A Tale of Two Mergers: Searching for Strangeness in Compact Stars

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We contrast the evolution of gravitational binary mergers for the two cases of a black hole and a normal neutron star, and a black hole and a self-bound strange quark matter star. In both cases inspiral continues until the Roche limit is reached, at which point it is expected that stable mass transfer to the black hole ensues. Whereas a neutron star would then outspiral, the strange quark matter star barely moves out at all. Eventually, the strange quark star loses all its mass to the black hole, as compared to the neutron star whose outspiral continues until stable mass transfer terminates and inspiral resumes. These scenarios result in distinctly different gravitational wave signatures.

I. INTRODUCTION

Several proposals concerning the physical state and the internal constitution of matter at supranuclear densities have been put forth (see [1, 2] for recent accounts). Figure 1 shows many exciting possibilities for the composition of compact stars including (1) strangeness-bearing matter in the form of hyperons, kaons, or quarks, (2) Bose (pion or kaon) condensed matter, and (3) so-called self-bound strange quark matter (SQM). Fermions, whether they are in the form of baryons or deconfined quarks, are expected to additionally exhibit superfluidity and/or superconductivity. In the case of quarks, this could occur either as two-flavor superconductivity (2SC) or in the Color-Flavor-Locked (CFL) phase in which all three flavors (up, down, and strange) participate.

The possibility of such exotic phases brings attendant changes to the predictions of maximum masses and radii, since the presence of multiple components or new phases of matter generally lessens the pressure for a given energy density. The fact that compact objects are the only objects in which such phases could occur underscores the importance of precise determinations of basic observables:

(i) masses and radii. Simply put, a precise determination of the mass and radius of the same

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neutron star would not only be a first achievement for observational astronomy, but would also hold the promise of delineating Quantum Chromo-Dynamics (QCD) in regimes of high baryon density heretofore not possible.

(ii) surface temperatures versus age. Detections of photons from cooling neutron stars could constrain the pairing gaps of baryons and the star’s mass. However, establishing the sizes of quark gaps or the presence of Goldstone bosons in the CFL phase of quarks in a hybrid star will be difficult because the layers of normal matter surrounding the quark core continues to cool through significantly more rapid processes. In contrast, the mean energy of emitted photons from the bare surface of a strange quark star would be significantly larger than that from a normal cooling neutron star ($30 < E/\text{keV} < 500$ versus $0.1 < E/\text{keV} < 2.5$). Due to its distinctive spectrum and time evolution, such an observation would constitute an unmistakable detection of a strange quark star and shed light on color superconductivity at “stellar” densities.

(iii) anomalous behavior in the spin-down rates of ms radio pulsars. Possible hysteresis (reversal) in the normal spindown rate of pulsars would signal the appearance of a mixed-phase of matter containing normal hadronic matter with more compressible Bose condensed or quark matter.

(iv) neutrino luminosities from future galactic core collapse supernovae. The main new feature that
has emerged from studies of neutron stars at birth is the possible metastability and subsequent collapse to a black hole of a proto-neutron star containing quark matter, or other types of matter including hyperons or a Bose condensate, which could be observable in the neutrino signal.

Examples of ongoing and planned observations of solar mass compact objects that could shed light on QCD at high baryon density include:

(i) multi-wavelength photon observations with the HST, Chandra, XMM, Integral, etc.,
(ii) spectral and temporal studies of supernova neutrinos with the SuperK, SNO, UNO, etc., and
(iii) gravity wave detections from coalescing compact object binaries though LIGO, VIRGO, etc.

In this work, we contrast the evolution of binary star mergers for two distinct cases:

(1) A black hole (BH) and a normal star. For the discussion at hand, this refers to a star with a surface of normal matter in which the pressure vanishes at vanishing baryon density. The interior of the star, however, may contain any or a combination of the many exotica depicted in Fig. 1

(2) A BH and a self-bound star which is exemplified by Witten’s Strange-Quark Matter (SQM) star; see [9] for a review. Such a star has a bare quark matter surface in which the pressure vanishes at a finite but supra-nuclear baryon density [24].

Prototypes of the mass versus central baryon density ($n_c$) and mass versus radius for these cases are shown in Fig. 2. Quantitative variations from these generic behaviors can be caused by uncertainties in the underlying strong interaction models (see the compendium of results in Fig. 2 of Ref. [1]). Qualitative differences in the outcomes of mergers with a black hole emerge, however, because of the gross differences in the mass-radius diagram.

**FIG. 2:** (a) Mass versus central density, and (b) mass versus radius, for normal and self-bound stars.
A normal star and a self-bound star represent two quite different possibilities (see Fig. 3) for the quantity

\[
\alpha \equiv \frac{d \ln R}{d \ln M} \begin{cases} 
\leq 0 & \text{for a normal neutron star (NS)} \\
\geq 0 & \text{for a self-bound SQM star}
\end{cases},
\]

(1) where \(M\) and \(R\) are the star’s mass and radius, respectively. For small to moderate mass self-bound stars, \(R \propto M^{1/3}\) so that \(\alpha \approx 1/3\); only for configurations approaching the maximum mass does \(\alpha\) turn negative.

![Comparison of \(\alpha\) and radii \(R\) for the normal star (NS) and self-bound strange quark matter (SQM) star as functions of mass.](image)

FIG. 3: Comparisons of \(\alpha\) and radii \(R\) for the normal star (NS) and self-bound strange quark matter (SQM) star as functions of mass.

Our objective is to explore the astrophysical consequences of these distinctive behaviors in \(R\) versus \(M\) as they affect mergers with a black hole. Note that \(\alpha\) is intimately connected with the dense matter equation of state (EOS), since there exists a one-to-one correspondence between \(R(M)\) and \(P(n_B)\), where \(P\) is the pressure and \(n_B\) is the baryon density. Gravitational mergers in which a compact star loses its mass (either to a companion star or to an accretion disk) during evolution is one of the rare examples in which the \(R\) versus \(M\) (or equivalently, \(P\) versus \(n_B\)) relationship of the same star is sampled. Although we focus here on the coalescence of a compact star with a BH, the theoretical formalism and our principal findings apply also to mergers in which both objects are compact stars.
II. THE MERGER OF A COMPACT STAR WITH A LOW-MASS BLACK HOLE

The general problem of the origin and evolution of systems containing a neutron star and a black hole was first detailed by Lattimer & Schramm [12, 13]. Compact binaries form naturally as the result of evolution of massive stellar binaries (e.g., [14]). The estimated lower mass limit for supernovae (which produce neutron stars or black holes) is approximately 8 M⊙. Observationally, the number of binaries formed within a given logarithmic separation is approximately constant, but the relative mass distributions are uncertain. There is some indication that the distribution in binary mass ratios might also be flat. Most progenitor systems do not survive the first, more massive star becoming a supernova. In the absence of a “kick” velocity from the explosion, it is easily found that the loss of more than half of the mass from the system will unbind it. However, the fact that pulsars are observed to have mean velocities in excess of a few hundred km/s implies that neutron stars are usually produced with large kicks. In the cases that the kick, which is thought to be randomly directed, opposes the star’s orbital velocity, the chances that the post-supernova binary remains intact increases. In addition, the orbital separation in a surviving binary will be reduced significantly. Subsequent evolution then progresses to the supernova explosion of the companion. Those systems that survive the second explosion should both have greatly reduced separations and orbits with high eccentricity.

Gravitational radiation causes the binary’s orbit to decay [15], such that a system with masses M₁ and M₂ with initial semimajor axes a satisfying

\[ a < 2.8[M₁M₂(M₁ + M₂)/M₁^3]^{1/4}R_☉, \]  

will fully decay within the age of the Universe (\( \sim 10^{10} \) yr). This limit is for circular orbits; those with highly eccentric orbits will decay much faster [16]. Ref. [12] argued that (1) mergers of neutron stars and black holes coupled with the subsequent ejection of a few percent of the neutron star’s mass, could easily account for all the r-process nuclei in the cosmos, and (2) compact object binary mergers could be associated with gamma-ray bursts (see also [17]).

When the less massive inspiralling compact star reaches its Roche limit (see Fig. 4), mass can be stripped from it. The radius of the compact object will quickly adjust to its new mass. If the radius increases more quickly than the Roche limiting radius, mass transfer to the BH will be stable, and the inspiral will be halted due to angular momentum conservation. The classical Roche limit is based upon an incompressible fluid of density \( ρ \) and mass M₂ in orbit about a mass M₁.
In Newtonian gravity, this limit is

\[ R_{\text{Roche,Newt}} = \left( \frac{M_1}{0.0901 \pi \rho} \right)^{1/3} = 19.2 \left( \frac{M_1}{M_\odot \rho_{15}} \right)^{1/3} \text{ km}, \]  

(3)

where \( \rho_{15} = \rho/10^{15} \text{ g cm}^{-3} \). Using general relativity, Fishbone \(^8\) found that the number \(0.0901\) in Eq. (3) becomes \(0.0664\), even for rotating BHs. In geometrized units, \( R_{\text{Roche}}/M_1 = 13(14.4)(M_1^2 \rho_{15}/M_\odot^2)^{-1/3} \), where the numerical coefficient refers to the Newtonian (GR) case. In other words, if the neutron star’s mean density is \( \rho_{15} = 1 \), the Roche limit is encountered beyond the last stable orbit (\( R = 6M_1 \) for a non-rotating BH) if \( M_1 < 5.9 \text{ M}_\odot \), leading to mass overflow and mass transfer. And, as now discussed, the mass transfer may proceed stably for some considerable time. This would lengthen the lifetime the BH would accrete matter from its companion, which, in itself, could be an observational signature.

### III. EVOLUTION OF MERGERS

The final evolution of a compact binary is now discussed (see Fig. for a schematic illustration). Define \( q = m_{cs}/M_{BH}, \mu = m_{cs}M_{BH}/M, \) and \( M = M_{BH} + m_{cs} \), where \( m_{cs} \) and \( M_{BH} \) are the compact star (normal star or self-bound SQM star) and black hole masses, respectively. The
orbital angular momentum is

\[ J^2 = G \mu^2 Ma = GM^3 a q^2 / (1 + q)^4. \]  \hspace{1cm} (4)

We can employ Paczyński’s \cite{19} formula for the Roche radius of the secondary:

\[ R_\ell/a = 0.46 [q / (1 + q)]^{1/3}, \]  \hspace{1cm} (5)

or a better fit by Eggleton \cite{20}:

\[ R_\ell/a = 0.49 \{0.6 + q^{-2/3} \ln(1 + q^{1/3})\}^{-1}. \]  \hspace{1cm} (6)

The orbital separation \( a \) during mass transfer is obtained by setting \( R_\ell = R \), the compact star

\[ \begin{align*}
M &= m_{cs} + M_{BH} \\
\mu &= \frac{m_{cs} M_{BH}}{M} \\
L_{GW} &= \frac{32}{5} \frac{G \mu^2 M^3}{c^5 a^{7/2}}
\end{align*} \]

FIG. 5: A schematic illustration of a compact star merger with a black hole (Courtesy: M.W. Carmell).

radius. For stable mass transfer, the star’s radius has to increase more quickly than the Roche radius as mass is transferred \cite{25}. Thus, we must have, using Paczyński’s formula,

\[ \frac{d \ln R}{d \ln m_{cs}} \equiv \alpha \geq \frac{d \ln R_\ell}{d \ln m_{cs}} = \frac{d \ln a}{d \ln m_{cs}} + \frac{1}{3} \]  \hspace{1cm} (7)

for stable mass transfer. If the mass transfer is conservative \cite{26}, \( \dot{J} = \dot{J}_{GW} \), where

\[ \dot{J}_{GW} = -\frac{32}{5} \frac{G^{7/2} \mu^2 M^{5/2}}{c^5 a^{7/2}} = -\frac{32}{5} \frac{G^{7/2} \mu^2 M^{5/2}}{c^5 a^{7/2}} \frac{q^2 M^{9/2}}{(1 + q)^4 a^{7/2}} \]  \hspace{1cm} (8)

and

\[ \frac{\dot{J}}{J} = \frac{\dot{a}}{2a} + \frac{\dot{q}(1 - q)}{q(1 + q)}, \]  \hspace{1cm} (9)
where the dots on various symbols denote time derivatives. This leads to

\[ \dot{q} \left( \frac{\alpha}{2} + \frac{5}{6} - q \right) \geq -\frac{32}{5} \frac{G^{\frac{3}{2}}}{c^5} \frac{q^2 M^3}{(1 + q)a^4}. \]  

(10)

Since \( m_{cs} < M_{BH} \), \( \dot{q} \leq 0 \), and the condition for stable mass transfer is simply

\[ q \leq \frac{5}{6} + \frac{\alpha}{2}. \]  

(11)

This condition is achieved in all normal neutron star - BH or self-bound star - BH binaries. In both cases, the luminosities of gravity wave emission can be estimated using

\[ L_{GW} = \frac{32}{5} \frac{G^{\frac{3}{2}} M^3 \mu^2}{a^5}. \]  

(12)

IV. RESULTS AND DISCUSSION

In addition to Eq. (11), a number of other conditions must hold for stable mass transfer to occur (cf. [21], and references therein). The discussion in [21] is relevant to the stellar merger of a self-bound star as well. We now compare the mergers of normal and self-bound stars with a BH. For definiteness, we choose \( m_{ns} = m_{SQM} = 1.5M_\odot \) and \( M_{BH} = 3.5M_\odot \). Results for other choices are straightforward to obtain. Fig. 6 (left panel) shows the time development of the orbital separation \( a \) and the compact star’s mass and radius during the stable mass transfer (outspiral) phases. The time evolutions during stable mass transfer are obtained from Eq. (10), using \( \dot{m}_{cs} = \dot{q} M/(1 + q)^2 \).

In the case of SQM, the inspiral continues to smaller separations since \( R_{SQM} < R_{NS} \). As a consequence, mass transfer is accelerated and the duration of the stable mass transfer outspiral phase is shortened considerably. In the SQM case the star barely spirals out since \( 1/3 - \alpha \simeq 0 \) (see Eq. 7). The star eventually loses all its mass. In contrast, for a neutron star \( 1/3 - \alpha > 0 \) and the star outspirals significantly, until \( \alpha \simeq -5/3 \) when stable mass transfer terminates (Eq. 11). This occurs above the minimum stable neutron star mass, \( \sim 0.08 \ M_\odot \), so the remnant mini-neutron star will resume spiralling in, concluding in a final merger. In either case, if stable mass transfer occurs, it proceeds for much longer than an orbital period, perhaps up to a few tenths of a second. The merger of neutron star and self-bound star with a BH will significantly differ in their gravity wave signatures (shown in the right panel of Fig. 6). The self-bound star case will have a shorter timescale of emission, with probably higher peak luminosity, and will lack a final increase due to resumption of inspiral in the NS case.

An additional contrast between mergers involving only NSs or only SQM stars (but not BHs) is that the condition for stable mass transfer might not be generally achievable for binary NSs.
FIG. 6: **Left panel**: The evolution of a 1.5 M_⊙ SQM star (solid line) is compared with that of a 1.5 M_⊙ normal neutron star (dashed line) with a 3.5 M_⊙ black hole during merger. The compact stars’ masses, radii, and outspiral orbital separations during stable mass transfer are shown. (For simplicity, the inspiral evolutions are not indicated.) **Right panel**: Luminosities of gravity wave emission for the two cases.

Note that moderate mass NSs have \( \alpha \approx 0 \), so Eq. (11) requires \( q < 5/6 \). Had we used the more exact formula of Eggleton, Eq. (6), we would have found an even greater restriction \( q < 0.78 \). For example, the binary pulsar PSR1913+16 has \( q = 0.96 \). In contrast, for small to moderate mass self-bound SQM stars \( \alpha \approx 1/3 \), so that stable mass transfer can occur for \( q < 1 \), which is its entire domain!

The evolution of self-bound SQM star - black hole merger including the effects of non-conservative mass transfer, tidal synchronization, the presence of an accretion disk, etc., together with extensions to include further effects of general relativity, will be reported separately.

In conclusion, compact star mergers offer a tantalizing possibility for the detection via gravitational waves of strange quark matter in the form of self-bound stars.

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[23] Not to be confused with the Canadian flick “Exotica.”
[24] In the context of the MIT bag model with first order corrections due to gluon exchange, the baryon density at which pressure vanishes is given by $n_b(P = 0) = (4B/3\pi^{2/3})^{3/4}(1 - 2\alpha_c/\pi)^{1/4}$, where $B$ is the bag constant and $\alpha_c = g_c^2/(4\pi)$ is the quark-gluon coupling constant. This density is not significantly affected by the finite strange quark mass or by pairing gaps in the CFL or 2SC phases.
[25] Mass extraction from a self-bound SQM star has been estimated by Madsen by requiring that the gravitational tidal force exceeds that due to surface tension, i.e., $G m_{cs} M_{BH} R/a^3 > \sigma R$, where $\sigma$ is the surface tension of SQM. This leads to $A > \sigma a^3/(GM_{BH}) = 4 \times 10^{38} (\sigma/20 \text{ MeV fm}^{-2}) (a/30 \text{ km})^3 (M_\odot/M_{BH})$, upon using $m_{cs} = A m_u$, where $m_u$ is the atomic mass.
[26] Some aspects of non-conservative mass transfer are discussed in and references therein.