Relative quadrupole moments of exotic shapes at ultrahigh spin in \(^{154}\text{Er} \): calibrating the TSD/SD puzzle

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Abstract. Transition quadrupole moments, \(Q_t\), of two ultrahigh-spin, collective structures in \(^{154}\text{Er} \) have been measured for the first time using the Doppler Shift Attenuation Method (DSAM). Data were acquired at the ATLAS accelerator facility of Argonne National Laboratory, using the Gammasphere detector array. A thick, gold-backed \(^{110}\text{Pd} \) foil was bombarded by a beam of \(^{48}\text{Ti} \) ions at 215 MeV. The \(Q_t\) for each band was determined from the Doppler shift of gamma rays emitted by the resulting recoil nuclei. The extracted transition quadrupole moments are significantly different in magnitude, suggesting the two structures in \(^{154}\text{Er} \) represent distinct exotic nuclear shapes, namely axial superdeformed (SD) with \(Q_t \approx 20\) eb, and triaxial strongly deformed (TSD) with \(Q_t \approx 11\) eb. Indeed, the results calibrate the quadrupole moments of TSD bands recently measured in light erbium nuclei, \(^{157,158}\text{Er} \).

1. Introduction

Triaxial nuclei, i.e. nuclei with no axis of symmetry, are not as well understood as their axially symmetric counterparts. This is in part because static triaxial shapes are not usually the most energetically favoured, and are therefore difficult to observe experimentally. However, in certain nuclei, the occupation of shape-driving orbitals can act to stabilise the triaxial nucleus [1, 2]. The first experimental evidence for static triaxial shapes was seen in \(^{163}\text{Lu} \) [3], which was observed to have a TSD band resulting from a ‘wobbling mode’, a unique indicator of triaxiality. Many more triaxial bands have since been discovered. The quadrupole moments of four TSD bands - two in \(^{157}\text{Er} \) and two in \(^{158}\text{Er} \) - have recently been measured, yielding values of ~11 eb [4]. These
values are significantly greater than those theoretically predicted by cranked Nilsson-Strutinsky (CNS) calculations [4, 5]. However, the quadrupole moments were determined using the Doppler shift attenuation method (DSAM) which relies in part on theoretical stopping powers, hence the need for some form of calibration. The nucleus \(^{154}\text{Er}\) was known to have two strongly deformed bands [6, 7]. One has been proposed to be a TSD band from its dynamic moment of inertia, while the other has been proposed to be a superdeformed (SD) band [7]. The SD band corresponds to an axially symmetric shape, which is theoretically well-understood, hence its quadrupole moment can be predicted accurately. The measurement of the quadrupole moment of the SD band in \(^{154}\text{Er}\) using DSAM therefore acts as a calibration for the other band in \(^{154}\text{Er}\) and the \(^{157,158}\text{Er}\) results.

2. Experiment
The experiment was performed at the ATLAS facility of Argonne National Laboratory. A self-supporting target foil was bombarded by a 215 MeV beam of \(^{48}\text{Ti}\) ions. The target foil consisted of a 1 mg/cm\(^2\) \(^{110}\text{Pd}\) layer backed by a 10 mg/cm\(^2\) \(^{197}\text{Au}\) layer. A 0.07 mg/cm\(^2\) \(^{27}\text{Al}\) layer between the target and backing layers was used to prevent the migration of palladium atoms into the backing. The nucleus \(^{154}\text{Er}\) was produced at high spin via the \(4n\) reaction channel. Gamma rays emitted by the recoil nuclei as they slowed down in the gold backing were detected with the Gammasphere array [8, 9]. A total of \(3.9 \times 10^9\) events comprising four gamma-ray coincidences (fold four) or higher were collected over 7 days of beam time. A further \(\sim 8 \times 10^8\) events of fold three or higher were recorded using two stacked self-supporting 500 \(\mu\text{g/cm}^2\) \(^{110}\text{Pd}\) foils.

Measurement of the lifetimes of the strongly deformed states was carried out using DSAM [10]. In order to carry out the DSAM analysis, data were sorted into spectra by placing gates on the transitions within the strongly deformed bands. Sorting was performed using the MTsort package [11]. Typical angular spectra for the TSD band (band 1) and the SD band (band 2) are shown in Fig. 1.

3. Results
In the centroid shift analysis approach to DSAM employed here, the data are sorted such that there is a spectrum for each ring grouping of detectors. The centroid for each transition is then shifted according to:

\[
E_\gamma = E_\gamma^0 [1 + F(\tau)\beta_0 \cos(\theta)],
\]

where \(E_\gamma^0\) is the unshifted gamma-ray energy, \(F(\tau)\) is the recoil velocity as a fraction of the initial velocity \(\beta_0\), and \(\theta\) is the detector angle. A linear fit of \(\cos(\theta)\) versus centroid energy yielded an \(F(\tau)\) value for each transition within the band, shown in Fig. 2.

The SRIM Monte Carlo program [12] was used to simulate 10000 \(^{154}\text{Er}\) nuclei slowing down within the target. The package MLIFETIME [13] was then used in combination with the simulated ion data to determine \(Q_\beta\) using a \(\chi^2\) minimisation procedure. With each iteration, a value of \(Q_\beta\) was tested and the resulting lifetimes were used together with the ion slowing-down histories to calculate a theoretical \(F(\tau)\) value for each state in the band. A \(\chi^2\) value was then calculated from the experimental and theoretical \(F(\tau)\) values. Fitted \(F(\tau)\) curves for bands 1 and 2 are shown in Fig. 2. Band 1 has a \(Q_\beta\) of 10.9 ± 0.4 eb, consistent with the TSD bands in \(^{157,158}\text{Er}\). Band 2 has a \(Q_\beta\) of 19.1\(^{+1.7}_{-1.2}\) eb, consistent with an SD band.

Two new bands have been discovered in \(^{154}\text{Er}\) resulting from searches of four-dimensional histograms (hypercubes) built from the data using the RADWARE software package [14]. Statistics were too low to measure the lifetimes of these bands. However, some of their properties may be inferred from their dynamic moments of inertia \(\mathcal{J}^{(2)}\), plotted in Fig. 3, together with comparable bands in \(^{152}\text{Dy}\).
Figure 1. Triple-gated, background-subtracted coincidence spectra for (a) band 1 and (b) band 2 of $^{154}$Er. In-band transitions are labelled with their unshifted energies in keV.

Figure 2. Measured fraction Doppler shift values, $F(\tau)$, shown as a function of $\gamma$-ray energy, together with best fit curves. The two horizontal dashed lines show the range of initial recoil velocities within the $^{110}$Pd target layer.

Bands 1 and 3 have very similar dynamic moments of inertia, suggesting that band 3 is also a TSD band. Indeed, the $J^{(2)}$ of both bands follows very closely that of band 3 in $^{152}$Dy, which has also been proposed to be TSD [15]. Band 4 has a much lower dynamic moment of inertia, of similar magnitude to the so-called triaxially deformed (TD) bands in $^{152}$Dy and $^{153}$Ho [15, 16], suggesting that it too may be a TD band. It is also apparent from Fig. 3 that the $J^{(2)}$ of Band 2 is close to that of the SD band in $^{152}$Dy (band 1), reinforcing the assignment of an SD shape for this band.

4. Conclusion
The transition quadrupole moment of the SD band in $^{154}$Er has been measured, providing a calibration for quadrupole moment measurements of TSD bands in $^{157,158}$Er. The value of $\sim 20$ eb is in agreement with the theoretically predicted value, supporting the $Q_t$ measurements for
157, 158 Er obtained using a similar experimental setup. As in 157, 158 Er, the $Q_t$ of the TSD band in 154 Er is $\sim 11$ eb, which is greater than predicted for the theoretical TSD energy minimum.

Two additional strongly deformed bands have been discovered in 154 Er. From an analysis of their dynamic moments of inertia, Band 3 is proposed to be a TSD band, while it is suggested that Band 4 is a TD band similar to those in 152 Dy and 153 Ho.

These findings reinforce indications from the study of 157, 158 Er [4, 17] that the theoretical description of triaxial bands is incomplete, and call for further work on theoretical models of the triaxial nucleus. In addition, such work would be greatly facilitated by further experimental studies of similar bands in the neutron-deficient rare earth region.

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