Axion model in gauge-mediated supersymmetry breaking and a solution to the $\mu/B\mu$ problem

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ABSTRACT: We present a simple supersymmetric axion model that can naturally explain the origin of the Higgs $\mu$ and $B\mu$ terms in gauge mediation while solving the strong CP problem. To stabilize the Peccei-Quinn scale, we consider mixing between the messenger fields that communicate the supersymmetry and Peccei-Quinn symmetry breaking to the visible sector. Such mixing leads to the radiative stabilization of the Peccei-Quinn scale. In the model, a Higgs coupling to the axion superfield generates the $B$ parameter at the soft mass scale while a small $\mu$ term is induced after the Peccei-Quinn symmetry breaking. We also explore the phenomenological and cosmological aspects of the model, which crucially depend on the saxion and axino interactions with the ordinary particles induced by the Higgs coupling to the axion superfield.

KEYWORDS: Supersymmetry breaking, Supersymmetric Standard Model
1. Introduction

Gauge mediation of supersymmetry (SUSY) breaking \[1, 2, 3\] is an attractive mechanism to generate soft terms in the minimal supersymmetric Standard Model (MSSM). In particular, since the transmission occurs through the gauge interactions, models of gauge mediation solve the SUSY flavor and CP problems. However, gauge mediation has difficulty in explaining the origin of the Higgs $\mu$ and $B\mu$ terms, and thus requires an extension. Since the $\mu$ term breaks the Peccei Quinn (PQ) symmetry \[4\], an interesting possibility is that the presence of a $\mu$ term has the same origin as the invisible axion \[5\] solving the strong CP problem \[6, 7, 8, 9\]. The size of the $B\mu$ term is then determined by how the saxion, the scalar partner of the axion, is stabilized. A potential for the saxion is generated only after SUSY breaking because the PQ symmetry makes the scalar potential flat along the saxion direction in the supersymmetric limit. This indicates that it is non-trivial to stabilize the saxion, which is a gauge singlet, within gauge mediation where soft terms receive contribution proportional to the gauge couplings.
In this paper, we consider a simple axion model in the framework of gauge mediation that provides a natural solution both to the $\mu/B\mu$ problem and the strong CP problem. The model contains matter fields that communicate SUSY breaking from a hidden sector to the MSSM, and also those that transmit the PQ symmetry breaking. They are charged under the Standard Model (SM) gauge groups, and become massive either by directly coupling to the hidden sector SUSY breaking field or by coupling to the axion superfield. The most important property of the model is that there is mixing between these two classes of messengers. It is through the mixing that the saxion feels SUSY breaking at the loop level and acquires soft mass comparable to those of the gauge-charged sparticles. As a result, the saxion is radiatively stabilized at a scale below or around the scale of gauge mediation. In this model, the $\mu$ term arises from an appropriate coupling of the Higgs doublets to the axion supermultiplet either in the superpotential or in the Kähler potential. Remarkably, the $B\mu$ term is then generated at the correct mass scale, thanks to the SUSY breaking in the axion supermultiplet induced by the mixing between the two messenger sectors.

The MSSM soft terms receive negligible threshold corrections at the PQ scale, but their renormalization group (RG) evolutions are affected by the PQ messengers. In particular, if the saxion has a vacuum expectation value rather close to the scale of gauge mediation, MSSM scalar masses can receive a sizable contribution through the hypercharge trace term because the PQ messenger scalars acquire additional soft masses due to the mixing. This contribution can make the stau lighter than the gauginos. In the model, the lightest superparticle (LSP) is given either by the axino, the fermionic partner of the axion, or by the gravitino depending on the scale of gauge mediation. The ordinary sparticles dominantly decay into axinos, not into gravitinos, through the interactions suppressed by the PQ scale. Meanwhile, the saxion properties are constrained by various cosmological considerations. In case that the Universe is dominated by the saxion, the axion energy density produced by the saxion decay should be less than that of one neutrino species to be consistent with the Big Bang nucleosynthesis. In addition, LSPs from the saxion decay should not overclose the Universe. To satisfy these constraints, one needs to enhance the saxion coupling to the SM particles. This is naturally achieved when the $\mu$ term is generated by a superpotential interaction between the axion supermultiplet and Higgs doublets.

This paper is organized as follows. In section 2, we examine how the saxion direction is lifted in the presence of mixing between the messengers that transmit SUSY breaking and PQ symmetry breaking to the MSSM sector. We then show in section 3 that the model, where the PQ scale is radiatively stabilized, can naturally generate the correct mass scale not only for $\mu$ but also for $B\mu$ in gauge mediation. Sections 4 and 5 are devoted to the discussion of phenomenological and cosmological aspects. We will examine the pattern of sparticle masses, the decay of sparticles into axinos or gravitinos, and the cosmological constraints on the saxion properties. An important role is played by the saxion/axino interactions with the MSSM particles induced by the Higgs coupling to the axion superfield. The last section is for the conclusion.
2. Axion in gauge mediation

To invoke the PQ mechanism within the framework of gauge mediation, we introduce heavy matter superfields that form vector-like pairs under the SM gauge groups. These fields are classified as

$$\Phi + \bar{\Phi} : \text{SUSY breaking messengers},$$
$$\Psi + \bar{\Psi} : \text{PQ messengers},$$

depending on the way of getting massive. The $\Phi + \bar{\Phi}$ are vector-like also under $U(1)_{\text{PQ}}$ and directly couple to hidden sector fields that participate in SUSY breaking. They are the usual messengers for gauge mediation. On the other hand, the PQ messengers couple to the axion superfield $S$ through Yukawa interaction and thus acquire heavy mass after PQ symmetry breaking.

One might think that minimal field content is prepared for the PQ mechanism\(^1\) to work in gauge mediation. However, previous studies \([12, 13, 14]\) have noticed that there should be new interactions transmitting SUSY breaking to the PQ sector in order to fix the PQ scale\(^2\). This feature is observed under the assumptions that $U(1)_{\text{PQ}}$ is spontaneously broken by a single field, $S$, and that soft terms only receive gauge-mediated contribution. Hence, one may extend the model to include extra SM singlet fields carrying PQ charge, or add an additional source of SUSY breaking such as gravity mediation \([13, 14]\). Another interesting approach we would like to pursue here is to consider the case that some messengers of the two sectors, say $\Phi$ and $\bar{\Phi}$, have the same charge under all the symmetries of the theory. Then, there arises mixing between them, which makes $S$ feel SUSY breaking at the same loop level as SM-charged scalars do through gauge mediation. Since SUSY breaking generates a potential for the saxion, such mixing can play an important role in determining the PQ scale.

In this paper, we consider a simple axion model\(^3\) within minimal gauge mediation where the messengers $\Phi + \bar{\Phi}$ and $\Psi + \bar{\Psi}$ belong to $5 + \bar{5}$ representation of the SU(5) into which the SM gauge groups are embedded. The gauge coupling unification is thus preserved. To allow mixing between the $5$ messengers, we simply take the PQ charge assignment such that $\Psi$ carries a charge opposite to that of $S$ while all the other messengers are neutral. The model is then described by the superpotential

$$W = W_0(X) + y_\Phi X \Phi \bar{\Phi} + y_\Psi S \Psi \bar{\Psi} + y_X X \Phi \bar{\Psi} + y_S S \Psi \bar{\Phi}, \quad (2.1)$$

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\(^1\)To solve the strong CP problem, the PQ symmetry should be anomalous under QCD interactions. If $\Psi + \bar{\Psi}$ are charged under QCD, one obtains a KSVZ-type (hadronic) axion model \([10]\). A DFSZ-type axion model \([11]\) is otherwise obtained for the Higgs bilinear $H_u H_d$ charged under $U(1)_{\text{PQ}}$.

\(^2\)See also \([16]\) for other interesting observations on axions in gauge mediation.

\(^3\)Though we are assuming that the model has the PQ symmetry to solve the strong CP problem, it is also possible to consider other cases where $S$ corresponds to a flaton field driving thermal inflation. In such case, $U(1)_{\text{PQ}}$ needs not be exact, but $U(1)_{\text{PQ}}$-breaking terms should be small enough so that the potential for $S$ can remain approximately flat in the supersymmetric limit. Most of our discussion can apply to these flaton models \([14]\). The property of the angular scalar of $S$ would however be quite different because its mass is sensitive to the $U(1)_{\text{PQ}}$-breaking terms.
in the field basis where the Kähler metric is diagonal, ignoring Planck-suppressed operators. The above superpotential includes all renormalizable couplings consistent with the SM gauge invariance and the PQ symmetry. The effects of hidden sector SUSY breaking are parameterized by a background singlet field $X$, whose Kähler potential should be included to correctly compute the anomalous dimension of operators depending on $X$.

The PQ symmetry ensures that $S$ corresponds to a flat direction in the supersymmetric limit for all the messengers fixed at the origin. Transmitted to the PQ sector by $\Phi + \bar{\Phi}$, the SUSY breaking effects will lift this flat direction and fix the vacuum expectation value (VEV) of the saxion, i.e. the PQ scale. Meanwhile, the axion remains massless until the QCD instanton effects are turned on. To examine the saxion potential, it is convenient to construct an effective theory with messengers integrated out. Here we assume $|F_X| \ll |X|^2$ so that the mass of $\Phi + \bar{\Phi}$ is dominated by supersymmetric contribution. Before proceeding to the analysis, one should note that the $\bar{\bar{5}}$ messengers can be redefined further, without loss of generality, so that either $X \Phi \bar{\Psi}$ or $S \Psi \bar{\Phi}$ is removed in the superpotential while keeping the Kähler metric diagonal. It is thus natural to assume

$$\frac{|y_X|}{|y_\Phi|} + \frac{|y_S|}{|y_\Psi|} = O(1), \quad (2.2)$$

in the canonical basis, but $y_\Phi$ and $y_\Psi$ may be hierarchically different from each other. The theory has thresholds at scales $\Lambda_\Phi = y_\Phi |X|$ and $\Lambda_\Psi = y_\Psi |S|$. For fixed $\Lambda_\Phi$, redefining the 5 messengers appropriately, messenger mixing can be treated perturbatively along the flat direction with $\Lambda_\Psi$ far from the scale $\Lambda_\Phi$.

### 2.1 Saxion potential

Let us first examine the saxion potential at the region with $\Lambda_\Psi = y_\Psi |S| \ll \Lambda_\Phi$. To derive the effective action for $S$, the SUSY breaking messengers can first be integrated out taking the field basis where the superpotential is written as

$$W = W_0(X) + y_\Phi X \Phi \bar{\Phi} + y_\Psi S \Psi \bar{\Psi} + y_S S \Phi \bar{\Phi}, \quad (2.3)$$

for the canonical Kähler potential. This approximately corresponds to the mass basis where the term $S \Psi \bar{\Phi}$ gives small mixing between $\bar{\bar{5}}$ messengers. The $\Phi + \bar{\Phi}$, which are integrated out below the scale $\Lambda_\Phi$, communicate the SUSY breaking to SM-charged fields via gauge interactions. In addition, the communication does occur also through the Yukawa interaction $S \Psi \bar{\Phi}$ in two ways:

1. The anomalous dimensions of $S$ and $\Psi$ are discontinuous at $\Lambda_\Phi$. As a consequence, the saxion acquires soft mass at the two-loop level \[2, 17\]

$$\frac{m_S^2(\Lambda_\Phi)}{M_0^2} = \sum_\Psi \left( \frac{8\pi^2 dy_\Psi^2}{d \ln Q} \bigg|_{\Lambda_\Phi} - \frac{8\pi^2 dy_\Phi^2}{d \ln Q} \bigg|_{\Lambda_\Phi^+} \right) + \sum_\Psi \frac{8\pi^2 dy_{\bar{\Phi}}^2}{d \ln Q} \bigg|_{\Lambda_\Phi^+} \approx N_\Phi N_\Psi \left( 5((5N_\Phi N_\Psi + N_\Phi + N_\Psi)g_{\bar{\Phi}}^2 + g_{\bar{\Phi}}^2) - 2(8g_{\bar{\Phi}}^2 + 3g_\Phi^2 + g_{\bar{\Psi}}^2) \right) y_{\bar{\Phi}}^2, \quad (2.4)$$

where we have neglected the splitting between the Yukawa couplings of doublet and triplet messengers. The scalar components of $\Psi + \bar{\Psi}$ also receive additional soft mass terms at two loops, besides the ordinary gauge-mediated contributions.
2. The effective Kähler potential for $\Psi$ receives a correction

$$\delta K = N_\Phi \frac{y_S^2 |S|^2 |\Psi|^2}{y_\Phi^2 |X|^2} , \quad (2.5)$$

from the tree-level exchange of the SUSY breaking messengers. Hence, soft mass terms for the scalar components of $\Psi$ receive additional contribution

$$\frac{\delta m^2(\Lambda_\phi^-)}{M_0^2} = -N_\Phi (16\pi^2)^2 y_S^2 |S|^2 \frac{\Lambda_\Phi}{\Lambda_\phi^2}, \quad (2.6)$$

The above contribution is always tachyonic and becomes sizable or even more dominant than the gauge-mediated soft mass for a saxion value close to $\Lambda_\Phi$.

Here $Q$ denotes the renormalization scale, $N_\Phi (N_\Psi)$ is the number of $\Phi + \bar{\Phi}$ ($\Psi + \bar{\Psi}$) pairs, and $M_0$ sets the overall scale of soft terms generated by the loops of SUSY breaking messengers

$$M_0 = \frac{1}{16\pi^2} \frac{\mathcal{F}^X}{X}, \quad (2.7)$$

Note that trilinear couplings for the scalar components of $\Psi + \bar{\Psi}$ also receive contribution mediated through the Yukawa coupling $y_S$. The explicit expressions for soft terms are given in the appendix.

With the knowledge of how SUSY breaking is transmitted to the PQ sector, we further integrate out the remaining messengers $\Psi + \bar{\Psi}$ under a large background value of $S$. The effective action is then determined by the running wave function of $S$

$$\mathcal{L}_{\text{eff}} = \int d^4 \theta Z_S (Q = y_\Psi |S|) |S|^2 , \quad (2.8)$$

from which the equation of motion for $F^S$ reads

$$\frac{F^S}{S} \simeq -\frac{1}{2} \left( \gamma^+ - \gamma^- \right) \frac{\mathcal{F}^X}{X} = -5N_\Psi N_\Phi y_S^2 M_0, \quad (2.9)$$

neglecting corrections suppressed by $\Lambda_\Phi^2/\Lambda_\phi^2$. Here $\gamma^\pm_i$ are the anomalous dimensions above and below $\Lambda_\Phi$, respectively. Hence, the scalar potential for the saxion is generated as

$$V = V_0 + m_S^2 (Q = y_\Psi |S|) |S|^2, \quad (2.10)$$

where $m_S^2$ is the running soft mass of $S$ in the theory between $\Lambda_\Phi$ and $\Lambda_\phi$, and a constant $V_0$ has been added to cancel the cosmological constant. It should be noted that $m_S^2 (Q)$ depends on $|S|$ itself because the Kähler correction generates soft terms for the scalar components of $\Psi + \bar{\Psi}$ that affect the running of $m_S^2$. Thus, in the region with $y_\Psi |S| \ll \Lambda_\Phi$, the saxion potential has a slope approximately given by

$$\frac{1}{2M_0^2 |S|} \frac{dV}{d|S|} \simeq \frac{m_S^2 (\Lambda^-)}{M_0^2} + \frac{5N_\Phi N_\Psi}{8\pi^2} \left[ C_\Psi y_\Phi^2 - 2y_S^2 \left( 16\pi^2 y_\Psi |S| \Lambda_\phi / \Lambda_\Phi \right)^2 \right] \ln \left( \frac{y_\Psi |S|}{\Lambda_\Phi} \right), \quad (2.11)$$
where the Yukawa couplings are evaluated at $\Lambda_\Phi$, and $C_\Psi$ depends on $y_{\Psi}^2 S$ and gauge couplings. The logarithmic dependence originates from running between $\Lambda_\Phi$ and $\Lambda_\Psi$ through the Yukawa interaction with $\Psi + \bar{\Psi}$. Due to the radiative effects from the color-charged scalars of $\Psi + \bar{\Psi}$, $C_\Psi$ has a positive value of order unity.

The slope of potential (2.11) shows that the saxion can be stabilized by the balance between two effects, i.e. the SUSY breaking mediated by $\Phi + \bar{\Phi}$ at $\Lambda_\Phi$, and the renormalization effect through the Yukawa interaction with $\Psi + \bar{\Psi}$. The second part of the slope monotonically increases as a function of $|S|$ for $y_\Psi |S| \leq O(0.1 \Lambda_\Phi)$, and crosses zero at $y_\Psi |S| = O(\Lambda_\Phi / 8\pi^2)$ because the two contributions in the bracket have the opposite sign. Hence, depending on the value of $m_\Psi^2$ at $\Lambda_\Phi$, the potential develops a minimum along the saxion direction as follows. If $m_\Psi^2(\Lambda_\Phi)$ has a positive value of $O(M_0^2)$ or less, the saxion is stabilized at $y_\Psi |S| \leq O(\Lambda_\Phi / 8\pi^2)$. On the other hand, for negative $m_\Psi^2(\Lambda_\Phi)$, a minimum appears at a scale rather close to $\Lambda_\Phi$ where the Kähler correction (2.5) becomes important.

In fixing the VEV of the saxion, the crucial role is played by the messenger mixing as can be seen from that the potential only has a negative slope in the limit $y_\Psi \to 0$.

Let’s move on to the opposite region with $\Lambda_\Psi \gg \Lambda_\Phi$ along the saxion direction. In this region, the correct procedure for constructing the effective theory is to first integrate out the PQ messengers at the scale $\Lambda_\Psi$. For this, we take the field basis such that

$$W = W_0(X) + y_\Phi X \Phi \bar{\Phi} + y_\Psi S \bar{\Psi} + y_X X \bar{\Phi},$$

for the canonically normalized fields. Integrating out $\Psi + \bar{\Psi}$, one obtains the effective action for $X$ determined by its running wave function. In the effective theory, $S$ does not have any renormalizable interactions, and the equation of motion for $F_S$ gives

$$\frac{F_S}{S} \simeq -5N_\Phi N_\Psi \frac{y_X^2 |X|^2}{|S|^2} M_0,$$

where corrections suppressed by $\Lambda_\Phi^2/\Lambda_\Psi^2$ have been neglected. Hence, the leading contribution to the saxion potential comes from the dependence on $S$ of the effective wave function of $X$. Because the anomalous dimension of $X$ is discontinuous at the scale $\Lambda_\Phi$, the slope of the potential is derived as

$$|S| \frac{dV}{d|S|} \simeq \frac{5N_\Phi N_\Psi}{8\pi^2} \left[ y_X^2 - \frac{C_\Phi}{(8\pi^2)^2} y_\Phi^2 \right] |F_X|^2,$$

where a positive constant $C_\Phi = O(g_\alpha^4)$ parameterizes the contribution induced at the three-loop level. The potential thus increases at $y_\Psi |S| \gg \Lambda_\Phi$ as a function of $|S|$, unless $y_X$ is smaller than $O(y_\Psi / 8\pi^2)$. This property is cosmologically favorable because the saxion may be displaced far from the minimum at the end of inflation. If this happens, the positive slope will make the saxion roll down toward the true minimum.

The relation (2.14) also gives information about the potential at saxion values close to $\Lambda_\Phi$, for which messenger mixing can no longer be treated as a perturbation. Instead of constructing an effective theory, we use the property that the slope at $\Lambda_\Psi \gg \Lambda_\Phi$ is positive for $y_X = O(y_\Phi)$. This implies that there must exist a minimum below or near $\Lambda_\Phi$ since
is driven negative at low scales by the Yukawa coupling with $\Psi + \bar{\Psi}$. For the saxion stabilized far below $\Lambda_\Phi$, the vacuum structure can easily be examined treating the mixing perturbatively. Note that the potential at $y_\Psi |S| \ll \Lambda_\Phi$ is essentially determined by $y_\Psi S$ at $\Lambda_\Phi$ and insensitive to the details of the potential at large $|S|$. Another possibility is that a minimum lies close to $\Lambda_\Phi$, which generically requires a rather small $y_\Sigma^2$. The existence of minimum is ensured by the positive slope at $\Lambda_\Psi \gg \Lambda_\Phi$.

It is worth discussing the situation that there is no mixing between the messengers, as usually assumed in gauge mediation. This corresponds to the limit that $y_{X,S}$ vanish. From the relations (2.11) and (2.14), one then finds that the potential runs off to infinity along the saxion direction. Hence, additional SUSY breaking effects are needed to stabilize the saxion. A natural candidate for this is gravity mediation since the saxion potential is generated at three-loop level. Indeed, a higher dimensional operator $\propto |X|^2|S|^2/M_{Pl}^2$ in the Kähler potential gives the gravity-mediated contribution

$$\delta V = k |S|^2 |F^X|^2,$$

which can compete with the gauge-mediated one to stabilize the saxion for a positive $k$ of order unity [13]. In the presence of messenger mixing, however, the saxion potential has a slope as (2.11) at $|S| \ll \Lambda_\Phi$ as long as $X$ is smaller than $O(10^{-3} M_{Pl})$, and the above contribution becomes important only at $|S| \geq O(y_X M_{Pl}/\sqrt{8\pi^2} k)$.

We complete this subsection by summarizing the role of mixing between SUSY breaking and PQ messengers. Such mixing indicates that there exist some SM-charged heavy fields that directly couple both to the SUSY breaking fields and to the PQ breaking field. It is the SUSY breaking effects transmitted by these fields that radiatively generate a potential for the saxion and fix the PQ scale. Moreover, the mixing prevents a runaway behavior of potential at large saxion values in gauge mediation. This would be cosmologically relevant for the saxion to settle down to the true vacuum.

### 2.2 Vacuum structure

Messenger mixing can be treated as a perturbation at $y_\Psi |S| \ll \Lambda_\Phi$ to construct the effective theory for $S$. Taking into account that $\Psi$ receives a Kähler correction (2.3) that contributes to its effective wave function, we examine the vacuum structure focusing on the case that the saxion is stabilized at $y_\Psi |S| \leq O(\Lambda_\Phi/\sqrt{8\pi^2})$. From the effective action for $S$, the saxion $\sigma$ and the axino $\tilde{a}$ are found to acquire SUSY breaking mass as

$$m_\sigma^2/M_0^2 \simeq \frac{5 N_\Psi N_\Phi}{4 \pi^2} \left[ C_\Psi y_\Psi^2 - 4 y_\Sigma^2 \left( 16 \pi^2 y_\Psi S_0 \Lambda_\Phi \right)^2 \ln \left( \frac{y_\Psi S_0}{\Lambda_\Phi} \right) \right],$$

$$m_{\tilde{a}}/M_0 \simeq \frac{N_\Psi}{8 \pi^2} \left[ 3 y_\Psi^2 A_q/M_0 + 2 y_\ell^2 A_\ell/M_0 + \frac{5 N_\Psi y_\Sigma^2}{8 \pi^2} \left( 16 \pi^2 y_\Psi S_0 \Lambda_\Phi \right)^2 \ln \left( \frac{y_\Psi S_0}{\Lambda_\Phi} \right) \right],$$

for the axion superfield expanded around its VEV, $S = (S_0 + \sigma/\sqrt{2}) e^{i a/\sqrt{2} S_0} + \sqrt{2} \theta \tilde{a} + \theta^2 F S$. Here the couplings are evaluated at $Q = y_\Psi S_0$, and $A_{q,\ell} = O(10 N_\Phi N_\Psi y_\Sigma^2 M_0)$ are the trilinear couplings associated with the Yukawa couplings of the PQ triplet and doublet.
messengers, respectively. For $y^2_{\Psi,S} = \mathcal{O}(0.1)$, which are the plausible values, the saxion mass lies in the range

$$\mathcal{O}\left(\frac{M_0}{\sqrt{8\pi^2}}\right) \leq m_{\sigma} \leq \mathcal{O}\left(\sqrt{\ln(8\pi^2)}M_0\right),$$

(2.17)

where the upper bound is obtained when $y_{\Psi}S_0 = \mathcal{O}(\Lambda_{\Phi}/\sqrt{8\pi^2})$. Though radiatively stabilized, $\sigma$ can be as heavy as the color-charged MSSM sparticles. This is because messenger mixing induces a correction to the Kähler potential for $\Psi$ whose loops contribute to the saxion potential. On the other hand, the axino mass is rather insensitive to the Kähler correction (2.3), and has the value

$$m_{\tilde{a}} = \mathcal{O}\left(\frac{M_0}{8\pi^2}\right),$$

(2.18)

for $y^2_{\Psi,S} = \mathcal{O}(0.1)$, thereby lighter than the MSSM sparticles.

To illustrate how the PQ scale is fixed after SUSY breaking, we provide some examples. Fig. 1 shows the scalar potential along the saxion direction in the model with $N_{\psi} = 1$, $N_{\Phi} = 2$, $\Lambda_{\Phi} = 10^{13}$GeV and $y_{\Phi}(\Lambda_{\Phi}) = 0.3$. Depending on the Yukawa couplings $y_{\Psi,S}$ at the scale $\Lambda_{\Phi}$, the saxion is stabilized in the following ways:

$$(y_{\Psi},y_{S}) = (0.3,0.5): S_0 \simeq 1.3 \times 10^{10}$GeV, $V_0 \simeq (0.4M_0)^2S_0^2$, $m_{\sigma} \simeq 0.5M_0$, $$

$$(y_{\Psi},y_{S}) = (0.5,0.4): S_0 \simeq 4.2 \times 10^{11}$GeV, $V_0 \simeq (0.9M_0)^2S_0^2$, $m_{\sigma} \simeq 1.7M_0,$$

(2.19)

where $m^2_S(\Lambda_{\Phi})$ is positive for $(y_{\psi},y_{S}) = (0.3,0.5)$, while negative for the other case. For $m^2_S(\Lambda_{\Phi}) < 0$, the saxion potential develops a minimum at a scale near $\Lambda_{\Phi}$ while providing a rather large mass to the saxion. As discussed already, the Kähler correction (2.3) to the
PQ messenger generates a potential for the saxion that becomes important at saxion values close to $\Lambda_{\Phi}$. In Fig 1, one can also see how the saxion VEV is fixed depending on $y_{\Psi,S}$ at $\Lambda_{\Phi}$. For $y_{S} < 0.42$ where $m_{S}^{2}(\Lambda_{\Phi})$ is negative, the contribution from the Kähler correction (2.5) stabilizes the saxion at $|S| \geq O(10^{-2}\Lambda_{\Phi})$. It is also possible to obtain $S_{0} = O(\Lambda_{\Phi})$, for instance, by taking $y_{\Psi} = 0.2$ and $y_{S}$ smaller than 0.3. In this case, the effective theory for $\Psi + \bar{\Psi}$ constructed by integrating out $\Phi + \bar{\Phi}$ is not reliable. Nonetheless, the relation (2.14) tells that the potential has a minimum as long as $y_{X} = O(y_{\Phi})$.

3. Higgs $\mu$ and $B\mu$ terms

Although it is an attractive mechanism for generating flavor and CP conserving soft terms, gauge mediation requires some additional structure to account for the origin of the $\mu$ and $B\mu$ terms in the MSSM. In particular, it is quite non-trivial to obtain an acceptable value of $B$ in theories with gauge mediation. If one introduces a direct coupling of the Higgs bilinear $H_{u}H_{d}$ to the SUSY breaking field $X$ in the superpotential, one obtains $B = O(8\pi^{2}M_{0})$ and thus needs an unnatural fine-tuning to achieve the electroweak symmetry breaking. One may instead consider an effective Higgs coupling in the Kähler potential

$$\int d^{3}\theta f(X, X^{*})H_{u}H_{d},$$

(3.1)

which relates $\mu$ to the SUSY breaking parameters$^{4}$. However, this operator generically gives $B$ of $O(8\pi^{2}M_{0})$ again. To avoid large $B$, $f$ should have a particular dependence on $X$ such that generates $\mu$ but not $B\mu$ $^{19}$, unless there are other SUSY breaking fields. For example, one can use $f = X^{*}/\Lambda$ with some mass scale $\Lambda$ $^{20}$. The dynamics that connects the Higgs sector to the SUSY breaking sector in such a particular way would generally affect other MSSM soft terms generated by gauge mediation.

Here, we take an alternative approach to solving both the $\mu$ and $B\mu$ problems, which is provided by the PQ mechanism incorporated into the gauge mediation. In fact, a natural solution is to consider a coupling between $H_{u}H_{d}$ and the axion superfield $S$. The $\mu$ term is then induced only after the PQ symmetry is broken. Furthermore, because the radiative stabilization of the saxion leads to

$$\frac{F}{S_{0}} \simeq -5N_{\Phi}N_{\Psi}y_{S}^{2}M_{0},$$

(3.2)

a generic coupling of $H_{u}H_{d}$ with $S$ is naturally expected to give $B$ of the order of MSSM sparticle masses for $y_{S}^{2} = O(0.1)^{5}$. The above relation is a consequence of mixing between SUSY breaking and PQ messengers. In the absence of such mixing, though the saxion can still be stabilized by adding additional SUSY breaking effects such as gravity mediation, $F/S_{0}$ would have a value much smaller than $M_{0}$ since $S$ couples to $X$ at more than two-loop level. The relation (3.2) implies that there are simple mechanisms operative to generate $\mu$ and $B\mu$ terms required for proper electroweak symmetry breaking:

$^{4}$This is a generalization of the Giudice-Masiero mechanism $^{18}$ that generates $\mu$ by SUSY breaking effect.

$^{5}$A similar idea to suppress $B$ was considered by $^{21}$ in a different model.
• **Kim-Nilles (KN) mechanism**: After the PQ symmetry is broken, the non-renormalizable term in the superpotential

\[ \mathcal{L}_{\text{KN}} = \int d^2 \theta \frac{\lambda S^2}{\Lambda_{\text{Pl}}} H_u H_d + \text{h.c.} \]  

(3.3)

generates \( \mu \) and \( B \mu \) terms with

\[ \mu = \lambda \frac{S^2}{\Lambda_{\text{Pl}}} , \quad B = -2 \frac{F^S}{S}. \]  

(3.4)

For the saxion stabilized at a scale around \( 10^{11} \text{GeV} \), which is well within the invisible axion window consistent with astrophysical and cosmological bounds, \( \mu \) is generated at the soft mass scale with \( \lambda = \mathcal{O}(0.1) \). In addition, we obtain \( B \) of the correct order of magnitude.

• **Giudice-Masiero (GM) mechanism**: For the Higgs fields that couple to \( S \) through the effective Kähler potential term

\[ \mathcal{L}_{\text{GM}} = \int d^4 \theta \kappa \frac{S^*}{S} H_u H_d + \text{h.c.}, \]  

(3.5)

both \( \mu \) and \( B \) are induced by SUSY breaking effects

\[ \mu = \kappa \frac{F^S}{S^*} , \quad B = \frac{F^S}{S^*}. \]  

(3.6)

The \( \mu \) and \( B \mu \) terms are thus of the desired size. The above coupling in the Kähler potential can arise, for instance, by integrating out heavy fields in the model with the superpotential terms \( \Sigma H_u H_d + \Sigma S S' \) for \( S' \) having a wave-function mixing with \( S \) in the Kähler potential.

It is important to note that, since the phase of \( F^S / S_0 \) is aligned with \( M_0 \), the \( B \mu \) term does not introduce new source of CP violation in either mechanism. Notice also that the Higgs coupling to \( S \) fixes the PQ charges of the MSSM matter fields. In order for \( a \) to play the role of the QCD axion, \( U(1)_{\text{PQ}} \) should be anomalous under QCD. This requires non-zero value for the QCD anomaly coefficient, \( N = N_{\Psi} \pm 6, + \) for the GM while \( - \) for the KN mechanism\(^7\).

\(^6\)A non-renormalizable superpotential coupling of \( H_u H_d \) to the PQ breaking field has been considered to explain the \( \mu \) term in various SUSY breaking schemes\(^4\). The size of \( B \) however depends on the mechanism stabilizing the PQ scale.

\(^7\)Models that incorporate the KN mechanism with \( N = \pm 1 \) are free from the domain wall problem. For example, in the model with \( N_{\Psi} = 5 \) and \( N_a = 1 \), one obtains \( N = -1 \), and the MSSM gauge couplings remain perturbative up to the unification scale as long as \( 10^6 \text{GeV} \lesssim \Lambda_{\Psi} \lesssim 0.1 \Lambda_a \). Meanwhile, for the case that \( \mu \) is generated by the KN or GM mechanism with \( N \neq \pm 1 \), one can consider the situation that the saxion is displaced far from the origin during the inflation. Then, the reheating would not restore \( U(1)_{\text{PQ}} \) after inflation. In addition, provided that the fluctuation around the initial displacement due to the quantum fluctuation during the de-Sitter expansion is small enough, the saxion will settle down to one of the \( |N| \) degenerate vacua. This will provide a solution to the domain wall problem (see\(^22\), for a similar consideration).
Another important consequence of the Higgs coupling to $S$ is that mixing between the saxion (axino) and neutral Higgs (Higgsino) fields is induced after electroweak symmetry breaking. Through the mixing, the saxion and axino interact also with other MSSM particles. The saxion mixing term with neutral Higgs bosons is obtained from

$$\mathcal{L}_{\text{Higgs}} = -|\mu|^2(|H_0^u|^2 + |H_0^d|^2) + |B\mu|(H_0^u\bar{H}_d^u + \text{c.c.}),$$

by making the replacement

$$|\mu| \to \frac{C_\sigma|\mu|\sigma}{S_0 \sqrt{2}} \text{ with } C_\sigma = \left. \frac{\partial \ln |\mu|}{\partial \ln |S|} \right|_{S=S_0},$$

where we have used that $F^S/S$ does not depend on $S$, as can be seen from (2.4). Therefore, the saxion slightly mixes with the neutral CP even Higgs bosons through the interaction suppressed by $v/S_0$ with $v^2 = (|H_0^u|^2 + |H_0^d|^2)$. Similarly, the axino has tiny mixing with the neutral Higgsinos determined by

$$\mathcal{L}_{\tilde{a}} = \frac{C_{\tilde{a}} H_0^u \tilde{H}_d^u + H_0^d \tilde{H}_u^d}{S_0} + \text{h.c.},$$

with

$$C_{\tilde{a}} = \left. \frac{\partial \ln \mu}{\partial \ln S} \right|_{S=S_0}.$$  

Note that the coefficient $C_\sigma$ crucially depends on the mechanism for generating the $\mu$ term:

$$C_\sigma|_{\text{KN}} = 2, \quad C_\sigma|_{\text{GM}} = Q \frac{d \ln \kappa}{d Q} \bigg|_{Q=y_{\psi S_0}} = \mathcal{O} \left( \frac{1}{8\pi^2} \right),$$

whereas the size of $C_{\tilde{a}}$ is insensitive to the form of the Higgs coupling to $S$:

$$C_{\tilde{a}}|_{\text{KN}} = 2, \quad C_{\tilde{a}}|_{\text{GM}} = -1.$$  

Let us examine the saxion/axino couplings to the MSSM sector. First, there are the interactions, $\sigma |H_0^u| H_0^{u,d} + \sigma \tilde{H}_d^u \tilde{H}_u^d + H_0^{u,d} \tilde{H}_d^{u,d} \tilde{a}$, that are induced directly from the Higgs coupling to $S$. The couplings for these interactions are non-vanishing even in the limit $v \to 0$. To derive other interactions between the saxion/axino and the MSSM particles, the small mixing terms from (3.7) and (3.9) should be removed by performing an appropriate field redefinition. Indeed, the saxion and axino couplings can be read off from the MSSM Lagrangian by the substitutions

$$H_0^{0,d} \rightarrow -\frac{C_\sigma v}{S_0} \frac{|\mu|^2}{m_h - m_\sigma^2} \frac{N_{d,u}^\sigma}{\sqrt{2}},$$

$$\langle \tilde{B}, \tilde{W}_d^0, \tilde{H}_d^0 \tilde{H}_u^0 \rangle \rightarrow -\frac{C_\tilde{a} v}{S_0} N_{B,W^0,\tilde{H}_d^0 \tilde{H}_u^0} \tilde{a},$$

where $m_h$ is the mass of the lightest CP even neutral Higgs boson $h$, and $N_{B,W^0,\tilde{H}_d^0 \tilde{H}_u^0}$ are non-vanishing because $\tilde{H}_d^{u,d}$ mix with the bino and neutral wino. The mixing parameters are
presented in the appendix. After the replacement, one must diagonalize the mass matrix for \((\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0)\). In addition to those proportional to \(C_{\sigma, a}\), interactions between the saxion/axino and MSSM particles are also generated by the loops involving PQ messengers that acquire mass from the VEV of \(S\). These couplings can be derived from the dependence on \(S\) of the MSSM gauge couplings

\[
\frac{1}{g_a^2} = -\frac{N_\Psi}{16\pi^2} \ln(S^*S) + (S\text{-independent part}),
\]

in the effective theory with \(\Psi + \bar{\Psi}\) integrated out. The saxion decay to MSSM particles and the decay of heavy sparticles into axino will be discussed later, after examining MSSM sparticle masses.

We stress that the axion superfield can play an important role not only in solving the strong CP problem but also in explaining the presence of the \(\mu\) and \(B\mu\) terms within theories with gauge mediation. This nice feature stems from the mixing between SUSY breaking and PQ messengers, through which the SUSY breaking is communicated to the PQ sector and radiatively stabilizes the PQ scale. In fact, an appropriate PQ charge assignment is the only thing that was needed to allow such mixing between the messengers.

### 4. Phenomenological implications

In this section, we discuss the phenomenological aspects of the model.

#### 4.1 Sparticle masses

To derive soft terms for the MSSM fields, we begin by summarizing the possible range of threshold scales in the theory. The PQ scale, which is radiatively stabilized in the presence of mixing between messengers, is constrained by various astrophysical and cosmological observations \cite{ref}. On the other hand, an upper bound is put on \(\Lambda\Phi = y\Phi |X|\) to suppress gravity mediation that in general generates flavor-violating soft terms with size \(FX/M_{Pl}\). These constraints lead to

\[
10^9\text{GeV} \lesssim S_0 \lesssim 10^{12}\text{GeV}, \quad 10S_0 \lesssim |X| < 10^{15}\text{GeV},
\]

where the lower bound on \(X\) has been put to concentrate on the case that messenger mixing can be treated perturbatively. Since the theory contains heavy messengers, the MSSM soft terms at TeV scale are determined by the parameters \(\{M_0, \Lambda_\Phi, N_\Phi, \Lambda_\Psi, N_\Psi\}\), while \(\mu\) and \(B\) are generated by the KN or GM mechanism.

At the higher threshold \(\Lambda_\Phi\), the SUSY breaking is transmitted to the MSSM sector by \(\Phi + \bar{\Phi}\) through ordinary gauge mediation \[3\]. The threshold effects induced by these messengers generate gaugino and scalar soft masses as

\[
\frac{M_a(\Lambda_\Phi)}{M_0} = N_\Phi g_a^2(\Lambda_\Phi),
\]

\[
\frac{m_i^2(\Lambda_\Phi)}{M_0^2} = 2N_\Phi C_i^a g_a^4(\Lambda_\Phi),
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for $C_{\alpha}^{\alpha}$ being the quadratic Casimir of the corresponding field. Soft trilinear terms arise at two-loop order, and thus are negligible at $\Lambda_{\Phi}$. Since the radiative corrections due to the SM gauge interaction play the dominant role for the mediation, soft terms preserve flavor and CP. The gauge-mediated soft parameters are subsequently RG evolved down to $\Lambda_{\Psi} = y_{\Psi} S_0$ in the presence of SM-charged fields $\Psi + \bar{\Psi}$.

Low energy soft terms below $\Lambda_{\Psi}$ can be computed integrating out $\Psi + \bar{\Psi}$. Since the saxion is stabilized with $F^S/S_0 = \mathcal{O}(M_0)$, the PQ messengers induce negligible threshold effects for gaugino masses and trilinear couplings. However, depending on the saxion VEV, threshold effect provides flavor-universal soft scalar masses because the PQ scalars acquire soft mass also from the Kähler correction (2.5). The soft mass terms for the MSSM scalar fields can receive sizable threshold corrections. This is because the PQ scalars acquire soft mass also from the Kähler correction \cite{23}. The threshold effect provides flavor-universal soft scalar masses

$$\frac{\Delta m_i^2(\Lambda_{\Psi})}{M_0} \simeq 4 N_{\Phi} N_{\Psi} \left[C_i^{\alpha} g_\alpha - Y_i g^2 \right] \frac{y^2 S_0^2}{\Lambda_{\Phi}^2} \ln \left( \frac{\Lambda_{\Phi}}{\Lambda_{\Psi}} \right),$$

where the gauge couplings are evaluated at $Q = \Lambda_{\Psi}$, and corrections suppressed by the two-loop factor have been neglected. The above contribution includes the hypercharge trace term, which is the part proportional to the hypercharge $Y_i$. This term is non-vanishing due to the Yukawa splitting of doublet and triplet components in $\Psi + \bar{\Psi}$:

$$R = 8 \pi^2 \frac{y^2 S_0 (\Lambda_{\Phi}) - y^2 S_i (\Lambda_{\Phi})}{y^2 S_i (\Lambda_{\Phi})} \sim \ln \left( \frac{M_{\text{GUT}}}{\Lambda_{\Phi}} \right),$$

with $y^2 S = (y^2 S_0 + y^2 S_i)/2$, and $M_{\text{GUT}}$ being the unification scale. The other term in the bracket arises from the loops of $\Psi + \bar{\Psi}$ since the supertrace of their mass matrix is non-vanishing \cite{23}. For example, depending on the values of $y_{\Psi, S}$ at $M_{\text{GUT}}$, the model with $N_{\Phi} = N_{\Psi} = 1$ and $\Lambda_{\Phi} = 10^{12}\text{GeV}$ leads to

$$(y_{\Psi}, y_S) = (0.34, 0.68) : R \simeq 14.5, S_0 \simeq 3.5 \times 10^9\text{GeV},$$

$$(y_{\Psi}, y_S) = (0.24, 0.48) : R \simeq 14.3, S_0 \simeq 8.7 \times 10^9\text{GeV},$$

where we have assumed $y_{\Phi} \ll 1$ to evaluate the running of $y_{\Psi, S}$ from $M_{\text{GUT}}$ to $\Lambda_{\Phi}$. In the case with $(y_{\Psi}, y_S) = (0.24, 0.48)$ at $M_{\text{GUT}}$, the PQ scale is close to $\Lambda_{\Phi}$, and thus the MSSM scalars receive non-negligible threshold correction $\Delta m_i^2 \simeq -0.13 Y_i M_0^2$ at $\Lambda_{\Psi} \simeq 2 \times 10^{10}\text{GeV}$. It is worth noting that one can simply change the sign of $R$ by considering mixing between 5 messengers instead of between 5 ones. Hence, $R$ can be of either sign.

The threshold effect at $\Lambda_{\Psi}$ for soft scalar masses becomes important when the saxion VEV is close to $\Lambda_{\Phi}$, as can be deduced from its origin. For instance, if $y_{\Psi} S_0 = \mathcal{O}(\Lambda_{\Phi}/\sqrt{8\pi^2})$, the hypercharge trace term makes $\Delta m_i^2$ comparable to the gauge-mediated soft mass for scalars that are charged only under $U(1)_{Y}$. This indicates that the lightest ordinary sparticle (LOSP) can be provided by the stau even for small $N_{\Phi}$, if $R$ is positive and the saxion is stabilized at a scale near $\Lambda_{\Phi}$. However, for the saxion VEV much lower than $\Lambda_{\Phi}$, the PQ messengers only give negligible threshold to soft terms. Meanwhile, the gauge-mediated sfermion masses satisfy two sum-rules

$$\sum_i Y_i m_i^2 = \sum_i (B_i - L_i) m_i^2 = 0,$$  \hfill (4.6)
where the sum is over one generation of sfermions, and $B(L)$ is the baryon (lepton) number. Affected by the PQ threshold $\Delta m_i^2$, these sum rules can be used to extract information about the PQ sector.

Finally, we examine which particle is the LSP. The theory contains two light fermionic sparticles, the axino and the gravitino. To determine the gravitino mass, we assume that $X$ is the goldstino superfield whose $F$-term cancels the cosmological constant. The gravitino then absorbs the fermionic component of $\tilde{X}$, $\tilde{\chi}$, to become massive with

$$\frac{m_{3/2}}{M_0} = \frac{16\pi^2}{\sqrt{3}} \frac{X}{M_{Pl}},$$

which lies in the range $10^{-6} M_0 \lesssim m_{3/2} < 10^{-1} M_0$ for the theory with (4.2). Using that the axino acquires mass of $\mathcal{O}(M_0/8\pi^2)$ for $y_{\tilde{q},S}^2 = \mathcal{O}(0.1)$, one also arrives at the relation

$$\frac{m_{3/2}}{m_{\tilde{a}}} \sim \frac{X}{10^{14}\text{GeV}}.$$  

This shows that the axino becomes the LSP if $X \geq \mathcal{O}(10^{14})\text{GeV}$. The LSP would otherwise be given by the gravitino. Though we will not consider it here, there is a possibility to have an axino LSP even for $X < 10^{14}\text{GeV}$. One way is to consider $y_{\tilde{q},S}^2 < 0.1$. For instance, in models with $y_{\tilde{q},S}^2 = \mathcal{O}(10^{-2})$, we obtain $m_{\tilde{a}} = \mathcal{O}(10^{-3} M_0)$ and $m_{\sigma} \geq \mathcal{O}(0.1 M_0)$. For $y_{S}^2 < 0.1$, a Higgs coupling to $S$ will give $|B| \ll \mathcal{O}(M_0)$, with which it is still possible to achieve the correct electroweak symmetry breaking. One can also consider other models where $\tilde{X}$ is not the main component of the goldstino. Then, $F^X$ will have a VEV less than $\mathcal{O}(m_{3/2} M_{Pl})$, and thus the gravitino can be heavier than the axino for $X < 10^{14}\text{GeV}$. In this case, one would need $m_{3/2} \ll m_{\sigma}$ in order not to destabilize the PQ scale, because gravity mediation provides soft mass typically of $\mathcal{O}(m_{3/2})$ to the saxion as well as to the MSSM scalars.

4.2 Decay of sparticles

The heavy MSSM sparticles rapidly decay into the LOSP, denoted by $\tilde{\chi}$, which subsequently decays into lighter sparticles, i.e. into the axino or gravitino. The decay of $\tilde{\chi}$ occurs more slowly because the axino and gravitino are very weakly coupled to other particles. Here we are assuming R-parity conservation. Measurement of the decay length of $\tilde{\chi}$ will give direct information either about the SUSY breaking scale or about the PQ scale, depending on which of axino and gravitino is the main decay product\footnote{However, the decay length alone would not allow us to distinguish between SUSY breaking scenarios where the LSP is given either by the axino or by the gravitino [24].}. In fact, the axino and gravitino have similar type of interactions that mediate the decay of heavy sparticles.

At energy scales much higher than $m_{3/2}$, the gravitino $\tilde{G}$ effectively behaves as a goldstino. To study the decay of heavy sparticles into gravitino, the effective interaction Lagrangian for the goldstino component can be written in non-derivative form [23]. Using the relation $F^X = -16\pi^2 M_0 X \simeq \sqrt{3} m_{3/2} M_{Pl}$, one obtains

$$\mathcal{L}_{\text{int}}^{\tilde{G}} \equiv \frac{i}{16\pi^2 X} \left( \frac{m_{\phi}^2 - m_{\psi}^2}{M_0} \phi^* \psi \tilde{X} + \frac{1}{4\sqrt{2} M_0} \tilde{X} \sigma^{\mu \nu} \lambda F_{\mu \nu} \right) + \text{h.c.},$$

\begin{equation}
\text{(4.9)}
\end{equation}
independently of how the SUSY breaking is mediated. Here $\lambda$ stands for the gaugino, and $F_{\mu \nu}$ is the corresponding field strength, while $\phi$ is the scalar, and $\chi$ is its fermionic partner. The gravitino interactions are proportional to the mass splitting in the supermultiplet, and inversely proportional to $F^X$.

Since the $\mu$ term is generated after the PQ symmetry breaking, $C_\mu$ has a value of order unity, and the axino interacts with MSSM particles through

$$H^0_{u,d}\tilde{H}^0_{u,d}\tilde{a} + \left(H^0_{u,d}\tilde{B}\tilde{H}^0_{u,d} + \tilde{f}^* f \tilde{B} f + \tilde{f}^* \tilde{W}^0 f + \tilde{H}^0_{u,d}\sigma^\mu \tilde{H}^0_{u,d}\tilde{Z}_{\mu}\right),$$  \hspace{1cm} (4.10)

where we have omitted the couplings. For those in parenthesis, the axino couplings are obtained after the replacement (3.14), and thus would vanish in the limit $v \rightarrow 0$. Here $Z_\mu$ is the Z-boson with mass $M_Z$, while $f$ denotes the SM fermion, and $\tilde{f}$ is its scalar partner. In addition, there are axino interactions induced at the loop level

$$H^0_{u,d}\tilde{H}^0_{u,d}\tilde{a} + \tilde{f}^* f \tilde{a} + \tilde{a}\sigma^\mu \lambda F_{\mu \nu},$$  \hspace{1cm} (4.11)

whose couplings are determined by the dependence on $S$ of the gauge couplings after integrating out $\Psi + \bar{\Psi}$. Because of the axino mixing with $\tilde{H}^0_{u,d}$, the above couplings also receive contribution from the loops involving MSSM particles that become massive after the electroweak symmetry breaking.

Using the Lagrangian for axino/gravitino interactions, one can estimate the lifetime of $\tilde{\chi}$, which is also subject to cosmological constraints. In the model, the LOSP can be provided by the stau or bino. Let us first examine the case that the bino is the LOSP. The coupling for the interaction $h\tilde{B}\tilde{a}$ receives contribution from that of $H^0_{u,d}\tilde{B}\tilde{H}^0_{u,d}$ due to the axino component of neutral Higgsinos, and from $H^0_{u,d}\tilde{H}^0_{u,d}\tilde{a}$ through the mixing between $\tilde{B}$ and $\tilde{H}^0_{u,d}$. This coupling is of $O(M_Z/S_0)$, and thus the bino mainly decays into $h$ and $\tilde{a}$ with decay width

$$\Gamma_{\tilde{B} \rightarrow \tilde{h} \tilde{a}} \sim \frac{1}{10^{-8}\text{sec}} \left(\frac{M_{\tilde{B}}}{200\text{GeV}}\right) \left(\frac{10^{10}\text{GeV}}{S_0}\right)^2,$$  \hspace{1cm} (4.12)

for $M_{\tilde{B}} > m_h + m_{\tilde{a}} = m_h + O(M_0/8\pi^2)$, assuming that other neutral Higgs bosons are very heavy. It is easy to see that the bino decay into gravitino is highly suppressed. For a bino with $M_{\tilde{B}} < m_h + m_{\tilde{a}}$, the dominant decay channel is $\tilde{B} \rightarrow Z\tilde{a}$. The coupling for this decay process is additionally suppressed by $O(M_Z/\mu)$ compared to that of $h\tilde{B}\tilde{a}$, because it requires bino-Higgsino mixing as well as the axino-Higgsino mixing. As it is mediated by the interaction induced at the loop level, the decay $\tilde{B} \rightarrow \tilde{a}\gamma$ has a small branching ratio. On the other hand, in the case of a stau LOSP, the decay takes place via the interaction $\tilde{\tau}^* \tilde{\tau} a$ with coupling of $O((M_Z N^2_{\tilde{B}} + m_f N^2_\tilde{H}_d / \cos \beta)/S_0)$. The decay rate is estimated as

$$\Gamma_{\tilde{\tau} \rightarrow \tilde{a}} \sim \frac{1}{10^{-7}\text{sec}} \left[\left(\frac{M_Z \cos 2\beta}{0.3 M_{\tilde{B}}}\right)^2 + \frac{(1 - n_{\tilde{a}} \tan \beta)^2}{10^2}\right] \left(\frac{m_{\tilde{a}}}{200\text{GeV}}\right) \left(\frac{10^{10}\text{GeV}}{S_0}\right)^2,$$  \hspace{1cm} (4.13)

with $n_{\tilde{a}} = O(M_Z^2/\mu M_{\tilde{B}})$, and $\tan \beta = \langle |H^0_u| \rangle / \langle |H^0_d| \rangle$. Therefore, for a bino or stau LOSP, the LOSP will decay mainly into axinos, rather than into gravitinos. Measuring its decay
length, one can thus extract information about the PQ scale. It is interesting to note
that, depending on the PQ scale, a bino LOSP can decay inside the detector while leaving
displaced vertices. For a stau LOSP, there is a possibility that decaying staus can appear
in the detector often enough to measure their charged tracks. Since $\tilde{\chi}$ has a lifetime much
shorter than a second, the nucleosynthesis does not place any significant bound.

The LSP, into which all the sparticles eventually decay, can be either the axino or the
gravitino depending on the scale of gauge mediation, and constitutes the dark matter of
the Universe. If it is lighter than the gravitino, the axino becomes a good candidate for
the cold dark matter [26, 27]. The axino can become the LSP when $X \geq \mathcal{O}(10^{14})$ GeV for
$y_{\Psi,S}^{2} = \mathcal{O}(0.1)$. In this case, the gravitino decay occurs through the interaction

$$\mathcal{L}_{\text{int}} = \frac{i}{2M_{Pl}} \tilde{a} \gamma^{\mu} \gamma^{\nu} \tilde{G}_{\mu} \partial_{\nu} a + \text{h.c.},$$

which leads to

$$\Gamma_{\tilde{G} \rightarrow \tilde{a}a} \sim \frac{1}{0.5 \times 10^{13} \text{sec}} \left( \frac{8\pi^{2} m_{3/2}}{M_{0}} \right)^{3} \left( \frac{M_{0}}{500 \text{GeV}} \right)^{3}.$$  (4.14)

The gravitino decay will thus produce an axino and axion. On the other hand, if heavier
than the gravitino, axinos produced by the decay of $\tilde{\chi}$ will decay into gravitino. Since it is
mediated by the effective interaction

$$\mathcal{L}_{\text{int}}^{\text{eff}} = \frac{i}{8\pi^{2} \chi} \frac{m_{a}}{M_{0}} \tilde{X} \sigma^{\mu} \tilde{a} \partial_{\mu} a + \text{h.c.},$$

for $m_{a} > m_{3/2}$, the decay $\tilde{a} \rightarrow \tilde{G} a$ occurs with

$$\Gamma_{\tilde{a} \rightarrow \tilde{G} a} \sim \frac{1}{0.5 \times 10^{13} \text{sec}} \left( \frac{m_{a}}{m_{3/2}} \right)^{2} \left( \frac{8\pi^{2} m_{a}}{M_{0}} \right)^{3} \left( \frac{M_{0}}{500 \text{GeV}} \right)^{3},$$

where we have used the relation (4.7). The axino will decay with a long lifetime, producing
gravitinos together with axions. Note that late decay of axino/gravitino produces LSPs and
axions, which may be warm or even hot at present unless $\tilde{a}$ and $\tilde{G}$ are highly degenerate in
mass. In fact, having a free-streaming length much larger than $\mathcal{O}(10) \text{Mpc}$, LSPs produced
by such late decays will behave like a hot dark matter. The energy density of hot dark
matter is severely constrained by the CMBR and structure formation [28, 29]. We will
return to this issue in the next section.

5. Cosmological aspects

The theory contains the saxion that has a rather flat potential generated after SUSY
breaking and interacts with other particles with coupling suppressed by the PQ scale. This
scalar may play some non-trivial role in cosmology as its potential can receive additional
sizeable contribution at early Universe. The relic abundance of dark matter depends on the
cosmological evolution of the saxion. It is thus of importance to understand the saxion
properties.
5.1 Saxion decay

Because it acquires mass as $\mathcal{O}(0.1M_0) \leq m_\sigma \leq \mathcal{O}(M_0)$ depending on $S_0/\Lambda_\Phi$, the saxion will decay into axino, gravitino, and light MSSM particles. The interaction relevant for its decay can be derived from the effective action \[ \mathcal{L}_{\text{int}}^a = \frac{\sigma}{\sqrt{2}S_0} \left[ (\partial^\mu a) \partial_\mu a + \left( \frac{c_3}{2} m_3 \tilde{a} a + \frac{1}{16\pi^2} \frac{m_2^2}{M_0} S_0 \tilde{X} \tilde{X} + \text{h.c.} \right) \right], \]

(5.1)

where we have included the effective interaction with the goldstino. The coefficient $c_3 = (\partial \ln m_3 / \partial \ln |S|)|_{S = S_0}$ has a value of $\mathcal{O}(0.1)$ or less for $y_3^2 S_0^2 \leq \mathcal{O}(10^{-3}) \Lambda_\Phi$. If the saxion is stabilized at $y_3 |S| = \mathcal{O}(0.1 \Lambda_\Phi)$, the Kähler correction (2.7) becomes important and leads to $c_3 = \mathcal{O}(1)$ and $m_\sigma = \mathcal{O}(M_0)$. Besides the above interactions, the saxion also has couplings to MSSM particles when the $\mu$ term arises after PQ symmetry breaking:

$$\sigma \tilde{H}_u^0 \tilde{H}_d^0 + \sigma |H_{u,d}^0| H_{u,d}^0 + \left( \tilde{H}_{u,d}^0 \tilde{f} + H_{u,d}^0 \tilde{f} \tilde{c} + H_{u,d}^0 Z, Z, \right) \right),$$

(5.2)

where the first two terms come from the Higgs coupling to $S$, while the other interactions require the saxion component of $H_{u,d}^0$. The ellipsis contains the interactions with $W^\pm$ and $W^{0,\pm}$. There are also saxion interactions induced at the loop level, which include

$$\sigma \tilde{f} \tilde{f}^* + \sigma F_{\mu\nu} F_{\mu\nu},$$

(5.3)

where the couplings can be derived using the relation (3.17), and making the replacement (3.13). Though it is suppressed by the loop factor, the interaction $\sigma \tilde{f} \tilde{f}^*$ can be important because the saxion coupling from $H_{u,d}^0 \tilde{f} \tilde{c}$ is suppressed by $\mathcal{C}_\sigma A_f / M_0 = \mathcal{O}(\mathcal{C}_\sigma / 8\pi^2)$ in gauge mediation. Here $A_f$ is the soft trilinear coupling.

Before examining the partial decay widths of the saxion, we note that the color-charged sparticles are much heavier than uncolored sparticles in minimal gauge mediation. This results in that the electroweak symmetry breaking is achieved under a fine tuning of a few percent between the Higgs mass parameters. So we shall consider the $\mu$ term with $9$

$$\frac{m_h^2}{M_0}, \frac{m_Z^2}{M_0^2}, \frac{M_{\tilde{B}_0}^2}{M_0^2} \ll \frac{\mu^2}{M_0^2} \leq \mathcal{O}(1).$$

(5.4)

For later discussion, it is convenient to define the total decay width of the saxion as

$$\Gamma_\sigma = \frac{1}{64\pi B_a} \frac{m_\sigma^2}{S_0^2},$$

(5.5)

with $B_a$ being the branching ratio into axions. Let us now examine the saxion decay. From (5.3), the decay widths for $\sigma \to \tilde{a} \tilde{a}$ and $\sigma \to a \tilde{G}$ are determined by

$$\left( \Gamma_{\sigma \to \tilde{a} \tilde{a}}, \Gamma_{\sigma \to a \tilde{G}} \right) \approx B_a \Gamma_{\sigma} \left( \frac{c_7^2 m_7^2}{2 m_\sigma^2} + \frac{1}{(8\pi^2)^2} \frac{m_\sigma^2 S_0^2}{M_0^2 \tilde{X}^2} \right).$$

(5.6)

---

9 Using that the combination $M_\beta / g_\tilde{b}^2$ is invariant under the RG evolution at the one-loop level, one obtains $M_\beta \simeq 0.22 N_\Phi M_0$ at $Q = 1 \text{TeV}$. The bino mass is thus about 100 GeV for $N_\Phi = 1$ and $M_0 = 500 \text{GeV}$. In the subsequent discussion, we will treat $M_\beta$ as a free parameter satisfying $M_\beta \ll M_0^2$. 
The saxion also decays into MSSM particles via the interactions (5.2) and (5.3), and thus crucially depends on the form of the Higgs coupling to \( S \). In particular, for \( C_\sigma = \mathcal{O}(1) \), the saxion decay occurs with

\[
\left( \Gamma_{\sigma \rightarrow hh}, \Gamma_{\sigma \rightarrow ff}, \Gamma_{\sigma \rightarrow ZZ} \right) \simeq 4\mathcal{O}_\sigma^2 B_a \Gamma_{\sigma} \frac{|\mu|^4}{m_\sigma^2} \left( \frac{N_f k_f^2 m_f^2}{(m_h^2 - m_\sigma^2)^2}, \frac{k_Z M_Z^4}{(m_h^2 - m_\sigma^2)^2} \right), \tag{5.7}
\]

if the processes are kinematically allowed, where \( k_h = 1 + |B|\sin 2\alpha/2|\mu| \), \( k_Z = N_u \sin \beta + N_d \cos \beta \), and we have neglected the mass of the decay products. In the decay width into the SM fermions, \( N_f = 3 \) (1) for quarks (leptons), and \( k_f = N_f^\dagger / \sin \beta \) for up-type quarks while \( k_f = N_f^\dagger / \cos \beta \) for down-type quarks and leptons. Here \( k_{h,f,Z} \) have a value of order unity. Furthermore, if it is heavy enough, the saxion will decay into MSSM sparticles with the decay widths roughly estimated as

\[
\left( \Gamma_{\sigma \rightarrow \bar{B}\tilde{B}}, \Gamma_{\sigma \rightarrow \tilde{\tau}\tilde{\tau}} \right) \sim 4B_a \Gamma_{\sigma} \frac{|\mu|^2}{m_\sigma^2} \left( \frac{C_s^2 M_Z^4}{(m_h^2 - m_\sigma^2)^2}, \frac{N_f k_f^2}{(8\pi^2)^2 m_\sigma^2 |\mu|^2} \right), \tag{5.8}
\]

where \( \bar{B} \) is a bino-dominant neutralino with small Higgsino component, and the stau has \( k_\tau = 4N_f^2 C_s^2 (g_0^6 (\Lambda) - g_0^6 (M_0))/b_a \). To estimate the decay width into MSSM sparticles, we have used that the interaction \( \sigma \bar{B} \tilde{B} \) arises from \( \sigma \tilde{H}_u^0 \tilde{H}_d^0 \) and \( H_{u,d}^0 \bar{B} \tilde{H}_{u,d}^0 \) due to the bino-Higgsino mixing, and that gauge mediation gives \( A_\tau = \mathcal{O}(M_0/8\pi^2) \).

### 5.2 Cosmological constraints

During the inflationary epoch, the saxion is displaced from the true vacuum because it obtains a Hubble-induced mass term. After inflation, as the Universe is reheated, the PQ messengers generate a thermal potential for the saxion, \( \delta V \sim g_0^2 T^2 |S|^2 \) at \( |S| \ll T \). For \( |S| \gg T \), the saxion begins to oscillate about the true minimum with an amplitude of \( \mathcal{O}(S_0) \). This implies that thermal inflation \([10]\) occurs when the potential energy \( V_0 \sim m_\sigma^2 S_0^2 \) dominates the Universe. After thermal inflation, the saxion begins to oscillate about the true minimum with an amplitude of \( \mathcal{O}(S_0) \). It is also possible that the saxion is shifted far from the origin during primordial inflation. In this case, the coherent oscillation of the saxion starts with an amplitude less than \( M_{Pl} \) when the Hubble parameter becomes comparable to \( m_\sigma \). Since it behaves like matter, the oscillation energy would dominate the energy of the Universe if the initial amplitude is large enough.

Let us examine cosmological constraints of the saxion properties in the case that the saxion dominates the Universe at an early time, which is a plausible possibility as discussed above. Since a late-time entropy production would alleviate the constraints placed on the saxion, we further assume that there is no additional entropy generation after the saxion decay. An important constraint then comes from the axions produced by the saxion decay as they behave like neutrinos \([30]\). If there is no coupling between \( H_u H_d \) and \( S \), the saxion mainly decays into two axions with \( B_a \sim 1 \). Obviously, the produced axions would spoil the Big Bang nucleosynthesis. However, the situation can be different if the \( \mu \) term is
generated after $U(1)_{\text{PQ}}$ breaking. From (5.5), one sees that the saxion couplings to the MSSM particles can easily suppress $B_a$ below 0.1 for

$$m_\sigma^2 \ll |\mu|^2,$$

(5.9)

when the $\mu$ term is generated by the KN mechanism. We assume that this is the case. Using (5.5), one then finds that $B_a$ has a value between $O(10^{-4})$ and $O(10^{-2})$ for $m_\sigma > 2M_Z$. Depending on $m_\sigma$, the decay is dominated by $\sigma \rightarrow hh$, $\sigma \rightarrow ZZ$, or $\sigma \rightarrow t\bar{t}$ with $t$ being the top quark. On the other hand, if $m_\sigma < 2M_Z$, the saxion mainly decays into bottom quarks, while giving $O(10^{-3}) \leq B_a < 0.1$ for $m_\sigma^2 \ll 0.1|\mu|^2$. Therefore, one can naturally avoid production of too many axions from the saxion decay.$^{10}$

There is also a constraint from the dark matter abundance. First of all, the total abundance of LSPs should not exceed the measured abundance of the dark matter.$^{11}$ In addition, an attention should be paid to the LSPs produced from late decays of the axino/gravitino because they will contribute to the energy density of hot dark matter. The current energy density of cold dark matter is $\Omega_{\text{CDM}} \simeq 0.2$, while the density of hot dark matter is bounded from above as $\Omega_{\text{HDM}} \lesssim 10^{-3}$ [4, 35]. In the situation under consideration, the dark matter abundance essentially depends on the decay temperature of the saxon

$$T_\sigma = \left( \frac{90}{\pi^2 g_*(T_\sigma)} \right)^{1/4} \sqrt{\Gamma_\sigma M_{\text{Pl}}} \simeq 6\text{GeV} \left( \frac{0.01}{B_a} \right)^{1/2} \left( \frac{m_\sigma}{10^2 \text{GeV}} \right)^{3/2} \left( \frac{10^{11} \text{GeV}}{S_0} \right),$$

(5.10)

for $g_*(T_\sigma) = O(10^2)$, with $g_*$ being the effective degrees of freedom of the radiation. Note that there are two main processes for the production of axinos and gravitinos, (i) the decay of the saxion $\sigma$, and (ii) the decay of the LOSP $\tilde{\chi}$. If $m_\sigma > 2m_\tilde{\chi}$, LOSPs will be produced directly from the saxion decay. In addition, there are thermally generated LOSPs when $T_\sigma$ is higher than $T_f$. Here $T_f \sim m_\chi/20$ denotes the freeze-out temperature of $\tilde{\chi}$, below which $\tilde{\chi}$ decouples from the thermal bath.

We first concentrate on the direct production of axinos and gravitinos from the saxion decay. The yield of axinos produced from the saxion decay is the sum of the axino yield from $\sigma \rightarrow \tilde{a}\tilde{a}$, $Y_0^\sigma$ and that from $\sigma \rightarrow \tilde{a}\tilde{G}$, $Y_3^\sigma/2$:

$$Y_0^\sigma = Y_0^\sigma + Y_3^\sigma/2.$$  

(5.11)

Using the relation (5.6) and $m_\tilde{a} = O(M_0/8\pi^2)$, each term is evaluated as

$$Y_0^\sigma = \frac{3}{2} \frac{T_\sigma}{m_\sigma} \frac{\Gamma_\sigma \rightarrow \tilde{a}\tilde{a}}{\Gamma_\sigma} \sim 4 \times 10^{-10} \left( \frac{c_\alpha}{0.1} \right)^2 \left( \frac{B_a}{0.01} \right)^{1/2} \left( \frac{m_\tilde{a}}{1\text{GeV}} \right)^2 \left( \frac{10^2 \text{GeV}}{m_\sigma} \right)^{3/2} \left( \frac{10^{11} \text{GeV}}{S_0} \right),$$

(5.12)

$^{10}$Models in [31, 32] have used this property to suppress the axion production from the saxion decay. In the model of [32], where the $\mu$ term is generated by the GM mechanism, the authors introduced additional SM singlet to achieve $C_\sigma = O(1)$. On the other hand, in axionic mirage mediation [33] which also incorporates the GM mechanism, the large entropy released by the modulus decay dilutes the axions produced by the saxion decay with $B_a \simeq 1$.

$^{11}$The axion also contributes to the energy density of dark matter, $\Omega_a \sim 0.4 \theta_2^2 (S_0/10^{12} \text{GeV})^{1.18}$ with $\theta_2$ being the initial misalignment angle.
and
\[ Y_{3/2}^\sigma = \frac{3}{4} \frac{T_\sigma}{m_\sigma} \frac{\Gamma_{\sigma \to a \tilde{G}}}{\Gamma_\sigma} \sim 10^{-4} \left( \frac{0.1}{c_a} \right)^2 \left( \frac{m_\sigma}{0.3 M_0} \right)^4 \frac{S_0}{10^{-2} X} Y_0^\sigma \ll Y_0^\sigma, \]
the latter of which is the same as the gravitino yield from \( \sigma \to a \tilde{G} \). Note that, when
the saxion is stabilized by \( y_5^2 S_0^2 \leq O(10^{-3} \Lambda_5^2) \), one generically obtains \( c_a = O(0.1) \) and
\( m_\sigma = O(0.1 M_0) \). For \( y_5 S_0 = O(0.1 \Lambda_5) \), \( c_a \) is of order unity, but the saxion acquires mass
of \( O(M_0) \). As discussed already, \( B_\alpha < 0.1 \) can be achieved for \( m_\sigma^2 \ll |\mu|^2 \). To be consistent
with the cosmological observations, \( Y_{a,3/2}^\sigma \) should satisfy the constraints
\[ Y_{a}^\sigma \leq 3.6 \times 10^{-10} \left( \frac{1 \text{GeV}}{m_\alpha} \right) \text{ and } Y_{3/2}^\sigma \leq 1.8 \times 10^{-12} \left( \frac{1 \text{GeV}}{m_\alpha} \right), \]
for the axino LSP, where the latter one comes from the constraint on the hot component.
For the gravitino LSP, the produced axinos decay into gravitinos, yielding the hot dark
matter. Thus, we obtain the constraint
\[ Y_{a}^\sigma \lesssim 1.8 \times 10^{-12} \left( \frac{1 \text{GeV}}{m_\alpha} \right) \] (5.14)
It is interesting to note that, if the axino is the LSP, the above cosmological constraints
can be satisfied easily for \( S_0 > 10^{11} \text{GeV} \). In addition, the axino can naturally explain
the dark matter component of the Universe today. On the other hand, in the case of the
gravitino LSP, the constraint is severer but not difficult to satisfy. For instance, models
with \( S_0 \sim 10^{11} \text{GeV} \) and \( m_{3/2} \sim 10 \text{MeV} \) will survive the constraint. The axino yield is
further suppressed when \( B_\alpha \) is less than 0.1. Here, one should note that the relation \( [18] \)
leads to \( (m_\alpha/m_{3/2}) S_0 \leq 10^{13} \text{GeV} \) for \( S_0 \leq 0.1 X \).

Another process for the production of axino/gravitino is the LOSP decay. This becomes
important either when \( T_\sigma > T_f \), or when \( T_\sigma < T_f \) and \( m_\sigma > 2 m_\tilde{\chi} \). For \( S_0 < 0.1 X \), since \( \tilde{\chi} \)
will dominantly decay into axinos with \( \Gamma_{\tilde{\chi} \to \gamma \tilde{a}} \ll 10^{-3} \Gamma_{\tilde{\chi} \to \tilde{a}} \), the gravitino production from
its decay can safely be ignored. Let us examine the case that \( T_\sigma \) is above \( T_f \), for which
LOSps can be in thermal equilibrium with SM particles. The axino production is then
determined by the total decay width of \( \tilde{\chi} \)
\[ \Gamma_{\tilde{\chi}} = \frac{r_{\tilde{\alpha}}^2 m_{\tilde{\chi}}^2}{4 \pi S_0^2}. \]
A stau LOSP has \( r_{\tilde{\alpha}}^2 = O(10^{-3}) \), whereas a bino LOSP has \( O(10^{-4}) \leq r_{\tilde{\alpha}}^2 \leq O(10^{-2}) \),
depending on their mass. The decay rate \( \Gamma_{\tilde{\chi}} \) is thus smaller than the Hubble parameter
at \( T = T_\sigma \), for \( S_0 \geq 10^{10} \text{GeV} \). If the decay temperature is below \( m_{\tilde{\chi}} \), the yield of axinos is
naively estimated as \( Y_{a,0}^\tilde{\chi} = Y_{a,0}^0 + Y_{a,1}^\tilde{\chi} \), with \( Y_{a,0}^\tilde{\chi} \) given by
\[ Y_{0}^{\tilde{\chi}} \sim \frac{45}{2 \pi^3 \sqrt{2} \pi g_*(T_f)} \left( \frac{m_{\tilde{\chi}}}{T_f} \right)^{3/2} e^{-m_{\tilde{\chi}}/T_f}, \]
\[ Y_{1}^{\tilde{\chi}} \sim \frac{45}{2 \pi^3 \sqrt{2} \pi} \int_{T_f}^{T_\sigma} \frac{dt}{g_*(T)} \left( \frac{m_{\tilde{\chi}}}{T} \right)^{3/2} e^{-m_{\tilde{\chi}}/T} \]
\[ \sim 3 \times 10^{-9} \left[ \int_{x_f}^{x_\sigma} dx \left( \frac{100}{g_*(x)} \right)^{3/2} e^{-x/x_f} \right] \left( \frac{r_{\tilde{\alpha}}^2}{200 \text{GeV}} \right) \left( \frac{m_{\tilde{\chi}}}{10^{11} \text{GeV}} \right)^2, \] (5.17)
for $x \equiv T/m_{\tilde{\chi}}$ with $x_\sigma = T_\sigma/m_{\tilde{\chi}}$ and $x_f = T_f/m_{\tilde{\chi}}$. After freeze-out of $\tilde{\chi}$, all the remaining LOSPs will decay into axinos. This contribution gives $Y_0^{\tilde{\chi}}$, which is less than $O(10^{-13})$ for $T_f \leq m_{\tilde{\chi}}/20$. Hence, a constraint is placed on $Y_0^{\tilde{\chi}}$. Substituting $Y_0^a$ by $Y_1^{\tilde{\chi}}$ in the equation (5.14), one can find that the LOSP decay would not cause cosmological problems for an axino LOSP if $T_\sigma \lesssim m_{\tilde{\chi}}/10$, or $S_0 \gtrsim 10^{11}\text{GeV}$. For $S_0 \sim 10^{10}\text{GeV}$, it is still possible to achieve $T_\sigma$ below $m_{\tilde{\chi}}/10$ when the saxion acquires mass, $m_\sigma = O(0.1M_0)$. On the other hand, if the gravitino is the LSP, one would need $T_\sigma \lesssim m_{\tilde{\chi}}/15$ and $m_{3/2} \ll 1\text{GeV}$ to suppress the abundance of hot LSPs. Notice that the case with $T_\sigma > m_{\tilde{\chi}}$ should obviously be excluded\(^{12}\).

Let’s move on to the case that $T_\sigma$ is lower than $T_f$, for which there are only a negligible number of LOSPs in thermal bath. However, LOSPs will be produced abundantly from the saxion decay if the decay process is kinematically allowed. In this case, the annihilation process can be effective to reduce their number density, depending on the decay width of the saxion into $\tilde{\chi}$:

$$
\Gamma_{\sigma \to \tilde{\chi}} \equiv r_\sigma^2 B_\sigma \Gamma_\sigma.
$$

From (5.7), one obtains $O(10^{-3}) \leq r_\sigma \leq O(10^{-1})$ for a bino LOSP, while $O(10^{-6}) \leq r_\sigma \leq O(10^{-3})$ in the stau LOSP case. The LOSPs produced by the saxion decay will annihilate with each other if the interaction rate is much larger than $\Gamma_{\tilde{\chi}}$. This condition translates into

$$
\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_{\tilde{\chi}} \gtrsim \frac{10^{-18}}{\text{GeV}^2} \left( \frac{g_*}{B_\sigma} \right) \left( \frac{r_\sigma^2}{r_{\tilde{\chi}}^2} \right) \left( \frac{m_\sigma}{2m_{\tilde{\chi}}} \right) \left( \frac{m_{\tilde{\chi}}}{25 T_\sigma} \right)^4 \left( \frac{10^{11}\text{GeV}}{S_0} \right)^2,
$$

where $\langle \cdots \rangle$ represents the thermal average of the annihilation cross section times the relative velocity of $\tilde{\chi}$. For a bino or stau LOSP, the above condition is indeed satisfied well, implying that LOSPs are so abundant. Therefore, the annihilation process occurs quite effectively until the Hubble parameter becomes comparable to the annihilation rate. After annihilation, $\tilde{\chi}$ decays into an axino and axion, and thus the axino abundance is determined by

$$
Y_{a'}^{\tilde{\chi}} \sim Y_{\tilde{\chi}}^{\sigma} \approx \frac{1}{2^{1/2}} \frac{90}{\tau_{\tilde{\chi}}^2 g_* (T_\sigma)} \left( \frac{m_{\tilde{\chi}}}{25 T_\sigma} \right)^4 \frac{1}{\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_{\tilde{\chi}} T_\sigma M_{Pl}}.
$$

For a stau LOSP, the annihilation cross section is roughly given by $\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_{\tilde{\chi}} \sim 10 \alpha_{\text{em}}^2 m_{\tilde{\chi}}^2 \left[ 37 \right]$, and has a value of $O(10^{-8})\text{GeV}^{-2}$ for $m_{\tilde{\chi}} = 200\text{GeV}$. A bino LOSP has $\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_{\tilde{\chi}} \sim 4 \alpha_{\text{em}}^2 m_{\tilde{\chi}}^2 m_{\tilde{\chi}}^{-4} \left[ 38 \right]$, which is of $O(10^{-9})\text{GeV}^{-2}$ for $m_{\tilde{\chi}} = 500\text{GeV}$. In addition, $T_\sigma$ cannot be much lower than $0.1\text{GeV}$ for $10^{10}\text{GeV} \lesssim S_0 \lesssim 10^{12}\text{GeV}$. It is thus not difficult to make $Y_{a'}^{\tilde{\chi}}$ less than $Y_{a'}$. This implies that, even for $T_\sigma$ below $T_f$, the saxion can have mass $m_\sigma > 2m_{\tilde{\chi}}$ without causing cosmological difficulties.

\(^{12}\)Axinos can also be produced by thermal scattering \cite{24} after saxion decay. But the thermal production would be negligible when the saxion decay temperature is much lower than the MSSM sparticle masses.
We close this section by summarizing the implications of the Higgs coupling to $S$. As discussed already, the coupling between $H_uH_d$ and $S$ naturally explains the $\mu$ and $B\mu$ terms in the MSSM. Furthermore, it makes the saxion and axino interact with the MSSM particles via various couplings suppressed by the PQ scale. Mediated by these interactions, the LOSP decay into axinos can occur inside the detector, depending on the PQ scale. The decay length will give us a direct information on the PQ scale. On the other hand, the saxion products might cause cosmological problems, once the Universe is dominated by the saxion. Since the saxion decays into axions, axinos, and gravitinos, there are constraints from the Big Bang nucleosynthesis and the dark matter abundance. However, the cosmological constraints can naturally be satisfied when the $\mu$ term is generated via the KN mechanism, for which the saxion coupling to the MSSM particles is stronger than those induced by the PQ messenger loops.

6. Conclusion

In this paper, we have studied a simple axion model that establishes a connection between the origin of the Higgs $\mu/B\mu$ term and the solution to the strong CP problem within the framework of gauge mediation. Such a connection is possible if the model possesses the PQ symmetry under which the Higgs bilinear $H_uH_d$ is charged. The PQ symmetry breaking is governed by SUSY breaking effects. We pointed out that a crucial role is played by the mixing between the messengers transmitting the SUSY breaking and the PQ symmetry breaking to the MSSM sector. In the presence of such mixing, the PQ scale is radiatively stabilized at a scale below the gauge mediation scale. This stabilization mechanism can apply to other cases as well, such as models with a generalized messenger sector, or flaton models where $S$ corresponds to a flaton field.

Also important is that the model provides a natural explanation for the presence of both $\mu$ and $B\mu$. They are generated with the correct size from a coupling between $H_uH_d$ and $S$, which also induces the saxion/axino interactions with the MSSM particles. Furthermore, the phase of $B$ is aligned with that of the gaugino masses, thereby not spoiling the nice property of gauge mediation that the induced soft terms do not lead to excessive flavor and CP violations. In the model, the LSP is either the axino or the gravitino depending on the scale of gauge mediation, while the LOSP can be the bino or the stau. The Higgs coupling to $S$ leads to that the LOSP mainly decays into axinos with coupling suppressed by the PQ scale. Thus, the collider signature highly depends on the PQ scale. If the saxion is stabilized at a scale around $10^{10}\text{GeV}$ or less, the LOSP can decay within the detector while giving distinct signals. On the other hand, for $S_0$ larger than $10^{10}\text{GeV}$, the LOSP will decay with a rather long lifetime, but still a non-negligible amount of the LOSPs will decay inside the detector unless $S_0$ is out of the axion window.

We have also investigated the cosmological constraints placed on the saxion when it dominates the energy density of the Universe. In order for the saxion decay not to conflict with the successful predictions of the Big Bang nucleosynthesis, its branching ratio into axions should be suppressed. Moreover, the LSPs from the saxion decay should not overclose the Universe. In particular, the hot LSPs from the late decay of axino/gravitino
should be small enough to be consistent with the cosmological observations. All these constraints can be satisfied well for the PQ scale around $10^{11}$ GeV when the $\mu$ term is generated via the KN mechanism, i.e. from a superpotential Higgs coupling to $S$. This is because the saxion is coupled to the SM particles more strongly compared to the case of the GM mechanism. If there is an extra entropy production, models that incorporate the GM mechanism can still be cosmologically viable.

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### A. PQ sector soft terms

In this appendix, we provide the expressions for PQ sector soft terms for the case that the saxion is stabilized at $y_{\Psi}|S| \leq \mathcal{O}(\Lambda_{\Phi}/\sqrt{8\pi^2})$. The soft terms are parameterized in terms of $\{M_0, \Lambda_{\Phi}, N_{\Phi}, N_{\Psi}\}$, and the supersymmetric couplings $g_\alpha$ and $y_{q,\ell}$. The Yukawa couplings are written as

$$y_{\Psi} S \bar{\Psi} = y_{q} S \bar{q} + y_{\ell} S \bar{\ell},$$

(A.1)

for the PQ triplet $q$ and doublet $\ell$. Integrating out $\Phi + \bar{\Phi}$, soft terms are generated as

$$A_{q,\ell}(\Lambda_{\Phi}^-) \sim \frac{N_{\Phi}}{M_0} \left( 10N_{\Psi} + 1 + 16\pi^2\frac{|S|^2}{\Lambda_{\Phi}^2} \right) y_{S}^2,$$

$$m_{q}^2(\Lambda_{\Phi}^-) \sim \frac{N_{\Phi}}{M_0^2} \left( \frac{8}{3}g_2^4 + \frac{2}{15}g_1^4 - (5N_{\Psi} + 1)y_{\Psi}^2y_{S}^2 \right),$$

$$m_{\ell}^2(\Lambda_{\Phi}^-) \sim \frac{N_{\Phi}}{M_0^2} \left( \frac{3}{2}g_2^4 + \frac{3}{10}g_1^4 - (5N_{\Psi} + 1)y_{\Psi}^2y_{S}^2 \right),$$

$$m_{\ell}^2(\bar{\Lambda}_{\Phi}^-) \sim \frac{N_{\Phi}}{M_0^2} \left( \frac{8}{3}(g_2^2 - 2y_{S}^2)g_3^2 + \frac{2}{15}(g_1^2 - 2y_{S}^2)g_1^2 + \xi y_{S}^2 \right),$$

$$m_{\ell}^2(\bar{\Lambda}_{\Phi}^-) \sim \frac{N_{\Phi}}{M_0^2} \left( \frac{3}{2}(g_2^2 - 2y_{S}^2)g_2^2 + \frac{3}{10}(g_1^2 - 2y_{S}^2)g_1^2 + \xi y_{S}^2 \right),$$

(A.2)

at the scale just below $\Lambda_{\Phi}$, with

$$\xi = (5N_{\Phi}N_{\Psi} + N_{\Phi} + N_{\Psi})y_{S}^2 + y_{\Psi}^2 - (16\pi^2)^2\frac{|S|^2}{\Lambda_{\Phi}^2},$$

(A.3)

where the gauge and Yukawa couplings are evaluated at $\Lambda_{\Phi}$, neglecting the splitting in PQ Yukawa couplings.
At scales between $\Lambda_\Psi$ and $\Lambda_\Phi$, soft terms are RG evolved as

\[
\begin{align*}
\frac{dA_q}{d\ln Q} &= \frac{1}{8\pi^2} \left\{ (3N_\Psi + 2)y_q^2 A_q + 2N_\Psi g_\ell^2 A_\ell - 2 \left( \frac{8}{3}g_3^2 M_3 + \frac{2}{15}g_\ell^2 M_1 \right) \right\}, \\
\frac{dA_\ell}{d\ln Q} &= \frac{1}{8\pi^2} \left\{ 3N_\Psi y_\ell^2 A_q + (2N_\Psi + 2)y_\ell^2 A_\ell - 2 \left( \frac{3}{2}g_2^2 M_2 + \frac{3}{10}g_\ell^2 M_1 \right) \right\}, \\
\frac{dm_\tilde{q}^2}{d\ln Q} &= \frac{1}{8\pi^2} \left\{ y_q^2 P_q - 2 \left( \frac{8}{3}g_3^2 |M_3|^2 + \frac{2}{15}g_\ell^2 |M_1|^2 \right) \right\}, \\
\frac{dm_\tilde{\ell}^2}{d\ln Q} &= \frac{1}{8\pi^2} \left\{ y_\ell^2 P_\ell - 2 \left( \frac{3}{2}g_2^2 |M_2|^2 + \frac{3}{10}g_\ell^2 |M_1|^2 \right) \right\}, \\
\frac{dm^2}{d\ln Q} &= \frac{N_\Psi}{8\pi^2} (3y_q^2 P_q + 2y_\ell^2 P_\ell),
\end{align*}
\]

for $P_q = m_S^2 + m_\tilde{q}^2 + m_{A_\ell}^2$, and $P_\ell = m_S^2 + m_\tilde{\ell}^2 + m_{A_\ell}^2$. On the other hand, the running of PQ Yukawa couplings is determined by

\[
\begin{align*}
\frac{dy_q^2}{d\ln Q} &= \frac{y_q^2}{8\pi^2} \left\{ (3N_\Psi + 2)y_q^2 + 2N_\Psi g_\ell^2 - 2 \left( \frac{8}{3}g_3^2 + \frac{2}{15}g_\ell^2 \right) \right\}, \\
\frac{dy_\ell^2}{d\ln Q} &= \frac{y_\ell^2}{8\pi^2} \left\{ 3N_\Psi y_\ell^2 + (2N_\Psi + 2)y_\ell^2 - 2 \left( \frac{3}{2}g_2^2 + \frac{3}{10}g_\ell^2 \right) \right\},
\end{align*}
\]

with gauge couplings given by

\[
\frac{1}{g_\ell^2(Q)} \approx 2 + \frac{N_\Psi}{8\pi^2} \ln \left( \frac{\Lambda_\Psi}{Q} \right) + \frac{b_\alpha}{8\pi^2} \ln \left( \frac{M_{\text{GUT}}}{Q} \right),
\]

at $\Lambda_\Psi < Q < \Lambda_\Phi$. Here $b_\alpha$ are the beta function coefficients for the MSSM.

**B. Mixing parameters**

To derive the saxion/axino couplings to MSSM particles, one can make the replacements (3.13) and (B.1). The mixing between the saxion with neutral CP even Higgs bosons, $h$ and $H$, is parameterized by $N_{d,a}^\alpha$:

\[
(N_d^\alpha, N_a^\alpha) = (-n_\sigma \sin \alpha + n'_\alpha \cos \alpha, n_\sigma \cos \alpha + n'_\alpha \sin \alpha),
\]

where $n_\sigma$ and $n'_\sigma$ are given by

\[
\begin{align*}
n_\sigma &= 2 \sin(\beta - \alpha) - \frac{|B|}{|\mu|} \cos(\beta + \alpha), \\
n'_\sigma &= \frac{m_h^2 - m_Z^2}{m_H^2 - m_W^2} \left( 2 \cos(\beta - \alpha) - \frac{|B|}{|\mu|} \sin(\beta + \alpha) \right),
\end{align*}
\]

with $\alpha$ being the mixing angle for $h$ and $H$. On the other hand, the parameters $N_i^\tilde{\alpha}$ for the axino mixing with the neutral gauginos and Higgsinos are determined by

\[
\begin{align*}
(N_B^\tilde{\alpha}, N_W^\tilde{\alpha}) &\simeq \frac{\cos 2\beta}{1 - n_\tilde{\alpha} \sin 2\beta} \left( \frac{M_Z}{M_B} \sin \theta_W, \frac{M_Z}{M_W} \cos \theta_W \right), \\
(N_R^\tilde{\alpha}, N_H^\tilde{\alpha}) &\simeq \frac{1}{1 - n_\tilde{\alpha} \sin 2\beta} \left( \cos \beta - n_\tilde{\alpha} \sin \beta, \sin \beta - n_\tilde{\alpha} \cos \beta \right),
\end{align*}
\]
for $n_{\tilde{a}}$ defined by

$$n_{\tilde{a}} = \frac{M_Z}{\mu} \left( \frac{M_Z}{M_B} \sin^2 \theta_W + \frac{M_Z}{M_W} \cos^2 \theta_W \right), \quad (B.4)$$

where $M_Z$ is the Z boson mass, and $\theta_W$ is the weak mixing angle. Here we have neglected corrections suppressed by $m_{\tilde{a}}/M_{\tilde{B}, \tilde{W}}$ or by $m_{\tilde{a}}/\mu$, and have used that there is mixing between the neutral Higgsinos and gauginos

$$\mathcal{L}^{\tilde{H}}_{\text{mix}} = M_Z \left( \tilde{H}_d^0 \cos \beta - \tilde{H}_u^0 \sin \beta \right) \left( \tilde{B} \sin \theta_W - \tilde{W}_0^0 \cos \theta_W \right) + \text{h.c.}, \quad (B.5)$$

which arises after the electroweak symmetry breaking.

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