Simulation Research on Damage Identification of Plate Weld Based on Correlation Vector Amplitude Vector

Lizong LIN¹, Xin ZHANG*, Haoyuan CHANG

¹School of Mechanical and Power Engineering, East China University of Science and Technology, Shanghai, 200237, China
*e-mail: 2221471646@qq.com

Abstract: A method for detecting the quality of the plate weld based on the cross-correlation function magnitude vector is proposed. In the large-scale plate welding process, the weld is the most vulnerable part of the damage. This paper proposes a method to accurately determine the location of the weld defect. In this paper, the finite element software ABAQUS is used for numerical simulation to establish a plate weld model, and the weld area is divided into 19 units. In order to avoid noise interference, a 10 second 1000N stationary random excitation is used, and 10 excitation reference points are arranged around the random excitation point, and the best reference point is determined by the simulation result. The damage of each weld unit is characterized as the decrease of Young's modulus, the acceleration response is collected at the joint of each weld unit, and the single defect, double defects and multi-defects conditions are simulated respectively. The numerical results show that the cross-correlation amplitude vector method can always accurately locate the defects under various damage conditions.

1. Preface
The welded structure has been widely used in the fields of automobile, aviation, construction, etc, and in these fields, the excitation is mostly random excitation, such as earthquakes, wind loads, and vehicle vibration caused by uneven ground. Under the long-term effect of these random excitations, the welding structure with damage or defects will be destroyed with great probability. Therefore, the research on random excitation has received more and more attention in recent decades.

Random vibration is non-deterministic and cannot be accurately predicted. It cannot be described by a certain function, and can only be expressed by mathematical statistics. Stochastic process theory is the basis for analyzing random vibration, so random vibration can also be described using methods that study stochastic processes, such as mean, variance, power spectral density (frequency domain description), correlation function (time domain description), and so on. In the research of structural damage identification method under stationary random excitation, Dr. Yu Zhefeng¹ proposed the damage identification method based on Cross Correlation Function Amplitude Vector (CorV) for the first time. The method is a comparative damage identification method, which requires a non-damaged model or structure as a reference state. According to the relative variation of CorV proposed, the damage unit can be identified and the degree of damage can be determined. In its paper, the confidence criterion for the magnitude vector of the cross-correlation function is defined with reference to the modal confidence criterion (MAC). At the same time, the noise, random excitation spectrum and the influence of measurement time on the detection result are analyzed in detail in the frame structure.
The structural damage identification method based on the cross-correlation function magnitude vector was first proposed by Professor Yang Zhichun\cite{5,6} and Dr. Yu Zhefeng. The research team proposed the definition of the correlation vector of the cross-correlation function and the confidence criterion. The validity of the method in structural damage identification was verified by theoretical and experimental studies. Literature\cite{7,8} studied the magnitude vector of cross-correlation function combined with wavelet transform, which was applied to aircraft siding fastener damage and composite materials. Through the combination of theory and experimental research, the location of structural damage was obtained and the results confirm that the method can be applied to the damage identification of the above two structures. In his research, Lei Jiayan\cite{9} derived the expression of the cross-correlation function of displacement, velocity and acceleration response under ideal white noise excitation, and used this expression to explain the solid principle of the magnitude vector of cross-correlation function and supplemented with the research content of Yu Zhefeng. Hu Weibing\cite{10} applied the magnitude vector of cross-correlation function to study the damage detection in ancient wood structures. Although the application of the structural damage identification method based on this theory has not been widely used, the previous theory and experiments have proved its effectiveness in structural damage identification, which provides a new idea for the damage identification method based on random excitation. The method has the advantages of simple detection of structural damage, it only needs a reference state of the non-destructive structure, then the corresponding structural damage position can be detected by one test, and the degree of damage can be determined by comparing the amplitude of the detection result.

2. Basic principle of cross-correlation function magnitude vector

2.1 Definition of cross-correlation function magnitude vector

The equation of motion of a linearly constant structure with N degrees of freedom can be expressed by formula (1), in this formula, \([M], [C], [K]\) respectively represent the mass matrix, the damping matrix, and the stiffness matrix, \(\{\ddot{x}\}, \{\dot{x}\}, \{x\}\) respectively representing the acceleration, velocity, and displacement vector, \(\{F\}, [B]\) respectively representing the magnitude and position of the applied external force.

\[
[M]\ddot{x} + [C]\dot{x} + [K]x = [B]F
\]  

Select a node \(k\) in the structure as a reference point, collect the time domain response of the remaining nodes \(i\) in the structure, respectively generate a cross-correlation function \(R_{ik}(\tau)\) with the response of the reference node, take the maximum value of \(R_{ik}(\tau)\), and form a vector, this vector is the cross-correlation function magnitude vector, which can be expressed by the following formula.

\[
\Phi_{corrK} = \{\rho_{1k}, \rho_{2k}, \ldots, \rho_{nk}\}
\]  

In the formula, \(\rho_{ik}\) represents the maximum amplitude of the cross-correlation function, \(\rho_{ik} = \max(R_{ik}(\tau))\), and the two time-domain response vectors that generate the cross-correlation function can be displacement, velocity or acceleration responses.

2.2 Definition of cross-correlation function

The cross-correlation function is a method for studying the statistical correlation between any two random signals \(X(t_1)\) and \(Y(t_2)\), and is a function of the time difference between the two signals, which can be expressed by formula (3).

\[
R(t_1, t_2) = E[X(t_1)Y(t_2)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} XY\rho(X, Y, t_1, t_2) dXdY
\]  

In the formula: \(\rho(X, Y, t_1, t_2)\) represents the joint probability distribution of signals \(X(t_1)\) and \(Y(t_2)\).

For the ergodic signals of various states, when obtaining the cross-correlation function, it is not necessary to observe an infinite number of times in order to obtain a sufficient number of sample
functions, but can be estimated from a single sample of the signal, and can be calculated by the formula (4).

\[ R_{xy}(\tau) = E[X(t_1)Y(t_2)] = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)y(t + \tau)dt \]  

(4)

2.3 Relationship between cross-correlation function and vibration parameters

Now suppose that \( x_i(t) \) and \( x_k(t) \) are respectively the time-domain responses of linearly-constant structural node \( i \) and reference node \( k \), and their Fourier transforms are respectively \( X_i(\omega) \) and \( X_k(\omega) \), which can be represented by formulas (5) and (6).

\[ X_i(\omega) = \int_{-\infty}^{+\infty} x_i(t)e^{-j\omega t}dt \]  

(5)

\[ X_k(\omega) = \int_{-\infty}^{+\infty} x_k(t)e^{-j\omega t}dt \]  

(6)

The inverse Fourier transform of formulas (5) and (6) can be expressed by formulas (7) and (8).

\[ x_i(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X_i(\omega)e^{j\omega t}d\omega \]  

(7)

\[ x_k(t + \tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X_k(\omega)e^{j\omega (t+\tau)}d\omega \]  

(8)

Multiply m and n, and the result is averaged over time, then combined with formulas (7) and (8), formula (9) is obtained.

\[ <x_i(t)x_k(t + \tau)> = \frac{1}{T} \int_{0}^{T} x_i(t)x_k(t + \tau)dt = \frac{1}{T} \int_{0}^{T} x_i(t) \left( \frac{1}{2\pi} \int_{-\infty}^{+\infty} X_i(\omega)e^{j\omega t}d\omega \right) \left( \frac{1}{2\pi} \int_{-\infty}^{+\infty} X_k(\omega)e^{j\omega (t+\tau)}d\omega \right) dt = \frac{1}{2\pi} \int_{0}^{T} X_k(\omega)e^{j\omega t} \left[ \int_{-\infty}^{+\infty} X_i(\omega)e^{j\omega t}d\omega \right] \frac{1}{2\pi} \int_{0}^{T} \left( \int_{-\infty}^{+\infty} X_k(\omega)e^{j\omega (t+\tau)}d\omega \right) d\omega = \frac{1}{2\pi} \int_{0}^{T} X_i(\omega)X_k(\omega)e^{j\omega t}d\omega \]  

(9)

In the formula: * represents conjugate.

For the formula (4), when \( T \to \infty \), \( R_{ik}(\tau) \) can be calculated by the formula (10).

\[ R_{ik}(\tau) = E[x_i(t)x_k(t + \tau)] = \frac{1}{T} \int_{0}^{T} \lim_{T \to \infty} E \left[ \left( \frac{1}{T} \int_{-\infty}^{+\infty} X_i(\omega)X_k(\omega) \right) e^{j\omega t}d\omega, \omega \right] \]  

(10)

According to the vibration theory, the frequency response function, input and output have such a relationship in the frequency domain, as shown in formulas (11) and (12).

\[ X_i(\omega) = H_i(\omega)F(\omega) \]  

(11)

\[ X_k(\omega) = H_k(\omega)F(\omega) \]  

(12)

In the formula: \( \mathbf{H}(\omega) \) represents the frequency response function, where the \( j \)-th column is \( \{H_{j1}(\omega) \quad H_{j2}(\omega) \quad \cdots \quad H_{jn}(\omega)\}^T \), and \( \mathbf{F}(\omega) \) represents the input stimulus, \( F(\omega) = \{F_1(\omega) \quad F_2(\omega) \quad \cdots \quad F_m(\omega)\}^T \).

By bringing formulas (11) and (12) into formula (10), the relationship between the cross-correlation function and the frequency response function can be derived, as shown in formula (13).

\[ R_{ik}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \lim_{T \to \infty} E \left[ \left( \frac{1}{T} \int_{-\infty}^{+\infty} H_i(\omega)F(\omega) \right)^* \left( \int_{-\infty}^{+\infty} H_k(\omega)F(\omega) \right) \right] e^{j\omega t}d\omega \]  

(13)

It can be concluded from formula (13) that the cross-correlation function between the structural response detection point \( i \) and the reference response detection point \( k \) is related to three factors, namely the input spectrum, the frequency response function of the response detection point \( i \), and the frequency response of the reference response detection point \( k \). From this, we can argue that when \( m \) is determined, there will be a certain relationship between each \( n \), which is relatively fixed or stable. For a complete defect-free structure, assuming that we are able to detect a sufficiently long response history, the CorV obtained relative to the reference point should be relatively fixed. However, in actual tests, this time is often not long enough, so CorV in a lossless structure obtained by different excitation samples in the same spectrum will vary within a certain range. In order to make the variation range of CorV under the lossless structure as small as possible, the amplitude of the excitation sample can be made smaller.

When \( \tau = \tau_1 \), the magnitude of the cross-correlation function is as shown in formula (14).

\[ \Phi_{ik}(\tau_1) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \lim_{T \to \infty} E \left[ \left( \frac{1}{T} \int_{-\infty}^{+\infty} H_i(\omega)F(\omega) \right)^* \left( \int_{-\infty}^{+\infty} H_k(\omega)F(\omega) \right) \right] e^{j\omega \tau_1}d\omega \]  

(14)
In the actual detection process, only a limited length of time can be detected, and it is impossible to have an infinite length of time like formula (14), so formula (14) is an approximate equation. Therefore, for a good structure, the CorV obtained from different samples may not be exactly the same, but vary within a certain range. After the structure is damaged, $R_{ik}(\tau)$ does not necessarily take the maximum value at $\tau = \tau_1$, so in order to make the CorV obtained each time comparable, the value at time $\tau_1$ is uniformly taken.

2.4 CorV confidence criteria

In order to judge whether damage has occurred in the structure, it is necessary to construct an index to quantitatively assess the overall damage degree. With reference to the modal confidence criterion, a corresponding confidence criterion CVAC (Cross-correlation function amplitude vector assurance criterion) can be constructed. It can be calculated by formula (15).

$$CVAC = \frac{[\text{MAX}(R_u(t))\{\text{MAX}(R_d(t))\}]^2}{[\text{MAX}(R_u(t))\{\text{MAX}(R_d(t))\}][\text{MAX}(R_u(t))\{\text{MAX}(R_d(t))\}]^T}$$ (15)

When the structure is non-destructive, $\text{MAX}(R_u(t))$ and $\text{MAX}(R_d(t))$ should be approximately equal, so the value of CVAC should be very close to 1. Conversely, when the structure is completely damaged, the value of CVAC should be 0. However, if the structure is completely damaged, that is, the structure is completely broken, it is meaningless to use this criterion, because the damage is too large, it is absolutely unnecessary to judge the degree of damage in this way, which is directly visible to the naked eyes. Therefore, it is important to use the sensitivity of the CVAC criterion in the case of minor injuries. Since the CorV is not very stable when the structure is not damaged, the CVAC change is small and difficult to discriminate when judging the minor damage. Furthermore, the essence of the criterion is to pass a value that reflects the CorV correlation before and after the injury, and the specific location of the damage cannot be obtained. In summary, this criterion can only be used as a preliminary basis for judgment, and cannot be used as the final basis for discrimination.

2.5 CorV normalization

Normalization is to better observe or process data within a certain range. After the data is normalized, it can greatly facilitate the processing of data and ensure the running speed of subsequent data processing programs. There are many methods of normalization at present, such as the normalization method of the implementation interval, the Z-score normalization method, and the Sigmoid function normalization method. This paper chooses to normalize by the modulus of the vector, as shown in formula (16).

$$X_{\text{normalization}} = \frac{X_i}{\sqrt{\sum X_i^2}}$$ (16)

When the dynamic parameters of the structure are fixed, CorV is only affected by the input excitation. Although the random excitation spectrum is fixed and the power spectral density is approximately uniform, the amplitude of the input excitation will affect the amplitude of CorV. Therefore, in order to eliminate the influence of the input excitation amplitude, CorV needs to be normalized. The normalized CorV is constructed to be better at identifying damage than the original vector.

2.6 Construction of damage factor

In order to describe the location of the damage, it is necessary to construct a damage factor. Since CorV has a relatively stable characteristic under random excitation, the amplitude of the cross-correlation function of the random response between arbitrary nodes $j$ and $k$ is relatively stable in the same structure. If the structure between the node $j$ and the node $k$ is damaged, the magnitude of the cross-correlation function will also change. Therefore, the relative variation of the normalized CorV can be used to describe the degree of damage and the location of the damage. The calculation formula of the constructed damage factor can be expressed by the formula (17), which is
the relative change of the CorV of the lossless structure and the lossy structure, expressed as a percentage, and the composition of the CorV is the same measurement point and reference point in the two structures.

\[ I_{\text{CorV}} = \left| \frac{\Phi_{\text{CorV}, \text{loss}} - \Phi_{\text{CorV}, \text{lossy}}}{\Phi_{\text{CorV}, \text{loss}}} \right| \]  

(17)

In the formula: \( \Phi_{\text{CorV}, \text{loss}} \) represents CorV of a lossless structure, \( \Phi_{\text{CorV}, \text{lossy}} \) represents CorV of a lossy structure.

3. Numerical simulation of plate weld

3.1 Measuring point arrangement

In order to enable CorV-based defect detection and recognition, the model is divided into regions, as shown in Figure 1. The black dots and numbers in the figure represent the measurement points of the response, and the circled numbers represent the reference points needed to generate the cross-correlation function.

![Figure 1 Model of butt welding structure and measurement point arrangement](image-url)

In the welded structure, the weld is most prone to defect. Therefore, it is more appropriate to arrange the measuring points around the weld. It can pay attention to the change of the mechanical properties of the weld zone and analyze the effect of random excitation on weld defects. Since the acceleration response is more sensitive to random excitation signals than displacement and velocity, the acceleration response is used to generate CorV.

In order to verify the effectiveness of the CorV-based method in the identification of defects in the plate weld structure, it will be divided into single defect, double defects and multiple defects. The numerical simulation is carried out under the finite element software ABAQUS.
3.2 Single reference point single defect conditions
This section numerically simulates the conditions in which welds have a single defect and analyzes the effect of different reference points on the recognition results. In order to detect weld defects, it is only necessary to select the measurement points numbered 1 to 19 in Figure 1. Since the response from these points is through the weld, compared with the 20~38 measuring points, they are more reflective of the problems in the structure.

In order to verify that the method can identify the single damage condition of the weld, 50% of the defect is respectively set for each weld unit (simulated by reducing the elastic modulus of the weld unit), and then a sample function of a stationary random excitation is applied to the excitation point, the excitation sample is shown in Figure 2. The recognition result of selecting reference point 2 is shown in Figure 3.

![Figure 2: The time domain function of the stimulus samples](image)

![Figure 3: Results of various conditions with single 50% defect (damaged element 14~19)](image)

The magnitudes of the various graphs in Figure 3 are the relative changes in the CorV of the defective condition and the CorV of the flawless condition. This relative change is the damage factor. It can be clearly seen from the figure that the maximum value of the relative variation of CorV varies with weld cell defects, showing a relatively obvious law.

From formula (15), the confidence criterion CVAV can be calculated. This value actually reflects the influence of defects at different positions of the weld on the magnitude vector of the cross-correlation function. When the defect is far away from the excitation point, the influence is small, so its CVAC is high, conversely, when the defect is closer to the excitation point, the influence on the magnitude vector of the cross-correlation function is larger and the CVAC value is smaller. In fact, it is also possible to judge the approximate defect position by this value, that is, the closer to the excitation point, the smaller the CVAC value. In Figure 1, the excitation point is near the measurement point 10, so when there is a defect in the 10th unit, its CAVC value should be the smallest, as shown in Figure 4.
3.3 Single defect conditions at different reference points

The above result is based on the cross-correlation function amplitude vector constructed by reference point 2. As can be seen from Figure 3, the identification of the 50% defect of the 15th unit is the most obvious, so this case is selected for research. In order to study the influence of different reference points on the identification of individual weld defects, the recognition results of the correlation function amplitude vector constructed using reference points 1~10 in Figure 1 are compared and analyzed, as shown in Figure 5.

It can be seen from the test results in the above figure that selecting different reference points can determine the defective unit, but reference points 1~6 can better identify the defects, while reference points 7~10 are relatively inferior, this is because the variation of many similar cross-correlation functions are included in the detection results. When selecting the reference point of the construction, you should select a measurement point similar to the position of the reference point 3, 4, which can have a better detection effect.
3.4 Double defect conditions
This section will test the welds with double defect conditions, as shown in Table 1, where the degree of unit defects is all 50%, by reducing the elastic modulus of the unit.

| Condition 1 | Condition 2 | Condition 3 | Condition 4 |
|-------------|-------------|-------------|-------------|
| Unit 10、11  | Unit 2、18  | Unit 2、3   | Unit 9、18  |

The above are four representative working conditions. The method of constructing the cross-correlation function magnitude vector with the reference point is still adopted, and the 3rd point is selected as the reference point. The detection result of the double defect condition is shown in Figure 6.

![Image of Figure 6 showing results of conditions with double defects](image)

It can be seen from Figure 6 that the working condition 2 and the working condition 3 can well detect the position of the defective unit, and in contrast, the detection results of the working condition 1 and the working condition 4 can obtain the position of the defect to some extent. However, it is not so obvious. The commonality is that there are defective units close to the excitation point in both cases. The existence of this defect has a great influence on the response of other detection points, which has a certain degree of influence on the detection results. The solution is to change the position of the excitation point, thereby increasing the distance between the weld defect unit and the excitation point, and improving the recognition degree of the detection.

3.5 Multiple defect conditions
This section will test the situation of multiple defects in the weld and analyze the results. The multi-defective conditions are shown in Table 2. The degree of defects is still 50% of the elastic modulus of the weld.

| Condition 1 | Condition 2 |
|-------------|-------------|
| Unit 3、7、12| Unit 1、2、7、18|
When constructing the cross-correlation function, the 3rd point is still selected as the reference point, and the test result is shown in Figure 7.

![Figure 7 Results of conditions with multiple defects](image)

As can be seen from the above figure, the cross-correlation function amplitude vector method can detect the defects of working condition 1 and working condition 2.

4. Conclusion
In this paper, a method based on the cross-correlation function amplitude vector to detect the defects of the plate weld structure is proposed. The defect in the plate weld is regarded as the decrease of Young's modulus. By applying a smooth random excitation to the plate, the vibration acceleration response of each measurement point of the welded joint unit is collected, and the single defect, double defects and multiple defects of the plate weld are respectively tested, then the following conclusions are obtained.

In a single defect condition, the defect of each weld unit can be identified, and the relative damage factor of the damaged unit is always the highest, but the detection point closer to the damaged unit is easily affected, and sometimes it is misjudged. When the position of the defect is different, it will affect the detected CVAV value. The closer to the excitation point, the greater the influence on the magnitude vector of the cross-correlation function is, and the smaller the CVAV value is. Therefore, based on the result of the detection, the damaged unit can be accurately positioned. When detecting multiple defect conditions, the detection result can also determine the position of the defect, but when the defect is too close to the excitation point, it will affect the detection of the adjacent unit.

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