Reinforcement Learning Based Temporal Logic Control with Soft Constraints Using Limit-deterministic Generalized Büchi Automata

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Abstract—This paper studies the control synthesis of motion planning subject to uncertainties. The uncertainties are considered in robot motion and environment properties, giving rise to the probabilistic labeled Markov decision process (PL-MDP). A model-free reinforcement learning (RL) is developed to generate a finite-memory control policy to satisfy high-level tasks expressed in linear temporal logic (LTL) formulas. One of the novelties is to translate LTL into a limit deterministic generalized Büchi automaton (LDGBA) and develop a corresponding embedded LDGBA (E-LDGBA) by incorporating a tracking-frontier function to overcome the issue of sparse accepting rewards, resulting in improved learning performance without increasing computational complexity. Due to potentially conflicting tasks, a relaxed product MDP is developed to allow the agent to revise its motion plan without strictly following the desired LTL constraints if the specified tasks can only be partially fulfilled. An expected return composed of violation rewards and accepting rewards is developed. The designed violation function quantifies the differences between the revised and the desired motion planning, while the accepting rewards are proposed to enforce the satisfaction of the acceptance condition of the relaxed product MDP. Rigorous analysis shows that any RL algorithm that optimizes the expected return is guaranteed to find policies that, in decreasing order, can 1) satisfy the acceptance condition of relaxed product MDP and 2) reduce the violation cost over long-term behaviors. Also, we validate the control synthesis approach via simulation and experimental results.

I. INTRODUCTION

Formal logic is capable of describing complex high-level tasks beyond traditional go-to-goal navigation for robotic systems. As a formal language, linear temporal logic (LTL) has been increasingly used in the motion planning of robotic systems\textsuperscript{[1]–[3]}. Since robotic systems are often subject to a variety of uncertainties arising from the stochastic behaviors of the motion (e.g., an agent may not exactly follow the control inputs due to potential sensing noise or actuation failures) and uncertain environment properties (e.g., there exit mobile obstacles or time-varying areas of interest), Markov decision processes (MDPs) are often used to model the probabilistic motion of robotic systems\textsuperscript{[4]}. Based on probabilistic model checking, control synthesis of MDP with LTL motion specifications has been widely investigated (cf.\textsuperscript{[5]–[19]}). In particular, the topic about partial satisfaction of high-level tasks in deterministic and stochastic systems is investigated in\textsuperscript{[9], [11]–[14]}. Yet, new challenges arise when considering motion and environment uncertainties. Hence, learning to find a satisfying policy is paramount for the robot to operate in the presence of motion and environment uncertainties.

Reinforcement learning (RL) is a sequential decision-making process in which an agent continuously interacts with and learns from the environment\textsuperscript{[15]}. For motion planning with LTL specifications, model-based RL has been employed when full knowledge of MDP is available\textsuperscript{[16]}. The work of\textsuperscript{[17]} extends model-based RL to temporal logic constrained control of stochastic systems with unknown parameters by model approximation. In\textsuperscript{[18] and [19]} transition probabilities are learned to facilitate the satisfaction of LTL specifications. However, these aforementioned works have a scalability issue due to the high complexity of model learning. In contrast, model-free RL approaches with LTL-based rewards generate desired policies by optimizing the Q-values\textsuperscript{[20]–[24]}. However, these works are based on a key assumption that there exists at least an accepting maximum end component (AMEC) in a standard product MDP\textsuperscript{[1]}, which may not be true in practice. For instance, some areas of interest to be visited can be probabilistically prohibitive to the agent in practice (e.g., potentially surrounded by water due to heavy rain that the ground robot cannot traverse), resulting in part of the user-specified tasks cannot be achieved and AMECs do not exist in the product MDP. Although minimal revision of motion plans in a potentially conflicting environment has been investigated in the works of\textsuperscript{[12], [25]–[28]}, only deterministic transition systems are considered, and it is not yet clear how to address the above-mentioned issues in stochastic systems.

A. Contributions

Motivated by these challenges, this work considers the interaction of environment and the mobile robot as a black box, and we study a learning-based motion planning subject to uncertainties, where control objectives are defined as high-level LTL formulas to express complex tasks. The contributions of this work are multi-fold summarized as:

- From task perspective, both motion and workspace uncertainties are considered, leading to potentially conflicting tasks (i.e., the pre-specified LTL tasks cannot be fully satisfied). To model motion and environment uncertainties, probabilistic labeled MDP is employed to abstract the overall uncertain systems.
- From the aspect of automaton theory, we develop an embedded limit deterministic generalized Büchi automaton

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(E-LDGBA) that has several accepting sets to maintain a dense reward and allows applying deterministic policies to achieve high-level objectives.

- The novelty of combining the MDP model and the E-LDGBA is to construct a relaxed product MDP so that the RL agent is allowed to revise its motion plan without strictly following the desired LTL constraints. An expected return composed of violation rewards and accepting rewards is developed. The designed violation function quantifies the differences between the revised and the desired motion plans, while the accepting rewards enhance the satisfaction of the acceptance condition of the relaxed product MDP.

- Analytically, with proper design of accepting and violation rewards, an RL-based control synthesis approach is developed to generate finite-memory policies that fulfill pre-specified tasks as much as possible by optimizing the expected return (utility) of the relaxed product MDP.

II. PRELIMINARIES

A. Probabilistic Labeled MDP

Definition 1. A probabilistic labeled finite MDP is a tuple $\mathcal{M} = (S, A, p_S, (s_0, l_0), \Pi, L, p_L)$, where $S$ is a finite state space; $A$ is a finite action space; $p_S : S \times A \times S \rightarrow [0, 1]$ is the transition probability function; $\Pi$ is a set of atomic propositions; and $L : S \rightarrow 2^\Pi$ is a labeling function. Let $\xi$ be an action function, which can be either deterministic such that $\xi : S \rightarrow A$ maps a state $s \in S$ to an action in $A(s)$, or randomized such that $\xi : S \times A \rightarrow [0, 1]$ represents the probability of taking an action in $A(s)$ at $s$. The pair $(s_0, l_0)$ denotes an initial state $s_0 \in S$ with an initial label $l_0 \in L(s_0)$. The function $p_L(s, l)$ denotes the probability of $l \subseteq L(s)$ associated to $s \in S$ satisfying $\sum_{l \in L(s)} p_L(s, l) = 1, \forall s \in S$. The transition probability $p_S$ captures the motion uncertainties of the agent while the labeling probability $p_L$ captures the environment uncertainties. It is assumed that the agent can fully observe its current state and the associated labels.

The probabilistic labeled finite MDP can be regarded as an advanced MDP and we use the MDP to denote it in the following sections. The MDP $\mathcal{M}$ evolves by taking actions $\xi_i$ at each stage $i$, and thus the control policy $\xi = \xi_0 \xi_1 \ldots$ is a sequence of actions, which yields a path $s = s_0 s_1 s_2 \ldots$ over $\mathcal{M}$ with $p_S(s_i, a_i, s_{i+1}) > 0$ for all $i$. If $\xi_i = \xi$ for all $i$, then $\xi$ is called a stationary policy. The control policy $\xi$ is memoryless if each $\xi_i$ only depends on its current state and $\xi$ is called a finite memory policy if $\xi_i$ depends on its past states.

Let $\text{Post}(s, a) = \{ s' \in S | p_S(s, a, s') > 0 \}$ denote the set of states that can be transitioned to by taking $a \in A(s)$. A sub-MDP of $\mathcal{M}$ is denoted by $\mathcal{M}(S', A')$, where $S' \subseteq S$, $A' \subseteq A$, and, for any $s \in S'$, $A'(s) \subseteq A(s)$ and $\text{Post}(s, a) \subseteq S'$ by taking $a \in A'(s)$. The directed graph induced by $\mathcal{M}(S', A')$ is denoted by $\mathcal{G}(S', A')$. An end component (EC) of $\mathcal{M}$ is a sub-MDP $\mathcal{M}(S', A')$ such that $\mathcal{G}(S', A')$ is strongly connected. An EC $\mathcal{M}(S', A')$ is called a maximal end component (MEC) if there is no other EC $\mathcal{M}(S'', A'')$ such that $S' \subseteq S''$ and $A'(s) \subseteq A''(s), \forall s \in S$. It should be noted that, once a path enters an EC, it will stay within the EC as the transitions are constrained. A detailed treatment of labeled MDP can be found in [1].

Let $A : S \rightarrow \mathbb{R}$ denote a reward function over $\mathcal{M}$. Given a discount factor $\gamma \in (0, 1)$, the expected return under policy $\xi$ starting from $s \in S$ can be defined as

$$U^\xi(s) = \mathbb{E}^\xi \left[ \sum_{i=0}^{\infty} \gamma^i A(s_i) | s_0 = s \right]$$

(1)

The optimal policy $\xi^*$ is a policy that maximizes the expected return for each state $s \in S$ as $\xi^* = \arg \max_{\xi} U^\xi(s)$. 

B. LTL and Limit-Deterministic Generalized Büchi Automaton

An LTL is built on a set of atomic propositions $\Pi$, standard Boolean operators such as $\land$ (conjunction), $\neg$ (negation), and temporal operators $\bigcup$ (until), $\bigcirc$ (next), $\bigtriangledown$ (eventually), $\square$ (always). The semantics of LTL formula are defined over
words, which form an infinite sequence \( o = o_0 o_1 \ldots \) with \( o_i \in 2^\Omega \). Denote by \( o \models \varphi \) if the word \( o \) satisfies the LTL formula \( \varphi \). More expressions can be achieved by combing temporal and Boolean operators. Detailed descriptions of the syntax and semantics of LTL can be found in [1].

Given an LTL formula that specifies the missions, the satisfaction of the LTL formula can be evaluated by an LDGBA [34].

Definition 2. A GBA is a tuple \( A = (Q, \Sigma, \delta, q_0, F) \), where \( Q \) is a finite set of states, \( \Sigma = 2^\Omega \) is a finite alphabet; \( \delta : Q \times \Sigma \rightarrow 2^Q \) is a transition function, \( q_0 \in Q \) is an initial state, and \( F = \{F_1, F_2, \ldots, F_f\} \) is a set of acceptance conditions with \( F_i \subseteq Q, \forall i \in \{1, \ldots, f\} \).

Denote by \( q = q_0 q_1 \ldots \) a run of a GBA, where \( q_i \in Q, i = 0, 1, \ldots \). The run \( q \) is accepted by the GBA, if it satisfies the generalized Büchi accepting sets, i.e., \( \inf(q) \cap F_i \neq 0, \forall i \in \{1, \ldots, f\} \), where \( \inf(q) \) denotes the set of states that is visited infinitely often.

Definition 3. A GBA is a Limit-deterministic Generalized Büchi automaton (LDGBA) if \( \delta : Q \times (\Sigma \cup \{\epsilon\}) \rightarrow 2^Q \) is a transition function and the states \( Q \) can be partitioned into a deterministic set \( Q_D \) and a non-deterministic set \( Q_N \), i.e., \( Q_D \cup Q_N = Q \) and \( Q_D \cap Q_N = \emptyset \), where

- the states transitions in \( Q_D \) are total and restricted within it, i.e., \( \delta(q, \alpha) = 1 \) and \( \delta(q, \alpha) \subseteq Q_D \) for every state \( q \in Q_D \) and \( \alpha \in \Sigma \),
- the \( \epsilon \)-transition is not allowed in the deterministic set, i.e., for any \( q \in Q_D \), \( \delta(q, \epsilon) = 0 \), and
- the accepting states are only in the deterministic set, i.e., \( F_i \subseteq Q_D \) for every \( F_i \in F \).

In Def. [3] the \( \epsilon \)-transitions are only defined for state transitions from \( Q_N \) to \( Q_D \), which do not consume the input alphabet. To convert an LTL formula to an LDGBA, readers are referred to [35] for algorithms with free implementations. Compared to the widely used deterministic Rabin Automaton (DRA), LDGBA generally has a smaller size and is more efficient in implementation [34]. It should be noted that the state-based LDGBA is used in this work for demonstration purposes. Transition-based LDGBA can be constructed based on basic graph transformations. More discussions about state-based and transition-based LDGBA in HOA format can be found in Owl [36].

III. PROBLEM STATEMENT

The task specification to be performed by the agent is described by an LTL formula \( \phi \) over \( \Pi \). Given \( \phi, \mathcal{M} \), and \( \xi = \xi_0 \xi_1 \ldots \), the induced infinite path is denoted by \( s_\xi = s_0 s_1 s_{i+1} \ldots \) which satisfies \( s_{i+1} \in \{s \in S | p_S (s_i, a, s) > 0\} \). Let \( L(s_\xi) = \{l_0 l_1 \ldots \} \) be the sequence of labels associated with \( s_\xi \) such that \( l_i \in L(s_i) \) and \( p_L(s_i, l_i) > 0 \). Denote by \( L(s_\xi) \models \phi \) if \( s_\xi \) satisfies \( \phi \). The product automaton is constructed as follows.

Definition 4. Given the labeled MDP \( \mathcal{M} \) and the LDGBA \( A_\phi \) corresponding to \( \phi \), the product MDP is defined as \( \mathcal{P} = (Y, U^P, p^P, x_0, F^P) \), where \( Y = S \times 2^\Pi \times Q \) is the set of labeled states, i.e., \( y = (s, l, q) \in Y \) with \( l \in L(s) \) satisfying \( p_L(s, l) > 0 \); \( U^P = A \cup \{\epsilon\} \) is the set of actions, where the \( \epsilon \)-actions are only allowed for transitions of LDGBA components from \( Q_N \) to \( Q_D \); \( x_0 = (s_0, l_0, q_0) \) is the initial state; \( F^P = \{F^P_1, F^P_2, \ldots, F^P_f\} \) where \( F^P_i = \{(s, l, q) \in X | q \in F_i \} \), \( j = 1, \ldots, f \), is the set of accepting states; \( p^P : Y \times U^P \times Y \rightarrow [0, 1] \) is a transition probability defined as: 1) \( p^P(y, u^P, y') = p_L(s', l') \cdot p_S(s, a, s') \) if \( \delta(q, l) = q' \) and \( u^P = a \in A(s) \); 2) \( p^P(y, u^P, y') = 1 \) if \( u^P \in \{\epsilon\} \); \( q' \in \delta(q, \epsilon) \), and \((s', l') = (s, l)\); and 3) \( p^P(y, u^P, y') = 0 \) otherwise.

The product MDP \( \mathcal{P} \) captures the intersections between all feasible paths over \( \mathcal{M} \) and all words accepted to \( A_\phi \), facilitating the identification of admissible agent motions that satisfy the task \( \phi \). Let \( \pi \) denote a policy over \( \mathcal{P} \) and denote by \( y^\pi_\infty = y_0 y_1 y_2 \ldots \) the infinite path generated by \( \pi \). A path \( y^\pi_\infty \) is accepted if \( \inf(y^\pi_\infty) \cap F^P_i \neq \emptyset \), \( \forall i \in \{1, \ldots, f\} \). If \( y^\pi_\infty \) is an accepting run, there exists a policy \( \xi \) in \( \mathcal{M} \) that satisfies \( \phi \).

Consider a sub-product MDP \( \mathcal{P}'(Y', U') \), where \( Y' \subseteq X \) and \( U' \subseteq U^P \). If \( \mathcal{P}'(Y', U') \) is a maximum end component of \( \mathcal{P} \) and \( Y' \cap F^P_i \neq \emptyset, \forall i \in \{1, \ldots, f\} \), then \( \mathcal{P}'(Y', U') \) is called an accepting maximum end component (AMEC) of \( \mathcal{P} \). Once a path enters an AMEC, the subsequent path will stay within it by taking restricted actions from \( U' \). There exist policies such that any state \( x \in X' \) can be visited infinitely often. As a result, satisfying the task \( \phi \) is equivalent to reaching an AMEC. Moreover, an MEC that does not contain any accepting sets is called a rejecting accepting component (RMEC) and an MEC with only partial accepting sets contained is called a neutral maximum end component (NMEC).

It should be noted that most existing results (cf. [18], [22], [24]) assume the existence of at least one AMEC in \( \mathcal{P} \), which may not be true in practice if the task is only partially feasible. We consider such a challenge, we consider the following problem.

Problem 1. Given an LTL-specified task \( \phi \) and an MDP with unknown transition probabilities (i.e., motion uncertainties) and unknown probabilistic label functions (i.e., workspace uncertainties), the objective is to find a policy that mostly fulfills the desired task \( \phi \) (i.e., soft constraints) over an infinite horizon.

IV. AUTOMATON ANALYSIS

To solve Problem 1 Section IV-A first presents how the LDGBA in Def. [3] can be extended to E-LDGBA to keep tracking the non-visited accepting sets. Section IV-B presents the construction of a relaxed product MDP to handle soft LTL constraints. The benefits of incorporating E-LDGBA with relaxed product MDP are discussed in Section IV-C.

A. E-LDGBA

In order to find the desired policy in PL-MDP \( \mathcal{M} \) to satisfy the user-specified LTL formula \( \phi \), one can construct the standard product MDP between \( \mathcal{M} \) and the LDGBA of \( \phi \) as
described in [1]. Then, the problem becomes finding the policy of LDGBA. Thus, the E-LDGBA is introduced as follows.

Algorithm 1 Procedure of E-LDGBA

1: procedure INPUT: (LDGBA \( \mathcal{A}, f_v, T \) and length \( L \))
2: Output: A valid run \( \mathbf{q} \) with length \( L \) in \( \mathcal{A}_\phi \)
3: \( \mathbf{q}_\text{next} = \delta(q, \sigma_0) \)
4: \( \mathbf{q}_\text{count} = 1 \)
5: while \( \mathbf{q}_\text{count} \leq L \) do
6: \( \mathbf{q}_\text{next} = \delta(q, \sigma) \)
7: \( \mathbf{q}_\text{count} = \mathbf{q}_\text{count} \)
8: if \( \mathbf{q}_\text{next} \in \mathcal{Q}_\text{accept} \) then
9: \( T = f_v(q, \mathbf{q}_\text{count}) \)
10: add state \( \mathbf{q}_\text{next} \) to \( \mathbf{q} \)
11: end if
12: end while
13: end procedure

Algorithm 2 Policy Execution over \( \mathbf{R} = \mathcal{M} \otimes \mathcal{A}_\phi \)

1: procedure INPUT: (policy \( \pi, \mathcal{M}, \mathcal{A}_\phi, N \))
2: Output: a sequence \( x_N \) with length \( N \) under policy \( \pi \)
3: \( x_0 = x_0 \)
4: while \( t < N \) do
5: Obtain \( u_0^N \) based on \( \pi(x_t) \)
6: Obtain \( x_{\text{next}} \) using \( \text{Relax-product} \left( u_0^N, \mathcal{M}, \mathcal{A}_\phi, x_N \right) \)
7: \( t \leftarrow t + 1 \)
8: \( x_t = x_{\text{next}} \)
9: Add \( x_t \) to \( x_N \)
10: end while
11: end procedure

Remark 1. In Definition [5] we abuse the tuple structure since the frontier set \( T \) is synchronously updated after each transition, and the state-space is augmented with the tracking-frontier set \( T \) that can be practically represented via one-hot encoding based on the indices of the accepting set. The acceptance state is determined based on the current automaton state and the frontier set \( T \). Such property is the innovation of E-LDGBA, which encourages all accepting sets to be visited in each round.

Alg. 1 demonstrates the procedure of obtaining a valid run \( q_\mathbf{T} \) (consisting of all visited states) within E-LDGBA. Given \( \mathcal{A}_\phi \) and \( \mathcal{A}_\phi \), for the same LTL formula, the E-LDGBA keeps track of previously accepted states of \( \mathcal{A}_\phi \) by incorporating \( f_v \) and \( T \), which can distinguish and enforce the procedure of acceptance satisfaction related to different accepting sets. \( T \) will be reset when all the accepting sets of \( \mathcal{A}_\phi \) have been visited. Let \( \mathcal{L}(\mathcal{A}_\phi) \subseteq \Sigma^* \) and \( \mathcal{L}(\mathcal{A}_\phi) \subseteq \Sigma^* \) be the set of all infinite words accepted by \( \mathcal{A}_\phi \) that satisfies LTL formula \( \phi \).

**Lemma 1.** For any LTL formula \( \phi \), we can construct LDGBA \( \mathcal{A}_\phi = (Q, \Sigma, \delta, q_0, F) \) and E-LDGBA \( \mathcal{A}_\phi = (Q, \Sigma, \delta, q_0, F, f_v, T) \). Then it holds that
\[
\mathcal{L}(\mathcal{A}_\phi) = \mathcal{L}(\mathcal{A}_\phi).
\]

**Proof:** We refer the readers to check more details of the proof in our work [30] that has shown \( \mathcal{L}(\mathcal{A}_\phi) \supseteq \mathcal{L}(\mathcal{A}_\phi) \) and \( \mathcal{L}(\mathcal{A}_\phi) \subseteq \mathcal{L}(\mathcal{A}_\phi) \).

**Lemma 1** indicates that both E-LDGBA and LDGBA accept the same language. Consequently, E-LDGBA can also be applied to verify the satisfaction of LTL specifications, and incorporating E-LDGBA into RL based model checking will not affect the convergence of optimality. Based on \( \mathcal{A}_\phi \), the embedded product MDP (EP-MDP) \( \mathcal{P} = \mathcal{M} \otimes \mathcal{A}_\phi \) can be constructed similarly as in Def. 3 by replacing LDGBA with E-LDGBA. We denote the EP-MDP by \( \mathcal{P} = \mathcal{M} \otimes \mathcal{A}_\phi = (X, U, \mathcal{P}, x_0, f_v, F, T) \).

**B. Relaxed Probabilistic Product MDP**

For the product MDP \( \mathcal{P} \) or \( \mathcal{P} \) introduced above, the satisfaction of \( \phi \) is based on the assumption that there exists at least one AMEC. Otherwise, learning-based motion plans...
that rely on AMECs are no longer applicable. To address this challenge, the relaxed product MDP is designed to allow the agent to revise its motion plan without strictly following the desired LTL constraints.

**Definition 6.** The relaxed product MDP is constructed from $\mathcal{R}$ as a tuple $\mathcal{R} = \mathcal{M} \otimes \mathcal{A}_\phi = (X, U_\mathcal{R}, \mathcal{F}_\mathcal{R}, \mathcal{c}_\mathcal{R}, f_V, T)$, where $X$, $x_0$, and $F_\mathcal{R} = F' \mathcal{R}$ are the same as in $\mathcal{P}$, $U_\mathcal{R}$ is the set of extended actions, $\mathcal{F}_\mathcal{R} : X \times U_\mathcal{R} \times X \rightarrow [0, 1]$ is the transition function, and $\mathcal{c}_\mathcal{R} : X \times U_\mathcal{R} \times X \rightarrow \mathbb{R}$ is the violation cost.

It’s worth to point out that $\mathcal{R}$ and $\mathcal{P}$ for the same $\mathcal{A}_\phi$ share the same states $X$. In Def. 6, the actions jointly consider the actions of $\mathcal{M}$ and the input alphabet of $\mathcal{A}_\phi$. Specifically, given a state $x = (s, l, \eta) \in X$, the actions available at $x$ in $\mathcal{R}$ are denoted by $\{a, i\} | a \in A(s), i \in (2^I \cup \emptyset \}$.

Given an action $u_\mathcal{R} = (a, i) \in U_\mathcal{R}(x)$, we further denote by $u_{\mathcal{M}}|_M$ and $u_{\mathcal{R}}|_A$ the projections of $u_\mathcal{R}$ to $A(s)$ in $\mathcal{M}$ and to $2^I \cup \{\}$ in $\mathcal{A}_\phi$, respectively, i.e., $u_{\mathcal{R}}|_A = a$ and $u_{\mathcal{R}}|_M = i$. The transition probability $p_\mathcal{R}$ from a state $x = (s, l, \eta)$ to a state $x' = (s', l', \eta')$ is defined as: 1) $p_\mathcal{R}(x, u_\mathcal{R}, x') = p_L(s', l') \cdot p_S(s, a, s')$ with $a = u_{\mathcal{M}}|_M$, if $\eta$ can be transition to $\eta'$ and $\eta' = \delta(s, \eta, \epsilon)$, and (s', l') = (s, l); 2) $p_\mathcal{R}(x, u_\mathcal{R}, x') = 1$, if $u_{\mathcal{R}}|_A = \epsilon$, $\eta' \in \delta(s, \epsilon)$, and (s', l') = (s, l); 3) $p_\mathcal{R}(x, u_\mathcal{R}, x') = 0$ otherwise. That is, if under a corresponding LTL mission $\pi$ that satisfies the constraints imposed by $\phi$. The transition probability $p_\mathcal{R}$ jointly considers the probability of an event $p_L(s', l')$ and the transition $p_S$. Minimizing the expected return of violation cost of $c_\mathcal{R}$ will enforce the planned path towards more fulfilling the LTL task $\phi$ by penalizing $w_\mathcal{V}$. A run $r_\mathcal{R}$ induced by a policy $\pi$ that satisfies the accepting condition of the relaxed product MDP $\mathcal{R}$ completes the corresponding LTL mission $\phi$ exactly if and only if the expected return of violation cost is equal to zero.

The on-the-fly policy execution over $\mathcal{R}$ during learning is outlined in Alg. 2 and Alg. 3. In each transition in Alg. 3, the next state is jointly determined by the constraints of $\delta$ in LDGBA $A$ and the tracking-frontier function $f_V$.

The relaxed product MDP between MDP and LDGBA can be also constructed as the procedure Def. 6 by replacing E-LDGBA. Here we denote it as $\mathcal{R} = \mathcal{M} \otimes \mathcal{A}_\phi = (Y, U_\mathcal{R}, \mathcal{F}_\mathcal{R}, x_0, F_\mathcal{R}, c_\mathcal{R}, f_V, T)$ from $\mathcal{P}$. Note, the product and relaxed product are the procedures of constructing the interaction between the automaton structure and MDP model, which can be applied to any of the automaton. Similarly, $\mathcal{R}$ and $\mathcal{P}$ for the same $\mathcal{A}_\phi$ share the same states $Y$.

### C. Properties of Relaxed Product MDP

This section discusses the properties of the relaxed product MDP $\mathcal{R} / \mathcal{R}$ compared with the corresponding product MDP $\mathcal{P} / \mathcal{P}$. Based on Def. 6, the relaxed product MDP and its corresponding product MDP share the same states. Hence, we can regard $\mathcal{R} / \mathcal{R}$ and $\mathcal{P} / \mathcal{P}$ as two separate directed graphs that have the same nodes. From the aspect of graph, the MEC can be regarded as a bottom strong connected component (BSCC). Similarly, let ABSCC denote the BSCC that contains at least one accepting state in $\mathcal{P}$ or $\mathcal{R}$.

Since the relaxed product MDP and product MDP can be adopted with LDGBA and E-LDGBA. The E-LDGBA is an extension of LDGBA to enable the recording ability that will not influence the overall product process. For simplicity purpose, we first verify the properties of $\mathcal{R}$, which can be also hold by $\mathcal{R}$.

**Theorem 1.** Given an MDP $\mathcal{M}$ and an LDGBA $\mathcal{A}_\phi$ corresponding to the desired LTL task specification $\phi$, the designed relaxed product MDP $\mathcal{R} = \mathcal{M} \otimes \mathcal{A}_\phi$ has the following properties compared with corresponding $\mathcal{P}$:

1. the directed graph of traditional MDP $\mathcal{P}$ is sub-graph of the directed graph of $\mathcal{R}$,
2. there always exists at least one ABSCC in $\mathcal{R}$,
3. if the LTL formula $\phi$ is feasible over $\mathcal{M}$, any ABSCC of $\mathcal{P}$ is the sub-graph of an ABSCC of $\mathcal{R}$.

**Proof.** The proof is the same as our previous work [14] by replacing the LDGBA into LDGBA.

**Example 1.** To illustrate the benefits of the designed relaxed product MDP, Fig. 1 (a) and (b) provide an example of an LDGBA $\mathcal{A}_\phi$ with $\phi = \Diamond \Box a \land \Diamond \Box b$, and an abstraction process of MDP $\mathcal{M}$ from a robotic system where $\phi$ is infeasible. Fig. 1 (c) and (d) show the corresponding product MDP $\mathcal{P}$ and the relaxed product MDP $\mathcal{R}$, respectively. Clearly, the product MDP in Fig. 1 (c) has MECs that only contain subset (the accepting set indexed with 1) of all accepting sets, while the relaxed product MDP in Fig. 1 (d) has at least one AMEC (the MEC in blue rectangular). In addition, the transitions with non-zero violation costs are marked with gray dash transition lines in Fig. 1 (d). Note that due to the complicated graph structure, we take the relaxed product instead of E-LDGBA in the example to illustrate the novel designed structure of the relaxed product.
Figure 1. (a) The LDBA of $\phi = \Box a \land \Box b$. (b) The abstraction process from a robotic sytem into MDP $M$, where the region labeled with $b$ is surrounded by the obstacles and can not be accessed. As a result, $\phi$ is infeasible in the case. (c) product MDP. There’s no criteria to evaluate a policy related to task satisfaction, since we can not find a path satisfying the acceptance condition. (d) Partial structure of relaxed product MDP. The gray dashed lines represent the transitions with non-zero violation cost, and the graph component in blue rectangular contains the path that satisfies the acceptance condition.

Lemma 2. Given an MDP $M$ and an E-LDGBA $A_\phi$ corresponding to the desired LTL task specification $\phi$, the designed relaxed product MDP $\overline{M} = M \otimes A_\phi$ has the following properties compared with corresponding $P$:

1) the directed graph of traditional product $P$ is sub-graph of the directed graph of $\overline{M}$,
2) there always exists at least one ABSCC in $\overline{M}$,
3) if the LTL formula $\phi$ is feasible over $M$, any ABSCC of $P$ is the sub-graph of an ABSCC of $\overline{M}$.

Lemma 2 can be verified via employing the proof of Theorem 1. Both of them demonstrate the advantages of applying the procedure of relax product, which allow us the handle the infeasible situations.

Given a relaxed product MDP $\overline{M}$, let $MC^{\pi}_{\overline{M}}$ denote the Markov chain induced by the policy $\pi$ on $\overline{M}$, whose states can be represented by a disjoint union of a transient class $T_\pi$ and $n$ closed irreducible recurrent sets $R^i_\pi$, $j \in \{1, \ldots, n\}$, i.e., $MC^{\pi}_{\overline{M}} = T_\pi \cup R^1_\pi \cup R^2_\pi \ldots R^n_\pi$ [37].

Lemma 3. Given a relaxed product MDP $\overline{M} = M \otimes A_\phi$, the recurrent class $R^j_\pi$ of $MC^{\pi}_{\overline{M}}$, $j \in \{1, \ldots, n\}$, induced by $\pi$ satisfies one of the following conditions:

1) $R^i_\pi \cap F^i_{\overline{M}} \neq \emptyset$, $\forall i \in \{1, \ldots f\}$, or
2) $R^i_\pi \cap F^i_{\overline{M}} = \emptyset$, $\forall i \in \{1, \ldots f\}$.

Proof: The following proof is based on contradiction. Assume there exists a policy such that $R^j_\pi \cap F^i_{\overline{M}} \neq \emptyset$, $\forall k \in K$, where $K$ is a subset of $2^\{1, \ldots, f\} \setminus \{\{1, \ldots f\}, \emptyset\}$. As discussed in [37], for each state in recurrent class, it holds that $\sum_{n=0}^{\infty} p^n(x,x) = \infty$, where $x \in R^j_\pi \cap F^i_{\overline{M}}$ and $p^n(x,x)$ denotes the probability of returning from a transient state $x$ to itself in $n$ steps. This means that each state in the recurrent class occurs infinitely often. However, based on the embedded tracking frontier function of E-LDGBA in Def. 5, the tracking set $T$ will not be reset until all accepting sets have been visited. As a result, neither $q_k \in F^i_{\overline{M}}$ nor $x_k = (s,q_k) \in R^j_\pi \cap F^i_{\overline{M}}$ with $s \in S$ will occur infinitely, which contradicts the property $\sum_{n=0}^{\infty} p^n(x_k,x_k) = \infty$.

Lemma 3 indicates that, for any policy, all accepting sets will be placed either in the transient class or the recurrent class. As a result, the issue of NMEC as in many existing methods can be avoided. Based on Theorem 1 and Lemma 3, Problem 1 can be reformulated as follows.

Problem 2. Given a user-specified LTL task $\phi$ and an MDP with unknown transition probabilities (i.e., motion uncertainties) and unknown labeling probabilities (i.e., environment uncertainties), the objective is to find a policy in decreasing order of priority to 1) satisfy the acceptance condition of the relaxed product MDP, and 2) reduce the violation cost of the expected return.
V. Learning Based Control Synthesis

In this section, reinforcement learning is leveraged to identify policies for Problem 2. Specifically, a model-free multi-objective reinforcement learning is designed.

A. Reward Design

The accepting reward function $\Lambda : X \rightarrow \mathbb{R}$ is designed as

$$\Lambda(x) = \begin{cases} r_\Lambda & \text{if } \exists i \in \{1, \ldots, f\} \text{ such that } x \in F_i, \\ 0 & \text{otherwise}, \end{cases}$$

(4)

where $r_\Lambda > 0$. The violation function $V : X \times U \rightarrow \mathbb{R}$ is designed as

$$V(x, u) = -c_u(x, u).$$

(5)

The non-negative $\Lambda(x)$ enforces the accepting condition of $\mathcal{R}$, while the non-positive function $V(x, u)$ indicates the penalties of violations. Let $U_\pi = [U_\pi(x_0), U_\pi(x_1), \ldots] \in \mathbb{R}^N$ denote the stacked expected return induced by $\pi$ over $\mathcal{R}$ with $N = |X|$, the expected return is designed as

$$U = \sum_{n=0}^{\infty} \gamma^n P_\pi^n (A_\pi + \beta V_\pi \cdot 1_N),$$

(6)

where $0 < \gamma < 1$ is a discount factor, $P_\pi \in \mathbb{R}^{N \times N}$ is a matrix with entries representing the probabilities $P_\pi(x, \pi(x), x')$ under $\pi$ for all $x, x' \in X$, $A_\pi = [A_\pi(x_0), A_\pi(x_1), \ldots] \in \mathbb{R}^N$ is the stacked state-rewards, $\beta \in \mathbb{R}^+$ is a weight indicating the relative importance, $1_N$ is an $N$-dimensional vector of ones, and $V_\pi = P_\pi \circ V$ is the Hadamard product of $P_\pi$ and $V$, i.e., $V_\pi = [p(x, \pi(x), x') \cdot V(x, u)]_{N \times N}$ and $a_\pi(x)$ represents taking action at $x$ from $\pi$.

The objective is to identify a stationary policy $\pi^*$ that maximizes the expected return

$$\pi^* = \arg \max_{\pi} \sum_{n=0}^{\infty} \gamma^n P_\pi^n (A_\pi + \beta V_\pi \cdot 1_N),$$

(7)

$$U_\pi(x) \leq U_{\pi^*}(x) \text{ for all } x \in X \text{ if } \pi^* \text{ in (7) is optimal.}$$

Theorem 2. Consider a relaxed MDP product $\mathcal{R} = M \otimes \mathcal{A}_\phi$. If there exists a policy $\pi^*$ such that an induced run $r_\pi^*$ satisfies the acceptance condition of $\mathcal{R}$, any optimization method that solves (7) can find the policy $\pi^*$.

Proof: For any policy $\pi$, let $MC_{\mathcal{R}}^\pi$ denote the Markov chain induced by $\pi$ on $\mathcal{R}$. Since $MC_{\mathcal{R}}^\pi$ can be written as $MC_{\mathcal{R}}^\pi = T_\pi \sqcup R_{\pi} \sqcup R_{\pi}^2 \ldots R_{\pi}^N$, (6) can be reformulated as

$$\begin{bmatrix} U_{\pi}^r \\ U_{\pi}^{rec} \end{bmatrix} = \sum_{n=0}^{\infty} \gamma^n \begin{bmatrix} P_\pi(T, T) & P_\pi^r \\ 0_{\sum_{n=1}^{\infty} N \times N} & P_\pi(R, R) \end{bmatrix}^n \begin{bmatrix} A_\pi^r \\ A_\pi^{rec} + \beta \begin{bmatrix} O_\pi^r \\ O_\pi^{rec} \end{bmatrix} \end{bmatrix},$$

(8)

where $U_{\pi}^r$ and $U_{\pi}^{rec}$ are the utilities of states in the transient and recurrent classes, respectively. In (8), $P_\pi(T, T) \in \mathbb{R}^{r \times r}$ denotes the probability transition matrix between states in $T_\pi$. $P_\pi^r = [P_\pi^{r_1} \ldots P_\pi^{r_N}] \in \mathbb{R}^{r \times \sum_{n=1}^{\infty} N}$ is a probability transition matrix where $P_\pi^{r_i} \in \mathbb{R}^{r \times N_i}$ represents the probability of transiting from a transient state in $T_\pi$ to the states of $R_{\pi}^i$. The $P_\pi(R, R)$ is a diagonal block matrix, where the $i$th block is an $N_i \times N_i$ matrix containing transition probabilities between states within $R_{\pi}^i$. $P_\pi^{rec} \otimes \mathcal{R}$ is a stochastic matrix since each block matrix is a stochastic matrix $[37]$. The rewards vector $A_\pi$ can also be partitioned to $A_\pi^r$ and $A_\pi^{rec}$. Similarly, $O_\pi = [O_\pi(x_0), O_\pi(x_1), \ldots]^T = V_\pi \cdot 1_N$ can be divided into transient class $O_\pi^r$ and recurrent class $O_\pi^{rec}$.

We prove this theorem by contradiction. Suppose there exists an optimal policy $\pi^*$ not satisfying the acceptance condition of $\mathcal{R}$. Based on Lemma 3, the following is true: $F_k^\pi \subseteq T_\pi^f, \forall i \in \{1, \ldots, f\}$. As a result, for any $j \in \{1, \ldots, n\}$, we have $R_{\pi}^j \cap F_{i}^\pi = \emptyset, \forall i \in \{1, \ldots, f\}$.

The strategy is to show that there always exists a policy $\pi^*$ with greater utility than $\pi^*$, which contradicts to the optimality of $\pi^*$. Let’s consider a state $x_R \in R_{\pi}^f$ and let $P_{\pi}^{r_i} R_{\pi}^R$ denote a row vector of $P_{\pi}^{r_i} (R, R)$ that contains the transition probabilities from $x_R$ to the states in the same recurrent class $R_{\pi}^R$ in $n$ steps. The expected return $U_{\pi}^{rec}(x_R)$ is then obtained from (8) as: $U_{\pi}^{rec}(x_R) = \sum_{n=0}^{\infty} \gamma^n \begin{bmatrix} 0_{r \times 1} & P_{\pi}^{r_i} R_{\pi}^R & 0_{r \times r} \end{bmatrix} (A_\pi^{rec} + \beta O_\pi^{rec})$ where $k_1 = \sum_{i=1}^{N} N_i$, $k_2 = \sum_{i=1}^{N_i} N_i$. Since $R_{\pi}^{rec} \cap F_{i}^\pi = \emptyset, \forall i \in \{1, \ldots, f\}$, all the elements of $A_\pi^{rec}$ are equal to zero and each entry of $O_\pi^{rec}$ is non-positive. We can conclude $U_{\pi}^{rec}(x_R) \leq 0$. To prove that optimal policy $\pi^*$ raises a contradiction, the following analysis will show that $U_{\pi}^{rec}(x_R) \geq U_{\pi^*}^{rec}(x_R)$ for some policies $\pi$ that satisfy the acceptance condition of $\mathcal{R}$. Thus we have $R_{\pi}^f \cap F_{i}^\pi = \emptyset, \forall i \in \{1, \ldots, f\}$.

Case 1: If $x_R \in R_{\pi}^f$, by Lemma 3 and (4), there exist a minimum of $f$ accepting states such that $X_\pi = \{x_i : x_i \in R_{\pi}^f \cap F_{i}^\pi, \forall i \in \{1, \ldots, f\}\} \in R_{\pi}^f$ with positive rewards $r_\Lambda$. From (8), $U_{\pi}^{rec}(x_R)$ can be lower bounded as $U_{\pi}^{rec}(x_R) \geq \sum_{n=0}^{\infty} \gamma^n \left( P_{\pi}^{r_i} A_{\pi}^{rec} + \beta P_{\pi}^{r_i} V_{\pi} \cdot 1_N \right)$, where $P_{\pi}^{r_i} A_{\pi}^{rec}$ is the transition probability from $x_R$ to $x_A$ in $n$ steps, and $V_{\pi} \in \mathbb{R}^{N_i \times N_i}$ represents the violation cost states in $R_{\pi}^i$. Since $x_R$ and $x_A$ are recurrent states, there always exists a lower bound $P_{\pi}^{r_i} A_{\pi}^{rec}$ of the transition probability $P_{\pi}^{r_i} A_{\pi}$. We can select a positive reward $r_\Lambda$ such that

$$U_{\pi^*}^{rec}(x_R) \geq P_{\pi}^{r_i} A_{\pi}^{rec} + \beta N_i^2 V_{\pi} \cdot 1_N \geq 0$$

where $V_{\pi} \in \mathbb{R}^r$ represents the minimal entry in $V_{\pi}$.

By selecting $r_\Lambda$ to satisfy (9), we can conclude in this case $U_{\pi}^{rec}(x_R) \geq U_{\pi^*}^{rec}(x_R)$.

Case 2: If $x_R \in T_\pi$, we know $T_{\pi} \cap F_{i}^\pi = \emptyset, \forall i \in \{1, \ldots, f\}$. As demonstrated in [37], for a transient state $x_{tr} \in T_{\pi}$, there always exists an upper bound $\Delta < \infty$ such that

$$\begin{bmatrix} p^n(x_{tr}, x_{tr}) < \Delta, \text{ where } p^n(x_{tr}, x_{tr}) \text{ denotes the probability of returning from a transient state } x_{tr} \text{ to itself in } n \text{ time steps.} \end{bmatrix}$$
For a recurrent state $x_{rec} \in \overline{K}_{\pi}$, it is always true that
\[
\sum_{n=0}^{\infty} \gamma^n p^n(x_{rec}, x_{rec}) > \frac{1}{1 - \gamma_\pi \tilde{p}}
\]
where there exists $\pi$ such that $p^n(x_{rec}, x_{rec})$ is nonzero and can be lower bounded by $\tilde{p}$ \[37\]. From (8), one has
\[
U_{tr}^\pi > \sum_{n=0}^{\infty} \gamma^n P_{\pi}^n(T,T) (A_{tr}^\pi + \beta O_{tr}^\pi)
+ \sum_{n=0}^{\infty} \gamma^n P_{\pi}^n (\overline{K}, \overline{K}) (A^{tec} + \beta O_{tec}^\pi)
\]
Let $\max (\cdot)$ and $\min (\cdot)$ represent the maximum and minimum entry of an input vector, respectively. The upper bound $\tilde{m} = \max (\overline{M})$, where $\overline{N} = \sum_{j=1}^{m} N_j$ and $\overline{P}$ is a block matrix whose nonzero entries are derived similarly to the $\tilde{p}$ in (10). Using the fact $\sum_{n=0}^{\infty} \gamma^n P_{\pi}^n(T,T) \leq \Delta 1_{\overline{r} \times \overline{r}} \[37\]$, where $1_{\overline{r} \times \overline{r}}$ is a $\overline{r} \times \overline{r}$ matrix of all ones, the utility $U_{tr}^\pi (x_R)$ can be lower bounded from (10) and (11) as
\[
U_{tr}^\pi (x_R) > \Delta \cdot r \cdot \beta V_{tr}^\pi x + \frac{1}{1 - \gamma_\pi} \tilde{m}
\]
Since $U_{tec}^\pi (x_R) = 0$, the contradiction $U_{tr}^\pi (x_R) > 0$ in this case will be achieved if $\Delta \cdot r \cdot \beta V_{tr}^\pi x + \frac{1}{1 - \gamma_\pi} \tilde{m} > 0$. Because $\Delta r (\bar{w} + \beta V_{tr}^\pi x) < 0$, it needs $\tilde{m} > 0$ as
\[
\bar{m} > r_A + \beta \bar{N} V_{tec}^\pi > 0.
\]
Thus, by choosing $\gamma$ to be sufficiently close to 1 with $\tilde{m} > 0$, we have $U_{tr}^\pi (x_R) > 0 \geq U_{tec}^\pi (x_R)$. The above procedure shows the contradiction of the assumption that $\pi^*_t$ is optimal.

To solve Problem 2 by optimizing the expected return of \[6\], $r_A$, $\beta$, $\gamma$ can be determined as follows. Firstly, we can choose a fixed $r_A$. Then, $\gamma$ can be obtained by solving (12). Finally, the range of $\beta$ can be determined by solving (9) and (13). In order to minimize the violation cost, a great value of $\beta$ is preferred.

Remark 2. Problem 2 is a multi-objective RL (MORL) problem, and Theorem 2 shows how it can be addressed to ensure the acceptance satisfaction while reducing the long-term violation. In literature, MORL based approaches often seek to identify Pareto fronts \[38\]. However, the problem in our case considers a trade off between two possibly conflicting objectives, and there are no existing MORL methods to provide formal guarantee of both acceptance satisfaction and least violation, which might have the issue of safety during the process of executing optimal policies. To overcome the challenges, we can always divide the task into two parts: hard and soft constraints with two parallel automata. The hard task should be always satisfied that can be applied with the traditional product MDP as Def. 4 while the soft part can be partially feasible that can be relaxed as Def. 6. Such an idea can be found in our previous results \[13\].

It is worth pointing out that $V^\pi_x$ is a sparse matrix, because most transitions have zero violation cost, and Theorem 2 provides a performance guarantee for the worst cases of $\beta$ for acceptance satisfaction. In addition, by applying E-LDGBA, the sparse reward issue can also be improved compared with LDGBA.

B. Model-Free Reinforcement Learning

Based on the Q-learning \[15\], the agent updates its Q-value from $x$ to $x'$ according to
\[
Q(x,u_{\overline{r}}) \leftarrow (1 - \alpha) Q(x,u_{\overline{r}}) + \alpha \left[ R(x,u_{\overline{r}},x') + \gamma \max_{u_{\overline{r}}} Q(x,u_{\overline{r}}) \right]
\]
where $Q(x,u_{\overline{r}})$ is the Q-value of the state-action pair $(x,u_{\overline{r}})$. $0 < \alpha \leq 1$ is the learning rate, $0 \leq \gamma \leq 1$ is the discount factor, and
\[
R(x,u_{\overline{r}},x') = A(x) + \beta V(x,u_{\overline{r}},x')
\]
denotes the immediate reward from $x$ to $x'$ under $u_{\overline{r}}$. With standard learning rate $\alpha$ and discount factor $\gamma$ as in \[15\], Q-value will converge to a unique limit $Q^\ast$. Therefore, the optimal expected utility and policy can be obtained as $U^\pi_\overline{r} (x) = \max_{u_{\overline{r}}} Q^\ast (x,u_{\overline{r}})$ and $\pi^\ast (x) = \arg \max_{u_{\overline{r}} \in U_{\overline{r}}(x)} Q^\ast (x,u_{\overline{r}})$.

In (14) and (15), the discount $\gamma$ is tuned to improve trade-off in between immediate and future rewards.

The learning strategy is outlined in Alg. 4. The number of states is $|S| \times |Q|$, where $|Q|$ is determined by the original LDGBA $\mathcal{A}_o$ because the construction of E-LDGBA $\mathcal{A}_e$ will not increase the size and $|S|$ is the size of the labeled MDP model. Due to the consideration of a relaxed product MDP and extended actions in Def. 6, the maximum complexity of actions available at $x = (s,i,q)$ is $O(|A(s)| \times (|\Sigma| + 1))$, since $U_{\overline{r}}^\pi (x)$ are created from $A(s)$ and $\Sigma \cup \{e\}$.

VI. CASE STUDIES

The developed RL-based control synthesis is implemented in Python. Owl \[36\] is used to convert LTL specifications into LDGBA and P-MDP package \[8\] is used to construct state transition models. All simulations are carried out on a laptop with 2.60 GHz quad-core CPU and 8 GB of RAM. For Q-learning, the optimal policies of each case are generated using $10^5$ episodes with random initial states. The learning rate $\alpha$ is determined by Alg. 4 with $\gamma = 0.999$ and $r_A = 10$. To validate the effectiveness of our approach, we first carry out simulations over grid environments. We then validate the approach in a more realistic office scenario with the TIA Go robot.

A. Simulation Results

Consider a partitioned $10m \times 10m$ workspace as shown in Fig. 2, where each cell is a $2m \times 2m$ area. The cells are marked with different colors to represent different areas of interest, e.g., Base1, Base2, Base3, Obs, Sp1y, where Obs and Sp1y are shorthands for obstacle and supply, respectively. To model environment uncertainties, the number associated with a
Algorithm 4 Model-free RL-based control on MDPs under potentially soft LTL specifications.

1: procedure INPUT: (M, ϕ, A) 
2: Output: optimal policy π∗
3: Initialization: Set episode = 0 , iteration = 0 and τ (maximum allowed learning steps) 
4: while iteration < τ do 
5: determine πc , β , γ
6: for all x ∈ X do
7: U (x) = 0 and Q (x, u) = 0 for all x and u ∈ U (x)
8: Count (x, u) = 0 for all u ∈ U (x)
9: end for
10: while U are not converged do
11: episode + γ;
12: ϵ = 1/episode;
13: while iteration < τ do
14: iteration + γ;
15: Select u curr = arg maxQ (x curr, u) with probability
16: 1 − ϵ and select u curr as a random action in U (x curr)
17: Obtain u curr of M from u curr
18: Execute u curr
19: Observe s next , A (x curr, u curr)U (x curr, u next)
20: Q (x curr, u curr) = (1 − α) Q (x next, u curr) + α γ + u next , max Q (x next, u)
21: x next = x curr
22: end while
23: end while
24: for all x ∈ X do
25: U (x) = maxQ (x, u)
26: π∗ (x) = arg maxQ U (x)
27: end for
28: end procedure

Figure 2. Simulated trajectories of 50 time steps under corresponding optimal policies. (a) Feasible workspace with a simple task. (b) Feasible workspace with a relatively complex task. (c) Workspace with partially low-risk conflicting tasks. (d) Workspace with partially high-risk conflicting tasks.

Cell represents the likelihood that the corresponding property appears at that cell. For example, 0obs : 0.1 indicates this cell is occupied by the obstacles with probability 0.1. The robot dynamics follow the unicycle model, i.e., ˙x = v · sin (θ), ˙y = v · cos (θ), and ˙θ = ω, where x, y, θ indicate the robot positions and orientation, and the linear and angular velocities are the control inputs, i.e., u = (v, ω).

To model motion uncertainties, we assume the action primitives can not always be successfully executed. For instance, action primitives “FR” and “BR” mean the robot can successfully move forward and backward 2m with probability 0.9, respectively, and fail with probability 0.1. Action primitives “TR” and “TL” mean the robot can successfully turn right and left for an angle of π 2 exactly with probability 0.9, respectively, and fail by an angle of π 2 (undershoot) with probability 0.05, and an angle of 3π 2 (overshoot) with probability 0.05. Action primitive “ST” means the robot remains at its current cell. The resulting MDP model has 25 states.

Case 1: As shown in Fig. 2 (a), we first consider a case that user-specified tasks can all be successfully executed. The desired surveillance task to be performed is formulated as

ϕcase1 = (□(Base1) ∧ (□(Base2) ∧ (□(Base3)) ∧ ◊¬Obs),

which requires the mobile robot to visit all base stations infinitely while avoiding the obstacles. Its corresponding LDGBA has 2 states with 3 accepting sets and the relaxed product MDP has 50 states. In this case, each episode terminates after τ = 100 steps and β = 8. The generated optimal trajectory is shown in Fig. 2 (a), which indicates ϕcase1 is completed.

Case 2: We validate our approach with more complex task specifications in Fig. 2 (b). The task is expressed as

ϕcase2 = ϕcase1 ∨ (Sply → ◊ ((¬Sply) U ϕone1)),

where ϕone1 = Base1 ∨ Base2 ∨ Base3. ϕcase2 requires the robot to visit the supply station and then go to one of the base stations while avoiding obstacles and requiring all base stations to be visited. Its corresponding LDGBA has 24 states with 4 accepting sets and the relaxed product MDP has 600 states. The generated optimal trajectory is shown in Fig. 2 (b).

Case 3: We consider more challenging workspaces in Fig. 2 (c) and (d), where user-specified tasks might not be fully feasible due to potential obstacles (i.e., environment uncertainties). The task specification is

ϕcase3 = ϕcase1 ∧ (ϕone1 → ◊ (¬ϕone1) U Sply)),

where ϕcase3 requires the robot to visit one of the base stations and then go to one of the supply stations while avoiding obstacles. Its corresponding LDGBA has 24 states with 4 accepting sets and the product-MDP has 600 states. For ϕcase2
To model motion uncertainties, it is assumed that the robot crossing other regions using obstacle-avoidance navigation. Free path from the center of one region to another without by $S_7$ as shown in Fig. 3, which consists of $B$. Experimental Results carried out in larger workspaces. It is also verified that the given task in the case of Fig. 2 (c) while the robot decides to not visit $Base2$ to avoid the high risk of running into obstacles in the case of Fig. 2 (d).

To show the computational complexity, the $\varphi_{case1}$ over workspaces of various sizes (each grid is further partitioned). The simulation results are listed in Table I which consists of the number of MDP states, the number of relaxed product MDP states. The steps in Table I indicate the time used to converge to an optimal satisfaction planing when applying reinforcement learning. It is also verified that the given task $\varphi_{case1}$ can be successfully carried out in larger workspaces.

### B. Experimental Results

Consider an office environment constructed in ROS Gazebo as shown in Fig. 3 which consists of 7 rooms denoted by $S_0, S_2, S_3, S_5, S_9, Obs$ and 5 corridors denoted by $S_1, S_4, S_7, S_8, S_{12}$. The TIAGo robot can follow a collision-free path from the center of one region to another without crossing other regions using obstacle-avoidance navigation. To model motion uncertainties, it is assumed that the robot can successfully follow its navigation controller moving to a desired region with probability 0.9 and fail by moving to the adjacent region with probability 0.1. The resulting MDP has 12 states. The service to be performed by TIAGo is expressed as $\varphi_{case4} = \varphi_{all} \land \neg \varphi_{0}$ (19)

where $\varphi_{all} = S_0 \land S_2 \land S_3 \land S_5 \land S_9 \land S_{10}$. In (19), $\varphi_{all}$ requires the robot to always service all rooms (e.g. pick trash) and return to $S_0$ (e.g. release trash), while avoiding $Obs$. Its corresponding LDGBA has 6 states with 6 accepting sets and the relaxed product MDP has 72 states.

All room doors are open, except the doors of room $S_5, S_{10}$ in Fig. 4 (b). As a result, $\varphi_{case4}$ can not be fully completed in the case of Fig. 4 (b) and the task is infeasible. It also worth pointing out there don’t exist AMECs in corresponding product automaton $P$ or $\bar{P}$ in Fig. 4 (a), because the motion uncertainties rise the robot has the non-zero probability to entering $Obs$ at state $S_7$. The optimal policies for the two cases are generated and each episode terminates after $\tau = 150$ steps with $\beta = 4$. The generated satisfying trajectories (without collision) of one round are shown in Fig. 4 (a) and (b), and the robot tries to complete the feasible part of task $\varphi_{case4}$ in Fig. 4 (b).

### VII. Conclusions

In this paper, we present a learning-based control synthesis of motion planning subject to motion and environment uncertainties. The LTL formulas are applied to express complex task via the automaton theory. Differently, in this work, an LTL formula is converted into a designed E-LDGBA to improve the performance of mission accomplishment and resolve the drawbacks of DRA and LDGBA. The innovative relaxed product MDP and utility schemes consisting of the accepting reward and violation reward are proposed to accomplish the satisfaction of soft tasks. In order to provide formal guarantees of achieving the goals of multi-objective optimizations, future research will consider more advanced multi-objective learning methods. In addition, problems over continuous state and/or action spaces will be studied by incorporating deep neural networks.

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