Asteroseismology of luminous red giants with \textit{Kepler}: Long Period Variables with radial and non-radial modes

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Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

While long period variables (LPVs) have been extensively investigated, especially with MACHO and OGLE data for the Magellanic Clouds, there still exist open questions in their pulsations regarding the excitation mechanisms, radial order and angular degree assignment. Here, we perform asteroseismic analyses on LPVs observed by the 4-year \textit{Kepler} mission. Using a cross-correlation method, we detect unambiguous pulsation ridges associated with radial fundamental modes ($n = 1$) and overtones ($n \geq 2$), where the radial order assignment is made by using theoretical frequencies and observed frequencies. We find that the amplitude of the dominant pulsation modes starts to increase more significantly with period at a period of $P = 4.3$ days, which can be a result of the transition of dominant modes between overtones and may suggest significant variations in the mode lifetime. Our results confirm that the amplitude variability seen in semiregulars is consistent with oscillations being solar-like. We identify that the dipole modes, $l = 1$, are dominant in the radial orders of $3 \leq n \leq 6$, and that quadrupole modes, $l = 2$, are dominant in the first overtone $n = 2$. A test of seismic scaling relations using Gaia DR2 parallaxes reveals the possibility that the relations break down when $\nu_{\text{max}} \lesssim 3 \mu\text{Hz}$ ($R \gtrsim 40 R_{\odot}$, or $\log L/L_{\odot} \gtrsim 2.6$). Our homogeneous measurements of pulsation amplitude and period for 3214 LPVs will be very valuable for probing effects of pulsation on mass loss, in particular in those stars with periods around 60 days, which has been argued as a threshold of substantial pulsation-triggered mass loss.

Key words: stars: oscillations, stars: evolution, stars: late-type, techniques: photometric

1 INTRODUCTION

Long Period Variables (LPVs)\textsuperscript{1} are cool evolved stars on the asymptotic giant branch or near the tip of the red giant branch. They are generally divided into Semiregular Variables (SRs) and Mira variables, based on the regularity and amplitudes of their light curves. Major advances in the understanding of pulsations in LPVs have been achieved from studying their period–luminosity (P–L) diagrams, using ground-based surveys such as MACHO (Wood et al. 1999), EROS (Lebzelter et al. 2002), and OGLE (Soszynski et al. 2004; Soszynski et al. 2009), and space missions like \textit{Hipparcos} (Beding & Zijlstra 1998; Tabur et al. 2010), CoRoT (Lebzelter 2011; Ferreira Lopes et al. 2015), \textit{Kepler} (Bánya et al. 2013; Mosser et al. 2013; Hartig et al. 2014; Stello et al. 2014), and Gaia (Mowlavi et al. 2018; Lebzelter et al. 2018). While the pulsation sequences of LPVs have

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\textsuperscript{1} In this work, we use these three terms interchangeably: LPVs, M giants, and high-luminosity red giants, though the first is extended to include pulsators with periods down to a few days, and the second is extended to include some late K giants.
been extensively studied, the nature of the pulsations is still not fully understood.

The first open question is linked to the driving mechanism of the pulsations in SRs: self-excitation via a heat-engine mechanism like Mira variables, or stochastically excited as solar-like oscillations in G and K stars? One method to investigate excitation mechanisms of LPVs is to analyze the relation between the pulsation amplitude and period, and compare with less-luminous red giants that are well-established to be sun-like oscillators (e.g. Tabur et al. 2010). Soszynski et al. (2007) proposed that stars falling along their so-called sequences b2 and b3 are sun-like pulsators (see the sequences in Figure 4a). Mosser et al. (2013) argued that all the P–L relations for LPVs can be explained by solar-like oscillations, in that the P–L sequences are an extension of a global oscillation pattern in less-evolved red giants. This is consistent with the findings by Dziembowski & Soszyński (2010) and Takayama et al. (2013). However, Bányai et al. (2013) argued that Mira/Semiregular variables may have a pulsation nature different from sun-like oscillations. This is based on their findings of a significant pulsation-amplitude transition at a period of ~10 days, a dividing point between SRs and shorter-period solar-like pulsators. We note that Ferreira Lopes et al. (2015, their Figure 3) found a similar amplitude-transition feature, which, however, may be attributed to the different amplitude definitions between their work and the comparison reference. In this work, we find evidence in support of solar-like oscillations in SRs.

The second question concerns assigning radial orders to the pulsation sequences on the P–L diagram. Soszynski et al. (2007), Dziembowski & Soszyński (2010), and Mosser et al. (2013) interpreted the sequences C′, B, and A as the radial fundamental mode, first overtone, and second overtone, respectively (see the sequences in Figure 4a). This means the longer-period sequence C, containing Mira variables, has no interpretation in terms of radial orders if we assume that two adjacent sequences differ by one radial order. However, a distinct set of radial order assignment from Wood et al. (1999), Soszyński & Wood (2013), Takayama et al. (2013), and Wood (2015) state that sequences C, C′, and B are associated with the radial fundamental mode, first overtone, and second overtone, respectively. Thus, the two sets of modal assignments differ by one radial order. Recently, Trabucchi et al. (2017) re-examined the observed P–L sequences and gave an intermediate solution that C and B both correspond to the first overtone, but include fundamental mode pulsations at lower luminosities of the two sequences. They suggested that sequences C, A, and A′ correspond to the radial fundamental mode, second overtone, and third overtone, respectively. We discuss this issue in Section 6.

The third question concerns the angular degree of the modes. Do LPVs exhibit radial and non-radial pulsations? And which modes are dominant, radial modes (l = 0), dipole modes (l = 1), or quadrupole modes (l = 2)? Soszynski et al. (2004) discovered that the sequence A consists of three closely separated parallel subsequences in the so-called Petersen diagram, where the ratio of a shorter period to a longer period is plotted against the longer period. The subsequences were later also found in the sequence B and A′ by Soszynski et al. (2007). Stello et al. (2014) found a triplet frequency pattern in Kepler M giants that is made up of l = 1, 2, 0 modes, sorted in the decreasing period order. This triplet pattern explains the parallel subsequences in the Petersen diagram. Mosser et al. (2013) further argued that dipole modes dominate in stars oscillating at higher frequency (≥1.0 µHz), while radial modes dominate at lower frequency (≤1.0 µHz). However, Stello et al. (2014) found that the pulsations of luminous Kepler stars with a characteristic oscillation frequency down to 0.2 µHz are dominated by dipole modes. The findings from Wood (2015, Figure 3) using the OGLE III catalog of LPVs in the LMC is in agreement with the findings by Stello et al. (2014). We address this issue in Section 7.

The fourth question is related to the asteroseismic scaling relations. The relations have been widely used to characterize oscillating dwarfs and giants (see Chaplin et al. 2013; Hekker & Christensen-Dalsgaard 2017, for reviews). Moreover, asteroseismically derived parallaxes have been used as references to calibrate the Gaia DR2 parallaxes (e.g. Zinn et al. 2019). Although the seismic scaling relations have been extensively tested on main-sequence stars, subgiants, and less-luminous red giants (see a recent review, Hekker 2019), it remains an open question if the relations work for high-luminosity red giants (see Section 8).

In this work we address these four open questions using a sample of 3214 Kepler LPVs, which includes pulsators with periods P ≥ 1 day. Our asteroseismic analyses are based on the light curves collected by the 4-year Kepler space mission (Borucki et al. 2010; Koch et al. 2010) and on Gaia DR2 parallaxes (Lindegren et al. 2018).

2 SAMPLE SELECTION AND DATA REDUCTION

To construct a sample of LPVs, we selected 4296 Kepler red giants from Mathur et al. (2017) with surface gravity log g < 2.0 dex, equivalent to a period ≥ 1 day. We added known M giants from the literature, namely, Bányai et al. (2013), Stello et al. (2014), and Yu et al. (2018). For the latter, we applied a cut of v max ≤ 15 µHz. We discarded targets observed for fewer than two Kepler quarters, given that long light curves are required to resolve multiple pulsation modes in LPVs. We excluded the stars with marginal pulsation detection by visual inspection of individual power spectra. Our final sample comprised 3214 LPVs, listed in Table 1.

Kepler long-cadence photometry is well-suited for exploring pulsations in LPVs, given its temporal sampling rate (29.4 min) and long baseline (~4 years). We used PDCSAP light curves, which have been corrected for systematic errors in each observing quarter using “cotrending basis vectors” (Stumpe et al. 2012; Smith et al. 2012). For some M giants pulsating at a long period, such as Mira variables, PDCSAP time series were over-corrected, by treating intrinsic pulsations as “systematic errors”. For these stars, we adopted “Simple Aperture Photometry” (SAP) light curves. To determine the stars for which the PDCSAP light curves were safe to use, we used a measure, P DC S A P max , which is easy to compute and approximates a typical period of a light curve. It is defined as

$$P_{\text{extrema}} = \frac{2N\delta t}{N_{\text{extrema}}},$$  

where N_{extrema} is the number of turning points (extrema), N is the total number of data points of a light curve, and δt is
Figure 1. Light curves for two representative Kepler LPVs: KIC 7624629 (upper) and KIC 2715041 (lower). For each star, SAP (blue), PDCSAP (green), and jump-corrected SAP (red) light curves are shown. Jump-corrected SAP time series are adopted for slowly pulsating stars, like KIC 7624629, while PDCSAP light curves are used for fast pulsating stars, like KIC 2715041, respectively. A $P_{\text{extrema}}$ of 6 days was adopted as a dividing point for this decision (see the text for more details). The bottom panel of each star shows the power spectrum, computed from the adopted time series.

the sampling interval of the long-cadence Kepler data (29.4 min).

To count the turning points in a light curve, we calculated the point-to-point difference of the light curve. The number of the zero crossings of the difference time series gives the number of turning points. Since there are quarter gaps in Kepler light curves, which will introduce turning points between quarter edges because the corresponding flux usually jumps dramatically, we then performed iterative 4-$\sigma$ clipping to discard outliers of the difference time series. Finally, we found $P_{\text{extrema}} = 6$ days is an appropriate threshold to select the light curve source (PDCSAP versus SAP). For PDCSAP time series with $P_{\text{extrema}} < 6$ days, we divided each quarter of time series by its median flux, and concatenated them together.

For SAP data, we used a Gaussian Process method (Rasmussen & Williams 2006) to remove jumps between two adjacent quarters for stars with $P_{\text{extrema}} > 6$ days. A step function was used to model jumps and a covariance function was used to approximate the residuals, due to actual physical brightness variations as well as noise. We implemented the Gaussian process fit using Celerite and the kernel SHOTerm (Foreman-Mackey et al. 2017). Note that systematic perturbations, such as spacecraft safe-mode events and/or long-term drifts, could affect light curves but were not corrected considering their much lower amplitudes than the intrinsic stellar variability. We carefully inspected the light curves and power spectra that output a period of approximate a year (one Kepler orbit) and removed the contamination.

Figure 1 shows the jump-corrected SAP light curve (red curve) and its power spectrum (black curve), for a representative slowly pulsating M giant, KIC 7624629 (top-3 panels), with $P_{\text{extrema}} = 198.41$ days. The PDCSAP time series for this star was clearly over-corrected (green curve). Figure 1 also shows the SAP and PDCSAP light curves for a typical fast pulsating star, KIC 2715041 (bottom-3 panels), with $P_{\text{extrema}} = 3.36$ days. For this star, we can see that systematic annual perturbations in the PDCSAP light curve have been nicely removed. The jump-corrected SAP light curves for this stars is clearly dominated by annual instrumental drifts.

For the fast and slow samples, we used different methods to measure the pulsation period and amplitude of the dominant mode. For pulsators in the slow sample, we first measured the period of the highest peak in the power spectrum of the difference time series. Note that a difference time series is nearly free from the quarter jumps in the associated light curve, since the jumps manifest themselves as a few outliers that can be easily clipped. In order to measure the amplitude, we then searched the power spectrum of the time series for the highest peak in a window with a width of 10 times the frequency resolution and centered at the frequency measured from the difference time series. The height of the highest peak was used as a proxy for the amplitude.

For stars in the fast sample, we first used the SYD pipeline (Huber et al. 2009) to measure the frequency of maximum oscillation power, $\nu_{\text{max}}$. We subsequently found the highest peak within a window centered at $\nu_{\text{max}}$. The width of this window was the full-width-at-half-maximum of a Gaussian fitted to the auto-correlation of the oscillation power excess. These steps are necessary, because for a fast oscillator the dominant mode generally is not the highest peak in the power spectrum, due to 1/f noise in the lower frequency regime. This is unlike a slow pulsator, such as a Mira variable, for which the highest peak generally is the dominant mode. Again, we used the height of the highest peak to approximate the amplitude. We note that the amplitude of a dominant mode contains the contributions from the granulation background, which is the case for both fast and slow pulsators. Considering the amplitude ratio between
Figure 2. (a) Relation between the period and amplitude of the dominant mode, the highest peak in a power spectrum. Periods and amplitudes were extracted from either PDCSAP time series shown in green circles, or jump-corrected SAP time series indicated in red diamonds (see the text). A piecewise linear line fitted to the period-amplitude relation shows a kink at period $P = 4.3$ days, indicated in the vertical dashed line. (b) Similar to (a) now color-coded by the radial order of the dominant mode.

Table 1. Asteroseismic parameters and stellar properties of Kepler M giants

| KIC  | LCs  | Q    | $P_{\text{excess}}$ | $T_{\text{eff}}$ | $\text{amp}$ | period | order | $v_{\text{max}}$ | $\Delta v$ | $\pi$ | $d$ | $L$ | $L_\odot$ | $A_V$ |
|------|------|------|---------------------|------------------|-------------|--------|-------|-----------------|-----------|------|-----|-----|--------|------|
| 892738 | PDCSAP | 18   | nan       | 4544±135    | 308.24     | 1.44   | 5     | 7.47±0.25       | 1.31±0.01 | 0.40±0.02   | 2.490±0.137 | 193.96±21.69 | 0.29   |
| 893210 | PDCSAP | 17   | nan       | 4204±127    | 1036.10    | 5.38   | 4     | 2.62±0.05       | 0.51±0.01 | 0.23±0.03   | 4.44±0.563 | 574.52±13.14 | 0.25   |
| 893223 | PDCSAP | 8    | nan       | 4207±147    | 903.45     | 1.94   | 5     | 6.16±0.08       | 1.18±0.01 | 0.41±0.03   | 2.33±0.151 | 227.36±29.88 | 0.28   |
| 1026309 | PDCSAP | 18   | nan       | 4114±80     | 97.86      | 0.77   | 6     | 16.09±0.91      | 1.92±0.01 | 0.67±0.02   | 1.30±0.048 | 178.02±12.19 | 0.28   |
| 1026892 | PDCSAP | 18   | nan       | 3900±80     | 737.91     | 4.72   | 4     | 2.78±0.11       | 0.54±0.01 | 0.81±0.02   | 2.14±0.036 | 445.59±27.16 | 0.24   |
| 1027110 | PDCSAP | 18   | nan       | 4190±80     | 466.93     | 1.62   | 5     | 6.67±0.14       | 1.15±0.01 | 0.31±0.02   | 3.01±0.194 | 187.73±24.44 | 0.24   |
| 1027707 | PDCSAP | 18   | nan       | 4254±148    | 872.85     | 3.87   | 4     | 3.01±0.04       | 0.54±0.01 | 0.17±0.03   | 5.73±0.841 | 654.15±198.08 | 0.23   |
| 1160655 | PDCSAP | 18   | nan       | 3740±130    | 1730.28    | 9.46   | 4     | 1.63±0.02       | 0.37±0.01 | 0.38±0.03   | 2.63±0.181 | 253.08±36.65 | 0.61   |
| 1160867 | PDCSAP | 18   | nan       | 4000±80     | 550.43     | 2.56   | 5     | 4.68±0.08       | 0.89±0.01 | 1.12±0.03   | 0.89±0.022 | 240.36±12.64 | 0.70   |
| 1160986 | PDCSAP | 4    | nan       | 4474±80     | 281.38     | 1.34   | 6     | 8.85±0.62       | 1.54±0.02 | 0.21±0.02   | 4.68±0.334 | 134.51±19.42 | 0.71   |
| 12600259 | PDCSAP | 18   | nan       | 4286±150    | 312.54     | 1.53   | 5     | 7.69±0.02       | 1.36±0.02 | 0.35±0.02   | 2.87±0.304 | 162.86±23.63 | 0.15   |
| 12600062 | PDCSAP | 18   | nan       | 4056±141    | 1210.53    | 3.98   | 4     | 2.86±0.04       | 0.57±0.02 | 0.27±0.02   | 3.80±0.351 | 348.10±65.53 | 0.13   |
| 12601840 | SAP | 32.30 | nan       | 3277±114    | 56380.99   | 67.45  | 2     | nan             | nan       | 0.26±0.06   | 3.99±0.918 | 1548.26±759.13 | 0.11   |
| 12602404 | PDCSAP | 10   | nan       | 4417±80     | 315.54     | 0.88   | 6     | 12.97±0.13      | 1.95±0.01 | 0.30±0.02   | 3.53±0.241 | 129.36±18.96 | 0.27   |
| 12602421 | SAP | 7.80 | nan       | 4175±124    | 2346.49    | 7.94   | nan   | nan             | nan       | 0.66±0.02   | 1.52±0.052 | 473.81±33.76 | 0.27   |
| 12644223 | PDCSAP | 18   | nan       | 4986±143    | 893.23     | 2.63   | 5     | 4.18±0.13       | 0.84±0.02 | 0.16±0.01   | 6.21±0.456 | 281.87±41.99 | 0.20   |
| 12645224 | SAP | 12.66 | nan       | 4035±141    | 5338.09    | 11.78  | nan   | nan             | nan       | 0.40±0.03   | 2.51±0.168 | 857.84±16.16 | 0.16   |
| 12688798 | PDCSAP | 6    | nan       | 4245±80     | 395.85     | 0.92   | 6     | 13.05±0.21      | 1.97±0.01 | 0.31±0.01   | 3.26±0.150 | 86.88±8.27  | 0.15   |
| 12690711 | SAP | 18   | nan       | 3900±189    | 1422.41    | 7.26   | nan   | nan             | nan       | 0.23±0.03   | 4.54±0.495 | 657.43±149.19 | 0.21   |
| 12984227 | SAP | 18   | nan       | 3489±122    | 9606.24    | 20.37  | nan   | nan             | nan       | 0.28±0.04   | 3.67±0.534 | 880.49±264.38 | 0.20   |

Note. (1) KIC ID; (2) Type of light curve used in this work; (3) Number of quarters of Kepler light curves; (4) $P_{\text{excess}}$ (see Section 2 for its definition); (5) Source: Mathur et al. (2017); (6) Dominant mode amplitude; (7) Dominant mode period; (8) Radial order; (9) The frequency of maximum power; (10) Mean larger frequency separation; (11) Gaia DR2 parallax with an offset of 0.03 mas added (Lindegren et al. 2018); (12) (13) (14) Distance, Luminosity, and Extinction, respectively. (This table is available in its entirety in machine-readable form.)
oscillation and granulation at a given frequency is nearly constant (Mosser et al. 2013, see Figure 4), and to make the amplitude measured in a consistent way for the two types of pulsators, the contributions from the granulation background were not removed.

3 PERIOD–AMPLITUDE RELATION OF LONG PERIOD VARIABLES

Figure 2a shows our measured amplitudes versus periods for the entire sample. At the longest periods we see a number of Mira variables with periods $P > 100$ days and amplitude near 1.0 mag (also shown in the pink asterisks in Figure 2b). We also see SRs, with periods typically longer than 20 days and lower amplitudes. Note that in this work we measured the amplitude of a sinusoidal wave, which is half of the peak-to-peak amplitude. Miras are characterized by pulsation periods longer than 100 days and peak-to-peak amplitudes greater than 2.5 mag at visual wavelengths. Here, for Mira variables a typical measured peak-to-peak amplitude is 2.0 mag, smaller than the 2.5 mag definition, which is because of the redder broad Kepler bandpass. Both period and amplitude together with additional parameters are listed in Table 1.

Figure 2b illustrates the period-amplitude relation color-coded by the radial order of the dominant mode, which was determined as discussed in Section 6.1. We provide radial order assignment only down to $n = 2$ using model frequencies from Stello et al. (2014). The radial order $n$ for a given star corresponds to the radial mode closest to $\nu_{\text{max}}$. It is the dominant mode in general but not necessarily, since it can be a non-radial mode, and can be a few orders away from the radial mode closest to $\nu_{\text{max}}$ due to the stochastic driving nature. Fortunately, the clear boundaries between adjacent orders imply the accuracy of radial order assignment to the dominant modes. A detailed analysis on the assignment of radial order and angular degree for each dominant mode is beyond the scope of this work.

Figure 2 reveals a break point at period $P = 4.3$ days, determined by a piecewise linear model fit. Pulsation amplitude increases more rapidly for $P > 4.3$ days. This might be caused by a transition of the dominant modes from $n = 5$ to
Figure 4. (a) Period-$M_K$ diagram of OGLE LPVs in the LMC (dominant mode only, Soszyński et al. 2009). (b) Similar to panel a now including the Kepler LPVs (Note the difference in the scale of vertical axes). Symbol colors have the same meaning as Figure 2b. Miras with Gaia DR2 parallaxes better than 30% are highlighted in the dark blue circles. The red line denotes the Period-$M_K$ relation for Miras (Feast 1996). (c) Uncertainties of $M_K$ for the OGLE LPVs. (d) Uncertainties of $M_K$ for the Kepler LPVs.

$n = 4$. Note that Báanyai et al. (2013) found a break point at $P \approx 10$ days, and interpreted it as an indication of a transition of the driving mechanisms. We suggest that this could instead be another transition of the dominant modes from $n = 4$ to $n = 3$. For dominant modes, a higher overtone has a longer mode lifetime, and thus has a higher amplitude. The break points from $n = 5$ to $n = 4$ suggest mode lifetimes vary significantly between different radial orders. Note that the breakpoints are independent of our decision of where the sample is divided into slow/fast pulsators.

Figure 3 shows the dominant mode amplitude as a function of period and $P_{\text{extrema}}$ for $P_{\text{extrema}} > 6$ days, respectively. We can see that the parameter $P_{\text{extrema}}$ has a tight relation with the amplitude, and is a very good proxy of the dominant mode period (note the horizontal axes). All the Miras, two Long Secondary Period (LSP) variables, and three representative SRs are highlighted. We can see that the SRs and LSPs are significantly shifted toward the left in the lower panel but the Miras are much less shifted. This is because SRs and LSPs show more variations in addition to their main periodicity, and thus have more turning points. Figure 3a shows a significant amplitude decrease at $P \approx 40$ days (the black arrow), which could be caused by a dominant mode transition from pulsation sequence A to B (see Figures 2 and 4). Figure 3b reveals a sharper lower boundary along the global trend than Figure 3a. This feature is mainly due to outliers (data points near the lower boundary in Panel a) that are contaminated by nearby or foreground/background stars, leading to lower amplitude and smaller $P_{\text{extrema}}$. Note that $P_{\text{extrema}}$ can be very easily computed from light curves and are hardly affected by Kepler quarter jumps (by sigma clipping). It could be a robust measure for searching for SRs and Miras observed by the TESS mission (Ricker et al. 2015), in particular those LPVs in the continuous viewing zones. We emphasize that this large and homogeneous sample of LPVs is excellent to study mass-loss triggered by pulsation. A period of 60 days has been argued as a threshold above which substantial dust mass loss is expected (see MNRAS 000, 1–16 (2019)).
4 PERIOD–LUMINOSITY RELATION

Over the past two decades, one of the major advances in the investigation of LPVs has been the detection of pulsation sequences on the P–L diagram using MACHO and OGLE data. Here, we combine the Kepler and OGLE LPVs as shown in Figure 4. Figure 4a shows a Period–\(M_K\) diagram of the LPVs in the LMC, where only the dominant period from the OGLE-III catalogue is used (Soszyński et al. 2009). The absolute 2MASS K magnitude, \(M_K\), was computed from the Gaia DR2 parallaxes using the same method as Huber et al. (2017) and Berger et al. (2018). We adopted a LMC distance modulus of 18.54 mag and an extinction \(A_V = 0.38\) mag (Imara & Blitz 2007). Sequences \(A', A, B, C,\) and \(C'\) are labeled, following the nomenclature of Soszyński et al. (2007). For OGLE small amplitude red giants, sequences \(a_2, a_3,\) and \(a_4\) denoting AGB stars are shown with blue lines, while sequences \(b_2\) and \(b_3\) denoting RGB stars are shown with red lines (the line parameters were adopted from Table 1 of Soszyński et al. 2007). Figure 4b shows that the vast majority of the Kepler LPVs occupy the sequences \(A, A',\) and typically exhibits shorter periods. They do not show the well-defined sequences of OGLE LPVs, due to the approximate six times larger \(M_K\) uncertainties, as revealed by Figures 4c and 4d. We do expect the pulsation sequences in Kepler LPVs to be present in Figure 4, given that we have detected radial and non-radial sequences over several radial orders (plotted in a difference way, see Section 6). Figure 4b shows that sequence \(A\) of the OGLE stars possibly corresponds to the second overtone by comparing to the Kepler LPVs, while the sequences \(B\) and \(C'\) could be the first overtone. Kepler Miras are near the sequence \(C\). Our findings are consistent with the theoretical results by Trabucchi et al. (2017), which provided a compromise solution of radial order assignment over the two sets of contradictory suggestions (see Section 1).
Figure 6. Stacked power spectra of high-luminosity red giants with $0.15 \mu$Hz $\leq \nu_{\text{max}} \leq 10.54 \mu$Hz. The stacked spectra are shown in four panels so as to highlight in different $\nu_{\text{max}}$ ranges clear ridges associated with multiple angular degrees over several radial orders. Each horizontal band represents one power spectrum with the power color-coded. The ordinate axis is not linear in $\nu_{\text{max}}$, hence the different ridge curvatures in the different panels. For each radial order $n \geq 3$, as indicated at the top of each panel, $l = 1, 2, 0$ modes lie along the left, middle, and right ridge, respectively.

5 STOCHASTIC VS MIRA-LIKE EXCITATION IN SEMIREGULAR VARIABLES

To address the question of mode excitation in SRs, we consider the properties characterizing solar-like oscillations. If a detected mode is resolved into a Lorentzian profile, its amplitude in the Fourier spectrum decreases with increasing length of the time series. However, if a detected mode is unresolved, and hence can be described by a sinc function, its amplitude does not depend on the length of the time series. Bearing this in mind, we cut the total light curve for each star into three segments with equal length, and selected the one with the highest duty cycle to measure the pulsation period and amplitude in the same way as before (see Section 3).

Figure 5 shows a comparison of the dominant mode amplitudes, measured from the full-length and 1/3-length light curves. We observe clearly a systematic offset in the amplitude ratios when the amplitude is less than 0.1 mag, or $P \approx 70$ days. Above this boundary, the scatter in the amplitude ratios drops significantly, particularly in the top panel. The offset and scatter confirm that solar-like oscillations can be used to explain the pulsations in SRs (Christensen-Dalsgaard et al. 2001; Bedding 2003). Indeed, four years of Kepler light curves are capable of resolving, or at least partially resolving, solar-like oscillations. For the Miras (the pink asterisks), the amplitude ratios are almost unity, and the scatter of the ratios is much smaller than those of SRs. It means for the Miras the pulsation amplitudes do not depend on the length of the light curves, which supports the self-excited driving mechanism in Miras. The offset in the amplitude ratio as a gradually decreasing function of amplitude tells us that the mode lifetime in Miras are much longer than in SRs and the transition starting at amplitude $\approx 0.1$ mag, or $P \approx 70$ days, appears continuous.
6 RADIAL ORDER ASSIGNMENT

6.1 Radial order assignment by theoretical frequencies

Figures 6 and 7 show the stacked power spectra of 2000 stars with clearly detected oscillations, for which the $v_{\text{max}}$ values are in the range $0.15 \mu Hz \leq v_{\text{max}} \leq 10.54 \mu Hz$ (period 1.1-77 d). For this, we used dedicated procedures (see below) to detect the clear pulsation ridges corresponding to multiple angular degrees $l = 0, 1, 2$ for a number of radial orders $1 \leq n \leq 7$. The basic idea in constructing these ridges is to construct a series of template power spectra as references, and then shift an observed target power spectrum to align with the references. The method is summarized as follows.

(i) We used the SYD pipeline (Huber et al. 2009) to prepare background-divided power spectra, and to measure global seismic parameters, $v_{\text{max}}$ and $\Delta v$.

(ii) We then created template spectra from model frequencies computed by Stello et al. (2014) for a fixed stellar mass of $1M_\odot$. Both $v_{\text{max}}$ and $\Delta v$ were derived from seismic scaling relations for each stellar model. From the frequencies of a given stellar model, we built a template spectrum, where each mode was described by a Lorentzian profile and its height was modulated by a Gaussian envelope. The Gaussian envelope was centered at $v_{\text{max}}$ with a standard deviation of $\Delta v$, hence FWHM $\approx 2.4 \Delta v$. Both $v_{\text{max}}$ and $\Delta v$ were subsequently interpolated with 100-times finer step sizes, as well as the associated model frequencies. Since the model frequencies were only available for models with $v_{\text{max}} \geq 0.2 \mu Hz$, we linearly extrapolated the model frequencies with associated $v_{\text{max}}$ down to $0.1 \mu Hz$.

(iii) For each target spectrum, we searched for its best-matching template spectrum by choosing the one whose maximum cross-correlation with the target spectrum was the greatest. We then shifted the target spectrum with respect to the best-matching template spectrum by an offset equal to the lag of the maximum cross-correlation. When
Figure 8. Oscillation patterns for (a) $l = 1$, (b) $l = 2$, and (c) $l = 0$ modes of high-luminosity red giants. (d) The combination of the $l = 1, 2, 0$ oscillation patterns. The horizontal axes are the frequency divided by measured $\Delta \nu$, while the vertical axes are frequency. For each star only one $l = 1, 2$, and 0 mode are shown.

The shifted spectra were stacked they formed clear ridges, as seen in Figure 6 and 7. Note that a spectrum was not used if the shift was greater than $1/2 \Delta \nu$, to avoid the observed and template spectra being mismatched by one or more radial orders (only 7 stars were discarded in this step).

(iv) Lastly, we sorted the shifted observed spectra by $\nu_{\text{max}}$ for Figure 6 and $\nu_{\text{max}}/\Delta \nu$ for Figure 7. Here, the values of $\nu_{\text{max}}$ and $\Delta \nu$ were not the ones measured using the SYD pipeline, but the ones corresponding to the best-matching template spectra.

Figure 6 displays the radial orders of $l = 0$ modes measured from the model frequencies as indicated at the top of each panel (the following mentioned $n$ values refer to the radial order of $l = 0$ modes, unless specifically stated). We have used an independent method to confirm this radial order assignment, which will be presented in the next section. Stello et al. (2014) firstly recognized triplet structures that consist of dipole ($l = 1$) modes to the left, quadrupole ($l = 2$) modes in the middle, and radial ($l = 0$) modes to the right. The so-called $f$-mode ridge (Stello et al. 2014), which would lie to the left of the ridge $n = 1&l = 0$, is not clearly detected. In Figure 6 we can see the triplet structure is gradually resolved towards larger $\nu_{\text{max}}$ values and higher radial orders.

We again show the stacked power spectrum in Figure 7, now with the horizontal axes divided by $\Delta \nu$. Figure 7 shows more clearly sub-ridges related to multiple angular degrees. Here, we can see the ridge of the first radial overtone ($n = 2$) is marginally resolved in the range $0.45 \mu \text{Hz} \leq \nu_{\text{max}} \leq 0.64 \mu \text{Hz}$, and the individual components of the triplet structure gradually merge towards the lower-$\nu_{\text{max}}$ end. Longer light curves are thus required to resolve triplet structures for the radial order $n = 2$ and $\nu_{\text{max}} < 0.45 \mu \text{Hz}$, and for the entire ridge of the radial fundamental mode. The red lines in the lower panels of Figure 7 indicate the model frequencies, which match the observations well.

The well-resolved ridges separated by the so-called small and large frequency separations for various radial orders at such late evolutionary phase ($\nu_{\text{max}}$ down to 0.14 $\mu \text{Hz}$) resemble the ridges seen in less-luminous red giants (e.g. Mosser et al. 2011). This implies that the asymptotic relation of acoustic modes remains helpful for assigning radial orders and angular degrees, although the relation is expected to break down at low radial orders. The asymptotic relation is given as (Tassoul 1980):

$$\nu_{n,l} = \Delta \nu (n + \frac{l}{2} + \epsilon) - \delta \nu_{0,l}.$$
Figure 9. (a) Same as Figure 8c now only for two ridges for which we aim to measure their radial orders. (b) The relation between $\Delta \nu$ and $\epsilon$, where $\epsilon$ is measured via Equation 2 by assigning a radial order of 6 to the ridge in red and 7 to the ridge in blue. (c) Same as (b) now color-coded by effective temperature.

where $\nu_{el}$ is the eigenfrequency at radial order $n$ and angular degree $l$, $\epsilon$ is an offset parameter, and $\delta \nu_{0,l}$ is the small frequency separation between radial and non-radial modes.

6.2 Radial order assignment by peak-bagging

In order to confirm the radial order assignment given by model frequencies, we aim to determine the radial orders from the observations by measuring mode frequencies, $\Delta \nu$ and $\epsilon$. For this, one of the most important steps is to make good initial guesses of mode frequencies. Here, for each star we used the frequencies from the best-matching template spectrum, as defined in Section 6.1. We then fitted three Lorentzian profiles to the $l = 1, 2, 0$ modes. Only the triplet structure with the largest power, thus closest to the $\nu_{max}$, was picked.

Figure 8 shows oscillation patterns of $l = 1, 2, 0$ modes for stars with $\nu_{max} > 1.0 \mu Hz$. This is analogous to the pattern seen in higher-$\nu_{max}$ red giants (Bedding et al. 2010; Huber et al. 2010; Mosser et al. 2011), here restricted to low-$\nu_{max}$ stars. The horizontal scatter increases with decreasing radial order, which is mainly because the mean large frequency separation $\Delta \nu$ is less well defined towards lower-$\nu_{max}$ stars.

To assign a radial order using Equation 2, we started with the two ridges on the far right in Figure 8c ($n = 6, 7$). This is because (1) for radial modes, the small frequency separation term, $\delta \nu_{0,l}$, is zero; (2) the associated $\Delta \nu$ can be measured more precisely, as indicated by the much smaller scatter, compared to the $n = 4$ and 5 ridges; (3) the asymptotic relation works more accurately at higher radial orders; and (4) the lower radial order can be easily deduced once higher radial orders are identified.

Figure 9a shows the two radial-mode ridges with $\nu_{max}$ in the range $4 \mu Hz < \nu_{max} < 17 \mu Hz$. Figure 9b shows $\Delta \nu$ as a function of the offset $\epsilon$ that was computed using Equation 2 and by assuming $n = 6$ for the red ridge and $n = 7$ for the blue ridge. Clearly, the $\epsilon$ values are collectively smaller than unity, which is in agreement with Mosser et al. (2011) and Kallinger et al. (2012) for the stars in their samples that overlap in $\nu_{max}$ with ours. This result confirms the ridges in red and blue correspond to the radial orders of 6 and 7, respectively. This radial order assignment is thus nicely consistent with the assignment given by the model frequencies as shown above. Figure 9c shows that for our sample the offset $\epsilon$ is an increasing function of $\Delta \nu$, and $\Delta \nu$ is an increasing function of effective temperature (see the colourbar). This means $\epsilon$ increases with increasing effective temperature. This relation between the offset $\epsilon$ and effective temperature has also been found in both dwarfs and giants (White et al. 2011, 2012; Lund et al. 2017). With $n = 6$ and $n = 7$ confirmed, the identifications for $n = 1$ to 5 follows from the discussions in Section 6.1.

To summarize, from the analyses in Section 6 and the
Figure 10. Relative amplitude of $l = 1, 2, 0$ modes across various radial orders, as indicated by the legends. As an example, the three peaks from left to right in Panel (b) correspond to $(n, l) = (1, 1), (1, 2), \text{ and } (2, 0)$ modes, respectively. Dipole modes are dominant in the radial orders of $n = 3, 4, 5, 6$, while quadrupole modes are dominant in the radial order of $n = 2$. The triplet structure shrinks gradually toward lower radial orders, and merges eventually in the radial orders of $n = 1$.

ARE DIPOLE MODES DOMINANT AMONG THE PULSATIONS OF LPVS?

Which modes are dominant, radial or non-radial modes? To answer this question, we measured relative amplitudes of $l = 0, 1, 2$ modes for the radial orders $1 \leq n \leq 6$.

In order to measure the amplitude for a given $n$ and $l$, we used the stacked power spectrum, as shown in Figure 7, and summed up the amplitude along the associated ridge indicated by the red lines in Figures 7c and 7d. The collapsed total amplitude was evaluated over the $\nu_{\text{max}}$ ranges equal to the length of the red lines. For each star, its background-divided power spectrum was normalized so that the amplitude of the highest peak was unity. Lastly, for each radial order, the collapsed amplitude was normalized to set its highest peak to unity. The results are shown in Figure 10.

Figure 10a shows only a single peak for $n = 1$. Interestingly, for $n = 2$, the middle peak ($l = 2$ modes) is globally the highest. This property is distinct from the higher radial orders $3 \leq n \leq 6$, for which all the collectively dominant modes are dipole modes. Note that, as shown in Figure 7c, for radial order $n = 2$ the dominant $l = 2$ mode is marginally visible at $\nu_{\text{max}} = \approx 0.55 \mu$Hz, and the $l = 2, 0$ ridges are gradually merged as $\nu_{\text{max}}$ decreases.

Our findings of the dominant $l = 1$ modes with radial order $n \geq 3$ are consistent with the results by Mosser et al. (2013) (see their Figure 9) and Stello et al. (2014). However, the findings of the dominant $l = 2$ modes in radial order $n = 2$ (at least in a higher $\nu_{\text{max}}$ range) is inconsistent with Mosser et al. (2013) who argued that radial modes are dominant when $\nu_{\text{max}} \lesssim 1.0 \mu$Hz. We note Stello et al. (2014) clearly detected the $n = 2$ ridge but did not resolve the associated sub-ridges.

Another interesting feature shown in Figure 10 is that the triplet structure gets more narrow with decreasing radial order. This feature makes it difficult to resolve multiple angular degrees, given the 4-year baseline of Kepler light curves. The closely spaced triplet structures are very different in the LPVs than in less-luminous red giants. For the latter, $l = 1$ modes are nearly located at the midpoint of adjacent $l = 0$ modes (e.g. Huber et al. 2010, see their Figure 10). OGLE data will be valuable for studying the unresolved comparison between the Kepler and OGLE LPVs (Figure 4b), we provide a solution to the open question on the radial order assignment of LPVs as detailed in the introduction. Our results confirm that the radial orders of $n = 1, 2, 3, \text{ and } 4$ can be used to explain the sequences C, C$'$ and B, A, and A$'$ in the P–L diagram, respectively, which is consistent with the recent theoretical explanations by Trabucchi et al. (2017).
or marginally resolved triplet structures, thanks to 4 years of data set from the OGLE-II project and 8 years of observations from the OGLE-III project. This will be presented in a future paper.

8 TESTING SEISMIC SCALING RELATIONS FOR HIGH-LUMINOSITY RED GIANTS

Since the seismic scaling relations provide an efficient way to derive stellar fundamental properties, such as mass and radius, their validity has been extensively tested on dwarfs and giants (see Chaplin & Miglio 2013; Hekker & Christensen-Dalsgaard 2017, for reviews). It seems inevitable that the seismic scaling relations should break down at a certain evolutionary stage, at least in Mira variables in which pulsations are self-excited via a heat-engine mechanism, which is different from solar-like oscillations. We will test the seismic scaling relations for high-luminosity red giants.

Figure 11 shows a comparison between radii calculated from the seismic scaling relations and radii derived from the Gaia DR2 parallaxes (Lindegren et al. 2018). To calculate the seismic radii, we use the following relation:

\[ \frac{R}{R_\odot} \approx \left( \frac{v_{\text{max}}}{v_{\text{max},\odot}} \right) \left( \frac{\Delta v}{\Delta v_\odot} \right)^{-2} \left( \frac{T_{\text{eff}}}{T_{\text{eff},\odot}} \right)^{1/2}, \]

where the solar references are \( v_{\text{max},\odot} = 3090 \ \mu\text{Hz}, \Delta v_\odot = 135.1 \ \mu\text{Hz} \) (Huber et al. 2011), and \( T_{\text{eff},\odot} = 5777 \ \text{K} \). The global seismic parameters, \( v_{\text{max}} \) and \( \Delta v \), were measured using the SYD pipeline (Huber et al. 2009), which gave very good agreement with Yu et al. (2018) (The mean differences were 0.8% in \( v_{\text{max}} \) and 0.2% in \( \Delta v \) for 692 stars in common) and Pinsonneault et al. (2018) (The mean differences were 1.3% in \( v_{\text{max}} \) and 0.2% in \( \Delta v \) for 531 stars in common). Effective temperatures in this work were adopted from Mathur et al. (2017), which were mainly based on photometry, and are also consistent with APOGEE spectroscopic temperatures from Pinsonneault et al. (2018). We then derived radii from Gaia DR2 parallaxes using the same method as Huber et al. (2017) and Berger et al. (2018). We applied a cut to fractional parallax uncertainty, namely, \( \sigma_\pi/\pi < 0.6 \). From this, we obtained a sample of 2241 LPVs with both seismic and parallax-based radii available.

Figure 11 shows the relative radius difference as a function of \( v_{\text{max}} \), colour-coded by the fractional uncertainty of Gaia DR2 parallax, \( \sigma_\pi/\pi \), and distance. We can see that
Figure 12. (a) Seismic H-R diagram ($\nu_{\text{max}}$ vs $T_{\text{eff}}$), color-coded by the oscillation amplitude per radial mode. The estimates of $\nu_{\text{max}}$ and oscillation amplitude were adopted from Huber et al. (2011) for main-sequence and subgiant stars, Yu et al. (2018) for low/intermediate luminosity red giants, and this work for high-luminosity red giants ($\nu_{\text{max}} < 15 \mu\text{Hz}$). We take effective temperatures from Mathur et al. (2017), but update them wherever temperature is cooler than 3200 K (for the scheme see the text). The dashed line indicates the long-cadence Nyquist frequency, below which it is very challenging to detect the oscillations using the Kepler long cadence data. (b) H-R diagram. The values of luminosity are calculated from Gaia DR2 parallaxes, either from this work or Berger et al. (2018).

9 HERTZSPRUNG-RUSSELL DIAGRAM OF KEPLER OSCILLATORS

Figure 12a shows a seismic H-R diagram, color-coded by the oscillation amplitude per radial mode. We note that for the stars in our sample with $T_{\text{eff}} < 32000$K from Mathur et al. (2017), $T_{\text{eff}}$ was poorly determined. For this, we updated their temperatures by using g-K$_s$ colour calculated from SDSS g and 2MASS K$_s$ magnitude and following the empirically calibrated scheme from Huang et al. (2015). Extinctions were calculated using the method by Huber et al. (2017) and Berger et al. (2018) and corrected by adopting the extinction laws from Yuan et al. (2013). Since $\nu_{\text{max}}$ and amplitude cannot be measured for all the stars in our high-luminosity red-giant sample in the same way as for lower-luminosity stars, we used the frequency and height of the highest peak to represent $\nu_{\text{max}}$ and the oscillation amplitude.

We note that the group of Miras (green dots) have higher amplitudes than expected from the trend of the stars with higher $\nu_{\text{max}}$. This is because pulsations in Miras are driven differently from the rest of the sample. This seismic H-R diagram indicates that both $\nu_{\text{max}}$ and the amplitude span more than six orders of magnitude, which represents so far the largest parameter ranges measured only from observations.
Considering the availability of Gaia DR2 parallaxes for almost all of the stars shown in Figure 12a, we plot a H-R diagram as shown in Figure 12b, where luminosities were computed from Gaia DR2 parallaxes either from this work for our sample or from Berger et al. (2018) for the rest of the stars shown in Figure 12b. We can see a relatively larger scatter present in luminosity than $\nu_{\text{max}}$. For the stars with $T_{\text{eff}} \leq 3600$ K their luminosities appear to be underestimated. We suspect this offset could be caused by possible under-estimated interstellar reddening and/or un-corrected substantial circumstellar extinction due to mass loss.

10 CONCLUSIONS

We carried out asteroseismic analyses of high-luminosity Kepler red giants with pulsation periods $P \geq 1$ day. We attempted to address open questions regarding the excitation mechanisms, radial order assignment, and dominant mode nature (radial or non-radial). We also investigated the relation between pulsation amplitude and period for low-$\nu_{\text{max}}$ sun-like oscillators ($\nu_{\text{max}} \lesssim 10 \mu$Hz), SRs, and Mira variables. For the first time we performed a test on the validity of the seismic scaling relations with high-luminosity Kepler red giants using Gaia DR2 parallaxes. The main results are summarized below:

(i) Pulsation amplitude varies significantly with period as pulsations undergo the transition from $n = 5$ to $n = 4$. This suggests a variation of mode lifetime across different radial orders (see Figure 2).

(ii) By comparing the amplitudes measured from full-length Kepler light curves and 1/3 shorter segments of the light curves, SRs are confirmed to be stochastically excited as solar-like oscillators, which is different from self-excited Mira variables. Using the same method, we find Mira variables have much longer mode lifetime than SRs, and the lifetime of SRs changes continuously (see Figure 5).

(iii) We have made an unambiguous detection of well-resolved pulsation ridges, corresponding to radial fundamental mode and overtones ($2 \leq n \leq 7$), and sub-ridges, linked to $l = 0, 1, 2$ modes (see Figure 6). Our radial order assignment from the two ways (model frequencies and peak-bagging) is consistent with Stello et al. (2014), and Mosser et al. (2013) for $n \geq 3$ but not for $n = 2$ (see Figure 7 and 8).

(iv) Clear pulsation sequences on the P-L diagram have not been detected in Kepler LPVs, which are expected to be present as for the OGLE LPVs in the LMC. The approximate six times larger uncertainty in absolute magnitude, $\sigma_{M_K}$, for Kepler LPVs makes the ridges difficult to be detected (see Figure 4).

(v) We show that the $l = 1$ modes are dominant in the overtones of $n=3, 4, 5, 6$, while the $l = 2$ modes appear to be dominant in the first overtone $n = 2$ (see Figure 10). Since the triplet structure gets gradually closer with decreasing pulsation frequency, longer time series are required to resolve multiple angular degrees. OGLE light curves with a typical baseline of 8 years, with an extension to 12 years for some pulsators, are thus very valuable to resolve the first overtone and radial fundamental modes.

(vi) A comparison of radii computed from the scaling relations to those derived from Gaia DR2 parallaxes shows good agreement, with an increasing systematic offset when $\nu_{\text{max}} \lesssim 3 \mu$Hz ($R \gtrsim 40 R_{\odot}$, or $\log L/L_{\odot} \gtrsim 2.6$). This suggests the seismic scaling relations could break down in this regime. On the other hand, the comparison shows an excellent agreement where $\nu_{\text{max}} \gtrsim 3 \mu$Hz, implying that the scaling relations are still accurate. This also confirms the existence of the 0.03 mas systematic offset in Gaia DR2 parallaxes (see Figure 11).

**ACKNOWLEDGEMENTS**

We thank Jennifer van Saders for comments that improved the quality of this paper. We gratefully acknowledge the entire Kepler team and everyone involved in the Kepler mission for making this paper possible. Funding for the Kepler Mission is provided by NASA’s Science Mission Directorate. This work was supported in part by the German space agency (Deutsches Zentrum für Luft- und Raumfahrt) under PLATO data grant 500O1501. The computational resources were provided by the German Data Center for SDO through a grant from the German Aerospace Center (DLR). D.H. acknowledges support by the National Science Foundation (AST-1717000). D.S. is the recipient of an Australian Research Council Future Fellowship (project number FT1400147). The research leading to the presented results has received funding from the European Research Council under the European Community’s Seventh Framework Programme (FP7/2007-2013) / ERC grant agreement no 338251 (StellarAges).

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**Kepler Long Period Variables**

We thank Jennifer van Saders for comments that improved the quality of this paper. We gratefully acknowledge the entire Kepler team and everyone involved in the Kepler mission for making this paper possible. Funding for the Kepler Mission is provided by NASA’s Science Mission Directorate. This work was supported in part by the German space agency (Deutsches Zentrum für Luft- und Raumfahrt) under PLATO data grant 500O1501. The computational resources were provided by the German Data Center for SDO through a grant from the German Aerospace Center (DLR). D.H. acknowledges support by the National Science Foundation (AST-1717000). D.S. is the recipient of an Australian Research Council Future Fellowship (project number FT1400147). The research leading to the presented results has received funding from the European Research Council under the European Community’s Seventh Framework Programme (FP7/2007-2013) / ERC grant agreement no 338251 (StellarAges).
