The investigation of stability of wheeled vehicle model in curved sections of overpass

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Abstract. It deals with the reassignment of vertical and transverse reactions of wheel assembly of monorail vehicle. Characteristics of force interaction of monorail wheels with overhead road surface at passage in curved sections of overpass were investigated. Also it was determined the intervals of vertical stiffness of bearing wheels and transverse stiffness of guide wheels with the safety movement of monorail vehicle.

1. Introduction

Metropolitans are crowded with vehicles that create traffic jams. The high concentration of cars reduces not only negative effects, such as traffic noise, air pollution and traffic accidents. All this has a negative environmental impact on the quality of life. The appearance of monorail transport in metropolitan solves a number of the above-mentioned problems. This mode of transport carries passengers without delay during rush hours, compared to common urban transport, thanks to the placement of monorail overpasses at some distance above the ground. The main advantage of the monorail road is that it, like the metro, does not occupy a place on the crowded highways of the city, but unlike the metro, is cheaper to build. The monorail stricter can overcome steeper vertical slopes compared to double rail transport and is much quieter.

The monorail with the "Alweg" support system [1,2] kindles practical interest. The structural features of the monorail car were considered in V.P. Sahno and A.N. Yefymenko’s works [3,4,5]. The mathematical model of the wheeled vehicle is described in N.L. Korotenko [6], A.N. Yefymenko and V.G. Verbitsky’s works [7].

The monorail vehicle moves on pneumatic tyres (bearing wheels). Stability of monorail in longitudinal and transverse direction is provided by guide wheels. Bearing and guide wheels are structurally fixed on truck (Figure 1).

The previous article presents the results of the study of vehicle stability in straight movement [8]. The purpose of this investigation is to check the possibility of implementing the least loaded mode of movement of the monorail car in curves of rounding of the specified radius $R$ on the basis of the proposed spatial model of the monorail vehicle.
2. Mathematical model of spatial motion
During the passing of a section with a certain radius of curvature, inertia force is added to all active forces and reactions of connections. The value of inertia force is determined by the product of mass and the value of normal acceleration of the car mass center (Figure 2).
The magnitude of normal acceleration is determined by the radius of curvature in the trajectory that the mass center is passing now.

\[ a_n = \frac{V^2}{R}; \quad (1) \]

\( V \) is longitudinal velocity of the mass center, m/s;
\( R \) is radius of curvature of mass center trajectory, m.

Following is a system of differential motion equations (2) linking the plane-parallel motion equations and the vertical dynamics equations (jump \( z(t) \), pitch \( \phi(t) \), roll \( \psi(t) \)).

\[ z(t): m \cdot \ddot{z}(t)+k_{11} (-a \cdot \dot{\phi}(t)+h \cdot \dot{\psi}(t)+z(t))+k_{12} (-a \cdot \dot{\phi}(t)+h \cdot \dot{\psi}(t)+z(t))+ \]
\[ +k_{21} (b \cdot \dot{\phi}(t)-h \cdot \dot{\psi}(t)+\dot{z}(t))+k_{22} (b \cdot \dot{\phi}(t)-h \cdot \dot{\psi}(t)+\dot{z}(t))+k_{d1} \times \]
\[ \times(-a \cdot \dot{\phi}(t)-h \cdot \dot{\psi}(t)+\dot{z}(t)) \times k_{d2} \times (-a \cdot \dot{\phi}(t)+h \cdot \dot{\psi}(t)+\dot{z}(t)) \times \]
\[ +k_{d1} \times (b \cdot \dot{\phi}(t)-h \cdot \dot{\psi}(t)+\dot{z}(t)) +k_{d2} \times (b \cdot \dot{\phi}(t)-h \cdot \dot{\psi}(t)+\dot{z}(t)) = 0; \]

\[ \phi(t): Jy \cdot \ddot{\phi}(t)+\left(z(t)+hm\right) m \cdot u(t) \cdot \omega(t-k_{11} (-a \cdot \dot{\phi}(t)-h \cdot \dot{\psi}(t)+z(t))-k_{12} \times \]
\[ \times(-a \cdot \dot{\phi}(t)+h \cdot \dot{\psi}(t)+z(t)) a+k_{21} (b \cdot \dot{\phi}(t)-h \cdot \dot{\psi}(t)+\dot{z}(t)) b+k_{22} (b \cdot \dot{\phi}(t)-h \cdot \dot{\psi}(t)+\dot{z}(t)) \times \]
\[ \times k_{d1} \times (-a \cdot \dot{\phi}(t)+h \cdot \dot{\psi}(t)+\dot{z}(t)) a+k_{d2} \times (-a \cdot \dot{\phi}(t)+h \cdot \dot{\psi}(t)+\dot{z}(t)) a+k_{d2} \times \]
\[ \times (b \cdot \dot{\phi}(t)-h \cdot \dot{\psi}(t)+\dot{z}(t)) b+k_{d2} \times (b \cdot \dot{\phi}(t)-h \cdot \dot{\psi}(t)+\dot{z}(t)) b = 0; \]

\[ \psi(t): Jx \cdot \ddot{\psi}(t)+\left(z(t)+hm\right) m \cdot u(t) + 2KK \cdot \dot{\psi}(t-k_{11} (-a \cdot \dot{\phi}(t)-h \cdot \dot{\psi}(t)+z(t))+k_{12} \times \]
\[ \times(-a \cdot \dot{\phi}(t)+h \cdot \dot{\psi}(t)+z(t)) h+k_{21} (b \cdot \dot{\phi}(t)-h \cdot \dot{\psi}(t)+\dot{z}(t)) h+k_{22} (b \cdot \dot{\phi}(t)-h \cdot \dot{\psi}(t)+\dot{z}(t)) \times \]
\[ \times k_{d1} \times (-a \cdot \dot{\phi}(t)-h \cdot \dot{\psi}(t)+\dot{z}(t)) h+k_{d2} \times (-a \cdot \dot{\phi}(t)+h \cdot \dot{\psi}(t)+\dot{z}(t)) h+k_{d2} \times \]
\[ \times (b \cdot \dot{\phi}(t)+h \cdot \dot{\psi}(t)+\dot{z}(t)) h+k_{d2} \times (b \cdot \dot{\phi}(t)+h \cdot \dot{\psi}(t)+\dot{z}(t)) h = 0. \]

Initial equations system of "two-wheeled" vehicle motion along straight section of overpass is transferred toward inertial coordinate system (3), that moves at constant speed (parameter \( v \)) along center line of overpass.

Figure 3. Calculated bicycle scheme of a wheeled vehicle

Equations of plane-parallel motion take into account rescheduling of vertical loads on bearing wheels, with conditional absence of guide wheels. Vehicle motion is described by two half degrees of freedom: transverse component of mass center velocity \( u \) and angular velocity \( \omega \) relative to vertical axis.
Forces of bearing wheels slip $Y_i$ in contact spot are considered. During the movement of monorail radially of rounding $R$, front and rear trucks turn through angle $\theta = l / (2R)$ and $\theta = - l / (2R)$, accordingly, that is taken into account in determination of slip angles $\delta_{ii}$ (5).

\[
m(\omega v + \dot{u}) = \cos \theta (Y_{i1} + Y_{i2}) + \cos \theta (Y_{i1} + Y_{i2});
\]

\[
J_2 \dot{\omega} = - b \cdot \cos \theta (Y_{i2} + Y_{i2}) + a \cdot \cos \theta (Y_{i1} + Y_{i2}).
\]

The pull factor depends on the vertical load of the bearing wheel that is determined by the following ratios for each bearing wheel:

\[
ky_{i1} = (-\alpha Z_{i1} + k_{ky}) \cdot Z_{i1}; \quad ky_{i2} = (-\alpha Z_{i2} + k_{ky}) \cdot Z_{i2};
\]

\[
ky_{21} = (-\alpha Z_{21} + k_{ky}) \cdot Z_{21}; \quad ky_{22} = (-\alpha Z_{22} + k_{ky}) \cdot Z_{22};
\]

$k_{ky}$ and $\alpha$ - coefficients defined empirical for bearing wheels; $Z_{i1}$, $Z_{i2}$, $Z_{21}$, $Z_{22}$ - vertical load of bearing wheel of $i$-th truck;

\[
Z_{i1} = \frac{mg}{l} + k_{i1} \left(-a\phi(t) - h\psi(t) + z(t)\right); \quad Z_{i2} = \frac{mg}{l} + k_{i2} \left(-a\phi(t) + h\psi(t) + z(t)\right);
\]

\[
Z_{21} = \frac{mg}{l} + k_{21} \left(b\phi(t) - h\psi(t) + z(t)\right); \quad Z_{22} = \frac{mg}{l} + k_{22} \left(b\phi(t) + h\psi(t) + z(t)\right);
\]

$k_{i1}$, $k_{i2}$, $k_{21}$, $k_{22}$ - radial stiffness of bearing wheels of $i$-th truck/

The total vertical load of the twin bearing wheels $Z_1$ and $Z_2$ was defined according to the structural arrangement of the bearing wheels and their rigidity.

\[
Z_1 = Z_{i1} + Z_{i2}; \quad Z_2 = Z_{21} + Z_{22}
\]

Quiescent value of vertical reactions on bearing axes:

\[
Z_{10} = \frac{mg}{l}; \quad Z_{20} = \frac{mg}{l}.
\]

Pull angels of bearing wheels of $i$-th truck:

\[
\delta_{i1} = \theta - \arctan \left(\frac{a\omega + u}{-h\omega + v}\right); \quad \delta_{i2} = \theta - \arctan \left(\frac{a\omega + u}{h\omega + v}\right);
\]

\[
\delta_{21} = -\theta + \arctan \left(\frac{b\omega - u}{-h\omega + v}\right); \quad \delta_{22} = -\theta + \arctan \left(\frac{b\omega - u}{h\omega + v}\right);
\]

$h$ is $\frac{1}{2}$ distance between longitudinal symmetry planes of bearing wheels.
The approximation of the nonlinear dependencies of pull forces can be featured as (6).

\[
Y_{11} = \frac{0.5 \cdot ky_{11} \cdot \delta_{11}}{\sqrt{1 + (0.5 \cdot ky_{11} \cdot \delta_{11} / (\kappa \cdot \delta_{11}))^2}}; \quad Y_{12} = \frac{0.5 \cdot ky_{12} \cdot \delta_{12}}{\sqrt{1 + (0.5 \cdot ky_{12} \cdot \delta_{12} / (\kappa \cdot \delta_{12}))^2}}
\]

\[
Y_{21} = \frac{0.5 \cdot ky_{21} \cdot \delta_{21}}{\sqrt{1 + (0.5 \cdot ky_{21} \cdot \delta_{21} / (\kappa \cdot \delta_{21}))^2}}; \quad Y_{22} = \frac{0.5 \cdot ky_{22} \cdot \delta_{22}}{\sqrt{1 + (0.5 \cdot ky_{22} \cdot \delta_{22} / (\kappa \cdot \delta_{22}))^2}}
\]  

(6)

3. Results of mathematical simulation of wheel vehicle spatial model

The following numerical values of the design parameters of the wheeled vehicle were used for the research:

- \( m_1 = m_2 = 1320 \text{ kg} \) and \( m_B = 16000 \text{ kg} \) – the mass of the truck and the car accordingly;
- \( m = 18640 \text{ kg} \) – total mass of wheeled vehicle;
- \( J_y = 10000 \text{ kg} \cdot \text{m}^2 \) – the central cross moment of inertia;
- \( J_x = 10000 \text{ kg} \cdot \text{m}^2 \) – the central longitudinal moment of inertia;
- \( J_z = 234864 \text{ kg} \cdot \text{m}^2 \) – the central vertical moment of inertia;
- \( L = 12 \text{ m} \) – base of wheeled vehicle;
- \( a = b = 6 \text{ m} \) – the distance from car mass center to hinge pivots "A" and "B";
- \( h_m = 1.2 \text{ m} \) – the distance between the mass center of the car and the bearing overpass area;
- \( h_b = 0.43 \text{ m} \) – the distance between longitudinal symmetry planes of bearing wheels;
- \( l_1 = 0.204 \text{ m} \) – the distance between longitudinal symmetry plane of upper row guide wheels and bearing overpass area;
- \( l_2 = 1.296 \text{ m} \) – the distance between longitudinal plane of symmetry of lower row guide wheels and bearing overpass area;
- \( \kappa = 640000 \text{ N/m} \) – vehicle stiffness factor assigned to one truck;
- \( k_{11} = k_{12} = k_{21} = k_{22} = 477700 \text{ N/m} \) – radial stiffness of bearing wheels;
- \( k_{d11} = k_{d12} = k_{d21} = k_{d22} = 7500 \text{ N/m} \) – total coefficient of damping effect suspension over each bearing wheel;
- \( h = 0.215 \text{ m} \) – \( \frac{1}{2} \) distances between longitudinal symmetry planes of bearing wheels;
- \( R = 300 \text{ m} \) – trajectory curve radius;
- \( \kappa_1 = \kappa_2 = 0,8 \) – wheel friction coefficient in transverse direction;
- \( V = 10.20 \text{ m/s} \) – motion speed of wheeled vehicle;
- \( k_{ky} = 1.222692064 \) – empirical coefficient;
- \( \alpha = 1.19130658 \cdot 10^{-6} \text{1/N} \) – empirical coefficient.

In response to more rational selection of stiffness characteristics, it is possible to reach of significant reduction of reassignment vertical loads and comfort criteria on the speed interval up to 20 m/s when moving along curve with radius of curvature 300 m. The graph shows the corresponding characteristics of reassignment of vertical loads on double bearing wheels of front and rear trucks.
Figure 4. Vertical reactions of double bearing wheels of front (a) and rear (b) truck

Figure 5. Time dependence of roll angle (a) and sliding angle (b)

4. Conclusions
It was determined a speed range by numerical analysis of vertical dynamics monorail vehicle model in curves of constant bend radius, where the motion of the monorail circular curve is realized with the least pressure on the side of guide wheels (without them), that will affect only realized roll angles. So for radius $R = 300$ m the possible interval of travel speed is up to 20 m/s. Simulating the spatial model of the monorail vehicle in circular curves, it was taking into account the effects of load reassignment on the factors of run out of bearing wheels’ model, the given results illustrating the rearrangement of vertical support pressure and the measure of the roll and pitch angle in the system. Proposed model of
spatial motion allows at design stage to make rational selection of elastic and damping characteristics of wheel vehicle suspension providing required level of safety and comfort during exploitation.

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