Radiative-conductive inverse problem for lumped parameter systems

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Abstract. The purpose of this paper is to introduce a iterative regularization method in the research of radiative and thermal properties of materials with applications in the design of Thermal Control Systems (TCS) of spacecrafts. In this paper the radiative and thermal properties (emissivity and thermal conductance) of a multilayered thermal-insulating blanket (MLI), which is a screen-vacuum thermal insulation as a part of the (TCS) for perspective spacecrafts, are estimated. Properties of the materials under study are determined in the result of temperature and heat flux measurement data processing based on the solution of the Inverse Heat Transfer Problem (IHTP) technique. Given are physical and mathematical models of heat transfer processes in a specimen of the multilayered thermal-insulating blanket located in the experimental facility. A mathematical formulation of the inverse heat conduction problem is presented too. The practical testing were performed for specimen of the real MLI.

1. Introduction
The non-stationary state and non-linearity (considerable, at times) of heat transfer phenomena can be referred to special features of thermal conditions of modern space structures and specially TCS. These factors considerably reduce the possibility of using many traditional theoretical and experimental methods. So it became urgent to develop new approaches to thermal engineering studies. Amongst such approaches are methods based on the solution of inverse problems, in which it is required, through measurements of the system or process state, to specify one or several characteristics which cause this state (in other words, to find not causal-sequential, as in direct problems, but rather sequential-causal quantitative relations). The advantage of these methods is that they help to carry out experimental studies under conditions very similar to full-scale tests or in the operation of the considered systems, in particular in flight tests. This is very important for structures used in the space industry, and we observe this situation in the field of practical applications where the first formulations and methods of solving the inverse heat transfer problems have appeared. Experimental-and-computational methods based on solving coefficient inverse heat transfer problems form an intensively developing direction in the field of studies of heat transfer processes [1-5].

Very often steady state cases are insufficient for accurate numerical simulation of heat transfer processes in MLI, and requirements of thermal design of TCS drives us to a necessity of using a transient heat transfer mathematical model (in particularly with lumped parameters). One of the main difficulties here is how to determine the coefficients of the mathematical model, providing its adequacy to real actions. Direct measurement of the most heat transfer characteristics is usually impossible, and their theoretical estimates are often far from being true and often contradictory. That
is why a problem arises to determine the radiative and thermal properties of space structures by combining the calculations and the results of experiments. The algorithms suggested for specifying the unknown radiative and thermal parameters of MLI for spacecraft TCS are based on the methodology of IHTP, which, at present, are widely used in the study of heat transfer processes. The computational methods for solving the boundary inverse heat conduction problems are now effectively used in experimental investigations of thermal processes occurred between the solids and the environment. The inverse problem methods allow the use of mathematically proved search algorithms for unknown heat transfer characteristics, estimated from the results of indirect measurements.

Formulations of the transient heat transfer problem usually assume variations in the temperature both in time and in position. However there are many engineering applications in which the variation of temperature within the medium can be neglected and temperature is then considered as a function of time only. Such a formulation of the problem, called a lumped system formulation, provides greater simplification in the analysis of transient heat transfer although the range of applicability is rather restricted. In this situation, a complex heat transfer in a spacecraft MLI is considered for a system of layers, which exchange thermal energy with the environment and between them. The basic heat transfer equation is obtained from the analysis of heat balance under the assumption that the MLI can be considered as a finite number $L$ of isothermal layers:

$$\frac{d}{d\tau}c_l(T)\rho_l d_l\frac{dT_l}{d\tau} = A_s(T)(q_s(T) + q_R(\tau)) + \varepsilon_l(T)q_c(\tau) - \varepsilon_l(T)\sigma T_i^4 + \varepsilon_{l-1,l}^e(T)\sigma(T_i^4 - T_{i-1}^4)$$

$$+ k_{l-1,l}(T)(T_{i-1} - T_i)$$

$$\tau \in (\tau_{\min}, \tau_{\max})$$

$$l = 2, L-1$$

$$c_l(T)\rho_l d_l\frac{dT_l}{d\tau} = \varepsilon_{L-1,L}^e(T)\sigma(T_{L-1}^4 - T_L^4) + k_{L-1,L}(T)(T_{L-1} - T_L) + k_{in}(T)(T_{in} - T_L)$$

$$\tau \in (\tau_{\min}, \tau_{\max})$$

$$T_i(\tau_{\min}) = T_{i0}, \ l = 1, L$$

where

$$\varepsilon_{l-1,l}^e(T) = \frac{\varepsilon_{l-1,l}(T)\varepsilon_l(T)}{\varepsilon_{l-1,l}(T) + \varepsilon_l(T)}$$

where $T_i$ is the temperature of the $l$-th layer, $\tau$ is the time, $c_l$ is the heat capacity of the $l$-th layer material, $d_l$ is the thickness of the $l$-th layer material, $\rho_l$ is the density of the $l$-th layer material, $k_{l-1,l}$ is the thermal conductance between elements $l-1$ and $l$, $\varepsilon_{l-1,l}^e$ is the effective emissivity, $\sigma$ is the Stephan-Boltzmann constant, $q_s$ is the direct solar radiation, $q_R$ is the solar radiation reflected from the Earth, $q_R$ is the Earth's irradiation [6], $A_s$ is the absorptivity of the 1st layer, $k_{in}$ is the thermal conductance between the MLI and internal wall.

In the general case, the heat transfer process covered by (1)-(4) is determined by the parameters of the boundary heat balance equations, by conductive and radiative heat transfer, by thermal properties, densities and thickness of layers, as well as by the system's initial thermal state. One should note that a simultaneous determination of all parameters of (1)-(4) is possible only with an accuracy up to a
constant multiplier. The uniqueness of solutions is extremely important in our studies involving the inverse problem solutions and the experimental merits ensure the determination of the solution. Moreover, in view of Tikhonov's classical theorem the uniqueness theorems ensure the computation stability if the solution of inverse problems is sought on the compact. The uniqueness of the corresponding problem solutions was analyzed by Nenarokomov [7].

2. Numerical algorithm

Let us suppose that in a real situation there are some unknown characteristics \( u_i, \ i = 1, 2, ..., N \) among elements of vectors \( \{e_1\}, \{e_i\}, \{k_{i, j+1}\} \) (where \( k_{L, L+1} = k_m \)). In addition, the results of temperature measurements in the system's separate elements are available.

One of the most effective directions in solving the inverse heat transfer problems is to reduce them to extremal formulations and apply numerical methods of the optimization theory [1,7-10]. In the exact extremal statement, the definition of functions \( u_i, \ i = 1, 2, ..., N \) corresponds to a minimization of the residual functional characterizing the deviation of temperature \( T_i(\tau_m) \) calculated for certain estimates of \( u_i, \ i = 1, 2, ..., N \) from known temperature \( f_{lm} \) in the metric of space of the input data (the mean-square deviation of experimental \( f_{lm} \) and theoretical \( T_i(\tau_m) \) temperatures can be used as a functional):

\[
J(u) = \min_{u \in L_2} \sum_{i=1}^{M} \sum_{m=1}^{L} \left( T_i(\tau_m) - f_{lm} \right)^2
\]

where \( J(u) = \sum_{i=1}^{M} \sum_{m=1}^{L} \left( T_i(\tau_m) - f_{lm} \right)^2 \).

A theoretical temperature \( T \) is calculated using the mathematical model (1)-(4). Such an inverse problem is known to belong to a class of ill-posed problems in their classical sense. More often the ill-posedness is stipulated by the instability of the problem solution with respect to small perturbations of the input data. Despite this feature, it is possible to solve an inverse problem using regularization method. Among them an iterative regularization method is one of the most universal and efficient [1]. The method is based on gradient iterative algorithms, in which, and this is very important, the last iteration number is chosen according to the residual principle [1]. To solve the inverse problem (6), the following iterative method of unconstrained minimization can be used:

\[
\begin{align*}
\beta^s &= \left( J_u(\beta^s) - J_u(u^{s-1}) \right)_{L_2} \left( \beta^s \right)_{L_2}, \\
g^s &= -J_u + \beta^s g^{s-1}, \\
\beta^0 &= 0
\end{align*}
\]

The last iteration number \( s^* \) is chosen according to the iterative residual principle. It is possible to suppose that methods enabling effective initiation of the iterative process from distant approximations \( u_j, \ i = 1, 2, ..., N \) and the sharp slow down in approaching the functional minimum would appear useful when solving inverse heat transfer problems. If iterations are stopped on the basis of residual criterion, these methods are the regularization algorithms, that is, they give stable approximate solutions whose accuracy increases steadily as the errors of the input data are reduced. These rigorous mathematical
results were obtained for a linear case [1]. Thus, let us bound the iterative sequence (7) according to the condition

$$J(u^j) \leq \delta_j^2$$

where $\delta_j^2$ is the mean-square temperature-measurement error, namely $\delta_j^2 = \sum_{l=1}^{N_x} \sum_{m=1}^{M_x} \sigma_{lm}^2$.

The descent parameter $\gamma^j$ is determined from a condition

$$\gamma^j = \arg \min_{\gamma \in R^n} (J(u^j + \gamma \gamma^j))$$

Accordingly in the approach suggested in [1] the unknown coefficients in equations (1)-(4) can be approximated by some systems of basic functions (in particular B-splines), for example

$$C_i(T) = \sum_{l=1}^{N_x} \epsilon_i^l \rho_i \phi_i^C(T)$$
$$\epsilon_i(T) = \sum_{l=1}^{N_x} \epsilon_i^l \phi_i^\epsilon(T)$$
$$k_{i,j+1}(T) = \sum_{k=1}^{N_k} k_{i,j+1}^l \phi_i^k(T)$$

The gradient of the minimized functional is computed using the solution of a boundary-value problem for an adjoint variable:

$$J'_{\psi_{ml}} = -\sum_{m=1}^{M_x} \left( \int \psi_{ml}(\tau_m) d_i d_j \rho_i \rho_j \phi_i^C(T(\tau_m)) \frac{dT}{d\tau}(\tau_m) \right)$$
$$J'_{\epsilon_i} = \sum_{m=1}^{M_x} \left( \int \psi_{ml}(\tau_m) \frac{d}{dE_i} \psi_{m}^{\epsilon}(T(\tau_m)) \right)$$
$$J'_{k_{i,j+1}} = \sum_{m=1}^{M_x} \left( \int \psi_{ml}(\tau_m) T_{i,j+1}(\tau_m) \phi_i^k(T(\tau_m)) \right)$$

where $\psi_{ml}$ is the solution of the following adjoint problem:

$$-c_i \rho_i d \psi_{lm}/d\tau = \frac{\partial A_k}{\partial \tau}(q_s(\tau) + q_R(\tau)) + \frac{\partial E_i}{\partial \tau}(T) q_{e}(\tau) - 4\epsilon_i \sigma T_i^3 + \frac{\partial \epsilon_{i,j+1}}{\partial E_i}(T_2^3 - T_i^3)$$
$$+ 4\epsilon_{i,j+1} \sigma(T_2^3 - T_i^3) + \frac{\partial k_{i,j+1}}{\partial E_i}(T_2 - T_i)$$
$$\tau \in [\tau_{min}, \tau_{max}]$$
3. Practical implementation

The purpose of this study is to estimate radiative and thermal properties (emissivity and thermal conductance) of a multi-layered thermal-insulating blanket. Consider a physical model of heat transfer process in the specimen located in the experimental facility (Figure 1). For the given tests a heating element was used in the experimental facility, which was in the form of a refractory stainless steel foil. The experimental specimen of the blanket is a multi-layer slab in the form of a rectangular parallelepiped. At zero time, the uniform temperature distribution is realized in the specimen. The initial data for estimating the radiative and thermal properties of MLI are formed based on the results of heat flux and temperature measurements and include external and internal heat fluxes time-temperature dependence at both surfaces of the specimen. Data on the a-priori known properties of materials composing a considered MLI are given in Table 1.

| Material                  | Aluminized polymer | Spacer material |
|---------------------------|--------------------|-----------------|
| Number of layers in blanket | 14                 | 13              |
| Emissivity, $\varepsilon$ | 0,06               | /               |
| Material’s layer thickness ($\times 10^{-6}$), $m$ | 7,5                | 25,4            |
| Density of material’s layer, kg / $m^3$ | 1576               | 5,66            |
| Heat capacity of material’s layers, J / (kg K) | 1130               | 223             |

Based on the given physical model, a corresponding mathematical model of heat transfer process in the material’s specimen can be considered. For this case the direct problem of heat transfer can be presented as

\[
-c_I \rho d_I \frac{d\psi_{ml}}{d\tau} = \frac{\partial E_{i,j}^{\text{eff}}}{\partial T} \sigma(T_{i,j}^4 - T_i^4) + 4 E_{i,j}^{\text{eff}} \sigma(T_{i,j}^3 - T_i^3) + \frac{\partial k_{i,j}}{\partial T}(T_{i,j} - T_i)
\]

\[
+ \frac{\partial E_{i+1,j}}{\partial T} \sigma(T_{i+1,j}^4 - T_i^4) + 4 E_{i+1,j}^{\text{eff}} \sigma(T_{i+1,j}^3 - T_i^3) + \frac{\partial k_{i+1,j}}{\partial T}(T_{i+1,j} - T_i)
\]

\[
\tau \in (\tau_{\text{min}}, \tau_{\text{max}})
\]

\[
l = 2, L - 1
\]

\[
-c_L \rho L d_L \frac{d\psi_{ml}}{d\tau} = \frac{\partial E_{L-1,L}^{\text{eff}}}{\partial T} \sigma(T_{L-1,L}^4 - T_L^4) + 4 E_{L-1,L}^{\text{eff}} \sigma(T_{L-1,L}^3 - T_L^3) + \frac{\partial k_{L-1,L}}{\partial T}(T_{L-1,L} - T_L)
\]

\[
\tau \in (\tau_{\text{min}}, \tau_{\text{max}})
\]

\[
c_L \rho L d_L \frac{d\psi_{ml}}{d\tau} = 2(T_i(\tau_m) - f_{ml}), m = 1, M, l = 1, L
\]

\[
\psi_{M+1,l}(\tau_m) = 0, l = 1, L
\]
\[ c(T) \rho L dT_i \frac{d}{d\tau} = \mathcal{E}^\text{eff}(T) \sigma \left( T_{i-1}^4 - 2T_i^4 + T_{i+1}^4 \right) + k(T) \left( T_{i-1} - 2T_i + T_{i+1} \right) \quad (18) \]

\[ \tau \in (\tau_{\text{min}}, \tau_{\text{max}}), \quad l = 2, L - 1 \]

\[ c(T) \rho L dL \frac{d}{d\tau} = \mathcal{E}^\text{eff}(T) \sigma \left( T_{L-1}^4 - T_L^4 \right) + k(T) \left( T_{L-1} - T_L \right) - q_2(\tau) \quad (19) \]

\[ \tau \in (\tau_{\text{min}}, \tau_{\text{max}}) \]

\[ T_i(\tau_{\text{min}}) = T_0, \quad l = 1, L \quad (20) \]

where \( \mathcal{E}^\text{eff}(T) = \frac{\varepsilon(T)}{2 - \varepsilon(T)} \).

There are three unknown parameters in (17)-(20): \( c, \varepsilon, k \). For complimentary information needed for solving the inverse problem prescribed are the results of temperature measurement at the two surfaces of the specimen:

\[ T_i^{\text{exp}}(\tau_m) = f_{i,m}, \quad m = 1, M, \quad l = 1, L \quad (21) \]

The uniqueness conditions of IHTP solution usually determine a minimum wanted volume of measurements necessary in one experiment. For simultaneous determination of \( c, \varepsilon, k \) it is necessary to measure a heat flux density passing through the specimen surface differing from zero at least at one boundary and perform unsteady temperature measurements not less than in three time instants.

In the process of non-steady heating of specimens by means of an automatic system, recording of temperatures inside the specimen in places of thermocouple positioning, heater's temperature and also electric power released on it were performed [10]:

\[ Q_{\text{elecr}} = U * I \quad (22) \]

where \( U \) is the electrical voltage on the heater, \( I \) is the magnitude of the current passing through the heating element. The heat flux supplied to a specimen due to symmetry is determined as [10]

\[ q_1(\tau) = \left( \frac{Q_{\text{elecr}}}{A} - c_h \rho_h d_h \frac{\partial T_1}{\partial \tau} \right) / 2 \quad (23) \]

where \( A \) is heater's surface area; \( \rho_h \) is stainless steel density; \( d_h \) is heating element thickness; \( c_h \) is specific heat capacity of stainless steel. The back heat flux \( q_2(\tau) \) was calculated using the temperature measurements on the surfaces of the thermal-insulated holder from a solution of the direct problem for the insulated slab with known thermal properties and the first kind (temperature) boundary condition on the surfaces.

Comparisons of the calculated and measured temperatures on the specimens' surfaces are presented in Figure 2. The resulting estimated values for the parameters are presented in Table 2. Table 3 includes the obtained values of the least squares and the maximum deviation of the calculated temperatures from that measured in the experiments.
Figure 1. A testing scheme for specimens a and b: 1 – electrical heating element; 2 – specimen A; 3 – specimen B; 4 – elements of thermal-insulating holder using as 1-D calorimeters for specimens; $T_1, T_2, T_3, T_4, T_5$ - thermocouples at the specimens.

Table 2: The estimated parameters

| Parameter          | Specimen a | Specimen b |
|--------------------|------------|------------|
| $c$, $J/(kg\,K)$   | 1221       | 1192       |
| $\varepsilon$      | 0.061      | 0.062      |
| $k$, $W/(m^2\,K)$ | 0.012      | 0.011      |

Figure 2. Comparing the calculated and measured temperatures (specimen a), 1 - calculated temperatures, 2 - experimental temperatures (both for heated surface), 3 - calculated temperatures, 4 - experimental temperatures (both for opposite surface).
Table 3: The deviation of the calculated temperatures.

| Specimen | Least-square temperature deviation (K) | Maximum temperature deviation (K) |
|----------|----------------------------------------|----------------------------------|
| a        | 3.54                                   | 10.5                             |
| b        | 7.45                                   | 12.8                             |

4. Conclusions
The paper aims to describe the algorithm developed to process the data of thermal experiments and hence to find the radiative and thermal properties of MLI. The algorithm is suggested for estimation of these unknown properties of the surface as a solution of the nonlinear IHTP in extreme formulation. The paper aims to describe the algorithm developed to process the data of thermal experiments and hence to find the radiative and thermal properties of MLI. The illustrative examples are presented to make a judgment on the convergence for real technical problems.

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