Direct test of composite fermion model in quantum Hall systems

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Abstract

We show that neutron scattering and Raman scattering experiments can unambiguously determine a composite fermion parameter, viz., the effective number of Landau Levels filled by the composite fermions. For this purpose, one needs partially polarized or more preferably unpolarized quantum Hall states. We further find that spin correlation function acts as an order parameter in the spin transition.

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I. INTRODUCTION

In recent years, fractional quantum Hall effect (FQHE) which is believed to arise due to complicated electron electron interactions in the presence of high magnetic field \( B \) (perpendicular to the plane of two dimensional electron system) has drawn much interest of physicists. The composite fermion model (CFM), which is proposed by Jain [1], is by now fairly well established in these systems. In this model the interaction of one electron with all the others is replaced by attaching an even number \( (2s) \) of flux quanta (in the units of \( 2\pi/e \)) to each electron. In the mean field (MF) approximation, these fluxes produce a uniform magnetic field such that the effective Landau levels (LL) formed by the effective magnetic field \( \tilde{B} = B - (2\pi/e)\rho(2s) \), (where \( \rho \) is the mean particle density), can accommodate the particles in an integral number \( (p) \) of effective LL. The integer quantum Hall effect (IQHE) at integer filling \( p \) by these composite fermions (CF) leads to FQHE at the filling fraction \( \nu = p/(2sp+1) \) in the original electronic system. Later Lopez and Fradkin [2] have developed a formalism to study FQHE within this model by the introduction of an appropriate Chern-Simons (CS) gauge field.

Successful though the model is, Laughlin [3] has criticised the model on the grounds that it does not make any reference either to fractionally charged quasi-particles [4] or their fractional statistics [5] with which one could construct the hierarchial FQHE states by their condensations [5,6]. In this hierarchial picture, the elementary excitations in the state with filling fraction \( \nu = p/(2p+1) \) have charge \( \pm e/(2p+1) \) [4–6]. Notice that on the other hand, in the CFM, quasi-particles have charge \( -e \) with 2s vortices [7].

On the other hand, good experimental evidence for the existence of CF in FQHE systems has emerged recently [9–15]. As Halperin, Lee and Read (HLR) [8] emphasized, the single particle excitation gap of CF, corresponding to the state with filling fraction \( \nu = p/(2sp+1) \), is the effective cyclotron frequency \( \bar{\omega}_c \) which is determined by the effective field \( \tilde{B} \). Du et al [9] find that their results on the activation of the diagonal resistivity \( \rho_{xx} \) is consistent with the above interpretation. More significantly, CFM makes the remarkable prediction that at
\( \nu = 1/2s \), the effective field \( \bar{B} = 0 \). The properties of the half-filled LL have been studied extensively by HLR [8] employing the CF picture. Indeed at \( \nu = 1/2s \), the CF should have a well defined Fermi surface which has been verified experimentally by Willett et al [10] and Kang et al [11] by observing cyclotron motion of CF near \( \nu = 1/2 \). Three recent experiments [12–14] have treated the oscillations in \( \rho_{xx} \) around \( \nu = 1/2 \) as Shubnikosov-de Haas oscillations (SDHO) of CF, in analogy to SDHO of free electrons near \( B = 0 \). However, Leadley et al [12] have reported a finite effective mass \( m^* \) of CF at \( \bar{B} = 0 \), and \( m^* \) increases linearly with \( |\bar{B}| \), while Du et al [13] and Manoharan et al [14] have observed ‘drastic enhancement’ of CF mass as \( \nu \to 1/2 \), indicating a novel Fermi liquid at \( \nu = 1/2 \). Goldman et al [15] have reported the confirmation of the existence of CF by observing negatively charged carriers to form a Fermi sea near \( \nu = 1/2 \) in a magnetic focussing experiment, and they have also found that the charge carriers experience an effective magnetic field \( \bar{B} \). The main conclusion of the above experiments is that the dynamics of the charged particles is governed by \( \bar{B} \), rather than the applied magnetic field \( B \).

All the above experiments which are strongly in favour of the existence of CF are still rather incomplete in the sense that none of them determine either of the composite fermion parameters, viz, the effective number of LL (\( p \)) or the number of flux quanta (\( 2s \)) attached to each electron directly. The gap measurements do not determine \( p \) unambiguously as the parametrization of activation of \( \rho_{xx} \) is not unique and \( m^* \) also changes with \( \bar{B} \). Here we propose experiments which would determine \( |p| \) unambiguously. The other parameter \( 2s \) can be found out from the knowledge of filling fraction \( \nu = |p|/(2s|p| \pm 1) \). To that end, we need the FQHE states which are either partially polarized or unpolarized. Indeed, they are central to our analysis because the wave functions for fully polarized quantum Hall states (QHS) depend solely on \( \nu \) [16], while on the other hand, as we have shown recently [17], the wave functions for unpolarized or partially polarized QHS depend on any two parameters among \( p, s \), and \( \nu \).

We compute here the spin density correlation (SDC) in QHS and find that it is, in fact, an order parameter in the spin transitions from spin unpolarized or partially polarized
phases to their fully polarized phase. In fact, the ratio of SDC for unpolarized and fully polarized phase would determine the effective number of LL which are filled. The static charge density structure factor depends on $p$ only in the unpolarized phase. These are independent of $m^*$. Therefore experiments like neutron scattering would determine $p$ avoiding any complexity arising from the dependence of effective mass on $\tilde{B}$. We also determine the collective excitations from the poles of charge density correlations (CDC) and SDC. For the former, there is no mode near $\bar{\omega}_c$ but near the actual cyclotron frequency $\omega_c$, irrespective of the spin phase. On the other hand, SDC is shown to possess an undispersed pole exactly at $\bar{\omega}_c$ in unpolarized and partially polarized phases. Therefore, depolarized Raman scattering would again determine the exact value of $\bar{\omega}_c$ which is also a measure of $p$.

II. BRIEF REVIEW

Recently we have developed an abelian doublet model \[18\] employing a doublet of CS gauge fields, by which we can account for all the known filling fractions with different possible spin polarizations. Further, we have extracted the wave functions \[17\] as well from the correlations for arbitrarily polarized QHS. We, therefore, do not repeat the details of either the model or the computation of correlation functions, but present only the essential features.

A. The Model

To describe in brief, consider a two-dimensional system of spin-1/2 interacting electrons in the presence of uniform magnetic field perpendicular to the plane. The complicated interaction among electrons is represented by the interaction of electrons with CS gauge fields and weak (short-ranged) fermion-fermion interaction as we discuss below. We consider quantum Hall effect in low but non-zero Zeeman energy limit. The dynamics of the system is represented by the Lagrangian density
\[ \mathcal{L} = \bar{\psi}_\uparrow \mathcal{D}(A_\mu^\uparrow + a_\mu^\uparrow) \psi_\uparrow + \bar{\psi}_\downarrow \mathcal{D}(A_\mu^\downarrow + a_\mu^\downarrow) \psi_\downarrow + \frac{1}{2} \tilde{a}_\mu \epsilon^{\mu \lambda \nu} \partial_\nu a_\lambda - eA_\mu^0 \rho + \frac{1}{2} \int d^3 x' A_0^\text{in}(x)V^{-1}(x - x')A_0^\text{in}(x') . \]  

Here, \( \psi \) is the fermionic field and \( \uparrow (\downarrow) \) represents spin-up (down),

\[ \mathcal{D}(A_\mu^r + a_\mu^r) = iD_\mu^r + (1/2m^*)D_k^2 + \mu + (g/2)\mu_B(B + B^r + b^r)\sigma , \]

with \( D_\mu^r = \partial_\mu - ie(A_\mu^r + a_\mu^r) \) where \( A_\mu^r \) is the external electro-magnetic field which interacts with all the electrons while \( a_\mu^r \) are the external probe and the CS gauge field respectively, interacting with only the particles having spin indices \( r = \uparrow, \downarrow \).

The field \( A_0^\text{in} \) is identified as an internal scalar potential. Fixed mean particle density \( \rho \) is represented by the chemical potential \( \mu \) which acts as a Lagrange multiplier. Note that the Zeeman term includes all the three kinds of magnetic fields. \( \mu_B \) is the Bohr-magneton, and \( \sigma = +1(-1) \) for spin-up (down) electrons. We have introduced an abelian doublet of CS gauge fields in (4) as

\[ a_\mu = \begin{pmatrix} a_\mu^\uparrow \\ a_\mu^\downarrow \end{pmatrix} , \]

and the strength of the real symmetric matrix valued CS parameter is taken to be

\[ \Theta = \begin{pmatrix} \theta_1 & \theta_2 \\ \theta_2 & \theta_1 \end{pmatrix} . \]

\( \tilde{a}_\mu \) is the transpose of the doublet field \( a_\mu \). The fourth term in Eq. (1) describes the charge neutrality of the system. Finally, \( V^{-1}(x - x') \) is the inverse of the electron interaction potential (in the operator sense). The usual fermion interaction term in quartic form would be achieved by an integration over \( A_0^\text{in} \) field. The values of \( \theta_1 \) and \( \theta_2 \) must be consistent with the composite fermion requirement.

We then diagonalize the matrix \( \Theta \), with the eigen values \( \theta_{\pm} = \theta_1 \pm \theta_2 \). In the eigen basis, by simple rescalings, Eq. (1) may be written as

\[ \mathcal{L} = \bar{\psi}_\uparrow \mathcal{D}(A_\mu^\uparrow + a_\mu^\uparrow) \psi_\uparrow + \bar{\psi}_\downarrow \mathcal{D}(A_\mu^\downarrow + a_\mu^\downarrow) \psi_\downarrow + \theta_{\pm} \epsilon^{\mu \lambda \nu} a_\mu^\mp \partial_\nu a_\lambda^\pm + \theta_{\pm} \epsilon^{\mu \lambda \nu} a_\mu^\pm \partial_\nu a_\lambda^\mp - eA_\mu^0 \rho + \frac{1}{2} \int d^3 x' A_0^\text{in}(x)V^{-1}(x - x')A_0^\text{in}(x') . \]  

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This incorporates the idea that each electron, in general, has two kinds of vortices associated with it — while they interact in phase with spin up particles, spin down particles get their out of phase contributions.

Consider the case $\theta_- = 0$. Here, $a^-_\mu$ decouples dynamically and merely plays the role of a Lagrange multiplier: $(\partial L/\partial a^-_\mu) = \rho_\uparrow - \rho_\downarrow \equiv 0$, where $\rho_\uparrow(\rho_\downarrow)$ is the density for spin-up (down) particles. We then parametrize $\theta = (e^2/2\pi)(1/2s)$ ($s$ is an integer) in order to impose the composite fermion picture – fermions are attached with $2s$ vortices. In the mean field (MF) ansatz, these vortices produce an average CS magnetic field $\langle b^+ \rangle = -e\rho/\theta_+$. These choice of the parameters lead to the unpolarized QHS.

On the other hand, for obtaining partially polarized QHS, we parametrize $\theta_\pm = (e^2/2\pi)(1/s_\pm)$ and set $s_+ = 2s$ and $s_- = 0$. In this case, the field $a^-_\mu$ provides a vanishing mean magnetic field $\langle b^- \rangle$, and does not contribute to tree level (in contrast to the unpolarized case where $a^-_\mu$ is completely nondynamical). Composite fermion picture is enforced by the choice of $s_+ = 2s$. Thus in the MF ansatz, CS magnetic field produced by the particles is $\langle b^+ \rangle = -e\rho/\theta_+$. In both the above cases, mean magnetic field for all the particles, irrespective of their spin, is given by $\bar{B} = B + \langle b^+ \rangle$. Let $p_\uparrow(p_\downarrow)$ be the number of effective Landau levels (LL) formed by $\bar{B}^+$ filled by spin up (down) particles. This leads to the actual filling fraction and the spin density to be

$$\nu = \frac{p_\uparrow + p_\downarrow}{2s(p_\uparrow + p_\downarrow) + 1}; \quad \Delta \rho = \rho \left( \frac{p_\uparrow - p_\downarrow}{p_\uparrow + p_\downarrow} \right).$$  \hspace{1cm} (6)$$

Note that $p_\uparrow$ and $p_\downarrow$ can be negative integers as well in which case $\bar{B}^+$ is antiparallel to $B$. The effective cyclotron frequency $\tilde{\omega}_c = e\bar{B}/m^*$ is related to $\omega_c = eB/m^*$ by $\tilde{\omega}_c = \omega_c[2s(p_\uparrow + p_\downarrow) + 1]$. For unpolarized QHS, $p_\uparrow = p_\downarrow = p$ (say) and therefore the states with filling fraction $\nu = 2p/(4sp + 1)$ are spin unpolarized in the limit of small Zeeman energy.

In this limit, $p_\uparrow = p_\downarrow + 1$ for partially polarized states with $\nu = (p_\uparrow + p_\downarrow)/(2s(p_\uparrow + p_\downarrow) + 1)$ and $\Delta \rho/\rho = 1/(p_\uparrow + p_\downarrow)$. Fully polarized Laughlin states are obtained for $p_\uparrow = 1, p_\downarrow = 0$. 


B. Effective Action

Employing the above MF ansatz, we then evaluate one-loop effective action for the gauge fields to be

\[
S_{\text{eff}} = -\frac{1}{2} \int \frac{d^3q}{(2\pi)^3} (A^{\mu^+}_\mu + a^{\mu+}_{\mu} + a^{\mu-}_{\mu}) \Pi^{\mu\nu}_{\mu\nu}(\omega, \mathbf{q}) (A^{\nu^+}_\nu + a^{\nu+}_{\nu} + a^{\nu-}_{\nu})
\]

\[
-\frac{1}{2} \int \frac{d^3q}{(2\pi)^3} (A^{\mu^+}_\mu + a^{\mu+}_{\mu} - a^{\mu-}_{\mu}) \Pi^{\mu\nu}_{\mu\nu}(\omega, \mathbf{q}) (A^{\nu^+}_\nu + a^{\nu+}_{\nu} - a^{\nu-}_{\nu})
\]

\[
+ \frac{i}{2} \int \frac{d^3q}{(2\pi)^3} \left[ \frac{\theta_+}{2} \epsilon^{\mu\nu\lambda} a^+_{\mu} q_{\nu} a^+_{\lambda} + \frac{\theta_-}{2} \epsilon^{\mu\nu\lambda} a^-_{\mu} q_{\nu} a^-_{\lambda} \right]
\]

\[
+ \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} A^{\mu\nu}_{0i} V^{-1}(|q|) A^{\mu\nu}_{0i}.
\]  

(7)

Here \( a^{\mu\pm}_{\mu} \) and \( A^{\mu\nu}_{0i} \) are fluctuating part of the corresponding gauge fields. Note that the field \( a^{\mu-}_{\mu} \) does not exist for unpolarized states and hence Eq. (7) reduces appropriately. The polarization tensors \( \Pi^{\mu\nu}_{\mu\nu} \) have the following form,

\[
\Pi^{\mu\nu}_{\mu\nu} = \Pi^{\mu\nu}_{\mu\nu}(\omega, \mathbf{q}) (q^2 g^{\mu\nu} - q^\mu q^\nu) + (\Pi^{\mu\nu}_{\mu\nu} - \Pi^{\mu\nu}_{\mu\nu}) (\omega, \mathbf{q})
\]

\[\times \left( q^2 \delta^{ij} - q^i q^j \right) \delta^{\mu i} \delta^{\nu j} + i \Pi^{\mu\nu}_{\mu\nu}(\omega, \mathbf{q}) (q^2) \epsilon^{\mu\nu\lambda} q_\lambda. \]  

(8)

Integrating out all the internal gauge fields, the effective action for the external probes turns out to be

\[
S_{\text{eff}} \left[ A^{\mu^+}_\mu, A^{\nu^+}_\nu \right] = \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} A^{\mu\nu}_\nu(q) K^{\mu\nu}_{rr'}(\omega, \mathbf{q}) A^{\mu\nu}_{r'}(-q),
\]

where the indices \( r, r' = \uparrow, \downarrow \). \( K^{\mu\nu}_{rr'} \) measures linear response of the system to weak external probes. Recall that \( K^{00}_{\uparrow\uparrow}, K^{00}_{\uparrow\downarrow}, K^{00}_{\downarrow\uparrow} \) and \( K^{00}_{\downarrow\downarrow} \) represent the density-density correlations among spin up-up, up-down, down-up and down-down species of the particles respectively. These are given by

\[
K^{00}_{\uparrow\uparrow} = \frac{q^2}{\Pi^\uparrow_0 + \Pi^\downarrow_0} \left[ \Pi^\uparrow_0 \Pi^\downarrow_0 - \frac{\left( \Pi^\uparrow_0 \Pi^\uparrow_0 - \Pi^\downarrow_0 \Pi^\downarrow_0 + \Pi^\theta_0 \right)^2}{D(\omega, \mathbf{q})} \right],
\]

(10a)

\[
K^{00}_{\uparrow\downarrow} = \frac{q^2}{\Pi^\uparrow_0 + \Pi^\downarrow_0} \left[ \Pi^\uparrow_0 \Pi^\downarrow_0 - \frac{\left( \Pi^\uparrow_0 \Pi^\uparrow_0 - \Pi^\downarrow_0 \Pi^\downarrow_0 + \Pi^\theta_0 \right)^2}{D(\omega, \mathbf{q})} \right],
\]

(10b)

\[
K^{00}_{\downarrow\uparrow} = K^{00}_{\uparrow\downarrow} = -\frac{q^2}{\Pi^\uparrow_0 + \Pi^\downarrow_0} \left[ \Pi^\uparrow_0 \Pi^\downarrow_0 + \frac{\left( \Pi^\uparrow_0 \Pi^\uparrow_0 - \Pi^\downarrow_0 \Pi^\downarrow_0 + \Pi^\theta_0 \right)}{D(\omega, \mathbf{q})} \left( \Pi^\uparrow_0 \Pi^\uparrow_0 - \Pi^\downarrow_0 \Pi^\downarrow_0 + \Pi^\theta_0 \right) \right],
\]

(10c)
III. RESULTS AND EXPERIMENTAL CONSEQUENCES

A. Correlations

The charge density correlation can now be obtained as

\[ K_{00}(\omega, q^2) \equiv \sum_{r,r'} K_{r r'}^{00}(\omega, q^2) = -q^2 \frac{D_{0}^{\dagger} + D_{0}}{D(\omega, q^2)} \cdot (12) \]

On the other hand the spin density correlation is given by

\[ \Sigma(\omega, q^2) = \sum_{r,r'} \left[ K_{r r'}^{00} \delta_{rr'} - K_{r r'}^{00}(1 - \delta_{rr'}) \right] \]

(13)

For unpolarized states, \( \Pi_0^\dagger = \Pi_0^\dagger\equiv \Pi_0^\dagger \). Thus \( \Sigma \) gets the simpler form,

\[ \Sigma_{\text{unp}}(\omega, q^2) = \Pi_0(\omega, q^2)q^2 \cdot (14) \]

Note at the outset that the charge density excitations (CDE) will be very different from spin density excitations (SDE) (spin-zero excitations) especially for unpolarized states, because, CDE are determined by the poles of \( K_{00}(\omega, q^2) \), while SDE are determined by the poles of \( \Pi_0 \). The collective CDE and SDE will be discussed below. We note that the leading order term in \( q^2 \) of \( K_{00} \) saturates the f-sum rule [19].

The above results are valid in the thermodynamic limit. For as Lopez and Fradkin [19] have argued in a similar case, we note that we have evaluated the effective action by neglecting higher order response functions, viz, the correlations of three or more currents or densities. These higher order correlations are of higher order in \( q^2 \) compared to the quadratic term in Eqs. (10, 12 – 14). These higher order terms would not be negligible for a finite system since the minimum allowed value of the momentum is then determined by
the linear size of the system $L$, i.e., $|\mathbf{q}| > 1/L$. On the other hand, in the thermodynamic limit, $L \to \infty$ and the minimum allowed value of $|\mathbf{q}|$ goes to zero. Therefore one is allowed to keep only the quadratic term in effective action and neglect the higher order corrections for an infinite system.

In the limit of low $\mathbf{q}^2$, CDC and SDC are respectively given by

\begin{align}
K^{00}(\omega, \mathbf{q}^2) &= -\left(\frac{e^2}{m^*}\right) \frac{1}{\omega^2 - \omega^2_c} \mathbf{q}^2 + \mathcal{O}(\mathbf{q}^4), \quad (15a) \\
\Sigma(\omega, \mathbf{q}^2) &= -\left(\frac{e^2}{m^*}\right) \left[\frac{(p_\uparrow - p_\downarrow)^2}{(p_\uparrow + p_\downarrow)^2} \frac{1}{\omega^2 - \omega^2_c} + \frac{p_\uparrow p_\downarrow}{p_\uparrow + p_\downarrow} \frac{1}{\omega^2 - \bar{\omega}^2_c}\right] \mathbf{q}^2 + \mathcal{O}(\mathbf{q}^4). \quad (15b)
\end{align}

We see from Eq. (15) that CDC preserves the Kohn mode of excitation. On the other hand, SDC shows a new mode of excitation at $\bar{\omega}_c$ apart from the actual cyclotron energy $\omega_c$. Interestingly, in the case of unpolarized QHS for which $p_\uparrow = p_\downarrow$, only the mode at $\bar{\omega}_c$ survives. This, in fact, gives the measure of energy scale for CF.

**B. Spin Transition**

At $\omega = 0$, SDC (15) can be written as

\begin{equation}
\Sigma(0, \mathbf{q}^2) = \mathbf{q}^2 \left(\frac{e^2 m^*}{4\pi^2 \rho}\right) \left[\frac{(p_\uparrow - p_\downarrow)^2}{(p_\uparrow + p_\downarrow)^2} \nu^2 + p_\uparrow p_\downarrow\right]. \quad (16)
\end{equation}

We see that $\Sigma(0, \mathbf{q}^2)$ given by the above expression plays an important role in the spin transitions. Eisenstein et al [21] and Engel et al [22] have observed spin transitions in QHS with filling fractions $\nu = 2/3$ and 3/5. By the increase of Zeeman energy, QHS at $\nu = 2/3$ ($p_\uparrow = p_\downarrow = -1, s = 1$) and $\nu = 3/5$ ($p_\uparrow = -2, p_\downarrow = -1, s = 1$) undergo a spin transition from their respective phase of no polarization and partial polarization to fully polarized phase ($p_\uparrow = 0$). In this context, we note that the effective number of LL acts as an order parameter in spin transition. Indeed, the ratio of the values of $\Sigma$ between the unpolarized and fully polarized phases is given by $\Sigma_{\text{unp}}/\Sigma_p = p_\uparrow^2/\nu^2$. Therefore, the ratio of $\Sigma(0, \mathbf{q}^2)$ in unpolarized and fully polarized phase would determine $p_\uparrow(= p_\downarrow)$ in the unpolarized phase unambiguously; the ratio does not depend on other parameters such as...
\( m^* \) which has complicated dependence on the magnetic field \([12–14]\). Similarly, the ratio of \( \Sigma(0, \mathbf{q}^2) \) in partially polarized and fully polarized would also determine \( p_\uparrow \) and \( p_\downarrow \) in partially polarized phase unambiguously. The order parameter shows a discontinuity in the spin transitions.

**C. Neutron Scattering**

In the standard neutron scattering experiment \([23]\), (in this case, the scattering is in the plane of the sample), the differential scattering cross section is given by

\[
\frac{d\sigma}{d\Omega} \propto \frac{k_f}{k_i} \left[ S_c(q) + \frac{\sigma_\Sigma}{\sigma_c} S_\Sigma(q) \right],
\]  

where \( k_i \) and \( k_f \) are the momentum of the incident and scattered neutrons, \( q = k_f - k_i \) is the momentum transfer. \( S_c(q) \) and \( S_\Sigma(q) \) are static charge and spin structure factors which are frequency integrated imaginary part the corresponding correlation functions. These are evaluated in this case, from Eq. (15), as

\[
S_c(q) = q^2 \left( \frac{e^2}{2} \right) \nu ,
\]  

\[
S_\Sigma(q) = q^2 \left( \frac{e^2}{2} \right) \left[ \frac{p_\uparrow p_\downarrow}{p_\uparrow + p_\downarrow} + \frac{(p_\uparrow - p_\downarrow)^2}{(p_\uparrow + p_\downarrow)^2} \nu \right].
\]  

Note that unlike the parent expressions in Eq. (15), the above expressions are free from the dependence on \( m^* \) which by now is known to possess a dependence on the magnetic field \([12–14]\). In the unpolarized phase, \( S_\Sigma(q) \propto p_\uparrow \). \( S_c(q) \) is proportional to \( \nu \) irrespective of the phase. (In the fully polarized phase, \( S_c(q) = S_\Sigma(q) \)). In Eq. (17), \( \sigma_\Sigma/\sigma_c \) is the ratio of the spin and charge dependent total cross sections. One can determine \( S_c \) and \( S_\Sigma \) in unpolarized or partially polarized phases by two different ways — (i) By the measurement of cross section in fully polarized phase, one will be able to extract \( S_c(q) \) since cross section is proportional to \( S_c(q) \). It is same in all phases. The same experiment in unpolarized or partially polarized phase, whichever is the relevant, has to be performed to know \( S_\Sigma(q) \). (ii) X-ray scattering experiment will measure \( S_c(q) \) and then neutron scattering would determine
\( S_\Sigma(q) \) with the knowledge of \( S_c(q) \). Particularly in the unpolarized phase, \( S_\Sigma(q) \) determines the composite fermion parameter \( p_\uparrow \). In summary, neutron scattering experiment provides a direct unambiguous test of CF. The accuracy of Eq. (18) lies on the region of small angle scattering as it is valid only for low \( q^2 \).

**D. Excitations and Raman Scattering**

We now determine the collective modes of CDE and SDE with respective spectral weights. We discuss the excitations for both unpolarized and fully polarized phases of the quantum Hall state with \( \nu = 2/3 \) only in detail, as the state is observed in both the phases \[21,22\], and partly for simplicity. The calculation for other states will follow a similar treatment. We use the same procedure as Lopez and Fradkin \[19\] who have worked out for fully polarized QHS. It should be possible to observe the modes by polarized and depolarized resonant Raman scattering. In this context, we note that in inelastic light scattering experiments, the magnetoplasmon modes of IQHE and FQHE state at \( \nu = 1/3 \) have been observed \[24,25\].

We first consider CDE for fully polarized phase of \( \nu = 2/3 \) \((p_\uparrow = -2, p_\downarrow = 0, s = 1)\) state. The modes are determined from the poles of \( K^{00} \). We look for the solutions of the form \[13\] \( \omega^2 = (k\bar{\omega}_c)^2 + \beta(q^2)\gamma \), where \( \bar{q}^2 = q^2l_0^2/2 \) with \( l_0 = (eB)^{-1/2} \) being the effective magnetic length, \( \beta \) and \( \gamma \) are two constants to be determined for the corresponding mode characterized by \( k \) (an integer). The values of \( k \) runs from 1 to 3. We find there are two modes for \( k = 2 \) whose dispersion relations are given by

\[
\omega^2_{2\pm} = (2\bar{\omega}_c)^2 + \beta_\pm \bar{q}^2 \tag{19}
\]

with the corresponding residues in \( K^{00} \) being

\[
\text{Res}(K^{00})|_{\omega_{2\pm}} = \mp \frac{1}{\pi\bar{\omega}_c} \frac{\beta_\pm}{\beta_+ - \beta_-} \left[ \beta_\pm - 3\bar{\omega}_c^2 \right] q^2 \bar{q}^2 , \tag{20}
\]

where

\[
\beta_\pm = \frac{\bar{\omega}_c^2}{10} \left( 180 \pm \sqrt{(180)^2 - 15360} \right) . \tag{21}
\]
We do not find any mode whose zero momentum gap is at $\bar{\omega}_c$. On the other hand, there are two modes at $\omega_c$ (for $q^2 = 0$) with the dispersion relations

$$\omega^2_\pm = \omega_c^2 - \bar{\omega}_c^2 \left[ \left( 14 + \frac{2m^* V(0)}{2\pi} \right) \mp \sqrt{ \left( 14 + \frac{2m^* V(0)}{2\pi} \right)^2 + 2700} \right] q^2$$  \hspace{1cm} (22)

provided the interaction potential $V(q)$ is a regular function at $q^2 = 0$. The residues in $K^{00}$ for the modes are proportional to $q^2$. The residues are given by

$$\text{Res}(K^{00})|_{\omega_\pm} = \pm q^2 \nu \frac{\omega_c}{8\pi} \frac{\sqrt{14 + \frac{2m^* V(0)}{2\pi} + \sqrt{\left( 14 + \frac{2m^* V(0)}{2\pi} \right)^2 + 2700}}}{\sqrt{\left( 14 + \frac{2m^* V(0)}{2\pi} \right)^2 + 2700}}.$$  \hspace{1cm} (23)

These modes have higher spectral weights compared to the modes $\omega_{2\pm}$.

In the unpolarized phase of $2/3 \ (p_\uparrow = p_\downarrow = -1, \ s = 1)$ state, the CDE modes for $k = 2$ are given by

$$\omega^2_{2\pm} = (2\bar{\omega}_c)^2 + \alpha_{\pm} q^2$$  \hspace{1cm} (24)

with the residues in $K^{00}$ are proportional to $q^4$ and they are given by

$$\text{Res}(K^{00})|_{\omega_{2\pm}} = \pm \frac{1}{\pi \bar{\omega}_c} \frac{\alpha_{\pm}}{\alpha_+ - \alpha_-} \left[ \alpha_{\pm} - 3\bar{\omega}_c^2 \right] q^2 q^2,$$  \hspace{1cm} (25)

where

$$\alpha_{\pm} = \frac{\bar{\omega}_c^2}{10} \left( 48 \pm \sqrt{(48)^2 - 1920} \right),$$  \hspace{1cm} (26)

The other two modes for which the zero momentum gaps are at $\omega_c$ follow

$$\omega^2_{\pm} = \omega_c^2 - \frac{\bar{\omega}_c^2}{20} \left[ \left( 206 + 20 \frac{2m^* V(0)}{2\pi} \right) \mp \sqrt{ \left( 206 + 20 \frac{2m^* V(0)}{2\pi} \right)^2 + 61 \times (120)^2} \right] q^2$$  \hspace{1cm} (27)

with the corresponding spectral weights are proportional to $q^2$. The residues of $K^{00}$ corresponding to these modes are given by

$$\text{Res}(K^{00})|_{\omega_{\pm}} = \pm q^2 \nu \frac{\omega_c}{8\pi} \frac{\sqrt{206 + 20 \frac{2m^* V(0)}{2\pi} + \sqrt{\left( 206 + 20 \frac{2m^* V(0)}{2\pi} \right)^2 + 61 \times (120)^2}}}{\sqrt{\left( 206 + 20 \frac{2m^* V(0)}{2\pi} \right)^2 + 61 \times (120)^2}}.$$  \hspace{1cm} (28)
Similar to the fully polarized phase, no mode exists for CDE at $\bar{\omega}_c$ (for $q^2$).

We now determine SDE in the unpolarized phase from the poles of $\Sigma_{\text{unp}}(\omega, q^2)$ in Eq. (14). Interestingly, SDE are at $\omega_k = k\bar{\omega}_c$ ($k$ an integer) which do not disperse with $|q|$. Note that, unlike the CDE, SDE have a mode at $\omega = \bar{\omega}_c$. The residue in $\Sigma$ for the mode $\omega = \bar{\omega}_c$ is $\text{Res}(\Sigma) = \omega_c \nu \pi q^2$. The spectral weights corresponding to other modes are proportional to $q^2 k$.

We report here that for unpolarized QHS, the SDE have only one dispersionless mode at $\omega = \bar{\omega}_c$. All the other modes $\omega_k$ disperse with $|q|$ from the zero momentum value $k\bar{\omega}_c$. The residue in $\Sigma$ for the mode $\omega = \bar{\omega}_c$ is proportional to $q^2$ and for all other dispersed modes $\omega_k (k \neq 1)$, they are down by a factor $q^{2(k-1)}$. Therefore in the unpolarized and partially polarized phase, unlike in the fully polarized phase, SDE are very different from CDE. Similarly for IQHE states, as have been obtained by Kallin and Halperin [26], CDE and SDE are same for fully polarized states, but they differ for partially polarized and unpolarized states. Longo and Kallin [27] have studied spin-flip and spin-wave excitations (which we do not consider here) recently.

By polarized and depolarized Raman scattering experiments, the modes of CDE and SDE can respectively be found out. The Raman intensity $I(\omega)$ is proportional to the imaginary part of the corresponding correlation functions [28].

In the limit $q^2 l_0^2 \ll 1$, most of the weight of CDC is in the cyclotron modes i.e., at $\omega_\pm$ (22 and 27) for both unpolarized and fully polarized phases. The accumulated contributions of these modes, in fact, saturate the f-sum rule. The modes are degenerate in the limit $q^2 \to 0$. The relative intensity for these modes is given by $I(\omega_+) / I(\omega_-) \sim 1$. The splitting between the two modes $\Delta \omega^2 = \omega_+^2 - \omega_-^2$ is proportional to $q^2$. The pole in CDC for the excitation frequencies $\omega_{2\pm}$ may be read off from Eqs. (14) and (24). Thus the intensities corresponding to these modes $\omega_{2\pm}$ will be suppressed by a factor of $q^2$ than the same for $\omega_{\pm}$ modes.

The situation for SDE in fully polarized and unpolarized phases are very different. Depolarized Raman scattering experiment in fully polarized phase creates a spectra very similar to the one in the polarized Raman scattering experiment because CDE and SDE are same
in this phase. On the other hand, in depolarized Raman scattering in the unpolarized phase, the highest intensity will be observed for the mode which is \textit{exactly} at $\bar{\omega}_{c}$. The intensity corresponding to the next higher mode at $2\bar{\omega}_{c}$ is suppressed by a factor of $\mathbf{q}^2$ than the mode at $\bar{\omega}_{c}$. Similarly the intensity for other modes are further down by factors of $\mathbf{q}^2$ compared to the previous lower mode. Although we have discussed depolarized Raman spectra only for $\nu = 2/3$ state in unpolarized phase, it is easy to check that the characteristics of the spectra will be similar for all other unpolarized QHS. Indeed, in all unpolarized and partially polarized QHS, depolarized Raman spectra have the highest intensity corresponding to the frequency $\omega = \bar{\omega}_{c}$, the effective cyclotron frequency for CF. In other words, the total effective number of LL ($p_{\uparrow} + p_{\downarrow}$) filled by CF can be determined by the depolarized Raman scattering experiments in the unpolarized or partially polarized phases of relevant QHS \cite{18} as $\bar{\omega}_{c} = (2\pi \rho / m^{*})(1/(p_{\uparrow} + p_{\downarrow}))$. This determination will become exact if the effective mass $m^{*}$ is determined independently. In any case, $\bar{\omega}_{c}$ as the relevant scale would again be established by this experiment. Importantly, note that this mode at $\bar{\omega}_{c}$ is $\mathbf{q}^2$ independent.

We remark that polarized Raman scattering in any of the quantum Hall phases and depolarized Raman scattering in fully polarized phase measure the actual filling fraction $\nu$. On the other hand in unpolarized and partially polarized phase, depolarized Raman scattering measures the filling fraction of CF.

\section*{IV. CONCLUSION}

In summary, we state the most important results. Spin density correlation (16) represents an order parameter in the spin transitions from unpolarized or partially polarized phases to the fully polarized phase of the relevant quantum Hall states \cite{18} as the Zeeman energy is increased. Spin density correlation shows an undispersed mode at $\bar{\omega}_{c}$ in the unpolarized and partially polarized phases. The spin density excitations in these phases are very different from charge density excitations. Neutron scattering and depolarized Raman scattering experiments would directly determine one of the composite fermion parameters,
viz, the effective number of filled Landau levels by CF. The other parameter may be find out from the knowledge of the former.

Finally, Lopez and Fradkin [29] have studied recently the bilayered QHS employing a similar model as ours. However, the two models leads to certain different physical consequences. (For detailed comparison between the two models, see Ref. [17]). It might be of interest to examine whether there is some experimental procedure that can determine the composite fermion parameters in bilayered systems as well.
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creates a net charge of magnitude \( -e/(2p+1) \) locally and pushes the missing charge
\( -2pe/(2p+1) \) to the the boundary of the system as the addition of two flux quanta
creates a local charge deficit of \( -2pe/(2p+1) \). The charges at the boundary get cancelled
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