Spin-density induced by electromagnetic wave in two-dimensional electron gas with both Rashba and Dresselhaus spin-orbit couplings

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We consider the magnetic response of a two-dimensional electron gas (2DEG) with both Rashba and Dresselhaus spin-orbit coupling to a microwave excitation. We generalize the results of Ref. [1], where pure Rashba coupling was studied. We observe that the microwave with the in-plane electric field and the out-of-plane magnetic field creates an out-of-plane spin polarization. The effect is more prominent in clean systems with resolved spin-orbit-split subbands. Considered as response to the microwave magnetic field, the spin-orbit contribution to the magnetization far exceeds the usual Zeeman contribution in the clean limit. The effect vanishes when the Rashba and the Dresselhaus couplings have equal strength.

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The spin-orbit effects in semiconductors have been studied for a long time [2, 3, 4]. The discussion was revived in relation to the spin-Hall effect [5, 6] in hope of applying spin-orbit related effects to spintronics [7]. Initially the effect was considered for conductors, e.g., for a 2DEG. Although the attention of the community has now mostly switched to the quantum spin-Hall effect in "spin-Hall insulators" [8, 9], we consider here the spin-Hall related effects in a 2DEG with both Rashba and Dresselhaus spin-orbit couplings.

The promise of the spin-Hall effect is in the possibility of generation of non-equilibrium spin polarization by means of a DC electric field. However, after some discussions it was concluded that in two-dimensional electron gases with the Rashba and the Dresselhaus spin-orbit interactions the spin-Hall effect vanishes for constant and homogeneous electric field [10, 11, 12, 13]. At finite frequencies the spin-Hall effect is non-zero [11].

While for the homogeneous spin-Hall effect (uniform applied electric field) the out-of-plane spin polarization is expected to accumulate only at the edges of the sample [11], there is an alternative possibility that we explore here: To create an out-of-plane inhomogeneous spin density in the bulk in response to a spatially-modulated field. Such bulk accumulation is free from the uncertainties associated with the charge and spin transport near the sample boundaries, and thus may provide an unambiguous method to detect the spin-Hall effect.

Alternatively, one can consider our results as providing the spin response to a long wave (spatially homogeneous), out-of-plane, oscillating magnetic field. According to the Faraday induction law such a field creates spatially inhomogeneous, in-plane electric field which in turn is responsible for the effect.

In this paper we generalize the results of Ref. [1] where the magnetic response of a 2DEG with pure Rashba spin-orbit coupling to microwaves was studied. Here we consider a more general situation where both couplings are present.

The spin-orbit coupling reads

\[ H_{SO} = \alpha_R (-p_x \sigma_y + p_y \sigma_x) + \alpha_D (p_x \sigma_x - p_y \sigma_y), \]

where \( \alpha_R \) and \( \alpha_D \) are the strengths of the Rashba and Dresselhaus spin-orbit couplings respectively. It is convenient to perform a \( \pi/4 \) rotation in both the momentum and the spin spaces. That is \( p'_x = \frac{p_x + p_y}{\sqrt{2}} \), \( p'_y = \frac{p_x - p_y}{\sqrt{2}} \), and \( \sigma'_x = \frac{\sigma_x + \sigma_y}{\sqrt{2}} \), \( \sigma'_y = \frac{-\sigma_x + \sigma_y}{\sqrt{2}} \). Then the spin-orbit coupling reads

\[ H_{SO} = -(\alpha_R + \alpha_D) p'_x \sigma'_y + (\alpha_R - \alpha_D) p'_y \sigma'_x. \]

In what follows we work in the rotated basis and omit the primes.

Introducing the angle \( \phi_\sigma \) via \( p_\sigma = |p| (\cos \phi_\sigma, \sin \phi_\sigma) \) we obtain the energies of the two sub-bands given by

\[ e^\pm(p) = \frac{p^2}{2m^*} \pm \alpha_\sigma(\phi_\sigma)|p|, \]

where \( m^* \) is the electron band mass, \( \alpha_\sigma(\phi_\sigma) \equiv \sqrt{(\alpha_R^2 + \alpha_D^2)(1 + T_\sigma \cos 2\phi_\sigma)} \), and \( T_\sigma \equiv \frac{2\alpha_R \alpha_D}{\alpha_R^2 + \alpha_D^2} \). For purely Rashba (Dresselhaus) coupling \( T_\sigma = 0 \), while in the case of equal coupling strengths \( T_\sigma = 1 \).

We consider a linearly polarized in-plane microwave field \( A = A_0 \exp(iqr - i\Omega t) \), where \( A_0 = A_0(\cos \alpha, \sin \alpha, 0) \) and \( q = q(\sin \alpha, -\cos \alpha, 0) \). The signs are chosen so that for positive \( A_0 \) and \( q \) the vectors \( q, A_0, e_z \) form a right-handed basis. As usual \( E = (i\Omega/c)A \) and \( B = iq \times A \).

**Kinetic equation.** We follow the route of Refs. [11] and use the standard linear response Keldysh technique to determine the dynamics of the charge and spin densities. For introduction into the Keldysh technique see Ref. [12]. In the dirty limit (to be defined below)
this technique leads to diffusion equations for the spin and charge densities \[11, 16\]. We concentrate here on the clean limit although our results are valid also in the dirty limit.

We consider only the s-wave disorder scattering, that is \( V_{\text{disorder}} = \sum_k u_d(r - r_k) \) where \( r_k \) are random locations with the average density \( n_{\text{imp}} \). We employ the linear response, \( H = H_0 + H_1 \), with

\[
H_0 = \frac{\mathbf{p}^2}{2m^*} + \mathbf{n}\mathbf{p} + V_{\text{disorder}} ,
\]

where \( \mathbf{n} = \{-(\alpha_R + \alpha_D)\sigma_y, (\alpha_R - \alpha_D)\sigma_z\} \), and

\[
H_1 = -\frac{e}{2c} \{\mathbf{v}, \mathbf{A}\} + \frac{1}{2} g\mu_B B\mathbf{\sigma} ,
\]

where \( \mathbf{v} \equiv \frac{\mathbf{p}}{m^*} + \mathbf{n} \).

The zeroth order in \( \mathbf{A} \) Green’s functions, \( G_0 \), reflect the standard disorder broadening. We introduce the inverse momentum relaxation time \( \tau^{-1} = 2\pi n_{\text{imp}} a^2 v \), where \( n_{\text{imp}} \) is the density of impurities and \( v = m^*/(2\hbar^2) \) is the electronic density of states per spin (strictly speaking \( v \) is the density of states in absence of the spin-orbit coupling). We obtain

\[
G_0^R = \left( \frac{1}{2} + \frac{1}{2} \frac{\mathbf{n}\mathbf{p}}{\alpha_0|\mathbf{p}|}\right) G_0^{R+} + \left( \frac{1}{2} - \frac{1}{2} \frac{\mathbf{n}\mathbf{p}}{\alpha_0|\mathbf{p}|}\right) G_0^{R-} ,
\]

where \( G_0^{R+} (\mathbf{p}, \omega) \equiv (\omega - \epsilon^\pm (\mathbf{p}) + i(2\pi)^{-1})^{-1} \). In equilibrium \( G_0^R = h(\omega) (G_{1}^R - G_{1}^A) \), where \( h(\omega) \equiv \text{tanh}(\frac{\omega - E_F}{2T}) \).

Within the self-consistent Born approximation we find the linear (in \( \mathbf{A} \)) correction to the Green’s function, \( G_1 \). Having \( G_0 \) we can calculate any single-particle quantity, i.e., density or current. As usual in the linear response theory the Keldysh component \( G_1^{K} \) splits into two parts, \( G_1^{K} = G_1^{K,I} + G_1^{K,II} \). The first part, \( G_1^{K,I} \), corresponds to the retarded (A-R) combinations in the Kubo formula, while \( G_1^{K,II} \) stands for the R-R and A-A combinations \[13\]. The standard Keldysh perturbation theory gives the spin-charge density matrix as \( \rho = \frac{1}{2} n(\mathbf{q}, \Omega) + s(\mathbf{q}, \Omega) \mathbf{\sigma} = \int \frac{d\epsilon}{2\pi} \int \frac{d^2 p}{(2\pi)^2} [\rho G_1^I] = \frac{i}{2} \int \frac{d\epsilon}{2\pi} \int \frac{d^2 p}{(2\pi)^2} (-i G_1^I) \), where \( n(\mathbf{q}, \Omega) \) is the charge density while \( s(\mathbf{q}, \Omega) \mathbf{\sigma} \) is the spin density. It splits as follows \( \rho = \rho^I + \rho^II \), where \( \rho^I = -\frac{i}{2} \int \frac{d\epsilon}{2\pi} \int \frac{d^2 p}{(2\pi)^2} G_1^{K,I} \) and \( \rho^II = -\frac{i}{2} \int \frac{d\epsilon}{2\pi} \int \frac{d^2 p}{(2\pi)^2} G_1^{K,II} \).

Introducing the average spin-orbit band splitting \( \Delta_F \equiv p_F \sqrt{\alpha_R^2 + \alpha_D^2} \), we can define three regimes: (i) “super-clean” \( \tau^{-1} < \Delta_F^2 m^*/p_F^2 \) \( = m^* (\alpha_R^2 + \alpha_D^2) \); (ii) clean \( m^* (\alpha_R^2 + \alpha_D^2) < \tau^{-1} < \Delta_F \); (iii) dirty \( \tau^{-1} > \Delta_F \). Our results below apply both in the clean and in the dirty regimes, but not in the “super-clean” one, i.e., our results apply for \( \tau^{-1} > \Delta_F^2 m^*/p_F^2 \).

We obtain

\[
\frac{(1 - I)}{\tau} \rho^I = i\Omega \rho^I \left[ \frac{e}{2c} \{\mathbf{v}, \mathbf{A}\} + \frac{1}{2} g\mu_B B\mathbf{\sigma} \right] ,
\]

The functional \( \tilde{I} \) is defined as

\[
\tilde{I}[X(\mathbf{p})] =
\frac{1}{m^*\tau} \int \frac{d^2 p}{(2\pi)^2} G_0^R (\mathbf{p} + \mathbf{q}/2, E_F + \Omega/2) \cdot X(\mathbf{p}) \cdot \cdot G_0^A (\mathbf{p} - \mathbf{q}/2, E_F - \Omega/2) ,
\]

while \( I \) is a \( 4 \times 4 \) matrix defined by its action on the 4-vector \( \hat{\rho} \) as \( I \hat{\rho} = \tilde{I} \hat{\rho} \) (we just use the fact that \( \hat{\rho} \) is independent of \( \mathbf{p} \) to represent the functional \( \tilde{I} \hat{\rho} \) as a product of a \( 4 \times 4 \) matrix \( I \) and a vector \( \hat{\rho} \)).

The second contribution to the density, \( \hat{\rho}^II \) is given by

\[
\hat{\rho}^II = \frac{1}{2} g\mu_B B\mathbf{\sigma} .
\]

We allow for arbitrary external frequency \( \Omega \), including \( \Omega > \tau^{-1} \). However, we limit ourselves to the experimentally relevant regime \( \nu \tau |\mathbf{q}| \ll \tau^{-1} \). Recently the spin and charge response functions to the longitudinal fields were calculated for arbitrary values of \( \mathbf{q} \) in Ref. \[17\].

We expand the matrix \( I \) in powers of \( \mathbf{q} \), \( I = I^{(0)} + I^{(1)} + \ldots \). In zeroth order in \( \mathbf{q} \) the matrix \( I \) is diagonal and its elements are given by

\[
I^{(0)}_{00} = \frac{1}{a} , \quad I^{(0)}_{zz} = \frac{a}{\sqrt{R}} ,
\]

\[
I^{(0)}_{xx} = \frac{(Q + D) - b^2 (1 + T_a)}{2a \sqrt{R}} ,
\]

\[
I^{(0)}_{yy} = \frac{(Q - D) - b^2 (1 - T_a)}{2a \sqrt{R}} ,
\]

where \( R \equiv (a^2 + b^2)^2 - b^4 T_a^2 = (a^2 + (1 + T_a)b^2)(a^2 + (1 - T_a)b^2) \), \( D \equiv (a^2 + b^2 - \sqrt{R})/T_a \), and \( Q \equiv (a^2 + b^2 + \sqrt{R}) \). We have introduced \( a \equiv 1 - \sqrt{\tau} \) and \( b \equiv 2\Delta_F \). Analyzing the path of the complex function \( R(\Omega) \) we conclude that for \( \sqrt{R} \) we have to choose the branch-cut along the positive semi-axis \( R > 0 \). Alternatively we can use the usual definition of \( \sqrt{\tau} \) (with the branch-cut along the negative semi-axis) but replace \( \sqrt{R} \to -i\sqrt{-R} \).

For the part linear in \( \mathbf{q} \), \( I^{(1)} \), we obtain the following matrix elements

\[
I^{(1)}_{xx} = -I^{(1)}_{zz} = \frac{p_F |\mathbf{q}| \tau \sin \alpha}{m^*} \frac{1}{a} ,
\]

\[
I^{(1)}_{yy} = -I^{(1)}_{yx} = \frac{p_F |\mathbf{q}| \tau \cos \alpha}{m^*} \frac{1}{a} ,
\]

where

\[
i^{(1)}_{xx} = \frac{-iab \sqrt{1 + \frac{2\Delta_F}{R}}}{a^2 + b^2 (1 + T_a)} ,
\]

\[
i^{(1)}_{yy} = \frac{-iab \sqrt{1 + \frac{2\Delta_F}{R}}}{a^2 + b^2 (1 - T_a)} .
\]
We have neglected terms mixing the charge density with the spin density, since they contain a small parameter $m/(p_F^2 \tau)$ as compared to the spin-spin terms.

Expanding the RHS of Eq. (1) in powers of $q$ up to the terms linear in $q$ and neglecting again the charge density term for the same reason as above we obtain for the orbital term

$$i \Omega \hat{\mathcal{I}} \left[ \frac{\mathbf{e}(\mathbf{v} \mathbf{A})^+}{2e} \right] = C^{(0)}_{E,x} \sigma_x + C^{(0)}_{E,y} \sigma_y + C^{(0)}_{E,z} \sigma_z \quad \text{and} \quad \nu e |\mathbf{E}| \sin \alpha c_{E,x}^{(0)} \sigma_x + \nu e |\mathbf{E}| \cos \alpha c_{E,y}^{(0)} \sigma_y + \nu e |\mathbf{E}| \sin \alpha c_{E,z}^{(1)}(\alpha) \sigma_z ,$$

(11)

where $E = \frac{\Omega}{c} \mathbf{A}$. The dimensionless coefficients in the zeroth order in $q$ contributions are given by

$$c_{E,x}^{(0)} = \frac{b(1+T_\alpha) \sqrt{1-T_\alpha}}{4a} (b^2 - D),$$

$$c_{E,y}^{(0)} = \frac{b(1-T_\alpha) \sqrt{1+T_\alpha}}{4a} (b^2 + D),$$

(12)

while the coefficient in the linear in $q$ contribution is

$$c_{E,z}^{(1)}(\alpha) = i \sqrt{1-T_\alpha^2} \times \left( b^4(1-T_\alpha^2) - a^4 \right) b^2 - (2T_\alpha a^2 b^4 + RD) \cos 2\alpha \right).$$

Finally, the Zeeman term in the RHS of Eq. (1) reads

$$\left[ 1 - (1-i\Omega \tau)I \right] \frac{\nu g \mu_B}{2\tau} \mathbf{B} \sigma = \nu g \mu_B B \sigma = \nu g \mu_B \sum_{\alpha=xyz} \left[ 1 - a I_{\alpha\alpha}^{(0)} \right] B_\alpha \sigma_\alpha .$$

(14)

Note that the magnetic terms ($\propto \mathbf{B} = i\mathbf{q} \times \mathbf{A}$) are already of first order in $q$.

We are now in a position to calculate the spin density. First we obtain the zeroth-order in $q$ contribution. It is given by

$$s_{x}^{(0)} = \frac{\tau}{1-I_{zz}^{(0)}} c_{E,x}^{(0)} = \frac{\nu e E_y}{2p_F} \frac{b(1+T_\alpha) \sqrt{1-T_\alpha}}{2a \sqrt{R}} (b^2 - D) + b^2(1+T_\alpha),$$

$$s_{y}^{(0)} = \frac{\tau}{1-I_{yy}^{(0)}} c_{E,y}^{(0)} = \frac{\nu e E_x}{2p_F} \frac{b(1-T_\alpha) \sqrt{1+T_\alpha}}{2a \sqrt{R}} (b^2 + D) + b^2(1-T_\alpha),$$

$$s_{z}^{(0)} = \frac{\nu e E_z}{2p_F} \frac{b(1+T_\alpha) \sqrt{1-T_\alpha}}{2a \sqrt{R}} (b^2 - D).$$

(15)

This is a generalization of the well known result [2, 4] meaning that there is an in-plane spin polarization "perpendicular" to the applied electric field. We observe that, for $T_\alpha \neq 0$, it is only perpendicular to $E$ if the electric field is along one of the main axes $x$ or $y$.

Next we calculate the first order orbital contribution. We only calculate the $z$ component of the spin density, as it was zero in the zeroth order. We obtain

$$s_{z}^{(1)\text{orbital}} = \frac{\tau}{1-I_{zz}^{(0)}} C_{E,z}^{(1)} = \frac{\nu g \mu_B}{2\tau} \sum_{\alpha=xyz} \left[ 1 - a I_{\alpha\alpha}^{(0)} \right] B_\alpha s_\alpha,$$

(17)

$$= \frac{\nu g \mu_B}{2\tau} \sum_{\alpha=xyz} \left[ 1 - a I_{\alpha\alpha}^{(0)} \right] B_\alpha \sigma_\alpha .$$

Discussion. We observe that both parts of the out-of-plane spin polarization $s_z$, i.e., the orbital part given by Eq. (17) and the Zeeman part given by Eq. (11) can be regarded as linear response to the out-of-plane magnetic field $B_z$. Thus our analysis amounts to a calculation of the susceptibility $\chi(\omega, q)$ so that $s_z = \chi B_z$.

We observe that in the clean limit, i.e., for $\Delta_F > 1^{-1}$, the orbital susceptibility greatly dominates over the Zeeman one. As one can see in Figs. 1 and 2 for experimentally relevant parameters, the susceptibility $\chi$ exceeds the Pauli susceptibility by a factor of order hundreds.

In the vicinity of $T_\alpha = 0$ we reproduce the results of Ref. [1] and the susceptibility is peaked around $\Omega = 2\Delta_F$. For $T_\alpha$ substantially different from zero a double peak structure develops with the positions $\Omega = 2\sqrt{1 \pm I_{zz} \Delta_F}$. Here the pole singularity present at $T_\alpha = 0$ splits into two square-root singularities corresponding to zeros of...
function $R(\Omega)$ at $\Omega = 2\sqrt{1 \pm T_\alpha \Delta_F - i/\tau}$. One or the other peak are emphasized depending on the angle $\alpha$ as seen in Fig. 2.

![Figure 1](image1.png)

**FIG. 1:** Real and imaginary parts of the total (orbital plus Zeeman) spin susceptibility $\chi \equiv s_z/B_z$ for $2\Delta_F \tau = 5$. Solid lines: $T_\alpha = 0.1$, dashed lines: $T_\alpha = 0.5$, dotted lines: $T_\alpha = 0.9$. Parameters assumed as in GaAs: $m_e/m^* \approx 15$, $g = -0.44$. The results are plotted for $\alpha = \pi/4$ which also corresponds to the averaged over $\alpha$ susceptibility.

We obtained our results for a microwave with a given direction of the wave-vector $\mathbf{q}$, i.e., for a given angle $\alpha$. The most obvious way to observe the orbital contribution to the spin susceptibility would be by applying a homogeneous oscillating magnetic field $B_z$, e.g., by putting the sample into a magnetic coil. Such a field corresponds to an equal superposition of plane waves with all possible wave vectors $\mathbf{q}$ laying in the $xy$ plane. To obtain the orbital spin response in this case one should just average over $\alpha$, i.e., substitute $\langle \cos 2\alpha \rangle = 0$ and $\langle \cos^2 \alpha \rangle = \langle \sin^2 \alpha \rangle = 1/2$. This is what we plot in Fig. 1.

Note, that for $T_\alpha = 0$, i.e., for pure Rashba or Dresselhaus coupling, the response is $\alpha$-independent and the averaging brings nothing new. On the other hand, when the two couplings are of comparable strength, the susceptibility strongly depends on $\alpha$ (see Fig. 2) and averaging over $\alpha$ can change the result considerably. At $T_\alpha = 1$ the orbital susceptibility vanishes.

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