Noise reduction of continuous wave radar and pulse radar using matched filter and wavelets

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Abstract

This paper analyzes noise reduction using matched filter and wavelet transform in the signals of continuous wave radar and pulse radar. The denoising application of wavelets has been used in spectrum cleaning of atmospheric radar signals. Matched filter has a strong anti-noise ability; it can also achieve accurate pulse compression in a very noisy environment. This paper analyzes the algorithms of matched filter and wavelets that are used in radar signal processing to reduce the noise. The simulation results indicate that matched filter has a strong anti-noise ability for pulse radar and wavelet for continuous wave radar.

Keywords: Denoising; Matched Filter; Radar Noise; SNR; Wavelets

1 Introduction

Chaos is the very complicated behavior of a low-order dynamical system, because it is both nonlinear and deterministic [1,2]. It demonstrates a strong notion, permitting the use of a simple deterministic system to illustrate highly irregular fluctuations exhibited by physical phenomena encountered in nature. Recently, some engineering applications of chaos have been reported in literature [3-6]. These can be grouped under two broadly defined categories [3,4]. One group is synthesis of chaotic signals, which includes signal masking and spread-spectrum communications. Another is analysis of chaotic signals. It exploits the fact that some physical phenomena allow the use of a chaotic model.

For more than 20 years, many methods for smoothing denoising filters have been proposed. Various standard spatial domain filters have been proposed to reduce noise [7]. The 2-D Gaussian filter [8] and the median filter [9] have also been widely applied. The most commonly used is the Lee filter [10], but it requires knowledge of interferometric coherence. The adaptive contoured-window filter method [11] and the two-stage filtering method [12] can also reduce noise effectively.

Wavelet analysis [5] and matched filter [6] are two of the most significant tools in the field of signal processing in the last few decades. We analyze both techniques and show how such an individual can improve the quality of radar-received signals in a noisy environment for both types of radar, i.e., continuous wave radar (radars continuously transmit a high-frequency signal and the reflected energy is also received and processed continuously) and pulse radar (transmits high power, high-frequency pulses toward the target, and it waits for the echo of the transmitted signal before it transmits a new pulse). The problem addressed here concerns the denoising of the radar-received signal immersed in noise. Several simulations have been performed to verify the algorithm for both types of radar. All of the simulations give the same time delay, and even the received signal is attenuated more than 90%. The simulation results indicate that the algorithm is effective and robust even when the receiver receives a very weak signal.

The remainder of the paper is organized as follows: In Section 2, we briefly describe the literature of matched filter and the characteristics of matched filter related to radar noise reduction. Section 3 presents wavelets denoising technique, while the simulations and data analysis are described in Section 4. Finally, we make some conclusions about our comparison related to noise reduction.
2 Noise reduction by matched filter

2.1 Theory of matched filter

Matched filter is not a specific type of filter, but a theoretical framework. It is an ideal filter that processes a received signal to minimize the effect of noise. Therefore, it optimizes the signal-to-noise ratio (SNR) of the filtered signal [13].

The original matched filter was proposed by D. O. North [14]. Later, many authors [15-17] tried to improve the performance of matched filter. However, the conventional matched-filter detector has some drawbacks [18]. To reduce the drawbacks, the exhaustive-search matched filter (EMF) detector and the near-optimal MF (NMF) detector are proposed in [19]. Recently, [20] proposed a developed NMF (D-NMF) to improve the performance of NMF.

The radar-received signal \( r(t) \) contains two components \( s_i(t) \) and \( n_i(t) \) that represent the certain signal (e.g., targets) and noise, respectively, i.e., \( r(t) = s_i(t) + n_i(t) \). The matched filter \( h(t) \) yielding the output \( y(t) = s_o(t) + n_o(t) \) is to generate the maximal ratio of the filtered signal and has an impulse response that is a reverse time-shifted version of the input signal. So, to obtain the maximum SNR, we need the time delay, \( D \). With the use of this time delay, \( D \), we obtain the output of the cross-correlation between transmitted signals and received signals.

The output of the matched filter is

\[
F_s(\omega) = \int_{-\infty}^{+\infty} s_i(t) e^{-j\omega t} dt
\]  

The average power of the noise equals the value of autocorrelation function, which is

\[
E[n^2_\omega(t)] = \frac{1}{2\pi} \int_0^{+\infty} |H(\omega)|^2 d\omega
\]  

Now, according to the definition of SNR, time \( T \) is

\[
\text{SNR} = \frac{|s_o(T)|^2}{E(n^2_\omega)}
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{+\infty} |H(\omega) F_{\text{target}}(\omega)|^2 d\omega
\]

\[
\leq \frac{1}{2\pi} \int_{-\infty}^{+\infty} |H(\omega)|^2 d\omega \int_{-\infty}^{+\infty} |F_{\text{target}}(\omega)|^2 d\omega
\]

Hence,

\[
\text{SNR} \leq \frac{1}{2\pi} \int_{-\infty}^{+\infty} |H(\omega)|^2 d\omega \int_{-\infty}^{+\infty} F_{\text{target}}(\omega) d\omega
\]

The numerator of the above equation denotes power of the signal according to Parseval’s theorem. From Equation 5, the matched filter maximizes the SNR of the filtered signal and has an impulse response that is a reverse time-shifted version of the input signal. So, to obtain the maximum SNR, we need the time delay, \( D \).

\[
\text{SNR} \leq \frac{1}{2\pi} \int_{-\infty}^{+\infty} |H(\omega)|^2 d\omega \int_{-\infty}^{+\infty} |F_{\text{target}}(\omega)|^2 d\omega
\]

Figure 1 Matched filter system.
2.2 Cross-correlation to find time delay $D$

Correlation is the process to determine degree of ‘fit’ between two waveforms and to determine the time at which the maximum correlation coefficient or ‘best fit’ occurs [21-23]. For the radar system, if we correlate between the transmitted signal and the received signal, then we get the time difference between the transmitted and received signals. We consider the transmitted signal to be $x(n)$, and then the returned signal $r(n)$ may be modeled as:

$$r(n) = ax(n-D) + w(n)$$  \hspace{1cm} (6)

where $w(n)$ is assumed to be the additive noise during the transmission, $a$ is the attenuation factor, and $D$ is the delay which is the time taken for the signal to travel from the transmitter to the target and back to the receiver.

The cross-correlation between the transmitted signal, $x(n)$, and the received signal, $r(n)$, is [21]

$$C_{x'r}(l) = aC_{xx}(l-D)$$  \hspace{1cm} (7)

From Equation 7, the maximum value of the cross-correlation will occur at $l = D$, which is our interest in cross-correlation from which we can get the time delay, $D$. For the multiple targets, we get the multiple number of $D$ from Equation 7. For example, if there are $n$ targets then we can get $n$ number of delays such as $D_1, D_2, D_3, ..., D_n$.

2.3 Matched filter for radar

For pulse radars, consider pulse width $T_p$ and $T_c$ is the time that a target is illuminated by the radars. Thus, we can write $r(t)$ as

$$r(t) = V \text{rect} \left[ \frac{t - T_p}{T_c} \right]$$  \hspace{1cm} (8)

From Equation 5, and for Equation 8, we can write the pulse radar as [24]

$$(\text{SNR})_{\text{pulse}} = \frac{P_i G^2 \lambda^2 \sigma T_p}{(4\pi)^3 K T_0 BFLR^2}$$  \hspace{1cm} (9)

where $P_i$ is the peak transmitted power of radar, $G$ is the antenna gain, $\sigma$ is the radar cross section (RCS), $R$ is the range which electromagnetic wave transmits, $\lambda$ is the wavelength, $K = 1.38 \times 10^{-23} \text{ J/K}$ is Boltzmann’s constant, $B$ is bandwidth, $T_0 = 290$ K is the operating temperature of antenna, $F$ is the noise figure of receiver, and $L$ denotes as radar losses.

For continuous wave radars, radar equation can be written as [24]

$$(\text{SNR})_{\text{cw}} = \frac{P_{\text{cw}} T_{\text{Dwell}} G^2 \lambda^2 \sigma}{(4\pi)^3 K T_0 BFLR^2}$$  \hspace{1cm} (10)

where $P_{\text{cw}}$ is the continuous wave average transmitted power and $T_{\text{Dwell}}$ is the dwell interval.

3 Wavelet denoising

For removing noise and extracting signal from any data, wavelet analysis is one of the most important methods. The wavelet denoising application has been used in spectrum cleaning of the atmospheric signals. There are different types of wavelets available like Morlet, Coiflet, Mexican hat, Symlet, Biorthogonal, and Haar, which have their own specifications such as filter coefficients and reconstruction filter coefficients. In this paper, to eliminate noise embedded in the radar signal ‘sym8’, wavelets have been used. The goal of this study is to denoise the radar signal. One often encounters the term ‘denoising’ in recent wavelet literature, described in an informal way with various schemes that attempt to reject noise by damping or thresholding in the wavelet domain [25,26]. The threshold of wavelet coefficient has near optimal noise reduction for different kinds of signals. Wavelets have many advantages over fast Fourier transform. Fourier analysis has a major drawback, which is that time information is lost, when transforming to the frequency domain. Thus, it is impossible to tell when a particular event took place under Fourier analysis. Wavelet analysis is capable of revealing aspects of data that other signal analysis techniques, aspects such trends, breakdown points, discontinuities in higher derivatives, and self-similarity, are unable to reveal. Wavelet analysis can often denoise a signal without appreciable degradation. Wavelet transform performs a correlation analysis. Therefore, the output is expected to be maximal when the input signal most resembles the mother wavelet.

3.1 Wavelet transform

According to the definition of wavelet transform [27], for function $f(t)$, wavelet transform coefficient $W_A(a,r)$

$$W_A(a,r) = \left\langle f(t), \psi_{(a,r)}(t) \right\rangle = \frac{1}{\sqrt{a}} \int f(t) \psi^* \left( \frac{t-r}{a} \right) \, dt$$  \hspace{1cm} (11)

Here, $f(t)$, $\psi_{(a,r)}(t)$ is the wavelet basis function, $\psi^* \left( \frac{t-r}{a} \right)$ is a conjugate of wavelet basis function, $r$ is the amount of shift, and $a$ is scale.

3.2 Wavelet denoising

The wavelet denoising procedure proceeds in three steps:

Step 1 Signal decomposing: Choose the wavelet basis function, and to determine the decomposition level $N$, get the coarse and detail coefficients by DWT.
Figure 2 The effect of pulse repetition frequency for the pulse radar. (a) Pulse radar transmitted signal. (b) Received signal (echo + noise), no attenuation. (c) Recovery signal from received signal by matched filter.

Figure 3 The effect of pulse repetition frequency for the continuous wave radar. (a) Continuous wave radar transmitted signal. (b) Received signal (echo + noise), no attenuation. (c) Recovery signal from received signal by matched filter.
Step 2 Threshold detail coefficients: For each level from 1 to \( N \), compare the detail coefficients to threshold values.

Step 3 Reconstructing the signal: Reconstruct the denoised signal based on the original approximation coefficients of level \( N \) and the modified detail coefficients of levels from 1 to \( N \).

3.3 Traditional threshold function

The major signal information mainly concentrates in the low frequency sub-band of wavelet transform domain.

Figure 4 Results of pulse radar signal using matched filter. (a) Pulse radar transmitted signal. (b) Received signal (echo + noise), 90% attenuation. (c) Recovery signal from received signal by matched filter.

Figure 5 Results of continuous wave radar signal using matched filter. (a) Continuous wave radar transmitted signal. (b) Received signal (echo + noise), 90% attenuation. (c) Recovery signal from received signal by matched filter.
Noise equally distributes in all wavelet coefficients, so the wavelet transform factor should be larger than the wavelet transform factor of the noises after wavelet decomposition. Therefore, the selection of wavelet threshold is an important step, which also directly impacts on the effect of noise reduction. Different methods have been proposed to choose the threshold. The frequently used thresholding of wavelet coefficients is governed mainly by either soft or hard thresholding function, proposed by Donoho [28]. The soft thresholding is generally referred to as wavelet shrinkage, since it ‘shrinks’ the coefficients with high amplitude toward zero, whereas the hard thresholding is commonly referred to simply as wavelet thresholding. Given that \( d_{jk} \) indicates the value of wavelet coefficient, \( \tilde{d}_{jk} \) implies the value of \( d_{jk} \) after thresholding function, and \( T \) is the threshold value.

The soft thresholding function is defined as

\[
\tilde{d}_{jk} = \begin{cases} 
0 & \text{if } |d_{jk}| \leq T \\
 d_{jk} - T & \text{if } |d_{jk}| > T \\
 d_{jk} + T & \text{if } |d_{jk}| > T
\end{cases}
\]  

Equation 12 indicates that the reconstructed signal faces oscillation, since the estimated wavelet coefficients are not continuous at position \( \pm T \) [31]. Although the estimated wavelet coefficients of Equation 13 have good continuity, these coefficients include constant errors [31], which directly influence the accuracy of the reconstructed signal.

### 3.4 An improved threshold function

To overcome the limitations of hard thresholding and soft thresholding denoising methods, an improved thresholding is proposed as follows:

\[
\tilde{d}_{jk} = \begin{cases} 
0 & \text{if } |d_{jk}| \leq T \\
 d_{jk} - \frac{T(e^{|d_{jk}|} - 1)}{2(e^{|d_{jk}|} + 1)} & \text{if } |d_{jk}| > T
\end{cases}
\]  

Equation 14 improves the reconstruction precision, since it reduces the constant errors. Hence, it enhances the denoising effect. This thresholding function also assures the continuity of estimated wavelet coefficients.

### 4 Simulation and performance assessment

The denoising of the received radar signal is simulated in the presence of white Gaussian noise. The effect of

![Figure 6 Plots of SNR versus detection range RCS = 1,000 m².](image)

Table 1 SNR of pulse radar and continuous wave radar

| Range in km | Pulse radar (matched filter) | CW radar (matched filter) | CW radar (traditional wavelet) | CW radar (proposed wavelet) |
|------------|-------------------------------|---------------------------|-------------------------------|----------------------------|
| 0          | 117                           | 107                       | 114                           | 119                        |
| 20         | 42                            | 30                        | 40                            | 45                         |
| 40         | 32                            | 17                        | 22                            | 25                         |
| 60         | 24                            | 10                        | 14                            | 16                         |
| 80         | 18                            | 6                         | 9                             | 11                         |
| 100        | 15                            | 1                         | 4                             | 5                          |
| 120        | 12                            | −3                        | 1                             | 2                          |

\( T \) is the threshold and generally can be a function of \( J \) and \( K \). The hard thresholding function is defined as

\[
\tilde{d}_{jk} = \begin{cases} 
0 & \text{if } |d_{jk}| \leq T \\
 d_{jk} & \text{if } |d_{jk}| > T
\end{cases}
\]  

Soft thresholding provides smoother results in comparison with the hard thresholding whereas thresholding technique provides better edge preservation in comparison with the soft thresholding technique.

Soft thresholding and hard thresholding have some limitations in denoising of signal [29,30]. The Equation 12 indicates that the reconstructed signal faces oscillation, since the estimated wavelet coefficients are not continuous at position \( \pm T \) [31]. Although the estimated wavelet coefficients of Equation 13 have good continuity, these coefficients include constant errors [31], which directly influence the accuracy of the reconstructed signal.
Figure 7: The noise reduction by traditional wavelet for continuous wave radar. (a) Continuous wave radar transmitted signal; (b) received signal (echo + noise), no attenuation; and (c) recovery signal from received signal by traditional wavelet.

Figure 8: Improvement in noise reduction by proposed wavelet. (a) Continuous wave radar transmitted signal; (b) received signal (echo + noise), no attenuation; and (c) recovery signal from received signal by proposed wavelet.
signal parameter changes on the algorithm has been investigated. These parameters include the SNR of the signal. The SNR is defined as the ratio of the signal power to the noise power in the entire period. The following parameters are assumed: sampling frequency = 10 GHz, pulse duration = 8 ns, pulse repetition frequency = 0.24 GHz for the pulse radar (Figure 2) and transmitted frequency = 10 GHz for the continuous wave radar (Figure 3). The receiver receives the return from the targets in the presence of AWGN.

We recover our transmitted signal from the received signal using the matched filter for both pulse radar and continuous wave radar. For the pulse radar, the recovery signal is almost similar to the transmitted signal shown in Figure 2c. But in the case of continuous wave radar, it is almost impossible to understand the shape of the transmitted signal from the recovery signal (Figure 3). Figures 4 and 5 are similar to Figures 2 and 3, respectively, except the received signals are attenuated more than 90%. Figure 4 indicates that the matched filter has a strong advantage in anti-noise ability for pulse radar. Unfortunately, the matched filter for continuous wave radar is not superior to recover the transmitted signal (Figure 5c). The reason for this is that the transmitted average power of the continuous wave radar is lower, and the thermal noise relevant to the operating bandwidth and the radar’s structure design, so the signal detection threshold of continuous wave radar is lower, and this demands higher receiver sensitivity (Table 1). This is shown in Figure 6 using Equations 9 and 10. Also, applying the matched filter for continuous wave radar limits the performance in three major areas [32]: the effect of nonlinearities in the frequency transmission and the matched filter, variations in the transmitted frequency, and relative velocity of target.

We also apply the wavelet denoising technique to remove the noise from received signal. This technique overcomes the problem of continuous wave radar. Figure 7c shows the noise reduction by traditional wavelet for continuous wave radar. By comparing this with Figure 3c, we can say that more noise is reduced in Figure 7c. Figure 8c shows the improvement in noise reduction by our proposed method. The reason for this is that the SNR is increased (Table 1) when we use the wavelet denoising technique for continuous wave radar. Our proposed wavelet threshold function even further reduced more noise for continuous wave radar. This is shown in Figure 9.

5 Conclusions
This paper presented a comprehensive comparison between matched filter and wavelet in terms of noise reduction abilities of pulse radar and continuous wave radar. The simulation results show that a significant reduction in noise is achieved for pulse radar by matched filter, but employing matched filter for continuous wave radar increases the difficulty of detection. The use of wavelet denoising technique instead of matched filter for continuous wave radar reduced more noise. Our proposed wavelet threshold function even further reduced more noise for continuous wave radar.

Competing interests
The authors declare that they have no competing interests.

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References

1. H Simon, L Xiao, Bo, Detection of signals in chaos. Proc. IEEE 83, 1 (1995)
2. TS Parker, LO Chua, Chaos: a tutorial for engineers. Proc. IEEE 75(8), 982–1008 (1987)
3. AV Oppenheim, GW Wornell, SH Isabelle, KM Cuomo, Signal processing in the context of chaotic signals, in Proc. ICASSP-92 vol. 4 (San Francisco, 1992), pp. 117–120
4. S Haykin, Chaotic signal processing: new research directions and novel applications, in IEEE Workshop on SSP (Victoria, 1992)
5. AS Grisipino, GO Petracca, AE Dominguez, Comparative analysis of wavelet and EMD in the filtering of radar signal affected by brown noise. IEEE Lat. Am. Trans. 11, 1 (2013)
6. MA Govoni, LI Hongbin, Range-Doppler resolution of the linear-Fm noise radar waveform. IEEE Trans. Aerosp. Electron. Syst. 49, 1 (2013)
7. G Kingsbury, Complex wavelets for shift invariant analysis and filtering of signals. J. Appl. Comput. Harmonic Anal. 10(3), 234–253 (2011)
8. D Geudtner, M Schwabich, R Winter, SAR-interferometry with ERS-1 data. Proc. PERS, 11–15 (1994)
9. ALB Candeias, LV Dutra, JC Mura, Interferogram phase noise reduction using morphological and modified median filters. Proc. IGARSS 1, 166–168 (1995)
10. JS Lee, Digital image enhancement and noise filtering by use of local statistics. IEEE Trans. Pattern Analysis Machine Intell. 2(2), 165 (1980)
11. Q Yu, X Yang, S Fu, X Liu, X Sun, An adaptive contoured window filter for interferometric synthetic aperture radar. IEEE Geosci. Remote Sens. Lett. 4(1), 23–26 (2007)
12. D Meng, Y Sethu, E Ambikairajah, L Gr, A novel technique for noise reduction in SAR images. IEEE Geosci. Remote Sens. Lett. 4(2), 165 (1980)
13. MD Saiful Islam, H Hyungsob, L Myung, J Gook, Small target detection and noise reduction in marine radar systems. IERI Procedia 51, 168–173 (2013)
14. DO North, An analysis of the factors which determine signal/noise discrimination in pulsed carrier systems. Proc. IEEE 51, 1016–1027 (1963)
15. WS Chen, IS Reed, A new CFAR detection test for radar. Digital Signal Process 1, 198–214 (1991)
16. FC Robey, DR Fuhrmann, EJ Kelly, R Nitzberg, A CFAR adaptive matched filter detector. IEEE Trans. Aerosp. Electron. Syst. 28(1), 208–216 (1992)
17. JR Roman, M Rangaswamy, DW Davis, Q Zhang, B Himed, JH Michels, Parametric adaptive matched filter for airborne radar applications. IEEE Trans. Aerospace Electron. Syst. 36, 2 (2000)
18. J Jeganathan, A Ghrayeb, L Szczecinski, Spatial modulation: optimal detection and performance analysis. IEEE Commun. Lett. 12(8), 545–547 (2008)
19. S Sugura, C Xu, SX Ng, L Hanzo, Reduced-complexity coherent versus non-coherent QAM-aided space-time shift keying. IEEE Trans. Commun. 59(11), 3090–3101 (2011)
20. Y Ping, X Yue, L Lei, T Qian, L Shaoqian, An improved matched-filter based detection algorithm for space-time shift keying systems. IEEE Signal Process Lett. 19, 5 (2012)
21. MD Saiful Islam, C Uljol, Detection of uncooperative targets using cross-correlation in oceanic environment. Int. J. Digital Content Technol. Appl. 7, 12 (2013)
22. H Sheng, Y Hongqi, T Wenhui, Z Zheng, Study on the auto-correlation and cross-correlation properties of hybrid bridge function sequence. Adv. Inf. Sci. Ser. Sci. 4, 7 (2012)
23. S Krill, E Ekaterina, S Boris, Echo delay estimation using algorithms based on cross-correlation. J. Convergence Inf. Technol. 6, 4 (2011)
24. MA Richards, Fundamentals of Radar Signal Processing, 1st edn. (McGraw-Hill, New York, 2005), pp. 88–91
25. L Yuan, Wavelet Analysis for Change Points and Nonlinear Wavelet Estimates in Time Series (Statistics Press, Beijing, 2001)
26. D Yinlong, L Yingmin, X Mengkui, M Lai, Analysis of earthquake ground motions using an improved Hilbert–Huang transform. Soil Dyn. Earthq. Eng. 28(1), 7–19 (2008)
27. Z Zhi Qiang, Z Guo Wei, P Yu, S Wei, L Cheng, L Jin Zhao, Study on pulse wave signal noise reduction and feature point identification. J. Convergence Inf. Technol. 8, 9 (2013)
28. DL Donoho, Denoising by soft-thresholding. IEEE Trans. Inf. Theory 41(3), 613–627 (1995)
29. RR Colman, DL Donoho, Translation-invariant de-noising, in Wavelets and Statistics, Springer Lecture Notes in Statistics 103 (Springer, New York, 1994), pp. 125–150
30. S Qin, C Yang, B Tang, S Tan, The denoise based on translation invariance wavelet transform and its applications. Conf. Struct. Dyn. Los Angeles 1, 783–787 (2002)
31. G Song, R Zhao, Three novel models of threshold estimator for wavelet coefficients, 2nd International Conference on Wavelet Analysis and its Applications (Springer, Berlin, 2001), pp. 145–150
32. MJ Withers, Matched filter for frequency-modulated continuous wave radar systems. Proc. IEEE 113, 3 (1966)