On Fuzzy $(p, \alpha, p\alpha)$-compact subspaces

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Abstract: In this paper, we study the relationships among fuzzy (compact, pre-compact, $\alpha$-compact and $p\alpha$-compact) subspaces; also we will discuss the relationships between fuzzy (compact, $p$-compact, $\alpha$-compact and $p\alpha$-compact) spaces and fuzzy (compact, precompact, $\alpha$-compact and $p\alpha$-compact) subspaces.

1. Introduction:

The fuzzy issue has entered nearly all mathematics branches, fuzzy set (FS) has membership degree elements and such sets issue was widespread 1st via Professor Lotfi A. Zada [1] in 1965. In 1968, Chang [2] announced the fuzzy topological Description of spaces and stretched in a straightforward fashion few crisp topological spaces issues to spaces as fuzzy topologically. The fuzzy topology (FT) was initiating in 1968 along article of Chang [2], as well might be regarded as a new mathematics branch, so several extra structures were considered via utilizing (FS)s and the correlated problems in applied and pure mathematics. In 1974, Wong debated and generalized few spaces properties of fuzzy topologically. In 1980, Ming and Ming utilized (FT) to describe the neighborhood fuzzy point (FP) structure. In 1991, Sharna described the $\alpha$-open and pre-open issue in fuzzy topological space (FTC) [3]. Rubasri and Palanisamy are reviewing the (FSs) of $p\alpha$-open (2017) [4].

1.1. Description [5, P.211-220], [6, P. 137-150]

A (FP) $x_\alpha$ in $X$ is (FS) described as following:

$$x_\alpha(y) = \begin{cases} 
\alpha & \text{if } y = x \\
0 & \text{if } y \neq x 
\end{cases}$$

Since $0 < \alpha \leq 1$: $\alpha$ is termed its value and $x$ is backing of $x_\alpha$.

All (FPs) set in $X$ will be signified via $FP(X)$.

1.2. Desscription [5, P.211-220], [7, P. 295-316]

A (FP) $x_\alpha$ in $X$ is assumed as belonging to a (FS) $A$ (signified via: $x_\alpha \in A$) when and only when $\alpha \leq A(x)$.

1.3. Desscription [5, P.211-220], [7, P. 295-316]
A (FS) A in X is termed quasi–coincident of a (FS) B in X signified via $AqB$ if and only if $A(x) + B(x) \geq 1$, for few $x \in X$. When A is not with quasi–coincident, so $A(x) + B(x) \leq 1$, for each $x \in X$ and signified via $\bar{q}B$.

1.4. Lemma [6, P. 137-150]
Suppose A and B are (FSs) in X. So:
(a) When $A \cap B = 0$, then $AqB$
(b) $AqB$ if and only if $A \leq B^c$

1.5. Proposition [6, P. 137-150]
When A a (FS) in X, so $x_a \in A$ when and only when $x_a \bar{q} A^c$.

1.6. Description [2, P.182-190]
A (FT) on a set X is a (FSs) collection $T$ in X satisfactory:
(1) $0 \in T$ and $1 \in T$,
(2) If $A$ and $B$ belong to $T$ so $A \cap B \in T$,
(3) If $A_i$ belong to $T$ for each $i \in I$, so does $\bigvee_{i \in I} A_i$

When $T$ is a (FT) on X, so is the pair $(X,T)$ is termed a (FTC). T Members are termed as open sets being fuzzy. (FSs) forms $1 - A = A^c$, since A is as an open sets being fuzzy are termed as closed sets being fuzzy.

1.7. Description [3, P.303-308]
Suppose $(X,T)$ is a (FTC). So:

i) The A fuzzy interior, signified via $A^\circ$ is the union of all open sets being fuzzy in X that are enclosed in A. ($A^\circ = \bigvee\{B : B \subseteq A, B \text{ is as an open sets being fuzzy in X}\}$

ii) The A fuzzy closure, signified via $\bar{A}$ is the intersection of all closed sets being fuzzy in X enclosed in A. ($\bar{A} = \bigwedge\{B : A \subseteq B, B^c \text{is fuzzy open set in X}\}$

1.8. Description [5, P.211-220]
A (FS) A in $ft(X)$ is termed quasi-neighborhood of (FP) $x_a$ in X when and only when there presents $B \in T$ so $x_a \bar{q} B \leq A$.

1.9. Description [5, P.211-220]

Suppose $(X,T)$ be a (FTC) and $x_a$ be a (FP) in X. So the family $N^q_{x_a}$ containing of all quasi-neighborhood (q-nbd) of a (FP) $x_a$ is assumed to be the quasi-neighborhood system of $x_a$.

1.10. Theorem [4, P.2395-4396], [11]

Suppose $(X,T)$ be a (FTC) and A,B are 2 (FS)s in X. So:

i) $0 = \bar{0}$,

ii) $\bar{A} \cup B = \bar{A} \cup B$ and $\bar{A} \cap B \leq \bar{A} \cap B$.

iii) $(A \cap B)^\circ = A^\circ \cap B^\circ$, $i(A \cup B)^\circ \geq A^\circ \cup B^\circ$.

iv) $\bar{A} = \bar{A}$, $A^\circ = A^\circ$.

v) $A^\circ \leq A \leq \bar{A}$.

vi) If $A \leq B$ then $A^\circ \leq B^\circ$ and $\bar{A} \leq \bar{B}$.

1.11. Description [4, P.2395-4396]

In a (FTC) $(X, T)$

(i) A (FS) A is assumed to be a fuzzy p-open set when $A \leq \bar{A}$.

(ii) A (FS) A is assumed to be a fuzzy $\alpha$-open set when $A \leq \bar{A}^\circ$. 
(iii) A (FS) is assumed to be a fuzzy $\alpha$-open set when $A \leq A^\alpha$.

The above fuzzy complement open sets respectively are defined as:

(i) A fuzzy p-closed.

(ii) A fuzzy $\alpha$-closed.

(iii) A fuzzy $p\alpha$-closed

1.12. Remarks:

(i) All fuzzy family as sets of p-open in $X$ is signified via $FPO(X)$.

(ii) All fuzzy family as sets of $\alpha$-open in $X$ is signified via $F\alpha O(X)$.

(iii) All fuzzy family as sets of $p\alpha$-open in $X$ is signified via $T_{p\alpha}$.

1.13. Theorem [3, P. 303-308]

All fuzzy family of sets of $p\alpha$-open $(T_{p\alpha})$ in a (FTC) $(X, T)$ form a (FT) on $X$.

1.14. Theorem [4, P.2395-4396]

Suppose $(X, T)$ is a (FTC), a subset being fuzzy $A$ of $X$ is assumed to be fuzzy $\alpha$-open set when a fuzzy open (FO) set (FOS) $U$ is there so $U \subseteq A \subseteq \overline{U}$.

1.15. Remarks:

Suppose $A$ is set being (FO) in a (FTC) $(X, T)$, so:

(1) Each set as (FO) be a fuzzy $\alpha$-open.
(2) Each set as (FO) be a fuzzy p-open.
(3) Each set as fuzzy $\alpha$-open be a fuzzy p-open.

Evidence: Suppose $X$ be a (FTC) and $A$ is a (FS) in $X$.

(1) If $A$ be a (FOS) in a fits $X$, so $A \subseteq \overline{A}$.

Since $A' \subseteq A$, thus $A' \subseteq \overline{A}$ put $U = A' \Rightarrow U \subseteq A \subseteq \overline{U}$. Hence, via (1.14. Theorem) $A$ is a fuzzy $\alpha$-open in $X$.

(2) If $A$ is set as (FO) in a fits $X$, $A \subseteq \overline{A} \Rightarrow A' \subseteq \overline{A}'$, but $A' = A \Rightarrow A \subseteq \overline{A}'$. Hence $A$ is a fuzzy p-open.

(3) It is straightforward.

1.16. Examples

The following are examples related to the remarks above:

Suppose $X = \{a, b\}$, $T = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$.

So $(X, T)$ is a (FTC).

C$(X) = \{\emptyset, \{a\}, \{a, b\}\}$ is the fuzzy closed family sets in $X$.

$A_2 = \{\emptyset, \{a\}\}$, $A_2$ is a fuzzy p-open set in $X$ so $x \in [0.9, 1] U [0, 0.5]$.

$A_2 = \{\emptyset, \{a\}\}$

* If we take the set $A_{0.2} = \{a, b\}$ which is a fuzzy $\alpha$-open set.

Where $A_{0.2}^\alpha = \{a_{0.2}, b_0\}$

$A_{0.2}^\alpha = \{a_{0.2}, b_0\}$

$A_{0.2}^\alpha = \{a_{0.2}, b_0\}$

But $A_{0.2}^\alpha = \{a_{0.2}, b_0\}$

This is mean $A_{0.2}$ is not (FOS).

* If we take the set $A_{0.99} = \{a_{0.99}, b_0\}$ which is a fuzzy p-open set.

Where $A_{0.99}^\alpha = 1$

But $A_{0.99}^\alpha = 1$

This is mean $A_{0.99}$ is not fuzzy open set.

* If we take the set $A_{0.99} = \{a_{0.99}, b_0\}$ which is a fuzzy p-open set.

Where $A_{0.99}^\alpha = 1$

But $A_{0.99}^\alpha = 1$

This is mean $A_{0.99}$ is not fuzzy open set.

* If we take the set $A_{0.99} = \{a_{0.99}, b_0\}$ which is a fuzzy p-open set.

Where $A_{0.99}^\alpha = 1$
Suppose \( A_{0.99}^a = [a_0.5, b_0] \)
\[ A_{0.99}^p = [a_0.5, b_1] \]
\[ A_{0.99}^{\alpha} = [a_0.5, b_0] \]
\[ A_{0.99}^s \not\subseteq A_{0.99}^{\alpha} \]
It’s mean \( A_{0.99} \) is not fuzzy \( \alpha \)-open set.

1.17. Description [8, P.131-139]
A (FS) \( A \) in \( \text{fts}(X) \) is assumed to be fuzzy \( p \)-(quasi-neighborhood) \([p-(q-nbd)] \) of \( x_a \in FP(X) \), when set of fuzzy \( p \)-open \( B \) is there in \( X \), so \( x_a \in B \leq A \). All (FP) \( p \)-(quasi-neighborhood) \( ox_{\alpha} \) family is assumed to be \( x_a p - \) (quasi - neighborhood) system and signified via \( N_{x_a}^{pQ} \).

1.18. Description
A (FS) \( A \) in \( \text{fts}(X) \) is assumed to be fuzzy \( p \alpha \)-(quasi-neighborhood) \([p\alpha-(q-nbd)] \) of \( x_a \in FP(X) \), when set as fuzzy \( p \alpha \)-open \( B \) in \( X \) is there , so \( x_a \in B \leq A \). All \( p\alpha \)-(quasi-neighborhood) family of (FP) \( x_a \) is assumed to be \( x_a p \alpha - \) (quasi-neighborhood) system and signified via \( N_{x_a}^{p\alpha Q} \).

1.19. Description
A (FS) \( A \) in \( \text{fts}(X) \) is assumed to be fuzzy \( \alpha \)-(quasi-neighborhood) \([\alpha-(q-nbd)] \) of \( x_a \in FP(X) \), when set as fuzzy \( \alpha \)-open \( B \) in \( X \) is there , so \( x_a \in B \leq A \). All \( \alpha \)-(quasi-neighborhood) family of (FP) \( x_a \) is assumed to be \( x_a \alpha - \) (quasi-neighborhood) system and signified via \( N_{x_a}^{\alpha Q} \).

1.20. Description
Suppose \( A \) be a (FS) in a \( \text{fts}(X) \). So:

i. A fuzzy \( p\alpha \) - interior, signified via \( A^{p\alpha} \) is all \( X \) \( p\alpha \)-open subsets union that is enclosed in \( A \).

ii. A fuzzy \( p\alpha \) - closure , signified via \( \overline{A}^{p\alpha} \) is all fuzzy \( X \) \( p\alpha \)-closed subset intersection comprises \( A \).

iii. A fuzzy \( \alpha \) - interior , signified via \( A^{\alpha} \) is all \( X \) \( \alpha \)-open subsets union that are enclosed in \( A \).

iv. A fuzzy \( \alpha \) - closure , signified via \( \overline{A}^{\alpha} \) is all fuzzy \( X \) \( \alpha \)-closed subset intersection comprises \( A \).

v. A fuzzy \( p \) - interior , signified via \( A^{p} \) is all \( X \) \( p \)-open subsets union that are enclosed in \( A \).

vi. A fuzzy \( p \) - closure , signified via \( \overline{A}^{p} \) is all \( X \) fuzzy \( p \)-closed subset intersection comprises \( A \).

1.21. Comment
Suppose \( A, B \) are 2 (FSs) in a \( \text{fts}(X) \), so:

\( A^s = 1 - (1 - A) \),
\( A^{p\alpha} = 1 - (1 - A)^{p\alpha} \),
\( A^{\alpha} = 1 - (1 - A)^{\alpha} \),
\( A^p = 1 - (1 - A)^p \).

Evidence: \( b \cdot A^{p\alpha} = \bigvee \{B: B \leq A, B \in T^{p\alpha}\} \)
\[ = 1 - \bigwedge (1 - B; (1 - A) \leq (1 - B), B \in T^{p\alpha}) \]
\[ = 1 - (1 - A)^{p\alpha} \]
a,c and d are clear.

1.22. Proposition [4, P.2395-4396]
Suppose \( (X,T) \) is a (FTC) and \( A, B \leq X \). So:

1) \( A^s \leq A^{p\alpha} \leq A^{\alpha} \leq A^p \leq A \);
2) \( A \leq A^{p\alpha} \leq A^{\alpha} \leq A^p \leq \overline{A} \);

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3) $A$ is a fuzzy closed iff $\bar{A} = A$;
4) $A$ is a fuzzy $p\alpha$-closed iff $\bar{A}^{p\alpha} = A$;
5) $A$ is a fuzzy $\alpha$-closed iff $\bar{A^\alpha} = A$;
6) $A$ is a fuzzy $p$-closed iff $\bar{A}^{p} = A$;
7) $\bar{A} = \bar{A^{p\alpha}} = \bar{A^\alpha} = \bar{A}^{p} = \bar{A}^{p\alpha}$;
8) If $A \leq B$, so $\bar{A} \leq \bar{B}$, $\bar{A^{p\alpha}} \leq \bar{B^{p\alpha}}$, $\bar{A^\alpha} \leq \bar{B^\alpha}$, $\bar{A}^{p} \leq \bar{B}^{p}$;
9) $\bigvee_{j \in J} \bar{U_j}^{\alpha} \leq \bigvee_{j \in J} \bar{U_j}^{\beta}$;
10) $x_\alpha \in \bar{A}^{p}$ iff $U \wedge A \neq 0$, $\forall U \in FP0(X)$, $x_\alpha \in U$.

1.23. Remarks
Suppose $A$ be (FOS) in a (FTC) $(X, T)$, so:

i) The fuzzy set being open in a (FTC) is a fuzzy $p\alpha$-open.
ii) The fuzzy set being $p\alpha$-open in a (FTC) is a fuzzy $\alpha$-open.
iii) The fuzzy set being $p\alpha$-open in a (FTC) is a fuzzy $p$-open.

Evidence: i) Suppose $A$ is set being (FO) in a (FTC) $(X, T)$, so $A = A'$.

Since $A \leq \bar{A}^{p} = \bar{A}^{p\alpha}$, thus $A \leq \bar{A}^{p\alpha} = \bar{A}^{p\alpha}$. Therefore $A$ is a fuzzy $p\alpha$-open in $X$.

ii) Suppose $A$ is set being fuzzy $p\alpha$-open in a (FTC) $(X, T)$, so $A \leq \bar{A}^{p\alpha}$.

Via (1.22. Proposition (2)) $\bar{A}^{p\alpha} \leq \bar{A}^{p}$, thus $A \leq \bar{A}^{p\alpha} \leq \bar{A}^{p\alpha}$. Therefore $A$ is a fuzzy $\alpha$-open.

iii) It is straightforward.

1.24. Example
Suppose $X = \{a, b\}$, $T = \{0, 1, \{a_{0.1}, b_0\}, \{a_{0.3}, b_0\}, \{a_{0.5}, b_0\}\}$
So (X, T) is a (FTC).

$C(X) = \{0, 1, \{a_{0.9}, b_1\}, \{a_{0.7}, b_1\}, \{a_{0.5}, b_1\}\}$ is the family of fuzzy closed sets in $X$.

$A_0 = \{0, 1, \{a_{0.1}, b_0\}\}$, $A_0$ is set being fuzzy $p$-open in $X$ so $\lambda \in [0.9, 1] \cup [0, 0.5]$.

$A_1 = \{0, 1, \{a_{1.3}, b_1\}\}$

• If we take the set $A_{0.2} = \{a_{0.2}, b_0\}$ which is set as fuzzy $\alpha$-open.

Where $A_{0.2} = \{a_{0.1}, b_0\}$

\[
\begin{align*}
\bar{A}_{0.2} &= \{a_{0.5}, b_1\} \\
\bar{A}_{0.2}^{\alpha} &= \{a_{0.5}, b_0\} \Rightarrow A_{0.2} \leq \bar{A}_{0.2}^{\alpha}.
\end{align*}
\]

But $\bar{A}_{0.2}^{\alpha} = \{a_{0.1}, b_0\}$

\[
\begin{align*}
\bar{A}_{0.2} &= \{a_{0.1}, b_0\} \\
\bar{A}_{0.2}^{\alpha} &= \{a_{0.1}, b_0\} \\
A_{0.2} &\leq \bar{A}_{0.2}^{\alpha} \text{ it's mean $A_{0.2}$ is not $p\alpha$-open set.}
\end{align*}
\]

• If we take the set $A_{0.99} = \{a_{0.99}, b_0\}$ which is set as fuzzy $p$-open

Where $\bar{A}_{0.99} = 1$

\[
\begin{align*}
\bar{A}_{0.99}^{\alpha} &= 1 \Rightarrow A_{0.99} \leq \bar{A}_{0.99}^{\alpha}.
\end{align*}
\]

But $\bar{A}_{0.99}^{\alpha} = \{a_{0.5}, b_0\}$

\[
\begin{align*}
\bar{A}_{0.99} &= \{a_{0.5}, b_1\} \\
\bar{A}_{0.99}^{\alpha} &= \{a_{0.5}, b_0\} \\
A_{0.99} &\leq \bar{A}_{0.99}^{\alpha} \text{ it's mean $A_{0.99}$ is not fuzzy $\alpha$-open set.}
\end{align*}
\]

• If we take the set $A_{0.4} = \{a_{0.4}, b_0\}$ which is a fuzzy $p\alpha$-open set

Since $A_{0.4} = \{a_{0.3}, b_0\}$

\[
\begin{align*}
\bar{A}_{0.4} &= \{a_{0.5}, b_0\} \\
\bar{A}_{0.4}^{\alpha} &= \{a_{0.5}, b_0\}
\end{align*}
\]
\[ A_{0.2}^p = \{ a_{0.5}, b_0 \} \Rightarrow A_{0.4} \neq A_{0.4}^p \]

But \( A_{0.4} \neq A_{0.4}^p \) so it’s mean \( A_{0.4} \) is set as not (FO)

1.25. Formula
In a (FTC) \((X, T)\),
When \( V \) is set as (FO), so \( V \wedge \overline{A} \leq \overline{V \wedge A} \) for every (FS) \( A \) in \( X \).

Evidence: Suppose \( x_a \in FP(X) \) and \( V \) is a (FO) in \( X \). When \( x_a \in (V \wedge \overline{A}) \), so \( x_a \bar{q}(V \wedge \overline{A})^{c} \Rightarrow x_a \bar{q}((V)^{c} \vee (\overline{A})^{c}) \Rightarrow x_a \leq V \) and \( x_a \leq \overline{A} \Rightarrow U \wedge A \neq 0 , \forall U \in T , x_a \notin U \). Since \( U \wedge V \) is (FOS), therefore \( U \wedge (V \wedge A) \neq 0 \) and \( x_a q \overline{V \wedge A} \). Hence \( V \wedge \overline{A} \leq \overline{V \wedge A} \).

1.26. Remark
In a (FTC) \((X, T)\), the set as (FO) intersection with fuzzy \( p \)-open set is fuzzy \( p \)-open set.

Evidence: Suppose \( A \) be a (FOS) in \( X \) and \( B \) is a fuzzy \( p \)-open. Thus \( A \wedge B \leq A \wedge \overline{B}^{p} = [A^{p} \wedge \overline{B}^{p}]^{\circ} \) via \( (1.25. \text{Theorem}) \leq \overline{A \wedge B}^{p} \). Thus \( S \wedge B \) is \( p \)-open in \( X \).

1.27. Formula
In a (FTC) \((X, T)\).
When \( V \) be a (FOS), so \( V \wedge \overline{A}^{p} \leq \overline{V \wedge A}^{p} \) for every (FS) \( A \) in \( X \).

Evidence : (i) Suppose \( x_a \in FP(X) \) and \( V \) is a (FO) in \( X \). When \( x_a \in (V \wedge \overline{A}^{p}) \), so \( x_a \bar{q}(V \wedge \overline{A}^{p})^{c} \Rightarrow x_a \bar{q}((V)^{c} \vee (\overline{A})^{c})^{c} \Rightarrow x_a \leq V \) and \( x_a \leq \overline{A}^{p} \). Via \( (1.22. \text{Proposition (10)}) \) \( U \wedge A \neq 0 , \forall U \in FPO(X) , x_a \in U \). Since \( U \wedge V \) is fuzzy \( p \)-open set via \( (1.26. \text{Remark}) \), therefore \( U \wedge (V \wedge A) \neq 0 \) and \( x_a q (\overline{V \wedge A})^{p} \). Hence \( V \wedge \overline{A}^{p} \leq \overline{V \wedge A}^{p} \).

1.28. Description [9, P.549-564]
In a (FTC) \((X, T)\), if \( A \leq B < X \), so a (FS) \( A \) is termed (FO) in \( B \) when there exist a (FO) \( H \) in \( X \) so \( A = H \wedge B \).

1.29. Description
In a (FTC) \((X, T)\), if \( A \leq B < X \), so a (FS) \( A \) is termed fuzzy \( \alpha \)-open in \( B \) if there exist a fuzzy \( \alpha \)-open \( H \) in \( X \) such that \( A = H \wedge B \).

1.30. Description
In a (FTC) \((X, T)\), if \( A \leq B < X \), so a (FS) \( A \) is termed fuzzy \( \alpha \)-open in \( B \) when there exist a fuzzy \( \alpha \)-open \( H \) in \( X \) such that \( A = H \wedge B \).

1.31. Description
In a (FTC) \((X, T)\), if \( A \leq B < X \), so a (FS) \( A \) is termed fuzzy \( p \)-open in \( B \) if there exist a fuzzy \( p \)-open \( H \) in \( X \) such that \( A = H \wedge B \).

1.32. Proposition
In a (FTC) \((X, T)\), if \( A \leq B < X \), so:
(i) A (FS) \( A \) is a fuzzy \( \alpha \)-open in \( B \) when \( A \) is a fuzzy \( \alpha \)-open in \( X \).
(ii) A (FS) \( A \) is a (FO) in \( B \), if \( A \) is a (FO) in \( X \).
(iii) A (FS) \( A \) is a fuzzy \( \alpha \)-open in \( B \), if \( A \) is a fuzzy \( \alpha \)-open in \( X \).
(iv) A (FS) \( A \) is a fuzzy \( p \)-open in \( B \), if \( A \) is a fuzzy \( p \)-open in \( X \).

Evidence: (i) We have \( A = A \wedge B \), but \( A \) is fuzzy \( \alpha \)-open in \( X \). Hence, via \( (1.26. \text{Description}) \) \( A \) is a fuzzy \( \alpha \)-open in \( B \).
(ii), (iii) and (iv) are straightforward.

1.33. Suggestion
Suppose \( A \leq B < X \), when \((X, T)\) is a (FTC), so:
(i) If \( B \) is a fuzzy \( \alpha \)-open in \( X \). So \( A \) is a fuzzy \( \alpha \)-open in \( B \) when and only when \( A = S \wedge B \), since \( S \) be a (FO) in \( X \).
(ii) If \( B \) is a fuzzy \( \alpha \)-open in \( X \). So \( A \) is a fuzzy \( \alpha \)-open in \( B \) when and only when \( A = S \wedge B \), since \( S \) be a (FO) in \( X \).
(iii) If $B$ is a fuzzy p-open in $X$. So $A$ is a fuzzy p-open in $B$ when and only when $A = S \land B$, since $S$ be a (FO) in $X$.
(iv) If $B$ is a fuzzy $\alpha$-open in $X$. So $A$ is a fuzzy $\alpha$-open in $B$ when and only when $A = S \land B$, since $S$ be a fuzzy $\alpha$-open in $X$.
(v) If $B$ is a fuzzy p-open in $X$. So $A$ is a fuzzy p-open in $B$ when and only when $A = S \land B$, since $S$ be a fuzzy p-open in $X$.
(vi) If $B$ is a fuzzy p-open in $X$. So $A$ is a fuzzy p-open in $B$ when and only when $A = S \land B$, since $S$ be a fuzzy $\alpha$-open in $X$.

Evidence: (i) $\Rightarrow$ For proving $A$ is a fuzzy $\alpha$-open in $B$, we should verify $S \land B$ is a fuzzy $\alpha$-open

in $X$ (such as $S \land B \leq (S \land B)^{p\alpha}$). Where $S \land B \leq S \land B^{p\alpha} = [S \land B^{p\alpha}] = [S \land B^{p\alpha}]$ via (1.26. Theorem) $\leq (S \land B)^{p\alpha} \leq (S \land B)$. Thus $S \land B$ is $\alpha$-open in $X$. Thus, $A$ is a fuzzy $\alpha$-open in $B$ via (1.31. Proposition (ii)).

($\Leftarrow$) As a result $A = S \land B$, where $S$ is (FO) in $X$, so via (1.23. Remarks) (i) $S$ is a fuzzy $\alpha$-open. Hence via (1.29. Description) $A$ is a fuzzy $\alpha$-open in $B$.

(ii) $\Rightarrow$ For proving $A$ is a fuzzy $\alpha$-open in $B$, we should verify $S \land B$ is a fuzzy $\alpha$-open in $X$ (i.e., $S \land B \leq (S \land B)^{\alpha}$).

While $S \land B \leq S \land B^{\alpha} = (S \land B^{\alpha}) = (S \land B^{\alpha})$ via (1.25. Theorem) $\leq (S \land B)^{\alpha} \leq (S \land B)^{\alpha}$. Thus $S \land B$ is $\alpha$-open in $X$. Thus, $A$ is a fuzzy $\alpha$-open in $B$ via (1.32. Proposition (ii)).

($\Leftarrow$) As a result $A = S \land B$, where $S$ is (FO) in $X$, so via (1.23. Remarks) $S$ is a fuzzy $\alpha$-open. Thus, via (1.30. Description) $A$ is a fuzzy $\alpha$-open in $B$.

(iii) $\Rightarrow$ For proving $A$ is a fuzzy $\alpha$-open in $B$, we should verify $S \land B$ is a fuzzy $\alpha$-open in $X$. Via (1.23. Remark) $S \land B$ is $p$-open in $X$. Thus, $A$ is a fuzzy $p$-open in $B$ via (1.32. Proposition (iv)).

($\Leftarrow$) As a result $A = S \land B$, where $S$ is (FO) in $X$, so via (1.23. Remarks) $S$ is a fuzzy $p$-open. Hence via (1.31. Description) $A$ is a fuzzy $p$-open in $B$.

(iv) $\Rightarrow$ For proving $A$ is a fuzzy $\alpha$-open in $B$, we should verify $S \land B$ is a fuzzy $\alpha$-open in $X$ (such as, $S \land B \leq (S \land B)^{\alpha}$). Where $S \land B \leq (S \land B)^{\alpha} \leq S \land B = (S \land B)^{\alpha}$ via (1.25. Theorem) $\leq (S \land B)^{\alpha} \leq (S \land B)^{\alpha}$. Thus $S \land B$ is $\alpha$-open in $X$. Hence $A$ is a fuzzy $\alpha$-open in $B$ via (1.32. Proposition (ii)).

($\Leftarrow$) As a result $A = S \land B$, since $S$ is (FO) in $X$, so via (1.23. Remarks) $S$ is a fuzzy $p$-open. Hence via (1.31. Description) $A$ is a fuzzy $p$-open in $B$.

(v) $\Rightarrow$ For proving $A$ is a fuzzy $p$-open in $B$, we should verify $S \land B$ is a fuzzy $p$-open in $X$ (i.e., $S \land B \leq (S \land B)^{p\alpha}$). Where $S \land B \leq (S \land B)^{p\alpha} \leq S \land B = (S \land B)^{p\alpha}$ via (1.25. Theorem) $\leq (S \land B)^{p\alpha} \leq (S \land B)^{p\alpha}$. Thus $S \land B$ is $\alpha$-open in $X$. Thus, $A$ is a fuzzy $\alpha$-open in $B$ via (1.32. Proposition (iii)).

($\Leftarrow$) As a result $A = S \land B$, where $S$ is fuzzy $\alpha$-open in $X$, so via (1.23. Remarks) $S$ is a fuzzy $p$-open. Thus, via (1.31. Description) $A$ is a fuzzy $\alpha$-open in $B$.

(vi) $\Rightarrow$ For proving $A$ is a fuzzy $p$-open in $B$, we should verify $S \land B$ is a fuzzy $p$-open in $X$. (i.e., $S \land B \leq S \land B^{p\alpha}$). Where $S \land B \leq S \land B^{p\alpha} = (S \land B^{p\alpha})$ via (1.25. Theorem) $\leq S \land B^{p\alpha} \leq S \land B^{p\alpha}$. Thus $S \land B$ is $\alpha$-open in $X$. Hence $A$ is a fuzzy $\alpha$-open in $B$ via (1.32. Proposition (iii)).

($\Leftarrow$) As a result $A = S \land B$, since $S$ is fuzzy $\alpha$-open in $X$, so via (1.23. Remarks) $S$ is a fuzzy $\alpha$-open. Thus, via (1.30. Description) $A$ is a fuzzy $\alpha$-open in $B$. 

7
1.34. Description [8, P.131-139]
(FSS) family \( V \) has property as fixed intersection when and only when the members intersection of \( V \) every finite sub-family is as not-empty.

2- Fuzzy \((p\alpha, a, p)\) compact subspaces

2.1. Description [10]
X as fuzzy space \( A \) is termed fuzzy compact when each (FO) of cover \( X \) of sub-cover being finite.

2.2. Description
(FSS) family \( B \) in a \((FTC) (X, T)\) is assumed to be a fuzzy \( p\alpha \)-open (FS) cover \( A \) when and only when \( A \leq \bigvee \{G: G \in B\} \) and every \( B \) member is \( p\alpha \)-open (FS). \( B \) Sub-cover is a sub-family that is covered as well.

2.3. Description
(FSS) family \( B \) in a \((FTC) (X, T)\) is assumed to be a fuzzy \( \alpha \)-open (FS) cover \( A \) when and only when \( A \leq \bigvee \{G: G \in B\} \) and every \( B \) member is \( p\alpha \)-open (FS). \( B \) Sub-cover is a sub-family that is covered as well.

2.4. Description
(FSS) family \( B \) in a \((FTC) (X, T)\) is assumed to be a fuzzy \( p \)-open cover \( A \) when and only when \( A \leq \bigvee \{G: G \in B\} \) and every \( B \) member is \( p\alpha \)-open (FS). \( B \) Sub-cover is a sub-family that is covered as well.

2.5. Description
X as fuzzy space is termed fuzzy \( p\alpha \)-compact when every cover \( X \) fuzzy \( p\alpha \)-open of sub-cover being finite.

2.6. Description
X as fuzzy space is termed fuzzy \( \alpha \)-compact when every cover \( X \) fuzzy \( \alpha \)-open of sub-cover being finite.

2.7. Description
X as fuzzy space is termed fuzzy \( p \)-compact when every cover \( X \) fuzzy \( p \)-open of sub-cover being finite.

2.8. Theorem
A \((FTC) (X, T)\) is a fuzzy \( p\alpha \)-compact, when and only when whichever collection \( \{B_j: j \in \mathbb{I}\} \) of sets being fuzzy \( p\alpha \)-closed in \( X \) of property as a limited intersection.

Evidence: \( \implies \) Assume that \( X \) is space of \( p\alpha \)-compact being fuzzy and \( \{B_j: j \in \mathbb{I}\} \) is \( X \) fuzzy \( p\alpha \)-closed sets collection of property as a limited intersection. To illustrate \( \{B_j: j \in \mathbb{I}\} \) of intersection as non-empty (such as to illustrate \( \bigwedge_{j \in \mathbb{I}} B_j \neq 0 \)).

Adopt that \( \bigwedge_{j \in \mathbb{I}} B_j = 0 \), so \( \bigvee_{j \in \mathbb{I}} B_j^c = 1 \) and every \( B_j^c \) is fuzzy set as \( p\alpha \)-open, therefore there exist \( j_1, j_2, ... j_n \) such that \( \bigvee_{i=1}^n B_{j_i}^c = 1 \) via \( 2.5.\) Description), thus, \( \bigwedge_{i=1}^n B_{j_i} = 0 \) that is contradiction and thus \( \bigwedge_{j \in \mathbb{I}} B_j \neq 0 \).

(\( \Leftarrow \) On the contrary, suppose \( \{A_j: j \in \mathbb{I}\} \) is \( X \) fuzzy \( p\alpha \)-open cover and every set as fuzzy being \( p\alpha \)-closed collection in \( X \) of property as a limited intersection as not empty. To illustrate that \( X \) is a fuzzy space as \( p\alpha \)-compact. Where \( \bigvee_{j \in \mathbb{I}} A_j = 1 \), so \( \bigwedge_{j \in \mathbb{I}} A_j^c = 0 \) and every \( A_j^c \) is fuzzy set as \( p\alpha \)-compact...
closed that involves that \( \{A_j^f : j \in \mathcal{J} \} \) fuzzy \( \alpha \) — closed collection sets of intersection being empty and so via proposition such collection have no property of limited intersection. Therefore, there (FS)s \( A_j^f_i, i = 1, 2, ..., n \), finite member exist where \( \bigvee_{i=1}^n A_j^f_i = 0 \), that indicates \( \bigvee_{i=1}^n A_j^f_i = 1 \) and \( \bigvee_{i=1}^n A_j^f_i = 1 \), is space \( X \) finite sub-cover belong to a fuzzy pre open cover \( \{A_j^f : j \in \mathcal{J} \} \), where, \( X \) is space as \( \alpha \)-compact being fuzzy.

2.9. Proposition
(i) Every \( \alpha \)-compact fuzzy space be compact as fuzzy.
(ii) Every \( \alpha \)-compact fuzzy space be compact as fuzzy.
(iii) Every \( \alpha \)-compact fuzzy space is compact as fuzzy.
(iv) Every \( \alpha \)-compact fuzzy space be \( \alpha \)-compact as fuzzy.
(v) Every \( p \)-compact fuzzy space be \( \alpha \)-compact as fuzzy.
(vi) Every \( p \)-compact fuzzy space be \( \alpha \)-compact as fuzzy.

Evidence: (i) Suppose that \( \mathcal{A} = \{A_j^f : j \in \mathcal{J} \} \) is a (FO) fuzzy space \( X \) cover and \( 1 = \bigvee_{j \in \mathcal{J}} A_j^f \). But, via each (FOS) in \( X \) is a fuzzy \( \alpha \)-open and \( X \) is a fuzzy \( p \)-compact space, so there presents \( j_1, j_2, ..., j_n \in \mathcal{J} \) such that \( 1 = \bigvee_{i=1}^n A_{j_i}^f \), thus \( X \) is space as fuzzy compact.
(ii), (iii), (iv), (v) and (vi) in the same way.

2.10. Suggestion
Suppose \( (X, T) \) be a (FTC).
(i) When \( B \) is a (FS) in \( X \) and \( A \leq B \), so \( A \) is a fuzzy compact set in \( X \) if and only if \( A \) is a fuzzy compact in \( B \).
(ii) If \( B \) is a (FS) in \( X \) and \( A \leq B \), so \( A \) is a fuzzy \( \alpha \)-compact set in \( X \) if and only if \( A \) is a fuzzy \( p \)-compact in \( B \).
(iii) If \( B \) is a (FS) in \( X \) and \( A \leq B \), so \( A \) is a fuzzy \( \alpha \)-compact set in \( X \) when and only when \( A \) is a fuzzy \( \alpha \)-compact in \( B \).
(iv) If \( B \) is a (FS) in \( X \) and \( A \leq B \), so \( A \) is a fuzzy \( p \)-compact set in \( X \) when and only when \( A \) is a fuzzy \( p \)-compact in \( B \).

Evidence: (ii) \( \Rightarrow \) Assume that \( \mathcal{A} = \{A_j^f : j \in \mathcal{J} \} \) be \( \alpha \) fuzzy cover via \( \alpha \)-open sets in \( B \). Via (1.29. Description), \( A_j^f = S_j \cap B \) for every \( j \in \mathcal{J} \), since \( S_j \) is a fuzzy \( \alpha \)-open in \( X \). Therefore, \( S = \{S_j : j \in \mathcal{J} \} \) be \( \alpha \) as fuzzy cover via \( \alpha \)-open sets in \( X \), but \( A \) is a fuzzy \( \alpha \)-compact in \( X \), so there presents \( j_1, j_2, ..., j_n \in \mathcal{J} \) so \( A \leq \bigvee_{i=1}^n (S_{j_i} \cap B) = \bigvee_{i=1}^n (A_{j_i}) \). Hence, \( A \) is a fuzzy \( \alpha \)-compact in \( B \).
\( \Leftarrow \) It is straightforward.
(i), (iii) and (iv) are clear.

2.11. Suggestion
Suppose \( (X, T) \) is a (FTC).
(i) When \( B \) be a fuzzy \( \alpha \)-open set in \( X \) and \( A \leq B \), so \( A \) be a fuzzy compact in \( X \) if and only if \( A \) be a fuzzy \( \alpha \)-compact in \( B \).
(ii) When \( B \) be a fuzzy \( \alpha \)-open set in \( X \) and \( A \leq B \), so \( A \) be a fuzzy compact in \( X \) if and only if \( A \) be a fuzzy \( \alpha \)-compact in \( B \).
(iii) When \( B \) be a fuzzy \( p \)-open set in \( X \) and \( A \leq B \), so \( A \) be a fuzzy compact in \( X \) when and only when \( A \) is a fuzzy \( p \)-compact in \( B \).
(iv) When \( B \) be a fuzzy \( \alpha \)-open set in \( X \) and \( A \leq B \), so \( A \) is a fuzzy \( \alpha \)-compact in \( X \) when and only when \( A \) is a fuzzy \( p \)-compact in \( B \).
(v) When \( B \) be a fuzzy \( p \)-open set in \( X \) and \( A \leq B \), so \( A \) is a fuzzy \( \alpha \)-compact in \( X \) when and only when \( A \) is a fuzzy \( p \)-compact in \( B \).
(vi) If \( B \) is a fuzzy \( p \)-open set in \( X \) and \( A \leq B \), so \( A \) is a fuzzy \( \alpha \)-compact in \( X \) when and only when \( A \) is a fuzzy \( p \)-compact in \( B \).
Evidence: (i) \( \Rightarrow \) assume that \( \mathcal{A} = \{A_j: j \in J\} \) is a fuzzy cover of p\( \alpha \)-open in \( B \). Via (1.33, Proposition (i)), \( A_j = S_j \land B \) for every \( j \in J \), since \( S_j \) is a (FO) in \( X \). Therefore, \( \mathcal{S} = \{S_j: j \in J\} \) is a fuzzy cover via (FOS) s in \( X \), wherease \( A \) is a fuzzy compact in \( X \), so there presents \( j_1, j_2, \ldots, j_n \in J \) such that \( A \leq \bigvee_{i=1}^{n}(S_{j_i} \land B) = \bigvee_{i=1}^{n}(A_{j_i}) \). Thus, \( A \) is a fuzzy p\( \alpha \)-compact in \( B \).

(\( \Leftarrow \)) Assume that \( \mathcal{S} = \{S_j: j \in J\} \) be a (FO) cover in \( X \). So \( \mathcal{A} = \{S_j \land B: j \in J\} \) is a fuzzy cover via (1.33, Proposition (i)) \( S_j \land B \) is a fuzzy p\( \alpha \)-open in \( B \) for all \( j \in J \). Via supposition \( A \) is a fuzzy p\( \alpha \)-compact in \( B \), so there presents \( j_1, j_2, \ldots, j_n \in J \) such that \( A \leq \bigvee_{i=1}^{n}(S_{j_i} \land B) \leq \bigvee_{i=1}^{n}(S_{j_i}) \). Hence, \( A \) is a fuzzy compact in \( X \).

(ii), (iii), (iv), (v) and (vi) in the same way.
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