Diffraction-free and dispersion-free pulsed beam propagation in dispersive media

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Pulsed Bessel beams of light propagating in free-space experience diffraction effects that resemble those of anomalous dispersion on pulse propagation. It is then shown that a pulsed Bessel beam in a normally dispersive material can remain diffraction- and dispersion-free due to mutual cancellation of diffraction and group velocity dispersion. The size of the Bessel transversal profile for localized transmission is determined by the dispersive properties of the material at the pulse carrier frequency.

Two of the biggest obstacles to the transmission of localized electromagnetic energy over large distances are diffraction and material dispersion. Generally speaking, diffraction makes waves to spread transversally to the intended propagation direction, and dispersion temporally (longitudinally). Many methods have been proposed and experimentally demonstrated, to diminish, even eliminate either diffraction spreading in free-space, by using diffraction-free Bessel beams, or their generalizations, focus wave modes of various types, optical missiles, or dispersion spreading effects in dispersive media, by exploiting the nonlinear properties of the medium, suitably designed Bessel-X waves, or the pseudo-dispersion-free behavior of specific pulse temporal forms.

In the propagation of a transversally and temporally localized wave in a dispersive material, both diffraction and dispersion effects act together, and lead, in general, to an enhanced deterioration of the wave depth of field. However, as shown in this paper, it is also possible to play off diffraction against dispersion during propagation of a pulsed beam: by suitably designing its transversal profile, the produced diffraction effects cancel, to a great extent, dispersion spreading, and vice versa, leading to dispersion-free and diffraction-free localized propagation in the dispersive medium. Specifically, diffraction changes in a pulse with Bessel transversal profile (do not confuse with the more known nondiffracting X-Bessel waves), and temporal spreading due to normal material dispersion mutually cancel if the transversal size of the Bessel profile is properly chosen.

It is possible to arrive at this result by thinking of diffraction of pulses as a dispersive phenomenon. Whenever the pulse has a transversal profile, diffraction causes its redder frequencies to spread at larger angles than its bluer frequencies, and hence to propagate at different effective velocities along the beam axis. A detailed investigation on the dispersive nature of free-space diffraction of pulses, including the description of diffraction forerunners, can be found in Ref. [1]. Here we consider the light disturbance \( E(x_\perp, t) = g(x_\perp)A(t) \exp(-i\omega_0 t) \), with \( x_\perp \equiv (x, y) \), representing a pulse of carrier frequency \( \omega_0 \), envelope \( A(t) \), and transversal profile \( g(x_\perp) \), at the entrance plane \( z = 0 \) of a dispersive material of refraction index \( n(\omega) \), which fills the half-space \( z > 0 \). The spatial-frequency spectrum of the transversal profile \( g(x_\perp) \) is \( \hat{g}(k_\perp) \), with \( k_\perp = (k_x, k_y) \), and the temporal-frequency spectrum of the pulse temporal form is \( \hat{A}(\omega - \omega_0) \). The propagated disturbance \( E(x_\perp, z, t) \) at any plane \( z > 0 \) inside the material can be seen as the result of superposing the monochromatic plane waves \( \hat{g}(k_\perp)\hat{A}(\omega - \omega_0) \exp[-i\omega t + ik_\perp \cdot x_\perp + ik_z(\omega)z] \) emitted by the source plane, of different frequencies \( \omega \), wavevectors \( |k_\perp, k_z(\omega)| \), with

\[
k_z(\omega) = \sqrt{k^2(\omega) - |k_\perp|^2},
\]

and \( k(\omega) = (\omega/c)n(\omega) \), and amplitudes \( \hat{g}(k_\perp)\hat{A}(\omega - \omega_0) \). These monochromatic plane waves are homogeneous if \( |k_\perp| < k(\omega) \), and evanescent otherwise. To perform this superposition, we first sum, for convenience, all monochromatic plane waves of different frequencies \( \omega \) but same value of \( k_\perp \),

\[
E_{k_\perp}(z, t) = \frac{\hat{g}(k_\perp)}{2\pi} \int_{-\infty}^{\infty} d\omega \hat{A}(\omega - \omega_0) \exp[-i\omega t + ik_z(\omega)z],
\]

and then superpose these partial fields,

\[
E(x_\perp, z, t) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dk_\perp \exp(i k_\perp \cdot x_\perp) E_{k_\perp}(z, t).
\]

In this way, the propagated the pulsed beam appears as the superposition of many pulses \( E_{k_\perp} \) associated to the different spatial-frequencies \( k_\perp \) of the initial transversal profile. The propagation of these subpulses is dispersive, not only in dispersive materials, but also in free space \( |k(\omega) = \omega/c| \), since their propagation constant \( k_z(\omega) \) is a nonlinear function of \( \omega \).
As in the usual theory of dispersive pulse propagation, we can expand $k_z(\omega)$ around the carrier frequency, $k_z(\omega) = k_{z,0} + k'_{z,0}(\omega - \omega_0) + k''_{z,0}(\omega - \omega_0)^2/2 + \ldots$ (where the prime sign denotes differentiation with respect to $\omega$, and the subscript 0 evaluation at $\omega_0$), to rewrite Eq. (3), up to second order in dispersion, as
\[
E_{k_z}(z, t) = \hat{g}(k_{\perp}) \exp(-i\omega_0 t + ik_{z,0}z) \times \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \hat{A}(\omega - \omega_0) \exp\left[\frac{i}{2} k'_{z,0} (\omega - \omega_0)^2 z\right] \times \exp[-i(\omega - \omega_0) (t - k''_{z,0} z)],
\]
where, from Eq. (1),
\[
k_{z,0} = \sqrt{k_0^2 - |k_{\perp}|^2},
\]
\[
k'_{z,0} = \frac{k_0 k'_{b}}{k_{z,0}},
\]
\[
k''_{z,0} = \frac{k_0 k''_{b} - |k_{\perp}|^2 k_0^2 + k_0 k_{b}^2}{k_{z,0}^3}.
\]
When $|k_{\perp}| < k_0$, $E_{k_z}(z, t)$ is a propagating pulse, whose carrier oscillations travel at the phase velocity $v_p = \omega_0/k_{z,0}$, while the group velocity $v_g = (k'_{z,0})^{-1}$, at the same time that it broadens due to the GVD of Eq. (3).

In free-space, for instance, the phase and group velocities are $v_p = c/\sqrt{1 - (c|k_{\perp}|/\omega_0)^2} > c$, $v_g = c/\sqrt{1 - (c|k_{\perp}|/\omega_0)^2} < c$, respectively, and the GVD $k''_{z,0} = -c|k_{\perp}|^2/\omega_0^3 \sqrt{1 - (c|k_{\perp}|/\omega_0)^2 < 0}$ is anomalous. Free-space dispersion originates from angular dispersion [see Fig. 3(a)]: different frequencies $\omega$ composing the pulse $E_{k_z}(z, t)$ propagate at different angles $\sin \theta(\omega) = |k_{\perp}|/\omega/c$ with respect to the z axis, and then travel at different effective velocities along the z direction. A geometrical picture of $v_p$ and its dependence on frequency is shown in Fig. 3(a). Free-space dispersion exists whenever there exists spatial-frequencies $k_{\perp} \neq 0$, i.e., the initial pulse has a transversal profile, and is responsible for diffraction changes in the pulsed beam during propagation. Indeed, if we neglect this kind of dispersion in Eq. (3) by approaching $\sqrt{(\omega/c)^2 - |k_{\perp}|^2} \sim \omega/c$, Eq. (3) would yield $E(x_{\perp}, z, t) = g(x_{\perp}) A(t) \exp(-i\omega_0 t)$, i.e., the pulsed beam would propagate without any change in free-space.

In a dispersive material, the total GVD of Eq. (3) has two contributions, originating from material dispersion and diffraction-induced dispersion. The remarkable point here is that for normal material dispersion ($k''_b > 0$), both types of GVD cancel mutually for pulses $E_{k_z}(z, t)$ with
\[
|k_{\perp}|^2 = K^2 = \frac{k_0^2 k''_b}{k''_b + k_0 k'_b},
\]
These pulses are not evanescent ($|k_{\perp}| = K < k_0$), their propagating fields being given, from Eq. (3) with $k''_{z,0} = 0$, by
\[
E_{k_z}(z, t) = \hat{g}(k_{\perp}) \exp(-i\omega_0 t + ik_{z,0}z) A(t - k'_{z,0} z),
\]
where $k_{z,0} = \sqrt{k_0^2 - K^2}$ and $k'_{z,0} = k_0 k'_b/\sqrt{k_0^2 - K^2}$. Material and diffraction-induced GVD cancelation is illustrated in Fig. 3(b).

Dispersion-free, diffraction-free pulsed beam propagation in a dispersive material can then be achieved if the initial transversal profile contains only spatial frequencies satisfying condition (8). The simplest example would be a single spatial-frequency $k_{\perp}$ of modulus $K$, but it does not represent a transversally localized wave, but a tilted plane pulse. This is equivalent to the result of Ref. 3 for material GVD suppression by reflection of a plane pulse in a grating of constant $k_{\perp}$. A second example, leading to transversal localization, is the Bessel profile $g(x_{\perp}) = J_0(K|x_{\perp}|)$, whose spectrum $\hat{g}(k_{\perp}) = \pi^{-1/2} \delta(|k_{\perp}|-K)$ is an annulus of radius $K$. Indeed, Eqs. (8) and (3) for this spectrum yield the pulsed beam with nonspreading envelope and transversal profile
\[
E(x_{\perp}, z, t) = J_0(K|x_{\perp}|) A(t - k'_{z,0} z) \exp(-i\omega_0 t).
\]

To illustrate these results, Fig. 3 shows the propagation of the pulsed Bessel beam of Gaussian envelope $J_0(K|x_{\perp}|) \exp(-t^2/b^2) \exp(-i\omega_0 t)$ in fused silica (solid curves), with refraction index given by Sellmeier relation. The pulse duration and carrier frequency have been chosen arbitrarily to be $b = 12$ fs and $\omega_0 = 1.9$ fs$^{-1}$ ($T_0 = 2\pi/\omega_0 = 3.3$ fs), respectively. Since $k_0 = 9193$ mm$^{-1}$, $k'_b = 4881$ mm$^{-1}$ fs and $k''_b = 21.78$ mm$^{-1}$ fs$^2$ at this frequency, we have taken from Eq. (3) $K = 839.4$ mm$^{-1}$ for invariant propagation, yielding the beam width (first zero of the Bessel profile) 2.404/K = 2.864 µm, or about three times the carrier wavelength. For comparison, we also show the propagation of the same pulse without transversal modulation in fused silica (open dots), and of the same pulsed Bessel beam in free-space (dots). It can be seen that the plane pulse in silica, under the only action of dispersion, and the pulsed Bessel beam in free-space, under the effects of diffraction only, have significantly spread at the dispersion length $2D = b_0^2/2|k''_b| = 3.3$ mm [Fig. 3(b)]. However, the pulsed Bessel beam propagating in silica under the joint action of dispersion and diffraction does not experience significant change up to $4z_D \simeq 13.2$ mm [Fig. 3(c)]. This limitation is due to the total third-order dispersion $k''_{z,0}$, whose effect becomes noticeable at the third-order dispersion length $b_3^2/2|k''_{z,0}| = 11.85$ mm.
Obviously, higher-order Bessel profiles, or the “cos” beam (one dimensional version of the Bessel beam) will also yield undeformable transmission. Spreading reduction is also expected to occur with other transversal profiles having annular spatial-frequency spectrum (though of finite thickness), as the Bessel-Gauss, other windowed Bessel profiles, and the so-called elegant Laguerre-Gauss beams. In these cases, invariant propagation will occur within the diffraction-free distance (within which these profiles resemble the Bessel one).

The above results must be clearly distinguished, despite some coincidences, from the Bessel-X dispersionless propagation reported in Refs. [6], whose GVD cancellation scheme is shown in Fig. 3, for comparison with Fig. [6]. Sonajalg’s dispersionless pulse is built from a supraluminal Bessel-X pulse [Fig. 3(a)] having the nonseparable initial disturbance $E(x,\omega) = S(\omega)\delta(k_{\perp} - k(\omega)\sin\theta)$ ($k_{\perp}$ depends on frequency), instead from our separable pulsed Bessel beam [Fig. 3(a)] $E(x,\omega) = S(\omega)\delta(k_{\perp} - K_{\perp})$ ($k_{\perp}$ takes a fixed value). With the Bessel-X pulse, normal material GVD (for instance) can be cancelled by slightly raising the cone angle $\theta$ of the monochromatic Bessel beam components with increasing frequency [Fig. 3(b)]. Dispersion in the cone angle is supplied by an appropriate optical system, such as an annular slit with frequency-dependent radius and a lens, an axicon, a lensaon plus a telescope, depending on the dispersive material behind, or introducing some defocusing in the lensaon. Here angular dispersion is inherent to the Bessel profile: monochromatic Bessel beams of same size but different frequencies have the different cone angles $\sin\theta(\omega) = K/k(\omega)$ [Fig. 3(b)]. It is also to be noted that Sonajalg’s pulse reduces to a pure nondistorted Bessel-X wave in the limiting case of zero material GVD, whereas our pulsed Bessel beam degenerates into a plane pulse ($K \to \infty$).

We have shown, in conclusion, that diffraction and material dispersion spreading effects can cancel one to another during propagation of a pulsed beam in a dispersive material, leading to dispersion-free, diffraction-free localized wave transmission, if the transversal profile of the pulse is suitably selected and scaled. This result can find application in ultrafast spectroscopy, large distance optical communications and electromagnetic energy delivery systems.

[1] J. Durnin, J.J. Miceli and J.H. Eberly, Phys. Rev. Lett. 58, 1499 (1987).
[2] R.W. Ziolkowski, Phys. Rev. A 39, 2005 (1989).
[3] M.A. Porras, F. Salazar-Bloise and L. Vazquez, Phys. Rev. Lett. 85, 2104 (2000).
[4] See for example G.P. Agrawal, Nonlinear Fiber Optics (Academic, San Diego, 1995).
[5] S. Szatmári, P. Simon and M. Feuerhake, Opt. Lett. 21, 1156 (1996).
[6] H. Sonajalg and P. Saari, Opt. Lett. 21, 1162 (1996); H. Sonajalg, M. Ratsep and P. Saari, Opt. Lett. 22, 310 (1997); P. Saari and K. Sonajalg, Laser Physics 7, 32 (1997).
[7] J. Rosen, B. Salik and A. Yariv, Opt. Lett. 20, 423 (1995).
[8] Z. Liu and D. Fan, J. Mod. Opt. 45, 17 (1998).
[9] J. Lu and J.F. Greenleaf, IEEE Trans. Ultrason. Ferroelectr. Freq. Control 39, 19 (1992).
[10] E.M. Belenov and A.V. Nazarkin, J. Opt. Soc. Am. A 11, 168 (1994).
[11] F. Gori, G. Guattari and C. Padovani, Opt. Commun. 156, 359 (1987).
[12] M.A. Porras, R. Borghi and M. Santarsiero, J. Opt. Soc. Am. A 17, 177 (2001).

FIGURES

FIG. 1: (a) Illustration of the angular dispersion and the free-space diffraction-induced anomalous dispersion in the group velocity $v_g(\omega) = c\sqrt{1-c^2(k_{\perp}/\omega)^2} < c$. Higher frequencies propagate at greater group velocities. (b) Cancellation of material GVD dispersion with diffraction-induced GVD. Provided that material dispersion is normal, i.e., $K'(\omega_1) < K'(\omega_2)$ for two close frequencies $\omega_1 < \omega_2$, there exists a particular value $K$ of $|k_{\perp}|$ for which the effective group velocities at $\omega_1$ and $\omega_2$ are equal.

FIG. 2: Propagation of the pulsed Bessel disturbance $J_0(K|x_{\perp}|) \exp(-t^2/b^2) \exp(-i\omega_0 t)$ in fused silica (solid curves) and in vacuum (dots), and of the pulsed plane wave $\exp(-t^2/b^2) \exp(-i\omega_0 t)$ in fused silica (open dots). Numerical values of the parameters are $b = 12$ fs, $K = 839.42$ mm$^{-1}$ and $\omega_0 = 1.9$ fs$^{-1}$. At this frequency the material constants are $k_0 = 9193$ mm$^{-1}$, $k'_0 = 4881$ mm$^{-1}$fs and $k''_0 = 21.78$ mm$^{-1}$fs$^2$. On-axis pulse forms at (a) $z = 0$, (b) $z = z_D = b^2/2|k''_0| = 3.3$ mm, and (c) $4z_D$.

FIG. 3: (a) Illustration of the superposition scheme of a free-space Bessel-X wave and its superluminal group velocity $v_g = c/\cos\theta > c$. All frequencies travel at the same angle $\theta$, and hence $k_{\perp}$ is proportional to frequency. (b) Cancellation of material GVD by slightly distorting the cone angle of the Bessel-X wave.
Figure 1

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Figure 2
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