Rotation and Mixing in Massive Stars: Principles and Uncertainties

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Abstract. The main instabilities induced by rotation in stellar interiors are described. We derive from first principles the general equation describing the transport of the angular momentum. The case of the transport of the chemical species is also discussed. As long as the mass loss rates are not too important, meridional currents, by advecting angular momentum from the inner regions to the outer layers, accelerate the stellar surface during the Main Sequence phase. A 9 M$_\odot$ stellar model at solar metallicity with an equatorial velocity at the beginning of the core H–burning phase equal to 340 km s$^{-1}$ reaches the break–up limit during the MS phase. The model with an initial velocity of 290 km s$^{-1}$ approaches this limit without reaching it. The models with 290 km s$^{-1}$ and 340 km s$^{-1}$ predict enhancements of the N/C ratio at the end of the MS phase equal to 2.8 and 3.2 times the initial value respectively.

1. The main instabilities in a rotating star

Recent discussions of the various instabilities induced by rotation may be found in Maeder & Meynet (2000), Heger & Langer (2000), and Talon (2004). Among the most important instabilities are the secular shear instability and the meridional circulation. These instabilities drive the transport of the chemical species and of the angular momentum. Let us briefly recall the physical principles underlying these two instabilities.

In a rotating star local radiative equilibrium cannot be achieved (Von Zeipel 1924; Eddington 1925; Vogt 1925). As a result some parts of the star are heated while others are cooled. The buoyancy forces then drive a large scale motion, called the meridional circulation. Meridional circulation, contraction/expansion of the stellar layers, and convection create gradients of the angular velocity inside the star. These gradients produce instabilities known as shear instabilities. The physical reason for this instability lies in the fact that the minimum energy state of a differentially rotating fluid is solid body rotation. The star will tend to approach this state by homogenizing the angular velocity by turbulent mixing.

In a radiative zone, the vertical stable density stratification counteracts both the shear and meridional instability. In that respect the $\mu$–gradients play a key role as a stabilizing agent. These gradients may even, depending on the physics involved in the model, completely inhibit the mixing (Meynet & Maeder 1997). There are different methods in the literature for accounting for the effects of the $\mu$–gradients on the mixing. Some authors choose a parametric approach consisting in multiplying the $\mu$–gradient by a free parameter, $f_\mu$, smaller than one in order to weaken the stabilizing effect of the $\mu$–gradient. The value of $f_\mu$ is chosen in order to enable the stellar models to reproduce the observed surface

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enrichments (Heger & Langer 2000). The very small values chosen (of the order of 0.05) well illustrates the strong inhibiting effects of the \( \mu \)-gradients. Other methods devised by Maeder (1997), Talon & Zahn (1997) take into account of the effects of the strong horizontal turbulence (see below) and of the heat exchanges which can occur during the mixing process. This reduces the vertical drawback force and makes easier the development of the instability. In that case, the energy available in the shear can always be used for performing some mixing. Expressions for the shear diffusion coefficients can then be deduced without the need of artificially reducing the inhibiting effect of the \( \mu \)-gradients.

In the horizontal direction, i.e. on a isobar, in contrast to what happens in the vertical direction, the shear instability is not inhibited by a stable density stratification and the turbulence can develop without difficulty. Thus as long as horizontal gradients are continuously built up by e.g. meridional circulation, a strong horizontal turbulence develops in the star (Zahn 1992). This strong horizontal turbulence will also erode very efficiently any horizontal gradient of \( \Omega \). Thus the star can always be considered to be in a state of shellular rotation, characterized by constant values of \( \Omega \) on isobars. Starting from this a priori, but reasonable hypothesis, Zahn (1992) proposed a consistent theory of the interaction between the shear instability and the meridional circulation. The theory can be kept one dimensional thanks to the hypothesis of strong horizontal turbulence.

2. Equations for the transport of the angular momentum

Let us express the rate of change of the angular momentum, \( \frac{d\mathcal{L}}{dt} \), of the element of mass in the volume ABCD represented in Fig. 1:

\[
\frac{d\mathcal{L}}{dt} = \mathbf{M},
\]

where \( \mathbf{M} \) is the momentum of the forces acting on the volume element. We assume that angular momentum is transported only through advection (by a velocity field \( \mathbf{U} \)) and through turbulent diffusion, which may be different in the radial (vertical) and tangential (horizontal) direction. The component of the angular momentum aligned with the rotational axis is equal to \(^1\)

\[
\frac{pr^2 \sin \theta d\theta d\varphi dr}{\text{Mass of ABCD}} \quad \frac{r \sin \theta \Omega}{\text{velocity}} \quad r \sin \theta
\]

where \( \Omega = \dot{\varphi} \). Since the mass of the volume element ABCD does not change, the rate of change of the angular momentum can be written

\[
pr^2 \sin \theta d\theta d\varphi dr \frac{d}{dt}(r^2 \sin^2 \theta \Omega)_{M_r}.
\]

\(^1\)The components perpendicular to the rotational axis cancel each other when the integration is performed over \( \varphi \).
Due to shear, forces apply on the surfaces of the volume element. The force on the surface AB is equal to

$$\eta v \frac{r \sin \theta}{dr} \frac{\partial \Omega}{dr} \sin \theta \sin \partial r \partial \varphi \cdot$$

The component of the momentum of this force along the rotational axis is

$$\eta v^3 \sin^2 \theta \frac{\partial \Omega}{dr} dr d\varphi \cdot$$

The component along the rotational axis of the resultant momentum of the forces acting on AB and CD is equal to

$$\frac{\partial}{\partial r} (\eta v^4 \sin^3 \theta dr d\varphi \frac{\partial \Omega}{dr}) dr. \quad (2)$$

The force on the surface AC due to the tangential shear is equal to

$$\eta h \frac{r \sin \theta}{dr} \frac{\partial \Omega}{dr} \sin \varphi \sin \frac{1}{dr} \cdot$$

The component along the rotational axis of the resultant momentum of the forces acting on AC and BD is equal to

$$\frac{\partial}{\partial \theta} (\eta h^2 \sin^3 \theta \frac{\partial \Omega}{dr}) \sin \varphi \cdot \quad (3)$$

Using Eqs. 1, 2 and 3, simplifying by $dr d\theta d\varphi$, one obtains the equation for the transport of the angular momentum

$$\rho \frac{d}{dt} \frac{r^2 \sin^2 \theta \partial}{dt} = \frac{\partial}{\partial t} (\rho v r^4 \sin^3 \theta \frac{\partial \Omega}{dr}) + \frac{\partial}{\partial \theta} (\rho h^2 \sin^3 \theta \frac{\partial \Omega}{dr}) \sin \varphi \cdot \quad (4)$$

Setting $\eta v = \rho D_v$ and $\eta h = \rho D_h$ and dividing the left and right member by $r^2 \sin \theta$, one obtains

$$\rho \frac{d}{dt} (r^2 \sin^2 \theta \partial) = \frac{\sin^2 \theta}{r^2} \frac{\partial}{\partial r} (\rho v r^4 \frac{\partial \Omega}{dr}) + \frac{\sin \varphi}{\sin \theta \frac{\partial}{\partial \theta} (\rho h^2 \sin^3 \theta \frac{\partial \Omega}{dr}) \sin \varphi \cdot \quad (5)$$

Now, the left–handside term can be written

$$\rho \frac{d}{dt} (r^2 \sin^2 \theta \partial) = \frac{\partial}{\partial t} (\rho v) r^2 \sin^2 \theta \partial + \nabla \cdot (\rho v^2 \sin^2 \theta \partial) - \rho \sin^2 \theta \frac{\partial \rho}{\partial t} \partial \cdot \quad (6)$$

Using the relation between the Lagrangian and Eulerian derivatives, one has

$$\rho \frac{d}{dt} (r^2 \sin^2 \theta \partial) = \frac{\partial}{\partial t} (\rho v^2 \sin^2 \theta \partial) + \nabla \cdot (\rho v^2 \sin^2 \theta \partial) - \rho \sin^2 \theta \frac{\partial \rho}{\partial t} \partial \cdot \quad (6)$$

Due to shear, forces apply on the surfaces of the volume element. The force on the surface AB is equal to
Figure 1. The momentum of the viscosity forces acting on the element ABCD is derived in the text and the general form of the equation describing the change with time of the angular momentum of this element is deduced. The star rotates around the vertical axis with the angular velocity $\Omega$; $r$ and $\theta$ are the radial and colatitude coordinates of point A.

Using

$$\frac{d\rho}{dt}|_{Mr} = \frac{\partial \rho}{\partial t}|_r + U \cdot \nabla \rho,$$

and the continuity equation

$$\frac{\partial \rho}{\partial t}|_r = - \text{div}(\rho U),$$

one obtains $d\rho/dt|_{Mr} + \rho \text{div}U = 0$, which incorporated in Eq. 6 gives

$$\rho \frac{d}{dt}(r^2 \sin^2 \theta \Omega)|_r = \frac{\partial}{\partial t}(r^2 \sin^2 \theta \Omega)_r + \nabla (U r^2 \sin^2 \theta \Omega).$$

Developing the divergence in spherical coordinates and using Eq. 5 one finally obtains the equation describing the transport of the angular momentum (Maeder & Zahn 1998; Mathis & Zahn 2004)

$$\frac{\partial}{\partial t}(r^2 \sin^2 \theta \Omega)_r + \frac{1}{r^2} \frac{\partial}{\partial r}(r^4 \sin^2 \theta w_r \Omega) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}(r^2 \sin^2 \theta w_\theta \Omega) = \frac{\sin^2 \theta}{r^2} \frac{\partial}{\partial r}(\rho D_r r^4 \frac{\partial \Omega}{\partial r}) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta}(\rho D_\theta \sin^2 \theta \frac{\partial \Omega}{\partial \theta}),$$

(7)

where $w_r = U_r + \dot{r}$ is the sum of the radial component of the meridional circulation velocity and the velocity of expansion/contraction, and $w_\theta = U_\theta$, where $U_\theta$ is the horizontal component of the meridional circulation velocity. Assuming, as in Zahn (1992), that the rotation depends little on latitude due to strong horizontal diffusion, we write

$$\Omega(r, \theta) = \tilde{\Omega}(r) + \hat{\Omega}(r, \theta),$$
with $\tilde{\Omega} \ll \Omega$. The horizontal average $\Omega$ is defined as being the angular velocity of a shell rotating like a solid body and having the same angular momentum as the considered actual shell. Thus

$$\tilde{\Omega} = \frac{\int \Omega \sin^3 \theta d\theta}{\int \sin^3 \theta d\theta}.$$ 

Any vector field whose Laplacian is null can be decomposed in spherical harmonics. Thus, the meridional circulation velocity can be written (Mathis & Zahn 2004)

$$U = \sum_{l>0} \frac{U_l(r)}{r} P_l(\cos \theta) e_r + \sum_{l>0} V_l(r) \frac{dP_l(\cos \theta)}{d\theta} e_\theta,$$

where $e_r$ and $e_\theta$ are unit vectors along the radial and colatitude directions respectively. Multiplying Eq. 7 by $\sin \theta d\theta$ and integrating it over $\theta$ from 0 to $\pi$, one obtains (Maeder & Zahn 1998)

$$\frac{\partial}{\partial t} \left( \frac{r^2 \tilde{\Omega}}{5} \right) = \frac{1}{5r^2} \frac{\partial}{\partial r} \left( r^4 \Omega \left[ U_2(r) - 5\dot{r} \right] \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left( \rho D_v r^4 \frac{\partial \tilde{\Omega}}{\partial r} \right). \quad (8)$$

It is interesting to note that only the $l = 2$ component of the circulation is able to advect a net amount of angular momentum. As explained in Spiegel & Zahn (1992) the higher order components do not contribute to the vertical transport of angular momentum. Note also that the change in radius $\dot{r}$ of the given mass shell is included in Eq. 8 which is the Eulerian formulation of the angular momentum transport equation. In its Lagrangian formulation, the variable $r$ is linked to $M_r$ through $dM_r = 4\pi r^2 \rho dr$, and the equation for the transport of the angular momentum can be written

$$\rho \frac{\partial}{\partial t} \left( r^2 \tilde{\Omega} \right)_{M_r} = \frac{1}{5r^2} \frac{\partial}{\partial r} \left( r^4 \Omega U_2(r) \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left( \rho D_v r^4 \frac{\partial \tilde{\Omega}}{\partial r} \right). \quad (9)$$

The characteristic time associated to the transport of $\Omega$ by the circulation is (Zahn 1992)

$$t_\Omega \approx t_{KH} \left( \frac{\Omega^2 R}{g_s} \right)^{-1}, \quad (10)$$

where $g_s$ is the gravity at the surface and $t_{KH}$ the Kelvin–Helmholtz timescale, which is the characteristic timescale for the change of $r$ in hydrostatic models. From Eq. 10 one sees that $t_\Omega$ is a few times $t_{KH}$, which itself is much shorter than the Main Sequence lifetime.

In order to resolve Eq. 9 one needs expressions for $U_2(r)$, $D_v$, and $D_h$. In the following we indicate the general lines of the reasoning and give the references where more complete mathematical derivations can be found. The velocity of meridional circulation is derived from the equation of energy conservation (Mestel 1953)

$$\rho T \left[ \frac{\partial S}{\partial t} + (\dot{r} e_r + U) \cdot \nabla S \right] = \text{div}(\chi \nabla T) + \rho e - \text{div} F_h \quad (11)$$
where $S$ is the entropy per unit mass, $\chi$ the thermal conductivity, $\epsilon$ the rate of nuclear energy per unit mass and $F_h$ the flux of thermal energy due to horizontal turbulence. All the quantities are expanded linearly around their average on a level surface or isobar, using Legendre function of order 2 $P_2(\cos \theta)$. For instance

$$T(P, \theta) = \bar{T} + \bar{T}P_2(\cos \theta).$$

Then Eq. 11 is linearized and an expression for $U_2$ can be deduced (Zahn 1992). Using the same method Maeder & Zahn (1998) revised the expression for $U_2$ to account for expansion and contraction in non–stationary models. They also studied the effects of the $\mu$–gradients (mean molecular weight gradients), of the horizontal turbulence and considered a general equation of state. They obtained

$$U_2(r) = \frac{P}{\rho g C_P T [\nabla_{ad} - \nabla + (\varphi/\delta) \nabla_\mu]} \times \left[ \frac{L}{M_*} (E_\Omega + E_\mu) + \frac{C_P}{\delta} \frac{\partial \Theta}{\partial t} \right],$$

(12)

where $M_* = M \left( 1 - \frac{\Omega^2}{2 \pi G \rho} \right)$ is the reduced mass and the other symbols have the same meaning as in Zahn (1992) and Maeder & Zahn (1998). The driving term in the square brackets in the second member is $E_\Omega$. It behaves mainly like $E_\Omega \approx \frac{5}{3} \left[ 1 - \frac{\Omega^2}{2 \pi G \rho} \right] \left( \frac{\Omega^2 r^3}{GM} \right)$. The term $\overline{\varphi}$ means the average on the considered equipotential. The term with the minus sign in the square bracket is the Gratton–Opik term, which becomes important in the outer layers when the local density is small. This term produces negative values of $U_2(r)$ (noted $U(r)$ from now), meaning that the circulation is going down along the polar axis and up in the equatorial plane. This makes an outward transport of angular momentum, while a positive $U(r)$ gives an inward transport. At lower $Z$, the Gratton–Opik term is negligible, which contributes to make larger $\Omega$–gradients in lower $Z$ stars.

Recently Mathis & Zahn (2004) rederived the system of partial differential equations, which govern the transport of angular momentum, heat and chemical elements. They expand the departure from spherical symmetry to higher order and include explicitly the differential rotation in latitude, to first order. Boundary conditions for the surface and at the frontiers between radiative and convective zones are also explicitly given in this paper.

In the above equation $D_v$ is the shear diffusion coefficient whose expression is taken as in Talon & Zahn (1997). The usual estimate of $D_h = \frac{1}{c_h} \left| 2V(r) - \alpha U(r) \right|$ was given by Zahn (1992). Recent studies suggest that this coefficient is at least an order of magnitude larger (Maeder 2003; Mathis et al. 2004).

3. Equations for the transport of the chemical species

The derivation of the equation for the transport of chemical species proceeds much like that of angular momentum: the mass fraction $X_i$ of a given element $i$ obeys an advection–diffusion equation which can be written

$$\frac{\partial (\rho X_i)}{\partial t} + \text{div}(\rho X_i U) =$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( \rho D_v r^2 \frac{\partial X_i}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \rho D_h \frac{\partial X_i}{\partial \theta} \right),$$

(13)
Splitting the mass fractions in their horizontal average \( \langle \bar{X}_i \rangle \) and their fluctuation on the isobar, assuming that \( D_h \gg D_v \), Chaboyer & Zahn (1992) showed that Eq. 13 can be written

\[
\rho \left( \frac{d \bar{X}_i}{dt} \right)_{M_r} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( \rho(D_v + D_{\text{eff}})r^2 \frac{\partial \bar{X}_i}{\partial r} \right)
\]

with the effective diffusivity

\[
D_{\text{eff}} = \frac{|rU(r)|^2}{30D_h}.
\]

\( D_{\text{eff}} \) expresses the effects of the meridional circulation and of the strong horizontal turbulence. This expression of \( D_{\text{eff}} \) tells us that the vertical advection of chemical elements is inhibited by the strong horizontal turbulence characterized by \( D_h \). For simplicity in Eq. 14, the terms due to nuclear reactions are omitted, as well as the effects of element separation through radiation and gravitational settling. Let us finally add that various numerical procedures for resolving the diffusion equation are described and compared in Meynet et al. (2004).

4. Effects of rotation on the stellar winds

The taking into account of the von Zeipel theorem in the frame of the radiative driven wind theory has many interesting consequences (Owocki et al. 1996; Maeder 1999; Maeder & Meynet 2000):

1. The maximum luminosity of a rotating star is reduced by rotation.

2. The expression for the critical velocity in a rotating star is different if the star is near or far from the Eddington limit. Far from the Eddington limit, \( v_{\text{crit},1} = \left( \frac{2GM}{3R_{\text{pb}}} \right)^{\frac{1}{2}} \), where \( R_{\text{pb}} \) is the polar radius at break–up. When the Eddington factor is bigger than 0.639, the expression for \( v_{\text{crit}} \) is different (Maeder & Meynet 2000). It is equal to 0.85, 0.69, 0.48, 0.35, 0.22, 0 times \( v_{\text{crit},1} \) for \( \Gamma = 0.70, 0.80, 0.90, 0.95, 0.98 \) and 1.00 respectively.

3. Rotation induces anisotropies of the stellar winds (Maeder & Desjacques 2001; Dwarkadas & Owocki 2002).

4. Rotation increases the mass loss rates. We may estimate the mass loss rates of a rotating star compared to that of a non–rotating star at the same location in the HR diagram. The result is (Maeder & Meynet 2000)

\[
\frac{\dot{M}(\Omega)}{\dot{M}(0)} \simeq \left( \frac{1 - \Gamma}{\Gamma} \right)^{\frac{1}{\alpha} - 1} \left[ 1 - \frac{4}{9} \left( \frac{v}{v_{\text{crit},1}} \right)^2 - \Gamma \right]^{\frac{1}{\alpha} - 1},
\]

where \( \Gamma \) is the electron scattering opacity for a non–rotating star with the same mass and luminosity, \( \alpha \) is a force multiplier (Lamers et al. 1995). For B–stars far from the Eddington Limit, \( \frac{\dot{M}(\Omega)}{\dot{M}(0)} \simeq 1.5 \). For stars close to \( \Gamma = 1 \) the increase of the mass loss rates may reach orders of magnitude.
It is interesting to note that three kinds of break–up limit can be defined depending on which mechanism, radiative acceleration, centrifugal acceleration or both contribute to counterbalance the gravity: 1.– The $\Gamma$–Limit occurs when radiation effects largely dominate; 2.– The $\Omega$–Limit, when rotation effects are essentially determining break–up; 3.– The $\Omega\Gamma$–Limit, when both rotation and radiation are important for the critical velocity. In the cases of Luminous blue variables the $\Gamma$– and $\Omega\Gamma$–Limits likely occurs, while in the case of Be stars, the $\Omega$–Limit is the relevant limit.

5. Some successes of rotating models

The inclusion of rotation in massive star models improves in many respects the theoretical predictions. In particular, rotating models can reproduce the chemical enrichments observed at the surface of OBA stars (Heger & Langer 2000; Meynet & Maeder 2000), the number ratio of blue to red supergiants in the Small Magellanic Cloud (Maeder & Meynet 2001), the variation with the metallicity of the number ratio of Wolf–Rayet to O type stars (Meynet & Maeder 2003) as well as of the type Ibc to type II supernovae (Prantzos & Boissier 2003; Meynet & Maeder 2004). Rotating models predict new chemical yields (Hirschi et al. 2004) and naturally lead to the production of primary nitrogen at low metallicity (Meynet & Maeder 2002), they also bring new views on the evolution of pop III stars (Meynet et al. 2004a). Comparisons with the observed rotational velocities of stars become possible and interesting consequences for the rotation rate of young pulsars and the progenitors of Gamma Ray Burst can be obtained (Woosley & Heger 2004; Meynet et al. 2004b).

6. Conditions for reaching break–up during the MS phase

Recently Townsend et al. (2004) have given strong arguments indicating that the $v\sin i$ measured for Be stars might be systematically underestimated. An important implication of their work is that these stars may be rotating much closer the critical velocity than generally assumed. As the authors note, rotation close to critical effectively reduces the effective equatorial gravity and could make material to easily leak into a disc. Typically, when $v/v_{\text{crit}} \sim 0.95$, the velocity necessary to launch material into orbit is of the same order as the speed of sound in the outer layers. Therefore mechanisms, as nonradial pulsation, begin to be effective for orbital ejection (Owocki 2004). In view of these considerations, it appears interesting to investigate with our rotating stellar models, which are the initial conditions required for a star to reach the critical velocity during the MS phase. This is the object of this section.

Figs. 2 show the evolution of the rotational velocities and of the fraction $\Omega/\Omega_c$ of the angular velocity to the critical angular velocity at the surface of star models of different initial masses between 9 and 120 $M_\odot$ with account taken of anisotropic mass loss during the MS phase. The evolution of the rotation velocities at the stellar surface depends mainly on 2 factors, the internal coupling and the mass loss.

1.– The coupling mechanisms transport angular momentum in the stellar interiors. The extreme case of strong coupling is the classical case of solid body
rotation. In this case when mass loss is small, the star reaches the critical velocity during the MS phase more or less quickly depending on the initial rotation as shown by Sackman & Anand (1970) (see also Langer 1997). In the case of no coupling, i.e. of local conservation of the angular momentum, rotation becomes more and more subcritical. In the present models, with initial velocities at the beginning of the MS phase of $\sim 260 \, \text{km s}^{-1}$ the situation is intermediate, with a moderate coupling due mainly to meridional circulation, which is more efficient than shear transport, as far as transport of angular momentum is concerned.

2.- For a given degree of coupling, the mass loss rates play a most critical role in the evolution of the surface rotation. As shown by the comparison of the models at $Z = 0.02$ and $Z = 0.004$ (Maeder & Meynet 2001), for masses greater than $\sim 40 \, \text{M}_\odot$ the models with solar composition have velocities that decrease rather rapidly, while at $Z = 0.004$ the velocities go up. Thus, for the most massive stars with moderate or strong coupling, the mass loss rates are the main factor influencing the evolution of rotation. The effect of mass loss is also well illustrated in Fig. 3 which shows the evolution of $\Omega/\Omega_{\text{crit}}$ at the surface of a 15 $\text{M}_\odot$ solar metallicity model. One sees that during the MS phase, the ratio increases as a function of time, until $\log T_{\text{eff}}$ becomes inferior to 4.40. According to Vink et al. (2000; 2001), below this value there is a sudden increase of the mass loss rates, which makes the rotation velocities rapidly decrease.

It is interesting to note that below a mass of about 12 $\text{M}_\odot$, the mass loss rates are smaller and the internal coupling plays the main role in the evolution of the rotational velocities. This provides an interesting possibility of tests on
Figure 3. Three dimensional evolutionary track of a 15 $\text{M}_\odot$ model at solar metallicity, with a velocity at the beginning of the MS phase of 260 km s$^{-1}$. The evolution in the theoretical HR diagram is plotted on the horizontal plane. The vertical axis shows the evolution of the surface angular velocity divided by the critical or break–up angular velocity ($R_c = \Omega/\Omega_{\text{crit}}$).

the internal coupling by studying the differences in rotational velocities for stars at different distances of the ZAMS. In particular, such a study could allow us to test the role of magnetic coupling in radiative envelopes, which is now a major open question in stellar rotation studies (Spruit 2002).

From Fig. 2, we see that the stars which have a velocity around 260 km s$^{-1}$ at the beginning of the MS phase, do not reach the break–up limit. Taking into account of the wind anisotropies induced by rotation, a 9 $\text{M}_\odot$ stellar model at solar metallicity with an equatorial velocity at the beginning of the core H–burning phase equal to 340 km s$^{-1}$ reaches the break–up limit during the MS phase. This corresponds to values of $\Omega/\Omega_{\text{crit}}$ and of $v/v_{\text{crit}}$ equal to 0.81 and 0.61 respectively. The model with with an initial equatorial velocity of 290 km s$^{-1}$ ($\Omega/\Omega_{\text{crit}} = 0.71$, $v/v_{\text{crit}} = 0.52$), approaches but does not reach the break–up limit during the MS phase. At the end of the MS phase, the 290 and 340 km s$^{-1}$ models present N/C ratios at the surface equal to 2.8 and 3.2 times the initial value.

In Fig. 4, evolutions of $v/v_{\text{crit}}$ for different initial metallicities and masses are shown. Interestingly, one notes that the 5 $\text{M}_\odot$ stellar model at $Z = 0.040$ reaches the break–up limit well before the model at $Z=0.020$. This comes essentially from the dependence on density of the meridional velocity as given by the Gratton–Opik term. At higher metallicity, the radii of stars are bigger and thus the
density in the outer layers are smaller. At a given metallicity (see right panel of Fig. 4), the higher initial mass star reach the break-up limit before the lower initial mass models. Again this can be explained through the density dependence of the meridional circulation velocity.

Thus these numerical experiments show the following:

1) For stars suffering little mass loss, the main effect which can bring these stars near the break-up limit is the outwards transport of angular momentum by the meridional circulation. Since the velocity of the meridional currents in the outer layers scales with the inverse of the density, the process becomes more efficient for stars of higher initial mass and/or higher initial metallicity.

2) When the metallicity increases however, mass loss becomes more and more important and can prevent stars to reach the break-up limit.

How the fraction of stars which reach the break-up limit varies as a function of the initial metallicity requires a good knowledge of the distribution of the initial velocities as a function of the mass and of the metallicity. Large and extended surveys are being now performed with this purpose (see e.g. North et al. 2004, Royer et al. 2004). Also the study of the time evolution of the surface velocities on the Main Sequence for stars undergoing little mass loss would provide very important hints on the efficiency of the internal transport mechanisms of the angular momentum.

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References
Chaboyer, B., Zahn, J.-P. 1992, A&A 253, 173
Dwarkadas, V.V., Owocki, S.P. 2002, ApJ 581, 1337
Eddington, A.S. 1925, The Observatory 48, 73
Heger, A., & Langer, N. 2000, ApJ, 544, 1016
Hirschi, R., Meynet, G., Maeder, A. 2004, A&A, in press
Lamers, H.J.G.L.M., Snow, T.P., Lindholm, D.M. 1995, ApJ 455, 269
Langer, N. 1997, The Eddington Limit in Rotating Massive Stars. In: Nota A., Lamers H. (eds.) Luminous Blue Variables: Massive Stars in Transition. ASP Conf. Series, 120, p. 83
Maeder, A. 1997, A&A 321, 134 (Paper II)
Maeder, A. 1999, A&A, 347, 185 (Paper IV)
Maeder, A. 2003, A&A, 399, 263
Maeder, A., Desjacques, V. 2001, A&A 372, L9
Maeder, A., & Meynet, G. 2000, ARA&A, 38, 143
Maeder, A., & Meynet, G. 2000, A&A, 361, 159, (Paper VI)
Maeder, A., & Meynet, G. 2001, A&A, 373, 555, (Paper VII)
Maeder, A., Zahn, J.–P. 1998, A&A 334, 1000
Mathis, S., Zahn, J.–P. 2004, A&A, 425, 229
Mathis, S., Palacios, A., Zahn, J.–P. 2004, A&A, 425, 243
Mestel, L. 1953, MNRAS, 113, 716
Meynet, G., Maeder, A. 1997, A&A 321, 465 (Paper I)
Meynet, G., & Maeder, A. 2000, A&A361, 101, (Paper V)
Meynet, G., & Maeder, A. 2002, A&A, 390, 561, (Paper VIII)
Meynet, G., & Maeder, A. 2003, A&A404, 975, (Paper X)
Meynet, G., & Maeder, A. 2004, A&A, in press, (Paper XI)
Meynet, G., Maeder, A., Ekström, S. 2004a, in “The Fate of the Most massive Stars”, ASP Conf. Ser., in press [astro-ph/0408322]
Meynet, G., Maeder, A., Hirschi, R. 2004b, in “1604–2004: Supernovae as Cosmological Lighthouses”, ASP Conf. Ser., in press [astro-ph/0409508]
North, P., Royer,F., Melo, C. et al. 2004, in “Stellar Rotation”, IAU Symp. 215, A. Maeder & Ph. Eenens (eds.), ASP, in press
Owocki, S.P. 2004, in “Stellar Rotation”, IAU Symp. 215, A. Maeder & Ph. Eenens (eds.), ASP, in press
Prantzos, N., Boissier, S. 2003, A&A, 406, 259
Royer, F., Melo, C., Mermilliod, J.–C. et al. 2004, in “Stellar Rotation”, IAU Symp. 215, A. Maeder & Ph. Eenens (eds.), ASP, in press
Vink, J.S., de Koter, A., & Lamers, H.J.G.L.M. 2000, A&A, 362, 295
Vink, J.S., de Koter, A., & Lamers, H.J.G.L.M. 2001, A&A, 369, 574
Vogt, H. 1925, Astron. Nachr. 223, 229
Von Zeipel, H. 1924, in Probleme der Astronomie, Festschrift für H. v. Seeliger, ed. by H. Kienle (Springer, Berlin), p. 144
Woosley, S.E., Heger, A. 2004, in IAU Symp.215, A. Maeder & P. Eenens (Eds.), in press
Zahn, J.–P. 1992, A&A 265, 115