Universal quantum computation with the orbital angular momentum of a single photon

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Abstract

We prove that a single photon with quantum data encoded in its orbital angular momentum can be manipulated with simple optical elements to provide any desired quantum computation. We will show how to build any quantum unitary operator using beamsplitters, phase shifters, holograms and an extraction gate based on quantum interrogation. The advantages and challenges of these approach are then discussed, in particular the problem of the readout of the results.

Keywords: optical quantum computation, orbital angular momentum of light

1. Optical implementations of quantum computing

Quantum information processing offers a new model of computation and communications. Certain tasks which can only be performed with a limited efficiency in a classical computer can be carried out efficiently in the quantum case [1]. For that reason, there is great interest in finding a suitable physical implementation of a quantum computer.

The DiVincenzo criteria give an important guide to the conditions a physical realization of a quantum computer should meet [2]. A practical quantum computer should be implemented over a system which is scalable with the input size and can be easily initialized and read. Additionally, the system must be able to carry out any desired quantum operation. This means that we must be able to implement any logic function and that the coherence time of the system (the time in which the quantum properties of the system are maintained) is long enough to finish the computation.

Optical implementations seem particularly attractive. Photons can be initialized and read (generated and detected) with relative ease and have possibly the longest coherence time of all the quantum information candidates. They are also very well suited for communications. However, the interaction between different photons is complicated and needs to be mediated by nonlinear processes. One proposed solution has been the use of measurement, like in the Linear Optics Quantum Computer of Knill, Laflamme and Milburn [3].

Most notably, if we only have a single photon, there is always a way to perform any desired quantum computation with linear optics. A linear optics multiport can be described by a scattering matrix that gives the relationship between the amplitudes of the fields at the different input and output modes. For a single photon in $2^n$ input spatial modes, the scattering matrix corresponds to the unitary operator that gives the quantum evolution of a system of $n$ qubits ($n$ quantum information units). Any desired unitary operator of this kind can be implemented using only beamsplitters and phase shifters [4]. Consequently, any desired quantum computation can be implemented on a single photon at the cost of having a number of paths which grows exponentially with the number of qubits of the input [5–7]. In this case, we can obtain universal logic but cannot meet the scalability requisite.

In this paper, we propose a compact variation of single-photon quantum computation with orbital angular momentum encoding. We will show how using only a limited set of optical elements it is possible to provide any unitary operator without needing an exponential number of paths in space. However, there will be a conflict with the readout criterion. We will discuss some possible ways out of this dilemma.
2. OAM encoding

An n-qubit quantum computer needs 2^n orthogonal states to encode all the possible values of n bits as well as superposition states. Instead of the usual option of having n two-level quantum systems, we will base our encoding on the orbital angular momentum (OAM) states of light. For a single photon, we can define orthogonal OAM states |ℓ⟩ that correspond to the photon carrying an OAM of ℓℏ, for an integer ℓ [8]. A generic state will be of the form |ψ⟩ = \sum_{\ell=0}^{2^n-1} α_\ell |ℓ⟩. The integer ℓ encodes the binary sequence \ell = b_{n-1}b_{n-2}⋯b_0. There are methods which can produce these states with current technology [9]. The primary advantage of this encoding is its compactness. The same photon in the same spatial mode can be in a superposition of 2^n OAM modes.

3. Building blocks

In our single-photon computer we will use standard optical elements. The basic devices and operations are:

- **Phase shifters.** A phase shifter will introduce the same phase shift on all input states crossing it so that \(PS_\theta |\ell⟩ = e^{i\theta |\ell⟩}\).

- **Holograms.** Different holograms can be designed to produce an increase or decrease of the OAM index |ℓ⟩ [10]. We will represent this operation as \(H_\ell |\ell⟩ = |\ell + k⟩\), where k is an integer.

- **Beamsplitters.** The different OAM inputs of different ℓ will be orthogonal and will not interfere in a beamsplitter. We will have the same operation as in a single photon for each of the OAM states. As we only have one photon, all the inputs will have an OAM state and the vacuum state |vac⟩. The evolution is given by \(BS_\ell |\ell⟩|\text{vac}⟩ = \cos \theta |\ell⟩|\text{vac}⟩ + \sin \theta |\text{vac}⟩|\ell⟩\) and \(BS_\ell |\text{vac}⟩|\ell⟩ = -\sin \theta |\ell⟩|\text{vac}⟩ + \cos \theta |\text{vac}⟩|\ell⟩\). Figure 1 shows our representation for the beamsplitter.

- **OAM filters.** We will need filters that can absorb all the photons except those of a particular OAM. Although it might be more efficient to describe tailor-made filters for any |ℓ⟩ state, for our proof of principle we only need filters for the |0⟩ state. Using holograms we can produce any other necessary filter. The filter will be a nonlinear element. We have the evolution \(F_0 \sum_{\ell=0}^{2^n-1} \alpha_\ell |\ell⟩ = |0⟩\) with a probability \(|\alpha_0|^2\) and the photon is absorbed with probability 1 − \(|\alpha_0|^2\). There can be different ways to implement this kind of filter. In our model, the filter is achieved by coupling the photon to a single-mode optical fibre (SMF). This coupling will only be efficient for the ℓ = 0 mode [11]. All the other modes will disperse.

Figure 1 shows the representation of all these elements.

3.1. The extraction gate

The main element of our construction is a gate that is able to separate a state of a given value of OAM from a superposition and take it to a different spatial mode. The operation can be seen as: \(E_m (\sum_{\ell=0}^{2^n-1} \alpha_\ell |\ell⟩|\text{vac}⟩) = \sum_{\ell \neq m} \alpha_\ell |\ell⟩|\text{vac}⟩ + \alpha_m |\text{vac}⟩|0⟩\). In the proposed gate, the extracted state is converted to the |0⟩ OAM state. This is more practical for the construction of the two-level units used in the proof of universality. If needed, a +m hologram can produce the more natural gate in which the second spatial mode carries an |m⟩ state.

Our extraction gate is based on quantum interrogation, where frequent measurement results in a directed quantum evolution [12]. Figure 2 shows the basic set-up. At the input, we have a −m hologram. From that point, we can study the evolution for two cases, the |0⟩ state (the former |m⟩ state) and all the other |ℓ⟩ states. The filter separates the evolution of these two groups of states.

Imagine we have an input |ℓ⟩|vac⟩ with ℓ ≠ 0. The first beamsplitter will have at its output the state \(\cos \theta |\ell⟩|\text{vac}⟩ + \sin \theta |\text{vac}⟩|\ell⟩\). The second spatial mode is then directed to an F0 filter. At the filter, the state is projected to |ℓ⟩|vac⟩ with probability \(\cos^2 \theta\). This will happen at each of the N beamsplitters. For our angle \(\theta = \frac{\pi}{N}\) the final state will be |ℓ⟩|vac⟩ with a probability \(\cos^{2N} \theta ≈ 1 - \frac{\theta^2}{N}\) which can be made arbitrarily close to one. If the photon was absorbed, the gate fails and we would have to start the computation again.

The F0 filter has no effect on the |0⟩ OAM state. The input sees N beamsplitters so that \(E_m |\text{vac}⟩|0⟩ = BS_N^0 |\text{vac}⟩|0⟩\). To prove this we only need to notice that the beamsplitter matrix is a rotation by an angle \(\theta\) and that \(BS_N^0 = BS_{N\theta} = BS_{\pi}\).

The conjunction of both evolutions gives us the desired operation. Notice that, despite the nonlinear measurements, the gate is reversible. If we repeat the same configuration at the output of an extraction gate, the extracted term will be reintegrated into the superposition.

There are two practical details worth commenting on. Reflections take the OAM state from |ℓ⟩ to |−ℓ⟩. For this reason, we need to make sure that all the paths have an even total number of reflections. Apart from that, we have to adjust the length of the optical fibre so that the |0⟩ states from both paths arrive at the beamsplitters with no phase shift.
Um the extraction gate. First, we extract a space-efficient universal set of gates. Of the operation, we have again only one path for the photon, which corresponds to the desired two-level unitary. At the end of the operation, we have again only one path for the photon.

Another example of two-level unitary is a GHZ state creation operation. For this gate, the global unitary operation $U$ and the two-level submatrix $U_{2}^{3,4}$ are

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad U_{2}^{3,4} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Another example of two-level unitary is a GHZ state creation gate. From an initial state $|0\rangle$, we can create the superposition $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, which corresponds to the qubit entangled state $|\alpha\rangle|\ell\rangle + |\beta\rangle|\ell\rangle$.

To prove that these gates can be used to implement any unitary operation we only need to show we can build two-level unitary matrices $U_{m,n}$ [4]. These matrices redistribute the probability of two of the $2^n$ states of its input superposition. The $U_{m,n}$ operation only affects states of OAM $mh$ and $nh$ so that $U_{m,n}|m\rangle = u_{m,n}|m\rangle + u_{m,n}|n\rangle$, $U_{m,n}|n\rangle = u_{n,m}|m\rangle + u_{n,n}|n\rangle$ and $U_{m,n}|\ell\rangle = |\ell\rangle$ for $\ell \neq m, n$. The matrix

$$U_{2}^{m,n} = \begin{pmatrix} u_{m,m} & u_{m,n} \\ u_{n,m} & u_{n,n} \end{pmatrix},$$

must be unitary. One example of a two-level gate is the CNOT operation. For this gate, the global unitary operation $U$ and the two-level subunitary $U_{2}^{3,4}$ are

Another example of two-level unitary is a GHZ state creation gate. From an initial state $|0\rangle$, we can create the superposition $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, which corresponds to the qubit entangled state $|\alpha\rangle|\ell\rangle + |\beta\rangle|\ell\rangle$.

Figure 3 shows a simple OAM implementation based on the extraction gate. First, we extract the $|m\rangle$ and $|n\rangle$ terms of the input state and take them to the $|0\rangle$ OAM mode of two separate spatial modes. Then, using linear optics elements (a beamsplitter and a phase shifter), we can give any $2 \times 2$ unitary [4]. Finally, the new superposition is integrated back into the spatial mode of the rest of the OAM states. The resulting evolution is

$$\sum_{\ell=0}^{2^{n-1}} a_\ell |\ell\rangle \rightarrow \sum_{\ell \neq m,n} a_\ell |\ell\rangle + (u_{m,m}a_m + u_{m,n}a_n)|m\rangle + (u_{n,m}a_m + u_{n,n}a_n)|n\rangle, \quad (1)$$

which corresponds to the desired two-level unitary. At the end of the operation, we have again only one path for the photon.

This completes the proof that OAM encoding is enough to have a space-efficient universal set of gates.

5. The readout problem

We have seen we can perform any desired quantum computation with the OAM of a single photon. However, the computation cannot be said to have finished until we have read the solution from the resulting OAM state. OAM sorters are devices that can discriminate between different OAM states of a single photon [13]. In a sorter, the photon is directed through different paths depending on its OAM. There are $n$ binary decision points in a branching optical set-up with $2^n$ arms. This branching takes us back to an exponential growth problem. Alternative approaches to OAM sorting share this exponential growth problem. In them, we either have an increasing number of spatial paths or we need to discriminate between an exponentially high number of frequencies [14, 15]. This limitation is a consequence of the destructive nature of photon measurement. A series of $n$ quantum nondemolition measurements of the photon would determine the OAM state efficiently, but these measurements are as experimentally challenging as achieving interaction between photons and, in many respects, are equivalent to them.

There are different solutions to this problem. On the one hand, if we have an algorithm where the result is always, or with very high probability, the same state $|L\rangle$, like in a simple Grover search, we can run the single-photon computer $n$ times and obtain $n$ copies of $|L\rangle$. The sorter architecture can be used without the need of exponential branching. We can measure the position of the photon after each point of branching. In the first measurement we can determine whether $L$ is even or odd. Subsequent measurements on the copies of $|L\rangle$ reduce by one-half the possible states. After the $n$th measurement, we have the value of the $n$ bits which give $L$.

If this is not the case, we would need to have optical CNOT gates. One option would be using OAM demultiplexers [16]. In a multiplexer we have the same OAM-dependent branching. At each point of branching the photon is separated into two different paths. Using an optical CNOT gate, we can change the path of an ancillary photon if, for instance, the original photon took the second path. This takes the path information to another photon. A second CNOT gate controlled by the new photon can restore the OAM state to a single path. If we repeat this procedure $n$ times, we can take the information from the OAM of a single photon and put it into the path of $n$ photons. In this new encoding, the readout is efficient. In order to build an OAM demultiplexer we would need $2n$ path CNOT gates, which are the kind of gate that requires the strong photon–photon interaction we are trying to avoid. Nevertheless, this would only impose a fixed price for any arbitrary computation.
which only grows linearly with the size of the input. This can be an important saving when compared with alternative optical models.

6. Alternatives for the extraction gate

In the previous sections, we have described a general method to perform any quantum computation using the OAM of light. The suggested implementation is not unique. Quantum interrogation systems can be very sensitive to losses and it will be useful to have different implementation choices for the extraction gate.

In the extraction gate we have assumed that the filter \( F_0 \) is completely transparent to the \( |0\rangle \) states. In fact, coupling to the fibre is bound to have some losses. Losses alter the evolution in two ways. If there is an absorption, the photon can be lost. Additionally, losses can also collapse the state of the photon to the upper path. For a probability of absorption \( A \), we can compute the final probability of finding the photon in each path by studying the density matrix of the system. The evolution can be modelled numerically [17]. A low value of stages (small \( N \)) makes the effects of the losses smaller, but increments the probability of absorbing one of the OAM states with \( \ell \neq 0 \). We can choose a value of \( N \) for which the probability of survival of an \( \ell \neq 0 \) state, \( \cos^{2N}(\frac{\pi}{N}) \), is approximately equal to the probability that the \( |0\rangle \) state takes the correct path. For a probability of absorption \( A = 1\% \), this happens for \( N = 20 \) and the global probability of success is around 88%. For \( A = 2\% \), \( N = 14 \) and the gate works 83.5% of the times.

There are alternatives to fibre coupling which could be worth exploring. Certain optical elements convert OAM into a lateral displacement [18]. These elements can also work as an \( F_0 \) filter. Interferometric configurations like the one of figure 2 need careful adjusting. A small displacement will ruin interference and, eventually, the \( \ell \neq 0 \) components will gradually disperse into the environment.

Finally, we can replace filters by selective absorbers. A material which only absorbs \( |0\rangle \) state photons can replace the \( F_0 \) filters. In this case, it is the \( |0\rangle \) state which remains in the upper path and the \( \ell \neq 0 \) states change their path. Selective absorption could appear if there are selection rules which forbid the absorption of certain photon states, similarly to what happens in polarizers.

7. Discussion

We have presented a scheme for universal computation using the OAM of a single photon. The quantum gates are compact in space, but the readout of the final state requires either repeated runs of the same computation or the use of a limited number of optical CNOT gates. The scheme is not fully scalable, but it could be used for a small quantum computer.

There are a few challenges to an efficient experimental realization. The main obstacle is the number of elementary gates needed in a concrete implementation. Although we have given a proof of universality, the number of optical gates used to implement a particular operation can be exponentially high. All universal sets of gates have this problem but, in many implementation proposals, there are interesting circuits (for instance, circuits for Shor’s or Grover algorithms) which are efficient in the number of elementary gates. For OAM it would be an important advance to find a simple optical implementation for a ‘dispersing’ gate, like the Hadamard gate or the quantum Fourier transform, which appear in many quantum algorithms which offer a speed-up with respect to classical ones. In these gates, the new probability of each state comes from the interference of the probability amplitudes of most of the possible logical states. An element that takes a photon from any single OAM state into a superposition of many others would be an important step towards useful single-photon quantum computers with OAM. If the ‘dispersing’ operations can be moved to the beginning of the computation, there can be a solution. State generation schemes are efficient and could be used to create highly dispersed initial states (like a uniform superposition).

One important challenge to a practical implementation is the high losses of the individual elements, particularly the holograms and the fibre filters. The proposed set-ups can have many chained steps and the total loss can be excessively high. If we have a quantum algorithm which always produces the same output, we can just send more photons. However, having many imperfect elements one after another can produce other undesirable effects like distortion of the wavefront. If we use Laguerre–Gauss beams, radial modes can also become important. In any case, there are practical limits to the maximum value of OAM that can be obtained. Resolution of the holograms, for instance, will grow with the state space. Nonetheless, a quantum computer of a few qubits should be possible.

All these problems notwithstanding, the proof that universal logic is, indeed, attainable with an OAM encoding suggests it is worth further investigation into this new kind of optical quantum computer. The experimental realization of a single-photon quantum computer, at least for certain tasks, seems feasible, although problems like overcoming losses in the optical elements, achieving interferometric stability in the extraction gate, and generating and processing states of a high OAM can limit the practical implementation.

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