Implications of the NANOGrav result on primordial gravitational waves in nonstandard cosmologies

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Abstract

Recently, the NANOGrav collaboration has reported the evidence for a common-spectrum stochastic process, which might be interpreted as the first ever detection of stochastic gravitational wave (GW) background. We discuss the possibility of the signal arising from the first and second order GWs in nonstandard cosmological history. We show that NANOGrav observation can be explained by the first order GWs in the nonstandard thermal history with an early matter dominated era, whereas the parameter space required to explain NANOGrav observation in the standard cosmology or in the nonstandard epoch of kination domination is ruled out by the BBN and CMB observations. For the second order GWs arising from the large primordial scalar fluctuations with a broad Gaussian power spectrum, we study two specific cases to achieve abundant primordial black hole (PBH) production. We find that the NANOGrav observation can be explained with standard radiation domination, or with a dust-like epoch where the gain in the latter case is the lower requirement of primordial amplitude. In this nonstandard epoch, for a broad power spectrum, PBH are produced in a wide mass range in the planetary mass regime. A nonstandard epoch of kination domination cannot produce enough PBH if NANOGrav result is to be satisfied.
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1 Introduction

The North American Nanohertz Observatory for Gravitational Waves (NANOGrav) searches for an isotropic stochastic gravitational wave background (SGWB) by analyzing the cross-power spectrum of pulsar timing residuals [1]. The recent 12.5 year pulsar timing array (PTA) data released by NANOGrav has reported the discovery of a stochastic common-spectrum process [2], which can be fitted into a power law in a narrow range of frequencies. At this moment, they report that the angular correlations are inconclusive and therefore, the collaboration does not claim a detection of GWs. However, if the signal is interpreted to be a gravitational wave (GW) background then it is interesting to explore the primordial universe with the NANOGrav constraints. Assuming the NANOGrav signal to be GW background, the source of such a signal can be supermassive black hole merger events [2,3], cosmic strings [4–7], phase transition of a hidden sector [8–10] from inflationary perturbations in the first [11] or second orders [12–15] etc.

The LIGO/VIRGO detection of GWs from binary black hole mergers has opened up a new window towards early universe cosmology [16–22]. The probability that the observed black holes of supersolar masses are of primordial origin has also rekindled the interest in primordial black holes (PBH) [23]. Other than such merger events, GW of a stochastic nature can originate from primordial tensor perturbations, generated during inflation, in the first or higher orders of perturbation theory [24,25]. The amplitude of the primordial tensor power spectrum at the CMB pivot scale ($k_{\text{pivot}} = 0.05 \text{ Mpc}^{-1}$) is constrained by Planck 2018 and BICEP2/Keck Array [26,27] in terms of the tensor-to-scalar ratio $r < 0.056$ with 95% confidence limit. However, if some feature in the inflationary dynamics induces a blue-tilt in the tensor power spectrum at smaller scales, then GWs of larger amplitude are possible [28].

Moreover, in the second and higher orders of perturbation theory, GWs are sourced by first order curvature perturbations [29,31]. If the curvature perturbations have a red-tilted power spectrum throughout inflation, then such second order GW are suppressed with respect to the first order contribution. However, if the primordial scalar power spectrum has a blue-tilt then large primordial curvature perturbations can induce second order GW of large amplitudes. If the curvature perturbations are larger than the critical value for gravitational collapse, then it leads to copious production of PBH. Hence, the phenomenology of PBH and GW in the second order are always related [32–34]. PBH abundance with NANOGrav result has been studied in [12–15].

Currently the constraints on the effects of neutrinos on the CMB, as well as Big Bang Nucleosynthesis (BBN) constraints demand that the universe was radiation dominated at temperature $T_{\text{BBN}} \approx 5 \text{ MeV}$. CMB constraints on $r$ reveals that the upper bound on the energy scale of inflation is $\approx 10^{16} \text{ GeV}$ [35,36]. For a standard slow roll-inflation, the energy scale of inflation remains nearly constant until the end of inflation. The epoch between end of inflation and BBN, which can span a large number of e-folds, is not well

\footnote{For comparison of our results to these papers, see Sec. 3}
constrained by observations. In the standard theoretical description, the universe quickly becomes radiation dominated after the end of inflation and the standard radiation epoch sustains until the matter-radiation equality ($T_{\text{eq}}$). However, it is possible that the equation of state of the universe deviates from radiation after the end of inflation and before BBN\(^2\). An epoch of nonstandard post-inflationary evolution can be motivated by slow reheating ($w$ slowly changes from 0 to 1/3) \[^{38}\]; early matter dominated epoch ($w = 0$) with energy dominated by moduli field \[^{39,41}\]; kinetic energy domination ($w = 1$) in case of quintessential inflation models \[^{42}\]; general stiff domination \[^{43}\] etc. The dynamics of formation and evolution of PBH in such epochs is also different and their abundance is related to the amplitude of the second order GW. Any deviation from the standard radiation epoch before BBN affects the PBH and GW dynamics, which is widely studied in literature \[^{44–51}\]. In this paper, we inspect first and second order primordial GWs in a non-standard post-inflationary epoch (i.e. equation of state $w \neq 1/3$) that is active at the scales probed by NANOGrav.

The rest of the paper is organised as follows: in Sec.\(^2\) we represent the result from NANOGrav in terms of the theoretical quantity: the GW spectrum $\Omega_{GW}$. In Sec.\(^2.1\) we discuss the effect of a general equation of state $0 \leq w \leq 1$ on the first order GW and constraint the relevant parameters using the power law nature of the observed GW signal, fitted by NANOGrav using the frequency in the first five bins. Sec.\(^2.2\) relates the second order GW to the observation and translates the constraints in terms of PBH abundance. In Sec.\(^3\) we discuss our results and conclude.

## 2 NANOGrav Result and Gravitational waves

PTA experiments typically describe the obtained result for GW background in terms of the characteristic strain spectrum $h_c(f)$. The quantity $h_c(f)$ is typically fitted with a power law dependence on frequency $f$ \[^{1,52,53}\]

$$h_c(f) = A_{\text{GWB}} \left( \frac{f}{f_{\text{yr}}} \right)^{\alpha_{\text{GWB}}},$$  \hspace{1cm} (2.1)

with $\alpha_{\text{GWB}} = (3 - \gamma)/2$, where $\gamma$ is the timing-residual cross-power spectral density. The reference frequency is $f_{\text{yr}} = yr^{-1} = 3.1 \times 10^{-8}$ Hz. The recent 12.5 yr observation from NANOGrav \[^{2}\] measured the characteristic strain in the frequency range $f \in (2.5 \times 10^{-9}, 1.2 \times 10^{-8})$ Hz and found evidence of a stochastic common-spectrum process (CP). The data is fitted as a power law and the corresponding GW energy density $\Omega_{GW}(f)$ is given as \[^{1,52,53}\]

$$\Omega_{GW}(f) = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f) = \frac{2\pi^2 f_{\text{yr}}^2}{3H_0^2} A_{\text{CP}}^2 \left( \frac{f}{f_{\text{yr}}} \right)^{5 - \gamma_{\text{CP}}}. \hspace{1cm} (2.2)$$

PTA data from NANOGrav give a joint $A_{\text{CP}} - \gamma_{\text{CP}}$ posterior distribution. Therefore, early universe theories with a prediction for a stochastic GW signal can be constrained using this result in terms of the amplitude $A_{\text{CP}}$ and slope $\gamma_{\text{CP}}$.

In a perturbed spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe, the tensor perturbations $h_{ij}$ are traceless and transverse. The tensor modes are given as:

$$h_{ij}(\eta, x) = \sum_{\lambda} \int \frac{d^3k}{(2\pi)^3} e^{i k \cdot x} \epsilon_{ij}^\lambda(k) h_{k}^\lambda(\eta), \hspace{1cm} (2.3)$$

\(^2\text{For a review, see}^{[37]}\).
where \( \eta \) is conformal time, \( k \) is the comoving wavenumber and \( \epsilon^{\lambda}_{ij}(k) \) are the polarization tensors for the two polarization states \( \lambda = +, \times \). The tensor power spectrum \( P_h \) is defined as:
\[
\frac{k^3}{2\pi^2} \langle h^{\lambda}_{k}(\eta) h^{\lambda'}_{k'}(\eta) \rangle = \delta_{\lambda\lambda'} \delta^3(k + k') P_h(k, \eta). \tag{2.4}
\]

The GW energy density spectrum \( \Omega_{GW} \) is defined as the GW energy density \( \rho_{GW} \) in a comoving wavenumber interval \( (k, k + dk) \), normalised with the critical density \( \rho_c(\eta) \).
\[
\Omega_{GW}(\eta, k) \equiv \frac{\rho_{GW}(\eta, k)}{\rho_c(\eta)} = \frac{1}{24} \left( \frac{k}{H} \right)^2 P_h(k, \eta). \tag{2.5}
\]

Observationally relevant quantity is the GW spectrum at present \( \Omega_{GW}(\eta_0, k) \).

As discussed in Sec. 1, for a given theory, GW spectrum can be evaluated at first and higher orders of perturbation theory. Thus, theoretical predictions for GW resulting in different orders can be constrained with the NANOGrav observation. Moreover, if the scalar fluctuations give rise to PBH, then the PBH abundance can also be probed in terms of the observed GW spectra, assuming subdominant contribution from the first order.

In this work, we focus on the analysis of the first and second order GW in a nonstandard cosmological history, where the background evolution deviates from radiation domination (RD) before BBN. We separately derive the predictions for different orders and compare with the NANOGrav observation to discuss bounds on the relevant primordial quantities.

### 2.1 Primordial gravitational waves in first order

Inflation predicts a stochastic GW background arising from the tensor fluctuations \([24][25]\). In general, the evolution of these tensor fluctuations is source free at the first order of perturbations. Moreover, GW background depends on the primordial tensor power spectrum, which can be written as:
\[
P_h(k, \eta) = T(k, \eta) \Delta^2_{h, \text{inf}}(k), \tag{2.6}
\]
where \( \Delta^2_{h, \text{inf}}(k) = \frac{2}{\pi^2} \left( \frac{H_{\text{inf}}}{M_P} \right)^2 \left( \frac{k}{k_p} \right)^{n_t} \) is primordial tensor power spectrum at horizon re-entry and \( T(k, \eta) \) is the transfer function. Here \( n_t \) is spectral index, \( k_p \) is a pivot scale and \( H_{\text{inf}} \) is Hubble rate when mode \( k_p \) exited the horizon during inflation. For a mode \( k \) re-entering the horizon at time \( \eta(k) \), the transfer function depends on the background evolution from the time of its horizon re-entry until the time of observation. In case of standard cosmological evolution history, radiation domination (RD) starts just after inflation ends (assuming instant reheating) followed by the matter domination (MD). Whereas, in alternate cosmological histories, there can be a pre-BBN epoch with EoS other than the radiation (\( w \neq 1/3 \)). In such an alternate cosmological histories, the transfer function will be different for the modes re-entering the horizon during this epoch. In this work, we have assumed a period with EoS \( w \neq 1/3 \) from the end of inflation till the onset of RD at temperature \( T_1 \). GW energy density in such a nonstandard cosmological thermal history at the first order of perturbations is \([50][54]\):
\[
\Omega^{(1)}_{GW,0}(k) = \frac{\Omega_{\text{rad},0}}{12\pi^2} \left( \frac{g_{s,k}}{g_{s,0}} \right)^{4/3} \left( \frac{H_{\text{inf}}}{M_P} \right)^2 \frac{\Gamma^2(\alpha + 1/2)}{2^{2(1-\alpha)} \alpha \Gamma^2(3/2)} W(\kappa) \kappa^{2(1-\alpha)} \left( \frac{k}{k_p} \right)^{n_t}, \tag{2.7}
\]
where \( \alpha = \frac{2}{1 + 3\omega}, \kappa = \frac{k}{\sqrt{k_T}} = \frac{f(T_1)}{T_1} \) and
\[
W(\kappa) = \frac{\pi \alpha}{2 \kappa} \left[ \kappa J_{\alpha+1/2}(\kappa) - J_{\alpha-1/2}(\kappa) \right] + \frac{\kappa^2 J_{\alpha-1/2}(\kappa)}{J_{\alpha-1/2}(\kappa)}, \tag{2.8}
\]
where $J_i$ is the Bessel function of order $i$. Also, $\Omega_{\text{rad},0} = 9 \times 10^{-5}$ is the radiation energy fraction in universe at present, $g_s$ and $g_f$ are the number of relativistic degrees of freedom for energy and entropy respectively. We also find that $\mathcal{V}(\kappa) \approx \alpha$ for $f > f(T_1)$. It is clear from (2.7) that GW spectrum today depends on $H_{\text{inf}}$, $w$, $n_t$, and $f(T_1)$ (for detailed discussion see ref [50][51][54]). In order to do the analysis with NANOGrav result, it is useful to relate $A_{\text{CP}}$, $\gamma_{\text{CP}}$, parameters related to NANOGrav result, to the set of parameters: ($H_{\text{inf}}$, $w$, $n_t$, $f(T_1)$). Using the equation (2.1), (2.2) and (2.7), we get

$$A_{\text{CP}} = \left(\frac{3H_0^2 \Omega_{\text{rad},0}}{2\pi^2} \frac{\Gamma^2(\alpha + 1/2)}{2^{2(1-\alpha)} \alpha^2 \Gamma^2(3/2)} \right)^{1/2} \frac{H_{\text{inf}}}{M_P} \left(\frac{f_7}{f_p}\right)^{n_t/2} f_7 \frac{\alpha}{f_{T1}^{1-\alpha}}$$

(2.9)

$$\gamma_{\text{CP}} = 3 + 2\alpha - n_t.$$

(2.10)

Evidently, (2.9) and (2.10) show that $A_{\text{CP}}$ depends on $H_{\text{inf}}$, $w$, $n_t$, and $f(T_1)$, whereas $\gamma_{\text{CP}}$ depends only on $w$ and $n_t$. In fig. 1a $A_{\text{CP}}$ is plotted against EoS $w$ for $H_{\text{inf}} = 10^{11}$ and $10^{13}$ GeV, $n_t = 1, 0, -0.1$, and $f(T_1) = 10^{-10}$ Hz, which corresponds to temperature $T_1 = 10$ MeV. We have also plotted $\gamma_{\text{CP}}$ against EoS $w$ for $n_t = 1, 0, -0.1$ in fig. 1b. Gray shaded band in both the fig. 1a and 1b represents the region allowed from NANOGrav 12.5-yr result. It is clear from the fig. 1a that a positive $n_t$ or a large $H_{\text{inf}}$ value is required for to get $A_{\text{CP}}$ in the NANOGrav allowed range. GWs generated during inflation leaves their imprint on CMB anisotropy and polarisation, which put a constraint on tensor-to-scalar ratio $r < 0.056$ at 95\% confidence limit at the CMB pivot scale $k_{\text{pivot}}=0.05$ Mpc$^{-1}$ from the joint analysis of BICEP2/Keck Array and Planck CMB data [35]. This bound in turn puts constraint on the value $H_{\text{inf}} < 6.2 \times 10^{13}$ GeV at 95\% confidence limit [26][27][35][55][56]. Therefore, it is evident from fig. 1a that a positive value of $n_t$ is required to explain the allowed $A_{\text{CP}}$ values. On the other hand, it can be seen from fig. 1b that $\gamma_{\text{CP}}$ value decreases with increasing $n_t$ and $w$ values. Therefore, for large $w$ values, $\gamma_{\text{CP}}$ values go outside the allowed region from NANOGrav observation.

Figure 1: (a) $A_{\text{CP}}$ is plotted against EoS $w$ for $H_{\text{inf}} = 10^{11}$ and $10^{13}$ GeV, $n_t = 1, 0, -0.1$. (b) $\gamma_{\text{CP}}$ is plotted against EoS $w$ for $n_t = 1, 0, -0.1$. Gray shaded band in both the plots is the allowed region from NANOGrav observation.
In this work, our aim is to find the allowed region of $H_{inf} - n_t$ parameter space required to explain the NANOGrav observation for different values of EoS. We do the analysis for three different values EoS, $w = 0, 1/3, 1$. For each value of $w$, we do a parameter scan for $n_t \in (-2, 5)$ and $\log_{10} \left( \frac{H_{inf}}{\text{GeV}} \right) \in (2, 14)$ and compute the $A_{CP}$ and $\gamma_{CP}$ values. Out of all these computed values of $A_{CP}$ and $\gamma_{CP}$, only those values which lie within 2-$\sigma$ region of NANOGrav results have been retained. In addition, constraint on $H_{inf}$ from CMB is used to put the upper limit on the $H_{inf}$ values. It is evident from fig. 2 that there is very small parameter space of $H_{inf} - n_t$ which is allowed for $w = 1/3$ and corresponding values of $A_{CP}$ and $\gamma_{CP}$ is shown in the orange shaded region in fig. 3a. On the other hand, we find that a somewhat large parameter space of $H_{inf} - n_t$ is allowed for $w = 0$ (see fig. 2) corresponding values of $A_{CP}$ and $\gamma_{CP}$ is shown in the blue shaded region in fig. 3b. However, the parameter space required to explain NANOGrav observation for $w = 1$ is excluded from the bound on $H_{inf}$. Therefore, a blue tilted primordial power tensor spectrum is required to explain the NANOGrav observation in case of standard cosmological history and in the nonstandard history with an early MD era.

As GW contributes to the radiation energy density in the universe, it will contribute to the effective number of relativistic degrees of freedom ($N_{eff}$) in the universe. An overly abundant GW can change the

![Figure 2: Allowed $H_{inf} - n_t$ parameter space required to explain NANOGrav result by first order GW in the case of standard cosmological thermal history is shown in orange shaded contour. Blue shaded contour shows the $H_{inf} - n_t$ parameter space required to explain NANOGrav by first order GW in the nonstandard cosmological thermal history with a pre-BBN matter dominated era. Dashed and solid blue lines corresponds to the $H_{inf} - n_t$ parameter space allowed within the 1-$\sigma$ and 2-$\sigma$ of NANOGrav observation respectively. Magenta line shows the bound on $H_{inf}$ from Planck CMB and BICEP2 observations, with the region to the right of the line not allowed. Dotted lines (purple for RD and red for MD) represents the bound from the contribution of the GW to the radiation energy density in the early Universe, with the region above the lines excluded. Long Dashed lines (cyan for RD and green for MD) gives the constraints from LIGO, with the region above the lines excluded.](image-url)
Figure 3: (a) Orange shaded region shows the $\gamma_{CP} - A_{CP}$ parameter space which corresponds to the allowed $H_{inf} - n_t$ parameter space in the standard cosmological thermal history. (b) Blue shaded region shows the $\gamma_{CP} - A_{CP}$ parameter space which corresponds to the allowed $H_{inf} - n_t$ parameter space in the nonstandard cosmological thermal history with a pre-BBN MD era.

expansion rate too much in the early universe and therefore, can affect the abundance of light elements produced during the BBN [57–61]. BBN and Planck CMB observations constrain the contribution to $N_{eff}$, $\Delta N_{eff}$ severely and to evade these constraints it is required that

$$\int df \frac{\Omega_{GW}(f) h^2}{f} \leq 5.6 \times 10^{-6} \Delta N_{eff},$$

where lower limit of the integration is equal to the frequency corresponding to the mode entering the horizon at the time of BBN and the upper limit is set to $10^7$ Hz approximately corresponding to the comoving horizon at the time of end of the inflation at temperature $T \approx 10^{15}$ GeV [53, 60, 62]. Planck CMB measurements and BBN observations severely constrain the $\Delta N_{eff} \lesssim 0.4$ [56, 63, 66]. We then compute the allowed values of $H_{inf} - n_t$ parameter space for which $\Delta N_{eff}$ satisfies the constraints given by Eq. (2.11). The resulting constraint on $H_{inf} - n_t$ parameter space is shown by the dotted lines in fig.2, where the region above that line is ruled out. Purple dotted line gives the constraints on first order GWs in the standard cosmological thermal history, whereas red dotted line represents the bound on first order GWs in nonstandard thermal history with an early MD era. It is evident from the fig.2 that the entire allowed region of $H_{inf} - n_t$ parameter space required to explain NANOGrav result in the standard RD history is ruled out from the bound on $\Delta N_{eff}$ from BBN and Planck CMB observations. However, there a finite parameter space of $H_{inf} - n_t$ which can evade the BBN and CMB constraints and can also explain the NANOGrav result in case of nonstandard thermal history with an early MD epoch. In addition to the bound on GWs from their contribution to the $N_{eff}$, non-observation of stochastic GW background in LIGO O1 and O2 runs can also put constraints on the $H_{inf} - n_t$ parameter space [67]. The resulting constraint on $H_{inf} - n_t$ parameter space are shown by the long dashed lines (cyan for standard RD and green for nonstandard early MD) in fig.2, where the region above that line is excluded. In order to calculate the bound from LIGO, primordial power spectrum is assumed to retain the
power law form till the LIGO frequencies and it is evident from the fig. 2 that the bound from LIGO further constrain the allowed $H_{inf} - n_t$ parameter space in case of nonstandard thermal history with an early MD epoch. However, there is still a certain $H_{inf} - n_t$ parameter space left which can explain the NANOGrav result and simultaneously evade the constraints from LIGO, BBN and CMB.

2.2 Induced primordial gravitational waves from curvature perturbations

Scalar and tensor perturbations do not evolve independently at the second order of perturbation theory. Evolution of the second order tensor perturbation is given as

$$h''_{k} + 2\mathcal{H} h'_{k} + k^2 h_{k} = S(k, \eta),$$

where $S(k, \eta)$ is the source term which depends on the first order scalar perturbation. The scalar induced second order tensor power spectrum is [30],

$$\bar{P}_h(k, \tau) = 2 \int_0^\infty dv \int_{|1 - v|}^{1 + v} dv \left[ \frac{4v^2 - (1 + v^2 - u^2)^2}{4uv} \right]^2 \frac{P_\zeta(kv)P_\zeta(ku)T^2(v, u, x)}{32\pi^2},$$

where the bar denotes oscillation average. Here $v \equiv q/k$ and $u \equiv |k - q|/k$ and $P_\zeta$ is the primordial curvature power spectrum and $x = k\eta$. The integration kernel $I(u, v, x)$ contains the source information (complete expression can be found in ref [30]) which can be written with a simple analytical expression for very late time, i.e. $x \gg 1$. For the modes entering the horizon during RD era

$$I_{RD}(x \gg 1, u, v) = \frac{9(u^2 + v^2 - 3)^2}{32u^6v^6x^2} \left\{ \pi^2(u^2 + v^2 - 3)^2\Theta(u + v - \sqrt{3}) \right.$$ 

$$+ \left( -4uv + (u^2 + v^2 - 3)\ln \left( \frac{3 - (u + v)^2}{3 - (u - v)^2} \right) \right) \right\}.$$  

For a general EoS ($0 < w \leq 1$), the expression for $I(u, v, x)$ is given as [68]

$$I(x \gg 1, u, v) = 2^\beta \frac{3\sqrt{2}}{w\pi\alpha^3} \frac{1 + w}{1 + 3w} \Gamma^2[\beta + 2](uvx)^{-\beta - 1/2} \times \left\{ \frac{\pi}{2} \sin \left( x - \frac{\beta\pi}{2} - \frac{\pi}{4} \right) I_f(u, v, w) + \cos \left( x - \frac{\beta\pi}{2} - \frac{\pi}{4} \right) I_Y(u, v, w) \right\}$$

where $I_f(u, v, w)$ and $I_Y(u, v, w)$ are expressed in terms of associated Legendre polynomials and Legendre polynomials on the cut (complete expression can be found in ref [68]). In general, it is numerically challenging to calculate the second order tensor power spectrum for an arbitrary value of EoS ($0 < w \leq 1$) due to the complicated dependence of $I_f$ and $I_Y$ on $w$. Hence, we chose two different values to interpret the dynamics in nonRD epochs:(i) $w = 1$, where the energy density falls faster than radiation and (ii) $w = \frac{1}{9}$, where it falls slower. We do the rest of the analysis for these two nonRD EoS only. EoS $w = 1$ can arise in models of quintessential inflation [42], where, after the end of inflation, the inflaton fast rolls (kinetic energy dominated (KD) universe) to reach the potential for quintessence dark energy. An epoch with $w = 1/9$ is a nearly dust-like EoS, which can arise during very slow reheating process of the inflaton or the moduli such that the effective EoS can lie between MD and RD for some considerable time. The particular choice of $w = 1/9$ is because the expressions for $I_f$ and $I_Y$ are much simplified in this case and can be solved analytically for $x \gg 1$. The nonstandard epochs terminate at $T_1 < T_{BBN}$ and we choose $T_1 = 10$ MeV.
The integral $I_{KD}$ for $w = 1$ is estimated as:

$$I_{KD}^w(x \gg 1, u, v) = \frac{9}{16\pi u^4 v^4 x} \left\{ \frac{(3(u^2 + v^2 - 1)^2 - 4u^2 v^2)^2}{4u^2 v^2 - (u^2 + v^2 - 1)^2} + 9(u^2 + v^2 - 1)^2 \right\}. \tag{2.16}$$

Similarly, the expression for $I_{1/9}$ can be derived for $w = \frac{1}{9}$ and given as

$$I_{1/9}^w(x \gg 1, u, v) = \frac{25}{16\pi u^4 v^8 x^3} \left\{ (u^2 + v^2 - 9)^2 \left[ 5(u^2 + v^2 - 9)^2 - 6u^2 v^2 \right] \
+ |4u^2 v^2 - (u^2 + v^2 - 9)^2| (4u^2 v^2 + 5(u^2 + v^2 - 9)^2)^2 \
+ 2(u^2 + v^2 - 9) \left[ 5(u^2 + v^2 - 9)^2 - 6u^2 v^2 \right] \sqrt{|4u^2 v^2 - (u^2 + v^2 - 9)^2|} \right\} \times (4u^2 v^2 + 5(u^2 + v^2 - 9)^2) \Theta(\sqrt{9} - u - v) \right\}. \tag{2.17}$$

Now, using the expressions for the integration kernels in Eq.s (2.14), (2.16) and (2.17) with the Eq.s (2.5) and (2.13), we can determine the $\Omega_{GW}^{(2)}$ for RD and the two nonstandard EoS. The resulting GW spectrum, normalised with $A_s^2$ is given in Fig. 4 for a Gaussian primordial power spectrum with a width $\sigma_p = 2$:

$$P_\zeta(k) = A_s \times \exp \left[ -\frac{(\log(k/k_\ast))^2}{2\sigma_p^2} \right]. \tag{2.18}$$

Figure 4: The normalised $\Omega_{GW}^{(2)}$ is shown as function of $k/k_\ast$ for the three $w$ cases we consider.

Evidently, for $w = 1$, there is an enhancement in the normalised GW spectra and for $w = 1/9$, it is diminished. Moreover, the peak of the spectrum shifts towards $k > k_\ast$ for $w = 1$ and towards $k < k_\ast$ for $w = 1/9$. Now, NANOGrav analysis fits the observed stochastic spectrum in the frequency range $f_0 \in (2.5 \times 10^{-9}, 1.2 \times 10^{-8})$ Hz. Thus, if we assume that the $\Omega_{GW}^{(2)}$ peak for each $w$ occurs at the scale (wavenumber $k_\ast = 2\pi f_0$) observed by NANOGrav , then for $w = 1/3$, $k_0/k_\ast \simeq 1$, but for $w = 1$, $k_0/k_\ast > 1$ and for $w = 1/9$, $k_0/k_\ast < 1$. However, in that case, the amplitudes required for each case to fit NANOGrav result will be different: $A_s(w = 1/3) = 6 \times 10^{-3}$, $A_s(w = 1) = 5.5 \times 10^{-4}$ and $A_s(w = 1/9) = 5.1 \times 10^{-3}$ to have $\Omega_{GW}^{(2)}$ peak $\simeq 10^{-9}$.
If the amplitude of the primordial spectrum is such that when these modes enter the horizon at the post-inflationary epoch, the density fluctuations $\delta$ are larger than the critical density for collapse ($\delta_c$), then PBH can be produced with mass $M = \epsilon M_H$, where $M_H$ is the horizon mass at collapse. The PBH formation process depend on the background EoS through the dependence of $\delta_c$ on $w$ and the evolution of PBH abundance in this epoch depends on the modified expansion rate. Taking the numerically evaluated expression in [69]:

$$\delta_c(w) = \frac{3(1+w)}{(5+3w)} \sin^2 \left( \frac{\pi \sqrt{w}}{(1+3w)} \right).$$

(2.19)

Considering the Press-Schechter formalism for gravitational collapse, present abundance of PBH in an interval of mass $M$ to $M + dM$ for general $w$ is given as [51]

$$\psi_w(M) = \frac{e}{T_{eq}} \left( \frac{g_s(T_1)}{g_s(T_{eq})} \right)^{\frac{1}{2}} \left( \frac{\Omega_m h^2}{\Omega_c h^2} \right) \left( \frac{90 M_P^2}{\pi^2 g_s(T_1)} \right)^{\frac{\tau_w}{1+w}} (4\pi \epsilon M_P^2)^{\frac{2w}{1+w}} T_1^{\frac{1-3w}{1+w}} \frac{\beta(M)}{M^{\frac{3w+1}{1+w}}},$$

(2.20)

where $M_P$ is the reduced Planck mass. The PBH mass fraction is given as:

$$\beta(M) = \text{erfc} \left[ \frac{\delta_c(w)}{\sqrt{2\sigma_\delta^2}} \right],$$

(2.21)

where $\sigma_\delta^2$ is the variance of the density power spectrum and calculated as:

$$\sigma_\delta^2 = \frac{4(1+w)^2}{(5+3w)^2} \int \frac{dk}{k} (kR)^4 W^2(k, R) P_c(k).$$

(2.22)

We choose the window function $W^2(k, R) = \exp \left( \frac{k^2 R^2}{4} \right)$ to smooth the perturbations on the comoving scale $R$ at formation. The mass $M$ of the PBH produced is related to the comoving wavenumber $k$ at formation for general EoS as:

$$M(k) = 4\pi \epsilon M_P^2 \left( \frac{\pi g_s^{eq}}{45 M_P^2} \right)^{\frac{1}{2}} \left( \frac{g_s^{eq}}{g_s(T_1)} \right)^{\frac{3w-1}{3w+1}} (a_{eq} T_{eq})^{\frac{3(1+w)}{3w+1}} T_1^{-\frac{3w-1}{3w+1}} k^{-\frac{3(1+w)}{3w+1}}.$$

(2.23)

$\psi_w(M)$ can be calculated from Eq. (2.20) using Eqs. (2.19), (2.21), (2.22) and (2.23). $\psi_w(M)$ for $w = 1/3$, 1 and 1/9 are evaluated and given in Fig. 5 for the same value of $A_s$. This figure evidently shows an enhancement in PBH abundance for nonRD epochs and peak shift with respect to the RD peak position. These peak shifts are correlated with the shifts in GW in Fig 4. The ratio of the PBH abundance for a general $w$ dominated epoch and RD is:

$$g_w(M, w) \equiv \frac{\psi_w(M)}{\psi_{RD}(M)} = \frac{\beta(M)}{\beta_{RD}(M)} \frac{g_s(T)^{w-1/3}}{g_s(T_1)^{w-1/g_s(T_{eq})^{2/3}} T_1^{-3w-1} T^{3w-1}}.$$

(2.24)

The dependence of $\delta_c$ on $w$ is such that $\delta_c^{(w=1/3)} < \delta_c^{(w=1/9)}$ and therefore, the gain $g_w(M, w) > 1$ for both the nonstandard epochs. This means that to form same total abundance $f_{PBH} = \int dM \psi_w(M)$, the two nonstandard epochs will require a lower value of $A_s$ than that required in RD. In this work, we check if the low value of $A_s$ required for abundant PBH formation in each nonstandard epoch can satisfy the NANOGrav bounds.

For the curvature power spectrum given in Eq. (2.18) with $\sigma_p = 1, 2$ and 3, we evaluate $\Omega^{(2)}_{GW}$ with $k_s = 3.6 \times 10^6$ Mpc$^{-1}$ which corresponds to $f_s = 5.5$ nHz, and fit the result in the frequency range of NANOGrav for each of the three $w$ epochs. For each case, we chose $A_s$ such that $f_{PBH} = 0.1$. The resulting $A_{CP}$ and $\gamma_{CP}$ are plotted with the NANOGrav 1$\sigma$ and 2$\sigma$ contours in Fig. 6. For RD (red circle, square and
Figure 5: PBH mass function is shown for a typical $A_s = 0.007$ for different $w$ epochs. Colour specifications are same as Fig. 4.

Figure 6: $A_{CP}$ and $\gamma_{CP}$ fitted for $f_{PBH} = 0.1$ for each case of a Gaussian primordial spectrum: $\sigma_p = 1, 2$ and 3 are plotted with circle, square and triangles respectively; for $w = 1/3$ in red, $w = 1$ in blue and $w = 1/9$ in green. Points for $f_{PBH} = 0.1$ for a constant primordial power spectrum for these three $w$ are plotted as pink stars. This constant power spectrum generates PBH masses in the range: $0.1 < M/M_{\odot} < 5$. The dashed gray line signifies the open edge of the $2\sigma$ contour in NANOGrav analysis (orange contour in Fig.1 of 2).
triangle), the resulting three points for different $\sigma_p$ are close together inside the contour, therefore, abundant PBH production is consistent with NANOGrav. However, for the other two cases ($w = 1$ in blue and $w = 1/9$ in green), points for different $\sigma_p$ are far apart with the value closest to the contours given by $\sigma_p = 2$ for both cases. Therefore, in the rest of our analysis, we use $\sigma_p = 2$ to check the consistency of abundant PBH formation in this nonRD epochs with NANOGrav result. In this figure, we also show the points corresponding to a constant power spectrum of the following form:

$$P_\zeta(k) = A_s \times \Theta(k - k_l) \times \Theta(k_s - k),$$

(2.25)

where $k_l$ and $k_s$ correspond to the scales between which the power spectrum is constant. We have chosen $k_l$ and $k_s$ values for each $w$ case such that the resulting PBHs from (2.25) are always in the mass range $M/M_\odot \in (0.1, 5)$. We find that for the constant power spectrum, PBH production with $f_{PBH} = 0.1$ (stars in Fig. 6) is not consistent with the NANOGrav result in these nonstandard epochs under consideration.

![Figure 7: $A_{CP}$ and $\gamma_{CP}$ fitted for $A_s$ varied in different ranges for $w = 1/3$ (magenta shaded), $w = 1$ (cyan shaded) and $w = 1/9$ (green shaded) with variation in the pivot scale in the range $k_s = 2 \times 10^6$ Mpc$^{-1}$ to $k_s = 7 \times 10^6$ Mpc$^{-1}$ for the Gaussian primordial spectrum with $\sigma_p = 2$. The dashed lines inside each shaded region are contours for $f_{PBH} = 0.1, 10^{-5}$ and $10^{-10}$ from top to bottom. The additional dotted cyan line in the $w = 1$ case just enters the 2$\sigma$ contour of observation, but does not motivate abundant PBH (here $f_{PBH} \simeq 10^{-22}$). For the $w = 1/9$ case, the upper edge of the green shaded region motivates $f_{PBH} = 1$, but observations constrain $f_{PBH}$ to a much lower value.

Table 1: Range of amplitude $A_s$ chosen for the three cases with different EoS used in Fig. 7

|        | $w = 1/3$ | $w = 1$ | $1 = 1/9$ |
|--------|-----------|---------|-----------|
| $A_s^{\text{max}}$ | 0.015     | 0.007   | 0.0082    |
| $A_s^{\text{min}}$ | 0.002     | 0.001   | 0.003     |

Fixing $\sigma_p = 2$, we vary $A_s$ (range given in Table 1) and vary $k_s$ in the range ($2 \times 10^6$ Mpc$^{-1}$ - $7 \times 10^6$ Mpc$^{-1}$) for each $w$ scenario. The corresponding variations in $A_{CP}$-$\gamma_{CP}$ space are given by magenta ($w = 1/3$), cyan ($w = 1$) and green ($w = 1/9$) shaded regions in Fig. 7. We show the contours for PBH abundance.
in dashed lines inside these contours with $f_{PBH} = 0.1$, $10^{-5}$ and $10^{-10}$ from top to bottom. It is evident from Fig. 7 that for RD, 10% PBH as DM is possible. For $w = 1$, All these dashed contours remain outside of the NANOGrav $2\sigma$ contour; the most hopeful case in this $k_*$ range is represented by the cyan dotted contour at the bottom, which represents $f_{PBH} \simeq 10^{-22}$. Therefore, abundant PBH production in $w = 1$ epoch is not consistent with the NANOGrav observation. Increasing $k_*$ will lead to the $\Omega_{GW}^{(2)}$ peak being closer to the NANOGrav frequency range, and the $f_{PBH}$ contours come down to lower $A_{\gamma CP}$ values. However, the slope of $\Omega_{GW}^{(2)}$ is such that $f_{PBH} = 0.1$ contour comes within the NANOGrav $A_{\gamma CP}$ range only when $\gamma_{CP}$ is very small. Moving towards lower values of $k_*$ does not help since power at $k_0$ goes further down and will also influence the power spectrum at CMB scales for a broad primordial spectrum.

For $w = 1/9$, the $f_{PBH} = 0.1$ contour enters the NANOGrav contour only for $k_* \simeq 7 \times 10^6$ Mpc$^{-1}$, and therefore it is possible achieve abundant PBH production for the $w = 1/9$ case for $k_*$ larger than the NANOGrav probed range. Therefore, we proceed with the case $w = 1/9$ now and vary $k_*$ in the range $(2 \times 10^7$ Mpc$^{-1}$ - $7 \times 10^7$ Mpc$^{-1}$) for a variation in $A_s$ in the range $0.005 - 0.0082$ for $w = 1/9$. For the RD case, $A_s$ is varied in the same $k_*$ range as before, but for a smaller range in $A_s$ between $0.008 - 0.015$. The resulting variation in $A_{\gamma CP}$-$\gamma_{CP}$ space are shown in Fig. 8 in green for $w = 1/9$ and in magenta for $w = 1/3$. The relevant PBH contours for $f_{PBH} = 0.1$ are shown in dashed lines. We conclude that PBH and second order GW production in a nonstandard epoch with EoS $w = 1/9$ is consistent with the NANOGrav result when $k_*$ is away from the NANOGrav probed range. The gain in the $w = 1/9$ over RD epoch is that for this nonstandard epoch, abundant PBH and observed GW can both be explained with a lower value of $A_s$: $A_s(w = 1/9)|_{f_{PBH}=0.1} \simeq 0.007$ and $A_s(w = 1/3)|_{f_{PBH}=0.1} \simeq 0.01$.

![Figure 8](image.png)

Figure 8: For a Gaussian primordial power spectrum, $A_{\gamma CP}$ and $\gamma_{CP}$ fitted for $A_s$ varied in different ranges for $w = 1/3$ (magenta shaded) and $w = 1/9$ (green shaded) with variation in the pivot scale in the range $k_*= 2 \times 10^6$ Mpc$^{-1}$ to $k_*= 7 \times 10^6$ Mpc$^{-1}$ for RD and $k_*= 2 \times 10^7$ Mpc$^{-1}$ to $k_*= 7 \times 10^7$ Mpc$^{-1}$ for $w = 1/9$. The dashed line inside each shaded region is the $f_{PBH} = 0.1$ contour. $\sigma_p = 2$ is fixed.

$\sigma_p = 2$ Gaussian power spectrum has a broad peak, therefore we quote a range of the PBH mass near the peaks for each case for the lowest and highest $k_*$ under consideration in Table 2.
Table 2: Ranges of PBH mass $M$ for each EoS

|                        | $w = 1/3$                                | $w = 1$                                | $1 = 1/9$                                |
|------------------------|------------------------------------------|----------------------------------------|------------------------------------------|
| Range of $k_*$ in Mpc$^{-1}$ | $2 \times 10^6 - 7 \times 10^6$        | $2 \times 10^6 - 7 \times 10^6$        | $2 \times 10^7 - 7 \times 10^7$          |
| Range of $M/M_{\odot}$ at $k_{*,\min}$ | 0.2-2                                   | 1-10                                   | $10^{-4} - 5 \times 10^{-3}$             |
| Range of $M/M_{\odot}$ at $k_{*,\max}$ | 0.01 - 0.33                             | 0.08-2                                 | $3 \times 10^{-6} - 3 \times 10^{-4}$    |

3 Discussions and conclusions

The recent NANOGrav 12.5 yr result can be the first putative signal for stochastic GW originating in the early universe; further re-analysis of the data will lead to a more precise statement about whether GW has been detected or not. Therefore, it is necessary to check the early universe theories that are consistent with the NANOGrav result interpreted as GW.

One of the possible explanations for NANOGrav result can come from the stochastic GW produced during inflation. Recently, it was shown in ref. [11] that if NANOGrav result is explained by GW produced during inflation with the standard thermal history, a blue tilted power spectrum will be required. However, the required parameter space to explain NANOGrav result is completely excluded by the GW’s contribution to effective number of relativistic degrees of freedom during BBN. In this work, we have explored the idea if the NANOGrav signal can be explained by the GWs generated during inflation with a nonstandard cosmological history. While it is well established that BBN must have taken place during RD era, there is no direct observational probe of EoS for the epoch between the end of the inflation and BBN. In this work, we have assumed a pre-BBN epoch with equation of state $0 \leq w \leq 1$. More specifically, in the analysis for the first order GW, we have considered three different EoS, $w = 0$, $1/3$ and $1$, which corresponds to an early MD era, RD era and KD era respectively.

Our main results are shown in fig. 2 from which it is evident that a blue tilted power spectrum is required to explain the NANOGrav result for both standard thermal history and nonstandard thermal history with an early MD epoch. Whereas, required parameter space to explain NANOGrav result in nonstandard thermal history with a KD epoch is ruled out by the bound on $H_{\text{inf}}$ from the BICEP2/Keck Array and Planck CMB data. Our results for standard thermal history are consistent with ref [11] as the entire parameter space is ruled out by BBN and LIGO bound. Interestingly, GWs in a nonstandard thermal history with an early MD epoch can explain the NANOGrav result and can also evade the BBN and LIGO bounds successfully as shown in Fig. 2. While calculating the bounds from BBN upper limit of the integration given by (2.11) is set to $10^7$ Hz which corresponds to temperature of universe $10^{15}$ GeV. The upper limit is related to the scale of the end of the inflation and if the scale of the end of the inflation decrease, the upper limit will also decrease. Therefore, for a blue tilted power spectrum, this bound will be further relaxed for a smaller value of scale corresponding to the end of the inflation. We have also assumed that the early MD era spans through from the end of the inflation until the onset of RD era at temperature $T_1 = 10$ MeV. However, there can be a scenario where this MD era last for a shorter period, still enveloping the NANOGrav frequency range. In that case, this bound on GWs from BBN will only be true if the total contribution of $\Omega_{GW}$ from the standard RD era (lasting in between end of inflation and onset of early MD) is smaller than the total contribution from MD era.

Our main underlying assumption is that the primordial power spectrum is a pure power law for all the frequencies ranging from CMB scale to the end of the inflation. Therefore, in future, if LISA, which will
be observing in the frequency range $f \sim 10^{-2}$ Hz, makes some positive or null detection, that can be used to further constrain the nonstandard pre-BBN epoch. In conclusion, we have explored the possibility of NANOGrav signal coming from GWs generated during inflation and evolving in a nonstandard thermal history. In the fig. 3a and 3b, we have shown the $\gamma_{\text{CP}} - A_{\text{CP}}$ parameter space (shaded region) which corresponds to the allowed $H_{\text{inf}} - n_t$ parameter space required to explain the NANOGrav result. If the parameter space for $\gamma_{\text{CP}} - A_{\text{CP}}$ shrinks further in the future with further constraints from NANOGrav and other PTA observatories in future, it will help us in further narrowing down the value EoS of a pre-BBN epoch.

For the second order GW phenomenology in Sec. 2.2, we consider three separate scenarios for the EoS, standard RD ($w = 1/3$), early kinetic energy domination ($w = 1$) and a near-dust scenario ($w = 1/9$). We quote the typical amplitudes required for each case and then proceed to the relevant implications for PBH. We find that the analysis to attain a viable scenario that satisfies NANOGrav constraints on $\Omega_{\text{GW}}^{(2)}$ while also producing abundant PBH crucially depends on not only the amplitude $A_s$ of the primordial curvature power spectrum in Eq. (2.18), but also on $\sigma_p$ and $k_*$ for a given $w$. These latter dependences mainly result from the relative shift of both $\Omega_{\text{GW}}^{(2)}$ and $\psi_w(M)$ peaks with respect to the peak of $P_{\zeta}(k)$ at $k_*$. For the RD case, both the $\Omega_{\text{GW}}^{(2)}$ peak and $\psi_w(M)$ peak occur very close to $k_*$, therefore the dependence of the PBH abundance and $\Omega_{\text{GW}}^{(2)}$ on $\sigma_p$ is mild, as evident from the clustering of the three red points in Fig. 6. For $w \neq 1/3$, the peak of $\Omega_{\text{GW}}^{(2)}$ is far away from $k_*$ and therefore, if the scale of NANOGrav observation is at $k_0 = k_*$ then the peak of $\Omega_{\text{GW}}^{(2)}$ is not being observed. Hence, such cases have considerable variations in the $A_{\text{CP}} - \gamma_{\text{CP}}$ space with change in $\sigma_p$. Now, when we put an additional constraint for obtaining $f_{\text{PBH}} = 0.1$, we notice the variation for points with different $\sigma_p$ for the two different nonstandard epochs under consideration as the green and blue circles, squares and triangles in Fig. 6. This variation shows that given a particular value of $k_*$ in the NANOGrav observed range, there is a value of $\sigma_p \simeq 2$ where the nonstandard cases come closest to the NANOGrav $2\sigma$ contour. This feature in the $A_{\text{CP}} - \gamma_{\text{CP}}$ space with varying $\sigma_p$ is a result of the competition between the two quantities $\frac{dA_{\text{CP}}}{d\sigma_p}$ and $\frac{dP_{\text{PBH}}}{d\sigma_p}$. It is anaytically challenging to check the variation of these two terms since both of them involve rigorous numerical integrations.

Now, fixing $\sigma_p = 2$ and varying $A_s$ as per Table 1 and $k_*$ in the NANOGrav range, we show the contours (dashed lines in Fig. 7) for abundant PBH production or less. We conclude from Fig. 7 that the $w = 1$ case cannot produce copious amount of PBH and satisfy the NANOGrav constraints simultaneously. We notice that for the $w = 1/9$ case, the contour for $f_{\text{PBH}} = 0.1$ just enters the NANOGrav $2\sigma$ contour in this range of $k_*$. Guided by our previous understanding of the relative shift in peak positions, we find that for $w = 1/9$, choosing a different $k_*$ range such that NANOGrav observes closer to the peak of the $\Omega_{\text{GW}}^{(2)}$ spectrum makes the nonstandard $w = 1/9$ case consistent with the NANOGrav $2\sigma$ contour. The gain in a $w = 1/9$ case over a pure RD epoch is that both abundant PBH and NANOGrav consistency are reached with a smaller $A_s \sim 0.007$ in the $w = 1/9$ case compared to $A_s \sim 0.01$ required in the RD case.

The reason for the consistency of the $w = 1/9$ case and the inconsistency of the $w = 1$ case in our analysis can also be explained in terms of the slope $\gamma_{\text{CP}}$. In a general $w$-dominated epoch, the slope of the $\Omega_{\text{GW}}^{(2)}$ spectrum is $5 - \gamma_{\text{CP}} = \frac{6w - 2}{3w + 1} + 2n_2$, where $n_2$ is the slope of $P_{\zeta}(k)$. For a Gaussian primordial power spectrum given in Eq. (2.18), $n_2 = -\frac{1}{\sigma_p^2} \log_{10}(k/k_*)$. Taking the $1\sigma$ limit on $\gamma_{\text{CP}}$ from NANOGrav $4.5 \lesssim \gamma_{\text{CP}} \gtrsim 6.5$, we find no real limit for $\sigma_p$ for the $w = 1$ case. However, for the $w = 1/9$ case, the lower limit on $\gamma_{\text{CP}}$ gives $\sigma_p \gtrsim 1.155$ with $k = k_0 = 3.6 \times 10^6$ Mpc$^{-1}$ and $k_* \simeq 3.6 \times 10^7$ Mpc$^{-1}$. Hence, the scenario where a nonstandard $w = 1/9$ epoch is consistent with NANOGrav result and abundant PBH production (green dashed contour in Fig. 8), $n_2|_{k_0} \lesssim 0.5$. Thus, the $w = 1/9$ is consistent when its $\Omega_{\text{GW}}^{(2)}$ peak at $k_0$ is close to $k_*$, and rises with a small slope.
We have checked that our results satisfy BBN bound on $\Omega_{GW}^{(2)}$ given in Eq. (2.11). The range of possible PBH masses are different for different $w$ and $k_*$ values. For the nonstandard case $w = 1/9$ that is finally consistent in our analysis can generate PBH in the mass range $5 \times 10^{-6}$ to $3 \times 10^{-3}$ depending on $k_*$. The observational constraints from lensing experiments such as Subaru HSC [70], EROS/MACHO [71] and OGLE [72] put upper bound on $f_{PBH}$ to be 0.1 in a few points across this mass range. However, the $f_{PBH} \simeq 10^{-3}$ contour will still be inside NANOGrav $2\sigma$ in this case, which is observationally valid throughout this mass range. For the RD epoch, near solar mass PBHs can be generated with $f_{PBH} = 0.1$, which is the maximum observationally allowed abundance in this range, constrained mainly with EROS/MACHO [71].

After the announcement of the NANOGrav 12.5 yr data release, many works have studied the consistency of abundant PBH formation with NANOGrav observation in terms of $\Omega_{GW}^{(2)}$. References [12–14] have studied this in RD epochs. Ref. [13] considered a wide constant primordial power spectrum and found high abundance in the PBH mass range $10^{18} - 10^{21}$ gm, while still satisfying the NANOGrav constraints. Ref. [12] studied different primordial curvature power spectra, including a tilted Gaussian one, which they discarded because its NANOGrav consistent amplitude does not work to generate abundant PBH. However, this analysis was done using critical collapse for PBH. Ref. [14] motivates a narrow Gaussian curvature power spectrum using the Press-Schechter formalism to generate abundant near solar mass PBHs to be consistent with the NANOGrav results in RD. In our work, we also consider Press-Schechter formalism for PBH analysis and found our $\sigma_p = 1$ case to be consistent with [14] for RD. The recent work [15] discusses implications of the NANOGrav result on the formation of planetary mass PBHs in nonstandard dust-like epochs with $-0.091 < w < 0.048$. They use narrow peak for their analysis, specifically a Dirac delta function, and translate the constraints on $\Omega_{GW}^{(2)}$ to that of the amplitude which generates monochromatic mass function for PBH, whereas our $P_{\zeta}(k)$ with $\sigma_p = 2$ can be noted as a broad spectrum.

For a broad primordial power spectrum in general, the resulting $\Omega_{GW}^{(2)}$ spectrum will also be broad. Therefore, future GW detectors close to the NANOGrav frequency range will further constrain the width through (non-)observation of the stochastic GW background. These future constraints combined with NANOGrav limits will motivate a thorough understanding of the primordial (inflationary) and pre-BBN cosmologies and also constrain the PBH abundances more stringently.

For our case the mass function is extended, however include planetary masses for PBH as well. The PBH analysis also depends on the formalism under consideration so that abundances from peak theory analysis or critical collapse analysis may differ considerably from the Press-Schechter analysis used here. We have used a broad window function which substantially helps in enhancing PBH. We note here that for a broad power spectrum, the integration kernels have to be evaluated on a case by case basis for different EoS and therefore, it is challenging to show constraints as functions of $w$ directly. Also, the $\Omega_{GW}^{(2)}$ for pure matter dominated case $w = 0$ should be calculated separately for a broad primordial power spectrum since one needs to take care of the scales where non-linear growths become important. For $w = 0$, the PBH formation mechanism is also quite different from $w > 0$ cases due to the absence of pressure and often result in PBHs with nonzero-spins and aspherical shapes. We hope to come back to these issues in near future.

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