Fixing the EW scale in supersymmetric models after the Higgs discovery.

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Abstract

TeV-scale supersymmetry was originally introduced to solve the hierarchy problem and therefore fix the electroweak (EW) scale in the presence of quantum corrections. Numerical methods testing the SUSY models often report a good likelihood \( L \) (or \( \chi^2 = -2 \ln L \)) to fit the data \textit{including} the EW scale itself \( (m_0^2) \) with a \textit{simultaneously} large fine-tuning i.e. a large variation of this scale under a small variation of the SUSY parameters. We argue that this is inconsistent and we identify the origin of this problem. Our claim is that the likelihood (or \( \chi^2 \)) to fit the data that is usually reported in such models does not account for the \( \chi^2 \) cost of fixing the EW scale. When this constraint is implemented, the likelihood (or \( \chi^2 \)) receives a significant correction \( (\delta \chi^2) \) that worsens the current data fits of SUSY models. We estimate this correction for the models: constrained MSSM (CMSSM), models with non-universal gaugino masses (NUGM) or higgs soft masses (NUHM1, NUHM2), the NMSSM and the general NMSSM (GNMSSM). For a higgs mass \( m_h \approx 126 \) GeV, one finds that in these models \( \delta \chi^2/n_{df} \geq 1.5 \) (\( \approx 1 \) for GNMSSM), which violates the usual condition of a good fit (total \( \chi^2/n_{df} \approx 1 \)) already before fitting observables other than the EW scale itself (\( n_{df} = \)number of degrees of freedom). This has (negative) implications for SUSY models and it is suggested that future data fits properly account for this effect, if one remains true to the original goal of SUSY. Since the expression of \( \delta \chi^2 \) that emerges from our calculation depends on a familiar measure of fine-tuning, one concludes that fine-tuning is an intrinsic part of the likelihood to fit the data that includes the EW scale \( (m_0^2) \).

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1 A correction to the likelihood from fixing the EW scale.

The main motivation for introducing low-energy (TeV-scale) supersymmetry (SUSY) was to solve the hierarchy problem and therefore fix the electroweak (EW) scale which remains stable under the addition of quantum corrections up to the Planck scale. This is achieved without a dramatic tuning of the parameters specific to the Standard Model (SM). SUSY is now under intense scrutiny at the LHC and the recent Higgs-like particle discovery of mass $m_h$ near 126 GeV brings valuable information on the physics beyond the SM. In this work we consider popular SUSY models and investigate the impact on their data fits of the requirement of fixing the EW scale (vev or $m_Z$), that SUSY was supposed to enforce, for the recently measured value of $m_h$.

Current numerical methods that test SUSY models against experimental data involve a remarkable amount of technical expertise and work, backed by impressive computing power, see for example [3, 4, 5, 6, 7]. Somewhat surprisingly, in the case of a "frequentist" approach that we discuss in this work, there seems to be a problem that is overlooked when computing the likelihood $L$ (or $\chi^2 \equiv -2 \ln L$) to fit the data. Let us detail. Numerical methods sometimes report a good likelihood (or $\chi^2$) to fit the data that includes the EW scale itself (mass of Z boson, $m_Z^0$), and also a simultaneous large fine tuning, i.e. a large variation of this scale under a small, fixed variation of SUSY parameters ($\gamma_i$) of the model. This suggests an inconsistency. To see this, consider a Taylor expansion of the theoretical value of $m_Z$ about its very well measured value ($m_Z^0$):

$$m_Z = m_Z^0 + \left( \frac{\partial m_Z}{\partial \gamma_i} \right)_{\gamma_i = \gamma_i^0} (\gamma_i - \gamma_i^0) + \cdots$$  \hspace{1cm} (1)

The values $\gamma_i^0$ correspond to the EW ground state, with $\gamma_i$ the SUSY parameters that define the model, such as $m_0$, $m_{1/2}$, $\mu$, $A_0$, $B_0$, $\cdots$, in a standard notation. A large fine-tuning means a large partial derivative, then $m_Z - m_Z^0$ suffers a large variation and that impacts significantly on the value of total $\chi^2$. So one should expect a poor fit in the models with large fine-tuning, however a good fit $\chi^2/\text{ndf} \approx 1$ is often reported in these models.

How do we clarify this puzzle? ($\text{ndf} = $ number of the degrees of freedom).

Our claim, that answers this issue, is that in such cases the currently reported likelihood to fit the data (or its $\chi^2$) does not account for the $\chi^2$ "cost" of the condition of fixing the EW scale to its well-measured value (and that motivated the initial idea of supersymmetry). This leads to an underestimate of the overall value of $\chi^2/\text{ndf}$ in all popular models used at

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1 A good fit should have $\chi^2/\text{ndf} \approx 1$, where $\text{ndf} \equiv n_O - n_p$, where $n_O$ is the number of observables fitted and $n_p$ is the number of parameters. For a set of observables $O_i$, one has total $\chi^2 = \sum_i (O_i - O_i^{\exp})^2/\sigma_i^2$.

2 In the following we do not use a particular mathematical definition of fine-tuning.

3 In some studies $m_Z$ is an input, so fixing the EW scale is expected to be respected, but the argument based on eq. (1) suggests otherwise. More details are given in Section 3 and eq. (15).

4 As we prove later in the text, eq. (19), the variation of $m_Z$ corresponding to an EW fine-tuning of $O(1000)$ that is found in most models, is well beyond a $\pm 2\sigma$ variation (around central $m_Z^0$) that is usually taken for an input or fitted observable and that actually corresponds to fine-tuning $<10$.
the LHC. In this work we evaluate the induced correction to $\chi^2/n_{df}$ due to this condition, for popular SUSY models. The models analyzed include MSSM-like models with different boundary conditions for gaugino and higgs soft masses, NMSSM and a generalized version of it, the so-called GNMSSM.

To these purposes, we use the approach of [8] which we extend to apply beyond its original restrictive setup that calculated only the \textit{integrated} likelihood over the nuisance variables ($y$). Instead, it is the likelihood itself that is required for a traditional, conservative frequentist approach used here. We generalize this method so that our calculation of the total likelihood function (or $\chi^2$) keeps explicit its dependence on the nuisance variables such as Yukawa couplings. This is important since: a) it allows one to subsequently maximize (profile) the likelihood wrt these variables, which is needed in numerical applications, and b) it avoids the likelihood dependence on the (chosen) measure under the integral over nuisance variables.

We show that in all SUSY models, the likelihood (or $\chi^2$) receives a correction due to the condition of fixing the EW scale to the measured value ($m_Z^0$) that worsens significantly the currently reported $\chi^2/n_{df}$. We then estimate this correction, hereafter denoted $\delta\chi^2$, by using the numerical results of [3, 9, 10, 11] for a fine tuning measure (denoted $\Delta_q$) that emerges in our calculation of $\delta\chi^2 \propto \ln \Delta_q$ (see later). Note that the value of $\delta\chi^2$ depends strongly on the value of the higgs mass $m_h$. Using the recent LHC result $m_h \approx 126$ GeV, we find in the models other than the GNMSSM, a correction $\delta\chi^2/n_{df} > 1.5$ without including the usual $\chi^2$ cost due to fixing observables other than the EW scale itself. Therefore, under the assumption of a simultaneous minimization of both $\delta\chi^2$ and the “usual” $\chi^2$ (assumed to respect $\chi^2/n_{df} \approx 1$) these models have a total ($\chi^2 + \delta\chi^2)/n_{df} > 2.5$, hardly compatible with the data (in the GNMSSM ($\chi^2 + \delta\chi^2)/n_{df} \approx 2$).

Given its implications for these models and for SUSY in general, it is thus suggested that this correction be included in future EW data fits that compute $\chi^2/n_{df}$. In the following we substantiate these claims and analyze the consequences for the viability of SUSY models.

2 The calculation of the correction to $\chi^2$.

Let us briefly review the numerical calculation of the likelihood in a SUSY model (see for example [12, 13]). One usually chooses a set of observables $O_i$ well measured such as: the W-boson mass, the effective leptonic weak mixing angle $\theta_{lep}^{eff}$, the total Z-boson decay width, the anomalous magnetic moment of the muon, the mass of the higgs ($m_h$), the dark matter relic density, the branching ratios from B-physics, $B_s - \bar{B}_s$ mass difference and also additional bounds (not shown) which should be counted in $n_{df}$, too:

$$m_W, \sin^2 \theta_{lep}^{eff}, \Gamma_Z, \delta a_\mu, m_h, \Omega_{DM} h^2, BR(B \to X_s \gamma), BR(B_s \to \mu^+ \mu^-), BR(B_u \to \tau \nu), \Delta M_{B_s}, \text{etc.}$$ (2)
Note that fixing the EW scale to its accurately measured value \((m_Z^0)\) is not on this list, even though this motivated SUSY in the first place. However, data fits often include \(m_Z^0\) as an *input* observable, so one could argue that this observable is indeed being fixed. We return to this issue later in the text\(^5\). For each observable \(O_i\) the corresponding probability is often taken a Gaussian \(P(O_i|\gamma, y)\) where by \(\gamma\) we denote the set of SUSY parameters that define the model, while by \(y\) we denote nuisance variables such as Yukawa (of top, bottom, etc) and other similar couplings. One then assumes that the observables are independent and multiplies their probability distributions to obtain a total distribution, which regarded as a function of \(\gamma, y\) (with “data” fixed), defines the likelihood \(L\):

\[
L = \prod_j P(O_j|\gamma, y); \quad \gamma = \{m_0, m_{1/2}, \mu_0, m_0, A_0, B_0, \cdots\}; \quad y = \{y_t, y_b, \cdots\}
\]  

(3)

in a standard notation for the SUSY parameters, that are components of the set \(\gamma\).

To work with dimensionless parameters, all \(\gamma\) should be “normalized” to some scale (e.g. the EW scale vev \(v_0 = 246\) GeV). In the “frequentist” approach one maximizes \(L\) or equivalently minimizes the value of \(\chi^2\) that, under a common assumption of Gaussian distributions, is defined as

\[
\chi^2 = -2 \ln L.
\]  

(4)

One then seeks a good fit, such that \(\chi^2/\text{ndf} \approx 1\) at the minimum, by tuning \(\gamma, y\) to fit observables \(O_i\). Fixing the EW scale \((m_Z)\) to its measured value \((m_Z^0)\) is not something optional in our opinion, but an intrinsic part of the likelihood to fit the EW data, and in the following we evaluate the corresponding \(\chi^2\) “cost” of doing so.

In other approaches like the “Bayesian” method one further combines \(L\) with priors (probabilities for \(\gamma, y\)) to obtain the posterior probability \(9 \ 12 \ 14 \ 15\). In this method one searches for the point with the largest probability in parameter space, given the data.

While the observables \(O_i\) are independent, the SUSY set of parameters \(\gamma\) used to fit them, are not. They are usually constrained (correlated) by the EW minimum conditions. Indeed, in MSSM-like models, there are two minimum conditions of the scalar potential\(6\), one of them is determining the EW scale \(v\) as a *function* of the parameters \(\gamma\) of the model (it is not fixing \(v\) to any numerical value). In practice however, when doing data fits, one usually replaces \(v\) *by hand*, by the measured mass of Z boson \((m_Z^0)\) and solves this minimum condition for a SUSY parameter instead (usually \(\mu_0\)). Doing so can miss the impact on \(L\) of the relation between the distribution fixing the observable \(m_Z \propto v\) to its measured \(m_Z^0\), and the distribution fixing \(\mu_0\). This is discussed shortly. The second minimum condition is fixing one parameter (say \(\tan \beta\)) as a function of the remaining \(\gamma\).

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5See discussion near eq.(15).
6There is an extra condition in the case of NMSSM and GNMSM models.
7One can choose another parameter (instead of \(\tan \beta\)) like \(B_0\), but the effect is just a change of variables.
Let us then consider the scalar potential for MSSM-like models and evaluate the total likelihood, hereafter denoted $L_w(\gamma, y)$. The potential has the following standard expression:

$$V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - (m_3^2 H_1 H_2 + h.c.) + (\lambda_1/2)|H_1|^4 + (\lambda_2/2)|H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_1|^2 |H_2|^2 + [(\lambda_5/2)(H_1 H_2)^2 + \lambda_6 |H_1|^2 (H_1 H_2) + \lambda_7 |H_2|^2 (H_1 H_2) + h.c.]$$  \hfill (5)

We denote

$$\lambda \equiv \lambda_1/2 \cos^4 \beta + \lambda_2/2 \sin^4 \beta + (\lambda_3 + \lambda_4 + \lambda_5)/4 \sin^2 2\beta + (\lambda_6 \cos^2 \beta + \lambda_7 \sin^2 \beta);$$

$$m^2 \equiv m_1^2 \cos^2 \beta + m_2^2 \sin^2 \beta - m_3^2 \sin 2\beta,$$  \hfill (6)

$\lambda$ is the effective quartic higgs coupling and $m^2$ is a combination of the higgs soft terms.\footnote{The EW min conditions are $v^2 = -m^2/\lambda, 2\lambda (\partial m^2/\partial \beta) - m^2 (\partial \lambda/\partial \beta) = 0$. See also \cite{8,10}.}

To compute the total likelihood $L_w(\gamma, y)$ that also accounts for the effect of fixing the EW scale, we use the original idea in \cite{8} which we extend to a general case. The result in \cite{8} is too restrictive since it only provided the integral of $L_w$ over nuisance variables like the Yukawa couplings $(y)$, giving $L(\gamma) \sim \int dy L(\gamma, y)$. Such integrated likelihood is not appropriate for a traditional, conservative “frequentist” approach. To avoid this shortcoming, we compute $L_w(\gamma, y)$ itself, rather than its integral over the set $y$ (or corresponding masses). This allows one to subsequently maximize $L_w(\gamma, y)$ with respect to nuisance variables for a fixed set of $\gamma$ parameters (to find the “profile likelihood”).

It is useful to write the two EW minimum conditions as Dirac delta of two functions $f_{1,2}$ defined below

$$f_1 \equiv v - (-m^2/\lambda)^{1/2},$$

$$f_2 \equiv \tan \beta - \tan \beta_0(\gamma, y, v), \quad \text{where} \quad f_i = f_i(\gamma; y, v, \beta), \quad i = 1, 2. \hfill (7)$$

$\beta_0$ is the root of the second minimum condition; $\beta_0, f_{1,2}$ depend on the arguments shown; $\lambda, m^2$ also depend on $\gamma, y, \beta$. Taking account of constraints \cite{7} the total (“constrained”) likelihood $L_w(\gamma, y)$ is

$$L_w(\gamma, y) = m_0^2 \int dv \, d(\tan \beta) \delta\left[f_1(\gamma; y, v, \beta)\right] \delta\left[f_2(\gamma; y, v, \beta)\right] \delta(m_Z - m_0^2) \, L(\gamma; y, v, \beta)$$

$$= v_0 \, L(\gamma; y, v_0, \beta_0(\gamma, y)) \delta\left[f_1(\gamma; y, v_0, \beta_0(\gamma, y))\right]$$

$$= L(\gamma; y, v_0, \beta_0(\gamma, y)) \delta(1 - \tilde{v}/v_0); \quad \tilde{v} \equiv (-m^2/\lambda)^{1/2}|_{\beta=\beta_0(\gamma,y),v=v_0}. \hfill (8)$$

Here $v_0 = 246 \text{ GeV}, m_0^2 = g v_0/2 \approx 91.2 \text{ GeV}, m_Z = g v/2, g^2 = g_1^2 + g_2^2$ with $g_{1,2}$ couplings
for U(1), SU(2). To simplify notation, we did not display the numerical argument $v_0$ of $\beta_0$, i.e. we denoted $\beta_0(\gamma, y) \equiv \beta_0(\gamma, y, v_0)$. $m_Z^0$ in rhs compensates the dimension of $\delta(m_Z - m_Z^0)$. $L(\gamma, y, v, \beta)$ in the first line of eq. (8) denotes the usual likelihood associated with fitting the observables other than the EW scale ($m_Z$), see eq. (2), before $v$, $\tan \beta$ are fixed by the EW min conditions. Integrating over other than the observables i.e. we denoted $j$ for U(1), SU(2). To simplify notation, we did not display the numerical argument $v_0$ instead of $z$. $\delta(f_1)$ is just a formal way to solve these EW minimum constraints and eliminate these parameters in terms of the rest. We also introduced a $\delta(m_Z - m_Z^0)$ distribution, which fixes the EW scale by enforcing the measured mass of Z boson ($m_Z^0$) and thus the replacement $v \to v_0$. Since $m_Z^0$ is very well measured, using $\delta(m_Z - m_Z^0)$ is indeed justified (however, the result can be generalized to a Gaussian).

A comment about normalization in eq. (8): one must ensure that the two constraints ($f_{1,2}$) are indeed normalized to unity wrt the variables over which we integrate them, in this case $\tan \beta$ and $v$ (this condition is indeed respected in our case). Therefore there is no freedom for any additional factors to be present in eq. (8).

The last two lines in eq. (8) show that the original likelihood $L$, evaluated on the ground state, is now multiplied by $v_0 \delta(f_1(\ldots, v_0, \ldots)) = \delta(1 - \bar{v}/v_0)$. Further, the Dirac delta of a function $f_1(z_i)$, $i = 1, 2, \ldots, n$, can be written as (after a Taylor expansion of $f_1$):

$$\delta(f_1(z_i)) = \delta[f_1(z_i^0) + (\nabla f_1)_0 \cdot (\bar{z}_i - z_i^0) + \cdots] = \frac{1}{|\nabla f_1|_0} \delta[n_j (z_j - z_j^0)],$$

A summation over repeated index $j$ is understood and the subscript “o” of $|\nabla f_1|_0$ means this quantity is evaluated at the point $z_i = z_i^0$ where $f_1 = 0$; $n_j$ are components of the normal $\bar{n}$ to the surface $f_1(z_i^0) = 0$, so $\bar{n} = (\nabla f_1/|\nabla f_1|)_0$. Eqs. (8), (9) tell us that it is not enough for the parameters of the model to respect the constraint $f_1 = 0$, and that there is instead an additional factor generated, represented by the gradient of the constraint.

The “constrained” likelihood of eq. (8) becomes, after using eq. (9):

$$L_w(\gamma, y) = \frac{\delta[n_i (\ln z_i - \ln z_i^0)]}{\Delta \gamma(\gamma^0, y^0)} L(\gamma; y, v_0, \beta_0(\gamma, y)), \quad z_i = \{\gamma_j, y_k\}$$

(10)

Here $\gamma_j$, $y_k$ denote components of the sets $\gamma$ and $y$ defined in eq. (3) and we used (9) for $\ln z_i$ instead of $z_i$ as variables, to ensure dimensionless arguments for $\delta$ function. $\Delta \gamma$ denotes the absolute value of the gradient of $f_1$ evaluated at $z_i^0 = \{\gamma_j^0, y_k^0\}$ that is a solution to the EW min condition $f_1(\gamma^0, y^0, v_0, \beta_0(\gamma^0, y^0)) = 0$, and has the value:

$$\Delta^2 \gamma(\gamma^0, y^0) = \sum_{z_i = \{\gamma_j, y_k\}} \left(\frac{\partial \ln \bar{v}}{\partial \ln z_i}\right)^2_o = \sum_{\gamma_j} \left(\frac{\partial \ln \bar{v}}{\partial \ln \gamma_j}\right)^2_o + \sum_{y_k} \left(\frac{\partial \ln \bar{v}}{\partial \ln y_k}\right)^2_o.$$  

(11)

$^9$ Additional distributions can also be considered for nuisance variables, like top, bottom masses, etc, and assumed to factorize out of $L$ in the rhs, with corresponding integrals over Yukawa couplings, see [8]. 

$^{10}$ $\delta(f_{1,2})$ are just distributions for $v$, $\tan \beta$ (albeit special ones), so they also must be normalized to unity. 

$^{11}$ The gradient measures how the constraint “reacts” to the variations of the parameters.
The subscript “o” stands for evaluation on the ground state \((\gamma_i = \gamma_i^0, \ k = y_k^0)\), \(\Delta_q\) that emerged above has some resemblance to what is called the fine tuning measure \(^{12}\) wrt all parameters, both \(\gamma\) and \(y\). The arguments of \(\Delta_q(\gamma^0, y^0)\) denote the parameters wrt which is computed and includes SUSY parameters and nuisance variables (Yukawa, etc).

Eq. \(\text{(10)}\) is the result in terms of distributions and can be written in an alternative form. The \(\delta\) in the rhs of \(\text{(10)}\) tells us that the lhs is non-zero when \(z_i = z_i^0\) for all \(i\) (i.e. \(\gamma_j = \gamma_j^0, \ y_k = y_k^0\)). Another way to present this eq is to do a formal integration of \(\text{(10)}\) in the general direction \(\ln \tilde{z} = n_j \ln z_j\), (sum over \(j\) understood, \(j\) running over the sets \(\gamma, y\)); this allows all independent parameters to vary simultaneously. \(^{13}\) After this, eq. \(\text{(10)}\) becomes:

\[
L_w(\gamma^0, y^0) = \frac{L(\gamma^0; y^0, v_0, \beta_0(\gamma^0, y^0))}{\Delta_q(\gamma^0, y^0)}. \quad \text{(12)}
\]

\(L\) in the rhs is exactly the usual, “old” likelihood computed in the data fits and evaluated at the EW minimum (reflected by its arguments) but without fixing the EW scale, while the lhs does account for this effect. Note that due to the minimum condition \(f_1(\gamma^0, y^0, v_0, \beta_0(\gamma^0, y^0)) = 0\), one element of the set \(\gamma^0\), say \(\gamma_0^0\), becomes a function of the remaining, independent \(\gamma_0^0, \ i \neq \kappa\). Usually \(\gamma_0^0\) is taken to be \(\mu_0\).

The result in eq. \(\text{(12)}\) shows that for a good fit (i.e. maximal \(L_w\), one has to maximize not the usual likelihood in the rhs, but actually its ratio to \(\Delta_q\). Let us introduce the notation \(\chi_w^2 \equiv -2 \ln L_w\) and also \(\chi^2 \equiv -2 \ln L\), then eq. \(\text{(12)}\) becomes

\[
\chi_w^2(\gamma^0, y^0) = \chi^2(\gamma^0, y^0) + 2 \ln \Delta_q(\gamma^0, y^0). \quad \text{(13)}
\]

Therefore, after fixing the EW scale the usual \(\chi^2\) receives a positive correction that depends on \(\Delta_q\) and that is not included in the current precision data fits. This result extends the validity of its counterpart in \(\text{[8]}\), in the presence of the nuisance variables \(y\). Unlike in \(\text{[8]}\), nuisance variables \(y\) are present in eq. \(\text{(13)}\) i.e. were not integrated out. This has the advantage of respecting the traditional, conservative frequentist approach and one can compute numerically from eq. \(\text{(12)}\) the profile likelihood, by maximizing \(L_w\) wrt nuisance variables \(y^0\), for a fixed set of \(\gamma^0\). This profile likelihood is then \(L_w(\gamma^0, y_{\max}^0(\gamma^0))\).

Let us remark that in eq. \(\text{(10)}\) we used \(\ln z_i\) as arguments under the Dirac delta function, which implicitly assumes that these are more fundamental parameters than \(z_i\) themselves, (here \(z_i = \{\gamma_j, y_k\}\)). In principle this is a choice, motivated here by the fact that it ensured dimensionless arguments for the delta function in \(\text{(10)}\), (unlike \(y_k, \gamma_j\) are dimensionful parameters). Going from these parameters to their log’s is a one-to-one change that does not affect the minimal value of total \(\chi_w^2\). If one insists in working with \(z_i\) as fundamental parameters, one simply changes \(\ln z_i \rightarrow z_i\) in eqs. \(\text{(10)}, \ (11)\), after “normalizing” \(\gamma_i\) to some

\(^{12}\)This was introduced in \(\text{[16]}\), with \(\Delta_{\max} = \max_i |\partial \ln \tilde{L}/\partial \ln \gamma_i|\); see \(\text{[17]}\) for an interpretation of \(1/\Delta_{\max}\).

\(^{13}\)Such integral is just a formal way of saying we solve \(f_1 = 0\) in favour of one particular \(\gamma_0^0\) (usually \(\mu_0\)).

\(^{14}\)In a Bayesian language, this would correspond to choosing log priors instead of flat ones for the parameters. Ultimately this may reflect a problem of measure that is beyond the purpose of this work.
scale (e.g. \( v_0 \)), to ensure dimensionless arguments for \( \delta \) in eq. (10). The result is that \( \Delta q \) is then computed wrt \( \gamma_j \) and \( y_k \) instead of their logarithms, so in eqs. (12), (13) one replaces

\[
\Delta_q^2 \rightarrow \Delta_q'{}^2 = \sum_{\gamma_j} \left( \frac{\partial \ln \tilde{v}}{\partial \gamma_i} \right)^2 + \sum_{y_k} \left( \frac{\partial \ln \tilde{v}}{\partial y_k} \right)^2.
\]

(14)

Compared to its counterpart in eq. (11), the second sum above is actually larger in this case since \( y_k < 1 \). In the following, for numerical estimates we shall work with \( \Delta_q \). A detailed investigation of the correction to \( \chi^2 \) is beyond the purpose of this paper and in the following we restrict the study to a numerical estimate. The main point is that \( \chi^2 \) has a correction that needs to be taken into account.

As argued in the introduction, numerical methods to fit the data seem to miss the above effect, leading to the puzzle mentioned near eq. (1), of having simultaneously a good fit \( \chi^2/n_{df} \approx 1 \) but a large fine-tuning (that should actually worsen this fit!). Usually in the numerical methods one often uses \( m_Z = m_Z^0 \) as an input, i.e. with a Dirac delta distribution, to compute instead the corresponding value \( \mu_0 \), which itself has a similar distribution; there is however a relative normalization factor between these, that is relevant in answering this issue. To see this, consider that all parameters \( \gamma_i, y_k, i \neq 1 \) are fixed to some numerical values \( (\gamma_i^0, y_k^0) \), except \( \gamma_1 \equiv \mu_0 \). Then

\[
L_w = L \delta(1 - \tilde{v}/v_0) = L \delta(1 - m_Z/m_Z^0) = L \frac{\delta(1 - \mu/\mu_0)}{\frac{\partial \ln m_Z}{\partial \ln \mu}} \bigg|_0
\]

(15)

The denominator is a particular version of our more general \( \Delta_q \) when only one parameter varies, and is just a normalization factor. When missing this factor, the likelihood of the model does not account for the “cost” of fixing the EW scale (i.e. \( m_Z^0 \)). This factor affects the minimal value of \( \chi^2 \) and must be included in total \( \chi^2/n_{df} \), as shown in eq. (13) when all parameters vary. The above discussion answers the puzzle and clarifies how a numerical method can be adapted to account for the \( \chi^2 \) “cost” of fixing the EW scale. The discussion can be extended to more general likelihood functions (beyond Dirac delta type).

The “frequentist” approach shown so far can be extended to the Bayesian case which is just a global version (in parameter space) of eq. (12). In this case one assigns, in addition, some initial probabilities (priors) to the parameters of the model \( (\gamma, y) \) then integrates over them the likelihood \( L_w \) multiplied by the priors. This gives the global probability of the model or “evidence”, \( p(\text{data}) \), that must be maximized. The result is (see [9, 18]):

\[
p(\text{data}) = \int_{f_1=0}^{f_2=0} dS \frac{1}{\Delta_q(\gamma, y)} L(\gamma, y, v_0, \beta) \times \text{priors}(\gamma, y)
\]

(16)

where the integral is over a surface in the parameter space \( \gamma, y \) defined by \( f_1 = f_2 = 0 \).  

\[\text{under the assumption of a unique root for } \mu.\]
In this case the factor $1/\Delta_q$ is itself an “emergent”, naturalness prior, in addition to the original priors of the model. Unlike the frequentist approach, the effect of fixing the EW scale (associated with $1/\Delta_q$) is indeed taken into account in the Bayesian approach \cite{14,15} (under some approximation). For details on the Bayesian case, see \cite{12,9,14,15,18,19,20}.

3 A numerical estimate of the correction to $\chi^2$ in SUSY.

In this section we estimate in some models the correction $\delta\chi^2$ obtained from eq. (13)

$$\delta\chi^2/n_{df} \equiv (2/n_{df}) \ln \Delta_q$$

By demanding that the correction $\delta\chi^2/n_{df}$ be small enough not to affect the current data fits results for $\chi^2/n_{df}$, one has a model-independent upper bound:

$$\Delta_q \ll \exp \left( n_{df}/2 \right).$$

When this bound is reached, then $\chi^2_{w}/n_{df} = 1 + \chi^2/n_{df}$, and assuming values of “usual” $\chi^2/n_{df} \approx 1$ i.e. a good fit without fixing the EW scale, gives $\chi^2_{w}/n_{df} \approx 2$.

Using (17), one can find the equivalent numerical value of $\Delta_q$ that corresponds to one observable having a given deviation from the central value. This can be read below:

\[ 2 \sigma \leftrightarrow \Delta_q \approx 8. \]
\[ 3 \sigma \leftrightarrow \Delta_q \approx 100. \]
\[ 3.5 \sigma \leftrightarrow \Delta_q \approx 1000. \]
\[ 5 \sigma \leftrightarrow \Delta_q \approx 100000. \]

This gives, in a model independent way, a different perspective and a probabilistic interpretation to the values of $\Delta_q$ (related to fine-tuning/naturalness) that avoid subjective criteria about this topic. As a result, any model with $\Delta_q > 100$ ($\Delta_q > 200$) such as those discussed shortly, is more than a $3\sigma$ (3.25$\sigma$) away from fixing the EW scale ($m_Z$). This supports our initial discussion (near eq. (11)), that one cannot have a good fit of $m_Z$ with a simultaneous, large EW fine tuning (assuming $m_Z$ is independent of other observables).

For our numerical estimates of $\delta\chi^2$ we restrict the analysis to using minimal values of $\Delta_q$ in SUSY models. Further, we only evaluate $\Delta_q$ wrt $\gamma$ parameters; notice that

$$\Delta_q(\gamma, y) > \Delta_q(\gamma).$$

where the arguments are the variables wrt which $\Delta_q$ is actually computed. That is, we shall ignore the contribution to $\Delta_q$ due to variations wrt Yukawa couplings or other nuisance parameters. This underestimates $\delta\chi^2$, but has the advantage that we can use the numerical values of $\Delta_q$ already available, in \cite{19} (also \cite{10}) for MSSM models with different boundary conditions (two-loop results) and in \cite{8,11} for NMSSM, GNMSM (one loop).
We consider the most popular SUSY models used for searches at the LHC, listed below:

- the constrained MSSM model (CMSSM); this is the basic scenario, of parameters \( \gamma \equiv \{m_0, m_{1/2}, \mu_0, A_0, B_0\} \), in a standard notation. Then \( \Delta_q \) is that of eq. (11) with summation over these parameters only:
\[
\Delta_q^2 = \sum_{\gamma_j} \left( \frac{\partial \ln \hat{v}}{\partial \ln \gamma_j} \right)_0^2
\]  

- the NUHM1 model: this is a CMSSM-like model in which one relaxes the Higgs soft masses in the ultraviolet (uv), to allow values different from \( m_0 \): \( m_{h_1}^{uv} = m_{h_2}^{uv} \neq m_0 \), with \( \gamma \equiv \{m_0, m_{1/2}, \mu_0, A_0, B_0, m_{h_1}^{uv}, m_{h_2}^{uv}\} \). Then \( \Delta_q \) is that of eq. (21) with summation over this set.

- the NUHM2 model: this is a CMSSM-like model with non-universal Higgs soft masses, \( m_{h_1}^{uv} \neq m_{h_2}^{uv} \neq m_0 \), with independent parameters \( \gamma \equiv \{m_0, m_{1/2}, \mu_0, A_0, B_0, m_{h_1}^{uv}, m_{h_2}^{uv}\} \). Then \( \Delta_q \) is that of eq. (21) with summation over this set.

- the NUGM model: this is a CMSSM-like model with non-universal gaugino masses \( m_{\lambda_i} \), \( i = 1, 2, 3 \), with \( \gamma = \{m_0, \mu_0, A_0, B_0, m_{\lambda_1}, m_{\lambda_2}, m_{\lambda_3}\} \). \( \Delta_q \) is that of eq. (21) with summation over this set.

- the NUGMd model: this is a particular case of the NUGM model with a specific relation among the gaugino masses \( m_{\lambda_i} \), \( i = 1, 2, 3 \), of the type \( m_{\lambda_i} = \eta_i m_{1/2} \), where \( \eta_{1,2,3} \) take only discrete, fixed values. Such relations can exist due to some GUT symmetries, like SU(5), SO(10), etc [21]. The particular relation we consider is a benchmark point with \( m_{\lambda_3} = (1/3) m_{1/2} \), \( m_{\lambda_1} = (-5/3) m_{1/2} \), \( m_{\lambda_2} = m_{1/2} \), corresponding to a particular GUT (SU(5)) model, see Table 2 in [21]. \( \Delta_q \) is that of eq. (21) with \( \gamma = \{m_0, m_{1/2}, A_0, B_0, \mu_0\} \).

- the next to minimal MSSM model (NMSSM): the model has an additional gauge singlet. Its parameters are \( \gamma = \{m_0, \mu_0, A_0, B_0, m_{1/2}, m_s\} \), with \( m_s \) the singlet soft mass. For an estimate of \( \Delta_q \) we use the results in [3, 11]. Notice that these papers evaluate instead \( \Delta_{max} \) which is the largest fine tuning wrt to any of these parameters (instead of their sum in “quadrature” as in \( \Delta_q \)). As a result, \( \Delta_{max} \) is usually slightly smaller, by a factor between 1 and 2 as noticed for the other models listed above [9] and thus \( \Delta_q \) is underestimated.

- the general NMSSM (GNMSSM) model: this is an extension of the NMSSM in the sense that it contains a bilinear term in the superpotential for the gauge singlet, \( M S^2 \), see for example [11]. So the singlet is massive at the supersymmetric level [17] and we have an additional parameter \( M \) to the set we have for the NMSSM. Again, \( \Delta_{max} \) is used here [11] instead of \( \Delta_q \), so \( \Delta_q \) is again underestimated.

Our estimates for \( \delta \chi^2 / n_{d.f} \) for different higgs mass values are shown in Tables [1] and [2].

The results present the minimal value of \( \Delta_q \) evaluated in the above models as a function of the higgs mass, after a scan over the entire parameter space (all \( \gamma \), \( y \) and also tan \( \beta \) of the

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16Parameters \( \gamma \) are those following from [10, 11] and this is why \( \mu_0 \) is quoted instead of usual \( \text{sgn}(\mu_0) \). Similar for the other models. Also \( B_0 \) is quoted instead of tan \( \beta \), the difference is a change of variables.

17Its mass can be of few (5-8) TeV, so one can integrate it out and work near the decoupling limit [22].
The correction \( \delta \chi^2 \equiv 2 \ln \Delta_q(\gamma) \), the number of parameters \( n_p \) and degrees of freedom \( n_{df} \) in SUSY models, for \( m_h \approx 115 \) GeV corresponding to the pre-LHC bound of \( m_h \). Notice that in this case \( \delta \chi^2 / n_{df} < 1 \). \( n_{df} \) may vary, depending on the exact number of observables fitted. The numerical values of \( \Delta_q \) are from [9] for the first 5 models and [11] for NMSSM, GNMSSM (see also [10] for CMSSM).

| Model     | \( n_p \) | Approx | \( \Delta_q \) | \( \delta \chi^2(115) \) | \( n_{df} \) |
|-----------|-----------|--------|----------------|----------------------|------------|
| CMSSM     | 5         | 2-loop | 15             | 5.42                 | 9          |
| NUHM1     | 6         | 2-loop | 100            | 9.21                 | 8          |
| NUHM2     | 7         | 2-loop | 85             | 8.89                 | 7          |
| NUGM      | 7         | 2-loop | 15             | 5.42                 | 7          |
| NUGMd     | 5         | 2-loop | 12             | 4.97                 | 9          |
| NMSSM     | 6         | 1-loop | 12             | 4.97                 | 8          |
| GNMSSM    | 7         | 1-loop | 12             | 4.97                 | 7          |

Table 1: The correction \( \delta \chi^2 \), the number of parameters \( n_p \) and degrees of freedom \( n_{df} \) in SUSY models, for \( m_h \approx 115 \) GeV corresponding to the pre-LHC bound of \( m_h \). Notice that in this case \( \delta \chi^2 / n_{df} < 1 \). \( n_{df} \) may vary, depending on the exact number of observables fitted. The numerical values of \( \Delta_q \) are from [9] for the first 5 models and [11] for NMSSM, GNMSSM (see also [10] for CMSSM).

For \( m_h \approx 115 \) GeV, one could still have \( \delta \chi^2 / n_{df} < 1 \) for some models, that could have allowed a corrected \( \chi^2 \) without considering the original contribution due to the “usual” \( \chi^2 / n_{df} \) coming from fitting observables other than the EW scale. The above correction is too large by the usual criteria that total \( \chi^2 / n_{df} \approx 1 \). Further, assume that in the above models one finds a point in the parameter space for which \( \chi^2 / n_{df} \approx 1 \) and then add to it the effect of \( \delta \chi^2 \). One then finds that \( \chi^2 / n_{df} \) is close to or larger than 2, see Table 2.

In all models, for the currently measured \( m_h \approx 126 \) GeV, \( \delta \chi^2 / n_{df} \) alone is larger than unity (or close to 1 for GNMSSM), without considering the original contribution due to the “usual” \( \chi^2 / n_{df} \) coming from fitting observables other than the EW scale. The above correction is too large by the usual criteria that total \( \chi^2 / n_{df} \approx 1 \). Further, assume that in the above models one finds a point in the parameter space for which \( \chi^2 / n_{df} \approx 1 \) and then add to it the effect of \( \delta \chi^2 \). One then finds that \( \chi^2 / n_{df} \) is close to or larger than 2, see Table 2. In this Table, the values of reduced \( \chi^2 \) increase (decrease) by \( \approx 0.1 \) for an increase (decrease) of \( m_h \) by 1 GeV, respectively, except in the GNMSSM where this is even smaller (0.04 for 1 GeV). The log dependence \( \delta \chi^2 = 2 \ln \Delta_q \) means that uncertainties in evaluating \( \Delta_q \) are reduced and \( \delta \chi^2 \) values listed in Table 2 and used in Table 3 are comparable, even though corresponding \( \Delta_q \)'s are very different.

Our estimates for \( \chi^2 / n_{df} \) shown in Table 2 are hardly acceptable for a good fit\(^{19}\). Interestingly, increasing the number of parameters of a model (decrease \( n_{df} \)) could decrease \( \Delta_q \).

\(^{18}\)The scan included a range of \([-7, 7] \) TeV for \( A_0, m_0 \) and \( m_{1/2} \) up to 5 TeV, and \( 2 \leq \tan \beta \leq 62 \) and also allowed a 2\(\sigma\) deviation for fitted observables and for a 3\(\sigma\) for \( \Omega h^2 \), see for details [9].

\(^{19}\)Assuming a \( \chi^2 \) distribution, the \( p \)-value in these models would be \( < 1\% \) (and 5\% for GNMSSM).
Table 2: As for Table 1, with $\Delta_q$ and corresponding $\delta \chi^2 \equiv 2 \ln \Delta_q(\gamma)$ for $m_h$ equal to 123, 125, 126 and 127 GeV (shown within brackets). This can also show the impact of the 2-3 GeV error in the theoretical calculation of $m_h$. $\Delta_q$ grows $\approx$ exponentially with $m_h$; a 1 GeV increase of $m_h$ induces about 1 unit increase of $\delta \chi^2$. In all cases except GNMSSM, fixing the EW scale brings a correction $\delta \chi^2/n_{df} > 1$. Note $\delta \chi^2/n_{df}$ can be larger if one also includes the impact of Yukawa couplings on $\Delta_q$. Same loop approximation, number of parameters and degrees of freedom apply as in Table 1. The values of $\Delta_q$ are from [9] for the first five models and from [11] for NMSSM, GNMSSM (see also [10] for CMSSM).

| Model   | $\Delta_q$ | $\delta \chi^2(123)$ | $\Delta_q$ | $\delta \chi^2(125)$ | $\Delta_q$ | $\delta \chi^2(126)$ | $\Delta_q$ | $\delta \chi^2(127)$ |
|---------|------------|------------------------|------------|------------------------|------------|------------------------|------------|------------------------|
| CMSSM   | 380        | 11.88                  | 1100       | 14.01                  | 1800       | 14.99                  | 3100       | 16.08                  |
| NUHM1   | 500        | 12.43                  | 1000       | 13.82                  | 1500       | 14.63                  | 2100       | 15.29                  |
| NUHM2   | 470        | 12.31                  | 1000       | 13.82                  | 1300       | 14.34                  | 2000       | 15.20                  |
| NUGM    | 230        | 10.88                  | 700        | 13.10                  | 1000       | 13.82                  | 1300       | 14.34                  |
| NUGMd   | 200        | 10.59                  | 530        | 12.55                  | 850        | 13.49                  | 1300       | 14.34                  |
| NMSSM   | >100       | 9.21                   | >200       | 10.59                  | >200       | 10.59                  | >200       | 10.59                  |
| GNMSSM  | 22         | 6.18                   | 25         | 6.43                   | 27         | 6.59                   | 31         | 6.87                   |

Table 3: An estimate for total $\chi^2_w/n_{df}$ in various SUSY models for $m_h \approx 126$ GeV and $\chi^2/n_{df} \approx 1$.

| $m_h=126$ GeV: | CMSSM | NUHM1 | NUHM2 | NUGM | NUGMd | GNMSSM |
|---------------|-------|-------|-------|------|-------|--------|
| $\chi^2_w/n_{df}$ | 2.66  | 2.83  | 3.05  | 2.97 | 2.49  | 1.94   |

and $\delta \chi^2$, but this reduction may not be enough to reduce $\chi^2_w/n_{df}$, since $n_{df}$ is now smaller. This is seen by comparing the reduced $\chi^2$ in NUGM and CMSSM. This is because $\delta \chi^2$ depends only mildly (log-like) on $\Delta_q$, so only a significant reduction of $\Delta_q$ can compensate the effect of simultaneously reducing $n_{df}$. For such case compare CMSSM with GNMSSM.

The values of $\chi^2_w/n_{df}$ could be higher than our estimates above, since they ignore that: a) we used only minimal values of $\Delta_q$ over the whole parameter space. b) Yukawa effects on $\delta \chi^2$ were ignored and these can be significant. There is also an argument in favour of a smaller $\chi^2_w/n_{df}$, that current theoretical calculations of $m_h$ may have a 2-3 GeV error. Assuming this, for $m_h \approx 123$ GeV (instead of 126 GeV), in GNMSSM one obtains a small change: $\chi^2_w/n_{df} = 1.8$ while for the other models this ratio is $\geq 2.3$. Another reduction may emerge in numerical analysis if using eq.(14) instead of (11), but the impact of its larger Yukawa contributions makes this possibility less likely.

Regarding the NMSSM model, Table 2 only provided a lower bound on $\Delta_q$. However we can do a more accurate estimate, using recent data fits that evaluated both $\chi^2$ and $\Delta_q$. To this purpose, we use the minimal value for $\chi^2 = 6.4$ in [3] (last two columns of

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20 In CMSSM, $\Delta_q$ wrt top Yukawa alone is larger than that wrt to any SUSY parameter, see fig.2 in [10].
21 For other recent data fits see [4, 5, 6, 7].
their Table 3) together with its corresponding $\Delta_q = 455$. This gives a $\chi^2_w/n_{df} = 18.64/8$, which is similar to the other models discussed above. Further, using instead our Table 2 where $\delta \chi^2 > 10.59$ we find $\chi^2_w/n_{df} > 2.32$ (for $\chi^2/n_{df} \approx 1$), which is in agreement with the aforementioned value derived from accurate data fits.

To conclude, the requirement of fixing the EW scale brings a significant contribution to the value of $\chi^2_w/n_{df}$, with negative impact on the data fits and on the phenomenological viability of these models. While our numerical results are just an estimate of the correction $\delta \chi^2$, the effect is nevertheless present and demands a careful re-consideration of this correction by the precision data fits that should include it in future analysis.

4 Some implications for model building.

Let us discuss some implications of the above result for model building.

a). A natural question is how to reduce $\delta \chi^2/n_{df}$. Here are three ways to attempt this: i) additional supersymmetric terms in the model, which unlike SUSY breaking ones, are less restricted by experimental bounds; ii) additional gauge symmetry, iii) additional massive states coupled to the higgs sector. All these directions have in common a possible increase of the effective quartic higgs coupling ($\lambda$) so one can more easily satisfy an EW minimum condition $v^2 = -m^2/\lambda$ for $v \sim O(100\text{GeV})$, $m \sim O(1\text{TeV})$, that demands a larger $\lambda$. As a result one can obtain a smaller $\delta \chi^2 \propto \ln \Delta_q$. As mentioned, the increased complexity of the model (more parameters) is to be avoided, since then $n_{df}$ can decrease and $\delta \chi^2/n_{df}$ may not change much (or even increase). The GNMSSM model is an example of i), where a supersymmetric mass term for the additional singlet essentially enabled a smaller $\delta \chi^2/n_{df}$ than in other MSSM-like models. Similar but milder effects exist in the NMSSM. An increase of $\lambda$ could also be generated by using idea ii) by adding more gauge symmetry (e.g. [26]). Regarding option iii), one can consider additional massive states that couple to the higgs sector, and that in the low energy generate corrections to the higgs potential and effective $\lambda$ and $m_h$ with similar effects. These ideas may indicate the direction for SUSY model building.

b). The above negative implications for some SUSY models remind us about the real possibility that no sign of TeV-scale supersymmetry may be found at the LHC. If so, this can suggest its scale is significantly larger than few TeV. Alternatively, one could attempt to forbid the existence of asymptotic supersymmetric states while trying to preserve some

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22 $\Delta_q$ is even higher as it is not computed according to but reports max values wrt each parameter.

23 Other values quoted in bring an even larger value for this ratio.

24 A tension in the relation $v^2 = -m^2/\lambda$ translates into a larger $\delta \chi^2 \sim \ln \Delta_q$. Recall that in MSSM $\lambda$ is very small and fixed by gauge interactions (at tree level) and this is one source for the above problems.

25 For a large value of the supersymmetric mass term of the singlet ($M$ of few TeV, 5-8), the correction to the higgs effective quartic coupling $\lambda$ is $\delta \lambda \sim (2\mu/M) \sin 2\beta$ already at tree level [22], with impact on $\Delta_q$ and $\delta \chi^2$. [11]. For a recent study of the GNMSSM and its LHC signatures see [23].

26 The correction to $\lambda$ and $m_h$ is in this case restricted by perturbativity in the singlet coupling $\tilde{\lambda}$ (of $\tilde{\lambda}SH_1H_2$) and also its proportionality to $\tilde{\lambda}\sin^2 2\beta$ instead, while in the GNMSsM is $\propto \sin 2\beta$. 

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of the nice advantages of SUSY. In such scenario, superpartners would be present only as internal lines in loop diagrams, they would not be real, asymptotic final states. One could describe this situation by using some variant of non-linear supersymmetry that can be described in a superfield formalism endowed with constraints (see examples in [28]). The hope would be to preserve SUSY results like fixing the EW scale, gauge couplings unification, radiative EW symmetry breaking, for which the superpartners in the loops play a crucial role[27]. However, it is difficult to realize this idea in practice. One also recalls the supersymmetric quantum mechanics case where SUSY is used only as a tool for performing complex calculations [30], which could suggest ideas for the field theory case.

c). The remaining possibility is to abandon (low-energy) SUSY and eventually consider a different symmetry instead. One can use model building based on SM extended with the (classical) scale symmetry, thus forbidding a tree level higgs mass. This symmetry is broken at the loop level by anomalous dimensions, which would bring in only log-like dependence on the mass scales [31]. Additional requirements (unitarity, etc) could be added. For model building along this direction see some examples in [32] and references therein.

5 Conclusions

The main motivation for TeV-scale supersymmetry was to solve the hierarchy problem and therefore fix the electroweak scale (vev $v$ or $m_Z$) in the presence of the quantum corrections. Rather surprisingly, the numerical methods that evaluate the likelihood (or its $\chi^2 \equiv -2 \ln L$) to fit the data in SUSY models do not account for the $\chi^2$ “cost” that is due to fixing the EW scale to its measured value ($m^0_Z$). When this condition is properly imposed, one finds that $\chi^2$ receives a positive correction, $\delta \chi^2 = 2 \ln \Delta_q > 0$, where $\Delta_q$ has some resemblance to a “traditional” EW fine-tuning measure in “quadrature”[28]. The correction $\delta \chi^2$ must be included in the analysis of the total $\chi^2$ of the SUSY models.

Our analysis also showed the contradiction that is present in those data fits of SUSY models that report a good fit $\chi^2/n_{df} \approx 1$ of the data including the EW scale itself while at the same time have a large EW fine-tuning. A large fine-tuning suggests a significant variation of the EW scale ($m_Z$) away from the measured value ($m^0_Z$) under a small variation of the SUSY parameters; this impacts on the value of total $\chi^2/n_{df}$ and worsens it, in contradiction with its good value (total $\chi^2/n_{df} \approx 1$) that is often reported in such data fits. The solution to this puzzle was mentioned above: our claim is that in these data fits the likelihood to fit the data ($\chi^2$) does not account for what we identified as the $\delta \chi^2$ “cost” (with $\delta \chi^2/n_{df} > 1$) of fixing the EW scale to its measured value. For this reason current data fits underestimate the total value of $\chi^2/n_{df}$ in SUSY models.

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[27] A related idea exists [29], based on a similarity to gauge fixing in gauge theories and the subsequent emergence of the ghost degrees of freedom of different statistics. This would have an analogue in the above SUSY scenario in “fixing the gauge” in the Grassmann space. Similar to the emergence of ghosts in gauge theories as non-asymptotic states one could attempt to obtain non-asymptotic superpartners states.

[28] For this reason one can say that the “traditional” fine-tuning is an intrinsic part of the likelihood to fit the data that includes the EW scale value ($m^0_Z$).
For the recently measured value of the higgs mass ($\approx 126$ GeV), the correction $\delta \chi^2$ was estimated and was shown to be significant in most popular SUSY models: constrained MSSM (CMSSM), models with non-universal higgs soft masses (NUHM1, NUHM2) or with non-universal gaugino masses (NUGM) and in the NMSSM and it was milder in the generalized version of NMSSM (GNMSSM). This correction has negative implications for the data fits of SUSY models. Our estimates show that for $m_h \approx 126$ GeV, this correction alone is $\delta \chi^2 / n_{df} > 1.5$, which violates the traditional condition for a good fit already before fitting observables other than the EW scale. Adding this contribution to that due to these observables assumed to bring the “usual” $\chi^2 / n_{df} \approx 1$, would give a total $\chi_w^2 / n_{df} = (\chi^2 + \delta \chi^2) / n_{df} > 2.5$, hardly acceptable. Another way to express this result is that in these models a good fit of $m_Z^0$ and current EW data (i.e. $\chi_w^2 / n_{df} \approx 1$), and a simultaneous large EW fine tuning (i.e. $\delta \chi^2 / n_{df} = 2 \ln \Delta_q / n_{df} > 1$) are not simultaneously possible. Further contributions to $\delta \chi^2$ also exist from the Yukawa couplings, but their effect was not discussed in this work. Let us mention that these results rely on the assumption made in the calculation of the total $\chi_w^2$ that the EW scale ($m_Z$) is independent of the other observables. Therefore it is possible that correlations effects between these can modify the above result and even relax the upper value of $\Delta_q$ that is still consistent with a good fit ($\chi_w^2 / n_{df} \approx 1$).

To conclude, the requirement of fixing the EW scale in SUSY models brings a correction to the likelihood ($\chi^2$) that can have significant negative implications for the quality of the data fits. We provided an estimate of this correction and argued that it must be included in the total likelihood to fit the EW data, when testing the viability of SUSY models.

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