A discussion on supersymmetric cosmic strings with gauge-field mixing

C N Ferreira\textsuperscript{1,2,3}, C F Godinho\textsuperscript{2,3} and J A Helayel-Neto\textsuperscript{2,3}

\textsuperscript{1} Departamento de Física, Universidade Federal do Rio de Janeiro, PO Box 68528, 21945-910, Rio de Janeiro, RJ, Brazil
\textsuperscript{2} Centro Brasileiro de Pesquisas Físicas, Rua Dr. Xavier Sigaud 150, Urca 22290-180, Rio de Janeiro, RJ, Brazil
\textsuperscript{3} Grupo de Física Teórica JoséLeite Lopes (GFT), Petrópolis, RJ, Brazil
E-mail: crisnfer@if.ufrj.br, godinho@cbpf.br and helayel@cbpf.br

New Journal of Physics 6 (2004) 58
Received 6 January 2004
Published 10 June 2004
Online at http://www.njp.org/
doi:10.1088/1367-2630/6/1/058

Abstract. Following a stream of investigation on supersymmetric gauge theories with cosmic-string solutions, we contemplate the possibility of building up a $D$ and $F$ term cosmic string by means of a gauge-field mixing in connection with a $U(1) \times U(1)'$ symmetry. The spontaneous break of both gauge symmetry and supersymmetry are thoroughly analysed and the fermion zero modes are worked out. The role of the gauge-field mixing parameter is elucidated in connection with the string configuration that comes out. As an application of the model presented here, we propose the possibility that the supersymmetric cosmic-string yields the production of fermionic charge carriers that may eject, at their late stages, particles that subsequently decay to produce cosmic rays of ultra-high energy. In our work, it turns out that massive supersymmetric fermionic partners may be produced for a SUSY breaking scale in the range $10^{11} - 10^{13}$ GeV, which is compatible with the phenomenology of a gravitino mass at the TeV scale. We also determine the range of the gauge-field mixing parameter, $\alpha$, in connection with the mass scales of the present model.
1. Introduction

Supersymmetry has been of interest to explain a number of problems in cosmology, including observations and astrophysical questions. In this context, supersymmetry may appear to offer a version to accommodate the ‘dark matter’ problem. There is evidence that most of the mass in the Universe is non-luminous and of unknown composition, probably non-baryonic. The supersymmetry framework predicts the existence of new stable elementary particles (neutralinos) having a mass less than a few TeV and weak interactions with ordinary matter. The neutralinos are linear combinations of the SUSY partners of the photon (photino), $Z^0$ and Higgs bosons. If such a weakly interacting massive particle (WIMP) [1] exists, then it has a cosmological abundance such as observed today, and we could therefore account for the dark matter in the Universe.

Another important result is the success of duality in supersymmetric Yang–Mills theories that may, by means of the physics of non-perturbative solutions, such as topological solitons, be more natural to understand than for non-supersymmetric theories. For these reasons, supersymmetric extensions of cosmic-string models are especially important to our understanding of the early Universe.

In cosmology, the common belief is that, at high temperatures, symmetries that are spontaneously broken today have already been exact in the primordial stages of the Universe. During the evolution of the Universe, there were various phase transitions, associated with the chain of spontaneous breakdowns of gauge symmetries. Cosmic strings [2]–[5] are a by-product of a series of symmetry-breaking phase transitions [6], which appear in some GUTs and carry a huge energy density. They may also carry enormous currents [7] and may provide a sensible justification for many astrophysical phenomena, such as the origin of primordial magnetic fields [8], charged vacuum condensates [9] and sources of ultra-high-energy cosmic rays [10]–[13], among others. They could also enforce the possible origin for the seed fluctuation density perturbations that imprint in the cosmic microwave background radiation (CMBR), which became the large-scale structure of the Universe as observed today [14]–[17]. In view of the possibility that supersymmetry was realized in the early Universe and was broken before, or at the same time, as cosmic strings were formed, many recent works investigate cosmic string in connection with a supersymmetric framework in inequivalent form [18]–[20].

In this work, we analyse the possibility that a $D$ and $F$ cosmic string may be generated in a supersymmetric environment. The mixing proposed for the gauge-field kinetic terms has
already been discussed in different contexts [21, 22]. This paper is outlined as follows. In section 2, we start by presenting the model under consideration, as well as some of its basic characteristics. In section 3, we show that the model admits a stable static cosmic-string configuration. The potential that rules the symmetry breaks is analysed and the ground state is read off. We also discuss the mixing terms and the masses of the bosonic excitations. We show that supersymmetry is spontaneously broken in the core. In section 4, we discuss the mixing terms in connection with the masses of the fermionic excitations; with the help of the supersymmetry transformations of the component fields, we compute the fermionic zero modes and tackle the problem of the fermionic charged carriers. Next, in section 5, we propose possible cosmological implications of the model analysed here. Finally, in section 6, our concluding remarks are summarized.

2. Supersymmetric extension of the $U(1) \times U(1)'$ model and the spontaneous breaking

In this section, we set up the superspace and the component-field version of the $U(1) \times U(1)'$ gauge theory in which framework we seek for a cosmic-string solution. (For the algebraic manipulations with the Grassmann-valued spinorial coordinates and fields, we refer to the conventions adopted in [23].) As our working model, we propose the supersymmetric extension of a $U(1) \times U(1)'$-Higgs model described by the superspace Lagrangian:

$$
\mathcal{L} = \Phi_i e^{2\alpha \Phi_i} \Phi_i + \alpha_1 (X^a X_a + \tilde{X}^a \tilde{X}_a) + \alpha_2 (Y^a Y_a + \tilde{Y}^a \tilde{Y}_a) + 2\alpha_3 (X^a Y_a + \tilde{X}^a \tilde{Y}_a) + W(\Phi) + \bar{W}(\Phi) + k D + \bar{k} \bar{D},
$$

(1)

where the mixing parameters, $\alpha_1$, $\alpha_2$ and $\alpha_3$, will be connected with the cosmic string, $i = 1, \ldots, N$, and the superfields appearing herein being defined in the sequel. $D$ and $\bar{D}$ are component fields accommodated in the superfields $A$ and $\mathcal{V}$, respectively.

This model consists of a family of chiral superfields, $\Phi_i$, that read as

$$
\Phi_i(x, \theta) = \phi_i(y) + \sqrt{2} \theta^a \psi_{i,a}(y) + \bar{\theta}^2 F_i(y),
$$

(2)

where $y^\mu = x^\mu + i \theta \sigma^\mu \tilde{\theta}$. The so-called vector superfield, $\mathcal{V}$, in the Wess–Zumino gauge is given by the $\theta$-expansion as follows:

$$
\mathcal{V} = -\theta \sigma_\mu \tilde{\theta} A_\mu^1(x) - i \bar{\theta}^2 \bar{\theta} \lambda_1(x) + i \bar{\theta}^2 \bar{\theta} \lambda_1 + \frac{1}{2} \bar{\theta}^2 \bar{\theta}^2 \bar{D},
$$

(3)

$$
\mathcal{A} = -((\theta \sigma_\mu \tilde{\theta} A_\mu^2(x) + i \bar{\theta}^2 \bar{\theta} \lambda_2(x) - i \bar{\theta}^2 \bar{\theta} \lambda_2 + \frac{1}{2} \bar{\theta}^2 \bar{\theta}^2 D),
$$

(4)

where the gauge-field strength superfields are $X_a = -\frac{1}{4} \bar{D}^2 D_a \mathcal{V}$ and $Y_a = -\frac{1}{4} \bar{D}^2 D_a \mathcal{A}$, with $D_a$ and $\bar{D}_a$ standing for the supersymmetry covariant derivatives.

In component-field form, our complete Lagrangian density is split into a bosonic piece $\mathcal{L}_B$, a fermionic contribution $\mathcal{L}_F$, the Yukawa part $\mathcal{L}_Y$ and the potential $U$:

$$
\mathcal{L} = \mathcal{L}_B + \mathcal{L}_F + \mathcal{L}_Y - U,
$$

(5)
where $\mathcal{L}_B$ and $\mathcal{L}_F$ are, respectively, the bosonic and fermionic Lagrangian densities, $\mathcal{L}_Y$ encompasses all Yukawa-type couplings and $U$ stands for the potential

$$U = \frac{\alpha_1}{2} \tilde{D}^2 + \frac{\alpha_2}{2} D^2 + \alpha_3 D \tilde{D} + \sum_i F_i \bar{F}_i. \quad (6)$$

The Fayet–Illiopoulos $D$ term provides a possible way of spontaneously breaking SUSY [25]. In this case, the superpotential is

$$W = m \Phi_1 \Phi_-, \quad (7)$$

where we replace the label $i = 1, 2$ by $i = +, -$, where $+ \text{ and } -$ refer to the $U(1)$ charges of the superfields. These charges rule the gauge transformations of $\Phi_+$ and $\Phi_-$. At this point, we mention that, by virtue of the mixing between the auxiliary fields $D$ and $\tilde{D}$ along with the Fayet–Illiopoulos (F-I) terms in (1), we are able to attain a superconducting cosmic-string configuration with four-chiral superfields, without the need of introducing an extra neutral matter supermultiplet:

$$D = \frac{1}{\beta} \left[ \frac{\alpha_3}{2} \sum_i q Q_i |\phi_i|^2 + \alpha_3 k - \alpha_1 k \right],$$

$$\tilde{D} = \frac{1}{\beta} \left[ -\frac{\alpha_2}{2} \sum_i q Q_i |\phi_i|^2 + \alpha_3 k - \alpha_2 \tilde{k} \right], \quad (8)$$

$$\bar{F}_i = -\frac{\partial W}{\partial \phi_i} = -m \phi_i,$$

with $\beta = \alpha_1 \alpha_2 - \alpha_3^2$. Now, let us analyse the positivity of the potential. For this, let us write the $D$ and $F$ terms in the Lagrangian as

$$-U = \frac{1}{2} \mathcal{D}' M \mathcal{D} + \frac{1}{2} \mathcal{D}' K + \frac{1}{2} K' \mathcal{D} - \sum_i \bar{F}_i F_i, \quad (9)$$

where

$$\mathcal{D} = \begin{pmatrix} D \\ \tilde{D} \end{pmatrix}, \quad K = \begin{pmatrix} k \\ \tilde{k} \end{pmatrix} \quad \text{and} \quad M = \begin{pmatrix} \alpha_1 & \alpha_3 \\ \alpha_3 & \alpha_2 \end{pmatrix},$$

where the latter is positive-definite. By adopting the re-definition, $\tilde{D} = \mathcal{D} + M^{-1} K$, we find that

$$U = \frac{1}{2} K' M^{-1} K + \sum_i \bar{F}_i F_i. \quad (10)$$

If we consider the $F$ term, and we choose $\tilde{k} \neq 0$ and $k \neq 0$, the potential (6) can be split as

$$U_{cs} = \frac{1}{\beta^2} \left[ \frac{\alpha_2}{8} q^2 (|\phi_+|^2 - |\phi_-|^2)^2 + \left( \beta m^2 + \frac{\alpha_2}{2} q \tilde{k} - \frac{\alpha_3}{2} q k \right) |\phi_+|^2 + \left( \beta m^2 - \frac{\alpha_2}{2} q \tilde{k} + \frac{\alpha_3}{2} q k \right) |\phi_-|^2 + \frac{\alpha_1}{2} k^2 + \frac{\alpha_2}{2} \tilde{k}^2 - \alpha_3 k \tilde{k} \right]. \quad (11)$$
Now, we analyse the delicate issue of gauge symmetry and supersymmetry breakings, and the consequent formation of a cosmic-string configuration. By minimizing the potential of equation (6), we shall focus on the possibility of finding a supersymmetric cosmic string.

For our purposes, we can work with the potential to cosmic string as given below:

\[
U(\phi_+, \phi_-) = \frac{\lambda_{\phi_+}}{4}(|\phi_+|^2 - \eta^2)^2 - f|\phi_+|^2|\phi_-|^2 + \frac{\lambda_{\phi_-}}{4}|\phi_-|^4 + \frac{m_\phi^2}{2}|\phi_-|^2,
\]

(12)

with \(\lambda_{\phi_+} = q^2/2\beta\), \(\lambda_{\phi_-} = q^2/2\beta\), \(f = q^2/4\beta\) and \(m_\phi^2 = 4\alpha qk + 4q^2\eta^2/4\beta\).

A minimum vacuum configuration for a static vortex may be obtained if we have \(\phi_+ = \eta\), \(\phi_- = 0\) and \(\eta^2 = (2k/q)v\) where \(v = \sqrt{1 - (2k/k)(\alpha k - \bar{k}/2)}\) and in potential (11) the \(m^2\) parameter is given by

\[
m^2 = \frac{q\alpha k}{2\beta} \left(1 - \frac{v + \bar{k}/k}{\alpha}\right),
\]

(13)

with \(\alpha_1 = \alpha_2 = 1\) and \(\alpha_3 = \alpha\) with \(\beta = 1 - \alpha^2\).

Another important feature of the cosmic-string configuration is its core. It is described by \(\phi_+ = \phi_- = 0\). The \(U(1)\) gauge symmetry is exact and the \(U(1)'\) gauge symmetry is broken. Supersymmetry is broken in the core.

We note that the cosmic-string exists only for \(m^2 > 0\); then we have \(k > 0\) with \(\bar{k} < (\alpha - v)k\), which can be fine-tuned. But the important feature is that for \(k \neq 0\) and \(\bar{k} \neq 0\) we have the cosmic-string potential with \(D\) and \(F\) term.

Nevertheless, the final comment is that our choice for \(m\) does eliminate the flat directions from our model, which will appear when \(m = 0\) (for a review on flat directions, see [27]). Thus the choice of \(m\) ensures the stability of our string configuration.

With our potential, in spite of the fact that we have two \(U(1)\)s at play, the flat directions disappear whenever the parameter \(m\) is non-vanishing. Usually, the flat directions inherent to supersymmetric section gauge theories appear because a single \(U(1)\)-symmetry factor is involved [27]. Our model, however, is based on two Abelian factors and this is crucial to ensure that, if only \(m\) were non-vanishing, the flat directions problem would be bypassed. Nevertheless, as a final comment we mention that, even in the presence of eventual flat directions, stability would not be jeopardized, since the 1-loop corrections do not change the conditions for SUSY breaking.

3. Bosonic cosmic-string configuration

In this section, we analyse the possibility of obtaining a stable bosonic configuration for a cosmic string in our supersymmetric approach. We notice that a mixing term can only occur when there are two or more field-strength tensors, \(F_{\mu\nu}^1, F_{\mu\nu}^2\). This only arises for Abelian groups. Thus, the simplest gauge model with a mixing in the kinetic terms is a model with gauge group \(U(1) \times U(1)'\). Let us denote the field strengths of the two \(U(1)\) fields by \(F_{\mu\nu}\) and \(H_{\mu\nu}\), respectively.

In the supersymmetric version [21], this mixing appears to be naturally given by

\[
-\frac{1}{4} F_{\mu\nu} F_{\mu\nu}^1 - \frac{1}{4} F_{\mu\nu} F_{\mu\nu}^2 - \frac{\alpha}{2} F_{\mu\nu}^1 F_{\mu\nu}^2.
\]

(14)
The constant $\alpha$ is a physical parameter and cannot be completely scaled away in the presence of interactions, as will be discussed in the next section. Now, we discuss possible string configurations in connection with the choice of basis in field space. We get two possibilities for cosmic strings.

In this section, we consider the case where the kinetic term can be diagonalized by performing the orthogonal transformation,

$$H^\mu = A_1^\mu + \alpha A_2^\mu, \quad A^\mu = \sqrt{1 - \alpha^2} A_2^\mu. \quad (15)$$

The requirement of a positive kinetic energy implies that $|\alpha| < 1$. Although the diagonalization of equation (15) eliminates the mixing term, the effect of this mixing is present in the couplings, after the field redefinitions $A^\mu$ and $H^\mu$ are adopted. This means that the elimination of the mixing has a physical implication: it yields a relative strength between the coupling constants that govern the interactions of $A^\mu$ and $H^\mu$ with matter. This is why, as we have anticipated above, the $\alpha$-parameter cannot be completely absorbed into field reshufflings. We replace these transformations in the bosonic Lagrangian,

$$\mathcal{L}_B = D_\mu^i \phi_i \bar{D}^i_\mu \phi^i - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{4} H_{\mu \nu} H^{\mu \nu} - U, \quad (16)$$

where the gauge-covariant derivatives read as follows:

$$D_\mu^i = \partial_\mu + i \frac{q}{2} Q^i H_\mu - i \frac{\alpha q}{2 \sqrt{\beta}} Q^i A_\mu. \quad (17)$$

The field-strengths are defined as usual by $F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $H_{\mu \nu} = \partial_\mu H_\nu - \partial_\nu H_\mu$ with $A^\mu$ and $H^\mu$ being the gauge fields.

Our ansatz for the supersymmetric generalization of the Nielsen–Olesen string configuration [28] is proposed as follows:

$$\phi_+ = \varphi_+(r) e^{i \theta}, \quad \phi_- = \varphi_-(r) e^{-i \theta}, \quad H_\mu = \frac{2}{q} (H(r) - 1) \delta_\mu^\theta, \quad (18)$$

parametrized in cylindrical coordinates $(t, r, \theta, z)$, where $r \geq 0$ and $0 \leq \theta < 2\pi$, with $H_\mu$ as the gauge field. The boundary conditions for the fields $\varphi_\pm(r)$ and $H(r)$ are the same as those for the ordinary cosmic strings:

$$\varphi_+(r) = \eta, \quad \varphi_-(r) = 0, \quad r \to \infty,$$

$$\varphi_+(r) = 0, \quad \varphi_-(r) = 0, \quad r \to 0,$$

$$H(r) = 0, \quad r \to \infty,$$

$$H(r) = 1, \quad r \to 0, \quad (19)$$

$$A_\mu = 2\sqrt{\beta} A(r) \delta_\mu^Z,$$

$$A(r) = 0, \quad r \to \infty, \quad A(r) = c, \quad r = 0. \quad (20)$$
The Euler–Lagrange equations for the $\phi_\pm$-fields are given by

$$\phi_+'' + \frac{1}{r} \phi_+'' - \phi_+ \left( H^2 + \frac{\alpha^2 q^2}{r^2} A^2 \right) + \lambda_\phi \left( \phi_+^2 - \phi_-^2 \right) = 0,$$

$$\phi_-'' + \frac{1}{r} \phi_-'' + \phi_- \left( H^2 + \frac{\alpha^2 q^2}{r^2} A^2 \right) - \lambda_\phi \left( \phi_+^2 - \phi_-^2 \right) = 0. \quad (22)$$

Now, the equations for the gauge fields read as

$$H'' - \frac{1}{r} H' + q^2 (\phi_+^2 - \phi_-^2) H = 0, \quad (23)$$

$$A'' + \frac{1}{r} A' + \frac{q^2 \alpha^2}{4 \beta} (\phi_+^2 - \phi_-^2) A = 0. \quad (24)$$

We choose the basis given by (15). These particular combinations simplify the discussion of the fermionic current and the breaking of SUSY in the string core. The extra gauge field plays a crucial role in connection with the breaking of the gauge symmetry outside the string core.

4. Fermionic current-carrying cosmic string

At this stage, some highlights on the fermionic configurations are worthwhile. This issue has already been discussed in a previous paper [24]. All we have done in the previous sections concerns the bosonic sector of the $U(1) \times U(1)'$ theory; to introduce the fermionic modes, which have partnership with the vortex configurations, we take advantage from SUSY invariance. By this, we imply that the non-trivial configurations for the fermionic degrees of freedom may be determined by SUSY transformations on the bosonic sector. This procedure is clearly discussed in [19] and we use the same procedure in this paper.

The SUSY transformations of the component fields for the $U(1) \times U(1)'$-model may be found in [25], where the component-field transformations of the matter and gauge supermultiplets are explicitly given. Following the steps presented in [24], where SUSY transformations act on the string bosonic configuration, we get the fermionic zero-mode configurations which turn out to be

$$\psi_\pm a = i \sqrt{2} \sigma_\mu^{\alpha \dot{\alpha}} \epsilon^a D_\mu \phi_\pm + \sqrt{2} \epsilon_a F_\pm^{\alpha \dot{\alpha}}, \quad \xi_a = \sigma_\mu^{\alpha \dot{\alpha}} \epsilon^a H_{\mu \nu} + i \epsilon_a \tilde{D}, \quad (25)$$

$$\lambda_a = \sigma_\mu^{\alpha \dot{\alpha}} \epsilon^a F_{\mu \nu} + i \epsilon_a D. \quad (26)$$

It is worthwhile to note that the SUSY transformations lead to a vector supermultiplet ($\mathcal{V}$) that is no longer in the Wess–Zumino gauge; to reset such a gauge for $\mathcal{V}$, we have to supplement the SUSY transformation by a suitable gauge transformation that has to act on the matter fields as well. All these facts have already been taken into account in the zero-modes of equations (25) and (26).

Now, we come to the question of the mass for the fermionic excitations. The gaugino-mixing fermionic term reads as

$$-i \lambda_1^{\alpha \dot{\alpha}} \sigma_\mu^{\alpha \dot{\alpha}} \partial_\mu \tilde{\lambda}_1^{\dot{\alpha}} - i \lambda_2^{\alpha \dot{\alpha}} \sigma_\mu^{\alpha \dot{\alpha}} \partial_\mu \tilde{\lambda}_2^{\dot{\alpha}} - \lambda_1^{\alpha \dot{\alpha}} \sigma_\mu^{\alpha \dot{\alpha}} \partial_\mu \tilde{\lambda}_2^{\dot{\alpha}} - i \alpha (\lambda_2^{\alpha \dot{\alpha}} \sigma_\mu^{\alpha \dot{\alpha}} \partial_\mu \tilde{\lambda}_1^{\dot{\alpha}}). \quad (27)$$
The field reshufflings below diagonalize the fermion mass matrix:

\[ \xi^a = \lambda^a_1 + \alpha \lambda^a_2, \quad \lambda^a = \sqrt{\beta} \lambda^a_2. \] (28)

With this field basis, we obtain the fermionic Lagrangian

\[ L_F = i \bar{\psi}_i \sigma^\mu D_\mu \psi_i - i \bar{\xi} a a \sigma^\mu \bar{\partial}_\mu \xi a - i \bar{\lambda} a a \sigma^\mu \bar{\partial}_\mu \lambda a \] (29)

with the covariant derivative reading as

\[ D_\mu = \partial_\mu + i \frac{q}{2} Q^I H_\mu - i \frac{\alpha q}{2 \sqrt{\beta}} Q^IA_\mu, \] (30)

and the Yukawa potential

\[ L_Y = -i q \sqrt{2} \phi_i \bar{\psi}_i \left( \bar{\xi} - \alpha \sqrt{\beta} \bar{\lambda} \right) - \phi_i \psi_i \left( \xi - \alpha \sqrt{\beta} \lambda \right). \] (31)

We propose our fermion solutions as given by

\[ \psi_\pm = \psi_i(r, \theta) e^{\zeta(z, t)}, \quad \xi = \xi(r, \theta) e^{-\zeta(z, t)}. \] (32)

Note that the left-moving superconducting current presents the function \( \zeta(z, t) \), which is taken to be the same for all fermions.

The symmetry breaking also modifies the spinor mass terms; in this case, we have, after the breaking, the following fermionic mass term:

\[ L_{\text{mass}} = i q \eta \sqrt{2} \left[ \bar{\psi}_+ \left( \bar{\xi} - \alpha \sqrt{\beta} \bar{\lambda} \right) - \psi_+ \left( \xi - \alpha \sqrt{\beta} \lambda \right) \right]. \] (33)

In fact, we see that only \( \psi_+ \) acquires mass, because \( \phi_- = 0 \) in the vacuum. In this form, this model describes one Goldstone fermion, one scalar and a vector field with mass \( (\alpha / \sqrt{2} \beta) q \eta \). Notice that these masses fit the well-known supersymmetry mass formula for the spontaneously broken case [29].

Before applying to cosmology, we study the currents of fermionic particles. For this, we shall consider our string to be describable by means of a surface action and accordingly integrate the action over the transverse degrees of freedom, i.e. \( (r, \theta) \) coordinates (for a review of the procedures, see [4]). In this case, let us consider by using the symmetry of the problem that \( A_\mu(\vec{x}) = A_a(t) \), where \( a = z, t \). The massless fermions interact with the electromagnetic gauge field in the core. The gauge field, \( A_z \), as we saw, is \( z \)-independent; it couples to the fermionic particles of the matter supermultiplet. The latter are massless in the core and massive outside, as given by equation (33). The analysis of the fermionic current is better carried out in the two-dimensional sheet. There is a current induced in a \( z \)-directed string by a homogeneous electric field, \( E \). The symmetry of the problem suggests that the current \( J^a \) can be also \( z \)-independent. Then,

\[ \frac{d J_z}{d t} = \frac{\alpha^2 q^2}{2 \beta} E_z. \] (34)
So, we can interpret that the electrical field $E_z$ is responsible for the orientation of the charges in the $z$-direction.

5. Possible cosmological implications of a supersymmetric superconducting cosmic string

In this section, we point out that cosmic strings associated with phase transitions in our model can, through their collapse, annihilation, or other processes, be sources for heavy fermions with masses in the range $10^{11}$–$10^{13}$ GeV, whose decay products may be observable in the Universe today.

Recent works have investigated extragalactic $\gamma$-rays and ultra-high-energy cosmic rays in connection with supersymmetric cosmic strings [30]–[32]. In this approach, the latter may be sources of Higgs particles with mass comparable with the (explicit) supersymmetry breaking scale (TeV), and superheavy gauge bosons of mass of order $\eta$, where $\eta$ might be of the order of the GUT scale.

In view of the scenario mentioned above, we propose here that our supersymmetric superconducting cosmic string, for which supersymmetry is spontaneously broken in the core, may be a source of fermionic particles with mass comparable with the spontaneous supersymmetry breaking range $\sim 10^{11}$–$10^{13}$ GeV. The gauge symmetry breaking is usually associated to massive particle production [4]; in our approach, where the string becomes superconducting, the fermionic carriers responsible for superconductivity may play a significant role in the process of massive particle production.

Next we consider fermions that carry charges under both the original $U(1)$ which is spontaneously broken and the electromagnetic $U(1)'$ which remains unbroken. For a particle of charge $q$, indeed, consider the current induced in the $z$-directed string by a homogeneous ($z$-independent) electric field, $E_z$. The string develops an electric current which grows in time,

$$\frac{dJ_z}{dt} \sim q^2 \alpha^2 \beta E_z,$$

where $E_z$ is the component along the string. We analyse the case of fermionic superconductivity in our model; the charged fermions, $\psi_i$, and the neutral fermions, $\lambda$, acquire their mass as a result of gauge symmetry breaking, which is responsible for the string formation; then, they are massive outside, but massless inside the string, as in (33). These fermions are electrically charged, then the strings have massless charged carriers which travel along the string at the speed of light, as given by (35). This current in the core is generated by the supersymmetry breaking scale, and Goldstone fermions appear. The fermion mass outside the string is

$$M_F = g \eta,$$

where $g = q \alpha$ is the Yukawa coupling of the fermion to the Higgs field of the string, given by (33), and $\alpha$ is the fine structure constant that fits the experimental data.

If we consider that loops may be formed by interaction between our long cosmic strings, we have that, with fermionic charge carriers, they are expected to eject, in their late stages,
high-mass particles which subsequently decay to produce ultra-high cosmic rays, neutrino and hadronic radiation (particles inside the string can be thought of as a one-dimensional Fermi gas) [10]. When an electric field is applied, the Fermi momentum grows as \( \frac{dp_F}{dt} = \pi qE \), where \( p_F \) and the number of fermions per unit length [10], \( N^F = Lp_F/\pi \), also grows:

\[
\frac{dN^F}{dt} \sim qEL. \tag{37}
\]

Now, since the constant are compatible, we can use (35) to get

\[
J^F = q \frac{\alpha}{\sqrt{2\beta L}} N^F. \tag{38}
\]

The current grows until it reaches a critical value,

\[
J^F_C \sim q \frac{\alpha}{\sqrt{2\beta}} M_F, \tag{39}
\]

where

\[
p_F = M_F = q \frac{\alpha}{2\beta} \eta. \tag{40}
\]

At this point, particles at the Fermi level have sufficient energy to leave the string. The fermion mass, \( M_F \), does not exceed the supersymmetry breaking scale, which is compatible with gauge symmetry vortex formation around 10^{16} \text{ GeV}. This result is compatible with the supersymmetry breaking in the core and with the fermion solutions given by the supersymmetry transformations (25), i.e., the solutions to \( \psi \) and \( \xi \) fall off to zero at infinity. Hence,

\[
J_C < J_{\text{max}} \sim q \frac{\alpha}{2\beta} M_F. \tag{41}
\]

The value of \( \alpha \) may be expected to be less than 1, with \( q^2 \sim 10^{-2} \), because \( \beta \sim 1 \) [4] and \( M_F \) and \( \eta \) may take values as below:

\[
\begin{align*}
M_F & \sim 10^{13} \text{ GeV}, & g & \sim 10^{-3}, & \alpha & \sim 10^{-2}, \\
M_F & \sim 10^{12} \text{ GeV}, & g & \sim 10^{-4}, & \alpha & \sim 10^{-3}, \\
M_F & \sim 10^{11} \text{ GeV}, & g & \sim 10^{-5}, & \alpha & \sim 10^{-4}.
\end{align*}
\]

It is a major issue to justify, on the basis of first principles, the way our parameters can be chosen with the values displayed above. First of all, the fact that \( M_F \) may be taken in the range 10^{11–10^{13}} \text{ GeV} is supported by our knowledge that, if SUSY is broken and the gravitino mass is to be of the order of TeV, then the SUSY breaking scale lies at the intermediate scale between 10^{11} and 10^{13} \text{ GeV}. As for the \( \beta \)-parameter, the values we have chosen
are compatible to reproduce cosmic-string phenomenology. On the other hand, being SUSY spontaneously (and, therefore, softly) broken, stability against quantum correction legitimates the values proposed in our calculations above. Then, we can state that it is supersymmetry and its power in the renormalization programme that mainly support the choice of our parameters.

Consequently, in this simplified picture, the growth of the current ends at $J_C$ and the string starts producing particles at the rate (37). In our model, we may accommodate the production of extremely energetic photons, quarks and neutrinos through decays mediated by superheavy bosons (typically of GUT-scale mass $10^{16}$ GeV). We may also include the production of heavy fermions with mass in the range $10^{11}$–$10^{13}$ GeV, compatible with spontaneous supersymmetry breaking as discussed in this section.

6. Concluding remarks

The goal of this work is the investigation of the structure of a cosmic string in the framework of a supersymmetric Abelian Higgs model. We showed that it is possible to set up a $U(1) \times U(1)^\prime$-Lagrangian with the property of a fermionic current in the core of the string generated by a $D$ term and the superpotential $W$. Supersymmetry is spontaneously broken in the core, while it is preserved outside the string. We found that the introduction of a mixed kinetic term involving the two Abelian factors appears as the only way to construct the bosonic cosmic string with $D$ and $F$ term.

The propagation of these fermionic excitations should be understood in connection with SUSY breaking and gives us an interesting result on the zero-modes related to the field that are not confined to the string. There are interesting questions that remain to be contemplated, concerning supersymmetric topological defects and their implication to particle physics in a supersymmetric version of the extended standard model, with a very general Lagrangian based on $SU(2) \times U(1) \times U(1)^\prime$[33], whose phenomenological implications have been currently discussed in the literature.

Finally, our discussion on a possible cosmological scenario for supersymmetric cosmic strings with superconductivity opens up a viable way of trying to fit the phenomenology concerning the production of extragalactic $\gamma$-rays and ultra-energetic cosmic rays. In fact, the fermionic current inside the vortex which can give us supermassive particles outside the string, is given by a supersymmetry breaking as justified in the last section; the connection between the supersymmetry spontaneous breaking scales ($\sim 10^{11}$–$10^{13}$ GeV) with massive particles in our model is given by the $\beta$-mixing parameter. These supersymmetric fermionic particles may have been detectable in Fly’s Eye and its successor HiRes, as well as in the Akeno Giant Air Shower Array (AGASA) experiments [34]. Interesting investigation about our model may be done in future works [35] using the fragmentation method for these supermassive particles. The air-showers produced by primaries can be studied from the fragmentation of the X-supersymmetric particles as in [31]. An interesting option is to perform the Feynman expansional operator method [36] that can be extended to include other contributions, such as Earth’s curvature, etc. So, the mechanism discussed in this paper gives us the possibility to obtain these energies if the scale for cosmic-string scale formation is of the order of $10^{16}$ GeV. This issue may offer a very rich set up to probe the details of our model.
Acknowledgments

The authors would like to thank (CNPq, Brazil) for invaluable financial support. C E C Lima is acknowledged for a careful reading of the manuscript.

References

[1] Jungman G, Kamionkowski M and Griest K 1996 Phys. Rep. 267 195
[2] Vilenkin A 1981 Phys. Rev. D 23 852
    Hiscock W A 1985 Phys. Rev. D 31 3288
    GottI II J R 1985 Astrophys. J. 288 422
    Garfinkel D 1985 Phys. Rev. D 32 1323
[3] Hindmarsh M B and Kibble T W B 1995 Rep. Prog. Phys. 58 477
[4] Vilenkin A and Shellard E P S 1994 Cosmic Strings and other Topological Defects (Cambridge: Cambridge University Press)
[5] Kibble T W B 1980 Phys. Rep. 67 183
[6] Kibble T W 1976 J. Phys. A: Math. Gen. 9 1387
[7] Witten E 1985 Nucl. Phys. B 249 557
[8] Vachaspati T 1985 Phys. Lett. B 249 557
[9] Nascimento J R S, Cho I and Vilenkin A 1999 Phys. Rev. D 60 083505
[10] Hill C T, Schramm D N and Walker T P 1987 Phys. Rev. D 36 1007
[11] Bhattacharjee P 1989 Phys. Rev. D 40 3968
[12] Bhattacharjee P 1992 Phys. Rev. Lett. 69 567
[13] Brandenberger R H 1990 Nucl. Phys. B 331 153
[14] Stebbins A 1986 Astrophys. J. Lett. 303 L21
[15] Sato H 1986 Prog. Theor. Phys. 75 1342
[16] Turok N and Brandenberger R H 1986 Phys. Rev. D 33 2175
[17] Bezerra V B and Ferreira C N 2002 Phys. Rev. D 65 084030
[18] Morris J R 1996 Phys. Rev. D 53 2078
[19] Davis S C, Davis A C and Trodden M 1997 Phys. Lett. B 405 257
[20] Edelstein J D, Fuertes W G, Mas J and Guilarte J M 2000 Phys. Rev. D 62 065008
[21] Dienes K R, Kolda C and Russell J M 1997 Nucl. Phys. B 492 104
[22] Foot R and He X G 1991 Phys. Lett. B 267 509
[23] Piguet O and Sibold K 1986 Renormalized Supersymmetry (Progress in Physics vol 12)
[24] Fereira C N, Helayël-Neto J A and Porto M B D S M 2002 Nucl. Phys. B 620 181
[25] Wess J and Bagger J 1992 Supersymmetry and Supergravity 2nd edn (Princeton Series in Physics) (Princeton, NJ: Princeton University Press)
[26] Penin A A, Rubakov V A, Tiyakov P G and Troitsky S V 1996 Phys. Lett. B 389 13
[27] Zumino B 1981 Spontaneous Breaking of Supersymmetry (Lecture Notes in Physics vol 160)
[28] Nielsen H B and Olesen P 1973 Nucl. Phys. B 61 45
[29] Ferrara S, Giradello L and Palumbo F 1979 Phys. Rev. D 20 403
[30] Bhattacharjee P, Shaif Q and Stecker F W 1998 Phys. Rev. Lett. 80 3698
[31] Berezinsky V and Kachelrieß M 1998 Phys. Lett. B 34 61
[32] Berezinsky V, Kachelrieß M and Vilenkin A 1992 Phys. Rev. Lett. 79 4302
[33] Babu K S, Kolda C and March-Russell J 1998 Phys. Rev. D 57 6788
[34] Berezinsky V 1998 Talk presented at XXXIII Rencontres de Moriond: Electroweak Interactions and Unified Theories
[35] Ferreira C N, Lima C E C, Helayël-Neto J A and Portella H M in preparation

New Journal of Physics 6 (2004) 58 (http://www.njp.org/)
[36] Bellandi Filho J et al 1989 Hadron. J. 12 13
Feynman R P 1948 Rev. Mod. Phys. 20 367
Feynman R P 1951 Phys. Rev. D 84 108
Portella H M, Gomes A S, Maldonado R H C and Amato N 1998 J. Phys. A: Math. Gen. 31 6861
Portella H M, Castro F M O and Arata N 1988 J. Phys. G: Nucl. Part. Phys. 14 1157
Portella H M, Oliveira L C S, Lima C E C and Gomes A S 2002 J. Phys. G: Nucl. Part. Phys. 28 415
Gomes A S 1999 PhD Thesis Centro Brasileiro de Pesquisas Físicas
Lima C E C 2003 PhD Thesis Centro Brasileiro de Pesquisas Físicas
Kato T 1966 Perturbation Theory for Linear Operators 1st edn (Berlin: Springer)