Radiation-induced magnetotransport in high-mobility two-dimensional systems: Role of electron heating

X.L. Lei and S.Y. Liu
Department of Physics, Shanghai Jiaotong University, 1954 Huashan Road, Shanghai 200030, China

Effects of microwave radiation on magnetoresistance are analyzed in a balance-equation scheme that covers regimes of inter- and intra-Landau level processes and takes account of photon-assisted electron transitions as well as radiation-induced change of the electron distribution for high-mobility two-dimensional systems. Short-range scatterings due to background impurities and defects are shown to be the dominant direct contributors to the photoresistance oscillations. The electron temperature characterizing the system heating due to radiation, is derived by balancing the energy absorption from the radiation field and the energy dissipation to the lattice through realistic electron-phonon couplings, exhibiting resonant oscillation. Microwave modulations of Shubnikov de Haas oscillations amplitudes are produced together with microwave-induced resistance oscillations, in agreement with experimental findings. In addition, the suppression of the magnetoresistance caused by low-frequency radiation in the higher magnetic field side is also demonstrated.

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I. INTRODUCTION

The discovery of microwave induced magnetoresistance oscillations (MIMOs) and zero-resistance states (ZRS) in high-mobility two-dimensional (2D) electron gas (EG) has stimulated tremendous experimental and theoretical interest in radiation related magneto-transport in 2D electron systems. Since theoretically it has been shown that the ZRS can be the result of the instability induced by absolute negative resistivity, the majority of microscopic models focus mainly on MIMOs in spatially uniform cases and identify the region where an negative dissipative magnetoresistance develops as that of measured zero resistance. Most of previous investigations concentrated on the range of low magnetic fields $\omega_c/\omega \leq 1$ ($\omega_c$ stands for the cyclotron frequency) subject to a radiation of frequency $\omega/2\pi \leq 100\text{GHz}$, where MIMOs show up strongly and Shubnikov-de Haas oscillations (SdHOs) are rarely appreciable. In spite of the fact that both MIMOs and SdHOs are magnetoresistance related phenomena appearing in overlapping field regimes, little attention was paid to the influence of a microwave radiation on SdHO until a recent experimental finding and theoretical investigation was previously considered to contribute to Landau-level broadening or to act as a damping for the orbit movement, providing a mechanism for the suppression of MIMOs when the lattice temperature increases. Besides the inelastic electron-phonon scattering also plays another important role to dissipate energy from the electron system to the lattice. The energy absorption rate is indeed small in high-mobility electron systems at low temperature as in the experiments. This, however, does not imply a negligible electron heating, since the electron energy-dissipation rate is also small because of weak electron-phonon scattering at temperature $T \leq 1\text{K}$. To deal with SdHO, which is very sensitive to the smearing of the electron distribution, one has to carefully calculate the electron heating due to microwave irradiation in a uniform model.

On the other hand, microwave irradiation heats the
electrons and thus greatly strengthens the thermalizing trend of the system by enhancing the electron-electron scattering rate at this low temperature regime. This enables us to describe these high-mobility 2D electron systems with a quasi-equilibrium distribution in a moving reference frame.

In this paper we pursue a theoretical investigation on MIMOs and SdHOS taking account of the electron heating under microwave irradiation. We generalize the balance equation approach to radiation-induced magnetotransport in high mobility two-dimensional electron systems. By carefully calculating the electron heating based on the balance of the energy absorption from the radiation field and the energy dissipation to the lattice through electron-phonon interactions in a typical GaAs-based heterosystem and taking into account the electron-dynamic effect, we are able not only to reproduce the interesting phenomena of MIMOs in quantitative agreement with experiments in amplitudes, phases and radiation dependence of the oscillation, but also to obtain SdH modulations observed in the experiments.

## II. FORMULATION

### A. Force- and energy-balance equations

This paper is concerned with the magnetotransport in a microscopically homogeneous 2D system, and refers the measured zero resistance to the macroscopic consequence of a microscopically homogeneous 2D system, and refers the quasi-2D system in the SdHO modulations observed in the experiments.

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The center-of-mass momentum and coordinate of the 2D electron system are defined as $p_j = \sum_j p_{j\parallel}$ and $R \equiv N_e^{-1} \sum_i r_{i\parallel}$ with $p_{j\parallel} = (p_{jx}, p_{jy})$ and $r_{j\parallel} = (x_j, y_j)$ being the momentum and coordinate of the $j$th electron in the 2D plane, respectively, and the relative electron momentum and coordinate are defined as $p_{j\parallel} \equiv p_{j\parallel} - \frac{P}{N_e}$ and $r_{j\parallel} \equiv r_{j\parallel} - R$, respectively. In terms of these variables, the Hamiltonian of the system, $H$, can be written as the sum of a center-of-mass part $H_c$ and a relative electron part $H_r$ ($A(r)$ is the vector potential of the $B$ field),

$$H_c = \frac{1}{2N_e m}(P - N_e eA(R))^2 - N_e e(E_0 + E(t)) \cdot \mathbf{R}$$

$$H_r = \sum_j \left[ \frac{1}{2m} \left( p_{j\parallel}^2 - eA(r_{j\parallel}) \right)^2 + \frac{p_{jz}^2}{2m_z} + V(z_j) \right] + \sum_{i<j} V_e(r_{i\parallel} - r_{j\parallel}, z_i, z_j),$$

(3)

together with electron-impurity and electron-phonon interactions

$$H_{ei} = \sum_{j,a,q_i} u(q_{i\parallel}, z_a) e^{i q_{i\parallel} \cdot (r + r' - r_a)}$$

(4)

$$H_{ep} = \sum_{j,q_i} M(q_{i\parallel}, z) (b_q + b_q^{\dagger}) e^{i q_{i\parallel} \cdot (r + r_{i\parallel})}.$$  

(5)

Here $m$ and $m_z$ are, respectively, the electron effective mass parallel and perpendicular to the 2D plane, and $V_e$ stands for the electron-electron Coulomb interaction; $u(q_{i\parallel}, z_a)$ is the potential of the $a$th impurity locating at $(r_{i\parallel}, z_a)$; $b^\dagger_q(b_q)$ are the creation (annihilation) operators of the bulk phonon with wavevector $q = (q_1, q_2)$ and $M(q_{i\parallel}, z)$ is the matrix element of the electron-phonon interaction in the 3D plane-wave representation. Note that the uniform electric field (dc and ac) appears only in $H_c$, and that $H_r$ is just the Hamiltonian of a quasi-2D system subjected to a magnetic field without an electric field. The coupling between the center-of-mass and the relative electrons appears only in the exponential factor $\exp(iq_{i\parallel} \cdot R)$ inside the 2D momentum $q_{i\parallel}$ summation in $H_c$ and $H_{ep}$. The balance equation treatment starts with the Heisenberg operator equation for the rate of change of the center-of-mass velocity $V = -i[V, H] + \partial V/\partial t$ with $V = -i[H, H]$, and that for the rate of change of the relative electron energy $H_r = -i[H_r, H]$. Then we proceed with the determination of their statistical averages.

As proposed in Ref. [55], the c.m. coordinate operator $R$ and velocity operator $V$ can be treated classically, i.e. as the time-dependent expectation values of c.m. coordinate and velocity, $R(t)$ and $V(t)$, such that $R(t) - R(t') = \int_{t'}^{t} V(s) ds$. We are concerned with the steady transport state under an irradiation of single frequency and focus on the photon-induced dc resistivity and the energy absorption of the HF field. These quantities are directly related to the time-averaged and/or base-frequency oscillating components of the c.m. velocity. Although higher harmonics of the current may affect the dc and lower harmonic terms of the drift velocity through entering the damping force and energy exchange rates
in the resulting equations, in an ordinary semiconductor the power of even the third harmonic current is rather weak as compared to the fundamental current. For the HF field intensity in the MIMO experiments, the effect of higher harmonic current is safely negligible. Hence, it suffices to assume that the c.m. velocity, i.e. the electron drift velocity, consists of a dc part \(v_0\) and a stationary time-dependent part \(v(t)\) of the form

\[
V(t) = v_0 - v_1 \cos(\omega t) - v_2 \sin(\omega t).
\]  

(6)

With this, the exponential factor in the operator equations can be expanded in terms of Bessel functions \(J_n(x)\),

\[
e^{i\mathbf{q} \cdot \mathbf{j} f_s x} = \sum_{n=-\infty}^{\infty} J_n^2(\xi) e^{i(q_\| \cdot v_0 - n\omega)(t - t')} + \sum_{m \neq 0} e^{im(\omega t - \varphi)} \sum_{n=-\infty}^{\infty} J_n(\xi) J_{n-m}(\xi) e^{i(q_\| \cdot v_0 - n\omega)(t - t')}.
\]

Here the argument in the Bessel functions

\[
\xi \equiv \frac{1}{\omega}\left[(q_\| \cdot v_1)^2 + (q_\| \cdot v_2)^2\right]^\frac{1}{2},
\]

(7)

and \(\tan \varphi = (q_\| \cdot v_2)/(q_\| \cdot v_1)\).

Under the influence of a modest-strength HF electric field the electron system is far from equilibrium. However, the distribution function of relative electrons, which experience no electric field directly, may be close to an quasi-equilibrium distribution function. For the experimental GaAs-based ultra-clean 2D electron systems having carrier mobility of the order of 2000 m²/Vs, the elastic momentum scattering rate is around \(\tau_m^{-1} \sim 10\) mK. In these systems, the thermalization time \(\tau_{th}\) (i.e. the time for system to return to its internal equilibrating state when it is deviated from), estimated conservatively using electron-electron (e-e) interaction related inelastic scattering time \(\tau_{sc}\) calculated with an equilibrium distribution function at temperature \(T = 1\) K, is also around \(\tau_{th}^{-1} \sim \tau_{sc}^{-1} \sim 10\) mK. The illumination of microwave certainly heats the electrons. Even an electron heating comparable to a couple of degrees temperature rise would greatly enhance \(\tau_{sc}^{-1}\), such that the thermalization time \(\tau_{th}\) would become much shorter than the momentum relaxation time \(\tau_m\) under microwave irradiation.\(^{17}\)

The relative electron systems subject to a modest radiation would rapidly thermalize and can thus be described by Fermi-type distribution function at an average electron temperature \(T_e\) in the reference frame moving with the center-of-mass. This allows us to carry out the statistical average of the operator equations for the rates of changes of the c.m. velocity \(V\) and relative electron energy \(H_{\epsilon r}\) to the leading order in \(H_{ei}\) and \(H_{ep}\) with succinct forms.

For the determination of unknown parameter \(v_0, v_1, v_2,\) and \(T_e\), it suffices to know the damping force up to the base frequency oscillating term \(F(t) = F_0 + F_s \sin(\omega t) + F_c \cos(\omega t)\), and the energy-related quantities up to the time-average terms. We finally obtain the force and energy balance equations,

\[
N_e eE_0 + N_e e(v_0 \times B) + F_0 = 0,
\]

(8)

\[
v_1 = \frac{eE_0}{m\omega} + \frac{F_s}{N_e m \omega} - \frac{e}{m\omega}(v_2 \times B),
\]

(9)

\[
-v_2 = \frac{eE_0}{m\omega} + \frac{F_c}{N_e m \omega} - \frac{e}{m\omega}(v_1 \times B),
\]

(10)

\[
N_e eE_0 \cdot v_0 + S_p - W = 0.
\]

(11)

Here

\[
F_0 = \sum_{q_\|} |U(q_\|)|^2 \sum_{n=-\infty}^{\infty} q_\| J_n^2(\xi) \Pi_2(q_\|, \omega_\|, n\omega)
\]

\[
+ \sum_{q} |M(q)|^2 \sum_{n=-\infty}^{\infty} n\omega J_n^2(\xi) \Lambda_2(q, \omega_\| + \Omega_q - n\omega)
\]

(12)

is the time-averaged damping force,

\[
S_p = \sum_{q_\|} |U(q_\|)|^2 \sum_{n=-\infty}^{\infty} n\omega J_n^2(\xi) \Pi_2(q_\|, \omega_\|, n\omega)
\]

\[
+ \sum_{q} |M(q)|^2 \sum_{n=-\infty}^{\infty} n\omega J_n^2(\xi) \Lambda_2(q, \omega_\| + \Omega_q - n\omega)
\]

(13)

is the time-averaged rate of the electron energy absorption from the HF field, and

\[
W = \sum_{q} |M(q)|^2 \sum_{n=-\infty}^{\infty} \Omega_q J_n^2(\xi) \Lambda_2(q, \omega_\| + \Omega_q - n\omega)
\]

(14)

is the time-averaged rate of the electron energy dissipation to the lattice due to electron-phonon scatterings.

The oscillating frictional force amplitudes \(F_s \equiv F_{22} - F_{11}\) and \(F_c \equiv F_{21} + F_{12}\) are given by \((\mu = 1, 2)\)

\[
F_{1\mu} = -\sum_{q_\|} q_\| \eta_\| U(q_\|)^2 \sum_{n=-\infty}^{\infty} [J_n^2(\xi)]^{\frac{1}{2}} \Pi_2(q_\|, \omega_\|, n\omega)
\]

\[
- \sum_{q} q_\| \eta_\| M(q)^2 \sum_{n=-\infty}^{\infty} [J_n^2(\xi)]^{\frac{1}{2}} \Lambda_1(q, \omega_\| + \Omega_q - n\omega),
\]

(15)

\[
F_{2\mu} = \sum_{q_\|} q_\| \eta_\| U(q_\|)^2 \sum_{n=-\infty}^{\infty} 2nJ_n^2(\xi) \Pi_2(q_\|, \omega_\|, n\omega)
\]

\[
+ \sum_{q} q_\| \eta_\| M(q)^2 \sum_{n=-\infty}^{\infty} 2nJ_n^2(\xi) \Lambda_2(q, \omega_\| + \Omega_q - n\omega).
\]

(16)

In these expressions, \(\eta_\| \equiv q_\| \cdot v_\mu /\omega \xi; \omega_\| = q_\| \cdot v_\|; U(q_\|)\) and \(M(q)\) are effective impurity and phonon scattering potentials (including effects of the spatial distribution of impurities and the form factor of quasi-2D electrons)\(^{16}\) \(\Pi_2(q_\|, \Omega)\) and \(\Lambda_2(q, \Omega) = 2\Pi_2(q, \Omega)[n(\Omega_q/T) - n(\Omega/T_e)]\) (with \(n(x) = 1/(e^{x} - 1)\)) are the imaginary parts of the electron density correlation
function and electron-phonon correlation function in the presence of the magnetic field. \( \Pi_1(\mathbf{q}_1, \Omega) \) and \( \Lambda_1(\mathbf{q}, \Omega) \) are the real parts of these two correlation functions.

Effects of a microwave radiation on electron transport first come from the HF field induced c.m. motion (electron drift motion) and the related change of the electron distribution. In addition to this, the HF field also enters via the argument \( \xi \) of the Bessel functions in \( F_0, F_{\mu\nu}, W \) and \( S_p \). Compared with that without a HF field, we see that in an electron gas having impurity and/or phonon scatterings (otherwise homogeneous), a HF field of frequency \( \omega \) opens additional channels for electron transition: an electron in a state can absorb or emit one or several photons of frequency \( \omega \) and scattered to a different state with the help of impurities and/or phonons. The sum over \( |n| \geq 1 \) represents contributions of real single and multiple photon participating processes. The role of these processes is two folds. On the one hand, they contribute additional damping force to the moving electrons, giving rise directly to photoresistance, and at the same time, transfer energy from the HF field to the electron system, resulting in electron heating, i.e. another change in the electron temperature, transfer energy from the HF field to the electron giving rise directly to photoresistance, and at the same time, transfer energy from the HF field to the electron system, making the radiation field, showing up in the term with \( J_0(\xi) \) in \( F_0, F_{\mu\nu} \) and \( W \), gives rise to another effective change of damping forces and energy-loss rate, without emission or absorption of real photons. This virtual photon process also contributes to photoresistance. All these effects are carried by parameters \( v_0, v_1, v_2 \) and \( T_c \). Eqs. (2)-(11) form a closed set of equations for the determination of these parameters when \( \mathbf{E}_0, \mathbf{E}_c \) and \( \mathbf{E}_v \) are given in a 2D system subjected to a magnetic field \( B \) at temperature \( T \).

### B. Longitudinal and transverse resistivities

The nonlinear resistivity in the presence of a high-frequency field is easily obtained from Eq. (9). Taking \( v_0 \) to be in the \( x \) direction, \( v_0 = (v_{0x}, 0, 0) \), we immediately get the transverse and longitudinal resistivities,

\[
R_{xy} \equiv \frac{E_{0y}}{\epsilon_0 e v_{0x}} = \frac{B}{\epsilon_0 e v_{0x}} \quad \text{(17)}
\]

\[
R_{xx} \equiv \frac{E_{0x}}{\epsilon_0 e v_{0x}} = \frac{F_0}{\epsilon_0 e v_{0x} N_c^2 e^2 v_{0x}} \quad \text{(18)}
\]

The linear magneto-resistivity is the weak dc current limit \( (v_{0x} \to 0) \):

\[
R_{xx} = \sum_{\mathbf{q}_1} \left[ \frac{U(\mathbf{q}_1)}{N_c^2 e^2} \right] \sum_{n=-\infty}^{\infty} J_n^2(\xi) \left| \frac{\partial \Pi_1}{\partial \Omega} \right|_{\Omega=\omega} + \sum_{\mathbf{q}} \left[ \frac{M(\mathbf{q})}{N_c^2 e^2} \right] \sum_{n=-\infty}^{\infty} J_n^2(\xi) \left| \frac{\partial \Lambda_1}{\partial \Omega} \right|_{\Omega=\omega \xi_{q+\omega}} \quad \text{(19)}
\]

Note that although according to Eqs. (12), (18) and (19), the longitudinal magneto-resistivity \( R_{xx} \) can be formally written as the sum of contributions from various individual scattering mechanisms, all the scattering mechanisms have to be taken into account simultaneously in solving the momentum- and energy-balance equations (9), (10) and (11) for \( v_1, v_2 \) and \( T_c \), which enter the Bessel functions and other parts in the expression of \( R_{xx} \).

### C. Landau-level broadening

In the present model the effects of interparticle Coulomb screening are included in the electron complex density correlation function \( \Pi(\mathbf{q}_1, \Omega) = \Pi_1(\mathbf{q}_1, \Omega) + i\Pi_2(\mathbf{q}_1, \Omega) \), which, in the random phase approximation, can be expressed as

\[
\Pi(\mathbf{q}_1, \Omega) = \frac{\Pi_0(\mathbf{q}_1, \Omega)}{\epsilon(\mathbf{q}_1, \Omega)}, \quad \text{(20)}
\]

where

\[
\epsilon(\mathbf{q}_1, \Omega) = 1 - V(\mathbf{q}_1) \Pi_0(\mathbf{q}_1, \Omega) \quad \text{(21)}
\]

is the complex dynamical dielectric function,

\[
V(\mathbf{q}_1) = \frac{e^2}{2\epsilon_0 \kappa q_1} H(q_1) \quad \text{(22)}
\]

is the effective Coulomb potential with \( \kappa \) the dielectric constant of the material and \( H(q_1) \) is a 2D wavefunction-related overlapping integration:

\[
\Pi_2(\mathbf{q}_1, \Omega) = \Pi_0(\mathbf{q}_1, \Omega) + \Pi_0(\mathbf{q}_1, \Omega) \Pi_2(\mathbf{q}_1, \Omega) \quad \text{(23)}
\]

the complex density correlation function of the independent electron system in the presence of the magnetic field. With this dynamically screened density correlation function the collective plasma modes of the 2DES are incorporated. Disregard these collective modes one can just use a static screening \( \epsilon(\mathbf{q}_1, 0) \) instead.

The \( \Pi_2(\mathbf{q}_1, \Omega) \) function of a 2D system in a magnetic field can be written in terms of Landau representation:

\[
\Pi_2(\mathbf{q}_1, \Omega) = \frac{1}{2\pi |B|_{n,n'}} \sum C_{n,n'}(l_B^2 q_1^2/2) \Pi_2(n, n', \Omega), \quad \text{(24)}
\]

where \( l_B = \sqrt{|1/|eB|} \) is the magnetic length,

\[
C_{n,n'+\gamma}(Y) = n!(n+l)!^{-1} Y^{l+1} e^{-Y} \left[ L_{n+\gamma}^{l+1}(Y) \right] \quad \text{(25)}
\]

with \( L_{n+\gamma}^{l+1}(Y) \) the associate Laguerre polynomial, \( f(\varepsilon) = \{\exp[(\varepsilon - \mu)/T_n] + 1\}^{-1} \) the Fermi distribution function, and \( \text{Im} G_n(\varepsilon) \) is the imaginary part of the electron Green’s function, or the density of states (DOS), of the Landau level \( n \). The real part function in \( \Pi_0(\mathbf{q}_1, \Omega) \) and corresponding \( \Delta_{n+\gamma}(\mathbf{q}_1, \Omega) \) function can be derived from their imaginary parts via the Kramers-Kronig relation.
In principle, to obtain the Green’s function \( \text{Im} G_n(\epsilon) \), a self-consistent calculation has to be carried out from the Dyson equation for the self-energy with all the impurity, phonon and e-e scatterings included. The resultant \( G_n \) is generally a complicated function of the magnetic field, temperature, and Landau-level index \( n \), also dependent on the different kinds of scatterings. Such a calculation is beyond the scope of the present study. In this paper we model the DOS function with a Gaussian-type form \( (\epsilon_n \text{ is the energy of the } n-\text{th Landau level})^{40,41} \)

\[
\text{Im} G_n(\epsilon) = -\frac{\sqrt{2\pi}}{T} \exp \left[ -\frac{2(\epsilon - \epsilon_n)^2}{T^2} \right]
\]

with a broadening width given by

\[
\Gamma = \left( \frac{8\epsilon_0\alpha}{\pi m_0} \right)^{1/2},
\]

where \( \mu_0 \) is the linear mobility in the absence of the magnetic field and \( \alpha \) is a semiempirical parameter to take into account the difference of the transport scattering time \( \tau_m \) determining the mobility \( \mu_0 \), from the single particle lifetime \( \tau_s \) related to Landau level broadening. The latter depends on elastic scatterings of different types and their relative strengths, as well as contributions of electron-phonon and electron-electron scatterings. \( \alpha \) will be served as the only adjustable parameter in the present investigation. Unlike the semielliptical function, which can model only separated Landau-level case, a Gaussian-type broadening function can reasonably cover both the separated-level and overlapping-level regimes.

### D. Effect of radiative decay

The HF electric field \( \mathbf{E}(t) \) appearing in Eqs. (8) and (9) is the total (external and induced) field really acting on the 2D electrons. Experiments are always performed under the condition of giving external radiation. In this paper we assume that the electromagnetic wave is incident perpendicularly (along \( z \)-axis) upon 2DEG from the vacuum with the incident electric field of

\[
\mathbf{E}_i(t) = E_{ix}\sin(\omega t) + E_{ic}\cos(\omega t)
\]

at plane \( z = 0 \). The relation between \( \mathbf{E}(t) \) and \( \mathbf{E}_i(t) \) is easily obtained by solving the Maxwell equations connecting both sides of the 2DEG which is carrying a sheet current density \( N_e\mathbf{v}(t) \). If the 2DEG locates under the surface plane at \( z = 0 \) of a thick (treated as semi-infinite) semiconductor substrate having a refraction index \( n_s \), we have\(^{42,43} \)

\[
\mathbf{E}(t) = \frac{N_e \mathbf{v}(t)}{(1 + n_s)c} + \frac{2}{1 + n_s} \mathbf{E}_i(t).
\]

If the 2DEG is contained in a thin sample suspended in vacuum at the plane \( z = 0 \), then

\[
\mathbf{E}(t) = \frac{N_e \mathbf{v}(t)}{2\epsilon_0 c} + \mathbf{E}_i(t).
\]

In the numerical calculation of this paper we consider the latter case and use Eq. (30) for the total selfconsistent field \( \mathbf{E}(t) \) in Eqs. (9) and (11). This electrodynamic effect recently referred as radiative decay\(^{46} \) gives rise to an additional damping in the 2DEG response to a given incident HF field. The induced damping turns out to be much stronger than the intrinsic damping due to scattering-related forces \( \mathbf{F}_s \) and \( \mathbf{F}_e \) for the experimental high-mobility systems at low temperatures. For almost all the cases pertinent to MIMO experiments we can neglect the forces \( \mathbf{F}_s \) and \( \mathbf{F}_e \) completely in solving \( \mathbf{v}_1 \equiv (v_{1x}, v_{1y}) \) and \( \mathbf{v}_2 \equiv (v_{2x}, v_{2y}) \) from Eqs. (34) and (31) for given incident fields \( \mathbf{E}_{is} \) and \( \mathbf{E}_{ic} \), and obtain explicitly

\[
\begin{align*}
\mathbf{v}_1 &= \left( \frac{a\chi_{xx} + b\chi_{yy}}{\Delta}, \frac{a\chi_{xy} - b\chi_{yx}}{\Delta} \right) \\
\mathbf{v}_2 &= \left( \frac{-a\chi_{xx} - b\chi_{yy}}{\Delta}, \frac{-a\chi_{xy} + b\chi_{yx}}{\Delta} \right)
\end{align*}
\]

with \( \Delta = (1 - \delta_\omega^2 + \gamma_\omega^2)^2 + (2\gamma_\omega\delta_\omega)^2 \), and

\[
\begin{align*}
\chi_{xx} &= \nu_{xx} - \delta_{\nu}\nu_{yy} + \gamma_{\nu}\nu_{cx} \\
\chi_{xy} &= \nu_{xy} + \delta_{\nu}\nu_{cy} + \gamma_{\nu}\nu_{cx} \\
\chi_{cx} &= \nu_{cx} - \delta_{\nu}\nu_{sx} - \gamma_{\nu}\nu_{xy} \\
\chi_{cy} &= \nu_{cy} - \delta_{\nu}\nu_{sy} + \gamma_{\nu}\nu_{xy}
\end{align*}
\]

Here

\[
\nu_{et} = \frac{\epsilon E_{in}}{m_0 \omega} \quad (\eta = sx, sy, cx, cy),
\]

\[
\delta_{\omega} \equiv \omega_c/\omega \quad \text{and} \quad \gamma_{\omega} \equiv \gamma/\omega
\]

With these \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \), the argument \( \xi \) entering the Bessel functions is obtained. All the transport quantities, such as \( S_p \), \( W \) and \( R_{xx} \), can be calculated directly with the electron temperature \( T_e \) determined from the energy balance equation (11).

### III. NUMERICAL RESULTS FOR GAAS-BASED SYSTEMS

As in the experiments, we focus our attention on high mobility 2DEGs formed by GaAs/AlGaAs heterojunctions. For these systems at temperature \( T \leq 1 \text{ K} \), the dominant contributions to the energy absorption \( S_p \) and photoresistivity \( R_{xx} - R_{xx}(0) \) come from the impurity-assisted photon-absorption and emission process. At different magnetic field strength, this process is associated with electron transitions between either inter-Landau level states or intra-Landau-level states. According to \(^{40} \), the width of each Landau level is about \( 2\Gamma \). The condition for inter-Landau level transition with impurity-assisted single-photon process\(^{42} \) is \( \omega > \omega_c - 2\Gamma \), or \( \omega_c/\omega < \alpha_{\text{inter}} = (\beta + \sqrt{\beta^2 + 4})^2/4 \); and that for impurity-assisted intra-Landau level transition is \( \omega < 2\Gamma \).
or $\omega_c/\omega > a_{\text{intra}} = \beta^{-2}$, here $\beta = (32e\alpha/\pi m_0\omega)\uparrow$. However, since the DOS of each Landau level is assumed to be Gaussian rather than a clear cutoff function and the multi-photon processes also play roles, the transition boundaries between different regimes may be somewhat smeared.

As indicated by experiments, although long range scattering due to remote donors always exists in the 2D heterostructures, in ultra-clean GaAs-based 2D samples having mobility of order of $10^4 \text{m}^2/\text{Vs}$, the remote donor scattering is responsible for merely $\sim 10\%$ or less of the total momentum scattering rate. The dominant contribution to the momentum scattering rate comes from short-range scatterers such as residual impurities or defects in the background. Furthermore, even with the same momentum scattering rate the remote impurity scattering is much less efficient in contributing to microwave-induced magnetoresistance oscillations than short-ranged background impurities or defects. Therefore, in the numerical calculations in this paper we assume that the elastic scatterings are due to short-range impurities randomly distributed throughout the GaAs region. The impurity densities are determined by the requirement that electron total linear mobility at zero magnetic field equals the giving value at lattice temperature $T$. Possibly, long-range remote donor scattering may give rise to important contribution to the Landau-level broadening. This effect, together with the role of electron-phonon and electron-electron scatterings, is included in the semiempirical parameter $\alpha$ in the expression [27].

In order to obtain the energy dissipation rate from the electron system to the lattice, $W$, we take into account scatterings from bulk longitudinal acoustic (LA) and transverse acoustic (TA) phonons (via the deformation potential and piezoelectric couplings), as well as from longitudinal optical (LO) phonons (via the Fröhlich coupling) in the GaAs-based system. The relevant matrix elements are well known. The material and coupling parameters for the system are taken to be widely accepted values in bulk GaAs: electron effective mass $m = 0.068\, m_e$ ($m_e$ is the free electron mass), transverse sound speed $v_{st} = 2.48 \times 10^3 \text{m/s}$, longitudinal sound speed $v_{sl} = 5.29 \times 10^3 \text{m/s}$, acoustic deformation potential $\Xi = 8.5 \text{eV}$, piezoelectric constant $e_{14} = 1.41 \times 10^9 \text{V/m}$, dielectric constant $\kappa = 12.9$, material mass density $\rho = 5.31\, \text{g/cm}^3$.

### A. 100 GHz

Figure 1 shows the calculated energy absorption rate $S_p$, the electron temperature $T_e$ and the longitudinal magnetoresistivity $R_{xx}$ as functions of $\omega_c/\omega$ for a 2D system having an electron density of $N_e = 3.0 \times 10^{15} \text{m}^{-2}$, a linear mobility of $\mu_0 = 2000 \text{m}^2/\text{Vs}$ and a broadening parameter of $\alpha = 10$, subject to linearly $x$-direction polarized incident microwave radiations of frequency $\omega/2\pi = 100 \text{GHz}$ having four different amplitudes $E_{i\parallel} = 2.2, 3.4$ and $5 \text{V/cm}$ at a lattice temperature of $T = 1 \text{K}$. The energy absorption rate $S_p$ exhibits a broad main peak at cyclotron resonance $\omega_c/\omega = 1$ and secondary peaks at harmonics $\omega_c/\omega = 1/2, 1/3, 1/4$. The electron heating has similar feature: $T_e$ exhibits peaks around $\omega_c/\omega = 1, 1/2, 1/3, 1/4$. For this GaAs system $\beta = 0.65$, $a_{\text{inter}} = 1.6$ and $a_{\text{intra}} = 4.7$. We can see that, at lower magnetic fields, especially $\omega_c/\omega < 1.4$, the system absorbs enough energy from the radiation field via inter-Landau level transitions and $T_e$ is significantly higher than $T$, with the maximum as high as 21 K around $\omega_c/\omega = 1$. With increasing strength of the magnetic field the inter-Landau level transition weakens (impurity-assisted single-photon process is mainly allowed when...
\( \omega_r/\omega < a_{\text{inter}} = 1.6 \) and the absorbed energy decreases rapidly. Within the range \( 2 < \omega_r/\omega < 4 \) before intra-Landau level transitions can take place, \( S_\parallel \) is two orders of magnitude smaller than that in the low magnetic field range. Correspondingly the electron temperature \( T_e \) is only slightly higher than the lattice temperature \( T \).

The magnetoconductivity \( R_{xx} \) showing in the upper part of Fig. 1, exhibits interesting features. MIMOs (with fixed points rather than extrema at \( \omega_c/\omega = 1, 1/2, 1/3, 1/4 \)) clearly appear at lower magnetic fields, which are insensitive to the electron heating even at \( T_e \) of order of 20K. SdHOs appearing in the higher magnetic field side, however, are damped due to the rise of the electron temperature \( T_e > 1 \) K as compared with that without radiation. With an increase in the microwave amplitude from \( E_{\text{is}} = 2.2 \) V/cm to 5 V/cm, MIMOs become much stronger and SdHOs are further damped. But the radiation-induced SdHO damping is always relatively smaller within \( 2.4 < \omega_r/\omega < 4 \) between allowed ranges of inter- and intra-Landau level transitions.

It is worth noting that the predicted MIMOs here exhibit much improved agreement with experiments over previous theoretical models. The maxima of \( R_{xx} \) oscillation locate at \( \omega/\omega_c = j - \delta_\perp \) and minima at \( \omega/\omega_c = j + \delta_\perp \), with \( \delta_\perp \sim 0.23 - 0.25 \) for \( j = 2, 3, 4 \ldots \) and \( \delta_\perp \sim 0.16 - 0.18 \) for \( j = 1 \) (see Fig. 2). These phase details, as well as the absolute (rather than reduced) magnitudes of the oscillation amplitudes and the required incident microwave strengths to induce oscillations are in good quantitative agreement with experiments.\(^{3,4,5,6,7}\).

The MIMOs depend on the polarization of the incident microwave field in respect to the dc field \( E_0 \). Physically this is clear in the present model since it is through the c.m. motion that a HF field affects the photore sistivity of the 2D electron system. Under the influence of a magnetic field perpendicular to the plane, the c.m. performs a cyclic motion of frequency \( \omega_r \) in the 2D plane. A perpendicularly incident circularly-polarized microwave would accelerate or decelerate this cyclic motion depending on the HF electric field circling with or against it. Thus, at fix incident power, a left-polarized microwave would yield much stronger effect on the \( R_{xx} \) oscillation than a right-polarized one and this effect is apparently strongest in the vicinity of cyclotron resonance \( \omega_c/\omega = 1 \). The difference between the \( x \)-direction linearly polarized wave and the \( y \)-direction linearly polarized wave, however, comes mainly from the different angle of radiation-induced c.m. motion with respect to the dc current, and thus not so sensitive to that of the \( \omega_r/\omega_c \) range. In Fig. 2 we plot the calculated \( R_{xx} \) versus \( \omega/\omega_c \) for the same system as described in Fig. 1, subject to a 100 GHz microwave radiation having a fixed incident power of \( P_i = 210 \) W/m\(^2\) (equivalent to an incident amplitude \( E_{\text{is}} = 4 \) V/cm of linear polarization) but four different polarizations: linear \( x \)-polarization, linear \( y \)-polarization, left circular polarization and right circular polarization. Their difference is clearly seen.

**FIG. 3:** The magnetoconductivity \( R_{xx} \), electron temperature \( T_e \) and energy absorption rate \( S_p \) of a GaAs-based 2DEG with \( \mu_0 = 2500 \) m\(^2\)/Vs and \( \alpha = 12.5 \), subjected to 50 GHz linearly \( x \)-polarized incident HF fields \( E_{\text{is}} \sin(\omega t) \) of four different strengths. The lattice temperature is \( T = 1 \) K.

**FIG. 4:** The magnetoconductivity \( R_{xx} \) versus \( \omega/\omega_c \) for the same system as described in Fig. 3, subject to 50GHz linearly \( x \)-polarized incident HF fields of four different strengths.

**B. 50 GHz and lower frequency**

Figure 3 shows the energy absorption rate \( S_p \), the electron temperature \( T_e \) and the longitudinal magnetoconductivity \( R_{xx} \) as functions of \( \omega_c/\omega \) for a 2D system having an electron density of \( N_e = 3.0 \times 10^{13} \) m\(^{-2}\), a linear mobility of \( \mu_0 = 2500 \) m\(^2\)/Vs and a broadening parameter of \( \alpha = 12.5 \), subject to linearly \( x \)-direction polarized incident microwave radiations of frequency \( \omega/2\pi = 50 \) GHz having four different amplitudes \( E_{\text{is}} = 0.8, 1.2, 2.0 \) and 3.5 V/cm at a lattice temperature of \( T = 1 \) K. For this GaAs system at 50GHz \( a_{\text{inter}} = 1.9 \) and \( a_{\text{intra}} = 2.4 \). The intra-Landau level single-photon transitions are allowed when \( \omega_c/\omega > 2.4 \), yielding, at the high \( \omega_c/\omega \) side, an absorption rate \( S_p \) somewhat larger, an electron...
temperature $T_e$ somewhat higher, and a SdHO damping stronger than those in the 100-GHz case (Fig. 1). On the other hand, at equivalent HF field strength the multiphoton processes are more important at lower frequency. This helps to enhance the absorption $S_p$ in the range $1.9 < \omega_c/\omega < 2.4$, where the single-photon process is forbidden and to increase the two-photon resonance in $S_p$ and $T_e$ around $\omega/\omega_c = 1.5, 2.5$ and $3.5$ (see $S_p$ and $T_e$ curves corresponding to $E_{is} = 3.5 \text{ V/cm}$ in Fig. 3). The effect of the two-photon process can also be seen clearly in the $R_{xx}$-vs-$\omega/\omega_c$ curves as shown in Fig. 4, where the $R_{xx}$ curve of $E_{is} = 3.5 \text{ V/cm}$ exhibits obvious shoulders around $\omega/\omega_c = 1.5, 2.5$ and $3.5$, and the descends down around $\omega/\omega_c = 0.6$. This kind of two-photon process was clearly seen in the experiments.

At even lower frequency, such as 30 GHz and 20 GHz, the ranges for intra-Landau level and inter-Landau level single-photon transitions overlap. The enhanced effect of the virtual photon process, together with enhanced multiphoton-assisted electron transition, pushes the resistivity $R_{xx}$ remarkably down below the average of its oscillatory curve without radiation, resulting in a strong suppression of dissipative magnetoresistance across a wide magnetic field range as shown in Fig. 5, in agreement with experimental observations.\textsuperscript{13,14}

### C. 150 and 280 GHz

The radiation-induced SdHO modulation can be seen clearly in the low magnetic field region $\omega/\omega_c > 1$ with higher radiation frequency. Figure 6 shows the calculated electron temperature $T_e$ and magnetoresistivity $R_{xx}$ as functions of $\omega/\omega_c$ for a 2D system of electron density $N_e = 3.0 \times 10^{15} \text{ m}^{-2}$, linear mobility $\mu_0 = 2000 \text{ m}^2/\text{Vs}$ and $\alpha = 3$, subject to a 150-GHz microwave radiation of three different amplitudes $E_{is} = 0.1, 0.6$ and $2 \text{ V/cm}$ at a lattice temperature of $T = 0.5 \text{ K}$. Low-power microwave illumination ($E_{is} = 0.1 \text{ V/cm}$) already yields sufficient $T_e$ oscillation with maxima at $\omega/\omega_c = 1, 2, 3, 4$, giving rise to clear SdHO modulations having nodes at $T_e$ maxima. At higher microwave power ($E_{is} = 0.6 \text{ V/cm}$) when the MIMO shows up, the $T_e$ maxima gets higher, suppressing the SdHO in the vicinities of $\omega/\omega_c = 1, 2, 3, 4$, but a strong amplitude modulation of SdHOs is still seen. In the case of $E_{is} = 2 \text{ V/cm}$, $R_{xx}$ shows strong MIMO and the electron temperature further grows so that most SdHOs almost disappear in the range of $\omega/\omega_c > 2$. Note that the small $T_e$ peaks at $\omega/\omega_c = 1.5$ and $2.5$ are due to the absorption rate $S_p$ maxima induced by two-photon processes, which gives rise to additional nodes in the SdHOs.

Another example of the SdHO modulation appearing simultaneously with MIMO is plotted in Fig. 7, where the energy absorption rate $S_p$, the electron temperature $T_e$, and the magnetoresistivity $R_{xx}$ are shown as functions of $\omega/\omega_c$ for a 2D system having an electron density of $N_e = 3.0 \times 10^{15} \text{ m}^{-2}$, a linear mobility of $\mu_0 = 1000 \text{ m}^2/\text{Vs}$, and a broadening parameter of $\alpha = 2$, subject to linearly $x$-direction polarized incident microwave radiations of frequency $\omega/2\pi = 280 \text{ GHz}$ and amplitude $E_{is} = 3.5 \text{ V/cm}$. The energy absorption rate $S_p$ has broad large peaks at $\omega/\omega_c = 1, 2, 3, 4, 5$ (due to single-photon resonant process) and small peaks at $\omega/\omega_c = 1.5, 2.5$ (due to two-photon resonant process), giving rise to the oscillation of the electron temperature $T_e$. One can clearly see the peaks of the electron temperature $T_e$ and the nodes of SdHO modulation at $\omega/\omega_c = 1, 2, 3, 4$ and $5$, together with MIMOs. These are in agreement with the experi-
ment observation reported in Ref.\[11].

D. Discussion

Note that in GaAs-based systems at a temperature around $T \sim 1\,\text{K}$, LA phonons generally give larger con-

tribution to the electron energy dissipation $W$ than that from TA phonons and LO phonons are usually frozen. However, in the case of high radiation power or in the vicinity of $\omega \sim \omega_c$, where the resonantly absorbed en-

gy can be relatively large and the electron temperature can rise up above 20 K, a weak emission of LO phonons takes place. Though at this temperature the number of excited LO phonons is still very small and their contribu-
tion to momentum relaxation (resistivity) is negligible in comparison with acoustic phonons, they can already provide an efficient energy dissipation because each excited LO phonon contributes a huge energy transfer of $\Omega_{LO} \sim 400 \,\text{K}$. With a continuing rise of electron tem-

erature the LO-phonon contribution increases rapidly. This effectively prevents the electron temperature from going much higher than 20 K, such that the $T_e$ vs. $\omega_c/\omega$ curve of large incident microwave power in Fig.1 exhibits a flat top around $\omega_c/\omega = 1$.

In this paper, we did not consider the role of surface or interface phonons in the GaAs heterostructure. Depending on sample geometry, the surface phonons may be important in dissipating electron energy thus decreasing the electron temperature.

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