Unifying Scalars and Light Mesons by Holographic QCD

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We show that the universality of the infrared dynamics of string modes dual to $q\bar{q}$ states within a Dynamical AdS/QCD model of coupled dilaton-gravity background implies in the Regge-like spectrum of $f_0$'s scalars and higher spin mesons consistent with experimental data. The pion mass and its trajectory were also described with a scale deformation of the metric and a rescaled string mass. The available experimental decay widths of the $S \rightarrow PP$ decays provided a complementary check of the proposed classification scheme for $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ as radial excitations of $f_0(600)$. For $f_0(980)$ we estimated a mixing angle of $\pm 20^\circ$ with other structures.

The two-pion partial decay width for $f_0(600)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$, for which experimental information on the two-pion decay width exists are qualitatively consistent with the model results, using one normalization parameter. In particular for $f_0(600)$ a width of about 500 MeV is found. The experimental value of $\Gamma_{\pi\pi}$ for $f_0(1500)$, known experimentally with a few % error, is used for normalization of the effective action for the decay amplitude in our holographic model.

Although $f_0(980)$ has a mass identified with the first excitation of the string mode dual to $q\bar{q}$ state, it has a too large $\Gamma_{\pi\pi}$ compared to the range of the experimental values. It is known that $f_0(980)$ should mix strongly with other structures such as $s\bar{s}$ states (see e.g. [6]). Indeed we got a mixing angle of about $\pm 20^\circ$ [1] to be consistent with the experimental data for the partial decay width. (We do not determine the sign of the mixing angle.) Below we briefly discuss the basis of present holographic model and substantiate quantitatively our claims.

The search for string dual models of Gauge theories was pursued since the pioneer work by t’Hooft [7] and vigourously developed after the Maldacena’s Conjecture [8]. Applications of AdS/CFT to describe QCD began with the Hard Wall model [9] that provides a good description of form factors at high $Q^2$ but does not give the linear Regge trajectory presented by the mesonic data. This phenomenological result can be obtained by a spin dependence in the metric [10] or by introducing a dilaton field [11]. In particular, scalar mesons were analyzed in [12] and [13]. However, both works do not include the sigma meson in their studies. Those models are not solutions of Einstein equations and also do not confine by the Wilson Loop criteria. We proposed a Dynamical AdS/QCD model [4] that is a solution of the Einstein equations, confines by the Wilson loop criteria and describe the scalar sector. We start from the Einstein-Hilbert action of five-dimensional gravity coupled to a
we showed that the leading infrared contribution for the chosen state configuration \( [9] \). The five-dimensional mass field equations in the fifth dimension found in \([4]\) are given by \( \Phi' = 3\sqrt{A^2 + A''} \) and \( V(\Phi(z)) = 3e^{2A(z)} [A''(z) - 3A^2(z)] / 2 \). The gauge/gravity dictionary formal invariance is broken in the IR by \( \Lambda_{QCD} \). The infrared (IR) behavior is chosen to obtain Regge-trajectories consistent with the Wilson-loop area-law for the gravity dual of a confining theory. The conformal invariance is broken in the IR by \( \Lambda_{QCD} \).

To calculate the spectrum of scalar mesons \( \varphi \) we start from the action

\[
I = \frac{1}{2} \int d^4xdz \sqrt{|g|} \left( \partial_\mu \varphi(x,z) \partial^\mu \varphi(x,z) - \frac{M^2}{\Lambda^2_{QCD}} \varphi^2 \right),
\]

that describes a scalar mode propagating in the dilaton-gravity background. We factorize the holographic coordinate dependence as \( \varphi(x,z) = e^{iP_\mu x^\mu} \varphi(z) \), \( P_\mu P^\mu = m^2 \).

The string modes of the massive scalar field \( \varphi \) can be rewritten in terms of reduced amplitudes \( \psi_n = \varphi_n \times e^{-(3A+\Phi)/2} \) which satisfy the Sturm-Liouville equation

\[
[-\partial_z^2 + \mathcal{V}(z)] \psi_n = m^2_n \psi_n
\]

where the string-mode potential is

\[
\mathcal{V}(z) = \frac{B^2(z)}{4} - \frac{B''(z)}{2} + M_5^2 e^{-2A(z)},
\]

with \( B = 3A + \Phi \). (Note that \( B = (2S - 1)A + \Phi \) for the spin nonzero states \([11]\).) The gauge/gravity dictionary identifies the eigenvalues \( m^2_{nS} \) with the squared meson mass spectrum of the boundary gauge theory.

The AdS/CFT correspondence states that the wave function should behave as \( z^\tau \), where \( \tau = \Delta - \sigma \) (conformal dimension minus spin) is the twist dimension for the corresponding interpolating operator that creates the given state configuration \([6]\). The five-dimensional mass chosen as \([14] \) \( M^2_5 = \tau(\tau - 4) \), fixes the UV limit of the dual string amplitude with the twist dimension.

In \([4]\), in the context of higher spins mesonic states we showed that the leading infrared contribution for the metric as

\[
A(z) = \text{Log}(z\Lambda_{QCD}) + \frac{(\xi z\Lambda_{QCD})^2}{1 + e^{(1-\xi z\Lambda_{QCD})}}
\]

with \( \xi = 0.58 \) from the fit (see figure 1). The slope of the Regge trajectory is decreased for the lower scalar excitations (see figure 1) in respect to \( S = 1 \) as \( \xi < 1 \) and also the ground state \( f_0(600) \) is large in the holographic coordinate compared to other light mesons. Therefore, \( f_0(600) \) comes as a broad resonance in pionic channels owing to a large overlap between the corresponding amplitudes. More on that will come on what follows (see figure 2 and Table I).

Solving the Sturm-Liouville equation \([3]\) we obtain the spectrum and the wave functions of the scalar mesons states. We used the lowest conformal dimension operator of a scalar, twist \( 2 \) \([13]\). The trajectories found were in agreement of experimental data of \( f_0 \) family as shown in figure 1. In this picture the sigma meson is the fundamental state and the other \( f_0 \)'s are radial excitation of the sigma. We have included in the plot all \( f_0 \)'s present in the Particle Listing of PDG \([1]\). That agreement may be
In our model, the fifth dimensional mass was rescaled according to 
\( M_5^n = \frac{1}{\xi} \), where \( p_\pi \) is the pion momentum in the meson rest frame. The Sturm-Liouville amplitudes of the scalar (pseudoscalar) modes are normalized just as a bound state wave function in quantum mechanics \( \| \psi \| = 1 \), which also corresponds to a normalization of the string amplitude

\[
\int_0^\infty dz \psi_m(z) \psi_n(z) = \int_0^\infty dz \varphi_m(z) e^{-i(\Phi + 3A)} \varphi_n(z) = \delta_{mn} .
\]

The overlap integral for the transition amplitude is naturally damped by increasing the \( f_0 \) excitation as destructive interference comes into scene within the holographic view. A qualitative understanding of that effect can be seen in figure 2, by observing that within the range of the pion wave function nodes of the higher scalar excitation takes place.

The two-pion partial decay width for the \( f_0 \)’s present in the particle listing of PDG, are calculated with Eq. (8) and shown in Table I. The width of \( f_0(1500) \) is used as normalization. In particular for \( f_0(600) \) the model gives a width of about 500 MeV, while its mass is 860 MeV. The range of experimental values quoted in PDG for the sigma mass and width are quite large as depicted in Table I. A recent analysis of the sigma pole in the \( \pi \pi \) scattering amplitude from ref. \( [18] \) gives \( m_\sigma = 441^{+15}_{-8} \) MeV and \( \Gamma_\sigma = 544^{+18}_{-25} \) MeV, which in comparison to our results the width seems consistent while the model mass appears somewhat larger. The analysis of the E791 experiment gives \( m_\sigma = 478^{+24}_{-23} \pm 17 \) MeV and \( \Gamma_\sigma = 324^{+42}_{-40} \pm 21 \) MeV \( [19] \), and the CLEO collaboration \( [20] \) quotes \( m_\sigma = 513 \pm 32 \) MeV and \( \Gamma_\sigma = 335 \pm 67 \) MeV, both values of the width smaller than our result. Other analysis of the \( \sigma \)-pole in the \( \pi \pi \rightarrow \pi \pi \) scattering amplitude present in the decay of heavy mesons indicates a mass around 500 MeV \( [21] \). A rescaling of the string mass as seen necessary for the pion case can lower the sigma mass. We just observe that coupling with higher twist string duals can be a source for the effective decrease of the string mass as in the pion case. The width is not strongly affected as the shift in the string mass mainly dislocates the squared meson mass by a constant. We do not attempt to fine tune the model at this stage.

The \( f_0(980) \) is identified with the first excitation of the string model dual to the \( q\bar{q} \) state (see Table I). The model mass is shifted to a value above the experimental one, i.e., 1.1 GeV compared to 0.98 GeV. The shift of -0.12 GeV can be attributed to a rescaling of the string mass as in the sigma case. By increasing the excitation of the scalar meson this shift tends to decrease (see \( f_0(1500) \) in Table I). The experimental values of \( \Gamma_{\pi\pi} \) for \( f_0(980) \) is too small compared to our result. We introduced a mixing angle for \( f_0(980) \) of \( \pm 20^\circ \), that corresponds to a composite nature by mixing, e.g., \( s\bar{s} \) with light non-strange quarks \( [3] \).
mixing angle absolute value between ~ 12° to 28° fits $\Gamma^{\pi\pi}$ within the experimental range.

| Meson | $M_{\exp}(\text{GeV})$ | $M_{th}(\text{GeV})$ | $\Gamma_{\pi\pi}^{\exp}(\text{MeV})$ | $\Gamma_{\pi\pi}^{th}(\text{MeV})$ |
|-------|----------------------|------------------|-------------------------------|-------------------------------|
| $f_0(600)$ | 0.4 - 1.2 | 0.86 | 600 - 1000 | 535 |
| $f_0(980)$ | 0.98 ± 0.01 | 1.10 | $\sim$ 15 - 80 | $\sim$ 6 |
| $f_0(1370)$ | 0.78 - 0.80 | 1.60 | $\sim$ 41 - 141 | 141 |
| $f_0(1500)$ | 1.505 ± 0.006 | 1.52 | $\sim$ 38 ± 3 | $\sim$ 38 |
| $f_0(1700)$ | 1.720 ± 0.006 | 1.70 | $\sim$ 0 - 6 | 5 |
| $f_0(2020)$ | 1.992 ± 0.016 | 1.88 | $\sim$ 0 - 0 | 0 |
| $f_0(2100)$ | 2.103 ± 0.008 | 2.04 | $\sim$ 1 - 2 | 1.2 |
| $f_0(2200)$ | 2.189 ± 0.013 | 2.19 | $\sim$ 2 - 5 | 2.5 |
| $f_0(2330)$ | 2.29 - 2.35 | 2.33 | $\sim$ 3 - 5 | 2.8 |

The mass of the higher scalar excitations are consistent with the experimental data in Table I and figure 1. The two-pion partial decay widths of $f_0(1370)$ and $f_0(1710)$ are compared to the experimental values, and particularly for $f_0(1370)$ we get a result of 141MeV within the experimental range, without any further assumption.

In summary, we provide the basic framework to study the $f_0$ family given by excitations of the sigma meson as the string duals to $q\bar{q}$ states. The classification, spectroscopy and decay is guided by an Holographic view of the scalar string modes obtained from a Dynamical AdS/QCD model of coupled dilaton-gravity background solution of the Einstein equations. The deformation of the anti-de Sitter metric encodes confinement by the area law behavior of the Wilson loop. Assuming the universality of the metric, apart a scale deformation, for the scalar, pseudoscalar and higher spin string modes dual to $q\bar{q}$ states, the Regge-like spectrum of the radial excitations of $f_0$ was obtained and the slope about 0.5 GeV$^2$ fitted to the data. The pion trajectory was also fitted to the data. The pion trajectory was also fitted to the data. The pion trajectory was also fitted to the data.

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