GUT Scale Fermion Mass Ratios

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Abstract. We present a series of recent works related to group theoretical factors from GUT symmetry breaking which lead to predictions for the ratios of quark and lepton Yukawa couplings at the unification scale. New predictions for the GUT scale ratios $y_\mu/y_s$, $y_\tau/y_b$ and $y_t/y_b$ in particular are shown and compared to experimental data. For this comparison it is important to include possibly large supersymmetric threshold corrections. Due to this reason the structure of the fermion masses at the GUT scale depends on TeV scale physics and makes GUT scale physics testable at the LHC. We also discuss how this new predictions might lead to predictions for mixing angles by discussing the example of the recently measured last missing leptonic mixing angle $\theta_{13}$ making this new class of GUT models also testable in neutrino experiments.

1. Introduction
The Standard Model of particle physics (SM) has been tremendously successful in describing the available data about the fundamental building blocks of nature. To achieve this only nineteen parameters are needed but thirteen of them are related to the flavour sector. Adding neutrino masses to the SM introduces on top of that at least seven additional parameters. Such a large number of parameters seems not quite appropriate for the fundamental theory of nature. Furthermore, these parameters exhibit an unexpected pattern. The masses are strongly hierarchical and the mixing angles in the quark sector are small, whereas two of the three mixing angles in the lepton sector are large.

There are various attempts to reduce the number of parameters in the flavour sector. One very popular attempt is to introduce additional symmetries like flavour symmetries. We will focus here on models of supersymmetric Grand Unified Theories (SUSY GUTs), where some of the Yukawa couplings can be related to each other due to the underlying group structure. This is in fact not the first and biggest motivation for them. SUSY GUTs follow the old dream of a unified theory of everything. They contain candidates for dark matter and they can explain in a natural way the discretizing of electric charge and the light Higgs mass.

2. Fermion Mass Ratios
The main focus here lies on the flavour sector of SUSY GUTs. We will start our discussion with some theoretical considerations where we will discuss large low energy corrections to the fermion masses which have a significant impact on the fermion masses at the GUT scale. Then we will discuss how SUSY GUTs do predict certain ratios for the fermion mass ratios and present predictions from SU(5) and Pati-Salam. We will conclude this section with a brief description on how this might be related to the recently measured neutrino mixing angle $\theta_{13}$. 

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2.1. Theoretical Considerations

Before we go into details we want to remark that we treat the running fermion masses as the product of the running Yukawa coupling times the low energy Higgs vev. Therefore there is a linear relation between fermion masses and Yukawa couplings and we will use the two terms interchangeably. Especially, if we take ratios of masses the Higgs vev drops out.

SUSY threshold corrections [1] can have a large impact on the high scale Yukawa couplings. After integrating out the SUSY particles at the SUSY scale $M_{\text{SUSY}}$ one has to match the Yukawa couplings of the SM with the Yukawa couplings of the minimal supersymmetric standard model (MSSM). This is shown diagrammatically in Figure 1 and gives the relation

\[ y^\text{MSSM}_i = \frac{y^\text{SM}_i}{\cos \beta (1 + \epsilon_i \tan \beta)} \]  

(1)

where $i$ stands for $d$, $s$, $b$, $e$, $\mu$ and $\tau$ and $\epsilon_i$ is a one loop factor. Explicit expressions for $\epsilon_i$ can be found in [2], where the importance of SUSY threshold corrections for the GUT scale Yukawa coupling ratios has been recently pointed out again. Since the one-loop factors $\epsilon_i$ appear with the possible large $\tan \beta$ the corrections can be quite significant, easily up to 30% and depending on the SUSY parameters they can appear with both signs.

We will present some results for the GUT scale fermion mass ratios in the next section. For the rest of this section we will discuss which predictions can be obtained within GUTs. On the renormalizable level in $SU(5)$ one can get only $(Y_e)_{ij}/(Y_d)_{ij} = 1$ [3] or $(Y_e)_{ij}/(Y_d)_{ij} = -3$ [4], where $(Y_{d/e})_{ij}$ stands for the $(i,j)$-entry of the down-type quark/charged lepton Yukawa matrix. The same ratios with further relations to the up-quark sector are obtained in Pati-Salam (PS) models [5].

This options are very limited and in a Froggat-Nielsen approach [6] one would expect the smaller Yukawa couplings to be generated by higher-dimensional effective operators. Within this higher-dimensional operators one can add non-singlet fields which receive a higher vev. Below this symmetry breaking scale one could then end up with new predictions for Yukawa coupling ratios.

We studied this systematically for dimension five operators in $SU(5)$ and PS [7, 8]. The considerations are based on the diagram in Figure 2. The fields $A$ and $C$ contain the MSSM matter fields and either $B_1$ or $B_2$ contains the MSSM Higgs fields. The other $B$ field is a field...
Figure 2. Diagrammatic representation of the operators giving Yukawa coupling ratios at the GUT scale. A and C are matter fields while $B_1$ and $B_2$ are Higgs fields. $\langle \Lambda \rangle$ represents the messenger mass term. If $\Lambda$ is a total singlet one could directly write down a mass like in [7]. Otherwise $\Lambda$ should transform as an adjoint of the GUT symmetry splitting the masses of the components of the messenger fields.

Table 1. Summary of possible SU(5) predictions for the GUT scale Yukawa coupling ratios $(Y_e)_{ij}/(Y_d)_{ij}$ taken from [8].

| Operator Dimension | $(Y_e)_{ij}/(Y_d)_{ij}$ |
|--------------------|--------------------------|
| 4                  | 1                        |
|                    | -3                       |
| 5                  | 1/6                      |
|                    | -1/2                     |
|                    | -2/3                     |
|                    | 1                        |
|                    | $\pm 3/2$                |
|                    | -3                       |
|                    | 9/2                      |
|                    | 6                        |
|                    | -18                      |

Table 2. Summary of possible PS predictions for the GUT scale Yukawa coupling ratios $((Y_e)_{ij}/(Y_d)_{ij}, (Y_u)_{ij}/(Y_d)_{ij}, (Y_e)_{ij}/(Y_u)_{ij})$ taken from [8].

| Operator Dimension | $((Y_e)_{ij}/(Y_d)_{ij}, (Y_u)_{ij}/(Y_d)_{ij})$ |
|--------------------|--------------------------------------------------|
| 4                  | $(1, \mp 1)$                                     |
|                    | $(-3, \mp 1)$                                    |
| 5                  | $(0, \pm 1)$                                     |
|                    | $(-1/3, \pm 1)$                                  |
|                    | $(1, \pm 1)$                                     |
|                    | $(3/2, \pm 1)$                                   |
|                    | $(-3, \pm 1)$                                    |
|                    | $(9, \pm 1)$                                     |
getting a heavy vev. In the first publication [7] we assumed the messenger fields $X$ and $\bar{X}$ to have a direct mass term while in the second publication [8] we assumed the mass term to be generated dynamically, possibly from a non-singlet field offering even more options. The results are collected in Tables 1 and 2.

In the next section we will compare the theoretical calculations and possibilities with existing data.

2.2. Comparison to Data

In [7] we implemented a RGE running of the fermion masses to the GUT scale including the SUSY threshold corrections. For the SUSY breaking parameters we assumed three popular simplified scenarios, minimal anomaly mediated SUSY breaking (mAMSB), minimal gauge mediated SUSY breaking (mGMSB) and the constrained MSSM (CMSSM). Furthermore, we considered certain experimental constraints which were up to date at that time, for a detailed discussion of these constraints and the discussion of an additional dark matter constraint, see the original paper [7] or the author’s PhD thesis [9].

The first thing to notice about our results summarized in Figure 3 is, that the common GUT assumptions of the GJ relation, $m_\mu/m_s=3$ and complete third family Yukawa coupling unification, $m_t=m_b=m_\tau$, are somewhat challenged in our setup. Whereas the GJ relation is possible only for the mAMSB scenario for $\tan \beta \sim 30$, complete third family Yukawa coupling unification is incompatible with our results in all three scenarios. In the mAMSB case, where the corrections point in the right direction to achieve this relation without fine tuning, unification cannot be achieved because of non successful electroweak symmetry breaking and tachyons in the spectrum.

Nevertheless, we find valid alternatives to the standard assumptions. For the second generation we find the SU(5) relations $y_\mu/y_s=9/2$ for $\tan \beta \sim 20$ and $y_\mu/y_s=6$ for $\tan \beta \sim 50$ (not for mAMSB). The first of these is even valid for all three SUSY breaking scenarios (and smaller $\tan \beta$), which is simply due to the fact, that it emerges when the threshold corrections are small and therefore the dependence on the spectrum is weak.

For the third generation the SU(5) and PS relation $y_\tau/y_b=1$ can be realized in mAMSB for $\tan \beta \sim 30$. The SU(5) relation $y_\tau/y_b=3/2$ appears in mGMSB for $\tan \beta \sim 30-45$ and in CMSSM for $\tan \beta \sim 20-50$. There are also alternatives in PS models, namely the relations $2y_t=2y_b=y_\tau$ for $\tan \beta \sim 60$ in mGMSB and CMSSM and $y_t=2y_b=2y_\tau$ for $\tan \beta \sim 30$ in mAMSB. These relations are however generated from dimension six operators and have been obtained in [11], where a slightly different setup has been used.

Nevertheless, the discovery of a light Higgs boson [12] had a huge impact on the allowed MSSM parameter space. And the question for the naturalness of the Higgs mass in the MSSM became more pressing. In this context we studied the naturalness of the MSSM in a pMSSM like parametrization of the parameter space [13].

It turned out that the most effective way to reduce the amount of fine-tuning is to allow for non-universal gauging masses at the GUT scale. In Figure 4 we compare possible GUT scale ratios $y_\tau/y_b$ and $y_t/y_b$ from a parameter scan before and after applying the Higgs mass constraint. The color coding gives the amount of fine-tuning. Without the Higgs mass constraint very low fine-tuning of order one is still allowed while after applying the constraint the fine-tuning is at least of order ten.

From a fine-tuning point of view one can also see that there is no clear preference for $b-\tau$ Yukawa coupling unification or $y_\tau/y_b=3/2$. But it is very obvious how strong the Higgs mass constraint is. This shows again how important the LHC results are even for the possible physics at very large scales like the GUT scale.

1 In our study we have focused on $\tan \beta$ between 20 and 60. We note that there can also be $b-\tau$-unification of Yukawa couplings for $\tan \beta \sim 1$ (see e. g. [10]).
Figure 3. Results from a phenomenological scan for the mAMSB, mGMSB and CMSSM scenarios taken from [7]. The (red) black points are the (excluded) allowed points after applying experimental constraints. The grey regions indicate the experimental quark mass errors. The green lines are predictions from SU(5), the dashed lines from SU(5) and PS and the (light) blue points from PS (dimension-six operators). The yellow squads are the GUT scale Yukawa coupling ratios without including SUSY threshold corrections for tan β = 20, 30, 40, 50 and 60 from top to bottom.
Figure 4. Lowest fine-tuning in the $y_t/y_b$-$y_{\tau}/y_b$ plane consistent with the experimental bounds as described in [13]. The Higgs mass constraint $m_h = 125.3 \pm 0.6$ GeV [12] and a theoretical uncertainty of $\pm 3$ GeV [14] is (not) included in the (left) right plot. The $1\sigma$ errors on the quark masses [15] are taken into account by scaling the data points in the last plot correspondingly.

2.3. Connection to $\theta_{13}$

In GUTs cannot only the fermion masses be related to each other but also relations between mixing angles are quite common. In this section we will discuss how such a connection can arise in SU(5) and its phenomenological implications based on [16].

In SU(5) the down-type quark and charged lepton matrix are identical up to transposition and order one Clebsch-Gordan (CG) coefficients, especially for the first two generations we find

$$\hat{\lambda}^D_{[12]} = \begin{pmatrix} a & b' \\ b & c \end{pmatrix} \quad \text{and} \quad \hat{\lambda}^E_{[12]} = \begin{pmatrix} \alpha a & \beta b' \\ \beta b & \gamma c \end{pmatrix}. \quad (2)$$

For the CG coefficients we can use the new relations from Table 1. Then one has to fit the four parameters $a$, $b$, $b'$ and $c$ to the masses of the down-type quarks and leptons of the first two generations. Furthermore, under the assumption that the up-quark mass matrix is very hierarchical the Cabibbo angle is dominated by the mixing in the down-quark sector so that we can also use this information.

Usually, the system is then (over)determined for every choice of CG coefficients and we can predict the value of the 1-2 mixing angle in the lepton sector, $\theta^e_{12}$. And now if the 1-3 mixing in the neutrino sector itself vanishes - which happens in certain well-motivated models - we can use the approximate relation for the physical $\theta_{13}$ mixing angle

$$\sin \theta_{13} = |U_{e3}| = \frac{\sin \theta_{12}^e \tan \theta_{23}}{\sqrt{1 + \tan^2 \theta_{23} - \sin^2 \theta_{12}^e}} \approx \sin \theta_{12}^e \sin \theta_{23}, \quad (3)$$

which appears in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. In Figure 5 we show a histogram with the results of our scan over the possible combinations of CG coefficients. Indeed, we found several realistic combinations. And in [18] we have given an explicit SU(5)$\times$T$'$ flavour model with $\alpha = -3/2$, $\beta = 1$ and $\beta' = \gamma = 6$.

Finally, if the neutrino mixing matrix coincides with the bi-maximal (BM) [19] mixing matrix one has:

$$\sin^2 \theta_{12} \simeq \frac{1}{2} + \cos \delta \, \sin \theta_{13}, \quad (4)$$
Figure 5. Graphical presentation of the results obtained. The vertical straight (dashed) lines denote the 1σ (2σ) allowed ranges of \( \sin \theta_{13} \) taken from [17]. The colors correspond to certain simplifying assumptions, for more details, see [16].

and in the case of the tri-bimaximal mixing (TBM) form [20]:

\[
\sin^2 \theta_{12} \simeq \frac{1}{3} + \frac{2\sqrt{2}}{3} \cos \delta \sin \theta_{13} .
\]

Since \( \theta_{12} \) and \( \theta_{13} \) are experimentally determined we get a constraint on the Dirac CP violating phase \( \delta \) in the lepton sector.

We have displayed this constraint in Figure 6 in terms of \( \cos \delta \) and the Jarlskog-Determinant \( J_{\text{CP}} \). While the TBM case prefers large CP violation, \( \cos \delta \) close to zero, the BM case prefers a small amount of CP violation, \( \cos \delta \approx -1 \). Therefore a precise measurement of \( \delta \) can distinguish the two cases or exclude them altogether.

3. Summary and Conclusions

We have presented here a new class of GUT models where fermion mass ratios are predicted not from renormalizable, but non-renormalizable operators. This is natural because it can explain at the same time the smallness of most of the Yukawa couplings.

We have seen that these new predictions in very good agreement with current data and in fact a large portion of the MSSM parameter space points towards these new ratios instead of the conventional Yukawa unification or Georgi-Jarlskog relation.

One can read this ratios also from both directions. In a bottom-up approach it points towards the right fundamental theory which might involve the new fermion mass ratios. In a top-down approach the high-scale theory puts constraints on the low energy SUSY spectrum to be in agreement with the data.

Since we have entered now the LHC data and discovered a light Higgs boson the experimental constraints on the allowed parameter space become stronger and stronger so that we can hope to have soon a very concrete idea of the right fermion mass ratios at the GUT scale. Even more so, when we really discover SUSY at the LHC.

Beyond that we have also demonstrated how these ratios can have an impact on neutrino physics. While in SU(5) models employing the GJ relations \( \theta_{13} \) often turns out to be to too small by a factor of three we can get the right \( \theta_{13} \) by employing certain combinations of the new CG coefficients.
Figure 6. The cosine of the Dirac CP phase $\delta$ (upper panels) and the rephasing invariant $J_{CP}$ (lower panels) as a function of $\sin \theta_{13}$ in the cases of tri-bimaximal (left panels) and bimaximal mixing (right panels) arising from the diagonalisation of the neutrino mass matrix taken from [16]. The green, yellow, orange regions correspond to the 1, 2, 3 $\sigma$ allowed ranges of $\sin^2 \theta_{12}$. The vertical straight, dashed, dotted lines denote the 1, 2, 3 $\sigma$ allowed ranges of $\sin \theta_{13}$. The values of $\sin^2 \theta_{12}$ and $\sin \theta_{13}$ are taken from [17].

Therefore we live in exciting times. Since the LHC started operating and since neutrino physics has entered the precision era we have made already huge progress in understanding the fundamental structure of nature. And in the near future we will learn even more from all the experimental efforts going on, like for instance the measurement of CP violation in the lepton sector.

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