Non(anti)commutative $\mathcal{N} = 2$ Supersymmetric $U(N)$ Gauge Theory and Deformed Instanton Equations

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Abstract

We study deformed supersymmetry in $\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theory in non(anti)commutative $\mathcal{N} = 1$ superspace. Using the component formalism, we construct deformed $\mathcal{N} = (1,1/2)$ supersymmetry explicitly. Based on the deformed supersymmetry, we discuss the $C$-dependence of the correlators. We also study the $C$-deformation of the instanton equation for the gauge group $U(2)$.
Supersymmetric field theories in non(anti)commutative superspace \[1, 2\] has been attracted much interests from the viewpoint of effective field theories on D-branes in the graviphoton background \[3, 4, 5\]. Superstrings in this background provide some interesting low-energy physics in \(\mathcal{N} = 2\) supersymmetric field theories \[6\] and their \(\mathcal{N} = 1\) deformations \[7\]. It would be important to study \(\mathcal{N} = 2\) supersymmetric gauge theories in non(anti)commutative superspace in order to understand graviphoton effects in the low-energy effective theories from the microscopic point of view.

It is convenient to use \(\mathcal{N} = 2\) extended non(anti)commutative superspace for studying non(anti)commutative gauge theories where supersymmetry is manifestly realized \[8, 9, 10, 11, 12, 13, 14\]. In previous papers \[11, 15, 16\], \(\mathcal{N} = 2\) supersymmetric \(U(1)\) gauge theory in non(anti)commutative \(\mathcal{N} = 2\) harmonic superspace has been studied. We have computed the deformed Lagrangian up to the first order in the deformation parameter \(C\) of the superspace and examined their deformed symmetries. It is, however, difficult to calculate higher order \(C\)-corrections and extend the \(U(1)\) gauge group to \(U(N)\).

There exist two cases such that the deformed Lagrangian of \(\mathcal{N} = 2\) supersymmetric \(U(N)\) gauge theory becomes simple. One is the case of the singlet deformation where the deformation parameter belongs to the singlet representation of the \(R\)-symmetry group \(SU(2)\) \[9, 10, 17, 13, 14\]. The other is the case that one introduces only deformation into \(\mathcal{N} = 1\) subsuperspace of \(\mathcal{N} = 2\) superspace. In a recent paper \[16\], it is shown that the \(O(C)\) Lagrangian of the \(U(1)\) theory defined in non(anti)commutative \(\mathcal{N} = 2\) harmonic superspace leads to the theory in the non(anti)commutative \(\mathcal{N} = 1\) superspace \[2\] by the reduction of deformation parameters and some field redefinitions. It is also shown that the theory has \(\mathcal{N} = (1, 1/2)\) supersymmetry consistent with the Poisson structure of the theory. Here \(\mathcal{N} = (1, 1/2)\) means that there are two chiral and one antichiral supercharges, as in \[10\].

In this paper, we will study \(\mathcal{N} = 2\) supersymmetric \(U(N)\) gauge theory in the deformed \(\mathcal{N} = 1\) superspace, whose Lagrangian has been constructed in \[18\]. We will construct deformed \(\mathcal{N} = (1, 1/2)\) supersymmetry explicitly. Based on this symmetry, we will study the \(C\)-deformed correlators of the observables. We will also examine the the \(C\)-deformed instanton equations for the gauge group \(U(2)\). The instanton solutions in non(anti)commutative \(\mathcal{N} = 1\) gauge theory have been investigated in \[19, 20\]. In the case
of the gauge group $U(2)$, it has been found that the $SU(2)$ part of the instanton equations is not deformed and the $U(1)$ part is deformed only. In the non(anti)commutative $\mathcal{N} = 2 U(2)$ theory, we will show that the $SU(2)$ part of the instanton equations is not deformed except for one of the fermions.

Let $(x^m, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$ $(m = 0, \ldots, 3, \alpha, \dot{\alpha} = 1, 2)$ be supercoordinates of $\mathcal{N} = 1$ superspace and $\sigma^m_{\alpha \dot{\alpha}}$ and $\bar{\sigma}^{m\dot{\alpha}}$ Dirac matrices. We will study Euclidean spacetime so that chiral and antichiral fermions transform independently under the Lorentz transformations.

Let

$$\begin{align*}
Q_\alpha &= \frac{\partial}{\partial \theta^\alpha} - i \sigma^m_{\alpha \dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_m \\
\bar{Q}^{\dot{\alpha}} &= -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i \theta^\alpha \bar{\sigma}^{m\dot{\alpha}} \partial_m
\end{align*}$$

are supercharges. $D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma^m_{\alpha \dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_m$ and $\bar{D}^{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i \theta^\alpha \bar{\sigma}^{m\dot{\alpha}} \partial_m$ are the supercovariant derivatives. $\sigma^{mn} = \frac{1}{4} (\sigma^m \bar{\sigma}^n - \sigma^n \bar{\sigma}^m)$, $\bar{\sigma}^{mn} = \frac{1}{4} (\bar{\sigma}^m \sigma^n - \bar{\sigma}^n \sigma^m)$ are the Lorentz generators. Here we will follow the conventions of Wess and Bagger.

The non(anti)commutativity in $\mathcal{N} = 1$ superspace is introduced by the $\ast$-product:

$$f \ast g(x, \theta, \bar{\theta}) = f(x, \theta, \bar{\theta}) \exp \left( -\frac{1}{2} Q_\alpha C^{\alpha \beta} Q^\beta \right) g(x, \theta, \bar{\theta}).$$

Using this $\ast$-product, the anticommutation relations for $\theta$ become

$$\{ \theta^\alpha, \theta^\beta \}_\ast = C^{\alpha \beta}$$

while the chiral coordinates $y^m = x^m + i \theta \sigma^m \bar{\theta}$ and $\bar{\theta}$ are still commuting and anticommuting coordinates, respectively.

$\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theory in this deformed superspace was formulated in [13]. It can be constructed by vector superfields $V$, chiral superfields $\Phi$ and an antichiral superfields $\bar{\Phi}$, where $\Phi$ and $\bar{\Phi}$ belong to the adjoint representation of $U(N)$. We introduce the basis $t^a$ $(a = 1, \ldots, N^2)$ of Lie algebra of $U(N)$, normalized as $\text{tr}(t^a t^b) = k \delta^{ab}$. The Lagrangian is

$$\mathcal{L} = \frac{1}{k} \int d^2 \theta d^2 \bar{\theta} \text{tr}(\bar{\Phi} \ast e^V \ast \Phi \ast e^{-V}) + \frac{1}{16 k g^2} \text{tr} \left( \int d^2 \theta W^a \ast W_a + \int d^2 \bar{\theta} \bar{W}_\dot{\alpha} \ast \bar{W}^{\dot{\alpha}} \right)$$

where $g$ denotes the coupling constant. $W_a = -\frac{1}{4} \bar{D}^2 e^{-V} D_a e^V$ and $\bar{W}_\dot{\alpha} = \frac{1}{4} D^2 e^{-V} \bar{D}\dot{\alpha} e^V$ are the chiral and antichiral field strengths. Note that multiplication of superfields are defined by the $\ast$-product.

This Lagrangian is invariant under the gauge transformations $\Phi \rightarrow e^{-i \Lambda} \ast \Phi \ast e^{i \Lambda}$, $\bar{\Phi} \rightarrow e^{-i \Lambda} \ast \bar{\Phi} \ast e^{i \Lambda}$ and $e^V \rightarrow e^{-i \Lambda} \ast e^V \ast e^{i \Lambda}$. To write down the Lagrangian in terms of component
fields, it is convenient to take the Wess-Zumino(WZ) gauge as in the commutative case. Since the *-product deforms the gauge transformation, it is necessary to redefine the component fields such that these transform canonically under the gauge transformation[2, 18]. For \( \mathcal{N} = 2 \) \( U(N) \) theory, these superfields in the WZ gauge are

\[
\Phi(y, \theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta \theta F(y),
\]

\[
\bar{\Phi}(\bar{y}, \bar{\theta}) = \bar{A}(\bar{y}) + \sqrt{2}\bar{\theta}\bar{\psi}(\bar{y}) + \bar{\theta} \bar{\theta} \left( \bar{F} + iC^{mn} \partial_m \{ v_n, \bar{A} \} - \frac{1}{4} C^{mn} \left[ v_m, \{ v_n, \bar{A} \} \right] \right)(\bar{y}),
\]

\[
V(y, \theta, \bar{\theta}) = -\theta \bar{\sigma}^{\mu} v_{\mu}(y) + i\theta \bar{\theta} \bar{\theta} \lambda(y) - i\theta \bar{\theta} \theta \alpha \left( \lambda_\alpha + \frac{1}{4} \delta_{\alpha \beta} \partial^{\mu} \left( \sigma^{\mu} \bar{\lambda} \right) v_{\mu} \right)(y) + \frac{1}{2} \theta \bar{\theta} \bar{\theta} (D - i\partial^{\mu} v_{\mu})(y).
\]

Here \( \bar{y}^m = x^m - i\partial \sigma^m \bar{\theta} \) are the antichiral coordinates and \( C^{mn} = C^{\alpha \beta} \delta_{\alpha \beta} (\sigma^{mn})_\gamma \). Since \( \sigma^{mn} \) is self-dual, \( C^{mn} \) is also self-dual. Substituting (4) into the Lagrangian (3), we obtain the deformed Lagrangian written in terms of component fields. In this expression, however, normalizations of two fermions \( \psi \) and \( \lambda \) are different. In order to see symmetries between two fermions manifestly, it is useful to rescale \( V \) to \( 2gV \) and \( C^{\alpha \beta} \) to \( \frac{-1}{2g} C^{\alpha \beta} \). Then the Lagrangian takes the form \( \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 \). Here \( \mathcal{L}_0 \) is the undeformed Lagrangian with the topological term:

\[
\mathcal{L}_0 = \frac{1}{k} \text{tr} \left( -\frac{1}{4} F^{mn} F_{mn} - \frac{1}{4} F^{mn} \bar{F}_{mn} - i\bar{\lambda} \sigma^m D_m \lambda + \frac{1}{2} \bar{D}^2 \right) - (D^m \bar{A}) D_m A - i\bar{\psi} \bar{\sigma}^m D_m \psi + \bar{F} F - i\sqrt{2} g [\bar{A}, \psi] \lambda - i\sqrt{2} g [A, \bar{\psi}] \bar{\lambda} - \frac{g^2}{2} [A, \bar{A}], (5)
\]

where \( F_{mn} = \partial_m v_n - \partial_n v_m + i g[v_m, v_n], \bar{F}_{mn} = \frac{1}{2} \epsilon_{mnpq} F^{pq} \) and \( D_m \lambda = \partial_m \lambda + i g[v_m, \lambda] \) etc. We have also introduced an auxiliary field \( \bar{D} \) defined by \( \bar{D} = D + g[A, \bar{A}] \) in order to see undeformed \( \mathcal{N} = 2 \) supersymmetry in a symmetric way. \( \mathcal{L}_1 \) is the \( C \)-dependent part of the Lagrangian:

\[
\mathcal{L}_1 = \frac{1}{k} \text{tr} \left( -\frac{i}{2} C^{mn} F_{mn} \bar{\lambda} \lambda + \frac{1}{8} |C|^2 (\bar{\lambda} \lambda)^2 + \frac{i}{2} C^{mn} F_{mn} \{ \bar{A}, F \} - \sqrt{2} C^{\alpha \beta} \left\{ D_m \bar{A}, (\sigma^m \lambda)_\alpha \right\} \psi_\beta - \frac{1}{16} |C|^2 [\bar{A}, \lambda][\bar{\lambda}, F] \right). (6)
\]

Here \( |C|^2 = C^{mn} C_{mn} \).

In the case of \( C = 0 \), the action is invariant under \( \mathcal{N} = 2 \) supersymmetry transformations, where only \( \mathcal{N} = 1 \) supersymmetry generated by \( Q_\alpha \) and \( \bar{Q}^\alpha \) are manifestly realized.
in $\mathcal{N} = 1$ superspace. Other $\mathcal{N} = 1$ supersymmetry would be realized manifestly when we use $\mathcal{N} = 2$ extended superspace. In particular $\mathcal{N} = 2$ harmonic superspace \cite{22} provides very efficient tools to study off-shell $\mathcal{N} = 2$ supersymmetric field theories. The most general non(anti)commutative deformations are studied by using extended superspace. We expect that this deformed theory is derived from the non(anti)commutative $\mathcal{N} = 2$ harmonic superspace by the reduction of deformation parameters, which will be discussed in a separate paper.

In previous papers\cite{11, 15, 16}, we have studied $\mathcal{N} = 2$ supersymmetric $U(1)$ gauge theory and their deformed supersymmetry structure in the component formalism. In particular, for generic deformation, $\mathcal{N} = (1, 0)$ deformed supersymmetry has been constructed up to the first order of the deformation parameters. When the deformation parameters are reduced such that only $\mathcal{N} = 1$ subspace becomes non(anti)commutative, the deformed supersymmetry is enhanced to $\mathcal{N} = (1, 1/2)$ supersymmetry. This is because supersymmetries other than $\bar{Q}\dot{\alpha}$ is consistent with the Poisson structure of the deformed superspace \cite{10}. In this reduced case, it is shown that the $O(C)$ Lagrangian defined in the $\mathcal{N} = 2$ harmonic superspace is equal to that of the deformed $\mathcal{N} = 1$ superspace by the field redefinitions.

We now study the $U(N)$ case. The undeformed superfield action is invariant under $\mathcal{N} = 1$ supersymmetry generated by $\xi Q + \xi\bar{Q}$. Since this transformation does not preserve the WZ gauge, we need to do gauge transformation to retain the WZ gauge. Then the (undeformed) supersymmetry transformations $\delta^0_\xi$ and $\delta^0_\bar{\xi}$ of the component fields in the WZ gauge are

$$
\begin{align*}
\delta^0_\xi v^m &= i\xi\sigma^m\bar{\lambda}, \\
\delta^0_\xi \lambda &= i\xi\bar{D} - ig[\lambda, \bar{A}] + \sigma^{mn}\xi F_{mn}, \quad \delta^0_\xi\bar{\lambda} = 0, \\
\delta^0_\xi\bar{D} &= -\xi\sigma^mD_m\bar{\lambda} + \sqrt{2}g[\lambda, \bar{A}], \\
\delta^0_\xi A &= \sqrt{2}\xi\bar{\psi}, \quad \delta^0_\xi\bar{\psi} = \sqrt{2}\xi F, \quad \delta^0_\xi F = 0, \\
\delta^0_\xi\bar{A} &= 0, \quad \delta^0_\xi\bar{\psi} = \sqrt{2}i\sigma^m\xi D_m\bar{\lambda}, \quad \delta^0_\xi F = i\sqrt{2}\xi\sigma^mD_m\bar{\psi} - 2gi\lambda[\bar{A}, \lambda], \quad (7)
\end{align*}
$$

$$
\begin{align*}
\delta^0_\bar{\xi} v^m &= i\bar{\xi}\sigma^m\lambda, \\
\delta^0_\bar{\xi} \lambda &= 0, \quad \delta^0_\bar{\xi}\bar{\lambda} = -i\bar{\xi}\bar{D} + ig[\lambda, \bar{A}] + \bar{\sigma}^{mn}\xi F_{mn},
\end{align*}
$$
\[ \begin{align*}
\delta^0_\xi \bar{D} &= \bar{\xi} \sigma^n D_n \lambda + \sqrt{2} g [A, \bar{\xi} \bar{\psi}], \\
\delta^0_\xi A &= 0, \quad \delta^0_\xi \bar{\psi} = \sqrt{2} i \sigma^m \bar{D}_m A, \quad \delta^0_\xi F = \sqrt{2} \bar{\xi} \bar{\sigma}^m D_m \psi + 2 g i \bar{\xi} [\bar{\lambda}, A], \\
\delta^0_\xi \bar{A} &= \sqrt{2} \bar{\xi} \bar{\psi}, \quad \delta^0_\xi \bar{\psi} = \sqrt{2} \bar{\xi} \bar{F}, \quad \delta^0_\xi F = 0. 
\end{align*} \] (8)

The remaining \( \mathcal{N} = 1 \) supersymmetry denoted by \( \delta^0_\eta \) and \( \delta^0_\bar{\eta} \) can be obtained from (7) by using the \( R \)-symmetry: \( \xi \to \eta, \lambda \to -\psi, \psi \to \lambda, \bar{D} \to -\bar{D}, \ F \to \bar{F} \).

Now we will construct the deformed \( \mathcal{N} = (1,1/2) \) supersymmetry which keeps the \( U(N) \) Lagrangian \( \mathcal{L} \) invariant up to the total derivatives. The term \( \mathcal{L}_1 \) is not invariant under the undeformed supersymmetry transformations \( \delta^0_\xi, \delta^0_\eta \) and \( \delta^0_\bar{\eta} \). Since the deformed term \( \mathcal{L}_1 \) is a polynomial in \( C \), we denote \( \mathcal{L}_1^{(n)} \) (\( n \geq 1 \)) by its \( n \)-th order term in \( C \). The deformed supersymmetry transformations can be expanded in the form \( \delta = \delta^0 + \delta^1 + \cdots \).

Here \( \delta^n \) is the \( n \)-th order term in \( C \). \( \delta^n \) is determined recursively by solving the conditions \( \delta^1 \mathcal{L}_0 + \delta^0 \mathcal{L}_1^{(1)} = 0 \) and \( \delta^2 \mathcal{L}_0 + \delta^1 \mathcal{L}_1^{(1)} + \delta^0 \mathcal{L}_2^{(1)} = 0 \) and so on.

The deformed transformation \( \delta_\xi \), which was calculated in [18], takes the form \( \delta_\xi = \delta^0_\xi + \delta^1_\xi \) and is given by

\[ \begin{align*}
\delta_\xi v_m &= i \xi \sigma_m \bar{\lambda}, \\
\delta_\xi \lambda_\alpha &= i \xi_\alpha \bar{D} - i g \xi_\alpha [A, \bar{A}] + (\sigma^{mn} \xi)_\alpha \left( F_{mn} + \frac{i}{2} C_{mn} \bar{\lambda} \right), \quad \delta_\xi \bar{\lambda} = 0, \\
\delta_\xi \bar{D} &= -\xi \sigma^m D_m \bar{\lambda} + \sqrt{2} g [\xi \psi, \bar{A}], \\
\delta_\xi A &= \sqrt{2} \xi \psi, \quad \delta_\xi \bar{\psi} = \sqrt{2} \bar{\xi} F, \quad \delta_\xi F = 0, \\
\delta_\xi \bar{A} &= 0, \\
\delta_\xi \bar{\psi} &= \sqrt{2} i \sigma^m \xi D_m \bar{A}, \\
\delta_\xi \bar{F} &= i \sqrt{2} \xi \sigma^m D_m \bar{\psi} - 2 g i \bar{\xi} [\bar{A}, \xi \lambda] + C^{mn} D_m \left\{ \bar{A}, \xi \sigma_n \bar{\lambda} \right\}. \tag{9}
\end{align*} \]

Note that the transformation of \( \Phi \) is undeformed. The deformed transformation \( \delta_\eta \), which relate the gauge field \( v_m \) to chiral fermion \( \psi \), can be calculated in a similar way. But as in the analysis of \( U(1) \) case, it is necessary to calculate up to the order \( O(C^2) \). The result is

\[ \begin{align*}
\delta_\eta v_m &= -i \eta \sigma_m \bar{\psi} - \frac{\sqrt{2}}{2} C^{\alpha \beta} \eta_\alpha \left\{ \bar{A}, (\sigma_m \bar{\lambda})_\beta \right\}, \\
\delta_\eta \lambda_\alpha &= \sqrt{2} \eta^\alpha F \\
&\quad - \frac{\sqrt{2}}{2} C^{\alpha \beta} \eta_\beta \left\{ \bar{D}, \bar{A} \right\} - \frac{\sqrt{2} i}{2} C^{\alpha \beta} (\sigma^{mn} \eta)_\beta \left\{ F_{mn}, \bar{A} \right\} - \frac{\sqrt{2} g}{2} C^{\alpha \beta} \eta_\beta \left\{ \bar{A}, [\bar{A}, A] \right\}.
\end{align*} \]
\[ + \frac{\sqrt{2}}{4} \det C \left( \{ \bar{\lambda} \bar{\lambda}, \bar{A} \} + 2 \bar{\lambda}_{\alpha} \bar{A} \bar{\lambda}^\alpha \right) \eta^\alpha, \]

\[ \delta_\eta \bar{\lambda} = \sqrt{2} i \bar{\sigma}^m \eta D_m \bar{A}, \]

\[ \delta_\eta \bar{D} = -\eta \sigma^m D_m \bar{\psi} - \sqrt{2} g[\eta \lambda, \bar{A}] - \frac{\sqrt{2}}{2} i C^{\alpha \beta} \eta_\beta D_m \left\{ \bar{A}, (\sigma^m \bar{\lambda})_\alpha \right\} - i g C^{\alpha \beta} \eta_\beta \left\{ \bar{A}, [\bar{A}, \psi_\alpha] \right\}, \]

\[ \delta_\eta A = \sqrt{2} \eta \lambda + i C^{\alpha \beta} \eta_\beta \left\{ \psi_\alpha, \bar{A} \right\}, \]

\[ \delta_\eta \bar{\psi} = i \eta^\alpha \bar{\psi} + i g \eta^\alpha [A, \bar{A}] - \bar{\varepsilon}^{\alpha \beta} (\sigma^m \eta)_\beta F_{mn} - i C^{\alpha \beta} \eta_\beta \left\{ (\bar{\lambda} \lambda) - \{ \bar{A}, F \} \right\}, \]

\[ \delta_\eta \bar{F} = i \sqrt{2} \eta \sigma^m D_m \bar{\lambda} + 2 g i [A, \eta \psi], \]

\[ \delta_\eta \bar{A} = 0, \]

\[ \delta_\eta \bar{\psi}_\alpha = C^{\alpha \beta} \eta_\beta \sigma^m \eta \left\{ \bar{A}, D_m \bar{A} \right\}, \]

\[ \delta_\eta \bar{\psi}_{\bar{\alpha}} = C^{\alpha \beta} \eta_\beta \sigma^m \left\{ \bar{A}, D_m \bar{A} \right\}, \]

\[ \delta_\eta \bar{F} = \sqrt{2} g C^{\alpha \beta} \eta_\beta \left\{ \bar{A}, [\bar{A}, \lambda_\alpha] \right\} + \frac{\sqrt{2} i}{4} \det C \left[ 3 \left\{ \bar{A}, \{ \eta \sigma^m \bar{\lambda}, D_m \bar{A} \} \right\} \right. \]

\[ + 2 D_m \bar{A} \bar{\eta} \sigma^m \bar{\lambda} + 2 \eta \sigma^m \bar{\lambda} A D_m \bar{A} + 2 \left\{ \bar{A}, \{ \eta \sigma^m D_m \bar{\lambda}, \bar{A} \} \right\} \right] \quad (10) \]

Here we have used the formula \( \det C = |C|^2/4 \). Note that there is an ambiguity to determine the \( \delta_\eta \) transformation as noticed in the \( U(1) \) case \[16\]. In fact, for arbitrary functions \( f_1(\bar{A}) \) and \( f_2(\bar{A}) \) of \( \bar{A} \), the transformation

\[ \delta_\eta \lambda^\alpha = \eta^\alpha f_1 F + F f_2 \eta^\alpha, \]

\[ \delta_\eta \bar{F} = i (\eta \sigma^m D_m \lambda) f_1 + i f_2 (\eta \sigma^m D_n \lambda) + i \sqrt{2} g [\bar{A}, \eta \psi] f_1 + \sqrt{2} i f_2 [\bar{A}, \eta \psi] \quad (11) \]

leaves the action invariant. In formulas \([10]\), we have chosen \( f_1 \) and \( f_2 \) such that we recover the \( U(1) \) result. This ambiguity would be fixed if we use non(anti)commutative \( \mathcal{N} = 2 \) harmonic superspace, which will not be discussed in this paper.

The deformed transformation \( \delta_\eta \) is found to be

\[ \delta_\eta v_m = -i \bar{\eta} \bar{\sigma}_m \psi \]

\[ \delta_\eta \lambda^\alpha = \sqrt{2} i \bar{\varepsilon}^{\alpha \beta} (\sigma^m \eta)_\beta D_m \lambda + i C^{\alpha \beta} \left\{ \bar{\eta} \lambda, \psi_\beta \right\}, \quad \delta_\eta \bar{\lambda} = \sqrt{2} \eta F, \]

\[ \delta_\eta \bar{D} = \bar{\eta} \sigma^m D_m \psi - \sqrt{2} g [A, \bar{\eta} \lambda], \]

\[ \delta_\eta A = 0, \quad \delta_\eta \psi = 0, \quad \delta_\eta F = 0, \]

\[ \delta_\eta \bar{A} = \sqrt{2} \eta \lambda, \]

\[ \delta_\eta \bar{\psi} = -i \bar{\eta} \bar{D} - i g \bar{\eta} [A, \bar{A}] - \sigma^{mn} \eta F_{mn}, \]
In the low-energy effective theory, this can be expressed in terms of anti-holomorphic vacuum is.

\[
\delta \eta \bar{F} = \sqrt{2i}\bar{\eta}\sigma^m D_m \lambda - 2gi [\bar{\eta}\tilde{\psi}, A] + C^{\alpha\beta}(\sigma^m \bar{\eta})_\alpha D_m \{\psi_\beta, \bar{A}\} \\
- \frac{\sqrt{2}}{4} \det C \left\{3\{\bar{\eta}\lambda, \bar{\lambda}\lambda\} + \bar{\eta}\lambda\bar{A} F + \bar{A}\bar{\eta}\lambda F - 2\bar{\eta}\lambda F\bar{A} + 2\bar{\lambda}_\alpha (\bar{\eta}\lambda)\bar{\lambda}^\alpha\right\}. \tag{12}
\]

Note that if we set \(N = 1\), the cubic terms in \(\bar{\lambda}\) and the commutators vanish. We then recover the \(U(1)\) results obtained in \([16]\).

We discuss some general properties obtained from the deformed supersymmetry. Firstly we examine the \(C\)-dependence of the correlation functions. Let us deform \(C_{mn} \to C_{mn} + \delta C_{mn}\) with keeping the self-dual condition. The variation of the Lagrangian is

\[
\delta \mathcal{L} = \frac{1}{k} \text{tr} \delta C^{mn} \left\{-\frac{i}{2} F_{mn} \bar{\lambda}\lambda + \frac{1}{4} C_{mn} (\bar{\lambda}\lambda)^2 + \frac{i}{2} F_{mn} \{\bar{A}, F\} \right. \\
- \frac{\sqrt{2}}{4} \varepsilon^{\alpha\gamma}(\sigma_{mn})_{\gamma}^\beta \left\{D_p \bar{A}, (\sigma^p \bar{\lambda})_\alpha \right\} \psi_\beta \right. \\
- \frac{1}{8} C_{mn}(\bar{\lambda}\lambda) \{F, \bar{A}\} - \frac{1}{4} C_{mn} \bar{A}\lambda_\alpha F\bar{\lambda}^\alpha\right\}. \tag{13}
\]

Writing \(\delta \xi = \xi^\alpha Q_\alpha\), the \(Q^\alpha\) action to the component fields are

\[
\{Q^\beta, \lambda_\alpha\} = -i\sigma^\beta_\alpha \bar{D} + ig\sigma^\beta_\alpha [A, \bar{A}] + (\sigma^{mn})_\alpha^\beta \left(F_{mn} + \frac{i}{2} C_{mn}(\bar{\lambda}\lambda)\right), \\
\{Q^\beta, \psi_\alpha\} = -\sqrt{2}\delta_\beta^\alpha F, \\
\{Q^\beta, \bar{\psi}_\alpha\} = -i\sqrt{2} D_m \bar{A}\varepsilon^{\beta\gamma}\tau^m_\gamma. \tag{14}
\]

Then we find that \(\delta \mathcal{L}\) is \(Q\)-exact, \(\delta \mathcal{L} = \{Q^\alpha, \Lambda_\alpha\}\), where

\[
\Lambda_\alpha = \text{tr} \delta C_{mn} \left\{\frac{i}{4}(\sigma_{mn})_\alpha^\beta \bar{\lambda}\lambda - \{\bar{A}, F\}\right\} - \frac{i}{4} (\sigma_{mn})_\alpha^\beta (\bar{\psi}\lambda + \bar{\lambda}\tilde{\psi}) \psi_\beta \right. \\
- \frac{1}{\sqrt{2}} \left\{\frac{1}{8} C_{mn}\{\psi_\alpha, \bar{A}\}(\bar{\lambda}\lambda) + \frac{1}{4} C_{mn} \bar{\lambda}_\alpha \bar{A}\bar{\lambda}^\alpha \psi_\alpha \right\}. \tag{15}
\]

In the case of non(anti)commutative \(\mathcal{N} = 1\) super Yang-Mills theory\([19]\), \(\delta \mathcal{L}\) is shown to be \(Q\)-exact, which means that the correlator \(\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle\) for \(Q\)-invariant operators \(\mathcal{O}_i\) is \(C\)-independent. The antichiral gluino condensates \(\langle \text{tr} \bar{\lambda}\lambda \rangle\), in particular, does not have \(C\)-correction. In the \(\mathcal{N} = 2\) case, the correlator \(\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle\) is also \(C\)-independent if the vacuum is \(Q\)-invariant: \(Q|0\rangle = |0\rangle Q = 0\) \([19]\). For example, the antichiral scalar field \(\bar{A}\) is a \(Q\)-invariant operator. Therefore, the vacuum expectation value \(\langle \text{tr} \bar{A}^2 \rangle\) is \(C\)-independent.

In the low-energy effective theory, this can be expressed in terms of anti-holomorphic prepotential \(\bar{F}(\bar{a})\) \([23]\) such as

\[
\langle \text{tr} \bar{A}^2 \rangle \sim \sum_i \bar{a}_i \frac{\partial \bar{F}(\bar{a})}{\partial \bar{a}_i} - 2\bar{F}(\bar{a}), \tag{16}
\]
where $\bar{a}_i$ is the vacuum expectation value of the low-energy $U(1)^N$ Higgs fields. Since $\bar{F}(\bar{a})$ is expanded in the QCD scale parameter, (14) implies that the instanton corrections to the anti-holomorphic prepotential are not $C$-deformed. On the other hand, the holomorphic prepotential would have $C$-corrections.

We next study the (constrained) instanton equations of the deformed $\mathcal{N} = 2$ supersymmetric $U(N)$ gauge theory. Since the deformed part of the gauge field in the action is the same as that of the $\mathcal{N} = 1$ theory[19], the deformed instanton equation for the gauge field is $F_{mn}^- = 0$ in the self-dual case and $F_{mn}^+ + \frac{i}{2}C_{mn}\bar{\lambda}\bar{\lambda} = 0$ in the anti-self-dual case[19] 20. Here $F_{mn}^\pm = \frac{1}{2}(F_{mn} \pm \tilde{F}_{mn})$. The other fields must satisfy the equation of motion in the instanton background.

We begin with considering the case where the vacuum expectation values of $A$ and $\bar{A}$ are zero. In this case we expect that the solutions are exact. From the Lagrangian (3), we obtain the equations of motion

$$D_m F_{mn}^\alpha - g\{\bar{\psi}, \sigma^m \lambda\} - g\{\bar{\psi}, \sigma^m \psi\} - ig[\bar{A}, D^n A] + ig[D^n \bar{A}, A]$$

$$+ iD_m(C_{mn} \bar{\lambda}\bar{\lambda}) + \frac{i}{\sqrt{2}}g[\bar{A}, C^{\alpha\beta}[(\sigma^m \bar{\lambda})_\alpha, \psi_\beta]] = 0,$$

$$\bar{\sigma}^m D_m \lambda + \sqrt{2}g[A, \bar{\psi}] + C_{mn}(F_{mn} + \frac{i}{2}C_{mn}\bar{\lambda}\bar{\lambda})\bar{\lambda} = 0,$$

$$\sigma^m D_m \bar{\lambda} + \sqrt{2}g[A, \psi] = 0, \quad \sigma^m D_m \psi - \sqrt{2}g[A, \bar{\lambda}] = 0,$$

$$(D_m \bar{\psi}\bar{\sigma}^m)^\alpha + \sqrt{2}g[A, \lambda^\alpha] + \frac{i}{\sqrt{2}}C^{\alpha\beta}D_m(A, (\sigma^m \bar{\lambda})_\beta) = 0,$$

$$D^2 A - i\sqrt{2}g\{\psi, \lambda\} - g^2[[A, \bar{A}], A] + \frac{1}{\sqrt{2}}C^{\alpha\beta}D_m[(\sigma^m \bar{\lambda})_\alpha, \psi_\beta] = 0,$$

$$D^2 \bar{A} - i\sqrt{2}g\{\bar{\psi}, \bar{\lambda}\} - g^2[[A, \bar{A}], \bar{A}] = 0,$$

(17)

where the auxiliary fields $F$, $\tilde{F}$ and $\tilde{D}$ have been eliminated. In the anti-self-dual background, if we set $\psi = \lambda = 0$, the equation of motion for $A$ becomes $D^2 A - g^2[[A, \bar{A}], A] = 0$, from which $A = 0$ is an exact solution as in the undeformed case. From $A = 0$ and absence of chiral zero-modes in the anti-self-dual background, $\psi = \lambda = 0$ is shown to be also an exact solution of the equations of motion for $\psi$ and $\lambda$. By a similar argument, $\bar{\lambda} = \bar{\psi} = 0$ and $\bar{A} = 0$ are an exact solution of the equation of motion in the self-dual background. Then the first equation of (17) is satisfied. In this paper we discuss the case of the gauge group $U(2)$ since the structure of the equations of motion (17) becomes simple as in
We decompose $U(2)$ into the $SU(2)$ and the $U(1)$ parts: $v_m = v_{SU(2),m} + v_{U(1),m}$ where

$$v_{SU(2),m} = v^a_m t^a, \quad (a = 1, 2, 3), \quad v_{U(1),m} = v^4_m t^4,$$

and $t^a = \tau^a$ and $t^4 = 1$. $\tau^a$ are the Pauli matrices. Other fields can be decomposed similarly. The equations of motion of $\bar{\lambda}_{U(1)}$, $\psi_{U(1)}$ and $\bar{A}_{U(1)}$ become

$$\sigma^m \partial_m \bar{\lambda}_{U(1)} = \bar{\sigma}^m \partial_m \psi_{U(1)} = \partial^2 \bar{A}_{U(1)} = 0,$$

(19)

Since eqs. (19) do not have any localized solution in four-dimensional Euclidean space, we have

$$\bar{\lambda}_{U(1)} = \psi_{U(1)} = 0, \quad \bar{A}_{U(1)} = \text{const}.$$

(20)

Using (20), one finds that only the equation of motion of $\lambda_{SU(2)}$ is deformed:

$$\bar{\sigma}^m D_m \lambda_{SU(2)} + \sqrt{2} g [A_{SU(2)}, \bar{\psi}_{SU(2)}] + \frac{1}{2} C^{mn} [F_{SU(2),mn}, \bar{\lambda}_{SU(2)}] = 0,$$

(21)

while the equations of motion of the other $SU(2)$ fields are not deformed. Therefore we find that the instanton equations for the $SU(2)$ gauge field are not deformed. In the case where the vacuum expectation values of $A$ and $\bar{A}$ are zero, we obtain the exact self-dual solution

$$v_{SU(2),m} = g^{-1} v_m^{(0)}, \quad \lambda_{SU(2)} = g^{-1/2} \lambda^{(0)}, \quad \psi_{SU(2)} = g^{-1/2} \psi^{(0)},$$

$$A_{SU(2)} = A^{(0)}, \quad \bar{\lambda}_{SU(2)} = \bar{\psi}_{SU(2)} = \bar{A}_{SU(2)} = 0,$$

(22)

which is not deformed by $C$. Here $\lambda^{(0)}$ and $\psi^{(0)}$ are the zero-modes in the self-dual instanton background. $A^{(0)}$ satisfies the equation $D^2 A^{(0)} - i \sqrt{2} \{ \psi^{(0)}, \lambda^{(0)} \} = 0$. Since $\bar{\lambda}_{SU(2)} = 0$, the $C$-dependent term in eq. (21) vanishes and $\lambda_{SU(2)}$ is the same as the solution of the undeformed theory. The anti-self-dual solution for the $SU(2)$ part becomes

$$v_{SU(2),m} = g^{-1} v_m^{(0)}, \quad \bar{\lambda}_{SU(2)} = g^{-1/2} \bar{\lambda}^{(0)}, \quad \bar{\psi}_{SU(2)} = g^{-1/2} \bar{\psi}^{(0)},$$

$$\bar{A}_{SU(2)} = \bar{A}^{(0)}, \quad \lambda_{SU(2)} = \psi_{SU(2)} = A_{SU(2)} = 0,$$

(23)

which are not also $C$-deformed.\footnote{In the case of singlet deformation, it is shown in \cite{13} that the anti-self-dual solution is not deformed by $C$.} Here $\bar{\lambda}^{(0)}$ and $\bar{\psi}^{(0)}$ are the zero-modes in the anti-self-dual instanton background. $\bar{A}^{(0)}$ fulfills $D^2 \bar{A}^{(0)} - i \sqrt{2} \{ \bar{\psi}^{(0)}, \bar{\lambda}^{(0)} \} = 0$. Since $v_{SU(2),m} = g^{-1} v_m^{(0)}$
is anti-self-dual, we get \( C^{mn} F_{SU(2),mn} = 0 \). Therefore we find that (21) has the solution \( \lambda_{SU(2)} = 0 \). From eq. (20), the equations of motion of other \( U(1) \) fields become

\[
\begin{align*}
\partial^m \left( F_{mn}^4 + iC_{mn}\bar{\lambda}^a\lambda^a \right) &= 0, \\
\bar{\sigma}^m \partial_m \lambda^4 + C^{mn} F_{mn}^a \bar{\lambda} &= 0, \\
\left( \partial_m \bar{\psi}^4 \bar{\sigma}^m \right)^\alpha + i\sqrt{2}C^{\alpha\beta} \partial_m \left( \bar{A}^a \sigma^m \bar{\lambda}^a \right)_\beta &= 0, \\
\partial^2 A^4 + \sqrt{2}C^{\alpha\beta} \partial_m \left\{ (\sigma^m \bar{\lambda}^a)_{\alpha} \psi^a_{\beta} \right\} &= 0.
\end{align*}
\]

Eqs. (24)–(27) are regarded as the free field equations with the source made of the \( SU(2) \) fields. The solution of (21) in the Lorentz gauge is obtained in [20] as \( v_m^4 = iC_{mn}\bar{\lambda}^a \lambda^a \), where \( \Psi \) obeys \( \partial^2 \Psi = \bar{\lambda}^a \lambda^a \). In the self-dual background (22), eqs. (24)–(27) have trivial solutions \( F_{mn}^4 = \lambda^4 = \bar{\psi}^4 = 0 \), \( A^4 = \text{const.} \) since \( \bar{\lambda} = 0 \). As a consequence, the self-dual instanton equation \( F_{mn}^{4-} = 0 \) is satisfied. In the anti-self-dual background (23), \( v_m^4 \) satisfies the deformed anti-self-dual instanton equation \( F_{mn}^{4+} + i\sqrt{2}C_{mn}\bar{\lambda}^a \lambda^a = 0 \). Eqs. (25) and (27) have trivial solutions \( \lambda^4 = 0 \), \( A^4 = \text{const.} \) due to \( C^{mn} F_{mn}^a = \psi^a = 0 \). On the other hand, from eq. (26), \( \bar{\psi}^4 \) depends on \( C \).

We next consider the case where the vacuum expectation values of \( A \) and \( \bar{A} \) are nonzero, which corresponds to the constrained instanton solutions (see [24] for a review). When the \( SU(2) \) instantons are self-dual, we expand the fields in the coupling constant \( g \):

\[
\begin{align*}
v_{SU(2),m} &= g^{-1}v_m^{(0)} + gv_m^{(1)} + \cdots, \\
\lambda_{SU(2)} &= g^{-1/2}\lambda^{(0)} + g^{3/2}\lambda^{(1)} + \cdots, \\
\bar{\psi}_{SU(2)} &= g^{-1/2}\bar{\psi}^{(0)} + g^{3/2}\bar{\psi}^{(1)} + \cdots, \\
A_{SU(2)} &= g^0 A^{(0)} + g^2 A^{(1)} + \cdots, \\
\bar{A}_{SU(2)} &= g^0 \bar{A}^{(0)} + g^2 \bar{A}^{(1)} + \cdots,
\end{align*}
\]

(28)

where \( v_m^{(0)} \) is self-dual \( F_{mn}^{(0)} = \bar{F}_{mn}^{(0)} \). We are interested in the leading terms of the \( SU(2) \) fields. The equations of motion of the fields are

\[
\begin{align*}
\bar{\sigma}^m D_m \lambda^{(0)} + \frac{1}{2} C^{mn}[F_{mn}^{(0)}, \bar{\lambda}^{(0)}] &= 0, \\
\sigma^m D_m \bar{\lambda}^{(0)} + \sqrt{2}[\bar{A}^{(0)}, \psi^{(0)}] &= 0, \\
\sigma^m D_m \bar{\psi}^{(0)} - \sqrt{2}[ar{A}^{(0)}, \lambda^{(0)}] &= 0, \\
\bar{\sigma}^m D_m \psi^{(0)} &= 0, \\
D^2 \lambda^{(0)} &= \frac{i}{\sqrt{2}}\{\psi^{(0)}, \lambda^{(0)}\} = 0.
\end{align*}
\]

(29)–(31)
Only the eq. (29) is deformed by \( C \). For example, the \( C \)-deformed supersymmetric zero-mode can be obtained from the supersymmetry transformation (10) as

\[
\lambda^{(0)} = -\sqrt{2}i(\bar{A}_{U(1)}C\eta)\beta(\sigma^{mn})_{\beta}^{\alpha}F^{(0)}_{mn}.
\]  

(32)

We note that \( \bar{\psi}(0) \) and \( A(0) \) are also deformed since eqs. (31) contain \( \lambda^{(0)} \) as a source.

In the case of anti-self-dual background \( F^{(0)}_{mn} = -\tilde{F}^{(0)}_{mn} \), the \( g \)-expansion of the \( SU(2) \) fields becomes

\[
v_{SU(2),m} = g^{-1}v_{(0)}^{(0)} + g^{1/2}v_{(1)}^{(0)} + \cdots, \quad F_{SU(2),mn} = g^{-1}F_{mn}^{(0)} + gF_{mn}^{(1)} + \cdots;
\]

\[
\lambda_{SU(2)} = g^{1/2}\lambda^{(0)} + g^{5/2}\lambda^{(1)} + \cdots, \quad \bar{\lambda}_{SU(2)} = g^{-1/2}\bar{\lambda}^{(0)} + g^{3/2}\bar{\lambda}^{(1)} + \cdots;
\]

\[
\psi_{SU(2)} = g^{1/2}\psi^{(0)} + g^{5/2}\psi^{(1)} + \cdots, \quad \bar{\psi}_{SU(2)} = g^{-1/2}\bar{\psi}^{(0)} + g^{3/2}\bar{\psi}^{(1)} + \cdots;
\]

\[
A_{SU(2)} = g^{0}A^{(0)} + g^{2}A^{(1)} + \cdots, \quad \bar{A}_{SU(2)} = g^{0}\bar{A}^{(0)} + g^{2}\bar{A}^{(1)} + \cdots.
\]  

(33)

The leading terms of the equations of motion are

\[
\bar{\sigma}^{m}D_{m}\lambda^{(0)} + \sqrt{2}[A^{(0)},\bar{\psi}^{(0)}] + \frac{1}{2}C^{mn}[F_{mn}^{(1)},\bar{\lambda}^{(0)}] = 0,
\]  

(34)

\[
\sigma^{m}D_{m}\bar{\lambda}^{(0)} = 0, \quad \bar{\sigma}^{m}D_{m}\psi^{(0)} - \sqrt{2}[A^{(0)},\bar{\lambda}^{(0)}] = 0,
\]  

(35)

\[
\sigma^{m}D_{m}\bar{\psi}^{(0)} = 0, \quad D^{2}A^{(0)} = 0, \quad D^{2}\bar{A}^{(0)} - i\sqrt{2}[\bar{\psi}^{(0)},\bar{\lambda}^{(0)}] = 0.
\]  

(36)

Eqs. (35) and (36) can be solved as in the undeformed theory. But the \( C \)-deformed source in \( \lambda^{(0)} \) comes from the order \( g \) contributions of the gauge fields. At the leading order in \( g \), the \( SU(2) \) anti-self-dual solutions are not modified by the deformation parameter \( C \). This is consistent with the result obtained from the \( Q \)-exactness of the \( C \)-variation of the action. In the \( U(1) \) part, the equations of motions (24)–(27) would have \( C \)-dependent solutions. A detailed analysis of a instanton calculus will be discussed in a separate paper.

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