\[ B \to X_s + \text{missing energy} \] in models with large extra dimensions

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Abstract

We study the neutral current flavour changing rare decay mode \( B \to X_s + \text{missing energy} \) within the framework of theories with large extra spatial dimensions. The corresponding Standard Model signature is \( B \to X_s + \nu \bar{\nu} \). But in theories with large extra dimensions, it is possible to have scalars and gravitons in the final state making it quite distinct from any other scenario where there are no gravitons and the scalars are far too heavier than the B-meson to be present as external particles. We give an estimate of the branching ratio for such processes for different values of the number of extra dimensions and scale of the effective theory. The predicted branching ratios can be comparable with the SM rate for a restrictive choice of the parameters.

Keywords: Extra dimensions, Rare B decay

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1 Introduction

The investigations of flavour changing neutral current (FCNC) transitions of b-quark offer excellent opportunities to test the basic structure of the underlying theory. These processes are quite sensitive to QCD and possible long distance corrections as well as contributions from new particles in the loops. Consequently, such processes become useful tools for testing new physics as well. The measurement of \( B \to X_s \gamma \) (and subsequently the exclusive channel \( B \to K^{*} \gamma \)) by CLEO \[ \Pi \] has been used to stringently constrain the parameter space of supersymmetric (SUSY) and various other theories (see for example \[ \Pi \] and references therein). The same is true for any other FCNC process. In particular, the quark level transition \( b \to s \nu \bar{\nu} \) can turn out to be a very successful place for putting strong and meaningful bounds on the underlying theory. The Standard Model (SM) experimental signature in this case is the observation of the decay process \( B \to X_s + \text{missing energy} \) in the form of neutrinos. In any extension of the SM, decay of a b-quark to an s-quark and neutral, ultra light

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particle(s) can mimic the SM process.

It is well known that SM is plagued with the hierarchy problem and many possible solutions have been proposed in the past. However, the idea that the fundamental scale of gravitational interaction being quite distinct from the Planck scale \( \sim 10^{19} \text{ GeV} \) and possibly as low as \( \mathcal{O}(\text{TeV}) \) has attracted a lot of attention. In the simplest version, as put forward by Arkani-Hamed et al. (referred to as ADD model/scenario from now on) \cite{3}, the basic idea is the existence of \( n \) spatially compact large dimensions. The spacetime is a direct product of the four dimensional Minkowski space and the compact space spanned by the \( n \) spatial extra dimensions. The SM fields are all restricted to a 3-brane while gravity is free to propagate in the bulk. Seen from a four dimensional point of view, it simply means that apart from the usual massless graviton mode, there is a tower of Kaluza-Klein (KK) excitations from the gravity sector due to compactification. Also, if \( M_\ast \) and \( M_{P Pl} \) denote the effective scale of gravity and the four dimensional Planck scale respectively and if \( R \) is the radius of compactification of the extra dimensions (assuming the same radius for all the \( n \) dimensions), then it turns out that they are related as follows

\[
M_{P Pl}^2 \sim R^n M_\ast^{n+2} \quad (1)
\]

Therefore, for large enough values of \( R \), the effective scale \( M_\ast \) can be as low as \( \mathcal{O}(\text{TeV}) \) and still be consistent with Eq. (1). Although the individual KK modes couple to the SM fields on the 3-brane by the ordinary gravitational strength but the presence of a large number of them makes the effective coupling appear to be of the order of \( \text{TeV}^{-1} \) rather than Planck scale inverse. This means that gravity starts to become a strong force at TeV scales in sharp contrast to the situation in any ordinary theory of gravity.

In the present study, we investigate the inclusive decay \( B \to X_s + \text{missing energy} \) in the context of the ADD scenario. In the SM or most of its extensions, the missing energy is in the form of the neutrinos that escape without being detected and it is only new contributions to the relevant Wilson coefficients that are induced by interactions beyond SM. However, there is a sharp contrast in the case at hand. In the present scenario, there can be very light gravitons and associated scalars (the dilatons) that can be present in the final state. The situation is clearly distinct from SM or its usual extensions as in all such cases there are no spin-2 gravitons involved and the scalars, both neutral and charged, are far too massive compared to the b-quark to be present in the external legs. But this is possible in theories with large compact extra dimensions. It is precisely this advantageous aspect that we would like to exploit in the present study and see whether we get any meaningful results.

2 \( b \to s + KK \) modes

The effective Hamiltonian for \( b \to s \) transition in the SM is \cite{4}

\[
\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i C_i(\mu) O_i(\mu),
\]

(2)
with

\[ O_1 = (\bar{s}_i c_j)_{V-A}(\bar{c}_j b_i)_{V-A}, \]
\[ O_2 = (\bar{s}_i c_i)_{V-A}(\bar{c}_j b_j)_{V-A}, \]
\[ O_3 = (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A}, \]
\[ O_4 = (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A}, \]
\[ O_5 = (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A}, \]
\[ O_6 = (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A}, \]
\[ O_7 = \frac{e}{16\pi^2} \bar{s}_i \sigma^{\mu\nu} (m_s P_L + m_b P_R) b_i F_{\mu\nu}, \]
and
\[ O_8 = \frac{g}{16\pi^2} \bar{s}_i \sigma^{\mu\nu} (m_s P_L + m_b P_R) T_{ij}^a b_j G_{\mu\nu}^a. \]

Apart from these there are semi-leptonic operators as well

\[ O(ee)_V = (\bar{s}_i b_i)_{V-A}(\bar{e}e)_V \]
\[ O(ee)_A = (\bar{s}_i b_i)_{V-A}(\bar{e}e)_A \]
and finally
\[ O(\nu\bar{\nu}) = (\bar{s}_i b_i)_{V-A}(\bar{\nu}\nu)_{V-A} \]

It is this last operator that is responsible for the relevant decay channel in SM. In order to calculate any process beyond these known operators, one will have to either write down all possible new operators respecting the symmetries of the low energy theory and then indirectly fix the associated coefficients or explicitly make the calculation. We follow the latter route.

The coupling of the gravitons and the dilatons can be obtained from the low energy effective action \[5, 6\]. As expected, the gravitons couple to the energy-momentum tensor of the SM fields and the dilaton couples to the trace of the energy-momentum tensor. It is then straightforward to get the Feynman rules and compute the individual contributions. We consider the dilaton/scalar-KK emission first and comment on the analogous graviton emission later. The diagrams contributing to the dilaton emission process are shown in Fig. 1. It may be important to mention here that: (i) there is no momentum dependent piece in the trace of energy-momentum tensor for the fermions and (ii) the dilaton-gauge boson-fermion-fermion vertex does not exist. However, both these are shown to be present in \[5\]. The fact that it is indeed so is not hard to see. The trace of the energy-momentum tensor for the fermionic plus gauge bosonic parts (including interaction term) is nothing but

\[ T^\mu_\mu = -m_f \bar{f} f + m^2 V A^2 \]  

(5)
In obtaining this, one needs to consider the interacting Heisenberg field equations rather than the free ones. The use of full interacting equations of motion for the fermion and the gauge boson results only in the mass terms for each of them (similar expression is obtained for the trace of the energy-momentum tensor in [7]) and thus there is no momentum dependence for the fermions involved in the Feynman rules. Also, the gauge boson-fermion-fermion-dilaton vertex will never be present (although a similar vertex for the gravitons will be there as in [5]). Thus, we differ on these points with the rules given in [5] and it may be important to emphasize again that in obtaining the energy-momentum tensor or its trace, nowhere has the on-shell condition been imposed. Therefore, the result Eq. (5) is exact and true in general. Similar remarks and results were quoted [8] in the context of a loop calculation in the Randall-Sundrum scenario, where the scalar in the theory (called the radion) couples, at the first order, with the trace of the energy-momentum tensor of the SM fields and is very similar to the dilaton in the ADD scenario as far as the couplings with the SM fields are concerned. There are less number of diagrams contributing to the dilaton emission process as compared to the graviton emission process. Also, in the limit of neglecting the strange quark mass, the diagram with dilaton being emitted from the s-quark line also vanishes.

Clearly, all the diagrams are divergent and there is no underlying symmetry like the one present in SM that ensures cancellations between the individual diagrams to render a finite result. By naive power counting, one finds that there are hard divergences (see Appendix) which are quadratic, quartic and possibly even higher in the case of graviton emission. It now brings the question of handling such divergences in the context of effective field theories. It has been very strongly advocated [9] that in any sensible effective field theory all the quadratic and higher divergences are cancelled by counter-terms arising out of the complete underlying high-energy theory. At the one loop level, it is thus the logarithmic dependence on the cut-off only that can be extracted from the low energy effective theory. Thus, it makes more sense to keep only the logarithmic pieces as far as the one loop calculation within an effective theory is concerned. We adhere to this approach in our calculations and thus use dimensional regularization throughout and at the end replace $\frac{1}{\epsilon}$ by $\ln \left( \frac{\Lambda^2}{m_W^2} \right)$ with the identification $\Lambda = M_*.$

As mentioned earlier, it is expected that the presence of the scalars and gravitons in the external legs makes the situation more interesting as new operators are introduced. The operators are of the form (denoting the dilaton and the graviton fields by $\Phi$ and $h_{\mu\nu}$ respectively)

$$O(\text{dilaton}) \sim (\bar{s}_i(1 \pm \gamma_5)b_i)\Phi$$

and

$$O(\text{graviton}) \sim (\bar{s}_i\gamma_\mu p_{\nu}(1 \pm \gamma_5)b_i)h^{\mu\nu}$$

(see Appendix for details)

The invariant matrix element for the quark level process $b \rightarrow s\Phi^{(n)}$ for a dilaton of mass $m_n$
is (the individual; contributions are listed in Appendix)

\[ \mathcal{M}(b \to s\Phi) = \left( \frac{i}{16\pi^2} \right) \frac{G_F \omega \kappa}{\sqrt{2}} \left( \frac{1}{\epsilon} \right) \left( \sum_i \xi_i m_i^2 \right) \left( \frac{m_b}{3} \right) \]

(8)

\[
\left[ 18 \left( \frac{m_s^2}{m_b^2} \right) - 1 \right] \bar{s} \left( 1 + \gamma_5 \right) b
\]

where \( \xi_i \) is the CKM factor, \( m_i \) is the mass of the up-type quark in the loop. We have neglected \( \mathcal{O}(\frac{m^2_s}{m_W}) \) and higher terms in obtaining this expression. Using this matrix amplitude it is now trivial to compute the decay rate for the emission of a single dilaton of mass \( m_n \). For the inclusive process, it suffices to use the quark level amplitude to get the decay rate. One should sum over the tower of these dilatons till the b-quark mass scale. Following the summing techniques illustrated in [5], we get the following expression for the decay rate

\[ \Gamma(B \to X_s\Phi) = \left( \frac{1}{16\pi^2} \right)^2 G_F^2 \ln \left( \frac{M_s^2}{m_W^2} \right) \left( \sum_i \xi_i m_i^2 \right)^2 \left( \frac{4m_b}{27} \right) \]

(9)

\[
\times \left( \frac{1}{n+2} \right) \left[ \frac{1}{n} + \frac{1}{n+4} - \frac{2}{n+2} \right] \left( \frac{m_b}{M_s} \right)^{n+2}
\]

where we have made the replacement

\[ \frac{1}{\epsilon} \to \ln \left( \frac{M_s^2}{m_W^2} \right) \]

The expression for the graviton emission rate will look similar in its final form (see Appendix) but the rate itself is expected to be higher because of two reasons. Firstly, there are two extra diagrams (with the graviton being attached to either of the fermion-fermion-gauge boson vertex) and secondly, the graviton coupling does not have the \( \omega \) factor suppression present in the dilaton coupling. Thus, the rate is expected to be

\[ \Gamma(B \to X_sG) \sim \mathcal{F} \Gamma(B \to X_s\Phi) \]

(10)

where \( \mathcal{F} \) is a numerical factor expected to be of the order 10 or higher. However, it is instructive to go through the complete steps for the graviton emission process as well. The procedure is completely in analogy with the dilaton emission. The total rate is obtained by summing over various massive modes in the final state. Using the expression for the rate corresponding to a single graviton of mass \( m_n \), the summation essentially involves integrating over the tower of the massive modes till the b-mass scale. When this is done, it is found that there is a divergent behaviour for small \( n \), the number of extra dimensions. This is like the soft emission rates diverging for some particular values of \( n \) and arises due to the structure of the integrand. We thus quote the values only for \( n \geq 4 \) (in particular \( n = 4, 6 \)). In fact, the limit \( n \to 4 \) has also to be taken with care and in some sense in the spirit of analytic continuation or something similar.
3 Results

The total contribution to the process $B \rightarrow X_s + KK$ modes is

$$\Gamma(B \rightarrow X_s \Phi) + \Gamma(B \rightarrow X_s G)$$

Below we quote the branching fraction for the dilaton mode for different values of $n$ and $M_s$. The branching ratio for the graviton mode for $n = 4$ and $n = 6$ are as follows:

Table 1: $Br(B \rightarrow X_s + \Phi)$

| $n$ | $M_s$ (TeV) | Branching ratio |
|-----|-------------|-----------------|
| 2   | 1           | $1.28 \times 10^{-3}$ |
| 3   | 1           | $2.25 \times 10^{-6}$ |
| 4   | 1           | $4.93 \times 10^{-9}$ |
| 2   | 5           | $5.50 \times 10^{-6}$ |
| 3   | 5           | $1.93 \times 10^{-9}$ |
| 4   | 5           | $8.45 \times 10^{-13}$ |
| 2   | 10          | $4.69 \times 10^{-7}$ |
| 3   | 10          | $8.23 \times 10^{-11}$ |
| 4   | 10          | $1.80 \times 10^{-14}$ |
| 2   | 50          | $1.33 \times 10^{-9}$ |
| 3   | 50          | $4.68 \times 10^{-14}$ |
| 4   | 50          | $2.05 \times 10^{-18}$ |

Table 2: $Br(B \rightarrow X_s + G)$

| $n$ | $M_s$ (TeV) | Branching ratio |
|-----|-------------|-----------------|
| 4   | 1           | $1.02 \times 10^{-4}$ |
| 6   | 1           | $3.2025 \times 10^{-11}$ |
| 4   | 5           | $1.72 \times 10^{-8}$ |
| 6   | 5           | $2.2 \times 10^{-16}$ |
| 4   | 10          | $3.67 \times 10^{-10}$ |
| 6   | 10          | $1.17 \times 10^{-18}$ |
| 4   | 50          | $4.17 \times 10^{-14}$ |
| 6   | 50          | $5.33 \times 10^{-24}$ |

These numbers are to be compared with the SM expectation, $Br(b \rightarrow s \nu \bar{\nu}) = 5 \times 10^{-5}$ and the experimental limits on the same, $Br(b \rightarrow s \nu \bar{\nu})_{exp} < 6.2 \times 10^{-4}$ at 90% confidence level [10]. From the Table 1 it is clear that for $n = 2$ and $M_s = 1$ TeV, the contribution far exceeds the SM prediction and the experimental limits and should have been observed.
Moreover, one encounters some divergent results for the graviton emission process for smaller values of $n$ which may be looked upon as arising due to something similar to gravi-strahlung typical to these smaller values of $n$. We therefore cannot say anything very satisfactorily for these values unless a more careful and detailed analysis is done. Thus we restrict ourselves to dilaton rates for these values of $n$ and don’t consider graviton rates for those values.

Also, evident is the fact that for higher values of $n$ and/or $M_*$, the branching fraction is smaller. The constraints obtained from the collider experiments [13] allow even smaller values of $M_*$ than quoted, thus allowing for the possibility of some more enhancement in the branching ratio predictions. However, strong constraints from the supernova SN1987A observations [11] and cosmology [12] rule out such choices of $n$ and $M_*$. For $n = 2$, SN1987A constraints imply $M_* \sim 50$ TeV. Even if one takes care of all the uncertainties entering the supernova calculations, $M_* = 1$ TeV seems to be far from reachable. The other values quoted in the table are more or less allowed by these constraints but seem to be contributing very little. However, for $n = 4$ and $M_*$ greater than a few TeV, the sum of the dilaton and graviton emission rates can be sizeable and may compete with the SM rate. The constraints coming from other astrophysical and cosmological processes [13, 14] like cosmic diffuse gamma ray background, early matter domination etc are even stronger than quoted above and rule out any chances of extra dimensional contribution to the decay process being comparable to the SM rate and thus being observable at the future B-factories. Also, given such strong constraints, the idea of having different radii for different compact dimensions, or atleast partially, does not seem to be of much significance.

It may be useful to mention here that the mode $B \to X_s G$ can contribute to SM process $B \to X_s \nu \bar{\nu}$ via $G \to \nu \bar{\nu}$ and the sum of the SM and the extra contribution will form the complete matrix element. But, in the case of the scalar mode, $B \to X_s \Phi$, there can be no cascade decay of $\Phi$ to $\nu \bar{\nu}$. Hence, until and unless the produced $\Phi$ decays into low mass particles and escapes as missing energy, the energy and angular spectrum of the hadronic junk, $X_s$, will be markedly different from the corresponding distribution observed in the case of $B \to X_s \nu \bar{\nu}$ coming from any theory. The same is true for the graviton emission process provided it does not decay further into $\nu \bar{\nu}$ or any other pair of low mass particles. From the structure of $O(graviton)$ it is evident that the operator corresponding to $b \to sv \bar{\nu}$ will have the form

$$O(\nu \bar{\nu})_{graviton} \sim (\bar{s}_i b_i)_{V \pm A}(\bar{\nu} \nu)_V$$

which is pure vector current for the $\nu \bar{\nu}$ pair, a feature different from the SM again where the intermediate particle responsible is the Z-boson, making the angular distribution different. Also observe that if instead of following the approach advocated in [9], we had retained quadratic and higher divergences, we would have obtained enormously large, intolerable numbers. This again justifies the retaining of only logarithmic terms and in turn justifies our use of dimensional regularization in carrying out the calculations, which otherwise would have to be carried out with a hard ultra-violet cut-off.

In conclusion, we can say that for two extra dimensions and with $M_* \geq$ few TeV (as
marginally allowed by the constraints from astrophysics and cosmology), the desired branching ratio can be sizeable. Other choices of \( n \) seem to be too small to compete with the SM rate. Also, for \( n > 2 \), the radius of compactification, \( R \), becomes too small for its effects to be probed at future mechanical experiments testing the gravitational law at small length scales (eg. see [15] for details of these experiments). Thus, theories with large extra dimensions predict a competing value of the branching ratio for the process \( B \rightarrow X_s + \text{missing energy} \) (in the form of soft dilatons and gravitons) for a very restrictive choice of the parameters. One can thus hope that future observation of the decay mode \( B \rightarrow X_s + \text{missing energy} \), along with the corresponding exclusive modes, will be able to testify theories with large extra dimensions and if not completely ruled out, yield severe constraints on the number of extra dimensions and the effective scale of gravitational interactions in these theories.

Appendix

We outline the steps involved in the evaluation of a typical loop diagram and quote the expressions for individual matrix elements. For the sake of illustration of the calculation, we evaluate the loop diagram (b), where the dilaton is emitted from the W-propagator. We call this matrix element \( \mathcal{M}_{(b)} \). Label the momenta as \( p_b, p_s \) and \( p_\Phi \) corresponding to b-quark, s-quark and the dilaton and denote the loop momentum variable by \( k \). The matrix element can be written as

\[
\mathcal{M}_{(b)} = \left[ \int \tilde{d}k \frac{\eta^{\mu\alpha} - \frac{(k-p_\Phi)^\mu (k-p_\Phi)^\alpha}{m_W^2}}{(k^2 - m_W^2)[(k - p_\Phi)^2 - m_W^2]((p_b - k + m_i)^2 - m_i^2)} \eta^{\nu\beta} - \frac{k^\nu k^\beta}{m_W^2} \right] \times \eta_{\alpha\beta} \bar{s}(p_s)[\gamma_{\mu a}(1 - \gamma_5)(\not{p}_b - k + m_i)\gamma_{\nu}(1 - \gamma_5)]b(p_b)
\]

where

\[
\left[ \right] = \left[ -\sum_i \xi_i \left( \frac{g}{2\sqrt{2}} \right)^2 \omega k m_W^2 \right] \quad \tilde{d}k = \frac{d^4k}{(2\pi)^4}
\]

In the above expression, \( g \) is the weak coupling constant, \( \xi_i \) is the relevant CKM factor, \( m_i \) is the mass of the up-type quark in the loop, \( \kappa^2 = 16\pi G_N \) gives the gravitational coupling in terms of Newton’s constant \( G_N \) and \( \omega \) is a number that depends only on the number of extra dimensions, \( n \), and is given by

\[
\omega^2 = \frac{2}{3(2 + n)}
\]

Clearly, \( \omega < 1 \) and gives a suppression factor. Also, it can be seen very clearly from the above expression for the matrix element that the \( k \)-integral is not logarithmically divergent by power counting but there are higher divergences present. The reason for the presence of such hard divergences is the fact that there is no underlying symmetry, like the one present in the SM, to ensure nice cancellations and render a finite or at the most logarithmically divergent result. This can be attributed to the non-renormalizable nature of these interactions. The calculation has been done in teh unitary gauge and it may seem that these power divergences are arising due to the form of the massive gauge boson propagator. However,
it should be noted that if instead one works in some other convenient gauge, say Feynman
gauge, the massive gauge boson propagators have a more well behaved high energy structure
but there are additional diagrams involving the associated ghosts which essentially are the
badly behaved longitudinal components of these massive gauge bosons and also that their
interaction vertices contain momentum dependence. Therefore, with some effort, various
terms in the calculation in some other gauge can be combined to give the same results. This
is also evident from the Feynman rules in [3] where the vertices have an additional gauge
dependence. But we follow the thumb rule [9] that only logarithmically divergent terms
be retained and thus use dimensional regularization (in the full spirit of an effective theory
calculation) throughout our calculation. The integral can be evaluated using standard proce-
dure of Feynman parameterization, shifting the variables and carrying out the integrations.
We don’t evaluate the finite pieces and only retain divergent terms. After the Feynman
parameterization and shifting of momentum variables, the terms that are not multiplied by
$m_i$ (or its powers) drop out due to the unitarity of the CKM matrix (the GIM mechanism).
Retaining just the various divergent terms in the integral and employing dimensional reg-
ularization (neglecting terms $\mathcal{O}(m_i^2)$ and higher), we arrive at the following expression for
the matrix element corresponding to the diagram (b):

$$
\mathcal{M}_b = \left( \frac{i}{16\pi^2} \right) \left[ 4 \sum_i \xi_i \left( \frac{g}{2\sqrt{2}} \right)^2 \omega \kappa \frac{m_i^2}{m_W^2} \right] \left( \frac{1}{\epsilon} \right) \bar{s}(p_s) \left[ \frac{3}{4} m_s(1 - \gamma_5) + \frac{2}{3} m_b(1 + \gamma_5) \right] b(p_b)
$$

where $m_b$ and $m_s$ are the masses of the b- and s-quark respectively. Also, using the relation
$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$ we can express the result in terms of Fermi-constant $G_F$.

The expressions for other diagrams are as follows:

$$
\mathcal{M}_a = \left( \frac{i}{16\pi^2} \right) \left[ 4 \sum_i \xi_i \left( \frac{g}{2\sqrt{2}} \right)^2 \omega \kappa \frac{m_i^2}{m_W^2} \right] \left( - \frac{3}{4} \right) \left( \frac{1}{\epsilon} \right) \bar{s}(p_s) \left[ m_s(1 - \gamma_5) + m_b(1 + \gamma_5) \right] b(p_b)
$$

$$
\mathcal{M}_c = \left( \frac{i}{16\pi^2} \right) \left[ 4 \sum_i \xi_i \left( \frac{g}{2\sqrt{2}} \right)^2 \omega \kappa \frac{m_i^2}{m_W^2} \right] \left( - \frac{3}{4} \right) \left( \frac{1}{\epsilon} \right) \frac{m_b m_s}{(m_b^2 - m_s^2)} \bar{s}(p_s) \left[ m_b(1 - \gamma_5) - m_s(1 + \gamma_5) \right] b(p_b)
$$

$$
\mathcal{M}_d = \left( \frac{i}{16\pi^2} \right) \left[ 4 \sum_i \xi_i \left( \frac{g}{2\sqrt{2}} \right)^2 \omega \kappa \frac{m_i^2}{m_W^2} \right] \left( \frac{3}{4} \right) \left( \frac{1}{\epsilon} \right) \frac{m_b m_s}{(m_b^2 - m_s^2)} \bar{s}(p_s) \left[ m_b(1 - \gamma_5) + m_s(1 + \gamma_5) \right] b(p_b)
$$
Adding all these contributions we arrive at Eq. (9).

Similar to the dilaton emission process, we take a look at the graviton emission process and the structure that is obtained. We compute the graviton emission process corresponding to diagram (a). Following the same steps as outlined above we get for the matrix element and neglecting $m_s$

$$\mathcal{M}^{\text{graviton}}_{(a)} = \left(\frac{i}{16\pi^2}\right) \left[ -\sum_i \xi_i \left(\frac{g}{2\sqrt{2}}\right)^2 \kappa \frac{m_i^2}{m_W^2} \right] \left(\frac{1}{3}\right) \left(\frac{1}{\epsilon}\right)$$

where $\epsilon^{\alpha\beta}$ is the polarization tensor for the graviton field. Other diagrams give similar contributions. Due to momentum dependent factors in the Feynman rules (when a graviton is involved), the divergence structure is even worse and naive power counting shows that divergences higher than quadratic are present. We don’t work in the cut-off scheme and the use of dimensional regularization again ensures that we retain only the logarithmic divergences. Note that there is no factor of $\omega$ appearing in the above expression and thus the suppression due to this factor is absent in the graviton emission process and depending on the number of extra dimensions etc., the rate for the graviton emission process can be larger than the dilaton emission rate by a factor of 10 or more. This extra multiplicative factor has been identified as $F$ in Eq. (10). The graviton emission rate (for a single graviton of mass $m_n$) is

$$\Gamma(B \to X_s G^{(n)}) = \left(\frac{1}{16\pi^2}\right) \frac{G^2_F}{2} \kappa^2 \left[ \ln \left(\frac{m^2}{m_W^2}\right) \right]^2 \left(\sum_i \xi_i m_i^2\right)^2 \left(\frac{5}{27}\right) \left(\frac{1}{32\pi}\right)\left(1 - \frac{m_b^2}{m^2}\right) \left[ \frac{(m_b^2 - m_n^2)^3(2m_b^2 + 3m_n^2)}{m_n^4} \right]$$

Summing over all the states lying till the b-quark mass scale gives the final expression for graviton emission rate. Clearly, the form for the expression is similar to that of the dilaton case and there are no factors of $\omega$ present. It is important to remark that from the above expressions, it is clearly evident that the operator structure for the dilaton or graviton emission processes is indeed the ones quoted in Eqs. (6)-(7).

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Figure 1: The diagrams contributing to the dilaton emission. We label the diagrams as (a), (b), (c) and (d) moving clock-wise from top left. For the graviton emission there are two more diagrams with the graviton being hooked to either of the two ends of the loop.

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