Collision times in $\pi\pi$ and $\pi K$ scattering and spectroscopy of meson resonances

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Abstract

Using the concept of collision time (time delay) introduced by Eisenbud and Wigner and its connection to on-shell intermediate unstable states, we study mesonic resonances in $\pi\pi$ and $\pi K$ scattering. The time-delay method proves its usefulness by revealing the spectrum of the well-known $\rho$- and $K^*$- mesons and by supporting some speculations on $\rho$-mesons in the 1200 MeV region. We use this method further to shed some light on more speculative meson resonances, among others the enigmatic scalars. We confirm the existence of chiralons below 1 GeV in the unflavoured and strange meson sector.

PACS numbers: 13.75.Lb, 11.55.-m, 14.40.Aq, 25.40.Ny
1 Introduction

The study of hadronic resonances, be it baryons or mesons, has never ceased (with seasonal fluctuations) to interest the particle and nuclear physics community for several specific and one global reason [1, 2, 3, 4]. The latter reason is to be found in the fact that hadrons and their interaction are the low energy manifestations of QCD. The specific reasons include among others: (i) the wish to improve upon standard results (e.g. resonance parameter extraction), (ii) the attempt to understand inconsistencies and puzzles, if any, (iii) possible new resonances (of speculative status, enigmatic nature, predicted by models and/or resurrection of previously refuted results), (iv) exotic resonances like glueballs, hybrids (named also hermaphrodites and meiktons), multiquark states such as $q^2\bar{q}^2$, mesonic molecules and $J^{PC}$-exotics, (v) the enigmatic scalar sector with the return of the $\sigma$-meson into the data book of the Particle Data Group (PDG) [5]. We shall come back to (i)-(v) in the next section. Here it suffices to note that it is very often the existence of a resonance which is at stake. Insufficient data, weak signal and different methods applied to analyze data can result in different conclusions. The analytical methods to extract resonances and their parameters are well known and well documented. It then comes as a mild surprise to know that not all known analytical methods have been fully exploited. For instance, it is now fifty years since the concept of collision time (time delay) was introduced by Eisenbud and Wigner [6, 7, 8]. Its connection to resonances is best explained in Wigner’s own words [6]: “Close to resonances, where the incident particle is in fact captured and retained for some time by the scattering centre, $\frac{d\eta}{dk}$ [this is the collision time where $\eta$ is the scattering phase shift] will assume large positive values”. Not much has changed since then, except that the concept of time delay has been further elaborated by many authors [9-12], incorporated into text books [13-18] and newly rediscovered in a different context [19, 20]. In all these references the connection to resonance physics is well documented and stressed. Nowadays, one might not look upon a resonance as a particle being retained by another for a while, but rather as an unstable intermediate state characterized by several quantum numbers. To clarify this point, we quote ref. [12]: “Finally we remark that a sharp maximum in the time delay (essentially condition (iii) [the specific time delay $\frac{d\delta}{dk}$ has a sharp maximum]) is sufficient for the existence of a resonance; the argument is given in Ref. 12 [our reference [15]]”. We could continue quoting text books and papers on time delay, but it would mean repeating more or less one and the same phrase. The surprise then grows when we learn that only recently the time delay concept has been put to a test in connection with hadron resonances [21, 22], with reasonable success. By this we do not mean that time delay has not been extensively discussed in literature (see earlier references and for some criticism see [23]), but we are not aware of any work which has made a practical use of the advocated time delay in connection with hadron resonances. Time delay analysis for $\Sigma$-hypernuclear states has been successfully performed in [24]. A related and less documented but somewhat popular concept, namely the speed plot,
is occasionally found in literature [5, 25, 26]. However, speed plots are positive by definition whereas time delay (as obvious from the citation from Wigner’s paper) can assume positive as well as negative values. Hence, regions with a negative bump in the collision time, could come out as positive peaks in the speed plot. In this context, we quote Wigner once again: “One would expect (on the basis of Liouville theorem or completeness relation) that the two effects [positive and negative values of time delay], on the whole, balance each other, i.e. that the integral of \( \frac{d\eta}{d\epsilon} \) (in Wigner’s notation, \( \eta \) is phase shift, which is denoted as \( \delta \) in the present work) over the whole energy range is close to zero, ..... Hence, if the cross section shows a resonance behaviour, one will expect \( \eta \) to decrease slowly between resonances and increase fast at resonances, increase and decrease almost exactly balancing if considered over the whole energy spectrum.” A demonstration of the above statement can actually be found in [27] for a constructed example of seven consecutive Breit-Wigner resonances. A detailed comparison between time delay and speed plots can be found in [28]. We supplement the above by noting that the connection between time delay and the statistical density of states in scattering has been given in [29]. We also note that not every peak in the cross section can be attributed to a resonance [12].

In view of the above discussion, it seems worth undertaking an examination of time delay in scattering processes with data, encountering several resonances over a wide energy range. In [21, 22] we have applied the time delay method for baryon resonances in \( \pi N \) scattering. In the present paper, we do the analog for mesonic resonances in \( \pi\pi \) and \( \pi K \) collisions. Indeed, we get very sharp positive peaks for all established resonances which confirms the statements made by Wigner and others in literature. Agreement with the known resonances adds support to the interpretation of the additional peaks we find as new resonances (indeed, these additional peaks have been a matter of debate for some time).

Our paper is organized as follows: in the next section we give some necessary facts about mesonic resonances. These facts will be necessary to interpret our results. In the subsequent section we briefly discuss the relevant formulae on time delay. In sections 4 and 5 we present the collision times calculated for \( \pi\pi \) and \( \pi K \) scattering. The last section is devoted to conclusions.

## 2 A short survey of relevant mesonic resonances

Given the seniority of the subject, the number of possible topics worth mentioning, even with the restriction to the points (i)-(v) mentioned in the Introduction in context with the continuing interest in mesonic resonances, is of course too large to be covered here. Therefore the short survey below is coloured by what we thought to be relevant for the results of the present paper.

As an example of the points (i) and (ii) from the previous section, we quote the case of the \( \rho \) mesons. Till the year 2000, the PDG listed a group of \( J^{PC} = 1^{--} \) mesons with masses around 1100 – 1200 MeV discovered in \( e^+ e^- \) collisions [30]. In
the 2002 edition [5] this entry has been removed due to the lack of further evidence supporting the old data. In the same year this controversy has been revived by an experimental indication of an isovector state with mass around 1200 MeV [31, 32, 33]. As noted in [33] this is also supported by the \( \gamma p \rightarrow (\omega \pi)p \) reaction, where one finds a mass enhancement in the \( \omega \pi \) system which is partly attributed to additional \( \rho \) meson [34].

Additional conclusions regarding the \( \rho \) mesons are drawn from 'inconsistencies' in \( e^+e^- \) collisions and \( \tau \) decays [35, 36]. These seem to indicate the necessity of a vector hybrid [37, 35] which serves as a good example for the points (iii)-(iv) in the \( \rho \) mesonic sector. It is beyond the scope of this work to describe in detail the situation of exotic mesons [38]. However, one of the exotics, namely the glueball, has indirectly to do with the findings of our paper. We say so because, according to lattice theory and other models, the lightest glueball is predicted to be a true scalar with a mass around 1700 MeV [39]. Opinion as to which one of the low lying scalars has the largest admixture of glue differs [40, 41, 42]. This adds to the other problems encountered in the scalar sector (point (v)). Regarding the glue, in the scalar sector, the interpolating field for \( \sigma \) could be \( F \cdot \tilde{F} \) [43]. The situation is not unlike the \( \eta \) mesons, where through the \( U(1)_A \) anomaly, there is a connection between \( F \cdot \tilde{F} \) and the \( \eta_0 \) field [44] making us think about a glue content of eta mesons and \( \eta_0-\eta_8 \)-glue mixing [45]. In any case, the most famous problem beside the glue in the scalar sector is the \( \sigma \) meson itself, which was removed from PDG in 1974 and reappeared there much later. This metamorphosis [46] (of appearance and disappearance) is typical for this meson in other respects too. From the experimental point of view, it is sometimes claimed that this meson behaves differently in different physical situations [47] i.e. displaying different masses and lifetimes. This is also reflected by the wide mass range quoted by PDG: 400 – 1200 MeV. From the theoretical side, we can view the \( \sigma \) meson as a Higgs particle in the context of the linear sigma model [48] after the spontaneous breaking of chiral symmetry, i.e., as the real part of this Higgs field whose vacuum expectation value breaks the chiral symmetry (the imaginary part resulting into pseudoscalars). Alternatively, one can look upon \( \sigma \) as a low energy manifestation of the scale invariance breaking in the strong interaction [49] (the 'identification' \( \sigma \sim F \cdot \tilde{F} \) mentioned above is motivated by this scale invariance breaking since the latter is connected to the trace anomaly in QCD by which the dilaton current is not conserved; note again the analogy to the eta mesons where the connection to the gluon fields is through another anomaly, namely the \( U(1)_A \)). Hence the question: chiralon, dilaton or both?

In spite of these theoretical insights, there is no general consensus about the parameters of the \( \sigma \) meson. The Nambu-Jona-Lasinio model [50] would require a mass of roughly twice the constituent quark mass i.e. 600 – 700 MeV. Weinberg’s prediction in the framework of mended symmetry is \( m_\sigma \approx m_\rho \) [51, 46]. The latter seems also to agree with values required to satisfy the Adler sum rule [52, 46]. Calculations using Bethe-Salpeter approach [53] give \( m_\sigma \sim 750 \) MeV which, however, is rejected by the
authors themselves [54] to be the mass of the $\sigma$ meson on account of it being too heavy. Indeed, the pole values predicted in the unitarized chiral perturbation theory [55] and the unitarized quark model [56], give the $\sigma$ mass around 400 MeV, which is roughly in agreement with potential like models [57, 58]. An even lower value for $m_\sigma$, namely $m_{\sigma}^{BW} \simeq 390^{+60}_{-36}$ MeV is reported by a very recent observation of $J/\Psi \rightarrow \sigma \omega \rightarrow \pi \pi \omega$ [59]. Last but not least, we mention a mysterious low $\pi \pi$ mass enhancement at 310 MeV which dates back to 1961 [60] and is since then coined ABC-effect. It has not vanished during the years, but continues to leave its fingerprints in several scattering processes [61] till today.

Above 1 GeV, a resonance is often mentioned around 1200$\pm$1300 MeV and was earlier known as $\varepsilon(1300)$ [4, 62]. It is often stressed that this is not the $f_0(1370)$ scalar because of different inelasticities into $\pi \pi$ and $4\pi$. This problematic region borders to $f_0(400 \pm 1200)$ on the one hand and $f_0(1500)$ on the other. Therefore strong interference effects are to be expected. A strong support for a resonance around 1300 is given in [42]. In [41] the status of $f_0(1370)$ as a genuine resonance is doubted.

Putting everything together, we recognize three problematic regions in the $I(J^{PC}) = 0(0^{++})$ sector: 300$\pm$450 MeV, 700$\pm$800 MeV and 1200$\pm$1300 MeV, where one finds strong hints/evidences for unusual activities/resonances. Actually these regions are often attributed to a single resonance, the $\sigma$. Reproducing the figure in [47] with new data in Fig. 1, one sees that most data points are grouped around these three regions.

![Figure 1: Masses and widths of the scalar $\sigma$ meson as determined by different groups as listed in PDG.](image)

Having discussed the unflavoured mesons which we might find in the $\pi \pi$ sys-
tem, we now turn our attention to strange resonances encountered in $\pi K$ scattering. We are not aware of any problems/controversies in the $I(J^P) = 1/2(1^-)$ sector and this problem free zone is also confirmed nicely by our analysis. There are some uncertain candidates in the $I(J^P) = 1/2(2^+)$ spectrum around 2 GeV [5] and we add to this uncertainty two never-heard-of ‘candidates’ around 1.7 and beyond 2 GeV. We will discuss the reliability of these results in section 5. However, more importantly, it is again the scalar mesons $I(J^P) = 1/2(0^+)$ which stir up controversies and it is again about the lightest strange scalar which right now goes under the name $\kappa$. Whereas the first suggestions of $\sigma$ date back to early 60’s [63], one can trace back the $\kappa$ to early 70’s [64]. As with the $\sigma$, the $\kappa$ meson too has met with either an outright refutal for the basis of its existence [65] or support together with determination of its mass and width. Two often quoted values for its mass are: $700 - 750$ MeV [66] and close to $900$ MeV [67]. This resembles very much the situation of $\sigma$.

3 Time-delay and phase shift

In this section we give the main formulae concerning time delay without derivation and refer the reader to literature for a more detailed account [6, 10, 14]. Intuitively, a positive time delay is a measure of how much a reaction is delayed, say due to an unstable intermediate state. Close to the resonance region we would expect to find a positive peak in the energy distribution of time delay. Indeed, since in a resonant process we produce the resonance on-shell, the process itself can be viewed in steps. The first is the resonance production at the space-time point $P_1 = (t_1, x_1)$ followed by the resonance decay at $P_2 = (t_2, x_2)$ with $P_1 \neq P_2$ and $\Delta t = t_2 - t_1 > 0$ (non-localized processes can also be encountered in t-channel [68] where one of the initial particles is a resonance). Negative time delay occurs when the interaction is repulsive and/or a new decay channel opens up [21]. The measure for both has been found by Wigner and Eisenbud [6, 8] in the form of the energy derivative of the phase shift $\delta$ as,

$$\Delta t = 2\hbar \frac{d\delta}{dE}$$

which is valid for elastic processes [8]. Smith [10] generalized this to calculate the time delay directly from the $S$ matrix (including also inelastic processes $i \rightarrow j$) giving,

$$\Delta t_{ij} = \Re\left[\frac{-i\hbar}{S_{ij}}\frac{dS_{ij}}{dE}\right].$$

With

$$S_{kj} = \delta_{kj} + 2iT_{kj}$$

and

$$T_{kj} = \Re T_{kj} + i\Im T_{kj},$$

6
equation (2) in the elastic case \((i = j)\) can be recast into

\[
S_{ii}^* S_{ii} \Delta t_{ii} = 2 \hbar \left[ \Re \left( \frac{dT_{ii}}{dE} \right) + 2 \Re T_{ii} \Im \left( \frac{dT_{ii}}{dE} \right) - 2 \Im m T_{ii} \Re \left( \frac{dT_{ii}}{dE} \right) \right],
\]

where obviously \(\Delta t_{ii}\) is the same as in equation (1). This can be checked by substituting \(S = \eta e^{i \delta}\) (where \(\delta\) is the real scattering phase shift and \(\eta\) the inelasticity parameter \((0 < \eta \leq 1)\)) which gives,

\[
\Delta t_{ii} = \Re \left[ -i \hbar \left( 2i \frac{d\delta}{dE} + \frac{d\eta}{dE} \frac{1}{\eta} \right) \right] = 2\hbar \frac{d\delta}{dE} \tag{6}
\]

Smith’s formalism shows that Wigner’s expression (1) is valid for elastic reactions even if there are non-zero inelasticities. The generalization of time delay with \(N\)-body interaction was done later in [11]. The statistical connection to time delay mentioned in the introduction is given by [29]

\[
\sum_l \Delta n_l(E) = \sum_l \frac{2l + 1}{\pi} \frac{d\delta_l(E)}{dE} \tag{7}
\]

where \(\Delta n_l\) is the difference in the density of states with and without interaction and \(\delta_l\) is the phase shift in the \(l\)th partial wave. Since \(\Delta t\) can be positive as well as negative, some references call it collision time reserving the name time delay for the case \(\Delta t > 0\) and time advancement for \(\Delta t < 0\).

It is well known that the lifetime of an unstable state depends in principle, on its preparation, i.e. on the energy spread \(\Delta E\) of the initial particles which produce the state. Most of the experiments producing hadron resonances certainly operate in the region where the inequality

\[
\frac{\Delta E}{m} << \frac{\Gamma}{m} \tag{8}
\]

with \(\Gamma\), the width of the resonance, holds true. In case of broad resonances, the Breit-Wigner distribution is not necessarily a good parametrization. Moreover, in such cases the mean lifetime is not \(1/\Gamma\) [17, 18], but rather \(\Delta t(E)\) (where \(E\) is the energy available in the centre of mass system), as given above [19]. One can also put it in different words. Since for a narrow resonance, the Breit-Wigner is the spectral function [69, 70] which leads to the exponential decay law, it is reasonable to say that \(\Delta t(E)\) is the right spectral function for a broad resonance (the Fourier transform of this spectral function gives the survival amplitude). It is then clear that time delay is so to say tailored to study broad hadronic resonances, especially the mesonic cases mentioned in the last section (which we shall confirm in the next two sections).

Before applying the time delay method to the broad mesonic resonances, we give here two words of caution. Since the collision time is given as derivative of the
phase shift, a rather good quality of data is required to avoid ‘false’ bumps, i.e. signals in the time delay plots which are not genuine resonances but rather an artifact of the fit. Secondly, the phase shift necessary to calculate the collision time, cannot be uniquely determined through data. Indeed, one usually gets several solutions which then have to pass certain tests (involving crossing symmetry, unitarity, analyticity etc.) in order to decide which solution is the physical one. Even then it is not so clear if one can avoid the so-called continuum ambiguity [71]. In our case, using time delay, a good check will always be if we can reproduce the well established resonances.

In passing, we note that Wigner’s intention in [6] was to find a criterion to distinguish between physical and unphysical phase shifts. He found

$$\Delta t = 2\hbar \frac{d\delta}{dE} > -a$$

which is a causality condition since a time advancement cannot be arbitrarily large. In (9) $a$ is interpreted as interaction range.

A legitimate question is about the relation between resonance parameters defined as poles of the T-matrix and positive peaks in time delay. In this connection we quote ref. [72] where the author says, “Moreover, if there is an appreciable time delay $(t'' > t')$, the latter should be interpretable as arising from the propagation of an unstable intermediate particle. The above requirements, which are readily generalized to multiple scattering processes, are sufficient to derive the pole structure of the S-matrix and the existence of antiparticles” and ref. [13] where the authors state, “However, the information that gives an indication of a large time delay - typically the fast traversal by the amplitude of a resonance circle as described in the sections “Elastic Resonances” and “Inelastic Resonances” - is the same information whose extrapolation on to the unphysical sheet informs us as to the existence of a pole”. We can also perform a practical test by evaluating time delay using phase shifts calculated within a model and comparing the time delay peaks with the pole values of the T-matrix within that model. For instance, using the model of Kaminski, Lesniak and Loiseau described in [73], for the $\pi\pi$ elastic scattering phase shifts, we get the resonance peaks shown in Fig. 2, which are in good agreement with the pole values found within the model (see Table 3, of [73]).

We would also like to draw the reader’s attention to ref. [27], where a simple model of a coherent sum of seven resonances was made to display the virtues of the time delay method. In the Argand diagram plot, these resonances showed up misleadingly as one resonance whereas the time delay plot could differentiate between these resonances with the peak values coinciding with the pole positions (see Fig. 1 and Fig. 3 in [27]). This work indicates that the time delay method is then well-suited for overlapping resonances.
Figure 2: Time delay plot of the scalar meson resonances evaluated using the s-wave phase shifts from the model calculation of [73] for ππ elastic scattering.

4 Resonances in the ππ system

We start our exploration of time delay and resonances, examining the ππ phase shifts. These phase shifts have been extracted with care and passed several tests [74] (among others the Roy equations [75]). Hence we have confidence that they represent the true physical situation (this is also justified because we do confirm all well-established resonances in this case). A note about error bars is in order here. The errors in the phase shifts get transformed into the errors in time delay and in principle can shift the positions of the time delay peaks. However, the aim of the present work is not a precise determination of resonance parameters but rather showing the usefulness of the time delay method. Hence we do not quote the errors in the peak positions, but we do make a double check by using two different sets of phase shifts whenever possible.

ρ mesons: This is an example of the analytical power of the time delay method applied
to resonance physics. As Fig. 3 nicely demonstrates, we find a prominent peak for the \( \rho \) meson where one would expect it to be, at 765 MeV. The phase shifts are from [76]. We identify the peak at 1380 MeV with \( \rho(1450) \). The PDG’s estimate (‘educated guess’) for this resonance is [5] 1465 MeV. However, in the \( \pi\pi \) mode the mass range listed is 1292–1406 MeV with an average value of 1370 MeV. A similar analysis can be applied to the next peak at 1725 MeV which is due to \( \rho(1700) \). The peak value is close to the PDG estimate of 1720 ± 20 MeV. According to PDG, the mass for this resonance in the \( \pi\pi \) mode ranges between 1590 and 1838 MeV. Having confirmed the three known mesons in the time delay plot, it was worrying to find an additional peak at 1166 MeV. It does not seem that this is an artifact of the fit or due to poor quality of the data and we had to accept this signal as a genuine one. We found independent support in [77, 31, 32, 33] resulting in an accumulated evidence for a \( \rho \) resonance around 1200 MeV. Note that the sources of this evidence are quite different.

Scalar (\( f_0 \)) mesons: The phase shifts in Fig. 4 are from [78] and [79]. The time delay

Figure 3: Time delay plot of the \( \rho \) resonances evaluated from a fit to the p-wave phase shifts in \( \pi\pi \) elastic scattering.
calculated from the two different sets of phase shifts gives a similar qualitative picture which shows that the underlying physics should be taken seriously. The peak on the far right 1.42 (1.47) GeV is a signal of $f_0(1500)$ whose possible mass values listed in PDG are between 1.4 and 1.6 GeV. This resonance is overlapping from the left with $f_0(1370)$ for which we find peaks at 1.23 (1.34) GeV (PDG quotes 1.2 – 1.5 for the pole position and also for the Breit-Wigner value). The sharp peaks at 0.982 (0.986) are of course the well established $f_0(980)$ bordering at a resonance structure at 0.7 GeV, which in turn in one of the cases overlaps with a bump at 0.35 GeV. The individual peaks are not well separated due to the large widths of scalar resonances. Nevertheless, the time delay method is able to distinguish between these overlapping cases. Due to the successful separation of standard cases, we do not think that the 2 peaks below 1 GeV (350 and 700 MeV) are accidental or due to the fit and/or data. These two regions at 400 and 700 MeV are also the most quoted in connection with the $\sigma$ meson mass. It is worth stressing that we have recovered these regions here in one and the same reaction.

At the end of this section, it is instructive to subject the time delay method to yet another test. The resonances found in the elastic scattering process, $\pi\pi \rightarrow \pi\pi$, should also manifest themselves in the coupled channel reactions, $\pi\pi \rightarrow K\bar{K}$ and
Figure 5: Energy dependence of the s-wave phase shifts and time delay in the (a) $\pi\pi \rightarrow KK$ and (b) $KK \rightarrow K\bar{K}$ reactions. The phase shifts $\delta_{KK}$ (solid lines in (b)) have been determined by subtracting the fit to the CERN-b $\pi\pi$ phase shift data in Fig. 4 from $\phi_{\pi K}$ in the above figure (a). Dashed lines show the same quantities in the model calculation of ref. [73].

$KK \rightarrow K\bar{K}$, provided they have an appreciable branching ratio for decaying into $K\bar{K}$. The plots in Fig. 5 are just the results of such a test in the mass range from 1 to 1.5 GeV. In Fig. 5a, we show the data on $\phi_{\pi K}$ ($\phi_{\pi K} = \delta_{\pi\pi} + \delta_{K\bar{K}}$, where $\delta_{\pi\pi}$ and $\delta_{K\bar{K}}$ are the elastic scattering phase shifts in the $\pi\pi$ and $KK$ channels) obtained in [80, 81]. We make a fit to this data and subtract from it, the fit made earlier in Fig. 4 to the CERN-b $\pi\pi$ data, to obtain the $K\bar{K}$ phase shift (solid line) shown in Fig. 5b. The dashed lines in this figure are the phase shifts obtained in a model calculation in [73]. The resonance found at 1.42 GeV (peaks in shaded regions) using $\phi_{\pi K}$ and $\delta_{K\bar{K}}$ is to be associated with $f_0(1370)$. We can see that by using the model phase shifts of [73] which give a peak at 1.4 GeV. Note that the data sets of $\pi\pi \rightarrow \pi\pi$ and $\pi\pi \rightarrow K\bar{K}$ reactions are different and we do not expect the mass values from the time delay peaks in $\pi\pi \rightarrow \pi\pi$ (Fig. 4) and $KK \rightarrow K\bar{K}$ (Fig. 5) to be exactly the same. Note also that nothing can be inferred about $f_0(980)$ and $f_0(1500)$ since both are just at the edge of the data set in Fig. 5.

The data in Fig. 5 has the advantage that one can compare the result from time delay with the speed plot shown in [80, 81]. The speed plot is actually the energy distribution, $SP(E)$, defined as,

$$SP(E) = \left|\frac{dT}{dE}\right|$$ (10)
where $T$ is the energy dependent complex $T$-matrix. In [80, 81], $SP(E)$ has been calculated (Fig. 2 in [80] and Fig. 28(b) in [81]) and the speed plot peak appears at 1.425 GeV, which is in excellent agreement with our finding of the time delay peak in Fig. 5.

5 Resonances in the $\pi K$ system

We now perform a similar analysis as above in the strange meson sector. The results confirm all established mesons, the $\kappa$ meson and hint in addition to the existence of new strange mesons.

Strange ($K^*_0$) scalar mesons: The only well established strange scalar is $K_0^*(1430)$. However, over the years there have been claims about the existence of a lighter strange scalar called $\kappa$ [64, 66, 67]. Its mass is thought to be within the range $660 - 900$ MeV.

Figure 6: Time delay plots evaluated from the phase shifts in s-wave, isospin $1/2$ $K\pi$ scattering, displaying the strange scalar mesons.

[66, 67]. The situation resembles the case of the light unflavoured mesons, not only regarding its existence, but also its wide mass range thought to be possible. It should then come as no surprise, when the broad mass range gets resolved into several peaks. Two sets of phase shifts from [82] and [83] are available to study these scalar mesons. Though both of them cover only a small energy region, we can see that they are complementary in a sense. In Fig. 6a we see a clear signal for $K_0^*(1430)$, whereas in Fig. 6b, the tendency for a peak in this mass range is present. The peak at 1 GeV (Fig. 6a) is also accompanied by a tendency towards a lower peak. This is confirmed
by the other time delay plot in Fig. 6b where we see two peaks at 0.8 GeV and 0.96 GeV. These two peaks can be interpreted as a signal of the \( \kappa \) meson in time delay plots. Since the two peaks are very close to each other, they could have been confused as one single resonance.

Strange \((K^*)\) vector mesons: The strange vector mesons have been so far a no-problem zone for resonance spectroscopy. The time delay plots in Fig. 7 calculated from the phase shift given in [82] do confirm this. Clear signals for \( K^*(892) \), \( K^*(1410) \) and \( K^*(1680) \), with PDG mass estimates of \( m = 892, 1414 \) and 1717 MeV respectively, are seen.

Strange \((K^*)\) tensor mesons: Alone a visual inspection of the two phase shift solutions from [84] reveals that this case is different from the previously discussed cases. The visual inspection also reveals that there will be four peaks in the corresponding time delay plots which is indeed confirmed in Fig. 8. Note that the two solutions give us within errors the same spectrum of the strange spin-two resonances. The first peaks in Figs 8a and 8b can be clearly attributed to \( K_{1}^*(1430) \). The peaks at 2.0 and 2.18 GeV are a confirmation of \( K_{2}^*(1980) \) which is listed in PDG, but omitted from its Summary Table (see also [85]). We find additional peaks around 1.73 and 2.4 GeV. In [84] the authors note that “... , but above 1 GeV, there is suggestive evidence for phase motion, though little structure in the magnitude.” We believe that we have now quantified the expression ‘phase motion’ through our time delay analysis. The

![Graph showing K mesons](image_url)

Figure 7: The strange \( K^* \) mesons as observed in time delay evaluated from a fit to the p-wave phase shifts in the isospin 1/2, \( K\pi \) scattering.
additional peaks which we found cannot be easily waived away once we accept the phase shifts where the quality of the data is sufficiently solid to support them. In the $K\pi$ system, one sees more regions of negative time delay. This happens very often in the presence of increased inelasticities. However, it was also pointed out by Wigner [6] that negative time delay is unavoidable over the whole energy region.

![Graph showing $\pi\pi$ and $\pi K$ reactions](image)

Figure 8: The strange $K_2^*$ tensor mesons as observed in time delay evaluated from a fit to the d-wave phase shift solutions of ref. [84] for the isospin 1/2, $K\pi$ scattering.

### 6 Conclusions

In the present work, we have used the method of time delay to investigate the resonances occurring in $\pi\pi$ and $\pi K$ reactions. Time delay (in connection with hadronic resonances) has been advocated since fifty years, but to our knowledge never used in practice. Given the available phase shifts, the outcome of such an analysis is beyond our control, and in principle, it was possible that our conclusions regarding the usefulness of time delay in resonance physics would come out to be negative. However, the contrary is the case. In our time delay plots, the well established resonances always appeared as prominent peaks, in all the cases we examined, which we believe is already quite remarkable. Apart from this we also find additional information. In the scalar sector, the signals at 350 and 720 MeV are in regions claimed also by other authors/groups to be regions of resonance activity. In the strange scalar sector we confirm the $\kappa$ meson; however, we get two neighbouring peaks. Since these peaks are very close to each other, we cannot say with certainty if they are really two resonances.
or artifacts of the fit. In the $\rho$ mesonic sector, we find evidence of a resonance around 1200 MeV which again has been and is a matter of debate (though it has been omitted by PDG in 2002 for lack of evidence, in the same year several groups found the evidence for its existence using different methods). The $K^*$ ($J = 1$) resonances would be the case par excellence, since no problems are known in this sector and all that we found are the well established cases too. The only sector where we found never-heard-of resonances (again in addition to known cases) is the $K^*_2$ group. The phase shifts in this particular case look distinctly different from the other cases we have examined. Taking the given phase shifts at face value, it would certainly be worthwhile looking into the possibility of two new strange tensor resonances. This is supported by the fact that two different phase shift solutions yield the same resonances in the time delay plot.

In general, it seems to us that the time delay method is a useful analytical tool in studying resonances. It certainly is not an exclusive tool, but taken together with other methods it allows us to have additional insight into the complicated matter of overlapping and/or broad resonances, as it can apparently resolve individual peaks. For instance, in the enigmatic scalar sector, two phase shifts from different groups (not two different solutions) show the same behaviour: three peaks around 400, 700 and 1200 MeV, rather than one broad resonance structure.

Acknowledgments
We wish to thank V. Krey and J. C. Sanabria for useful information and discussions. We are very thankful to Prof. R. Kaminski for providing the programs and phase shifts generated in their model [73].

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