Reaction cross section of proton scattering consistent with PREX-II

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**Background:** The neutron skin thickness \( R_{\text{skin}} \) of PREX-II is presented in Phys. Rev. Lett. **126**, 172502 (2021). The reaction cross section \( \sigma_R \) is useful to determine the matter radius \( R_m \) and \( R_{\text{skin}} \). For proton scattering, the reaction cross section \( \sigma_R \) are available for \( E_{\text{lab}} \geq 400 \text{ MeV} \).

**Method and results:** We determine \( R_{\text{PV}}^{\text{exp}} \) = 5.727 ± 0.071 fm and \( R_{\text{PV}}^{\text{exp}} \) = 5.617 ± 0.044 fm from \( R_{\text{PV}}^{\text{exp}} = 5.444 \text{ fm} \) and \( R_{\text{PV}}^{\text{skin}} \). The \( R_{\text{PV}}^{\text{GHFB}} \) calculated with Gongny-D1S HFB (GHFB) with the angular momentum projection (AMP) agrees with \( R_{\text{PV}}^{\text{exp}} \). The neutron density calculated with GHFB+AMP is scaled so as to \( R_{\text{PV}}^{\text{scaling}} \) = 5.727 fm. The Love-Franey \( t \)-matrix model with the scaled densities reproduces the data on \( \sigma_R \).

**Aim:** Our aim is to find the \( \sigma_R \) of proton scattering consistent with \( R_{\text{PV}}^{\text{PV}} \).

**Conclusion:** The \( \sigma_R \) of proton scattering consistent with \( R_{\text{PV}}^{\text{PV}} \) are \( \sigma_R^{\text{PV}} \) at \( E_{\text{lab}} = 534.1, 549, 806 \text{ MeV} \).

I. INTRODUCTION AND CONCLUSION

**Background:** Horowitz et al. [1] proposed a direct measurement for neutron skin \( R_{\text{skin}} \). The measurement is composed of parity-violating (PV) weak scattering and elastic electron scattering. The neutron radius \( R_n \) is determined from the former experiment, whereas the proton radius \( R_p \) is from the latter.

Very recently, the PREX collaboration presented the PREX-II value [2]:

\[
R_{\text{PV}}^{\text{PV}} = 0.283 \pm 0.071 \text{ fm},
\]

combining the original Lead Radius EXperiment (PREX) result [3,4] with the updated PREX-II result. The \( R_{\text{PV}}^{\text{PV}} \) value is most reliable at the present stage, and provides crucial tests for the equation of state (EoS) of nuclear matter [5-9] as well as nuclear structure and reaction. In particular, Reed et al. [10] report a value of the slope parameter of the EoS and experiment on the impact of such a stiff symmetry energy on some critical neutron-star observables. The \( R_{\text{PV}}^{\text{PV}} \) value is considerably larger than the other experimental values which are significantly model dependent [11-14].

The nonlocal dispersive-optical-model (DOM) analysis of \( ^{208}\text{Pb} \) deduces \( R_{\text{PV}}^{\text{DOM}} = 0.25 \pm 0.05 \text{ fm} \) [15]. The chiral (Kyushu) \( g \)-matrix folding model determines \( R_{\text{PV}}^{\text{PV}} = 0.27 \pm 0.03 \text{ fm} \) from reaction cross section \( \sigma_R \) in 30 ≤ \( E_{\text{lab}} \) ≤ 100 MeV [16]. These values are consistent with \( R_{\text{PV}}^{\text{PV}} \).

**Aim:** The aim is to find the \( \sigma_R \) of \( p + ^{208}\text{Pb} \) scattering that supports \( R_{\text{PV}}^{\text{PV}} \) (PREXII).

**Method and results:** The reaction cross section \( \sigma_R \) is a powerful tool of evaluating the matter radius \( R_m \). We first determine \( R_{\text{PV}}^{\text{exp}} = 5.727 \pm 0.071 \text{ fm} \) and \( R_{\text{PV}}^{\text{exp}} = 5.617 \pm 0.044 \text{ fm} \) from \( R_{\text{PV}}^{\text{exp}} = 5.444 \text{ fm} \) and \( R_{\text{PV}}^{\text{PV}} \). The \( R_{\text{PV}}^{\text{GHFB}} \) calculated with Gongny-D1S HFB (GHFB) with the angular momentum projection (AMP) agrees with \( R_{\text{PV}}^{\text{exp}} \) of electron scattering.

The neutron density calculated with GHFB+AMP is scaled so as to \( R_{\text{PV}}^{\text{scaling}} \) = 5.727 fm. The Love-Franey \( t \)-matrix model with the scaled densities reproduces the data on \( \sigma_R \).

**Conclusion:** The \( \sigma_R \) of proton scattering consistent with \( R_{\text{PV}}^{\text{PV}} \) are \( \sigma_R^{\text{PV}} \) at \( E_{\text{lab}} = 534.1, 549, 806 \text{ MeV} \).

II. MODEL

Our model is the folding model based on Love-Franey (LF) \( t \)-matrix [18].

The formulation of the folding model is shown below. For proton-nucleus scattering, the potential \( U(R) \) between an incident proton and a target (T), has the direct and exchange parts, \( U^{\text{DR}} \) and \( U^{\text{EX}} \), as

\[
U^{\text{DR}}(R) = \sum_{\mu} \int \rho^\mu_T(\mathbf{r}_T) \frac{e^{i \mathbf{K} \cdot s}}{\mu - 1/2, \nu} \mathbf{d}\mathbf{r}_T, \quad (2a)
\]

\[
U^{\text{EX}}(R) = \sum_{\mu} \int \rho^\mu_T(\mathbf{r}_T, \mathbf{r}_T + s) \cdot \frac{e^{i \mathbf{K} \cdot s}}{\mu - 1/2, \nu} \mathbf{d}\mathbf{r}_T, \quad (2b)
\]

where \( \mathbf{K}(R) \) is the local momentum between an incident proton and T, and \( M = A/(1 + A) \) for the target mass number \( A \); see Ref. [20] for the validity of the localization.

The direct and exchange parts, \( t_{\mu \nu}^{\text{DR}} \) and \( t_{\mu \nu}^{\text{EX}} \), of the matrix
are described as
\[
\begin{align*}
\hat{\rho}_{\mu\nu}^{\text{DR}}(s) &= \frac{1}{8} \sum_{S,T} \hat{S}_T \hat{S}_S^{\mu\nu}(s) + \text{for } \mu + \nu = \pm 1 \\
&= \frac{1}{8} \sum_{S,T} \hat{S}_T \hat{S}_S^{\mu\nu}(s) + \text{for } \mu + \nu = 0 \\
\hat{\rho}_{\mu\nu}^{\text{EX}}(s) &= \frac{1}{8} \sum_{S,T} (-1)^{S+T} \hat{S}_T \hat{S}_S^{\mu\nu}(s) + \text{for } \mu + \nu = \pm 1 \\
&= \frac{1}{8} \sum_{S,T} (-1)^{S+T} \hat{S}_T \hat{S}_S^{\mu\nu}(s) + \text{for } \mu + \nu = 0
\end{align*}
\]
where \(\hat{S} = \sqrt{2S + 1}\) and \(\hat{S}_T \hat{S}_S^{\mu\nu}\) are the spin-isospin components of the \(t\)-matrix interaction. We apply the LF \(t\)-matrix folding model for \(p^{+208}\text{Pb}\) scaling in \(E_{\text{lab}} = 400, 534.1, 549, 806\) MeV.

The relative wave function \(\psi\) is decomposed into partial waves \(\chi_L\), each with different orbital angular momentum \(L\). The elastic \(S\)-matrix elements \(S_L\) are obtained from the asymptotic form of the \(\chi_L\). The total reaction cross section \(\sigma_R\) is calculable from the \(S_L\) as
\[
\sigma_R = \frac{\pi}{K^2} \sum_L (2L + 1)(1 - |S_L|^2),
\]
where \(hK\) is an incident momentum.

As proton and neutron densities, \(\rho_p = -\frac{1}{2}\) and \(\rho_n = \frac{1}{2}\), we use the densities calculated with GHFB+AMP \(21\). As a way of taking the center-of-mass correction to the densities, we use the method of Ref. \(22\), since the procedure is quite simple. The \(R_p^{\text{GHFB}}\) calculated with GHFB+AMP agrees with \(R_p^{\text{exp}} = 5.444\) fm \(17\). The neutron density calculated with GHFB+AMP is scaled so as to \(R_n^{\text{scaling}} = 5.727\) fm (the central value of \(R_n^{\text{exp}} = 5.727 \pm 0.071\) fm determined in Sec. \(1\)). The scaled densities based on \(R_n^{\text{skin}}\) and \(R_n^{\text{exp}}\) are used for analyses of \(p^{+208}\text{Pb}\) scattering.

Now we explain the scaling of density \(\rho(r)\). We can obtain the scaled density \(\rho_{\text{scaling}}(r)\) from the original density \(\rho(r)\) as
\[
\rho_{\text{scaling}}(r) = \frac{1}{\alpha^3} \rho(r/\alpha)
\]
with a scaling factor
\[
\alpha = \sqrt{\frac{\langle r^2 \rangle_{\text{scaling}}}{\langle r^2 \rangle}}.
\]

### III. RESULTS

The LF \(t\)-matrix folding model with the GHFB+AMP densities underestimates the \(\sigma_R\) data in \(400 \leq E_{\text{lab}} \leq 900\) MeV only by a factor of \(0.96\), as shown in Fig. \(1\). The LF \(t\)-matrix folding model with the scaled densities reproduces the data in \(E_{\text{lab}} = 534.1, 549, 806\) MeV. This indicates that the LF \(t\)-matrix folding model with the scaled densities is useful in \(400 \leq E_{\text{lab}} \leq 900\) MeV.

![FIG. 1. E_{lab} dependence of reaction cross sections \(\sigma_R\) for p+^{208}\text{Pb} scattering. Open circles stand for the results of the LF \(t\)-matrix folding model with GHFB+AMP densities, whereas closed circles correspond to those of the LF \(t\)-matrix folding model with the scaled densities. The data are taken from Refs. \(23, 24\).](image)

### IV. DISCUSSIONS

Now we discuss how good the LF \(t\)-matrix folding model with the scaled densities is for \(p^{+12}\text{C}\) scattering at \(E_{\text{lab}} = 800\) MeV and \(p^{+40}\text{Ca}\) scattering at \(E_{\text{lab}} = 700\) MeV.

For \(^{40}\text{Ca}\), Zenihiro \textit{et al.} determined neutron radius \(R_n^{\text{exp}} = 3.375\) fm, \(R_p^{\text{exp}} = 3.385\) fm and \(R_n^{\text{exp}} = -0.01 \pm 0.049\) fm from the differential cross section and the analyzing powers for \(p^{+40}\text{Ca}\) scattering \(25\). The GHFB+AMP densities are scaled so as to \(R_p^{\text{scaling}} = R_p^{\text{exp}}\) and \(R_n^{\text{scaling}} = R_n^{\text{exp}}\).

For \(^{12}\text{C}\), Tanihata \textit{et al.} determined matter radius \(R_m^{\text{exp}} = 2.35(2)\) fm from interaction cross sections \(\sigma_I\) \(26\). We deduce neutron radius \(R_n^{\text{exp}} = 2.37\) fm from the \(R_m^{\text{exp}}\) and the \(R_p^{\text{exp}} = 2.33\) fm of electron scattering. The GHFB+AMP densities are scaled so as to \(R_p^{\text{scaling}} = R_p^{\text{exp}}\) and \(R_n^{\text{scaling}} = R_n^{\text{exp}}\).

Figure \(2\) shows \(\sigma_R\) for \(p^{+40}\text{Ca}\) scattering at \(700\) MeV and \(p^{+12}\text{C}\) scattering at \(800\) MeV. The LF \(t\)-matrix folding model with the scaled densities is good for \(p^{+40}\text{Ca}\) scattering at \(E_{\text{lab}} = 700\) MeV, and almost reproduces the data for \(p^{+12}\text{C}\) scattering at \(E_{\text{lab}} = 800\) MeV.
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