Determination of an allowable value of internal uniform pressure on the underground horizontal working contour with a trapezoidal form of its cross-section

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Abstract. The paper presents the study results of the stress state at the contour points of an underground horizontal working carried out by the methods of the complex variable theory. This study is the basis for solving the problem of determining an allowable value of the uniform pressure transmitted to the contour of the working, with predetermined values of the depth of its laying. The condition proposed by the authors of the paper is used as a criterion of strength (stability), according to which the working’s contour strength is ensured if at any point the amount of the tangential normal stress does not exceed the tensile and compressive strength of the host soil (rock), i.e. \( \sigma_t \leq \sigma \leq \sigma_c \). As a result of the solution, an assessment of the stress distribution on the contour of an underground horizontal working has been made. The working has a cross-section in the form of a curved trapezoid, which is under the action of uniform all-round stretching of a given intensity with a varying depth and various values of the host mass lateral expansion coefficient. The allowable values of the tensile internal pressure on the contour of an underground horizontal working of a trapezoidal cross-section have been determined at the given values of its depth and a constant coefficient value of the lateral expansion of the host mass \( \mu = 0.25 \). The materials presented in the paper can be used when choosing the maximum or minimum allowable depth of the working used as a storage facility for liquid or gaseous hydrocarbons.

1. Introduction
One of the most urgent problems of geomechanics is the use of underground cavities as storage facilities for recovered reserves of liquid and gaseous minerals. This problem is directly related to studying the strength of underground, in particular, horizontal, workings [1-4]. It is known that after the exhaustion of mineral deposit reserves being developed by the underground method, there are many kilometers of underground mine networks left, which can be used to solve the problem mentioned above.

The use of such workings as storage facilities for gaseous hydrocarbons requires to analyze the stress state of the rock mass in the vicinity of the working and at the points of its contour by the elasticity theory methods. The most effective methods in this situation are the methods of the complex variable theory. In order to use them, it is necessary to specify the function of a complex variable that performs a conformal mapping of the interior or exterior of a unit circle onto an infinite simply-connected area, whose boundary is a simple closed curve imitating the contour of an underground horizontal working of the required configuration [5; 6].

The construction of mapping functions, as a rule, is a rather difficult problem and when solving it, one has to abandon complex analytical expressions, replacing them with the simple ones, for example,
polynomials. Such a replacement made it possible to develop convenient and effective methods for mapping function construction [7-9].

Let us consider the mapping function used in [10; 11], which has the form of

$$z = \omega(\zeta) = i(A \zeta^{-1} + B \zeta + C \zeta^2 + D \zeta^3),$$  

(1)

where \(A, B, C, D\) are real numbers.

This function applies a conformal mapping of the interior of the unit circle \(|\zeta|<1\) to the exterior of infinite simply connected areas whose boundaries is a family of simple closed curves.

Mapping function (1) has already been used in a number of works [12-14] by the authors of this paper when solving problems related to the study of stresses at the contour points of underground workings and determining their allowable depth.

2. Goal setting and investigation objectives

Let us consider an underground horizontal working in the elastically isotropic rock mass. The form of its cross-section is determined by the mapping function (1). We assume that the working is laid at a sufficiently large depth \(H\). What is more, along its contour (from inside), there is all-round uniform pressure of intensity \(p\), which makes it possible to use the working as an underground storage facility of gaseous hydrocarbons, for example.

The present study is aimed at determining an allowable value of uniform pressure transmitted to the contour at preset depth values based on the stress state analysis on the contour of the working. As a strength criterion we use the one proposed by the authors, according to which the strength of the contour will be ensured if the value of the tangential normal stress at any point does not exceed the rock strength of the host rock under tension and compression, i.e. \(\sigma_t \leq \sigma \leq \sigma_c\) [13].

To achieve the target goal two specific problems have been solved in the paper.

**Problem A.** To assess the stress distribution on the underground contour which has cross-section in the form of a curvilinear trapezoid being under the impact of all-round uniform tension with preset intensity at alternating depth and different values of the lateral thrust coefficient of host rock.

**Problem B.** To determine allowable values of tensile pressure stress on the underground horizontal contour of trapezoid cross-section at preset depth values and the lateral thrust coefficient constant values of host rock mass \(\mu = 0.25\).

**Problem solution.** Let us consider the working whose cross-section represents a curvilinear trapezoid with dimensions \(4 \times 3.2\) m.

Using an approach proposed in [15], we calculate the coefficients of mapping function (1).

As a result, we have \(A = 1.972; B = -0.222; C = 0.197; D = 0.195\). The coefficients of mapping function (1) provide the vertical section (contour) with the form of a curvilinear trapezoid. The contour of the working is given in figure 1.

![Figure 1](image.png)

**Figure 1.** Cross-section in the form of a curvilinear trapezoid.
\[
\sigma_v = -\gamma H (F + G \cos \theta + Q \cos 2\theta) - p(K - 4U + (L - 4V) \cos \theta +
\frac{(M - 4W) \cos 2\theta - N \cos 3\theta - R \cos 4\theta}{K + L \cos \theta + M \cos 2\theta + N \cos 3\theta + R \cos 4\theta},
\]

where

\[
F = (1 + \mu)(9D^2 + 4C^2 - A^2) + BS; \quad Q = (1 + \mu)(A + 3D)B + (3D - A)S; \quad G = 2C((1 + \mu)(B + 6D) + S);
\]

\[
S = \frac{(1 + \mu)(A + D)B - 2(1 - \mu)A^2}{A - D};
\]

\[
U = \frac{AB^2}{A - D} + 4C^2 + 9D^2; \quad V = \frac{2C}{A - D} \left[ AB + (A - D)(B + 6D) \right];
\]

\[
W = \frac{B}{A - D} \left[ 6AD - A^2 - 3D^2 \right];
\]

\[
M = 2B(3D - A), \quad N = -4AC, \quad R = -6AD.
\]

Where \( \gamma \) is rock volume-weight; \( \mu \) is lateral thrust coefficient; \( H \) is depth, \( p \) is the value of uniform pressure, applied to the contour of the hole. What is more, according to [4], we assume that at \( p > 0 \), the contour of the working sustained compression of the constant value of \( p \), and at \( p < 0 \), it sustains tension of the same intensity.

Formula (2) is derived under the condition that depth \( H \) is sufficiently great.

Following [4], we suppose that

\[
H \geq 50R_{\text{max}},
\]

where \( R_{\text{max}} \) is the maximum linear size of the contour.

Finding zeros of the tangential normal stress is reduced to the solution of equation

\[
8Rt^4 + 4Nt^3 + 2(QH + (4W - M - 4R)p)t^2 + (G\gamma H + (4V - L - 3N)p)t +
+(F - Q)\gamma H + (M - 4W + R - K + 4U)p = 0,
\]

where \( t = \cos \theta, |t| \leq 1 \).

Taking into account the results of [14], we note that the extreme values of function \( \sigma(\theta) \) could be derived from the following equations

\[
\sin \theta = 0,
\]

\[
32a_1 \cos \theta + 16a_1 \cos \theta + 8(a_1 - 4a_2) \cos \theta + 4(a_1 - 3a_2) \cos \theta +
+ 2(3a_1 - 2a_2 + a_3) \cos \theta + (a_1 - a_2 + a_3) = 0,
\]

where
Let us move onto considering the problem which is connected with the study of the stress state at the points of the trapezoid cross-section contour given in figure 1.

**Problem A.** Consider granite with the volume-weight of $\gamma=2.5$ t/m$^3$, ultimate tensile strength $R_{\text{ten}}=-17$ MPa and compressive strength $R_{\text{com}}=200$ MPa as host (wall) rock. Since the width being the largest linear size of the trapezoid working is equal to 4m, then taking into account (6), we will have $H=200m$. Then $\gamma H=50000$ kg/m$^2$.

When solving the problem, we use two values of the lateral thrust coefficient: $\mu_1=0.25$ and $\mu_2=1$. The first value corresponds to the Poisson ratio, which, on average, is equal to $\nu=0.20$ for rocks [3]. The second value corresponds to the Poisson ratio $\nu=0.5$ and assumes the hydrostatical distribution of stresses in rocks, which is taken when determining stresses at rather large depths [1].

So, let us consider the trapezoid cross-section working (figure 1), corresponding to mapping function (1), i.e.

$A = 1.972; B = -0.222; C = 0.197; D = 0.195.$

The values of coefficients (4) and (5) do not depend on the value of the lateral thrust coefficient. In both cases they are equal to

\[ U = 0.542, V = 0.294, W = -0.646; \]

\[ K = 4.436, L = 0.286, M = -1.135, N = 1.554, R = 2.307. \]

Case $\mu_1=0.25$.

Based on formulas (3), we obtain

\[ F_i = -4.786; G_i = 1.438; Q_i = 6.686. \]

Then equation (7) will be as follows

\[ 18.458p \cos \theta + 6.216p \cos \theta + (13.372\gamma H + 21.359p) \cos \theta + \\
+ (1.438\gamma H - 3.772p) \cos \theta - 11.472 \gamma H + 1.491p = 0. \]

To find zeros, it is necessary to set pressure value $p$. From the conditions of the problem, it follows that we should choose values $p < 0$. Suppose $p = -1$ MPa.

Let

\[ H_1 = 200 \text{ m} \quad H_2 = 300 \text{ m} \quad H_3 = 400 \text{ m} \quad H_4 = 500 \text{ m} \quad H_5 = 600 \text{ m} \quad H_6 = 700 \text{ m}. \]

Direct calculations by formula (12) taking into account (13) give the following values of zeros:

- at $H_1 = 200 \text{ m}$ \( \theta_1 = 1.053, \theta_2 = 2.271; \)
- at $H_2 = 300 \text{ m}$ \( \theta_1 = 0.951, \theta_2 = 2.372; \)
- at $H_3 = 400 \text{ m}$ \( \theta_1 = 0.869, \theta_2 = 2.452; \)
- at $H_4 = 500 \text{ m}$ \( \theta_1 = 0.804, \theta_2 = 2.515; \)
at $H_i = 600$ m $\theta_i = 0.755, \theta_z = 2.566$;

at $H_o = 700$ m $\theta_i = 0.716, \theta_z = 2.609$.

Let us move to calculating the extreme values of tangential normal stresses for $H_i = 200$ m and $H_s = 500$ m.

By formulas (8) and (9) with the account of (10), (11), find that

at $H_i = 200$ m we have $\theta_i = 0.740, \theta_z = 1.606, \theta_3 = 2.665$.

By adding $\theta_z = 0$ and $\theta_z = \pi$ to the obtained values, we have

\[
\sigma(\theta) = -2540577012, \sigma(\theta) = -85232293, \\
\sigma(\theta) = 918128439, \sigma(\theta) = -1084028298, \\
\sigma(\theta) = -1484436051,
\]

Then

\[
\sigma_{\text{max}}(\theta) = -2540577012, \sigma_{\text{max}}(\theta) = 918128439. \tag{14}
\]

Similarly to the above, we have $\theta_i = 0.613, \theta_z = 1.103, \theta_3 = 2.834$ at $H_s = 500$ m.

Adding $\theta_4 = 0$ and $\theta_5 = \pi$ to the obtained ones, we have

\[
\sigma(\theta) = -1210221946, \sigma(\theta) = -421348068, \\
\sigma(\theta) = 2286292648, \sigma(\theta) = -1175996330, \\
\sigma(\theta) = -1171809377,
\]

Then

\[
\sigma_{\text{max}}(\theta) = -1210221946, \sigma_{\text{max}}(\theta) = 2286292648. \tag{15}
\]

The calculations made allow us to define the areas which sustain tensile stresses at preset depth values, such as

at $H_i = 200$ m we have $\theta \in (0, 1.053) \cup (2.271, 4.012) \cup (5.234, 2\pi)$;

at $H_s = 300$ m we have $\theta \in (0, 0.951) \cup (2.372, 3.911) \cup (5.332, 2\pi)$;

at $H_i = 400$ m we have $\theta \in (0, 0.869) \cup (2.452, 3.831) \cup (5.414, 2\pi)$.

at $H_s = 500$ m we have $\theta \in (0, 0.804) \cup (2.515, 3.768) \cup (5.879, 2\pi)$;

at $H_s = 600$ m we have $\theta \in (0, 0.755) \cup (2.566, 3.717) \cup (5.528, 2\pi)$;

at $H_s = 700$ m we have $\theta \in (0, 0.716) \cup (2.609, 3.674) \cup (5.567, 2\pi)$.

In other areas of interval $(0, 2\pi)$ there are compressive stresses.

Diagrams of tangential normal stress distribution for underground trapezoid cross-section at preset depth values $H_i$ to $H_s$ in case $\mu_i = 0.25$ are given in figure 2.

Case $\mu = 1$.

On the basis of formulas (3) we get

\[
F_z = -6.70 \ell; G_z = 0.604; Q_z = -0.315. \tag{16}
\]

Then equation (7) will have the following form

\[
18.458 \rho \cos^3 \theta + 6.216 \rho \cos^2 \theta + (-0.630 \rho H + 21.359 \rho) \cos \theta + \\
+ (0.604 \rho H - 3.772 \rho) \cos \theta - 6.386 \rho H + 1.491 \rho = 0. \tag{17}
\]

Direct calculations by formula (17) with the account of (13) give the following values of zeros:

at $H_i = 200$ m $\theta_i = 0.505, \theta_s = 1.112, \theta_3 = 2.282$;

at $H_s = 300$ m $\theta_i = 0.687, \theta_s = 0.934$.
At other values of \( H_i, i = 3 – 6 \) there are no zeros. Let us make calculations of extreme values of tangential normal stresses for the same two depth values, i.e. for \( H_1 = 200 \) m and \( H_4 = 500 \) m.

By formulas (8) and (9) taking into account (10), (16), we get
\[
\theta_1 = 0.803, \theta_2 = 1.644, \theta_3 = 2.582 \text{ at } H_1 = 200 \text{ m.}
\]
Adding \( \theta_1 = 0 \) and \( \theta_4 = \pi \) to the obtained values, we find
\[
\sigma(\theta_1) = -1576288612, \sigma(\theta_2) = 569422445, \\
\sigma(\theta_3) = 602106025, \sigma(\theta_4) = -11358138. \\
\sigma(\theta_5) = -508326596.
\]

Then
\[
\sigma_{\text{max}}(\theta_1) = -1576288612, \sigma_{\text{max}}(\theta_2) = 602106025. \quad (18)
\]

Similarly to the above, at \( H_4 = 500 \) m, we have \( \theta_1 = 0.811, \theta_2 = 1.653, \theta_4 = 2.577 \).

Adding \( \theta_1 = 0 \) and \( \theta_4 = \pi \) to the obtained values, we find
\[
\sigma(\theta_1) = 2289444939, \sigma(\theta_2) = 1215288777, \\
\sigma(\theta_3) = 1198902332, \sigma(\theta_4) = 1505679070. \\
\sigma(\theta_5) = 1754267230.
\]

Then
\[
\sigma_{\text{max}}(\theta_1) = 1198902332, \sigma_{\text{max}}(\theta_2) = 2289444939. \quad (19)
\]

Similarly to the previous, we determine areas in which tensile stresses are in force at preset values of depth (13).
At \( H_1 = 200 \) m we have \( \theta \in (0.505, 1.112) \cup (2.282, 4.001) \cup (5.171, 5.778) \);

at \( H_1 = 300 \) m we have \( \theta \in (0.687, 0.934) \cup (5.349, 5.596) \).

In other areas of interval \((0, 2\pi)\), and also at all other depth values the compressive stresses are in force.

The diagrams of tangential normal stress distribution for underground trapezoid cross-section at preset depth values \( H_1 \) to \( H_6 \) in case \( \mu_2 = 1 \) are given in figure 3.

\[ \begin{align*}
(\text{a}) & \quad H_1 = 200 \text{ m} \\
(\text{b}) & \quad H_2 = 300 \text{ m} \\
(\text{c}) & \quad H_3 = 400 \text{ m} \\
(\text{d}) & \quad H_4 = 500 \text{ m} \\
(\text{e}) & \quad H_5 = 600 \text{ m} \\
(\text{f}) & \quad H_6 = 700 \text{ m}
\end{align*} \]

**Figure 3.** Diagrams of tangential normal stress distribution.

**Problem B.** Let \( H = 0 \). Then, inserting (10) and (11) into (2), we get an equation for extreme values of function \( \sigma(\theta) \). The roots of the equation are

\[ \theta_1 = 0.791, \theta_2 = 1.664, \theta_3 = 2.583. \]

Adding values \( \theta_4 = 0 \) и \( \theta_5 = \pi \) to them, we define that

\[ \sigma_{\text{min}}(\theta) = \sigma_{\theta}(\theta) = -4.117p. \]

Based on the results of [12, 13], we get

\[ p \approx -0.421341 \text{ MPa}. \]

Let the underground working be at a depth of \( H_1 = 200 \) m.

Consider equation

\[ (20) \]

\[ \gamma H(F + G\cos\theta + Q\cos 2\theta) - p(K - 4U + (L - 4V)\cos\theta + \frac{K + L\cos\theta + M\cos 2\theta + N\cos 3\theta + R\cos 4\theta}{(M - 4W)\cos 2\theta - N\cos 3\theta - R\cos 4\theta}) = \frac{R_i}{g}, \]

where \( R_i \) is ultimate tensile strength of rock, \( g \) is free-fall acceleration as pressure function \( p(\theta) \).
By differentiating equation (20), we get a trigonometric equation of degree 9 for determining extreme values of function \( p(\theta) \) together with equation \( \sin \theta = 0 \). We will look for the minimum value of function \( p(\theta) \) near the value of the argument at which the pressure function reaches its minimum at \( H = 0 \). This value at \( H = 200 \) m is \( \theta = 0.749 \). Then, the value of the allowable uniform tensile pressure is equal to \( p \approx -0.781994 \) MPa.

Let the underground working be now at a depth of \( H = 500 \) m. Making necessary calculations, we get that at \( \theta = 0.676 \) the amount of allowable tensile pressure is equal to \( p \approx -1.202117 \) MPa. Similarly, at a depth of \( H = 500 \) m at \( \theta = 0.634 \) the target value is equal to \( p \approx -1.413983 \) MPa.

The graphs of function \( p(\theta) \) at the values of the underground depth given above are presented in figure 4.

![Graphs of pressure function](image)

**Figure 4.** Graphs of pressure function \( p(\theta) \) at \( H = 200 \) m (a), \( H = 500 \) m (b), \( H = 500 \) m (c)

3. Conclusion

1. As a result of the stress state study at the boundary points of the underground horizontal trapezoidal cross-section working, the sections of the boundary with tensile and compressive tangential normal stresses were identified. Their minimum and maximum values at an alternating working depth for two values of the lateral thrust coefficient (\( \mu_1 = 0.25 \) and \( \mu_2 = 1 \)) were calculated. It was established that with the value of the lateral thrust coefficient \( \mu_2 = 1 \) and the working depth \( H > 322 \) m at all points of the working contour compressive normal tangential stresses occurred.

2. The allowable values of uniform tensile pressure on the working contour considered at different depths \( H = 200; 500; 700 \) m and the value of the lateral thrust coefficient equal to \( \mu_1 = 0.25 \) were determined. It was shown that an increase in the values of the depth entailed an increase in the maximum allowable value of the uniform all-round tensile internal pressure.

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