The new scenario of the initial evolution of the Universe

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Abstract

We propose that the Universe created from "nothing" with relatively small particles number and quickly relaxed to quasiequilibrium state at the Planck parameters. The classic cosmological solution for this Universe with Λ-term has two branches divided by the gap. The quantum process of tunneling between the cosmological solution branches and kinetic of the second order relativistic phase transition in supersymmetric SU(5) model on the GUT scale are investigated by numerical methods. Einstein equations was solved together with the equations of relaxation kinetics. Other quantum geometrodynamics process (the bounce from singularity) and the Wheeler-De Witt equation are investigated also. For the formation of observable particles number the model of the slowly swelling Universe in the result of the multiple reproduction of cosmological cycles is arised naturally.

The inflation model and its basic modifications is very attractive and explains many cosmological problems. Today it is the cosmological paradigm. The standard conception of first order cosmological relativistic phase transitions (RPT) from strong overcooled high symmetric (HS) phase develops well. For the realization of this conception it was necessary
the inflaton potential with quite concrete properties (as rule this potential must have a wide flat part). However, the problem of the formation of observable particles number is not investigated in detail and usually they speak that in the process reheating the energy of collisions of bubble walls converts into energy individual quanta of scalar field, which then decayed into normal particles. We discuss the case when the inflation potential does not possess by specific properties, the Universe is created from "nothing" and RPT is near to second order.

We proposed that:

I. Plasma and vacuum of the Universe, which was created from "nothing", after processes of relaxation taking place near Planck parameters (see 4), are in quasiequilibrium state. In our opinion the superearly Universe created from "nothing" in anisotropic state (for example IX type of Bianchi) with some number of particles and with some nonequilibrium state of vacuum condensate.II. The topology of the Universe is closed. This suggestion is caused by fact that only this Universe can be created from "nothing" (local properties of this Universe approach to local properties of the flat Universe if the cosmological scenario solves the problem of the flatness/entropy).III. After going out of the Universe from singularity the initial number particles $N_o$ is large in comparison with unit ($N_o \sim 10^4 \div 10^6$; but it is small in comparison with particles number in observed Universe ($N_{obs.} \sim 10^{88}$).IV. RPT on the GUT scale ($T \sim 10^{16}$ Gev) which is more near to Planck scale is not the first order RPT. This one is the second order RPT for which the generation of new phase occurs by continuous mean.

One from possible series of RPT in the early Universe for the symmetry breaking is:

$$G \Rightarrow [SU(5)]_{SU SY} \Rightarrow [U(1) \times SU(2) \times SU(3)]_{SU SY}$$

$$\Rightarrow U(1) \times SU(2) \times SU(3) \Rightarrow U(1) \times SU(3) \Rightarrow U(1)$$

The only trace of first RPT is the initial $\Lambda$-term (the vacuum energy density) connected with interactions of the local multidimensional supergravity. The rest RPT are described by modern theories of elementary particles. During RPT when cooling of cosmological plasma
the vacuum condensate with negative density of energy is produced. This condensate has the asymptotic state equation \( p_{\text{vac}} = -\epsilon_{\text{vac}} = \text{const.} \) Thus the RPT series are accompanied by generation of negative contributions in cosmological \( \Lambda \)-term standing in Einstein equations. Accordingly to observational data after all RPT the final \( \Lambda \)-term is zero practically. For simplicity we have proposed the exact compensation of \( \Lambda \)-term already on the SUSY GUT energy scale. Our paper devotes the quantitative model of the RPT \([SU(5)]_{\text{SUSY}} \Rightarrow [U(1) \times SU(2) \times SU(3)]_{\text{SUSY}}\) on the scale \( \sim 10^{16} \text{ GeV} \). Some particles acquire a rest mass which is proportional of an average value of Higgs field after this gauge symmetry spontaneous breaking. The considered system consists from third subsystems: 1) gas of massless particles, 2) gas of massive particles interacting with vacuum condensate, 3) vacuum condensate. The reactions of massless and massive particles on the cosmological expansion is different. The change of these particles energy spectra because of red shift happens on different laws. To get of evolution equations of nonequilibrium system we use: a) the order parameter (OP) origin as C number average of Higgs field; b) the method of getting of relaxation kinetics equations for subsystems particles analogous to described in\[6\]; c) the method of analysis of nonequilibrium relativistic systems analogous to described in\[7\]; d) the estimation of creation particles local rate in variable OP field which was got by the method analogous to described in\[8\].

The total system of equations of our theory involves: Einstein equations with nonequilibrium energy momentum tensor of heterogeneity system

\[
\frac{3}{\kappa} \left( \frac{\dddot{a}}{a^2} + \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} \right) = \frac{k \pi^2}{16} T^4 + w (J_2 + \eta^2 J_1) + \frac{1}{2g^2} \dot{\eta}^2 + \frac{1}{8g^2} (\eta^2 - m^2)^2, \]

\[
\frac{1}{\kappa} \left( \frac{2\dddot{a}}{a} + \frac{2\dot{a}^2}{a^2} + \frac{1}{a^2} \right) = -\frac{k \pi^2}{48} T^4 - \frac{w}{3} J_2 + \frac{\eta^2 (2J_1 + \eta^2 J_0)}{3(4J_2 + 5\eta^2 J_1 + \eta^4 J_0)} w D - \frac{1}{2g^2} \dot{\eta}^2 + \frac{1}{8g^2} (\eta^2 - m^2)^2 \]

the evolution equations for dissipative function \( D \) and order parameter \( \eta \)
\[ D + \left(4 \frac{\dot{a}}{a} + \frac{1}{\tau}\right) D = \]
\[ = \frac{k \pi^2 T^4 [(2J_1 + \eta^2 J_o)(\eta \frac{\dot{a}}{a} + \eta \dot{\eta}) + b T \eta^2]}{k \pi^2 T^4 / 4 + w (4J_2 + 5\eta^2 J_1 + \eta^4 J_o)} \left[ 1 + \frac{D}{4J_2 + 5\eta^2 J_1 + \eta^4 J_o} \right] \]  
\[ (2) \]

\[ \dot{\eta} + \left[ 3 \frac{\dot{a}}{a} + bwg^2 T \left( 1 + \frac{D}{4J_2 + 5\eta^2 J_1 + \eta^4 J_o} \right) \right] \dot{\eta} + \]
\[ + \left\{wg^2 \left[ J_1 + \frac{(2J_1 + \eta^2 J_o) D}{4J_2 + 5\eta^2 J_1 + \eta^4 J_o} \right] + \frac{1}{2}(\eta^2 - m^2) \right\} \eta = 0 \]  
\[ (3) \]

and also the entropy equation, which can be transformed to the equation for temperature of plasma:

\[ \dot{T} + \frac{\dot{a}}{a} = \frac{w T [(2J_1 + \eta^2 J_o)(\eta \frac{\dot{a}}{a} + \eta \dot{\eta}) + b T \eta^2]}{k \pi^2 T^4 / 4 + w (4J_2 + 5\eta^2 J_1 + \eta^4 J_o)} \left[ 1 + \frac{D}{4J_2 + 5\eta^2 J_1 + \eta^4 J_o} \right] \]  
\[ (4) \]

where \( \kappa \simeq (10^{19} \text{GeV})^{-2} \) is the Einstein constant; \( k \) is the number of bosons freedom degrees (exactly equal to the number of fermions freedom degrees) the rest mass of which is equal to zero both in the high symmetric (HS) phase and in the low symmetric (LS) phase; \( w \) is an analogous number of particles freedom degrees acquiring of mass \( \eta \) in LS phase; \( g \) is the gauge coupling constant of particles with vacuum condensate; \( m = \text{const} \) is the limited value of particles mass when \( T \to 0 \) in the LS phase; \( b \) is the numerical coefficient unity order; \( \tau \) is the time of relaxation between subsystems massless and massive particles (we use units in which \( \hbar = c = 1 \));

\[ J_n(T, \eta) = \frac{1}{2\pi^2} \int_{-\infty}^{\infty} \frac{dpp^n}{\omega} \left( \frac{1}{\exp \frac{p}{T} - 1} + \frac{1}{\exp \frac{p}{T} + 1} \right) \]
\[ n = 0, 1, 2 \quad \omega = (p^2 + \eta^2)^{1/2} \]

characteristic integrals through which observable magnitudes of our model are expressed. In the theory (1)-(4) the property of the early Universe depends on total number of particles in closed space the critical value of which is: \( N_{cr} \equiv 7 \xi(3)(\frac{k + w}{2})^{1/4}(\frac{12g}{\pi \kappa m^2})^{3/2} \simeq 5 \times 10^{11} \),

where \( \xi(3) \) is the zeta function of Riemann. If the value of initial total particles number in HS plasma \( N_o < N_{cr} \) then the cosmological solution contains two branches divided by the gap (Fig. 1). The branch I is the Friedmann solution which is distorted slightly by
Λ-term and the branch II is the de Sitter solution which is distorted slightly by matter. The investigation of the plasma evolution temperature regime shows that on the branch I the minimal temperature $T_{I(\text{min})}$ (corresponding to max radius $a_{I(\text{max})}$) is essentially more than critical temperature $T_c = (\frac{4m^2}{\sqrt{g}})^{1/2}$ defining the boundary of thermodynamical stability of the HS phase. On the branch II the maximal temperature $T_{II(\text{max})}$ (corresponding of min radius $a_{II(\text{min})}$ is essentially less than $T_c$. From this hierarchy of temperatures it follows that: 1) the Universe evolving on the branch I doesn’t undergo to RPT in the LS phase (it goes out from initial singularity and come to final one in the HS phase; 2) on the branch II the Universe can’t be in the HS phase in principle. The branch II is classically prohibited. There are two reasons for it: at first the branch II is separated from the branch I by classically nonovercome gap; at second the branch II is thermodynamicaly instable. Note that the branch II exists formally when nonzero Λ-term takes place. When plasma and vacuum are in the LS phase and Λ-term is equal to zero, the branch II doesn’t appear at all. From the classical point of view the branch II does not exit, i.e. in general it exists only virtualy as classical unrealized variant of evolution. Therefore if the Universe, which was created from "nothing", has particles number less than critical one, then this Universe can not transfer in the macroscopic object containing observed particles number.

But the situation is changed radically in quantum geometrodynamics (QGD) of the closed Universe. In this theory there is a small but nonzero probability of tunneling through the gap dividing the branch I and the branch II of the classical solution. Let us to discuss shortly of the mathematical model of this phenomenon.

The problem is to construct the quantum analogue of equations (1)-(4) i.e. the equation of Wheeler-De Witt (WDW) for the Universe wave function $\psi = \psi(a, \eta)$. However, the dissipative dynamics of system (1)-(4) isn’t Hamilton one since formal methods of quantization don’t apply to solution of this problem. The necessary of the new quantum theory, which must correlate with the second law of thermodynamics, was discussed by Penrose. The absent of this theory compells us to decide of this task in two steps. On the first stage the dissipative processes aren’t take into account. We hope the quantum nondissipative
geometrodynamics reflects approximately the properties of processes of tunneling through barrier and bounce from singularity. On the second stage the dissipative processes are described by the classical method on basic of the eqs. (1)-(4).

The quantization nondissipative system is the realization of the Lapchinsky-Rubakov’s idea suggested, to describe the present of matter in the closed Universe by methods of the effective potential involved in WDW equation. The fact of the existence of Hamiltonian bond is needed in another discussion. This bond is caused by the gauge invariance of the theory relatively the time transformation. The arbitrary in time choice is removed by an addition condition which is put on the gauge variable. This variable is the algebraic form of metric components containing the $g_{oo}$-component. For example, if the gauge variable is $g_{oo} = \lambda^2$ then the WDW equation doesn’t depend from choice of the additional condition $F(\lambda, a, \eta) = 0$. However, we can bring the local - conform transformation of time and gauge variable

$$dt = af(a)dt', \quad \lambda = \lambda'af(a),$$

(5)

where $f(a)$ is an arbitrary function. In this case the equation doesn’t depend on the gauge condition $F(\lambda', a, \eta) = 0$ but it depends on the generator of local-conform transformation $f(a)$. Formally from the mathematically point of view this dependence is caused by the nonlinear coupling of gauge variable $\lambda = \lambda'af(a)$ with square of generalized impulse $p \sim \dot{a}$. The every variant putting order of operators $p$ and $f(a)$ which coincide with the property of hermiticity of Hamiltonian generates additional members in the WDW equation. These members can be interpreted as additional contribution in potential of WDW equation:

$$U_f(a) = \frac{\kappa}{24\pi^2} \left[ \frac{1}{4f} \frac{d^2 f}{da^2} - \frac{3}{16f^2} \left( \frac{df}{da} \right)^2 \right].$$

(6)

Thus the effect of the spontaneous breaking of symmetry relatively locally-conform time transformations (5) takes place in the WDW theory for the closed isotropic Universe. The additional contribution (6) has the sense of energy of some gravitational vacuum condensate (GVC) the production of which fixes the breaking of discussed symmetry in whole space
of the closed Universe. We propose the GVC must secure the bounce from singularity i.e. prolong the time of the Universe existence. This result will take place if

\[ f(a) = a^{4(S+1)}, \quad U_{f(a)} \equiv U_{S(a)} = \frac{\kappa}{24\pi^2} \frac{S(S + 1)}{a^2}, \quad (7) \]

where \( S = \text{const} > 0 \) is the parameter of the GVC. The WDW equation in this case is written for the Universe wave function \( \psi = \psi_{NS}(a, \eta) \) depending on two variables \( a, \eta \) and on two parameters \( N \) and \( S \)

\[-\frac{\kappa}{24\pi^2} \partial^2 \psi_{NS} \partial \frac{a^2}{\partial a^2} + \frac{g^2}{4\pi^2 a^2} \partial^2 \psi_{NS} \partial \frac{\eta^2}{\partial \eta^2} + \]

\[ + \left[ \frac{\kappa}{24\pi^2} \frac{S(S + 1)}{a^2} + \frac{6\pi^2}{\kappa} a^2 - 2\pi^2 a^4 \epsilon_N(a, \eta) - \frac{\pi^2}{4g^2} a^4 (\eta^2 - m^2)^2 \right] \psi_{NS} = 0, \quad (8)\]

where \( \epsilon_N(a, \eta) \) is the energy density of subsystems of particles the mathematics form for which coincides with thermodynamical expression. As we have proved the equation (8) has the most important property: the solution located on the HS vacuum (near \( \eta = 0 \)) for every values of the Universe radius is among its solutions. The main dependence of a wave function quasilocated on the HS vacuum on the Universe radius can be factorized by the separate function. This function satisfies the equation which is formally similar to the Schrodinger one for stationar states of some conditional "particle" in potential field \( U(a) \) the shape has been shown on Fig. 2. The asymptotics of the potential \( U(a) \) in the small \( a \) region is defined by the GVC energy (7). From Fig. 2 one can see that the GVC provides the quantum bounce from singularity. The quantum bounce hypothesis being accepted leads to this that Universe with small particles number oscilates quasiclassically in the region I of potential \( U(a) \). The sector II of potential \( U(a) \) corresponds to the branch II of the classical cosmological solution (compare Fig. 1 and Fig. 2). The probability of tunneling through the barrier dividing the branches I and II is exponential small when a particles number is small. It is increased monotonously with increase the number of oscillation cycles in the region I. If the number of oscillation cycles is large then the causely-consequence connections have been set up among all its space-time points and in this case the problem of horizon is absent.
We propose that after tunneling the causely-consequence connections among different points of the Universe are conserved.

Thus our Universe was created from "nothing" with small particles number and performed exponentially long oscillations in the region I of effective potential $U(a)$. Here the Universe existed in the HS phase. After large number of oscillations the Universe has undergone to the tunneling transition in the HS thermodynamically instable phase. If after tunneling the Universe is appeared directly near the barrier i.e. on the left boundary of the region II then it size is increased multiplely:

$$\frac{a_{II(min)}}{a_{I(max)}} = 2\left(\frac{N_{cr}}{N_0}\right)^{2/3}$$

(9)

Accordingly to (9) when the initial particles number $N_0 = 5 \times 10^5$ radius increases in $2 \times 10^4$ times in the result of the quantum tunneling. This phenomenon can be considered as analogue of classical inflation. After tunneling the Universe will be found in strongly nonequilibrium state. The relaxation of nonequilibrium plasma and vacuum to new equilibrium state corresponding the stable of LS phase is coming in the relaxation kinetics regime and is accompanied by sharply entropy increase. This process is described by the eqs. (1)-(4).

The classical state corresponding to minimum curve $a_{II}$ must appear with the greatest probability. The evolution of the Universe with the initial state $a(t_0) = a_{II(min)}$ and with $N_0 \simeq 5 \times 10^5$ shows on Fig. 3-5. The RPT begins in fact immediately after tunneling and is accompanied by nonlinear OP vibrations wit the frequency $\sim T_c$ (Fig. 3) and particles creation (Fig. 4). After damping of these vibrations and the RPT finish the particles number in the Universe is increased in $2.6 \times 10^6$ times (for $N_o = 5 \times 10^5$). The numerical experiments have shown that the total particles number in the Universe after relaxation coincides approximately with $N_{cr}$ for different values $N_o \sim 10^2 \div 10^9$. Thus he particles number in the macroscopic Universe after tunneling and RPT is expressed through the fundamental constants:

$$N_{cr} \sim \kappa^{-3/2}m^{-3} = \left(\frac{10^{19}GeV}{m}\right)^3$$

(10)
The classical evolution of the closed Universe shown on Fig. 3-5 finishes in singularity. However the calculation of QGD effects transfers the enter in singularity on the quantum bounce at the Planck parameters. The number of particles is conserved during this bounce since next classical evolution cycle starts for $N > N_{cr}$. The numerical research of this Universe evolution cycle was performed also. The main result is the conclusion that in the Universe with $N > N_{cr}$ the second order RPT happens quasiadiabatically. Relative increase of particles number for total evolution cycle is smaller than one percent. Thus we got the model of the slowly swelling Universe.

Further detalization of the scenario of the Universe evolution requires the calculation of RPT on others energetic scales. The Universe during of evolution must overcome some potential barriers similar to the barrier which is shown on Fig. 2. Accordingly (10) after last RPT on scale $\sim 100Mev$ we obtain the macroscopic Universe with particles number $N \sim 10^{60}$. The problem is that $N \sim 10^{88}$ in the observable Universe. We can formulate some hypothesis explaining as to obtain this. 1. The observable particles number has been created after multiple reproduction of cosmological cycles containing all series of RPT. During every cycle the particles number is increased in comparison with a previous one because of dissipative processes accompanying of RPT. 2. After tunneling through some barrier on some cycles of evolution the Universe can appear in a point of trajectory which is sufficiently far from left boundary of the region II (see Fig. 2). From the solution of the WDW equations it follows that the probability of this process is decreased exponentially as far as moving away from barrier. The numerical experiments, have shows that particles number creating during of RPT which is delayed on time, increases exponentially with time delay. 3. Effects of strong nonlinear interaction of different vacuum condensates in more complex series of RPT lead to time delay of these RPT (as example, the series $G \Rightarrow E_6 \Rightarrow O(10) \Rightarrow SU(5)$ can be considered). 4. The fourth variant connects with the hypothetical possibility of dynamics chaos regime in the region of nonequilibrium RPT.
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FIGURES

Fig. 1. The cosmological solution for closed Universe for initial particles number $N_o \ll N_{cr}$. The units of time and scale factor are: $t_o \simeq 3.5 \times 10^{-38}$ sec, $a_o \simeq 1.5 \times 10^{-27}$ cm.

Fig. 2. The dependence of the Universe wave function factorized by the separate function $U(a)$ from scale factor $a$.

Fig. 3. The change of order parameter ($\eta$) during the Universe evolution. The units of time and scale factor are the same as Fig.1.

Fig. 4. The change of relative particles number ($N/N_o$) during the Universe evolution. The units of time and scale factor are the same as Fig.1.

Fig. 5. The change of scale factor ($a$) during the Universe evolution. The units of time and scale factor are the same as Fig.1.
Branch II

$T_{II(\text{max})}$

$a_{II(\text{min})} = 1.444$

$a_{I(\text{max})} = 0.707 \times 10^{-4}$

Branch I
