Observing Wind-Forced Flexural-Gravity Waves in the Beaufort Sea and Their Relationship to Sea Ice Mechanics

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Abstract: We developed and deployed two inertial measurement units on mobile pack ice during a U.S. Navy drifting ice campaign in the Beaufort Sea. The ice camp was more than 1000 km from the nearest open water. The sensors were stationed on thick (>1 m) first- and multi–year ice to record 3-D accelerations at 10 Hz for one week during March 2020. During this time, gale-force winds exceeded 21 m per second for several hours during two separate wind events and reached a maximum of 25 m per second. Our observations show similar sets of wave bands were excited during both wind events. One band was centered on a period of ~14 s. Another band arrived several hours later and was centered on ~3.5-s. We find that the observed wave bands match a model dispersion curve for flexural gravity waves in ~1.2-m ice with a Young’s modulus of 3.5 GPa under compressive stresses of ~0.3 MPa. We further evaluate the bending stress and load cycles of the individual wave bands and their potential role in break-up of sea ice. This work demonstrates how observations of waves in sea ice using these and similar sensors can potentially be a valuable field-based tool for evaluating ice mechanics. In particular, this approach can be used to observe and describe the combined mechanical behavior of consolidated floes relevant for understanding sea ice mechanical processes and model development.

Keywords: sea ice; flexural-gravity waves; inertial motion units

1. Introduction

Widespread Arctic sea ice decline and reduced ice extent [1,2] are increasing the area of ice-free water in the Arctic Ocean. More open water and fetch increase the area for momentum transfer and wave generation [3] that can carry substantial energy over long distances [4,5]. A clear example of significant wave generation in open water occurred during the record-breaking September 2012 sea ice minimum when strong winds (18 m s$^{-1}$) forced large (5-m) waves in the central Beaufort Sea that evolved into sea swell [6]. Swell can carry energy far into the ice pack [6,7] where it can induce ice fracture. This can potentially drive further ice retreat in a positive feedback loop that contributes to an increasingly ice-free Arctic Ocean.

The importance of wave propagation and attenuation in sea ice led to several scientific experiments in the 1970s and 1980s conducted from drifting sea ice [8–10]. These campaigns have greatly enhanced our knowledge about waves in sea ice and wave-generated ice fracture.

In this paper our focus is on flexural-gravity waves having a dispersion curve with two waves, a shorter-period flexural wave and a longer-period gravity wave. The flexural wave propagates in the ice (treated mathematically as an elastic plate [11]), and the gravity wave acts similar to swell in the open ocean [12]. The first measurements of “ice coupled” flexural-gravity waves were from Antarctic fast ice using wire strain meters [12]. Their sampling interval of one second allowed [12] to record frequencies below 0.15 Hz. Now,
high-sensitivity accelerometers such as the VN100 inertial measurement unit (IMU) manufactured by Vectornav Co with a sampling frequency up to 800 Hz are in use. Sensors designed with the VN100 IMU have been deployed near Svalbard [13] and in the Antarctic marginal ice zone to capture storm-generated wave signals propagating hundreds of kilometers into the ice [7]. Open-source designs using this IMU are available [7,14].

Additional direct observations are needed of waves propagating in ice under a broader range of conditions than acquired to date. We have thus applied the recent advances in sensor development and the available open-source design to produce our own sensors for deployment on the comparatively thick landfast ice north of Alaska and on mobile pack ice in the Beaufort Sea. The sensors, named Ice Wave Riders (IWRs), are built around the VN100 IMU and include Iridium telemetry and battery power for field deployments for up to 60 days [15].

In this work, we present measurements from two IWR sensors deployed opportunistically on drifting Beaufort Sea pack ice at a time when gale-force winds were measured up to 25 m s$^{-1}$. The results presented here, as far as we know, are the first high-resolution accelerometer measurements of flexural-gravity waves from Arctic pack ice under extreme wind forcing. We evaluate the sea ice response based on analysis of the measured vertical accelerations, and assess the agreement between the observational data and the dispersion relation for flexural-gravity waves.

This paper is organized as follows. The next section presents information related to wave propagation in sea ice and the modeling scheme used in this work. The following section includes details of the sensor design and data processing followed by a section describing the winds and ice conditions in the vicinity of the IWR deployments. The Results section presents the processed sensor data and includes the dispersion curve for flexural-gravity waves tuned with values for sea ice thickness, Young’s modulus, and compressive stress. In the Discussion we suggest possible wave forcing mechanisms consistent with the observations. The Conclusions follow.

2. Modeling Wave Propagation in Sea Ice

In the open ice-free ocean, a typical wave spectrum has an amplitude peak in the period range of 2 to 30 s, corresponding to wind driven waves and sea swell propagating as gravity waves [16]. As waves propagate into the marginal ice zone there is an increase in the peak period [17]. The wave energy is attenuated due to scattering and dissipation with dissipation narrowing and scattering broadening the wave spectra in general [17]. Thus, under an ice cover the wave periods typically range between 4 and 24 s, with swells passing through dense pack ice typically ranging from 11 to 23 s [18].

The excitement of flexural-gravity waves requires wind speeds above the minimum wave phase speed which can be found from the dispersion relation. If the wind speed is less than the minimum wave phase speed, no waves are excited. When the wind speed is greater than the phase speed minimum, waves with two different frequencies are excited, lower frequency $k^-$ and and higher frequency $k^+$, both with phase speeds equal to the moving pressure field or wind speed. Because the group speed of the $k^-$ wave is smaller than the $k^+$, wave energy is transferred away at two different group velocities.

Flexural-gravity waves decay exponentially [19]. Wave trains of flexural-gravity were identified in large scale (~50 km) NSIDC Arctic ice draft data to conclude that scattering dominates decay for periods less than 10 s, viscosity dominates for periods longer than 20 s, and both viscosity and scattering are significant from 10 to 25 s [20]. Where the ice is elevated, e.g., hummocks, wind forcing may generate lower-frequency flexural gravity waves that propagate along the hummock, decaying exponentially with distance from it [21]. Similar edge waves modes can be generated in the vicinity of ice cracks by a plane flexural-gravity wave carrying energy to the crack [22]. Inversely, if edge waves are forced by winds on the surface of the ice they will radiate plane flexural gravity waves carrying energy from cracks or ice ridges.
The presence of ice adds complexity to the open-water dispersion relation, \( \omega = \sqrt{gh \tan h(kH)} \) where \( g \) is gravity, \( H \) is water depth, and the wavenumber \( k \) is \( 2\pi/\lambda \), for wavelength \( \lambda \). The added complexity is due not only to its lower density and elasticity, but also because its mass distribution may be uneven from deformation such as ice ridging, rafting, and cracking. In this work, we simplify the ice cover and treat it as a thin homogeneous elastic plate, ignoring the heterogeneities mentioned above. The dispersion relation for waves in sea ice can then be described [4,12,13,23] as:

\[
\omega^2 = \frac{gh + Dk^3 - Qk^3}{\cot h(kH) + kM},
\]

where \( g \) and \( H \) are as above, \( \omega \) is the wave frequency \( 2\pi/T \) for period \( T \), \( k \) is the wavenumber \( 2\pi/\lambda \) for wavelength \( \lambda \), and \( D \) is the bending modulus. The bending modulus depends on the ice thickness and its rheological properties such that:

\[
D = \frac{Eh_i^3}{\rho_i 12(1 - \nu^2)}
\]

where \( E \) is Young’s modulus, \( h_i \) is the ice thickness, \( \rho_i \) the water density, and \( \nu \) is the Poisson ratio. For Young’s modulus, [12] used a range from 0.95 to 4.3 \( \times 10^9 \) N m\(^{-2}\), [24] used 6 \( \times 10^9 \) N m\(^{-2}\) and [13] used half of that, 3 \( \times 10^9 \) N m\(^{-2}\). For pure ice and small scales, larger values up to 9.5 \( \times 10^9 \) N m\(^{-2}\) have been suggested [25]. Here we use initially a value of \( E = 3 \times 10^9 \) N m\(^{-2}\).

The mass loading term, \( M \), is equal to \( h_i \rho_i / \rho_w \) and the ice compression term, \( Q \), is equal to \( P h_i \rho_w \) where \( P \) is the compressive stress. The compressive stress accumulates over distance through the drag coefficient and wind speed where \( P h_{ice} = \rho_a C_D V^2 L \) where \( h_{ice} \) is ice thickness, \( \rho_a \) is the density of the air (1.27 kg m\(^{-3}\)), \( C_D \) is a drag coefficient (0.002), \( V \) is the wind speed, and \( L \) is the accumulation distance.

Near the ice edge or on thin ice, the \( Q \)- and \( M \)-terms may be small compared to the gravity and bending terms and can potentially be neglected [13], and so we first use the following values \( Q = 0, M = 0, P = 0, \rho_w = 1025 \) kg m\(^{-3}\), \( \rho_i = 922.5 \) kg m\(^{-3}\), and \( \nu = 0.3 \) [4,13] to generate a dispersion curve.

It is important to note that \( Q \) and \( M \) are not negligible in the denser pack ice. For unbroken ice, the compressive strength is generally less than 1 MPa [26]. Observations from a 1986 R/V Polarstern cruise in the Weddell Sea suggested \( P > 5 \) MPa to account for a ~50% reduction in wavelength in the pack ice in comparison to open water [23]. In this work, we deployed sensors on ~1.2 to 3-m thick, mobile, pack ice where compression could potentially be significant. This is addressed in the Results.

Flexural stress arises from ice flexing and bending and becomes increasingly important if there is repeated loading from waves. The flexural stress, \( s \), can be found using \( s = 0.5 E h_{ice} a k^2 \) where \( E \) is the elastic modulus (3 \( \times 10^9 \) Pa), \( h_{ice} \) is the ice thickness (1.2 and up to 3 m), \( a \) is the wave amplitude, and \( k \) is the wavenumber. Flexural strength for sea ice ranges from 0.109 to 0.415 MPa [27,28]. We compute the stresses for the deployment conditions in the Results.

Compared to open water, an ice cover increases the phase \( (C_p = \omega/k) \) and group \( (C_g = \partial \omega / \partial k) \) speeds for the higher frequencies and larger wavenumbers (Figure 1), separating the flexural from the gravity wave. Under a 1-m ice cover, for example, the group speeds diverge for frequencies greater than ~0.06 Hz (blue vertical line in Figure 1a), and the phase speeds diverge for frequencies greater than ~0.075 Hz (red vertical line in Figure 1a). In the case of 1-m ice, when the frequency is greater than 0.145 Hz, the group speed, \( C_g \), becomes greater than the minimum phase speed, \( C_p \) (Figure 1).
3. Ice Wave Riders

The deployed sensors, called Ice Wave Riders (IWRs), measure 3-D accelerations and attitude heading references (yaw, pitch, and roll) at 10 Hz. This enables measurements of ice motion associated with ocean waves, ice ridging, and ice-ice collisions [15]. The IWRs are built to operate in the ice environment of the Chukchi and Beaufort Seas, functioning for up to two months unattended in temperatures reaching −40 °C. Each IWR telemeters at programmable intervals (generally one hour) summary ice motion statistics, UTC time, GPS position, and battery voltage via Iridium to our commercial partner, Pacific Gyre Inc., for temporary storage and online graphical display. The lowest-power Iridium Short Burst Data channel is sufficient to transmit all data as “alerts”, minimizing telemetry costs. Each IWR is enclosed in a Pelican Storm Case and the collected data are stored onto a USB Flash drive. Alkaline batteries power the unit and were chosen because they do not include hazardous materials, are easier to transport then lithium batteries, and eliminate the need for solar recharging which can be a problem during Arctic winters.

Each IWR uses the VectorNav VN100 IMU that includes a three-axis accelerometer, a three-axis gyroscope, a three-axis magnetometer, a temperature sensor, and an air pressure sensor. Temperature and air pressure were not stored during the deployments described here. Detailed VN100 performance information and sensor designs are found in [7,14]. The component integration of the IWR is broadly similar to both [7,14]. Inside each IWR, the VN100 communicates via a TTL serial interface with a Linux compatible BeagleBone computer that integrates all the sensor components. The VN100 operates internally at a factory-set rate of 800 Hz and by default subsamples the data output, storing every 80th value when sampling at 10 Hz. To improve signal-to-noise, we changed the default VN100 filter settings to average the 800 Hz signal in sequences of 80 values before storing at 10 Hz [29].

We found the spectral character of the response by partitioning the vertical acceleration time-series into 30-min segments with each new segment overlapping the previous one by 50%. Fourier analysis was applied to each timeseries segment, with the resulting spectral amplitudes stacked in time [30]. The resulting Welch periodogram provides spectral information of the signal over time.

We calculated the ice displacement associated with the measured vertical acceleration by double integration following the approach of [7] and further detailed in [14]. The double integration is performed in the frequency domain using a Fourier transform and then reverse-transformed back into the time domain. To prevent abrupt frequency cut-offs
which can add noise (amplitude) to the displacement, frequency response weights, \( H(f) \), are applied where \( f_1 = 0.02 \) Hz, \( f_2 = 0.03 \) Hz, and \( f_c \) is the Nyquist frequency [7,14].

\[
H(f) = \begin{cases} 
0, & 0 < f < f_1 \\
\frac{1}{2} \left[ 1 - \cos \left( \frac{\pi f}{f_2 - f_1} \right) \left( \frac{f - f_1}{f_2 - f_1} \right) \right], & f_1 \leq f \leq f_2 \\
\frac{1}{2\pi f}, & f_2 \leq f \leq f_c 
\end{cases}
\]  

(3)

4. IWR Deployments and Field Conditions

We deployed IWRs on pack ice adrift in the Beaufort Sea as part of the U.S. Navy’s ICEX 2018 and 2020 campaigns. The interest here is on the ICEX2020 Camp Seadragon deployments that acquired on-ice data ~125 km north of Alaska between 7 and 15 March 2020 (Figure 2).

Figure 2. (a) Arctic sea ice concentration for March 2020 with blue square outlining the region shown at right. The nearest open water is in the Bering Sea, more than 1000 km distant. (b) Drift track of Camp Seadragon and insets of 13 March 2020 and 14 March 2020 wind lines directed southwest to northeast [earth.nullschool.net].

The Camp Seadragon layout and the location of the two IWR sensors are shown in Figure 3. IWR5 was anchored to multi-year ice ~300 m north of a runway, and IWR6 was anchored to first year ice ~60 m south of the runway. The sensors were secured with ratchet straps to ice screws embedded in the ice.

Due to the relevance of sea ice thickness for our measurements, we acquired thickness measurements of the area surrounding the ice camp along the transect lines shown in Figure 3 using an electromagnetic conductivity meter (EM-31) that measures the snow + ice thickness. We use this combined measure as a proxy for ice thickness that likely overestimates the ice thickness. The transect lines do not provide a systematic survey of the region but do provide information related to the approximate ice thickness of the relatively level and ridged multiyear ice (1.5 to 6-m averaging ~3 m), first-year ice (~1.2–1.5 m), and hummock height (up to ~6 m).

We define here a characteristic length-scale (CLS) as the distance between elevated ice features. The CLS may help set the length-scales of the wind blowing immediately over the ice. We approximate the CLS using the ice thickness data and dividing the number of peaks by the total length of the transect. Over relatively level multiyear ice, the CLS is ~150 to ~200 m. On first year ice, the EM-31 survey shows a hummock “basin” with a CLS ranging from 300 to 400 m.
Winds at the local and mesoscale are important to understanding the response of the sea ice during ICEX2020. Wind speed measured at Camp Seadragon is plotted from 10 to 16 March 2020 in Figure 4. Two values of wind speed are marked by red lines. A speed of 16 m s$^{-1}$ marks the theoretical minimum wind speed needed to excite flexural gravity waves based on the dispersion relation for conditions at Camp Seadragon.
Figure 4. Wind speed (meters per second) measured at Camp Seadragon from 10–16 March 2020. Two wind events are marked by red vertical dashed lines when the wind speed exceeded the 16 m s\(^{-1}\) threshold needed to excite flexural-gravity waves. A 21 m s\(^{-1}\) wind speed is also marked. The black vertical lines mark the times that bound the wave response from Figure 5. All times are in UTC.

Figure 5. Welch periodogram for (a) IWR5 and (b) IWR6 with time (x-axis) from early 13 March through noon 15 March, 2020 versus normalized frequency (y-axis left). Selected periods (red lines) have the seconds labeled on the y-axis at right. Wave events appear from 13:30 on 13 March and persist for about 14 h then begin again around 21:00 h on 14 March and last for about 7 h. Black vertical lines mark the start and end of the wave events, labeled Event 1 and 2.

There are two time spans, labeled Event 1 and Event 2, when the measured wind speed exceeds the 16 m s\(^{-1}\) threshold. Event 1 begins about 13:00 on 13 March and lasts for ~16 h until the wind decreases to less than ~10 m s\(^{-1}\). Event 2 begins around 21:00 on 14 March, lasting for about 14 h. A wind speed of 21 m s\(^{-1}\) is also marked, and its importance is discussed later. During wind Event 1, on 13 March 2020 at 16:00 UTC, the winds are offshore, driving ice away from the coast and toward the ice camp. As the wind falls to less than 10 m s\(^{-1}\), they begin to blow toward the northeast, and by late 14 March 2020 at 13:00 UTC, the winds are blowing toward the east.
5. Results

We processed the vertical accelerations measured by the two IWR sensors in the frequency domain using Fourier analysis as described above. The resulting Welch periodograms display the wave amplitude at each frequency over time and are unremarkable except for the time span during the strongest winds from 13 to 15 March (Figure 5). In that time frame, two events are visible having periods that fall within the 2 to 30-s wave band. The first wave event begins at 13:30 on 13 March and ends at 04:00 on 14 March (vertical lines in Figure 5) and aligns quite well with the temporal window of wind Event 1 (black vertical lines in Figure 4). A second wave event begins at 21:00 on 14 March and ends at 04:30 15 March (vertical lines in Figure 5) and falls within the first half of wind Event 2 (black vertical lines in Figure 4).

The spectral response is bi-modal, indicating that two waves are generated. The two spectral bands are separated by a relatively quiet band about six-seconds wide. The lower frequency band ranges from ~10 to ~20 s with a dominant peak at ~14 s. The higher frequency band ranges from 2.7 to 3.8 s with a dominant peak at 3.5 s. For both wave events, the response in the higher frequency band is weaker at IWR5 (deployed on multi-year ice 1.5 to ~4-m thick) than at IWR6 (deployed on first-year ice 1.2 to 1.5 m thick). For both wave events, the higher frequency waves appear 4 to 6 h later than the lower frequency waves.

We calculated the vertical displacement for both bands as described above and plotted it from the afternoon of 13 March to early on 15 March (Figure 6). The largest amplitudes occur at IWR5 and reach 2 cm (wave height of 4 cm) during the times of the largest measured winds. Most of the displacement amplitude is contributed by the 14-s wave. We estimate the 14-s wave amplitude by band-pass filtering the displacement time series from 7.5 to 22 s, leaving amplitudes up to ~1.5 cm. To estimate 3.5-s wave amplitude, we band-pass filtered the displacement time-series from 1 to 7.5 s, leaving amplitudes up to 0.2 cm.

![Figure 6](image_url)

**Figure 6.** Vertical displacement timeseries in meters from early 13 March through noon 15 March 2020, for (a) IWR5 and (b) IWR6. The largest amplitudes exceed two centimeters. The beginning and ends have been tapered to zero.

We calculated attenuation coefficients, $\alpha$, following [31]:

$$\frac{A(x)}{A(x = 0)} = e^{-\alpha x}$$

(4)
where \( A \) is the wave amplitude and \( x \) is the distance between the two IWR sensors (350 m) with IWR6 at \( x = 0 \). For wave amplitude, \( A \), we used half the significant wave height calculated as four times the standard deviation of the displacements timeseries [32] over the durations of Event 1 and Event 2 described above. We bandpass filtered the displacements, between 1 and 7.5 s, and 7.5 and 22 s, to determine the 3.5 and 14-s wave amplitudes respectively. Table 1 shows the wave amplitudes and the resulting attenuation coefficients.

**Table 1.** Wave amplitudes and attenuation coefficients for 3.5 and 14-s waves for wind Event 1 and Event 2. Amplitude is computed as half the significant wave height (see text for details).

|        | 3.5 s |        | 14 s |
|--------|-------|--------|------|
|        | \( A, \text{cm} \) | \( \alpha, 10^{-4} \text{ M}^{-1} \) | \( A, \text{cm} \) | \( \alpha, 10^{-4} \text{ M}^{-1} \) |
| Event 1 | IWR5  0.10791 | 1.327 | 0.64690 | 0.836 |
|         | IWR6  0.11304 |       | 0.66611 |       |
| Event 2 | IWR5  0.10974 | 1.647 | 0.61243 | 0.551 |
|         | IWR6  0.11625 |       | 0.62436 |       |

The attenuation values in Table 1 for the 14-s wave are in close agreement with values of 0.522 and 0.536 \( 10^{-4} \text{ m}^{-1} \) reported for 2-m ice at nearly 100% concentration in the Sea of Okhotsk [31]. The amplitude attenuation values for the 3.5-s wave are 2 to 3 times larger than for the 14-s wave.

To find the wavelength of these waves we need the dispersion relation for flexural-gravity waves using parameters appropriate for ICEX2020. In the Introduction we set the compression term to zero, used \( 3 \times 10^9 \text{ N m}^{-2} \) for Young’s modulus, and choose an ice thickness of one meter. We now look specifically at the effect on the dispersion relation from the compression term, \( Q \), the Young’s modulus, \( E \), and the ice thickness.

The dispersion relation is very sensitive to ice thickness through both the bending modulus and the compression terms. Figure 7 shows the dispersion relation for 1.2 and 1.5-m ice, representative of the first-year ice around IWR5 and the thinner ice around IWR6. When an ice thickness of 2-m is used in the dispersion relation, the minimum wave phase speed, i.e., the threshold wind speed needed to excite the 3.5-s flexural waves, increases to 28 m s\(^{-1}\), beyond the maximum wind speeds measured at ICEX2020. For the 14-s waves, a 2-m ice thickness shifts the dispersion curve above 23 m s\(^{-1}\), a wind speed reached only a few times at ICEX2020. From the dispersion curve we find values for ice thickness from 1.2 to 1.3-m allow for the flexural-gravity waves observed here.

Increasing the Young’s modulus increases the group and phase speeds, particularly of the higher frequencies (Figure 7). A Young’s modulus of \( 6 \times 10^9 \text{ N m}^{-2} \) increases the phase speed of the 3.5-s wave to 24 m s\(^{-1}\), above most of the values for the observed wind speeds. The 14-s period waves are less sensitive to the Young’s modulus values used here. Based on the dispersion curve and the data, we adopt values for the Young’s modulus between 3 and \( 4 \times 10^9 \text{ N m}^{-2} \).

For the dispersion curves, we assume that the compressive stress is less than 1 MPa [26] because the ice was generally unbroken in the region surrounding the ICEX2020 camp. We can narrow this assumption by evaluating the length scale \( L \) over which the stress can accumulate. The ice camp at this time was ~125 km north of the Alaska coast (\( L = 125 \text{ km} \)) and the wind speeds were approximately from 16 to 21 m s\(^{-1}\) blowing in an offshore direction. We calculate the accumulated stress for 1 to 2-m ice to be in the range of 0.1 and 0.3 MPa. In the dispersion curve, compression lowers the phase speed for both the 3.5- and 14-s waves. The dispersion curves for \( P = 0 \) and 0.3 MPa are plotted in Figure 7. A compression stress of 2 MPa or greater yields dispersion curves with a wave speed minimum at or below 10 m s\(^{-1}\), inconsistent with observed wave generation above at least 16 m s\(^{-1}\).
Young’s modulus values of 3 × 10^6 N m⁻² (light lines) and 4 × 10^6 N m⁻² (heavy lines), and compression stress of zero (solid) and 0.3 MPa (dashed). A 21 m s⁻¹ wind speed is marked, along with the dominant wave periods, 3.5 and 14 s. The phase speed minimum for 1.2-m ice is 16–17 m s⁻¹ and falls at ~7.8 s.

Figure 7 shows multiple dispersion curves for the range of parameters described above. To allow for excitation of the 3.5-s and 14-s waves from the measured wind speeds at ICEX2020, the dispersion curve requires values of the Young’s modulus between 3 and 4 × 10^6 N m⁻², compression less than 0.3–0.6 MPa, and the measured ice thickness of ~1.2- to 1.3-m. From the dispersion relation with the above values for compression, Young’s modulus, and 1.2-m ice thickness, the 14-s wave has a wavelength of 305-m and the 3.5-s wave has a 74-m wavelength.

The flexural stress on the ice from the 3.5-s wave (wavelength of 74 m and an amplitude of 0.2 cm), a Young’s modulus of 3 × 10^6 N m⁻², and ice thickness of 1.2 m, is 27.4 kPa. This is smaller than the reported range for the strength of sea ice from 100 to 300 kPa [28]. The amplitude of the 3.5 s waves would have to be at least 4 times larger than measured to exceed the flexural strength of the ice. For the 14-s wave (wavelength of 305 m and an amplitude of 1.5 cm) the stress ranges from 11.2 kPa for 1.2-m ice up to 28.1 kPa for 3-m ice, values also below the reported flexural strength. For the 14-s waves, an amplitude closer to 15 cm would be needed to break the ice.

6. Discussion

The on-ice measurements of vertical acceleration are bi-modal with periods between ~2 and ~20 s, consistent with flexural-gravity waves propagating in sea ice from 1.2 to 1.3 m thick. We have found values for Young’s modulus, compressive stress, and a representative ice thickness to “tune” the dispersion curve for flexural-gravity waves to allow for waves in the observed frequency bands. It is fortuitous that gale-force winds arose during the IWR deployments to generate a robust flexural-gravity wave signal. We discuss next how such waves may arise.

No incoming wave spectra were taken near the ice camp as there was no open water for more than 1000 km. On 14 March 2020 waves with a period of 13.4 s are indicated for the open water of the Bering Sea at about 60° N [see earth.nullwchool.net], south of the ice edge in Figure 2. Using Equation (4), the attenuation values from Table 1, and a distance of 1000 km, we calculate an unrealistic initial wave amplitude at the source. We rule out the reduced ice region of the Bering Sea (and the Atlantic Ocean as well) as sources of the waves described here. Ref [33] show Barents Sea waves of about 10-s penetrate only several tens of km into the solid ice. We note that [34] used a 2-D model to show
long-distance propagation, with little shape change and weak dispersion, of solitary waves, and [35] showed infragravity wave propagation over long distances across the Arctic Ocean. However, based on the fit with the dispersion relation, we believe the waves we measured were generated within tens of kilometers of our sensors and arose within the ice pack.

Wind forcing of open water waves begins with ripples arising from an initially “aerodynamically-smooth surface” followed by wave growth of the longer waves with their wavenumbers depending on the surface roughness [36]. The energy transfer from winds to waves is through non-linear, resonant interactions of gravity waves over deep water [36]. However, it is not clear how, during wave generation, the water surface “selects” [36] from the wavenumbers of the atmospheric pressure distribution. We show (see Appendix A) that the pressure fluctuation term arising from the influence of surface waves on the air pressure near the surface can be ignored in the dispersion relation for flexural-gravity waves. This implies that, over rough sea ice, the stress fluctuations are produced from air flowing over the irregular surface.

We therefore seek characteristic scales of the ice consistent with the frequencies and wavelengths of the observed waves. For example, high frequency pressure fluctuations (of the order of 10 s) can arise above the ice due to the distortion induced by surface features such as sastrugi [37]. For example, [38] found sastrugi at Alert, Nunavut, with wavelengths between 3 and 10 m that generated surface pressure fluctuations of 5 Pa under moderate winds of 4–6 m s$^{-1}$. Although these wind speeds and wavelengths are less than observed at ICEX2020, similar processes may be in effect there.

Specific length scales may be “selected” by winds in contact with sea ice ridges. [39] analyzed ICESat-2 data to compute the power spectra of ridge frequency across the Arctic Ocean. In the central Beaufort Sea, the mean ridge frequency was 2 km$^{-1}$, (ranging from ~1 and 7 km$^{-1}$) corresponding to a mean ridge spacing of 500 m (ranging from ~150 to 1000 m). This is consistent with the ~300-m wavelength of the 14-s waves but slightly more than the local 100- to 200-m length scales from the EM-31 measurements at ICEX2020. Fourteen second waves are less dispersive than the higher frequency waves and can propagate long distances (10 s of km). We can reasonably expect wave generation at a distance from the sensors.

Flexural waves at 3.5 s are highly-dispersive, suggesting a more local generation. A high-frequency atmospheric signal can be generated through vortex shedding when wind blows past a cylinder and vortices at a specific frequency are generated downstream. This process relates the effects of wind at speed $U$ passing an ideal cylinder of diameter $d$ to the Strouhal number, $S_t$, where $S_t = f_s d / U$. The Strouhal number ranges from 0.18 to 0.22 for sub-critical flow and $f_s$ is the shedding frequency (s$^{-1}$). For a wind speed of 21 m s$^{-1}$ and a period of 3.5 s, the feature diameter is 11 to 16 m. Along a 1-km transect in the Beaufort Sea, [39] noted ridge widths ranging from 7.1–35.7 m, scales consistent with vortex shedding between 1.7 and 8.5 s, bracketing the observed 3.5-s period.

We noted above (see Figure 5) the later arrival of the 3.5-s waves by 3 to 4 h. The 3.5-s wave is the fourth harmonic of the 14-s wave, and resonant excitation, in addition to wind forcing, may have played a role in its generation and later arrival [40,41]. Over the several hours delay time, it is also possible that winds or waves weakened or loosened the ice enough for it to heave. We calculate the angular frequency of heave using the relation among the local conditions, ice vertical acceleration, $z_{tt}$, and ice displacement, $z$, such that $\rho_{ice} h_{ice} z_{tt} = -\rho_w g z$ where the other terms are as before. The angular frequency, $\omega$, is then equal to $\sqrt{\rho_w g / \rho_{ice} h_{ice}}$. For 1.2-m ice and density values as above, the resulting period is too small, ~2.1-s. To generate a 3.5-s signal requires ice thickness to be ~4-m. This is only possible if large ice features were heaving. In the area of first-year ice near IWR6, the EM-31 data shows hummocks reaching six meters. Their size and proximity make ice heave a compelling generating mechanism and in the future additional measurements near such features are needed.

The simplest explanation for a gap in the spectra between ~4 and ~10 s is a rapid increase in winds past ~16 m s$^{-1}$ so that no waves were excited. Figure 4 shows that
early on 15 March 2020 the winds rapidly increased from less than 15 m s\(^{-1}\) to more than 21 m s\(^{-1}\). We speculate that winds near 16 m s\(^{-1}\) were not sustained for a sufficiently long duration to excite flexural-gravity waves and fill the 4 to 10-s gap. It is also worth noting that resonance may have occurred where wind and phase and group speeds are the same and energy cannot easily escape the region of forcing \([12]\). If the measured waves were generated at a distance, as seems likely, the energy would have been trapped locally, unable to propagate to the sensors. \([42]\) suggested resonance conditions as an explanation for the 2011 extreme sea state in the ice-free North Atlantic where a storm moving at 20 m s\(^{-1}\) excited surface gravity waves with periods from 20 to 25 s and peak wavelengths of 700 m. Conditions for resonance and wind speeds can be calculated following \([43]\) and applying the parameter values used in Equation (1). For conditions at ICEX2020, 1.2-m ice and 16 m s\(^{-1}\) winds, the “trapped” resonant wave has a ~110-m wavelength. Additional sensor measurements placed over large regions would be needed to confirm this possibility. Future field experiments may benefit from more detailed ice thickness surveys of the surrounding region and deploy IMUs in proximity to prominent ice features as cracks, ridges, and hummocks.

7. Conclusions

We developed a new motion sensor to measure high-precision acceleration in sea ice. These sensors, named Ice Wave Riders (IWRs) were designed based on state-of-the-art sensor technology to characterize waves in sea ice in harsh Arctic conditions. We deployed two IWRs on drifting sea ice during the U.S. Navy ICEX2020 campaign in the Beaufort Sea and recorded vertical accelerations for a week in March 2020 when local winds reached gale-force. The nearest open water was more than 1000 km distant. Based on our observed attenuation rates, the measured waves were generated by winds acting directly on the sea ice rather than propagating from open water. At each of two different time spans when the winds were strongest, two waves were generated with periods that centered on ~3.5 and ~14-s. A gap in the bimodal frequency response falls at the phase speed minimum indicating that the sustained winds were above this speed, or the wave energy was trapped at the generating source by resonance. The dispersion relation for flexural-gravity waves fits the observations for values of the Young’s modulus between 3 and \(4 \times 10^9\) N m\(^{-2}\), compressive stress less than 0.3 MPa, and ice thickness of ~1.2-m. From the dispersion relation, the 14-s wave is ~300-m and the 3.5-s wave is its fourth harmonic with a wavelength of ~74 m. These new data add to the high precision accelerometer measurements already acquired by several investigators using the VN100 IMU. Continued improvements of these and similar sensors will allow for new observational data to be collected to better understand the sea ice environment and to constrain parameters critical to model development.

Author Contributions: Conceptualization, M.A.J. and A.R.M.; methodology, M.A.J. and A.R.M.; software, M.A.J. and A.V.M.; validation, M.A.J., A.V.M., D.O.D. and A.R.M.; formal analysis, M.A.J., A.V.M., D.O.D. and A.R.M.; investigation, M.A.J., A.V.M., D.O.D. and A.R.M.; data curation, M.A.J.; writing—original draft preparation, M.A.J.; writing—review and editing, M.A.J., A.V.M., D.O.D. and A.R.M.; visualization, M.A.J., A.V.M., D.O.D. and A.R.M.; project administration, M.A.J.; funding acquisition, M.A.J. and A.R.M. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Bureau of Ocean Energy Management, award number M16A00006, through the Coastal Marine Institute at the University of Alaska Fairbanks and by the Research Council of Norway, Funding ID: 280625, the Dynamics of Floating Ice Project at the University of Oslo. Support is also through The Arctic Engineering in Climate Change Project (AOCEC) funded by IntPart Programme of Research Council of Norway led by A.V.M. that supported M.A.J. and University of Oslo personnel. Current funding is through the Ice Radar Support for Observing and Predicting Coastal Ice Stability and Trafficability project funded by the U.S. Army Corp of Engineers through the University of Alaska Fairbanks.

Data Availability Statement: Data is available by contacting the corresponding author.
Acknowledgments: Many thanks to Graigory Sutherland for the generous introduction to the sensor work on the Svalbard ice with the AOCEC project. Thanks also to Jean Rabault and Atle Jensen for sharing ideas on sensor development, data processing, and support while M.A.J. was at the University of Oslo. Thanks to Chris Polashenski for discussions on sea ice conditions at the ICEX2020 Camp Seadragon and thanks also to its many participants for help during the IWR deployments and recoveries. We thank Andy Sybrandy and Glen Montgomery, Pacific Gyre Inc., for ongoing discussions and improvements to the Ice Wave Riders.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Propagation of waves with small amplitudes in water layer covered by an ice plate is investigated using the following model including the second order equation for the velocity potential \( \varphi \) \[44\]:

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \varphi = 0, \quad z \in (-H, 0)
\]  

and boundary conditions at the bottom and below the ice plate

\[
\frac{\partial \varphi}{\partial z} = 0, z = -H; \quad \frac{\partial \eta}{\partial t} = \frac{\partial \varphi}{\partial z} \quad \frac{\partial \eta}{\partial t} + g\eta + \frac{p_w}{\rho_w} = C, z = 0.
\]

where \( \eta \) is the elevation of ice plate, \( p_w \) is water pressure below the ice, \( \rho_w \) and \( H \) are the water density and water depth, \( g \) is the gravity acceleration, \( t \) is the time, \( x \) and \( z \) are the horizontal and vertical coordinates, and \( C \) is a constant in the Bernoulli integral.

The water pressure and atmosphere pressure are related by the formula

\[
p_w - p_a = \rho_i h_i g + \rho_i h_i \frac{\partial^2 \eta}{\partial t^2} + D \frac{\partial^4 \eta}{\partial x^4}, \quad D = \frac{Eh_i^3}{12(1 - \nu^2)}
\]

where \( \rho_i \) and \( h_i \) are the ice density and ice thickness, \( D \) is the ice rigidity, \( E \) and \( \nu \) are the elastic modulus and Poisson’s ratio of ice. Further, we set \( C = \rho_i h_i g \).

From (A2) and (A3) it follows

\[
\rho_w \left( \frac{\partial^2 \varphi}{\partial t^2} + \frac{8 \rho}{\rho} \frac{\partial \varphi}{\partial z} \right) + \rho_i h_i \frac{\partial^2 \varphi}{\partial t^2} \frac{\partial \varphi}{\partial z} + D \frac{\partial^4 \varphi}{\partial x^4} + \frac{\partial \delta p_a}{\partial t} = 0, z = 0,
\]

where \( \delta p_a \) is the perturbation of the atmosphere pressure near the ice surface.

The centrifugal force on the pressure fluctuations due to the air motion near the water surface curved by a harmonic wave with small amplitude is \( \eta = a \sin \theta \), where \( \theta = kx - \omega t \) is the wave phase, and \( a, \omega \) and \( k \) are the wave amplitude, the wave frequency and the wave number \[45\]. According to his theory the pressure fluctuations caused by a surface wave with wave number \( k \) are calculated by the formula

\[
\delta p = -\rho_a a k^2 \sin \theta \int_0^\infty (V_a - c)^2 e^{-2kz} dz, \quad \theta = kx - \omega t,
\]

where \( \rho_a \) is water density, \( V_a(z) \) is the wind velocity, and \( c = \omega / k \) is the phase velocity of the wave.

The vertical profile of the wind velocity is approximated by the formula

\[
V_a(z) = \begin{cases} 
V_0 z h_s^{-1}, & z \in (0, h_s) \\
V_0, & z > h_s
\end{cases},
\]

where \( h_s \) is the thickness of the boundary layer near the water surface.
Substitution of Formula (A6) into Formula (A5) leads to the expression

$$
\delta p = -\rho_a ak^2 \sin \theta
$$

(A7)

$$
\alpha = \frac{\omega^2}{2k^3} \left[ 1 + \frac{1}{2} \frac{(\omega_0/\omega)^2}{(1 - e^{-2kh_a})(1 + 2kh_a)} \right] - \frac{\omega_0}{\omega} \left( 1 - e^{-2kh_a} \right).
$$

(A8)

Further we estimate the influence of the wind pressure specified by Formula (A7) on the dispersion of flexural-gravity waves. The velocity potential corresponding to the harmonic wave is written as follows

$$
\varphi = \varphi_0 \cos \theta \frac{\cos h[k(z + H)]}{\cos h[kH]} - \varphi_0 = -\frac{\omega a}{k \tan h[kH]}
$$

(A9)

Substituting Formulas (A7) and (A9) into boundary condition (A4) we find the dispersion equation

$$
\omega^2 \left( 1 + \rho_a p_w^{-1}kh_a \tan h[kH] \right) = gh \tan h[kH] \left( 1 + \frac{D}{g p_w} k^4 - \frac{\rho_a \alpha k^2}{g p_w} \right)
$$

(A10)

Numerical simulations show that the influence of the term $\rho_a ak^2/(gp_w)$ on the dispersion of flexural-gravity waves is very small. The wave frequency can be calculated from the dispersion Equation (A10) with assumption that $\alpha = 0$ with high accuracy. Figure A1 shows the dependencies of $\rho_a ak^2/(gp_w)$ from the wave number $k$, where the value of wave frequency $\omega$ is calculated with $\alpha = 0$. One can see that $\rho_a ak^2/(gp_w)$ is much smaller than 1 and, therefore, the effect of the wind pressure on the dispersion of flexural-gravity waves with wave numbers smaller 0.1 m$^{-1}$ and wave periods greater about 3 s can be ignored.

Figure A1. Dependencies of the term $\rho_a ak^2/(gp_w)$ from the wave number $k$ calculated with $h_s = 1$ m and three different values of $H$ (a) and $H = 1000$ m and three different values of $h_s$ (b).

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