Lorentz Invariance, Vacuum Energy, and Cosmology

F.R. Klinkhamer
Institute for Theoretical Physics, University of Karlsruhe (TH), 76128 Karlsruhe, Germany

This contribution reviews recent work on a new approach to the cosmological constant problem, which starts from the macroscopic behavior of a conserved relativistic microscopic variable $q$. First, the statics of the vacuum energy density is discussed and, then, the dynamics in a cosmological context.

0. INTRODUCTION

The cosmological constant was introduced by Einstein [1] nearly a century ago and, in the years after, was interpreted as a possibly dynamic vacuum energy density $\varphi$ by Bronstein, Lemaître, and others [2]. The task, then, was to explain the apparently zero value of $\varphi$.

But, thanks to progress in observational cosmology over the last decennium, there are now really three cosmological constant problems (see, e.g., Ref. [3] for one review article out of many):

1. why is $\varphi$ Planck $4 \times 10^{28}$ eV $^4$?
2. why is $\varphi$ 0 ?
3. why is $\varphi$, present $10^{-3}$ eV $^4$ ?

Clearly, we first need to get a handle on the main cosmological constant problem No. 1. Only then can we start worrying about the more delicate problems Nos. 2 and 3.

This contribution gives a brief overview of ongoing work with G.E. Volovik [4,5,6,7]. As the title makes clear, the write-up consists of three parts and, correspondingly, has three conclusions (with the last one split up into Conclusions 3.1 and 3.2). Natural units with $c = \gamma = 1$ are used, except where stated otherwise.

1. LORENTZ INVARIANCE

Lorentz invariance (LI) has been tested by many experiments over the years and verified to ever greater precision (and, hopefully, accuracy). A particularly clean and compelling case occurs with the search of Lorentz-violating (LV) effects in the photonic sector. A priori, there can be $O(1)$ effects in the modified Maxwell action, which maintain gauge invariance, CPT, and renorm alizability. These effects are characterized by 19 real dimensionless parameters and LI holds only if all parameters vanish.

From a variety of methods, it is found that the absolute value of each of the 19 parameters is less than $10^{-15}$, with some even bounded down to the $10^{-32}$ level. This result would seem to suggest that all parameters would be strictly zero, so that local LI would be exact. The fundamental underlying principle would remain to be discovered. For further discussion and references, we refer to a recent review article [8(a)] and an update in these Proceedings [8(b)].

Conclusion 1: Lorentz invariance of the electromagnetic sector in particular has been verified to high precision.

2. VACUUM ENERGY

As mentioned in the Introduction, observational cosmology suggests a nonzero value of the cosmological constant, $\varphi > 0$, or gravitating vacuum energy density, $\varphi = \varphi_r > 0$. But what is the theory behind this result?

First, consider the statics of "dark energy," viewed as vacuum energy. A simple picture [4] has been suggested which is based on the assumption that the perfect quantum vacuum is:
1. a Lorentz-invariant state (cf. Sec. 3);

2. a self-sustained medium at zero external pressure (isolated from the environment);

3. characterized by a new type of conserved charge density \( q \), which is constant over spacetime.

The analog of free energy, charge conservation (\( qV = \text{const} \)), and pressure equilibrium \( (P = P_{\text{ext}}) \) gives an integrated form of the Gibbs (duhem) equation (in this context, rst discussed by Volovik [9]):

\[
e_{V}(q_{0}) + d_{q}(q)_{q=q_{0}} = P_{V}(q_{0}) = P_{\text{ext}} = 0 ;
\]

for the self-tuned equilibrium value \( q_{0} \) of the vacuum variable \( q \) and the microscopic energy density \( (q) \) from the action (the quantity \( d_{q} = dq \) plays the role of a chemical potential). The following remarks are in order:

- the effective energy density \( \varphi(q_{0}) \) is zero by cancelation of two terms which can each be of order \( E_{\text{Planck}} \);
- it can be argued that the effective energy density \( \varphi \) gravitates and not, so that \( \nu = e_{V}(q_{0}) \);
- explicit dynamical models are known [4], which give indeed the result \( \nu = e_{V}(q_{0}) \).

All in all, we have for a perfect LI quantum vacuum in equilibrium [3]

\[
\nu = e_{V} = 1 ; \quad P_{V} = P_{M} = P_{\text{ext}} = 0 ;
\]

with step 1 from thermodynamics and step 2 from pressure equilibrium. Result [4] points toward a possible solution of the cosmological constant problem No. 1, as mentioned in the Introduction.

Up till now, we considered the statics of a perfectly Lorentz-invariant vacuum state. But what happens if the ground state is Lorentz non-invariant (the physical laws are assumed to be exactly Lorentz invariant). For example, in the presence of dark matter (i.e., a Lorentz-non-invariant state), pressure equilibrium gives

\[
P_{V} + P_{M} = P_{\text{ext}} = 0 ;
\]

with the resulting relation \( P_{V} = R_{V} \), the previous chain [2] is then replaced by a new one [4]

\[
\nu = e_{V} = 1 ; \quad R_{V} = P_{M} = w_{M} > 0 ;
\]

for dark matter with energy density \( \omega_{M} \) and equation-of-state parameter \( w_{M} > 0 \). A similar connection between the value of the cosmological constant (vacuum energy density) and that of the averaged energy density of matter in the Universe was already found in the very rst paper on this topic [3]. Qualitatively, result [4] points toward a possible solution of the cosmological constant problem (Nos. 2 and 3) from the Introduction.

Conclusion 2: The gravitating vacuum energy density of a perfect (Lorentz-invariant) quantum vacuum in equilibrium vanishes by the self-tuning of a conserved relativistic variable \( q \) and the gravitating vacuum energy density of a perturbed in perfect (Lorentz-non-invariant) quantum vacuum in equilibrium remains all conserved to the microscopic energy density \( (q) \), namely, proportional to the Lorentz-violating perturbation.

3. COSMOLOGY

In Sec. [2], only the equilibrium situation was considered, which is static by de nition. But what about the vacuum energy density in Hubble’s expanding Universe? This is a difficult question as it concerns the exchange of energy between the deep vacuum (described in part by the microscopic variable \( q \)) and the low-energy degrees of freedom (described by the standard model of elementary particles and general relativity). There is no definite answer yet.

For the moment, two different approaches have been followed:
1. to investigate how, starting far away from equilibrium in a very early phase of the universe, the vacuum may reach an equilibrium state;

2. to investigate how, starting from a postulated equilibrium state at very late times, a universe can arise which resembles our present Universe.

Here, there is only space to sketch the basic ideas and to give the main results.

### 3.1. Vacuum Dynamics

Vacuum dynamics has been studied with a particular realization of the q variable. The idea is to start from the four-fermion field $F(x)$, which is known to have no propagating degrees of freedom in a 4-dimensional spacetime. Writing $F(x)/F(x)$, the scalar $F(x)$ has been found to play the role of our q variable.

A second ingredient for the study of nontrivial vacuum dynamics is the realization that Newton’s constant $G_N$ must be replaced by a gravitational coupling parameter $G$ which depends on the state of the vacuum and thus on the vacuum variable $F$. Such a $G(F)$ dependence can be expected to occur on general grounds. Moreover, a $G(F)$ dependence allows the cosmological constant to change with time, which is otherwise prohibited by the Bianchi identities and energy-momentum conservation.

With simple Ansätze for the microscopic energy density ($F$) and the gravitational coupling parameter $G(F)$, the field equations can be solved to determine the dynamical behavior of the vacuum energy density in the context of a Friedmann-Robertson-Walker (FRW) universe. As always, it is useful to work with dimensionless variables (e.g., a dimensionless comoving time $t$), where the scale is set by the microscopic energy density ($q$) which can be assumed to have a Planckian value. Recall $E_{\text{Planck}} = cG_N^{1/2} = 1.2 \times 10^{18}$ eV and $t_{\text{Planck}} = 5 \times 10^{44}$ s.

The results obtained may be relevant to the early history of the Universe and can be summarized as follows (see also Fig. 1 on page 5 of this contribution):

- the discovery of a mechanism of vacuum energy-density decay, which, starting from a natural Planck-scale value at very early times, leads to the correct order of magnitude for the present cosmological constant;
- the realization that a substantial part of the inferred cold dark matter may come from the oscillating part of the vacuum energy density;
- the identification of the important role of oscillations of the vacuum variable $q$ (with an oscillation period on the order of $t_{\text{Planck}}$), which drive the vacuum energy-density oscillations responsible for the two results;
- the derivation of an $f(R)$ modified-gravity model of a particular type that describes the overall gravitational effects of the rapidly oscillating vacuum variable close to equilibrium.

Conclusion 3.1: The dynamical vacuum variable $q$ allows for the vacuum energy density in a FRW universe to relax from a natural value $E_{\text{Planck}}^{4/3}$ to its equilibrium value $E_V = 0$, corresponding to Minkowski spacetime.

### 3.2. Closed-Universe Model

With the equilibrium physics of the quantum vacuum and the rapid equilibrium approach reasonably well understood (cf. Secs. 3 and 3.1), we now turn to the other side of the coin, namely, the behavior of the model universe at late times, including the present epoch (these times are, of course, huge in microscopic units). Then, the following question arises: is it possible at all to relate equilibrium boundary conditions for $V(t)$ to an expanding universe which matches the observations, even if we are free to choose the type of vacuum energy dynamics, $dV/dt = 0$?

Inspired by the $q$-theory approach, the following purely phenomenological Ansatz has been proposed for the time dependence of the vacuum energy density (and the corresponding energy exchange between vacuum and matter):

$$\frac{dV(t)}{dt} = V_M(t) \times M(t); \quad (5)$$
with a dimensionless function \( t(t) \) of the comoving time \( t \), normalized by \( (t_{eq}) = 1 \), and a new fundamental decay constant \( \nu M > 0 \). For pressureless matter, the density \( \rho \) in \( 3 \) can be interpreted as corresponding to the cold-dark-matter (CDM) energy density from observational cosmology, with the baryonic contribution neglected. Furthermore, the gravitational coupling parameter \( G \) is assumed to equal Newton’s constant \( G_N \). In fact, the model results sketched in the previous subsection give that \( G^{-1}(t)/F(t) \) and that fluctuations around \( G_N \) are, as present, suppressed by a factor \( \nu \_{\text{B, present}} = 10^{6} \); see the two panels with \( f() \) in Fig. 1 and Ref. 3 for further details.

With pressureless matter (e.g., CDM) and vacuum energy density governed by \( 3 \), the idea, now, is to use equilibrium boundary conditions at a coordinate time \( t = t_{eq} = 0 \) and to solve the Einstein equations of a closed FRW universe for time running in the negative direction (see Fig. 2 on page 6 of this contribution). It indeed turns out to be possible to choose an appropriate function \( t(t) \), so that a nearly standard Big Bang phase is recovered close to the time \( t = t_{eq} = 0 \) with vanishing 3-volume of the universe, \( V(t_{eq}) = 2^{2} a(t_{eq})^{3} = 0 \). (Note that the boundary conditions have been set at \( t = 0 \), so that having a Big Bang is a result of the calculation, not an input.) Moreover, it is possible to identify a moment \( t = t_{0} \) in the "life" of this model universe, which resembles the presently observed Universe remarkably well: \( (V_{0} = M_{0}) \), \( (V_{0} + M_{0}) \), and \( (t_{BB}) = 14 \text{ Gyr} \). It is flat from trivial that these more or less reasonable values can be produced at all in our approach.

The main points of this particular toy-m-odel universe can be summarized as follows:

- A Gibbs-Duhem-type boundary condition at \( t = t_{0} \) with finite \( V(t_{eq}) = (1 = 2)^{2} a(t_{eq})^{3} = 0 \) for \( w_{m} = 0 \).
- An accelerating phase with \( a(t) / (1 + \nu M)^{-3} \) for \( w_{m} = 0 \) [the power \( 2 = 3 \) changes to \( 1 = 2 \) for \( w_{m} = 1 = 3 \)].

**Conclusion 3.2**: An "existence proof" has been given for a model universe with both equilibrium boundary conditions and a Big Bang.

### 4. OUTLOOK

In the present contribution, we have reviewed one approach to the statics and dynamics of the quantum vacuum. This entirely new research topic waits for crucial input from:

- experiment (e.g., observational cosmology with a multitude of planned "dark-energy" experiments);
- theory (e.g., the emergent-symmetry approach inspired by condensed-matter physics).

The coming decade promises to be exciting for both particle astrophysics and theoretical physics.

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Figure 1: Flat FRW model universe with ultrarelativistic matter \((w_M = 1)\) and dynamical vacuum energy density \((w_V = 1)\), for initial boundary conditions \((a_0; h_0; f_0; r_{BB}) = (1; 1; 2; 1; 2; 1; 20)\) at \(t = 1\). The \(q\)-type variable \(F\) has been made dimensionless and this dimensionless variable \(f\) is seen to approach an equilibrium value \(f = 1\) via rapid (Planck-scale) oscillations. Further shown are the dimensionless Hubble parameter \(h(t)\) (da/dt)/a for scale factor \(a(t)\) and the dimensionless energy densities \(r_M(t)\) and \(r_V(t)\). The effective parameters \(r_B \) in the middle top-row panel has been set to the value \(3\). The three bottom-row panels display the asymptotic behavior \(f \sim 1/j^2, h \sim 1, r_V \sim 1^2\).
Figure 2: Closed FRW model universe \[ ] with pressureless matter \( (w_M = 0) \) and dynamic vacuum energy density \( (w_V = 1) \), for equilibrium boundary conditions at \( t = t_{eq} = 0 \). The assumed behavior of the vacuum-energy dynamics is given by \[ ] with the function \( (t) \) shown in the left-most panel on the second row and vacuum decay constant \( \rho_{V\text{M}} = 50 \), in units with \( 8G_N = 3 = c = 1 \). The present Universe, with "cosmic acceleration" and an energy density ratio \( \rho_V / \rho_M = 2.75 \), would correspond to this model universe at coordinate time \( t_B = t_{eq} = 0.584 \). The "Big Bang" with 3-sphere radius \( a(t_{BB}) = 0 \) would occur at coordinate time \( t_B = t_{BB} = 0.016 \). For the time interval \( 0s \text{ to } t_0 \), the model universe has nearly standard FLRW behavior, as shown by the three bottom-row panels. For the time interval \( 0s \text{ to } t_0 \), vacuum energy is transferred to the matter sector, so that \( \rho_M(t) \) increases with \( t \), even though the universe expands. For the time interval \( 0s \text{ to } t_0 \), some energy of the matter sector is transferred back to the vacuum and the expansion of the universe slows down to reach the equilibrium radius \( a = 10 \) at \( t = 0 \).