Measurement of the Ratio of $B^+$ and $B^0$ Meson Lifetimes

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The ratio of $B^+$ and $B^0$ meson lifetimes was measured using data collected in 2002–2004 by the DØ experiment in Run II of the Fermilab Tevatron Collider. These mesons were reconstructed in $B \to \mu^+\nu D^*-X$ decays, which are dominated by $B^0$, and $B \to \mu^+\nu D^0X$ decays, which are dominated by $B^+$. The ratio of lifetimes is measured to be $\tau^+/\tau^0 = 1.080 \pm 0.016 \text{(stat)} \pm 0.014 \text{(syst)}$.

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In the last few years, significant progress has been made, on both experimental and theoretical fronts, in the understanding of the lifetimes of hadrons containing heavy quarks. Charm and bottom meson (except $B_c$) lifetimes have been measured with precisions ranging from 0.5% to 4%, although lifetimes of heavy baryons are not known as well \footnote{On the theoretical front, predictions are being made using a rigorous approach based on the heavy quark expansion (in negative powers of the heavy quark mass) \cite{2}, where the large mass of the bottom quark considerably simplifies calculations. Theoretical uncertainties are further reduced for ratios of lifetimes. For instance, the ratio of the $B^+$ and $B^0$ lifetimes has been predicted to be $1.06 \pm 0.02$ \cite{3}. Experimentally, ratios of lifetimes have smaller uncertainties, since many common sources of systematics cancel.}. Experimental uncertainties are further reduced for ratios of lifetimes. For instance, the ratio of the $B^+$ and $B^0$ lifetimes has been predicted to be $1.06 \pm 0.02$ \cite{3}. Experimentally, ratios of lifetimes have smaller uncertainties, since many common sources of systematics cancel.
In this Letter, we present a measurement of the ratio of $B^+$ and $B^0$ lifetimes using a large sample of semileptonic $B$ decays collected by the DØ experiment at Fermilab in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV. The data correspond to approximately 440 pb$^{-1}$ of integrated luminosity. $B$ mesons were selected via their decays $B \rightarrow \mu^+\nu\bar{D}^0X$ and were classified into two exclusive groups: a $\bar{D}^{*-}\rightarrow \bar{D}^0\pi^-$ sample, containing all events with reconstructed $D^{*-}\rightarrow \bar{D}^0\pi^-$ decays, and a $\bar{D}^0$ sample, containing all remaining events. Both simulation and available experimental results show that the $D^{*-}$ sample is dominated by $B^0 \rightarrow \mu^+\nu\bar{D}^0X$ decays, while the $\bar{D}^0$ sample is dominated by $B^+ \rightarrow \mu^+\nu\bar{D}^0X$ decays.

The classification into these two samples was based on the presence of a slow pion from $D^{*-}\rightarrow \bar{D}^0\pi^-$ decay, and thus was independent of the $B$-meson lifetime. Therefore, the ratio of the number of events in the two samples, expressed as a function of the proper decay length, depends mainly on the lifetime difference between the $B^+$ and $B^0$ mesons. The influences of the selection criteria, detector properties, and some systematic uncertainties are significantly reduced.

The DØ detector is described in detail elsewhere \[5\]. The detector components most important to this analysis are the central tracking and muon systems. The tracking system consists of a silicon microstrip tracker and a central fiber tracker, both located within a 2 T superconducting solenoidal magnet. The resolution for the distance of closest approach as provided by the tracking system consists of a silicon microstrip tracker and matching them with muon track segments formed from hits in the muon system.

Events with semi-muonic $b$-hadron decays were selected using a suite of inclusive single-muon triggers in a three-level trigger system. Muons were identified by extrapolating tracks found in the central tracking system and matching them with muon track segments formed from hits in the muon system. Muons were required to have a transverse momentum $p_T > 2$ GeV/c and total momentum $p^\mu > 3$ GeV/c.

The primary vertex of the $p\bar{p}$ interaction was determined for each event. The average position of the beam-collision point was included as a constraint. The precision of the primary vertex reconstruction was on average about 20 $\mu$m in the plane perpendicular to the beam direction and about 40 $\mu$m along the beam direction.

$\bar{D}^0$ candidates were selected using their $\bar{D}^0 \rightarrow K^+\pi^-$ decay mode. All charged particles in an event were clustered into jets using the DURHAM clustering algorithm \[6\] with a jet $p_T$ cut-off parameter of 15 GeV/c. The $\bar{D}^0$ candidate was constructed from two particles of opposite charge belonging to the same jet as the reconstructed muon. Both particles were required to have $p_T > 0.7$ GeV/c and to form a common $\bar{D}^0$ vertex. The $p_T$ of the $\bar{D}^0$ was required to exceed 5 GeV/c. To reduce combinatorial background, we required the $\bar{D}^0$ vertex to have a positive displacement in the $xy$ plane, relative to the primary vertex, with at least 4$r$ significance. Although this last requirement can bias the lifetime distribution of a $B$ candidate, our analysis procedure of determining the ratio of $B^+$ and $B^0$ events in bins of proper time should remove this bias in the final result. The trajectory of the muon and $D^0$ candidates were required to originate from a common $B$ vertex. The $\mu^+\bar{D}^0$ system was required to have an invariant mass between 2.3 and 5.2 GeV/c$^2$.

The masses of the kaon and pion were assigned to the two tracks according to the charge of the muon, assuming the $\mu^+K^+\pi^-$ combination. The mass spectrum of the $K\pi$ system after these selections is shown in Fig. 1(a). The signal in the $\mu^+\bar{D}^0$ system was required to exceed 5 GeV/c. To reduce combinatorial background, we required the $\bar{D}^0$ vertex to have a positive displacement in the $xy$ plane, relative to the primary vertex, with at least 4$r$ significance. Although this last requirement can bias the lifetime distribution of a $B$ candidate, our analysis procedure of determining the ratio of $B^+$ and $B^0$ events in bins of proper time should remove this bias in the final result. The trajectory of the muon and $D^0$ candidates were required to originate from a common $B$ vertex. The $\mu^+\bar{D}^0$ system was required to have an invariant mass between 2.3 and 5.2 GeV/c$^2$.

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All reconstructed $\mu^+\bar{D}^0$ events were classified into three non-overlapping samples. For each $\mu^+\bar{D}^0$ candidate, a search was made for an additional pion. The mass difference $\Delta m = m(\bar{D}^0\pi^-) - m(D^0)$ for all such pions, when $1.8 < m(\bar{D}^0) < 1.9$ GeV/c$^2$, is shown in Fig. 1(b). The peak in this figure corresponds to the production of the $\mu^+D^{*-}$ system. All events containing a pion with a charge opposite to that of the muon (right-charge combination) and 0.1425 < $\Delta m$ < 0.1490 GeV/c$^2$ were included in the $D^{*-}\rightarrow \pi^-\nu$ sample. All events containing a pion with the same charge as the muon (wrong-charge combination) and 0.1425 < $\Delta m$ < 0.1490 GeV/c$^2$ were included in the auxiliary $D^{*-}\rightarrow \pi^-\nu$ sample. This sample contains true $\bar{D}^0$ but fake $D^{*-}$ events and gives an estimate of the combinatorial background for selected $\mu^+\bar{D}^0$ candidates. The $\Delta m$ distribution for such events is shown in Fig. 1(b) as the filled histogram. All remaining events were assigned to the $\bar{D}^0$ sample.

Since the final (semileptonic) state has missing particles, including the neutrino, the proper de-
cay length was not determined. Instead, for each reconstructed candidate, the measured visible proper decay length $x^M$ was computed as $x^M = m_B c (\mathbf{L}_T - \mathbf{p}_T (\mu^+D^0)) / |\mathbf{p}_T (\mu^+D^0)|^2$. $\mathbf{L}_T$ is the vector in the axial plane from the primary to the $B$-meson decay vertex, $\mathbf{p}_T (\mu^+D^0)$ is the transverse momentum of the $\mu^+D^0$ system and $m_B$ is the mass of the $B$ meson, for which the value 5.279 GeV/c$^2$ was used [3]. The pion from the $D^{*-}$ decay was not used for the computation of the transverse momentum and the decay length.

Candidates in each of the samples were divided into eight groups according to their $x^M$ value. The number of $\mu^+D^0$ events $N^i_R$ (from the $D^{*-}R$ sample), $N^i_W$ (from the $D^{*-}W$ sample), and $N^i_0$ (from the $D^0$ sample) in each interval $i$ (where $i$ ranges from one to eight) were determined from the fit of the $K\pi$ mass spectrum between 1.72 and 2.16 GeV/c$^2$ with the sum of a Gaussian signal function and a polynomial background function. The mean and width of the Gaussian function were fixed to the values obtained from the fit of the overall mass distribution in each sample. The fitting procedure was the same for all samples. Table I gives the numbers obtained for each $x^M$ interval.

The number of $\mu^+D^{*-}$ events for each interval $i$ of $x^M$ was defined as $N_i (\mu^+D^{*-}) = N^i_R - C \cdot N^i_W$, where $C \cdot N^i_W$ accounts for the combinatorial background under the $D^{*-}$ peak as shown in Fig. 1b). The coefficient $C = 1.27\pm0.03$ reflects the difference in the combinatorial background between $\mu^+D^0\pi^-$ and $\mu^+D^0\pi^+$ events. It was determined from the ratio of the numbers of these events in the interval $0.153 < m_{\pi^\pm} < 0.160$ GeV/c$^2$. The number of $\mu^+D^0$ events in each interval $i$ of $x^M$ was defined as $N_i (\mu^+D^0) = N^i_0 + N^i_W - C \cdot N^i_0$. The experimental observable $r_i$ is the ratio of $\mu^+D^{*-}$ and $\mu^+D^0$ events in interval $i$ of $x^M$, i.e., $r_i = N_i (\mu^+D^{*-}) / N_i (\mu^+D^0)$. Values of $r_i$ and statistical uncertainties are given in Table I. The measurement of the lifetime difference between $B^+$ and $B^0$ is given by $k \equiv \tau^+/\tau^0 - 1$. It was determined from the minimization of $\chi^2 (\varepsilon, k)$:

$$\chi^2 (\varepsilon, k) = \sum_i \frac{(r_i - r^0_i (\varepsilon, k))^2}{\sigma^2 (r_i)}.$$

where $r^0_i (\varepsilon, k)$ is the expected ratio of $\mu^+D^{*-}$ and $\mu^+D^0$ events, and $\varepsilon$ is the efficiency to reconstruct the slow pion in the $D^{*-} \rightarrow D^0\pi^-$ decay. $\varepsilon$ of $D^{*-}$ was assumed to be independent of $x^M$ and, along with $k$, was a free parameter in the minimization. We present evidence for the validity of this assumption in the discussion of systematic uncertainties. The sum $\sum_i$ was taken over all intervals with positive $x^M$.

Information used to determine the expected ratio, $r^0_i (\varepsilon, k)$, included both experimental measurements as well as results from Monte Carlo simulations. For the $j$th $B$-meson decay channel, the distribution of the visible proper decay length $x$ is given by $P_j (x) = \int dK D_j (K) \cdot \theta (x - K / c \cdot \tau_j) \cdot \frac{K^4}{c^4} \exp (- \frac{K^2}{c^4} / 2)$. $\tau_j$ is the lifetime of the $B$ meson, the $K$-factor, $K = \frac{p_j^2}{p_B^2}$, reflects the difference between the observed and true momentum of the $B$ meson, and $\theta (x)$ is the step function. The function $D_j (K)$ is the normalized distribution of the $K$-factor for the $j$th decay channel.

Transformation from the true value of $x$ to the experimentally measured value $x^M$ is given by $f_j (x^M) = \int dx R_j (x - x^M) \cdot \varepsilon_j (x) \cdot P_j (x)$, where $R_j (x - x^M)$ is the detector resolution, and $\varepsilon_j (x)$ is the reconstruction efficiency of $\mu^+D^0$ for the $j$th decay. It does not include $\varepsilon$ for channels with $D^{*-}$. Finally, the expected value $r^0_i (\varepsilon, k)$ is given by:

$$r^0_i (\varepsilon, k) = \frac{\varepsilon \cdot F^*_i (k)}{F^0_i (k)}.$$
TABLE I: Definition of the intervals in visible proper decay length, \( x^M \). For each interval \( i \), the number of events in the \( D^{*-} \), \( D^{*-W} \) and \( D^0 \) samples, the ratio \( r_i \), and the expected value \( r_i^\ast \) for \( \tau^+ / \tau^0 - 1 = 0.080 \) are given.

| \( i \) | \( x^M \) range (cm) | \( N^{+}_{i} \) | \( N^{+}_{i} \) | \( N^{0}_{i} \) | \( r_i \) | \( r_i^\ast \) |
|---|---|---|---|---|---|---|
| 1 | \(-0.1 - 0.0\) | 1714 \( \pm \) 53 | 89 \( \pm \) 22 | 5225 \( \pm \) 151 | 0.295 \( \pm \) 0.015 | 0.309 |
| 2 | 0.0 - 0.02 | 6213 \( \pm \) 94 | 200 \( \pm \) 28 | 18134 \( \pm \) 222 | 0.321 \( \pm \) 0.007 | 0.315 |
| 3 | 0.02 - 0.04 | 5941 \( \pm \) 91 | 169 \( \pm \) 22 | 17703 \( \pm \) 208 | 0.317 \( \pm \) 0.007 | 0.313 |
| 4 | 0.04 - 0.07 | 6424 \( \pm \) 94 | 213 \( \pm \) 23 | 19707 \( \pm \) 216 | 0.305 \( \pm \) 0.006 | 0.308 |
| 5 | 0.07 - 0.10 | 4292 \( \pm \) 74 | 115 \( \pm \) 17 | 12885 \( \pm \) 171 | 0.295 \( \pm \) 0.007 | 0.300 |
| 6 | 0.10 - 0.15 | 3459 \( \pm \) 68 | 106 \( \pm \) 16 | 11532 \( \pm \) 162 | 0.282 \( \pm \) 0.007 | 0.291 |
| 7 | 0.15 - 0.25 | 2253 \( \pm \) 57 | 58 \( \pm \) 13 | 7567 \( \pm \) 137 | 0.283 \( \pm \) 0.009 | 0.276 |
| 8 | 0.25 - 0.40 | 518 \( \pm \) 28 | 2 \( \pm \) 6 | 1875 \( \pm \) 75 | 0.274 \( \pm \) 0.019 | 0.256 |

FIG. 2: Points with the error bars show the ratio of the number of events in the \( \mu^+ D^{*-} \) and \( \mu^+ D^0 \) samples as a function of the visible proper decay length. The result of the minimization of Eq. (1) with \( k = 0.080 \) is shown as a histogram.

Using all these inputs, the minimization of the \( \chi^2 \) distribution, Eq. (1), gives: \( k = \tau^+ / \tau^0 - 1 = 0.080 \pm 0.016 \) (stat). The \( \chi^2 \) at the minimum is 4.2 for 5 d.o.f, \( \varepsilon_\pi \) is 0.864 \( \pm \) 0.006 (stat), and the global correlation coefficient between \( k \) and \( \varepsilon_\pi \) is 0.18. The simulation predicted \( \varepsilon_\pi = 0.877 \pm 0.003 \). The reasonable agreement in \( \varepsilon_\pi \) between data and simulation reflects good consistency of input efficiencies and branching fractions with experimental data. Figure 2 presents the \( r_i \) values together with the result of the fit.

The influence of various sources of systematic uncertainty on the final result is summarized in Table I. Different contributions can be divided into three groups. The first part includes uncertainties coming from the experimental measurements, e.g., branching fractions and lifetimes. All inputs were varied by one standard deviation. Only the most significant contributions are listed as individual entries in Table I; all remaining uncertainties are combined into a single entry “other contributions.”

The second group includes uncertainties due to the inputs taken from the Monte Carlo simulation. They were estimated as follows. The uncertainty due to the decay length dependence of the efficiencies \( \varepsilon(B \rightarrow \mu^+ \nu \bar{D}X) \) was obtained by repeating the analysis with decay length independent efficiencies used for all decay modes. This dependence almost cancels in the ratio of the number of events in the two samples, leading to the reduced systematic uncertainty in \( \tau^+ / \tau^0 \).

The variation of the efficiency from channel to channel arises from differences in the kinematics of \( B \)-meson decays and thus depends on their modeling in simulation. To estimate the uncertainty in the efficiency due to this effect, an alternative HQET model of \( B \)-meson decays \[13] was implemented, and the selection cuts on the \( p_T \) of the \( \mu^+ \) and \( D^0 \) were varied over a wide range.

The same alternative model and the variation of \( p_T \) cuts were used to study the model dependence of the \( K \)-factors. In all cases, the variation of the average value of \( K \)-factors did not exceed 2%. Distributions of \( K \)-factors were determined separately for \( B \rightarrow \mu^+ \nu \bar{D} \), \( B \rightarrow \mu^+ \nu \bar{D}^* \), \( B \rightarrow \mu^+ \nu \bar{D}^{**} \rightarrow \bar{D}X \), and \( B \rightarrow \mu^+ \nu \bar{D}^{**} \rightarrow \bar{D}^* \). To estimate the uncertainty due to the modeling of \( \bar{D}^{**} \) decays, which include both resonant and non-resonant components and are not yet well understood, the analysis was repeated with the distributions of \( K \)-factors from \( B \rightarrow \bar{D}^{**} \rightarrow \bar{D} \) (\( \bar{D}^{**} \)) decays set to be the same as for \( B \rightarrow \bar{D} \) (\( \bar{D}^{**} \)) decays.

The selection of the slow pion was made independently of the \( B \) lifetime, and the efficiency \( \varepsilon_\pi \) was assumed constant in the minimization. A dedicated study of \( K_S \rightarrow \pi^+ \pi^- \) decays showed good stability of the track reconstruction efficiency with the change of decay length over a wide range. The slope in the efficiency was estimated to be 0.0038 \( \pm \) 0.0059 cm\(^{-1}\). The independence of \( \varepsilon_\pi \) on the decay length was also verified in simulation, where no deviation from the constant value was detected within available simulation statistics.

The average decay length resolution, approximately 35 \( \mu m \) for this measurement, and the fraction of events with larger resolution, modeled by a Gaussian function with resolution of 1700 \( \mu m \), were varied over a wide range, significantly exceeding the estimated difference in resolution between data and simulation.

The ratio of events with negative decay length in \( D^{*+} \) and \( \bar{D}^0 \) samples (the first row in Table I) is sensitive to the differences in resolution of these two samples. The
TABLE II: Summary of systematic uncertainties.

| Source                      | \(\Delta(\tau^+ / \tau^0)\) |
|-----------------------------|-------------------------------|
| \(Br(B^0 \to \mu^+ \nu D^{*-})\) | 0.0005                       |
| \(Br(B^+ \to \mu^+ \nu D^{*0})\) | 0.0010                       |
| \(Br(B^+ \to \mu^+ \nu D^{*-*})\) | 0.0009                       |
| \(Br(B^+ \to \mu^+ \nu D^{*-} - \pi^+ X)\) | 0.0059                       |
| \(Br(B^0_s \to \mu^+ \nu D_s^{*-} X)\) | 0.0009                       |
| \(D_{s(*)}^{*-*} \to D^{*-*} X\) | 0.0020                       |
| \(\epsilon\bar{c} \to \mu^+ \nu D^0 X\) contribution | 0.0015                       |
| Other contributions         | 0.0006                       |
| \(\varepsilon(B \to \mu^+ \nu D^0 X)\), decay length dependence | 0.0014                       |
| \(\varepsilon(B \to \mu^+ \nu D^0 X)\), average value | 0.0030                       |
| \(\varepsilon_{\pi}, \) decay length dependence | 0.0036                       |
| decay length resolution     | 0.0024                       |
| Difference in \(D^{*-*}\) and \(\bar{D}^0\) resolution | 0.0053                       |
| \(K\)-factors, average value | 0.0032                       |
| \(K\)-factors, difference between channels | 0.0013                       |
| Fitting procedure           | 0.0086                       |
| Background level under \(D^{*-*}\) | 0.0004                       |
| **Total**                   | **0.0136**                   |

This result is the most precise measurement of this parameter, and agrees well with the world average value \(k = 0.086 \pm 0.017\) [1]. Improved precision of the ratio of \(B^+\) and \(B^0\) lifetimes will allow a better test of theoretical predictions, especially those inputs to the calculations that rely on lattice QCD or on other non-perturbative methods [2, 3].

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[1] Visitor from University of Zurich, Zurich, Switzerland.
[2] Visitor from Institute of Nuclear Physics, Krakow, Poland.
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