Chiral and Heavy Quark Symmetry Violation in B Decays

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The most general Lagrangian consistent with chiral, heavy quark, and strong interaction symmetries to order $\frac{1}{M}$ and to linear order in the $SU(3)$ vector and axial currents is presented. Two new dimensionful and five dimensionless couplings arise at this order. The heavy to light flavor changing current is derived to the same order, giving rise to two additional dimensionful constants and six dimensionless ones. The dimensionless parameters are shown to be irrelevant at $O(\frac{1}{M})$. Analytic SU(3) counterterms are also considered. Form factors for $D \to \pi l \nu$ and $B \to \pi l \nu$ are computed at $O(\frac{1}{M}, m_{s}^{0})$ and $O(M^{0}, m_{s}^{1})$. The eight decay constants $f_{D}, f_{D_{s}}, f_{D_{s}^{*}}, f_{B}, f_{B_{s}}, f_{B_{s}^{*}}$ and $f_{B_{s}^{*}}$ are computed at $O(\frac{1}{M}, m_{s})$ in terms of seven parameters which can be determined by $\{B, D\} \to \{K, \pi\} l \nu$ decays, and two undetermined counterterms. The ratio $R_{1} = \frac{f_{B_{s}}}{f_{B}} / \frac{f_{D_{s}}}{f_{D}}$ is expressed in terms of four parameters.

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I. INTRODUCTION

The incorporation of chiral and heavy quark symmetries into a Lagrangian governing the interactions of heavy mesons with pseudo-Nambu-Goldstone bosons leads to new relations among heavy meson leptonic and semi-leptonic form factors. Because the charm quark mass is not large, and the strange mass not small, one expects significant deviations from the heavy quark and chiral limits in many processes. It is therefore important to compute these corrections both to improve the accuracy of theoretical relations and to test the validity of the perturbative expansion.

SU(3) corrections to heavy meson decay constants have been found to be around 20% in a chiral log approximation\cite{1}. There is suggestive evidence that there are, in addition, large heavy quark symmetry violations\cite{2–4}, although the question remains open\cite{5}. However, the leading corrections to decay constants which violate both chiral and heavy quark symmetries have not, to our knowledge, been examined. That is the primary purpose of this paper.

Quantities which are sensitive only to violations of both symmetries, such as the ratio

\[ R_1 = \frac{f_{B_s}}{f_B} \frac{f_{D_s}}{f_D}, \]

(1)

can be predicted with greater accuracy\cite{6}, and can provide information on \(\frac{\Lambda_{\chi}}{M_B}\) and \(\frac{m_s}{\Lambda_{\chi}}\) coupling constants. These couplings also enter into such processes as \(B \to D\) and \(B \to K\) semileptonic decays, \(B \to \overline{B}\) mixing, and hyperfine mass splittings\cite{7,8}.

To see how \(R_1\) deviates from unity requires the heavy quark chiral Lagrangian to \(\mathcal{O}\left(\frac{1}{M}\right)\). We derive both the Lagrangian and the heavy-light current to this order in the next section. We also present analytic counterterms linear in the light quark mass matrix. Implications for \(B \to \pi l\nu\) decays are then considered, with emphasis on separately heavy quark or chiral symmetry violating quantities. The remainder of the paper is devoted to a computation of the leading heavy quark and chiral symmetry violating corrections to heavy meson decay constants, including both analytic and nonanalytic contributions.

The low momentum strong interactions of \(B\) and \(B^*\) mesons are governed by the chiral Lagrangian \cite{9}

\[ \mathcal{L} = - \text{Tr} \left[ \mathcal{P}_a(v) i v \cdot D_{ba} H_b(v) \right] \\
+ g \text{Tr} \left[ \mathcal{P}_a(v) H_b(v) A_{ba} \gamma_5 \right] . \]

(2)
Operators suppressed by powers of the heavy meson mass $1/M_B$, factors of a light quark mass $m_q$, or additional derivatives have been omitted. The field $\xi$ contains the octet of pseudo-Nambu-Goldstone bosons

$$ \xi = \exp \left( i \Pi / f \right), $$

where

$$ \Pi = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
\pi^- - \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
K^- \\
-\sqrt{\frac{2}{3}} \eta 
\end{pmatrix}. $$

The bosons couple to heavy fields through the covariant derivative and the axial vector field,

$$ D_{ab}^\mu = \delta_{ab} \partial^\mu + V_{ab}^\mu = \delta_{ab} \partial^\mu + \frac{1}{2} \left( \xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger \right), $$

$$ A_{ab}^\mu = \frac{i}{2} \left( \xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger \right) = -\frac{1}{f} \partial_\mu \Pi_{ab} + O(\Pi^3). $$

Both the vector and axial vector fields are traceless, $A_{aa}^\mu = 0 = V_{aa}^\mu$. Since $f_\pi$ and $f_K$ are equivalent at tree level, we will use $f_\pi \simeq 132$ MeV when pions are involved, and $f_K \simeq 161$ MeV otherwise.

Under chiral $SU(3)_L \times SU(3)_R$ transformations,

$$ \xi \rightarrow L \xi U^\dagger = U \xi R^\dagger, $$

where $L$ and $R$ are elements of $SU(3)_L$ and $SU(3)_R$, respectively, and $U$ is defined implicitly by Eq. (3). The $B$ and $B^*$ heavy meson fields are incorporated into the $4 \times 4$ matrix $H_a$:

$$ H_a = \frac{1}{2} (1 + \phi) \left[ \overline{B}_a^\mu \gamma_\mu - \overline{B}_a \gamma_5 \right], $$

$$ \overline{H}_a = \gamma^0 H_a^\dagger \gamma^0. $$

The four-velocity of the heavy meson is $v^\mu$, and the index $a$ runs over light quark flavor. The bar over $B$ will sometimes be omitted for notational simplicity. Under $SU(2)_c$ heavy quark spin symmetry and chiral $SU(3)_L \times SU(3)_R$ symmetry, $H_a$ transforms as

$$ H_a \rightarrow S(HU^\dagger)_a, $$

1 Unless the deviation of the expansion mass from $M_B$ is suppressed by $1/M$, the Lagrangian (2) must include an explicit mass term. We choose the spin-weighted average of the $B$ and $B^*$ masses as our expansion parameter, allowing us to use the equation of motion $v \cdot k \sim \frac{1}{M^2}$, where $k$ is the residual momentum of the meson.
where $S \in SU(2)_v$. The covariant derivative has simple transformation properties under chiral $SU(3)_L \times SU(3)_R$:

$$D \bar{H} \to U(D \bar{H}),$$

(9)

$$D \xi \to U(D \xi) \bar{R}^\dagger$$

and

$$D \xi^\dagger \to U(D \xi^\dagger) \bar{L}^\dagger.$$  

(10)

The left handed current is represented by

$$J^\lambda_a = \frac{i\alpha}{2} \text{Tr}[\Gamma H_0(v)\xi_{ba}]$$

(12)

where $\Gamma = \gamma^\lambda(1 - \gamma_5)$. Tree level matching to the definition of the decay constant $f_B$ gives $\alpha = \sqrt{M_B f_B}$.

All formulas hold for the D meson as well, after the substitution $M_B, f_B \to M_D, f_D$. Thus the axial coupling constant $g$ is responsible for $D^* \to D\pi$ decays.

II. 1/M CORRECTIONS

To go beyond leading order, we make some approximations based on the formal hierarchy $m_\pi \ll m_K \ll \Lambda \sim \Lambda_\chi \ll M$, where $\Lambda = M_B - M_b$, $M$ is a heavy meson mass, and $\Lambda_\chi$ is the chiral symmetry breaking scale. Theoretically this hierarchy holds arbitrarily well in the $m_{u,d} \ll m_s \to 0$ and $M \to \infty$ limits. However, the relation $\Lambda \sim \Lambda_\chi$ should be taken loosely; factors of three or four will be important numerically. Such factors can only be found by explicit calculation, or by comparing to experiment. Since $\Delta = M_{D^*} - M_D \sim m_\pi$, we do not make assumptions about the size of the ratio $m_\pi/\Delta$.

New parameters appear in the Lagrangian and current beyond leading order. Since there are more experimental observables than parameters, verifiable relations will exist. As a means to organize the calculation, we will use $B \to \pi l \nu$ and its symmetry-related decays to fix, in principle, as many of these parameters as possible, then describe the various $B$ and $D$ decay constants in terms of these parameters. Accordingly, we will examine corrections to $B \to \pi l \nu$ form factors which violate either $SU(3)$ or heavy quark symmetry, but not both. Since $SU(3)$ loop corrections have been computed elsewhere [10], this task requires only the use of tree graphs. Both loop graphs and analytic counterterms, however, enter into decay constant calculations. For decay constants, we include terms which violate $SU(3)$ and heavy quark symmetry simultaneously.
For $m_s = 0$, operators appearing in the Lagrangian at $\mathcal{O}(1/M)$ may be either dimension four, with dimensionful coupling constants of order $\Lambda$, or dimension five, with additional derivatives. As will become evident, dimension five operators containing multiple factors of the axial or vector fields $A$ or $V$ will be relevant to neither tree level $B \to \pi e\nu$ nor one-loop decay constant calculations, so they will be ignored. The remaining higher derivative operators formally give contributions to $B \to \pi e\nu$ of order $k\Lambda^2/M^0$, where $k$ is the pion momentum, so they can be neglected to this order in the chiral expansion. This approximation is best in the kinematic region $k \sim m_\pi$. Similar statements apply to higher derivative operators in the current. These operators could also enter the calculation of the decay constants, but their $\mathcal{O}(1/M)$ contributions vanish, as we shall see below explicitly. Such higher derivative operators will be included in our construction of the Lagrangian and current, but we will show that they play no role in the final results.

An important restriction on the Lagrangian is that it satisfy velocity reparametrization invariance (VRI). To ensure a velocity reparametrization invariant Lagrangian, one must use velocities and derivatives on heavy fields in the combination $v^\mu + iD^\mu M$. One should also use fields which transform by only a phase under velocity reparametrization. Such a field is

$$\tilde{H} = H + \frac{1}{2M}iD^\mu [\gamma^\mu, H]$$

where $D^\mu$ is the covariant derivative.

The VRI consistent Lagrangian resulting from generalizing Eq. (2) in this way is

$$\mathcal{L} = - \operatorname{Tr} \left[ H_a(v)(iv \cdot D_{ba} - \frac{1}{2M}D^2_{ba})H_b(v) \right]$$

$$- \frac{2}{M} \operatorname{Tr} \left[ H_a(v)(iv \cdot D)_{ba}H_b(v) \right] + g \operatorname{Tr} \left[ H_a(v)H_b(v)A_{ba}\gamma_5 \right]$$

$$+ \frac{g}{M} \operatorname{Tr} \left[ H_c(v)(i\delta_{bd}\tilde{D}^\mu_{ac} - i\delta_{ac}D^\mu_{bd})H_d(v)\gamma_\mu\gamma_5 \right] v \cdot A_{ba}$$

$$- \frac{g}{M} \operatorname{Tr} \left[ H_c(v)(i\delta_{bd}v \cdot \tilde{D}_{ac} - i\delta_{ac}v \cdot D_{bd})H_d(v)A_{ba}\gamma_5 \right].$$

VRI fixes the coefficients of the $D^2$ and $(i\tilde{D}^\mu - iD^\mu)$ operators, but the other operators (involving $v \cdot D$) are unconstrained because they obey VRI, to the order we are working.

Another restriction is that the Lagrangian must be time reversal invariant. Under time reversal, the pseudoscalar field transforms as $B_\nu(x) \to B_\bar{\nu}(-\bar{x})$ and the vector field transforms as $B_{\nu}^{*\mu}(x) \to B_{\bar{\nu}}^{\mu}(-\bar{x})$, where $\bar{x}, \bar{v}$ are the parity reflections of $x$ and $v$ (e.g. $\bar{v}^\mu = v_\mu$). Although the parameter $v$ is unchanged by time reversal, the field label in $B_{\nu}^{*\mu}$
alters to reflect the transformed transversality equation \( v \cdot B^* = 0 \to \bar{v} \cdot B^* = 0 \). From this and the anti-unitary nature of time reversal, it follows that

\[
H_v(x) \to TH_v(-\bar{x})T^{-1}
\]

and

\[
\overline{H}_v(x) \to T\overline{H}_v(-\bar{x})T^{-1},
\]

where \( T \) is a dirac matrix obeying \( T \gamma^\mu T^{-1} = \gamma^*_\mu \). Since the pseudoscalars transform as \( \Pi(x) \to -\Pi(-\bar{x}) \), the axial current transforms as \( A^\mu(x) \to A_\mu(-\bar{x}) \). The condition of time reversal invariance then forbids operators such as \( \text{Tr} \left[ \overline{H}_a(v)\sigma^{\mu\nu}H_b(v)\gamma_{\mu\gamma_5}A_{\nu ba} \right] \) and \( i\text{Tr} \left[ \overline{H}_a(v)\sigma^{\mu\nu}v \cdot D_{bc}H_c(v)\gamma_{\mu\gamma_5}A_{\nu ba} \right] \) from appearing in the Lagrangian.

The most general form of the heavy meson Lagrangian subject to the above constraints is

\[
\mathcal{L}_M = -(1 + \frac{\epsilon_1}{M}) \text{Tr} \left[ \overline{H}_a(v)i\nu \cdot D_{ba}H_b(v) \right] \\
+ (g + \frac{g_1}{M}) \text{Tr} \left[ \overline{H}_a(v)H_b(v)\bar{A}_{ba}\gamma_5 \right] \\
+ \frac{g_2}{M} \text{Tr} \left[ \overline{H}_a(v)\bar{A}_{ba}\gamma_5H_b(v) \right] \\
+ \frac{\lambda_2}{M} \text{Tr} \left[ \overline{H}_a(v)\sigma^{\mu\nu}H_a(v)\sigma_{\mu\nu} \right] \\
+ \frac{\epsilon_2}{M} \text{Tr} \left[ \overline{H}_a(v)\sigma^{\mu\nu}i\nu \cdot D_{ba}H_b(v)\sigma_{\mu\nu} \right] \\
- \frac{\delta_0}{M} \text{Tr} \left[ \overline{H}_a(v)(iD)^2_{ba}H_b(v) \right] \\
+ \frac{\delta_1}{M} \text{Tr} \left[ \overline{H}_c(v)(i\delta_{ba}\bar{D}_{ac} + i\delta_{ac}D^\mu_{db})H_d(v)\gamma_{\mu\gamma_5} \right] v \cdot A_{ba} \\
+ \frac{\delta_2}{M} \text{Tr} \left[ \overline{H}_a(v)H_b(v)\gamma_{\mu\gamma_5} \right] i\nu \cdot D_{bc}A^\mu_{ca} \\
+ \frac{\delta_3}{M} \text{Tr} \left[ \overline{H}_a(v)H_b(v)\gamma_{\mu\gamma_5} \right] D^\mu_{bc}i\nu \cdot A_{ca} \\
+ \frac{\delta_4}{M} \text{Tr} \left[ \overline{H}_a(v)i\nu \cdot D_{cb}H_c(v)\bar{A}_{ba}\gamma_5 \right] \\
+ \frac{\delta_5}{M} \text{Tr} \left[ \overline{H}_a(v)\sigma^{\mu\nu}(i\nu \cdot D)^2_{ba}H_b(v)\sigma_{\mu\nu} \right] \\
+ \frac{\delta_6}{M} \text{Tr} \left[ \overline{H}_a(v)(i\nu \cdot D)^2_{ba}H_b(v) \right]
\]

(15)

where all couplings are taken to be real. The effect of \( \lambda_2 = -\frac{M\Delta}{2} = -\frac{M}{2}(M_{B^*} - M_B) \) is merely to shift the \( B \) and \( B^* \) propagators to \( \frac{-i}{2(v \cdot k + \frac{4\Delta}{3})} \) and \( \frac{-i\nu v}{2(v \cdot k - \frac{4\Delta}{3})} \), respectively, so we will ignore the \( \lambda_2 \) term once we make this shift.
We may take $\epsilon_1$ and $\epsilon_2$ to be zero by making a spin and flavor dependent renormalization of the heavy meson fields and then redefining the coupling constants to absorb the $\epsilon$ dependence. The current is then constructed using the new, properly normalized, fields.

VRI implies $\delta_0 = \frac{1}{2}$ and $\delta_1 = \gamma$. However, we will eventually show that all the higher derivative terms (with coefficients $\delta_i$) contribute only at order $\frac{1}{M_F}$, so we will omit these terms for now.

For $m_s \neq 0$, the Lagrangian contains additional terms involving the light quark mass matrix $m_q = \text{diag}[0,0,m_s]$. To the order we are working, only operators linear in $m_q$ and inserted in tree graphs are relevant. The SU(3) violating Lagrangian contains

$$\mathcal{L}_m = 2\lambda_1 \text{Tr} \left[ \overline{H}_a(v) H_b(v) \right] \mathcal{M}_{ba}^+ + 2\lambda'_1 \text{Tr} \left[ \overline{H}_a(v) H_a(v) \right] \mathcal{M}_{bb}^+$$

$$+ \frac{g_{\kappa_1}}{\Lambda_x} \mathcal{M}_{ca}^+ \text{Tr} \left[ \overline{H}_a(v) H_b(v) A_{bc} \right] \gamma_5 + \frac{ig_{\kappa_2}}{\Lambda_x} \mathcal{M}_{ca}^- \text{Tr} \left[ \overline{H}_a(v) H_b(v) A_{bc} \right]$$

$$+ \frac{g_{\kappa_3}}{\Lambda_x} \mathcal{M}_{ce}^+ \text{Tr} \left[ \overline{H}_a(v) H_b(v) A_{ba} \right] \gamma_5 + \frac{ig_{\kappa_4}}{\Lambda_x} \mathcal{M}_{ce}^- \text{Tr} \left[ \overline{H}_a(v) H_b(v) A_{ba} \right]$$

$$+ \frac{g_{\kappa_5}}{\Lambda_x} \mathcal{M}_{dc}^+ \text{Tr} \left[ \overline{H}_a(v) H_a(v) A_{cd} \right] \gamma_5 + \frac{g_{\kappa_6}}{\Lambda_x} \mathcal{M}_{dc}^- \text{Tr} \left[ \overline{H}_a(v) H_a(v) A_{cd} \right]$$

$$+ \frac{g_{\kappa_7}}{\Lambda_x} \mathcal{M}_{cc}^+ \text{Tr} \left[ \overline{H}_a(v) i v \cdot D_{ba} H_b(v) \right] + \frac{g_{\kappa_8}}{\Lambda_x} \mathcal{M}_{ac}^+ \text{Tr} \left[ \overline{H}_c(v) i v \cdot D_{ba} H_b(v) \right]$$

$$+ ...$$

(16)

where $\mathcal{M}^\pm = \frac{1}{2}(\xi m_q \xi + \xi^\dagger m_q \xi^\dagger)$, and the ellipses denote operators suppressed by additional powers of $M$, $m_s$, or derivatives. The $O(\frac{1}{M})$ operators are exactly analogous to these, but with spin preserving or spin violating Dirac structures (two Dirac structures for each $\kappa$ term). Because chiral loops add a factor of $m_K^2$, terms with additional pions are not relevant to the processes and order we are working, so for our purposes, we may take $\xi \rightarrow 1$, $\mathcal{M}^+ \rightarrow m_q$, and $\mathcal{M}^- \rightarrow 0$. Of the $\kappa'$s, only $\kappa_1$ and $\kappa_5$ cannot then be absorbed by parameter and field redefinitions; neither enters into the calculation of decay constants. Note that the products $\kappa_i m_q$, rather than $\kappa_i$ and $m_q$ separately, enter the Lagrangian, so there are no ambiguities due to the definition of $m_s$. The $\lambda$ terms may be accounted for by shifting $v \cdot k \rightarrow v \cdot k - \delta$ in strangeness carrying heavy meson propagators, with $\delta = M_{D_s} - M_D = M_{B_s} - M_B + O(\frac{1}{M})$ being the SU(3) mass splitting.

Omitting operators which are irrelevant to the calculations at hand allows us to write the simplified Lagrangian

$$\mathcal{L}_{M+m} = - \text{Tr} \left[ \overline{H}_a(v) i v \cdot D_{ba} H_b(v) \right] + \frac{g_{\kappa_1}}{\Lambda_x} \mathcal{M}_{ca}^+ \text{Tr} \left[ \overline{H}_a(v) H_b(v) A_{bc} \right] \gamma_5$$

$$+ \frac{g_{\kappa_2}}{\Lambda_x} \mathcal{M}_{ca}^- \text{Tr} \left[ \overline{H}_a(v) H_b(v) A_{bc} \right]$$

$$+ \frac{g_{\kappa_5}}{\Lambda_x} \mathcal{M}_{dc}^+ \text{Tr} \left[ \overline{H}_a(v) H_a(v) A_{cd} \right] \gamma_5$$

(17)

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where

\[ \tilde{g} = \begin{cases} 
\tilde{g}_B^* = g + \frac{1}{M} (g_1 + g_2) & \text{for } B^* B^* \text{ coupling}, \\
\tilde{g}_B = g + \frac{1}{M} (g_1 - g_2) & \text{for } B^* B \text{ coupling}, 
\end{cases} \]

The two new flavor violating terms appearing at this order correspond to spin symmetric and spin dependent axial coupling renormalizations. As such, we expect them to affect heavy meson interactions generically at \( O(1/M) \). For example, they will enter into heavy quark and chiral symmetry violating corrections to semileptonic \( B \to D \) and \( B_s \to D_s \) form factors, as well as to hyperfine mass splittings. Previous work has neglected the effects of these couplings [8,12].

The chiral representation of the left handed current \( \bar{q}_a \Gamma b \) proceeds similarly. The SU(3) preserving current is

\[
J^\lambda_{a(M)} = \frac{i\alpha}{2} (1 + \frac{\rho_1}{M}) \text{Tr}[\Gamma H_b(v)\xi^\dagger_{ba}] + \frac{i\alpha}{2} \frac{\rho_2}{M} \text{Tr}[\gamma^\mu \gamma_\mu H_b(v)\xi^\dagger_{ba}]
\]

\[ + \frac{\varepsilon_1}{M} \text{Tr}[\gamma^\alpha H_b(v)D^\mu \xi^\dagger_{ba}] \\
+ \frac{\varepsilon_2}{M} \text{Tr}[\gamma^\alpha H_b(v)\gamma^\mu D^\mu \xi^\dagger_{ba}] \\
+ \frac{\varepsilon_3}{M} \text{Tr}[\Gamma D^\mu \xi^\dagger_{ba}] \\
+ \frac{\varepsilon_4}{M} \text{Tr}[\gamma^\alpha \gamma^\alpha D^\mu \xi^\dagger_{ba}] \\
+ \frac{\varepsilon_5}{M} \text{Tr}[\gamma^\alpha H_b(v)D^\mu \xi^\dagger_{ba}] \\
+ \frac{\varepsilon_6}{M} \text{Tr}[\gamma^\alpha D^\mu \xi^\dagger_{ba}] \\
+ \frac{i\alpha}{4M} \text{Tr} \left[ \left[ \Gamma, \gamma_\mu \right] iD^\mu H_b(v)\xi^\dagger_{ba} \right]
\]

where \( \Gamma = \gamma^\lambda (1 - \gamma_5) \). Coefficients with even subscripts multiply heavy quark spin violating operators. The operators \( \text{Tr}[\gamma_\mu H_b(v)D^\mu \xi^\dagger_{ba}] \) and \( \text{Tr}[\gamma_\alpha \gamma_\alpha \gamma_\mu H_b(v)D^\mu \xi^\dagger_{ba}] \) can be absorbed into \( \omega_1 \) and \( \omega_2 \). Terms proportional to the light quark mass matrix have been omitted because their contributions are suppressed by \( \frac{m^2}{\Lambda^2} \) compared to those in Eq. (18). Potential terms involving the axial current, such as \( \text{Tr}[\gamma_\alpha \gamma_\alpha H_c(v)\gamma_\mu v \cdot A_{bc} \xi^\dagger_{ca}] \), can be rewritten in terms of the listed operators by use of the identities \( iD^\mu \xi^\dagger = -A_\mu \xi^\dagger \), \( iD^\mu \xi = A_\mu \xi \).

The last term in Eq. (18) is given by VRI. At tree level, its sole effect is to turn parameters such as \( v \) into physical quantities like \( \overline{\tau} = \frac{\tau_{PB}}{M_B} \). For example, because of this VRI completion term, the axial vector part of the current contributes \( \frac{i\alpha}{2} \text{Tr}[\gamma^\lambda \gamma_5 \tilde{H}] = \frac{i\alpha}{2} (v + \frac{k}{M})^\lambda = \frac{i\alpha}{2} \overline{\tau}^\lambda \), while the vector part contributes \( \frac{i\alpha}{2} \text{Tr}[\gamma^\lambda \tilde{H}] = \frac{i\alpha}{2} (\overline{\epsilon}_v \cdot \gamma^\lambda - \overline{v}^\lambda \epsilon_v) = i\alpha \overline{\epsilon}_v^\lambda \),
where $\epsilon_v$ is the polarization vector for the effective field $H_v$. The last relation follows from transforming $\epsilon_v$ to $\epsilon_T$ via a Lorentz boost.[11]

We are now in a position to compute corrections to meson decays at $\mathcal{O}(\frac{1}{M}, m_0)$. At this order, the rate for $D^* \to D\pi$ is governed by $g + \frac{1}{M_D}(g_1 - g_2)$ instead of $g$. This is the quantity which is extracted from either the $D^{*+}$ width[13] or the $D^* \to D\gamma$ branching ratio[14]. This leads to

$$1 < g^2 + \frac{2}{M_D}(gg_1 - gg_2) < .5.$$ 

The decay constants defined by

$$\langle 0 | \bar{q}_a \gamma^\mu \gamma^5 b | B_a(p) \rangle = if_{B_a}p^\mu,$$

$$\langle 0 | \bar{q}_a \gamma^\mu b | B^*_a(p, \epsilon) \rangle = if_{B^*_a}\epsilon^\mu$$

are altered only by current corrections at this order:

$$\sqrt{M_D}f_D = \alpha[1 + \frac{\rho_1 + 2\rho_2}{M_D}]$$

(20)

$$\frac{1}{\sqrt{M_D}}f_{D*} = \alpha[1 + \frac{\rho_1 - 2\rho_2}{M_D}]$$

(21)

In principle, knowledge of $f_{B, B^*, D, D^*}$, and $\Gamma(D^{*+} \to D\pi)$ would give $\alpha, \rho_1, \rho_2$ and $\tilde{g}_B$. Only four couplings are determined because the decay constants must satisfy

$$\frac{f_{D^*}}{M_D f_D} - 1 = \frac{M_B}{M_D} \left( \frac{f_{B^*}}{M_B f_B} - 1 \right).$$

A lattice calculation of the pseudo-scalar decay constants[4][15], finds $\frac{f_B}{f_D} = .9$ to 5%, from which we estimate $\rho_1 + 2\rho_2 \approx -1.4$ GeV.

By matching a $\mathcal{O}(\frac{1}{M})$ calculation of $f_B$ to the same calculation[4] in the heavy quark effective field theory (HQEFT)[16], the current corrections can be related to matrix elements in the effective theory. The two HQEFT matrix elements $G_1$ and $G_2$, respectively related to $< 0|\bar{q}\Gamma h \quad \bar{T}D^2h|B >$ and $< 0|\bar{q}\Gamma h \quad \bar{T}D_{\mu\nu}G^{\mu\nu}h|B >$, where $h$ is the effective heavy quark field and $G^{\mu\nu}$ is the gluon field strength tensor, have been estimated using QCD sum rules[4]. Matching gives $\rho_1 = G_1 + 2G_2 - \frac{\Lambda}{6}$ and $\rho_2 = 2G_2 - \frac{\Lambda}{6}$, while sum rule estimates give $G_1 \approx -2.3$ GeV, $G_2 \approx .05$ GeV, and $\Lambda \approx .5$ GeV. This indicates $\rho_2 \ll \rho_1$. However, the same author has recently called into question the large value of $G_1$[3]. Moreover, an independent sum rule calculation gives[17] $\rho_1 = 2\rho_2 = -0.6$ GeV.
We can also compute the $O(\frac{1}{M_B})$, SU(3) symmetric corrections to semileptonic $B \to K$. The relevant matrix elements are

\[
\begin{align*}
\langle K(p_K) | \bar{s} \gamma^\mu b | B(p_B) \rangle &= f_+ (p_B + p_K)^\mu + f_- (p_B - p_K)^\mu, \\
\langle K(p_K) | \bar{s} \sigma^{\mu\nu} b | B(p_B) \rangle &= 2i h [p_K^\mu p_B^\nu - p_K^\nu p_B^\mu].
\end{align*}
\]

Only the form factors $h(p_K \cdot p_B)$ and $f_+(p_K \cdot p_B)$ enter into the differential partial decay rate.

The operators which match onto the heavy-light currents above are determined by equation (18) to be

\[
\begin{align*}
\mathcal{O}^\mu &= \frac{i}{4} \alpha \left\{ (1 + \frac{\rho_1 - 2\rho_2}{M_B}) \text{Tr} [\gamma^\mu H_b(v)(\xi^\dagger + \xi)_{ba}] \\
&\quad + (1 + \frac{\rho_1 + 2\rho_2}{M_B}) \text{Tr} [\gamma\gamma^\mu H_b(v)(\xi^\dagger - \xi)_{ba}] \right\}, \\
\mathcal{O}^{\mu\nu} &= \frac{i}{4} \alpha \left\{ (1 + \frac{\rho_1}{M_B}) \left\{ \text{Tr} [\sigma^{\mu\nu} H_b(v)(\xi^\dagger + \xi)_{ba}] \\
&\quad + \text{Tr} [\gamma\sigma^{\mu\nu} H_b(v)(\xi^\dagger - \xi)_{ba}] \right\} \right. \\
&\quad + \text{Tr} [\gamma\sigma^{\mu\nu} H_b(v)(\xi^\dagger - \xi)_{ba}] \right\}.
\end{align*}
\]

The derivative suppressed terms in the current have been ignored. They give no contribution, at tree level, to $f_B$, while for $B \to \pi l\nu$, they are down by $\frac{k_\pi}{\Lambda}$. The form factors implied by Fig. (1) and Eq. (23) are

\[
\begin{align*}
f_+ &= -\frac{\alpha}{2\sqrt{M_B f_\pi}} \left[ M_B - v \cdot k \right] \left( 1 + \frac{\rho_1 - 2\rho_2}{M_B} \right) \tilde{g}_B \\
&\quad + \left( 1 + \frac{\rho_1 + 2\rho_2}{M_B} \right) \tilde{g}_B f_B^* \\
&\quad + \frac{1}{2f_\pi} \left[ \frac{\tilde{g}_B f_B^*}{v \cdot k - \Delta} - \frac{\tilde{g}_B f_B^*}{M_B} + f_B \right].
\end{align*}
\]
and
\[
    h = -\frac{\alpha}{2\sqrt{M_B f_\pi}} \left[ \frac{1}{v \cdot k - \Delta} (1 + \frac{\rho_1}{M_B}) \bar{g}_B \right]
    = -\frac{1}{2f_\pi M_B} \left[ \frac{\bar{g}_B f_{B^*}}{v \cdot k - \Delta} (1 + \frac{2\rho_2}{M_B}) \right],
\]
(25)

where \( k \) is the pion momentum. The analogous formula for \( D \) decays apply with the substitution \( M_B, f_B \rightarrow M_D, f_D \). When expressed in terms of physical quantities the expression for the pole part of \( f_+^{(D)} \) is not modified at \( \mathcal{O}(\frac{1}{M}) \)[18],
\[
    f_+^{(D)} = -\frac{\bar{g}_D f_{D^*}}{2f_\pi} \frac{1}{v \cdot k - \Delta},
\]
(26)

but the relation between \( f_+^{(D)} \) and \( h \) is[19],
\[
    f_+^{(D)} = M_D h^{(D)} (1 - \frac{2\rho_2}{M_D}).
\]
(27)

To the extent that \( \rho_2 \) is small, we expect relations between, say, \( B \rightarrow K\mu^+\mu^- \) and \( B \rightarrow \pi\mu\nu \) form factors to be dominated by SU(3) rather than heavy quark corrections. In principle, measurements of \( f_+^{(B)} \), \( f_+^{(D)} \) and either \( h^{(B)} \) or \( h^{(D)} \), coupled with a precise measurement of \( D^* \rightarrow D\pi \), would determine all six unknown constants \( \alpha, \rho_1, \rho_2, g, g_1 \), and \( g_2 \).

Now consider the SU(3) breaking current, linear in \( m_q \):
\[
    J_{\lambda(m)} = \frac{i\alpha}{2} \left( \frac{\eta_0}{\Lambda} + \frac{\eta_1}{M} \right) \text{Tr}[\Gamma H_c(v)] M^+_{cb} s_{ba} \xi^\dagger + \frac{i\alpha}{2} \left( \frac{\eta_2}{\Lambda} + \frac{\eta_3}{M} \right) \text{Tr}[\gamma^\mu \Gamma \gamma_\mu H_c(v)] M^+_{cc} \xi^\dagger
\]
\[
    + \frac{i\alpha}{2} \left( \frac{\eta_4}{\Lambda} + \frac{\eta_5}{M} \right) \text{Tr}[\Gamma H_b(v) \xi^\dagger_{ba}] M^+_{cc} + \frac{i\alpha}{2} \left( \frac{\eta_6}{\Lambda} + \frac{\eta_7}{M} \right) \text{Tr}[\gamma^\mu \Gamma \gamma_\mu H_b(v)] \xi^\dagger_{ba} M^+_{cc}
\]
\[
    + \frac{i\alpha}{2} \left( \frac{\eta_8}{\Lambda} + \frac{\eta_9}{M} \right) \text{Tr}[\Gamma H_c(v) \gamma_5] M^-_{cb} s_{ba} \xi^\dagger + \frac{i\alpha}{2} \left( \frac{\eta_{10}}{\Lambda} + \frac{\eta_{11}}{M} \right) \text{Tr}[\gamma^\mu \Gamma \gamma_\mu H_c(v) \gamma_5] M^-_{cb} \xi^\dagger_{ba}
\]
\[
    + \frac{i\alpha}{2} \left( \frac{\eta_{12}}{\Lambda} + \frac{\eta_{13}}{M} \right) \text{Tr}[\Gamma H_b(v) \xi^\dagger_{ba} \gamma_5] M^-_{cc} + \frac{i\alpha}{2} \left( \frac{\eta_{14}}{\Lambda} + \frac{\eta_{15}}{M} \right) \text{Tr}[\gamma^\mu \Gamma \gamma_\mu H_b(v) \xi^\dagger_{ba} \gamma_5] M^-_{cc}
\]
(28)

For \( \xi \rightarrow 1 \), the last two lines vanish, while the second line, which is SU(3) symmetric, can be absorbed by redefinitions of \( \alpha, \rho_1, \) and \( \rho_2 \). Of the three remaining parameters relevant at \( \mathcal{O}(\frac{1}{M}, m_s) \), only \( \eta_0 \) contributes to the pole part of \( D \rightarrow Kl\nu \) form factors, so we can learn about it by looking at SU(3) violations among the form factors \( f_+ \) in the heavy quark limit.

The SU(3) loop corrections to these processes have been previously computed[10], so we will present only the counterterm contributions. In the Lagrangian (17), \( \kappa_1 \) enters into
decays involving both kaons and etas, while $\kappa_5$ enters exclusively into decays involving etas. Since the SU(3) properties of the $\eta_0$ and $\kappa_1$ terms differ, $\eta_0$ can be extracted, in principle, from the following two form factors:

$$f_{+}^{D \rightarrow KL} = -\frac{g\alpha\sqrt{M_B}}{2f_{\pi}v \cdot k}[1 + \frac{m_s}{\Lambda}(\eta_0 + \kappa_1)],$$  \hspace{1cm} (29)$$

and

$$f_{+}^{D_s \rightarrow K L} = -\frac{g\alpha\sqrt{M_B}}{2f_{\pi}v \cdot k}[1 + \frac{m_s}{\Lambda}\kappa_1].$$  \hspace{1cm} (30)$$

These relations, augmented by the known loop corrections, relate $\eta_0$ to decay constant corrections.

III. LOOP CORRECTIONS

The leading SU(3) and heavy quark symmetry violating contributions to $f_B$ come from one loop diagrams involving both virtual kaons and $O(\frac{\Lambda}{M})$ heavy quark violating contact terms (with coefficients $\rho_1, \rho_2, g_1, g_2$), and arise from nonanalytic dependence on the strange quark mass $m_s$. Since this nonanalytic dependence is only generated by chiral loops, we can compute chiral and heavy quark symmetry violation, at leading order, in terms of the six parameters $\alpha, \rho_1, \rho_2, g, g_1$ and $g_2$. The formally sub-leading counterterms $\eta_0, \eta_1,$ and $\eta_2$ are added to absorb the scheme and subtraction scale dependence of the loop calculation, and to make the expansion more reliable numerically.

The nonanalytic $\mu$ dependent parts of several integrals which arise during the loop computation have been compiled using dimensional regularization\[10\]. Here we retain the complete expression, including a pole contained in $\Delta = \frac{2}{\epsilon} - \gamma + \ln(4\pi) + 1$. We will use

$$i \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2} = \frac{1}{16\pi^2} I_1(m),$$

$$i \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m^2)(p \cdot v - \Delta)} = \frac{1}{16\pi^2} \frac{1}{\Delta} I_2(m, \Delta),$$  \hspace{1cm} (31)$$

and

$$J^{\mu\nu}(m, \Delta) = i \int \frac{d^4p}{(2\pi)^4} \frac{p^\mu p^\nu}{(p^2 - m^2)(p \cdot v - \Delta)}$$

$$= \frac{1}{16\pi^2} \Delta [I_1(m, \Delta)g^{\mu\nu} + J_2(m, \Delta)v^\mu v^\nu],$$  \hspace{1cm} (32)$$

where

$$I_1(m) = m^2 \ln(m^2/\mu^2) - m^2\Delta,$$

$$I_2(m, \Delta) = -2\Delta^2 \ln(m^2/\mu^2) - 4\Delta^2 F(m/\Delta) + 2\Delta^2 + 2\Delta^2\Delta^2.$$  \hspace{1cm} (33)$$
\[ F(x) = \begin{cases} \sqrt{1-x^2} \tanh^{-1} \sqrt{1-x^2}, & |x| \leq 1 \\ -\sqrt{x^2-1} \tan^{-1} \sqrt{x^2-1}, & |x| \geq 1 \end{cases} \]

\[ J_1(m, \Delta) = (-m^2 + \frac{2}{3} \Delta^2) \ln(m^2/\mu^2) + \frac{4}{3} (\Delta^2 - m^2) F(m/\Delta) \]
\[ -\frac{2}{3} \Delta^2 (1+\Delta) + \frac{1}{3} m^2 (2+3\Delta), \]

\[ J_2(m, \Delta) = (2m^2 - \frac{8}{3} \Delta^2) \ln(m^2/\mu^2) - \frac{4}{3} (4\Delta^2 - m^2) F(m/\Delta) \]
\[ + \frac{8}{3} \Delta^2 (1+\Delta) - \frac{2}{3} m^2 (1+3\Delta). \]

We choose \( \Delta = 0 \) for our calculations. The analytic terms from these integrals, which were dropped from the SU(3) loop corrections in reference [10], are particularly easily recovered with this scheme.

The demonstration that higher derivative operators give no \( O(\frac{1}{M}) \) contribution requires only the nonanalytic terms of several integrals which are easily derived from the above integrals. The needed integrals are

\[ L_{\mu\nu\lambda}(m, \Delta) = i \int \frac{d^{4-\epsilon} p}{(2\pi)^{4-\epsilon}} \frac{p^\mu p^\nu p^\lambda}{(p^2 - m^2)(p \cdot v - \Delta)} \]
\[ = \frac{1}{3} g^{(\mu \nu \lambda)} [L_1 - L_2] + v^\mu v^\nu v^\lambda [2L_2 - L_1], \]

where

\[ L_1 = i \int \frac{d^{4-\epsilon} p}{(2\pi)^{4-\epsilon}} \frac{p^2 (v \cdot p)}{(p^2 - m^2)(p \cdot v - \Delta)} \]
\[ = \frac{m^4}{16\pi^2} \ln(m^2/\mu^2) - \frac{2\Delta^2 m^2}{16\pi^2} \left[ \ln(m^2/\mu^2) + 2F(m/\Delta) \right], \]

\[ L_2 = i \int \frac{d^{4-\epsilon} p}{(2\pi)^{4-\epsilon}} \frac{(v \cdot p)^3}{(p^2 - m^2)(p \cdot v - \Delta)} \]
\[ = \frac{1}{4} \frac{m^4}{16\pi^2} \ln(m^2/\mu^2) + \frac{\Delta^2}{16\pi^2} \left[ (m^2 - 2\Delta^2) \ln(m^2/\mu^2) - 4\Delta^2 F(m/\Delta) \right], \]

and

\[ \frac{\partial}{\partial \Delta} L_{\mu\nu\lambda}(m, \Delta) = i \int \frac{d^{4-\epsilon} p}{(2\pi)^{4-\epsilon}} \frac{p^\mu p^\nu p^\lambda}{(p^2 - m^2)(p \cdot v - \Delta)^2} \]
\[ = g^{(\mu \nu \lambda)} \frac{2\Delta}{16\pi^2} \left[ \left( \frac{4}{3} \Delta^2 - m^2 \right) \ln(m^2/\mu^2) + \frac{2}{3} F\left( \frac{m}{\Delta} \right) \frac{2m^4 + 4\Delta^4 - 7m^2 \Delta^2}{\Delta^2 - m^2} \right] \]
\[ + v^\mu v^\nu v^\lambda \frac{2\Delta}{16\pi^2} \left[ (4m^2 - 8\Delta^2) \ln(m^2/\mu^2) + 2F\left( \frac{m}{\Delta} \right) \frac{8m^2 \Delta^2 - m^4 - 8\Delta^4}{\Delta^2 - m^2} \right]. \]
as well as

\[ M^{\mu\nu\alpha\beta}(m, \Delta) = i \int \frac{d^4 - \epsilon}{(2\pi)^4 - \epsilon} \frac{p^\mu p^\nu p^\alpha p^\beta}{(p^2 - m^2)(p \cdot v - \Delta)^2} \]

\[ = \frac{1}{30} g^{(\mu\nu} g^{\alpha\beta)} [M_1(m, \Delta) - 2M_2(m, \Delta) + M_3(m, \Delta)] \]

\[ + \frac{1}{15} g^{(\mu\nu} v^{\alpha\beta)} [-M_1(m, \Delta) + 7M_2(m, \Delta) - 6M_3(m, \Delta)] \]

\[ + \frac{1}{5} v^{\mu\nu} v^{\alpha\beta} [M_1(m, \Delta) - 12M_2(m, \Delta) + 16M_3(m, \Delta)], \]

where

\[ M_1(m, \Delta) = i \int \frac{d^4 - \epsilon}{(2\pi)^4 - \epsilon} \frac{p^4}{(p^2 - m^2)(p \cdot v - \Delta)^2} \]

\[ = -m^4 \left[ \frac{1}{16\pi^2} \left( 2 \ln \left( \frac{m^2}{\mu^2} \right) + \frac{4\Delta^2}{\Delta^2 - m^2} \frac{2\ln \left( \frac{m^2}{\Delta} \right)}{m^2 - \Delta^2} \right) \right] \]  

\[ (41) \]

\[ M_2(m, \Delta) = i \int \frac{d^4 - \epsilon}{(2\pi)^4 - \epsilon} \frac{p^2(v \cdot p)^2}{(p^2 - m^2)(p \cdot v - \Delta)^2} \]

\[ = \frac{1}{16\pi^2} (m^4 - 6m^2\Delta^2) \ln \left( \frac{m^2}{\mu^2} \right) + \frac{4\Delta^2m^2}{16\pi^2} F \left( \frac{m}{\Delta} \right) \frac{3\Delta^2 - 2m^2}{m^2 - \Delta^2}, \]  

\[ (42) \]

\[ M_3(m, \Delta) = i \int \frac{d^4 - \epsilon}{(2\pi)^4 - \epsilon} \frac{(v \cdot p)^4}{(p^2 - m^2)(p \cdot v - \Delta)^2} \]

\[ = \frac{1}{16\pi^2} \left( \frac{1}{4} m^4 + 3m^2\Delta^2 - 10\Delta^4 \right) \ln \left( \frac{m^2}{\mu^2} \right) + \frac{4\Delta^4}{16\pi^2} F \left( \frac{m}{\Delta} \right) \frac{4m^2 - 5\Delta^2}{\Delta^2 - m^2}, \]  

\[ (43) \]

and ( ) means symmetrization, e.g.,

\[ g^{(\mu\nu} v^{\alpha\beta)} = g^{\mu\nu} v^{\alpha\beta} + g^{\mu\alpha} v^{\nu} v^{\beta} + g^{\mu\beta} v^{\nu} v^{\alpha} \]

\[ + g^{\nu\alpha} v^{\mu} v^{\beta} + g^{\nu\beta} v^{\mu} v^{\alpha} + g^{\alpha\beta} v^{\mu} v^{\nu}. \]  

\[ (44) \]

The entire nonanalytic parts of these integrals have been retained, even though we will only use the leading terms. The analytic parts, including the divergent \( \epsilon \)-pole, have been discarded. It is worth pointing out that there are no singularities at \( m^2 = \Delta^2 \) in the \( M_i \): the apparent pole is cancelled by a zero in the function \( F \).
The diagrams of Fig. (2) contribute to wavefunction renormalization. The graph of Fig. (2a), after summing over intermediate states and ignoring the pion mass, is

$$\frac{6i g_B^2}{16\pi^2 f^2} \left[ J(m_K, \frac{\Delta}{4} + \delta - v \cdot k) + \frac{1}{6} J(m_\eta, \frac{\Delta}{4} - v \cdot k) \right]$$

(45)

for the $B^0$, and

$$\frac{6i g_B^2}{16\pi^2 f^2} \left[ 2J(m_K, \frac{\Delta}{4} - v \cdot k) + \frac{2}{3} J(m_\eta, \frac{\Delta}{4} + \delta - v \cdot k) \right]$$

(46)

for the $B_s$.

For the vector mesons, there are two graphs, Figs. (2b) and (2c). Together, they give (we drop the $\nu^\mu\nu^\nu$ terms)

$$\frac{-2i}{16\pi^2 f^2} \left[ 2g^2_{B^*} J(m_K, \frac{\Delta}{4} + \delta - v \cdot k) + \frac{1}{3} g^2_{B^*} J(m_\eta, \frac{\Delta}{4} - v \cdot k) 
+ \frac{1}{6} g^2_{B^*} J(m_\eta, \frac{\Delta}{4} + \delta - v \cdot k) \right] g^{\mu\nu}$$

(47)

for the $B^{0*}$, and

$$\frac{-2i}{16\pi^2 f^2} \left[ 4g^2_{B^*} J(m_K, \frac{\Delta}{4} - v \cdot k) + \frac{4}{3} g^2_{B^*} J(m_\eta, \frac{\Delta}{4} + \delta - v \cdot k) 
+ \frac{2}{3} g^2_{B^*} J(m_\eta, \frac{\Delta}{4} + \delta - v \cdot k) \right] g^{\mu\nu}$$

(48)

for the $B^*_s$, where $J(m, x) = xJ_1(m, x)$. 

Figure 2. Wavefunction renormalization diagrams for (a) pseudoscalar heavy mesons, and (b,c) vector mesons.
Differentiating the above self energies with respect to $2v \cdot k$ and evaluating on-shell gives the wavefunction renormalizations $Z$. Expanding the couplings $\tilde{g}$, applying the Gell-Mann-Okubo formula $m_{\eta}^2 = \frac{4}{3} m_K^2$, and noting $\Delta \sim \frac{1}{M}$, yields

$$Z_{B^0} = 1 - \frac{11}{3} \frac{m_K^2 (\ln \frac{m_K^2}{\mu^2} + \frac{2}{3} + \frac{2}{11} \ln \frac{4}{3})}{16\pi^2 f_K^2} g^2 + \frac{3}{8} \frac{(\delta + \Delta)^2 g^2}{\pi^2 f_K^2} \left[ \ln \frac{m_K^2}{\mu^2} - \frac{1}{3} + 2F\left(\frac{m_K}{\Delta + \delta}\right) \right]$$

$$- \frac{22}{3} \frac{m_K^2 (\ln \frac{m_K^2}{\mu^2} + \frac{2}{3} + \frac{2}{11} \ln \frac{4}{3})}{16\pi^2 f_K^2 M} (g_1 g - g_2 g)$$

(49)

$$Z_{B_s} = 1 - \frac{26}{3} \frac{m_K^2 (\ln \frac{m_K^2}{\mu^2} + \frac{2}{3} + \frac{4}{13} \ln \frac{4}{3})}{16\pi^2 f_K^2} g^2 + \frac{3}{4} \frac{(\delta - \Delta)^2 g^2}{\pi^2 f_K^2} \left[ \ln \frac{m_K^2}{\mu^2} - \frac{1}{3} + 2F\left(\frac{m_K}{\Delta - \delta}\right) \right]$$

$$- \frac{52}{3} \frac{m_K^2 (\ln \frac{m_K^2}{\mu^2} + \frac{2}{3} + \frac{4}{13} \ln \frac{4}{3})}{16\pi^2 f_K^2 M} (g_1 g - g_2 g)$$

(50)

$$Z_{B^0*} = 1 - \frac{11}{3} \frac{m_K^2 (\ln \frac{m_K^2}{\mu^2} + \frac{2}{3} + \frac{2}{11} \ln \frac{4}{3})}{16\pi^2 f_K^2} g^2 + \frac{1}{8} \frac{(\delta - \Delta)^2 g^2}{\pi^2 f_K^2} \left[ \ln \frac{m_K^2}{\mu^2} - \frac{1}{3} + 2F\left(\frac{m_K}{\Delta - \delta}\right) \right]$$

$$- \frac{22}{9} \frac{m_K^2 (\ln \frac{m_K^2}{\mu^2} + \frac{2}{3} + \frac{2}{11} \ln \frac{4}{3})}{16\pi^2 f_K^2 M} (3g_1 g + g_2 g)$$

(51)

$$Z_{B_s^*} = 1 - \frac{26}{3} \frac{m_K^2 (\ln \frac{m_K^2}{\mu^2} + \frac{2}{3} + \frac{4}{13} \ln \frac{4}{3})}{16\pi^2 f_K^2} g^2 + \frac{1}{4} \frac{(\delta + \Delta)^2 g^2}{\pi^2 f_K^2} \left[ \ln \frac{m_K^2}{\mu^2} - \frac{1}{3} + 2F\left(\frac{m_K}{\Delta + \delta}\right) \right]$$

$$- \frac{52}{9} \frac{m_K^2 (\ln \frac{m_K^2}{\mu^2} + \frac{2}{3} + \frac{4}{13} \ln \frac{4}{3})}{16\pi^2 f_K^2 M} (3g_1 g + g_2 g)$$

(52)

The chiral log terms are the leading SU(3) corrections because they are logarithmically enhanced in the chiral $m_K \to 0$ limit. Whether $F(m_K/\Delta)$ is similarly enhanced depends on how we take the combined heavy quark and chiral limit – the behavior of the ratio $m_K/\Delta$ affects the answer we get, including the coefficient of the chiral log. A consistent but non-unique scheme is to take $m_K \to 0$ holding $m_K/\Lambda \chi \Delta$ fixed. Rather than choosing a particular scheme, we retain nonanalytic terms such as $F(m/(\Delta + \delta))$. Note that while
\( F(m/x) \to -\frac{\pi}{2}m/x \to \infty \) as \( x \to 0 \), the function involved is always of the form \( x^2 F(m/x) \), which vanishes in the limit. Therefore, we do not drop terms of order \( \Delta^2 \) or \( \delta^2 \) when they appear in the combination \( (\Delta \pm \delta)^2 \) as a factor multiplying \( F(m/(\Delta \pm \delta)) \), even if they are formally of order \( 1/M^2 \) and \( m_K^4 \), respectively.

![Figure 3. Diagrams contributing to decay constants by modifying the current vertex.](image)

Vertex corrections arise from the diagrams in Fig. (3). The graph of Fig. (3b) vanishes at \( \mathcal{O}(\frac{\Lambda}{M}) \). Fig. (3a) gives,

\[
\frac{i\alpha v}{32\pi^2 f^2} (1 + \frac{\rho_1 + 2\rho_2}{M}) \left[ I_1(m_K^2) + \frac{1}{6} I_1(m_\eta^2) \right]
\]

(53)

for the \( B^0 \) meson,

\[
\frac{i\alpha v}{32\pi^2 f^2} (1 + \frac{\rho_1 + 2\rho_2}{M}) \left[ 2I_1(m_K^2) + \frac{2}{3} I_1(m_\eta^2) \right]
\]

(54)

for the \( B_s \),

\[
-\frac{i\alpha v}{32\pi^2 f^2} (1 + \frac{\rho_1 - 2\rho_2}{M}) \left[ I_1(m_K^2) + \frac{1}{6} I_1(m_\eta^2) \right]
\]

(55)

for the \( B^{0*} \) meson, and

\[
-\frac{i\alpha v}{32\pi^2 f^2} (1 + \frac{\rho_1 - 2\rho_2}{M}) \left[ 2I_1(m_K^2) + \frac{2}{3} I_1(m_\eta^2) \right]
\]

(56)

for the \( B^*_s \)
Figure 4. Vertices relevant to one loop diagrams by which higher derivative operators contribute to decay constants. (a) The $\mathcal{O}(1)$ current vertex is independent of loop or external momenta $\sim 1$. (b) Both the $\mathcal{O}(1)$ axial coupling and the derivative suppressed $\mathcal{O}(\frac{1}{M})$ current correction (shaded square) are linear in momentum, $\sim p^\mu$. (c) Derivative operators in the Lagrangian can modify the two-point function (cross), the axial coupling (solid dot), or the vector coupling (shaded dot) at $\mathcal{O}(\frac{1}{M})$. All are quadratic in momentum, $\sim p^\mu p^\nu$.

At $\mathcal{O}(\frac{1}{M})$, Fig. (3b) no longer vanishes because terms in equations \([\ref{eq:15}]\) and \([\ref{eq:18}]\) with extra derivatives contribute. To justify our claim that such terms are suppressed by $\frac{\Delta}{M} \sim \frac{1}{M^2}$ rather than $\frac{m_K}{M}$, we explicitly compute the contribution of the $\frac{\Delta}{M}$ VRI completion term of the current (the last term in Eq. (18)) to the graph in Fig. (3b). For an incoming $B^0$ turning into a $B_s^*K^0$ loop, the graph gives

$$\frac{3i\alpha g\Delta v^\lambda}{M_B 64\pi^4 f_K^2} J_1(m_K, \frac{\Delta}{4})$$

which is suppressed by $\frac{1}{M^2}$. The other higher derivative operators are similarly suppressed.
Figure 5. **Graphs through which higher derivative operators in the current or Lagrangian can contribute to decay constants.** Diagrams (a), (b) and (c) represent corrections to the current vertex. The diagrams of (d), (e) and (f) are wavefunction renormalizations.

We can outline why this is the case by classifying the graphs involving higher derivative operators according to their associated integrals. The relevant vertices are grouped in Fig. (4). The leading, $\mathcal{O}(1)$, part of the current shown in Fig. (4a) has no powers of
momentum in its vertex, independent of whether the incoming meson is spin zero or one. The $\mathcal{O}(M^0)$ axial coupling and the $\mathcal{O}(\frac{1}{M})$ current vertex in Fig. (4b) are linear in momentum, while the higher derivative Lagrangian insertion and pion couplings in fig. (4c) are quadratic in momentum. Since we are free to take the residual momentum of the incoming, on-shell, meson to be zero, only loop momentum will appear in the graphs of Fig. (5) (except for one power of residual momentum needed for the wavefunction renormalization graphs of Figs. (5d), (5e) and (5f)).

The vertex graph in Fig. (5a) involves the integral
\[
\int \frac{d^{4-\epsilon} p}{(2\pi)^{4-\epsilon}} \frac{p^\mu}{p^2 - m^2}
\]
which trivially vanishes. Vertex corrections involving derivative suppressed Lagrangian insertions, as in Fig. (5b), are proportional to $\frac{1}{M} \frac{\partial}{\partial \Delta} L^{\mu\nu\lambda}(m, \Delta)$. Since we treat $\frac{F(m/\Delta)}{M}$ as $\mathcal{O}(1)$, one can see from Eq. (39) that these corrections are $\mathcal{O}(\frac{1}{M})$. Both vertex loops of Fig. (5c) involve $\frac{1}{M} J^{\mu\nu}(m, \Delta)$, which is also subleading.

For wavefunction renormalization, we consider the diagram of Fig. (5d), containing an axial coupling correction, which involves $\frac{1}{M} L^{\mu\nu\lambda}(m, \Delta + v \cdot k)$, where $k$ is the heavy meson’s residual momentum. From Eq. (56), we see that the term proportional to $v \cdot k$ contains an additional power of $\Delta$, so it is down by $\mathcal{O}(\frac{1}{M})$. The loop graph in Fig. (5e) contains a factor of $\frac{1}{M} M^{\mu\nu\alpha\beta}$. Again, the terms with at least one factor of $v \cdot k$ automatically come with at least one factor of $\Delta$. For either graph, replacing one of the loop momenta in the numerator with the residual momentum fails to alter the $\frac{1}{M}$ suppression. The last graph, Fig. (5f), can contribute to wavefunction renormalization at $\mathcal{O}(\frac{1}{M})$ only if the four-point coupling is linear in the external momentum, in which case it is also linear in the pion momentum, causing the loop integral to vanish. Thus, none of the higher derivative operators contribute to the leading heavy quark and chiral symmetry violating terms.

The final results for the decay constants, valid to $\mathcal{O}(\frac{1}{M}, \frac{m}{M})$, are found by combining the wavefunction, vertex, and counterterm corrections:

\[
\sqrt{M_{B^0}} f_{B^0} = \alpha \left[ 1 + \frac{\rho_1 + 2\rho_2}{M} - \frac{11 m_K^2 \ln \frac{m^2}{\mu^2}}{6} (g^2 + \frac{1}{3}) 
- \frac{1}{3} \left( \frac{11}{3} + \ln \frac{4}{3} \right) \frac{m^2_K g^2}{16 \pi^2 f_K^2} 
+ \frac{3}{16} \left( \ln \frac{m_K^2}{\mu^2} - \frac{1}{3} \right) + 2 \frac{(\delta + \Delta)}{\pi^2 f_K^2} \right]
\]  
\[
- \frac{11 m_K^2 \ln \frac{m^2}{\mu^2}}{6} \frac{1}{16 \pi^2 f_K^2 M} \left( (\rho_1 g^2 + \frac{1}{3}) + 2 \rho_2 (g^2 + \frac{1}{3}) + 2 g g_1 - 2 g g_2 \right)
\]  
\[
- \frac{1}{3} \left( \frac{11}{3} + \ln \frac{4}{3} \right) \frac{m^2_K}{16 \pi^2 f_K^2 M} \left( (\rho_1 + 2 \rho_2) g^2 + 2 g g_1 - 2 g g_2 \right)
\]  
\[
(59)
\]
Equations (59) to (62) constitute the main results of this paper, and agree with Ref. [1] in the infinite mass limit.
The physical decay constants and masses are independent of the choice of renormalization point $\mu$. On the right hand side of Eqs. (59) to (62) the explicit dependence on the renormalization point $\mu$ is cancelled by implicit dependence in the parameters $\alpha, g_i, \rho_i$ and $\eta_i$. To see this one must recall that we have absorbed counterterms with $\mu$ and analytic $m^2_K$ dependence into these parameters.

It is worth pointing out that if we ignore analytic and $O(M^2, \delta^2/M)$ terms, the decay constants can be written in the simpler form

$$\sqrt{M_B f_B} = \alpha (1 + \frac{\rho_1 + 2\rho_2}{M}) \left\{ 1 - \frac{11}{6} \frac{m^2_K}{16\pi^2 f_K^2} g^2 + \frac{1}{3} \right\}$$

and analogous formulas for the other decay constants. Except for the last term, this form is simply the SU(3) correction to the infinite mass limit, but with $\alpha(1 + \frac{\rho_1 + 2\rho_2}{M})$ and $\hat{g}$ replacing $\alpha$ and $g$. For $f_B, \hat{g}$ is simply $\hat{g}_B$, but for $f_{B^*}$, we need $\hat{g} = g + \frac{g_1 + g_2}{M} \neq \hat{g}_{B^*}$.

The constants $g_1$ and $g_2$ do not simply enter through the Lagrangian parameters $g_B$ and $g_{B^*}$; their contributions to $f_{B^*}$ must be computed from the diagrams.

From these equations, it follows that the ratio of decay constants $R_1$ is

$$R_1 = 1 - \frac{9g^2\delta(\ln\frac{m^2_K}{\mu^2} - \frac{1}{3})}{8\pi^2 f_K^2} (\Delta^{(B)} - \Delta^{(D)})$$

$$+ \frac{3g^2}{8\pi^2 f_K^2} \left[ -(\Delta^{(B)} + \delta)F(\frac{m_K}{\Delta^{(B)} + \delta}) + 2(\Delta^{(B)} - \delta)F(\frac{m_K}{\Delta^{(B)} - \delta}) \right]$$

$$+ (\Delta^{(D)} + \delta)F(\frac{m_K}{\Delta^{(D)} + \delta}) - 2(\Delta^{(D)} - \delta)F(\frac{m_K}{\Delta^{(D)} - \delta})$$

$$- \frac{5m^2_K(\ln\frac{m^2_K}{\mu^2} + \frac{2}{3} + \frac{2}{3} \ln\frac{4}{3})}{16\pi^2 f_K^2} (\frac{1}{M_B} - \frac{1}{M_D}) g(g_1 - g_2) + m_s(\eta_1 + 2\eta_2)(\frac{1}{M_B} - \frac{1}{M_D})$$

This quantity is relevant to the extraction from $B - \bar{B}$ mixing of the Cabbibo-Kobayashi-Maskawa angle $V_{ts}$ [3].

We can estimate the size of corrections to $R_1$. Taking $\mu = 1$ GeV and inserting known masses and constants gives

$$R_1 - 1 = -0.11g^2 - 0.06\text{GeV}^{-1}g(g_1 - g_2) - 0.10(\eta_1 + 2\eta_2)$$

The first term contributes $-6\%$ to $R_1 - 1$ for $g^2 = 0.5$. 

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Information about $g_1$ and $g_2$ may be gleaned by studying $B_s \to D_s \nu \pi$ decays, hyperfine mass splittings, or $B \to \pi \ell \nu$. The presence of $g_1, g_2$ in the Lagrangian means they will contribute to any process involving $B$ or $D$ mesons which receive corrections due to pion loops. The counterterms $\eta_1$ and $\eta_2$ enter at one loop into $B \to \pi \ell \nu$ form factors. It remains to be seen if predictive power persists in this system at $\mathcal{O}(\frac{1}{M}, m_s)$. However, the presence of a large number of form factors (both the pole and constant parts) for the set of semileptonic decays $\{B, B_s, D, D_s\} \to \{K, \pi, \eta\}$ looks promising, since many of the counterterms listed here may be eliminated by redefinitions of fields and coupling constants.

IV. CONCLUSIONS

We have derived the most general form of the heavy quark chiral Lagrangian and the heavy to light current to linear order in the $SU(3)$ vector and axial fields and to $\mathcal{O}(\frac{1}{M})$. Four new unknown parameters $\rho_1, \rho_2, g_1, g_2$ arise at this order. Additional parameters, needed for terms suppressed by one derivative, have been presented, but the contribution of these terms to the decay constants are found to be $\mathcal{O}(\frac{1}{M^2})$. $\mathcal{O}(m_s)$ counterterms in the Lagrangian and current have also been presented, leading to three new parameters $\eta_0, \eta_1,$ and $\eta_2$ which enter into decay constant relations.

The $\mathcal{O}(\frac{1}{M}, m_s^0)$ heavy quark symmetry corrections to $B \to \pi \ell \nu$ and its flavor related decays are computed in terms of the four unknown parameters $\rho_1, \rho_2, g_1, g_2$. Derivative terms are suppressed by $k_\pi/\Lambda_\chi$. SU(3) splittings are examined at tree level, giving rise to relations which allow the extraction of $\eta_0$ from $D \to K \ell \nu$ and $D_s \to K \ell \nu$ form factors.

The $\mathcal{O}(\frac{1}{M}, m_s^0)$ corrections to $f_B$ involve only two of the parameters, which may be related to HQEFT matrix elements by comparison to the heavy quark effective theory.

Finally, the logarithmically enhanced contributions to heavy meson decay constants violating both chiral SU(3) and heavy quark symmetries can be expressed in terms of two leading order constants $\alpha$ and $g$, and four new parameters $\rho_1, \rho_2, g_1, g_2$. Three unenhanced counterterms $\eta_0, \eta_1$ and $\eta_2$ also enter. The ratio $R_1 = \frac{f_{B^s}}{f_B}/\frac{f_{D^s}}{f_D}$ is expressed in terms of two unknown axial parameters and two counterterms.

In principle, measurements of $f_{B^+}^{(B)}$, $f_{D^+}^{(D)}$ and either $h^{(B)}$ or $h^{(D)}$, coupled with a precise measurement of $D^* \to D \pi$, would determine six unknown constants $\alpha, \rho_1, \rho_2, g, g_1$ and $g_2$. Measurements of SU(3) violation in $f_\pi$ or $h$ determine, in addition, $\eta_0$. The observed decay constant $f_{D^+}$, the potentially measurable $f_D$, $f_B$ and $f_{B_s}$, and the inaccessible decay constants $f_{D^*}, f_{D^*_s}, f_{B^*}$ and $f_{B^*_s}$ are then fixed in terms of two parameters.
\( \eta_1 \) and \( \eta_2 \). Predictions of inaccessible quantities may be of use for lattice or hadron model computations. For example, neither quenched lattice nor non-relativistic quark models include the nonanalytic terms computed here, and should be augmented by our results.

An obvious extension of this work is the computation of \( \mathcal{O}\left(\frac{1}{M}, m_s \right) \) corrections to processes such as semileptonic \( \bar{B} \to D \) and \( \bar{B} \to KK \), and \( \bar{B} \to \pi \). These processes depend on many of the same parameters presented in this work. Combinations of such quantities sensitive only to simultaneous violations of chiral and heavy quark symmetry may be amenable to fruitful examination.

\begin{center}
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