Shedding light on electroweak phase transition from the exotic Higgs decays at the lifetime frontiers

Wei Liu\textsuperscript{a} Aigeng Yang\textsuperscript{b} Hao Sun\textsuperscript{b}

\textsuperscript{a}Department of Applied Physics, Nanjing University of Science and Technology, Nanjing, 210094, P.R.China
\textsuperscript{b}Institute of Theoretical Physics, School of Physics, Dalian University of Technology, No.2 Ling-gong Road, Dalian, Liaoning, 116024, P.R.China

E-mail: wei.liu@njust.edu.cn, aigengyang@mail.dlut.edu.cn, haosun@dlut.edu.cn

ABSTRACT: We study the scenarios where a strongly first-order electroweak phase transition (EWPT) is triggered by a light singlet scalar, which has feeble interactions to the Higgs. Since the singlet scalar is light and has weak couplings, it can decay at a macroscopic distance away from the collision point. Therefore, it can be regarded as a long-lived particles (LLP) in such scenarios. We perform the searches of the LLPs from the exotic Higgs decays, at the FASER, MAPP and CMS-Timing detectors of the 14 TeV HL-LHC, to probe the SFOEWPT. In certain scenarios, we show that the LLP searches can help to reach the parameter space of the strongly first-order EWPT remarkably, where the searches for promptly exotic Higgs decays are not valid.
1 Introduction

After the discovery of the Higgs boson at the LHC [1, 2], the pattern of the electroweak phase transition (EWPT) is still unknown, remained as one of the most important problems of the particle physics. Although the Standard Model predicts a smooth crossover of the EWPT in the early universe [3–5], there is another intriguing possibility that the EWPT can be strongly first-order (SFOEWPT). A SFOEWPT is indeed interesting, as it can accommodate the observation of the baryon asymmetry of the Universe via the electroweak baryogenesis mechanism (EWBG) [6–8].

In general, there are two ways to probe the EWPT experimentally, either via the phenomenology of the Higgs related physics at the colliders, or the primordial stochastic gravitational waves (GW) generated during the EWPT at the GW detectors [9–13]. Many studies have been done, either via focusing on one of the phenomena or comparing the both, based at different new physics models beyond the SM, including the real singlet extended SM [14–30], two-Higgs-doublet model [31–44], left-right symmetric model [45, 46], Georgi-Machacek model [47], composite Higgs models [48–56], as well as SUSY models [57], etc. Among these studies, Ref. [29] investigates the complementarity of the proposed multi-TeV muon colliders and the LISA GW detectors to the first-order EWPT based at the real singlet extended SM. In most cases, the mass of the BSM particles responsible for the interactions driving a SFOEWPT is taken to be larger than the electroweak scale.

As motivated by Ref. [15, 26, 30, 58], a light BSM particle with masses below the electroweak scale can also trigger a SFOEWPT. If its mass is below $m_Z$, it must be a singlet-like scalar (we denote the mass eigenstate as $h_1$) as argued by Ref. [58].
production of such scalar is controlled by the Higgs mixings $\theta$, which is tiny at the current limits via the Higgs measurements and exotic Higgs searches at the LHC and LEP [59], such as $\theta \lesssim 10^{-2}$. Nonetheless, a first-order EWPT scenario can lead to appreciable direct couplings between the singlet-like scalar and SM Higgs, results in sizeable $h_2 \to h_1 h_1$ branching ratio, where the $h_2$ is the mass eigenstate of the observed Higgs. Therefore, promptly exotic Higgs decay searches at the current LHC as well as the future Higgs machines can be used to probe the EWPT as summarised in Ref. [30]. There are still plenty of parameters open for the SFOEWPT, and the promptly exotic Higgs decay searches exclude the scenarios where $m_{h_2} \gtrsim 30$ GeV at the current LHC.

In search of additional sensitivity, one can consider another possible signature of their final states. Searches for the long-lived particles (LLP) is a hot topic in recent years. Specified detectors at the lifetime frontier for this purpose are proposed and designed, in which the FASER [60] and MoEDAL-MAPP (MAPP) [61], as well as the precision timing detector at the CMS (CMS-Timing) [62, 63] are already in installation. Benefited from the low SM background, they can probe new physics with high sensitivity. Studies about the possible candidates of the LLPs at the LHC is widely discussed, including the axion, heavy neutral lepton, dark photon as well as the light scalar, etc (see Ref. [64] and references therein). Ref. [65–68] treated the heavy neutrinos as the LLP candidate, and performed the detailed analyses of the LLP searches to test the type-I seesaw mechanisms. If the Higgs mixings are much smaller than the current limits, i.e. $\theta \lesssim 10^{-4}$, or the singlet-like scalar is light, it can decay at a macroscopic distance away from the collision point. Hence, the EWPT can also be probed by the searches of the LLPs. A study about the long-lived light singlet scalar via the CMS phase II track trigger has already been done at Ref. [69]. Due to the low SM background, the LLP searches can potentially be sensitive to much lower branching ratio of the $h_2 \to h_1 h_1$, probing much more cases of the EWPT.

This possibility is however rarely been investigated. We therefore focus on the searches of the exotic Higgs decays at the lifetime frontiers, and compare it to the LISA GW detector in probing the EWPT. As the real singlet extended SM is one of the simplest models which can introduce the SFOEWPT, we take it as our benchmark model. The most important features of the SFOEWPT induced by tree level barrier via renormalizable operators [70], is already captured by this model. We perform the studies of the collider phenomenology at the 14 TeV HL-LHC, with an integrated luminosity of 3 $ab^{-1}$, and compare the sensitivity from the searches of the LLP to those from the promptly exotic Higgs decays as summarised in Ref. [30].

This paper is organized in following order. In Section 2, we introduce the real singlet extended SM and the corresponding parameter space for SFOEWPT, as well as its GWs signals at the LISA detector. In Section 3, the phenomenology of the long-lived singlet-like scalar is studied, including its production cross section and decay length, as well as the detailed LLP analyses. The sensitivity to the EWPT is shown in the Section 4, where the projected limits from the promptly exotic Higgs decays at the HL-LHC is also discussed for comparison. In the end, we draw our conclusion in Section 5.
2 FOEWPT triggered by a light singlet scalar

2.1 The real singlet extended SM

In tree level, the scalar potential of the real singlet extended SM can be expressed as

\[ V = -\mu^2|H|^2 + \lambda|H|^4 + \frac{a_1}{2}|H|^2 S + \frac{a_2}{2}|H|^2 S^2 + b_1 S + \frac{b_2}{2} S^2 + \frac{b_3}{3} S^3 + \frac{b_4}{4} S^4, \]  

(2.1)

where \( H \) and \( S \) are the scalar fields for the SM Higgs doublet and singlet respectively. There are eight parameters, while the shift invariance of the singlet potential removes one degree of freedom as we take \( b_1 = 0 \), and the measurements of the Higgs mass and electroweak vacuum expectation value removes two. Therefore, we have five free parameters.

After the electroweak symmetry breaking, we can expand the fields around the vacuum in unitary gauge,

\[ H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad S = v_s + s, \]  

(2.2)

where \( v = 246 \) GeV is the vacuum expectation value (VEV) of the electroweak, and \( v_s \) is the VEV for the singlet. The physical scalar masses can be obtained via diagonalizing the Hessian matrix at the physical VEV,

\[ M_s^2 = \left( \begin{array}{cc} \frac{\partial^2 V}{\partial s^2} & \frac{\partial^2 V}{\partial s \partial h} \\ \frac{\partial^2 V}{\partial s \partial h} & \frac{\partial^2 V}{\partial h^2} \end{array} \right). \]  

(2.3)

Without additional symmetries, the two mass eigenstates are mixed as

\[ h_1 = s \cos \theta + h \sin \theta, \]

\[ h_2 = -s \sin \theta + h \cos \theta, \]  

(2.4)

where we choose \( M_{h_1} < M_{h_2} = M_h^{SM} = 125.09 \) GeV. We are more interested in the low mixing cases, where the \( h_1 \) is mostly singlet-like, and the \( h_2 \) is the observed Higgs. Note that we have defined the mixing angle \( \theta \) inversely comparing to Ref. [58].

As these parameters are directly linked to the experimental observable, we use \( M_{h_1} \) and \( \theta \) as input free parameters. The coefficients \( \mu, b_2, \lambda, a_1 \) and \( v_s \) can be expressed as

\[ \mu^2 = \lambda v^2 + \frac{v_s}{2} (a_1 + a_2 v_s), \]

\[ b_2 = -\frac{1}{4v_s} \left[ v^2 (a_1 + 2a_2 v_s) + 4v_s^2 (b_3 + b_4 v_s) \right], \]

\[ \lambda = \frac{M_{h_1}^{2} \sin^2 \theta + M_{h_2}^{2} c_\theta^2}{2v^2}, \]

\[ a_1 = \frac{4v_s}{v^2} \left[ v_s^2 \left(2b_4 + \frac{b_3}{v_s}\right) - M_{h_1}^{2} c^2_\theta - M_{h_2}^{2} s^2_\theta \right], \]

\[ v_s = \frac{1}{2a_2} \frac{s_\theta}{v} \left( M_{h_1}^{2} - M_{h_2}^{2} \right) - a_1, \]  

(2.5)

where \( s_\theta \) and \( c_\theta \) are in short of \( \sin \theta \) and \( \cos \theta \), respectively.

Therefore, we choose the following five free parameters,

\[ \{ M_{h_1}, \theta, a_2, b_3, b_4 \}. \]  

(2.6)
2.2 Strongly first order electroweak phase transition in the small-mixing limit

The scalar potential in the early universe receives thermal corrections, so Eq. 2.1 becomes

$$V_{\text{eff}} = V + V_T,$$  \hspace{1cm} (2.7)

where

$$V_T = - (\mu^2 - c_H T^2) |H|^2 + \lambda |H|^4 + \frac{a_1}{2} |H|^2 S + \frac{a_2}{2} |H|^2 S^2$$
$$+ (b_1 + m_1 T^2) S + \frac{b_2 + c_S T^2}{2} S^2 + \frac{b_3}{3} S^3 + \frac{b_4}{4} S^4,$$  \hspace{1cm} (2.8)

in the finite temperature \([29]\), we only keep the \(O(T^2)\) terms \([15, 19]\), and the coefficients are

$$c_H = \frac{3g^2 + g'^2}{16} + \frac{g^2}{4} + \frac{\lambda}{2} + \frac{a_2}{24}, \quad c_S = \frac{a_2}{6} + \frac{b_1}{4}, \quad m_1 = \frac{a_1 + b_3}{24}.$$  \hspace{1cm} (2.9)

According to Ref. \([29]\), the tadpole term for the \(s\) should be kept since it has considerable impact on the EWPT pattern.

The vacuum structure of the scalar potential is modified by the thermal corrections, especially at high temperatures. We define the critical temperature \(T_c\), at which there exists two degenerate vacua. At temperature below \(T_c\), the EW-broken vacuum is more stable, and the Universe decay to it spontaneously, with the decay rate per unit volume as \([71]\)

$$\Gamma(T) \sim T^4 \left( \frac{S_3(T)}{2\pi T} \right)^{3/2} e^{-S_3(T)/T}.$$  \hspace{1cm} (2.10)

Here, \(S_3(T)\) is the Euclidean action of the \(O(3)\)– symmetric bounce solution. First-order EWPT happens when the decay is strong in the expanding Universe, i.e. \(\Gamma(T)/H^4(T) \gtrsim O(1)\), where \(H(T)\) is the Hubble constant at temperature \(T\). The nucleation temperature \(T_n\) when the EW-broken vacuum just starts to nucleate can be defined as \(\Gamma(T_n) = H^4(T_n)\), and it can be solved approximately using \([72]\)

$$S_3(T_n)/T_n \approx 140,$$  \hspace{1cm} (2.11)

in the radiation-dominated Universe with phase transition at EW scale. To accommodate successful EWBG, the phase transition is required to be strong, i.e. SFOEWPT,

$$v_f/T_n \gtrsim 1,$$  \hspace{1cm} (2.12)

where \(v_f\) is the final VEV of the electroweak after phase transition.

Before we carry on the numerically scan to generate the data samples in the parameter space where SFOEWPT can happen, we noted that there exists semi-analytical bounds on them as described in Ref. \([58]\). This is realised by approximately adopting a \(Z_2\) symmetry for the \(S\) field. In this situation, the EWPT pattern can be treated approximately as

$$(h = 0, s \simeq 0) \rightarrow (h = 0, s \neq 0) \rightarrow (h \neq 0, s \simeq 0),$$  \hspace{1cm} (2.13)

so called two-step transitions.

- 4 -
A $Z_2$ symmetry can be taken, since the mixing $\theta$ are well-constrained, see Fig. 1 for the current limits on $(M_{h_1}, \theta)$ plane. For $M_{h_1} \gtrsim 10$ GeV, the measurements of the Higgs signal rates and light Higgs searches at the LEP and LHC [59, 73, 74] put stringent limits on the mixing. The measurements of the Higgs invisible decays put bound $\theta \lesssim 0.079$ from $\text{Br}_{h \to \text{inv}} \lesssim 0.11$ when $m_{h_1} \ll m_{h_2}$ [75–77]. When the $h_1$ is even lighter than mesons, the measurements of rare meson decays at the LHCb can be applied [78]. Therefore, from the figure, in most of the parameter space, we have $\theta \lesssim O(10^{-1})$, especially for $M_{h_1} < M_{h_1}/2 \approx 62.5$ GeV where the exotic decays of the Higgs $h_2 \to h_1h_1$ can happen. The HL-LHC with much larger integrated luminosity, and the proposed Higgs factories, can further constrain the mixings down to $O(10^{-2})$ [79].

From Ref. [58], adopting the two-step transitions, in order for SFOEWPT to happen in the small mixing limits, several conditions on the parameters of the scalar potential must be satisfied, such as

$$a_2 \gtrsim \frac{m_s^2 \Delta}{4v^2(1 - \Delta)},$$

$$|b_3| > \frac{\sqrt{9}}{4}b_4(2m_s^2 - a_2v^2 + 2T^2_{EW}/\beta),$$

$$b_4 \gtrsim \frac{m_4^2 \Delta}{4\lambda v^4(1 - \Delta)},$$

(2.14)

where $T_{EW} \approx 140$ GeV, $\beta \equiv \frac{1}{12}(2a_2 + 3b_4)$ and $\Delta \gtrsim 0.6 - 0.8$ empirically, while we take $\Delta = 0.7$ as in Ref. [58]. This can be understood as we need strong couplings between the $S$ and $H$ to switch the EWPT from cross-over to strongly first-order, and $a_2$ is the only such coupling in the $Z_2$ limits, therefore has lower bound.

Adopting these conditions, we numerically scan the parameter space in the following
After requiring the data points to satisfy the SM constraints, we further use the Python package \texttt{CosmoTransitions} \cite{80} to solve Eq. 2.11 to get $T_n$. Around 2\% of the data points can trigger a SFOEWPT. We realised that in our Eq. 2.8, only $O(T^2)$ terms are considered, whereas leading $O(T^3)$ terms from the gauge bosons are taken in Ref. \cite{58}. However, the $O(T^3)$ terms do not have appreciable effects, and our generated samples roughly matches with the results in Ref. \cite{58}.

### 2.3 Gravitational waves

Stochastic GWs can be generated by a first-order EWPT. There are three sources which can contribute to the GWs spectrum including the bubble collisions, sound waves in plasma and the magneto-hydrodynamics turbulence \cite{81}. As discussed in Ref. \cite{82}, the contribution from the bubble collisions is negligible due to the tiny energy transfer efficiency. Therefore, we only consider the latter two sources with the numerical expressions described in Ref. \cite{9, 83} \footnote{We also consider the suppression from the short duration of the sound wave period as discussed in Ref. \cite{82, 84}.}. These expressions mainly depends on three input parameters, the transition latent heat over the radiation energy $\alpha$, the Universe expansion time scale over the phase transition duration $\beta/H$, and the bubble expansion velocity $v_b$. Larger $\alpha$ (larger energy released) and smaller $\beta/H$ (faster transition) can lead to stronger GWs. We take $v_b = 0.8$ as a benchmark, while the former two parameters can be derived from the SFOEWPT pattern,

$$
\alpha = \frac{1}{g_\ast \pi^2 T_n^4 / 30} \left( T \frac{\partial \Delta V_T}{\partial T} - \Delta V_T \right) \bigg|_{T_n}; \quad \beta/H = T_n \frac{d(S_3/T)}{dT} \bigg|_{T_n},
$$

where $\Delta V_T = V_T|_{T_n(v_f, v_f')} - V_T|_{T_n(0, v_f)}$ is effective potential difference between the true and false vacua, and $g_\ast \sim 100$ is the number of relativistic degrees of freedom.

The GWs can be detected by the next-generation space-based interferometers, such as the LISA. The detectability can be characterised by the signal-to-noise ratio (SNR), expressed as

$$
\text{SNR} = \sqrt{T \int_{f_{\text{min}}}^{f_{\text{max}}} df \left( \frac{\Omega_{\text{GW}}(f)}{\Omega_{\text{LISA}}(f)} \right)^2},
$$

where $\Omega_{\text{LISA}}$ is the sensitivity curve of the LISA detector \cite{9}, and $T \approx 1.26 \times 10^8$ s = 4 years is the data-taking time. In the rest of the paper, we take SNR $> 10$ as the detection threshold for a six-link configuration LISA.
3 Long-lived light scalars at the HL-LHC

The SFOEWPT can also be probed at colliders. It can lead to signals of resonantly produced scalars and correction to the Higgs couplings. As already mentioned in the introduction, many existing studies have focused on the cases with heavy scalars, while we focus on a light singlet scalar, enabling the exotic Higgs decay $h_2 \rightarrow h_1 h_1$ to happen. Such light singlet scalars with sufficiently small mixings, can be regarded as a LLP candidate, therefore the SFOEWPT can be tested at the lifetime frontiers of the LHC. In this section, we introduce the production cross section and decay length of the light scalar, and the relevant LLP analyses at the HL-LHC.

3.1 Production and decay length of the light scalar

![Feynman diagram of the process $pp (gg) \rightarrow h_2 \rightarrow h_1 h_1 \rightarrow jjjj$.]

Once kinematically allowed, the light singlet scalar $h_1$ can be pair-produced by the exotic decays of the SM Higgs at colliders. We have shown the corresponding Feynman diagram with $h_1 \rightarrow jj$ leading to 4 jets final states in Fig. 2. The cross section of the process is

$$\sigma_{pp \rightarrow h_2 \rightarrow h_1 h_1 \rightarrow jjjj} = \frac{\lambda^2}{9} \times \sigma_{h_2}^{SM} \times \text{Br}_{h_2 \rightarrow h_1 h_1} \times \text{Br}_{h_1 \rightarrow jj} \approx \sigma_{h_2}^{SM} \times \text{Br}_{h_2 \rightarrow h_1 h_1} \times \text{Br}_{h_1 \rightarrow jj},$$  \(3.1\)

and

$$\text{Br}_{h_2 \rightarrow XX} \approx \frac{\Gamma_{h_2 \rightarrow XX}^{SM}}{\Gamma_{h_2}^{SM} + \Gamma_{h_2 \rightarrow h_1 h_1}} \times \frac{\Gamma_{h_2 \rightarrow h_1 h_1}}{\Gamma_{h_2}^{SM} + \Gamma_{h_2 \rightarrow h_1 h_1}},$$  \(3.2\)

where $\Gamma_{h_2}^{SM} \approx 4$ MeV. The partial decay

$$\Gamma_{h_2 \rightarrow h_1 h_1} = \frac{\lambda^2}{32\pi M_{h_2}} \sqrt{1 - \frac{4M_{h_1}^2}{M_{h_2}^2}},$$  \(3.3\)

and the $h_2 h_1 h_1$ coupling is defined by

$$V \supset \frac{1}{2!} \lambda_{h_2 h_1 h_1 h_2 h_1^2}.$$  \(3.4\)
At the tree level, in the low mixing limit [58],
\[
\lambda_{h_2h_1h_1} = \left( \frac{1}{2}a_1 + a_2v_s \right) s_\theta^2 + \frac{2}{3}a_2 (2a_2v - 6\lambda v) c_\theta s_\theta^2 + (6b_4v_s + 2b_3 - 2a_2v_s - a_1) c_\theta^2 s_\theta - a_2v c_\theta^3 s_\theta \approx -a_2v,
\]
(3.5)
hence $\text{Br}_{h_2\rightarrow h_1h_1}$ is only controlled by $a_2$. Determining $\text{Br}_{h_1\rightarrow jj}$ involves perturbative calculations of hadronic effects, as discussed in Ref. [69], and is shown in Fig. 3 left. For the $M_{h_1}$ of our interests, $\text{Br}_{h_1\rightarrow jj} \approx 100\%$ except the window where $h_1 \rightarrow \tau\tau$ just opens for $5\text{ GeV} \lesssim M_{h_1} \lesssim 10\text{ GeV}$. In general, the partial decay width of the $h_1$,
\[
\Gamma_{h_1\rightarrow XX} = s_\theta^2 \times \Gamma_{h_1\rightarrow XX}^{\text{SM}} \sim 0,
\]
(3.6)
where $\Gamma_{h_1\rightarrow XX}^{\text{SM}}$ is the partial decay width with the SM couplings. If the mixing is tiny, the $h_1$ can become a LLP.

![Figure 3](image.png)

**Figure 3.** Left Panel: Estimated branching ratios of the scalar $h_1$ from Ref. [69], where $m_s$ should be replaced by $M_{h_1}$. Right panel: The proper decay length of the $h_1$ calculated by Monte-Carlo simulation.

To simulate the signal events which can be used for analyses of LLP searches, we use the Monte-Carlo generator MadGraph5aMC@NLO v2.6.7 [85] with input of a Universal FeynRules Output (UFO) [86] file generated by FeynRules [87, 88] based on a scalar singlet model. The initial and final state parton shower, hadronization, heavy hadron decays, etc are handled by PYTHIA v8.235 [89]. The clustering of the events is performed by FastJet v3.2.1 [90]. Detector level simulation is not considered at the current stage, while geometrical acceptance of the detectors are roughly simulated by the analyses in the following subsection.

With the above setup, we show the proper decay length of the $h_1$ calculated by Monte-Carlo simulation in Fig. 3 right. We verify that our results roughly matches with Ref. [69], which only has $O(1)$ difference to Ref. [91]. For $10^{-7} \lesssim \theta \lesssim 10^{-3}$, the $h_1$ decays meters away from the interaction point, can be regarded as a LLP candidate. In the next subsection, we introduce several analyses at the HL-LHC for the searches of such LLPs at far detectors including the FASER and MAPP, as well as the CMS-Timing.
Detectors & $L_x$ [m] & $L_y$ [m] & $L_{xy}$ [m] & $L_z$ [m] & trigger \\
FASER & – & – & [0, 1] & [475, 480] & $E_{\text{vis}} > 100$ GeV \\
MAPP & [3, 6] & [−2, 1] & – & [48, 61] & $E_{\text{track}} > 0.6$ GeV \\
CMS-Timing & – & – & [0.05, 1.17] & [−3.04, 3.04] & $p_T(j_i, j) > 30$ GeV, $\Delta t > 0.3$ ns \\

| $L_x$ [m] | $L_y$ [m] | $L_{xy}$ [m] | $L_z$ [m] | trigger |
|-----------|-----------|-------------|-----------|---------|
| –         | –         | [0, 1]      | [475, 480]| $E_{\text{vis}} > 100$ GeV |
| [3, 6]    | [−2, 1]   | –           | [48, 61]  | $E_{\text{track}} > 0.6$ GeV |
| –         | –         | [0.05, 1.17]| [−3.04, 3.04]| $p_T(j_i, j) > 30$ GeV, $\Delta t > 0.3$ ns |

Table 1. The LLP analyses corresponding for the FASER [60], MAPP [61] and CMS-Timing [62, 63] detectors. $p_T(j_i, j)$ is the transverse momentum of the jet from initial state radiation, $\Delta t$ is the time delay between the jets from initial state radiation and the LLP decays.

### 3.2 LLP searches at the HL-LHC

Aiming at probing LLPs, several specialized far detectors are proposed including the FASER and MAPP, as well as the CODEX-b [92], FACET [93] and MATHUSLA [94]. The upgrade of the CMS detector also include a timing detector which can be used for the same purpose. Following them, a substantial number of analyses have been put forward. We only focus on the detectors which are already in installation, and adopt the analyses from Ref. [68, 95]. These analyses have already been introduced in details in the literature, a summary of them is provided in Table 1. The signal of LLPs is defined such the particle is decayed inside the detector volume, and the final states can pass the trigger. We use the phase 2 design of the FASER to maximize the geometrical acceptance. The CMS-Timing analysis requires one jet from initial state radiation (ISR) to timestamp the event, which we denote as $j_i$. The signal event is identified if the jets from the LLP decays are sufficiently delayed comparing to the ISR jet. And the time delay can be estimated as $\Delta t \approx \frac{L_{h_1}}{\beta_{h_1}} + \frac{L_{j}}{c} - \frac{L_{j_i}}{c}$ [63, 95].

After applying these analyses, the overall signal events can be expressed as,

$$N_{\text{signal}} = \sigma_{pp \rightarrow h_2}(M_{h_1}, s_{\theta}) \times L \times \text{Br}_{h_2 \rightarrow h_1 h_1}(a_2, M_{h_1}) \times \text{Br}_{h_1 \rightarrow jj}(M_{h_1}) \epsilon_{\text{kin}}(M_{h_1}) \epsilon_{\text{geo}}(M_{h_1}, s_{\theta}) \quad (3.7)$$

where $L$ is the integrated luminosity, $\epsilon_{\text{kin,geo}}$ are the efficiencies due to trigger requirements and geometrical acceptance, respectively. The reconstruction efficiency is not considered so far, assumed to be 100%.

The effects of $\epsilon_{\text{geo}}$ is prominent, varies at different LLP detectors, depend on the angle between the LLPs and the beam line $\alpha$, the time delay $\Delta t$, and the lab decay length of the LLP $L_{h_1}^{\text{lab}}$. We therefore display the distribution of these variables for the $h_1$ in Fig. 4. In the left panel, we show the $\alpha$ and $L_{h_1}^{\text{lab}}$ distribution for the LLP at a benchmark where $M_{h_1} = 20$ GeV and $\sin \theta = 10^{-6}$. The coverage of the CMS-Timing and MAPP are also shown for comparison. The CMS-Timing covers the region where the LLPs are highly distributed, which is much larger than the one from the MAPP, hence expected to yield higher $\epsilon_{\text{geo}}$. In spite of that, the FASER has failed to reach any coverage at all. Since the $\epsilon_{\text{geo}}$ of the CMS-Timing also depends on the $\Delta t$, we show the distribution of the $\Delta t$ and $L_{h_1}^{\text{lab}}$ in the right panel for the same benchmark. Most of the LLPs follow the line where $L_{h_1}^{\text{lab}} \sim c \Delta t$. And the CMS-Timing detector has shown substantial coverage as well.
The overall efficiencies $\epsilon_{DV} \equiv \epsilon_{\text{kin}} \times \epsilon_{\text{geo}}$ for the two detectors, CMS-Timing and MAPP, as a function of $M_{h_1}$ and $\theta$ are shown in Fig. 5 left and right, respectively. $\epsilon_{DV}$ only depends on the decay length and masses of the $h_1$, therefore do not depend on the production channels, can be used to determine the sensitivity for other channels as well. The efficiencies of the MAPP and CMS-Timing peak at where $L_{h_1} \sim 1 - 100$ m. As expected, $\epsilon_{DV}$ of the CMS-Timing can reach as high as $10^{-1}$, which is about four magnitude larger than the one from the MAPP.

**Figure 5.** Left panel: Detection efficiency of the MAPP as a function of $M_{h_1}$ and $\theta$. Right panel: Same but for CMS-Timing.
4 Sensitivity

Since we already obtain the number of the signal events above, now we need to estimate the background to derive the sensitivity. In general, the background for the LLPs mainly consists of the long-lived mesons decays in the SM, real particles produced via interactions with the detector, real particles originating from outside the detector, fake-particle signatures as well as the algorithmically induced fakes [64]. The first one can be cut away by the mass fit, while the other ones are hard to model but still suppressed. Given our selected analyses, the background events can be regarded as negligible [63, 96]. Therefore, we adopt a Poisson distribution, and only require

\[ N_{\text{signal}} = 3.09, \]  

at 95 % confidence level (C.L.) to obtain sensitivity at certain parameter points [97].

The SFOEWPT parameter space can be projected into the plane \((M_{h_1}, \text{Br}_{h_2 \to h_1 h_1})\), while the \(N_{\text{signal}}\) is additionally dependent on the \(\theta\), since it alters the decay length. In order to show this dependency, we selected three benchmark scenarios as defined below,

- Scenario A, fixed \(\theta = 10^{-4}\).
- Scenario B, fixed \(\theta = 10^{-6}\).
- Scenario C, running \(\theta \in [10^{-10}, 10^{-2}]\).

The other parameters are chosen as described in Eq. 2.15.

The resulting sensitivity for the SFOEWPT is shown in Fig. 6 for Scenario A and B. For Scenario A, only \(M_{h_1} \lesssim 10\) GeV can lead to LLPs, as pointed out by the vertical dashed black line. Due to the high detection efficiency as shown in Fig. 5, the CMS-Timing can reach \(\text{Br}_{h_2 \to h_1 h_1} \sim 10^{-6}\), which is four magnitude better than the MAPP. Hence, it can cover almost all the SFOEWPT points when \(M_{h_1} \lesssim 10\) GeV, where the promptly exotic Higgs decays at the HL-LHC (solid curves) and Higgs factories (dashed curves) [30] can not probe since the final states are no-longer prompt. However, it failed to reach any points for \(M_{h_1} \gtrsim 10\) GeV where the prompt searches come into rescue, and probe most of the parameter points there. The sensitivity from the prompt analyses is still shown for \(M_{h_1} \lesssim 10\) GeV, but the validity of it needs to be clarified. The limits on \(\text{Br}_{h_2 \to h_1 h_1}\) should be re-scaled since the detection efficiency drops when the \(h_1\) has sufficient large decay length, i.e. \(\epsilon_{\text{prompt}} \approx 1 - e^{-1\text{mm}/L_{h_1}^{\text{lab}}}\). Considering a average Lorentz factor of \(\mathcal{O}(10)\) from the boost of the \(h_1\) so \(L_{h_1}^{\text{lab}} \sim 10\) mm, \(\epsilon_{\text{prompt}} \sim 10^{-1}\). Therefore, the prompt searches should only reach \(\text{Br}_{h_2 \to h_1 h_1} \sim 10^{-2}\) at most for \(M_{h_1} \lesssim 10\) GeV. Due to the low detection efficiency and the requirement on the masses of the \(h_1\), the MAPP can merely probe any SFOEWPT points. As for the detectability of the GWs at the LISA, we use black, red colors of the SFOEWPT points to represent \(\text{SNR} \in [0, 10), [10, \infty)\), respectively. Almost all the SFOEWPT points can not lead to sufficient SNR at the LISA, except a few points around \(M_{h_1} \gtrsim 10\) GeV, which are already covered by the prompt searches at the HL-LHC, and the expected upper limit for invisible exotic Higgs decay branching ratio at the HL-LHC (2%) [98]. Hence, combing the CMS-Timing and prompt searches, the collider searches
for the exotic Higgs decays can probe SFOEWPT parameter space remarkably, where the LISA can merely cover.

As shown in the right panel of the Fig. 6, the Scenario B does not have significant difference in the GWs detectability comparing to the Scenario A. Nonetheless, the sensitivity of the LLP analyses have shown drastic changes owing to the lower $\theta$. Now the LLP analyses require $M_{h_1} \gtrsim 10$ GeV, since lighter $h_1$ escapes the detector volumes. The CMS-Timing and MAPP are now sensitive to $M_{h_1} \gtrsim 10$ GeV, and reach $\text{Br}_{h_2 \rightarrow h_1 h_1} \sim 10^{-4}$ and $10^{-2}$, respectively. In this scenario, within the whole parameter space, the $h_1$ is long-lived enough, cannot be captured by the prompt searches, therefore only the LLP analyses and the invisible Higgs exotic decay at the HL-LHC can probe the SFOEWPT.

Fig. 7 shows the sensitivity for the Scenario C. Since the $\theta$ varies, the detectability of the LLP analyses for the SFOEWPT points are different even at same $M_{h_1}$. Overall, the CMS-Timing have shown better sensitivity than the MAPP. Compared to the MAPP, the CMS-Timing can cover all of the SFOEWPT points which are probed by the MAPP, and probe an appreciable number of additional points especially for light $h_1$ with $M_{h_1} \lesssim 10$ GeV. There are still a substantial number of the points which are not probed by any of the LLP analyses. The sensitivity from the prompt searches and invisible Higgs exotic decays are not shown here, as the decay length of the $h_1$ varies, so the final states might not be prompt or invisible.
Figure 7. Same as Fig. 6, but for Scenario C. The blue, green and red colors of the scatter points represent whether they are probed by both the MAPP and CMS-Timing, only the CMS-Timing, or not any of them.

5 Conclusion

The pattern of the electroweak phase transition is an important question of the particle physics, as it can reveal the early history of the Universe. To answer this question, we concentrate on the SFOEWPT triggered by a light scalar, and it can be probed by the exotic Higgs decays to a pair of the scalar $h_1$ at colliders.

We generate the points for the SFOEWPT in this case. As already pointed out by Ref. [26, 30, 58], light $h_1$ can lead to sufficiently large $\text{Br}(h_2 \rightarrow h_1h_1)$ which can be probed by the searches for the promptly exotic Higgs decays at the HL-LHC and Higgs factories. In this article, we additionally show that for very light $h_1$, the scalar can be regarded as a LLP for certain Higgs mixings $\theta$, therefore can be probed by the existing LLP analyses based at the FASER, MAPP and CMS-Timing detectors. In such parameter space, the prompt analyses are not valid anymore, since the final states are no-longer prompt, LLP analyses are one of the only ways to test the SFOEWPT. Thanks to the low background and high detection efficiencies, we show that both the LLP analyses of the MAPP and CMS-Timing probe very low $\text{Br}(h_2 \rightarrow h_1h_1)$ for certain $\theta$, while the FASER fail to reach any parameter space due to the small geometrical acceptance. This helps to probe substantial part of the SFOEWPT points, especially using the analysis at the CMS-Timing detector.

The possibility of using the GWs to probe the SFOEWPT is also considered. However, it is shown for light $h_1$, the GWs detector, LISA is not likely to observe sufficient signals. Even though a few of the SFOEWPT points can be probed at the LISA, they are already covered by the projected limits from invisible exotic Higgs decays at the HL-LHC.

We have shown that the LLP and prompt analyses are complementary in probing the
SFOEWPT. The LLP analyses are in particular sensitive to the cases where the \( M_{\ell \ell} \) and \( \theta \) can lead to the scalar with meters of decay length. While the prompt analyses can probe SFOEWPT points only if the \( \theta \) is sufficiently large, so the scalar possess decay length less than millimeter.

Acknowledgments

We thanks Ke-pan Xie for sharing the codes and useful discussions. WL is supported by the 2021 Jiangsu Shuangchuang (Mass Innovation and Entrepreneurship) Talent Program (JSSCBS20210213). Hao Sun is supported by the National Natural Science Foundation of China (Grant No.12075043, No.12147205).

References

[1] ATLAS collaboration, G. Aad et al., Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. B 716 (2012) 1–29, [1207.7214].

[2] CMS collaboration, S. Chatrchyan et al., Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC, Phys. Lett. B 716 (2012) 30–61, [1207.7235].

[3] K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, A Nonperturbative analysis of the finite \( T \) phase transition in SU(2) x U(1) electroweak theory, Nucl. Phys. B 493 (1997) 413–438, [hep-lat/9612006].

[4] K. Rummukainen, M. Tsypin, K. Kajantie, M. Laine and M. E. Shaposhnikov, The Universality class of the electroweak theory, Nucl. Phys. B 532 (1998) 283–314, [hep-lat/9805013].

[5] M. Laine and K. Rummukainen, What’s new with the electroweak phase transition?, Nucl. Phys. B Proc. Suppl. 73 (1999) 180–185, [hep-lat/9809045].

[6] J. M. Cline, Baryogenesis, in Les Houches Summer School - Session 86: Particle Physics and Cosmology: The Fabric of Spacetime, 9, 2006. hep-ph/0609145.

[7] M. Trodden, Electroweak baryogenesis, Rev. Mod. Phys. 71 (1999) 1463–1500, [hep-ph/9803479].

[8] D. E. Morrissey and M. J. Ramsey-Musolf, Electroweak baryogenesis, New J. Phys. 14 (2012) 125003, [1206.2942].

[9] C. Caprini et al., Science with the space-based interferometer eLISA. II: Gravitational waves from cosmological phase transitions, JCAP 04 (2016) 001, [1512.06239].

[10] C. Caprini et al., Detecting gravitational waves from cosmological phase transitions with LISA: an update, JCAP 03 (2020) 024, [1910.13125].

[11] M. Cepeda et al., Report from Working Group 2: Higgs Physics at the HL-LHC and HE-LHC, CERN Yellow Rep. Monogr. 7 (2019) 221–584, [1902.00134].

[12] J. Alison et al., Higgs boson potential at colliders: Status and perspectives, Rev. Phys. 5 (2020) 100045, [1910.00012].

[13] FCC collaboration, A. Abada et al., FCC Physics Opportunities: Future Circular Collider Conceptual Design Report Volume 1, Eur. Phys. J. C 79 (2019) 474.
[14] J. McDonald, Electroweak baryogenesis and dark matter via a gauge singlet scalar, Phys. Lett. B 323 (1994) 339–346.

[15] S. Profumo, M. J. Ramsey-Musolf and G. Shaughnessy, Singlet Higgs phenomenology and the electroweak phase transition, JHEP 08 (2007) 010, [0705.2425].

[16] J. R. Espinosa, T. Konstandin and F. Riva, Strong Electroweak Phase Transitions in the Standard Model with a Singlet, Nucl. Phys. B 854 (2012) 592–630, [1107.5441].

[17] J. M. Cline and K. Kainulainen, Electroweak baryogenesis and dark matter from a singlet Higgs, JCAP 01 (2013) 012, [1210.4196].

[18] T. Alanne, K. Tuominen and V. Vaskonen, Strong phase transition, dark matter and vacuum stability from simple hidden sectors, Nucl. Phys. B 889 (2014) 692–711, [1407.0688].

[19] S. Profumo, M. J. Ramsey-Musolf, C. L. Wainwright and P. Winslow, Singlet-catalyzed electroweak phase transitions and precision Higgs boson studies, Phys. Rev. D 91 (2015) 035018, [1407.5342].

[20] A. Alves, T. Ghosh, H.-K. Guo, K. Sinha and D. Vagie, Collider and Gravitational Wave Complementarity in Exploring the Singlet Extension of the Standard Model, JHEP 04 (2019) 052, [1812.09333].

[21] V. Vaskonen, Electroweak baryogenesis and gravitational waves from a real scalar singlet, Phys. Rev. D 95 (2017) 123515, [1611.02073].

[22] F. P. Huang, Z. Qian and M. Zhang, Exploring dynamical CP violation induced baryogenesis by gravitational waves and colliders, Phys. Rev. D 98 (2018) 015014, [1804.06813].

[23] W. Cheng and L. Bian, From inflation to cosmological electroweak phase transition with a complex scalar singlet, Phys. Rev. D 98 (2018) 023524, [1801.00662].

[24] T. Alanne, T. Hugle, M. Platscher and K. Schmitz, A fresh look at the gravitational-wave signal from cosmological phase transitions, JHEP 03 (2020) 004, [1909.11356].

[25] O. Gould, J. Kozaczuk, L. Niemi, M. J. Ramsey-Musolf, T. V. Tenkanen and D. J. Weir, Nonperturbative analysis of the gravitational waves from a first-order electroweak phase transition, Phys. Rev. D 100 (2019) 115024, [1903.11604].

[26] M. Carena, Z. Liu and Y. Wang, Electroweak phase transition with spontaneous $Z_2$-breaking, JHEP 08 (2020) 107, [1911.10206].

[27] K. Ghorbani and P. H. Ghorbani, Strongly First-Order Phase Transition in Real Singlet Scalar Dark Matter Model, J. Phys. G 47 (2020) 015201, [1804.05798].

[28] P. H. Ghorbani, Electroweak Baryogenesis and Dark Matter via a Pseudoscalar vs. Scalar, JHEP 08 (2017) 058, [1703.06506].

[29] W. Liu and K.-P. Xie, Probing electroweak phase transition with multi-TeV muon colliders and gravitational waves, JHEP 04 (2021) 015, [2101.10469].

[30] M. Carena, J. Kozaczuk, Z. Liu, T. Ou, M. J. Ramsey-Musolf, J. Shelton et al., Probing the Electroweak Phase Transition with Exotic Higgs Decays, in 2022 Snowmass Summer Study, 3, 2022. 2203.08206.

[31] N. Turok and J. Zadrozny, Electroweak baryogenesis in the two doublet model, Nucl. Phys. B 358 (1991) 471–493.

[32] N. Turok and J. Zadrozny, Phase transitions in the two doublet model, Nucl. Phys. B 369 (1992) 729–742.
[33] J. M. Cline, K. Kainulainen and M. Trott, *Electroweak Baryogenesis in Two Higgs Doublet Models and B meson anomalies*, JHEP 11 (2011) 089, [1107.3559].

[34] G. Dorsch, S. Huber and J. No, *A strong electroweak phase transition in the 2HDM after LHC8*, JHEP 10 (2013) 029, [1305.6616].

[35] W. Chao and M. J. Ramsey-Musolf, *Catalysis of Electroweak Baryogenesis via Fermionic Higgs Portal Dark Matter*, 1503.00028.

[36] P. Basler, M. Krause, M. Muhlleitner, J. Wittbrodt and A. Wlotzka, *Strong First Order Electroweak Phase Transition in the CP-Conserving 2HDM Revisited*, JHEP 02 (2017) 121, [1612.04086].

[37] A. Haarr, A. Kvellestad and T. C. Petersen, *Disfavouring Electroweak Baryogenesis and a hidden Higgs in a CP-violating Two-Higgs-Doublet Model*, 1611.05757.

[38] G. Dorsch, S. Huber, K. Mimasu and J. No, *The Higgs Vacuum Uplifted: Revisiting the Electroweak Phase Transition with a Second Higgs Doublet*, JHEP 12 (2017) 086, [1705.09186].

[39] J. O. Andersen, T. Gorda, A. Helset, L. Niemi, T. V. I. Tenkanen, A. Tranberg et al., *Nonperturbative Analysis of the Electroweak Phase Transition in the Two Higgs Doublet Model*, Phys. Rev. Lett. 121 (2018) 191802, [1711.09849].

[40] J. Bernon, L. Bian and Y. Jiang, *A new insight into the phase transition in the early Universe with two Higgs doublets*, JHEP 05 (2018) 151, [1712.08430].

[41] L. Wang, J. M. Yang, M. Zhang and Y. Zhang, *Revisiting lepton-specific 2HDM in light of muon $g-2$ anomaly*, Phys. Lett. B 788 (2019) 519–529, [1809.05857].

[42] X. Wang, F. P. Huang and X. Zhang, *Gravitational wave and collider signals in complex two-Higgs doublet model with dynamical CP-violation at finite temperature*, Phys. Rev. D 101 (2020) 015015, [1909.02978].

[43] K. Kainulainen, V. Keus, L. Niemi, K. Rummukainen, T. V. Tenkanen and V. Vaskonen, *On the validity of perturbative studies of the electroweak phase transition in the Two Higgs Doublet model*, JHEP 06 (2019) 075, [1904.01329].

[44] W. Su, A. G. Williams and M. Zhang, *Strong first order electroweak phase transition in 2HDM confronting future Z & Higgs factories*, 2011.04540.

[45] V. Brdar, L. Graf, A. J. Helmboldt and X.-J. Xu, *Gravitational Waves as a Probe of Left-Right Symmetry Breaking*, JCAP 12 (2019) 027, [1909.02018].

[46] M. Li, Q.-S. Yan, Y. Zhang and Z. Zhao, *Prospects of gravitational waves in the minimal left-right symmetric model*, 2012.13686.

[47] R. Zhou, W. Cheng, X. Deng, L. Bian and Y. Wu, *Electroweak phase transition and Higgs phenomenology in the Georgi-Machacek model*, JHEP 01 (2019) 216, [1812.06217].

[48] J. R. Espinosa, B. Gripaios, T. Konstandin and F. Riva, *Electroweak Baryogenesis in Non-minimal Composite Higgs Models*, JCAP 01 (2012) 012, [1110.2876].

[49] M. Chala, G. Nardini and I. Sobolev, *Unified explanation for dark matter and electroweak baryogenesis with direct detection and gravitational wave signatures*, Phys. Rev. D 94 (2016) 055006, [1605.08663].

[50] M. Chala, M. Ramos and M. Spannowsky, *Gravitational wave and collider probes of a triplet Higgs sector with a low cutoff*, Eur. Phys. J. C 79 (2019) 156, [1812.01901].
[51] S. Bruggisser, B. Von Harling, O. Matsedonskyi and G. Servant, *Baryon Asymmetry from a Composite Higgs Boson*, Phys. Rev. Lett. **121** (2018) 131801, [1803.08546].

[52] S. Bruggisser, B. Von Harling, O. Matsedonskyi and G. Servant, *Electroweak Phase Transition and Baryogenesis in Composite Higgs Models*, JHEP **12** (2018) 099, [1804.07314].

[53] L. Bian, Y. Wu and K.-P. Xie, *Electroweak phase transition with composite Higgs models: calculability, gravitational waves and collider searches*, JHEP **12** (2019) 028, [1909.02014].

[54] S. De Curtis, L. Delle Rose and G. Panico, *Composite Dynamics in the Early Universe*, JHEP **12** (2019) 149, [1909.07894].

[55] K.-P. Xie, L. Bian and Y. Wu, *Electroweak baryogenesis and gravitational waves in a composite Higgs model with high dimensional fermion representations*, JHEP **12** (2020) 047, [2005.13552].

[56] K.-P. Xie, *Lepton-mediated electroweak baryogenesis, gravitational waves and the 4τ final state at the collider*, 2011.04821.

[57] W. Wang, K.-P. Xie, W.-L. Xu and J. M. Yang, *Cosmological phase transitions, gravitational waves and self-interacting dark matter in the singlet extension of MSSM*, 2204.01928.

[58] J. Kozaczuk, M. J. Ramsey-Musolf and J. Shelton, *Exotic Higgs boson decays and the electroweak phase transition*, Phys. Rev. D **101** (2020) 115035, [1911.10210].

[59] T. Robens and T. Stefaniak, *Status of the Higgs Singlet Extension of the Standard Model after LHC Run 1*, Eur. Phys. J. C **75** (2015) 104, [1501.02234].

[60] J. L. Feng, I. Galon, F. Kling and S. Trojanowski, *Forward Search ExpeRiment at the LHC*, Phys. Rev. D **97** (2018) 035001, [1708.09389].

[61] J. L. Pinfold, *The MoEDAL Experiment at the LHC—A Progress Report*, Universe **5** (2019) 47.

[62] CMS collaboration, J. N. Butler and T. Tabarelli de Fatis, *A MIP Timing Detector for the CMS Phase-2 Upgrade*, .

[63] J. Liu, Z. Liu and L.-T. Wang, *Enhancing Long-Lived Particles Searches at the LHC with Precision Timing Information*, Phys. Rev. Lett. **122** (2019) 131801, [1805.05957].

[64] J. Alimena et al., *Searching for long-lived particles beyond the Standard Model at the Large Hadron Collider*, J. Phys. G **47** (2020) 090501, [1903.04497].

[65] F. F. Deppisch, W. Liu and M. Mitra, *Long-lived Heavy Neutrinos from Higgs Decays*, JHEP **08** (2018) 181, [1804.04075].

[66] F. Deppisch, S. Kulkarni and W. Liu, *Heavy neutrino production via Z’ at the lifetime frontier*, Phys. Rev. D **100** (2019) 035005, [1905.11889].

[67] W. Liu, S. Kulkarni and F. F. Deppisch, *Heavy Neutrinos at the FCC-hh in the U(1)_{B–L} Model*, 2202.07310.

[68] W. Liu, J. Li, J. Li and H. Sun, *Testing the seesaw mechanisms via displaced right-handed neutrinos from a light scalar at the HL-LHC*, 2204.03819.

[69] Y. Gershtein, S. Knapen and D. Redigolo, *Probing naturally light singlets with a displaced vertex trigger*, Phys. Lett. B **823** (2021) 136758, [2012.07864].

[70] D. J. Chung, A. J. Long and L.-T. Wang, *125 GeV Higgs boson and electroweak phase transition model classes*, Phys. Rev. D **87** (2013) 023509, [1209.1819].
[71] A. D. Linde, Decay of the False Vacuum at Finite Temperature, *Nucl. Phys. B* **216** (1983) 421.

[72] M. Quiros, Finite temperature field theory and phase transitions, in ICTP Summer School in High-Energy Physics and Cosmology, pp. 187–259, 1, 1999. [hep-ph/9901312].

[73] P. Bechtle, S. Heinemeyer, O. Stål, T. Stefaniak and G. Weiglein, HiggsSignals: Confronting arbitrary Higgs sectors with measurements at the Tevatron and the LHC, *Eur. Phys. J. C* **74** (2014) 2711, [1305.1933].

[74] P. Bechtle, O. Brein, S. Heinemeyer, O. Stål, T. Stefaniak, G. Weiglein et al., HiggsBounds – 4: Improved Tests of Extended Higgs Sectors against Exclusion Bounds from LEP, the Tevatron and the LHC, *Eur. Phys. J. C* **74** (2014) 2693, [1311.0055].

[75] ATLAS collaboration, Combination of searches for invisible Higgs boson decays with the ATLAS experiment.

[76] P. S. B. Dev, R. N. Mohapatra and Y. Zhang, Stellar limits on light CP-even scalar, *JCAP* **05** (2021) 014, [2010.01124].

[77] P. S. B. Dev, J.-F. Fortin, S. P. Harris, K. Sinha and Y. Zhang, Light scalars in neutron star mergers, *JCAP* **01** (2022) 006, [2111.05852].

[78] LHCb collaboration, R. Aaij et al., Search for long-lived scalar particles in $B^+ \rightarrow K^+\chi(\mu^+\mu^-)$ decays, *Phys. Rev. D* **95** (2017) 071101, [1612.07818].

[79] P. Draper, J. Kozaczuk and S. Thomas, Precision inclusive Higgs physics at $e^+ e^-$ colliders with tracking detectors and without calorimetry, *JHEP* **09** (2020) 174, [1812.08289].

[80] C. L. Wainwright, CosmoTransitions: Computing Cosmological Phase Transition Temperatures and Bubble Profiles with Multiple Fields, *Comput. Phys. Commun.* **183** (2012) 2006–2013, [1109.4189].

[81] A. Mazumdar and G. White, Review of cosmic phase transitions: their significance and experimental signatures, *Rept. Prog. Phys.* **82** (2019) 076901, [1811.01948].

[82] J. Ellis, M. Lewicki and J. M. No, On the Maximal Strength of a First-Order Electroweak Phase Transition and its Gravitational Wave Signal, *JCAP* **04** (2019) 003, [1809.08242].

[83] J. Espinosa, T. Konstandin, J. M. No and G. Servant, Energy Budget of Cosmological First-order Phase Transitions, *JCAP* **06** (2010) 028, [1004.1817].

[84] H.-K. Guo, K. Sinha, D. Vagie and G. White, Phase Transitions in an Expanding Universe: Stochastic Gravitational Waves in Standard and Non-Standard Histories, *JCAP* **01** (2021) 001, [2007.08537].

[85] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer et al., The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations, *JHEP* **07** (2014) 079, [1405.0301].

[86] C. Degrande, C. Duhr, B. Fuks, D. Grellscheid, O. Mattelaer and T. Reiter, UFO - The Universal FeynRules Output, *Comput. Phys. Commun.* **183** (2012) 1201–1214, [1108.2040].

[87] A. Alloul, N. D. Christensen, C. Degrande, C. Duhr and B. Fuks, FeynRules 2.0 - A complete toolbox for tree-level phenomenology, *Comput. Phys. Commun.* **185** (2014) 2250–2300, [1310.1921].

[88] N. D. Christensen and C. Duhr, FeynRules - Feynman rules made easy, *Comput. Phys. Commun.* **180** (2009) 1614–1614, [0806.4194].

– 18 –
[89] T. Sjöstrand, S. Ask, J. R. Christiansen, R. Corke, N. Desai, P. Ilten et al., \emph{An introduction to PYTHIA 8.2}, \textit{Comput. Phys. Commun.} \textbf{191} (2015) 159–177, [,1410.3012].

[90] M. Cacciari, G. P. Salam and G. Soyez, \textit{FastJet User Manual}, \textit{Eur. Phys. J. C} \textbf{72} (2012) 1896, [,1111.6097].

[91] M. W. Winkler, \textit{Decay and detection of a light scalar boson mixing with the Higgs boson}, \textit{Phys. Rev. D} \textbf{99} (2019) 015018, [,1809.01876].

[92] V. V. Gligorov, S. Knapen, M. Papucci and D. J. Robinson, \textit{Searching for Long-lived Particles: A Compact Detector for Exotics at LHCb}, \textit{Phys. Rev. D} \textbf{97} (2018) 015023, [,1708.09395].

[93] S. Cerci et al., \textit{FACET: A new long-lived particle detector in the very forward region of the CMS experiment}, 2201.00019.

[94] J. P. Chou, D. Curtin and H. J. Lubatti, \textit{New Detectors to Explore the Lifetime Frontier}, \textit{Phys. Lett. B} \textbf{767} (2017) 29–36, [,1606.06298].

[95] A. Berlin and F. Kling, \textit{Inelastic Dark Matter at the LHC Lifetime Frontier: ATLAS, CMS, LHCb, CODEX-b, FASER, and MATHUSLA}, \textit{Phys. Rev. D} \textbf{99} (2019) 015021, [,1810.01879].

[96] M. Frank, M. de Montigny, P.-P. A. Ouimet, J. Pinfold, A. Shaa and M. Staelens, \textit{Searching for Heavy Neutrinos with the MoEDAL-MAPP Detector at the LHC}, \textit{Phys. Lett. B} \textbf{802} (2020) 135204, [,1909.05216].

[97] Particle Data Group collaboration, P. A. Zyla et al., \textit{Review of Particle Physics}, \textit{PTEP} \textbf{2020} (2020) 083C01.

[98] J. de Blas et al., \textit{Higgs Boson Studies at Future Particle Colliders}, \textit{JHEP} \textbf{01} (2020) 139, [,1905.03764].