Application of genetic algorithms to generate the geometry of elements with a given stiffness coefficient

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Abstract. Work presents the optimization process based on genetic algorithms of two mechanical systems. The first of these is the system of two springs, connected serially by a rigid medial element whose movement is forced in the direction transverse to the original axis of the springs. The goal of the optimization is to obtain a given characteristic describing the dependence of displacement of the medial element on the excitation force. The second part is devoted to multi-criteria optimization of geometry of statically indeterminate system of three rods, which is to ensure the required stiffness of the system while meeting the condition of mass minimization.

1. Introduction

The dynamic increase in the computing power of computers allows for developing new computational algorithms and adapt existing ones to solve common problems. An example of this trend may be genetic algorithms, presented for the first time in [1]. Since then, they have found application in sociology [2], economics [3] or many fields of technology, such as electronics [4], logistics [5] or mechanics [6].

The essence of genetic algorithms is to imitate the optimization process that occurs among living organisms [7]. The process begins with the generation of the a predetermined number of initial population of solutions with random features (parameters) presented numerically, often in binary form. At this stage, the key is to ensure the greatest possible diversity of individuals. Then each of the individuals is evaluated - the degree to which the characteristics of a given individual meet the goal function is defined, therefore an appropriate adaptation function is required, assigning a given set of features (individual) a fitness score from 0 to 1. The best-adapted individuals have the best chance to take part in the reproduction - a process aimed at identifying those between which the exchange of features will occur. It can be implemented, by using, among others, the roulette method - percentage expression of chances for the participation of the individual in the reproduction process is obtained by calculating the ratio of his fitness and the sum of fitness of all individuals. Such randomly selected solutions are coupled in random pairs in which crossover occurs - the creation of new individuals (children) on the basis of the exchange of some traits between individuals - parents (Figure 1).
In order to allow occurrence in the current population of features that did not appear in the previous population, the mutation process is implemented to the algorithm - it is determined with a certain probability of occurrence of a phenomenon in which random feature of a randomly selected individual is modified. In this way a new population is created. It can has the same number of population as the previous one or a different one, defined by the program. The new population is re-evaluated and the algorithm is repeated until the program termination condition, usually associated with obtaining a specific minimum match value or reaching the maximum number of iterations, is met. The diagram of genetic algorithms used in this paper is presented in Figure 2.

![Diagram of genetic algorithm](image)

**Figure 2. Diagram of genetic algorithm**

2. **Optimization tasks**
Genetic algorithms, as numerical methods, are a convenient tool for solving problems that are difficult or impossible to solve with analytical methods, which may occur in the case of nonlinear models and problems described by many dependent variables. Such problems include the task of optimizing the
stiffness of the structures presented in this section – simple but nonlinear spring system and rod system with mass minimization criteria.

2.1. Spring system
The system shown in Figure 3 will be optimized. It consists of a central plate and springs with stiffness \( k \) and initial length \( L_0 \), which engage the plate with the foundation and keep it in an equilibrium position. Despite the use of springs with linear characteristics, displacing the plate horizontally is strongly non-linear - the stiffness of the system increases with its deformation.

![Figure 3. Optimized system](image)

In devices such as bumpers, dampers or springboards the model of which may represent the presented system, the key is to choose the spring parameters so that the values of the forces that cause the individual strains are precisely defined. Let us assume that we are looking for a system with the following relationship between deformation \( s \) and their stiffness \( K(x) \) (Table 1).

| \( i \) | \( x_i \) [mm] | \( K(x) \) [N/mm] |
|-----|---------|--------|
| 1   | 5       | 0.02   |
| 2   | 30      | 0.16   |
| 3   | 70      | 0.20   |

Obtaining the desired effect with analytical methods, by approximating the function may turn out to be a very demanding process, therefore the problem will be solved using a genetic algorithm whose goal will be to find such parameters \( k \) and \( L_0 \) that the system stiffness curve will go through the points indicated in Table 1. Stiffness is defined as a derivative of the force after the deformation that this force caused. The function of force dependence on horizontal deformation is:

\[
P(x) = 2k\sqrt{L_0 \cos(\alpha)^2 + (L_0 \sin(\alpha) + x)^2} - L_0 \cos(\alpha)
\]

\[
\alpha = \arctan\left(\frac{x}{L_0}\right)
\]

hence the vertical stiffness of the system is therefore equal to:

\[
K(x) = \frac{\delta P(x)}{\delta x} = \frac{2kx \cos(\alpha)}{\sqrt{L_0^2 + x^2}}
\]

For the purposes of calculations, 100 stiffness curves were generated with random values of \( k \in (0; 50) \) and \( L_0 \in (0; 30) \) parameters, indicated with an accuracy of 0.01. The fitness to the goal function by each of them was calculated on the basis of equation (2.3):

\[
fit_i = \sum_{i=1}^{3} \frac{[K(x_i) - K(p_i)]}{3}
\]
where $K(p_i)$ means the stiffness of the currently tested pair of parameters $k$ and $L_0$. This definition of match means that each of the three points is treated equally and the full convergence of stiffness in one of the three required points guarantees a match at 0.33.

The next operation is to select individuals that will create a new population. This was done using the roulette method described in section 1, in such a way that the number of individuals did not change. The crossover process is carried out on the newly formed population. Individuals are selected in random pairs in which the random number of traits is exchanged. It was assumed that during crossover there is a five-percent probability of occurrence of a mutation - change of the value of a randomly selected parameter to a random value, selected from a previously set interval. For each individual of the newly created population, the match is recalculated and the entire algorithm is repeated until a 0.95 fitness is obtained.

Achieving the results with the assumed accuracy required seventeen iterations of the program. The objective function has been met with a fitting degree of 0.96 for parameters $k = 0.1$ and $L_0 = 20.1$. The graphical representation of the result is shown in (Figure 4). Interestingly, after reducing the population to 25 individuals, the algorithm returned the exact same result after the same number of iterations, but it is worth noting that the execution time of one iteration was significantly shortened.

![Figure 4](image)

**Figure 4.** The obtained function of stiffness and strain dependence for $k = 0.1$ N/mm and $L_0 = 20.1$ and the goal points

2.2. Rod system

Genetic algorithms are perfect for optimizing more complex structures. Figure 5 shows a convergent system of steel rods connected with a joint and loaded with horizontal force $P = 1000$ N at point O. The aim of optimization is to choose such geometric parameters of the structure (cross-sections $A_1$, $A_2$, $A_3$ and angles $\alpha$, $\beta$, $\gamma$) so that the force $P$ does not cause displacement point $O$ in the horizontal direction by a value greater than 0.7 mm and in the vertical direction by a value greater than 0.2 mm, while maintaining the condition of minimizing the mass and constant height $h = 1$ m of the system.
The system meets the following equations of equilibrium:

\[
\begin{align*}
\sum F_x &= 0; \quad P + S_1 \sin(\alpha) - S_2 \sin(\beta) + S_3 \sin(\gamma) \\
\sum F_y &= 0; \quad -S_1 \cos(\alpha) + S_2 \cos(\beta) + S_3 \cos(\gamma)
\end{align*}
\]  

(2.4)

where \(S_1, S_2, S_3\) are the internal forces of the respective bars. The analyzed system is a statically indeterminate design, so to calculate the displacement of the node \(O\) of the structure, in addition to the equilibrium equations, a geometric displacement equation will be required (Figure 6).

Figure 5. Scheme of considered rod system

Figure 6. Geometric displacement scheme
\[ \Delta L_3 = x + y \]  

where:

\[
x = \frac{\Delta L_2}{\cos(\beta + \gamma)} \quad a = \Delta L_2 \tan(\beta + \gamma)
\]

\[
y = z \cos(90 - (\beta + \gamma)) \quad c = \frac{\Delta L_2}{\cos(\alpha - \beta)}
\]

\[
z = u - a \quad b = \Delta L_2 \tan(\alpha - \beta)
\]

\[
u = \frac{\Delta L_1 + c}{\sin(\alpha - \beta)} - b \quad \Delta L_i = \frac{s_i L_i}{E A_i}, i = 1,2,3
\]

\[
L_1 = \frac{h}{\cos(\alpha)}, \quad L_2 = \frac{h}{\cos(\beta)}, \quad L_1 = \frac{h}{\cos(\gamma)}
\]

The values of vertical and horizontal displacements are then:

\[
u_H = \Delta L_2 \cos(\beta) + (z + a) \sin(\beta)
\]

\[
u_V = x \sin(\gamma) + z \cos(\beta)
\]

The first operation performed by the algorithm was to generate 100 sets of searched parameters meeting the condition of maximum displacements of the O point and additional conditions: \(\alpha > \beta; \beta, \gamma^\circ = 0\). For each of them, the mass \(m_i\) of the system and the value of the fit were calculated. As the task of the algorithm is to minimize the mass, and not to obtain a specific value, the fit is not a standardized parameter – it is a qualitative assessment referring to a given population:

\[
\text{fit}_i = \frac{\sum m_i - m_i}{\sum m_i}
\]

According to the roulette method, one hundred individuals were randomly selected, which were crossed with a mutation probability of 1%. Those of new individuals that meet the condition of maximum displacement of the O-point of the system are subjected to the assessment of the fit due to the mass and the algorithm is repeated until the mass of the system in ten consecutive iterations will not change by more than 1%.

Calculated geometric parameters, for which the system have the desired stiffness while maintaining the criterion of mass minimization, are presented in the Table 2. For these parameters, the appropriate displacements are \(u_H = 0.69\; \text{mm}, u_V = 0.17\; \text{mm}\), and its total mass is equal \(M = 6.67\; \text{kg}\). Convergence of algorithm has been obtained after 374 iterations.

**Table 2.** Obtained results

| \(\alpha [^\circ]\) | \(\beta [^\circ]\) | \(\gamma [^\circ]\) | \(A_1 [\text{mm}^2]\) | \(A_1 [\text{mm}^2]\) | \(A_1 [\text{mm}^2]\) |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 31,0                | 16,7                | 24,2                | 18,1                | 22,8                | 12,6                |
In order to confirm the correctness of calculations, the optimized system was modeled, in the COMSOL Multiphysics 5.1 environment and its load was simulated. Figure 7 demonstrate the obtained displacement map - it shows a large convergence with the obtained results.

As stated in [8], the size of the initial population may influence the number of iterations of the algorithm and the same calculation time. This phenomenon has been examined by re-performing the above calculations with different population sizes of 20, 50, 100, 150, 200, 300, 500 and 1000 individuals respectively. The obtained dependence is presented on Figure 8.

Figure 7. Results obtained by FEM analysis

Figure 8. Dependence of the number of algorithms iterations on the initial population
3. Conclusions
Genetic algorithms are a considerable tool for optimizing mechanical structures. The calculations carried out showed that the efficiency and accuracy of the obtained results goes hand in hand with their simplicity. Both examples indicated that an important feature of this type of algorithms is the appropriate selection of input parameters, including the size of the initial population. The analysis of the influence of the latter on the time of calculations of the second task showed, that there may be some optimal starting population number, for which the number of iterations of the algorithm required to achieve the set goal function is the shortest. In addition, for too small population sizes, convergence may not be possible - for its values of 10 and 20, the goal function has not been achieved. It is also worth noting that - as both examples show - the size of the population does not have a significant impact on the result of the calculation.

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