Hidden Patterns within the Yukawa Couplings of the Higgs boson

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The Large Hadron Collider (LHC) has measured the Higgs boson couplings with the heavier particles of the Standard Model (SM), and they seem to agree with the linear-mass relation predicted in the SM. However a complete test of this property must involve the lighter generations, and the coming measurements at LHC, or future colliders, may reveal hidden patterns associated with physics beyond the SM. In renormalizable multi-Higgs models, the Higgs-fermion couplings could still be linear, but they would lay on multiple lines, one for each different Higgs boson participating in the mass generation of a given fermion type. Then, the angle subtended by these lines, with respect to the SM line, can be used to characterize different multi-Higgs models, such as the Two-Higgs doublet Model (2HDM). Models where fermion masses arise from higher-dimensional operators, may also result in large deviations from the SM values for the Higgs couplings, which would show an irregular pattern as a function of the fermion masses. In the case of neutrino masses, when they arise from the See-saw mechanism, one finds that Higgs Yukawa couplings will show a non-linear mass relation.

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I. INTRODUCTION

The physics of the Higgs boson post-discovery \cite{1,2} has entered a new level of precision, with new decay modes and properties expected to be studied at the LHC \cite{3}. Within the SM only one Higgs doublet gives masses to gauge bosons and all type of fermions, thus their couplings are proportional to the particle mass, and when plotted as function of the masses, they lay on a single line. In particular, the Yukawa couplings of the Higgs boson with the 3rd generation fermions (t, b, τ), seem to be in agreement with the linear relation predicted in the SM. Furthermore, the LHC has started to probe the Higgs couplings for the lighter generations \cite{8}, which will open the possibility to test this SM prediction with better precision. Different patterns or relations associated with models of new physics, could be hidden in the data, an this may show up only after using the appropriate Higgs observables. Then, one could compare the predictions for these observables associated with different extensions of the SM Higgs sector \cite{4}.

A variety of signal strengths $\mu_X$ for different channels have been measured at LHC, these include the Higgs boson production through gluon fusion, in association with a gauge boson ($W, Z$) or with top quark pairs, and with the Higgs boson decays into $XX$ with $X = b, \tau, W, Z, \gamma$. The SM Higgs boson couplings for third generation fermions and gauge bosons, have been extracted from these channels. Moreover, it is expected that LHC will be able to probe the Higgs couplings with charm, strange quarks and muons, with different levels of precision. The table I shows some of the expected limits on the signal strengths at the current or future phases of the LHC \cite{5,6}.

| Method            | $\mu_{cc}$ | $\mu_{ss}$ | $\mu_{\mu\mu}$ |
|-------------------|------------|------------|----------------|
| LHC bound         | 110        | 200        | 7              |
| Theory/HL-LHC     | 16         | 100        | $O(1)$         |

TABLE I. Signal strength $\mu_X$ for 2nd generation fermions obtained or expected from LHC or theory estimates.

In this letter we look at the dependence of the Higgs couplings on the fermion masses in models with extended Higgs sector. We shall consider first renormalizable multi-Higgs models, which also display a linear relations for the Higgs couplings \cite{7}. When more than one Higgs multiplet participates in the fermion mass generation, the single Higgs Coupling Line (HCL) obtained from LHC Higgs data, would only be apparent, and when looked more closely it would reveal some sub-structure. For instance, within the two-Higgs doublet model types all the fermion couplings would lay on a single line (2HDM-I) or two lines (2HDM-II). There are also models where each fermion type gets its mass from a private Higgs \cite{8,9}.

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for the realization of this idea, one needs at least 3 Higgs
doublets, (4 doublets in the SUSY case) [10, 11]; here
the Higgs boson couplings would lay on three different
lines. In all these models, the angle subtended by those
HCL and the SM one, can be calculated in terms of the
fermion masses and Higgs couplings, and then one can
also study their behavior in the alignment or decoupling
limits.
We could also consider models where fermion masses
arise from higher-dimensional operators, which have been
considered as an attempt to explain the hierarchies in the
flavor parameters (masses and mixing angles) with an
scale of $O$(TeV) [12]. In this case the dependence of the
Higgs couplings on fermion masses would still be linear,
but they would not appear laying on definite lines, rather
they would look dispersed. For neutrinos the situation is
less clear, and it would depend on the mechanism of mass
generation (See-saw of type III, III, etc), or weather it is
of Dirac or Majorana type [13,14]. As it will be discussed
at the end of our paper, when the neutrino masses are
of the Dirac-type, the Higgs couplings would be linear
in the neutrino masses, but when they arise from the
See-saw mechanism, the corresponding Higgs couplings
would obey a non-linear mass relation.

II. HIGGS COUPLINGS IN
RENORMALIZABLE MULTI-HIGGS MODELS

Let us study an extension of the SM with $n$ Higgs
doublets $\Phi_a = (\phi_a^{+}, \phi_a^{0})^{T}$, with $a = 1, 2, ..., n$, and write
the Yukawa Lagrangian for the fermion types $f = u, d, \ell$
as follows:

$$\mathcal{L} = \sum_{a=1}^{n} \tilde{F}_{Li}\Phi_{a}\lambda_{ij}^{a}f_{Rj} + h.c. \quad (1)$$

It involves the left-handed fermion doublet $F_{Li} =
(\tilde{f}_{i}^{L}, f_{i}^{L})^{T}$ (with $f_{i}^{L}, \tilde{f}_{i}^{L}$ denoting the fermions with
isospin $T_{3} = 1/2, -1/2$, respectively) and right-handed
fermion singlet $f_{Rj}$, with $i, j = 1, 2, 3$ labeling the fermion
generations. Here $\lambda_{ij}^{a}$ denotes the matrix of Yukawa
couplings. Further, let us consider first the class of multi-
Higgs models that respect the Glashow-Weinberg theo-
rem [15], where each fermion type $f$ couples at most
with one Higgs doublet. For the up-type quarks $f = u$, we
shall assume that it only couples to $\Phi_1$. After sponta-
nous symmetry breaking (SSB), i.e. $< \Phi_1 > = v_{1}/\sqrt{2}$, the eq. (1) is written as follows:

$$\mathcal{L} = \lambda_{f,ij}\tilde{u}_{Li}\nu_{Rj}\left(\frac{v_{1} + \phi_{1}^{0}}{\sqrt{2}}\right) + \sum_{a=2}^{N} \tilde{F}_{Li}\Phi_{a}\lambda_{ij}^{a}f_{Rj} + h.c. \quad (2)$$

The fermion mass is then given by $m_{a} = \lambda_{f}v_{1}/\sqrt{2}$, while
the coupling of the fermion with $\phi_{1}^{0}$ is $g_{\phi_{1}uu} = m_{a}/v_{1}$, i.e. it is a linear relation as a function of the mass. As
$\phi_{1}^{0}$ is not a mass eigenstate in general, one need to rotate
to the Higgs mass basis, which will introduce elements
of the rotation matrix into the HYCs. Since we are as-
suming that $\Phi_1$ only couples to the up-type quarks, then
the remaining Higgs doublets could at most couple to $d$-
type quarks or leptons $\ell$. Specifying which Higgs doublet
couples to a given fermion type, defines the particular
multi-Higgs model.

Thus, for these models multi-Higgs models with natu-
ral flavor conservation, the Higgs couplings as function
of the masses, could lay on one or more HCL. For instance,
in the 2HDM-I, all charged fermions obtain their masses
from one of the Higgs doublets, say $\Phi_1$, and all the HYC
will lay on a single line, as in the SM. But in general, this
line will not coincide with the SM one. In the 2HDM-
II, one Higgs doublet ($\Phi_2$ within our conventions) gives
masses to $d$-type quarks and leptons, and the other ($\Phi_1$)
gives mass to up-type quarks. Thus, the model would
have two HCL: one for $d$-type quarks and leptons, and
another one for up-type quarks.

We have argued that the HCL’s predicted in Multi-
Higgs models may provide interesting information about
such models. Furthermore, it could be interesting to cal-
culate the angle subtended by these lines with respect to
the SM one, as this observable may help to discriminate
between the SM and those models.

In the multi-Higgs models under consideration, the
Higgs boson couplings with fermions ($y_{f}$) are functions
of the masses $m_{f},$ where $m_{f} = m_{f}/v$. In the plane $m_{f} - y_{f}$, we can
define the vectors $\vec{w}_{fj} = (\tilde{m}_{fj}, y_{fj})$ for models
of type $K$. Then we build the vectors $\vec{r}_{fj} = \vec{w}_{fj} - \vec{w}_{ff}$, for
two types of fermions $i$ and $j$, which may be from differ-
fent families or different flavors within the same family.

In order to determine the opening angle between the
HY lines associated with SM and model $K$, we define the
SM vector $\vec{r}_{f}^{SM} = \vec{w}_{fj}^{SM} - \vec{w}_{ff}^{SM}$ = $(\tilde{m}_{fj} - \tilde{m}_{ff}, y_{fj}^{SM} - y_{ff}^{SM})$, and
$\vec{r}_{f}^{K} = (\tilde{m}_{fj} - \tilde{m}_{ff}, y_{fj}^{K} - y_{ff}^{K})$, for model the $K$. Then,
by evaluating their inner product, one finds the following
expression for the opening angle between those lines:

$$\cos \Psi_{K} = \frac{(\Delta \tilde{m}_{ij})^{2} + \Delta y_{ij}^{K} \Delta y_{ij}^{SM}}{\sqrt{(\Delta \tilde{m}_{ij})^{2} + (\Delta y_{ij}^{K})^{2}} \sqrt{(\Delta \tilde{m}_{ij})^{2} + (\Delta y_{ij}^{SM})^{2}}} \quad (3)$$

where: $\Delta \tilde{m}_{ij} = \tilde{m}_{ij} - \tilde{m}_{ff}$, $\Delta y_{ij}^{K} = y_{ij}^{K} - y_{ij}^{K}$, $\Delta y_{ij}^{SM} =
y_{ij}^{SM} - y_{ij}^{SM}$. Then we are ready to calculate $\Psi_{K}$ for the
2HDM of different types.

III. THE OPENING ANGLE $\Psi_{K}$ IN THE
2HDM-I, II

We can use the master formula (11) in order to evalu-
ate the angles predicted in the 2HDM of type I and II.
Within the 2HDM-I, the Higgs-fermion couplings is given
by: $y_{f}^{K} = \eta_{f}y_{SM} = \eta_{f}\tilde{m}_{f}$, with $\eta_{f} = \cos \alpha / \sin \beta$, then the
The expression for the opening angle $\Psi_I$ simplifies as:

$$\cos \Psi_I = \frac{1 + \eta_f}{\sqrt{2(1 + \eta_f^2)}}^{1/2}. \quad (4)$$

The angles $\Psi_I$ must be in agreement with the most up-to-date Higgs measurements, for this purpose we shall use the current reports on the Higgs boson properties from the ATLAS collaboration [16] (while CMS results are from [17]). In addition, in order to confront our analysis with future expectations, we shall also consider the analyses for the HL-LHC [18]. Table I shows the corresponding results of the fits for Higgs boson coupling modifiers $\kappa_X$.

We show in Fig. 1 the contour lines for the angle $\Psi_I$ within the 2HDM-I in the plane $c_{\beta-\alpha} - t_\beta$ (where $\cos(\beta - \alpha) = c_{\beta-\alpha}$ and $\tan \beta = t_\beta$), as well as the allowed regions by LHC signal strengths $\mu_X$ with $X = \gamma, \beta, \tau, W, Z$, our analysis takes into account both the LHC Higgs data as well as the low-energy constraints (e.g., rare b-decays, muon magnetic moment, etc). We also notice that the couplings modifiers $\kappa_F$ and $\kappa_V$ are measured assuming no-BSM contributions to the Higgs boson decays. The allowed region by current LHC data corresponds to the area encircled by the red line; within that region we observe that $\Psi_I \lesssim 0.034$. When the expected measurements at the HL-LHC are considered, the allowed region is now delimited by the segmented green line, in this case we find that $\Psi_I \lesssim 0.021$.

Then, we choose a few particular points in that plane, namely: $c_{\beta-\alpha} = 1$ and $t_\beta = 1, 5$ and 30, and look for the plot of the HCL. This is shown in Fig. 2 which includes the HCL for both the SM and 2HDM-I cases. When the LHC results are considered (shaded pink area), it exclude values of $t_\beta \lesssim 1$, but values with $t_\beta > 1$ are allowed. The corresponding HCL $\Psi_I$ of the 2HDM-I get closer to the SM line as $t_\beta$ grows. Figure 2 also shows the best fit for the HCL, which is clearly within the errors. A slight difference is observed when the signal strengths $\mu_X$ is considered, in that case $t_\beta \lesssim 1.2$ is the excluded. On the other hand, when the HL-LHC option is considered we notice that a significant reduction of possible allowed HCL is obtained (shaded green area).

Far the 2HDM of type II, the HCL associated with up-type quarks subtends an opening angle $\Psi_I^{F_I}$, while d-type quarks and leptons define a second angle $\Psi_I^{V_I}$. The corresponding expressions for $\Psi_I^{F_I}$ are similar to eq. (4), but with $\eta_f$ replaced by $\eta_u = \cos \alpha/\sin \beta$ and $\eta_d = -\sin \alpha/\cos \beta = \eta_l$. We also notice that in the alignment limit, when $\tan \beta \simeq 1$, and $\alpha \simeq -\pi/4$, one gets: $\cos \Psi_I^{F_I} \simeq 1$, i.e. $\Psi_I^{F_I} = 0$. For up-type quarks it is not possible to get a value of the opening angle $\Psi_I^{F_I}$, as so far we only have a precise measurement of the top...
quark Yukawa coupling in the up-quark sector. However, we for the $d$-type quarks and leptons, we can combine the corresponding measurements in order to calculate $\Psi^d_{II} = \Psi^l_{II}$.

The contour lines of the opening angle $\Psi^d_{II}$ are shown in Fig. 3, again in the plane $c_{\beta-\alpha} - t_\beta$, for points that satisfy all $\mu_X$ tests. Current LHC data allows two areas, which are encircled by the red lines, but one of them disappears when the HL-LHC option is considered (segmented green lines). We also notice that when the region $c_{\beta-\alpha} \to 0$ is favored, it allows values up to $t_\beta = 50$. Within the region close to $c_{\beta-\alpha} \to 0$, we find that $\Psi^l_{II} \lesssim 0.74$, whereas in the other area we have $1.5 \Psi^l_{II} \lesssim 1.6$. Then, in order to display the HCL, we choose particular points in that plane, namely:

- $c_{\beta-\alpha} = 0.05$ and $\tan \beta = 1, 2$,
- $c_{\beta-\alpha} = -0.05$ and $\tan \beta = 1, 2$.

The corresponding HCL’s for 2HDM-II are shown in Fig. 4. After considering the LHC constraints (shaded pink area), we notice that all of the above points are allowed (small box with bottom, tau an muon couplings). However, when one takes into account the HL-LHC option, the point $c_{\beta-\alpha} = 0.05$ and $\tan \beta = 2$ is no longer allowed, which shows how powerful are the constraints obtained from the opening angle in the 2HDM-II.

IV. HIGGS COUPLINGS IN EFFECTIVE THEORIES AND NEUTRINO MASSES

It is also possible to construct models where some of the fermion masses are forbidden at the lowest order (dim-4 operators), and rather involve higher-dimensional operators $[12]$. In particular, one can use higher-powers of the bilinear Higgs field, i.e. $(\Phi^\dagger \Phi)^n$, with $n$ being some integer adjusted to reproduce the fermion mass. For the complete 3-family case one needs a set of integers aimed to explain the hierarchy of the fermion masses and mixing angles. For a single fermion type, the effective lagrangian can be written, and expanded, as follows:

$$\mathcal{L} = \lambda_f \left( \frac{\Phi^\dagger \Phi}{\Lambda^2} \right)^n \overline{F}_L \Phi f_R + \ldots + h.c.$$  \quad (5)

where $\lambda_f$ is an O(1) coefficient, $\Lambda$ is the scale at which these operators are induced from a UV-completed theory. In the second step, we identified $m_f = \frac{\pi}{\sqrt{2}} \left[ \lambda_f \left( \frac{\mu^2}{\Lambda^2} \right)^0 \right]$. The second term implies that the HYC is given by: $g_{hff} = (2n+1)g_{hff}^{SM}$. Thus, we can see that for $n = 0$ one reproduces the SM relation for the Higgs couplings and the fermion mass, which is certainly correct for the top quark case. But for the lighter fermions, powers $n \geq 1$ are needed for $\Lambda \simeq 1 \, \text{TeV}$, which then result in O(1) deviation from the SM case. This idea was implemented in ref. [12] for the 3-family case, and the values of $n$ employed there are already excluded by LHC data.

More realistic models, in agreement with LHC has been constructed in ref. [19], but this achieved by considering
two Higgs doublets. In general the expressions for the HUY will have the same form as before, i.e. with the factors \((2n+1)\), but we can suppress them with the vevs \(v_2 < v_1\), or \(\tan \beta\). For instance in the type-2 model considered in ref. [19], the kappa factor for the d-type quarks is given by:

\[ \kappa_{di} = n_{di} \frac{\cos \alpha}{\sin \beta} - (n_{di} + 1) \frac{\sin \alpha}{\cos \beta} \]  

(6)

The b-quark mass is reproduced with \(n_{dib} = 1\), but in order to satisfy current LHC bounds on deviations from the SM Higgs couplings with b-quark, one may require special values of \(\alpha\) and \(\beta\). By considering that the Higgs coupling with massive vector bosons \(W\) and \(Z\) deviates by a little from the SM case, i.e. \(\cos(\beta - \alpha) = 0.1\) and \(\tan \beta \approx 1\), we have that the positive solution for \(\alpha\) gives a value \(\kappa_b \approx 2\), which is clearly excluded by the data shown in Table II. However, the negative solution for \(\alpha\) implies a value of \(\kappa_b \approx 1.07\), which is allowed by the Higgs LHC data. We leave a detailed numerical analysis of this model for the future, including the possibility of having flavor violation [20].

Thus, so far we have seen that the Higgs couplings with quarks and charged leptons display a linear relationship with the fermion masses. But, is this the only possibility? What happens in the case of neutrinos? Here it depends on the type of neutrino masses. When their masses are of the Dirac type, the corresponding relationships are also of the linear type. However, when neutrinos are of Majorana type, and its smallness is associated with the see-saw mechanism, it is possible to have a non-linear relationship. We can illustrate this by considering a single family, with a lagrangian that includes both a Yukawa couplings of the type: \(y_L \Phi \nu_R\), as well as a Majorana mass term for the right-handed neutrinos, \(M \nu_R \nu_R\). Then the well known see-saw mass formula gives the light neutrino mass of the form: \(\frac{(y_L \nu)^T}{M}\), which then implies that the light neutrino coupling with the Higgs is given by:

\[ g_{\nu \nu} = \frac{(m_\nu M)^{1/2}}{\nu} \]

Thus, for neutrinos its coupling with the Higgs goes like the square root of the neutrino mass, i.e. in this case we have a non-linear relation.

V. CONCLUSIONS

Although, the LHC has provided bounds on the new physics scale (\(\Lambda\)), that are already entering into the multi-TeV range, some of the motivations for new physics are so deep, that it seems reasonable to wait for the next LHC runs, with higher energy and luminosity, in order to have stronger limits, both in the search for new particles, such as heavier Higgs bosons, and for precision tests of the SM properties. The Higgs coupling with fermions and gauge bosons, as a function of the particle mass, are predicted to lay on a single straight line within the SM. So far, the LHC has measured these couplings for the heavier particles, which are found to satisfy this prediction, but a complete test of this property must also involve the lighter generations.

We have studied models with an extended Higgs sector, where there could be one or more Higgs Couplings lines (HCL), such as the 2HDM or the Private Higgs Models (in this case there could be three HCL’s). In this letter, we used the fermion masses and Yukawa couplings obtained at LHC, to calculate the angle \(\Psi_K\) subtended by those lines and the SM one. By considering the Higgs data from two flavors (t and b, for instance) we can calculate the angle subtended by the SM prediction, with respect to the best fit line, and we obtained the value \(\Psi_{ex} = \pm O(10^{-4})\). We also studied the predictions for these angles for several versions of the 2HDM, which are compared with the values of these angles obtained using current data on the fermion masses and couplings. Our results indicate that already LHC data constraints significantly the parameters of the models. Models where the fermion masses hierarchy arise from higher-dimensional operators defined with the electroweak scale, may also result in large deviations from the SM values for the Higgs couplings, but in this case we would have an irregular pattern for the couplings, as a function of the fermion masses. In the case of neutrino masses, it happens that when the masses arise from the See-saw mechanism, the corresponding neutrino-Higgs couplings display a non-linear mass relation.

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