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Microscopic analysis of $^{10,11}\text{Be}$ elastic scattering on protons and $^{12}\text{C}$ and breakup processes of $^{11}\text{Be}$ within the $^{10}\text{Be}+n$ cluster model

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Abstract. The elastic scattering cross-sections of $^{10,11}\text{Be}$ on protons and $^{12}\text{C}$ at energy $E < 100$ MeV/nucleon using microscopically calculated optical potentials (OP) are presented. The real OP is obtained by a folding procedure with effective NN interactions, while the imaginary OP is estimated within the high energy approximation (HEA). The spin-orbit part of the OP is also included. The characteristics of the breakup processes of $^{11}\text{Be}$ on different nuclear targets are also considered. The cross-sections of diffractive breakup and stripping reactions of $^{11}\text{Be}$ on $^{9}\text{Be}$, $^{93}\text{Nb}$, $^{181}\text{Ta}$ and $^{238}\text{U}$ at energy $E = 63$ MeV/nucleon and the longitudinal momentum distributions of $^{10}\text{Be}$ fragments produced in the breakup of $^{11}\text{Be}$ on these nuclei are presented. The results are in a good agreement with the available experimental data, in particular the obtained widths of about 50 MeV/c are closed to the empirical ones.

In the present work we study the elastic scattering cross section of $^{10,11}\text{Be}$ on protons and $^{12}\text{C}$ at incident energies $E < 100$ MeV/nucleon using microscopically calculated OP’s within the hybrid model. Following our previous works, considering processes with exotic He and Li isotopes [1]-[6], we apply this model in which $^{11}\text{Be}$ consists of a core of $^{10}\text{Be}$ and a halo formed by a motion of a neutron in its periphery. Also we estimate important characteristics of the reactions with $^{11}\text{Be}$, such as the breakup cross sections and momentum distributions of fragments in breakup processes.

The real part of the OP (ReOP) is calculated by a folding of a nuclear density and the effective NN potentials and includes both direct and exchange parts. The imaginary part of the OP (ImOP) is obtained within the HEA model. There are only two or three fitting parameters $N$’s in the hybrid model that are related to the depths of the ReOP, ImOP and the spin-orbit part of the OP and are obtained by a fitting procedure to the available experimental data. In the calculations of the microscopic OPs for the scattering of $^{10,11}\text{Be}$ on protons and nuclei we used realistic density distributions of $^{10}\text{Be}$ calculated within the quantum Monte Carlo model (QMC) [7] and of $^{10,11}\text{Be}$ from the generator coordinate method (GCM)[8].

The OP has the form:

$$U(r) = N_R V^F(r) + i N_I W(r) - 2\lambda_2 \left[ N_R^{ls} V^{ls}_R \frac{1}{r} \left( \frac{df_R(r)}{dr} \right) + i N_I^{ls} W^{ls}_I \frac{1}{r} \left( \frac{df_I(r)}{dr} \right) \right] (\vec{l} \cdot \vec{s}),$$

(1)
where \(2\lambda^2 = 4 \text{ fm}^2\) with the squared pion Compton wave length \(\lambda^2 = 2 \text{ fm}^2\). We denote the values of the ReOP and ImOP at \(r = 0\) by \(V_R(\equiv V^F(r = 0))\) and \(W_I(\equiv W(r = 0))\). The spin-orbit part of the OP contains real and imaginary terms with the parameters \(V^I_{ls}\) and \(W^I_{ls}\) related to \(V_R\) and \(W_I\) by the \(V^I_{ls} = V_R/4\) and \(W^I_{ls} = W_I/4\), correspondingly. The ReOP \(V^F(r)\) is a sum of isoscalar \((V^F_{ls})\) and isovector \((V^F_{IV})\) components and each of them has its direct \((V^D_{ls})\) and \(V^D_{IV}\) and exchanged \((V^E_{ls})\) and \(V^E_{IV}\) parts.

The ImOP is chosen either to be in the form of the microscopically calculated \(W(r) = V^F(r)\) or in the form \(W(r) = W^H(r)\) obtained in Ref. [1, 9] within the HEA.

The elastic scattering of light nuclei is rather sensitive to their periphery, where transfer and breakup processes also take place. In our the previous papers has been pointed out [3, 4], that the inclusion of a surface imaginary term to the OP [Eq. (1)] leads to a better agreement with the experimental data. As known, this contribution can be considered as the so-called dynamical polarization potential, which allows one to simulate the surface effects caused by the latter. In fact, the imaginary part of the \(ls\) term in our OP [see Eq. (1)] plays effectively this role. However, sometimes one needs to increase the absorption in the surface region and thus, one adds a derivative of the ImOP (surface term):

\[
W^{sf}(r) = -iN_f^sf r \frac{dW(r)}{dr},
\]

where \(N_f^sf\) is also a fitting parameter.

All calculations of elastic scattering using the obtained OPs are performed by using the DWUCK4 code. The results for the elastic \(^{10}\text{Be}+p\) and \(^{11}\text{Be}+p\) scattering cross sections are given in Figs. 1 and 2, respectively, and are compared with the experimental data. In general, the account for the spin-orbit term in the volume OP gives a trend of increasing the cross sections at larger angles.

The problem of the ambiguity of the values of \(N\)'s arises when the fitting procedure concerns a limited number of experimental data. We impose a physical constraint, namely choosing those ReOP and ImOP that give volume integrals with a correct dependence on the energy [12]. It is known [12] that the volume integrals (their absolute values) for the ReOP decrease with increasing the energy, while for the ImOP they increase up to a plateau and then decrease. The values of the \(N\)'s parameters from the fitting procedure are given in the Table 1. It can be seen that the tendency (the decrease of \(J_R\) and the increase of \(J_W\)) is generally confirmed. A fair agreement between the calculated \(^{10}\text{Be}+p\) angular distributions and the experimental data is obtained only when both \(ls\)- and surface contributions to the OP are included.

For a more complete analysis of the elastic scattering cross sections, we extend the incident energy region to lower energies on the example of the scattering of \(^{10}\text{Be}\) on protons that has been recently studied by Schmitt et al. [13]. Moreover, this could be a test of our hybrid model at low energies. The calculated results for the differential cross sections, shown as a ratio to Rutherford scattering, are given and compared with the data [13] in Fig. 3 for energies of 7.5 MeV and 10.7 MeV. The values of the \(N\)'s parameters from the fitting procedure and the corresponding total reaction cross sections and volume integrals are listed in Table 1. The results shown in Fig. 3 when including in the calculations only the \(ls\) term demonstrate a fairly good agreement with the data. The values of the parameters \(N_R\) deduced from the fitting procedure for both energies in the case of GCM density of \(^{10}\text{Be}\) are quite large that indicates for the specific peculiarities of the elastic scattering at low energies with account for the spin-orbit term. We also calculated the \(^{10}\text{Be}+p\) elastic scattering cross sections at the same proton energies taking into account the surface term [Eq. (2)]. In this case, only the QMC density of \(^{10}\text{Be}\) is tested.

The elastic scattering cross sections of \(^{11}\text{Be}+^{12}\text{C}\) (their ratios to the Rutherford one) calculated within the hybrid model are presented in the Fig. 4. The results are obtained with the GCM densities and they are in a good agreement with the available data. The values of the
Figure 1. $^{10}\text{Be}+p$ elastic scattering cross sections with both $ls$ and surface terms. Solid lines: calculations with GCM density of $^{10}\text{Be}$; dashed lines: calculations with QMC density of $^{10}\text{Be}$. Experimental data for 39.1 MeV/nucleon and 59.4 MeV/nucleon are from Refs. [10] and [11], respectively.

Figure 2. $^{11}\text{Be}+p$ elastic scattering cross sections. Calculations are performed with GCM density of $^{11}\text{Be}$. Solid line: OP with both $ls$ and surface terms [Eqs. (1) and (2)]; dashed line: OP with $ls$ term [Eq. (1)]; dotted line: the volume part of OP from Eq. (1). Experimental data for 38.4 MeV/nucleon and 49.3 MeV/nucleon are taken from Refs. [10] and [11], respectively.

parameters $N$ and the volume integrals $J_V$ and $J_W$ are given in the Table 2. It is seen from the figure that it is difficult to determine the advantage of the use for the ImOP $W = W^H$ or $W = V^F$, because the differences between the theoretical results start at angles for which the experimental data are not available.

In the paper we also consider the characteristics of breakup processes of the $^{11}\text{Be}$ nucleus, namely diffraction and stripping reaction cross sections and the momentum distributions of the fragments. We use a simple model in which $^{11}\text{Be}$ consists of a core of $^{10}\text{Be}$ and a halo of a single neutron. In this model the density of $^{10}\text{Be}$ has to be given. We use the QMC [7] and GCM [8] density distributions of $^{10}\text{Be}$. The hybrid model is applied to calculate the OP of the interaction of $^{10}\text{Be}$ with the target, as well as OP for the $n+$target interaction. In the
Figure 3. $^{10}\text{Be}+p$ elastic scattering cross sections as a ratio to Rutherford scattering at proton energies of 7.5 MeV (left panel) and 10.7 MeV (right panel). The solid and dashed lines show the results with QMC and GCM density of $^{10}\text{Be}$, respectively, and with $ls$ term in OP. The dotted lines show the QMC results obtained by accounting for both the $ls$- and surface terms in OP. Experimental data are taken from Ref. [13].

Figure 4. $^{11}\text{Be}+^{12}\text{C}$ elastic scattering cross sections. Solid lines: $W = W^H$; dashed lines: $W = V^F$. Experimental data are from Refs. [10] and [11], respectively.
Table 1. The renormalization parameters \( N_R, N_I, N_{ls}^R, N_{ls}^I, \) and \( N_{sf}^I, \) the total reaction cross sections \( \sigma_R \) (in mb), and the volume integrals \( J_V, J_W^{(a)}, \) and \( J_W^{(b)} \) (in MeV.fm\(^3\)) as functions of the energy \( E \) (in MeV) for: (a) \(^{10}\text{Be} + p\) with \( ls \) and \( surf \) terms; (b) \(^{11}\text{Be} + p\) with \( ls \) term; (c) \(^{11}\text{Be} + p\) with \( ls \) and \( surf \) terms.

| Model | \( E \) | \( N_R \) | \( N_I \) | \( N_{ls}^R \) | \( N_{ls}^I \) | \( N_{sf}^I \) | \( \sigma_R \) | \( J_V \) | \( J_W^{(a)} \) | \( J_W^{(b)} \) |
|-------|------|------|------|------|------|------|------|------|------|------|
| (a) GCM | 39.1 | 0.995 | 0.266 | 0.095 | 0.082 | 0.004 | 298.65 | 394.161 | 117.040 | 122.321 |
| QMC   | 1.194 | 0.260 | 0.075 | 0.025 | 0.018 | 333.71 | 425.971 | 115.286 | 139.235 |
| GCM   | 59.4 | 0.970 | 0.000 | 0.365 | 1.000 | 0.373 | 400.26 | 323.404 | 0.000 | 367.802 |
| QMC   | 1.043 | 0.281 | 0.000 | 1.000 | 0.270 | 389.27 | 311.325 | 93.053 | 361.343 |
| (a) QMC | 7.5 | 1.483 | 0.000 | 0.442 | 0.208 | 0.044 | 306.28 | 719.605 | 0.000 | 148.453 |
| QMC   | 10.7 | 1.354 | 0.098 | 0.178 | 1.000 | 0.193 | 636.50 | 632.936 | 100.685 | 695.676 |
| (b) GCM | 38.4 | 0.787 | 0.799 | 0.000 | 0.507 | 0.000 | 458.63 | 324.148 | 355.844 | 355.844 |
| 49.3 | 0.793 | 0.867 | 0.123 | 0.316 | 0.000 | 426.85 | 296.301 | 301.184 | 301.184 |
| (c) GCM | 38.4 | 0.849 | 0.106 | 0.102 | 0.380 | 0.152 | 493.01 | 349.685 | 47.208 | 269.903 |
| 49.3 | 0.801 | 0.000 | 0.213 | 0.394 | 0.200 | 436.46 | 299.280 | 0.000 | 246.162 |

Table 2. The renormalization parameters \( N_R \) and \( N_I \), the total reaction cross sections \( \sigma_R \) (in mb), and the volume integrals \( J_V \) and \( J_W \) (in MeV.fm\(^3\)) as functions of the energy \( E=38.4 \) and 49.3 MeV/nucleon for the \(^{11}\text{Be} + ^{12}\text{C}\) elastic scattering.

| Nucleus | Model | \( E \) | \( W \) | \( N_R \) | \( N_I \) | \( \sigma_R \) | \( J_V \) | \( J_W^{(a)} \) |
|---------|------|------|------|------|------|------|------|------|
| \(^{11}\text{Be}\) | GCM | 38.4 | \( W^H \) | 0.769 | 0.711 | 127.123 | 216.879 | 287.235 |
|      | \( V^F \) | 0.708 | 0.521 | 126.825 | 199.676 | 146.937 |
|      | 49.3 | \( W^H \) | 0.820 | 0.883 | 124.406 | 213.754 | 300.193 |
|      | \( V^F \) | 0.743 | 0.574 | 123.302 | 193.682 | 149.628 |

The \( s \)-state \( (l = 0, n = 1, 2) \) of the relative motion of two clusters and the corresponding density distribution have the forms:

\[
\phi^{(n)}_{00}(s) = \phi^{(n)}_0(s) \frac{1}{\sqrt{4\pi}}, \quad \rho^{(n)}_{00}(s) = |\phi^{(n)}_{00}(s)|^2 = \frac{1}{4\pi} |\phi^{(n)}_0(s)|^2, \quad n = 1, 2. \tag{3}
\]

Within the \(^{10}\text{Be}+n\) cluster model, in order to calculate the \(^{11}\text{Be}\) breakup in its collision with the protons and nuclear targets, one should calculate two OPs of \(^{10}\text{Be}+A\) and \(n+A\) scattering [6]:

\[
U^{(b,n)}(r) = V^{(b,n)} + iW^{(b,n)} = \int ds \rho^{(n)}_0(s) \left[ U^{(n)}_c(r + (1/11)s) + U^{(n)}_n(r - (10/11)s) \right] \tag{4}
\]
Figure 5. Cross sections of diffraction breakup and stripping reaction in $^{11}$Be+ nuclei scattering at $E = 63$ MeV/nucleon. Experimental data are from Ref. [15]

In Eq. (4) $r - (10/11)s \equiv r_n$ and $r + (1/11)s \equiv r_c$ give the distances between the centers of each of the clusters and the target, and $s = s_1 + s_2 = (10/11)s + (1/11)s$ determines the relative distance between the centers of the two clusters, where $s_1$ and $s_2$ are the distances between the centers of $^{11}$Be and each of the clusters, correspondingly. The respective OPs for the $^{10}$Be+$A$ and $n+A$ scattering are calculated within the microscopic model of OP. The differential and total cross sections (for elastic scattering, as well as for diffractive breakup and absorption) all require calculations of the probability functions $d^3P(\Omega, k)/dk$ that depend on the impact parameter $b$. The general expression for the probability functions can be written as in Ref.[14]:

$$
\frac{d^3P_\Omega(b, k)}{dk} = \frac{1}{(2\pi)^3} \left| \int ds \phi_k^*(s) \Omega(b, r_\perp) \phi_{\langle 00 \rangle}^{(n)}(s) \right|^2 ,
$$

(5)

where $\Omega(b, r_\perp)$ depends on the two profile functions $S_i(b_i)$, $(i = c, n)$, $\phi_k$ is the continuum wave functions, $k$ is the relative momentum of both clusters in their center-of-mass frame. One can integrate over the transverse angle of the momenta $\varphi_k$ to get the double-differential probability $d^2P_{\Omega}(b, k)/dk_L dk_\perp$.

The diffraction breakup cross section has the form

$$
\left( \frac{d\sigma}{dk_L} \right)_{diff} = b_n \int_0^\infty d\varphi_n \int_0^{2\pi} d\varphi_n \int_0^\infty dk_\perp \frac{d^2P_{\Omega}(b, k)}{dk_L dk_\perp} .
$$

(6)

The cross sections of the stripping reaction when the neutron leaves the elastic channel is [14]:

$$
\left( \frac{d\sigma}{dk_L} \right)_{str} = \frac{1}{2\pi^2} \int_0^\infty b_n d\varphi_n \left[ 1 - |S_n(b_n)|^2 \right] \times \int d\rho d\varphi_\rho |S_c(b_c)|^2 \times \left[ \int_0^\infty dz \cos(k_L z) \phi_0 \left( \sqrt{\rho^2 + z^2} \right) \right]^2 .
$$

(7)
The results for the diffraction and stripping cross sections (when a neutron leaves the elastic channel) for the reactions $^{11}\text{Be}+^{9}\text{Be}$, $^{11}\text{Be}+^{93}\text{Nb}$, $^{11}\text{Be}+^{181}\text{Ta}$, and $^{11}\text{Be}+^{238}\text{U}$ are presented in the Fig. 5. We note the good agreement with the experimental data from light and heavy breakup targets. The obtained cross sections for the diffraction and stripping have a similar shape. The values of the widths are around 50 MeV in agreement with the experimental ones. Our results confirm the observations that the width almost does not depend on the mass of the target and as a result, it gives information basically about the momentum distributions of two clusters.

We can conclude that, in general, the hybrid model for microscopic calculations of the OPs gives the basic important features of the scattering cross sections and can be recommended and applied to calculate more complex processes such as breakup reactions, momentum distributions of fragments and others.

Future measurements of such reactions are highly desirable for the studies of the exotic $^{11}\text{Be}$ structure.

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