Perturbative and non-perturbative effects in ultraperipheral production of lepton pairs

I.M. Dremin\[1\]
Lebedev Physical Institute, Moscow, Russia

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Abstract

Perturbative and non-perturbative terms of the cross sections of ultraperipheral production of lepton pairs in ion collisions are taken into account. It is shown that production of low-mass $e^+e^-$ pairs is strongly enhanced (compared to perturbative estimates) due to the non-perturbative Sommerfeld-Gamow-Sakharov (SGS) factor. Coulomb attraction of the non-relativistic components of those pairs leads to the finite value of their mass distribution at lowest masses. Their annihilation can result in the increased intensity of 511 keV photons. It can be recorded at the NICA collider and is especially crucial in astrophysical implications regarding the 511 keV line emitted from the Galactic center. The analogous effect can be observed in lepton pairs production at LHC. Energy spectra of lepton pairs created in ultraperipheral nuclear collisions and their transverse momenta are calculated.

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Interaction of two photons from electromagnetic fields of colliding ions can lead to production of lepton pairs. They are produced in grazing ultraperipheral collisions. This process was first treated perturbatively by Landau and Lifshitz in 1934 [1]. It was shown that its total cross section rapidly increases with increasing energy $E$ as $\ln^3 E$ in asymptotics. This is still the strongest asymptotical energy dependence known in particle physics. Moreover, the numerical factor $Z^4\alpha^4$ in the total cross section compensates the effect of the small electromagnetic coupling $\alpha$ for heavy ions with large charge $Ze$. Therefore, the ultraperipheral production of $e^+e^-$-pairs (as well as $\mu^+\mu^-$ etc) in ion collisions can become the dominant mechanism at very high energies. It is already widely studied at colliders. The heuristic knowledge

\[1\]dremin@lpi.ru
of these processes is helpful in understanding some important astrophysical phenomena as well.

Even earlier, almost a century ago, in 1924, Fermi \[2, 3\] considered the general problem of interaction of charged objects with matter: "Let’s calculate, first of all, the spectral distributions corresponding to those of the electric field created by a particle with electric charge, \(e\), passing with velocity, \(v\), at a minimum distance, \(b\), from a point, \(P\).” Fermi obtained the formula for the intensity of the electromagnetic field created by a moving charged object. It was used in 1934 by Weizsäcker \[4\] and Williams \[5\] for their formulation of the method of equivalent photons widely applied nowadays for treatment of the ultraperipheral interactions. Solutions of Maxwell equations for the electromagnetic fields of the moving charge are used to derive the Poynting vector. The component of the Poynting vector along the direction of colliding electromagnetic charges defines the total energy of equivalent photons and their energy distribution.

The distribution of equivalent photons with a fraction of the nucleon energy \(x\) generated by a moving nucleus with the charge \(Ze\) can be obtained from the expression for the Poynting vector as

\[
\frac{dn}{dx} = \frac{2Z^2\alpha}{\pi x} \ln \frac{u(Z)}{x}
\]  

if integrated over the transverse momentum of scattered nuclei up to some value (see, e.g., Ref. \[6\]). The physical meaning of the ultraperipherality parameter \(u(Z)\) is the ratio of the maximum adoptable transverse momentum to the nucleon mass as the only massless parameter of the problem. Its value is determined by the form factors of colliding ions responsible for their entity (see, e.g., \[7\]). It is clearly seen from Eq. (1) that soft photons with small fractions \(x\) of the nucleon energy dominate in these fluxes.

Pairs of leptons with opposite charges are produced in grazing collisions of interacting ions where two photons from their electromagnetic clouds interact. Abundant creation of pairs with rather low pair-masses is the typical feature of ultraperipheral interactions \[8\]. Two-photon fusion production of lepton pairs has been calculated with both the equivalent photon approximation proposed in \[4, 5\] and via full lowest-order QED calculations \[1, 9, 10, 11, 12\] reviewed recently in \[13\]. According to the equivalent photon approximation, the general expression for the total cross section looks like

\[
\sigma_{\text{up}}(X) = \int dx_1 dx_2 \frac{dn}{dx_1} \frac{dn}{dx_2} \sigma_{\gamma\gamma}(X).
\]
Feynman diagrams of ultraperipheral processes contain the subgraphs of two-photon interactions leading to production of some final states $X$ (e.g., $e^+e^-$ pairs). These blobs can be represented by the cross sections of the corresponding processes. Therefore, $\sigma_{\gamma\gamma}(X)$ in (2) denotes the total cross section of production of the state $X$ by two photons from the electromagnetic clouds surrounding colliding ions and $dn/dx_i$ describe the densities of photons carrying the share $x_i$ of the ion energy. The spectra of lepton pairs created in ultraperipheral collisions can be obtained from Eq. (2) by omitting the relevant integrations.

Both bound (e.g., para- and ortho-positronia for $e^+e^-$) and unbound pairs can be created. However, the bound pairs are less intensively produced than the unbound ones (see, [8]). Therefore we consider here the processes with unbound pairs only. The cross section usually inserted in (2) in case of creation of the unbound pairs $X = e^+e^-$ was calculated by Breit and Wheeler in the lowest order perturbative approach and looks [6, 14] as

$$\sigma_{\gamma\gamma}^{BW}(X) = \frac{\pi \alpha^2}{m^2} (1 - v^2) \left[ \frac{3}{1 - v^4} \ln \frac{1 + v}{1 - v} - 2v(2 - v^2) \right], \quad (3)$$

where $v = \sqrt{1 - \frac{4m^2}{M^2}}$ is the relative velocity of a pair component to the pair center of mass, $m$ and $M$ are the electron and pair masses, correspondingly. The Breit-Wheeler cross section tends to 0 at the threshold of pair production $M = 2m$ ($v = 0$) and decreases as $\frac{1}{M^2} \ln M$ at very large $M$ ($v \to 1$).

However, one must take into account the specific attractive long-range Coulomb forces acting non-perturbatively between the leptons with opposite charges. At the production point, the components of pairs with low masses $M$ close to $2m$ move very slowly relative to one another. They are strongly influenced by the attractive Coulomb forces. In the non-relativistic limit, these states are transformed by mutual interactions of the components to form effectively a composite state whose wave function is a solution of the relevant Schroedinger equation. The normalization of Coulomb wave functions plays an especially important role at low velocities. It differs from the normalization of free motion wave functions used in the perturbative derivation of Eq. (3).

The normalization of the unbound pair wave function reads [15]

$$|\psi(\vec{r} = 0)|^2 = \frac{\pi \xi}{\pi \xi} e^{\pi \xi} = \frac{2\pi \xi}{1 - e^{-2\pi \xi}}; \quad \xi = \frac{\alpha}{v}, \quad (4)$$
This is the widely used Sommerfeld-Gamow-Sakharov (SGS) factor [16, 17, 18, 19] which is responsible for the non-perturbative contribution to the matrix element. It results in the so-called “$\frac{1}{v}$-law” of the enlarged outcome of the reactions with extremely low pair masses. This factor is described in the standard textbooks (see, e.g., the 4-th and later editions of the Landau-Lifshitz book on non-relativistic quantum mechanics [15]) and used in various publications (e.g., [20, 21, 22, 23]). The Sakharov recipe of its account for production of $e^+e^−$-pairs described in [19] consists in direct multiplication of the Breit-Wheeler cross section by the SGS-factor

$$T = \frac{2\pi\alpha}{v(1 - \exp(-2\pi\alpha/v))},$$

such that the cross section $\sigma_{\gamma\gamma}(X)$ in (2) is

$$\sigma_{\gamma\gamma}(X) = \sigma_{BW}^{\gamma\gamma}(X)T.$$

The differential distributions of leptons are easily computed from the integrands of (2). For example, the distribution of the velocity $v$ of pair components with account of the SGS-factor was first obtained in [24]:

$$\frac{d\sigma}{dv^2} = \frac{16Z^4\alpha^5}{3m^2v} \left(3 - v^4\right) \ln \frac{1 + v}{1 - v} \frac{2v(2 - v)}{1 - \exp(-2\pi\alpha/v)} \ln^3 u \sqrt{s_{nn}} \frac{(1 - v^2)}{2m}.$$

Here $\sqrt{s_{nn}}$ is the total energy of two colliding nucleons in the center of mass system. It can be represented by the following expressions $s_{nn} = 4m_n^2\gamma_r^2 = 2m_n^2(\gamma_r + 1) = 2m_n(E_k + 2m_n)$ where $m_n$ is a nucleon mass, $\gamma_c$ and $\gamma_r$ are the Lorentz-factors of the nucleon in the center of mass and rest (of another nucleon) systems and $E_k$ is the excess of the total energy of an impinging proton in the rest system of a target proton over its mass that corresponds to the nucleon kinetic energy in the non-relativistic domain. The threshold of $e^+e^−$-pair creation in the rest system of one of the nucleons is $E_{k,t} = 4m(1 + \frac{m}{2m_n}) \approx 2.05$ MeV.

The distribution (7) is shown in the left-hand side of Fig. 1.

Its most interesting feature is not clearly seen in the left-hand side and, therefore, it is deciphered at the larger scale in the right-hand side of the same Figure for small velocities $v$. It demonstrates the crucial difference between the distributions with (a) and without (b) account of the SGS factor. At low velocities $v$ (i.e., small masses $M = 2m/\sqrt{1 - v^2}$) the two curves tend
Fig. 1. The distribution of the velocities $v$ in $e^+e^-$-pairs produced in ultraperipheral Au-Au-collisions at NICA energy $\sqrt{s_{nn}}=11$ GeV with (a) and without (b) account of the SGS-factor [24]. For the region of small velocities (masses) they are shown in the right-hand side. Their difference (a-b) is shown by the dashed line. They tend to a constant at $v \to 0$ with accounted SGS-factor and to 0 without its account.

Note the factors $10^{-3}$ at the abscissa scale and $10^6$ at the ordinate.

to different values. It is finite with account of SGS factor and vanishes without it. This is a clear signature of its $1/v$-law. In general, the non-relativistic nature of the pair of annihilating particles separates the short-distance annihilation process (taking place at distances up to $O(1/m)$) from the long-distance interactions (characterized by the Bohr radius of the pair $O(1/m\alpha)$), responsible for the SGS-effect. The same peculiar feature of finite $d\sigma/dM^2$ at $M$ close to $2m$ is seen in the distribution of masses $M$ (see Ref. [24]).

The integral contribution of the non-perturbative factor is small as discussed in Ref. [24] but it is crucial for production of pairs with low velocities of their components (see Fig. 1). This remarkable effect of the mutual attraction of the created components is well known.

Protons are the main component of the ion flows in star explosions. The energy behaviour of the total cross section of the ultraperipheral production of $e^+e^-$-pairs in proton-proton collisions is shown in Fig. 2. It is obtained by the integration of Eq. (7) over all admissible velocities $v$ at a given energy $s_{nn}$. They are defined by the requirement to the argument of the logarithm being larger than 1. The cross section does not vanish at the very threshold of pair production $\sqrt{s_t} = 2(m_p + m) \approx 1.88$ GeV but stays finite.

$^2$The ion-ion cross sections are $Z_1^2Z_2^2$-times larger.
Fig. 2. The energy behaviour of the total cross section of ultraperipheral proton-proton interactions from the threshold of $e^+e^-$-production to NICA energies. ($s ≡ s_{nn}$ for $pp$-collisions.)

if the energy dependence in (7) is boldly extended to the threshold. Surely, this is an artefact of its extrapolation down to low energies. It must tend to zero at the threshold because the small masses close to $2m$ (i.e., small velocities $v$) are important near it. At low energies one has to integrate up to small velocities. The finiteness of values of $d\sigma/dv^2$ at the very threshold is provided by the $1/v$-law of the SGS-factor. Similar to shown in Fig. 1, it implies that the total cross section must increase linearly with $s - s_t$ there. The non-perturbative factor is most important near the threshold.

The knowledge of values of the ultraperipheral cross sections at low energies is especially important for astrophysical applications where the energy spectrum of charged hadrons created in star explosions peaks at rather low energies. Correspondingly, the upper abscissa axis shows the values of the relativistically invariant variable $E_k = (s - 4(m_p + m)^2)/2m_p$ which coincides with the kinetic energy of an impinging proton in the non-relativistic
region of collisions treated in the rest system of another proton. The total cms-energy of the colliding protons \( \sqrt{s} \) is shown at the lower abscissa axis.

The flows of created slow electrons and positrons can become rather high to provoke numerous annihilations and emission of 511 keV photons. Further knowledge of these flows can be got from theoretical distributions of energies and transverse momenta of produced electrons and positrons. The energy distributions of electrons and positrons created in ultraperipheral processes coincide with the corresponding distributions of photons in the clouds around the colliding nuclei. Therefore they are directly obtained from Eq. (2) by omitting the \( x_i \)-integrations. One gets

\[
\frac{d^2\sigma}{dE_1dE_2} = \frac{4(Z\alpha)^4 \alpha(1-v^2)^2[(3-v^4) \ln \frac{1+v}{1-v} - 2v(2-v^2)]}{m^4 v(1-\exp(-\frac{2\pi\alpha}{v}))} \ln \frac{u\sqrt{s_{nn}}}{E_1} \ln \frac{u\sqrt{s_{nn}}}{E_2}. \tag{8}
\]

As usual, here \( v = \sqrt{1-\frac{4m^2}{M^2}} = \sqrt{1-\frac{m^2}{E_1E_2}}. \) The low-energy electrons and positrons are favoured.

The transverse momentum \( p_t \)-distribution of leptons can also be obtained from the general formula (2). The non-perturbative effects are prevailing in the region of small masses. Their integral contribution is rather low. Therefore, the main features can be seen if the differential Breit-Wheeler distribution is inserted there. One gets:

\[
\frac{d\sigma_{BW}}{dp_t} = \frac{2\pi\alpha^2(1-v^2)}{m^2 p_T} \frac{1 - p_t^2(1-v^2)/2m^2}{\sqrt{1-p_t^2(1-v^2)/m^2}}. \tag{9}
\]

The transverse momenta of leptons are strongly limited in ultraperipheral production for \( p_t \geq m \) as

\[
\frac{d\sigma_{up}}{dp_t} = \frac{16(Z\alpha)^4}{9\pi p_t^3} \ln^3 \frac{u^2 s_{nn}}{p_t^2}. \tag{10}
\]

Summarizing, it is shown that the total cross sections and differential distributions of lepton production in ultraperipheral nuclear collisions can be calculated with account of both perturbative (Breit-Wheeler) and non-perturbative (Sommerfeld-Gamov-Sakharov) contributions. The rapid energy increase and the finite values of the differential cross sections at the threshold are the most peculiar features of these processes. Their particular values are much larger for heavy nuclei with large charge \( Z e \) surrounded...
by stronger electromagnetic fields. It results in the factor $Z^4$ for the cross sections of identical ions. Emission of low-mass pairs of leptons is favoured.

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