Recent theoretical results on $|\Delta I| = 3/2$ decays of hyperons

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We present a discussion of the $|\Delta I| = 3/2$ amplitudes of the hyperon decays $B \rightarrow B'\pi$ in the context of chiral perturbation theory. We evaluate the theoretical uncertainty of the lowest-order predictions by calculating the leading non-analytic corrections. We find that the corrections to the lowest-order predictions are within the expectations of naive power-counting and, therefore, that this picture can be examined more quantitatively with improved measurements.

Hyperon nonleptonic decays have been much studied within the framework of chiral perturbation theory ($\chi$PT). The decay modes are $\Sigma^+ \rightarrow n\pi^+$, $\Sigma^+ \rightarrow p\pi^0$, $\Sigma^- \rightarrow n\pi^-$, $\Lambda \rightarrow p\pi^-$, $\Lambda \rightarrow n\pi^0$, $\Xi^- \rightarrow \Lambda\pi^-$, and $\Xi^0 \rightarrow \Lambda\pi^0$. Most of the calculations have dealt with the dominant $|\Delta I| = 1/2$ amplitudes of these decays, and the results have been mixed [1, 2, 3, 4, 5, 6, 7]. The $|\Delta I| = 3/2$ amplitudes have not been well studied in $\chi$PT, whereas the $|\Delta S| = 1$, $|\Delta I| = 3/2$ weak transitions are described by an effective Hamiltonian that transforms as $(27_L, 1_R)$ under chiral rotations. At lowest order in $\chi$PT, the Lagrangian for such weak interactions of baryons that has the required transformation properties is [8, 12]

$$L^w = \beta_{27} T_{ij,kl} (\xi B_{i,s} \xi^\dagger_{k,l}) (\xi B_{s} \xi^\dagger_{l,j})$$

$$+ \delta_{27} T_{ij,kl} \xi_{kl} \xi_{bd} \xi_{le} \xi_{cj} (\gamma^\mu)(\gamma^\nu)(\gamma^\rho)(\gamma^\sigma)_{abc}$$

$$+ h.c.,$$

where $\beta_{27}$ ($\delta_{27}$) is the coupling constant for the octet (decuplet) sector, $T_{ij,kl}$ is the tensor that project out the $|\Delta S| = 1$, $|\Delta I| = 3/2$ transitions, and further details are given in Ref. [8].

One can now calculate the decay amplitudes. In the heavy-baryon approach, the amplitude for $B \rightarrow B'\pi$ can be written as [8]

$$iM_{B \rightarrow B'\pi} = G_F m^2 \times$$

$$\bar{u}_{B'} \left( A^{(S)}_{BB'} + 2k \cdot S_v A^{(P)}_{BB'} \right) u_B, \quad (2)$$

where the superscripts refer to S- and P-wave contributions, the $u$’s are baryon spinors, $k$ is the outgoing four-momentum of the pion, and $S_v$ is the velocity-dependent spin operator [8].

At tree level, $O(1)$ in $\chi$PT, contributions to the amplitudes come from diagrams each with a weak vertex from $L^w$ in (3) and, for the P-waves, a vertex from the lowest-order strong Lagrangian. At next order in $\chi$PT, there are amplitudes of order $m_s$, the strange-quark mass, arising both from one-loop diagrams with leading-order vertices and from counterterms. Currently, there is
not enough experimental input to fix the counterterms. For this reason, we follow the approach that has been used for the $|\Delta I| = 1/2$ amplitudes $|\Delta I| = 1/2$ and calculate only nonanalytic terms up to $O(m_\Lambda \ln m_\Lambda)$. These terms are uniquely determined from the one-loop amplitudes because they do not arise from local counterterm Lagrangians. It is possible to do a complete calculation at next-to-leading order and fit all the amplitudes (as was done in Ref. [13]) for the $|\Delta I| = 1/2$ sector, without explicitly including the decuplet baryons in the effective theory), but then one loses predictive power, given the large number of free parameters available. Here, we want to limit ourselves to studying the question of whether the lowest-order predictions are subject to large higher-order corrections.

To compare our theoretical results with experiment, we introduce the amplitudes $|\Delta I| = 1/2$ in the rest frame of the decaying baryon. From these amplitudes, we can extract for the S-waves the $|\Delta I| = 3/2$ components

$$s = A^{(S)}(\Lambda), \quad p = -|k|A^{(P)}(\Lambda)$$

in the $|\Delta I| = 3/2$ sector, without explicitly including the decuplet baryons in the effective theory, but then one loses predictive power, given the large number of free parameters available. Here, we want to limit ourselves to studying the question of whether the lowest-order predictions are subject to large higher-order corrections.

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We can see that some of the loop corrections in Table 3 are comparable to or even larger than the lowest-order results even though they are expected to be smaller by about a factor of $M_K^2/(4\pi f_K)^2 \approx 0.2$. These large corrections occur when several different diagrams yield contributions that add up constructively, resulting in deviations of up to an order of magnitude from the power-counting expectation. This is an inherent flaw in a perturbative calculation where the expansion parameter is not sufficiently small and there are many loop-diagrams involved. We can, therefore, say that these numbers are consistent with naive expectations.

Although the one-loop corrections are large, they are all much smaller than their counterparts in the $|\Delta I| = 1/2$ sector, where they can be as large as 30 times the lowest-order amplitude $|\bar{f}|$ in the P-wave in $\Lambda \rightarrow p\pi^-$. In that sector, the loop dominance in the P-waves was due to an anomalously small lowest-order prediction arising from the cancellation of two nearly identical terms $|\bar{f}|$. Such a cancellation does not happen in the $|\Delta I| = 3/2$ case because each of the lowest-order P-waves has only one term $|\bar{f}|$.

In conclusion, we have discussed $|\Delta I| = 3/2$ amplitudes for hyperon nonleptonic decays in $\chi$PT. At leading order, these amplitudes are described in terms of only one weak parameter. We have fixed this parameter from the observed value of the S-wave amplitudes in $\Sigma$ decays. Then we have predicted the P-waves and used our one-loop calculation to discuss the uncertainties of the lowest-order predictions. Our predictions are not contradicted by current data, but current experimental errors are too large for a meaningful conclusion. We have shown that the one-loop nonanalytic corrections have the relative size expected from naive power-counting. The combined efforts of E871 and KTeV experiments at Fermilab could give us improved accuracy in the measurements of some of the decay modes that we have discussed and allow a more quantitative comparison of theory and experiment.

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### Table 1
Experimental values of ratios of $|\Delta I| = 3/2$ to $|\Delta I| = 1/2$ amplitudes.

| Amplitude | Experiment | $S_3^{(A)} / S_1^{(A)}$ | $S_3^{(B)} / S_1^{(B)}$ | $S_3^{(D)} / s_{\Sigma^- \rightarrow n\pi^-}$ | $P_3^{(A)} / P_1^{(A)}$ | $P_3^{(B)} / P_1^{(B)}$ | $P_3^{(D)} / P_1^{(D)}$ |
|-----------|------------|-------------------------|-------------------------|---------------------------------|-------------------------|-------------------------|-------------------------|
|           |            | 0.026 ± 0.009           | 0.042 ± 0.009           | −0.055 ± 0.020                  | 0.031 ± 0.037           | −0.045 ± 0.047          | −0.059 ± 0.024          |

### Table 2
Summary of results for $|\Delta I| = 3/2$ components of the S- and P-wave amplitudes to $O(m_s \ln m_s)$. We use the parameter values $\beta_{27} = \delta_{27} = -0.068 \sqrt{2} f_\pi G_F m_\pi^2$ and a subtraction scale $\mu = 1 \text{ GeV}$.

| Amplitude | Experiment | Theory |
|-----------|------------|--------|
|           |            | $O(1)$ | $O(m_s \ln m_s)$ | $O(m_s \ln m_s)$ |
| $S_3^{(A)}$ | -0.047 ± 0.017 | 0      | 0               | 0               |
| $S_3^{(B)}$ | 0.088 ± 0.020 | 0      | 0               | 0               |
| $S_3^{(D)}$ | -0.107 ± 0.038 | -0.107 | -0.089          | -0.084          |
| $P_3^{(A)}$ | -0.021 ± 0.025 | 0.012  | 0.005           | -0.060          |
| $P_3^{(B)}$ | 0.022 ± 0.023 | -0.037 | -0.024          | 0.065           |
| $P_3^{(D)}$ | -0.110 ± 0.045 | 0.032  | 0.015           | -0.171          |
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