DETERMINING THE HUBBLE CONSTANT FROM HUBBLE PARAMETER MEASUREMENTS

YUN CHEN1, SURESH KUMAR2, AND BHARAT RATRA3

1 Key Laboratory for Computational Astrophysics, National Astronomical Observatories, Chinese Academy of Sciences, Beijing, 100012, China; chenyun@bao.ac.cn
2 Department of Mathematics, BITS Pilani, Pilani Campus, Rajasthan-333031, India; suresh.kumar@pilani.bits-pilani.ac.in
3 Department of Physics, Kansas State University, 116 Cardwell Hall, Manhattan, KS 66506, USA; ratra@phys.ksu.edu

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ABSTRACT

We use 28 measurements of the Hubble parameter, H(z), at intermediate redshifts 0.07 ≲ z ≲ 2.3 to determine the present-day Hubble constant H0 in four cosmological models. We measure H0 = 68.3±2.6, 68.4±3.3, 65.0±6.5, and 67.9±2.4 km s⁻¹Mpc⁻¹ (1σ errors) in the ΛCDM (spatially flat and non-flat), ΩCDM, and ΩCDM models, respectively. These measured H0 values are more consistent with the lower values determined from recent data on the cosmic microwave background and baryon acoustic oscillations, as well as with the value found from a median statistical analysis of Huchra’s compilation of H0 measurements, but include the higher local measurements of H0 within the 2σ confidence limits.

Key words: cosmological parameters – dark energy

1. INTRODUCTION

The current value of the cosmological expansion rate, the Hubble constant H0, is an important cosmological datum. Although one of the most measured cosmological parameters, it was more than seven decades after Hubble’s first measurement before a consensus value for H0 started to emerge. In 2001 Freedman et al. (2001) provided H0 = 72 ± 8 km s⁻¹Mpc⁻¹ (1σ error including systematics) as a reasonable summary of the H0 value from the Hubble Space Telescope Key Project. In the same year Gott et al. (2001) applied median statistics to 331 H0 estimates tabulated by Huchra and determined H0 = 67 ± 3.5 km s⁻¹Mpc⁻¹. During the following decade median statistics was applied to larger compilations of H0 measurements from Huchra, in 2003 to 461 measurements by Chen et al. (2003), who found H0 = 68 ± 3.5 km s⁻¹Mpc⁻¹, and in 2011 to 553 measurements by Chen & Ratra (2011a), who found H0 = 68 ± 2.8 km s⁻¹Mpc⁻¹.

Many more recent H0 determinations are consistent with these results. For instance, the final Wilkinson Microwave Anisotropy Probe (WMAP) measurement is H0 = 70.0 ± 2.2 km s⁻¹Mpc⁻¹ (Hinshaw et al. 2013), while the Atacama Cosmology Telescope and the WMAP seven-year data on the anisotropy of the cosmic microwave background (CMB) give H0 = 70.0 ± 2.4 km s⁻¹Mpc⁻¹ (Sievers et al. 2013); baryon acoustic oscillations (BAO), Type Ia supernovae, and CMB data result in H0 = 67.3 ± 1.1 km s⁻¹Mpc⁻¹ (Aubourg et al. 2015; also see Ross et al. 2015; Bernal et al. 2016; L’Huillier & Shafieloo 2016; Luković et al. 2016), with the Planck 2015 CMB data value being H0 = 67.8 ± 0.9 km s⁻¹Mpc⁻¹ (Ade et al. 2015; but see Addison et al. 2016).

While the consistency of these results is encouraging, some recent local estimates of H0 are larger. Riess et al. (2011) find H0 = 73.8 ± 2.4 km s⁻¹Mpc⁻¹ (but see Efstathiou 2014, who argues that H0 = 72.5 ± 2.5 km s⁻¹Mpc⁻¹ is a better representation), Freedman et al. (2012) find H0 = 74.3 ± 2.1 km s⁻¹Mpc⁻¹, while Riess et al. (2016) give H0 = 73.24 ± 1.74 km s⁻¹Mpc⁻¹.

It is important to understand the reasons for this difference. For instance, the value and uncertainty of H0 affect observational constraints on other cosmological parameters (see, e.g., Samushia et al. 2007; Chen et al. 2016). Given current cosmological data, the standard model of particle physics with three light neutrino species is more compatible with the lower H0 value and difficult to reconcile with the higher value (see, e.g., Calabrese et al. 2012); and the difference between the local and global H0 values might be an indication that the ΛCDM model needs to be extended (see, e.g., Di Valentino et al. 2016).

Here we use measurements of the Hubble parameter, H(z) (where z is redshift), to determine the Hubble constant. H(z) data have previously been used to constrain other cosmological parameters (see, e.g., Samushia & Ratra 2006; Chen & Ratra 2011b; Farooq et al. 2013b, 2015; Farooq & Ratra 2013a; Capozziello et al. 2014; Chen et al. 2015; Meng et al. 2015; Alam et al. 2016; Guo & Zhang 2016; Mukherjee 2016; Solà et al. 2016), including measuring the redshift of the cosmological deceleration–acceleration transition between the earlier nonrelativistic matter-dominated epoch and the current dark-energy-dominated epoch (see, e.g., Farooq & Ratra 2013b; Moresco et al. 2016). See Verde et al. (2014) for an early attempt at measuring H0 from H(z) data. Here we use more data (28 versus 15 measurements) to higher redshift (2.30 versus 1.04) than Verde et al. (2014) used and so find tighter constraints on H0.

We find that our H0 values obtained from H(z) are more consistent with the lower values determined using median statistics or from CMB anisotropy or BAO measurements, and with the predictions of the standard model of particle physics with only three light neutrino species and no “dark radiation.” Systematic errors affecting H(z) measurements are largely different from those affecting CMB and BAO measurements. In addition, median statistics does not make use of the error

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5 For applications and discussions of median statistics see Podariu et al. (2001), Chen & Ratra (2003), Mamajek & Hillenbrand (2008), Croft & Dalley (2015), Andreon & Hurn (2012), Farooq et al. (2013a), Crandall & Ratra (2014, 2015), Ding et al. (2015), Crandall et al. (2015), and Zheng et al. (2016). Median statistics does not make use of the error bars of the individual measurements.

5 https://www.cfa.harvard.edu/~dffabricant/huchra/
while in the general (non-flat) ΛCDM model it is
\[ H(z) = H_0 \sqrt{\Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0} - \Omega_{\Lambda})(1 + z)^2 + \Omega_{\Lambda}}, \]

where \( \Omega_{m0} \) is the current value of the nonrelativistic matter density parameter and \( \Omega_{\Lambda} \) is the cosmological constant density parameter.

In the spatially flat \( \omega \)CDM parameterization we have
\[ H(z) = H_0 \sqrt{\Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})(1 + z)^{(1+wx)}}, \]

where \( \omega_X \) is the constant, negative, equation-of-state parameter relating the (dark energy) \( X \)-fluid pressure and energy density through \( p_X = \omega_X p_X \). The \( \omega \)CDM parameterization is incomplete and does not consistently describe inhomogeneities. However, \( \phi \)CDM, discussed next, is a consistent dynamical dark energy model.

The Friedmann equation of the spatially flat \( \phi \)CDM model is
\[ H^2(z) = \frac{8\pi}{3m_p^2}(\rho_m + \rho_\phi), \]

where \( m_p \) is the Planck mass, \( \rho_m \) is the nonrelativistic matter energy density, and the energy density of the scalar field \( \phi \) is
\[ \rho_\phi = \frac{m_p^2}{32\pi}(\dot{\phi}^2 + \kappa m_p^2 \phi^{-\alpha}). \]

Here an overdot denotes a time derivative, \( \kappa (m_p, \alpha) \) are positive constants, and we have picked an inverse-power-law potential energy density of the scalar field \( V(\phi) = \kappa m_p^2 \phi^{-\alpha}/2 \). The equation of motion of the scalar field is
\[ \ddot{\phi} + \frac{3}{a} \dot{\phi} + \frac{dV}{d\phi} = 0 \]

where \( a \) is the scale factor. These equations are numerically integrated to provide \( H(z) \) in the \( \phi \)CDM model (Peebles & Ratra 1988; Samushia 2009; Farooq 2013).

3. ANALYSIS AND RESULTS

We constrain cosmological parameters by minimizing \( \chi^2_H \):
\[ \chi^2_H(p) = \sum_{i=1}^{N} \frac{[H_{\text{th}}(z_i; p) - H_{\text{obs}}(z_i)]^2}{\sigma_{H,i}^2}, \]

for \( N \) measured values of \( H_{\text{obs}}(z_i) \) with variance \( \sigma_{H,i}^2 \) at redshift \( z_i \) where \( H_{\text{th}} \) is the predicted value of \( H(z) \) in the cosmological model. \( p \) represents the free parameters of the cosmological model under consideration, \( H_0 \) and \( \Omega_{m0} \) in all four cases, with one additional parameter in three of the cases: \( \Omega_{\Lambda} \) in non-flat ΛCDM, \( \omega_X \) in the spatially flat \( \omega \)CDM parameterization, and \( \alpha \) in the spatially flat \( \phi \)CDM model. We use the compilation of 28 \( H(z) \) data points from Farooq & Ratra (2013b) as reproduced here in Table 1 to constrain the model parameters under consideration by using the Markov chain Monte Carlo method coded in the publicly available package CosmoMC (Lewis & Bridle 2002).

Our results are summarized in Table 2 and Figures 1–5. The limits on cosmological parameters shown in Table 2 are derived from the corresponding one-dimensional likelihood.

| \( z \) | \( H(z) \) (km s\(^{-1}\) Mpc\(^{-1}\)) | \( \sigma_H \) (km s\(^{-1}\) Mpc\(^{-1}\)) | Reference |
|---|---|---|---|
| 0.070 | 69 | 19.6 | 5 |
| 0.090 | 69 | 12 | 1 |
| 0.120 | 68.6 | 26.2 | 5 |
| 0.170 | 83 | 8 | 1 |
| 0.179 | 75 | 4 | 3 |
| 0.199 | 75 | 5 | 3 |
| 0.200 | 72.9 | 29.6 | 5 |
| 0.270 | 77 | 14 | 1 |
| 0.280 | 88.8 | 36.6 | 5 |
| 0.350 | 76.3 | 5.6 | 7 |
| 0.352 | 83 | 14 | 3 |
| 0.400 | 95 | 17 | 1 |
| 0.440 | 82.6 | 7.8 | 6 |
| 0.480 | 97 | 62 | 2 |
| 0.593 | 104 | 13 | 3 |
| 0.600 | 87.9 | 6.1 | 6 |
| 0.680 | 92 | 8 | 3 |
| 0.730 | 97.3 | 7.0 | 6 |
| 0.781 | 105 | 12 | 3 |
| 0.875 | 125 | 17 | 3 |
| 0.880 | 90 | 40 | 2 |
| 0.900 | 117 | 23 | 1 |
| 1.037 | 154 | 20 | 3 |
| 1.300 | 168 | 17 | 1 |
| 1.430 | 177 | 18 | 1 |
| 1.530 | 140 | 14 | 1 |
| 1.750 | 202 | 40 | 1 |
| 2.300 | 224 | 8 | 4 |

References. (1) Simon et al. (2005), (2) Stern et al. (2010), (3) Moresco et al. (2012), (4) Busca et al. (2013), (5) Zhang et al. (2014), (6) Blake et al. (2012), (7) Chuang & Wang (2013).
function that results from marginalizing over all of the other parameters. The constraints listed in Table 2 are roughly in line with those now under discussion. The small reduced $\chi^2$ values that follow from the entries in the last line of the table are not unexpected given the results of Farooq et al. (2013a).

The $H_0$ values listed in Table 2 are in good accord with the lower recent values determined by using median statistics on Huchra’s compilation and from CMB and BAO data, as well as with what is expected in the standard model of particle physics with only three light neutrino species and no additional “dark radiation.”

There are two high-weight data subsets in our analysis: the cosmic chronometer data from Moresco et al. (2012) and the Ly$\alpha$ data from Busca et al. (2013). Since both of these results are based on relatively new approaches to measuring $H(z)$, it is informative to see an analysis of $H_0$ when one and then the other of these data sets is omitted from the analysis. When we drop the data of Moresco et al. (2012) from the compilation, we find $H_0 = 67.5_{-3.7}^{+8.0}$ km s$^{-1}$ Mpc$^{-1}$, while by dropping the point of Busca et al. (2013) we obtain $H_0 = 66.9_{-2.8}^{+5.2}$ km s$^{-1}$ Mpc$^{-1}$. Comparing these with the full-data result $H_0 = 68.3_{-2.6}^{+5.1}$ km s$^{-1}$ Mpc$^{-1}$, we observe a minor

| Parameter          | $\Lambda$CDM | Non-flat $\Lambda$CDM | $\sigma$CDM | $\phi$CDM |
|--------------------|--------------|------------------------|--------------|-----------|
| $H_0$              | $68.5_{-2.7}^{+5.2}$ | $68.4_{-3.3}^{+5.4}$ | $65.8_{-6.6}^{+9.4}$ | $67.9_{-7.4}^{+4.7}$ |
| $\Omega_{m0}$      | $0.276_{-0.032}^{+0.072}$ | $0.267_{-0.080}^{+0.110}$ | $0.300_{-0.080}^{+0.114}$ | $0.275_{-0.032}^{+0.063}$ |
| $\Omega_{\Lambda}$ | ...          | $0.708_{-0.176}^{+0.219}$ | ...          | ...       |
| $w_X$              | ...          | ...                    | $-0.780_{-0.292}^{+0.414}$ | ...       |
| $\alpha$           | ...          | ...                    | ...          | ...       |

$\chi^2_{\text{min}}$ | 17.0 | 16.9 | 17.0 | 17.0 |
parameters are also displayed.

Huchra with the value found from a median statistics analysis of determined from the recent CMB and BAO data, as well as values we.

shift in the central values and larger error bars when one or other data subset is omitted from the compilation.

4. CONCLUSIONS

We have used the $H(z)$ data tabulated in Farooq & Ratra (2013b) as reproduced here in Table 1 to measure $H_0$. The $H_0$ values we find are more consistent with the lower values determined from the recent CMB and BAO data, as well as with the value found from a median statistics analysis of Huchra’s compilation of $H_0$ measurements.

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