Constraints from Nucleosynthesis and SN1987A on Majoron Emitting Double Beta Decay

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Abstract

We examine whether observable majoron emission in double beta decay can be compatible with the big-bang nucleosynthesis (NS) and the observed neutrino flux from SN1987A. It is found that the NS upper bound on $^4\text{He}$ abundance implies that the majoron-neutrino Yukawa coupling constant $g \leq 9 \times 10^{-6}$ and its maximal value is allowed only when the scalar quartic coupling constant $\lambda$ is extremely small, $\lambda \leq 100g^2$. It is also observed that, although quite less restrictive, SN1987A also provides independent constraints on coupling constants.
Majoron is the massless Goldstone boson associated with spontaneous lepton number violation [1]. If exists, it would lead to many interesting phenomenological consequences. Amongst them, one particularly interesting phenomenon is “majoron emitting neutrinoless double beta decay” ($\beta\beta_J$) [2]. Recently it was pointed out that the potential anomaly in the double beta decay spectra of several elements may be explained by $\beta\beta_J$ [3]. The desired value of the Yukawa coupling constant is roughly $g_{ee} \simeq 10^{-4}$. Regardless of this observation, if $g_{ee}$ is not very small, e.g. not less than $10^{-5}$, one may be able to observe $\beta\beta_J$ in the near future. As was shown in ref. [4], it is not difficult to construct majoron models which provides such a value of $g_{ee}$ while satisfying all known experimental constraints. However in view of the strong cosmological and astrophysical implications of majoron [5], it is desirable to examine whether observable $\beta\beta_J$ can be compatible with the big-bang cosmology and also with astrophysical observations. In this paper, we consider possible constraints from the big-bang nucleosynthesis and the supernova SN1987A on majoron models in which observable $\beta\beta_J$ is possible.

If the lepton number symmetry $L$ is spontaneously broken at an energy scale $v$, majoron-neutrino Yukawa coupling matrix $g_{\alpha\beta}$ ($\alpha,\beta = e, \mu, \tau$) is related to the neutrino mass matrix $m_{\alpha\beta}$ as $g_{\alpha\beta} = m_{\alpha\beta}/v$. Current data on $\nu$-less $\beta\beta$ decay implies $m_{ee} \leq 1$ eV and $g_{ee} \leq \text{few} \times 10^{-4}$ with uncertainties arising from nuclear matrix elements [2]. For $\beta\beta_J$ to be observable in the near future, we may need $g_{ee} \geq 10^{-5}$. This implies that $L$-breaking scale is very small compared to the Fermi scale,

$$v \leq 100 \text{ keV},$$

leading to a fine tuning problem in general [6]. Here we will not concern this theoretical difficulty. We rather concentrate on models with such a low $L$-breaking scale to see whether observable $\beta\beta_J$ can be compatible with the standard nucleosynthesis model [7] and the observed neutrino flux from SN1987A [8].
In majoron models adopting low energy $L$-violation (1) to provide observable $\beta\beta_J$, low energy majoron-neutrino interactions can be described by the effective lagrangian:

$$\mathcal{L}_{\text{eff}} = \bar{\nu}_\alpha \gamma \cdot \partial \nu_\alpha + \frac{1}{\sqrt{2}} |\partial_\mu \chi|^2 - \frac{1}{\sqrt{2}} (g_{\alpha \beta} \chi^* \bar{\nu}_\alpha \nu_\beta + \text{h.c.}) - \lambda (\chi \chi^* - \frac{\nu^2}{2})^2 + ...$$  (2)

where $\chi$ is a (mostly) gauge singlet Higgs field carrying the lepton number two, and the ellipsis denotes generic nonrenormalizable interactions suppressed by the powers of cutoff scale $\Lambda$ which is usually taken to be the Fermi scale. The majoron field $J$ appears in $\chi$ as

$$\chi = \frac{1}{\sqrt{2}} (v + \rho + iJ),$$  (3)

and the mass eigenstates neutrino $\nu_i = \sum_{\alpha} U_{i\alpha} \nu_\alpha (i = 1, 2, 3)$ has the Yukawa interaction with $J$ and $\rho$ whose coupling constant is given by $g_i = |\sum_{\alpha \beta} U_{i\alpha} U_{i\beta} g_{\alpha \beta}|$ for a unitary mixing matrix $U_{i\alpha}$.

Let us now consider possible constraints on majoron-neutrino interactions from the big-bang nucleosynthesis (NS) [9]. As is well known, in the standard model of NS, the energy density of exotic particles at the NS epoch ($T_{NS} \simeq 1 \text{ MeV}$) is severely constrained [10]. In the case that $L$ is restored and also there is no sizable lepton number excess, the observed $^4\text{He}$ abundance implies

$$Y(T_{NS}) = \rho_\chi(T_{NS}) / \rho_\nu(T_{NS}) \leq 0.3,$$  (4)

where $\rho_\chi$ and $\rho_\nu$ denote the energy density of $\chi$ and a single species of left-handed neutrino respectively [11]. Since $v < T_{NS}$, before the NS the ratio of the Hubble expansion rate $H$ to the $\nu$-$\chi$ interaction rate behaves as $\Gamma_{\text{int}} / H \sim T^{-1}$. Then $\chi$’s would be rare at high temperature, but eventually enter into a thermal equilibrium with neutrinos at some temperature $T_{\text{eq}}$ at which $\Gamma_{\text{int}} \simeq H$. For the NS constraint (4) to be satisfied, we need $T_{\text{eq}} < T_{NS}$. Here we will directly evaluate $\rho_\chi$ and apply the constraint (4), rather than using more naive condition $T_{\text{eq}} < T_{NS}$.
To evaluate $\rho_\chi$, we first need to know whether $L$ is restored around the NS epoch. The effective mass of $\chi$ in the early universe is given by \[ m_{\text{eff}}^2 = -\lambda v^2 + \int \frac{d^3k}{(2\pi)^3} E (4\lambda f_\chi + 2 \text{tr}(gg^\dagger)f_\nu) \]

\[ = -\lambda v^2 + 4\lambda n_\chi \langle 1/E_\chi \rangle + \text{tr}(gg^\dagger)T^2/12, \quad (5) \]

where $f_X (X = \chi, \nu)$ denotes the phase space distribution function of $X$ whose number density is defined as $n_X = \int d^3k f_X/(2\pi)^3$, and neutrinos are assumed to be in thermal equilibrium. To proceed, let us set

\[ n_\chi \langle 1/E_\chi \rangle = \xi Y \omega n_\nu \langle 1/E_\nu \rangle = \xi Y^\omega T^2/24, \quad (6) \]

where $Y = \rho_\chi/\rho_\nu$. Clearly the average energy of $\chi$ do not exceed that of $\nu$ and thus $n_\chi/n_\nu \geq Y$, implying that $\xi Y^\omega \geq Y$. Then for $v \leq 100$ keV and $Y$ saturating the NS bound (4), which is the most interesting case for us, $L$ is restored around the NS epoch regardless of the value of $\text{tr}(gg^\dagger)$ \[ [13]. \]

It may be necessary to further discuss on the parameters $\xi$ and $\omega$. Self interactions among $\chi$’s do not change $\rho_\chi$, but can increase $n_\chi$ through the processes like $\chi\chi \to \chi\chi\chi^\ast$. The values of $\xi$ and $\omega$ depend on the strength of such “number changing self interaction process” (NCP). In our case, NCP’s can occur through the quartic coupling $\lambda \chi^2 \chi^*^2$ (loop effects) or through nonrenormalizable interactions like $\kappa \chi^3 \chi^*^3/\Lambda^2$. If the NCP rate is weaker than the expansion rate, the average energy of $\chi$ is roughly equal to that of $\nu$, and then $\xi \simeq \omega \simeq 1$. In the opposite case, $\chi$’s would achieve a thermal distribution, giving $\xi \simeq 2$ and $\omega \simeq 1/2$. In the subsequent analysis, we will simply set $\xi \simeq 2$ since as we have argued $Y \leq \xi Y^\omega$ and then this choice gives more conservative result for the $\chi$-production.

Clearly $\chi$’s are mainly produced by the heaviest mass eigenstate neutrino $\nu$ which has the largest Yukawa coupling $g \equiv \max(g_i)$. For interaction terms in (2), the
processes which can dominantly produce $\chi$ are as follows: (A) $\nu\nu \to \chi$; (B) $\nu\bar{\nu} \to \chi\chi^*$; (C) $\nu\nu \to \chi\chi\chi^*$ and $\bar{\nu}\bar{\nu} \to \chi\chi^*\chi^*$. Before $\chi$'s enter into an equilibrium with $\nu$'s, particularly when the NS constraint (4) is satisfied, we can safely ignore the inverse processes annihilating $\chi$'s. Then the Boltzmann equation describing the evolution of $\rho_\chi$ before the onset of the NS is given by

$$\frac{d}{dt}\rho_\chi + 4H\rho_\chi = \sum_I \int [dX](2\pi)^4\delta^4(p_i - p_f)E_I|M_I|^2f,$$

where $[dX] = \prod_a d^3k_a/(2\pi^3)2E_a$ (a runs over particle species participating in the process), $E_I$ ($I = A, B, C$) is the total energy of $\chi$'s in the final state of the process $I$, $|M_I|^2$ is the amplitude squared including the symmetry factor for identical particles, and finally $f = \prod_i f_i$ for the phase space distribution functions $f_i$ of particles in the initial state. After a straightforward computation, this Boltzmann equation can be cast into the following form:

$$-H\frac{d}{dT}Y = 6.7 \times 10^{-4}g^2\lambda[(Z)_A + (1.6\frac{g^2}{\lambda}\ln(5/\lambda Z))_B + (10^{-2}\lambda)_C],$$

where $Z = Y + (g^2/2\lambda) - 6(\nu/T)^2$. Here the subscripts in the brackets denote the contributing process. Note that the rate of (A) is proportional to $m_{\text{eff}}^2 \propto Z$, while that of (B) includes the factor $\ln(m_{\text{eff}}^2)$. Of course the above equation is valid only when $L$ is restored, viz $Z > 0$. At any rate, it indicates that if $\lambda \geq g^2\ln g^{-2}$, $\chi$’s are mainly produced around the NS epoch by the inverse decay process (A) whose rate is dominated by the background matter induced piece which is proportional to $Y$. However for $\lambda \leq g^2\ln g^{-2}$, the process (B) dominates over other processes. If $\lambda \gg g$, the process (C) can be important at the very early stage of $Y \ll 1$.

The Boltzmann equation (8) can be integrated to obtain $Y(T_{NS})$. Applying the NS constraint (4), we then find

$$\lambda g^2 \leq 7.2 \times 10^{-19}\ln(1 + \epsilon\lambda/g^2),$$

(9)
where \( \epsilon = 30/(\lambda^2/g^2 - 160 \ln(g^2/5 + 0.1\lambda)) \). This gives

\[
g \leq 9 \times 10^{-6} R
\]  

(10)

where \( R \simeq 1 \) for \( \lambda \leq 100g^2 \), while \( R \simeq (r^{-1} \ln r)^{1/4} \) for \( r = \lambda/100g^2 \gg 1 \). Note that in the case of \( \lambda > 100g^2 \), the \( \chi \)-production is dominated by the process (A) whose rate is enhanced by the factor \( \lambda/(g^2 \ln g^{-2}) \) compared to the rate of the process (B). This is the reason why we get a stronger bound on \( g \) in this case.

The implication of our NS result for \( \beta\beta_J \) is clear. Since \( g \) is the majoron Yukawa coupling constant of the heaviest mass eigenstate neutrino, \( g_{ee} \leq g \) and thus any upper limit on \( g \) applies also to \( g_{ee} \). Then taking into account possible uncertainties of our analysis, we can conclude that \( g_{ee} \simeq 10^{-5} \) for which observable \( \beta\beta_J \) is barely possible can be consistent with the standard NS model, but only under very unlikely conditions that the quartic coupling constant \( \lambda \) is extremely small (\( \leq 100g^2 \)) and also the majoron Yukawa couplings with \( \nu_\mu \) and \( \nu_\tau \) do not exceed that with \( \nu_e \). This conclusion is valid as long as \( v \) is small enough, \( e.g. \) less than few hundreds keV, for \( L \) to be restored around the NS epoch.

Although widely accepted and very natural, the standard NS model is not a unique model explaining observed cosmological data. There may be other successful models in which the constraint (4) is not valid any more \([4]\), implying that the NS limit on \( g \) does not have a strictly firm foundation. In this regard, it is still worthwhile to consider other implications of observable \( \beta\beta_J \) with \( g_{ee} \geq 10^{-5}, \ e.g. \) for supernova dynamics. It has already been studied how majoron-like particles can affect the explosion and the subsequent cooling of supernovae \([14, 15]\), but the results of these studies do not show any meaningful implication for \( \beta\beta_J \). Just after the observation of SN1987A, Kolb and Turner \([14]\) derived a limit on the interactions between supernova neutrinos and “cosmic background majorons” (CBM). Since the observed neutrino
pulse from SN1987A is that of \( \nu_1 \), the mass eigenstate neutrino which is mostly \( \nu_e \), the relevant Yukawa coupling constant here is \( g_1 \). Although in principle the parameter \( g_{ee} \) describing \( \beta \beta_J \) can be significantly different from \( g_1 \) (note that \( g_{ee} = \sum_i U^2_{ie} g_i \)), it is somewhat natural to assume that neutrino mixing is small enough for \( g_1 \) to be close to \( g_{ee} \). In this regard, the consideration of CBM’s may provide a useful information on \( \beta \beta_J \).

If \( \nu_1 \)'s from the supernova are scattered off by CBM’s, there would be a substantial decrease in the average neutrino energy, leading to an effective loss of detectable flux. The observed data [8] indicates that the mean free path \( l \) of \( \nu_1 \) through the CBM’s should be comparable to or greater than the distance to the supernova, viz

\[
l \geq 1.7 \times 10^{23} \text{ cm.}
\]  (11)

Using this, it was found in ref. [16] that \( g_1 \leq 10^{-3} \). However as we will see, one can in fact obtain a significantly stronger limit unless \( \lambda \) is significantly less than \( 10^{-3} \).

Clearly for \( g_{ee} \) in the range of observable \( \beta \beta_J \), i.e. \( g_{ee} \geq 10^{-5} \), there was a period in the early universe when \( \rho \) and \( J \) were at thermal equilibrium with neutrinos. Later that, the relic majoron number density can be increased by the decays of \( \rho \) and \( \nu \) and also by the \( \nu \) annihilations. With these observations, we will set the majoron temperature at present as \( T_J \approx 1.9 \text{ K} \), which is a somewhat conservative choice. Then the inverse mean free path of \( \nu_1 \) propagating through CBM’s is given by

\[
l^{-1} \approx \frac{\sqrt{2}}{4\pi^2} T_J^3 \int_0^\infty dw \frac{w^2}{e^{w} - 1} \int_{-1}^1 dz (1 - z)^{1/2} \sigma(s),
\]  (12)

where \( \sigma(s) \) is the total cross section for the reaction \( \nu_1 J \rightarrow \nu_1 J \) with the total energy-momentum squared \( s = 2ET_J w (1 - z) + m_1^2 \). Here \( E (\approx 10 \text{ MeV}) \) and \( m_1 \) denote the energy and the mass of the incident \( \nu_1 \). There are three diagrams responsible for the reaction \( \nu_1 J \rightarrow \nu_1 J \). Two of them are the Compton-type ones, while the rest one
involves the $\rho$-exchange. The resulting cross section can be written as

$$\sigma(s) = \frac{g_1^4}{16\pi s} \left[ F(s) + y^{-1} G(s) \right]$$  \hspace{1cm} (13)

where

$$F(s) \simeq \frac{5}{2} + \frac{2}{x} - \frac{2(1 + 2x)^2 \ln(1 + x)}{x^2(1 + x)} + \frac{4 + (1 - x) \ln y}{1 + x}$$

$$G(s) \simeq \frac{\ln(1 + x)}{x^2} - \frac{1}{x(1 + x)},$$

for $y = m_1^2/s$ and $x = s/m_\rho^2 = s/2\lambda v^2$. Here $F(s)$ represents the contribution from the Compton-type diagrams, $y^{-1}G(s)$ is from the $\rho$-exchange diagram.

If $\lambda$ is so small, e.g. $\lambda \ll g_1\sqrt{s}/v$, that the $\rho$-exchange diagram can be ignored, we have $\sigma \simeq g_1^4 \ln(s/m_1^2)/16\pi s$. Then applying the supernova constraint (11) yields $g_1 \leq 10^{-3}$ as was obtained in ref [16]. However in the other case of $\lambda \gg g_1\sqrt{s}/v$, the cross section is largely enhanced by the $\rho$-exchange diagram, allowing us to obtain a more stringent limit on $g_1$. Our results summarized in fig. 1 shows that, unless $\lambda \leq 10^{-3}$, some portion of the parameter region giving $g_1 \leq 3 \times 10^{-4}$ is ruled out by the supernova constraint (11). As was mentioned, if neutrino mixing is small, so that $g_{ee} \simeq g_1$, this supernova result can be directly applied to $\beta\beta_J$.

To conclude, we find that the standard nucleosynthesis model strongly constrains the coupling constants in majoron models in which observable “majoron emitting $\nu$-less double beta decay” ($\beta\beta_J$) is possible through a spontaneous lepton number violation below few hundreds keV. Our results directly applies to the Yukawa coupling constant $g_{ee}$ governing $\beta\beta_J$, yielding $g_{ee} \leq 9 \times 10^{-6}$. Here the maximal value $9 \times 10^{-6}$ is allowed only when (i) the scalar quartic coupling is extremely weak, roughly $\lambda \leq 10^{-8}$, and (ii) $g_{ee}$ is rather close to the majoron Yukawa coupling constant of the heaviest neutrino species. We also find that the consideration of the scatterings between cosmic background majorons and supernova neutrinos provides a constraint on the majoron
Yukawa coupling constant $g_1$ of the mass eigenstate neutrino which is mostly $\nu_e$. Compared to the nucleosynthesis constraint, this supernova constraint is much less restrictive, but is independent of the validity of the standard nucleosynthesis model. If $g_1 \simeq g_{ee}$ and $\lambda$ is not less than $10^{-3}$, which is somewhat plausible, some part of the parameter region giving observable $\beta\beta_J$ is excluded by the supernova data.

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Figure 1: Supernova constraint depicted on the plane of the $L$ breaking scale $v$ and the electron neutrino mass $m_1$. The solid curves from bottom to top are obtained by applying the condition (11) for $\lambda = 10^{-3}, 10^{-2}, 10^{-1}$, and 1 respectively, and the regions below them are excluded. The two dotted lines correspond to $g_1 = 10^{-3}$ and $10^{-5}$. 