Gauge Theories with Lorentz-Symmetry Violation by Symplectic Projector Method

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The violation of Lorentz symmetry is studied from the point of view of a canonical formulation. We make the usual analysis on the constraints structure of the Carroll-Field-Jackiw model. In this context we derive the equations of motion for the physical variables and check out the dispersion relations obtained from them. Therefore, by the analysis using Symplectic Projector Method (SPM), we can check the results about this type of Lorentz breaking with those in the recent literature: in this sense we can confirm that the configuration of $v^\mu$ space-like is stable, and the $v^\mu$ time-like carry tachyonic modes.

I. INTRODUCTION

Investigations about repercussions of spontaneous symmetry breaking in context of a fundamental theory has gain special attention in the last years. The immediate consequence is the non equivalence between particle and observer Lorentz transformations [1]. In the last decade some works [2] explored this breaking in the context of string theories. These models with Lorentz and CPT breaking were used as a low-energy limit of an extension of the standard model, valid at the Plank scale [3]. An effective action that incorporates CPT and Lorentz violation is obtained and it keeps unaffected the $SU(3) \times SU(2) \times U(1)$ gauge structure of the underlying theory. In the context of $N = 1$ supersymmetric models, there have appeared two proposals: the one which violates the algebra of supersymmetry was first addressed by Berger and Kostelecky [4], and other that preserve the SUSY algebra gives, after integration on Grassman variables, the Carroll-Field-Jackiw model [5]. More recently, in the work of ref. [6], the authors addressed the discussion of Lorentz symmetry breaking in the context of exact SUSY by working out a whole list of manifestly supersymmetric Lorentz-violating operators. The observation of ultra-high energy cosmic rays with energies beyond the Greisen-Zatsepin-Kuzmin (GZK) cutoff ($E_{\text{GZK}} \simeq 4 \cdot 10^{19} \text{eV}$) [7, 8], could be potentially taken to be as an evidence of Lorentz-violation. The rich phenomenology of fundamental particles has also been considered as a natural environment in the search for indications of breaking of these symmetries [9, 10], revealing up the moment stringent limitations on the factors associated with such violation. Another point of interest refers to ancient issue of space-time varying coupling constants [11], which has been reassessed in the light of Lorentz-violating theories, with interesting connections with the construction of supergravity models. Moreover, measurements of radio emission from distant galaxies and quasars put in evidence that the polarization vectors of the radiation emitted are not randomly oriented as naturally expected. This peculiar phenomenon suggests that the space-time intervening between the source and observer may exhibit some sort of optical activity (birefringence), whose origin is unknown [12]. There are some different proposals of the Lorentz violation: one of these consists in obtain this breaking from spontaneous symmetry breaking of a vector matter field [13].

Our approach to the Lorentz breaking consist in adopting the 4-dimensional version of a Chern-Simons topological

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term, namely $\epsilon_{\mu\nu\kappa\lambda}v^\mu A^\nu F^\kappa\lambda$, where $\epsilon_{\mu\nu\kappa\lambda}$ is the 4-dimensional Levi-Civita symbol and $v^\mu$ is a fixed four-vector acting as a background. This idea was first settled down in the context of QED in [2]. A study of the consequences of such breaking in QED is extensively analyzed in [13], [14]. An extension of the Carroll-Field-Jackiw model in $(1 + 3)$ dimensions, including a scalar sector that yields spontaneous symmetry breaking (Higgs sector), was recently developed and analyzed, resulting in an Abelian-Higgs gauge model with violation of Lorentz symmetry [16].

The dimensional reduction (to 1+2 dimensions) of the Carroll-Field-Jackiw model was successfully realized [17], [18], resulting in a planar theory composed by a Maxwell-Chern-Simons gauge field, a massless scalar field, and a mixing term (responsible by the Lorentz violation) that couples both of these fields.

In this paper we explore the physical content of this model from the viewpoint of a canonical formulation; incidentally, as far as we know, it has not yet been treated in the literature. So, in section II, we make the usual analysis on the constraints structure of the model. There we get restrictions on the vector $v^\mu$ still as a condition to have a consistent constrained (or gauge) theory. Next, in section III, we apply the so-called Symplectic Projector Method (SPM), a recently developed alternative to Dirac’s most traditional methods, in which the physical viewpoint is most hardly lost. In this context we derive the equations of motion for the physical variables and check out the dispersion relations obtained here with that in [2]. Finally, in section IV we draw our conclusions and make some general remarks.

II. THE MAXWELL - CHERN-SIMONS MODEL

One starts from the Maxwell Lagrangian in 1+3 dimensions supplemented with a term that couples the dual electromagnetic tensor to a fixed 4-vector, $v^\mu$, as it appears in ref. [16]:

$$\mathcal{L}_{1+3} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \epsilon_{\mu\nu\kappa\lambda} v^\mu A^\nu F^{\kappa\lambda}. \quad (1)$$

This model, as discussed in [2], is gauge invariant but does not preserve the Lorentz and CTP symmetries. It is interesting to remark that this four-vector $v^\mu$ is rigid when we apply a rotation or generic observable Lorentz’s boost. The phase velocity of light changes and we have two modes of propagation moving with different velocities. As consequence we have a rotation of the polarization plane (vacuum birefringence). The change in the wave equation by the Chern-Simons term also opens the possibility of obtain imaginary frequencies. These unstable modes can be avoided establishing condition in our four-vector. In the last section we analyze the dispersion relation of the model and return to these questions. Now we are going to make another analysis under the canonical formalism. Taking as starting point, we assume no restriction on the four-vector $v^\mu$ The restriction $v^\mu$ constant will be obtained as consistence condition of the model viewed as constrained theory.

A. The Constraints Analysis

The lagrangian (1) carries the primary constraint

$$\Phi_1 = \pi^0 \approx 0. \quad (2)$$

Together with the the primary Hamiltonian

$$\mathcal{H}_p = \frac{1}{2} \pi_i \pi^i + \frac{1}{4} F^{ij} F_{ij} - \frac{1}{2} \epsilon_{0ijk} (\pi^i v^j A^k - v^0 A^k \partial^i A^j) + \frac{1}{8} (v_j v^j A_k A^k - v_j A_j v_k A^k) + A^0 \left( \partial_i \pi_i - \frac{1}{2} \epsilon_{0ijk} v^k \partial^j A^i \right), \quad (3)$$

the consistency condition on eq.2

$$\{ \Phi_1, \mathcal{H}_p \} \approx 0, \quad (4)$$

generates the secondary constraint

$$\Phi_2 = \partial_i \pi_i - \frac{1}{2} \epsilon_{0ijk} v^k \partial^j A^i \approx 0. \quad (5)$$

Now, analyzing the consistency condition on eq. (5) we conclude that the Dirac’s algorithm turns out to produce a restriction on $v^\mu$,

$$\epsilon^{0ijk} \partial_i v_j - \epsilon^{0ijk} \partial_i A_j \partial_k v_0 - \frac{1}{4} A_k \partial^k v^2 - v_j A^k \partial^j v_k = 0, \quad (6)$$
that must be preserved in order that a consistent constraint theory can be built. So, with the stronger restriction
\[ \partial_\mu v^\mu = 0, \]  
(7)
as made by Jackiw, we have a set of two, first-class, constraints \((\text{eq. } 2)\) and \((\text{eq. } 5)\), and the canonical Hamiltonian
\[ H = \frac{1}{2} \pi^i \pi^i + \frac{1}{4} F^{ij} F_{ij} - \frac{1}{2} \varepsilon_{0ijk} \left( \pi^i v^j A^k - v^0 A^k \partial^i A^j \right) + \frac{1}{8} \left( v_j v^j A^k - v_j A^k v_k A^k \right). \]  
(8)
As we are going to apply the SPM, a consistent set of gauge-fixing conditions must be imposed. We propose to work on the radiation conditions:
\[ \Phi_3 = A_0 \approx 0, \]  
\[ \Phi_4 = \partial^\mu A_\mu \approx 0. \]  
(9)
So the counting of degrees of freedom indicates that, after applying the SPM, we can expect a 4-dimensional reduced, or physical, phase-space.

### III. THE SYMPLECTIC PROJECTOR METHOD

Basically, two approaches are possible to treat problems with gauge symmetries in the canonical framework, since the presence of spurious coordinates is the mark of such theories: one can either reduce or expand the original space where act the constraints. Although the latter is the choice where the problem have been treated in a more complete and successful way \((\text{eq. } 11)\), the first one may offer, at least in simpler models, an alternative where neither strong mathematical tools are needed nor the physical viewpoint may be lost. In this sense, in recent years, the SPM have been developed and applied to several multidimensional problems \((\text{eq. } 24, 24, 24, 26, 27, 28)\). However, since this is not a very widespread method, we report to \((\text{eq. } 24)\), where a recent and brief review has been made on it’s construction, so as we can avoid here a very exhaustive paper.

The matrix whose elements are \( g^{ij} \equiv \{ \Phi^i, \Phi^j \} \) is easily calculated, and it’s inverse \( g^{-1}(x, y) \) is
\[
g^{-1} = \begin{bmatrix} 0 & 0 & \delta^3(x - y) & 0 \\ 0 & 0 & 0 & \nabla^2 \\ -\delta^3(x - y) & 0 & 0 & 0 \\ 0 & -\nabla^2 & 0 & 0 \end{bmatrix}. \]  
(10)
The general form of the symplectic projector is given by \((\text{eq. } 24)\)
\[
\Lambda^\nu_\mu(x, y) = \delta^\nu_\mu \delta^3(x - y) - \epsilon^{\mu\alpha} \int d^3\tau d^3\varpi g_{ij} (\tau, \varpi) \delta_{\alpha(x)} \Phi^i (\tau) \delta_{\nu(y)} \Phi^j (\varpi), \]  
(11)
with \( \delta_{\alpha(x)} \Phi^i (\tau) \equiv \frac{\delta \Phi^i (\tau)}{\delta \xi^\alpha(x)} \) and \( \epsilon^{\mu\alpha} \) is the symplectic matrix element. Let’s make the following correspondence:
\[ (A^0, A^1, A^2, A^3, \pi_0, \pi_1, \pi_2, \pi_3) \longleftrightarrow (\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6, \xi_7, \xi_8). \]  
(12)
After some long and straightforward calculations, we find the projector matrix below. It’s a matter of following the prescription \((\text{eq. } 24)\)
\[
\xi^\nu(x) = \int dy \; \Lambda^\nu_\mu(x, y) \; \xi_\mu(y) \]  
(13)
to get the projected coordinates:
\[ \xi_1^\nu(x) = 0, \]  
\[ \xi_2^\nu(x) = A^1^\nu(x), \]  
\[ \xi_3^\nu(x) = A^2^\nu(x), \]  
\[ \xi_4^\nu(x) = A^3^\nu(x), \]  
\[ \xi_5^\nu(x) = 0, \]  
\[ \xi_6^\nu(x) = \pi_1^\nu(x) + a A^2^\nu + b A^3^\nu, \]  
\[ \xi_7^\nu(x) = \pi_2^\nu(x) - a A^1^\nu + c A^3^\nu, \]  
\[ \xi_8^\nu(x) = \pi_3^\nu(x) - b A^1^\nu - c A^2^\nu. \]  
(14)
\[
\Lambda = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \delta - \partial_1^2 \partial_1^2 \nabla^{-2} & -\partial_1^2 \partial_1^2 \nabla^{-2} & -\partial_1^2 \partial_2^2 \nabla^{-2} & 0 & 0 & 0 & 0 \\
0 & -\partial_1^2 \partial_1^2 \nabla^{-2} & \delta - \partial_1^2 \partial_1^2 \nabla^{-2} & -\partial_1^2 \partial_2^2 \nabla^{-2} & 0 & 0 & 0 & 0 \\
0 & -\partial_1^2 \partial_1^2 \nabla^{-2} & -\partial_1^2 \partial_2^2 \nabla^{-2} & \delta - \partial_1^2 \partial_2^2 \nabla^{-2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a & b & -2 \left( v^2 \partial_1^2 - v^3 \partial_2^2 \right) \nabla^{-2} (x, y) & \delta + \partial_1^2 \partial_1^2 \nabla^{-2} & \partial_1^2 \partial_2^2 \nabla^{-2} & \partial_2^2 \partial_2^2 \nabla^{-2} & \nabla^{-2} \\
0 & -a & 0 & c & -2 \left( v^2 \partial_1^2 - v^3 \partial_2^2 \right) \nabla^{-2} (x, y) & \partial_1^2 \partial_1^2 \nabla^{-2} & \delta + \partial_1^2 \partial_2^2 \nabla^{-2} & \partial_2^2 \partial_2^2 \nabla^{-2} & \nabla^{-2} \\
0 & -b & -c & 0 & -2 \left( v^2 \partial_1^2 - v^3 \partial_2^2 \right) \nabla^{-2} (x, y) & \partial_1^2 \partial_1^2 \nabla^{-2} & \delta + \partial_1^2 \partial_2^2 \nabla^{-2} & \partial_2^2 \partial_2^2 \nabla^{-2} & \nabla^{-2} \\
\end{bmatrix}
\]

\[
a = \left[ -v^3 \left( \partial_1^2 \partial_2^2 + \partial_1^2 \partial_2^3 \right) + v^1 \partial_1^2 \partial_2^5 + v^2 \partial_1^2 \partial_2^3 \nabla^{-2} (x, y) \right],
\]

\[
b = \left[ v^2 \left( \partial_1^2 \partial_2^2 + \partial_1^2 \partial_2^3 \right) - v^1 \partial_1^2 \partial_2^5 - v^3 \partial_1^2 \partial_2^3 \nabla^{-2} (x, y) \right],
\]

\[
c = \left[ -v^1 \left( \partial_1^2 \partial_2^2 + \partial_1^2 \partial_2^3 \right) + v^3 \partial_1^2 \partial_2^5 + v^2 \partial_1^2 \partial_2^3 \nabla^{-2} (x, y) \right].
\]
Due to the presence of transverse fields, we simplify, without any lost of generality, the set of coordinates by choosing a direction of propagation. So, making $\mathbf{k} = (0, 0, k)$ we get the simpler set:

$$
\begin{align*}
\xi_2^* &= A^1 \rightarrow q_1, \\
\xi_3^* &= A^2 \rightarrow q_2, \\
\xi_6^* &= \pi_1 + v^2 A^3 \rightarrow p_1, \\
\xi_7^* &= \pi_2 - v^2 A^3 \rightarrow p_2, \\
\xi_8^* &= -v^2 \xi_2^* + v^1 \xi_3^* \rightarrow -v^2 q_1 + v^1 q_2.
\end{align*}
$$

The notation $(q, p)$ turns it more evident that we are in 4-dimensional phase space.

The canonical Hamiltonian, written with symplectic coordinates is

$$
\mathcal{H}(\xi) = \frac{1}{2} \xi_{i+5} \xi_{i+5} + \frac{1}{2} \varepsilon_{ijk} \partial^j \xi_{j+1} \varepsilon_{lmk} \partial^l \xi_{m+1} + \frac{1}{2} \varepsilon_{0ijk} (\xi_{i+5} v^j \xi_{j+1} + v^0 \varepsilon_{0jk} \xi_{k+1} \partial^j \xi_{j+1}) + \frac{1}{8} v_j (v^j \xi_{k+1} \xi_{k+1} + \xi_{j+1} v_k \xi_{k+1}),
$$

so that the physical Hamiltonian is

$$
\mathcal{H}^*(\xi^*) = \frac{1}{2} \xi_{i+5}^* \xi_{i+5}^* + \frac{1}{2} \varepsilon_{ijk} \partial^j \xi_{j+1}^* \varepsilon_{lmk} \partial^l \xi_{m+1}^* + \frac{1}{2} \varepsilon_{0ijk} (\xi_{i+5}^* v^j \xi_{j+1}^* + v^0 \xi_{k+1}^* \partial^j \xi_{j+1}^*) + \frac{1}{8} v_j (v^j \xi_{k+1}^* \xi_{k+1}^* + \xi_{j+1}^* v_k \xi_{k+1}^*).
$$

Written in $(q, p)$ notations it is:

$$
\mathcal{H}^*(q, p) = \frac{1}{2} (p_1^2 + p_2^2) + \frac{1}{2} (\partial^3 q_2)^2 + \frac{1}{2} (\partial^3 q_1)^2 + \frac{1}{2} v^3 \left( p_1 q_2 - p_2 q_1 - \frac{1}{4} \beta (q_1^2 + q_2^2) - \frac{1}{2} v_0 (q_1 \partial^3 q_2 - q_2 \partial^3 q_1),
\right)
$$

$$
\beta = v_1^2 + v_2^2 + \frac{1}{2} v_3^2.
$$

The equations of motion are thus:

$$
\dot{q}_1 = p_1 - \frac{1}{2} v_3 q_2,
$$

$$
\dot{q}_2 = p_2 + \frac{1}{2} v_3 q_1,
$$

$$
\dot{p}_1 = \frac{1}{2} v_3 p_2 - \frac{1}{2} \beta q_1 - \partial^3 \partial^3 q_1 - v_0 \partial^3 q_2,
$$

$$
\dot{p}_2 = -\frac{1}{2} v_3 p_1 - \frac{1}{2} \beta q_2 - \partial^3 \partial^3 q_2 + v_0 \partial^3 q_1,
$$

or,

$$
\Box q_1 = \frac{1}{2} v_j v^j q_1 - v_0 \partial^3 q_2,
$$

$$
\Box q_2 = \frac{1}{2} v_j v^j q_2 + v_0 \partial^3 q_1.
$$

To solve this system we decouple the equations and we obtain:

$$
\left[ \left( \Box - \frac{1}{2} v_j v^j \right)^2 + v_0^2 (\partial^3)^2 \right] q_1 = 0.
$$

The dispersion relation obtained is

$$
\left( k^\alpha k_\alpha - \frac{1}{2} v_j v^j \right)^2 + v_0^2 (i k^3)^2 = 0.
$$
We would like to compare this equation with the dispersion relation of the work \cite{2}

\[(k^\alpha k_\alpha)^2 + (v^\beta v_\beta) = (k^\alpha v_\alpha)^2.\]  

(24)

To this end we rewrite it, first, with the prescription \(k^\mu = (k^0, 0, 0, k^3)\) and \(v^\mu = (0, 0, 0, v^3)\); we have:

\[(k_0)^2 = \frac{(2(k_3)^2 + (v_3)^2) \pm v_3 \sqrt{(v_3)^2 + 4(k_3)^2}}{2}.\]  

(25)

This choice of \(v^\mu\) is convenient because we can avoid unstable solutions. In our case we have, from eq. \ref{23}:

\[(k_0)^2 = (k_3)^2 + \frac{1}{2}(v_3)^2.\]  

(26)

In both cases we have stable solutions. The configuration of a \(v^\mu\) space-like do not generate imaginary frequencies. On the other hand, if we adopt the prescription: \(k^\mu = (k^0, 0, 0, k^3)\) and \(v^\mu = (v^0, 0, 0, v^3)\), to eq. \ref{24} we have:

\[(k_0)^2 = (k_3)^2 \pm |v_0 k_3|,\]  

(27)

and in our case, eq. \ref{25} we have

\[(k_0)^2 = (k_3)^2 \pm |v_0 k_3|.\]  

(28)

With this comparison we can extract similar conclusion which has been discussed about this type of Lorentz breaking, i. e., that we must take care with the \(v_0\) component. If we observe the last solution above we conclude that we can obtain imaginary frequencies. This situation characterizes tachionic modes and certainly we have problems with the causality of this model. On the other hand, the conclusions about the results in both dispersion relations \ref{25,26} with \(v^\mu\) space-like is similar. In this case we don’t have problems with imaginary frequencies and the causality in this case is completely assured. Therefore, by the analysis using SPM, we can arrive to the results about this type of Lorentz breaking in the recent literature: the configuration of \(v^\mu\) space-like is stable, and the \(v^\mu\) time-like carry tachionic modes.

\[\text{IV. CONCLUDING COMMENTS}\]

The central motivation of this work was to revisit the model of Lorentz and CPT violation proposed by Jackiw et. al \cite{2} in the context of the canonical formalism, since, as far we know, it had not been worked out up to present days. To this end, we applied the recently developed SPM, so that some relevant physical aspects could be easily read.

The first interesting point to remark is that the conditions on the vector \(v_\mu\), imposed by \cite{2} after an analysis in order to preserve gauge invariance, here turns out to be a condition that comes up naturally still in the context of Dirac’s algorithm: we can’t get a consistent constrained theory unless condition \ref{6} is obeyed.

After processed the physical variables of the model we got the dispersion relations to be checked with that of \cite{2}. Using SPM, we arrive to results that agree with those in the literature: the configuration of \(v^\mu\) space-like is stable, and the \(v^\mu\) time-like carry tachionic modes. An interesting discussion that emerge of this analysis is about the investigations of this scenarios with spontaneous symmetry breaking of gauge fields. In a Maxwell Electrodynamics we have a wave propagation with only two transverse modes moving with the same velocity \(c\) (two degrees of freedom). When we gain a mass term in the Lagrangian \cite{11} provided by spontaneous symmetry breaking, the longitudinal mode can not be eliminated. Therefore we gain plus one degree of freedom. We have an interaction mediating massive fotons (short range), and we have a screening of the electromagnetic radiation.

In Maxwell-Chern-Simons model \((1 + 2 d)\) \cite{41} we have only one d. f. What’s going on in our model? We do not have a massive term coming from the Higgs Mechanism. Therefore we should expect to have a model with only transversal modes. In fact, the two transverse fields, \(q_1\) and \(q_2\) in the eq. \cite{15} are the physical variables we had got.

The scenarios with spontaneous symmetry breaking is under investigation and we shall soon be reporting on it in \cite{42}. 
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