Gravitational Baryogenesis in Anisotropic Universe

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Abstract

The interaction between Ricci scalar curvature and the baryon number current, dynamically breaks CPT in an expanding universe and leads to baryon asymmetry. Using this kind of interaction and study the gravitational baryogenesis in the Bianchi type I universe. We find out the effect of anisotropy of the universe on the baryon asymmetry for the case which the equation of state parameter, \( \omega \), is dependent to time.

Keywords: Baryon Asymmetry; Baryogenesis; Gravitational Interaction.

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1 Introductions

Theoretical prediction of antimatter is one of the most impressive discoveries of quantum field theory which made by Paul Dirac about 80 years ago [1]. Some scientist thought that "maybe there exists a completely new universe made of antimatter" because they believe that there is a symmetry between matter and antimatter. Our present point of view on being symmetry between matter and antimatter is very much different, even opposite. The absence of γ-ray emission from matter-antimatter annihilation [2] and the theory of Big Bang nucleosynthesis [3], the measurements of cosmic microwave background [4], indicate that there is more matter than antimatter in the universe. So that, we sure that antimatter exists but believe that there is an asymmetry between matter and antimatter. The origin of the difference between the number density of baryons and anti-baryons is still an open problem in particle physics and cosmology. Observational results yield that the ratio of the baryon number to entropy density is approximately $n_b/s \sim 10^{-10}$. The standard mechanism of baryogenesis is based on the following three principles as it was formulated in 1967 by A. D. Sakharov [5]:

1. Non-conservation of baryons. It is predicted theoretically by grand unification [6] and even by the standard electroweak theory [7].

2. Breaking of symmetry between particles and antiparticles, i.e. C and CP. CP-violation was observed in experiment in 1964 [8]. Breaking C-invariance was found earlier immediately after discovery of parity non-conservation [9].

3. Deviation from thermal equilibrium. This is fulfilled in nonstationary, expanding universe for massive particles or due to possible first order phase transitions.

Similarly, in [10], a mechanism for baryon asymmetry was proposed. They introduced an interaction between Ricci scalar curvature and any current that leads to net $B - L$ charge in equilibrium ($L$ is lepton number) which dynamically violates CPT symmetry in expanding Friedmann Robertson Walker (FRW) universe. The proposed interaction shifts the energy of a baryon relative to an antibaryon, giving rise to a non-zero baryon number density in thermal equilibrium. The author of [11] studied the some mechanism of baryon asymmetry which was proposed in [10] for the case which the equation of state parameter of the universe, $\omega$, is dependent to time.
Some of another investigations about gravitational baryogenesis are done in [11], [12], [13], [14], [15], [16], [17].

In this paper, we study the gravitational baryogenesis in the Bianchi type I universe. We assume the universe is filled with to components of perfect fluid and study this model for different cases. We will study the effect of anisotropy and interaction between two different components of perfect fluid on $\dot{R}$ and consequently on gravitational baryogenesis.

## 2 Preliminary

The gravitational field in our model is given by a Bianchi type I metric as

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2,$$

where, the metric function, $A$, $B$, $C$, being the function of time, $t$, only. We assume the matter is perfect fluid, then the energy momentum tensor is given by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu},$$

where $u^\nu$ is the four vector satisfying

$$u^\nu u_\nu = -1,$$

and $\rho$ is the total energy of a perfect fluid and $p$ is the corresponding pressure. $p$ and $\rho$ are related by an equation of state as

$$p = \omega \rho.$$  

One can obtain the Einstein field equations form the BI space time as

$$8\pi Gp = -\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC},$$

$$8\pi Gp = -\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC},$$

$$8\pi Gp = -\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} - \frac{\dot{A}\dot{B}}{AB},$$

$$8\pi Gp = \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC},$$

3
where $G$ is the Newtonian gravitational constant and over-dot means differentiation with respect to $t$. Using Eqs. (5-8), we can obtain the Hubble parameter as

$$H = \frac{1}{3} \frac{\dot{\theta}}{\theta} = \frac{1}{3} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = \frac{\dot{a}}{a},$$

(9)

$$H^2 = \frac{1}{3} \left( 8\pi G \rho + \sigma^2 \right),$$

(10)

$$\dot{H} = -4\pi G (\rho + p) - \sigma^2,$$

(11)

$$\dot{\sigma} + \sigma \theta = 0,$$

(12)

where $a = (ABC)^{\frac{1}{3}}$ is the scale factor, and $\sigma^2 = 1/2\sigma_{ij}\sigma^{ij}$ in which $\sigma_{ij} = u_{i,j} + \frac{1}{2}(u_{i;k}u^k u_j + u_{j;k}u^k u_i) + \frac{1}{2}\theta(g_{ij} + u_{i}u_{j})$ is the shear tensor, which describes the rate of distortion of the matter flow, and $\theta = u_{i}^{;j}$ is the scalar expansion. The equation of state parameter, $\omega$, can be expressed in terms of the Hubble parameter and shear tensor as

$$\omega = -1 - \frac{2(\dot{H} + \sigma^2)}{3H^2 - \sigma^2}.$$  

(13)

We obtain the Ricci scalar as

$$R = 3H^2(1 - 3\omega) + \sigma^2(3\omega - 1).$$

(14)

By derivating of $R$ and use of Eq. (11) and (12), it is shown that

$$\dot{R} = \frac{\sqrt{3}}{M_p^2}(1 + \omega)(3\omega - 1)\rho \sqrt{\rho + \sigma^2 M_p^2} - \frac{3}{M_p^2} \rho \dot{\omega},$$

(15)

where, $M_p \simeq 1.22 \times 10^{19}\, GeV$ is the Planck mass. If the space time will be isotropic, $\sigma = 0$, Eq. (15) reduce to the result of [11]. Also, if $\dot{\omega} = 0$, only the first term reminds and it is zero at $\omega = 1/3$ and at $\omega = -1$. Therefore by taking into account $\dot{\omega}$, we have baryon asymmetry at $\omega = 1/3$ and at $\omega = -1$, because $R \neq 0$

3 Perfect Fluid with Interaction.

In the following we consider our study with universe dominated by two interacting perfect fluids with equation of states as

$$p_d = \gamma_d \rho_d,$$

(16)
\[ p_m = \rho_m \gamma_m. \tag{17} \]

We assume that the conservation relation of energy for these two components are
\[
\dot{\rho}_d + \theta(\rho_d + p_d) = \Gamma_1 \rho_d + \Gamma_2 \rho_m, \tag{18}
\]
\[
\dot{\rho}_m + \theta(\rho_m + p_m) = -\Gamma_1 \rho_d - \Gamma_2 \rho_m, \tag{19}
\]

where \( \Gamma_1 \rho_d + \Gamma_2 \rho_m \) is the source term of interaction and \( \Gamma_1 \) and \( \Gamma_2 \) may be time dependent [19], [20], [21], [22], [23]. Although Eq. (18), and (19) do not satisfy the conservation equation, but we have
\[ \dot{\rho} + \theta(1 + \omega)\rho = 0. \tag{20} \]

Here \( \rho = \rho_d + \rho_m \) and \( p = p_d + p_m \) and
\[ \omega = \frac{\gamma_d + \gamma_m r}{1 + r}, \tag{21} \]

where \( r = \rho_m / \rho_d \). Using Eqs. (16-20), we obtain
\[ \dot{\omega} = \frac{\dot{\gamma}_d + r \gamma_m}{1 + r} - \frac{(\gamma_m - \gamma_d)(\Gamma_1 + r \Gamma_2)}{1 + r} - \frac{\theta(\gamma_m - \gamma_d)^2 r}{(1 + r)^2}. \tag{22} \]

From the third term of Eq. (22), it is seen that even for constant equation of state, \( \omega \) varies with time. This is due to that the universe is supposed to be filled of components with different equation of state parameters. Substituting Eq. (22) into Eq. (15), give
\[ \dot{R} = -\frac{3 \rho(\gamma_d + \gamma_m r)}{M_p^2(1 + r)} - \frac{\sqrt{3} \rho \sqrt{\rho + M_p^2 \sigma^2}}{M_p^2} \times \left[ (1 + \gamma_m)(1 - 3 \gamma_m)r^2 + 2(1 - \gamma_m - \gamma_d - 3 \gamma_m \gamma_d)r + (1 - 3 \gamma_d)(1 + \gamma_d) \right] \]
\[ + \frac{3 \rho}{M_p^2} \left( \frac{(\gamma_m - \gamma_d)(\Gamma_1 + \Gamma_2 r)}{(1 + r)} + \frac{\sqrt{3} r(\gamma_m - \gamma_d)^2 \sqrt{\rho + M_p^2 \sigma^2}}{M_p(1 + r)^2} \right). \tag{23} \]

We want to check this result for some components.
3.1 Radiation Dominant

In this subsection we suppose one of the fluid components correspond to radiation. To do this, we take $\gamma_m = 1/3$ so that $\dot{\gamma}_m = 0$ and therefore Eq. (23) reduces to

$$
\dot{R} = \frac{\sqrt{3}}{M_p^3(1+r)} \rho \left[ \sqrt{\rho + M_p^2 \sigma^2} (1 + \gamma_d) (3\gamma_d - 1) - \frac{3\rho \gamma_d}{M_p^2(1+r)} \right] + \frac{\rho (1 - 3\gamma_d)(\Gamma_1 + \Gamma_2 r)}{1+r}, \tag{24}
$$

choosing, $\Gamma_1 = \lambda_1 \theta$ and $\Gamma_2 = \lambda_2 \theta$, $\lambda_1, \lambda_2 \in \mathbb{R}$ [20], [21], [22], [23] and one can achieve as

$$
\dot{R} = \frac{\sqrt{3}}{M_p^3(1+r)} \rho \left[ \sqrt{\rho + M_p^2 \sigma^2} (1 + \gamma_d) (3\gamma_d - 1) - \frac{3\rho \gamma_d}{M_p^2(1+r)} \right] + \frac{\rho \theta (1 - 3\gamma_d)(\lambda_1 + \lambda_2 r)}{1+r}. \tag{25}
$$

We assume that the other components which fill the universe, is a massive scalar field of mass $m$, with a time dependent equation of state parameter interacting with radiation. The time dependent equation of state parameter of the massive scalar field as an universe anisotropic universe is define as

$$
\gamma_d = \frac{p_d}{\rho_d} = \frac{1}{2} \dot{\phi}^2 - V(\phi). \tag{26}
$$

Where $V(\phi) = (1/2)m^2 \phi^2$. The interaction between scalar field and radiation is given by Eq. (18), and (19) with $\gamma_m = 1/3$. By defined $z = (1 - \gamma_d)\rho_d/2$ and using $\dot{z} = m\rho_d \sqrt{1 - \gamma_d^2}$, which was defined in [24], we can obtain

$$
\dot{\gamma}_d = -2m \sqrt{1 - \gamma_d^2} + \theta(1 - \gamma_d)[\lambda_1 + r\lambda_2 - (1 + \gamma_d)]. \tag{27}
$$

At last by substituting Eq. (27) into Eq. (25), we get

$$
\dot{R} = \frac{6m \sqrt{1 - \gamma_d^2}}{M_p^3(1+r)} + \frac{2\sqrt{3}}{M_p^3(1+r)} \rho \sqrt{\rho + M_p^2 \sigma^2} (\gamma_d - \lambda_1 - r\lambda_2 + 1). \tag{28}
$$

For the scalar field dominant, which is equivalent with, $r \to 0$, we have

$$
\dot{R}_\phi = \frac{6m \rho \sqrt{1 - \gamma_d^2}}{M_p^3} + \frac{2\sqrt{3}}{M_p^3} \rho \sqrt{\rho + M_p^2 \sigma^2} (\gamma_d - \lambda_1 + 1). \tag{29}
$$
For the case that $\dot{\phi}^2 \gg m^2 \phi^2$ ($\dot{\phi}^2 \ll m^2 \phi^2$) we have $\gamma_d = 1(-1)$ so that $\gamma_d = 0$ therefore we have

$$\dot{R}_\phi = \frac{2\sqrt{3}}{M_p^3} \rho \sqrt{\rho + M_p^2 \sigma^2(2 - \lambda_1)}, \quad (30)$$

or

$$\dot{R}_\phi = -\frac{2\sqrt{3}}{M_p^3} \rho \sqrt{\rho + M_p^2 \sigma^2 \lambda_1}, \quad (31)$$

Eq. (31) shows that if there is no interaction source term with dark matter, i.e. $\Gamma_1 = 0$, then $\dot{R}_\phi \simeq 0$ and in this case there is no any gravitational source for asymmetry in baryon number. On the other hand for the radiation dominant, i.e., $r \to \infty$ we obtain

$$\dot{R}_R = -\frac{2\sqrt{3}}{M_p^3} \rho_R \sqrt{\rho_R + M_p^2 \sigma^2 \lambda_2}, \quad (32)$$

we see that, $\dot{R} \neq 0$ if $\lambda_2 \neq 0$. It is seen that isotropic universe, $\sigma = 0$, Eq. (32) reduce to the result which is obtained in [11]. We can obtain $\rho_R$ as a function of equilibrium temperature, $T$. The radiation energy density is related to the $T$ as [17], [18].

$$\rho_R = K_R T^4, \quad (33)$$

where $K_R$ is proportional to the total number of effectively degree of freedom. So we have

$$\dot{R}_R = -\frac{2\sqrt{3}}{M_p^3} K_R T^4 \lambda_2 \sqrt{K_R T + M_p^2 \sigma^2}. \quad (34)$$

4 Gravitational Baryogenesis in Anisotropic Universe

The author of [10] introduced a mechanism to generate baryon asymmetry. Their mechanism is based on an interaction between the derivative of the Ricci scalar and the baryon number current, $J^\mu$, as

$$\frac{\epsilon}{M^2_*} \int d^4 \sqrt{-g} (\partial_\mu R) J^\mu; \quad (35)$$
where $M_s$ is a cut off characterizing the scale of the energy in the effective theory and $\epsilon = \pm 1$. This interaction violate CP. The baryon number density in the thermal equilibrium, has been worked out in detail in [10]. It lead to
\begin{equation}
  n_b = n_B - n_{\bar{B}} = \frac{g_b T^3}{6\pi^2} \left( \frac{\pi^2 \mu_B}{T} + \left( \frac{\mu_B}{T} \right)^3 \right),
\end{equation}
where $\mu_B$ is a chemical potential and $\mu_B = -\mu_B = -\epsilon \dot{R}/M_s^2$ and $g_b \sim 1$ is the number of internal degrees of freedom of baryons. According to [17], the entropy density of the universe is given by $S = (2\pi^2/45)g_s T^3$, where $g_s \simeq 106$. The ratio $n_b/S$ in the limit $T \gg m_b$ and $T \gg \mu_b$ is given by
\begin{equation}
  \frac{n_b}{S} = -\epsilon \frac{15g_b}{4\pi^2 g_s} \frac{\dot{R}}{M_s^2 T^3}|_{T_D},
\end{equation}
where $T_D$ is called the decoupled temperature and in the expanding universe the baryon number violation decouples at the $T_D$ temperature. Therefore the baryon asymmetry in terms of temperature can be determined from Eq. (34) and (37) as
\begin{equation}
  \frac{n_b}{S} \simeq 2.5 \frac{\lambda_2 g_b T_D^5}{\alpha^2 M_p^3} + 0.04 \frac{\lambda_2 g_b \sigma^2}{\alpha^2 (M_p T_D)^3},
\end{equation}
where $\alpha = M_s/M_p$.

4.1 Baryogenesis with out Interaction

In this subsection we assume $\gamma_m = \gamma_R = 1/3$. We assume $\gamma_d > 1/3$ which is corresponding to non-thermal component and it decrease more rapidly then radiation [10]. If there is no any interaction between these two component of universe, i.e. $\Gamma_1 = \Gamma_2 = 0$ then we have
\begin{equation}
  \dot{\rho}_d + \theta (1 + \gamma_d) \rho_d = 0,
\end{equation}
\begin{equation}
  \rho_R + \frac{4}{3} \theta \rho_R = 0.
\end{equation}
In this case we can arrive at
\begin{equation}
  \dot{R} = \frac{\sqrt{3} \rho \sqrt{\rho + M_p^2 \sigma^2}}{M_p^3} \frac{1 + r}{1 + r} (1 + \gamma_d)(1 - 3\gamma_d),
\end{equation}
and
\begin{equation}
  \dot{r} = r \theta (\gamma_d - \frac{1}{3}),
\end{equation}
it is clearly seen that for $\gamma_d > 1/3$, $i > 0$. This means that $\rho_d$ decreases faster than $\rho_R$. From Eq. (30) we can obtain $\rho_R \propto (ABC)^{-4/3} \propto a^{-4}$ and then the temperature red shift is as $T \propto a^{-1} \propto (ABC)^{-1/3}$ and also from Eq. (39) one can obtain $\rho_d \propto T^{3(1+\gamma_d)}$ [18]. We suppose at $T = T_R$, $\rho_R = \rho_d$ we have

$$\rho_d = \epsilon_R T_R^4 \left(\frac{T}{T_R}\right)^{3(1+\gamma_d)},$$

$$\rho_R = \epsilon_R T_R^4 \left(\frac{T}{T_R}\right)^4,$$

therefore we obtain

$$\frac{n_b}{S} = \frac{g_b(1 + \gamma_d)(1 - 3\gamma_d)}{M^2 M_p} \frac{T^2_R}{T_D} \left(\frac{T_D}{T_R}\right)^{1.5(1+\gamma_d)} \times \left[1.28 \frac{T_R^4}{M_p^2} \left(\frac{T_D}{T_R}\right)^{3(1+\gamma_d)} \sqrt{1 + \left(\frac{T_D}{T_R}\right)^{1-3\gamma_d}} + \frac{0.018\sigma^2}{\sqrt{1 + \left(\frac{T_D}{T_R}\right)^{1-3\gamma_d}}}\right].$$

We assume $T_D = \eta T_R$. The case which $\eta \gg 1$ is equivalence with the state which $r \rightarrow 0$. Hence we have

$$\frac{n_b}{S} \approx \frac{(1 + \gamma_d)(3\gamma_d - 1)}{M^2 M_p} \frac{T^2_R}{T_D} \left(\frac{T_D}{T_R}\right)^{1.5(1+\gamma_d)} \times \left[1.28 \frac{T_D^3}{M_p^2} \left(\frac{T_D}{T_R}\right)^{\gamma_d - 1} + 0.018\sigma^2\right].$$

For $\eta \ll 1$, we can arrive at

$$\frac{n_b}{S} \approx (3\gamma_d - 1) \left[\frac{g_b 4 + 3\gamma_d}{\alpha^2} \left(\frac{T_R}{M_p}\right)^5 + \left(\frac{\sigma}{\alpha}\right)^2 \frac{T_R}{M_p} \eta^{3\gamma_d}\right],$$

it is clearly seen that if $\gamma_d > 1/3$, $n_b/s > 0$.

## 5 Conclusion

The main purpose of the present work has been to explore the consequences of using the anisotropy of metric, (11), as input in Einstein’s equation, assuming that the cosmic fluid is endowed with a perfect fluid. The expression for the energy-momentum tensor $T_{\mu\nu}$ is given in (22). The cosmological constant $\Lambda$ has been set equal to zero. We have obtained the following result, for our studies.
1. We show that the universe which dominated by two interacting perfect fluids has a curvature that varied with time and the effect of anisotropic space time is remarkable.

2. We have obtained $\dot{R}$ for radiation dominant regime and the effect of anisotropy of space time obviously is seen in it.

3. We assume one of the components which fill the universe, is a massive scalar field. We have shown that the effect of shear tensor in $\dot{R}$ is notable and also for scalar field dominant and for the case which kinetic term is negligible with respect to potential term, have obtained $\dot{R} = f(\rho_R, \sigma)\lambda_1$. This result shows that in this case, if $\Gamma_1 = 0$, there is no any gravitational source for asymmetry in baryon number.

4. We have studied the gravitational baryogenesis in anisotropic universe and have obtained the quantity $n_b/s$ for some typical example. We have shown that the baryon asymmetry in anisotropic universe is larger than the baryon asymmetry in Friedmann Robertson Walker (FRW) space time.

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