CHIRAL PERTURBATION THEORY FOR DAΦNE PHYSICS

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Abstract

This talk is an overview of Chiral Perturbation Theory for Kaon physics as far as not covered by other speakers. It includes a short introduction to the strengths and weaknesses of Chiral Perturbation Theory and its applications to semileptonic Kaon decays and $K \rightarrow \pi \pi(\pi)$. 
1 Introduction

The topic of this talk is a review of Chiral Perturbation Theory (CHPT) for DAΦNE physics. This title is at the same time too broad and too narrow for what I will discuss. It is too broad because I will restrict myself here to the topics related to Kaon decays only and even there I do not discuss the parts which have been covered by other speakers in this meeting. It is too narrow because Kaon physics is a very alive subject and is done at many places besides the experiments at DAΦNE.

The full explanation of most of what I will present are in “The Second DAΦNE Physics Handbook” [1]. So I have left out from DAΦNE physics the following topics

1. $\gamma\gamma \rightarrow \pi\pi$ and $\gamma\gamma \rightarrow \pi\pi\pi$.
2. All $\eta$ physics.
3. Hypernuclear physics and Kaon-Nucleon scattering. This is the domain of the FINUDA detector.
4. Vector meson properties.
5. All topics where CHPT is not relevant.

Even within Kaon physics I have left out all discussion related to mixings and theoretical estimates of CHPT parameters [2]. The $K \rightarrow \pi\pi$ aspects have been discussed at length here as well [3] and finally the rare decays were discussed by A. Pich [4].

In section 3 I discuss the semileptonic decays and $\pi\pi$ scattering. $K \rightarrow 3\pi$ is treated in section 4.

2 Chiral Perturbation Theory

Chiral Perturbation Theory is a systematic way to use the constraints of chiral symmetry and its spontaneous breakdown. It is a systematic way to go beyond the PCAC method to reach higher orders in the expansion in quark masses and momenta. Its basic principles were laid out by S. Weinberg in a nicely written paper [5]. A systematic derivation and the use of the external field method then provided the basis for the revival of these techniques [3].

It is an approach based on:

- The Chiral Symmetry $SU(3)_L \times SU(3)_R$ of QCD and its spontaneous breakdown to the vector subgroup $SU(3)_V$.

- The Goldstone Bosons from this spontaneous breakdown are the only relevant degrees of freedom at low energies.

- Analyticity, causality, cluster expansion and relativity.
A proof that these are the only assumptions involved was given by H. Leutwyler\cite{7}.

So there are only two questions (apart from the technical part of performing the calculation) when we apply CHPT:

- Does the expansion in $m_q$ and $p^2$, where $p$ is a generic momentum or energy, converge?

- If yes and the higher orders are important, do we have enough data to determine all relevant parameters or do we have a sufficiently reliable way of estimating them?

### 3 Semileptonic Decays

The lowest order lagrangian is

\[
\mathcal{L}_2 = \frac{F^2}{4} \text{tr} \left( D_\mu D^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \right) .
\]

It contains 2 free parameters, $F$, related to the pion decay constant $F_\pi$, and $B_0$, which is related to the vacuum expectation value $\langle \bar{q}q \rangle$, plus the ratios of the quark masses.

The next-to-leading, $O(p^4)$, Lagrangian was derived in Ref.\cite{6} and contains an additional 10 parameters labelled $L_i$. All of these are at present determined from experiment and most can be in principle determined in Kaon semileptonic decays. In table \[1\] the present best values for these parameters, the source of this value and to which Kaon decays they contribute are listed. $L_4$ and $L_5$ occur in all decays but the crosses in table \[1\] signify a different combination than in $K_{l2}$ decays.

For more extended discussions and more references see Ref.\cite{9}.

#### 3.1 $K_{l2}$

The main use of these decays is to determine $F_K$. From the ratio of the muon to the electron decay we can also test electron-muon universality. This assumes we can reliably calculate the relevant electromagnetic radiative corrections. For the absolute value this is rather unsure but most of the unsure contributions seem to cancel in the ratio. For a discussion with an optimistic estimate of the error involved here see Ref.\cite{10}. An improvement by a factor of 10 over the present error of about 5% on this ratio seems feasible from the statistical point of view.

#### 3.2 $K_{l2\gamma}$

This decay has two formfactors depending on the lepton-neutrino mass squared. The prediction for the axial, $A(W^2)$, and the vector, $V(W^2)$, formfactors are to order $p^4$\cite{11,12}:

\[
A = -\frac{4}{F} (L_9 + L_{10}) = \frac{-0.030}{m_K} .
\]

\footnote{I will use standard CHPT where we count $p^2$ as the same order as $m_q$. An alternative view can be found in talk by M. Knecht and references therein.}
\[ L_i(M_\rho) \]

| \( L_i \) | \( 10^3 \cdot L_i(M_\rho) \) | Source | \( K_{l2\gamma} \) | \( K_{l2ll} \) | \( K_{l3} \) | \( K_{l3\gamma} \) | \( K \rightarrow \pi^+\pi^-e^+\nu \) |
|---|---|---|---|---|---|---|---|
| \( L_1 \) | 0.4 ± 0.3 | \( K_{e4}, \pi \pi \) | ✗ | ✗ | ✗ | ✗ | ✗ |
| \( L_2 \) | 1.35 ± 0.3 | \( K_{e4}, \pi \pi \) | ✗ | ✗ | ✗ | ✗ | ✗ |
| \( L_3 \) | −3.5 ± 1.1 | \( K_{e4}, \pi \pi \) | ✗ | ✗ | ✗ | ✗ | ✗ |
| \( L_4 \) | −0.3 ± 0.5 | \( 1/N_c \) | ✗ | ✗ | ✗ | ✗ | ✗ |
| \( L_5 \) | 1.4 ± 0.5 | \( F_K/F_\pi \) | ✗ | ✗ | ✗ | ✗ | ✗ |
| \( L_9 \) | 6.9 ± 0.7 | \( r^2_{\pi V} \) | ✗ | ✗ | ✗ | ✗ | ✗ |
| \( L_{10} \) | −5.5 ± 0.7 | \( \pi \rightarrow e\nu\gamma \) | ✗ | ✗ | ✗ | ✗ | ✗ |

Anomaly | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ |

Table 1: Occurrence of the low–energy coupling constants \( L_1, \ldots, L_{10} \) and of the anomaly in Kaon semileptonic decays. In \( K_{\mu 4} \) decays, the same constants as in the electron mode (displayed here) occur. In addition, \( L_6 \) and \( L_8 \) enter in the channels \( K^+ \rightarrow \pi^+\pi^-\mu^+\nu_\mu \) and \( K^+ \rightarrow \pi^0\pi^0\mu^+\nu_\mu \).

\[
V = -\frac{1}{8\pi^2 F} = \frac{-0.067}{m_K}. \quad (2)
\]

The form factor \( V(W^2) \) is known to order \( p_6^6 \). The correction is of order 10 to 20% in the relevant region of phase space and the formfactor has become \( W^2 \) dependent to a similar amount.

Present data are:

\[
m_K |A + V| = 0.105 \pm 0.008 \quad \text{and} \quad m_K |A - V| \leq 0.35. \quad (3)
\]

The \( W^2 \) dependence of both form factors has never been measured and data on \( A - V \) are rather poor.

### 3.3 \( K_{l2ll} \)

In these decays there are 3 axial formfactors and one vector one. The vector one is known to order \( p_4^4 \). Most of the rate and distributions are determined by the three axial form factors. These are known to order \( p_4^4 \). Especially in the modes with \( e^+\nu \) in the final state the rate is very much enhanced over the helicity suppressed Bremsstrahlung part. Extrapolation to full phase space is rather difficult for the \( e^+e^- \) final states due to the presence of very small invariant masses for the pair. Some predictions of Ref.\[12\] compared with experiment can be found in table\[4\]. There the effects mentioned are clearly visible.
\[ K^+ \to \mu^+\nu^-e^- \quad e^+\nu^-e^- \quad \mu^+\nu^+\mu^- \quad e^+\nu^+\mu^- \]

| \( K^+ \to \) | \( \mu^+\nu^-e^- \) | \( e^+\nu^-e^- \) | \( \mu^+\nu^+\mu^- \) | \( e^+\nu^+\mu^- \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Experiment      | (1.23 \pm 0.32) \times 10^{-7} | (2.8_{-4.4}^{+2.8}) \times 10^{-8} | \leq 4.1 \times 10^{-7} | - |
| Tree, cuts      | 4.98 \times 10^{-8} | 2.1 \times 10^{-12} | 3.8 \times 10^{-9} | 3.1 \times 10^{-12} |
| 1-loop, cuts    | 8.5 \times 10^{-8} | 3.4 \times 10^{-8} | 1.35 \times 10^{-8} | 1.1 \times 10^{-8} |
| 1-loop, full    | 2.49 \times 10^{-5} | 1.8 \times 10^{-7} | 1.35 \times 10^{-8} | 1.1 \times 10^{-8} |

Table 2: Branching ratios from experiment and theory. The cuts are \( m_{e^+e^-} \geq 140 \text{ MeV} \). No cuts for the decays with \( \mu^+\mu^- \). The last row is with integrating over all of phasespace.

3.4 \( K_{l3} \)

This was calculated to order \( p^4 \) by Gasser and Leutwyler\[{15}\). There are two form factors here which for \( K^{+,0} \to \pi^0,\gamma^+\nu \) are parametrized as

\[
\langle \pi(p')|V_{\mu}^{4-i5}|K(p)\rangle = \frac{1}{\sqrt{2}}\left[ (p' + p)_\mu f_+(t) + (p - p')_\mu f_-(t) \right], \tag{4}
\]

with \( t = (p - p')^2 \). In the analysis of experimental data usually one uses instead of \( f_- \) the scalar formfactor \( f_0 \).

\[ f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2}f_-(t). \tag{5} \]

These are both parametrized in a linear fashion.

For \( f_+ \) the \( p^4 \) expression fits the linear parametrization very well over the relevant range. Using \( f_+(t) = f_+(0)[1 + \lambda_+ t/m_\pi^2] \) theory and experiment agree very well. Data give \( \lambda_+ = 0.029 \pm 0.002 \) while \( p^4 \) predicts from the value of \( L_0 \) in table \[1\] \( \lambda_+ = 0.031 \). Even the isospin breaking predicted from CHPT is observed in the measured values of \( f_+(0) \) for both decays.

For \( f_0 \) the linear parametrization \( f_0(t) = f_+(0)[1 + \lambda_0 t/m_\pi^2] \) fits satisfactorily the \( p^4 \) expression. The prediction depends only on \( F_K/F_\pi \) and is \( \lambda_0 = 0.017 \pm 0.004 \). The experimental situation needs clarification. Trying to average incompatible measurements leads to 0.025 \pm 0.006 for \( K_{\mu3}^0 \) and 0.004 \pm 0.007 in \( K_{\mu3}^+ \). In addition to the interest for the strong interaction effects these decays are our major source of knowledge of \( |V_{us}| \).

3.5 \( K_{l3\gamma} \)

This process was calculated to order \( p^4 \) in Ref.\[{12}\). There are 10 formfactors possible here which are all nonzero at order \( p^4 \). The calculations shows a rather complicated interplay between all the various contributions. In the rates the final effect is rather small. Various distributions might show more sensitivity to the higher order contributions. In other processes like \( K_{l4} \) there are large corrections so the prediction of small corrections to the rate is definitely nontrivial. As an example with cuts \( E_\gamma \geq 30 \text{ MeV} \) and \( \theta_{e}\gamma \geq 20^\circ \) we
Decay | $l = e$ | $l = \mu$
---|---|---
$K^+ \to \pi^+\pi^- l^+\nu$ | $F, G, H$ | $F, G, R, H$
$K^+ \to \pi^0\pi^0 l^+\nu$ | $F$ | $F, R$
$K^0 \to \pi^0\pi^- l^+\nu$ | $G, H$ | $G, H, R$

Table 3: The contributions of the various formfactors to the different $K_{l4}$ decays.

obtained for the branching ratio for $K^+_{e3\gamma}$

$$\text{tree } 2.8 \cdot 10^{-4} + \frac{L_4}{p^4} 3.2 \cdot 10^{-4} + \text{loops} 3.0 \cdot 10^{-4}.$$  \hfill (6)

Afterwards there was a new result from NA31 [16] for $K^+_{Le3\gamma}$ with the same cuts

$$BR = (3.61 \pm 0.14 \pm 0.21) \cdot 10^{-3}$$ \hfill (7)

to be compared with [12]

$$\text{tree } 3.6 \cdot 10^{-3} + \frac{L_4}{p^4} 4.0 \cdot 10^{-3} + \text{loops} 3.8 \cdot 10^{-8}.$$ \hfill (8)

These are in excellent agreement. All possible decay modes should in fact be observable in the near future so the other predictions will also be tested.

### 3.6 $K_{l4}$

In these decays there are 4 formfactors possible. They all have a quite different behaviour under chiral perturbation theory. The four form factors are 3 axial ones, $F$, $G$ and $R$, and one vector one $H$. They contribute to the various decays as shown in table [3]. The other form factors also contribute but are very small. Good measurements at present exist for the formfactors mainly of $K_{e4}$ only. $F, G$ were calculated to $p^4$ in Refs. [17] and $R$ in Ref. [18]. The formfactor $H$ is known to order $p^6$ [13]. In Ref. [18] higher order effects were also estimated using dispersion relations. Conclusions of these papers were:

- $F$ and $G$ get large corrections from their tree level value of $m_K/(\sqrt{2}F_{\pi})$ and allow for an accurate measurement of $L_1$, $L_2$ and $L_3$. These then allow for a clean prediction of the total rates [18] reproduced in table [4]. The errors in the predictions are in fact dominated by the most accurate measurement now available [19]. There is excellent agreement with all available experimental results and several predictions remain to be tested.

- $R$ is a relatively small effect even in $K_{\mu4}$ decays. With accurate measurements of $F$ and $G$ in $K_{e4}$ and distributions in $K_{\mu4}$ a good determination might still be possible. This would allow a test of the $1/N_c$ assumption used in setting $L_4$ essentially to zero. For other possible relevance of measurements of $R$ see Ref. [8].
Table 4: Predictions from Ref.[18] for the various $K_{l4}$ decay widths. The last two columns are normalized to $K_L$ decays. Full includes the unitarization estimates. Exp. are the experimental values. Errors are in brackets and all values are in $s^{-1}$.

| $\pi\pi$ charge | $++$ | $00$ | $+-$ | $00$ | $0-$ | $0-$ |
|------------------|------|------|------|------|------|------|
| Leptons          | $e^+\nu$ | $e^+\nu$ | $\mu^+\nu$ | $\mu^+\nu$ | $e^+\nu$ | $\mu^+\nu$ |
| Tree             | 1297  | 683  | 155  | 102  | 561  | 55   |
| $p^4$            | 2447  | 1301 | 288  | 189  | 953  | 94   |
| Full input       | 1625(90) | 333(15) | 225(11) | 917(170) | 88(22) |
| Exp.             | 3160(140) | 1700(320) | 1130(730) | -- | 998(80) | -- |

- $H$ has relatively small higher order corrections if $F_\pi$ is used in its $p^4$ expression. The slope also is very small[13].

This was all for the real part of the formfactors essentially. The imaginary part allows us to extract more information, see the next subsection.

3.7 $\pi\pi$ scattering

$K_{l4}$ decays also allow an accurate measurement of some $\pi\pi$ scattering angles. These are now known to order $p^6$ or two-loops in chiral perturbation theory[20] and in generalized CHPT[21]. They can be easily obtained using the Pais-Treiman asymmetry methods[22]. A comparison of the calculation of Ref.[20] with the present data is shown in Fig. 1. $\pi\pi$ scattering will allow for a clean test of generalized versus standard CHPT. The generalized CHPT result allows for a somewhat larger range of scattering angles. That allows for results between our 2-loop calculation and a curve roughly following the top of the last three error bars in fig. 1.

4 Nonleptonic Decays

A more extended version of the present discussion is present in Ref. [23]. The main CHPT calculation in this respect is Ref. [24]. The number of free parameters in the weak chiral lagrangian is rather large but still relevant predictions can be made. To lowest order, $p^2$, there are two parameters, $c_2$ and $c_3$, that are essentially the strength of the octet and 27 transitions. At $p^4$ we now restrict to those in $K \rightarrow \pi\pi(\pi)$ decays. For the leading octet parameters to order $p^4$ there are 3 more. One of these can not be disentangled from $c_2$ within these decays. For the 27 there are 4 more one of which appears always in the same combination with $c_3$ in these decays and is hence also not relevant. The total number of parameters is thus 7.

The number of observables in the CP-conserving part is two rates (notice I assume isospin throughout) for $K \rightarrow 2\pi$ and 14 in the $K \rightarrow 3\pi$ rates and Dalitz plots. The latter
Figure 1: The combination of $\pi\pi$ phase shifts that can be measured in $K_{l4}$ decays at two loops in CHPT. Also shown are the present data. The curves are: tree level (dashed), $p^4$ (dot-dashed) and $p^6$ (full).
Table 5: The parameters of the Dalitz plot in $K \to 3\pi$ decays. The first 4 are octet and the last 6 are 27. The quadratic ones are not present at tree level. Experimental values are taken from Ref. [24].

| Variable | Tree   | $p^4$   | Exp.   |
|----------|--------|---------|--------|
| $\alpha_1$ | 74     | input   | 91.71 ± 0.32 |
| $\beta_1$  | −16.5  | input   | −25.68 ± 0.27 |
| $\zeta_1$  | −0.47 ± 0.18 |         | −0.47 ± 0.15 |
| $\xi_1$    | −1.58 ± 0.19 |         | −1.51 ± 0.30 |
| $\alpha_3$ | −4.1   | input   | −7.36 ± 0.47 |
| $\beta_3$  | −1.0   | input   | −2.43 ± 0.41 |
| $\gamma_3$ | 1.8    | input   | 2.26 ± 0.23  |
| $\xi_3$    | 0.092 ± 0.030 |       | −0.12 ± 0.17 |
| $\xi'_3$   | −0.033 ± 0.077 |       | −0.21 ± 0.51 |
| $\zeta_3$  | −0.011 ± 0.006 |       | −0.21 ± 0.08 |

are using a quadratic representation of the Dalitz plot and a linear one for the phases:

Constant part : $1$(Octet) + $1$(27) + $1$(phase)
Linear : $1$(Octet) + $2$(27) + $3$(phases)
Quadratic: $2$(Octet) + $3$(27)

The phases in principle should be calculable in CHPT since they are at very low $\pi\pi$ center of mass energies. They have not been measured at present. In table 5 I give the list of parameters, see Ref. [23] for their definition, the present experimental values, the CHPT tree level predictions and the $p^4$ tree level predictions. The $K \to 2\pi$ rates have always been used as input.

Notice that there is in fact a direct relation between several of the parameters predicted in CHPT. Counting the number of observables and parameters there should be 5 relations. These are:

Octet : $\alpha_1 \to \zeta_1$
$\beta_1 \to \xi_1$

27 : $\alpha_3 \to \zeta_3$
$\beta_3 \to \xi_3$
$\gamma_3 \to \xi'_3$

These are clean predictions of CHPT and should be more stringently tested. The measurements, especially in the smaller 27 sector, should be relatively easy to improve using the new facilities. The agreement at present is very good within the errors.

The CP-violating asymmetries in the Dalitz plot are expected to be of order $10^{-6}$. The main reason for this is that in order for the CP phase to be observable it has to interfere
with the final state phases. These are small since the pions in $K \to 3\pi$ are at very low energies. In addition to order $p^4$ the interference happens only with the suppressed 27 amplitudes. So at order $p^4$ one expects asymmetries of order $10^{-6}$. The final number might be significantly enhanced by $p^6$ effects where interference with the dominant octet amplitudes becomes possible. A CHPT inspired estimate of this effect is in Ref. [25].

5 Conclusions

Chiral Perturbation Theory for Kaons is in very good shape as can be seen from this talk and various others in this meeting.

In the semileptonic sector all parameters to order $p^4$ are determined and various good tests have already been obtained and we look forward to more tests in the near future. On the theoretical side the push beyond $p^4$ has slowly started, e.g. in $\pi\pi$ scattering, and more data are very welcome.

In the nonleptonic sector it has so far been most powerful in rare decays [4]. Restricting to $K \to 2\pi$ and $K \to 3\pi$ the present experimental tests of the $p^4$ relations are only relevant in the octet part. Chiral symmetry does however provide a simple explanation for the various sizes of the Dalitz plot parameters, with the exception of $\Delta I = 1/2$ rule. We look forward to more stringent tests here in the future.

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