Non-Contact Spin Pumping by Microwave Evanescent Fields

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The angular momentum of evanescent light fields has been studied in nano-optics and plasmonics, but not in the microwave regime. Here we predict non-contact pumping of electron spin currents in conductors by the evanescent stray fields of excited magnetic nanostructures. The coherent transfer of the photon to the electron spin is proportional to the $g$-factor, which is large in narrow-gap semiconductors and surface states of topological insulators. Electron correlation may lead to chirality of the spin injection.

\begin{equation}
\mathcal{D} = \frac{1}{4\omega} \text{Im}(\varepsilon_0 \mathbf{E}^* \times \mathbf{E} + \mu_0 \mathbf{H}^* \times \mathbf{H}),
\end{equation}

where $\mu_0/\varepsilon_0$ are the vacuum permeability/permittivity, and in the microwave regime the magnetic field component $\sim \text{Im}(\mathbf{H}^* \times \mathbf{H})$ dominates the contribution of the electric field $\mathbf{E}$. The evanescent fields at boundaries can have local angular momentum even when the (linearly-polarized) propagating ones have not. A distinguishing feature of such evanescent fields is the locking between the linear and angular momentum. The chiral electrical near-field of a rotating electrical dipole, e.g., unidirectionally excites surface plasmon polaritons. Metallic striplines or coplanar waveguides biased by currents in the GHz regime also emit chiral magnetic near-fields, which is of considerable interest for magnonics, since chiral excitation is a robust and switchable mechanism to pump a DC unidirectional magnon current by an AC field.

Spin pumping by exchange interaction is established when the magnet and conductor form a good electric contact, which is difficult to achieve between metals and semiconductors including graphene because of Schottky barriers and electronic structure mismatch. Even when a good contact to a magnet can be established, results may be difficult to interpret due to proximity effects. Spin pumping at a distance by microwaves solves these issues since it does not require direct contact between the magnet and the system of interest. In this Letter, we address the non-contact angular momentum transfer to an electric conductor by stray magnetic fields emitted by an excited magnet, whereby generalizing the concept of spin pumping by a contact exchange interaction. We are motivated by the significant near fields that couple magnetic nanowires and ultrathin magnetic insulating films, causing several chiral magnon transport phenomena. Here we demonstrate that a magnetodipolar field pumps electron spins into a conductor in a non-chiral fashion without the need of an electric contact. We illustrate the physics for a simple yet realistic model system of a magnetic nanowire on top of a two-dimensional electron gas (2DEG) as illustrated in Fig. 1. We first be graphene, but the effect is strongly enhanced by spin-orbit interaction, such as a large $g$-factor in InAs or InSb quantum wells (QWs) or the surface states of 3D topological insulator. Electron interaction effects may recover the chirality of the spin injection.

Transverse spin density of microwaves.—We first demonstrate that the evanescent magnetodipolar field of a magnetic nanowire carries transverse angular momentum or “spin”. The nanowire with width $w$, thickness $d$ and equilibrium magnetization $M_s$ along the wire ($y$-) direction, on top of an electron gas confined in the $z$-direction on a length scale $s$ as illustrated in Fig. 1 acts as an antenna for external microwaves with frequency tuned to the ferromagnetic resonance $\omega_R$. In the following we use a quantum mechanical notation for convenience, but in the classical limit operators can be simply replaced by field amplitudes. The magnetization dynamics expressed by the spin operator $\hat{S}(r,t)$ generates a magnetic field $\mathbf{h}$ by Coulomb’s Law (disregarding retardation):

\begin{equation}
\mathbf{h}_\beta(r,t) = -\frac{\gamma h}{4\pi} \partial_\beta \partial_\alpha \int d\mathbf{r'} \frac{\hat{S}_\alpha(r',t)}{|\mathbf{r} - \mathbf{r'}|},
\end{equation}
in the summation convention over repeated spatial (or spin) indices \(\{\alpha, \beta\} = \{x, y, z\}\). \(\gamma\) is the gyromagnetic ratio of the nanowire. For sufficiently weak excitation the spin operators in the wire can be expanded into magnon field operators \(\hat{\alpha}_k\) and their amplitudes across the nanowire \(m_{x,z}(x, z)\):

\[
\hat{S}_{x,z}(r, t) = \sqrt{2S} \sum_k (m_{x,z}^k(x, z)e^{i k_y y} \hat{\alpha}_k(t) + \text{H.c.}) ,
\]

\[
\hat{S}_y(r, t) = -S + (\hat{S}_x^2 + \hat{S}_z^2)/(2S) ,
\]

where \(S = M_s/(\gamma h)\). The static stray field is negligibly small for sufficiently long nanowires. The dynamic stray field \(h_\beta(\rho, z) = \sum_k e^{i k \rho - i \omega t} h_\beta(z, k) + \text{H.c.} = \sum_k e^{i k \rho} h_\beta(z, k, t) = \hat{h}_\beta(z, k) e^{-i \omega t} + \hat{h}_\beta(z, -k) e^{i \omega t}\) is generated by spin waves with momentum \(k_y y\). Below the nanowire \((z < 0)\) it has Fourier components for \(k = (k_x, k_y, 0)^T\) [16,21,22]

\[
\begin{align*}
\hat{h}_x(z, k) &= F_k \left( m_{x^y}^k + \frac{i k_x}{k} m_{y^x}^k \right) , \\
\hat{h}_y(z, k) &= \frac{i k_x}{k} m_{y^x}^k , \\
\hat{h}_z(z, k) &= \frac{1}{k} m_{y^x}^k ,
\end{align*}
\]

\[e^{i k z} \mid \hat{\alpha}_{k_y} \rangle ,
\]

where \(F_k = -\gamma h_\beta \sqrt{2S}(1 - e^{-k \rho}) \sin(k_x w/2)/k_x\) is the form factor of the rectangular wire.

\(h = (\hat{h})\) is the response to \(\langle \hat{\alpha}_{k_y} \rangle\), the coherent amplitude of magnons excited by external microwaves. \(h_\beta(k)\) decays exponentially \(\sim e^{-k |z|}\) on a scale governed by the complex momentum \(\kappa = k_x x + k_y y - i k z\) [31]. We focus here on the circularly polarized spin waves with \(m_x \to im_z\) in nanowires with circular/square cross sections. External microwaves excite the Kittel magnon [25] with \(k_y = 0\), such that \(\hat{h}_y(k)\) vanishes and \(\hat{h}_z(k) = \text{sgn}(k_x)\hat{h}_z(k)\). Nevertheless, \(\hat{h}(k)\) = 0 for \(k_x > 0\), since magnons precess preferentially in one direction, which distinguishes the chirality found here from the polarization-momentum locking in optics [2] [3]. The photon spin density under the nanowire \(D(x, z) = \mu_0 \text{Im} \left[ \hat{h}^*(x, z) \times \hat{h}(x, z) \right] / (4\omega)\) is purely transverse since \(D \cdot \hat{h} = 0\), as illustrated in Fig. 2 for \(w = d = 60\) nm. \(D\) is symmetric with respect to the center of the nanowire. At finite distances from the wire the near singularity at the edges is smeared out, but the average amplitude remains significant. The photon magnetic field couples to the electron spins by the Zeeman interaction. Absorption transfers the photon spin over distances limited by the evanescent decay length in contrast to conventional spin pumping, which happens directly at the interface.

**Formalism.—**The photon field derived above can excite spins into any conductor in its proximity. Here we illustrate the concept by a 2D electron gas with Hamiltonian \(\hat{H}_0\) in which only the lowest subband with envelope wavefunction \(\phi(z)\) is occupied (see Supplemental Material Sec. III for a 1D quantum wire [36]). The Zeeman coupling between the conduction electron spin \(\hat{s}\) and the evanescent (near) field amplitude \(\hat{h}(r, t)\) reads [34]

\[\dot{\hat{H}}_Z = \eta \int dr |\phi(z)|^2 \hat{s}(\rho, t) \cdot \hat{h}(r, t) ,
\]

where \(\eta = \mu_0 \gamma_e, \rho = \rho + iz, \gamma_e = -g_e \mu_B / h\), with \(\mu_B\) and \(g_e\) being the Bohr magneton and (effective) electron g-factor, respectively. In the strictly 2DEG limit \(|\phi(z)|^2 \to \delta(z)\) and \(\hat{h}(\rho, z) \approx \hat{h}(\rho, z = 0)\). The excited spin density in the linear response reads

\[\langle \hat{s}_{\alpha}(\rho, t) \rangle_l = -\eta \sum_k e^{ik \rho - i \omega t} \chi_{\alpha\beta}(k, \omega) \hat{h}_\beta(k, \omega) ,
\]

\[\chi_{\alpha\beta}(\rho - \rho', t - t') = i\Theta(t-t') \langle [\hat{s}_\alpha(\rho, t), \hat{s}_\beta(\rho', t')] \rangle .
\]

The nearly homogeneous microwave fields excite the magnetic resonance of the wire but not the electron gas and may be disregarded here. \(\langle \hat{s}\rangle\) decays with the dipolar field on the scale of the wire width. Assuming that the spin diffusion length, which can be of the order of micrometers in 2DEGs [37] (and even longer in graphene [40]), exceeds the field decay length (tens of nanometer), we can compute \(\chi_{\alpha\beta}\) by straightforward linear response theory (see Supplemental Material [36]).

The excited spin density is a source term for the kinetic equations, from which we can calculate spin dynamics and transport, and when the spin-orbit coupling is sufficiently weak, the spin current. For the spin dynamics in the 2DEG we need not only \(s_i(\rho, t) = \langle \hat{s}(\rho, t) \rangle\), from Eq. (6), but also its time derivative (in interaction representation "\(T\)" below) [1] [17] [41] [43], which is derived in...
the Supplemental Material [36]:
\[
\left\langle \frac{\partial \mathbf{s}_f (\mathbf{p}, t)}{\partial t} \right\rangle = \frac{i}{\hbar} \left\langle \left[ \mathbf{H}_0, \mathbf{s}_f (\mathbf{p}, t) \right] \right\rangle = \frac{\partial \mathbf{s}_f (\mathbf{p}, t)}{\partial t} + \eta \mathbf{s}_f (\mathbf{p}, t) \times \mathbf{h}(\mathbf{p}, t).
\]
(8)

This relation recovers that in the conventional spin pumping [17, 18] when replacing the microwave field \( \mathbf{h} \) by the magnetization \( \mathbf{m} \) at the interface in Eqs. [9] and [57, 92], in which case \( \left\langle \mathbf{s}_f \right\rangle \) can be interpreted as the spin injection rate \( \mathbf{R} (t) \) or spin current (see below) across the interface.

When the spin-orbit coupling is negligible, the spin current operator is defined through the commutator
\[- \nabla_{\rho} \cdot \mathbf{J}_I = (i/\hbar) \left[ \mathbf{H}_0, \mathbf{s}_f (\mathbf{p}, t) \right],\]
leading to [41, 42]
\[- \nabla_{\rho} \cdot \mathbf{J} (\mathbf{p}) = \frac{\partial \mathbf{s}_f (\mathbf{p}, t)}{\partial t} + \eta \mathbf{s}_f (\mathbf{p}, t) \times \mathbf{h}(\mathbf{p}, t),\]
(9)
where \( \mathbf{J} (\mathbf{p}) \) is the spin current tensor with elements \( J^\alpha_\beta (\rho, z) \) (\( \alpha \) and \( \delta \) are the spatial and spin indexes).

Substituting Eq. (9) into Eq. (9) leads to DC and AC spin currents. When the susceptibility is well behaved at low frequencies and assuming that the Fermi energy \( E_F \gg \hbar \omega \), we may use the adiabatic approximation for the excited spin density [37, 41, 44, 45].

\[
\left\langle s_\alpha (\mathbf{p}, t) \right\rangle_I = -\eta \sum_k e^{ik_\rho \mathbf{p}} \text{Re} \chi_{\alpha\beta} (\mathbf{k}, \omega \rightarrow 0) \hbar \beta (\mathbf{k}, t) + \eta \sum_k e^{ik_\rho \mathbf{p}} \left\langle \frac{\partial \text{Im} \chi_{\alpha\beta} (\mathbf{k}, \omega) \partial \omega}{\partial \omega} \right\rangle \hbar \beta (\mathbf{k}, t),
\]
(10)
In the long wavelength limit in which \( \text{Re} \chi_{\alpha\beta} (\mathbf{k} \rightarrow 0, \omega \rightarrow 0) \) is constant the first, reactive term on the r.h.s. of Eq. (10) causes a pure AC contribution \( \sim \hbar \) to the spin current through the first term \( \mathbf{s}_f \) in the r.h.s. of Eq. [9]. The second, dissipative term contributes to a DC spin current. We may disregard the external microwave field that excites the wire FMR since it does not contribute to the DC response. We also assume that a small static magnetic field that align the magnetization in the wire has a negligible effect on the 2DEG spins. The spin current injected under the nanowire can be used as a boundary condition for a spin transport theory [46].

The mechanism is most transparent for an isotropic system with spin susceptibility tensor \( \chi_{\xi\xi} = \chi_{\delta\xi} \) (see examples below [36]). The DC spin current
\[
d\mathbf{J}_x^{\text{DC}} (x) / dx = -\eta \sum_{k_x} e^{ik_x x} \text{Re} \chi (k_x, \omega) \left\langle \mathbf{h} (k_x, t) \times \mathbf{h} (x, t) \right\rangle_{\text{DC}}
+ \eta \sum_{k_x} e^{ik_x x} \left\langle \frac{\partial \text{Im} \chi (k_x, \omega)}{\partial \omega} \right\rangle \left\langle \mathbf{h} (k_x, t) \times \mathbf{h} (x, t) \right\rangle_{\text{DC}}.
\]
(11)
is evaluated below for two model systems with large g-factors, viz. the 2DEGs in narrow-gap semiconductor heterostructures and topological surface states and for the 1D electron gas in the Supplemental Material [36].

**Dipolar spin pumping.**—For the free electron gas, the spin susceptibility is isotropic [37, 38],
\[
\chi (k, \omega) = \frac{\hbar^2}{2} \sum_q \frac{f (\xi_q) - f (\xi_{k+q})}{\hbar \omega + i \theta + \xi - \xi_{k+q}}.
\]
(12)
where \( \xi_k = \hbar^2 k^2 / 2m^* - \mu \) is the electron energy with effective mass \( m^* \), relative to the chemical potential \( \mu \), and \( f (\xi_k) = \{ \exp (\xi_k / k_B T) \} + 1 \) is the Fermi-Dirac distribution at temperature \( T \). In the microwave regime for the nanowire, \( |k_x| < 2k_F \), where \( k_F \) is the Fermi vector in semiconductors \( k_F = \text{O} (\text{nm}^{-1}) \) and \( \text{Re} \chi (k_x, \omega \rightarrow 0) = N(0) = m^* / (\pi \hbar^2) \) and the reactive first term in Eq. (11) vanishes. The DC spin current then reduces to
\[
\mathbf{J}_x^{\text{DC}} (x) = \eta^2 \int_0^x dx \sum_{k_x} e^{ik_x x} \partial_x \text{Im} \chi (|k_x|, \omega) \left\langle \mathbf{h} (k_x, t) \times \mathbf{h} (x, t) \right\rangle_{\text{DC}},
\]
(13)
where we used the symmetry relations \( \mathbf{J}_x^{\text{DC}} (k_x) = -\mathbf{J}_x^{\text{DC}} (-k_x) \) and \( \mathbf{J}_x^{\text{DC}} (x = 0) = 0 \). Assuming for the moment that \( \chi (|k_x|, \omega) \approx \chi (k_{\text{ave}}, \omega) \) with \( k_{\text{ave}} \sim \pi / (2\omega) \) and with \( \mathbf{h} (x, t) = \mathbf{h} (x, \omega \mathbf{K}) e^{-i\omega t} + \mathbf{h}^* (x, \omega \mathbf{K}) e^{i\omega t} \), we obtain the simplified expression
\[
\mathbf{J}_x^{\text{DC}} (x) \approx -2\eta^2 \omega \mathbf{K} \partial_x \text{Im} \chi (|k_x| \rightarrow k_{\text{ave}}, \omega) \left\langle |k_x| \times \mathbf{h} (x, \omega \mathbf{K}), \mathbf{h}^* (x, \omega \mathbf{K}) e^{i\omega t} \right\rangle \int_0^x dx \text{Im} \left[ \mathbf{h}^* (x') \times \mathbf{h} (x') \right].
\]
(14)
Hence, the DC spin-current below the nanowire is (approximately) proportional to the transverse spin of the magnetic field, implying transfer of the photon spin angular momentum to the electron spin with an efficiency governed by \( \partial_x \text{Im} \chi (|k_x| \rightarrow k_{\text{ave}}, \omega) \left|_{\omega=0} \right. \). The spin current is polarized in the \( -y \)-direction, i.e., opposite to the magnetization direction of the nanowire.

Since \( \mathbf{J}_x (x) \propto \text{sgn} (x) \), the excited spin is not chiral, but flows into both directions on both sides of the nanowire as indicated by the blue arrow in Fig. [1] just as in conventional spin pumping [17, 41, 43, 45]. Although excited by the same field, this result is in stark contrast to the magnon spin current [10, 16, 21, 22] or the chiral energy currents of surface plasmon polaritons excited by a rotating electric dipole [13, 7], which are both unidirectional and flow in half space. We can trace the different physics to the collective nature of magnons/plasmons with a well-defined dispersion relation that in the present geometry are symmetric in \( k \)-space, but of which a chiral dipolar field selects only one. The susceptibility of the non-interacting electron gas, on the other hand, is made
up by a broad spectrum of electron-hole pair excitations at the Fermi energy, and chirality vanishes in the integral over wave vectors at a given frequency. Results for non-interacting electrons in a quantum wire or metallic carbon nanotube crosswise to the magnetic wire leads to similar conclusions [30].

Electron-electron interactions in the 1D electron gas transform the ground state into Tomonaga-Luttinger liquid with a sharp bosonic excitation spectrum and singular spin susceptibility [36, 37]

\[
\chi(k, \omega) = \frac{-|k|L}{\pi} \left( \frac{1}{\omega + \omega_k + i0^+} + \frac{1}{-\omega + \omega_k - i0^+} \right),
\]

(15)

where \( L \) is the system length. By contour integration

\[
s_j(x, t) = \begin{cases} \frac{i2\pi k_F}{v_F} \mathbf{h}(k, \omega_k) \mathrm{e}^{ik x - i\omega_k t} + \text{H.c.} & \text{for } x < 0 \\ 0 & \text{for } x > 0 \end{cases},
\]

(16)

where \( v_F \) is the Fermi velocity, implying that the excited spin density lives only in half of the nanowire. The DC spin current

\[
\mathbf{J}_s(x, t) = \begin{cases} -\eta \int_0^x dx' s_j(x', t) \times \mathbf{h}(x', t)|_{\text{DC}} & \text{for } x < 0 \\ 0 & \text{for } x > 0 \end{cases}
\]

(17)

flows in the same half space, recovering the chiral excitation of a spin-density current [36] found earlier in magnetic films. This example proves that quite generally chiral excitation by dipolar radiation is not caused by a hidden symmetry, but requires collective excitations such as plasmons, magnons, phonons, of a rigid ground state.

We now estimate the magnitude of the DC spin current and/or spin-injection rate by the dipolar field from an excited magnetic nanowire. We choose a symmetric QW with \( s = 20 \text{ nm} \) of a semiconductor with small effective mass such as InSb with \( m^* = 0.015 \text{me}_0 \) [25] and electron density \( n_e = 3 \times 10^{11} \text{ cm}^{-2} \) (corresponding to a Fermi energy \( E_F \approx 50 \text{ meV} \) and Fermi temperature \( 560 \text{ K} \)), such that only the lowest band is populated even at room temperature. The Dresselhaus-type spin-orbit coupling with coefficient \( \gamma_D = 220 \text{ eVÅ}^3 \) [28, 47] causes a small correction \( \gamma_D (\pi/s)^2 k_F \sim 0.7 \text{ meV} \ll E_F \) that we disregard. The \( g \)-factor of electron is \( g_s = -36 \) [28, 29], but the sign is not important here. At temperature \( T = 100 \text{ K} \) the system is degenerate with subband splitting \( h^2(\pi/s)^2/(2m^*) = 63 \text{ meV} \gg k_BT \). For a Co or CoFeB nanowire with \( d = 60 \text{ nm} \) and \( \mu_0M_s = 1.2 \text{ T} \) [22], we assume a coherent magnon density \( \rho_m = |\langle \alpha_{kz=0} \rangle|^2 = 10^9 \text{ m}^{-2} \) in Eq. (3) that corresponds to a transverse magnetization amplitude \( M_{x,y} \approx 2\sqrt{2\gamma HM_s \rho_m m_{x,y}}_{kz=0} \), i.e., a small precession cone angle \( \sim 3.2 \times 10^{-3} \text{ degrees} \) that is easily excited by ferromagnetic resonance.

We plot the DC spin current under the nanowire from Eqs. (11) and (14) in Fig. 3(a), in which the bidirectional spin current is indicated by the black arrows, and the spin-injection rate in Fig. 3(b). The simplication Eq. (14) describes the pumped current by Eq. (11) well. With the same conditions, the pumped spin current is four times in magnitude smaller in InAs QWs \( m^* = 0.023 \text{me}_0 \) and \( |g_s| = 14.3 \) [28]. The spin current is of the same order as the spin Hall current generated by an electric field of 0.1 kV/cm and a spin Hall conductivity \( \sigma_y = 10^9 \text{ (Ω m)}^{-1} \), which should be easily measurable [48]. Under the same conditions, the spin current pumped by the dipolar interaction is comparable with that from interfacial exchange interaction with an exchange splitting \( JM_s h \sim 10 \text{ meV} \) [49], but does not require good electric contact between magnet and semiconductor. The spin pumping current is less efficient for graphene and other semiconductors because of the smaller \( g \)-factor. Nevertheless, we cannot exclude that the observations in graphene/YIG [20, 50] are caused by dipolar, not exchange interactions.

The excited spin current under the transducer drives diffusive spin transport over the spin diffusion length scale [46]. The spin signal can be converted to a transverse voltage by the inverse spin Hall effect in the 2DEG itself or by heavy metal contacts [48] or the inverse Edelstein effect [51]. The cyclotron resonance excited by evanescent microwave magnetic fields in the quantum Hall regime could be an interesting extension of the present work.

Finally, we estimate the efficiency of the dipolar spin pumping for surface states of the \( n \)-doped topological insulator Bi\(_3\)Se\(_3\) [52] at a low temperature \( T = 30 \text{ K} \) for which a good exchange interaction with magnetic contacts is difficult to achieve [51] and perhaps not desired because of an associated proximity effect. Since the spin current is not conserved, we focus on the DC spin injec-
tion rate $R_{DC}$ defined in Eq. (57) and compared with the semiconductor case in Fig. 3(b). Only the diagonal terms of the susceptibility tensor $\chi^{zz/xx}(k_x,\omega) = \frac{\hbar^2}{8} \sum_i (1 - \cos \phi_i) \frac{n_F(\xi_{q-k}) - n_F(\xi_{q})}{\omega + i\gamma + \xi_{q-k} - \xi_{q}}$, where $\xi_k = hvF_k - \mu$ with $v_F$ being the Fermi velocity, contribute to the DC spin injection (see Supplemental Material [26]). With $n_i = 10^{11} \text{cm}^{-2}$, $v_F = 10^5 \text{m/s}$, and $|g_e| = 20$ [30], the spin injection rate is of the same order as that of the InSb semiconductor 2DEG.

Discussion.—We demonstrate transfer of transverse spin angular momentum from an evanescent microwave field to an electron gas, thereby establishing a new spin pumping mechanism into electric conductors. The photon angular momentum is inherent to the evanescent stray fields of a precessing magnetization, but it also exists in microwave cavities or waveguides. The dipolar spin pumping is contactless and avoids possible artifacts by the magnetic proximity effect. The excited spin current is not chiral, in contrast to the spin pumping of spin waves into magnetic films [16], but chirality can be recovered by electron-electron interactions. Handedness in transport by dipolar radiation is therefore not associated to a hidden symmetry, but is a consequence of long-lived collective excitations or rigidity of the transported carriers. The spin pumping by a magnetic transducer into a 2DEG and a surface state of a topological insulator are estimated large enough to be observable. Our study bridges the concepts and understandings in different fields including spintronics [1] [11] [12] [18] [54], nano-optics [5] and plasmonics [24] [5].

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Supplemental Material

SPIN INJECTION RATE

Here we derive the rate of change of the spin density in the presence of spin-non-conserving drive \[17, 42\]. The time-dependent Zeeman Hamiltonian \( \hat{H}_Z(t) = \eta \int \hat{s}(x, t) \cdot \mathbf{h}(x, t) dx \) perturbs the system Hamiltonian \( \hat{H}_0 \), leading to the Heisenberg equation of motion for the electron spin density \( \hat{s} \)

\[
\frac{\partial \hat{s}(x, t)}{\partial t} = \frac{i}{\hbar} \left[ \hat{H}_0 + \hat{H}_Z, \hat{s}(x, t) \right] = \frac{i}{\hbar} \left[ \hat{H}_0, \hat{s}(x, t) \right] - \eta \hat{s}(x, t) \times \mathbf{h}(x, t),
\]

(S1)

where we use the commutator \( [\hat{s}_\beta(x', t), \hat{s}_\alpha(x, t)] = i\hbar \varepsilon_{\alpha\beta} \hat{s}_\delta(x) \delta(x - x') \). In the Heisenberg representation the wave function does not depend on time, so

\[
\langle \frac{\partial \hat{s}(x, t)}{\partial t} \rangle = \frac{\partial \langle \hat{s}(x, t) \rangle}{\partial t}.
\]

(S2)

In the interaction representation \[37, 38\], operators \( \hat{A}_I(t) = e^{i\hat{H}_0 t/\hbar} \hat{A}_Z e^{-i\hat{H}_0 t/\hbar} \) evolve only by the unperturbed, time-independent Hamiltonian \( \hat{H}_0 \). The spin-density rate of change now contains two terms

\[
\langle \frac{\partial \hat{s}(x, t)}{\partial t} \rangle = \frac{\partial \langle U^\dagger(t) \hat{s}_I(x, t) U(t) \rangle}{\partial t} = \frac{i}{\hbar} \left\langle U^\dagger(t) \left[ \hat{H}_Z(t), \hat{s}_I(x, t) \right] U(t) \right\rangle + \left\langle U^\dagger(t) \frac{\partial \hat{s}_I(x, t)}{\partial t} U(t) \right\rangle,
\]

(S3)

where

\[
U(t) = T_t \exp \left( -\int_0^t dt' \hat{H}_Z(t') \right)
\]

(S4)

is (time-ordered) time evolution operator governed by the perturbation.

In the interaction representation the pump field does not appear in the operators, so for the free electron gas \( \hat{H}_0 = -\frac{\mathbf{p}^2}{2m} \) we find the spin-conservation relation

\[
\frac{\partial \hat{s}_I(x, t)}{\partial t} = \frac{i}{\hbar} \left[ \hat{H}_0, \hat{s}_I(x, t) \right] = -\frac{\partial}{\partial x} \mathbf{J}_I(x, t),
\]

(S5)

where by the Fermion operator \( \hat{f}^I_\alpha(x) \) with spin \( \alpha \)

\[
\mathbf{J}_I = \frac{\hbar^2}{4im} \hat{f}^I_\alpha(x) \sigma_{\alpha\beta} \partial_x \hat{f}_\beta^I(x) + \text{H.c.}
\]

(S6)

is the spin current operator. The expectation value for the spin current is therefore

\[
-\frac{\partial}{\partial x} \mathbf{J}(x, t) = \frac{\partial \langle U^\dagger(t) \hat{s}_I(x, t) U(t) \rangle}{\partial t} = \frac{i}{\hbar} \left\langle U^\dagger(t) \left[ \hat{H}_Z(t), \hat{s}_I(x, t) \right] U(t) \right\rangle,
\]

(S7)

which is formally exact. We may now expand the spin current divergence to first order in the microwave field \( \mathbf{h} \). The first term

\[
\langle U^\dagger(t) \hat{s}_\alpha(x, t) U(t) \rangle \rightarrow \langle \hat{s}_\alpha(x, t) \rangle_t = \frac{i}{\hbar} \int_0^t dt' \left\langle \left[ \hat{H}_Z(t'), \hat{s}_\alpha(x, t) \right] \right\rangle
\]

\[
= -\eta \int dx' dt' \chi_{\alpha\beta}(x - x', t - t') h_\beta(x', t') + O(\mathbf{h}^2),
\]

(S8)

is the response in terms of the linear \[37, 38\]

\[
\chi_{\alpha\beta}(x - x', t - t') = i \Theta(t - t') \left\langle [\hat{s}_\alpha^I(x, t), \hat{s}_\beta^I(x', t')] \right\rangle
\]

(S9)
and quadratic $\chi^{(2)}$ spin susceptibilities. The second term

$$-rac{i}{\hbar} \langle U^\dagger(t) \left[ \hat{H}_Z(t), \hat{s}^\dagger_\alpha(x,t) \right] U(t) \rangle$$

$$\to \frac{\eta}{\hbar} i\varepsilon_{\beta\alpha\delta} h_\beta(x,t) \int_0^t dt' \left\langle \left[ \hat{H}_Z(t'), \hat{s}_\delta(x,t) \right] \right\rangle = \eta \varepsilon_{\beta\alpha\delta} h_\beta(x,t) \hat{s}_\delta(x,t) |t|.$$  (S10)

scales like $\sim \chi^2$. It directly affects the transverse dynamics in contrast to the $\chi^{(2)} \hbar^2$ term that we therefore disregard. We thereby recover the spin-current to leading order in the microwave field used in the main text

$$- \nabla \cdot \mathbf{J}(x,t) = \frac{\partial \langle s \rangle}{\partial t} + \eta \langle s \rangle \times \mathbf{h}.$$  (S11)

The torque term is second order in $\mathbf{h}$. Physically, the microwave field generates a spin polarization with a (mainly dissipative) component $\langle s \rangle \propto \mathbf{h}$ which instantaneously precesses in the field. We could redefine the spin current by including this external torque into Eq. (S6) [?], which would reduce to the conventional one outside the reach of the evanescent field $\mathbf{h}$.

Spin-orbit coupling also leads to an additional spin-nonconserving torque on the right hand side that can be included into the definition of the spin current [?], leading to an ambiguity of the latter concept. $\mathbf{R}(t) = \langle U^\dagger(t) \partial_t \hat{s}_I(\mathbf{\rho},t) U(t) \rangle$ can still be interpreted as a spin injection rate, however.

**SPIN SUSCEPTIBILITY OF TOPOLOGICAL INSULATOR**

Here we rederive the spin susceptibility for the 2DEG [37 38]. Including the free kinetic energy $\varepsilon_k$ and chemical potential $\mu$ by $\xi_k = \varepsilon_k - \mu$, the Rashba Hamiltonian reads

$$H_k = \xi_k + \mathbf{h}_k \cdot \boldsymbol{\sigma}.$$  (S12)

In the normal electron gas, $\mathbf{h}_k = 0$, while for a topological surface state, $\varepsilon_k = 0$ and $\mathbf{h}_k \cdot \boldsymbol{\sigma} = h v_F (k_y \sigma_x - k_x \sigma_y)$ where $v_F$ is the Fermi velocity. The energy spectra is given by

$$\xi_{k \pm} = \xi_k \pm |\mathbf{h}_k|.$$  (S13)

With the electron (annihilation) operator $\hat{f}^\dagger_{k\xi}$ with momentum $\mathbf{k}$ and spin $\xi$ and vector of Pauli matrices $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)^T$, the spin density operator reads

$$\hat{s}_\alpha(\mathbf{\rho}) = \frac{\hbar}{2} \sum_{\mathbf{k}\mathbf{k}'} e^{i(\mathbf{k} - \mathbf{k}') \cdot \mathbf{\rho}} \rho_{\alpha\beta} \hat{f}_{\mathbf{k}\xi}^\dagger \hat{f}_{\mathbf{k}'\xi'},$$  (S14)

with which the spin susceptibility is given by

$$\chi_{\alpha\beta}(\mathbf{\rho} - \mathbf{\rho}', t - t') = i \Theta(t - t') \langle \{ \hat{s}_\alpha(\mathbf{\rho}, t), \hat{s}_\beta(\mathbf{\rho}', t') \} \rangle.$$  (S15)

In the Matsubara representation,

$$\chi_{\alpha\beta}(\mathbf{k}, i\omega_n) = - \int e^{-i\mathbf{k} \cdot \mathbf{\rho} + i\omega_n \tau} \langle T_\tau \hat{s}_\alpha(\mathbf{\rho}, \tau) \hat{s}_\beta(0, 0) \rangle d\tau d\mathbf{\rho}$$

$$= \frac{\hbar^2}{4} \frac{1}{\beta} \sum_{\mathbf{q}} \sum_{\omega_{n'}} \text{Tr} \{ \sigma_\alpha \mathcal{G}_{\mathbf{k}+\mathbf{q}}(\omega_{n'}) \sigma_\beta \mathcal{G}_{\mathbf{k}}(\omega_n + \omega_{n'}) \},$$  (S16)

where $\beta = 1/(k_B T)$, and $\mathcal{G}_{\mathbf{k}}(\omega_n)$ is the electron Green function. From Eq. (S12),

$$\mathcal{G}_{\mathbf{k}}(\omega_n) = \frac{P_{k+}}{i\omega_n - \xi_{k+}} + \frac{P_{k-}}{i\omega_n - \xi_{k-}},$$  (S17)

with projection operators

$$P_{k\pm} = \frac{1}{2} \left( \begin{array}{cc} 1 & \pm \mathbf{l}_k \\ \pm \mathbf{l}_k & 1 \end{array} \right).$$  (S18)
where \( l_k = ie^{-i\phi_k} \) by the angle of momentum \( \phi_k = \arctan(k_x/k_y) \) for topological surface state.

For the free electron gas Eq. (S17) reduces to

\[
G_k(\omega_n) \rightarrow \frac{1}{i\omega_n - \xi_k}.
\] 

(S19)

The spin susceptibility Eq. (S16) then becomes

\[
\chi^{(F)}_{\alpha\beta}(k, i\omega_n) = \frac{\hbar^2}{2} \delta_{\alpha\beta} \sum_q n_F(\xi_q) - n_F(\xi_{k+q}),
\]

(S20)

leading to the retarded one by the analytical continuation

\[
\chi^{(F)}_{\alpha\beta}(k, \omega) = \frac{\hbar^2}{2} \delta_{\alpha\beta} \sum_q \frac{n_F(\xi_q) - n_F(\xi_{k+q})}{\omega + i\delta + \xi_q - \xi_{k+q}}.
\]

(S21)

At low temperatures \( T \to 0 \) or in the degenerate regime [37, 38],

\[
\text{Im} \chi^{(F)}(k, \omega) = -N(0) \frac{\hbar^2 k_F}{\pi k} \left( \Theta(1 - v^2) \sqrt{1 - v^2} - \Theta(1 - v_F^2) \sqrt{1 - v_F^2} \right),
\]

(S22)

where the electron density of states \( N(0) = m^*/(2\pi\hbar^2) \) and \( v_{\pm} = \omega/(kv_F) \pm k/(2k_F) \) with \( v_F = \hbar k_F/m^* \). Thus,

\[
\frac{\partial \text{Im} \chi^{(F)}(k, \omega)}{\partial \omega} \bigg|_{\omega \to 0} = -\frac{m^*}{4\pi\hbar k_F} \Theta(1 - v^2) \frac{1}{\sqrt{1 - v^2}},
\]

(S23)

where \( v = k/(2k_F) \).

With spin-orbit interaction, the retarded spin susceptibility Eq. (S16) becomes [33]

\[
\chi_{\alpha\beta}(k, \omega) = \frac{\hbar^2}{4} \sum_{ij \in \{+, -\}} \Gamma_{\alpha\beta}^{(ij)}(k, q) \sum_q \frac{n_F(\xi_q) - n_F(\xi_{k+q})}{\omega + i\delta + \xi_q - \xi_{k+q}}.
\]

(S24)

where

\[
\Gamma_{\alpha\beta}^{(ij)}(k, q) = \text{Tr} (\sigma_\alpha P_{k+q+i} \sigma_\beta P_{k+q-j}).
\]

(S25)

We assume \( n \)-doped topological insulator at low temperatures [32] and focus on its \((+)\)-branch. Since the dipolar field has only \( x \) - and \( z \)-components, \( \beta = \{x, z\} \) in the following. With \( 2P_{k, +} = 1 + \sigma_x \sin \phi_k - \sigma_y \cos \phi_k \),

\[
2\text{Tr} (\sigma_\alpha P_{k+q+i} \sigma_\beta P_{k+q}) = \delta_{\alpha\beta} \left( 1 - \sin \phi_{k+q} \sin \phi_k (-1)^{\delta_{xz}} \cos \phi_{k+q} \cos \phi_k \right)
+ i(-\sin \phi_{k+q} (-1)^{\delta_{xz}} + \sin \phi_k) \varepsilon_{\alpha\beta x}
+ i(\cos \phi_{k+q} - \cos \phi_k) \varepsilon_{\alpha\beta y}
- (\sin \phi_{k+q} \cos \phi_k (-1)^{\delta_{xz}} - \cos \phi_{k+q} \sin \phi_k) \varepsilon_{\alpha\beta z},
\]

(S26)

where \( \varepsilon_{\alpha\beta\gamma} \) is the Levi-Civita symbol with \( \varepsilon_{xyz} = 1 \). The components of the spin susceptibility of interest read

\[
\chi_{xx}(k, \omega) = \frac{\hbar^2}{8} \sum_q (1 + \sin \phi_{k+q} \sin \phi_k - \cos \phi_{k+q} \cos \phi_k) A_{k}(q),
\]

\[
\chi_{yx}(k, \omega) = \frac{\hbar^2}{8} \sum_q (1 + \sin \phi_{k+q} \sin \phi_k - \cos \phi_{k+q} \cos \phi_k) A_{k}(q),
\]

\[
\chi_{zz}(k, \omega) = \frac{\hbar^2}{8} \sum_q (1 - \sin \phi_{k+q} \sin \phi_k - \cos \phi_{k+q} \cos \phi_k) A_{k}(q),
\]

\[
\chi_{zx}(k, \omega) = i \frac{\hbar^2}{8} \sum_q (\cos \phi_{k+q} - \cos \phi_k) A_{k}(q),
\]

\[
\chi_{xz}(k, \omega) = -i \frac{\hbar^2}{8} \sum_q (\cos \phi_{k+q} - \cos \phi_k) A_{k}(q),
\]

\[
\chi_{yz}(k, \omega) = i \frac{\hbar^2}{8} \sum_q (\cos \phi_{k+q} - \cos \phi_k) A_{k}(q),
\]

(S27)
where we define

$$A_k(q) = \frac{n_F(\xi_{q+}) - n_F(\xi_{q+})}{\omega + i0^+ + \xi_{q+} - \xi_{q+}}.$$ 

In these components, $\chi_{xx}(k, \omega)$, $\chi_{yx}(k, \omega)$ and $\chi_{zy}(k, \omega)$ are even function of momentum $k$, while $\chi_{zz}(k, \omega)$ and $\chi_{yz}(k, \omega)$ are odd of $k$. As $\chi_{\alpha\beta}(r, t)$ is real, we have $\chi_{\alpha\beta}(k, \omega) = \chi_{\alpha\beta}^*(-k, -\omega)$, leading to $\text{Re}\chi_{\alpha\beta}(k, \omega) = \text{Re}\chi_{\alpha\beta}^*(-k, -\omega)$ and $\partial_\omega \text{Im}\chi_{\alpha\beta}(k, \omega)|_{\omega = \delta} = \partial_\omega \text{Im}\chi_{\alpha\beta}^*(-k, \omega)|_{\omega = -\delta}$. Thus,

$$\begin{align*}
\text{Re}\chi_{xx}(k, \omega) = 0, & \quad \partial_\omega \text{Im}\chi_{xx}(k, \omega)|_{\omega = 0} = 0, \\
\text{Re}\chi_{xz}(k, \omega) = 0, & \quad \partial_\omega \text{Im}\chi_{xz}(k, \omega)|_{\omega = 0} = 0, \\
\text{Re}\chi_{yx}(k, \omega) = 0, & \quad \partial_\omega \text{Im}\chi_{yx}(k, \omega)|_{\omega = 0} = 0, \\
\text{Re}\chi_{yy}(k, \omega) = 0, & \quad \partial_\omega \text{Im}\chi_{yy}(k, \omega)|_{\omega = 0} = 0,
\end{align*}$$  

(S28)

because $\chi_{xx}$, $\chi_{xz}$ and $\chi_{yx}$ are odd of $k$. Furthermore $\chi_{yx}(k, \omega) = 0$ with $\phi_{k, k} = 0$:

$$\chi_{yx}(k, \omega) = \frac{\hbar^2}{8} \sum_q \sin(\phi_k + \phi_q)A_k(q - k)$$

$$= \frac{\hbar^2}{8} \sum_q \sin(\phi_{k, q})A_k(q - k)$$

since the integral is odd in $\phi_q$. Only two terms contribute to the dipolar spin pumping, viz. 53

$$\chi_{zz}(k, \omega) = \chi_{xx}(k, \omega) = \frac{\hbar^2}{8} \sum_q (1 - \cos \phi_q)A_k(q - k).$$  

(S30)

**CHIRAL SPIN PUMPING INTO TOMONAGA-LUTTINGER LIQUID**

Here we illustrate the chiral spin pumping 16 into a one-dimensional electron gas (along the $x$-direction) perpendicular to a nearby rectangular magnetic nanowire (along the $y$-direction) in order to emphasize the importance of interactions on chirality. The spin density pumped by the Kittel magnon with frequency $\omega_K$ reads

$$\langle \hat{\delta}_\alpha(x, y) \rangle = -\eta \sum_k e^{iky - i\omega_Kt} \chi(k, \omega_K) h_\alpha(k, \omega_K) + \text{H.c.},$$  

(S31)

where $h_\alpha(k, \omega_K) = 0$ and $h_\alpha(-k, \omega_K) \neq 0$ are chiral in momentum space.

The non-interacting case in one-dimension is similar to the 2DEG. The Pauli spin susceptibility is an integral over single-electron-hole pair excitation,

$$\chi_F(k, \omega) = \frac{\hbar^2}{2} \sum_q \frac{n_F(\xi_q) - n_F(\xi_{q+k})}{\omega + i0^+ + \xi_q - \xi_{q+k}}.$$  

(S32)

Far below the Fermi temperature $T \ll T_F$ 38

$$\begin{align*}
\text{Re}\chi_F(k, \omega) = -\frac{\hbar^2}{2} N(0) \frac{k_F}{2k} \left( \ln \left| \frac{\nu_- - 1}{\nu_- + 1} \right| - \ln \left| \frac{\nu_+ - 1}{\nu_+ + 1} \right| \right), \\
\text{Im}\chi_F(k, \omega) = -\frac{\hbar^2}{2} N(0) \frac{\pi k_F}{2k} \left( \Theta(1 - \nu_-^2) - \Theta(1 - \nu_+^2) \right),
\end{align*}$$  

(S33)

where the density of state $N(0) = m^*/(\pi\hbar^2 k_F)$ and $\nu_\pm = \omega/(k_F v_F) \pm k/(2k_F)$. In contrast to the 2D case, the real part of the spin susceptibility depends on momentum also for $|k| < 2k_F$ and diverges logarithmically at $|k| = 2k_F$. With $k \sim \pi/(2w)$, $m^* \omega_K/h \ll k^2/2$ and using the parameters of InSb from the main text, $m^* \omega_K/h \approx 0.05k^2/2$. Close to the origin $\chi_F(k, \omega)$ is well behaved and only weakly depends on $\omega$. In the adiabatic limit $\text{Im}\chi_F(k, \omega \to 0) = 0$

$$\chi_F(k, \omega \to 0) \approx -\frac{\hbar^2}{2} N(0) \frac{k_F}{k} \ln \left| \frac{2k_F + k}{2k_F - k} \right| \approx -\frac{\hbar^2}{2} N(0) \frac{\left| k \right|}{k_F}.$$  

(S34)
The excited spin density in real space follows the dipolar field Eq. (S31)

\[ \langle \hat{s}_\alpha(x,t) \rangle_t \approx -\frac{\eta\hbar^2}{2} N(0) h_\alpha(x,t). \] (S35)

Figure S1 is a snapshot of the excited spin density calculated by using the spin susceptibility in Eqs. (S33) and (S34), respectively, in which the latter describes the feature of former well (apart from the fine structure caused by the singularity at \( |k| = 2k_F \)), confirming the validity of the adiabatic approximation. The DC spin current reads

\[ \mathcal{J}_x(x) = \mathcal{J}_x(x=0) - \eta \int_0^x s_l(x,t) \times h(x,t), \] (S36)

where we show below that \( \mathcal{J}_x(x=0) \) is in practice very small.

![Graph](image)

**FIG. S1.** (Color online) A snapshot of the spin density vector components \( s_x(x,t) \) and \( s_z(x,t) \) of the non-interacting one-dimensional electron gas excited by the dipolar field of a crossed magnetic nanowire, calculated from the spin susceptibilities in Eqs. (S33) and (S34), respectively. Results are normalized to the maximum value of the blue curve.

When interaction is switched on, the electrons close the Fermi energy can be modeled by the Tomonaga Hamiltonian

\[ \hat{H}_0 = v_F \sum |k| \hat{f}^\dagger_{k\sigma} \hat{f}_{k\sigma} + \frac{1}{2L} \sum_k V_k \rho(k) \hat{\rho}(-k), \] (S37)

where \( v_F \) is the Fermi velocity, \( V_k \) is the Fourier component of Coulomb potential in one dimension, and the Fourier component of the density operator reads

\[ \hat{\rho}(k) = \sum_{q\sigma} \hat{f}^\dagger_{q-k,\sigma} \hat{f}_{q+k,\sigma}. \] (S38)

This model supports two kinds of Bosonic excitations [37], i.e. charge and spin density waves with different group velocity that causes a spin-charge separation. The spin density excitations have a dispersion \( \omega_k = v_F |k| \) and susceptibility (cf. Chap. 4 of Ref. [37])

\[ \chi_{\alpha\beta}(k,\omega) = -\delta_{\alpha\beta} |k| L \pi \left( \frac{1}{\omega + \omega_k + i0^+} + \frac{1}{-\omega + \omega_k - i0^+} \right). \] (S39)

The Coulomb interaction dramatically modifies the spectrum from a broad area of single electron-hole pair excitation in the \((k,\omega)\) plane to a sharp collective one when the interaction is switched on, leading to a singular response to external perturbations. For positive \( \omega \), the two poles \( k_{\pm} = \pm(\omega/v_F + i0^+) \) are in the upper and lower complex plane, respectively. The Fourier components of the spin density are

\[ s_l(k,t) = -\frac{2\eta \omega_K}{v_F} h(k_-,\omega_K) \frac{1}{k + \omega_K/v_F + i0^+} e^{-i\omega_k t} + \frac{2\eta \omega_K}{v_F^2} h^*(k_-,\omega_K) \frac{1}{k - \omega_K/v_F + i0^+} e^{i\omega_k t}. \] (S40)
When $x > 0$ we close the contour in the upper complex plane, which leads to a vanishing spin density, while for $x < 0$

$$\langle \hat{s}_\alpha (x,t) \rangle_t = i \frac{2 \eta \omega_k}{i \omega} h_{\alpha}(k_-, \omega_k) e^{i k x - i \omega_k t} + \text{H.c.}, \quad x < 0.$$  

(S41)

Thus, the excited spin density lives only in half of the nanowire. Also the DC spin current

$$\mathcal{J}^\text{DC}_x(x) = \mathcal{J}^\text{DC}_x(x = \frac{w}{2}) + \left\{ \begin{array}{ll} - \eta \int_0^x dx' \ s_i(x', t) \times \mathbf{h}(x', t) |\text{DC} \quad & \text{for } x < 0 \\ 0 & \text{for } x > 0 \end{array} \right.$$

(S42)

flows only in the same half space because $\mathcal{J}^\text{DC}_x(x = \frac{w}{2}) \to 0$ as we show now. The Fourier components of the spin current are

$$\mathcal{J}_x(k) = - \frac{\eta}{i k} \sum_{k'} s_i(k', t) \times \mathbf{h}(k - k', t).$$  

(S43)

Since the poles of $s_i(k, t)$ lie in the lower complex plane and for $x = w/2 > 0$

$$\mathcal{J}_x \left(x = \frac{w}{2}, t\right) = \sum_k \ e^{i k x} \mathcal{J}_x(k, t)$$

$$= i \eta \sum_k \sum_{k'} e^{i(k+k')x} \frac{1}{k + i0 + k'} s_i(k', t) \times \mathbf{h}(k, t) = 0,$$

by closing the integral path of $k'$ in the upper complex plane. The singularities in Eq. (S39) at low frequencies do not allow making an adiabatic approximation as in non-interacting systems.

Comparing the interacting and non-interacting cases, we may conclude that real-space chiral excitations are caused by singularities in the spin susceptibility that select the $k$-space chirality of the exciting field.

**SPIN CURRENT AT ORIGIN**

Here we demonstrate that for the 2DEG $\mathcal{J}_x(x=0) = 0$ in the adiabatic approximation. In momentum space, the spin current is given by $\mathcal{J}_x(x) = \sum_{k_z} e^{i k_z x} \mathcal{J}_x(k_z)$, and $\mathcal{J}_x(x=0) = \sum_{k_z} \mathcal{J}_x(k_z) \Rightarrow \mathcal{J}_x(k_z) = \mathcal{J}_x^*(-k_z)$ since the current is real. The Fourier components of the dissipative contribution are

$$\mathcal{J}_x(k_z) = \frac{\eta^2}{i \omega k_z} \sum_{k_z'} \frac{\partial \text{Im} \chi(|k_z'|, \omega)}{\partial \omega} \bigg|_{\omega=0} \frac{d\mathbf{h}(k_z', t)}{dt} \times \mathbf{h}(k_z - k_z', t).$$  

(S45)

Following the main text, $\mathbf{h}_\beta(k, t) = \tilde{\mathbf{h}}_\beta(k) e^{-i \omega_k t} + \tilde{\mathbf{h}}^*_\beta(-k) e^{i \omega_k t}$, leading to

$$\mathcal{J}_{x}^\text{DC}(k_z) = \frac{\eta^2 \omega_k}{i k_z} \sum_{k_z'} \frac{\partial \text{Im} \chi(|k_z'|, \omega)}{\partial \omega} \bigg|_{\omega=0} \left( \tilde{\mathbf{h}}^*(-k_z') \times \tilde{\mathbf{h}}(k_z - k_z') - \tilde{\mathbf{h}}(k_z') \times \tilde{\mathbf{h}}^*(-k_z + k_z') \right).$$  

(S46)

By the main text, $\tilde{\mathbf{h}}_z$ is purely real and $\tilde{\mathbf{h}}_{k'_z}$ is purely imaginary, we then conclude that $\mathcal{J}_{x}^\text{DC}(k_z)$ is purely imaginary. Thus, $\mathcal{J}_{x}^\text{DC}(k_z)$ is odd with respect to $k_z$, by $\mathcal{J}_{x}^\text{DC}(k_z) = -\mathcal{J}_{x}^\text{DC}(-k_z)$, leading to

$$\mathcal{J}_{x}^\text{DC}(x=0) = \sum_{k_z} \mathcal{J}_{x}^\text{DC}(k_z) = 0.$$  

(S47)

The spin current vanishes because the right and left propagating contributions cancel at $x = 0$.

However, the real part of the spin-susceptibility does contribute, rendering a finite DC spin current $\mathcal{J}_x^R$ at $x = 0$, at least in principle. The DC contributions of its Fourier components

$$\mathcal{J}_x^R(k_z) = - \frac{\eta^2}{i k_z} \sum_{k_z'} \text{Re} \chi(|k_z'|, \omega \to 0) \mathbf{h}(k_z', t) \times \mathbf{h}(k_z - k_z', t)$$

$$= - \frac{\eta^2}{i k_z} \sum_{k_z'} \text{Re} \chi(|k_z'|, \omega \to 0) \left( \tilde{\mathbf{h}}(k_z') \times \tilde{\mathbf{h}}^*(-k_z + k_z', t) + \tilde{\mathbf{h}}^*(-k_z') \times \tilde{\mathbf{h}}(k_z - k_z', t) \right),$$  

(S48)
are real and \( \mathbf{J}_x(k_x) = \mathbf{J}_x(-k_x) \) is even, leads to a \( \mathbf{J}_x(x = 0) \neq 0 \). However, for the non-interacting 1D (2D) electron gases, the real part of the spin susceptibility is approximately (exactly) constant at long wave length and the DC spin current is small (zero) because \( \sum_{k_x'} \mathbf{h}(k_x', t) \times \mathbf{h}(k_x - k_x', t)|_{\text{DC}} = 0 \). The spin polarization follows the magnetic field, so \( \mathbf{s}_t(x = 0, t) \neq 0 \).