Quadratic Programming for Optimizing the Diversified Shariah Stock Portfolio

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ABSTRACT
Risk is a challenging module in evaluating stock investment prospects that are often taken into account by investors. This paper presents the method of Quadratic Programming to optimize the risk of Shariah stock portfolio. The dataset deals with the weekly close price of all active issuers listed in FTSE Bursa Malaysia Hijrah Shariah Index from January 2016 to December 2018. In general, there are two issues that are highlighted: portfolio selection and portfolio optimization. Portfolio selection is carried out in several phases, namely grouping the issuers into two portfolios by considering the technical and fundamental aspects, nominating feasibility of each portfolio using a Constant Correlation Model, and finally selecting the most diversified Shariah stock portfolio. Furthermore, the selected portfolio risk optimization is formulated by Quadratic Programming. The results of this study show that the optimum portfolio is the portfolio B which includes 42.73% of BTKW, 8.1% of GENP, 13.5% of IHHH, 0.84% of IOIB, 5.04% of PCGB, 15.48% of PEPT, 4.86% of PGAS, and 9.46% of TENA with a minimum value of risk 0.58%. Since the variance of portfolio A is 0.81%, it implies that a maximum diversified Shariah portfolio provides better risk.

Keywords: quadratic programming; constant correlation model; diversified shariah stock portfolio.

1. INTRODUCTION

Naturally, investment deals with the activity on allocating goods or funds that are not consumed today but are used in the future to create wealth. This mechanism is always faced with two conflicting problems, namely, return and risk. Both return and the risk have a direct relationship and key role in most decision making process [1]. As the risk level of an investment increases, the potential return usually increases as well. Surprisingly, some investors tend to reduce investment uncertainty, especially on the issue of low risk.

In the last decade, Islamic investment has been grown as a form of alternate financial activities on any securities, including stock. Basically, it required to conduct financial transactions that conform to Islamic tents or Islamic law, so called Shariah. The investors concerned with adhering to the Islamic way nevertheless have an expectation of gaining wealth of their investment on Shariah stock [2]. In order to encounter these targets, investors need to have some strategies not only to concern about how to obtain return but also in reducing risk.

There are some scholars concerned with the risk of the stock portfolio. In terms of portfolio selection, [3] did a comparative study between the single index model and constant correlation model. A few years later, [4] applied a Grey Relational Analysis (GRA) approach to select portfolio. Furthermore, in terms of portfolio optimization, [5] uses the Lagrange multiplier approach to select the optimum portfolio of the Malaysian stock exchange, [6] optimizes securities (stock, bond, and money market) using Mean-Variance Optimization, and [7] employs a pivoting-based method for solving convex quadratic programming.

In this paper, we extend the paper of [4], [5] by adding the screening procedure of stock portfolio using diversification ratio initiated by [8]. It will ensure investors to hold well diversified portfolio instead of investing their entire fund in a single stock. Moreover, we also extend the paper of [5]–[7] by modelling the portfolio problem and constructing computer programming of the standard quadratic programming that enables users to obtain the result with minimum elapsed time. Last but not least, the dataset used on those studies are mainly referring to the conventional stock portfolio index. However, this paper will observe one of Shariah stock indexes in Malaysia.

The objective of this paper is to construct the most diversified Shariah stock portfolio through some procedures such as grouping the index into two opposite aspects, correlation selection and end up with the final selection of diversification. In addition, we will also minimize the risk of the selected portfolio by implementing Quadratic Programming and consider the stock weighting of each stock for fund allocation.
2. MATERIALS AND METHODS

2.1. Shariah Stock Portfolio

Stock is a share in the ownership of a company while a group of stock which refer to a financial asset is called a stock portfolio. In recent times, Shariah Stock, Islamic ethics-based stock, has grown significantly. It is due to the greater potential of growth and profitability in several countries [9]. For instance, Malaysia leads the Islamic Asset Under Management (AUM) with 36.5% share of global Islamic funds as reported by the Securities Commission in 2017 [10]. In addition, the total AUM increased by 11.5% from RM696.3 billion in 2016 became RM776.2 billion at the end of December 2017. Some researchers also revealed that performance of Shariah stock has a trend to be higher than the conventional platform in terms of return ([11], [12]) and steady during the financial crisis ([13]).

This type of financial investment refers to the Shariah principles of transactions (Muamalat). As mentioned in [14], Shariah Advisory Council (SAC) of the Securities Commission Malaysia categorizes stocks as Shariah compliant if the issuer company declares that its business activities, as well as its business management, are conducted based on the Shariah principles, and it is not involved in any of the following businesses: (1) Riba as practised by conventional financial institutions; (2) Gambling; (3) The production and sale of goods and services which covers processing, producing and marketing alcoholic drinks, supplying non-halal meat like pork, and providing immoral services like prostitution; (4) Trading of risk that contains gharar (uncertainty), e.g. conventional insurance.

2.2. Portfolio Selection

There are several methods in portfolio selection, one of which is Constant Correlation Model (CCM). It is accepted as the best way to forecast correlation coefficients and assumed that the correlation between all pairs of issuers is the same [15]. The selection of issuers which are included in the portfolio is based on the excess return to standard deviation value (ERS) as a risk indicator and cut-off rate ($C^*$) as an optimal threshold. Let $R_i$ be return on issuer $i$, $R_j$ risk free rate, and $\sigma_i$ the standard deviation of the return on issuer $i$, then $ERS_i$ is determined by

$$ ERS_i = \frac{R_i - R_j}{\sigma_i} \quad (2.1) $$

Moreover, the general expression of $C_i$ is used to consider the cut-off rate. It can be found from

$$ C_i = \frac{\rho}{1 - \rho + i\rho} \sum_{j=1}^{n} \frac{R_i - R_j}{\sigma_j} \quad (2.2) $$

where $\rho = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} \hat{\rho}_{ij}}{N}$ is the correlation coefficient-assumed constant for all issuers. The subscript $i$ indicates that $C_i$ is calculated using data on the first $i$ issuers. Here, cut-off rate $C^*$ is the optimal value of $C_i$ all issuers, which have a higher excess return to standard deviation are included into the portfolio candidate.

After having the portfolio candidate, there exists an extra step on portfolio construction so-called portfolio diversification, a technique that reduces risk and investment uncertainty by allocating investments among various financial issuers or categories. [8] introduced the concept of maximum diversification via a formal definition of the diversification ratio (DR). The basic idea behind the maximum diversification approach is to construct a portfolio that maximizes the benefits from diversification. Basically, DR can be defined as the ratio of a portfolio weighted average volatility of individual assets divided by the volatility of the overall portfolio. Formally, given a portfolio composed of $N$ issuers ($X_1, ..., X_N$) and weights ($w_1, ..., w_n$) with $\sum_{i=1}^{N} w_i = 1$, then

$$ DR = \frac{\sum_{i=1}^{N} w_i \sigma_i}{\sigma_p} \quad (2.3) $$

where

$$ \sigma_p = \sqrt{\sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=i+1}^{N} w_i w_j \text{cov}(X_i, X_j)} $$

is the overall portfolio volatility and $\sigma_i$ the volatility of issuer $i$. According to [8], the result of equation (2.3) measures the essence of diversification event though different asset classes are not perfectly correlated to each other (DR > 1).

2.3. Optimization and Quadratic Programming

Optimization can be described as the collection of techniques, methods, procedures and algorithms that can be used to find the optima. In the past, the application of optimization was mainly in the area of Linear Programming but now many experts use Nonlinear Programming particularly on handling more complex problem. Consider the following Nonlinear Programming problem
Minimize $f(x)$
subject to $g_i(x) \leq 0$ for $i = 1,\ldots,m$
$h_i(x) = 0$ for $i = 1,\ldots,l$
$x \in X$.

(2.4)

where $f, g_1,\ldots,g_m,h_1,\ldots,h_l$ are functions defined on $\mathbb{R}^n$, $X$ is a subset of $\mathbb{R}^n$, and $x$ is a vector of $n$ components $x_1,\ldots,x_n$. The equation (2.4) must be solved for the values of the variables $x_1,\ldots,x_n$ that satisfy the constraints function (both inequality $g_i(x)$ and equality $h_i(x)$) and meanwhile minimize the objective function $f$. The set $X$ might typically include lower and upper bounds on the variables [16].

A more particular Nonlinear Programming problem is the Quadratic Programming (QP) that frequently encountered in the optimization model. Observe that the below quadratic program represents a special class of Nonlinear Programming problems in which the objective function is quadratic and the constraints are linear. As mentioned in [6], [16], suppose $c$ is an $n$–vector, $b$ is an $m$–vector, $A$ is $m\times n$ matrix, and $H$ is an $n\times n$ symmetric matrix, the standard QP is defined as

$$
\begin{align*}
\min & \quad \frac{1}{2} x^T H x + c^T x \\
\text{s.t.} & \quad A x = b \\
& \quad x \geq 0
\end{align*}
$$

(2.5)

As can be seen in equation (2.5), since the feasible set is a polyhedral set and $H$ is symmetric and positive semidefinite, the objective function is convex.

3. METHODS

The method of this research is descriptive quantitative research. The sample of research is issuers listed on FTSE Bursa Malaysia Hijrah Shariah Index. This index is constituent of the FTSE Bursa Malaysia EMAS Shariah Index and has been designed to be used as a basis of Shariah-compliant investment products that meet the screening requirements of international Islamic investors. Companies in the index are screened by the Malaysian Securities Commission’s Shariah Advisory Council (SAC) and the leading global Shariah consultancy, Yasaar Ltd, against a clear set of guiding principles.

The data analysis of this research deals with two main phases, namely, portfolio selection and portfolio optimization. The detailed procedure of data analysis is given in Figure 1. The objective of the model is to reduce the variance or the risk of Shariah stock portfolio investment by supposing the investor as risk averse, i.e. an investor who prefers lower return with known risk rather than higher return with unknown risk. In addition, short selling, an act that speculates on the decline in a stock price, is not allowed.
4. RESULTS AND DISCUSSION

4.1. Data and relevant descriptive statistics

The dataset collected in this research is secondary data in the form of weekly closing stock price of 30 issuers listed in FTSE Bursa Malaysia Hijrah Shariah Index (published by investing.com) during January 2016–December 2018. The criterion of stock selection is the stock which is joined in FTSE Bursa Malaysia Hijrah Shariah Index issued by the annual report of SAC in the year of 2018 (Table 1). In addition, an interest rate data released by Bank Nasional Malaysia is employed as the proxy of risk free rate during the same period. Moreover, for the fundamental aspect, the top ten issuers on market capitalization was collected from FTSE Russel for the year 2018. Based upon the weekly prices, weekly geometric returns were calculated and average return $R$ for 157 weeks was calculated for each stock. Furthermore, standard deviation $\sigma_i$ as a risk measure and variance $\sigma_i^2$ were calculated as well for each stock.
Table 1. 30 Issuers listed on FTSE Bursa Malaysia Hijrah Shariah Index 2018 and relevant descriptive statistics

| Rank | Code | Issuers                        | \( R_i \)  | \( \sigma_i \)  | \( \sigma_i^2 \) |
|------|------|--------------------------------|------------|-----------------|-----------------|
| 1    | BTKW | Batu Kawan Bhd                 | -0.0003022 | 0.01125258      | 0.00012662      |
| 2    | KLLK | Kuala Lumpur Kepong Bhd        | 0.00046685 | 0.01221049      | 0.000149096     |
| 3    | PEPT | PPB Group Bhd                  | 0.00189257 | 0.01430498      | 0.000204632     |
| 4    | TENA | Tenaga Nasional Bhd            | 0.00021282 | 0.01776935      | 0.00031575      |
| 5    | GENP | Genting Plantations Bhd        | -0.0004    | 0.0183641       | 0.00033724      |
| 6    | PCGB | Petronas Chemicals Group Bhd   | 0.00135881 | 0.01851344      | 0.00038077      |
| 7    | PETR | Petronas Dagangan Bhd          | 0.00055999 | 0.01951334      | 0.000383125     |
| 8    | IOIB | IOI Corporation Bhd            | 0.00011651 | 0.02044225      | 0.000417886     |
| 9    | IHHH | IHH Healthcare Bhd             | -0.0011643 | 0.02044877      | 0.000418152     |
| 10   | PGAS | Petronas Gas Bhd               | -0.0008928 | 0.0220794       | 0.000527316     |
| 11   | FRAS | Fraser Neave Holdings Bhd      | 0.00380455 | 0.02258529      | 0.000510095     |
| 12   | NESM | Nestle (Malaysia) Bhd          | 0.00448945 | 0.02296335      | 0.000527316     |
| 13   | MXSC | Maxis Bhd                      | -0.0012721 | 0.02369335      | 0.000527316     |
| 14   | WPHB | Westports Holdings Bhd         | -0.0005761 | 0.02464123      | 0.000598366     |
| 15   | DSOM | Digi.Com Bhd                   | -0.0009242 | 0.02503219      | 0.000598366     |
| 16   | SIPL | Sime Darby Plantation Bhd      | -0.0015657 | 0.0257945       | 0.000716068     |
| 17   | MISC | MISB Bhd                       | -0.0021343 | 0.02673414      | 0.000716068     |
| 18   | DIAL | Dialog Group Bhd               | 0.00426578 | 0.02699099      | 0.000728519     |
| 19   | SWAY | Sunway Bhd                     | 0.00179011 | 0.02700844      | 0.000729456     |
| 20   | QRES | QL Resources Bhd               | 0.00287626 | 0.02985219      | 0.000891153     |
| 21   | IOIP | IOI Properties Group Bhd       | -0.001915  | 0.03385029      | 0.001145842     |
| 22   | TLMM | Telekom Malaysia Bhd           | -0.0059164 | 0.03750376      | 0.001406532     |
| 23   | AXIA | Axiata Group Bhd               | -0.0029382 | 0.03730478      | 0.001406532     |
| 24   | HTHB | Hartalega Holdings Bhd         | -0.0045976 | 0.04130281      | 0.001705922     |
| 25   | UMWS | UMW Holdings Bhd               | -0.0022243 | 0.04142417      | 0.001715962     |
| 26   | SETI | SP Setia Bhd                   | -0.0013899 | 0.0416418       | 0.001734039     |
| 27   | TPGC | Top Glove Corporation Bhd      | 0.00323138 | 0.04322279      | 0.00186821      |
| 28   | IJMS | IJM Corporation Bhd            | -0.0047432 | 0.04670911      | 0.00204632      |
| 29   | FGVH | FGV Holdings Bhd               | -0.0053951 | 0.05490791      | 0.00214395      |
| 30   | PMET | Press Metal Bhd                | 0.00512971 | 0.09969103      | 0.009938301     |

The first step on portfolio selection is grouping among all issuers, namely portfolio A represents for the best ten issuers based on market capitalization (refer to FTSE Russel annual report for 2018) and portfolio B represents for the best ten issuers due to the smallest risk (refer to historical price). Table 2 displays portfolio A while Table 1 shows the portfolio B (shaded cells).

4.2. Empirical Results

4.2.1. Portfolio Selection

Table 2. FTSE Bursa Malaysia Hijrah Shariah Index 2018 Top 10 Issuers

| Rank | Code | Issuers                        | Market Capitalization (MYRm) |
|------|------|--------------------------------|------------------------------|
| 1    | TENA | Tenaga Nasional Bhd            | 53,631                       |
| 2    | PCGB | Petronas Chemicals Group Bhd   | 26,210                       |
| 3    | AXIA | Axiata Group Bhd               | 17,938                       |
| 4    | SIPL | Sime Darby Plantation Bhd      | 16,743                       |
| 5    | DSOM | Digi.Com Bhd                   | 16,439                       |
| 6    | IHHH | IHH Healthcare Bhd             | 15,010                       |
| 7    | PGAS | Petronas Gas Bhd               | 14,856                       |
| 8    | MXSC | Maxis Bhd                      | 14,622                       |
| 9    | DIAL | Dialog Group Bhd               | 14,191                       |
| 10   | IOIB | IOI Corporation Bhd            | 14,064                       |
The risk-free rate $R_p$ used in this study is obtained from the interest rate issued by official website of Bank Nasional Malaysia during the period of 2016–2018. Due to the observation, the average interest rate is 3.0793% per year. Since the historical price of issuers analyzed is weekly data, the annual interest rate must be divided by 52 so that a weekly interest rate of 0.059217% is obtained.

Furthermore, in order to investigate the correlation between stocks in each of portfolio A and portfolio B, correlation selection is performed using CCM by considering ERS and cut-off rate (Equation 2.1 and 2.2). The constant correlation between stocks is calculated to identify how strong the correlation between stocks. In CCM, the correlation between stocks is assumed to be constant and will be used to calculate the cut-off rate. The constant correlation $\rho$ for 10 issuers in portfolio A is 0.1932, while the constant correlation $\rho$ for 10 shares in portfolio B is 0.2133. Table 3 indicates that the highest ERS for portfolio A is belong to DIAL which means the return of DIAL 13.61% higher than its risk. By the same analogy, return of PEPT 9.09% higher than its risk (see Table 4).

### Table 3. CCM analysis on Portfolio A with $\rho = 0.1932$

| Code | ERS | $C_i$ | Status    |
|------|-----|-------|-----------|
| DIAL | 0.136104333 | 0.026297543 | Selected Portfolio |
| PCGB | 0.041414128 | 0.01600375 | Selected Portfolio |
| SIPL | 0.036415417 | 0.021108131 | Selected Portfolio |
| DSOM | -0.061537684 | -0.071340415 | Selected Portfolio |
| PGAS | -0.07262046 | -0.098220076 | Selected Portfolio |
| MXSC | -0.081182613 | -0.125486279 | Selected Portfolio |
| IHHH | -0.085925207 | -0.149419172 | Selected Portfolio |
| AXIA | -0.094130093 | -0.181874461 | Selected Portfolio |
| TENA | -0.021348955 | -0.016499844 | Not selected |
| IOIB | -0.024300701 | -0.023476429 | Not selected |

### Table 4. CCM analysis on Portfolio B with $\rho = 0.2133$

| Code | ERS | $C_i$ | Status    |
|------|-----|-------|-----------|
| PEPT | 0.099090496 | 0.021422718 | Selected Portfolio |
| PCGB | 0.041414128 | 0.019519357 | Selected Portfolio |
| TENA | -0.021348955 | -0.025155538 | Selected Portfolio |
| IOIB | -0.024300701 | -0.034360308 | Selected Portfolio |
| GENP | -0.054026888 | -0.08912406 | Selected Portfolio |
| PGAS | -0.07262046 | -0.136910255 | Selected Portfolio |
| BTKW | -0.07948504 | -0.168583446 | Selected Portfolio |
| IHHH | -0.085925207 | -0.202491867 | Selected Portfolio |
| PETR | -0.00185452 | 0.021422718 | Not selected |
| KLKK | -0.01026382 | 0.019519357 | Not selected |

Moreover, from the CCM simulation of portfolio A (see Table 3), it is known that from 10 stocks, there are only eight stocks which have more ERS value than $C$ with cut-off rate $C = -0.1818$. Hence, those eight stocks become the last candidate which will be counted within the selected portfolio. On the other hand, Table 4 shown that there are eight stocks of portfolio B which fulfilled CCM criterion with cut-off rate $C = -0.2024$.

After having the selected portfolio as shown on Table 3 and Table 4, the final step would be the diversification. The last step on this study is optimizing portfolio B as the most diversified Shariah stock portfolio with respect to the Standard Quadratic Programming. The common examination. The objective of this stage is to choose the maximum diversification between those two portfolios. By conducting the diversification ratio (Equation 2.3), portfolio B has a ratio value of 2.0934 which is slightly higher than portfolio A with ratio 1.9433. Hence, portfolio B that denotes the smallest risk, was chosen as the most diversified Shariah stock portfolio.

### 4.2.2. Portfolio Optimization

mathematical model of equation (2.5) can be adapted to the problem of portfolio optimization by determining objective and constraints functions. Suppose stocks
According to problem (4.1), let the standard deviation of the proportion of the total fund invested in portfolio (set of stock $i$). For $i \neq j$, $\rho_{ij}$ denotes the correlation coefficient of stocks $S_i$ and $S_j$. Let $\mu = [\mu_1, ..., \mu_n]^T$, and $H = (\sigma_{ij})$ be the $n \times n$ symmetric covariance matrix with $\sigma_{ii} = \sigma_i^2$ and $\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j$ for $i \neq j$. One can represent the expected return and the variance of the resulting portfolio $x = (x_1, ..., x_n)$ as follows:

$$E[x] = x_1 \mu_1 + x_2 \mu_2 + ... + x_n \mu_n = \mu^T x,$$

and

$$Var[x] = \sum_{i,j} \rho_{ij} \sigma_i \sigma_j x_i x_j = x^T H x,$$

where $\rho_{ii} = 1$.

Since variance is always nonnegative, it follows that $x^T H x \geq 0$ for any $x$, i.e., $H$ is positive semidefinite. Nevertheless, it will be assumed in fact as positive definite, which is essentially equivalent to assuming there are no redundant assets in our collection $S_1, S_2, ..., S_n$. Also assume that the set of admissible portfolio is nonempty polyhedral set and represent it as $X := \{x: Ax = b, Cx \geq d\}$, where $A$ is $m \times n$ matrix, $b$ is $m$-dimensional vector, $C$ is $p \times n$ matrix and $d$ is $p$-dimensional vector. In particular, one of the constraints in the set $X$ is $\sum_{i=1}^n x_i = 1$.

Furthermore, since it is assumed that $H$ positive definite, the variance is a strictly convex function of the portfolio variables and there exists a unique portfolio in $X$ that has the minimum variance. Let denote this portfolio with $x_{\text{min}}$ and its return $\mu^T x_{\text{min}}$ with $R$. Note that $x_{\text{min}}$ is an efficient portfolio. The target is to find the minimum risk portfolio of the securities 1 to $n$ that yields at least a target value of expected return. Mathematically, this formulation together with (4.1) produces a quadratic programming problem for portfolio optimization:

$$\min \quad \frac{1}{2} x^T H x + c^T x,$$

subject to

$$\sum_{i=1}^n x_i = 1, \quad x \geq 0,$$

The first constraint indicates that the expected return is no less than the target value $R$. Due to problem (4.2) and, the objective function is referred to variance-covariance matrix while the first constraint is expressed by geometric mean. Moreover, portfolio optimization in this section will be focused on portfolio B as the result of maximum diversified Shariah stock portfolio. Table 5 and Table 6 sequentially present the variance-covariance matrix and geometric mean of portfolio B which will be used as variables in the objective function and one of the constraint functions in equation (4.2).

### Table 5. Variance-Covariance Matrix of Portfolio B

|          | BTKW | GENP | IHHH | IOIB | PCGB | PEPT | PGAS | TENA |
|----------|------|------|------|------|------|------|------|------|
| BTKW     | 0.0001269 | 0.0000289 | 0.0001513 | 0.000239 | 0.000285 | 0.0000272 | 0.0000997 | 0.0000332 |
| GENP     | 0.0000289 | 0.0003385 | 0.0000339 | 0.0001801 | 0.0001276 | 0.000691 | 0.0001330 | 0.0000827 |
| IHHH     | 0.00001513 | 0.0000385 | 0.0001801 | 0.0004182 | 0.0000282 | 0.0000132 | 0.0001345 | 0.0000891 |
| IOIB     | 0.000239 | 0.0001801 | 0.0000282 | 0.0003880 | 0.0001345 | 0.000979 | 0.0001337 | 0.0000873 |
| PCGB     | 0.0000285 | 0.0001276 | 0.0000013 | 0.0001345 | 0.0003431 | 0.0000790 | 0.0001306 | 0.0000891 |
| PEPT     | 0.0000272 | 0.0000691 | 0.0000323 | 0.0000979 | 0.0000790 | 0.0002111 | 0.0000535 | 0.0000683 |
| PGAS     | 0.0000098 | 0.0000130 | 0.0000768 | 0.0001337 | 0.0001306 | 0.0000535 | 0.0000420 | 0.0000652 |
| TENA     | 0.0000332 | 0.0000827 | 0.0000122 | 0.0000873 | 0.0000891 | 0.0000683 | 0.0000652 | 0.0003161 |

### Table 6. Geometric Mean of Portfolio B

|          | BTKW | GENP | IHHH | IOIB | PCGB | PEPT | PGAS | TENA |
|----------|------|------|------|------|------|------|------|------|
| $\mu$    | -0.0003466 | -0.0004892 | -0.001214 | -0.000587 | 0.0013087 | 0.0016893 | -0.0010858 | 0.0001640 |

Additionally, conducting equation (4.2) into the data, i.e., portfolio B which represent eight stocks. Assume that BTKW, GENP, IHHH, IOIB, PCGB, PEPT, PGAS, and TENA are represented by variables $x_a, x_b, x_c, x_d, x_e, x_f, x_g$, and $x_h$. 
The portfolio problem can be adapted based on the assumption that the variance as objective function and expected return as one of constraint functions. The computational experience in portfolio B as the most diversified portfolio produces a minimum risk of 0.58%.

Equation (4.3) can be solved technically by MATLAB software. The construction of computer programming deals with m-file MATLAB through the help of optimization toolbox particularly optionitops and quadprog [17]. The output of the intended m-file program that consumes elapsed time less than 2 seconds is shown in Figure 2.

| Fill in the number of issuers: 8 |
|----------------------------------|
| INPUT                           |
| --------------------------------|
| Variance-covariance matrix:     |
| 1.0e-03 *                       |
| 0.1269  0.0289  0.0153  0.0239  |
| 0.0289  0.3385 -0.0339  0.1801  |
| 0.0153  -0.0339  0.4182  0.0282  |
| 0.0239  0.1801  0.0282  0.3880  |
| 0.0285  0.1276  0.0013  0.1345  |
| 0.0272  0.0691  0.0323  0.0979  |
| 0.0098  0.1330  0.0768  0.1337  |
| 0.0332  0.0827  -0.0122  0.0873  |
| Geometric mean:                  |
| -0.0003  -0.0005  -0.0011  -0.0001 |
| Minimum found that satisfies the constraints. Optimization completed |
| to within the value of the optimality tolerance, and constraints are satisfied to |
| within the value of the constraint tolerance. |
| (stopping criteria details)      |

| OUTPUT                          |
|---------------------------------|
| --------------------------------|
| Minimum value of risk: 0.58%    |
| Solution (portfolio weight of x): |
| 42.73%  8.10%  13.50%  0.84%    |
| The number of iterations: 7     |
| The algorithm converged to the local minimum x. |
| Elapsed time is 1.794379 seconds. |

**Figure 2. Output of Program**

![Figure 2: Output of Program](image)

Figure 2 and Figure 3 summarize the minimum value of risk and the fund proportion of investment for each stock on portfolio B. As can be seen that BTKW hold the biggest allocation with 42.73%. It was then followed by other stocks such as PEPT, IHHH, TENA, GENP, PCGB, and PGAS with 15.48%, 13.5%, 9.46%, 8.1%, 5.04%, and 4.86% respectively. Finally, only 0.84% of total fund is distributed to IOIB.

**Figure 3. Stock weighting of portfolio B**

![Figure 3: Stock weighting of portfolio B](image)

**Figure 4. Solution of Problem**

5. **CONCLUSION**

Portfolio B, the set of eight stocks with the smallest risk, has a correlation coefficient that is closer to 1 when compared to portfolio A. This implies it has a stronger correlation between any two stocks. Moreover, portfolio B has higher diversification ratio and the amount of portfolio risk is smaller compared to each individual stock. This concludes that portfolio B establishing the well-diversified Shariah stock portfolio in portfolio selection.

The portfolio problem can be adapted based on the Quadratic Programming by assuming variance as objective function and expected return as one of constraint functions. The computational experience in portfolio B as the most diversified portfolio produces a minimum risk of 0.58%.
This value is almost three quarters more efficient than the risk in portfolio A.

REFERENCES

[1] Poornima, S., Swathiga, P. (2017). A study on relationship between risk and return analysis of selected stocks on NSE using capital asset pricing model. International Journal of Applied Research. 3(7): 375-378.

[2] Lusyana, D., Sherif, M. (2017). Shariah-compliant investments and stock returns: evidence from the Indonesian stock market. Journal of Islamic Accounting and Business Research.

[3] Pratiwi, D. A., Yunita, I. (2015). Optimal portfolio construction (A case study of LQ45 Index in Indonesia Stock Exchange). International Journal of Science and Research. 4(6): 2525-2530.

[4] Škrinjarić, T., Šego, B. (2019). Using Grey Incidence Analysis Approach in Portfolio Selection. International Journal of Financial Studies. 7(1): 1.

[5] Yousfat, A. (2015). The Portfolio Selection By using Quadratic Programming Approach Case Study of Malaysia Stock Exchange. Int. J. Eng. Technol. 7(4): 1361–1369.

[6] Mussafi, N. S. M. (2012). OPTIMISASI PORTOFOLIO RESIKO MENGGUNAKAN MODEL MARKOWITZ MVO. Jurnal Ilmiah AdMathEdu. 1(1).

[7] Zhang, Z., Zhang, H. (2011). An Efficient and Concise Algorithm for Convex Quadratic Programming and Its Application to Markowitz’s Portfolio Selection Model. Technology and Investment. 2(4): 229-239.

[8] Choueifaty, Y., Froidure, T., Reynier, J. (2013). Properties of the Most Diversified Portfolio. J. Invest. Strateg. 2(2): 49–70.

[9] C. S. F. Ho, N. A. Abd Rahman, N. H. M. Yusuf, and Z. Zanzamin, “Performance of global Islamic versus conventional share indices: International evidence,” Pacific Basin Financ. J., vol. 28, no. June, pp. 110–121, 2014.

[10] Securities Commission of Malaysia, “Annual Report,” 2017.

[11] Karim, B. A., Datip, E., Shukri, M. H. M. (2014). Islamic Stock Market Versus Conventional Stock Market. International Journal of Economics, Commerce and Management. 2(11): 1-9.

[12] Reddy, K., Fu, M. (2014). Does shariah compliant stocks perform better than the conventional stocks? A comparative study of stocks listed on the Australian Stock Exchange. Asian Journal of Finance & Accounting. 6(2): 155-170.

[13] Bakar, M. A. A. A., Ali, N. (2014). Performance Measurement Analysis: Shariah-Compliant Vs. Non Shariah-compliant Securities. Management & Accounting Review (MAR). 13(1): 75-108.

[14] Securities Commission Malaysia, Resolutions of the Securities Commission Shariah Advisory Council. 2007.

[15] Goetzmann, W. N., Brown, S. J., Gruber, M. J., Elton, E. J. (2014). Modern portfolio theory and investment analysis. John Wiley & Sons. 237.

[16] Bazaraa, M. S., Sherali, H. D., Shetty, C. M. (2013). Nonlinear Programming: Theory and Algorithms. John Wiley & Sons.

[17] Brandimarte, P. (2013). Numerical methods in finance and economics: a MATLAB-based introduction. John Wiley & Sons.