The optimal parameters technique for the vertical ground heat exchangers of the geothermal heat pump systems

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Abstract. A scientifically substantiated methodology for determination the optimal parameters of vertical heat exchangers of a underground circuit of the geothermal heat pump systems (GHPS) by the condition of reaching the maximum integral effect or Net Present Value (NPV) is presented. The calculations have confirmed the maximum economic efficiency of the vertical ground heat exchangers can be achieved when observing the optimal values of two defining parameters: the total length of U-shaped heat-receiving pipe and the consumption of the heat carrier passing on it. The optimal values of these parameters are calculated depending on the type and temperature of the ground, the diameter and grade of polyethylene pipes, the initial temperature of the heat carrier at the inlet, tariffs for heat and electricity, specific capital investments in all elements of the ground exchangers, including earthworks, taking into account the real discount rate and a number of additional initial data. The technique allows, to determine the optimal construction and operating parameters of vertical ground heat exchangers under various climatic conditions and types of ground. Also, this technique makes it possible to quantitatively evaluate the maximum achievable economic efficiency of using underground circuit with vertical U-shaped exchangers for the extraction of low-potential ground heat. The use of given technique is necessary for a real assessment of the feasibility of using GHPS as alternative sources of thermal energy in each case.

Key words: geothermal heat pump, ground contour, low-temperature heat carrier, vertical ground heat exchanger, optimal parameters, maximum integral effect.

1. Introduction

Research in the sector of modern energy, aimed at expanding the field of using alternative and renewable sources of thermal energy, is especially relevant today for Russia.

The search for inexhaustible, economically profitable and environmentally safe energy sources is due to the need to reduce the consumption of traditional energy resources, which are only becoming more expensive every year, and the mineral resource themselves and the combustion products obtained as a result of their combustion are not environmentally safe for the environment.

Therefore, the energy strategy of Russia defines as one of the important directions of the industry development defines the wider use of non-traditional and renewable energy sources [1].

Along with solar and wind energy, which are mainly used to generate electricity, geothermal resources, which represent the accumulated low-potential thermal energy of the upper layers of the ground, can serve as an alternative source of heat. Therefore, the geothermal heat pumps with the vertical heat exchangers with low-grade heat from an unlimited ground massif, in some cases, may turn out to be promising alternative heat sources of heat and cold supply for detached buildings or objects remote from centralized heat sources [2].

One of the main elements of the GHPS is a ground heat exchanger used directly for the selection of low-potential ground heat. Structurally, the GHPS’s heat exchangers is U-shaped vertical ground heat exchangers (VGHE), which are made of cross-linked polyethylene, is practically not subject to natural decomposition during prolonged contact with the ground. Such exchangers are installed in wells up to 150 m deep, the free space of which is filled with a heat-conducting filler.

It is known that, taking into account the cost of earthworks, about 50% of the one-time capital investments in the GHPS fall on the underground circuit. However, there is still no normative method for calculating the required length of vertical geothermal heat exchangers. There is also no official
methodology for optimizing constructive parameters and operating modes of ground circuits with U-shaped ground heat exchangers. The absence of these techniques significantly complicates the design of energy-efficient and economical ground heat pump unit (HPU), which in turn does not contribute to the widespread practical use of such units as autonomous sources of heat energy in Russia [1, 3-7].

Purpose of the study: scientific substantiation and development of a methodology for calculation of the optimal design parameters and hydraulic modes of operation of vertical ground heat exchangers geothermal heat pump systems on the condition for achieving the maximum integral effect (Net Present Value = max).

2. Formulation of the problem

A useful effect from the use of autonomous heat supply sources with GHPS could be considered a decrease in the annual operating costs of subscribers to pay for the total amount of heat consumed during the heating period. A geothermal heat taken from the ground replaces the heat, which, with an alternative heat supply option, could be supplied to the subscriber from external heat sources with payment at the current tariff. Therefore, in this case, the cost of a unit of heat being replaced can be quantified by the tariff set for heat energy in the alternative heat supply from an external heat source.

However, for the ground heat exchanger operation, forced circulation of the low-temperature heat carrier along with U-shaped circuit is required. Therefore, the overall reduction in the variable part of the annual operating costs of a subscriber using the geothermal HPS could be estimated by the difference between the cost of replaced heat and the operating costs of electricity consumed by the electric motor of the ground contour circulation pump. Taking this into account, the variable part of the integral effect, $ΔI_{int}$, rub., depending only on the design parameters and the hydraulic mode of operation of the ground contour, could be determined using the UNIDO recommended methodology for assessing the economic efficiency of investment projects [10], as the difference between discounted results and costs for expression:

$$ΔI_{int} = (ΔC_{Q,year} - E_{e,year}) \cdot α - K,$$

where $ΔC_{Q,year}$ is the decrease in year operating costs of subscribers to pay for substituted heat, rub./year; $E_{e,year}$ is the year cost of paying for the electricity consumed by the drive of the circulation pump, rub./year; $K$ is the capital investments in the device of a vertical ground heat exchanger, rub.; $α$ is the discount factor, years, determined by the expression:

$$α = E_{\beta}^{-1} \left[ 1 - (1 + E_{\beta})^{-T_S} \right],$$

where $T_S$ is the service life of the ground heat exchanger, years; $E_{R}$ is the real rate of return, year$^{-1}$, determined taking into account inflation using Fisher's formula by the expression:

$$E_{\beta} = (1 + E)(1 + i) - 1,$$

where $E$ is the nominal discount rate, year$^{-1}$; $i$ is the inflation rate, year$^{-1}$.

The research objectives are: identification of the main factors influencing the variable part of the integral effect; obtaining the calculated dependences of all components of equation (1) on the design parameters and hydraulic modes of operation of U-shaped ground heat exchangers; analytical substantiation of the conditions for achieving the global maximum of the function (1) and the development of a technique for optimizing vertical ground exchangers, taking into account real ground and climatic conditions, tariffs for heat and electricity and other initial data determined by the technical of task.

3. Theory

The decrease in the year operating costs of subscribers to pay for substituted heat, included in expression (1), can be determined by the product:

$$ΔC_{Q,year} = Q^h \cdot Q^{int},$$
where $C_{Q}$ is the tariff for heat energy from the replaced heat source, rub./MJ; $Q_{\text{h.p.}}^{\text{int}}$ is the required integral heating capacity of the ground heat exchanger HPU, determined by the subscriber's need for the entire heating period, MJ/year.

In fact, the heat demand of the users (heating load) varies depending on the outside temperature during the heating period. Therefore, the value, $Q_{\text{h.p.}}^{\text{int}}$, MJ/year, could be determined through the installed thermal power of the ground heat exchangers and the duration of the heating period, taking into account the correction for the ratio of temperature differences:

$$Q_{\text{h.p.}}^{\text{int}} = 3600 \cdot 24 \cdot 10^{-6} \cdot z_{\text{h.p.}} \cdot \frac{t_{\text{int}} - t_{\text{av.h.p.}}}{t_{\text{int}} - t_{\text{out.c.h}}} \cdot Q_{\text{max}}^c,$$

where $Q_{\text{max}}$ is the installed heat power of the ground heat exchangers, W, equal to the calculated heat flow taken from the ground in the mode of maximum heat consumption; $z_{\text{h.p}}$ is the heating period, days/year; $t_{\text{int}}$ is the normative temperature of outer air in a heated room, °C; $t_{\text{av.h.p}}$ is the average outer air temperature during the heating period according to climatological data, °C; $t_{\text{out.c.h}}$ is the estimated outer heating temperature in a given area, °C.

The dependence of the calculated heat flux $Q_{\text{max}}^c$, W, perceived from the ground in the mode of maximum heat consumption, on the length of a single ground heat exchanger $L_x$, m, can be represented as follows:

$$Q_{\text{max}}^c = c \cdot G \cdot \Delta t_0 \left[ 1 - \exp \left( -\eta_U \cdot \frac{k_p \cdot L_x}{c \cdot G} \right) \right],$$

where $c$ is the specific heat capacity of the low-temperature heat carrier, J/(kg⋅°C); $G$ is the mass consumption of the heat carrier in the tubular U-shaped element of the ground exchanger, kg/s; $\Delta t_0$ is the calculated temperature difference between the background temperature of the ground (at an infinite distance) and the heat carrier at the entrance to the tubular U-shaped element, °C; $\eta_U$ is the coefficient of decrease in the intensity of heat perception during the interaction of temperature fields of the forward and reverse sections of the pipe of the U-shaped element in the well; $k_p$ is the calculated value of the linear heat transfer coefficient, W/(m⋅°C), numerically equal to the average value of $k(\tau)$ for the five coldest days with a provision of 0.92.

Substitution of expressions (6) and (5) in (4) makes it possible to obtain a dependence for a quantitative assessment of the decrease in annual operating costs, $\Delta C_{Q,\text{year}}$, rub./year, relative to the base case, that is, in the case of replacement of the heat energy paid for by the tariff by geothermal heat of the ground:

$$\Delta C_{Q,\text{year}} = 86.4 \cdot 10^{-3} \cdot z_{\text{h.p.}} \cdot \frac{t_{\text{int}} - t_{\text{av.h.p.}}}{t_{\text{int}} - t_{\text{out.c.h}}} \cdot C_{Q} \cdot c \cdot G \cdot \Delta t_0 \cdot \eta_U \cdot \left[ 1 - \exp \left( -\eta_U \cdot \frac{k_p \cdot L_x}{c \cdot G} \right) \right].$$

The analysis shows that for a given temperature difference between the ground and the heat carrier at the entrance to the U-shaped heat-sensing element $t_0 = \text{const}$, and the known value of $k_p$, which is calculated using a previously developed method depending on the type of ground and the calculated Fourier number, the obtained dependence (7) is a function of only two independent variables: the length of the ground heat exchanger, $L_x$, and the consumption of the heat carrier circulating in it, $G$. These variables are selected as optimization parameters, since all other quantities included in dependence (7) are uniquely are determined by the given external conditions, being constants that cannot vary arbitrarily.

To simplify the mathematical notation of equation (7), we introduce the notation of the constant complexes $A$, rub-s/(kg-year), and $B$, kg/(s-m), formed by the products of constants that do not depend on the selected optimization parameters (length of the ground heat exchanger and heat carrier consumption):
Using expression (9), with known numerical values of complexes (8), it is possible to quantitatively determine the reduction in annual operating costs for paying for heat energy, that is, to estimate the useful effect of the device of a U-shaped ground heat exchanger with a length of $L_x$, m, at a heat carrier consumption $G$, kg/s.

Additional costs included in expression (1) for payment of electricity consumed by the circulating pump drive, $E_{e\text{-year}}$, rub./year, can be determined as:

$$E_{e\text{-year}} = 24 \cdot 10^{-3} \cdot N_{e\text{-inst}} \cdot z_{h}\cdot C_e,$$  \hspace{1cm} (10)

where $N_{e\text{-inst}}$ is the installed power of the circulation pump electric motor, W; $z_{h}$ is the duration of the heating period, days/year; $C_e$ is the tariff for electricity, rub/(kW-h).

The required installed power of the circulating pump can be calculated as:

$$N_{e\text{-inst}} = 1.2 \cdot \frac{\Delta P \cdot G}{\rho \cdot \eta_{e\text{-m}}},$$  \hspace{1cm} (11)

where $G$ is the consumption of the circulating heat carrier, kg/s; $\eta_{e\text{-p}}$ and $\eta_{e\text{-m}}$ are the respectively: efficiency of the circulation pump and electric motor in fractions of a unit; $\rho$ is the density of the coolant, kg/m$^3$; 1.2 is the standard safety factor; $\Delta P$ is the total pressure loss, Pa, in the circulation circuit of the ground heat exchanger and the HPU evaporator, which can be defined as:

$$\Delta P = \lambda \frac{L_x}{d_{\text{int}}^2} \cdot \frac{w^2 \cdot \rho}{2} + \Delta P_{\text{evap}},$$  \hspace{1cm} (12)

where $w$ is the velocity of the heat carrier, m/s; $\Delta P_{\text{evap}}$ is the pressure drop in the heat pump evaporator (along the low-temperature heat carrier circuit), Pa; $\lambda$ is the dimensionless coefficient of friction; $d_{\text{int}}$ is the inner diameter of the pipe of the ground heat exchanger, m.

The heat carrier velocity in the cross section of the pipe of the ground heat exchanger, $w$, m/s, is determined by the expression:

$$w = \frac{4 \cdot G}{\pi \cdot d_{\text{int}}^2 \cdot \rho}.$$  \hspace{1cm} (13)

Since vertical ground heat exchangers are subjected to aggressive action from the ground during operation, special pipes made of "cross-linked" polyethylene are used for their manufacture, which are distinguished by the greatest durability compared to pipes made of other materials. In particular, the passport service life of such pipes under conditions of contact with aggressive media is at least 50 years [9].

According to the instructions for the design and installation of polyethylene pipelines [9], the numerical values of the dimensionless coefficients of friction of pipes made of "cross-linked" polyethylene $\lambda$ are calculated by the expression:

$$\lambda = \left[ \frac{0.5}{\lg(3.7 \cdot d_{\text{int}}/k_{eq})} \left( \frac{b}{2} + 1.312(2-b) \frac{\lg(3.7 \cdot d_{\text{int}}/k_{eq})}{\lg(Re_{\lambda})-1} \right) \right]^2,$$  \hspace{1cm} (14)

where $k_{eq}$ is the coefficient of equivalent roughness, taken for polyethylene pipes equal to $2 \cdot 10^{-5}$ m; $b$ is the number of similarity of flow regimes; $Re_{\lambda}$ is the actual Reynolds number, determined by the ratio [8]:

$$Re_{\lambda} = \frac{w \cdot d_{\text{int}}}{v} = \frac{4}{\pi \cdot d_{\text{int}} \cdot v \cdot \rho} \cdot G,$$  \hspace{1cm} (15)

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$$Re_{\lambda} = \frac{w \cdot d_{\text{int}}}{v} = \frac{4}{\pi \cdot d_{\text{int}} \cdot v \cdot \rho} \cdot G,$$  \hspace{1cm} (15)
where \( \nu \) and \( \rho \) are the respectively: kinematic viscosity, \( \text{m}^2/\text{s} \), and the density of the low-temperature heat carrier, \( \text{kg/m}^3 \), at the temperature of the exchanger inlet.

The number of flow regimes similarity \( b \) is determined by the formula:

\[
 b = 1 + \frac{\lg(Re_{\text{quad}})}{\lg(Re_1)} ,
\]

(16)

where \( Re_{\text{quad}} \) is the Reynolds number corresponding to the beginning of the quadratic range of hydraulic resistances during turbulent movement of the heat carrier, which is determined by the expression:

\[
 Re_{\text{quad}} = 500 \frac{d_{\text{int}}}{k_{\text{eq}}}.
\]

(17)

Specific pressure losses due to friction per 1 m of pipe length, \( R_{sp} \), \( \text{Pa/m} \), taking into account expression (13), can be calculated as:

\[
 R_{sp} = \frac{\lambda}{d_{\text{int}}} \cdot \frac{w^2 \cdot \rho}{2} = 8 \cdot \frac{\lambda}{\pi^2} \cdot \frac{G^2}{\rho \cdot d_{\text{int}}}.
\]

(18)

With the known value of the specific indicator \( R_{sp} \), \( \text{Pa/m} \), the total pressure loss in the closed circulation loop of the U-shaped ground heat exchanger is determined by the expression:

\[
 \Delta P = R_{sp} \cdot L_x + \Delta P_{\text{evap}},
\]

(19)

where \( \Delta P_{\text{evap}} \) is the hydraulic pressure loss in the heat pump evaporator, \( \text{Pa} \), determined by the expression:

\[
 \Delta P_{\text{evap}} = \frac{\xi_{\text{evap}}}{2} \cdot \frac{w_{\text{evap}}^2 \cdot \rho}{2} = \frac{\xi_{\text{evap}}}{2} \cdot \frac{(m \cdot G)^2}{f_{l.s.\text{evap}} \cdot \rho},
\]

(20)

where \( \xi_{\text{evap}} \) is the coefficient of the local resistance of the evaporator along the circuit of the low-temperature heat carrier; \( w_{\text{evap}} \) is the velocity of the low-temperature heat carrier in the air section of the heat pump evaporator, \( \text{m/s} \); \( f_{l.s.\text{evap}} \) is the area of the free cross-section of the evaporator in the low-temperature heat carrier circuit, \( \text{m}^2 \); \( m \) is the total number of heat-receiving U-shaped elements connected in parallel to the HPU evaporator, pcs.

Substituting expressions (19) and (20) in (11), after transformations taking into account (18), we write the following expression to determine the required installed power of the circulating pump:

\[
 e.m.\text{p}_{\text{c},vp} = \frac{\lambda}{\pi^2} \cdot \frac{G^3}{\rho \cdot d_{\text{int}}^3 \cdot \eta_{cp} \cdot \eta_{cm}} \cdot L_x + 0.6 \cdot \frac{\xi_{\text{evap}} \cdot (m \cdot G)^2}{f_{l.s.\text{evap}} \cdot \rho \cdot \pi^3 d_{\text{int}} \cdot \eta_{cp} \cdot \eta_{cm}}.
\]

(21)

Substituting (21) into (10), we obtain an expression for determining the year cost of electricity consumed by the circulation pump:

\[
 E_{\text{year}} = 24 \cdot 10^{-3} \cdot G^{3} \cdot \frac{C_v \cdot G^{3}}{\rho^3 \cdot \eta_{cp} \cdot \eta_{cm}} \left( L_x + 0.6 \cdot \xi_{\text{evap}} \cdot \frac{m^2}{f_{l.s.\text{evap}}} \right).
\]

(22)

In order to simplify the mathematical notation of the obtained equation (22), we introduce the following notation for the constant complexes \( D \), \( \text{rub}\cdot\text{s}^3/(\text{kg}^3\cdot\text{m} \cdot \text{year}) \), and \( F \), \( \text{rub}\cdot\text{s}^3/(\text{kg}^3\cdot\text{year}) \), independent of the length of the ground heat exchanger and the consumption heat carrier:

\[
 D = 230.4 \cdot 10^{-3} \cdot G^{3} \cdot \frac{C_v}{\rho^3 \cdot \eta_{cp} \cdot \eta_{cm}};
\]

\[
 F = 24 \cdot 0.6 \cdot 10^{-3} \cdot G^{3} \cdot \frac{m^2}{f_{l.s.\text{evap}}} \cdot \frac{C_v}{\rho^3 \cdot \eta_{cp} \cdot \eta_{cm}}.
\]

(23)

Taking into account these designations, expression (21) will take the form:

\[
 E_{\text{year}} = D \cdot \lambda \cdot G^{3} \cdot L_x + F \cdot G^{3}.
\]

(24)

Analysis shows that the values of the dimensionless coefficient of friction \( \lambda \) included in equation (24) depend on the consumption and pipe diameter. However, due to the fact that this dependence determined by expression (14) has a rather complex character, for practical engineering calculations a
A simpler two-factor approximating function was obtained in the range variation of consumption $0.05 \leq G \leq 1.05$ kg/s:

$$\lambda = \frac{1}{a \cdot G^{0.085} - b}, \quad (25)$$

where $a$ and $b$ are the coefficients depending on the diameter of standard polyethylene pipes:

$$\begin{cases} a = 81.395 + 9.046 \cdot d_c^{0.36} \quad \text{at outer pipe diameters: } 0.025 \leq d_{\text{out}} \leq 0.05 \text{ m}, \quad (26) \\ b = 38.558 + 63.273 \cdot d_c^{0.36} \quad \end{cases}$$

where $d_c$ is the calculated values of the internal diameters of the line of standard cross-linked polyethylene pipes, m.

Approximating dependence (25) with coefficients (26) was obtained on the basis of mathematical processing of the results of multivariate calculation of the values of $\lambda$, performed according to expression (14) in the indicated ranges of variation of factors. A 25% aqueous solution of propylene glycol with a freezing point of minus 10 °C was used as a low-temperature heat carrier in the calculations. Thermophysical parameters ($\rho = 1030$ kg/m$^3$; $c = 3950$ J/kg·ºС; $\nu = 6 \cdot 10^{-6}$ m$^2$/s) corresponded to the average operating temperature of the heat carrier in the circuit $t = 0$ °C. The maximum relative error of the obtained approximating formula within the specified intervals of variation of the factors does not exceed 1%.

For illustration, the graph in figure 1 shows the curves of friction coefficient change calculated by expression (14) and indicated by the symbol (P) depending on the consumption of the heat carrier, as well as individual points calculated from the obtained approximating dependence (25), which are indicated by the symbol (A).

![Graph showing friction coefficient change](image)

**Figure 1.** Dependence of the friction coefficient on the consumption of a low-temperature heat carrier by the example of an aqueous solution of propylene glycol with a concentration of 25% at $t = 0$ °C.
Graphs figure 1 show that the dimensionless coefficient of friction significantly depends on the consumption of the heat carrier at any chosen constant pipe diameter \( d = \text{const} \). In this case, the close coincidence of the points (A) obtained from the approximating dependence (25) with the calculated curves (P) visually confirms the high accuracy of the simpler approximating formula, which is quite sufficient for performing engineering and technical and economic calculations.

Capital investments in the device of a vertical ground heat exchanger, \( K \), rub., in the first approximation could be considered as consisting of four main components: the cost of a plastic pipe of a given diameter with a length of \( L_x \), m; the cost of a low-temperature heat carrier filling the internal volume of a tubular element of length \( L_x \), m; the cost of excavation work for drilling a well (the depth of which, when using one U-shaped tubular element in the well, is equal to half the length \( L_x \), and with two U-shaped elements, one quarter of this length) and the cost of a heat-conducting filler (bentonite) filling the intertubular space of the well after installation of U-shaped pipe elements. All of the listed components of capital investments are proportional to the length of the U-shaped pipe, so you can write:

\[
K = L_x \cdot C_{sp},
\]

where \( C_{sp} \) is the specific value of the total capital investments per 1 m of the full length of the pipe of one of several U-shaped ground exchangers placed in one well, provided they are connected in parallel by the heat carrier, rub./m, defined as:

\[
C_{sp} = C_{pipe} + C_{lh} \cdot \rho \cdot \frac{\pi \cdot d_{wall}^2}{4} \cdot \frac{C_{exc} + \pi \cdot d_{wall}^2}{2 \cdot n} \cdot \left( \frac{d}{d_{wall}} \right)^2 \cdot C_{h-ag},
\]

where \( C_{pipe} \) is the cost of 1 m of a cross-linked polyethylene pipe of a given diameter, rub./m; \( C_{lh} \) is the cost of 1 kg of low-temperature heat carrier (aqueous solution of propylene glycol), rub./kg; \( \rho \) is the density of an aqueous solution of propylene glycol kg/m\(^3\); \( d_{wall} \) is the required borehole diameter, m; \( C_{exc} \) is the specific cost of earthworks (cost of drilling 1 meter of a well), rub./(m depth); \( n \) is the number of U-shaped exchangers connected in parallel on the heat carrier in one well, pcs./well; \( C_{h-ag} \) is the cost of heat-conducting filler (bentonite) for pouring into the annular space of the well, rub./m\(^3\).

Substituting (9), (24) and (27) into (1) after transformations, we obtain the objective response function in the form most convenient for analytical research on the extremum:

\[
\Delta I_{int} = \alpha \cdot A \cdot G - \alpha \cdot F \cdot G^3 - \alpha \cdot A \cdot G \cdot \exp\left(-B \cdot \frac{L_x}{G}\right) - L_x \cdot C_{ad} - \alpha \cdot D \cdot \lambda \cdot G^3 \cdot L_x.
\]

The maximum of the objective function (29), in this case \( \Delta I_{int} = \text{max} \), can be found from the condition that the partial derivatives of this function are equal to zero for each of the two optimization parameters:

\[
\frac{\partial \Delta I_{int}}{\partial L_x} = \alpha \cdot A \cdot B \cdot \exp\left(-B \cdot \frac{L_x}{G}\right) \cdot \left(\frac{C_{ad} + \alpha \cdot \lambda \cdot D \cdot G^3}{G}\right) = 0;
\]

\[
\frac{\partial \Delta I_{int}}{\partial G} = \alpha \cdot A - 3 \alpha \cdot F \cdot G^2 - 3 \alpha \cdot D \cdot \psi_G \cdot G^2 L_x - \alpha \cdot A \left(\frac{B \cdot L_x}{G} + 1\right) \cdot \exp\left(-B \cdot \frac{L_x}{G}\right) = 0,
\]

where \( \psi_G \) is the a differential function that additionally takes into account the dependence of the friction coefficient in pipes made of cross-linked polyethylene on the consumption of the heat carrier \( G \), obtained by analytical differentiation of the product \( \lambda \cdot G \) taking into account the approximating dependence (25) and the numerical coefficients of this dependence determined by expressions (26):

\[
\psi_G = \frac{1}{(a \cdot G^{0.085} - b)} \left( 1 - \frac{0.085}{1 - b \cdot a^{-1} \cdot G^{0.085}} \right).
\]

Having solved the first equation of system (30) with respect to the length of the ground heat exchanger \( L_x \), we obtain:

\[
L_{opt} = -\frac{G}{B} \ln \left( \frac{C_{ad} + \alpha \cdot D \cdot \lambda \cdot G^3}{\alpha \cdot A \cdot B} \right).
\]
The length calculated according to equation (32) will determine the particular extremum of function (29), being optimal only at one given value of consumption of the heat carrier \( G = \text{const.} \).

Substituting expression (32) into the second equation of system (30), we obtain after transformations:

\[
\frac{\partial \Delta I_{\text{int}}}{\partial G} = A \cdot \left( 1 - \left[ 1 - \ln \left( Z \right) \right] \right) G + \frac{3}{A} \left( G \cdot \Psi(G) \cdot \frac{D}{B} \ln \left( Z \right) - F \right) \cdot G^2 = 0, \tag{33}
\]

where \( Z \) is the a dimensionless variable introduced to simplify the mathematical notation, which depends on consumption of the heat carrier - \( G \), kg/s, and the dimensionless friction coefficient - \( \lambda \), which, according to (25), also depends from of the consumption:

\[
Z = \left( \frac{C_{\text{sp}} + \alpha \cdot D \cdot G \cdot C_{\text{sp}}^2}{\alpha \cdot A \cdot B} \right). \tag{34}
\]

Due to the transcendence of equation (33), its general solution with respect to the consumption \( G \) cannot be obtained algebraically. However, in each specific case (with known numerical values of the complexes \( \alpha, A, C_{\text{sp}}, D, F \), formed from constants specified by the conditions of the problem), the numerical solution of this equation can be obtained by an iterative method of successive approximations.

The algorithm for implementing the iterative method is as follows: by varying the values of the consumption of the heat carrier, the values of the variables \( \lambda, \Psi(G) \) and \( Z \) are calculated, which are substituted into expression (33) until the condition \( \frac{\partial \Delta I_{\text{int}}}{\partial G} = 0 \) is met with a given accuracy. The consumption of the heat carrier at which expression (33) vanishes is optimal for this particular case - \( G_{\text{opt}}, \) kg/s.

4. Experimental results

Calculation example. As an example of the implementation of the developed technique, we will determine the optimal length and optimal consumption of the heat carrier for a vertical heat exchanger with the following initial data: \( t_{\text{gr}} = 8 ^\circ \text{C}; t_0 = 0 ^\circ \text{C}; t_{\text{int}} = 20 ^\circ \text{C}; t_{\text{n.r.o}} = -25 ^\circ \text{C}; t_{\text{av.o.p}} = -3.5 ^\circ \text{C}; z = 188 \) days/year; pipe 32x2.9 (\( d_{\text{out}} = 0.032 \) m; \( d_{\text{int}} = 0.026 \) m; price page = 174.9 rub./m); low-temperature heat carrier water solution of 25% propylene glycol (\( c = 3952 \) J/(kg ⋅ °C); \( \rho = 1040 \) kg/m³; price of \( C_{\text{sp}} = 41.3 \) rub./kg); the tariff to replaceable heat \( Q = 0.456 \) rub./MJ; electricity tariff \( C_{\text{E}} = 3.55 \) rub./(kW ⋅ h); \( \eta_{\text{em}} = 0.98; \eta_{\text{cp}} = 0.8; \) borehole diameter \( d_{\text{well}} = 0.3 \) m; the price of heat-conducting placeholder \( C_{\text{hp}} = 5000 \) rub./m³; the cost of earthworks \( C_{\text{cw}} = 2000 \) rub./(m well); the number of exchangers in one well \( n = 1; \) free area of the HPU evaporator \( f_{\text{en}} = 0.02 \) m²; \( \zeta_{\text{evap}} = 5; \) number of parallel exchangers connected to the evaporator \( m = 6; \) nominal discount rate \( E = 0.1 \) year⁻¹; inflation rate \( i = 0.06 \) year⁻¹; calculation horizon (service life \( T_s = 50 \) years). Given linear coefficient heat transfer depending on the thermophysical characteristics of the ground and taking into account the interaction of temperature fields between the pipe \( \eta_{1}: k_p = 2.75 \) W/(m⋅°C).

The values of the constant complexes calculated by expressions (8) and (23): \( A = 122350 \) rub./s/(kg-year); \( B = 695.85 \times 10^{-6} \) kg/(s⋅m); \( D = 1591 \) rub./s²/(kg³⋅m-year); \( F = 5.1 \) rub./s³/(kg³⋅year).

Optimization calculation results: optimal pipe length \( L_{\text{opt}} = 218.6 \) m; optimal heat carrier flow \( G_{\text{opt}} = 0.4 \) kg/s; maximum integral \( \Delta I_{\text{int}} = 61.37 \) thousand rubles for one exchanger.

For clarity, figure 2 shows the combined graphs of the dependences \( \frac{\partial \Delta I_{\text{int}}}{\partial G} \) of the integral effect and its derivative on the consumption of the heat carrier \( G \), constructed according to equations (29) and (33), provided that the optimal length of the heat-receiving pipe is observed, determined by expression (32).
5. Discussion of results

The presented graphs show that the coordinates of the maximums of energy and economic efficiency do not coincide.

In particular, under the conditions of this example, calculations have established that the maximum heat perception $Q_{\text{max}} = 4278.9 \text{ W}$ is achieved at the consumption $G_{Q_{\text{max}}} = 0.5 \text{ kg/s}$ and the length $L_{Q_{\text{max}}} = 226.7 \text{ m}$, and the maximum optimal length $L_{\text{opt}}^{\text{max}} = 227.7 \text{ m}$ – with $G_{L_{\text{opt}}^{\text{max}}} = 0.48 \text{ kg/s}$. However, both in that and in the other case, the maximum of the integral effect is not achieved, which under these conditions, at $\eta U \cdot k_r = 2.75 \text{ W/(m}^2\text{°C)}$, can be $\Delta I_{\text{int}}^{\text{max}} = 61.4 \text{ thousand rubles}$ at compliance with the optimal parameters: pipe length $L_{\text{opt}} = 218.6 \text{ m}$ and optimal heat carrier flow $G_{\text{opt}} = 0.4 \text{ kg/s}$.

The optimal parameters of U-shaped exchanger for other values of the reduced linear heat transfer coefficient and temperature drops $t_0$, calculated with all other initial data unchanged from the condition of achieving the maximum of the integral effect, are shown in the form of graphs in figure 3 and 4.
Figure 3. Dependence of the optimal pipe length of the U-shaped exchanger on the reduced linear heat transfer coefficient

Figure 4. Dependence of the optimal heat carrier consumption on the reduced linear heat transfer coefficient

In figure 5 and 6 show the dependences of the maximum integral effect and the corresponding calculated heat perception of the exchanger on the reduced linear heat transfer coefficient determined by the type of ground, obtained provided that the optimal length and consumption of the heat carrier are observed.

Figure 5. Dependence of the maximum integral effect achieved at the optimal length and consumption of the heat carrier on the reduced linear heat transfer coefficient

Figure 6. Dependence of the calculated heat perception of the exchanger at the optimal length and consumption of the heat carrier on the reduced linear heat transfer coefficient

In figure 7 and 8 show the dependences of the relative temperature drops on the linear heat transfer coefficient while observing the optimal length of the U-shaped pipe and the consumption of the heat carrier.
Figure 7. Dependence of the relative temperature increment of the heated heat carrier on the reduced linear heat transfer coefficient at the optimal exchanger parameters

Figure 8. Dependence of the relative value of the mean logarithmic temperature head on the reduced linear heat transfer coefficient at the optimal exchanger parameters

The presented graphs are calculated as an example for two values of the differences between the background temperature of an unlimited ground massif and the temperature of the heat carrier at the exchanger inlet \( t_0 = 8 \, ^\circ C \) and \( t_0 = 10 \, ^\circ C \). For other values of \( t_0 \), the obtained dependences should be recalculated according to the developed method.

Analysis of the graphs in figure 3-8 shows that all the indicators of the optimal mode presented on them increase with an increase in the reduced linear heat transfer coefficient, which depends on the thermophysical characteristics of the ground, and the initial temperature difference \( t_0 \) between the ground and the heat carrier. At the same time, a particularly significant increase in the integral effect is observed.

However, it should be understood that in cases where the optimal length of the U-shaped exchanger, obtained by calculation, exceeds 200 m, the well depth must exceed 100 m, which is impractical from a technological point of view, since drilling deep wells requires special equipment and cost earthworks at the same time increases several times. In addition, when using deep wells, the static pressure of the heat carrier on the walls increases significantly, which can exceed the limit value established by the technical regulations of the used grade of polyethylene pipes.

Therefore, in order to comply with the optimal parameters in such cases, it is possible to recommend placing two probes in one well connected in series by the heat carrier, with a total length equal to the calculated optimal value. But at the same time, due to the more intense inter-tube interaction of temperature fields, the value of the reduced linear heat transfer coefficient may significantly decrease, which will require additional research.

6. Conclusions

The calculations have established the maximum economic efficiency of ground heat exchangers in each specific case could be achieved while observing the optimal values of two main parameters: the total length of the pipe of the U-shaped ground exchanger and the consumption of the heat carrier.

The optimal values of these parameters can be calculated according to the developed methodology, depending on the type and temperature of the ground, the diameter and grade of polyethylene pipes (taking into account the limitations on the working pressure), the initial temperature of the heat carrier at the inlet, tariffs for heat and electricity, specific capital investments in all elements of the ground exchanger, including excavation, taking into account the real discount rate and other input data.

Under the conditions of the considered example, graphs of the dependences of the optimal parameters of the ground exchanger on the reduced linear heat transfer coefficient, determined by the
thermophysical characteristics of the ground, and the initial temperature difference between the
ground and the heat carrier are obtained.

The developed optimization technique and the obtained dependencies could be used for
engineering calculation of ground exchangers of the optimal design.

References

[1] Zhurmilova I A 2016 *Improvement of Heating and Cooling Systems for Buildings Using
Ground Heat Exchangers (dissertation for the degree of candidate of technical sciences)*
(Vladivostok) p 155

[2] Semenov B A and Soloviev V A 2009 Problems and Features of Using Ground Heat Pumps for
Autonomous Heat Supply of Facilities in the Central Regions of Russia *Bulletin of the
Saratov State Technical University Press* 38 (Saratov) pp 166–71

[3] Sannera B, Mands E and Sauer M K 2003 Larger geothermal heat pump plants in the central
region of Germany *Geothermics* 32 (Germany) pp 589–602

[4] Samson M, Dallaire J and Gosselin L 2018 Influence of groundwater flow on cost minimization
of ground coupled heat pump systems *Geothermics* 73 (Canada) pp 100–10

[5] Freedman V L, Waichler S R, Mackley R D and Horner J A 2012 Assessing the thermal
environmental impacts of an groundwater heat pump in southeastern Washington State
*Geothermics* 42 (United States) pp 65–77

[6] Koroneos C J and Nanak E A 2017 Environmental impact assessment of a ground source heat
pump system in Greece *Geothermics* 65 (Greece) pp 1–9

[7] Smith D C and Elmore A C 2019 Characterizing lithological effects on large scale borehole heat
exchangers during cyclic heating of the subsurface *Geothermics* 77 (United States) pp 166–74

[8] Mikheev M A and Mikheeva I M 1977 *Heat Transfer Basics* vol 2 (Moscow: Energy) p 155

[9] Kulikhin N G 1999 *Cross-Linked Polyethylene Pressure Pipes for Cold and Hot Water Supply
and Heating Systems* TU 2248-039-00284581-99 (Moscow: NIsantekhnika) p 19

[10] Shakhnazarov G 1994 *Guidelines for Assessing the Effectiveness of Investment Projects and
Their Selection for Financing* (Moscow: Ministry of Economy and Finance of the Russian
Federation) p 81