Thermodynamic model for lattice point defect-mediated semi-coherent precipitation in alloys

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The formation of precipitates with an atomic volume different from their parent phase eventually leads to a loss of the lattice continuity at the matrix-precipitate interface. Here, we show the creation or removal of lattice sites mediated by lattice point defects is an accommodation mechanism of the coherency loss and even a precipitation driving force. We introduce a thermodynamic approach that rationalizes the selection of phases resulting from chemical and crystallographic constraints in relation to point defect properties. The resulting semi-coherent phase diagram and the precipitation kinetic model depend on the equilibrium phase diagram, the eigenstrain of the precipitating phase, and the chemical potential of point defects. From a joint experimental and modeling study, we uncover the prominent role of excess point defects in unforeseen phase transformations of the Fe–Ni metallic system under irradiation. By addressing the fundamental role of lattice point defects in the accommodation mechanisms of precipitation, we provide a step towards the understanding of semi-coherent phase transformations occurring in solid materials upon synthesis and in use.

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Secondary phase precipitates contribute to the microstructure of a material. In presence of precipitates, a material never really reaches its equilibrium state. It evolves by getting around or overcoming energy barriers it encounters on its way to equilibrium. Energy barriers can be related to the nucleation of finite-size secondary phases, resulting from the balance between the volume precipitation driving force and the energetic cost of forming a nuclei/matrix interface. The system activates accommodation mechanisms that reduce the nucleation barrier by decreasing the interface energy. Such mechanisms usually decrease the bulk precipitation driving force. A well-known example is the generation of an elastic strain preserving the lattice continuity between the precipitating phase and the parent phase along with specific directions. When precipitates have a small eigenstrain (stress-free strain) and the same crystallographic structure as the parent one, an elastic strain accommodation mechanism may lead to fully coherent precipitate/matrix interfaces. As outlined in Fig. 1a, b, the obtained two-phase equilibrium domain is reduced by the constraint of coherency1,2.

Whenever the precipitate eigenstrain is not compensated by an elastic strain, precipitates feature a loss of coherency at the interface, i.e., the number of lattice sites across the interface is not conserved. It occurs through sequences of kinetic mechanisms involving lattice point and dislocation defects, such as the absorption of point defects condensing in precipitates and forming extra atomic planes (dislocations)3,4 (Fig. 1c). In Al–Cu, the nucleation of the incoherent $\theta^\prime$ phase on the coherent $\theta^{0}$ precipitates occurs through the injection of vacancies into $\theta^{0}$5 (as in Fig. 1d). In these examples, dislocations and point defects act as local sources and sinks of lattice sites. Thanks to their non-conservative nature, point defects may appear, transform, or annihilate. Although a thermodynamic formulation including defects is lacking, experimental observations clearly show that the selection of precipitating phases and their evolution is governed by the interplay between the chemistry of phases, interface constraints6,7, diffusion properties8–10, and lattice defects11,12. Vacancy as a thermodynamic protagonist has been invoked to explain phase transformations of irradiated systems12,13. Surprisingly, no one has ever considered the thermodynamic potential of the self-interstitial atom (SIA). In systems submitted to thermal quenching, severe mechanical solicitation, or irradiation, point defects are in excess. The removal of these point defects resulting from semi-coherent precipitation stabilizes the system, therefore increases the precipitation driving force12,14.

By introducing the concept of a semi-coherent phase diagram, we challenge the standard formulation of the phase diagram. We add to the fundamental Gibbs free energy of solid phases, an energy term modeling the interplay between point defects and the volume constraints of a finite-size precipitate. Either as a precipitation driving force (in case of an excess of point defects) or as an accommodation mechanism (in thermal conditions), the effect

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**Fig. 1 Concept of semi-coherent phase diagram accounting for the point defect accommodation mechanism.** Illustrations of (a) the final equilibrium state between infinite volume phases (blue and red domains), (b) a deformed precipitate (in red) in a supersaturated solution, with an elastic strain ensuring the continuity of the lattice at the precipitate/matrix interface, and reducing the two-phase domain of the phase diagram; (c, d) the semi-coherent precipitation in an undersaturated solid solution triggered by respectively an excess of self-interstitial atoms (SIA) and an excess of vacancies (Va). In (c), the precipitate eigenstrain is negative (dense precipitate). The transformation of SIAs into substitutional atoms of the precipitate increases the volume of the inclusion resulting from the phase transformation of the parent phase inclusion and decreases the excess Gibbs free energy $\Delta G_{\text{SIA}}$ generated by the excess of SIA. In precipitate, the extra atoms condense into finite-size atomic planes leading to misfit dislocations. In (d), the precipitate eigenstrain is positive (less dense precipitate), the exchange of Va with the atom of the precipitate reduces the volume of the transformed inclusion, and the excess Gibbs free energy $\Delta G_{\text{Va}}$ generated by the excess of Va. The excess of point defects enlarges the two-phase equilibrium domains so that a solid solution initially undersaturated in solute may become unstable with respect to phase separation.
of this interplay on the precipitation driving force, has been completely underestimated up to now.

Our focus is on the Fe–Ni system because it is an excellent model alloy of ferritic steels at low concentrations of Ni and of austenitic steels at a higher concentration of Ni, and its behavior under irradiation raises questions. To investigate properly the austenitic steels at a higher concentration of Ni, and its behavior model alloy of ferritic steels at low concentrations of Ni and of precipitation of austenite in ferrite.

Results

Experimental evidence of the role of point defects on the precipitation of austenite in ferrite. The investigated material is a bcc (body-centered cubic)-based Fe-3.3 at% Ni alloy of high purity. The evolution of the microstructural damage upon self-irradiation as a function of depth, was evaluated by high-resolution transmission electron microscopy (HRTEM) and plotted in Fig. 2. The irradiated sample exhibits a heterogeneous microstructure attributed to the spatial radiation dose variation. Interestingly, void density increases together with precipitate density and begins to drop when the precipitate density reaches a plateau.

Microstructural characterization of one face-centered cubic (fcc) γ precipitate is presented in Fig. 3 (HRTEM images of four other precipitates are presented in Supplementary Note 1 and Fig. 1). The fast Fourier transform (FFT) of the HRTEM image, in Fig. 3f, shows a co-linearity of both the close-packed directions [1 1 0]γ and [1 1 1]γ, indicating that plane remained invariant (not rotated) upon the phase transformation. The invariant-plane crystallographic model was thus developed assuming Kurjumov–Sachs orientation relationship between fcc precipitates and the bcc matrix. It reproduces the experimental FFT within 1% in length and within 7% in angle for the close-packed direction vectors (see colored circles superimposed on diffraction spots in Fig. 3f and Supplementary Note 2 and Table 1 for more details). We deduce, from the invariant planes’ reflections, the fcc/bcc ratio of the crystal lattice parameters: $\chi \equiv 1.247$. The latter is in very good agreement with the X-ray lattice ratio determined in macroscopic phases, spanning from $\chi = 1.249$ at 20 at% of Ni, to $\chi = 1.246$ at 50 at% of Ni in phase $\gamma$ at $T = 400 ^\circ C$. Hence, the elastic strain of the two-phase lattice is negligible. Fig. 3 also shows that twin-related Kurjumov–Sachs crystallographic variants formed to share a mirror symmetry that coincides with the $\{1 1 0\}_M$ twin-boundary plane. Figure 3c displays the experimental inverse FFT (iFFT) obtained from $\{1, 0, 1\}_M$, (0, 0, 2)$_{\gamma}$, and (1, 1, 1)$_{\gamma}$ reflections, where Moiré fringes’ width and misfit dislocations’ periodicity along with the interfaces (in red) can be easily inferred. Misfit dislocations can also be seen on the direct HRTEM images (Fig. 3b). The crystallographic model assumes a perfect fcc lattice within both variants of the precipitate. Consequently, the good agreement between this model and the HRTEM observations on both iFFT and the misfit dislocation periodicity tends to prove that the eigenstrain has been compensated by the creation of fcc lattice sites in the precipitate. Based on our crystallographic model, we estimate the atomic fraction of these extra sites $\nu_1$ (cf Eq. (1) of the “Methods”). For a Ni content of $y$ around 30 at%, $\nu_1$ is equal to $\nu_1 = 1.5%$.

As the dislocation line density is very low ($<10^3 \text{cm}^{-2}$) and dislocation loops absent in the precipitate region (Fig. 2a), dislocations are unlikely involved in the mechanism of coherency loss, at least when precipitates grow. These extra sites might result from the transformation of SIAs of the host matrix into substitutional atoms of precipitates. A scheme of this mechanism is given in Fig. 1c, where one SIA attached to a single lattice site of the parent phase is transformed into two atoms seating on two perfect lattice sites of the precipitate when transferred. Besides, voids coexist with precipitates. Their growth with radiation dose proves that they absorb more vacancies than SIAs. The mass balance implies precipitates absorb SIAs.

Dislocation loops take a part in the nucleation process of precipitates. Indeed, in the lower dose region, (<1 dpa), there are dislocation loops and no precipitates. Radiation-induced segregation takes place through point defect and solute flux coupling, leading to solute enrichment on dislocation loops. They contain up to 17 at% Ni. By acting as local reservoirs of strain, lattice sites, and Ni atoms, Ni-decorated loops likely facilitate $\gamma$ phase nucleation, similarly to what occurs in a complexion-mediated martensitic phase transformation.

Misfit dislocations certainly control the shape of precipitates. If we assume that interface energies are proportional to the number of misfit dislocations, we may elucidate their shape by relying on a 2D-Wulff construction (Fig. 3e). (121)$_M$ planes (in green) are...
Fig. 3 The demonstration that additional lattice sites are created within semi-coherent precipitates of irradiated Fe-Ni. a HR-TEM image of the fcc variants formed under irradiation at 400 °C at a flux of 9.6 × 10⁻⁶ dpa s⁻¹ within the Fe-3.3 at% Ni bcc matrix. fcc variants and bcc matrix are respectively in [110] and [111] zone axis. The schematic shape of the precipitate is superimposed. Interfaces with the invariant plane orientation, (110)ₘ, are in violet, (112)ₘ and (110)ₘ orientated interfaces are respectively in green and black. b Enlargement of two series of dislocation misfits between (A1) the (011)ₘ matrix planes and the (111)ₐ planes of variant 1 (half top of the precipitate); (A2) the (011)ₘ matrix planes and the (002)ᵥ planes of variant 1 (half top of the precipitate); (B1) the (011)ₘ matrix planes and the (002)ᵥ planes of variant 2 (half bottom of the precipitate); (B2) the (011)ₘ matrix planes and the (112)ᵥ planes of variant 2 (half bottom of the precipitate). c Inverse Fast Fourier Transform (iFFT) of the (101)ₘ matrix planes and (002)ᵥ planes of variant 1 (half top of the precipitate in pink) and the (112)ᵥ planes of variant 2 (half bottom of the precipitate in violet). d, e Simulated iFFT corresponding to the image (c). The representation of (101)ₘ matrix planes, (002)ᵥ planes of variant 1 and (112)ᵥ planes of variant 2 in black, pink, and violet, highlight the atomic plane discontinuities, which may lead to the formation of dislocation misfits at interfaces (the extra planes are indicated as red lines and discontinuities at interfaces as red open circles). The superposition of the same matrix and precipitate planes yields Moiré fringes, whose periodicity along an interface corresponds to the dislocation misfit periodicity. f Superposition of the experimental and simulated FFT of both the matrix and the precipitate. The Wulff-radius of interface normalized by the (110)ₘ interface radius is equal to: E₁₁₁ = E₂₁₁ = 0.6, and E₁₁₂ = E₁₁₁ = E₁₀₁ = 1.5.

Fe-Ni phase diagram under an excess of self-interstitials. As mentioned previously, when lattice deformation is not sufficient to compensate for the eigenstrain, removal or creation of lattice sites operates. This mechanism can be achieved by point defect reactions between the parent and the precipitating phases. If point defects are in excess, their chemical potentials are non-zero and fixed by external forces. Removing point defects through an accommodation mechanism of precipitation strongly reduces the overall free energy, hence increases the precipitation driving force. When initially there is no excess of point defects, the driving force of a thermal phase decomposition mediated by the production of point defects is reduced, similarly to what occurs in elastic-constrained coherent precipitation. A point defect-mediated precipitation driving force is, thereby, either increased or decreased by an energetic term proportional to the point defects chemical potential multiplied by the eigenstrain (see the “Methods”). Consequently, not only are the phase boundaries modified, but a non-linear variation of either point defect chemical potential or eigenstrain with composition may cause the formation of unexpected phases.

In Fe-Ni systems, radiation-induced phase transformations in both ferrite and austenite systematically involve the precipitation of denser precipitates. Therefore, we assume the eigenstrain is compensated by the creation of lattice sites with an atomic favored against (110)ₘ (in magenta) and (011)ₘ planes (in black): their interface energy, normalized by the (110)ₘ one, are E₁₂₁ = E₂₁₁ = 0.7 and E₀₁₁ = 1.6. These ratios are very close to the experimental ones (Fig. 3c). In particular, the low energy of the (211)ₘ and (121)ₘ interface planes might be at the origin of their large contribution to the precipitate/matrix interface, leading to the strongly peaked shape of the precipitates along the [112]ₘ direction.

Twelve proxigrams obtained from atom probe tomography (APT) volumes, among which two are represented in Fig. 4, reveal two Ni-rich γ precipitates (~45 at% Ni), together with ten Ni-poor γ precipitates (~29 at% Ni). According to the equilibrium phase diagram in Fig. 5a, at the irradiation temperature of T = 400 °C, the alloy is undersaturated and should not decompose, and the γ phase, which would thermally precipitate in a super-saturated phase α has a Ni content of around 47 at%. The average Ni composition of the matrix close to the precipitate/matrix interface is 1.55 ± 0.6 at%, which is far below the thermal equilibrium solubility limit of 9 at% Ni, whereas the interface composition should be close to the solubility limit. In parallel, a Fe-1 at% Ni alloy submitted to the same irradiation conditions shows no phase decomposition. Hence, we expect the metastable solubility limit of Ni to be ~1.5 at%.
fraction of $\nu_{I}$ through the transformation of SIAs into substitutional atoms of the precipitating phase. As explained in “Methods”, we calculate constrained Gibbs free energies and deduce metastable phase diagrams taking into account the contribution of SIA reactions.

The most striking feature is the apparition of an austenitic Fe-rich metastable phase, $\gamma_{m}$, an fcc solid solution with a Ni atomic fraction roughly equal to $c_{Ni} \approx 25-30\%$ (Fig. 5). This composition is related to the atypical INVAR properties of the $\gamma$ lattice parameter whose composition is plotted in Fig. 5a. At the INVAR composition, $c_{Ni} \approx 35\%$, the lattice parameter is maximum. When decreasing the Ni content from the position of the lattice maximum, the lattice parameter falls into a local minimum, whose position is $c_{Ni} \approx 23\%$ at $T = 100^\circ$C, and slightly shifting to higher nickel content with respect to temperature. This minimum produces a local maximum of the absolute eigenstrain (or equivalently $\nu_{I}$ in Fig. 5a). The greater the absolute eigenstrain, the greater the amount of absorbed SIAs by precipitate and the released Gibbs free energy. This produces a local minimum of the constrained free energy, leading to the formation of $\gamma_{m}$ (Figs. 5b).

Under irradiation, whether at low and high temperature, in bcc or fcc matrix, there has been experimental evidence of this metastable phase. The precipitation of a Fe-rich $\gamma$ phase was observed during ferrite-to-austenite phase transformations (ref. 16 and Fig. 4), high-temperature decomposition of $\gamma_{m}$15,26, or low-temperature $L1_0$-ordering of austenite15,27.

The ternary phase diagram of Fig. 5c calculated at $T = 400^\circ$C, highlights the formation of two-phase equilibria involving the metastable phase $\gamma_{m}$; the $\gamma_{m}$-$\gamma$ equilibrium at low concentration of SIAs, the $\alpha$-$\gamma_{m}$ and $\gamma_{m}$-$L1_2$ equilibria at higher supersaturation of SIAs. The supersaturation of SIAs, as well as phase thermodynamics and lattice parameters (Fig. 5a), vary with temperature. From a diffusion-reaction model of SIAs (cf “Methods” and the legend of Fig. 6), we relate the supersaturation of SIAs to the temperature and the flux of irradiation. The resulting metastable phase diagrams together with the equilibrium phase diagram are presented in Fig. 6.

In particular, Fig. 6 demonstrates that the excess of SIAs produced by irradiation induces the apparition of $\gamma_{m}$ through the decomposition of austenite $\gamma$ at high temperatures. Although our objective is not to model the kinetics of decomposition of the austenite, our continuous formulation of the constrained Gibbs free energy of phase $\gamma$ (Fig. 4b) gives indications on the range of temperature and composition for which we expect a spinodal-like decomposition regime. In this regime, the variation of the composition of phases is continuous, up to their metastable equilibrium values. The spinodal limits that separate the kinetic domains between the nucleation-and-growth and the spinodal regimes are given by the zero second derivatives of the constrained Gibbs free energy.28 Whereas phase boundaries and the spinodal domain do not vary much around the eutectic temperature of the $\gamma_{m}$-$L1_2$ equilibrium, the height of the miscibility gap is largely increased with the irradiation flux (Fig. 6b, d). Spinodal decomposition was observed in irradiated solid solutions at temperatures between 450 and 725°C and nearby the INVAR composition.15 This decomposition systematically features a periodic composition modulation along the [100] direction. It spans from 25 to 50% Ni with a dominant wavelength on the order of hundreds of nanometers, whatever the temperature and the incident particle (neutron, or ion). The [100] direction is the preferential direction of decomposition in cubic crystals with coherency constraints at the precipitate/matrix interface.29 Therefore, we expect, at an early stage of decomposition, an elastic strain partially releasing the eigenstrain, and ensuring a lattice coherency at the (100) interface (cf “Methods”). Applied to our study, simulations based on the [100] direction of decomposition effectively show that this spinodal decomposition still occurs when taking into account the coherency constraint, as reported in Fig. 6c. In this system, the elastic contribution to the Gibbs free energy is small compared to the SIA-releasing energy. Hence, adding an elastic constraint is almost equivalent to reducing the volume mismatch contribution, thereby the radiation flux, as shown by the similarities between the low flux (Fig. 6b) and high flux-coherent (Fig. 6c) phase diagrams. Ignoring the coherency constraint enlarges the two-phase domain.
Therefore, we expect, at a higher dose where such coherency is lost, an increase of the separation driving force. In Fig. 6c, the minimal and maximal compositions of the measured modulation (empty blue squares) are close to the calculated lower and upper boundary limits of the $\gamma_m$-coherent phase domain.

Modeling of the ferrite to austenite phase transformation kinetics. As shown in Fig. 6, irradiation below $T = 700$ °C largely decreases the solubility limit of Ni in the $\alpha$ phase. At temperatures above $T = 700$ °C, there is no experimental data for the $\gamma$ phase. Nevertheless, an extrapolation of the fcc lattice parameter from lower temperature lattice parameters, predicts a supersaturated ferritic phase might decompose into a bcc phase $\alpha$ more concentrated in Ni than the equilibrium one. This is mostly due to the high increase of the lattice parameter at low Ni content. In the undersaturated solid solution Fe-3.3 at% Ni, at $T = 400$ °C, irradiation caused the austenitic phase precipitation. The bulk composition in most precipitates is $\approx 29$ at%, rather close to the calculated composition of the metastable $\gamma_m$ phase ($\approx 23$ at% Ni).

(Fig. 6d). Therefore, we expect, at a higher dose where such coherency is lost, an increase of the separation driving force. In Fig. 6c, the minimal and maximal compositions of the measured modulation (empty blue squares) are close to the calculated lower and upper boundary limits of the $\gamma_m$-$\gamma$ coherent phase domain.

Fig. 5 The formation of the metastable phase $\gamma_m$ stems from the nonlinear variation of its lattice parameter vs. Ni composition. a Lattice parameter $a_\gamma$ of phase $\gamma$ and related atomic fractions of SIAs, $\nu_I$, deduced from $a_\gamma$ and the bcc lattice parameter of phase $\alpha$ at $c_{\alpha Ni}^0 = 0.01$, with respect to the atomic fraction of Ni and temperature. Fifth order polynomial interpolation of the X-Ray experimental lattice parameter of phase $\gamma$ at $T = 15, 100, 200, 300, 400, 500, 600, 730$ °C (crosses are extracted from ref. 17 and squares from ref. 47). Dashed lines are obtained from a linear extrapolation of the polynomial coefficients between the two previous temperatures. Based on experimental X-ray data, we assume the lattice parameter of the L1$_2$ ordered phase is the $\gamma$ one. Composition interpolation of the X-ray fcc lattice parameter of phase $\alpha$ is adjusted on a second-order polynomial, $a_\alpha(T = 300 \text{ K}) = 0.286654 + 0.007055 c_{\alpha Ni} - 0.0359296 (c_{\alpha Ni})^2$. Its temperature variation is deduced from an average thermal expansion coefficient, $\alpha = 12 \times 10^6 \text{ K}^{-1}$. Gibbs-associated grand potential with respect to the atomic fraction of Ni at $T = 400$ °C, at various supersaturation of SIAs, $s = c_I / c_{eq}^\alpha$, $s = 10^{18}$ (in green, corresponding to the averaged experimental radiation flux), $s = 10^8$ (in blue), $s = 10^4$ (in magenta). The metastable phase $\gamma_m$ associated with a local minimum of the grand potential comes from the local minimum of $a_\gamma$ just below the INVAR composition.

c Ternary phase diagram representing the phase boundary limits with respect to the atomic fraction of Ni and the supersaturation of SIA at $T = 400$ °C. Since the atomic fraction of SIAs is very small, we directly deduce the atomic fraction of Fe ($c_{\alpha Fe}^0 = 1 - c_{\alpha Ni}^0$). Thick black lines indicate three-phase constrained equilibria and colored stars are the supersaturations used in panel (b).
Ferrite decomposition does not follow a spinodal-like but rather a nucleation and growth kinetic regime, since the product fcc phase has not the same crystallographic structure as the parent one. The last step undertaken in this work aimed at retrieving numerically the growth rate of γ precipitates. We developed a diffusion-reaction model to compute precipitate and void volume fractions (cf "Methods"). This kinetic model relies on the constrained relationship between solute and SIAs precipitation driving forces, which is mainly controlled by the number of transformed SIAs into substitutional atoms of the precipitates (equal to $v_1$ multiplied by the number of precipitate lattice sites). First, regarding SIAs, as mentioned previously, the precipitation driving force according to Fig. 5a, at $T = 400^\circ C$ (green curve), $v_1$ is almost constant ($v_1 \approx 0.015$) between 25 and 50 at% Ni. Considering the distribution of precipitates (Fig. 2a) together with their $v_1$-dependent sink efficiency (Eq. (10)), we could determine the SIA stationary atomic fraction ($c_{I}^s \approx 10^{-13}$ at%) and thereby, the metastable solubility limit of Ni: $c_{Ni}^{eq} \approx 1.5$ at% (dashed red lines in Figs. 2b and 4). This value is in excellent agreement with the APT estimation of Ni composition at the matrix/precipitate interface (dotted red line in Fig. 4). Note that the increase of $c_{I}^s$ with the radiation flux is too little to have an effect on $c_{Ni}^{eq}$.

Eventually, the growth rate of precipitates mainly depends on (i) the radiation dose, (ii) the excess of Ni with respect to the metastable solubility limit, and (iii) a ratio of diffusion coefficients. The latter, according to point defect properties in Fe–Ni, comes down to the ratio of diffusion coefficients of Ni and vacancy since Ni diffusion is not mediated by SIAs at all. From a matter balance, we relate the evolution of void volume fractions to the growth of precipitates. Figure 2b reports the good agreement between the simulated (solid lines) and measured (squares and triangles) growth of precipitates and voids. From an estimation of the initial precipitate volume fraction (1 dpa) and considering the amount of Ni atoms already distributed in precipitates at this starting observation dose, we set the initial Ni composition of ferrite to 2.7 at%. The model reproduces the rate of precipitate growth and the saturation dose at which the growth stops, though the simulated kinetics are slightly faster. This might be due to an overestimation of the vacancy concentration. Part of vacancies might be consumed to release the interface constraints, leading to a reduction of the point defect absorption bias between precipitates and voids and a slow down of the kinetics.

**Discussion**

In the present work, we highlight the tight relation between the point defects and atoms’ fate. This relation lasts as long as there is an excess of solute atoms in the matrix. SIAs are usually much more super-saturated than vacancies due to their higher formation energy. Thus, the precipitation of dense phases followed by the precipitation of low-density phases might occur, resembling the usual successive formation of SIA dislocation loops followed by voids in irradiated samples. Among radiation damage phenomena to be revisited, let us mention swelling and the ones naturally related to a phase transformation such as stress corrosion cracking and amorphization. The primary mechanism of swelling is the preferential elimination of SIAs at dislocations which make them climb and form new lattice sites. As long as SIAs are preferentially absorbed by dense precipitates, the creation of lattice sites does not lead to an increase of the total volume, consequently, no onset of swelling. This may explain the superior swelling resistance of bcc materials compared to fcc ones. Indeed, the open crystallographic structure of a bcc lattice widens the possibilities of hosting SIAs, that is, forming high-density precipitates. Precisely, a number of puzzling phase transformations occur in bcc nuclear materials. Let us mention...
the surprising sequence of intermetallics precipitation in both W–Re\textsuperscript{13} and ferritic-martensitic steels\textsuperscript{34–37}.

Conclusions
Building semi-coherent phase diagrams and deducing kinetic laws of precipitation can now be intended using the constrained thermodynamic model developed in the present work. Our results on the Fe–Ni system demonstrate that specific partitioning of atoms and point defects between phases may move the phase boundary limits. Furthermore, it may cause the precipitation of chemical phases which do not even exist in equilibrium phase diagrams, when the variation of the phase lattice parameter or the formation energy of point defects with composition is non-monotonous.

We could produce constrained phase diagrams dedicated to successive kinetic stages of the phase transformation, an early-stage decomposition phase diagram accounting for an elastic coherency constraint along a specific lattice direction of the matrix, then a later-stage decomposition phase diagram releasing the elastic coherency constraint. Though in the present work, the thermodynamic model has been applied to semi-coherent precipitation of dense phases promoted by supersaturation of SIAs, it can be applied to the precipitation of phases less dense than the matrix promoted by supersaturation of vacancies and extended further to systems with initially no excess of point defects. Whenever semi-coherent precipitation involves a driving force well above the point defect formation enthalpy, point defects can be emitted from precipitates subsequent to a thermal phase transformation, as evidenced in semi-conductors\textsuperscript{38,39}. The newly point defect generated microstructure may have a dramatic impact on both functional and structural properties of materials, as for example the generation of dislocation loops leading to light-induced degradation of solar cells\textsuperscript{40}. Sources of point defects such as surfaces and dislocations, by reducing the point defect formation enthalpy and their characteristic time to reach the precipitates, could also enhance semi-coherent precipitation. We expect this mechanism to be totally relevant in nanoscale materials with precipitates systematically surrounded by surfaces\textsuperscript{8}.

To go further, we should consider the possibility of forming non-equilibrium phases including point defects such as the ones discovered by systematic ab-initio searches of chemical phases\textsuperscript{37,41}. We could add more complex accommodation mechanisms involving for example the punching of dislocations loops\textsuperscript{3,4}.

We are only at the beginning of paving the way to incorporate kinetic constraints and accommodation mechanisms in constrained phase diagrams.

Methods
Crystallographic model
We represent the distortion of the bcc parent phase during the plane strain transformation, with the matrix \(P_L\), \(P_S\) is the product of a Bain strain matrix and a rotation matrix keeping the initial orientation of the (1, 1, 0)\textsubscript{bcc} atomic plane and the \(A = [1, 1, 1]_{\text{hkl}}\) direction\textsuperscript{1}. This transformation leads to the first variant of a twin-variant precipitate. The second variant has the same invariant plane and direction, but the Bain transformation is made along another axis of the cubic axis, leading to the strain transformation matrix, \(P_S\). Both variants share a twin-boundary and have a Kurjumov–Sachs orientation relationship with the atomic lattice of the host matrix (see Supplementary Note 2 for more details). From the calculation of the pure strain component of the strain-rotation operations leading to variants 1 and 2, we obtain the volume mismatch of the precipitate. If there is no elastic strain, it exactly corresponds to the atomic fraction of additional sites \(v_1\) of the precipitates

\[
\nu_1 = \frac{1}{2} \left( \alpha_3 \alpha_3 \alpha_F \right)^2 + \frac{1}{2} \alpha_3 \alpha_F - 2. \tag{1}
\]

We compute the diffraction pattern produced by variant 1 of a precipitate by applying the matrix \(P_L\) to the reciprocal lattice vectors of the matrix. We obtain the Moiré fringe periodicity and the density of misfit dislocations produced by a reciprocal lattice vector \(H^\text{ML}\) and its variant \(P_L H^\text{ML}\) by computing \(G^\text{ML} = H^\text{ML} - P_L H^\text{ML}\). The modulus of the inverse of \(G^\text{ML}\) is equal to the value of the fringe

\[
\delta_\text{fringe} = -N \mu_f. \tag{6}
\]
where \( N_{\gamma} \) is the number of lattice sites of the precipitated phase \( \gamma \). We then obtain the phase boundary limits by applying the common tangent construction between this effective Gibbs free energy of phase \( \gamma \) and the Gibbs free energy of phase \( \alpha \). A common tangent construction is equivalent to subtracting from the free energies the equation of a straight line until the minima of the phases are aligned on a horizontal line. The slope of this straight line corresponds to the chemical potential and the aligned free energies are here called grand potentials (Fig. 5b).

**Concentration of point defects in Fe–Ni under irradiation.** Under irradiation, Frenkel pairs are created with a production rate \( S \), the so-called radiation flux in dpa s\(^{-1}\). Vacancies and SIAs of the Frenkel pairs may mutually recombine and annihilate at the rate \( R_{sv} \), or diffuse to form point defects clusters such as dislocation loops and voids, or solute-SIA \( \gamma \) precipitates\(^{44} \). Voids and precipitates when formed, become the primary point defects sinks in Fe–Ni. Far from the point defects sinks and assuming steady-state fluxes, we may write the diffusion equation of SIAs as a reaction equation

\[
\frac{dc_\gamma}{dt} = S - R_{sv} - (k_{p1} + k_{c1})dc_\gamma ,
\]

where \( k_{p1} \) is the diffusion coefficient of SIA, \( k_{c1} \) \( \gamma \) and \( k_{c1} \) \( \gamma \) are respectively the sink strengths of precipitates and voids. Note that, we ignore the elimination of SIA at dislocation loops because their corresponding sink strength is very small. Although, precipitates of phase \( \gamma \) are not spherical, for the sake of simplicity, we rely on the spherical model of point defects sinks to write the sink strength of both the voids and the precipitates

\[
k_{p1} = k_{p1}^\gamma Z_\gamma ,
\]

\[
k_{c1} = k_{c1}^\gamma Z_\gamma ,
\]

with

\[
k_{p1}^\gamma = 4\pi R_p^3 \rho_p .
\]

\[
k_{c1}^\gamma = \frac{Z_\gamma}{\gamma} \left( \frac{\alpha_{p,V}}{\gamma} \right) = \frac{Z_\gamma}{\gamma} \left( \frac{\alpha_{p,V}}{\gamma} \right) .
\]

where \( \alpha_{p,V} \) is the composition of the \( \gamma \) precipitates. When the solubility limit is reached, we have \( Z_{\gamma} = Z_{\gamma}^{eq} \), where \( Z_{\gamma} \) is the sink efficiency of precipitates with respect to the vacancy. Note that the latter is different from because the precipitate-matrix interfaces at the precipitate tips are semi-coherent and probably act as a recombination center between SIA and vacancies, maintaining a very low concentration of vacancies and SIA at the semi-coherent interface. Hence, we may write for the vacancy rate equation

\[
\frac{dc_\gamma}{dt} = S - R_{sv} - k_{p1}^\gamma Z_\gamma V \alpha_{p,V} \alpha_{\gamma,V} ,
\]

we assume the kinetics of SIA and vacancy are much more rapid than the solute one. Therefore, we set to zero Eqs. (7) and (12). From Eq. (7), and by assuming recombination reactions are negligible, we deduce the stationary SIA atomic fraction

\[
\alpha_{\gamma,V} = \frac{S}{k_{p1}^\gamma Z_\gamma V \alpha_{p,V}} ,
\]

and from the difference of Eqs. (7) and (12), the stationary atomic fraction of vacancy

\[
\alpha_{p,V} = \frac{S}{k_{c1}^\gamma Z_\gamma V} ,
\]

For the sake of simplicity, we assume \( Z_{\gamma} \equiv 1 \), as if the overall precipitate interface was acting as a perfect sink for the vacancy.

**Growth rate of precipitates.** The rate equation associated with the matrix solute content reads

\[
\frac{dc_p}{dt} = -k_{p1}^\gamma \left( c_{p,V}^{eq} + c_{p,V}^{eq} \right) c_\gamma - c_{p,V}^{eq} ,
\]

where \( d_1 \) and \( d_0 \) are the solute diffusion coefficients respectively mediated by SIA. Note that we ignore the effect of an SIA excess on the diffusion driving force of solutes towards precipitates because its atomic fraction never exceeds the ppm, hence its effect on the solute chemical potential is very small. We ignore as well the drag of solutes by vacancies induced by a flux coupling mechanism because the flow of solute driven by the precipitation driving force is a lot greater. Based on the assumption that every solute atom leaving the parent phase contributes to the growth of precipitates, we deduce the evolution of the precipitate volume fraction \( f_p \)

\[
\frac{df_p}{dt} = -\frac{1}{\gamma} \frac{df_{p1}}{dt} .
\]

Since the kinetics of precipitates and voids steadily follows a matter balance, we easily relate the growth rate of voids to the growth rate of precipitates

\[
\frac{df_p}{dt} = \frac{df_v}{dt} .
\]

The number of vacancies contributing to the growth of voids per time unit is equal to the number of SIAs absorbed by the precipitates per time unit.

**Point defect data of Fe–Ni.** The most stable configuration of SIA in Fe–Ni is a split-interstitial configuration, also called dumbbell configuration. Relying on DFT calculations of a dumbbell in pure Fe, their formation enthalpy is \( H_{p,V} = 3.96 \text{ ev} \), their migration enthalpy is \( E_{m,V} = 0.34 \text{ eV} \), and the prefactor of the interstitial diffusion coefficient is \( d_1 = 3.3 \times 10^{16} \text{ m}^{2} \text{s}^{-1} \). We do not account for the effect of Ni on the formation and migration energies because the interaction of the dumbbell with Ni is negligible\(^{21} \). Hence, we obtain at \( T = 400^\circ \text{C} \), \( d_1 = 8.9 \times 10^{-9} \text{ m}^2 \text{s}^{-1} \) and \( d_0 \approx 0.0 \). We ignore the impact of Ni on the diffusion of vacancies and from a DFT diffusion database\(^{46} \), we obtain at \( T = 400^\circ \text{C} \), \( d_{v,V} = 5.6 \times 10^{-11} \text{ m}^2 \text{s}^{-1} \) and \( d_{v,V} = 5.5 \times 10^{-12} \text{ m}^2 \text{s}^{-1} \). Due to a strong thermodynamic and kinetic interaction between vacancies and Ni, the vacancy diffusion coefficient is very close to the Ni one.

**Experimental details.** The Fe-3.3 at% Ni alloy was continuously irradiated for 45 h at 400\(^\circ \text{C} \) at the JANNUS-Saclay facility. In total, 27 MeV Fe\(^{2+} \) ions with a flux of \( 8.2 \pm 2.2 \times 10^{10} \text{ ions cm}^{-2} \text{s}^{-1} \) up to a fluence of \( 1.4 \pm 0.4 \times 10^{10} \text{ ions cm}^{-2} \) were used. The damage profile was calculated with the stopping ranges of ions in matter (SRIM) 2008 code using Kinchin–Pease option with a displacement threshold of 40 eV. APT needles and TEM thin foils were prepared using a focused ion beam equipped on an FEI Helios 650 NanoLab dual-beam scanning electron microscopy. APT needles were taken out at a depth of 750 nm corresponding to a radiation dose rate of 7.41.9 \times 10^{-6} \text{ dpa s}^{-1} \) and at 1200 nm with a radiation dose rate of 1 \text{ 1.9 \times 10^{-5} dpa s}^{-1} \). HRTEM images were performed with a JEOL ACCEL ARM 200F Cs image corrected operating at 200 kV equipped with a Camera GATAN OneView. The APT analysis was performed using a CAMECA LEAP 4000XHR at a set-point temperature of 50 K in laser-pulsing mode at a wavelength of 382 nm, 200 kHz pulse repetition rate, and 26 p) pulse energy. For 3D atom reconstruction, visualization, and data post-treatments, the software IVAS by CAMECA was employed. Atomic fraction profiles at interfaces were obtained with a 5 nm diameter cylinder using the algorithm 1D concentration profile with a fixed bin of 0.5 nm.

**Data availability**

The raw image files of Figs. 3 and 4, and the data that support the findings of this study, are available on request.

**Code availability**

Our code computing the phase diagram from a Calphad database and the point defect thermodynamic driving force is available on reasonable request.

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