Surrogate runner model for draft tube losses computation within a wide range of operating points

R Susan-Resiga1,2, S Muntean2, T Ciocan1, T de Colombel3 and P Leroy3

1 Politehnica University Timișoara, Romania
2 Romanian Academy – Timișoara Branch, Romania
3 Alstom Power Hydro, Grenoble, France

E-mail: romeo.resiga@upt.ro

Abstract. We introduce a quasi two-dimensional (Q2D) methodology for assessing the swirling flow exiting the runner of hydraulic turbines at arbitrary operating points, within a wide operating range. The Q2D model does not need actual runner computations, and as a result it represents a surrogate runner model for a-priori assessment of the swirling flow ingested by the draft tube. The axial, radial and circumferential velocity components are computed on a conical section located immediately downstream the runner blades trailing edge, then used as inlet conditions for regular draft tube computations. The main advantage of our model is that it allows the determination of the draft tube losses within the intended turbine operating range in the early design stages of a new or refurbished runner, thus providing a robust and systematic methodology to meet the optimal requirements for the flow at the runner outlet.

1. Introduction

Modern hydraulic turbines are required to operate over a significantly wider range of regimes, extending quite far from the best efficiency point, as a result of the variable demand on the energy market and the increase in highly fluctuating renewable energy sources, respectively. In particular, Francis turbines experience efficiency loss and pressure fluctuations at off-design operating regimes. For low head Francis turbines or propeller turbines, the hydraulic losses within the draft tube practically drive the shape of the efficiency hill chart. Moreover, the draft tube of hydraulic reaction turbines ingests the swirling flow exiting the runner which may generate low-frequency phenomena as reviewed by Dörfler et al. [1]. They recall that at the design point water turbines generally operate with little swirl entering the draft tube and no flow separations, but at off-design, at both high and low load, the flow leaving the runner has a large swirling component. Such flow configurations develop swirling flows that produce pressure pulsation in the draft tube. The exit velocity field of the particular runner is recognized as the main factor that influences such phenomena.

When refurbishing the hydraulic turbines usually the main focus is on modifying the guide vanes and the runner, while generally keeping unchanged the spiral case and the draft tube. However, in some refurbishment cases installing a new runner on an old facility while keeping all the stationary components may lead to a pressure recovery ‘accident’ near the best operating point, with an
unfortunate sudden efficiency drop [2]. Tridon et al. [3] show that such unwanted phenomena could sometimes be mitigated, up to some extent, by changing the draft tube shape.

The approach we advocate for is to focus on the swirling flow ingested by the draft tube in order to design the runner that is best suited for the existing draft tube. While studying the swirling flow at the draft tube inlet for a set of operating points with variable discharge, Susan-Resiga et al. [4] developed an three-vortex analytical representation for the velocity field that can be successfully fitted to experimental data. This analytical model was later used to examine the decelerated swirling flow in the discharge cone [5] and it was employed by Galván et al. [6][7] for optimizing the draft tube inlet flow at the design operating point in order to minimize the hydraulic losses. However, although this empirical model offers a convenient and accurate representation of some experimental data, it is valid only in the neighborhood of the best efficiency point. Moreover, it cannot provide an a-priori estimation of the velocity field exiting the runner while changing the operating point. As a result, Susan-Resiga et al. [8] developed a mathematical model aimed at predicting the swirling flow at the draft tube inlet for a wide range of turbine operating points. The usefulness of such models is proved in [9], where the swirl ingested by the draft tube is optimized by simultaneously considering a set of operating points with variable discharge. Muntean et al. [10] show that this swirling flow model can be successfully used at the Kaplan runner outlet, and also introduce a rigorous analytical expression for the radial velocity which is not negligible [11]. It is clear that the model could be further employed to represent the velocity data downstream a propeller turbine runner, as provided by Gagnon et al. [12]. The axial and circumferential velocity profiles computed with such simplified swirling flow models can also be used to assess the main parameters associated with flow unsteadiness, such as the dominant frequency and pressure fluctuation amplitude, Kuibin et al. [13]. The present paper is devoted to further developments of these models, by improving the mathematical formulation as well as by employing information directly related to the runner blades outlet geometry.

Let us now state the aim and benefits of the methodology further detailed in the paper. First of all, the swirling flow model is a surrogate model for the runner of hydraulic turbines since it accounts for the kinematic constraint of the swirl at the runner outlet corresponding to the relative flow angle. As a result, one can use this model to compute the swirling flow at the runner outlet without knowing the detailed runner geometry, or the runner blade for that matter, and use it to assess the draft tube performances. Second, the model accounts for the two main integral parameters that characterize the swirling flow downstream the runner at each operating point: the discharge and the moment of momentum flux. In consequence, the model predicts the swirling flow ingested by the draft tube for arbitrary operating points, within a wide range of operating points. The correlation between the moment of momentum flux downstream the runner and the turbine discharge and head is an intrinsic characteristic of the turbine, and it can be described analytically [8]. In conclusion, the surrogate runner model presented in this paper aims at a-priori assessment of the swirling flow exiting the runner, in the early design stages. As a result, one can optimize the swirling flow ingested by a given draft tube, to minimize the weighted average losses within an operating range, then design the runner that provides that flow. Of course, the runner is going to be designed for one operating point, but the methodology a priori guarantees that the operation in conjunction with a given draft tube will be optimal within a range of operating points.

The paper is organized as follows. Section 2 presents the problem formulation within the framework of the present simplified swirling flow model. The mathematical formulation and corresponding computing algorithm are presented in Section 3. A numerical example is presented in Section 4, with the main conclusions summarized in Section 5.

2. The problem setup

The typical geometry for a Francis turbine runner is shown in Fig.1, where two revolution surfaces are considered upstream and downstream the runner, generated by the Upstream Diagonal Probing (UDP) and Downstream Diagonal Probing (DDP) lines. In addition, a Downstream Radial Probing (DRP) line is considered, corresponding to a cross-section through the discharge cone. The choice of these
probing lines depends on the particular configuration of the turbine meridian cross-section, as shown in Fig.1. Our main goal is to compute the velocity components on the DDP, to be used as input for draft tube computations, without actually computing the three-dimensional runner flow. As a result, within the framework of the methodology to be further detailed one does not need to know the runner geometry. Although our focus is on the swirling flow downstream the runner, corresponding to the grey area in the meridian half-plane, Fig.1, some information is required on the flow non-uniformity from hub to shroud on UDP. Currently, our methodology considers straight probing lines, but it can be further developed to consider arbitrary probing lines. In particular, the DDP can be considered as close as possible to the runner blade trailing edge, as employed in [14] for axisymmetric swirling flow computations downstream a Francis runner.

![Figure 1. Downstream Diagonal Probing (DDP) and Downstream Radial Probing (DRP) lines.](image)

All physical quantities involved in the model are made dimensionless using a reference radius, \( R_{ref} \), chosen as the runner outlet radius, and a reference velocity \( V_{ref} = \Omega R_{ref} \), where \( \Omega \) is the runner angular velocity. As a result, the dimensionless velocity is \( v = V/V_{ref} \), the dimensionless pressure is \( p = P/(\rho V_{ref}^2) \), and the dimensionless discharge and turbine head coefficients are defined as

\[
q = \frac{Q}{\pi R_{ref}^2 V_{ref}}, \quad h_r = \frac{2gH}{V_{ref}^2},
\]

where \( Q[m^3/s] \) is the turbine discharge and \( H[m] \) is the turbine head. In addition, we define the flux of moment of momentum, and its dimensionless form, as:

\[
M = \int (\Omega RV_{r}) \mathbf{V} \cdot \mathbf{n} \, dA, \quad m = \frac{M}{\pi R_{ref}^2 V_{ref}^3}.
\]

The swirling flow downstream the runner is characterized for each operating point by a pair \( q, m_r \), and the dependence \( m_r(q, h_r) \) can be described analytically as shown in [8]. Here \( m_r \) denotes the dimensionless moment of momentum flux downstream the runner, and according to Kelvin’s theorem for inviscid flows it has the same value for both DPL and RPL. If \( m \) is the corresponding value on UDP, then for loss-free flows the Euler equation of turbomachines simply reads as \( q h_r / 2 = m_1 - m_2 \).

### 2.1. The swirl-free velocity concept

Since we aim at computing the swirling flow downstream the runner, it is expected that some information of the flow kinematics at runner outlet be embedded into the model. Usually, the runner blade geometry at the trailing edge gives the relative flow angle at the runner outlet. However, instead
of using the relative flow angle, $\beta$, it is more convenient to define the fictitious swirl free velocity 
[8], as $V' = \Omega R \tan \beta$. This quantity is valid at the runner blades trailing edge, or in its immediate 
neighborhood, on the DDP from Fig.1. As a result, we have the dimensional, $V_d$, and dimensionless, 
$v_d$, swirl-free velocity defined as,

$$V_d = \frac{\Omega RV_n}{\Omega R - V'_o}, \quad v_d = \frac{V_d}{\Omega R v_o} = \frac{r v_o}{r - v_o},$$

where the subscript $n$ denotes the meridian velocity component normal to the DDP, and the subscript $T$
denotes the circumferential component. The swirl-free velocity defined in Eq.(3) can be 
approximated for most turbines as having a linear variation from hub to shroud, thus providing a two-
parameter framework for optimizing the runner outlet. An important development in this paper with 
respect to our previous work [8][9] is to consider the $v_d$ on the DDP, where it is valid, instead of a 
DRP where its validity is questionable. The main assumption behind the swirl-free velocity concept is 
that the $v_d$ profile from hub to shroud is practically independent of the operating point. This is true if 
the flow does not detach from the runner blades, at least in the neighborhood of the trailing edge. A 
proper blade design, with a gradual blade loading decrease towards the trailing edge in order to 
comply with the Kutta-Jukowski condition, will insure the validity of the above assumption.

3. Mathematical model for swirling flow exiting the runner

The swirling flow model further employed in this paper assumes an inviscid and incompressible fluid 
and an axisymmetrical flow. Neglecting the viscosity is acceptable given the high Reynolds number 
values specific to hydraulic turbines, but also because we are concerned with the early design stages 
when the main design decisions should be based on the most relevant aspects of the flow. The axial-
symmetry assumption is valid because we consider the swirling flow only in an axisymmetric segment 
of the flow passage from the runner outlet and further downstream into the discharge cone. Even when 
the swirling flow develops self-induced instabilities, such as the precessing vortex rope, and the flow 
becomes unsteady and three-dimensional in spite of geometrical axial symmetry, the axisymmetric 
flow model can correctly capture the circumferentially averaged flow [14].

Thanks to the axial symmetry, all flow quantities depend only on two spatial coordinates, $z$ and $r$, 
in axial and radial directions, respectively. In this case, the velocity vector can be written in cylindrical 
coordinates with unit vectors $\hat{z}, \hat{r}, \hat{\theta}$ in axial, radial and circumferential directions, respectively, as 
$v = v_z \hat{z} + v_r \hat{r} + v_\theta \hat{\theta}$, where $v$ is the Stokes’ streamfunction for axisymmetric 
flows, and $\kappa = rv_\theta$ is the circulation function. In bladeless regions, this circulation function depends 
only on the streamfunction, i.e. we have $\kappa(\psi)$. The steady, axisymmetric, swirling flow equations for 
an incompressible and inviscid fluid can be condensed in one single partial differential equation, 
known as Bragg-Hawthorne [15] or Long – Squire [16][17] equation,

$$\nabla \left( \frac{\nabla \psi}{r^2} \right) + \frac{\kappa(\psi)}{r^2} \frac{d \kappa(\psi)}{d \psi} - \frac{dh(\psi)}{d \psi} = 0,$$

where $h = p + \left( v_z^2 + v_r^2 + v_\theta^2 \right)/2$ is the dimensionless total head. Thanks to the Bernoulli’s theorem, we 
have $h(\psi)$, i.e. the total head is constant along streamtubes of constant $\psi$. Equation (4) is a nonlinear 
elliptic PDE for the unknown function $\psi(z,r)$, given the functions $\kappa(\psi)$ and $h(\psi)$, respectively, 
together with proper essential or natural boundary conditions.

For our swirling flow model we consider the 1D simplified form of Eq.(4) along the probing lines. 
Moreover, instead of the differential formulation (4) we employ the equivalent variational formulation 
given by Benjamin [18]. As a result, one should look for the streamfunction $\psi$ that minimizes the 
functionals,
The subscript BH denotes that the above functionals correspond to the boundary-value problem for the Bragg-Hawthorne equation (4), while the superscript indicates the corresponding probing line.

Note that the boundary conditions for \( \psi \) insure that the overall discharge \( q \) is preserved. However, the integral constraint for the moment of momentum flux must be explicitly enforced on DDP as,

\[
F_{BH}^{DRP}(\psi, r) = \int_{r_*}^{r_{h}} \left[ \frac{1}{2} \left( \frac{1}{r} \frac{d\psi}{dr} \right)^2 - \frac{\kappa^2(\psi)}{2r^2} + h(\psi) \right] 2r \, dr, \quad \text{on DRP, with } \psi(r_*) = 0 \text{ and } \psi(r_{h\text{wall}}) = q/2 \quad (5)
\]

\[
F_{BH}^{DDP}(\psi, s) = \int_{s_*}^{s_{h\text{wall}}} \left[ \frac{v_s^2}{2} - \frac{v_\theta^2}{2} + h(\psi) \right] 2r \, ds, \quad \text{on DDP, with } \psi(s_*) = 0 \text{ and } \psi(s_{shroud}) = q/2 . \quad (6)
\]

The subscript BH denotes that the above functionals correspond to the boundary-value problem for the Bragg-Hawthorne equation (4), while the superscript indicates the corresponding probing line.

In Eq.(5), the lower integration limit \( r_* \geq 0 \) is a problem unknown, and it corresponds to the extent of the possible quasi-stagnant region developed near the axis when vortex breakdown occurs. As a matter of fact, this is the case when \( \kappa(0) \neq 0 \), which is the general situation in hydraulic turbines; \( \kappa(0) \) eventually vanishes only for one operating point. As a result, \( F_{BH}^{DRP} \) must be minimized, given \( r_* \).

In Eq.(6), \( s \) represents the curvilinear coordinate along DDP, running from \( s_{hub} = 0 \) up to \( s_{shroud} \). Again, the model assumes that the lower integration limit, \( s_* \geq 0 \), is a problem unknown, since the stagnant region may develop near the hub even on the DDP, when operating far enough from the design operating regime. It is clear that the functional \( F_{BH}^{DDP} \) in Eq.(6) follows from the functional \( F_{BH}^{DRP} \) in Eq.(5), since \( v_\theta = (1/r)(d\psi/ds) \) and \( v_\theta = \kappa/r \), respectively. However, in Eq.(6) the circumferential velocity is related to the normal component of the meridian velocity through the swirl-free velocity defined in Eq.(3),

\[
v_\theta(s) = r \left( 1 - \frac{v_s(s)}{v_{id}(s)} \right) . \quad (8)
\]

Actually, Eq.(8) embeds into the \( F_{BH}^{DDP} \) functional the kinematic constraint corresponding to the relative flow direction, expressed via the swirl-free velocity profile \( v_{id}(s) \).

The total head in Eq.(6), is estimated using the Euler equation of turbomachines written along the streamtubes, with the assumption that on the UDP the total head is practically constant, while the circulation function varies linearly from hub to shroud with the slope \( \kappa_{id}^{UDP} \). This coefficient is of order of unity for medium-high specific speed turbines, and it decreases with the specific speed until it practically vanishes for low specific speed turbines or pump-turbines. When the meridian flow in the guide vanes also stars turning from radial to axial in the meridian plane, as for example in Fig.1, the circulation function increases from hub to shroud, thus \( \kappa_{id}^{UDP} \) is positive. As a result, in Eq.(6) we have

\[
h(\psi(s)) = r v_\theta(s) - \kappa_{id}^{UDP} \psi(s) + \text{constant} . \quad (9)
\]

The arbitrary constant in Eq.(9) should not influence the solution. As a matter of fact, \( h(\psi) \) is defined up to an arbitrary additive constant which vanishes in the differential formulation, Eq.(4), thanks to the differentiation with respect to \( \psi \).

3.1. The stagnant region model

We now turn our attention to the determination of the lower integration limits \( r_* \) in Eq.(5) and \( s_* \) in Eq.(6). In order to do that, we add to the functional the contribution of the stagnant region developed as a result of the vortex breakdown. The main assumption, in agreement with the flow physics, is that the boundary between the central stagnant region and the annular main swirling flow is a vortex sheet that may have a jump in velocity components while the pressure is continuous across this boundary.
The pressure continuity follows from the fact that the vortex sheet is a fluid boundary, and in order to remain in equilibrium for a steady flow it cannot support a jump in the static pressure. Inside the stagnant region the velocity vanishes, and the pressure remains constant. This stagnant region model has been validated in [14] with respect to measured velocity profiles in swirling flow at partial turbine discharge, where a vortex rope is developed. The resulting functionals are called ‘flow force functional’ according to the name coined by Benjamin [18], thus the subscript FF,

\[
F_{FF}^{DRP}(r) = \min_{\psi} F_{BH}^{DRP}(\psi, r) + \left[ h(0) - \frac{\psi'(r)}{2} - \frac{\psi'^2(0)}{2r^2} \right] r^2
\]  

(10)

\[
F_{FF}^{DDP}(s) = \min_{\psi} F_{BH}^{DPP}(\psi, s) + \left[ h(0) - \frac{\psi'(s)}{2} - \frac{\psi'^2(s)}{2} \right] (r_{hub} + r_s) s
\]

(11)

Once again, the functional from Eq.(11) reduces to the one in Eq.(10) by observing that on the DRP we have \(v_n \rightarrow v_z\), \(v_{\rho} = \kappa/r\), \(s \rightarrow r\) and obviously \(r_{hub} \rightarrow 0\).

In the end, the lower integration limits are found by maximizing the corresponding functionals,

\[
\max_{r} F_{FF}^{DRP}(r) \quad \text{and} \quad \max_{s} F_{FF}^{DPP}(s)
\]

(12)

The physical interpretation of functional maximization is that in doing so we actually find the swirling flow configuration that minimizes the swirl number. The swirl number is the ratio between the flux of angular momentum and the flux of axial momentum. The numerator is kept constant by prescribing the integral quantity \(m_z\), while the denominator corresponds to the flow force functional and it is maximized. Although we do not have a rigorous proof of the above result, it is intuitively correct.

### 3.2. The numerical algorithm

In order to summarize the numerical algorithm, let is recall the input information for the problem. First of all we have the geometrical data corresponding to the end-points coordinates for both DDP and DRP. The reason for using both probing lines will be explained later in this section. Second, we have the operating point parameters, given by the numerical values for dimensionless discharge, \(q\) , moment of momentum angular flux, \(m_z\) , and the slope coefficient \(\kappa_{hub}\) . Third, we have the coefficients of the swirl-free velocity linear approximation \(v_d(s) = v_{dhub} + v_{dslo} s\). These slope, \(v_{dslo}\), and intercept, \(v_{dhub}\), coefficients are the blueprint for the runner outlet geometry.

The numerical algorithm has three main steps. The first step is the computation of the normal and circumferential velocity profiles on DDP, say \(v_n^{DPP}(s)\) and \(v_{\rho}^{DPP}(s)\) together with the stagnant region extent \(s\), by maximizing the functional from Eq.(11) while minimizing the functional from Eq.(6) with the integral constraint from Eq.(7). As a post-processing step for this computation, we obtain the streamfunction \(\psi^{DPP}(s)\) as well as the functions \(\kappa(\psi)\) and \(h(\psi)\). The second step is the computation of the axial and circumferential velocity profiles on DDP, say \(v_z^{DPP}(r)\) and \(v_{\rho}^{DPP}(r)\) together with the stagnant region extent \(r\), by maximizing the functional from Eq.(10) while minimizing the functional from Eq.(5). Note that the functional \(F_{BH}^{DPP}\) uses the functions \(\kappa(\psi)\) and \(h(\psi)\) previously found on the DDP, thanks to Kelvin’s and Bernoulli’s theorems, respectively. This computation also provides the streamfunction \(\psi^{DPP}(r)\). The third step provides the velocity components on DDP to be used for the draft tube computations. The circumferential velocity component is readily available. However, we have only the normal projection of the meridian velocity while we need both axial and radial velocity components. As a result, we need to estimate the streamline direction between DDP and DRP. This is the reason why we need both these probing lines within the present methodology. Although the computations on either DDP or DRP are one-dimensional, this third step makes the overall algorithm
quasi two-dimensional. The streamlines are approximated as straight lines between points on DDP and DRP, respectively, with the same \( \psi \) value. The corresponding normal velocity at the streamline starting point on DDP is divided by the scalar product of the unit normal vector on DDP and the unit vector along the streamline, thus obtaining the meridian velocity magnitude. Once both the magnitude and direction of the meridian velocity are known, we project the meridian velocity vector on axial and radial directions, thus obtaining the corresponding velocity components.

The above three-step algorithm is implemented into the FORTRAN code called TurboSwirlQ2D, developed and validated within a joint research project by the authors of this paper.

4. Numerical example
The numerical example further presented in this section corresponds to the Francis turbine model investigated in the FLINDT project [2], with the specific speed \( v = 0.56 \) [19, p.477]. The turbine model has a spiral casing of double curvature type, with a stay ring of 10 stay vanes, a distributor made of 20 guide vanes, a 17-blade runner of a 0.4 m outlet diameter, and a symmetric elbow draft tube with one pier.

The probing lines have the end-points dimensionless coordinates given in Tab.1. Note that the axial coordinates have reversed sign with respect to the \( z \)-axis in Fig.1.

| Probing line | Point 1 \((z,r)\) | Point 2 \((z,r)\) |
|--------------|-----------------|-----------------|
| DDP          | (0.394,0.171)   | (1.060,1.04)    |
| DRP          | (1.382,0.000)   | (1.382,1.091)   |

Five operating points are considered for this example, corresponding to constant head and variable discharge as shown in Tab.2.

Table 2. Operating points parameters.

| Guide vane opening angle | Discharge coefficient \( q \) [-] | Head coeff. \( h_i \) [-] | Moment of momentum flux \( m_2 \) [-] | Circulation slope \( k_{\text{UDP}} \) [-] |
|-------------------------|---------------------------------|------------------|----------------------|---------------------|
| OP1 17.0'               | 0.2749018                       | 1.18             | 0.1603683 \( q \)    | 0.7                 |
| OP2 19.0'               | 0.3106692                       | 1.18             | 0.1187753 \( q \)    | 0.7                 |
| OP3 21.0'               | 0.3442632                       | 1.18             | 0.0810984 \( q \)    | 0.7                 |
| OP4 23.0'               | 0.3714126                       | 1.18             | 0.0513976 \( q \)    | 0.7                 |
| OP5 25.0'               | 0.3941436                       | 1.18             | 0.0278549 \( q \)    | 0.7                 |

We consider the simplest approximation for the swirl-free velocity, according to Fig. 2.

\[ v = q^{0.55} / h_i^{0.35} = 0.4^{0.55} / 1.18^{0.35} \]

\( q \) and \( h_i \) defined according to Eq.(1).
Figure 3 shows the numerical results obtained with the present Q2D swirling flow model, using the information from Tab. 1, Tab. 2 for OP1, and Fig. 2. The figure illustrates the three algorithmic steps described in §3.2, corresponding to the 1D computation on DDP (subfigure a), the 1D computation on DRP (subfigure b), and the Q2D approximation of the streamlines direction (subfigure c). In the end, the dimensional velocity components on DDP – axial, radial and circumferential – are shown in subfigure d), and further used as inlet conditions for draft tube computations in §4.1.

One can see from Fig. 3 that the Q2D model captures the main characteristics of the swirling flow downstream the runner. Although, the Q2D velocity profiles have a simplified shape as the model does not account for viscous effects or for three-dimensional flow features developed in the inter-blade channels, by construction the model preserves the discharge and the moment of momentum flux. At low discharge, Fig. 3, the flow detaches already from the runner crown, and develops a large stagnant region further downstream.

Figure 4 qualitatively shows that the streamlines converge towards the axis as the discharge increases, leading to the flow acceleration in the central region, as expected.
Although it is not intended to compete with the full 3D runner flow simulations usually employed for \textit{a posteriori} evaluation of a runner design, the present Q2D methodology is robust enough to be used for \textit{a priori} assessment of the possible design options for the runner outlet, as shown in Fig. 5.

![Figure 5. Q2D dimensional velocity components on DDP for the operating points defined in Tab. 2.](image)

### 4.1. Draft tube computations

The main purpose of the Q2D model presented in this paper is to provide inlet velocity profiles for draft tube computations, such that one can \textit{a priori} assess how various runner outlet configurations will influence the draft tube performance within a range of operating regimes. As a result, we performed a series of 3D turbulent flow computations for the draft tube model shown in Fig. 6. The inlet section is a cone generated by the DDP line, and the computational domain is further extended downstream the geometrical draft tube outlet section. A structured discretization with approximately $10^6$ cells is used. Numerical simulations are performed using the FLUENT 6.3 expert code, using a realizable $k-\varepsilon$ turbulence model, with an average 10% inlet turbulence intensity. For each operating point we compute the hydraulic losses using both inlet conditions from 3D runner computations (dashed lines in Figs. 3 and 5) and from Q2D computations (solid lines in Figs. 3 and 5). Note that the above dimensional velocity profiles correspond to a reference radius $R_{ref} = 0.2\, m$ and a runner speed of 750 rpm, corresponding to the FLINDT Francis turbine model.

![Figure 6. Model draft tube with inlet section corresponding to DDP (left) and head loss relative to the turbine head versus the discharge coefficient (right). The dashed line corresponds to the inlet velocity from 3D runner computations, and the solid line corresponds to the inlet velocity computed with the present Q2D model.](image)

The hydraulic losses are computed as the difference between the flux-weighted average total pressure on inlet/outlet sections, being normalized with respect to the model turbine head. The results for the five operating points given in Tab. 2 are shown in Fig. 6. One can see that the inlet velocity profiles predicted by the Q2D model lead to draft tube losses quite close to the values obtained by prescribing the inlet velocity from 3D runner computations, in spite of their significantly simpler shape.
5. Conclusions
The paper introduces a novel quasi two-dimensional (Q2D) methodology (mathematical model, numerical algorithms) for evaluating the swirling flow exiting the hydraulic turbines at arbitrary operating regimes that can be quite far from the best efficiency point. This is a surrogate model for the turbine runner in the sense that it replaces the runner computations, thus providing an a-priori assessment of the swirling flow ingested by the draft tube within a wide operating range. As a result, one can optimize the runner outlet before actually designing to runner in order to minimize the weighted averaged hydraulic losses in the draft tube within a given operating range. This is a typical requirement when re-designing the runner in the refurbishment projects.

Our Q2D model employs a novel variational formulation for the steady axisymmetric swirling flows that includes the contribution of the quasi-stagnant region developed as a result of the vortex breakdown. A constrained swirling flow problem is solved on a diagonal probing line located immediately downstream the runner blades trailing edge (DDP), where the kinematic constraint corresponding to the relative flow direction is accounted for via the so-called swirl-free velocity. Then, an unconstrained swirling flow problem is solved on a cross-section located in the discharge cone (DRP) using the circulation and head dependence on the streamfunction determined from the DDP computation. In the end, the streamlines direction is estimated between DDP and DRP, and the axial, radial and circumferential velocity profiles are computed on DDP to be further used for draft tube calculations.

A numerical example is presented, and the Q2D velocity profiles computed for a model Francis runner are compared with the corresponding data obtained from a full 3D runner computation. Obviously, the Q2D results display simplified axial, radial and circumferential velocity profiles since the model does not account for viscous and three-dimensional effects associated with the complex real runner flow. However, the Q2D model is not intended to compete in accuracy with 3D flow analysis, but rather to provide an a-priori estimation before (or without) actually knowing the runner geometry.

The evaluation of the hydraulic losses in a draft tube model shows that the results obtained by using the Q2D velocity profiles at the draft tube inlet are quite close to the corresponding values obtained by using the inlet velocity from 3D runner computations. We conclude that the present Q2D methodology provides a robust assessment of the draft tube behavior within a range of operating regimes prior to runner design, and as a result it offers the possibility to optimize the runner outlet requirements in the early design stages.

Acknowledgements
The mathematical model development has been supported by a grant of the Romanian Ministry of National Education, CNCS-UEFISCDI, project number PNII-ID-PCE-2012-4-0634. The TurboSwirlQ2D computer code was developed and validated with the support from Alstom Power Hydro within the framework agreement between the Alstom Hydro France and the Politehnica University Timișoara concerning the joint research and development project related to assessing, developing and using a mathematical and numerical modeling of the hydraulics of water turbines in a view to improve the performances of such turbines.

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