Building oscillations based on a plate model

M Usarov \(^1\)\((0000-0001-7848-3696)\), G Ayubov \(^1\)\((0000-0002-8948-3808)\), G Mamatisaev \(^2\)\((0000-0001-6517-0287)\)
and B Normuminov \(^3\)\((0000-0002-5018-3533)\)

\(^1\)Institute of Mechanics and Seismic Stability of Structures of the Academy of Sciences of the Republic of Uzbekistan, Tashkent, Uzbekistan
\(^2\)Fergana Polytechnic Institute, Fergana, Uzbekistan
\(^3\)Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, Tashkent, Uzbekistan

umakhamatali@mail.ru

Abstract. The paper is devoted to the numerical solution of the problem of transverse oscillations of a multi-storey building within the framework of a continuous plate model under seismic effects. Cantilevers anisotropic plate is proposed as a building dynamic model, the theory of which is developed in the framework of a three-dimensional dynamic theory of elasticity and takes into account not only structural forces and moments but also the bimoments. The proposed plate model of a building allows us to take into account and study all types of different spatial oscillations of the building structure under the impacts different in direction. Formulas are given for the reduced density, elastic moduli, and shear of the plate model of the building. The base acceleration, given in time by a harmonic law, is taken as a seismic impact. The problem is solved by the finite difference method. Examples are considered and numerical results are obtained. Waveform, displacements, and accelerations distribution diagram of multi-storey high-rise buildings under transverse oscillations are plotted.

1. Introduction

At the present stage of earthquake-resistant construction, the underground and surface shell and plate structures interacting with soil are widely used. Buildings and structures with various constructive solutions make up a complex spatial mechanical system "structure-soil".

At the present stage of science development, the issues of seismic resistance of hydro-technical structures interacting with a soil base are relevant and present a difficult problem in the mechanics of a deformable rigid body. Design models of underground structures should reflect the real conditions of their interaction with the soil. In the studies in [1–3], a dynamic theory of soil resistance of underground structures interacting with soil is proposed.

In [4–6], various calculated linear and nonlinear models of ground-based hydro-technical structures such as water dams, levees, reservoirs, etc., constructed and operated in seismically active regions and subjected to static and dynamic (seismic) loads, are proposed.

In calculations on earthquake resistance of underground and aboveground structures, an important role is played by consideration of physically nonlinear properties of the structure elements material and the soil base and the structure-soil interaction. Such studies include the works of the authors [7–9], who investigated the elastic-plastic behavior of structural materials under complex loading conditions, the studies in [10–13] devoted to the determination of elastic-plastic law of the state of soils and soil structures.
In [14] the solution to the problem of optimizing the projects of industrial buildings designed in seismically hazardous areas was considered. Economic efficiency was taken as an optimality criterion, depending on certain variable parameters adopted at the design stage.

In [15], the problem of evaluating the impact of wall panel fractures when estimating large-panel structures strength was solved. The forces exceeding the permissible values in structural elements were calculated.

Calculation methods for earthquake resistance of multi-storey buildings are developing as one of the urgent tasks of the dynamics of structures. In the design schemes of structural elements of buildings, the methods and equations of plate theory are successfully applied. It was shown in [16, 17] that using the theory of beams and plates it is possible to construct a spatial dynamical model of the box-like structure of buildings and structures, taking into account the real conditions of contact joints of beam and plate elements.

One of the important tasks of the modern theory of earthquake resistance of structures is the development of design continuum models of high-rise buildings that adequately describe their vibrations under seismic impacts. A continual plate model of buildings developed on the basis of the bimoment theory of thick plates [18] was proposed in [19, 20]. It should be noted that many engineering structures could be represented as a plate model. For example, hydro-technical structures that are close in configuration to thick plates of varying thickness.

2. Methods

In this paper, we proposed a cantilever thick anisotropic plate as a dynamic model of a building, the theory of which is developed in the framework of the three-dimensional dynamic theory of elasticity and takes into account not only structural forces and moments but bimoments as well. The proposed plate model of the building allows us to take into account and explore all types of different spatial vibrations of the building structure under different in direction impacts. Formulas for reduced density, elastic moduli and shear of the plate model of a building are given.

To describe the movement of the plate building, we introduce a Cartesian coordinate system with variables $x_1$, $x_2$, and $z$. The origin is located in the lower left corner of the middle surface of the continuum plate. We direct the axes $OX_1$ and $OX_2$ along the length and height, and the axis $OZ$ - along the thickness (the width of the building) of the plate model.

Assume that the seismic motion of soil occurs in the direction of the $OZ$ axis (the width of the building). Based on this, as an external influence on the lower fixed edge, we set the acceleration to the base $\ddot{u}_b(t)$ in the form:

$$\ddot{u}_0(t) = a_0 \cos(\rho_0 t)$$ \hspace{1cm} (1)

Where $a_0 = k_c g$ and $\rho_0 = 2\pi\omega_0$ are the maximum acceleration and the frequency of earth base, respectively.

At the base of the building, the boundary conditions for bending-shear vibrations are:

$$\ddot{\varphi}_1 = 0, \quad \ddot{\varphi}_2 = 0, \quad \ddot{\beta}_1 = 0, \quad \ddot{\beta}_2 = 0, \quad \ddot{\gamma}_1 = 0, \quad \ddot{\gamma}_2 = 0, \quad \ddot{\gamma} = 0, \quad \ddot{\gamma}_1 = 0, \quad \ddot{\gamma}_2 = 0, \quad \ddot{\gamma} = 0, \quad \ddot{W} = 0$$ \hspace{1cm} (2)

On the free side faces of the building, we have the conditions: forces, moments and bimoments and force factors equal to zero [18–20]:

$$M_{11} = 0, \quad M_{12} = 0, \quad P_{11} = 0, \quad P_{12} = 0, \quad Q_{13} = 0,$$

$$\ddot{P}_{13} = 0, \quad \ddot{\sigma}_{11} = 0, \quad \ddot{\sigma}_{12} = 0, \quad \ddot{\sigma}_{11} = 0$$ \hspace{1cm} (3)

On the free upper edge of the building we have the conditions:
\[ M_{12} = 0, \quad M_{22} = 0, \quad P_{12} = 0, \quad P_{22} = 0, \quad Q_{23} = 0, \]
\[ \ddot{\rho}_{23} = 0, \quad \ddot{\sigma}_{11} = 0, \quad \ddot{\sigma}_{12} = 0, \quad \sigma_{22} = 0 \]

The problem is solved by the finite difference method. The finite-difference equations of motion of the transverse vibrations of buildings have the following form:

\[ \frac{\partial M_{11}}{\partial x_1} + \frac{\partial M_{12}}{\partial x_2} - Q_{13} + Hq_{12} + \ddot{F}_1 = \frac{H^2}{2} \rho \ddot{\psi}_1, \]
\[ \frac{\partial M_{21}}{\partial x_1} + \frac{\partial M_{22}}{\partial x_2} - Q_{23} + Hq_{22} + \ddot{F}_2 = \frac{H^2}{2} \rho \ddot{\psi}_2, \]
\[ \frac{\partial Q_{13}}{\partial x_1} + \frac{\partial Q_{21}}{\partial x_2} + 2\ddot{q}_3 + \ddot{F}_3 = \rho H \ddot{\theta} + \rho H u_0(t), \]
\[ \frac{\partial P_{11}}{\partial x_1} + \frac{\partial P_{12}}{\partial x_2} - 3Hq_{11} + Hq_{12} + \ddot{G}_1 = \frac{H^2}{2} \rho \ddot{\beta}_1, \]
\[ \frac{\partial P_{21}}{\partial x_1} + \frac{\partial P_{22}}{\partial x_2} - 3Hq_{21} + Hq_{22} + \ddot{G}_2 = \frac{H^2}{2} \rho \ddot{\beta}_2, \]
\[ \frac{\partial \ddot{p}_{13}}{\partial x_1} + \frac{\partial \ddot{p}_{23}}{\partial x_2} - \frac{4\ddot{p}_{33}}{H} + \frac{2\ddot{q}_3}{H} + \ddot{g}_3 = \rho \ddot{\gamma} + \frac{1}{3} \rho \ddot{u}_0(t). \]

Where \( M_{ij}, \) \( Q_{ij} \) \((i, j = 1,2)\) are the bending moments and shear forces; \( P_{ij}, \) \((i, j = 1,2)\) are the longitudinal bimoments, \( p_{ij} \) \((i, j = 1,2)\), \( p_{33} \) are the intensities of transverse bimoments.

The expressions of forces, moments, and bimoments are determined in the framework of the bimoment theory of the plate model of buildings and structures [18–20].

Expressions of bending and torsional moments have the form

\[ M_{11} = \frac{H^2}{2} \left( E_{11} \frac{\partial \ddot{\psi}_1}{\partial x_1} + E_{12} \frac{\partial \ddot{\psi}_2}{\partial x_2} - E_{13} \frac{2(\dddot{r} - \dddot{W})}{H} \right), \]
\[ M_{12} = G_{12} \frac{H^2}{2} \left( \frac{\partial \ddot{\psi}_1}{\partial x_2} + \frac{\partial \ddot{\psi}_2}{\partial x_1} \right), \]
\[ M_{22} = \frac{H^2}{2} \left( E_{12} \frac{\partial \ddot{\psi}_1}{\partial x_1} + E_{22} \frac{\partial \ddot{\psi}_2}{\partial x_2} - E_{23} \frac{2(\dddot{r} - \dddot{W})}{H} \right). \]

The expressions for the shear forces are written as

\[ Q_{13} = G_{13} \left( 2\ddot{u}_1 + H \frac{\partial \ddot{\gamma}}{\partial x_1} \right), \quad Q_{23} = G_{23} \left( 2\ddot{u}_2 + H \frac{\partial \ddot{\gamma}}{\partial x_2} \right). \]

The bimoments \( P_{11}, P_{22}, P_{12} \) have the form
Expressions of the intensity of bimoments $\tilde{p}_{13}$, $\tilde{p}_{23}$ are defined as

$$\tilde{p}_{13} = G_{13} \left( \frac{2\tilde{u}_1 - 4\tilde{\psi}_1}{H} + \frac{\tilde{\psi}}{\xi_1} \right), \quad \tilde{p}_{23} = G_{23} \left( \frac{2\tilde{u}_2 - 4\tilde{\psi}_2}{H} + \frac{\tilde{\psi}}{\xi_2} \right).$$

(12)

Expressions of the intensity of bimoments $\tilde{r}_{33}$, $\tilde{\tau}_{33}$, depending on stress $\sigma_{33}$, have the form

$$\tilde{r}_{33} = E_{31} \frac{\hat{\psi}_1}{\xi_1} + E_{32} \frac{\hat{\psi}_2}{\xi_2} - E_{33} 2(\tilde{\gamma} - \tilde{W}) H,$$

$$\tilde{\tau}_{33} = E_{31} \frac{\hat{\psi}_1}{\xi_1} + E_{32} \frac{\hat{\psi}_2}{\xi_2} - E_{33} 2(\tilde{\gamma} - \tilde{W}) H.$$  

(13)

The system of equations of motion concerning three generalized functions $\tilde{u}_1, \tilde{u}_2, \tilde{W}$ is approximated in the form

$$\tilde{u}_1 = \frac{1}{2} \left( 2(\tilde{\beta}_1 - 7\tilde{\psi}_1) - \frac{1}{30} H \frac{\partial \tilde{W}}{\xi_1} + \frac{1}{30} G_{13} H \tilde{q}_1 \right),$$

$$\tilde{u}_2 = \frac{1}{2} \left( 2(\tilde{\beta}_2 - 7\tilde{\psi}_2) - \frac{1}{30} H \frac{\partial \tilde{W}}{\xi_2} + \frac{1}{30} G_{23} H \tilde{q}_2 \right),$$

$$\tilde{W} = \frac{1}{4} \left( 2(\tilde{\gamma} - 3\tilde{\tau}) - \frac{1}{20} H \left( E_{31} \frac{\partial \tilde{u}_1}{\xi_1} + E_{32} \frac{\partial \tilde{u}_2}{\xi_2} \right) + \frac{H \tilde{q}_3}{20 E_{33}} \right).$$

(14)

(15)

3. Results and Discussion

We conventionally assume that the mechanical and geometric characteristics of the materials of the room panels are the same: the elastic modulus is $E = 20 000$ MPa, the density $\rho = 2700$ kg/m$^3$ and the Poisson's ratio $\nu = 0.3$.

Present the results of calculations of the building forced vibrations in the framework of a thick plate model for the following plate and building sizes: length and width of the building $a=30$ m, $H=11$ m, the height of one floor of the building $b_1=3$ m, the thickness of the bearing external and internal walls $h_1=0.25$ m and $h_2=0.2$ m, ceiling thickness $h_{ce} = 0.2$ m.

Based on the above initial data and the expressions for determining the density and elastic modulus of the plate model of the building, given in [18], we find the following characteristics of the building materials:

- $E_1^{bd} = 2600$ MPa, $E_2^{bd} = E_3^{bd} = 2000$ MPa, $G_{12}^{bd} = 480$ MPa,
- $G_{13}^{bd} = 520$ MPa, $G_{23}^{bd} = 200$ MPa,
- $\rho_{bd} = 451$ kg/m$^3$, $\nu_{21} = 0.3$, $\nu_{31} = \nu_{23} = 0.4$.

In calculations, the seismicity coefficient and the base acceleration frequency are set as $k = 0.1$ and $\omega_0 = 9.5$ Hz. The dimensionless variables are $x = x/a$, $y = y/b$, and $t = ct/H$, where $c = \sqrt{E/\rho}$. 

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The calculation step for dimensionless coordinates is adopted as $\Delta x=1/60$, $\Delta y=1/30$. The stability of calculation in dimensionless time is ensured according to the explicit scheme at a step $\Delta=0.01$.

Table 1 shows the maximum values of the generalized displacement and acceleration $\vec{r}$ and $\ddot{r}$, obtained under forced transverse vibrations of multi-storey buildings at extreme points at the upper levels of floors.

| Number of floors | $\vec{r}$, $10^{-3}$ m | $\ddot{r}$, m/s$^2$ |
|------------------|-------------------------|---------------------|
| 16               | 0.8984                  | -2.5240             |
| 20               | 1.5506                  | -2.5451             |
| 22               | -2.2687                 | -5.1227             |
| 24               | -1.6073                 | 2.9966              |
| 28               | -3.4151                 | -4.0074             |
| 32               | 2.8451                  | 3.0118              |

Figure 1 shows the oscillation modes of a sixteen-story building, and Fig. 2 shows the diagram of acceleration distribution in a sixteen-story building. Figures 3 and 4 show oscillation graphs of a sixteen-story building in displacements and accelerations of the upper floors in the dimensionless time. From figures 3 and 4 it is seen that the maximum displacement of the upper floor is $r = 0.8984 \times 10^{-3}$ m, and the acceleration of the upper floors of the sixteen-story building is $\ddot{r} = -2.5240$ m/s$^2$. 
Figures 5 and 6 show the oscillation modes and the diagram of acceleration distribution, respectively, in a twenty-story building. Figures 7 and 8 show the graphs of the generalized displacement $r$ and acceleration $\ddot{r}$ in time of a twenty-story building, obtained under transverse vibrations located at the level of the building floors. As can be seen, the maximum displacement of the upper floor of the building is $r = 1.5506 \times 10^{-3}$ m, and the acceleration is $\ddot{r} = -2.5451$ m/s$^2$.

Figures 9 and 10 show vibration modes of a twenty-two-story building and an acceleration distribution diagram in a twenty-two-story building, respectively. Figures 11 and 12 show the oscillation graphs of displacement and acceleration of the upper floors in the dimensionless time of a twenty-two-story building. As seen, the maximum displacement of the upper floor of the building is $r = 2.2687 \times 10^{-3}$ m, and the acceleration is $\ddot{r} = -5.1227$ m/s$^2$. 
Figure 9. Mode of vibrations of a twenty-two-story building

Figure 10. Acceleration distribution diagram in a twenty-two-story building

Figure 11. Displacement of the upper floor of a twenty-two-story building, m

Figure 12. Acceleration of the upper floor of a twenty-two-story building, m/s²

Figure 13 shows the vibration mode of a twenty-four-story building, and Figure 14 shows the diagram of acceleration distribution in a twenty-four-story building.

The graphs in Figures 15 and 16 show the oscillations of displacements and accelerations of the upper floors in the dimensionless time of a twenty-four-story building. From figs. 15 and 16 it is seen that the maximum displacement of the upper floor is \( r = -1.6073 \times 10^{-3} \) m, and the acceleration of the upper floors of the twenty-four-story building is \( \ddot{r} = 2.9966 \) m/s².
Figures 17 and 18 show the oscillation mode and the diagram of acceleration distribution, respectively, in a twenty-eight-story building.

Figures 19 and 20 show the graphs of the generalized displacement $r$ and time acceleration $\ddot{r}$ of a twenty-eight-story building located at the level of the building’s floors. As can be seen, the maximum displacement of the upper floor of the building is $r = -3.415 \times 10^{-3}$ m, and the acceleration is $\ddot{r} = -4.0074$ m/s$^2$.

Figure 21 shows the vibration modes of a thirty-two-story building, and Figure 22 shows the diagram of acceleration distribution in a thirty-two-story building. Figures 23 and 24 show the graphs of the oscillation of displacements and accelerations of the upper floors in the dimensionless time of a thirty-two-story building. From figs. 23 and 24 it is seen that the maximum displacement of the upper floor is $r = 2.8451 \times 10^{-3}$ m, and the acceleration of the upper floors of the thirty-two-story building is $\ddot{r} = 3.0118$ m/s$^2$. 
Conclusions

Based on the analysis of numerical results, it can be noted that the maximum displacement values are reached at the upper level of buildings. The advantage of the proposed methodology for the dynamic analysis of buildings based on the continuous plate model is to reduce the three-dimensional problem to a two-dimensional one, as well as the possibility of high accuracy calculation of numerical results for the plate structure in the form of a spatial model of multi-storey buildings.
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