A family of braneworld wormholes

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A wormhole solution is presented on a braneworld scenery, the material source is given by a perfect fluid on brane that does not violate the energy conditions. We propose an ansatz for the state equation resulting in a family of solutions that satisfy necessary conditions over the free parameters of the solution. The effect of projecting the 5D Weyl tensor components on the brane, gives the conditions to have a wormhole satisfying the NEC, besides simultaneously guaranteeing the absence of horizons. Furthermore, our model predicts an astronomical bound for the wormhole throat.

PACS numbers:

I. INTRODUCTION

Space-time’s represented of wormholes are associated with matter fields that violate the null energy condition \((c^2ρ + P < 0)\) or as well as the violation of the null energy condition by the effective stress–energy tensor \(\rho\). This due to the existence of a minimal area region known like throat that connect two spacetime regions, on static case the throat is a stable minimum close surface embedded in the orthogonal hypersurface to the temporal Killing vector field connecting two space time regions. Wormholes in the beginning are the result of a theoretical approach that has attracted the attention of researches given their attractive properties of connecting two regions of space-time and its possible use for time travel.

Although a numerical analysis suggest that quantum effects may be sufficient to keep the throat of a wormhole. The possible existence is sustained due to the observational results of the accelerating universe \(1,11\), which implies the existence of dark energy, which violates the strong energy condition \((c^2ρ + 3P) < 0\), an example of this is the called ghost energy that have the state equation \(P = -w c^2 ρ, w > 1\). For this state equation also we have the violation of the null energy condition \((c^2ρ + P) < 0\). Diversity of wormholes have been presented, in some cases based on the standard gravitational theory of Einstein and assuming a fluid, some people have built various asymptotically flat models, or by the cutting and paste technique having a layer associated with the wormhole \(13,14\), with matter sources and some scalar field \(15\) or simply with scalar fields \((\text{ghost fields}) \ 16,17\). In other cases it is considered the construction of wormholes in different gravitational theories to Einstein gravitational theory, like \(f(R)\) theory \(18\) or the Finsler geometry \(19,20\).

One of the themes that has been developed in different researches, is the construction of wormholes with matter sources that does not violate the null energy condition, but as we know the effective stress–energy tensor in fact violate the null energy condition \(5\). In the \(f(R)\) theory, have been built wormholes in which the fluid considered, anisotropic, does not violate the energy conditions WEC and NEC, this is possible due the election of \(f(R)\) and the state equation \(18\) or the Einstein–Cartan theory \(21\) between others.

An alternative option to formulate models made by the standard GR are the braneworlds \(22\), which emerge as an alternative geometric solution to the problem of mass hierarchy \(23\). The braneworlds can be defined as 3+1 hypersurfaces called branes where all the matter fields are confined except gravity, which is free to propagate in the 5D bulk. The physics our universe can be recovered from the braneworld, making a dimensional reduction of the effective 5D theory to the 4D, as shown in \(24\). the 4D gravitational field equations on the brane contain corrections to the GR, and therefore worth exploring new wormhole solutions \(25,26\) and study the phenomenology that the braneworlds bring to the problem of exotic matter. The gravitational field equations on the brane are recovered from the 5D warped geometry, which contributes to the 4D theory with an effective stress-energy tensor, in particular for a perfect fluid we can recover the conventional GR, plus corrections given in terms of \(P\) and \(ρ\), that contain the 5D Weyl ten-
II. THE EQUATIONS ON THE BRANE

The components of the Einstein equations in five dimensions are given by [22]

\[ (5) G_{AB} = -\Lambda (5) g_{AB} + \kappa_5^2 (5) T_{AB} \tag{1} \]

in our case we will not consider matter fields on the bulk 

\[ (5) T_{AB} = 0 \]

the field equations projected on brane are given by

\[ (4) G_{\mu\nu} = -\Lambda g_{\mu\nu} + k T_{\mu\nu} + \frac{6k}{\Lambda} S_{\mu\nu} + \frac{6}{k\Lambda} W_{\mu\nu} \tag{2} \]

where

\[ k = \frac{\lambda k_5^2}{6}, \quad \Lambda = \frac{1}{2} (\Lambda_5 + k\lambda) \tag{3} \]

where \( k \) and \( k_5^2 \) are the gravitational coupling constants, \( \Lambda \) and \( \Lambda_5 \) the cosmology constants on brane and the bulk, respectively; \( \lambda \) it is the tension of the brane. \( T_{\mu\nu} \) is the stress-energy tensor confined on the brane. In order to have a realistic braneworld scenario in concordance with RSII, we have fixed \( \Lambda_5 = -k\lambda \). While that

\[ S_{\mu\nu} = \frac{1}{12} TT_{\mu\nu} - \frac{1}{4} T_{\mu\alpha} T_{\alpha\nu} + \frac{1}{8} g_{\mu\nu} \left[ T_{\alpha\beta} T^{\alpha\beta} - \frac{1}{3} T^2 \right] \tag{4} \]

are the first correction to Einstein equations of general relativity. Meanwhile the terms \( \frac{6}{k\Lambda} W_{\mu\nu} = -\delta^\mu_\alpha \delta^\nu_\beta \) \( \left( C_{ABCD} n^B n^D \right) \), are the 5D Weyl tensor projections that correspond to corrections of second order on the braneworld. We assume a stress-energy tensor of a perfect fluid

\[ T_{\mu\nu} = (P + c^2 P) u_\mu u_\nu + P g_{\mu\nu} \tag{5} \]

where the Weyl tensor projections are given by

\[ W_{\mu\nu} = \frac{1}{3} (4U - P) u_\mu u_\nu + P X_{\mu} X_{\nu} + \frac{1}{3} (U - P) g_{\mu\nu} \tag{6} \]

For a static and spherically symmetric space-time we will consider the metric in Schwarzschild coordinates

\[ ds^2 = -g(r) dt^2 - \frac{dr^2}{1 - b(r)/r} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \tag{7} \]

with the range of coordinates \((t, \theta, \phi)\) ass the usual \( r \geq r_0 \), where \( r_0 \) correspond to the throat and

\[ u_\mu = \delta^\mu_\nu \tag{8} \]

Additionally must satisfy \( b(r_0) = r_0 \), \((b - b')r/b^2 > 0 \), or equivalently \( b'(r_0) < 1 \), this implies that the effective stress-energy tensor violates the null energy condition, for values of \( r > r_0 \), \( 1 - b/r \geq 0 \). Furthermore, the absence of horizons is satisfied if \( r(r_0) > 0 \).

The field equations in this coordinates for a perfect fluid have the form:

\[ \frac{b'}{r^2} = k c^2 \rho + \frac{k c^4 \rho^2}{2 \lambda} U, \]

\[ -\frac{b}{r^2} = \left( \frac{b}{r} \right)^2 \frac{2y'}{ry} + kP + \frac{k c^2 \rho (2P + c^2 \rho)}{2 \lambda} + \frac{4P + 2U}{k \lambda}, \]

\[ kP = \frac{4x - (b - b') (y'' + y')}{2 x^3 y} \frac{y r^2}{2 \lambda} - \frac{k c^2 \rho (2P + c^2 \rho)}{2 \lambda} - \frac{2(4P + 2U)}{k \lambda}, \]

\[ -\rho' = \frac{4(2U + P) y'}{k^2 c^2 (P + c^2 \rho) y} \frac{y r^2}{2 \lambda} - \frac{(4P + 2U) r + 12 P}{k^2 c^2 (P + c^2 \rho) \lambda} \tag{11} \]

\[ P' = \frac{(P + c^2 \rho) y'}{y}, \tag{12} \]

where \( ' \) denotes the derivative with respect to the coordinate \( r \). Below in the next section we will build the solution for the wormhole.

III. THE SOLUTION

A. Integrating the equations

Given the complexity to found a general analytic solution to the equations \([5, 9, 10] \) and \([12] \), we have chosen to propose an ansatz for the equation of state as follow

\[ P = \frac{bc^4 \rho^2}{2 \lambda} \left( 1 - \frac{b c^2 \rho}{\lambda} \right)^{-1} \]
likewise

\[ \rho = \frac{\beta r_0^3}{k c^2 r^5}, \quad (14) \]

where \( r_0 \) is the radius of the throat and \( \mu \) is an arbitrary positive constant. To build the solution we proceed as follows: We replace (13) in the conservation equation for the perfect fluid (12) to obtain the following equation

\[ \frac{y'}{y} = \frac{c^2 \mu \rho'}{\mu c^2 \rho - \lambda}, \quad (15) \]

Integrating we obtain the following expression for \( y(r) \)

\[ y = C_1 \left( 1 - \frac{\mu c^2 \rho}{\lambda} \right), \quad (16) \]

where \( C_1 \) is the arbitrary constant of integration, which we will fix as 1 in order to impose an asymptotically flat solution (see Figure 1). Replacing \( \rho \) in the equation (8) we obtain an expression for \( \mathcal{U}(r) \)

\[ \mathcal{U} = \frac{k \beta y}{6 r^2} - \frac{\beta r_0^3 \left( 2 k \lambda r^5 + \beta r_0^3 \right)}{12 r^{10}}. \quad (17) \]

Replacing (13), (16) and (17) in the equation (10) we obtain \( \mathcal{P} \)

\[ \mathcal{P} = \frac{\beta r_0^3 \left( 2 \lambda k^2 r^{10} - \beta r_0^3 \left( 5 \mu + 2 \right) \lambda k r^5 - \mu \beta r_0^3 \right)}{24 r^{10} \left( k \lambda r^5 - \mu \beta r_0^3 \right)} + \frac{5 \left( r - b \right) \lambda \mu \beta r_0^3 k}{2 r^3 \left( k \lambda r^5 - \mu \beta r_0^3 \right)} - \frac{k \lambda \left( b' r + 3 b \right)}{12 r^3} \quad (18) \]

Replacing (13), (16) and (18) (17) in the equation (9) we obtain the next differential equation:

\[ \left( 2 k \lambda r^5 + 3 \mu \beta r_0^3 \right) b' \right) - 45 \beta \mu r_0^3 b r + 5 \beta r_0^3 \left( 40 \mu - k \lambda r^2 \right) + \frac{\beta^2 \beta r_0^3 \left( 5 \mu + 2 \right)}{2 r^3} + \frac{\beta^3 r_0^9 \mu}{2 k \lambda r^8} = 0. \quad (19) \]

Integrating we obtain the expression for \( b(r) \)

\[ b = -\frac{r^{15} C_2}{(2 k \lambda r^5 + 3 \mu \beta r_0^3)^3} + \frac{1}{(2 k \lambda r^5 + 3 \mu \beta r_0^3)^3} - \frac{1}{2 r^{11} \lambda^2 \beta r_0^3 k^2 (k \lambda r^2 - 20 \mu)} + \frac{2 r^6 \lambda \beta^2 r_0^6 k (3 k \lambda (\mu - 2) r^2 - 560 \mu^2)}{21} + \frac{9 \beta^5 r_0^{15} \beta}{44 k r^7 \lambda} + \frac{7 \beta^3 k \lambda \mu r_0^9 \left( 3 \mu + 2 \right) r^3}{15 \beta^3 r_0^3 \beta^2 \mu} + \frac{12}{34 r^2} \left( 3 \mu + 2 \right) \quad (20) \]

In this case the integration constant was fixed imposing the condition \( b(r_0) = r_0 \), for obtain

\[ C_2 = -2 k^3 \lambda^3 \left( \beta + 4 \right) r_0 - \frac{2 \beta k^2 \lambda^2 ((\mu - 2) \beta - 14 \mu)}{7 r_0} + \frac{3 \beta^2 \lambda \mu (7 (3 \mu + 2) \beta - 8 \mu)}{12 r_0^3} + \frac{9 \beta^5 \mu^3}{44 k \lambda r_0^7} \]

\[ + \frac{3 \beta^3 \mu^2 (35 (3 \mu + 2) \beta - 102 \mu)}{238 r_0^5}. \quad (21) \]

With this, we have managed to integrate the system, the expressions for \( \mathcal{U} \) and \( \mathcal{P} \) will remain indicated in terms of \( b \) and \( b' \) because they are a overlong expressions.

![FIG. 1: The shape for \( y(r) \) is traced in natural units, considering \( r_0 = 1, \mu = 1, \lambda = 0.36 \), the blue, orange, green and red lines are plotted with \( \beta = 3, 4, 5, 6 \) respectively.](image)

### B. Analysis of the solution

In order to know the asymptotic behavior of our solution, we will expand \( g^{rr} \) around of \( r = \infty \) as follow

\[ 1 - \frac{b}{r} = 1 + \frac{C_2}{8 k^3 \lambda r^3} + \frac{\beta r_0^3}{4 r^3} - \frac{5 \mu \beta r_0^3}{k \lambda r^5} - O \left( r^{-6} \right) \quad (22) \]

Together with the previous expansion the expression (16) show that the obtained solution is asymptotically flat when \( r \rightarrow \infty \) (see Figure 2). Near of or in the throat \( b = r \) we require that our wormhole is connected with an asymptotically flat surface, this should be satisfied for the flare-out condition, which implies that \( b' < 1 \), clearing \( b' \) from (19) we obtain

\[ b' = \frac{2 k^2 \lambda^2 r_0^4 - k \lambda (5 \mu \beta + 2 \beta - 10 \mu) r_0^2 - \beta^2 \mu)}{2 r_0^2 k \lambda (2 r_0^2 k \lambda + 3 \mu \beta)} \quad (23) \]

Applying the flare–out condition to previous expression, and after some algebra we obtain the following inequality

\[ - \beta \left( r_0^2 k \lambda (5 \beta - 4) + \beta^2 \mu \right) - 2 k \lambda r_0^2 \left( \beta^2 - r_0^2 k \lambda (\beta - 2) \right) < 0. \quad (24) \]
the density $\rho$ introduce the following redefinition parameters in our solution. For our convenience we will existence of horizons, reveals possible bounds on the parameters in our solution. For our convenience we will introduce the following redefinition $x = r_0^2 k \lambda$. From the density $\rho$ in [13], we conclude that $\beta > 0$, together with the condition of absence of horizons near the throat $y(r_0) > 0$ we obtain the following restriction
\[
\frac{\beta \mu}{x} < 1. \quad (25)
\]
Under the previous redefinition we seek to satisfy conveniently the flare-out condition, also taking into account the absence of horizons as shown below
\[
\frac{2 x (\beta - 2) - 2 \beta^2}{x (5 \beta - 4) + \beta^2} < \frac{\beta \mu}{x}. \quad (26)
\]
In order to use a realistic estimates for the parameter $\lambda > 0$ we must demand that $\mu > 0$, to do this we must also impose the following restriction on the left side of [26]
\[
\frac{2 x (\beta - 2) - 2 \beta^2}{x (5 \beta - 4) + \beta^2} > 0. \quad (27)
\]
To satisfy inequality [27] is necessary impose $x > \frac{\beta^2}{\beta - 2}$, from this restriction is evident that $\beta > 2$ and considering previous restrictions, we can establish directly from [26] and [20] the valid range for the parameter $\mu$ as follow
\[
x \left( \frac{1}{\beta} - \frac{3 (\beta + x)}{\beta^2 + (5 \beta - 4) x} \right) < \mu < \frac{x}{\beta}. \quad (28)
\]
It is noted that, the absence of additional zeros in the function $g''$, should be done by a fine-tuning on the constant $\mu$, because an extra condition for the constant $\mu$ taking into account the regularity of $g''$ is difficult to obtain. Upon returning to the original variable, we can set a lower bound for the size of the throat of wormhole in terms of the brane tension and the parameter $\beta$, as shown below
\[
r_0 > \frac{\beta}{\sqrt{\beta - 2) k \lambda}}. \quad (29)
\]
Although our model can be considered a toy model, we can estimate in a first approximation the smaller radius $r_0$ permitted by our model, considering the lower bound for $\lambda > 0$, restricts the radial pressure of the perfect fluid to be define positive for $r \geq r_0$ as shown in the following expression
\[
P(r) = \frac{\mu \beta^2 r_0^6}{2 k^2 r^{10} \left( \lambda - \frac{\beta \mu r_0^3}{k r^5} \right)}. \quad (30)
\]
Contributions made by the 5D Weyl tensor projections to the brane decay rapidly to zero, to ensure that the anisotropic fluid generated by the wormhole decreases rapidly away from the throat as $r \to \infty$; below is shown the asymptotic form for $U$ and $P$
\[
U = -\frac{\beta r_0^3 k \lambda}{12 r^3} - \frac{10 \beta \mu r_9^3}{3 r^7} - \frac{15 \beta r_0^3 C_2 \mu}{32 k^3 \lambda^2 r_5^6} + O(r^{-10}), \quad (31)
\]
\[
P = \frac{C_2}{32 k^3 \lambda^2 r_3^6} + \frac{5 \beta r_0^3 k \lambda}{48 r^3} + \frac{35 \beta \mu r_9^3}{12 r^5} + O(r^{-8}), \quad (32)
\]
where the constant $C_2$ is defined in [21]. Comparing with other works in the literature, where wormhole solutions are explore, exploiting the fact of having $R \neq 0$ using only dust, as in the case of [26], our exact solution is more general. The regularity of spacetime required as a condition of the wormhole is satisfied as can be seen in the form of scalar curvature
\[
R = -\frac{\beta r_0^3}{2 k^2 \lambda r^{15}} \times
\frac{\beta^2 \mu r_9^6}{\lambda} - 2 k^2 \lambda r_9^3 + 5 \beta k \mu r_5^3 r_9^6 + 2 \beta k r_5^3 r_9^3, \quad (33)
\]
is regular for $r \geq r_0$ as $y(r) > 0$, meanwhile for $r \to \infty R = 0$. The Kretschmann scalar $K = R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu}$ also is regular and tends to zero when $r$ tends to infinity.

Finally, we will analyze the expression related to the energy condition given by the following expression
\[
c^2 \rho + P = \frac{\beta r_0^3}{k r^5} \left[ 1 + \left( 1 - \frac{\beta \mu r_9^3}{k \lambda r^5} \right)^{-1} \right]. \quad (34)
\]
Analyzing the expression [34] according to [24], we find that, in order to have the energy density defined positive around the throat $r = r_0$, it requires that $\mu < \frac{k \lambda r_5^3}{\beta}$,
or \( \mu > \frac{2k\Omega r_0^2}{\rho} \), the first inequality is the condition \( \Omega > 0 \), the second inequality must be discarded, because it does not guarantee the absence of horizons in our solution. Imposing the condition of absence of horizons result clear then, that around the throat the null energy condition is defined positive and tends asymptotically to zero when \( r \to \infty \) as show in the following Figure 3.

![Figure 3](image)

**Figure 3**: The plot shows how the NEC is satisfied for different values of \( \beta \), considering \( r_0 = 1, \lambda = .36 \) and \( \mu = 1 \).

**IV. COMMENTS AND CONCLUSIONS**

In this paper we construct an exact solution for a wormhole from an effective anisotropic fluid, using corrections of the braneworld to gravitational 4D field equations. Our solution was built with the help of an ansatz for the state equation of a perfect fluid 4D, it should be noted that this solution meets the necessary conditions to form a wormhole. In addition, it does not violate the NEC by the perfect fluid for this particular solution. Also this solution allows us to establish an astronomical bound for the radius of the throat in terms of tension of the brane, for that reason worth further explored the scope of ansatz used in this work in more realistic cosmological scenarios.

**V. ACKNOWLEDGEMENTS**

We gratefully acknowledge support from CIC–UMSNH and RRML acknowledge support from SNI–CONACyT.

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