Hydrodynamic Flow of the Quark-Gluon Plasma and Gauge/Gravity Correspondence (lecture 3)

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1. Boost-invariant flow – reminder of lectures 1+2 by Robi

2. Going beyond perfect fluid
   - Viscosity, relaxation time etc.

3. Going beyond boost-invariance
   - General hydrodynamic equations from AdS/CFT

4. Going beyond hydrodynamics
   - Modeling a heavy-ion collision
   - A 1+1 dimensional toy model
   - The 3+1 dimensional problem

5. Physics in the expanding plasma
   - Fundamental flavours and mesons in AdS/CFT
   - Fundamental flavours and mesons in an expanding plasma system

6. Summary
Bjorken ’83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.

- Pass to proper-time/spacetime rapidity coordinates \((\tau, y, x_1, x_2)\).
- In a conformal theory, \(T^\mu_\mu = 0\) and \(\partial_\mu T^{\mu\nu} = 0\) determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function \(\varepsilon(\tau)\).

\(\varepsilon(\tau)\) is the energy density at mid-rapidity.

Previous lectures \(\rightarrow\) late-time asymptotics of \(\varepsilon(\tau)\).

Here \(\rightarrow\) subasymptotic behaviour of \(\varepsilon(\tau)\).
What is the physics of $\varepsilon(\tau)$?

- Weak coupling – free streaming
  \[ \varepsilon(\tau) = \frac{1}{\tau} \]

- Perfect fluid assumption
  \[ \varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} \]

- Fluid with viscosity $\eta = \frac{\eta_0}{\tau}$
  \[ \varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} \left( 1 - \frac{2\eta_0}{\tau^{\frac{2}{3}}} + \ldots \right) \]

- Second order viscous hydrodynamics: $\eta, \tau \Pi$
  \[ \varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} \left( 1 - \frac{2\eta_0}{\tau^{\frac{2}{3}}} + \frac{B(\eta, \tau \Pi)}{\tau^{\frac{4}{3}}} + \ldots \right) \]
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How to determine $\varepsilon(\tau)$?

1. Consider $\varepsilon(\tau) = 1/\tau^s + \ldots$
2. Construct the dual geometry

$$\varepsilon(\tau) \rightarrow ds^2 = \frac{g_{\mu\nu}(z,\tau)dx^\mu dx^\nu + dz^2}{z^2}$$

3. Take the scaling limit $\tau \rightarrow \infty$, $z \rightarrow \infty$ keeping $v = \frac{z}{\tau^{3/4}}$ fixed
4. The curvature invariant $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ is nonsingular in the scaling limit only for $s = \frac{4}{3}$
5. This corresponds to perfect fluid behaviour
6. The resulting geometry is the evolving black hole described in previous lectures

Is this an exact perfect fluid?
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- At subleading order we find 4th order pole singularities in the curvature
  
  \[ R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} = R_0(v) + \frac{1}{\tau^{4/3}} R_2(v) + \ldots \]

  \[ \text{nonsingular} + \text{singular!} \]

- Set $\varepsilon(\tau) = \frac{1}{\tau^{4/3}} \left( 1 - \frac{2A}{\tau^r} \right)$

- Solve for geometry and compute the curvature \[ \text{[Nakamura, Sin; RJ]} \]

  \[ R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} = R_0(v) + \frac{1}{\tau^r} R_1(v) + \frac{1}{\tau^{2r}} \tilde{R}_2(v) + \frac{1}{\tau^{4/3}} R_2(v) + \ldots \]

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- The singular terms may cancel with each other only when $r = \frac{2}{3}$ and $A = 2^{-\frac{1}{2}} 3^{-\frac{3}{4}}$

- This correspond exactly to corrections coming from viscosity with the numerical coefficient exactly corresponding to $\eta/s = \frac{1}{4\pi}$ \[ \text{[See Son’s lecture]} \]

- This is a very nontrivial consistency check that the nonlinear dynamics is given by viscous hydrodynamics
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Go to higher order

\[ \varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} \left( 1 - \frac{2\eta_0}{\tau^{\frac{2}{3}}} + \frac{B}{\tau^{\frac{4}{3}}} + \ldots \right) \]

Curvature

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\( R_3(v) \) has 4\(^{th} \) order poles which can be cancelled by a definite choice of \( B \) which fixes the relaxation time \( \tau_\Pi \).

\( B \) is uniquely fixed. The value of \( \tau_\Pi \) depends on the type of 2\(^{nd} \) order hydrodynamic theory used to describe \( \varepsilon(\tau) \). Subsequent work fixed uniquely this theory → Son's lectures

After fixing \( R_3(v) \) there remains a logarithmic singularity. This is probably due to a pathology of Fefferman-Graham coordinate expansion → see talk by M. Heller
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\[\text{nonsingular}\]

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Assumptions

- We picked boost-invariant setup with full transverse symmetry
- Energy-momentum tensor completely expressed in terms of $\varepsilon(\tau)$

AdS/CFT computation

- Construct dual geometry – solve Einstein’s equations
- Fix $\varepsilon(\tau)$ from nonsingularity

Link with hydrodynamics

- Take $\varepsilon(\tau)$ from AdS/CFT
- Plug it into phenomenological hydrodynamic equations
- Find that $\varepsilon(\tau)$ can be a solution
- Fix parameters in these equations (viscosity, relaxation time etc.)
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Question

- Can one lift the symmetry assumptions?
- Is it possible to see hydrodynamic equations more directly?
The approach of [Bhattacharyya, Hubeny, Minwalla, Rangamani]

- Start from a static black hole with fixed temperature $T$ which describes a fluid at rest, $u^\mu = (1, 0, 0, 0)$ with constant energy density
- Perform a boost to obtain a uniform fluid moving with constant velocity $u^\mu$
- The resulting metric (in Eddington-Finkelstein coordinates) is

$$ds^2 = -2u_\mu dx^\mu dr - r^2 \left( 1 - \frac{T^4}{\pi^4 r^4} \right) u_\mu u_\nu dx^\mu dx^\nu + r^2 \left( \eta_{\mu\nu} + u_\mu u_\nu \right) dx^\mu dx^\nu$$

where $r = \infty$ corresponds to the boundary, $r = T/\pi$ is the horizon while $r = 0$ is the position of the singularity.

Promote $T$ and $u^\mu$ to (slowly-varying) functions of $x^\mu$

Caveat: The metric is no longer an exact solution of Einstein’s equations
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where $r = \infty$ corresponds to the boundary, $r = T/\pi$ is the horizon while $r = 0$ is the position of the singularity.

Promote $T$ and $u^\mu$ to (slowly-varying) functions of $x^\mu$.

Caveat: The metric is no longer an exact solution of Einstein’s equations.
Perform an expansion of the Einstein equations in gradients of spacetime fields.

Find corrections to the metric at first and second order

Require nonsingularity to fix integration constants

Read off the resulting energy-momentum tensor $T_{\mu\nu}$

$T_{\mu\nu}$ is expressed in terms $u^\mu$ and $T$ and their derivatives

$$
T^{\mu\nu}_{\text{rescaled}} = (\pi T)^4 (\eta^{\mu\nu} + 4 u^\mu u^\nu) - 2 (\pi T)^3 \sigma^{\mu\nu} + \underbrace{(\pi T^2) \left( \log 2 T^{\mu\nu}_{2a} + 2 T^{\mu\nu}_{2b} + (2 - \log 2) \left( \frac{1}{3} T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e} \right) \right)}_{\text{second order hydrodynamics}}
$$

Full nonlinear hydrodynamic equations follow now from $\partial_\mu T^{\mu\nu} = 0$
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$$+ \left( \pi T^2 \right) \left( \log 2 T_{2a}^{\mu\nu} + 2T_{2b}^{\mu\nu} + (2 - \log 2) \left( \frac{1}{3} T_{2c}^{\mu\nu} + T_{2d}^{\mu\nu} + T_{2e}^{\mu\nu} \right) \right)$$

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Full nonlinear hydrodynamic equations follow now from $\partial_\mu T^{\mu\nu} = 0$
Going beyond hydrodynamics

The most interesting (and most difficult) open problems are beyond the reach of hydrodynamics.

Key questions:
- why is thermalization/isotropisation so fast?
- can we understand the thermalization time?
- how does thermalization occur?
Modeling a heavy-ion collision

Find a model for the projectile

- Assume no dependence on transverse coordinates
- The configuration should depend only on one light cone coordinate
- Tracelessness and conservation of energy-momentum tensor leads to

$$T_{--} = f(x^-)$$

all other components vanish

- The dual geometry can be found exactly [RJ, Peschanski]

$$ds^2 = \frac{-2dx^+dx^- + z^4f(x^-)dx^{-2} + dx_{\perp}^2 + dz^2}{z^2}$$

- When $$f(x^-) = \mu\delta(x^-)$$ we are dealing with a shock-wave

Consider the collision of two such shockwaves
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A 1 + 1 dimensional toy model

- [Kajantie, Louko, Tahkokallio] considered a collision in a 1+1 dimensional CFT dual to 2+1 dimensional gravity
- Advantage (but also a disadvantage...) is that 2+1 gravity is often easy to solve exactly
- Consider two shockwaves (or more general projectiles) coming on two light-cone directions:

\[ T_{--} = f(x^-) \quad T_{++} = g(x^+) \]

- Dual geometry can be found exactly:

\[
ds^2 = \frac{-(2 + \frac{z^4}{2} f(x^-) g(x^+)) dx^- dx^- + z^2 f(x^-) dx^- dx^2 + z^2 g(x^+) dx^+ dx^+ + dz^2}{z^2}
\]

Problem: The projectiles pass through each other and after the collision are unaffected by the presence of each other

Reason: The distribution (\(*\)) is the most general one possible in a 1+1D CFT. No place for nontrivial dynamics of thermalization etc.
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- In order to study more realistic physics we have to tackle the full 3+1 dimensional setup
- No longer solvable
- The existence of exact plane-wave collision metrics in 4D general relativity does not help – one more coordinate!
- Pioneering work by Grumiller, Romatschke: small time expansion – but problems with energy positivity
- Different expansion: Albacete, Kovchegov, Taliotis
- Entropy production through a trapped surface formation [Gubser, Pufu, Yarom] (somewhat different setup)

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It is interesting to ask questions about gauge theory physics influenced by the expanding and cooling plasma system.

- Use the ‘moving black-hole’ geometry instead of the usual $AdS_5$ or $AdS$ black hole.
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Consider $\mathcal{N} = 4$ SYM + one additional flavour ($\mathcal{N} = 2$)

- No chiral symmetry breaking!

AdS/CFT description:
  Embed a $D7$ brane in the geometry

Lightest mesons $\equiv$ fluctuations of the $D7$ brane embedding (or $D7$ gauge fields)
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Two types of embeddings:

1. ‘Minkowski embeddings’ : the D7 brane does not reach the horizon – heavy quarks – stable mesons
2. ‘Black hole embeddings’ : the D7 brane touches the horizon – light quarks – mesons dissociate

Procedure:

1. Fix the current quark mass $m$ by boundary conditions for the embedding at the boundary
2. Solve for the embedding from DBI EOM
3. Read off $\langle \bar{\psi} \psi \rangle$ from subasymptotics of the embedding
4. Study fluctuations of the embedding

$$\delta \phi(x^\mu, \rho) = e^{ik_\mu x^\mu} f_{k_\mu}(\rho)$$

5. Obtain meson masses from $M^2 = k^2$ for which the solution $f_{k_\mu}(\rho)$ is nonsingular
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  1. ‘Minkowski embeddings’ : the $D7$ brane does not reach the horizon – heavy quarks – stable mesons
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- Procedure:
  1. Fix the current quark mass $m$ by boundary conditions for the embedding at the boundary
  2. Solve for the embedding from DBI EOM
  3. Read off $\langle \bar{\psi} \psi \rangle$ from subasymptotics of the embedding
  4. Study fluctuations of the embedding
     \[ \delta \phi(x^\mu, \rho) = e^{ik_{\mu}x^\mu} f_{k_{\mu}}(\rho) \]
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Finite temperature case (static plasma)

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Fundamental flavours and mesons in an expanding plasma system

- Study the setup in the expanding plasma
  - Use late time expansion $\rightarrow$ ‘Minkowski embedding’
  - Embed $D7$ brane into plasma geometry (including viscosity)

$$y_6(\rho, \tau) = m - \frac{f_1(\rho)}{\tau^{\frac{8}{3}}} + \frac{f_2(\rho)}{\tau^{\frac{10}{3}}} + \ldots$$

- Read off the condensate

$$\langle \bar{\psi} \psi \rangle \propto \frac{\varepsilon^2(\tau)}{m^5}$$

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  - Major complication: Lack of separability
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Ansatz for mesonic fluctuations:

\[ \delta \phi = \sqrt{\int \frac{\omega(\tau) d\tau}{\omega(\tau) \tau}} J_0 \left( \int \omega(\tau) d\tau \right) \left\{ g_0(\rho) + \frac{1}{\tau^{\frac{4}{3}}} g_1(\rho) + \frac{1}{\tau^2} g_2(\rho) + \ldots \right\} \]

Determine \( \tau \)-dependent frequency

\[ \omega(\tau) = \frac{4\pi}{\lambda} \left( m + \ldots + \frac{4}{\tau^{\frac{4}{3}}} + \frac{2}{\tau^2} + \ldots \right) \]

Time-dependent frequencies suggest particle production – but difficult to control in this approximation scheme...
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- AdS/CFT predicts hydrodynamic behaviour of $\mathcal{N} = 4$ plasma
  - This extends to the nonlinear regime!
  - Expanding plasma is stable against small perturbations
  - Very general framework for studying time-dependent dynamical processes or out-of-equilibrium configuration
  - Use gravity backgrounds to study physics influenced by the expanding plasma system

Outlook

- Early stage of the collision??
- Initial thermalization??
- Isotropisation?? Plasma instabilities??
- Lifting the constraints of $\mathcal{N} = 4$ SYM...

see talk by P. Witaszczyk
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