Channel Estimation and Detection in FBMC/OQAM System with Affine Precoding and Decoding

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Abstract—We derive the mathematical equations required for channel estimation and data detection of filter bank multi-carrier (FBMC) offset quadrature amplitude modulation (OQAM) systems with affine precoding and decoding. The mean square error (MSE) in least square (LS) channel estimation and bit error rate (BER) of the system is found for different training power coefficients. The proposed system gives better BER performance compared to other cutting edge FBMC systems. The optimum training power coefficient ($\sigma^2_{\text{opt}}$) is also found from the simulation results. The band width efficiency of the system is also calculated.

Index Terms—FBMC, OQAM, affine precoding and decoding, MSE, LS, BER.

I. INTRODUCTION

Implementation of prototype filters in filter bank multi-carrier (FBMC) offset quadrature amplitude modulation (OQAM) systems decrease side lobes strength and hence the adjacent channel interference is reduced [1]. The prototype filters employ the time-frequency localization feature that makes FBMC systems immune to dispersive channels. Thus, FBMC has become as an alternate to OFDM as it is a robust system and is also immune to multipath fading. In the same time, FBMC produces intrinsic imaginary interference (II). This is because FBMC supports orthogonality to real signals only. Therefore, the channel estimation in FBMC has become an interesting research area. The absence of training or pilot in blind estimation methods demand long statistics of received data to estimate the channel. In the FBMC system based on the method of interference approximation (IAM), first, the III is estimated approximately from adjacent symbols. Then, the estimation and detection is done [2]. The odd symmetry property of the prototype filters cancels the III in interference cancellation method (ICM) method [3]. Concatenation of data and training sequence is a feature of complex training sequence decomposition (CTSD) method, which leads to spectrum inefficiency [4]. This loss is dealt with by superimposing data on the training sequence [5]. However, superimpose results in interference between data and training in estimation as well as demodulation process. This issue is addressed in this proposed method. The contribution of our work is listed below.

- Necessary mathematical equations are derived for channel estimation and data detection in FBMC/OQAM system with affine precoding and decoding.
- The optimum training sequence power coefficient ($\sigma^2_{\text{opt}}$) is found from the simulation results.
- Spectrum efficiency of the proposed method is also calculated.

Notations: Bold face capital letters (A) represents matrices and $a_{i,j}$ represents the $i^{th}$ row and $j^{th}$ columnn element of matrix $A$. $*$, $(\cdot)^*$, $(\cdot)^H$, $E(\cdot)$, $E(\| \cdot \|^2)$ represents convolution, complex conjugate, Hermitian, expectation and Frobenius norm respectively. $j = \sqrt{-1}$, $\Re\{\cdot\}$ and $\Im\{\cdot\}$ represents the real and imaginary part of a complex number respectively. $I_N$ and $0_{a \times b}$ represents identity matrix of size $N \times N$ and zero matrix of dimension $a \times b$ respectively.

II. SISO FBMC/OQAM SYSTEM MODEL

A. Transmitted Signal

We consider FBMC/OQAM system having $N$ sub-carriers with sub-carrier spacing of $1/T_s$. Here, $T_s$ and $T_s/2$ represents the interval of complex symbols and real QAM symbols respectively. The prototype filter used in this model is Bellanger’s Phydas project filter having impulse response $p[k]$, whose coefficients are symmetrical and real. The length of the filter is $L_p$ and the energy is unity, due to which the energy of the original signal remains unaltered [6]. The matrix $S' \in \mathbb{R}^{N \times K/2}$ contains the raw data elements which are independent and identically distributed (i.i.d.) having zero mean and unit variance. The elements of $S'$ are QPSK modulated and their complex constellation points are stored in matrix $S \in \mathbb{C}^{N \times K}$ at their respective locations. The real and imaginary parts of elements of $S$ are extracted according to OQAM concept and are stored in $X \in \mathbb{R}^{N \times K}$ as given below.

$$
\begin{align*}
x_{m,2n} &= \begin{cases} 
\Re(s_{m,n}), & m \text{ even} \\
\Im(s_{m,n}), & m \text{ odd}
\end{cases} \\
x_{m,2n+1} &= \begin{cases} 
\Im(s_{m,n}), & m \text{ even} \\
\Re(s_{m,n}), & m \text{ odd}
\end{cases}
\end{align*}
$$

(1)

where the sub-carrier index $m = 0, \cdots ,N-1$ and the OQAM symbol index $n = 0, \cdots ,K/2-1$. The precoder matrix $P \in \mathbb{R}^{K \times (K+n)}$, training sequence matrix $C \in \mathbb{R}^{N \times (K+n)}$, estimator matrix $E \in \mathbb{R}^{(K+n) \times N}$ and detector
matrix $ \mathbf{D} \in \mathbb{R}^{(K+n) \times K}$ are designed using orthogonal matrix $ \Phi \in \mathbb{R}^{(K+n) \times (K+n)}$ as given in [7].

$$
P = \sqrt{(K+n) / K} \Phi((N+1): (N+K),:),$$

$$
\mathbf{E} = \Phi(1:N,:)^H,$$

$$
\mathbf{C} = \sqrt{K+n} \Phi(1:N,:),$$

$$
\mathbf{D} = \mathbf{P}^H (\mathbf{P} \mathbf{P}^H)^{-1},$$

where $n$ is a number such that $n \geq N$. The above matrices are designed so as to satisfy conditions given below.

$$
\mathbf{P} = \mathbf{I}_K, \quad \mathbf{PE} = \mathbf{0}_{K \times N}, \quad \mathbf{CE} = \mathbf{I}_N \quad \text{and} \quad \mathbf{CD} = \mathbf{0}_{N \times K}. \quad (3)
$$

The covariance matrices of $\mathbf{C}$, $\mathbf{P}$, $\mathbf{E}$ and $\mathbf{D}$ can be obtained as follows:

$$
\mathbb{E}\{ \mathbf{C} \mathbf{C}^H \} = (K+n) \mathbf{I}_N, \quad \mathbb{E}\{ \mathbf{P} \mathbf{P}^H \} = (K+n) / K \mathbf{I}_K,$$

$$
\mathbb{E}\{ \mathbf{E}^H \mathbf{E} \} = 1 / (K+n) \mathbf{I}_N, \quad \mathbb{E}\{ \mathbf{D}^H \mathbf{D} \} = K / (K+n) \mathbf{I}_K. \quad (4)
$$

Now, matrix $\mathbf{X}$ is post multiplied with precoder matrix $\mathbf{P}$ and then the training matrix $\mathbf{C}$ is superimposed to get affine precoded matrix $\mathbf{Z} \in \mathbb{R}^{N \times (K+n)}$ as given below.

$$
\mathbf{Z} = \sigma_x \mathbf{X} \mathbf{P} + \sigma_r \mathbf{C}, \quad (5)
$$

where $\sigma_r^2$ is the power of training symbols and $\sigma_x^2$ is the data power coefficient such that $\sigma_r^2 + \sigma_x^2 = 1$. In [7] it is proved that the variance of each element of $\mathbf{X}$ and $\mathbf{XP}$ remains same i.e. unity. Therefore, the discrete time FBMC signal at the $k^{th}$ sample index is represented as [1]

$$
s[k] = \sum_{m=0}^{N-1} \sum_{n \in \mathbb{Z}} z_{m,n} \chi_{m,n}[k],$$

where we can define the FBMC basis function $\chi_{m,n}[k]$ as $\chi_{m,n}[k] = e^{j2\pi m \cdot \frac{n}{N} p[k - n \frac{N}{N}]}$. The prototype filter $p[k]$ used is symmetrical, real valued and is of length $N_p = bN$ where the integer $b$ is known as overlapping factor. Phase factor is defined as $\phi_{m,n} = \pi / 2 (m + n) - \pi mn$.

**B. Received Signal**

We consider frequency selective time invariant wireless channel having $L_h$ tap length and impulse response $h[k] = [h_1, h_2, \cdots, h_{L_h}]$. The received signal $y[k]$ is

$$
y[k] = s[k] * h[k] + \eta[k],$$

$$
y[k] = \sum_{q=0}^{L_h-1} h[q] \sum_{m=0}^{N-1} \sum_{n \in \mathbb{Z}} z_{m,n} \chi_{m,n}[k-q] + \eta[k]. \quad (7)
$$

where $\eta[k]$ represents additive white Gaussian noise (AWGN) having zero mean and variance $\sigma_r^2$. After FBMC demodulation, the received signal $y_{\hat{m},\hat{n}}$ at the $\hat{m}^{th}$ sub-carrier and $\hat{n}^{th}$ QAM symbol time is given by

$$
y_{\hat{m},\hat{n}} = \sum_{k=0}^{N-1} y[k] \chi_{\hat{m},\hat{n}}^*[k] = \sum_{m=0}^{N-1} \sum_{n \in \mathbb{Z}} z_{m,n} H_m \xi_{\hat{m},\hat{n}} + \eta_{\hat{m},\hat{n}}, \quad (8)
$$

where $H_m = \sum_{q=0}^{L_h-1} h[q] e^{-j2\pi mq N}$ is the $m^{th}$ element of $N$-point DFT of $h[q]$, $\eta_{\hat{m},\hat{n}} = \sum_{k=0}^{N-1} \eta[k] \chi_{\hat{m},\hat{n}}^*[k]$ and $\xi_{\hat{m},\hat{n}} = \sum_{k=0}^{N-1} \chi_{m,n}[k] \chi_{\hat{m},\hat{n}}^*[k]$. It is known that [1]

$$
\xi_{\hat{m},\hat{n}} = \begin{cases} 1, & \text{if } (m,n) = (\hat{m},\hat{n}) \\ j \langle \xi_{\hat{m},\hat{n}} \rangle, & \text{if } (m,n) \neq (\hat{m},\hat{n}) \end{cases}, \quad (9)
$$

where $\langle \xi_{\hat{m},\hat{n}} \rangle = \Re \{ \sum_{k=0}^{N-1} \chi_{m,n}[k] \chi_{\hat{m},\hat{n}}^*[k] \}$ represents the imaginary part of the trans-multiplexer. Therefore, (5) can be re-written as

$$
y_{\hat{m},\hat{n}} = H_{\hat{m}} z_{\hat{m},\hat{n}} \xi_{\hat{m},\hat{n}} + \sum_{m=0}^{N-1} H_m z_{m,n} \xi_{\hat{m},\hat{n}} + \eta_{\hat{m},\hat{n}}$$

$$
= H_{\hat{m}} z_{\hat{m},\hat{n}} + j \sum_{(m,n) \neq (\hat{m},\hat{n})} H_m z_{m,n} \xi_{\hat{m},\hat{n}} + \eta_{\hat{m},\hat{n}}. \quad (10)
$$

Without loss of generality it can be assumed that with a well localized prototype filter in time-frequency domain, the III is contributed from first order neighborhood of $(\hat{m},\hat{n})$ i.e. $\Omega_{\hat{m},\hat{n}} = \{(\hat{m} \pm 1,\hat{n} \pm 1), (\hat{m} \pm 1,\hat{n}), (\hat{m},\hat{n} \pm 1)\}$. Assuming the channel frequency response (CFR) to be constant over this neighborhood, it can be written as

$$
y_{\hat{m},\hat{n}} \approx H_{\hat{m}}$$

$$
y_{\hat{m},\hat{n}} \approx H_{\hat{m}} \left\{ z_{\hat{m},\hat{n}} + j \sum_{(m,n) \in \Omega_{\hat{m},\hat{n}}} z_{m,n} \xi_{\hat{m},\hat{n}}^* \right\} + \eta_{\hat{m},\hat{n}} \quad \text{(11)}
$$

Here III represents the intrinsic imaginary interference which arises due to both precoded data $(\sigma_x \mathbf{XP})$ as well as superimposed training sequence $(\sigma_r \mathbf{C})$ whose contributions in matrix form can be denoted as $\Theta_1$ and $\Theta_2$ respectively. The received samples $y_{\hat{m},\hat{n}}$ in (11) can be represented in matrix form $\mathbf{Y} \in \mathbb{C}^{N \times (K+N)}$ as

$$
\mathbf{Y} = \mathbf{H} \{ \mathbf{Z} + j(\Theta_1 + \Theta_2) \} + \mathbf{W}, \quad (12)
$$

where $\mathbf{H} \in \mathbb{C}^{N \times N}$ is a diagonal matrix containing elements $\{H_0, H_1, \cdots, H_{N-1}\}$. Using (5) we get

$$
\mathbf{Y} = \sigma_x \mathbf{H} \mathbf{X} \mathbf{P} + \sigma_r \mathbf{H} \mathbf{C} + j\mathbf{H}(\Theta_1 + \Theta_2) + \mathbf{W}. \quad (13)
$$

**C. Channel Estimation**

Using the received signal matrix $\mathbf{Y}$, the performance of the channel estimator will be poor. This is because, the estimator will be affected by $\sigma_x \mathbf{H} \mathbf{P}$. Therefore, to nullify this effect estimator matrix $\mathbf{E}$ is post-multiplied with $\mathbf{H}$. In [7] it is proved that post multiplication of $\mathbf{E}$ does not amplify the power of the signal. Now, the modified received signal matrix is

$$
\mathbf{YE} = \sigma_x \mathbf{H} \mathbf{PE} + \sigma_r \mathbf{H} \mathbf{CE} + j\mathbf{H}(\Theta_1 + \Theta_2) \mathbf{E} + \mathbf{WE}$$

$$
= \mathbf{E} + j\mathbf{H}(\Theta_1 + \Theta_2) \mathbf{E} + \mathbf{WE}. \quad (14)
$$
The second term in (17) is due to intrinsic imaginary interference from precoded data as well as superimposed training which is not present in OFDM. So the channel estimation in FBMC has become challenging. In conventional superimposed training based FBMC systems, data and training sequence interference is present in channel estimation which is eliminated in affine precoding and decoding method because of orthogonality property. Now, the least square (LS) channel estimator can be obtained as

\[ \hat{YD} = \sigma_c \hat{H} \sigma_c + \sigma_c \hat{H} \Theta_1 + \Theta_2 \sigma_c \hat{D} + \sigma_c \hat{W} \sigma_c \nu, \]

(17)

where \( \sigma_c \) follows \( \nu \). The QAM symbols are detected by pre-multiplying (17) with \( \hat{H}^{-1} \) and taking the real part only as given below.

\[ \hat{X} = \Re \{ \hat{H}^{-1} YD \} = \Re \{ \hat{H}^{-1} \sigma_c \hat{H} \sigma_c + \sigma_c \hat{H} \Theta_1 + \Theta_2 \sigma_c \hat{D} + \sigma_c \hat{W} \sigma_c \nu, \}
\]

\[ \approx \Re \{ \sigma_c \hat{X} + j \sigma_c \hat{X} + \sigma_c \hat{D} + \sigma_c \hat{W} \sigma_c \nu, \} . \]

The second term in (17) is due to intrinsic imaginary interference from precoded data as well as superimposed training which is not present in OFDM. However, its effect on data detection in FBMC is reduced after taking the real part of the signal as in (18). In conventional superimposed training based FBMC systems, data and training sequence interference is present in data detection which is eliminated in affine precoding and decoding method because of orthogonality property. Rearrangement of the elements of \( \hat{X} \) to get back the complex symbols \( S \in C^{N \times K/2} \) is given by

\[ \hat{s}_{m,n} = \begin{cases} \hat{x}_{m,2n} + j \hat{x}_{m,2n+1}, & m \text{ even} \\ \hat{x}_{m,2n+1} + j \hat{x}_{m,2n}, & m \text{ odd} \end{cases} . \]

Then by taking QPSK demodulation of \( \hat{S} \) we will get the actual message transmitted.

### E. Band Width Efficiency

After implementation of OQAM concept, the data matrix is \( X \in R^{(N \times K)} \). If no precoding and decoding is applied, then to transmit \( N \times K \) OQAM symbols we need \( K \) symbol instants. But because of affine precoding and decoding, the matrix becomes \( Z \in R^{N \times (K+n)} \). So \( K+n \) symbol instants are required for transmission of same data matrix. So the band width efficiency (BW\(_{\text{eff}}\)) of the proposed method can be defined in terms of number of raw OQAM symbols transmitted in a single affine precoded frame. Therefore, we can write

\[ BW_{\text{eff}} = \frac{NK}{N(K+n)} = \frac{K}{K+n} . \]

### III. SIMULATION RESULTS

The simulations are done with the parameters like number of sub-carriers \( N = 256 \), prototype filter length \( L_p = 4N \), channel tap length \( L_h = 12 \) and QPSK modulation. The real and imaginary parts of complex constellation points after QPSK modulation are separately extracted using OQAM concept. The simulation is carried out for different values of \( \sigma_c^2 \) and signal to noise ratio (SNR) without channel encoding. 100 number of iterations are done to carry out the simulation. The usefulness of the proposed system is validated in terms of BER.

Fig.1 represents (19) i.e. the MSE in LS channel estimation for different training power coefficients \( \sigma_c^2 = 0.1, 0.2, 0.3, 0.5, 0.7, 0.9, 1.0 \). From (19) it’s clear that as \( \sigma_c^2 \) increases from 0 to 1, the error decrases which is validated in this figure. Right hand side of (15) contains three terms: first one is useful for channel estimation, second is related to III...
BER performance improves as $n$ is increased. However, as $n$ value increases, the computational complexity of the system increases and spectrum efficiency decreases (from (20)).

Fig.4 represents BER of this system with $n = N$ for different $\sigma_c^2$ values. It is noticed that the BER performance is best for $\sigma_c^2 = 0.2$ for any value of SNR. Hence, affine precoding and decoding based FBMC system performs better than other FBMC systems and is best with $\sigma_c^2 = 0.2$.

IV. CONCLUSION

We derived the mathematical equations needed for channel estimation and data detection of SISO-FBMC/OQAM system with affine precoding and decoding. With the simulation results we proved that the BER of this method is better than other cutting edge methods. The optimum training power coefficient ($\sigma_c^{opt}$) is also found from the simulation results. Though the proposed method is giving band width loss, from BER point of view the proposed FBMC system model is having practical importance.

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