Linear wave dynamics explains observations attributed to dark-solitons in a polariton quantum fluid

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We investigate the propagation and scattering of polaritons in a planar GaAs microcavity in the linear regime under resonant excitation. The propagation of the coherent polariton wave across an extended defect creates phase and intensity patterns with identical qualitative features previously attributed to dark and half-dark solitons of polaritons. We demonstrate that these features are observed for negligible nonlinearity (i.e. polariton-polariton interaction) and are therefore not sufficient to identify dark- and half-dark solitons. A linear model based on the Maxwell equations is shown to reproduce the experimental observations.

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Solitons are solitary waves that preserve their shape while propagating through a dispersive medium [1, 2] due to the compensation of the dispersion-induced broadening by the nonlinearity of the medium [3]. Over the years, spatial solitons have been observed by employing a variety of nonlinearities ranging from Kerr nonlinear media [4] to photorefractive [5] and quadratic [6] materials. Apart from their potential application in optical communications [7, 8], solitons are important features of interacting Bose-Einstein condensates (BECs) and superfluids. The nonlinear properties of BEC can give rise to the formation of quantized interacting vortices and solitons, the latter resulting from the cancellation of the dispersion by interactions, for example in atomic condensates. A special class of solitons are the so-called dark solitons, which feature a density node accompanied by a π phase jump. Since the first theoretical prediction in the context of Bose-Einstein condensates (BECs) [9], dark solitons were studied and observed first in the field of nonlinear optics [10] and then in cold-atom BECs [11]. The experimental observation of BEC [12] and superfluidity [13, 14] of exciton-polaritons, has sparked interest in the quantum-hydrodynamic properties of polariton fluids. In particular, the nucleation of solitary waves in the wake of an obstacle (i.e. defect) has been claimed recently [15–21]. Here, the source of nonlinearity, essential for the formation of such a solitary wave, has been identified in the repulsive polariton-polariton interactions [15–21]. In these previous works, the observation of dark notches in the intensity profiles together with a π shift in the phase have been used as sufficient signatures for dark solitons in microcavities. In addition, half-dark solitons have been found to carry a non-zero degree of circular polarization in presence of the TE–TM splitting of the cavity mode [20, 21].

In this Letter we demonstrate that these features previously used as dark-soliton fingerprints [15–21] can also be observed without the presence of nonlinearity, which is the fundamental ingredient differentiating solitons from linear wave propagation. Specifically we investigate the propagation of polaritons with a small exciton fraction and at low polariton densities, excluding a relevant influence of nonlinearities. We show that polariton propagation in this linear regime across an extended defect can create deep notches in the intensity profile accom-

Figure 1: Experimental (a),(b) and simulated (c),(d) real space intensity and interference patterns showing the two “soliton fingerprints” generated by the scattering of a beam with a point-like defect: a dark notch in the intensity pattern together with π phase dislocations. In the images the polaritons propagate downwards, along the y-axis, and are injected with a wavevector of 1.5 μm−1. Color scales are given.

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panied by a $\pi$ phase shift. We model the observation using linear wave propagation, clarifying that these features are not indicative for a nonlinear interaction between polaritons, but are interference patterns created by scattering from the defect. Moreover, we show that the appearance/disappearance of these features for different in-plane kinetic energies is reproduced in the linear regime and thus does not provide evidence of an interacting quantum fluid. Therefore, the previous reports of the observation of dark-solitons \cite{15-19} and half-dark-solitons \cite{20, 21} which were based on these features have to be reconsidered.

The investigated sample is a bulk $\lambda$ GaAs microcavity surrounded by 27 (top) and 24 (bottom) distributed GaAs/AlAs Bragg reflector pairs. The sample is held in a cold-finger cryostat at a temperature of 15 K and is illuminated by a narrow linewidth single-mode continuous wave Ti:Sapphire laser, tuned to the resonance of the cavity at about 1.485 eV. The measurements were performed in transmission configuration. The phase was measured using a shearing Mach-Zehnder interferometer (see Supplementary Information, S1). Our experiments were performed in the linear regime, facilitated by the large negative detuning of $-29$ meV of the cavity photon mode from the exciton resonance at $1.514$ eV, resulting in a small exciton fraction of the polariton of about 1%. To verify the linear regime, we studied the excitation density dependence of our results (see Supplementary Information, S3). We find that they are independent of the excitation density over a range of four orders of magnitude, and they persist at an exciton density as low as $1.4 \times 10^5$ cm$^{-3}$, which is 6 orders of magnitude lower than the nonlinearity threshold of bulk GaAs \cite{22, 23}. On the other hand, the density of polaritons, calculated from the detected photons transmitted through the sample, is $2.7 \times 10^3$ cm$^{-2}$, 5 orders of magnitude lower than the lasing threshold observed in standard microcavities \cite{24}. The real space intensity and interference of a polariton wave propagating across a defect are shown in Fig. 1. The experimental results show the presence of two dark notches in the intensity pattern along with a $\pi$ phase shift visible in Fig. 1(b) as paths of vortices merging in succession with alternating topological charge $\pm 1$. Simulations of the measurements using the realistic experimental parameters are shown in Fig. 1(c) and Fig. 1(d).

Solitons are predicted to appear in polariton microcavities as the result of the nonlinearity due to the polariton-polariton interactions \cite{15}. Since our experiments are in the linear regime, it is important to understand how the nature and the size of the defect affects the formation process of these soliton-like features. In a recent study \cite{25} of the structural and optical properties of GaAs/AlAs microcavities grown by molecular beam epitaxy it was shown that the most common point-like defects (PD) were characterized by a circular or elliptical shape \cite{26}, due to Gallium droplets emitted occasionally during the growth \cite{27, 28}. The presence of the defect has the effect of modifying the effective thickness of the cavity layer, which typically results in an attractive potential for the cavity mode inside the defect \cite{26}. Consequently, the wavevector of the photonic mode in the region of the defect is higher than in the rest of the cavity. The polariton scattering by the defect depends on the wave vector mismatch between the polaritons outside and inside the defect at the energy of excitation. When the energy shift of the defect photon mode with respect to the unperturbed cavity mode is large enough to make the coupling between them inefficient, the defect behaves like a hard scatterer and the spatial intensity distribution is similar to the complementary case of a single-slit diffraction \cite{29}. In our case, however, there is a finite transmission through the defect, producing dark and bright traces with a more complicated phase pattern. As it has been shown by Berry et al. \cite{30, 31}, wavefronts of waves resulting from interference can contain dislocation lines. In the case of a scattered beam, dislocations are composed of phase shifts at positions where the amplitude of the electromagnetic wave and thus the intensity vanishes, representing nodes of the wave. It is worth mentioning that nonlinearities are negligible close to nodes also in the nonlinear regime, and phase dislocations at zero intensity (i.e. at the dark notches) are features of both linear \cite{32, 33} and nonlinear waves.

Beyond the qualitative discussion above, we performed simulations of the experiments, based on a numerical solution of the linear scattering problem using the classical theory of electromagnetism. The choice of such a
model is justified by the fact that we operate in the linear regime and with a small exciton fraction of about 1%, such that the polariton dispersion is dominated by the cavity mode. In the model, we consider the propagation of quasi two-dimensional photons with a parabolic dispersion in a cavity with a fixed width. The incident wave has been treated as coming from a linearly polarized point-like source with polarization in the plane of the cavity. Defects have been modeled as disk-shaped perturbations of the cavity thickness resulting in an energy shift of the photon dispersion (see supplementary information, Fig. S2). To model the defect parameters, which are not experimentally known, we use a disk shape with a radius of 3 μm and a polariton potential of −2.3 meV (in agreement to Ref. 26). Maxwell’s equations are then solved using expansion of the fields into the planar cavity eigenmodes in cylindrical coordinates fulfilling the boundary conditions for tangent component of electric and magnetic field on the interface between the cavity and the defect (see Supplementary Information S2). This linear wave dynamics model reproduces the intensity notch and the phase dislocation previously used as dark-soliton fingerprints. The results show a marked dependence on the geometry of the scattering problem, as shown in the Supplementary Information S4. In particular, the phase jump visible in the interference pattern depends on the direction of the incoming polariton wave relative to the defect (see Fig. S4). On the other hand, the size of the defect relative to the polariton wavelength affects the formation of high-order phase dislocations (see Fig. S5).

In a nonlinear cavity-polariton system, a polariton fluid has been predicted [15] to flow almost unperturbed around the defect (i.e. disappearance of the features) or experience the nucleation of vortices and/or solitons at the position of the defect (i.e. appearance of the features), depending on the excitation density. We evaluated the possibility of observing these features, ascribed in the literature to dark-solitons resulting from the interaction within the polariton fluid, in the absence of non-linearities. Fig. 2 (a) and (b) show the phase and the intensity of soliton-like fingerprints in real space. Instead of increasing the excitation power, which has no effect in the linear regime, we tune the energy of the excitation beam, and observe the appearance and disappearance of soliton-like features. As discussed above, the appearance of the intensity minima and phase dislocations is a result of interference which is sensitive to the intensity and relative phase of contributing waves. The increase of the energy of the excitation beam by 2 meV causes an increase of the in-plane wave vector of the propagating polariton mode that, in turn, changes the interference condition so that the straight dark notches [Fig. 2 (c)] and the phase dislocations [Fig. 2 (d)] disappear. Intensity profiles measured at a fixed distance from the defect [Fig. 2 (e) and (f)] confirm the observed transition. Thus it becomes apparent that the appearance/disappearance of soliton-like features, although independent of the excitation density, strongly depends on the wave vector of the propagating mode. It is worth noting that in the case of polariton condensates, the increase of the excitation density corresponds to an energy blue shift of the condensate that, consequently, results in an increase of the in-plane wave vector in the lower density region behind the defect (i.e. the number of fringes increases with the excitation density [16]). Specifically in non-resonantly excited experiments [34], this blueshift is dominated by the exciton density in the reservoir at high

Figure 3: Experimental intensity pattern (a-b) and real space interference (c-d) showing two half-soliton features as indicated by the arrows. The blue and red arrows indicate respectively the position of the σ− and σ+ soliton-features: a dark-notch with an associated phase jump present in only one circular component. The green vertical line is a guide for eyes to distinguish the two different regions while the dashed circle in (a) indicates the defect. (e) The intensity profile extracts from the yellow dotted line displaying the two dark notches present respectively in only one of the opposite polarization basis, as indicated by the arrows.
wavevectors. The interaction with the exciton reservoir is not a polariton-polariton interaction within the condensate which could provide the non-linearity needed for the formation of solitons, but instead represents an external potential sculpting the polariton energy and gain landscape.

In a different experiment, we address the observation of half-soliton fingerprints, which requires polarization-resolved measurements. The intensity images [Fig. 3(a) and (b)] are measured using an excitation linearly polarized parallel to the $y$-direction. The interferograms [Fig. 3(c) and (d)], are obtained by selecting the same polarization for the excitation and reference beam (see section S1 for details). The signature of an oblique dark half-soliton (ODHS) is a notch in only one circular polarization component [20, 21]. We excite the sample with a linearly polarized beam and detect the two circular polarization components ($\sigma_-$, $\sigma_+$) separately. The measurements are performed with the same excitation energy (1.485 eV) and negative detuning ($-29$ meV) as in the previous case. The measured intensity and the interferogram for the $\sigma_-$ component are given in Fig. 3(a) and Fig. 3(c) respectively. The images show the presence of a $\sigma_-$ soliton fingerprint, indicated by the blue arrows, that is absent in the $\sigma_+$ component [Fig. 3 (b), (d)]. The same applies to the $\sigma_+$ counterpart, where a half-soliton fingerprint is observed only on the right side of the image. By calculating the degree of circular polarization, given by $S_c = (I_{\sigma_-} - I_{\sigma_+}) / (I_{\sigma_-} + I_{\sigma_+})$, with $I_{\sigma_+}$ and $I_{\sigma_-}$ being the measured intensities of the two components, we measure the pseudospin state inside the cavity [Fig. 4]. Here, if we look at the same position where the soliton features have been observed [Fig. 3], indicated by the black dotted lines in Fig. 4 (a), we note the presence of a pair of oblique traces with opposite circular polarization, resembling the predictions and observations attributed to a polariton superfluid [20, 21]. The high degree of circular polarization that we observe is due to the polarization splitting of transverse electric and transverse magnetic optical modes (TE-TM splitting) [35] (see Supplementary Information S5). The latter gives rise to the optical spin Hall effect [36] that has been observed in both polariton [37] and purely photonic microcavities [38]. In our simulations [Fig. 4 (b)] a linearly polarized incoming beam propagates along the $y$-direction and is scattered by a defect positioned at $25 \mu m$ away from the excitation spot, inducing the formation of two traces propagating in oblique directions. The detected field is a superposition of the incoming linearly polarized wave and the scattered wave. The TE-TM splitting of the optical mode in a photonic cavity is responsible for an anisotropy in the polarization flux, as previously shown on the same sample in Ref. [38]. Here the same values of the TE-TM splitting have been used to perform the simulations. The polaritons scatter from the defect with wave vectors of equal modulus but in different directions both in the real and momentum space. Because of the birefringence induced by the TE-TM splitting, polaritons propagating in different directions experience different polarization rotation and shift. Polaritons traveling to the right gain a $\sigma_+$ component while polaritons traveling to the left gain a $\sigma_-$ component. The anisotropy of the effect manifests itself in the intensity pattern, where it is possible to observe the features of an oblique soliton in one circular component and not in the other.

In conclusion, we have shown that the previously reported experimental signatures of oblique dark-solitons and half-solitons in polariton condensates can be observed in the case of polaritons propagating in the linear regime. We find that these features are the result of the interference of the incoming wave with the waves scattered by the defect. In the case of the polarized counterpart (i.e. half-soliton-like features) the intrinsic TE-TM splitting of the cavity dispersion gives rise to oblique straight traces with opposite polarization.

Our results clarify that phase vortex lines in polariton propagation together with dark notches in the intensity patterns, used as fingerprints of oblique-dark solitons and half-solitons in the literature, are present in the linear propagation regime. Consequently, these features are necessary but not sufficient evidence to identify solitons.

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**Supplementary Information**

**I. EXPERIMENTAL SETUP**

The investigated sample is a $\lambda$ bulk GaAs microcavity surrounded by 27 (top) and 24 (bottom) distributed GaAs/AlAs Bragg reflector pairs. The sample is held in a cold-finger cryostat at a temperature of 15 K and is illuminated by a narrow linewidth ($\approx 30$ kHz) single-mode continuous wave Ti:Sapphire laser, tuned to the resonance of the cavity (1.485 eV).

![Mach-Zehnder interferometer](image)

**Figure 5:** Sketch of the Mach-Zehnder interferometer used in the experiments. List of the optical components: BS$_1$ and BS$_2$ are non-polarizing beam splitters; M$_1$ and M$_2$ are mirrors; Obj$_1$ is the excitation objective with a 20x magnification and 0.4 numerical aperture; Obj$_2$ is the objective used for collection, with 10x magnification and 0.25 numerical aperture; L$_1$a, L$_1$b and L$_2$ are convex lenses; $\lambda/4$ and LP$_2$ are respectively the quarter-wave plate and the linear polarizer used to measure the circular Stokes parameters while $\lambda/2$ e LP$_1$ are respectively a half-wave plate and a linear polarizer used to control the excitation power. The optical elements enclosed in dashed box ($\lambda/4$ and LP$_2$) are introduced in the setup only in the case of polarization measurements (i.e. half-soliton like features) corresponding to Fig. 3 (a-d) of the manuscript.

The data reported in the manuscript have been acquired by using the experimental setup represented in Fig. 5. The optical elements enclosed in the dashed box ($\lambda/4$ and LP$_2$) are introduced in the setup only in the case of polarization measurements (i.e. half-soliton like features) corresponding to Fig. 3 (a-d) of the manuscript. The intensity measurements have been performed in transmission configuration, by blocking Arm 2 of the interferometer. The phase, on the other hand, is acquired using both Arm 1 and Arm 2. In this arrangement the setup correspond to a Mach-Zehnder interferometer, in which a laser is split into two arms: one is used to excite the sample (Arm 1), while the other with a flat phase is used as the reference (Arm 2). The excitation beam (Arm 1) is focused by a 0.4 NA microscope objective to a 3 $\mu$m diameter spot on the sample, resulting in a circular distribution in $k$-space with a diameter of 3 $\mu$m$^{-1}$. The light transmitted through the sample is then collected using a 0.25 NA microscope objective and focused on a CCD (charge-coupled device) camera by a convex lens ($L_2$). The reference beam (Arm 2) is expanded by a telescope (formed by $L_{1a}$ and $L_{1b}$ in Fig. 5) so that a bigger area of the sample could be investigated and then interfered with the transmitted beam on the CCD camera. The incidence angle of the reference beam was adjusted in order to obtain interference fringes along $y$. The power of the excitation beam is adjusted by means of a half-wave plate ($\lambda/2$) and a linear polarizer (LP$_1$).

**Polarization measurements.** The investigation of half-soliton features, shown in Fig. 3 (a-d) of the manuscript, requires a polarization-resolved measurement. To this end we use a linear polarizer (LP$_1$) to prepare the excitation beam in the linearly polarized basis (parallel to the $y$-axis in Fig. 3 of the manuscript) and we introduce in the setup a polarimeter composed by a quarter-wave plate ($\lambda/4$) and a linear-polarizer (LP$_2$), oriented at 45° with respect to one another, to measure the circular Stokes parameter of the transmitted signal. In this way, by rotating the wave-plate it is possible to select the component of the Stokes parameter of which one wants to measure the relative intensity. Then, using...
with the measured intensities $I_{\sigma_+}$ and $I_{\sigma_-}$ of the two circularly polarized components, we calculate the circular component of the Stokes vector (Fig. 4 (a) of the manuscript).

Fig. 6 shows a sketch of the linear wave dynamics in the $x$-$y$-plane of the microcavity. The polaritons propagate along the positive direction of the $y$-axis and are scattered by a circular defect giving rise phase singularities at points where the amplitude vanishes, i.e. at the dark notches of the intensity profile. The total detected polariton field is given by the interference of the incoming wave and the scattered wave.

Figure 6: Sketch of the linear wave dynamics in the $x$-$y$-plane of the microcavity.

II. THEORY FOR THE CAVITY MODE SCATTERING BY A POINT DEFECT

The classical theory of electromagnetism is used in order to calculate the distribution of electric and magnetic fields inside the cavity in the presence of a disk-shaped defect and illumination of the cavity by a monochromatic laser beam. As already mentioned in the manuscript, the choice of such a model is justified by the fact that we operate in the linear regime with a polariton dispersion dominated by the cavity mode. In the model, we consider the propagation of two-dimensional photons with a quadratic dispersion in the microcavity plane, as shown in Fig. 7. A quadratic dispersion is found for all planar microcavity polaritons for small in-plane momenta plane. In our case, the large negative detuning provides a large range over which the dispersion is to a good approximation quadratic, covering all the relevant excitation wavevectors used.

The field distribution in a bare cavity obeys Maxwell’s equations for the electric field $E(x, y, z, t)$ and magnetic field $H(x, y, z, t)$. Symmetry of the planar cavity allows one to separate the solutions as follows:

\[
E(x, y, z, t) = E_\omega(x, y)\chi(z)\exp[-i\omega t]
\]

\[
H(x, y, z, t) = H_\omega(x, y)\xi(z)\exp[-i\omega t]
\]

The subscript $\omega$ denotes that the in-plane components of the fields depend on the energy of radiation while the normal components $\chi(z)$ and $\xi(z)$ are independent of energy under consideration of small in-plane wave vector $k_\parallel$:

\[
k_\parallel \ll \frac{n_{\text{cav}}\omega}{c}
\]

where $n_{\text{cav}}$ is the refractive index of material of the cavity and $c$ is the vacuum speed of light. The normal components $\chi(z)$ and $\xi(z)$ of the fields can be estimated inside the cavity where the most of light energy is concentrated: $\chi(z) \propto \xi(z) \propto \cos(n_{\text{cav}}\omega_0 z/c)$ where $\omega_0$ is the cavity resonance frequency at normal incidence. The in-plane wave vector then can be deduced as

\[
k_\parallel(\omega) = \frac{n_{\text{cav}}}{c}\sqrt{\omega^2 - \omega_0^2}.
\]
The cavity defect is given by a change of the Bragg mirror composition by the presence of additional GaAs due to the Ga droplet formation during the growth process [27, 28]. The presence of the defect has the effect to modify the effective thickness of the cavity layer, resulting in a red-shift of the photonic dispersion inside the defect [26]. As a result, the resonance frequency inside the defect shifts from $\omega_0$ to $\omega'_0$ with respect to the bare cavity and accordingly the in-plane wave vector $k_\parallel$ to $k'_\parallel$, with $k'_\parallel > k_\parallel$ (Fig. 7). The energy shift of the cavity mode represents an attractive potential in the two-dimensional polariton propagation.

Figure 7: Theoretical dispersion inside (red) and outside (blue) the defect. The presence of the defect has the effect to modify the effective thickness of the cavity layer, resulting in a red-shift of the photonic dispersion inside the defect. Consequently, for a fixed energy, the wavevector of the photonic mode in the region of the defect is higher than in the rest of the cavity ($k' > k$). The black dashed line indicate the excitation energy used in the experiment.

Besides the change of the resonance condition, also the normal components of the fields vary the spatial distribution and become $\chi(z) \rightarrow \chi'(z)$ and $\xi(z) \rightarrow \xi'(z)$. In our model, however, we assume that these changes are small (the relative change of the cavity energy considered in our case is only about 0.1%) and therefore we neglect them. Within this approximation, the solution of the problem of light propagation through a cavity with arbitrarily shaped defect is reduced to the solution solely in the $xy$ plane because boundary conditions are independent of the position on the axis $z$. First we find two basis sets of solutions of Maxwell’s equations for the bare cavity and the perturbed cavity. We denote these sets as $E_{cav,\omega,j,m}$, $H_{cav,\omega,j,m}$ and $E_{def,\omega,j,m}$, $H_{def,\omega,j,m}$ respectively. Here the index $j$ stands for polarization (TE or TM) and $m$ is the discrete index of the mode in the expansion.

The two respective sets of fields defined above are local solutions of Maxwell’s equations outside and inside the defect area. In order to solve the whole problem of scattering, we have to find a solution on the boundary between the bare cavity and the perturbed cavity. The boundary behaves like an ordinary boundary between two dielectrics, i.e. we require continuous tangent components of all fields. Let us write the fields in the bare cavity and in the defect area in the following form:

$$E_{\omega} = E_{\text{incident}} + \sum_{j,m} c_{j,m}^c E_{cav,\omega,j,m}$$  \hspace{1cm} (1)

$$H_{\omega} = H_{\text{incident}} + \sum_{j,m} c_{j,m}^c H_{cav,\omega,j,m}$$  \hspace{1cm} (2)

$$E_{\omega} = \sum_{j,m} c_{j,m}^d E_{def,\omega,j,m}$$  \hspace{1cm} (3)

$$H_{\omega} = \sum_{j,m} c_{j,m}^d H_{def,\omega,j,m}$$  \hspace{1cm} (4)
The coefficients $c_{\text{cav}}$ and $c_{\text{def}}$ are finally set so that the boundary conditions are fulfilled. If the basis sets are chosen properly, the solution is unambiguous. For the case of a circular defect, it is convenient to use the basis of fields in cylindrical coordinates [3] whose boundary conditions reduce to simple algebraic equations for the unknown coefficients. Once the coefficients are known, the spatial field distribution is evaluated using the definitions above, performing the summation on right hand side. To include the TE–TM splitting in cylindrical coordinates, it suffices to discriminate between the in-plane wave vectors $k_{\parallel,\text{TE}}$ and $k_{\parallel,\text{TM}}$ and the same inside the defect.

### III. CALCULATION OF THE EXCITONIC AND POLARITON DENSITIES

To verify the linear regime, we studied the excitation density dependence of our results by performing power dependence measurements.

**Exciton density.** The polariton-polariton interaction in our experiments is suppressed in the investigated sample by the large negative detuning of -29 meV of the cavity photon mode from the exciton resonance (1.514 eV), resulting in an exciton fraction of the polariton mode of less than 1%. In order to verify that nonlinear effects are negligible in our experiments, we varied the excitation density over 4 orders of magnitude and did not observe changes in the detected propagation at an excitation density as low as $D_{\text{exc}} = 1.4 \times 10^6 \text{cm}^{-3}$, which is six orders of magnitude lower than the nonlinearity threshold of the bulk GaAs, as reported in the literature [22, 23].

For continuous wave excitation we calculate the exciton density $D_{\text{exc}}$ excited in bulk GaAs by using the following formula:

$$D_{\text{exc}} = \frac{P_{\text{pump}}}{E_{\text{pump}}} \times (1 - R) \times \tau \times \alpha_{\text{bulk}}$$

where $P_{\text{pump}} = 31 \text{ W/cm}^2$ is the power density of the beam, $E_{\text{pump}}$ is the energy of the pump, $R = 0.6$ is the reflectivity of the sample at pump energy, $\tau = 1.3 \text{ ps}$ is the carrier lifetime in the bulk GaAs [39] and $\alpha_{\text{bulk}} = 2 \text{ cm}^{-1}$ is the bulk GaAs absorption coefficient (estimated from Fig. 2 of Ref. [39]).

**Polariton density.** The polariton density inside the cavity has been estimated from the number of photons transmitted through the sample and detected on the CCD camera. For a microcavity, in fact, the polariton population is proportional to the detected intensity. At the excitation density of 31 W/cm$^2$, the polariton density inside the microcavity is estimated to be $D_{\text{pol}} = 2.7 \times 10^3 \text{ cm}^{-2}$, 5 orders of magnitude lower than the lasing threshold observed in standard microcavities [24].

The density of polaritons have been estimated by using the following formula:

$$D_{\text{pol}} = \Phi_{hv} \times \tau,$$

where $\Phi_{hv} = 2.7 \times 10^{14} \text{ cm}^{-2} \text{ s}^{-1}$ is the flux of photons transmitted through the sample and $\tau = 10 \text{ ps}$ is the polariton lifetime. The flux of photons $\Phi_{hv}$ have been calculated as:

$$\Phi_{hv} = \frac{C_{px} \times \alpha_{\text{phe}}}{t \times QE \times A_{px} \times \eta_{\text{obj}} \times \eta_{\text{lens}}},$$

where $C_{px} = 12733$ is the maximum pixel counts corrected for the backgrounds, $\alpha_{\text{phe}} = 7.3$ is the number of photoelectrons per count (assuming shot-noise limit), $t = 1500 \text{ ms}$ is the integration time, $QE = 0.3$ is the quantum efficiency of the CCD camera at the wavelength used in the experiment and $A_{px} = 0.1225 \mu \text{m}^2$ is the real-space area of a single CCD pixel on the sample, $\eta_{\text{obj}} = 0.7$ and $\eta_{\text{lens}} = 0.9$ are the assumed intensity transmission factors due respectively to the objective ($\Omega_{j2}$) and the lens ($L_2$) used in the experiment (see Fig. 5).

### IV. DEPENDENCE OF THE SOLITON-LIKE FEATURES ON THE SCATTERING GEOMETRY

The observed features depend on the shape and size of the defect and the direction and polarization of the incoming polariton wave relative to the defect. For an elliptical defect, the phase and amplitude of the scattering depend on the direction of the incoming wave. Also the polarization contributes to the anisotropy of the effect because for a given absolute polarization direction a different angle of incidence corresponds to a different polarization relative to the defect.

Fig. 8 shows an example of the beam incident on the defect at an angle in the experiment. We use the same parameters of Fig. 1(c) and Fig. 1(d) of the manuscript to perform the simulations, but we change the direction of the incoming
beam. In the previous case (Fig. 1 of the manuscript), the excitation beam is polarized orthogonal to the incidence direction, while in Fig. 8 the beam direction has a 28 degree angle to its polarization ($y$), and generates a phase dislocation only in the upper dark line but not in the lower one, as indicated by the arrow in Fig. 8(c).

Moreover, we have investigated the case of a larger defect. The number of dark lines is increasing with increasing defect size, allowing the formation of quadruplet soliton-like features. This is confirmed by the simulations shown in Fig. 9(b) and Fig. 9(d). Once again we refer to the simulations shown in Fig. 1 of the manuscript to simulate high-order dislocations. In particular, Fig. 9(b) and Fig. 9(d) have been calculated by using the same parameters as Fig. 1(c) and Fig. 1(d) of the manuscript except for increasing the radius of the defect from 3 $\mu$m to 5 $\mu$m.

In Fig. 9(a) and Fig. 9(c) the experimental observation of a high order soliton-like features is shown in both intensity and phase. In the case of a bigger defect, it is possible to note how the wave appears to bend around the edges of the defect.

Figure 8: Experimental (a),(c) and simulated (b),(d) real-space intensity and interference pattern showing soliton-like fingerprints generated by the interaction of the beam with a defect. Unlike Fig. 1 of the manuscript, the phase shift is only present in correspondence of the upper soliton-like feature as indicated by the light blue arrow in (c).

Figure 9: Experimental (a),(c) and simulated (b),(d) real-space intensity and interference pattern showing higher-order soliton features generated by the interaction of the beam with a defect bigger than the one present in Fig. 1 of the manuscript.
V. HALF-SOLITON-LIKE FEATURES CAUSED BY TE-TM SPLITTING

In our simulations a linear $y$-polarized incoming beam, propagates along the $y$-direction and is scattered by a defect positioned at 25 $\mu$m away from the excitation spot, inducing the formation of two traces propagating in oblique directions. In the case of half-soliton features in the circular polarisation basis, we found that the birefringence in the scattering by the defect is due to the intrinsic TE-TM splitting of the polariton dispersion. This is confirmed by the simulations shown in Fig. 10 where the scattered field, produced by the wave hitting the defect, is calculated in absence, Fig. 10(a), or in presence, Fig. 10(b) of the TE-TM splitting. In the latter case we use $\vec{k}_{L}/\vec{k}_{T} = 1.004$ which is the same value that has been used in reference [38] for the same sample. In order to simplify the theoretical discussion, we consider the TE-TM splitting constant across the whole cavity including the defect and no additional splitting in the defect is considered.

![Figure 10: Simulated circular Stokes parameters showing half-soliton features. The images have been calculated by considering a beam hitting a circular defect in absence (a) and in presence (b) of the TE-TM splitting.](image-url)

[1] G. I. Stegeman and M. Segev, Science 286, 1518 (1999), ISSN 0036-8075, 1095-9203, URL http://www.sciencemag.org/content/286/5444/1518.
[2] M. Segev and G. Stegeman, Physics Today 51, 42 (1998), URL http://link.aip.org/link/?PTO/51/42/1.
[3] Z. Chen, M. Segev, and D. N. Christodoulides, New J. Phys. 75, 086401 (2012), ISSN 0034-4885, 1361-6633, URL http://iopscience.iop.org/0034-4885/75/8/086401.
[4] M. Hercher, J. Opt. Soc. Am. 54, 563 (1964).
[5] M. Segev, B. Crosignani, A. Yariv, and B. Fischer, Phys. Rev. Lett. 68, 923 (1992), URL http://link.aps.org/doi/10.1103/PhysRevLett.68.923.
[6] K. Hayata and M. Koshina, Phys. Rev. Lett. 71, 3275 (1993), URL http://link.aps.org/doi/10.1103/PhysRevLett.71.3275.
[7] M. Nakazawa and K. Suzuki, Electronics Letters 31, 1076 (1995), ISSN 0013-5194.
[8] P. D. Miller, Phys. Rev. E 53, 4137 (1996), URL http://link.aps.org/doi/10.1103/PhysRevE.53.4137.
[9] T. Tsuzuki, J. Low Temp. Phys. 4, 441 (1971), ISSN 0022-2291, URL http://www.springerlink.com/content/w57665640h03131/abstract/.
[10] Y. Kivshar, J. Quantum. Electron. 29, 250 (1993), ISSN 0018-9197.
[11] J. Denschlag, J. E. Simsarian, D. L. Feder, C. W. Clark, L. A. Collins, J. Cubizolles, L. Deng, E. W. Hagley, K. Helmerson, W. P. Reinhardt, et al., Science 287, 97 (2000), ISSN 0036-8075, 1095-9203, URL http://www.sciencemag.org/content/287/5450/97.
[12] J. Kasprzak, M. Richard, S. Kundermann, A. Baas, P. Jeambrun, J. M. J. Keeling, F. M. Marchetti, M. H. Szyma˚nska, R. André, J. L. Staehli, et al., Nature 443, 409 (2006), ISSN 0028-0836, URL http://www.nature.com/nature/journal/v443/n7110/abs/nature05131.html.
[13] A. Amo, J. Lefr`ere, S. Pigeon, C. Adrados, C. Ciuti, I. Carusotto, R. Houdr´e, E. Giacobino, and A. Bramati, Nature Physics 5, 805 (2009), ISSN 1745-2473, URL http://www.nature.com/nphys/journal/v5/n11/full/nphys1364.html.
[14] A. Amo, D. Sanvitto, F. P. Laussy, D. Ballarini, E. d. Valle, M. D. Martin, A. Lemaitre, J. Bloch, D. N. Križhanovskii, M. S. Skolnick, et al., Nature 457, 291 (2009), ISSN 0028-0836, URL http://www.nature.com/nature/journal/v457/n7227/full/nature07640.html.
[15] S. Pigeon, I. Carusotto, and C. Ciuti, Phys. Rev. B 83, 144513 (2011), URL http://link.aps.org/doi/10.1103/PhysRevB.83.144513.

[16] A. Amo, S. Pigeon, D. Sanvitto, V. G. Sala, R. Hivet, I. Carusotto, F. Pisanello, G. Leménager, R. Houdré, E. Giacobino, et al., Science 332, 1167 (2011), ISSN 0036-8075, 1095-9203, URL http://www.sciencemag.org/content/332/6034/1167.

[17] G. Grosso, G. Nardin, F. Morier-Genoud, Y. Léger, and B. Deveaud-Plédran, Phys. Rev. Lett. 107, 245301 (2011), URL http://link.aps.org/doi/10.1103/PhysRevLett.107.245301.

[18] G. Grosso, G. Nardin, F. Morier-Genoud, Y. Léger, and B. Deveaud-Plédran, Phys. Rev. B 86, 020509 (2012), URL http://link.aps.org/doi/10.1103/PhysRevB.86.020509.

[19] B. Deveaud, G. Nardin, G. Grosso, and Y. Léger, in Physics of Quantum Fluids, edited by A. Bramati and M. Modugno (Springer Berlin Heidelberg, 2013), vol. 177 of Springer Series in Solid-State Sciences, pp. 99–126, ISBN 978-3-642-37568-2, URL http://dx.doi.org/10.1007/978-3-642-37569-9_6.

[20] H. Flayac, D. Solnyshkov, and G. Malpuech, Phys. Rev. B (2011), h. Flayac, D. S. Solnyshkov, and G. Malpuech, Phys. Rev. B 83, 193305 (2011), URL http://arxiv.org/abs/1103.4516.

[21] R. Hivet, H. Flayac, D. S. Solnyshkov, D. Tanese, T. Boulier, D. Andreoli, E. Giacobino, J. Bloch, A. Bramati, G. Malpuech, et al., Nature Physics 8, 724 (2012), ISSN 1745-2473, URL http://www.nature.com/nphys/journal/v8/n10/full/nphys2406.html.

[22] A. C. Schaefer and D. G. Steel, Phys. Rev. Lett. 79, 4870 (1997), URL http://link.aps.org/doi/10.1103/PhysRevLett.79.4870.

[23] K. Akiyama, N. Tomita, Y. Nomura, and T. Isu, Appl. Phys. Lett. 75, 475 (1999), ISSN 00036951, URL http://apl.aip.org/resource/1/applab/v75/i4/p475_s1.

[24] H. Deng, G. Weihs, D. Snoke, J. Bloch, and Y. Yamamoto, Proceedings of the National Academy of Sciences 100, 15318 (2003), URL http://www.pnas.org/content/100/26/15318.full.pdf+html, URL http://www.pnas.org/content/100/26/15318.abstract.

[25] J. Zajac, W. Langbein, M. Hugues, and M. Hopkinson, Phys. Rev. B 85 (2012), URL http://prb.aps.org/abstract/PRB/v85/i16/e165309.

[26] J. Zajac and W. Langbein, Phys. Rev. B 86 (2012).

[27] K. Fujiwara, K. Kanamoto, Y. Ohta, Y. Tokuda, and T. Nakayama, Journal of Crystal Growth 80, 104 (1987), ISSN 0022-0248, URL http://www.sciencedirect.com/science/article/pii/002202487990529X.

[28] N. Chand and S. Chu, Journal of Crystal Growth 104, 485 (1990), ISSN 0022-0248, URL http://www.sciencedirect.com/science/article/pii/002202489090151A.

[29] E. Hecht and A. Zajac, Optics (Addison-Wesley, 1982).

[30] M. V. Berry, R., waves and phase: a picture book of cusps (Elsevier Science Publishers B.V., Amsterdam, 1992), 1992nd ed., URL http://www.phy.bris.ac.uk/people/berry_mv/the_papers/Berry228.pdf.

[31] M. V. Berry, J. F. Nye, and F. J. Wright, Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences 291, 453 (1979), ISSN 0080-4614, ArticleType: research-article / Full publication date: Apr. 12, 1979 / Copyright © 1979 The Royal Society, URL http://www.jstor.org/stable/75150.

[32] S. V. P. Senthilkumar, Opt Commun 283, 2767 (2010), URL http://202.114.89.42/resource/pdf/5496.pdf.

[33] G. Ruben and D. M. Paganin, Phys. Rev. E 75, 066613 (2007), URL http://link.aps.org/doi/10.1103/PhysRevE.75.066613.

[34] G. Tosi, G. Christmann, N. G. Berloff, P. Tsotsis, T. Gao, Z. Hatzopoulos, P. G. Savvidis, and J. J. Baumberg, Nat Phys 8, 190 (2012), URL http://dx.doi.org/10.1038/nphys2182.

[35] G. Panzarini, L. C. Andreani, A. Armitage, D. Baxter, M. S. Skolnick, V. N. Astratov, J. S. Roberts, A. V. Kavokin, M. R. Vladimirova, and M. A. Kaliteevski, Phys. Rev. B 59, 5082 (1999), URL http://link.aps.org/doi/10.1103/PhysRevB.59.5082.

[36] A. Kavokin, G. Malpuech, and M. Glazov, Phys. Rev. Lett. 95, 136601 (2005), URL http://link.aps.org/doi/10.1103/PhysRevLett.95.136601.

[37] C. Leyder, M. Romanelli, J. P. Karr, E. Giacobino, T. C. H. Liew, M. M. Glazov, A. V. Kavokin, G. Malpuech, and A. Bramati, Nature Physics 3, 628 (2007), ISSN 1745-2473, URL http://www.nature.com/nphys/journal/v3/n9/full/nphys676.html.

[38] M. Maragkou, C. E. Richards, T. Ostatnický, A. J. D. Grundy, J. Zajac, M. Hugues, W. Langbein, and P. G. Lagoudakis, Opt. Lett. 36, 1095 (2011), URL http://ol.osa.org/abstract.cfm?url anywhere=bib&uri=ol-36-7-1095.

[39] M. D. Sturges, Phys. Rev. 127, 768 (1962), URL http://link.aps.org/doi/10.1103/PhysRev.127.768.