Investigation of Seismic Behaviour of Low-Temperature Tanks Taking into Account Loose Perlite Insulation

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Abstract. This paper explores some specific features of seismic analysis of low-temperature tanks for storage of large amounts of liquefied gases under near-atmospheric pressure. Major construction types of this kind of tanks are composed of two vertical cylindrical containers with loose perlite insulation in between. The research is carried out to estimate the influence of the presence of loose isolation on a dynamic interaction between inner and outer vessels and stored liquid in case of a full tank and in case of an empty tank. It is concluded that the effects of this interaction can be neglected in case of a full tank and can’t be neglected in case of an empty tank.

1. Introduction
The increasing use of liquefied gases require significant amount of large facilities in order to keep cryogenic liquids. Low-temperature tanks (including cryogenic tanks) are objects that stand out from other liquid storages both in terms of design complexity, dimensions, and role in the country's fuel and energy industry and technological hazards. LNG and LPG facilities, as well as other facilities that require storage of liquefied gases, are frequently built in seismic areas. Thus, it is needed to conduct a proper study of the seismic behaviour of these tanks.

Numerous research works were published in a field of seismic analysis of liquid storage tanks, starting from the 1930s. A significant contribution to the solution of this problem was made by Housner [1], Haroun [2–4], Veletsos [5], Malhotra [6,7], Manos [8], Nikolaenko [9]. A detailed review of these works is given in [10].

The most frequent variant of low-temperature tanks is composed of two vertical cylindrical containers with loose perlite insulation in between. Seismic effects of low-temperature tanks were examined in [11–16]. The effects of perlite insulation in double-wall tanks were hardly investigated. Issa et al[17] investigated the compaction of perlite insulation for relatively small NASA cryogenic containers, using the end chronic theory of plasticity. It was concluded that the effect of perlite compaction is negligible. Shigapov and Kovalchuk [18] conducted a study on the effects of perlite insulation during operation and made an estimation of additional inertial pressure of loose perlite body. Thus, the influence of loose insulation on dynamic interaction needs more detailed research.

2. Formulation of problem
2.1. Formulation of FSI problem
In order to analyze the seismic behaviour of low-temperature tanks, one shall determine, whether load transfer between inner and outer vessels occurs. The second purpose of the analysis is to find out if
patterns and frequencies of “flexible modes” of inner wall with and without the perlite insulation differ from each other.

A variation functional of coupled problem, including effects of perlite, takes the form:

\[
\Lambda = \int_{V} \left( T - \Pi + \frac{E}{2} \int (\nabla \phi \nabla \phi) dV - \rho \int (\phi \dot{\phi}) dS \right) + W_p \right) d\tau, \tag{1}
\]

where \( T \) is the shell kinetic energy, \( \Pi \) is the shell strain energy, \( W_p \) is the work, done by perlite passive pressure, \( S_2 \) is wall surface and \( \phi \) is the fluid velocity potential.

Applying Hamilton’s Principle to the functional and using Finite Element Method for the discretization of a wall and Boundary Element Method for the fluid domain as given in \[19\], we obtain the following equation for a full tank without perlite for every circumferential mode \( n \) as:

\[
([M_s] + [DM])\{\bar{u}\} + [K_s]\{\bar{u}\} = 0 \tag{2}
\]

where \([M_s], [K_s]\) are mass and stiffness matrices, respectively, and \(\{\bar{u}\}\) is the global displacement vector. Basic functions for FEM are Hermitian polynomials. Basic functions for BEM are:

\[
\tilde{N}_j(r, \theta, z) = \sum_{n=1}^{L_n} \left( \frac{V_r}{H_i} \right) \cos \left( \frac{V_z}{H_i} \right) \cos(n\theta), \tag{3}
\]

where \( L_n \) is the modified Bessel function of the first order, \( H_i \) is the height of liquid and

\[
V_j = \frac{(2j+1)\pi}{2}. \tag{4}
\]

### 2.2. Formulation of an eigenvalue problem

As shown in \[18\], there are two states of perlite body – compacted and uncompacted. The compacted state can be approximately modelled with the elastic body with Young Modulus, equal to 500 kPa. Compacted state of perlite is used for estimation of eigenmodes and their frequencies via eigenvalue problem.

Passive pressure, in this case, can be expressed as:

\[
p_{p,\phi}^E(z) = \frac{E_p}{d_p} w(\theta, z, t), \tag{5}
\]

where \( d_p \) is the perlite body width, \( E_p \) is the Young Modulus and \( w \) is radial displacement.
Table 1. Formulation of eigenvalue problem

| Case                              | Equation                                      |
|-----------------------------------|-----------------------------------------------|
| Empty tank without perlite        | $[M_s](\vec{u}) + [K_s](\vec{u}) = 0$        |
| Empty tank with perlite           | $[M_s](\vec{u}) + ([K_s] + [K_p])(\vec{u}) = 0$ |
| Full tank without perlite         | $([M_s] + [DM])(\vec{u}) + [K_s](\vec{u}) = 0$ |
| Full tank with perlite            | $([M_s] + [DM])(\vec{u}) + ([K_s] + [K_p])(\vec{u}) = 0$ |

2.3. Formulation of dynamic problem

The uncompacted state can be described with nonlinear “stress-strain” curves, obtained in [20]. Relations for this curve for an initial density of perlite $\rho_0$ take the form:

$$
\varepsilon = 1 - \frac{1}{A_1 \exp \left[ B_1 p_s^\rho \right] - (A_1 - 1) \exp \left[ -100B_1 p_s^\rho \right]}
$$

where $A_1 = 0.2 \left( \frac{\rho_0}{58} \right)^{-5.5} + 1.13; B_1 = 0.003 \left( \frac{\rho_0}{58} \right)^{-0.34}$.

Specific work, done by passive pressure, can be approximated by the following expression:

$$
w_p = \int_0^w p_s^\rho(w) dw \bigg|_{w>0} = -1.9 e^{27w} \bigg|_{w>0};
$$

Seismic inertia pressure can be found by the expression [18]:

$$
p_s^\rho(z) = \rho_s^\rho(z) \int_S d_s \left( \frac{z_s}{z_p} \frac{R_s}{d_p} + 1 \right) \left[ \frac{k \sin \theta - \cos \theta}{k^2 + 1} + \cos \theta \right]
$$

where

$$
k = \frac{\xi \tan(\phi_p)(R_1 + R_2)}{d_p}.
$$

$\xi_s$ is the lateral pressure coefficient, $\phi_p$ is the angle of internal friction and $\rho_s^\rho$ is the perlite density. According to [20], $\xi_s = 0.55; \phi_p = 29^\circ$.

Integrating this expression by circumferential direction and applying discretization procedure, we obtain load vectors $\{P_{p,NL}^\rho\}$ and $\{P_p^\rho\}$, respectively.
The dynamic problem requires expressions for work, done by external excitation. Following the procedure, given \[19\], we obtain expressions for impulsive pressure vector \(\{P_i\}\) and shell inertia force vector \(\{P_s\}\) for \(n = 1\). Symmetrical modes \((n > 1)\) does not produce resulting force in case of perfect shell, so these modes were omitted. Thus, motion equations for \(n=1\) take the following form (table 2).

**Table 2. Formulation of dynamic problem**

| Case                           | Equation                                                                                                                                 |
|--------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| Empty tank without perlite     | \([M_s]\{\ddot{u}\} + [K_s]\{\ddot{u}\} = -\{P_i\}\)                                                                                     |
| Empty tank with perlite        | \([M_s]\{\ddot{u}\} + [K_s]\{\ddot{u}\} = \{P_i^p\} + \{P_{p,NL}\} - \{P_i\}\)                                                          |
| Full tank without perlite      | \(([M_s] + [DM])\{\ddot{u}\} + [K_s]\{\ddot{u}\} = -\{P_i\} - \{P_s\}\)                                                                |
| Full tank with perlite         | \(([M_s] + [DM])\{\ddot{u}\} + [K_s]\{\ddot{u}\} = \{P_i^p\} + \{P_{p,NL}\} - \{P_i\} - \{P_s\}\)                                  |

Input seismic data, used in the analysis, is a model of seismic excitation "SA-482"[21], which represents a set of universal characteristics intended for seismic calculations of on-ground facilities, consisting of generalized spectra of dynamic response factors and a synthesized accelerogram SA-482(figure 1). This input signal combines the parameters of a commonly used set of analogue accelerograms. It has low selectivity to the eigenfrequencies of the structure and allows to omit damping in the analysis.

**Figure 1.** Input time-history data, \(\%g\)
A multi-paradigm numerical computing environment MATLAB was used for computation procedure. Symbolic computation was used to minimize cumulating computational error. Eigenfrequencies and eigenvectors were obtained by using Cholesky decomposition.

A numerical implicit-explicit integration method by Hughes [22] was implemented in MATLAB for the integration of motion equations with the following algorithm:

1) determination of input acceleration and external load vector for (i+1) step.
2) determination of displacement and velocity vectors for (i+1) step:

\[
\begin{align*}
\{u(i+1)\} & = \{u(i)\} + \{\dot{u}(i)\} \Delta t + 0.25 \Delta t^2 \{\ddot{u}(i)\} \\
\{\ddot{u}(i+1)\} & = \{\ddot{u}(i)\} + 0.5 \Delta t \{\dddot{u}(i)\}
\end{align*}
\]  \hspace{1cm} (10)

3) determination of corrected acceleration, displacement and velocity vectors for (i+1) step:

\[
\begin{align*}
\{\dddot{u}(i+1)\} & = \left(\left[\mathbf{M}\right] + 0.25 \Delta t^2 \left[\mathbf{K}\right]\right)^{-1}\left(-\left[\mathbf{M}\right] \dddot{u}_s(i+1) - \left[\mathbf{K}\right] \dddot{u}(i+1) - \mathbf{P}_{ext}\right) \\
\{\dddot{u}(i+1)\} & = \left(\dddot{u}(i+1)\right) + 0.5 \Delta t \{\dddot{u}(i+1)\}
\end{align*}
\]  \hspace{1cm} (11)

4) go to the next step.

3. Results

Results, obtained from MATLAB were analyzed and translated into graphic form.

Graphical representation of some results of the eigenvalue problem solution is given below (table 3 and figure 2,3). The following observation can be made:

- period of free oscillations of first mode \((n = 1, m = 1)\) reduced by 11-13%;
- periods of free oscillations of higher circumferential modes significantly change \((n \geq 2, m = 1 \div 5)\) in both cases;
- high axial modes \(m > 3\) make an extremely small contribution to the total modal mass.

### Table 3. Comparison of periods for cases with and without perlite

| Axial mode | Circumferential mode, \(n\) | Period change of Empty Tank, % | Period change of Full Tank, % |
|------------|-----------------------------|-------------------------------|-------------------------------|
|            | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  |
| 1          |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 2          |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 3          |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 4          |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 5          |     |     |     |     |     |     |     |     |     |     |     |     |     |

-11.0 -25.3 -41.8 -55.2 -64.4 -70.4 -74.2 -76.6 -78.1 -79.0 -79.6 -80.0 -80.2

-12.7 -27.7 -44.3 -57.6 -66.7 -72.5 -76.1 -78.3 -79.6 -80.3 -80.7 -80.9 -80.9

-6.5 -10.0 -4.4 -5.8 -6.9 -7.7 -8.4 -9.2 -10.0 -10.9 -11.7 -12.6 -13.5

-6.8 -6.6 -6.3 -6.7 -7.1 -7.7 -8.3 -9.0 -9.7 -10.5 -11.4 -12.2 -13.0
**Empty tank without perlite**

- Figure 2. Solution of the eigenvalue problem for the empty tank

**Empty tank with perlite**

- Figure 3. Solution of the eigenvalue problem for the full tank

**Full tank without perlite**

**Full tank with perlite**
Graphical representation of displacement vector for key time steps is given below (figure 4). The following observation can be made:
– presence of perlite insulation has a negligible effect on displacements of the wall of the full tank;
– presence of perlite insulation significantly changes displacements of the wall of the empty tank.

Figure 4. Solution of direct integration of motion equation

4. Conclusions
The following conclusions were made as a result of an investigation:

1. As a result of eigenvalue problem solution for the case of compacted perlite, it was found, that presence of insulation can significantly change periods of high flexible modes while periods of first mode change by 11-13%. Assumption of compacted state of perlite is very conservative, thus, a real change of period of the first mode will be less than 10%. In accordance with calculations based on a simplified mechanical model, only vibration forms with \( n = 1 \) are taken into account (symmetric vibration shapes do not have a net effect), therefore it must be concluded that for practical calculations from linear-spectral characteristics, the influence of perlite thermal insulation can be neglected.

2. The results of direct integration show that the presence of perlite insulation has virtually no effect on the oscillations of the wall of the filled tank. The values of the passive pressure in the filled tank are quite small, therefore, the transfer of the load from the inner tank to the outer one does not occur. Thus seismic calculations of the outer tank and the filled inner tank can be made separately.

3. However, the presence of perlite insulation significantly affects the oscillations of the walls of an empty tank. This means that it is impossible to ignore the effects of perlite insulation when calculating an empty tank.
Further research is needed in order to more accurate estimation of the influence of the perlite insulation on higher flexible modes. Complicated dynamic properties of loose body are omitted in this study, therefore, an experimental verification of results is needed.

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