Research Article

Representation and Reasoning of Three-Dimensional Spatial Relationships Based on R5DOS-Intersection Model

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1. Introduction

The reasoning of spatial relationship, a.k.a. spatial reasoning, can be implemented quantitatively or qualitatively. Qualitative spatial reasoning, aiming to represent and analyze spatial information, is an important tool in artificial intelligence (AI), machine vision, robot navigation [1, 2], and geographic information system [3].

Over three decades, many theories and models have been developed for spatial reasoning. For instance, Randell et al. [4, 5] put forward the region connection calculus (RCC) theory. Egenhofer and Franzosa [6, 7] proposed the theory of 4-intersection model and 9-intersection model. Li [8] derived a dynamic reasoning method for azimuth relationship.

In recent years, spatial reasoning has evolved rapidly, thanks to the emerging AI applications in image processing [9, 10], computer vision [11, 12], and model prediction [13]. However, most studies on spatial reasoning focus on the spatial relationships on two-dimensional (2D) planes rather than those in three-dimensional (3D) spaces. The 3D space contains too many information elements to be handled by ordinary reasoning methods.

At present, the relationships between objects in the 3D space are mostly solved by compound reasoning. The common approaches of compound reasoning include the compound reasoning of directional and topological relationships [14, 15] and the compound reasoning of directional and distance relationships [16]. Liu et al. [17] designed a 3D improved composite spatial relationship model (3D-ICSRM) in a large-scale environment and proposed a reasoning algorithm to solve that model. The accuracy of the 3D-ICSRM is very limited, and it considers the relationship between qualitative distance and direction. In 2016, Hou et al. [18] extended the convex tractable subalgebra into 3D space and used the BCD algorithm to calculate it. In 2019, Wang et al. [19] extended the oriented point relation algebra (OPRAm) model to 3D and proposed oriented point relation algebra in three-dimensional (OPRA3Dm) algorithm, which...
has certain practical significance. These two papers consider the direction relationship. In recent years, the literature mainly studies the relationship between the direction and qualitative distance, while there is less research on the direction and topological relationship. This article will focus on the direction and topological relationship to fill the gaps in this field.

This paper aims to disclose the compound topological and directional relationships of three simple regions in the 3D space. Firstly, the RCC-5 model was combined with a strong directional relationship model for two simple regions, based on the extended 4-intersection theory and spatial orientation relationship in RCC5. The combined model was used to identify the compound topological and azimuth relationships between two simple regions, and solved by a self-designed algorithm. Through programming, a total of 65 topological and directional relationships were obtained in the 3D space.

On this basis, the extended 4-intersection matrix was replaced with an 8-intersection matrix to represent the 3D spatial topological and directional relationships between three simple regions. Then, it was found that the topological and directional relationships between the R5DOS-intersection model of two regions and three regions are complete and mutually exclusive. Further programming reveals a total of 11,038 topological and azimuth relationships between three simple regions in the 3D space and derives a simple multiple dimensions. Compared with the 8-direction cone model, the 16-direction cone model is also consistent with the human cognition of directions, but too complicated to express. Hence, the 8-direction cone model is more suitable for the reasoning of spatial relationships.

Considering its excellence in spatial segmentation, the 8-direction cone model was coupled with the RCC-5 intersection model for compound reasoning of topological and azimuth relationships in the 3D space.

2.2. Orientation Model. Minimum bounding rectangle (MBR) is a commonly used model for directional relationship in space [18–20]. The MBR model, 8-direction model, and 16-direction model are shown in Figure 3 below. The MBR model is not consistent with human cognition of directions.

In 2010, He and Bian [21] came up with a special 8-direction cone model (Figure 4), which divides the space into eight regions: NW, NE, EN, ES, SE, SW, WS, and WN. Among them, NW and NE belong to the N direction, EN and ES belong to the E direction, SE and SW belong to the S direction, and WS and WN belong to the W direction.

The 8-direction cone model is easy to describe and recognize and is flexible in dealing with relationships in topological and directional relationship $R(A, C)$ from two sets of two simple regions $R(A, B)$ and $R(B, C)$.

2. Materials and Methods

2.1. RCC Theory. In 1992, Randell et al. [4, 5] proposed the RCC theory and established the RCC-8 intersection model, which is a boundary-sensitive model. Based on the boundary-sensitive conditions, the RCC-5 intersection model can be derived (Figure 1).

In 1991 and 1995, Egenhofer et al. constructed an extended 4-intersection matrix, which covers two space objects $A$ and $B$, with $A^o$ being the interior of $A$:

$$\begin{pmatrix}
A^o \cap B^o \\ (A^o)\cap B^o \\ A^o \cap (B^o) \\ (A^o)\cap (B^o)
\end{pmatrix}.$$ (1)

The value of each position set is either empty or non-empty. Then, the five kinds of relationships in the RCC-5 intersection model can be represented as the matrix in Table 1 and expressed as a set $R_5 = \{ (0 0 0 1),(0 1 0 1),(1 0 0 1)\}$. For three simple areas $A, B$, and $C$, $R = (\partial A \cup \partial B \cup \partial C)$ can be partitioned into 8 parts (Figure 2).

The eight parts can be illustrated by an 8-intersection matrix:

$$\begin{pmatrix}
(A^o)\cap B^o \\ (A^o)\cap (B^o) \\ A^o \cap (B^o) \\ (A^o)\cap (B^o)
\end{pmatrix}. \quad (2)$$

The RCC theory fuels the research on spatial relationship models in the past three decades, giving birth to many new theories. Nonetheless, most of these theories target the 2D plane rather than the 3D space. Recently, there is a growing interest in the spatial relationship models of the 3D space, especially the compound reasoning of directional and topological relationships, and that of directional and distance relationships.

2.3. Model Construction. Any object in space is wrapped by an outer sphere $\partial A$ with a radius $r_A$ (Figure 5), that is, $\forall (x_A, y_A, z_A) \in \partial A$.

Taking the center of $\partial A$ as the origin of the rectangular coordinate system in space, the spatial Cartesian coordinate system can be established and the reference space can be divided into eight intervals by the $x$-, $y$-, and $z$-axes. Each interval is called a hexagon limit Oct $= \{1, 2, 3, 4, 5, 6, 7, 8\}$ (Figure 6).

Suppose $n$ points $B_i(x_i, y_i, z_i) \in \{i = 1, 2, \ldots, n\}$ are scattered in the space. The centroid $b(x_b, y_b, z_b)$ of the point set $B = \{B_1, B_2, \ldots, B_n\}$ can be obtained by $k$-means clustering (KMC) [20] and treated as the center of the sphere of point set $B$.
Figure 1: The relationships between RCC-8 and RCC-5 intersection models.

Table 1: Matrix representation of the RCC-5 relationships.

| RCC5 relationships | DR (A, B) | PO (A, B) | PP (A, B) | EQ (A, B) | PPI (A, B) |
|-------------------|-----------|-----------|-----------|-----------|------------|
| Extended 4-intersection matrix representation | \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} |
\[ x_C = \frac{x_{BC} + x_{PC}}{2}, \]
\[ y_C = \frac{y_{BC} + y_{PC}}{2}, \quad \forall (x_{C}, y_{C}, z_{C}) \in \mathbb{C}, \quad (4) \]
\[ z_C = \frac{z_{BC} + z_{PC}}{2}. \]

If it is impossible to find the outer sphere of the space object, the object can be treated as an irregular convex object. Then, five planes \( \pi_1: y = 0, \pi_2: x = 0, \pi_3: z = 0, \pi_4: y = z, \) and \( \pi_5: y = -z, \) can be inserted into the rectangular coordinate system in space (Figure 7).

Then, the 3D space can be represented as \( \mathbb{D} = \{\text{NE}, \text{EN}, \text{ES}, \text{SE}, \text{SW}, \text{WS}, \text{WN}, \text{NW}\} \). The angle corresponding to each region can be described as follows:

\[ \theta_{\text{NE}} \in \left[0, \frac{\pi}{4}\right), \]
\[ \theta_{\text{EN}} \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right), \]
\[ \theta_{\text{ES}} \in \left[\frac{\pi}{2}, \frac{3\pi}{4}\right), \]
\[ \theta_{\text{SE}} \in \left[\frac{3\pi}{4}, \pi\right), \]
\[ \theta_{\text{SW}} \in \left[\pi, \frac{5\pi}{4}\right), \]
\[ \theta_{\text{WS}} \in \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right), \]
\[ \theta_{\text{WN}} \in \left[\frac{3\pi}{2}, \frac{7\pi}{4}\right), \]
\[ \theta_{\text{NW}} \in \left[\frac{7\pi}{4}, 2\pi\right). \]

where \( \theta \) is the dihedral angle of the plane \( \pi_i \) (\( i = 1, 2, 3, 4, 5 \)).

Adding the set of hexagram limits \( \mathbb{O} = \{1, 2, 3, 4, 5, 6, 7, 8\} \), the space can be divided into 16 regions:

\[ \mathbb{D}_O = \begin{pmatrix}
1\text{NE} & 2\text{NE} & 3\text{NW} & 4\text{NW} \\
1\text{EN} & 2\text{EN} & 3\text{NW} & 4\text{NW} \\
5\text{ES} & 6\text{ES} & 7\text{WS} & 8\text{WS} \\
5\text{SE} & 6\text{SE} & 7\text{SW} & 8\text{SW}
\end{pmatrix}, \quad (6) \]

where \( \mathbb{D}_O \) is the set of 3D regions and their hexagram limits. If the center of outer sphere \( B \) exists in region 1NE, then \( B \) strongly exists in that region, denoted as s1NE. If outer sphere \( B \) partly exists in region 2NE, then \( B \) weakly exists in that region, denoted as w2NE. We let “0” indicate that there is no object in the area, “1” indicates that the object “strongly

The outer sphere \( B \) completely covers the \( n \) points: \( \forall (x_{Bi}, y_{Bi}, z_{Bi}) \in \mathbb{O}. \) Similarly, the outer sphere \( C \) for point set \( \mathbb{C} \) can be defined as follows:
exists” in this area, and “2” indicates that the object “weakly exists” in this area. An example is shown in Figure 8:

\[
\text{DOS} = \begin{pmatrix}
1 & 2 & 0 & 0 \\
2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

(7)

For simplicity, only strong existence scenarios were considered. Then, the set of regions, where \( B \) strongly exists, can be defined as follows:

\[
\begin{align*}
\text{DOS} &= \begin{pmatrix}
s1NE & s2NE & s3NW & s4NW \\
s1EN & s2EN & s3NW & s4NW \\
s5ES & s6ES & s7WS & s8WS \\
s5SE & s6SE & s7SW & s8SW
\end{pmatrix}, \\
\text{where} \\
s1NE: x_b \geq 0, y_b \geq 0, z_b \geq 0, \quad \theta_{ab} \in \left[0, \frac{\pi}{4}\right), \\
s2NE: x_b < 0, y_b \geq 0, z_b \geq 0, \quad \theta_{ab} \in \left[0, \frac{\pi}{4}\right), \\
s1EN: x_b \geq 0, y_b \geq 0, z_b \geq 0, \quad \theta_{ab} \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right), \\
s2EN: x_b < 0, y_b \geq 0, z_b \geq 0, \quad \theta_{ab} \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right), \\
s5ES: x_b \geq 0, y_b \geq 0, z_b < 0, \quad \theta_{ab} \in \left[\frac{\pi}{2}, \frac{3\pi}{4}\right), \\
s6ES: x_b < 0, y_b \geq 0, z_b < 0, \quad \theta_{ab} \in \left[\frac{\pi}{2}, \frac{3\pi}{4}\right), \\
s5SE: x_b \geq 0, y_b \geq 0, z_b < 0, \quad \theta_{ab} \in \left[\frac{3\pi}{4}, \pi\right), \\
s6SE: x_b < 0, y_b \geq 0, z_b < 0, \quad \theta_{ab} \in \left[\frac{3\pi}{4}, \pi\right), \\
s8SW: x_b \geq 0, y_b \geq 0, z_b < 0, \quad \theta_{ab} \in \left[\frac{\pi}{2}, \frac{5\pi}{4}\right), \\
s7SW: x_b \geq 0, y_b \geq 0, z_b \geq 0, \quad \theta_{ab} \in \left[\frac{\pi}{2}, \frac{5\pi}{4}\right), \\
s8WS: x_b \geq 0, y_b < 0, z_b < 0, \quad \theta_{ab} \in \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right], \\
s7WS: x_b < 0, y_b < 0, z_b < 0, \quad \theta_{ab} \in \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right], \\
s4WN: x_b \geq 0, y_b < 0, z_b \geq 0, \quad \theta_{ab} \in \left[\frac{3\pi}{2}, \frac{7\pi}{4}\right), \\
s3WN: x_b < 0, y_b < 0, z_b \geq 0, \quad \theta_{ab} \in \left[\frac{3\pi}{2}, \frac{7\pi}{4}\right), \\
s4NW: x_b \geq 0, y_b < 0, z_b \geq 0, \quad \theta_{ab} \in \left[\frac{7\pi}{4}, 2\pi\right), \\
s3NW: x_b < 0, y_b < 0, z_b \geq 0, \quad \theta_{ab} \in \left[\frac{7\pi}{4}, 2\pi\right),
\end{align*}
\]

(8)
where $\theta_{ob}$, the dihedral angle formed by planes $\pi_{ob}$ and $\pi_1$, which is perpendicular to the $x$-axis and passes the straight line $ab$ (Figure 9).

For two regions, the extended 4-intersection matrix can be introduced to the DOS:

$$R_{52}^{\text{DOS}} = \begin{pmatrix}
A^0 \cap B^0 & A^0 \cap (B')^0 & (A')^0 \cap B^0 & (A')^0 \cap (B')^0 \\
1 \text{NE} & 2 \text{EN} & 3 \text{NW} & 4 \text{NW} \\
1 \text{EN} & 2 \text{EN} & 3 \text{NW} & 4 \text{NW} \\
5 \text{ES} & 6 \text{ES} & 7 \text{WS} & 8 \text{WS} \\
5 \text{SE} & 6 \text{SE} & 7 \text{SW} & 8 \text{SW}
\end{pmatrix}. \quad (10)$$

For three regions, the 8-intersection matrix can be introduced to the DOS:

$$R_{53}^{\text{DOS}} = \begin{pmatrix}
A^0 \cap B^0 \cap C^0 & A^0 \cap B^0 \cap (C')^0 & A^0 \cap (B')^0 \cap C^0 & A^0 \cap (B')^0 \cap (C')^0 \\
(A')^0 \cap B^0 \cap C^0 & (A')^0 \cap B^0 \cap (C')^0 & (A')^0 \cap (B')^0 \cap C^0 & (A')^0 \cap (B')^0 \cap (C')^0 \\
1 \text{NE} & 2 \text{NE} & 3 \text{NW} & 4 \text{NW} \\
1 \text{EN} & 2 \text{EN} & 3 \text{NW} & 4 \text{NW} \\
5 \text{ES} & 6 \text{ES} & 7 \text{WS} & 8 \text{WS} \\
5 \text{SE} & 6 \text{SE} & 7 \text{SW} & 8 \text{SW}
\end{pmatrix}. \quad (11)$$

Our model consists of two layers: the first layer is the topological relationship $R_3$ layer, and the second layer is the orientation relationship $\text{DOS}$ layer. Then, the following definition can be derived.

**Definition 1.** For the orientation relationship layer, there is

$$\epsilon(\text{DOS}) = \begin{cases} 1, & \text{the enter of the outer sphere } B \text{ exists in this region,} \\
0, & \text{otherwise.} \end{cases} \quad (12)$$

Suppose

$$\epsilon(\text{R}_{52}^{\text{DOS}}) = \begin{pmatrix}
\epsilon(A^0 \cap B^0) & \epsilon(A^0 \cap (B')^0) & \epsilon((A')^0 \cap B^0) & \epsilon((A')^0 \cap (B')^0) \\
\epsilon(1 \text{NE}) & \epsilon(2 \text{EN}) & \epsilon(3 \text{NW}) & \epsilon(4 \text{NW}) \\
\epsilon(1 \text{EN}) & \epsilon(2 \text{EN}) & \epsilon(3 \text{NW}) & \epsilon(4 \text{NW}) \\
\epsilon(5 \text{ES}) & \epsilon(6 \text{ES}) & \epsilon(7 \text{WS}) & \epsilon(8 \text{WS}) \\
\epsilon(5 \text{SE}) & \epsilon(6 \text{SE}) & \epsilon(7 \text{SW}) & \epsilon(8 \text{SW})
\end{pmatrix}. \quad (13)$$

For any two simple regions $A$ and $B$, it is possible to obtain a $5 \times 4'$0-1 matrix. In theory, a total of $2^{20}$ matrices could be acquired, which correspond to $2^{20}$ topological and directional relationships in the 3D space:

$$R_{52}^{\text{DOS}} = \begin{pmatrix}
\epsilon(A^0 \cap B^0 \cap C^0) & \epsilon(A^0 \cap B^0 \cap (C')^0) & \epsilon(A^0 \cap (B')^0 \cap C^0) & \epsilon(A^0 \cap (B')^0 \cap (C')^0) \\
\epsilon((A')^0 \cap B^0 \cap C^0) & \epsilon((A')^0 \cap B^0 \cap (C')^0) & \epsilon((A')^0 \cap (B')^0 \cap C^0) & \epsilon((A')^0 \cap (B')^0 \cap (C')^0) \\
\epsilon(1 \text{NE}) & \epsilon(2 \text{NE}) & \epsilon(3 \text{NW}) & \epsilon(4 \text{NW}) \\
\epsilon(1 \text{EN}) & \epsilon(2 \text{EN}) & \epsilon(3 \text{NW}) & \epsilon(4 \text{NW}) \\
\epsilon(5 \text{ES}) & \epsilon(6 \text{ES}) & \epsilon(7 \text{WS}) & \epsilon(8 \text{WS}) \\
\epsilon(5 \text{SE}) & \epsilon(6 \text{SE}) & \epsilon(7 \text{SW}) & \epsilon(8 \text{SW})
\end{pmatrix}. \quad (14)
Based on the topological relationship between outer spheres B and C, the existence of the centers of the two spheres can be described in two cases.

Case 1. Only the center of one outer sphere exists in the current region:

\[ \varepsilon(\text{DOS}) = \begin{cases} 
0, & \text{no center exists in the current region,} \\
1, & \text{the center of outer sphere } B \text{ exists in the current region,} \\
2, & \text{the center of outer sphere } C \text{ exists in the current region.} 
\end{cases} \] (15)

Case 2. The centers of both outer spheres exist in the current region:

\[ \varepsilon(\text{DOS}) = \begin{cases} 
0, & \text{no center exists in the current region,} \\
1, & \text{the centers of both outer spheres exist in the current region.} 
\end{cases} \] (16)

According to the above conditions, \(2^8 \times 3^{16}\) matrices could be obtained theoretically, which correspond to \(2^8 \times 3^{16}\) topological and directional relationships in the 3D space.

2.4. Model Properties

Definition 2. In layer \(R_5\), any \(m \times n\) -order 0-1 matrices \(A = (a_{ij})_{m \times n}\) and \(B = (b_{ij})_{m \times n}\) can be defined as \(A \cup B = (a_{ij} \lor b_{ij})_{m \times n}\). Then, a 0-1 diagonal matrix can be established as Table 2.

The following proposition can be derived from Table 2:

Proposition 1. \(\varepsilon(A \cup B) = \varepsilon(A) \lor \varepsilon(B)\).

For \(R_5 = \{(0 \ 1 \ 1 \ 1), (1 \ 1 \ 1 \ 1), (1 \ 0 \ 1 \ 1), (1 \ 0 \ 0 \ 1), (1 \ 1 \ 0 \ 1), (1 \ 0 \ 0 \ 1), (1 \ 1 \ 1 \ 1), (1 \ 0 \ 1 \ 1), (1 \ 1 \ 1 \ 1), (1 \ 0 \ 1 \ 1)\}\), \(R(A, B)\) is the element that corresponds to the topological relationship \(R_5\) between any two simple regions \(A\) and \(B\).

Then, the following theorem can be obtained.

Theorem 1. For simple regions \(A, B,\) and \(C, there exists \(R(A, B) = (A \cap B) \setminus (A \cap B)\)

\[ \begin{cases} 
\varepsilon(A \cap B) \lor \varepsilon((A \cap B)) \lor \varepsilon((A \cap B) \cap (B \cap C)) = (A \setminus B) \in R_5. 
\end{cases} \]

Similarly, there exists

\[ \begin{cases} 
\varepsilon(A \cap C) \lor \varepsilon((A \cap C) \lor \varepsilon((A \cap C) \lor (B \cap C)) = (A \cap B) \in R_5. 
\end{cases} \]

Theorem 2. In the 3D space given by the R5DOS-intersection matrix, the topological relationship between the three simple regions is mutually exclusive and complete. The DOS space of the R5DOS-intersection matrix, which consists of 16 regions, is a half-open, half-closed interval with mutual exclusion. That is, for any three simple regions \(A, B,\) and \(C\) in the 3D space, there exists only one relationship satisfied by the ordered pair \(<A, B, C>\).

Proof. For any three simple regions \(A, B,\) and \(C\), the 8 and 16 regions divided by the 8-intersection matrix are disjoint. The

2.5. Constraints on Two Simple Regions. Theoretically, two simple regions might correspond to \(2^{20}\) matrices, but there must be 0-1 matrices that cannot be realized. Therefore, the following constraints were designed on two simple regions.

Constraint 1: to correspond to a real-world topological relationship, the 0-1 matrix of layer \(R_5\) must belong to one of the five cases: \(R_5 = \{(0 \ 1 \ 1 \ 1), (1 \ 1 \ 1 \ 1), (1 \ 0 \ 1 \ 1), (1 \ 0 \ 0 \ 1), (1 \ 1 \ 0 \ 1)\}\).

Constraint 2: if layer \(R_5\) satisfies \(R_5 = (1 \ 0 \ 0 \ 1)\), that is, outer spheres \(A\) and \(B\) are equal, then the 0-1 matrix of the DOS layer is \(\text{DOS} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\). This means, when outer spheres \(A\) and \(B\) are equal, the center \(b\) of outer sphere \(B\) is \(G\), which coincides with that of outer sphere \(A: b(x_b, y_b, z_b) = (0, 0, 0)\).

Constraint 3: since the 16 regions are disjoint, they must be mutually exclusive and complete. If \(R(A, B)\) does not fall at the center of outer sphere \(B,\) it can only exist in one of these regions. In the DOS layer, an outer sphere only exists in one of the 16 intervals. In this way,
2.6. Constraints on Three Simple Regions. The following constraints were designed on three simple regions.

\[
R(A, B) = \begin{cases} 
  \epsilon(A^o \cap B^o \cap C^o) & \epsilon(A^o \cap (B^o)^o \cap (C^o)^o) \\
  \epsilon((A^o)^o \cap B^o \cap C^o) & \epsilon((A^o)^o \cap (B^o)^o \cap (C^o)^o) \\
  \epsilon((A^o)^o \cap B^o \cap C^o) & \epsilon((A^o)^o \cap (B^o)^o \cap (C^o)^o) \\
  \epsilon(A^o \cap (B^o)^o \cap (C^o)^o)
\end{cases} \in R_3,
\]

\[
R(A, C) = \begin{cases} 
  \epsilon(A^o \cap C^o) & \epsilon(A^o \cap (C^o)^o) \\
  \epsilon((A^o)^o \cap C^o) & \epsilon((A^o)^o \cap (C^o)^o) \\
  \epsilon(A^o \cap (B^o)^o \cap C^o) & \epsilon(A^o \cap (B^o)^o \cap (C^o)^o) \\
  \epsilon((A^o)^o \cap B^o \cap (C^o)^o)
\end{cases} \in R_3,
\]

\[
R(B, C) = \begin{cases} 
  \epsilon(B^o \cap C^o) & \epsilon(B^o \cap (C^o)^o) \\
  \epsilon((B^o)^o \cap C^o) & \epsilon((B^o)^o \cap (C^o)^o) \\
  \epsilon((B^o)^o \cap B^o \cap C^o) & \epsilon((B^o)^o \cap B^o \cap (C^o)^o) \\
  \epsilon((B^o)^o \cap (B^o)^o \cap (C^o)^o)
\end{cases} \in R_3.
\]

Constraint 1: to uniquely correspond to the topological and directional relationships in the 3D space, a $R_3DOS$ matrix must satisfy the following conditions.

**Definition 3**

\[
\begin{align*}
\begin{array}{c}
\begin{cases}
\{D \in \mathbb{R}^3 \mid D \in \Phi \}
\end{cases}
\end{array}
\end{align*}
\]
Constraint 2: since all three simple regions are bounded, \((A^C) \cap (B^C) \cap (C^C)\) is always 1.

From Constraints 1 and 2, it can be inferred that layer \(R_5\) has 109 topological relationships for any three simple regions in the 3D space.

Constraint 3: after adding the orientation relationship, some topological relationships are not satisfied in the orientation regions. In some topological relationships, the center of an outer sphere will change with that of the other outer spheres. For instance, if layer \(R_5\) is \((A, B)\) or \((A, C)\) or \((B, C)\) and the center of outer sphere \(B\) was assumed to fall into 1NE or 2NE, this situation does not exist in the real world. Under Constraints 2 and 4, there is no solution to this situation. However, the \(R_5\)-DOS-intersection model can explain the situation that cannot be realized in the 3D space.

For a ternary reference object in the 3D space, there are theoretically \(2^8 \times 3^{16}\) matrices. Under the above constraints, a total of 11,038 matrices were obtained after removing the nonexistent scenarios.

2.7. Topological Relationship Algorithm for 3 Simple Regions in the 3D Space. The topological relationship algorithm for 3 simple regions in the 3D space can be implemented in the following steps.

1. Assign each object a row vector \([a_1, a_2, \ldots, a_{24}]\).
2. Generate a theoretical object of the type \(2^8 \times 3^{16}\), i.e., a matrix \(A\) of \(2^8 \times 3^{16}\) row vectors.
3. Scan each row of matrix \(A\), and mark all row vectors that satisfy the constraints.
4. Save all the marked row vectors as a matrix \(B\) and output the matrix as the final result.

The pseudocode of the algorithm is displayed as follows:

**Algorithm 1**

```plaintext
R5:DOSaALL ← 2^8 \times 3^{16} basic topological relationships // All basic topological relationships
R5:DOSa ← null // TR empty Test
for each x in R5:DOSaALL
    if x satisfies Constraint 1 // if t satisfies Constraint 1 Test
        if x satisfies Constraint 2 // if t satisfies Constraint 2 Test
            if x satisfies Constraint 3 // if t satisfies Constraint 3
                R5:DOSa ← {R5:DOSa, x} // If the constraint is satisfied, t is placed in TR
            end if
        end if
    end if
end for
return R5:DOSa // Return result
```

3. Results and Discussion

3.1. Comparison between R5:DOS-Intersection Model and MBR Model. This section proves that the R5:DOS-intersection model has stronger expressive power than the MBR model in the 3D space [21–23].

First, layer \(R_5\) was defined as \(R (A, B) = \text{PPI}, R (A, C) = \text{PPI}\), and \(R (B, C) = \text{PPI}\), and the center of outer sphere \(B\) was assumed to fall into 1NE or 2NE. This situation does not exist in the real world. Under Constraints 2 and 4, there is no solution to this situation. However, the R5:DOS-intersection model can explain the situation that cannot be realized in the 3D space.

Next, the R5:DOS-intersection model was found capable of expressing situation that cannot be illustrated by the MBR model through the analysis of the following example. For any three external spheres \(A–C\) in the 3D space, it is assumed that the topological and azimuth relationships between them are known, and these spheres are separated from each other.
For the MBR model, Example 1: (a) dir(A, B)=(1, 1, 1) and (b) dir(A, C)=(1, 1, 1) were obtained for the two examples (Figures 11 and 12).

For the R53DOS-intersection model, layer R5 can be described as:

For the R53DOS-intersection model, layer R5 can be described as:

\[
R(A, B) = \left( \begin{array}{ccc}
\varepsilon(A \cap B) & \varepsilon(A \cap (B')^o) & \varepsilon(A \cap C) \\
\varepsilon((A')^o \cap B) & \varepsilon((A')^o \cap (B')^o) & \varepsilon((A')^o \cap C) \\
\varepsilon((A')^o \cap (B')^o) & \varepsilon((A')^o \cap (B')^o \cap (C')^o)
\end{array} \right) = (0, 1, 1),
\]

(22)

\[
R(A, C) = \left( \begin{array}{ccc}
\varepsilon(A \cap C) & \varepsilon(A \cap (C')^o) & \varepsilon(A \cap (C')^o) \\
\varepsilon((A')^o \cap C) & \varepsilon((A')^o \cap (C')^o) & \varepsilon((A')^o \cap (C')^o) \\
\varepsilon((A')^o \cap (B')^o \cap (C')^o)
\end{array} \right) = (0, 1, 1),
\]

\[
R(B, C) = \left( \begin{array}{ccc}
\varepsilon(B \cap C) & \varepsilon(B \cap (C')^o) & \varepsilon(B \cap (C')^o) \\
\varepsilon((B')^o \cap C) & \varepsilon((B')^o \cap (C')^o) & \varepsilon((B')^o \cap (C')^o)
\end{array} \right) = (0, 1, 1).
\]

Without changing the positions of A–C, the images of the R53DOS-intersection model in the two examples can be obtained as Figures 13 and 14, where green, blue, and red balls are the outer spheres A–C, respectively.
Example 2. (a) \( \text{dir}(A, B) = (0, 0, 1) \) and (b) \( \text{dir}(A, C) = (0, 1, 1) \) were obtained for the two examples (Figures 15 and 16).

In the same way, we can get the corresponding \( R_3 \text{DOS-intersection model} \) (Figures 17 and 18):

\[
R_{3\text{DOS}}_1 = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad (23)
\]

\[
R_{3\text{DOS}}_2 = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 \\
2 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}. \quad (24)
\]

Through the above comparison, it can be seen that the \( R_3 \text{DOS-intersection model} \) can represent the topological relationship of space objects \( A, B, \) and \( C \), and it can accurately represent the spatial situation that the MBR model cannot represent.

3.2. Compound Relationship Reasoning Based on \( R_3 \text{DOS-Intersection Model} \). This section applies the \( R_3 \text{DOS-Intersection Model} \) to the reasoning of the compound relationships between simple regions in the 3D space. It is assumed that the topological and azimuth relationships between simple regions \( A \) and \( B \) and those between simple regions \( B \) and \( C \) are known in advance. Then, the goal is to deduce the possible topological and azimuth relationships between simple regions \( A \) and \( C \).

According to Section 2.3, we have
Figure 15: MBR model in Example 2(a).

Figure 16: MBR model in Example 2(b).

Figure 17: R5$_3$DOS-intersection model in Example 2(a).

Figure 18: R5$_3$DOS-intersection model in Example 2(a).

$$R5_3\text{DOS}_3 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 \\
2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}$$

$$R5_3\text{DOS}_4 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 \\
2 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}$$
therealworld. Hence, it is possible to obtain $65 \times 1$ matrices to obtain topological and azimuth relationshipswere obtained from the $R_5$ DOS-intersection model, a total of 65 topological and azimuth relationships were obtained from the real world. Hence, it is possible to obtain $65 \times 1$ matrices.

Using the $R_5$ DOS-intersection model, a total of 65 topological and azimuth relationships were obtained from the real world. Hence, it is possible to obtain $65 \times 1$ matrices of 5 rows and 4 columns, which is denoted as $\Omega_1 = \{R_i; i = 1, \ldots, 65\} R_i$. Targeting at region A, the topological and directional relationships between A and C and those between B and C were taken into account.

Since the topological and azimuth relationships between simple regions A and B and those between simple regions B and C are known in advance, we have $R(B, C) \in \Omega_1$. Then, the possible topological and orientation relationships between A and C were derived from the $R_5$ DOS-intersection model. According to Definition 2, we have
Suppose the real-world 0-1 matrices satisfy
\[ M = \{ M_i = 1, \ldots, n \}, \]
Then, all 0-1 matrices must meet:
\[ m_i = R(A, B) \lor R(B, C), \quad i = 1, \ldots, n. \]
Hence, the matrix that does not satisfy the condition belongs to the empty set, namely, \( M \in \emptyset \). This shows the topological and directional relationships \( R(A, B) \) and \( R(A, C) \) cannot be compounded. Then, all 0-1 matrices represented in the R5DOS-intersection model were judged one by one. The duplicates in the set \( \{ M_i = 1, \ldots, n \} \) were removed, leaving the possible topological and azimuth relationships between \( A \) and \( C \).

In theory, there are a total of \( 65 \times 65 = 4,225 \) topological-azimuth relationships \( R(A, B) \) and \( R(B, C) \). On this basis, the compound relationship reasoning table was set up (Table 4).

4. Conclusions

This paper extends the compound directional and topological relationships on the 2D plane to the 3D space and then creates the R53DOS-intersection model. Based on the model, a total of 11,038 directional and topological relationships were calculated. Compared with the MBR model, the proposed model can describe the relationships between simple regions accurately and express the relationships with sufficient clarity. To further improve the model, the future research will consider the impact of simple area boundaries on the model and apply the R5DOS model to the formation control of UAV formations.

Data Availability

The data used to support the findings of the study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Supplementary Materials

This code is the screening algorithm of the R5DOS-intersection model. The purpose is to screen several matrices theoretically in the model according to the constraints and finally get the algorithm of the matrix that meets the requirements, the result of running the code needs simple processing, not the result of the article. The code is developed based on MATLAB software. (Supplementary Materials)

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