Research Article

$B_c \rightarrow BP, BV$ Decays with the QCD Factorization Approach

Junfeng Sun, 1 Na Wang, 1,2 Qin Chang, 1 and Yueling Yang 1

1 Institute of Particle and Nuclear Physics, Henan Normal University, Xinxiang 453007, China
2 Institute of Particle and Key Laboratory of Quark and Lepton Physics, Central China Normal University, Wuhan 430079, China

Correspondence should be addressed to Qin Chang; changqin@htu.cn

Received 7 January 2015; Accepted 17 March 2015

Academic Editor: Michal Kreps

Copyright © 2015 Junfeng Sun et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. The publication of this article was funded by SCOAP3.

We studied the nonleptonic $B_c \rightarrow BP, BV$ decays with the QCD factorization approach. It is found that the Cabibbo favored processes of $B_c \rightarrow B_\pi, B_\rho, B_\phi K$ are the promising decay channels with branching ratio larger than 1%, which should be observed earlier by the LHCb collaboration.

1. Introduction

The $B_c$ meson is the ground pseudoscalar meson of the $b\bar{c}$ system [1]. Compared with the heavy unflavored charmonium $c\bar{c}$ and bottomonium $b\bar{b}$, the $B_c$ meson is unique in some respects. (1) Heavy quarkonia could be created in the parton-parton process $ij \rightarrow QQ$ at the order of $\alpha_s^2$ (where $ij = gg$ or $gq\bar{q}$, $Q = b, c$), while the production probability for the $B_c$ meson is at least at the order of $\alpha_s^4$ via $ij \rightarrow B_c^{(*)} + b\bar{c}$, where the gluon-gluon fusion mechanism is dominant at Tevatron and LHC [2]. The $B_c$ meson is difficult to produce experimentally, but it was observed for the first time via the semileptonic decay mode $B_c \rightarrow J/\psi \ell \nu$ in $p\bar{p}$ collisions by the CDF collaboration in 1998 [3, 4], which showed the realistic possibility of experimental study of the $B_c$ meson. One of the best measurements on the mass and lifetime of the $B_c$ meson is reported recently by the LHCb collaboration, $m_{B_c} = 6276.28 \pm 1.44 \pm 0.36$ MeV [5] and $\tau_{B_c} = 513.4 \pm 11.0 \pm 5.7$ fs [6]. With the running of the LHC, the $B_c$ meson has a promising prospect. It is estimated that one could expect some $10^{10}$ $B_c$ events at the high-luminosity LHC experiments per year [7, 8]. The studies on the $B_c$ meson have entered a new precision era. (2) For charmonium and bottomonium, the strong and electromagnetic interactions are mainly responsible for annihilation of the $QQ$ quark pair into final states. The $B_c$ meson, carrying nonzero flavor number $B = C = \pm 1$ and lying below the $BD$ meson pair threshold, can decay only via the weak interaction, which offers an ideal sample to investigate the weak decay mechanism of heavy flavors that is inaccessible to both charmonium and bottomonium. The $B_c$ weak decay provides great opportunities to investigate the perturbative and nonperturbative QCD, final state interactions, and so forth.

With respect to the heavy-light $B_{d,s,d}$ mesons, the doubly heavy $B_c$ meson has rich decay channels because of its relatively large mass and that both $b$ and $c$ quarks can decay individually. The decay processes of the $B_c$ meson can be divided into the following three classes [2, 9–11]: (1) the $c$ quark decays with the $b$ quark as a spectator; (2) the $b$ quark decays with the $c$ quark as a spectator; (3) the $b$ and $c$ quarks annihilate into a virtual $W$ boson, with the ratios of $\sim$70%, 20% and 10%, respectively [2]. Up to now, the experimental evidences of pure annihilation decay mode [class (3)] are still nothing. The $b \rightarrow c$ transition, belonging to the class (2), offers a well-constructed experimental structure of charmonium at the Tevatron and LHC. Although the detection of the $c$ quark decay is very challenging to experimentalists, the clear signal of the $B_c \rightarrow B_\pi \pi$ decay is presented by the LHCb group using the $B_c^+ \rightarrow D_\pi \pi$ and $B_c^0 \rightarrow J/\psi \phi$ channels with statistical significance of 7.7σ and 6.1σ, respectively [12].

Anticipating the forthcoming accurate measurements on the $B_c$ meson at hadron colliders and the lion’s share of the $B_c$...
decay width from the $c$ quark decay [31–33], many theoretical papers were devoted to the study of the $B_c \rightarrow BP, BV$ decays (where $P$ and $V$ denote the SU(3) ground pseudoscalar and vector mesons, resp.), such as [17, 34–37] with the BSW model [38, 39] or IGSW model [40], [18, 19, 41] based on the Bethe-Salpeter (BS) equation, [20–24, 42] with potential models, [25] with constituent quark model, [26–28] with QCD sum rules, [43] with the quark diagram scheme, [29, 30] with the perturbative QCD approach (pQCD) [44–49], and so on. The previous predictions on the branching ratios for the $B_c \rightarrow BP, BV$ decays are collected in Table 3. The discrepancies of previous investigations arise mainly from the different model assumptions. Recently, several phenomenological methods have been fully developed to cope with the hadronic matrix elements and successfully applied to the nonleptonic $B_c$ decay, such as the pQCD approach [44–49] based on the $k_T$ factorization scheme, the soft-collinear effective theory [50–57] and the QCD-improved factorization (QCDF) approach [58–63] based on the collinear approximation and power counting rules in the heavy quark limit. In this paper, we will study the $B_c \rightarrow BP, BV$ decays with the QCDF approach to provide a ready reference to the existing and upcoming experiments.

This paper is organized as follows. In Section 2, we will present the theoretical framework and the amplitudes for the $B_c \rightarrow BP, BV$ decays within the QCDF framework. Section 3 is devoted to numerical results and discussion. Finally, Section 4 is our summation.

2. Theoretical Framework

2.1. The Effective Hamiltonian. The low energy effective Hamiltonian responsible for the nonleptonic bottom-conserving $B_c \rightarrow BP, BV$ decays constructed by means of the operator product expansion and the renormalization group (RG) method is usually written in terms of the four-quark interactions [64, 65]. Consider

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{ud} V_{cd}^* \left[ C_1^u (\mu) Q_1^u (\mu) + C_2^u (\mu) Q_2^u (\mu) \right] \right. \\
+ \sum_{q_1, q_2} V_{uq_1} V_{c q_2}^* \left[ C_1 (\mu) Q_1 (\mu) + C_2 (\mu) Q_2 (\mu) \right] \\
+ \left. \sum_{q_1, q_2} V_{uq_1} V_{c q_2}^* \sum_{k=3}^{10} C_k (\mu) Q_k (\mu) \right] + \text{h.c.},
\]  

(1)

where the Fermi coupling constant $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$ [1]; $Q_1, Q_2,$ and $Q_{3-10}$ are the relevant local tree, annihilation, and penguin four-fermion operators, respectively, which govern the decays in question. The Cabibbo-Kobayashi-Maskawa (CKM) factor $V_{uq_1} V_{c q_2}^*$ and Wilson coefficients $C_i$ describe the coupling strength for a given operator.

Using the unitarity of the CKM matrix, there is a large cancellation of the CKM factors

\[
V_{ud} V_{cb}^* + V_{ut} V_{cs}^* = V_{ub} V_{cb}^* \sim \mathcal{O} (\lambda^5),
\]

(2)

where the Wolfenstein parameter $\lambda = \sin \theta_c = 0.22537 (61)$ [1] and $\theta_c$ is the Cabibbo angle. Hence, compared with the tree contributions, the contributions of annihilation and penguin operators are strongly suppressed by the CKM factor. If the CP-violating asymmetries that are expected to be very small due to the small weak phase difference for $c$ quark decay are prescinded from the present consideration, then the penguin and annihilation contributions could be safely neglected.

The local tree operators $Q_{1,2}$ in (1) are expressed as follows:

\[
Q_1 = [\bar{q}_2 \gamma_{\mu} q_1 (1 - \gamma_5) c_a] \left[ \bar{\psi}_b \gamma^\mu (1 + \gamma_5) \gamma_\alpha q_{1,\beta} \right],
\]

\[
Q_2 = [\bar{q}_2 \gamma_{\mu} q_1 (1 - \gamma_5) c_\beta] \left[ \bar{\psi}_b \gamma^\mu (1 - \gamma_5) \gamma_\alpha q_{1,\alpha} \right],
\]

(3)

where $\alpha$ and $\beta$ are the SU(3) color indices.

The Wilson coefficients $C_1 (\mu)$ summarize the physics contributions from scales higher than $\mu$. They are calculable with the RG improved perturbation theory and have properly been evaluated to the next-to-leading order (NLO) [64, 65]. They can be evolved from a higher scale $\mu \sim \mathcal{O} (m_{b'})$ down to a characteristic scale $\mu \sim \mathcal{O} (m_c)$ with the functions including the flavor thresholds [64, 65]

\[
\bar{C} (\mu) = U_{1, \tau, \mu_c} M (\mu_c) U_{2, \tau, \mu_b} M (\mu_b) \bar{C} (\mu_b),
\]

(4)

where $U_{f, \mu_f, \mu_c}$ is the RG evolution matrix converting coefficients from the scale $\mu_c$ to $\mu_f$, and $M (\mu)$ is the quark threshold matching matrix. The expressions of $U_{f, \mu_f, \mu_c}$ and $M (\mu)$ can be found in [64, 65]. The numerical values of LO and NLO $C_{1,2}$ with the naive dimensional regularization scheme are listed in Table I. The values of NLO Wilson coefficients in Table I are consistent with those given by [64, 65] where a trick with "effective" number of active flavors $f = 4.15$ rather than formula (4) is used.

To obtain the decay amplitudes, the remaining work is how to accurately evaluate the hadronic matrix elements $\langle BM (Q, \mu) | B \rangle$ which summarize the physics contributions from scales lower than $\mu$. Since the hadronic matrix elements involve long distance contributions, one is forced to use either nonperturbative methods such as lattice calculations and QCD sum rules or phenomenological models relying on some assumptions. Consequently, it is very unfortunate that hadronic matrix elements cannot be reliably calculated at present, and that the most intricate part and the dominant theoretical uncertainties in the decay amplitudes reside in the hadronic matrix elements.

2.2. Hadronic Matrix Elements. Phenomenologically, based on the power counting rules in the heavy quark limit, Beneke et al. proposed that the hadronic matrix elements could be written as the convolution integrals of hard scattering kernels and the light cone distribution amplitudes with the QCDF master formula [58–63]. The QCDF approach is widely applied to nonleptonic $B$ decays and it works well [66–76].
which encourage us to apply the QCDF approach to the study of $B_c \to B \eta$ decays. Since the spectator is the heavy $b$ quark who is almost always on shell, the virtuality of the gluon linked with the spectator should be $\sim \mathcal{O}(A_{QCD}^2)$. The dominant behavior of the $B_c \to B$ transition form factors and the contributions of hard spectator scattering interactions are governed by soft processes. According to the basic idea of the QCDF approach [69, 70], the hard and soft contributions to the form factors entangle with each other and cannot be identified reasonably, so the physical form factors are used as the inputs. The hard spectator scattering contributions are power suppressed in the heavy quark limit. Finally, the hadronic matrix elements can be written as

$$
\langle BM | Q_{12} | B_c \rangle = \sum_i F_i^{B_c \to B} \int dx H_i(x) \Phi_M(x) \propto \sum_i F_i^{B_c \to B} f_M \{ 1 + \alpha_i r_1 + \cdots \},
$$

(5)

where $F_i^{B_c \to B}$ is the transition form factor and $\Phi_M(x)$ is the light-cone distribution amplitudes of the emitted meson $M$ with the decay constant $f_M$. The hard scattering kernels $H_i(x)$ are computable order by order with the perturbation theory in principle. At the leading order $\alpha_s^0$, $H_i(x) = 1$, that is, the convolution integral of (5) results in the meson decay constant. The hadronic matrix elements are parameterized by the product of form factors and decay constants, which are real and renormalization scale independent. One goes back to the simple “naive factorization” (NF) scenario. At the order $\alpha_s$ and higher orders, the information of strong phases and the renormalization scale dependence of hadronic matrix elements could be partly recuperated. Combined the nonfactorizable contributions with the Wilson coefficients, the scale independent effective coefficients at the order $\alpha_s$ can be obtained [58–63] as follows:

$$
a_1 = C_1^{NLO} + \frac{1}{N_c} C_2^{NLO} + \frac{\alpha_s}{4\pi N_c} C_2^{\text{LO}} V_1
$$

$$
a_2 = C_2^{NLO} + \frac{1}{N_c} C_1^{NLO} + \frac{\alpha_s}{4\pi N_c} C_1^{\text{LO}} V_1
$$

(6)

where the expressions of vertex corrections are [58–63]

$$
V = 6 \log \left( \frac{m_t^2}{\mu^2} \right) - 18 - \frac{1}{2} + i3\pi a_0^M
$$

$$
+ \frac{11}{2} i3\pi a_1^M a_2^M + \cdots,
$$

(7)

with the twist-2 quark-antiquark distribution amplitudes of pseudoscalar $P$ and longitudinally polarized vector $V$ meson in terms of Gegenbauer polynomials [14–16]. One has

$$
\phi_M(x) = 6x \sum_{n=0}^{\infty} a_n^M C_n^{3/2} (x - \bar{x}),
$$

(8)

where $\bar{x} = 1 - x$; $a_n^M$ is the Gegenbauer moment and $a_0^M = 1$.

From the numbers in Table I, it is found that (1) for the coefficient $a_1$ the nonfactorizable contributions accompanied by the Wilson coefficient $C_2$ can provide $\geq 10\%$ enhancement compared with the NF’s result, and a relatively small strong phase $\leq 5^\circ$; (2) for the coefficient $a_2$, the nonfactorizable contributions assisted with the large Wilson coefficient $C_1$ are significant in addition, a relatively large strong phase $\sim 155^\circ$ is obtained; (3) the QCDF’s values of $a_{1,2}$ agree basically with the real coefficients $a_1 = 1.20$ and $a_2 = -0.317$ which are used by previous studies on the $B_c \to B \eta$ in [17, 18, 20–28, 34–37, 42], but with more information on the strong phases.

2.3. Decay Amplitudes. Within the QCDF framework, the amplitudes for $B_c \to BM$ decays are expressed as

$$
s^f_B (B_c \to BM) = \langle BM | \mathcal{H}_{\text{eff}} | B_c \rangle = \frac{G_F}{\sqrt{2}} V_{ud} V_{cd}^* a_1 \langle M | J^\mu | 0 \rangle \langle B | f_\mu | B_c \rangle,
$$

(9)

The matrix elements of current operators are defined as

$$
\langle P (p) | \bar{\psi} Y^\mu (1 - \gamma_5) \psi | 0 \rangle = -if_P p^\mu,
$$

$$
\langle V (c, p) | \bar{\psi} Y^\mu (1 - \gamma_5) \gamma_5 \psi | 0 \rangle = f_V m_V c^\mu,
$$

(10)

where $f_P$ and $f_V$ are the decay constants of pseudoscalar $P$ and vector $V$ mesons, respectively; $m_V$ and $c$ denote the mass and polarization of vector meson, respectively.

For the mixing of physical pseudoscalar $\eta$ and $\eta'$ meson, we adopt the quark-flavor basis description proposed in [13] and neglect the contributions from possible gluonium and $c\bar{c}$ compositions; that is,

$$
\left( \begin{array}{c} \eta \\ \eta' \end{array} \right) = \left( \begin{array}{cc} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{array} \right) \left( \begin{array}{c} \eta_\eta \\ \eta_{\eta'} \end{array} \right),
$$

(11)

where $\eta_\eta = (\mu \bar{u} + d \bar{d})/\sqrt{2}$ and $\eta_{\eta'} = s \bar{s}$; the mixing angle $\phi = (39.3 \pm 1.0)^\circ$ [13]. We assume that the vector mesons are ideally mixed; that is, $\omega = (\mu \bar{u} + d \bar{d})/\sqrt{2}$ and $\phi = s\bar{s}$.
The transition form factors are defined as \[ (B(k) | \gamma^\mu (1 - \gamma_5) c | B_c (p)) \]

\[
= \left[ p + k - \frac{m_B^2 - m_{B_c}^2}{q^2} q \right]^\mu \mathcal{F}_1^{B_c \rightarrow B} (q^2)
+ \frac{m_B^2 - m_{B_c}^2}{q^2} q^\mu \mathcal{F}_0^{B_c \rightarrow B} (q^2)
\]

where \( q = p - k \), and the condition of \( \mathcal{F}_0^{B_c \rightarrow B} (0) = \mathcal{F}_1^{B_c \rightarrow B} (0) \) is required compulsorily to cancel the singularity at the pole \( q^2 = 0 \).

For the decay of the recoiled meson, the velocity of the recoiled meson is very low in the rest frame of the meson, the wave functions of the recoiled meson overlap strongly. It is believed that the form factors \( F_{0,1}^{B_c \rightarrow B} \) should be close to the result using the nonrelativistic harmonic oscillator wave functions with the BSW model [17]. Consider

\[
\mathcal{F}_0^{B_c \rightarrow B} = \left( \frac{2m_B m_{B_c}}{m_B^2 + m_{B_c}^2} \right)^{1/2} = 0.99.
\]

The flavor symmetry breaking effects on the form factors are negligible in (13). For simplification, we take \( F_{0,1}^{B_c \rightarrow B} = 1.0 \) in our numerical calculation to give a rough estimation.

### 3. Numerical Results and Discussions

The branching ratios of nonleptonic two-body \( B_c \) decays in the rest frame of the \( B_c \) meson can be written as

\[
\mathcal{B}r (B_c \rightarrow BM) = \frac{\tau_{B_c} p}{8 \pi m_{B_c}^3} |\mathcal{A} (B_c \rightarrow BM)|^2,
\]

where the lifetime of the \( B_c \) meson \( \tau_{B_c} = 513.4 \pm 11.0 \pm 5.7 \text{ fs} \) [6] and \( p \) is the common momentum of final particles. The decay amplitudes \( \mathcal{A} (B_c \rightarrow BM) \) are listed in the Appendix.

The input parameters in our calculation, including the CKM Wolfenstein parameters, decay constants, and Gegenbauer moments of distribution amplitudes in (8), are collected in Table 2. If not specified explicitly, we will take their central values as the default inputs. Our numerical results on the CP-averaged branching ratios are presented in Table 3, where theoretical uncertainties of the "QCDF" column come from the CKM parameters, the renormalization scale \( \mu = (1 \pm 0.2)m_{B_c} \), decay constants, and Gegenbauer moments, respectively. For comparison, previous results calculated with the fixed coefficients \( a_1 = 1.22 \) and \( a_2 = -0.4 \) are also listed. There are some comments on the branching ratios.

1. From the numbers in Table 3, it is seen that different branching ratios for the \( B_c \rightarrow BP, BV \) decays were obtained with different approaches in previous works, although the same coefficients \( a_{1,2} \) are used. Much of the discrepancy comes from the different values of the transition form factors. If the same value of the form factor is used, then the disparities on branching ratios for the \( a_1 \)-dominated \( B_c \) decays will be highly alleviated. For example, all previous predictions on \( \mathcal{B}r(B_c \rightarrow B \pi) \) will be about 10% with the same form factor \( F_0^{B_c \rightarrow B} = 1.0 \), which is generally in line with the QCDF estimation within uncertainties and also agrees with the recent LHCb measurement [12].

2. There is a hierarchical structure between the QCDF's results on branching ratios for the \( B_c \rightarrow BP \) and \( BV \) decays with the same final \( B_q \) meson, for example,

\[
\mathcal{B}r (B_c \rightarrow B_q \pi) > \mathcal{B}r (B_c \rightarrow B_q \rho),
\]

\[
\mathcal{B}r (B_c \rightarrow B_q K) \geq 5 \mathcal{B}r (B_c \rightarrow B_q K^*),
\]

which differs from the previous results. There are two decisive factors. One is the kinematic factor. The phase space for the \( B_c \rightarrow BV \) decays is more compressed than that for the \( B_c \rightarrow BP \) decays, because the mass of the light pseudoscalar meson (except for the exotic \( \eta' \) meson) is generally less than the mass of the corresponding vector meson with the same valence quark components. The other is the dynamical factor. The
Table 3: The CP-averaged branching ratios for the $B_c \to BP, BV$ decays.

| Decay mode | Case | Reference | Reference | Reference | Reference | Reference | Reference | Reference | Reference | QCDF |
|------------|------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|------|
| $B_c \to B_s^0 \pi^+$ | 1-a | $1.0 \times 10^{-3}$ | $6.8 \times 10^{-2}$ | $1.8 \times 10^{-2}$ | $2.9 \times 10^{-2}$ | $4.0 \times 10^{-2}$ | $4.3 \times 10^{-2}$ | $4.6 \times 10^{-2}$ | $1.3 \times 10^{-1}$ | $1.9 \times 10^{-1}$ | $8.8 \times 10^{-2}$ |
| $B_c \to B_s^0 K^+$ | 1-b | $7.6 \times 10^{-3}$ | $4.9 \times 10^{-3}$ | $2.0 \times 10^{-3}$ | $2.4 \times 10^{-3}$ | $3.3 \times 10^{-3}$ | $3.3 \times 10^{-3}$ | $8.5 \times 10^{-3}$ | $3.4 \times 10^{-3}$ | $1.2 \times 10^{-2}$ | $5.2 \times 10^{-3}$ |
| $B_c \to B_s^0 \rho^+$ | 1-a | $6.3 \times 10^{-2}$ | $5.2 \times 10^{-2}$ | $4.6 \times 10^{-2}$ | $1.6 \times 10^{-2}$ | $2.7 \times 10^{-2}$ | $3.0 \times 10^{-2}$ | $2.7 \times 10^{-2}$ | $8.4 \times 10^{-2}$ | $3.2 \times 10^{-2}$ | $(4.44)^{\pm (0.00 \pm 0.13)}_{(0.00 \pm 0.23 \pm 0.13)} \times 10^{-2}$ |
| $B_c \to B_s^0 K^{*+}$ | 1-b | $3.8 \times 10^{-4}$ | $1.5 \times 10^{-3}$ | $3.5 \times 10^{-5}$ | $1.5 \times 10^{-4}$ | $1.0 \times 10^{-5}$ | $8.0 \times 10^{-5}$ | $9.7 \times 10^{-4}$ | $1.3 \times 10^{-4}$ | $(1.25)^{\pm (0.00 \pm 0.22 \pm 0.07)} \times 10^{-4}$ |
| $B_c \to B_s^0 \pi^0$ | 1-b | $7.4 \times 10^{-3}$ | $3.8 \times 10^{-3}$ | $1.2 \times 10^{-3}$ | $1.2 \times 10^{-3}$ | $1.3 \times 10^{-3}$ | $1.8 \times 10^{-3}$ | $1.8 \times 10^{-3}$ | $1.2 \times 10^{-2}$ | $6.9 \times 10^{-3}$ | $(7.83)^{\pm (0.00 \pm 0.20 \pm 0.06)_{(0.00 \pm 0.41 \pm 0.04)} \times 10^{-3}$ |
| $B_c \to B_s^0 K^*$ | 1-c | $3.0 \times 10^{-4}$ | $1.2 \times 10^{-4}$ | $1.0 \times 10^{-4}$ | $1.1 \times 10^{-4}$ | $1.5 \times 10^{-4}$ | $1.5 \times 10^{-4}$ | $5.9 \times 10^{-4}$ | $1.8 \times 10^{-4}$ | $8.1 \times 10^{-4}$ | $(5.29)^{\pm (0.00 \pm 0.31 \pm 0.07)_{(0.00 \pm 0.28 \pm 0.07)} \times 10^{-4}$ |
| $B_c \to B_s^0 \rho^*$ | 1-b | $8.3 \times 10^{-3}$ | $6.9 \times 10^{-3}$ | $3.3 \times 10^{-3}$ | $1.5 \times 10^{-3}$ | $1.6 \times 10^{-3}$ | $2.2 \times 10^{-3}$ | $2.2 \times 10^{-3}$ | $2.4 \times 10^{-3}$ | $1.1 \times 10^{-2}$ | $(5.32)^{\pm (0.00 \pm 0.20 \pm 0.15)_{(0.00 \pm 0.28 \pm 0.15)} \times 10^{-3}$ |
| $B_c \to B_s^0 K^{*0}$ | 1-c | $2.1 \times 10^{-4}$ | $1.5 \times 10^{-4}$ | $4.6 \times 10^{-5}$ | $4.4 \times 10^{-5}$ | $4.9 \times 10^{-5}$ | $3.5 \times 10^{-4}$ | $3.5 \times 10^{-4}$ | $5.7 \times 10^{-5}$ | $1.7 \times 10^{-4}$ | $(1.06)^{\pm (0.00 \pm 0.10 \pm 0.05)_{(0.00 \pm 0.28 \pm 0.15)} \times 10^{-3}$ |
| $B_c \to B_s^0 \bar{K}^0$ | 2-a | $2.1 \times 10^{-3}$ | $1.2 \times 10^{-2}$ | $4.9 \times 10^{-3}$ | $4.2 \times 10^{-3}$ | $4.4 \times 10^{-3}$ | $6.0 \times 10^{-3}$ | $2.3 \times 10^{-2}$ | $6.8 \times 10^{-3}$ | $3.6 \times 10^{-2}$ | $(1.97)^{\pm (0.00 \pm 0.11 \pm 0.05)_{(0.00 \pm 0.54 \pm 0.05)} \times 10^{-2}$ |
| $B_c \to B_s^0 K^{*0}$ | 2-c | $1.6 \times 10^{-5}$ | $1.3 \times 10^{-5}$ | $1.6 \times 10^{-5}$ | $1.6 \times 10^{-5}$ | $1.6 \times 10^{-5}$ | $1.6 \times 10^{-5}$ | $1.6 \times 10^{-5}$ | $1.6 \times 10^{-5}$ | $1.6 \times 10^{-5}$ | $(6.3 \times 10^{-6})^{\pm (5.7 \times 10^{-6} \pm 1.0)}_{(5.6 \times 10^{-6} \pm 1.5)} \times 10^{-5}$ |
| $B_c \to B_s^0 \bar{K}^{*0}$ | 2-a | $7.8 \times 10^{-3}$ | $8.5 \times 10^{-3}$ | $5.8 \times 10^{-3}$ | $1.6 \times 10^{-3}$ | $1.6 \times 10^{-3}$ | $1.9 \times 10^{-3}$ | $1.3 \times 10^{-2}$ | $2.1 \times 10^{-3}$ | $2.0 \times 10^{-4}$ | $(3.72)^{\pm (0.00 \pm 0.20 \pm 0.10)_{(0.00 \pm 0.10 \pm 0.04)} \times 10^{-3}$ |
| $B_c \to B_s^0 K^{*0}$ | 2-c | $5.0 \times 10^{-6}$ | $3.7 \times 10^{-6}$ | $5.0 \times 10^{-6}$ | $5.0 \times 10^{-6}$ | $5.0 \times 10^{-6}$ | $5.0 \times 10^{-6}$ | $5.0 \times 10^{-6}$ | $5.0 \times 10^{-6}$ | $5.0 \times 10^{-6}$ | $(1.07)^{\pm (0.00 \pm 0.08 \pm 0.06)_{(0.00 \pm 0.08 \pm 0.06)} \times 10^{-5}$ |
| $B_c \to B_s^0 \rho^0$ | 2-b | $4.0 \times 10^{-4}$ | $2.1 \times 10^{-4}$ | $6.4 \times 10^{-5}$ | $6.2 \times 10^{-5}$ | $6.7 \times 10^{-5}$ | $9.7 \times 10^{-5}$ | $4.5 \times 10^{-4}$ | $1.3 \times 10^{-4}$ | $6.6 \times 10^{-4}$ | $(4.23)^{\pm (0.02 \pm 0.17 \pm 0.07)_{(0.02 \pm 0.17 \pm 0.07)} \times 10^{-4}$ |
| $B_c \to B_s^0 \rho^0$ | 2-b | $4.4 \times 10^{-4}$ | $3.7 \times 10^{-4}$ | $1.7 \times 10^{-4}$ | $8.7 \times 10^{-5}$ | $8.9 \times 10^{-5}$ | $1.2 \times 10^{-4}$ | $7.4 \times 10^{-4}$ | $1.3 \times 10^{-4}$ | $5.5 \times 10^{-4}$ | $(2.86)^{\pm (0.00 \pm 0.10 \pm 0.05)_{(0.00 \pm 0.08 \pm 0.05)} \times 10^{-4}$ |
| $B_c \to B_s^0 \omega$ | 2-b | $4.1 \times 10^{-4}$ | $9.0 \times 10^{-5}$ | $7.0 \times 10^{-5}$ | $1.3 \times 10^{-5}$ | $1.3 \times 10^{-5}$ | $1.3 \times 10^{-5}$ | $1.3 \times 10^{-5}$ | $1.3 \times 10^{-5}$ | $1.3 \times 10^{-5}$ | $(2.05)^{\pm (0.01 \pm 0.15 \pm 0.12)_{(0.01 \pm 0.37 \pm 0.12)} \times 10^{-4}$ |
Table 3: Continued.

| Decay mode | Case | Reference [17] | Reference [18] | Reference [19] | Reference [20] | Reference [21, 22] | Reference [23] | Reference [24] | Reference [25] | Reference [26–28] | Reference [29, 30] | QCDF |
|------------|------|----------------|----------------|----------------|----------------|---------------------|----------------|----------------|----------------|-------------------|------------------|------|
| $B_c \to B_s^+ \eta$ |      | 5.0 × 10^{-4} | (4.1 × 10^{-4}) |                      |                     | 1.4 × 10^{-4} | (1.46^{+0.01+0.02+0.13}_{-0.01-0.01-0.12}) × 10^{-3} |                     |                     |                     |                   |      |
| $B_c \to B_s^{*+} \eta'$ | | 6.7 × 10^{-6} | (5.6 × 10^{-6}) |                      |                     | 4.2 × 10^{-6} | (7.28^{+0.04+0.05+0.16}_{-0.04-0.04-0.14}) × 10^{-5} |                     |                     |                     |                   |      |

*It is estimated with the form factors $F_{B_c \to B_s}^{B_c} = 0.925$, $F_{B_c \to B}^{B_c} = 0.91$ and parameter $\omega = 1$ GeV [17] based on the BSW model.

*It is estimated with the instantaneous nonrelativistic approximation and the potential model based on the BS equation.

*It is estimated in a relativistic model with a one-gluon interaction plus a scalar confinement potential based on the BS equation.

*It is estimated with the form factors $F_{0}^{B_s \to B_s} = 0.5$, $F_{B_s \to B}^{B_s} = 0.39$ using a quasipotential in the relativistic quark model [20].

*It is estimated with the form factors $F_{0}^{B_s \to B_s} = 0.58$, $F_{B_s \to B}^{B_s} = 0.39$ in the nonrelativistic constituent quark model [21, 22].

*It is estimated with the form factors $F_{0}^{B_s \to B_s} = 0.573$ (0.571), $F_{B_s \to B}^{B_s} = 0.467$ (0.426) in the light-front quark model based on the Coulomb plus linear (harmonic oscillator) potential, together with the hyperfine interaction [23].

*It is estimated with the form factors $F_{0}^{B_s \to B_s} = 1.03$, $F_{B_s \to B}^{B_s} = 1.01$ in the relativistic independent quark model [24].

*It is estimated within a relativistic constituent quark model [25].

*It is estimated with the form factors $F_{0}^{B_s \to B_s} = 1.3$, $F_{B_s \to B}^{B_s} = 1.27$ in the QCD sum rules [26–28].

*It is estimated with the perturbative QCD approach based on the $k_T$ factorization scheme [29, 30].
Table 4: Hierarchy of amplitudes among the QCDF’s branching ratios for $B_c$ decay.

| Case | Coefficient | CKM factor | Branching ratio | Decay modes |
|------|-------------|-------------|-----------------|-------------|
| 1a   | $a_1$       | $|V_{ud}|V_{c1}| \sim 1$ | $\geq 10^{-3}$  | $B_\pi \pi, B_\rho \rho$ |
| 1b   | $a_1$       | $|V_{ud}|V_{c2}|, |V_{us}|V_{c1}| \sim \lambda$ | $\geq 10^{-3}$  | $B_K, B_\rho \pi, B_\rho \rho$ |
| 1c   | $a_1$       | $|V_{us}|V_{c1}| \sim \lambda^2$ | $\geq 10^{-5}$  | $B_\rho K^*, B_\rho K^{*0}$ |
| 2a   | $a_2$       | $|V_{ud}|V_{c2}| \sim 1$ | $\geq 10^{-3}$  | $B_\rho K^*, B_\rho K^{*0}$ |
| 2b   | $a_2$       | $|V_{us}|V_{c2}|, |V_{us}|V_{c1}| \sim \lambda$ | $\geq 10^{-4}$  | $B_\rho K^*, B_\rho K^{*0}$ |
| 2c   | $a_1$       | $|V_{us}|V_{c1}| \sim \lambda^2$ | $\geq 10^{-4}$  | $B_\rho K^*, B_\rho K^{*0}$ |

(1) There are many uncertainties on the QCDF’s results. The first uncertainty from the CKM factors is small due to the high precision on Wolfenstein parameter $\lambda$ with only 0.3% relative errors [1]. Large uncertainty comes from the renormalization scale, especially for the $a_1$ dominated $B_c \rightarrow B_\rho \rho$ decays. It has been showed [77, 78] that the branching ratios for the Cabibbo favored $B_c \rightarrow B_\rho \rho$ decays are expected to have the largest branching ratio, ~100%, within the QCDF framework. In addition, the branching ratios, which might be promisingly detected at experiments, come from hadron parameters, such as the transition form factors, which is expected to be cancelled from the rate of branching ratios. For example,

$$\frac{\mathcal{B}(B_c \rightarrow B_K)}{\mathcal{B}(B_c \rightarrow B_\rho)} \approx \frac{|V_{ud}|^2}{|V_{us}|} \frac{f_K^2}{f_\rho},$$

$$\frac{\mathcal{B}(B_c \rightarrow B_\rho \rho)}{\mathcal{B}(B_c \rightarrow B_\rho)} \approx \frac{|V_{us}|^2}{|V_{us}|^2} \frac{f_\rho^2}{f_\rho^2} \frac{\mathcal{B}(B_c \rightarrow B_\rho \rho)}{\mathcal{B}(B_c \rightarrow B_\rho \rho)},$$

$$\frac{\mathcal{B}(B_c \rightarrow B_\pi \pi)}{\mathcal{B}(B_c \rightarrow B_\rho \rho)} \approx \frac{1}{2} \frac{|a_1|^2}{|a_1|^2} \frac{\mathcal{B}(B_c \rightarrow B_\rho \rho)}{\mathcal{B}(B_c \rightarrow B_\rho \rho)}.\quad (16)$$

(17) Particularly, the relation of (18) might be used to give some information on the coefficients $a_{1,2}$ and to provide an interesting feasibility research on the validity of the QCDF approach for the charm quark decay. Finally, we would like to point out that many uncertainties from other factors, such as the final state interactions, which deserve the dedicated study, are not considered here. So one should not be too serious about the numbers in Table 3. Despite this, our results will still provide some useful information to experimental physicists; that is, the Cabibbo favored $B_c \rightarrow B_\rho \pi, B_\rho \rho, B_\rho K$ decays have large branching ratios $\geq 1%$, which could be detected earlier.

4. Summary

In prospects of the potential $B_c$ meson at the LHCb experiments, accurate and thorough studies of the $B_c$ decays will be accessible very soon. The carefully theoretical study on the $B_c$ decays is urgently desired. In this paper, we concentrated on the nonfactorizable contributions to hadronic matrix elements within the QCDF framework, while the transition form factors are taken as nonperturbative inputs, which is different from previous studies. It is found that the branching ratios for the Cabibbo favored $B_c \rightarrow B_\rho \pi, B_\rho \rho, B_\rho K$ decays are very large and could be measured earlier by the running LHCb experiment in the forthcoming years.

Appendix

Decay Amplitudes

$$\mathcal{A}(B_c^+ \rightarrow B_\pi^0 \pi^-) = -\frac{G_F}{\sqrt{2}} f_{B_\pi^0} \sum_{m_B} \epsilon_{m_B} \chi_{m_B} \chi_{\pi^-} \langle 0 | V_{ud} | m_{B_c} \rangle.$$

$$\mathcal{A}(B_c^+ \rightarrow B_\rho^0 \rho^-) = -\frac{G_F}{\sqrt{2}} f_{B_\rho^0} \sum_{m_B} \epsilon_{m_B} \chi_{m_B} \chi_{\rho^-} \langle 0 | V_{ud} | m_{B_c} \rangle.$$

$$\mathcal{A}(B_c^+ \rightarrow B_\rho^0 \rho^-) = -\frac{G_F}{\sqrt{2}} f_{B_\rho^0} \sum_{m_B} \epsilon_{m_B} \chi_{m_B} \chi_{\rho^-} \langle 0 | V_{ud} | m_{B_c} \rangle.$$
(B^+_c \rightarrow B^0_d K^+) = -i G_F F_0^{B_c \rightarrow B} f_K \left( m_{B_c}^2 - m_{B^0_d}^2 \right) V_{ud} V_{cd}^* a_1,

\mathcal{A}(B^+_c \rightarrow B^0_d \rho^+)
= \sqrt{2} G_F F_1^{B_c \rightarrow B} f_\rho \left( \epsilon_\rho \cdot p_B \right) V_{ud} V_{cd}^* a_1,

\mathcal{A}(B^+_c \rightarrow B^0_d K^{*+})
= \sqrt{2} G_F F_1^{B_c \rightarrow B} f_K m_K \left( \epsilon_K \cdot p_B \right) V_{ud} V_{cd}^* a_1,

\mathcal{A}(B^+_c \rightarrow B^+_d K^0)
= -i G_F F_0^{B_c \rightarrow B} f_K \left( m_{B_c}^2 - m_{B^+_d}^2 \right) V_{ud} V_{cd}^* a_2,

\mathcal{A}(B^+_c \rightarrow B^+_d K^0)
= -i G_F F_0^{B_c \rightarrow B} f_K \left( m_{B_c}^2 - m_{B^+_d}^2 \right) V_{ud} V_{cd}^* a_2,

\mathcal{A}(B^+_c \rightarrow B^+_d K^{*0})
= \sqrt{2} G_F F_1^{B_c \rightarrow B} f_K m_K \left( \epsilon_K \cdot p_B \right) V_{ud} V_{cd}^* a_2,

\mathcal{A}(B^+_c \rightarrow B^+_d \pi^0)
= +G_F F_1^{B_c \rightarrow B} f_\pi \left( \epsilon_\pi \cdot p_B \right) V_{ud} V_{cd}^* a_2,

\mathcal{A}(B^+_c \rightarrow B^+_d \omega)
= +G_F F_1^{B_c \rightarrow B} f_\omega \left( \epsilon_\omega \cdot p_B \right) V_{ud} V_{cd}^* a_2,

\mathcal{A}(B^+_c \rightarrow B^+_d \eta)
= -i G_F F_0^{B_c \rightarrow B} f_\eta \left( m_{B_c}^2 - m_{B^+_d}^2 \right) V_{ud} V_{cd}^* a_2,

\mathcal{A}(B^+_c \rightarrow B^+_d \eta)
= -i G_F F_0^{B_c \rightarrow B} f_\eta \left( m_{B_c}^2 - m_{B^+_d}^2 \right) V_{ud} V_{cd}^* a_2,

\mathcal{A}(B^+_c \rightarrow B^+_d \eta)
= -i G_F F_0^{B_c \rightarrow B} f_\eta \left( m_{B_c}^2 - m_{B^+_d}^2 \right) V_{ud} V_{cd}^* a_2,

\mathcal{A}(B^+_c \rightarrow B^+_d \eta)
= -i G_F F_0^{B_c \rightarrow B} f_\eta \left( m_{B_c}^2 - m_{B^+_d}^2 \right) V_{ud} V_{cd}^* a_2,

\mathcal{A}(B^+_c \rightarrow B^+_d \eta)
= -i G_F F_0^{B_c \rightarrow B} f_\eta \left( m_{B_c}^2 - m_{B^+_d}^2 \right) V_{ud} V_{cd}^* a_2,

\mathcal{A}(B^+_c \rightarrow B^+_d \eta)
= \cos \phi \mathcal{A}(B^+_c \rightarrow B^+_d \eta) - \sin \phi \mathcal{A}(B^+_c \rightarrow B^+_d \eta),

\mathcal{A}(B^+_c \rightarrow B^+_d \eta)
= \sin \phi \mathcal{A}(B^+_c \rightarrow B^+_d \eta) + \cos \phi \mathcal{A}(B^+_c \rightarrow B^+_d \eta).

(A.1)

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The work is supported by the National Natural Science Foundation of China (Grant nos. 11475055, 11275057, and U1232101). N. Wang thanks the support from CCNU-QLPL Innovation Fund (QLPL201411). Q. Chang is also supported by the Foundation for the Author of National Excellent Doctoral Dissertation of China (Grant no. 201317) and the Program for Science and Technology Innovation Talents in Universities of Henan Province (Grant no. 14HASTIT036).

References

[1] K. Olive, K. Agashe, C. Amsler et al., “Review of particle physics,” Chinese Physics C, vol. 38, no. 9, Article ID 090001, 2014.

[2] N. Brambilla, M. Krämer, R. Mussa et al., “Heavy quarkonium physics,” http://arxiv.org/abs/hep-ph/0412158.

[3] F. Abe, H. Akimoto, A. Akopian et al., “Observation of Bc mesons in ρ̅ρ̅ collisions at √s = 1.8 TeV,” Physical Review D, vol. 61, no. 9, Article ID 112004, 29 pages, 1998.

[4] F. Abe, H. Akimoto, A. Akopian et al., “Observation of the Bc meson in ρ̅ρ̅ collisions at √s = 1.8 TeV,” Physical Review Letters, vol. 81, p. 2432, 1998.

[5] R. Aaij, C. A. Beteta, B. Adeva et al., “Observation of the Bc decay modes,” Physical Review D, vol. 87, Article ID 112012, 2013.

[6] R. Aaij, B. Adeva, M. Adinolfi et al., “Measurement of the lifetime of the Bc meson using the Bc decay mode,” Physics Letters B, vol. 742, pp. 29–37, 2015.

[7] I. P. Gouz, V. V. Kiselev, A. K. Likhoded, V. I. Romanovskiy, and O. P. Yushchenko, “Prospects for the Bc studies at LHCB,” Physics of Atomic Nuclei, vol. 67, no. 8, pp. 1559–1570, 2004.

[8] A. Likhoded and A. Luchinsky, “Light hadron production in B_c → B_s( killers) + X decays,” Physical Review D, vol. 82, Article ID 014012, 2010.

[9] M. Lusignoli and M. Masetti, “B_c decays,” Zeitschrift für Physik C: Particles and Fields, vol. 51, no. 4, pp. 549–555, 1991.

[10] C. Chang and Y. Chen, “Decays of the B_c meson,” Physical Review D, vol. 49, no. 7, pp. 3399–3411, 1994.

[11] S. S. Gershtein, V. V. Kiselev, A. K. Likhoded, and A. V. Tkabladze, “Reviews of topical problems: physics of Bc-mesons,” Physics-Uspekhi, vol. 38, no. 1, pp. 1–37, 1995.

[12] R. Aaij, B. Adeva, M. Adinolfi et al., “Observation of the Bc decay B_c → B^0_d π^+,” Physical Review Letters, vol. 112, Article ID 181801, 2013.

[13] T. Feldmann, P. Kroll, and B. Stech, “Mixing and decay constants of pseudoscalar mesons,” Physical Review D, vol. 58, Article ID 140006, 1998.

[14] P. Ball, “Theoretical update of pseudoscalar meson distribution amplitudes of higher twist: the nonsinglet case,” Journal of High Energy Physics, vol. 1999, no. 1, article 10, 1999.

[15] P. Ball, V. M. Braun, and A. Lenz, “Higher-twist distribution amplitudes of the K meson in QCD,” Journal of High Energy Physics, vol. 2006, no. 5, article 4, 2006.
Advances in High Energy Physics 9

[16] P. Ball and G. Jones, “Twist-3 distribution amplitudes of K* and ϕ mesons,” Journal of High Energy Physics, vol. 2007, no. 3, article 069, 2007.

[17] D. Du and Z. Wang, “Predictions of the standard model for B* weak decays,” Physical Review D, vol. 39, no. 5, pp. 1342–1348, 1989.

[18] C.-H. Chang and Y.-Q. Chen, “Decays of the Bpheric meson,” Physical Review D, vol. 49, no. 7, Article ID 3399, 1994.

[19] A. El-Hady, J. Muñoz, and J. Vary, “Semileptonic and nonleptonic B* decays,” Physical Review D, vol. 62, Article ID 014009, 2000.

[20] D. Ebert, R. N. Faustov, and V. O. Galkin, “Weak decays of the B* meson to B and B mesons in the relativistic quark model,” European Physical Journal C, vol. 32, no. 1, pp. 29–43, 2003.

[21] E. Hernández, J.Nieves, and J. Verde-Velasco, “Study of exclusive semileptonic and nonleptonic decays of B* in a nonrelativistic quark model,” Physical Review D, vol. 74, Article ID 074008, 2006.

[22] E. Hernández, J. Nieves, and J. M. Verde-Velasco, “Study of semileptonic and nonleptonic decays of the Bc-Meson,” European Physical Journal A, vol. 31, no. 4, pp. 714–717, 2007.

[23] H. Choi and C. Ji, “Nonleptonic two-body decays of the B* meson in the light-front quark model and the QCD factorization approach,” Physical Review D, vol. 80, Article ID 114003, 2009.

[24] S. Naimuddin, S. Kar, M. Priyadarshini, N. Barik, and P. C. Dash, “Nonleptonic two-body B*-meson decays,” Physical Review D, vol. 86, Article ID 094028, 2012.

[25] M. Ivanov, J. Körner, and P. Santorelli, “Exclusive semileptonic and nonleptonic decays of the B* meson,” Physical Review D, vol. 73, Article ID 054024, 2006.

[26] V. V. Kiselev, A. E. Kovalsky, and A. K. Likhoded, “B*- decays and lifetime in QCD sum rules,” Nuclear Physics B, vol. 585, no. 1-2, pp. 353–382, 2000.

[27] V. V. Kiselev, A. E. Kovalsky, and A. K. Likhoded, “Bc-Meson decays and lifetime within QCD sum rules,” Physics of Atomic Nuclei, vol. 64, no. 10, pp. 1860–1875, 2001.

[28] L. P. Gouz, V. V. Kiselev, A. K. Likhoded, V. I. Romanovsky, and O. P. Yushchenko, “Prospects for the B studies at LHCb,” Physics of Atomic Nuclei, vol. 67, no. 8, pp. 1559–1570, 2004.

[29] J. Sun, Y. Yang, Q. Chang, and G. Lu, “Phenomenological study of the Bc → BP, BV decays with perturbative QCD approach,” Physical Review D, vol. 89, no. 11, Article ID 114019, 17 pages, 2014.

[30] J. Sun, Y. Yang, and G. Lu, “Study of the Bc → Bπ decay with the perturbative QCD approach,” Science China Physics, Mechanics & Astronomy, vol. 57, no. 10, pp. 1891–1897, 2014.

[31] M. Beneke and G. Buchalla, “B*- meson lifetime,” Physical Review D, vol. 53, article 4991, 1996.

[32] C. Chang, S. Chen, T. Feng, and X. Li, “Lifetime of the Bc meson and some relevant problems,” Physical Review D, vol. 64, Article ID 014003, 2001.

[33] C.-H. Chang, S.-L. Chen, T.-F. Feng, and X.-Q. Li, “Study of the Bc-Meson Lifetime,” Communications in Theoretical Physics, vol. 35, no. 1, p. 57, 2001.

[34] M. Lusignoli and M. Masetti, “B* decays,” Zeitschrift für Physik C: Particles and Fields, vol. 51, no. 4, pp. 549–555, 1991.

[35] A. Y. Anisimov, P. Y. Kulikov, I. M. Narodetski, and K. A. Ter-Martirosyan, “Exclusive and inclusive decays of the Bc meson in the light-front ISGW model,” Physics of Atomic Nuclei, vol. 62, no. 10, pp. 1739–1753, 1999.
leading power," *Nuclear Physics B*, vol. 643, no. 1–3, pp. 431–476, 2002.

[56] M. Beneke and T. Feldmann, "Multipole-expanded soft-collinear effective theory with non-Abelian gauge symmetry," *Physics Letters B*, vol. 553, no. 3–4, pp. 267–276, 2003.

[57] M. Beneke and T. Feldmann, "Factorization of heavy-to-light form factors in soft-collinear effective theory," *Nuclear Physics B*, vol. 685, no. 1–3, pp. 249–296, 2004.

[58] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, "QCD factorization for $B \to \pi \pi$ decays: strong phases and $\mathcal{C}\mathcal{P}$ violation in the heavy quark limit," *Physical Review Letters*, vol. 83, no. 10, pp. 1914–1917, 1999.

[59] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, "QCD factorization for exclusive non-leptonic $B$-meson decays: general arguments and the case of heavy-light final states," *Nuclear Physics B*, vol. 591, no. 1–2, pp. 313–418, 2000.

[60] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, "QCD factorization in $B \to \pi K$, $\pi \pi$ decays and extraction of Wolfenstein parameters," *Nuclear Physics B*, vol. 606, no. 1–2, pp. 245–321, 2001.

[61] D. Du, D. Yang, and G. Zhu, "Infrared divergence and twist-3 distribution amplitudes in QCD factorization for $B \to PP$," *Physics Letters B*, vol. 509, no. 3–4, pp. 263–272, 2001.

[62] D. Du, D. Yang, and G. Zhu, "Analysis of the decays $B \to \pi \pi$ and $\pi K$ with QCD factorization in the heavy quark limit," *Physics Letters B*, vol. 488, no. 1, pp. 46–54, 2000.

[63] D. Du, D. Yang, and G. Zhu, "QCD factorization for $\bar{B}PP$," *Physical Review D*, vol. 64, Article ID 014036, 2001.

[64] G. Buchalla, A. Buras, and M. Lautenbacher, "Weak decays beyond leading logarithms," *Reviews of Modern Physics*, vol. 68, p. 1125, 1996.

[65] G. Bell, "NNLO vertex corrections in charmless hadronic $B$ decays revisited," *Physical Review D*, vol. 73, no. 11, Article ID 114026, 2006.

[66] H. Cheng and C. Chua, "Revisiting charmless hadronic $B_{ud}$ decays in QCD factorization," *Physical Review D*, vol. 80, no. 11, Article ID 114008, 33 pages, 2009.

[67] H. Cheng and C. Chua, "QCD factorization for charmless hadronic $B$ decays revisited," *Physical Review D*, vol. 80, no. 11, Article ID 114026, 25 pages, 2009.

[68] H. Cheng and C. Chua, " Charmless hadronic $B$ decays into a tensor meson," *Physical Review D*, vol. 83, Article ID 034001, 2011.

[69] J. Sun, G. Zhu, and D. Du, "Phenomenological analysis of charmless decays $B_s \to PP, PV$ with QCD factorization," *Physical Review D*, vol. 68, Article ID 094025, 2003.

[70] H. Cheng and C.-K. Chua, "Resolving $B-\mathcal{C}\mathcal{P}$ puzzles in QCD factorization," *Physical Review D*, vol. 80, Article ID 074031, 2009.
Submit your manuscripts at
http://www.hindawi.com