Residual proton-neutron interactions and the $N_pN_n$ scheme

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We investigate the correlation between integrated proton-neutron interactions obtained by using the up-to-date experimental data of binding energies and the $N_pN_n$, the product of valence proton number and valence neutron number with respect to the nearest doubly closed nucleus. We make corrections on a previously suggested formula for the integrated proton-neutron interaction. Our results demonstrate a nice, nearly linear, correlation between the integrated p-n interaction and $N_pN_n$, which provides us with a firm foundation of the applicability of the $N_pN_n$ scheme to nuclei far from the stability line.

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The importance of the residual proton-neutron interaction in the nuclear structure has long been emphasized[1-3] in evolution of single-particle structure, the development of collectivity, phase transition and deformation. Based on this scenario, Casten suggested[4,5] that a simple quantity, $N_pN_n$, the product of valence proton number $N_p$ and valence neutron number $N_n$ with respect to the nearest doubly closed nucleus, is a reasonable measure for the strength of the residual proton-neutron interaction, and that this simple quantity is both interpretive in classification of collective motion for low-lying states and predictive in studying unknown regions. See Ref.[6] for a comprehensive review. Recent developments and applications along this line can be found in refs.[7-9].

In ref.[10] Zhang et al. suggested a very simple approach to extract the proton-neutron interaction of the last proton with the last neutron ($\delta V_{pn}$) for odd-odd nuclei by experimental data of binding energies of their even-even neighboring nuclei, via a double difference procedure. This approach attracted much attention since then[8,11,12]. In the same paper Zhang et al. also suggested a formula of the integrated proton-neutron interaction ($V_{pn}$). They plotted the $V_{pn}$ versus $N_pN_n$ for nuclei with mass number around 100 and 130. Their results showed that $V_{pn}$ is approximately proportional to the value of $N_pN_n$, with deviations for small $N_pN_n$.

Ref.[10] was published more than two decades ago. At that time experimental data of binding energies were restricted to a limited number of nuclei. In the last two decades piles of new experimental data of binding energies became available[17], due to the new radioactive beam facilities worldwide. It would be interesting to revisit the integrated proton-neutron interactions and to investigate whether or not there exist similar relations between $V_{pn}$ and $N_pN_n$ for unstable nuclei. In doing so, we suggest other formulas to extract the integrated proton-neutron interactions.

Let us begin with the famous formula of the residual proton-neutron interaction of the last proton with the last neutron ($\delta V_{pn}$) for odd-odd nuclei suggested in ref.[10],

$$
\delta V_{pn}(Z, N) = \frac{1}{4} \{ [B(Z + 1, N + 1) - B(Z + 1, N - 1)] \\
- [B(Z - 1, N + 1) - B(Z - 1, N - 1)] \},
$$

where $B$ is the nuclear binding energy, both proton number $Z$ and neutron number $N$ are odd.

By summing $\delta V_{pn}$ over all the valence protons and valence neutrons, the integrated proton-neutron interaction $V_{pn}$ for an odd-odd nucleus ($Z + \delta_p, N + \delta_n$) was obtained by Zhang et al. in Ref.[10]. Their formula is given as follows.

$$
V_{pn}(Z + \delta_p, N + \delta_n) = \delta_p \delta_n \cdot \sum_{Z_x=\delta_p}^{Z} \sum_{N_x=\delta_n}^{N} 4 \cdot \delta V_{pn}(2Z_x + \delta_p, 2N_x + \delta_n) \\
= \delta_p \delta_n \{ [B(Z + 2\delta_p, N + 2\delta_n) - B(Z + 2\delta_p, N_0)] \\
- [B(Z_0, N + 2\delta_n) - B(Z_0, N_0)] \},
$$

where $Z_0$ and $N_0$ are the nuclear numbers of the nearest doubly closed nucleus.
where $Z_0$ and $N_0$ are the nearest magic numbers, $\delta_p \ (\delta_n)$ is +1 if the valence protons (neutrons) are particle-like and −1 if the valence protons (neutrons) are hole-like, both $Z$ and $N$ are even numbers.

The results of $V_{pn}$ by applying eq. (2) to odd-odd nuclei are shown in panels (a) in Figs. 1-3 (a), for nuclei with $28<Z<50$ and $50<N<82$, those with $50<Z<82$ and $50<N<82$, and those with $50<Z<82$ and $82<N<126$, respectively. The experimental data of binding energies are taken from Ref. [17] (i.e., here up-to-date experimental data of binding energies are included in comparison to Ref. [10]). These results of $V_{pn}$ are consistent with those shown in Ref. [10]: $V_{pn}$ is proportional to $N_p N_n$ for large $N_p N_n$, and there are deviations from the linear correlation for small $N_p N_n$. The inserts of Figs. 1-2 (a) highlight the details of such deviations.

We would like to point out that there are inadequacies in the construction of the formula for $V_{pn}$ (see eq. (2) above) in ref. [10], which can be important for nuclei with smaller numbers of valence nucleons. To see this, let us apply eq. (2) to the $^{128}\text{I}$ nucleus.

$$V_{pn}(^{128}\text{I}) = V_{pn}(53, 75) = V_{pn}(52 + 1, 76 - 1)$$
$$= -\{[B(52 + 2, 76 - 2) - B(52 + 2, 82)]$$
$$- [B(50, 76 - 2) - B(50, 82)]\}$$
$$= B(54, 82) - B(54, 74) - B(50, 82) + B(50, 74). \ (3)$$

One sees that the above $V_{pn}$ for $^{128}\text{I}$ actually corresponds to even-even nucleus $^{128}\text{Xe}$ which has four valence protons and eight valence holes with respect to $^{132}\text{Sn}$. Assuming the proton-neutron interaction is equal for all valence particles, this $V_{pn}$ equals $32 \times \delta V_{np}$, while $^{128}\text{I}$ has three valence protons and seven valence neutrons and its $V_{pn}$ should equal $21 \times \delta V_{np}$.

In order to correct the above inadequacies with eq (2), i.e., for example, $V_{pn}$ for $^{128}\text{I}$ approximately equals $21 \times \delta V_{np}$ but eq. (2) gives $32 \times \delta V_{np}$, we suggest the...
We obtain odd-odd but also for even-even and odd-mass nuclei. For eq. (3). We also note that eq. (5) works not only for interactions from the linear $V_{np}$-nuclei correlation in Figs. 1-2 (b), respectively. Very interestingly, one sees that the deviations from the linear $V_{np}$-nuclei correlation in Figs. 1-2 (a) does not arise in the $V_{np}$-nuclei plot in Figs. 1-2 (b), after our refinements.

Similarly, we show in Fig. 3 (b) the results of $|V'_{pn}|$ versus $N_pN_n$ for the case of 50$<Z<82$, 82$<N<126$. One sees that $|V'_{pn}|$ versus $N_pN_n$ remains to be good, except slight deviations from the linear correlation arise for nuclei with proton number or neutron number close to the nearest magic number. For these “anomalous” nuclei (i.e., $Z$=80 and 81 isotopes, see Fig. 3(b) for details) in Fig. 3(b), the values of $V'_{pn}$ are slightly smaller than those of “normal” nuclei.

To summarize, in this paper we examine the linear correlation between the integrated proton-neutron interaction obtained by using the up-to-date experimental data of binding energies for relevant nuclei and the $N_pN_n$, the product of valence proton number $N_p$ and valence neutron number $N_n$ with respect to the nearest doubly closed nucleus. We suggest refinements to the formula of the proton-neutron interaction suggested in ref. [10]. The new formula works not only for odd-odd nuclei but also for even-even and odd-mass nuclei.

The proton-neutron interactions obtained in the present paper exhibit excellent linear correlations in terms of $N_pN_n$ for nuclei with 28$<Z<50$, 50$<N<82$, and those with 50$<Z<82$, 82$<N<126$. For nuclei with 50$<Z<82$ and 82$<N<126$, the linearity remains to be good in general, although proton-neutron interactions for $Z=80$ and 81 exhibit very slight deviations.

Thus our results provide us with a firm foundation for the applicability of the $N_pN_n$ scheme to nuclei far from the stability line.

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