Abstract. There is a big wind energy potential in supplying the power in an island and most of the islands are off-grid. Due to the limited area in island(s), there is need to find appropriate layout / location for wind turbines suited to the local wind conditions. In this paper, we have considered the wind resources data of an island in Trøndelag region of the Northern Norway, situated on the coastal line. The wind resources data of this island have been analysed for wake losses and turbulence on wind turbines for determining appropriate locations of wind turbines in this island. These analyses are very important for understanding the fatigue and mechanical stress on the wind turbines. In this work, semi empirical wake model has been used for wake losses analysis with wind speed and turbine spacings. The Jensen wake model used for the wake loss analysis due to its high degree of accuracy and the Frandsen model for characterizing the turbulent loading. The variations of the losses in the wind energy production of the down-wind turbine relative to the up-wind turbine and, the down-stream turbulence have been analysed for various turbine distances. The special emphasis has been taken for the case of wind turbine spacing, leading to the turbulence conditions for satisfying the IEC 61400-1 conditions to find the wind turbine layout in this island. The energy production of down-wind turbines has been decreased from 2 to 20% due to the lower wind speeds as they are located behind up-wind turbine, resulting in decreasing the overall energy production of the wind farm. Also, the higher wake losses have contributed to the effective turbulence, which has reduced the overall energy production from the wind farm. In this case study, the required distance for wind turbines have been changed to 6 rotor diameters for increasing the energy gain. From the results, it has been estimated that the marginal change in wake losses by moving the down-stream wind turbine by one rotor diameter distance has been in the range of 0.5 to 1% only and it is insignificant. In the full-length paper, the wake effects with wind speed variations and the wind turbine locations will be reported for reducing the wake losses on the down-stream wind turbine. The Frandsen model has been used for analysing turbulence loading on the down-stream wind turbine as per IEC 61400-1 criteria. In larger wind farms, the high turbulence from the up-stream wind turbines increases the fatigue on the turbines of the wind farm. In this work, we have used the effective turbulence criteria at a certain distance between up-stream and down-stream turbines for minimizing the fatigue load level. The sensitivity analysis on wake and turbulence analysis will be reported in the full-length paper. Results from this work will be useful for finding wind farm layouts in an island for utilizing effectively the wind energy resources and electrification using wind power plants.
turbulence $\sigma_{eff}$ must be lower or equal than the value of the turbulence standard deviation $\sigma_1$ that is given by the 90-percentile value of the wind turbulence intensity $I_{90}$ for the given hub height wind speed $V_{hub}$ [3, 4], such that:

$$\sigma_{eff} \leq \sigma_1,$$  

(1)

where, $\sigma_1$ is the standard deviation of the longitudinal component of the wind velocity, and $\sigma_{eff}$ is effective turbulence intensity.

We consider $V_{hub}$ wind speed at hub height 15 m/s as per IEC standards. The comparison is only necessary in a wind range from 60% of rated wind speed at hub height $V_{hub} < 0.6V_r$. Where, $V_r$ is the rated wind speed averaged over 10 min.

To predict the wind velocity deficit, many studies have been done to compare different engineering wake models to analyse the performance. In a recent case study, Frandsen and Jensen models are explained in detailed, have been compared with Computational Fluid Dynamics (CFD) simulations (see in Methods) [5].

In the following sections, a detailed study has been done on both Jensen and Frandsen analytical models. Both the models are compared to check wake loss sensitivity as well as effective turbulence criteria, respectively. The total power of the downstream wind turbine is compared with upstream turbine to analyse the amount of wake loss effect. Wake loss sensitivity is demonstrated by effective turbulence criteria.

2 Methods

Two models N.O. Jensen and Frandsen [6] are used to calculate the normalised wake loss for downstream wind turbine and effective turbulence intensity $I_{eff}$.

2.1 N.O. Jensen wake model

Wake loss is estimated by using N.O. Jensen Wake Model. The model uses the linear expansion of the wake radius with the downstream spread distance [7], the model describes the single turbine wake.

$$1 - \frac{U_w}{U_{in}} = \frac{1 - \sqrt{1 - C_L(V_{hub})}}{(1 + 2k_w x)^{2}},$$  

(2)

where, $U_w$ is the in-wake velocity, $U_{in}$ is the ambient free stream velocity, $C_L$ is the thrust coefficient, $k_w$ the wake decay coefficient [10] (Onshore value of $k_w = 0.075$ and for offshore value of $k_w = 0.04$ [11]) and $x$ is the turbine spacing, normalized by the rotor diameter.

2.2 Frandsen model

The Frandsen model [12] defines the effective turbulence as a combination of ambient and wake generated turbulence integrated overall directions in a way that accounts for accumulation of fatigue using material properties. The effective turbulence is calculated using the 90th percentile of ambient turbulence as per IEC61400-1 edition-3 2010 amendment [13].

For each wind Turbine Generation position in the calculation, the Frandsen model needs the following inputs[12]:

1. $\bar{\sigma}(\theta, V_{hub})$ and $\bar{\sigma}_f(\theta, V_{hub})$ – Ambient turbulence (mean and standard deviation functions of direction and wind speed)
2. $W(A_i, k_i)$ and $f(\theta)$ – Weibull distributions and sector-wise frequencies
3. $C_T$ – Turbine thrust curve and park geometry
4. $m$ – Relevant material fatigue property

Wöhler exponent [14].

Input 1 is used to calculate the ambient characteristic turbulence, i.e. the 90th percentile.

Input 2 is used to calculate the directional wind speed distribution conditioned on wind speed.

Input 3 is used to calculate the wake generated contribution to turbulence.

Input 4 is used in the fatigue weighted combination model of single directions to obtain an omnidirectional effective turbulence as a function of wind speed only.

Effective turbulence is calculated as function of wind speed only. This is done by integrating the directional variation of turbulence over all directions for each wind speed bin. However, effective turbulence is not a measurable quantity as it combines the directional contributions with a special weighting that accounts for material fatigue via use of the material parameter, the Wöhler exponent. The estimated wake added turbulence ($\sigma_{wake}$) contribution is combined with the 90th percentile of the ambient turbulence at each wind turbine generation. The normal turbulence model is illustrated below for each of the three turbulence classes. To solve for effective turbulence as a function of wind speed at hub height $\sigma_{eff}(V_{hub})$:

$$\sigma_{eff}(V_{hub}) = (\bar{\sigma} + \sigma_{total}(s, V_{hub})^m).f(s))^{m},$$  

(3)

where, $s$ is the number of sectors, (in our case $s=1$), $f$ is the frequency sector-wise, $m$ is the material parameter Wöhler exponent (in our case $m=10$), and $\sigma_{total}$ is the total turbulence.

![Fig. 1. Schematic of Jensen Wake Model](image)
If we consider only one wind direction and Wöhler coefficient (\(m=10\)), then the effective turbulence, \(\sigma_{\text{eff}}\), reduces as:

\[
\sigma_{\text{eff}}(V_{\text{hub}}) = \sigma_{\text{tot}}(V_{\text{hub}})
\]  

(4)

2.3 Wake added turbulence (\(\sigma_{\text{wake}}\))

Wind turbines in their normal operation periods within wind farm changes the ambient turbulence that causes wake from the neighbouring or nearest turbines. We calculate the wake added turbulence (\(\sigma_{\text{wake}}\)), for distance with less than 10RD, to verify with \(\sum_{8}^{\text{50/28/38/32}}=0.14\). We verify the effective turbulence variations over distance less than 10RD, such as:

\[
\sigma_{\text{wake}} = \left\{ \begin{array}{l l}
\frac{V_{\text{hub}}}{1.5+0.8x}, & x < 10\text{RD} \\
\sqrt{c_{f}(V_{\text{hub}})}, & x > 10\text{RD}
\end{array} \right.
\]  

(5)

where, RD is rotor diameter.

The turbulence criteria given by equation (5) solve for the distance that satisfies the effective turbulence criteria of the turbine class (in our case class B) at the given wind speed (\(=15\text{ m/s}\)) [12], and we calculate the wake loss at this distance. Then we verify the changes in wake loss if we increase the distance by one rotor diameter.

2.4 Total Turbulence

The total turbulence \(\sigma_{\text{tot}}\) is different from effective turbulence \(\sigma_{\text{eff}}\), is calculated in each direction combining of characteristic turbulence (\(\sigma_{c}\)) and calculated wake added turbulence \(\sigma_{\text{wake}}\) [12],

\[
\sigma_{\text{tot}} = \sqrt{\sigma_{c}^2 + \sigma_{\text{wake}}^2}
\]  

(6)

The characteristic turbulence (\(\sigma_{c}\)) is calculated as the 90th percentile of the estimated turbulence. The correction factor 1.28 is estimated from the normal distribution curve of standard deviation, where the value of 90% percentile is sought [15].

\[
\sigma_{c} = (\bar{\sigma} + 1.28\delta_{\sigma})
\]  

(7)

The measured turbulence (\(\bar{\sigma}\)) and standard deviation of the measured turbulence (\(\delta_{\sigma}\)) estimated from time series data on 10-min interval at wind speed of 15 m/s. Downstream turbine is positioned such that the effective turbulence criteria is satisfied.

2.5 Turbine Classes

The external conditions to be specified for the design of wind turbines, i.e., extreme wind conditions, material properties etc. In general, when designing a wind farm, it is important to choose a certified turbine as per IEC 61400-1 edition 3 [13, 16], which defines the standard wind turbine design classes as:

| Wind Turbine Class | I     | II    | III   | S     |
|--------------------|-------|-------|-------|-------|
| \(V_{\text{ref}}\) [m/s] | 50.0  | 42.5  | 37.5  | Values specified by the designer |
| A \(I_{\text{ref}}\) [-]  | 0.16  |       |       |       |
| B \(I_{\text{ref}}\) [-]  | 0.14  |       |       |       |
| C \(I_{\text{ref}}\) [-]  | 0.12  |       |       |       |

The selection of wind turbines must be made in terms of its capacity of withstanding with severe wind conditions, and structural component of wind turbine such as the rotor blades, must comply with certain load capacity, that is basically choosing a suitable turbine model to develop a wind farm.

2.6 Estimation of annual energy production

To calculate the annual energy production, it is important to know the power curve. The power curve is function of wind speed, is obtained from given wind turbine model S-108 3MW is shown in Fig. 3.

Fig. 3. Power curve of the Siemens 108 3MW wind turbine, for a range of wind speeds. The turbine starts producing power at cut-in wind speed of 3 m/s, but turbine reaches at its maximum power at wind speed of 13 m/s. When turbine reaches at maximum wind speed of 25 m/s, it stops operating.

The main reason for stopping the wind turbine at cut out speed is safety, because components of the turbines are not designed to handle the loads created by wind speed higher than the cut-out speed.

Fig. 4. The thrust coefficient curve, typically available from the manufacturer. Both the wind turbine power curve and thrust coefficient curve for the Siemens 108 turbine are also used for the study of wake loss effects.

Then, the overall annual energy \((E)\) production of each wind turbine can be calculated as:

\[
E(V_{\text{hub}}) = \int_{3}^{25} P(V_{\text{hub}}) W(V_{\text{hub}}, A, k) dV,
\]  

(8)
where, \( P \) is the Power curve of turbine as a function of wind speed at hub height, \( W \) is the Weibull distribution function, the \( k \) is Weibull shape parameter (equal to 2), \( A \) is Weibull scale parameter that is scaled at mean wind speed, 8.11 m/s and calculated as:

\[
A = \frac{\bar{U}}{\Gamma(1+\frac{1}{k})}, \quad (9)
\]

where, \( \bar{U} \) is the mean wind speed and \( \Gamma \) the gamma function.

The annual energy loss is estimated over the whole range of wind speed from 3-25 m/s. The energy loss is calculated for the downstream turbine due to the wake effects.

\[
E_w(V_{def}) = \int_3^{25} P(V_{hub})W(V_{hub},A_w,k) dV \quad (10)
\]

Jensen wake model uses the assumption that the momentum is conserved inside the wake. Weibull parameter \( A_w \) is corrected by using sensitivity of the wake loss \( (\Delta U) \) for downstream turbine to check the energy loss. Whereas, sensitivity of the wake loss \( \Delta U \) or the velocity in the fully developed wake is calculated as:

\[
\Delta U = \bar{U} \times \frac{1-\tau(V_{hub})}{(1+2kk)^2} \quad (11)
\]

The total power loss is estimated by:

\[
\Delta E(V) = E(V_{hub}) - E_w(V_{def}), \quad (12)
\]

where, \( E \) is the energy from upstream turbine [kWh] and \( E_w \) the energy from downstream turbine [kWh].

3 Results and discussion

3.1 Effective Turbulence Criteria

Effective turbulence is a simplified way of performing load calculations, recommended in the IEC 61400-1 standard. It is based on Frandsen’s model. The effective turbulence criteria is checked by using normal turbulence model (NTM) as per IEC 61400-1 [13]. The parameter for the effective turbulence \( \sigma_{eff} \) is calculated and compared with value of the turbulence standard deviation \( \sigma_t \), that is given by the 90 percentiles for the given hub height wind speed, \( V_{hub}(=15m/s) \), see Fig. 5. This value of the standard wind turbine class is described as [13]:

\[
\sigma_t = I_{ref}(0.75V_{hub} + 5.6), \quad (13)
\]

where, \( I_{ref} \) is referenced value of turbulence intensity at certain hub height at 10 min average wind speed of 15 m/s. All values of the \( V_{hub} \) must be between the wind speed \( 0.2V_{ref} - 0.4V_{ref} \), \( (V_{ref} = 42.5 \, \text{for class B turbines}) \).

The effective turbulence \( \sigma_{eff} \) is verified with standard deviation turbulence intensity \( \sigma_t \) over distance less than 10RD, see Fig. 6. Increase in loading are result of wake effects, and that is accounted for using effective turbulence.

![Fig. 5. Turbulence standard deviation for the normal turbulence model (NTM).](image)

The value of turbulence is larger due to the higher wake of upstream turbine from \( x = 2RD - 6RD \), where it violates the condition of \( \sigma_t \), is in order of 4.5 to 2, see Fig. 6. The effective turbulence criteria \( \sigma_{eff} \) is satisfied when turbine spacing at \( x = 8.4RD \).

![Fig. 6 Effective turbulence criteria satisfy the optimum turbine distance at \( x = 8.4RD \).](image)

Table 2. Wake Loss at satisfied turbulence criteria

| Distance | Wake Loss | Wake Loss |
|----------|-----------|-----------|
| 1417     | 1368      | 3.38      |
| 8.40RD   | 2.359     |           |

Wake Loss with respect to distance is demonstrated where the total power production from freestream or upstream turbine is 1417 kWh, while power production from downstream turbine is decreased to 49 kWh, due to the wake loss effects at normalized distance \( x = 5RD \).
Table 3. Wake Loss with Respect to Distance

| E(V)  | E(V_{deficit}) | ΔE [%] | Distance [RD] |
|-------|----------------|--------|---------------|
| 1417  | 1368           | 7.26   | 5             |
| 1417  | 1337           | 5.6    | 6             |
| 1417  | 1353           | 4.5    | 7             |
| 1417  | 1365           | 3.65   | 8             |
| 1417  | 1373           | 3      | 9             |
| 1417  | 1380           | 2.6    | 10            |

Fig. 7. Wake loss with respect to turbine spacing in total, the energy loss at the distance of 5RD, is 7.26 %. The marginal change in wake losses is 0.5 to 1 % at 5RD. The losses are not significant.

In complex terrain, the "standard" wake decay coefficient $k_w = 0.075$ is lower, because of more turbulent mixing, which results in a faster decay. WindPro [16] has included the relationship of wake decay coefficient as a linear relationship of ambient turbulence $\sigma$ and wake decay coefficient, $k_w = 0.37$ at 8.11 m/s [12].

$$k_w = 0.47 \cdot \sigma(V_{hub} \cdot x) / V_{hub} + 0.04. \quad (14)$$

Fig. 8. The marginal change in wake loss, by moving the turbine one RD further downstream is $dW/dx$, where, $W$ is the wake loss function or a given mean wind speed (in our case 8.11 m/s). The marginal change in wake losses by accounting for wake losses at rotor distances around 5 RD, is in the order of less than 1 %.

Wake losses as function of rotor distances are in the order of 5-15%. The results assume Weibull distributions, and the SWT-108 power curve. If we consider an optimized layout without accounting for wake effects, moving the turbine downstream will gain some more energy (in the order of 2% from reduced wake losses), but will also lose some energy due to lower ambient wind conditions, and it is quite probable that this would reduce the potential gain from including wake effects down to 1% or less.

The other consideration is that positioning the turbine downstream from an optimization that does not consider the wake loss will be a less windy spot. The marginal change in energy is the sum of the reduced wake loss from moving the turbine downstream, plus the reduced energy production from a less windy location. We would assume this effect by reducing the effect of wake losses by 50%.

Fig. 9. Shows the wake loss effect at the optimum distance satisfied by effective turbulence criteria.

4 Conclusion

This paper demonstrates an analysis tools for windfarms in view of both the energy gain and the restriction of the fatigue loads within acceptable limits. As example, for a two turbine along-wind case the dependence of both wake losses and turbulence conditions to turbine spacing is analysed. As a result, it can be stated that for the case analysed the turbine spacing that results in turbulence conditions follow IEC 61400-1 minor wake losses. However, the required distance is elevated with 6 rotors diameters.

Wind turbines are sensitive to the wake losses at distances when they are operating in a large wind farm. N.O. Jensen wake model is used to check the wake effects from the upstream turbines. From the experimental results, we estimated that the marginal change in wake loss by moving downstream turbine by one rotor diameter distance is in order from 0.5 to 1% only. This marginal change is insignificant. On the other hand, if wake effects are not considered to optimize the wind farm layout, result can be less windy place for turbine, that will reduce the wake losses by half margin.

By using Frandsen model, we analysed the increased loads on downstream turbine during their normal operations as per IEC 61400-1 criteria (WFDs uses the same approach). In large wind farm, the high turbulence from upstream turbine increases the fatigue damage levels. We satisfy the effective turbulence criteria at a certain distance between upstream and downstream turbines to minimize the fatigue load level.
To sum up, it can be estimated from the analytical results that wake losses are not so important in load compliant optimizations. However, we must satisfy the effective turbulence criteria at certain distance to avoid fatigue for lifetime damages.

In more general applications, this tool can be used to optimize the wind farm layout that provide fast and accurate wind turbine suitability assessment, maximizing the annual energy production including load constraints as per IEC 61400-1 standard.

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