Particle-based Simulation of Rayleigh-Taylor Instability

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Abstract. Rayleigh-Taylor instability is a typical benchmark in a computational fluid dynamics, which show interpenetrated materials happened when a heavy liquid is above a lighter liquid. In particle method, Shirakawa et al. had been proposed a correction term on modelling a buoyancy force to show the phenomena of Rayleigh-Taylor instability (RTI) in the simulation. By adjust a specific value of correction term for a specific mass density ratio, Rayleigh-Taylor instability phenomena had been observed on MPS method. Later, the phenomena is also observed on FVP methods using the same modelling. In this study, several mass density ratio and viscosity ratio of liquids have been proposed as parametric simulation to observe Rayleigh-Taylor instability. The results show the correlations between mass density ratio and viscosity ratio in the specific correction term value of buoyancy model. This results confirmed that the RTI phenomena can be observed in specific mass density ratio.

1. Introduction

The Rayleigh Taylor Instability is one of fingering instabilities of an interface between two fluids of different densities. It occurs, in many natural and engineering fields, when the light fluids accelerates the heavy fluid. In, computational fluid dynamics, the Rayleigh Taylor Instabilities is a typical benchmark to simulate the buoyancy force.

There are a wide variety of particle-based simulation method; Smoothed Particle Hydrodynamics (SPH), Moving Particle Semi-implicit (MPS), and Finite Volume Particle (FVP). These particle-based simulation methods are powerful on simulate particular shape and geometry since no mesh needed on modelling. In this particle method, we use deterministic approach to solve governing equations. The continuum is represented by particle, where the convection and deformation are simulated by particle’s movement.

In this study, the influence of density ratio and viscosity ratio to the Rayleigh Taylor Instability (RTI) phenomena were analysed using particle based simulation.

2. Finite Volume Particle method

In this study, the Rayleigh-Taylor Instability is analysed using a fully Lagrangian approach, based on the FVP method, for incompressible flows. In FVP methods, particle variables such as velocity and pressure are obtained by solving governing equation; the Navier-Stokes equation and the continuity equation.
\[
\rho \frac{D\vec{u}}{Dt} = -\nabla p + \nabla(\mu \nabla \cdot \vec{u}) + \vec{f}_e + \vec{f}_s \tag{1}
\]

\[
\nabla \cdot \vec{u} = 0 \tag{2}
\]

where \( \rho \) is the density, \( \vec{u} \) is the velocity, \( t \) is the time, \( p \) is the pressure, \( \mu \) is the dynamic viscosity. The third term \( \vec{f}_e \) in the right-hand side of Eq. (1) represents other forces per unit volume, such as surface tension and gravity.

In the FVP method, the numerical particles are assumed to occupy a certain volume, where the control volume of one moving particle is a circle in 2D simulations.

\[
S = 2\pi R, V = \pi R^2 = (\Delta l)^2 \tag{3}
\]

where \( S, V, R, \) and \( \Delta l \) are the particle surface area, the particle control volume, the radius of particle control volume and the initial particle distance, respectively. According to Gauss’s law, the gradient and Laplacian operators acting on arbitrary scalar function \( \phi \) in the FVP method can be approximated as

\[
(\nabla \phi)_i = \frac{1}{V} \int_S \phi n_i dS = \frac{1}{V} \sum_{j \neq i} \Phi_{zu} \cdot \vec{n}_{ij} \cdot \Delta S_{ij} \tag{4}
\]

\[
(\nabla^2 \phi)_i = \frac{1}{V} \int_S \nabla \phi \cdot \vec{n} dS_i = \frac{1}{V} \sum_{j \neq i} \left( \frac{\phi_j - \phi_i}{|\vec{r}_{ij}|} \right) \cdot \Delta S_{ij} \tag{5}
\]

In the present study, the natural convection flows, which is driven by the buoyancy force, was considered by the Boussinesq approximation for the gravity term involved in \( f \) of Eq. (1).

\[
\vec{f}_g, \beta, T_0, \text{ and } c_p \text{ are the gravity, the volumetric thermal expansion coefficient, the reference temperature, and coefficient correction, respectively.}
\]

3. Simulation conditions

Figure 1 shows the geometrical simulation of Rayleigh-Taylor Instability using FVP method. We performed 2D calculations by assumed adiabatic geometrical boundaries, neglecting the effect of heat transfer from the system to surroundings. The initial particle distance is setted at 1.0 mm. The simulation were conducted for 1.5 s with the time step size 1.0 x 10^{-4} s.
To estimate essential effect of density and viscosity ratio to the Rayleigh Taylor Instability (RTI) phenomena, in the present study, we varies mass density ratio and viscosity ratio as shown in table 1.

**Table 1.** Parametric simulations

|       | Mass density $^3\text{kg/m}^3$ | Surface tension (N/m) | Viscosity (m $^2$/s) |
|-------|---------------------------------|------------------------|-----------------------|
| A     | $0.90 \times 10^3 - 0.98 \times 10^3$ | 0.0527                | $1.0 \times 10^{-6} - 96.0 \times 10^{-6}$ |
| B     | $1.0 \times 10^3$                | 0.0728                | $1.0 \times 10^{-6}$  |

4. Results and Discussion

Figure 2 shows conditions where the Rayleigh-Taylor instability could be observed. There is no effect varying viscosity ratio to the Rayleigh-Taylor instability. Only density ratio which has effect to the phenomena. The Rayleigh-Taylor Instability (RTI) can be observed clearly in specific density ratio $0.91 - 0.97$. 

![Figure 2. Simulation result (1)](image-url)
The effect of density ratio can be seen in Table 2. For viscosity ratio 60, the Rayleigh-Taylor instability occurs clearly at density ratio 0.97. Smaller density ratio shows smaller amplitude of fingering instability. In other hand, the Rayleigh-Taylor phenomena could not be observed for higher density ratio (0.98).

**Table 2. Effect of density ratio**

| Viscosity ratio | Density ratio |
|-----------------|---------------|
| 60              | 0.98, 0.97, 0.91 |

Figure 3. Show the correlation of peak amplitude time and density ratio. This correlation can be approximated by polynomial regression.

**Figure 3. Simulation result (2)**

5. Conclusions
The Rayleigh-Taylor Instability (RTI) phenomena can be observed clearly in specific density ratio 0.91-0.97. In smaller density ratio (than 0.91), the buoyancy force is too large so the RTI almost cannot be observed. In larger density ratio (than 0.98), the buoyancy force is too small so the RTI also cannot be observed.

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