EXPERIMENTAL EVIDENCE FOR A POWER LAW
IN ELECTROENCEPHALOGRAPHIC $\alpha$–WAVE DYNAMICS

Y. Georgelin$^a$, L. Poupard$^b$, R. Sartène$^b$ and J.C. Wallet$^a$

$^a$ Division de Physique Théorique†, Institut de Physique Nucléaire
F-91406 ORSAY Cedex, France

$^b$ Laboratoire d’Explorations Fonctionnelles,
Hôpital Robert Ballanger, F-93602 AULNAY-SOUS-BOIS

Abstract: We perform an experimental study of the time behavior of the $\alpha$-wave events occurring in human electroencephalographic signals. We find that the fraction of the time spent in an $\alpha$-burst of time size $\tau$ exhibits a scaling behavior as a function of $\tau$. The corresponding exponent is equal to $1.75 \pm 0.13$. We therefore point out the existence of a new power law appearing in physiology. Furthermore, we show that our experimental result may have a possible explanation within a class of Self-Organized Critical (SOC) models recently proposed by Boettcher and Paczuski. In particular, one of these models, when properly re-interpreted, seems to be consistent both with our experimental result and a commonly accepted physiological description of the possible origin of $\alpha$-wave events.

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Trying to understand and encode in rather simple models the fundamental properties underlying the richness and complexity of biological systems and functions has become a major topic in modern biology. Despite their apparent complexity, some of these systems/functions exhibit, among other features, a tendency for organization as well as self-organization which can occur at various levels. A simple example in morphogenesis is provided by the organized variability observed in the branching structure of the lung which can be explained by scaling arguments, first introduced a long time ago in [1] and further developed in [2]. The concept of scaling is now well established in biology and physiology (for a review see e.g. [3]) and appears to be a useful tool to understand features of many processes. In particular, scaling shows up in the power law behavior of some observables.

The human brain is one of the most complex physiological systems. It involves billions of interacting physiological and chemical processes giving rise to the experimentally observed neuroelectrical activity. The corresponding dynamics exhibits a complicated behavior which reflects itself in electrophysiological recordings, namely the electroencephalographic recordings (EEG), which, roughly speaking, are assumed to capture the mean/global electrical activity of the neurons located in the cortex, that is, the outermost (2 mm thick) layer of the brain. The attempts to extract relevant information from the neuroelectrical activity have generated a large amount of work for more than 20 years. Former investigations were mainly based on the Fourier analysis of the time series stemming from the EEG signal [4]. More recently, the use of more powerful methods inherited from nonlinear physics have provided a deeper insight into the fundamental properties ruling the observed neuroelectrical dynamics [5] and, in particular, the possible occurrence of self-organization in the cortical electrical activity has been suggested in recent works [5], but so far no evidence for scaling laws in the corresponding dynamics has been reported.

One of the major difficulties to observe a power law in human neuroelectrical activity is to determine relevant observables from the EEG signal. Recall that the evolution from a deep sleep to an (active) awakening level reflects itself into four dominant regimes of the EEG signal which are conventionally classified according to their frequency range [6]. These four regimes are called δ-waves (\([0.5Hz,4Hz]\)), θ-waves (\([4Hz,8Hz]\)), α-waves (\([8Hz,12Hz]\)) and β-waves (\([13Hz,19Hz]\)) (the lowest frequency range δ corresponding to a deep sleep level). It is known that α-waves occur when human awakening level drops down slowly towards sleep while the eyes are kept open [7]. α-waves represent therefore an electroencephalographic landmark of drowsiness. Successive α-wave events/bursts can be observed for a rather long period (up to a few hours) with typical time size (lifetime) from \(\mathcal{O}(100)\)msec up to \(\mathcal{O}(10)\)sec. They can be easily isolated from the background EEG activity so that they are good candidates for study. Figure 1 shows successive α-wave events with different lifetimes. Notice the irregular variations of the occurrence times between the onset of two successive α events.
In this letter, we study the lifetime of $\alpha$-wave events occurring in EEG signals. The signal processing is performed using a standard wavelet transform analysis [8] which appears to be well-suited to deal with the transients involved in the EEG’s and in particular to extract reliably the various $\alpha$-wave events [9]. For each EEG signal, we measure the cumulated time for $\alpha$-events with fixed time size $\tau$, normalized to the total duration of the EEG signal (which basically represents the fraction of the time spent in an $\alpha$-burst of time size $\tau$), hereafter denoted by $P_{\text{exp}}(\tau)$. We find that $P_{\text{exp}}(\tau)$ has a power law form given by $P_{\text{exp}}(\tau) \sim \tau^{-\omega}$ with $\omega = 1.75 \pm 0.13$. This provides a new example of a power law with fractional exponent appearing in this area of physiology. Furthermore, we show that this experimental result may have a possible explanation within a class of Self-Organized Critical (SOC) models recently discussed in the physics literature [10]. In particular, one of these models, when properly re-interpreted, seems to be consistent both with our experimental result and a physiological description of the possible origin of $\alpha$-wave events.

Let us first describe briefly the pure experimental part of this work (i.e. the data recording). The experimental procedure consists in recording the EEG activity of 10 subjects who all have had a four hours sleep deprivation during the previous night. It is known that sleep deprivation (and thus drowsiness) reinforces the appearance of $\alpha$-wave events. Each subject was installed in the sitting posture for a two-hour EEG recording and had to keep himself awake. Each EEG signal was obtained from temporal and occipital electrode location and was further filtered through a $[0.5\text{Hz},30\text{Hz}]$-bandpass and digitally converted at a rate of 200 samples/sec.

In order to get more insight into the dynamics governing the occurrence of $\alpha$-bursts, we choose the $\alpha$-events lifetime as a representative physical observable [11]. The various $\alpha$-events (and corresponding lifetimes) are easily extracted from any EEG signal $s(t)$ using standard wavelet analysis [8,9]. In particular, $\alpha$-events correspond to those part of the signal whose wavelet transform modulus is maximum in the $\alpha$-frequency range $[8\text{Hz},12\text{Hz}]$. Recall that the continuous one-dimensional wavelet transform is given by [8]

\[
(W_{\psi}s)(b,a) = |a|^{-\frac{1}{2}} \int_{-\infty}^{+\infty} dt \ s(t) \psi^*\left(\frac{t-b}{a}\right)
\]

where the real parameters $a$ ($a > 0$) and $b$ are respectively the scale and time parameter, $\psi(t)$ is the so-called mother function and $^*$ denotes complex conjugation. In what follows, we choose $\psi(x) = \pi^{1/4} \exp(i\theta_0x) \exp(-x^2/2)$ where $\theta_0$ is a numerical constant [12], which is particularly suitable for frequency characterization and offers a good compromise between frequency resolution and time localization [9]. In the numerical analysis, we consider the discrete version of (1) which can be written as

\[
(W_{\psi}s)(n,a) = \left(\frac{\delta t}{a}\right)^{1/2} \sum_{n'=0}^{N-1} s(n')\psi^*\left(\frac{(n'-n)\delta t}{a}\right)
\]

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for any EEG signal \( s(n\delta t) \) (\( n \) integer) of total duration \( N\delta t \), where \( \delta t \) is the time step.

It is convenient to consider the time average of the square modulus of (2). The corresponding expression is given by

\[
< |(W_{\psi_s})(m\delta t', a)|^2 > = \sum_{n=mk}^{(m+1)k-1} \frac{1}{k} |(W_{\psi_s})(n\delta t, a)|^2, \quad k = \frac{\delta t'}{\delta t}; \quad m = 0, 1, \ldots, \left( \frac{N}{k} - 1 \right) \tag{3}
\]

where \( k \) is a reduction factor from \( \delta t \) to \( \delta t' \) [13], this later being identified with the uncertainty in time localization. This permits one to disregard the events whose time duration is shorter than \( \delta t' \) (and also to eliminate spurious effects due to EEG background noise). Then, any \( \alpha \)-burst will correspond to the part of the signal for which (3) is maximum when the scale parameter \( a \) belongs to a range associated with the \( \alpha \)-frequency range \([8Hz,12Hz]\). The corresponding lifetime can then be straightforwardly obtained from (3).

We have extracted all the \( \alpha \)-events from the EEG activity in each of the 10 EEG signals and determined the corresponding lifetimes. This allows us to define \( P_{exp}(\tau) \), the fraction of the time spent in an \( \alpha \)-burst of time size \( \tau \). As shown in Fig.2, this quantity exhibits a scaling behaviour, \( P_{exp}(\tau) \sim \tau^{-\omega} \). The corresponding exponent is found to be

\[
\omega = 1.75 \pm 0.13 \tag{4}
\]

where the second term in (4) (standard deviation) reflects both the inter individual variability and artifacts such as eyes motions and/or muscular activity (which are inherent to EEG measurement).

We now try to identify a simple model capturing some features of the usually accepted physiological description and whose predictions are in good agreement with our experimental result. To do this, we adopt a phenomenological viewpoint. Let us first start with physiological considerations. Although \( \alpha \)-wave occurrence is an important feature of the EEG activity, the corresponding generating mechanisms are far from being understood. It is commonly accepted that \( \alpha \)-waves have a cortical origin and are driven by presynaptic inputs from the thalamic level to cortical neurons [6]. Now, when drowsiness occurs, the transmission of information from the thalamus to the cortex may be partially altered by some sleep inducing mechanisms so that small clusters of neurons (involving \( O(1000-10000) \) neurons) may be prevented from receiving information from the thalamus. Then, any \( \alpha \)-burst will start when such a cluster becomes isolated from the rest of the surrounding cortex and will last until information can be transmitted again to the cluster, due to some reactivation mechanism.

This specific feature, where basically peculiar changes are concentrated in time intervals interrupting periods of inactivity, is somehow similar to a punctuated equilibrium behavior which appears in particular within a class of SOC models [10], called multi-trait models, which can be viewed as extensions of the original Bak-Sneppen model [14]. These
models are defined as follows (for more details see [10] and ref. therein): each site of a d-dimensional lattice is labelled by $M$ numbers belonging to the unit interval. At every time step, the smallest number in the lattice is replaced by a new number randomly chosen from a flat distribution in the unit interval, whereas one of the $M$ numbers on each neighboring site is also randomly replaced by a new random number taken from the flat distribution. Now assume crudely that the relevant part of the cortex that gave rise to the $\alpha$ activity that we have observed here can be modeled by a 1-dimensional lattice, each site of which is identified with a cluster of neurons. Owing to the fact that each cluster is actually controlled by a large number of parameters (stemming from (external) neuronal inputs, ion channels,...), it is reasonable to consider the limit $M \to \infty$. The corresponding multi-trait model has been considered in detail in [10] and is known to represent a different universality class than the Bak-Sneppen model. Its punctuated equilibrium behavior has been characterized in particular through $P_F(\tau)$ the distribution of (time) sizes of periods of inactivity ($\sim$ isolation) for a given site, which can then be identified with $P_{exp}(\tau)$, keeping in mind the physiological considerations developed above together with the fact that $P_{exp}(\tau)$, due to its very definition, is nothing but the distribution of time size of periods of isolation of some neuronal cluster. The distribution $P_F(\tau)$ has been show [10] to obey a power law given by

$$P_F(\tau) \sim \tau^{-7/4}$$

(5)

whose exponent is in good agreement with the one (4) characterizing the scaling behavior of $P_{exp}(\tau)$ that we have determined experimentally, therefore indicating that the $M \to \infty$ multi-trait model may well be of some relevance to describe the dynamics of the $\alpha$-bursts in the EEG activity. One remark is in order. The corresponding exponent in the $d=2$ Directed Percolation model is equal to 1.84 [15]. Strictly speaking, this model is still consistent with our experimental result although the existence of a preferred direction in the cortex is difficult to reconcile with the present physiological knowledge. We therefore consider this model as rather unsuitable for describing the $\alpha$-wave dynamics.

Summarizing, we have pointed out the existence of a new power law occuring in the $\alpha$-wave dynamics. Our experimental result may be understood in the framework of a particular SOC model, namely the limit $M \to \infty$ of the multi-trait model [10]. This suggests that this model may be successfully applied to describe (some of) the dynamics of the $\alpha$-bursts for which, consequently, self-organization and punctuated equilibrium behavior may well play a salient role.

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11) The amplitude of the α-waves, which is frequently used as a relevant observable, depends strongly on the electrode positions on the scalp, whereas the corresponding lifetime dependance is rather weak.
12) In the present numerical analysis, θ0=5.5. For a discussion on the Morlet-Grossmann wavelet that we choose, see ref. [10]; see also D. Gabor, J. of the IEE., vol.93 (1946) 429.
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FIGURE CAPTIONS

Figure 1: On panel (a) is depicted the time average of the square modulus of the wavelet transform of a typical EEG signal whose maxima, indicated by the darkest areas, correspond to $\alpha$-events. The corresponding lifetimes are collected on panel (b). An example of successive $\alpha$-events occurring in the EEG signal is presented in panel (c).

Figure 2: Log-log plot of the fraction of time spent in an $\alpha$-burst of lifetime $\tau$ versus $\tau$. All the data for the subjects are collected on Fig.2a. The straight line depicted on Fig.2a corresponds to an exponent equal 1.75, obtained by first fitting the data for each subject by using the mean square method (see Fig.2b for a typical example for a given subject) and then averaging the results over the 10 subjects. The corresponding standard deviation is equal to 0.13.
