev S M and Part A A 2019 Solvability of hyperbolic systems with distributed parameters on the graph in the weak formulation Vestnik of Saint Petersburg University Applied Mathematics Computer Science Control Processes \textbf{14}(1) 107–17
[2] Provotorov V V 2015 Boundary control of a parabolic system with distributed parameters on a graph in the class of summable functions Automation and Remote Control \textbf{76}(2) 318-22
[3] Borisoglebskaya L N and Sergeev S M 2017 Model of assessment of the degree of interest in business interaction with the university Journal of Applied Economic Sciences \textbf{XII 8}(54) 2423-48
[4] Podvalny S L and Provotorov V V 2015 The questions of controllability of a parabolic systems
with distributed parameters on the graph International Conference "Stability and Control Processes" in Memory of V.I. Zubov (SCP) 117-9

[5] Iliashenko O, Krasnov S and Sergeev S 2018 Calculation of high-rise construction limitations for non-resident housing fund in megacities E3S Web of Conferences High-Rise Construction 2017 03006.

[6] Kravets O J, Barkalov S A, Butyrina N A, Sekerin V D and Gorokhova A E 2018 Processes of multidimensional classification of scoring objects with heterogeneous features based on the neural networks modeling International Journal of Pure and Applied Mathematics 119(7a) 875-9

[7] Kamachkin A M and Yevstafyeva V V 2000 Oscillations in a relay control system at an external disturbance Control Applications of Optimization 2000 Proceedings of the 11th IFAC Workshop 2 459-62

[8] Potapov D K 2009 Optimal control of higher order elliptic distributed systems with a spectral parameter and discontinuous nonlinearity Journal of Computer and System Sciences International 52(2) 180-5

[9] Sergeev S, Kirillova T and Krasyuk I 2019 Modelling of sustainable development of megacities under limited resources. TPACEE-2018 E3S Web of Conferences 91 05007

[10] Aleksandrov A, Aleksandrova E and Zhabko A 2014 Asymptotic stability conditions for certain classes of mechanical systems with time delay WSEAS Transactions on Systems and Control 9 388-97

[11] Provotorov V V, Ryazhskikh V I and Gnilitskaya Yu A 2017 Unique weak solvability of a nonlinear initial boundary value problem with distributed parameters in a netlike region Vestnik of Saint Peterburg University Applied Mathematics Computer Science Control Processes 13(3) 264-77

[12] Borisoglebskaya L N, Provotorov V V, Sergeev S M and Kosinov E S 2019 Mathematical aspects of optimal control transference processes in spatial networks IOP Conf. Ser.: Mater. Sci. Eng 537 042025

[13] Provotorov V V 2008 Eigenfunctions of the Sturm-Liouville problem on astar graph Shornik: Mathematics 199(10) 1523-45

[14] Krasyuk I A, Bakharev V V, Kozlova N A and Mirzoeva D D 2018 Staffing in the sphere of trade: the main issues and prospects of solution Proceedings of 2017 IEEE 6th Forum Strategic Partnership of Universities and Enterprises of Hi-Tech Branches (Science Education Innovations) 6 48-50

[15] Aleksandrov A and Zhabko A 2014 Delay-independent stability of homogeneous systems Applied Mathematics Letters 34(1) 43-50

[16] Aleksandrov A, Zhabko A. and Hu G-D 2014 Delay-independent stability conditions for some classes of nonlinear systems IEEE Transactions on Automatic Control 59(8) 2209-14

[17] Provotorov V V 2015 Boundary control of a parabolic system with delay and distributed parameters on the graph International Conference "Stability and Control Processes" in Memory of V. I. Zubov (SCP) 126-8

[18] Podvalny S L, Provotorov V V and Podvalny E S 2017 The controllability of parabolic systems with delay and distributed parameters on the graph Procedia Computer Science 12 324-30

[19] Kapustina I V, Kirillova T V, Ilyina O V, Razzhivin O A and Smelov P A 2017 Features of Economic Costs of Trading Enterprise: Theory and Practice International Journal of Applied Business and Economic Research 15(11) 1-10

[20] Krasyuk I, Kirillova T, Bakharev V and Lyamin B 2019 Life cycle management in network retail enterprise based on introduction of innovations IOP Conference Series: Materials Science and Engineering 491 012125

[21] Grishchenko O V, Kireev V S, Dubrova L I, Yanenko M B and Vakulenko R Y 2016 Organization, planning and control of marketing logistics International Journal of Economics and Financial Issues 6(8) 166-72