Ghost Number Cohomologies and M-theory Quantum States

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Abstract

We review and develop the formalism of ghost number cohomologies, outlined in our previous work, to classify the quantum states of M-theory. We apply this formalism to the matrix formulation of M-theory to obtain NSR superstring action from dimensionally reduced matrix model. The BPS condition of the matrix theory is related to the worldsheet reparametrizational invariance in superstring theory, underlining the connection between unbroken supersymmetries in M-theory and superstring gauge symmetries.

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1. Introduction

Extended superalgebras appear to be a potentially powerful tool in describing the
dynamics of branes in M-theory [1,2,3,4,5]. In the recently proposed matrix approach,
which supposedly describes the M-theory apart from its low-energy limit [3,7] the
non-perturbative superalgebras play the crucial role. Non-perturbative superalgebras are
superalgebras with p-form central terms, with p-branes accounting for p-forms. In the matrix
theory superalgebra, a membrane and a non-covariant fivebrane have been shown to appear
as central terms. M-theory is a strongly coupled limit of type IIA superstring theory [8],
therefore one may look for the string-theoretic origin of the p-forms.

In the recent paper we have pointed out the connection existing between picture-changing
gauge symmetry in superstring theory and central terms of non-perturbative superalgebras.
Namely, the certain singular limit of picture-changing transformation referred to as the “picture-changing at the infinite-momentum”, plays the role of “map” between
non-perturbative and perturbative superalgebras.

The central terms, generated by this singular version of the picture-changing, are
essentially zero momentum parts of some peculiar bosonic open string vertex operators.
These vertices appear to have rather unusual properties - they are not BRST-trivial, at the
same time they do not describe emissions of any massless particles in perturbative open-
string theories. While their s-matrix elements vanish among elementary string states,
they do interact with Ramond-Ramond charges, i.e. their matrix elements are non-zero in
the presence of D-branes; the examples of such matrix elements have been computed in [4].
These vertex operators may also be interpreted as “brane-emitting vertices”, or creation
operators for non-perturbative brane-like states. The example of such an operator is
a 5-form $e^{-3\phi} \psi_{a_{1}} \ldots \psi_{a_{5}}$ where $\phi$ is a bosonized superconformal ghost and $\psi_{a_{i}}, a = 0, \ldots, 9$
are superstring worldsheet fermions. The crucial feature of such “brane-emitting” vertices
is that they all appear to have essentially non-zero ghost numbers which cannot be re-
moved by picture-changing transformations - and this situation is quite contrary to the
perturbative string theory where it is always possible to choose a picture zero for vertex
operators. These observations led us to introduce the notion of ghost number cohomolo-
gies, which we will develop here along with improving some of the definitions contained in
the previous work. The ghost number cohomologies may be used to classify perturbative
and non-perturbative string states; moreover, we shall argue that the elements of these co-
homologies correspond to the M-theory quantum states. Ghost number cohomologies may
be used to study the relation between matrix theory and strings, which has been suggested
Namely, in this paper we shall demonstrate how one can obtain the worldsheet supersymmetric action for the $D = 10$ NSR superstring theory from the matrix theory by using the correspondence between the M-theory and ghost number cohomologies. The analysis will point at significantly different roles played by supersymmetries in the matrix theory and in superstring theory. The worldsheet reparametrizational invariance in superstring theory will be related to the BPS condition in matrix theory.

**Review of Ghost Number Cohomologies**

We start with recalling how the presence of branes modifies the SUSY algebra. As is now well-known, p-branes lead to the appearance of p-form central charges in the superalgebra, also known as Page charges:

$$\{Q_\alpha, Q_\beta\} = \Gamma^{m}_{\alpha\beta} P_m + \sum_p \Gamma^{m_1...m_p}_{\alpha\beta} Z_{m_1...m_p}$$

(1)

Strictly speaking, these p-forms are not the central charges since they may have non-trivial commutation relations with other generators, as well as with themselves.

Particularly, for the $D = 11$ supermembrane the corresponding two-form charge is given by:

$$Z_{m_1m_2} = \int d^2\sigma \epsilon^{0ik}\partial_iX_{m_1}\partial_kX_{m_2}$$

(2)

where $X^m$'s are coordinates in the eleven-dimensional space-time, and the integral is taken over the surface of the membrane. This charge does not vanish if a membrane configuration defines non-trivial two-cycles in the space-time. The presence of this charge is closely related to the fact that the Wess-Zumino term in the supermembrane action is supersymmetric only up to total derivative. The integration of the boundary term over the membrane then gives (2). In $D = 10$ there exists a surprising connection between the central terms in the non-perturbative superalgebra (1) and the singular limit of perturbative open string gauge symmetry - the picture-changing. Namely, consider the anticommutator of two supercharges $\{Q_\alpha, Q_\beta\}$ in ten dimensions, where the supercharges are given by [13]:

$$Q_\alpha = \oint \frac{dz}{2\pi i} e^{-\frac{1}{2}\phi} \Sigma_\alpha$$

(3)

Again, here $\phi$ stands for the bosonized superconformal ghost field and $\Sigma_\alpha$ is spin operator for matter fields in a space-time. The O.P.E. between two such spin operators is given by:

$$\Sigma_\alpha(z)\Sigma_\beta(w) = \frac{\epsilon_{\alpha\beta}}{(z - w)^4} + \sum_p \frac{\Gamma^{a_1...a_5}_{\alpha\beta}\psi_{a_1}...\psi_{a_5}}{(z - w)^{4-p}} + \text{derivatives}$$

(4)
Then, the straightforward evaluation of the anticommutator gives:

\[
\{Q_\alpha, Q_\beta\} = \Gamma^a_{\alpha\beta} P_a
\]

\[
P_a = \oint \frac{dz}{2i\pi} e^{-\phi} \psi_a
\]

Here \(P_a\) is a momentum operator in \(-1\)-picture. In other words, evaluation of the anticommutator with two supercharges (3) taken in the standard \(-1/2\)-picture gives an usual perturbative superalgebra (5) with no central terms. Now, consider another space-time fermionic generator

\[
T_\alpha = \oint \frac{dz}{2i\pi} e^{-\frac{3}{2}\phi} \Sigma_\alpha
\]

Contrary to what one may naively suspect, this is not the perturbative supercharge (3) in another picture, as the straightforward application of picture-changing operator \(\Gamma_1 := \delta(\gamma)(S_{\text{matter}} + S_{\text{ghost}})\) : (with \(S\) being the worldsheet supercurrent and \(\beta, \gamma\) the superconformal ghosts) to (6) gives zero rather than (3). Nevertheless, as we will show, the generator (6) is related to the space-time supercharge (3) in a rather subtle way. The operator (6) has some interesting physical properties. First of all, as we have pointed out some time ago, its integrand generates, up to picture-changing, the \(\kappa\)-symmetry transformations in the Green-Schwarz superstring theory. Also, in the context of extended \(D = 10\) space-time superalgebras [1,3] it may be understood, as is easy to check, to be a r.h.s. of the commutator

\[
\{Q_\alpha, Q_\beta\} = \Gamma^a_{\alpha\beta} T_\beta
\]

Evaluating the anticommutator of two \(T\)'s gives the result:

\[
\{T_\alpha, T_\beta\} = \oint \frac{dz}{2i\pi} \left[ \frac{1}{2} \Gamma^a_{\alpha\beta} e^{-3\phi} \psi_a \left( \frac{9}{8} \partial \phi \partial \phi - \frac{3}{2} \partial^2 \phi \right) - \frac{3}{2} \partial \psi_a \partial \phi - \frac{1}{2} \partial^2 \psi_a \right. \\
\left. + \Gamma^{a_1 \cdots a_3} \partial (e^\phi \psi_{a_1} \cdots \psi_{a_3}) + \Gamma^{a_1 \cdots a_5} e^{-3\phi} \psi_{a_1} \cdots \psi_{a_5} \right]
\]

What is the structure of the r.h.s. of this anticommutator? As one may check, terms proportional to \(\Gamma^a_{\alpha\beta}\) constitute the momentum operator in the \(-3\)-picture: applying the picture-changing operator to these terms twice, one finds, up to unimportant ghost terms:

\[
(\Gamma_1)^2 \oint \frac{dz}{2i\pi} e^{-3\phi} \psi_a \left( \frac{9}{16} \partial \phi \partial \phi - \frac{3}{4} \partial^2 \phi \right) - \frac{3}{2} \partial \psi_a \partial \phi - \frac{1}{2} \partial^2 \psi_a := \\
\frac{1}{16} \oint \frac{dz}{2i\pi} e^{-\phi} \psi_a \sim P_a
\]
The numerical factor of $\frac{1}{16}$ is unessential as it can always be absorbed by choosing an alternative normalization for $T$. Therefore we find that the anticommutator $\{T_\alpha, T_\beta\}$ reproduces the non-perturbative superalgebra (1) with the fivebrane central term proportional to $e^{-3\phi} \psi_a ... \psi_a$. At the same time, the presence of this five-form central term in the non-perturbative $D = 10$ superalgebra is required by the $M$-theory. There is also a total derivative 3-form term in (8); it is known that a threebrane in $D = 10, 11$ is not fundamental, but rather just an intersection of two fivebranes. Therefore we interpret the 3-form of (8) as an intersection term, with the derivative possibly reflecting the fact of the intersection. This intersection term, though not of a fundamental origin, may be related to monopole dynamics, in the light of the recently discovered correspondence between three-dimensional gauge theories and moduli spaces of magnetic monopoles [14].

We see that $T_\alpha$ may be interpreted as a “non-perturbative supercharge”, generating a superalgebra with branes, with the map between $T_\alpha$ and $Q_\alpha$ being essentially the transformation of $S$-duality. Before going further to explain what this map is and how it is related to the picture-changing gauge transformation, we note that, although the two-form term (corresponding to another M-brane - a membrane) is absent in the anticommutator $\{T_\alpha, T_\beta\}$, it does appear in the anticommutator of $T_\alpha$ with $Q_\beta$:

$$\{T_\alpha, Q_\beta\} = \Gamma_{\alpha\beta}^{a_1 a_2} \oint \frac{dz}{2i\pi} e^{-2\phi} \psi_{a_1} \psi_{a_2} + \frac{1}{2} \oint \frac{dz}{2i\pi} \partial e^{-2\phi} \epsilon_{\alpha\beta}$$

with the second term apparently related to a $D0$-brane. In other words, we have obtained a chain of anticommutators

$$\{Q_\alpha, Q_\beta\} \rightarrow \{T_\alpha, Q_\beta\} \rightarrow \{T_\alpha, T_\beta\}$$

where the first anticommutator is just a perturbative superalgebra without central terms, the last one represents the superalgebra with a fivebrane, and the

“cross-anticommutator” $\{T, Q\}$ contains a membrane and a $D0$-brane. In order to get a superalgebra which includes both M-branes at once, one should, of course, simply consider the supercharge being a sum of $T$ and $Q$:

$$S_\alpha = T_\alpha + Q_\alpha$$

with $T$ being a “strongly coupled” part and $Q$ a “weakly coupled”. What precisely relates $T$ and $Q$? As we have already mentioned, it is not the picture-changing. Rather, they are connected through quite a peculiar transformation, of which one may think as a singular
limit of the picture-changing transform at infinite (or zero) momentum. Namely, noticing that $T_\alpha$ and $Q_\alpha$ are zero momentum parts of some fermionic vertex operators, consider the following vertices:

$$
V(k) = \oint dz \frac{dz}{2i\pi} u^\alpha(k) e^{-\frac{i}{2} \phi \Sigma_\alpha e^{ikX}}
$$

$$
W(k, \bar{k}) = \oint dz \frac{dz}{2i\pi} v^\alpha(k, \bar{k}) e^{-\frac{i}{2} \phi \Sigma_\alpha e^{ikX}}
\quad \text{where}\quad v_\alpha(k, \bar{k}) = (\Gamma \bar{k})_{\alpha\beta} u_\beta(k)
$$

$$
(kk) = k^2 = (\bar{k}\bar{k}) = (\bar{k})^2 = 0
\quad \text{and}\quad (k\bar{k}) = 1
$$

Here $k$ is a momentum of the fermionic emission vertex in the $-\frac{1}{2}$-picture, and $\bar{k}$ is an auxiliary momentum, analogous to the one present in the dilaton vertex operator; $u_\alpha(k)$ is some constant space-time spinor, satisfying the on-shell Dirac equation. The obvious difficulty with our definition of the vertex operator $W(k, \bar{k})$ is that it is not BRST-invariant because of our choice of its polarization spinor $v_\alpha(k, \bar{k})$. In the limit $\bar{k} \to \infty$ and $k \to 0$, however, its BRST-invariance is restored and the operation of picture-changing is again well-defined. Applying the picture-changing operator now gives:

$$
limit_{\bar{k} \to \infty} : \Gamma_1 W(k, \bar{k}) := i(\Gamma \bar{k})(\Gamma k)_{\alpha\gamma} \oint dz \frac{dz}{2i\pi} e^{-\frac{i}{2} \phi \Sigma_\gamma e^{ikX}}
= 2i u_\alpha(k) \oint dz \frac{dz}{2i\pi} e^{-\frac{i}{2} \phi \Sigma_\alpha e^{ikX}} = 2i V(k)
$$

where we have used the identity $(\Gamma \bar{k})(\Gamma k) + (\Gamma k)\Gamma \bar{k} = 2(k\bar{k}) = 2$ and the on-shell condition for the spinor $u_\alpha(k)$.

Therefore the relation between $T_\alpha$ and $Q_\alpha$ is given by:

$$
N_{\alpha\beta} T_\beta = ((\Gamma \bar{k})(\Gamma k))_{\alpha\beta} Q_\beta
$$

where the S-duality generator $N_{\alpha\beta}$ is defined as

$$
N_{\alpha\beta} = \lim_{\bar{k} \to \infty} [(\Gamma \bar{k})_{\alpha\beta} : \Gamma_1 : e^{ikX}]
$$

Another role of this generator is that it “improves” the operation of picture-changing, in general not well-defined at $k = 0$. It should be emphasized that the operation $N_{\alpha\beta}$ is only defined for those zero momentum fermionic vertices belonging to the kernel of $\Gamma_1$. 

For the operators not belonging to \( ker(\Gamma_1) \), \( N_{\alpha\beta} \) should be replaced by ordinary picture-changing. Let us now analyze the central \( p \)-form terms appearing in the non-perturbative superalgebra (8). The 5-form \( e^{-3\phi}\psi_{a_1}...\psi_{a_5} \) appears to be a zero momentum part of a rather peculiar vertex operator, which, while not being BRST trivial, does not appear to describe an emission of any massless particle in perturbative string theory. Its S-matrix elements vanish among elementary string states, but are non-zero in the presence of D-branes due to the interaction of this vertex with Ramond-Ramond charges. This property prompts us to interpret this 5-form as a brane-emitting vertex (versus particle-emitting vertices in perturbative string theory). Another crucial property of this vertex operator is that it has no analogue in the picture zero (although it does have an analogue in the +1-picture), in other words its nonzero ghost number appears to be its indispensable feature - and again this is quite contrary to the properties of perturbative string vertices for which the representation in the picture of ghost number zero always exists. The above considerations lead us to introduce the following notion of ghost number cohomologies.

Let \( \{V_n\} \) be a set of physical states (vertex operators), perturbative or non-perturbative, having a ghost number \( n \leq 0 \). For \( n < 0 \), let us further define the subset \( \{\tilde{V}_n\} \subset \{V_n\} \) of the operators of ghost number \( n \) for which there exists a picture-changing transformation relating them to vertices of some ghost number \( m \), \( n < m \leq 0 \), i.e.

\[
V_m =: (\Gamma_1)^{m-n} : V_n \in \{\tilde{V}_n\}
\]

By definition, we put \( \{\tilde{V}_0\} = \emptyset \). The ghost number \( n \leq 0 \) cohomologies \( [H_n] \) are then defined as

\[
[H_n] = \frac{\{V_n\}}{\{\tilde{V}_n\}}
\]

All the states not belonging to any of \( [H_n] \) of \( n \neq 0 \) are by definition set to belong to \( H_0 \). The fivebrane central term \( \oint \frac{dz}{2i\pi} e^{-3\phi}\psi_{a_1}...\psi_{a_5} \) is then the element of \( [H_{-3}] \); the membrane \( \oint \frac{dz}{2i\pi} e^{-2\phi}\psi_{a_1}\psi_{a_2} \) of the anticommutator \( \{T_\alpha, Q_\beta\} \) belongs to \( [H_{-2}] \). However, for instance the operator \( \oint \frac{dz}{2i\pi} e^{-\phi}\psi_a \) does not belong to to \( [H_{-1}] \), as the picture-changing operator \( \Gamma_1 \) transforms it into \( \oint \frac{dz}{2i\pi} \partial X_a \) of \( [H_0] \). The cohomology \( [H_{-4}] \) contains the non-dynamical state defined by \( \oint \frac{dz}{2i\pi} e^{-4\phi}\psi_{a_1}...\psi_{a_{10}} \) which arguably may be attributed to cosmological constant. Other ghost number cohomologies seem to be redundant; our conclusion is that \([H_{-4}], [H_{-3}], [H_{-2}], [H_0]\) form a basis for the quantum states of \( M \)-theory. All the elementary string states belong to \([H_0]\), while the non-perturbative physics is hidden in the cohomologies of non-zero ghost numbers. Thus, the cohomologies \([H_{-2}], [H_{-3}]\) contain
D0-branes and M-brane states (including intersecting branes), and \([H_{-4}]\) accounts for the cosmological constant. Let us present this schematically:

\[
\begin{align*}
[H_0] & \rightarrow \text{particles} \\
[H_{-2}] & \rightarrow \text{membranes} + D0 - \text{branes} \\
[H_{-3}] & \rightarrow 5 - \text{branes} \\
[H_{-4}] & \rightarrow \text{cosm. constant}(?)
\end{align*}
\]

This classifies the M-theory quantum states in the formalism of ghost number cohomologies. Dualities are contained in maps between these cohomologies; the example of such a map is the \(N\)-operator (16), which relates elementary and non-perturbative superalgebras.

In the following section we will explore the application of this formalism to the matrix theory.

**Application to Matrix Theory**

The action of the matrix model is given by the \(N \times N\) matrix [6]:

\[
\frac{1}{2g} \int dt ((D_0 X^a)^2 + \theta^\alpha D_0 \theta^\alpha + \frac{R}{4} [X^a, X^b]^2 + \frac{i R}{2} [\theta^\beta, [X^a, \theta^\alpha]] \Gamma^a_{\alpha\beta})
\]

where \(D_0 = \partial_0 - i[A_0,]\) and \(\theta^\alpha (\alpha = 1,...,16), X^a, a = 1,...,10\) and \(A_0\) are hermitian \(N \times N\) matrices. The dynamical SUSY transformations are given by:

\[
\begin{align*}
\delta X^a &= -2 \epsilon^\alpha \theta^\beta \Gamma^a_{\alpha\beta}, \\
\delta \theta^\alpha &= \frac{1}{2} (D_0 X_a \Gamma^a_{\alpha\beta} \epsilon^\beta + [X_a, X_b] \Gamma^a_{\alpha\beta} \epsilon^\beta) \\
\delta A_0 &= -2 \epsilon^\alpha \theta^\alpha
\end{align*}
\]

and the trivial kinematic SUSY is given by

\[
\delta \theta^\alpha = \epsilon^\alpha, \delta X^a = \delta A_0 = 0
\]

Now, let us reduce the Lagrangian (20) to \(D = 0\), i.e. to a zero-dimensional \(N \times N\) matrix model. Such a reduction implies choosing the gauge \(A_0 = 0\) and dropping the kinetic terms in (20). The dimensionally reduced Lagrangian is given by

\[
\frac{1}{2g} tr(\sum_{a<b} ([X_a, X_b])^2 - \sum_a [\theta^\alpha \Gamma^a_{\alpha\beta} [\theta^\beta, X_a]])
\]
The superalgebra corresponding to the supersymmetry transformations (21), (22) contains the following 2-form central charge:

\[ Z_{ab} = \frac{i}{2} Tr[X_a, X_b] \] (24)

The energy of the membrane state is proportional to the square of the membrane charge:

\[ E \sim (Z_{ab})^2 = -\frac{1}{4} (Tr[X_a, X_b])^2 \] (25)

In the matrix-superstring relation this expression for the membrane energy should correspond to the stress-energy tensor in superstring theory (here the appropriate dimensional reduction from \( D = 11 \) to \( D = 10 \) is implied, of course, to produce a string out of a membrane). To construct a superstring stress-energy tensor from (25) we therefore need a “glossary” which relates the matrix variables of (20), (23) to the superstring variables in the NSR formalism, under the reduction to \( D = 10 \). So what is the counterpart of the commutator \([X_a, X_b]\) in the strongly coupled superstring theory? Since \([X_a, X_b]\) is the membrane topological charge in matrix theory, it corresponds, in the formalism of ghost number cohomologies, to the membrane of \([H_{-2}]\), i.e. we require

\[ [X_a, X_b] \rightarrow e^{-2\phi} \psi_a \psi_b \] (26)

The drawback of such an ansatz is that it does not seem to preserve the identity satisfied by the matrices \( X_a \) due to the BPS property:

\[ [X_a, X_b] = \frac{i}{2} \epsilon_{abcd} [X_c, X_d] \] (27)

Nevertheless, while the right-hand side of (26) does not satisfy the BPS identity (27) in a straightforward way, we will see later that in superstring theory the analogue of (27) does hold, corresponding to the stress-energy tensor vanishing (which is equivalent, in turn, to the worldsheet reparametrizational invariance condition). The Sugawara stress-energy tensor corresponding to (25) under such a matrix-superstring correspondence is then given by:

\[ T^{(-4)}(z) = e^{-2\phi} \psi_a \psi_b e^{-2\phi} \psi_a \psi_b : (z) = \]

\[ e^{-4\phi} \left\{ \left( \frac{1}{2} \psi_a \partial^2 \psi_a \psi_b \partial^2 \psi_b + \frac{1}{3} \psi_a \partial^3 \psi_a \psi_b \partial \psi_b \right) - \frac{1}{15} \psi \partial^5 \psi + \frac{4}{45} P^{(6)}(-2\phi) \right\} \]

\[ + \left( \psi_a \partial \psi_a \psi_b \partial \psi_b \right) (\partial^2 \phi - 2 \partial \phi \partial \phi) + \left( \psi_a \partial^2 \psi_a \psi_b \partial \psi_b \partial \phi - \frac{2}{3} \psi \partial^4 \psi \right) \] (28)
where \((-4)\) stands for the total ghost number \(-4\) of this tensor.

At first glance this expression does not seem at all to resemble the NSR stress-energy tensor. To understand its relevance to superstring theory, it is crucial to point out the role played by the fermionic part of the matrix theory Lagrangian (23), which is given by the double commutator \(\Gamma^a_{\alpha\beta}[\theta^\alpha, [\theta^\beta, X_a]]\). It has been shown [15] that the integration over fermionic variables \(\theta^\alpha\) in the partition function with the static part of the action (3) yields the determinant \(Pfaff(\frac{i}{2g} f_{ijk} \sum_a \gamma^a_{\alpha\beta} X^k_a)\) This Pfaffian has been shown to identify the Nicolai map [16, 17] for the bosonic potential \(Tr[X_a, X_b]\). That is, the Nicolai map: \(W^c = \gamma^c_{k\dot{k}} [X_k, X_{\dot{k}}]\) defines the new variable \(W^c\) in terms of the membrane charge \([X, X]\), and the Jacobian of this map precisely cancels the Pfaffian coming out of the integration over fermions. The partition function then reduces to the finite-dimensional Gaussian integral over \(W^c\). Next, we observe the following important connection between Nicolai map and picture-changing gauge transformation. Namely, consider the gauge-fixed \(D = 10\) supersymmetric fivebrane Lagrangian in a light-cone gauge: [10]

\[
I = \frac{T_5}{2} DX^a DX^a - det(\partial_r X^a \partial_s X^a) + i \bar{\theta} D\theta \\
+ \frac{i}{4} \epsilon^{rstuv} \partial_r X^{a_1} \partial_s X^{a_2} \partial_t X^{a_3} \partial_u X^{a_4} \partial_v X^{a_5} \bar{\theta} \gamma_{a_1 a_2 a_3 a_4} \theta 
\]

For this fivebrane action the equilibrating Nicolai map is given by [18]:

\[
|\eta_a| = \frac{dX_a}{d\tau} + \frac{1}{5!} \epsilon^{rstuv} \epsilon_{a a_1 \ldots a_5} \partial_r X^{a_1} \ldots \partial_v X^{a_5} 
\]

The quintic-like Wess-Zumino term in the fivebrane action (29) is understood to give rise to the five-form central term in the superalgebra (1), with \(p = 5\). After having performed the Nicolai transform (30) we obtain an essentially quadratic action in terms of new variables \(\eta\), without the Wess-Zumino term - and accordingly, the superalgebra in the Nicolai-transformed theory should no longer include the five-form central term. In other words, the Nicolai transform maps the superalgebra with the central 5-form term to the one without central terms - but this is exactly what the \(N\)-operator (16) also does. This motivates our conjecture that the string-theoretic counterpart of the fermionic term in (23) must be the picture-changing operator: \(\Gamma_1 := e^{\phi}(S_{\text{matter}} + S_{\text{ghost}}): \) of conformal dimension zero. Beside that, the following heuristic argument may be given. Suppose that the counterpart of the fermionic matrix variable \(\theta^\alpha\) is the Green-Schwarz fermion \(\theta^\alpha(z)\). Then it is related to the NSR variables through

\[
\theta^\alpha = e^{\frac{\phi}{2}} \Sigma^\alpha + \text{ghosts} 
\]
where $\Sigma^\alpha$ is a spin operator for matter fields. Then, the GSO projected string-theoretic counterpart of the fermionic term, having conformal dimension zero is

$$\Gamma^{a}_{\alpha\beta} [\theta^\alpha, [\theta^\beta, X_a]] \rightarrow \Gamma^{a}_{\alpha\beta} : e^{\frac{c}{2} \Sigma^\alpha(z)} e^{\frac{c}{2} \Sigma^\beta(w)} X_a : = e^c Tr (\Gamma^a \Gamma^m) \psi_m \partial X_a$$

$$= 8 e^c \psi_a \partial X_a + \text{ghosts} \sim \Gamma_1$$

(32)

i.e. it is proportional to the picture-changing operator $\Gamma_1$. The superstring partition function corresponding to the reduced matrix theory action (23) is then given by

$$Z = \int D[X]D[X]D[ghosts] e^{\Gamma_1}e^{-S^{(-4)}}$$

(33)

where $S^{(-4)}$ is the action corresponding to the Sugawara tensor $T^{(-4)}$ of (28). Then, the partition function may be written as:

$$Z = \int D[X]D[X]D[ghosts] \left\{ \sum_m \left[ (\Gamma_1)^m \right] \sum_n \frac{(-S^{(-4)})^n}{n!} \right\}$$

$$= \int D[X]D[X]D[ghosts] \sum_n \frac{(-1)^n (\Gamma_1)^4 S^{(-4)})^n}{n!}$$

(34)

$$= \int D[X]D[X]D[ghosts] e^{-S^{(0)}}$$

where $S^{(0)} = (\Gamma_1)^4 S^{(-4)}$: due to ghost number conservation. It is easy to check now that the stress-energy tensor $T^{(0)}$ corresponding to the “effective” action $S^{(0)}$ is given by

$$T^{(0)} = (\Gamma_1)^4 T^{(-4)} :$$

(35)

Indeed, since $T_{ik} = \frac{\delta S}{\delta \gamma_{ik}}$, (35) simply follows from the fact that variation of the picture-changing operator with respect to the worldsheet metric vanishes. Applying the picture-changing operator $\Gamma_1$ four times to $T^{(-4)}$ we obtain

$$T^{(0)} = (\Gamma_1)^4 T^{(-4)} := \frac{1}{2} (\partial X^m \partial X_m + \partial \psi^m \psi_m + \partial \sigma \partial \sigma + 3 \partial^2 \sigma$$

$$- \partial \phi \partial \phi - 2 \partial^2 \phi + \partial \chi \partial \chi)$$

(36)

where $\sigma$ is bosonized fermionic ghost: $c = e^\sigma, b = e^{-\sigma}$ Thus $T^{(0)}$ is exactly the expression for the full matter+ghost stress-energy tensor of the NSR superstring theory in ten dimensions and therefore $S^{(0)}$ is the NSR superstring in the superconformal gauge. We see that the supersymmetry in the matrix theory and the worldsheet supersymmetry in the NSR superstring theory appear on essentially different grounds. That is, naively one may expect
the supersymmetries of the matrix theory (21),(22) to translate into the worldsheet supersymmetry of the NSR theory in this matrix-superstring correspondence through ghost number cohomologies. However, it appears that the entire worldsheet supersymmetry of superstring theory is contained in just the bosonic part of the matrix theory Lagrangian, given by the term $\sim ([X_a, X_b])^2$ as this term accounts for the Sugawara tensor (28), of which the NSR stress-energy tensor (36) is obtained by the four-fold application of the picture-changing operator $\Gamma_1$. Now, the fermionic part of the matrix theory action (23) is exactly the one which gives us the picture-changing transformation we need. Let us now address the question of compatibility of the matrix theory BPS condition (27) and the ansatz (26) dictated by ghost number cohomology arguments. At first glance, there seems to be a disagreement between (26) and (27), but this disagreement may be resolved due to what we find to be an intriguing fact in the matrix-superstring correspondence. Let us multibly both the left-hand and the right-hand sides of (27) by $[X_a, X_b]$, substitute the ansatz (26) and apply the four-fold picture-changing ($\Gamma_1)^4$ to both the l.h.s. and the r.h.s. of the identity obtained. Then, as we have already shown, the left-hand side becomes the worldsheet NSR stress-energy tensor $T(z)$. As to the right-hand side, due to the ansatz (26) it becomes:

$$-\frac{1}{4} \epsilon_{abcd}[X_a, X_b][X_c, X_d] = -\frac{1}{4} \epsilon_{abcd}e^{-4\phi}\{\partial^4(\psi_a \psi_b)\psi_c \psi_d - \frac{1}{3} \partial \phi \partial^3(\psi_a \psi_b)\psi_c \psi_d$$

$$+ (\partial \phi \partial \phi - \frac{1}{2} \partial^2 \phi) \partial^2(\psi_a \psi_b)\psi_c \psi_d - \frac{4}{3} \partial \phi \partial \phi \partial \phi - \frac{4}{3} \partial^2 \phi \partial \phi + \frac{1}{3} \partial^3 \phi) \partial(\psi_a \psi_b)\psi_c \psi_d$$

$$+ \frac{1}{24}(-2\partial^4 \phi + 8(\partial^2 \phi \partial^2 \phi + \partial^3 \phi \partial \phi) - 24\partial^2 \phi \partial \phi \partial \phi + 16\partial \phi \partial \phi \partial \phi \partial \phi)\psi_a \psi_b \psi_c \psi_d\}$$

Making the operator : ($\Gamma_1)^4 :$ to the right-hand side of (37) one finds that the picture zero counterpart of (37) vanishes. Therefore, we find that the BPS relation (27) in the matrix theory translates into the condition $T = 0$ in the superstring theory, i.e. the condition of reparametrizational invariance on the worldsheet. This result points at the connection between the unbroken supersymmetries of the matrix model and worldsheet gauge symmetries in superstring theory.

**Conclusion**

In this paper, only those $p$-form fields of $n \neq 0$ ghost number cohomologies corresponding to ground states of branes have been considered. The next logical step would be to extend this analysis to involve other p-brane modes of $[H_n] n \neq 0$. The method of ghost number cohomologies appears to be helpful in analyzing the non-perturbative
spectrum of superstring theory and M-theory. The non-perturbative physics is hidden in cohomologies of non-zero negative numbers, while $[H_0]$ contains the elementary states of superstring theory. The conformal field theory may be used to analyze the scattering amplitudes of branes. Applying this method to the matrix theory emphasizes the connection between matrix M-theory and superstrings. Some novel features arise - such as the worldsheet reparametrizational invariance corresponding to the matrix theory BPS condition and the supersymmetry of the dimensionally reduced matrix model being the analogue of picture-changing in superstring theory. In general, dualities may be understood as maps between ghost number cohomologies. We hope that studying the structure of ghost number cohomologies may prove helpful to understand the interplay between M-theory and non-commutative geometry, suggested originally in [6]. We hope to elaborate on that in our future works. Very roughly, one may consider ghost number cohomologies as foliated spaces with picture-changing and $N$-operators playing the role of foliations. P-branes then may be interpreted as leaves of integrable foliations. Given the string-theoretic origin of the $p$-form terms in (1) the proper question is where the eleventh dimension comes from. Recently the discussion in [13] has pointed at the role that the twistor-like superstring variable plays in “building the bridge” to $D = 11$. If the cohomologies of higher ghost numbers are to adequately describe the non-perturbative physics of M-theory, and the fivebrane dynamics in particular, their structure shall somehow involve the 2-form non-Lagrangian field propagating on the fivebrane worldvolume, which partition function has been determined in [20]. At present, finding such a correspondence is an open question.

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