Proposal of an experimental scheme for realising a translucent eavesdropping on a quantum cryptographic channel

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Abstract

Purpose of this paper is to suggest a scheme, which can be realised with today’s technology and could be used for entangling a probe to a photon qubit based on polarisation. Using this probe a translucent or a coherent eavesdropping can be performed.

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In the last years quantum mechanics properties of states are assuming a technological relevance. Among different applications of quantum mechanics to technology the possibility of transmitting absolutely confidential messages has been of great interest. This is due to the possibility of creating a key for encoding and decoding secret messages by transmitting single quanta between two parties (usually dubbed Alice and Bob). The underlying principle of quantum key distribution (QKD) is that nature prohibits gaining information on the state of a quantum system without disturbing it. Thus possible eavesdropping by a third party (usually dubbed Eve) can be identified. Since the original proposal of quantum cryptography [1], many different protocols for this kind of transmission have been suggested [2].

For example in the BB84 scheme [3] single photons are transmitted from Alice to Bob, preparing them at random in four partly orthogonal polarisation states (at 0° and 90°, 45° and 135° for example). Bob selects the bases for measuring the photons polarisation at random too. Then Alice and Bob communicate on a classical channel the bases they have used (but not the results of course): when they have used the same base Bob knows what polarisation was selected by Alice and can build a key. If a spy (Eve) tries to intercept the message, she will inevitably introduce errors, which Alice and Bob can detect by comparing a subsample of the generated key.

In Ekert’s protocol [4] entangled pairs are used. Both Alice and Bob receive one particle of the entangled pair. Then they perform a measurement choosing among at least three different directions. Again, Alice and Bob communicate on a classical channel the bases they have used: if measurements were performed along parallel axes they are used for generating the secret key. The other measurements can be used for a test of Bell inequalities. If Eve tries to eavesdrop, she inevitably affects the entanglement between the two particles leading to a reduction of the violation of the Bell inequalities, which allows Alice and Bob to recognise the presence of the spy.

Finally, in the B92 protocol [5], Alice sends a state chosen between two non-orthogonal
states to Bob, who performs a measurement using a projection on the subspaces orthogonal to the two states. Then Bob publicly informs Alice when he obtained positive results (but not, of course, which measurement he made). Finally they retain only bits corresponding to these results.

Other schemes have also been proposed [6]. Common characteristics of all these schemes are the presence of a quantum channel, where different quantum states can be transmitted, and of a classic channel which is used for selecting a subsample of the transmitted states and for testing the presence of an eavesdropper. If the classical channel is eliminated, Eve could simply cut the quantum channel and substituting herself as Bob for Alice and as Alice for Bob creating in this way two keys with the two other parties.

Many different experiments have been realised using the former schemes, demonstrating the feasibility of QKD up to a distance of many kilometers [7].

All of them are based on transmission of single photon states, where the alphabet is based either on photon polarisation or on photon phase.

Concerning the strategy of Eve for eavesdropping, the simplest one is when she simply intercepts the state, which Alice has sent. Then she performs a measurement on this state and finally she sends a new state to Bob preparing it according to the result of the measurement.

However, more elaborated strategies have been proposed, where Alice does not stop the transmitted state, but causes it to interact with a second state (an ancilla) and then obtains information about the transmitted qubit thanks to the result of a measurement on the ancilla. This procedure is known as translucent eavesdropping [8].

Finally, Alice can use eavesdropping schemes where her probe interacts with more than one of Alice’s qubits. These schemes are known as coherent or joint attacks [9].
Many interesting studies [10] have been devoted to understanding general conditions for obtaining a secure transmission between Alice and Bob in the presence of an eavesdropper, even using non-ideal channels.

However, as far as we know, no practically realisable scheme for translucent or coherent eavesdropping has been proposed yet.

The purpose of this paper is to suggest a scheme which can be realised with today’s technology and could be used for entangling a probe to a photon qubit based on polarisation. Using this probe Eve can then perform a translucent or a coherent eavesdropping (causing the ancilla interact with more than one transmitted qubit).

The scheme of the eavesdropping apparatus, shown in the figure, is a Mach-Zender interferometer with a Kerr cell on one of the arms inserted on the quantum channel (see the figure).

The input port of the first beam splitter is fed with a single photon of vertical polarisation, which splits on the two interferometer arms. On arm 1 it interacts with the transmitted qubit inside the Kerr cell.

This has no effect except when both the photons interacting in the cell have vertical polarisation (\(|V⟩|V⟩ \rightarrow |V⟩|V⟩e^{iφ}\) (in the following \(H\) will denote the horizontal polarisation).

Let us suppose that the transmitted qubit is in the general form:

\[
|u⟩ = \cos(θ)|H⟩ + \sin(θ)|V⟩ \tag{1}
\]

If we denote the probe photon with \(|p⟩\), the final state is in the entangled form:

\[
|Ψ⟩ = \frac{1}{\sqrt{2}} \left[ \cos(θ)|H⟩|p⟩_1 + \sin(θ)|V⟩|p⟩_1e^{iφ} + i|u⟩|p⟩_2 \right] \tag{2}
\]

where the suffixes after the probe \(|p⟩\) denotes the path followed and where we have considered
a 50% : 50% beam splitter (which allows Eve to obtain the largest information on Alice-Bob transmission).

We have thus obtained the desired entanglement between the probe and the transmitted qubit, which can be used for translucent or coherent eavesdropping.

The possibility of realising this scheme, and thus the interest of it, derives by the fact that, although admittedly very difficult, the Quantum Non Demolition (QND) detection of a single photon is at present possible \[12,13\]. QND measurements of welcher Weg (which path) have already been achieved using 100 meter long optical fiber \[11\]. Of course, the implementation of the present scheme using such devices would be, even though not impossible in theory, almost impossible in practice. The recent discovery of new materials with very high Kerr coupling, could however allow an easier and more realistic implementation of this scheme. Two candidates as Kerr cell with ultra-high susceptibility to be used for this scheme are the Quantum Coherent Atomic Systems (QCAS) \[14,12\] and the Bose-Einstein condensate of ultracold (at nanoKelvin temperatures) atomic gas \[15\]. These are recent great technical improvements which could permit the realisation of small Kerr cells, capable of large phase shift, even with a single photon probe. In fact, both exhibit extremely high Kerr couplings compared to more traditional materials. In particular, the QCAS is rather a simple system to be realised (for a review see \[16\]) and thus represents an ideal candidate in this role. Incidentally, one can notice that Kerr coupling can be further enhanced by enclosing the medium in a cavity \[17\]. The scheme that we propose in this paper could, in principle, be used with relatively small phase shifts too. However, the maximal efficiency is reached when a phase shift of \(\pi\) is produced on the probe by a single photon. Recently a Lukin and Imamoglu’s paper has shown that this result can be effectively reached \[18\]. Experiments addressed to single photon QND, using a Kerr cell, are in progress \[19\].

This recent development of high coupling Kerr cell has already been applied to the proposal of schemes for complete teleportation \[20\], for generating Schrödinger cats and
modulating quantum interference \cite{21,22}, for generating \cite{22} GHZ states \cite{23} and for realizing quantum gates \cite{20,24}.

Incidentally, the use of a Bose condensate could also allow Eve to ”stock” her photon till when the other has reached Bob, using the very low propagation velocity of light inside a suited Bose condensate (as low as few tens of meters per second) \cite{15}.

For the sake of exemplification in the following we consider the application of this scheme to a simple procedure of eavesdropping on a quantum channel where the BB84 protocol is used.

Let us begin with the simple example where Bob performs measurement on the base H, V or in the one at 45° degrees.

If Alice sends H (θ = 0), Bob measures H and Eve has the state

\[
\frac{|p\rangle_1 + i|p\rangle_2}{\sqrt{2}}
\]  

(3)

and thus after the second beam splitter the photon will be detected by the photodector D3. On the other hand, if Alice send V, Bob measures V and, for Eve, only photodetector D4 clicks. In this case no transmission error is inserted on the quantum line by Eve presence, while she obtains a perfect identification of the transmitted qubit.

On the other hand, if Alice sends a photon \(|+\rangle = \frac{|H\rangle + |V\rangle}{\sqrt{2}}\), after the Kerr cell the entangled state is:

\[
|\Psi\rangle = \frac{1}{2\sqrt{2}} \left[ |+\rangle ([1 + e^{i\phi}] |p\rangle_1 + 2i |p\rangle_2) + |-\rangle [1 - e^{i\phi}] |p\rangle_1 \right]
\]  

(4)

where \(|-\rangle = \frac{|H\rangle - |V\rangle}{\sqrt{2}}\).

If \(\phi = \pi\), Eve has a 50 % probability of observing the photon at D3 and a 50 % probability of observing the photon at D4. Also, a 50 % error on Bob measurement is introduced.
The same situation happens when Alice sends a $|−\rangle$ photon.

Altogether, the probability of a successful eavesdropping for Eve is $p = 3/4$, which leads to a capacity of the channel (Alice-Eve) \[ I_{AE} = 1 + p \log_2 p + (1 - p) \log_2 (1 - p) = 0.189 \] \[ (5) \]

As the error rate for Bob is $1/4$, the capacity of the channel Alice-Bob is $I_{AB} = 0.189$ as well.

Of course, a careful spy will not use such a procedure which produces asymmetric errors for the two bases, which can be easily identified.

In general, if Alice sends a generic state $|u\rangle$ the probability that Bob measures $|u\rangle$ as well and Eve sees a photon at D3 is

$$P_{uu}^3 = \frac{1}{2}[1 - \cos^2(\theta) \sin^2(\theta)(1 - \cos(\phi)) + \cos^2(\theta) + \cos(\phi) \sin^2(\theta)]$$ \[ (6) \]

Similarly, in the same situation Eve could observe a photon at D4 with probability:

$$P_{uu}^4 = \frac{1}{2} \sin^4(\theta)(1 - \cos(\phi))$$ \[ (7) \]

With similar notations, if Alice sends the orthogonal state $|v\rangle$:

$$|v\rangle = \cos(\theta)|V\rangle - \sin(\theta)|H\rangle$$ \[ (8) \]

one has

$$P_{vv}^3 = \frac{1}{2}[1 - \cos^2(\theta) \sin^2(\theta)(1 - \cos(\phi)) + \sin^2(\theta) + \cos(\phi) \cos^2(\theta)]$$ \[ (9) \]

and

$$P_{vv}^4 = \frac{1}{2} \cos^4(\theta)(1 - \cos(\phi))$$ \[ (10) \]
Finally, the probabilities corresponding to introducing an error on the Alice-Bob communication are:

\[ P_{uv}^3 = P_{uv}^4 = P_{vu}^3 = P_{vu}^4 = \sin^2(\theta)\cos^2(\theta)\sin^2(\phi/2) \] (11)

From Eqs. 6-11 it follows that if Eve uses the base bisecting the two used by Alice and Bob and intercepts a fraction \( \alpha \) of the transmitted qubits (in order to reduce the errors introduced in Bob measurements), higher information is obtained. In fact, being the error rate on the Alice-Eve channel

\[ q_{AE} = \frac{(P_{uu}^3 + P_{uv}^3 + P_{vu}^4 + P_{uv}^4)}{2} \] (12)

one obtains \( I_{AE} = 0.40\alpha \) (when \( \phi = \pi \)). Furthermore, as this procedure generates symmetric errors for the two bases, it is much more difficult for Bob to distinguish if these derive from noise in the quantum channel or by Eve’s presence.

Let us now analyse the security threshold. The error rate in the Alice-Bob channel is unsafe if the mutual information in the Alice-Bob channel does not exceed the minimum of the mutual information in the Alice-Eve and Eve-Bob channels [26]:

\[ I_{AB} \leq \min(I_{AE}, I_{EB}) \] (13)

This means that whenever \( I_{AB} \) is greater than either \( I_{AE} \) or \( I_{EB} \), then at least in principle there is a way for Alice and Bob to distribute a string of secret information. On the other hand, if \( I_{AB} \) is smaller than \( I_{AE} \) or \( I_{EB} \), no sifting procedure can make the transmission safe.

In [26] it was suggested that this condition may be overly cautious, however in [27] it has been shown that this is not the case for the entangled translucent eavesdropping scenario.

In our case the information on the Eve-Bob and Alice-Eve channels are \( I_{EB} = I_{AE} = \)
0.4\(\alpha\), whilst the error introduced in the Alice-Bob channel becomes 
\[ q_{AB} = \alpha(P_{vu}^3 + P_{uv}^3 + P_{uv}^4 + P_{vu}^4) / 2 = \alpha / 4. \]

Thus, the transmission cannot be considered safe if Eve intercepts a fraction \(\alpha = 0.755\), or larger, of the transmitted photons. This corresponds to an error rate on the Alice-Bob channel \(q_{AB} = 0.189\). Of course, this error is relatively large, but one should not forget that nowadays quantum channels, both in fibers or air, have huge losses. Furthermore, with other protocols this value could be smaller.

If Ekert or BB92 protocols are used, a similar analysis can be carried out. Many papers with general results about eavesdropping and security of quantum channels are already published [10]. We refer to them for general discussion about application of translucent or joint eavesdropping and conditions for obtaining a safe communication in the presence of eavesdropping.

In conclusion, we think that the proposed scheme represents an interesting chance for an experimental realisation of eavesdropping on a quantum channel and, as far as we know, this is the first proposal for a practical realisation of translucent eavesdropping. Of course, this scheme could also be used in any other situation where interaction with a qubit (represented by polarisation properties of a photon) and an ancilla qubit is required.

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Figure Caption

The Alice - Bob cryptographic channel, with the eavesdropping apparatus of Eve, which is constituted of a Mach-Zender interferometer and a Kerr cell. A probe photon can follow the arm of the interferometer where the Kerr cell is posed or the other. If the transmitted photon has a vertical polarisation the probe photon phase is changed. The observation of the probe photon at photodetector D3 or D4 gives Eve information on the transmitted photon polarisation.
