Degree based Topological indices of Hanoi Graph

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Abstract

There are various topological indices for example distance based topological indices and degree based topological indices etc. In QSAR/QSPR study, physiochemical properties and topological indices for example $ABC$ index, $ABC_4$ index, Randic connectivity index, sum connectivity index, and so forth are used to characterize the chemical compound. In this paper we computed the edge version of $ABC$ index, $ABC_4$ index, Randic connectivity index, sum connectivity index, $GA$ index and $GA_5$ index of Double-wheel graph $DW_n$ and Hanoi graph $H_n$. The results are analyzed and the general formulas are derived for the above mentioned families of graphs.

Key words: Topological indices; Double-wheel graph $DW_n$; Hanoi graph $H_n$

1. Introduction and preliminary results

A single number, in graph theoretical terms, representing a chemical structure, is named as topological descriptor. A topological descriptor when correlates with a molecular property, it can be demonstrated as molecular index or topological index. Good correlation with the structure was found between the molecular properties for: thermodynamic properties (for example boiling points, heat of combustion, enthalpy of formation, etc.) and several boiling properties. Consequently, a topological index renovates a chemical structure into a particular number, beneficial in QSPR/QSAR studies.

In this paper all molecular graphs are considered to be connected, finite, loopless and deprived of parallel edges. Let $F$ be a graph with $n$ vertices and $m$ edges. The degree of a vertex is the number of vertices adjacent to $q$ and is signified as $d(q)$. By these terminologies, certain topological indices are well-defined in the following way.

The oldest degree based topological index is Randic index signified as $\chi(F)$ and presented by Randic [11]. He suggested this index for calculating the degree of branching of the carbon-atom skeleton of saturated hydrocarbons.

Definition 1. For a molecular graph $F$, the Randic index is defined as

$$\chi(F) = \sum_{q\in E(F)} \frac{1}{\sqrt{d_q d_r}}.$$
There is a relationship among Randic index and certain physiochemical properties of alkanes: surface area, boiling points, energy level, etc.

A variation of Randic connectivity index is the sum-connectivity index. It was presented by Zhou and Trinajstic [15]. They determined upper and lower bounds of this index for trees in terms of other graph invariants.

**Definition 2.** For a molecular graph $F$, the sum-connectivity index is defined as

$$S(F) = \sum_{qr \in E(F)} \frac{1}{d_q + d_r}.$$

Estrada et al. in [3] proposed a degree based topological index of graphs, which is said to be atom-bond connectivity index. It can be used as tool to model the thermodynamic properties of organic compounds.

**Definition 3.** Let $F$ be molecular graph, then $ABC$ index is defined as

$$ABC(F) = \sum_{qr \in E(F)} \sqrt{\frac{d_q + d_r - 2}{d_q d_r}}.$$

Geometric-arithmetic index is associated with a variation of physiochemical properties. It can be used as possible tool for QSPR/QSAR research. Vukicevic and Furtula in [14] introduced the geometric-arithmetic ($GA$) index.

**Definition 4.** Let $F$ be molecular graph, then geometric-arithmetic index is defined as

$$GA(F) = \sum_{qr \in E(F)} \frac{2 \sqrt{d_q d_r}}{d_q + d_r}.$$

Ghorbani and Hosseinzadeh in [7] presented the fourth atom-bond connectivity index.

**Definition 5.** Let $F$ be molecular graph, then $ABC_4$ index is defined as

$$ABC_4(F) = \sum_{qr \in E(F)} \sqrt{\frac{S_q + S_r - 2}{S_q S_r}}.$$

Where $S_q$ is the summation of degrees of all the neighbors of vertex $r$ in $F$.

Recently Graovac et al. in [9] proposed fifth $GA$ index. As an essential topological index, the fifth geometric index is used to check the chemical properties of chemical compounds, nanomaterial and drugs.

**Definition 6.** Let $F$ be molecular graph, then $GA_5$ index is defined as

$$GA_5(F) = \sum_{qr \in E(F)} \frac{2 \sqrt{S_q S_r}}{S_q + S_r}.$$

Das and Trinajstic in [2] associated the $ABC$ and $GA$ indices for molecular graphs and chemical trees also compared there two indices for general graphs. Later on Gan et al. in [8] introduced some sharp lower and upper bounds on $ABC$. Chen and Li in [1] gave sharp lower bound for sum-connectivity index having $n$-vertex unicyclic graphs by $k$ pendant vertices. Farhani in [4] investigated numerous topological indices in polyhex nanotubes: Randic connectivity index, sum connectivity index, atom-bond connectivity index, geometric-arithmetic index, first and second Zagreb indices and Zagreb polynomials. After
that Farahani in [5] proposed $ABC_4$ index for Nanotori and V-Phenylenic Nanotube. After that Farhani in [6] gave explicit formulas for $GA_5$ index of a family of Hexagonal Nanotubes namely: Armchair Polyhex Nanotubes. Later on Sridhara et al. in [12] computed Randic index, $ABC$ index, $ABC_4$ index, sum connectivity index, geometric-arithmetic index and $GA_5$ index of Graphene.

Inspired by recent work on Graphene (Sridhara et al., 2015) of computing topological indices, Shigehlli and Kanabur in [13] proposed new topological indices, namely, Arithmetic-Geometric index ($AG_1$ index), $SK$ index, $SK_1$ index and $SK_2$ index of a molecular graph $G$ and found the explicit formulae of these indices for Graphene. After that Kanna et al. in [10] investigated Randic, ABC, sum connectivity, $ABC_4$, $GA$ connectivity and $GA_5$ indices of Dutch windmill graph.

2. Main results for Double-wheel graph

A double-wheel graph $DW_n$ of size $n$ can be composed of $2C_n + K_1, n \geq 3$, that is it contains two cycles of size $n$, where all the points of the two cycles are associated to a common center.

The degree based topological indices like Randic, sum, atom-bond, geometric-arithmetic, fourth atom-bond, $GA_5$ index for Double-wheel graph are calculated in this section.

Figure 1: A representation of Double-wheel graph $DW_n$. 
Theorem 1

The Randic connectivity index of Double-wheel graph is \( \chi(DW_n) = \frac{2n}{3} + \frac{2n}{\sqrt{6n}} \).

Proof. Consider the Double-wheel graph \( DW_n \). The edges of \( DW_n \) can be partition into edges of form \( E_{(d_i,d_j)} \), where \( uv \) is an edge. We develop the edges of the form \( E_{(3,3)} \) and \( E_{(3,2n)} \). In figure 1 \( E_{(3,3)} \) is colored in red. These forms for the sum of edges are given in the table 1.

We know that \( \chi(F) = \sum_{q \in E(F)} \frac{1}{\sqrt{d_q d_r}} \).

\( \chi(DW_n) = |E_{(3,3)}| \sum_{q \in E_{(3,3)}(F)} \frac{1}{\sqrt{d_q d_r}} + |E_{(3,2n)}| \sum_{q \in E_{(3,2n)}(F)} \frac{1}{\sqrt{d_q d_r}} \)

From table 1 and figure 1,

\[ \chi(DW_n) = \frac{2n}{3} + \frac{2n}{\sqrt{6n}}. \]

Here the proof of the theorem 1 is completed.

Table 1: Edge partition created by sum of adjacent vertices of every line.

| Edge of the form \( E_{d_i,d_j} \) | Sum of edges |
|-----------------------------------|-------------|
| \( E_{(3,3)} \)                  | 2n          |
| \( E_{(3,2n)} \)                 | 2n          |

Theorem 2

The sum connectivity index of Double-wheel graph is \( S(DW_n) = \frac{2n}{\sqrt{6}} + \frac{2n}{\sqrt{3}+2n} \).

Proof. We know that \( S(F) = \sum_{q \in E(F)} \frac{1}{\sqrt{d_q + d_r}} \).

\[ S(DW_n) = |E_{(3,3)}| \sum_{q \in E_{(3,3)}(F)} \frac{1}{\sqrt{d_q + d_r}} + |E_{(3,2n)}| \sum_{q \in E_{(3,2n)}(F)} \frac{1}{\sqrt{d_q + d_r}} \]
From table 1 and figure 1,
\[
S(DW_n) = \frac{2n}{\sqrt{6}} + \frac{2n}{\sqrt{3+2n}}.
\]
Here the proof of the theorem 2 is completed.

**Theorem 3**

The \( ABC \) index of Double-wheel graph is \( ABC(DW_n) = \frac{4n}{3} + 2n\sqrt{\frac{1+2n}{6n}} \).

Proof. We know that \( ABC(F) = \sum_{qr \in E(F)} \sqrt{\frac{d_q + d_r - 2}{d_q d_r}} \).

\[
ABC(DW_n) = \left| E_{(3,3)} \right| \sum_{qr \in E(3,3)(F)} \sqrt{\frac{d_q + d_r - 2}{d_q d_r}} + \left| E_{(3,2n)} \right| \sum_{qr \in E(3,2n)(F)} \sqrt{\frac{d_q + d_r - 2}{d_q d_r}}
\]

From table 1 and figure 1,
\[
= 2n\sqrt{\frac{3+3-2}{9}} + 2n\sqrt{\frac{3+2n-2}{6n}}
\]
\[
= 2n\sqrt{\frac{4}{9}} + 2n\sqrt{\frac{1+2n}{6n}}
\]
\[
ABC(DW_n) = \frac{4n}{3} + 2n\sqrt{\frac{1+2n}{6n}}.
\]

Here the proof of the theorem 3 is completed.

**Theorem 4**

The \( GA \) index of Double-wheel graph is \( GA(DW_n) = 2n + \frac{4n\sqrt{6n}}{3+2n} \).

Proof. We know that \( GA(F) = \sum_{qr \in E(F)} \frac{2\sqrt{d_q d_r}}{d_q + d_r} \).

\[
GA(DW_n) = \left| E_{(3,3)} \right| \sum_{qr \in E_{(3,3)}(F)} \frac{2\sqrt{d_q d_r}}{d_q + d_r} + \left| E_{(3,2n)} \right| \sum_{qr \in E_{(3,2n)}(F)} \frac{2\sqrt{d_q d_r}}{d_q + d_r}
\]

From table 1 and figure 1,
\[
GA(DW_n) = 2n + \frac{4n\sqrt{6n}}{3+2n}.
\]
Here the proof of the theorem 4 is completed.

**Theorem 5**

The $ABC_4$ index of Double-wheel graph is $ABC_4(DW_n) = \frac{4n}{3} + 2n\sqrt{\frac{1+2n}{6n}}$.

Proof. Consider the Double-wheel graph $DW_n$. The edges of $DW_n$ can be partition into edges of form $E_{d_q,d_r}$, where $qr$ is an edge. We develop the edges of the form $E_{(2n+6,2n+6)}$ and $E_{(2n+6,6n)}$ that are shown in table 9. For convenience these edge kinds are colored by bright green and blue respectively, as shown in figure 2.

**Table 2:** Edge partition created by sum of degrees of neighbors of the head-to-head vertices of every edge.

| Edge of the form $E_{d_q,d_r}$ | Sum of edges |
|--------------------------------|--------------|
| $E_{(2n+6,2n+6)}$             | $2n$         |
| $E_{(2n+6,6n)}$               | $2n$         |

We know that $ABC_4(F) = \sum_{qr \in E(F)} \sqrt{S_q + S_r - 2}$. 

$$ABC_4(DW_n) = \left| E_{(2n+6,2n+6)} \right| \sum_{qr \in E_{(2n+6,2n+6)}} \sqrt{S_q + S_r - 2} + \left| E_{(2n+6,6n)} \right| \sum_{qr \in E_{(2n+6,6n)}} \sqrt{S_q + S_r - 2}$$

From table 2 and figure 2,

$$= 2n \sqrt{\frac{2n+6+2n+6-2}{(2n+6)(2n+6)}} + 2n \sqrt{\frac{2n+6+6n-2}{(2n+6)(6n)}}$$

$$= \frac{2n}{2n+6} \sqrt{4n+10} + \frac{2n.2\sqrt{2n+1}}{2.\sqrt{3n^2 + 9n}}$$

$$ABC_4(DW_n) = \frac{2n}{2n+6} \sqrt{4n+10} + 2n \sqrt{\frac{2n+1}{3n^2 + 9n}}.$$

Here the proof of the theorem 5 is completed.
Theorem 6

The $GA_5$ index of Double-wheel graph is $GA_5(DW_n) = 2n + \frac{4n\sqrt{3n^2 + 9n}}{4n + 3}$. 

Proof. We know that $GA_5(F) = \sum_{q \in E(F)} \frac{2 \sqrt{S_q S_r}}{S_q + S_r}$. 

$GA_5(DW_n) = \left| E_{(2n+6, 2n+6)}(F) \right| \sum_{q \in E_{(2n+6, 2n+6)}(F)} \frac{2 \sqrt{S_q S_r}}{S_q + S_r} + \left| E_{(2n+6, 6n)}(F) \right| \sum_{q \in E_{(2n+6, 6n)}(F)} \frac{2 \sqrt{S_q S_r}}{S_q + S_r}$

From table 2 and figure 2,

$= 2n \frac{2 \sqrt{(2n+6)^2}}{2n+6+2n+6} + 2n \frac{2 \sqrt{12n^2 + 36n}}{2n+6+6n}$

$= \frac{4n(2n+6)}{2(2n+6)} + \frac{8n\sqrt{3n^2 + 9n}}{2(4n+3)}$. 

$GA_5(DW_n) = 2n + \frac{4n\sqrt{3n^2 + 9n}}{4n + 3}$

Here the proof of the theorem 6 is completed.

3. Main results for Hanoi graph

The Hanoi graph $H_n$ can be created by taking the vertices to be the odd binomial coefficients of pascal’s triangle calculated on the integers from 0 to $2^n - 1$ and drawing a line when coefficients are together diagonally or horizontally. The graph $H_n$ has $3^n$ vertices and
\[
\frac{3(3^n - 1)}{2}\text{ edges. Every Hanoi graph has a unique Hamiltonian cycle.}
\]

The degree based topological indices like Randic, sum, atom-bond, geometric-arithmetic, fourth atom-bond, \(GA_4\) index for Hanoi graph \(H_n\) are computed in this section.

Theorem 7

The Randic connectivity index of Hanoi graph is \(\chi(H_n) = \sqrt{6} + \frac{3^{n+1}}{6} - \frac{5}{2}\).

Proof. Consider the Hanoi graph \(H_n\). We partition the edges of \(H_n\) into edges of form \(E_{dq,dr}\), where \(qr\) is an edge. We develop the edges of the form \(E_{(2,3)}\) and \(E_{(3,3)}\). In figure 4, \(E_{(2,3)}\) and \(E_{(3,3)}\) are colored in lavender and bright green, respectively. The sum of edges of these forms is given in the table 3.

We know that \(\chi(F) = \sum_{qr \in E(F)} \frac{1}{\sqrt{dd}}\).

\[
\chi(H_n) = |E_{(2,3)}| \sum_{qr \in E_{(2,3)}(F)} \frac{1}{\sqrt{dd}} + |E_{(3,3)}| \sum_{qr \in E_{(3,3)}(F)} \frac{1}{\sqrt{dd}}
\]

From table 3 and figure 3,

\[
= \frac{6}{\sqrt{6}} + \frac{3^{n+1} - 15}{2} \times \frac{1}{\sqrt{9}}
\]
\[ n + \frac{3^{n+1} - 15}{6} \]

\[ \chi(H_n) = \sqrt{6} + \frac{3^{n+1} - 5}{2}. \]

Here the proof of the theorem 7 is completed.

**In general case for** \( H_n \), **when** \( n \geq 3 \):

**Table 3:** Edge partition created by sum of adjacent vertices of every line.

| Edge of the form \( E_{d_i,d_j} \) | Sum of edges |
|----------------------------------|-------------|
| \( E_{(2,3)} \)                  | 6           |
| \( E_{(3,3)} \)                  | \( \frac{3^{n+1} - 15}{2} \) |

**Theorem 8**

The **Sum-connectivity index** of Hanoi graph is

\[ S(H_n) = \frac{6}{\sqrt{5}} + \frac{3^{n+1} - 15}{2 \sqrt{6}}. \]

**Proof.** We know that

\[ S(F) = \sum_{q \in E(F)} \frac{1}{\sqrt{d_q + d_r}}. \]

\[ S(H_n) = \left| E_{(2,3)} \right| \sum_{q \in E_{(2,3)}(F)} \frac{1}{\sqrt{d_q + d_r}} + \left| E_{(3,3)} \right| \sum_{q \in E_{(3,3)}(F)} \frac{1}{\sqrt{d_q + d_r}} \]

From table 3 and figure 3,

\[ = \frac{6}{\sqrt{2} + 3} + \frac{3^{n+1} - 15}{2} \times \frac{1}{\sqrt{3} + 3} \]

\[ S(H_n) = \frac{6}{\sqrt{5}} + \frac{3^{n+1} - 15}{2 \sqrt{6}}. \]

Here the proof of the theorem 8 is completed.
Theorem 9

The $ABC$ index of Hanoi graph is $ABC(H_n) = 3\sqrt{2} + 3^n - 5$.

Proof. We know that $ABC(F) = \sum_{qr \in E(F)} \sqrt[2]{d_q + d_r - 2} / d_q d_r$.

$$ABC(H_n) = |E_{(2,3)}| \sum_{qr \in E_{(2,3)}(F)} \sqrt[2]{d_q + d_r - 2} / d_q d_r + |E_{(3,3)}| \sum_{qr \in E_{(3,3)}(F)} \sqrt[2]{d_q + d_r - 2} / d_q d_r$$

From table 3 and figure 3,

$$= 6 \sqrt{\frac{2 + 3 - 2}{6} + \frac{3^{n+1} - 15}{2} \sqrt{\frac{3 + 3 - 2}{9}}}$$

$$= \frac{6}{\sqrt{2}} + \frac{3^{n+1} - 15}{2} \frac{4}{\sqrt{9}}$$

$$= \frac{6}{\sqrt{2}} + \frac{3^{n+1} - 15}{3}$$

$$= \frac{6}{\sqrt{2}} + \frac{3^{n+1} - 15}{3}$$

$$ABC(H_n) = 3\sqrt{2} + 3^n - 5$$

Here the proof of the theorem 9 is completed.

Theorem 10

The $GA$ index of Hanoi graph is $GA(H_n) = \frac{12\sqrt{6}}{5} + \frac{3^{n+1} - 15}{2}$.

Proof. We know that $GA(F) = \sum_{qr \in E(F)} \frac{2\sqrt{d_q d_r}}{d_q + d_r}$.

$$GA(H_n) = |E_{(2,3)}| \sum_{qr \in E_{(2,3)}(F)} \frac{2\sqrt{d_q d_r}}{d_q + d_r} + |E_{(3,3)}| \sum_{qr \in E_{(3,3)}(F)} \frac{2\sqrt{d_q d_r}}{d_q + d_r}$$

From table 3 and figure 3,
\[ GA(H_n) = \frac{12\sqrt{6}}{5} + \frac{3^{n+1} - 15}{2}. \]

Here the proof of the theorem 10 is completed.

**Theorem 11**

The \( ABC_4 \) index of Hanoi graph \( ABC_4(H_n) = 3\sqrt{\frac{7}{32}} + 6\sqrt{\frac{5}{24}} + \frac{2(3^{n+1})}{9} - \frac{13}{3}. \)

Proof. Consider the Hanoi graph \( H_n \). We partition the edges of \( H_n \) into edges of form \( E_{d_q,d_r} \), where \( qr \) is an edge. We develop the edges of the form \( E_{(6,8)}, E_{(8,8)}, E_{(9,8)} \) and \( E_{(9,9)} \) that are shown in table 4. For convenience these edge kinds are colored by green, blue, pink and red, respectively, as shown in figure 4.

**Table 4:** Edge partition created by sum of degrees of neighbors of the head-to-head vertices of every edge.

| Edge of the form \( E_{d_q,d_r} \) | Sum of edges |
|---------------------------------|-------------|
| \( E_{(6,8)} \)                | 6           |
| \( E_{(8,8)} \)                | 3           |
| \( E_{(9,8)} \)                | 6           |

We know that \( ABC_4(F) = \sum_{qr \in E(F)} \sqrt{\frac{S_q + S_r - 2}{S_q S_r}} \).

\[
ABC_4(H_n) = |E_{(6,8)}| \sum_{qr \in E_{(6,8)}(F)} \sqrt{\frac{S_q + S_r - 2}{S_q S_r}} + |E_{(8,8)}| \sum_{qr \in E_{(8,8)}(F)} \sqrt{\frac{S_q + S_r - 2}{S_q S_r}} + |E_{(9,8)}| \sum_{qr \in E_{(9,8)}(F)} \sqrt{\frac{S_q + S_r - 2}{S_q S_r}} + |E_{(9,9)}| \sum_{qr \in E_{(9,9)}(F)} \sqrt{\frac{S_q + S_r - 2}{S_q S_r}}.
\]

From table 4 and figure 4,

\[
= 6 \sqrt{\frac{6+8-2}{48}} + 3 \sqrt{\frac{8+8-2}{64}} + 6 \sqrt{\frac{9+8-2}{72}} + \frac{3^{n+1} - 33}{2} \sqrt{\frac{9+9-2}{81}}
\]

\[
= 6 \sqrt{\frac{12}{48}} + 3 \sqrt{\frac{14}{64}} + 6 \sqrt{\frac{15}{72}} + \frac{3^{n+1} - 33}{2} \sqrt{\frac{16}{81}}
\]

\[
= 3 + 3 \sqrt{\frac{7}{32}} + 6 \sqrt{\frac{5}{24}} + \frac{2(3^{n+1} - 33)}{9}
\]
\[ \sqrt[3]{\frac{7}{32}} + 6\sqrt[3]{\frac{5}{24}} + \frac{2(3^{n+1})}{9} - \frac{22}{3} + 3 \]

\[ ABC_3(H_n) = 3\sqrt[3]{\frac{7}{32}} + 6\sqrt[3]{\frac{5}{24}} + \frac{2(3^{n+1})}{9} - \frac{13}{3}. \]

Here the proof of the theorem 11 is completed.

**Figure 4**

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**Theorem 12**

The fifth geometric-arithmetic connectivity index of Hanoi graph is

\[ GA_5(H_n) = \frac{12^{3/2}}{7} + 3 + \frac{72\sqrt{2}}{17} + \frac{3^{n+1} - 33}{2}. \]

Proof. We know that \( GA_5(F) = \sum_{qr \in E(F)} \frac{2\sqrt{S_q S_r}}{S_q + S_r} \).

\[ GA_5(H_n) = |E_{(6,8)}| \sum_{qr \in E_{(6,8)}(F)} \frac{2\sqrt{S_q S_r}}{S_q + S_r} + |E_{(8,8)}| \sum_{qr \in E_{(8,8)}(F)} \frac{2\sqrt{S_q S_r}}{S_q + S_r} + |E_{(9,8)}| \sum_{qr \in E_{(9,8)}(F)} \frac{2\sqrt{S_q S_r}}{S_q + S_r} + \sum_{qr \in E_{(9,9)}(F)} \frac{2\sqrt{S_q S_r}}{S_q + S_r}. \]

From table 4 and figure 4,

\[ = 6\frac{2\sqrt{48}}{14} + 3 + 6\frac{2\sqrt{64}}{16} + 6\frac{2\sqrt{72}}{17} + \frac{3^{n+1} - 33}{2} \times \frac{2\sqrt{81}}{18} \]

\[ GA_5(H_n) = \frac{12^{3/2}}{7} + 3 + \frac{72\sqrt{2}}{17} + \frac{3^{n+1} - 33}{2}. \]

Here the proof of the theorem 12 is completed.
4. Conclusion

The problem of finding the general formula for edge version of $ABC$ index, $ABC_4$ index, Randic connectivity index, sum connectivity index, $GA$ index and $GA_5$ index of Double-wheel graph $DW_n$ and Hanoi graph $H_n$ is solved here analytically.

References

[1] Chen, J., & Li, S. (2011). On the sum-connectivity index of unicyclic graphs with k dependent vertices. Mathematical Communications, 16(2), 359-368.

[2] Das, K. C., & Trinajstić, N. (2010). Comparison between first geometric–arithmetic index and atom-bond connectivity index. Chemical physics letters, 497(1), 149-151.

[3] Estrada, E., Torres, L., Rodríguez, L., & Gutman, I. (1998). An atom-bond connectivity index: modelling the enthalpy of formation of alkanes. Indian journal of chemistry. Sect. A: Inorganic, physical, theoretical & analytical, 37(10), 849-855.

[4] Farahani, M. R. (2012). Some connectivity indices and Zagreb index of polyhex nanotubes. Acta Chim. Slov, 59, 779-783.

[5] Farahani, M. R. (2013). Computing fourth atom-bond connectivity index of V-Phenylenic Nanotubes and Nanotori. Acta Chimica Slovenica, 60(2), 429-432.

[6] Farahani, M. R. (2014). Computing $GA_5$ index of armchair polyhex nanotube. Le Matematiche, 69(2), 69-76.

[7] Ghorbani, M., & Hosseinzadeh, M. A. (2010). Computing ABC4 index of nanostar dendrimers. Optoelectron. Adv. Mater. Rapid Commun, 4, 1419-1422.

[8] Gan, L., Hou, H., & Liu, B. (2011). Some results on atom-bond connectivity index of graphs. MATCH Commun. Math. Comput. Chem, 66(2), 669-680.

[9] Graovac, A., Ghorbani, M., & Hosseinzadeh, M. A. (2011). Computing fifth geometric-arithmetic index for nanostar dendrimers. J. Math. Nanosci, 1(1), 32-42.

[10] Kanna, M. R., Kumar, R. P., & Jagadeesh, R. (2016). Computation of Topological Indices of Dutch Windmill Graph. Open Journal of Discrete Mathematics, 6(02), 74.

[11] Randic, M. (1975). Characterization of molecular branching. Journal of the American Chemical Society, 97(23), 6609-6615.

[12] Sridhara, G., Kanna, M. R., & Indumathi, R. S. (2015). Computation of topological indices of graphene. Journal of Nanomaterials, 16(1), 292.
[13] Shigehalli, V. S., & Kanabur, R. (2016). Computation of New Degree-Based Topological Indices of Graphene. Journal of Mathematics, 2016.

[14] Vukičević, D., & Furtula, B. (2009). Topological index based on the ratios of geometrical and arithmetical means of end-vertex degrees of edges. Journal of mathematical chemistry, 46(4), 1369-1376.

[15] Zhou, B., & Trinajstić, N. (2009). On a novel connectivity index. Journal of mathematical chemistry, 46(4), 1252-1270.