String Theory: Progress and Problems

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String theory builds on the great legacy of Yukawa and Tomonaga: New degrees of freedom and control of the UV are two important themes. This talk will give an overview of some of the progress and some of the unsolved problems that characterize string theory today. It is divided into two parts:

- Connecting String Theory to the Real World
- Gauge Theory/String Theory Duality

Two other major subjects, which I will omit, are Black Holes in String Theory and The Impact of String Theory on Mathematics.

§1. Connecting string theory to the real world

String theory became a hot subject in the mid-1980s when it became clear that it might give a deeper understanding of the origins of the standard model and a consistent quantum theory containing gravity.\(^1\)–\(^3\) At that time five consistent string theories were known, each of which requires ten spacetime dimensions and supersymmetry. They are called:

- Type I
- Type IIA
- Type IIB
- \(SO(32)\) heterotic
- \(E_8 \times E_8\) heterotic

Each of these theories is entirely free of adjustable dimensionless parameters. All dimensionless parameters arise in either of two possible ways:

- dynamically as the expectation values of scalar fields
- as integers that count something such as topological invariants, physical objects (branes), or quantized fluxes.

1.1. Calabi-Yau compactification

One scheme looked particularly promising in the mid-1980s. Specifically, the \(E_8 \times E_8\) heterotic theory has consistent vacuum solutions in which six spatial dimensions form a compact Calabi-Yau manifold, which has \(SU(3)\) holonomy, and the other four dimensions form Minkowski spacetime.\(^4\) Thus, the ten-dimensional spacetime \(\mathcal{M}_{10}\) is a direct product

\[
\mathcal{M}_{10} = CY_6 \times M_{3,1}.
\]

The effective four-dimensional theories that characterize such solutions at low energies have the following attractive features:

- They have the structure of supersymmetric grand unified theories. The well-known advantages of low-energy supersymmetry and grand unification are therefore naturally incorporated.
• Each solution has a definite number of families of quarks and leptons determined by the topology of the CY space.

• The standard model gauge symmetry is embedded in one $E_8$ factor, and there is a hidden sector, associated to the second $E_8$ factor. Supersymmetry can break dynamically (by gluino condensation) in the hidden sector. This breaking is communicated gravitationally to the visible sector. Such schemes suppress unwanted flavor-changing neutral currents, though possibly not as much as is required.

• There are several good dark-matter candidates: The lightest supersymmetric particle, called the LSP (perhaps a neutralino) is absolutely stable if there is an unbroken R symmetry, as generally supposed to be the case. A stable neutralino has the right properties for weakly interacting cold dark matter (a WIMP). Other possibilities include gravitinos, axions, and hidden-sector particles.

Despite the exuberance that was in the air in 1985, there was much that was not yet understood. The successes were qualitative, and there were many problems and puzzling questions. Some of these problems and questions were the following:

• Hundreds of Calabi-Yau manifolds were known (now there are many thousands). Which one of them, if any, is the right one? Is there a principle (other than agreement with observations) by which the right one can be determined? Are there string-theory based schemes other than CY compactification of the $E_8 \times E_8$ heterotic theory that can give quasi-realistic solutions?

• Why are there four other consistent superstring theories? After all, we only need one fundamental theory. Are some of them inconsistent, or else, could some of them somehow be equivalent?

• The CY compactification scenario was analyzed using perturbation theory, but there is no good reason to believe that the string coupling should be small. What new nonperturbative features appear at strong coupling? Does the same qualitative picture continue to hold at strong coupling?

• The CY solutions typically give many massless scalars (called moduli). Since the moduli have gravitational strength interactions, they are ruled out by standard tests of GR. How can we get rid of them? The effective potential does not depend on the values of the moduli, so they describe flat directions. One should somehow stabilize the moduli by generating an effective potential with isolated minima and no flat directions.

• What ensures that the vacuum energy density (or dark energy) is sufficiently small, namely of order $10^{-120}$ in Planck units? In 1985 it was generally believed to be zero, so one popular idea was to look for a symmetry principle that would enforce this. It is just as well that such a symmetry was never found, since

\*I felt at the time that the subject had undergone an almost instantaneous transition from being vastly under-appreciated to being over-optimistically regarded as being on the verge of providing a theory of everything.\*
we now know that the vacuum energy density is not exactly zero. A generic nonsupersymmetric vacuum is expected to give a vacuum energy of order one. Supersymmetry, broken at the TeV scale, cuts this down to $10^{-60}$, which is only half way to the right value on a logarithmic scale.

- What does string theory have to say about cosmology?

### 1.2. Lessons of the second superstring revolution

In the mid-1990s there was a remarkable burst of progress in addressing some of the issues listed above. The main lessons were the following:

- There is just one theory! What had been viewed as five theories are actually five different (highly symmetric) corners of a space of solutions to a unique underlying theory. The five superstring theories are related by various dualities: T-duality ($R \rightarrow \ell_s^2/R$) relates the two type II superstring theories, and also the two heterotic string theories. Here, $R$ represents the radius of a circular extra dimension and $\ell_s$ is the fundamental string length scale. There are analogous dualities involving pairs of CY manifolds. The two CY spaces are said to be related by mirror symmetry.

S-duality ($g_s \rightarrow 1/g_s$) relates the type I superstring theory and the $SO(32)$ heterotic string theory. Here, $g_s$ denotes the dimensionless string coupling constant, whose value is determined by the vacuum expectation value of a certain scalar field called the dilaton. The type IIB superstring theory has an $SL(2, Z)$ duality group, which contains this transformation as a special case. The type I theory can be derived from the type IIB theory by a procedure called orientifold projection. This projection essentially enforces left-right symmetry on the world-sheet theory.

- New objects, called $p$-branes, arise nonperturbatively. Here $p$ is an integer that represents the number of spatial dimensions of the brane. (Brane is a made-up word derived from membrane.) Stable $p$-branes carry conserved charges and satisfy generalized Dirac quantization conditions. There are also unstable $p$-branes that do not carry conserved charges. The main categories of stable $p$-branes are D-branes, M-branes, and NS5-branes. NS5-branes are magnetic duals of fundamental strings in the heterotic and type II theories. (Type I strings do not carry a conserved charge, which is one way of understanding why they can break.)

- D-branes are characterized by Dirichlet boundary conditions for open strings. Also, the stable D-branes in type I and type II superstring theories carry conserved charges of the Ramond-Ramond type. The relation of D-branes to open strings has the crucial consequence that the world-volume theories of $Dp$-branes are Yang-Mills gauge theories in $p + 1$ dimensions. $p$ takes even values in the type IIA theory and odd values in the type IIB theory. A system of $N$ coincident $Dp$-branes gives a $U(N)$ gauge theory. Other gauge groups can also be obtained, if the D-branes coincide with certain types of singularities.
• In units where the fundamental superstring has tension (energy density) of the form

\[ T_{F1} \sim m_s^2, \]

the Dp-branes have tension

\[ T_{Dp} \sim m_s^{p+1}/g_s. \]

This is in contrast with more conventional nonperturbative excitations, such as monopoles in gauge theory or NS5-branes in superstring theory, which have mass or tension proportional to \( 1/g_s^2 \).

• The strong-coupling limit of the Type IIA superstring theory or the \( E_8 \times E_8 \) heterotic string theory gives 11-dimensional M-theory. The size of the 11th dimension is

\[ R_{11} \sim g_s \ell_s. \]

In the \( E_8 \times E_8 \) heterotic case the 11th dimension is a line interval, so the 11-dimensional spacetime has two 10-dimensional boundaries. One set of \( E_8 \) gauge fields is localized on each boundary. This gauge group is singled out by quantum consistency (absence of anomalies).

After Calabi-Yau compactification of the strongly coupled \( E_8 \times E_8 \) heterotic string theory, it is possible for the volume of the CY manifold to vary along the 11th dimension, which opens up new possibilities for phenomenology. This approach is referred to as heterotic M-theory.

1.3. Intersecting D-brane models

The fact that D-branes carry Yang-Mills gauge theories suggests another approach to model-building — this time based on type II superstrings. One again starts with a compactification of the form \( K_6 \times M_{3,1} \). Then one introduces D-branes that fill the four-dimensional spacetime. The way to do this is to introduce D(3 + n)-branes that wrap \( n \)-dimensional cycles of \( K_6 \), while the other dimensions are spacetime filling.

For example, consider D6-branes in the type IIA theory compactified on a six-torus. Suppose that a set of \( N_1 \) D6-branes wraps three compact dimensions and a second set of \( N_2 \) D6-branes wraps a different three-cycle. The two stacks of branes give a \( U(N_1) \times U(N_2) \) gauge theory in four dimensional spacetime. Open strings connecting the two stacks of branes at their intersection points give chiral matter transforming as \( (N_1, N_2) \). This is roughly the type of structure that one has in the standard model and many of its possible extensions. If certain restrictions are satisfied, these constructions can give supersymmetric theories.

There are many schemes of this type. The successes and difficulties in this approach to model building are comparable to those of CY compactification of the heterotic string.
1.4. Flux compactifications

Type II superstrings contain various massless antisymmetric tensor gauge fields (in the RR sector). These are conveniently described as differential forms

$$A_n = \frac{1}{n!} A_{\mu_1\mu_2...\mu_n} dx^{\mu_1} \wedge dx^{\mu_2} \wedge ... \wedge dx^{\mu_n}.$$  

The integer $n$ is odd in the type IIA theory and even in the type IIB theory. These have gauge-invariant field strengths of the form

$$F_{n+1} = dA_n,$$

generalizing Maxwell theory ($n = 1$). D-branes are sources for these fields. There is also a two-form $B_2$ (in the NS sector) for which the fundamental strings are electric sources and NS5-branes are magnetic sources.

If the compact dimensions contain nontrivial $n$-cycles, $C_n$, it is possible to have (quantized) flux threading the cycle:

$$\int_{C_n} A_n = 2\pi N.$$  

Such possibilities greatly increase the number of possible quantum vacua, though there are constraints that must be satisfied. Mike Douglas has analyzed one particular CY compactification of the type IIB theory, and he has estimated the number of distinct flux vacua to be about $10^{500}$.  

1.5. Warped compactification

One of the important properties of flux compactifications is that they give rise to warped geometries. This means that the ten-dimensional geometry is no longer a direct product of the form

$$M_{10} = K_6 \times M_{3,1}.$$  

Instead, the 4d Minkowski metric part of the 10d metric is multiplied by a warp factor $h(y)$ that depends on position $y$ in the internal manifold

$$ds_{10}^2 = h(y) dx \cdot dx + ds_6^2.$$  

Also the internal six-manifold is no longer Calabi-Yau, since it is also multiplied by a warp factor.

In the brane-world picture one can have spacetime-filling D3-branes that are localized at points in the internal manifold. Randall and Sundrum have proposed that a large ratio between the warp factors at the position of a standard model brane and a Planck brane could provide a solution of the hierarchy problem, accounting for the large ratio between the Planck scale and the weak scale. The RS scenario can be made rather precise in the context of flux compactifications with branes.

1.6. Moduli stabilization and the landscape

The moduli problem — the occurrence of massless scalar fields $\phi_i$ with continuously adjustable vacuum expectation values — can be solved in the context of
flux compactification. The fluxes induce a nontrivial potential energy function for the moduli \( V(\phi_i) \). This landscape has isolated minima, which are what Douglas counted. The moduli are massive at a minimum. The details are a bit complicated, as some moduli are stabilized by perturbative effects and others are stabilized by nonperturbative effects.

There are many issues. For example, one of the moduli fields is the dilaton \( \Phi \), whose vacuum value determines the string coupling constant:

\[
g_s = \langle e^\Phi \rangle.
\]

If this is stabilized at a nonzero value, then the system is inherently nonperturbative, and perturbation theory (an expansion about the free theory value \( g_s = 0 \)) does not make sense. Even though the qualitative picture may survive, this is a serious technical problem.

The proliferation of vacua raises many questions: Is it completely hopeless to find the right one? Is it meaningful or useful to assign probabilities to the vacua and study them statistically? One proposal is that such a probability distribution is determined by the wave function of the Universe, which might be determined by a suitable initial value boundary condition.

1.7. The cosmological constant

The values of the cosmological constant in the various (metastable) vacua seem to be more or less randomly distributed over a range of order unity (in Planck units). If this is the case, then about 1 in \( 10^{120} \) would have roughly the right value. If there really are \( 10^{500} \) or more vacua, this is a very large number even though it is a very small fraction.

Weinberg has argued that if the cosmological constant were more than an order of magnitude or two larger than it is, galaxies would not form, and then we would not be here to discuss the question. This type of reasoning may be correct, but it leaves me with an unsatisfied feeling. I hope that we can find the right vacuum, or the right cosmological solution, and use it to answer many of the questions that motivated us to become physicists.

Even if string theory provides the correct theory, and it is completely unique, it is still a big question how predictive the theory is. If the number of consistent vacua (or solutions) is too large, and there is no principle for making an informed choice among them, it might not be possible (even in principle) to compute fundamental constants such as the fine-structure constant and the electron-muon mass ratio. However, I remain optimistic that this is not the case.

1.8. Brane inflation

There is a lot of evidence that supports the hypothesis that the very early Universe underwent a period of inflation during which the scale factor grew exponentially by a factor of at least \( e^{60} \) (i.e. 60 e-foldings). In simple field theory models this is described in terms of a "slowly rolling" scalar field called an inflaton.

The subject of string cosmology has become very active in the past several years. Proposals for the string-based origins of inflation, or possible alternatives, are being
explored extensively. Let me sketch a specific scenario due to Kachru et al.\textsuperscript{8,9} It takes place in the setup of CY compactification with flux and warped throats in the geometry.

Inflation takes place as a D3-brane moves down a throat, attracted to an anti-D3-brane at the bottom until they collide and annihilate. A scalar mode of an open string connecting the branes is the inflaton. The annihilation releases the tension energy stored in the branes. It heats up the Universe to start the hot big-bang epoch. All sorts of strings are produced, and some might survive to be observable as cosmic superstrings. Their tensions could be small enough to have avoided detection so far, due to the warping of the geometry. So, while there can be no definitive prediction at this time, the search for linear lenses in the sky is one way in which fundamental strings might be discovered.

§2. Gauge theory/string theory duality

A class of dualities — referred to as AdS/CFT dualities or holographic dualities — was proposed by Maldacena.\textsuperscript{10} A very symmetrical example is the duality between $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in four dimensions,\textsuperscript{11} with gauge group $SU(N)$, and type IIB superstring theory\textsuperscript{12} in an $AdS_5 \times S^5$ background\textsuperscript{13} with $N$ units of RR flux. $S^n$ denotes an $n$-dimensional sphere, and $AdS_n$ refers to anti de Sitter space, a maximally symmetric spacetime geometry with negative curvature, in $n$ dimensions. This geometry is certainly unrealistic, but it is an excellent theoretical laboratory.

One class of generalizations relates various superconformal field theories in four dimensions to type IIB superstring configurations with spacetime backgrounds of the form $AdS_5 \times X_5$. Here $X_5$ is a Sasaki-Einstein space, which has the property that a six-dimensional cone over $X_5$ is a noncompact Calabi-Yau space. Another class of generalizations introduces deformations such that the gauge theory no longer has conformal symmetry, and the dual $AdS_5$ geometry is deformed in a corresponding manner. Therefore, the subject of gauge theory/string theory duality entails much more than AdS/CFT duality.

2.1. An example

The discussion that follows is restricted to the $AdS_5 \times S^5$ example. The most obvious test of the duality is to check that both theories have the same symmetry. The symmetry in each case is known to be the supergroup $PSU(2,2|4)$, so this test is passed. Exhibiting the dynamical details of the duality is much more challenging for a number of reasons. For one thing, the gauge theory is a nontrivial interacting field theory. However, it is somewhat special: there are indications that it may be integrable in the planar approximation. The planar approximation corresponds to the leading terms in the ’t Hooft large-$N$ expansion,\textsuperscript{14} which is carried out for fixed ’t Hooft parameter

$$\lambda = g_{YM}^2 N.$$
The large-$N$ expansion is dual to the string-theory perturbation expansion, since the string coupling constant $g_s$ is given by

$$g_s = \frac{g_{YM}^2}{4\pi} = \frac{\lambda}{4\pi N}.$$ 

Even in the planar approximation, in which the strings do not interact, the string theory analysis is very challenging. The spectrum of string excitations is easy to compute in Minkowski spacetime, but very hard in a curved spacetime geometry. The string world-sheet theory (a two-dimensional QFT) is believed to be integrable in the case of $\text{AdS}_5 \times S^5$. The geometry (and hence the string spectrum) depends on the dimensionless parameter $\alpha'/R^2$, where $\alpha' = \ell_s^2$ is the string Regge slope parameter and $R$ is the radius of the $S^5$ and the $\text{AdS}_5$. This ratio corresponds to $1/\sqrt{\lambda}$. Thus, the supergravity approximation ($\alpha' \ll R^2$) corresponds to large 't Hooft coupling $\lambda$.

There is a lot of effort underway trying to solve the gauge theory in the planar approximation, as well as the string world-sheet theory, and to understand how they are related. Specifically, one wants to relate the anomalous dimensions of gauge-invariant operators in the gauge theory to the energies of type IIB string excitations. Much ingenious work has been done, but this seems to be a very challenging program.

An important aspect of the characterization of these operators and excitations is the amount of supersymmetry that they possess. The ground state preserves all of the supersymmetry. Beyond that, there are states (or operators) that preserve one-half, one-quarter, one-eighth, one-sixteenth or none of the supersymmetry. As the amount of supersymmetry is decreased, the complexity of the analysis increases. The states that preserve half of the supersymmetry, for example, are fully understood from the gauge theory perspective.\footnote{\cite{15}} They are also understood from the type IIB superstring theory perspective in a supergravity/semiclassical approximation.\footnote{\cite{16}}

### 2.2. D-brane model of QCD

As mentioned earlier, an interesting approach to particle physics phenomenology is based on intersecting D-brane models. There are a lot of issues to contend with, and it is difficult to address them all at once. A more modest approach than trying to find a system that accounts for all known interactions in a realistic way is to isolate a subset of the features that one would like to implement and analyze how they can be achieved.

A good example is to restrict to hadron physics and try to construct a D-brane model of QCD. This brings one back to the original goal of string theory research in the late 1960s and early 1970s — the construction of a theory of the strong interactions. But now we can approach the problem with many more tools than were available at that time.

$N_c$ coincident D-branes give an $SU(N_c)$ gauge theory, so that is a good start. One way to break supersymmetry is to take one dimension to be a circle of radius $R$ and give fermions antiperiodic boundary conditions on the circle. If one chooses D4-branes (in the type IIA theory) that wrap the circle, following Witten,\footnote{\cite{17}} then their five-dimensional world-volume theory reduces to pure four-dimensional gauge
theory at energies $E \ll 1/R$. The dual geometry can be treated in a supergravity approximation for large $N_c$ and large ’t Hooft coupling.

The addition of quark-like matter to this system was considered by Sakai and Sugimoto. Their proposal is to add $N_f$ coincident D8-branes and $N_f$ coincident anti-D8-branes that are localized on the circle. These D-branes are introduced using a probe approximation. This means that their effect on the spacetime geometry created by the D4-branes is neglected. The D4-D8 open strings give $N_f$ quark flavors in the gauge theory. At low energies this results in a Born-Infeld type action in a curved background.

The gauge symmetry of the D8-branes results in a

$$U(N_f)_L \times U(N_f)_R$$

global chiral symmetry in the 4d gauge theory. As is well-known from studies of QCD, this should break spontaneously to a vectorlike $SU(N_f)$ subgroup with the appearance of adjoint massless Nambu-Goldstone bosons (pions).

Studying the BI equations of motion, SS show that the branes reconnect into a single set of D8-branes in a U-like configuration. This beautifully accounts for the required chiral-symmetry breaking and gives the requisite massless pions as internal components of the higher-dimensional gauge fields. Moreover, the effective low-energy theory of the pions has the correct structure.

This construction captures the physics of QCD correctly at low energies, where the pion Lagrangian is the whole story. However, it differs from QCD at higher energies. The key unphysical feature is that the QCD scale $\Lambda$ is comparable to the Kaluza-Klein scale $1/R$. In a more realistic scheme the dual geometry would have string-scale curvature, and therefore it would not admit a supergravity approximation. This is one reason why the string theory dual of QCD is not yet known. At least we now know that it involves more than four dimensions.

### 2.3. Application of AdS/CFT to RHIC physics

RHIC collides gold nuclei with 200 GeV per nucleon. This presumably produces a quark-gluon plasma (QGP) with $T \approx 250$ MeV and $\alpha_s \approx 0.5$. This is a strongly coupled nonabelian plasma. The collisions produce an enormous number of particles, which makes it difficult to extract significant physics results. One way of probing of what is happening is to observe the rate and spectrum of D-meson production. From this one infers how the energy of a charm quark is dissipated as it moves through the hot plasma. This is characterized by the viscous drag.

In order to utilize AdS/CFT to study the strongly coupled theory, one replaces QCD by $\mathcal{N} = 4$ SYM. This sounds like a terrible thing to do, but it is probably not as bad for high temperature as it would be at zero temperature. (At high temperature, both theories both are deconfined, for example.) The temperature is taken account in the dual picture by replacing $AdS_5$ with an $AdS_5$ black hole with Hawking temperature $T$. Since $\lambda \sim 10$, a supergravity approximation can be used.

Bulk thermodynamic quantities have a finite limit for $\lambda \to \infty$. In particular, the
dimensionless ratio
\[
\frac{\text{shear viscosity}}{\text{entropy density}} \to \frac{1}{4\pi}
\]
was computed by Policastro, Son, Starinets.\textsuperscript{19})  This very low value is in rough agreement with deductions based on measurements of the \textit{elliptic flow} of collisions. The viscous drag
\[
\frac{dp}{dt} = -\frac{\pi}{2} \sqrt{\lambda T^2} \frac{v}{\sqrt{1 - v^2}},
\]
obtained by analyzing open strings connecting a D7-brane to the black hole, also gives rough agreement with observations.

There are many uncertainties in relating theory and experiment, but I don’t think there is a better approach on the market. I find it quite amazing that some nuclear physicists have become interested in black holes in five-dimensional anti de Sitter space!

2.4. \textit{Matching critical exponents}

In 1993 Choptuik discovered by numerical studies that if one forms a black hole by gravitational collapse of a free massless scalar field and varies the conditions of the collapse by changing some control parameter \(f\), there is a critical value of \(f\) above which a black hole forms. The mass of the resulting black hole scales as
\[
M_{\text{BH}} \sim (f - f_c)^\gamma.
\]
Choptuik found that \(\gamma_4 \approx 0.372\) in four dimensions. Sorkin and Oren\textsuperscript{20}) found that in five dimensions
\[
\gamma_5 = 0.408 \pm 2\%.
\]

The dual gauge theory process is the high energy Regge domain, which is dominated by Pomeron exchange — the BFKL (Balitsky-Fadin-Kuraev-Lipatov) Pomeron, to be precise. One-Pomeron exchange violates the unitarity bound above a certain energy (since the Regge intercept is \(\alpha(0) > 1\)), but this is cured at high energy by the nonlinear evolution of the BFKL kernel, which takes account of multi-Pomeron effects. This is characterized by a critical exponent
\[
\gamma_{\text{BFKL}} = 0.409552.
\]
This is in striking agreement with \(\gamma_5 = 0.408 \pm 2\%\).\textsuperscript{21}) It will be interesting to obtain a more accurate computation of \(\gamma_5\) and see whether this agreement continues to hold.

\section*{§3. Conclusion}

There has been a lot of progress in understanding string theory and its possible connections to the real world, but many problems remain. Even if progress continues to be made at the current rapid pace, I do not expect the subject to be completely understood by the time of the Yukawa-Tomonaga bicentennial. I do not consider this to be a pessimistic viewpoint. After all, it would be sad if we were so successful that we put ourselves out of business.
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Discussions

Q: Questioner  A: Answerer

Q(‘t Hooft): You said that most of the theories are perturbation expansions with some coupling constant $g$. We know from field theories that such expansions tend to diverge fundamentally. So strictly speaking, they mean nothing at all, using asymptotic (behavior) only, which means for various values of $g$, you don’t really know what these theories are describing.

A: Like QED?

Q: Like QED. So what do you think is necessary to overcome the problem, and get a rigorous and accurate definition of such a theory beyond whatever difficulties arise because of the finite-dimension coupling constant? You get another theory with a coupling constant.

A: What the dualities have done for us is that they allow us to also define asymptotic expansions about $g = \infty$, because you can relate a theory with a coupling constant $g$ to another theory with a coupling constant $1/g$. Then you can worry about what do you do in the middle? That’s hard. There are many properties that we can confidently extrapolate from one to the other because they are protected by supersymmetry. That is why we know about these dualities in the first place, because there are things that can be confidently extrapolated from weak coupling to strong coupling, using supersymmetry. Of course there are many quantities of interest that are not protected by supersymmetry, and so it is a challenge to explore them. The bizarre thing I just told you at the end here about these critical exponents has nothing to do with supersymmetry. If it is true and not just some fluke, then it is some non-supersymmetric probe of some funny thing, though I don’t know what.

Q: I’m not sure that’s an answer to the question.

A: Well, that’s the best I can do. Maybe David (Gross) wants to take it.

Gross: In QFT, we have non-perturbative definitions of the theory, say lattice gauge theory, which one can prove yields the same asymptotic expansion as perturbation theory, and are independently well defined. As I remarked in my talk, with string theory we still don’t have a theory. We just have these perturbative constructions, and there is nothing like a Lagrangian or a Hamiltonian, or any way, even in principle, of reducing any question one might ask to a limit of a well-defined computer calculation. In the absence of that, there is still wonder.

A: The only thing that I would add to what David just said is that for those cases in which we have a duality between a gauge theory and a string theory solution, if you accept the duality, you do the lattice gauge calculations that dual gauge theory presents about the string theory of arbitrary values. But you would have to accept the duality, for those parameters, exactly.

Q: I wanted to know how you met with Michael Green? How did you conquer your most difficult time?

A: How I met Michael Green? It’s really two different questions. (laughter) I met Michael Green when we were post-docs in Princeton, though we didn’t work together until about 10 years later, when we met up at a certain cafeteria and started our collaboration on string theory. That was 1979, and we had a good collaboration.
for about five or six years. We started another collaboration about three days ago. The other question is about the hard days. The proposal that string theory should be used for unification that I may have mentioned was proposed by Joel Scherk and myself, and there was related work by Yoneya here in Japan in the mid-70s. This idea did not take hold in the theoretical community until the mid-80s, which is where I sort of started this talk. We had confidence in these ideas as I pursued them during that period.

Q: In the conclusion of the final talk of this centennial symposium, you gave this expectation that the final theory will be given ···

A: I do not expect that. I do not expect that the subject will be completely understood, even in 100 years from now.

Q: Isn’t that too pessimistic?

A: There are a lot of hard problems here, and I think some of them will take a long time to sort out.

Q: To me, the uniqueness of the vacuum is not important at all, but rather that only what I want is that there is a vacuum in which this real world is explained. If there is just one vacuum that can explain this real world, that would be enough for me.

A: The even more ambitious thing, of course, is to ask for a cosmological solution that asymptotes to a specific vacuum.

Q: Maybe David could ···. There is no vacuum?

A: So I didn’t emphasize that in this landscape, when I say we can get the right cosmological constant, that is a positive energy. So there’s not a bottom, and there are only meta-stable vacua. But the energy gaps are very small, and the barriers are pretty big. So the lifetimes are undoubtedly far in excess of the universe.

Q: I am not an expert in this field, but after hearing the talk by Professor Go, you also mentioned the landscape of the united field. But in the story of molecules, you may be satisfied with some macro state, and a coarse-grained vacuum energy. If you have fine tuning as Professor Go told us, then you can see a landscape. Can such a scheme work?

A: There are many wild speculations that one might make. But this is not serious. So for example, he talked about getting to a nice protein through evolution. So you might ask, can we get to a nice vacuum through evolution? Well, what would that mean? You would through go some sequence of universes using chaotic inflation or something. But why should some be made preferentially to others? Is this completely nuts? I don’t know. I’m not advocating that. In your spare time, it is fun to think about things like that.

A: Let me thank the organizers for a wonderful conference. I think they still have some closing remarks of their own. I’ve certainly enjoyed this.