QCD Aspects of the NuTeV Anomaly

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The weak mixing angle $\sin^2 \theta_W$ measured in neutrino scattering differs from the world average of other measurements by about $3\sigma$. I discuss QCD corrections of perturbative and nonperturbative (parton structure) origin to the underlying neutrino observables.

1 Introduction

An important open question in particle physics in recent years has been the significance of the “NuTeV anomaly”—a $3\sigma$ deviation of the measurement of $\sin^2 \theta_W$ ($0.2277 \pm 0.0013 \pm 0.0009$) in neutrino scattering$^1$ from the world average of other measurements$^2$ ($0.2277 \pm 0.0004$). Possible sources of the NuTeV anomaly, both within and beyond the standard model, have been examined by Davidson et al.$^3$. The NuTeV measurement was based on a correlated fit to the ratios $R_{\nu, \bar{\nu}}^{\exp}$ of “long” and “short” events [dominated by CC and NC interactions, respectively] in sign-selected neutrino and anti-neutrino scattering on a (primarily) iron target at Fermilab. Here and below, the subscript $\exp$ will label observables defined in terms of the experimental event length, to distinguish them from the (unlabeled) theory ratios defined in terms of CC and NC interactions. The differences between the experimentally accessible observables $R_{\nu, \bar{\nu}}^{\exp}$ and the fundamental ratios $R_{\nu, \bar{\nu}}$ that can be calculated in theory are significant, e.g. $R_{\nu}^{\nu}$ and $R_{\bar{\nu}}^{\nu}$ differ by $\sim 20\%$ when the accuracy of $R_{\exp}^{\nu}$ is $< 1\%$. The complexity of the Monte Carlo analysis$^4$ in terms of event length limits the conclusiveness of a theory assessment of the anomaly in several respects. E.g. the NuTeV analysis procedure of simultaneously fitting $R_{\exp}^{\nu, \bar{\nu}}$ is related but not identical to measuring the Paschos-Wolfenstein ratio$^5$ $R^-$

$$R^- \equiv \frac{\sigma_{\nu, \bar{\nu}}^{NC} - \sigma_{\bar{\nu}}^{NC}}{\sigma_{\nu}^{CC} - \sigma_{\bar{\nu}}^{CC}} \simeq \frac{1}{2} - \sin^2 \theta_W + \delta R^-_A + \delta R_{QCD}^- + \delta R_{EW}^- \quad (1)$$
where the three correction terms are due to the non-isoscalarity of the target \((\delta R_A^-)\), NLO and nonperturbative QCD effects \((\delta R_{QCD}^-)\), and higher-order electroweak effects \((\delta R_{EW}^-)\). We can split the QCD corrections up some more and write

\[
\delta R_{QCD}^- = \delta R_s^- + \delta R_I^- + \delta R_{NLO}^- \quad (2)
\]

where the three terms on the right-hand side are due to possible strangeness asymmetry \((s^- = s - \bar{s} \neq 0)\) and isospin violation \((u_{p,n} \neq d_{n,p})\) effects in the parton structure of the nucleon, and NLO corrections, respectively. The original NuTeV analysis was carried out at LO in QCD and assumed \(\delta R_a^- = 0 = \delta R_I^-\).

2 Perturbative Corrections:

At sufficiently high neutrino energy, the total neutrino cross section

\[
\sigma^\nu \equiv \sigma^{\nu N \rightarrow lX} = \int d^3p_l \frac{d^3\sigma^{\nu N \rightarrow lX}}{d^3p_l} \quad (3)
\]

can be calculated in QCD perturbation theory. The differential cross section in Eq. (3) factorizes into a sum of convolutions of parton distribution functions and partonic cross sections

\[
d^3\sigma^{\nu N \rightarrow lX} = \sum_{f=q,g} f \otimes d^3\sigma^{f \rightarrow lX} . \quad (4)
\]

This calculation has been performed at NLO accuracy \(6\). The analysis included target and charm mass effects. Also included are the non-isoscalarity of the target material (iron), i.e., \(\delta R_A^-\) in \(1\); energy averaging over the neutrino and anti-neutrino flux spectra; and cuts in hadronic energy as used in the experimental analysis \(1\).

Table 1 summarizes the QCD corrections to \(R^\nu, \bar{R}^\nu\). (Deviations from moment estimates \(3, 8\) can be traced back \(3\) to the approximations required to apply the moment technique.) The NLO corrections to \(R^\nu, \bar{R}^\nu\) are of comparable size than the experimental errors assigned to \(R_{exp}^\nu\). Because of the above mentioned mismatch between \(R^\nu, \bar{R}^\nu\) and \(R_{exp}^\nu\) it is impossible to predict how these corrections will shift the central value of the Weinberg angle \(\sin^2 \theta_W\). The perturbative QCD corrections to \(R^\nu, \bar{R}^\nu\) largely cancel in \(R^\nu\) but it remains unclear to what extent the NuTeV LO Monte Carlo analysis is equivalent to measuring this ratio.

In the context of perturbative corrections it should also be noted that a reanalysis \(9\) of electroweak corrections to \(R^\nu\) points at potentially significant effects as well.

3 Nuclear Effects and Higher Twists

In principle, the parton distribution functions in Eq. (4) should be those of nuclear targets. Our calculation is done as an incoherent sum of contributions from parton densities of unbound nucleons. Experimental information on nuclear PDFs is relatively scarce, and nuclear PDFs
only account for leading twist 2 ($\tau = 2$) effects. Higher twists, whether they relate to nuclear modifications or not, are generally difficult to handle consistently. By limiting ourselves to $\tau = 2$, our error estimates may be underestimates. Flavour non-diagonal higher twists ("cat’s ears") can have an impact on the measurement of the weak mixing angle in neutrino scattering off a non-isoscalar target.\textsuperscript{10}

4 Strangeness Asymmetry:

An asymmetric strange sea in the nucleon ($s^- \neq 0$) contributes to a correction term to $R^-$ at LO. Neglecting some other effects, we have approximately that\textsuperscript{3}

$$\delta R_s^- \simeq -\left(\frac{1}{2} - \frac{7}{6}\sin^2 \theta_W \right) \frac{[S^-]}{[Q^-]},$$

(5)

where the strangeness asymmetry is quantified by the second moment integral

$$[S^-] \equiv \int x [s(x) - \bar{s}(x)] \, dx ;$$

(6)

and $[Q^-]$ represents the isoscalar up and down quark combination. A positive moment $[S^-]$ works in the direction to improve the anomaly and a value of about half a percent would remove it, altogether. While the first moment

$$\int [s(x) - \bar{s}(x)] \, dx = 0,$$

(7)

has to be zero as an exact sum rule there is not even an approximate symmetry [CP conjugation turns $s(x)$ into $\bar{s}(x)$ – the anti-strange sea of the anti-proton] that would protect the second moment $[S^-]$. It is, therefore, bound to be nonzero. As an even moment of a $q - \bar{q}$ difference $[S^-]$ is not among the local quark operators that are probed in DIS. Being not accessible to the lattice, it affords a phenomenological determination. A recent global CTEQ PDF analysis\textsuperscript{7} has included data ("dimuon events")\textsuperscript{11} on the neutrino- and antineutrino-production of charm

$$W^+ + s \to c$$

(8)

$$W^- + \bar{s} \to \bar{c}$$

(9)

It finds a central value $[S^-] \simeq 0.002$ and conservative bounds

$$-0.001 < [S^-] < 0.004.$$  

(10)

Via the NLO calculation described above, this translates into

$$-0.005 < \delta(\sin^2 \theta_W) < +0.001.$$  

(11)

The shift in $\sin^2 \theta_W$ corresponding to the central fit bridges a substantial part ($\sim 1.5 \sigma$) of the original $3 \sigma$ discrepancy between the NuTeV result and the world average of other measurements of $\sin^2 \theta_W$. For PDF sets with a shift toward the negative end, such as $-0.004$, the discrepancy is reduced to less than $1 \sigma$. On the other hand, for PDF sets with a shift toward the positive end, such as $+0.001$, the discrepancy remains.
5 Isospin Violations:

Isospin symmetry violation effects at the parton level contribute a shift of $R^-$ that can be gauged by the second moment $[D_N^2 - U_N^2]$, where $N = (p+n)/2$. The MRST collaboration has recently made a first attempt to separate proton and neutron PDFs where isospin for the valence quarks is broken by a function with a single parameter $\kappa$. We have applied the candidate MRST PDFs to our NLO calculation and find that the range of allowed $\kappa$ parameter, $-0.7 < \kappa < 0.7$, implies

$$-0.007 \lesssim \delta R_I^- \lesssim 0.007.$$  \hspace{1cm} (12)

The best fit value of $\kappa = -0.2$ corresponds to a shift of $\delta R_I^- = -0.0022$, bringing the anomaly down to $\sim 1.5 \sigma$.

6 Conclusions:

The uncertainties in the parton structure of the nucleon that relate to $R^-$ will not decrease substantially any time soon. The uncertainties in the theory that relates $R^-$ to $\sin^2 \theta_W$ are substantial on the scale of precision of the high statistics NuTeV data. Within their bounds, the results of this theory study suggest that the new dimuon data, the Weinberg angle measurement, and other global data sets used in QCD parton structure analysis can all be consistent within the standard model of particle physics. The central value of the Weinberg angle measured in neutrino scattering will also depend on perturbative QCD and electroweak corrections to the experimentally observable $\nu$ and $\bar{\nu}$ cross section ratios $R_{\nu,\bar{\nu}}$ and a definitive statement can only be obtained from an experimental re-analysis of the NuTeV data. Before a careful re-assessment of all theoretical corrections & uncertainties (pQCD & non-pQCD & electroweak), the significance of the $\sim 3 \sigma$ discrepancy with the standard model remains questionable.

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