On formalizing fairness in prediction with machine learning

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ABSTRACT

Machine learning algorithms for prediction are increasingly being used in critical decisions affecting human lives. Various fairness formalizations, with no firm consensus yet, are employed to prevent such algorithms from systematically discriminating against people based on certain attributes protected by law. The aim of this article is to survey how fairness is formalized in the machine learning literature for the task of prediction and present these formalizations with their corresponding notions of distributive justice from the social sciences literature. We provide theoretical as well as empirical critiques of these notions from the social sciences literature and explain how these critiques limit the suitability of the corresponding fairness formalizations to certain domains. We also suggest two notions of distributive justice which address some of these critiques and discuss avenues for prospective fairness formalizations.

KEYWORDS

fairness, fairness-measures, discrimination, machine learning, survey

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1 INTRODUCTION

Discrimination refers to unfavourable treatment of people due to the membership to certain demographic groups that are distinguished by the attributes protected by law (henceforth, protected attributes). A discriminatory action may be direct or indirect [81]. A direct discrimination is said to occur when a person is treated less favorably than another is, has been or would be treated in a comparable situation on account of a protected attribute. An indirect discrimination is said to occur where an apparently neutral provision, criterion or practice would put persons with protected attribute(s) at a particular disadvantage compared with other persons. Discrimination can occur simultaneously based on two or more attributes [26].

Discrimination, based on many attributes and in several domains, is prohibited by international legislation. The European Union (EU) have formalized legislation (e.g. Directive 2000/43/EC, Directive 2000/78/EC, Directive 2002/73/EC, Article 21 of the Charter of Fundamental Rights and Protocol 12/Article 14 of the European Convention on Human Rights) to achieve non-discrimination on account of the protected attributes such as race, ethnicity, religion, nationality, gender, sex, 4 disability, marital status, genetic features, language and age [25, 46, 47]. The domains included in the legislation include education, employment, access to housing, public services, health care, adoption, credit and insurance. In the USA, a number of federal non-discrimination laws exist to prevent discrimination in similar domains [72–76]. Similar legal frameworks are present in many other countries too [64, 70, 71].

Nowadays, machine learning algorithms are increasingly being used in high-impact domains such as credit, employment, education, and criminal justice which are prone to discrimination. If the observed human behaviors that dictate how an algorithm transforms input into output are flawed, the ability of the modern computing devices to swiftly replicate decision-making can lead to magnifying erroneous behavior [9]. Barocas and Selbst [6], Citron and Pasquale [12], Edelman and Luca [24], Kay et al. [42] and Sweeney [65] show that algorithms can make discriminatory decisions even if the computing process is well-intentioned. This is because, as Barocas and Selbst [6] assert, the training data "may simply reflect the widespread biases that persist in society at large ... data mining can discover surprisingly useful regularities that are really just preexisting patterns of exclusion and inequality”. A report released by the US Government [27] states that discrimination can sometimes "be the inadvertent outcome of the way big data technologies are structured and used” and points toward “the potential of encoding discrimination in automated decisions”. The goal of fairness in prediction with machine learning is to design algorithms that make fair predictions devoid of discrimination.

The aim of this article is to survey how fairness is formalized in the machine learning literature for the task of prediction and present these formalizations with their corresponding notions from the social sciences literature. Given that the outcomes of the prediction are decisions about social benefits like education, employment and health-care, the fairness formalizations in the machine learning literature correspond to the notions of distributive justice from the social sciences literature. Since, some formalizations of fairness can be conflicting with others, the predictions produced by the algorithms using them would vastly differ as well. As these machine learning prediction algorithms are being used in high-impact domains, their predictions have far-reaching effects on the society. Therefore, from the practical point of view, it is important to study how fairness is formalized in the machine learning literature and the implications of various formalizations. To this end, we present theoretical as well as empirical critiques of their corresponding notions from the social sciences literature. The co-presentation of the formalizations from the machine learning literature and the corresponding critiques from the social sciences literature is with the intention to assist in the following two tasks:

(1) To determine the suitability of the existing formalizations of fairness in machine learning literature.
(2) To build newer formalizations of fairness in prediction with machine learning.

4 Throughout this article, we follow the Publication Manual of the American Psychological Association, "Gender is cultural... Sex is biological."
To forge the way for newer formalizations, we nominate two notions from the social sciences literature which answer some of the critiques of the existing formalizations in the machine learning literature.

For how the fairness (or alternatively discrimination) is measured, we point the readers to Zliobaite [82]. For a multidisciplinary survey on discrimination analysis, we recommend Romei and Ruggieri [58]. In the context of this article, we are mainly interested in the theories of distributive justice from the social science literature. Covering them all in detail is beyond the scope of this article. Roemer [57] provide an overview of this topic from the perspective of both economics and philosophy. From the point of view of legislation, an overview and analysis of fairness is provided by Franck [28].

Figure 1: Disciplines involved in fairness in prediction with machine learning and the place of this article.

In Section 2, we state the mathematical formulation of the problem of prediction with machine learning. In Section 3, we review the fairness formalizations in the literature of prediction with machine learning with their corresponding notions of distributive justice from the social sciences literature. We also provide critiques and analyses of these notions and explain how these critiques limit the suitability of these formalizations to certain domains. In Section 4, we illustrate the differences among the surveyed fairness formalizations with the help of a prediction scenario. As an attempt to answer some of critiques of previous fairness formalizations, we propose two prospective notions of fairness in Section 5. Lastly, in Section 6, we discuss avenues for prospective fairness formalizations.

2 MATHEMATICAL FORMULATION OF THE PROBLEM

Let \( X \) be a set of individuals. We term \( X \) as the population. The individuals in the population are divided into several groups depending upon the protected attributes. Each of the individuals can be assigned an outcome. We denote the set of outcomes by \( A \). Formally, the outcome set could be continuous, but in practice the case with finite number of outcomes is of importance in many applications. Consequently, most of the formalisms that have focused on problem settings with finite number of outcomes and so will this article. In the simplest case, the outcome is binary i.e. \( A = \{0, 1\} \). Some of the prediction outcomes are considered to be more beneficial or desirable than others. If that is not the case, all the outcomes are equivalent and there can be no discrimination in assigning them to the individuals.

For an individual \( x_i \in X \), let \( y_i \) be the true outcome (also called as label, in the literature) to be predicted. A (possibly randomized) predictor can be represented by a mapping \( \mathcal{H} : X \rightarrow A \) from the population \( X \) to the set of outcomes \( A \), such that \( \mathcal{H}(x_i) \) is the predicted outcome for individual \( x_i \).

**Definition 1.** (Group-conditional predictor) A group-conditional predictor consists of a set of mappings, one for each group of the population:\(^2\)

\[
\mathcal{H} = \{\mathcal{H}_S\} \text{ for all } S \subset X.
\]

The accuracy of a predictor \( \mathcal{H} \) is computed as

\[
\text{Accuracy} = \mathbb{P}\{\mathcal{H}(x_i) = y_i\}
\]

where the probability is computed over a distribution of the population. As maximizing the accuracy of a predictor is in the benefit of the decision-maker using it, it is also termed as utility ([79]).

Next, we will see the attempts to formalize fairness with the above definition of the problem.

3 WHAT IS FAIR? (FORMALIZATIONS OF FAIRNESS IN PREDICTION WITH MACHINE LEARNING)

The first step in formalizing fairness in prediction with machine learning is to answer the following two questions:

- **Parity or preference?** : whether fairness means achieving parity or satisfying the preferences.
- **Treatment or impact?** : whether fairness is to be maintained in treatment or impact (results).

Parity-based fairness formalizations typically correspond to egalitarianism, which is a school of thought that equates fairness with some form of equality of all people. Within egalitarianism, there are various notions of fairness in the social sciences literature, which in turn elicit the corresponding formalizations in the machine learning literature. Preference-based fairness formalizations, on the other hand, correspond to non-egalitarian doctrine of distributive justice.

\(^2\)For the sake of simplicity, assume that the groups induce a partition of the population.
Next, we will see the existing formalizations of fairness in the machine learning literature and their corresponding notions from the social sciences literature. Table 1 summarizes how these formalizations answer the two questions presented above.

### 3.1 Treatment parity

Treatment parity as the name suggests, and as Table 1 shows, follows a particular notion of egalitarian fairness in treatment. It is formalized by the following definition.

**Definition 2.** (Treatment parity) A predictor is said to achieve treatment parity if the protected attributes are not explicitly used in the prediction process.

Any predictor which is not group-conditional thus satisfies this formalization of fairness. Treatment parity is appealing because, from the point of view of decision-maker using the predictor, it is easier to deploy rather than proving parity in treatment. However, when the attributes most relevant for prediction overlap with the protected attributes, treatment parity can lead to low accuracy and hence sub-optimal utility to the decision-maker.

A number of proposed predictors in the machine learning literature satisfy treatment parity ([22, 45]), while some don’t ([10, 34, 41]). Pedreshi et al. [52] show that satisfying treatment parity is not a sufficient condition to avoid discrimination when other background knowledge is available. Especially with the ubiquity of social media, there are many avenues for personal information including protected attributes to be leaked from the information readily available in the public domain. Indeed, Korolova [43] note that using data available on Facebook profiles, one can infer user age, sexual orientation, religious affiliation, etc. Kosinski et al. [44] show that it is possible to accurately predict a range of highly sensitive personal attributes including: sexual orientation, ethnicity, religious views, age, and gender from easily accessible digital records of behaviour.

Furthermore, Calders and Žliobaitė [11] show how some of the assumptions made during the construction of a predictor might not hold in real-life scenarios with the help of fictitious examples. This mismatch leads to discrimination even when information about the protected attributes is removed and prediction is guided by neutral arguments such as accuracy only.

From the point of view of distributive justice, treatment parity can be considered a naive way to achieve egalitarianism. In practice, treatment parity corresponds chiefly to the approach of being “blind” to counter racial/gender discrimination. Bonilla-Silva [8] submit that discrimination is still possible against the protected group because of the structural barriers which hinder the protected groups. In particular, Bonilla-Silva [8] analyze the notion of race-blindness in terms of an important frame of abstract liberalism which is stated as the practice of invoking abstract ideals while ignoring concrete proposals to reduce inequality on the ground which results from systematic discrimination. Moreover, Bonilla-Silva [8] also extensively provide studies to document various discriminatory practices in the domains like education, housing, credit etc. Fryer et al. [29] provide a study to compare the efficiency of race-blind approach. The study is based on a hypothetical experiment supposing that seven selected colleges would have had to admit only a fraction of as many students as were, in fact, admitted. The problem is to choose which of the students to retain while achieving the goal of maintaining the original racial representation. The study shows that, while in the short run, race-blind approach is as efficient as race-conscious approach, in the long run, the former is less efficient than the latter. Taslitz [66] and the corresponding essays in the symposium articulate how discrimination is perpetuated in the American criminal justice system despite using the race-blind approach. Alternatively some studies show that a blind approach can sometimes work. For example, Goldin and Rouse [33] show that using blind auditions help in reducing the discrimination against women while hiring musicians in symphony orchestras.

The above critiques challenge the suitability of treatment parity to domains in which,

- protected attributes can be deduced readily from easily available non-protected attributes,
- structural barriers, which hinder the protected groups, are shown to be present by credible surveys.

### 3.2 Group fairness (Statistical/demographic parity)

As shown by Table 1, group fairness follows a particular notion of egalitarian fairness in impact. It imposes the condition that the predictor should predict a particular outcome for individuals across groups with almost equal probability.

**Definition 3.** (Group fairness) A predictor \( \mathcal{H} : X \rightarrow A \) is said to achieve group fairness with bias \( \epsilon \) with respect to the groups \( S, T \subseteq X \):

| Parity       | Preference                  |
|--------------|-----------------------------|
| Treatment    | Treatment parity            | Preferred treatment |
| Impact       | Group fairness              | Preferred impact    |
|              | Individual fairness         |                    |
|              | Equality of opportunity     |                    |

Table 1: The surveyed notions of fairness from the literature of prediction with machine learning.
well-documented. Therefore, disproportionality in equivalent to the stated notion of group fairness unlike the affirmative action program of group fairness is the application of affirmative action (17) to collectivist egalitarianism for distributive justice. In practice, the individuals from the group S should not receive beneficial outcomes insured by some criteria, an argument can be made to "justify" why receiving beneficial outcomes cause reduced performance (measured individuals from group T. If the unqualified individuals beneficial outcomes to unqualified individuals from group S and this finding goes against the opinion that affirmative action policies might harm the intended beneficiaries by placing them in academic situations for which they are poorly suited. To the contrary, the study finds positive change in the test scores of the beneficiaries of the affirmative action after the first year of the college. Galanter [32] provide another assessment of the affirmative action program.

The major findings of interest are as follows ([16]):

- Affirmative action has helped in substantially redistributing the access to education and employment to wider spectrum of the society than earlier, although the redistribution is not evenly spread among the protected groups.
- Affirmative action has brought a significant increase in the number of people from the protected group having access to prominent social roles (e.g. in legislative bodies) that would otherwise be lacking. Further evidence for this was also provided by Pande [51].
- In the short run, people from protected group availing social benefits through affirmative action might experience social rejection in workplace and educational institutes. Although, such social rejection exists independently and affirmative action may only magnify it. Moreover, in the long run, an increased level of education and employment weaken the stereotypical association of protected groups with incompetence and ignorance.

Two of the standard objections to group fairness are:

- it is not meritocratic,
- the increased representation of the protected group often comes at the cost of reduced efficiency.

Regarding the first claim, the equivalence of aptitude or merit with test scores can be questioned and furthermore social capital also contributes towards these test scores[20]. Moreover, the underlying assumption behind this claim is that the allocation of social benefits without affirmative action is meritocratic. However, several studies [7, 18, 40, 50, 63, 68] have confirmed discrimination on the basis of protected attributes.

For the second claim, several studies have been conducted to examine the supporting evidence with respect to the affirmative action policies in the USA. A survey of such studies by Holzer and Neumark [36] concludes that "the empirical case against Affirmative Action on the grounds of efficiency is weak at best". In India, Deshpande and Weisskopf [19] study the effect of affirmative action policies on the efficiency of the workforce of the Indian Railways - the world’s largest employer subject to affirmative action. This study found no evidence to support the claim that increasing the proportion of the protected group by affirmative action policies leads to loss in efficiency.

iff

\[ \Pr(\mathcal{H}(x_i) \in O \mid x_i \in S) - \Pr(\mathcal{H}(x_i) \in O \mid x_i \in T) \leq \epsilon \]

where \( O \subseteq A \) is any subset of outcomes.

From the above definition it is clear that, group fairness, in fact, imposes the condition of statistical parity on the predictor. Statistical parity ensures that an individual belonging to a group of population is equally likely (up to bias \( \epsilon \)) to receive a particular outcome as an individual belonging to another group of the population. If we choose \( S \) to be the group of individuals with protected attributes and \( T = X \setminus S \), statistical parity stipulates that the predictor treats the general population statistically similarly to the protected group. Statistical parity is also called as demographic parity as it makes the demographics of those receiving beneficial outcome almost identical to the demographics of the population as a whole.

Note that unlike some of the other formalizations of fairness, group fairness is independent of the “ground truth” i.e. the label information. This is particularly useful when reliable ground truth information is not available. This might be true in scenarios where the aggregation of historical manual decisions is used for training an automated predictor which will be used to predict the outcome for future cases. In domains like employment, housing, credit and criminal justice, discrimination against protected groups has been well-documented [13, 14, 49, 77]. Therefore, disproportionality in the respective outcomes for individuals from protected group and non-protected group can not be justified based on historical data.

Alternatively in the cases where disproportionality in the respective outcomes can be justified by using non-protected attributes, imposing statistical parity leads to incorrect outcomes and may amount to discrimination against qualified candidates. To illustrate this, let us consider the example given by Luong et al. [45]. Consider a job that requires a special driving license (in legal terms, a genuine occupational requirement). The predictor in this scenario takes resumes of the applicants and produces a binary outcome indicating whether they should be called for an interview. If most of the younger applicants have the required special driving license and most of the elder applicants do not have it, then unequal positive outcome rates across the age groups by the predictor can be justified by legally admissible attributes.

Dwork et al. [22] indicate another deficiency of group fairness by noting that predictor is not stipulated to select the most “qualified” individuals within the groups as long as it maintains statistical parity. This might lead to the predictor being calibrated to give beneficial outcomes to unqualified individuals from group S and qualified individuals from group T. If the unqualified individuals receiving beneficial outcomes cause reduced performance (measured by some criteria), an argument can be made to "justify" why individuals from the group S should not receive beneficial outcomes in the future.

The formalization of group fairness follows from the notion of collectivist egalitarianism for distributive justice. In practice, the biggest (in terms of the number of people affected) implementation of group fairness is the application of affirmative action ([17]) to address discrimination on the basis of caste ([21]) and gender 3. See

\[ \text{Pratik Gajane} \]

\[ \text{Weisskopf [78, Chapter 2] for arguments made for and against affirmative action polices in both India and the USA. Here we examine} \]

\[ \text{the evidence for some of them.} \]

\[ \text{Bagde et al. [4] study the effects of the affirmative action program} \]

\[ \text{which maintains group fairness with respect to the protected groups in admissions to educational institutes. The study shows that affirmative action has little impact on on-time graduation which is the} \]

\[ \text{considered a measure of the performance of an admitted student.} \]

\[ \text{This finding goes against the opinion that affirmative action policies might harm the intended beneficiaries by placing them in academic} \]

\[ \text{situations for which they are poorly suited. To the contrary, the} \]

\[ \text{study finds positive change in the test scores of the beneficiaries of} \]

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\[ \text{The major findings of interest are as follows ([16]):} \]

\[ \text{- Affirmative action has helped in substantially redistributing the access to education and employment to wider spectrum of the society than earlier, although the redistribution is not evenly spread among the protected groups.} \]

\[ \text{- Affirmative action has brought a significant increase in the number of people from the protected group having access to prominent social roles (e.g. in legislative bodies) that would otherwise be lacking. Further evidence for this was also provided by Pande [51].} \]

\[ \text{- In the short run, people from protected group availing social benefits through affirmative action might experience social rejection in workplace and educational institutes. Although, such social rejection exists independently and affirmative action may only magnify it. Moreover, in the long run, an increased level of education and employment weaken the stereotypical association of protected groups with incompetence and ignorance.} \]

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\[ \text{- it is not meritocratic,} \]

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3.3 Individual fairness

As Table 1 shows, individual fairness follows a particular notion of egalitarian fairness in impact. It ascertains that a predictor is fair if it produces similar outputs for similar individuals. Formally, it is defined as follows:

**Definition 4.** (Individual fairness) A predictor is said to achieve individual fairness iff

\[ \mathcal{H}(x_i) \approx \mathcal{H}(x_j) \mid d(x_i, x_j) \approx 0 \]

where \( d : X \times X \rightarrow \mathbb{R} \) is a distance metric for individuals.

Dwork et al. [22] use this notion of fairness. In their formulation of the problem, a predictor takes an individual \( x \) from the population as an input and outputs a probability distribution \( \mathcal{H}(x) \) over the set of outcomes \( A \). In this formulation, individual fairness can be interpreted to mean that the distributions assigned to similar people are similar.

The notion of individual fairness can be then captured by \((D, d)\)-Lipschitz property which states that

\[ D(\mathcal{H}(x_i)_A, \mathcal{H}(x_j)_A) \leq d(x_i, x_j) \]

where \( D \) is a distance measure for distributions. Furthermore, Dwork et al. [22] prove that if a predictor satisfies \((D, d)\)-Lipschitz property, then it also achieves statistical parity with certain bias.

Luong et al. [45] use a similar notion for detecting discrimination. For each member in the protected group with negative outcome, they look for individuals with "similar" non-protected attributes. If the outcomes for individuals from the protected group and non-protected group having "similar" non-protected attributes are significantly different, then discrimination is said to have occurred. As in Dwork et al. [22], the similarity is modeled by a distance metric.

In the social sciences literature, this notion of fairness can be traced back to Book 5 of the Nicomachean Ethics. It can be called individualism egalitarianism. According to Sacksteder [59], this is the formal principle of justice. The idea of treating similar cases similarly is the central theme of the legal framework of most countries. As given by Definition 4, this notion delegates the responsibility of ensuring fairness from the predictor to the distance metric. If the distance metric uses the protected attributes directly or indirectly to compute the distance between two individuals, a predictor satisfying Definition 4 would be deemed to be discriminative. Therefore, the potency of this notion of fairness to prevent discrimination depends largely upon the distance metric used. Hence, individual fairness as stated above, can not be considered suitable for domains where reliable and non-discriminating distance metric is not available.

3.4 Equality of opportunity

As shown in Table 1, equality of opportunity follows a particular notion of egalitarian fairness in impact. In the literature of machine learning, the formalization of equality of opportunity was introduced by Hardt et al. [34]. An equivalent formalization was also proposed concurrently and independently by Zafar et al. [80].

Let us consider the case of binary outcomes with a single beneficial outcome. For example, if the prediction task is to whether approve a loan application then “approving the loan” is the beneficial outcome. Let us denote the beneficial outcome by \( y = 1 \).

**Definition 5.** (Equal opportunity) A predictor is said to satisfy equal opportunity with respect to group \( S \) iff

\[ \mathbb{P}(\mathcal{H}(x_i) = 1 \mid y_i = 1, x_i \in S) = \mathbb{P}(\mathcal{H}(x_j) = 1 \mid y_j = 1, x_j \in X \setminus S) \]

It can be considered as a stipulation which states that the true positive rate should be the same for all the groups. The equivalent notion by Zafar et al. [80], called disparate mistreatment, asks for the equivalence of misclassification rates across the groups.

In the social sciences literature, the corresponding notion was presented by Rawls [56]. The central feature of Rawlsian theory is the original position in which all the parties are deprived of their knowledge of their personal attributes and social and historical circumstances (the so called “veil of ignorance”).

The critique to this notion of fairness was provided by Arneson [2]. This essay states that equality of opportunity would not be able to cope with the following problems.

- Stunted ambition: In many parts of the world, members of the protected group are socially conditioned to accept that it is inappropriate for them to aspire to social benefits including education, jobs and wealth. For example, women are shunted by their socialization toward fields of graduate study that are less well-funded, and that frequently offer poorer professional employment prospects [38]. Such impediments to social advantages are presented on account of other protected attributes as well. For example, Thorat and Neuman [69] argue, on the basis of field surveys and interviews, that protected caste groups face discrimination due to underlying attitudinal orientations. It is not obvious how equality of opportunity could avoid discrimination by social conditioning.
- Selection by merit and bigotry: Bigotry among members of society might influence what count as qualifications for positions of advantage.

The notion of equality of opportunity has also been criticized for not considering the effect of discrimination due to protected attributes like gender [48] and race [62] as they are not included in the list of attributes affecting an individual’s life prospects. It has been shown that the protected attributes like race and gender affect one’s access to opportunities in domains such as education, business, politics in many parts of the world. For example, a comprehensive recent study states that 155 of the 173 economies covered have at least one law impeding women’s economic opportunities [37].

The exclusion of attributes like race and gender from the list of attributes deemed to be affecting an individual’s life prospects in the notion of equality of opportunity thus calls into question its suitability to the domains in which there exists vast evidence that such attributes do indeed affect one’s prospects.

3.5 Preference-based fairness

Zafar et al. [79] introduce two formalizations of fairness which follow particular notions of non-egalitarian fairness in impact and in treatment. In order to provide the definitions for the same, the authors first introduce the notion of group benefit.
Definition 6. (Group benefit) Group benefit of a predictor $\mathcal{H}$ for a particular group $S$ of the population is defined as expected proportion of individuals in the group $S$ for whom the predictor $\mathcal{H}$ predicts the beneficial outcome.

$$E_S(\mathcal{H}) = \mathbb{P}\{\mathcal{H}(x_i) = 1 | x_i \in S\}$$

Group benefit can also be defined as the expected proportion of individuals from the group who receive the beneficial output for whom the true label is the same i.e.

$$E_S(\mathcal{H}) = \mathbb{P}\{\mathcal{H}(x_i) = 1, y_i = 1 | x_i \in S\}$$

Based on the above notion of group benefit, Zafar et al. [79] provide a formalization of fairness called preferred treatment which follows a notion of non-egalitarian fairness in treatment.

Definition 7. (Preferred treatment) A group-conditional predictor $\mathcal{H} = \{\mathcal{H}_S\}_{S \subseteq X}$ is said to satisfy preferred treatment if each group of the population receives more benefit from their respective predictor than they would have received from any other predictor i.e.

$$E_S(\mathcal{H}_S) \geq E_S(\mathcal{H}_T) \quad \text{for all } S, T \subseteq X$$

If a classifier is not group-conditional then, it by default satisfies preferred treatment as $\mathcal{H} = \mathcal{H}_X$ for all $S \subseteq X$.

To compare the predictors, Zafar et al. [79] suggest the formalization of Preferred impact which follows a particular notion of non-egalitarian fairness in impact.

Definition 8. (Preferred impact) A predictor $\mathcal{H}$ is said to have preferred impact as compared to another predictor $\mathcal{H}'$ if $\mathcal{H}$ offers at-least as much benefit as $\mathcal{H}'$ for all the groups.

$$E_S(\mathcal{H}) \geq E_S(\mathcal{H}') \quad \text{for all } S \subseteq X$$

In certain applications, there might not be a single universally accepted beneficial outcome. It is possible that a few individuals from a group may prefer another outcome than the one preferred by the majority of the group. In order to alleviate their concerns, the collectivist definition of group benefit needs to be extended to account for individual preferences.

In the social sciences literature, the above notion corresponds to envy-freeness [3]. This notion of fairness is attractive because it can be defined in terms of ordinal preference relations of the utility values of the predictors. On the other hand, Holcombe [35] show that freedom from envy is neither necessary nor sufficient for fairness. For many real-world problems, one needs to find fair and efficient solutions amongst the groups. An efficient solution ensures the greatest possible benefit to the groups. In collective decision making problems, like the domain applications of prediction with machine learning, it can be formally expressed by the notion of Pareto-efficiency. A Pareto-efficient solution is such that there can be no increase in the benefit of one group without strictly decreasing the benefit of another group. However, it should be noted that deciding whether there is a Pareto-efficient envy-free allocation is computationally very hard even with simple additive preferences [15].

These critiques indicate that the suitability of such envy-free formalizations is limited only to the domains where such an effective and envy-free allocation can be computed easily.

### 4 PREDICTORS USING VARIOUS FAIRNESS NOTIONS IN A FICTITIOUS SCENARIO

To demonstrate the fairness formalizations described above, let us see how predictors satisfying them would behave on a fictitious scenario. Assume that a population of $\{A, B, C, D, E, F\}$ is evenly divided into two groups : group $S$ with attribute value $s$ and group $T$ with attribute value $t$. Predictors $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$ and $\mathcal{H}_4$ are employed to predict outcome for which reliable label information is available. Note that $\mathcal{H}_4$ is group-conditional i.e. it uses $\mathcal{H}_4_S$ for an individual from group $S$ and $\mathcal{H}_4_T$ for an individual from group $T$. Assume that a distance metric on the individuals is available for the predictors. Refer to Figure 2 for further exposition of this scenario.

Next, we shall see which of the predictors satisfy the considered fairness formalizations:

1. $\mathcal{H}_1, \mathcal{H}_2$ and $\mathcal{H}_3$ satisfy treatment parity because they don’t use the protected attribute while $\mathcal{H}_3$ doesn’t satisfy treatment parity because it is group-conditional.
2. $\mathcal{H}_1$ satisfies group fairness as,

$$\mathbb{P}\{\mathcal{H}_1(x_i) = 1 | x_i \in S\} = \mathbb{P}\{\mathcal{H}_1(x_i) = 1 | x_i \in T\} = 1/3$$

while the rest don’t.
### 5 PROSPECTIVE NOTIONS OF FAIRNESS

In this section, we describe two prospective notions of fairness which have not been considered in the literature of machine learning so far. Our intention to propose these notions is to address the following critique of the considered formalizations. As seen in Section 3, many of the past formalizations in the machine learning literature do not offset for the fact that social benefits, for which the proposed prediction algorithms are used, are being allocated unequally among the people owing to the attributes they had no say in. We suggest two of the notions from the literature of social sciences which address this concern.

#### 5.1 Equality of resources

Dworkin [23] propose the notion of equality of resources in which unequal distribution of social benefits is only considered fair when it results from the intentional decisions and actions of the concerned individuals. This notion stipulates that the distribution of social benefits to satisfy the following two properties:

- ambition-sensitive: each individual's ambitions and choices that follow them are recognized as the benefits they receive.
- endowment-insensitive: each individual's unchosen circumstances including the natural endowments should be offset. If such circumstances affect one's chances of achieving the social benefits, then they should be compensated.

In the second property, equality of resources differs from equality of opportunity as the latter considers differences in natural endowments (including the protected attributes such as sex) as facts of nature which need not be adjusted to achieve fairness.

#### 5.2 Equality of capability of functioning

Sen [61] extend the insight that people should not be held responsible for attributes they had no say in to include personal attributes which cause difficulty in developing functionings. Functionings are states of "being and doing", that is, various states of existence and activities that an individual can undertake. For example, they vary from simple things from being well-nourished and being housed to more complex things such as being happy and having self-respect.

This notion calls for equalizing capabilities which are defined as the set of valuable functionings that an individual has effective access to. Put simply, functionings refer to what people really 'do and are'; capabilities denote what people potentially 'can do and can be'.

Sen [60, 61] argue that variations related to the protected attributes like age, sex, gender, race, caste give individuals unequal powers to achieve goals even when they have the same opportunities. In order to equalize capabilities, people should be compensated for their unequal powers to convert opportunities into functionings.

The second criticism applies to equality of resources as well and it makes exact mathematical formalizations of these notions a potentially difficult problem. However, the suitability of these prospective formalizations (unlike the current formalizations) to domains in which natural endowments or social endowments or both impede an individual's prospect to receive social benefits makes the open problem of formalizing them worthwhile.

### 6 DISCUSSION AND FURTHER DIRECTIONS

At the end of Plato's Republic I, Socrates says "So long as I do not know what the just is, I shall hardly know whether it is a virtue or not" [5]. To paraphrase Socrates, before conforming to a fairness formalization, we should accurately analyze it. As the field of fairness in machine learning prediction algorithms is evolving rapidly, it is important for us analyze the fairness formalizations considered so far.

To this end, we juxtaposed the fairness notions considered so far in the machine learning literature with their corresponding theories of distributive justice in the social sciences literature. We saw the theoretical critique and analysis of these fairness notions from the social sciences literature. We outlined the performance of these fairness notions when they were used in a policy employed on a large population (e.g. group fairness in affirmative action). Such critiques of the formalizations and experimental studies of their use in large-scale practice serve as guiding principles while choosing the fairness formalizations to use in particular domains.

We also proposed two prospective notions of fairness, which have been studied extensively in the social sciences literature, for prediction with machine learning. Of course, we do not claim that these notions will serve as panacea for all the critiques of the notions already considered in the machine learning literature. Our intention behind nominating these notions is to initiate a discussion about fairness formalizations in prediction with machine learning which recognize the following - since individuals should not be held responsible for the attributes they can not change or had no say in, the social benefits they receive, which in turn...
affect their prospects in life, should not depend upon those attributes. If the machine learning prediction algorithms are used to make decisions about the social benefits whose allocation exhibits discrimination for some people owing to the attributes they had no say in, then the fairness formalizations should offset the existing discrimination due to such attributes.

Of course, the obvious difficulty lies in determining which attributes that an individual has no say in. Education, at first glance, might seem like an attribute that an individual can choose. However, several studies show that the attributes that an individual has no say in (e.g. birth-place, race, caste) can impact the level of their education. For example, Jacobs [39] provide a comprehensive survey on education in the USA which demonstrates that women are particularly disadvantaged with respect to the outcomes of education. This can be seen in academia in the USA as women make up only 26% of full professors, 23% of university presidents and 14% of presidents of doctoral degree-granting institutions [54]. Similar gender disparity at higher echelons of positions is found in politics and business. Moreover, this is despite the fact that women fare relatively well in access to education in the USA. Indeed, 57% of all the college students in the USA are women [54]. However, easy access to education for women is not a universal phenomenon and discrimination is present in many parts of the world as confirmed by the Department of Economic and Social Affairs, the United Nations [67, Chapter 3]. This leads us to another key insight that the level of discrimination varies with the domain (or even for different issues within a single domain as seen above) and the place of interest. Thus while proposing notions of fairness, credible surveys appraising the discrimination caused by the protected attributes should be taken into consideration.

Which fairness formalizations should be used in for the prediction tasks corresponding to a particular social benefit should also depend upon whether the benefit in question can be considered as a basic human right. For domains like affordable housing, essential health-care and basic education, fairness formalizations which actively try to remove disparity and provide benefits to all the individuals should be considered. However, for other domains which require qualifications not evenly distributed in the population, a justification could be made for relaxing the stipulations. At the same time, independent efforts could be made to diffuse the ability to have such qualifications evenly in the population.

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