ABSTRACT

Effective Lagrangian describing gravitational source spin-particle spin interactions is given. Cosmological and astrophysical consequences of such interaction are examined. Although stronger than expected, the spin-spin interactions do not change any cosmological effect observed so far. They are important for background primordial neutrinos.
Introduction  It is not uncommon to find in the Universe rotating massive objects. This rotation may be rather slow, like in the case of Earth, or relatively rapid, like that of some neutron stars. Spinning of the source changes the resulting gravitational field and introduces new with respect to the case of simple static sources, angular momentum dependent, gravitational forces. We find it interesting to study and clarify the status of the source-spin dependent gravitational interactions between source and particles travelling through its field. We consider a simplest case of nonzero-spin particle, a spin-1/2 fermion. To be specific, in this note we shall discuss in some detail neutrino interacting with the spinning Sun, but in fact our effective Lagrangian introduced in Section 2. is a general one, valid for any kind of spin-half fermion and any rotating source, like a pulsar or a rotating black hole.

1. Gravitational field of a rotating body  The gravitational field of a spinning sphere of mass M and angular momentum \( \vec{J} = M\vec{a} \) is described by the Kerr metric, which is an exact solution to the Einstein equations. Since we are going to apply methods of Minkowski space field theory to interactions of spin \( \frac{1}{2} \) fermions with the spinning background, it is meaningful and sufficient to consider the asymptotic form of the Kerr metric obtained in the limit \( \frac{1}{r} \rightarrow 0 \).

As we are going to consider the effects of rotation, we give the asymptotic form of the Kerr metric up to terms \((\frac{r_g}{r})^2\) and \((\frac{r_g}{r^3})\), where \( r_g = 2M/M_{\text{Planck}}^2 \) is the Schwarzchild radius, (we use units and notation of [1])

\[
\begin{align*}
g_{00} &= 1 - \frac{r_g}{r} + \left(\frac{r_g}{2r}\right)^2 \\
g_{ij} &= -\delta_{ij} \left(1 + \frac{r_g}{r} + \frac{1}{2} \left(\frac{r_g}{r}\right)^2\right) \\
g_{0j} &= \frac{r_g}{r^3} \omega_j \\
\omega_j &= (\vec{a} \times \vec{x})_j
\end{align*}
\]

(1)

The reference frame has been fixed to the axis of rotation of the body and the metric is written in the so-called isotropic coordinates (the coordinate system in which the asymptotic Schwarzchild metric assumes the diagonal form). One should note that the exact Kerr metric gives the upper limit on the radius \( a, a < r_g/2 \) (cf. [1]).

The asymptotic metric (1) can also be obtained, without any reference to the Kerr metric, just by solving the linearized (in weak field approximation) Einstein equations [1]. Our
2. Effective lagrangian for spin–$\frac{1}{2}$ particle in the spinning background

The general invariant coupling of spin 1/2 fermions to gravity is given by (conventions we use are those of Bjorken and Drell [2])

$$\mathcal{L} = \sqrt{-g}(i\bar{\psi}\gamma^a D_a \psi - m\bar{\psi}\psi)$$

where $D_a = e^a_\mu(\partial_\mu + \frac{1}{2}\Sigma^{cb}e^c_\nu e_{b\nu;\mu}), \Sigma^{ab} = \frac{i}{4}[\gamma^c, \gamma^b], e_{b\nu;\mu} = (\delta^c_\mu \partial_\nu - \Gamma^c_{\nu\mu})e_{b\gamma}$. The $a, b, c, ...$ are flat indices and $\alpha, \beta, \gamma, ...$ are curved space indices.

In consistency with the metric (1), valid, to repeat, in weak field and slow rotation limit, we retain in the lagrangian (2) only terms of up to the order $O((\frac{r_g}{r})^2)$ and $O(\frac{r_g}{r})$. Then the final form of the relevant Dirac operator is

$$D = \gamma^a D_a = (i\gamma^a \partial_a - m) + i\gamma^0 \frac{r_g}{2r} \partial_0 +$$

$$+ \frac{if}{2} \gamma^a \partial_a - i \frac{r_g^2}{4r^4} \vec{\gamma} \vec{\omega} +$$

$$- \frac{r_g}{r^3} \gamma^5 \partial_0 +$$

$$+ \frac{r_g}{4r^3} \gamma^5 \vec{a} - \frac{3r_g}{4r^3} \gamma^5(\vec{\gamma} \vec{\omega})(\vec{a} \vec{\omega})$$

where $f = -r_g/r - 1/2(r_g/r)^2$. The operator (3) is not explicitly Lorentz-invariant (but it is $O(3)$ invariant). In fact, to write explicitly the interactions with the background we had to choose the specific coordinate system, the one where the source of the background stays at the origin. Another issue is the gauge invariance (the invariance under small coordinate reparametrizations, e.g. the change from isotropic to Schwarzschild coordinates). One can check that as long as one considers only terms which are lowest order in expansion parameters (i.e. $O(\frac{r_g}{r}), O(\frac{r_g}{r})$), and one restricts oneself to small gauge transformations, the changes in the interaction terms are higher order ones.

Various terms in (3) have a straightforward interpretation. The first term is the usual “flat” Dirac operator, the second term describes the central attractive force and the spin-orbit interaction. The terms in the second line don’t cause any spin-flip as they are spin-independent operators. The term from the third line describes the interaction of the orbital angular
momentum of the particle with the spin of the background\textsuperscript{[1]} and the last line contains the
tensorial operator describing spin-dependent gravitational interactions of the particle in ques-
tion.

3. Effects of interaction with gravitational field

In this section we estimate the
magnitude of several physical effects on spin 1/2 particle interacting with gravitational back-
ground. We are primarily interested in the effects of the spin-spin interactions, hence we
neglect the terms involving the angular momentum operator. Physically, we can imagine the
particle travelling (or emitted) radially, at some angle $\theta_i$ with respect to the rotation axis.
One should notice at this point that the contribution to the relevant cross sections coming
from orbital momentum interactions can be added incoherently to the spin–spin cross sections
at the level of tree-graph processes.

Secondly, we are of course interested in the coherent interaction of the particle spin with
the total, macroscopic spin of the body, as this can in principle enhance the gravitational
strength interactions which we consider. Hence, we have to assume some reasonable ultra-
violet momentum cut-off for the allowed range of momentum transfer during the scattering
event. In the following we assume as the cut-off the inverse Schwarzchild radius

$$q^2 < \frac{1}{r_s^2} \quad (4)$$

where the momentum transfer is $q^2 = 4p^2 \cos^2(\theta/2)$, $p$ being the momentum of the incoming
particle and $\theta$ the scattering angle. We note that in any case our metric (1) is not expected
to hold below the Schwarzchild radius.

We shall discuss the following effects:

\begin{enumerate}
\item Energy level shift for a particle with positive and negative helicities;
\item The cross section for spin flip in the process of scattering in the spinning background;
\item The alignment of spin along the direction of the rotation axis.
\end{enumerate}

To estimate the effects (2)–(3) we imagine the particle to be scattered from an asymptotic $in$
state to an asymptotic $out$ state (both corresponding to the flat Minkowski background far
\textsuperscript{3}One can see that the leading order in the nonrelativistic expansion of the hamiltonian density corresponding
to this term is proportional to $a\tilde{E}E_r_g/((E + m)r^3)$.}
from the source) and apply the standard rules for calculating the scattering cross sections in the external field described by

\[ \delta L = \frac{r g}{4 r^3} \gamma^5 \vec{\gamma} \vec{\alpha} - \frac{3 r g}{4 r^5} \gamma^5 (\vec{\gamma} \vec{\alpha}) (\vec{a} \vec{x}) \]  

(5)

Although this may be not more than a crude approximation to the effects expected for a particle e.g. produced in the rotating body, hopefully it provides the correct order of magnitude estimate for the actual interaction.

Let us note that interaction (5) resembles closely the interaction between a magnetic dipole moment and the magnetic field produced by another dipole \( \vec{m} \)

\[ \delta H_{mag} = -\vec{\mu} \vec{B} \]  

(6)

where \( \vec{B}_m(\vec{x}) = \frac{3(\vec{a} \vec{m}) - \vec{m}}{4|\vec{x}|^3} \). In the present case we can write (5) as

\[ \delta H = -\vec{S} \vec{B} = -\frac{1}{2} \vec{\Sigma} \vec{B} \]  

(7)

where \( \vec{B}(\vec{x}) = \frac{3(\vec{a} \vec{J}) - \vec{J}}{|\vec{x}|^3} \) with angular momentum of the background \( \vec{J} = M \vec{a} \) corresponding to the magnetic moment \( \vec{m} \) from (5). For a massive Majorana neutrino, which is not allowed to have static magnetic or electric dipole moments, the interaction (5),(7) is the only possible dipole-dipole type interaction (up to the order of magnitude considered here).

**Shift of energy levels between opposite chirality states**

We calculate the shift of energy levels

\[ \delta E = < \tilde{\Psi} | -\gamma^0 \delta L | \Psi > \]  

(8)

with \( \delta L \) given by (5) and the wavefunction of the fermion given by a well localized packet

\[ \Psi(p, \lambda) = \int_x f(x, p) u(p, \lambda) \]  

(9)

We assume that the function \( f \) is sufficiently well localized to pull the factors of \( 1/r \) outside the space integral. Spinors \( u \) are normalized as \( \bar{u} u = 2m \) hence the normalization of the packet profile \( f \) is \( \int d^3 x \bar{f} f = \frac{1}{2m} \). Let’s assume that the direction of \( \vec{a} \) coincides with the \( z \)-axis. Then the energy shifts \( \delta E^{+/-} \) of the states with positive (negative) helicity are

\[ \delta E^{(+)} = -\delta E^{(-)} = \frac{r g a}{4 r^3} (\cos(\theta) - 3 \frac{\vec{a} \vec{x}}{ar} \frac{x^1}{r} \sin(\theta) + \frac{x^3}{r} \cos(\theta)) \]  

(10)

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In the specific case of “radial” emission from the rotating star, \( \vec{x} = \vec{p}_i = \vec{n} \),

\[
\delta E^{(+)} = -\delta E^{(-)} = -\frac{r_g a}{2r^3} \cos(\theta) \tag{11}
\]

This can be interpreted as the mass shift of the helicity eigenstates, \( \delta m = \frac{r_g a}{r^5} |\cos(\theta)|. \) In the case of the Sun, which will be our standard example, this mass shift is

\[
\delta m = 0.7 \times 10^{-27} \left( \frac{r}{R_\odot} \right)^{-3} |\cos(\theta)| \text{eV} \tag{12}
\]

For \( r = r_\odot \) one gets \( \delta m = 8.9 \times 10^{-12} |\cos(\theta)| \text{eV}. \)

The scattering cross section

The differential cross section for scattering in the field (5) is given by

\[
d\sigma = \frac{1}{16\pi^2} |M_{fi}|^2 d\Omega \tag{13}
\]

where

\[
M_{fi} = \langle f | \delta L | i \rangle \tag{14}
\]

The fourier transform of (5) is

\[
F(\delta L) = \int_{-\infty}^{\infty} d^3x e^{i\vec{q}\cdot\vec{x}} \delta L = T(q) \frac{r_g a}{8} \gamma_5 \gamma_\vec{3} \tag{15}
\]

with \( \vec{3} = 3(\vec{n}_a \vec{n}_q - \vec{n}_a) \) and \( T = 4\pi \left( \frac{\cos(qr_g)}{qr_g} - \frac{\sin(qr_g)}{(qr_g)^2} \right), \vec{n}_a = \vec{a}/a, \vec{n}_q = \vec{q}/q. \) T approaches the constant value \(-4\pi/3\) for small \( q \) and oscillates approaching zero as the function of the natural variable \( qr_g = 2pr_g \sin(\theta/2). \) As we do not want to penetrate the inside of the Schwarzchild radius, we restrict the allowed range of \( q \) by the condition \( qr_g < 1. \)

We are now ready to calculate the differential cross section for transition between different spin states. Let \( \theta_i(f) \) denote the angle between the incoming (outgoing) momentum and the axis of rotation, and \( \theta \) the scattering angle. The final formulae for helicity flip (\( \sigma_{LR} \)), no-flip (\( \sigma_{LL} \)) and the total spin-spin interaction (\( \sigma_{TOT} \)) cross-sections are

\[
\frac{d\sigma_{LR}}{d\Omega} = \frac{1}{2\pi^2} 2a^2 m^2 r_g^2 |T|^2 \left( \frac{3}{2} \cos(\theta_f) - \cos(\theta_i) \right)^2
\]

\[
\left( 1 + 2 \cos^2(\theta/2) + 2 \cos^2(\theta/2) - 2 \cos(\theta_f) \cos(\theta_i) \right) \tag{16}
\]

\[
\frac{d\sigma_{LL}}{d\Omega} = \frac{1}{2\pi^2} 2a^2 m^2 r_g^2 |T|^2 \frac{E^2}{m^2} (1 - \cos(\theta) + 2 \cos(\theta_f) \cos(\theta_i)) \tag{17}
\]

\[
\frac{d\sigma_{TOT}}{d\Omega} = \frac{d\sigma_{LR}}{d\Omega} + \frac{d\sigma_{LL}}{d\Omega} \tag{18}
\]
where $\Omega$ is the solid angle around the direction of the scattered particle and the angles $\theta_i$, $\theta_f$, $\theta$ and the azimuthal angle $\phi$ are related through $\cos(\theta_f) = -\sin(\theta_i) \sin(\phi) + \cos(\theta) \cos(\theta_i)$.

For the benefit of the further discussion we have collected in the Appendix the values of the parameters entering the cross section formulae relevant for the case of the Sun.

4. Discussion and Conclusions

Let us discuss the spin-flip effect first. The differential cross section $d\sigma_{LR}$ we have got has the general form

$$d\sigma_{LR} = \kappa T^2 (r_g q) A(\theta_i, \theta, \theta_f) d\Omega$$

(19)

where $\kappa = 8 \times 10^{-4} a^2 r_g^2 m^2$, $q r_g = 2 p r_g \sin(\theta/2)$ and $A$ is some function of angular variables only. Moreover, when calculating the total cross sections we have to restrict our angular integration to scattering angles smaller than $\theta_{\text{max}} = \frac{1}{2} pr_g$. The integrated spin-flip scattering cross section as a function of the variable $R = 2 p r_g$ for fixed values of the angle $\theta_i$ can be found numerically. Using the analytic formula for $T(q)$ and formula (14) one can find an approximate expression for the magnitude of the integrated spin flip cross section

$$\sigma_{LR} = \begin{cases} 
10^{-13} (p/GeV)^{-2} (a GeV)^2 (m/keV)^2 GeV^{-2} & pr_g > 1 \\
10^{-13} (r_g GeV)^2 (a GeV)^2 (m/keV)^2 GeV^{-2} & pr_g < 1 
\end{cases}$$

(20)

(this is assuming $a \approx r_g$). Numerical values of the gravitational spin flip cross section should be compared to the average weak spin flip cross section assuming that the spin 1/2 particle in question interacts weakly (like a massive neutrino)

$$\sigma_{Z, \text{flip}} = 1.6 \times 10^{-23} (m/keV)^2 GeV^{-2}$$

(21)

Let us check when the gravitational spin flip becomes comparable to the weak spin flip

$$\sigma_{LR}/N = \sigma_{LR}/(M/m_{\text{proton}}) \geq \sigma_{Z, \text{flip}}$$

(22)

($N$ is the number of scattering centers in the body.) Using the formula (20) one can obtain the condition

$$M \geq M = 10^{47} GeV$$

(23)

(with $p < \frac{1}{r_g}$), which is fullfilled in the case of the Sun (however, the neutrinos actually coming from the Sun have much larger momenta), and the other condition

$$M \geq 10^{-10} (M_{\text{Planck}}/GeV)^4 (p/GeV)^2 GeV$$

(24)
which is relevant for relativistic fermions. This last condition is also fulfilled in the case of
the Sun as long as, approximately, $p \leq 10$ keV (which is still too small to be seen). One should
also note that according to (20) for larger $p$ the cross section falls off like $m^2/p^2$. A possibility
for the larger cross section is hidden in the factor $a^2 (\sigma_{LR} \approx \frac{a^2 m^2}{p^2})$, but the relation $a > r_g$
implies presence of the naked singularity for a rotating black hole, and also leads beyond the
range of validity of the perturbative expansion of the metric employed in our calculation.
From the point of view of the analogy [6], the conditions on the mass $M$ of the source or/and
on its angular momentum ($M \vec{a}$) correspond to determination of critical "magnetic fields"
which make the effect of gravitational dipole-dipole interaction important in comparison with
other forces in given contexts.

To discuss the possibility of the alignment of spin of the scattered particles along the axis
of rotation, let us define the average angle between the spin of the scattered particles and the
rotation axis (we work in helicity basis and assume that the incoming beam of particles has
a definite helicity)

$$
<\cos(\theta_s)> = \frac{1}{\sigma_{TOT}} \int d\Omega \cos(\theta_f) \left( \frac{d\sigma_{LR}}{d\Omega} - \frac{d\sigma_{LL}}{d\Omega} \right)
$$

(25)

This average angle should be compared with the angle between the spin of the incoming
particle and the rotation axis. We have to point out that in the relativistic case the effect
of the spin flip onto the spin alignment is negligible. The no-flip cross section is related to
spin-flip one through the approximate relation

$$
\sigma_{LR} \approx (1 + \frac{p^2}{m^2})^{-1} \sigma_{LL}
$$

(26)

and in the ultrarelativistic case $\sigma_{LL} >> \sigma_{LR}$. Thus, in this extreme case any observable
deflection of spin outwards or towards the rotation axis is due to the nontrivial distribution
of final momentum with respect to this axis, determined solely by the no-flip cross section.
However, as $p$ falls closer to $m$, both components of the cross section do contribute, as
discussed later.

Similarly, to discuss the alignment of the final momentum one defines

$$
<\cos(\theta_p)> = \frac{1}{\sigma_{TOT}} \int d\Omega \cos(\theta_f) \left( \frac{d\sigma_{LR}}{d\Omega} + \frac{d\sigma_{LL}}{d\Omega} \right)
$$

(27)
which, again, has to be compared with θ. Let us note that, taking into account (25), (26), (27), in the ultrarelativistic limit $<\cos(\theta_p)> = -<\cos(\theta_s)>$. Fortunately, one can write down approximate expressions for $<\theta_s>$ and $<\theta_p>$ valid when $pr_g > 1$, which covers most cases of practical interest:

$$<\cos(\theta_s)> = \cos(\theta_i) \left( -1 + \frac{2 \sin^2(\theta_i)}{\sin^2(\theta_i) + \frac{E^2}{m^2} \cos^2(\theta_i)} - \frac{0.24}{p^2 r_g^2} \right)$$

$$\times \frac{1}{(\sin^2(\theta_i) + \frac{E^2}{m^2} \cos^2(\theta_i))^2} \left( 4 \sin^4(\theta_i) + \frac{E^2}{m^2} \sin^2(2\theta_i) \right)$$

$$+ 2 \frac{E^2}{m^2} \left( 2 + 2 \sin^2(\theta_i) - 9 \sin^2(\theta_i) \cos^2(\theta_i) \right)$$

(28)

$$<\cos(\theta_p)> = \cos(\theta_i) \left( 1 + \frac{0.24}{p^2 r_g^2} \frac{\left( \frac{E^2}{m^2} - 1 \right) \sin^2(\theta_i)}{\sin^2(\theta_i) + \frac{E^2}{m^2} \cos^2(\theta_i)} \right)$$

(29)

(28)

(where using the approximate expressions one should take care that the corrections are smaller than the leading contributions over the whole range of $\theta_i$, the relevant condition being $r < p^2 r_g^2/0.24$).

The distributions of $\theta_s$ and $\theta_p$ as functions of the angle $\theta_i$ between initial momentum and the rotation axis are shown in the Figure 1 for $x = E^2/m^2$ equal to 4 and 100. Figure 1(a) shows the case when the momentum of scattered particle lies in the low energy range i.e. is negligible with respect to the planck scale, Figure 1(b) shows the case when momentum is comparable to the planck scale. It is clearly visible that the average final spin gets deflected towards the direction of the source spin, $\vec{a}$, when $\theta_i < \pi/2$ and outwards when $\theta_i > \pi/2$. These deviations from the incoming direction of spin become smaller the more relativistic the particle is (the larger $x$ becomes), and have maxima, one for $\theta_i < \pi/2$ and one for $\theta_i > \pi/2$, which are approaching $\pi/2$ as the ratio $x$ grows. As seen from (27) the deflection of the momentum is a second-order effect in the expansion we use, proportional to $p^{-2} r_g^{-2}$, and for typical momenta encountered in terrestrial or solar physics it is practically zero. To be able to draw lines visible on the picture 1(b) we have assumed an exotic value of the order parameter, $pr_g = 6$ (which implies planck scale $p$ and also very large $m$ for chosen values of $x$). One can see that the outgoing momentum gets deflected towards the direction of the source spin.

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2In the case of the Sun this inequality means $p > 10^{-10}$ eV.
Figure 1. Distributions of the mean angles between average outgoing spin, \( \theta_s(\theta_i) \), average outgoing momentum, \( \theta_p(\theta_i) \), and the rotation axis. Plots are given for two values of the ratio \( x = E^2/m^2 \), \( x = 4 \) (solid lines) and \( x = 100 \) (thick solid lines). Figure (a) shows the case of terrestrial-scale momentum, when the \( \theta_p \) deviation from \( \theta_i \) is heavily suppressed and invisible on the picture. Figure (b) shows the distribution of \( \theta_p - \theta_i \) visible when momentum enters the planck scale region – it is obtained with \( pr_g = 6 \) (initial spin polarization is assumed antiparallel to incoming momentum).

when \( \theta_i < \pi/2 \) and away from the vector \( \vec{a} \) when \( \theta_i > \pi/2 \). Again, there are maxima in the deflection angle \( |\theta_p - \theta_i| \) which come closer to \( \theta_i = \pi/2 \) as \( x \) grows. The visible tendency for the deflection in momentum distribution in Figure 1(b) to grow with growing \( x \) whereas the deflection in spin distribution in Figure 1(a) gets suppressed for larger \( x \) stems from the fact, that Figures 1(a) and 1(b) are drawn for vastly different values of momentum \( p \). Hence, the two distributions are determined by expressions which are of different order in parameter \( 1/(pr_g) \) and have different \( x \)-dependence. The leading, \( o\left(\frac{1}{(pr_g)^0}\right) \) term in spin asymmetry (28) behaves as \( o\left(\frac{1}{x}\right) \) when \( x \) grows. But the nonleading, \( o\left(\frac{1}{(pr_g)^2}\right) \) terms in both (28) and (29) contain \( x \) both in numerators and denominators. They increase with \( x \) and in the limit of very large \( x \) they behave as \( O(x^0) \). In fact, (28) and (29) become the same expression, up to a sign, in the limit \( x \to \infty \) as they should in agreement with the remark after (27).
Finally, one should compare the magnitude of the effects due to the spin-spin interaction with the effects of spin-orbit interaction, which arises due to the existence of the second term in the Dirac operator as the standard Thomas-precession term. The total spin-flip cross section caused by L-S interaction is computed to be (we consider the example of the Sun again)

\[ \sigma_{LR-LS} = \frac{\pi E^2 m^2 r_g^2}{2p^4 \log(2pR_\odot)} \] (30)

The spin-orbit cross section falls down for large momenta at the same rate as the spin-spin flip cross section (like \( \frac{m^2}{p^2} \)) with similar numerical coefficients, so for the relativistic neutrinos (fulfilling however the conditions formulated above) the two effects can be comparable. Also, the spin-orbit cross section has an obvious peak at small momenta whereas the spin-spin cross section approaches a constant value as \( p \) goes to 0. One has to admit that, in general, the spin-orbit interaction forms a significant background hiding the spin-spin effects. However, there are certain kinematical conditions which are in favour of spin-spin interaction, for instance the situation of quasi-radial emission of neutrinos from the Sun, from cores of supernovae or neutron stars, when the orbital angular momentum is naturally suppressed.

Let us discuss briefly possible cosmological implications of our results. First, we have to say that as far as solar neutrinos are concerned, that have energies between 0.1 – 10 MeV and in typical models masses between \( 10^{-3} \) and 1 eV, our effect is subdominant with respect to weak or magnetic moment spin-flipping interactions, although it is not as dramatically small as one would be tempted to claim naively. If there would be in the solar spectrum neutrinos with energies smaller than approximately 10 keV, then spin-solar-spin interactions of these particles would be important. The domain where spin-dependent interactions of neutrinos are important, if there are massive ones, is physics of background primordial neutrinos, for review cf. [6]. These primordial neutrinos have today the average momentum of \( 5.2 \times 10^{-4} \) eV and, with a source of the solar or larger mass, they typically interact reasonably strongly via gravitational spin-spin interactions as seen from (22), (23), (24) – at least stronger than weakly. Of course, also in this case for the flipping to be comparable to no-flip interactions

3For the Sun the spin-orbit interaction was considered in ref. [4]
these particles should be nonrelativistic, i.e. sufficiently massive. Hence, one expects the primordial neutrino sea to be partially polarized (spins aligned along the background angular momentum) in the vicinity of the Sun, and generally in the vicinity of any massive, rotating body in the Universe. The exact nature of the final spin state of the local neutrino sea would depend on the local kinematics, in particular on the degree of anisotropy of the momentum distribution of the neutrinos in the source’s center of mass rest frame. However, at present we are not aware of any real or “gedanken” experiment which can see and test the neutrino background. Similar conclusions hold also for other fermionic primordial relics, in fact the effect should be particularly important for massive warm or cold relics, like background gravitinos, if they exist.

At last, let us discuss spin-spin interactions in the context of the supernova physics. As pointed out in the context of weak or magnetic moment interactions if the spin flipping is to efficient, then the sterile right-handed neutrinos stream freely from the supernova core and the supernova cooling is too rapid. In fact in the case of the supernova SN 1987A the stream of neutrinos was observed over a period of the order of 10 seconds, which gives a direct limit on the flipped neutrino luminosity from the supernova, $L < 4 \times 10^{-22} \text{GeV}^2$. We can compute the flipped neutrino luminosity due to the gravitational spin-flip

$$L_g \approx 1.8 N_\nu \sigma_{LR} T^4$$

where $N_\nu$ is the number of neutrino flavours considered. If one demands the $L_g$ to be smaller than the limiting value quoted above and taking the standard reference value for the temperature $T$, $T_o = 30 \text{ MeV}$, then one obtains the limit

$$\left( \frac{a}{r_g} \right)^2 \left( \frac{m}{\text{keV}} \right)^2 \left( \frac{T}{T_o} \right)^4 < 0.7 \times 10^{10}$$

If one takes $a/r_g \approx 1$ then one gets an upper limit on $m$, $m < 30 \text{ MeV}$. This number coincides with similar limits coming from weak and magnetic moment interactions. Unfortunately, our

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4One should note at this point that, for instance, Sun moves with respect to the background with a velocity $v_S \approx 10^{-3}$. But, if neutrino masses are in the typical model range, i.e. at most of the order of a few eV, cf. then in solar rest frame they have momenta at most of a few times $10^{-3}$ eV, so they are still nonrelativistic if they were so in the background frame. The same applies to other kinds of nonrelativistic relics.
limit is in fact weaker. The analysis of possible models for rapidly rotating pulsars, §, has shown that reasonable values of $a$ are rather $0.30 - 0.34 r_g$, hence our limit probably becomes an order of magnitude weaker.

In conclusion, we have described and examined the interaction of spin-one half fermions with the spin of the local gravitational field due to rotation of some massive body. These interactions, although more effective than one would naively guess, do not seem to change any cosmological effect observed so far. They can become important in case of unusually fast rotating and massive pulsars, and they are important for background primordial neutrinos and for other primordial fermionic relics from the Big-Bang epoch.

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Appendix The vierbein which reconstructs the metric (I) through $g_{\mu\nu} = \eta_{\alpha\beta}e_{\mu}^{\alpha}e_{\nu}^{\beta}$ we choose in the form

$$
e_0^\mu = (1 - \frac{r_g}{2r}, \frac{r_g^2}{8r^2}, \frac{r_g}{r^3} \vec{\omega})$$

$$
e_1^\mu = (0, 1 + \frac{r_g}{2r} + \frac{r_g^2}{4r^2}, 0, 0)$$

$$
e_2^\mu = (0, 0, 1 + \frac{r_g}{2r} + \frac{r_g^2}{4r^2}, 0)$$

$$
e_3^\mu = (0, 0, 0, 1 + \frac{r_g}{2r} + \frac{r_g^2}{4r^2})$$

(A.1)

The inverse vierbein is

$$
e_0^\alpha = ((1 - \frac{r_g}{2r} - \frac{r_g^2}{4r^2})^{-1}, -\frac{r_g}{r^3} \vec{\omega})$$

$$
e_1^\alpha = (0, (1 + \frac{r_g}{2r} + \frac{r_g^2}{4r^2})^{-1}, 0, 0)$$

$$
e_2^\alpha = (0, 0, (1 + \frac{r_g}{2r} + \frac{r_g^2}{4r^2})^{-1}, 0)$$

\( e^3_\alpha = (0, 0, 0, (1 + \frac{r \rho}{2 r} + \frac{r^2}{4 r^2})^{-1}) \)  

(A.2)

One can easily find Christoffel symbols for the metric (1), we list below the nonvanishing ones

\[
\begin{align*}
\Gamma^0_{0l} &= \frac{1}{2r^3} r \rho x^l \\
\Gamma^i_{00} &= \frac{1}{2r^3} r \rho x^i \\
\Gamma^0_{kl} &= -\frac{3r \rho}{2r^3} (x^l \omega_k + x^k \omega_l) \\
\Gamma^i_{0j} &= \frac{r \rho}{r^3} \epsilon_{jkl} a^k - \frac{3r \rho}{2r^3} (x^i \omega^j - x^j \omega_i) \\
\Gamma^i_{kl} &= \frac{1}{2} (f^i_{jl} \delta_{kl} - f^j_{il} \delta_{lk} - f^k_{jl} \delta_{il})
\end{align*}
\]

(A.3)

where \( f^i_{jl} = \frac{\partial f}{\partial x^j} \), \( f = -\frac{\rho}{r} - \frac{r^2}{2r^2} \), \( f^i_{jl} = \left( \frac{\partial f}{\partial x^i} + \frac{r^2}{2r^2} \right) x^j \), \( i, k, l \) denote 3-d indices, \( \Gamma^i_{ij} = \Gamma^i_{ji} \) and the 3-d antisymmetric tensor is normalized so that \( \epsilon_{123} = 1 \).

Parameters entering the cross section formulae relevant for the case of the Sun are

\[
\begin{align*}
M_\odot &= 1.1 \times 10^{57} \text{ GeV} & J_\odot &= 0.2 \times 10^{76} \\
R_\odot &= 3.5 \times 10^{24} \text{ GeV}^{-1} & M_{\text{Planck}} &= 1.2 \times 10^{19} \text{ GeV} \\
a_\odot &= 0.2 \times 10^{19} \text{ GeV}^{-1} & r_g &\odot &= 1.5 \times 10^{19} \text{ GeV}^{-1}
\end{align*}
\]

(A.4)

References

[1] L.D. Landau, E.M. Lifshitz, The Classical Theory of Fields, Pergamon Press 1975.

[2] J.D. Bjorken, S.D. Drell, Relativistic Quantum Mechanic, McGraw-Hill Book Company 1964.

[3] K.J.F. Gaemers, R. Ganhdii, J.M. Lattimer, Phys.Rev. D40, 309 (1989).

[4] D. Choudhury, N.D. Hari Dass, M.V.N. Murthy, Class.Quantum Grav. 6, L167 (1989).

[5] P. Langacker, Solar Neutrinos, U. of Penn. prep. UPR-0640T, 1994.
[6] P. Langacker, J.P. Leveille, J. Sheiman, Phys.Rev. D27, 1228 (1983).

[7] J. Lattimer, J. Cooperstein, Phys.Rev.Lett. 61, 23 (1988).

[8] J.L. Friedman, J.R. Ipser, L. Parker, Phys.Rev.Lett. 62, 3015 (1989).