Superconducting islands, phase fluctuations
and the superconductor-insulator transition

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Abstract

Properties of disordered thin films are discussed based on the viewpoint that superconducting islands are formed in the system. These lead to superconducting correlations confined in space, which are known to form spontaneously in thin films. Application of a perpendicular magnetic field can drive the system from the superconducting state (characterized by phase-rigidity between the sample edges) to an insulating state in which there are no phase-correlations between the edges of the system. On the insulating side the existence of superconducting islands leads to a non-monotonic magnetoresistance. Several other features seen in experiment are explained.

\textit{Key words:} Disordered superconductors, phase, fluctuations, superconductor, insulator, transition

1 Introduction

Superconductivity (SC) in disordered thin films has been a subject of intense study for more than a decade \cite{1}. Nevertheless, even elucidation of one of the most fundamental property of these systems, namely the superconductor-insulator transition (SIT), remains a puzzle. Specifically, it is still unclear whether this is a truly quantum phase-transition, what is the role of the magnetic field in the transition, and even its universality class remains undetermined. Another profound feature which is still in debate is the non-monotonic magneto-resistance (MR), which in some systems can reach several orders of magnitude \cite{3}. The non-monotonic MR is accompanied by a unique temperature, magnetic field and disorder dependence of the resistance \cite{3\textit{a}}. On
the theoretical front, the adequacy of treating the system in terms of strictly bosonic excitations (so called ”dirty boson” models [2]) is still questionable.

We adopt a perspective within which the system is composed of SC islands, a structure implying local SC correlations. The notion of SC islands appeared nearly a decade ago in two contexts. Galitski and Larkin [5] suggested that in a strongly disordered system the SC order parameter \( \langle \text{and hence also } T_c \text{ and } H_c \rangle \) fluctuate in space. As a result, at a given temperature and magnetic field (close to \( H_c \) of the clean system) there will be areas in the sample where the local critical field exceeds the external field. Therefore these domains still display SC correlations. Ghosal et.al. [6] used a locally self-consistent solution of the Bogoliubov-de-Gennes equations to show that even in the presence of extremely strong disorder there remain regions in space where the SC order parameter is finite, surrounded by regions of vanishingly small order parameter (and hence dubbed SC islands).

As it is shown below, the concept of SC islands might explain numerous experimental observations. In section 2 we describe the nature of the SIT as it emerges from the existence of SC islands with fluctuating phases. In section 3 we show how SC islands lead to non-monotonic MR and explain its dependence on temperature and magnetic field. Section 4 is devoted to a summary and outlook.

2 From Superconducting islands to the superconductor – insulator transition

In this section we describe the nature of the SIT, based on our previous investigations [7]. On the SC side of the transition, the system has a complex SC order parameter, and its amplitude strongly fluctuates in space. When a perpendicular magnetic field is turned on and then monotonically increased, it begins to penetrate the sample in the form of disordered vortices [8]. As a consequence, there are regions in which the SC order parameter vanishes, and SC islands are formed. However, these islands are still interconnected via a Josephson coupling (JC), and hence their phases are well correlated. This means that Cooper-pairs can coherently traverse through the sample and carry super-current, i.e. the system is in a macroscopic SC state.

When the JC between two islands becomes smaller than the temperature, thermal phase fluctuations overcome the phase-locking induced by the JC, and the islands become separate (in terms of SC wave functions). Now, as the magnetic field is increased the JC between the different islands decreases. However, as long as there is a phase-stiff path between the edges of the sample (that is a path of islands with JC larger than temperature) the system will
maintain its SC nature. At a certain magnetic field $B_c$, the phase-stiff path is broken and the system is no longer in a SC state. Since it is a highly disordered two-dimensional system, one expects that it will become insulating, as indeed observed.

An immediate consequence of the above scenario is that SC correlations still survive on the insulating side of the transition. This idea is corroborated by a number of experiments. For instance, measurements of the AC conductance \[9\] show that the superfluid stiffness remains finite well above the transition. Another example is magnetic-field induced conductance oscillations with $2\phi_0$ period observed in a sample with a fabricated nano-hole lattice \[10\], indicating the presence of Cooper pairs.

To support the above picture we have performed numerical calculations of phase correlations between the two edges of a highly disordered superconducting thin film. We used a Monte-Carlo scheme \[11\] on-top of a self-consistent solution of the Bogoliubov-de-Gennes mean-field equations to obtain both the order parameter amplitude and phase, and calculated the correlation function $F_{LR} = \langle \cos(\delta \theta_i - \delta \theta_j) \rangle$ (where $\delta \theta_i$ is the shift of the order parameter phase from its mean-field value) as a function of magnetic field (in terms of the flux per plaquette, $\phi/\phi_0$). Here $i(j)$ are lattice sites on the left (right) edge of the sample. Details of the calculations are described elsewhere \[7\]. In Fig. 1 we show $F_{LR}$ (diamonds) as a function of magnetic field (or flux per plaquette) for a single disorder realization of a system of size $20 \times 5$, with strong disorder $W/t = 1.4$ (where $t$ is the hopping integral), attractive interaction strength $U/t = 2$, average electron density (per site) $\langle n \rangle = 0.92$ and temperature $T = 0.04$. In addition we plot the average value of the SC order parameter amplitude $|\Delta|$ (triangles), for the same magnetic field. As can be seen, at a certain (disorder and density dependent) magnetic field the phase correlation vanish, and hence this value corresponds to $B_c$. However, the order parameter amplitude is finite beyond this point and vanishes only at higher fields. This can also be seen from the density of states (DOS), plotted in the inset of Fig. 1 for two values of the magnetic field. The left inset displays the results for vanishing magnetic field, where a clear BCS-like DOS is observed. The right inset pertains to magnetic field of $\phi/\phi_0 = 0.06$, that is above the SIT. However, since there are still SC correlations a pseudogap develops. This may indicate that our picture is relevant also to the physics of high Tc superconductors \[12\]. We point that similar results were obtained from tunneling measurements \[13\], where the broadened BCS peak was accounted for by SC order parameter amplitude fluctuations.

Although the transition is described in terms of thermal fluctuations, this description does not rule out the possibility that the transition is quantum in nature. All one needs to do is to replace thermal fluctuations with quantum fluctuations and temperature with an energy scale associated with quantum
Fig. 1. Phase correlations between the two sides of the system $F_{LR}$ (diamonds) and the SC order parameter amplitude $|\Delta|$ (triangles) as a function of magnetic field. At a certain magnetic field $B_c$, phase-correlations vanish, indicating the SIT. However, $|\Delta|$ is still finite after this point, indicating the existence of SC islands.

Insets: the density of states for vanishing magnetic field (left inset) and just above the transition (right inset), demonstrating the appearance of a "pseudogap" above the transition.

fluctuations. In fact, recent measurements [9,14] indicate that the transition crosses smoothly from a thermal to a quantum phase transition, the phenomenology of the two classes being very similar.

3 Superconducting islands and non-monotonic magneto-resistance

In a recent set of experiments, Sambandamurthy et.al. [3] showed that the magneto-resistance is non-monotonic above the SIT and develops a peak at a certain magnetic field $B_{\text{max}}$. Surprisingly, the resistance at the peak can be several orders of magnitude larger than the resistance at the transition (which is found not to be universal, see e.g. [15,16]). Perhaps even more surprising is the fact that with increasing the magnetic field beyond $B_{\text{max}}$ the resistance drops several orders of magnitude and comes close to its value at the SIT. While non-monotonicity in the MR was observed more than a decade ago [17], there it was a miniscule effect. Here, however, this effect is huge and cannot be overlooked. In addition, the resistance develops an activation dependence on temperature, which vanishes at low enough temperatures [3]. An experimental study of MR for different values of disorder [4] shows that with increasing disorder the value of the resistance at the MR peak increases, and at the same time the values of both $B_c$ and $B_{\text{max}}$ decreases.

How all these phenomena can be explained in terms of the formation of SC islands [18] is discussed below. The model is based on three assumptions. The first is that disorder induces the formation of SC islands due to fluctuations in the amplitude of the SC order parameter. The second assumption is that as the magnetic field is increased, the concentration and size of these SC islands
decrease. This does not mean, however, that the physical size of the islands (that is, the spatial extent to which the order parameter amplitude is finite) decreases (although it might), but rather that islands become disconnected in the sense of phase-fluctuations described in the previous section. The third assumption is that the SC islands have a charging energy, and thus, a Cooper pair entering a SC island (via an Andreev tunneling process) has to overcome it. This charging energy is expected to be inversely proportional to the island size, and thus to increase with increasing magnetic field. All three assumptions have been corroborated by numerical calculations described elsewhere [8].

Consider now such a system in the strong magnetic field regime, \( B >> B_{\text{max}} \). Due to the strong magnetic field the SC islands are small and have a large charging energy. Now the electrons can traverse through the system in two types of trajectories: those which follow normal areas of the sample and do not cross the SC islands ("normal paths") and those in which an electron tunnels into a SC island via the Andreev channel ("island paths"). The resistance of the normal paths, \( R_N \), has some value (which may depend on e.g. length, temperature, disorder etc.) and is assumed to be weakly affected by magnetic field. Due to Coulomb blockade, transport through the "island paths" is thermally activated, and hence the resistance of the island paths is of the form \( R_I \sim \exp(E_c/T) \), where \( E_c \) is the typical charging energy of the island. If \( E_c \) is large then the main contribution to the conductance is due to transport along the normal paths. Consistent with experiment, the MR in this regime is small.

As the magnetic field is decreased (but still in the regime \( B > B_{\text{max}} \)), more SC islands are created and their size increases, but they are still small enough such that transport along normal paths is favourable. However, some paths which were normal at higher fields now become island paths and hence unavailable for electron transport. Thus, the effective density of normal electrons which contribute to the resistance diminishes, resulting in a negative MR. Eventually, at a certain magnetic field \( B = B_{\text{max}} \) some SC islands are large enough so that their charging energy is small and the resistance through them is comparable to the resistance through normal paths, i.e. \( R_N \approx R_I \). At this point the resistance reaches its maximum value, since as the magnetic field is further decreased the SC islands are so large that transport through them is always preferred over transport through normal paths. An increase in number and size of the SC islands will thus result in a decrease in the resistance. At the critical field \( B_c \) a phase-stiff path percolates through the system, resulting in the SIT.

The above model was encoded into a numerical calculation using a lattice network model of normal resistors, SC resistors and insulating resistors (to mimic the Coulomb blockade) [18]. Resistance of a normal resistor connecting two sites on the lattice is given by \( R_{ij} = R_0 \exp \left( \frac{2r_{ij}}{\xi_{\text{loc}}} + \frac{|e_i| + |e_j| + |e_i - e_j|}{2kT} \right) \), where \( R_0 \) is a constant, \( r_{ij} \) is the distance between sites \( i \) and \( j \), \( \xi_{\text{loc}} \) is the localization
length, $e_i$ is the energy of the $i$-th site measured from the chemical potential (taken from a uniform distribution $[-W/2, W/2]$) and $T$ is the temperature. The resistance between two (neighboring) SC sites is taken to be very small compared with that of a normal resistor, but still not zero and temperature dependent, in such a way that it vanishes as $T \to 0$ (distant SC sites are disconnected). The resistance between a normal site and a SC site (i.e. an insulating resistor) is taken to be $R_{NS} \propto \exp(E_c/kT)$, where $E_c$ is the charging energy of the island.

In Fig. 2 we plot the resistance as a function of the concentration of the SC islands (which corresponds to the magnetic field) for different values of temperature. The quantitative resemblance to the experimental data (inset of Fig. 2) is self-evident. In addition, this simple phenomenological theory is capable of accounting for the breakdown of activation behaviour at low temperatures, the dependence of disorder and other experimental observations.

![Fig. 2. Magneto-resistance as a function of concentration of SC islands for different temperatures (taken from Ref. [18]), showing good quantitative resemblance between the theory and experimental data (inset, taken from Ref. [3]) is evident.](image)

4 Summary and outlook

In this contribution we have demonstrated that many of the properties of disorder SC thin films may be accounted for by the formation of SC islands. Specifically, we have demonstrated that phase-fluctuations between different islands lead to the SIT, and that above the transition formation of SC islands leads to a non-monotonic magneto-resistance.

Still, there are many puzzles left in elucidating the physics of these systems. For instance, an unusual disorder and magnetic-field dependent anisotropy in the magneto-resistance was recently measured [19] and is up-to-date unexplained, although it seems that a percolation theory similar to that presented here may account for it [20]. Even more challenging is the appearance of a seemingly
correlated [21] and perhaps metallic state at high magnetic field [22]. A final example is the fact that some materials seem to exhibit a metallic intermediate state at the SIT while others do not [23]. These and other questions are still waiting to be resolved.

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