A reduced symmetric 2D photonic crystal cavity with wavelength tunability

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Abstract
In this paper, we propose a microcavity supported by a designed photonic crystal structure that provides both tunability of cavity modes and the cavity’s quality factor. A low symmetric defect region provides a trigger effect for the frequency shifting by means of rotational manipulation of small symmetry elements. Deviation of the effective filling ratio as a result of rotational modification within the defect region results in the emanation of cavity modes at different frequencies. Here, we numerically demonstrate the frequency shifting for each obtained mode with respect to the defect region architecture. In addition to wavelength tunability, quality factor, mode volume, and Purcell factors are analyzed for the slightly modified structures. Besides, electric field distributions and polarization properties of each mode that emerge at distinct frequencies have been studied at adjusted frequency modes which are observed for all rotational modification scenarios as \( \theta_{\text{Rot}} = [0^\circ, 15^\circ, 30^\circ, 45^\circ] \). After the investigations in 2D of silicon material (\( \varepsilon_r = 12 \)), 3D simulations are performed and the collected data is used for the height approximation of 3D structures to approach the results consistent with 2D ones, thus the cross-checking of the quality factor acquired from the 2D simulation can be executed by comparison with 3D. Moreover, 2D and 3D simulations of alumina material (\( \varepsilon_r = 9.61 \)) in terms of mode analysis and quality factor have been repeated considering the microwave experiments. Therefore, experimental analysis is compared with the numerical results and good agreement between the two is found.

Keywords: optical resonators, optical device, photonic crystal cavities, photonic crystals

(Some figures may appear in colour only in the online journal)

1. Introduction
Photonic crystals (PhC) are novel structures which have the ability to manipulate light. A conceived PhC structure can manipulate the light behavior in many appealing ways, and the cavity effect is an intriguing one, with a surge of interest in recent years. A PhC cavity provides a medium for the localization of light inside the structure by utilizing localized defects. Light is gathered into a small mode volume approximately in optical-wavelength dimensions whilst it is passing through the PhC, this behavior results in a horizontal and vertical localization effect which is triggered by distributed Bragg reflection (DBR) and total internal reflection (TIR). As is known, TIR can be described by the way of the \( k \) vector placement according to the light cone, which is directly associated with the photonic band structure. In accordance with approximation, frequencies which take part above the light line are not allowed to be guided inside the structure, and these modes are known as radiation modes, whereas the frequencies below the light line represent allowed modes and they are named guided modes. The contribution of the TIR to performance of any cavity is related to the allowed mode portion. Vertically confined guided modes, resulting from TIR are controllable for all structures and they come to exist owing to defects belong to the structure. Otherwise, 3D structures cannot trap the light due to the failure of totally providing the TIR requirements although small mode volumes are still consistent. 3D structures sense this phenomenon as a loss mechanism and consequently these out of plane losses cause a drastic decline of the vertical cavity performance compared
to 2D systems, which ensure the vertical localization [1–5]. However, the opportunities in the field of performance come at a price, namely fabrication irregularities and sustainability issues such as material and surface-state absorptions and surface roughness, which are the main obstacles for 2D PhCs [6]. These 2D microscale structures need rigorous fabrication processes to maintain the same performance, and slight defect disarrangements cause serious losses. Several optical applications rely mostly on TIR, like microspheres or microdisks, and 3D systems work by exposure to the more configurable DBR mechanism [7]. Laterally confined guided modes emerged by way of Bragg reflection of the light from the surrounding PhC layers of the structure. Each layer of the PhC works like a mirror, and symmetric architecture provides an efficient localization effect from each direction of the cavity region. If the existing modes fulfill the requirements and are confined laterally and vertically after disposing of the defect region within a regular structure, a PhC cavity is generated [8, 9]. Applying structural modifications to the cavity region provides acquisition of targeted enhancement, as well as being a method to control the cavity modes [10–13]. Besides regular and simple structures, complex ones have also been investigated, like heterostructures or quasicrystals to acquire further cavity information [14–17]. The mentioned cavity studies above have opened up new opportunities to many applications for integrated devices such as filters [15, 18], all-optical switches [19, 20], sensors [21–24] and optical storing devices [25, 26].

In the present work, a 2D square lattice PhC possessing defect region is investigated, such that symmetry reduced configuration allows resonance mode tunability via rotational symmetry without any change in the filling ratio of the defect site. In detail, it is a well-known attribute of transverse magnetic (TM) modes that the localization of the electric field distribution occurs on the dielectric. In accordance with this information it can be claimed that the TM modes distribute depending on the dielectric constant variation. The alleged shift property of the presented structure can be attributed to the modifications of the dielectric distribution by means of the smaller symmetry elements rotation. Rotation of the smaller symmetry elements changes the dielectric distribution, and the electric field is affected by this distribution. Thereby, the mode generation is triggered at different frequencies, and this situation is commented as a frequency shift. Here, we trigger the mode generation by fulfilling the circumstances at frequencies different from each other and observe the resonant peaks at shifted frequencies. In addition to 2D investigations, 3D examinations along with the experimental case are also performed. Thereby, a rich point of view about light confinement at cavities been provided in the study.

In this paper, we discuss the following. First, 2D frequency domain analyses have been performed to have an insight into the resonant modes localized in 2D structure and correlatively 3D one. After identification of modes, time domain analyses have been applied to the localized modes, and the responses to the angular modifications have been investigated. The quality factor, Purcell effect, polarization properties, and electric field distributions analyses have carried out for all modes; then, an ideal mode has been chosen to focus on a specific situation. Here, mode analyses such as the investigation of the optimum thickness approach to achieve the maximum quality value and the response of the mode to the angular manipulation have been carried out. Before performing the experimental measurement, a material exchange was needed and time domain analyses have been applied to the new material. Finally, the main issues that are wavelength tunability and quality factor investigations have observed for the experiment of alumina consistent with 2D and 3D simulations.

2. Numerical study of low-symmetric photonic cavity

2.1. Proposed design and frequency domain analysis

In this paper, we claim that the low symmetric PhC structures are good candidates to gather the cavity modes tunability owing to rotational tailoring without a decrease in the performance of the cavity. In this stage, it has been observed that the equal x and y dimensions of the structure help the light confinement and enhance the cavity performance as a result of an effect which can be defined as symmetric mirror effect. By considering this, an equal number of layers along x and y axes are reached for the cavity design after sufficient iterations. Also, one of the further requirements to the occurrence of cavity effect is verification of an optimum layer number which surrounds the cavity region. After a number of layer iterations, the optimum layer number is obtained as $15\alpha$ in this study. Eventually, the structural iterations which find the ideal condition to establish a cavity region show that it has been achieved with a high-performance cavity design with a relatively compact array with $15\alpha \times 15\alpha$ size.

![Figure 1](image_url)
Figure 2. Band diagrams of the \((5a \times 5a)\) supercell according to the rotation angle of the small rod within defect region, \(\theta_{\text{rot}} = 0^\circ\) (a), \(\theta_{\text{rot}} = 15^\circ\) (b), \(\theta_{\text{rot}} = 30^\circ\) (c) and \(\theta_{\text{rot}} = 45^\circ\) (d) Shaded parts which emphasize those above the light line are given as an intuition for allowing normalized frequencies of 3D analysis of 2D PhC that will be investigated in the following section.

The mentioned regular PhC structure with a low symmetric defect region is shown in figure 1(a), also the zoomed version is given in figure 1(b). As it is given, this region formed by four dielectric silicon rods with \(R_1 = 0.20a\) and it is supported with smaller dielectric rods with \(R_2 = 0.10a\) radii and \(\varepsilon_r = 12\) dielectric constant, as a defect region. It is demonstrated that the active zone is designed on the basis of the smaller rods rotating and getting the reference to the nearest bigger rods at specific angles, \(\theta_{\text{rot}} = [0^\circ, 15^\circ, 30^\circ, 45^\circ]\) [27].

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all existing modes for the designed structure. The mode identification has performed in an exact way with parallel analyses of electric field distributions, that we will discuss later. Without any hesitation, a big portion of peak shifting is observed at the second mode ($M_2$) which is used for telecom wavelength, given as $\lambda_2 = [1561 \text{ nm}, 1571 \text{ nm}, 1589 \text{ nm}, 1597 \text{ nm}]$ for $\theta_{\text{Rot}} = [0^\circ, 15^\circ, 30^\circ, 45^\circ]$ rotational angles, respectively, with the maximum deviation at $\lambda_{\text{shift,2}} = [1561 \text{ nm}, 1597 \text{ nm}]$ which corresponds to 36 nm shifting as represented in figure 3(b) for $\theta_{\text{Rot}} = [0^\circ – 45^\circ]$ angles. A similar trend exists for the third mode ($M_3$) which has a wavelength range of $\lambda_3 = [1381 \text{ nm}, 1366 \text{ nm}, 1346 \text{ nm}, 1355 \text{ nm}]$ for $\theta_{\text{Rot}} = [0^\circ, 15^\circ, 30^\circ, 45^\circ]$, respectively. A reasonable shift is valid between $\theta_{\text{Rot}} = [0^\circ – 30^\circ]$ rotational angle deviation, and maximum peak shift $\lambda_{\text{shift,3}} = [1346 \text{ nm}, 1381 \text{ nm}]$ with 35 nm shifting value, results are given in figure 3(c). Between the modes that show nearly the same attitude to peak shifting with respect to rotational alteration ($M_2$ and $M_3$), the existence of a common behavior of some wavelengths towards higher rotational angles provides an insight for the mode evaluation of wavelengths. However, there is almost no shift for the first mode ($M_1$) which has a range of $\lambda_1 = [1956 \text{ nm}, 1956 \text{ nm}, 1956 \text{ nm}, 1955 \text{ nm}]$ seen in figure 3(a) whilst a slight shifting exists for the fourth mode ($M_4$) that belongs to the following wavelengths $\lambda_4 = [1362 \text{ nm}, 1365 \text{ nm}, 1372 \text{ nm}, 1375 \text{ nm}]$, for $\theta_{\text{Rot}} = [0^\circ, 15^\circ, 30^\circ, 45^\circ]$, respectively. The slight change of 13 nm for $M_4$ exists between $\theta_{\text{Rot}} = [0^\circ – 45^\circ]$ with the approximate values of $\lambda_{\text{shift,4}} = [1362 \text{ nm}, 1375 \text{ nm}]$ for another telecom wavelength which is illustrated in figures 3(d). By considering the various type of wavelength tuning cavity studies in terms of modifications; position [28, 29], thickness [30], material [31] etc, it can be claimed that our study, which uses the rotational modification in the cavity region, has a good result without any drop in quality performance. As expected from the band diagrams, wavelengths are consistent with each other at every used rotational angle for band diagram and transmission spectrum outcomes. In addition to frequency domain investigations of the cavity, we also focus on time domain analyses as an additional tool requirement for $H$-polarization which is considered as a TM mode. In the scope of time domain study, some performance criteria like quality and Purcell factors have to be taken into consideration. Here, electric field distributions of the modes existing for the designed structure are given with quality and Purcell factors according to rotation angles.

The quality factor is the decaying rate of electric field in a cavity region, it can be identified as a criterion to determine the performance of the light confinement within the cavity.
We can describe our designed cavity as a high-quality microcavity, according to the results that can be shown in figure 4(a). We obtained high quality values for each of configurations by performing the finite difference time domain method (FDTD), but the most satisfying results are obtained for $M_2$ with high values of $Q = \{2.296 \times 10^8, 1.997 \times 10^8, 1.405 \times 10^8, 1.043 \times 10^8\}$ by showing a decaying trend with regard to increasing rotational angle that can we relate the angles as $\theta_{Rot} = [0^\circ, 15^\circ, 30^\circ, 45^\circ]$, respectively. As can be seen from the obtained values the highest quality factor value in 2D is $Q = 2.296 \times 10^8$, and it exists at $\theta_{Rot} = [0^\circ]$. Relatively small values are obtained for other modes $M_1$ and $M_3$ in consequence of bigger mode areas by comparison to $M_2$, an intuitive thought can be formed from figure 4(b) [13, 32].

![Figure 4](image_url)

Figure 4. (a) Presentation of the quality factor $Q = \{Q_1, Q_2, Q_3, Q_4\}$ and Purcell factor $F_p = \{F_{p1}, F_{p2}, F_{p3}, F_{p4}\}$ values according to rotational angles, subscripts indicates the first ($M_1$), second ($M_2$), third ($M_3$) and fourth modes ($M_4$) of the designed structures, respectively. (b) E-field distributions and polarization labeling of each mode with respect to rotational angles $\theta_{Rot} = [0^\circ, 15^\circ, 30^\circ, 45^\circ]$.

The mentioned $F_p$ values for each rotational angle $\theta_{Rot} = [0^\circ, 15^\circ, 30^\circ, 45^\circ]$ can be ordered as follows: $F_p = [281.165, 331.412, 325.910, 310.605]$ in order to visually demonstrate the concentrated accumulation of electromagnetic waves, which is correlated with the low mode volume and high Purcell factor in the designed microcavity, the mode profiles are needed. When E-field distributions are investigated based on the mode profiles, the results illustrated in figure 4 are obtained. According to the outcomes, it can be referred that the light strongly confines on some specific points which are around the defect region, the claimed points closely correspond to the dielectric rods. The localization on the dielectric rods shows identical E-field distributions, which are called modes and appear at separate wavelengths. We come across four types of modes for our designed structure, these modes are supported by our cavity structure at some exact wavelengths and are described above as $M_1$, $M_2$, $M_3$, and $M_4$. The mentioned existing modes can be investigated in terms of polarization properties as well. E-field distributions of the localized states can be attributed to the polarization properties. In the well-established cavity regions, the optical cavity modes can be classified as even or odd in the manner of the field profile with respect to the mirror planes [9–38]. When we investigated $M_1$, for the proposed structure with the orientation angle of $\theta_{Rot} = [0^\circ, 15^\circ, 30^\circ, 45^\circ]$ with respect to the diagonal symmetry plane, the mode can be named as odd. In the case of the orientation angle of $15^\circ$, the mode is also odd, as distinct from...
the other modes according to the different diagonal plane orthogonal to the first one. Furthermore, $M_{1}$ can be classified as a dipole (DI) state for all the defects with a different arrangement [9–39]. As $M_{2}$ is discussed, the mode is odd based on the $x = 0$ and $y = 0$ reflection planes. On the other hand, because the mode has four nodals, it can be allocated as quadrupole (QUAD). The third localized quadrupole state $M_{3}$ is an odd mode according to the diagonal symmetry planes for the orientation angle of $\theta_{rot} = [0^\circ, 15^\circ, 30^\circ, 45^\circ]$. Finally, the last mode $M_{4}$ is a monopole (MON) even state for all the cases of the rod array. The orientation of the modes is depicted with subscripts ‘0’, ‘45’, ‘−45’ and ‘90’. To summarize, the proposed defect regions support the modes with symmetries odd-dipole, odd-quadrupole, even-quadrupole and even-monopole. On the other hand, band diagrams give some clues about the mode behaviors that expected. All the supported modes appearing at the band diagrams are seen at the transmission spectrum as well. This observation indicates that the results of frequency and time domain analyses are consistent with each other. The most reasonable results are obtained for the $M_{2}$, this mode takes part in a location which is far away from the edge of the band gap for all applied defect region modifications. This type of positioning is an inducement for showing the ideal attitude of the mentioned wavelength for all cases. The other operating wavelengths that belong to $M_{3}$ and $M_{4}$ are observed around the positions of band gap edge. The destructive effects of the edge conditions are violently sensed for the cases of the nearest positioning existing on the $\theta_{rot} = [30^\circ, 45^\circ]$ rotational angles for mentioned modes. The drop of cavity effect and the lack of the localization attitude of the modes are an expected consequence of getting closer to the band edge.

Investigations show that the most promising results are obtained for the $M_{2}$ which is supported by the wavelengths as $\lambda_{2} = [1561\ nm, 1571\ nm, 1589\ nm, 1597\ nm]$ for $\theta_{rot} = [0^\circ, 15^\circ, 30^\circ, 45^\circ]$ rotational angles. Therefore, more detailed investigations on this wavelength are performed and three-dimensional (3D) analyses are carried out by considering the finite length of the structure along the third dimension specifically out-of-plane direction.

Before presenting the 3D analyses of the cavity structure, we should briefly mention the mode volume of the electric field distributions, which is related to the Purcell and quality factors. To emphasize the relationship between these figures of merit, it will be beneficial to present the correlation equation which is called the Purcell factor [36].

$$ F_{p} = \frac{3}{4\pi^{2}} \left( \frac{\lambda_{c}}{n_{c}} \right)^{3} \left( \frac{Q}{V_{eff}} \right), $$

where $\lambda_{c}$ is the free space wavelength, $n_{c}$ is the corresponding refractive index of the using material in the cavity region at the operation wavelength, $Q$ is the quality factor, and $V_{eff}$ is the effective mode volume where the light localized. According to equation 1, it is clear that Purcell factor has a linear relation with the quality factor while it is inversely proportional to the mode volume. As an important expectation of the cavity performance, mode volume should be as small as possible while the quality factor should have high values to fulfill the requirement of a high Purcell factor, which is the enhancement parameter of the cavity region. Here, we present the performance results of one-layer 3D simulations for each rotational angle at the mode $M_{2}$, which gives the best results for prior analyses, see table 1. It can be easily seen that the light is confined in a small volume as $V(\lambda/n)^{3} = [4.3\theta_{rot} = 0^\circ, 4.2\theta_{rot} = 15^\circ, 3.8\theta_{rot} = 30^\circ, 6.3\theta_{rot} = 45^\circ]$ and has small quality factors $Q = [13058.0\theta_{rot} = 0^\circ, 12544.0\theta_{rot} = 15^\circ, 10211.0\theta_{rot} = 30^\circ, 7256.7\theta_{rot} = 45^\circ]$ for the rotational angles $\theta_{rot} = [0^\circ, 15^\circ, 30^\circ, 45^\circ]$, respectively. These results can be commented as small mode volumes by quoting references to cavity studies in the literature [6, 13, 40–42]. However, the quality and Purcell factor values are not as higher as those of the 2D.

Table 1. Quality $Q$, mode volume $V(\lambda/n)^{3}$ and Purcell factors $F_{p}$ of silicon for mode $M_{2}$ for 3D analyses.

| $\theta_{rot}$ | $Q$   | $V(\lambda/n)^{3}$ | $F_{p}$ |
|---------------|-------|------------------|--------|
| $0^\circ$     | 13058.0 | 4.3              | 228.7  |
| $15^\circ$    | 12544.0 | 4.2              | 224.8  |
| $30^\circ$    | 10211.0 | 3.8              | 203.2  |
| $45^\circ$    | 7256.7   | 6.3              | 87.8   |

![Figure 5](image-url) Quality factor trend of 3D structures with respect to normalized height.

The reason for the dramatically lower quality factor results of the 3D compared to 2D is the leakage of the light into the air from the top and bottom interfaces of the structure, as mentioned in the introduction. This leakage is caused by the inability of the light satisfying the TIR condition between the layers along the $z$-axis. Therefore, it leaks instead of getting trapped inside the cavity region. To define a structure as 2D, it is required to compare the operating wavelength and the structural length on the basis of the investigated axis. Ensuring the convergence of 3D structure to 2D structure in terms of the quality factor, the proper thickness approach is needed [43]. To approach the maximum quality despite the
leakage mechanism mentioned above, the thickness approach has been applied to the ideal case in terms of wavelength and rotational angle. It is decided that the analyses are performed by applying the stacking method to small heights which are 1/16 of one layer for the ideal mode \( M_2 \). Along the way of the thickness analyses, it has been observed that the quality values exhibit peaks and valleys for definite heights, as shown in figure 5. As can be seen from the figure, which is scaled in the lattice constant, the quality value for 1/16 = 0.0625 × (layer height) = 0.0625 × 5500 nm = 343.75 nm = 0.612 \( a \), which can be confirmed as a reasonable value by comparing with the other studies in the literature. Similar behavior is encountered in the following studies [44, 45].

2.3. Additional time domain analysis

To be able to compare the numerical mode analyses with real-world conditions, experimental analysis has been needed. When taking into consideration the experimental setup that includes alumina rods, new numerical analysis implementation obligation has been raised to alumina material \( \varepsilon_r = 9.61 \) for \( H \)-polarization. Thus, a new opportunity has emerged in terms of the evaluation of two different dielectric material \( \varepsilon_r = 12 \) and \( \varepsilon_r = 9.61 \) behaviors for the same cavity structure. Unlike the silicon dielectric material, the designed cavity structure with alumina supports only two modes. The disappearance of the rest of the modes, which has emerged near the bandgap, can be explained as a shifting of these modes towards the band region. According to the given numerical transmission results of the structure consisting of the \( \varepsilon_r = 9.61 \) rods are given in figure 6(a), transmission peaks appear at \( \lambda = [1770 \text{ nm } \lambda_1, 1418 \text{ nm } \lambda_2] \) wavelengths and these peaks can be represented by \( M_1 \) and \( M_2 \), respectively. It can be seen that two modes exist with approximately

200 nm shift to the lower wavelengths with respect to silicon at \( \theta_{\text{rot}} = [0^\circ] \) as a simple configuration. The difference of the mode existing wavelengths between silicon and alumina can be associated with the refractive index ratio. In terms of the quality factors of the mentioned wavelengths, they indicate a similar attitude with silicon material and the results show that \( Q = [6.742 \times 10^7, 1.419 \times 10^7] \) for \( \lambda_1 = 1770 \text{ nm} \) and \( \lambda_2 = 1418 \text{ nm} \), respectively. As is performed in the silicon case, the ideal mode determination has been done based on high quality and wavelength tunability, for two wavelengths in alumina. As a result of this investigation, the ideal mode is determined as \( M_2 \). After that, wavelength tunability has been analyzed for the ideal mode at \( \theta_{\text{rot}} = [0^\circ, 15^\circ, 30^\circ, 45^\circ] \) rotational angles, see figure 6(b).

According to the analyses, results for the ideal mode show that a significant wavelength tune is possible in terms of the designed cavity structure, \( \lambda_2 = [1418 \text{ nm}, 1428 \text{ nm}, 1446 \text{ nm}, 1455 \text{ nm}], \) for \( \theta_{\text{rot}} = [0^\circ, 15^\circ, 30^\circ, 45^\circ] \), respectively. The wavelength change of 37 nm for \( M_2 \) exists between \( \theta_{\text{rot}} = [0^\circ - 45^\circ] \) with the approximate values of \( \lambda_{\text{shift}} = [1418 \text{ nm } \theta_{\text{rot}} = 0^\circ \text{, } 1.562 \times 10^5 \text{ nm } \theta_{\text{rot}} = 15^\circ \text{, } 1.419 \times 10^5 \text{ nm } \theta_{\text{rot}} = 30^\circ \text{, } 1.209 \times 10^5 \text{ nm } \theta_{\text{rot}} = 45^\circ] \). Furthermore, quality analyses of the ideal mode at specified rotational angles are also conducted, and result in \( Q = [1.607 \times 10^7, 1.149 \times 10^7, 1.209 \times 10^7] \) for \( \theta_{\text{Rot}} = [0^\circ, 15^\circ, 30^\circ, 45^\circ] \), respectively. Besides 2D simulations, 3D analyses are performed in terms of quality and Purcell factors to compare the results with that of experimental ones. The results show that the cavity structure confines the light for the ideal mode \( M_2 \) in a similar way as the silicon results, see table 2. Quality and Purcell factors can be

| \( \theta_{\text{rot}} \) (°) | \( Q \) | \( V(\lambda/n)^3 \) | \( F_p \) |
|-----------------|-----|----------------|-----|
| 0°              | 5560.9 | 2.6          | 159.0 |
| 15°             | 5305.6 | 2.6          | 156.3 |
| 30°             | 4217.0 | 2.2          | 144.2 |
| 45°             | 2971.9 | 3.1          | 71.5  |
After the numerical investigation is conducted, microwave experiments are performed to verify the overlap between the real environment and our design. In the prepared setup that is for performing transmission measurement, an Agilent E5071C type network analyzer, standard monopole antennas as receiver and transmitter are used. Antennas are positioned at the cavity region of the 15 × 15 sized PhC structure the same as for the simulations, according to circumstances adequate distances are arranged to be able to diagnose the operation wavelengths at which the cavity effect emerges, see figure 7(a). In contrast, for the experimental spectra measurement of the structure without defect has been performed by using a monopole antenna and a horn antenna as transmitter and receiver, respectively. Cylindrical alumina rods that have \( \varepsilon_r = 9.61 \) dielectric constant, \( R_1 = 6.35 \text{ mm} \) \( R_2 = 3.17 \text{ mm} \) diameters, and \( h = 15.3 \text{ cm} \) height are utilized in the PhC structure, these rods construct a square structure with dimensions, \( w = 23.8 \text{ cm} \) widths, and \( l = 23.8 \text{ cm} \) lengths and according to the given dimensions the lattice constant is calculated as \( a = 15.85 \text{ mm} \).

First, a structure without defect (SWD) experiment has been performed to be able to show the bandgap before the cavity effect does not exist. It is expected that SWD does not support any guided mode. This expectation is met according to the results which are represented in figure 7(b) as an inset. Next, the experimental operation range is calculated based on the numerical cavity mode wavelengths. Conforming to the wavelength-frequency transition, the predicted operation frequency interval is 6 GHz–7.5 GHz. As it is mentioned above, experimental measurements are given at \( \theta_{\text{Rot}} = [0^\circ] \) due to aiming for the main goal of a simple configuration without any performance metrics like wavelength tuning and quality factor. In this manner, we could also compare experimental results with 3D stacking layer approximation. Transmission measurement results in the microwave regime are given in figure 7(b), and experimental center frequency results \( f = [6.169 \text{ GHz}, 7.520 \text{ GHz}] \) are correlated with that of numerical ones at \( \theta_{\text{Rot}} = [0^\circ] \). It should be also mentioned that there is a slight frequency deviation \( f_{\text{dev}} = [0.17 \text{ GHz}, 0.02 \text{ GHz}] \) interval for first and second frequencies, respectively. The deviation is caused by the environmental perturbation and array issues of the structure, also possible irregularities of the rods can be added as a reason.

To find out a way from general to specific, we focused on the most utilizable wavelength in terms of the wavelength tunability and quality factor. There is a need to investigate the experimental reaction of the modes to angular modifications as a comparison. In addition to the previous angular modification on the alumina PhC structure. Taking the \( a/\lambda \) ratio constant for experimental and numerical cases, numerical data has converted into experimental frequencies for ideal mode \( M_2 \), \( f_{\text{exp}}=[7.496 \text{ GHz}, 7.444 \text{ GHz}, 7.351 \text{ GHz}, 7.305 \text{ GHz}] \), responding to \( \theta_{\text{Rot}} = [0^\circ, 15^\circ, 30^\circ, 45^\circ] \) rotational angles. Mentioned data is finely correlated with obtained experimental frequencies \( f_{\text{exp}}=[7.520 \text{ GHz}, 7.468 \text{ GHz}, 7.388 \text{ GHz}, 7.368 \text{ GHz}] \), see figure 7(c) for comparison. Similar to the numerical results, maximum shifting is observed between \( \theta_{\text{Rot}} = [0^\circ, 45^\circ] \) interval with 0.152 GHz shifting value for \( f_{\text{shift}}=[7.520 \text{ GHz}, 7.468 \text{ GHz}, 7.388 \text{ GHz}, 7.368 \text{ GHz}] \) frequencies. After the ideal mode gives satisfying wavelength tunability.
results, the investigation of the quality factor emerges as a requirement for an efficient cavity effect. In contrast to the numerical quality factor investigation, a low quality analysis represented in equation (2) is used for obtaining experimental results [46]

\[ Q = \frac{f_{\text{center}}}{f_2 - f_1}. \]  

Equation (2) includes \( f_{\text{center}} \) and \( f_2 - f_1 \) as representing the central value of operating frequency and bandwidth of the curve, respectively. All the existing modes are analyzed to be sure that the ideal mode is also valid in terms of quality factor-like wavelength tunability. According to the results, \( Q = [1258.97, 1074.81] \), the ideal mode is compatible with numerical ones. With high wavelength tunability and quality factor, \( M_2 \) is approved as the most efficient one, this mode shows \( Q = [1074.81, 1166.87, 1159.81, 1051.06, 1083.74] \) quality factors for \( \theta_{rot} = [0^\circ, 15^\circ, 30^\circ, 45^\circ] \) angular modifications, respectively. In addition to the comparison between 2D and 3D in terms of quality factors, a similar analogy can be applied for 3D and experimental quality values, for achieving further information about the behavior of PhC structure. According to the outcomes, there is an appreciable consistency for each rotation angle for 3D and experimental results. Even though the mentioned results show a decrease with respect to simulation 3D results, this discrepancy can be described as experimental conditions which are caused by the structural imperfections of the cavity or active regions. Perfectly placing each alumina rod at correct locations and aligning them in parallel is very difficult to achieve. Hence, an acceptable deviation between experimental and numerical results is observed [14].

### 4. Conclusion

This study shows a roadmap to achieve the most ideal case for a cavity mode by investigating the performance in terms of various criteria and also includes a comparison between two different dielectric materials in the manner of resonance mode manipulations. Silicon takes part in the center of the study, and the investigations are conducted with frequency and time domain analyses. For a silicon-based structure, frequency domain analysis exhibits that the obtained four resonance modes shifts demonstrated better results for specified rotational angles \( \theta_{rot} = [0^\circ, 15^\circ, 30^\circ, 45^\circ] \) compared to other studies. The mentioned frequency shift is caused by the deviation of the dielectric distribution by means of the rotation of the smaller symmetry elements, thus, the structure can fulfill the requirement of mode generation at shifted frequencies. On the other hand, by using time domain analysis, such a high, \( \sim 10^6 \), quality is achieved for 2D. In terms of 3D analysis, an ideal mode has been chosen in terms of providing a maximum wavelength shift 36 nm, and the highest quality factor \( Q = 2.296 \times 10^8 \) at \( \theta_{rot} = 0^\circ \), among to the other modes. According to 3D results, maximum quality factor reduces down to \( Q = 1.365 \times 10^5 \), as expected. To perform the experimental investigation, these analyses have been repeated for alumina and quite similar outcomes have been acquired such as a \( 1.607 \times 10^7 \) quality factor and a 37 nm wavelength shift. Additionally, 3D analyses have been conducted for quality performance, Purcell factor and mode volume investigations to be able to do a reasonable comparison with silicon. Then, the experimental stage was carried out for two cavity modes, ideal mode selection was compatible with that of numerical in terms of the high quality factor and maximum wavelength shift \( Q = 1074.81 \), and \( f_{\text{shift}} = 0.152 \text{ GHz} \), respectively. All the results show that our respectively compact and simply designed microcavity structure provides high tunability and control on several modes which operate mostly on telecom wavelengths. Thereby, it is enables the establishment of well-controlled devices for photonic integrated circuits like filters, sensors, and optical storing applications.

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