Redshift-dependent lag–luminosity relation in 565 BATSE gamma-ray bursts

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ABSTRACT
We compared redshifts \( z_Y \) measured from the Yonetoku relation and \( z_{\text{lag}} \) from the lag–luminosity relation for 565 BATSE gamma-ray bursts (GRBs) and were surprised to find that the correlation between these two redshifts is very low. Assuming that the luminosity is a function of both \( z_Y \) and the intrinsic spectral lag \( \tau_{\text{lag}} \), we found a new redshift-dependent lag–luminosity relation \( L = 7.5 \times 10^{50} \text{ erg s}^{-1} (1 + z)^{2.53} \tau_{\text{lag}}^{-0.282} \) with the correlation coefficient of 0.77 and the chance probability of \( 7.9 \times 10^{-75} \). Although the spectral lag is computed from two channels of the Burst and Transient Source Experiment (BATSE), our new lag–luminosity relation suggests that a future lag–luminosity relation defined in the Swift data should also depend on the redshift.

Key words: gamma-rays: bursts – gamma-rays: observation.

1 INTRODUCTION
Several luminosity indicators have been proposed (see Fenimore & Ramirez-Ruiz 2000; Norris, Marani & Bonnell 2000; Amati et al. 2002; Yonetoku et al. 2004; Ghirlanda, Ghisellini & Lazzati 2004; Liang & Zhang 2005; Firmani et al. 2006; Schafer 2007) for possible evolution and bias effects. The first luminosity indicator is the variability–luminosity relation suggested by Fenimore & Ramirez-Ruiz (2000). It is based on the fact that the highly variable (spiky) gamma-ray bursts (GRBs), in temporal history, are much brighter than smoother ones. Norris et al. (2000) first reported the spectral time-lag, which is the time difference of arrival photon in separate energy range (25–50 keV and 100–300 keV) of the Burst and Transient Source Experiment (BATSE), and used as a luminosity indicator based on six BATSE GRBs with the optically determined redshifts. Using the BeppoSAX data, Amati et al. (2002) found the correlation between the total isotropic energy and the peak energy \( E_p \) of the prompt emission (the so-called Amati relation). Then Yonetoku et al. (2004) independently proposed a \( E_p \)-luminosity relation (the Yonetoku relation). If we use these luminosity indicators under the standard cosmological model, we can estimate the probable redshifts of GRBs without known redshifts only from the gamma-ray data, even if the spectroscopic observations are not performed by the ground-based telescopes. (Schafer, Deng & Band 2001; Yonetoku et al. 2004; Band et al. 2004).

These luminosity indicators, such as the Yonetoku relation and the lag–luminosity relation, are independent of each other, but we expect to obtain the similar redshifts for the same GRB if both empirical relations are true. The consistency between independent empirical relations is a key of their reliability as empirical luminosity indicators. Therefore we have to investigate deeply the characteristics for the proposed relations. In this Letter, we first examine the correlation of two redshifts derived from the Yonetoku relation (\( z_Y \)) and the lag–luminosity relation (\( z_{\text{lag}} \)) for 565 BATSE GRBs. In Section 2, we show that the correlation between \( z_Y \) and \( z_{\text{lag}} \) is very low. This is a surprising matter, and we must investigate the reason of this discrepancy between redshifts estimated from two independent methods.

In Section 3 we introduce a new redshift-dependent lag–luminosity relation, and show an advantage compared with the original lag–luminosity relation by Norris et al. (2000). In Section 4, we discuss possible physical origins of the new lag–luminosity relation, using the thermal model (Ryde 2004) and the sub-jet model (Ioka & Nakamura 2000). Section 5 will be devoted to discussions for future observations. Throughout the Letter, we assume the flat-isotropic universe with \( \Omega_m = 0.30, \Omega_{\Lambda} = 0.70 \) and \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \), respectively.

2 COMPARISON OF TWO REDSHIFTS
Yonetoku et al. (2004) proposed the \( E_p \)-luminosity relation and used it as a cosmological probing tool. They estimated the redshifts (\( z_Y \)) of 689 GRBs without known redshift observed by the BATSE detectors aboard the Cosmic Gamma-Ray Observatory satellite. Recently, Tanabe et al.(in preparation) revised the relation with more GRBs...
and obtained the functional form as
\[
\frac{L}{10^{52} \text{ erg s}^{-1}} = 7.90 \times 10^{-3} \left[ \frac{F_p(1+z)}{1 \text{ keV}} \right]^{1.82}.
\] (1)

Here the luminosity is defined as \( L \equiv 4\pi d_L^2 F \), with the observed energy flux \( F \) in units of \( \text{erg cm}^{-2} \text{s}^{-1} \), and the luminosity distance described as
\[
d_L = \frac{(1+z)c}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_k}}.
\] (2)

For a large amount of GRBs without known redshift, once we obtained \( F \) and \( E_p \) in the observer’s rest frame, we can estimate their redshifts and luminosities with Yonetoku relation. Although the power-law index is \( \sim 0.2 \) smaller than the original function by Yonetoku et al. (2004), the relation is essentially the same so that we adopt equation (1) in this Letter.

Norris et al. (2000) found the lag–luminosity relation to be
\[
\frac{L}{10^{51} \text{ erg s}^{-1}} = 2.18 \left[ \frac{\tau_{\text{lag}}}{0.35 s(1+z)} \right]^{-1.15}.
\] (3)

for only six GRBs with known redshift at that time. Using this lag–luminosity relation, Band et al. (2004) estimated the peak luminosity and the redshifts (\( z_{\text{lag}} \)) from the time-lags and flux observed with BATSE. Therefore we now have two kinds of data bases of the redshifts \( z_Y \) and \( z_{\text{lag}} \) estimated by Yonetoku relation and the lag–luminosity relation. We can perform direct comparison between these estimated redshifts, and it is very important to verify whether these luminosity indicators are reliable for GRB cosmology or not.

Yonetoku et al. (2004) treated 745 GRBs at first. They found 21 GRBs have \( z > 12 \) and 35 have no solution satisfying the Yonetoku relation, so they analyzed the remaining 689 GRBs. In these 689 GRBs, 23 GRBs have \( E_{\text{iso}}/L < 1 \). In this Letter, we compare two redshifts \( z_Y \) and \( z_{\text{lag}} \) for the remaining 666 GRBs. We use lags published as a data base for 1430 BATSE bursts. We found that 621 GRBs are included in both data. 56 GRBs have negative lags so that equation (3) cannot be used for these GRBs. The number of GRBs is now 565, and hereafter we use these samples. Fig. 1 shows the data distribution on \((z_Y, z_{\text{lag}})\) plane with the solid line being \( z_Y = z_{\text{lag}} \). Surprisingly enough there are many GRBs with (1) large \( z_Y \) and small \( z_{\text{lag}} \) as well as (2) small \( z_Y \) and large \( z_{\text{lag}} \). Although we expect to be \( z_Y = z_{\text{lag}} \) if the both relations are really universal scaling lows, we can recognize that the correlation between \( z_Y \) and \( z_{\text{lag}} \) is very low. At this point, we can image three reasons for large discrepancy between the redshifts estimated from two independent methods; (i) the lag–luminosity relation, (ii) the Yonetoku relation, or (iii) both relations may be responsible for this low correlation.

We consider the first possibility (i). This is because Tanabe et al. (in preparation) argued the evolution effect as well as the observational selection bias in the revised Yonetoku relation. They concluded that these contributions are very small, and the correlation in equation (1) is indeed reliable. Adopting \( z_Y \) into the lag–luminosity relation of equation (3), we show the data distribution on \( \log(\tau_{\text{lag}}) \) versus \( \log(L_{\text{iso}}) \) plane in Fig. 2. The solid line is the original lag–luminosity relation by Norris et al. (2000). The correlation coefficient is 0.38 and the chance probability is \( 1.7 \times 10^{-19} \), which is rather poor considering the large number of samples.

Then we tested what will happen if we adopt \( z_Y \) into the other luminosity indicator, such as the Amati relation. In Fig. 3, we show a distribution on \([E_p(1+z_Y), E_{\text{iso}}]\) plane assuming \( z = z_Y \). The correlation coefficient is 0.92, and the chance probability is \( 5.2 \times 10^{-16} \). Therefore, we can say that the Amati relation is compatible with the Yonetoku relation while the lag–luminosity relation is not so. We here assume that the Yonetoku relation itself is a better indicator of redshift as the Amati relation is derived from the Yonetoku relation if \( E_{\text{iso}} \) is in proportion to \( L \).

**Figure 1.** The distribution of \((z_Y, z_{\text{lag}})\) for 565 BATSE GRBs where \( z_Y \) and \( z_{\text{lag}} \) are redshifts determined by the Yonetoku and the lag–luminosity relations, respectively. The correlation is very low. The solid line is \( z_Y = z_{\text{lag}} \). It is expected that \((z_Y, z_{\text{lag}})\) is distributed around the solid line.

**Figure 2.** \( \tau_{\text{lag}} \) vs \( L_{\text{iso}} \) using \( z_Y \) for 565 BATSE GRBs. The correlation coefficient is 0.38. The chance probability is \( 1.7 \times 10^{-19} \), so that the correlation coefficient is low.

**Figure 3.** Amati relation in 565 BATSE GRBs. Redshifts derived from the Yonetoku relation \( z_Y \) are used to estimate \( E_{\text{iso}} \) and \( E_p(1+z_Y) \). The correlation coefficient is 0.92. The chance probability is \( 5.2 \times 10^{-16} \), so that the correlation is tight. The solid line is the functional form of the Amati relation (Amati et al. 2006).
new lag–luminosity relation

3 NEW LAG–LUMINOSITY RELATION

Fig. 2 shows that there is a large variance around the original functional form. To seek the origin of this large scatter, especially the redshift dependence, we divide the data in Fig. 2 into eight redshift groups as: $0 \leq z < 1; 1 \leq z < 2; 2 \leq z < 3; 3 \leq z < 4; 4 \leq z < 5; 5 \leq z < 6; 6 \leq z < 7; 7 \leq z < 9.3$. In each redshift group we tested the relation $L = a \tau_{\text{lag}}^b$. The best-fitting values of $a$ and $b$ are shown in each figure. The solid lines are the best-fitting power-law models for each redshift group. We see that $b$ is almost the same while $a$ increases as a function of redshift.

As for the correlation coefficients and the chance probability, we found that the new lag–luminosity relation for the same six GRBs on the lag–luminosity plane. The correlation coefficient is 0.94, and the chance probability is 0.021. In Fig. 7, we show the new lag–luminosity relation in the lag–luminosity plane. The correlation coefficient is 0.90, and the chance probability is 0.027. As for the correlation coefficients and the chance probability, we found no significant difference between two correlations for the original six samples. Therefore we conclude that our new lag–luminosity relation is consistent with the original one from Norris et al. (2000).

4 POSSIBLE INTERPRETATION OF THE NEW LAG–LUMINOSITY RELATION

Ryde (2004) studied five GRBs whose spectra are consistent with thermal blackbody radiation throughout their entire duration, and

\[ L_{52} = 10^{-1.12 \pm 0.07} (1 + z)^{2.53 \pm 0.10} \times 0.28 \pm 0.03 \] (4)

The standard deviation of the relation is $\sigma = 0.473$. In Fig. 5 we plot $L_{52}$ as a function of $0.0758 (1 + z)^{2.53} \tau_{\text{lag}}^{-0.282}$. We found that the correlation coefficient is 0.77 and the chance probability is $7.9 \times 10^{-75}$. This result is obviously improved and is much better than the original lag–luminosity relation shown in Fig. 2.

In the new lag–luminosity relation, the power-law index for $\tau_{\text{lag}}$ is about a factor of 4 smaller than that in the original lag–luminosity relation. One may ask the reason for the difference. We consider the same six GRBs as in Norris et al. (2000). Fig. 6 shows the original lag–luminosity relation. The correlation coefficient is 0.94, and the chance probability is 0.021. In Fig. 7, we show the new lag–luminosity relation for the same six GRBs on the luminosity and $-0.0758 (1 + z)^{2.53} \tau_{\text{lag}}^{-0.282}$ plane. We found that the correlation coefficient is 0.90, and the chance probability is 0.027. As for the correlation coefficients and the chance probability, we found no significant difference between two relations for the original six samples. Therefore we conclude that our new lag–luminosity relation is consistent with the original one from Norris et al. (2000).
found the temperature $kT$ can be well described by broken power-law as a function of time. Fig. 11 in Ryde (2004) shows the time-evolution of the temperature as

$$kT_{\text{obs}} \approx 100 \text{ keV} \times t_{\text{obs}}^{-0.25}.$$  

in the relevant early time to the spectral lag. If the peak energy of GRB is determined by the temperature of blackbody spectrum (Thompson, Mészáros & Rees 2007; Rees & Mészáros 2005), we can approximately identify $kT_{\text{obs}}$ as $E_{\text{p, obs}}$, and then the $E_p$ evolves like equation (5). The differential value $\frac{dE_p}{dE_{\text{p, obs}}}$ should be in proportion to the spectral time-lag $\tau_{\text{lag}}$, so combining these functions with the Yonetoku relation of $L \sim E_p^{3}$, we have

$$L_{52} \propto (1+z)^{2.67} \tau_{\text{lag}}^{-0.33}.$$  

This equation behaves in a similar way to power-law indices of both the redshift evolution and the $\tau_{\text{lag}}$ value to our new lag–luminosity relation in equation (4).

Ioka & Nakamura (2001) suggested an idea that the origin of the lag–luminosity relation is caused by the viewing angle to the jet axis. They adopted the following spectral shape

$$f(\nu) = \left( \frac{\nu}{\nu_0} \right)^{1+\alpha_B} \left[ 1 + \left( \frac{\nu}{\nu_0} \right)^{\beta_B} \right]^{\frac{2}{2-B}}$$

which is approximately equivalent to the Band spectrum in the comoving frame. Here $\nu$ is a parameter which controls the smoothness of the transition from the low- to the high-energy power-laws with the indices of $\alpha_B$ and $\beta_B$ in the Band function, respectively. They adopted $l = 2$ in their application to explain the original lag–luminosity relation. For general $l$, we can derive the following lag–luminosity relation

$$L \propto \nu F_{\nu} \propto \nu^l \Delta T_{\text{p}} \propto \nu^{l+1+\alpha_B},$$

where $\Delta T_p$ is the spectral time-lag, and $\nu$ is the intrinsic frequency. When we determine the observed frequency as $\nu_{\text{obs}}$ which is fixed by the BATSE energy channels, using $\nu = (1+z)\nu_{\text{obs}}$, we can rewrite equation (8) as

$$L \propto (1+z)^{l+0.3} \tau_{\text{lag}}^{0.3}$$

for $l = 6$. This is qualitatively consistent with our new lag–luminosity relation in equation (4).

5 DISCUSSIONS

The spectral time-lag must depend on the redshift, because it is calculated with a cross-correlation method between the data observed in different two channels fixed as 25–50 and 100–300 keV of the BATSE instruments. For example, in the rest frame of GRBs at the redshift of $z = 4$, these two channels correspond to 125–250 and 500–1500 keV. Then this lower channel at $z = 4$ is equivalent to the higher channel for the local events ($z \sim 0$). Therefore it is natural to consider that the lag–luminosity relation should depend on redshift such as our new lag–luminosity relation shown in equation (4).

If the lag–luminosity relation does not depend on the redshift, the spectral lag should not depend on the intrinsic photon energy. However, the concept of the spectral lag comes from the fact that the peak arrival time is related to the photon energy. In reality, Norris et al. (2000) showed that the time-lag between channels 4 (>300 keV) and 1 (25–50 keV) is about three times larger than the one between channels 3 (100–300 keV) and 1.

In this Letter, we found the new lag–luminosity relation from 565 GRBs while Norris’s original relation was derived from only six events. The only assumption we used is that the Yonetoku relation is free from serious evolution and selection bias effects. Our new lag–luminosity relation is, of course, compatible with the original lag–luminosity relation by Norris et al. (2000), and moreover has lower chance probability. As shown in Fig. 1, Norris’s lag–luminosity relation is incompatible with the Yonetoku relation because there is no correlation between $z_{E_{\gamma}}$ and $\tau_{\text{lag}}$ estimated from both relations. However, if we include the effect of the redshift evolution shown in equation (4), we can solve this problem. We strongly claim that our new lag–luminosity relation is consistent with Yonetoku relation. We think this is a powerful agreement about reliability for the redshift estimation with the empirical relations of GRBs.

Finally we discuss redshifts determined by the new lag–luminosity relation. Equation (4) can be rewritten as

$$\frac{d_{L,26}^2}{(1+z)^{2.81}} = 0.0758 \left( \tau_{\text{lag}}^{0.3} \frac{t_{\text{obs}}}{4\pi F} \right)^{0.282}$$

for each GRB, where $d_{L,26}$ is the luminosity distance and $F$ is the photon energy flux in units of cm and erg cm$^{-2}$ s$^{-1}$, respectively. The left-hand side of equation (10) as a function of $z$ begins from zero, and has a maximum at $z \sim 4$. After that, the value decreases toward higher redshift. Therefore we may have two solutions for some particular events. The new lag–luminosity relation has a 1σ deviation of 0.47 in a log10 scale. The right-hand side of equation (10) then changes by a factor of 3 so that the accuracy of the redshifts is not so good. It often occurs that there is no solution for $z$, as is also experienced in the Amati relation.

We need a tighter lag–luminosity relation if we are to use it as the redshift estimator. The spectral lag for BATSE GRBs are defined from two channels fixed in BATSE instruments. However, the energy range of the BAT detector does not completely cover the BATSE’s range. Therefore a new definition of spectral lag is needed in the Swift era, so that we may construct the tighter lag–luminosity relation with the large amount of Swift data with known redshifts. Then our result predicts the existence of redshift evolution in a future lag–luminosity relation in the Swift era.

So far Swift has observed ∼200 GRBs, but only ∼50 events have had their redshifts determined by the spectroscopic observation made with ground-based telescopes. If we can promptly estimate redshifts only from gamma-ray data just after the Swift trigger time, we may enable to select high-redshift candidates. Although the small energy range of BAT is a weak point to measure the peak energy $E_p$, we...
the future lag–luminosity relation will be a powerful tool, instead of \( E_p \), for the selection of high-redshift events.

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