Everett’s Missing Postulate and the Born Rule

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Abstract

Everett’s Relative State Interpretation (aka Many Worlds Interpretation) has gained increasing interest due to the progress understanding the role of decoherence. In order to fulfill its promise as a realistic description of the physical world, two postulates are formulated. In short they are 1) for a system with continuous coordinates $x$, discrete variable $j$, and state $\psi_j(x)$, the density $\rho_j(x) = |\psi_j(x)|^2$ gives the distribution of the location of the system with the respect to the variables $x$ and $j$; 2) an equation of motion for the state $\frac{i\hbar}{2}\partial_t \psi = H\psi$.

The first postulate gives a connects the mathematical description to the physical reality, which has been missing in previous versions. The contents of the standard (Copenhagen) postulates are derived including the appearance of Hilbert space and the Born rule. The approach to probabilities earlier proposed by Greaves replaces successfully the classical probability concept in the Born rule. The mistakes and misunderstandings of other proofs of the Born rule are discussed too.

1 Introduction

Early on, quantum mechanics was judged unable to describe the measurement process. The spatial spread of the wave function conflicted with that each recorded particle was found in a definite place or direction. Born gave the rule that the wave function absolute squared $|\psi|^2$ gives the probability distribution [1]. Bohr took the view that quantum mechanics can be used for the process between preparation and measurement [2]. However, at the act of measurement, a transition is necessary to “classical” description of the macroscopic detector. Heisenberg inferred that the wavefunction collapses to the component that is compatible with the measurement result [3]. These views have been called the Copenhagen Interpretation. They did not explain the transition from quantum to classical or an acceptable mechanism for the collapse. Heisenberg [3] suggested that the measurement give
random phases to the terms of the quantum state corresponding to different measurement values. This effect could explain the lack of interference due to 'which way' measurements, but not the state collapse. In 1958, Heisenberg advocated that the environment of the detector play need to be taken into account, without giving any details \[4\]. The dichotomy between quantum and classical description or unexplained collapse has been problematic and led Feynman to write “I think I can safely say that nobody understands quantum mechanics” \[5\]. Mermin \[6\] wrote “shut up and calculate” to sum up what the Copenhagen Interpretation meant to him.

It has served us well to calculate system properties with the quantum equations and using Born’s probability rule when applicable. Further investigations of the foundation of quantum mechanics have been given a low priority within the physics community. The initial acceptance of the Copenhagen interpretation may be understood from that a partial description is better than no description at all of nature. Nevertheless, a full description is more desirable than a partial description.

Einstein refused to believe that the fundamental physics is probabilistic \[7\]. The quantum physics could at best be an effective theory, covering up a deeper reality.

The wave function amplitude is sometimes called the probability amplitude \[3\] as if its sole purpose is to give probabilities \[1\]. That terminology suggests that a classical particle is lurking behind the scenes. This thinking is difficult to reconcile with the superposition principle, which is of ultimate importance. For this hypothetical classical particle to be revealed there has to be a deviation from the state vector description. However, we have no evidence for a mechanism that produces any deviation from the unitary evolution of the quantum state vector. The unitary evolution always gives the proper detailed description of the interaction between a quantum system and a detector. Notably, Bohr believed that the measurement apparatus is correctly described by quantum mechanics, though had to be viewed as if it was a classical apparatus in the actual measurement situation \[9\].

Quantum physics has been enormously successful in describing the detailed physics of solids, molecules, atoms, nuclei and particle physics. Is it sensible to claim that this theory only describes what happens in between preparation and measurement, but not the measurement process? When you sit on a chair, you do a continuous observation of its stability that is ex-

\[1\] Bohr was not comfortable with the term “In this connection, it should also be emphasized that the term probability amplitude for the amplitude functions of the matter waves is part of a mode of expression which, although often convenient, can, nevertheless, make no claim to possessing general validity.” \[8\]
plained by the wave nature of electrons. If the observation makes quantum physics invalid, how come the chair still holds you up? As we do observations all the time of phenomena that are explained by quantum mechanics, it seems that quantum mechanics also describes what happens during measurements.

Everett took an essential step towards erasing the dichotomy between state vector evolution and measurement in 1957 [10]. In Everett’s interpretation, the state vector completely describes the state of the physical world. Everett noted that a measurement would lead to entanglement between the system being measured and the detector. An observer that reads of the detector can be viewed as being another detector that measures this entangled state and hence gets entangled with it.

After the measurement, the total state is (still) a sum over all possibilities of the measured system. Each term describes the detector in a state of the precise value of the measured property. The measured system and the observer are in a state corresponding to that value. These terms were denoted branches and, according to Everett, the observer has experienced that one particular value was measured. However, the observed value differs between the branches. Within the branches our experiences of consistency between observers and between the findings in measurements and further investigations of the measured system [2] The problems of previous interpretations seem to be resolved, though some essential aspects were missing or insufficiently treated.

Firstly, it was not shown that the branches could not interfere, which is necessary for the branching to be well defined. Zeh [12] noted this problem and realized that its solution was what we now call decoherence [13, 14, 15]. It is the effect of entanglement with and within the environment. Decoherence also explains why macroscopic objects stay localized and obey the laws of classical physics.

Secondly, as Everett’s theory aims to describe the process of measurement in quantum mechanical terms, the Born rule probabilities should be derived from the theory. Everett attempted this, but his argumentation was insufficient, see section 6.

Thirdly, there is no physical interpretation of the state vector. This lack is a fundamental shortcoming of Everett’s theory. He writes [10] “The wave function is taken as the basic physical entity with no a priori interpretation.

\footnote{Already 1939 London and Bauer [11] analyzed measurements as entangling between system and detector and eventually with the observer. They seem to have failed to observe the ‘objectivity’ within a branch}
Interpretation comes after an investigation of the logical structure of the theory. By several proponents of Everett’s quantum mechanics is given a purely mathematical character by defining the theory as vectors of Hilbert space that evolve according to the quantum equations \[16, 17, 18\]. This formulation gives the impression that the theory is purely mathematical and has no physical content. The following quote from Tegmark [17] illustrates “... postulates in English regarding interpretation would be (mathematically) derivable and thus redundant”. Kent [19] and Maudlin [20] has found the theory to be empty of physical content. Maudlin writes “Taking away the measurement postulates ... robs the textbook recipe of any empirical consequences”.

The choice of explicit Hilbert space gives the physical content. By defining the meaning of the variables of the wavefunctions regarding position variables for specific types of particles, or the corresponding definition of the quantum fields, the physical content can be achieved. However, it requires additional explanation which has been missing.

A notable exception is the works of Vaidman, where he introduced a postulate that defines what the values of the probabilities [21] for the different branches or the corresponding betting preferences [22]. This postulate avoids the criticism but introduces the notions of probability for a sentient being or betting preferences, which seems alien to Everett’s project. Sentient beings should have no special role, as it is assumed that they can be described as quantum systems. Not only does it make the theory less convincing, but it also fails to justify the use of decoherence theory to define the branches.

Equations that describe physical processes always need to be interpreted. Ballantine [23] makes this clear in the following quote “from the formalism \( f = ma \), one cannot deduce that \( f \) is a force, \( m \) is a mass, and \( a \) is an acceleration”. Without an interpretation, there can be no meaning to expressions derived from the wave function. There has to be: (1) a correspondence between the quantities that enter into the equations and well defined physical phenomena and observations; (2) an understanding of what the equations can describe, their region of applicability.

The purpose of this article is to give an interpretation of the quantum state, suitable to describe the measurement as a quantum unitary evolution including how the Born rule arises. For this end, the quantum theory is supplied with a new set of postulates replacing the traditional postulates. The new postulates are presented in section 2. How the traditional measurement postulates, except for the Born rule, are derived from the new postulates is presented in section 3. In section 4 it is presented the description of mea-
measurements on many equally prepared systems. The perception of the Born rule for a typical observer is presented in 5. A discussion of other attempts to prove the Born rule is given in section 6. Conclusions and final remarks are presented in section 8.

2 Postulates

In Heisenberg’s article 24 that came to be the starting point of quantum mechanics, he aimed to replace the notion of a definite position of the electron with a quantity that could give transition probabilities using the classical dipole radiation formulas. He also aimed to reach a theory that could be generalized to more systems than the Bohr-Sommerfeld orbits could be applied to. Thus quantum mechanics is about position, though the particle position concept is different from classical mechanics.

Schrödinger 25 sought to find an equation for a (wave) function of space, that could give the quantized energies. At the large quantum number limit, there is a clear correspondence between wave function and the classical mechanical orbit, at least for integrable systems. Thus, the wave function replaces the where and how of the classical orbit.

**Postulate EQM 1** The meaning of the quantum state: The state is a set of complex functions of positions

\[ \Psi = \{\psi_j(t, x_1, x_2, \ldots)\} \]  

where \( j \) is a discrete index, for example spin and gauge components. Its basic interpretation is given by that the density

\[ \rho_j(t, x_1, x_2, \ldots) = |\psi_j(t, x_1, x_2, \ldots)|^2 \]  

answers where the system is in position, spin, etc. It is absolute square integrable and normalized to one

\[ \int \int \cdots dx_1 dx_2 \cdots \sum_j |\psi_j(t, x_1, x_2, \ldots)|^2 = 1. \]  

This requirement signifies that the system has to be somewhere, not everywhere. If the value of the integral is zero, the system does not exist anywhere.

With the usual way of denoting the norm \( \| \cdot \| \), equation 3 can be written \( \|\Psi\| = 1. \)
If something is possible to measure, then it is possible to separate such a small part from the rest. The separated part will act as a system of its own, thus cannot have zero norm. The difference between two states $\Psi$ and $\Psi'$ for which $\|\Psi - \Psi'\| = 0$ can have no measurable consequences, as $\Psi - \Psi'$ is, according to EQM1, physically equivalent to a function being zero everywhere. This equivalence implies that the state of the system can be viewed as a vector in the Hilbert space of functions of the type (1), see the appendix.

If the index $j$ contains gauge components, these may be summed over in equation (2) to get a gauge independent density. The state vector $\psi$ is not directly observable as it is gauge dependent.

The EQM1 gives us a necessary element of understanding of the quantum state. For example, from the hydrogen groundstate wavefunction, we get where we can find that electron. From some parts of the quantum state of the experiment as a whole we can conclude what and where is the detector. Neither what nor where could be answered without any interpretation in the Copenhagen interpretation; the Born rule gives the corresponding information. For anyone that has learned quantum mechanics from standard textbooks, it has become the intuitive understanding of the wavefunction. Formally, it is nevertheless necessary to define its meaning. The criticisms [19, 20] clearly proves that.

The density gives how much the system is present at a location in configuration space $x_1, x_2, \ldots$, with the discrete index $j$. The density can be denoted the position distribution or the presence distribution, and both will be used here. The quantity presence has previously been denoted measure of existence by Vaidman [21] and caring measure by Greaves [26, 27], but they have not fully clarified its meaning.

EQM1 gives the relation between the theory and what is going on in the laboratory, the world or the universe. It also turns out that EQM1 becomes a powerful tool to uncover the appearance of the Born rule. The interpretation that EQM1 provides can also be given by postulating that the wavefunction belongs to the Hilbert space of functions of the particular variables and indexes. The Hilbert norm is then the measure of position or presence. The disadvantage with such a postulate is the difficulty in explaining the physical meaning of the norm. This difficulty is illustrated by that no one seems to have fully understood it. Additionally, it is backward to start with postulating which mathematics should be used and then derive the physics from that.

Geroch [28] suggested a related interpretation of the quantum state which states that a region of configuration space is ‘precluded’ if the wave-
function is very small there. This suggestion corresponds to ignoring contributions from configuration space where the system is hardly present.

Postulate EQM 2 The equation of motion: There is a linear and unitary time development of the state, e.g.,

\[ i\hbar \partial_t \Psi = H \Psi, \]

where \( H \) is the hermitian Hamiltonian. The term unitary signifies that the value of the left hand side in (3) is a constant of motion for any state \( \Psi \) of the system.

When investigating how the theory describes the world we observe the Hamiltonian has to have realistic features.

The quantum world around us is understood in terms of local interactions. The standard model of particle physics is formulated in terms of locally interacting fields. These properties imply that we only have to understand and interpret quantum mechanics with local interactions. In particular, measurement processes are physical processes confined to the interactions available. In connection with measurements, it can be assumed that there are locally conserved currents.

In EQM1 there is no mention of any relation between the density (presence) \( \rho \) and probability. When the propagation of different parts is dependent on each other due to coherence, the concept of probability is not relevant. However, the density \( \rho_j(t,x_1,x_2,\ldots) \) as a distribution of the particles positions is always relevant. It is similar to Schrödinger’s original interpretation of quantum mechanics \([29]\), in which for a single electron \(-e\rho(x)\) was assumed to be a (classical) charge density. Schrödinger wrongly assumed it could be used in Maxwell’s equations.

The following relations lends support to the interpretation of the density \( \rho \) as the distributed position. For the sake of simplicity, the discrete index \( j \), as well as the time dependence are omitted here.

(i) A single particle in a local potential obey the continuity equation,

\[ \partial_t \rho + \nabla \cdot j = 0, \]

where \( j \) is the conventional current. This shows that the particle position distribution changes in a continuous manner.

(ii) The correction to the energy for a bound single particle disturbed by a local static potential \( V(x) \) is to first order

\[ \Delta E = \int d^3x \rho(x) V(x). \]
An outside agent that interacts with the system weakly enough not to essentially change the state will find that it interacts with a distribution, not with a particle in a definite position.

(iii) We can define the average position as the first moment of the density distribution

\[ \langle x \rangle = \int d^3x \, x \rho(x). \]  

(7)

According to Ehrenfest theorem, if the force \( F = \partial_x V \) is essentially constant in the region where the density is appreciable, the average position will move according to Newton’s Law,

\[ m \frac{d^2}{dt^2} \langle x \rangle = F \]  

(8)

If the width of the density distribution is “small” it gives the position of a particle moving along as classical particle.

(iv) The particles are not at positions where the density is zero.

(v) From molecular, atomic, nuclear and particle physics it is well established that the single particle density of \( N \) electrons, protons or quarks

\[ \rho(x) = N \int d^3x_2 d^3x_3 \cdots \rho(x, x_2, x_3, \ldots) \]  

(9)

gives the charge density if multiplied with the charge a single particle.

In the Oppenheimer-Born approximation, the nuclei interact with the (instantaneous) charge distribution of the electrons as given by equation 9.

In nuclear physics, the comparison between calculated charge distribution and experimental is an important method to test theories, see [30].

(vi) The dependence of \( \rho_j(x_1, x_2, \ldots) \) on all the different degrees of freedom in strongly interacting systems is impossible to uncover in a non-destructive method as in (ii). Information about correlations can be extracted from inelastic scattering. We can also get a clue to them from theoretical considerations of the energy and other static properties of bound states. For example, atoms with half-filled valence subshell, with fixed \( n \) and \( l \) quantum numbers, have maxim spin value and maxim total orbital angular momentum as the antisymmetric spatial electron wavefunction minimizes the Coulomb repulsion.
The position of separate compound subsystems can be measured in a similar way as for a single particle in (ii).

(vii) Position is the fundamental quantity of classical mechanics and from that all other classical physics concepts. The classical physics can be discussed from where things are without having to understand exactly how the properties of macroscopic objects emerge.

This listing shows that if we wish to interpret the meaning of \( \rho_j(x_1, x_2, \ldots) \) without any attention to the measurement process, the interpretation that is given by EQM1 or something to the same effect seems unavoidable.

Below are listed the Copenhagen Interpretation postulates.

**C1** The state of a physical system is a normalized vector \( |\Psi\rangle \) in a Hilbert space \( H \) which evolves unitarily with time.

**C2** Every measurable quantity is described by a Hermitian operator (observable) \( B \) acting in \( H \).

**C3** The only possible result of measuring a physical quantity is one of the eigenvalues of the corresponding observable \( B \).

**C4** The probability for obtaining eigenvalue \( b \) in a measurement of \( B \) is
\[
P(b) = \langle \Psi | \pi_b | \Psi \rangle,
\]
where \( \pi_b \) is the projector onto the eigen-subspace of \( B \) having eigenvalue \( b \).

**C5** The post-measurement state is (the result of the unitary development during the measurement of) \( \pi_b |\Psi\rangle / P(b)^{1/2} \).

Some modern formulations of the postulates allow for positive operator value measurements, but that generalization offers nothing extra here. It is the same as the projection value measurement postulates C2-5 up to a unitary transformation [31].

In the discussion of EQM1 above, it was shown that it is a consequence that the state belongs to a Hilbert space so that EQM1 and 2 imply the content C1.

As the measurement process is a physical process described by the dynamics (EQM2), no new postulates are corresponding to C2-5. How do real physical measurement processes correspond to the C-postulates? In order to answer that question, this article will not address everyday observations but confine the discussion to measurements in designated experimental setups.
3 Basics of Measurements

It is difficult to analyze, which quantities can be measured based on all conceivable experimental setups. However, an understanding of the fundamentals of measurements can be achieved from the fundamentals of detectors. Detectors can typically record that a particle entered it, which can be used to create position information. The momentum of a charged particle can be transformed into a measurement of position. The measurement of photon energy can be transformed into the measurement of position using a grating. The measurement of angular momentum of an atom can be transformed into a photon energy measurement by the Zeeman effect or position by a Stern-Gerlach apparatus. These are examples of measurements of physical quantities which correspond to Hermitian operators. Even the measurement of the time for an event is in principle transformable to a position measurement.

It is reasonable to assume that all types of measurements transform the property in question to measure position or simply counting particles, primary or secondary. Alternatively, the measurement procedure is related to that in the way it is calibrated. The following discussion of measurements will be confined to the recording of a particle entering a detector. This detector may be a part of an array of detectors in order to get position information from which detector was hit.

Particle recording detectors react when a particle is entering a particular volume or area. There is an infinite set of states with support inside the volume (area) and another infinite set of orthonormal states with support only outside. Together they make up a complete basis. The Hermitian operator that corresponds to measurements with this detector can be defined such that all the inside states are eigenstates with a common eigenvalue and the outside with another value. This detector can only tell whether a particle came into it or not. A less crude position detector may be constructed by placing several such particle recorders at different positions. The Hermitian operator for this composite detector may be constructed by associating the same value for all states inside one particle recorder, but different values for different recorders. Additionally, another value should be attributed to the outside of all particle recorders. This detector records if any of the individual particle recorders fired and which fired and there is a Hermitian operator with eigenvalues corresponding to the different recordings.

Obviously, the detector described so far is highly idealized. For example, it is unrealistic that a particle recording detector can register particles at any energy. However, at a specific experiment, the energy range of the particles
is limited. The described model is relevant as long the efficiency is close to 100\% in the real experiment.

It is assumed that the measurement setup is such that, which particle recorder the particle reaches is given by its value of the property being measured. There is a unitary operator

\[ U = \exp(-iH_e t/\hbar) \]  \hspace{1cm} (10)

corresponding to the Hamiltonian \( H_e \) that describes this part of the experimental setup.

Denote the Hermitian operator that corresponds to the position detector with \( Y \). The operator \( B \) being measured by \( Y \) and the unitary evolution \( U \) is given by

\[ B = U^\dagger Y U. \]  \hspace{1cm} (11)

The eigenstates \( |b\rangle \) of \( B \) are related to eigenstates of \( Y \) by

\[ |y\rangle = U|b\rangle. \]  \hspace{1cm} (12)

As described above, each eigenvalue of \( Y \) is typically degenerate. According to (11), the same applies to \( B \), but as noted above in an actual experiment only a small number of states are involved. For simplicity, it is assumed that only one state per particle recorder is relevant.

![Diagram](image)

Figure 1: The position detector consisting is the particle recorders D1-D3 receive the different components of the wave function \( |\psi\rangle \) due to the unitary transformation \( U \). The state \( |a_n\rangle \) transforms to \( |y_n\rangle \) by \( U \).

The state to be measured is written in the eigenstates to \( B \),

\[ |\psi\rangle = \sum_b c_b |b\rangle. \]  \hspace{1cm} (13)
The state that enters the position detector system is
\[ \sum_b c_b U|b\rangle = \sum_b c_b |y_b\rangle. \] (14)

This expresses that the different eigenstates \(|b\rangle\) enters separate particle
recorders and is there represented by \(|y_b\rangle\), see figure 1. As the functions
\(y_b(j, x)\) with differing value of \(b\) have disjoint spatial support, the density
of the state (14) is
\[ \rho_j(x) = \sum_b |c_b|^2 |y_b(j, x)|^2. \] (15)

It describes where the system is according to EQM1. Summation over
the spin and integration over the volume of one of the particle recorders
will give the value \(|c_b|^2\), where \(b\) is the eigenvalue of \(B\) associated with that
recorder. The interpretation of this result is that
\[ \rho_b = |c_b|^2 \] (16)
as a function of the discrete variable \(b\) tells where the system is for the
eigenvalue of \(B\). Note that this result is an important step towards replacing
the old axioms C2 and C3 with EQM1 and EQM2.

So far, the interaction between the particle and the detector has been
ignored. The decoherence necessary for a measurement to happen relies on
this interaction, see section 3.1.

In order to simplify the notation, it will be assumed that the state \(|\psi\rangle\)
(13), instead of the transported state (14), directly interacts with the (com-
posite) detector. Then, the interaction with the detector \(M\) is described by
\[ (\sum_b c_b|b\rangle)|M\phi\rangle \rightarrow \sum_b c_b|b\rangle'||M_b\rangle. \] (17)

The detector changes its state from its nothing registered state \(|M\phi\rangle\) to a
state \(|M_b\rangle\) consistent with having registered the state \(|b\rangle\). The state of the
system before the measurement \(|b\rangle\) and after \(|b\rangle'\), may be the same. In
reality, there are a set of states of the detector that all correspond to the
value \(b\). This and similar complications are henceforth ignored.

According to Everett, the observation process is described by
\[ (\sum_b c_b|b\rangle'||M_b\rangle)|O\phi\rangle \rightarrow \sum_b c_b|b\rangle'||M_b\rangle|O_b\rangle, \] (18)

\footnote{For simplicity, the terms of the lefthand side (17) are written as if the incoming state \(|b\rangle\) stops interacting with the detector. It can be captured for a long time in which case it is not a product state.}
where the state of the state of the observer $O$ is altered to having observed the value that the detector has measured. Equally, a friend $F$ will also get entangled, if $F$ is told what value $O$ has measured, or reads the detector value. This entanglement guarantees the consistency between observers. The distribution $\rho_b$ gives the position of the total system over the branches. Another way to express this, the value of $\rho_b$ gives the presence at the branch with outcome $b$ of the observer and everything else entangled with the measurement result.

The particular measurement setup which the present discussion uses may seem to be too specialized. There exist other measurement methods, they are usually calibrated using the type of measurement discussed here. Other methods are accepted if they agree with the type of measurements of figure 1.

Another way energy detectors may be calibrated is to deposit a macroscopic number of particles known to have the same energy (distribution) into some container and measure the deposited macroscopic energy by classical (calorimetric) means. Also, in this case, is the measured value is at some point converted to a position.

Note, that a current or a voltage is at some stage converted to one or several positions in terms of a pointer or positions of charges in a digital memory or display. The general feature is that measurements results are sooner or later manifested as positions, which emphasizes the prudence of EQM1.

3.1 Decoherence: Selector and Protector

There is an ambiguity in the transformation (17). If $|\psi\rangle$ is written in another basis $|x\rangle$ that are eigenstates to operator $X$, then we get

$$\left( \sum_x d_x |x\rangle |M_\phi\rangle \right) \rightarrow \sum_x d_x |x\rangle' |M'_x\rangle.$$  \hspace{1cm} (19)

where the detector states $|M'_x\rangle$ are linear combinations of the states $|M_b\rangle$. From (19) it might look like it the quantity $X$ that has been measured. However, the assumed experimental setup, figure 1, with realistic properties of the particle recorders guarantees that $B$ is measured, as expression (17) suggests.

When a particle recorder is excited because the system enters it, very many degrees of freedom get excited. Very soon after the initial excitation, any operator $T$ that can give a non-zero matrix element between different
\( |M_b\rangle \) states
\[
\langle M_b | T | M_{b'} \rangle \neq 0, \ b \neq b',
\] (20)

need to be extremely complicated. It has to be an operator that touches as many particles as has been excited in the two different particle recorders corresponding to \( b \) and \( b' \). Additionally, \( T \) has to depend on the exact initial state of the particle recorders. The probability that nature, with or without the involvement of humans, will supply a process corresponding to such an operator \( T \) is FAPP zero. This property implies that we cannot experience any of the detectors states \( |M_{b'}\rangle \) as an observation of such a state, entails an interference between \( |M_b\rangle \) states. The measurement process creates decoherence between terms with different values of \( B \).

As the particle recorders are in contact with its environment, it will influence the environment such that it will reflect its state. Hence, also particles in the near environment change their state which further strengthens the decoherence. That will continue to parts of the environment even further out and so on. The readout of the state of the detector that the experimenter has made to happen is one such process that increases the decoherence by having more physical systems to change their state in a way that reflects the initial excitation of the detector.

Due to the lack of any interference effects between the terms in the state that results from the structure of the detector and the coupling to the environment, \( \epsilon \),
\[
\sum_b c_b |b\rangle' |M_b\rangle |\epsilon_b\rangle,
\] (21)
can be thought of as isolated systems. DeWitt called them “worlds” but Everett named them “branches” which seems more appropriate.

Once, the measured data is stored into some memory constructed to be resilient and with considerable redundancy, that itself is enough to hinder any coherence between the possible measurement values. For example, if the data that is written on a piece of paper, the molecules of ink or ‘lead’ that attach to the paper are not likely to lose their position by quantum spreading. They attach to the paper and each other, forming macroscopic structures. It is well known that macroscopic structures are measured continuously by their surroundings [13]. The quantum Zeno effect then implies that the quantum uncertainty of the position of the writing will be minimal. If nothing else protects from coherence between the terms of (21), the way we store the data guarantees that we will not notice any effects of coherence.

Vaidman [32] has defined ‘worlds’ as having different macroscopic structure. This might not be the appropriate general definition of a branch.
However, successful measurement results give rise to different macroscopic structures, so the branches that are discussed here is precisely Vaidman ‘worlds’.

There is a suggestion from Hanson [33, 34] that low-density branches can get ‘mangled’ by large density branches. The mangling is caused by residual coherence of branches. However, as soon a branch is created in which values are recorded and given well protected macroscopic manifestations, any effects of recoherence are negligible. The problem of the ‘competition’ between small and relatively large density branches is a practical experimental issue related to the creation of the branches. It is a well-known difficulty to measure values \( u \) that have very low density \( \rho_u \). Large density branches \( \rho_b \gg \rho_u \) tend to interfere with the observation of the low-density value. The general term for this experimental problem is ‘cross-talk’.

The derivation of the decoherence mechanism is based on the traditional interpretation of quantum mechanics [12, 35]. Joos [36] and Tappenden [37] questioned the use of decoherence theory to infer the Born rule as it is already assumed. However, decoherence theory primarily relies on the Born rule to conclude that the environmental particles are ‘measured’ somewhere after that they scattered off ‘macroscopic’ systems that are then found to be localized. The interpretation (ontology) given by EQM1 serves well to replace the Born rule in decoherence theory.

3.2 The Measurement Result

So far, it has been established that the measurement setup as in figure 1 can create one well-defined branch for every possible measurement value, which are eigenvalues to a Hermitian operator. The quantum state of the branch is that of the eigenstate (with amplitude \( c_b \)) entering the detector. The Copenhagen postulates C1, C2, and C3, section 2, are fulfilled in each branch, as well as C5 if the current branch is renormalized to norm 1 by the observer within a branch.

Looking at the many branches from the outside the question “What reading did the observer get?” is equivalent to “What is the distribution of observer readings?” - The answer is given by the distribution \( \rho_b \) (16). This value can also be arrived at calculating the total density (the norm) of the \( b \)-term in the final state of (17) or (18). Note that once decoherence has

\[^4\]Zurek [38] proved under the assumption of non-disturbing measurement that the measured basis states have to be orthogonal in order for the measurement apparatus to differentiate them.
taken place, the created branches evolve independently keeping their norms conserved.

4 Repeated Measurements

Suppose the detector is able to record several subsequent measurements of identically prepared systems (13). Further, assume that the way the detector interacts with the next system is not essentially affected by previous measurements. The second measurement is described by the transition

$$\left( \sum_{b_2} c_{b_2} |b_2\rangle \right) \sum_{b_1} c_{b_1} |b_1\rangle' |M_{b_1}\rangle \rightarrow \sum_{b_1b_2} c_{b_2} c_{b_1} |b_2\rangle' |b_1\rangle' |M_{b_1b_2}\rangle.$$  \hspace{1cm} (22)

When the interaction with the observer is included the final state becomes

$$\sum_{b_1b_2} c_{b_2} c_{b_1} |b_2\rangle' |b_1\rangle' |M_{b_1b_2}\rangle |O_{b_1b_2}\rangle.$$ \hspace{1cm} (23)

Each sequence of readings belong to different branches. The distribution of observer reading sequences is now

$$\rho_{b_1b_2} = |c_{b_1}|^2 |c_{b_2}|^2.$$ \hspace{1cm} (24)

After N measurements, the sequences of observer readings are distributed according to

$$\rho_{b_1b_2...b_N} = |c_{b_1}|^2 |c_{b_2}|^2 \cdots |c_{b_N}|^2.$$ \hspace{1cm} (25)

When N is large, the relative frequencies of the values of b became interesting. To focus on the value $b = u$, denote the summed density of all the other values of b by

$$\rho_{\sim u} = \sum_{b \neq u} |c_{b}|^2$$ \hspace{1cm} (26)

and $\rho_u = |c_u|^2$. The sum of the densities (25) over all sequences where $b = u$ appears precisely m times out of N measurements

$$\rho(m: N|u) = \frac{N!}{(N - m)!m!} (\rho_u)^m (\rho_{\sim u})^{N-m}.$$ \hspace{1cm} (27)

This gives the total summed density of the branches in which the observer has found the value $u$, m times. Hence, the question 'how many times
have the observer measured the value $u'$ is answered by $\rho(m : N|u)$ as a distribution over $m$-values.

For large number of measured systems $N$, the distribution (27) may be approximated by a gaussian, see Feller [39],

$$\rho(m : N|u) \approx \frac{1}{(2\pi N\rho_u \rho_{-u})^{1/2}} \exp\left(-\frac{(m - N\rho_u)^2}{2N\rho_u \rho_{-u}}\right).$$  \hfill (28)

The distribution (28) may be represented as function of the relative frequency $z = m/N$ taken as a continuous variable. The properly normalized position or presence distribution with respect to $z$ is

$$\rho(z|u) = \left(\frac{N}{2\pi\rho_u \rho_{-u}}\right)^{1/2} \exp\left(-\frac{N(z - \rho_u)^2}{2\rho_u \rho_{-u}}\right).$$  \hfill (29)

As $N \to \infty$ this density approaches the delta function $\delta(z - \rho_u)$. This relation says that at infinitely large $N$ there is only one value of the frequency $z = \rho_u$. It might look like as a big stride towards proving Born’s probability rule, but $\rho(z|u)$ is an approximate result.

To get from the exact expression for $\rho(m : N | u)$ (27) to the continuous frequency distribution, the interval $[0, 1]$ is divided into a set of intervals $\{I_k\}$,

$$I_k = [0, 1] \cap [z_{k - \Delta z/2}, z_k + \Delta z/2], z_k = \rho_u + k\Delta z. \quad \hfill (30)$$

The index $k$ belongs to the minimal set of integers such that $\{I_k\}$ covers $[0, 1]$. Define $\tilde{\rho}(k)$ as the sum of densities $\rho(m : N | u)$ with $m/N$ in the interval $I_k$. Set

$$\rho_{\Delta z}(z|u) = \tilde{\rho}(k)/\Delta z \text{ if } z \in I_k. \quad \hfill (31)$$

This is a histogram type piece-wise constant function. If $\Delta z = \Delta z_1/N^{-1/2}$ and $\Delta z_1$ is small and $N$ is large, then $\rho_{\Delta z}(z|u)$ can be arbitrarily close to $\rho(z|u)$.

In order to adequately justify the use of the frequency distribution (29), an operator should be found that is closely related to this distribution. The first guess may be the frequency operator

$$F_N = \frac{1}{N} \sum_{i=1}^{N} f_i \quad \hfill (32)$$

where $f_i$ operates on the $i$-th system being measured with $f|u\rangle = |u\rangle$ and $f|b\rangle = 0$ if $b \neq u$. The eigenvalues of $F_N$ are $z = m/N$, $m = 1, ..., N$. The density related to $F_N$ acting on this state is given by (27) with $m$ replaced by
As pointed out by Squires [40], the values of this discrete distribution approaches zero as $N \to \infty$.

The operator $F_{N\Delta z}$ defined by its action on products of eigenstates to the operator $B$. If the frequency of the eigenvalue $u$ is in the interval $I_k$ with midpoint $z_k$, then

$$F_{N\Delta z}|b_N\rangle|b_{N-1}\rangle...|b_1\rangle = z_k|b_N\rangle|b_{N-1}\rangle...|b_1\rangle.$$  \hspace{1cm} (33)

The density of this operator is $\tilde{\rho}(k)$. As the eigenvalues $z_k$ of $F_{N\Delta z}$ is a discrete set its density distribution $\rho_{z_k} = \tilde{\rho}(k)$ is represented be a bar graph rather than the histogram that represents $\rho_{\Delta z}(z|u)$.

To see the behavior of these densities as $N$ approaches infinity, the Chebyshev’s inequality [39] can be applied to the distribution $\rho(m : N | u)$ \hspace{1cm} (27). The result can be written as

$$\sum_{|m/N - \rho_u| > \Delta z/2} \rho(m : N | u) \leq \frac{4\rho_u\rho - u}{(\Delta z)^2 N}.$$  \hspace{1cm} (34)

From this follows that $\sum_{k \neq 0} \tilde{\rho}(k) \to 0$ as $N \to \infty$ and that $\tilde{\rho}(0)$ approaches one for any value of $\Delta z$. The delta function limit of $\rho(z|u)$ is confirmed by the exact calculation.

The quantity $\rho(z|u)$ is a continuous approximate representation of $\rho(m : N | u)$, which is a sum of the densities of several branches. The interpretation is that $\rho(z|u)$ gives the position distribution with respect to the relative frequency $z$ of everything entangled with the measurement result. The presence of the observer within an interval in the relative frequency of length $dz$ is $\rho(z|u) dz$.

5 The Born Rule

The frequentists may find that the Born rule is derived from section 4, the discussion on currents arriving to the detectors 5.1. The pros and cons of the frequentist view when applied to Everettian quantum probabilities and the Bayesian view is presented in 5.2. In subsection 5.3, it is shown that the present theory implies, that in the significant part of the presence, physicists believe in the Born rule, which should be the most persuasive argument for the Born rule. The subsection 5.3 discuss the appearance of the Born rule using the global perspective of all the branches and the distribution of the observer with respect to her observation of the Born rule. How a rational agent’s subjective probabilities get confined to the Born rule is shown in subsection 5.4.
5.1 The Current View

Born first suggested his rule in connection with scattering theory [1]. The most common way to derive cross sections involves an incoming current. Everett’s description of measurement is that the measured system and the detector scatter against each other. A combination of these ideas give a description of repeated measurements in terms of currents.

For repeated measurements set up as in figure 1, there is a stream of particles going in the directions indicated by the arrows. The magnitude of the currents in each stream is equal to \( J_b = \rho_b n \), where \( n \) is the number of measurements per time unit. The current \( J_b \) equals the average number of recordings in detector \( b \), per time unit.

After time \( t \), the number of measurements \( N \) equals \( nt \). The average number of recordings in detector \( u \) is \( \bar{m} = \rho_u N \). The number of recordings varies between branches. The distribution \( \rho(m : N|u) \) gives the total presence of the branches with \( m \) recordings. The discreteness of the number of recordings \( m \) and its variation may be thought of as ‘shot noise’.

The average over branches weighted with their presence of the frequency \( \bar{m} = \rho_b N \) shows that the average relative frequency is \( z = \rho_b \). This relation is the Born rule! Can this be considered sufficient proof of the Born rule in EQM?

5.2 Proving Probabilities

Probability is a complicated concept. If a deterministic process gives a single outcome, probabilities are used to describe and evaluate the situation when details are not known to with certainty predict the result. The numerical value of the probability equals the expected relative frequency of a repeated process. A statistical analysis gives increased knowledge and is used to constrain probability values.

The common frequentist view is that the probability is the relative frequency from infinitely many repetitions [39, 41]. The following criticism against the common frequentist view, adapted from Appleby [42, 43] and Wallace [18], should be responded before accepting a proof relying on the common frequentist view.

1. C: After infinitely many repetitions, it is possible for the value of the relative frequency to deviate from the probability. The probability of such sequences tends to zero, but the use of the notion of probability makes this definition of probability circular. R: In EQM, all possible sequences of measurement results together constitute the reality.
Not only a single sequence as in the case of a single outcome. After infinitely many repetitions, the universal wavefunction is only located at the relative frequency \( z = \rho_u \). The observer sees a random sequence which suggests an analysis in terms of probability, which, in view of the behavior of the presence distribution, will be taken to be \( \rho_u \). This analysis creates no circularity as presence is a quantity on its own, not derived from the probability concept. However, the probability concept does not directly appear from such an analysis.

2. C: It is impossible to make infinitely many repetitions. R: In a theoretical analysis, it is possible to consider thought experiments in which there are infinitely many repetitions.

3. C: There is no well-defined frequency of a particular outcome in an infinite sequence, as a reordering can change the value. R: The universe of all of the branches is left unchanged under reordering if all branches are reordered in the same way with respect their ordinal number in the sequence. Any other reordering would violate the branches being the result of repeated experiments.

4. C: Any particular infinite sequence has zero probability, so how can those with the ‘right frequency’ be favored against the one with another frequency? R: All sequences are present and contribute with its presence.

5. C: The first (finite) part of an infinite sequence is a vanishingly small compared with the rest and give no reliable information about the infinite sequence. R: This is the problem of that statistics of finite sequences can be misleading. When all branches are studied together, which we can do theoretically, this will not be a problem.

An alternative to the frequentist view of probabilities is the subjective view. Even in frequentists description of probability theory, probabilities are about expectations as evident from the term ‘expectation value’. Probabilities reflect an agent’s estimate of how likely is a particular outcome. It is often denoted the Bayesian theory of probability and has been advocated by de Finetti [44] and Savage [45]. The theory concerns the beliefs of rational agents for which Savage identified necessary requirements (postulates). These give a foundation on which the theory of probability could be built. Most notable in this theory is the update of probabilities a rational agent
will make on the discovery of new information,

\[ P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}. \]  

(35)

Here, \( P(A|B) \) is the probability of \( A \) when the agent know \( B \) to be true, \( P(A \cap B) \) is the agent’s probability for both \( A \) and \( B \), and \( P(B) \) is the total probability for \( B \). It originates from the identity \( P(A \cap B) = P(A|B)P(B) = P(B|A)P(A) \). In order to evaluate the update expression, the agent has to analyze the process that leads to an outcome. The theory is only a skeleton, which has abstracted away the world which the agent has beliefs about. In order to get any values of the probabilities, the nature of the world has to be taken into account by the agent.

### 5.3 Statistics and One-Worlders

Consider from the EQM point of view an observer, who believes there is no branching, only one world exists at every time. She is involved in a deliberate measurement process of a phenomenon where the outcome is uncertain due to the observed quantum state containing more than one possible value. After a long sequence of measurements, the observer is distributed over very many branches. In each branch, a random sequence is observed which call for a statistical analysis by the observer. The observer will assume that there is a probability \( P_u \) for measuring \( u \) in a single measurement. The probability of the measured relative frequency \( z \) after \( N \) repeated measurements, for this value of \( P_u \), is then

\[ P(z|u) = \left( \frac{N}{2\pi P_u(1-P_u)} \right)^{1/2} \exp \left( -\frac{N(z-P_u)^2}{2P_u(1-P_u)} \right). \]  

(36)

As this is a very narrow distribution for large \( N \), a frequentist analysis would give that \( P_u \) is in some narrow interval around the measured value of the relative frequency, with some low \( p \)-value. The Bayesian analysis, assuming de Finetti’s infinite exchangeability, gives rise to the probability distribution for the value of \( P_u \) conditioned on the measured relative frequency \( z \),

\[ P(P_u|z) = \frac{P(z|u)P(P_u)}{\int_0^1 dP_u P(z|u)P(P_u)}. \]  

(37)

Here, \( P(P_u) \) is the prior distribution of \( P_u \). If there is no previous information, it will be constant and the dependence of \( P(P_u|z) \) on \( P_u \) will be given by \( P(z|u) \).
The relative frequency \( z \) is distributed over all branches according to (29), see figure 2. Hence, the distribution of \( P_u \) over the branches may be seen as the folding of the two distributions (29) and (36).

Figure 2: The solid line shows the density \( \rho(z|u) \) for \( \rho_u = 0.3 \) and \( N = 1000 \). The dotted line shows where an observer in a typical branch may estimate the probability \( P(z|u) \) to be from the observed sequence alone.

As the number of repeated measurements \( N \) grows the width of the probability distribution \( P(z|u) \), tends to zero as does the position distribution \( \rho(z|u) \). After a large number of repeated measurements, the observer sees a relative frequency close to \( \rho_u \), and the large value of \( N \) implies that the value of \( P_u \) is probably close to the observed frequency. Hence, the observer believes that the probability \( P_u \) is very close to the value of \( \rho_u \).

There is one additional effect that can make the difference between the values of \( \rho_u \) and \( P_u \) go away. That happens if the observer knows what quantum state the system is in. When the observer sees a relative frequency close to \( \rho_u \) the observer will likely assume that \( P_u \) is precisely \( \rho_u \), as stated by the Born rule. This might well be assumed even if the Born rule is previously not known to the observer, as inventing the Born rule is known to be possible.

To summarize, the observer distribution in relative frequency (29) is narrowing in precisely the same fashion as for a classic probability (36). The integral of \( \rho(z|u) \) is dominated by the peak, which implies that the observer’s position is mostly where the relative frequency is close to \( \rho_u \). If the observer believes in a single world interpretation, the observer has arguments for the statistical analysis. This observer’s position is dominantly where she
believes in the Born rule. From the point of view of EQM1, where do we expect to find ourselves? If we are to expect anything, our expectation to be near the peak of $\rho(z|u)$ will be high, and our expectation to be in the far tails will be low. Our expectation agrees with observation, physicists believe in the Born rule.

Expectation and probability are very similar concepts, but the word probability has many precise meanings that need not be considered here. Nevertheless, what have been addressed here is the self-locating probability that Barrett have discussed [46]. The situation of self-location uncertainty proposed by Vaidman [21, 22], where the observer is not aware of the measured result, is something different.

When the standard interpretation quantum mechanics is verified on bases of the Born rule, the data is compared with the expectation we have from the Born rule. As has been demonstrated, EQM gives rise to the same expectation as the Born rule. This implies that EQM is equally well verified as the standard interpretation quantum mechanics.

5.4 Decision Theoretic Probabilities

We have seen that according to the postulates EQM, the observer will typically have observed a random sequence with a relative frequency close to the Born rule value. An observer that assumes a single world theory will believe that there is a probability for the different outcomes given by the Born rule. What about the believer in the branching world? No doubt, there is a challenge to argue for the appearance of probability when all alternatives with non-zero amplitudes are going to be realized.

In classical situations where probabilities apply, there is a level of uncertainty, and else it is known what happens. Greaves [26] has taken the view that before a measurement there is no uncertainty for the observer who believes in EQM. The observer knows that every possibility with non-zero amplitude is represented in its branch. When observing the outcome, she will also branch, each of her ‘descendants’ seeing the value of that branch. Although there is no uncertainty, thus no classical probability, the situation before the measurement warrants much the same decision theory as when classical uncertainty is at hand.

In the Bayesian theory [44] the probabilities on a fundamental level are taken to be subjective beliefs. Probability is related to decisions an agent

\footnote{Deutsch [17] seems to assert that statistical evidence is not admissible for the verification of a theory, but considering the ultimate importance of such evidence in quantum physics, he is too restrictive.}
is willing to make. Deutsch pioneered the use of decision theory to understand Everettian quantum probabilities. Wallace and Greaves have continued this work in their own directions. Both base their analysis on the axioms Savage formulated. These axioms define rules that the beliefs a rational agent should follow. Savage proves that the rational decision an agent makes is equivalent to the agent choosing the action that maximizes the expected utility,

$$\langle U \rangle = \sum_i P_i U_i.$$  

(38)

Here $P_i$ is the probability, with $\sum_i P_i = 1$, and $U_i$ is the numerical value of the utility that the agent will get on outcome $i$. From considering the decisions an agent will make under a variety of situations, and a variety of utilities the agent might get at the different outcomes, the agent’s subjective probabilities $P_i$ are uniquely determined, and the utilities $U_i$ are determined up to an affine transformation. It is not assumed that the agent consciously considers the probabilities or that she gives the utilities numerical values.

Lewis acknowledges that even if probabilities are primarily subjective, there are instances like radioactive decay where probabilities are objective features. He formulated the link between subjective probabilities and objective probabilities in the Principal Principle. It implies that an agent who knows that there is an objective probability $P$ for a particular outcome sets her subjective probability to $P$.

Greaves and Myrvold have reformulated Savage postulates for rationality, to suit the case of experiments performed in branching as well as non-branching universes. The following quote formulated the purpose of their investigation. – “The problem is not one of deriving the correct probabilities within the theory; it is one of either making sense of ascribing probabilities to outcomes of experiments in the Everett interpretation, or of finding a substitute on which the usual statistical analysis of experimental results continues to count as evidence for quantum mechanics.”

Greaves and Myrvold arrived at the same expression as Savage for an agent’s expected utility that the agent should seek to maximize the expected utility

$$\langle U \rangle = \sum_b w_b U_b,$$  

(39)

where $U_b$ is the numerical value of the utility the agent gets at the outcome $b$. The $w_b$ is a weight that the subjects assigns to the outcome $b$. In the case of

6The $\langle \rangle$ notation can be read as probabilistic expectation value or a quantum average.
that, there is a single outcome, and it is the subject's subjective probability or credence of the outcome. In the case of a branching universe, Greaves and Myrvold call it quasi-credence. Fundamentally, it has to be a subjective property, precisely as the probability is. In both cases, the values of the weights are well-defined if the agent is rational. The weights are subject to the condition $\sum_b w_b = 1$.

If new information is presented, they are updated according to the Bayesian update expression,

$$w_{c|b} = \frac{w_{b\land c}}{w_b}. \tag{40}$$

The value of $w_{c|b}$ gives the agent's belief about the weight of outcome $c$ in a measurement under the condition that $b$ has been measured in another experiment, while $w_{b\land c}$ is the weight for the combined outcome $b$ and $c$. One particular updating situation is when a particular value has been measured. If the concept of probability is believed to apply, the probability of seeing that value is updated to one. That is the agent believes there is an only possible outcome when she knows the outcome. According to Everett, after the branching the observer's descendants experience as if its branch is the world. After the measurement value is known, the rational agent/observer will set the corresponding quasi-credence to one, $w_{b|b} = 1$. Likewise, for practical reasons the agent will after a branching normalize the quantum state of her branch to have amplitude one if she is to calculate future events.

Greaves and Myrvold assume that de Finetti's infinite exchangeability can be used for repeated experiments. By this, they have arrived at we are entitled to use the same Bayesian statistical analysis of the branching world as for the non-branching. For example, the expression for the probability of $P_u$, is applicable. They argue that de Finetti's theorem, contrary to de Finetti's position, provides us with the notion of objective weights. In the case of a single outcome, they are called chance, while in the branching world case Greaves and Myrvold use the term branching-weights.

Greaves and Myrvold assume that the EQM Born rule was already proven, which gives the branching-weights equal to $\rho_u$. They use the Bayesian updating of the quasi-credence on the results of the collective data, which has been taken as evidence of Copenhagen style quantum mechanics.

Here, the Born rule is not assumed, but it is taken as a possibility that an agent considers some weights that only depend on the state there are weights for branches that only depend on the part of the measured state that ends up in the branch. The agent is supposed to be rational as defined by the axioms that Greaves Myrvold defined. The agent might be unsure about
which is the quantum state at hand such that a mixed state density matrix gives the weights. The branching-weights and classical probabilities are two kinds of weights, but they obey the same mathematical relations. The mixed state is trivial to handle once the branching-weights are understood. It is henceforth assumed that the wavefunction is known to the agent.

The power of using decision theory is particularly evident when it is used consistently in many decisions. For simplicity, consider an agent that anticipates many identical branchings with identical utilities. If in a single event the expected utility is given by

\[ \langle U \rangle_1 = w_u U_u + w_{-u} U_{-u} \]  

(41)

then the expected utility after \( N \) branchings is,

\[ \langle U \rangle_N = \sum_{m=0}^{N} w(m : N|u)(mU_u + (N - m)U_{-u}) \]  

(42)

\[ = N(w_u U_u + w_{-u} U_{-u}) = N \langle U \rangle_1. \]  

(43)

Here, it has been assumed that the utilities \( U_u \) and \( U_{-u} \) can be summed. The quantity \( w(m : N|u) \) is the agent’s total weight for the collection of branches where the value \( u \) appears \( m \) times,

\[ w(m : N|u) = \frac{N!}{(N - m)!m!}(w_u)^m(w_{-u})^{N-m}. \]  

(44)

The multiplicative form of weights of branches after multiple independent branchings can be seen from the expression (40), with \( w_c|b = w_c \) for independent branchings. From \( w(m : N|u) \) a frequency distribution of the weights \( w(z|u) \) is arrived at in the same way as \( \rho(z|u) \) (29).

When the agent makes a rational decision, she has to make a forecast about the properties of the future. In a single world theory, this amounts to finding the probability distribution of the different outcomes. In EQM it is about finding the presence distribution for the different branches, which answers where the world will be in the future.

The functional form of \( w(m : N|u) \) and \( w(z|u) \) are identical to that of \( \rho(m : N|u) \) and \( \rho(z|u) \), respectively. A rational agent that believes in EQM will have to put \( w_u \), the weight of branch \( u \), equal to \( \rho_u \), the presence of branch \( u \), at least for the wavefunction she thinks is the likely one. If she puts her weights \( w_u \) different from \( \rho_u \) and \( \rho_{-u} \), for large \( N \) values she will have hardly any presence where she expected to have most of her presence.

The frequency distribution from repeated measurements gives what the ‘probabilities’ have to be if they exist. The Greaves-Myrvold theory explains
that there is a quantity that plays the same role in decision making as subjective probabilities do, quasi-credence. Correspondingly, there is a quantity that corresponds to the objective probability of the Born rule of the Copenhagen or the standard interpretation. It is the presence $\rho_u$, the amount at which the system is at that value. There is a property $w$ called weight, that can be a credence, a subjective probability, or a quasi-credence, a subjective presence. Whether it is credence or quasi-credence, it is updated in the same way and thus give rise to the same mathematical appearance. It walks and looks and smells like a probability, so some might be bold and call it a probability. Its version in the branching situation, quasi-credence or presence are more precise terms, but quantum probability is the generic term that can be used independently of interpretation.

There is no need for a classical uncertainty when the agent is considering different possible decisions related to a branching situation. The agent has a situation that warrants similar considerations as if the agent is uncertain about a single outcome. Greaves and Myrvold showed, how the similarity implies that the rational agent will maximize the expected utility. This process makes the values of the weights and well-defined and the numerical values of the utilities well-defined up to an affine transformation.

In section 5.3 it was shown that the postulate EQM1 produces a theory such that all the evidence for the Born rule is evidence for this theory. In this section, it has been shown that presence is the quantity that corresponds to the objective probability present in the Born rule. From those considerations, it should be clear that all aspects of statistical analysis, such as determining an unknown quantum state, can be performed using the concepts and steps presented above. All aspects of the Born rule that are necessary, are present.

The picture that emerges is the equality of equal amount of presence, in a similar way to Laplace principle of fundamentally equal probabilities.

The dynamics are represented by current densities (presence currents) in configuration space. How the presence currents go into the different branches when they emerge, give the presence distribution of the branches. Thus, gives the apparent probabilities. This ties to the current view of section 5.1.

5.5 Responses to the Criticism of the Decision Theory by Greaves and Myrvold

As the argumentation relies on the work by Greaves and Myrvold, the criticism against it need to be addressed.

Kent [55] criticizes Savage postulates that Greaves and Myrvold used,
which identified as necessary requirements for an agent to be rational. Kent argues that there are many possible strategies that are in conflict with the postulates, but are rational. He lists eight different strategies, one of them the Savage rational, the others are alternative strategies that do not conform to Savage postulates. He claimed that the alternative strategies are rational, but he only argued that those strategies can be applied consistently, which is not the same as rational. It is possible to consistently act in an irrational manner.

Two of his alternative strategies are not sufficiently well-defined enough but the others can easily be seen to be irrational. For example, the strategy to choose the option that maximizes the minimum reward that is given in any branch. Consider the following two offers: 1) the agent is offered a very low reward \(u = 1\) in an outcome which has extremely low weight \(w = 10^{-100}\), and a very high reward \(u = 10^6\) for all other outcomes; 2) the agent is offered a reward slightly higher than the low reward \(u = 1.1\) for all outcomes. The suggested strategy implies the agent chooses alternative 2, even if more extreme values of weights and rewards are given. It cannot be considered rational to stick with this strategy, no matter what the weights and the rewards are. Additionally, this is a strategy for which the weights are irrelevant. The other well-defined alternative strategies can be shown to be irrational in a similar way. They are all partly insensitive to the values of the weights. If an agent does not care about the probabilities we cannot learn about them from her actions. If no one cares, the Born rule is unimportant.

With the intent to criticize the concept of branch-weight, Kent suggests five different computer-generated branching worlds CBU\(_{1-4}\) and CBU-qualia. All the worlds have a machine with a red button and tape on which the machine writes numbers. Pressing the button causes a new number to be written on the tape. The problem with Kent’s analysis of these worlds is illustrated by his claims about CBU\(_{1-3}\) and CBU-qualia. Here \(N\) new worlds are copies of the previous world but get an additional number on the tape, \(0 - (N - 1)\), respectively. Kent claims there is no branch-weights\(\footnote{Branch-weight seems the most appropriate term here. It is the term that Kent uses in the article.}\) in these models. It might have been his intention when the models were conceived, but upon examination the opposite seems to be true. Kent agrees that after many branchings the inhabitants can make statistics. Looking at the worlds from the outside one will find that in the overwhelming majority of the worlds the inhabitants will find that a new branch has weight close
to $1/N$. In fact, all new branches have the objective branch-weight $1/N$. In CBU$_4$ the number of new worlds varies at ‘the whim of the simulators’ such that it is impossible for the inhabitants to make any sense of it. In CBU$_3$ there is a number written in the sky which corresponds to the weight of that branch while in CBU$_{2,4}$ the number in the sky is irrelevant. In CBU-qualia the inhabitants are given an enhanced feeling in favor of one outcome. Kent’s different models of branching worlds only shows that it is conceivable that the universe could have been impossible or more or less difficult to understand. However, his arguments do not imply that our world is such.

Kent’s conclusion from Greaves’ and Myrvold’s work is that “Everettians cannot give an explanation that says that all observers in the multiverse will observe confirmation of the Born rule, or that very probably all observers will observe confirmation of the Born rule.” In EQM, It is true that will be some branches with low presence where the statistics disconfirm the Born rule. However, in a single world interpretations the Born rule implies that there is a finite probability that we will fail to confirm the Born rule. EQM gives that the total presence of the branches in which we should have seen the Born rule is overwhelmingly large. For a more extended argumentation against Kent on this point, see [56].

Price [57] is skeptical towards the existence of a situation where there is an analog of uncertainty. As has been shown above, there is no actual uncertainty, but there is a distribution situation to which Savage decision theory is applicable. Uncertainty turns out not to be a necessary requirement because the classical concept of probability is successfully replaced by the concept presence.

Further, Price erroneously regards a person’s descendants in the different branches, as if they are different persons. In the example ‘Legless at Bondi beach’, he discusses the misfortune that swimmer’s choice causes to one of his descendants as if the swimmer caused harm to another person in an unethical way. However, the choice corresponds to a gamble that in a one world scenario could, if unlucky, give a disastrous result. There is actually no reason to view the decision more or less ethical to cause your self harm with a low probability or with a low presence.

Price also questions the use of Savage type decision theory. He argues that the decision strategy he calls “social justice” is rational but in conflict with the decision theory Greaves and Myrvold defined. That Price strategy should be rational is argued from the rationality of the principle for organizing societies with the same name. Again Price views the descendants as if they are different individuals that exist together, but that is not a correct view of the situation. See section [7] for a related discussion.
Albert \cite{58} has been critical towards the combined efforts by Saunders, Wallace, Greaves and Myrvold to understand probability and the Born rule in EQM. It seems that much of the critique concerned the earlier work by Greaves \cite{26} on the ‘caring measure’ (=quasi-credence) replacing uncertainty related probability and Wallace early attempts to prove the Born rule purely from decision theory. The latter has problems which Albert discusses at length and this criticism is further mentioned in section 6. He concludes that also the Greaves Myrvold analysis is “unmotivated and wrong” but his arguments rely exclusively on the lack of arguments for the values of the (objective) quasi-credence that the Born rule give. They used the hypothesis that the Born rule to have been proven, for example by Wallace. None of Albert’s arguments seems to be valid against how the article by Greaves and Myrvold enter into the present theory. On the contrary, Albert’s request for arguments related to the relative frequencies from long sequences of measurements is answered in the previous sections.

6 Other Approaches to the Born Rule

To get a solid argumentation for Born’s probability rule, one cannot assume the properties of the rule beforehand. Every statement should either serve as a postulate or be derived from possible postulates.

In this section, several attempts to prove Born’s rule in ways that could suit EQM are discussed with respect to what is proved and what is explicitly or implicitly assumed.

6.1 Finite repetitions

In Everett’s own proof \cite{10}, he shows that the observer in a repeated measurement will see a random series of results in each branch. He then concludes that “we must put some sort of measure (weighting) in the elements of a final superposition”. Everett assumes that the measure only depends on the (absolute) value of the amplitudes $c_b$ \cite{13}, which implies that the measure is independent of other properties of the state. Further, he assumes the measure of a state should be the sum of measures of its orthonormal basis states. By this, Everett arrives at the measure $\mu$ which is the Born rule probability measure $\mu_b = \rho_b$. He compares with the probability measure of statistical mechanics and finds that is equally justified in view of the “conservation of probability” that the sum of the measure of the future branches equals the measure of the current branch. In addition to that the theory
has not been given any contact with physical reality, to simply show that there is a measure falls short of explaining the appearance of probabilities.

Graham [59] has tried to derive Born’s rule using arguments that in effect are based on the narrowing of the relative frequency distribution [29]. He shows that in the limit \( N \to \infty \) the variance of the frequency operator \( F_N \) (32), \( (\Delta N F_N)^2 = N \langle \psi | (F_N - \rho_u)^2 | \psi \rangle^N \), tends to zero. As the Born rule postulate C4 have been abandoned, \( \rho(z|a) \) is left without interpretation which implies \( (\Delta N F_N)^2 \) has no interpretation either. To let the Born rule give it meaning, would result in a circular proof [60, 61, 62]. Note that, the procedure of taking the limit \( N \to \infty \) only deals with quantities at finite \( N \) values. Without a physical interpretation of \( (\Delta N F_N)^2 \) at finite \( N \) the limiting procedure will have no physical significance.

6.2 Infinite Repetitions

To prove the Born rule, Hartle [63] and Farhi, Goldstone, Gutmann [61, 64] start from C1-3. The postulate C4, the Born rule, have been replaced with (C4'): When the system state is an eigenstate to the operator B corresponding to the property to be measured, the measurement result will with certainty be the eigenvalue to the operator.

They take a frequentist approach and prove that

\[
|\psi\rangle^\infty = \prod_{i=1}^{\infty} |\psi\rangle_i
\]  

(45)

is an eigenstate to the corresponding frequency operator,

\[
F_\infty = \lim_{N \to \infty} F_N.
\]  

(46)

The state (45) belongs to the Hilbert space \( H^{\otimes \infty} \), the tensor product of infinitely many single system Hilbert spaces. The proofs are complicated as \( H^{\otimes \infty} \) is non-separable, see the appendix. For example, Squires [40] mistakes the fact that all the \( \rho(m : N|u) \) values approach zero as \( N \) goes to infinity, to show that we cannot deduce anything about the properties of the state \( |\psi\rangle^\infty \) with regard to the frequency observable.

Caves and Schack [60] have correctly criticized the proof by Fahri, Goldstone and Gutmann for lack of rigor, but their criticism of the proofs by Gutmann [64] and Hartle [63] seems not correct. Since, an additional proof along a more mathematically complicated route has been given by van We sep [65]. He also disproved the claim by Caves and Schack that another norm, then the Hilbert norm, would give another result.
Wada avoided the use of the frequency operator by focusing on the density as function of the relative frequency \( \rho(z|u) \). He considered the state \( |\psi\rangle^\infty \), but only from the first \( N \) states to then take the limit \( N \to \infty \) and proved that in the limit, the density vanishes if \( z \neq \rho_u \). In order for this result to suffice, Wada formulated an alternative somewhat vague postulate, where \( u \) need not correspond to a state in Hilbert space. What the ‘variable’ \( u \) instead corresponds to was not defined.

As Caves and Schack notes, even if the mathematics is correct, there are difficulties to deduce probability from the relative frequency of infinite sequences. What does an infinite sequence say about a finite sequence? How is it possible to derive a probability from a certainty? This depends on the context ultimately given by the set of postulates. The postulate C4’ gives no information when the quantum state is not an eigenstate to the operator of the measurement, which is the situation where the Born rule applies. What the proofs give is that, if probabilities apply, then the Born rule gives the right values. There is no information about how or why probabilities apply.

The mathematical proofs, can probably be extended to an infinite product of states entangled with the environment. This would add another element of consistency to the theory presented in this article.

### 6.3 Envariance

Zurek use the invariance that exists in systems entangled with the environment that he denotes envariance. Zurek’s proof is based on four postulates (o)-(iii). In short, (o) and (i) corresponds to C1, (ii) the time evolution is unitary, and (iii) subsequent measurements on the same system yield the same outcome. The last postulate invites the same question that the Copenhagen interpretation fails to answer, what is meant by ‘measurement’. Zurek evades standard decoherence theory as it relies on the Born rule. There are several different proofs given in all based on assumptions, that Zurek largely left unproven. The proof given in the Physics Today article is discussed here.

In short, Zurek’s proof fokus on the entangled system that appears when the state \( |\phi\rangle = (|a\rangle + |b\rangle)/\sqrt{2} \) is entangled with the measurement apparatus \( A \). The entangled state is transformed with the unitary transformation \( U_S = |a\rangle\langle b| + |b\rangle\langle a| \),

\[
|\Psi\rangle = \frac{1}{\sqrt{2}}(|a\rangle|A_1\rangle + |b\rangle|A_2\rangle) \rightarrow (47)
\]
\[ \frac{1}{\sqrt{2}} (|b\rangle |A_1\rangle + |a\rangle |A_2\rangle) = |\Psi\rangle. \]  

(48)

In \(|\Psi\rangle\) the probability for \(|a\rangle\) is as probable as \(|A_1\rangle\) and \(|b\rangle\) is as probable as \(|A_2\rangle\). In \(|\overline{\Psi}\rangle\), Zurek claims it is the other way around. The state \(|b\rangle\) is as probable as \(|A_1\rangle\) and \(|a\rangle\) is as probable as \(|A_2\rangle\). This seems to make sense if a non-disturbing measurement is followed by a transformation, but it relies on that probabilities are not changed by the transformation. This need to be derived from the postulates, but Zurek gives no valid argumentation for that. Zurek considers the possibility that the probabilities of the two terms in \(|\Psi\rangle\) are different. Then, the transformation \(U_S\) changes the probabilities of the states \(|a\rangle\) and \(|b\rangle\) as he argues that the probabilities of \(|A_1\rangle\) and \(|A_2\rangle\) are unchanged because \(A\) is “untouched”.

Zurek continues with a unitary transformation of the measurement apparatus \(U_A = |A_1\rangle\langle A_2| + |A_2\rangle\langle A_1|\), which transforms back again \(|\Psi\rangle \rightarrow |\overline{\Psi}\rangle\). Zurek concludes that \(U_A\) did not change the probabilities for the states \(|a\rangle\) and \(|b\rangle\) as that system is untouched. But the entangled state is back to its original form so the probabilities must be the same as before the swaps. Then he concludes that the probabilities of \(|a\rangle\) and \(|b\rangle\) are “exchanged yet unchanged. Therefore, they must be equal to 1/2.”

The second transformation is very strange. If \(A\) is really a measurement apparatus, it would violate the irreversibility of decoherence. It is a transformation of a branch to become the other branch and vice versa. If it is a pre-measurement, then Zurek discussed a system to be measured for which probabilities not yet apply. Anyhow, Zurek give no valid argument that can be used to conclude how probabilities are affected or not by the unitary transformations.

Perhaps Zurek had in mind transformations that happen before the entanglement with the apparatus. In this case one could argue that the probabilities are equal for \(|a\rangle\) and \(|b\rangle\), if one assumes that unitary transformations do not change the probabilities. It may seem reasonable to assume that probabilities are not changed by unitary transformations. But, if that is assumed, it has to be derived from the postulates, which requires that the postulates are chosen such that it is possible. As the norm is the only invariant, the Born rule follow easily if probabilities are known to be unchanged under a general unitary transformation and they are properties of the quantum state.\(^8\)

Zurek relies on Laplace idea about probabilities which is that certain states of the system have equal probability. This misses out that proba-

\(^8\)Gleason’s theorem [72] is a more general statement, the proof of which is not trivial.
bilities pertain to the possible results of a process and its properties may important. It makes a big difference if I toss a dice or if I simply put it down [18].

Zurek only views the different ways to understand the concept of probability in classical contexts. He never considers how the notion of probability can be understood when ‘everything happens’, which is the case according to his postulates.

6.4 Self-Locating Uncertainty

Vaidman [21, 22] has argued that the observer might come into a situation of true uncertainty, after the measurement has been recorded by the detector and before the observation of its result. In the situations that Vaidman discussed the observer’s location in space varied with the measurement result. As the detector has recorded the measurement, the splitting into new branches has already occurred. The observer has not yet observed her location and it is argued that she is then uncertain in which branch she is in, or more accurately, her descendants are uncertain in which branch they are located.

This post-measurement and pre-observation uncertainty is denoted ‘self-locating uncertainty’, a term taken from discussions on classical probability [73]. Vaidman has argued that the observer’s self-locating uncertainty is the necessary and sufficient requisite for the notion of (classical) probabilities to apply.

6.4.1 Carroll and Sebens

Carroll and Sebens [74, 75] has taken Vaidman’s analysis and combined it with a principle they formulate called Epistemic Separability Principle, ESP. In their shorter article [74], they give a simplified description ESP “the probability assigned post-measurement/pre-observation to an outcome of an experiment performed on a specific system shouldnt depend on the physical state of other parts of the universe”. One objection against ESP is that it assumes that EQM describes the world as we see it, while that is what should be proven. Another objection is, to distinguish between different parts as system, detector and environment, a fundamental interpretation of the quantum state is needed.

Leaving that aside, there is still severe problems with their proof of the Born rule. Two ‘alternate scenarios’ are considered in which there is a spin-$\frac{1}{2}$ particle with its spin in the $x$-direction and an environment consisting
of a coin$^9$ and the rest $\Omega$. The environment detect the spin value in the $z$-direction. Before the observation, the state is either

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}|O\rangle|\uparrow\rangle|H\rangle|\Omega_1\rangle + \frac{1}{\sqrt{2}}|O\rangle|\downarrow\rangle|T\rangle|\Omega_2\rangle,$$  \hspace{1cm} (49)$$

or

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}|O\rangle|\uparrow\rangle|T\rangle|\Omega_2\rangle + \frac{1}{\sqrt{2}}|O\rangle|\downarrow\rangle|H\rangle|\Omega_1\rangle.$$

(50)

where $O$ is the observer that have not yet observed and $H$ and $T$ are the two states of the coin. Similarly to Zurek they assume that a unitary transformation of the measured system does not change the probabilities. As noted previously, this cannot be assumed but need to be derived.

Kent$^{[76]}$ questions the interpretation Carroll and Sebens give to the states (49) (50), that the observer $O$ is in different branches, though the observer is not entangled

$$|\Psi_1\rangle = |O\rangle\left(\frac{1}{\sqrt{2}}|\uparrow\rangle|H\rangle|\Omega_1\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle|T\rangle|\Omega_2\rangle\right).$$

(51)

Sebens’ and Carroll’s view is that the state (51) describes an observer that is located in one or the other branch. Their view is that “branching happens throughout the whole wave function whenever it happens anywhere”. However, how the words ‘branch’ or ‘branching’ are used does not change the physical content of the theory. The physical state (51) is not that of an observer that is classically uncertain but of somebody whose state will ‘fission’ at the time of observation.

There is a situation which Sebens and Carroll bring up, where it is less obvious that Kent’s criticism apply but McQueen and Vaidman argues it does. The point they bring up turns out to be of particular interest, as it actually constitutes an important argument against Born rule proof of McQueen and Vaidman, section 6.4.2.

Sebens and Carroll argued that there is a moment of self-locating uncertainty during the short time between that the observer’s retina has registered a result and that the observer is mentally aware of the result. McQueen and Vaidman argues that the system $O$ should only be the observers “cognitive system”. For clarity and later reference lets write this state vector, where the observer $O$ is viewed as two systems, $M$ for mind and $B$ for body

$$|\Psi_1\rangle = |M\rangle\left(\frac{1}{\sqrt{2}}|B_\uparrow\rangle|\uparrow\rangle|H\rangle|\Omega_1\rangle + \frac{1}{\sqrt{2}}|B_\downarrow\rangle|\downarrow\rangle|T\rangle|\Omega_2\rangle\right).$$

(52)

$^9$In$^{[75]}$ there are two detectors. One of them corresponds to the coin.
The observer’s mind does not belong to any of the two branches yet. The situations that Carroll and Sebens consider are quantal in nature and classical notions of uncertainty and probability cannot be applied without substantial further reasoning.

### 6.4.2 McQueen and Vaidman

Vaidman [21, 32] has previously postulated the Born rule and he has stated that it seems not possible to derive the Born rule [22]. In his article together with McQueen [77] the postulate reads “The probability of self-location in a world with a given set of outcomes is the absolute square of that world’s amplitude.” They motivate it by “This postulate is needed to explain observed frequencies of outcomes of quantum experiments.” However, in spite of postulating the Born rule, McQueen and Vaidman also attempt to prove the Born rule, why we should regard the postulate as a theorem instead.

They consider the very special situation in which the observer is asleep during measurement and is placed in different identically looking rooms depending on the outcome of the measurement. When the observer wakes up, she is ignorant about in which room she is located and, correspondingly, what the outcome was.

The criticism that McQueen and Vaidman made against Carroll and Sebens view that the observer’s mind is in self-locating uncertainty when the retina has registered the measurement result, but the observer is not yet aware of the result, also constitutes an argument against the appearance of classical uncertainty for their observer as well. The state (52) also applies to the situation where the body is in different positions. That the mind has the same state but is located in two different positions is, from a fundamental point of view, no different from an electron being distributed onto two different locations with the same spin state at both positions.

Albert [58] and Lewis [78] criticize that the self-location uncertainty arguments give probabilities that come too late in the game. They do not properly address the pre-measurement situation. McQueen and Vaidman answer this question with that Vaidman’s ‘measurement of existence’ “which may make pre-measurement agents care more about descendants that exist more”. That agrees with the point of view of EQM1 and section 5.4, but they refer to Vaidman’s article [21] where he postulates that the ‘measure of existence’ gives the probability, but that is quite different from explaining it.

McQueen and Vaidman assume that measurements that take place at one location do not affect what happens at another location, this is also an
essential part of the argumentation in the present analysis of measurements, section 3 which was also present in an early version [79].

6.5 Wallace Decision Theory

Wallace has vigorously pursued to derive the Born rule in EQM inspired by Deutsch attempt to derive the Born rule from decision theory. In his book on EQM [18], Wallace reproduces his very elaborate attempt to prove the Born rule, first published in 2010 [51]. He defines EQM as a theory of states that belong to a Hilbert space, which is defined by a set of operators that act on it, for example, position operators for individual particles. More generally, he refers to quantum field theories and their definition [80].

He argues against the circularity of proving the Born rule from decoherence that Joos [36] identified. Wallace states that the Hilbert norm is “telling us when some emergent structure really is robustly present” and “a perfectly objective feature of the physics, prior to any considerations of probability”. Wallace also refers to relations between dynamical features and the norm. Here, Wallace interprets the norm to have a physical significance. This interpretation is in accordance with EQM1, but his theory is not defined from such a statement, but instead from assuming Hilbert space. As noted in section 2 it is conceivable to derive postulate EQM1 from the Hilbert space structure. The backward character of starting with mathematics to get physics makes it difficult to be firmly convinced by such a derivation. Wallace does not derive EQM1, and he seems not to appreciate the significance of the quantity $\rho(\cdot)$, as can be seen from his arguments against a frequentist derivation of the Born rule. He states “mod-squared amplitude of all branches on which the relative frequencies are not approximately correct will tend to zero. And of course this is circular: it proves not that mod-squared amplitude equals relative frequency, but only that mod-squared amplitude equals relative frequency with high mod-squared amplitude.” There is no reference here to the norm, ‘mod-squared’, as a significant quantity from which conclusions on physics can be drawn. Wallace seems not sufficiently convinced about the significance of the norm, to also use it for conclusions about probabilities. This uncertainty is the consequence of postulating the Hilbert space structure, rather than EQM1, the physical significance of the quantity $\rho(\cdot)$ is not easily understood.

The decision theory that is used to prove the Born rule proof is based on six rationality ‘axioms’ labeled: ordering, diachronic consistency, microstate indifference, branching indifference, state supervenience, solution continuity and four richness axioms that assumes that the world is sufficiently versatile.
The axioms and the proof are quite complex. It has been examined \cite{81} and so far, no objections have been raised on the proof. However, the axioms have been questioned \cite{82, 83, 84}. Here is the trouble of Wallace’ approach, if this is going be a valid proof of the Born rule, then the ‘axioms’ should be as easily acceptable as Savage axioms or those of Greaves and Myrvold.

Savage axioms of rational are rules that a rational agent will follow, but they do not directly define the values of the probabilities. It is the agent’s view of the world around her that result in probability values. The combination of rational behavior and knowledge about the quantum structure is enough to prove the Born rule, as shown in section \ref{5}. For the Deutsch-Wallace program to become convincing, it is necessary to prove that Savage type axioms and EQM lead to Wallace axioms, or similar.

Kent \cite{55} and Dawid and Thébault \cite{85} criticized Wallace Born rule derivation for the ‘fuzziness’ of the branch definition. The point is that the observer’s beliefs can then not give a well-defined Born rule. The answer is important, in EQM the measurement is described as a physical process. The fuzziness of branches is the fuzziness of actual measurements. It is generally assumed that the experiments can be made arbitrarily exact, in principle. If that is not the case, several interpretations will be in trouble.

7 What is Real?

Some criticism of the derivations of the Born rule goes deep into how EQM describes the world. For example, Hemmo and Pitowsky \cite{86} contrast the standard quantum mechanics where one alternative become “realized” with EQM where “all of them are real”. Price \cite{57} wrote about EQM that “all possible outcomes of a quantum measurement are treated as equally real.” With such views, it is no wonder that they come to that the Born rule is a logical impossibility. If we know for sure that something will happen, then it has probability one.

The EQM description of a single particle is a spatially varying amplitude for each spin component. Its absolute square $\rho_j(x)$ gives the locations of the particles, which is a distribution. If the particle is in a bound state, it can be probed with forces. The strength of the interactions will reveal the values of $\rho$ in different regions (and spins).

Is the quantum particle equally real at all positions where the amplitude is non-zero? The question is based on a mistake of category. A quantum particle is not that kind of thing which is localized to a single spot. The same is true for complex systems as well. They are not localized to a single
point in configuration space and spin, but a distributed quantity.

When a system becomes entangled with the environment, the combined system becomes very complex indeed. Nevertheless, it is still a distributed system. It is still a category error to state that it is equally real at all the positions where the amplitude (or $\rho$) is non-zero, as the system is not localized to a specific position. If the entanglement with the environment leads to many different branches, it is still a category error to think that the system is equally real at the different branches. It is distributed, and it is a mistake to view it as if there are copies of it in the different branches. This is so as long as we view the whole set of branches as the total system.

When the perspective is taken to be that of an observer that observed and so become entangled in the same way as the system is, the perspective needs to be that within a single branch in order to understand the observer’s continued observations. The perception the observer’s descendants have is that of the branch she is in, where everything consistently is as if the measured system’s initial state was the corresponding component $|b\rangle$. On the other hand, before the measurement our view is that of the system as a whole. We envisage all the possibilities, all the future branches that constitute the positions over which the (single) system will be distributed.

From the pre-measurement perspective, the future observer’s experience of the system being in a specific branch is a mirage. The views expressed by Hemmo, Pitowsky, and Price that “all of them are real” corresponds to viewing mirages as if they are real.

8 Conclusions and Final Remarks

The postulate EQM1 gives a physical foundation to Everett’s quantum mechanics. It gave that the quantum state belongs to a Hilbert space and an extraordinary simplicity of the postulates. Additionally, the relation with observation with macroscopic-classical phenomena is equally simple.

The relation to the observation of microscopic-quantal phenomena is a complex process as it involves decoherence. The existing formulations of decoherence theory are based on the Born rule. EQM1 interprets the quantity appearing in the Born rule, $\rho_b$ to give to what extent the system is present at $b$. When decoherence theory gives that the amplitude of coherent effects is vanishingly low, that means that the presence of coherent effects is vanishingly small.

The physical description of measurements assumes that interactions are essentially that of the standard model of particle physics. This assumption
allows for the assumption of that particle recording detectors only react to
the part of the measured state that falls upon the detector. There is no
need to explain the measurement process under various hypothetical types
of interactions that might not even allow for the construction of detectors.
The need for assumptions in order to address probabilities have been dis-
cussed by Barrett [46]. Much of the discussions of measurements and the
Born rule have gone astray in unnecessarily general and abstract reasonings.
The cause for this may be the heritage of classical mechanics and quantum
mechanics textbooks where the mechanics and the interactions are two com-
pletely independent entities.

The analysis of the statistics from repeated measurements implies that
physicist in the typical branch believes in the Born rule. Observations con-
firm that physicist believes in the Born rule. The possibility that we are
not in a typical branch is, of course, possible, precisely as it is possible that
the actual probabilities are entirely different from the Born rule, the data in
support of it are a ‘statistical mishap’.

From the theory, we get the expectations that we should see the Born
rule, and we have. All the data that verify the Born rule also verify EQM.

Greaves and Myrvold introduced the idea of a quantity quasi-credence
that is the pre-measurement analog to the credence an agent has to the
different outcomes. The agent/physicist that believes in EQM and is aware
of the statistical analysis will put her quasi-credence values equal to the
presence distribution $\rho_b$, which is the same as values as the Born rule prob-
abilities. Operationally, there is no difference between Born rule probability
in a single world theory and presence. The presence appears in statistical
analysis and decision making in the way as the notion of probability does
in single world theories. In order to make the notation more similar in be-
tween interpretations, the presence may be called ‘Everettian probability’
or simply probability, when ‘Everettian’ can be tacitly understood.

It is possible to take the quantum state as the full description of the
physical world without any additional degrees of freedom or mechanisms
that select a single value in a measurement. All aspects of the measurement
process are fully understood using Everett’s interpretation with EQM1 and
EQM2. This fact explains the elusive character of the measurement problem
that made Feynman doubt its existence and some to suggest that a selection
happens without a cause [87].

Interpretations, like Everett’s, that describe the measurement process
are more complete theories than those that do not. They are also more
vulnerable, as the description can be disproven. EQM can be disproven if
measurements can be performed without decoherence. On the other hand,
all other interpretations will be disproven if effects of recoherence are detected. Experimental tests of quantum mechanical processes related to measurements and locality and other factors are necessary.

I wish to acknowledge Ben Mottelson, David Wallace, Robert Geroch and Lev Vaidman for stimulating discussions and useful suggestions.

APPENDIX

Hilbert Spaces

A Hilbert space \( H \) is a normed linear space \([88]\). The members are often called vectors. The norm \( \| \cdot \| \) attributes a real number \( \geq 0 \) for any member in \( H \). The normed space has the property that

\[
\|x\| = 0 \iff x = 0. \tag{53}
\]

This implies that

\[
\|x - x'\| = 0 \iff x = x'. \tag{54}
\]

An example, the equation for an eigenvalue \( a \) of a linear operator \( A \) is

\[
Ax = ax \iff \|Ax - ax\| = 0. \tag{55}
\]

For a Hilbert space consisting of functions \((\mathbb{R}^n \rightarrow \mathbb{C}, n \in \mathbb{N} \text{ or } n = \infty)\) the functions make up equivalence classes. Two functions \( \psi \) and \( \psi' \) belong to the same equivalence class if they are equal almost everywhere in the sense \( \|\psi - \psi'\| = 0 \). The vectors of the Hilbert space are the equivalence classes. If it is clear that we deal with vectors in Hilbert space, then the statement \( \psi = \psi' \) means \( \|\psi - \psi'\| = 0 \).

The most important property of Hilbert spaces is the existence of an inner product \( \langle x|y \rangle \) which is related to the norm by \( \|x\|^2 = \langle x|x \rangle \). Furthermore, \( \langle x|y + \lambda z \rangle = \langle x|y \rangle + \lambda \langle x|z \rangle \) and \( \langle y|x \rangle = \langle x|y \rangle^* \).

Hilbert spaces are complete, and spaces with an inner product can be completed to become a Hilbert space. Any Hilbert space has a complete orthonormal basis set.

In separable Hilbert spaces, the basis set is countable. Then any vector \( \psi \) can be written as a sum of the basis states \( \phi_b \),

\[
\psi = \sum_b \langle \phi_b|\psi \rangle \phi_b \tag{56}
\]
This equation holds only in the sense of the equivalence $\|\psi - \psi'\| = 0$, if $\|\psi - \psi'\| = 0$, then the matrix elements $\langle \phi_b | \psi \rangle = \langle \phi_b | \psi' \rangle$. Both $\psi$ and $\psi'$ give rise to the same lefthand side.

The Hilbert space for a finite number of particles is separable. The Hilbert space for infinitely many particles $H^{\otimes \infty} = H \otimes H \otimes \ldots$, which is an infinite tensor product of separable Hilbert spaces. This space is not separable so that any complete basis set will be uncountable. The non-separable character can result in that the inner product $\langle \phi | \psi \rangle$ between normalized basis states $\phi$ and the normalized state $\psi$ all are zero. At most a countable set of inner products can be non-zero.

The left hand side of (3) defines a norm and there is a unique inner product

$$\langle \Psi | \Phi \rangle = \int \int \cdots d x_1 d x_2 \cdots \sum_j \psi_j(t, x_1, x_2, \ldots)^* \phi_j(t, x_1, x_2, \ldots).$$

(57)

As stated in the comment to EQM1, there is no observable difference between $\Psi$ and $\Psi'$ if $\|\Psi - \Psi'\| = 0$. This equivalence shows that the Hilbert space vector represents all (observable) physical properties of the system.

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