Conformal sparse metasurfaces for wavefront manipulation

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While flat metasurfaces are intensively studied, theoretical modelling of conformal metasurfaces appears to be exceptionally challenging where it demands accurate analysis of the metasurface geometry. Here, it is shown how numerical calculation of Green’s function can be employed to design conformal sparse metasurfaces of arbitrary geometries within the same framework and without any accommodation. Furthermore, following the presented approach, it can be sufficient to realize only electric response to perform an efficient control of wavefronts that might simplify the design and fabrication of conformal metasurfaces.

In the last two decades, metasurfaces that are thin two-dimensional metamaterials have proven themselves as a powerful tool to tailor wavefronts [1,2]. A myriad of both passive and reconfigurable metasurface-based planar devices has been proposed and validated. Nowadays there is an increasing interest in conformal metasurfaces to perfectly match curved shapes [3–11]. As expected, the design and fabrication of conformal metasurfaces are more demanding compared to planar metasurfaces [6]. Adopting flexible substrates to implement metasurfaces opens a new way for integration with other elements and non-planar designs including lightweight wearable devices [6,8,10]. In addition, for example, microwave antennas based on conformal metasurfaces do not only allow one to meet the aerodynamic specifications of aircrafts and satellites [3,11] but also break the fundamental constraint of their planar counterparts. For instance, the aperture of a flat metasurface antenna vanishes when the beam steering angle increases [12].

The design of a conformal metasurface is generally based on geometrical optics approach [3,9,11], where a proper spatial distribution of local reflection (transmission) coefficient should be established along the reflecting (transmitting) metasurface. It has been revealed that this heuristic approach has strong limitations in terms of efficiency and versatility [13,14]. On the other hand, more rigorously, metasurfaces can be described by means of continuous surface impedances [15,16]. Unfortunately, theoretical modelling of conformal metasurfaces appears to be exceptionally challenging, demanding accurate analysis of the metasurface geometry and dealing with curvilinear coordinates [10].

The contribution of this Letter is twofold. First, it is shown how numerical calculation of Green’s function can be employed to design conformal sparse metasurfaces capable of creating arbitrary field patterns for arbitrary external excitations. The proposed approach does not use any complex local coordinate system matched to a particular geometry such that arbitrarily-shaped metasurfaces can be considered without any accommodation. Sparse metasurfaces possess strongly non-local response [17–24] and can be described in terms of neither surface impedance nor local reflection and/or transmission coefficients being not subject to fundamental efficiency limitations as their “dense” counterparts [13,14,21,24,25]. At the same time, the sparseness allows to establish a global theoretical model and get a microscopic insight into the theoretical analysis of conformal metasurfaces. Secondly, it is detailed how to realize these conformal sparse metasurfaces. To describe the design procedure, three semi-cylindrical sparse metasurfaces illuminated by an arbitrary complex wave configuration are experimentally demonstrated at microwave frequencies.

Without loss of generality, we consider the case of TE polarization and a two-dimensional (2D) geometry. A translation symmetry is assumed along one of the three spatial dimensions. We compose a sparse metasurface by a finite set of N loaded wires distributed along the surface of an arbitrarily shaped dielectric substrate. The wires are oriented along the translation-invariant direction. An example presented in Fig. 1(a) illustrates an array of loaded wires on a semi-cylindrical substrate. Microscopically, a loaded wire represents itself as a chain of subwavelength meta-atoms as shown in Fig. 1(b). On the other hand, macroscopically, we model the loaded wire as uniform and having a deeply subwavelength effective radius $r_{eff}$.

Electric field directed along $x$-axis of a background wave radiated by external sources excites polarization currents in the loaded wires. In Fig. 1(a), external sources are depicted as an array of printed dipoles printed on a planar substrate. However, here we do not impose any condition on the external sources unlike plane wave excitation used for majority of metasurfaces presented in literature. The polarization currents $I_q$ excited in the wires can shape the field radiated by a sparse metasurface in accordance with the following equation:

$$E_x(r, \phi) = E_x^{ext}(r, \phi) + \sum_{q=1}^{N} G_{xx}(r, \phi; r_q) I_q,$$  \hspace{1cm} (1)

where $r$ and $\phi$ are polar coordinates: radius and polar angle, respectively. The total field $E_x(r, \phi)$ is represented by the superposition of the wave radiated by external
sources $E_x^{(ext)}(r, \phi)$ and waves $G(r, \phi; \mathbf{r}_q)I_q$ scattered by the wires, with $G(r, \phi; \mathbf{r}_q)$ being the Green’s function corresponding to the $q$th wire at $\mathbf{r}_q$. As the considered system does not have a translational symmetry in the 2D plane, a Green’s function $G(r, \phi; \mathbf{r}_q)$ is a function of the observation point $\mathbf{r}$ and the position of the wire $\mathbf{r}_q$ (not their difference). Importantly, $E_x^{(ext)}(r)$ and Green’s functions should be calculated in the presence of the substrate and all other non-engineered elements of the sparse metasurface (such as for e.g., the substrate supporting external sources).

Sparse configuration of the metasurface allows one to accurately take into account the interactions between the wires via Ohm’s law

$$Z_qI_q = E_x^{(ext)}(\mathbf{r}_q) - Z_q^{(in)}I_q - \sum_{p=1, p \neq q}^N Z_q^{(m)}I_p. \quad (2)$$

A load-impedance density $Z_q$ is a characteristic of a loaded wire and can be engineered for instance by tuning the geometrical parameters of meta-atoms constituting a wire. The right-hand side of Eq. (2) represents the total electric field at the position of the $q$th wire, where $Z_q^{(m)} = -G_{xx}(\mathbf{r}_q, \mathbf{r}_p)$ is the mutual-impedance density (the electric field created by the $p$th wire at the position of the $q$th wire). The separation between two neighboring wires can be arbitrary as long as polarization currents in wires can be approximated by a 2D delta function. As it is demonstrated further, this simple model works surprisingly well even for complex designs. The self-action of the $q$th wire is accounted via the input-impedance density defined as $Z_q^{(in)} = -G_{xx}(\mathbf{r}_q + r_{eff}\hat{\mathbf{r}}_q, \mathbf{r}_q)$, with $r_{eff}$ being the effective radius of the wire. Although being very simple, Eq. (2) has an important practical implication: it allows one to know in advance the impact of one polarization current on another and to accordingly adjust the load-impedance densities. Conceptually, it means that the developed approach is global, being in strong contrast with conventional theoretical models of metasurfaces which are essentially local. Interaction between wires described via the matrix of mutual-impedance densities indicates that sparse metasurfaces are intrinsically strongly non-local.

Meanwhile, by appropriately choosing $Z_q$ and the number of wires $N$, we are then able to construct desirable radiation patterns. Each term on the right-hand side of Eq. (1) at a given distance $r$ (for all $q$) can be approximated by a partial Fourier sum over the polar angle $\phi$: $E_x^{(ext)}(r, \phi) = \sum_{n=-M}^M C_n^{(q)} e^{in\phi}$ and $G_{xx}(r, \phi; \mathbf{r}_q) = \sum_{n=-M}^M C_n^{(q)} e^{in\phi}$. $M$ is the maximum Fourier harmonic defined as the minimum number $M$ such that

$$|C_n^{(ext,q)}|/\max|C_n^{(ext,q)}| \ll 1 \quad (3)$$

for all $r = 1, 2, \ldots$. The number $M$ might affect many parameters (such as geometrical parameters and material properties of a sample, the distance $r$) but the most important one is the physical aperture max$|\mathbf{r}_q - \mathbf{r}_p|$. The larger the aperture the greater is $M$. Evidently, the total electric field $E_x(r, \phi)$ can also be represented by a partial Fourier sum $\sum_{n=-M}^M C_n e^{in\phi}$ and the relation between the Fourier coefficients is as follows:

$$C_n = C_n^{(ext)} + \sum_{q=1}^N C_n^{(q)}I_q, \quad (4)$$

where $C_n^{(ext)}$ and $C_n^{(q)}$ are known. When a sparse metasurface is composed of at least $N = 2M+1$ loaded wires one can establish arbitrary azimuthal field distributions within the functional space of the $2M+1$ Fourier harmonics by adopting the Fourier coefficients $C_n$. Corresponding load-impedance densities can be found from Eq. (2) after solving Eq. (4) with respect to $I_q$, which in this case has a single solution [26]. As a matter of fact, there is no guarantee that analytically found $Z_q$ would not require implementing active and/or lossy elements since $\Re[Z_q] \neq 0$ in a general case. In order to additionally deal with only reactive load-impedance densities $Z_q = i\delta[Z_q]$, one might need a number of wires $N \geq 2M+1$ for constructing arbitrary radiation patterns, as recently discussed in Ref. [20]. Essentially, the procedure described in this paragraph allows one to know in advance all possible configurations of the azimuthal field for a given geometry of metasurface and number of wires. It includes practical parameters such as beamwidth and sidelobes level.

Let us consider a simple example of a cylindrical sparse metasurface for far-field manipulation. The radius of the...
to find the parameter Green’s function and of the background wave in order one has to analyse the Fourier decomposition of only one spectrum to each other (as function of φ on the cylindrical substrate are simply shifted with respect to each other (as function of φ). It means that one has to analyse the Fourier decomposition of only one Green’s function and of the background wave in order to find the parameter M. Following the definition in Eq. (3), from Figs. 2(c) and (d), it can be seen that it is enough to have N = 2 × 10 + 1 = 21 wires in order to be able to construct all possible far-field radiation patterns within the Fourier space of 21 harmonics exp[i(nφ)]. As an illustrative example, one can reconstruct in the far-field the shape of a “heart” after the required corresponding load-impedance densities are found from Eqs. (4) and (2). The required load-impedance densities are plotted in Fig. 2(e) and correspond to passive elements (ℜ[Zq] > 0). The real part of Zq can be engineered in a similar fashion as proposed in in Ref. 27. Resulted far-field pattern is depicted in Fig. 2(f) and compared to the ideal shape of a “heart”. Figure 2(a) shows the corresponding profile of the electric field in the proximity to the metasurface. In order to improve the accuracy of approximating some ideal curve, one needs to increase M which can be done by increasing the size of the metasurface (the radius in the considered case). Finally, it should be noted that, to best of our knowledge, there is no analytical formula for a Green’s function of a system with a cylindrical substrate. Additional details on this example are given in Supplemental Material 28.

To summarize, after establishing the geometry of a sparse metasurface (flat, cylindrical, or any other shape), excitation type and positions of N wires, we are able to calculate the Green’s functions $G_{xx}(r, φ; r_q)$ and the background field $E_{x}^{(ext)}(r, φ)$ radiated by external sources. With Eqs. (1) and (2), a relation between the radiated field and load-impedance densities is established and the procedure related to Eq. (4) is used to find the number of wires and to determine possible functional characteristics of a sparse metasurface such as beamwidth and sidelobes level in beamforming applications. An optimization procedure is brought to solve the inverse scattering problem assuming only reactive load-impedance densities of wires. A crucial point of the above analysis is the Green’s functions which, being defined for an arbitrary finite-size system, do not have any applicable analytical form. Instead, it is suggested to compute them numerically with the help of full-wave numerical simulations.

FIG. 2. (a) Profile of the electric field created by a cylindrical sparse metasurface. The metasurface is represented by 21 loaded wires uniformly distributed along a cylindrical substrate of thickness $λ_0/120$ and having the relative permittivity 2.2. The metasurface is excited by a point source placed in its center. (b) Zoom of a part of the sparse metasurface. (c) $|C_{0}^{(q)}|/\max[|C_{n}^{(q)}|]$ (d) $|C_{n}^{(xx)}|/\max[|C_{n}^{(xx)}|]$ vs. the number n of Fourier harmonic. (e) Load-impedance densities required to approximate the “heart” shaped far-field pattern. (f) Comparison of the far-field pattern created by the sparse metasurface (solid curve) and the ideal “heart” shape (dashed curve).
FIG. 3. (a) A photography of a fabricated sparse metasurface on a thin flexible substrate. (b) Schematics of a printed capacitance (left) and inductance (right), \( w = 0.25 \text{ mm} \) (\( r_{\text{eff}} = w/4 \), see Ref. [29]). (c) Retrieved load-impedance densities (only imaginary parts are shown) of capacitively- and inductively-loaded wires at 5 GHz. (d) A photography of an assembled experimental prototype. (e) Schematic illustration of the experimental setup used to measure the far-field patterns. (f) Measured far-field radiation patterns of the different fabricated samples.

wavefront at the distance \( r \). In beamforming applications, far-field calculations should be performed and the electric field in the far-field as a function of the azimuthal angle is recorded.

To validate the approach, we designed and fabricated three sparse metasurfaces using extremely thin (\( \lambda_0/240 = 0.25 \text{ mm} \)) flexible substrates F4BM220, as highlighted by the photography in Fig. 3(a). The samples operating in the microwave frequency range with a central frequency fixed at 5 GHz were fabricated by means of conventional printed circuit board (PCB) technology. Each sample is composed of twenty nine wires equidistantly distributed along a 5.25\( \lambda_0 \approx 315 \text{ mm} \) long substrate, making the separation between two neighbouring wires approximately equal to \( \lambda_0/5 \). The design of loaded wires is performed within the local periodic approximation developed in [30] which allows one to retrieve load-impedance density. Although this method was originally developed for planar periodic metagratings, it can be successfully applied for curved sparse metasurfaces as well. The wires are built up from printed capacitors (left design of Fig. 3(b)) and inductors (right design of Fig. 3(b)), which provide a wide range of accessible load-impedance densities as shown in Fig. 3(c).

The ultra-thin samples are then conformed to create semi-cylindrical surfaces of 100 mm radius. The external source exciting the samples is made of two microstrip dipole antennas printed on a metal-backed substrate [31], whose design is detailed in the Supplemental Material [28]. The distance between the source and sparse metasurface being 100 mm \(( \approx 1.7\lambda_0 \) ), the background field pattern is neither a plane wave nor a cylindrical wave, as shown in the Supplemental Material [28].

A photography of the assembled prototype is presented in Fig. 3(d).

The three samples were designed to show different beamforming examples: a single broadside beam at \( 0^\circ \), a steered beam at \( 40^\circ \) and a multibeam configuration with two beams at \( \pm 30^\circ \) from broadside. The inverse scattering problem was solved by maximizing the power in the desired direction and minimizing the side lobes level (with respect to the geometrical parameters \( A \) and \( B \) of the loaded wires) radiated by the sparse metasurface in desired directions, and the maximization procedure was implemented as particle swarm optimization [32]. The different configurations were experimentally validated by radiation patterns measurements performed in an anechoic chamber. A horn antenna used a receiver is kept fixed and the assembled prototype is mounted on a rotating platform as shown in Fig. 3(e). Figure 3(f) presents the experimental results obtained from the three samples. The level of spurious scattering (at the operating frequency) does not exceed \(-12 \text{ dB}\) for the first sample, \(-9 \text{ dB}\) for the second one and \(-13 \text{ dB}\) for the third one.

It is important to note that in the demonstrated numerical and experimental examples, the sparse metasurfaces are transmitting while possessing only electric response. On the other hand, conventional phase gradient approach to design transmitting metasurfaces demands implementing additionally to the electric response an effective magnetic one. It is necessary to suppress reflection and achieve 2\( \pi \)-range phase response to be able to establish a required phase gradient along the metasurface [1, 3, 7, 8, 11]. A more rigorous approach is certainly based on engineering of electric and magnetic surface impedances [2, 10, 16, 24] (and sometimes electro-
magnetic coupling \(25\, 33\, 34\) to manipulate wavefronts according to the equivalence theorem \(2\, 35\). Following the theory presented in this study, realizing only electric response can be sufficient for an efficient control of wavefronts that might significantly simplify the design and fabrication of wavefront manipulation devices. Furthermore, intrinsic strongly non-local response of sparse metasurfaces overcomes the fundamental efficiency constraint of conventional metasurfaces imposed by the conservation of normal power flow density \(17\, 18\, 20\, 23\).

To conclude, we have presented a theoretical approach that opens the way to consequently design conformal sparse metasurfaces without appealing to a complex theory. Due to the versatility of the approach, one can consider different metasurface geometries and arbitrary excitation sources within the same framework. The theoretical analysis represented by Eqs. (3) and (4) allows one to approach problems of superdirectivity \(36\) and subdiffraction focusing \(37\). Although the experiments have been performed at microwave frequencies, the theory is valid in any frequency range and may inspire research on novel applications of conformal metasurfaces. Particularly, sparse metasurfaces implemented on flexible substrates can be advantageous for realizing a reconfigurability mechanism based on mechanical deformations \(6\). It can represent a fruitful approach to create an adaptive response without complicating a design with tunable elements (which also often bring additional ohmic losses) and bias networks.

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[28] See Supplemental Material at [URL will be inserted by publisher] for additional information on the numerical and experimental examples.

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