Bonsai: Synthesis-Based Reasoning for Type Systems

KARTIK CHANDRA, Stanford University
RASTISLAV BODIK, University of Washington

We describe algorithms for symbolic reasoning about executable models of type systems, supporting three queries intended for designers of type systems. First, we check for type soundness bugs and synthesize a counterexample program if such a bug is found. Second, we compare two versions of a type system, synthesizing a program accepted by one but rejected by the other. Third, we minimize the size of synthesized counterexample programs.

These algorithms symbolically evaluate typecheckers and interpreters, producing formulas that characterize the set of programs that fail or succeed in the typechecker and the interpreter. However, symbolically evaluating interpreters poses efficiency challenges, which are caused by having to merge execution paths of the various possible input programs. Our main contribution is the Bonsai tree, a novel symbolic representation of programs and program states which addresses these challenges. Bonsai trees encode complex syntactic information in terms of logical constraints, enabling more efficient merging.

We implement these algorithms in the Bonsai tool, an assistant for type system designers. We perform case studies on how Bonsai helps test and explore a variety of type systems. Bonsai efficiently synthesizes counterexamples for soundness bugs that have been inaccessible to automatic tools, and is the first automated tool to find a counterexample for the recently discovered Scala soundness bug SI-9633 [2].

1 INTRODUCTION

Today’s type system designers strive to develop typecheckers that balance expressiveness and convenience with powerful static guarantees. This has led to a variety of innovations, such as polymorphism, path-dependent types, and ownership types. On their own, these features promise programmers strong static guarantees on their programs. Unfortunately, combining such features often creates intricate soundness bugs, some of which have gone unnoticed for many years. For example, soundness bugs were caused by the confluence of assignment and polymorphism in ML [55, 61], and of path-dependent types and nullable values in Scala [2].

To automate checking of type systems, we present a set of symbolic algorithms that aid type system designers in reasoning about executable language models. Using bounded program synthesis techniques, we demonstrate how users can compute answers to queries such as these:

• Soundness. If the type system is not sound, synthesize a counterexample, a program that passes the typechecker but fails in the interpreter.
• Comparison. Given two versions of a typechecker, synthesize a program accepted by one version but rejected by the other, elucidating the impact of changes to a type system.
• Minimization. For either of the above queries, produce the smallest possible counterexample.

Successfully answering these queries requires us to efficiently explore the extremely large space of candidate programs that are complex enough to demonstrate a harmful interaction. Fuzzers, which are commonly used to answer the first query, attempt to search these large spaces by avoiding programs that would be rejected somewhere along the parser-typechecker-interpreter pipeline (see Figure 1). Early fuzzers explored the space of all programs, some of which may have failed in the parser [63]. More recent “syntax fuzzers” such as Redex [28] generate only syntactically correct programs, thus shrinking the candidate space. Further advancements allow us to generate only type-safe programs [14, 15, 19, 43]. These “type fuzzers” use constraint solvers to search for a
typesafe program in a goal-directed fashion; only the interpreter is executed forward. However, even among type-safe programs, counterexamples are scarce.

The natural next step, then, is to reason about the entire pipeline in a goal-directed fashion, so that a counterexample can be directly constructed by the constraint solver by reasoning backwards from the possible failure points in the interpreter. We develop such goal-directed algorithm by symbolically compiling the entire parser-typechecker-interpreter pipeline into a single logical formula. This reduction also allows us to make the other queries mentioned above by modifying the structure of the formula.

Our contributions are:

- **The Bonsai tree** (Sections 2 and 3): We describe Bonsai trees, a symbolic representation of a bounded space of input programs and tree-shaped program states. Bonsai trees enable symbolic evaluation of language models into logical formulas. Symbolic evaluation can be performed on standard trees, of course, but Bonsai-powered symbolic evaluation is more efficient and more effective, generating formulas that are smaller and easier to solve. Bonsai trees reduce formula sizes by encoding the syntactic information in logical constraints rather than in the shape of the data structure.

  We demonstrate several interesting properties of Bonsai trees (Sections 3 and 4). First, Bonsai trees make it easier to avoid symbolically evaluating the parser-typechecker-interpreter pipeline as a monolithic composition of the three components. Instead, we evaluate them separately, producing three simpler formulas that in turn represent syntactically correct programs; programs that pass the typechecker even though they may not be syntactically correct; and programs that succeed (or fail) in the interpreter even though they may syntactically incorrect or not typesafe. These three sets, illustrated in Figure 2, are then intersected by conjuncting the formulas, to obtain the set of counterexamples.

  Second, depending on what sets we intersect, we can formulate other queries. For example, synthesis of correct programs that are rejected by the typechecker is also shown in Figure 4. The Compare query is formed analogously (see the bullet “Case studies” below for more details). The Minimize query simply adds the requirement that the solver returns a solution that minimizes some user-supplied metric, such as the tree size.

  Third, Bonsai trees can be added to standard symbolic evaluators. In particular, we formalize symbolic evaluation of Bonsai trees on a small core language (Section 3) and outline how we added this functional core language to Rosette [58], a symbolic evaluator for a subset of Racket with assignments, objects, and other features (Section 4). This integration of Bonsai symbolic evaluation into Rosette is at the heart of the BONSAI tool.
Fig. 2. Bonsai performs three independent symbolic evaluations, executing the interpreter on trees that are both syntactically and type-incorrect.

- **Case studies** (Section 5): We study a variety of type systems with Bonsai, gauging its utility during a type system design. We perform three case studies, with Featherweight Java [26], Ownership Java [11], and DOT, the dependently-typed model of Scala [5].

  We first examine how counterexamples can provide a clear explanation of soundness bugs. We demonstrate that by understanding how a counterexample fails in the interpreter, in our case by examining a stack trace of a program crash, we can better understand the cause of a soundness bug.

  Second, we investigate what language specification patterns are friendly to symbolic evaluation. Interestingly, we find that Bonsai trees can be efficiently used to represent not only ASTs, but also other auxiliary data structures such as environments and class tables.

  Third, we experiment with the Compare query: we ask Bonsai to explain a static type rule that appeared to us unnecessary (subsumed by other checks) or unnecessarily restrictive (rejecting correct programs). We removed the rule, creating a more permissive version of the type system and asked Bonsai “Is there a program (i) rejected by the old type system, (ii) accepted by the new type system, and (iii) that nevertheless runs without error?” Bonsai synthesized such a program, suggesting that it may be useful to design a less restrictive type system.

  Finally, we use the DOT model to stress Bonsai’s expressiveness. DOT is a rich calculus with dependent types, function types, records, intersection types, and recursive types. We find DOT easy to reformulate to Bonsai patterns. We were able to express all features of DOT except for recursive types: early on we decided to use call-by-value semantics, only to realize later that DOT’s definition of recursive types is only compatible with call-by-name semantics. Overall, implementing DOT features did not hamper symbolic evaluation. In fact, in minutes, we synthesize on DOT a counterexample for the Scala soundness error SI-9633 [2].

- **Performance evaluation** (Section 6): First, in under an hour, we synthesize a counterexample for the language with assignments and polymorphic references [54], which has been inaccessible to automatic tools [19]. Second, we show that Bonsai trees allow exploration of vastly larger spaces than the standard symbolic tree. In our experiments with lambda calculus, Bonsai explores $10^{50}$ vs. $10^{25}$ candidate programs in 20 seconds. Third, on a subset of Redex benchmarks, Bonsai is about 600-times faster than a syntactic fuzzer and 12-times faster than a type fuzzer. We also find Bonsai superior in the size of counterexamples, synthesizing counterexamples about 10-times smaller, on average, than the type fuzzer.

Implementations of many of the case studies and evaluations can be found at https://bitbucket.org/bonsai-checker/.
(define arithmetic-syntax
  '([exp zero (suc exp) (if exp exp exp) (zero? exp) ]))

(define check t)
  (tree-match t
    'zero (lambda () 'nat)
    'succ (lambda (x) (+ (execute x) 1))
    'if (lambda (c t f)
         (assert (eq? (check c) 'bool))
         (define \t+ (check t))
         (define \f+ (check f))
         ;; (assert (eq? \t+ \f+))
         ;; omitted!
         \t+)'
    'zero? (lambda (x) (= 0 (execute x))))

Fig. 3. An executable specification of the arithmetic language [41] that we would like to check for soundness. tree-match is a pattern-matching macro, which checks an input AST against the given patterns, and, upon finding a match, executes the accompanying lambda with the contents of the blanks as arguments.

Overall, we believe that Bonsai complements the theorem-prover-based mechanization of a soundness proof, especially during design space exploration. Bonsai’s strengths are automation and queries that go beyond soundness checking. Among weaknesses is the inability to guarantee absence of bugs. This limitation is somewhat compensated by the Bonsai’s efficiency: we observe that during an overnight run, it exhaustively explores candidates 2- to 4-times larger than the counterexamples constructed by human experts (Table 3), providing a safety margin. The relationship with other tools is given in Section 7.

2 BACKGROUND AND MOTIVATION

Figure 3 shows a simple executable specification of the simply-typed arithmetic language [41]. This particular specification has an unsound typechecker, due to an omitted type rule—it does not verify that the two branches of an “if” term have the same type. We would like to automatically synthesize a counterexample to soundness for this language by symbolically translating the typechecker and interpreter to SMT formulas, and then querying a solver for a program that passes “check” but fails in “execute.” In this section, we explain the scalability challenges presented by this strategy, motivating a new symbolic representation of abstract syntax trees. In the following section, we address those challenges by designing such a symbolic representation, the Bonsai Tree. Finally, equipped with the Bonsai Tree, we return to the example above and demonstrate how to synthesize a counterexample.

We begin with a brief overview of symbolic evaluation techniques, focusing in particular on the “branch-and-merge” strategy for evaluating conditional statements. We then explain how currently-used symbolic data structures like ASTs are created by representing “symbolic unions” of concrete trees, and why this structure makes branch-and-merge operations prohibitively expensive. Finally, we introduce the key idea behind our new tree encoding, “Bonsai,” which allows the symbolic
evaluator to efficiently branch and merge, dramatically decreasing the time needed to create the SMT formula.

2.1 Background: Symbolic Evaluation

Symbolic evaluation was first described by King [27]: the goal is to translate a program $P$ with inputs $x_1, x_2, \ldots, x_n$ into an equivalent SMT formula $f(x_1, x_2, \ldots, x_n)$, which can then be processed using an SMT solver. The inputs $x_i$ are called *symbolic constants*; they can be booleans, integers, bitvectors, or members of any other theory supported by the SMT solver. Importantly, substituting a concrete value $v_i$ for each $x_i$ in the formula should simplify to the same result as evaluating $P$ with arguments $v_i$ directly. For example, the symbolic evaluation of the program

```plaintext
P(x1 : int) : int { 
  int k = 0;
  for (int i=0; i<3; i++) { k += x1; }
  return k;
}
```

yields the formula $(((0 + x_1) + x_1) + x_1)$.

A program may also make *assertions* about its symbolic constants. For example, the statement `assert P(x1) = 9` would create the formula $(((0 + x_1) + x_1) + x_1) = 9$. Such an assertion can then be sent as part of a query to the SMT solver, which would report the solution model $\{x_1 = 3\}$. Symbolic evaluators generally collect all assertions encountered during the program’s execution in a set called the *assertion store*. The final query is the conjunction of all assertions in the store.

The symbolic evaluation of most terms is a straightforward application of partial evaluation. However, branching statements require some care. Consider the program below.

```plaintext
P(x1 : int) : string { 
  if (x1 > 5) { return "big"; }
  else { return "small"; }
}
```

Notice that the value of the condition $x1 > 5$ is not known to the symbolic evaluator. Thus, the symbolic evaluator must *branch* to try both paths, and then *merge* the results to create a formula expressing both alternatives. Such a formula is known as an *ite* for *if-then-else*, and the entire process is known as a "branch-and-merge operation." Here, the branch-and-merge operation yields ite($x_1 > 5$, "big", "small"). The conditions $x_1 > 5$ and $¬(x_1 > 5)$ are called the *path conditions* of their respective values "big" and "small".

Operations on an ite are mapped over its branches. For example, the program `strLen P(x1)` produces ite($x_1 > 5$, `strLen "big"`, `strLen "small"`), which is then simplified to ite($x_1 > 5$, 3, 5). Thus, one could query the solver for $x_1$ such that `assert strLen P(x1) = 5`. The solver may report ‘2’ as a solution. Following Torlak and Bodik [58], we generalize the ite to the *symbolic union*, which represents a sequence of $n$ chained ites. We represent ite($\phi_1, v_1, \text{ite}($$\phi_2, v_2, \ldots$), $\ldots, \phi_n, v_n$) as the symbolic union $\{\phi_1 \to v_1, \phi_2 \to v_2, \ldots, \phi_n \to v_n\}$. Each $\phi_i$ is a path condition to its corresponding value $v_i$; as with ites, operations on symbolic unions are mapped over each such value.

One final nuance lies in assertions made in the bodies of conditionals. Consider the program

```plaintext
P(x1 : int) : void { 
  if (x1 > 5) { assert x1 = 10; } // A
  else { assert x1 = 0; } // B
}
```

A naive execution of this program creates the unsatisfiable assertion store $x_1 = 10 \land x_1 = 0$. However, both 10 and 0 should be valid values for $x_1$. To allow this, the symbolic evaluator maintains a *global* path condition, which reflects the conditions under which an assertion is made. The path condition
is updated every time the program branches with a symbolic condition. For example, when the first assertion ("A") is made, \( x_1 > 5 \) must hold. Thus, the path condition is \( x_1 > 5 \), and the actual assertion recorded by the symbolic evaluator is the implication \( x_1 > 5 \implies (x_1 = 10) \). Similarly, the second assertion ("B") is recorded as \( \neg(x_1 > 5) \implies (x_1 = 0) \). Thus, the query to the solver becomes \( (x_1 > 5 \implies x_1 = 10) \land (\neg(x_1 > 5) \implies x_1 = 0) \), which is satisfiable with both 10 and 0 as solutions for \( x_1 \).

2.2 Symbolic Syntax Trees

To symbolically evaluate typecheckers and interpreters, we must represent programs in terms of a set of symbolic constants. The classical approach to representing symbolic ASTs uses symbolic unions to represent choosing between production rules. For the remainder of this section, we will consider a simple grammar for the lambda calculus, with the production rules

\[
e \rightarrow x \mid \lambda x.e \mid (e e)
\]

where \( x \) ranges over variable names.

**Structure.** To construct a symbolic AST, one might begin by creating a symbolic union over a symbolic constant \( c \), which chooses among the three types of depth-1 trees, i.e. the three production rules:

\[
\{ c = 1 \rightarrow x, c = 2 \rightarrow \lambda y.x, c = 3 \rightarrow (x y) \}
\]

This symbolic union is depicted in Figure 4. Larger ASTs can then be created recursively using the same process, using smaller ASTs as subtrees.

Notice that the structure of these trees depends on the syntax of the language. We thus call such trees "syntax trees." Syntax trees are appealing because of their simplicity: indeed, they are the most commonly-used symbolic representations of ASTs. However, they can become extremely inefficient when symbolically evaluated.

**Scalability challenges.** The inefficiency of symbolic syntax trees can be revealed by analyzing their "branch-and-merge" operations. Recall that the "branch" operation takes place on a symbolic condition: in the case of an operation on ASTs, this condition determines whether a tree matches a given pattern. To perform a pattern-match on a symbolic syntax tree, the symbolic evaluator must examine the members of the symbolic union and individually check whether each member matches. Furthermore, when pattern-matching, we usually wish to extract subtrees that match our pattern’s metavariables. The symbolic evaluator must therefore perform this extraction on each member of the symbolic union and then merge the results into a fresh symbolic union. This situation is depicted in Figure 5 on the left. For a symbolic union with \( n \) members, this operation takes at least \( O(n) \) time.

Analyzing the "merge" operation reveals that such a merge often grows the size of the symbolic union. To merge two symbolic unions, a symbolic evaluator must add together the members of each symbolic union, updating the respective path conditions as required. In the worst case, this can double the size of the symbolic union. This is depicted in Figure 5 on the right. Over the course of many branch-and-merge operations, the symbolic unions that represent symbolic syntax trees grow. Over time, symbolic evaluation slows down, and ultimately becomes impractically inefficient.
Possible solutions. A crucial insight is that syntax trees are inefficient because their structure depends on the grammar of the language. Representing a set of differently-shaped trees requires us to create a symbolic union, which is expensive to match against. This intuition guides us to the key idea behind the Bonsai tree: maintain a single tree that may or may not be syntactically-valid, and push the constraints of the syntax to assertions in the assertion store. Such a “clean” data structure, unencumbered by syntactic constraints, should be efficiently manipulable by a symbolic evaluator. In the remainder of this paper, we demonstrate that this is indeed the case.

3 BONSAI TREES

We now formalize the intuition above in terms of the Bonsai Tree, an efficient alternative to the syntax tree. Our discussion is guided by the Bonsai Core language, a minimal calculus that describes programs that manipulate symbolic Bonsai trees via “match” and “merge” operations (Figure 6). An empirical evaluation of the Bonsai encoding is presented in Section 6.3.

Structure. Bonsai trees are embedded in perfect binary trees. Figure 7 depicts three such “concrete” trees, and their embeddings in perfect binary trees. As shown in the Bonsai Core rule SE-TREE, the embedding is represented by assigning two symbolic constants to each node of the binary tree. The first determines whether the node is internal or a leaf, and the second determines the terminal in case it is a leaf. Figure 8 depicts one such representation.

More formally, a Bonsai tree $G$ is a tuple $(T, \mathcal{L}, m, \text{internal}, \text{leaf})$ where $T = \langle N, E \rangle$ is a perfect binary tree of depth $d$ and size $m = 2^d - 1$, such that $N$ is the set of nodes $\{n_1, \ldots, n_m\}$ and $\mathcal{L}$ is the set of potential leaf symbols. The predicates internal : $N \to \text{Bool}$ and leaf : $(N \times \mathcal{L}) \to \text{Bool}$ specify what tree is embedded in $G$. In a symbolic Bonsai tree, these guards are logical formulas composed of symbolic constants.

Branch-and-merge operations. Bonsai carefully maintains the following invariant: each ite created in the language has the property that if its condition is true, the result is a leaf, else it is an inner node. This invariant can easily be verified for the rules SE-TREE and SE-MERGE, the only sources of ites in Bonsai Core.

A useful result of this invariant is that a pattern matcher need not branch when matching against an ite: depending on whether the pattern is a leaf or an inner node, it is clear which path of the ite must be taken. Thus, symbolic Bonsai trees can be matched in a single operation. In contrast, recall that symbolic syntax trees required a match operation for each member of their symbolic unions. Pattern-matching a Bonsai tree is depicted in Figure 9 on the left. The path condition $\phi$ represents the conditions required for that Bonsai tree to match the pattern. Notice that since the pattern-matcher never branches, any subtree can be extracted directly. In contrast, the symbolic syntax tree must merge the extracted subtrees from each individual match into another, larger symbolic union.

A merge of two Bonsai trees under path conditions $\phi_1, \phi_2 : = \neg \phi_1$ is depicted in Figure 9 on the right. No matter what the guards for these individual trees are, we can merge them by joining them.
Fig. 6. The Bonsai Core language and reduction rules. Here, $t$, $e$, $f$ range over terms; $p$, $q$ range over patterns; $x$, $y$ range over pattern variables; $c$ ranges over $\mathbb{L}$, the set of possible leaf values; $\alpha$ ranges over $\mathbb{S}$, the set of symbolic constants; $\pi$, $\phi$, $\psi$ range over boolean formulas with equality defined on $\mathbb{S} \times \mathbb{L}$. $(t, \pi, \phi)$ denotes term $t$ with path condition $\pi$ and assertion store $\phi$. fresh($\mathcal{A}$) denotes a fresh symbolic constant ranging over set $\mathcal{A}$. merge is defined in its most general form: a merge of two ite values. Definitions for merging non-ite values can be derived by setting $\phi$ and/or $\psi$ to $T/F$. 

![Concrete Bonsai trees for terms x, $\lambda y.x$, and ($x \ y$)](image_url)

Fig. 7. Left: Concrete Bonsai trees for terms $x$, $\lambda y.x$, and ($x \ y$). Right: and their embeddings in perfect binary trees.

node-wise under these path conditions. Thus, though we require $O(m)$ operations to merge trees of size $m$, the trees never grow: the output of the merge is the same size as each input tree, and the $m$
operations are a constant time cost. In contrast, the symbolic syntax tree grow at each merge, and thus the time cost grows as more branch-and-merge operations are carried out.

Notice that while the tree itself does not grow, the formulas for the guards do indeed grow at each merge due to the addition of path conditions. This, however, is not a concern for two reasons. First, symbolic evaluation engines represent formulas as DAGs. Creating a disjunction of two formulas does not require us to duplicate each formula in memory, which would be an expensive operation. Instead, we only need to allocate a fresh “∨” operator node, and create cheap pointers to existing formulas for the operands. Second, we never have to “look inside” these formulas. We only construct them and, when finished, pass them on to the SMT solver. As a result, the growing formulae do not affect the efficiency of the symbolic evaluation.

Example. Figure 10 depicts a small worked example of pattern-matching against and merging symbolic Bonsai trees. The mini-language has three terms: id(x), which evaluates to its argument, swap(x, y) which swaps its arguments when evaluated, and a, a constant that evaluates to itself. The grammar, as well as its embedding in binary trees, is presented in the top half of the figure. The bottom half depicts the symbolic evaluation of the Bonsai Core term

\[
\text{match (tree 3) on a for a [swap, [x, y]] for [swap, [y, x]] [id, x] for x end}
\]

We first perform each individual match, creating path conditions and results, and then merge the results under each path condition.

4 IMPLEMENTATION

This section illustrates how to implement Bonsai trees in a specific language, Racket [22], using a specific symbolic evaluator, Rosette [56]. Of course, guided by the rules in Figure 6, these ideas can be implemented in other symbolic evaluation platforms as well [48, 49]. After implementing Bonsai trees, we demonstrate how to use them to check the arithmetic language in Figure 3.

Constructing a symbolic tree. Recall rule SE-TREE from Figure 6. Two fresh symbolic constants on each node dictate whether it is an internal node or a leaf, and if it is a leaf, which symbol it represents. We can construct such a node by symbolically evaluating the call to binary-tree!
Grammar $G$:
\[
e ::= a \mid id e \mid swap e e
\]

Program $P$:
\[
\text{match } t \text{ on } \\
a \text{ for } a \\
id x \text{ for } x \\
swap x y \text{ for } swap y x \\
\text{end}
\]

A bonsai embeddings for grammar $G$:
\[
\text{id} \quad \text{swap} \\
a \quad a 
\]

Two tree transformations made by $P$. Left: on concrete syntax trees. Right: on concrete bonsai trees.

case:
\[
a \text{ for } a \\
id x \text{ for } x \\
swap x y \text{ for } swap y x
\]

path condition:
\[
\phi_a: n_1 = a \\
\phi_{id}: \text{inner}(n_1) \land n_2 = id \\
\phi_{swap}: \neg(\phi_a \land \phi_{id}) \text{ otherwise}
\]

matched tree:
\[
\]

result per case
\[
\]

result after merge:
\[
\phi'_1: \{\phi_a \Rightarrow a, \phi_{id} \Rightarrow \phi_5, \phi_{swap} \Rightarrow \text{internal}\} \\
\phi'_2: \{\phi_{id} \Rightarrow \phi_6, \phi_{swap} \Rightarrow \phi_5\} \\
\phi'_3: \{\phi_{swap} \Rightarrow \phi_6\} \\
\phi'_4: \{\phi_{id} \Rightarrow \phi_7, \phi_{swap} \Rightarrow \phi_2\} \\
\phi'_5: \{\phi_{swap} \Rightarrow \phi_7\} \\
\phi'_6: \{\phi_{swap} \Rightarrow \phi_5\}
\]

- internal node
- leaf node
- unused node
- symbolic
  (any of the above)

The effect of $P$ on symbolic bonsai trees. $\phi_i$ and $\phi'_i$ are the conditions of $n_i$ on entry/exit of $P$.

Fig. 10. An example of pattern-matching against a symbolic Bonsai tree and merging the results.
(define (binary-tree! depth)
  (if (> depth 0)
      ; SE-Tree
      (if
       (eq? 'leaf (fresh-symbolic! '(leaf inner)))
       (fresh-symbolic! all-leaves)
       (cons
        (binary-tree! (- depth 1))
        (binary-tree! (- depth 1))))
      ; SE-Leaf
      (fresh-symbolic! all-leaves)))

(define all-leaves '(zero succ zero? if))
; for arithmetic language

Fig. 11. A procedure that generates a fresh symbolic Bonsai tree (left), and its output (right).

shown in Figure 11 (left). The call is recursively unrolled by the symbolic evaluator until the depth
limit reaches zero, at which point we follow SE-Leaf by creating a single symbolic constant to
represent which symbol is represented by the leaf. fresh-symbolic! is a macro that symbolically
selects from a set $S$, analogous to fresh($S$) in BONSAI Core. Internally, it is implemented by creating
symbolic integer constants and maintaining a map from integers to members of $S$. Inner nodes
are represented by cons pairs so that we can take advantage of existing Racket primitives for
manipulating S-expressions.

The result of symbolically evaluating binary-tree! is a data structure that embeds the symbolic
constants $s_i$, pictured in Figure 11 (right). Note that this tree contains no syntactic information:
concrete assignments of values to each $s_i$ may create syntactically-invalid ASTs.

Pattern-matching. The BONSAI pattern-matching macro, tree-match, is implemented by following
the rules SE-Test-{ Const-Pass, Const-Fail, Sym, Pattern-Variable, Inner-Node, Trans } as
expected. Since the pattern-matcher is written within a symbolically-evaluated language, SE-Test-
ITE follows automatically from the symbolic evaluator’s behavior on operations on ites. Figure 12
illustrates the behavior of the pattern-matcher using the example of syntactic constraints (see
below). Note that as per SE-Match-NONEMPTY, the pattern-matcher is sensitive to the order in
which patterns are presented. If a tree matches two patterns $p_1$ and $p_2$, the earlier one ($p_1$) is given
priority. The path condition for the $p_2$ match will stipulate that the $p_1$ match fails. This resolves
ambiguities in case the patterns are not mutually exclusive.

Syntactic constraints. To assert that the BONSAI tree embeds only syntactically correct trees, we
can take advantage of pattern-matching. We invoke (assert (is-program? tree)), which traverses
the tree and generates constraints required by the grammar defined in Figure 3. This call expands
into the procedures in Figure 12. More generally, we provide the macro syntax-matches?, which
converts a grammar in BNF to a syntax checker in the style of is-program?.

Merge. Recall from Section 2.1 that symbolic evaluators already perform merges on conditional
branches. Rosette in particular also performs some simplification: when merging two cons pairs, it
first individually merges the heads and tails, and then creates a fresh cons pair of the two merges.
This gives us half of the merge operation "for free": we must only be careful to arrange for Rosette
to merge pairs separately from leaves, since the simplification does not apply to merging a leaf.
(define (is-program? p)
  (tree-match p
    'zero (lambda () #t)
    '(succ _) (lambda (x) (is-program? x))
    '(if _ _ _) (lambda (c t f)
                  (and (is-program? c)
                       (is-program? t)
                       (is-program? f)))
    '(zero? _) (lambda (x) (is-program? x))
    '_' (lambda () #f)))

Fig. 12. Placing syntactic constraints on the Bonsai tree. The code results from macro expanding the arithmetic grammar (left). One more level of expansion (right) reveals the internal Bonsai tree structure.

with an inner node. This, however, is always the case as long as the ites being merged satisfy the invariant described in Section 3. Thus, by carefully regulating the creation of ite values in binary-tree!, we force Rosette to always perform a Bonsai merge on trees.

4.1 The Bonsai Checker

Equipped with the Bonsai Tree, we now return to our goal of symbolically reasoning about the language in Figure 3. We begin by searching for counterexamples to soundness. The Bonsai Tree allows us to efficiently execute the algorithm shown earlier in Figure 2. First, create a symbolic representation of the set of all ASTs up to some maximum size \( m \). Next, compute symbolic representations of trees that (a) are syntactically valid; (b) pass the typechecker; (c) fail in the interpreter. Finally, ask the solver to find a tree in the intersection of these sets. If it exists, this program is a counterexample.

Recall that the typechecker in Figure 3 has a soundness bug, failing to verify that the two clauses of 'if' expressions have the same type. Bonsai catches this error by executing the algorithm described above:

```scheme
(define program (binary-tree! 10))
(assert-pass (syntax-matches? arithmetic-syntax program))
(assert-pass (check program))
(assert-fail (execute program))
(define solution-model (solve))
(諸 (evaluate program solution-model))
```

The first line creates the formula characterizing the set \( A \) of all abstract syntax trees of depth 10 or less that adhere to the grammar. The second line creates the formula \( S \), which describes the subset of programs in \( A \) that are syntactically-valid. The third line evaluates the formula on the typechecker to create the formula \( T \), which describes the set of programs that pass the typechecker. The fourth line does the same with the interpreter, negating the formula generated to produce \( I' \).
The fifth line queries the solver for a model that satisfies the formula $S \land T \land I'$, and the sixth line simply interprets the model to yield a concrete program, which is then echoed to the user. **Bonsai** returns the following counterexample in less than 2 seconds.

'\(\text{succ (if (zero? (succ zero)) zero (zero? zero))}\)'

Indeed, this program passes the typechecker but causes the interpreter to crash by attempting to increment a boolean value. If we reintroduce the omitted check in our typechecker, our system fails to find a counterexample, showing that this check fixes the bug.

### 4.2 A Variation on the Query

A small variation on the counterexample-finding algorithm above allows us to compute a “diff” of two typecheckers by querying for programs that are accepted by one but rejected by the other. Suppose a student submits the unsound arithmetic typechecker above to a teacher. The teacher can compare it to a correct reference implementation and give the student feedback in the form of a program accepted by the former but rejected by the latter.

To synthesize such a program, we initialize $T$, the set of all programs that pass the student’s implementation, and $U$, the set of all programs that pass the teacher’s implementation. Then, we query the SAT solver for any member of the set $S \land T \land U'$.

```
(define program (binary-tree! 10))
(assert-pass (syntax-matches? arithmetic-syntax program))
(assert-pass (student-check program))
(assert-fail (teacher-check program))
(define solution-model (solve))
(echo (evaluate program solution-model))
; ==> '\(\text{if (iszero? zero) zero (iszero? zero)}\)'
```

Note that this program is not a counterexample to soundness because it does not fail at runtime. However, it behaves differently on the two typecheckers, and thus represents a member of their “diff.”

### 5 CASE STUDIES

We now demonstrate how **Bonsai** can be used to explore three more advanced type systems: an object-oriented, an ownership-based, and a dependent type system. Besides looking for soundness issues, we also run queries that help us better understand the type system’s restrictions.

#### 5.1 Featherweight Java

Featherweight Java [26] is a calculus that models essential features of the Java programming language. Featherweight Java has classes with methods, as well as inheritance via subclasses.

We test **Bonsai’s** ability to identify the classical bug in function subtyping, which is to insist that function arguments are covariant, i.e. to assume that $s <: t$ implies $(s \rightarrow u) <: (t \rightarrow u)$. The correct approach is to require contravariance in function arguments, i.e. if $s <: t$ then $(s \rightarrow u) :> (t \rightarrow u)$. This bug is of historical interest because it appeared in early versions of the Eiffel programming language, and its discovery came as a surprise to the Eiffel community [12].

After we introduced the bug into our Featherweight Java implementation, **Bonsai** reported the following counterexample in a few seconds. This counterexample has been manually formatted to Java syntax:

```
class A extends Object {
    B bar(A arg) {
        return arg.bar(this);
    }
}
class B extends A {
```
B quux(C arg) {
    return new A().bar(arg);
}

class C extends B {
    B bar(B arg) {
        return arg.quux(this);
    }
}

new C().quux(new C());

The call stack execution trace:

Error: class A has no method quux
in new2 A().quux(new1 C()); // line 11
...new1 C().bar(new2 A()); // line 3
...new2 A().bar(new1 C()); // line 7
...new0 C().quux(new1 C()); // line 13
...[init]

In the stack trace, we can see that the failure is caused by new C().bar(new A()). This call occurs because the method C.bar is unsoundly allowed to be used in place of A.bar because of their covariant argument types. This explains why the covariant argument rule is problematic: whenever C <: A, the rule promises that we can safely use a C wherever we use an A. This promise is violated because A.bar accepts inputs that C.bar does not. Note that this issue does not apply to true Java, because shadowing an inherited method with a different argument type overloads that method name, creating a distinct method.

The symbolic evaluation of Featherweight Java is much more demanding than that of the arithmetic language. To typecheck and evaluate Featherweight Java, we must make lookups into a class table in order to traverse the subclass hierarchy; when the class table is symbolic, a lookup could potentially be any of the entries in the table. Representing the class table as a Bonsai Tree allows us to efficiently merge all possible results of a lookup into small, compact formulas that can easily be manipulated by the interpreter.

5.2 Ownership Java

Our next language augments Featherweight Java with an object-oriented ownership type system, Ownership Java [9, 11]. The safety guarantee made by Ownership Java is that well-typed objects must access only objects that they own. The type system statically enforces this encapsulation guarantee by keeping track of the binary ownership relation on objects, which is declared in annotations on each class, method, and field.

A canonical example of Ownership Java is the implementation of a stack. The stack object is allowed to access its inner linked list but not the objects stored in the linked list; those can only be accessed by the client of the stack. The stack class is thus defined with two owner parameters. The first parameter is special because it is the owner of the stack object at runtime. The second owner parameter denotes the client who owns the data in the list; this parameter is used by the stack class implementation to annotate its methods and the encapsulated linked list. For example, the return value from the pop method is annotated with the second owner parameter, allowing the client to access objects retrieved from the stack.

To prove soundness of Ownership Java, Boyapati et al. [11] introduce an “ownership tree,” a transitive view of the ownership relation: $o_2$ is a direct descendant of $o_1$ if $o_1$ owns $o_2$. The ownership tree is rooted at the special owner world. Using the ownership tree, the encapsulation theorem is stated: object $x$ can access object $o$ only if (1) $x = o$, (2) $x$ is a descendant of $o$ in the ownership tree, or (3) $x$ is an inner class object of $o$. This invariant can be checked by tools like Pipal (Section 7).
With the Bonsai Checker, however, we do not need to formulate any such invariant. Instead, we only need one straightforward dynamic check to detect unsoundness. The check asserts that the owner stored in the accessed object (\texttt{object}) is owned by the receiver of the accessor method (\texttt{this}):

\begin{verbatim}
(define (evaluate-expression ...) ... (assert (owns? this object) "Error: attempted invalid access!") ...)
\end{verbatim}

This check is easy to implement because it is a direct statement of the desired guarantee. We believe that a counterexample that violates this assertion provides a more intuitive understanding of the issue than a counterexample that violates the encapsulation theorem because a trace shows the origins of the unsafe state. We reproduce one such counterexample below, found by Bonsai in about 90 seconds when we disabled the static owner check of method calls:

\begin{verbatim}
class Main<O1, O2> extends Foo {
    Main<O1, O1> meth3(Main<O1, O1> arg) {
        (arg.meth3(arg)).main(arg);
    }
}
class Foo<O1, O2> extends Object {
    Main<O2, O2> main(Main<O2, O2> arg) {
        (new Main<O2, O2>().meth3(arg);
    }
}
new Main<world, world>().main(new Main<world, world>);
\end{verbatim}

This program fails at runtime because \texttt{Foo<O1, O2>} should not be allowed to access methods of a \texttt{Main<O2, O2>} on line 7. It should only be allowed to access an instance of \texttt{Main} that it owns, which could be a \texttt{Main<this, O1>} or a \texttt{Main<this, world>}.

\textbf{Why have additional constraints on owners?} Next, we illustrate Bonsai’s ability to compare type systems by formulating queries involving multiple typecheckers and interpreters.

When instantiating a new object with owner parameters \(o_0...n\), the Ownership Java type system insists that the object’s owner \(o_0\) is a descendant of all other owner parameters \(o_1...n\) in the ownership tree. It is not immediately obvious why this condition is imposed — it is dictated by features interacting with subtyping [10].

A language designer could thus be posed the question, “What are the consequences of this extra check? Specifically, does adding the check reject any correct programs that were accepted prior to adoption of this rule?” We answer this question with Bonsai. First, we create a version of Ownership Java (OJ) called Reduced Ownership Java (ROJ) where subtyping is prohibited and the additional constraint on owners is removed. As expected, Bonsai fails to find a soundness counterexample in the ROJ language.

Now we are ready to query for the program of interest. This program must (1) fail the OJ typechecker, (2) pass the ROJ typechecker, and (3) succeed at runtime. This program is rejected by OJ, does not use subtyping, and is correct, per the three conditions in the query. Bonsai produces the following counterexample in just over a minute.

\begin{verbatim}
class Main<O1, O2> {
    Main<world, O2> main(Main<O1, O2> arg) {
        return new Main<world, O2>();
    }
}
new Main<world, world>.main(new Main<world, world>);
\end{verbatim}
This program fails the Ownership Java typechecker because `world` is not a descendant of `O2` in the ownership tree. However, this program is otherwise correct: it passes our Reduced Ownership Java typechecker and does not fail at runtime. While the counterexample itself is not particularly profound, it provides evidence that suggests that a less restrictive static type rule could be designed.

The comparison between OJ and ROJ is made possible by the novel way in which Bonsai allows users to compose sets of constraints when querying the solver. Crucially, by comparing two versions of a typechecker, we can better understand how the difference in type rules affects which programs are rejected.

**Simplifying Counterexamples with Minimization.** Ownership Java programs can easily grow extremely large, due to the presence of unused classes and methods. In order to have an easily-understandable counterexample, we present our SMT solver with an additional goal: to minimize the size of the generated program. Thus, the counterexamples listed above are minimal: no smaller program satisfies the constraints we imposed.

### 5.3 Dependent Object Types (DOT)

*Dependent Object Types*, or DOT [4] is both a formalization of the essence of Scala [40] and the basis for dotty [38], an experimental Scala compiler. This makes DOT a good candidate for a real-world type system that underlies a common, modern, and practical programming language.

DOT features *path-dependent types*. For example, the signature

```
feed(pet : Animal, food: pet.FoodType) : Boolean
```

describes a function whose second argument has a type `pet.FoodType` that depends on the first argument `pet`, hence the name path-dependent types. DOT also provides record types such as

```
{ name : String }
```

and intersection types such as `type Nat = Int ∧ Positive`.

In this section, we examine the Scala soundness issue SI-9633 [2, 3]. The issue was discovered by reasoning about extending DOT with some features of full Scala, namely the addition of the `null` object. In this section, we demonstrate how to add `null` to DOT and use the Bonsai Checker to find a counterexample to the issue.

With the Bonsai Checker library, we need just over 400 lines of code to implement DOT. The implementation closely follows the specification by Amin et al. [5]. For example, three of the subtyping rules are below:

\[
\begin{align*}
\Gamma \vdash T <: \top \\
\Gamma \vdash \bot <: T \\
\Gamma \vdash S <: T \\
\Gamma \vdash S <: U \\
\Gamma + S <: T \land U
\end{align*}
\]

Their corresponding implementations in Bonsai are almost direct translations:

```
(define/rec (dot-subtype? sub sup)
  (tree-match '(_,sub . ,sup)
    (Var Any) (lambda (T) #t)
    (Var _) (lambda (T) #t)
    (Var (and . (_ . _))) (lambda (S T U)
      (and (dot-subtype? sub T)
           (dot-subtype? sub U)))
    ...
  ))
```

The **NanoDOT language**. Figure 13 shows the syntax of the version of DOT we use to investigate the issue SI-9633. There are two deviations: first, for convenience, we add a global variable \( \varphi \)

---

Our Bonsai models of NanoDOT and NanoScala are available at https://bitbucket.org/bonsai-checker/dot/.

---
When given this program, the interpreter crashes, finding that \( \phi \) correct bounds. So, one can create the badly-bounded type

\[
\{ \text{val } a = \{ \text{val } a = \varphi \} \}
\]

much like how a single contradiction allows proving anything in an inconsistent logical system \[5\]. Otherwise, we would be able to collapse the subtype hierarchy and prove all types equivalent.

We now discuss two counterexamples found using BONS\textsc{ai}. The first explains a restriction imposed by DOT, while the second is a variant on the counterexample in SI-9633.

Disjoint domains in intersection types. DOT restricts intersection types and aggregate definitions to field-disjoint terms. That is, ANDDEF-I reads \[5\]:

\[
\Gamma \vdash d_1 : T_1 \\
\text{dom}(d_1) \cap \text{dom}(d_2) = \emptyset \\
\Gamma \vdash d_1 \land d_2 : T_1 \land T_2
\]

where \( d_i \) are definitions and \( \text{dom}(d) \) is the set of fields bound in \( d \).

A reader lacking expertise in type systems might not immediately see the rationale for this restriction. Such a reader can simply remove ANDDEF-I from our implementation of NanoDOT and observe the results. BONS\textsc{ai} constructs the following useful counterexample:

\[
\lambda ( b : \{ \text{val } a : \{ \text{val } a : T \} \}) : T \{ \\
b \cdot a \cdot a \\
\{ \text{val } a = \varphi \} \land \{ \text{val } a = \{ \text{val } a = \varphi \} \}
\}
\]

When given this program, the interpreter crashes, finding that \( b \cdot a \) is undefined. Since \( b \) is bound to an intersection where both sides have a value for the field \( a \), the value \( b \cdot a \) could be either \( \varphi \) or \( \{ \text{val } a = \varphi \} \). If the left side is chosen, we get a runtime error. If the right side is chosen, we get \( \varphi \). Since our implementation gives precedence to the left member of an intersection, our interpreter throws a runtime error, making this program a counterexample. In summary, this example points out that if the interpreter is to make a greedy choice among intersected values, these values must have no conflicting fields, which explains ANDDEF-I.

Collapsing the subtype hierarchy using bad bounds (SI-9633). A common soundness issue with dependent types relates to bad bounds, which allows creating uninhabitable types. For instance, the type \( \{ \text{type } S :> T <: \bot \} \), which means that \( T <: S <: \bot \), should clearly be uninhabitable. Otherwise, we would be able to collapse the subtype hierarchy and prove all types equivalent, much like how a single contradiction allows proving anything in an inconsistent logical system \[5\].

DOT does not check the bounds of dependent types but ensures that values have types with correct bounds. So, one can create the badly-bounded type \( \{ \text{type } S :> T <: \bot \} \) but values are...
syntactically constrained to have coinciding lower and upper bounds, using the construct \( \{ \text{type } S = \top \} \), which expands to \( \{ \text{type } S : \top <: \top \} \). As a result, while types with bad bounds can be created in DOT, it is impossible to instantiate values with badly-bounded dependent types. This restriction makes it easy to check the bounds of a value upon its instantiation.

In Scala, however, it is possible to create a value without going through a constructor, which makes bounds checking hard. Odersky \[39\] describes three situations where this is possible: using `lazy` for delayed instantiation, using `null` for uninstantiated values, and using type projection. In these cases, the compiler cannot check for bad bounds of dependent types. In Scala, it is possible to instantiate values with badly-bounded dependent types. This is the basis for SI-9633.

Scala soundness bugs, including SI-9633, are traditionally demonstrated with an example that casts an arbitrary value to the type `Nothing`. Since `Nothing` is a subtype of all other types, this allows us to cast a value to any other value, for example, via the sequence `Number → Nothing → String`. In practice, such an operation leads to the runtime `ClassCastException` in the JVM.

To explore bugs related to bad bounds, we extend NanoDOT to NanoScala with a `null` value of type \( \bot \). We also add a term to allow casting. The term `cast t to T` is typechecked in the usual manner, by ensuring that \( t : S \land S <: T \). At runtime, the cast performs the same check to detect counterexample programs. This is sufficient to model the relevant Scala issues in NanoScala.

We are now ready to query Bonsai for a NanoScala equivalent of the SI-9633 counterexample. This program was found in around 4 minutes:

```scala
cast
  (λ (a: { type b :> : \top <: : \bot }): a.b {
    \varphi
  })(cast null to { type b :> : \top <: : \bot})
to \bot
```

Why does this program typecheck? The inner cast typechecks because `null` is of type \( \bot \), which is a subtype of all types — this includes the type \( \{ \text{type } S : \top <: \bot \} \), regardless of its bad bounds. The outer cast to \( \bot \) typechecks because the type \( a.b \) is bounded above by \( \bot \); this is allowed because badly-bounded types can be created in DOT.

Why does the program fail at runtime? At runtime, the issue occurs when `null` is cast to \( \{ \text{type } S : \top <: \bot \} \). This type is uninhabited, yet, modeling Scala, NanoScala nevertheless casts `null` to it. The execution continues with a value with badly-bounded dependent types. As a consequence, we can now invoke the lambda expression synthesized by Bonsai. This lambda should not be called, since its argument is of an uninhabited type. However, when called anyway, it returns \( \varphi : \top \) which is cast to \( \bot \). This raises an exception.

In summary, NanoScala allows us to cast a non-null value to \( \bot \), which is at the heart of the SI-9633 counterexample. This is because the NanoScala `null` and cast terms violate NanoDOT’s property that all dependent types are backed with an instantiated value.

There are two important notes regarding our implementation of DOT: First, to prevent Bonsai from synthesizing counterexamples related to null dereferencing, we must tell Bonsai to disregard bugs involving the selection operator. Second, our current implementation of NanoDOT does not maintain type information at runtime. Thus, we only raise an exception on casts of non-null values to \( \bot \), which can be detected without any runtime type information. This is sufficient to catch the runtime errors we are interested in; the Scala example `Number → String` would fail when `Number` is cast to `Nothing`, which we model as DOT \( \bot \).

Finally, we can translate the NanoScala counterexample to Scala. We must manually make two minor changes. First, unlike DOT, Scala does check type annotations that have bad bounds. A
trait A { type L <: Nothing }
trait B { type L >: Any }
def toL(b: A with B): b.L = 123
val p: B with A = null
toL(p): Nothing

trait A { type L <: Nothing }
trait B { type L >: Any }
def toL(b: B)(x: Any): b.L = x
val p: B with A = null
println(toL(p)("hello"): Nothing)

Discussion. We demonstrate the use of Bonsai in studying DOT, a language with a complex type
system. We provide two subtle bugs that can be introduced in DOT by modifying the language, and
show that in each case Bonsai produces elegant counterexamples after only a couple of minutes.
The latter of these counterexamples reproduces a soundness counterexample for the Scala compiler,
SI-9633.

6 EMPIRICAL EVALUATION OF PERFORMANCE
In this section, we evaluate the efficiency of Bonsai. We consider the speed of symbolic evaluation
and solving, the sizes of the counterexamples, and the sizes of program spaces that are explored. We
find that Bonsai reliably finds bugs in a few seconds or minutes. When left to run for longer periods
of time, Bonsai explores spaces containing programs much larger than the minimal counterexample,
providing a margin of assurance.

6.1 Comparing Bonsai with Fuzzers
Here, we compare Bonsai to the fuzzers mentioned in Section 7: in particular, the syntactic fuzzer
built into Redex [30], and the type fuzzer based on judgement trees [18].
The benchmark. We implemented the “stlc-lists” language from the Redex benchmark [45] in Bonsai and introduced each of the nine bugs to our implementation (these were all one-line changes). We compared the performance of Bonsai to the performance of the syntactic fuzzer and type fuzzer from the Redex benchmark in two respects: first, the time taken to find a counterexample, and second, the average size of counterexamples found. Note that all tests were run on the same machine.

Results. Our results are plotted in Figure 15 (left), and the times and sizes, correlated by bug, are listed in Table 1. We make two important observations below.

First, we note Bonsai’s consistency. While fuzzers are slightly faster than Bonsai on some benchmarks, they are several orders of magnitude slower on others, taking anywhere from a few minutes to over an hour. Bonsai, on the other hand, takes between 1 and 5 seconds on every benchmark. Consider, for example, bugs 4 and 5, which were the hardest for fuzzers to find, despite both being labeled “shallow” errors in the benchmark suite. Bug 5 changes the interpreter to return the head of a list when \texttt{tail} is applied, while bug 4 assigns the return type of \texttt{cons} to \texttt{int}, rather than \texttt{Listof int} as expected. Bonsai finds these bugs just as fast as it finds other bugs. However, these bugs are many orders of magnitude more difficult for fuzzers to reveal. This is because these bugs require a specific set of interactions in the counterexample program, and thus the probability of randomly generating such a program is extremely low.

Second, we compare the sizes of counterexamples produced by the two tools. The type fuzzer, though efficient, generates extremely large and complex programs as counterexamples. This likely contributes to its effectiveness, since larger programs are likelier to uncover soundness bugs [62]. However, such programs are extremely difficult for users to reason about, with (for example) dozens of layers of nested lambdas. In contrast, Bonsai consistently produces small counterexamples. In almost all cases, Bonsai’s counterexamples were identical to the ones suggested by the Redex benchmark authors — one was smaller.

Inspired by the latter observation, we evaluated Bonsai’s efficiency on constructing minimal counterexamples by querying the solver for a solution that minimizes the size of the counterexample. Our results are shown in Table 2. We find that the most of the counterexamples produced by Bonsai were already minimal, even without the minimization query. However, for the three bugs that could be further minimized, the minimization query only took 10%-30% longer than the standard query.

As a final demonstration of Bonsai’s efficiency compared to fuzzers, we consider a historic bug involving let-polymorphism in the presence of mutable references, in e.g. ML [55, 61]. This “classic let+references” bug is included in the Redex benchmark as let-poly-2. However, the syntactic fuzzer has been run on it for several days with no results. The type fuzzer cannot model let-poly due to the presence of polymorphism, which requires the generator to make parallel choices that must match up, and CPS-transformed type judgements, which impede its termination heuristics [18]. Bonsai, on the other hand, finds this bug in just over twenty minutes\(^3\), with a counterexample almost identical to the one presented in literature [55].

6.2 Comparing Bonsai with Pipal

We compare the scalability of Bonsai to the the scalability of Pipal as described by Roberson et al. [44]. This comparison is only an approximation because Bonsai and Pipal encode different search spaces (symbolic programs and symbolic intermediate states, respectively). Thus, for example,

\(^2\)Our Bonsai model of stlc-lists, with patchfiles to introduce each bug, is available at https://bitbucket.org/bonsai-checker/stlc-benchmark/.

\(^3\)Our Bonsai model of let-poly-2 is available at https://bitbucket.org/bonsai-checker/let-poly/.
Table 1. The performance of Bonsai and the syntax and type fuzzers described in the text. Note that fuzzer statistics are provided as ratios against Bonsai statistics. "*" indicates a lower bound due to a one-hour timeout. Size is measured in nodes (n) as per the Redex benchmark: "the number of pairs of parentheses and atoms in the s-expression representation of the term" [45]. Figure 15 (left) represents this data on a scatter plot. Note that the type fuzzer cannot model let-poly-2 due to polymorphism and CPS-transformed judgement rules [18].

| Bug     | Bonsai (sec) | Syntax/Bonsai | Type/Bonsai | Bonsai | Syntax/Bonsai | Type/Bonsai |
|---------|--------------|---------------|-------------|--------|---------------|-------------|
| stlc-1  | 0.86s        | 0.61          | 1           | 5n     | 1             | 57          |
| stlc-2  | 0.99s        | 5.6           | 7.7         | 9n     | 1             | 14          |
| stlc-3  | 0.84s        | 0.25          | 0.054       | 5n     | 3.6           | 51          |
| stlc-4  | 3s           | 1200+         | 34          | 17n    | N/A           | 1.8         |
| stlc-5  | 0.99s        | 3600+         | 40          | 9n     | N/A           | 14          |
| stlc-6  | 2s           | 1000          | 15          | 17n    | 0.76          | 5.7         |
| stlc-7  | 1s           | 8             | 0.07        | 9n     | 2             | 19          |
| stlc-8  | 4.6s         | 14            | 0.061       | 29n    | 0.88          | 6.2         |
| stlc-9  | 1.2s         | 0.4           | 0.049       | 15n    | 1.4           | 13          |
| Full suite | 16s         | 600+          | 12          |        |               |             |

Table 2. The performance of Bonsai on constructing minimal counterexamples. The star (*) indicates that the minimized counterexample is smaller than the one produced without the minimization query.

| Bug     | Time (sec) | Min. Size |
|---------|------------|-----------|
| stlc-1  | 0.92s      | 5n        |
| stlc-2  | 1.1s       | 9n        |
| stlc-3  | 0.99s      | 5n        |
| stlc-4  | 2.7s       | 17n       |
| stlc-5  | 1.0s       | 9n        |
| stlc-6  | 2.6s       | 13n*      |
| stlc-7  | 1.2s       | 5n*       |
| stlc-8  | 4.9s       | 21n*      |
| stlc-9  | 1.3s       | 15n       |

Bonsai must model all heap objects, while Pipal must limit the exploration to four heap objects and n integer literals. Furthermore, the Pipal experiments were performed using different symbolic execution frameworks and solvers. Nevertheless, Pipal results for comparable trials are listed in Table 3.

These measurements illustrate that Bonsai and Pipal generally scale to around the same order of magnitude. Thus, we find that even though Bonsai performs symbolic execution on both the typechecker and the interpreter, it still has comparable performance to Pipal, which only performs symbolic execution on the typechecker. That is, compared to Pipal, Bonsai does not sacrifice efficiency for its versatility and ease-of-use.

6.3 The Bonsai Encoding

Here, we compare the classical “syntax tree” described in Section 3 (with subtree sharing) against the Bonsai tree. We evaluate the time required to check a type system as a function of the program
space size. We break down the time into symbolic evaluation and solving. We find that the Bonsai tree scales significantly better than the classical syntax tree, allowing us to explore much larger search spaces in the same amount of time.

The benchmark. We implemented a simple recursive-descent interpreter for the lambda calculus, bounded at 4 recursive calls. We then added a ‘typechecker’ that checks for free variables. We searched for counterexamples using both encodings, with the same typechecker and interpreter. Thus, the only variable between the trials is the symbolic tree encoding used and its size.

In this setup, there are no possible counterexamples. This is intentional: a satisfiable formula requires the solver to reach any valid model, whereas an unsatisfiable formula requires the solver to visit all models and prove that none are satisfiable. Thus, an unsatisfiable formula represents a more taxing stress test for the solver. In practice, we find that satisfiable queries almost always take less than half the time that unsatisfiable queries take, and are usually even faster.

Since tree ‘size’ has different meanings for the classical and Bonsai encodings, we normalize by the number of syntactically valid ASTs represented by a symbolic tree of a given depth.

Results. The graph in Figure 15 (right) shows that the Bonsai encoding allows us to scale to a much larger number of ASTs than the classical encoding in the same amount of time. For example, if we impose a time limit of approximately 20 seconds, then the classical encoding explores less than $10^{25}$ ASTs while the Bonsai encoding explores more than $10^{50}$ ASTs. Similarly, if we want to explore $10^{25}$ trees, then Bonsai is roughly a thousand times faster than the classical encoding. In our experience, the classical encoding does not scale to be able to catch most of the counterexamples discussed in this paper.

Note that the Bonsai encoding consistently spends less time in the solver than the classical encoding, even though Bonsai pushes the complexity of the formal grammar to the solver. This suggests that even though Bonsai often creates larger symbolic trees than the classical encoding to represent the same AST, it has a performance advantage in both symbolic execution and the solver.

In practice, we rarely search ASTs beyond depth 10; smaller trees are always enough to synthesize counterexamples identified by experts (see Table 3). Nevertheless, we provide data for much larger trees in this graph to show that the encoding is scalable to much larger spaces if needed.
| Language       | Description                                                                 | Parameters                  | $|S|$ | $|ce|$ | $t_{Bonsai}$ | $t_{Pipal}$ | $|S_+|$ | $|ce_+|$ | $t_+$ |
|---------------|----------------------------------------------------------------------------|-----------------------------|------|-------|-------------|-------------|-------|-------|-------|
| Arithmetic    | Typed arithmetic [41]                                                      | depth = 9, no bound        | $10^4$ | 13     | 1s          | 0.5s        | $10^{65}$ | 99     | 11m   |
| Simply-typed λ-calculus | From Redex [45], bug 5                                                   | depth = 5, bound = 4        | $10^4$ | 9      | 0.97s       | —           | $10^{21}$ | 43     | 1.7m  |
| Featherweight Java       | A minimal core calculus for Java with inheritance [26]                   | # classes = 3, # methods = 3 | $10^{27}$ | 42     | 3s          | 2.1s        | $10^{119}$ | 163    | 13m   |
| Ownership Java         | Featherweight Java augmented with ownership types [11]                   | # classes = 2, # methods = 4 # owners = 2, depth = 8, bound = 7 | $10^{63}$ | 72     | 17s         | >250s       | $10^{342}$ | 158    | 30m   |
| NanoDOT               | Subset of dependent object type calculus [5]                             | depth = 10, obj depth = 6, no bound | $10^{19}$ | 23     | 211s        | —           | $10^{39}$  | 49     | 8m    |
| NanoScala            | NanoDOT augmented with casts and null                                    | Same as NanoDOT            | $10^{20}$ | 24     | 175s        | —           | $10^{64}$  | 39     | 48m   |

Table 3. An overview of the languages implemented with Bonsai. Here, $t_{Bonsai}$ is the time required to catch the soundness bug described in the text, $|ce|$ is the size of the counterexample measured as in Table 1, and $|S|$ is the number of syntactically-correct programs explored. $t_+$, $|S_+|$, and $|ce_+|$ are analogous, but for long-running trials. We show relevant comparisons with Pipal where possible.

6.4 Summary of Case Studies

The benchmark. We implemented several different languages with Bonsai, and searched for counterexamples by planting bugs in the typecheckers. We collected data for the smallest parameters needed to catch these bugs, as well as for scaled-up trials. The scaled-up trials were run with the same queries, but parameters modified in order to search larger spaces.

Results. Table 3 lists data collected for several languages implemented with Bonsai. We find that each of the counterexamples, including those found by experts, can be synthesized by Bonsai within a few minutes. This suggests that Bonsai is suitable for interactive use while developing a type system.

Additionally, we find that Bonsai scales to much larger search spaces effectively, searching through larger, more complex programs. Indeed, the new ‘scaled-up’ counterexamples were often several times as large as the ‘minimal’ counterexamples. Thus, while bounded model checking is not a substitute for a formal proof, these results suggest that Bonsai can be used to search through large spaces of programs efficiently to provide confidence that the type system is most likely sound—or, alternatively, reveal a large counterexample that may not be found by a human.

Finally, we note that Bonsai performs well on a wide variety of languages, allowing users to easily implement features such as closures, environments, classes/objects, ownership types, dependent types, records, and polymorphism in their languages.
7 RELATED WORK
Most existing tools for type system designers are focused on checking for soundness bugs. Here, we discuss the various techniques used to detect such bugs. Empirical evaluations of BONSAI against these tools are presented in Section 6. Additionally, we discuss additional related work in theorem-proving and symbolic execution.

7.1 Fuzzing
A type system fuzzer generates random programs and tests them on the typechecker and interpreter until it finds one that (1) passes the typechecker, and (2) fails in the interpreter. If such a program is found, it witnesses a soundness bug.

Fuzzing presents serious scalability challenges. Recall Figure 1, which shows a high-level overview of the fuzzing process. At each step, the parser, typechecker, or interpreter rejects the vast majority of its inputs, making the probability of a random program witnessing a soundness error very small. Modern fuzzers address this problem by carefully generating random programs that are likelier to be counterexamples. For example, syntactic fuzzers such as the one used by PLT Redex [30] generate random syntactically-valid programs, bypassing the parser. Syntactic fuzzers have been shown to find a wide variety of bugs in real languages [29]. However, they are still not scalable enough to catch many simple bugs [18].

A type fuzzer generates random well-typed programs, bypassing both the parser and the typechecker. Type fuzzing can be done in many ways: for example, by using constraint logic programming to express the typechecker declaratively (this has been used to fuzz Rust [15]) or by generating random type judgment trees and then using that to derive a program (this has been used to fuzz GHC [18]).

Fuzzing is an effective technique for two reasons. First, a fuzzer can easily check actual implementations of languages. This eliminates the need to formalize the language, and allows users to catch implementation-dependent bugs that would be missed by a formalization. Second, a fuzzer produces a concrete counterexample program, which makes it easy to diagnose and fix the soundness issue. This claim is supported by a recent study, which used NanoMaLy [47] to produce counterexamples that cause ill-typed ML programs to crash. Students who were given NanoMaLy’s counterexamples in an exam setting were 10% to 30% more likely to correctly explain and fix the type error than students who were given only a printout of the compiler error.

However, fuzzers have a critical weakness: by their nature, random fuzzers are non-exhaustive. If after 24 hours no soundness error has been discovered, we cannot assume that the typechecker is sound. Indeed, neither syntactic fuzzers nor type fuzzers are able to discover certain simple bugs in the PLT Redex benchmark [29] after many hours [18], because those bugs are only witnessed by a small set of programs that are extremely unlikely to be generated randomly.

7.2 Handling non-exhaustiveness with Pipal
One specialized kind of type fuzzer does make some exhaustiveness guarantees. Pipal [43, 44] requires a typechecker to be encoded as a set of constraints imposed on a finite set of intermediate program states. At each iteration, Pipal queries a constraint solver for a well-typed state and performs a single step of evaluation to see whether progress or preservation were violated.

In order to search the space of intermediate states efficiently, Pipal carefully monitors how the intermediate state was manipulated during the single step of evaluation. If the program is not a counterexample, Pipal uses this information to add more constraints to the set of programs, effectively pruning the search space for the next iteration. Eventually, if no counterexample is found,
the solver returns UNSAT, which implies that the search space has been exhaustively checked. That is, no state within that space witnesses a soundness bug.

While Pipal is scalable, it requires typecheckers to be manually rewritten as a set of declarative constraints and carefully-formulated invariants on intermediate states. Not only is this tedious for the user, but it also prevents users from directly checking the implementations of typecheckers. Thus, Pipal loses one of the primary benefits of fuzzing, which is to check executable implementations directly.

The Bonsai algorithm, on the other hand, combines the ease-of-use of a fuzzer with the scalability of Pipal. Bonsai uses symbolic execution to automatically convert an executable language implementation into constraints for a solver, and it directly solves for a counterexample with a single call to the solver. In this sense, Bonsai may be regarded as a final successor to the type fuzzer, deriving many of Pipal’s scalability benefits “for free” without any additional effort on the part of the user. Importantly, Bonsai searches for full executable programs as counterexamples: these programs are easier to understand than the intermediate states reported by Pipal. Finally, unlike Pipal’s iterative algorithm, Bonsai makes only one query to the solver. This property allows us to ask the solver a variety of questions beyond soundness, and is at the heart of Bonsai’s versatility.

7.3 Theorem Proving

If a fuzzer or a bounded model checker fails to find a witness to a soundness bug, then we cannot claim to have a proof of soundness: there is always the possibility that a program larger than the bound might expose a bug. In practice, the small-scope hypothesis [6] conjectures that all soundness bugs will be revealed by small witness programs, and thus for a sufficiently large search bound the failure to find a counterexample is usually compelling evidence (but not proof) that the type system is correct.

Unlike fuzzing or bounded model checking, however, a mathematical proof of soundness provides complete assurance that a type system is sound. Such proofs can be developed manually or using proof assistants such as [53] and [37]. For instance, Drossopoulou and Eisenbach [17] prove soundness of a subset of Java manually, while Nipkow and von Oheimb [36] do so using Isabelle/HOL [37].

Unfortunately, theorem-proving requires a formalized model of the language, such as Featherweight Java [26] for Java and DOT [4] for Scala. Most modern languages are too complex to be fully formalized, and the models that do emerge often take years of development. Furthermore, the actual proofs are often long and tedious, requiring significant manual effort. For example, the mechanized proof of the DOT language [4] using the Coq proof assistant [53] consists of several thousand lines of code [46], even though the language can be formalized in a couple of pages [5]. We hope that Bonsai may aid in model design in a fashion similar to PLT Redex [29].

7.4 Symbolic execution and synthesis

Bounded verification tools [1, 13, 16, 24, 31, 32, 50, 52, 59] encode the concrete semantics of the language and translate the program (or just one execution path) into a logical formula. The formula represents constraints whose solution answers queries about symbolic inputs to the program. Symbolic execution has become competitive in advanced analysis tasks, such as analysis of high-order contracts [35].

Generation of a counterexample program is related to program synthesis because we can think of the type system as the specification: the desired program passes the typechecker and fails in the interpreter. Synthesizers can be roughly divided into three kinds: rewriting [20, 23, 42], deductive [33, 34, 51], and those based on searching a space of programs with constraint solvers [7,
25, 50, 51, 60]. While the first two categories derive a program from a specification, the last category searches a space of programs, conceptually evaluating each against the specification. This search is analogous to Bonsai’s search for the counterexample program.

Reducing the path explosion during symbolic evaluation has been previously addressed in ESC/Java [21] and Rosette [57].

Bonsai trees are a so-called symbolic data structure [8]: despite producing an unusual encoding when symbolically evaluated, they offer programmers the usual AST interface, facilitating specification of compact (big step) typecheckers and interpreters.

8 CONCLUSION

Bonsai is a type system designer’s assistant, which uses symbolic evaluation to convert typecheckers and interpreters into constraints. By combining these constraints, Bonsai can query a constraint solver for programs with specific properties. Such queries can be used to find soundness errors (including intricate bugs that took experts many years to discover), compare two type systems, find unnecessary restrictions, and even synthesize simple typecheckers automatically.

Bonsai uses a novel data structure to encode sets of abstract syntax trees. This allows it to scale to extremely large search spaces, while still being fast enough to use interactively while designing a type system. Together, these results suggest that Bonsai can significantly aid type system designers in developing novel type systems.

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