Neutrino Induced Charged Current $1\pi^+$ Production At Intermediate Energies

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The charged current one pion production induced by $\nu_\mu$ from nucleons and nuclei like $^{12}\text{C}$ and $^{16}\text{O}$ nuclei has been studied. The calculations have been done for the incoherent and the coherent processes from nuclear targets assuming the $\Delta$ dominance model and take into account the effect of Pauli blocking, Fermi motion of the nucleon and renormalization of $\Delta$ properties in a nuclear medium. The effect of final state interactions of pions has been taken into account. The theoretical uncertainty in the total cross sections due to various parameterizations of the weak transition form factors used in literature has been studied. The numerical results for the total cross sections are compared with the recent preliminary results from the MiniBooNE collaboration on $^{12}\text{C}$ and could be useful in analyzing future data from the K2K collaboration.

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\section{I. INTRODUCTION}

The study of neutrino induced pion production from nucleons and nuclei has a long history starting with the neutrino experiments performed at CERN \cite{2} and Serpukhov \cite{2} with the bubble chambers filled with heavy liquid like propane and freon. However in the intermediate energy region of 1-3 GeV, most of the data have been obtained from the later experiments performed at ANL \cite{3} and BNL \cite{4} with hydrogen and deuterium filled bubble chambers. Theoretically, the weak production of pions induced by neutrinos from the free nucleons have been studied by many authors \cite{5}-\cite{14} using various approaches like multipole analysis, effective Lagrangian and Quark model. Recent interest in the study of these processes has been generated by the ongoing neutrino oscillation experiments being performed at the intermediate neutrino energies by the MiniBooNE and the K2K collaborations using $^{12}\text{C}$ and $^{16}\text{O}$ as the nuclear targets in the detector \cite{15}, \cite{17}. Furthermore, many high precision neutrino experiments in the intermediate energy region of 1-3 GeV using neutrino beams from neutrino factories, superbeams and $\beta$-beams have been recently proposed \cite{18}, \cite{21}. These experiments are planned to be performed with the nuclear targets like $^{12}\text{C}$, $^{16}\text{O}$, $^{40}\text{Ar}$, $^{56}\text{Fe}$, etc. In order to analyze these neutrino oscillation experiments, a study of neutrino induced pion production from nuclei is very important. It is, therefore, desired that various nuclear effects in the weak pion production processes induced by neutrinos be studied in the energy region of these experiments. There exist some calculations in the past where these studies have been made \cite{22}-\cite{28} which are relevant for neutrino oscillation experiments with atmospheric neutrinos. In view of the recent data on some weak pion production processes already available \cite{10} and new data to be expected soon from MiniBooNE and K2K collaborations, the subject has attracted much attention and many calculations have been made for these processes \cite{24}, \cite{25}.

In the energy region of low and intermediate neutrino energies, the dominant mechanism of single pion production from the nucleus arises through the excitation of a baryon resonance which then decays into a nucleon and a pion. In a nucleus, the target nucleus can stay in the ground state leading to the coherent production of pions or can be excited and/or broken up leading to the incoherent production of pions. The excitation of the $\Delta$ resonance is the dominant resonance excitation at these energies contributing to one pion production and many authors have used the delta dominance model to calculate the one pion production. However, neutrino generators like NUANCE and NEUGEN which are used to model low energy neutrino nucleus interactions to analyze the neutrino oscillation experiments include higher resonance states as well \cite{31}, \cite{32}. However, these generators do not include any nuclear effects in their resonance production model for the single pion production and take into account the pion absorption effects in some adhoc way \cite{34}. These nuclear effects are quite important in the energy region of 1 GeV, corresponding to K2K and MiniBooNE experiments and should be included in the numerical codes of various neutrino generators.

In this paper, we have studied the neutrino induced charged current incoherent and coherent single pion production from $^{12}\text{C}$ and $^{16}\text{O}$ at intermediate energies relevant for the MiniBooNE and the K2K experiments using the delta dominance model developed by Oset and his collaborators \cite{35}. In section-II, we describe the formalism for single $\pi^+$ production from the nucleons in the $\Delta$ dominance model and describe the nuclear medium and the final state interaction effects in section-III. In section-IV, we present and discuss the numerical results for the total cross section for $\pi^+$ production and their $Q^2$ distribution and compare them with the preliminary results available from the MiniBooNE experiment \cite{10}. In section-V, we provide a summary and conclusion of our

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work.

II. WEAK PION PRODUCTION FROM NUCLEONS

In the intermediate energy region of about 1 GeV the neutrino induced pion production from nucleon is dominated by the \( \Delta \) excitation in which a \( \Delta \) resonance is excited which subsequently decays into a pion and a nucleon through the following reactions:

\[
\nu_{\mu}(k) + p(p) \rightarrow \mu^{-}(k') + \Delta^{++}(p')
\]

(1)

\[
\nu_{\mu}(k) + n(p) \rightarrow \mu^{-}(k') + \Delta^{+}(p')
\]

(2)

In this model of the \( \Delta \) dominance the neutrino induced charged current one pion production is calculated using the Lagrangian in the standard model of electroweak interactions given by

\[
L = \frac{G_F}{\sqrt{2}} l_\mu(x) J^\mu(x) + h.c., \text{ where}
\]

(3)

\( l_\mu(x) = \bar{\psi}(k') \gamma_\mu (1 - \gamma_5) \psi(k) \) and \( J^\mu(x) = \cos \theta_c (V^\mu(x) + A^\mu(x)) \), \( \theta_c \) being the Cabibbo angle.

The matrix element of the vector current \( V^\mu \) and the axial vector current \( A^\mu \) of the hadronic current \( J^\mu \) for the \( \Delta \) excitation from proton target is written as:

\[
< \Delta^{++} | V^\mu | p > = \sqrt{3} \bar{\psi}_\alpha(p') \left( \frac{C_4^V(q^2)}{M} (g^{\alpha \mu} \hat{q} - q^\alpha \gamma^\mu) ight.
\]

\[
+ \frac{C_4^V(q^2)}{M^2} (g^{\alpha \mu} q \cdot p' - q^\alpha p'^\mu) 
\]

\[
+ \frac{C_5^V(q^2)}{M^2} (g^{\alpha \mu} q \cdot p - q^\alpha p^\mu) 
\]

\[
+ \frac{C_6^V(q^2)}{M^2} (g^{\alpha \mu} q^\mu) \gamma_5 u(p)
\]

(4)

and

\[
< \Delta^{++} | A^\mu | p > = \sqrt{3} \bar{\psi}_\alpha(p') \left( \frac{C_4^A(q^2)}{M} (g^{\alpha \mu} \hat{q} - q^\alpha \gamma^\mu) ight.
\]

\[
+ \frac{C_4^A(q^2)}{M^2} (g^{\alpha \mu} q \cdot p' - q^\alpha p'^\mu) 
\]

\[
+ \frac{C_5^A(q^2) g^{\alpha \mu} + C_6^A(q^2) q^\mu}{M^2} \gamma_5 u(p)
\]

(5)

A similar expression is used for the \( \Delta^+ \) excitation from the neutron target. Here \( \psi_{\alpha}(p') \) and \( u(p) \) are the Rarita Schwinger and Dirac spinors for the \( \Delta \) and the nucleon of momenta \( p' \) and \( p \) respectively. \( q(= p' - p = k - k') \) is the momentum transfer, \( Q^2(= -q^2) \) is the momentum transfer square and \( M \) is the mass of the nucleon. \( C_i^V(q^2) \) are the vector and \( C_i^A(q^2) \) are the axial vector transition form factors. The vector form factors \( C_i^V(q^2) \) are determined by using the conserved vector current(CVC) hypothesis which gives \( C_i^V(q^2) = 0 \) and relates \( C_i^V(i = 3-6) \) to the electromagnetic form factors which are determined from the analysis of experimental data on the photoproduction and electroproduction of \( \Delta \)s. They are generally parameterized in a dipole form [12]:

\[
C_i^V(q^2) = C_i^V(0) \left( 1 - \frac{q^2}{M_V^2} \right)^{-2}; \quad i = 3, 4, 5.
\]

(6)

where \( M_V \) is the vector dipole mass.

However, some authors [12], [29], [33], [38], [39] have recently proposed modified dipole form factors while others use quark models without or with some pion dynamics. In the case of dipole form factors various modifications have been proposed. For example, Lalakulich et al. [38] use

\[
C_i^V(q^2) = C_i^V(0) \left( 1 - \frac{q^2}{M_V^2} \right)^{-2} D_i; \quad i = 3, 4, 5.
\]

(7)

while Paschos et al. [29] and Leitner et al. [32] use

\[
C_i^V(q^2) = C_i^V(0) \left( 1 - \frac{q^2}{M_V^2} \right)^{-2} D_i; \quad i = 3, 4, 5.
\]

(8)

Similarly, the axial vector form factors are determined using PCAC which gives \( C_i^A(q^2) = C_5^A(q^2) \frac{M^2}{M_A^2 - q^2} \) and the other form factors are defined from the analysis of neutrino induced pion production from hydrogen and deuteron targets. They are generally parameterized in a modified dipole form and are given as

\[
C_i^A(q^2) = C_i^A(0) \left( 1 - \frac{q^2}{M_A^2} \right)^{-2} D_i; \quad i = 3, 4, 5.
\]

(9)

\[
D_i = 1 + \frac{a_i q^2}{(b_i - q^2)}; \quad i = 3, 4, 5.
\]

\[
a_i = -1.21 \text{ and } b_i = 2.0 \text{ GeV}^2
\]

by Schreiner and von Hippel [12], while Paschos et al. [29], Leitner et al. [33] and Lalakulich et al. [38] use

\[
C_i^A(q^2) = C_i^A(0) \left( 1 - \frac{q^2}{M_A^2} \right)^{-2} D_i; \quad i = 3, 4, 5.
\]

(10)

where \( M_A \) is the axial vector dipole mass and \( m_\pi \) is the pion mass.
Various parameters occurring in these form factors used by these authors are summarized in table-1.

The differential scattering cross section is given by

\[
d\sigma/dE_k d\Omega_k = \frac{1}{64\pi^3} \frac{1}{MM_\Delta} \frac{1}{E_k} (W - M_\Delta)^2 + F^2(W) |M|^2
\]

where \(\Gamma\) is the delta decay width and \(|M|^2 = \frac{G_F^2 L_{\mu\nu} J^{\mu\nu}}{2}\), with

\[
L_{\mu\nu} = \bar{\Sigma} \Sigma \mu \nu \rho \nu = L_{\mu\nu}^S + iL_{\mu\nu}^A
\]

\[
= 8(k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}k \cdot k' + i\epsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta),
\]

and

\[
J^{\mu\nu} = \bar{\Sigma} \Sigma J^{\mu\nu} J^\nu
\]

which is calculated with the use of spin \(\frac{1}{2}\) projection operator \(P^{\mu\nu}\) defined as

\[
P^{\mu\nu} = \sum_{\text{spins}} \psi^\mu \bar{\psi}^\nu
\]

and is given by:

\[
P^{\mu\nu} = -\frac{g' + M_\Delta}{2M_\Delta} \left( g^{\mu\nu} - \frac{2g^{\mu\rho}g^{\nu\sigma}}{M^2} + \frac{1}{3} \frac{g^{\mu\nu}g^{\rho\sigma}}{M^2} - \frac{1}{3} \gamma^{\mu} \gamma^{\nu} \right)
\]

In eq.(11), the delta decay width \(\Gamma\) is taken to be an energy dependent P-wave decay width given by \(37\):

\[
\Gamma(W) = \frac{1}{6\pi} \left( \frac{f_\pi M_\Delta}{m_\pi} \right)^2 \frac{M}{W} |q_{cm}|^3 \Theta(W - M - m_\pi),
\]

where}

\[
|q_{cm}| = \sqrt{(W^2 - m_\pi^2 - M^2)^2 - 4m_\pi^2 M^2} / 2W
\]

and \(M\) is the mass of nucleon. The step function \(\Theta\) denotes the fact that the width is zero for the invariant masses below the \(N\pi\) threshold. \(|q_{cm}|\) is the pion momentum in the rest frame of the resonance.

### III. WEAK PION PRODUCTION FROMNUCLEI

#### A. Incoherent Pion Production

When the reactions given by eq.1 or 2 take place in the nucleus, the neutrino interacts with a nucleon moving inside the nucleus of density \(\rho(r)\) with its corresponding momentum \(p\) constrained to be below its Fermi momentum \(k_F\), \(F\) is the center of mass energy i.e. \(W = \sqrt{(p + q)^2}\) and \(M_\Delta\) is the mass of \(\Delta\).

In eq.(11), the delta decay width \(\Gamma\) is taken to be an energy dependent P-wave decay width given by \(37\):

\[
\Gamma(W) = \frac{1}{6\pi} \left( \frac{f_\pi M_\Delta}{m_\pi} \right)^2 \frac{M}{W} |q_{cm}|^3 \Theta(W - M - m_\pi),
\]

where}

\[
|q_{cm}| = \sqrt{(W^2 - m_\pi^2 - M^2)^2 - 4m_\pi^2 M^2} / 2W
\]

and \(M\) is the mass of nucleon. The step function \(\Theta\) denotes the fact that the width is zero for the invariant masses below the \(N\pi\) threshold. \(|q_{cm}|\) is the pion momentum in the rest frame of the resonance.

\[
\frac{d^2\sigma}{dE_k d\Omega_k} = \frac{1}{64\pi^3} \frac{1}{MM_\Delta} \frac{1}{E_k} (W - M_\Delta)^2 + F^2(W) |M|^2
\]

where \(\Gamma\) is the delta decay width and \(|M|^2 = \frac{G_F^2 L_{\mu\nu} J^{\mu\nu}}{2}\), with

\[
L_{\mu\nu} = \bar{\Sigma} \Sigma \mu \nu \rho \nu = L_{\mu\nu}^S + iL_{\mu\nu}^A
\]

\[
= 8(k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}k \cdot k' + i\epsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta),
\]

and

\[
J^{\mu\nu} = \bar{\Sigma} \Sigma J^{\mu\nu} J^\nu
\]

which is calculated with the use of spin \(\frac{1}{2}\) projection operator \(P^{\mu\nu}\) defined as

\[
P^{\mu\nu} = \sum_{\text{spins}} \psi^\mu \bar{\psi}^\nu
\]

and is given by:

\[
P^{\mu\nu} = -\frac{g' + M_\Delta}{2M_\Delta} \left( g^{\mu\nu} - \frac{2g^{\mu\rho}g^{\nu\sigma}}{M^2} + \frac{1}{3} \frac{g^{\mu\nu}g^{\rho\sigma}}{M^2} - \frac{1}{3} \gamma^{\mu} \gamma^{\nu} \right)
\]

In eq.(11), the delta decay width \(\Gamma\) is taken to be an energy dependent P-wave decay width given by \(37\):

\[
\Gamma(W) = \frac{1}{6\pi} \left( \frac{f_\pi M_\Delta}{m_\pi} \right)^2 \frac{M}{W} |q_{cm}|^3 \Theta(W - M - m_\pi),
\]

where}

\[
|q_{cm}| = \sqrt{(W^2 - m_\pi^2 - M^2)^2 - 4m_\pi^2 M^2} / 2W
\]

and \(M\) is the mass of nucleon. The step function \(\Theta\) denotes the fact that the width is zero for the invariant masses below the \(N\pi\) threshold. \(|q_{cm}|\) is the pion momentum in the rest frame of the resonance.

| TABLE I: Weak vector and axial vector couplings at \(q^2 = 0\) and the values of \(M_V\) and \(M_A\) used in the literature. |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Author         | \(C_V\) \((0)\)| \(C_A^0\) \((0)\)| \(C_V^0\) \((0)\)| \(C_A^0\) \((0)\)| \(C_V^0\) \((0)\)| \(M_V\) \((\text{GeV})\)| \(M_A\) \((\text{GeV})\) |
| Schreiner & von Hippel | \(0.0\) | \(-0.25\) | \(0.0\) | \(0.0\) | \(-0.3\) | \(1.2\) | \(0.73\) | \(1.05\) |
| Singh et al. \[27\] | \(1.95\) | \(-0.25\) | \(0.0\) | \(0.0\) | \(-0.25\) | \(1.2\) | \(0.84\) | \(1.05\) |
| Paschos et al. \[29\] | \(1.95\) | \(-0.25\) | \(0.0\) | \(0.0\) | \(-0.25\) | \(1.2\) | \(0.84\) | \(1.05\) |
| Leitner et al. \[33\] | \(2.13\) | \(-1.51\) | \(0.48\) | \(0.0\) | \(-0.25\) | \(1.2\) | \(0.84\) | \(1.05\) |

W is the center of mass energy i.e. \(W = \sqrt{(p + q)^2}\) and \(M_\Delta\) is the mass of \(\Delta\).
which $\Delta$ disappear in the nuclear medium without producing a pion, while a two body $\Delta$ absorption process like $\Delta N \rightarrow \pi NN$ gives rise to some more pions. These nuclear medium effects on the $\Delta$ propagation are included by describing the mass and the decay width in terms of the self energy of $\Delta$. These considerations lead to the following modifications in the width $\Gamma$ and mass $M_\Delta$ of the $\Delta$ resonance.

\[
\frac{\Gamma}{2} \to \frac{\Gamma}{2} - Im\Sigma_\Delta \quad \text{and} \quad M_\Delta \to M_\Delta + Re\Sigma_\Delta. \quad (18)
\]

The expressions for the real and the imaginary parts of $\Sigma_\Delta$ are [37]:

\[
Re\Sigma_\Delta = 40 \frac{\rho}{\rho_0}M eV \quad \text{and} \quad
-Im\Sigma_\Delta = C_Q \left( \frac{\rho}{\rho_0} \right)^\alpha + C_{A2} \left( \frac{\rho}{\rho_0} \right)^\beta + C_{A3} \left( \frac{\rho}{\rho_0} \right)^\gamma \quad (19)
\]

In the above equation $C_Q$ accounts for the $\Delta N \rightarrow \pi NN$ process, $C_{A2}$ for the two-body absorption process $\Delta N \rightarrow NN$ and $C_{A3}$ for the three-body absorption process $\Delta NN \rightarrow NNN$. The coefficients $C_Q, C_{A2}, C_{A3}$ and $\alpha, \beta$ and $\gamma$ are taken from Ref. 37.

With these modifications the differential scattering cross section described by eq.(15) modifies to

\[
\frac{d^2\sigma}{dE_k d\Omega_{k'}} = \frac{1}{64\pi^3} \int d\rho_p(r) \frac{|k|}{E_k} \frac{1}{MM_\Delta} \times \frac{\frac{\Gamma}{2} - Im\Sigma_\Delta}{(W - M_\Delta - Re\Sigma_\Delta)^2 + (\frac{\Gamma}{2} - Im\Sigma_\Delta)^2} |M|^2 \quad (20)
\]

For one $\pi^+$ production process $\tilde{\Gamma}$ and $C_Q$ term in $Im\Sigma_\Delta$ give contribution to the pion production. For $\pi^+$ production on the neutron target, $\rho_p(r)$ in the above expression is replaced by $\frac{\rho}{\rho_0}(r)$, where the factor $\frac{\rho}{\rho_0}$ comes due to suppression of $\pi^+$ production from the neutron target as compared to the $\pi^+$ production from the proton target through process of $\Delta$ excitation and decay in the nucleus.

The total scattering cross section for the neutrino induced charged current one $\pi^+$ production in the nucleus is given by

\[
\sigma = \frac{1}{64\pi^3} \int \int d\rho_p(r) \frac{1}{E_k E_{k'}} \frac{1}{MM_\Delta} \times \frac{\frac{\Gamma}{2} - C_Q \left( \frac{\rho}{\rho_0} \right)^\alpha}{(W - M_\Delta - Re\Sigma_\Delta)^2 + (\frac{\Gamma}{2} - Im\Sigma_\Delta)^2} \times \left[ \rho_p(r) + \frac{1}{\beta} \rho_n(r) \right] |M|^2 \quad (21)
\]

For our numerical calculations we take the proton density $\rho_p(r) = \frac{Z}{A} \rho(r)$ and the neutron density $\rho_n(r) = \frac{A-Z}{A} \rho(r)$, where $\rho(r)$ is nuclear density which we have taken as 3-parameter Fermi density given by:

\[
\rho(r) = \rho_0 \left( 1 + \frac{r^2}{c^2} \right) / \left( 1 + \exp \left( \frac{r-c}{z} \right) \right)
\]

and the density parameters $c = 2.355$ fm, $z = 0.5224$ fm and $w = -0.149$ for $^{12}$C and $c = 2.608$ fm, $z = 0.513$ fm and $w = -0.051$ for $^{16}$O are taken from Ref. [13].

The pions produced in these processes inside the nucleus may rescatter or may produce more pions or may get absorbed while coming out from the final nucleus. We have taken the results of Vicente Vacas [44] for the final state interaction of pions which is calculated in an eikonal approximation using probabilities per unit length as the basic input. In this approximation, a pion of given momentum and charge is moved along the z-direction with a random impact parameter $b$, with $|b| < R$, where $R$ is the nuclear radius which is taken to be a point where nuclear density $\rho(R)$ falls to $10^{-3} \rho_0$, where $\rho_0$ is the central density. To start with, the pion is placed at a point $(b, z_{in})$, where $z_{in} = -\sqrt{R^2 - |b|^2}$ and then it is moved in small steps $\delta l$ along the z-direction until it comes out of the nucleus or interact. If $P(p_{\pi}, \lambda)$ is the probability per unit length at the point $r$ of a pion of momentum $p_{\pi}$ and charge $\lambda$, then $P\delta l \ll 1$. A random number $x$ is generated such that $x \in [0, 1]$ and if $x > P\delta l$, then it is assumed that pion has not interacted while traveling a distance $\delta l$, however, if $x < P\delta l$ then the pion has interacted and depending upon the weight factor of each channel given by its cross section it is decided that whether the interaction was quasielastic, charge exchange reaction, pion production or pion absorption [44]. For example, for the quasielastic scattering

\[
P_{N(\pi^+, \pi^{+\prime})N'} = \sigma_{N(\pi^+, \pi^{+\prime})N'} \times \rho_N
\]

where $N$ is a nucleon, $\rho_N$ is its density and $\sigma$ is the elementary cross section for the reaction $\pi^+ + N \rightarrow \pi^{+\prime} + N'$ obtained from the phase shift analysis.

For a pion to be absorbed, $P$ is expressed in terms of the imaginary part of the pion self energy $\Pi$ i.e. $P_{abs} = \frac{Im\Pi_{\pi}^{\pi}}{\bar{P}}$, where the self energy $\Pi$ is related to the pion optical potential [15].

**B. Coherent Pion Production**

The coherent production of pion has been calculated earlier in this model [16], where $\Delta$ resonance excitations and their decays are such that the nucleus stays in the ground state. The matrix elements for $\Delta$ excitations are calculated using the hadronic transition current given in eqs.4 and 5 with the nuclear modification in $\Delta$ properties as described in eqs.(18) and (19).

With the incorporation of the nuclear medium effects as discussed in section-IIIA, the $\Delta$-dependent hadronic factors become density dependent and the hadronic transition operator $J^\mu$ is written as

\[
J^\mu = cos \theta_c \sum_{i=s,u} T^\mu(i) \frac{M^2}{P_i^2 - M_\Delta^2 + i\Gamma M_\Delta} \rho_j(r) e^{i(q - q_j) \cdot \vec{r}} d\vec{r}
\]

(22)
obtained with the N-Δ transition form factors given by Schreiner and von Hippel [12]. The total cross sections predicted by the NUANCE [16] are larger than the cross sections obtained by using the Schreiner and von Hippel [12], Paschos et al. [29] and Lalakulich et al. [38] parameterization. The uncertainty in the total cross section for 1π⁺ production associated due to the uncertainty in the transition form factors is seen from these figures to be about 10-20% in this energy region.

In Fig.2, we show the total cross section for charged current single π⁺ production from 12C using the N-Δ transition form factors given by Lalakulich et al. [38] for the incoherent(Fig.2a) and the coherent(Fig.2b) processes. We have presented the results for total scattering cross section σ(Eν) without the nuclear medium effects, with the nuclear medium modification effects, and with nuclear medium and pion absorption effects. For the incoherent process, we find that the nuclear medium effects lead to a reduction of around 12-15% for neutrino energies Eν=0.7-2 GeV. When pion absorption effects are taken into account along with the nuclear medium effects the total reduction in the cross section is around 30-40%. For the coherent process, the nuclear medium effects lead to a reduction of around 45% for Eν=0.7 GeV, 35% for Eν=1 GeV, 25% for Eν=2 GeV. The pion absorption effects taken into account along with the nuclear medium effects lead to a very large reduction in the total scattering cross section. The suppression in the total cross section due to nuclear medium and pion absorption effects in our model is found to be 80% for Eν around 1 GeV and 70% for Eν around 2 GeV [40]. Due to large reduction in the total cross section for the coherent process its contribution to the total charged current 1π⁺ production (< 4–5%) in the neutrino energy region of

where P is the momentum of the Δ resonance, Tµ is the non-pole part of the kinematic factors involving transition form factors G^A_v (q²), ρ(r) is the linear combination of proton and neutron densities incorporating the isospin factors for one pion production from proton and neutron targets.

In this case the final state interactions involve the interaction of the outgoing pions with the final nucleus in the ground state. This has been calculated using a distorted wave pion wave function in the field of the final nucleus. The distortion of the pion has been calculated in the eikonal approximation [47] using a pion nucleus optical potential which is given in terms of the self energy of pions in the nuclear matter calculated in the local density approximation. The nuclear form factor corresponding to the coherent pion production is calculated using a final state pion wave function given by [46]

\[
\tilde{\phi}_\pi (\tilde{r}) = e^{-i\tilde{q}_\pi \cdot \tilde{r}} e^{-i \int_0^{\infty} \Pi(\rho(z')) dz'}
\]

(33)

where \(\tilde{r} = (\tilde{b}, z)\). \(\Pi(\rho)\) is the self energy of pion calculated in the local density approximation of the delta hole model and is taken from Ref. [37].

The numerical results for the coherent pion production cross sections from 12C are recently presented in Ref. [60]. For the sake of completeness, these are also included here in the total cross sections along with the cross sections for the incoherent pion production and are discussed in section-IV while comparing with the experimental results on the total one π⁺ production from nuclei.

IV. RESULTS AND DISCUSSION

We have calculated the total scattering cross section for the charged current 1π⁺ production for the incoherent and coherent processes using different N-Δ transition form factors given by Schreiner and von Hippel [12], Paschos et al. [29] and Lalakulich et al. [38] as discussed in section-II. The numerical results for the total scattering cross section σ(E_ν) for νμ induced reaction on a free proton target i.e. νμ + p → μ⁻ + p + π⁺ are presented in Fig.1 along with the experimental results from the ANL and the BNL experiments [23-25]. The various theoretical curves show the cross sections calculated using N-Δ transition form factors given by Schreiner and von Hippel [12], Paschos et al. [29] and Lalakulich et al. [38]. We see from this figure that the BNL measurements are around 40% larger than the ANL measurements and our theoretical results are closer to the ANL measurements. The total cross sections predicted by the NUANCE [16] Monte Carlo generator which are used in the analysis of the MiniBooNE experiment are also shown in Fig. 1. We have also studied the uncertainty in the total cross sections due to the use of various parameterizations of the weak form factors used in literature. We find that in the neutrino energy region of 0.7-2.0 GeV the cross sections obtained with the N-Δ transition form factors given by Paschos et al. [29] and Lalakulich et al. [38] are larger than the cross sections obtained by using the Schreiner and von Hippel [12] parameterization. The uncertainty in the total cross section for 1π⁺ production associated due to the uncertainty in the transition form factors is seen from these figures to be about 10-20% in this energy region.
1-2 GeV is found to be smaller than the predictions of the NUANCE neutrino generator [34].

We have calculated the ratio of the cross sections for charged current $1\pi^+(\text{CC 1}$π$^+)$ production to charged current quasielastic scattering (CCQE) cross sections. For this purpose the cross section for quasi-elastic charged current lepton production is calculated in this model [48]-[49] for this purpose the cross section for quasielastic scattering (CCQE) cross sections. For $\pi^+$ charged current the NUANCE neutrino generator [34]. and pion absorption effects.

The result with(without) the nuclear medium modification effects and the solid line is the result with the medium modification and pion absorption effects.

FIG. 2: Charged current one pion production cross section induced by neutrinos on $^{12}\text{C}$ target using the Lalakulich’s $^{38}$ N-$\Delta$ weak transition form factors for the incoherent(Fig.2a) and the coherent(Fig.2b) processes. The dashed(dashed dotted) line is the result with(without) the nuclear medium modification effects and the solid line is the result with the medium modification and pion absorption effects.

![FIG. 2]  

FIG. 3: Ratio of $\frac{\text{CC 1}$π$^+}{\text{CCQE}}$ total scattering cross section for the $\nu_\mu$ induced reaction on $^{12}\text{C}$. The experimental points are taken from Wascko [16]. The various theoretical curves show the ratio of the cross sections for the charged current $1\pi^+$ production to the charged current quasielastic scattering (CCQE) scattering using Schreiner and von Hippel [12] (double dashed-dotted line), Paschos et al. [23] (dashed line) and Lalakulich et al. $^{38}$ (solid line) weak N-$\Delta$ transition form factors for C.C.1$\pi^+$ production and Bradford et al. [51] weak nucleon form factors for CCQE.

![FIG. 3]  

and vector dipole mass $M_V$ are used. However, recently the K2K collaboration [51] has analyzed their low energy quasielastic lepton production data using dipole parameterization for the axial vector form factor with the axial dipole mass $M_A=1.2$ GeV. If this value of the axial dipole mass is used then the cross section for the quasielastic lepton production increases by 12% for $E_\nu=1$ GeV as compared to the cross section calculated by using dipole parameterization with $M_A=1.05$ GeV.

The numerical values of the total cross sections for...
$1\pi^+$ production shown in Figs.2(a) and 2(b) with nuclear medium effects and final state interaction effects and the total cross sections for quasielastic lepton production as discussed above have been used to calculate the ratio which is shown in Fig.3. The quasielastic lepton production cross section is calculated using BBBA05 weak nucleon form factors and various parameterizations for N-Δ transition form factors given by Schreiner and von Hippel [12], Paschos et al. [29] and Lalakulich et al. [38] have been used to calculate the total cross sections for $1\pi^+$ production. We also show in this figure the experimental results for for this ratio reported by the MiniBooNE collaboration [16]. We see that in our model, the experimental results for the ratio are described satisfactorily below $E_\nu = 1.0$ GeV. For neutrino energies higher than $E_\nu = 1.0$ GeV the theoretical value of this ratio underestimates the experimental value. It is very likely that, at higher neutrino energies($E_\nu > 1.0$ GeV) the contributions from the excitation of higher mass resonances is important and should be taken into account. We will like to emphasize that the nuclear medium and pion absorption effects in pion production processes as shown in Fig. 2 play an important role in bringing about this agreement. For a given choice of the electroweak nucleon form factors in the quasielastic sector, there is a theoretical uncertainty of 10-20% in this ratio due to use of various parameterisations for the N-Δ transition form factors. However there is a further uncertainty of 2-3% in this ratio due to the various form factors used in the calculations of the total cross section for the quasielastic production.

In Fig.4, we have shown the variation in the total cross section for the charged current $1\pi^+$ production for $\nu_\mu$ induced reaction in $^{12}$C due to the variation in the axial vector dipole mass $M_A$ in the N-Δ transition form factors using the parametrization given by Lalakulich et al. [38]. The results are shown for $M_A=1.0$ GeV, $M_A=1.1$ GeV and $M_A=1.2$ GeV. We find that a 20% change in $M_A$ results in a change of around 20% in the cross section which increases with $M_A$. In this figure we have also shown the results for the total cross section for charged current $1\pi^+$ production reported by the MiniBooNE collaboration [16] along with the results predicted by the NUANCE [34] and NEUGEN [35] neutrino event generators. The theoretical predictions for the total cross sections by the neutrino generators like NUANCE [34] and NEUGEN [35] over estimate the experimental cross sections as they do not include the nuclear effects appropriately which are known to reduce the cross sections. For example, the nuclear effects lead to a reduction of 30-40% for the dominant process of incoherent production in this energy region as shown in Fig.2(a) which is large compared to 10% reduction considered in the T=3/2 channel in the NUANCE generator [34]. On the other hand, a microscopic description of nuclear medium and final state interaction effects considered in the present model under estimates the experimental cross sections. This is not surprising considering the fact that we are calculating the pion production only due to the Δ excitations. It seems that even in the intermediate energy of 1 GeV the role of higher resonance excitations are important and should be considered accordingly. Quantitatively similar results have also been recently obtained for the neutrino induced pion production from $^{12}$C by Cassing et al. [32] using a different model for the treatment of nuclear medium and final state interaction effects.

In Fig.5, we have presented the results for the differential cross section $<\frac{d\sigma}{dQ^2}> vs Q^2$ for charged current $1\pi^+$ production for the incoherent process averaged over the MiniBooNE and K2K spectrum for $\nu_\mu$ induced reaction in $^{12}$C (Fig.5a for MiniBooNE) and $^{16}$O (Fig.5b for K2K). The various curves show the results with the nuclear medium modification and final state interaction effects obtained by using the different N-Δ transition form factors given by Schreiner and von Hippel [12], Paschos et al. [29] and Lalakulich et al. [38]. We find that for the incoherent process in the peak region, $<\frac{d\sigma}{dQ^2}>$ obtained by using Paschos et al. [29] and Lalakulich et al. [38] N-Δ transition form factors are respectively 4 – 5% and 10% larger than the differential cross section obtained by using Schreiner and von Hippel [12] N-Δ transition form factors. In the inset of these figures we have also shown the effect of nuclear medium and pion absorption on $<\frac{d\sigma}{dQ^2}>$ using N-Δ transition form factors given by Lalakulich et al. [38]. We find that for the incoherent process, the nuclear medium effects lead to a reduction in the differential cross section of around 14% in the peak region. When nuclear medium and final state interaction effects are taken into account the total reduction in the
FIG. 5: $\langle \frac{d\sigma}{dQ^2} \rangle$ vs $Q^2$ for $\nu_\mu$ induced reaction on $^{12}\text{C}$ averaged over the MiniBooNE spectrum (Fig. 5a) and on $^{16}\text{O}$ averaged over the K2K spectrum (Fig. 5b) for the incoherent process. The various curves are the differential cross sections for the charged current $1\pi^+$ production with nuclear medium and final state interaction effects and calculated by using Schreiner and von Hippel [12] (double dashed-dotted line), Paschos et al. [29] (dashed line) and Lalakulich et al. [38] (solid line) weak N-$\Delta$ transition form factors. In the inset we have also shown the nuclear medium modification effects on $\langle \frac{d\sigma}{dQ^2} \rangle$ vs $Q^2$ averaged over the MiniBooNE and K2K spectra using the Lalakulich’s [38] N-$\Delta$ weak transition form factors. The dashed-dotted (dashed double dotted) line is the result with (without) the nuclear medium modification effects and the solid line is the result with the medium modification and pion absorption effects.

FIG. 6: $\langle \frac{d\sigma}{dQ^2} \rangle$ vs $Q^2$ for $\nu_\mu$ induced reaction on $^{12}\text{C}$ averaged over the MiniBooNE spectrum (Fig. 6a) and on $^{16}\text{O}$ averaged over the K2K spectrum (Fig. 6b) for the coherent process using the Lalakulich’s [38] N-$\Delta$ weak transition form factors. The dashed-dotted (dashed double dotted) line is the result with (without) nuclear medium modification effects and the solid line is the result with medium modification and pion absorption effects.

cross section is around 38%.

In Fig. 6, we have presented the results for the coherent process and shown the effect of nuclear medium and pion absorption effects on $\langle \frac{d\sigma}{dQ^2} \rangle$ averaged over the MiniBooNE and K2K spectrum for $\nu_\mu$ induced reaction in $^{12}\text{C}$ (Fig. 6a for MiniBooNE) and $^{16}\text{O}$ (Fig. 6b for K2K) using N-$\Delta$ transition form factors given by Lalakulich et al. [38]. We find that the reduction in the differential scattering cross section $\langle \frac{d\sigma}{dQ^2} \rangle$ in the peak region, when nuclear medium effects are taken into account is around 35% and the total reduction is 85% when pion absorption effect is also taken into account. The uncertainty due to the use of various parameterizations of the transition form factors is small in the case of the coherent process as it is dominated by the low $Q^2$ behavior of the form factor $C_3^A(Q^2)$ which is fixed by the generalised Goldberger Treiman relation at $Q^2=0$.

V. SUMMARY AND CONCLUSION

We have studied neutrino induced charged current $1\pi^+$ production from proton, $^{12}\text{C}$ and $^{16}\text{O}$ at the intermediate neutrino energies relevant for the MiniBooNE and
the K2K experiments. The energy dependence of the total scattering cross sections for the charged current one pion production induced by $\nu_e$ is studied. We have done the calculations for the incoherent and coherent production of pions from nuclear targets in the $\Delta$ dominance model which incorporates the modification of the mass and the width of $\Delta$ resonance in the nuclear medium and takes into account the final state interaction of pions with the final nucleus. We have presented the results for the total cross section for $1\pi^+$ production from $^{12}$C and studied the energy dependence of the ratio of single $\pi^+$ production to the quasielastic reaction. The results have been compared with the preliminary results available from MiniBooNE experiment. We have also presented the numerical results for $Q^2$ distribution i.e. $<\frac{d\sigma}{dQ^2}>$ in $^{12}$C and $^{16}$O averaged over the MiniBooNE and K2K spectra respectively.

From this study we conclude that:

1. The total cross sections for neutrino induced $1\pi^+$ production from free proton are closer to the $\pi^+$ production cross sections obtained by the ANL experiment and are smaller than the $\pi^+$ production cross sections obtained by the BNL experiment in the intermediate energy region. In this energy region, there is a $10-20\%$ theoretical uncertainty in the total cross section due to use of various parameterization of N-$\Delta$ transition form factors.

2. The total cross sections for $1\pi^+$ production is dominated by the incoherent process. The contribution of the coherent pion production is about 4-5\% in the energy region of 0.7-1.4 GeV.

3. In the neutrino energy region of 0.7-1.4 GeV, the results for the ratio of cross section of $1\pi^+$ production to the quasielastic lepton production is described quite well for $E_\nu < 1.0$ GeV, when nuclear effects in both the processes are taken into account. However, for energies higher than $E_\nu > 1.0$ GeV, the theoretical value of the ratio underestimates the experimental value. This might be due to $1\pi^+$ contribution coming from the excitation of higher resonances which are not included in the present calculations.

4. The role of nuclear medium effects is quite important in bringing out the good agreement between the theoretical and experimental value of the ratio for the total cross sections for $1\pi^+$ production and quasielastic lepton production for neutrino energies upto 1.0 GeV. For $E_\nu = 1$ GeV, the nuclear medium effects reduce the charged current quasielastic scattering cross section by 18\%, while $1\pi^+$ production cross section is reduced by 40\%.

5. The results for $<\frac{d\sigma}{dQ^2}>$ vs $Q^2$ in $^{12}$C and $^{16}$O averaged over the MiniBooNE and K2K spectra have been presented for the incoherent and coherent charged current one pion production with various N-$\Delta$ transition form factors. We have also presented the results for the nuclear medium and the final state interaction effects on the $Q^2$ distribution.

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