Micromechanical approach to effective viscoelastic behavior of jointed rocks

Samir Maghous a, Cássio Barros de Aguiar a,*, Rodrigo Rossi b

a Department of Civil Engineering, Federal University of Rio Grande do Sul, Porto Alegre, RS, Brazil
b Department of Mechanical Engineering, Federal University of Rio Grande do Sul, Porto Alegre, RS, Brazil

ABSTRACT

Assessing the overall instantaneous behavior and strength properties of jointed materials have been the subject of important investigations in the last decades, including phenomenological or micromechanics-based contributions. However, less attention has been dedicated to delayed component of deformation in such media. This issue is addressed in this paper, which is devoted to the formulation of a micromechanical approach to effective viscoelastic properties of jointed rocks with consideration of constituents aging. At the scale of representative elementary volume (REV), the joints are modeled as planar interfaces whose behavior is described by means of generalized viscoelastic state equations under normal and shear loading conditions. Closed-form expressions for the homogenized creep tensor are derived from solving an appropriate viscoelastic concentration problem stated on the REV. The local strain and displacement jump fields are analyzed by extending the concept of strain concentration to relate the components of joint displacement jump to macroscopic strain. Main features of the theoretical overall creep behavior, such as the anisotropy associated with the privileged joint orientations, are highlighted through explicit formulations in some particular configurations of the jointed medium. Finally, the ability of the approach to accurately reproduce the creep behavior of jointed media is assessed by comparison with experimental data as well as with finite element solutions derived in the context of multilayered stratified composite modeling.

1. Introduction

It is commonly well established from laboratory and in situ observations that rock masses always display at different scales discontinuity surfaces with various sizes and orientations. Ranging from the fine (crystalline) to large (geodetic) scale, these discontinuities are usually referred to as joints or fractures and correspond to thin layers along which the physical and mechanical properties of the intact rock matrix significantly degrade. Since joints exhibit much poorer mechanical properties than the intact rock matrix while increasing the permeability by providing preferential channels for fluid flow, their presence strongly affects the overall behavior of rock media and constitute notably an essential weak component for deformability and failure patterns. It follows therefore that, from a geomechanical point of view, the structural and design analyses of engineering rock works, such as tunnels and underground excavations, dam foundations, oil wells or waste storage repositories, should absolutely incorporate a comprehensive and accurate modeling of such weakness surfaces. It should be however emphasized that the presence of joints drastically complicates the formulation of the constitutive behavior for the jointed rock mass as a whole. The first difficulty is related to the assessment of representative constitutive models for the joints and the intact rock matrix separately by relying mainly on experimental identification, which has often been associated with theoretical analyses.1–35 The second difficulty is to compute these constitutive models altogether in order to formulate the overall behavior based either on phenomenological approaches or upscaling-based methods and related micromechanical tools.

In the context of rock engineering modeling, most contributions have mainly addressed the impact of joints on the overall instantaneous behavior and strength properties of the material. It has been, however, recognized that the delayed behavior represents a fundamental component of rock deformation in many practical rock engineering projects in which the behavior of the rock mass is controlled by the creep and long-term strength properties.36–46 As far as the viscoelastic behavior of jointed rocks is concerned, several studies have been devoted to experimentally identify and assess the creep properties of the
intact and jointed rocks, as well as those of the individual rock joints. Based on the experimental data reported in the literature, models have been therefore formulated with the aim to describe the viscoelastic behavior of jointed rock masses. Most of the modeling approaches fall within two categories: (i) models accounting for small scale fractures (i.e., micro-fractures): and which are specifically devised for assessing local creep damage and rupture of the material, and (ii) models dedicated to large scale joints that are relevant to the analysis of deformation and failure of engineering rock structures.

The approach to viscoelastic behavior of jointed rocks developed in the paper specifically addresses the situation of rock media with long joints, which is frequently encountered in rock engineering problems. The adjective ‘long’ refers to discontinuities that cut through the representative elementary volume of the material, the existence of the latter being implicitly assumed in the analysis. As far as the modeling of the material constitutive behavior is concerned, joints are viewed as 2D interfaces endowed with specific mechanical properties in normal and tangential directions. Provided that the rock medium involves a high density of long joint families, it appears advisable to resort to the homogenized-based approaches and related micromechanical tools for the prediction of overall (also referred to as effective or ‘equivalent’) constitutive model of such materials. Conceived as a potential alternative to discrete structural and design oriented methods that consider the jointed rock as an assemblage of deformable intact rock blocks in mutual interaction along the separating joint interfaces (see for instance Refs. among others), the homogenization approaches stem from the heuristic idea that a densely jointed rock can be regarded at macroscopic scale as a homogeneous anisotropic continuum. The anisotropic constitutive properties of the homogenized material expected from the preferential joint orientations are implicitly accounted for in the upsampling process. This concept has actually been empirically conveyed in the pioneering works by Hoek and Brown dedicated to the formulation of a homogenized failure criterion for isotropic jointed rocks.

In the context of homogenization-based approaches, have long been a matter of investigations seeking to predict the essential features of the equivalent properties of jointed rock media related to strength, deformation in both linear and non-linear frameworks, fluid transport and poromechanical couplings. Restricting the purpose to works that addressed the situation of long joints, one may quote references to cite a few. From the mechanical modeling standpoint, most contributions handled the joints as two-dimensional interfaces, even though joints are sometimes viewed as continuum 3D thin layers for computational convenience or either with the objective to explicitly account for the interaction between intersecting joints. These contributions were intended to mainly assess the impact of joints on the overall instantaneous constitutive properties of the medium, while little attention in comparison has been devoted to investigate the effect of delayed component of the jointed material deformation. The latter may significantly affect the intermediate and long-term behavior and even controls the stability of rock masses (such as sedimentary rocks or salt rocks), as widely reported from laboratory and field measurements.

As far as the formulation of overall viscoelastic properties of jointed materials is concerned, the approaches relying upon micromechanics techniques have essentially addressed the case of a matrix with embedded micro-cracks or micro-fractures under the assumption of non-linear viscoelasticity for the constituents. The main interest consists in taking advantage of the classical results established within the context of elasticity in conjunction with the strain-softening of the material. The theoretical framework of random media upsampling is commonly used to evaluate the effective elastic properties in the Laplace-Carson space by resorting to the Eshelby-based homogenization schemes. Extending these methods to the configuration of long joints is not straightforward and may prove a complex task, in the case of non-aging constituents. As a matter of fact, the classical Eshelby’s solution for the inhomogeneity problem and related homogenized stiffness estimates are relevant to short discontinuities regarded as ellipsoidal inclusions, which cannot be representative of the morphology of crossing long joints. Referring to the latter configuration, few works have addressed the formulation of viscoelastic behavior of rock masses involving specifically long joints. In addition, these approaches developed in non-aging viscoelasticity were restricted to either particular non-aging rheological models for the constituents or the particular situation of a rock with two mutually orthogonal joint sets handled in a 2D setting.

In this specific context, the primary purpose of the present paper is to derive a rigorous formulation for the homogenized linear viscoelastic behavior of a jointed rock mass with aging constituents from the knowledge of the joints and rock material respective properties. Particular emphasis is devoted to the assessment of anisotropy and creep compliance increase induced at macroscopic scale by the presence of joints. The three-dimensional formulation of the creep properties of the jointed rock is achieved by solving in the time domain a viscoelastic concentration problem stated on the representative elementary volume with prescribed stress loading history. The closed-form expressions thus derived for the contribution of joints do not require any restricting assumption regarding the joint orientations or associated constitutive model. The explicit evaluation and discussion of the local stress and strain fields prevailing at the representative elementary volume scale provide relevant elements that could explain some aspects of rock deformation observed at macroscopic scale. An important feature of the homogenized model is related to its ability to cover a wide range of configurations while remaining simple to compute, then becoming possible to incorporate it into finite element procedures specifically devised to deformation analysis in jointed rock structures.

The paper content can be summarized as follows. Section 2 briefly introduces the general framework of micromechanics applied to jointed materials, together with the fundamental assumptions of the upsampling modeling. The formulation of overall elastic properties of the rock material with several sets of crossing joints is detailed in Section 3. The step of viscoelastic homogenization is then developed in Section 4 with derivation of closed-form solutions for the homogenized creep tensor. This section also includes explicit analyses of some particular configurations of the joint network. Finally, Section 5 describes some numerical applications and provides a comparison of the model predictions with available experimental data and finite element solutions.

The following notations are used throughout the paper: \( \xi \) will refer to a vector, \( \xi \) to a second-order tensor, \( C \) to a fourth-order tensor. Products denoted by dots: \( \xi \cdot n \) and \( C : \xi \) in terms of components in an orthonormal frame.

2. Modeling framework

The micromechanical reasoning developed in the subsequent analysis for applying the upsampling procedure relies upon the central concept of a representative elementary volume (REV) for the densely jointed medium. Despite this concept should be reconsidered in some situations of natural diffuse fractured media, we assume in this paper the existence of such a REV for the rock material with randomly distributed joints. More precisely, let \( \Omega \) stands for the REV of homogeneous material matrix cut by a discrete distribution of \( N \) joint families \( \omega = \sum_{j=1}^{N} \omega_j \) (Fig. 1). Each family \( \omega_j \), \( 1 \leq j \leq N \), contains a large number of long joints crossing the REV and exhibiting the same orientation and mechanical properties. From the geometrical viewpoint, each joint is described as a flat plane, thus assuming that the joint thickness is negligible when compared to the joint spacing and that its curvature can be neglected at the scale of the REV.

In the specific situation of joints crossing the REV that is addressed herein, the average spacing of the \( j \)-th joint family can be viewed as the characteristic dimension of the family and should be
very small with respect to the joint extension and to the size of the REV. The latter scale separation condition is required for the REV to be statistically representative of the densely jointed rock medium. At the REV—very small with respect to the joint extension and to the size of the REV—associated loading mode conditions.

As regards the deformation of jointed medium, two components contribute to the macroscopic strain $\varepsilon$, in terms of components with respect to an orthonormal base. In the above relationship (4), the first contribution to the macroscopic strain $\varepsilon$ refers to the matrix average strain whereas the second term is related to the joints average deformation.

In the context of jointed materials modeling, the generalized form of Hill’s lemma, which plays a central role in homogenization theory, express as follows:

$$\Sigma : \varepsilon = (\sigma : \varepsilon) + \frac{1}{\Omega} \int_{\partial\Omega} t^i n_i (\varepsilon) \, dS$$

Prior to further developments, some comments deserve to be introduced. A fundamental assumption of the modeling is related to the homogeneity of the rock matrix at the REV scale (microscopic scale). This assumption expresses that the joints are the only heterogeneities considered for the medium at this scale. The rock matrix should therefore be regarded as a homogenized material whose constitutive properties result from a previous homogenization procedure, which accounts for micro-heterogeneities present within the intact rock at a finer scale than the microscopic scale (see for instance Refs. 96–100, to cite a few). It should be emphasized that the homogenized behavior of the rock matrix at the REV scale could be anisotropic. This possible anisotropy is generally induced by the micro-heterogeneity orientations.

The analysis is restricted to the situation of joints that cross through the REV. This means that the joint extension is much longer than the characteristic length of the whole rock structure under consideration. In that respect, rock media exhibiting intermittent joints are not included in the micromechanical formulations developed in this paper. In addition, the formulation described herein explicitly considers a discrete distribution of joint sets, which implies that discrete values for the associated geometrical and mechanical properties are defined. However, continuous distribution of joint families could straightforwardly be addressed by means of the same homogenization procedure, provided that the joint characteristics are defined by means of probability density functions.

Periodicity condition for the joint spatial distribution is not required for the validity of the present micromechanical constitutive modeling.

3. Effective elastic properties the jointed medium

The analysis starts by the formulation of overall elastic properties of the jointed rock within the framework of infinitesimal linear elasticity. A more comprehensive formulation including the non-linearity associated with elastic behavior of the rock matrix or/and the joints has been presented in Ref. 32. This section actually aims to briefly describe the main ideas of the micromechanics-based reasoning in the linear elastic range. In the context of infinitesimal strain analysis, the stress-strain elastic relationship for the solid reads

$$\varepsilon = \varepsilon' : \sigma$$

where $\varepsilon'$ stands for the fourth-order compliance tensor associated with rock matrix behavior. This relationship relates the local stress and strain fields at any point of the domain $\Omega_{\omega_j}$.

As regards the elastic behavior of the joints modeled as interfaces, it is expressed by means of linear relationships between the stress vector and the displacement jump. More precisely, it is assumed that any element of the family $\omega_j$ is characterized by the joint stiffness $k_j$ relating

$$\sigma \cdot n = \Sigma \cdot n$$
the displacement jump \( \mathbf{\tilde{u}} \) to the stress vector \( \mathbf{T} = \mathbf{\bar{g}} \mathbf{d} \) through

\[
\mathbf{T} = \mathbf{k} \mathbf{n} \mathbf{\tilde{u}} \quad \text{or} \quad \mathbf{\tilde{u}} = \mathbf{\bar{g}}^{-1} \mathbf{T} \tag{7}
\]

where \( \mathbf{\bar{g}}^{-1} = \left( \mathbf{k} \mathbf{n} \right)^{-1} \) is the second-order compliance tensor of the joints of family \( \omega_j \). The joint stiffness \( \mathbf{k} \mathbf{n} \) (or alternatively the joint compliance \( \mathbf{g} \)) is classically evaluated from appropriate laboratory tests carried out on joint rock specimen. The works by Goodman and Bandis et al.\(^{101} \) represent reference contributions regarding the experimental identification of joint properties. We shall refer in the sequel to the orthonormal local frame \( \left( \mathbf{n}, \mathbf{t}_j, \mathbf{t}_j' \right) \), defined for each joint of the set \( \omega_j \) by the unit normal vector \( \mathbf{n} \) and two orthogonal unit vectors \( \left( \mathbf{t}_j, \mathbf{t}_j' \right) \) of the plane parallel to joint direction (Fig. 1). Referring to this orthonormal basis and disregarding the coupling between joint normal and shear behavior, the components of \( \mathbf{k} \mathbf{n} \) and \( \mathbf{g} \) can be expressed as

\[
\mathbf{k} = \mathbf{k}_n \mathbf{n} \mathbf{n} + \mathbf{k}_t \mathbf{t}_j \mathbf{t}_j + \mathbf{k}_t' \mathbf{t}_j' \mathbf{t}_j' \quad \mathbf{g} = \mathbf{g}_n \mathbf{n} \mathbf{n} + \mathbf{g}_t \mathbf{t}_j \mathbf{t}_j + \mathbf{g}_t' \mathbf{t}_j' \mathbf{t}_j' \tag{8}
\]

with \( \mathbf{g}_n = \frac{1}{\mathbf{k}_n} \), \( \mathbf{g}_t = \frac{1}{\mathbf{k}_t} \), \( \mathbf{g}_t' = \frac{1}{\mathbf{k}_t'} \) where \( \mathbf{k}_n \) is the joint normal stiffness, whereas \( \mathbf{k}_t \) and \( \mathbf{k}_t' \) stand for tangential stiffness parameters. Note that in most rock engineering applications, isotropy is considered for the joint shear response shear \( \mathbf{k}_t = \mathbf{k}_t' \), which means that \( \mathbf{k} = \mathbf{k}_n \mathbf{n} \mathbf{n} + \mathbf{k}_t \mathbf{t}_j \mathbf{t}_j \mathbf{t}_j' \mathbf{t}_j' \).

The homogenized elastic behavior stems from solving the concentration problem stated on the REV as follows. The macroscopic stress \( \mathbf{\bar{g}} \mathbf{d} \) being prescribed, the problem consists in determining the local fields

\[
\begin{aligned}
\mathbf{\bar{g}} \mathbf{d} = \mathbf{\bar{g}} \mathbf{d} \quad & \text{solution to} \\
\text{div} \mathbf{g} = 0 \quad & (\boldsymbol{\Omega} \omega) \\
\mathbf{\bar{g}} \mathbf{d} = \mathbf{\bar{g}} \mathbf{d} \quad & \text{is continuous when crossing } \omega_j \\
\mathbf{\bar{g}} \mathbf{n} = \mathbf{\bar{g}} \mathbf{n} \quad & (\partial \boldsymbol{\Omega}) \\
\mathbf{\bar{g}} = \mathbf{\bar{g}} \quad & (\boldsymbol{\Omega} \omega) \\
\mathbf{\tilde{u}} = \mathbf{\tilde{u}} \quad & (\omega_j) \\
\end{aligned} \tag{9}
\]

It arises from the analysis of the above problem that the stress solution is homogeneous within the REV

\[
\mathbf{\bar{g}} \mathbf{d} = \mathbf{\bar{g}} \mathbf{d} \quad \forall \mathbf{\bar{g}} \in \partial \boldsymbol{\Omega} \omega \tag{10}
\]

This means that the stress concentration tensor \( \mathbf{B} \), classically introduced in micromechanics to relate the stresses at local and macroscopic levels through \( \mathbf{\bar{g}} \mathbf{d} = \mathbf{B} \mathbf{\bar{g}} \mathbf{d} \mathbf{\tilde{u}} \), actually reduces to the fourth-order identity \( \mathbf{B} = 1 \) (Reuss type concentration rule). It is important to emphasize that the homogeneity of the stress field within the REV is the closely related to three fundamental considerations, namely: (a) the homogeneity of rock matrix assumed at the REV scale, (b) the “homogeneity” of each joint belonging to family \( \omega_j \) explicitly expressed by the spatial uniformity of \( \mathbf{\bar{g}} \) along the joint, and (c) the modeling of the joints as planar interfaces. The homogeneity of the stress field implies that of the strain distribution within the rock matrix \( \mathbf{\bar{g}} = \mathbf{\bar{g}}' = \mathbf{\bar{g}}' \) which in turn indicates that the displacement field is piecewise linear in the rock matrix. The corresponding displacement jump is thus uniform along each joint of family \( \omega_j \)

\[
\mathbf{\tilde{u}} = \mathbf{\tilde{u}}'(\mathbf{\bar{g}}') \quad (\omega_j) \tag{11}
\]

The macroscopic state equation is then obtained from applying the strain average rule (4)

\[
\mathbf{E} = \mathbf{\bar{g}}' \mathbf{\bar{g}}' + \sum_{j=1}^{N} \omega_j \mathbf{g}_t \mathbf{t}_j \mathbf{t}_j' \mathbf{t}_j' \tag{12}
\]

where \( \mathbf{d} = \frac{1}{B} \int_{\omega_j} \mathbf{d} \) is the specific area of the joint family \( \omega_j \), which could be evaluated by the value of the inverse of average joint spacing \( \mathbf{d} \). Equation (12) can be rearranged as

\[
\mathbf{E} = \mathbf{\bar{g}}_{\text{hom}} + \sum_{j=1}^{N} \omega_j \mathbf{g}_t \mathbf{t}_j \mathbf{t}_j' \mathbf{t}_j' \tag{13}
\]

in which \( \mathbf{\bar{g}}_{\text{hom}} \) is the macroscopic compliance tensor

\[
\mathbf{\bar{g}}_{\text{hom}} = \mathbf{\bar{g}}' + \sum_{j=1}^{N} \omega_j \mathbf{g}_t \mathbf{t}_j \mathbf{t}_j' \mathbf{t}_j' \tag{14}
\]

The fourth-order tensor \( \mathbf{\bar{g}}' \) represents the contribution of joint family \( \omega_j \) to the overall compliance. Its expression reads

\[
\mathbf{\bar{g}}'(\mathbf{d}) = \left( \omega_j \mathbf{d} \mathbf{d} \mathbf{d} \mathbf{d} + \mathbf{g}_t \mathbf{t}_j \mathbf{t}_j' \mathbf{t}_j' + \mathbf{g}_t' \mathbf{t}_j' \mathbf{t}_j' \mathbf{d} \mathbf{d} \mathbf{d} \mathbf{d} \right) \tag{15}
\]

where the joint compliance components \( \omega_j \mathbf{d} \mathbf{d} \mathbf{d} \mathbf{d} \) are defined in (8). Expression (14) of the macroscopic compliance tensor shows that the elastic anisotropy of the jointed medium is induced either by the elastic anisotropy of the constituents or by the privileged orientations of the joints. It is also noted that expression of \( \mathbf{\bar{g}}_{\text{hom}} \) is obtained by adding the individual contributions of all joint sets, without accounting at macroscopic level for the interaction between intersecting joints that may arise at the REV scale. This is mainly due to the modeling of joints as infinite planar interfaces.\(^{28} \)

In the context of linear elasticity, it is noted that several works based on purely macroscopic reasoning have led to expressions similar to (15) for the contribution of joints to the overall rock mass compliance. Among these contributions, one may quote the method relying on the concept of joint deformation tensor proposed in Jiang et al.,\(^{35} \) which extends the concept of fabric tensor originally introduced by Oda\(^{102} \) for discontinuous geological materials to integrate both geometry and deformability properties of the joints. A main interest of such approach is the ability to deal in a straightforward way with several joint networks within a probabilistic framework.

It is important to underline that expression (15) giving the contribution of joints to overall compliance is theoretically valid only for long joints crosscutting the REV. In order to assess the effects of joint size in rocks with joints of different lengths, an approximate approach directly inspired by that proposed in Ref. 35 can be adopted. It consists of introducing a reducing factor \( \eta \) that affects the specific area \( \mathbf{d} \) of the joint family \( \omega_j \) in expression (15). Accordingly, term \( \mathbf{d} \) should be replaced by \( \eta \mathbf{d} \) in expression (15), where parameter \( \eta \), with \( 0 < \eta \leq 1 \), is analogous to the joint persistence ratio introduced in Ref. 35 for formulating the concept of joint deformation tensor.

4. Effective viscoelastic behavior of the jointed medium

Based upon the ideas introduced in the preceding section for elastic behavior, we move now to the formulation of viscoelastic effective properties of the jointed medium. The analysis is restricted to the
framework of linear viscoelasticity with account for aging.

4.1. Formulation of macroscopic state equation

In this context, the local strain field in the rock matrix is related to the stress history starting at \( t = 0 \) by means of the hereditary Boltzmann integral

\[
\mathbf{\varepsilon}(t) = \mathbf{F}^t \circ \mathbf{\Sigma} = \mathbf{F}^t(t, t) \mathbf{\varepsilon}(t) + \int_0^t \frac{\partial \mathbf{F}^s}{\partial t}(t, \tau) \mathbf{\varepsilon}(\tau) d\tau (\mathbf{\Omega} \omega)
\]

which is the viscoelastic counterpart of relationship (6). The fourth-order tensor \( \mathbf{F}^t \) refers to creep tensor of the rock matrix, arguments \( t \) and \( \tau \) stand respectively for the delayed and current (instantaneous) times, and symbol \( \circ \) is the hereditary Boltzmann operator. When non-aging viscoelasticity is assumed, the creep tensor explicitly depends only on the loading time \( t - \tau \) (i.e., \( \mathbf{F}^t(t, \tau) = \mathbf{F}^t(t - \tau) \)) and the operator \( \circ \) reduces to a Riemann convolution product.

Likewise, the viscoelastic behavior of the joints of family \( \omega_j \) relates the displacement jump to the stress vector history through

\[
\mathbf{\xi}(t) = \mathbf{F}^t \circ \mathbf{\Sigma} = \mathbf{F}^t(t, t) \mathbf{\xi}(t) + \int_0^t \frac{\partial \mathbf{F}^s}{\partial t}(t, \tau) \mathbf{\xi}(\tau) d\tau (\mathbf{\Omega} \omega)
\]

In the above relationship, \( \mathbf{F}^F \) stands for the second-order creep tensor of the joints belonging to set \( \omega_j \). Referring as in elasticity to the ortho-normal local frame \( (\mathbf{n}^j, \mathbf{t}^j, \mathbf{\xi}^j) \) attached to joint set \( \omega_j \), the components of \( \mathbf{F}^F \) (creep functions) read

\[
\mathbf{F}^F = \mathbf{F}_{\omega}^{s} \mathbf{n} \otimes \mathbf{n} + \mathbf{F}_{\omega}^{t} \mathbf{t} \otimes \mathbf{t} + \mathbf{F}_{\omega}^{\xi} \mathbf{\xi} \otimes \mathbf{\xi}
\]

where coupling between joint normal and shear behavior is disregarded. The creep function \( \mathbf{F}_{\omega}^{s} \) characterizes the joint creep function in normal direction, relating the normal displacement jump \( \mathbf{\xi}^n \) to the normal stress \( \sigma^s = \mathbf{N} \cdot \mathbf{t}^j \) history. Component \( \mathbf{F}_{\omega}^{t} \) (resp. \( \mathbf{F}_{\omega}^{\xi} \)) is the tangential creep function, which relates the displacement jump \( \mathbf{\xi}^t \) (resp. \( \mathbf{\xi}^\xi \)) to the shear stress \( \sigma^t = \mathbf{N} \cdot \mathbf{t}^j \) (resp. \( \sigma^\xi = \mathbf{N} \cdot \mathbf{\xi}^j \)). Appropriate creep tests are necessary to assess the joint creep functions \( \mathbf{F}_{\omega}^{s} \), \( \mathbf{F}_{\omega}^{t} \) and \( \mathbf{F}_{\omega}^{\xi} \). The creep components in shear are taken equal \( \mathbf{F}_{\omega}^{t} = \mathbf{F}_{\omega}^{\xi} \) when isotropy is admitted for the joint shear response, leading to \( \mathbf{F}^F = \mathbf{F}_{\omega}^{s} \mathbf{n} \otimes \mathbf{n} + \mathbf{F}_{\omega}^{t} \left( \mathbf{t} \otimes \mathbf{t} + \mathbf{\xi} \otimes \mathbf{\xi} \right) \).

The formulation of the aging effective viscoelastic behavior requires solving the following upscaling problem stated on the REV. Given a prescribed macroscopic stress history \( \tau \rightarrow \mathbf{\Sigma}(\tau) \) such that \( \mathbf{\Sigma}(\tau) = 0 \) for \( \tau \leq 0 \), the problem consists in evaluating the couple of microscopic fields \( \left( \mathbf{\sigma}^\omega, \mathbf{\varepsilon}^\omega \right) \) satisfying at any instant \( t \geq 0 \) the following conditions

\[
\begin{align*}
\text{div } \mathbf{\sigma}^\omega &= 0 \quad (\mathbf{\Omega} \omega) \\
\mathbf{\tau}^\omega &= \mathbf{\sigma}^\omega \mathbf{n} \quad \text{(continuity when crossing } \omega_j \}) \\
\mathbf{\sigma}^\omega &= \mathbf{\Sigma}^\omega 
\end{align*}
\]

\[
\mathbf{\varepsilon} = \mathbf{F}^t \circ \mathbf{\sigma}^\omega \quad (\mathbf{\Omega} \omega)
\]

\[
\mathbf{\xi}^\omega = \mathbf{F}^t \circ \mathbf{\xi}^\omega \quad (\mathbf{\Omega} \omega)
\]

(19)

The strain average rule (4) together with the constitutive viscoelastic equations (16) and (17) allow computing the macroscopic strain as

\[
\mathbf{\varepsilon}(t) = \left( \mathbf{\varepsilon}^\omega + \frac{1}{\mathbf{\Omega} \omega} \int_{\mathbf{\Omega} \omega} \mathbf{\varepsilon}^\omega \circ \mathbf{\xi}^\omega \right) \mathbf{\xi}^\omega \ dS
\]

Similarly to the elastic upscaling problem, the viscoelastic concentration problem (20) admits the following homogeneous stress solution

\[
\mathbf{\sigma}^\omega(x,t) = \mathbf{\Sigma}^\omega(t) \quad \forall \mathbf{x} \in \mathbf{\Omega} \omega
\]

meaning that the evolution of local stress distribution coincides with that of the macroscopic stress. The concept of stress concentration tensor \( \mathbf{\Sigma}^\omega \) relating the local and macroscopic stories within the REV stress histories can be extended as follows

\[
\mathbf{\sigma}^\omega(x,t) = \mathbf{\Sigma}^\omega \quad (\mathbf{\Omega} \omega)
\]

which according to (21), it indicates that \( \mathbf{\Sigma}(x,t) = \mathbf{Y}(t) \), where \( \tau \rightarrow \mathbf{Y}(\tau) \) denotes the Heaviside step function at time \( \tau \). The spatial homogeneity of \( \mathbf{\Sigma}^\omega \) in the rock matrix implies that of \( \mathbf{\Sigma}^\omega \). The corresponding displacement jump is thus uniform along each joint of family \( \omega_j \)

\[
\mathbf{\xi}^\omega = \mathbf{F}^t \circ \left( \mathbf{\Sigma}^\omega \right) \quad (\omega_j)
\]

It therefore follows that:

\[
\mathbf{\xi}^\omega = \mathbf{F}^t \circ \left( \mathbf{\Sigma}^\omega \right) \quad (\omega_j)
\]

which implies that the expression of homogenized creep tensor \( \mathbf{F}^{\text{hom}} \) reads

\[
\mathbf{F}^{\text{hom}}(t, \tau) = \mathbf{F}^t(t, \tau) + \sum_{j=1}^N a_j \mathbf{F}^J(t, \tau) \mathbf{F}^{\text{hom}}(t, \tau) \mathbf{F}^{\text{hom}}(t, \tau)
\]

where \( a_j \) is attributed to the assumption of homogeneity for the rock matrix and to the modeling of joints as infinite planar interfaces. As a matter of fact, the original anisotropy of the constituents, a strong anisotropy of the macroscopic viscoelastic properties shall be associated with the privileged joint orientations.

Similarly to the result established in elasticity, the overall creep properties are obtained by adding separately the individual contributions of the rock matrix and of each joint family. Once again, this feature is attributed to the assumption of homogeneity for the rock matrix and to the modeling of joints as infinite planar interfaces. As a matter of fact, the assessment of interaction between intersecting joint families would require modeling the joints at the scale of the REV as 3D thin layers. As a consequence of the uncoupled contributions of the jointed rock constituents, it appears that creep spectrum associated with the overall behavior is closely controlled by the original memories of the constituents, namely the rock matrix and joints. It turns out that the long-range memory effects, which are generally induced by the upscaling process in
viscoelastic composite materials, \textsuperscript{23,104,105} actually do not appear in the case of jointed materials. From a theoretical viewpoint, these memory effects that are induced at macroscopic scale by the homogenization process reflect the compatibility restrictions present at microscopic level along the interfaces of dissimilar materials. The mechanical framework, in which the joints are regarded as interfaces and not as 3D heterogeneities, is not adequate for the assessment of such effects. Another important consequence of the uncoupled contributions of the jointed rock constituents to overall creep properties is that aging of either the rock matrix or of the joints will be integrally (without amplification or attenuation) transposed at macroscopic scale.

As previously commented in section 3 for the elastic context, the above formulation can be extended in an approximate manner to handle the situation of joints with different lengths by replacing in expression (26) the term \( \bar{a} \) by \( \bar{a}' \), where \( \bar{a}' \) is a reducing factor analogous to the joint persistence ratio.\textsuperscript{32}

Finally, it is important to emphasize that consideration of aging for the constituent materials invalidates operating in the Laplace or Boltzmann operator

The deviation of the macroscopic strain \( \text{macroscopic strain} \) in the effective creep properties of a jointed rock mass made of a standard Zener rheological matrix and joints. As expected, the general viscoelastic behavior is valid. The case of linear non-aging viscoelasticity can be readily deduced from this formulation \( \text{homogenized creep tensor} \) in time domain is therefore obtained by performing the inverse of obtained homogenized creep tensor \( \text{homogenized creep tensor} \).

The above relationships extend to aging viscoelasticity the expressions proposed in the context of elasticity by Maghous et al.\textsuperscript{34}

Remarks. Referring to the loading applied to the REV, the two upscaling problems defined respectively by homogeneous stress boundary conditions (adopted in the analysis) and homogeneous strain boundary conditions are not equivalent. However, this equivalence is assumed to hold provided the scale separation conditions are fulfilled. In that case, the homogenized relaxation tensor can be evaluated from the inverse of obtained homogenized creep tensor \( \text{homogenized creep tensor} \).

The general viscoelastic formulation described by relationships (24) to (26) includes any rheology of the jointed material constituents, including the aging effects, provided the assumption of linearity of the behavior is valid. The case of linear non-aging viscoelasticity can be directly deduced from this formulation \( \text{homogenized creep tensor} \). The stress-strain relation

where the creep functions \( \text{creep functions} \) associated with the joint viscoelastic properties are defined in (26). As emphasized by the above relationship, the deviation of \( \text{deviation of} \) from identity \( \text{identity} \) corresponds to the fraction of macroscopic strain that localizes along the joints.

The displacement jump along the joints of family \( \text{displacement jump along the joints of} \) is directly deduced from relationship (23). For this purpose, it is also convenient to relate the normal and tangential components of displacement jump to the macroscopic strain

The tensors \( \text{tensors} \) play the role of “concentration tensors” for the normal and tangential components of the displacement jump along the joints of \( \text{displacement jump along the joints of} \) although this concept refers to homogeneous strain type boundary conditions on the REV. Their expressions are obtained from some algebraic developments that make explicitly use of the macroscopic state equation (24)

\[
\begin{align*}
\text{macroscopic strain} & = \text{macroscopic strain} \\
\text{macroscopic strain} & = \text{macroscopic strain} \\
\text{macroscopic strain} & = \text{macroscopic strain}
\end{align*}
\]

4.2. The case of a rock mass with a single family of joints

The general expression of homogenized creep tensor is particularized in the case of a rock matrix with a distribution of parallel crosscutting joints. For the sake of clarity, the components of \( \text{components of} \) will be expressed in

![Fig. 2. Rock mass cut by a single family of parallel joints: REV and reference frame.](image-url)
the fixed orthonormal base \( \{ \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \} \) attached to the joints as displayed in Fig. 2.

\[
\hat{\mathbf{F}}^\text{hom} = \hat{\mathbf{F}}' + a^1 \left( \mathbf{F}'_1 \mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{F}'_2 \mathbf{e}_2 \otimes \mathbf{e}_2 + \mathbf{F}'_3 \mathbf{e}_3 \otimes \mathbf{e}_3 \right)
\]

(31)

where the dependence of creep functions with respect to \((t, \tau)\) has been omitted.

If isotropy is assumed for the viscoelastic behavior of the matrix material, the associated creep tensor \(\mathbf{F}\) can conveniently be expressed as

\[
\mathbf{F}' = \frac{1}{3} \mathbf{F}'_+ J + \frac{1}{6} \mathbf{F}'_+ \mathbf{K}
\]

(32)

where \(\mathbf{F}'_+\) and \(\mathbf{F}'_-\) are respectively the creep function in bulk and the creep function in shear of the solid matrix. The fourth-order tensors \(\mathbf{J}\) and \(\mathbf{K}\) are defined as \(\mathbf{J} = \frac{1}{3} \mathbf{I} \otimes \mathbf{I}\) and \(\mathbf{K} = \mathbf{I} - \mathbf{J}\). The tensor \(\hat{\mathbf{F}}^\text{hom}\) containing the creep functions can thus be expressed in a classical matrix notation as

\[
\hat{\mathbf{F}}^\text{hom} = \begin{bmatrix}
\frac{F'_1}{9} & \frac{F'_2}{6} & \frac{F'_3}{3} \\
\frac{F'_1}{9} & \frac{F'_2}{6} & \frac{F'_3}{3} \\
\frac{F'_1}{9} & \frac{F'_2}{6} & \frac{F'_3}{3} \\
\end{bmatrix}
\]

(33)

which is defined by five independent parameters and characterizes an orthorhombic medium. It is noted that the above representation is consistent with the following matrix notation for stress and strain

\[
\begin{bmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} \\
\sigma_{31} & \sigma_{32} & \sigma_{33} \\
\end{bmatrix}
= \begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\
\varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\
\varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \\
\end{bmatrix}
\]

(34)

where subscript ‘\(T\)’ stands for the transposition of a vector.

4.3. Rock medium with two families of joints

We address in this section the particular configuration of a rock mass intersected by two joint families \(\omega_1\) and \(\omega_2\). The relative angular inclination between the corresponding joint normal directions is denoted by \(\theta\) (Fig. 3). As in the preceding section, the explicit calculation of \(\hat{\mathbf{F}}^\text{hom}\) will be achieved by expressing its components in the fixed frame \(\{ \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \}\), \(\mathbf{e}_4\) = \(\left( \mathbf{e}_1', \mathbf{e}_2', \mathbf{e}_3' \right)\) attached to the joints of family \(\omega_1\), choosing the unit vector \(\mathbf{e}_3\) parallel to the direction defined by the intersection line between a joint of \(\omega_1\) and a joint of \(\omega_2\). The general expression of \(\hat{\mathbf{F}}^\text{hom}\) defined by (25) and (26) reduces in the particular situation of two sets of families to

\[
\hat{\mathbf{F}}^\text{hom} = \hat{\mathbf{F}}' + a^1 \left( \mathbf{F}'_1 \mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{F}'_2 \mathbf{e}_2 \otimes \mathbf{e}_2 + \mathbf{F}'_3 \mathbf{e}_3 \otimes \mathbf{e}_3 \right)
\]

(35)

where the vectors \(\mathbf{e}_1', \mathbf{e}_2\) and \(\mathbf{e}_2'\) that define the contribution of the joint family \(\omega_2\) are expressed in the reference frame \(\{ \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \}\) by means of the angle \(\theta\)

\[
\begin{bmatrix}
\mathbf{e}_1' \\
\mathbf{e}_2' \\
\mathbf{e}_3'
\end{bmatrix} = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(36)

Assuming once again isotropic viscoelastic properties for the matrix material, the creep tensor \(\hat{\mathbf{F}}'\) is then defined through (32) by means of the creep functions \(\mathbf{F}'_1\) and \(\mathbf{F}'_2\). The matrix containing the homogenized creep functions reads

\[
\hat{\mathbf{F}}^\text{hom} = \begin{bmatrix}
\frac{F'_1}{9} & \frac{F'_2}{6} & \frac{F'_3}{3} \\
\frac{F'_1}{9} & \frac{F'_2}{6} & \frac{F'_3}{3} \\
\frac{F'_1}{9} & \frac{F'_2}{6} & \frac{F'_3}{3} \\
\end{bmatrix}
\]

(37)

where the non-null terms corresponding to the joints contribution are

\[
\begin{align*}
\alpha_{11} &= a^1 F'_1 + a^2 \left( F'_1 \cos^2 \theta + F'_2 \sin^2 \theta \right) \\
\alpha_{22} &= a^2 \left( F'_2 \sin^4 \theta + F'_3 \sin^2 \theta \right) \\
\alpha_{33} &= a^3 F'_3 \\
\alpha_{66} &= a^4 \left( F'_1 \cos^2 \theta + F'_2 \sin^2 \theta \right) \\
\alpha_{12} &= a^4 \left( F'_3 \sin^2 \theta - F'_2 \cos^2 \theta \right) \\
\alpha_{16} &= a^4 \left( -F'_1 \cos^2 \theta \sin \theta + F'_2 \sin^2 \theta \right) \\
\alpha_{26} &= a^4 \left( -F'_2 \sin^2 \theta \sin \theta + F'_1 \cos^2 \theta \right) \\
\alpha_{45} &= a^5 \left( F'_1 \cos 2\theta + F'_2 \sin 2\theta \right) \\
\end{align*}
\]

(38)

It should be pointed out that the above expression of is closely connected with the particular choice of the reference orthonormal frame \(\{ \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \}\) taken with the choice (36) for defining \(\mathbf{e}_3\) together with the choice (36) for defining \(\mathbf{e}_3\).

4.4. Rock medium with isotropic distribution of joints

In many rock engineering applications, the rock mass exhibits discontinuities of various orientations so that it macroscopically behaves like a quasi-isotropic material, provided the rock matrix is itself isotropic. In such situations where the anisotropy induced by the joint orientations can be reasonably disregarded, the assumed isotropy for the
jointed rock material behavior allows for more simplified experimental procedures aimed at identifying the homogenized properties, while leading to more tractable engineering problems at the macroscopic level.

In this context, we shall consider the case of a rock matrix with randomly oriented crosscutting joint families (isotropic distribution). The density of jointing is characterized by a total joint specific area \( a \).

Referring to a fixed orthonormal Cartesian base \( (\xi_1, \xi_2, \xi_3) \), the orientation of a generic joint family \( a_j \) is prescribed by two spherical angular coordinates \( \theta \in [0, \pi] \) and \( \phi \in [0, 2\pi] \) defining the normal vector \( \hat{n} = \sin \theta \left( \cos \phi \, \xi_1 + \sin \phi \, \xi_2 \right) + \cos \theta \, \xi_3 \) as depicted in Fig. 4 (subscript ‘j’ is omitted in this section).

Considering that all joint families exhibit the same creep functions in bulk as well as in shear (identical material properties) and assuming isotropy for the joint shear response \( (F_i = F_j) \), the creep tensor of joints belonging to family \( a_j \) reads

\[
F(\theta, \phi) = F_{i\hat{n}} \hat{n} \otimes \hat{n} + F_{i\hat{t}} \left( 1 - \hat{n} \otimes \hat{n} \right) = \left( F_{i\hat{n}} - F_{i\hat{t}} \right) \hat{n} \otimes \hat{n} + F_{i\hat{t}} \hat{t} \hat{t} \tag{39}
\]

where the dependence of the creep functions with respect to \((t, \tau)\) has been omitted.

The homogenized creep tensor is deduced from (25) and (26) by integration over all joint family orientations

\[
f_{\text{hom}} = F^s + \int_0^\phi \int_0^{2\pi} \frac{a}{4\pi} F(\theta, \phi) \, d\theta \, d\phi \tag{40}
\]

with

\[
F(\theta, \phi) = (F_{i\hat{n}} - F_{i\hat{t}}) \hat{n} \otimes \hat{n} \otimes \hat{n} \otimes \hat{n} + F_{i\hat{t}} \sum_{k=1}^3 \hat{\xi}_k \otimes \hat{\xi}_k \otimes \hat{\xi}_k \otimes \hat{\xi}_k \tag{41}
\]

The following relationships are conveniently used for the integration term involved in (40)

\[
\int_0^\pi \int_0^{2\pi} n_i n_j \sin \theta \, d\theta \, d\phi = \frac{4\pi}{5} \delta_{ij} \tag{42}
\]

and

\[
\int_0^\pi \int_0^{2\pi} n_i n_j n_k n_l \sin \theta \sin \phi d\theta d\phi = \frac{4\pi}{5} \left( \delta_{ij}\delta_{kl} + \delta_{il}\delta_{jk} + \delta_{jl}\delta_{ik} \right) \tag{43}
\]

where \( \delta_{ij} \) is the Kronecker delta function, which takes the value 1 if \( i = j \) and 0 otherwise.

It follows from the calculation of the integral term in (40) that the contribution of the joints to homogenized creep tensor reduces to an isotropic term

\[
g_{\text{hom}} = F^s + \frac{1}{3} F_{\text{joint}}^s \mathbb{J} + \frac{1}{2} F_{\text{joint}}^\mathcal{E} \mathcal{K} \tag{44}
\]

where \( F_{\text{joint}}^s \) and \( F_{\text{joint}}^\mathcal{E} \) denote the bulk and shear creep functions due to the joints and express as

\[
F_{\text{joint}}^s = a \, F_{\tau} \quad ; \quad F_{\text{joint}}^\mathcal{E} = \frac{2}{15} a \left( 2 F_{\tau} + 3 F_{\tau} \right) \tag{45}
\]

When isotropic creep properties are assumed for the matrix material (i.e., \( \frac{\mathcal{E}}{\mathcal{K}} = \frac{2}{3} \frac{F_{\tau}}{F_{\tau}} \mathbb{J} + \frac{1}{2} F_{\tau}^\mathcal{E} \mathcal{K} \)), the creep behavior at macroscopic scale is also isotropic and \( F_{\text{hom}} \) reduces to

\[
g_{\text{hom}} = \frac{1}{3} \left( F_{\tau}^s + F_{\text{joint}}^s \right) \mathbb{J} + \frac{1}{2} \left( F_{\tau}^\mathcal{E} + F_{\text{joint}}^\mathcal{E} \right) \mathcal{K} \tag{46}
\]

5. Illustrative examples

This section aims at applying the proposed formulation to predict the homogenized viscoelastic properties in four distinct situations of jointed medium. Based on experimental data related to the behavior of the rock constituents in creep (first situation) or upon a comparison with numerical solutions derived for the jointed rock regarded as a multilayered medium (second situation), the two first applications address the formulation of homogenized creep properties without consideration of aging. The third example refers to the configuration of a jointed medium involving an aging matrix material, with specific emphasis given to the effect of aging on the creep behavior at macroscopic level. The last part of this section presents a preliminary validation of the proposed approach trough comparison of the micromechanical predictions against available experimental results.

It should be emphasized that, although the formulation developed in the paper can conceptually cover a large range of rock jointing configurations regarding the orientations of the joints as well as the associated constitutive models, its practical implementation is still limited due to the lack of experimental data related to the aging behavior in creep of the jointed rock constituents (rock matrix, joints).

5.1. Creep behavior of a jointed shale rock under pure shearing

As first illustrative application, we examine the overall creep prop-

![Fig. 4. Angular coordinates of a joint.](image)

![Fig. 5. Shale-like rock under shearing parallel to joints direction.](image)
erties in shear of a shale rock cut by a single family of crosscutting joints as schematized in Fig. 5 (subscript $j$ referring to joints is omitted in this section).

Based on viscoelastic shear creep experimental data, Chin and Rogers\cite{Chin1987} and Yang and Chang\cite{Yang1988} proposed rheological Kelvin-Voigt models (standard three-element viscoelastic models) to describe the individual non-aging viscoelastic behavior in shear of the shale rock matrix (Fig. 6-a) and of the joints (Fig. 6-b). The values of the parameters (spring stiffness and dashpot viscosity) defining the shear relaxation modulus of the shale matrix $\mu^s(t)$ and that of the joints $k_j(t)$ provided in the above mentioned works are shown in Table 1.

For comparison purposes, the mathematical representation of the homogenized component of creep tensor associated with the simple shearing parallel to the joint direction $\dot{\epsilon}_j$ is deduced from the expression (33) of $\dot{\epsilon}_{j\text{em}}$. Assuming isotropy for the joint shear response (i.e., $k_j = k_l$), the shear creep component reads therefore

$$F_{\text{shear}}^{\text{hom}} = F_s^0 + a F_l,$$  \hspace{1cm} (47)

where the creep functions $F_s^0$ and $F_l$ of the rock matrix and the joints are obtained from the relaxation counterparts $\mu^s$ and $k_l$ associated with the respective Kelvin-Voigt models by inverse Boltzmann operator.

Assuming a joint specific area equal to $a = 10 \text{ m}^{-1}$, which correspond to an average joint spacing $d = 0.1 \text{ m}$, Fig. 7 displays the time variations of the homogenized creep function in shear together with that of the shale rock matrix (intact rock).

The most direct conclusion from Fig. 7 refers to the significant increase in the shear creep function induced by the presence of the joints when compared to that of the intact material. This result is corroborating the laboratory and field observations that the time-dependent properties of rocks with high joint density, such as shale-like rocks, are strongly affected by the presence of discontinuities, which increase the overall deformability of the rock mass (e.g., Ref. 83). It is also observed that for the considered materials properties, the presence of joint also accentuates the evolution of homogenized creep function at early stages ($t < 100 \text{ h}$). Although only the component associated with shearing parallel to the joints was analyzed, similar observations could be expected for the other components of the overall creep components.

5.2. Rock mass with two orthogonal joint sets: a verification example

The predictive capabilities of the micromechanics-based approach to creep behavior of jointed rocks is assessed by comparison with the numerical solutions derived by\cite{Ferri1987} in the context of an alternative formulation, which relies upon the assumption the jointed rock can be regarded as a multilayered stratified composite.

In the conceptual two-dimensional model developed in Ref. 80, the rock matrix is intersected by a periodic network of two orthogonal families of joints with identical thickness $\beta l$, where 1 refers to the in-plane dimension of the elementary cell (Fig. 8-a). As regards the constitutive behavior of the constituents, both the rock matrix and the joints exhibit isotropic and non-aging viscoelastic properties, defined by the Zener rheological models respectively sketched in Fig. 8-a for the uniaxial relaxation modulus $E'(t)$ of the rock matrix and in Fig. 8-c for the uniaxial relaxation modulus $E'(t)$ of the joints (identical material properties are assumed for the orthogonal joint families):

$$E'(t) = \frac{\nu_j}{k_j + \eta_j} \left( E^s_0 e^{-t/\tau_s} \right) Y(t)$$

$$E'(t) = \frac{\nu_j}{k_j + \eta_j} \left( E^j_0 e^{-t/\tau_j} \right) Y(t)$$  \hspace{1cm} (48)

$E^s_0$ and $E^j_0$, with $j \in \{ s, j \}$, are the springs stiffness in the rock matrix and joints related Zener models, $\nu_j$ is the dashpot viscosity, $\tau_s = E^s_0 / \nu_j$ is the relaxation characteristic time, and $Y(t)$ stands for the Heaviside step function at origin. Note that $E'(0) = E^s_0$ represents the instantaneous relaxation modulus, whereas $E'(0) = \frac{E^s_0}{E^s_0 + k_j}$ refers to the asymptotic one. In addition, the same constant Poisson ratio $\nu$ is considered for the jointed rock constituents:

$$\nu'(t) = \nu'(0) = \nu Y(t)$$  \hspace{1cm} (49)

Restricting the analysis to the two-dimensional setting (plane stress or plane strain conditions), the numerical approach developed by these authors to evaluate the equivalent constitutive model for the orthogonal

---

**Table 1**

| Parameter | Value |
|-----------|-------|
| $\mu_s^0$ | 0.181 GPa |
| $\mu_j^0$ | 0.299 GPa |
| $\eta_s^0$ | 7.539 GPa h |
| $k_{s0}$ | 6.338 GPa/m |
| $k_{j0}$ | 5.333 GPa/m |
| $\eta_j$ | 5.792 GPa h/m |

---

**Fig. 6.** Kelvin-Voigt rheological models used for the constituents of the jointed rock.
viscoelastic jointed may be summarized as follows. The first step is the formulation of an approximate viscoelastic concentration problem stated on the elementary cell that assumes that the stress and strain fields are piecewise homogeneous within the cell. More precisely, these fields take uniform values at any time $t$ within the four domains shown in Fig. 8-a: (I) rock matrix, (II)-(III) joints, (IV) joints intersection zone. Referring to an incremental time discretization, the local as well as the overall stress and strains fields are approximated in a second step by piecewise polynomial expansions with respect to time variable, the coefficients in the respective expansions defining therefore the problem unknowns. In the last step, the macroscopic strain prescribed, the local stress field and associated macroscopic value are evaluated along each time interval by numerically solving a recursive homogeneous problem defined in the elementary cell.

Unlike the fully analytical formulation developed in this paper, which makes use of the expression (37) for creep tensor $C^{hom}$ (or relaxation tensor $\mathcal{R}^{hom} \approx (C^{hom})^{-1}$) developed in section 4.2 for two arbitrary oriented joint families, the numerical recursive modeling in Ren and Yang (2010) intends to mainly correlate the overall stress and strain tensors and not to directly evaluating the homogenized relaxation or creep tensor.

Prior to performing comparisons between the predictions based on

![Diagram](image1)

**Fig. 8.** The orthogonal jointed rock regarded as a stratified composite (Ren and Yang, 2010).

![Diagram](image2)

(b) Zener model for the rock matrix

(c) Zener model for the joints

![Diagram](image3)

(a) Elementary cell of the orthogonal jointed rock

![Diagram](image4)

Fig. 9. Zener rheological model for the relaxation functions of the joints viewed as interfaces.
the closed-form expression (37) of $F$-hom (considered herein for $\theta = \pi/2$) and the numerical solutions derived from Ren and Yang’s approach, a key issue regarding the modeling of joints should be first addressed. While in the latter approach the joints are viewed as interlayer with finite thickness $\beta > 0$, they are modeled in the present formulation as interfaces that are geometrically described as surfaces in the three-dimensional context. In that respect, consistent comparisons therefore require to preliminary correlate the viscoelastic properties of the joint modeled as an interface with those of the joint viewed as an interlayer material with vanishing thickness (i.e., when $\beta \to 0$).

Referring to the present framework of modeling, the non-aging viscoelastic behavior of the joints shall be defined by means of the relaxation functions $\tau \to k_1^j(t)$ and $\tau \to k_2^j(t) = k_2^j(t)$ in normal and tangential directions, respectively. In a similar manner to that adopted in the Ren and Yang’s approach, Zener rheological models are also introduced to describe these relaxation functions (Fig. 9).

The reasoning behind the correlation between the two kinds of joint modeling relies then upon the following considerations, referring respectively to macroscopic and microscopic scales: (i) the homogenized instantaneous and asymptotic relaxation moduli predicted by both the joint modeling (interlayer or interface) for the jointed rock are equal when $\beta \to 0$, and (ii) the relaxation characteristic times associated with the Zener models of Figs. 8 and 9, which were adopted for the viscoelastic behavior of the joints in the two approaches, are also equals.

The above conditions in conjunction with the assumption (49) yield the sought relationships

$$\frac{E_j}{\beta} = \frac{k_1^j}{\alpha^2} \quad \text{and} \quad k_2^j = \frac{k_2^j}{2(1+\nu)} \quad \text{when} \quad \beta \ll 1$$

$$\eta_j = \frac{\eta_k^j}{\alpha^2} \quad \text{and} \quad \eta_j^\nu = \frac{\eta_k^\nu}{2(1+\nu)}$$

with $q \in \{0, 1\}$.

Since the approach of $\theta^0$ operates in the time domain with a recursive stress/strain relationship for the numerical evaluation of the equivalent viscoelastic response of the orthogonal jointed rock, evaluation of the effective relaxation or creep functions cannot be directly assessed. Thus, we analyze for illustrative comparisons the mechanical response under effective relaxation or creep functions cannot be directly assessed. Thus, the Zener models of Figs. 8 and 9, which were adopted for the viscoelastic properties of the jointed rock, is defined by a matrix material exhibiting aging viscoelastic behavior in Fig. 8, the first loading case simulates a standard relaxation test and is defined by imposing a deviatoric strain loading $E_2(t) = \varepsilon(t)$, which is the macroscopic strain is maintained constant for $t \geq 0$.

$$F(t, \tau) = \left[ 0.14 + 0.69 \tau^{-0.5} + \sum_{k=1}^m \left( 0.025 + 0.175 \tau^{-0.25} \right) \left( 1 - e^{-\frac{\tau}{m}} \right) \right] Y_i(t) \times 10^4 \text{psi}$$

0: $\varepsilon(t) = \varepsilon_0 \text{ Y}(t)$. The second shear loading history refers to a constant prescribed strain rate $\dot{\varepsilon}$, that is, $E_2(t) = \varepsilon(t)$, which is $\varepsilon(t) = \dot{\varepsilon} \text{ Y}(t)$. The materials parameter data provided in Table 2 were used for the simulations, together with joint specific area $a' = 1 \text{ m}^{-1}$. The remaining material properties are deduced from (50).

For the numerical applications, the loading parameters were fixed to $\varepsilon_0 = 10^{-3}$ and $\dot{\varepsilon} = 10^{-3} \text{ s}^{-1}$. The whole simulations along $0 \leq t \leq 5 \text{ s}$ carried out in the context of Ren and Young’s recursive model considered 1000 intervals for the time discretization together with piecewise linear expansions, which proved sufficient for the algorithm convergence.

Fig. 10-a and 10-b summarize the predictions for macroscopic shear stress component $\Sigma_{21}(t)$ obtained numerically from Ren and Young’s recursive model and computed analytically using the homogenized relaxation tensor $F^{\text{hom}}$ associated with creep tensor $F^{\text{hom}}$ given by (37). Fig. 10-a and 10-b referring to the macroscopic loadings defined respectively by constant shear strain and constant shear strain rate indicate a very good agreement between the model analytical predictions and the numerical solutions. The discrepancy observed in Fig. 10-b as the loading time increases is attributed to the piecewise linear expansions adopted for the time variations of local strain and stress fields, and it is expected to be reduced by resorting to higher order polynomial expansions. It should however be kept in mind that, unlike that developed in Ref. 80, the present formulation applies to more general situations regarding the relative orientation of the joint families and the constitutive behavior of the constituents.

5.3. Jointed rock mass with aging viscoelastic rock matrix

The evaluation of overall creep properties of a jointed rock medium, which is defined by a matrix material exhibiting aging viscoelastic properties and two families of crossing joints, is addressed in this section. Special emphasis will be given to the analysis of aging effects induced at macroscopic scale by considering a concrete-like creep behavior for the matrix constituent.

The REV of the jointed rock is that schematized in Fig. 3. The general problem definition and notations are analogous to those introduced for the case studied in section 4.3. In particular, angle $\theta$ will denote the relative inclination of the joints belonging to family $a_2$ with respect to those of family $a_1$. The density of the joint network in the rock mass is characterized by either the joint specific areas ($a_1, a_2$) or corresponding average joint spacing ($d_1, d_2$). The homogenized creep tensor $F^{\text{hom}}$ will be once again conveniently expressed through its components in the reference frame $(\xi_1, \xi_2, \xi_3) = (n, t^1, t^2)$ attached to the joints of family $a_1$ in which the unit vector $\xi_3$ is chosen parallel to the intersection line between a joint of $a_1$ and a joint of $a_2$. Vectors forming the frame $(n, t^1, t^2)$ attached to the joint family $a_2$ are still given by (36).

The uniaxial linear viscoelastic behavior of the rock matrix is modeled by means of the creep function $F$ introduced by 108 for aging concrete (Ross dam concrete).

**Table 2**

| Material parameters used for the simulations (from Ren and Yang 108) |
|---------------------------------|----------|-----------------|-----------------|
| Solid Matrix                   | $E_0 = 25 \text{ GPa}$ | $E_1 = 0.1 \text{ GPa}$ |
| Joints                         | $k_{1,0} = 1 \text{ GPa/m}$ | $k_{1,0} = 0.01 \text{ GPa/m}$ |
| Poisson Ratio                  | $\nu = 0$ | $\nu = 0.3$ |

11
where arguments \( t \) and \( \tau \) stand for the elapsed time from a reference date and the material loading age, respectively. It is observed that the resulting creep function is given in \( 10^{-6} \text{psi} / \text{day} \) (1 psi = 6.89 KPa). The three-dimensional viscoelastic behavior is then formulated assuming isotropy and constant Poisson ratio \( \nu = 0.18 \).

As regards the constitutive properties of the joints, a non-aging viscoelastic behavior described in normal and tangential directions by the Zener rheological models previously sketched in Fig. 9 are adopted for both sets of joints. Accordingly, the creep functions of the joints read

\[
F_1(t - \tau) = \frac{k_{n1}}{k_{n1} + k_{n1}(1 - e^{(-\bar{\tau} - \bar{\tau})/\bar{\tau}_c})} Y_i(t)
\]

\[
F_2(t - \tau) = \frac{k_{n1}}{k_{n1} + k_{n1}(1 - e^{(-\bar{\tau} - \bar{\tau})/\bar{\tau}_c})} Y_i(t)
\]

with \( \bar{\tau}_c = k_{n1}/k_{n1} \) and \( \bar{\tau}_c = k_{n1}/k_{n1} \).

Table 3 summarizes the parameters values used for the numerical illustrations.

| Parameter | Value         |
|-----------|---------------|
| \( k_{n0} \) | 0.919 × 10^{-6} psi/m |
| \( k_{n1} \) | 0.773 × 10^{-6} psi/m |
| \( \bar{\tau}_c \) | 350 × 10^{-6} psi × day/m |
| \( k_{n0} \) | 0.368 × 10^{-6} psi/m |
| \( k_{n1} \) | 0.309 × 10^{-6} psi/m |
| \( \bar{\tau}_c \) | 140 × 10^{-6} psi × day/m |

5.4. Preliminary model validation

The micromechanics-based predictions defined by the relationships (24) to (26) are now compared to the experimental data reported in Yang and Chen. The viscoelastic creep behavior of shale rock samples originating from dam base of Longtan Hydropower Project has been investigated through direct shear tests. The Samples were elaborated from shale block with the joint plane collected from the field engineering. The shale samples of size 150 mm × 150 mm × 150 mwere submitted during the shear creep testing to different shear stress levels (ranging from \( \tau = 0.45 \) MPa to \( \tau = 1.45 \) MPa) under normal stress \( \sigma = 2.49 \) MPa. It is noted that the applied shear stresses remain lower than the short-term strength of the material, which was evaluated to about 2.65 MPa for the considered value of normal stress, thus justifying the analysis of material response within the framework of viscoelasticity. Fig. 12 shows the obtained shear creep test results of shale (diamond symbols) for two shear stress levels, namely \( \tau = 1.20 \) MPa and \( \tau = 1.45 \) MPa. This figure displays the evolution of measured shear displacement \( u \) with respect to loading time \( t \).

The experimental data are compared to the model predictions relying upon expression (47) of homogenized shear creep component \( \bar{\tau}_{\text{hom}} \). Note that the joint specific area is equal to \( a = 1/150 \) mm^{-1}, corresponding to average joint spacing \( d = 150 \) mm. Proceeding as in

![Fig. 10. Evolution of macroscopic stress induced by application of constant shear strain and constant shear strain rate.](image-url)
section 5.1, the individual linear viscoelastic behavior in shear of the intact rock as well as of the joints is described by means of Kelvin-Voigt rheological models shown in Fig. 6-a and Fig. 6-b. The first step of validation procedure is the model calibration, which basically consists in adjusting the constitutive parameters associated with the rock matrix and joints Kelvin-Voigt models, namely $\mu_0$, $\mu_1$, $\eta$, $k_0$, $k_1$ and $\eta_t$. The model calibration is carried out by considering the experimental data represented by black diamond symbols in Fig. 12, which were taken from the creep test performed at shear level $\tau = 1.20$ MPa. The obtained fitted model parameters are provided in Table 4.

Fig. 12 displays the model predictions of shear displacement versus...
loading time obtained with the adjusted parameters of Table 4 (continuous lines), together with the experimental results (diamond symbols). It rises from the present comparison that, although the model parameters have been adjusted using short-term creep test data from lower shear level \( \tau = 1.20 \text{ MPa} \) experiment (black diamond symbols in Fig. 12), the theoretical curve perfectly fits the experimental results at short, intermediate and long times. The relative error between model predictions and experimental results is less than 6% during the analyzed loading time interval. Finally, it should be pointed out that the comparison between model predictions and experimental results was restricted to the highest shear stress levels \( \tau = 1.45 \text{ MPa} \) experiment. In this latter case, the maximum relative error between model predictions and experimental results is less than 6% during the analyzed loading time interval. Finally, it should be pointed out that the comparison between model predictions and experimental results was restricted to the highest shear stress levels \( \tau = 1.45 \text{ MPa} \). This limitation is mainly due to the non-linearity of shear creep behavior observed for the studied shale rock at lower shear stress levels,\(^{10}\) which cannot be appropriately modeled in the context of linear viscoelasticity.

Beyond the encouraging agreement obtained between the theoretical predictions and experimental data, a more comprehensive validation against laboratory experiments or in situ observation remains to be achieved, notably in rocks involving aging creep properties.

6. Conclusions
Relying upon an upscaling approach developed in the context of micromechanics of randomly heterogeneous materials, the homogenized viscoelastic properties of a joined rock have been assessed. The formulation explicitly addresses the situation of rock masses with a dense network of crosscutting joints and extends early formulations to aging linear viscoelasticity. In the framework of modeling, the joints are viewed as planar interfaces whose constitutive behavior relates the local stress vector and displacement jump histories.

The micromechanical reasoning consisted in solving the viscoelastic concentration problem stated on the representative elementary volume of the jointed material. General closed-form expressions for the homogenized creep properties have been thereby derived from the knowledge of the viscoelastic behavior of the individual constituents, namely the rock matrix and the joint interfaces. The creep anisotropy induced by the existence of the privileged directions associated with the joint orientation distribution is automatically, and explicitly, accounted for in the homogenization process. An important feature of the formulation lies on the fact that the overall creep tensor expresses as the sum of the individual contributions of rock matrix and each joint family, thus disregarding the possible interaction between the components of the jointed rock. This is clearly attributed to the assumed homogeneity of rock matrix at the REV scale and the modeling of the joints as infinite planar interfaces. The analysis provided specific insight into the local fields prevailing within the REV submitted to macroscopic loading, through the introduction of generalized strain concentration tensors for the joint interfaces.

A natural continuation of the analysis developed in the paper would consist in implementing the analytical homogenized model for the aging viscoelasticity of jointed rock material into a finite element code specifically devised for computational analysis of short-term as well as transient and long-term structural behaviors of rock engineering structures involving one or more sets of joints, such as the convergence analysis in tunnels driven in jointed rock masses or the simulation of load-settlement law of shallow foundations lying on a jointed rock. From a computational viewpoint, the modeling based on homogenized viscoelastic medium appears suitable for addressing this class of engineering problems. The main advantage of the approach based on homogenization lies precisely on the fact it becomes unnecessary to operate with intricate geometry discretizations (finite element meshes). In line of a recent work dedicated to computational implementation of micromechanics-based viscoelastic model for geomaterials with distributed micro-fractures,\(^{11}\) the incorporation of the homogenized creep behavior within a finite element tool is the matter of ongoing research.

While failure of jointed rocks has been addressed in previous micromechanics-based works,\(^{22,24,25,29,32,107}\) extending the modeling to account for load-induced damage in rock joints is a fundamental issue that still needs to be foreseen in the future. The modeling developed for assessing the overall creep properties of jointed rocks does not account for the damage effects arising at microscale. However, several works have pointed out that shearing deformation of joint may induce a damage process (e.g., Refs. 85,108–115), which reflects the joint surface degradation. In this context, a possible alternative to address this feature would consist in resorting to continuum damage mechanics concepts to simulate the damage process leading to degradation of joint properties under creep conditions. For this purpose, a damage model should be first formulated providing the creep tensor of the joint as a function \( F = (d) \) of a damage parameter. Tensor \( d \) is similar to that introduced by Xu et al.\(^{85}\) and stands for the damage variable that accounts for properties degradation of the joints belonging to set \( \omega_j \). Damage criterion as well as damage evolution law should be formulated combining both the micromechanical reasoning and thermodynamics arguments. The second step of the formulation consists in assuming that the additive rule (25) remains valid in the context of damage analysis, which in turns leads to the expression of homogenized creep tensor \( F^{hom} \) as a function of the joint damage parameters \( \tau \equiv \tau^j + \sum_{j=1}^{N} \tau^j \equiv \sum_{j=1}^{N} \frac{d}{d_j} \). The key issue is therefore to define a macroscopic damage tensor for the jointed rock that is linked to the set of joint damage parameters \( \{ d_j \}, j = 1 \ldots N \) through micromechanics.

Finally, it should be emphasized that effective validation of the theoretical modeling remains to be achieved through comparison of the model predictions against experimental data from laboratory or field tests. In that respect, a major concern is the lack in available data regarding the creep properties of joints. A possible strategy to overcome this difficulty could rely upon inverse analyses developed at macroscopic scale with the aim to identify the viscoelastic properties of the constituents (i.e., rock matrix and joints) from direct comparisons with experimental results referring to the material or structure levels.

Funding sources
This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Declaration of competing interest
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments
The authors gratefully appreciate the support provided by the Brazilian Research Council (CNPq) and the Federal University of Rio Grande do Sul (UFRGS).

References
\(^1\) Goodman RE, Shi GH. Block Theory and its Application in Rock Engineering. Englewood Cliffs, New Jersey: Prentice-Hall; 1985.
