Mathematical Problem Proving and Problem Posing

T Y E Siswono¹, S Hartono¹, and A W Kohar¹

¹Faculty of Mathematics and Natural Science, Universitas Negeri Surabaya, Indonesia

*e-mail: tatag siswono@unesa.ac.id

Abstract. This article describes students’ tasks that concerned to assess students’ problem proving and problem posing on triangle tasks. This descriptive-exploratory research involves 24 prospective mathematics teachers at a state university in Surabaya, Indonesia. They had to answer three questions. The first question related to problem proving of triangle theorem. The second question related to their perception of the first question, and the last question related to the problem-posing task. A Newman analysis is used to analyse the data. The analysis of the data focused on students’ problem proving and students’ problem posing of triangle task. The results showed that most of them were correct to answer the question as many as 41%. It can be seen of the first question was performed by the highest number of them. However, some of them still have a perception of the first question is a problem, because it shows as many as 95% of 24 students choose it. They also have some weaknesses in proving problems, mainly misinformation or ambiguity about the problem, it is not a category as a problem proving. They constructed proof by formal proof with a low level of problem difficulty as many as only 38% of them.

1. Introduction

Today learning mathematics should have oriented to a problem [1][2]. Naturally, a mathematical problem is not just finding a solution but try to convict that proposition is true. There is a problem to finding and a problem to proving. Proving has become a highlight topic in mathematics education. Proving as one of the five standard competencies in mathematics mentioned by NCTM (National Council of Teachers of Mathematics) [3] not only develops individuals’ conception of the aspects of mathematics, but also it helps to show the trueness or falseness of statements in their lives. However, it is relatively difficult among students and even college’s students. Many students do not know how to prove a mathematical statement. There is a critical question, whether students lack knowledge of proof construction or the proving problem that is given above the ability of students? To answer this question, one way that can be done is giving a freedom to students to pose a problem related to proof. As consequent, students are faced two tasks, namely problem proving and problem posing.

2. Literature Review

2.1. Problem proving

In mathematics, problem proving is a series of logical arguments that explain the truth of a problem. This is stated by Hanna [4] that problem proving is the application of an infinite number of logical steps of what is known (axioms, principles or results that have been proven before) and applying the principles of logic, to create deductive arguments valid for reaching a conclusion of a problem using acceptable inference rules.
Therefore, an explanation of problem proving is a key component in improving mathematics learning as a whole. There are several reasons why teaching problem proving should be given, namely: 1) evidence is an integral part of mathematics, 2) for verification and discovery of facts, 3) for the development of students’ logical and critical thinking skills, and 4) accelerating and improving students’ mathematical understanding [5]. In addition, the need for problem proving of mathematics was introduced in several schools recommended by NCTM [3] that problem proving is part of the mathematics curriculum at all levels. The "Reasoning and Proof" section of this NCTM document states that students should be able to: recognize reasoning and proof as aspects of fundamental aspects of mathematics; make a conjecture and check the truth of that conjecture; develop and evaluate mathematical arguments and proofs; choose and use various types of reasoning and methods of proof in solving a problem.

As a recommendation from NCTM [3] it indicates that the problem of proving in mathematics is one aspect that must be considered in learning mathematics in schools. Some students experience in preparing a proof in high school will have an impact on the ability to prove when they attend college at the first level, as stated by Moore [6]; Weber[7]; Sandie[8] that one of the reasons why students encounter difficulties in proof is experience they in constructing evidence is limited to proof of school geometry. Accordingly, based on the results of a study conducted by Sabri [9] on the concept of problem proving in mathematics that prospective teacher students are advised that the high school curriculum should prepare students better in learning problem-based mathematical proof.

2.2. Problem Posing
In learning mathematics, problem posing can occupy strategic positions [10]. Problem posing is said to be the most important core in the discipline of mathematics and in the nature of mathematical reasoning thinking [11]. Graeimeijer [12] mentions the problem posing is a form of creative activity that can operate within tasks involving structured "rich contexts" and uses real-life documents and human interactions [13][14]. Furthermore, Stoyanova and Ellerton [15] said that mathematical problems posing as the process by which, based on mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems.

In other words, problem posing is a process that is based on mathematical experience then students construct their own interpretations of concrete situations and formulate them into meaningful mathematical problems. Silver [11] says the problem posing refers to both the generation of new problems and the re-formulation of given problems. Problem posing refers to the formation of new problems and to reformulate given problems. Spillane [16] also defines problem posing as a task designed by the teacher that requires students to make one or more questions. Furthermore, problem posing is seen as a task that asks students to submit or make mathematical problems or problems based on the information provided, as well as solving problems or problems created.

Ulusoy [17] gives the term problem posing applied to three different forms of mathematical cognitive activity, which are as follows: presolution posing is a student makes questions about the situation (information) that is held, within-solution posing is a student reformulates a questions such as those that are being solved, and post solution posing is a student modifies the goals or conditions of the questions that have been solved to create new questions. In addition, Stoyanova and Ellerton [15] also provide a classification of situations in problem posing, namely as follows free problem posing where students are asked to make questions from a given situation or information, situations created, or natural situations, then semi-structured problem posing namely students are given an open situation and invited to explore the structure of the situation and complete it using knowledge, skills, concepts, and relationships from mathematical experiences that have been held, and structured problem posing namely problem posing activities based on a problem special.

3. Method
This is descriptive-exploratory research that aims to describe students’ task that concerned to assess students’ problem proving and problem posing on triangle task.
3.1. Participants
The participants of our study were 24 prospective teachers who studied at Department of Mathematics of Universitas Negeri Surabaya in Surabaya city, Indonesia. The research participants were 24 mathematics prospective teacher’s students that consist of 10 males and 14 females at Universitas Negeri Surabaya.

3.2. Data collection and Analysis
The data were collected using a simple task of mathematical problem "(1) Prove that sum of measures of triangles are 1800, (2) What do you think it is the mathematical problem? Explain it, and (3) Create other problems that challenges in triangles proving". All students had to answer three questions. The first question related to problem proving of triangle theorem. The second question related to their perception of the first question, and the last question related to the problem posing task. A Newman analysis is used to analyse the data. Analysis focused on students’ problem proving and students’ problem posing of triangle task. Each prospective teacher has been given the task for 25 minutes. Then the results of data from participants’ answers about the correct answer were analysed by classification problem proving and posing framework. The classification problem proving and posing framework, mainly:

| Question 1 |
| --- |
| No | Criteria |
| 1 | Comprehension error |
| 2 | Transformation/devising strategies error |
| 3 | Mathematical Processing error |
| 4 | Interpretation error |
| 5 | Full understanding |

| Question 2 |
| 1 | Students perception that the question is not a problem |
| 2 | Students perception that the question as problem |

| Question 3 |
| --- |
| 1 | A posed problem is not category as problem proving |
| 2 | Misinformation/ambiguity (unsolved problem) |
| 3 | A problem is not related to the initial problem |
| 4 | Levels of problem difficulty: |
| a. | Low |
| b. | Medium |
| c. | High |

In these cases, the coding was carried out to all the prospective teachers’ answers. Because there was more than one possibility of coding given to each answer with different categories, we selected the most significant feature of the response category that emerged from the answer. Thus, each answer only has one code of category.

4. Results and Discussion
In this chapter, the data obtained from the participants were analysed, discussed, and then the results are presented in Table 1.
Table 1. The number of prospective teachers’ answers toward task problem

| Question 1 | Criteria                              | No | Quantity | Total | Percentage |
|------------|---------------------------------------|----|----------|-------|------------|
|            | Comprehension error                   | 1  | 0        | 0     | 0%         |
|            | Transformation/devising strategies    | 2  | 3        | 6     | 13%        |
|            | Mathematical processing error         | 3  | 6        | 6     | 25%        |
|            | Interpretation error                  | 4  | 5        | 5     | 21%        |
|            | Full understanding                    | 5  | 10       | 10    | 41%        |

| Question 2 | Students perception that the question is not a problem | 1  | 1        | 1     | 5%         |
|            | Students perception that the question as problem       | 2  | 23       | 23    | 95%        |

| Question 3 | A posed problem is not category as problem proving     | 1  | 4        | 4     | 16%        |
|            | Misinformation/ambiguity (unsolved problem)            | 2  | 3        | 3     | 13%        |
|            | A problem is not related to the initial problem        | 3  | 2        | 2     | 8%         |
|            | Levels of problem difficulty:                         |    |          |       |            |
|            | a. Low                                                | 4  | 9        | 9     | 38%        |
|            | b. Medium                                             | 5  | 6        | 6     | 25%        |
|            | c. High                                               | 6  | 0        | 0     | 0%         |

Table 1 shows that at question 1 shows that full understanding is the most found in the prospective teachers’ answer, mainly 41% and 0% for comprehension error. Then, at question 2 shows that student’s perception that the question is not a problem obtained as many as 5% of 24 students and 95% have a perception is a problem. However, if we look at question 3, there will be shows that level low of problem difficulty is very high than other, mainly 38% of 24 students.

Furthermore, most students able to prove the triangle theorem completely. They constructed proof by formal proof, although they obtained as many as only 10 students. However, the students’ proof construction has weakness. In addition, mostly student assumed that the question is classified as a problem. They supposed that the proving task as problem. In fact, this question is not category as problems because the question is commonly used in introducing triangle problem task. Students pose some problems are classified as low level of problem difficulty. Most students proposed questions of similarity problem on a trapezoid. Apart of problems could not be understood and could not be solved.

5. Conclusion
We conclude that most prospective teachers were correct to answer the question as many as 41%. It can be seen at question 1 was performed by the highest number of prospective teachers. However, some of the prospective teachers still have a perception question 1 is a problem, because it shows as many as 95% of 24 students choose it. Furthermore, the students have some weaknesses in proving problems, mainly misinformation or ambiguity about problem, it is not category as problem proving. They constructed proof by formal proof with low level of problem difficulty as many as only 38% of 24 students. This suggestion of this study is that teachers should emphasize learning mathematics through problem proving and problem posing task.

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