Research on the Test-times Selection Method for the Accuracy Evaluation of Inertial Navigation System

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Abstract—Aiming at the unreasonable test-times selection method in the precision evaluation of inertial navigation system, this paper puts forward the test-times selection method. The test conclusion confidence, test condition confidence, and the sample confidence limit are included in this test-times selection method for quantitative test-times. In addition, the selection process is illustrated through an example.

1. Introduction
The inertial navigation system is the main navigation equipment and attitude reference of all kinds of carriers, especially for submarines. Therefore, the results of the acceptance, finalization and identification tests of the inertial navigation system reflect the performance of the system, and the test results should be authentic [1]. The selection of test-times is directly related to the credibility of the accuracy evaluation results of inertial navigation system, which should be reasonably selected according to scientific methods.

Zheng Zizhen et al. systematically discussed the design and requirements of the ship inertial navigation test [2-3]. The credibility of the circular probability error index of ship inertial navigation system was analyzed by Chen Rongjuan et al[4]. Fan Yonghua et al. have developed a multi-channel data acquisition and precision evaluation system of integrated navigation system, and successfully applied the test and precision evaluation system to a novel type of marine INS test[5]. At present, there is no data in foreign countries that can guide the strict selection method of test-times for precision evaluation of inertial navigation system in actual sea trials. In the fields of domestic ships and aviation, 8 times are often used in the accuracy evaluation tests of inertial navigation system[1]. Reference [1] pointed out that choosing 8 times as test-times is mainly due to the factors such as long test period of inertial navigation system, difficulty in field test measurement, and the serious constraints of weather factors, so as to minimize the number of voyages, which was the result of multiple trade-offs. However, the practice of selecting 8 times as test-times only took into account the acceptability of both parties of the trial, without considering the different test samples and test conditions in various trials, which might result in the demand for the selection of different test times. Therefore, according to the specific conditions of the tests and the statistical theory, it is of great practical value to find a method to determine the test-times.
2. Selection method of test-times

In view of the above problems, the test-times selection method is proposed. Starting from the test conclusion confidence, the method comprehensively considers the test sample confidence and the test condition confidence. Then, according to the acceptable sample confidence limit, the test-times should be selected for accuracy evaluation of the inertial navigation system, which is calculated through the quantitative relationship.

The test-times selection is as follows.

(1) Determine the confidence of the test conditions according to the test conclusion confidence set by the receiver, analyze the test conditions according to the test conditions, and calculate the confidence of the test samples.

(2) Establishes the relationship among test sample confidence, test sample confidence limit and the number of trials.

(3) According to the confidence level of the test sample calculated in (1) and the confidence limit of the test samples that the receiver can obtain, select the test-times according to the corresponding relationship.

As mentioned in the above test number selection method, the test conclusion confidence refers to the confidence degree of test conclusion acceptable to the product receiver, which is determined by test sample confidence and test condition confidence. Its value is the product of the test sample confidence and test condition confidence, which is expressed as a percentage. Test condition confidence refers to the possibility that the true error will be covered by the test samples under certain test conditions, which are expressed as a percentage, and are jointly determined by the relative accuracy of the measurement system, the test environment and the accuracy of the admission equipment. Test sample confidence refers to the probability that the error confidence intervals estimated by multiple test samples selected for the accuracy assessment can cover the true error of the system, which is expressed as a percentage. The computational expression for the confidence of the test samples is:

\[
\text{Test sample confidence} = \frac{\text{test conclusion confidence}}{\text{test condition confidence}}
\]

3. Correspondence among confidence, confidence limit and number of test samples

Test sample confidence limit refers to the deviation degree among systematic error estimation, random error estimation and random error in the test samples. The extent to which the systematic error estimation deviates from the systematic error is called the mean confidence limit of the test samples, and the extent that the random error estimate deviates from the random error is called the standard deviation of the test sample confidence’s limit. The correspondence among the confidence, confidence limit and number of trials is:

3.1 Correspondence among confidence, mean confidence limit and test-times\(^{[8]}\)

\[
\frac{\bar{X} - S_{t(\alpha/2)}(N-1)}{\sqrt{N}} \leq \mu_x \leq \frac{\bar{X} + S_{t(\alpha/2)}(N-1)}{\sqrt{N}}
\]

In the formula:

- \(\mu_x\) - Systematic error of inertial navigation system.
- \(\bar{X}\) - Estimated error of inertial navigation system.
- \(S\) - Estimated value of standard deviation of inertial navigation system.
- \(N\) - Test-times;
- \(\alpha\) - Significance level, which is obtained by test sample confidence \(\lambda\), where \(\alpha = 1 - \lambda\).
- \(t_{\alpha/2}(N-1)\) - The t distribution function when the degree of freedom is N-1 and the test sample confidence is \(\lambda\), which is available from the t distribution table; where \(t_{\beta}(n)\) satisfies \(P\{t(n) > t_{\beta}(n)\} = \beta\).
\[ t_{\alpha/2}(N-1)\sqrt{N} \] -Mean confidence limit of test samples.

3.2 Correspondence among confidence, standard deviation confidence limit and test-times\[5\]

\[ S\sqrt{(N-1)/\chi^2_{\alpha/2}(N-1)} \leq \sigma_x \leq S\sqrt{(N-1)/\chi^2_{1-\alpha/2}(N-1)} \tag{3} \]

where,

\( \sigma_x \) - Standard deviation of inertial navigation system;

\( \chi^2_{\alpha/2}(N-1) \) \( \chi^2_{1-\alpha/2}(N-1) \) - The \( \chi^2 \) distribution function when the degree of freedom is N-1 and the test sample confidence is \( \lambda \), which is available from the \( \chi^2 \) distribution table;

\( \sqrt{(N-1)/\chi^2_{\alpha/2}(N-1)} \) \( \sqrt{(N-1)/\chi^2_{1-\alpha/2}(N-1)} \) - Standard deviation confidence limit of test samples.

The test sample confidence limit accepted by the receiver is determined by the deviation degree between the systematic error estimation and the random error accepted by the receiver. The test sample confidence limit is set by the receiver and there is only one of the mean confidence limit and the test sample, which is usually on the mean confidence limit of the test sample.

4. Selection process of test-times

The accuracy evaluation of inertial navigation system is independent of each trial, so it can be assumed that each trial error conforms to the normal distribution (but the less the number of trials, the greater the difference from the normal distribution). The system error of inertial navigation system meets the t distribution and the standard deviation \( \chi^2 \)\[8\].

When the system standard deviation is unknown, the sample systematic error is the t distribution and the test statistic is as follows:

\[ t = \left( \overline{X} - \mu_x \right) \sqrt{N}/S \tag{4} \]

In the formula:

\( \mu_x \) - Systematic error of inertial navigation system.

\( \overline{X} \) - Estimated error of inertial navigation system system.

\( S \) - Estimated value of standard deviation of inertial navigation system.

\( N \) - Test-times.

Among them:

\[ \overline{X} = \frac{1}{m} \sum_{i=1}^{m} \Delta x_i \tag{5} \]

\[ S = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} (\Delta x_i - \overline{X})^2} \tag{6} \]

where,

\( \Delta x_i \) - Inertial navigation system error at \( i \) sampling point.

\( m \) - Number of measuring points for per valid test.

When the degree of freedom is N-1, and the confidence of the test sample is \( \lambda \), the acceptance region is:

\[ |x| \leq t_{\alpha/2}(N-1) \tag{7} \]

That is:

\[ -t_{\alpha/2}(N-1) \leq x \leq t_{\alpha/2}(N-1) \tag{8} \]
Substitute the $t$ to:

$$\begin{align*}
-t_{\alpha/2}(N-1) & \leq \left( \bar{X} - \mu \right) \sqrt{N/S} \leq t_{\alpha/2}(N-1) \\
\bar{X} - St_{\alpha/2}(N-1)/\sqrt{N} & \leq \mu \leq \bar{X} + St_{\alpha/2}(N-1)/\sqrt{N}
\end{align*}$$

where:

- $\alpha$ - Significance level, which is obtained by test sample confidence $\lambda$, where $\alpha = 1 - \lambda$;
- $t_{\alpha/2}(N-1)$ - The t distribution function when the degree of freedom is N-1 and the test sample confidence is $\lambda$, which is available from the t distribution table; where $t_\beta(n)$ satisfies $P\{t(n) > t_\beta(n)\} = \beta$;
- $t_{\alpha/2}(N-1)/\sqrt{N}$ - Mean confidence limits of test samples.

The above relation gives the degree of the systematic error estimation of the inertial navigation system deviating from the systematic error, which is determined by the mean confidence limit of the test samples, and the relationship between the test sample confidence limits and the test number is given.

Similarly, when the mean value of the system is unknown, the sample standard deviation conforms to $\chi^2$ distribution, and the test statistic is as follows:

$$\chi^2 = (N-1)S^2 / \sigma_x^2$$

where,

- $\sigma_x$ - Standard deviation of inertial navigation system;
- $S$ - Estimated value of standard deviation of inertial navigation system, where

$$S = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} (\Delta x_i - \bar{X})^2}$$

$N$ - Test-times;

When the standard deviation of the inertial navigation system is $\sigma_x$, the acceptance domain is:

$$\chi^2_{1-\alpha/2}(N-1) \leq \chi^2 \leq \chi^2_{\alpha/2}(N-1)$$

Substitute $\chi^2$ into the above formula:

$$\begin{align*}
\chi^2_{1-\alpha/2}(N-1) & \leq (N-1)S^2 / \sigma_x^2 \leq \chi^2_{\alpha/2}(N-1) \\
(N-1)S^2 / \chi^2_{\alpha/2}(N-1) & \leq \sigma_x^2 \leq (N-1)S^2 / \chi^2_{1-\alpha/2}(N-1)
\end{align*}$$

where,

- $\alpha$ - Significance level, obtained by test sample confidence $\lambda$, where $\alpha = 1 - \lambda$;
- $\chi^2_{\alpha/2}(N-1)$ - The $\chi^2$ distribution function when the degree of freedom is N-1 and the test sample confidence is $\lambda$, which is available from the $\chi^2$ distribution table;
- $\sqrt{(N-1)/\chi^2_{\alpha/2}(N-1)}$, $\sqrt{(N-1)/\chi^2_{1-\alpha/2}(N-1)}$ - Confidence limits of standard deviation of test samples.

Similarly, the above relation gives the degree of the standard deviation estimated from the standard deviation of the standard deviation confidence limit of the test samples, and the confidence limit between the test samples and the number of test samples.
One effective test was recorded as generating one test sample, and n effective tests were executed to generate n test samples. Test sample confidence represents the probability that the error confidence intervals estimated by the n test samples can cover the real error of the system, representing the probability that the actual error of the system falls within the error range of sample estimation.

When the test sample confidence is fixed, the test sample confidence limits narrow with the increase number of trials, indicating that the credibility of the test confidence increases, and the change stabilizes after a certain number of times. Therefore, the above relationship can quantitatively determine the test-times to be selected when determining the confidence limit of the test samples and the confidence limit of the test samples.

5. Select examples of the test-times
The following example describes the method and procedure of test number selection:

(1) Calculate the confidence of the test samples. It is assumed that the receiver specifies that the test conclusion confidence is 95% in the general documents, such as the general development requirements. At the same time, there is a big gap between the relative accuracy of measurement system and the test system, and the good test environment and high accuracy of the admission equipment are large, with the 100% of assessed confidence of the test conditions. Therefore, the confidence is 95% between the confidence of test conclusion and test sample confidence and test condition confidence.

(2) Establish the mutual relationship among the test sample confidence, the confidence limit of the test samples and the number of trials. Based on the confidence limit of the test sample and the relational expression between the confidence limit and the average confidence limit of the test sample, the confidence limit of the test sample is 95%. For 90% and 95% confidence levels of common test samples, the common relationship table between test sample mean confidence limit and test sample standard deviation confidence limit can be established. For confidence common test samples, a general relational table of confidence limits of test samples mean and standard deviation can be established.

(3) Select the number of trials. When the receiver proposes the acceptable confidence limit of the mean value of the test samples, the number of tests selected shall be the minimum confidence limit of the mean value of the test number on the curve. For example, with the 95% of test sample confidence, and when the receiver proposed that the acceptable test sample mean confidence limit of ±1, six trials can be determined by the correspondence or graph check (corresponding to Figure 1). Similarly, when the receiver proposes the confidence limit of the acceptable standard deviation of the test sample, the number of tests to be selected can be determined by correspondence or chart inspection (corresponding to Figure 2). For 90% and 95% confidence of commonly used test samples, the number of tests can also be completed by checking the table for selection (corresponding to Table 1).

Fig.1 Relationship curve between the number of trials and the mean confidence limit of the test samples at 95% confidence level
6. Conclusions
Through solving the unreasonable test selection method for accuracy evaluation of inertial navigation system, a quantitative test selection method based on the test conclusion confidence set by the receiver, the confidence of the test condition for the actual evaluation, and the test samples accepted by the receiver is proposed.

(1) The influence of different test samples and different test conditions on the number of tests is fully considered.
(2) The number of times to complete the test can be selected according to the receiver's confidence in the test and the actual test conditions.

(3) The number of tests completed by this method can improve the cost-effectiveness ratio of accuracy assessment.

References

[1] Z.Z. Zheng and D.Y. Liu. (2006) Sea Experiment for Marine Inertial Navigation System (in Chinese). National Defense Industry Press. Beijing.

[2] R.J. Chen, H.B. Wang, H.C. Hu, et al. (2015) Simulation Analysis on Reliability of CEP of Ship INS (in Chinese). Navigation of China. pp,38(2),3-5.

[3] G.Q. Shi, X.G. Gao, M.S. Wu. (2011) Reliability Evaluation of Inertial Navigation Simulation System Based on Grey Clustering Method (in Chinese). Journal of Northwestern Polytechnical University. pp, 29(06),960-964.

[4] Y.H. Fan, F. Zha and J.S. Li. (2012) Multi-channel Data Acquisition and Precision Evaluation System for Integrated Navigation System (in Chinese). Ship Electronic Engineering. pp,32(09):135-137+143.

[5] S. Zhou, S.Q. Xie and C.Y. Pan. (2008) Probability Theory and Mathematical Statistic (in Chinese). Higher Education Press. Beijing.