Omnidirectional Quasi-Orthogonal Space-Time Block Coded Massive MIMO Systems

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Abstract — Common signals in public channels of cellular systems are usually transmitted omnidirectionally from the base station (BS). In recent years, both discrete and consecutive omnidirectional space-time block codings (STBC) have been proposed for massive multiple-input multiple-output (MIMO) systems to ensure cell-wide coverage. In these systems, constant received signal power at discrete angles, or constant received signal sum power in a few consecutive time slots at any angle is achieved. In addition, equal-power transmission per antenna at any time and full spatial diversity can be achieved as well. In this letter, we propose a new consecutive omnidirectional space-time block codings (STBC) design, where the received signal sum power in 4 consecutive time slots at any angle is constant, equal-power transmission per antenna at any time and full diversity order of 4 can be achieved.

Index Terms — Massive MIMO, QOSTBC, omnidirectional transmission, orthogonal complementary codes.

I. INTRODUCTION

To meet the challenging capacity requirement of the fifth generation (5G), massive multiple-input multiple-output (MIMO) system with tens to hundreds of antennas deployed at base station (BS) has attracted substantial attentions [1]. Public channels play important roles since many essential common messages or services are provided to users from BS through public channels.

In order to broadcast common information from BS, discrete and consecutive omnidirectional space-time coding have been recently proposed in [2]-[6]. In [2]-[5], by utilizing the Zadoff-Chu (ZC) sequence and its properties, at any time signals across all transmit antennas have constant power, and the received signal power keeps constant at finite discrete angles as well, where the number of discrete angles is the same as the number of transmit antennas. In [6], a design of consecutive omnidirectional space-time coding is proposed, where the orthogonal space-time block code (OSTBC), Alamouti code (AC), is used for 2 information symbol (data) streams, the sum of received signal power at 2 consecutive time slots is constant at any angle, and equal-power transmission per antenna at any time and diversity order of 2 are achieved as well. However, the design in [6] can only be applied to AC for 2 data streams with diversity order 2. Although quasi OSTBCs (QOSTBC) for 4 data streams of diversity order 4 are designed in [5], the omnidirectionality is only for a number of discrete angles.

In this letter, to further increase the diversity order over the AC coding in [6], a new consecutive omnidirectional QOSTBC design is proposed, where received signal sum power in 4 consecutive time slots at any angle is constant, equal-power transmission per antenna at any time and full diversity order of 4 are achieved as well.

II. PROBLEM DESCRIPTION

A. System Model

In this letter, we consider STBC transmission for common information broadcasting. A BS is equipped with a uniform linear array (ULA) of M antennas and serves K users each with a single antenna. The common information is mapped to an STBC $\mathbf{S} \in \mathbb{C}^{M \times T}$ with $M \geq T$ and transmitted from M antennas of BS within T time slots to all the users. The received signal of user $k$ can be written as

$$[y_{k,1}, y_{k,2}, \ldots, y_{k,T}] = \sqrt{P_k} h_{k}^H \mathbf{S} + [z_{k,1}, z_{k,2}, \ldots, z_{k,T}]$$

where $P_k$ denotes the total transmit power, the channel $h_k \in \mathbb{C}^{M \times 1}$ is assumed to keep constant within these T time slots, $(\cdot)^H$ stands for the Hermitian operation, and $z_{k,t} \sim CN(0, \sigma_z^2)$ is the additive white Gaussian noise (AWGN) at time slot $t \ (t = 1, 2, \ldots, T)$.

To decode the transmitted information symbols in codeword $\mathbf{S}$, the instantaneous channel state information (CSI) $h_k$ must be known at the user side. However, in massive MIMO systems, since the number of BS antennas is very large and the number of downlink resources needed for pilots is proportional to the number of BS antennas, the downlink channel estimation becomes challenging. In order to reduce the pilot overhead, a dimensional-reduced STBC is utilized in, for example, [2]-[6], where a high dimensional STBC is composed by a precoding matrix $\mathbf{W}$ and a low dimensional STBC $\mathbf{X}$, i.e., $\mathbf{S} = \mathbf{WX}$, where $\mathbf{W} \in \mathbb{C}^{M \times N}$ is a tall precoding matrix independent of the channel or the information data, and $\mathbf{X} \in \mathbb{C}^{N \times T}$ is a low dimensional STBC modulated by the common information data. To decode the common information data, users only need to estimate the effective channel $\mathbf{W}^H h$ of dimension $N \times 1$ with $N \ll M$ instead. To normalize the total average transmission power at the BS side, we assume that $\mathrm{E} (\mathbf{XX}^H) = T/N \cdot \mathbf{I}_N$ and $\mathrm{tr}(\mathbf{WW}^H) = N$ where $\mathrm{tr} (\cdot)$ stands for the trace of a matrix.
B. Criteria of Consecutive Omnidirectional STBC

For consecutive omnidirectional STBC design, the following three criteria should be guaranteed [6].

1. Criterion of Constant Instant Transmission Power at Each Antenna at Any Time Slot: Assume \( x_t \) is the \( t \)-th column vector of the low dimensional STBC \( X \), then \( Wx_t \) is the transmitted signal in BS at time slot \( t \). To sufficiently utilize all the power amplifiers (PA) capacities of BS antennas, the precoding design should satisfy the following constraint

\[
|Wx_t|_m = \frac{1}{\sqrt{M}} \quad m = 1, \ldots, M
\]

at any time slot \( t \), where \( |Wx_t|_m \) denotes the transmitted signal on the \( m \)-th antenna at time slot \( t \).

2. Criterion of Full Diversity Order \( T \): The STBC \( S = WX \) satisfies the full column rank, i.e., for any two distinct STBC codewords \( S_1 \) and \( S_2 \) of \( S \), the difference \( S_1 - S_2 \) has full column rank \( T \).

3. Criterion of Constant Received Signal Sum Power at Any Angle: The corresponding transmitted signal in the angle domain can be written as

\[
S_i(\omega) = a(\omega) \cdot |Wx_t|,
\]

where \( a(\omega) = [1, e^{-j\omega}, \ldots, e^{-j(M-1)\omega}] \) is the antenna array response vector under the ULA setup and \( \omega = 2\pi d \sin(\theta) / \lambda \) with carrier wavelength \( \lambda \), antenna spacing \( d \) and angle of departure (AoD) \( \theta \). The criterion is

\[
\sum_{i=1}^{L} |S_i(\omega)|^2 = c, \forall \omega \in (-\pi, \pi)
\]

for a positive constant \( c \).

From (3), one can see that, \( S_i(\omega) \) is the Fourier transform (FT) of \( Wx_t \). Note that the received signal power of \( S_i(\omega) \) at a time slot \( t \) cannot be constant for any angle \( \omega \) as mentioned in [6]. It is the motivation in [6] to consider a sum of a few consecutive signal powers as [4]. Based on this idea, in [6] the precoded STBC, Alamouti coding, was fully studied to satisfy the above three criteria for \( T = 2 \) data streams with spatial diversity order of 2.

In this letter, we design precoded QOSTBC for \( T = 4 \) data streams with spatial diversity order of 4. Note that QOSTBC for \( T = N = 4 \) can accommodate at most 3 data streams [7], although it can achieve the spatial diversity order of 4.

C. Orthogonal Complementary Codes

Some useful mathematical results are reviewed here to help the omnidirectional STBC design. Let \( \{c_{i,j}, 1 \leq i \leq p, 1 \leq j \leq q\} \) be a set of orthogonal complementary codes (OCC), where each code \( c_{i,j} \) is a sequence of length \( L \). Every \( i \)-th subset \( \{c_{i,1}, c_{i,2}, \ldots, c_{i,q}\} \) is a complementary set of \( q \) sequences, and for \( i \neq i' \), the \( i \)-th and \( i' \)-th subsets are mutually orthogonal complementary [8][9]. The OCC has the following properties.

\[
\begin{align*}
\sum_{j=1}^{q} R_{c_{i,j}}(\tau) &= 0, \forall \tau \neq 0, 1 \leq i \leq p \\
\sum_{j=1}^{q} R_{c_{i,j} c_{i',j}}(\tau) &= 0, \forall \tau, 1 \leq i \neq i' \leq p
\end{align*}
\]

where

\[
R_{c_{i,j}}(\tau) = \sum_{l=1}^{L} |c_{i,j}|_{l}^2 e^{j\tau}
\]

III. OMNIDIRECTIONAL QOSTBC DESIGN

QOSTBCs have both advantages of complex symbol-wise maximum-likelihood (ML) decoding and full diversity. However, their symbol rates are upper bounded by 3/4 for more than two antennas for complex symbols [7] as mentioned earlier. Therefore, QOSTBCs are proposed in [10][11] where the orthogonality is relaxed to achieve high symbol transmission rate but with a more complex symbol pair-wise ML decoding. By rotating the constellations of half of the complex symbols, the QOSTBCs can further achieve full diversity [12][13]. We next want to study the consecutive omnidirectional STBC design based on the QOSTBCs and orthogonal complementary sets.

Consider the low dimensional QOSTBC of Tirkkonen, Boariu, and Hottinen (TBH) scheme [11] as an example, and constellation rotations [12][13] are applied to achieve the diversity order of \( T = 4 \).

\[
X = X_Q \Delta = \begin{bmatrix} x_1 & x_2^* & x_3 & x_4 \\ x_2 & -x_4^* & x_3^* & x_2^* \\ x_3 & x_4^* & x_1 & x_3 \\ x_4 & -x_2^* & x_2 & x_4^* \end{bmatrix}
\]

where \( x_1, x_2 \) are taken from a symbol constellation \( S \), and \( x_3, x_4 \) are taken from the rotated symbol constellation \( e^{j\theta}S \). The correlation matrix of \( X_Q \) in (7) is

\[
X_QX_Q^H = \begin{bmatrix} \alpha & 0 & \beta & 0 \\ 0 & \alpha & 0 & \beta \\ \beta & 0 & \alpha & 0 \\ 0 & \beta & 0 & \alpha \end{bmatrix} = \alpha I_4 + \beta \Pi_2
\]

where

\[
\alpha = \frac{4}{L} |x_1|^2, \quad \beta = 2\Re(x_1x_3^* + x_2x_4^*)
\]

the real part of a complex number, and \( \Pi_2 = \begin{bmatrix} 0 & I_2 \\ I_2 & 0 \end{bmatrix} \) where \( I_2 \) is the \( 2 \times 2 \) identity matrix. Note that while \( x_1, x_2, x_3, x_4 \) are independent with each other, \( \beta \) is not always

| Table I: Binary OCC of length 16 |
|-----------------------------|
| \( c_1 \) | 1 | 1 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 |
| \( c_2 \) | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 |
| \( c_3 \) | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 |
| \( c_4 \) | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 |
possible to be 0. Then, (4) can be rewritten as
\[
\sum_{i=1}^{4} |S_i(\omega)|^2 = \sum_{i=1}^{4} a(\omega)Wx_i[Wx_i]^H a^H(\omega)
\]
\[
= a(\omega)WX_QX_Q^H W^H a^H(\omega)
\]
\[
= \alpha a(\omega)WW^H a^H(\omega) + \beta a(\omega)Wl_2W^H a^H(\omega)
\]
\[
= \alpha \sum_{n=1}^{4} a(\omega)w_n w_n^H a^H(\omega)
\]
\[
+ \beta \sum_{n=1}^{2} a(\omega) (w_n w_{n+2}^H + w_{n+2} w_n^H) a^H(\omega)
\]
\[
= \alpha \sum_{n=1}^{4} |W_n(\omega)|^2 + 2\beta \text{Re}\left(\sum_{n=1}^{2} W_n(\omega)W_n^*(2\omega)\right)
\]
where \(w_n\) is the \(n\)-th column vector of \(W\), and \(W_n(\omega)\) is the FT of \(w_n\) for \(n = 1, \cdots, 4\).

**Theorem 1.** If the sequence sets \(\{w_1, w_2\}\) and \(\{w_3, w_4\}\) are mutually orthogonal complementary, and \(\alpha\) is constant for signal constellation \(S\), then, the criterion 3, i.e., (4), of constant received signal sum power with \(T = 4\) holds.

**Proof.** See Appendix A.

In addition, the QOSTBC design should satisfy the criterion of constant instantaneous transmission power at each antenna as well. Consequently, we propose the precoding matrix \(W = W_Q\) as follows.

Assume that the number of BS \(M\) antennas is an integer multiple of 4, and let \(\{c_1, \ldots, c_4\}\) be a set of binary orthogonal complementary codes of length \(M_0 = M/4\), in which \(\{c_1, c_2\}\) and \(\{c_3, c_4\}\) are two sets of complementary pairs of components either 1 or -1 and they are mutually orthogonal complementary. The precoding matrix \(W_Q\) is

\[
W_Q = \sqrt{\frac{4}{M}} [c_1 \otimes u_1, c_2 \otimes u_2, c_3 \otimes u_3, c_4 \otimes u_4]
\]  

(10)

where \(u_i\) is the \(i\)-th column vector of the \(4 \times 4\) identity matrix \(I_4\), and \(\otimes\) denotes the Kronecker product. Clearly, \(W_Q\) has rank 4, i.e., it has full column rank. The signal constellation \(S\) is selected to be a phase shift keying (PSK), i.e., \(x_i \in S_{PSK} = \{1, e^{j2\pi/3}, \ldots, e^{j2\pi(i-1)/3}\}\) for some positive integer \(\Omega\). The optimal rotation angle for PSK signal \(x_3\) and \(x_4\) is \(\pi/\Omega\) when \(\Omega\) is even and \(\pi/(2\Omega)\) or \(3\pi/(2\Omega)\) when \(\Omega\) is odd \([13]\). In this way, the precoding design will satisfy all the three criteria as we shall see below.

First, let \(c_{n,i}\) be the \(i\)-th element of \(c_n\), we have

\[
S = W_QX_Q
\]

\[
= \sqrt{\frac{4}{M}} [c_{1,1}x_1, c_{1,2}x_2, c_{1,3}x_3, c_{1,4}x_4, c_{2,1}x_2, c_{2,2}x_3, c_{2,3}x_4, c_{2,4}x_1, c_{3,1}x_1, c_{3,2}x_2, c_{3,3}x_3, c_{3,4}x_4, c_{4,1}x_2, c_{4,2}x_3, c_{4,3}x_4, c_{4,4}x_1]
\]  

(11)

\[
= \sqrt{\frac{4}{M}} [c_{1,0}x_1, c_{1,0}x_2, c_{1,0}x_3, c_{1,0}x_4, c_{2,0}x_2, c_{2,0}x_3, c_{2,0}x_4, c_{2,0}x_1, c_{3,0}x_1, c_{3,0}x_2, c_{3,0}x_3, c_{3,0}x_4, c_{4,0}x_2, c_{4,0}x_3, c_{4,0}x_4, c_{4,0}x_1]
\]

Since \(c_n\) is a sequence of 1’s and -1’s, and \(x_i\) is constant even with the rotations, it is easy to see that, all the elements in \(S\) have the same amplitude, so the criterion 1 of equal-power transmission per antenna at any time holds.

Since the low dimensional QOSTBC \(X_Q\) in (7) has diversity order of 4 after the optimal angle rotation, and the pre-coding matrix \(W_Q\) is a constant full column rank matrix, it is clear that the low dimensional QOSTBC \(S = W_QX_Q\) has diversity order \(T = 4\), i.e., the criterion 2 of full diversity order holds.

Then, let us prove that it satisfies the constant received signal sum power criterion 3. When PSK signals are adopted, we have \(a = \sum_{i=1}^{4} |x_i|^2 = 1\). Let \(C_n(\omega)\) be the FT of \(c_n\), so \(W_n(\omega) = \sqrt{\frac{4}{M}} e^{-j(n-1)\omega}C_n(4\omega)\). According to (9) and Theorem 1, we have

\[
\sum_{i=1}^{4} |S_i(\omega)|^2 = \frac{4\alpha}{M} \sum_{n=1}^{4} |C_n(4\omega)|^2 + \frac{4\beta}{M} \text{Re}\left(e^{-j2\omega} \sum_{n=1}^{2} C_n^*(4\omega)C_n+2(4\omega)\right) = 4
\]  

(12)

which is constant for all \(\omega \in (-\pi, \pi]\).

In summary, the above precoded QOSTBC satisfies all the three criteria presented in Section II. Similarly, the omnidirectional QOSTBC based on the Jafarkhani scheme \([10]\) can be done and we omit the details here.

Note that the length of a binary OCC may not be arbitrary. The construction of binary OCC is possible with any length \(L = 2^n \cdot 10^b \cdot 26^c\) for all integers \(a, b, c \geq 0\), and there is no binary OCC containing two sequences whose length cannot be expressed as a sum of two squares \([9]\). Therefore, the number of BS antennas may not be arbitrary either for our proposed designs in this letter.

**IV. Numerical Results**

In this section, we will evaluate the performance of the proposed omnidirectional QOSTBC design. We consider that the BS is a ULA of \(M = 64\) antennas in a 120° sector, where the antenna space is \(d = \lambda/\sqrt{3}\), and serves \(K = 300\) users each with a single antenna. The channel model here refers to the model in \([5]\). To represent that users have different angles with respect to the ULA of the BS within the sector, the mean AoD \(\theta_0\) is randomly selected in uniform distribution on \([-60^\circ, 60^\circ]\), while the angle spread (AS) \(\sigma = 5^\circ\). The signal constellation in all the schemes is QPSK, while for either the discrete \([5]\) or the proposed precoded QOSTBC, the optimal rotation angle of \(\pi/4\) is adopted \([13]\). The binary OCC, which is shown in Table I \([8,9]\), is used to obtain the proposed precoding matrix.

First, we have the average bit error rate (BER) performance with respect to the signal-to-noise ratio (SNR) value, i.e., \(P_b/\sigma_n^2\), as shown in Fig. 1. We can see that although all the schemes can achieve full diversity order of 2 or 4, consecutive precoded STBCs (either AC \([6]\) or proposed QOSTBC) always outperform the discrete ones obtained in \([5]\) with the same low dimensional STBCs. This is because when the antenna number
is limited, the angle resolution of the discrete precoded STBCs is not enough.

Then, we evaluate the BER performance with respect to the mean AoD $\theta_0$ to verify the ability of omnidirectional transmission for different STBC designs which is shown in Fig. 2. We can see that compared with the discrete precoded STBCs, the consecutive precoded STBCs have flatter BER performance for different values of mean AoDs, since the sum of the received signal powers of consecutive precoded STBCs is constant at any angle, rather than just at finite discrete angles.

V. CONCLUSION

In this letter, a consecutive omnidirectional QOSTBC design is proposed to guarantee omnidirectional transmission, i.e., the sum of $T=4$ consecutive received signal powers is constant at any angle, equal instantaneous power on each transmit antenna, and achieve the full diversity of the low-dimensional QOSTBCs. Compared with the discrete omnidirectional precoding design, our proposed omnidirectional QOSTBCs has flatter BER performance for all angles of DoA.

APPENDIX A

PROOF OF THEOREM 1

Let $\mathbf{w}_n = (w_{n,m})_{1 \leq m \leq M}$ be the $n$-th column vector of precoding matrix $\mathbf{W}$, the Fourier transform of $\mathbf{w}_n$ is

$$ W_n(\omega) = \sum_{m=1}^{M} w_{n,m} e^{-j(m-1)\omega}. \tag{13} $$

Thus, $W_n(\omega)W_k^*(\omega)$ can be written as

$$ W_n(\omega)W_k^*(\omega) = \sum_{m=1}^{M} \sum_{l=1}^{M} w_{n,m} w_{k,l}^* e^{-j(m-l)\omega} $$

$$ \sum_{a=0}^{M-1} \sum_{m=1}^{M} w_{n,m} w_{k,m+a} e^{ja\omega} $$

$$ \sum_{a=1-M}^{1} \sum_{m=1}^{M} w_{n,m} w_{k,m+a} e^{ja\omega} $$

$$ \sum_{a=0}^{M-1} R_{\mathbf{w}_n,\mathbf{w}_k}(a) e^{ja\omega} \tag{14} $$

where (a) follows by letting $a = l - m$, and (b) is due to the definition of AACF and ACCF in (6). Letting $\{\mathbf{w}_1, \mathbf{w}_2\}$ and $\{\mathbf{w}_3, \mathbf{w}_4\}$ be mutually orthogonal complementary, with the properties (5) of OCC, it is easy to know that

$$ \sum_{n=1}^{4} |W_n(\omega)|^2 = \sum_{a=1-M}^{M-1} \sum_{n=1}^{4} R_{\mathbf{w}_n}(a) e^{ja\omega} = \sum_{n=1}^{4} R_{\mathbf{w}_n}(0), $$

$$ 2 \sum_{n=1}^{4} W_n(\omega)W_{n+2}^*(\omega) = \sum_{a=1-M}^{M-1} \sum_{n=1}^{2} R_{\mathbf{w}_n,\mathbf{w}_{n+2}}(a) e^{j2a\omega} = 0. $$

Therefore, (9) can be rewritten as

$$ \sum_{t=1}^{4} |S_t(\omega)|^2 = \sum_{n=1}^{4} \phi_{\mathbf{w}_n}(0) = \alpha \cdot tr \left( \mathbf{W} \mathbf{W}^H \right) = 4\alpha \tag{15} $$

which proves the theorem.

Fig. 1: BER performance versus SNR for 2 bps/Hz.

Fig. 2: BER performance versus mean AoD for 2 bps/Hz and SNR=10dB.

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