Magnetic moments of heavy baryons in the relativistic three-quark model

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The magnetic moments of ground state single, double and triple heavy baryons containing charm or bottom quarks are calculated in a relativistic three-quark model, which, in the heavy quark limit, is consistent with Heavy Quark Effective Theory and Heavy Hadron Chiral Perturbation Theory. The internal quark structure of baryons is modeled by baryonic three-quark currents with a spin-flavor structure patterned according to standard covariant baryonic wave functions and currents used in QCD sum rule calculations.

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I. INTRODUCTION

Electromagnetic properties of baryons are an important source of information on their internal structure. The success of quark models for the description of static properties (masses, magnetic moments, etc.) and the results from deep inelastic lepton scattering are clear indication for the three-quark structure of baryons.

Magnetic moments of heavy baryons (mostly with a single heavy quark) have been considered in different approaches. In Refs. [1]-[6] the magnetic moments of charmed baryons have been computed using naive quark models based on different realizations of spin-flavor symmetry. In Refs. [7]-[10] the magnetic moments of charmed and bottom hyperons have been calculated in quark models incorporating the ideas of hadronization, confinement, chiral symmetry and Poincaré covariance. In Refs. [11]-[13] soliton-type approaches were applied in the analysis of the magnetic moments of heavy baryons. In Refs. [14] QCD spectral sum rules in the presence of the external electromagnetic field have been used to calculate the magnetic moments of the Σ and Λ baryons. In [15] the method of light-cone QCD sum rules has been used to calculate the magnetic moments of the Λ_b and Λ_c baryons.

Heavy hadron chiral perturbation theory (HHChPT) [16] has been applied in Refs. [17]-[20] to derive model-independent expressions for the magnetic moments of heavy baryons containing a single heavy quark. In HHChPT [17]-[20] heavy quark symmetry (HQS) and chiral symmetry (χS) have been combined in order to describe the soft hadronic interactions of hadrons containing a heavy quark. The underlying Lagrangian deals with heavy hadrons, light mesons and external fields (photons).

In the heavy quark limit (HQL), when the heavy quark mass goes to infinity (m_Q → ∞), baryons containing a single heavy quark can be classified according to the spin of the light degrees of freedom: i) the antitriplet 3 of baryons for Λ-type baryons (isosinglet Λ_Q with quark content Q[ud] and the isodoublet Σ_Q with quark content Q[us] and Q[ds]) and ii) the sextet 6 of baryons or Σ-type baryons (isotriplet triplet Σ_Q with quark content Q{uu}, Q{ud} and Q{dd}, and the isodoublet Σ_Q' with quark content Q{us} and Q{ds}). Here the symbols [ ] and { } denote antisymmetric and symmetric flavor index combinations. For the 3 states the total spin of the light diquark system is s_l = 0, while for the 6 states the total spin of the light diquark system is s_l = 1. In the case of the 3 states, the spin of the

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baryon is made up entirely by the heavy quark and, therefore, the magnetic moment of these baryons vanishes in the HQL \[17, 18\] (the magnetic moment is a spin–flip transition down by a factor of \(1/m_Q\)). The leading \(O(1/m_Q^2)\) long-distance contribution to the magnetic moments of \(A\)-type baryons arises from spin-symmetry breaking which also leads to the \(\Sigma_Q - \Sigma_Q\) mass splitting \[18\], where \(\Lambda_\chi = 4\pi F_\pi \approx 1.2\) GeV is the scale parameter of the spontaneously broken chiral symmetry and \(F_\pi\) is the pion decay constant. In the case of the \(\Sigma\)-type baryons the magnetic moments are nonzero in the HQL due to the contribution of the light-quark system \[17\]. In particular, the leading contributions from the light- and heavy-quark magnetic interactions are of order of \(O(1/\Lambda_\chi)\) and \(O(1/m_Q)\) \[17\], respectively. The next-to-leading chiral corrections to the magnetic moments of \(\Sigma\)-type and \(A\)-type baryons are of order \(O(1/\Lambda_\chi^2)\) and \(O(1/m_Q^2)\), respectively, the contributions of which have been calculated in Ref. \[19\].

The present paper focuses on the magnetic moments of the ground state single, double and triple heavy baryons in the relativistic three-quark model (RQM) which has been developed both for light \[21\] and heavy baryons \[22–27\]. This model can be viewed as a quantum field theory approach based on an interaction Lagrangian of light and heavy baryons interacting with their constituent quarks. The coupling strength of the baryons with the three constituent quarks is determined by the compositeness condition \(Z_B = 0\), where \(Z_B\) is the wave function renormalization constant of the hadron. The condition \(Z_B = 0\) has been proposed in \[28\] and extensibly used in \[29\]. The compositeness condition enables one to unambiguously and consistently relate approaches with quark and hadron degrees of freedom to the effective Lagrangian approaches formulated in terms of hadron variables only. Our strategy is as follows. We start with an interaction Lagrangian written down in terms of quark and hadron variables. Then, by using Feynman rules, the \(S\)-matrix elements describing hadron-hadron interactions are given in terms of a set of quark diagrams. The compositeness condition ensures that there is no double counting of the quark and hadron degrees of freedom. One of the corollaries of the compositeness condition is the absence of a direct interaction of the dressed charged particle with the electromagnetic field. The RQM model contains only a few model parameters: the masses of the light and heavy quarks and certain size parameters that define the size of the distribution of the constituent quarks inside the hadron. The RQM approach has been previously used to compute the exclusive semileptonic, nonleptonic, strong and electromagnetic decays of single heavy baryons \[22–25\] in the heavy quark limit \(m_Q \to \infty\) always employing the same set of model parameters. In Ref. \[20\] the approach has been extended to the study of heavy baryon transitions at finite values of the heavy quark mass without using an explicit \(1/m_Q\) expansion. Later, in Ref. \[26\] the RQM has been generalized to the study of double heavy baryons. In the present manuscript we extend our approach to the triple heavy baryons. We also check the consistency of our approach with HHChPT.

We proceed as follows. First, in Section II we briefly explain the basic ideas of the RQM. Next, in Section III we discuss matrix elements for the baryon mass operator and the electromagnetic vertex function. In Section IV we discuss the matching of our model to Heavy Hadron Chiral Perturbation Theory (HHChPT). In Section V we compare our numerical results with the results of other theoretical approaches. Finally, we summarize our results in Section V.

II. RELATIVISTIC THREE-QUARK MODEL

We will consistently employ the relativistic three-quark model (RQM) \[22–27\] to compute the magnetic moments of single, double and triple heavy baryons. In the following we will present details of the model which is essentially based on an interaction Lagrangian describing the coupling between baryons and their constituent quarks.

The coupling of a baryon \(B(q_1q_2q_3)\) to its constituent quarks \(q_1, q_2\) and \(q_3\) is described by the Lagrangian

\[
\mathcal{L}_{\text{int}}^{\text{str}}(x) = g_B B(x) \int dx_1 dx_2 dx_3 F_B(x, x_1, x_2, x_3) J_B(x_1, x_2, x_3) + \text{H.c.}
\]

(1)

where \(J_B(x_1, x_2, x_3)\) is the three-quark current with the quantum numbers of the relevant baryon \(B\). One has

\[
J_B(x_1, x_2, x_3) = \epsilon^{a_1a_2a_3} \Gamma_1 q_1^{a_1}(x_1) q_2^{a_2}(x_2) C \Gamma_2 q_3^{a_3}(x_3),
\]

(2)

where \(\Gamma_{1,2}\) are Dirac structures, \(C = \gamma^0\gamma^2\) is the charge conjugation matrix and \(a_i, i = 1, 2, 3\) are color indices.

The function \(F_B\) is related to the scalar part of the Bethe-Salpeter amplitude and characterizes the finite size of the baryon. To satisfy translational invariance the function \(F_B\) has to fulfill the identity

\[
F_B(x + a, x_1 + a, x_2 + a, x_3 + a) = F_B(x, x_1, x_2, x_3)
\]

(3)

for any 4-vector \(a\). In the following we use a particular form for the vertex function

\[
F_B(x, x_1, x_2, x_3) = \delta^4(x - \sum_{i=1}^3 w_i x_i) \Phi_B \left( \sum_{i<j}(x_i - x_j)^2 \right)
\]

(4)
where $\Phi_B$ is the correlation function of three constituent quarks with masses $m_1$, $m_2$, $m_3$. The variable $w_i$ is defined by $w_i = m_i/(m_1 + m_2 + m_3)$ and therefore depends only on the relative Jacobi coordinates $(\xi_1, \xi_2)$ as $\Phi_B(\xi_1^2 + \xi_2^2)$, where

$$x_1 = x - \frac{\xi_1}{\sqrt{2}} (w_2 + w_3) + \frac{\xi_2}{\sqrt{6}} (w_2 - w_3),$$

$$x_2 = x + \frac{\xi_1}{\sqrt{2}} w_1 - \frac{\xi_2}{\sqrt{6}} (w_1 + 2w_3),$$

$$x_3 = x + \frac{\xi_1}{\sqrt{2}} w_1 + \frac{\xi_2}{\sqrt{6}} (w_1 + 2w_2),$$

and where $x = \sum_{i=1}^3 w_i x_i$ is the center of mass (CM) coordinate. In terms of the CM and quark coordinates the Jacobi coordinates are simply given by

$$\xi_1 = \frac{1}{\sqrt{2}} (x_2 + x_3 - 2x_1), \quad \xi_2 = \frac{\sqrt{3}}{2} (x_3 - x_2).$$

Note, that the choice of Jacobi coordinates is not unique. We choose the most convenient ansatz defined by Eqs. (5) and (6). Expressed in relative Jacobi coordinates and the center of mass coordinate, the Fourier transform of the vertex function reads [22], [23]:

$$\Phi_B(\xi_1^2 + \xi_2^2) = \int \frac{d^4p_1}{(2\pi)^4} \int \frac{d^4p_2}{(2\pi)^4} e^{-ip_1\xi_1 - ip_2\xi_2} \Phi_B(-p_1^2 - p_2^2)$$

In the numerical calculations we consider two specific limits for the correlation functions (CF) of heavy baryons: i) the exact form with no approximations (full) and ii) the heavy quark limit (HQL) for the heavy quark masses in the baryonic correlation functions (BCF). We will refer to the second model as the HQL BCF model. In particular, the HQL BCF model for single heavy baryons means that: 1) they are treated as bound states of a heavy quark and the light diquark system; 2) a heavy quark is located in the center of the system and is surrounded by the light degrees of freedom. We apply the heavy quark limit $m_1 = m_Q \rightarrow \infty$ in Eq. (6), i.e. $w_1 \rightarrow 1, w_2 \rightarrow 0$ and $w_3 \rightarrow 0$ and, therefore, one has $x_1 \rightarrow x$. Double heavy baryons are treated as bound states of a heavy diquark and a light quark. In the HQL BCF model the heavy diquark is located at the center of the double heavy baryon: $m_2 = m_Q \rightarrow \infty, m_3 = m_Q' \rightarrow \infty$ and, therefore, one has $w_1 \rightarrow 0, w_2 \rightarrow 1/2$ and $w_3 \rightarrow 1/2$. In the HQL BCF model the triple heavy baryon $\Omega_{cbb}^+$ is treated as a bound state of a heavy bottom quark and a relatively “light” charm diquark with $m_1 = m_b \rightarrow \infty$ and, therefore, $w_1 \rightarrow 1, w_2 \rightarrow 0, w_3 \rightarrow 0$ and $x_1 \rightarrow x$ (its structure is similar to the single heavy baryon in HQL). Finally, the HQL BCF limit for the $\Omega_{cbb}^+$ baryon means that the $\Omega_{cbb}^+$ is a bound state of a heavy bottom diquark and a relatively “light” charm quark with $m_2 = m_3 = m_b \rightarrow \infty$ and, therefore, $w_1 \rightarrow 0, w_2 \rightarrow 1/2$ and $w_3 \rightarrow 1/2$ (its structure is similar to the double heavy baryon in the HQL).

We consider two types of heavy baryons: $\Lambda$-type baryons which are bound states of a quark and diquark system with spin 0 and $\Sigma$-type baryons which are bound states of a quark and a diquark system carrying spin 1. In general, for the $\Lambda$-type baryons one can construct three types of currents without derivatives - pseudoscalar $J_P^\Lambda$, scalar $J_S^\Lambda$ and axial-vector $J_A^\Lambda$ and two types of currents (the vector $J_V^\Sigma$ and the tensor $J_T^\Sigma$ form) for $\Sigma$-type baryons (see Refs. [30]-[32] and [22]-[27]):

$$J_{q_1[q_2q_3]}^\Lambda = \epsilon^{a_1a_2a_3} q_1^{a_1} q_2^{a_2} C\gamma_5 q_3^{a_3},$$

$$J_{q_1[q_2q_3]}^\Sigma = \epsilon^{a_1a_2a_3} q_1^{a_1} q_2^{a_2} Cq_3^{a_3},$$

$$J_{q_1[q_2q_3]}^A = \epsilon^{a_1a_2a_3} \gamma^\mu q_1^{a_1} q_2^{a_2} C\gamma_\mu q_3^{a_3},$$

$$J_{q_1[q_2q_3]}^V = \epsilon^{a_1a_2a_3} \gamma^\mu q_1^{a_1} q_2^{a_2} C_{\mu a_3},$$

$$J_{q_1[q_2q_3]}^T = \epsilon^{a_1a_2a_3} \sigma^{\mu\nu} \gamma^\mu q_1^{a_1} q_2^{a_2} C_{\mu\nu} q_3^{a_3}.$$ 

The symbols [...] and {...} denote antisymmetrization and symmetrization over flavor indices of the second and the third quark, respectively.

In the following we restrict ourselves to the simplest baryonic currents - pseudoscalar $J_P^\Lambda$ for the $\Lambda$-type and vector $J_V^\Sigma$ for the $\Sigma$-type baryons. Note, that these currents for the $\Lambda$ and $\Sigma$-type baryons have the correct nonrelativistic
limit (NL). In particular, in the case of the Λ-type baryons the scalar current goes to zero in the nonrelativistic limit whereas the pseudoscalar and axial-vector currents become degenerate in this limit with the following naive quark model baryon spin-flavor function:

$$|\Lambda_{q_1,q_2,q_3}\rangle = \frac{1}{2} |q_1(q_2q_3 - q_3q_2)\rangle \uparrow (\uparrow \downarrow - \downarrow \uparrow)\rangle. \quad (10)$$

In the case of the Σ-type baryons the vector and tensor currents also become degenerate in the nonrelativistic limit. Their naive quark model spin-flavor wave functions read:

$$|\Sigma_{q_1,q_2,q_3}\rangle = \frac{1}{2\sqrt{3}} |q_1(q_2q_3 + q_3q_2)\rangle \uparrow (\uparrow \uparrow + \downarrow \downarrow) - 2 \downarrow \uparrow\rangle. \quad (11)$$

The classification of the heavy baryon states (spin-parity, flavor content and quantum numbers, mass spectrum) is given in Table 1 (single charm baryons), Table 2 (single bottom baryons) and Table 3 (double and triple heavy baryons). We use the data from Refs [33]-[35] and restrict ourselves to $\frac{1}{2}^+$ baryons.

The heavy-baryon quark coupling constants $g_B$ are determined by the compositeness condition [22]-[27] (see also [28],[29]). The compositeness condition implies that the renormalization constant of the hadron wave function is set equal to zero:

$$Z_B = 1 - \Sigma'_B(m_B) = 1 - g_B^2 \Sigma'_B(m_B) = 0 \quad (12)$$

where $\Sigma'_B$ is the derivative of the baryon mass operator described by the diagram Fig.1 and $m_B$ is the heavy baryon mass. To clarify the physical meaning of this condition, we first want to remind the reader that the renormalization constant $Z_B^{1/2}$ can also be interpreted as the matrix element between the physical and the corresponding bare state. For $Z_B = 0$ it then follows that the physical state does not contain the bare one and is described as a bound state.

The interaction Lagrangian Eq. (1) and the corresponding free parts describe both the constituents (quarks) and the physical particles (hadrons) which are taken to be the bound states of the constituents. As a result of the interaction, the physical particle is dressed, i.e. its mass and its wave function have to be renormalized. The condition $Z_B = 0$ also effectively excludes the constituent degrees of freedom from the physical space and thereby guarantees that there is no double counting for the physical observable under consideration. In this picture the constituents exist in virtual states only. One of the corollaries of the compositeness condition is the absence of a direct interaction of the dressed charged particle with the electromagnetic field. Taking into account both the tree-level diagram and the diagrams with the self-energy insertions into the external legs (that is the tree-level diagram times ($Z_B - 1$)) one obtains a common factor $Z_B$ which is equal to zero.

We use the standard free fermion Lagrangian for the baryons and quark fields:

$$\mathcal{L}_{\text{free}}(x) = \bar{B}(x)(i \not \partial - m_B)B(x) + \sum_q \bar{q}(x)(i \not \partial - m_q)q(x), \quad (13)$$

where $m_q$ is the constituent quark mass. This leads to the free fermion propagator for the constituent quark:

$$i S_q(x - y) = \langle 0 | T \{ q(x) \bar{q}(y) \} | 0 \rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} \hat{S}_q(k) \quad (14)$$

where

$$\hat{S}_q(k) = \frac{1}{m_q - k^2 - i\epsilon} \quad (15)$$

is the usual free fermion propagator in momentum space. We shall avoid the appearance of unphysical imaginary parts in Feynman diagrams by postulating the condition that the baryon mass must be less than the sum of the constituent quark masses $M_B < \sum_q m_q$. As was mentioned before, we treat heavy quark masses in the baryonic correlation functions in two specific limits: i) using their finite values and ii) applying the heavy quark limit (HQL). In the case of single heavy baryons we also consider the heavy quark limit for the heavy quark propagator. Therefore, in the case of single heavy baryons we consider three models: i) exact calculations (full); ii) exact form of the heavy quark propagator (with a finite value for the heavy quark mass) and heavy quark limit for the baryonic correlation function (HQL BCF); 3) full heavy quark limit for both the heavy quark propagator and the baryonic correlation function (HQL BCF+HQP).

The interaction with the electromagnetic field is introduced in two ways. The free Lagrangians of quarks and hadrons are gauged in the standard manner by using minimal substitution:

$$\partial^\mu B \rightarrow (\partial^\mu - ie_B A^\mu)B, \quad \partial^\mu \bar{B} \rightarrow (\partial^\mu + ie_B A^\mu)\bar{B}, \quad \partial^\mu q_i \rightarrow (\partial^\mu - ie_q A^\mu)q_i, \quad \partial^\mu \bar{q}_i \rightarrow (\partial^\mu + ie_q A^\mu)\bar{q}_i, \quad (16)$$

where $e_B$ and $e_q$ are the electromagnetic coupling constants of the baryon and quark, respectively.
where \( e_B \) is the electric charge of the baryon \( B \) and \( e_q \) is the electric charge of the quark with flavor \( q \). The interaction of the baryon and quark fields is then specified by minimal substitution. The interaction Lagrangian reads

\[
\mathcal{L}_{\text{int}}^{\text{em}}(x) = e_B \bar{B}(x) A(x) + \sum_q e_q \bar{q}(x) A(x) q(x). \tag{17}
\]

We remind the reader that the electromagnetic field does not directly couple to the baryon fields in the relativistic three quark model as explicitly shown in the following. The gauging of the nonlocal Lagrangian Eq. (11) proceeds in a way suggested and extensively used in Refs. [21, 36, 37]. In order to guarantee local invariance of the strong interaction Lagrangian one multiplies each quark field \( q(x) \) in \( \mathcal{L}_{\text{int}}^{\text{str}} \) with a gauge field exponential. One then has

\[
\mathcal{L}_{\text{int}}^{\text{str}+\text{em}(2)}(x) = g_B \bar{B}(x) \int dx_1 \int dx_2 \int dx_3 F_B(x_1, x_2, x_3) e^{a_i a_3} \Gamma_3 e^{-ie_{q_i} I(x_1, x_2, P)} q^{a_1}(x_1) \times e^{-ie_{q_2} I(x_2, x, P)} q^{a_2}(x_2) \Gamma_2 e^{-ie_{q_3} I(x_3, x, P)} q^{a_3}(x_3) + \text{H.c.} \tag{18}
\]

where

\[
I(x_1, x, P) = \int_0^{x_i} d\mu A^\mu(z). \tag{19}
\]

Finally, the Lagrangian suitable for the calculation of the electromagnetic properties of heavy baryons is given by:

\[
\mathcal{L}_{\text{full}}(x) = \mathcal{L}_{\text{free}}(x) + \mathcal{L}_{\text{int}}^{\text{em}(1)}(x) + \mathcal{L}_{\text{int}}^{\text{str}+\text{em}(2)}(x). \tag{20}
\]

It is readily seen that the full Lagrangian is invariant under the transformations

\[
A^\mu(x) \to A^\mu(x) + \partial^\mu f(x), \quad q_i(x) \to e^{ie_{q_i} f(x)} q_i(x), \quad \bar{q}_i(x) \to \bar{q}_i(x) e^{-ie_{q_i} f(x)}, \tag{21}
\]

where \( e_B = \sum_{i=1}^3 e_{q_i} \).

When expanding the gauge exponential up to a certain power of \( A_\mu \) relevant to the desired order of perturbation theory in the given process the second term of the electromagnetic interaction Lagrangian \( \mathcal{L}_{\text{int}}^{\text{em}(2)} \) arises. At first sight it appears that the results will depend on the path \( P \) taken to connect the end-points in the path integral in Eq. (19). However, one needs to know only the derivatives of the path integral expressions when calculating the perturbative series. Therefore, we use the formalism suggested in [21, 36, 37] which is based on the path-independent definition of the derivative of \( I(x, y, P) \):

\[
\lim_{dx^\mu \to 0} \int dx^\mu \frac{\partial}{\partial x^\mu} I(x, y, P) = \lim_{dx^\mu \to 0} [I(x + dx, y, P') - I(x, y, P)] \tag{22}
\]

where the path \( P' \) is obtained from \( P \) by shifting the end-point \( x \) by \( dx \). The definition (22) leads to the key rule

\[
\frac{\partial}{\partial x^\mu} I(x, y, P) = A_\mu(x) \tag{23}
\]

which in turn states that the derivative of the path integral \( I(x, y, P) \) does not depend on the path \( P \) originally used in the definition. The non-minimal substitution (13) is therefore completely equivalent to the minimal prescription as is evident from the identities (22) or (24). In Appendix A we demonstrate explicitly how to derive the Feynman rules for a non-local coupling of hadrons to photons and quarks considering the case of the coupling of a baryon to three quarks and a single photon (see Fig.2).

In the next step we have to specify the vertex function \( \tilde{\Phi}_B \), which characterizes the finite size of the baryons. In principle, its functional form can be calculated from the solutions of the Bethe-Salpeter equation for the baryon bound states [38, 39]. In Refs. [40] it was found that, using various forms for the vertex function, the basic hadron observables are insensitive to the details of the functional form of the hadron-quark vertex form factor. We will use this observation as a guiding principle and choose a simple Gaussian form for the vertex function \( \tilde{\Phi}_B \). Any choice for \( \tilde{\Phi}_B \) is appropriate as long as it falls off sufficiently fast in the ultraviolet region of Euclidean space to render the Feynman diagrams ultraviolet finite. We employ a Gaussian form for the vertex function

\[
\tilde{\Phi}_B(k_{1E}^2, k_{2E}^2) = \exp(-[k_{1E}^2 + k_{2E}^2]/\Lambda_B^2), \tag{24}
\]
where $k_1E$ and $k_2E$ are the Euclidean momenta. Here $\Lambda_B$ is a size parameter parametrizing the distribution of quarks inside a given baryon. In fact we shall use only a reduced set of size parameters, namely $\Lambda_{B_{qqq}}$ for the light baryons, $\Lambda_{B_{qQQ}}$ for single heavy baryons, $\Lambda_{B_{QQQ}}$ for double heavy baryons and $\Lambda_{B_{QQQ}}$ for triple heavy baryons $\Lambda_{B_{QQQ}}$. We use the following set of parameters:

$$
\begin{array}{cccc}
  m_u(d) & m_s & m_c & m_b \\
  0.42 & 0.57 & 1.7 & 5.2 \text{ GeV}
\end{array}
$$

and

$$
\begin{array}{cccc}
  \Lambda_{B_{qqq}} & \Lambda_{B_{qQQ}} & \Lambda_{B_{QQQ}} & \Lambda_{B_{QQQ}} \\
  1.25 & 1.8 & 2.5 & 5 \text{ GeV}
\end{array}
$$

The size parameters $\Lambda_{B_{qqq}}$, $\Lambda_{B_{qQQ}}$ and $\Lambda_{B_{QQQ}}$ and the constituent quark masses $m_u = m_d$, $m_s$, $m_c$ and $m_b$ have been taken from a fit in a previous analysis of the properties of single and double heavy baryons [26, 27]. In the present paper we have one remaining free parameter $\Lambda_{B_{QQQ}}$ characterizing the triple heavy baryons which will be fixed at 5 GeV using the heuristic relation for heavy baryons: $\Lambda_{B_{qqq}} : \Lambda_{B_{qQQ}} : \Lambda_{B_{QQQ}} \approx B_{qqq} : B_{qQQ} : B_{QQQ}$, where $B_{qqq}$, $B_{qQQ}$ and $B_{QQQ}$ are the typical masses of single, double and triple heavy baryons. The value of $\Lambda_{B_{QQQ}} = 5$ GeV is considered as preliminary because we need some data on triple heavy baryons. In Section 5 we also discuss the sensitivity of the magnetic moments of triple heavy baryons on the variation of the size parameter $\Lambda_{B_{QQQ}}$ in the region $3 - 7$ GeV.

### III. BARYON MASS OPERATOR AND THE BARYON MATRIX ELEMENT OF THE ELECTROMAGNETIC CURRENT

We begin by expanding the baryon matrix element of the electromagnetic current in terms of the Dirac $F_D$ and the Pauli $F_P$ form factors:

$$
M_\mu = \bar{u}_B(p') \Lambda_\mu(p,p') u_B(p)
$$

$$
\Lambda_\mu(p,p') = \gamma_\mu F_D(q^2) + \frac{i}{2m_B} \sigma_{\mu\nu} q^\nu F_P(q^2)
$$

where $u_B(p)$ is the baryon spinor with normalization $\bar{u}_B(p) u_B(p) = 2m_B$. The momenta of the incoming photon, incoming and outcoming baryon are denoted, respectively, by $q$, $p$ and $p'$ with $q = p' - p$.

The magnetic moment of the baryon is defined by

$$
\mu_B = [F_D(0) + F_P(0)] \frac{e}{2m_B}.
$$

We have set $\hbar = 1$. In terms of the nuclear magneton (n.m.) $\mu_N = \frac{e\hbar}{2m_p}$ the baryon magnetic moment is given by

$$
\mu_B = [F_D(0) + F_P(0)] \frac{m_p}{m_B} \quad \text{(in units of n.m.)}
$$

where $m_p$ is the proton mass.

We continue with a summary of some useful analytical results. The baryon mass operator is described by the Feynman diagram Fig.1. There are three diagrams that contribute to the electromagnetic vertex of the baryon: the triangle diagram Fig.3a and the two bubble diagrams Fig.3b and Fig.3c. In the triangle diagram the coupling of the photon to each of the three quark lines is implied. The bubble diagrams are generated by the nonlocal coupling of the photon to the baryon made up of three quark fields as described by the the Lagrangian (18) after expansion of the gauge exponential. Other possible diagrams at this order, where the photon couples directly to the baryons, are excluded due to the compositeness conditions $Z_B = 0$ as discussed before.

In general for off-shell baryons, it is convenient to write down the electromagnetic vertex function for the transition $\Lambda_\mu(p,p')$ in the form

$$
\Lambda_\mu(p,p') = \frac{q_\mu}{q^2} e_B \left[ \Sigma_B(p') - \Sigma_B(p) \right] + \Lambda^\perp_\mu(p,p')
$$
where $\Lambda^\perp_\mu(p, p')$ is the part of the vertex function which is orthogonal to the photon momentum $q^\mu\Lambda^\perp_\mu(p, p') = 0$. The explicit expression for $\Lambda^\perp_\mu(p, p')$ results from the sum of the gauge-invariant parts of the triangle ($\Delta$) in Fig.3a and of the bubble (φ) diagrams in Figs.3b and 3c:

$$\Lambda^\perp_\mu(p, p') = \Lambda^\perp_\mu, \Delta(p, p') + \Lambda^\perp_\mu, \phi(p, p')$$

(30)

The separation (29) can be achieved in the following manner. For the $\gamma^\perp$ representation:

$$\gamma^\mu = \gamma^\mu + q^\mu \frac{q^\perp}{q^2}, \quad k^\mu = k^\perp + q^\mu \frac{k^\perp}{q^2},$$

(31)

such that $\gamma^\perp q^\mu = 0$ and $k^\perp q^\mu = 0$. The orthogonal vertex function $\Lambda^\perp_\mu(p, p')$ is expressed in terms of $\gamma^\perp_\mu$ and $k^\perp_\mu$.

Then the Ward-Takahashi identity [41] relating the baryon electromagnetic vertex and the mass operator is satisfied according to

$$q^\mu \Lambda_\mu(p, p') = e_B \left[ \Sigma_B(p') - \Sigma_B(p) \right].$$

(32)

In particular, if $q = 0$ and both baryons are on their mass-shell, then the following Ward identity is satisfied:

$$\Lambda_\mu(p, p) = e_B \frac{\partial}{\partial p_\mu} \Sigma_B(p).$$

(33)

The identities 32 and 33 have to be satisfied in our approach since gauge invariance is fulfilled by construction of a gauge invariant Lagrangian.

The expressions for the baryon mass operator $\Sigma_B(p)$ (Fig.1), the triangle $\Delta_\mu, \Delta$ (Fig.3a) and the bubble diagrams $\Lambda^\perp_\mu, \phi_L$ (Fig.3b) and $\Lambda^\perp_\mu, \phi_R$ (Fig.3c) read (here and in the following we omit the flavor coefficients):

$$\Sigma_B(p) = \alpha_B \int d^4k_{123} \tilde{\Phi}^2(z_0) R_\Sigma(k^+_1, k^+_2, k^+_3),$$

$$\Lambda^\perp_\mu, \Delta(p, p') = \alpha_B \int d^4k_{123} \sum_{i=1}^{3} e_i \tilde{\Phi}(z_0) \tilde{\Phi}[z_0 + z_i(q)] R^\perp_\mu, \Delta_i(k^+_1, k^+_2, k^+_3, q),$$

$$\Lambda^\perp_\mu, \phi_L(p, p') = -\alpha_B \int d^4k_{123} \sum_{i=1}^{3} e_i L^\perp_{\mu i} \tilde{\Phi}(z_0) \int_0^1 dt \tilde{\Phi}'[z_0 + t z_i(-q)] R_\Sigma(k^+_1, k^+_2, k^+_3, q),$$

$$\Lambda^\perp_\mu, \phi_R(p, p') = -\alpha_B \int d^4k_{123} \sum_{i=1}^{3} e_i L^\perp_{\mu i} \tilde{\Phi}(z_0) \int_0^1 dt \tilde{\Phi}'[z_0 + t z_i(q)] R_\Sigma(k^+_1, k^+_2, k^+_3, q),$$

(34)

where for convenience we have introduced the notation:

$$\alpha_B = 6 g_B^2, \quad k^+_i = k_i + p^i \omega_i, \quad k^{'+}_i = k_i + p'^i \omega_i, \quad z_0 = -6(k^2_1 + k^2_2 + k^2_3)$$

$$d^4k_{123} = \frac{d^4k_1 d^4k_2 d^4k_3}{(2\pi)^3 \sqrt{i^4}} \delta^4(k_1 + k_2 + k_3), \quad L_i = 12(k_i - \sum_{j=1}^{3} k_j \omega_j),$$

(35)

$$z_1(q) = -12q^2(\omega^2_2 + \omega^2_3 + \omega^2_1) - L_1 q,$$

$$z_2(q) = -12q^2(\omega^2_1 + \omega_1 \omega_3 + \omega^2_3) - L_2 q,$$

$$z_3(q) = -12q^2(\omega^2_1 + \omega_1 \omega_2 + \omega^2_2) - L_3 q.$$
and
\[ R^\perp_\mu,\Delta_1(r_1, r_2, r_3, q) = \Gamma_1 S_{q_1}(r_1 + q) \gamma^\perp_\mu S_{q_1}(r_1) \Gamma_1 \text{tr} \left[ \Gamma_2 S_{q_2}(r_2) \Gamma_2 S_{q_3}(-r_3) \right], \]
\[ R^\perp_\mu,\Delta_1(r_1, r_2, r_3, q) = \Gamma_1 S_{q_1}(r_1) \Gamma_1 \text{tr} \left[ \Gamma_2 S_{q_2}(r_2 + q) \gamma^\perp_\mu S_{q_2}(r_2) \Gamma_2 S_{q_3}(-r_3) \right], \]
\[ R^\perp_\mu,\Delta_1(r_1, r_2, r_3, q) = -\Gamma_1 S_{q_1}(r_1) \Gamma_1 \text{tr} \left[ \Gamma_2 S_{q_2}(r_2) \Gamma_2 S_{q_3}(-r_3) \gamma^\perp_\mu S_{q_3}(-r_3 - q) \right], \]
\[ R_\Omega(r_1, r_2, r_3) = \Gamma_1 S_{q_1}(r_1) \Gamma_1 \text{tr} \left[ \Gamma_2 S_{q_2}(r_2) \Gamma_2 S_{q_3}(-r_3) \right]. \]

The prime in Eq. (34) on \( \tilde{\Phi} \) denotes the derivative:
\[ \tilde{\Phi}'(s) = \frac{d\tilde{\Phi}(s)}{ds}. \] (37)

Note that the expressions for the left (Fig.3a) and right (Fig.3b) bubble diagrams are related to each other via exchange of the external momenta \( p \leftrightarrow p' \) with
\[ \Lambda^\perp_\mu,\sigma_1(p, p') \equiv \Lambda^\perp_\mu,\sigma_µ(p', p). \] (38)

In Appendix B we describe the calculation of the baryon mass operator and the vertex functions.

IV. HEAVY QUARK LIMIT AND MATCHING TO HEAVY HADRON CHPT

In this section we check the consistency of our approach with the model-independent predictions of Heavy Hadron Chiral Perturbation Theory (HHChPT) for the magnetic moments of single heavy baryons. Note, that HHChPT is the combination of Chiral Perturbation Theory (ChPT) and Heavy Quark Effective Theory (HQET) which is thus well suited for the description of the soft interactions of hadrons containing a single heavy quark with light pseudoscalar mesons and photons.

In particular, HHChPT predicts the following structure of the magnetic moments of the \( \Lambda_Q \) and \( \Sigma_Q \) baryons including the leading and next-to-leading order terms in \( 1/m_Q \) and \( 1/\Lambda_\chi \):
\[ \mu_{\Lambda_Q}(q_2q_3) = \frac{e_Q}{2m_Q} + \frac{c_\chi_{\Lambda_Qq_2q_3}}{m_\chi} \Lambda_\chi + \frac{d_{\Lambda_Qq_2q_3}}{m_\chi^2} \Lambda_\chi^2 + \ldots, \] (39)
\[ \mu_{\Sigma_Q}(q_2q_3) = -\frac{e_Q}{6m_Q} + \frac{c_\chi_{\Sigma_Qq_2q_3}}{\Lambda_\chi} + \frac{d_{\Sigma_Qq_2q_3}}{\Lambda_\chi^2} + \ldots, \]

where \( e_Q \) is the heavy quark charge, and \( c_{B_{\Lambda_Qq_2q_3}} \) and \( d_{B_{\Lambda_Qq_2q_3}} \) are unknown coupling factors in the HHChPT approach. As we shall see further on their numerical values can be determined in our approach. Here \( \Lambda_\chi = 4\pi F_\pi \approx 1.2 \) GeV is the scale parameter of spontaneously broken chiral symmetry. It is known that the leading contribution \( (e_Q/2m_Q) \) to the magnetic moments of the \( \Lambda_Q \) baryons comes from the coupling of the heavy quark to the photon. Therefore, \( \mu_{\Lambda_Q}(q_2q_3) \) should vanish in the heavy quark limit. On the other hand, the leading contribution to the magnetic moment of the \( \Sigma_Q \) type baryon, which survives in the heavy quark limit, comes from the coupling of the light quarks to the photon. The leading contribution to \( \mu_{\Sigma_Q} \) due to the coupling of the heavy quark to the photon field is also proportional to \( 1/m_Q \) in the case of the \( \Lambda_Q \) type baryons. It should be clear that any phenomenological quark model should be able to reproduce these model independent predictions of HHChPT.

Note that the terms proportional to the coupling factors \( c_{\Lambda_Qq_2q_3} \), \( d_{\Lambda_Qq_2q_3} \) and \( d_{\Sigma_Qq_2q_3} \) originate from the meson-cloud (chiral) corrections. In the near future we intend to evaluate these corrections using the dressing formalism of the electromagnetic quark operator recently developed in Ref. [12]. The dressing formalism is consistent with baryon ChPT [13-15] due to the matching of the physical amplitudes of both approaches at the baryonic level. In particular, we intend to perform a comprehensive analysis of the magnetic moments of light and heavy baryons including the contributions of valence and sea quarks.

In the present manuscript we restrict ourselves to the contributions of the valence quark degrees of freedom to the magnetic moments of heavy baryons. We first reproduce the leading contributions to the \( \Lambda_Q \) and \( \Sigma_Q \) baryons and, second, estimate the coupling factors \( c_{\Sigma_Qq_2q_3} \).
To this end we expand the heavy quark propagator in powers of the inverse heavy quarks mass and keep only the leading term in the expansion. One has

$$\hat{S}_Q(k+p) = \frac{1}{m_Q-k-\not{p}} = -\frac{1+\frac{v^2}{2(kv+\Lambda_{q_2q_3})}}{m_Q} + O(1/m_Q)$$

(40)

where $v = p/m_B$ is the four-velocity of the single heavy baryon and $\Lambda_{q_2q_3}$ is the difference between the masses of the single heavy baryon and the heavy quark in the heavy quark limit given by

$$m_{BQ_{q_2q_3}} = m_Q + \Lambda_{q_2q_3} + O(1/m_Q).$$

(41)

We use the same values of the $\Lambda_{q_2q_3}$ parameters as in our previous papers [22, 27]:

$$\begin{align*}
\Lambda_{uu} &= \Lambda_{ud} = \Lambda_{dd} = 600 \text{ MeV}, \\
\Lambda_{us} &= \Lambda_{ds} = 750 \text{ MeV}, \\
\Lambda_{ss} &= 900 \text{ MeV}.
\end{align*}$$

(42)

Using the calculational technique discussed in Appendix B we find that the contributions of the triangle diagram in Fig.3a (coupling of the photon to the heavy quark) to the magnetic moments of the $\Lambda$- and $\Sigma$-type baryons is in exact agreement with HHChPT:

$$\begin{align*}
\mu_{\Lambda_{q_2q_3}}^{\text{heavy}} &= \frac{e_Q}{2m_Q}, \\
\mu_{\Sigma_{q_2q_3}}^{\text{heavy}} &= -\frac{e_Q}{6m_Q}.
\end{align*}$$

(43, 44)

These results are independent of the form of the baryon correlation function and of the flavor content of the light diquark system.

We stress that the bubble diagrams in Figs.3b and 3c do not contribute to the magnetic moment at the order of accuracy that we are interested in. For the contribution of the triangle diagrams describing the coupling of the photon to the light quarks we obtain the following results. In agreement with HHChPT there is no contribution to magnetic moments of $\Lambda$-type baryons at order $O(1/m_Q)$. The leading contribution to $\mu_{\Sigma_{q_2q_3}}$ which survives in the heavy quark limit is given by

$$\mu_{\Sigma_{q_2q_3}}^{\text{light}} = \frac{1}{\Lambda_{BQ}} \frac{I_2(m_q^2, m_{q_3})}{I_1(m_q^2, m_{q_3})},$$

(45)

where the integrals $I_1$ and $I_2$ depend on the model parameters (the baryon correlation function and the light flavors through their constituent quark masses $m_q$ and $m_{q_3}$):

$$\begin{align*}
I_1(m_q^2, m_{q_3}) &= 3(e_Q + e_{q_2} + e_{q_3}) \int_0^\infty \int_0^\infty d\alpha_1 \int_0^\infty d\alpha_2 \int_0^\infty d\alpha_3 \frac{\Phi^2(12z)}{|\det A|^2} \Phi^2(12z) \left(\alpha_2 e_{q_2} + \alpha_3 e_{q_3}\right) \left[\mu_{q_2} \mu_{q_3} + \frac{\alpha_2^2}{\det A} - \frac{\tilde{\Lambda}_{\alpha_1}}{2\det A}\right], \\
I_2(m_q^2, m_{q_3}) &= \int_0^\infty \int_0^\infty \int_0^\infty d\alpha_1 \int_0^\infty d\alpha_2 \int_0^\infty d\alpha_3 \frac{\Phi^2(12z)}{|\det A|^2} \Phi^2(12z) \left(\alpha_2 e_{q_2} + \alpha_3 e_{q_3}\right) \left[\mu_{q_2} \mu_{q_3} + \frac{\alpha_2^2}{\det A} \left(1 + \frac{1 + \alpha_{23}}{2\det A}\right) - \frac{\tilde{\Lambda}_{\alpha_1}}{\det A}\right],
\end{align*}$$

(46)

where

$$\begin{align*}
\mu_{q_i} &= \frac{m_{q_i}}{\Lambda_{BQ}}, \\
\tilde{\Lambda} &= \frac{\Lambda_{BQ}}{\Lambda_{BQ}}, \\
\alpha_{ij} &= \alpha_i + \alpha_j, \\
\det A &= \frac{3}{4} + \alpha_{23} + \alpha_2 \alpha_3, \\
z &= \mu_{q_2}^2 \alpha_2 + \mu_{q_3}^2 \alpha_3 + \frac{\alpha_2^2}{\det A} \left(1 + \alpha_{23}\right) - 2\tilde{\Lambda}_{\alpha_1}
\end{align*}$$

(47)

and $\Lambda_{BQ}$ is the size parameter appearing in the correlation function of the single heavy baryons $\Phi(z)$.

Our prediction for the unknown HHChPT coupling $c_{\Sigma_{q_2q_3}}$ is

$$c_{\Sigma_{q_2q_3}} = \frac{\Lambda_{BQ}}{I_1(m_q^2, m_{q_3})},$$

(48)
We complete our analysis of the magnetic moments of the $\Lambda_Q^-$ and $\Sigma_Q^-$ type baryons by deriving the magnetic moments $\mu_{\Lambda_Q}$ and $\mu_{\Sigma_Q}$ in the framework of the naive nonrelativistic quark model. We use the baryonic spin-flavor wave functions $\Pi$ and $\Theta$ arising from the relativistic three-quark currents in the nonrelativistic limit. After some simple algebra we get

$$
\mu_{\Lambda_Q[q_2q_3]} = \frac{e_Q}{2m_Q} \sum_{i=1}^{3} \frac{e_{q_i}}{2m_{q_i}} \sigma_{3i} |\Lambda_{q_1[q_2q_3]}|,
$$

$$
\mu_{\Sigma_Q[q_2q_3]} = \frac{e_Q}{6m_Q} + \frac{e_{q_2}}{3m_{q_2}} + \frac{e_{q_3}}{3m_{q_3}}.
$$

V. NUMERICAL RESULTS AND DISCUSSION

Our results for magnetic moments of heavy baryons are given in Tables 4-6. First, we discuss the numerical results for the magnetic moments of single heavy baryons for three models (see Table 4): 1) exact results (full) without any approximations; 2) heavy quark limit (HQL) for the baryonic correlation function (BCF) and 3) full HQL for both the BCF and the heavy quark propagator (HQP). We refer to these models as full, HQL BCF and HQL BCF+HQP, respectively. For the convenience we separate the contributions coming from the coupling of the heavy quark ($c$ or $b$) to the photon (heavy quark contribution) and the same for the light degrees of freedom (light quark contribution).

In the numerical calculations we use the Gaussian form of the baryonic correlation function (24). The last column contains the results of the nonrelativistic quark model (NRQM) which uses the spin-flavor baryonic wave functions described in the previous sections. The basic notions of the NRQM are given in Appendix C. In particular, in Table 7 we present the wave functions and magnetic moments of heavy baryons in the NRQM.

Next we predict the unknown HHChPT couplings $c_{\Sigma_Q[q_2q_3]}$ using Eqs. (89) and (118) restricting ourselves to the model HQL BCF+HQP for $\Sigma$-type baryons. For the sake of comparison we following Ref. [19] in the construction of the flavor coefficients in $c_{\Sigma_Q[q_2q_3]}$. One has

$$
c_{\Sigma_Q[q_2q_3]} = \frac{4}{9} c_S \mu_{\Sigma_Q[q_2q_3]}^S
$$

where

$$
\mu_{\Sigma_Q^+} = \mu_{\Sigma_Q^0} = 2, \quad \mu_{\Sigma_Q^+} = \mu_{\Sigma_Q^0} = \frac{1}{2}, \quad \mu_{\Sigma_Q^+} = \mu_{\Sigma_Q^0} = -1,
$$

$$
\mu_{\Xi_{0}^-} = \mu_{\Xi_{0}^+} = -1, \quad \mu_{\Xi_{0}^+} = \mu_{\Xi_{0}^+} = \frac{1}{2}, \quad \mu_{\Xi_{0}^-} = \mu_{\Xi_{0}^-} = -1.
$$

In Table 5 we present our predictions for the HHChPT coupling factor $c_S$. In the approach of this value we predict the coupling factor depends on the light quark content of the heavy baryon. It is also flavor dependent because we break SU(3) flavor symmetry. Therefore, there are different predictions (see Table 5) for the $c_S$ coupling factors depending on whether one is dealing with nonstrange, single strange or double strange states. Even for the cascade states $\Xi_Q^{(1s)}$ and $\Xi_Q^{(1s)}$ we get different predictions, because the contributions of $u$ and $d$-quarks enter with different coefficients. While the coupling factors $c_S$ do not depend on the heavy flavor (as stressed in Ref. [19]) they do depend on the light quark flavor. The coupling factors $c_S$ in Table 5 vary from 0.26 to 0.55 depending on the $SU(3)$ flavor content. In Table 6 we present our results for the magnetic moments of double and triple heavy baryons for two models: 1) exact calculation (full); 2) heavy quark limit (HQL) for the baryonic correlation function (BCF). We also perform a comparison with the results of the NRQM.

Finally we compare our predictions for the magnetic moments of heavy baryons with the results of other theoretical approaches: QCD sum rules [14, 15], soliton approaches [11, 13] and quark models [7, 9, 10]. First of all, we stress again that in the sector of single heavy baryons we are consistent with HHChPT in the heavy quark limit. We reproduce the leading terms in the expansion of the magnetic moments in powers of $1/m_Q$ and $1/\Lambda$. Moreover, we were able to predict the unknown HHChPT coupling factors $c_S = 0.26 \text{--} 0.55$.

The magnetic moments of $A$-type single heavy baryons are practically the same in all three models, because the leading contribution comes from the coupling of the photon to the heavy quark. This is not the case for the $\Sigma$-type baryons: the contribution of the heavy quarks remains unchanged but the light quark contribution is increased in the “full” scheme while it is suppressed in the HQL BCF and HQL BCF+HQP schemes. We mention that the contribution of the bubble diagrams Fig.3b and Fig.3c is suppressed. In magnitude it is less than 5%. Note, that the predictions of the HQL BCF and HQL BCF+HQP models are very similar.
Our “full” model is close to the predictions of the naive quark model as well as the results obtained in the quark models \[8\], \[9\] given by

\[
\mu_{\Lambda_0^+} = \mu_{\Xi_c^0} = \mu_{\Xi_b^0} = 0.35 \\
\mu_{\Sigma^+_c} = 2.37 - 2.45, \quad \mu_{\Sigma^+_b} = 0.50 - 0.52, \quad \mu_{\Sigma^-_b} = -(1.36 - 1.40) \\
\mu_{\Xi_c^0} = 0.75 - 0.78, \quad \mu_{\Xi_b^0} = -(1.12 - 1.15), \quad \mu_{\Omega_b^0} = -(0.88 - 0.89) \\
\mu_{\Sigma^+_b} = 2.50 - 2.59, \quad \mu_{\Sigma^-_b} = 0.64 - 0.66, \quad \mu_{\Sigma^-_b} = -(1.22 - 1.26) \\
\mu_{\Xi_c^0} = 0.88 - 0.92, \quad \mu_{\Xi_b^0} = -(0.98 - 1.01), \quad \mu_{\Omega_b^0} = -(0.74 - 0.75)
\]

and \[8\]

\[
\mu_{\Lambda_0^+} = \mu_{\Xi_c^0} = \mu_{\Xi_b^0} = 0.38 \\
\mu_{\Sigma^+_c} = 2.33, \quad \mu_{\Sigma^+_b} = 0.49, \quad \mu_{\Sigma^-_b} = -1.35 \\
\mu_{\Xi_c^0} = 0.65, \quad \mu_{\Xi_b^0} = -1.18, \quad \mu_{\Omega_b^0} = -1.02
\]

The “full” scheme results can be compared to the predictions of the QCD sum rule approach \[14\]:

\[
\mu_{\Sigma^+_c} = 2.1 \pm 0.3, \quad \mu_{\Sigma^+_b} = 0.6 \pm 0.1, \\
\mu_{\Sigma^-_b} = -(1.6 \pm 0.2), \quad \mu_{\Lambda_0^+} = 0.15 \pm 0.05
\]

and \[15\]:

\[
\mu_{\Lambda_0^+} = 0.40 \pm 0.05, \quad \mu_{\Lambda_b^0} = -(0.18 \pm 0.05).
\]

Let us stress that the agreement with the NRQM is based on the use of the specific spin-flavor wave functions of baryons which correspond to our relativistic baryonic currents in the nonrelativistic limit. The use of other NRQM spin-flavor structures give very different results.

A detailed analysis of the magnetic moments of single heavy baryons has been performed in solitonic (Skyrme) approaches \[11,12,13\]. The most recent calculation \[13\] gives the following numbers for \(\Lambda\)-type baryons

\[
\mu_{\Lambda_0^+} = 0.12 - 0.13, \quad \mu_{\Lambda_b^0} = -0.02,
\]

which are smaller than our numbers. Their results for the \(\Sigma\)-type baryons are

\[
\mu_{\Sigma^+_c} = 2.45 - 2.46, \quad \mu_{\Sigma^-_b} = -1.96, \\
\mu_{\Sigma^+_b} = 2.52, \quad \mu_{\Sigma^-_b} = -(1.93 - 1.94).
\]

are larger than ours in average. Recently, the magnetic moments of charmed baryons have been calculated in a relativistic quark model \[10\] exploiting three different forms of relativistic kinematics. In the case of single heavy baryons it was found that there is only a small dependence on the kinematics for \(\Lambda\)-type baryons. For the magnetic moments of the \(\Lambda\)-type baryons they quote:

\[
\mu_{\Lambda^+_c} = 0.39 - 0.52, \quad \mu_{\Xi_c^0} = 0.39 - 0.47, \quad \mu_{\Xi_b^0} = 0.39 - 0.47.
\]

Contrary to this there is a strong dependence on the form of the relativistic kinematics for \(\Sigma\)-type baryons. For these they quote

\[
\mu_{\Sigma^+_c} = 0.90 - 3.07, \quad \mu_{\Sigma^-_c} = -(0.74 - 1.78), \quad \mu_{\Omega_b^0} = -(0.67 - 1.03).
\]

Our predictions in both our HQL schemes are similar to the results of the MIT bag model \[\tilde{\text{H}}\], for which one obtains

\[
\mu_{\Sigma^+_c} = 0.70, \quad \mu_{\Sigma^-_c} = -0.44, \quad \mu_{\Xi_c^0} = -0.35, \\
\mu_{\Sigma^+_b} = 0.8, \quad \mu_{\Sigma^-_b} = -0.40, \quad \mu_{\Omega_b^0} = -0.30.
\]

Now we turn to the results for the magnetic moments of double and triple heavy baryons (see Table 6). One can see that the results of the “full” model are close to the results of the NRQM. Note that the predictions for the HQL BCF
model are similar to the results of the relativistic quark model \cite{10} if one uses the “point” form of the relativistic kinematics, e.g.: 

$$\mu_{\Xi_{c}^{++}} = 0.29 - 0.30, \quad \mu_{\Xi_{c}^{+}} = 0.68 - 0.69, \quad \mu_{\Omega_{c}^{+}} = 0.66.$$ \hspace{1cm} (61)

Also there is an agreement for some states with the relativistic quark potential model \cite{8}: 

$$\mu_{\Xi_{c}^{+}} = 0.78 - 0.79, \quad \mu_{\Omega_{c}^{+}} = 0.66, \quad \mu_{\Xi_{bb}^{0}} = -0.71 - 0.73,$$ \hspace{1cm} (62)

$$\mu_{\Xi_{cb}^{+}} = 0.23 - 0.24, \quad \mu_{\Omega_{cb}^{+}} = 0.11, \quad \mu_{\Xi_{bb}^{0}} = 1.50 - 1.54.$$ \hspace{1cm} (63)

We found the agreement with Ref. \cite{8} for the magnetic moments of triple heavy baryons: 

$$\mu_{\Omega_{c}^{+}} = 0.49, \quad \mu_{\Omega_{c}^{0}} = -0.20.$$ \hspace{1cm} (64)

Finally, we show the sensitivity of the magnetic moments of triple heavy baryons to a variation of the size parameter $\Lambda_{BQQ}$ in the region $3 - 7$ GeV. When the value of $\Lambda_{BQQ}$ is varied from 3 to 7 GeV the values of $\mu_{\Omega_{c}^{+}}$ and $\mu_{\Omega_{c}^{0}}$ are changed as: 

$$\mu_{\Omega_{c}^{+}} = 0.58 - 0.50, \quad \mu_{\Omega_{c}^{0}} = -0.21 - 0.20$$ \hspace{1cm} (65)

in the “full” model and 

$$\mu_{\Omega_{c}^{+}} = 0.09 - 0.16, \quad \mu_{\Omega_{c}^{0}} = -0.11 - 0.14$$ \hspace{1cm} (66)

in the HQL BCF scheme.

It will be interesting to compare our result for the magnetic moments of double and triple heavy baryons with possible future results in the framework of nonrelativistic QCD (NRQCD) recently extended on the sector of baryons containing two and three heavy quarks \cite{47,48}.

VI. CONCLUSION

We have employed the relativistic three-quark model to calculate the magnetic moments of single, double and triple heavy baryons. We have used a Gaussian shape for the baryon-quark vertex. For the propagators we have used free quark propagators. The electromagnetic vertex functions of the heavy baryons are described by a set of three-quark (triangle and bubble) diagrams. The parameters of the model are the constituent quark masses and the size parameters $\Lambda_{B}$ which appear as free parameters in the baryonic correlation functions. We have presented a detailed analysis of the magnetic moments of heavy baryons. We have shown that our results have the correct structure predicted by the model independent approach of HHChPT. This allowed us to fix the values of some of the coupling factors that appear in HHChPT. Finally, we have compared our numerical results to the results of a variety of other approaches.

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APPENDIX A: FEYNMAN RULE FOR THE NONLOCAL ELECTROMAGNETIC VERTEX

In the following we derive the Feynman rules for the nonlocal vertex of Fig.1 describing the coupling of a baryon, the three quarks and the photon field. This vertex contains the path integral over the gauge field

\[ I(x, y, P) = \int \frac{dz}{y} A^\mu(z) . \tag{A1} \]

The crucial point is to calculate the expression

\[ \Gamma_{B^{3qg}} = \int d^4x_1 \int d^4x_2 \Phi(x_1^2 + x_2^2) e^{ip_1x_1 + ip_2x_2} I(x_+, x, P) \tag{A2} \]

where \( x_+ = x \pm a_1x_1 \pm a_2x_2 \) and where \( p_1 \) and \( p_2 \) are linear combination of the momenta. We do not specify the parameters \( a_{1,2} \) and momenta \( p_{1,2} \) because their specific form is not necessary for the further derivation.

We use the operator identity

\[ \Phi(x_1^2 + x_2^2) = \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \Phi(-k_1^2 - k_2^2) e^{ik_1x_1 + ik_2x_2} , \]

\[ \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \Phi(\partial_{x_1}^2 + \partial_{x_2}^2) e^{ik_1x_1 + ik_2x_2} = \delta^4(x_1) \delta^4(x_2) \Phi(\partial_{x_1}^2 + \partial_{x_2}^2) , \tag{A3} \]

where \( \Phi \) is the Fourier transform of the vertex function \( \Phi \). We then get

\[ \Gamma_{B^{3qg}} = \int d^4x_1 \int d^4x_2 \Phi(x_1^2 + x_2^2) e^{ip_1x_1 + ip_2x_2} I(x_+, x, P) \]

It is readily seen that

\[ \Phi(\partial_{x_1}^2 + \partial_{x_2}^2) e^{ip_1x_1 + ip_2x_2} I(x_+, x, P) = e^{ip_1x_1 + ip_2x_2} \Phi(D_{x_1}^2 + D_{x_2}^2) I(x_+, x, P) \]

where \( D_{x_i} \equiv \partial_{x_i} + ip_i \).

In order to evaluate

\[ \Phi(D_{x_1}^2 + D_{x_2}^2) I(x_+, x, P) = \sum_{n=0}^{\infty} \frac{\Phi^{(n)}(0)}{n!} [D_{x_1}^2 + D_{x_2}^2]^n I(x_+, x, P) \]

we make use of the key equation \( \Phi^{(n)}(0) = \int (D x)^n \Phi(x) \). One obtains

\[ \partial_{x_i}^n I(x, y, P) = A^n(x) . \tag{A7} \]

resulting in

\[ [D_{x_1}^2 + D_{x_2}^2] I(x_+, x, P) = L(A) - (p_1^2 + p_2^2) I(x_+, x, P) \]

where

\[ L(A) \equiv (\partial_{x_1} + \partial_{x_2}) A(x) + 2i(p_1 + p_2) A(x) . \tag{A8} \]

The iteration of the last expression gives the sequence

\[ (D_{x_1}^2 + D_{x_2}^2)^2 I(x_+, x, P) = [D_{x_1}^2 + D_{x_2}^2 - (p_1^2 + p_2^2)]L(A) - (p_1^2 + p_2^2)^2 I(x_+, x, P) \]

\[ (D_{x_1}^2 + D_{x_2}^2)^3 I(x_+, x, P) = [(D_{x_1}^2 + D_{x_2}^2)^2 - (D_{x_1}^2 + D_{x_2}^2)(p_1^2 + p_2^2) + (p_1^2 - p_2^2)^2]L(A) - (p_1^2 - p_2^2)^3 I(x_+, x, P) \]

\[ \ldots \]

\[ (D_{x_1}^2 + D_{x_2}^2)^n I(x_+, x, P) = \sum_{k=0}^{n-1} (D_{x_1}^2 + D_{x_2}^2)^{n-1-k} (p_1^2 - p_2^2)^k L(A) + (p_1^2 - p_2^2)^n I(x_+, x, P) \]

\[ = n \int_0^1 dt \ [(D_{x_1}^2 + D_{x_2}^2)t - (p_1^2 + p_2^2)(1-t)]^{n-1} L(A) + (p_1^2 - p_2^2)^n I(x_+, x, P) . \tag{A9} \]
One finally has
\[
\hat{\Phi}(D_{x_1}^2 + D_{x_2}^2)I(x_+, x, P) = \int_0^1 dt \hat{\Phi}'[(D_{x_1}^2 + D_{x_2}^2)t - (p_1^2 + p_2^2)(1 - t)] L(A) + \hat{\Phi}(-p_1^2 - p_2^2) I(x_+, x, P)
\]
\[
= \int \frac{d^4 q}{(2\pi)^4} \hat{A}_\mu(q) \left\{ iK^\mu e^{-iqx_+} \int_0^1 dt \hat{\Phi}'[w(t)] + \hat{\Phi}[w(0)] \right\},
\]
(A11)
where
\[
K^\mu = a_1[2p_1 - q] + a_2[2p_2 - q] - p_1^2 - p_2^2(1 - t),
\]
\[
w(t) = -(p_1 - a_1 q)^2 t - (p_2 - a_2 q)^2 t - (p_1^2 + p_2^2)(1 - t),
\]
\[
\hat{A}_\mu(q) \text{ is the Fourier-transform of the electromagnetic field and } \hat{\Phi}'(z) = d\hat{\Phi}(z)/dz. \text{ The last term in Eq. (A11) contains an integration from } x \text{ to } x_+ \text{ in the path integral. This term vanishes due to the delta functions } \delta^4(x_1) \text{ and } \delta^4(x_2) \text{ in Eq. (A3).}
\]
Finally, we get
\[
\Gamma_{B3q_2} = i \int \frac{d^4 q}{(2\pi)^4} e^{-iqx} \hat{A}_\mu(q) K^\mu \int_0^1 dt \hat{\Phi}'[w(t)].
\]
(A12)

APPENDIX B: DETAILS OF THE CALCULATION OF MATRIX ELEMENTS

As an example of the evaluation of matrix elements we explicitly calculate the baryon mass operator of Eq. (34). The generic integral reads:
\[
I_B(p) = \int d\mathbf{k}_{123} \hat{\Phi}^2(z_0) R_{S\Sigma}(k_1^+, k_2^+, k_3^+).
\]
(B1)

For simplicity we set \( w_1 = 1 \) and \( w_2 = w_3 = 0 \) in Eq. (B1). Then the integral \( I_B(p) \) can be written as:
\[
I_B(p) = \int \frac{d^4 k_1}{\pi^2} \int \frac{d^4 k_2}{\pi^2} \hat{\Phi}^2(12[k_1^2 + k_1 k_2 + k_2^2]) \Gamma_{1f} S_{q_1}(k_1 + p) \Gamma_{1i} \text{tr}[\Gamma_{2f} S_{q_2}(k_2) \Gamma_{2i} S_{q_1}(k_1 + k_2)].
\]
(B2)

The technique we use is based on the following main ingredients:

- use of the Laplace transform of the vertex function, its derivative and integral:
\[
\hat{\Phi}(z_0) = \int_0^\infty ds \Phi_L(s) e^{-sz_0},
\]
\[
\hat{\Phi}'(z_0) = -\int_0^\infty ds \Phi_L(s) e^{-sz_0},
\]
\[
\int_0^\infty d\alpha \alpha^n \hat{\Phi}(z_0 + \alpha) = \Gamma(n + 1) \int_0^\infty \frac{ds}{s^n} \Phi_L(s) e^{-sz_0},
\]

- \(\alpha\)-transform of the propagator functions \( S_{q_1}, S_{q_2} \) and \( S_{q_3} \):
\[
\frac{1}{m_q^2 - (k + p)^2} = \int_0^\infty d\alpha e^{-\alpha(m^2 - (k + p)^2^2)},
\]


- differential representation of the numerator

\[(m + k + p) e^{kq} = \left( m + \gamma^\mu \frac{\partial}{\partial q^\mu} + p \right) e^{kq}.
\]

- Gaussian integral over virtual momenta \(k_1\) and \(k_2\)

\[
\prod_{j=1}^n \int \frac{d^4k_j}{\pi^{2n}} \exp[kAk + 2Bk] = \frac{1}{|\text{det}A|^n} \exp[-BA^{-1}B]
\]

where, in a general approach, \(A\) is a \(n \times n\) matrix and \(B\) a \(n\)-component vector. In the present application we have \(n = 2\).

After some algebra we get the following expressions for the structure integral \(I_B(p)\)

\[
I_B(p) = \int_0^\infty d\alpha_1 \int_0^\infty d\alpha_2 \int_0^\infty d\alpha_3 \int_0^\infty d\beta \tilde{\Phi}^2[-12(z + \beta)]
\]

\[
\times \left\{ \Gamma_{1f} D_1 \Gamma_{1i} \text{tr}[\Gamma_{2f} D_2 \Gamma_{2i} D_3] - \beta \Gamma_{1f} D_1 \Gamma_{1i} \text{tr}[\Gamma_{2f} \gamma^\mu \Gamma_{2i} \gamma_\mu] (A^{-1}_{i1} + A^{-1}_{i2}) \right\}
\]

\[
- \beta \Gamma_{1f} \gamma^\mu \Gamma_{1i} \text{tr}[\Gamma_{2f} \gamma_\mu \Gamma_{2i} D_3 A^{-1}_{i2} + \Gamma_{2f} D_2 \Gamma_{2i} \gamma_\mu (A^{-1}_{i1} + A^{-1}_{i2})] \right\}
\]

where

\[
D_i = m_{qi} + P_i, \quad P_1 = p - B_1 A^{-1}_{i1}, \quad P_2 = -B_1 A^{-1}_{i2},
\]

\[
P_3 = -B_1 (A^{-1}_{i1} + A^{-1}_{i2}), \quad z = -\sum_{i=1}^3 \alpha_i m_{qi}^2 + p^2 \alpha_1 - B_1^2 A^{-1}_{i1}.
\]

Here \(B_1 = p\alpha_1\) and \(A^{-1}_{ij}\) are the elements of the inverse matrix \(A_{ij}\):

\[
A = \begin{pmatrix}
1 + \alpha_1 + \alpha_3 & \frac{1}{2} + \alpha_3 & \frac{1}{2} + \alpha_3 \\
\frac{1}{2} + \alpha_3 & 1 + \alpha_2 + \alpha_3 & \frac{1}{2} + \alpha_3 \\
\frac{1}{2} + \alpha_3 & \frac{1}{2} + \alpha_3 & 1 + \alpha_1 + \alpha_3
\end{pmatrix}; \quad A^{-1} = \frac{1}{\text{det}A} \begin{pmatrix}
1 + \alpha_2 + \alpha_3 & -\frac{1}{2} + \alpha_3 & -\frac{1}{2} + \alpha_3 \\
-\frac{1}{2} + \alpha_3 & 1 + \alpha_1 + \alpha_3 & -\frac{1}{2} + \alpha_3 \\
-\frac{1}{2} + \alpha_3 & -\frac{1}{2} + \alpha_3 & 1 + \alpha_1 + \alpha_3
\end{pmatrix}.
\]

The advantage of choosing this order of integration (made possible by the use of Laplace transforms) is that the specification of the baryonic correlation function \(\tilde{\Phi}\) can be left to the last step after having integrated over the virtual momenta. All further calculations have been done by using computer programs written in FORM for the manipulations of the Dirac matrices and in FORTRAN for the final numerical evaluation.

**APPENDIX C: NONRELATIVISTIC QUARK MODEL: SPIN-FLAVOR WAVE FUNCTIONS AND MAGNETIC MOMENTS**

In this Appendix we present the results for the magnetic moments of heavy baryons within the nonrelativistic quark model. As emphasized before the nonrelativistic quark model is based on the spin-flavor wave functions which arise in the nonrelativistic limit of the relativistic covariant three-quark currents carrying the quantum numbers of 1/2+ baryons. Using Eqs. (10) and (11) we specify the wave functions of all baryonic states involved in our calculations. Then we derive the expressions for the baryonic magnetic moments using the master formula:

\[
\mu_{B_{q1q2q3}} = \langle B_{q1q2q3} | \sum_{i=1}^3 \frac{e_{qi}}{2m_{qi}} \sigma_{3i} | B_{q1q2q3} \rangle
\]

where \(\sigma_{3i}\) is the third component of the spin operator of the \(i\)-th quark.

In Table 7 we display our results for the wave functions and our predictions for the magnetic moments, where we use the following notation for the antisymmetric \(\chi_A\) and symmetric \(\chi_S\) spin wave functions:

\[
\chi_A = \sqrt{\frac{7}{2}} \left\{ \uparrow \downarrow - \downarrow \uparrow \right\}; \quad \chi_S = \sqrt{\frac{7}{6}} \left\{ \uparrow \downarrow + \downarrow \uparrow \right\} - 2 \downarrow \uparrow \uparrow \right\}.
\]

\[
(C2)
\]
## List of Tables

**Table 1.** Single charm 1/2$^+$ baryons

| Notation | Content | $J^P$ | SU(3) | $I_3$ | S | C | Mass (GeV) |
|----------|---------|-------|-------|-------|---|---|-----------|
| $\Lambda_c^+$ | $c[ud]$ | 1/2$^+$ | 3     | 0     | 0 | 1 | 2.286     |
| $\Xi_c^+$ | $c[us]$ | 1/2$^+$ | 3     | 1/2   | -1| 1 | 2.466     |
| $\Xi_c^0$ | $c[sd]$ | 1/2$^+$ | 3     | -1/2  | -1| 1 | 2.472     |
| $\Sigma_c^{++}$ | $suu$ | 1/2$^+$ | 6     | 1     | 0 | 1 | 2.453     |
| $\Sigma_c^+$ | $c[ud]$ | 1/2$^+$ | 6     | 0     | 0 | 1 | 2.451     |
| $\Sigma_c^0$ | $cdd$ | 1/2$^+$ | 6     | -1    | 0 | 1 | 2.452     |
| $\Xi_c^+$ | $c[su]$ | 1/2$^+$ | 6     | 1/2   | -1| 1 | 2.574     |
| $\Xi_c^0$ | $c[sd]$ | 1/2$^+$ | 6     | -1/2  | -1| 1 | 2.579     |
| $\Omega_c^0$ | $css$ | 1/2$^+$ | 6     | 0     | -2| 1 | 2.698     |

**Table 2.** Single bottom 1/2$^+$ baryons

| Notation | Content | $J^P$ | SU(3) | $I_3$ | S | B | Mass (GeV) |
|----------|---------|-------|-------|-------|---|---|-----------|
| $\Lambda_b$ | $b[ud]$ | 1/2$^+$ | 3     | 0     | 0 | 1 | 5.624     |
| $\Xi_b^0$ | $b[us]$ | 1/2$^+$ | 3     | 1/2   | -1| 1 | 5.80      |
| $\Xi_b^-$ | $b[sd]$ | 1/2$^+$ | 3     | -1/2  | -1| 1 | 5.80      |
| $\Sigma_b^+$ | $bvu$ | 1/2$^+$ | 6     | 1     | 0 | 1 | 5.82      |
| $\Sigma_b^0$ | $b[ud]$ | 1/2$^+$ | 6     | 0     | 0 | 1 | 5.82      |
| $\Sigma_b^-$ | $b[bd]$ | 1/2$^+$ | 6     | -1    | 0 | 1 | 5.94      |
| $\Xi_b^0$ | $b[su]$ | 1/2$^+$ | 6     | 1/2   | -1| 1 | 5.94      |
| $\Xi_b^-$ | $b[sd]$ | 1/2$^+$ | 6     | -1/2  | -1| 1 | 5.94      |
| $\Omega_b^0$ | $bss$ | 1/2$^+$ | 6     | 0     | -2| 1 | 6.04      |

**Table 3.** Double and triple heavy 1/2$^+$ baryons

| Notation | Content | $J^P$ | $I_3$ | S | C | B | Mass (GeV) |
|----------|---------|-------|-------|---|---|---|-----------|
| $\Xi_{cc}^{++}$ | $u\{cc\}$ | 1/2$^+$ | 1/2   | 0 | 2 | 0 | 3.519     |
| $\Xi_{cc}^{+}$ | $d\{cc\}$ | 1/2$^+$ | -1/2  | 0 | 2 | 0 | 3.519     |
| $\Omega_{cc}^{+}$ | $s\{cc\}$ | 1/2$^+$ | 0     | -1| 2 | 0 | 3.59      |
| $\Xi_{bb}^{0}$ | $u\{bb\}$ | 1/2$^+$ | 1/2   | 0 | 0 | 2 | 10.09     |
| $\Xi_{bb}^{-}$ | $d\{bb\}$ | 1/2$^+$ | -1/2  | 0 | 0 | 2 | 10.09     |
| $\Omega_{bb}^{-}$ | $s\{bb\}$ | 1/2$^+$ | 0     | -1| 0 | 2 | 10.18     |
| $\Xi_{cb}^{+}$ | $u\{cb\}$ | 1/2$^+$ | 1/2   | 0 | 1 | 1 | 6.82      |
| $\Xi_{cb}^{0}$ | $d\{cb\}$ | 1/2$^+$ | -1/2  | 0 | 1 | 1 | 6.82      |
| $\Omega_{cb}^{0}$ | $s\{cb\}$ | 1/2$^+$ | 0     | -1| 1 | 1 | 6.91      |
| $\Xi_{ccb}^{+}$ | $u\{ccb\}$ | 1/2$^+$ | 1/2   | 0 | 1 | 1 | 6.85      |
| $\Xi_{cbb}^{0}$ | $d\{cbb\}$ | 1/2$^+$ | -1/2  | 0 | 1 | 1 | 6.85      |
| $\Omega_{cbb}^{0}$ | $s\{cbb\}$ | 1/2$^+$ | 0     | -1| 1 | 1 | 6.93      |
| $\Omega_{cccb}^{+}$ | $bcc$ | 1/2$^+$ | 0     | 0   | 2 | 1 | 8.0       |
| $\Omega_{ccbb}^{0}$ | $cbb$ | 1/2$^+$ | 0     | 0   | 1 | 2 | 11.5      |
## Table 4. Magnetic moments of single heavy baryons (in units of $\mu_N$)

| Baryon | RQM                  | NRQM                  |
|--------|-----------------------|-----------------------|
|        | full                 | HQL BCF               | HQL BCF+HQP          |
| $\Lambda^+_c$ | 0.42 (0.41; 0.01)     | 0.38 (0.38; 0.003)    | 0.37 (0.37; 0)       | 0.37 (0.37; 0)     |
| $\Lambda^+_b$ | -0.06 (-0.06; 0.002)  | -0.06 (-0.06; 0.001)  | -0.06 (-0.06; 0)    | -0.06 (-0.06; 0)  |
| $\Xi^+_c$       | 0.41 (0.40; 0.01)     | 0.37 (0.37; 0.01)     | 0.37 (0.37; 0)       | 0.37 (0.37; 0)     |
| $\Xi^+_b$       | 0.39 (0.40; -0.01)    | 0.37 (0.37; -0.004)   | 0.37 (0.37; 0)       | 0.37 (0.37; 0)     |
| $\Xi^0_c$       | -0.06 (-0.06; 0.002)  | -0.06 (-0.06; 0.001)  | -0.06 (-0.06; 0)    | -0.06 (-0.06; 0)  |
| $\Xi^0_b$       | -0.06 (-0.06; -0.003)| -0.06 (-0.06; -0.001)| -0.06 (-0.06; 0)    | -0.06 (-0.06; 0)  |
| $\Xi^{++}_c$    | 0.47 (-0.11; 0.58)    | 0.10 (-0.11; 0.21)    | 0.08 (-0.12; 0.20)  | 0.51 (-0.12; 0.63) |
| $\Xi^0_c$       | -0.95 (-0.11; -0.84)  | -0.38 (-0.11; -0.27)  | -0.37 (-0.12; -0.25)| -0.98 (-0.12; -0.86)|
| $\Xi^{0}_b$    | 0.66 (0.02; 0.64)     | 0.22 (0.02; 0.20)     | 0.22 (0.02; 0.20)   | 0.65 (0.02; 0.63)  |
| $\Xi^{0}_b$    | -0.91 (0.02; -0.93)   | -0.23 (0.02; -0.25)   | -0.23 (0.02; -0.25) | -0.84 (0.02; -0.86)|
| $\Sigma^{++}_c$| 1.76 (-0.11; 1.87)    | 0.58 (-0.11; 0.69)    | 0.53 (-0.12; 0.65)  | 1.86 (-0.12; 1.98) |
| $\Sigma^+_c$    | 0.36 (-0.11; 0.47)    | 0.06 (-0.11; 0.17)    | 0.04 (-0.12; 0.16)  | 0.37 (-0.12; 0.49) |
| $\Sigma^+_b$    | -1.04 (-0.11; -0.93)  | -0.46 (-0.11; -0.35)  | -0.44 (-0.12; -0.32)| -1.11 (-0.12; -0.99)|
| $\Sigma^0_c$    | 2.07 (0.02; 2.05)     | 0.68 (0.02; 0.66)     | 0.67 (0.02; 0.65)   | 2.01 (0.02; 1.99)  |
| $\Sigma^0_b$    | 0.53 (0.02; 0.51)     | 0.18 (0.02; 0.16)     | 0.18 (0.02; 0.16)   | 0.52 (0.02; 0.50)  |
| $\Sigma^-_b$    | -0.10 (0.02; -1.03)   | -0.31 (0.02; -0.33)   | -0.30 (0.02; -0.32) | -0.97 (0.02; -0.99) |
| $\Omega^0_c$    | -0.85 (-0.11; -0.74)  | -0.32 (-0.11; -0.21)  | -0.31 (-0.12; -0.19)| -0.85 (-0.12; -0.73)|
| $\Omega^0_b$    | -0.82 (0.02; -0.84)   | -0.17 (0.02; -0.19)   | -0.17 (0.02; -0.19) | -0.71 (0.02; -0.73) |

## Table 5. Predictions for the HHChPT coupling constant $c_S$

| Baryon | $\Sigma_{Q(yy')}$ | $\Xi_{Q(ux)}$ | $\Xi_{Q(dx)}$ | $\Omega_{Q(zz)}$ |
|--------|-------------------|----------------|----------------|-----------------|
| $c_S$  | 0.45              | 0.55           | 0.35           | 0.26            |
Table 6. Magnetic moments of double and triple heavy baryons (in units of $\mu_N$)

| Baryon  | RQM full | HQL BCF  | NRQM full |
|---------|----------|----------|-----------|
| $\Xi^{++}_{cc}$ | 0.13 (0.52; -0.38) | 0.25 (0.51; -0.26) | -0.01 (0.49; -0.50) |
| $\Xi^+_{cc}$ | 0.72 (0.52; 0.20) | 0.64 (0.51; 0.13) | 0.74 (0.49; 0.25) |
| $\Xi^0_{bb}$ | -0.53 (-0.06; -0.47) | -0.42 (-0.08; -0.34) | -0.58 (-0.08; -0.50) |
| $\Xi^-_{bb}$ | 0.18 (-0.06; 0.24) | 0.09 (-0.08; 0.17) | 0.17 (-0.08; 0.25) |
| $\Omega^+_{cc}$ | 0.67 (0.53; 0.14) | 0.60 (0.50; 0.10) | 0.67 (0.49; 0.18) |
| $\Omega^-_{bb}$ | 0.04 (-0.08; 0.12) | 0.14 (-0.06; 0.20) | 0.10 (-0.08; 0.18) |
| $\Xi^+_{cb}$ | 1.52 (0.002; 1.52) | 0.75 (0.001; 0.75) | 1.49 (0; 1.49) |
| $\Xi^0_{cb}$ | -0.76 (0.002, -0.76) | -0.38 (0.001; -0.38) | -0.74 (0; -0.74) |
| $\Xi^+_{cb}$ | -0.12 (0.24; -0.36) | 0.18 (0.42; -0.24) | -0.29 (0.21; -0.50) |
| $\Xi^0_{cb}$ | 0.42 (0.24; 0.18) | 0.54 (0.42; 0.12) | 0.46 (0.21; 0.25) |
| $\Omega^0_{cb}$ | -0.61 (0.002; -0.61) | -0.26 (0.001; -0.26) | -0.55 (0; -0.55) |
| $\Omega^+_{cb}$ | 0.45 (0.25; 0.20) | 0.50 (0.42; 0.08) | 0.39 (0.21; 0.18) |
| $\Omega^+_{cc}$ | 0.53 (0.02; 0.51) | 0.14 (0.02; 0.12) | 0.51 (0.02; 0.49) |
| $\Omega^0_{cc}$ | -0.20 (-0.08; -0.12) | -0.13 (-0.05; -0.08) | -0.20 (-0.08; -0.12) |
Table 7. Heavy baryon wave functions and magnetic moments in the nonrelativistic quark model, where \( q, q' = u \) or \( d \) and \( Q, Q' = c \) or \( b \).

| Baryon          | Wave function                          | Magnetic moment                     |
|-----------------|----------------------------------------|-------------------------------------|
| \( \Lambda_{Q(ud)} \) | \( \frac{1}{\sqrt{2}} Q(ud - du) \chi_A \) | \( \frac{e_Q}{2m_Q} \)            |
| \( \Xi Q(qs) \)  | \( \frac{1}{\sqrt{2}} Q(qs - sq) \chi_A \) | \( \frac{e_Q}{2m_Q} \)            |
| \( \Sigma Q(qq') \) | \( \frac{1}{\sqrt{2}} Q(qq' + q'q) \chi_S \) | \( \frac{e_Q}{6m_Q} + \frac{e_q}{3m_Q} + \frac{e_{q'}}{3m_{q'}} \) |
| \( \Omega Q_{(ss)} \) | \( Qss \chi_S \)                       | \( -\frac{e_Q}{6m_Q} + \frac{2e_s}{3m_s} \) |
| \( \Xi s(QQ') \)  | \( \frac{1}{\sqrt{2}} s(QQ' + Q'Q) \chi_S \) | \( -\frac{e_q}{6m_q} + \frac{e_{Q}}{3m_{Q}} + \frac{e'_{Q}}{3m_{Q}} \) |
| \( \Omega s(QQ) \) | \( sQQ \chi_S \)                       | \( -\frac{e_s}{6m_s} + \frac{2e_{Q}}{3m_{Q}} \) |
| \( \Xi q(cb) \)   | \( \frac{1}{\sqrt{2}} q(cb - bc) \chi_A \) | \( \frac{e_q}{2m_q} \)            |
| \( \Xi s(cb) \)   | \( \frac{1}{\sqrt{2}} q(cb + bc) \chi_S \) | \( -\frac{e_s}{6m_s} + \frac{e_c}{3m_c} + \frac{e_b}{3m_b} \) |
| \( \Omega s(cb) \) | \( s(cb - bc) \chi_A \)               | \( \frac{e_s}{2m_s} \)            |
| \( \Omega s(cb) \) | \( s(cb + bc) \chi_S \)               | \( -\frac{e_s}{6m_s} + \frac{e_c}{3m_c} + \frac{e_b}{3m_b} \) |
| \( \Omega s(bc) \) | \( bbc \chi_S \)                      | \( -\frac{e_b}{6m_b} + \frac{2e_c}{3m_c} \) |
| \( \Omega s(bb) \) | \( cbb \chi_S \)                      | \( -\frac{e_c}{6m_c} + \frac{2e_b}{3m_b} \) |
Fig. 1 Baryon mass operator

Fig. 2 Coupling vertex of baryon, photon and three quarks

Fig. 3 Diagrams contributing to the baryon electromagnetic vertex function: triangle (a), bubble (b) and (c) diagrams.