Spin-orbit coupling measurement by the scanning gate microscopy

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We propose a procedure for extraction of the Fermi surface for a two-dimensional electron gas with a strong Rashba spin-orbit coupling from conductance microscopy. Due to the interplay between the effective spin-orbit magnetic field and the external one within the plane of confinement, the backscattering induced by a charged tip of an atomic force microscope located above the sample, leads to the spin precession, and thus to the spin mixing of the incident and reflected modes. This mixing leads to a characteristic angle-dependent beating pattern visible in the conductance maps.

We show that the structure of the Fermi level, bearing signatures of the spin-orbit coupling, can be extracted from the Fourier transform of the interference fringes in the conductance maps as a function of the magnetic field direction. We propose a simple analytical model which can be used to fit the experimental data in order to obtain the spin-orbit coupling constant.

Introduction. Charge carriers in semiconductors are subject to spin-orbit (SO) interactions [1] due to electric fields or anisotropy of the crystal lattice. The consequences of these interactions, including spin relaxation and dephasing [2–4], spin Hall effects [5,6], topological insulators [7], persistent spin helix states [9,11], Majorana fermions [12] etc., are extensively studied e.g., A spin-active devices, including spin-filters based on quantum point contacts (QPCs) [13], spin transistors [14–18], exploiting the precession of the electron spin in the SO effective magnetic field [19], are well known examples. The most popular playground for spin effects and spin-active devices is the two-dimensional electron gas (2DEG) in III–V heterostructures, which show a strong built-in electric fields in the confinement layer, giving rise to the Rashba SO coupling [20]. The knowledge of the SO interaction strength is of fundamental importance for description of spin devices and phenomena. The measurements of the SO coupling constant are usually analyzed from the Shubnikov-de Haas [21–23] oscillations, antilocalization as observed in the magnetotransport [24], photocurrents [25], or precession of optically polarized electron spins as a function of their drift momentum [26].

The SO coupling produces a shift of the spin-up and spin-down dispersion relations on the wave vector scale [1] that is a linear function of the SO coupling constant. In this Letter we propose a way to extract the structure of the dispersion relation near the Fermi level [1] using spin-dependent scattering and the resulting interference with the scanning gate microscopy [32–34] (SGM) applied to systems with QPCs [34,35]. In this technique, the tip acts as a floating perturbation of the potential landscape as seen by the Fermi level electrons. As a result the recorded SGM images contain interference fringes due to the incident and backscattered electron waves [36,37]. In presence of an in-plane magnetic field the fringes form beating pattern due to spin-dependence of the Fermi wavelengths [38]. In this Letter we analyze the beating patterns that appear for SO-coupled systems. The electron – when scattered – experiences precession of its spin due to rotation of the momentum-dependent effective magnetic field [31], and the interference of the incident and reflected electron waves potentially involves spin-mixing effects. However, we find that in the absence of the external magnetic field the backscattering involves a pure inversion of the effective field with no precession effect. The latter are triggered by an external in-plane magnetic field, and lead to an appearance of the dependence of the beating patterns on the orientation of the magnetic field. We demonstrate that the shape of the Fermi level structure and thus the SO coupling constant can be traced back from the beating patterns by Fourier transform analysis.

Theory. We consider Fermi level transport in a 2DEG with a local constriction QPC as depicted in Fig. 1. The Fermi level electrons travel from the electron reservoir placed at $x < 100$ nm through a channel modeled with an infinite potential step and an additional potential tuned by gates (gray areas of the scheme). A negatively charged tip acts as a backscatterer to the right of the QPC. The conductance maps as functions of the tip position resolve the coherent interference fringes as observed in a number of experiments [34,36,37,39,40]. The part of the system to the right of the QPC is considered open such that electron may freely propagate without reflections. Transparent boundary conditions for the electron flow are introduced with a method described in Ref. [41].

We adopt a standard two-dimensional model assuming that all the electrons of 2DEG occupy a strongly localized lowest-energy state of the vertical quantization. The Hamiltonian accounts for the Rashba SO interaction and a presence of the external magnetic field applied within the plane of confinement

$$H = \left[ \frac{\hbar^2}{2m}\mathbf{k}^2 + eV_{\text{ext}}(\mathbf{r}) \right] \mathbf{I} + \frac{1}{2} g_{\mu_B} \mathbf{B} \cdot \mathbf{\sigma} + H_{\text{Oh}}$$  \hspace{1cm} (1)
The scattering problem is solved within the finite difference approach \[47\], with spatial discretization \[46\] with spatial discretization. Zeeman term. The magnetic field enters the Hamiltonian only via the spin form of the potential results from the screening of the electrostatic confinement of the 2DEG in the growth. The Rashba Hamiltonian \[H_{\text{rsb}} = \gamma \{ \sigma_x k_y - \sigma_y k_x \} \] in Eq. \[1\] comes from the electrostatic confinement of the 2DEG in the growth direction \[15\]. We apply the symmetric gauge \[\mathbf{A} = (B_y z, -B_z x, 0)\]. By choosing the plane of the 2DEG confinement to be located at \(z = 0\), we get \(\mathbf{A} = 0\), and the magnetic field enters the Hamiltonian only via the spin Zeeman term.

The scattering problem is solved within the finite difference approach \[16\] with spatial discretization \(\Delta x = \Delta y = 6\) nm using the wave function matching (WFM) method \[17\]. Then we calculate conductance \(G\) using the Landauer approach by evaluating \(G = G_0 \sum_{\tau} T_{\tau}\) at the Fermi level (with \(G_0 = \frac{\pi e^2}{2 h}\)). For simplicity, we consider the case of single mode transmitting through the QPC \((G \leq 2G_0)\) (see the inset to Fig. 1). We set \(E_F = 20\) meV (for \(\gamma = 0\) the Fermi wavelength is \(\lambda_F = 40\) nm), and the tip potential \(V_t = 40\) meV for which a strict depletion of the electron density below the tip is obtained (see dashed circle in Fig. 1). Landé factor is assumed to be \(g = 9\) and effective mass \(m_{\text{eff}} = 0.0465m_0\) as for InGaAs.

**Results and Discussion.** Figs. 2(a-f) show spatial derivatives of SGM images \(dG/dx_{\text{tip}}\) obtained for QPC tuned to the first QPC conductance plateau. For \(B = 0\) and \(\gamma = 0\) [Fig. 2(a)] a pronounced interference pattern of the incident and backscattered wave is observed \[34\, 37\] with the period of \(\lambda_F/2\) for both \(\gamma = 0\) [Fig. 2(a)] and \(\gamma \neq 0\) [Fig. 2(b)]. A beating pattern \[38\] appears at non-zero \(B\) [Fig. 2(c)], which depends on the orientation of the in-plane field for \(\gamma \neq 0\) (Figs. 2(d-f)).
where \( \alpha_{x/y} = \frac{1}{2} g \mu_B B_{x/y} \) and \( E_{\text{kin}} = \frac{\hbar^2 k^2}{2m_{\text{eff}}} \). Plain wave solution for the Schrödinger equation gives two eigenvalues

\[
E_\sigma = \frac{\hbar^2 k^2}{2m_{\text{eff}}} + \sigma |p|,
\]

where \( p = (\gamma k_y + \alpha_x, -\gamma k_x + \alpha_y) \), with \( \sigma = \{+, -\} \) and

\[
|k_\sigma^\pm| = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \sigma \frac{p^x + ik_y}{p^2} \\ \sigma \frac{p^x - ik_y}{p^2} \end{array} \right),
\]

where \( p^\pm \) denotes the value of \( p \) vector, are eigenvectors for incoming + and outgoing directions – of an electron. Due to the assumed infinite potential generated by the SGM tip, the scattering wave function in Eq. (2) has to vanish at \( r = 0 \) (see Fig. 2)

\[
\Psi_\sigma (r = 0) = |k_\sigma^+\rangle + \sum_{\sigma'} a_{\sigma \sigma'} |k_\sigma^-\rangle = 0.
\]

By substituting Eq. (5) to this equation one evaluates the scattering amplitudes \( a_{\sigma \sigma'} \).

For the simplest case when SOI and magnetic field are not present in the Hamiltonian (3) the propagating modes in Eq. (5) reduce to

\[
|k_\sigma^+\rangle = |k_+\rangle = \left( \begin{array}{c} 1 \\ 0 \end{array} \right), \quad |k_\sigma^-\rangle = |k_-\rangle = \left( \begin{array}{c} 0 \\ 1 \end{array} \right),
\]

with \( k_\sigma^\pm = k \) and scattering amplitudes \( a_{\sigma \sigma'} = -\delta_{\sigma \sigma'} \), from which one finds that reflection does not change the spin orientation. The scattering wave function from Eq. (2) is then

\[
|\Psi_\sigma\rangle = e^{ikr} |k_\sigma^+\rangle - e^{-ikr} |k_\sigma^-\rangle = (e^{ikr} - e^{-ikr}) |k_\sigma\rangle,
\]

and the scattering density is given by \( \rho_\sigma = \langle \Psi_\sigma | \Psi_\sigma \rangle \propto \cos (2kr) \), and the variation of the \( G \) map follows the pattern of the density [38]. The SGM conductance pattern can be approximated by \( G(r_{\text{tip}}) \propto \cos (2kr) \). The SGM image obtained with this model is presented in Fig. 3(a) and is consistent with the simulated image obtained in Fig. 2

![Figure 3](image-url)

Figure 3. Sketch of considered scattering process. The electron wave leaves QPC in one of two spin states, propagates to the right and is backscattered at position \( r = (0,0) \) by the potential wall barrier induced by the SGM tip. Here we assume a hard wall potential profile (i.e. \( V_{\text{tip}} = +\infty \) inside the circle).

For \( B = 0 \) and \( \gamma \neq 0 \) one may easily check that the propagating modes [Eq. (5)] still satisfy orthogonality relations \( \langle k_{\sigma'}^+ | k_\sigma^- \rangle = \delta_{\sigma \sigma'} \), and \( |k_\sigma^\pm\rangle \), which leads to the spin conserving reflection \( a_{\sigma \sigma'} = -\delta_{\sigma \sigma'} \). However in this case \( k_\sigma^\pm \neq k_\sigma^- \) and the scattering wave function is given by \( |\Psi_\sigma\rangle = (e^{ikr} - e^{-ikr}) |k_\sigma\rangle \). The electron density is then proportional to \( \rho_\sigma \propto \cos (2kr) \), and the variation of the density [38] discussed in this paper.

The third possible configuration of parameters i.e. \( \gamma = 0 \) and \( B \neq 0 \) was recently discussed in Ref. [38]. In this case the same orthogonality relation is still satisfied \( \langle k_{\sigma'}^+ | k_\sigma^- \rangle = \delta_{\sigma \sigma'} \), and \( a_{\sigma \sigma'} = -\delta_{\sigma \sigma'} \). However, the resulting electron density is now proportional to \( \rho_\sigma \propto \cos (2kr) \), and depends on the spin via the Zeeman term in Eq. (4) inducing shifts of \( k_\sigma \). The approximated SGM map \( G = G_0 \sum_\sigma T_\sigma \cos (2kr) \) gives a signal being a superposition of two frequencies \( \omega_\sigma = 2k_\sigma \) resulting in the beating pattern visible in Fig. 4(c). The present reasoning explains the findings of Ref. [38].

In a general case of \( B \neq 0 \) and \( \gamma \neq 0 \) the eigenvalues of \( H \) depend on both the direction of magnetic field and the propagation vector, thus the spin will not be conserved anymore during the backscattering process, since the orthogonality relations between the incident and backscattered modes no longer hold \( \langle k_{\sigma'}^+ | k_\sigma^- \rangle \neq \delta_{\sigma \sigma'} \), and \( a_{\sigma \sigma'} \neq -\delta_{\sigma \sigma'} \). The resulting electron density will be then a composition of four different possible superposition of the Fermi vector waves \( k_i = \{ k_+ + k_\gamma, k_+ + k_\gamma, k_+ + k_\gamma, k_+ + k_\gamma \} \). The SGM images obtained for this general case for three different orientation of magnetic field \( \alpha = \{0^\circ, 45^\circ, 90^\circ \} \) are depicted in Figs. 4(d-f). Although, the images differ somewhat
from Fig. 4 (d-f), still both the model and the full simulation allow for extraction of the wave vectors and their dependence on the orientation of the magnetic field in the Fourier analysis (see below).

The form of Eq. (3) indicates that rotation of a SGM tip position along the arc centered at the QPC entrance is equivalent to a rotation of the in-plane magnetic field (in an opposite direction) for a fixed tip position. For a practical implementation of an experiment it should be more efficient to perform a SGM scan along a straight line, where the longest electron branch \[ | \lambda k/2 \rangle \] is present and rotate the magnetic field instead (see Fig. 5(a)).

In Fig. 5(b-c) we present the Fourier transform (FT) of (a) remapped from \( k \) space to \( \lambda \) space measurement of conductance as a function of the tip position involving spin-scattering in a crossed external and built-in magnetic fields. The backscattering taken along the axis of the QPC involves \( k_y = 0 \) and we find in general four various values of \( k_x \) visible as four lines in FT images. However, when \( B_y = 0 \), Eq. (4) reduces to

\[
E_x = \frac{h^2 k_x^2}{2m} + \sigma \sqrt{\alpha_x^2 + (\gamma k_x)^2},
\]

which is symmetric with respect to electron reflection \( E_x(k_x) = E_x(-k_x) \), which implies the symmetry of the scattering process that \( k_x^+ = k_x^- = k_x \) (see Fig. 6(a)), and thus reducing the number of resonance lines in FT image to two. For other cases presented in Figs. 6(b-c) this symmetry is not satisfied and all four frequencies are visible.

Summary. In summary, we have shown that SGM imaging can be used to extract the Fermi surface properties by Fourier analysis of the beatings due to the SO interaction and an in-plane magnetic field. The analysis allows for deduction of the Rashba constant from the real space measurement of conductance as a function of the tip position involving spin-scattering in a crossed external and built-in magnetic fields.

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