COMPUTATIONAL STUDY OF IMMERSED BOUNDARY - LATTICE BOLTZMANN METHOD FOR FLUID-STRUCTURE INTERACTION

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Abstract. In this article, we deal with the numerical immersed boundary-lattice Boltzmann method for simulation of the fluid-structure interaction problems in 2D. We consider the interaction of incompressible, Newtonian fluid in an isothermal system with an elastic fiber, which represents an immersed body boundary. First, a short introduction to the lattice Boltzmann and immersed boundary method is presented and the combination of these two methods is briefly discussed. Then, the choice of the smooth approximation of the Dirac delta function and the discretization of the immersed body is discussed. One of the significant drawbacks of immersed boundary method is the penetrative flow through the immersed impermeable boundary. The effect of the immersed body boundary discretization is investigated using two benchmark problems, where an elastic fiber is deformed. The results indicate that the restrictions placed on the discretization in literature are not necessary.

1. Introduction. In the last decades, the combination of the immersed boundary method (IBM) [16] and the lattice Boltzmann method (LBM) [9] arose as an efficient numerical method for simulating fluid-structure interaction problems. LBM is a numerical method which is based on the mesoscopic description of the fluid dynamics using discrete probability density functions, see [13]. Although LBM deals with the mesoscopic description, it can be shown that LBM can be used to solve Navier-Stokes equations [18]. The advantage of LBM lies in the algorithm, which can be massively parallelized. Because of its efficiency, LBM has rapidly evolved during the last years and several submethods emerged such as SRT-LBM1 [9], MRT-LBM2 [5], CLBM3 [7], CuLBM4 [8], ELBM5 [3], KBC6 [11] etc.

2010 Mathematics Subject Classification. Primary: 76D99, 65N99; Secondary: 74B20.
Key words and phrases. Cascaded lattice Boltzmann method, immersed boundary method, penalty immersed boundary method, computational study, Lagrangian point spacing.
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1LBM with single relaxation time
2LBM with multiple relaxation time
3Cascaded LBM
4Cumulant LBM
5Entropic LBM
6Entropic multi-relaxation time LBM
In principle, IBM deals with the interaction between the Eulerian and the Lagrangian coordinate systems, where the former is used to describe the fluid flow dynamics while the latter allows to deal with the evolution of the (elastic) immersed body boundary, see [16]. The principal advantage of this description is the possibility of describing complex boundaries accurately. On the other hand, one of the significant drawbacks of IBM is the penetrative flow through the immersed impermeable boundary, which influences the resulting behavior of the immersed boundary. There are several aspects that cause the penetration, e.g., the discretization of the immersed boundary, the approximative model of the interaction, etc.

The present study aims to determine the effect of the immersed boundary discretization on the numerical solution. To prevent the penetrative flow, various implicit models for the force computation were suggested, see [20, 10]. On the other hand, the implicitness of those methods often brings several drawbacks. First, it increases the computational time and second, implicit models usually depend on the numerical model for the fluid simulation and, thus, it can be difficult to combine them with an arbitrary numerical method for the fluid flow simulation, in our case with LBM. Another way, how to ensure impermeability of the immersed boundary, is by increasing the number of discrete Lagrangian points. In [16], a condition which imposes the upper bound on the Lagrangian node spacing is formulated as

\[
\left| \bar{X}_a - \bar{X}_b \right| < \frac{h}{2},
\]

where \( \bar{X}_a \) and \( \bar{X}_b \) denote the Lagrangian coordinates of two neighboring nodes discretizing the immersed body boundary and \( h \) [m] is the Eulerian uniform grid spacing parameter. Since the computational cost increases with the number of discrete Lagrangian points, there is a question, whether the fulfillment of the condition given by Eq. (1) is necessary. In other words, can satisfactory results be obtained with a lower number of discrete Lagrangian points?

In [6], a computational study showed that the condition given by Eq. (1) is not necessary if the immersed body is solid and immobile. It was shown that if the distance between neighboring Lagrangian nodes is close to \( h \), sufficiently accurate results can be obtained. Similarly to the study of spherical capsules deformation in [14], we investigate the influence of the Lagrangian point discretization on the numerical results. For this, we choose three test problems to demonstrate that similar results can also be obtained for the case of an elastic mobile immersed boundary.

To achieve good computational stability of LBM in 2D, CLBM with the D2Q9 velocity model is used. The CLBM algorithm is efficiently implemented in parallel on GPU\(^7\). The interaction of the fluid with the elastic body with mass is modeled using PIBM\(^8\) proposed in [12], which is explicit in time.

2. Mathematical and numerical model.

2.1. Mathematical model. The flow of an incompressible, Newtonian fluid in a domain \( \Omega \subset \mathbb{R}^2 \) within a time interval \([0, T]\) can be described by the following set of partial differential equations [21]:

\[
\rho(\bar{x}, t) \left( \frac{\partial \bar{u}(\bar{x}, t)}{\partial t} + \bar{u}(\bar{x}, t) \cdot \nabla \bar{u}(\bar{x}, t) \right) + \nabla p(\bar{x}, t) = \mu \Delta \bar{u}(\bar{x}, t) + g(\bar{x}, t),
\]

\(\Omega\) is the computational domain, \(\rho\) is the fluid density, \(\bar{u}\) is the fluid velocity, \(p\) is the pressure, \(\mu\) is the fluid viscosity, and \(g\) is the external body force, respectively. The transport of the immersed body boundary is described by a system of partial differential equations [16].

\(^7\)Graphic Processing Unit
\(^8\)penalty IBM
\[ \nabla \cdot \bar{u}(\bar{x}, t) = 0, \quad (2b) \]

where \( \bar{x} \) [m], \( \bar{\Omega} \) is the position vector, \( t \) [s], \( t \in [0, T] \) is the time, \( \rho \) [kg m\(^{-3}\)] is the fluid density, \( \bar{u} \) [m s\(^{-1}\)] is the fluid velocity, \( p \) [kg m\(^{-1}\)s\(^{-2}\)] is the fluid pressure, \( \mu \) [kg m\(^{-1}\)s\(^{-1}\)] is the dynamic viscosity and \( \bar{g} \) [kg m\(^{-2}\)s\(^{-2}\)] is the volumetric force acting on the fluid.

The elastic fiber immersed in \( \bar{\Omega} \) is represented by a domain \( \Gamma \) and its dynamics is described by the following partial differential equation \([4]\)

\[ \frac{\partial^2 \bar{X}(l, t)}{\partial t^2} + k_B \frac{\partial^4 \bar{X}(l, t)}{\partial l^4} - k_S \frac{\partial}{\partial l} \left[ \left( 1 - \left\| \frac{\partial \bar{X}(l, t)}{\partial l} \right\|^2 \right)^{-1} \frac{\partial \bar{X}(l, t)}{\partial l} \right] = \bar{F}(l, t), \quad (3) \]

where \( \bar{X}(l, t) \in \Gamma \) is the Lagrangian point with the initial arc-length coordinate \( l \) [m], \( \varrho \) [kg m\(^{-2}\)] is the (constant) density of the elastic fiber, \( k_B \) [kg m\(^2\)s\(^{-2}\)] is the bending coefficient, \( k_S \) [kg s\(^{-2}\)] is the stiffness coefficient and \( \bar{F} \) [kg m\(^{-1}\)s\(^{-2}\)] is the force acting on the fiber.

In order to transfer variables from Lagrangian to Eulerian coordinates and vice versa, a simple layer and a convolution with Dirac delta function are used, respectively. If arbitrary variables \( g_b \) in the Lagrangian coordinates and \( g \) in the Eulerian coordinates are denoted, they satisfy for some arbitrary test function \( \vartheta \) \([16, 24]\)

\[ (g_b, \vartheta) = (g(\bar{x}) \ast \delta(\bar{x} - \bar{X}), \vartheta), \quad (4a) \]

\[ (g, \vartheta) = (g_{\delta_{\Gamma}}, \vartheta) = \int_{\Gamma} g_b(\bar{X}(l)) \vartheta(\bar{X}(l)) d\bar{X}(l), \quad (4b) \]

where \( g_{\delta_{\Gamma}} \) denotes the simple layer \([24]\). Note that the arguments in Eqs. \((4a)\) and \((4b)\) are given here to emphasize the corresponding coordinate system for respective quantities.

2.2. **Numerical model.** Let the computational domain \( \Omega \) be rectangular and discretized by a regular mesh

\[ \hat{\Omega} = \left\{ \bar{x}_{i,j} = (ih, jh)^T \mid i \in \{1, \ldots, N_x\}, j \in \{1, \ldots, N_y\} \right\}, \]

where \( h \) is the distance parameter between neighboring discrete Eulerian points and \( N_x, N_y \in \mathbb{N} \) define the number of discrete Eulerian points in the \( x \) and \( y \) direction, respectively. Let the elastic fiber \( \Gamma \) be discretized by a finite set of Lagrangian points

\[ \hat{\Gamma} = \left\{ \bar{X}_l = \bar{X}(\sigma l) \mid l \in \{1, \ldots, N_l\} \right\}, \]

where \( \sigma \) denotes the initial arc-length between neighboring discrete Lagrangian points at time \( t = 0 \) and \( N_l \in \mathbb{N} \) is the total number of discrete Lagrangian points discretizing \( \Gamma \).

According to \([16]\) and \([24]\), the formulae \((4a)\) and \((4b)\) are discretized as follows:

\[ g_b(\bar{X}_l) \approx \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} g(\bar{x}_{i,j}) D(\bar{x}_{i,j} - \bar{X}_l) h^2, \quad (5a) \]

\[ g(\bar{x}_{i,j}) \approx \sum_{l=1}^{N_l} g_b(\bar{X}_l) D(\bar{x}_{i,j} - \bar{X}_l) \Delta s, \quad (5b) \]
where $D$ is the smooth approximation of the Dirac delta function and $\Delta s$ is the arc-length around $\vec{X}_l$. Based on [16], $D(\vec{z}) = \frac{1}{h^2} \phi(z_1)\phi(z_2)$, where $\vec{z} = (z_1, z_2)^T$ and $\phi$ denotes some one-dimensional approximation of the Dirac delta function. Based on [6, 25], we consider

$$\phi(r) = \begin{cases} 
0 & \text{for } |r| \geq 2, \\
\frac{1}{h^2} \left( 5 - 2|r| - \sqrt{-7 + 12|r| - 4r^2} \right) & \text{for } 2 \leq |r| \leq 1, \\
\frac{1}{h^2} \left( 3 - 2|r| + \sqrt{1 + 4|r| - 4r^2} \right) & \text{for } 1 \leq |r| \leq 0,
\end{cases} \quad (6)$$

for $r \in \mathbb{R}$.

2.2.1. Lattice Boltzmann method. The lattice Boltzmann method describes the fluid dynamics using the discrete probability density functions $f_k$, $k = 0, 1, \ldots, q - 1$, where $q$ corresponds to the total number of discrete directions, in which the information can propagate [13]. Here, we consider $q = 9$. The evolution of the discrete probability density functions $f_k$, $k = 0, 1, \ldots, 8$, is given by the discrete Boltzmann transport equation

$$f_k(\vec{x}_{i,j} + \Delta t \vec{\xi}_k, t + \Delta t) - f_k(\vec{x}_{i,j}, t) = C_k(\vec{x}_{i,j}, t) + S_k(\vec{x}_{i,j}, t), \quad \forall \vec{x}_{i,j} \in \Omega, \ \forall t \in [0, T], \quad (7)$$

where $\Delta t$ is the discrete time step, $C_k$ is the discrete collision operator, $S_k$ is the discrete forcing term, and $\vec{\xi}_k$ is the discrete microscopic velocity given by,

$$\vec{\xi}_k = \begin{cases} 
(0, 0)^T & \text{for } k = 0, \\
(1, 0)^T, (0, 1)^T, (-1, 0)^T, (0, -1)^T & \text{for } k = 1, 2, 3, 4, \\
(1, 1)^T, (-1, 1)^T, (-1, -1)^T, (1, -1)^T & \text{for } k = 5, 6, 7, 8,
\end{cases} \quad (8)$$

see [19]. Note that all arguments in Eq. (7) are non-dimensional.

Based on the functions $f_k$, the information about the fluid properties such as the fluid density and the fluid velocity can be recovered as:

$$\rho = \sum_{k=0}^{8} f_k, \quad (9a)$$

$$\rho \vec{u} = \sum_{k=0}^{8} f_k \vec{\xi}_k + \frac{\Delta t}{2} \vec{g}. \quad (9b)$$

Here, we use the CLBM approach for $C_k$ based on [7, 17] and the model for $S_k$ reported by [17].

2.2.2. Penalty immersed boundary method. Based on [16], IBM considers variations in the fluid density caused by the presence of the immersed body. However, LBM is sensitive to significant variations in the fluid density and, therefore, it is difficult to couple IBM and LBM together directly. To overcome the effects of the variable fluid density, the penalty immersed boundary method (PIBM) was proposed in [12] that assumes two bodies in the fluid which are mutually connected. The first body is the original body interacting with the fluid but considered as mass-less. The second, also called a ghost body, has a mass but does not directly interact with the fluid. In order to add inertial forces of the body to the fluid model, additional force
term $\vec{F}_I$ is added, which describes the interaction between the ghost body and the immersed body. The formula for $\vec{F}_I$ is given by

$$\vec{F}_I(\vec{X}_l(t), t) = K \left[ \vec{Y}(\sigma_l, t) - \vec{X}_l(t) \right], \quad l = 1, 2, \ldots, N_l,$$

where $K$ [kg m$^{-2}$ s$^{-2}$] is the PIBM stiffness parameter, $\vec{X}_l(t)$ [m] is the discrete Lagrangian point describing the original body and $\vec{Y}(\sigma_l, t)$ [m] is the discrete Lagrangian point describing the ghost body. The fluid-body interaction is governed by Eq. (3) with $\varrho = 0$, the evolution of the ghost filament is given by

$$\varrho \frac{\partial^2 \vec{Y}(\sigma_l, t)}{\partial t^2} = -\vec{F}_I(\vec{X}_l(t), t),$$

and the evolution of the original body is then treated by

$$\frac{\partial \vec{X}(\sigma_l, t)}{\partial t} = \vec{U}(\sigma_l, t), \quad l \in \{1, 2, \ldots, N_l\},$$

where the velocity $\vec{U}$ is obtained from the convolution given by Eq. (4a) in discrete form. Eq. (12) is discretized using either the forward Euler scheme, or the third order Runge-Kutta scheme, or the second order Adams-Bashforth scheme [1].

Note that the parameter $K$ should be chosen sufficiently large to approximate the inertial forces as much as possible [12].

2.3. Implementation remarks. The LBM algorithm allows for massive parallel implementation on GPUs, c.f. [23]. The LBM-PIBM numerical scheme is implemented in C++ with the support of the CUDA software architecture that allows using GPUs for parallel computations. The LBM part of LBM-PIBM can be efficiently implemented on GPU and the rest of the algorithm is treated by CPU which involves relatively slow communication between CPU and GPU in every time step. All results were obtained using our “in-house” code and for the simulation presented in this paper, GPUs Nvidia Tesla P100, Nvidia Tesla K40, Nvidia Quadro P6000, and Nvidia Tesla V100 were used.

3. Results and discussion. In this section, three benchmarks problems in 2D are used to investigate the influence of the immersed body discretization on the penetrative flow. The first problem in Section 3.1, inspired by [22], is used to validate our implementation of PIBM-LBM.

Then, the problems in Sections 3.2 and 3.3, inspired by [2] and medical applications, investigate the influence of $\sigma/h$ on the immersed fiber penetration, where

$$\sigma = \max \left\{ \left| \vec{X}_l - \vec{X}_{l+1} \right| : l = 1, 2, \ldots, N_l - 1 \right\}.$$  

3.1. Flapping fiber in the Kármán vortex street region. The first computational study investigates the problem of the elastic fiber deformation in the Kármán vortex street region. The reason for this study is the justification of our implementation through a comparison with the results in [22].

Let $\Omega$ be a rectangular domain with dimensions shown in Fig. 1. On the boundary $\partial \Omega_{in}$, a parabolic velocity profile is prescribed with the maximum velocity $U = 50$ m s$^{-1}$. The free stream boundary condition is prescribed on the boundary $\partial \Omega_{out}$ and a zero velocity condition is prescribed on the remaining boundary $\partial \Omega_{z}$. $\Omega$ is discretized by a regular mesh $\hat{\Omega}$ described by Eq. (2.2) with the mesh parameter $h = 1/700$ m.
Figure 1. The setup of the computational domain in the case of the Kármán vortex flow problem. Two obstacles are immersed in the fluid. The first one is a rigid cylinder with diameter $d$. The last one is an elastic fiber with one fixed end (filled circle) and one free end (empty circle).

In $\Omega$, two obstacles are placed as depicted in Fig. 1. The first obstacle is a rigid cylinder with a diameter $d = 0.1 \text{ m}$. The second obstacle is an elastic fiber with one fixed and one free end. The fixed end is placed at a distance $G_b = 3d$ from the center of the cylinder. The length of the fiber is $L_b = 4d$.

In the numerical model, bounce back boundary condition [13] is used to simulate the interaction with the cylindrical obstacle. The deformation of the elastic fiber is determined by Eq. (3) with $k_S = 10^4 \text{ kg s}^{-2}$, $k_B = 10^{-1} \text{ kg m}^2 \text{ s}^{-2}$, $\varrho = 0.1 \text{ kg m}^{-2}$, $K = 10^6 \text{ kg m}^{-2} \text{ s}^{-2}$ and $\sigma = 2.5 \cdot 10^{-2} \text{ m}$. For the fluid, we set $\nu = 10^{-2} \text{ m}^2 \text{ s}^{-1}$ and $\nu_{LB} = 10^{-3}$. The final time is $T = 2 \text{ s}$.

In the following, the trajectory of the fiber’s free end is investigated based on the numerical method used for the solution of the evolution Eq. (11). We used either the forward Euler method or the third order Runge-Kutta method. The results with the trajectory of the free end are presented in Figs. 2a and 2b. The trajectory of the whole fiber is demonstrated in Fig. 2c. The results are comparable with the results in [22]. Both methods (forward Euler and the third order Runge-Kutta) give similar results. In Figs. 2a and 2b, the trajectories contain small oscillations (i.e., the trajectory appears as “thick”) that are caused by the inertial forces modeled by PIBM. Without the inertial forces, the trajectory is “thin” as shown in Figs. 3a and 3b and the trajectory of the whole fiber is demonstrated again in Fig. 3c.

From the previous results, we can conclude that our implementation gives similar results compared to the results given in [22] and that the inclusion of the inertial forces into our model with PIBM caused small oscillations in the free end trajectory.

3.2. Cavity flow deformation. A square domain $\Omega$ is considered as shown in Fig. 4. On the top boundary $\partial \Omega_t$, according to [26], the Dirichlet boundary condition for the fluid velocity is prescribed as $\vec{u}_{in} = (u_{in}, 0)^T$, where

\[
\begin{align*}
u_{in}(x) = \begin{cases} 
\sin^2 \left( \frac{\pi x}{0.3} \right) & \text{for } x \in [0, 0.3], \\
1 & \text{for } x \in [0.3, 1.7], \\
\sin^2 \left( \frac{\pi(x-2)}{1.7} \right) & \text{for } x \in [1.7, 2],
\end{cases}
\end{align*}
\]
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(a) Forward Euler method. 

(b) The third order Runge-Kutta method.

(c) A superposition of many flexible filaments in various times obtained by the forward Euler method.

Figure 2. The study of the elastic fiber deformation in the Kármán vortex street region. In Figs. 2a and 2b, the free end trajectory is shown for two methods: (A) Forward Euler method (red), (B) The third order Runge-Kutta method (blue), and the results are compared with those in [22] (black circles). In Fig. 2c, the superposition of the elastic fibers is shown.

with units [cm s$^{-1}$]. On the remaining boundaries of $\Omega$, zero velocity Dirichlet boundary conditions are prescribed. In the fluid domain $\Omega$, the elastic fiber $\Gamma$ is initially tightened parallel to the x-axis with fixed ends as shown in Fig. 4.

To investigate the influence of the immersed body discretization on the numerical solution, the relative kinetic energy loss function $E_{rkl}$ is introduced in the form

$$E_{rkl}(t) = \frac{\int_{\Omega_u} \frac{1}{2} \| \tilde{u}(t) \|^2 d\vec{x}}{\int_{\Omega} \frac{1}{2} \| \tilde{u}(t) \|^2 d\vec{x}},$$

where $\Omega_u$ denotes the part of $\Omega$ below the elastic fiber $\Gamma$ for all $t \in [0, T]$. For the numerical approximation of Eq. (12), Adams-Bashforth scheme is used.

The physical properties are chosen as follows: $\nu = 2 \cdot 10^{-5}$ m$^2$s$^{-1}$, $k_B = 10^{-7}$ kg m$^2$s$^{-2}$, $K = 10^5$ kg m$^{-2}$s$^{-2}$ and $\varrho = 0.1$ kg m$^{-3}$. For the fiber stiffness, four different values are considered: (A) $k_S = 100$ kg s$^{-2}$, (B) $k_S = 10$ kg s$^{-2}$, (C) $k_S = 1$ kg s$^{-2}$, and (D) $k_S = 0.1$ kg s$^{-2}$. The LBM non-dimensional viscosity is $\nu_{LB} = 0.01$. The final time is $T = 10$ s when the steady state is reached.

In Fig. 5, the evolution of $E_{rkl}$ is shown with respect to $\sigma/h$. As expected, the values of $E_{rkl}$ are decreasing as $\sigma/h \to 0+$ for all choices of $k_S$. The penetrative flow through the fiber $\Gamma$ causes larger values of $E_{rkl}$, which is illustrated in Fig. 6, where fluid velocity vectors and the fluid velocity magnitude $\| \tilde{u} \|$ [cm s$^{-1}$] are shown. For
(a) Forward Euler method. (b) The third order Runge-Kutta method.

(c) A superposition of many flexible filaments in various times with neglected inertial forces obtained by the forward Euler method.

Figure 3. The study of the elastic fiber deformation in the Kármán vortex street region with neglected inertial forces of the elastic fiber. In Figs. 3a and 3b the free end trajectory is shown for two methods: (A) Forward Euler method (red), (B) The third order Runge-Kutta method (blue), and the results are compared with results in [22] (black triangles). In Fig. 3c, the superposition of the elastic fibers is shown.

Figure 4. The setup of the computational domain for the cavity flow problem.

large $\sigma/h$ in Figs. 6a and 6b, the flow penetrates $\Gamma$ as depicted by the fluid velocity vector field. For moderate $\sigma/h \leq 2$, $\Gamma$ behaves as almost impermeable as shown in Figs. 5 and Figs. 6c. From the previous results in Fig. 5, we can conclude that if $\sigma/h \approx 2$, satisfactory results can be obtained, and, thus the condition given by
Eq. (1) is not necessary. Furthermore, the discrete number of Lagrangian nodes influence the resulting deformation of the elastic fiber as shown in Fig. 7.

For $\sigma/h \leq 1/2$, small oscillations occur in the numerical results. These oscillations indicate that the choice of the time step $\Delta t$ is beyond the Courant–Friedrichs–Lewy stability threshold for the explicit finite difference scheme [15]. However, smaller time steps $\Delta t$ produce numerical noise (or instabilities) in the LBM numerical scheme since for a fixed Eulerian parameter $h$, the physical time step $\Delta t$ is directly proportional to the LBM viscosity $\nu_{LB}$ and proportional to the relaxation time $\tau$ [13].

3.3. Elastic bump deformation. The last computational study is inspired by the blood flow through stenotic vessels, i.e., vessels with abnormal narrowings. We propose the following test problem in a rectangular computational domain $\Omega$ with a semi-circular bump mimicking an elastic narrowing of a vessel as shown in Fig. 8.

The dimensions of $\Omega$ are $H = 0.04$ m and $W = 0.016$ m. On the boundary $\partial\Omega_{in}$, uniform velocity $U_\infty$ is prescribed, on $\partial\Omega_{out}$, free outflow condition is prescribed, and on $\partial\Omega_z$, no-slip boundary condition is prescribed using the bounce-back boundary condition [13]. Then, the elastic bump represented by $\Gamma$ is a semi-circle with the center in $C = [0.08, 0]$ m and radius $R = 0.015$ m. The kinematic viscosity is $\nu = 3.8 \cdot 10^{-6}$ m$^2$s$^{-1}$ and the LBM kinematic viscosity is $\nu_{LB} = 10^{-4}$. Two values

![Graphs showing the evolution of $E_{rkl}$ w.r.t. $\sigma/h$ for different values of the stiffness coefficient $k_S$ and for different mesh spacings $h$: $h_1 = 1.6 \cdot 10^{-4}$ m, $h_2 = 7.84 \cdot 10^{-5}$ m, and $h_3 = 5.24 \cdot 10^{-5}$ m. The results computed at the final time $T = 10$ s when the steady state is reached.](image-url)
Figure 6. Schematic visualization of the fluid penetration through the flexible fiber for three values of $\sigma/h$. In the figures, the velocity vectors (white arrows) and values of the fluid velocity magnitude (color scale) are shown. The results are at the final time $T = 10$ s when the steady state is reached. All results are computed using $k_S = 100$ kg s$^{-2}$ and $h = 5.24 \times 10^{-5}$ m.
of $U_\infty$ are considered, $U_\infty = 0.5 \text{ m s}^{-1}$ and $U_\infty = 0.05 \text{ m s}^{-1}$. For the numerical approximation of Eq. (12), the Adams-Bashforth scheme is used. We investigate the permeability of the boundary $\Gamma$ using relative kinetic energy lost function defined in Eq. (15), where $\Omega_u = \Omega_u(t)$ denotes the interior of the semi-circle for all $t \geq 0$. Additionally, the surface area of $\Omega_u$ is also investigated. If $\sigma/h$ is too large ($\sigma/h > 2$), the penetrative flow through the boundary $\Gamma$ causes the bump to collapse as shown in Figs. 9a, 9b, where the surface area of $\Omega_u$ is very small or almost zero. As further shown in Figs. 9a and 9b, a finer Lagrangian mesh discretization is required for larger $U_\infty$ to achieve similar sufficient surface area values.

**Figure 7.** Schematic visualization of the fiber deformation at the final time $T = 10$ s and for $h = 5.24 \cdot 10^{-5}$ m. Different colors of the fibers represent different values of $\sigma/h$. 

\[ k_S = 100 \text{ kg s}^{-1} \]
\[ k_S = 10 \text{ kg s}^{-1} \]
\[ k_S = 1 \text{ kg s}^{-1} \]
\[ k_S = 0.1 \text{ kg s}^{-1} \]
Similar to the previous test case described in Section 3.2, the relative kinetic energy lost $E_{rkl}$ can be used to indicate undesired permeability of the Lagrangian discretization. As shown in Figs. 9c, 9d, and 10, the values of $E_{rkl}$ decrease as $\sigma/h$ decreases and the loss of the kinetic energy can be considered sufficiently small for $\sigma/h < 2$. For $U_\infty = 0.5 \, \text{m s}^{-1}$, the $E_{rkl}$ was time averaged over the time interval of $(1, 9.5) \, \text{s}$ and is labeled as $E_{rkl}$.

Finally, we can again conclude, that satisfactory results can be obtained for $\sigma/h \in (0.5, 1)$ and for higher Reynolds number, finer Lagrangian mesh is needed to moderate the penetrative flow through the boundary.

Figure 8. The setup of the computational domain in the case of elastic bump deformation.

4. Conclusion. The principal aim of this work was to investigate the condition (1) proposed by [16] in the case of PIBM for the fluid interaction with the elastic body and how the density of the immersed body boundary discretization influences the numerical. Three benchmark tests were used to investigate the influence of the immersed body discretization.

The first benchmark problem deals with the elastic fiber deformation in the Kármán vortex street region behind a cylindrical obstacle inspired by [22]. In this problem, two different numerical methods for the numerical solution of Eq. (11) were considered. Both methods gave similar results. Next, the influence of the addition of inertial forces through the penalty IBM was tested and showed that small oscillations appeared in the numerical solution. These cases were demonstrated in Figs. 2 and 3.

The second benchmark problem, inspired by [26], represents the elastic fiber deformation in the fluid domain with the cavity flow. The relative kinetic energy loss function $E_{rkl}$ was evaluated with respect to $\sigma/h$ for four different values of the stiffness coefficient $k_S$. For the values of $\sigma/h \geq 10$, significant penetration was observed, which was illustrated also in Fig. 6. On the other hand, if $\sigma/h \leq 2$, no penetration was observed in all cases.

The last benchmark problem studies the deformation of an elastic bump with respect to $\sigma/h$ for two values of $U_\infty$. It was found that the penetration depends both on the ratio $\sigma/h$ and $U_\infty$. For higher $U_\infty$, a finer Lagrangian mesh is needed. Nevertheless, in both cases, if $\sigma/h \leq 2$, satisfactory results are obtained.

To conclude, the numerical method provides comparative results to the results shown in [22] and that the condition given by Eq. (1) proposed in [16] is not a necessary condition in the case of simulations of the fluid-elastic body interaction.
Figure 9. Investigation of fluid flow around the bump. In Figs. (A) and (B), the relation between the surface area $\mu(\Omega_u)$ and $\sigma/h$ is shown. Figs. (C) and (D) illustrate the evolution of $E_{rkl}$ with respect to $\sigma/h$. For $U_\infty = 0.5$ m s$^{-1}$, the values are time-averaged over the time interval of (1, 9.5) s. For $U_\infty = 0.5$ m s$^{-1}$, the values are computed at the final time $T = 10$ s.

Acknowledgments. The work was supported by the Czech Science Foundation project No. 18-09539S, by the project No. 15-27178A of the Ministry of Health of the Czech Republic, and by project Research Center for Informatics No. CZ.02.1.01/0.0/0.0/16_019/0000765. The authors thanks Ing. Jakub Klinkovský for the arrangement of computational resources and gratefully acknowledge the support of NVIDIA Corporation with the donation of the Quadro P6000 GPU used for this research.

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Figure 10. Schematic visualization of the fluid flow around an impermeable flexible fiber in time $t = 9.5\, \text{s}$. In the figures, the velocity vectors (white arrows) and the values of the fluid velocity magnitude (color scale) are shown.

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Received January 2019; revised December 2019.

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