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Contribution of Nuclear Field Theory medium effects to the pairing gap

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Abstract. We calculate the different contributions to the pairing gap for the case of the superfluid nucleus $^{120}$Sn, making use of the Dyson equation (also known as the Nambu-Gor’kov equation in the case of superfluid systems) to propagate leading medium polarization Nuclear Field Theory (NFT) processes to all orders. Starting from a mean field obtained with the SLy4 effective force ($m_k \approx 0.7$), we renormalize the single-particle states and the pairing gap obtained from $^{1}$S$_0$ component of the Argonne $v_{14}$ $N - N$ interaction, through the interweaving of single-particle motion and collective vibrations of the system. The results provide a detailed description of the fragmentation of the quasiparticle strength and of the state-dependent pairing gap.

1. Dyson equation as a framework for Self Energy renormalization

Nuclear Field Theory (NFT) provides an exact theoretical framework to describe nuclear structure (see Refs. [1,2] and references therein and within the context of the present contribution, see also Ref.[3]) and reactions [4] in terms of elementary modes of excitation which, in the case of spherical nuclei, correspond to single-particle (quasiparticle) motion and collective vibrations. In keeping with the overcompleteness of the basis (vibrations are built out of the same quasiparticle degrees of freedom as those involved in independent single-particle motion), there is a (coupling) term $H_c$ in the NFT Hamiltonian which depends both on the single-particle and on the collective coordinates. Following the NFT theory rule stating that the collective vibrations are to be calculated within the framework of the RPA, the radial part of the matrix element associated with scattering process which brings a nucleon from a single-particle state of quantum numbers $a \equiv \{n_a, l_a, j_a\}$ to a state $b$ (mean field solutions) and a phonon vibration, can be written as [5]

$$h(a, b\lambda_\nu) = \frac{i^{l_a - l_b + \lambda}}{\sqrt{4\pi}} \langle j_a | 1 2 \lambda \lambda \frac{1}{2} \lambda \lambda \frac{1}{2} | j_b \rangle \beta_{\lambda_\nu} \langle n_a l_a j_a | R_0 \frac{\partial U}{\partial r} | n_b l_b j_b \rangle,$$ (1)

In the above expression $U(r)$ is the mean field, $\nu$ labels the various phonons of angular momentum $\lambda$ and $\beta_{\lambda_\nu}$ is the (dynamical) deformation parameter associated with the vibration. One can now systematically calculate, making use of (1), the variety of NFT contributions to the matrix elements of $H_c$, to any selected order of perturbation. Diagonalizing the resulting
matrix gives rise to dressed particles and vibrations (self-energy and vertex corrections), as well as to renormalized interactions.

We first perform a HF+BCS calculation, employing the Argonne $v_{14}$ bare $N - N$ interaction as the pairing force in the $1^1S_0$ channel. In this way we obtain a set of quasiparticles with energies $E_a$ and amplitudes $u_a, v_a$. We then renormalize the quasiparticle properties carrying out the diagonalization of $H_e$ making use of the Dyson equation. Because in superfluid nuclei one has to take into account both the normal and anomalous self-energies namely $\Sigma^{11}$ and $\Sigma^{12}$ respectively [6], the Dyson equation becomes a set of two coupled equations known as the Nambu-Gor’kov equations:

\[
\begin{pmatrix}
E_a & 0 \\
0 & -E_a
\end{pmatrix}
+ \begin{pmatrix}
\Sigma^{11}(a, \tilde{E}_a(n)) & \Sigma^{12}(a, \tilde{E}_a(n)) \\
\Sigma^{21}(a, \tilde{E}_a(n)) & \Sigma^{22}(a, \tilde{E}_a(n))
\end{pmatrix}
\begin{pmatrix}
x_{a(n)} \\
y_{a(n)}
\end{pmatrix}
= \tilde{E}_a(n)\begin{pmatrix}
x_{a(n)} \\
y_{a(n)}
\end{pmatrix},
\]

(2)

where

\[
\Sigma^{11}(a, \tilde{E}_a(n)) = -\Sigma^{22}(a, -\tilde{E}_a(n)) = \\
\sum_{b,m,\lambda,\nu} \left[ \frac{|h(a, b \lambda)(u_a \tilde{v}_b(m) - v_a \tilde{u}_b(m))|^2}{\tilde{E}_a(n) - E_b(m) - i\hbar \omega_{\lambda,\nu}} + \frac{|h(a, b \lambda)(u_a \tilde{v}_b(m) + v_a \tilde{u}_b(m))|^2}{\tilde{E}_a(n) + E_b(m) + i\hbar \omega_{\lambda,\nu}} \right],
\]

(3)

and

\[
\Sigma^{12}(a, \tilde{E}_a(n)) = \Sigma^{21}(a, \tilde{E}_a(n)) = \Sigma^{12}(a, -\tilde{E}_a(n)) = \\
\sum_{b,m} (u_a \tilde{v}_b(m) - v_a \tilde{u}_b(m))(u_a \tilde{v}_b(m) + v_a \tilde{u}_b(m)) \cdot \\
\left[ \sum_{\lambda,\nu} \left( \frac{|h(a, b \lambda)|^2}{\tilde{E}_a(n) - (E_b(m) + i\hbar \omega_{\lambda,\nu})} - \frac{|h(a, b \lambda)|^2}{\tilde{E}_a(n) + (E_b(m) + i\hbar \omega_{\lambda,\nu})} \right) \right],
\]

(4)

from which the amplitudes $x_{a(n)}; y_{a(n)}$ and the renormalized quasiparticle energies $\tilde{E}_a(n)$ can be obtained self-consistently considering the following relation for the renormalized quasiparticle amplitudes $\tilde{u}_a(n); \tilde{v}_a(n)$:

\[
\tilde{u}_a(n) = [x_{a(n)} u_a - y_{a(n)} v_a], \quad \tilde{v}_a(n) = [x_{a(n)} v_a + y_{a(n)} u_a].
\]

(5)

These quasiparticle amplitudes obey the normalization condition

\[
1 = x^2_{a(n)} + y^2_{a(n)} - \frac{\partial \Sigma^{11}}{\partial E} x^2_{a(n)} - \frac{\partial \Sigma^{22}}{\partial E} y^2_{a(n)} - 2 \frac{\partial \Sigma^{12}}{\partial E} x_{a(n)} y_{a(n)}.
\]

(6)

The above relation arises from the energy-dependence of Eq. (2), embodied in the energy denominators of Eqs. (3) and (4). Consequently, the quasiparticle strength associated with a given fragment $a(n)$, namely $N_{a(n)} = x^2_{a(n)} + y^2_{a(n)} = \tilde{u}_{a(n)}^2 + \tilde{v}_{a(n)}^2$, can be smaller than one. In other words, the original HF single-particle strength, after the interweaving with collective vibrations in terms of NFT processes, can become fragmented over a number quasiparticle states.

Making use of these elements one can define, for each quasiparticle fragment $a(n)$, a state-dependent pairing gap (see Ref.[6],[7]) according to

\[
\tilde{\Delta}_{a(n)} = Z_{a(n)} \tilde{\Sigma}^{12}(a, \tilde{E}_a(n)) = Z_{a(n)} \left( \Delta_a^{BCS} + \tilde{\Sigma}^{12, pho}(a, \tilde{E}_a(n)) \right) = \tilde{\Delta}_{a(n)}^{bare} + \tilde{\Delta}_{a(n)}^{pho},
\]

(7)
where
\[ Z_{a(n)} = \left( 1 + \frac{\Sigma^{11}(a, E_{a(n)}) + \Sigma^{22}(a, E_{a(n)})}{2E_{a(n)}} \right)^{-1}, \]  
provides a measure of the discontinuity of the single-particle strength at the Fermi energy. The term \( \Sigma^{12, pho}(a, E_{a(n)}) \) is given by
\[ \Sigma^{12, pho}(a, E_{a(n)}) = \Sigma^{12}(a, E_{a(n)})(u_{a}^{2} - v_{a}^{2}) + u_{a}v_{a}(\Sigma^{11}(a, E_{a(n)}) - \Sigma^{22}(a, E_{a(n)})). \]  

1.1. Mean Field and QRPA
The mean field and the single-particle levels used in the present contribution were obtained from a HF calculation based on the effective SLy4 interaction [8]. The collective modes have been calculated in QRPA using the gap extracted from the odd-even mass difference (\( \Delta \approx 1.4 \text{MeV} \)), together with a separable interaction associated with a surface peaked formfactor given by \( R_{0dU/dr} \) (see Eq.(1)). The coupling constants have been determined so as to reproduce the experimental energy and transition strength of the low-lying \( 2^+, 3^-, 4^+ \) and \( 5^- \) collective surface vibrations. Of course this is not consistent with NFT in keeping with the fact that phonons should also be renormalized to the same order to which particles and interactions are. In other words this should be done through self-energy and vertex corrections (see e.g. [3] and refs. therein). In principle, this would have consequences not only on the properties of the calculated phonons, but also on their ability to renormalize the single-particle strength and the bare \( N-N \) interaction.

In keeping with the fact that the work presented here can be considered as a pilot calculation to quantitatively assess the degree of quasiparticle fragmentation resulting from the interweaving of quasiparticle and vibrational states, we have preferred in this first step to use phenomenological phonons, constraining the effective interaction to reproduce the experimental properties of the low-lying collective vibrational states [13] (more precisely, we impose that the experimental value of the polarizability \( \beta_{N}^{2}/\hbar\omega_{\lambda_{1}} \) is reproduced).

We are of course aware of the dangers of treating at different levels of accuracy the renormalization processes of the different elementary modes of excitation entering in NFT, and will remedy these shortcomings in a future publication. Once this will be done, we will have the elements to calculate the absolute single-particle transfer cross sections associated with each quasiparticle fragment, and thus be able to directly compare experimental observations with theoretical predictions, avoiding the question of whether or not one can define spectroscopic factors (see e.g. [9] and refs. therein).

2. Results for the \( ^{120}\text{Sn} \) nucleus
As mentioned above, the coupling of nucleons to collective vibrations leads to a fragmentation of the quasiparticle states. In most cases and near the Fermi energy, the quasiparticle strength remains concentrated in a single peak carrying typically 70\% of the single particle strength. Thus, in such cases, the one-pole (quasiparticle) approximation is reasonably good, as for the \( h_{11/2} \) orbital (see figure 1 (a)). It may however happen, that a level becomes strongly fragmented, as in the case of the quasiparticle strength associated with the \( d_{5/2} \) orbital (see figure 1 (b)). In such a case the calculation becomes rather dependent on the mean field input. Arguably, this result may reflect the lack of consistency at the NFT level, implied by the use of phenomenological phonons but, most likely, to the fact of not having considered the coupling of single-particle states to vibrational modes associated with \( \vec{\sigma} \cdot \vec{\sigma}, \vec{\tau} \cdot \vec{\tau} \) and \( (\vec{\sigma} \cdot \vec{\sigma})(\vec{\tau} \cdot \vec{\tau}) \) (spin, isospin, and spin-isospin) degrees of freedom (see below). If this was the case, one would have a unique opportunity to assess, from a non-conventional point of view, the role of these degrees of freedom in the nuclear spectrum.
Figure 1. (Color online) Strength distribution of $\tilde{u}_a^2$ and $\tilde{v}_a^2$ for the $h_{11/2}$ (a) and $d_{5/2}$ (b) orbitals resulting from the coupling to collective vibrations compared with available experimental spectroscopic factors [14, 15, 16].

As seen from figure 2, the quasiparticle energy spectrum resulting from the fragments carrying the largest single-particle strength is considerably denser than that obtained from Hartree-Fock plus BCS theory, in overall agreement with the experimental findings.

Figure 2. Single-particle spectrum obtained from HF theory with the SLy4 interaction (first column) and used as starting point for solving the BCS equations (second column) with the Argonne $v_{14}$ pairing interaction. In the third and fourth column we give respectively the solution of the Dyson equation which takes into account the particle-vibration coupling at the first iteration and at convergence. The experimental results [14, 15, 16] for the energy of the main quasiparticle peaks of $^{119}\text{Sn}$ and $^{121}\text{Sn}$ are displayed in the last two columns.
Figure 3. (Color online) (a) State-dependent pairing gap obtained from the BCS calculation performed with the $v_{14}$ residual pairing interaction (circles); this is compared with the induced (squares) and bare (diamonds) contributions to the total renormalized paring gap (triangles) calculated by the Dyson equation. The labels indicate the quantum number of the five valence orbitals of $^{120}$Sn. (b) $Z-$ (circles) and $N-$ factors (blue triangles). The red stars report, with due caution, the experimental spectroscopic factors [14, 15, 16].

Figure 4. (Color online) Comparison between different contributions to the pairing gap. The circles show the bare BCS-$v_{14}$ pairing gap, while the empty triangles show the gap renormalized by the coupling to $S=0$ (density) modes, already shown in figure 3(a); the brown diamonds show the gap obtained including also the coupling to $S=1$ (spin) modes.
Of notice that the pairing gap receives contributions from both the bare interaction and the induced one (phonon-mediated). In the case under discussion the induced and the bare contributions each account for about one half of the total gap, respectively (see figure 3(a)). In the determination of the pairing gap the $Z$-factor plays a crucial role, since its value is about 0.75 (see figure 3(b)), thus renormalizing $\Sigma^{12}$ in an important way (see Eq.(1), see also [10, 11]).

Another effect to be considered is the coupling with the spin modes. In fact, it is rather well established in the literature that this coupling represents the leading renormalization effect in uniform nuclear matter, providing a repulsive contribution to the pairing gap. While the effect in finite nuclei seems to be not as crucial as it is in nuclear matter, it has been estimated in [12] that the contribution of spin modes to the induced pairing interaction leads to a reduction by about 30% of the pairing gap obtained renormalizing the bare pairing interaction by only including density fluctuations (see figure 4).

3. Conclusions
The Dyson equation constitutes a powerful framework to propagate to all orders the basic NFT processes which renormalize the motion of nucleons and their interaction. This is particularly important in connection with self-energy processes, which give the distribution of the full single-particle strength and thus of the quasiparticle lifetime, the abnormal component of the self-energy providing information concerning the phonon-induced interaction. Quasiparticle fragmentation and the induced pairing interaction can affect the structure of nuclear Cooper pairs and thus nuclear superfluidity in an important way. Such effects can be specifically probed through two-particle transfer reactions and made quantitative, and thus eventually subject to experimental test, in terms of the associated absolute differential cross sections.

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