An Imaging Method for Electrical Capacitance Tomography Based on Projections Multiplication

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Abstract. In this work, a novel method for the formation of Electrical Capacitance Tomography (ECT) images is presented. This method is based on the modification of the classical algebraic superposition of sensitivity maps, weighted with the inter-electrode capacitance measurements, which is the basis of most of the existing image reconstruction algorithms. The proposed approach replaces this superposition with the scalar multiplication of partial images generated considering only the data corresponding to fixed source electrodes. Results obtained with this proposed method show that the quality of the images can be improved without any image post-processing, which is very interesting especially for real-time applications.

1. Introduction

Electrical Capacitance Tomography (ECT) is an imaging technique for the visualization of the instantaneous distribution in a multiphase flow composed of elements of different permittivity [1]. ECT is based on the measurement of the electrical capacitance between all the different electrode pairs surrounding a pipe when an electric field is applied. From these data, a reconstructed image of the permittivity distribution inside the vessel can be obtained [2].

The reconstruction algorithm selected for a particular application implies a trade-off between imaging velocity and image quality, since improving the quality of a reconstructed image usually requires the image to be processed by filtering techniques, error minimization protocols, etc., which increases the computation time required for obtaining that image. Therefore, in real-time applications, where the elapsed time for the image reconstruction is a limiting factor, simple algorithms are usually selected, for example the Linear Back Projection (LBP) or the Tikhonov regularization [3, 4], which generate images very rapidly, but also with poor quality.

In this work, an image composition method is proposed, which is suitable for being applied with any reconstruction algorithm while providing a better image quality and saving computation time without any additional

2. Image Composition Method

As described above, the aim of any ECT system is to obtain images of the instantaneous phase distribution inside a vessel through a set of capacitance measurements. The forward problem in ECT, i.e., the calculus of the capacitance values for a given permittivity distribution, can be written as a...
linearized and discrete form \[ A = S \times G, \]
where \( A \) is the normalized capacitance vector, \( S \) is the normalized permittivity vector, and \( S \) is the sensitivity matrix, formed by all the sensitivity maps corresponding to the different electrode pairs in the sensor [6]. The inverse problem in ECT consists of the generation of the instantaneous permittivity distribution from the measured capacitance values, i.e., the resolution of the forward problem stated above, being \( G \) the unknown parameters that describe the distribution. The direct solution could be expressed as \( G=S^\dagger \times \lambda \) However, in general, the inverse of the sensitivity matrix does not exist, and the solution must be obtained as

\[
g = S \times \lambda
\]

where \( g \) is the approximation to the real solution and \( \hat{S} \) is a modified sensitivity matrix that represents the inverse of \( S \). Equation (1) is a common expression for many reconstruction algorithms, being the way \( \hat{S} \) is constructed what establishes the difference between them. In the LBP algorithm, \( \hat{S} \) is taken as the transposed sensitivity matrix, i.e., \( \hat{S} = S^T \) [5]. In the Tikhonov regularization, this matrix has a more complex expression [7] \( \hat{S} = (S^T S + \beta I)^{-1} S^T \), where \( I \) is the identity matrix and \( \beta \) is a regularization parameter. For the Singular Value Decomposition method, this matrix is written as \( \hat{S} = V \Sigma U^T \), where the matrices \( V, U \) and \( \Sigma \) are obtained by decomposing \( S \) in singular values [8]. If Equation (1) is further developed, it can be written as:

\[
g = S^1 \times \lambda^1 + S^2 \times \lambda^2 + ... + S^n \times \lambda^n = g^1 + g^2 + ... + g^n = \sum_{k=1}^{n} S^k \times \lambda^k = \sum_{k=1}^{n} g^k
\]

where \( n \) is the number of electrodes in the sensor. The term \( \hat{S} \) represents the modified normalized Jacobian matrix, i.e., the modified sensitivity matrix, corresponding to the \( k \)-th source electrode with respect to the rest of the detecting electrodes, and \( \lambda^k \) is the normalized capacitance vector between the same source electrode \( k \) and the rest of electrodes. Each factor in the form \( \hat{S} \times \lambda^k \) can be seen as a partial image reconstruction for a fixed source electrode, as expressed in Equation (2), which is added to the other partial images in order to generate a final reconstruction. It is obvious that a partial image, formed considering only the data corresponding to a fixed source electrode, will contain an important amount of incorrect pixels, compared to the original distribution. Thus, most of the partial image will be wrong, and only some pixels will represent the real distribution, as it can be seen in Figure 1, where some partial images obtained with the LBP algorithm for a simulated distribution are depicted. If these images are added to form the final reconstruction the original permittivity distribution will be formed by the sum of the pixels of the partial images with a high value (marked in black in Figure 1A), whereas the addition of the pixels with low values will generate a background noise, i.e., a background color different from white, as illustrated in Figure 1A (d).

This noise is inherent to the so-called non-iterative reconstruction methods and should be removed to generate clearer images, and this can be done using some iterative method, which is usually based on the minimization of the image error [4], or applying some image filtering. In any case, this image processing implies an increment of the computation time required for obtaining the image, which can be unacceptable, especially in real-time applications. Moreover, iterative algorithms or filtering techniques are usually experimentally fitted, and the fitting parameters can be useful for certain distributions but not for others, which introduces a serious problem in real applications where the phase distribution is unknown, and can also vary with time.

To solve this problem, an image composition method is proposed in this paper, which implies a modification of Equation (2). Instead of forming the final image by superposing the partial images, that is, by adding them, the permittivity reconstruction can be generated by the scalar multiplication of these partial images, as expressed in the following:

\[
g = g^1 \cdot g^2 \cdot ... \cdot g^n = \left( S^1 \times \lambda^1 \right) \left( S^2 \times \lambda^2 \right) \cdot ... \cdot \left( S^n \times \lambda^n \right) = \prod_{k=1}^{n} \left( S^k \times \lambda^k \right) = \prod_{k=1}^{n} g^k
\]
where the scalar product, denoted as a dot, means the multiplication of the elements in different matrices occupying the same position. Thus, every pixel in the final image is obtained as the product of the pixels with the same index in the partial images, instead of being obtained as the sum of these, as it was done in Equation (2).

This approach implements a process of mask-based image filtering, in which every partial image obtained for fixed electrode positions acts as a mask for the rest of partial reconstructions. This can be understood if the concept of back-projection reconstruction is revised. This technique typically consists of the addition of all the capacitance measurements, or back-projections, normalized through a sensitivity matrix, as stated in Equation (2) [9, 10]. By summing up all these partial images over the cross section of the pipe, the grey level in the areas where objects with higher permittivity are found will be enhanced, while in other areas the grey level will be lower. This can be observed in Figure 1B, where the scheme of a back-projection reconstruction of two objects is shown. In this simplified example, only three measurements, or projections, are considered, as illustrated in Figure 1B(b). The addition of these back-projections results in the reconstruction depicted in Figure 1B (c), where the areas with higher grey levels correspond to the positions of the original objects in Figure 1B (a), while the rest of the low-grey zones are artifacts or reconstruction errors that do not correspond to any real object. If the normalized back-projections are pixel-to-pixel multiplied, as expressed in (3), instead of added, the reconstruction of Figure 1B (d) is obtained. In this way, only the common grey areas for the three projections remain in the final image, showing the positions of the real objects, while the rest of the pipe cross section presents a white tone, i.e., it is free of reconstruction errors.

This simple idea has been translated to back-projection based reconstructions in capacitance tomography, and it is carried out following two steps: first, the classical back-projection reconstruction is applied to every set of capacitance measurements corresponding to a fixed position of the source electrode, obtaining the partial images $g^k$, defined in (2). Secondly, these partial images are pixel-to-pixel multiplied in order to eliminate the areas with low grey level, as defined in (3) and illustrated in Figure 1B. Thus, these areas with low grey level act as masks for the reconstruction, resulting in an enhanced final image free of background noise. This characteristic leads to a better quality of the image reconstruction, since the background image error is removed by the reconstruction algorithm itself, while little or no further processing is required. Therefore, the image processing required to
obtain a better resolution and removing the background noise is included in the reconstruction algorithm itself, as it is discussed in the following section.

3. Results and Discussion

The image composition method described in the previous section has been applied for the reconstruction of different permittivity distributions, which have been processed through simulations based on the Finite Element method [11]. Results presented in Figures 2 and 3 compare the images obtained with both methods (addition and multiplication composition), corresponding to equations (2) and (3), respectively, and for three non-iterative and one iterative image reconstruction algorithms, namely the Linear Back Projection, the Singular Value Decomposition algorithm, the Tikhonov regularization and the Landweber iteration. The parameters $\alpha$ and $\beta$ correspond to the regularization parameters of the SVD and Tikhonov algorithms, respectively. It must be remarked that the images shown in these figures correspond to the direct result of the reconstruction algorithms, with no additional processing. These simulations have been carried out using a 12-electrode sensor.

Fig. 2. Image reconstructions using the LBP and SVD algorithms

The reconstruction error is indicated below every image, and it has been calculated as the relationship between the norms of the vectors corresponding to the original simulated distribution $G$ and the obtained with the reconstruction algorithm $g$ [8]:

$$\text{Image error} = \frac{\|g - G\|}{\|G\|}$$

(4)
From the results shown in figures 2, and 3, it can be seen that the reconstructions obtained with the proposed image composition method are free of the background noise described above, while this noise is observed in every reconstruction carried out with the traditional composition methods. This fact leads to a better quality of the images, as it is confirmed by the lower reconstruction error generated in the multiplication-based reconstructions. Moreover, as indicated in the previous section, the images obtained with the SVD and Tikhonov algorithms have a strong dependence on the value of the regularization parameter selected, and it should be adjusted for every permittivity distribution, as it can be seen in Figures 2 and 3, where the best value generating the lowest image error has been selected in every case. Nevertheless, for the new image composition method, results are independent of these parameters and the same image is obtained when the corresponding value is changed. This characteristic represents a great advantage for real applications, where there is no previous information on the real distribution available for adjusting the regularization parameter. The iterative Landweber algorithm has been applied with 300 iterations and, as it can be derived from results of Figure 3, the image errors have decreased down to similar values to those provided with the one-step LBP reconstruction based on the proposed multiplication-based image composition. Nevertheless, the computation time required to achieve this image quality is higher when using the Landweber iteration; in this case, the reconstructions of Figure 3 have been carried out using custom-developed Matlab® code based on Finite Elements, which run on a PC laptop platform with a Centrino® Duo (2.2 GHz) processor and 2 Gbytes of RAM memory. The elapsed time for the generation of images based on the Landweber iteration is 3.2s, while the images reconstructed using the proposed multiplication-based composition can be completed in 45 ms. Therefore, although the same image quality, or even better, can be achieved using an iterative reconstruction algorithm, it could not be applied if real-time images are required, while the proposed image composition method is able to provide images with low error in a short time, which makes it appropriate for on-line applications.
4. Conclusions

The proposed image composition method, which represents a modification of the classic algebraic combination of the partial images that form the final reconstruction, is a technique suitable for application in every reconstruction algorithm, and generates better images with no need of additional data processing, such as error minimization or filtering methods. This quality improving is the result of the image filtering inherent to the proposed algorithm. Moreover, this composition method is completely independent of the value of the fitting parameters corresponding to any reconstruction algorithm based on regularization, such as the Singular Value Decomposition or the Tikhonov algorithm. All these benefits make the proposed method suitable for real applications, especially in situations where real-time images are required, since higher quality images are produced with no need of periodically adjusting or fitting the reconstruction algorithm.

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