TESTING THE TRANSITION LAYER MODEL OF QUASI-PERIODIC OSCILLATIONS IN NEUTRON STAR X-RAY BINARIES

XUE-BING WU1,2

Received 2000 May 8; accepted 2001 January 9

ABSTRACT

We compare the theoretical predictions of the transition layer model with some observational features of quasi-periodic oscillations (QPOs) in neutron star X-ray binaries. We found that the correlation between horizontal branch oscillation (HBO) frequencies and kilohertz (kHz) QPO frequencies, the difference between the low-frequency QPOs in atoll sources and HBOs in Z sources, and the correlation between the frequencies of low-frequency QPOs and break frequencies can be well explained by the transition layer model, provided the neutron star mass is around 1.4 $M_\odot$ and the angle between magnetosphere equator and accretion disk plane is around 6°. The observed decrease of peak separation between the two kHz QPO frequencies with the increase of the kHz QPO frequencies and the increase of QPO frequencies with the increase of inferred mass accretion rate are also consistent with the theoretical predictions of the transition layer model. In addition, we derive a simple equation that can be adopted to estimate the angle ($\delta$) between magnetosphere equator and accretion disk plane by use of the simultaneously observed QPO frequency data. We estimate this angle, in the range of $4^\circ$–$8^\circ$, for five Z sources and two atoll sources. The nearly constant $\delta$ value for each source, derived from the different sets of simultaneously observed QPO frequency data, provides a strong test of the theoretical model. Finally, we suggest that similar transition layer oscillations may be responsible for the observed QPOs in accretion-powered millisecond X-ray pulsars and Galactic black hole candidates.

Subject headings: accretion, accretion disks — binaries: general — stars: neutron — stars: oscillations — X-rays: stars

1. INTRODUCTION

Quasi-periodic oscillations (QPOs) have been found to be a very common feature of accreting systems around compact objects. In neutron star low-mass X-ray binaries (LMXBs) with low magnetic fields, at least four types of QPOs, namely, the 15–60 Hz horizontal branch oscillations (HBOs; van der Klis et al. 1985), the 6–20 Hz normal branch oscillations (NBOs; Middleditch & Priedhorsky 1986), and the 300–1200 Hz lower and upper kilohertz (kHz) QPOs (van der Klis et al. 1996), have been discovered by several X-ray satellites, including EXOSAT, Ginga, and the Rossi X-Ray Timing Explorer (RXTE). In spite of the different evolution tracks in the X-ray color-color diagram (Hasinger & van der Klis 1989), the low-frequency QPOs in atoll sources seem to have features similar to those of the HBOs in Z sources (Strohmayer et al. 1996; Wijnands & van der Klis 1997). With the advantage of RXTE, 20 sources have now shown kHz QPOs, and 18 of them have shown two simultaneous kHz peaks (see van der Klis 2000 for a review). The peak separations of the two kHz QPOs have been observed to decrease considerably in five sources when the kHz QPO frequencies increase (van der Klis et al. 1997; Méndez et al. 1998a, 1998b; Ford et al. 1998; Méndez & van der Klis 1999; Markwardt, Strohmayer, & Swank 1999). Other sources may show similar variations (Psaltis et al. 1998). It has been found that the HBO frequencies of Z sources and the frequencies of low-frequency QPOs in atoll sources are well correlated with the kHz QPO frequencies (Psaltis, Belloni, & van der Klis 1999a). Similar correlation of low- and high-frequency QPOs might also exist for some other neutron star systems and Galactic black hole candidates. In addition, tight correlations of the HBO frequencies of Z sources and of the frequencies of low-frequency QPOs in atoll sources and black hole candidates with the break frequencies shown in their power density spectra have also been found recently (Wijnands & van der Klis 1999). These two correlations strongly imply that similar mechanisms may be responsible for the break frequency, the low-frequency QPOs, and the high-frequency QPOs in both neutron stars and black hole X-ray binaries.

There are several QPO models that are currently in debate. Due to the difficulty in producing kHz QPOs, the previous magnetospheric beat-frequency model proposed by Alpar & Shaham (1985) has been modified as the sonic point beat-frequency model (Miller, Lamb, & Psaltis 1998). In this model the upper kHz QPOs arise from the clumps moving with the Keplerian frequency at the sonic point that is near the innermost stable circular orbit of a neutron star. The lower kHz QPO frequency is thus the beat frequency between the upper kHz QPO frequency and the neutron star spin frequency. However, this model, if without further modifications, predicts a constant peak separation between the two kHz QPO frequencies, which is contrary to the observations. Moreover, recent investigation of the boundary layer accretion onto neutron stars indicates that the accretion flow is always subsonic and that there is probably no such sonic point at all near the neutron star surface (Popham & Sunyaev 2001). This leads to further doubt about the sonic-point beat-frequency model. Another attractive model is the relativistic precession model (RPM) proposed by Stella & Vietri (1998, 1999), who identified the upper kHz QPO frequency with that of a Keplerian orbit in the disk and the lower kHz QPO frequency and HBO frequency with, respectively, the periastron precession fre-
frequency and twice the nodal precession frequency of this orbit. This model can qualitatively explain the observed correlations between these three QPO frequencies (Stella, Vietri, & Morsink 1999), but the precise match requires some fine tuning of additional free parameters such as the orbital eccentricity and the periastron distance. Moreover, in order to explain the HBOs and their correlation with orbital eccentricity and the periastron distance. Moreover, some fine tuning of additional free parameters such as the neutron star equation of state (Psaltis et al. 1999b).

In addition, the RPM model requires neutron star mass in the range of 1.9–2.2 \( M_\odot \) in order to explain the correlation between low and high QPO frequencies, which is significantly larger than the measured value, 1.1 \( M_\odot < M < 1.6 \ M_\odot \), for neutron stars in radio pulsar binaries (Thorsett & Chakrabarty 1999; Finn 1994). The problem of having a neutron star with mass larger than 2.2 \( M_\odot \) arises because it would require accretion of material of at least 0.8 \( M_\odot \), which means that these X-ray binaries would have to be rather old and would have to have spun up rapidly for 90% of their lifetime (Lai, Lovelace, & Wasserman 1999). The same problem probably exists for the new model proposed by Psaltis & Norman (2000), who derived characteristic QPO frequencies similar to those in RPM in the inner accretion disk.

Another alternative model for QPOs in neutron star X-ray binaries is the disk transition layer model (TLM) proposed recently by Titarchuk & Osherovich (1999), Osherovich & Titarchuk (1999a, 1999b), and Titarchuk, Osherovich, & Kuznetsov (1999). The geometry of the transition layer model has been clearly shown in Figure 1 of Titarchuk et al. (1999). In this two-oscillator model, the lower kHz QPO frequency is identified with the Keplerian frequency in a viscous transition layer between the Keplerian disk and neutron star surface. Assuming that the magnetic field of the neutron star is low (in the range of \( 10^8-10^9 \) G), the Alfvén radius, depending mainly on the magnetic field and accretion rate, is at most several radii of the neutron star. The size of the magnetosphere, as well as that of the transition layer, is thus very small. Viscous oscillations in this transition layer produce the observed noise break and a low-frequency QPO. In addition, the adjustment of the Keplerian disk to the sub-Keplerian layer may create conditions favorable for the formation of a hot blob in the transition layer. This blob, when thrown out into the rotating magnetosphere from the transition layer, participates in the radial oscillations with Keplerian frequency. Under the influence of the Coriolis force, such a blob, assumed to be a Keplerian oscillator, oscillates both radially and perpendicular to the disk. This produces two harmonics of another low-frequency QPO (HBO in Z sources) and the upper kHz QPO. These six different frequencies have been identified in two atoll sources (4U 1728–24 and 4U 1702–42) and Z source Sco X-1 (Titarchuk et al. 1999; Osherovich & Titarchuk 1999a, 1999b). The observed correlations of QPO frequencies for these sources seem to be consistent with the theory of TLM.

In this paper, we derive more theoretical predictions for TLM and compare them with the available observational data of QPOs in neutron star X-ray binaries. The consistency between model predictions and observational data without assuming larger neutron star mass and a stiff equation of state suggests that TLM is perhaps a more competitive model than the others. We also suggest that a similar model may be applied to the cases of millisecond X-ray pulsars and Galactic black hole candidates.

### 2. CHARACTERISTIC QUASI-PERIODIC OSCILLATION FREQUENCIES IN THE TRANSITION LAYER MODEL

As indicated by Titarchuk & Osherovich (1999) and Osherovich & Titarchuk (1999b), there are six characteristic QPO frequencies in TLM. First, the fundamental frequency is assumed to be the Keplerian frequency, namely,

\[
v_k = \frac{1}{2\pi} \left( \frac{GM}{R^3} \right)^{1/2},
\]

where \( G \) is the gravitational constant, \( M \) is the mass of the neutron star, and \( R \) is the radius of an orbit in the transition layer. The linear Keplerian oscillator with frequency \( \Omega_k/2\pi \) in the frame of reference rotating with rotational frequency \( \Omega/2\pi \) (not perpendicular to the disk plane) is known to have an exact solution describing two branches of oscillations (see § 39 in Landau & Lifshitz 1960). Assuming \( \Omega \) is constant, the dispersion relation of the frequency of oscillation \( \omega \) can be derived as

\[
\omega^4 - (\Omega_k^2 + 4\Omega^2)\omega^2 + 4\Omega^2\Omega_k^2 \sin^2 \delta = 0,
\]

where \( \delta \) is the angle between \( \omega \) and the normal to the Keplerian oscillation. If \( \delta \) is assumed to be small, one can obtain simple expressions of two characteristic oscillation frequencies from the dispersion relation. The radial eigenmode has a frequency

\[
v_r = \frac{\Omega}{\pi} v_k \sin \delta.
\]

These three characteristic frequencies, \( v_k, v_h, \) and \( v_L \), are assumed to account for the frequencies of lower kHz QPOs, upper kHz QPOs, and HBOs in Z sources, respectively. Because the second harmonics of HBOs are often seen in Z sources, they are interpreted as \( 2v_L \), counting as the other characteristic frequency.

From equations (3) and (4), we can derive several very useful relations among these characteristic frequencies. First, the correlation between the HBO frequency and the two kHz QPO frequencies can be expressed as

\[
v_L = v_k \sqrt{1 - \left( \frac{v_k}{v_h} \right)^2 \sin \delta},
\]

Second, the difference between the two kHz QPO frequencies can be related to \( v_h \) as

\[
\Delta v = v_h - v_k = v_k \left[ \sqrt{1 + \left( \frac{\Omega}{\pi v_k} \right)^2} - 1 \right].
\]
Third, if the Keplerian oscillation is aligned with the accretion disk plane, the angle between the magnetosphere equator and the disk plane can be determined by

\[
\delta = \arcsin \left( \frac{v_k v_L}{v_k \sqrt{v_k^2 - v_r^2}} \right). \tag{7}
\]

Because the three frequencies in the right of equation (7) are all observational quantities, this equation perhaps suggests a useful method to determine \(\delta\) for neutron star X-ray binaries.

The other two characteristic frequencies in TLM are related to the two typical timescales in the transition layer. One is the radial drift timescale of matter through the layer bounded between the disk and neutron star, namely, \(t_d \sim (R_{\text{out}} - R_0)/v_r\), where \(R_0\) is the radius of the neutron star (taken to be 6\(GM/c^2\) in this paper) and \(v_r\) is the radial drift velocity. This timescale gives the characteristic viscous frequency as

\[
v_v \simeq \frac{\gamma v}{(R_{\text{out}} - R_0) R} \approx 2\pi \gamma v_K \left( \frac{H}{R} \right)^2 \frac{r_{\text{out}}}{r_{\text{out}} - 1}, \tag{8}
\]

where \(r_{\text{out}}\) is defined as \(r_{\text{out}} = R_{\text{out}}/R_0\) and \(\gamma\) is the Reynolds number given by \(\gamma = M/4\pi \rho v^2 = v_r/R v_K\), where \(M\) is the mass accretion rate and \(\rho\) is the mass density. We adopt the standard viscosity prescription \(v = \alpha c_s H = \alpha \Omega_c H^2\), where \(c_s\) is the local sound speed and \(H\) is the scale height (Shakura & Sunyaev 1973). The relation between \(\gamma\) and \(r_{\text{out}}\) can be obtained by considering the radial transport of angular momentum and the boundary conditions at \(R_0\) and \(R_{\text{out}}\) (Titarchuk & Osherovich 1999). Assuming that at \(R_{\text{out}}\), \(\Omega = \Omega_{\text{out}}\) and at \(R_0\), \(\Omega = \Omega_0\) and \(d\Omega/dt = d\Omega_0/dt\) and defining \(A = \Omega_{\text{out}}^2/\Omega_0^2\) (where \(\Omega_0\) is the Keplerian angular velocity at \(R_0\)), \(r_{\text{out}}\) can be determined by the following equation:

\[
\left( \frac{3}{2} - \gamma \right) A r_{\text{out}}^2 - (\gamma - 2) r_{\text{out}}^2 + \frac{1}{2} A = 0. \tag{9}
\]

Another characteristic timescale in the transition layer is the diffusion timescale given by \(t_{\text{diff}} \sim (R_{\text{out}} - R_0)^2/\ell_d v_r\), where \(\ell_d\) is the diffusion length scale in the transition layer. This gives the break frequency as

\[
v_{\text{break}} \simeq \frac{l_d}{(R_{\text{out}} - R_0) v_r} = \frac{l_d}{R_0} \frac{v_r}{r_{\text{out}} - 1}. \tag{10}
\]

This expression of break frequency is similar to that given in equation (29) of Psaltis & Norman (2000) if \(l_d\) is replaced by \(R\). Similar to the argument given by Psaltis & Norman (2000), the reason that \(v_{\text{break}}\) is observed as a frequency break may be that the transition layer acts as a filter band so that the response is constant if the oscillation frequency is lower than the inverse diffusion timescale and the response decreases above it. Titarchuk & Osherovich (1999) described \(l_d\) as the mean free path of a particle; however, this introduced another uncertain quantity in the TLM theory. In this paper, using a similar approach adopted in the standard accretion disk model (Shakura & Sunyaev 1973), we assume that \(l_d\) is smaller than the vertical and radial size of the transition layer and take \(l_d/R_0\) as a constant less than unity for simplicity (in § 3.3 we will show that this assumption is consistent with the observed data of the break frequency). Equations (8), (9), and (10) can be used to calculate the correlations between other low-frequency QPO (besides HBO) frequencies, break frequencies, and Keplerian frequencies. The original descriptions of viscous and break frequencies shown in equations (12) and (13) in Titarchuk & Osherovich (1999) have been improved here (see eqs. [8] and [10]) by considering a more explicit expression of the radial drift timescale.

### 3. Comparisons of Model Predictions with the Observational Data

There are many observations made by several X-ray satellites on QPOs in neutron star X-ray binaries. But, only very recently have the correlations among the frequencies of low- and high-frequency QPOs, the variation of kHz QPO frequencies, and the difference between HBOs and other low-frequency QPOs been systematically studied with plenty of QPO data. In this section we try to compare the predictions from TLM, as indicated by the equations in the section above, with the observational results.

#### 3.1. Correlation between HBOs and kHz QPOs in Z Sources

Recently, Psaltis et al. (1999a) have compiled the low- and high-frequency QPO data of Z sources and atoll sources, as well as black hole candidates. They found a tight correlation between HBO frequency and the frequency of lower kHz QPOs. The density points for the Z sources are consistent with the empirical relation \(v_{\text{HBO}} \sim (42 \pm 3\text{ Hz})[v_1/(500\text{ Hz})]^{0.95 \pm 0.16}\) when the frequency of lower kHz QPO \(v_1\) is less than 550 Hz. When \(v_1\) is larger than 500 Hz, \(v_{\text{HBO}}\) increases slowly with \(v_1\). This feature seems to be difficult to fit by RPM.

In Figure 1, we plot \(v_{\text{HBO}}\) and \(v_1\) data for five Z sources and compare this with the predictions from both RPM and TLM. The fit parameters for RPM \((M = 1.95\ M_\odot, a/M = 0.22)\) are taken from Stella et al. (1999), and \(a\) is the spin parameter of the neutron star. It is evident that RPM...

---

**Figure 1**—Correlation between HBO frequencies and lower kHz QPO frequencies of Z sources. The solid, short-dashed, and long-dashed lines represent TLM predictions with parameters \(\Delta v = 300\text{ Hz}\) and \(\delta = 5.5, 6.5\), and 6.5, respectively. The dotted line represents the prediction of RPM with \(M = 1.95\ M_\odot\) and \(a/M = 0.22\). The error bars, if not shown, are comparable to the size of the symbols.
predicts a steeper correlation than the observed one and cannot explain the flatness of such a correlation when \( \nu_1 > 500 \) Hz. Any fine tuning of the fit parameters \( M \) and \( a/M \) cannot provide a better fit. However, when we plot the theoretical curves of TLM according to equation (5) by choosing quite reasonable parameters \( \nu_b - \nu_K = 300 \) Hz and \( \delta \) around 6°, the match of model prediction and observational data is quite good. In addition, we also plot the observed correlation between \( \nu_{\text{HBO}} \) and upper kHz frequency \( \nu_2 \) for five Z sources in Figure 2. In this case, RPM cannot fit the observational data if we still choose the parameters \( M = 1.95 \, M_\odot \), \( a/M = 0.22 \). Even when we choose the fit parameters \( M = 1.95 \, M_\odot \), \( a/M = 0.30 \), the fit is still not satisfactory. Therefore, our comparison shows that RPM, at least in its current version, is difficult to fit to the correlations between simultaneously observed HBO frequencies and kHz QPO frequencies. On the contrary, from Figures 1 and 2 we can see clearly that TLM predicts rather flatter slopes than RPM. With the same parameters, TLM can provide good fits to both the \( \nu_{\text{HBO}}-\nu_1 \) and \( \nu_{\text{HBO}}-\nu_2 \) correlations.

For some sources (especially GX 17 + 2 and GX 5 – 1) in Figure 1, it seems that the slope is steeper than that predicted by TLM and perhaps matches better with the RPM prediction. However, we must note that the TLM predictions plotted in Figures 1 and 2 were based on a very simple assumption \( (\nu_b - \nu_K = 300 \) Hz). In fact, the peak separation between the two kHz QPO frequencies, \( \nu_b - \nu_K \), is certainly not constant for individual sources (see § 3.4). If we take the observed values of \( \nu_b \) and \( \nu_K \) directly and use equation (5) to predict \( \nu_b \), the agreement with the observations will be closer for the individual sources plotted in Figures 1 and 2. On the other hand, with reasonable parameters, we note that RPM is unable to fit both the observed correlations shown in Figures 1 and 2 consistently. Even if the RPM prediction could better match the slopes of GX17 + 2 and GX 5 – 1 in Figure 1, it would fail to match the observed slopes for these sources in Figure 2 if the same parameters were used.

3.2. Difference between Low-Frequency QPOs in Atoll Sources and HBOs in Z Sources

Although there are some reports of the similarity of QPOs in atoll sources to HBOs in Z sources (Strohmayer et al. 1996; Wijnands & van der Klis 1997), it can be clearly seen from Figure 2 in Psaltis et al. (1999a) that the frequencies of these low-frequency QPOs (10–50 Hz) of atoll sources have a steeper dependence on the lower kHz QPO frequency than those of HBOs in Z sources. In Figure 3, we plot this correlation and compare it with the empirical relation found for HBOs in Z sources. Obviously, the difference between these low-frequency QPOs and HBOs is large. It is therefore quite possible that these low-frequency QPOs may have an origin different from that of HBOs in Z sources. Actually, similar low-frequency QPOs (usually seen as extra noise components) have been found simultaneously with HBOs in Z sources Sco X-1 and GX 17 + 2 (van der Klis et al. 1997; Titarchuk et al. 1999; Wijnands & van der Klis 1999), while similar HBO-like frequencies have been identified in two atoll sources, 4U 1728 – 34 and 4U 1702 – 42 (Titarchuk et al. 1999; Osherovich & Titarchuk 1999b).

In Figure 3, it can be clearly seen that these low-frequency QPOs in Z source Sco X-1 follow the same correlation with \( \nu_1 \) as other atoll sources. This probably suggests that the low-frequency QPOs in both Z and atoll sources have the same physics origin, and they are obviously different from HBOs. In TLM, the frequency of this kind of QPO is described as the viscous frequency in the transition layer; therefore, the mechanism for producing the low-frequency QPOs is clearly different from that of HBOs. Using equations (8) and (9) and assuming \( \alpha(H/R)^2 = 7.0 \times 10^{-4}, M \)
around 1.4 $M_\odot$, and $r_{\text{out}}$ in the range of 1.2–1.8 (namely, $\gamma$ in the range of 6–20), we plot the model prediction of TLM in Figure 3 (the result is insensitive to the spin frequency of the neutron star; $v_0 = 300$ Hz is assumed to make the plot). We can see the close agreement between theory and observations. For KS 1731–26 and 4U 1735–44, the data points are a little away from the others. They are likely associated with the NBOs of Sco X-1 (Psaltis et al. 1999a), though they can still be fitted by assuming a smaller value of $\alpha(H/R)^2$. However, we note that for 4U 1735–44, recent observations indicate the existence of a 67 Hz QPO together with a possible 900 Hz lower kHz QPO (Wijnands et al. 1998c). If we plotted this point in Figure 3, it would also be located in the range of the TLM prediction.

In Figure 3, only the data of 1996 February 16 for 4U 1728–34 are plotted because in these observations the low kHz QPOs can be clearly distinguished from the simultaneously observed upper kHz QPOs (Ford & van der Klis 1998). The agreement of the observational data with our predictions assuming $M$ around 1.4 $M_\odot$ suggests that the compact object in 4U 1728–34 may not be a strange star as argued by Li et al. (1999). We note that their conclusion is based on the fit result $d_k = 1.03$ by Titarchuk & Osherovich (1999), who used less accurate expression for the viscous frequency (see § 2) and ambiguous data for the lower kHz QPO frequencies of 4U 1728–34.

3.3. Correlation between Frequencies of Low-Frequency QPOs and Break Frequencies in Z Sources and Atoll Sources

A recent detailed study on the broadband power density spectra of X-ray binaries indicates that the frequency (1–60 Hz) of low-frequency QPOs (including HBOs) have a strong correlation with the break frequency ($v_{\text{break}}$; 0.1–30 Hz) of both Z sources and atoll sources, as well as, possibly, black hole candidates (Wijnands & van der Klis 1999). However, the HBO frequencies of Z sources follow a relationship with $v_{\text{break}}$ different from that of the low-frequency QPOs of other sources. Such a difference has not been satisfactorily explained so far (see, however, Titarchuk et al. 1999 for a possible explanation, but note that they mistakenly plotted the HBO frequencies of Z sources as the viscous frequencies in their Fig. 4).

In TLM, because HBO and viscous frequencies come from different mechanisms, it is natural that they follow different correlations with the break frequencies. According to equation (5) and equations (8)–(10), we can derive the predicted correlation of HBO and viscous frequencies with break frequencies. In Figure 4, we plot these predictions and compare them with the observational data. For atoll sources, our prediction closely agrees with the relation $v_{\text{break}} = 0.044 v_{L\text{F}}^{1.63}$, which was derived from the observed correlations $v_{L\text{F}}-v_{\text{break}}$ and $v_{\text{break}}-v_{\text{Hz}}$ for atoll source 4U 1728–34 (Ford & van der Klis 1998). The viscous frequencies of other atoll sources and Z source Sco X-1 also follow nearly the same $v_{L\text{F}}-v_{\text{break}}$ correlation. We adopted $I_p/R_0 = 0.3$ in equation (9) and the same parameters $[\alpha(H/R)^2 = 7.0 \times 10^{-4}, M = 1.4 M_\odot]$ as above to derive this correlation. The value $I_p/R_0 = 0.3$ is appreciated by the observed data and also consistent with our assumption that the diffusion length scale is smaller than the vertical and radial size of the transition layer. Note that $I_p = 0.3 R_0 < H$ means that $H/R$ may be larger than 0.3. This is quite possible if the magnetic field is low and the radial size of the transition layer is small. If we take $\alpha(H/R)^2 = 7.0 \times 10^{-4}$, this requires that $\alpha$ should be less than 0.01. Such a lower value of $\alpha$ is still well within the range of the viscosity parameter discussed in the accretion disk model with a boundary layer. Assuming $v_{\text{break}} = 300$ Hz and $\delta = 4.5, 6, 7, 5$, respectively. The solid line represents the prediction with $\alpha(H/R)^2 = 7.0 \times 10^{-4}$ and $M = 1.4 M_\odot$. The term $I_p/R_0 = 0.3$ was assumed to derive the theoretical break frequency. The dotted line represents the empirical relation $v_{\text{break}} = 0.044 v_{L\text{F}}^{1.63}$ derived for 4U 1728–34. Other atoll sources are plotted as open triangles, diamonds, and squares.

3.4. Variation of Peak Separation of the Two kHz QPOs

The observed variation of difference of the two kHz QPO frequencies in at least one Z source, Sco X-1 (van der Klis et al. 1997) and four atoll sources, 4U 1728–34, 4U 1608–52, 4U 1735–44, and 4U 1702–43 (Mendez et al. 1998a, 1998b; Ford et al. 1998; Mendez & van der Klis 1999; Markwardt et al. 1999), seems to exclude any QPO model that predicts constant peak separation. The magnetospheric beat-frequency model, as well as the sonic-point beat-frequency model, probably cannot be responsible for the kHz QPOs, unless further modifications are made.

Recent precise measurements of kHz QPO frequencies show that the peak separation $\Delta v = v_2 - v_1$ decreases considerably when kHz QPO frequencies increase (van der Klis 2000). This feature cannot be well fitted by RPM, which predicts a much steeper decrease of $\Delta v$ when $v_2$ increases to more than 1000 Hz and a significant decrease of $\Delta v$ when $v_2$ decreases to less than 700 Hz (Stella & Vietri 1999). The latter effect has not been observed yet. A precise match between RPM and observations requires some additional assumptions such as the dependence of orbital eccentricity on the orbital frequency. In TLM, however, the decrease of...
\(\Delta \nu\) with the increase of kHz QPO frequency is a natural result even if we simply take \(\Omega = \) constant (see eq. [6]). In Figure 5, we show the \(\Delta \nu - \nu_1\) relation for all five sources observed with \(\Delta \nu\) variations. When we plot the TLM prediction assuming constant \(\Omega/2\pi\) (340, 365, and 380 Hz) in equation (6), we see that even at this simple assumption the theoretical predictions are in close agreement with most observational data of Sco X-1, 4U 1708–52, 4U 1735–44, and 4U 1702–43. However, it is clear that this simple assumption cannot explain the data of 4U 1728–34. In fact, the matter in the magnetosphere probably experiences differential rotation, and \(\Omega\) may not be a constant. If we assume that the angular velocity could be expressed as \(\Omega/2\pi = v_0 - (c_1 v_K^{1/2} - c_2 v_2^{1/2})^2\) (Osherovich & Titarchuk 1999a), where \(v_0\), \(c_1\), and \(c_2\) are constants, and take \(v_0\) = 405 Hz, \(c_1 = 0.26 \text{ Hz}^{-1/\nu}\), and \(c_2 = 3.4 \times 10^{-5} \text{ Hz}^{-3/2}\), we can see that the prediction of peak variation of twin kHz QPO frequencies is close to the observational data for 4U 1728–34. In summary, it seems that the variation of peak separation of the two kHz QPOs is also consistent with the theoretical prediction of TLM.

4. OBSERVATIONAL DETERMINATION OF MISALIGNMENT OF NEUTRON STAR MAGNETOSPHERE AND ACCRETION DISK

There are only a few methods that can be used to estimate the magnetic inclination angle between the magnetic axis and neutron star rotation axis for radio pulsars (Lyne & Manchester 1988). It may be more difficult to measure these angles for X-ray pulsars and low-mass X-ray binaries. However, according to TLM (see eq. [7]), if we have simultaneous observational data for frequencies of both kHz QPOs and HBOs, we may estimate the angle between magnetosphere equator and disk plane for neutron star X-ray binaries. We note that equation (7) is derived under the assumption that \(\delta\) is small. In fact, a more accurate equation, which is also applicable to arbitrary values of \(\delta\), can be directly obtained from the dispersion relation of the Keplerian oscillator (see eq. [2]), namely,

\[
\delta = \arcsin \left( \frac{v_2 v_{\text{HBO}}}{v_1 \sqrt{v_2^2 + v_{\text{HBO}}^2 - v_1^2}} \right),
\]

where \(v_1\), \(v_2\), and \(v_{\text{HBO}}\) are the observed lower and upper kHz QPO and HBO frequencies, respectively, and we assume that they correspond to the Keplerian frequency and the two characteristic oscillation frequencies derived from the dispersion relation (see §2). In the limit \(v_{\text{HBO}} \ll v_2\), equation (11) is identical to equation (7). Note that equation (11) can also be used as a test of TLM because the derived values of \(\delta\) using a different set of simultaneously observed QPO frequency data for each source should be nearly the same (Titarchuk & Osherovich 2000).

Up to now, simultaneous frequency data of both kHz QPOs and HBOs have been obtained with RXTE for five Z sources. In the two atoll sources with simultaneous data for the two kHz QPO frequencies, HBO-like frequencies have recently been identified (Titarchuk et al. 1999; Osherovich & Titarchuk 1999b). Therefore, the misalignment of magnetosphere and accretion disk can be estimated for these sources according to equation (11). In Table 1, we list the observational data and the derived average values of \(\delta\) for them. It is clear that all of them have smaller \(\delta\)-values and are therefore consistent with the assumptions in §2. Especially for Z sources, their \(\delta\)-values are in a very narrow range from 5.4 to 6.4. Moreover, the standard derivations

| Name | Type | \(v_2\) (Hz) | \(v_1\) (Hz) | \(v_{\text{HBO}}\) (Hz) | Reference | \(\langle \delta \rangle\) (deg) |
|------|------|-------------|-------------|----------------|----------|-------------|
| Cyg X-2 | Z | 856 | 532 | 46 | 1, 2 | 6.34 ± 0.14 |
| GX 17+2 | Z | 640–1090 | 480–780 | 25–61 | 1, 3 | 6.30 ± 0.37 |
| Sco X-1 | Z | 870–1080 | 550–660 | 41–48 | 1, 4 | 5.44 ± 0.22 |
| GX 5-1 | Z | 500–890 | 210–660 | 17–51 | 1, 5 | 5.80 ± 0.62 |
| GX 340+0 | Z | 560–820 | 250–500 | 22–45 | 1, 6 | 6.40 ± 0.45 |
| 4U 1728–34 Atoll | 930–1130 | 600–790 | 52–90 | 1, 7, 8 | 8.01 ± 1.06 |
| 4U 1702–42 Atoll | 1000–1080 | 660–770 | 33–40 | 9, 10 | 3.96 ± 0.23 |

REFERENCES.—(1) Psaltis et al. 1999a; (2) Wijnands et al. 1998a; (3) Wijnands et al. 1997; (4) Van der Klis et al. 1997; (5) Wijnands et al. 1998b; (6) Jonker et al. 1998; (7) Strohmayer et al. 1996; (8) Ford & van der Klis 1998; (9) Markwardt et al. 1999; (10) Osherovich & Titarchuk 1999b.
neutron star mass (around 1.4 and radius around 12 km. Indeed, the in TLM can still be a normal neutron star with mass cession model (Stella & Vietri 1998, 1999). The compact star frequency model (Miller et al. 1998) and the relativistic pre-

a larger neutron star mass and a sti equation of state as frequency. Therefore, it is not necessary in TLM to involve the upper kHz QPO frequency, is described as a Keplerian treatment of peak separation of the two kHz QPO frequencies, TLM is so far explaining the correlation of HBO frequencies with both the lower and upper kHz QPO frequencies, TLM is far the only model that can self-consistently explain these observed correlations. In addition, TLM suggests some physics mechanisms for producing low-frequency QPOs and break frequencies in X-ray binaries and can explain the observed correlation between the frequencies of low-frequency QPOs and the break frequencies. On the con-

and break frequencies in X-ray binaries and can explain the physics mechanisms for producing low-frequency QPO frequencies in X-ray binaries. Because the size of magneto-

quencies with lower kHz QPO frequencies than that lead to a steeper correlation of low-frequency QPO fre-

We note that similar QPO frequencies in the range of 1–400 Hz have been observed in the X-ray millisecond pulsar SAX J1808.4–3658 (Wijnands & van der Klis 1998a) and Galactic black hole candidates GRO J1655–40 (Remillard et al. 1999b), GRS 1915+105 (Morgan, Remillard, & Greiner 1997), XTE J1550–564 (Remillard et al. 1999a), and XTE J1859+226 (Cui et al. 2000). It is still not clear whether the 67–400 Hz QPOs observed in these objects are identical to the kHz QPOs in neutron star LMXBs. But if we assume that these hundred hertz QPOs are similar to the lower kHz QPOs, the correlations of their frequencies of low-frequency QPOs (1–20 Hz) and break frequencies with the frequencies of these kHz QPOs seem to be similar as found for atoll sources (see Psaltis et al. 1999a; Wijnands & van der Klis 1999). In fact, if we plotted the QPO frequency data for SAX J1808.4–3658 in Figures 3 and 4, these points would be located in the lower left parts of both figures but would still be consistent with the correlations for atoll sources. Their positions in these two figures would also support that SAX J1808.4–3658 is similar to the low-luminosity LMXBs with low accretion rate (Wijnands & van der Klis 1998a). Its estimated magnetic field strength, \( B < (2–6) \times 10^6 \) G (Wijnands & van der Klis 1998b), is indeed similar to that of atoll sources. For black hole candidates, a transition layer similar to that in LMXBs may also exist between a geometrically thin cold disk and a geometrically thick hot disk (or advection-dominated accretion flow); see Narayan & Yi (1994) due to the sub-Keplerian rotating feature of the inner hot disk. Because of the lack of a magnetosphere near the black hole, we can only predict three characteristic frequencies (\( v_K, v_s, \) and \( v_{\text{break}} \)) from TLM. This may help us to understand why we did not observed QPO frequencies larger than 400 Hz for black hole candidates and why their power density spectra and correlations between low and high kHz QPO frequencies are more similar to those for atoll sources than those for Z sources (van der Klis 1994; Psaltis et al. 1999a). A detailed comparison of these features, however, is beyond the scope of this paper.

We should also mention that as in other QPO models, TLM requires that the blob be lifted from the inner part of the Keplerian disk. It is still unclear which mechanism could lead to such lifting. In addition, most theoretical predictions of TLM are made by assuming that the angular velocity of magnetosphere \( \Omega \) is constant. This may not be the case in reality. Considering a nonconstant \( \Omega \) will lead to the decrease of \( R_{\text{out}} - R_0 \). According to equations (8) and (10), both the viscous frequency and the break frequency will increase with the increase of accretion rate. Indeed, many observations have shown the increases of the kHz QPO frequencies, HBO frequency, and break frequency when the sources move along the tracks in the color-color diagram, and even the sense of increasing inferred accretion rate (e.g., Méndez & van der Klis 1999). These results are also consistent with the predictions of TLM. Recently, Cui (2000) proposed that the disappearance of the kHz QPOs may be explained by the “disengagement” between magnetosphere and Keplerian disk. Campana (2000) suggested that the van-

Another clear difference between TLM and the other models is that the lower kHz QPO frequency, rather than the upper kHz QPO frequency, is described as a Keplerian frequency. Therefore, it is not necessary in TLM to involve a larger neutron star mass and a sti equation of state as required in other models such as the sonic-point beat-frequency model (Miller et al. 1998) and the relativistic pre-

cess model (Stella & Vietri 1999, 1999). The compact star in TLM can still be a normal neutron star with mass around 1.4 \( M_\odot \) and radius around 12 km. Indeed, the observed correlation of QPO frequencies can be well explained by taking these parameters in TLM. A larger neutron star mass (\( > 2 M_\odot \)) is not appreciated since it will lead to a steeper correlation of low-frequency QPO frequencies with lower kHz QPO frequencies than that observed for atoll sources (see Fig. 2).

The variations of the size of magnetosphere and the size of transition layer due to the change of mass accretion rate may account for the observed variations of QPO frequencies in X-ray binaries. Because the size of magnetosphere can be described approximately by the Alfvén radius \( R_A \), which is inversely proportional to \( M^{-1/7} \), the magnetosphere will shrink if accretion rate \( M \) increases. This may lead to increases of the kHz QPO frequencies (\( v_s, v_K \)) and the HBO frequency (\( v_s \)). The size of the transition layer can be described by \( R_{\text{out}} - R_0 \), which is directly related to the Reynolds number of the accretion flow (see eq. [9]). The increase of accretion rate will cause the increase of the Rey-

nolds number and therefore lead to the decrease of \( R_{\text{out}} - R_0 \). According to equations (8) and (10), both the viscous frequency and the break frequency will increase with the increase of accretion rate. Indeed, many observations have shown the increases of the kHz QPO frequencies, HBO frequency, and break frequency when the sources move along the tracks in the color-color diagram, and even the sense of increasing inferred accretion rate (e.g., Méndez & van der Klis 1999). These results are also consistent with the predictions of TLM. Recently, Cui (2000) proposed that the disappearance of the kHz QPOs may be explained by the “disengagement” between magnetosphere and Keplerian disk. Campana (2000) suggested that the van-

ishing of the magnetosphere may lead to the stopping of the kHz QPO activity. This could also be the natural result of TLM since the Keplerian oscillator has no chance to enter the rotating frame of reference at all if it loses direct contact with the magnetosphere.
complexity of the dispersion relation and therefore some different characteristic oscillation frequencies. Moreover, the lack of explicit knowledge about the diffusion process in the transition layer may result in uncertainty in estimating the break frequency. In spite of these questions, TLM should be considered a very competitive model for QPOs in X-ray binaries.

I am very grateful to Dimitrios Psaltis, Eric Ford, Michiel van der Klis, Rudy Wijnands, Mariano Méndez, and Lev Titarchuk for kindly providing me the data files. I thank Wei Cui and the anonymous referee for helpful comments, and Lev Titarchuk, Wenfei Yu, and Chengmin Zhang for stimulating discussions.

REFERENCES

Akhiezer, A. I., Akhiezer, I. A., Polovin, R. V., Sitenko, A. G., & Stepanov, K. N. 1975, Plasma Electrodynamics (Oxford: Pergamon)

Alpar, M. A., & Shaham, J. 1985, Nature, 316, 239

Benson, R. F. 1977, Radio Sci., 12, 861

Campana, S. 2000, ApJ, 534, L79

Cui, W. 2000, ApJ, 534, L31

Cui, W., Shramer, C. R., Haswell, C. A., & Hynes, R. I. 2000, ApJ, 535, L123

Finn, L. S. 1994, Phys. Rev. Lett., 73, 1878

Ford, E. C., & van der Klis, M. 1998, ApJ, 506, L39

Ford, E. C., van der Klis, M., van Paradijs, J., Méndez, R., & Kaaret, P. 1998, ApJ, 508, L155

Hasinger, G., & van der Klis, M. 1989, A&A, 225, 79

Jonker, P., Wijnands, R., van der Klis, M., Psaltis, D., Kuulkers, E., & Lamb, F. K. 1998, ApJ, 499, L191

Lai, D., Lovelace, R., & Wasserman, I. 1999, ApJ, submitted (astro-ph/9904111)

Landau, L. D., & Lifshitz, E. 1960, Mechanics (2d ed.; New York: Pergamon)

Li, X. D., Ray, S., Dey, J., Dey, M., & Bombaci, I. 1999, ApJ, 527, L51

Lyne, A. G., & Manchester, R. N. 1988, MNRAS, 234, 477

Markwardt, C. B., Strohmayer, T. E., & Swank, J. H. 1999, ApJ, 512, L125

Méndez, M., et al. 1998a, ApJ, 494, L65

Méndez, M., & van der Klis, M. 1999, ApJ, 517, L51

Méndez, M., van der Klis, M., Wijnands, R., Ford, E. C., van Paradijs, J., & Vaughan, B. A. 1998b, ApJ, 505, L23

Middleditch, J., & Priedhorsky, W. C. 1986, ApJ, 306, 230

Miller, M. C., Lamb, F. K., & Psaltis, D. 1998, ApJ, 508, 791

Morgan, E. H., Remillard, R. A., & Greiner, J. 1997, ApJ, 482, 993

Narayan, R., & Yi, I. 1994, ApJ, 428, L13

Osherovich, V., & Titarchuk, L. 1999a, ApJ, 522, L113

Psaltis, D., et al. 1999a, ApJ, 520, 262

Psaltis, D., et al. 1998, ApJ, 501, L95

——. 1999b, ApJ, 520, 763

Psaltis, D., & Norman, C. 2000, ApJ, submitted (astro-ph/0001391)

Remillard, R. A., McClintock, J. E., Sobczak, G., Bailyn, C. D., & Orosz, J. A. 1999a, ApJ, 517, L127

Remillard, R. A., Morgan, E. H., McClintock, J. E., Bailyn, C. D., & Orosz, J. A. 1999b, ApJ, 522, 397

Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337

Stella, L., & Vietri, M. 1998, ApJ, 492, L59

——. 1999, Phys. Rev. Lett., 82, 17

Stella, L., Vietri, M., & Morsink, S. M. 1999, ApJ, 524, L63

Strohmayer, T. E., Zhang, W., Swank, J. H., Smale, A., Titarchuk, L., & Day, C. 1996, ApJ, 469, L9

Thorstens, S. E., & Chakrabarty, D. 1999, ApJ, 512, 288

Titarchuk, L., & Osherovich, V. 1999, ApJ, 518, L95

——. 2000, ApJ, 537, L39

Titarchuk, L., Osherovich, V., & Kuznetsov, S. 1999, ApJ, 525, L129

van der Klis, M. 1994, A&A, 283, 469

——. 2000, ARA&A, 38, 717

van der Klis, M., et al. 1996, ApJ, 469, L1

van der Klis, M., Jansen, F., van Paradijs, J., Lewin, W. H. G., & van den Heuvel, E. P. J. 1985, Nature, 316, 225

van der Klis, M., Wijnands, R., Horne, K., & Chen, W. 1997, ApJ, 481, L97

Wijnands, R., et al. 1997, ApJ, 490, L157

——. 1998a, ApJ, 493, L87

Wijnands, R., Méndez, M., van der Klis, M., Psaltis, D., Kuulkers, E., & Lamb, F. K. 1998b, ApJ, 504, L35

Wijnands, R., & van der Klis, M. 1997, ApJ, 482, L65

——. 1998a, ApJ, 507, L63

——. 1998b, Nature, 349, 344

——. 1999, ApJ, 514, 939

Wijnands, R., van der Klis, M., Méndez, M., van Paradijs, J., Lewin, W. H. G., Lamb, F. K., & Vaughan, B. 1998c, ApJ, 495, L39