Tit for Tat: Cooperation, communication, and how each could stabilize the other

by

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Abstract

Explanations for altruism, such as kin selection, reciprocity, indirect reciprocity, punishment, and genetic and cultural group selection, typically involve mechanisms that make altruists more likely to benefit from the altruism of others. In the case of kin altruism and reciprocity, individuals use private information to identify targets of their altruism. In the case of indirect reciprocity, where individuals cooperate with high-reputation individuals, outside information is required: unless agents can directly observe every interaction, communication between agents disseminates reputation information. But most accounts of indirect reciprocity take as given a truthful and misunderstanding-free communication system. In this paper, we seek to explain how such a communication system could remain evolutionarily stable in the absence of exogenous pressures. Specifically, we present three conditions that together allow signaling and (altruistic) cooperation to interact in a way that maintains both the effectiveness of the signal and the prevalence of cooperation. The conditions are that individuals (1) can signal about who is truthful, requiring a vital conceptual slippage between cooperation/defection and truthfulness/deceit, (2) make occasional mistakes, demonstrating how error can create stability by expressing all information about agents’ strategies, and (3) use a norm that rewards defection against defectors, confirming that the norms encoded by a communication system determine its stability.

Keywords

signaling · indirect reciprocity · cooperation · prisoner’s dilemma · norms

Highlights

• We find a state in which cooperation and communication stabilize each other

• A signaling system intended initially for stabilizing cooperation can be co-opted to also stabilize itself

• Errors in cooperation decisions bring out unexpressed information that would otherwise allow for the invasion and demise of cooperative equilibria

• Norms, when followed by gossipers, behave differently from norms followed by direct observers. In the latter case, the norms of invaders affect only their own actions, but in the former, they can affect the actions of conforming agents with whom they gossip

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I Introduction

Explanations for the evolution of altruism (helping others at a cost to oneself) including kin altruism [Hamilton, 1964], reciprocity [Trivers, 1971], indirect reciprocity [Nowak and Sigmund, 1998], punishment [Boyd et al., 2003], and group selection [Bowles et al., 2003], all ensure that altruists are more likely than non-altruists to benefit from the altruism of others [Henrich, 2004, Queller, 1992b].

Kin altruism rests on the correlation between an individual’s altruistic tendencies and that of their “social circle,” which presumably consists disproportionately of genetic relatives [Queller, 1992a]. Direct reciprocity leads individuals to withhold their (altruistic) cooperation from defectors [Brown et al., 1982]. Under indirect reciprocity, individuals cooperate with people who have cooperated with others, obviating the need for repeated interactions [Panchanathan and Boyd, 2003]. Altruistic punishers coerce compatriots into altruistic behavior, making the punishers more likely to receive benefits of altruism [Boyd and Richerson, 1992]. And with group selection, groups that happen to have high numbers of altruists are the ones most likely to spread, so most altruists will be concentrated in groups with high densities of altruists [Odouard et al., 2021]. In all these ways, altruism produces a second-order benefit compensating for its first-order cost, making it potentially evolutionarily viable.

Large groups hinder the ability to direct altruism towards altruists, because (1) their interactions are more likely to occur between non-kin, weakening kin selection, (2) they cluster closer to the population average, reducing between-group variance that group selection relies on [Maynard Smith, 1976, Nowak, 2006], and (3) they make repeated interactions rare, weakening reciprocity [Brown et al., 1982, Henrich, 2004].

Indirect reciprocity mitigates the large-group problem, as individuals do not need to interact directly to know each other’s reputations. But vitally, if direct observation of every interaction is infeasible, communication is required to disseminate reputations.

Most models simply assume the existence of an informative and truthful communication system. In this paper, by contrast, we don’t assume that such a communication system exists, and instead consider the conditions under which communication and cooperation can stabilize each other. Specifically, we ask

under what conditions does the interaction between signaling and cooperation stabilize both high levels of (altruistic) cooperation and a truthful and informative communication system?

To answer this question, we searched for stable, high-cooperation states in an indirect reciprocity model where agents evolve both rules for how to act in a prisoner’s dilemma (i.e., behavior) and rules for how to communicate about the actions of others (i.e., language). In such a model, “lying” is expressed by individuals with communication strategies that deviate from the norm. Under the conditions that allowed for a stable cooperative-communicative state, agents

1. exert normative pressure on each other’s signals, which allows benefits not only to be disproportionately distributed to cooperators in the prisoner’s dilemma, but also to truthful communicators.
2. occasionally deviate from their strategy, which prescribes particular actions in particular situations. If everyone employs the same strategy, the environment becomes homogenized to the
point where agents become unprepared for novel threats. But occasionally deviating from said strategy keeps the environment sufficiently variable that agents can remain prepared (it also decreases the sensitivity of the outcome to initial conditions).

3. use a strategy that encodes an “aligned” norm, that is, a norm that rewards the actions that it prescribes. In this case, that means that with individuals in bad standing, the norm will reward defection and punish cooperation (and the opposite for agents in good standing). This provides an incentive to pay attention to each others’ score when deciding whether to cooperate.

In Section 2, we will provide background on relevant concepts, which those familiar may choose to skip. In Section 3 we lay out our model of the interaction of communication and cooperation, which extends an indirect reciprocity model. We then identify the possible strategies that could conceivably lead to a stable and cooperative state and find the conditions under which they are stable in Section 4. Finally, in Section 5 we discuss the broader significance of our results.

2 Background

2.1 Cooperation in the Prisoner’s Dilemma

In this paper, when we refer to cooperation, we mean altruistic cooperation, which we define as paying a cost \( \gamma \) for the benefit \( \beta \) of others, restricting to \( \beta - \gamma > 0 \), so that there is a net benefit to the altruistic act. In a two-way interaction, this leads to a prisoner’s dilemma, since \( \beta > \beta - \gamma > 0 > -\gamma \) (see Table 1, which shows agent A’s payoff in an interaction with B).

|       | Agent B |
|-------|---------|
| Agent A | d       | c       |
| d      | 0       | \( \beta \) |
| c      | \( -\gamma \) | \( \beta - \gamma \) |

Table 1: Agent A’s payoff in the prisoner’s dilemma. d stands for defect and c stands for cooperate.

The dominant strategy for both agents in a one-shot prisoner’s dilemma is defection — (d, d). But if you’re likely to meet your partner again, the thinking goes, it might pay to cooperate. This leads to the “iterated prisoner’s dilemma,” but even here defecting is an equilibrium when there is a common-knowledge upper bound on the number of iterations (shown by a backwards induction) [Kuhn, 2009]. However, this equilibrium requires strong assumptions about human rationality [Bicchieri, 1989] that, when relaxed, might allow for rational cooperation (for instance, if one player believes that the other player might act irrationally [Kreps et al., 1982]).

One not-quite-rational strategy that does particularly well is tit-for-tat, a short-memory reciprocity: cooperate on the first round, and cooperate with those who cooperated in the previous round. This strategy performed well in tournaments [Axelrod, 1981], and in a world dominated by tit-for-taters, no
strategy can do better [Axelrod and Hamilton, 1981]. However, *tit-for-tat* does not satisfy Maynard Smith’s conditions for stability [Maynard Smith and Price, 1973], meaning that *tit-for-tats* cannot resist the population growth of alternative strategies (see Section 5.3). This is because one can always construct a competing strategy that is indistinguishable from *tit-for-tat* when playing against a *tit-for-tater*, and such strategies perform equally well and thus can grow in population by drift (e.g. unconditional cooperators and *tit-for-taters* are indistinguishable playing against each other, since they both cooperate all the time). In fact, a similar construction exists for any pure strategy, so no pure strategy is stable in the iterated prisoner’s dilemma [Boyd and Lorberbaum, 1987].

One reason *tit-for-tat* performs so well, but is not quite evolutionarily stable, is that it can initiate long chains of mutual cooperation, where both parties do very well but where *tit-for-tat* does not strictly outperform the other strategy. In fact, in our prisoner’s dilemma formulation, *tit-for-tat* never beats another strategy head-to-head (Figure 1).

| TIT-FOR-TAT | c | c | d | c | d | d | c | d |
|-------------|---|---|---|---|---|---|---|---|
| OTHER       | c | d | c | d | c | d | c | d |

Figure 1: *Tit for taters always cooperate as many or more times as their partners.* A possible sequence of moves between a *tit for tater* and a randomly chosen strategy. Every *tit-for-tat* defection is the result of the opponent’s defection in the previous round–so the opponent will always have defected at least as many times as the *tit-for-tater*.

2.2 Extending to Large Groups with Indirect Reciprocity

Indirect reciprocity can explain altruism in large groups where acts are unlikely to be directly reciprocated, as *third parties* can withhold cooperation from those with bad reputations (thereby punishing them). There is substantial empirical and ethnographic evidence for the practice. Empirically, people often cooperate disproportionately with those who have cooperated with others, when there is no possibility of having their act directly reciprocated, suggesting that reputation is not just an estimator for the likelihood of having one’s altruistic act directly reciprocated [Wedekind and Milinski, 2000]. And ethnographically, some societies tolerate stealing from families with bad reputations [Henrich and Muthukrishna, 2021; Bhui et al., 2019].

It is useful to define two crucial terms: norms and strategies. A strategy is a set of rules for acting and signaling. A norm is a good/bad assignment on actions, and a strategy is said to *encode* a particular norm when it prescribes cooperating with the “goods” and defecting with the “bads” of that norm. The simplest norm in indirect reciprocity is image-scoring, which says that “cooperation is good, defection is bad” [Nowak and Sigmund, 1998]. However, the strategy that encodes this norm punishes those who defect against defectors, even though this is precisely what maintains cooperation [Nowak, 2005]. A
more nuanced approach might be to introduce the notion of *standing* (someone is in good standing if they cooperated, or if they defected against a defector), and to cooperate with agents if and only if (iff) they are in ‘in good standing’. A stricter strategy than standing, called *stern judging*, also punishes cooperation with defectors [Pacheco et al., 2006].

Both standing and stern judging create an incentive to heed an opponent’s reputation when choosing how to act, unlike with image scoring case, where cooperation is good and defection is bad, regardless of your opponent’s score [Leimar and Hammerstein, 2006] [Panchanathan and Boyd, 2003]. Both also require the communication system to do more work: now, in order to determine how to act, an agent must know not just her partner’s previous action, but also his previous partner’s score, or standing. In the second-order case, the communication system must impart both “moral” information (the “score” of a previous partner) along with just “the facts” (the action of the agent). See Table 2 for three such second-order norms [Ohtsuki and Iwasa, 2003].

| Partner’s score | Actor’s action | Image Scoring | Standing | Stern Judging |
|-----------------|----------------|---------------|----------|---------------|
| good            | c(cooperate)   | good          | good     | good          |
| good            | d(defect)      | bad           | bad      | bad           |
| bad             | c              | good          | good     | bad           |
| bad             | d              | bad           | good     | good          |

Table 2: Three potential second-order norms. A norm is a good/bad assignment on a set of actions. We say a strategy that “encodes” a norm is one that cooperates with “good” and defects with “bad.” Second-order norms can take into account the standing of the actor’s partner (the person affected by the action). Image-scoring does not care about the partner’s score, while standing and stern-judging do.

2.3 The puzzle of communication

Indirect reciprocity is possible because a communication system disseminating reputations allows agents to withhold their cooperation specifically from defectors, which is vital for the stability of cooperation [Boyd and Richerson, 1988]. In this paper, we seek to understand how such a communication system, which embeds both factual and moral information, remains stable.

Communication is stable when there is pressure on the signaler to be truthful [Oliphant, 1996]: when there is a common interest between receiver and signaler [Blume et al., 2001], when signals are costly [Cintis et al., 2001], when there is a cost differential between truthful and non-truthful signaling (even if the equilibrium is cost-free) [Lachmann et al., 2001], when the interaction is a coordination game (where both parties benefit from knowing the world-state of the other) [McElreath et al., 2003] [Young, 1998], or when direct observation and partner-choice supplement the signals themselves [Robinson-Arnall, 2018].
In its basic form, communicating the reputations of others places no such pressure on the signaler. Solutions have been offered: for instance, a truth-for-truth reciprocity system was successful in sparking the rise of a truthful communication system [Oliphant, 1996]. The problem here, however, is that reputations and indirect reciprocity become important precisely when repeated interactions become unlikely. But perhaps the very indirect reciprocity mechanism that maintains reputations regarding agents’ actions also maintains reputations about their truthfulness. We will investigate this possibility later.

3 Model

Model 0: To answer our question—can cooperation and communication maintain each other’s stability—we consider various modifications to the baseline image-scoring indirect reciprocity model (“Model 0”):

1. Pair up with a partner at random, and make a choice about whether to cooperate or defect in a prisoner’s dilemma. Your choice can depend on your partner’s score (next step and Figure 2).

2. Your partner signals your new “score” based on your action: 0 if you defect and 1 if you cooperate (this is the image-scoring norm), overwriting whatever score you had previously. This can be imagined as your partner writing a 0 or 1 on your forehead for your next partner to see. You do the same for your partner (Figure 2).

3. Pair up with a new partner and restart the process (Figure 3).

![INDIRECT RECIPROCITY](image)

Figure 2.: Indirect reciprocity model. In each round, agents pair up and (1) act according to their strategies, (2) signal the other’s score, overwriting any previous score. Then, they (3) pair up with new partners and repeat. The discriminator defects with 0s and cooperates with 1s.

Model 1: In the basic version of our model (“Model 1”), which was first formulated by Smead [Smead, 2010], we modify step 2. Instead of preordaining that 0 means defector and 1 means cooperator, we allow for agents to use any arbitrary mapping. Perhaps 1 means defector and 0 means cooperator, or perhaps 0 means both cooperator and defector.
There are four possibilities for these signaling strategies. Similarly, there are four possible action strategies: unconditional defectors, unconditional cooperators, discriminators (d with 0, c with 1), and reverse discriminators (the opposite). See Table 3.

This gives sixteen possible action-signal strategy pairs, which we will denote by the juxtaposition of their action and signal strategy IDs (e.g. dc01 means defect with 0s, cooperate with 1s, score defectors with a 0, and score cooperators with a 1). Our task is to find one strategy that, if followed by everyone, leads to a stable world of (i) effective communication and (2) high cooperation. If such a stable strategy exists, then we can answer our question in the affirmative: yes, cooperation and communication can maintain each other’s stability. In what follows, we will analytically determine which strategies could fulfill these criteria, but before that, we will need to (1) define what effective communication is, (2) make approximations to make the mathematics tractable, and (3) define stability.

### Action Strategies

| Name         | ID | Action for 0s | Action for 1s |
|--------------|----|---------------|---------------|
| Defector     | dd | d             | d             |
| Discriminator| dc | d             | c             |
| Reverse-discriminator | cd | c             | d             |
| Cooperator   | cc | c             | c             |

### Signal Strategies

| Name        | ID | Signal for defectors | Signal for cooperators |
|-------------|----|----------------------|------------------------|
| All-zero    | 00 | 0                    | 0                      |
| Separator   | 01 | 0                    | 1                      |
| Reverse-separator | 10 | 1                    | 0                      |
| All-one     | 11 | 1                    | 1                      |

Table 3: **Action and signal strategies** Each row represents a possible action or signal strategy. In the case of action strategies, the first and second bit in the ID specify how to act towards a partner with scores of 0 and 1, respectively. In the case of signal strategies, they specify how to signal about a defector and the second how to signal about a cooperator, respectively. Strategies will be denoted by the juxtaposition of their action and signal strategy IDs (e.g. dc01)
3.1 Effective communication

We aim to understand whether a stable state exists in which both effective communication and high cooperation are maintained without exogenous pressures, but first we must define the former. We say that a communication system is a set of mappings from meanings to symbols, and we define an effective communication system as one that is

1. uniform, that is, everyone agrees on the mapping from meanings to symbols, and

2. informative, that is, everyone’s mapping distinguishes between at least two meanings (in our case, the “meanings” are facts about the actions of other agents), guaranteeing that at least some information gets transmitted.

Truthfulness in an effective communication system, then, is simply relative to whatever arbitrary mapping choice everyone agrees on. If everyone agrees that 0 means “bad standing,” an agent that uses 0 to map to good standing is deceiving the others and can be called “untruthful.”

3.2 Justification of approximations

To enable the derivation of mathematical results about our model, we assume an (1) infinite and (2) well-mixed population that interacts in (3) infinitely many rounds per generation. These assumptions also (counter-intuitively) have the side effect of positioning our model within the explanatory voids in previous models:

- **infinite rounds** - humans face cooperation choices daily, and live for tens of thousands of days. By contrast, many simulations of the evolution of cooperation simulate tens or at most hundreds of interactions per generation (e.g. [Smead, 2010], [Yamamoto et al., 2017]). For cases in which it takes time to reach a long-run equilibrium (e.g. Section 4.2.3), taking the first ten or hundred rounds would not give a result representative of the long run average. Approximating infinite rounds, by contrast, takes this long run equilibrium into account much more faithfully.

- **infinite population** - kin altruism, reciprocity, and group selection work best when group sizes are small [Maynard Smith, 1976], [Brown et al., 1982]. Because the mathematical analysis of our model is feasible for very small and very large (infinite) groups, we chose the latter in order fill out the explanatory void for cooperation in large groups.

- **well-mixed groups** - This means that any two individuals are equally likely to interact–agents do not preferentially interact with a subset of the group. Many times, selective assortativity, in which cooperators preferentially interact with other cooperators, is used as an explanatory mechanism for cooperation in groups, since it reduces the payoff disadvantage of cooperators when compared to defectors. Similarly, spatial structure, in which agents interact with nearby agents, has the effect of lowering effective group size [Bowles et al., 2003], [Wang et al., 2012]. These kinds of mechanisms are generally a tailwind in favor of cooperation, and by keeping our model well-mixed, we can examine the effect of indirect reciprocity in isolation, without the help of these other forces.
In many ways, therefore, our approximations, while making the math easier, make the evolution of cooperation harder. This is not true in all respects, however: a zero probability of invasion in the infinite case translates to a very small probability of invasion in the finite case (but this becomes negligible as \( n \) becomes large).

### 3.3 Modifying the Definition of Stability

Strategy \( A \) is stable if no other strategy can invade it, that is, no other strategy can proliferate in a population of 100% \( A \)-type agents (when \( A \) is said to “predominate”).

We formalize stability in Appendix B, but we will include a short version here. Maynard Smith’s definition of stability \([\text{Maynard Smith and Price, 1973}]\) says that, where \( R(A, B) \) is the payoff (or reward) of strategy \( A \) against strategy \( B \), \( A \) is stable against \( B \) when

1. \( R(A, A) > R(B, A) \) or
2. \( R(A, A) = R(B, A) \) and \( R(A, B) > R(B, B) \),

This definition requires two modifications to make sense in our model, in which \( R(A, B) \) is determined not only by the strategies of the interacting agents (\( A \) and \( B \)) but also by their scores, which depend on the strategies of their previous partners. This necessitates two changes:

- Because \( R(A, B) \) depends on factors beyond agent strategies, it is a random variable. We therefore take the expectation \( E(A, B) \).
- Because the probability distribution on \( R(A, B) \) depends on who \( A \) and \( B \) might have interacted with previously, payoffs must specify the population compositions: \( R_{A,1-q}(A, B) \), for instance, is the expected payoff of \( A \) against \( B \) when \( A \) makes up \((1 - q)\)-fraction of the population.

Further, sometimes it will be useful to refer to \( R_{A,1-q}(B) \), which is the expected payoff of \( B \) when \( A \) makes up \((1 - q)\)-fraction of the population, but \( B \)’s opponent’s strategy is unknown, but chosen uniformly at random from the population.

We start by modifying Maynard Smith’s first condition, the scenario where \( A \) (when predominant) outcompetes \( B \) from the start. Say \( A \) starts off dominant and we introduce a finite number of \( B \). Then, using the infinite population approximation, all agents will meet an agent of type \( A \) with probability 1, and since \( B \) does worse than \( A \) in this scenario, it will never increase as a proportion of the population. So for the first condition, the only relevant case is where \( A \) constitutes 100% of the population.

With the second condition, \( B \) does equally well against \( A \) as the other \( A \)-s, so it could grow in population. The key is to make sure that the potential growth is self-defeating. Maynard Smith in condition \([\text{Maynard Smith and Price, 1973}]\) expresses this by saying that if the \( Bs \) do worse against \( Bs \) than the \( As \) do against \( Bs \), then as soon as the \( Bs \) grow in population enough that they interact with other \( Bs \) with non-negligible frequency, they will start to do worse, on the whole, than the \( As \).

We state this self-defeating condition directly in our second condition, by asserting that when \( B \) makes up a small but nonzero percentage of the population, \( q \), we need for the expected payoff of \( B \) to be lower than \( A \). Altogether, this gives us

1. \( R_{A,1}(A, A) > R_{A,1}(B, A) \), or
2. \( R_{A,1}(A, A) = R_{A,1}(B, A) \) and for all small \( q \), \( R_{A,1-q}(A) > R_{A,1-q}(B) \),

which we derive in Appendix B.2. In the case where payoffs do not depend on population composition, this definition is equivalent to Maynard Smith’s (Appendix B.3).

Of course, this definition only analyzes strategies pairwise, and, if the first but not the second condition is satisfied, it is possible that a coalition of strategies might invade if they all happen to grow by drift at the same time (also a problem with Maynard Smith’s original definition). For this reason, we call a strategy satisfying only the second condition weakly stable, and a strategy satisfying the first condition as strongly stable. In the end, we will find a strongly stable, highly cooperative state with effective communication, so we will not have to worry about a potential coalition of invaders.

4 Results

Having defined stability as a state in which no alternative strategies can grow in population, we are ready to attempt to find a state that is

1. stable,
2. cooperative, and
3. effectively communicative.

The analysis will occur in two steps: first, we will find a candidate state that could fulfill these criteria—we settle on the state in which everyone employs the \( dc01 \) strategy (\( d \) with \( 0 \), \( c \) with \( 1 \), score defectors with \( 0 \), and score cooperators with \( 1 \)). Then, we will identify a minimal set of mechanisms (metasignaling, error, and aligned norms, see Figure 5) for which that candidate state will actually be stable.

4.1 Searching for a Candidate Stable and Cooperative State

We define a state as a distribution of strategies in the population, and we will confine our analysis to the sixteen simplest ones: 100% of each of the sixteen strategies. These states fall into one of four categories (Figures 3 and 4):

1. **Full cooperation** (predominant strategies \( cc, dc11, cd00 \) - \( cc \)s cooperate with everyone, and the other two create a world in which everyone has a score with which the strategy cooperates. These worlds, however, are unstable, since everyone will cooperate with defectors, allowing them to do just as well as the predominant strategy. (Figure 3.1).

2. **Full defection**: (predominant strategies \( dd, dc00, cd11 \) - \( dds \) defect with everyone, and the other two create a world in which everyone receives a score with which the strategy defects (Figure 3.2).

3. **Oscillating**: (predominant strategies \( cd01, dc10 \) - These states oscillate between \( p \)-fraction of cooperation and \( (1-p) \)-fraction of cooperation, because agents who cooperate receive a score that leads their subsequent partner to defect against them, and vice-versa. Averaging across all rounds, there is just 50% cooperation, so these worlds are not quite cooperative (Figure 3.3).
4. **Sticking**: (predominant strategies dc01, cd10) - These worlds “stick” at whatever level of cooperation they started with, because agents who cooperate receive a score that leads others to cooperate with them, and analogously for the defection case. They could be stable, and could be cooperative, if everyone started cooperative (Figure 4).

Thus, sticking states, with predominant strategies dc01 or cd10, are the only possibilities for stably cooperative states. Note that both of these worlds have an effective communication system (both are informative and uniform, as defined in section 5.1), implying that in this model, if a world is stable and cooperative, it must be effectively communicative.

By symmetry, it is irrelevant which one we choose to analyze, so we chose dc01. We will also employ the shorthand notation $\mathbb{R}_{dc01,q} \rightarrow \mathbb{R}_q$, as unless otherwise noted, dc01 will be the focal strategy.

Importantly, our analysis will initially assume that agents started off cooperative, as sticking states are only cooperative when individuals start off cooperating. However, we will be able to relax this assumption later.

4.2 **Determining a minimal set of mechanisms for stability**

The state in which all agents follow dc01 could be stable and cooperative (we’ve not yet shown that it isn’t), but our next step is to determine a minimal set of social mechanisms (e.g. third-party observation, error-prone strategies, and different kinds of norms) for which that state is actually stable. Each mechanism that we include is rooted in observed human behavior, and we will add them to the model one by one such that we find a minimal set for which the state is stable (Figure 5).

4.2.1 **Instabilities in the base model**

Starting with the basic model (Model 1), in which agents simply pair up, act according to their action strategy, and signal 0 or 1 depending on their partner’s action (section 3), eight out of the sixteen strategies invade the state predominated by dc01 agents (Figure 6). Two instabilities (that is, vulnerabilities that allow alternative strategies to perform just as well) lie at the root of these invading strategies:

1. **Unpunishability of language** - In the base model, there is no payoff difference between individuals with the same action strategy but different languages. This is because agents’ payoffs depend on two things: their own actions and their partners’ actions, neither of which depend on their own signal strategies (see Figure 7C). Thus, given an action strategy that can perform as well as dc01, any of its variants can, too (e.g. cc01 invades means that cc00 also does). This fact remains true no matter which strategy dominates the population.

2. **Unexpressed strategy information** - We know that in the dc01 world, everyone is cooperative, and therefore everyone receives a score of 1. Therefore, no one ever must decide how to act against
Figure 3: Proportions of cooperators and defectors in different world states. Each bar represents the balance of cooperators and defectors in a given round. The arrow-connected sequences show agents’ actions, their resulting scores, the actions those scores will garner, the scores those resulting actions will garner, and so forth. For instance, in sticking states cooperators receive a score that leads their subsequent partner to cooperate with them, while in oscillating states, the opposite is true.

In (1) and (2), no matter the initial balance of cooperators, all end up at either 100% cooperation or defection. (3) and (4) both depend completely on the starting state. The only two varieties that can lead to cooperative states are (1) and (4), but in (1), defectors do better than cooperators, so the state cannot be stable. This leaves (4) as the only potential stably cooperative state.
Figure 4.: **What happens when everyone follows a particular strategy.** Each cell represents a possible strategy. The coloring of the box indicates the outcome when everyone follows that strategy.

Figure 5.: **Iterations of the model, and the instabilities in each.** In every section, we identify instabilities, that is, properties of the system that leave it vulnerable to invaders. The three instabilities identified at various points in this section are the unpunishability of language (agents’ payoffs didn’t depend on their signaling strategy), unexpressed strategy information (certain components of the strategy were never expressed), and perverse norms (cooperation-enhancing behaviors were punished). For each instability, we add a mechanism—grounded in observed human behavior—that addresses it, but sometimes reveals others. We finish by adding stern judging, which completes a minimal set of three mechanisms required to stabilize a cooperative-communicative state. Importantly, the order in which we add these mechanisms makes no difference to our final stability claim.
an agent with a score of 0, and no one must decide how to score an agent who has defected. This means that both the first signaling bit and the first communication bit are inactive, giving four clusters of indistinguishable strategies: \(-c-1, -c-0, -d-1, \) and \(-d-0\). This is an instability because when a strategy is indistinguishable from the predominant strategy, it can likely invade. We originally called this phenomenon unexpressed genes but a colleague noted that our model was general enough to encompass both genetically and culturally transmitted strategies.

Referring to Figure 6B, we can see that any strategy in the \(-c-1\) cluster performs as well as \(dc01\) (unexpressed strategy information). Furthermore, the \(-c-1\) cluster spans two columns, and because all strategies in those columns perform equally well (unpunishability of language), strategies in both the \(dc\) and \(cc\) columns will do as well as \(dc01\).

This allows us to conclude that, at the very most, \(dc01\) is weakly stable, since it fails to satisfy the first condition for stability against the \(dc\) and \(cc\) strategies. However, by the unpunishability of language, payoffs in the model so far are completely independent of signaling strategy, and from that we can conclude that any \(dc\) type will invade, since all \(dc\) agents, no matter the population composition, will do equally well. In fact, as we prove in Appendix C, it also fails to be weakly stable for the \(cc\) strategies (Figure 6C).

By contrast, the \(-d-1\) and \(-d-0\) clusters do not invade, since their defections earn them scores of 0, which means discriminators will defect against them, garnering a payoff of zero (compared to \(\beta - \gamma\) for \(dc\) agents).

4.2.2 Making language punishable with meta-signaling

Model 2: In this section, we add a mechanism, meta-signaling, to the model to make language relevant by folding the truthfulness of an agent into their reputation (creating “Model 2”). We achieve this by adding an observer to each signaling act with some probability \(p\). If the signal conforms to the observer’s language, the observer gives the signaler the score of a cooperator, overwriting whatever score they already have. If, however, the signal diverges, the observer scores the signaler as they would a defector.

This co-opts the same cooperation-enforcing reputation mechanism to enforce truthfulness. With meta-signaling in place, it is always the case that conforming signalers do at least as well as their non-conforming counterparts (e.g. among the \(dd\) strategies, \(dd01\) will do at least as well as the others). This is because either

1. The agent was scored based on their action, in which case both the conforming and non-conforming signaler would receive the same score (since they have the same action strategy), or
2. The agent was scored based on their signal (by an observer), in which case the conforming signaler will never receive a 0 score, since they always signal in a conforming fashion. As for the non-conforming signaler, either
   a) They signaled using a conforming bit (e.g. \(dc11\) signals about a cooperator), in which case they would receive a 1 score, or
Figure 6.: (A) Factors influencing an agent’s present payoff. Many factors from previous rounds determine an agent’s payoff in the current round. However, none of them have to do with their own signaling strategy.

(B) Instabilities allow alternative strategies to perform just as well. Because there is no selection pressure on language in the base model, each of the four columns contains strategies that all perform equally well. Furthermore, in a population composed entirely of dc01 agents, the first bit of both the action and signal strategies will be unexpressed, creating clusters of four strategies in the corners that perform equally well.

(C) Eight strategies invade the base model. The four –c−1 strategies invade due to unexpressed information, and the strategies with whom they share a column invade due to unpunishability of language.
b) They signaled using a nonconforming bit (e.g. \( dc11 \) signals about a defector), in which case they would receive a 0 score.

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**Figure 7:** (A), (B) Conforming signalers do better. There are two components of the payoff: the actor component (which is determined by how one acts) and the recipient component (which is determined whether one’s partner cooperated). If agents have the same action strategy, their donor component will always be the same (A). However, they may have different recipient components if they are scored based on their signal (B).

(C) Invaders in the meta-signaling model. Now that language is relevant, some \( dc \) and \( cc \) strategies are cut out from the list of possible invaders. But since both the first action bit and the first signal bit are unexpressed, all of the \( \sim c \sim 1 \) strategies can still invade.

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In cases (i) and (2a), the agents will do equally well, and in case (2b), the non-conformer will do worse (Figure 7B). It is, however, possible that case (2b) will have zero probability—that is, when one of the bits of the signal strategy is unexpressed—which is why the conforming signaler does not necessarily do strictly better.

So now, with meta-signaling, unpunishability of language is no longer an issue, but there remains unexpressed information on both an agent’s action and signal strategies (since everyone in the \( dc01 \) world is a cooperator with score 1). Referring back to Figure 6B, we can deduce that the \( \sim c \sim 1 \) cluster of strategies will all perform just as well as \( dc01 \) due to these unexpressed bits, but the \( \sim c \sim 0 \) cluster will no longer perform as well, since meta-signalers will sometimes punish them for signaling non-conformingly. And in fact (as shown in section 4.2.2.1) it is these four strategies that invade (Figure 7C).

### 4.2.2.1 Payoffs

In this section we will calculate the payoffs for each class of strategies, but it may be skipped without loss of continuity. Strategy \( \sim c \sim 0 \) are sometimes scored as defectors due to their non-conforming signal,
and this takes a toll on their payoff (calculated fully in Appendix C):

\[
R_1(-c-0, dc01) = \beta - 2\gamma < \beta - \gamma = R_1(dc01, dc01) \tag{1}
\]

However, the \(-c-1\) cluster can continue to invade. Any world consisting fully of \(-c-1\) agents that starts off cooperative will remain fully cooperative with everyone scoring 1 (precisely because (2b) has zero probability). Thus,

\[
R_1_q(-c-1, -c-1) = \beta - \gamma = R_1_q(dc01, dc01) \tag{2}
\]

and

\[
R_1_q(-c-1) = \beta - \gamma = R_1_q(dc01) \tag{3}
\]

So, by equation [2] dc01 fails to satisfy the first condition of stability, and by equation [3] it fails to satisfy the second. It is therefore not even weakly stable.

Lastly, as we show in Appendix C, as long as \(p < \frac{\beta - \gamma}{\beta} \) (e.g. if \(\beta = 3 \) and \(\gamma = 2\), then \(p < 1/3\)), \(-d-1\) cannot invade. Then, by the fact that conforming communicators do at least as well as their non-conforming counterparts, \(-d-0\) cannot invade either, since their communication strategy does not conform. It follows, then, that \(-c-1\) is the only class of strategies that can invade.

### 4.2.3 Expressing all information through error

**Model 3:** The unexpressed strategy information problem allows \(-c-1\) individuals to invade the meta-signaling model, since half of the strategy bits are latent as a result of the lack of defectors and 0-agents. This suggests a simple modification to the model: let agents make mistakes (deviate from their strategy) with some probability \(\epsilon\), to expose latent elements of the genome (“Model 3”). Indeed, this creates a situation in which, given an action strategy, *conforming signalers do strictly better*. In section 4.2.2 we showed that conforming signalers do at least as well. However, it is now true that conforming signalers do strictly better, since, due to error, the case in which a non-conforming signaler signals in a deviant fashion (case (2b) in section 4.2.2) *always* has nonzero probability, as there will be some (erroneous) defection mixed in with all the cooperation.

But a “defection explosion” occurs with the introduction of error. This phenomenon is easiest to describe in the non-meta-signaling case, but it also occurs (as we will see) with meta-signaling. In every round, \(\epsilon\)-fraction of agents who previously cooperated will suffer an erroneous defection, and \(\epsilon\)-fraction of agents who previously defected will benefit from an erroneous cooperation. As a result, as long as cooperation is above 50%, the erroneous defections will outnumber the erroneous cooperations, gradually increasing the percentage of defections until it reaches 50%, *regardless of the initial distribution of cooperators* (Figure 8A).

Not only does this decrease payoffs across the board, but it decreases the relative payoff of discriminators (dc) against cooperators, because discriminators are punished for having punished the (ever-increasing) number of defectors. Thus, when error is low enough, cooperators do best. But when error is high enough, enough agents cooperate mistakenly with defectors that defectors perform the best. Nowhere, however, do discriminators outperform all others (Figure 8B). As we calculate in Appendix C.3.2

\[
R_1(dd01, dc01) > R_1(dc01, dc01) \text{ when } \frac{\beta - \gamma - \beta p}{2\beta(1 - p)} < \epsilon < \frac{1}{2} \tag{4}
\]
Figure 8.: **(A) When there is error, defection increases until half of the population defects.** Each bar represents a round, and the segments of each bar represent the share of the population cooperating and defecting. At every step, the percentage of agents who accidentally cooperate against defectors (third bar, bottom segment) will be smaller than the share that accidentally defects against cooperators (third bar, second segment from top), until half the population defects.

**(B) Error means that discriminators never do the best.** Here we charted the payoffs of different agent types for different levels of error, with $\beta = 2$, $\gamma = 1$, $p = 0.1$. The discriminator strategy never does the best.
and
\[ R_1(\text{cc}01, \text{dc}01) > R_1(\text{dc}01, \text{dc}01) \text{ when } \epsilon < \frac{\beta - \gamma - \beta p}{2\beta(1 - p)} < \frac{1}{2} < \epsilon \] (5)

So there are no values of \( \epsilon \) for which \( \text{dc}01 \) does the best, and either \( \text{dd}01 \) or \( \text{cc}01 \) will invade. We take up this final issue in the next section.

### 4.2.4 Aligning norms with stern judging

One last issue, or instability, remains: perverse norms, or norms that punish the behaviors they prescribe. In this case, the image-scoring norm that prescribes defection against defections also punishes those very defections against defectors — while those defections are precisely what maintains cooperation. This makes punishment costly when it need not be.

**Model 4:** Our last addition to the model, then, is to adjust the signaling strategies so that they can encode norms that reward the discriminator action strategy at equilibrium (“Model 4”). We do this by distinguishing between four, rather than two, possible scenarios: cooperation with a cooperator, cooperation with a defector, defection with a cooperator, and defection with a defector. For each scenario, agents score their partners with either a 0 or 1, producing sixteen possible signal strategies, a superset of the four that existed previously.

One such norm is called stern judging: it does not punish defection against defectors, and in fact, it punishes cooperation with defectors (see Table 4). Because the \( \text{dc} \) agent always has the “correct” default action in this world, \( \text{dc} \) agents will receive a score of 1 as long as they do not err, which happens with probability \( 1 - \epsilon \). And since agents cooperate with 1s with probability \( 1 - \epsilon \) we get that the equilibrium share of non-erroneous cooperation is \( (1 - \epsilon)^2 \). In addition, agents erroneously cooperate with the \( \epsilon \)-fraction of 0s with probability \( \epsilon \), giving the share of erroneous cooperations as \( \epsilon^2 \). This gives a total cooperation share of \( (1 - \epsilon)^2 + \epsilon^2 \), which is higher than the \( \frac{1}{2} \) we had before (assuming \( \epsilon < \frac{1}{2} \), that is, that agents conform with the prescription of their strategy more often than not).

| Input | 1c | 1d | 0c | 0d |
|-------|----|----|----|----|
| Output| 1  | 0  | 0  | 1  |

**Table 4: Stern Judging (1001) strategy** The string represents how the signaler responds to various different scenarios (where 1c is “cooperation with an agent with score 1”)

On top of leading to relatively higher cooperation levels, the stern judging norm also portends that discriminators will outperform other strategies, because, whenever they do not err, their actions will garner cooperation from others. And, indeed, this turns out to be true. For reasonably small (but nonzero) error, no action strategy can invade the stern-judging discriminator. In fact, not even a coalition can do so, because this strategy is strongly stable (Figure 4).

We calculate in Appendix C.4.2 that there will be an interval,
\[ 0 < \epsilon < \frac{\beta - \gamma - \beta p}{2\beta(1 - p)} \]

(6)
in which dc outperforms all other conformingly-communicating strategies (e.g., for $\beta = 3$, $\gamma = 2$, and $p < 1/10$, the range is $0 < \epsilon < 0.129$).

Furthermore, as we’ve argued before, all non-conforming communication systems do worse when there is error (the same proof we used for the dc01 world goes through in the dc1001 world — see Appendix C.4.3 for a note on this). So non-conforming discriminators cannot invade, either, creating a strongly stable state of effective communication and high cooperation.

**Figure 9.** *Stern-judging creates a state where discriminators perform the best.* We chart payoffs at different levels of error, with $\beta = 2$, $\gamma = 1$, $p = 0.1$. At low levels of error, discriminators outperform all other strategies. This region thus represents a strongly stable state of effective communication and high cooperation.

5 Discussion

The three conditions under which communication and cooperation could remain stable, without assuming that any exogenous machinery was doing the work for us, were:

1. *Meta-signaling*, rewarding conforming signaling strategies
2. *Error*, ensuring that all components of the strategy specification (both signal and action) will find expression at least some of the time—important because when certain components of the
strategy remain unexpressed, there will be clusters of indistinguishable strategies, all of which will likely invade.

3. **Aligned norms**, rewarding conforming action strategies

Thus, each of the three mechanisms adds stabilizing pressure to a different component of the strategy. Furthermore, they apply equally to genetically and culturally transmitted strategies, and they are indifferent to whether alternative strategies invade an existing equilibrium by way of mutation, migration, or agents simply choosing to change their strategies. We will first examine these three stabilizers in more detail, and then go on to analyze the resulting equilibrium and its downstream consequences.

### 5.1 The three stabilizers

#### 5.1.1 Meta-signaling

The concept of meta-signaling makes it clear why it makes sense to study the evolution of communication in the context of cooperation. Research suggests that there must be some kind of pressure on the signaler for a uniform communication system to emerge \[\text{Oliphant, 1996}\]. This cost may be intrinsic to the signal, or it may result from differential allocations of social benefits and costs to truth-tellers and deceivers. And this differential allocation — whether through kin selection, reciprocity, punishment, or indirect reciprocity — is also how altruism becomes stable. Because the machinery for such differential allocation is already present for cooperation, it is plausible for it to have been co-opted for the purposes of communication. Meta-signaling co-opts the indirect reciprocity mechanism for the purposes of maintaining the communication system, but one can equally imagine a co-optation of other mechanisms, such as reciprocity (e.g., lying to those who have lied to you) and punishment (e.g., ostracizing those who miscommunicate) for the same purpose.

*How* meta-signaling accomplishes this co-optation is also vital. It is through “slippage” between the concepts of defection and cooperation, on the one hand, and deception and truthfulness, on the other. This kind of slippage is central to human cognition \[\text{Hofstadter, 1995}\]; in fact, such slippage has already occurred to define the concept of “cooperation”. Altruistic cooperation can take many forms—donating to charity, maintaining irrigation ditches, participating in a group hunt, going to war—which all share the property of producing a benefit to the group at a potential individual cost. An analogy between these disparate actions leads to the concept of cooperation. Honesty, by providing a service to the group (i.e., maintaining cooperation), has a similar character to all these other cooperative behaviors. So to declare truth-telling a subset of “cooperation” in the meta-signaling scheme described above is not so different from declaring any other behavior an instance of cooperation.

Meta-signaling helps point at why exaptation through analogy-making is so powerful. We humans build mechanisms that do a lot of heavy lifting: tool-making technologies, and yes, reputation systems to enforce cooperation. When we can co-opt existing systems \[\text{Villani et al., 2007}\], we save ourselves the effort of both *building* and *maintaining* a separate system. But here, something extra special is happening. The innovation was not simply horizontal—repurposing a technology for an analogous purpose—but vertical—repurposing a technology for the analogous purpose of maintaining itself. The slippage not only allows the system to perform a new task, but it also enhances its performance on its old task. This vertical co-optation is extra powerful because it sets up a positive feedback loop: enhancements to the system lead to further enhancements, since the system “gives back” to itself. For
instance, in the case where tools are used for tool-making, advances in tool-making technology not only lead to better tools, but better tool-making tools, which in turn could lead to even better tools. The same goes for compilers written in the language that they compile. For this reason, vertical co-optation seems to be at the root of leaps in complexity [Hofstadter, 1979].

\subsection{Error}

Error plays a vital role for two reasons. First, it pushes the system to a unique equilibrium that does not depend on initial conditions; and, second, it ensures that all of the information about an individual’s strategy, at least sometimes, finds expression.

When there is no error, “sticking” states (Figure 3) remain exactly at the level of cooperation they start with. This lacks realism, especially when deviations are likely to be in one direction over another (as will be the case if everyone is cooperating, since deviations can only be in the direction of defection). But the disturbance that error causes, in the end, creates regularity: in our case, no matter what level of cooperation the system starts at, it will converge to the same percentage of cooperation in the end.

Error also has the advantage of keeping the “environment” varied. As social beings, other people make up a large part of our environment, and as a result, our adaptations often deal with navigating social situations. But a homogeneous population creates homogeneous social situations and correspondingly brittle social adaptations that cannot handle situations outside the monoculture. In this case, when we live in a completely homogeneous environment of cooperators, absent is the selection pressure to maintain defense mechanisms against defectors. This homogeneity problem is the reason that tit-for-tat fails to be evolutionarily stable—tit-for-taters look like unconditional cooperators when interacting with each other, so cooperative (but nonconforming) strategies are indistinguishable, and can invade by drift. In our model, error, by introducing heterogeneity, provides the pressure needed to maintain defense mechanisms against potential invading strategies.

\subsection{Norms in communication}

This model demonstrates the benefits and dangers of communicating moral information rather than simply factual information. In the initial case, the simple morality “defection is bad, cooperation is good” had a one-to-one correspondence between factual states of the world (cooperate/defect) and moral states (good/bad). But with the more complex set of norms in “good” could mean cooperation with a cooperator, cooperation with someone who defected against a cooperator, defection with a someone who cooperated with someone who defected against a cooperator...

An infinite number of states like this collapse into “good” and “bad.” And this is the power of morality in communication: it collapses infinite factual information into (sometimes) a single bit. Agents need not know the infinite chain of who cooperated with which defectors and whether they defected against cooperators; they need only know two things: (i) what their partner did (cooperate or defect) and (2) whether that was against someone with a “good” or “bad” reputation. When Henrich objected to Boyd’s recursive punishment strategy (punish those who are in bad standing, where someone is in bad standing if they fail to cooperate, or if they fail to punish someone in bad standing) [Henrich, 2004; Boyd and Richerson, 1992], he objected to agents having to track an infinite chain of possible transgressions—but moral communication solves this issue, by collapsing that infinite chain at the second step.
However, embedding moral norms in the communication system comes with danger. When dealing with norms and cooperation, one can ask three questions:

1. Does a given norm lead to high levels of cooperation?
2. Is that norm stable against other norms?
3. Is that norm stable against other norms when embedded in a communication system?

Clearly the first question is of a different class that the next two. But even questions 2 and 3 deal with profoundly different dynamics. In the scenario 2, someone following different norms acts according to their private rule that deviates from the prevailing norm, but otherwise minds their own business. In scenario 3, however, a signaler subscribing to different norms “pollutes” everyone else’s reputation information with their own normative judgment, thus impacting not only their own actions but the actions of others. This can lead to domino effects that do not exist in case 2, which will therefore lead to different equilibria [Yamamoto et al., 2017].

### 5.2 The equilibrium

In the direct reciprocity case, tit-for-tat is only collectively stable; that is, it performs at least as well as any other strategy when it is dominant. Our equilibrium is much stronger: discriminating stern-judgers, under the conditions we laid out, perform strictly better than any other strategy when error is low enough. This is even stronger than standard evolutionary stability, which allows potential invaders to do as well as the dominant strain as long as their multiplication is self-defeating [Maynard Smith and Price, 1973].

In our model, we consider all the possible pure (but error-prone) strategies that take into account a certain restricted set of information: in the case of the action strategy, the partner’s score, and in the case of the signaling strategy, the action and the recipient’s score.

By contrast, in the direct reciprocity case, strategies are allowed to take into account the full history of cooperation and defection between two partners, allowing a much broader variety. So tit for tat must defend against many more possible invaders (and they can’t possibly defend against all of them [Boyd and Lorberbaum, 1987]).

The constraints on the information available for the strategies are justified, however, in the context of indirect reciprocity, which is most important in interactions with people one has not interacted with before. Thus, much less information will be available about one’s partner than in the case of direct reciprocity, where partners share a long history.

Importantly, the equilibrium we found is not the only stable point in the system. Trivially, there is the symmetrical reverse-discriminator state that would clearly also be stable (cd0110). This stable point is a non-issue because it is similarly cooperative and communicative. But even if there are stable states with defection, even weak group selection could lead to the propagation of the cooperative equilibrium. Usually, group selection is under intense time pressure, as it must quickly kill off groups that descend into defection before they propagate. In our case, groups do not tend towards defection, since the forces we laid out produce a cooperative equilibrium. Thus, group selection no longer has such time pressure, and it need only select, rather than maintain, the cooperative equilibrium [Henrich, 2004].
5.3 Implications for human communication

We have demonstrated a mechanism for the social enforcement of truthful communication, which has a number of advantages:

1. Social enforcement is more flexible in the determination of costs. This flexibility not only allows for the truthful signal to be the least costly, but also to be potentially cost-free. Furthermore, it allows for more complex, combinatorial communication systems (where meaning is determined as a function of a set of symbols, rather than having a straightforward symbol-to-world-state mapping [Lachmann et al., 2001]), because costs can be differentially determined on the level of a statement, rather than, say, associating each symbol with a cost. This kind of flexibility would be unlikely if costs were determined physiologically, for instance. [Lachmann et al., 2001]

2. It has stability benefits. When costs are not socially determined, but instead determined physiologically, there is selection pressure on the signaler to evolve ways of producing (perhaps untruthful) signals in a cheaper fashion. But in the social enforcement case, costs are determined by recipients, and therefore, they are less likely to be destabilized by natural selection [Lachmann et al., 2001]. This is not to say that they are completely immune to such disturbance, however—

There are constraints we place on our strategy set that make it easier for the social enforcement mechanism to function: for instance, signalers cannot adjust their signaling strategy based on whether they are being observed. This is not such a great problem. While there is sometimes an absence of pressure to signal in a conforming fashion, there is never any positive pressure for an individual to signal in a non-conforming fashion—there is no benefit to smearing or praising untruthfully, because it is not the reputation of others, but ones own reputation, that determines one’s fitness. Thus, even if an agent could signal differentially based on whether they were being observed, if they even entertained any minute probability of being caught, they would always signal in a conforming fashion.

5.4 The ubiquity of reputation

Reputation—and the communication system that underlies it—is vital for all kinds of decision-making. In indirect reciprocity, it indicates who to cooperate with. In third party punishment, reputation would likely be necessary for the third party to know who to punish (see Appendix A, and Boyd et al., 2010). And for cooperation to function as a costly signal for mate quality or alliance potential [Gintis et al., 2001], there must be some kind of reputation mechanism that propagates information about who is cooperating—unless every cooperative act is observed directly by everyone, which seems unlikely.

For these cases, our results provide two alternative explanations. One possibility is that a communication system for disseminating reputation information might have evolved for the indirect reciprocity case, and was then co-opted for use by third-party punishers. Another possibility is that selection pressures analogous to the indirect reciprocity case exist in these other situations, and our research can inform their exploration. Third party punishment is especially similar to indirect reciprocity, since the
latter is punishment by withholding of cooperation while the former is punishment by direct imposition of cost. While this creates a slightly different payoff matrix, it is likely that the three principles underlying the viability of communication in our case could be extended to third-party punishment.

5.5 Conclusions

Human relationships cannot be reduced to pairwise interactions. Instead, we are submerged in a tangle of observation and judgment, where people form opinions and base decisions on interactions they are not involved with. And since it is simply impossible to observe all interactions to which one is a third party, communication must play a vital role in virtually every decision we make. As institutions, legal systems, and governments develop, the role of the third party, and therefore, communication, only increases [Nowak, 2003] [Alexander, 1986].

It is therefore vital to understand how a communication system maintaining all of this social information could possibly be stable. Here we found three conditions of stability for a simple binary-valued communication system: meta-signaling, error, and stern judging norms.

There are a number of directions to pursue in further work.

1. Alternative equilibria - We have found an equilibrium point, but we have not scoured the full space for other potential equilibria. Knowing more about the full set of equilibria, in addition to the probabilities of fixating on each given some initial conditions, will shed light on how likely a society is to attain the equilibrium we’ve identified.

2. Additional strategies - mixed strategies might be analyzed, in addition to strategies that take into account information beyond the score of one’s partner (one’s own score, for instance, might be relevant [Nowak and Sigmund, 1998]).

3. Elaborate communication - we analyzed a basic language where every world state maps to a symbol (and a binary one, at that), but human language involves combining symbols to create meaning. This type of combinatorial communication lacks some of the stability properties of more basic communication and therefore needs to be analyzed further [Lachmann and Bergstrom, 2004]. Related to this, if communication is too elaborate, cognitive and informational bounds my place limits on communication and decision making [Price and Jones, 2020]. One intriguing way to address this in particularly complicated evolutionary models is to use reinforcement learning to set agent’s strategies [Köster et al., 2022].

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Appendix A  Background

Appendix A.1  Punishment

Punishment has extensively been studied as a potential stabilizer of cooperation, partially because it is so prevalent cross-culturally [Henrich et al., 2006, Henrich et al., 2001]. Much of the literature has focused on how it can survive as a practice despite being costly to the punisher [Boyd et al., 2010, Boyd et al., 2003, Henrich and Boyd, 2001, Boyd and Richerson, 1992], though we do believe the hyper-focus on costly punishment may stem from western-centric conceptions on what punishment looks like (e.g. police spending their time and risking their lives to enforce the laws) rather than what punishment must inherently be [Henrich and Muthukrishna, 2021]. Still the literature is informative for the cases in which punishment is actually costly. In these cases, the problem appears to be similar to the initial problem of costly cooperation. It is not quite the same, however, because when punishment is common, it becomes less costly, as there are fewer defectors to punish. Such is not the case with cooperation, which remains equally costly no matter how common it is [Boyd et al., 2003].

Partially due to this asymmetry, many solutions to the costly punishment problem have been proposed. If the punishers can recoup the costs of punishment by coercing those around them to cooperate, they can proliferate. If a recursive strategy exists that punishes non-cooperators, and punishes those that fail to punish non-cooperators, and so on, ad infinitum, then punishment can stabilize almost any behavior [Boyd and Richerson, 1992]. If some modicum of conformist transmission exists, that is, a tendency to absorb norms from the majority behavior of the group [Henrich and Boyd, 1998], punishment can also emerge [Henrich and Boyd, 2001]. Furthermore, if the costs of punishment are spread across the group, as is the case for ostracism, punishment can be stable [Hirshleifer and Rasmusen, 1989].

Appendix B  Stability

A strategy A (e.g. dc01) is evolutionarily stable when no other strategy B can start off rare and come to dominate the population. Of course, what exactly this requires depends on many details of the model in question—the size of the population, the determination of payoffs, the dependence of payoffs on population composition, the structure of the interactions, and the quantity of interactions per generation, to name a few important characteristics.

In our world, agents of various strategies interact in an infinite series of pairwise interactions, where payoffs for any given interaction are stochastic. Furthermore, the expected payoff of an agent does depend on the overall composition of the population—this fact means that Maynard Smith’s conditions will not suffice.

First, therefore, let us translate the requirement that “no other strategy can invade” into a statement about the payoffs. To do this, some random variables are necessary:

1. \( R(A) \) - the payoff obtained by an agent strategy A in a given round.
2. \( R(A, B) \) - the payoff of strategy A against strategy B in a given round.
3. \( S \) - the strategy of an agent’s opponent in a given interaction, either A or B, for our purposes.
Agents reproduce at the end of a generation according to their average payoff over the course of the generation, so agents with higher average payoffs will grow as a percentage of the population. If we define $\overline{R}(A)^k$ to be the average payoff of an agent of strategy $A$ after $k$ rounds, we know that, due to the law of large numbers,

$$\overline{R}(A)^k \to \mathbb{E}[R(A)] \text{ as } k \to \infty$$

(7)

where $\mathbb{E}(A) := \mathbb{E}[R(A)]$. It follows from this that the agents with the highest expected payoffs will have the highest average payoffs at the end of the generation, and will grow in population. Knowing this, what should the criteria for evolutionary stability be?

### Appendix B.1 Conditions

Let us then consider the possibility that

$$R_{A,1}(A) > R_{A,1}(B),$$

(8)

that is, $A$ outperforms $B$ when the population is homogeneous of type $A$. Then clearly $A$ will be stable against $B$, because if it starts dominant, and its agents outperform those with strategy $B$, the $B$ agents will never grow in population. But there is another way the strategy can be stable. If

$$R_{A,1}(A) = R_{A,1}(B),$$

(9)

that is, $A$ and $B$ perform equally well when $A$ is dominant, the population of $B$ can still grow by random drift. So suppose we have that the population of $B$ grows to $q$-fraction of the population. If it is the case that, for a range of small values $q \in (0, q_0)$, we have

$$R_{A,1-q}(A) > R_{A,1-q}(B),$$

(10)

in other words, for small values of $q$, $A$ outperforms $B$. Thus, in this range of values, $A$ will start to outcompete $B$, pushing their population back to zero, and regain its dominance. However, this might seem like a strange type of situation—why would $A$ outperform $B$ when the population of $B$ is small, but fail to outperform $B$ when the population of $B$ is negligibly small?

An example would help. Suppose, for instance, that an entire population of 100 birds feeds in the morning. Then along comes a single bird that feeds in the evening. There is 100x more food in the morning, but because initially there is only one bird feeding in the evening, the single invader does as well as the morning-feeders. However, as soon as there are even just two evening-feeders, they begin to severely underperform the morning-feeders. Here we see how the returns of a strategy can quickly fall as more agents practice it. And this pattern is common in a whole class of “strategies”—arbitrage loses its profitability as more people engage in it, getting up early to catch an empty gym is no longer advantageous if everyone does it, and so forth.

So these two scenarios both define cases in which strategy $A$ is stably dominant. If neither of them hold, however, we will have a situation where a small population of strategy $B$ can grow, either by drift or by outperforming $A$, to a significant portion of the population, destabilizing $A$. Thus these two conditions are necessary and sufficient for stability.
But we would like to express our criteria in terms of the easier-to-calculate $R(A, B)$, so we calculate a general formula for $R(A)$, using the law of total expectation:

$$R(A) = R(A|S = A)\Pr(S = A) + R(A|S = B)\Pr(S = B)$$

(11)

Our first possibility for stability, in equation (8) was that in the beginning, $R_{A,1}(A) > R_{A,1}(B)$, so using equation (8) we obtain

$$R_{A,1}(A) = R_{A,1}(A, A)\Pr(A) + R_{A,1}(A, B)\Pr(B)$$

(12)

since $\Pr(A) = 1$ and $\Pr(B) = 0$.

The second possibility, in equation (9) and (10) was that in the beginning, $R_{A,1}(A) = R_{A,1}(B)$ but for small $q = Pr(B)$ we have $R_{A,1−q}(A) = R_{A,1−q}(B)$. Observe that, by an exactly analogous argument as Equation (12).

$$R_{A,1}(A) = R_{A,1}(B) \iff R_{A,1}(A, A) = R_{A,1}(B, B).$$

(13)

We can then use (2) and (13) to simplify our conditions for evolutionary stability:

1. $R_{A,1}(A, A) > R_{A,1}(B, B)$, or
2. $R_{A,1}(A, A) = R_{A,1}(B, B)$ and for small $q$, $R_{A,1−q}(A) > R_{A,1−q}(B)$.

It is sometimes useful to put the second clause of the second condition in terms of $R(x, y)$. Using the well-mixed approximation, we can do that as follows:

$$R_{A,1−q}(A) = (1-q)R_{A,1−q}(A, A) + qR_{A,1−q}(A, B)$$

(14)

An analogous equation, of course, holds for $B$.

### Appendix B.3 Equivalence

The result is a slight modification of the familiar bipartite definition of evolutionary stability due to Maynard Smith [Maynard Smith and Price, 1973]. The main difference is in the second condition—while Maynard Smith’s second condition looks similar, it is more basic, since his version assumes $R(x, y)$ is constant no matter the relative populations of the strategies.

We will now show that this definition of stability is equivalent to Maynard Smith’s in the case that $R(x, y)$ is constant.
It is clear that both the first condition and the first clause of the second condition are equivalent to Maynard Smith’s in this case (we simply drop the subscripts). We can then simplify the second clause of the second condition by using Equation 14:

\[
R_{A,1-q}(A) > R_{A,1-q}(B) \\
\iff (1 - q)R(A, A) + qR(A, B) > (1 - q)R(B, A) + qR(B, B) \\
\iff R(A, B) > R(B, B)
\]

where the third line comes from the fact that \(R(A, A) = R(B, A)\) (according to the first part of the second condition).

Intuitively, since we already know that \(R(A, A) = R(B, A)\), the only way there can be a difference is if they perform differently against Bs. Thus, the only way for B to start doing worse than A as its population grows is for \(R(A, B) > R(B, B)\).

### Appendix C Calculating Payoffs

We calculate the expected payoffs of various agent types throughout the article. Here we will elaborate on how that was done.

The fundamental task is to calculate \(R_{A,1-q}(B)\). If we take \(abxy\) and \(cdzw\) to be general strategies, \(p_{abxy}^0\) to be the probability that an agent of type \(abxy\) will have score 0, and \(\pi_{ac}\) to be the expected payoff of an agent when their strategy says to do \(a\) and their opponent’s strategy says to do \(c\), we can write the general equation as follows:

\[
R_{cdzw,1-q}(abxy) = Pr(abxy|R_{cdzw,1-q}(abxy, abxy)) + Pr(cdzw|R_{cdzw,1-q}(abxy, cdzw))
\]

\[
= q(p_{abxy}^0 p_{cdzw}^1 \pi_{bb} + p_{abxy}^0 p_{cdzw}^0 \pi_{ab} + p_{abxy}^0 \pi_{ba} + p_{abxy}^0 \pi_{aa} + (1 - q)(p_{abxy}^1 p_{cdzw}^0 \pi_{bd} + p_{abxy}^1 p_{cdzw}^1 \pi_{ad} + p_{abxy}^0 p_{cdzw}^0 \pi_{bc} + p_{abxy}^0 \pi_{cdzw}^1 \pi_{ac} + p_{abxy}^0 \pi_{cdzw}^0 \pi_{ac} + (1 - q) (\pi_{cc} = \beta - \gamma \\
\pi_{cd} = \gamma \\
\pi_{dc} = \beta \\
\pi_{dd} = 0
\]

But when there is error, we must account for the possibilities that agents will act contrary to their strategies:
\[ \begin{align*}
\pi_{cc} &= (\beta - \gamma)(1 - \epsilon)^2 - \gamma(1 - \epsilon)\epsilon + \beta(1 - \epsilon)\epsilon \\
\pi_{cd} &= -\gamma(1 - \epsilon)^2 + (\beta - \gamma)(1 - \epsilon)\epsilon + \beta\epsilon^2 \\
\pi_{dc} &= \beta(1 - \epsilon)^2 + (\beta - \gamma)(1 - \epsilon)\epsilon + (\gamma)\epsilon^2 \\
\pi_{dd} &= \beta(1 - \epsilon)\epsilon + (\gamma)(1 - \epsilon)\epsilon + (\beta - \gamma)\epsilon^2
\end{align*} \] 

(18)

Where each one has four terms (one of the terms is hidden), corresponding to (1) both agents follow their strategy, (2) only the ego agent follows their strategy, (3) only the partner follows their strategy, and (4) neither agent follows their strategy. One term is always zero because one of these cases always involves two defections (which leads to a payoff of zero).

The final step in calculating payoffs is to calculate the probabilities that particular agents will have a particular score (for instance, \( p_{ab01} \)), which depend on population distributions and the version of the model we are discussing. Once we have those, however, we can simply plug the \( p \)-values (probabilities of having a given score) and \( \pi \)-values (the resulting payoffs) into equation (16) to find the payoff of any strategy.

### Appendix C.1 Basic Model

We show for which strategies \( dc01 \) satisfies the first stability condition in section [4]. Here we will analyze the second condition, starting with the \( cc11 \) and \( cc01 \) strategies, since they are straightforward. No matter what fraction \( q \) of the agents are of type \( cc-1 \), we can deduce that the world is fully cooperative, and all agents will have score 1 (the \( cc \) agents all cooperate, and all agents score everyone with 1s, meaning the \( dc \) agents will all cooperate). So every interaction will involve both agents cooperating and we conclude that

\[ \begin{align*}
R_{1-q}(dc01) &= (1 - q)(\beta - \gamma) + q(\beta - \gamma) = \beta - \gamma \\
R_{1-q}(cc-1) &= (1 - q)(\beta - \gamma) + q(\beta - \gamma) = \beta - \gamma
\end{align*} \]

So these strategies can invade.

Now we examine the more interesting cases of \( cc00 \) and \( cc10 \). For these strategies, the complication is that they introduce 0-scores into the population. For \( cc00 \), let us define \( p_{00}^k \) as the percentage of \( dc01 \) agents a score of 0 at the end \( k \)th round, when \( cc00 \) agents make up \( q \)-fraction of the population.

Now, to calculate this quantity, we see that there are two cases. Either our agent interacted with a \( dc01 \) agent, with probability \( (1 - q) \), or they interacted with a \( cc00 \) agent, with probability \( q \). Now, if they interact with a \( dc01 \) agent, with probability \( p_{00}^{k-1} \), that other agent will have score 0, in which case our agent will defect, and in turn receive a score of 0. Otherwise, if they interact with a \( cc00 \) agent, they will get a 0 no matter what. This gives

\[ p_{00}^k = (1 - q)(p_{00}^{k-1}) + q, \]

which has a fixed point of \( p_{00}^k = 1 \). So in the limit, we obtain that \( p_{00}^{dc01} \rightarrow 1 \). Now, for the \( cc00 \) agents, they will receive a 0-score against other \( cc00 \) agents and a 1-score against \( dc01 \) agents. So \( p_{00}^{cc00} = q \). So we can calculate the expected payoffs for each agent type, which are
\[ R_{1-q}(dc01) = (1 - q)(0) + q(1 - q)(\beta - \gamma) + q^2(\beta) \]
\[ R_{1-q}(cc00) = (1 - q)(1 - q)(\beta - \gamma) + (1 - q)q(-\gamma) + q(\beta - \gamma) \]

(21)

For what values of \( q \), then, does the payoff for cc00 exceed that of dc01? Well, if we take a tiny value of \( q \) (not zero, however, since in the zero case our calculations for \( p^0_{dc01} \) do not apply) then we see that the \( (1 - q) \) term dominates for dc01 and the \( (1 - q)^2 \) term dominates for cc00. So we have \( R_{1-q}^1(\text{cc00}) \approx \beta - \gamma > 0 \approx R_{1-q}^1(\text{dc01}) \). And this is true for a range of small \( q \)-values, so it is not true that for a range of \( q \)-values near 0, dc01 outperforms cc00, and cc00 invades.

Now we examine the cc10 case. Here, in any given interaction, the cc10 agent will either meet another cc10 with probability \( q \), or a dc01 with probability \( 1 - q \). Thus, since cc10 agents always cooperate, with probability \( q \) they will be scored 0, and with probability \( (1 - q) \), they will be scored 1. So \( p^0_{cc10} = q \).

Now, what of the dc01 agents? We have that
\[ p^0_k = (1 - q)p^0_{k-1} + q(1 - q) \]

(22)

That is, in the scenario where a dc01 agent meets another dc01, the only way for them to receive a 0 is if they defect, and they will only defect if their partner had a score of 0. So with probability \( p^0_{k-1} \), they will receive a 0. On the other hand, in the scenario where a dc01 agent meets a cc10 agent, the only way for them to receive a 0 is if they cooperate, which they will only do if the cc10 agent has a 1. This has probability \( (1 - q) \), from what we argued above. Solving for the fixed point of this series we get \( p^0_{dc01} \to (1 - q) \).

Now knowing all this information, we can calculate the expected payoffs of each agent type:

\[ R_{1-q}(dc01) = (1 - q)[q^2(\beta - \gamma) + q(1 - q)(-\gamma) + q(1 - q)\beta] + q[(1 - q)(\beta - \gamma) + q\beta] \]
\[ R_{1-q}(cc10) = (1 - q)((1 - q)(\beta - \gamma) + q(-\gamma)) + q(\beta - \gamma) \]

(23)

Now, as before, we find that when \( q \) is infinitesimally close to 0, the payoff for the dc01 agents will be zero, and the payoff for the cc10 agents will be \( (\beta - \gamma) \). Therefore, again, there is no range of small values for which dc01 outperforms cc10, and cc10 invades.

**Appendix C.2 Meta-signaling**

The payoff calculation for -c-0 agents is obtained by reasoning that, with \( (1 - p) \)-probability, they will receive a score of 1 (since they cooperate), but with \( p \)-probability, they will receive a score of 0 since their signal doesn’t conform. With the knowledge that dc01 agents always have a score of 1, we can plug in to equation (23) to obtain:

\[ R_1(-c-0, dc01) = (1 - p)(\beta - \gamma) + p(-\gamma) \]
\[ = \beta - 2\gamma \]
\[ < \beta - \gamma \]

(24)

Lastly, it remains to check that the dc01 world still fends off -d-1 types. We see that with \( 1 - p \) probability, -d-1s receive a score of 0 based on their action, which will be a defection, and with
probability, they receive a score of 1 based on their signal, which conforms. Where \( p_x^{abxy} \) is the probability that an agent of type \( abxy \) will have score \( x \), this gives:

\[
R_1(-d-1, dc01) = p_1^{-d-1} \beta + p_0^{-d-1} \cdot 0 = p\beta \tag{25}
\]

Therefore, as long as \( p < \frac{\beta - \gamma}{\beta} \) (e.g. if \( \beta = 3 \) and \( \gamma = 2 \), then \( p < 1/3 \)), \(-d-1\) cannot invade.

**Appendix C.3 Error**

In this section we will calculate the specific payoffs for the error case, with and without meta-signaling.

**Appendix C.3.1 No meta-signaling**

Returning to Equation 6, we can calculate the expected payoffs in the error-prone case without meta-signaling using the stochastic values of \( \pi \) presented in Equation 8. Plugging in, we obtain

\[
R_1(dd, dc01) = (2\beta(1 - \epsilon) - \gamma)\epsilon \tag{26}
\]

\[
R_1(dc, dc01) = \frac{1}{2}(\beta - \gamma) \tag{27}
\]

\[
R_1(cc, dc01) = \beta(1 - 2\epsilon + 2\epsilon^2) - \gamma(1 - \epsilon). \tag{28}
\]

For a quick sanity check, we can see what happens when we plug in \( \epsilon = 0 \) (the no error case). Equation 26 and Equation 28 simplify down to 0 and \( \beta - \gamma \), respectively. This is what we would expect, since Equation 26 should equal the payoff in the case where both agents defect, and Equation 28 the payoff where both cooperate. Unfortunately this sanity check doesn’t work for \( R_1(dc, dc01) \), since for this case there is a discontinuity at 0.

Then, solving for their points of intersection, we obtain that

\[
R_1(dd, dc01) > R_1(dc01, dc01) \text{ when } \frac{\beta - \gamma}{2\beta} < \epsilon < \frac{1}{2} \tag{29}
\]

and

\[
R_1(cc, dc01) > R_1(dc01, dc01) \text{ when } \epsilon < \frac{\beta - \gamma}{2\beta} \text{ and } \frac{1}{2} < \epsilon \tag{30}
\]

So the \( dc01 \) strategy never outperforms both of the others at the same time.

**Appendix C.3.2 With meta-signaling**

In this section we will calculate exact payoffs, showing that at the very point that discriminators begin to outperform cooperators, defectors begin to outperform discriminators.

Due to meta-signaling, probabilities \( p_x^{abxy} \) now depend on language, but as we’ve shown, conforming-language agents will outperform all others, so we will restrict our attention to them. We calculate \( p_1^{co1} \), for instance, by splitting into cases. If the score was determined by the agent’s signaling (this scenario has probability \( p \)), it will be a 1, since the signal conforms. On the other hand, if it was determined by
the agent’s action, it will be a 1 with probability \((1 - \epsilon)\), since cc agents defect only with probability \(\epsilon\). This gives us \(p^{cc01}_1 = p + (1 - p)(1 - \epsilon)\). Similar reasoning gives

\[
p^{cc01}_1 = p + (1 - p)(1 - \epsilon) \\
p^{dd01}_1 = p + (1 - p)\epsilon
\]  

Let us then find the equilibrium level of \(p^{dc01}_1\) (analogous to the iterative process in figure 8A). Suppose that \(p^k_1\) is the probability that a \(dc01\) agent will have score 1 on the \(k\)th round. Then for the \((k + 1)\)th round we get

\[
p^{k+1}_1 = p + (1 - p)((1 - \epsilon)p^k_1 + \epsilon(1 - p^k_1))
\]  

Where the first term is for the scenario in which the agent is scored according to their (conforming) signal, and the remaining “clump” deals with the scenario where agents are scored according to their action. The first term in this clump is the scenario where the agent employs their default action with the agent whose score is 1, and the second is the scenario in which the agent deviates from their default action with an agent whose score is 0.

Solving for the equilibrium level (by setting \(p^{k+1}_1 = p^k_1\)) we get

\[
p^{dc01}_1 = \frac{p + \epsilon - p\epsilon}{p + 2\epsilon - 2p\epsilon}.
\]  

Of note here is that when \(p = 0\), \(p^{dc01}_1 = 1/2\), which leads to the 50-50 cooperation rate described in subsection 4.2.5. In general, the equilibrium cooperation level \(t\) can be calculated from this, too:

\[
t = (1 - \epsilon)p^{dc01}_1 + \epsilon(1 - p^{dc01}_1) = \frac{p + \epsilon - 2p\epsilon}{p + 2\epsilon - 2p\epsilon}
\]  

The first term on the first line corresponds to cooperating with 1-agents, and the second term corresponds to accidentally cooperating with a 0-agent. This equilibrium cooperation level gets significantly perturbed from 1/2 when \(p > 0\); for instance, for \(p = 1/10\) and \(\epsilon = 1/20\), the equilibrium is 0.737.

Now, using Equation 16, the values of \(\pi\) in equation 18 and the \(p\)-values just calculated, we obtain

\[
R_1(dd01, dc01) = -\gamma \epsilon + (-1 + \epsilon)(-2\epsilon + p(-1 + 2\epsilon))
\]

\[
R_1(dc01, dc01) = (\beta - \gamma)(-\epsilon + p(-1 + 2\epsilon))
\]

\[
R_1(cc01, dc01) = \beta + \gamma(-1 + \epsilon) + \beta(-2 + p)\epsilon - 2\beta(-1 + p)\epsilon^2
\]  

By looking at the intersections of these, we obtain that

\[
R_1(dd01, dc01) > R_1(dc01, dc01) \text{ when } \frac{\beta - \gamma - \beta p}{2\beta(1 - p)} < \epsilon < \frac{1}{2}
\]  

and

\[
R_1(cc01, dc01) > R_1(cc01, dc01) \text{ when } \epsilon < \frac{\beta - \gamma - \beta p}{2\beta(1 - p)} < \frac{1}{2} \text{ and } \frac{1}{2} < \epsilon
\]  

Thus, for any given value of \(\epsilon\), either dd01 or cc01 will invade.
As for the case of error, we will analyze the aligned norm case both with and without meta-signaling.

**APPENDIX C.4.1 No Meta-signaling**

We begin by finding $p_{dc1001}^{*}$, which will not depend on the language since there is no meta-signaling yet. Because the dc1001 strategy always prescribes the “right” action (that is, the action that garners a score of 1 from other dc1001 agents), they will only ever receive a score of 0 when they deviate from their default strategies, which happens with probability $\epsilon$. So from this we can conclude that

$$p_{dc}^{1} = 1 - \epsilon$$  \hspace{1cm} (41)

In contrast to the dc agents, the cd strategy always prescribes the wrong strategy (the one that garners a zero score from dc1001 agents). So for them, we have the mirror image situation,

$$p_{cd}^{1} = \epsilon.$$  \hspace{1cm} (42)

Now we calculate the other quantities, $p_{cc}^{1}$ and $p_{dd}^{1}$, given what we know so far. Cooperators do the “right” thing with 1-agents and the “wrong” thing with 0-agents, so we have that

$$p_{cc}^{1} = (1 - \epsilon)p_{dc}^{1} + \epsilon p_{dc}^{0} = 1 - 2\epsilon(1 - \epsilon)$$  \hspace{1cm} (43)

So as long as $1 - \epsilon > \frac{1}{2}$, cooperators are less likely than discriminators to be scored favorably—another good sign for the discriminators, because we recall that in the simple error case the reverse was true. Similarly for $p_{dd}^{1}$, we obtain that

$$p_{dd}^{1} = (1 - \epsilon)p_{dc}^{0} + \epsilon p_{dc}^{1} = 2\epsilon(1 - \epsilon)$$  \hspace{1cm} (44)

Thus, as long as $\epsilon < \frac{1}{2}$, defectors are less likely than discriminators to be receive a score of 1. This gives us all the information we need to calculate the expected payoffs of each type, using equations [16] and [18]

$$\mathbb{R}_1(dc, dc1001) = (1 - 2\epsilon + 2\epsilon^2)(\beta - \gamma)$$  \hspace{1cm} (45)

$$\mathbb{R}_1(cc, dc1001) = (1 - \epsilon)(\beta - \gamma - 2\beta\epsilon + 4\beta\epsilon^2)$$  \hspace{1cm} (46)

$$\mathbb{R}_1(cd, dc1001) = 2\epsilon(1 - \epsilon)(\beta - \gamma)$$  \hspace{1cm} (47)

$$\mathbb{R}_1(dd, dc1001) = \epsilon(\beta(3 - 6\epsilon + 4\epsilon^2) - \gamma)$$  \hspace{1cm} (48)

And comparing all these quantities we obtain that

$$\mathbb{R}_1(dd, dc1001) > \mathbb{R}_1(dc, dc1001) \text{ when } \frac{\beta - \gamma}{2\beta} < \epsilon < \frac{1}{2},$$  \hspace{1cm} (49)

$$\mathbb{R}_1(cc, dc1001) > \mathbb{R}_1(dc, dc01) \text{ when } \frac{1}{2} < \epsilon < \frac{\beta + \gamma}{2\beta},$$  \hspace{1cm} (50)

and

$$\mathbb{R}_1(cd, dc1001) \leq \mathbb{R}_1(dc, dc1001)$$  \hspace{1cm} (51)
Appendix C.4.2  With Meta-signaling

Let us calculate \( p_1^{dc} \) (where the signaling strategy is implied to the conforming one). First, we split our calculation into two scenarios: the one in which the score is given based on action (probability \( 1 - p \)), and the one in which the score is given based on signal (probability \( p \)). In the first case, we see that the agent conforms with probability \( 1 - \epsilon \), and in the second, the agent conforms with probability 1. So we get that

\[
p_1^{dc} = (1 - p)(1 - \epsilon) + p = 1 - \epsilon + pe
\]  

(52)

Then, to calculate \( p_1^{cc} \), \( p_1^{dd} \), and \( p_1^{cd} \), we use the no-meta-signaling expressions (from equations \( \text{two.osf} \), \( \text{three.osf} \), and \( \text{four.osf} \)) but with the new value of \( p_1^{dc} \) in a term scaled by \( (1 - p) \), which corresponds to receiving a score based on action. As for receiving a score based on signal, we add \( p \), since we are calculating these values for conforming signalers. This gives

\[
p_1^{cc} = (1 - p)((1 - \epsilon)p_1^{dc} + \epsilon(1 - p_1^{dc})) + p
\]  

(53)

\[
p_1^{dd} = (1 - p)(p_1^{dc} + (1 - \epsilon)(1 - p_1^{dc})) + p
\]  

(54)

\[
p_1^{cd} = (1 - p)\epsilon + p
\]  

(55)

And using our expected payoffs for each scenario, we can calculate the expected payoffs for each agent type

\[
\mathbb{R}_1(\text{dc1001}, \text{dc1001}) = -(\beta - \gamma)(-1 - (-2 + p)e + 2(-1 + p)e^2)
\]  

(56)

\[
\mathbb{R}_1(\text{cc1001}, \text{dc1001}) = \gamma(-1 + \epsilon) + \beta(1 - (3 - 3p + p^2)\epsilon
\]  

+ \(2(3 - 5p + 2p^2)\epsilon^2 - 4(-1 + p)^2\epsilon^3)\)

(57)

\[
\mathbb{R}_1(\text{cd1001}, \text{dc1001}) = \gamma\epsilon(-2 + p + 2\epsilon - 2pe) + \beta(-1 + \epsilon)(-2\epsilon + p(-1 + 2\epsilon))
\]  

+ \(p(1 - 5\epsilon + 10\epsilon^2 - 8\epsilon^3))\)

(58)

\[
\mathbb{R}_1(\text{dd1001}, \text{dc1001}) = -\gamma\epsilon + \beta(p^2(1 - 2\epsilon)\epsilon + \epsilon(3 - 6\epsilon + 4\epsilon^2)
\]  

+ \(p(1 - 5\epsilon + 10\epsilon^2 - 8\epsilon^3))\)

(59)

These equations are unenlightening at first blush, but when we calculate the points of intersection, we see that they are slight \( p \)-related adjustments on what we had before.

\[
\mathbb{R}_1(\text{dd}, \text{dc1001}) > \mathbb{R}_1(\text{dc}, \text{dc1001}) \text{ when } \frac{\beta - \gamma - \beta p}{2\beta(1 - p)} < \epsilon < \frac{1}{2},
\]  

(60)

\[
\mathbb{R}_1(\text{cc}, \text{dc1001}) > \mathbb{R}_1(\text{dc}, \text{dc1001}) \text{ when } \frac{1}{2} < \epsilon < \frac{\beta + \gamma - \beta p}{2\beta(1 - p)},
\]  

(61)

and

\[
\mathbb{R}_1(\text{cd}, \text{dc1001}) \leq \mathbb{R}_1(\text{dc}, \text{dc1001}) \text{ when } \frac{\beta - \gamma - \beta p}{2(\beta - \gamma)(1 - p)} < \epsilon < \frac{1}{2}.
\]  

(62)

So the only two strategies that can outperform \( dc \) when \( \epsilon < \frac{1}{2} \) are \( dd \) and \( cd \). But we notice that in the interval that we are concerned with, where \( \beta > \gamma > 0 \),
\[
\frac{\beta - \gamma - \beta p}{2\beta(1-p)} < \frac{\beta - \gamma - \beta p}{2(\beta - \gamma)(1-p)},
\]

since the denominator of the latter is smaller \(\beta > \beta - \gamma > 0\). So the interval on which \(cd\) outperforms \(dc\) is a subset of the interval for \(dd\). Thus we focus on \(dd\). How small must \(p\) be so that there is an interval where \(dc\) outperforms \(dd\)? Solving for the \(p\)-value which makes \(\frac{\beta - \gamma - \beta p}{2\beta(1-p)} > 0\), we need that \(p < (\beta - \gamma)/\beta\). If this holds, there will be an interval,

\[
0 < \epsilon < \frac{\beta - \gamma - \beta p}{2\beta(1-p)},
\]

in which \(dc\) outperforms all defectors, cooperators, and reverse discriminators.

**Appendix C.4.3 An addendum on conforming signalers doing better**

There is one additional probabilistic note to be made. While we said that performing better in expectation guarantees performing better overall in the infinite round case, this is not true once we relax this assumption. So it is worth examining where the the result holds most strongly.

A language tells an agent how to signal in each of the four possible signal-action scenarios: 0d, 0c, 1d, and 1c. Depending on the world we are in, each of these signal-action scenarios has a different probability of occurring, and a language that deviates on the the less-likely scenarios will perform almost as well as the conforming language. If we relax the infinite-rounds assumption, this could mean that this non-conforming language spreads by random chance.

Let us take the \(dc1001\) world as an example. For the scenario 1c to occur, we need that the opponent has score 1 and that the ego agent does his default action. This therefore occurs with probability \(p_1(1 - \epsilon) = (1 - \epsilon)^2\). Similarly, scenario 1d happens with probability \(p_1\epsilon = (1 - \epsilon)\epsilon\), 0d with \(p_0(1 - \epsilon) = (1 - \epsilon)\epsilon\), and 0c with probability \(p_0\epsilon = \epsilon^2\). Thus, an agent whose communication bit does not conform to 0c \((dc1101\), in this case) is much less likely to be caught, and therefore does almost as well as \(dc1001\). In finite-round models, therefore, this could be a so-called “weak spot” vulnerable to invasion by chance, especially if \(\epsilon\) is very small (an epsilon value of \(\frac{1}{100}\) means that the c0 scenario occurs only with probability \(\frac{1}{10000}\), for instance).

However, for the infinite round case (and the finite-round case when \(n\) is large and \(\epsilon\) is not too small), this justifies restricting our attention to conforming languages.

**References**

[Alexander, 1986] Alexander, R. D. (1986). Ostracism and indirect reciprocity: the reproductive significance of humor. *Ethology and Sociobiology*, 7:253–270.

[Axelrod, 1981] Axelrod, R. (1981). The emergence of cooperation among egoists. *The American Political Science Review*, 75(2):306–318.

[Axelrod and Hamilton, 1981] Axelrod, R. and Hamilton, W. D. (1981). The evolution of cooperation. *Science*, 211(4489):1390–1396.
[Bhui et al., 2019] Bhui, R., Chudek, M., and Henrich, J. (2019). How exploitation launched human cooperation. *Behavioral Ecology and Social Biology*, 73(6):78.

[Bicchieri, 1989] Bicchieri, C. (1989). Self-refuting theories of strategic interaction: A paradox of common knowledge. *Erkenntnis*, 30:69–85.

[Blume et al., 2001] Blume, A., DeJong, D. V., Kim, Y.-G., and Sprinkle, G. B. (2001). Evolution of communication with partial common interest. *Games and Economic Behavior*, 37:79–120.

[Bowles et al., 2003] Bowles, S., Choi, J.-K., and Hopfensitz, A. (2003). The coevolution of individual behaviors and social institutions. *Journal of Theoretical Biology*, 223:135–147.

[Boyd et al., 2010] Boyd, R., Gintis, H., and Bowles, S. (2010). Coordinated punishment of defectors sustains cooperation and can proliferate when rare. *Science*, 328:617–620.

[Boyd et al., 2003] Boyd, R., Gintis, H., Bowles, S., and Richerson, P. J. (2003). The evolution of altruistic punishment. *Proceedings from the National Academy of Sciences*, 100(6):3531–3535.

[Boyd and Lorberbaum, 1987] Boyd, R. and Lorberbaum, J. P. (1987). No pure strategy is evolutionarily stable in the iterated prisoner’s dilemma. *Nature*, 327:58–59.

[Boyd and Richerson, 1992] Boyd, R. and Richerson, P. (1992). Punishment allows the evolution of cooperation (or anything else) in sizable groups. *Ethology and Sociobiology*, 13:171–195.

[Boyd and Richerson, 1988] Boyd, R. and Richerson, P. J. (1988). The evolution of reciprocity in sizable groups. *Journal of Theoretical Biology*, 132:337–356.

[Brown et al., 1982] Brown, J. S., Sanderson, M. J., and Michod, R. E. (1982). The evolution of social behavior by reciprocation. *Journal of Theoretical Biology*, 99:319–339.

[Gintis et al., 2001] Gintis, H., Smith, E. A., and Bowles, S. (2001). Costly signaling and cooperation. *Journal of Theoretical Biology*, 213:103–119.

[Hamilton, 1964] Hamilton, W. D. (1964). The genetical evolution of social behavior. *Journal of Theoretical Biology*, 7:1–16.

[Henrich, 2004] Henrich, J. (2004). Cultural group selection, coevolutionary processes, and large-scale cooperation. *Journal of Economic Behavior and Organization*, 53(1):3–35.

[Henrich and Boyd, 1998] Henrich, J. and Boyd, R. (1998). The evolution of conformist transmission and the emergence of between-group difference. *Evolution and Human Behavior*, 19:215–241.

[Henrich and Boyd, 2001] Henrich, J. and Boyd, R. (2001). Why people punish defectors. *Journal of Theoretical Biology*, 208:79–89.

[Henrich et al., 2001] Henrich, J., Boyd, R., Bowles, S., Camerer, C., Fehr, E., Gintis, H., and McElreath, R. (2001). In search of homo economicus. *The American Economic Review*, 91(2):73–78.
Henrich et al., 2006 Henrich, J., McElreath, R., Barr, A., Ensminger, J., Barrett, C., Bolyanatz, A., Cardenas, J. C., Gurven, M., Gwako, E., Henrich, N., Lesorogol, C., Marlowe, F., Tracer, D., and Ziker, J. (2006). Costly punishment across human societies. Science, 312(5781):1767–1770.

Henrich and Muthukrishna, 2021 Henrich, J. and Muthukrishna, M. (2021). The origins and psychology of cooperation. Annual Review of Psychology, 72:24.1–24.34.

Hirshleifer and Rasmusen, 1989 Hirshleifer, D. and Rasmusen, E. (1989). Cooperation in a repeated prisoners’ dilemma with ostracism. Journal of Economic Behavior and Organization, 12:87–106.

Hofstadter, 1979 Hofstadter, D. (1979). Godel, Escher, Bach: An eternal golden braid. Basic Books.

Hofstadter, 1995 Hofstadter, D. (1995). Fluid Concepts and Creative Analogies. Basic Books.

Hirshleifer and Rasmusen, 1989 Hirshleifer, D. and Rasmusen, E. (1989). Cooperation in a repeated prisoners’ dilemma with ostracism. Journal of Economic Behavior and Organization, 12:87–106.

Kuhn, 2019 Kuhn, S. (2019). Prisoner’s dilemma.

Kreps et al., 1982 Kreps, D. M., Milgrom, P., Roberts, J., and Wilson, R. (1982). Rational cooperation in the finitely repeated prisoner’s dilemma. Journal of Economic Theory, 27:245–252.

Leimar and Hammerstein, 2000 Leimar, O. and Hammerstein, P. (2000). Evolution of cooperation through indirect reciprocity. Proceedings from the Royal Society London, 268:745–753.

Maynard Smith and Price, 1973 Maynard Smith, J. and Price, G. R. (1973). The logic of animal conflict. Nature, 246(5427):15–18.

McElreath et al., 2003 McElreath, R., Boyd, R., and Richerson, P. J. (2003). Shared norms and evolution of ethnic markers. Current Anthropology, 44(1):122–129.

Nowak, 2005 Nowak, M. A. (2005). Evolution of indirect reciprocity. Nature, 437(7077):1291–1298.

Nowak, 2006 Nowak, M. A. (2006). Five rules for the evolution of cooperation. Science, 314(5805):1560–1563.

Nowak and Sigmund, 1998 Nowak, M. A. and Sigmund, K. (1998). Evolution of indirect reciprocity by image scoring. Nature, 393:573–577.
[Odouard et al., 2021] Odouard, V. V., Smirnova, D., and Edelman, S. (2021). Polarize, catalyze, stabilize: Conscience and the evolution of cooperation. arXiv, (2112):11664.

[Ohtsuki and Iwasa, 2005] Ohtsuki, H. and Iwasa, Y. (2005). The leading eight: Social norms that can maintain cooperation by indirect reciprocity. Journal of Theoretical Biology, 239:435–444.

[Oliphant, 1996] Oliphant, M. (1996). The dilemma of saussurean communication. Biosystems, 37:31–38.

[Pacheco et al., 2006] Pacheco, J. M., Santos, F. C., and Chalub, F. A. C. C. (2006). Stern judging: A simple, successful norm that promotes cooperation under indirect reciprocity. PLoS Computational Biology, 2(12):e178.

[Panchanathan and Boyd, 2003] Panchanathan, K. and Boyd, R. (2003). A tale of two defectors: the importance of standing for the evolution of indirect reciprocity. Journal of Theoretical Biology, 224:115–126.

[Price and Jones, 2020] Price, M. H. and Jones, J. H. (2020). Fitness-maximizers employ pessimistic probability weighting for decisions under risk. Evolutionary Human Sciences, 2.

[Queller, 1992a] Queller, D. C. (1992a). A general model for kin selection. Evolution, 46(2):376–380.

[Queller, 1992b] Queller, D. C. (1992b). Quantitative genetics, inclusive fitness, and group selection. The American Naturalist, 139(3):540–558.

[Robinson-Arnull, 2018] Robinson-Arnull, C. (2018). Moral talk and indirect reciprocity: direct observation enables the evolution of moral signals. Biology and Philosophy, 33(42).

[Smead, 2010] Smead, R. (2010). Indirect reciprocity and the evolution of “moral signals”. Biology and Philosophy, 25:33–51.

[Trivers, 1971] Trivers, R. L. (1971). The evolution of reciprocal altruism. Quarterly Review of Biology, 46(1):35–37.

[Villani et al., 2007] Villani, M., Banacini, S., Ferrari, D., Serra, R., and Lane, D. (2007). An agent-based model of exaptive processes. European Management Review, 4:141–151.

[Wang et al., 2012] Wang, J., Suri, S., and Watts, D. J. (2012). Cooperation and assortativity with dynamic partner updating. PNAS, 109(36):14363–14368.

[Wedekind and Milinski, 2000] Wedekind, C. and Milinski, M. (2000). Cooperation through image scoring in humans. Science, 288(5467):850–852.

[Yamamoto et al., 2017] Yamamoto, H., Okada, I., Uchida, S., and Sasaki, T. (2017). A norm knockout method on indirect reciprocity to reveal indispensable norms. Nature Scientific Reports, 7(1).

[Young, 1998] Young, H. P. (1998). Social norms and economic welfare. European Economic Review, 42:821–830.