Research on Harmonic Characteristic of Electronic Current Transformer Based on the Rogowski Coil

SHEN Diqiu1, HU Bei2, WANG Xufeng3, ZHUMingdong4, WANG Liang1 and LU Wenxing1

1China Southern Power Grid EHV Transmission Company, Liuzhou 545006, China
2China Electric Power Research Institute, Wuhan 430074, China
3China Southern Power Grid EHV Transmission Company, Guangzhou 510620, China
4XJ Electric Co., Ltd, Xuchang 461000, China

hubei2@foxmail.com

Abstract. The nonlinear load present in the power system will cause the distortion of AC sine wave and generate the harmonic, which have a severe impact on the accuracy of energy metering and reliability of relay protection. To satisfy the requirements of energy metering and relay protection for the new generation of intelligent substation, based on the working principle of Rogowski coil current transformer, mathematical model and transfer characteristics of Rogowski coil sensors were studied in this paper, and frequency response characteristics of Rogowski coil current transformer system were analyzed. Finally, the frequency response characteristics of the Rogowski coil current transformer at 2 to 13 harmonics was simulated and experimented. Simulation and experiments show that Rogowski coil current transformer could meet 0.2 accuracy requirements of harmonic power measurement of power system, and measure the harmonic components of the grid reliably.

1. Introduction
Power system harmonic is one of the important parameters of power quality. With the widely use of high-power electronic devices in modern industry, part of basewave will be converted into harmonic electrical energy when non-linear load and FACTS works, which will be fed back to the grid, causes serious harmonic pollution to the power grid, results in the increase of non-operating costs in electrical operating companies, and have impacts on the relay protection reliability and the electric energy metering accuracy of power department and user [1]. With the construction of strong smart grid, the number of used electronic transformers is growing sharply, in this context, the research on Harmonic Characteristic of electronic transformers has important theoretical significant and practice value.

Rogowski coil current transformer is with characteristics of good linearity, large dynamic range, wide frequency response and no magnetic saturation, so it meets the digital applications requirement of intelligent substation and is widely used [2-4]. International and domestic academics have made a deep research on the principle and structural design, distribution parameter influence, grouping modeling calculation methods, integral way, merging unit design and performance improvement of Rogowski coil current transformer [5-9], which promotes its practical process effectively. These studies showed the good steady-state and dynamic measurement performance of Rogowski coil current transformer from its measurement principle and structure design aspects, however, these
studies did not analyzed the problems existing in the transfer characteristics from the perspective of the actual power metering applications, it lacks of research in power system harmonics and transient measurements.

In this paper, from the application point view of the new generation of intelligent substation, we gave a detailed analysis on the sensing principle and structure parameters of Rogowski coil current transformer, made an intensive study of the whole transfer characteristics on Rogowski coils and its current transformers, analyzed the impact of coil resistance, distributed capacitance and other parameters on the frequency response characteristics. Combined simulation output amplitude-frequency and phase-frequency characteristics with the test results, we verified frequency response characteristic and power measurement accuracy of Rogowski coil current transformer.

2. The sensing principle and transfer characteristics of Rogowski coil

2.1. The sensing principle of Rogowski coil

Rogowski coil is a widely used and mature current sensing element, it has the advantages of small size, light weight, high sensitivity, easily measured and digital protection. The sensing principle is shown in Figure 1: Use a toroidal coil, which is uniform tightly wound in a non-ferromagnetic matrix (a rectangular cross section, uniform), as a current sensing element, the output voltage is proportional to the time rate of change of the measured primary current, it is integrated with external integration circuit, and restores the voltage signal which reflects the measured primary current[10-12].

Assuming that the cross-section of the Roche coil and the coil turns are uniform everywhere, Rogowski coil induced emf can be obtained by the law of electromagnetic induction:

\[ e(t) = -\frac{d\Phi}{dt} = -\frac{\mu_0 Nh}{2\pi} \ln \frac{R_2}{R_1} \frac{di}{dt} = -M \frac{di}{dt} \]  

(1)

Where: \( i \) is one of the instantaneous current flowing through the conductor; \( r \) is an arbitrary radius of Rogowski coil skeleton; \( \mu_0 \) is the vacuum permeability, \( 4\pi \times 10^{-7} \text{H/m} \); \( h \) is the height of the skeleton rectangular section; \( R_2 \) is the outer diameter of the skeleton; \( R_1 \) is the inner diameter of the skeleton; \( N \) is the total number of turns of the coil; \( M \) is the mutual inductance.

When Roche coil skeleton materials, dimensions, coil winding diameter, the average radius of coil and the coil turns are determined, you can calculate the mutual inductance. When the frequency of the measured current is low, the induced electromotive force \( e(t) \) is proportional to differential value of the measured current. Figure out the integral on (1), and then we get:

\[ i(t) = -\frac{1}{M} \int e(t) \, dt \]  

(2)

By integrating from \( e(t) \), we can restore the micro voltage signal which shows a linear relationship with the primary current; and by digitizing and calculating, we can obtain the measured current \( i(t) \). To measure the induced electromotive force of Rogowski coil, we usually use parallel sampling resistance.
in the out-port of the Rogowski coil. By measuring the voltage values across the resistor and through the integration process, we can calculate the measured current.

2.2. The transfer characteristics of Rogowski coil

According to the sensing principle and structure parameters of Rogowski coil, we established the equivalent circuit model of Rogowski coil, as shown in Figure 2 [13]. Where: \( i(t) \) is the primary current which current carrying conductors go through; \( M \) is the mutual inductance of Rogowski coil, \( R_0 \), \( L_0 \) and \( C_0 \) were the resistance, inductance and capacitance distribution of Rogowski coil respectively; \( R_L \) is the load resistance.

![Figure 2. Rogowski coil equivalent circuit.](image)

By the equivalent circuit, the transfer function of Rogowski coil is:

\[
H_i(s) = \frac{-Ms}{L_0C_0s^2 + \left( \frac{L_0}{R_L} + R_0C_0 \right) s + \left( \frac{R_0}{R_L} + 1 \right)}
\]  

(3)

When the primary side goes through the current \( I_N \), Rogowski coil EMF is \( E = \omega M I_N \), the rated secondary output is \( U_1(s) = H_i(s) * E \). If the rated primary current is 600A, set the corresponding rated secondary output is 2V. When designing the structure parameter Rogowski coil, we can determine the number of turns \( N \) based on the needed size of the mutual inductance \( M \), and the coil inductance is the product of the number of turns and the mutual inductance: \( L_0 = M N \), and the main factors that affect the frequency response characteristic of the Rogowski coil are the resistance \( R_0 \) and distributed capacitance \( C_0 \) of the coil.

For the cylindrical Rogowski coil whose cross-section is rectangular, the internal resistance \( R_0 \) is calculated as follows:

\[
R_0 = \rho \frac{4N \ast (b - a + 2c)}{\pi \ast \Phi^2}
\]  

(4)

Approximate calculation of distributed capacitance is:

\[
C_0 \approx \frac{2\pi^2 \ast (b - a + 2c)}{\lambda \ast \ln \left( \frac{b}{a} \right)} \ast \varepsilon_0 \ast \varepsilon_r
\]  

(5)

Where: \( b \) the outer diameter of the coil, \( a \) is the inner diameter of the coil, \( c \) is the height of the coil, \( \lambda \) is the diameter of the coil wire, \( \rho \) is winding resistivity, \( \Phi \) is winding diameter, \( \varepsilon_0 \) is the vacuum permittivity, \( \varepsilon_r \), is the relative dielectric constant of the winding insulation material.

By equation (4) and (5) we can know that, the main factors affecting the Rogowski coil resistance \( R_0 \)are the size of coil skeleton, the winding resistivity and the winding wire diameter; and the factors affecting the distributed capacitance \( C_0 \) are the size of coil skeleton, the uniformity of coil winding, the winding density, winding layers and the coil shielding grounding method, etc. This study showed the measured parameters of Rogowski coil, as shown in Table 1. If the internal resistance \( R_0 \) was taken as 180\( \Omega \), 380\( \Omega \) respectively with other parameters constant, amplitude-frequency and phase-frequency characteristics of Rogowski coil affected by the resistance changes was shown in Figure 3; if the distributed capacitance \( C_0 \) were taken as 12nF, 52nF respectively, amplitude-frequency and phase-frequency characteristics of Rogowski coil affected by the resistance changes was shown in Figure 4.
The frequency characteristics of Rogowski coil affected by the distributed capacity changes was shown in Figure 4.

**Table1. Model simulation parameters of Rogowski coil**

| $M$(μ H) | $R_d$(Ω ) | $L_0$(mH) | $C_0$(nF) | $R_L$(kΩ ) |
|----------|-----------|-----------|------------|------------|
| 10.25    | 280       | 58.6      | 32         | 32.4       |

*Figure 3.* Amplitude-frequency, phase-frequency characteristics by the resistance changes.

As shown in the Bode diagram, the magnitude of the gain of the Rogowski coil was -66.4dB when the power frequency is 50 Hz, corresponding to the magnitude of the gain $k=4.786e-4$, the phase difference generated was 89.97°; the magnitude of the gain was -39.9dB when the frequency is 1 kHz, corresponding to the magnitude of the gain $k=1.012e-2$, the phase difference generated was 89.38°; and within the measurement bandwidth, the gain of the output amplitude of the Rogowski coil is proportional to the frequency, the output of the phase shift increases with frequency increasing.

When the other parameters constant, we regulated the coil resistance and distributed capacitance respectively, and compared the amplitude-frequency phase-frequency characteristics, as shown in Fig. 3 with Fig.4, the resistance and distributed capacitance of Rogowski coil have little effect on its low frequency transfer characteristics, while the effect on the high frequency transfer characteristics can not be ignored. When designing the structure and parameters of Rogowski coil in high-frequency applications fields, we should reduce the resistance and distributed capacitance of Rogowski coil, improve measurement bandwidth.

3. **Current transformer system structure and frequency response characteristics of Rogowski coil**

The development direction of the principle and structure of electronic current transformer can be divided into two types: active type and passive type, and the active type has many benefits: simple structure, safe, reliable and long-term job stability etc. This study introduced an electric power system metric active electronic current transformer, as shown in Figure 5. The over-all structure of the transformer was divided into four portions by function: Rogowski coil sensors, data acquisition systems, merging unit and power supply.
According to the sensing principle of Rogowski coil, when the measured current is at low frequency, Rogowski coil secondary side is close to the open state, then the output voltage of Rogowski coil is proportional to the measured current to a time differential, it need extra integration processing.

Base on the difference of input signal processing, the integrator circuit can be divided into two kinds: analog integration and digital integration. The integration circuit performance determines the spectrum transformer, response time, and it is one of the important factors that impacts the accuracy and stability of the Rogowski coil electronic current transformer. In this paper, current transformers used high-performance operational amplifiers, constructed active analog integral circuit with inertial link, achieved continuous integration to differential input voltage signal, and was able to reduce the differential signal within a wide frequency range, to ensure the accuracy of the output transformer when the frequency changes. The integrating circuit diagram was shown in Figure 6.

Using the resonant circuit principle to calculate the cut-off frequency, the frequency band of the outer integral Rogowski coil current transformer is:

\[
 f_L = \frac{1}{2\pi R_1 C_1}
\]
\[ f_H = \frac{1}{2\pi \sqrt{L_0 C_0}} \left( \frac{R_0 + R_L}{R_L} \right)^{1/2} \]  

(9)

\( f_L \) is the lower limit frequency, \( f_H \) is the upper limit frequency, the working frequency band is:

\[ BW = f_H - f_L = \frac{1}{2\pi \sqrt{L_0 C_0}} \left( \frac{R_0 + R_L}{R_L} \right)^{1/2} - \frac{1}{2\pi R C_1} \]  

(10)

If the parameters of Rogowski coil was captured from Table 1, you can obtain the lower frequency of Rogowski coil \( f_L = 0.4 \) Hz, the upper limit frequency \( f_H = 3.7 \) kHz, the working frequency band is approximately 3.7 kHz, they were consistent with the amplitude-frequency phase-frequency characteristics of Bode diagram. It follows that Rogowski coil work in differential state, the lower limit frequency of measuring circuit depends on the time constant of the integrator circuit \( C_i R_i \), the upper limit frequency determined by the inherent resonance frequency of measuring circuits.

In this paper, the time constant \( C_i R_i \) of the Rogowski coil current transformer integrating circuit is 0.44s, if we change the integration time constant to 0.22s, 0.88s respectively, then compare and analyze the transient responses, we can get amplitude-frequency and phase-frequency characteristics of Rogowski coil current transformers at the frequency band of 10Hz ~ 100kHz base on the transfer function of the equation (7), as shown in Figure 7.

![Figure 7: Amplitude-frequency, phase-frequency characteristics of Rogowski coil current transformer.](image)

As shown in the Bode diagram, the amplitude gain of the Rogowski coil current transformer at the power frequency of 50Hz was -56.536dB, corresponding to the magnitude gain \( k = 1.49 \text{e}^{-3} \); the phase shift was 1.46 °, and the output phase shift of current transformer increases with the frequency increases. The amplitude gain at 650Hz is -56.528dB, corresponding to the magnitude gain \( k = 1.4914 \text{e}^{-3} \), the difference is -0.1 °. At this moment, the magnitude gain variation of the current transformer at the 2~13 harmonics is less than 0.094%, the phase shift is no more than 1.56 °.

Comparing the amplitude-frequency and phase-frequency characteristics as shown in Figure 7, we can know that the integration time constant mainly affects the high-frequency transfer characteristics of the current transformer, and the higher the integration time constant, the better the high-frequency transfer characteristics. But when the time constant is increased, the gain of the low-frequency noise signals increase, and the adverse effect of temperature drift weakens. So when we optimize the performance of harmonic current transformer we should increase the integration time constant reasonably while maintaining the stability of the systems.

4. Simulation and Test Verification

The power system has raised the requirement of power measurement accuracy for the power harmonic measurement, we need to measure the output ratio and phase error of the current transformer at 2 to 13
harmonic. In this paper, based on the mathematical model of Rogowski coil current transformer, we took the simulation analysis for the response output of the current transformer under the harmonic current, and proved by experiment.

Using Multisim 12.0 circuit simulation software, we can establish harmonic test system circuit model of Rogowski coil current transformer, as shown in Figure 8. The circuit simulation model, as used in this paper, conducted the simulation of the response of current transformers at 2 to 13 harmonics. Using this simulation model, we can also change the self inductance of the Rogowski coil, distributed capacitance, load resistance and other parameters, combined with practical design and production, to improve the process and optimize the frequency response characteristics of Rogowski coil. On this basis, we can optimize the integrating circuit design to improve the spectral range of the current transformer system, to improve the reliability and stability of the transformer for harmonic and transient measurement.

During the simulation, 2 to 5 times harmonic amplitude of the signal source is 10% of rated primary current 600A, 6 to 13 times harmonic amplitude is 5% of the rated primary current 600A, and the initial phase are 0 rad; frequency is 1 to 13 times of work frequency 50Hz. Output of ratio and phase difference of current transformer by the Single harmonic current is shown in Figure 9.

According to the simulation results, we can draw that the simulation output of Rogowski coil current transformer model in 2~13 harmonic current meets the harmonic power measurement accuracy requirements of 0.2.

To further verify the accuracy of the structure model and harmonic characteristics of the Rogowski coil current transformer, we built the harmonic test system block diagram of current transformer, as shown in Figure 10.
Equipment and main terminal

Frequency (Hz)
Standard current ratio means
0.5A Analog output standard
XL-807 electronic transformer calibrator
0-2 Output
The combined unit

Figure 10. Harmonic test system block diagram of current transformer.

Output of ratio and phase difference of current transformer by the Single harmonic current is shown in Figure 11.

Figure 11. Experimental output of ratio and phase difference by the Single harmonic current

Comparing the simulation and experimental results, the mathematical model of Rogowski coil current transformer system proposed by this paper is accurate and reliable, the response output at 2 to 13 times harmonic current meets harmonics power measurement accuracy requirements. It shows that the Rogowski coil current transformer can be operated from work frequency to high frequency in wide frequency band, the harmonic components of grid can be measured reliably.

In the vicinity of frequency 50Hz, Rogowski coil is in the open state, the output voltage and the measured current are in the same phase, the output voltage can accurately reflect the measured current through appropriate signal amplification and calibration; while at high frequencies, the sensor head cannot be equivalent to work in an open state, as limited by the resonant frequency and the passband upper cut-off frequency of the self-inductance for the sensor head and distributed capacitance, so that the frequency characteristics of transformer deteriorates, and the output ratio difference and the phase difference increases.

The other reason of Current transformer systems phase difference increased is the phase frequency characteristics of the integration circuit. Take the outer integral circuit, which is studied in this paper, for example, the output phase shift increases with increase of frequency, in a positive correlation. Based on the parameters of external integrating circuit, at 650Hz, the phase shift caused by integrating circuit is -324°, while the phase shift caused by integrating circuit has reached to -514° at 1kHz. The advantages of out-integrator work patterns are with multiple circuit structures and implementations, Rogowski coil current transformer can be optimized for performance as needed in the choice of the integral structure, to achieve a stable and reliable measurement of the current in various fields.
5. Conclusion
In this paper, based on the mathematical model and transfer characteristics of the Rogowski coil, we focused on the frequency response characteristics of current transformer system of Rogowski coil, and determined that the sensor's resistance and distributed capacitance change of Rogowski coil have a great impact on the high-frequency transfer characteristics. In this paper, we designed Multism12.0 simulation circuit, built a current transformer harmonic test system, verified frequency response characteristics of Rogowski coil current transformer under 2 to 13 times harmonic current by simulation and experiment. Simulation and experimental results showed that the Rogowski coil current transformer can meet 0.2 grade accuracy of the test requirements of the power system for harmonic power measurement.

Within a wider frequency range, when we carry out an energy metering using Rogowski coil current transformer, we need to further optimize the design of the current transformer to enhance its high frequency band frequency response performance. On the one hand, we can improve the manufacturing process of Rogowski coil, reduce its impacts on parameters such as Internal resistance and distributed capacitance; on the other hand, we can optimize the structure and implementation of the integration circuit, reduce the electronic circuits, decrease the harmonic measurement error in the digital sampling processing system.

References
[1] Li Yanqing, Chen Zhiye, Li Peng, et al. Technology of Power System Harmonic Suppression[J]. North China Electric Power University, 2001, 28(4): 19-22.
[2] Current Transformer and New Technology Applied In Digital Power Station[D]. Wuhan: Huazhong University of Science and Technology, 2011: 20-29.
[3] Long Zhuli. Design and performance research of junior high current Rogowski coil[J]. High Voltage Engineering, 2007, 33(7): 79-83.
[4] Y. Liu, F. Lin, Q. Zhang, H. Zhong. Design and construction of a Rogowski coil for measuring wide pulsed current[C]. IEEE Sensors J., vol. 11, no. 1, pp. 123-130, Wuhan, China, 2011.
[5] Luo Sunan, Tian Zhaobo, Zhao Xicai. Performance analysis of air-core current transformer[J]. Proceedings of the CSEE, 2004, 24(3): 108-113.
[6] Zhai Xiaoshe, Gen Yingsan, Song Zhengxiang. Packet modeling and numerical methods of Rogowski coil distribution parameters[J]. High Voltage Electrical Apparatus, 2007, 43(2): 102-105.
[7] Li Weibo, Mao Chengxiong, Lu Jiming, et al. Study of the influence of the distributed capacitance on dynamic property of Rogowski coil[J]. Electrical Technology, 2004, 19(6): 12-17.
[8] Zhang Mingming, Zhang Yan, Li Hongbin, et al. Research of integrator technology of Rogowski current transformer[J]. High Voltage Engineering, 2004, 30(9): 13-16.
[9] E. Suomalainen, J. Hallstrom. On-site calibration of a current transformer using a Rogowski coil[C]. Conference on Precision Electromagnetic Measurements Digest. IEEE, vol. 58, no. 4, pp. 1054-1058, Finland, 2009.
[10] V. Dubickas, H. Edin. High-frequency model of the Rogowski coil with a small number of turns[C]. IEEE Trans. Instrum. Meas., vol. 56, no. 6, pp. 2284-2288, Stockholm, Sweden, 2007.
[11] Li Weibo. Study of Sensor Theory Centered on Rogowski Coil for Heavy Current Measurement Application[D]. Wuhan: Huazhong University of Science and Technology, 2005: 39-48.
[12] Wang Baocheng, Wang Deyu, Wu Weiyang. Frequency characteristics analysis and design of Rogowski coil sensor[J]. Electrical Technology, 2009, 24(9): 21-27.
[13] X. Minjiang, G. Houlei, Z. Baoguang, W. Chengzhang, et al. Analysis on transfer characteristics of Rogowski coil transducer to travelling wave[C]. International Conference on Advanced Power System Automation and Protection (APAP), vol. 2, pp. 1056-1059, Jinan, China, 2011.