Adiabatic fluctuations from cosmic strings in a contracting universe

Robert H. Brandenberger
Department of Physics, McGill University, 3600 University Street, Montréal QC, H3A 2T8, Canada

Tomo Takahashi
Department of Physics, Saga University, Saga 840-8502, Japan

Masahide Yamaguchi
Department of Physics and Mathematics, Aoyama Gakuin University, Sagamihara 229-8558, Japan,
and
Department of Physics, Stanford University, Stanford CA 94305

Abstract. We show that adiabatic, super-Hubble, and almost scale invariant density fluctuations are produced by cosmic strings in a contracting universe. An essential point is that isocurvature perturbations produced by topological defects such as cosmic strings on super-Hubble scales lead to a source term which seeds the growth of curvature fluctuations on these scales. Once the symmetry has been restored at high temperatures, the isocurvature seeds disappear, and the fluctuations evolve as adiabatic ones in the expanding phase. Thus, cosmic strings may be resurrected as a mechanism for generating the primordial density fluctuations observed today.

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1. Introduction

The cosmic microwave background (CMB) anisotropies observed by the Wilkinson Microwave Anisotropy Probe (WMAP) have revealed that the primordial density fluctuations are almost adiabatic and scale invariant \[1, 2\]. Inflationary cosmology \[3\] is currently the favored scenario to explain such primordial fluctuations. According to it, density perturbations are produced as quantum vacuum fluctuations on sub-Hubble scales and then stretched to super-Hubble scales during the phase of accelerated expansion of space. However, inflationary cosmology is not without its problems (see e.g. \[4\]), and thus it is important to study scenarios alternative to inflation. In the 1980s, topological defect models such as those based on cosmic strings were investigated intensely as a possible alternative to generate primordial density fluctuations \[5\]. However, the fluctuations induced by defects in an expanding universe are isocurvature and, even if they might mimic the inflationary predictions for the temperature-temperature (TT) correlation of the CMB \[6\], observations of the anti-correlation of the temperature and the E-mode polarization (TE), precisely measured by WMAP, confirmed that such fluctuations could not be the dominant source of CMB anisotropies \[7, 1\]. Thus, causal scaling seed models are ruled out as a main component of primordial density fluctuations.

On the other hand, in recent years other types of scenarios alternative to inflation motivated by developments in string theory have been proposed. Examples are the Pre-Big-Bang model (PBB) \[8\] and the Cyclic/Ekpyrotic scenario \[9, 10\]. The common feature of these models is that the universe begins in a contracting phase before emerging into the expanding phase of Standard Big Bang cosmology after a cosmological bounce. In the contracting phase, comoving scales exit the Hubble radius unless the contraction is too rapid. Previous studies have considered quantum mechanical vacuum fluctuations of a scalar matter field evaluated when the matter scales exit the Hubble radius during the contracting phase. Of course, once they are produced in the contracting phase, the fluctuations must be coupled to fluctuations in the expanding phase after the bounce. The propagation of fluctuations through the bounce phase depends on the details of the bounce \[13\]. There are models which yield an almost scale invariant spectrum (see e.g. \[14\]) after the bounce.

In this paper, we suggest the possibility that primordial density fluctuations are produced by causal seeds such as cosmic strings in the contracting phase, and show that they could generate adiabatic, almost scale invariant, and super-Hubble curvature fluctuations in the expanding universe. One simple possibility to realize

\[‡\] One of the problems in the usually considered mechanism, where quantum fluctuations are the origin of the present cosmic fluctuations, is the transition from quantum quantities to classical observables. However, in the mechanism proposed in this paper, such a problem does not exist because they are classical fluctuations from the beginning.

\[§\] Another model is string gas cosmology \[11, 12\], in which even the dimensionality of the observable universe may be determined dynamically.

\[∥\] Another attempt to produce adiabatic fluctuations from cosmic strings is discussed in the context of
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Cosmic strings in the contracting universe is to embed our model into a so-called cyclic universe, in which cosmic strings are formed in the usual way during a phase transition in the expanding phase, if the matter Lagrangian admits cosmic strings. In our scenario, different from the cyclic scenario of Steinhardt and Turok [10] where quantum fluctuations source the primordial density fluctuations, here cosmic strings seed the perturbations. Of course, in the cyclic scenario, topological defects may be dangerous because they may dominate the energy density of the universe, as pointed out by Avelino et al. in Refs. [16, 17]. However, Avelino et al. also give a solution to that problem: they point out that a relatively long period of cosmic acceleration at low energies (late period of one cycle) can dilute topological defects in order that they do not overdominate the universe. A second possibility is to consider the birth of the universe in the contracting universe (not cyclic). If the universe has finite birth time, the correlated region at the birth of the universe does not necessarily cover the whole volume of the universe. Then, the randomness of the values of the underlying field beyond the correlation length leads to the formation of topological defects.

Density fluctuations produced by causal seed models naturally become super-Hubble in the contracting phase. More specifically, the key point is that the defect-seeded perturbations which are initially isocurvature in nature seed a growing adiabatic mode. At the time when the symmetry (whose breaking yields the topological defects) is restored, the seed term disappears and the fluctuations become frozen-in super-Hubble scale adiabatic perturbations. As long as no dominant isocurvature fluctuations are produced in the expanding phase, the fluctuations in the expanding phase will be adiabatic and thus able to explain the TE anti-correlation observed in the CMB. In the following we consider cosmic strings as a concrete example and investigate the nature of the density fluctuations in detail.

2. Adiabatic fluctuations from cosmic strings in a contracting universe

The evolution of cosmic strings in a contracting universe was investigated in Refs. [16, 17]. As in these references, we will assume that the distribution of strings on super-Hubble scale is like a random walk. We make the simplest assumption that the universe is initially matter and then radiation dominated in the contracting phase. In this context, it was shown that cosmic strings obey the scaling solution asymptotically both in the radiation and matter dominated epochs. Specifically, the correlation length $L$ is proportional to $a^2 \ln a \propto (-t) \ln(-t)$ in the radiation era ($a$ is the scale factor and $t(< 0)$ is cosmic time, with the bounce time taken as $t = 0$). If we take the string loop chopping efficiency $\tilde{c}$ to be a non-zero constant, then the ratio of the energy density in cosmic strings to the total one stays almost constant. Therefore, the density fluctuations produced by cosmic strings are almost scale invariant at least initially at two-metric theories of gravity [15].

There remains the logarithmic dependence in the radiation dominated era, which can lead to a small deviation from scale invariance of the final curvature perturbations.
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the Hubble radius exit.

On super-Hubble scales, the dynamics of the defect network sets up isocurvature fluctuations which in turn act as a seed for growing curvature perturbations. As the universe contracts, the temperature of radiation increases, and eventually leads to symmetry restoration and the disappearance of the topological defects. Thus, there will no longer be any isocurvature fluctuations, and the source on super-Hubble scales in the differential equation for the adiabatic mode vanishes. Hence, the fluctuations become frozen in as adiabatic ones.

On super-Hubble scales, the equation for the evolution of the curvature perturbation on uniform total density hypersurfaces, \( \zeta \), is given by

\[
\dot{\zeta} = -\frac{H}{\rho + P} \delta P_{\text{nad}},
\]

where \( H \equiv \dot{a}/a \) is the Hubble parameter, \( \rho \) and \( P \) are the total energy density and pressure, respectively. A dot represents a derivative with respect to the cosmic time.

The non-adiabatic pressure perturbation \( \delta P_{\text{nad}} \) is defined as \( \delta P_{\text{nad}} \equiv \delta P - c_s^2 \delta \rho \) with \( \delta \rho \) being the total density fluctuation and \( c_s^2 = \dot{P}/\dot{\rho} \) being the adiabatic sound speed. In the multi-fluid case, the total non-adiabatic pressure perturbation consists of two parts. The first part comes from intrinsic entropy perturbations which vanish for a barotropic fluid. Therefore, the intrinsic entropy perturbations of matter and radiation vanish. On the other hand, it is non-trivial that the intrinsic isocurvature perturbation for cosmic strings also vanishes as strings can be modeled as a simple fluid with equation of state \( w_{st} \equiv P_{st}/\rho_{st} = -1/3 \). However, we expect that the intrinsic entropy perturbation of cosmic strings is negligible and will assume this in the following. The second part \( \delta P_{\text{rel}} \) comes from the relative entropy perturbation between different fluids \( S_{\alpha\beta} \),

\[
\delta P_{\text{rel}} \equiv \frac{1}{6H\dot{\rho}} \sum_{\alpha,\beta} \dot{\rho}_\alpha \dot{\rho}_\beta (c_\alpha^2 - c_\beta^2) S_{\alpha\beta},
\]

where the relative entropy perturbation between different fluids \( S_{\alpha\beta} \) is given by

\[
S_{\alpha\beta} = -3H \left( \frac{\delta \rho_\alpha}{\dot{\rho}_\alpha} - \frac{\delta \rho_\beta}{\dot{\rho}_\beta} \right),
\]

and the adiabatic sound speed for each component, \( c_\alpha^2 \), is given by \( \dot{P}_\alpha/\dot{\rho}_\alpha \) with \( \rho_\alpha \) and \( P_\alpha \) being the energy density and pressure of the component.

Now, let us estimate the amplitude of the curvature perturbation \( \zeta \) for a comoving scale \( k \). First, we consider a scale \( k \) which exits the Hubble radius during a radiation dominated era. We neglect the curvature perturbations which are generated when the corresponding scale is sub-Hubble. This can be justified by assuming that string loop distribution is subdominant and the initial value of the curvature perturbation at \( t \to -\infty \) vanishes. In fact, we expect that the string loop distribution will be subdominant in a contracting universe compared to an expanding universe since comoving scales
perturbation from the epoch $t_H(k)$ when a comoving scale $k$ exits the Hubble radius until the time $t_{cs}$ when the symmetry is restored and the strings disappear.

The relative entropy perturbation between radiation and cosmic strings is

$$S_{rs} \simeq -3H \frac{\dot{\rho}_{st}}{\rho_{st}},$$

where we have used the fact that $\dot{\rho}_{st}/\dot{\rho}_{rad}$ is almost constant and much smaller than unity, and $|\delta \rho_{rad}|$ is at most comparable to $|\delta \rho_{st}|$. From the scaling solution, it follows that cosmic strings can be modeled as a random walk on scales larger than the Hubble radius with step length comparable to the Hubble radius. Then, the density fluctuations of cosmic strings for a super-Hubble scale can be easily estimated as

$$\delta \rho_{st}(k) \simeq N|H|\mu k_{\text{phys}},$$

where $\mu$ is the mass per unit length of a string, $N = O(1)$ is the number of long strings crossing any given Hubble volume, and $k_{\text{phys}} \equiv k/a$. Notice that $N$ can be different in radiation and matter dominated epochs although the difference is expected to be at most $O(1)$. Inserting these equations to Eq. (1) yields

$$\dot{\zeta} \simeq NG\mu k_{\text{phys}}.$$  

As stated above, cosmic strings disappear at the time $t_{cs}$ due to symmetry restoration. Once cosmic strings disappear, the curvature perturbation is conserved at least until the bounce. Thus, the final curvature perturbation before the bounce is estimated as

$$\zeta = \int_{t_H(k)}^{t_{cs}} dt \dot{\zeta} \sim N_{r}G\mu k(-t_H(k))^{\frac{1}{2}} \sim N_{r}G\mu.$$  

Here we have made use of $k(-t_H(k))^{\frac{1}{2}} = 1/2$ (in the radiation era), and $N_{r}$ is the value of $N$ (in the radiation epoch) which is $O(1)$. Thus, the curvature perturbations are independent of the comoving scale $k$ and hence scale invariant at least before the bounce. In the same way, the curvature perturbation for comoving scales whose physical scales exit the Hubble radius during the matter era is estimated as

$$\zeta = \int_{t_H(k)}^{t_{eq}} dt \dot{\zeta} + \int_{t_{eq}}^{t_{cs}} dt \dot{\zeta} \sim N_{m}G\mu k(-t_H(k))^{\frac{2}{3}} \sim N_{m}G\mu,$$

where $t_{eq}$ is the matter-radiation equality time in the contracting phase, we have made use of $k(-t_H(k))^{\frac{2}{3}} = 2/3$ (in the matter era), and $N_{m}$ is the value of $N$ in the matter epoch. Therefore, the curvature perturbation is scale invariant at least before the bounce also on these scales.

According to the often-used Hwang-Vishniac [20] (Deruelle-Mukhanov [21]) matching conditions for fluctuations across a space-like hypersurface, the curvature perturbation is conserved across the bounce. If we apply these matching condition, are exiting rather than entering the Hubble radius. Loops exit the Hubble radius before they collapse through emission of gravitational waves. This effect reduces the loop chopping efficiency $\tilde{c}$, and hence the number of produced loops is smaller in a contracting phase.
we conclude that the final curvature perturbations in the expanding phase are almost scale invariant and hence could be responsible for the present density fluctuations if $G\mu \sim 10^{-5}$. As emphasized in [13], there are problems with blindly applying these matching conditions. Subsequent studies have shown that the actual transfer of the fluctuations depends quite sensitively on the details of the bounce. There are cases where the curvature perturbation is conserved (see e.g. [22, 23, 24]), but there are other examples where this does not hold [25, 26, 27]. However, if the bounce time is short compared to the time scale of the fluctuations of interest, it can be rather rigorously shown that the spectrum of $\zeta$ is maintained through the bounce. This can be shown [27, 28] by modeling the background cosmology with three phases: the initial contracting radiation phase, the “bounce phase” during which $H = \alpha t$, where $\alpha$ is some constant, and the expanding radiation phase. The matching conditions at the two hypersurfaces between these phases can be consistently applied (since the background also satisfies the matching conditions, unlike what happens in the single matching between contracting and expanding phase which has been applied in the case of the singular Ekpyrotic bounce). However, we would like to point out that, even if the curvature perturbation is not conserved through the bounce, the scale invariance of the final curvature perturbation still holds true as long as its change in the amplitude of the fluctuations across the bounce is independent of the comoving scale. This can be reasonably expected for modes we are interested in because their momenta are much smaller than the maximal value of $|H|$ around the bounce point (assuming that the bounce is smooth), the only energy scale which can be set by the bounce.

Finally, we comment on some subtleties. First of all, cosmic strings may be formed again in the expanding phase. In this case, cosmic strings again produce isocurvature fluctuations in the expanding phase, which should be suppressed to less than 10% of the total curvature perturbations [32, 33]. Such a suppression may be realized in the case when the constant $N$ for cosmic strings in the contracting phase is much larger than that in the expanding phase. The fact that the loop chopping efficiency $\tilde{c}$ is smaller implies that the constant $N$ might be larger. Therefore, it is plausible that the constant $N$ for cosmic strings in the contracting phase is larger than that in the expanding phase. Another possibility is that the curvature perturbation sourced by fluctuations of cosmic strings in a contracting phase is amplified at the bounce. Another solution is to simply assume that cosmic string are not produced in the expanding phase because the symmetry breaking patterns of scalar fields are not necessarily the same in the contracting and the expanding phases.

Another issue is that cosmic strings generate not only density perturbations but also vector and tensor perturbations (gravitational waves). Gravitational waves are produced by oscillations of loops as well as long strings. As explained before, the radius of a loop could become larger than the Hubble before there has been a significant

$\uparrow$ Even in the case when the change through the bounce depends on the comoving scale, a scale invariant spectrum may be realized by considering the time varying tension of cosmic strings discussed in Refs. [29, 30, 31], which compensates for the variation of the curvature perturbation.
amount of gravitational radiation. Therefore, we expect that the relative amplitude of gravitational waves to scalar metric fluctuations will be smaller in the contracting phase than the standard scenario of cosmic strings in an expanding universe. Regarding the vector mode, it has been shown that vector perturbations exhibit growing mode solutions in the contracting phase \cite{34}. In particular, the metric perturbations always grow while the matter perturbations stay constant in the radiation dominated era. However, the vector perturbations will decrease in the expanding phase. If vector fluctuations are suppressed (even slightly) across the bounce (or the scalar fluctuations enhanced), then the vector modes will be sufficiently small today not to destroy the successful agreement with the CMB angular power spectrum. Small but not negligible vector fields could, in fact, be useful for generating the observed large scale magnetic fields, as pointed out in Ref. \cite{34}. As a final remark, we mention possible non-Gaussianities in the scenario. Since the distribution of cosmic string is highly non-Gaussian, the produced density fluctuations may give large non-Gaussianity, though the bispectrum for a simulated string model in the expanding universe is shown not to be so large for the diagonal contribution \cite{35}. All these topics are worth investigating.

3. Summary

We have shown that adiabatic, super-Hubble, and almost scale invariant density fluctuations can be produced by cosmic strings in a contracting universe. Although cosmic strings can only generate isocurvature fluctuations in an expanding universe, they can produce adiabatic fluctuations by considering them in a contracting universe because cosmic strings disappear due to symmetry restoration. Our findings open the possibility that topological defects could be resurrected as the main source of the current cosmic density fluctuations.

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