Approximation Algorithms for the Bottleneck Asymmetric Traveling Salesman Problem

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Abstract. We present the first nontrivial approximation algorithm for the bottleneck asymmetric traveling salesman problem. Given an asymmetric metric cost between \(n\) vertices, the problem is to find a Hamiltonian cycle that minimizes its bottleneck (or maximum-length edge) cost. We achieve an \(O(\log n / \log \log n)\) approximation performance guarantee by giving a novel algorithmic technique to shortcut Eulerian circuits while bounding the lengths of the shortcuts needed. This allows us to build on the recent result of Asadpour, Goemans, Madry, Oveis Gharan, and Saberi to obtain this guarantee. Furthermore, we show how our technique yields stronger approximation bounds in some cases, such as the bounded orientable genus case studied by Oveis Gharan and Saberi.

Keywords: Approximation algorithms, traveling salesman problem, bottleneck optimization.

1 Introduction

In this paper, we study the bottleneck asymmetric traveling salesman problem; that is, in contrast to the variant of traveling salesman problem most commonly studied, the objective is to minimize the maximum edge cost in the tour, rather than the sum of the edge costs. Furthermore, while the edge costs satisfy the triangle inequality, we do not require that they be symmetric, in that the distance from point \(a\) to point \(b\) might differ from the distance from \(b\) to \(a\). The triangle inequality is naturally satisfied by many cost functions; for example, minimizing the longest interval between job completions in the no-wait flow-shop reduces

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to the bottleneck asymmetric traveling salesman problem under a metric cost. The bottleneck asymmetric traveling salesman problem cannot be approximated within a reasonable factor without assuming the triangle inequality. Surprisingly, no approximation algorithm was previously known to deliver solutions within an \(o(n)\) factor of optimal, where \(n\) denotes the number of nodes in the input. We present the first nontrivial approximation algorithm for the bottleneck asymmetric traveling salesman problem, by giving an \(O(\log n / \log \log n)\)-approximation algorithm. At the heart of our result is a new algorithmic technique for converting Eulerian circuits into tours while introducing “shortcuts” that are of bounded length.

For any optimization problem defined in terms of pairwise distances between nodes, it is natural to consider both the symmetric case and the asymmetric one, as well as the min-sum variant and the bottleneck one. The standard (min-sum symmetric) traveling salesman problem (TSP) has been studied extensively \[18\], and for approximation algorithms, Christofides’ 3/2-approximation algorithm \[5\] remains the best known guarantee, and yet the strongest NP-hardness result, due to Papadimitriou and Vempala \[22\], states that the existence of a \(\rho\)-approximation algorithm with \(\rho < 220/219\), implies that P=NP. In contrast, for the bottleneck symmetric TSP, Lau \[17\], and Parker & Rardin \[23\], building on structural results of Fleischner \[7\], give a 2-approximation algorithm, and based on the metric in which all costs are either 1 or 2, it is easy to show that, for any \(\rho < 2\), the existence of a \(\rho\)-approximation algorithm implies that P=NP. For the asymmetric min-sum problem, Frieze, Galbiati, and Maffioli \[8\] gave the first \(O(\log n)\)-approximation algorithm, which is a guarantee that was subsequently matched by work of Kleinberg and Williamson \[16\], and only recently improved upon by work of Asadpour, Goemans, Madry, Oveis Gharan, and Saberi \[3\].

This cross-section of results is mirrored in other optimization settings. For example, for the min-sum symmetric \(k\)-median problem in which \(k\) points are chosen as “medians” and each point is assigned to its nearest median, Arya, Garg, Khadkacak, Meyerson, Munagala, and Pandit \[2\] give a \(\rho\)-approximation algorithm for each \(\rho > 3\), whereas Jain, Mahdian, Markakis, Saberi and Vazirani prove hardness results for \(\rho < 1 + 2/e\) \[14\]. In contrast, for the bottleneck symmetric version, the \(k\)-center problem, Hochbaum and Shmoys \[12\] gave a 2-approximation algorithm, whereas Hsu and Nemhauser \[13\] showed the NP-hardness of a performance guarantee of \(\rho < 2\). For the asymmetric \(k\)-center, a matching upper and lower bound of \(\Theta(\log^* n)\) for the best performance guarantee was shown by Panigrahy & Vishwanathan \[21\] and Chuzhoy, Guha, Halperin, Khanna, Kortsarz, Krauthgamer & Naor \[6\], respectively. In contrast, for the asymmetric \(k\)-median problem, a bicriterion result which allowed a constant factor increase in cost with a logarithmic increase in the number of medians was shown by Lin and Vitter \[19\], and a hardness tradeoff matching this (up to constant factors) was proved by Archer \[1\].

In considering these comparative results, there is a mixed message as to whether a bottleneck problem is easier or harder to approximate than its min-sum counterpart. On the one hand, for any bottleneck problem, one can