Microcausality violation of scalar field on noncommutative spacetime with the time-space noncommutativity

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Abstract

Quantum field theories on noncommutative spacetime have many different properties from those on commutative spacetime. In this paper, we study the microcausality of free scalar field on noncommutative spacetime. We expand the scalar field in the form of usual Lorentz invariant measure in noncommutative spacetime. Then we calculate the expectation values of the Moyal commutators for the quadratic operators, such as $\varphi(x)\star\varphi(x)$, $\pi(x,t)\star\pi(x,t)$, $\partial_i\varphi(x,t)\star\partial_i\varphi(x,t)$, and $\partial_i\varphi(x,t)\star\pi(x,t)$. We obtain that for space-space noncommutativity, i.e., $\theta^{0i} = 0$ in $\theta^{\mu\nu}$, microcausality of free scalar field on noncommutative spacetime is satisfied. For time-space noncommutativity, i.e., $\theta^{0i} \neq 0$ in $\theta^{\mu\nu}$, microcausality of free scalar field on noncommutative spacetime is violated.

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1 Introduction

Many years ago, the model of quantized and noncommutative spacetime was first constructed by Snyder [1]. The mathematical development on noncommutative geometry was carried out by Connes [2]. In Ref. [3], Doplicher et al. proposed the uncertainty relations for the measurement of spacetime coordinates from the Heisenberg’s uncertainty principle and Einstein’s gravitational equations. In recent years, spacetime noncommutativity was discovered again in superstring theories [4]. It has resulted a lot of researches on noncommutative field theories (NCFTs) [5,6].

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The coordinates of noncommutative spacetime become noncommutative operators. They satisfy the commutation relations

\[ [x^\mu, x^\nu] = i \theta^{\mu\nu}, \]  

(1.1)

where \( \theta^{\mu\nu} \) is a constant real antisymmetric matrix with the dimension of square of length. Field theories on noncommutative spacetime can be formulated through the Weyl-Moyal correspondence \([5,6]\). Every field \( \phi(x) \) defined on noncommutative spacetime is mapped to its Weyl symbol \( \phi(x) \) that defined on the corresponding commutative spacetime. At the same time, the products of field functions are replaced by the Moyal star-products of their Weyl symbols

\[ \phi(x) \psi(x) \rightarrow \phi(x) \star \psi(x), \]  

(1.2)

where the Moyal star-product is defined by

\[
\phi(x) \star \psi(x) = e^{i \frac{1}{2} \sum_{\alpha, \beta} \theta^{\alpha\beta} \partial_{\alpha} \phi(x + \alpha) \psi(x + \beta)} |_{\alpha = \beta = 0} \\
= \phi(x) \psi(x) + \sum_{n=1}^{\infty} \left( \frac{i}{2} \right)^n \frac{1}{n!} \theta^{\mu_1 \nu_1} \cdots \theta^{\mu_n \nu_n} \partial_{\mu_1} \cdots \partial_{\mu_n} \phi(x) \partial_{\nu_1} \cdots \partial_{\nu_n} \psi(x).
\]  

(1.3)

Thus, the Lagrangians of field theories on noncommutative spacetime can be formulated directly through replacing the products by the Moyal star-products in the Lagrangians of field theories on commutative spacetime. And the commutators of coordinate operators of Eq. (1.1) are equivalently replaced by the Moyal star-product commutators of the noncommutative coordinates

\[ [x^\mu, x^\nu]_* = i \theta^{\mu\nu}. \]  

(1.4)

Quantum field theories on noncommutative spacetime have many different properties from those on commutative spacetime \([5,6]\). In this paper, we study the microcausality of scalar field on noncommutative spacetime. In fact, such a problem need to be studied from several different aspects.

First, such a problem is related with the Lorentz invariance problem of NCFTs. It is well known that NCFTs violate Lorentz invariance because of their theoretical constructions. As pointed out in Ref. [7], there are two kinds of Lorentz transformations for NCFTs. One is the observer Lorentz transformation. In this case, one can suppose \( \theta^{\mu\nu} \) carries Lorentz indices, which means that \( \theta^{\mu\nu} \) transforms covariantly under the transformations of observer’s frame. This will leave the physics unchanged because both field operators and \( \theta^{\mu\nu} \) transform covariantly. The other is the particle Lorentz transformation. In this case, Lorentz transformations for field operators are taken in a fixed observer frame. Because \( \theta^{\mu\nu} \) are not fields, they are only certain parameters in the theory, particle Lorentz transformation leave \( \theta^{\mu\nu} \) unchanged and hence modify the physics.

Therefore NCFTs at least violate particle Lorentz transformation invariance. In Ref. [8], the authors have proposed that NCFTs satisfy the twisted Poincaré invariance. In Ref. [9],
the authors have pointed out that for Lorentz transformations that leave $\theta^{\mu\nu}$ unchanged, such as the particle Lorentz transformation mentioned above, the $SO(3, 1)$ Lorentz group is broken down to a subgroup $SO(1, 1) \times SO(2)$. Therefore in Ref. [9], the authors constructed the $SO(1, 1) \times SO(2)$ invariant spectral measure for the expansions of quantum fields on noncommutative spacetime. They have proposed that microcausality condition for quantum fields on noncommutative spacetime should be formulated with respect to the two dimensional light-wedge. Therefore the traditional concept of microcausality is violated generally for quantum fields on noncommutative spacetime [9-12].

On the other hand, corresponding to the observer Lorentz transformations for NCFTs, one should expand quantum fields on noncommutative spacetime in the form of their usual Lorentz invariant measures in order to study their microcausality properties. In fact, for such a problem, some results were given in Ref. [13]. Recent results were obtained by Greenberg [14]. In Ref. [14], Greenberg have obtained that for scalar field on noncommutative spacetime, $[: \varphi(x) \star \varphi(x) ; : ; \varphi(y) \star \varphi(y) :]$, is nonzero for a spacelike interval for the case $\theta^{0i} \neq 0$ and $\theta^{0i} = 0$. Thus microcausality is violated for scalar field on noncommutative spacetime generally. In this paper we will study this problem further. We obtain the results different from those of Ref. [14].

Besides, we need to investigate microcausality properties of interacting fields on noncommutative spacetime. Recently in Ref. [15], Haque and Joglekar have obtained that for the Yukawa interaction in noncommutative spacetime, causality is violated for both $\theta^{0i} \neq 0$ and $\theta^{0i} = 0$. In addition we can see from Refs. [16-19] that quantum and classical nonlinear perturbations have infinite propagation speed in noncommutative spacetime. While these phenomena should also have relations with the violation of causality of quantum fields on noncommutative spacetime.

Because in the following we need to calculate the commutation relations of two field operators defined at two different spacetime points, we need to generalize Eq. (1.3) for two fields defined at two different spacetime points. In fact, such a formula has been given in Ref. [6]. It reads

\[
\phi(x_1) \star \psi(x_2) = e^{i\theta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x^\nu} \phi(x_1 + \alpha) \psi(x_2 + \beta)|_{\alpha = \beta = 0}
\]

\[
= \phi(x_1)\psi(x_2) + \sum_{n=1}^{\infty} \left( \frac{i}{2} \right)^n \frac{1}{n!} \theta^{\mu_1\nu_1} \cdots \theta^{\mu_n\nu_n} \partial_{\mu_1} \cdots \partial_{\mu_n} \phi(x_1) \partial_{\nu_1} \cdots \partial_{\nu_n} \psi(x_2) .
\]

(1.5)

This formula can be derived through the standard Weyl symbol method as demonstrated in Ref. [6], supposing that we generalize the spacetime commutation relations (1.1) to two different spacetime points

\[
[x^\mu_1, x^\mu_2] = i\theta^{\mu\nu} .
\]

(1.6)

A simplified derivation for Eqs. (1.5) and (1.6) has been given by Chaichian et al. in Ref. [20] using the concept of quantum shift. Consequently, the Moyal star-product commutators
for the noncommutative coordinates of Eq. (1.4) become

\[ [x_1^\mu, x_2^\nu]_\star = i\theta^{\mu\nu}, \]  

(1.7)

which can be resulted from Eq. (1.5). Equation (1.5) has been used many where in the literature [14,15,21].

The content of this paper is organized as follows. In Sec. 2, we analyze the measurement of quantum fields on noncommutative spacetime and the criterion of microcausality violation. In Sec. 3, we calculate the vacuum state expectation value for the Moyal commutator \([\varphi(x) \star \varphi(x), \varphi(y) \star \varphi(y)]_\star\) and obtain that microcausality is violated for the case \(\theta^{0i} \neq 0\) of spacetime noncommutativity. In Sec. 4, we calculate the non-vacuum state expectation value for the Moyal commutator \([\varphi(x) \star \varphi(x), \varphi(y) \star \varphi(y)]_\star\) and obtain that microcausality is violated for the case \(\theta^{0i} \neq 0\) of spacetime noncommutativity. In Sec. 5, we calculate the expectation values for some other quadratic operators of scalar field on noncommutative spacetime and obtain the similar result as that of Sec. 3 and Sec. 4. In Sec. 6, we discuss some of the problems.

2 The criterion of microcausality violation

In this section, we first analyze the measurement of quantum fields on noncommutative spacetime and the criterion of microcausality violation.

For quantum field theories, as well as quantum mechanics, what the observer measures are certain expectation values. We suppose that there are two observers A and B situated at spacetime points \(x\) and \(y\), they proceed a measurement separately on the state vector \(|\Psi\rangle\) for the locally observable quantity \(\mathcal{O}(x)\) in the same occasion. However the time \(x_0\) may not equal to the time \(y_0\) generally. For the observer A, the state vector \(|\Psi\rangle\) has been affected by the measurement of the observer B at the spacetime point \(y\). Or we can say the observer B’s observation instrument has taken an action on the state vector \(|\Psi\rangle\). The state vector has become \(\mathcal{O}(y)|\Psi\rangle\). When the observer A takes his or her measurement on the state vector, his or her observation instrument will act on the state vector \(\mathcal{O}(y)|\Psi\rangle\) again. These two sequent actions should be represented by the product operation of the operators. However because now the spacetime is noncommutative, the product operation should be the Moyal star-product, while not the ordinary product. Or we regard that in noncommutative spacetime, the basic product operation is the Moyal star-product. Thus what the measuring result the observer A obtained from his or her instrument is \(\langle\Psi|\mathcal{O}(x) \star \mathcal{O}(y)|\Psi\rangle\). Similarly for the observer B, the state vector \(|\Psi\rangle\) has been affected by the action of the observer A’s instrument at the spacetime point \(x\). The state vector becomes \(\mathcal{O}(x)|\Psi\rangle\). What the measuring result the observer B obtained from his or her instrument is \(\langle\Psi|\mathcal{O}(y) \star \mathcal{O}(x)|\Psi\rangle\).

Supposing that microcausality is satisfied for NCFTs, this means that there do not exist the physical information and interaction with the transmission speed faster than the speed
of light. Thus when the spacetime interval between $x$ and $y$ is spacelike, the affection of the observer B’s measurement or the action of observer B’s instrument at spacetime point $y$ on the state vector $|\Psi\rangle$ has not propagated to the spacetime point $x$ when the observer A takes his or her measurement on the state vector $|\Psi\rangle$ at the spacetime point $x$. These two physical measurements do not interfere with each other. For the observer A, the state vector is still $|\Psi\rangle$, while not $O(y)|\Psi\rangle$. Therefore the measuring result what the observer A obtained is just $\langle\Psi|O(x)|\Psi\rangle$. Thus from the sense of experimental measuring discussed above, we have

$$\langle\Psi|O(x) \star O(y)|\Psi\rangle = \langle\Psi|O(x)|\Psi\rangle \quad \text{for} \quad (x-y)^2 < 0 . \quad (2.1)$$

The same reason as the observer A, the measuring result what the observer B obtained at the spacetime point $y$ is just $\langle\Psi|O(y)|\Psi\rangle$. Thus from the sense of experimental measuring discussed above, we have

$$\langle\Psi|O(y) \star O(x)|\Psi\rangle = \langle\Psi|O(y)|\Psi\rangle \quad \text{for} \quad (x-y)^2 < 0 . \quad (2.2)$$

We can suppose that the state vector $|\Psi\rangle$ is in the momentum eigenstate, thus it is also in the energy eigenstate. At the same time, we can suppose that the state vector $|\Psi\rangle$ is in the Heisenberg picture, therefore it does not rely on the spacetime coordinates. From the Heisenberg relations and the translation transformation, we have

$$\langle\Psi|O(x)|\Psi\rangle = \langle\Psi|O(y)|\Psi\rangle . \quad (2.3)$$

Therefore the condition

$$\langle\Psi|[O(x),O(y)]_\star|\Psi\rangle = \langle\Psi|[O(x) \star O(y)]|\Psi\rangle - \langle\Psi|[O(y) \star O(x)]|\Psi\rangle = 0 \quad \text{for} \quad (x-y)^2 < 0 \quad (2.4)$$

should be satisfied for a NCFT to satisfy the microcausality.

If microcausality is violated for a NCFT, then there may exist the physical information and interaction with the transmission speed faster than the speed of light. For the two measurements of the observer A and observer B located at $x$ and $y$ separated by a spacelike interval, the affection of the observer B’s measurement at spacetime point $y$ on the state

$$O(y) = \exp(ia_\mu P^\mu)O(x) \exp(-ia_\mu P^\mu) .$$

Because $a_\mu$ now is a constant four-vector, from Eq. (1.3) we can also write the above expression as

$$O(y) = \exp(ia_\mu P^\mu) \star O(x) \star \exp(-ia_\mu P^\mu) .$$

We use $P^\mu$ to represent the eigenvalues of the energy-momentum of the state vector $|\Psi\rangle$. Therefore we obtain

$$\langle\Psi|O(y)|\Psi\rangle = \langle\Psi|\exp(ia_\mu P^\mu) \star O(x) \star \exp(-ia_\mu P^\mu)|\Psi\rangle = \langle\Psi|\exp(ia_\mu P^\mu) \star O(x) \star \exp(-ia_\mu P^\mu)|\Psi\rangle$$

$$= \exp(ia_\mu P^\mu) \star \langle\Psi|O(x)|\Psi\rangle \star \exp(-ia_\mu P^\mu) = \exp(ia_\mu P^\mu) \langle\Psi|O(x)|\Psi\rangle \exp(-ia_\mu P^\mu) = \langle\Psi|O(x)|\Psi\rangle .$$

Therefore Eq. (2.3) is satisfied.

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1We note $y-x=a$. From the Heisenberg relations and the translation transformation, we have
vector $|\Psi\rangle$ will propagate to the spacetime point $x$ when the observer A takes his or her measurement on the state vector $|\Psi\rangle$ at the spacetime point $x$, and the affection of the observer A’s measurement at spacetime point $x$ on the state vector $|\Psi\rangle$ will propagate to the spacetime point $y$ when the observer B takes his or her measurement on the state vector $|\Psi\rangle$ at the spacetime point $y$. These two physical measurements will interfere with each other. For such case, Eqs. (2.1) and (2.2) cannot be satisfied, while we still have $\langle \Psi | O(x) | \Psi \rangle = \langle \Psi | O(y) | \Psi \rangle$ as that of Eq. (2.3). Therefore generally we have

$$\langle \Psi | [O(x), O(y)]_{\star} | \Psi \rangle \neq 0 \quad \text{for} \quad (x - y)^2 < 0 \quad (2.5)$$

for a NCFT to violate the microcausality. Thus we can judge whether the microcausality is maintained or violated for a NCFT according to Eq. (2.5).

Now we suppose that $O_1(x)$ and $O_2(y)$ are two different observable field operators, $x$ and $y$ are separated by a spacelike interval, two observers A and B situate at $x$ and $y$, and microcausality is satisfied for the field theory on noncommutative spacetime. Supposing that the observers A and B proceed a measurement separately on the state vector $|\Psi\rangle$ for the locally observable quantities $O_1$ and $O_2$ at $x$ and $y$ respectively, then from the above analysis, from the sense of experimental measuring, we have for the observer A

$$\langle \Psi | O_1(x) \star O_2(y) | \Psi \rangle = \langle \Psi | O_1(x) | \Psi \rangle \quad \text{for} \quad (x - y)^2 < 0 . \quad (2.6)$$

And we have for the observer B

$$\langle \Psi | O_2(y) \star O_1(x) | \Psi \rangle = \langle \Psi | O_2(y) | \Psi \rangle \quad \text{for} \quad (x - y)^2 < 0 . \quad (2.7)$$

Because now $O_1(x)$ and $O_2(y)$ are two different operators representing two different physical observable quantities, we have generally

$$\langle \Psi | O_1(x) | \Psi \rangle \neq \langle \Psi | O_2(y) | \Psi \rangle . \quad (2.8)$$

Therefore from Eqs. (2.6)-(2.8) we have

$$\langle \Psi | [O_1(x), O_2(y)]_{\star} | \Psi \rangle \neq 0 \quad \text{for} \quad (x - y)^2 < 0 \quad (2.9)$$

generally, even if $x$ and $y$ are separated by a spacelike interval, and the field theory satisfies the microcausality. Therefore we cannot deduce that a NCFT violates microcausality from Eq. (2.9) from the expectation values of the Moyal commutator of two different operators. Thus, in order to judge whether a NCFT violates microcausality, we need to analyze the expectation values of the Moyal commutator of the same operator as that of Eq. (2.5).

For quantum field theories on ordinary commutative spacetime, the analysis for the criterion of microcausality violation is similar to the above, except that we replace the Moyal star-products by the ordinary products in the above equations. Usually for quantum field theories on ordinary commutative spacetime, equation (2.4) can be simplified to

$$[O(x), O(y)] = 0 \quad \text{for} \quad (x - y)^2 < 0 \quad (2.10)$$
for the satisfying of the microcausality. For example for scalar field, the fundamental commutator is
\[ [\varphi(x), \varphi(y)] = i\Delta(x - y) . \] (2.11)

It is a \( c \)-number function. Therefore we have
\[ \langle \Psi | [\varphi(x), \varphi(y)] | \Psi \rangle = [\varphi(x), \varphi(y)] . \] (2.12)

Because the commutator \([\varphi(x), \varphi(y)]\) is zero for a spacelike interval, \( \langle \Psi | [\varphi(x), \varphi(y)] | \Psi \rangle \) is also zero for a spacelike interval. Similarly for the quadratic operator \( \varphi(x)\varphi(x) \), we have
\[ [\varphi(x)\varphi(x), \varphi(y)\varphi(y)] = \varphi(x)[\varphi(x), \varphi(y)]\varphi(y) + \varphi(y)[\varphi(x), \varphi(y)]\varphi(x) + \varphi(x)[\varphi(x), \varphi(y)]\varphi(y) + \varphi(y)[\varphi(x), \varphi(y)]\varphi(x) . \] (2.13)

Although the result of Eq. (2.13) is not a \( c \)-number function, from the fundamental commutator of Eq. (2.11) we know that \([\varphi(x)\varphi(x), \varphi(y)\varphi(y)]\) is zero for a spacelike interval. Therefore \( \langle \Psi | [\varphi(x)\varphi(x), \varphi(y)\varphi(y)] | \Psi \rangle \) is also zero for a spacelike interval. Hence for the quadratic operator \( \varphi(x)\varphi(x) \) of the scalar field theory on ordinary commutative spacetime, microcausality is satisfied.

For quantum field theories on noncommutative spacetime, because of the noncommutativity of spacetime coordinates, the Moyal commutators are not \( c \)-number functions generally, as can be seen in Ref. [22] for noncommutative scalar field and Dirac field. Thus we cannot move away the state vectors in Eqs. (2.4) and (2.5) for the criterion of the microcausality violation for NCFTs generally. We need to evaluate their expectation values.

### 3 Vacuum state expectation values

For scalar field on noncommutative spacetime, the Lagrangian for free field is given by
\[ \mathcal{L} = \frac{1}{2} \partial^\mu \varphi \star \partial_\mu \varphi - \frac{1}{2} m^2 \varphi \star \varphi . \] (3.1)

Its Hamiltonian density and momentum density are given by \([23,24]\)
\[ \mathcal{H}(\pi, \varphi) = \frac{1}{2} \left[ \pi(x, t) \star \pi(x, t) + \partial_i \varphi(x, t) \star \partial_i \varphi(x, t) + m^2 \varphi(x, t) \star \varphi(x, t) \right] \] (3.2)
and
\[ \mathcal{P}^i(\pi, \varphi) = -\frac{1}{2} \left[ \pi(x, t) \star \partial_i \varphi(x, t) + \partial_i \varphi(x, t) \star \pi(x, t) \right] , \] (3.3)

where \( \pi(x, t) = \hat{\pi}(x, t) \). In accordance with the observer Lorentz transformation invariance of NCFTs, which means that \( \Theta_{\mu\nu} \) is a tensor constant, it transforms covariantly under the
transformations of observer’s frame, we can expand the free scalar field on noncommutative spacetime according to its usual Lorentz invariant spectral measure \([25,26]\). Thus we have
\[
\varphi(x,t) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_k}} [a(k)e^{ik\cdot x - i\omega t} + a^\dagger(k)e^{-ik\cdot x + i\omega t}]
\]
(3.4)
for the Fourier expansion of the scalar field.

In Eq. (3.4), we take the spacetime coordinates to be noncommutative. They satisfy the Moyal star-product commutation relations (1.4) and (1.7). The commutation relations for the creation and annihilation operators are given by
\[
[a(k), a^\dagger(k')] = \delta^3(k - k') ,
[a(k), a(k)] = 0 ,
[a^\dagger(k), a^\dagger(k')] = 0 .
\]
(3.5)
They are the same as those of the commutative spacetime case. The commutator of the Moyal star-product for the scalar field is defined to be
\[
[\varphi(x), \varphi(y)]_* = \varphi(x) \star \varphi(y) - \varphi(y) \star \varphi(x) .
\]
(3.6)
In Ref. [22], the vacuum state and non-vacuum state expectation values for the Moyal commutator (3.6) were calculated. According to the result of Ref. [22], microcausality is satisfied for the linear operator \(\varphi(x)\) of the free scalar field on noncommutative spacetime. However, we need to study whether microcausality is satisfied or not for quadratic operators of free scalar field on noncommutative spacetime. In this section, we study the quadratic operator \(\varphi(x) \star \varphi(x)\), to see whether it satisfies the microcausality or not. Because we consider that in noncommutative spacetime, the basic product operation is the Moyal star-product, we need to study the commutators of Moyal star-products.

The Moyal commutator of the field operators \(\varphi(x) \star \varphi(x)\) and \(\varphi(y) \star \varphi(y)\) is given by
\[
[\varphi(x) \star \varphi(x), \varphi(y) \star \varphi(y)]_* = \varphi(x) \star [\varphi(x), \varphi(y)]_* + [\varphi(x), \varphi(y)]_* \star \varphi(x) + [\varphi(x), \varphi(y)]_* \star [\varphi(x), \varphi(y)]_* ,
= \varphi(x) \star [\varphi(x), \varphi(y)]_* \star \varphi(y) + [\varphi(x), \varphi(y)]_* \star \varphi(x) \star [\varphi(x), \varphi(y)]_*
= \varphi(x) \star [\varphi(x), \varphi(y)]_* \star \varphi(y) + [\varphi(x), \varphi(y)]_* \star \varphi(x) \star [\varphi(x), \varphi(y)]_*
= \varphi(x) \star \varphi(y) \star \varphi(x) - \varphi(y) \star \varphi(x) \star \varphi(y)
\]
(3.7)
Because the fundamental Moyal commutator \([\varphi(x), \varphi(y)]_*\) is not a \(c\)-number function, as shown in Ref. [22], we need to calculate the expectation value for Eq. (3.7) in order to investigate whether \(\varphi(x) \star \varphi(x)\) satisfies the microcausality condition. As analyzed in Sec. 2, this is also the demand of physical measurements. Therefore we need to calculate the function
\[
A(x,y) = \langle \Psi \left| [\varphi(x) \star \varphi(x) \star \varphi(y) \star \varphi(y) \star \varphi(x)]_* \right| \Psi \rangle ,
\]
(3.8)
where $|\Psi\rangle$ is a state vector of scalar field quantum system. In Eq. (3.8), we have adopted
the normal orderings for the field operators $\varphi(x) \star \varphi(x)$ and $\varphi(y) \star \varphi(y)$. This means that
an infinite vacuum energy has been eliminated in the corresponding commutative spacetime
field theory. To be the limit of physical measurements, we take the state vector $|\Psi\rangle$ in Eq.
(3.8) to be the vacuum state $|0\rangle$. Therefore in this section, we will first calculate the function
\[ A_0(x, y) = \langle 0 | : \varphi(x) \star \varphi(x) : :: \varphi(y) \star \varphi(y) : | 0 \rangle . \]  
(3.9)
For the non-vacuum state expectation value of Eq. (3.8), we will analyze it in Sec. 4.

We decompose $\varphi(x)$ into the creation (negative frequency) and annihilation (positive frequency) part
\[ \varphi(x) = \varphi^-(x) + \varphi^+(x) , \]  
(3.10)
where
\[ \varphi^-(x) = \int \frac{d^3k}{\sqrt{(2\pi)^32\omega_k}} a^\dagger(k)e^{-ik \cdot x + i\omega t} = \int \frac{d^3k}{\sqrt{(2\pi)^32\omega_k}} a^\dagger(k)e^{ikx} \]  
(3.11)
and
\[ \varphi^+(x) = \int \frac{d^3k}{\sqrt{(2\pi)^32\omega_k}} a(k)e^{ik \cdot x - i\omega t} = \int \frac{d^3k}{\sqrt{(2\pi)^32\omega_k}} a(k)e^{-ikx} . \]  
(3.12)
Here we define $kx = k_\mu x^\mu$. From Eq. (3.10) we have
\[ \varphi(x) \star \varphi(x) = \varphi^-(x) \star \varphi^-(x) + \varphi^+(x) \star \varphi^-(x) + \varphi^-(x) \star \varphi^+(x) + \varphi^+(x) \star \varphi^+(x) . \]  
(13.3)
The normal ordering of the operator $\varphi(x) \star \varphi(x)$ is given by
\[ : \varphi(x) \star \varphi(x) : = \varphi^-(x) \star \varphi^-(x) + 2\varphi^-(x) \star \varphi^+(x) + \varphi^+(x) \star \varphi^+(x) . \]  
(13.4)
Here we have made a simplified manipulation for the normal ordering of the Moyal star-product operator $\varphi^+(x) \star \varphi^-(x)$. This is because the result of the Moyal star-product between two functions is related with the order of the two functions. In the Fourier integral representation, we can see that $\varphi^-(x) \star \varphi^+(x)$ will have an additional phase factor $e^{ik \cdot ki'}$ relative to $\varphi^+(x) \star \varphi^-(x)$. However in Eq. (3.14) we have ignored such a difference. The reason is that the terms that contain $\varphi^-(x) \star \varphi^+(x)$ in the expansion of Eq. (3.9) will contribute zero when we evaluate the vacuum expectation values, as can be seen in the following. Thus we can ignore such a difference equivalently for convenience.

To expand $: \varphi(x) \star \varphi(x) : \star : \varphi(y) \star \varphi(y) :$, we obtain
\[ : \varphi(x) \star \varphi(x) : \star : \varphi(y) \star \varphi(y) : = \varphi^-(x) \star \varphi^-(x) \star \varphi^-(y) \star \varphi^-(y) + \varphi^-(x) \star \varphi^-(x) \star 2\varphi^-(y) \star \varphi^+(y) + \varphi^-(x) \star \varphi^+(y) \star \varphi^+(y) + 2\varphi^-(x) \star \varphi^+(y) \star \varphi^-(y) + 2\varphi^-(x) \star 2\varphi^-(y) \star \varphi^+(y) + 2\varphi^-(x) \star \varphi^+(y) \star \varphi^+(y) + \varphi^+(x) \star \varphi^+(x) \star \varphi^+(y) + \varphi^+(x) \star \varphi^+(x) \star 2\varphi^-(y) \star \varphi^+(y) + \varphi^+(x) \star \varphi^+(x) \star 2\varphi^-(y) \star \varphi^+(y) + \varphi^+(x) \star 2\varphi^-(y) \star \varphi^+(y) \star \varphi^+(y) + \varphi^+(x) \star \varphi^+(x) \star \varphi^+(y) \star \varphi^+(y) . \]  
(13.5)
From Eq. (3.15), we can see clearly that the non-zero contributions to the vacuum expectation value of $\varphi(x) \star \varphi(x) : \star : \varphi(y) \star \varphi(y) :$ come from the terms which the most right hand side components of the product operators are the negative frequency operators, and at the same time for these terms the number of the positive frequency component operators are equal to the number of the negative frequency component operators in the total product operators. Therefore we can see that there is only one such term $\varphi^{(+)}(x) \star \varphi^{(+)}(x) \star \varphi^{(-)}(y) \star \varphi^{(-)}(y)$ which will contribute to non-zero vacuum expectation value. Thus we have

$$\langle 0 | : \varphi(x) \star \varphi(x) : \star : \varphi(y) \star \varphi(y) : | 0 \rangle = \langle 0 | \varphi^{(+)}(x) \star \varphi^{(+)}(x) \star \varphi^{(-)}(y) \star \varphi^{(-)}(y) | 0 \rangle . \quad (3.16)$$

Similarly, the non-zero contribution to the vacuum expectation value of $\varphi(x) \star \varphi(x) : \star : \varphi(x) \star \varphi(x) :$ only comes from the part $\varphi^{(+)}(y) \star \varphi^{(+)}(y) \star \varphi^{(-)}(x) \star \varphi^{(-)}(x)$. We have

$$\langle 0 | : \varphi(y) \star \varphi(y) : \star : \varphi(x) \star \varphi(x) : | 0 \rangle = \langle 0 | \varphi^{(+)}(y) \star \varphi^{(+)}(y) \star \varphi^{(-)}(x) \star \varphi^{(-)}(x) | 0 \rangle . \quad (3.17)$$

If we do not use the normal orderings for the operators $\varphi(x) \star \varphi(x)$ and $\varphi(y) \star \varphi(y)$ as that of Eq. (3.7), then in the calculation of the vacuum expectation value for Eq. (3.7), we need to consider the additional four terms which will contribute non-zero results

$$\varphi^{(+)}(x) \star \varphi^{(-)}(x) \star \varphi^{(+)}(y) \star \varphi^{(-)}(y) - \varphi^{(+)}(y) \star \varphi^{(-)}(y) \star \varphi^{(+)}(x) \star \varphi^{(-)}(x) ,$$

$$\varphi^{(-)}(x) \star \varphi^{(+)}(x) \star \varphi^{(+)}(y) \star \varphi^{(-)}(y) - \varphi^{(-)}(y) \star \varphi^{(+)}(y) \star \varphi^{(+)}(x) \star \varphi^{(-)}(x) .$$

However we can obtain that the total vacuum expectation value of such four terms cancel at last. Thus to take the normal orderings for the operators $\varphi(x) \star \varphi(x)$ and $\varphi(y) \star \varphi(y)$ has simplified the calculation.

Through calculation we obtain

$$\langle 0 | : \varphi(x) \star \varphi(x) : \star : \varphi(y) \star \varphi(y) : | 0 \rangle = \langle 0 | \varphi^{(+)}(x) \star \varphi^{(+)}(x) \star \varphi^{(-)}(y) \star \varphi^{(-)}(y) | 0 \rangle - \langle 0 | \varphi^{(+)}(y) \star \varphi^{(+)}(y) \star \varphi^{(-)}(x) \star \varphi^{(-)}(x) | 0 \rangle$$

$$= \int \frac{d^3k_1}{(2\pi)^3 \omega_1} \int \frac{d^3k_2}{(2\pi)^3 \omega_2} \left[ e^{-ik_2x} \times e^{-ik_1x} \times e^{ik_2y} \times e^{ik_1y} + e^{-ik_1x} \times e^{-ik_2x} \times e^{ik_2y} \times e^{ik_1y} \right.$$

$$\left. - e^{-ik_2y} \times e^{-ik_1y} \times e^{ik_2x} \times e^{ik_1x} - e^{-ik_1y} \times e^{-ik_2y} \times e^{ik_2x} \times e^{ik_1x} \right]$$

$$= \int \frac{d^3k_1}{(2\pi)^3 \omega_1} \int \frac{d^3k_2}{(2\pi)^3 \omega_2} \left[ e^{-ik_2\times k_1} + 1 \right] \left[ e^{-i(k_2+k_1)x+y(i(k_2+k_1))} - e^{-i(k_2+k_1)y+i(k_2+k_1)x} \right]$$

$$= \int \frac{d^3k_1}{(2\pi)^3 \omega_1} \int \frac{d^3k_2}{(2\pi)^3 \omega_2} (-2i) \left[ 1 + e^{i(k_1\times k_2)} \right] \sin(k_1 + k_2)(x-y) . \quad (3.18)$$

In Eq. (3.18), the first term of the third line means that two scalar field quanta $|k_1\rangle$ and $|k_2\rangle$ are generated at spacetime point $y$ and annihilated at spacetime point $x$. Because Moyal star-products depend on the orders of the functions, there is the second term of the third line that is responsible to the first term of the second line. Similarly, the two terms of the
fourth line mean that two scalar field quanta $|k_1\rangle$ and $|k_2\rangle$ are generated at spacetime point $x$ and annihilated at spacetime point $y$. In Eq. (3.18), $\int \frac{d^3k_1}{(2\pi)^32\omega_1} \int \frac{d^3k_2}{(2\pi)^32\omega_2}$ is the Lorentz invariant volume element, $k_1 \times k_2 = k_{1\mu}\theta^{\mu\nu}k_{2\nu}$, and $(k_1+k_2)(x-y) = (k_1+k_2)_{\mu}(x-y)^{\mu}$. The total expression is Lorentz invariant if we suppose that $\theta^{\mu\nu}$ is a second-order antisymmetric tensor. In the above calculation, we have used Eq. (1.5) for the Moyal star-product of two functions defined at two different spacetime points.

We need to analyze whether the expression of Eq. (3.18) disappears for a spacelike interval. This can be seen through the vacuum expectation value of the equal-time commutator. Thus to take $x_0 = y_0$ in Eq. (3.18), we have

\begin{equation}
\langle 0 | [\varphi(x,t) \star \varphi(x,t) :, \varphi(y,t) \star \varphi(y,t) :]_s | 0 \rangle = \int \frac{d^3k_1}{(2\pi)^32\omega_1} \int \frac{d^3k_2}{(2\pi)^32\omega_2} (2i) [1 + e^{ik_1 \times k_2}] \sin(k_1 + k_2) \cdot (x - y) . \tag{3.19}
\end{equation}

We expand $k_1 \times k_2$ as

\begin{equation}
k_1 \times k_2 = (k_{10} - k_1) \times (k_{20} - k_2) = k_{1i}\theta^{ij}k_{2j} - k_{120}\theta^{0i}k_{2i} . \tag{3.20}
\end{equation}

For the case $\theta^{0i} = 0$ of the spacetime noncommutativity, we have

\begin{equation}
\langle 0 | [\varphi(x,t) \star \varphi(x,t) :, \varphi(y,t) \star \varphi(y,t) :]_s | 0 \rangle = \int \frac{d^3k_1}{(2\pi)^32\omega_1} \int \frac{d^3k_2}{(2\pi)^32\omega_2} (2i) [1 + e^{ik_1 \times \theta^{ij}k_{2j}}] \sin(k_1 + k_2) \cdot (x - y) . \tag{3.21}
\end{equation}

We can see that in Eq. (3.21), the integrand is an odd function to the arguments $(k_1,k_2)$. The integrand changes its sign when the arguments $(k_1,k_2)$ change to $(-k_1,-k_2)$, while the integral measure does not change. The integral space is symmetrical to the integral arguments $(k_1,k_2)$ and $(-k_1,-k_2)$. This makes the total integral of Eq. (3.21) to be zero. We have seen that the total expression of Eq. (3.18) is Lorentz invariant in the sense $\theta^{\mu\nu}$ being a second-order antisymmetric tensor. This means that for an arbitrary spacelike interval of $x$ and $y$, the integral of Eq. (3.18) vanishes. Thus we have

\begin{equation}A_0(x,y) = \langle 0 | [\varphi(x) \star \varphi(x) :, \varphi(y) \star \varphi(y) :]_s | 0 \rangle = 0 \quad \text{for} \quad (x-y)^2 < 0 \quad \text{when} \quad \theta^{0i} = 0 . \tag{3.22}\end{equation}

Therefore microcausality for the quadratic operator $\varphi(x) \star \varphi(x)$ of free scalar field is maintained for the case $\theta^{0i} = 0$ of spacetime noncommutativity.

For the case $\theta^{0i} \neq 0$ of spacetime noncommutativity, we write $e^{ik_1 \times k_2}$ as

\begin{equation}
e^{ik_1 \times k_2} = e^{i(k_{1i}\theta^{ij}k_{2j} - k_{10}\theta^{0i}k_{2i} - k_{100}k_{2j})} = e^{ik_{1i}\theta^{ij}k_{2j}} \left[ \cos(k_{1i}\theta^{0j}k_{2j}) + k_{10}\theta^{0i}k_{2i} \right] - i \sin(k_{1i}\theta^{0j}k_{2j}) . \tag{3.23}\end{equation}

From Eq. (3.19), we have

\begin{equation}\langle 0 | [\varphi(x,t) \star \varphi(x,t) :, \varphi(y,t) \star \varphi(y,t) :]_s | 0 \rangle \end{equation}
In order to evaluate Eq. (4.1), we first need to define the state vector field quantum system, the total energy is always finite. Because the occupation numbers
\[ N \]
take values of finitely integral numbers, the occupation numbers
\[ N \]
vanish. Thus we have
\[ A_0(x, y) = \langle 0 | [\varphi(x) \ast \varphi(x) : : \varphi(y) \ast \varphi(y) :] | 0 \rangle \neq 0 \quad \text{for} \quad (x - y)^2 < 0 \quad \text{when} \quad \theta^{0i} \neq 0. \] (3.26)

This means that microcausality is violated for the quadratic operator \( \varphi(x) \ast \varphi(x) : \) of the free scalar field for the case \( \theta^{0i} \neq 0 \) of spacetime noncommutativity. In the limit \( \theta^{0i} = 0 \), the integral of Eq. (3.25) vanishes and hence \( A_0(x, y) \) vanishes for \( (x - y)^2 < 0 \), which is in accordance with Eq. (3.22).

4 Non-vacuum state expectation values

In Sec. 3, we have calculated the expectation value \( A_0(x, y) \) for the Moyal commutator of the quadratic operator \( \varphi(x) \ast \varphi(x) : \). In this section we will analyze the non-vacuum state expectation value \( A(x, y) \) for the Moyal commutator of the quadratic operator \( \varphi(x) \ast \varphi(x) : \). As defined in Eq. (3.8), we write
\[ A(x, y) = \langle \Psi | [\varphi(x) \ast \varphi(x) : : \varphi(y) \ast \varphi(y) :] | \Psi \rangle. \] (4.1)

In order to evaluate Eq. (4.1), we first need to define the state vector |\( \Psi \rangle \) for a scalar field quantum system.

Supposing that the state vector |\( \Psi \rangle \) is in the occupation eigenstate, we can write
\[ |\Psi \rangle = |N_{k_1} N_{k_2} \cdots N_{k_i} \cdots, 0 \rangle, \] (4.2)

where \( N_{k_i} \) represents the occupation number of the momentum \( k_i \). For an arbitrary actual field quantum system, the total energy is always finite. Because the occupation numbers \( N_{k_i} \) take values of finitely integral numbers, the occupation numbers \( N_{k_i} \) should only be nonzero
on finite number separate momentums $k_i$. Otherwise, if $N_{k_i}$ take nonzero values on infinite number separate momentums $k_i$, or on a continuous interval of the momentum, the total energy of the scalar field quantum system will be infinite. While for a actual field quantum system, the total energy is always finite. These properties for the state vector $|\Psi\rangle$ are very useful for the following calculations. In Eq. (4.2), we use 0 to represent that the occupation numbers are all zero on the other momentums except for $k_i$. The state vector $|\Psi\rangle$ has the following properties [27]:

$$\langle N_{k_1}N_{k_2} \cdots N_{k_i} \cdots | N_{k_1}N_{k_2} \cdots N_{k_i} \cdots \rangle = 1 , \quad (4.3)$$

$$\sum_{N_{k_1}N_{k_2} \cdots} | N_{k_1}N_{k_2} \cdots N_{k_i} \cdots \rangle \langle N_{k_1}N_{k_2} \cdots N_{k_i} \cdots | = 1 , \quad (4.4)$$

$$a(k_i)|N_{k_1}N_{k_2} \cdots N_{k_i} \cdots \rangle = \sqrt{N_{k_i}}|N_{k_1}N_{k_2} \cdots (N_{k_i} - 1) \cdots \rangle , \quad (4.5)$$

$$a^\dagger(k_i)|N_{k_1}N_{k_2} \cdots N_{k_i} \cdots \rangle = \sqrt{N_{k_i} + 1}|N_{k_1}N_{k_2} \cdots (N_{k_i} + 1) \cdots \rangle . \quad (4.6)$$

In Eqs. (4.3)-(4.6), we have omitted the notation 0 of Eq. (4.2) in the state vectors for convenience. Equation (4.4) is the completeness expression. Thus Eq. (4.2) can represent an arbitrary scalar field quantum system.

In Ref. [22], we have obtained that the non-vacuum state expectation values for the Moyal commutator $[\varphi(x), \varphi(y)]_*$ of the scalar field and Moyal anticommutator $\{\psi_\alpha(x), \psi_\beta(x')\}_*$ of the Dirac field are just equal to their vacuum state expectation values. If such a property is still held for the quadratic operator $:\varphi(x) \star \varphi(y):$ of scalar field studied in this paper, we can obtain a direct answer for the function $A(x,y)$ of Eq. (4.1). As can be seen in the following, such a property is really held for the quadratic operator $:\varphi(x) \star \varphi(x):$. However we need to verify this point.

For the convenience of the analyzing, we decompose the state vector of Eq. (4.2) into two parts

$$|\Psi\rangle = |(k_1)(k_2) \cdots (k_i) \cdots 0\rangle + |N_{k_1}N_{k_2} \cdots N_{k_i} \cdots \rangle . \quad (4.7)$$

In Eq. (4.7), we use $|(k_1)(k_2) \cdots (k_i) \cdots 0\rangle$ to represent that the state is on the vacuum, while the finite number separate momentums $k_i$ are eliminated from the arguments of $k$ for such a vacuum state. And we use $|N_{k_1}N_{k_2} \cdots N_{k_i} \cdots \rangle$ to represent a non-vacuum state which the arguments only take the finite number separate values $k_i$. On these separate $k_i$, the occupation numbers are $N_{k_i}$, which take values of finite integrals. Then for Eq. (4.1), we can write

$$\langle \Psi | \{ :\varphi(x) \star \varphi(x) : : \varphi(y) \star \varphi(y) : \}_* | \Psi \rangle$$

$$= \langle (k_1)(k_2) \cdots (k_i) \cdots 0 | :\varphi(x) \star \varphi(x) : : \varphi(y) \star \varphi(y) : \}_* | (k_1)(k_2) \cdots (k_i) \cdots 0 \rangle$$

$$+ \langle N_{k_1}N_{k_2} \cdots N_{k_i} \cdots | :\varphi(x) \star \varphi(x) : : \varphi(y) \star \varphi(y) : \}_* N_{k_1}N_{k_2} \cdots N_{k_i} \cdots \rangle$$

$$+ \langle (k_1)(k_2) \cdots (k_i) \cdots 0 | :\varphi(x) \star \varphi(x) : : \varphi(y) \star \varphi(y) : \}_* | N_{k_1}N_{k_2} \cdots N_{k_i} \cdots \rangle$$

$$+ \langle N_{k_1}N_{k_2} \cdots N_{k_i} \cdots | :\varphi(x) \star \varphi(x) : : \varphi(y) \star \varphi(y) : \}_* (k_1)(k_2) \cdots (k_i) \cdots 0 \rangle . \quad (4.8)$$
We can see that the last two terms of Eq. (4.8) are all zero, because their arguments of the momentum $k$ will not match with each other for the bras and kets. Hence we have

$$
\langle \Psi | : \varphi(x) \star \varphi(x) \cdots : \varphi(y) \star \varphi(y) : | A | \rangle = 
\langle (k_1)(k_2) \cdots (k_i) \cdots 0 | : \varphi(x) \star \varphi(x) \cdots : \varphi(y) \star \varphi(y) : | (k_1)(k_2) \cdots (k_i) \cdots 0 \rangle
$$

+ \langle N_{k_1}N_{k_2} \cdots N_{k_i} \cdots | : \varphi(x) \star \varphi(x) \cdots : \varphi(y) \star \varphi(y) : | N_{k_1}N_{k_2} \cdots N_{k_i} \cdots \rangle .

(4.9)

For the first term of Eq. (4.9), its calculation is just like that of Eq. (3.18), except that the separate momentums $k_i$ should be eliminated from the final integral measure. Thus according to the result of Eq. (3.18), we obtain

$$
\langle (k_1)(k_2) \cdots (k_i) \cdots 0 | : \varphi(x) \star \varphi(x) \cdots : \varphi(y) \star \varphi(y) : | (k_1)(k_2) \cdots (k_i) \cdots 0 \rangle
$$

= \int \frac{d^3k_a}{(2\pi)^32\omega_a} \int \frac{d^3k_b}{(2\pi)^32\omega_b} (-2i) \left[ 1 + e^{ik_a \times k_b} \right] \sin(k_a + k_b)(x - y) .

(4.10)

In Eq. (4.10), we use $k_i$ eliminated to represent that in the integral for $k_a$ and $k_b$, a set of finite number separate points $k_i$ is eliminated from the total integral measure of $k_a$ and $k_b$ respectively. We can write Eq. (4.10) equivalently in the form

$$
\langle (k_1)(k_2) \cdots (k_i) \cdots 0 | : \varphi(x) \star \varphi(x) \cdots : \varphi(y) \star \varphi(y) : | (k_1)(k_2) \cdots (k_i) \cdots 0 \rangle
$$

= \int \frac{d^3k_a}{(2\pi)^32\omega_a} \int \frac{d^3k_b}{(2\pi)^32\omega_b} (-2i) \left[ 1 + e^{ik_a \times k_b} \right] \sin(k_a + k_b)(x - y)

- \int \frac{d^3k_a}{(2\pi)^32\omega_a} \int \frac{d^3k_b}{(2\pi)^32\omega_b} (-2i) \left[ 1 + e^{ik_a \times k_b} \right] \sin(k_a + k_b)(x - y) .

(4.11)

In Eq. (4.11), we use (only on $k_i$) to represent that in the second integral, the integral is only taken on a set of finite number separate points $k_i$ for $k_a$ and $k_b$ respectively. Because the integrand is a bounded function, while the integral measure is zero for the second integral, according to the theory of integration (for example see Ref. [28]), we obtain that the second part of Eq. (4.11) is zero. Therefore we obtain

$$
\langle (k_1)(k_2) \cdots (k_i) \cdots 0 | : \varphi(x) \star \varphi(x) \cdots : \varphi(y) \star \varphi(y) : | (k_1)(k_2) \cdots (k_i) \cdots 0 \rangle
$$

= \int \frac{d^3k_a}{(2\pi)^32\omega_a} \int \frac{d^3k_b}{(2\pi)^32\omega_b} (-2i) \left[ 1 + e^{ik_a \times k_b} \right] \sin(k_a + k_b)(x - y) .

(4.12)

Or we can write

$$
\langle (k_1)(k_2) \cdots (k_i) \cdots 0 | : \varphi(x) \star \varphi(x) \cdots : \varphi(y) \star \varphi(y) : | (k_1)(k_2) \cdots (k_i) \cdots 0 \rangle
$$

= \langle 0 | : \varphi(x) \star \varphi(x) \cdots : \varphi(y) \star \varphi(y) : | 0 \rangle = A_0(x, y) .

(4.13)
For the second term of (4.9), we have
\[
\langle N_{k_1}N_{k_2} \cdots N_{k_i}, \cdots | : \varphi(x) \star \varphi(x) : \cdots : \varphi(y) \star \varphi(y) : \cdots | N_{k_1}N_{k_2} \cdots N_{k_i}, \cdots \rangle \\
= \langle N_{k_1}N_{k_2} \cdots N_{k_i}, \cdots | : \varphi(x) \star \varphi(x) : \cdots : \varphi(y) \star \varphi(y) : \cdots | N_{k_1}N_{k_2} \cdots N_{k_i}, \cdots \rangle \\
- \langle N_{k_1}N_{k_2} \cdots N_{k_i}, \cdots | : \varphi(x) \star \varphi(x) : \cdots : \varphi(x) \star \varphi(x) : \cdots | N_{k_1}N_{k_2} \cdots N_{k_i}, \cdots \rangle .
\]
(4.14)

We first analyze the first term of Eq. (4.14). We can find from Eq. (3.15) that the nonzero contributions not only come from the operator \(\varphi^{(+)}(x) \star \varphi^{(+)}(x) \star \varphi^{(-)}(y) \star \varphi^{(-)}(y)\) like that of Eq. (3.16), but also come from the operators \(\varphi^{(-)}(x) \star \varphi^{(-)}(x) \star \varphi^{(+)}(y) \star \varphi^{(+)}(y)\) and \(2\varphi^{(-)}(x) \star \varphi^{(+)}(x) \star 2\varphi^{(-)}(y) \star \varphi^{(+)}(y)\) of Eq. (3.15) which have the equal numbers of negative frequency and positive frequency components. The situation for the second term of Eq. (4.14) is similar. For the total integrand generated from these operators, we can note it as \(G_{\text{vac}}(k_a, k_b, x, y)\). We can write Eq. (4.14) as
\[
\langle N_{k_1}N_{k_2} \cdots N_{k_i}, \cdots | : \varphi(x) \star \varphi(x) : \cdots : \varphi(y) \star \varphi(y) : \cdots | N_{k_1}N_{k_2} \cdots N_{k_i}, \cdots \rangle \\
= \int_{\text{only on } k_i} \frac{d^3k_a}{(2\pi)^32\omega_a} \int_{\text{only on } k_i} \frac{d^3k_b}{(2\pi)^32\omega_b} G(k_a, k_b, x, y) .
\]
(4.15)

The integrand \(G_{\text{vac}}(k_a, k_b, x, y)\) is not equal to the integrand of Eq. (4.12) generally. However we need not to obtain its explicit form in fact. This is because its contribution to the integral of Eq. (4.15) is zero. The reason is that the integral of \(G_{\text{vac}}(k_a, k_b, x, y)\) is only taken on finite number separate points of \(k_a\) and \(k_b\), their total integral measure is zero. While the integrand \(G_{\text{vac}}(k_a, k_b, x, y)\) should be a bounded function. This makes the total integral of Eq. (4.15) be zero according to the theory of integration (for example see Ref. [28]). Therefore we obtain
\[
\langle N_{k_1}N_{k_2} \cdots N_{k_i}, \cdots | : \varphi(x) \star \varphi(x) : \cdots : \varphi(y) \star \varphi(y) : \cdots | N_{k_1}N_{k_2} \cdots N_{k_i}, \cdots \rangle = 0 .
\]
(4.16)

To combine the results of Eqs. (4.12) and (4.16) together, we obtain
\[
\langle \Psi | : \varphi(x) \star \varphi(x) : \cdots : \varphi(y) \star \varphi(y) : \cdots | \Psi \rangle \\
= \int \frac{d^3k_1}{(2\pi)^32\omega_1} \int \frac{d^3k_2}{(2\pi)^32\omega_2} (-2i) [1 + e^{ik_1 \times k_2}] \sin(k_1 + k_2)(x - y) .
\]
(4.17)

Or we can write
\[
\langle \Psi | : \varphi(x) \star \varphi(x) : \cdots : \varphi(y) \star \varphi(y) : \cdots | \Psi \rangle = \langle 0 | : \varphi(x) \star \varphi(x) : \cdots : \varphi(y) \star \varphi(y) : \cdots | 0 \rangle .
\]
(4.18)

Therefore we obtain \(A(x, y) = A_0(x, y)\), which is a universal function for an arbitrary state vector of Eq. (4.2). Thus for the quadratic operator : \(\varphi(x) \star \varphi(x)\) : of free scalar field, its microcausality under the measurements of non-vacuum states is equivalent to the measurement of the vacuum state. Thus from the result of Sec. 3, we have
\[
A(x, y) = \langle \Psi | : \varphi(x) \star \varphi(x) : \cdots : \varphi(y) \star \varphi(y) : \cdots | \Psi \rangle = 0 \quad \text{for} \quad (x - y)^2 < 0 \quad \text{when} \quad \theta^{0i} = 0 ,
\]
(4.19)
which means that microcausality is maintained for the quadratic operator: \( \varphi(x) \ast \varphi(x) \): of free scalar field for the case \( \theta^{0i} = 0 \) of spacetime noncommutativity. For the case \( \theta^{0i} \neq 0 \), for the equal-time commutator, like that of Eq. (3.25), we have

\[
\langle \Psi | \langle \varphi(x,t) \ast \varphi(x,t) : ; \varphi(y,t) \ast \varphi(y,t) : , | \Psi \rangle = \int \frac{d^3k_1}{(2\pi)^32\omega_1} \int \frac{d^3k_2}{(2\pi)^32\omega_2} 2e^{ik_1,\theta^{0i}k_2} \sin(k_1,\theta^{0i}k_2_0 + k_10\theta^{0i}k_2_0) \sin(k_1 + k_2) \cdot (x - y).
\]

(4.20)

It is not zero because the integrand is an even function. If we suppose that \( \theta^{\mu\nu} \) is a second-order antisymmetric tensor, the total expression of Eq. (4.17) is Lorentz invariant, we have

\[
A(x,y) = \langle \Psi | \langle \varphi(x) \ast \varphi(x) : ; \varphi(y) \ast \varphi(y) : , | \Psi \rangle \neq 0 \quad \text{for} \quad (x - y)^2 < 0 \quad \text{when} \quad \theta^{0i} \neq 0,
\]

(4.21)

which means that microcausality is violated for the quadratic operator: \( \varphi(x) \ast \varphi(x) \): of free scalar field for the case \( \theta^{0i} \neq 0 \) of spacetime noncommutativity. In Eq. (4.20), \( \langle \Psi | \langle \varphi(x,t) \ast \varphi(x,t) : ; \varphi(y,t) \ast \varphi(y,t) : , | \Psi \rangle \) is also a universal function for an arbitrary state vector of Eq. (4.2). In the limit \( \theta^{0i} = 0 \), the integral of Eq. (4.20) vanishes and hence \( A(x,y) \) vanishes for \( (x - y)^2 < 0 \), which is in accordance with Eq. (4.19).

To summarize, we have obtained in Sec. 3 and this section that microcausality for the quadratic operator \( \varphi(x,t) \ast \varphi(x,t) \) of free scalar field on noncommutative spacetime is satisfied for the case \( \theta^{0i} = 0 \) of spacetime noncommutativity, and is violated for the case \( \theta^{0i} \neq 0 \) of spacetime noncommutativity.

5 Some other quadratic operators of scalar field on noncommutative spacetime

In this section, we analyze the microcausality of some other quadratic operators of free scalar field on noncommutative spacetime. These quadratic operators include \( \pi(x,t) \ast \pi(x,t), \partial_i \varphi(x,t) \ast \partial_i \varphi(x,t), \) and \( \partial_i \varphi(x,t) \ast \pi(x,t), \) which are composition parts of the energy-momentum density of Eqs. (3.2) and (3.3). We first calculate their vacuum expectation values. To be brief, some processes are omitted. From the result of Eq. (3.18), we can obtain

\[
\langle 0 | \langle \pi(x) \ast \pi(x) : ; \pi(y) \ast \pi(y) : , | 0 \rangle = \int \frac{d^3k_1}{(2\pi)^32\omega_1} \int \frac{d^3k_2}{(2\pi)^32\omega_2} (-2i\omega_1^2 \omega_2^2) \left[ 1 + e^{ik_1 \times k_2} \right] \sin(k_1 + k_2)(x - y),
\]

(5.1)

\[
\langle 0 | \langle \partial_i \varphi(x) \ast \partial_i \varphi(x) : ; \partial_i \varphi(y) \ast \partial_i \varphi(y) : , | 0 \rangle = \int \frac{d^3k_1}{(2\pi)^32\omega_1} \int \frac{d^3k_2}{(2\pi)^32\omega_2} (-2ik_1^2 k_2^2) \left[ 1 + e^{ik_1 \times k_2} \right] \sin(k_1 + k_2)(x - y),
\]

(5.2)
\begin{align}
\langle 0| : \partial_i \varphi(x) \ast \pi(x) ; : \partial_j \varphi(y) \ast \pi(y) : | 0 \rangle &= \int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3} \frac{(-2i k_2 \omega_1)}{(2\pi)^3} \left[ k_1 \omega_2 + k_2 \omega_1 e^{ik_1 \times k_2} \right] \sin(k_1 + k_2)(x - y) .
\end{align}

We can see that the differences of these integrals to Eq. (3.18) lie in the coefficient factors, such as \( \omega_1^2, k_1^2, k_2^2, k_1 k_2 \omega_1, \) and \( k_2 \omega_2 \). These coefficient factors make these integrals be not Lorentz invariant functions. The reason lies in the fact that these operators are not Lorentz invariant themselves.

We can also obtain the vacuum expectation values of the equal-time Moyal commutators for these operators respectively. They are given by

\begin{align}
\langle 0| : \partial_i \varphi(x, t) \ast \pi(x, t) ; : \partial_j \varphi(y, t) \ast \pi(y, t) : | 0 \rangle &= \int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3} \frac{1 + e^{ik_1 \times k_2}}{(2\pi)^3} \sin(k_1 + k_2) \cdot (x - y) ,
\end{align}

\begin{align}
\langle 0| : \partial_i \varphi(x, t) \ast \pi(x, t) ; : \partial_j \varphi(y, t) \ast \pi(y, t) : | 0 \rangle &= \int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3} \frac{2i k_2 \omega_1}{(2\pi)^3} \left[ k_1 \omega_2 + k_2 \omega_1 e^{ik_1 \times k_2} \right] \sin(k_1 + k_2) \cdot (x - y) ,
\end{align}

\begin{align}
\langle 0| : \partial_i \varphi(x, t) \ast \pi(x, t) ; : \partial_j \varphi(y, t) \ast \pi(y, t) : | 0 \rangle &= \int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3} \frac{2i k_2 \omega_1}{(2\pi)^3} \left[ k_1 \omega_2 + k_2 \omega_1 e^{ik_1 \times k_2} \right] \sin(k_1 + k_2) \cdot (x - y) .
\end{align}

As in Sec. 3, we write \( k_1 \times k_2 \) as

\begin{align}
k_1 \times k_2 &= (k_{10}, -k_1) \times (k_{20}, -k_2) = k_{11} \theta^i j k_{2j} - k_{1j} \theta^0 j k_{20} - k_{10} \theta^0 i k_{2i} .
\end{align}

For the case \( \theta^0 i = 0 \) of spacetime noncommutativity, we have

\begin{align}
\langle 0| : \pi(x, t) \ast \pi(x, t) ; : \pi(y, t) \ast \pi(y, t) : | 0 \rangle &= \int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3} \frac{(2i k_2 \omega_1)}{(2\pi)^3} \left[ 1 + e^{ik_1 \theta^i j k_{2j}} \right] \sin(k_1 + k_2) \cdot (x - y) ,
\end{align}

\begin{align}
\langle 0| : \partial_i \varphi(x, t) \ast \pi(x, t) ; : \partial_i \varphi(y, t) \ast \pi(y, t) : | 0 \rangle &= \int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3} \frac{(2i k_2 \omega_1)}{(2\pi)^3} \left[ 1 + e^{ik_1 \theta^i j k_{2j}} \right] \sin(k_1 + k_2) \cdot (x - y) ,
\end{align}

\begin{align}
\langle 0| : \partial_i \varphi(x, t) \ast \pi(x, t) ; : \partial_i \varphi(y, t) \ast \pi(y, t) : | 0 \rangle &= \int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3} \frac{(2i k_2 \omega_1)}{(2\pi)^3} \left[ 1 + e^{ik_1 \theta^i j k_{2j}} \right] \sin(k_1 + k_2) \cdot (x - y) .
\end{align}
We can see that in Eqs. (5.8)-(5.10), the integrands are all odd functions to the arguments \((k_1, k_2)\). The integrands change their signs when the arguments \((k_1, k_2)\) change to \((-k_1, -k_2)\), while the integral measures do not change. The integral spaces are symmetrical to the integral arguments \((k_1, k_2)\) and \((-k_1, -k_2)\). Thus the total integrals of Eqs. (5.8)-(5.10) are all zero. Although the integrals of Eqs. (5.1)-(5.3) are not Lorentz invariant functions, this does not mean that microcausality will violate for an arbitrary spacelike interval of \(x\) and \(y\), because the Lorentz un-invariance of these integrals comes from the fact that the considered operators are not Lorentz invariant themselves. Thus from these results we can draw conclusion that microcausality is maintained for these quadratic operators for the case \(\theta^{0i} = 0\) of spacetime noncommutativity.

For the case \(\theta^{0i} \neq 0\) of spacetime noncommutativity, we write \(e^{ik_1 \times k_2}\) as

\[
e^{ik_1 \times k_2} = e^{i(k_1^0 \theta^{i0} k_{2j} - k_1^0 \theta^{i0} k_{2j} + k_0 \theta^{0i} k_{2i})}
= e^{ik_1^0 \theta^{i0} k_{2j}} \left[ \cos(k_1^0 \theta^{i0} k_{20} + k_0 \theta^{0i} k_{2j}) - i \sin(k_1^0 \theta^{i0} k_{20} + k_0 \theta^{0i} k_{2j}) \right]. \tag{5.11}
\]

To substitute Eq. (5.11) in Eqs. (5.4)-(5.6), and to remove away the odd function parts in the integrands which will contribute zero to the whole integrals, we obtain

\[
\langle 0| [\pi(x, t) \ast \pi(x, t) \ast \pi(y, t) \ast \pi(y, t)]_{*} |0 \rangle
= \int \frac{d^3k_1}{(2\pi)^3 2\omega_1} \int \frac{d^3k_2}{(2\pi)^3 2\omega_2} (2\omega_1^2 \omega_2^2) e^{ik_1 \theta^{i0} k_{2j}} \sin(k_1^0 \theta^{i0} k_{20} + k_0 \theta^{0i} k_{2j}) \sin(k_1 + k_2) \cdot (x - y),
\tag{5.12}
\]

\[
\langle 0| [\partial_i \varphi(x, t) \ast \partial_i \varphi(x, t) \ast \partial_i \varphi(y, t) \ast \partial_i \varphi(y, t)]_{*} |0 \rangle
= \int \frac{d^3k_1}{(2\pi)^3 2\omega_1} \int \frac{d^3k_2}{(2\pi)^3 2\omega_2} (2k_1^2 \omega_1^2) e^{ik_1 \theta^{i0} k_{2j}} \sin(k_1^0 \theta^{i0} k_{20} + k_0 \theta^{0i} k_{2j}) \sin(k_1 + k_2) \cdot (x - y),
\tag{5.13}
\]

\[
\langle 0| [\partial_i \varphi(x, t) \ast \pi(x, t) \ast \partial_i \varphi(y, t) \ast \pi(y, t)]_{*} |0 \rangle
= \int \frac{d^3k_1}{(2\pi)^3 2\omega_1} \int \frac{d^3k_2}{(2\pi)^3 2\omega_2} (2k_2^2 \omega_1^2) e^{ik_1 \theta^{i0} k_{2j}} \sin(k_1^0 \theta^{i0} k_{20} + k_0 \theta^{0i} k_{2j}) \sin(k_1 + k_2) \cdot (x - y).
\tag{5.14}
\]

In Eqs. (5.12)-(5.14), the integrands are all even functions. This makes the whole integrals not vanished. As pointed out above, the integrals of Eqs. (5.1)-(5.3) are not Lorentz invariant functions, while the Lorentz un-invariance of these integrals comes from the fact that the considered operators are not Lorentz invariant themselves. Thus for an arbitrary spacelike interval of \(x\) and \(y\), equations (5.1)-(5.3) do not vanish either generally when \(\theta^{0i} \neq 0\). This means that microcausality is violated for these quadratic operators for the case \(\theta^{0i} \neq 0\) of
spacetime noncommutativity. In the limit $\theta^{\nu i} = 0$, the integrals of Eqs. (5.12)-(5.14) vanish, which is in accordance with Eqs. (5.8)-(5.10). To be complete, we also need to analyze the non-vacuum state expectation values for these operators. However, we can obtain that for the non-vacuum state expectation values of these operators, similar as that of Sec. 4 for the operator $\varphi(x, t) \star \varphi(x, t)$, their results are the same as the corresponding vacuum state expectation values. The conclusions of microcausality for the non-vacuum state expectation values of these operators are also the same as that for the vacuum state expectation values of these operators. Therefore we need not to write down them explicitly.

To summarize, we have obtained the conclusion that the microcausality properties for the quadratic operators $\pi(x, t) \star \pi(x, t)$, $\partial_i \varphi(x, t) \star \partial_i \varphi(x, t)$, and $\partial_i \varphi(x, t) \star \pi(x, t)$ are the same as that for the quadratic operator $\varphi(x, t) \star \varphi(x, t)$. For the case $\theta^{\nu i} = 0$ of spacetime noncommutativity, they satisfy the microcausality. For the case $\theta^{\nu i} \neq 0$ of spacetime noncommutativity, they violate the microcausality.

6 Conclusion

For the microcausality problem of quantum fields on noncommutative spacetime, it need to be studied from several different aspects. Because NCFTs cannot maintain particle Lorentz transformation invariance [7], no matter whether $\theta^{\mu \nu}$ is a tensor or not, it is necessary to investigate the properties of quantum fields on noncommutative spacetime with respect to a subgroup of the usual Lorentz group, which is the group $SO(1, 1) \times SO(2)$ that leaves $\theta^{\mu \nu}$ invariant [9]. Thus in this case, microcausality of quantum fields on noncommutative spacetime is formulated with respect to a two dimensional light-wedge [9]. In fact in this representation, the traditional meaning of microcausality is violated for quantum fields on noncommutative spacetime. Inside the two dimensional $SO(1, 1)$ light wedge, waves have infinite propagation speed.

On the other hand, to suppose that $\theta^{\mu \nu}$ is a Lorentz tensor, NCFTs will maintain the observer Lorentz transformation invariance [7]. Thus it is also necessary to study the microcausality properties of quantum fields on noncommutative spacetime with respect to their usual Lorentz invariant spectral measures. For such a case, some of the results were obtained in Refs. [13,14]. In Ref. [14], Greenberg have obtained the result that for scalar field on noncommutative spacetime, microcausality is violated generally, no matter whether $\theta^{\nu i} \neq 0$ or $\theta^{\nu i} = 0$. In this paper we have studied this problem further. We obtain the result different from that of Ref. [14]. We obtain the result that for free scalar field on noncommutative spacetime, microcausality is satisfied when $\theta^{\nu i} = 0$, and violated when $\theta^{\nu i} \neq 0$. The difference between the results of Ref. [14] and this paper lies in several reasons, such as the form of the Fourier expansion of scalar field and the criterion of microcausality violation. In Ref. [14], scalar field is expanded with respect to positive frequency (annihilation) part only. We consider that it is not the complete Fourier expansion for quantum fields. On the
other hand, in Ref. [14], some of the results are based on the commutators of two different operators. However as pointed out in Sec. 2 of this paper, the criterion of microcausality violation should be given by the commutators of the same operator, while not two different operators. These reasons result the different conclusions.

Besides, we need to investigate microcausality properties of interacting fields on noncommutative spacetime. Recently in Ref. [15], from the generalized Bogoliubov-Shirkov criterion of causality violation, Haque and Joglekar have obtained that for the Yukawa interaction in noncommutative spacetime, causality is violated for both $\theta^{0i} \neq 0$ and $\theta^{0i} = 0$. In addition we can see from Refs. [16-19] that quantum and classical nonlinear perturbations have infinite propagation speed in noncommutative spacetime. While these phenomena should also have relations with the violation of causality of quantum fields on noncommutative spacetime.

We need also to mention here that because what we have studied in this paper are the microcausality properties of free fields, we have only calculated the correlation functions, i.e., the expectation values of the Moyal commutators, while not analyzed their relations with the $S$-matrix amplitudes. On the other hand, because we consider that in noncommutative spacetime, the basic product operation is the Moyal star-product, we need to investigate the commutators of the Moyal star-products. Although what we have studied in this paper are the microcausality properties of free fields, the results are not trivial and obvious.

The problem of microcausality violation of quantum fields on noncommutative spacetime is very important, because it is related with the existence of infinite propagation speed of physical information as analyzed in Sec. 2 of this paper. Thus the violation of microcausality for quantum fields on noncommutative spacetime may have important applications in future information transmission.

As demonstrated in Refs. [29,30], unitarity of the $S$-matrix is lost for NCFTs with $\theta^{0i} \neq 0$. However, for NCFTs even if unitarity is satisfied, they still have many other properties different from field theories on commutative spacetime. Thus for the unitarity violation of NCFTs with $\theta^{0i} \neq 0$, it may need to be explained and understood from different approaches. Thus one may not exclude the case of time-space noncommutativity from their unitarity problems. In fact, it is more reasonable that time and space are in the equal position. They should be both quantized under a very small microscopic scale. On the other hand, for the unitarity problem of NCFTs with $\theta^{0i} \neq 0$, some authors have argued that they can be resolved through many different methods [31-33].

References

[1] H.S. Snyder, Phys. Rev. 71, 38 (1947).
[2] A. Connes, *Noncommutative geometry* (Academic Press, New York, 1994).
[3] S. Doplicher, K. Fredenhagen, and J.E. Roberts, Phys. Lett. B 331, 39 (1994);
   Commun. Math. Phys. 172, 187 (1995), hep-th/0303037.
[4] N. Seiberg and E. Witten, J. High Energy Phys. 09 (1999) 032, hep-th/9908142.
[5] M.R. Douglas and N.A. Nekrasov, Rev. Mod. Phys. 73, 977 (2001), hep-th/0106048.
[6] R.J. Szabo, Phys. Rep. 378, 207 (2003), hep-th/0109162.
[7] S.M. Carroll, J.A. Harvey, V.A. Kostelecky, C.D. Lane, and T. Okamoto, Phys. Rev.
   Lett. 87, 141601 (2001), hep-th/0105082.
[8] M. Chaichian, P.P. Kulish, K. Nishijima, and A. Tureanu, Phys. Lett. B 604, 98
   (2004), hep-th/0408069.
[9] L. Álvarez-Gaumé and M.A. Vázquez-Mozo, Nucl. Phys. B668, 293 (2003),
   hep-th/0305093.
[10] L. Alvarez-Gaumé, J.L.F. Barbón, and R. Zwicky, J. High Energy Phys. 05 (2001)
    057, hep-th/0103069.
[11] D.H.T. Franco and C.M.M. Polito, J. Math. Phys. 46, 083503 (2005),
    hep-th/0403028.
[12] C.S. Chu, K. Furuta, and T. Inami, Int. J. Mod. Phys. A 21, 67 (2006),
    hep-th/0502012.
[13] M. Chaichian, K. Nishijima, and A. Tureanu, Phys. Lett. B 568, 146 (2003),
    hep-th/0209008.
[14] O.W. Greenberg, Phys. Rev. D 73, 045014 (2006), hep-th/0508057.
[15] A. Haque and S.D. Joglekar, hep-th/0701171.
[16] S. Minwalla, M. Van Raamsdonk, and N. Seiberg, J. High Energy Phys. 02 (2000)
    020, hep-th/9912072.
[17] M. Van Raamsdonk, J. High Energy Phys. 11 (2001) 006, hep-th/0110093.
[18] A. Hashimoto and N. Itzhaki, Phys. Rev. D 63, 126004 (2001), hep-th/0012093.
[19] B. Durhuus and T. Jonsson, J. High Energy Phys. 10 (2004) 050, hep-th/0408190.
[20] M. Chaichian, K. Nishijima, and A. Tureanu, Phys. Lett. B 633, 129 (2006),
    hep-th/0511094.
[21] M. Chaichian, P. Prešnajder, and A. Tureanu, Phys. Rev. Lett. 94, 151602 (2006),
    hep-th/0409096.
[22] Z.Z. Ma, hep-th/0601046, hep-th/0601094.
[23] A. Gerhold, J. Grimstrup, H. Grosse, L. Popp, M. Schweda, and R. Wulkenhaar,
    hep-th/0512112.
[24] A. Micu and M.M. Sheikh-Jabbari, J. High Energy Phys. 01 (2001) 025,
    hep-th/0008057.
[25] C. Itzykson and J.-B. Zuber, *Quantum field theory* (McGraw-Hill Inc., 1980).
[26] J.D. Bjorken and S.D. Drell, *Relativistic quantum fields* (McGraw-Hill, 1965).
[27] L.D. Landau and E.M. Lifshitz, *Quantum mechanics* (Pergamon Press, 1977).
[28] G. de Barra, *Measure theory and integration* (Halsled Press, New York, 1981).
[29] J. Gomis and T. Mehen, Nucl. Phys. B591, 265 (2000), hep-th/0005129.
[30] A. Bassetto, F. Vian, L. Griguolo, and G. Nardelli, J. High Energy Phys. 07 (2001) 008, hep-th/0105257.
[31] D. Bahns, S. Doplicher, K. Fredenhagen, and G. Piacitelli, Phys. Lett. B 533, 178 (2002), hep-th/0201222.
[32] C.S. Chu, J. Lukierski, and W.J. Zakrzewski, Nucl. Phys. B632, 219 (2002), hep-th/0201144.
[33] N. Caporaso and S. Pasquetti, J. High Energy Phys. 04 (2006) 016, hep-th/0511127.