A Modeling of Battery Discharge Within Different Temperature Operating Conditions

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Abstract. The article presents the results of investigations including the battery characteristics in conditions of changing temperature conditions. The possibility of applying logistic functions for approximation depending on the capacity of the battery from the load current is substantiated. An analytical overview of approaches to power modeling depending on the type of battery is presented. The simulation of the discharge process of the CR2 battery in the MATLAB & Simulink™ environment was carried out.

Keywords: Battery, Electrical Capacity, Discharge Current, Operating Conditions, Logistic Function, MATLAB & Simulink™

1. Introduction
The development of devices using power sources in the form of rechargeable batteries is increasing annually. The global consumer battery market is projected to reach $ 50 billion by 2025 [1]. Electric batteries are actively used in portable electronics and industry. In that connection, power supplies for the Internet of Things (IoT) peripherals, smart homes have an interest for researches.

Manufacturers maximize capacity, service life while reducing weight and dimensions. The most promising in the near future are lithium-ion batteries (Figure 1). Compared to other rechargeable batteries, such as Ni-Cd, Ni-MH and lead-acid battery, the lithium-ion battery has a high energy density and power, long life [2].

When operating rechargeable power supplies, it is necessary to consider the discharge processes, including the evaluate energy reserves. Various analytical and empirical characteristics are developed and used to describe these processes. These models describe the change in parameters (State of Charge - SoC, State of Health - SoH, etc.) of autonomous power sources depending on the load and operating conditions.

For NiCd batteries, the following methods [3] for approximating discharge functions are typically used.

The Peukert law in general view [4,5] is:

\[ C = \frac{A}{i^n} \]

where: \( C \) is discharge capacity;
\( i \) is discharge current;
\( A, n \) are empirical coefficients.
This expression loses its significance when currents tend to zero. In this connection, in practice, the Peukert ratio is more often used:

$$C = \frac{A}{1+B\cdot i^n},$$  \[1\]

where: $C$ is discharge capacity;
$i$ is discharge current;
$A, B, n$ are empirical coefficients.

Libenov’s Equation [6] is:

$$C = \frac{A}{1+B\cdot i},$$  \[2\]

where: $C$ is discharge capacity (A*h);
$i$ is discharge current (A);
$A, B$ are empirical coefficients.

Aguf’s Equation [6] is:

$$C = a0 + \frac{a1}{i} + \frac{a2}{i^2} + \frac{a3}{i^3} + \cdots,$$  \[3\]

where: $C$ is discharge capacity (A*h);
$i$ is discharge current (A);
$a1, a2, a3$ are empirical coefficients.

Korovin-Skundin’s Equation [7] is:

$$C = \frac{A}{i^n} th\left(\frac{i^n}{B}\right),$$  \[4\]

where: $C$ is discharge capacity (A*h);
$i$ is discharge current (A);
$A, B$ are empirical coefficients.
To approximate the discharge functions of alkaline batteries, the dependences of Shepherd, Haskina-Danilenko, Romanova [6] are used.

The disadvantages of the above approximating functions of the lithium batteries discharge, which are currently the "standard" due to their high power density, include insufficient accuracy [8]. In this connection, it is of interest to study the logical function for approximating these dependencies in order to increase the accuracy of modeling this process.

2. Approximation of the battery discharge using the logistic function

2.1. Experimental data

For research, the claimed battery characteristics [9] Tekcell (Manufacture Vitzrocell Co. Ltd, r. Saint-Peterburg, Russian Federation) were selected.

Table 1. Technical specification of the CR2 battery

| Characteristic                                      | Value                |
|---------------------------------------------------|----------------------|
| Nominal Voltage                                   | 3,0 V                |
| Nominal Capacity (at the 10 mA, 20 °C ; cutoff 2,0| 850 mA*h             |
| Weight                                            | 11,5 g               |
| Operational temperature                          | -30°C … +60 °C       |

According to the claimed technical characteristics [9], this battery has the following parametric dependence of capacity on current consumption:

Figure 2. The diagram of the CR2 battery capacity and current [9]
2.2. Linearization of the logistic function

The logistic function (sigmoid) has the following form:

\[ y = 1 - \frac{K}{1 + b e^{-c x}}, \]  \[5\]

To convert the function to the linear form, it is necessary to perform several transformations:

\( \frac{1 - y}{K} - 1 = b e^{-c x}, \)

[6]

Logarithm of the expression [6], we obtain:

\[ \ln\left( \frac{1 - y}{K} - 1 \right) = \ln(b) - c \cdot \ln(x), \]  \[7\]

Introducing new variables \( z = \ln\left( \frac{1 - y}{K} - 1 \right); \) \( A = \ln(b); \) \( B = c; \) \( t = \ln(x) \) into expression [7], we obtain the classical linear regression equation:

\[ Z = A - B \cdot t. \]  \[8\]

The result of the linear function parameterization [8] by the least-squares method are the formulas:

\[ B = \frac{\bar{Y}\bar{X} - \bar{X}\bar{Y}}{\bar{X}^2 - (\bar{X})^2}; \quad A = \bar{Y} - B \cdot \bar{X} \]  \[9\]

where \( \bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i; \) \( \bar{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i; \) \( \bar{X} \cdot \bar{Y} = \frac{1}{n} \sum_{i=1}^{n} X_i Y_i. \)

After calculating the parameters A and B, the inverse transformation of the variables is performed to obtain the coefficients b and c of the function [5].

2.3. Verification of modeling results

To verify the obtained analytical dependencies MATHLAB & Simulink™ was used. Curve Fitting Tool was involved. This package has the following features:

- primary processing of data, such as division into regions and smoothing;
- parametric and nonparametric approximation with the possibility of using function libraries or creating your own function;
- standard linear least squares method, nonlinear least squares method, weighted least squares method, constrained least squares method and stable approximation methods;
- the ability to assess the quality of the approximation.

3. Comparative analysis of the obtained empirical dependence

Based on the results of the study, the following results were obtained:

Table 2. Experimental data for approximating the dependence of the relative capacitance on the load current at temperature -20 °C

| Approximate Graph | Goodness of fit |
|-------------------|-----------------|
| SSE: 0.03345      | R-square: 0.9596|
|                   | Adjusted R-square: 0.9546|
|                   | RMSE: 0.04573   |

![Approximate Graph](image-url)
Tables 2-5 present the simulation results of the dependence of the relative capacity on the load current value of the CR2 battery. To approximate the s-shaped lines (Figure 2), a logistic function was adopted.

Table 3. Experimental data for approximating the dependence of the relative capacitance on the load current at temperature 0 °C

| Approximate Graph | Goodness of fit |
|-------------------|-----------------|
| ![Approximate Graph](image) | SSE: 0.01244  
R-square: 0.9685  
Adjusted R-square: 0.9646  
RMSE: 0.02788 |

Table 4. Experimental data for approximating the dependence of the relative capacitance on the load current at temperature +20 °C

| Approximate Graph | Goodness of fit |
|-------------------|-----------------|
| ![Approximate Graph](image) | SSE: 0.003763  
R-square: 0.9663  
Adjusted R-square: 0.9621  
RMSE: 0.01534 |

Table 5. Experimental data for approximating the dependence of the relative capacitance on the load current at temperature +60 °C

| Approximate Graph | Goodness of fit |
|-------------------|-----------------|
| ![Approximate Graph](image) | SSE: 0.002947  
R-square: 0.9558  
Adjusted R-square: 0.9503  
RMSE: 0.01357 |
By the methodology (Section 2.2), linearization was carried out, the values of the coefficients were determined. The data obtained were investigated in the Curve Fitting Tool MATLAB & Simulink™.

To assess the accuracy of the approximating model, standard SSE (Sum Squared Error) criteria were used; R-square; Adjusted R-square, RMSE (Root Mean Square Error). The presented data show the preference for using the logistic function in comparison with other ratios (1-4). The standard deviation in assessing the relative capacitance is 0.013-0.046. This indicator is lower than in the case of applying the equations of Peikert (0.02-0.03), Libenov (0.03-0.05), Aguf (0.01-0.02) [10]. In this connection, we can conclude that the use of a logistic function for the approximation of discharge processes of rechargeable batteries.

4. Conclusion
The objective of the research presented in the article was a comparative analysis of the dependences of the parameters of electric batteries obtained during the simulation. In particular, modeling of the process of discharging the battery when changing the temperature conditions of operation was considered. Based on the analysis of open sources, the methods used for approximating these dependencies were identified, their advantages and disadvantages were determined. We continued to use the s-shaped logistic function, which generally describes these graphical dependencies, to model this process. The simulation of the discharge process of the CR2 battery was carried out in MATLAB & Simulink™.

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Reference:
[1] Anon Consumer Batteries: Global Market Insights and Projections to 2025
[2] Ahmadi L, Young S B, Fowler M, Fraser R A and Achachlouei M A 2017 A cascaded life cycle: reuse of electric vehicle lithium-ion battery packs in energy storage systems Int. J. Life Cycle Assess. 22 111–24
[3] Galushkin N E, Yazvinskaya N N and Galushkina I A 2012 Analysis of the use of empirical relations for assessing the capacity of a CFT NiCd battery of a long discharge mode Basic Res. 11 1180–4
[4] Xie J, Ma J and Chen J 2018 Peukert-equation-based state-of-charge estimation for LiFePO4 batteries considering the battery thermal evolution effect Energies 11 1–14
[5] Hausmann A and Depcik C 2013 Expanding the Peukert equation for battery capacity modeling through inclusion of a temperature dependency J. Power Sources 235 148–58
[6] Galushkin N E and Galushkina I A 2005 An analysis of empirical relationships describing a discharge of alkaline batteries Electrochem. Energy 5 43–50
[7] Korovin N V and Skudin A M 2003 Chemical Power Supply Sources: Handbook (Moscow: Moscow Power Energy Institute Publ.)
[8] Traub L 2016 Calculation of Constant Power Lithium Battery Discharge Curves Batteries 2 17
[9] Anon Datasheet CR2 Li Battery
[10] Galushkin N E, Yazvinskaya N N and Galushkina I A 2013 Generalized Model of the Dependence of the Capacity of NiCd Batteries on the Discharge Current Electrochem. Energy 13 96–102