Shell closure effects studied via cluster decay in heavy nuclei

Sushil Kumar¹, Ramna ¹, Rajesh Kumar²

¹ Department of Physics, Chitkara University, solan-174103, H.P., India
² Department of Applied Sciences and Humanities, N.I.T. Hamirpur-177005, H.P., India.

Abstract

The effects of shell closure in nuclei via the cluster decay is studied. In this context, we have made use of the Preformed Cluster Model (PCM) of Gupta and collaborators based on the Quantum Mechanical Fragmentation Theory. The key point in the cluster radioactivity is that it involves the interplay of close shell effects of parent and daughter. Small half life for a parent indicates shell stabilized daughter and long half life indicates the stability of the parent against the decay. In the cluster decay of trans lead nuclei observed so far, the end product is doubly magic lead or its neighbors. With this in our mind we have extended the idea of cluster radioactivity. We investigated decay of different nuclei where Zirconium is always taken as a daughter nucleus, which is very well known deformed nucleus. The branching ratio of cluster decay and α-decay is also studied for various nuclei, leading to magic or almost doubly magic daughter nuclei. The calculated cluster decay half-life are in well agreement with the observed data. First time a possibility of cluster decay in $^{218}$U nucleus is predicted.
1 Introduction

When a charged particle heavier than $\frac{1}{2}He$, but lighter than a fission fragment is emitted by an unstable nucleus, the process is called cluster radioactivity or heavy ion radioactivity. Cluster radioactivity was predicted theoretically in 1980 by Sândulescu, Poenaru and Greiner [1] on the basis of Quantum Mechanical Fragmentation Theory ($QMFT$). The first experimental identification of a case of radioactive decay of heavy nuclei by emission of a nuclear fragment heavier than $\frac{1}{2}He$ was done by Rose and Jones [2], who studied the decay of $^{223}Ra$ by $^{14}C$ emission with a half-life of $(3.7 \pm 1.1) \times 10^7$ yr. Rose and Jones measured a branching ratio of $B(^{14}C/\alpha) = 8.5(2.3) \times 10^{-10}$. This result was then confirmed by Alekshandrov et al. [3], Gale et al. [4] and Price et al. [5].

Cluster radioactivity has grown and there are more than 24 cases of cluster radioactivity with partial half-lives ranging from $\log_{10}T > 3.63(s)$ to $> 29.20(s)$ [6]. There exists a whole family of cluster decay modes like $^{14}C$ radioactivity, $^{24}Ne$ radioactivity, $^{28}Mg$ radioactivity and so on. The mother nuclei range from $^{221}Fr$ to $^{242}Cm$, all from trans-lead region and cluster radioactivity in this region indicates the presence of heavier clusters. Shell effects are clearly manifested in cluster radioactivity since experiments show that the cluster decay result in magic daughter products.

The study of $\alpha$-decay and cluster decay has been used for identifying the shell closure effects including even the very weak sub-shell closures [7, 8, 9, 10]. Both the cases of large and small decay half-lives are important. Small ones imply closed shell effects in daughter nucleus and the large ones indicate closed shell effects of the mother nucleus. In decay calculations presence of the known spherical or deformed daughter should result in a small decay half-life or a small decay half-life should refer to the existence of a known or new, spherical or deformed, closed shell for the daughter nucleus.

Superdeformation in medium-mass nuclei is a subject of interest and it was first discovered in the actinide fission isomers [11]. Later it was explained from the secondary minimum at very large deformation[12]. $\gamma$-ray transitions
between states in the $N = Z$ nucleus $^{80}\text{Zr}$ was studied to see the deformation, and it was observed that this is the most deformed nucleus in nature, with a quadrupole deformation $\beta_2 \approx 0.4$ [13]. Superdeformation band in the $N, Z = 40$ mass region has also been studied experimentally by the Baktash et.al., [14] and a significant subshell closure at the $N = 40$ neutron number has been studied in the $^{68}\text{Ni}$ [15].

We have not performed the cluster decay calculations, only to search the spherical and/or deformed magicity i.e. not to just study the stability /instability of the concerned nuclei, but also for looking the most probable cluster decay modes of $^{218}\text{U}$ nucleus that has recently been studied for $\alpha$-decay [16].

We are using the Preformed Cluster Model (PCM) of Gupta and Collaborators, where the cluster is assumed to be preformed in the mother nucleus. In this model the preformation probability (also known as spectroscopic factor in various models) for different possible clusters are calculated by solving the Schrodinger equation for the dynamical flow of mass and charge. Another such model based on the calculations of spectroscopic factors for different clusters, has also been given Blendowske and Walliser [17, 18]. In this model they have given parameterization of the spectroscopic factor and meanwhile, other contributions have extended these results [19]. Also, the so called ”Semimicroscopic Algebraic Cluster Model” [20, 21, 22, 23] contributes to parameterization of the spectroscopic factors. Deformations of the neutron rich clusters have effect on penetration probability as the inclusion of the deformations reduces the barrier height. This fact is incorporated into PCM through parameter $R$, which results in the lowering of barrier height[24]. Recently [25, 26], role of deformation on binary and ternary clusterization has been studied to investigate the exotic nuclear shapes, on the basis of $U(3)$selection rule .

The paper is organised as follows. The calculations are made by using the preformed cluster-decay model (PCM) of Gupta and collaborators [24, 27, 28] whose outline is presented in section II. Section III deals with the calculations and results obtained from this study.
The Preformed Cluster Model (PCM)

The Preformed Cluster Decay Model (PCM)\cite{24, 27, 28} is based on the Quantum Mechanical Fragmentation Theory \cite{29, 30, 31}. The dynamical collective coordinates of mass $\eta$ and charge asymmetries $\eta Z$ are in addition to the usual coordinates of relative separation $R$.

The decay constant $\lambda$ (or the decay half-life $T_{1/2}$) in PCM is defined as

$$\lambda = \frac{ln 2}{T_{1/2}} = P_0 \nu_0 P.$$  \hspace{1cm} (1)

Here $P_0$ is the cluster (and daughter) preformation probability and $P$ the barrier penetrability which refer, respectively, to the $\eta$ and R motions. The $\nu_0$ is the barrier assault frequency. The $P_0$ are the solutions of the stationary Schrödinger equation in $\eta$,

$$\left\{ -\frac{\hbar^2}{2\sqrt{B_m}} \frac{\partial}{\partial \eta} \sqrt{B_m} \frac{\partial}{\partial \eta} + V_R(\eta) \right\} \psi^{(\nu)}(\eta) = E^{(\nu)} \psi^{(\nu)}(\eta),$$  \hspace{1cm} (2)

which on proper normalization are given as

$$P_0 = \sqrt{B_m} \mid \psi^{(0)}(\eta(A_i)) \mid^2 (2/A),$$  \hspace{1cm} (3)

with i=1 or 2 and $\nu$=0,1,2,3.... Eq. (2) is solved at a fixed $R = R_a = C_i (= C_1 + C_2)$, the first turning point in the WKB integral for penetrability $P$ (Eq. 5). The $C_i$ are Süssmann central radii $C_i = R_i - (1/R_i)$, with the radii $R_i = 1.28 A_i^{1/3} - 0.76 + 0.8 A_i^{-1/3} fm$.

The fragmentation potential $V_R(\eta)$ in (2) is calculated simply as the sum of the Coulomb interaction, the nuclear proximity potential \cite{32} and the ground state binding energies of two nuclei,

$$V(R_a, \eta) = \sum_{i=1}^{2} B(A_i, Z_i) + \frac{Z_1 Z_2 e^2}{R_a} + V_F,$$  \hspace{1cm} (4)

with B’s taken from the 2003 experimental compilation of Audi et al., \cite{33} and from the 1995 calculations of Möller et al. \cite{34} whenever not available in \cite{33}. Thus, full shell effects are contained in our calculations that come from the experimental binding energies and/or from the calculations of Möller et al. \cite{34}. The charges $Z_1$ and $Z_2$ in (4) are fixed by minimizing the potential
in $\eta_z$ coordinate with $Z$. The Coulomb and proximity potentials in (4) are for spherical nuclei. The mass parameters $B_{\eta}(\eta)$, representing the kinetic energy part in (2), are the classical hydrodynamical masses [35].

The WKB tunnelling probability is $P = P_i P_b$ with

$$P_i = \exp\left[-\frac{2}{\hbar} \int_{R_a}^{R_i} \{2\mu[V(R) - V(R_i)]\}^{1/2} dR\right]$$

(5)

$$P_b = \exp\left[-\frac{2}{\hbar} \int_{R_i}^{R_b} \{2\mu[V(R) - Q]\}^{1/2} dR\right].$$

(6)

These integrals are solved analytically [28] for $R_b$, the second turning point, defined by $V(R_b) = Q$-value for the ground-state decay.

The assault frequency $\nu_0$ in (1) is given simply as

$$\nu_0 = (2E_2/\mu)^{1/2}/R_0,$$

(7)

with $E_2 = (A_1/A)Q$, the kinetic energy of the lighter fragment, for the $Q$-value shared between the two products as inverse of their masses. Here $R_0$ is the equivalent spherical radius of the mother nucleus.

3 \hspace{1em} Results and Discussion

In this paper we have performed three types of calculations. Firstly, we have focused on the search for the magicity (spherical and/or deformed) taking the Zirconium nucleus as daughter in the decay of mother nuclei, which are taken from mass region $A = 88$ to $A = 186$. Secondly, we have investigated the cluster decay of various nuclei taken from the heavy mass region and have also compared data with experimental results. Thirdly, we have searched for the possible cluster decay modes of $^{218}U$ nucleus, whose alpha decay has been studied recently [16], along with few other isotopes of Uranium ($^{230,232,234,236}U$).

In the first part of our calculation we studied the decay of different nuclei taken from the intermediate mass region. We considered the Zirconium nucleus as daughter product in the decay of all these nuclei. Mother of even mass number is chosen and it results in a daughter with different even neutron number. The decaying nuclei are from $^{88}Ru$, $^{86-90}Pd$, $^{90-96}Cd$, $^{100-108}Cd$, $^{110}Cd$. 
Selection of isotopes of different mother nuclei for the calculations has been made on the basis of Q value and the decay of only those was considered for which it is positive.

The histogram in Fig.(1), contains maxima and minima of half-lives of above mentioned parent nuclei with neutron number of the daughter. The daughter was fixed to be Zirconium nucleus i.e. $Z_1 = 40$. We considered all possible cluster decay channels of above mentioned nuclei. Half lives for different decaying parents giving $Z_1 = 40$ and corresponding neutron number $N_1$ of the daughter with the complementing cluster were calculated. There exists different combinations of parents and clusters giving rise to $Z_1 = 40$ with a particular value of $N_1$ i.e. a particular isotope of Zirconium, as a daughter product. Among different values of half lives of all such possible decays resulting into the given Zirconium isotope, we noted the maximum and minimum values only. Then these two extreme values of the half life were plotted for different isotopes of Zirconium.

In another histogram the Fig.(2), in which even neutron number of daughter nuclei vs half-lives of various mother nuclei are shown. The height of the bar corresponding to the daughter neutron number represents the stability/instability of the daughter nucleus. In this calculation the shell closure effects appear at $N_1 = 40$ and at $N_1 = 82$. Also at $N_1 = 70$, these appear, but weakly.

Shell effects can be seen in Fig.(3) also, where we have displayed half-lives for the following decay modes: $^{14}C$, $^{18-20}O$, $^{24}Ne$, versus the neutron number of the daughter, $N_d$. The half-lives shows the minima at the magic number for all the decay mode considered above. The $Q$-value is one of the physical quantity which plays a very important role in any spontaneous nuclear decay with emission of charged particles. The $Q$-value behavior has been noticed for these calculations, as the size of the cluster increases $Q$-value also increases [7]. The calculations belonging to the same atomic number of the daughter are joined with a line of a style mentioned in the figure for $Z_d = 82$ and 84. For the $^{14}C$ radioactivity where daughter is always with same atomic
number and neutron numbers vary, we have observed in our calculations that $N_d = 126$ appears as a magic number in all the cases, same is for the $^{20}O$ cluster when the daughters are $Z_d = 80, 82$. A strong shell effect can be seen in this figure. As a rule, the shortest value of the half-life is obtained when the daughter nucleus has a magic number of neutrons $N_d = 126$ and protons $Z_d = 82$.

Same calculation has been made but for different clusters taking the same atomic number of different daughter nuclei $Z_d = 80, 81, 82, 83, 84$. For the $^{18}O$ radioactivity where daughter is always with same atomic number and neutron number vary, we have observed that $N_d = 126$ appears as a magic number in all the cases, same is for the $^{24}Ne$ cluster when the daughter are $Z_d = 80, 81, 82$. Again a strong shell effect can be seen in Fig.3. The most probable cluster from heavy nuclei in PCM model and their characteristics (i.e., Q-value and Half-life) are compared with the experimental data (given in [6, 36, 37, 38] and the references therein) in Table 1.

In the third part of the calculations, we have predicted for the first time the possible cluster decay modes of $^{218}U$ which was recently investigated for alpha-decay [16]. The $\alpha$-decay calculation of $^{218}U$ in the Quantum Mechanical Fragmentation Theory has been studied and we have calculated the decay properties not only for $\alpha$-decay but also for other possible clusters. The half life for for $\alpha$-decay, using PCM calculations comes out to be $T_{1/2} = 4.36 ms$ and its experimental value is $T_{1/2} = 0.51 ms$. The calculated Q-value is, $E_{\alpha}^{PCM} = 8.788 MeV$ and its experimental value is $E_{\alpha}^{exp} = 8.612 MeV$. Shell closure effects play an important role in the cluster decay studies. Predicted half-lives $T_{1/2}(s)$ and other characteristics for this isotope of Uranium are given in table(2). $^8Be$, $^{12}C$ and $^{22-24}Ne$ appear as the most probable cluster decay modes in our calculations.
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Figure Captions

Fig.1: Predicted half-lives minimum and maximum with daughter (Zr) neutron number for various cluster decay mode.

Fig.2: Predicted half-lives (minimum only) plotted as a function of the daughter neutron number for various cluster decay mode.

Fig.3: Predicted half-lives for the following decay modes: $^{14}$C, $^{18-20}$O, $^{24}$Ne, versus the neutron number of the daughter, $N_d$. 
Table 1: Comparision of experimental $Q$-values, Half-lives with calculated $Q$-values, Half-lives in PCM $\alpha$-decay of cluster emitters.

| Parent nucleus $Z$ and $A$ | Emitted cluster $Z_e$ and $A_e$ | $Q^{exp.}$ | $Q^{PCM.}$ | $T^{exp.}_{1/2}$ | $T^{PCM.}_{1/2}$ |
|---------------------------|---------------------------------|----------|------------|-----------------|-----------------|
| 87-221                    | 6-14                            | 31.294   | 31.292     | 14.53           | 18.044          |
| 88-221                    | 6-14                            | 32.402   | 32.395     | 13.39           | 17.25           |
| 88-222                    | 6-14                            | 33.052   | 33.05      | 11.01           | 15.098          |
| 88-223                    | 6-14                            | 31.839   | 31.829     | 15.15           | 18.548          |
| 88-224                    | 6-14                            | 30.541   | 30.536     | 15.69           | 19.676          |
| 88-226                    | 6-14                            | 28.198   | 28.197     | 21.22           | 25.805          |
| 89-225                    | 6-14                            | 30.479   | 30.477     | 17.16           | 22.664          |
| 90-226                    | 8-18                            | 45.731   | 45.726     | >16.76          | 23.289          |
| 90-228                    | 8-20                            | 44.730   | 44.724     | 20.72           | 23.357          |
| 90-230                    | 10-24                           | 57.765   | 57.759     | 24.61           | 26.849          |
| 90-232                    | 10-24                           | 54.491   | >29.20     |                |                 |
| 90-232                    | 10-26                           | 55.973   | 55.964     | >29.20          | 29.831          |
| 91-231                    | 10-24                           | 60.413   | 60.409     | 22.88           | 24.999          |
| 91-231                    | 9-23                            | 51.854   | 51.844     | >26.02          | 26.463          |
| 92-230                    | 10-22                           | 61.390   |            | 19.57           |                 |
| 92-232                    | 10-24                           | 62.312   | 62.309     | 20.42           | 21.528          |
| 92-233                    | 10-25                           | 60.736   |            | 24.84           |                 |
| 92-233                    | 10-24                           | 60.490   |            | 24.84           |                 |
| 92-233                    | 12-28                           | 74.235   |            | 27.59           |                 |
| 92-234                    | 12-28                           | 74.118   | 74.109     | 25.74           | 27.746          |
| 92-234                    | 10-24                           | 58.831   | 58.825     | >25.92          | 28.775          |
| 92-235                    | 10-25                           | 57.717   | 57.756     | 27.42           | 31.173          |
| 92-235                    | 12-28                           | 72.162   | >28.09     |                |                 |
| 93-237                    | 12-30                           | 74.791   | 74.816     | >27.57          | 28.524          |
| 94-236                    | 12-28                           | 79.674   | 79.668     | 21.67           | 21.637          |
| 94-238                    | 14-32                           | 91.198   | 91.189     | 25.27           | 27.069          |
| 94-238                    | 12-28                           | 75.919   | 75.91      | 25.70           | 28.395          |
| 94-238                    | 12-30                           | 76.801   | 76.822     | 25.70           | 25.542          |
| 94-240                    | 14-34                           | 91.038   | 91.027     | >25.52          | 25.904          |
| 95-241                    | 14-34                           | 93.931   | 93.925     | >25.26          | 23.204          |
| 96-242                    | 14-34                           | 96.519   | 96.509     | 23.15           | 21.08           |
Table 2: Predicted half-lives $T_{1/2}(s)$ and other characteristics for cluster decays of Uranium nuclei. For the Q-value estimates the masses for these parents are taken from the Audi et. al. mass table [33] and Moller- Nix [34].

| Parent nucleus | Emitted cluster | Daughters | $\log T_{1/2}$ sec | $\log P_o$ | $\log 10^{-05}$ | $\log 10^{-16}$ | $\log 10^{-21}$ | $\log 10^{-33}$ | $\log 10^{-47}$ | $\log 10^{-43}$ |
|----------------|----------------|-----------|--------------------|------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $^{218}$U      | $^4$He         | $^{214}$Th | 2.36               | 2.50       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^8$Be         | $^{210}$Ra | 19.759             | 9.20       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{10}$Be      | $^{208}$Ra | 74.199             | 4.68       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{12}$C       | $^{206}$Rn | 24.992             | 1.07       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{18}$O       | $^{200}$Po | 41.371             | 7.68       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{20}$O       | $^{198}$Po | 58.519             | 4.09       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{22}$Ne      | $^{196}$Po | 39.92              | 2.99       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{24}$Ne      | $^{194}$Pb | 46.19              | 2.83       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{28}$Mg      | $^{190}$Hg | 42.169             | 3.85       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
| $^{230}$U      | $^4$He         | $^{226}$Th | 8.024              | 1.15       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{10}$Be      | $^{220}$Ra | 64.1               | 8.55       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{14}$C       | $^{210}$Rn | 30.109             | 3.04       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{20}$O       | $^{210}$Th | 28.69              | 2.06       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{23}$F       | $^{207}$Bi | 34.014             | 1.48       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{24}$Ne      | $^{206}$Pb | 24.286             | 7.45       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{26}$Ne      | $^{204}$Pb | 33.291             | 1.62       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
| $^{232}$U      | $^4$He         | $^{228}$Th | 10.913             | 5.41       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{10}$Be      | $^{222}$Ra | 74.01              | 4.14       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{14}$C       | $^{218}$Rn | 35.302             | 2.41       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{22}$O       | $^{210}$Po | 32.841             | 5.05       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{24}$Ne      | $^{208}$Pb | 21.528             | 3.78       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
|                | $^{26}$Ne      | $^{206}$Pb | 28.832             | 2.98       | 9.72            | 4.68            | 1.07            | 4.09            | 2.99            | 3.85            |
Table 2: Continued.....

| Parent nucleus | Emitted cluster | Daughter nucleus | Log $T_{1/2}$ sec | Preformation probability $P_o$ | Decay constant | Q value MeV |
|----------------|----------------|------------------|------------------|-------------------------------|----------------|-----------|
| $^{234}$U     | $^4$He         | $^{230}$Th       | 14.443           | $1.64 \times 10^{-09}$       | $2.50 \times 10^{-15}$ | 4.859     |
|               | $^{10}$Be      | $^{224}$Ra       | 86.579           | $1.04 \times 10^{-23}$       | $1.83 \times 10^{-87}$ | 6.713     |
|               | $^{14}$C       | $^{220}$Rn       | 42.214           | $9.38 \times 10^{-26}$       | $4.24 \times 10^{-43}$ | 24.514    |
|               | $^{22}$O       | $^{212}$Po       | 38.903           | $1.35 \times 10^{-25}$       | $8.66 \times 10^{-40}$ | 39.231    |
|               | $^{24}$Ne      | $^{210}$Pb       | 28.775           | $7.96 \times 10^{-26}$       | $1.16 \times 10^{-29}$ | 58.825    |
|               | $^{26}$Ne      | $^{208}$Pb       | 26.411           | $1.14 \times 10^{-23}$       | $2.69 \times 10^{-27}$ | 59.465    |
| $^{236}$U     | $^4$He         | $^{232}$Th       | 16.521           | $8.58 \times 10^{-10}$       | $2.09 \times 10^{-17}$ | 4.574     |
|               | $^{10}$Be      | $^{226}$Ra       | 93.871           | $1.06 \times 10^{-23}$       | $9.33 \times 10^{-95}$ | 6.171     |
|               | $^{14}$C       | $^{222}$Rn       | 47.102           | $1.57 \times 10^{-26}$       | $6.74 \times 10^{-48}$ | 23.054    |
|               | $^{22}$O       | $^{214}$Po       | 42.807           | $1.77 \times 10^{-26}$       | $1.08 \times 10^{-43}$ | 37.631    |
|               | $^{24}$Ne      | $^{212}$Pb       | 34.18            | $8.13 \times 10^{-28}$       | $4.58 \times 10^{-35}$ | 55.944    |
|               | $^{26}$Ne      | $^{210}$Pb       | 31.417           | $1.50 \times 10^{-25}$       | $2.65 \times 10^{-32}$ | 56.745    |
