η-nuclear bound states revisited

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Abstract

The strong energy dependence of the s-wave $\eta N$ scattering amplitude at and below threshold, as evident in coupled-channels $K$-matrix fits and chiral models that incorporate the $S_{11} N^*(1535)$ resonance, is included self-consistently in $\eta$-nuclear bound-state calculations. This approach, applied recently in calculations of kaonic atoms and $K$-nuclear bound states, is found to impose stronger constraints than ever on the onset of $\eta$-nuclear binding, with a minimum value of $\text{Re } a_{\eta N} \approx 0.9$ fm required to accommodate an $\eta$-4He bound state. Binding energies and widths of $\eta$-nuclear states are calculated within several underlying $\eta N$ models for nuclei across the periodic table, including $^{25}\eta$Mg for which some evidence was proposed in a recent COSY experiment.

Keywords: meson-baryon interactions, mesons in nuclear matter, mesic nuclei

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1. Introduction

Searches for meson-nuclear bound states have focused on $K^-$ and $\eta$ mesons, motivated by a general theoretical consensus that the near-threshold $\bar{K}N$ and $\eta N$ attraction generated by the s-wave resonances $\Lambda(1405)$ and $N^*(1535)$, respectively, translates into sufficiently attractive $K^-$-nucleus and $\eta$-nucleus interactions. A corollary of this resonance dominance is a strong energy dependence of the underlying $\bar{K}N$ and $\eta N$ interactions. Here we apply the lessons gained by handling the strong energy dependence of the near-threshold $\bar{K}N$ interaction in $K^-$-nuclear calculations [1] to $\eta$-nuclear

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bound-state calculations. Early calculations by Haider and Liu\cite{2,3}, predicted \(\eta\)-nuclear bound states beginning with nuclear mass number \(A \sim 12\). In these, as well as in a follow-up calculation\cite{4}, a fairly weak \(\eta N\) attraction input was used, with \(\text{Re } a_{\eta N} \lesssim 0.3\) fm, where \(a_{\eta N}\) is the \(\eta N\) scattering length. Several versions of coupled-channels chiral models\cite{5,6,7} give similar values as well, whereas other models, particularly those using \(K\)-matrix methods to fit \(\pi N\) and \(\gamma N\) reaction data in the \(N^*(1535)\) resonance region, e.g.\cite{8,9,10}, yield considerably stronger \(\eta N\) attraction with values of \(\text{Re } a_{\eta N} \approx 1\) fm\footnote{As for \(\text{Im } a_{\eta N}\), it is constrained by \(\pi N \rightarrow \eta N\) cross-section measurements, with values of \(\text{Im } a_{\eta N} \sim 0.2-0.3\) fm in most theoretical analyses.}.

This might suggest that the onset of \(\eta\)-nuclear binding occurs already in the He isotopes for which strong final-state interaction precursors have been noted in proton- and deuteron-initiated \(\eta\) production\cite{11,12}. A robust pattern of \(\eta\)-nuclear bound states could yield useful information on the size of SU(3) flavor \(\eta - \eta'\) mixing and about axial U(1) dynamics\footnote{As for \(\text{Im } a_{\eta N}\), it is constrained by \(\pi N \rightarrow \eta N\) cross-section measurements, with values of \(\text{Im } a_{\eta N} \sim 0.2-0.3\) fm in most theoretical analyses.}. To date, however, experimental searches for such bound states have been unsuccessful, e.g. the latest negative results for \(^3\eta\)He (in photoproduction on \(^3\)He\cite{14}) and for \(^4\eta\)He (in \(dd \rightarrow ^3\)He\( p\pi^-\)\cite{15}).
Regardless of the strong model dependence of Re $a_{\eta N}$, all studies of the $\eta N$ system near threshold, $\sqrt{s_{th}} = m_N + m_\eta \approx 1487$ MeV, agree that both real and imaginary parts of the $s$-wave center-of-mass (cm) scattering amplitude $f_{\eta N}$ decrease steeply in going subthreshold, as illustrated in Fig. 1. Since the in-medium $\eta N$ interaction relevant to the evaluation of $\eta$-nuclear bound states involves subthreshold $\eta N$ configurations, a procedure for going subthreshold is mandatory. Previous calculations focused on shifting the energy variable of $f_{\eta N}(\sqrt{s})$ or its in-medium version by a fixed amount below threshold: $\delta \sqrt{s} = -30$ MeV was found in Ref. [4] to provide a good approximation to a variety of off-shell effects, whereas $\delta \sqrt{s} = -B_\eta$, with $B_\eta$ the $\eta$-nuclear binding energy, was used in Refs. [16, 17]. The latter procedure requires a self-consistent calculation to ensure that the $B_\eta$ argument of the input $f_{\eta N}$ coincides with the $B_\eta$ output of the binding energy calculation. However, it was shown in our recent studies of $K^-\text{nuclear}$ dynamics [18, 19, 20, 21, 22, 23] that a more involved self-consistent calculation is required to correctly implement the subthreshold energy dependence, and it is this self-consistent procedure that is applied here to calculate $\eta$-nuclear bound states. This procedure results in imposing stronger constraints than ever on the onset of $\eta$-nuclear binding.

Table 1: $\eta N$ scattering length $a_{\eta N}$ (in fm) in three coupled-channels models used in the present work. M1 and M2 correspond to versions I and II, respectively, of $\eta N$ amplitudes from the recent chiral-model work by Mai et al. [7], and GW denotes the $K$-matrix $\eta N$ amplitude due to Green and Wycech [8] shown in Fig. 1.

| Model | Re $a_{\eta N}$ | Im $a_{\eta N}$ |
|-------|----------------|----------------|
| M1    | 0.22           | 0.24           |
| M2    | 0.38           | 0.20           |
| GW    | 0.96           | 0.26           |

Below we proceed to describe briefly the self-consistent procedure used to handle the subthreshold energy dependence of the $\eta N$ amplitude for bound nucleons, and its embedding into a dynamical Relativistic Mean Field (RMF) scheme which allows for the first time to consider the polarization of the core nucleus by the bound $\eta$ meson. To span a broad range of bound-state scenarios we apply our methodology to three distinct $\eta N$ amplitude models, with threshold values listed in Table 1. These amplitudes differ primarily
in the value of the real part, while their shape below threshold exhibits a substantial decrease particularly for \( \text{Im} \, \alpha_{\eta N} \), as illustrated for the GW amplitude model \([8]\) in Fig. 1. We have calculated \( \eta \)-nuclear bound states across the periodic table for these three amplitude models, as reported and discussed here for \( 1s_\eta \) states. Finally, we also confront our results with a recent experimental suggestion of a \( ^{25}_\eta \text{Mg} \) bound state \([24]\).

2. Methodology

In close analogy to the latest calculation of \( K^- \)-nuclear bound states \([21]\), we calculate \( \eta \)-nuclear bound states by solving self-consistently the Klein-Gordon (KG) equation

\[
\left[ \nabla^2 + \tilde{\omega}_\eta^2 - m_\eta^2 - \Pi_\eta(\omega_\eta, \rho) \right] \psi = 0,
\]

where \( \tilde{\omega}_\eta = \omega_\eta - i\Gamma_\eta/2 \) and \( \omega_\eta = m_\eta - B_\eta \), with \( B_\eta \) and \( \Gamma_\eta \) the binding energy and the width of the \( \eta \)-nuclear bound state. The self-energy operator \( \Pi_\eta \) is related to a density- and energy-dependent optical potential \( V_\eta \) which is given by the following “\( t\rho \)” form:

\[
\Pi_\eta(\omega_\eta, \rho) \equiv 2\omega_\eta V_\eta = -4\pi F_{\eta N}(\sqrt{s}, \rho) \rho,
\]

where \( s = (E_\eta + E_N)^2 - (\vec{p}_\eta + \vec{p}_N)^2 \) is the Lorentz invariant Mandelstam variable \( s \) which reduces to the square of the total \( \eta N \) energy in the two-body cm frame and \( F_{\eta N} \) is the in-medium \( \eta N \) s-wave scattering amplitude in the lab system. Note that for \( A \gg 1 \) the lab system approximates well the \( \eta \)-nucleus cm system. Our in-medium \( F_{\eta N} \) accounts for Pauli correlations in the Ericson-Ericson multiple-scattering approach, as reformulated in Ref. \([25]\) and used recently in Ref. \([23]\):

\[
F_{\eta N}(\sqrt{s}, \rho) = \frac{\tilde{f}_{\eta N}(\sqrt{s})}{1 + \xi(\rho)\tilde{f}_{\eta N}(\sqrt{s})\rho}, \quad \xi(\rho) = \frac{9\pi}{4p_F^2},
\]

where \( \tilde{f}_{\eta N}(\sqrt{s}) = (\sqrt{s}/m_N)f_{\eta N}(\sqrt{s}) \), with the kinematical factor \( \sqrt{s}/m_N \) transforming \( f \) from the two-body cm frame to the lab \( \tilde{f} \), and where \( p_F \) is the local Fermi momentum corresponding to density \( \rho = 2p_F^3/(3\pi^2) \). Note that \( F_{\eta N}(\sqrt{s}, \rho) \rightarrow \tilde{f}_{\eta N}(\sqrt{s}) \) upon \( \rho \rightarrow 0 \), as required by the low-density limit. Extensions of Eq. \([3]\) to coupled channels and inclusion of self-energies do not change the results presented here in any qualitative way and will be discussed elsewhere \([26]\).
In specifying the two-body cm energy $\sqrt{s}$ appearing in Eq. (3) we recall that $s = (\sqrt{s}_{\text{th}} - B_\eta - B_N)^2 - (\vec{p}_\eta + \vec{p}_N)^2$, where the momentum-dependent term provides additional downward energy shift to that arising from the sum of binding energies $B_N + B_\eta$. Unlike in the free-space $\eta N$ cm system where $(\vec{p}_\eta + \vec{p}_N)_{\text{cm}} = 0$, this term in the lab system was found to contribute substantially in realistic nuclear applications [18, 19]. It has been verified numerically by us that $(\vec{p}_\eta + \vec{p}_N)^2$ is well approximated by its angle-average $(p_\eta^2 + p_N^2)$. Near threshold, then, to leading order in binding energies and kinetic energies with respect to masses, one obtains

$$\sqrt{s} \approx \sqrt{s}_{\text{th}} - B_N - B_\eta - \xi_N \frac{p_N^2}{2m_N} - \xi_\eta \frac{p_\eta^2}{2m_\eta},$$

where $\xi_N(\eta) \equiv m_N(\eta)/(m_N + m_\eta)$. To transform momentum dependence into density dependence, the nucleon kinetic energy $p_N^2/(2m_N)$ is approximated within the Fermi gas model by $T_N(\rho/\rho_0)^2$, with average bound-nucleon kinetic energy $T_N = 23.0$ MeV, and the $\eta$ kinetic energy $p_\eta^2/(2m_\eta)$ is substituted within the local density approximation by $-B_\eta - \text{Re} V_\eta(\sqrt{s}, \rho)$. Thus, the in-medium $\sqrt{s} = \sqrt{s}_{\text{th}} + \delta\sqrt{s}$ energy argument of $F_{\eta N}$ in Eq. (3) is density-dependent, with a form adjusted to respect the low-density limit, $\delta\sqrt{s} \rightarrow 0$ with $\rho \rightarrow 0$, as used recently in $K^-$-atom studies [23]:

$$\delta\sqrt{s} \approx -B_N \frac{\rho}{\bar{\rho}} - \xi_N B_\eta \frac{\rho}{\rho_0} - \xi_N T_N(\frac{\rho}{\rho_0})^{2/3} + \xi_\eta \text{Re} V_\eta(\sqrt{s}, \rho).$$

Here $B_N \approx 8.5$ MeV is an average nucleon binding energy and $\rho_0$ ($\bar{\rho}$) is the maximal (average) nuclear density. The appearance of the $V_\eta$ term due to $p_\eta \neq 0$ in finite nuclei contrasts with the common assumption $p_\eta = 0$ made in nuclear matter calculations. The dependence of $V_\eta$ on energy through $\sqrt{s}$ and on density $\rho$ is explicitly marked in this expression. Note that for an attractive $V_\eta$ and as long as $\rho \neq 0$, the shift of the two-body energy away from threshold is negative definite, $\delta\sqrt{s} < 0$, even as $B_\eta \rightarrow 0$. For a given $B_\eta$, neither $\sqrt{s}$ nor $V_\eta$ can be evaluated separately, implying that $V_\eta$ is to be constructed self-consistently together with $\sqrt{s}$, which takes typically about 5 cycles of iteration. Once $V_\eta(\sqrt{s}, \rho)$ has been determined, it is used in the KG Eq. (1) to solve for the binding energy eigenvalue $B_\eta^{(n\ell)}$ in the $\eta$-nuclear $n\ell$ single-particle state. While varying the value of $B_\eta^{(n\ell)}$ in this process, the self-consistent requirement Eq. (5) is imposed at each step of the calculation of the eigenvalue.
Figure 2: Subthreshold \( \eta N \) energies probed by the \( \eta \)-nuclear potential as a function of the relative nuclear RMF density in Ca. Each of the three curves was calculated self-consistently according to Eq. (5) for a specific version of \( \eta N \) subthreshold amplitude model, see text.

\[
E - E_{\text{th}} \equiv \delta \sqrt{s} = E - E_{\text{th}}
\]

In Fig. 2 we show the downward subthreshold energy shift \( \delta \sqrt{s} \equiv E - E_{\text{th}} \) as a function of the nuclear RMF density \( \rho \) in Ca, calculated self-consistently according to Eq. (5) for \( \eta N \) amplitude models M1, M2 and GW (see Table 1). The hierarchy of the three curves reflects the strength of the input \( \text{Re} f_{\eta N} (\sqrt{s}) \) in the subthreshold region, with threshold values listed in Table 1. It is clear that downward energy shifts of up to \( \approx 55 \) MeV are involved in the present self-consistent calculations.

3. Results and discussion

The methodology described in the last section was used to solve the KG equation (1) for \( \eta \)-nuclear bound states across the periodic table. In this Letter we highlight the systematics of the \( 1s_\eta \) bound state and compare our treatment of subthreshold energy dependence with previous studies. A more detailed account plus extensions are given elsewhere [26]. Three representative \( \eta N \) amplitude models M1, M2 and GW (see Table 1) are employed here in order to span a wide range of \( \eta N \) interaction strengths. Our main results are shown in Fig. 3 for binding energies \( B_\eta \) and widths \( \Gamma_\eta \) calculated...
for $1s_\eta$ nuclear states in core nuclei from $^{12}\text{C}$ to $^{208}\text{Pb}$. RMF equations of motion, along with the KG equation [11], are solved self-consistently [27], thereby allowing for core polarization by the $\eta$ meson (see Ref. [19] for the latest application to $K^-$ mesons). The core polarization effect on $B_\eta$ and $\Gamma_\eta$ was found in all cases displayed here to be less than 1 MeV. Therefore, the use of static nuclear densities is acceptable for not-too-light nuclear cores.

Figure 3: Binding energies (left) and widths (right) of $1s_\eta$ nuclear states across the periodic table calculated self-consistently using the M1, M2 and GW subthreshold $\eta N$ scattering amplitudes within a dynamical RMF scheme, see text.

Inspection of the l.h.s. of Fig. 3 reveals that for each of the three input $\eta N$ amplitude models the binding energy increases with $A$ and appears to saturate for large values of $A$. As in Fig. 2 here too the hierarchy of the three curves reflects the strength of the Re $f_{\eta N}(\sqrt{s})$ input in the subthreshold region, with threshold values listed in Table 1. The M1 and M2 amplitudes are too weak to produce a $1s_\eta$ bound state in $^{12}\text{C}$, with the onset of binding for the weaker M1 amplitude deferred to around $^{40}\text{Ca}$. Of our three representative amplitudes, M1 is the closest one on shell to the Haider-Liu standard amplitude [2, 3, 4] which was used by these authors to argue for $^{12}\text{C}$ as the approximate onset of $\eta$-nuclear binding. In contrast, Re $f_{\eta N}(\sqrt{s})$ of the GW model is sufficiently strong to bind the $1s_\eta$ state in $^{12}\text{C}$ and in lighter core nuclei, in spite of the suppression it undergoes here by forming its in-medium version and dealing with its energy dependence. The GW amplitude model even admits a $1s_\eta$ bound state in $^4\text{He}$ with as low a binding energy as 1.2 MeV and a width of 2.3 MeV, both calculated using a static $^4\text{He}$ density.
Inspection of the r.h.s. of Fig. 3 reveals a trend for the three curves of calculated widths which is opposite to that observed on the l.h.s. for the calculated binding energies. Here, the GW model produces relatively small widths of order 4 MeV uniformly across the periodic table, whereas M1 and M2 give larger widths, particularly M1 with widths of order 20 MeV. This reflects partly the energy dependence of $\text{Im} \ f_{\eta N}(\sqrt{s})$ in the subthreshold region, which is quite distinct in each one of the three amplitude models, and partly the difference in the in-medium renormalization arising from the $\text{Re} \ f_{\eta N}(\sqrt{s})$ input. The latter point is readily understood by noting in Fig. 2 that the largest values of subthreshold downward energy shift are due to the GW subthreshold amplitude. This causes a particularly large reduction in the strength of the $\text{Im} \ f_{\eta N}(\sqrt{s})$ input for the GW amplitude model.

Table 2: Static calculations of $1s_\eta$ binding energy ($B$) and width ($\Gamma$) in $^{25}$Mg, using three $\eta N$ amplitude models (M1, M2, GW) with (YES) and without (NO) medium corrections from Eq. (3), for several procedures of treating the energy dependence of $f_{\eta N}$. Energies and widths are given in MeV.

| Eq. (3) subthreshold | M1 | M2 | GW |
|----------------------|----|----|----|
| B$_{\eta}$ | $\Gamma_{\eta}$ | B$_{\eta}$ | $\Gamma_{\eta}$ | B$_{\eta}$ | $\Gamma_{\eta}$ |
| NO | $\delta \sqrt{s} = 0$ | 3.2 | 37.4 | 17.3 | 37.8 | 81.8 | 62.7 |
| NO | $\delta \sqrt{s} = -30$ | – | – | 3.0 | 11.2 | 31.2 | 10.0 |
| NO | $\delta \sqrt{s}$ Eq. (5) | – | – | 3.2 | 10.6 | 23.8 | 7.4 |
| YES | $\delta \sqrt{s} = 0$ | 4.3 | 23.8 | 11.6 | 18.9 | 33.3 | 14.0 |
| YES | $\delta \sqrt{s} = -B_{\eta}$ | 3.8 | 21.7 | 8.3 | 13.2 | 19.4 | 5.8 |
| YES | $\delta \sqrt{s}$ Eq. (5) | – | – | 2.5 | 7.4 | 14.8 | 3.9 |

Focusing on a given core nucleus, we show in Table 2 results of static-density calculations in models M1, M2 and GW of the $1s_\eta$ state in $^{25}$Mg with (YES) and without (NO) employing the in-medium modification of Eq. (3). The first row in each of the YES and NO groups lists results of using threshold amplitudes: $F_{\eta N}(\sqrt{s_{th}}, \rho)$ for YES and $\tilde{f}_{\eta N}(\sqrt{s_{th}})$ for NO. The self-consistency requirement imposed by Eq. (4) is used and comparison is made within each group with another procedure applied in previous studies to incorporate energy dependence. These are (i) a fixed 30 MeV downward shift applied to the free-space $\eta N$ amplitude $f_{\eta N}(\sqrt{s})$ by Haider and Liu.
(HL) \[4\]; and (ii) shifting down self-consistently the energy argument of the in-medium $\eta N$ amplitude $F_{\eta N}(\sqrt{s}, \rho)$ by the resultant $B_\eta$, as implemented for example by García-Recio et al. (GR) \[16\].

The HL procedure is compared with ours in the second and third rows of the first (NO) group, using free-space amplitudes. Both do not produce binding for the weakest amplitude M1 and practically agree for M2, while disagreeing significantly for the strongest GW amplitude. By comparing each of these rows with the first row where free-space threshold amplitudes are used, it is seen the effects of accounting for energy dependence are substantial in both. The GR procedure is compared with ours in the second and third rows of the second (YES) group, using in-medium amplitudes. The GR procedure is found to give higher binding energies and widths than ours for all amplitude models tested here, particularly for the weaker M amplitudes where it is the only one that produces a $1s_\eta$ bound state for M1. The overall effects in this group of accounting for energy dependence with respect to using in-medium threshold amplitudes (first row of the YES group), however, are less substantial than in the preceding group, particularly for the M amplitudes.

Of the three models used by us with in-medium amplitudes in Table 2 (last line), only GW provides $B_\eta$ which is comparable with

$$B^{\exp}(^{25}\eta Mg) = 13.1 \pm 1.6 \text{ MeV}, \quad \Gamma^{\exp}(^{25}\eta Mg) = 10.2 \pm 3.0 \text{ MeV}, \quad (6)$$

deduced from the following $^{25}\eta Mg$ interpretation of a peak reported by the COSY-GEM Collaboration \[24\]:

$$p + ^{27}\text{Al} \rightarrow ^{25}\eta Mg + ^{3}\text{He} \rightarrow (\pi^- + p) + X, \quad (7)$$

with a decay induced by $\eta + n \rightarrow \pi^- + p$. Hence, if this peak assignment to a $1s_\eta$ state is correct\[2\] then the underlying threshold value $\text{Re } a_{\eta N}$ must be rather large, close to 1 fm. Other procedures listed in Table 2 for treating the subthreshold $\eta N$ energy dependence require considerably smaller values of $\text{Re } a_{\eta N}$. Finally, the relatively small value of width $\Gamma$ produced in the GW model should not be viewed as too restrictive since the total width must be larger than given in these models, owing to true $\eta NN$ absorption and two-pion production $\eta N \rightarrow \pi\pi N$ processes that are not accounted for by the models considered in the present work.

\[2\]This has been contested recently by Haider and Liu who offered a different interpretation of the reported peak \[28\].
4. Conclusions

In this work we have demonstrated the importance of, as well as the subtleties involved in constructing self-consistent $\eta$-nucleus optical potentials that incorporate the strong subthreshold energy dependence of the underlying $\eta N$ scattering amplitude. Of the three $\eta N$ amplitude models studied here self-consistently, even the relatively weak attraction in model M1 with a threshold value $\text{Re} a_{\eta N} \approx 0.2 \text{ fm}$ requires going down to about 35 MeV below threshold, as shown in Fig. 2 in order to calculate reliably the $\eta$-nuclear optical potential $V_\eta(\rho)$ at central nuclear densities. This downward energy shift exceeds by far the downward shifts $-B_\eta$, with $B_\eta \lesssim 20 \text{ MeV}$ encountered in the self-consistent calculations of García-Recio et al. [16]. The relatively large downward energy shifts in the present approach together with the rapid decrease of the free-space and in-medium $\eta N$ amplitudes lead to smaller than ever binding energies and widths with respect to those calculated in comparable models [4, 16, 17]. Thus, $^{12}\eta C$ bound states are unlikely in models with threshold values $\text{Re} a_{\eta N} \lesssim 0.5 \text{ fm}$, and as large a value as $\text{Re} a_{\eta N} \approx 0.9 \text{ fm}$ is required to reproduce the $^{25}\eta \text{Mg}$ bound-state hint from the recent COSY-GEM experiment [24]. Complementarily, for as sufficiently large values of $\text{Re} a_{\eta N}$ as provided by the GW amplitude model, the calculated widths come out smaller than in other calculations.

A value of $\text{Re} a_{\eta N} \sim 0.9 \text{ fm}$ is likely to yield a near-threshold $^4\eta \text{He}$ bound state, as found here using the GW amplitude model, but it is short of binding $^3\eta \text{He}$. Stretching the limits of optical potential usage down to these light systems is of course questionable, and corresponding few-body calculations are highly needed to resolve such issues. Nevertheless, if one applies our subthreshold self-consistency scheme to $\eta^3\text{He}$ low-energy scattering, then a very large imaginary part that might indicate a nearby virtual state is found for the $\eta^3\text{He}$ cm scattering amplitude when using the GW amplitude model. This large imaginary part might be associated with the strong final-state interaction effects observed for the $\eta^3\text{He}$ system [11]. In contrast to previous estimates that assigned a value of $\text{Re} a_{\eta N} \approx 0.5 \text{ fm}$ to describe such occurrence [29], in our self-consistent calculations it requires substantially larger values, more likely around 0.9 fm.

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