Caught Active Walkers

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Abstract

A discrete implementation on a lattice of the Active Walker Model is presented. After the model’s validity is shown in simple simulations, more complex simulations of walkers passing consecutively a lattice from an arbitrary starting point at the left border to a random destination on the right border are presented. It is found that walkers may be caught at a certain position by bouncing back and forth between two contiguous lattice sites. The statistical characteristics of this catchment effect are being studied. The probability distribution of the number of walkers having passed the lattice before the catchment occurred shows an exponential decrease to higher numbers. Furthermore the influence of some parameters of the model on the catchment phenomenon is discussed. Position and height of the maxima of these distributions show a linear dependency on a parameter.
1 INTRODUCTION

Studies of the interaction in many-particle systems has revealed a wide range of fascinating phenomena. Pattern formation and self-organization has been found in chemistry, physics and biology [1, 2, 3, 4]. The same ideas and principles can even be applied in sociology and economics [5, 6]. The analytical approach usually models a system by differential equations. Thus fluids, reaction-diffusion systems and many other systems have been studied. Since several years, the enhancing capacity of computers allows to pursue also the discrete, numerical approach. Attention has also been paid to the simulation of traffic and pedestrian motion [7].

Microsimulations of traffic, pedestrian and also granular flow compute the motion of each distinct object [8, 9]. For example, the motion of a granular bead is determined by its interaction with the environment, i.e. walls, other beads, electrical fields and possibly further more. Striking correspondence between simulation and experiment can be found [10].

2 Active Walker Model

The Active Walker Model is based on the social force concept [11, 12]. According to this idea, environmental influences causing behaviorial changes of an individual are modelled by a social force. Thus the change in velocity $\frac{dv}{dt}$ of a walker is determined by the social force $\vec{F}_s$. $\vec{F}_s$ depends not only on the environment but also on the preferences and aims of the individual. $\vec{F}_s$ is often calculated from a social or environmental potential $V_{env}$. The walker is called “active” because he is able to change the environment locally. For example, walkers may increase the walking comfort by their footprints and thus establish a beaten path [13, 14, 15, 16].

3 The Discrete Model

Let the world be a two-dimensional lattice and let time pass in discrete steps. At each time $t$, a walker has a certain position $\vec{x}(t)$. Furthermore, we assume that this walker is on his way to a distinct destination $\vec{d}$ on our lattice. If reaching his destination were his one and only will, he would take the direct path along $\vec{d} - \vec{x}$. But even in a two-dimensional lattice world life may become more complicated. Let each lattice site $\vec{x} = (x, y)$ has a set

$$\mathcal{N}(\vec{x}) = \begin{cases} 
\vec{n}_1 = (x-1, y), \\
\vec{n}_2 = (x+1, y), \\
\vec{n}_3 = (x, y-1), \\
\vec{n}_4 = (x, y+1) 
\end{cases}$$

(1)

of four neighbouring sites. If $\vec{x}(t)$ is the position of the walker at time $t$, $p(\vec{n})$ is the probability that $\vec{n} \in \mathcal{N}(\vec{x}(t))$ will become the next position $\vec{x}(t+1)$ of the walker. The neighbouring site with the highest value of $p$ becomes the position of the walker at time $t+1$. $p(\vec{n})$ is the mathematical expression of the walker’s aims. If the walker just wants to reach his destination without taking the environment into account, the following expression for $p(\vec{n})$ would be reasonable:

$$p(\vec{n}) = |\vec{x}(t) - \vec{d}| - |\vec{n} - \vec{d}|.$$  

(2)

In general, the influence of the environment can be modelled by a environmental potential $V_{env}(\vec{x}, t)$. Thus, the probability $p(\vec{n})$ becomes

$$p(\vec{n}) = w_{\text{dist}} \cdot (|\vec{x}(t) - \vec{d}| - |\vec{n} - \vec{d}|) + w_{\text{pot}} \cdot V_{env}(\vec{n}, t),$$

(3)
i.e. we calculate the weighted sum of the aim to reach the destination and to maximize the environmental potential. The condition $w_{\text{dist}} + w_{\text{pot}} = 1$ has to be fulfilled.

The environmental potential $V_{\text{env}}(\vec{x}, t)$ determines how comfortable the walker feels at site $\vec{x}$ at time $t$. It thus depends heavily on the individual conditions and aims of the walker. For example, if a walker runs down a street in order to reach a bus station at the end of this street and he is also interested in books, he will possibly try to pass book stores to catch a glimpse. So the environmental potential $V_{\text{env}}(\vec{x}, t)$ near book stores would be higher than near clothes stores (of course!).

In our model, $V_{\text{env}}(\vec{x}, t)$ reflects the walking comfort $C(\vec{x}, t)$ at site $\vec{x}$ at time $t$. If a walker passes a site $\vec{x}$, the walking comfort $C(\vec{x}, t)$ will increase (because hindering vegetation is being damaged). Several walkers running the same way may cause a beaten path on the long term. On the other hand, $C(\vec{x}, t)$ decreases by time as beaten paths vanish if they are not used. Thus, the following differential equation holds for $C(\vec{x}, t)$:

\[
\frac{dC(\vec{x}, t)}{dt} = \frac{1}{\tau} \cdot [C_{\text{min}} - C(\vec{x}, t)] + I(\vec{x}, t) \cdot [C_{\text{max}} - C(\vec{x}, t)].
\] (4)

$C_{\text{min}}$ is the minimum walking comfort and $C_{\text{max}}$ is the maximum one. $I(\vec{x}, t)$ describes the intensity site $\vec{x}$ is frequented by walkers at time $t$. $1/\tau$ quantifies how fast a beaten path weathers.

The environmental potential $V_{\text{env}}(\vec{x}, t)$ is mainly but not exclusively determined by the walking comfort $C(\vec{x}, t)$ at site $\vec{x}$. If we used only $C(\vec{x}, t)$, the walker would not be able to recognize a comfortable site at some distance and move to it. Clearly this is not realistic. So we calculate $V_{\text{env}}(\vec{x}, t)$ as the distance-weighted sum of the walking comforts of all lattice sites:

\[
V_{\text{env}}(\vec{x}, t) = \sum_{\vec{y} \neq \vec{x}} e^{-|\vec{y} - \vec{x}|/s(\vec{x}, t)} \cdot C(\vec{y}, t),
\] (5)

where $s(\vec{x}, t)$ indicates how far one can see at site $\vec{x}$ at time $t$.

## 4 First Results

![Figure 1](image.png)

Figure 1: A walker crosses a beaten path on the way to its destination. Paths resulting from different values for $w_{\text{pot}}$ are shown.

Fig. 1 illustrates how our model works. We use a $(40, 40)$–lattice and let a walker start at $(1,36)$ with destination $(40,6)$. All lattice sites has the same initial walking comfort $C_0$ apart from the sites on the beaten path in the middle between $(1,21)$ and $(40,21)$ shown
as a solid line in Fig. 1. If we use $w_{\text{pot}} = 0.005$ in our discrete active walker model, the walker’s path is hardly influenced by the environment. The walker looks for the fastest way to his destination. As Fig. 1 reveals, the “fastest” way on our two-dimensional lattice is not the same as in a continuous two-dimensional world. Initially, the walker moves horizontally. Afterwards, he follows a diagonal path to his destination. This strange behaviour results from the combination of restricted movements (horizontal and vertical steps only) and the usage of the Euclidean distance in equ. 2. Nevertheless, this effect is not important for the results presented in this paper. A close look on Fig. 1 shows that the walker follows the beaten path just for 3 steps.

If $w_{\text{pot}}$ is increased to 0.05, the walkers quits its “direct” way to the destination in order to follow the beaten path. One can also see the effect of the distance-weighted sum in equ. 3 for $V_{\text{env}}$: The walker is actually attracted by the beaten path. Finally, for $w_{\text{pot}} = 0.85$ the influence of the beaten path increases again.

We define the mean difference $\bar{\Delta}$ between the actual path of a walker and the “direct” path as

$$\bar{\Delta} = \frac{1}{n} \sum_{i=1}^{n} |\vec{x}_i - \vec{x}_{i,\text{dir}}|,$$

where $\vec{x}_i, i = 1, \ldots, n$ are the n actual positions of the walker on its way and $\vec{x}_{i,\text{dir}}, i = 1, \ldots, n$ are the positions on the direct path to the walker’s destination. $\vec{x}_{i,\text{dir}}$ depends on $\vec{x}_i$ by equal x values: $x_{i,\text{dir}} = x_i$. $\Delta$ describes how strong the walker swerved from the direct way to its destination. Fig. 1 shows the reasonable result that $\Delta$ increases if the influence of the environment becomes higher.

![Figure 2: $\bar{\Delta}$ for different values of $w_{\text{pot}}$ found in the above simulation (see fig. 1).](image)

Let us do now a more interesting simulation. Initially, all sites have the same walking comfort. We use a $(50, 50)$-lattice. A walker starts from a random position on the left edge to a random position on the right edge. He will definitely use the shortest way to his destination because there are no beaten paths so far. After the first walker has reached his destination, we continue with a second walker somewhere on the left border of the lattice and again a random destination on the right. This walker might be affected by the beaten path the first one has left on the lattice (see fig. 3). My initial interest was to study the lattice and the walkers after say 1000 iterations:

1. Are there persistent beaten paths? How long do they remain?
2. What quantity may describe the change of the beaten path pattern from one iteration to the next?
3. What is the influence of the parameters $s$, $\tau$ and $w_{\text{pot}}$?
At some point in my studies, I wondered why a walker seems to never reach its destination. I had a closer look and found out that he was bouncing from one site to a contiguous one and vice versa all the time. First, I thought of an error in the implementation of the model. But I could not find any. So I checked the algorithm by calculating \( p(\vec{n}) \) by hand - and astonishingly, my program was correct. The explanation is simple. In direction of his destination the sites offer much less walking comfort than the site he has been before. Although this artefact obviously diminishes the applicability of the model, it is worth a close look.

It was obvious that the number of walkers having reached their destinations successfully before a walker is being caught differed from one simulation run to the next. Thus I was interested in the probability distribution \( p(n) \) of the number of walkers \( n \) reaching their destination. Further on I investigated how this probability distribution \( p(n) \) depended on the parameters of the model, i.e. \( w_{\text{pot}}, \tau \) or even the size of the lattice. The distributions \( p(n) \) shown below are normalized:

\[
\int_0^{n_{\text{max}}} p(n) dn \approx \sum_{n=0}^{n_{\text{max}}} p(n) \cdot \Delta n = 1.
\]  

(7)

\( \Delta n = 1 \) is the bin width of the discrete probability distribution and \( n_{\text{max}} \) is the maximum number of walkers.
Fig. 4 elucidates the dependency of $p(n)$ on $w_{\text{pot}}$. Generally speaking, if the influence of the environment and thus $w_{\text{pot}}$ is increased, the walkers are being caught faster. $p(n)$ shows a distinct maximum. The plots for $w_{\text{pot}} = 0.055$ and 0.5 show a peak at $n = 220$. Because the simulation was stopped after 220 walkers had reached their destinations, the probability $p(n \geq 220)$ is aggregated in $p(220)$. Only for small values of $w_{\text{pot}}$ $p(n \geq 220)$ is neglectable.

The half-logarithmic plot in fig. 4a shows that $p(n)$ decreases exponentially for increasing $n$.

Where are the “prisons” of the walkers located on the lattice? In order to answer this question, I determined the normalized probability distribution $p(\vec{x})$ for the locations $\vec{x}$ of the prisons. $p(\vec{x} = (x|y))$ fulfills the equation

$$\int_{x} \int_{y} p(x, y) \, dx \, dy \approx \sum_{x=0}^{x_{\text{max}}} \sum_{y=0}^{y_{\text{max}}} p(x, y) \cdot \Delta x \Delta y = 1,$$

where $x_{\text{max}} = y_{\text{max}} = 40$ is the size of the lattice and $\Delta x = \Delta y = 1$ the size of the bins.

Fig. 5 shows that $p(\vec{x})$ is nearly symmetrical to $y = 20$. Please note that for $x < 12$ and $x > 32$ no prisons are found at all. The highest values for $p(\vec{x})$ are found for $x = 32$. Astonishingly, walkers are not being caught next to the right border of the lattice.
Figure 6: Dependency of $p(n)$ on $\tau$ (a) and the lattice size (b). The labels in (a) are the values for $\tau$ and in (b) the number of sites in each dimension. $n_{\text{max}} = 220$ in (a) and 180 in (b).

The probability distributions $p(n)$ for different $\tau$ and different lattice sizes are shown in fig. 6 in half-logarithmic plots. They are very similar to the curves discussed above and also show exponential decay for increasing $n$.

Now we have a closer look on the maxima of the plots in fig. 4a. The maxima are found at different positions $n_{\text{max}}$ and have different heights $p_{\text{max}}$. Fig. 7 shows $n_{\text{max}}$ and $p_{\text{max}}$ for different values of $w\text{pot}$. Obviously, $n_{\text{max}}$ decreases for higher $w\text{pot}$ whereas the maxima becomes higher. Especially $p_{\text{max}}$ shows nearly a linear dependency on $w\text{pot}$.

Figure 7: Position $n_{\text{max}}$ (a) and height $p_{\text{max}}$ (b) of the maximum of $p(n)$ for different values of $w\text{pot}$. The linear fits $y = a + m \cdot x$ are done with $a = 114$ and $m = -1403$ in (a) and $a = -0.1199$ and $m = 2.349$ in (b).

6 Outlook

The described “catchment” phenomenon is an artefact of the introduced discrete active walker model. Thus the model has to be improved in order to allow real-life simulations. On the other hand, it would be interesting to compare the described statistical characteristics of the catchment phenomenon with those of real-world catchment effects for example in particle physics.
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