We discuss the (non)-restoration of global and local symmetries at high temperature. First, we analyze a two-scalar model with $Z_2 \times Z_2$ symmetry using the exact renormalization group. We conclude that inverse symmetry breaking is possible in this kind of models within the perturbative regime. Regarding local symmetries, we consider the $SU(2) \otimes U(1)$ gauge symmetry and focus on the case of a strongly interacting scalar sector. Employing a model-independent chiral Lagrangian we find indications of symmetry restoration.
been investigated, through the study of gap equations, which are equivalent to a resummation of the super-daisy diagrams of the perturbative series. Large subleading corrections have been identified, which lead to a sizeable reduction of the parameter space where inverse symmetry breaking occurs. The question of inverse symmetry breaking has also been studied through the use of the renormalization group and a variational approach, with similar conclusions. Contrary to the results of the above studies, a large-$N$ analysis seems to indicate that symmetry is always restored at high temperature. However, the validity of this claim has recently been questioned. A finite-lattice calculation also supports symmetry restoration at sufficiently high temperature. Although the relevance of this result for the continuum limit is not clear, a Monte Carlo simulation in 2+1 dimensions seems to support this conclusion.

2 Inverse Symmetry Breaking and the Renormalization Group

In this section we consider the simplest model that exhibits inverse symmetry breaking: a two-scalar model with $Z_2 \times Z_2$ symmetry. The tree-level potential is given by

$$ V_{tr}(\phi_1, \phi_2) = \frac{1}{2} m_1^2 \phi_1^2 + \frac{1}{2} m_2^2 \phi_2^2 + \frac{1}{4} \lambda_1 \phi_1^4 + \frac{1}{4} \lambda_2 \phi_2^4 - \frac{1}{2} \lambda_{12} \phi_1^2 \phi_2^2. \quad (1) $$

This potential is bounded for $\lambda_{1,2} > 0$ and

$$ \lambda_1 \lambda_2 > \lambda_{12}^2. \quad (2) $$

In the high temperature limit, $|m_i| \ll T$, the thermal correction to the above potential at the one-loop level is given by

$$ \Delta V_T(\phi_1, \phi_2) \simeq \frac{T^2}{24} \left[ (3\lambda_1 - \lambda_{12}) \phi_1^2 + (3\lambda_2 - \lambda_{12}) \phi_2^2 \right] + \ldots \quad (3) $$

For the parameter range

$$ 3\lambda_1 - \lambda_{12} < 0, \quad (4) $$

which can be consistent with the stability condition of eq. (2), the thermal correction for the mass term of the $\phi_1$ field is negative. Notice that the stability condition (2) does not allow both mass terms to be negative. If the system is in the symmetric phase at zero temperature with $m_{1,2}^2 > 0$, there will be a critical temperature $T_{cr}^2 = 12m_i^2/(\lambda_{12} - 3\lambda_1)$ above which the symmetry will be broken. If the system is in the broken phase at $T = 0$, the symmetry will never be restored by thermal corrections.
Our aim is to discuss the above scenario in the context of the Wilson approach to the renormalization group. The main ingredient in this approach is an exact flow equation that describes how the effective action of the system evolves as the ultraviolet cutoff is lowered. We consider the lowest order in a derivative expansion of the effective action, which contains a general effective potential and a standard kinetic term. At non-zero temperature this approach can be formulated either in the imaginary-time or in the real-time formalism. In the latter formulation, the evolution of the potential lowering the cutoff scale $\Lambda$ is given by the partial differential equation

$$\Lambda \frac{\partial}{\partial \Lambda} V_\Lambda(\phi_1, \phi_2) = -T \frac{\Lambda^3}{2\pi^2} \text{Tr} \left\{ \log \left[ 1 - \exp \left( -\frac{1}{T} \sqrt{\Lambda^2 + M_\Lambda^2} \right) \right] \right\}, \quad (5)$$

where

$$[M_\Lambda^2(\phi_1, \phi_2)]_{i,j} = \frac{\partial^2 V_\Lambda(\phi_1, \phi_2)}{\partial \phi_i \partial \phi_j}, \quad i, j = 1, 2. \quad (6)$$

The initial condition for the above equation, at a scale $\Lambda_0 \gg T$, is the renormalized effective potential at zero temperature. We consider small quartic couplings, so that the logarithmic corrections of the zero-temperature theory can be safely neglected. The initial condition for the evolution is a zero-temperature effective potential given by eq. (1). Integrating the evolution equation (5), we obtain the non-zero-temperature effective potential in the limit $\Lambda \to 0$.

Finding the solution of eq. (5) is a difficult task. An approximate solution can be obtained by expanding the potential in a power series in the fields. In this way the partial differential equation (5) is transformed into an infinite system of ordinary differential equations for the coefficients of the expansion. This system can be solved approximately by truncation at a finite number of equations. That is, the potential is approximated by a finite-order polynomial. As a first step, we follow this procedure and define the running masses and couplings at the origin

$$m_{1,2}^2(\Lambda) = \left. \frac{\partial^2 V_\Lambda}{\partial \phi_{1,2}^2} \right|_{\phi_{1,2}=0}, \quad \lambda_{1,2}(\Lambda) = \left. \frac{1}{6} \frac{\partial^4 V_\Lambda}{\partial \phi_{1,2}^4} \right|_{\phi_{1,2}=0}, \quad \lambda_{12}(\Lambda) = \left. -\frac{1}{2} \frac{\partial^4 V_\Lambda}{\partial \phi_1^2 \partial \phi_2^2} \right|_{\phi_{1,2}=0}. \quad (7)$$

The corresponding evolution equations can be obtained by differentiating eq. (5) and neglecting the higher derivatives of the potential. We find

$$\Lambda \frac{\partial}{\partial \Lambda} m_{1,2} = -6C_{1,2} \lambda_{1,2} + 2C_{2,1} \lambda_{12}$$

$$\Lambda \frac{\partial}{\partial \Lambda} \lambda_{1,2} = -18D_{1,2} \lambda_{1,2}^2 - 2D_{2,1} \lambda_{12}^2 \quad (8)$$
\[ \Lambda \frac{\partial}{\partial \Lambda} \lambda_{12} = -6D_1 \lambda_1 \lambda_{12} - 6D_2 \lambda_2 \lambda_{12} + 8 \frac{C_1 - C_2}{m_1^2 - m_2^2} \lambda_{12}^2, \]

with

\[ C_{1,2} = \frac{\Lambda^3}{4\pi^2} \frac{N(\omega_{1,2})}{\omega_{1,2}}, \quad D_{1,2} = \frac{\partial C_{1,2}}{\partial m_{1,2}^2}, \quad \omega_{1,2}^2 = \Lambda^2 + m_{1,2}^2, \] (9)

and \( N(\omega) = [\exp(\omega/T) - 1]^{-1} \) the Bose-Einstein distribution function. For \( \omega_{1,2} \ll T \) we have

\[ C_{1,2} \rightarrow \frac{\Lambda^3}{4\pi^2} \frac{T}{\Lambda^2 + m_{1,2}^2}, \] (10)

and the above equations agree with those considered in ref. 5 in the same limit. For \( \omega_{1,2} \gg T \) there is no running, because of the exponential suppression in the Bose-Einstein function.

We have solved numerically the system of equations (8) and determined the range of zero-temperature parameters that lead to inverse symmetry breaking. In fig. 1 we present the results for a zero-temperature theory with positive mass terms \( m_1^2(\Lambda_0) = m_2^2(\Lambda_0) \) and \( \lambda_2(\Lambda_0) = 0.3 \). The temperature has been chosen much higher than the critical one \( (T = 500m_1(\Lambda_0)) \). The system (8) has been integrated from \( \Lambda_0 \gg T \) down to \( \Lambda = 0 \), where the thermally corrected masses and couplings at non-zero temperature have been obtained. A negative value for the mass term \( m_1^2 \) at \( \Lambda = 0 \) has been considered as the signal of inverse symmetry breaking. This has been achieved in the region above line (a) in fig. 1. We also plot the stability bound of eq. (2) (the allowed range is below line (b)), and the perturbative prediction for the range that leads to inverse symmetry breaking (above line (c)). The phenomenon of inverse symmetry breaking is confirmed by our study, in agreement with ref. 5, where the imaginary-time formulation of the renormalization-group approach has been used. We observe that the renormalization-group treatment eliminates a large part of the parameter space allowed by perturbative theory, in agreement with the results obtained by solving the gap-equations 4.

The reliability of our conclusions crucially depends on whether the solution of the system of truncated equations (8) provides an approximate solution to the full partial differential equation (5). We have checked that by numerical integration of eq. (8) through the algorithms discussed in ref. 15. Due to limitations in computer time, we restrict our discussion of eq. (8) along the \( \phi_1 \) axis, which is the direction of expected symmetry breaking for our choice of couplings. We approximate the potential by the expression

\[ V_\Lambda(\phi_1, \phi_2) = V_\Lambda(\phi_1) + \frac{1}{2} m_2^2(\Lambda) \phi_2^2 + \frac{1}{4} \lambda_2(\Lambda) \phi_2^4 - \frac{1}{2} \lambda_{12}(\Lambda) \phi_1^2 \phi_2^2. \] (11)
The evolution of $m_2^2(\Lambda)$, $\lambda_2(\Lambda)$ and $\lambda_{12}(\Lambda)$ is determined through the truncated eqs. (8). However, the full $\phi_1$ dependence is preserved through the numerical integration of eq. (5), with the eigenvalues of the mass matrix $M_2^{\Lambda}$ given by

$$[M_2^{\Lambda}]_1 = \frac{\partial^2 V_\Lambda(\phi_1)}{\partial \phi_1^2} \quad \text{and} \quad [M_2^{\Lambda}]_2 = m_2^2(\Lambda) - \lambda_{12}(\Lambda)\phi_1^2. \quad (12)$$

This treatment permits a reliable study of the order of the symmetry-breaking phase transition; we have found that it is governed by the Wilson-Fisher fixed point of the one-scalar three-dimensional theory, resulting in a second order phase transition.

3 \textbf{SU}(2) \otimes U(1) gauge symmetry with strongly coupled Higgs sector

A natural question to ask is whether the standard model gauge symmetry $SU(2) \otimes U(1)$ could remain broken at high temperatures. It is well known that the symmetry is restored in the minimal standard model, so we consider its simplest extension with two Higgs doublets. We find that due to the positive and large contribution of the gauge bosons to the scalar thermal masses, it is not possible to attain a negative mass term (needed for symmetry non-restoration) within the perturbative range of the scalar couplings.

A strongly coupled Higgs sector implies (at least naively) heavy physical scalar particles, which can be effectively removed from the physical low-energy
spectrum. To study the behaviour of the gauge symmetry in this case we use an effective Lagrangian which keeps only the light degrees of freedom, namely the gauge and Goldstone bosons together with the fermions. The resulting chiral Lagrangian is a non-renormalizable non-linear sigma model coupled in a gauge invariant way to the Yang-Mills theory at lowest order, it is model independent.

We propose to use the gauge boson magnetic mass as an indicator of symmetry (non)-restoration. It is defined as the transverse part of the corresponding self-energy, $\Pi_T(0,\vec{k})$, on-shell. A perturbative computation shows that it is exactly equal to zero at one loop in an unbroken gauge theory. For unbroken non-Abelian gauge theories, such as QCD, higher orders in perturbation theory suffer from infrared divergences, and a magnetic mass of order $g^2T$ is expected to be generated non-perturbatively. In spontaneously broken gauge theories, such as the standard electroweak model and its extensions, no such divergences are present. Thus, we expect that even in perturbation theory the magnetic mass (as computed in the broken phase) will show a tendency to vanish at high enough temperatures, whenever symmetry restoration occurs.

We have calculated the thermal gauge boson self-energies at one loop and leading order, $\mathcal{O}(T^2)$, from which we obtain the magnetic masses

\begin{align}
M^2_{W,\text{mag}} &= g^2 \frac{v(T)^2}{4}, \\
M^2_{Z,\text{mag}} &= (g^2 + g'^2) \frac{v(T)^2}{4},
\end{align}

with

\begin{equation}
v(T)^2 = v^2 \left[1 - \frac{T^2}{6v^2}\right],
\end{equation}

where the fact that the gauge couplings are not renormalized at this order has been used. We conclude thus that in models with strongly interacting Higgs sector the spontaneously broken $SU(2) \otimes U(1)$ gauge symmetry tends to be restored when the system is heated. Notice though that our calculation is only valid for temperatures below the electroweak scale. Details of the calculation and the approximations involved can be found in [17].

It is worth to remark that the thermal corrections to $v$ coincide with those of the pion decay constant $F_\pi$ in the non-linear sigma model. That is, at one loop and leading order $T^2$ all the temperature corrections to the scalar v.e.v. are due to the would-be Goldstone bosons; fermions and gauge boson transverse degrees of freedom will only contribute at higher order.
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References

1. S. Weinberg, Phys. Rev. D 9, 3357 (1974).
2. R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 42, 1651 (1979); G. Dvali and G. Senjanović, Phys. Rev. Lett. 74, 5178 (1995); G. Dvali, A. Melfo and G. Senjanović, Phys. Rev. Lett. 75, 4559 (1995) and references therein.
3. H. Haber, Phys. Rev. D 46, 1317 (1992); M. Mangano, Phys. Lett. B 147, 307 (1984); G. Dvali, K. Tamvakis, Phys. Lett. B 378, 141 (1996); B. Bajc, A. Melfo, G. Senjanović, Phys. Lett. B 387, 796 (1996); A. Riotto and G. Senjanović, Phys. Rev. Lett. 79, 349 (1996).
4. G. Bimonte and G. Lozano, Phys. Lett. B 366, 248 (1996); Nucl. Phys. B 460, 155 (1996).
5. T.G. Roos, Phys. Rev. D 54, 2944 (1996).
6. M. Pietroni, N. Rius, N. Tetradis, Phys. Lett. B 397, 119 (1997).
7. G. Amelino-Camelia, Phys. Lett. B 388, 776 (1996); Nucl. Phys. B 476, 255 (1996).
8. Y. Fujimoto and S. Sakakibara, Phys. Lett. B 151, 260 (1985); E. Manesis and S. Sakakibara, Phys. Lett. B 157, 287 (1985); K.G. Klimenko, Theor. Math. Phys. 80, 929 (1989).
9. J. Orloff, Phys. Lett. B 403, 309 (1997).
10. G. Bimonte and G. Lozano, Phys. Lett. B 388, 692 (1996).
11. G. Bimonte et al., DFTUZ 97/17, hep-lat/9707029.
12. L. Dolan and R. Jackiw, Phys. Rev. D 9, 3320 (1974).
13. N. Tetradis and C. Wetterich, Nucl. Phys. B 398, 659 (1993).
14. M. D’Attanasio and M. Pietroni, Nucl. Phys. B 472, 711 (1996).
15. J. Adams, J. Berges, S. Bornholdt, F. Freire, N. Tetradis and C. Wetterich, Mod. Phys. Lett. A 10, 2367 (1995).
16. T. Appelquist and C. Bernard, Phys. Rev. D 22, 200 (1980); A.C. Longhitano, Phys. Rev. D 22, 1166 (1980); Nucl. Phys. B 188, 118 (1981).
17. M.B. Gavela, O. Pène, N. Rius and S. Vargas-Castrillón, hep-ph/9801244.
18. J. Gasser, H. Leutwyler, *Phys. Lett.* B 184, 83 (1987); A. Schenk, *Phys. Rev. D* 47, 5138 (1993); A. Bochkarev, J. Kapusta, *Phys. Rev. D* 54, 4066 (1996); R.D. Pisarski and M. Tytgat, *Phys. Rev. Lett.* 78, 3622 (1997); C. Manuel, [hep-ph/9710208](http://arxiv.org/abs/hep-ph/9710208), to appear in *Phys. Rev. D.*