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1. Introduction

The adaptation of wireless technologies to the users rapidly changing demands is one of the main drivers of the wireless access systems development. New high-performance physical layer and multiple access technologies are needed to provide high speed data rates with flexible bandwidth allocation, hence high spectral efficiency as well as high adaptability. Multi carrier-code division multiple access (MC-CDMA) technique is candidate to fulfil these requirements, answering to the rising demand of radio access technologies for providing mobile as well as nomadic applications for voice, video, and data. MC-CDMA systems, in fact, harness the combination of orthogonal frequency division multiplexing (OFDM) and code division multiple access (CDMA), taking advantage of both the techniques: OFDM multi-carrier transmission counteracts frequency selective fading channels and reduces signal processing complexity by enabling equalization in the frequency domain, whereas CDMA spread spectrum technique allows the multiple access using an assigned spreading code for each user, thus minimizing the multiple access interference (MAI) (K. Fazel, 2003; Hanzo & Keller, 2006). The advantages of multi-carrier modulation on one hand and the flexibility offered by the spread spectrum technique on the other hand, let MC-CDMA be a candidate technique for next generation mobile wireless systems where spectral efficiency and flexibility are considered as the most important criteria for the choice of the air interface.

Two different spreading techniques exist, referred to as MC-CDMA (or OFDM-CDMA) with spreading performed in the frequency domain, and MC-DS-CDMA, where DS stands for direct sequence and the spreading is intended in the time domain. We consider MC-CDMA systems where the data of different users are spread in the frequency-domain using orthogonal code sequences, as shown in Fig. 1: each data symbol is copied on the overall sub-carriers or on a subset of them and multiplied by a chip of the spreading code assigned to the specific user.

The spreading in the frequency domain allows simple methods of signal detection; in fact, since the fading on each sub-carriers can be considered flat, simple equalization with one complex-valued multiplication per sub-carrier can be realized. Furthermore, since the spreading code length does not have to be necessarily chosen equal to the number of sub-carriers, MC-CDMA structure allows flexibility in the system design (K. Fazel, 2003).
2. Equalization techniques

The main impairment of this multiplexing technique is given by the MAI, which occurs in the presence of multipath propagation due to loss of orthogonality among the received spreading codes. In conventional MC-CDMA systems, the mitigation of MAI is accomplished at the receiver by employing single-user or multiuser detection schemes. In fact, the exploitation of suitable equalization techniques at the transmitter or at the receiver, can efficiently combine signals on different sub-carriers, toward system performance improvement.

We focus on the downlink of MC-CDMA systems and, after an overall consideration on general combining techniques, we consider linear equalization, representing the simplest and cheapest techniques to be implemented (this can be relevant in the downlink where the receiver is in the user terminal). The application of orthogonal codes, such as Walsh-
Hadamard (W-H) codes for a synchronous system (e.g., the downlink of a cellular system) guarantees the absence of MAI in an ideal channel and a minimum MAI in real channels.\(^1\)

### 2.1 Linear equalization

Within linear combining techniques, various schemes based on the channel state information (CSI) are known in the literature, where signals coming from different sub-carriers are weighted by suitable coefficients \(G_m\) (\(m\) being the sub-carrier index).

The equal gain combining (EGC) consists in equal weighting of each sub-carrier contribution and compensating only the phases as in (1)

\[
G_m = \frac{H_m^*}{|H_m|}
\]

where \(G_m\) indicates the \(m\)th complex channel gain and \(H_m\) is the \(m\)th channel coefficient (operation * stands for complex conjugate).

If the number of active users is negligible with respect to the number of sub-carriers, that is the system is noise-limited, the best choice is represented by a combination in which the sub-carrier with higher signal-to-noise ratio (SNR) has the higher weight, as in the maximal ratio combining (MRC)

\[
G_m = H_m^*.
\]

The MRC destroys the orthogonality between the codes. For this reason, when the number of active user is high (the system is interference-limited) a good choice is given by restoring at the receiver the orthogonality between the sequences. This means to cancel the effects of the channel on the sequences as in the orthogonality restoring combining (ORC), also known as zero forcing, where

\[
G_m = \frac{1}{H_m}.
\]

This implies a total cancellation of the multiuser interference, but, on the other hand, this method enhances the noise, because the sub-carriers with low SNR have higher weights. Consequently, a correction on \(G_m\) is introduced with threshold orthogonality restoring combining (TORC)

\[
G_m = u(|H_m| - \rho_{TH}) \frac{1}{H_m}
\]

where \(u(\cdot)\) is the unitary-step function and the threshold \(\rho_{TH}\) is introduced to cancel the contributions of sub-carriers highly corrupted by the noise.

However, exception made for the two extreme cases of one active user (giving MRC) and negligible noise (giving ORC) the presented methods do not represent the optimum solution for real cases of interest.

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\(^1\) In the uplink a set of spreading codes, such as Gold codes, with good auto- and cross-correlation properties, should be employed. However in this case a multi-user detection scheme in the receiver is essential because the asynchronous arrival times destroy orthogonality among the sub-carriers.
The optimum choice for linear equalization is the minimum mean square error (MMSE) technique, whose coefficient can be written as

\[ G_m = \frac{H^*_m}{|H_m|^2 + \frac{1}{N\gamma}} \tag{5} \]

where \( N_u \) is the number of active users and \( \gamma \) is the mean SNR averaged over small-scale fading. Hence, in addition to the CSI, MMSE requires the knowledge of the signal power, the noise power, and the number of active users, thus representing a more complex linear technique to be implemented, especially in the downlink, where the combination is typically performed at the mobile unit.

To overcome the additional complexity due to estimation of these quantities, a low-complex suboptimum MMSE equalization can be realized (K. Fazel, 2003). With suboptimum MMSE, the equalization coefficients are designed such that they perform optimally only in the most critical cases for which successful transmission should be guaranteed

\[ G_m = \frac{H^*_m}{|H_m|^2 + \lambda} \tag{6} \]

where \( \lambda \) is the threshold at which the optimal MMSE equalization guarantees the maximum acceptable bit error probability (BEP) and requires only information about \( H_m \). However, the value of \( \lambda \) has to be determined during the system design and varies with the scenario.

A new linear combining technique has been recently proposed, named partial equalization (PE), whose coefficient \( G_m \) is given by (Conti et al., 2007)

\[ G_m = \frac{H^*_m}{|H_m|^1 + \beta} \tag{7} \]

where \( \beta \) is the PE parameter having values in the range of \([-1, 1]\). It may be observed that, being parametric with \( \beta \), (7) reduces to EGC, MRC and ORC for \( \beta = 0, -1, \) and 1, respectively. Hence, (7) includes in itself all the most commonly adopted linear combining techniques.

Note also that, while MRC, and ORC are optimum in the extreme cases of noise-limited and interference-limited systems, respectively, for each intermediate situation an optimum value of the PE parameter \( \beta \) can be found to optimize the performance. Moreover, the PE scheme has the same complexity of EGC, MRC, and ORC, but it is more robust to channel impairments and to MAI-variations (Conti et al., 2007).

2.2 Non-linear equalization

Linear equalization techniques compensate the distortion due to flat fading, by simply performing one complex-valued multiplication per sub-carrier. If the spreading code structure of the interfering signals is known, the MAI could not be considered in advance as noise-like, yielding to suboptimal performance.

Non-linear multiuser equalizers, such as interference cancellation (IC) and maximum likelihood (ML) detection, exploit the knowledge of the interfering users’ spreading codes in the detection process, thus improving the performance at the expense of higher receiver complexity (Hanzo et al., 2003).
IC is based on the detection of the interfering users’ information and its subtraction from the received signal before the determination of the desired user’s information. Two kinds of IC techniques exist: parallel and successive cancellation. Combinations of parallel and successive IC are also possible. IC works in several iterations: each detection stage exploits the decisions of the previous stage to reconstruct the interfering contribution in the received signal. It can be typically applied in cellular radio systems to reduce intra-cell and inter-cell interference. Note that IC requires a feedback component in the receiver and the knowledge of which users are active.

The ML detection attains better performance since it is based on optimum maximum likelihood detection algorithms which optimally estimate the transmitted data. Many optimum ML algorithms have been presented in literature and we remind the reader to (Hanzo et al., 2003; K. Fazel, 2003) for further investigation which are out of the scope of the present chapter. However, since the complexity of ML detection grows exponentially with the number of users and the number of bits per modulation symbol, its use can be limited in practice to applications with few users and low order modulation. Furthermore, also in this case as for IC, the knowledge about which users are active is necessary to compute the possible transmitted sequences and apply ML criterions.

2.3 Objectives of the chapter

We propose a general and parametric analytical framework for the performance evaluation of the downlink of MC-CDMA systems with PE. In particular,

- we evaluate the performance in terms of bit error probability (BEP);
- we derive the optimum PE parameter \( \beta \) for all possible number of sub-carriers, active users, and for all possible values of the SNR;
- we show that PE technique with optimal \( \beta \) improves the system performance still maintaining the same complexity of MRC, EGC and ORC and is close to MMSE;
- we consider a combined equalization (CE) scheme jointly adopting PE at both the transmitter and the receiver and we investigate when CE introduces some benefits with respect to classical single side equalization.

3. System model

We focus on PE technique, that being parametric includes previously cited linear techniques and allows the derivation of a general framework to assess the performance evaluation and sensitivity to system parameters.

3.1 Transmitter

Referring to binary phase shift keying (BPSK) modulation and to the transmitter block scheme depicted in Fig. 1(a), the transmitted signal referred to the \( k \)th user, can be written as

\[
s^{(k)}(t) = \sqrt{\frac{2F_B}{M}} \sum_{m=-\infty}^{\infty} \sum_{i=0}^{M-1} c_{m}^{(k)} a^{(k)} \hat{f}(t - iT_B) \cos(\varphi_m)
\]

(8)
where $E_b$ is the energy per bit, $i$ denotes the data index, $m$ is the sub-carrier index, $c_m$ is the $m$th chip (taking value $\pm 1$), $a^{(k)}_i$ is the data-symbol transmitted during the $i$th time-symbol, $g(t)$ is a rectangular pulse waveform, with duration $[0,T]$ and unitary energy, $T_b$ is the bit-time, $\varphi_m = 2\pi f_m t + \phi_m$ where $f_m = f_0 + m \cdot \Delta f$ is the sub-carrier-frequency (with $\Delta f \cdot T$ and $f_0 T$ integers to have orthogonal frequencies) and $\phi_m$ is the random phase uniformly distributed within $[-\pi, \pi]$. In particular, $T_b = T + T_g$ is the total OFDM symbol duration, increased with respect to $T$ of a time-guard $T_g$ (inserted between consecutive multi-carrier symbols to eliminate the residual inter symbol interference, ISI, due to the channel delay spread). Note that we assume rectangular pulses for analytic purposes. However, this does not lead the generality of the work. In fact, a MC-CDMA system is realized, in practice, through inverse fast Fourier transform (IFFT) and FFT at the transmitter and receiver, respectively. After the sampling process, the signal results completely equivalent to a MC-CDMA signal with rectangular pulses in the continuous time-domain.

Considering that, exploiting the orthogonality of the code, all the different users use the same carriers, the total transmitted signal results in

$$s(t) = \sum_{k=0}^{N_u-1} \sum_{m=-\infty}^{M-1} c_m^{(k)} [i] g(t - iT_b) \cos(\varphi_m)$$

(9)

where $N_u$ is the number of active users and, because of the use of orthogonal codes, $N_u \leq M$.

### 3.2 Channel model

Since we are considering the downlink, focusing on the $n$th receiver, the information associated to different users experiments the same fading. Due to the CDMA structure of the system, each user receives the information of all the users and select only its own data through the spreading sequence. We assume the impulse response of the channel $h(t)$ as time-invariant during many symbol intervals.

We employ a frequency-domain channel model in which the transfer function, $H(f)$, is given by

$$H(f) = H(f_m) = \alpha_m e^{j\psi_m} \text{ for } |f - f_m| < \frac{W_s}{2}, \forall m$$

(10)

where $\alpha_m$ and $\psi_m$ are the $m$th amplitude and phase coefficients, respectively, and $W_s$ is the transmission bandwidth of each sub-carrier. The assumption in (10) means that the pulse shaping still remains rectangular even if the non-distortion conditions are not perfectly verified. Hence, the response $g'(t)$ to $g(t)$ is a rectangular pulse with unitary energy and duration $T' \triangleq T + T_d$, being $T_d \leq T_b$ the time delay. Note that this assumption is helpful in the analytical process and does not impact in the generality of the work.

We assume that each $H(f_m)$ is independent identically distributed (i.i.d.) complex zero-mean Gaussian random variable (r.v.) with variance, $\sigma^2_H$, related to the path-loss $L_p$ as $1/L_p = \mathbb{E}[\alpha^2] = \sigma^2_H$.  

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3 We assume orthogonal sequences $c^{(k)}$ for different users, such that:

$$<c^{(k)}_m, c^{(k')}_{m'}) = \begin{cases} M & k = k' \\ 0 & k \neq k' \end{cases}$$
3.3 Receiver

The received signal can be written as

\[ r(t) = \sqrt{\frac{2E_b}{M}} \sum_{k=0}^{N_u-1} \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} \alpha_m c_m^{(k)}(i)[i] g'(t - iT_b) \cos(\phi_m) + n(t) \] (11)

where \( n(t) \) is the additive white Gaussian noise with two-side power spectral density (PSD) \( \frac{N_0}{2} \), \( \phi_m = 2\pi f_m t + \theta_m \), and \( \theta_m \triangleq \phi_m + \psi_m \). Note that, since \( \theta_m \) can be considered uniformly distributed in \([-\pi, \pi]\), we can consider \( \angle H(f_m) \sim \theta_m \) in the following.

The receiver structure is depicted in Fig. 1(b). Focusing, without loss of generality, to the \( l \)th sub-carrier of user \( n \), the receiver performs the correlation at the \( j \)th instant (perfect synchronization and phase tracking are assumed) of the received signal with the signal \( c_l^{(n)} \sqrt{2} \cos(\tilde{\phi}_l) \), as

\[ z_l^{(n)}[j] = \frac{1}{\sqrt{T}} \int_{jT_b}^{(j+1)T_b} r(t) c_l^{(n)} \sqrt{2} \cos(\tilde{\phi}_l) dt. \] (12)

Substituting (11) in (12), the term \( z_l^{(n)}[j] \) results in (13)

\[ z_l^{(n)}[j] = \sqrt{\frac{E_b}{MT}} \sum_{i=-\infty}^{\infty} \int_{jT_b}^{(j+1)T_b} \sum_{k=0}^{N_u-1} \sum_{m=0}^{M-1} \alpha_m c_m^{(k)}(i)[i] g'(t - iT_b) \]

\[ \times \cos(\tilde{\phi}_m) \cos(\tilde{\phi}_l) dt + \int_{jT_b}^{(j+1)T_b} \frac{\sqrt{2} c_l^{(n)} \sqrt{T}}{T} n(t) \cos(\tilde{\phi}_l) dt \]

\[ = \sqrt{\frac{E_b}{M}} \alpha_l c_l^{(n)}[j] + \sqrt{\frac{E_b}{M}} \alpha_l \sum_{k=0, k \neq l}^{N_u-1} c_l^{(k)}[j] + n_l[j] \] (13)

where \( \delta_d \triangleq 1/(1 + T_d/T) \) represents the loss of energy caused by the time-spreading of the impulse.

4. Decision variable

The decision variable, \( v^{(n)}[j] \), is obtained by linearly combining the weighted signals from each sub-carrier as follows\(^4\)

\[ v^{(n)} = \sum_{l=0}^{M-1} |G_l| z_l^{(n)} \] (14)

where \( |G_l| \) is a suitable amplitude of the \( l \)th equalization coefficient. By considering PE, the weight for the \( l \)th sub-carrier is given by

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\(^4\) For the sake of conciseness in our notation, since ISI is avoided, we will neglect the time-index \( j \) in the following.
\[ G_l = \frac{H^*(f_l)}{|H^*(f_l)|^{1-\beta}}, \quad -1 \leq \beta \leq 1. \]  

(15)

Therefore, from (13) and (14) we can write

\[ v^{(n)} = \sqrt{\frac{U}{M}} \sum_{l=0}^{M-1} \alpha_l^{1-\beta} a_l^{(n)} + \sqrt{\frac{N}{M}} \sum_{l=0}^{N-1} \alpha_l^{1-\beta} n_l + \sqrt{\frac{I}{M}} \sum_{l=0}^{I-1} \sum_{k=0,k \neq n} \alpha_l^{1-\beta} c_l^{(n)(k)} a_l^{(k)}. \]  

(16)

At this point, the distribution of the test statistic can be obtained by studying the statistics of \( U, I \) and \( N \) in (16).

### 4.1 Interference term

Exploiting the properties of orthogonal codes, the interference term can be rewritten as

\[ I = \sqrt{\frac{E_b \delta_d}{M}} \sum_{k=0,k \neq n}^{M-1} a^{(k)} \left( \sum_{h=1}^{A_1} \alpha_1^{1-\beta} - \sum_{h=1}^{A_2} \alpha_2^{1-\beta} \right), \]  

(17)

where indexes \( x_h \) and \( y_h \) define the following partition

\[ c^{(n)}[x_h]c^{(k)}[x_h] = 1 \]  

(18)

\[ c^{(n)}[y_h]c^{(k)[y_h]} = -1 \]  

(19)

\[ \{x_h\} \cap \{y_h\} = 0,1,2,\ldots,M-1. \]  

(20)

For large \( M \), we can apply the central limit theorem (CLT) to each one of the internal sums in (17) obtaining

\[ A_1, A_2 \sim N\left(0, M \zeta_\beta(\alpha)\right) \]  

(21)

where \( \zeta_\beta(\alpha) \) indicates the variance of \( \alpha^{1-\beta} \) given by

\[ \zeta_\beta(\alpha) \triangleq \mathbb{E}\{ (\alpha^{1-\beta})^2 \} - (\mathbb{E}\{\alpha^{1-\beta}\})^2. \]  

(22)

Therefore, \( A \triangleq A_1 - A_2 \) is distributed as

\[ A \sim N\left(0, M \zeta_\beta(\alpha)\right). \]  

(23)

By exploiting the symmetry of the Gaussian probability density function (p.d.f.) and the property of the sum of uncorrelated (and thus independent) Gaussian r.v.’s (\( \tilde{A}_k = a^{(k)} A \sim (0,M \zeta_\beta(\alpha)) \)), the interference term results distributed as

\[ I \sim N\left(0, \sigma_I^2 \triangleq E_b \delta_d (N - 1) \zeta_\beta(\alpha)\right). \]  

(24)
4.2 Noise term

The thermal noise at the combiner output is given by

\[ N = \sum_{l=0}^{M-1} \alpha_l \beta n_l \]  

(25)

where terms \( \alpha_l \) and \( n_l \) are independent and \( n_l \) is zero mean. Thus, \( N \) consists on a sum of i.i.d zero mean r.v.’s with variance \( N_0/2 \ \mathbb{E}\{\alpha^{2\beta}\} \). By applying the CLT, we approximate the unconditioned noise term \( N \) as

\[ N \sim \mathcal{N}\left(0, \sigma_N^2 \triangleq M \frac{N_0}{2} \ \mathbb{E}\{\alpha^{-2\beta}\}\right). \]  

(26)

4.3 Useful term

By applying the CLT, the gain \( U \) on the useful term in (16) results distributed as

\[ U \sim \mathcal{N}\left(\sqrt{E_b \delta_d M \mathbb{E}\{\alpha_1^{-\beta}\}}, E_b \delta_d \zeta_\beta(\alpha)\right). \]  

(27)

4.3.1 Independence between each term

By noting that \( a^{(k)} \) is zero mean and statistically independent on \( \alpha \), \( A \), and \( n \), it follows that \( \mathbb{E}\{I N\} = \mathbb{E}\{I U\} = 0 \). Since \( n \) and \( \alpha \) are statistically independent, the \( \mathbb{E}\{N U\} = 0 \). The fact that \( I \), \( N \) and \( U \) are uncorrelated Gaussian r.v.’s implies they are also independent.

5. Bit error probability evaluation

From (24) and (26) we obtain

\[ I + N \sim \mathcal{N}\left(0, E_b \delta_d (N_u - 1) \zeta_\beta(\alpha) + M \mathbb{E}\{\alpha^{-2\beta}\} \frac{N_0}{2}\right) \]  

(28)

that can be applied to the test statistic in (16) to derive the BEP conditioned to the r.v. \( U \) as

\[ P_b |U=1 = \frac{1}{2} \text{erfc}\left(\frac{U}{\sqrt{2(\sigma_I^2 + \sigma_N^2)}}\right). \]  

(29)

By applying the law of large number (LLN), that is approximating \( \sum_{l=0}^{M-1} \alpha_l^{-\beta} \) with \( M \mathbb{E}\{\alpha^{-\beta}\} \), we can derive the unconditioned BEP as

\[ P_b = \frac{1}{2} \text{erfc}\left(\frac{E_b \delta_d (\mathbb{E}\{\alpha^{-1}\})^2}{2 E_b \delta_d \frac{N_u - 1}{M} \zeta_\beta(\alpha) + M \mathbb{E}\{\alpha^{-2\beta}\} N_0}\right) \]  

(30)

where it can be evaluated that
\begin{align}
\mathbb{E}\{\alpha^{1-\beta}\} &= (2\sigma_H^2)^{1-\beta} \frac{\Gamma \left( \frac{3-\beta}{2} \right)}{\Gamma(1-\beta)} \\
\mathbb{E}\{\alpha^{-2\beta}\} &= (2\sigma_H^2)^{-\beta} \frac{\Gamma(1-\beta)}{\Gamma(2-\beta) - \Gamma^2 \left( \frac{3-\beta}{2} \right)} \\
\zeta_\beta(\alpha) &= (2\sigma_H^2)^{1-\beta} \left[ \Gamma(2-\beta) - \Gamma^2 \left( \frac{3-\beta}{2} \right) \right]
\end{align}

being \( \Gamma(z) \) the Euler Gamma function. Hence, we can write

\begin{equation}
P_b = \frac{1}{2} \text{erfc} \left( \frac{\Gamma \left( \frac{3-\beta}{2} \right)}{\sqrt{2}} \sqrt{\frac{N_u - 1}{M} \left[ \Gamma(2-\beta) - \Gamma^2 \left( \frac{3-\beta}{2} \right) \right]} + \Gamma(1-\beta) \right)
\end{equation}

where

\begin{equation}
\overline{\gamma} = \frac{2\sigma_H^2 \mathbb{E}_b}{N_0}
\end{equation}

represents the mean SNR averaged over small-scale fading.

Note that the BEP expression is general in \( \beta \) and it is immediate to verify that results in the expressions for EGC (\( \beta = 0 \)) and MRC (\( \beta = -1 \)) as in (Yee et al., 1993). As a benchmark, note also that for MRC with one active user (i.e., \( N_u = 1 \)), (34) becomes

\begin{equation}
P_b = \frac{1}{2} \text{erfc} \sqrt{\overline{\gamma}}
\end{equation}

that is independent on the number of sub-carrier \( M \) and represents the well known limit of the antipodal waveforms in AWGN channel. This means that the approximation due to LLN is equivalent to assume that we have a number of sub-carriers (\( M \)) sufficiently high to saturate the frequency-diversity, then the transmission performs as in the absence of fading.

### 5.1 Optimum choice of the combining parameter

Now we will analyze the proposed PE technique with the aim of finding the optimum value of \( \beta \), defined as the value within the range \([-1,1]\] that minimizes the BEP

\begin{equation}
\beta^{(\text{opt})} = \arg \min_{\beta} \left\{ P_b(\beta, \overline{\gamma}) \right\}
\end{equation}

\begin{equation}
= \arg \max_{\beta} \left\{ \frac{\Gamma^2 \left( \frac{3-\beta}{2} \right) \overline{\gamma}}{2 \frac{N_u - 1}{M} \left[ \Gamma(2-\beta) - \Gamma^2 \left( \frac{3-\beta}{2} \right) \right]} \right\}.
\end{equation}

It will be shown in the numerical results that the approximation on the BEP does not significantly affect \( \beta^{(\text{opt})} \). By forcing to zero the derivative of the argument in (37), after some mathematical manipulations we obtain the following expression
\[ \left[ \Psi \left( \frac{3 - \beta}{2} \right) - \Psi(1 - \beta) \right] \left[ \frac{1}{\xi} + (1 - \beta) \right] - 1 = 0 \]  

(38)

where \( \Psi(x) \) is the logarithmic derivative of the Gamma function, the so-called Digamma-function defined as \( \Psi(x) \triangleq \frac{d \ln \Gamma(x)}{dx} \) (Gradshteyn & Ryzhik, 2000), and

\[ \xi = 2 \gamma \frac{N_u - 1}{M} = 2 \gamma S_L \]  

(39)

being \( S_L \) the system load. In (Zabini et al., to appear), the analysis has been extended also to derive the optimum \( \beta \) with imperfect channel estimation and correlated fading showing that the optimum PE parameter is not significatively affected by channel estimation errors meaning that it is possible to adopt the value of the PE parameter which would be optimum in ideal conditions even for estimation errors bigger than 1% (Zabini et al., 2007; to appear).

The parameter \( \xi \) quantifies how much the system is noise-limited (low values) or interference-limited (high values), and (38) represents the implicit solution, for the problem of finding the optimum value of \( \beta \) for all possible values of SNR, number of sub-carriers and number of users. Indeed, (38) open the way to an important consideration. In fact, the optimum \( \beta \) only depends, through \( \xi \), on slowly varying processes such as the SNR (averaged over fast fading then randomly varying according to shadowing), the number of users and the number of sub-carriers. This means that it could be reliable an adaptive partial equalization technique in which \( \beta \) is slowly adapted to the optimum value for the current set of \( \bar{\gamma}, N_u \) and \( M \).

6. Numerical results

In this Section, numerical results on the BEP and the optimum \( \beta \) in different system conditions are shown. Firstly, the goodness of the presented approach is proved by comparison with simulations. In particular, Fig. 2 shows the BEP as a function of \( \beta \) for different values of \( \bar{\gamma} \) (5 dB and 10 dB) and \( N_u = M = 1024 \). Analysis and simulations appear to be in a good agreement, in particular for what concerns the value of \( \beta \) providing the minimum for the BEP. Moreover, it can be noted that the choice of the optimum value of \( \beta \) guarantees a significant improvement in the performance with respect to the cases of MRC (\( \beta = -1 \)), EGC (\( \beta = 0 \)) and ORC (\( \beta = 1 \)); this improvement appears more relevant as the SNR increases.

The performance improvement of PE technique with optimum \( \beta \) with respect to classical MRC can be evaluated, for different system load \( S_L = (N_u - 1)/M \) and SNRs, by observing Fig. 3. As an example, at \( \bar{\gamma} = 8 \) dB with \( S_L = 20\% \) the BEP is about 0.005 with optimum \( \beta \) against 0.03 with MRC, whereas for \( S_L = 60\% \) is about 0.015 and 0.11, for optimum \( \beta \) and MRC, respectively. When the system is fully-loaded, Fig. 3 also shows a comparison with MMSE (from (Slimane, 2000)) and TORC detector. For TORC we checked that \( \rho_{TH} = 0.25 \) is a good value for the SNR range considered. As can be observed, MMSE always provides the better performance and it is about 1 - 1.5 dB away from that obtained with PE technique with optimum \( \beta \). Note also that the system with optimum \( \beta \) and system load 60% performs as fully-loaded MMSE.
Fig. 2. BEP as a function of the PE parameter $\beta$ for $\bar{\gamma} = 5$ and 10 dB in fully loaded system conditions. Comparison between analysis and simulation.

Fig. 3. BEP as a function of the mean SNR for system load $S_L = (N_u - 1)/M$ equal to 20%, 60% and fully-loaded when MRC or partial equalization with optimum $\beta$ are adopted. For the fully-loaded case, the comparison includes also MMSE (from (Slimane, 2000)) and TORC detector.
In Fig. 4 the impact of different equalization strategies on the BEP as a function of the number of active users, $N_u$, is reported for $\gamma = 10$ dB and $M = 1024$. First of all it can be noted that the optimum $\beta$ always provides the better performance; then, it can be observed that when few users are active MRC represents a good solution, approaching the optimum, crossing the performance of EGC for a system load about $1/64 \div 1/32$ (i.e., $N_u = 16 \div 32$) and the performance of a TORC detector with $\rho_{th} = 0.25$ for a system load about $1/16 \div 1/8$. Note that a fixed value of $\beta$ equal to 0.5 represents a solution close to the optimum for system loads ranging in $1/4 \div 1$ (i.e., $N_u = 256 \div 1024$) and the performance still remain in the same order for all system loads.

7. Combined equalization

Another approach to combine the sub-carriers contributions consists in applying pre-equalization at the transmitter in conjunction with post-equalization at the receiver, thereby splitting the overall equalization process on the two sides (Masini & Conti, 2009). We will call this process combined equalization (CE). The transmitter and receiver block schemes are depicted in Fig. 5.

A similar approach was proposed in (Cosovic & Kaiser, 2007), where the performance was analytically derived in the downlink for a single user case and in (Masini, 2008), where PE was considered at the transmitter and threshold ORC (TORC) at the receiver. For time division duplex direct sequence-CDMA systems a pre and post Rake receiver scheme was presented in (Barreto & Fettweis, 2000). Here we present a complete framework useful to evaluate the performance of CE (i) in a multiuser scenario; (ii) analytically evaluating optimal values for PE parameters; (iii) investigating when combined equalization introduces some benefits with respect to classical single side equalization techniques.
We assume CSI simultaneously available at both the transmitter and the receiver in order to evaluate the impact of a combined equalization at both sides on the system performance in terms of BEP with respect to single-side equalization. In particular we assume PE performed at both sides, thus allowing the derivation of a very general analytical framework for the BEP evaluation and for the explicit derivation of the performance sensitivity to the system parameters.

7.1 Transmitter

The signal transmitted in the downlink to the totality of the users can be written as

\[ s(t) = \sqrt{\frac{2E_b}{M}} \sum_{k=0}^{N_b-1} \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} c_m^{(k)}(t) \int_{\frac{iT}{M}}^{\frac{(i+1)T}{M}} \left[ a_{m,i}(t) \right] G_{m,\text{pre}} g(t - iT_b) \cos(\varphi_m). \] (40)

where \( G_{m,\text{pre}} \) is the pre-equalization coefficient given by
and $G_m$ is the pre-equalization coefficient without power constraint given by (7) and here reported

$$G_m = \frac{H_m^*}{|H_m|^{1+\beta_t}}$$  \hspace{1cm} (42)$$

with $\beta_t$ representing the PE coefficient at the transmitter.

The coefficient $G_{m,\text{pre}}$ has to be normalized such that the transmit power is the same as in the case without pre-equalization, that means

$$\sum_{m=0}^{M-1} |G_{m,\text{pre}}|^2 = M.$$  \hspace{1cm} (43)$$

Note that when $\beta_t = -1, 0, \text{ and } 1$, coefficient in (41) reduces to the case of MRC, EGC and ORC, respectively. Since we are considering the downlink we assume perfect phase compensation, the argument of $G_{m,\text{pre}}$ can be included inside $\phi_m$ in (40), explicitly considering only its absolute value.

Note that, to perform pre-equalization, CSI has to be available at the transmitter; this could be possible, for example, in cellular systems where the mobile unit transmits pilot symbols in the uplink which are used by the base station for channel estimation.

7.2 Receiver

By assuming the same channel model as in Sec. 3.2, the received signal results

$$r(t) = \sqrt{\frac{2E_b}{M}} \sum_{k=0}^{N_k-1} \sum_{i=-\infty}^{+\infty} \sum_{m=0}^{M-1} \alpha_m c_m^{(k)} a^{(k)}[i] g(t-iT_b) |G_{m,\text{pre}}| \cos(\varphi_m) + n(t).$$  \hspace{1cm} (44)$$

At the receiver side, the post-equalization coefficient has to take into account not only the effect of channel but also of pre-equalization in order to counteract additional distortion caused by the last one. (see Fig. 5). Hence, it is given by

$$G_{l,\text{post}} = \frac{(G_{l,\text{pre}}H_l)^*}{|G_{l,\text{pre}}H_l|^{1+\beta_R}}$$  \hspace{1cm} (45)$$

where $\beta_R$ is the post-equalization parameter. Note again that when $\beta_R = -1, 0 \text{ and } 1$, (45) reduces to MRC, EGC and ORC, respectively.

8. Decision variable for combined equalization

Adopting the same procedure as in Sec. 4 and, hence, by linearly combining the weighted signals from each sub-carriers, we obtain the decision variable

$$v^{(n)} = \sum_{l=0}^{M-1} |G_{l,\text{post}}| z_l^{(n)}$$  \hspace{1cm} (46)$$
where the received signal before combination can be evaluated as

\[
z^{(n)}[j] = \frac{E_b \delta_d}{M} \alpha_j^{1-\beta_t} \sqrt{\frac{M}{\sum_{i=0}^{M-1} \alpha_i^{-2 \beta_t}}} a^{(n)}[j] + \sqrt{\frac{E_b \delta_d}{M}} c_i^{(n)} \alpha_i^{1-\beta_t} \sum_{k=0, k \neq n}^{N-1} c_i^{(k)} d^{(k)}[j] + n_i[j].
\]  

(47)

After some mathematical manipulation

\[
v^{(n)} = \frac{E_b \delta_d}{M} \sum_{i=0}^{M-1} \alpha_i^{1-\beta_t}(1-\beta_k) a^{(n)} + \frac{E_b \delta_d}{M} \sum_{i=0}^{M-1} \sum_{k=0, k \neq n}^{N-1} \alpha_i^{1-\beta_t}(1-\beta_k) c_i^{(n)} c_i^{(k)} d^{(k)}
\]  

+ \sum_{i=0}^{M-1} \alpha_i^{1-\beta_k}(1-\beta_t) n_i \sqrt{\frac{\sum_{i=0}^{M-1} \alpha_i^{-2 \beta_t}}{M}}
\]  

(48)

where \(U\), \(I\), and \(N\) represent the useful, interference, and noise term, respectively and whose statistic distribution has to be derived to evaluate the BEP.

Following the same procedure adopted in Sec. 4, we obtain

\[
U \sim \mathcal{N} \left( \sqrt{E_b \delta_d M} \mathbb{E} \{ \alpha_i^{1-\beta_t}(1-\beta_k) \}, \sigma_U^2 \right)
\]  

(49)

\[
I \sim \mathcal{N} \left( 0, \sigma_I^2 = E_b \delta_d (N_u - 1)(2 \sigma_{II}^2)(\beta_t^{-1}) \right)
\]  

(50)

\[
\times \left[ \Gamma(2 + \beta_t (\beta_R - 1) - \beta_k - 1) \Gamma \left( \frac{3 + \beta_t (\beta_R - 1) - \beta_k}{2} \right) \right]
\]  

(51)

\[
N \sim \mathcal{N} \left( 0, \sigma_N^2 = M \frac{N_0}{2} (2 \sigma_{II}^2)^{\beta_t + \beta_k (\beta_t^{-1})} \Gamma(1 - \beta_t) \Gamma(1 + \beta_k (\beta_t - 1)) \right)
\]  

(52)

Also in this case, since \(d^{(k)}\) is zero mean and statistically independent of \(\alpha_i\) and \(n_i\), and considering that \(n_i\) and \(\alpha_i\) are statistically independent and zero mean too, then \(\mathbb{E}[IN] = \mathbb{E}[IU] = 0\). Since \(n_i\) and \(\alpha_i\) are statistically independent, then \(\mathbb{E}[NI] = 0\). Moreover \(I, N, \) and \(U\) are uncorrelated Gaussian r.v.'s, thus also statistically independent.

9. Bit error probability evaluation with combined equalization

By applying the LLN to the useful term, that is by approximating \(U\) with its mean value, the BEP averaged over small-scale fading results

\[
P_b = \frac{1}{2} \text{erfc} \sqrt{\Xi},
\]  

(53)
where $\Xi$ is the signal-to-noise plus interference-ratio (SNIR) given by

$$
\Xi = \frac{v \Gamma^2 \left[ 3 + \beta_T (\beta_R - 1) - \beta_R \right]}{\Gamma \left[ 1 - \beta_T \right] \Gamma \left[ 1 + \beta_R (\beta_T - 1) \right] + 2v \sum_{n=1}^{N_u} \left[ \Gamma \left[ 2 + \beta_T (\beta_R - 1) - \beta_R \right] - \Gamma^2 \left[ 3 + \beta_T (\beta_R - 1) - \beta_R \right] \right]^{\frac{3}{2}}.}
$$

(54)

Note that when one between $\beta_T$ or $\beta_R$ is zero, (53) reduces to (34).

### 10. Optimum combination with combined equalization

We aim at deriving the optimal choice of the PE parameters, thus the couple $(\beta_T, \beta_R)$ jointly minimizing the BEP

$$
(\beta_T, \beta_R)^{(\text{opt})} = \arg \min_{\beta_T, \beta_R} \{ P_b(\beta_T, \beta_R, \overline{\gamma}) \}. \quad (13)
$$

However, being in the downlink, the receiver is in the mobile unit, hence, it is typically more convenient, if necessary, to optimize the combination at the transmitter (i.e., at the base station), once fixed the receiver. Therefore, we find the optimum values of $\beta_T$ defined as that values within the range $[-1,1]$ that minimizes the BEP for each $\beta_R$

$$
\beta_T^{(\text{opt})} = \arg \min_{\beta_T} \{ P_b(\beta_T, \beta_R, \overline{\gamma}) \} = \arg \max_{\beta_T} \{ \Xi \}. \quad (14)
$$

By deriving (54) with respect to $\beta_T$ and after some mathematical manipulation, we obtain the implicit solution given by (15)

$$
\xi = \frac{\Gamma \left[ 1 - \beta_T \right] \Gamma \left[ 1 + \beta_R (\beta_T - 1) \right]}{(\beta_R - 1) \Gamma \left[ 2 + \beta_T (\beta_R - 1) - \beta_R \right]} \left[ \psi \left[ \frac{3 + \beta_T (\beta_R - 1) - \beta_R}{2} \right] - \psi \left[ 2 + \beta_T (\beta_R - 1) - \beta_R \right] \right] \times \left\{ \left[ - \left( \beta_R - 1 \right) \psi \left[ \frac{3 + \beta_T (\beta_R - 1) - \beta_R}{2} \right] - \psi \left[ 1 - \beta_T \right] + \beta_R \psi \left[ 1 + \beta_R (\beta_T - 1) \right] \right] \right\}. \quad (15)
$$

#### 11. Numerical results for combined equalization

In Fig. 6, the BEP is plotted as a function of $\beta_T$ for different values of $\beta_R$ and mean SNR $\overline{\gamma} = 10$ dB in fully loaded system conditions ($M = N_u = 1024$). Note that, in spite of the post-PE technique, there is always an optimum value of $\beta_T$ minimizing the BEP and this value depends on $\beta_R$. Moreover, the BEP is also drastically dependent on $\beta_R$, meaning that a not suitable post-PE technique can even deteriorate the performance, with respect to one side combination, rather than improving it. Simulation results are also reported confirming the analysis especially in correspondence to the optimal $\beta_R$ (note that the analysis is confirmed for 64 sub-carriers and thus it is expected to be even more accurate for higher number of sub-carriers).^5

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^5 Similar considerations can be drawn for time- and frequency correlated SUI-x channels as shown, by simulation, in (Masini et al., 2008) referred to PE at the receiver.
Fig. 6. BEP vs. the pre-equalization parameter $\beta_T$ for different post-equalization parameter values $\beta_R$ and $\bar{\gamma} = 10$ dB in fully loaded system conditions. Comparison between analysis and simulation. Figure reprinted with permission from B. M. Masini, A. Conti, “Combined Partial Equalization for MC-CDMA Wireless Systems”, IEEE Communications Letters, Volume 13, Issue 12, December 2009 Page(s):884 – 886. ©2009 IEEE.

In Fig. 7, the BEP is plotted as a function of the mean SNR, $\bar{\gamma}$, in fully loaded system conditions ($M = N_u = 1024$). The effect of the combining techniques at the transmitter and the receiver can be observed: a suitable choice of coefficients (such as $\beta_T = 0.5$ and $\beta_R = 0.5$) improves the performance with respect to single side combination ($\beta_T = 0$, $\beta_R = 0.5$); however, a wrong choice (such as $\beta_T = 0.5$ and $\beta_R = -1$) can drastically deteriorate the BEP.

In Fig. 8, the BEP as a function of the system load $S_L$ in percentage is shown for $\bar{\gamma} = 10$ dB and different couples ($\beta_T$, $\beta_R$). Note how a suitable choice of pre- and post-PE parameters can increase the sustainable system load. At instance, by fixing a target BEP equal to $4 \cdot 10^{-3}$, with combination at the transmitter only (i.e., $\beta_T = 0.5$, $\beta_R = 0$) we can serve the 45% of users, while fixing $\beta_T = 0.5$ and adaptively changing $\beta_R$ following the system variations (i.e., always setting $\beta_R$ at the optimum value minimizing the BEP), the 100% of users can be served. The same performance can be obtained by fixing the combination parameter at 0.5 at the transmitter or at the receiver and adaptively changing the combination parameter at the...
Fig. 7. BEP vs. the mean SNR $\bar{\gamma}$ for different couples of $\beta_t$ and $\beta_k$ in fully loaded system conditions.

other side. The same performance can also be obtained by exploiting the couple of fixed parameter ($\beta_t = 0.5$, $\beta_k = 0.5$), thus avoiding the complexity given by parameters adaptation. It is also worth noting that a not suitable choice of combination parameters, such as ($\beta_t = -0.5$, $\beta_k = 0$) or ($\beta_t = 0.5$, $\beta_k = -0.5$) can even deteriorate the performance with respect to single side combination.

12. Final considerations

We summarized the main characteristics of MC-CDMA systems and presented a general framework for the analytical performance evaluation of the downlink of MC-CDMA systems with PE.

We can conclude that MC-CDMA systems may be considered for next generation mobile radio systems for their high spectral efficiency and the low receiver complexity due to the avoidance of ISI and ICI in the detection process. The spreading code length can be dynamically changed and not necessarily equal to the number of sub-carriers enabling a flexible system design and further reducing the receiver complexity.
To enhance their performance, PE can be adopted in the downlink, allowing good performance in fading channels still maintaining low the receiver complexity. The optimal choice of the PE parameter is fundamental to improve the performance in terms of BEP averaged over small-scale fading.

When CE is adopted at both the transmitter and the receiver a proper choice of PE parameters is still more important, to significatively improve the performance with respect to single-side detection.

The gain achieved by a suitable combination of transmission and reception equalization parameters could be exploited to save energy or increase the coverage range (a similar approach was used for partial power control in cellular systems in (Chiani et al., 2001)).

In case of non-ideal channel estimation, the performance results to be deteriorated; however, it has been shown that the optimum PE parameter is not significatively affected by channel estimation errors. The analysis for correlated fading channels and imperfect CSI has been
performed in (Zabini et al., to appear). optimum PE parameter with perfect CSI This means that, in practical systems, it is possible to adopt the value of the PE parameter which would be optimum in ideal conditions (it is simple to evaluate and does not require the knowledge of the channel estimation error) without a significant loss of performance, even for estimation errors bigger than 1% (Zabini et al., 2007; to appear).

The effect of block fading channels and time and frequency correlated fading channel on the performance of MC-CDMA systems with PE has been investigated in (Masini & Zabini, 2009) and (Masini et al., 2008), respectively, still showing the goodness of PE as linear equalization technique and still demonstrating that the PE parameter that is optimum in ideal scenarios still represents the best choice also in more realistic conditions.

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