Type IIA orientifold compactification on SU(2)-structure manifolds

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Abstract

We investigate the effective theory of type IIA string theory on six-dimensional orientifold backgrounds with SU(2)-structure. We focus on the case of orientifolds with \textit{O}6-planes, for which we compute the bosonic effective action in the supergravity approximation. For a generic SU(2)-structure background, we find that the low-energy effective theory is a gauged \( \mathcal{N} = 2 \) supergravity where moduli in both vector and hypermultiplets are charged. Since all these supergravities descend from a corresponding \( \mathcal{N} = 4 \) background, their scalar target space is always a quotient of a \( SU(1,1)/U(1) \times SO(6,n)/SO(6) \times SO(n) \) coset, and is therefore also very constrained.
1 Introduction

The study of string backgrounds which are compactifications on manifolds with $G$-structure has been of interest for some time now. (For reviews see, for example, [1, 2] and references therein.) $G$-structure manifolds generalize Calabi-Yau spaces in that they also admit a number of globally-defined and nowhere vanishing spinors. These spinors, however, are no longer required to be parallel with respect to the Levi-Civita connection. Instead, they are parallel with respect to a connection with torsion [3, 4]. As a consequence the number of supersymmetries in the background is unchanged but they can be spontaneously broken. This in turn generates a potential and lifts (part of) the vacuum degeneracy [1, 2].

So far mainly compactifications with one globally defined spinor or in other words compactifications on SU(3)-structure manifolds were considered [1]. In type II theories, they lead to four-dimensional low-energy effective theories with $N = 2$ supersymmetry. Including D-branes and orientifold planes the supersymmetry can be further reduced to $N = 1$ [5]-[12].

If instead two spinors are globally defined the internal manifold has SU(2)-structure and generically the effective theory obtained from type II has $N = 4$ supersymmetry [13]-[18]. Including orientifold planes this supersymmetry can be further reduced to $N = 2$ or $N = 1$ [14, 19, 11, 20].

Aspects of the low energy effective action for type II string theory compactified on orientifolds of K3 $\times T^2$ have been computed in refs. [21, 22, 23, 24]. In this paper, we focus on type IIA, and calculate the bosonic $N = 2$ effective action for a background manifold with SU(2)-structure and with an O6 orientifold projection, within the supergravity approximation. This is the analogue of the analysis performed in [5, 8] where the $N = 1$ effective action for orientifolds of SU(3)-structure compactifications was determined. The low energy effective theory which we find is a gauged $N = 2$ supergravity where the scalar manifold $\mathcal{M}$ is particularly simple and the product of the three symmetric spaces

$$\mathcal{M} = \frac{SU(1,1)}{U(1)} \times \frac{SO(2,n)}{SO(2) \times SO(n)} \times \frac{SO(4,m)}{SO(4) \times SO(m)} ,$$

which descends from the scalar field space $SU(1,1)/U(1) \times SO(6,n)/SO(6) \times SO(n)$ of $\mathcal{N} = 4$ supergravity. The first two factors in (1.1) are a special Kähler manifold and spanned by the scalars in the vector multiplets while the last factor is quaternionic-Kähler and spanned by the scalars in the hypermultiplets. Furthermore we find that isometries of all three components can be simultaneously gauged when appropriate torsion components are present. To our knowledge, this situation has not been encountered previously in any $N = 2$ compactification of type II string theory[1].

The paper is organized as follows. In section 2, we consider the special case where the internal manifold is an orientifold of the Calabi-Yau space K3 $\times T^2$. In order to prepare the discussion for more general SU(2)-structure manifolds, we phrase our analysis in the formalism introduced in [13, 15]. In section 2.1 we briefly review the orientifold

\footnote{It does occur in certain heterotic SU(2)-structure compactification [25] and can probably also be arranged in appropriate generalizations of M-theory compactifications on SU(3)-structure manifolds considered in [26, 27].}
projection. Section 2.2 is concerned with finding a suitable orientifold projection on \( K3 \times T^2 \) which preserves half of the supersymmetry. In 2.3 we compute the massless spectrum of the orientifolded theory and in 2.4 determine the effective action via a Kaluza-Klein reduction. By performing a set of field redefinitions, we are able to show that the scalars fields are indeed coordinates on the scalar manifold \( \mathcal{M} \) given in (1.1). In section 3 we then turn to generic manifolds with SU(2)-structure. We first describe compactifications on G-structure manifolds following [28], and recall the properties of the moduli space of metrics on SU(2)-structure manifolds as discussed in [17]. We then see that most of our results from sections 2.2 and 2.3 still hold in this more general case, when we also generalize our Kaluza-Klein ansatz appropriately. Section 3.3 then contains the effective action for the general case, which is a gauged four-dimensional \( N = 2 \) supergravity. We then describe the obtained gaugings in terms of the variables introduced in section 2.4. The corresponding Killing prepotentials are computed in appendix C. In appendix D, we show that the potential obtained from compactification is consistent with the general formula for the potential in \( N = 2 \) supergravities. Appendix A contains details on the chosen spinor conventions, appendix B contains the gauge kinetic coupling function, and we give our conclusions in section 4.

2 Orientifolds of Type IIA on \( K3 \times T^2 \)

2.1 Type IIA Orientifolds

Let us start by recalling the orientifold projection for type IIA theories [12]. From a world-sheet perspective, orientifolds arise by modding out the string theory by a discrete involutive symmetry \( S \). This symmetry includes the map \( \Omega_p \) which inverts orientation of the string world sheet (parametrized by \( \sigma \) and \( \tau \)) according to

\[
\Omega_p : (\sigma, \tau) \rightarrow (2\pi - \sigma, \tau) .
\]

(2.1)

\( \Omega_p \) is such that it exchanges left- and right-moving string modes. For the fermionic modes in type IIA, this means spinors of opposite target-space chirality must be mapped to each other. In order to do this consistently, one has to combine \( \Omega_p \) with an involution \( \sigma \) of the target space, which inverts target space orientation [12]. Locally, such a \( \sigma \) can be thought of as an odd number of reflections along tangent space directions. Depending on the number of reflections, the transformation properties of the fermionic modes change and one may have to add an extra operator \((-1)^{F_L}\) in order to ensure \( S^2 = 1 \) for all states [12]. More straightforwardly, the number of flipped directions also determines the generic dimension of the orientifold planes, since these lie at the fixed-point loci of \( \sigma \). Thus, one arrives at the following possibilities for \( Op \)-planes and the corresponding projections:

\[
\begin{align*}
O2, O6: & \quad S = (-1)^{F_L} \Omega_p \sigma , \\
O0, O4, O8: & \quad S = \Omega_p \sigma .
\end{align*}
\]

(2.2)

(More details of this projection are given in appendix A.2.)

To be consistent with the standard notation we use \( \sigma \) to also denote this involution which, however, has nothing to do with the world-sheet coordinate \( \sigma \).
In type IIA string theory, the massless ten-dimensional bosonic spectrum consists of the metric $\hat{g}$, the dilaton $\hat{\phi}$, and the 2-form $\hat{B}$, all in the NS-sector, and a one- and three-form field $\hat{A}$ and $\hat{C}$ in the RR-sector. The orientifold map $S$ acts on these fields by the pull-back $\sigma^*$ of the target-space involution $\sigma$, combined with extra minus signs, which can be deduced from the world-sheet description of each field. Altogether one has

$$\Omega_p \sigma : \begin{cases} \hat{\phi} \rightarrow \sigma^*(\hat{\phi}) \\ \hat{g} \rightarrow \sigma^*(\hat{g}) \\ \hat{B} \rightarrow -\sigma^*(\hat{B}) \\ \hat{A} \rightarrow \sigma^*(\hat{A}) \\ \hat{C} \rightarrow -\sigma^*(\hat{C}) \end{cases}, \quad (-1)^{F_L} : \begin{cases} \hat{\phi} \rightarrow \hat{\phi} \\ \hat{g} \rightarrow \hat{g} \\ \hat{B} \rightarrow \hat{B} \\ \hat{A} \rightarrow -\hat{A} \\ \hat{C} \rightarrow -\hat{C} \end{cases}. \quad (2.3)$$

($-1)^{F_L}$ only acts on the R-R fields, since they are built from the tensor product of one left- and one right-moving world-sheet spinor. The transformation properties under $\Omega_p \sigma$ can be derived by writing the NS-NS modes as symmetric or antisymmetric products of left- and right-moving bosonic oscillators, and the R-R modes as spinor bilinears, and switching left- and right-moving modes [12].

### 2.2 Orientifold action on $K3 \times T^2$

Let us now consider backgrounds which include orientifolds of the six-dimensional Calabi-Yau manifolds $K3 \times T^2$ and first determine the appropriate orientifold projection for this case. $K3$ is the unique four-dimensional manifold with SU(2)-holonomy and thus $K3 \times T^2$ can be viewed as a special case of a six-dimensional manifold with SU(2) structure. We will see that it already exhibits many features of a generic SU(2)-structure manifold that we will analyze in section 3.

A Ricci-flat metric on $K3 \times T^2$ admits 2 covariantly constant spinors $\eta^i$, $i = 1, 2$. As a consequence the two parameters $\varepsilon^I, \varepsilon^I$ of the unbroken ten-dimensional type II supersymmetry transformations may be decomposed as

$$\varepsilon^{I0} = \varepsilon^I_+ \otimes \eta^i_+ + \varepsilon^I_- \otimes \eta^i_-, \quad i = 1, 2, \quad (2.4)$$

where the $\varepsilon_i$ and the $\eta^i$ are Spin(1,3), resp. Spin(6) Weyl spinors. The minus sign in the second line of (2.4) is due to our choice for the Majorana condition and the fact that $\varepsilon^I_{00}$ has chirality -1 (see appendix A for details on the chosen conventions). We see that, from a four-dimensional perspective, the unbroken supersymmetries feature the four parameters $\varepsilon^I_i, \varepsilon^I_i$. Therefore type IIA string theory in a $K3 \times T^2$ background is described at low energies by a four-dimensional $N = 4$ supergravity theory [29] [16].

In this section we aim at constructing an $N = 2$ theory by including an appropriate orientifold projection in this setup. Preserving $N = 2$ supersymmetry requires that $\sigma$ leaves half of the supersymmetries invariant. For concreteness we consider a background with $O6$-planes, for which $\sigma$ acts on the six-dimensional spinors $\eta^i$ as follows [8] [11]

$$\sigma^*(\eta^i_+) = \pm \eta^i_. \quad (2.5)$$

(We derive the form of this projection in our conventions in appendix A.) In general, one can add a multiplication by a phase $e^{i\theta}$ to the action of $\sigma$, which has the effect of
rotating the $O$-planes. However, in the case of a single $O$-plane, one can always choose suitable variables in which the phases disappear.

The Kähler form $J$ and the holomorphic 2-form $\Omega$ on K3 can be expressed in terms of the globally defined spinors $\eta^i$ as

$$J := \frac{i}{4} (\eta_1^1 \gamma_{mn} \eta_1^1 - \eta_2^2 \gamma_{mn} \eta_2^2) \, dY^m \wedge dY^n = \frac{1}{2} J_{ab} dz^a \wedge dz^b,$$
$$\Omega := \frac{i}{2} \eta_1^1 \gamma_{mn} \eta_1^2 \, dY^m \wedge dY^n = \frac{1}{2} \Omega_{ab} dz^a \wedge dz^b. \tag{2.6}$$

Using (2.5) one can infer the following transformation properties [12]:

$$\sigma^*(J) = -J, \quad \sigma^*(\Omega) = -\bar{\Omega}. \tag{2.7}$$

In addition, the basis one-forms on the torus can also be expressed as a bispinor in the $\eta^i$ via [15]

$$K := \eta_2^2 \gamma_m \eta_1^1 \, dY^m = dy^2 + idy^1, \tag{2.8}$$

where $\gamma_1, \gamma_2$ are the gamma-matrices in the torus directions. Here, the transformations (2.5) act as

$$\sigma^*(K) = \bar{K}, \tag{2.9}$$

which implies $\sigma^*(dy^1) = -dy^1$ and $\sigma^*(dy^2) = dy^2$.

### 2.3 Massless spectrum

Let us now determine the massless spectrum of the orientifolded theory and assign it to $N = 2$ multiplets. As in any Calabi-Yau compactification, the massless modes of the four-dimensional theory are obtained by expanding the ten-dimensional fields in harmonic modes on $K3 \times T^2$. On K3 there are the constant function, the 22 harmonic 2-forms $\omega^{\alpha}(y)$ and one harmonic 4-form. On $T^2$, the “harmonic modes” are just the constant functions and forms. Therefore the ten-dimensional (hatted) fields can be expanded as

$$\hat{A} = A + A_i \nu^i, \quad i = 1, 2 \tag{2.10}$$
$$\hat{B} = B + B_i \wedge \nu^i + \frac{1}{2} B_{ij} \nu^i \wedge \nu^j + B_\alpha \omega^\alpha(y), \quad \alpha = 1, \ldots, 22 \tag{2.10}$$
$$\hat{C} = C + (C_i - A \wedge B_i) \wedge \nu^i + \frac{1}{2} (C_{ij} - A B_{ij}) \wedge \nu^i \wedge \nu^j$$
$$+ (C_\alpha - A B_\alpha) \wedge \omega^\alpha + C_{i\alpha} \nu^i \wedge \omega^\alpha,$$

where the “vielbein” one-forms $\nu^i$ are defined as $\nu^i = dy^i - g^i_\mu dx^\mu$ with $g^i_\mu$ being the appropriate off-diagonal metric component. All other (unhatted) variables denote four-dimensional fields. In this basis the metric is block-diagonal and given by

$$\hat{d}s^2 = g_{\mu\nu} dx^\mu dx^\nu + g_{ij} (dy^i - g^i_\mu dx^\mu)(dy^j - g^j_\mu dx^\mu) + g_{ab} dz^a dz^b. \tag{2.11}$$

---

3Let us summarize our conventions for the different coordinates: ten-dimensional coordinates are labeled $X^M, M = 0, \ldots, 9$ while four-dimensional space-time coordinates are labeled by $x^\mu, \mu = 0, \ldots, 3$. The real coordinates on the internal manifold (here $K3 \times T^2$) are labeled $Y^m$ which are split into the coordinates $y^i, i = 1, 2$ on $T^2$ and the (real) coordinates $z^a, a = 1, \ldots, 4$ on K3.
where \( g_{\mu\nu}, g^i_\mu \) and \( g_{ij} \) only depend on \( x^a \) while \( g_{ab} \) is the metric on K3 which also depends on the K3 coordinates \( z^a \).

In order to determine the spectrum of the orientifolded theory, we have to project the spectrum onto the modes which are even under the map \( S = (-1)^{F_i} \Omega_\sigma \). The pull-back of \( \sigma \) splits the modes in the Kaluza-Klein expansion (2.10) into an eigenspace with eigenvalue +1, and an eigenspace with eigenvalue −1, which we will refer to as the even and odd eigenspaces. The forms \( dx^\mu \) are even, since \( \sigma \) does not act on the non-compact directions. For \( T^2 \) we determined below (2.9) that \( dy^1 \) is odd and \( dy^2 \) is even. The harmonic 2-forms \( \omega^\alpha \) on K3 split into an odd eigenspace \( H^{2-} \) of dimension \( n_- \) and an even eigenspace \( H^{2+} \), of dimension \( n_+ \). We can always choose a basis of forms in \( H^2 \) which consists of eigenvectors of the involution \( \sigma \). This basis then splits into a basis of \( H^{2+} \) and a basis of \( H^{2-} \), which we label as follows:

\[
H^{2+} = \text{span} \{ \omega^A \} , \quad A = 1, \ldots, n_+ ,
\]

\[
H^{2-} = \text{span} \{ \omega^P \} , \quad P = 1, \ldots, n_- ,
\]

(2.12)

with \( n_+ + n_- = 22 \). Since the wedge product and the pull-back \( \sigma^* \) commute, it is easy to determine the parity with respect to \( \sigma^* \) of products of the \( \omega^\alpha \) and \( dy^i \).

Using (2.3) together with the action of \( \sigma^* \) that we just determined we can now determine the orientifold spectrum by projecting onto those modes which are invariant under \( S = (-1)^{F_i} \Omega_\sigma \). Let us start with the components of the metric \( \hat{g} \) on \( \mathbb{R}^{1,3} \times T^2 \). It is slightly more transparent to do the projection for the metric components in the coordinate basis which uses \( dx^\mu, dy^i \) as differentials rather than the “vielbein” basis \( dx^\mu, \nu^j \) of (2.11). Let us define

\[
\hat{g}_{\mu\nu} = g_{\mu\nu} + g_{ij} g^j_\mu g^i_\nu , \quad \hat{g}_{ij} = g_{ij} , \quad \hat{g}_{i\mu} = -g_{ij} g^j_\mu , \quad \hat{\omega} = \hat{\omega} ,
\]

(2.13)

such that \( ds^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu + \hat{g}_{ij} dy^i dy^j + 2\hat{g}_{i\mu} dy^i dx^\mu + g_{ab} dz^a dz^b \).

\( S \) maps the metric \( \hat{g} \) to \( \sigma^*(\hat{g}) \), so we have to project out the modes with odd parity under the action of \( \sigma^* \). Since we are restricting ourselves to the metric on \( \mathbb{R}^{1,3} \times T^2 \) for now, this means that we only keep the even forms \( dx^\mu dx^\nu, dx^\mu dy^i, dy^i dy^j \) and \( dy^2 dy^2 \) or in other words the components \( \hat{g}_{\mu\nu}, \hat{g}_{ij}, \hat{g}_{i\mu} \) and \( \hat{\omega} \) remain in the spectrum. Using (2.13) to return to the “vielbein” frame, we see that we are left with the components \( g_{\mu\nu}, g_{i\mu} \) and \( g_{22} \). The \( g^i_\mu \), which are related to the metric \( \hat{g} \) by \( g^i_\mu = g^{ij} \hat{g}_{j\mu} \) are reduced to \( g^1_\mu = 0, g^2_\mu = g^{22} \hat{g}_{2\mu} \) by the orientifold projection. Now it is easy to see that \( \nu^1 = dy^1 \) is odd while \( \nu^2 = dy^2 - g^2_\mu dx^\mu \) is even.

Let us continue to impose the orientifold projection on the other fields in the spectrum. (2.3) implies that the two-form field \( \hat{B} \) has to transform as \( \hat{B} \to -\sigma^*(\hat{B}) \), so that only odd modes survive in the expansion of \( \hat{B} \) in (2.10). These are the coefficients of \( \nu^1 \wedge dx^\mu, \nu^1 \wedge \nu^2 \) and \( \omega^P \) or in other words the components \( B_{1\mu}, B_{12} \) and \( B_P \). \( \hat{A} \) similarly transforms as \( \hat{A} \to -\sigma^*(\hat{A}) \) so that again only the \( \sigma^* \)-odd component \( A_1 \) survives. The three-form \( \hat{C} \) transforms as \( \hat{C} \to \sigma^*(\hat{C}) \) which implies that the even modes \( C_{2\mu\nu}, C_{A\mu}, C_{1P}, C_{2A} \) and \( C_{\mu\nu\rho} \) remain in the spectrum. \( C_{\mu\nu\rho} \), however, contains no dynamical degrees of freedom.) Finally from the dilaton only the even \( x \)-dependent scalar field \( \phi \) is kept.

\(^{4}\)The numbers \( n_+ \) and \( n_- \) depend on the involution \( \sigma \), which we do not specify here.
This concludes the truncation of the modes coming from (2.10). The reduction of the action for the K3 metric \( g_{ab} \) is slightly more complicated. A Ricci-flat metric on K3 is determined, up to the global volume factor, by its hyperkähler structure. The hyperkähler structure in turn is determined by the subspace \( \Sigma \) of the second cohomology class \( H^2(K3, \mathbb{R}) \) spanned by the two-forms \( J, \text{Re}\Omega \) and \( \text{Im}\Omega \) defined in (2.6), or equivalently, the space of self-dual harmonic two-forms on K3. The second cohomology class of K3 is a 22-dimensional vector space, equipped with a metric of signature (3,19) via the intersection product. One can express the intersection product in terms of the basis \( \{\omega^\alpha\} \) of harmonic two-forms:

\[
\eta^{\alpha\beta} = \int_{K3} \omega^\alpha \wedge \omega^\beta, \quad \alpha, \beta = 1, \ldots, 22.
\] (2.14)

The space of self-dual forms \( \Sigma \) is then a 3-dimensional subspace of the second cohomology class, spanned by forms with a positive self-intersection number, which one can regard as a subspace of \( \mathbb{R}^{3,19} \) spanned by vectors with positive norm.\(^5\) As a consequence the moduli space of Ricci-flat metrics on K3 is the Grassmannian

\[
\mathcal{M}_{K3} = \frac{\text{SO}(3,19)}{\text{SO}(3) \times \text{SO}(19)} \times \mathbb{R}^+, \quad (2.15)
\]

up to a quotient by the discrete group of isomorphisms on K3.\(^3\) The factor \( \mathbb{R}^+ \) is the volume of the K3 surface, which we denote as

\[
e^{-\rho} = \int_{K3} \sqrt{\det(g_{ab})}.
\] (2.16)

The remaining moduli can be conveniently encoded in a matrix \( H^{\alpha\beta} \), which determines the action of the Hodge \(*\)-operator on the harmonic 2-forms via

\[
*\omega^\alpha = H^{\alpha\beta} \omega^\beta \quad (2.17)
\]

We can also describe the matrix \( H^{\alpha\beta} \) in terms of three orthonormal (with respect to \( \eta^{\alpha\beta} \)) vectors \( \xi^x_\alpha, x = 1, 2, 3 \), which parametrize the variations of the two-forms \( J \) and \( \Omega \):

\[
J = \sqrt{2} e^{-\frac{\rho}{2}} \xi^1_\alpha \omega^\alpha,
\]

\[
\Omega = \sqrt{2} e^{-\frac{\rho}{2}} (\xi^2_\alpha \omega^\alpha + i \xi^3_\alpha \omega^\alpha),
\] (2.18)

Since \( J \) and \( \Omega \) span the subspace \( \Sigma \) of self-dual harmonic two-forms, \( H^{\alpha\beta} \) takes the form

\[
H^{\alpha\beta} = -\delta^{\alpha\beta} + 2 \xi^x_\alpha \xi^x_\beta, \quad (2.19)
\]

where \( \xi^{x\alpha} = \eta^{\alpha\beta} \xi^x_\beta \).

The next step is to determine the \( S \)-invariant subspace of \( \mathcal{M}_{K3} \) or in other words determine the \( S \)-invariant deformations of the K3-metric. From the transformation properties given in (2.5), we learn that these are the deformation which are invariant under the action of \( \sigma^* \). Since \( \sigma \) is an isometry, it leaves the Hodge \(*\)-operator invariant. It follows that the

\(^5\)In the remainder of the article we call these subspaces “spacelike subspaces”.

6
Hodge $*$-operator only acts within each of the eigenspaces $H_{2}^{2\pm}$ defined in (2.12). From (2.17) we then immediately conclude that the matrix $H_{\alpha \beta}$ has to be block-diagonal, i.e.

$$H_{\alpha \beta} = \begin{pmatrix} H_{AB} & 0 \\ 0 & H^{PQ} \end{pmatrix},$$

(2.20)

where $A, B = 1, ..., n_{+}$ and $P, Q = 1, ..., n_{-}$ label the even and odd two-forms, respectively. The intersection product (2.14) is a topological invariant, so it remains unchanged under the action of any diffeomorphism, and thus, more specifically, under the involution $\sigma$. Therefore, also $\eta^{\alpha \beta}$ has the block-diagonal form

$$\eta^{\alpha \beta} = \begin{pmatrix} \eta^{AB} & 0 \\ 0 & \eta^{PQ} \end{pmatrix}.$$  

(2.21)

As we already recalled, $J, \text{Re}\Omega$ and $\text{Im}\Omega$ span $\Sigma$ and thus have positive self-intersection number. It follows from the transformation properties (2.7) that $\text{Im}\Omega$ lies in $H_{2}^{2, -}$, whereas $J$ and $\text{Re}\Omega$ lie in $H_{2}^{2, +}$. Together, these facts imply that the intersection form $\eta^{AB}$ on $H_{2}^{2, +}$ has signature $(1, n_{+} - 1)$, whereas $\eta^{PQ}$ has signature $(2, n_{-} - 2)$.

The reduction of $H_{\alpha \beta}$ to a block-diagonal form corresponds to the following reduction of the parameter space of the $\xi^{x}_{A}$: the choice of three orthonormal vectors $\xi^{x}_{A} \in \mathbb{R}^{3, 19}$ is reduced to a choice of one unit vector $\xi^{3}_{A} \in \mathbb{R}^{1, n_{+} - 1}$ (with $n_{+} - 1$ degrees of freedom) and two orthogonal unit vectors $\xi^{1}_{P}, \xi^{2}_{P} \in \mathbb{R}^{2, n_{-} - 2}$ (with $2(n_{-} - 2)$ degrees of freedom). In other words, the matrices $H_{AB}, H^{PQ}$ are given by

$$H_{AB} = -\delta_{AB} + 2\xi^{3}_{A}\xi^{3}_{B}, \quad H^{PQ} = -\delta^{PQ} + 2(\xi^{1P}\xi^{1Q} + \xi^{2P}\xi^{2Q}).$$

(2.22)

We see that, for the metric to be invariant under the orientifold projection, the spacelike three-plane $\Sigma$ must be a product of a one-dimensional spacelike subspace in $H_{2}^{2, +}$ and a two-dimensional spacelike subspace in $H_{2}^{2, -}$. This means that the Grassmannian in equation (2.15) is reduced to the product of two Grassmannians. Together with the volume factor, this accounts for the moduli space

$$\mathcal{M}_{\text{OK3}} = \frac{\text{SO}(1, n_{+} - 1)}{\text{SO}(n_{+} - 1)} \times \frac{\text{SO}(2, n_{-} - 2)}{\text{SO}(2) \times \text{SO}(n_{-} - 2)} \times \mathbb{R}^{+}.$$  

(2.23)

To summarize, we determined the massless bosonic modes which survive the orientifold projection and assembled them in Table 2.1. As we will discuss in more detail in the coming sections, these fields match the bosonic content of a four-dimensional $N = 2$ supergravity theory which contains, apart from the gravity multiplet, $n_{+} + 1$ vector multiplets and $n_{-}$ hypermultiplets. However, we can already anticipate the field content of these multiplets:

- The gravity multiplet contains the metric $g_{\mu\nu}$ and the graviphoton $g^{3}_{\mu}$.
- The $n_{+}$ vector fields $C^{A}$, the $n_{+}$ real scalars $C^{2A}$, the $(n_{+} - 1)$ degrees of freedom from $\xi^{3A}$, and $e^{-2\phi}g_{22}$ together form $n_{+}$ vector multiplets.
- The vector field $B_{1}$, the product $g_{11}g_{22} = e^{-2\eta}$ and $B_{12}$ form one more vector multiplet.
Table 2.1: This table lists the massless fields which survive the O6 orientifold projection. The hatted fields are the massless ten-dimensional fields while the unhatted fields are the massless modes in four space-time dimensions with $j$ indicating their (four-dimensional) spin. The indices $A, B = 1, ..., n_+$ label components from the expansion in even two-forms, the indices $P, Q = 1, ..., n_-$ correspond to odd two-forms.

- The $n_-$ scalars $B^P$, the $n_-$ scalars $C_{1P}$ and the $2(n_- - 2)$ degrees of freedom contained in $H^{PQ}$ assemble in $(n_- - 1)$ hypermultiplets.

- An additional hypermultiplet arises as the Poincaré dual of the tensor multiplet containing the scalars $A_1$ and $e^{-2\hat{\phi}} - \rho g_{11}$, the K3 volume factor $\rho$, and the two-form $C_2$.

We note that the dilaton $\hat{\phi}$ is a combination of scalars from the vector and hypermultiplets. This implies that both sectors receive string loop corrections, as is the case in type I compactifications [31].

2.4 Effective action

We can now compute the effective four-dimensional action for the orientifolded theory. The starting point is the bosonic action of ten-dimensional type IIA supergravity given by [32]

$$S_{IIA} = \int e^{-2\hat{\phi}} \left( \text{d}^{10} x \sqrt{-g} (\hat{\mathcal{R}} + 4 \partial_M \hat{\phi} \partial^M \hat{\phi}) + \frac{1}{2} \text{d} \hat{B} \wedge * \text{d} \hat{B} \right)$$

$$+ \frac{1}{2} \int \left( \text{d} \hat{A} \wedge * \text{d} \hat{A} + \hat{F}_4 \wedge * \hat{F}_4 \right) + \frac{1}{2} \int \hat{B} \wedge \text{d} \hat{C} \wedge \text{d} \hat{C} ,$$

with the field strength

$$\hat{F}_4 = \text{d} \hat{C} - \hat{A} \wedge \text{d} \hat{B} .$$

Substituting the Kaluza-Klein expansion of (2.10) together with the orientifold projection as determined in the previous section into the action (2.24) and integrating over K3 $\times T^2$,
we obtain\footnote{We do not give the computation here but refer the reader to ref. \textsuperscript{25} for further details of the reduction in the NS-sector and to ref. \textsuperscript{16, 18} for the reduction in the RR-sector. The six-dimensional action obtained from K3 compactifications of type IIA is given in \textsuperscript{29}, while background fluxes are turned on in \textsuperscript{33}.}

\[
S_{\text{kin}} = \int d^4 x \sqrt{-g} \left( \frac{1}{2} R - \partial_\mu (\dot{\phi} + \frac{1}{2} \rho + \frac{i}{2} \eta) \partial^\mu (\dot{\phi} + \frac{1}{2} \rho + \frac{i}{2} \eta) \\
+ \frac{1}{4} (\partial_\mu \epsilon^\nu \partial^\mu e^{-\rho} + \partial_\mu g_{11} \partial^\mu g^{11} + \partial_\mu g_{22} \partial^\mu g^{22}) \\
+ \frac{1}{16} (\partial_\mu H_A^B \partial^\mu H_B^A + \partial_\mu H_F^P \partial^\mu H_F^Q ) \right) \\
+ \frac{1}{2} \int \left( \frac{1}{2} e^\rho H^{PQ} (dB_P \wedge *dB_Q) + \frac{1}{2} e^{2\eta} dB_{12} \wedge *dB_{12} \\
+ e^{-2\dot{\phi} - \rho - \eta} g_{22} d\eta \wedge *d\eta \\
+ e^{-2\dot{\phi} - \rho - \eta} g^{11} (dB_1 + dg^2 B_{12}) \wedge *(dB_1 + dg^2 B_{12}) \right) \\
+ \frac{1}{2} \int \left( \frac{1}{2} e^{2\dot{\phi} + \rho} H^{PQ} g^{11} (dC_{1P} + A_1 dB_P) \wedge * (dC_{1Q} + A_1 dB_Q) \\
+ e^{-\eta} H^{AB} (dC_A - dg^2 C_{2A}) \wedge * (dC_B - dg^2 C_{2B}) \\
+ 2 e^{-2\dot{\phi} - 2\eta - 2\rho} g^{22} dC_2 \wedge *dC_2 \\
+ 4 e^{-4\dot{\phi} - 3\eta - 3\rho} (dC - dg^2 \wedge C_2) \wedge * (dC - dg^2 \wedge C_2) \right) \\
+ \frac{1}{2} \int \left( - \eta^{AB} B_{12} dC_A \wedge dC_B - 2 \eta^{PQ} dC_2 \wedge B_P dC_{1Q} \\
+ \eta^{AB} (dB_1 + dg^2 B_{12}) \wedge (2dC_A - dg^2 C_{2A}) C_{2B} \right).
\]

(2.26)

\eta encodes the volume of the torus $T^2$ and is defined by

\[
e^{-\eta} = \int_{T^2} \sqrt{\det(g_{ij})} = \sqrt{g_{11} g_{22}}.
\]

(2.27)

As was already mentioned in the previous section, the three-form $C$ does not carry any degrees of freedom in 4 dimensions, and we choose to integrate it out. The equation of motion for $C$ derived from (2.26) reads $d(C - g^2 \wedge C_2) = 0$. Its solution $C = g^2 \wedge C_2$ is then inserted back into (2.26). Similarly, the massless two-form field $C_2$ can be dualized to a massless scalar field $\gamma$. Following the well-known dualization prescription (see, for instance, \textsuperscript{34}), we arrive at the following action for $\gamma$

\[
S_\gamma = \frac{1}{16} \int e^{2\dot{\phi} + 2\rho} g^{11} (d\gamma + 2 B^P dC_{1P}) \wedge *(d\gamma + 2 B^Q dC_{1Q}),
\]

(2.28)

which replaces all terms containing $C_2$ in the action (2.26).
In the following two sections, we perform the necessary field redefinitions which bring the action into the canonical form of $N = 2$ supergravity. Let us start with the vector multiplets.

### 2.4.1 Vector multiplets

In order to display the supergravity basis we need to perform a set of field redefinitions which decouples the kinetic terms of the vector multiplet scalars from the hypermultiplet scalars. As was already mentioned in section 2.3, the scalars in the vector multiplets are the $n_+ - 1$ metric moduli in $\xi^{3A}$, the $C_{2A}, B_{12}, \eta$, and $e^{-2\hat{\phi} - \rho}g_{22}$. In order to exhibit the $N = 2$ special geometry, we assemble them into the following $n_+ + 1$ complex fields

\[
z^A = C_2^A + ie^{-\hat{\phi} + \frac{1}{2}\rho} \sqrt{2}g_{22} \xi^{3A} ,
\]

\[s = B_{12} + ie^{-\eta} ,
\]

where $\eta$ is defined in (2.27). We recall that $\xi^{3A}$ has unit norm and thus carries only $n_+ - 1$ degrees of freedom. The required extra degree of freedom turns out to be the factor $e^{-\hat{\phi} + \frac{1}{2}\rho} \sqrt{2}g_{22}$. In terms of these fields the kinetic terms are indeed block diagonal and read for the vector multiplet scalars $z^A$ and $s$

\[
S_{\text{vector}} = \int \frac{-1}{(s - \bar{s})^2} \, ds \wedge *d\bar{s} + G_{\bar{A}B} \, dz^A \wedge *d\bar{z}^B ,
\]

where the coupling $G_{\bar{A}B}$ is given by the expression

\[
G_{\bar{A}B} = -4 \frac{(z - \bar{z})_A(z - \bar{z})_B}{((z - \bar{z})^C(z - \bar{z})_C)^2} + \frac{2\eta_{AB}}{(z - \bar{z})^C(z - \bar{z})_C} .
\]

The combined metric defined by (2.30) is Kähler with the Kähler potential

\[
K = -\ln i(s - \bar{s}) - \ln[-\frac{1}{2}\eta_{AB}(z - \bar{z})^A(z - \bar{z})^B] .
\]

Note that $K$ can also be expressed in terms of geometrical quantities as

\[
K = -\ln e^{-2\hat{\phi}} \int_{K3 \times T^2} \text{Im}\Omega \wedge dy^2 \wedge *(\text{Im}\Omega \wedge dy^2) ,
\]

where we used (2.18). $K$ is a Kähler potential for the coset space

\[
\mathcal{M}_v = \frac{\text{SU}(1,1)}{U(1)} \times \frac{\text{SO}(2,n_+)}{\text{SO}(2) \times \text{SO}(n_+) .
\]

Consistent with $N = 2$ supergravity $\mathcal{M}_v$ is a special Kähler manifold in that $K$ can be written in the form $K = -\ln i[\bar{X}^I \bar{F}_I - X^I \bar{F}_I]$ for

\[
\bar{F}_I = \partial_I \mathcal{F} , \quad \mathcal{F} = -\frac{S_{\eta AB} Z^A Z^B}{2X^0} ,
\]

and a choice of special coordinates $X^I = (X^0, S, Z^A) = \frac{1}{2}(1, s, z^A), I = 0, \ldots, n_++1$. 10
This prepotential $F$ also determines the couplings of the field strengths of the graviphoton and the $n_+ + 1$ vector fields. We label the vector fields with the same index $I$ and define

$$F^I = dA^I = (dg^2, dB_1, dC^A).$$

In a consistent $N = 2$ supergravity Lagrangian, they should couple as

$$S_F = \frac{1}{2} \int \text{Re} \mathcal{N}_{IJ} F^I \wedge F^J - \text{Im} \mathcal{N}_{IJ} F^I \wedge *F^J,$$

where the matrix $\mathcal{N}$ is expressed in terms of the prepotential $F$

$$\mathcal{N}_{IJ} = \bar{F}_{IJ} + 2i \text{Im} F^I \text{Im} F^J X^K X^L \text{Im} F^M X^N.$$ 

In equation (B.1) we display the matrix $\mathcal{N}_{IJ}$ obtained from the effective action (2.26). It is indeed consistent with the matrix obtained by inserting $F$ given in (2.35) into (2.38).

### 2.4.2 Hypermultiplets

Let us now turn to the geometry of the scalar fields in the hypermultiplets. The field redefinition (2.29) decoupled the scalars in the vector multiplet so that the remaining scalar kinetic terms in the effective action (2.26) can be written as

$$S_{\text{hyper}} = \int -\frac{1}{16} dH^P Q \wedge * dH^Q P$$

$$+ \frac{1}{4} e^{2\hat{\varphi}} g^{11} \left( d(e^{-\varphi} g^{11}) \wedge * d(e^{-\varphi} g^{11}) + dA_1 \wedge * dA_1 \right)$$

$$+ \frac{1}{4} e^{2\hat{\varphi} + 2\rho} g^{11} (e^{-\varphi - \rho} g^{11}) \wedge * d(e^{-\varphi - \rho} g^{11})$$

$$+ \frac{1}{4} e^{\rho} B_{PQ} dC_{1P} \wedge * dC_{1Q}$$

$$+ \frac{1}{4} e^{2\hat{\varphi} + \rho} g^{11} (1/2 d\gamma + B^P dC_{1P} \wedge * (1/2 d\gamma + B^Q dC_{1Q}).$$

We will now show that this defines a metric on the quaternionic manifold

$$\mathcal{M}_h = \frac{\text{SO}(4, n_-)}{\text{SO}(4) \times \text{SO}(n_-)}.$$ 

To do so we use the fact that $\mathcal{M}_h$ is in the image of the c-map. More specifically, this implies that $\mathcal{M}_h$ can be viewed as a fibration over a special Kähler base space which, for the case at hand, is the manifold

$$\mathcal{M}_b = \frac{\text{SU}(1, 1)}{\text{U}(1)} \times \frac{\text{SO}(2, n_- - 2)}{\text{SO}(2) \times \text{SO}(n_- - 2)}.$$ 

Indeed, the first 2 lines of (2.39) are precisely the metric of $\mathcal{M}_b$. This follows from our discussion in section 2.3 and in particular from eq. (2.23). There we already argued
that the $H^P_Q$ can be viewed as the coordinates of the second factor of $\mathcal{M}_b$. Furthermore, we recognize the second line of \((2.39)\) as a standard parametrization of the coset $\text{SU}(1,1)/\text{U}(1)$ by combining $A_1$ and $e^{-\hat{\phi}}\sqrt{g_{11}}$ into the complex field

$$T := A_1 + ie^{-\hat{\phi}}\sqrt{g_{11}}.$$  \hfill (2.42)

In order to compare the action \((2.39)\) with the form given in \[37\] we define \[35\]

$$\mathcal{M}_{PQ} = \Re T \eta_{PQ} + i\Im T H_{PQ}.$$  \hfill (2.43)

and

$$\phi := 2e^{-\hat{\phi} - r}\sqrt{g_{11}}, \quad \tilde{\phi} := \gamma + B^P C_1 P.$$  \hfill (2.44)

With the help of \((2.42)-(2.44)\) we can recast the action \((2.39)\) into the form

$$S_{\text{hyper}} = \int \frac{-1}{(T - \bar{T})^2} dT \wedge *d\bar{T} - \frac{1}{16} dH^P_Q \wedge *dH^P_Q$$

$$+ \frac{1}{4\phi^2} d\phi \wedge *d\phi + \frac{1}{2\phi} (\Im \mathcal{M})_{PQ} dB^P \wedge *dB^Q$$

$$+ \frac{1}{2\phi} (\Im \mathcal{M})^{-1} \left( dC_{1P} + (\Re \mathcal{M})_{PR} dB^R \right) \wedge *\left( dC_{1Q} + (\Re \mathcal{M})_{QS} dB^S \right)$$

$$+ \frac{1}{4\phi^2} (d\tilde{\phi} + B^P dC_{1P} - C_{1P} dB^P) \wedge *(d\tilde{\phi} + B^Q dC_{1Q} - C_{1Q} dB^Q),$$  \hfill (2.45)

which exactly coincides with the explicit form of the $c$-map as given in \[37\].

This ends our discussion of type IIA supergravity compactified on orientifolds of $K3 \times T^2$. Our main result is that using a KK-reduction the scalar field space is determined to be

$$\mathcal{M} = \frac{\text{SU}(1,1)}{\text{U}(1)} \times \frac{\text{SO}(2, n_+)}{\text{SO}(2) \times \text{SO}(n_+)} \times \frac{\text{SO}(4, n_-)}{\text{SO}(2) \times \text{SO}(n_-)},$$  \hfill (2.46)

where $n_+ + n_- = 22$ and $n_+(n_-)$ count the number of even (odd) harmonic two-forms of $K3$.

### 3 SU(2)-structure orientifolds

We are now in a position to discuss the more general case of type IIA compactification on a generic SU(2) structure manifold. Before we move on to the orientifold projection and the effective action, we recall some facts about six-dimensional SU(2)-structure manifolds and briefly discuss the moduli space of metrics on these manifolds as determined in \[17\].

#### 3.1 SU(2)-structure manifolds

As stated before, the reduction of the structure group of a six-dimensional manifold $\mathcal{Y}$ to SU(2) is equivalent to the existence of two globally defined spinors $\eta^i$ on $\mathcal{Y}$. With the
help of these spinors, one can define a real 2-form $J$, a complex 2-form $\Omega$, and a complex one-form $K$ exactly as in section 2.2, i.e.

\[
J = \frac{i}{2} (\eta_1^+ \gamma_{mn} \eta_1^- - \eta_2^+ \gamma_{mn} \eta_2^-) \, dY^m \wedge dY^n
\]

\[
\Omega = \frac{i}{2} \eta_1^+ \gamma_{mn} \eta_2^- \, dY^m \wedge dY^n
\]

\[
K = \eta_2^+ \gamma_m \eta_1^- \, dY^m = K^2 + iK^1,
\]

where $Y^m, m = 1, \ldots, 6$ denote the coordinates on $\mathcal{Y}$. However, for a generic $\mathcal{Y}$ neither of these forms is necessarily closed as they are for $K3 \times T^2$. Using Fierz identities and the definitions (3.1), one can show that $J$, $\Omega$ and $K$ obey

\[
\iota_K J = 0, \quad \iota_K \Omega = \iota_K \bar{\Omega} = 0,
\]

\[
\Omega \wedge \bar{\Omega} = 2 J \wedge J \neq 0, \quad \Omega \wedge J = 0, \quad \Omega \wedge \Omega = 0,
\]

\[
K^1_m K^{1m} = 1 = K^2_m K^{2m}, \quad K^1_m K^{2m} = 0.
\]

A generic $\mathcal{Y}$ with SU(2)-structure will no longer be a direct product $M_4 \times M_2$, but it follows from the constraints (3.2), (3.3) that the tangent bundle still splits into two orthogonal sub-bundles: the 2-dimensional part $T_2 \mathcal{Y}$ spanned by the components of $K$, and its orthogonal complement $T_4 \mathcal{Y}$, or in other words, an almost product structure exists on $\mathcal{Y}$ [13 15 17 25 38]. As a further consequence the volume form splits according to

\[
\text{vol}_{\mathcal{Y}} = \text{vol}_2 \otimes \text{vol}_4 \sim K \wedge \bar{K} \otimes J \wedge J.
\]

We will make the extra assumption that the almost-product structure is integrable, which seems necessary in order to make the calculation of the effective action tractable [25 8].

There is no general procedure by which one can construct a set of light Kaluza-Klein modes on a general SU(2)-structure background. On Calabi-Yau manifolds, there is a clear distinction between the harmonic modes, which are massless, and the heavier modes, whose masses are at the Kaluza-Klein scale. In the non-Calabi Yau case, the distinction between light and heavy modes in the compactification is not obvious. The current procedure is to assume that, nevertheless, a suitable finite set of “light” modes exists, whose properties can then be constrained by various consistency conditions [28 39].

Using these assumptions, the scalar field space for the light modes of SU(2)-structure compactifications was determined in [16 17 25]. It was shown in [17] that in the absence of massive but light gravitino multiplets, the low-energy theory is determined by a set of $n$ two-forms $\omega^{\alpha}, \alpha = 1, \ldots, n$ which describe the deformations of $\Omega$, $J$, and the complex one-form $K = K^2 + iK^1$. These two-forms are the analogue of the 22 harmonic two-forms of K3. Furthermore, the intersection form $\eta^{\alpha \beta}$ is defined as in (2.14) and can be shown to have signature $(3, n - 3)$ instead of $(3, 19)$ for K3. The deformation space of $\Omega$ and $J$ is again a symmetric space analogous to the moduli space of K3 metrics given in (2.15).

---

7 A set of globally defined differential forms $K, J$ and $\Omega$ subject to the constraints (3.2), (3.3) is an equivalent characterization of an SU(2)-structure on a six-dimensional manifold.

8 By integrability of the almost-product structure, we mean that local coordinates $y^i, i = 1, 2, z^a, a = 1, \ldots, 4$ can be found in every neighborhood of $\mathcal{Y}$, such that $T_2 \mathcal{Y}$ is spanned by the $\partial / \partial y^i$, and $T_4 \mathcal{Y}$ is spanned by the $\partial / \partial z^a$. 

13
and was found to be \[17\]

\[
\mathcal{M}_{J,\Omega} = \frac{\text{SO}(3, n-3)}{\text{SO}(3) \times \text{SO}(n-3) \times \mathbb{R}^+}.
\] (3.6)

The deformations corresponding to the metric on \(T_2 \mathcal{Y}\) are parametrized by its components \(\hat{g}_{ij}\) defined by the line element \(ds_{T_2 \mathcal{Y}} = \hat{g}_{ij} K^i K^j\). They again span the coset space

\[
\mathcal{M}_K = \frac{\text{SU}(1, 1)}{\text{U}(1)}.
\] (3.7)

As we already said, \(K, \Omega\) and \(J\) are no longer necessarily d-closed on \(\mathcal{Y}\) and the exterior derivatives parametrize the (intrinsic) torsion of the manifold. Imposing that the truncation to a finite set of light modes is non-degenerate, constrains the structure of the torsion terms. For the case at hand one has \[16, 25\]

\[
d\omega^\alpha = D^\alpha_{i\beta} K^i \wedge \omega^\beta, \quad \alpha, \beta = 1, \ldots, n, \] (3.8a)

\[
dK^i = \theta^i K^1 \wedge K^2, \quad i = 1, 2, \] (3.8b)

where \(D^\alpha_{i\beta}\) and \(D^i_{jk}\) are constant\[9\] Imposing \(d^2 = 0\) and \(\int d(K^i \wedge \omega^\alpha \wedge \omega^\beta) = 0\) implies the following constraints

\[
D^\alpha_{i\beta} D^\gamma_{j\beta} - D^\alpha_{j\beta} D^\gamma_{i\beta} = \epsilon_{ij} \theta^k D^\alpha_{k\beta},
\] (3.9a)

\[
D^\alpha_{i\gamma} \eta^\gamma_{\beta} + \eta^\alpha_{i\beta} \epsilon_{ij} \theta^j = -\eta^\alpha_{\gamma} D^\gamma_{i\beta}.
\] (3.9b)

The constraint (3.9b) implies that we can define traceless \((n \times n)\) matrices \(T_i\) as

\[
T^\alpha_{i\beta} = D^\alpha_{i\beta} + \frac{1}{2} \epsilon_{ij} \theta^j \delta^\alpha_{\beta}.
\] (3.10)

In terms of the \(T_i\), the constraints (3.9) take the form

\[
[T_i, T_j] = \epsilon_{ij} \theta^k T_k ,
\] (3.11)

\[
T^\alpha_{i\gamma} \eta^\gamma_{\beta} = -\eta^\alpha_{\gamma} T^\beta_{i\gamma},
\]

which implies that the \(T_i\) are in the algebra of \(\text{SO}(3, n-3)\). For completeness, we also rewrite the exterior derivatives (3.8) in terms of the \(T_i\).

\[
d\omega^\alpha = T^\alpha_{i\beta} K^i \wedge \omega^\beta + \frac{1}{2} \theta^i \epsilon_{ij} K^j \wedge \omega^\alpha, \quad \alpha, \beta = 1, \ldots, n,
\] (3.12)

\[
dK^i = \theta^i K^1 \wedge K^2, \quad i = 1, 2.
\]

Let us now implement the orientifold projection on such generic SU(2) structure backgrounds.

\[9\text{Exterior derivatives of the form } dK^i = D^i_{\alpha} \omega^\alpha \text{ can be ruled out as a consequence of the integrable almost-product structure.}\]
3.2 Orientifold projection

We have argued in the previous section that under reasonable assumptions the space of light Kaluza-Klein modes in SU(2)-structure compactifications can be constructed, and that this space has a similar structure as its $K3 \times T^2$ counterpart. In particular the massless modes of $K3 \times T^2$ compactifications are replaced by a finite set of light modes with similar couplings.

Under these assumptions it is straightforward to also generalize the orientifold projection, which is the topic of this section. In particular in both cases ($K3 \times T^2$ and SU(2)-structure manifolds) we have given $J, \Omega$ and $K$ in terms of spinor bilinears in eqs. (2.6), (2.8) and (3.1). Furthermore the orientifold projections (2.7) and (2.9) were derived from the action of the orientifold map $\sigma$ on the two globally defined spinors $\eta^i$ given in (2.5). Therefore we can immediately conclude

$$\sigma^*(J) = -J, \quad \sigma^*(\Omega) = -\bar{\Omega}, \quad \sigma^*(K) = \bar{K}. \quad (3.13)$$

Correspondingly, the considerations from section 2.3 still apply. The generalized space of Kaluza-Klein modes is divided into $\sigma^*$-even and -odd modes as before, with signature of the intersection forms on the $H^{2,+}$ and $H^{2,-}$ equal to $(1, n_+ - 1)$ and $(2, n_- - 2)$. For our purposes, the only difference is that the number $n = n_+ + n_-$ of “light” two-forms in the Kaluza-Klein expansion is now arbitrary, depending on the details of the internal manifold $Y$. Thus, the moduli space of metrics (3.6) on $T^4 Y$ is reduced to

$$\mathcal{M} = \frac{\text{SO}(1, n_+ - 1)}{\text{SO}(n_+ - 1)} \times \frac{\text{SO}(2, n_- - 2)}{\text{SO}(2) \times \text{SO}(n_- - 2)} \times \mathbb{R}^+, \quad (3.14)$$

exactly as in (2.23). On $T^2 Y$ the metric degrees of freedom are again reduced to the diagonal components $g_{11}, g_{22}$.

To determine the projection of the remaining modes, we can truncate the Kaluza-Klein expansion exactly as we did in section 2.3. Therefore the structure of the light multiplets and their kinetic terms is completely unchanged. In particular the scalar field space is still given by (2.46) (again with $n_+ + n_-$ arbitrary). The difference only arises from the non-vanishing torsion components or in other words from the non-vanishing exterior derivatives given in (3.12).

All that remains to be done, then, is to specify the transformation properties of the exterior derivatives with respect to $\sigma^*$. Since $\sigma^*$ and $d$ commute, a $p$-form and its exterior derivative must have the same parity. This implies that the general form of the exterior derivatives given in (3.12) reduces to

$$d\omega^A = T_{2B}^A K^2 \wedge \omega^B + \frac{1}{2} \theta K^2 \wedge \omega^A + T_{1Q}^A K^1 \wedge \omega^Q, \quad A, B = 1, \ldots, n_+, \quad (3.15)$$

for the even forms, whereas for the odd forms we have

$$d\omega^P = T_{2Q}^P K^2 \wedge \omega^Q + \frac{1}{2} \theta K^2 \wedge \omega^P + T_{1B}^P K^1 \wedge \omega^B, \quad P, Q = 1, \ldots, n_-,$$

$$dK^1 = \theta K^1 \wedge K^2, \quad (3.16)$$

where we have omitted the index on $\theta^1$, since $\theta^2 = 0$. These exterior derivatives will induce a scalar potential and give charge to some of the scalar fields. These modifications are the subject of the next section.
3.3 Effective action

We are now prepared to discuss the effective action of type IIA supergravity compactified on a general SU(2)-structure manifold with orientifold projection. On a formal level, the only differences with the compactification on $K3 \times T^2$ are that the forms $K^i$ replace the differentials $dy^i$ in the Kaluza-Klein expansion (2.10), (2.11), as well as the fact that the expansion forms $K^i, \omega^\alpha$ are no longer required to be closed. Physically, the effect of choosing a background manifold with intrinsic torsion is that the fields parametrizing its deformations become charged. This leads to an effective action with gauge symmetries and a corresponding potential for the scalar fields. In case of an SU(2)-structure compactification, the effective action is an $\mathcal{N} = 4$ gauged supergravity [16, 18]. If in addition the orientifold projection discussed in the previous section is implemented this $\mathcal{N} = 4$ theory is reduced to a gauged $\mathcal{N} = 2$ supergravity.

We now substitute the Kaluza-Klein expansion (2.10) for the modes which survive the orientifold projection (and which are recorded in Table 2.1) into the type IIA effective action (2.24). Using the exterior derivatives given in eqs. (3.15) and (3.16), we obtain an effective action of the form

$$S = S^{(d\rightarrow D)}_{\text{kin}} + S_{\text{pot}},$$

where the first term $S^{(d\rightarrow D)}_{\text{kin}}$ coincides with the action given in eq. (2.26), but the ordinary derivatives for the following fields are replaced by the covariant derivatives

$$De^{-\eta} = d e^{-\eta} - g^2 \theta e^{-\eta},$$
$$Dg_{11} = dg_{11} - g^2 \theta g_{11},$$
$$De^{-\rho} = d e^{-\rho} + g^2 \theta e^{-\rho},$$
$$DH^A_B = dH^A_B - g^2 (T^A_{2C}H^C_B - H^A_C T^C_{2B}),$$
$$DH^P_Q = dH^P_Q - g^2 (T^P_{2R}H^R_Q - H^P_R T^R_{2Q}),$$
$$DB_{12} = dB_{12} - g^2 \theta B_{12} + B_1 \theta,$$
$$DB_P = dB_P + g^2 T^Q_P B_Q + \frac{1}{2} \theta B_P,$$
$$DA_1 = dA_1 - g^2 \theta A_1,$$
$$DC_{2A} = dC_{2P} + g^2 (T^C_{2A} - \frac{1}{2} \theta \delta^C_A) C_{2C},$$
$$DC_{1P} = dC_{1P} + g^2 (T^Q_{2P} - \frac{1}{2} \theta \delta^Q_P) C_{1Q} - C^A \eta_{AB} T^B_{1P}.$$

Furthermore, the Abelian vector field strengths $F^I$ given in eq. (2.36) are replaced by the non-Abelian field strengths

$$F^0 = dg^2,$$
$$F^1 = dB_1 - \theta g^2 \wedge B_1,$$
$$F^{A+1} = dC^A - g^2 \wedge (T^A_{2B} C^B - \frac{1}{2} \theta C^A).$$
The additional term $S_{\text{pot}}$ in (3.17) corresponds to the scalar potential

$$S_{\text{pot}} = -\int \frac{1}{8} e^{2\phi + 2\rho + n} \left( g^{22} H^{PQ} (T_{2P}^R + \frac{1}{2} \theta \delta_{R}^{P}) (T_{2Q}^S + \frac{1}{2} \theta \delta_{S}^{Q}) + g^{11} H^{AB} T_{1A}^{R} T_{1B}^{S} B_{R} B_{S} ight) - \frac{1}{32} e^{2\phi + \rho + \eta} g^{22} \left( [H, D_{2}]^{P} Q [H, T_{2}]^{Q}_{P} + [H, D_{2}]^{A} B_{2} [H, T_{2}]^{B} \right) - \frac{1}{16} e^{2\phi + \rho + \eta} g^{11} (T_{1Q}^{A} H^{P} - H^{A} B_{1P}) (T_{1C}^{P} H^{C}_{A} - H^{P} R T_{1A}) + \frac{5}{16} e^{2\phi + 3\eta + \rho} \theta^{2} (A_{1})^{2} + \frac{5}{8} e^{4\phi + 3\eta + 2\rho} H^{PQ} \left( C_{2A} T_{1P}^{A} + \theta C_{1P} (C_{1R} + A_{1} B_{R}) (T_{2Q}^{A} + \frac{1}{2} \theta \delta_{S}^{Q}) \right)$$

\[ \cdot \left( C_{2B} T_{1Q}^{B} + \theta C_{1Q} (C_{1S} + A_{1} B_{S}) (T_{2Q}^{S} + \frac{1}{2} \theta \delta_{S}^{Q}) \right) - \int (dC - dg^{2} \wedge C_{2}) \wedge B^{P} (T_{1P}^{B} C_{2B} - T_{2P}^{B} C_{1Q} + \frac{1}{2} \theta C_{1P}). \]

We can rewrite the last line of (3.20) into a more standard form if we integrate out the three-form $C$ as in section 2.4. Again, $C$ has no independent degrees of freedom, but due to the extra topological term which now arises in its action, a contribution to the potential remains after its elimination. Solving the equations of motion and substituting the result back into the action, we obtain the new term

$$-\frac{1}{8} \int e^{4\phi + 3\eta + 3\rho} \left( B^{P} (C_{1Q} T_{2P}^{Q} - \frac{1}{2} \theta C_{1P} - C_{2A} T_{1P}^{A}) \right)^{2} \quad (3.21)$$

As before, the next step is to rewrite the effective action (3.17) in terms of the canonical $N = 2$ field variables. Since the kinetic terms are unchanged, we use exactly the same redefinitions (2.29), (2.42), and (2.44) from section 2.4. The local gauge symmetries which are implicit in the covariant derivatives given in (3.18) can be related to an appropriate set of Killing vectors on the scalar manifolds (2.46). Let us start with the vector multiplets.

### 3.3.1 Vector multiplets

Using again the field redefinitions given in (2.29) the kinetic term for the scalar fields in the vector multiplets read

$$S_{\text{vector}}^{(d-d)} = \int \frac{-1}{(s - \bar{s})^{2}} D_{s} \wedge * D_{\bar{s}} + G_{AB} D z^{A} \wedge * D_{\bar{s}}^{B}, \quad (3.22)$$

where $G_{AB}$ is given in (2.31), and the covariant derivatives read

$$D_{\mu} s = \partial_{\mu} s - g_{\mu}^{2} \theta s + B_{1\mu} \theta, \quad D_{\mu} z^{A} = \partial_{\mu} z^{A} - g_{\mu}^{2} (T_{2B} z^{B} - \frac{1}{2} \theta z^{A}) + C_{\mu}^{B} T_{2B}^{A} - \frac{1}{2} C_{\mu}^{A} \theta. \quad (3.23)$$

We can combine these covariant derivatives into the form

$$D_{\mu} z^{i} = \partial_{\mu} z^{i} - A_{\mu}^{i} k_{i}^{j}, \quad (3.24)$$
where \( z^i \) denotes collectively all vector multiplet scalars \( z^i = (s, z^A) \) and \( A^I_\mu \) denotes all gauge fields, i.e. \( A^I_\mu = (g^I_\mu, B_{i\mu}, C^A_\mu) \). Comparing (3.24) with (3.23) we can read off the Killing vectors
\[
\begin{align*}
    k_0 &= \theta s \partial_s + T^B_2 z^B \partial z^A - \frac{1}{2} \theta z^A \partial z^A , \\
    k_S &= - \theta \partial_s , \\
    k_A &= - T^B_2 \partial z^B + \frac{1}{2} \theta \partial z^A .
\end{align*}
\]
This leads to the gauge algebra
\[
\begin{align*}
    [k_0, k_S] &= - \theta k_S , \\
    [k_0, k_A] &= -(T^B_2 - \frac{1}{2} \theta \delta^B_A) k_B , \\
    [k_S, k_A] &= [k_A, k_B] = 0 .
\end{align*}
\]
This solvable algebra is the semi-direct sum of the Abelian algebra of the translation generators \( k_S, k_A \) and the generator \( k_0 \). Furthermore, one can check that the non-Abelian field-strengths given in (3.19) are indeed of the form \( F^I = dA^I + f^I_{JK} A^J A^K \) for the structure constants defined via \([k_J, k_K] = f^I_{JK} k_I\).

For completeness let us also compute the (real) Killing prepotentials \( P_I \) which exist for all isometries of a special Kähler manifold. They are defined by
\[
    k^i_I = ig^i_\beta \partial_\beta P_I .
\]
Integrating (3.27) for the Killing vectors (3.25) we find
\[
\begin{align*}
    P_0 &= - \frac{i}{2} \theta \frac{s + \bar{s}}{s - \bar{s}} - 2i \frac{z^A T^B_2 z^B}{(z - \bar{z})^2} - \frac{1}{2} \theta \frac{z^2 - \bar{z}^2}{(z - \bar{z})^2} , \\
    P_S &= - i \theta \frac{1}{s - \bar{s}} , \\
    P_A &= - 2i \frac{(z - \bar{z}) B^A}{(z - \bar{z})^2} - i \theta \frac{(z - \bar{z})_A}{(z - \bar{z})^2} .
\end{align*}
\]

### 3.3.2 Hypermultiplets

The scalars in the hypermultiplets are also charged, as can be seen from the covariant derivatives given in (3.18). Using again the definitions (2.42)–(2.44) the kinetic terms of the hypermultiplet scalars are given by
\[
\begin{align*}
    S^{(d\to D)}_{\text{hyper}} &= \int \frac{-1}{(T - \bar{T})^2} DT \wedge * DT - \frac{1}{16} DH_P^Q \wedge * DH_Q^P \\
    &\quad + \frac{1}{4\theta^2} d\phi \wedge * d\phi + \frac{1}{2\phi} (\text{Im} \mathcal{M})_{PQ} DB^P \wedge * DB^Q \\
    &\quad + \frac{1}{2\phi} (\text{Im} \mathcal{M})^{-1} P^Q \left( DC_{1P} + (\text{Re} \mathcal{M})_{PR} DB^R \right) \wedge * \left( DC_{1Q} + (\text{Re} \mathcal{M})_{QS} DB^S \right) \\
    &\quad + \frac{1}{4\phi^2} (D\tilde{\phi} + B^P DC_{1P} - C_{1P} DB^P) \wedge * (D\tilde{\phi} + B^Q DC_{1Q} - C_{1Q} DB^Q) ,
\end{align*}
\]
with the covariant derivatives
\[ D_\mu \xi^P = \partial_\mu \xi^P - g^2 T_Q^P \xi^Q, \]
\[ D_\mu T = \partial_\mu T - g^2 \theta T, \]
\[ D_\mu B^P = \partial_\mu B^P - g^2 (T_Q^P B^Q - \frac{1}{2} \theta B^P), \]
\[ D_\mu C_{1P} = \partial_\mu C_{1P} + g^2 (T_Q^P C_{1Q} - \frac{1}{2} \theta C_{1P}) - C_\mu^A \eta_{AB} T^B_{1P}, \]
\[ D_\mu \tilde{\phi} = \partial_\mu \tilde{\phi} - C_\mu^A \eta_{AB} T^A_{1P} B^P. \]

(3.30)

These covariant derivatives can again be cast into the generic form
\[ D_\mu q^u = \partial_\mu q^u - A^I_\mu k^u_I, \]
where \( q^u \) collectively denote all scalars in the hypermultiplets. Comparing with (3.30) determines the Killing vectors \( k_I = k^u_I \partial_u \) on the quaternionic manifolds. We find that the non-trivial Killing vectors on \( \mathcal{M}_h \) are
\[ k_0 = T_Q^P \xi^Q \partial_\xi^P + (T_Q^P B^Q - \frac{1}{2} \theta B^P) \partial_B^P - (T_Q^P C_{1Q} - \frac{1}{2} \theta C_{1P}) \partial_C^P + \theta T \partial_T, \]
\[ k_A = \eta_{AB} T^B_{1P} (\partial_{C_{1P}} + B^P \partial_{\tilde{\phi}}). \]

(3.31)

Obviously, consistency requires that they form the same gauge algebra as the algebra \( \{ k_S \} \) of the Killing vectors on the special Kähler manifold. \( k_S \) does not act on the quaternionic space and therefore the only non-trivial commutator we need to check is \( [k_0, k_A] \). Using in turn the commutation property from (3.11) and the fact that the \( T_i \) are in the algebra of \( \mathrm{SO}(3,3-n) \), we obtain
\[ = \eta_{AB} T^B_{1P} (T_Q^P \partial_{C_{1P}} - \frac{1}{2} \theta \partial_{C_{1P}}) \]
\[ + (T_Q^P B^Q - \frac{1}{2} \theta B^P) \eta_{AB} T^B_{1P} \partial_{C_{1P}} \]
\[ = \eta_{AB} (T_Q^P \partial_{C_{1P}} - \frac{1}{2} \theta \partial_{C_{1P}}) \]
\[ = - (T_Q^P \partial_{C_{1P}} - \frac{1}{2} \theta \partial_{C_{1P}}) \eta_{BC} T^C_{1P} (\partial_{C_{1P}} + B^P \partial_{\tilde{\phi}}), \]

which is indeed the commutation relation (3.26a).

On a quaternionic Kähler manifold, there is an \( \mathrm{SU}(2) \) triplet of Killing prepotentials associated to each isometry. They are computed in appendix C. Finally, checking the agreement of the potential (3.20) with the corresponding expression of \( N = 2 \) is relegated to appendix D. This completes our discussion of the properties of the effective gauged supergravity.

4 Conclusions

In this paper, we have constructed an \( O6 \) orientifold projection of type IIA string theory, compactified on a background manifold with \( \mathrm{SU}(2) \) structure. In order to find the correct orientifold projection, we first studied the simpler case of compactification on \( K3 \times T^2 \), where all moduli remain massless. Having found the \( O6 \) orientifold projections that
leave intact half of the supersymmetry of these backgrounds, we found that they could be easily generalized to projections on generic SU(2) backgrounds. We then applied these orientifold projections to the effective $N = 4$ theory obtained from compactifications of type II string theory on SU(2) structure backgrounds \cite{16,18}. We have shown that the result corresponds to a standard gauged $N = 2$ supergravity by performing the appropriate field redefinitions. We have seen that in the supergravity field basis, the multiplets mix the Ramond and Neveu-Schwarz fields. The effective theory has a scalar target space

$$ \frac{SU(1,1)}{U(1)} \times \frac{SO(2,n_+)}{SO(2) \times SO(n_+)} \times \frac{SO(4,n_-)}{SO(4) \times SO(n_-)}, $$

where $n_\pm$ is the number of 2-forms with even/odd transformations under the orientifold involution $\sigma$. Thus, the scalar target space takes a simple form, but one expects that the last two factors of (4.1) both receive corrections at string loop order, since they both depend on the dilaton.

Isometries of all sectors of the scalar target space can become gauged, when the internal manifold has suitable torsion components. The gauge algebra which we found, is a solvable semi-direct sum of two Abelian sub-algebras, similar to the algebras found in other $G$-structure compactifications \cite{25,27}. These gaugings induce a potential, which is of the canonical form. An application would be to investigate moduli stabilization in these scenarios.

As a next step, we can combine multiple orientifold projections in order to arrive at a theory with $N = 1$ supersymmetry. If one could find a further orientifold projection which is still compatible with some of the gauge transformations, while at the same time it reduces the supersymmetry, the result would be a simple, yet non-trivial, $N = 1$ toy model.

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A  Spinor conventions

In this appendix we give a brief overview of the conventions used for the spinor representations in various dimensions, and discuss the transformation properties of those spinors under the orientifold map. This section is largely based upon [11], with some adaptations due to our slightly different conventions.

A.1 Representations

In agreement with the compactification ansatz, the ten-dimensional spinors transform in a representation of $\text{Spin}(1,3) \times \text{Spin}(6)$. The corresponding decomposition of the ten-dimensional gamma-matrices $\gamma_M$ is given by

$$\Gamma_\mu = \gamma_\mu \otimes 1, \quad \Gamma_m = \gamma_5 \otimes \gamma_m,$$

(A.1)

where the $\gamma_\mu$ and $\gamma_m$ are the four-dimensional, respectively six-dimensional gamma-matrices, and $\gamma_5$ is the four-dimensional chirality operator. The ten-dimensional chirality operator $\Gamma_{11}$ is the tensor product of the four-dimensional and six-dimensional chirality operators

$$\Gamma_{11} = \gamma_5 \otimes \gamma_7.$$

(A.2)

We work with four- and six-dimensional Weyl spinors, and use subscript $\pm$ to indicate their chirality. Complex conjugation changes the chirality, and we have the following Majorana conditions in four and six dimensions:

$$\zeta_{\pm} = B(4)\zeta_{\mp}^*, \quad \eta_{\pm} = B(6)\eta_{\mp}^*,$$

(A.3)

where the following relations hold

$$B^{-1}_{(4)}\gamma_\mu B_{(4)} = \gamma_\mu^*, \quad A.4a$$

$$B^{-1}_{(6)}\gamma_m B_{(6)} = -\gamma_m^*, \quad A.4b$$

The ten-dimensional spinors are Majorana-Weyl, and satisfy the Majorana condition

$$\varepsilon = B_{(10)}\varepsilon^*,$$

(A.5)

where $B_{(10)}$ is given by

$$B_{(10)} = \Gamma_{11} \cdot B_{(4)} \otimes B_{(6)},$$

(A.6)

and satisfies

$$B_{(10)}^{-1}\Gamma_M B_{(10)} = -\Gamma^*_M.$$

(A.7)

A.2 Transformation properties

Locally, the target space involution $\sigma$ is a combination of a number of reflections. Since we want to preserve all four-dimensional symmetry, these reflections will be along directions in the internal space $\mathcal{Y}$. A reflection that preserves the Majorana property and only acts
on the internal component of a ten-dimensional spinor should act on the spinors with the transformation
\[ R_m = i \Gamma_m \Gamma_{(10)} = i \mathbf{1} \otimes \gamma_m \gamma_{(6)}. \tag{A.8} \]
For an orientifold with \( O_p \)-planes, \( \sigma \) consists of \( l = 10 - (p + 1) \) reflections. Taking the square of \( \sigma = R_{m_1} \ldots R_{m_l} \), we get the following action on spinors
\[ (\sigma^*)^2 = (-1)^{\frac{l(l-1)}{2}} \mathbf{1}. \tag{A.9} \]
In the case of \( O_6 \) planes, we have \( \sigma^2 = -\mathbf{1} \), which demonstrates the need for the extra transformation \( (-1)^F \) in the orientifold projection \( S \).

If the orientifold projection is to preserve some of the supersymmetry, \( \sigma^* \) must map between the ten-dimensional supersymmetry parameters
\[ \sigma^* (\varepsilon^{I}_{10}) = \varepsilon^{I}_{10}, \tag{A.10} \]
\[ \sigma^* (\varepsilon^{II}_{10}) = \pm \varepsilon^{I}_{10}, \]
where the minus sign applies in the case of \( O_6 \) orientifolds, accounting for the fact that \( \sigma^2 = -1 \).

Since \( \sigma \) is a symmetry of our chosen background, it must preserve the spinors \( \eta^i \). Recalling that \( (\sigma^*)^2 = -\mathbf{1} \) in the case of an \( O_6 \) orientifold, we are led to the choice
\[ \sigma^* (\eta^i_\pm) = \pm \eta^i_\mp. \tag{A.11} \]
In principle, more general transformations are of course possible, but in the case of a single orientifold, we can bring the transformation into the form (A.11) by a suitable redefinition of the \( \eta^i \). Looking at the decomposition (2.3) of the ten-dimensional supersymmetry parameters, and using the transformation property (A.11), we see that dividing out the relation (A.10) forces
\[ \varepsilon^I_1 = \varepsilon^I_2, \tag{A.12} \]
reducing the available four-dimensional supersymmetry.

For completeness, we also mention the case of \( O4/O8 \) orientifolds. In this case, \( (\sigma^*)^2 = \mathbf{1} \), so we do not add \( (-1)^F \) to the orientifold action. With our conventions, we can choose the following action on the \( \eta^i \)
\[ \sigma^* (\eta^I_\pm) = \pm \eta^I_\mp, \tag{A.13} \]
\[ \sigma^* (\eta^{II}_\pm) = \mp \eta^{II}_\mp. \]
In the case of an \( O4/O8 \) orientifold, the ten-dimensional supersymmetry parameters are related as in equation (A.10), now without the minus sign. Using (A.13) in the decomposition (2.3), we see that the four-dimensional supersymmetries must satisfy
\[ \varepsilon^I_1 = \varepsilon^I_2, \tag{A.14} \]
\[ \varepsilon^{II}_1 = -\varepsilon^{II}_2. \]
We see that the presence of an extra internal spinor, i.e. SU(2) structure, is necessary to define the (supersymmetric) \( O4/O8 \) orientifold projection [11]. Thus this option is absent in the case of orientifolds of SU(3)-structure compactifications [5, 8].
B  Gauge field kinetic couplings

Applying equation (2.37) for the canonical form of the gauge field kinetic term to the effective action (2.26) obtained from the compactification, we find that the matrix $N$ has the following form

$$
N_{ij} = \begin{pmatrix}
-B_{12}C_2^A C_{2A} & \frac{1}{2} C_2^B C_{2A} & B_{12} C_{2B} \\
\frac{1}{2} C_2^A C_{2A} & 0 & -C_{2B} \\
B_{12} C_{2A} & -C_{2A} & -B_{12} \eta_{AB}
\end{pmatrix}
$$

$$
+ i \begin{pmatrix}
-e^{-2\phi - \rho - \eta}(g_{22} + g^{11}(B_{12})^2) & e^{-2\phi - \rho - \eta} g_{11} B_{12} & e^{-\eta} C_2^A H_{AB} \\
e^{-\eta} H_{AB} C_2^A C_2^B & -e^{-2\phi - \rho - \eta} g_{11} B_{12} & 0 \\
e^{-\eta} H_{AB} C_2^B & 0 & -e^{-\eta} H_{AB}
\end{pmatrix}, \tag{B.1}
$$

which can be written in terms of the complex scalars $s$ and $z^A$ using the field redefinitions (2.29). In terms of the $N = 2$ complex variables, the entries of $N$ become

$$
N_{00} = -B_{12} C_2^A C_{2A} - i \left( e^{-2\phi - \rho - \eta}(g_{22} + g^{11}(B_{12})^2) + e^{-\eta} H_{AB} C_2^A C_2^B \right)
$$

$$
= \frac{-1}{2(s - \bar{s})(z - \bar{z})^2} \left( \bar{z}^2 (2(z \bar{z})^2 - 2z^2 \bar{z}^2) \\
+ s \bar{s} (4z^2 \bar{z}^2 - 2z^2 (z \bar{z}) - \bar{z}^2 (z \bar{z})) + \frac{1}{2}s^2 (z^2 - \bar{z}^2)^2 \right),
$$

$$
N_{0S} = \frac{1}{2} C_2^A C_{2A} + i e^{-2\phi - \rho - \eta} g^{11} B_{12} = \frac{1}{4(s - \bar{s})}(s(z^2 + \bar{z}^2) - 2s\bar{s}z\bar{z}), \tag{B.2}
$$

$$
N_{0A} = B_{12} C_{2A} + i e^{-\eta} H_{AB} C_2^B = \frac{(s - \bar{s})(z^2 - \bar{z}^2)}{2(z - \bar{z})^2}(z - \bar{z}) A + \frac{1}{2}s(z + \bar{z}) A,
$$

$$
N_{SS} = -i e^{-2\phi - \rho - \eta} g^{11} = -\frac{(z - \bar{z})^2}{4(s - \bar{s})},
$$

$$
N_{SA} = -C_{2A} = -\frac{1}{2}(z + \bar{z}) A,
$$

$$
N_{AB} = -B_{12} \eta_{AB} - i e^{-\eta} H_{AB} = -\bar{s}\eta_{AB} - \frac{(s - \bar{s})}{(z - \bar{z})^2}(z - \bar{z}) A(z - \bar{z}) B,
$$

where we have abbreviated contractions of the $z^A$ and $\bar{z}^A$ with the form $\eta^{AB}$ as $z\bar{z}, z^2$ and $\bar{z}^2$. One can check that the expressions (B.2) agree with the result obtained when substituting the prepotential (2.35) into the equation (2.38).

C  Calculation of the Killing prepotentials

In this appendix, we give some details on the computation of the Killing prepotentials $P^I_f$ on the hypermultiplet target space $\mathcal{M}_h = \text{SO}(4, n_-)/\text{SO}(4) \times \text{SO}(n_-)$ following [35].

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One can parametrize the Grassmannian of spacelike 4-planes $\text{SO}(4, n_-)/\text{SO}(4) \times \text{SO}(n_-)$ using four orthonormal vectors of dimension $4 + n_-$, which span the spacelike 4-plane. Writing these four vectors as the rows of a $4 \times n_-$-matrix $Z_{ua}$, we obtain

$$Z_{ua}^T = \frac{1}{\sqrt{2}} \begin{pmatrix}
\frac{1}{2} \sqrt{\frac{T}{\widetilde{\phi} - T}} (\tilde{\phi} + BQ C_1 Q) & \frac{1}{2} \sqrt{\phi(T - T)} (A_1 \tilde{\phi} + C_1 Q C_1 Q + A_1 C_1 Q B Q) & -\sqrt{2} \xi^i C_1 Q \\
-\sqrt{\frac{\phi}{T - T}} A_1 & \frac{\phi}{\sqrt{\phi(T - T)}} & 0 \\
\sqrt{T - T} B_{2Q} & \frac{\phi}{\sqrt{\phi(T - T)}} (\tilde{\phi} - B Q (C_1 Q + A_1 B Q)) & -\sqrt{2} \xi^i B Q \\
-\sqrt{T - T} \sqrt{\frac{\phi}{T - T}} B P & \frac{\phi}{\sqrt{\phi(T - T)}} (2 A_1) & 0 \\
\sqrt{T - T} \sqrt{\frac{\phi}{T - T}} C_{1P} + A_1 B P & \frac{\phi}{\sqrt{\phi(T - T)}} (C_{1P} + A_1 B P) & \sqrt{2} \xi^i P \\
\end{pmatrix} \tag{C.1}$$

where $i = 1, 2$. From $Z$, we can compute the $\text{SO}(4)$ component of the connection on $\mathcal{M}_h$

$$\theta_{ab} = Z_{ua} \eta^{uv} dZ_{vb}, \tag{C.2}$$

where $\eta$ is the following metric of signature $(4, n_-)$:

$$\eta_{uv} = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 \\
0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & \eta_{PQ} \\
\end{pmatrix}. \tag{C.3}$$

We obtain the $\text{SU}(2)$ connection on $\mathcal{M}_h$ by decomposing $\theta$ with respect to the three self-dual 't Hooft matrices $J^{x+}$ given in $[35]$

$$\omega^x = -\frac{1}{2} \text{tr}(\theta J^{x+}), \quad x = 1, 2, 3. \tag{C.4}$$

Computing $\theta$ from $Z$ as given in (C.1) and extracting the different components according to (C.4), we find the $\text{SU}(2)$ connection components $\omega^x$

$$\omega^x = \begin{pmatrix}
\frac{i}{T - T} dA_1 - \frac{1}{2 \phi} (C_{1P} d B P - B P d C_{1P} - d \tilde{\phi}) \\
+ \frac{1}{2} (\xi^1 P d \xi^2 P - \xi^2 P d \xi^1 P) \\
\sqrt{T - T} \xi^1 P d B P - \sqrt{\phi(T - T)} (A_1 \xi^2 P d B P + \xi^2 P d C_{1P}) \\
- \sqrt{T - T} \xi^2 P d B P - \sqrt{\phi(T - T)} (A_1 \xi^1 P d B P + \xi^1 P d C_{2P}) \\
\end{pmatrix}. \tag{C.5}$$

The Killing prepotentials $P^i_I$, then, are the solutions to the set of differential equations

$$- k_I \omega (d \omega^z + \frac{1}{2} \epsilon^{xyz} \omega^y \wedge \omega^z) = d P^i_I + \epsilon^{xyz} \omega^y P^z_I, \tag{C.6}$$

where the left-hand side is the insertion of the $I$-th Killing vector into the $\text{SU}(2)$ curvature form, and the right-hand side is the $\text{SU}(2)$-covariant derivative acting on the triplet $P^i_I$. 

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Combining the connection from equation (C.5) with the Killing vectors \((k_0, k_A)\) on the quaternionic manifold, we obtain the following Killing prepotentials:

\[
P_0^1 = \frac{i}{T-T} A_1 \theta - \frac{1}{\phi} C_1 P(T_{2Q} - \frac{1}{2} \theta \delta^P Q) B^Q + \xi^1 P T_{2Q} \xi^{2Q},
\]

\[
P_0^2 = \sqrt{\frac{T-T}{2\phi}} \xi^1 P(T_{2Q} - \frac{1}{2} \theta \delta^P Q) B^Q - \sqrt{\frac{2i}{(T-T)}} (A_1 \xi^1 P(T_{2Q} - \frac{1}{2} \theta \delta^P Q) B^Q - C_1 P(T_{2Q} - \frac{1}{2} \theta \delta^P Q) \xi^{2Q}),
\]  

\[
P_0^3 = -\sqrt{\frac{T-T}{2\phi}} \xi^1 P(T_{2Q} - \frac{1}{2} \theta \delta^P Q) B^Q - \sqrt{\frac{2i}{(T-T)}} (A_1 \xi^1 P(T_{2Q} - \frac{1}{2} \theta \delta^P Q) B^Q - C_1 P(T_{2Q} - \frac{1}{2} \theta \delta^P Q) \xi^{1Q}),
\]

\[
P_A^1 = \eta AB \frac{1}{\phi} T_{1P} B^P,
\]

\[
P_A^2 = -\eta AB \sqrt{\frac{2i}{\phi(T-T)}} T_{1P} \xi^{2P},
\]

\[
P_A^3 = -\eta AB \sqrt{\frac{2i}{\phi(T-T)}} T_{1P} \xi^{1P}.
\]

We can also express these potentials by the following integrals over the internal manifold:

\[
P_0^1 = -\frac{1}{2} e^{\phi + \rho} \sqrt{g^{11}} \int Y \left( \frac{1}{2} (d(\text{Re}\Omega) \wedge \text{Re}\Omega + dJ \wedge J) \wedge A + dB \wedge C_+ \right.
\]

\[
+ \frac{1}{4} e^\rho \int Y (J \wedge d(\text{Re}\Omega) - \text{Re}\Omega \wedge dJ) \wedge K^1,
\]

\[
P_0^2 = -\frac{1}{2} e^{\phi + \rho} \sqrt{g^{11}} \int Y dB \wedge \text{Re}\Omega \wedge A + C_- \wedge d(\text{Re}\Omega)
\]

\[
+ \frac{1}{2} e^{\frac{3}{2} \rho} \int Y dB \wedge J \wedge K^1,
\]

\[
P_0^3 = -\frac{1}{2} e^{\phi + \rho} \sqrt{g^{11}} \int Y dB \wedge J \wedge A - C_- \wedge dJ
\]

\[
- \frac{1}{2} e^{\frac{3}{2} \rho} \int Y dB \wedge \text{Re}\Omega \wedge K^1,
\]

\[
P_A^1 = -\eta AB \frac{1}{2} e^{\phi + \rho} \sqrt{g^{11}} \int Y dB \wedge \Omega^B \wedge K^2,
\]

\[
P_A^2 = \eta AB \frac{1}{2} e^{\phi + \rho} \sqrt{g^{11}} \int Y d\text{Re}\Omega \wedge \Omega^B \wedge K^2,
\]

\[
P_A^3 = \eta AB \frac{1}{2} e^{\phi + \rho} \sqrt{g^{11}} \int Y dJ \wedge \Omega^B \wedge K^2,
\]

25
where $C_-$ represents those modes of $\hat{C}$ which are odd under the action of $\sigma^*$, e.g. $C_- = C_{1P}K^1 \wedge \omega^P$.

**D The potential**

In this appendix we check consistency of the potential obtained from the SU(2)-structure compactification with $N = 2$ supergravity. The latter requires that the potential takes the special form [35]:

$$V = e^KX^I \bar{X}^J (g_{ij}k_I^ik_J^j + 4h_{uv}k_I^uk_J^v) - \left(\frac{1}{2} (\text{Im}\mathcal{N})^{-1} IJ + 4e^KX^I \bar{X}^J\right) P_I^xP_J^x, \quad (D.1)$$

where $g_{ij}$ and $h_{uv}$ represent the metrics on the special Kähler, resp. quaternion-Kähler target spaces, the $k_I$ are the Killing vectors from equation (3.25), the $X^I$ are the homogeneous coordinates on the special Kähler manifold and the $P_I^x$ are the Killing prepotentials associated with the gauged isometries of the quaternion-Kähler manifold. We now verify that the potential obtained by Kaluza Klein reduction has this canonical form.

We start by simplifying the rightmost term in equation (D.1). Inverting the imaginary part of the $\mathcal{N}$ in (B.1) gives

$$(\text{Im}\mathcal{N})^{-1} = e^{2\hat{\phi} + \rho + \eta} g^{22} \begin{pmatrix} -g^{22} & -g^{22}B_{12} & -C_2^B g^{22} \\ -g^{22}B_{12} & -g^{22}(B_{12})^2 - g_{11} & -g^{22}B_{12}C_2^B \\ -C_2^A g^{22} & -g^{22}B_{12}C_2^A & -e^{-2\phi - 2\rho} H^{AB} - g^{22}C_2^A C_2^B \end{pmatrix}. \quad (D.2)$$

It follows from the formulas for $\mathcal{K}$ and the definition of the variables $s$ and $\zeta^A$ in section 2.4.1 that

$$\frac{1}{2}e^{2\phi + \rho + \eta} g^{22} = e^K. \quad (D.3)$$

Using equations (D.2) (D.3) and the definition of the coordinates $X^I$ in section 2.4.1 we see that the contribution from the Killing prepotentials $P_I^x$ given in (C.7), (C.8) reduces to

$$\left(\frac{1}{2} (\text{Im}\mathcal{N})^{-1} IJ + 4e^KX^I \bar{X}^J\right) P_I^xP_J^x = \frac{1}{2} \eta^{AB} P_A^x P_B^x, \quad (D.4)$$

where

$$\frac{1}{2} \eta^{AB} P_A^x P_B^x = \frac{e^n}{2\phi^2} \eta_{AB} T_{1P}^A B P T_{1Q}^B Q + i \frac{e^n}{\phi(T - T)} \eta_{AB} (T_{1P}^A \zeta^i P T_{1Q}^B \zeta^i Q)$$

$$= \frac{1}{8} e^{2\phi + 2\rho + \eta} g^{11} \eta_{AB} T_{1P}^A B P T_{1Q}^B Q$$

$$+ \frac{1}{8} e^{2\phi + 2\rho + \eta} g^{11} \eta_{AB} (T_{1P}^A \zeta^i P T_{1Q}^B \zeta^i Q). \quad (D.5)$$

In the last equation we rewrote the result in terms of the original variables for ease of comparison with the potential (3.20).

Calculating the contribution from the first term in (D.1) is straightforward. We insert the Killing vectors as given in equations (3.25), (3.31), and find

$$e^KX^I \bar{X}^J (g_{ij}k_I^ik_J^j + 4h_{uv}k_I^uk_J^v) = \frac{3}{16} e^{2\phi + \rho + \eta} g^{22} \theta^2 + \frac{1}{4} e^{2\phi + \rho + \eta} g^{22} (\zeta^3 A T_{2B}^A T_{2C}^B \zeta^C), \quad (D.6)$$
together with
\[ 4e^K X^I X^J h_{uv} k^I J = \frac{1}{8} e^{4\hat{\phi}+3\eta+\rho+2\theta(A_1)^2} + \frac{1}{8} e^{2\hat{\phi}+\rho+\eta} g^{22}\theta^2 \]
\[ -\frac{1}{32} e^{2\hat{\phi}+\rho+\eta} [H, T]^P Q [H, T]^Q \]
\[ + \frac{1}{8} e^{2\hat{\phi}+2\rho+\eta} g^{22} H^{PQ} B_R (T^R_{2P} + \frac{1}{2} \theta \delta^R_P) B_S (T^S_{2Q} + \frac{1}{2} \theta \delta^S_Q) \]
\[ + \frac{1}{8} e^{4\hat{\phi}+2\rho+3\eta} H^{PQ} \]
\[ \cdot \left( C_{2A} T^A_{1P} - C_{1R} (T^R_{2P} - \frac{1}{2} \theta \delta^R_P) - A_1 B_R (T^R_{2P} + \frac{1}{2} \theta \delta^R_P) \right) \]
\[ \cdot \left( C_{2B} T^B_{1Q} - C_{1S} (T^S_{2Q} - \frac{1}{2} \theta \delta^S_Q) - A_1 B_S (T^S_{2Q} + \frac{1}{2} \theta \delta^S_Q) \right) \]
\[ + \frac{1}{4} e^{2\hat{\phi}+\rho+\eta} H^{PQ} \xi^3_A D^A_{1P} \xi^3_B D^B_{1Q} \]
\[ + \frac{1}{8} e^{4\hat{\phi}+3\rho+3\eta} \left( B^P (C_{2A} T^A_{1P} - C_{1R} T^R_{2P} + \frac{1}{2} \theta C_{1P}) \right)^2 \]
\[ + \frac{1}{4} e^{2\hat{\phi}+2\rho+2\eta} g^{11} (\xi^3_A T^A_{1P} B^P)^2. \]

The total potential is now equal to the sum of the contributions (D.5), (D.6) and (D.7), and most of these terms can be recognized immediately in the potential (3.20) obtained from the compactification. The equivalence of the remaining terms can be shown by rewriting the $H^\alpha_{ij}$ in terms of the $\xi^{\alpha x}$ using (2.22), and using the constraints (3.11) on the parameters $T^\alpha_{ij \beta}$. 

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