1. Introduction

The spur gears are widely used in many mechanical constructions, that is why the tooth contact research of them is important because of the geometric development and optimization [5, 6, 10, 13] (Fig. 1).

The main steps of the gear generation process [1–7, 9, 10, 12, 13]:

- **Gear designing**: determination of the geometric parameters, calculation of the profile points, creation of the computer aided model (CAD), TCA analysis, etc.
- **Gear manufacturing**: selection of the necessary technology, selection of the necessary cutting
tool, determination of the main steps of the manufacturing process, etc.

- Gear measuring process: beat examination, real connection analysis, noise and vibration analysis, etc.

The aim of the TCA (Tooth Contact Analysis) is the determination and analysis of the mechanical parameters of the gear connection (normal stress, normal, deformation, normal, strain, etc.) [7–9]. Based on these results the evaluations of the disposition of the established mechanical parameters are very important because of the appropriate selection of the usable gear pair for a given technological task [7–9]. The results are given a way for the other construction development possibilities of the gears.

2. Designing of spur gears having different base profile angles

The tooth connection always occurs on the common normal line of the tooth arc. This line is called line of action. This line is a tangent line of both base circle di-

![Fig. 2. Definition of the base profile angle](image)

The main property of the x-zero gear drive is the standard centre distance along which the elements could be connected. In this case the rolling diameter and the pitch circle diameter are the same [6, 9, 10, 12, 13]. Knowing the pitch circle diameters the base circle diameters could be calculated [6, 12, 13]:

\[
d_{b_{1,2}} = d_{1,2}\cos\alpha_0.
\]

The equations of the involute profile curves (Fig. 3):

\[
x_i = \frac{d_1\cos\alpha_0}{2}(\cos\beta + \beta\sin\beta),
\]

\[
= \frac{\cos\alpha_0}{2}(\sin\beta - \beta\cos\beta).
\]

In case of the determination of the profile curve the \(\beta\) parameter is the changing parameter which has to be changed between the base circle diameter and the tip circle diameter of the given gear.

Five types of gear pairs have been designed by our developed computer software [1, 2]. Knowing the
necessary designing formulas and based on the input gear parameters (tooth numbers and module) the software could calculate the necessary gear parameters and determine the involute profile curve. This software was helpful in the modelling and designing process [1, 2] (Fig. 4).

Table 1. The calculated parameters of the gear pairs

| The main parameters of the gear pairs          | Gear drive I | Gear drive II | Gear drive III | Gear drive IV | Gear drive V |
|-----------------------------------------------|-------------|--------------|----------------|--------------|-------------|
| Base profile angle ($\alpha_0$) [°]           | 14.5        | 15           | 18             | 20           | 22          |
| Axial module (mm)                             |             |              |                |              |             |
| Number of tooth of the driving gear ($z_1$)   |             |              |                |              |             |
| Number of tooth of the driven gear ($z_2$)    |             |              |                |              |             |
| Standard centre distance ($a_0$) (mm)         |             |              |                |              |             |
| Addendum ($h_a$) [mm]                         |             |              |                |              |             |
| Bottom clearance ($c$) [mm]                   |             |              |                |              |             |
| Dedendum ($h_f$) [mm]                         |             |              |                |              |             |
| Circular pitch ($t_0$) [mm]                   |             |              |                |              |             |
| Backlash ($j_s$) [mm]                         |             |              |                |              |             |
| Whole depth ($h$) [mm]                        |             |              |                |              |             |
| Working depth ($h_{wp}$) [mm]                 |             |              |                |              |             |
| Tooth thickness ($S_{w1}$) [mm]               |             |              |                |              |             |
| Pitch circle diameter of the driving gear ($d_1$) [mm] | 193.62    | 193.18       | 190.21         | 187.93       | 185.43      |
| Tip circle diameter of the driving gear ($d_{a1}$) [mm] | 200        |              |                |              |             |
| Root circle diameter of the driving gear ($d_{r1}$) [mm] | 175        |              |                |              |             |
| Basic circle diameter of the driven gear ($d_n2$) [mm] | 290.44    | 289.77       | 285.31         | 281.90       | 278.15      |
| Pitch circle diameter of the driven gear ($d_2$) [mm] | 300        |              |                |              |             |
| Tip circle diameter of the driven gear ($d_{a2}$) [mm] | 320        |              |                |              |             |
| Root circle diameter of the driven gear ($d_{r2}$) [mm] | 275        |              |                |              |             |
| Transmission ratio ($i$)                       |             |              |                |              | 1.5         |
The base profile angles have been selected following the suggestions of [5, 6, 12, 13]. After the calculation and the profile determination the CAD model of the gear pairs has been generated by SolidWorks software (Fig. 5). Interpolation B spline curve has been fixed to the received profile points.

3. Analysis of the TCA parameters by the modification of the base profile angle

After the generation of the CAD models of the analyzed gear pairs the following step is the adoption of

![Fig. 5. Generation of the CAD model of the gear pairs](image)

![Fig. 6. Adoption of the finite element meshing](image)

![Fig. 7. Normal stress distribution on the contact zone and the surfaces of the gear pairs. \((m = 10 \text{ mm, } z_1 = 20, z_2 = 30, \alpha_0 = 18^\circ)\)](image)
the necessary coordination systems for the analysis. Two coordinate systems are needed on the axis of rotation of the gears and one other coordination system is needed on the tooth contact zone [7–10]. The one axis of the latter coordinate system would be perpendicular to the profile surface of the gear [7–10]. It is important because of the analysis of the normal mechanical parameters.

For the analyses fine triangle meshing has been applied on the tooth contact zone (Fig. 6) [7–9]. The applied meshing density has been 0.8 mm. On the contact zone structural meshing has been applied by sphere of influence’s structure. On the outside zones automatic meshing has been applied (Fig. 6).

The adopted surface meshing has been divided equally along the tooth length by the sweep operation (Fig. 6).

| Table 2. Parameters of the applied material (structural steel) |
|---------------------------------------------------------------|
| Density            | 7850 kg/m³             |
| Yield limit        | 250 MPa                |
| Ultimate strength  | 460 MPa                |

The driving tooth gear has been loaded by 700 Nm moment. Five degrees of freedom have been fixed on the driving gear, only the turning motion around the axis of rotation has been let. The driven gear has been totally fixed.

3.1. Analysis of the normal stress

The normal stress is interpreted on perpendicular direction of the tooth surface [7–9, 11, 14, 15]. This direction is the most determinative from the aspect of

![Fig. 8. Normal stress results for every gear pairs](image)

\( \sigma_n = -6.223 \text{ MPa} \)  \( \sigma_n = -5.065 \text{ MPa} \)

\( \alpha_0 = 14.5^\circ \)

\( \sigma_n = -3.299 \text{ MPa} \)  \( \sigma_n = -3.843 \text{ MPa} \)

\( \alpha_0 = 15^\circ \)

\( \sigma_n = -5.193 \text{ MPa} \)  \( \sigma_n = -5.135 \text{ MPa} \)

\( \alpha_0 = 18^\circ \)
tooth deformation. This normal stress has been calculated for the tooth contact zone (Fig. 7).

In Figs 8 and 9 the normal stress distributions can be seen in the function of the base profile angle. According to the normal stress the Gear drive II (Table 1) is the most appropriate because the normal stress in absolute value is the lowest in this case (Fig. 9). The highest normal stress values are applied in the case of the application of Gear drive I. (Fig. 9).

3.2. Analysis of the normal elastic strain

The normal elastic strain has been calculated for the tooth contact zone (Fig. 10). It is defined on perpendicular direction for the tooth surface [7–9, 11, 14, 15].

In Figs 11 and 12 the normal elastic strain distributions can be seen in the function of the base profile angle. According to the normal elastic strain the Gear drive II (Table 1) is the most appropriate because the

\[
\sigma_n = -5.556 \text{ MPa} \quad \sigma_n = -4.842 \text{ MPa}
\]

\[
\alpha_0 = 20^\circ
\]

\[
\sigma_n = -4.038 \text{ MPa} \quad \sigma_n = -4.133 \text{ MPa}
\]

\[
\alpha_0 = 22^\circ
\]
normal elastic strain in absolute value is the lowest in this case (Fig. 12). The highest normal stress values are applied in the case of the application of Gear drive IV. (Figure 12).

3.3. Analysis of the normal deformation

The normal deformation has been analyzed into the $x$ direction, which is perpendicular to the contact surfaces [14]. This direction is the most determinant because the main deformation is applied into the perpendicular direction of the contact surfaces [4, 9] (Fig. 13).

In Figs 14 and 15 the normal deformation distributions can be seen in the function of the base profile angle. According to the normal deformation the Gear drive II (Table 1) is the most appropriate because the normal deformation in absolute value is the lowest in...
Fig. 11. Normal elastic strain results for every gear pairs

| Driving gear | Driven gear |
|--------------|-------------|
| $\bar{\varepsilon}_n = -0.000024$ | $\bar{\varepsilon}_n = -0.0000202$ |
| $\alpha_0 = 14.5^\circ$ |
| $\bar{\varepsilon}_n = -0.0000134$ | $\bar{\varepsilon}_n = -0.0000184$ |
| $\alpha_0 = 15^\circ$ |
| $\bar{\varepsilon}_n = -0.0000223$ | $\bar{\varepsilon}_n = -0.0000197$ |
| $\alpha_0 = 18^\circ$ |
| $\bar{\varepsilon}_n = -0.0000225$ | $\bar{\varepsilon}_n = -0.0000193$ |
| $\alpha_0 = 20^\circ$ |
| $\bar{\varepsilon}_n = -0.0000237$ | $\bar{\varepsilon}_n = -0.0000205$ |
| $\alpha_0 = 22^\circ$ |
**Fig. 12a.** The normal stress results in the function of the base profile angle, driving gear

**Fig. 12b.** The normal stress results in the function of the base profile angle, driven gear

**Fig. 13.** Normal deformation distribution on the contact zone and the surfaces of the gear pairs. \( m = 10 \text{ mm}, z_1 = 20, z_2 = 30, \alpha_0 = 18^\circ \)
Fig. 14. Normal deformation results for every gear pairs

| Driving gear | Driven gear |
|--------------|------------|
| $\bar{u}_x = -0.00322$ mm | $\bar{u}_x = -0.00286$ mm |
| $\alpha_0 = 14.5^\circ$ | |
| $\bar{u}_x = -0.00216$ mm | $\bar{u}_x = -0.00203$ mm |
| $\alpha_0 = 15^\circ$ | |
| $\bar{u}_x = -0.00308$ mm | $\bar{u}_x = -0.00282$ mm |
| $\alpha_0 = 18^\circ$ | |
| $\bar{u}_x = -0.00311$ mm | $\bar{u}_x = -0.00281$ mm |
| $\alpha_0 = 20^\circ$ | |
this case (Fig. 14). The highest normal deformation values are applied in the case of the application of Gear drive III. and IV. (Fig. 15).

4. Conclusion

The aim of this research is the analysis of the TCA parameters (normal stress, normal elastic strain, normal...
deformation) in the function of the base profile angle. This angle influenced the shape of the connection impression and the mechanical parameters on the connection zone, that is why the appropriate selection is important in the case of geometric designing.

Previously, the geometric designing and the CAD modelling of the x-zero gear drives were needed for the designing of the analysis. We have worked out a new type computer program with which the designing process of the gear pairs could be eased.

After the calculation of the geometric parameters and the profile points the following step is the CAD modelling process. Interpolation B-spline curve has to be fit on the calculated profile points. Using of this designing process five types of x-zero gear drive have been designed with different base profile angles. All the other parameters have been the same on every gear.

Before the TCA the adoption of the necessary coordinate systems and the setting of the load and boundary conditions are necessary. The adoption of the finite element mesh is needed for the analysis, but dense meshing is suggested into the contact teeth because of the more accurate results.

Based on the received TCA parameters of the gear pairs we can determine that the application of the 15° base profile angle is the best choice in the case of the analyzed gear pairs. By using this angle the TCA parameters will be lower than in the other cases.

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