Robust $H_\infty$ reliable control for nonlinear delay switched systems based on the average dwell time approach

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This paper is concerned with the robust $H_\infty$ reliable control problem for a class of Lipschitz nonlinear delay switched systems. Considering the case of actuator fault, the sufficient condition is proposed to guarantee the global exponential stability with a guaranteed $H_\infty$ performance for the nonlinear delay switched systems by using the piecewise Lyapunov functional and average dwell time (ADT) approach. Then, the corresponding solvability condition for the desired robust $H_\infty$ reliable controllers is established, and the controller design is cast into a convex optimization problem which can be efficiently obtained by numerical software. A numerical example shows validity and feasibility of the proposed design method.

Keywords: nonlinear switched systems; global exponential stability; average dwell time; robust $H_\infty$ reliable control

1. Introduction

As an important abstraction form of hybrid systems, switched systems have received increasing attention within the past few decades. The so-called switched systems consist of a finite number of subsystems and a switching strategy deciding the switching sequence along the system trajectory at a time instant (Lu, Zhang, & Karimi, 2013). In general, the switching strategy of subsystems is considered as completely unknown in advance, and the switching times of subsystems are considered to be either arbitrary or constrained. Switched systems can describe a class of practical engineering. Many real-world processes and systems can be modelled as switched systems, such as the automobile direction-reverse systems, mechanical processes, multiple work points control systems of airplanes. Thus, switched systems have a wide engineering background and deserve a thorough study on theories (Wu & Lam, 2009). Zhai and Hu had investigated the stability of the linear switched systems containing unstable switching models by using the common quadratic Lyapunov function (Zhai, Hu, & Michel, 2001). The average dwell time (ADT) approach was introduced for linear switched systems to study their stability and stabilization (Balochian & Sedigh, 2012; Wu, Qi, & Feng, 2009). In Zhai, He, and Wu (2000), a class of switched systems, which could be described by differential, had been studied for stability analysis. And their sufficient conditions of the asymptotic stability could be obtained based on the Lyapunov function approach. Hu and Michel (2000) analysed the local asymptotic stability of nonlinear switched systems by using the dwell time approach. In Kim, Park, and Ko (2004), the nonlinear switched systems were studied for the robustness and the capacity to improve disturbance attenuation based on the $H_\infty$ optimal control approach. It has achieved lots of certain results about the study of linear switched systems. However, a further study of nonlinear switched systems remains to be investigated.

On the other hand, with the development of science and technology, control systems have become more and more complicated. It is inevitable that actuator failures may exist in some specific operating conditions which will affect the stability and other performance of the systems. Then, the demands for reliability, safety and efficiency of the systems become higher and higher (Du, Lin, & Li, 2013). Therefore, it is of practical importance to design control systems, which can revise malfunction actuators. At present, some achievements have been acquired by the study of reliable control problems. For example, reliable control problems had been studied for linear systems, see Wang, Sun, and Liu (2006) and the references therein. Yang, Zhengrong, Qingwei, and Weili (2006) researched a robust fault-tolerant control problem of delay switched systems, then the state feedback reliable controllers were designed by using the convex combination technique, in order that the closed-loop systems became asymptotically stable. In Lin and Antsaklis (2009), the robust reliable
control problem for linear switched systems with time-varying delay was researched. In Liu (2000), the reliable control problem is investigated for a class of stochastic nonlinear time-delay systems with multiplicative noises. For linear systems, some results of reliable control have been obtained. However, the study of reliable control for nonlinear switched systems has not been fully investigated, which motivates the present study. Therefore, the exploration of robust reliable control problem of nonlinear switched systems is still serious and challenging.

This paper deals with the problem of robust $H_{\infty}$ reliable control for nonlinear delay switched systems. Choose the continuous gain actuator failure model to describe the actuator failure part in this paper, which is more practical than the discrete actuator failure model. Based on piecewise Lyapunov stability theory and the ADT approach, the robust $H_{\infty}$ feedback reliable controllers are designed, in order that global exponential stability of the closed-loop systems will be ensured and the $H_{\infty}$ performance index will be satisfied when the actuator is running normally or malfunctioning. Finally, a numerical simulation shows that the designed control method can meet the control requirements.

2. System description and preliminaries

Consider the following uncertain nonlinear delay switched systems with disturbance:

\[\dot{x}(t) = A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t - d(t)) + B_{\sigma(t)}u_{\sigma(t)}^i(t) + D_{\sigma(t)}\omega(t) + L_{\sigma(t)}f(t)(x(t)),\]

\[z(t) = C_{\sigma(t)}x(t) + G_{\sigma(t)}u_{\sigma(t)}^i(t) + N_{\sigma(t)}\omega(t),\]

\[x(t) = \phi(t), t \in [-\rho, 0],\] (1)

where $x(t) \in \mathbb{R}^n$ is the state vector, $u_{\sigma(t)}^i(t) \in \mathbb{R}^l$ is the control input of the system with partial fault, $\omega(t) \in \mathbb{R}^q$ is the disturbance input which belongs to $L_2[0, \infty)$, $z(t) \in \mathbb{R}^p$ is the output signal to be controlled. The function $\sigma(t) : [t_0, +\infty) \to \mathbb{N} = \{1, 2, \ldots, n\}$ is the switching signal which is right continuous. $d(t)$ is the time-varying delay of the control systems satisfying $0 \leq d(t) \leq \rho < \infty$, $d(t) \leq \mu < 1$, and $\phi(t)$ is a continuous vector-valued initial function. For an arbitrary value $i \in \mathbb{N}$, $C_i$, $G_i$, $N_i$ are constant matrices. $A_i$, $A_{d_i}$, $B_i$, $D_i$, $L_i$ are uncertain real-valued matrices of appropriate dimensions. And the uncertain forms are as follows:

\[\begin{bmatrix}
A_i & A_{d_i} & B_i & D_i & L_i
\end{bmatrix} = \begin{bmatrix}
\tilde{A}_i & \tilde{A}_{d_i} & \tilde{B}_i & \tilde{D}_i & \tilde{L}_i
\end{bmatrix} + \begin{bmatrix}
E_{1i} & E_{2i} & E_{3i} & E_{4i}
\end{bmatrix},\] (2)

where $\tilde{A}_i$, $\tilde{A}_{d_i}$, $\tilde{B}_i$, $\tilde{D}_i$, $\tilde{L}_i$, $E_{1i}$, $E_{2i}$, $E_{3i}$, $E_{4i}$ are known constant matrices of appropriate dimensions. $F_i(t)$ are unknown time-varying matrices which are assumed to be norm-bounded, which satisfy the inequality: $F_i^T(t)F_i(t) \leq I$. $f_i(\cdot) : \mathbb{R}^m \to \mathbb{R}^m$ is the unknown nonlinear function and satisfies the global Lipschitz condition:

\[\| f_i(x(t)) \| \leq \| U_i(x(t)) \|,\] (3)

where $U_i$ is known as the Lipschitz constant matrix.

In actual control systems, there inevitably occur faults in the operation process of actuators. Thus, the control input of actuator fault can be described as

\[u_i^f(t) = M_iu_i(t), \quad i \in \mathbb{N},\] (4)

where $u_i(t)$ represents the normal control inputs, $u_i^f(t)$ represents the abnormal control inputs and $M_i$ is the actuator fault matrix with the following form:

\[M_i = \text{diag}[m_{i1}, m_{i2}, \ldots, m_{ik}], \quad 0 \leq m_{ik} \leq 1, \quad k = 1, 2, \ldots, l,\] (5)

Remark 2.1 For the $k$th actuator control signal of the $i$th subsystem, there are three cases as follows. When $m_{ik} = 1$, it means the normal operation. When $m_{ik} = 0$, it covers the case of the complete fault. When $0 < m_{ik} < 1$, it corresponds to the case of partial fault. It is noted that $m_{ik}$ ($k = 1, 2, \ldots, l$) cannot be 0 at the same time.

Construct the state-feedback reliable controllers with the following form:

\[u_i^f(t) = M_iK_ix(t),\] (6)

Then, the closed-loop switched systems (1) can be written as the following form:

\[\dot{x}(t) = \tilde{A}_i x(t) + \tilde{A}_{d_i}x(t - d(t)) + D_i\omega(t) + L_{fi}(x(t)),\]

\[z(t) = \tilde{C}_ix(t) + N_i\omega(t),\]

\[x(t) = \phi(t), t \in [-\rho, 0],\] (7)

where for any $i \in \mathbb{N}$, $\tilde{A}_i = A_i + B_iM_iK_i$, $\tilde{C}_i = C_i + G_iM_iK_i$.

Definition 2.2 For the switching law of the systems (1) and any $t_2 > t_1 > 0$, $\tau_{\sigma(t_1)}(t_2, t_2)$ is the number of switching between $t_1$ and $t_2$. If the inequality $\tau_{\sigma(t_1)}(t_1, t_2) \leq \tau_0 + (t_2 - t_1)/\tau_0$ holds for any $\tau_0 \geq 0$, $\tau_a > 0$, $\tau_0$ is said to be ADT of the switched systems (1), $\tau_0$ is vibration amplitude.

Definition 2.3 If there exist a switching law $\sigma(t)$ and the state trajectory of the systems (7) satisfying

\[\| x(t) \|_2 \leq \alpha \| x(t_0) \|_2 e^{-\beta(t-t_0)}\] (8)

then, the systems (7) are globally exponentially stable, where $\alpha \geq 0$, $\beta \geq 0$, $t \geq t_0$, and $\beta$ is exponential damping decrement.
Robust $H_\infty$ Reliable Control (RHRC) Problem. For a given constant scalar $\gamma > 0$, design the state-feedback controllers in the form of the equality (6) for systems (1) such that

1. Closed-loop systems are globally exponentially stable under the designed switching strategy;
2. Under zero-initial condition, the following inequality holds:

$$\|z(t)\|^2 \leq \gamma^2 \|\omega(t)\|^2, \quad \forall \omega(t) \in L_2[0, \infty).$$

In this case, the equality (6) is said to be robust $H_\infty$ reliable controllers and the systems (7) are globally exponentially stable with a $H_\infty$ disturbance attenuation level $\gamma$.

**Lemma 2.4** (Xie, Wang, & Hao, 2003) Let $L,M,W$ and $Z$ be real matrices of appropriate dimensions, and $Z$ satisfies $Z = Z^T$. Then for all $M$ satisfying $M^T M \leq I$, we have

$$Z + LMW + W^T M^T L < 0$$

if and only if there exists a scalar $\xi > 0$ such that

$$Z + \xi L L^T + \xi^{-1} W^T W < 0.$$  

3. **Main results**

We shall first investigate the global exponential stability with a guaranteed $H_\infty$ performance of the nonlinear delay switched systems (7). The following theorem presents an existence sufficient condition of state-feedback reliable controllers for the systems (7).

3.1. **Switched systems performance analysis**

**Theorem 3.1** Considering the nonlinear delay switched systems (7), for the given scalars $\rho > 0$, $\eta > 1$, $\alpha > 0$, if there exist scalars $\gamma > 0$, $\beta_i > 0$ and positive-definite symmetric matrices $P_i, Q_i$, such that the following inequalities hold:

$$\begin{bmatrix}
\Sigma_i & P_i A_{di} & P_i D_i + \tilde{C}_i^T N_i \\
\ast & -(1-\mu)e^{-\alpha p} Q_i & 0 \\
\ast & \ast & N_i^T N_i - \gamma^2 I
\end{bmatrix} < 0,$$

$$P_i \leq \eta P_j, \quad Q_i \leq \eta Q_j$$

and the ADT holds that

$$\tau_0 > \tau_a^* = \frac{\ln \eta}{\alpha}$$

then the closed-loop systems (7) are globally exponentially stable with an $H_\infty$ performance level $\gamma$ under the switching law $\sigma(t)$, where

$$\Sigma_i = \tilde{A}_i^T P_i + P_i \tilde{A}_i + Q_i + \alpha P_i + \beta_i P_i L_i^T P_i + \beta_i^{-1} U_i^T U_i + C_i^T \tilde{C}_i, \quad 0 < j < i, \quad i, j \in \mathbb{N}.$$
\[ e^{-1/2(\alpha - \ln \eta/\tau_s)(t-t_0)} \]. Thus, it can be obtained that if the ADT satisfies the inequalities (13), the switched systems are globally exponentially stable.

Based on the Schur Complement lemma,
\[
z^T(t)z(t) - y^2\omega^T(t)\omega(t) + V(t) + \alpha V(t)
\leq x^T(t)(A^T_iP_i + P_iA_i + Q_i + \beta_iP_iL_i^TP_i + \beta_i^{-1}U_i^TU_i + \alpha P_i + \vec{C}_i^T\vec{C}_i)x(t)
\]
\[- (1 - \mu)e^{-\omega}\xi x^T(t - d(t))Q_i x(t - d(t))
\]
\[+ 2x^T(t)P_iA_di(t - d(t))
\]
\[+ \omega^T(t)(N_i^TN_i - \gamma^2I)\omega(t) + 2x^T(t)(P_iD_i + \vec{C}_i^TN_i)\omega(t)
\]
\[= \left[ \begin{array}{c}
cx(t) \\
x(t - d(t)) \\
\omega(t)
\end{array} \right]^T \Psi_i \left[ \begin{array}{c}
x(t) \\
x(t - d(t)) \\
\omega(t)
\end{array} \right]
\]
where
\[
\Psi_i = \left[ \begin{array}{ccc}
\Sigma_i & P_iA_i & P_iD_i + \vec{C}_i^TN_i \\
0 & - (1 - \mu)e^{-\omega}Q_i & 0 \\
0 & 0 & N_i^TN_i - \gamma^2I
\end{array} \right].
\]

Obviously, the inequality (12) is equivalent to \( \Psi_i < 0 \), then
\[ z^T(t)z(t) - y^2\omega^T(t)\omega(t) + V_i(t) + \alpha V_i(t) < 0. \] (17)

Considering the zero-initial condition,
\[ J = \int_{t_0}^{\infty} (z^T(t)z(t) - y^2\omega^T(t)\omega(t)) \, dt \] (18)

Define the piecewise Lyapunov function \( V(t) = V_i(t) \), \( t \in [t_{v-1}, t_v] \), \( v \in \mathbb{N} \). Combine the inequality (17) and the equality (18),
\[ J = \int_{t_0}^{\infty} (z^T(t)z(t) - y^2\omega^T(t)\omega(t)) \, dt \]
\[
\leq \int_{t_0}^{t_1} (z^T(t)z(t) - y^2\omega^T(t)\omega(t) + V_{i\phi}(t) + \alpha V_{i\phi}(t)) \, dt
\]
\[+ \int_{t_1}^{t_2} (z^T(t)z(t) - y^2\omega^T(t)\omega(t) + V_{i\phi}(t) + \alpha V_{i\phi}(t)) \, dt
\]
\[+ \cdots < 0,
\]
\[\int_{t_0}^{\infty} z^T(t)z(t) \, dt \leq y^2 \int_{t_0}^{\infty} \omega^T(t)\omega(t) \, dt. \] (19)

Therefore, \( \| z(t) \|_2^2 \leq y^2 \| \omega(t) \|_2^2 \) will always hold for any nonzero external disturbance, \( \omega(t) \in L_2[0, \infty) \). The proof is concluded.

**Remark 3.2.** In Theorem 1, the Lipschitz condition has been applied on linearizing the nonlinear parts. For meeting the exact linearization conditions, we need to leave out some nonlinear characteristics. Then, the omitted parts, which are linearized approximately by Lipschitz functions, continue to exist in the linearized systems in order to improve the control accuracy of the systems.

**Remark 3.3.** The exponential stability has a fast convergence rate and good dynamic properties in contrast with the asymptotical case. On the other hand, we consider the relation between the parameters \( \alpha, \eta \) and systems stability. When we increase \( \eta \), it can be seen from Theorem 3.1 that the existence of the multiple Lyapunov functions will be increased. In other words, for a given \( \alpha \), the system stability directly depends on the choice of \( \eta \).

### 3.2. The design method of robust \( H_\infty \) reliable controllers

**Theorem 3.4** Considering the nonlinear switched systems (7), for the positive scalars \( \rho > 0, \eta > 1, \alpha > 0 \), the closed-loop system is globally exponentially stable with a \( H_\infty \) performance level \( \gamma \) for existing uncertainties, time-varying delays and actuator faults, if there exist scalars \( \gamma > 0, \epsilon_i > 0, \beta_i > 0 \), positive-definite symmetric matrices \( X_i, S_i \) and matrix \( Y_i \), that satisfying the inequalities (13), (14) and the following LMI:

\[
\begin{bmatrix}
\Lambda_i & \tilde{A}_iS_i & X_i & D_i & \beta_iL_i & X_iU_i^T & \Theta_i^T & \epsilon_i I & \Phi_i^T \\
* & - (1 - \mu)e^{-\omega}S_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & -S_i & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\gamma^2I & 0 & 0 & N_i^TN_i & E_i^TN_i & E_i^T \\
* & * & * & * & -\beta_iI & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\beta_iI & 0 & 0 & 0 \\
* & * & * & * & * & * & -\beta_iI & 0 & 0 \\
* & * & * & * & * & * & * & -\epsilon_i I & 0 \\
* & * & * & * & * & * & * & * & -\epsilon_i I \\
\end{bmatrix}
< 0,
\]

where \( \Lambda_i = \tilde{A}_iX_i + \tilde{B}M_iY_i + (\tilde{A}_iX_i + \tilde{B}M_iY_i)^T + \alpha X_i, \Theta_i = C_iX_i + G_iM_iY_i, \Phi_i = E_iX_i + E_2M_iY_i, \) Moreover, the parameters of the robust reliable controllers are given by

\[ K_i = Y_iX_i^{-1}. \] (21)

**Proof** For the \( i \)th subsystem of systems (7), considering the inequality (12), based on the Schur Complement lemma and Lemma 1, it is established that

\[
\begin{bmatrix}
\Pi_i & P_iA_i & \Delta_{1i} & P_iL_i & U_i^T & \tilde{C}_i^T \\
* & \Delta_{2i} & 0 & 0 & 0 & 0 \\
* & * & \Delta_{3i} & 0 & 0 & 0 \\
* & * & * & -\beta_iI & 0 \end{bmatrix}
< 0,
\]

where \( \Pi_i = \tilde{A}_iP_i + \tilde{P}_i \tilde{A}_i + Q_i + \alpha P_i, \Delta_{1i} = P_iD_i + \tilde{C}_i^TN_i, \Delta_{2i} = -(1 - \mu)e^{-\omega}Q_i, \Delta_{3i} = N_i^TN_i - \gamma^2I. \)
According to the equalities (2), based on Lemma 1, through equivalent transformation, it can be obtained that
\[
\gamma_i + \varepsilon_i \Gamma_i T + \varepsilon_i^{-1} \Xi_i \Sigma_i < 0,
\]
where
\[
\gamma_i = \begin{bmatrix}
\Omega_i & P_i \tilde{A}_{di} & \Delta_{di} & P_i \tilde{L}_i & U_i \bar{C}_i \\
\ast & \Delta_{2i} & 0 & 0 & 0 \\
\ast & \ast & \bar{d}_i & 0 & 0 \\
\ast & \ast & \ast & -\beta_i^{-1} & 0 \\
\ast & \ast & \ast & \ast & -\beta_i \\
\ast & \ast & \ast & \ast & \ast & -I
\end{bmatrix}
\]
\[
\Xi_i = [E_{i1} + E_{2i} M_i K_i \ E_{di} \ E_{3i} \ E_{\beta} \ 0 \ 0], \quad \Gamma_i = [H_i \Gamma_i \ 0 \ 0 \ 0 \ 0 \ 0],
\]
\[
\Omega_i = (\tilde{A}_i + \tilde{B}_i M_i K_i) \tilde{P}_i + \tilde{P}_i (\tilde{A}_i + \tilde{B}_i M_i K_i) + \alpha \tilde{P}_i + Q_i,
\]
\[
\Delta_{di} = P_i \tilde{D}_i + \bar{C}_i \bar{N}_i.
\]
Then, based on the Schur Complement lemma, it is converted to
\[
\begin{bmatrix}
\Omega_i & P_i \tilde{A}_{di} & \Delta_{di} & P_i \tilde{L}_i & U_i \bar{C}_i \\
\ast & \Delta_{2i} & 0 & 0 & 0 \\
\ast & \ast & \bar{d}_i & 0 & 0 \\
\ast & \ast & \ast & -\beta_i^{-1} & 0 \\
\ast & \ast & \ast & \ast & -\beta_i \\
\ast & \ast & \ast & \ast & \ast & -I
\end{bmatrix}
= \begin{bmatrix}
00000 \\
00000 \\
00000 \\
00000 \\
00000 \\
00000 \\
00000
\end{bmatrix}, \quad \Xi_i = [E_{i1} + E_{2i} M_i K_i \ E_{di} \ E_{3i} \ E_{\beta} \ 0 \ 0],
\]
\[
\Omega_i = (\tilde{A}_i + \tilde{B}_i M_i K_i) \tilde{P}_i + \tilde{P}_i (\tilde{A}_i + \tilde{B}_i M_i K_i) + \alpha \tilde{P}_i + Q_i,
\]
\[
\Delta_{di} = P_i \tilde{D}_i + \bar{C}_i \bar{N}_i.
\]
Make a congruence transformation for the inequality (22) via \(\text{diag}(P_i^{-1}, Q_i^{-1}, I, \beta_i I, I, I, \varepsilon_i I, I)\) and the Schur Complement lemma, and denote \(X_i = P_i^{-1}, S_i = Q_i^{-1}, Y_i = K_i P_i^{-1}\). Then, the inequality (22) is equivalent to the inequality (20). Obviously, it is easy to obtain the inequality (21). This proof is completed.

**Remark 3.5.** According to Theorem 2, if there exists a feasible solution of the inequality (20), the robust \(H_\infty\) reliable controllers can be designed in the form of the equality (21). In addition, we can also treat \(\gamma^2\) as the optimization variable to obtain the optimal disturbance attenuation level, that is,
\[
\min \gamma^2 \text{ subjected to } (13), (14), (20)
\]
by solving the above convex optimization problem, we can obtain the corresponding controller parameters.

### 4. Illustrative example

In this section, we provide an example to validate the effectiveness of the controller design method proposed in the previous section. Consider the nonlinear delay switched systems (1) with the following state-space matrices:

**Subsystem 1:**
\[
\begin{array}{ccc}
\tilde{A}_1 &=& \begin{bmatrix} -9 & 0.4 \\ 0.9 & -7 \end{bmatrix}, \\
\tilde{B}_1 &=& \begin{bmatrix} 6 \\ 1 \end{bmatrix}, \\
\tilde{A}_{d1} &=& \begin{bmatrix} -0.09 & 0 \\ -0.01 & -0.7 \end{bmatrix}, \\
\tilde{D}_1 &=& \begin{bmatrix} -0.1 & 0 \\ 0.1 & 0.1 \end{bmatrix}, \\
\tilde{L}_1 &=& \begin{bmatrix} -0.19 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
C_1 &=& \begin{bmatrix} -0.5 & 0 \\ -0.1 & 0 \end{bmatrix}, \\
G_1 &=& \begin{bmatrix} 1 & 0.3 & 0.2 \\ 0.3 & 0.2 & 0 \end{bmatrix}, \\
N_1 &=& \begin{bmatrix} 1 & 0.9 \\ 0.3 & 0.15 \end{bmatrix}, \\
U_1 &=& \begin{bmatrix} -0.01 & 0.1 \\ 0.1 & 0 \end{bmatrix}, \\
H_1 &=& \begin{bmatrix} -0.01 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \\
E_{11} &=& \begin{bmatrix} 0.35 & 0 \\ 0.1 & 0 \end{bmatrix}, \\
E_{21} &=& \begin{bmatrix} 0 & 0 \end{bmatrix}, \\
E_{d1} &=& \begin{bmatrix} 0 & -0.3 \\ 0 & 0.6 \end{bmatrix}, \\
E_{31} &=& \begin{bmatrix} 0 & 0.1 \\ 0 & 0.4 \end{bmatrix}, \\
\end{array}
\]

**Subsystem 2:**
\[
\begin{array}{ccc}
\tilde{A}_2 &=& \begin{bmatrix} 20 & 0.9 \\ 0.7 & -3 \end{bmatrix}, \\
\tilde{B}_2 &=& \begin{bmatrix} 8 \\ -1 \end{bmatrix}, \\
\tilde{A}_{d2} &=& \begin{bmatrix} -0.01 & -0.02 \\ -0.08 & -0.1 \end{bmatrix}, \\
\tilde{D}_2 &=& \begin{bmatrix} 0 & -0.5 \\ -0.1 & -0.3 \end{bmatrix}, \\
\tilde{L}_2 &=& \begin{bmatrix} 0 & -0.2 \\ -0.1 & -0.2 \end{bmatrix}, \\
C_2 &=& \begin{bmatrix} 0 & -0.1 \end{bmatrix}, \\
G_2 &=& \begin{bmatrix} -0.1 & 0.1 \\ 0.4 & 0.15 \end{bmatrix}, \\
N_2 &=& \begin{bmatrix} 1 & 0.1 \ 0.4 & 0.15 \end{bmatrix}, \\
U_2 &=& \begin{bmatrix} 0.1 & 0.1 \\ 0 & -0.01 \end{bmatrix}, \\
H_2 &=& \begin{bmatrix} 0.01 & 0 \\ 0 & -0.1 \end{bmatrix}, \\
E_{12} &=& \begin{bmatrix} 0 & 0.5 \\ 0 & 0.1 \end{bmatrix}, \\
E_{22} &=& \begin{bmatrix} 0.1 & 0.1 \\ 0.8 & 0 \end{bmatrix}, \\
E_{d2} &=& \begin{bmatrix} 0.1 & 0 \\ 0.5 & 0 \end{bmatrix}, \\
E_{32} &=& \begin{bmatrix} 0.2 & 0 \\ 0.6 & 0 \end{bmatrix}, \\
E_{f2} &=& \begin{bmatrix} 0.3 & 0 \\ 0 & 0.6 \end{bmatrix}.
\end{array}
\]

The minimum ADT of the switched systems (1) is \(\tau^*_a = \ln \eta/\alpha = \ln 4.6679/0.0800 = 1.925\), so we select the ADT \(\tau_o = 2\), and the uncertain time-varying matrices are \(F_1(t) = F_2(t) = \text{diag}(\sin t, \sin t)\).

Let the initial condition of the systems and the disturbance signal be
\[
x(0) = [0 \ 0] T, \quad \omega(t) = [8 \sin(-0.1 \pi t)e^{-0.1t} \\ 10 \sin(-0.02\pi t)e^{-0.1t}] T.
\]

The state response of the nonlinear control system within periodic switching signal, which is shown in Figures 1 and 2, can be acquired. It can be seen that, for
two cases that normal operation of the actuators and partial fault of the actuators, the designed robust $H_\infty$ reliable controllers can make the nonlinear switched systems globally exponentially stable. Figure 3 gives the state response of the control system within the arbitrary switching signal, which shows that the designed controllers are also effective on robust stabilization problem of nonlinear switched systems within arbitrary switching signal.
Figure 3. State response for partial fault of the actuators within the arbitrary switching signal.

5. Conclusion
In this paper, the robust $H_{\infty}$ reliable controllers have been investigated for the nonlinear delay switched systems. Based on the piecewise Lyapunov functional and ADT approach, sufficient conditions have been established for the existence of the reliable controllers that guarantee the nonlinear delay switched systems to be globally exponentially stable with a guaranteed $H_{\infty}$ performance. However, the nonlinear functions of switched systems in this paper are constrained by Lipschitz conditions. If the Lipschitz conditions are not satisfied any more, our designed control method may not be effective for the closed-loop systems. Thus, the reliable control problems for the switched systems, which have general nonlinear parts, will be good topics. For example, the study for the switched systems, which have stochastic nonlinear parts with multiplicative noise, may be meaningful researches in the future, and the explorations will be big challenges.

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References
Balochian, S., & Sedigh, A. K. (2012). Sufficient conditions for stabilization of linear time invariant fractional order switched systems and variable structure control stabilizers. *ISA Transactions, 51*, 65–73.
Du, H. B., Lin, X. Z., & Li, S. H. (2013). Finite-time stability and stabilization of switched linear systems. Joint 48th IEEE conference on decision and control and 28th Chinese Control Conference, Shanghai, pp. 1938–1943.
Hu, B., & Michel, A. N. (2000). Stability analysis of digital feedback control systems with time-varying sampling periods. *Automatica, 36*, 897–905.
Kim, D. K., Park, P. G., & Ko, J. W. (2004). Output-feedback $H_{\infty}$ control of systems over communication networks using a deterministic switching system approach. *Automatica, 40*, 1205–1212.
Lin, H., & Antsaklis P.J. (2009). Stability and stabilizability of switched linear systems: A short survey of recent results. *IEEE Trans on Automatic Control, 54*, 308–322.
Liu, Y. S. (2000). Reliable control and filtering for some class of stochastic nonlinear time-delay systems. Dalian: Dalian University of Technology.
Lu, Q. G., Zhang, L. X., & Karimi, H. R. (2013). $H_{\infty}$ control for asynchronously switched linear parameter-varying systems with mode-dependent average dwell time. *Control Theory and Applications, IET, 7*, 673–685.
Wang, L. M., Sun, C. C., & Liu, Y. Z. (2006). Robust fault-tolerant control of uncertain nonlinear delay switched systems. *Journal of Shenyang Normal University*, 24, 136–139.

Wu, L., & Lam, J. (2009). Weighted $H_{\infty}$ filtering of switched systems with time-varying delay: average dwell time approach. *Circuits System Signal Process*, 28, 1017–1036.

Wu, L., Qi, T., & Feng, Z. (2009). Average dwell time approach to $L_2 - L_\infty$ control of switched delay systems via dynamic output feedback. *IET Control Theory and Applications*, 10, 1425–1436.

Xie, D. M., Wang, L., & Hao, F. (2003). Robust stability analysis and control synthesis for discrete-time uncertain switched systems. Proceedings of the 42nd IEEE conference on decision and control, pp. 4812–4817. Retrieved from http://ieeexplore.ieee.org/xpl/login.jsp?tp=&arnumber=1272350&url=http%3A%2F%2Fieeexplore.ieee.org%2Fxpls%2Fabs_all.jsp%3Farnumber%3D1272350.

Yang, S., Zhengrong, X., Qingwei, C., & Weili, H. (2006). Robust reliable control of switched uncertain systems with time-varying delay. *International Journal of Systems Science*, 37, 1077–1087.

Zhai, C. L., He, W., & Wu, Z. M. (2000). Stability analysis of switched systems and the design method of controllers. *Information and Control*, 29, 21–26.

Zhai, G., Hu, B., & Michel, A. N. (2001). Stability analysis of switched systems with stable and unstable subsystems: an Average dwell time approach. *International Journal of Systems Science*, 32, 1055–1061.