Higgs Decay \( H \rightarrow \gamma\gamma \) through a \( W \) Loop: Difficulty with Dimensional Regularization

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Abstract

Since the photon has no mass, it does not couple directly to the Higgs particle. This implies that the one-loop correction to the decay \( H \rightarrow \gamma\gamma \) is necessarily finite. Therefore, this correction should be calculable without introducing either regularization or ghosts. Such a calculation is carried out in this paper for the case of one \( W \) loop. The result obtained this way turns out \textit{not to} agree with the previous, well-known one, and it is argued that the present result is to be preferred because it satisfies the decoupling theorem. The discrepancy can be traced to the use of dimensional regularization in the previous result.

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1. From the experimental data of the Large Electron-Positron colliding accelerator (LEP) at CERN \cite{1, 2}, there was first possible evidence for the Higgs particle \cite{3} at a mass of about 115 GeV/c^2. It will be important for the Large Hadron Collider (LHC) to either confirm or contradict this first possible evidence.

If the Higgs particle has indeed a mass around this value, then, at the LHC, one of the good ways to detect this particle is through the decay

\[ H \rightarrow \gamma\gamma \]  

because experimentally this decay mode can be seen cleanly.

Within the standard model of Glashow, Weinberg, and Salam \cite{4}, since the Higgs particle couples most strongly to heavy particles, this decay \cite{1} proceeds predominantly through a top loop and a W loop. The contribution from one top loop was first obtained by Rizzo \cite{5}; it is the purpose of the present paper to study the contribution from one W loop.

This decay \cite{1} through one W loop was already studied many years ago \cite{6, 7, 8}. A different point of view is taken here, and our result does not agree with the earlier one. A comparison of the results and the reason for discrepancy is to be given in Sec. 9.

2. In the Lagrangian of the standard model \cite{4}, there is no coupling of the Higgs particle to the photons. It therefore follows that the one-loop contribution to the decay \cite{1} must be finite. This fact then implies that no regularization of any sort is necessary for this calculation; in particular, dimensional regularization is not needed.

In the present paper, we take the attitude that concepts that are not necessary for the present calculation are to be avoided. As a first application of this point of view, throughout the present calculation,

\[ \text{the space-time dimension} = 4. \]  

In particular, there will be no non-integer dimensions.

Let this point of view be take one step further. The perturbative calculation for the decay \cite{1} through one W loop is to be carried out in the most straightforward way. What this means is that the present work is to be carried out in the unitary gauge.

This point requires some discussion. It is known that the unitary gauge has many desirable properties, but it has the major handicap of very serious difficulties with renormalization. Why? On the one hand, because of gauge invariance, the sum of the contributions from the various diagrams, properly interpreted, does not depend on the gauge, and is, in particular, the same for the unitary gauge and the renormalizable gauges, provided all the external lines are on mass shell. On the other hand, for the purpose of renormalization, the various diagrams cannot be treated together. It is the separate treatment of the various diagrams that makes the unitary gauge unsuitable for renormalization.

For the present problem of the W loop contribution to the Higgs decay \cite{1}, the amplitude, as discussed above, is convergent, and hence there is no need to be concerned with renormalization. Accordingly, there is no fundamental difficulty to carry out the theoretical considerations in the unitary gauge.
\[
\frac{i}{p^2 - M_W^2 + i\epsilon} \left[ -g^{\alpha\beta} + \frac{p^\alpha p^\beta}{M_W^2} \right].
\]

Figure 1: The Feynman rule for the \( W \) propagator in the unitary gauge with \( M_W \) the \( W \) mass.

![Diagram](image1)

(a) \( M_1 \)  
(b) \( M_2 \)  
(c) \( M_3 \)

Figure 2: The three diagrams in the unitary gauge for the decay \( H \rightarrow \gamma\gamma \) via a \( W \) loop.

3. We proceed to calculate, in the unitary gauge, the decay \( (1) \) through one \( W \) loop. Since calculations in the unitary gauge are not entirely familiar to everybody, the two salient features are to be reviewed in the present section.

The first salient feature is that, in the unitary gauge, there is no ghost of any sort; there are only the physical particles. For the present problem, since the only internal particle is the \( W \), the relevant Feynman rules consist of the \( W \) propagator, the \( HWW \) vertex, the \( WW\gamma \) vertex, and the \( WW\gamma\gamma \) vertex; the \( W \) propagator is shown in Fig. 1 the vertices being the same in all gauges. For the decay \( (1) \) through one \( W \) loop, the Feynman rules lead to only three diagrams, which are shown in Fig. 2.

The second salient feature is more complicated. Because of the \( W \) propagator of Fig. 1
each of the three diagrams of Fig. 2 is highly divergent, in fact quartically divergent. For any integral that is linearly divergent or worse, it is not allowed to shift the variable of integration. Since the quantity of interest is the sum of the contributions from the three diagrams of Fig. 2, this means that the choices of the momentum variables for the three diagrams are inter-related. Therefore, in order to use the unitary gauge, it is necessary to solve first the non-trivial problem of the proper choice of the momentum variable for the three diagrams of Fig. 2.

Issues of choosing the momentum variable are well known in quantum field theory.

(a) The best known case occurs in the context of the Ward identity in quantum electrodynamics [9]. This Ward identity gives a relation between the electron self-energy and the $ee\gamma$ vertex, and plays an important role in disentangling overlap divergences. The diagrammatic verification of the Ward identity requires that the momentum to be differentiated is taken along the electron line, i.e., the external momentum of the electron self-energy is routed through the diagram following the electron line.

(b) In order to disentangle overlap divergences in quantum electrodynamics, it is necessary to treat the photon self-energy in a similar way. For the photon self-energy, there is no longer a similar obvious routing for the external photon momentum. This problem was solved by Mills and Yang [10]: instead of a unique routing of the external photon momentum, the choice of the routing for one diagram constrains the allowed routing for other diagrams. The situation for the photon self-energy in quantum electrodynamics is therefore similar to that of the present problem, where the routings of the external momenta are inter-related for the three diagrams of Fig. 2.

(c) Such problems of the routings of the external momenta are in no way limited to quantum electrodynamics. For example, they are also present for the scalar $\phi^4$ theory [11]. In fact, for the present problem, there is actually more similarity to this $\phi^4$ theory.

4. Let the above general considerations be applied to the present specific problem of calculating the matrix element for the decay (1) through a $W$ loop. In other words, the momenta for the internal lines of the three diagrams of Fig. 2 are to be specified using the unitary gauge.

The starting point consists of the following four obvious but important observations.

(a) The diagrams of Fig. 2(a) and Fig. 2(c) can be obtained from each other by exchanging the two external photon lines. It is therefore sufficient, for the present purpose, to concentrate first on, say, the diagram of Fig. 2(a).

(b) Under the same exchange of the external photon lines, the diagram of Fig. 2(b) remains unchanged.

(c) The diagram of Fig. 2(b) can be obtained from that of Fig. 2(a) by “shrinking” the vertical $W$ line connecting the two external photon vertices.

(d) Let $k$ be the $W$ loop momentum to be integrated over, then the sign of this $k$, i.e., whether $k$ or $-k$ is used, is arbitrary.

Because of (b), it is convenient, for the study of the momentum assignments in the unitary gauge, to begin with the diagram of Fig. 2(b). In this case, the external momenta $k_1$ and $k_2$ must be routed symmetrically between the two internal $W$ lines, thus leading to the routing
Figure 3: The routing of the external momenta $k_1$ and $k_2$ for the diagram of Fig. 2(b).

Figure 4: The three diagrams in the unitary gauge for the decay $H \rightarrow \gamma \gamma$ via a $W$ loop.

In this way, the momentum assignment of Fig. 3(b) is obtained. Note that there is nothing that specifies whether $k$ or $-k$ should be used; see (d) above.

Because of (c), it is natural to route the external photon momenta so that the upper
internal $W$ line of Fig. 2(a) and Fig. 2(b) carry the same momentum. This leads to the momentum assignment of the upper internal $W$ line as shown in Fig. 4(a). By momentum conservation, the assignment of Fig. 4(a) is obtained. Finally, by (a), the assignment of Fig. 4(c) follows.

It is interesting to find that this momentum assignment is identical to that used for the scalar $\phi^4$ theory many years ago \cite{11}.

Once the internal momenta are specified as shown in Fig. 4, it is straightforward to write down the corresponding integrands for the matrix elements from these three diagrams. Because of the quartic divergences, it is not allowed to integrate them separately. Rather, these integrands have to be added together first and then integrated.

Let the integrands for these three diagrams be called $I_1$, $I_2$, and $I_3$. They are to be evaluated with the external lines on mass shell; in particular $(k_1 + k_2)^2 = M_H^2$, where $M_H$ is the Higgs mass.

The matrix element for the decay $H \rightarrow \gamma\gamma$ via a $W$ loop is given by

$$
\mathcal{M} = ie^2 g M_W \int \frac{d^4k}{(2\pi)^4} I,
$$

with

$$
I = I_1 + I_2 + I_3 + I_4,
$$

where

$$
I_4 = -(I_1 + I_2 + I_3) \bigg|_{k_1=k_2=0},
$$

is the Dyson subtraction \cite{12}. In eq. (3), $e$ is the electric charge and $g$ the SU(2) electroweak coupling constant.

5. As seen from Fig. 4, the $W$ propagator consists of two terms. For large momenta $p$, the first term is of the order $p^{-2}$, while the second term is of the order of $p^0$. This behavior of the second term is a feature of the unitary gauge, and is the underlying reason why the contributions of each of the three diagrams in Fig. 2 are separately so highly divergent, as already discussed in Sec. 3.

Roughly speaking, the more times the second term in the $W$ propagator is used, the more divergent is the $k$ integration of this term. Such highly divergent terms are expected to be canceled if the different $I$’s are summed first, as indicated by eq. (4). For the diagrams of Fig. 4(a) and Fig. 4(c), this second term may be used three times because there are three $W$ propagators. However, these terms are actually zero.

Next, for each of the three diagrams of Fig. 4, this second term of the $W$ propagator may be used twice. In the cases of Fig. 4(a) and Fig. 4(c), the two $W$ propagators so chosen must be those connected to the external Higgs vertex; otherwise, the contribution is again zero. These terms, with the second term used in two $W$ propagators, may be conveniently referred to as the $M_W^{-4}$ terms. These $M_W^{-4}$ terms from the three diagrams of Fig. 4 cancel each other, provided that the various internal momenta are chosen as indicated in this Fig. 4. See especially the consequence of this point (c) as discussed in Sec. 4. With these $M_W^{-6}$ (which
is 0) and the $M_W^{-4}$ terms taken care of, the sum $I_1 + I_2 + I_3$ can be written in the following form:

$$I_1 + I_2 + I_3 = I'_1 + I'_2 + I'_3,$$

where

$$I'_2 = -3 \, g_{\mu\nu} \, M_W^2 \left( \frac{1}{D_1 D_2 D_3} + \frac{1}{D_1 D'_2 D_3} \right)$$

and where the quantities $D$ are the denominators of the $W$ propagators.

In eq. (6), the quantities $I'_1$ and $I'_3$ are related by

$$I'_3 = I'_1 \bigg|_{k_1 \leftrightarrow k_2, \mu \leftrightarrow \nu};$$

it is convenient to separate $I'_1$ into $M_W^{-2}$ and $M_W^0$ terms as follows:

$$I'_1 = I''^{(2)} + I''^{(0)},$$

where

$$I''^{(2)} = \frac{1}{M_W^2 \, D_1 \, D_2 \, D_3}$$

$$\times \left\{ g_{\mu\nu} \left[ - \left( \left( k + \frac{k_1 + k_2}{2} \right) \cdot \left( k - \frac{k_1 + k_2}{2} \right) \right) \left( k - \frac{k_1 - k_2}{2} \right)^2 \right. 
+ \left. \left( k + \frac{k_1 + k_2}{2} \right)^2 \left( k - \frac{k_1 + k_2}{2} \right)^2 \right] 
+ 2 \left[ - \left( k + \frac{k_1 + k_2}{2} \right)^2 \left( k - \frac{k_2}{2} \right)\mu \left( k - \frac{k_1}{2} \right)\nu - \left( k - \frac{k_1 + k_2}{2} \right)^2 \left( k + \frac{k_2}{2} \right)\mu \left( k + \frac{k_1}{2} \right)\nu 
+ 4 \left( \left( k + \frac{k_1 + k_2}{2} \right) \cdot \left( k - \frac{k_1 + k_2}{2} \right) \right) \left( k + \frac{k_2}{2} \right)\mu \left( k - \frac{k_1}{2} \right)\nu \right] \right\}$$

and $I''^{(0)}$ consists of terms without a factor of $M_W^2$ in the denominator.

Even though the right-hand side of eq. (11) contains an overall factor of $M_W^{-2}$ while $I''^{(0)}$ does not, the splitting as given by eq. (10) for $I'_1$ is not quite what is needed. The underlying reason for this complication is that the integral

$$\int d^4 k \, I''^{(2)}$$

is linearly, not logarithmically, divergent. This point is to be discussed in the next section, Sec. 6.

6. Since linearly divergent integrals such as (12) are tricky to deal with, it is highly desirable to rewrite the $I''^{(2)}$ of eq. (11) in such a way that the integral (12) becomes logarithmically divergent. The basic idea of such rewriting is to average the integral under the change of sign $k \rightarrow -k$; see (d) of Sec. 4.
From eq. (11), the quantity $I'(2)M^2_W D_1 D_2 D_3$ contains the term cubic in $k$
\[ \left( k + \frac{k_1 + k_2}{2} \right) \cdot \left( k - \frac{k_1 + k_2}{2} \right) \left\{ g_{\mu\nu} \left[ k \cdot (k_1 - k_2) \right] + 2 \left( k_{2\mu} k_{\nu} - k_{\mu} k_{1\nu} \right) \right\} ; \quad (13) \]
this is the term responsible for the linear divergence mentioned above in Sec. 5.

The first factor in the bracket can be written as the sum of three terms: (a) $D_2 = (k - (k_1 - k_2)/2)^2 - M^2_W$; (b) $M^2_W$; and (c) $(k \cdot (k_1 - k_2))$. Of these three terms, (a) integrates to zero because the cancelation of the factor $D_2$ makes the integrand odd under $k \leftrightarrow -k$; (b) has an overall factor of $M^2_W$ and is to be combined with the $I'^{(0)}$ terms, this combination being called $I^{(0)}$, while (c) remains, leading to a logarithmically divergent integral. The result is therefore
\[ I'_1 = I^{(2)} + I^{(0)} + \text{terms that are odd under } k \rightarrow -k , \quad (14) \]
where
\[ I^{(2)} = \frac{1}{M^2_W D_1 D_2 D_3} \left\{ g_{\mu\nu} \left[ 2 k^2 (k_1 \cdot k_2) - 4 (k \cdot k_1) (k \cdot k_2) \right] + 2 \left[ -2 (k_1 \cdot k_2) k_{\mu} k_{\nu} + 2 (k \cdot k_2) k_{\mu} k_{1\nu} + 2 (k \cdot k_1) k_{2\mu} k_{\nu} - k^2 k_{2\mu} k_{1\nu} \right] \right\} ; \quad (15) \]
and
\[ I^{(0)} = \frac{1}{D_1 D_2 D_3} \left\{ g_{\mu\nu} \left[ 6 k_1 \cdot k_2 + 3 (k - \frac{k_1 - k_2}{2})^2 \right] + \left[ -12 (k + \frac{k_2}{2}) \mu (k - \frac{k_1}{2}) \nu - 6 k_{2\mu} k_{1\nu} \right] \right\} . \quad (16) \]

7. Consider first the quantity $I^{(2)}$ as given by eq. (15). Since this $I^{(2)}$ is to be integrated over $d^4k$, it is convenient to use the Feynman parameters, called $\alpha_1$, $\alpha_2$, and $\alpha_3$ for the three denominators $D_1$, $D_2$, and $D_3$ respectively. Thus, the appropriate change of variables is
\[ \ell = k + \frac{1}{2} [(\alpha_1 - \alpha_2 - \alpha_3) k_1 + (\alpha_1 + \alpha_2 - \alpha_3) k_2] . \quad (17) \]
From eq. (15), the numerator for $I^{(2)}$ is
\[ I^{(2)} M^2_W D_1 D_2 D_3 , \]
which should first be expressed as a polynomial in $\ell$, and then only the part even in $\ell$ is kept. This even part is given as
\[ g_{\mu\nu} \left[ 2 \ell^2 (k_1 \cdot k_2) - 4 (\ell \cdot k_1) (\ell \cdot k_2) \right] + 2 \left[ -2 (k_1 \cdot k_2) \ell_{\mu} \ell_{\nu} + 2 (\ell \cdot k_2) \ell_{\mu} k_{1\nu} + 2 (\ell \cdot k_1) k_{2\mu} \ell_{\nu} - \ell^2 k_{2\mu} k_{1\nu} \right] . \quad (18) \]
It is interesting to note that this expression is independent of the Feynman parameters \( \alpha_1, \alpha_2, \) and \( \alpha_3. \)

At this point, symmetric integration may be applied so that

\[
\ell_\alpha \ell_\beta \rightarrow \frac{1}{4} g_{\alpha\beta} \ell^2.
\]  

(19)

It should be emphasized that the factor on the right-hand side is \( \frac{1}{4} \), because the entire calculation is carried out in four dimensions, as expressed explicitly by eq. (2).

Application of this (19) for symmetric integration then leads to the nice result that

the numerator (18) \( \rightarrow 0 \).

(20)

In other words, the \( I^{(2)} \) as given by eq. (15) gives zero when integrated over \( d^4k. \)

The consequence of this simplification is that, of the various terms on the right-hand side of eq. (14), only one — \( I^{(0)} \) — gives a non-zero contribution when integrated over \( d^4k. \)

8. The rest of the calculation is completely straightforward although still somewhat tedious. The result of the present considerations using the unitary gauge is

\[
\mathcal{M} = -\frac{e^2 g}{8\pi^2 M_W} [k_{2\mu} k_{1\nu} - g_{\mu\nu} (k_1 \cdot k_2)] [3 \tau^{-1} + 3 (2 \tau^{-1} - \tau^{-2}) f(\tau)],
\]  

(21)

where

\[
\tau = \frac{M_W^2}{4 M_W^2}
\]  

(22)

and

\[
f(\tau) = \begin{cases} 
    \sin^{-1} \sqrt{\tau} & \text{for } \tau \leq 1, \\
    -\frac{1}{4} \left[ \ln \frac{1 + \sqrt{1 - \tau^{-1}}}{1 - \sqrt{1 - \tau^{-1}}} - i\pi \right]^2 & \text{for } \tau > 1.
\end{cases}
\]  

(23)

9. The most interesting and unexpected aspect of the present result as given by eq. (21) for the matrix element of the decay \( H \rightarrow \gamma\gamma \) through a \( W \) loop is that it disagrees with the previous one \([6, 7, 8]\). The previous result is

\[
\mathcal{M} = -\frac{e^2 g}{8\pi^2 M_W} [k_{2\mu} k_{1\nu} - g_{\mu\nu} (k_1 \cdot k_2)] [2 + 3 \tau^{-1} + 3 (2 \tau^{-1} - \tau^{-2}) f(\tau)],
\]  

(24)

The present answer (21) differs from this (24) by the term 2 in the second bracket. This implies that, under no circumstance, the two answers can agree exactly.

What is the basic difference between the present derivation of our result (21) and the previous one of (23)? As already mentioned in Sec. 2, the philosophy or the point of view for the present derivation is based on the following two related points:
(a) Since the Higgs particle does not couple directly to the massless photon, the decay $H \rightarrow \gamma\gamma$ to one-loop order must be finite. Under this circumstance, there should be straightforward calculation for the present process of this Higgs decay through a $W$ loop.

(b) For such a finite calculation, there is no reason to introduce complications such as regularization of any sort and/or the use of various ghost particles.

What has been carried out in this paper is just such a straightforward calculation. This is to be contrasted with the derivation of the previous result (24), where both dimensional regularization and ghosts are used [6, 7, 8].

When two results are derived on the basis of very different points of view, it is often difficult to pinpoint the reason for the different results. However, in the present case, there is one eminently reasonable explanation for the difference, and this difference is likely to have far-reaching consequences.

In Sec. 7, the conclusion is reached that $\int d^4 k I^{(2)}$ is zero. If dimensional regularization is used, then the integral should instead be $\int d^n k I^{(2)}$. With the dimension being $n$ instead of 4, eq. (19) must be replaced by

$$\ell_\alpha \ell_\beta \rightarrow \frac{1}{n} g_{\alpha\beta} \ell^2,$$

and, consequently, (20) is instead

$$\text{the numerator of } (18) \rightarrow 2 \left(1 - \frac{4}{n}\right) \ell^2 \left( g_{\mu\nu} (k_1 \cdot k_2) - k_{2\mu} k_{1\nu} \right),$$

which is not zero when $n \neq 4$.

This factor of $(4 - n)$ on the right-hand side of (26) is canceled by another factor of $1/(4 - n)$ coming from the $n$-dimensional integration over $d^n k$, leaving a finite answer instead of zero. Therefore, when dimensional regularization is used, this $I^{(2)}$ contributes a finite term instead of the zero of Sec. 7. Indeed, this contribution is precisely the term 2, which is the difference between the previous result for the decay $H \rightarrow \gamma\gamma$ via a $W$ loop and the present one.

As emphasized in (b) above, there is no justification to use dimensional regularization for the present problem. This is the first theoretical reason why the present result is to be preferred. This theoretical reason is supported by the second independent one as follows.

This is connected to the decoupling theorem [13]. In the present context of the Higgs decay, decoupling refers to the phenomenon that the decay $H \rightarrow \gamma\gamma$ becomes weaker and weaker when the Higgs mass increases without bound. While this “decoupling theorem” is not really a theorem in the sense that its validity has been established beyond doubt, it does make good physical sense. Within the present context, it is known that the decoupling theorem does hold for the decay $H \rightarrow \gamma\gamma$ through a top loop [5].

A major qualitative difference between (21) and (24) is that the present result (21) does satisfy the decoupling theorem while the previous result (24) does not. Thus there are two
independent theoretical arguments that the present formula \((21)\) for the decay matrix element of \(H \to \gamma\gamma\) through a \(W\) loop is to be preferred.

10. It only remains to add a few important comments.

(a) Even though the present study concerns mostly with triangle diagrams, there are significant differences from the anomalies, for example the Adler-Bell-Jackiw anomaly \([14]\). While the effect of this ABJ anomaly is limited entirely to the lowest-order diagrams, the present disagreement with the earlier result propagates to higher orders, including diagrams with a \(H\gamma\gamma\) vertex insertion. Thus, the present result should not be called an anomaly.

(b) In the pioneering paper of Ellis, Gaillard, and Nanopoulos \([6]\), the Higgs mass was considered to be small, as generally believed at that time. Compared with their result, in this limit, the present result is smaller by a factor of \(5/7\). For larger Higgs mass \([6,7,8]\), this ratio first increases but eventually decreases in absolute value to approach zero, consistent with the present result satisfying the decoupling theorem \([13]\).

(c) It was pointed out in Sec. 4 that, for this lowest-order one \(W\) loop diagram, the routing of the external momenta is identical to that used for the scalar \(\phi^4\) theory \([11]\). It is likely that this relation also holds for higher-order diagrams. Such a relation would be very interesting because there are a number of unanswered problems in this Ref. \([11]\), and the question may be raised whether the relation, if it indeed holds to higher orders, is of help in studying these open problems.

(d) The purpose of this paper is to give a correct and straightforward calculation of the matrix element for the decay \(H \to \gamma\gamma\) through one \(W\) loop. In a more general context, the significance of the present paper is probably in pointing out that dimensional regularization must be used with care: continuity in the space-time dimensionality is not always true.

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