Some New Gears Aspects

Florian Ion Tiberiu Petrescu

ARoTMM-IFToMM, Bucharest Polytechnic University, Bucharest, (CE), Romania

Abstract: In today's mechanical transmissions most widely used, the gears, are spread across all industries. For this reason, their importance has become overwhelming, which is why we want to recall in this paper some important aspects regarding toothed wheels. It is the geometry, cinematic, the forces and the yields of these mechanisms, which will be presented in the work in the form of newly synthesized relations on modern bases. Another important aspect in toothed wheels is their synthesis by modern methods that avoid tooth interference during operation. To avoid the interference between teeth, we must know the minimum number of teeth of the driving wheel, in function of the pressure angle (normal on the pitch circle, alpha0), in function of the tooth inclination angle (beta) and in function of the transmission ratio (i). In optimal and high-efficiency gearing, gears require a modern design with increased coverage. These achievements can only be achieved today in the context of lowering the value of the alpha engagement angle. Through all the aspects presented, which relate to the dynamics of gears, the work can be considered among those of the optimal dynamic synthesis of the gears.

Keywords: Gears, Gearboxes, Dynamic Synthesis, Yield

Introduction

Gears have spread today in all areas. They have the advantage of working with very high efficiency. In addition, tools can transmit large tasks. Regardless of their size, tools need to be synthesized carefully according to specific conditions.

This paper tries to present the main conditions that must be met for the correct synthesis of a tool.

The beginning of the use of pinion gears should be sought precisely in ancient Egypt at least a thousand years before Christ, where for the first time wheel drive units were used for irrigation and worm gear worm gears for cotton processing.

Then, 230 years BC, in Alexandria, Egypt, the toothed wheel was used again.

These tools have been built and used since ancient times to handle heavy anchors and catapults used on battlefields. These were then introduced into the wind and water mills (as a reduction or multiplication in wind or water pumps) (Fig. 1).

The Antikythera Mechanism is a name given to a complex astronomical device, a 32×16×10 cm device discovered in 1900 in a sunken ship near the coast of Antikythera, an island between Crete and the Greek continent, for which several types of evidence undoubtedly point to around 80 BC, for the date of the shipwreck. The device was made of bronze gears mounted in a wooden box, but due to the fact that it was crushed in the wreck, various parts of the faces were lost and the remainder was then covered with a hard limestone deposit in time at the same time as the corroded metal to a thin core covered with strong metal salts that retains much of the previous bronze shape during the 2000 years of the dive (See Antikythera 1 in Fig. 2).

The modern adventure of the toothed wheel began with the toothed wheel created by Leonardo da Vinci in the fifteenth century. He is also the founder of a new cinema and dynamics, stating, among other things, the principle of overlapping independent movements (Fig. 3).

Benz has created an original toothed and transmission chain engine (patented after 1882, Fig. 4), but the first patent of a toothed gear belongs to British British Starley & Hillman in 1870 (12 years before the Germans) being designed and built to be used for bicycle transmissions and later for motored tricycles.

In Cleveland (USA), begin after 1912 to produce industrial specialized wheels and gears (cylindrical, worm, conical, with straight teeth, inclined or curved; Fig. 5).
Fig. 1: Transmissions wheeled "spurred" to irrigate crops and worm gears to the cotton processing

Fig. 2: The Antikythera mechanism is the name given to an astronomical calculating device

Fig. 3: The modern adventure began with the gear wheel spurred of Leonardo da Vinci, in the fifteenth century
After 1912, in Cleveland (USA), begin to produce industrial specialized wheels and gears (cylindrical, worm, conical, with straight teeth, inclined or curved).

The gears are present today everywhere, in the mechanical world (in vehicle’s industries, in electronics and electro-technique types of equipment, in energetically industries, etc.; Fig. 6).
The paper presents how to accurately determine the mechanical performance of a gearbox. Based on these relationships, an optimal synthesis of the performance of a classic, mechanical, manual gearshift can be achieved regardless of its operating status (Frăţilă et al., 2011; Pelecudi, 1967; Antonescu, 2000; Comănescu et al., 2010; Aversa et al., 2016a; 2016b; 2016c; 2016d; 2017a; 2017b; 2017c; 2017d; 2017e; Mirsayar et al., 2017; Cao et al., 2013; Dong et al., 2013; De Melo et al., 2012; Garcia et al., 2007; Garcia-Murillo et al., 2013; He et al., 2013; Lee, 2013; Lin et al., 2013; Liu et al., 2013; Padula and Perdereau, 2013; Perumaal and Jawahar, 2013; Petrescu and Petrescu, 1995a; 1995b; 1997a; 1997b; 1997c; 2000a; 2000b; 2002a; 2002b; 2003; 2005a; 2005b; 2005c; 2005d; 2005e; 2016a; 2016b; 2016c; 2016d; 2016e; 2013; 2012a; 2012b; 2011; Petrescu et al., 2009; 2016a; 2016b; 2016c; 2016d; 2016e; 2017a; 2017b; 2017c; 2017d; 2017e; 2017f; 2017g; 2017h; 2017i; 2017j; 2017k; 2017l; 2017m; 2017n; 2017o; 2017p; 2017q; 2017r; 2017s; 2017u; 2017v; 2017w; 2017x; 2017y; 2017z; 2017aa; 2017ab; 2017ac; 2017ad; 2017ae; Petrescu and Calautit, 2016a-b; Reddy et al., 2012; Tabakovov et al., 2013; Tang et al., 2013; Tong et al., 2013; Wang et al., 2013; Wen et al., 2012; Antonescu and Petrescu, 1985; 1989; Antonescu et al., 1985a; 1985b; 1986; 1987; 1988; 1994; 1997; 2000a; 2000b; 2001; List the first flights, From Wikipedia; Chen and Patton, 1999; Fernandez et al., 2005; Fonod et al., 2015; Lu et al., 2015; 2016; Murray et al., 2010; Palumbo et al., 2012; Patre and Joshi, 2011; Sevil and Dogan, 2015; Sun and Joshi, 2009; Crickmore, 1997; Donald, 2003; Goodall, 2003; Graham, 2002; Jenkins, 2001; Landis and Dennis, 2005; Clément, Wikipedia; Cayley, Wikipedia; Coandă, Wikipedia; Gunston, 2010; Lamming, 2000; Norris, 2010; Goddard, 1916; Kaufman, 1959; Oberth, 1955; Cataldo, 2006; Gruener, 2006; Sherson et al., 2006; Williams, 1995; Venkataraman, 1992; Oppenheimer and Volkoff, 1939; Michell, 1784; Droste, 1915; Finkelstein, 1958; Gorder, 2015; Hewish, 1970).

**Materials and Methods; Gearings Synthesis**

In a cylindrical gearing, forces, speeds, powers and efficiency can be determined using relationships 2.1-2.6 and can be seen in Fig. 7:

\[
\begin{align*}
F_1 &= F_m \cdot \cos \alpha_i \\
F_2 &= F_m \cdot \sin \alpha_i \\
v_y &= v_i \cdot \cos \alpha_i \\
v_{z2} &= v_i \cdot \sin \alpha_i \\
F_{n2} &= F_r + F_{v} \\
\tau_1 &= \tau_r + \tau_{z2}
\end{align*}
\]

(2.1)
Where:
\[ F_m = \text{The motive force (the driving force)} \]
\[ F_\tau = \text{The transmitted force (the useful force)} \]
\[ F_\psi = \text{The slide force (the lost force)} \]
\[ v_1 = \text{The velocity of element 1, or the speed of wheel 1 (the driving wheel)} \]
\[ v_2 = \text{The velocity of element 2, or the speed of wheel 2 (the driven wheel)} \]
\[ v_{12} = \text{The relative speed of the wheel 1 in relation with the wheel 2 (this is a sliding speed)} \]

The consumed power (in this case the driving power):
\[ P_c = P_m = F_m \cdot v_1 \quad (2.2) \]

The useful power (the transmitted power from the profile 1 to the profile 2) will be written:
\[ P_\tau = P_\tau = F_\tau \cdot v_2 = F_m \cdot v_1 \cdot \cos^2 \alpha_i \quad (2.3) \]

The lost power will be written:
\[ P_\psi = P_\psi = F_\psi \cdot v_{12} = F_m \cdot v_1 \cdot \sin^2 \alpha_i \quad (2.4) \]

The momentary efficiency of couple will be calculated directly with the next relation:
\[ \eta = \frac{P_\tau}{P_m} = \frac{P_{\tau \omega}}{P_m} = \frac{F_m \cdot v_1 \cdot \cos^2 \alpha_i}{F_m \cdot v_1} \]
\[ \eta = \cos^2 \alpha_i \quad (2.5) \]

The momentary losing coefficient, will be written:
\[ \eta' = \frac{P_\psi}{P_m} = \frac{F_m \cdot v_1 \cdot \sin^2 \alpha_i}{F_m \cdot v_1} = \sin^2 \alpha_i \]
\[ \eta + \eta' = \cos^2 \alpha_i + \sin^2 \alpha_i = 1 \quad (2.6) \]

It can easily see that the sum of the momentary efficiency and the momentary losing coefficient is 1.

Now, one can determine the geometrical elements of gear. These elements will be used in determining the couple efficiency, \( \eta \).

The main geometric elements belonging to the external cylindrical gear (for straight teeth, \( \beta = 0 \)) can still be determined.

The radius of the basic circle of the wheel 1 (of the driving wheel), (2.7):
\[ r_{b1} = \frac{1}{2} \cdot m \cdot z_i \cdot \cos \alpha_i \quad (2.7) \]

The radius of the outside circle of wheel 1 (2.8):
\[ r_{s1} = \frac{1}{2} \left( m \cdot z_i + 2 \cdot m \right) = \frac{m}{2} \left( z_i + 2 \right) \quad (2.8) \]
It determines now the maximum pressure angle of the gear (2.9):

$$\cos \alpha_{\text{IRM}} = \frac{1}{2} \frac{m \cdot z_1 \cdot \cos \alpha_a}{\cos \alpha_a \cdot z_1 + 2}$$ (2.9)

And now one determines the same parameters for the wheel 2, the radius of basic circle (2.10) and the radius of the outside circle (2.11) for the wheel 2:

$$r_{s2} = \frac{1}{2} m \cdot z_2 \cdot \cos \alpha_a$$ (2.10)

$$r_{e2} = \frac{m}{2} (z_2 + 2)$$ (2.11)

Now it can determine the minimum pressure angle of the external gear (2.12, 2.13):

$$\tan \alpha_{\text{IRM}} = \frac{N}{N_1 - r_{e1}}$$

$$N = (r_{s1} + r_{s2}) \cdot \tan \alpha_a - \sqrt{r_{s1}^2 - r_{s2}^2}$$

$$= \frac{1}{2} \frac{m \cdot (z_1 + z_2) \cdot \sin \alpha_a}{\cos \alpha_a}$$

$$= \frac{m}{2} \left( (z_1 + z_2) \cdot \sin \alpha_a - \sqrt{z_1^2 \cdot \sin^2 \alpha_a + 4 \cdot z_1 + 4} \right)$$

$$\cot \alpha_{\text{IRM}} = \frac{1}{z_1} \frac{z_1 \cdot \cos \alpha_a}{\sqrt{z_1^2 \cdot \sin^2 \alpha_a + 4 \cdot z_1 + 4}} (2.13)$$

Now we can determine, for the external gear, the minimum (2.13) and the maximum (2.9) pressure angle for the right teeth. For the external gear with bended teeth ($\beta \neq 0$) it uses the relations (2.14 with 2.17, 2.18-A, or with 2.19, 2.20-B):

**A. When the Driving Wheel 1, Has External Teeth:**

$$\tan \alpha_{\text{IRM}} = \left[ \frac{(z_1 - z_2) \cdot \sin \alpha_a}{\cos \beta} \right.$$  

$$+ \sqrt{z_1 \cdot \sin^2 \alpha_a + 4 \cdot z_1 + 4} \cdot \frac{\cos \beta}{z_1 \cdot \cos \alpha_a} \left. \right]$$ (2.17)

$$\cos \alpha_{\text{IRM}} = \frac{z_1 \cdot \cos \alpha_a}{\cos \beta \cdot z_1 + 2}$$ (2.18)

**B. When the Driving Wheel 1, Have Internal Teeth:**

$$\tan \alpha_{\text{IRM}} = \left[ \frac{(z_1 - z_2) \cdot \sin \alpha_a}{\cos \beta} \right.$$

$$+ \sqrt{z_1 \cdot \sin^2 \alpha_a + 4 \cdot z_1 + 4} \cdot \frac{\cos \beta}{z_1 \cdot \cos \alpha_a} \left. \right]$$ (2.19)

The mechanical efficiency of the cylindrical gear shall be determined by integrating the instantaneous efficiency across all gear sections of the gear unit starting from the minimum pressure angle and going up to the maximum pressure angle as defined in expression (2.21):

$$\eta = \frac{1}{2} \int_{\alpha_m}^{\alpha_M} \frac{\eta \cdot d\alpha}{\Delta \alpha}$$

$$= \frac{1}{2} \times \left[ \frac{\sin (2 \cdot \alpha) + \alpha}{\Delta \alpha} \right]_{\alpha_m}^{\alpha_M}$$ (2.21)

For the internal gear with bended teeth ($\beta \neq 0$) it uses the relations (2.14 with 2.17, 2.18-A, or with 2.19, 2.20-B):

**A. When the Driving Wheel 1, Has External Teeth:**

$$\tan \alpha_{\text{IRM}} = \left[ \frac{(z_1 - z_2) \cdot \sin \alpha_a}{\cos \beta} \right.$$  

$$+ \sqrt{z_1 \cdot \sin^2 \alpha_a + 4 \cdot z_1 + 4} \cdot \frac{\cos \beta}{z_1 \cdot \cos \alpha_a} \left. \right]$$ (2.17)

$$\cos \alpha_{\text{IRM}} = \frac{z_1 \cdot \cos \alpha_a}{\cos \beta \cdot z_1 + 2}$$ (2.18)

**B. When the Driving Wheel 1, Have Internal Teeth:**

$$\tan \alpha_{\text{IRM}} = \left[ \frac{(z_1 - z_2) \cdot \sin \alpha_a}{\cos \beta} \right.$$  

$$+ \sqrt{z_1 \cdot \sin^2 \alpha_a + 4 \cdot z_1 + 4} \cdot \frac{\cos \beta}{z_1 \cdot \cos \alpha_a} \left. \right]$$ (2.19)

The mechanical efficiency of the cylindrical gear shall be determined by integrating the instantaneous efficiency across all gear sections of the gear unit starting from the minimum pressure angle and going up to the maximum pressure angle as defined in expression (2.21):

$$\eta = \frac{1}{2} \int_{\alpha_m}^{\alpha_M} \frac{\eta \cdot d\alpha}{\Delta \alpha}$$

$$= \frac{1}{2} \times \left[ \frac{\sin (2 \cdot \alpha) + \alpha}{\Delta \alpha} \right]_{\alpha_m}^{\alpha_M}$$ (2.21)

Electronic copy available at: https://ssrn.com/abstract=3306815
larger, more natural, stronger, more complete and without wear, noises, shocks, like in external gears. However, due to the fact that the internal gears are provided with additional conditions for avoiding the teeth interference in contact, the correct design is much more difficult and from a technological point of view it sometimes does not work correctly, leading to slight random interferences during the operation of the built-in gear, which in the course of time lead to premature wear, noises, or even blocking in operation, although their operation should have been much superior theoretically and for this reason the design difficulties most often give up the superiority internal gears preferring the choice of the outer ones.

To an external gearing, contact between profiles shall only be made to a single point, while at the internal gearing the contact between profiles is by winding each other (Fig. 9).

![Fig. 8: (a) An external gearing; (b) An internal gearing](image)

**Fig. 8:** (a) An external gearing; (b) An internal gearing

![Fig. 9: Contact between profiles](image)

**Fig. 9:** Contact between profiles
Results; Gears Synthesis by Avoid the Interferences

In order to avoid interference phenomenon, point $A$ must lie between $C$ and $K_1$ (the addendum circle of the wheel 2, $C_a2$ need to cut the line of action between points $C$ and $K_1$ and under no circumstances does not exceed the point $K_1$). Similarly, $C_a1$ addendum circle must cut the action line between points $C$ and $K_2$, resulting in point $E$, which in no circumstances, does not exceed the point $K_2$.

The conditions to avoid the phenomenon of interference can be written with the relations (3.1).

The basic conditions of interference, are the same ($CA<K1C; CE<K2C$), but the originality of this new presented method consist in the mode in which it was solved the classical relationship (see the system 3.1) (Fig. 10).

The system (3.3) represents a simple, unitary and general relationship capable of generating functional solutions for gears, giving the minimum number of teeth of wheel 1 (motor wheel) to avoid interference. In the appendix Table 1-15 an alpha0 value ($35^\circ$) will be chosen and the beta angles (from $0^\circ$ to $40^\circ$) and the transmission ratio $i$ (from 1 to 80) are incrementally incremented in order to thus getting the minimum number of teeth correctly.

Then, the alpha value (from $35^\circ$ to $5^\circ$) will be decreased successively.

At the internal gearbox, the interference avoidance condition is the same as for the external gear (relationship 3.3):
\[
CA < K_i C \quad \text{and} \quad CE < K_o C
\]
\[
CA = K_i A - K_o C = \sqrt{r_i^2 - r_0^2} - r_i \cdot \sin \alpha_0; \quad CA < K_o C
\]
\[
\Rightarrow \sqrt{r_i^2 - r_0^2} - r_i \cdot \sin \alpha_0 < r_i \cdot \sin \alpha_o \Rightarrow \sqrt{r_i^2 - r_0^2} < (r_i + r_j) \cdot \sin \alpha_o
\]
\[
\Rightarrow d_{i_n}^2 - d_{o_n}^2 < (d_i + d_o) \cdot \sin^2 \alpha_o
\]
\[
\Rightarrow m^2 \cdot (z_i + 2)^2 - m^2 \cdot z_i^2 \cdot \cos^2 \alpha_o < m^2 \cdot (z_i + z_j)^2 \cdot \sin^2 \alpha_o
\]
\[
z_i^2 + 4 \cdot z_i + 4 - z_j^2 < z_i^2 \cdot \sin^2 \alpha_o + 2 \cdot z_i \cdot z_j \cdot \sin^2 \alpha_o
\]
\[
\Rightarrow 4 \cdot z_i + 4 < z_j^2 \cdot \sin^2 \alpha_o + 2 \cdot z_i \cdot z_j \cdot \sin^2 \alpha_o
\]

from \( CE < K_o C \) \( \Rightarrow \) \( 4 \cdot z_i + 4 < z_j^2 \cdot \sin^2 \alpha_o + 2 \cdot z_i \cdot z_j \cdot \sin^2 \alpha_o \)

it obtains the system

\[
\begin{aligned}
4 \cdot z_i + 4 < z_j^2 \cdot \sin^2 \alpha_o + 2 \cdot z_i \cdot z_j \cdot \sin^2 \alpha_o \\
4 \cdot z_i + 4 < z_j^2 \cdot \sin^2 \alpha_o + 2 \cdot z_i \cdot z_j \cdot \sin^2 \alpha_o
\end{aligned}
\]

take \( i \equiv \frac{z_j}{z_i} \Rightarrow z_j = i \cdot z_i \); result the system

\[
\begin{aligned}
\sin^2 \alpha_o \cdot (1 + 2 \cdot i) \cdot z_i^2 - 2 \cdot 2 \cdot i \cdot z_i - 4 > 0 \\
\sin^2 \alpha_o \cdot (i^2 + 2 \cdot i) \cdot z_i^2 - 2 \cdot 2 \cdot z_i - 4 > 0 \quad \text{with the solutions:}
\end{aligned}
\]

\[
\begin{align*}
z_{i_n} &= \frac{2 \cdot i \pm 2 \cdot \sqrt{i^2 + \sin^2 \alpha_o + 2 \cdot i \cdot \sin^2 \alpha_o}}{(2 \cdot i + 1) \cdot \sin^2 \alpha_o} \\
z_{i_s} &= \frac{2 \cdot 2 \pm 2 \cdot \sqrt{1 + i^2 \cdot \sin^2 \alpha_o + 2 \cdot i \cdot \sin^2 \alpha_o}}{(2 \cdot i + i^2) \cdot \sin^2 \alpha_o} \quad \text{it keeps solutions +}
\end{align*}
\]

\[
\begin{align*}
z_i &= \frac{2 + \sqrt{i^2 + \sin^2 \alpha_o + 2 \cdot i \cdot \sin^2 \alpha_o}}{(2 \cdot i + 1) \cdot \sin^2 \alpha_o} \\
z_i &= \frac{2 + \sqrt{1 + i^2 \cdot \sin^2 \alpha_o + 2 \cdot i \cdot \sin^2 \alpha_o}}{(2 \cdot i + i^2) \cdot \sin^2 \alpha_o}
\end{align*}
\]

Relationship which generates \( z_{i_n} \) always gives lower values than the relationship which generates \( z_i \) so it is sufficient the condition (3.2) for finding the minimum number of teeth of the wheel 1, necessary to avoid interference:

\[
z_{\min} = z_i = 2 \cdot \frac{i + \sqrt{i^2 + \sin^2 \alpha_o + 2 \cdot i \cdot \sin^2 \alpha_o}}{(2 \cdot i + 1) \cdot \sin^2 \alpha_o} \tag{3.2}
\]

When we have inclined teeth, one takes \( z_{\min} \rightarrow z_{\min}/\cos \beta \) and \( \alpha_0 \rightarrow \alpha_0/\cos \beta \) and the relationship (3.2) takes the form (3.3). The minimum number of teeth of the driving wheel 1, is a function on some parameters: The pressure angle (normal on the pitch circle, \( \alpha_0 \)), the tooth inclination angle (\( \beta \)) and the transmission ratio (\( i = |z_i| = \frac{|z_i^*|}{|z_i|} \)), (see the relationship 3.3):

\[
\begin{align*}
z_{\min} &= z_i = 2 \cdot \cos \beta \cdot \frac{i + \sqrt{i^2 + \sin^2 \alpha_o + 2 \cdot i \cdot \sin^2 \alpha_o}}{(2 \cdot i + 1) \cdot \sin^2 \alpha_o} \\
\end{align*}
\]

\[
\begin{align*}
\text{where: } & \quad \tan \alpha_o = \frac{\tan \alpha_o}{\cos \beta} \Rightarrow \alpha_o = \arctan \left( \frac{\tan \alpha_o}{\cos \beta} \right) \\
& \quad (3.3)
\end{align*}
\]

In addition, the inner gear can also write the additional condition of the wheel with internal teeth (systems 3.4 and 3.5). If the mechanism is designed and built without checking these two additional conditions for the existence of an internal gear, it will not work properly. As it has already shown, the inner gear is much superior in operation to the outside, but only when rigorous design and construction, its manufacturing technology being much more difficult than that of the classic outer gear.
It should also be mentioned that additional relations (3.6) have also been used.

\[
\begin{align*}
  \tau_i &= \frac{1}{2} m \cdot z_i; \quad \tau_i = \frac{1}{2} m \cdot z_i; \quad \tau_i = \frac{1}{2} m \cdot z_i; \\
  r_i &= r_i + m = \frac{1}{2} m \cdot z_i + \frac{1}{2} m = \frac{m}{2} (z_i + 2); \\
  r_j &= r_j - m = \frac{1}{2} m \cdot z_j - \frac{1}{2} m = \frac{m}{2} (z_j - 2); \\
  r_k &= r_k - 1.25m = \frac{1}{2} m \cdot z_k - \frac{2.5}{2} m = \frac{m}{2} (z_k - 2.5); \\
  r_l &= r_l + 1.25m = \frac{1}{2} m \cdot z_l + \frac{2.5}{2} m = \frac{m}{2} (z_l + 2.5).
\end{align*}
\]

Discussion; Determining the Gearing Performance Depending on the Degree of Coverage

In this section, there is briefly presented a completely original method of determining the efficiency of parallel gear gears. Based on the computational relationships presented, the dynamic synthesis of the gears can be made so as to result in mechanisms with high efficiency in operation.

The originality of the method consists in determining the yield (which does not take into account the friction coefficient in the coupling, this being considered only an additional effect and not the main cause that produces the effective mechanical efficiency, the mechanical efficiency of a machine depends on the authors' mainly by the transmission angle of the main coupler of the mechanism).

Calculate the yield of a gear with a fixed spindle gear, considering that at a certain moment there are several pairs of drive drums, not just one.

It starts from the idea of having four pairs of drums in engagement (simultaneous). The first pair of teeth (which go on the right-to-left engagement line as it engages) are the engagement point i, defined by the radius of the \( r_{i1} \) and the angle of the position \( \alpha_i \); the forces at this point are the force of the \( F_{m1} \) motors, perpendicular to the point and position of the vector and the force transmitted from the wheel 1 to the second wheel by the point \( i \), \( F_{in} \) parallel to the engagement line and pointing from the wheel 1 to the wheel 2, the transmission force being basically the projection of the drive force on the engagement axis (line); the defined speeds are similar to the forces (for the original cinematic, precision); the same parameters will also be defined for the other three points of engagement, \( j, k, l \) (following the drawing in Fig. 11).

Write the relationships between speeds (4.1) first:

\[
\begin{align*}
  v_{ri} &= v_{re} \cdot \cos \alpha_r = r_i \cdot \omega_i \cdot \cos \alpha_r = r_i \cdot \omega_i; \\
  v_{ri} &= v_{re} \cdot \cos \alpha_r = r_i \cdot \omega_i \cdot \cos \alpha_r = r_i \cdot \omega_i; \\
  v_{ri} &= v_{re} \cdot \cos \alpha_r = r_i \cdot \omega_i \cdot \cos \alpha_r = r_i \cdot \omega_i; \\
  v_{ri} &= v_{re} \cdot \cos \alpha_r = r_i \cdot \omega_i \cdot \cos \alpha_r = r_i \cdot \omega_i.
\end{align*}
\]

From relations (4.1) one obtains the equality of tangential speeds (4.2) and we express the motor speeds (4.3):

\[
\begin{align*}
  v_{ri} &= v_{re} = v_{ri} = r_i \cdot \omega_i; \\
  v_{re} &= r_i \cdot \omega_i \cdot \cos \alpha_r = r_i \cdot \omega_i \cdot \cos \alpha_r = r_i \cdot \omega_i.
\end{align*}
\]

The forces simultaneously transmitted at the four points must be equal to each other (4.4):

\[
F_{ri} = F_{ri} = F_{ri} = F_{ri}.
\]

Engine forces shall be deducted (4.5):
The instantaneous yield in the expression (4.10) is for the gears to which the driving wheel 1 has external engagement. One starts in the relation (4.9) with the expression of the yield (4.6) for 4 pairs of engaging teeth, but immediately (even within the relation) we make a generalization by replacing the number 4 (four pairs of engaging teeth) with the variable \( E \), which represents the full side of the coverage +1 and after the expressions written in the form of sums narrow down, the variable \( E \) is replaced with the degree of coverage, thus reaching the final shape. The average yield is more interesting than the instantaneous one and is calculated (precisely by integrating the instantaneous one from the minimum pressure to the maximum angle) simply by the approximation which determines the average yield by replacing in the expression of the instantaneous yield of the variable pressure angle (\( \alpha_i \) with its average value given by the normal pressure angle (standardized, \( \alpha_i \), (4.10), where \( \varepsilon_{i1} \) represents the degree of coverage and is calculated with the expression (4.11) for the external engagement and the relation (4.12) for the inner engagement:

\[
\eta = \frac{P_e}{P_i} = \frac{F_e \cdot \varphi_{te} + F_{te} \cdot \varphi_{te} + F_{te} \cdot \varphi_{te} + F_{te} \cdot \varphi_{te}}{F_{te} \cdot \varphi_{te} + F_{te} \cdot \varphi_{te} + F_{te} \cdot \varphi_{te} + F_{te} \cdot \varphi_{te}} = \frac{4 \cdot F_e \cdot \varphi_{te} + F_{te} \cdot \varphi_{te} + F_{te} \cdot \varphi_{te} + F_{te} \cdot \varphi_{te}}{\cos^2 \alpha_i + \cos^2 \alpha_i + \cos^2 \alpha_i + \cos^2 \alpha_i} \tag{4.6}
\]

\[
\eta = \frac{1}{1 + \tan \alpha_i + \frac{2 \pi}{z_1} \cdot (\varepsilon_{i1} - 1)} \cdot \frac{1}{2 \cdot \pi \cdot \tan \alpha_i} \cdot (\varepsilon_{i1} - 1) \tag{4.10}
\]

\[
\varepsilon_{i1}^e = \frac{\sqrt{z_1^2 \cdot \sin^2 \alpha_i + 4 \cdot z_1 + 4} + \sqrt{z_1^2 \cdot \sin^2 \alpha_i + 4 \cdot z_1 + 4} + \sqrt{z_1^2 \cdot \sin^2 \alpha_i + 4 \cdot z_1 + 4} + \sqrt{z_1^2 \cdot \sin^2 \alpha_i + 4 \cdot z_1 + 4}}{2 \cdot \pi \cdot \tan \alpha_i} \sin \alpha_i \tag{4.11}
\]

\[
\varepsilon_{i1}^i = \frac{\sqrt{z_1^2 \cdot \sin^2 \alpha_i + 4 \cdot z_1 + 4} + \sqrt{z_1^2 \cdot \sin^2 \alpha_i + 4 \cdot z_1 + 4} + \sqrt{z_1^2 \cdot \sin^2 \alpha_i + 4 \cdot z_1 + 4} + \sqrt{z_1^2 \cdot \sin^2 \alpha_i + 4 \cdot z_1 + 4}}{2 \cdot \pi \cdot \tan \alpha_i} \sin \alpha_i \tag{4.12}
\]
There are wheels of the helical gears, which are used very often (relations 4.13, 4.14, 4.15). For helical gears, the calculations show a decrease in the efficiency of the with the tilt angle on the rise of the teeth ($\beta$). For given angle which does not exceed 25°, the efficiency of fishing gear is good enough. However, when the tilt angle is greater than 25°, speeds will suffer a significant decrease in the yield.

\[ \eta_m = \frac{z_i^2 \cdot \cos^2 \beta}{z_i^2 \cdot (tg^2 \alpha_i + \cos^2 \beta) + \frac{2}{3} \pi \cdot \cos^2 \beta \cdot (\varepsilon - 1) \cdot (2\varepsilon - 1) \pm 2\pi \cdot tga_i \cdot z_i \cdot \cos^2 \beta \cdot (\varepsilon - 1)} \]  

(4.13)

\[ \varepsilon^{+} = \frac{1 + tg^2 \beta}{2 \cdot \pi} \left\{ \sqrt{\left( z_i + 2 \cdot \cos \beta \right) \cdot tga_i} \right\}^2 + 4 \cdot \cos^2 \beta \cdot (z_i + \cos \beta) \]

\[ + \sqrt{\left( z_i + 2 \cdot \cos \beta \right) \cdot tga_i} \right\}^2 + 4 \cdot \cos^2 \beta \cdot (z_i + \cos \beta) - (z_i + z_i) \cdot tga_i \]  

(4.14)

\[ \varepsilon^{-} = \frac{1 + tg^2 \beta}{2 \cdot \pi} \left\{ \sqrt{\left( z_i + 2 \cdot \cos \beta \right) \cdot tga_i} \right\}^2 + 4 \cdot \cos^2 \beta \cdot (z_i + \cos \beta) \]

\[ - \sqrt{\left( z_i - 2 \cdot \cos \beta \right) \cdot tga_i} \right\}^2 - 4 \cdot \cos^2 \beta \cdot (z_i - \cos \beta) - (z_i - z_i) \cdot tga_i \]  

(4.15)

**Conclusion**

There are wheels of the helical gears, which are used very often. For helical gears, the calculations show a decrease in the efficiency of the with the tilt angle on the rise of the teeth ($\beta$). For given angle which does not exceed 25°, the efficiency of fishing gear is good enough. However, when the tilt angle is greater than 25°, speeds will suffer a significant decrease in the yield.

The highest yield that can be achieved with two gears is that of the inner gear, with the inner driving gear (the wheel becomes the driver and the smaller wheel with external gear will be driven); Conversely, when we form an inner gear with the small (outer) driving wheel,
the resulting yield is the smallest possible; When gearing is external, the output is higher for the high steering wheel; The more the normal engagement angle, $\alpha_0$, decreases, the degree of coverage increases and with it the engagement efficiency; when the normal angle of engagement drops to 5 degrees, the coverage reaches 6.5-7.3 and the output reaches theoretical values of 99-99.5%, meaning that the gear will actually work at 100%. Yield increases also with the number of teeth of the driving wheel.

In order to avoid interference phenomenon, point A must lie between C and $K_1$ (the addendum circle of the wheel 2, $C_a$), need to cut the line of action between points C and $K_1$, and under no circumstances does not exceed the point $K_1$. Similarly, $C_a$ addendum circle must cut the line of action between points C and $K_2$, resulting in point E, which in no circumstances, does not exceed the point $K_2$.

The conditions to avoid the phenomenon of interference can be written with the relations (3.1).

The basic conditions of interference, are the same (CA<K1; CE<K2C), but the originality of this new presented method consist in the mode in which it was solved the classical relationship (see the system 3.1).

The system (3.3) represents a simple, unitary and general relationship capable of generating functional solutions for gears, giving the minimum number of teeth of wheel 1 (motor wheel) to avoid interference. In the appendix Table 1-15 of Figure 12 an alpha0 value (35°) will be chosen and the beta angles (from 0° to 40°) and the transmission ratio $i$ (from 1 to 80) are incrementally incremented in order to thus getting the minimum number of teeth correctly.

Then, the alpha value (from 35° to 5°) will be decreased successively.

Acknowledgement

This text was acknowledged and appreciated by Dr. Veturia CHIROIU, Honorific member of Technical Sciences Academy of Romania (ASTR) PhD supervisor in Mechanical Engineering.

Funding Information

Research contract: Contract number 36-5-4D/1986 from 24IV1985, beneficiary CNST RO (Romanian National Center for Science and Technology) Improving dynamic mechanisms internal combustion engines. All these matters are copyrighted. Copyrights: 548-cgiywDssin, from: 22-04-2010, 08:48:48.

Ethics

This article is original and contains unpublished material. Authors declare that are not ethical issues and no conflict of interest that may arise after the publication of this manuscript.

References

Antonescu, P., 2000. Mechanisms and Handlers. 1st Edn., Printech Publishing House, Bucharest.

Antonescu, P. and F. Petrescu, 1985. Analytical method of synthesis of cam mechanism and flat stick. Proceedings of the 4th International Symposium on Mechanism Theory and Practice, (TPM’ 85), Bucharest.

Antonescu, P. and F. Petrescu, 1989. Contributions to cinetoelastodynamic analysis of distribution mechanisms. Bucharest.

Antonescu, P., M. Opren and F. Petrescu, 1985a. Contributions to the synthesis of oscillating cam mechanism and oscillating flat stick. Proceedings of the 4th International Symposium on Theory and Practice of Mechanisms, (TPM’ 85), Bucharest.

Antonescu, P., M. Opren and F. Petrescu, 1985b. At the projection of the oscillante cams, there are mechanisms and distribution variables. Proceedings of the 5th Conference for Engines, Automobiles, Tractors and Agricultural Machines, I-Engines and Automobiles, (AMA’ 85), Brasov.

Antonescu, P., M. Opren and F. Petrescu, 1986. Projection of the profile of the rotating camshaft acting on the oscillating plate with disengagement. Proceedings of the 3rd National Computer Assisted Designing Symposium in Mechanisms and Machine Bodies, (MOM’ 86), Brasov.

Antonescu, P., M. Opren and F. Petrescu, 1987. Dynamic analysis of the cam distribution mechanisms. Proceedings of the Seventh National Symposium of Industrial Robots and Spatial Mechanisms, (IMS’ 87), Bucharest.

Antonescu, P., M. Opren and F. Petrescu, 1988. Analytical synthesis of Kurz profile, rotating flat cam cam. Machine Build. Rev. Bucharest.

Antonescu, P., F. Petrescu and O. Antonescu, 1994. Contributions to the synthesis of the rotating cam mechanism and the tip of the balancing tip. Brasov.

Antonescu, P., F. Petrescu and D. Antonescu, 1997. Geometrical synthesis of the rotary cam and balance tappet mechanism. Bucharest.

Antonescu, P., F. Petrescu and O. Antonescu, 2000a. Contributions to the synthesis of the rotary disc-cam profile. Proceedings of the 8th International Conference on Theory of Machines and Mechanisms, (TMM’ 00), Liberec, Czech Republic, pp: 51-56.

Antonescu, P., F. Petrescu and O. Antonescu, 2000b. Synthesis of the rotary cam profile with balance follower. Proceedings of the 8th Symposium on Mechanisms and Mechanical Transmissions, (MMT’ 00), Timișoara, pp: 39-44.
García-Murillo, M., J. Gallardo-Alvarado and E. Castillo-Castaneda, 2013. Finding the generalized forces of a series-parallel manipulator. IJARS. DOI: 10.5772/53824

Goddard, J., 1916. Rocket apparatus patent. Smithsonian Institution Archives.

Goodall, J., 2003. Lockheed’s SR-71 "Blackbird" Family. 1st Edn., AeroFax/Midland Publishing, Hinckley, UK, ISBN-10: 1-85780-138-5.

Gorder, P.F., 2015. What’s on the surface of a black hole? Not a “firewall”—and the nature of the universe depends on it, a physicist explains.

Graham, R.H., 2002. SR-71 Blackbird: Stories, Tales and Legends. 1st Edn., Zenith Imprint, North Branch, Minnesota, ISBN-10: 1610607503.

Gruener, J.E., 2006. Lunar exploration (Presentation to ITEA Human Exploration Project Authors, at Johnson Space Center). Houston, TX.

Gunston, B., 2010. Airbus: The Complete Story. 1st Edn., Haynes Publishing UK, Sparkford, ISBN-10: 1844255589, pp: 288.

He, B., Z. Wang, Q. Li, H. Xie and R. Shen, 2013. An analytic method for the kinematics and dynamics of a multiple-backbone continuum robot. IJARS. DOI: 10.5772/54051

Hewish, A., 1970. Pulsars. Ann. Rev. Astronomy Astrophys., 8: 265-296.

Jenkins, D.R., 2001. Lockheed Secret Projects: Inside the Skunk Works. 1st Edn., Specialty Press, North Branch, Minnesota, ISBN-10: 1610607287.

Kaufman, H.R., 1959. Installations at NASA Glenn.

Laming, T., 2000. Airbus A320. 1st Edn., Zenith Press. ISBN-10: 1610607287.

Lee, B.J., 2013. Geometrical derivation of differential kinematics to calibrate model parameters of flexible manipulator. Int. J. Adv. Robot. Sys. DOI: 10.5772/55952

Lin, W., B. Li, X. Yang and D. Zhang, 2013. Modelling and control of inverse dynamics for a 5-DOF parallel kinematic polishing machine. Int. J. Adv. Robot. Sys. DOI: 10.5772/54966

List the first flights, From Wikipedia, free encyclopedia. https://ro.wikipedia.org/wiki/List%e2%80%93cu_prim ele_zboruri

Liu, H., W. Zhou, X. Lai and S. Zhu, 2013. An efficient inverse kinematic algorithm for a PUMA560-structured robot manipulator. IJARS. DOI: 10.5772/56403

Lu, P., L. Van Eykeren, E. van Kampen and Q. P. Chu, 2015. Selective-reinitialized multi-model adaptive estimation for fault detection and diagnosis. J. Guidance Control Dynam., 38: 1409-1424. DOI: 10.2514/1.G000587

Lu, P., L. Van Eykeren, E. van Kampen, C.C. de Visser and Q.P. Chu, 2016. Adaptive three-step Kalman filter for air data sensor fault detection and diagnosis. J. Guidance Control Dynam., 39: 590-604. DOI: 10.2514/1.G001313

Michell, J., 1784. On the means of discovering the distance, magnitude and c. of the fixed stars, in consequence of the diminution of the velocity of their light, in case such a diminution should be found to take place in any of them and such other data should be procured from observations, as would be farther necessary for that purpose. Philosophical Trans. Royal Society, 74: 35-57.

Mirsayar, M.M., V.A. Joneidi, R.V. Petrescu, F.I.T. Petrescu and F. Berto, 2017. Extended MTSN criterion for fracture analysis of soda lime glass. Eng. Fracture Mech., 178: 50-59. DOI: 10.1016/j.engfracmech.2017.04.018

Murray, K., A. Marcos and L.F. Penin, 2010. Development and testing of a GNC-FDI filter for a reusable launch vehicle during ascent. Proceedings of the AIAA Guidance, Navigation and Control Conference, Aug. 2-5, Toronto, Ontario Canada. DOI: 10.2514/6.2010-8195

Norris, G., 2010. Airbus A380: Superjumbo of the 21st Century. 1st Edn., Zenith Press.

Oberth, H., 1955. They come from outer space. Fly. Saucer Rev., 1: 12-14.

Oppenheimer, J.R. and G.M. Volkoff, 1939. On massive neutron cores. Phys. Rev., 55: 374-381.

Padula, F. and V. Perdereau, 2013. Selective-reinitialization multiple-model adaptive estimation with a particle filter for air data sensor fault detection and diagnosis. Proceedings of the 18th AIAA Guidance, Navigation and Control Conference, Aug. 24-28, Tours, France. DOI: 10.2514/6.2012-5857

Palumbo, R., G. Morani, M. De Stefano Fumo, C. Richiello and M. Di Donato et al., 2012. Concept study of an atmospheric reentry using a winged unmanned space vehicle. International Space Planes and Hypersonic Systems and Technologies Conference, Sept. 24-28, Tours, France. DOI: 10.2514/6.2012-5857

Patre, P. and S.M. Joshi, 2011. Accommodating sensor bias in MRAC for state tracking. Proceedings of the AIAA Guidance, Navigation and Control Conference, Aug. 8-11, Portland, Oregon. DOI: 10.2514/6.2011-6605

Pelecudi, C., 1967. The basics of mechanism analysis, Publishing house: Academy of the People's Republic of Romania.

Perumaal, S. and N. Jawahar, 2013. Automated trajectory planning of industrial robot for pick-and-place task. IJARS. DOI: 10.5772/53940

1234
Petrescu, F. and R. Petrescu, 1995a. Contributions to optimization of the polynomial motion laws of the stick from the internal combustion engine distribution mechanism. Bucharest.

Petrescu, F. and R. Petrescu, 1995b. Contributions to the synthesis of internal combustion engine distribution mechanisms. Bucharest.

Petrescu, F. and R. Petrescu, 1997a. Dynamics of cam mechanisms (exemplified on the classic distribution mechanism). Bucharest.

Petrescu, F. and R. Petrescu, 1997b. Contributions to the synthesis of the distribution mechanisms of internal combustion engines with Cartesian coordinate method. Bucharest.

Petrescu, F. and R. Petrescu, 1997c. Contributions to maximizing polynomial laws for the active stroke of the distribution mechanism from internal combustion engines. Bucharest.

Petrescu, F. and R. Petrescu, 2000a. Synthesis of distribution mechanisms by the rectangular (cartesian) coordinate method. University of Craiova, Craiova.

Petrescu, F. and R. Petrescu, 2000b. The design (synthesis) of cams using the polar coordinate method (the triangle method). University of Craiova, Craiova.

Petrescu, F. and R. Petrescu, 2002a. Motion laws for cams. Proceedings of the 7th National Symposium with International Participation Computer Assisted Design, (PAC’ 02), Braşov, pp: 321-326.

Petrescu, F. and R. Petrescu, 2002b. Camshaft dynamics elements. Proceedings of the 7th National Symposium with International Participation Computer Assisted Design, (PAC’ 02), Braşov, pp: 327-332.

Petrescu, F. and R. Petrescu, 2003. Some elements regarding the improvement of the engine design. Proceedings of the 8th National Symposium, Descriptive Geometry, Technical Graphics and Design, (GTD’ 03), Braşov, pp: 353-358.

Petrescu, F. and R. Petrescu, 2005a. The cam design for a better efficiency. Proceedings of the International Conference on Engineering Graphics and Design, (EGD’ 05), Bucharest, pp: 245-248.

Petrescu, F. and R. Petrescu, 2005b. Contributions at the dynamics of cams. Proceedings of the 9th IFToMM International Symposium on Theory of Machines and Mechanisms, (TMM’ 05), Bucharest, Romania, pp: 123-128.

Petrescu, F. and R. Petrescu, 2005c. Determining the dynamic efficiency of cams. Proceedings of the 9th IFToMM International Symposium on Theory of Machines and Mechanisms, (TMM’ 05), Bucharest, Romania, pp: 129-134.

Petrescu, F. and R. Petrescu, 2005d. An original internal combustion engine. Proceedings of the 9th IFToMM International Symposium on Theory of Machines and Mechanisms, (TMM’ 05), Bucharest, Romania, pp: 135-140.

Petrescu, F. and R. Petrescu, 2005e. Determining the mechanical efficiency of Otto engine’s mechanism. Proceedings of the 9th IFToMM International Symposium on Theory of Machines and Mechanisms, (TMM’ 05), Bucharest, Romania, pp: 141-146.

Petrescu, F.I. and R.V. Petrescu, 2013. Cinematics of the 3R Dyad. Engevista, 15: 118-124.

Petrescu, F.I.T. and R.V. Petrescu, 2012a. The Aviation History. 1st Edn., Books On Demand, ISBN-13: 978-3848230778.

Petrescu, F.I. and R.V. Petrescu, 2012b. Mechatronic-Sisteme Seriale si Paralele. 1st Edn., Create Space Publisher, USA, ISBN-10: 978-1-4756-0039-9, pp: 124.

Petrescu, F.I. and R.V. Petrescu, 2016a. Parallel moving mechanical systems kinematics, ENGEVISTA, 18: 455-491.

Petrescu, F.I. and R.V. Petrescu, 2016b. Direct and inverse kinematics to the Anthropomorphic Robots. ENGEVISTA, 18: 109-124.

Petrescu, F. and R. Petrescu, 2016c. An Otto engine dynamic model. IJM&P, 7: 038-048.

Petrescu, F.I. and R.V. Petrescu, 2016d. Otto motor dynamics, GEINTEC, 6: 3392-3406.

Petrescu, F.I. and R.V. Petrescu, 2016e. Dynamic cinematic to a structure 2R. GEINTEC, 6: 3143-3154.

Petrescu, F.I., B. Grecu, A. Comanescu and R.V. Petrescu. 2009. Some mechanical design elements. Proceeding of the International Conference on Computational Mechanics and Virtual Engineering, (MEC’ 09), Braşov, pp: 520-525.

Petrescu, R.V., R. Aversa, A. Apicella, M.M. Mirsayar and F.I.T. Petrescu, 2016a About the gear efficiency to a simple planetary train. Am. J. Applied Sci., 13: 1428-1436. DOI: 10.3844/ajassp.2016.1428.1436

Petrescu, R.V., R. Aversa, A. Apicella, S. Li and G. Chen et al., 2016b. Something about electron dimension. Am. J. Applied Sci., 13: 1272-1276. DOI: 10.3844/ajassp.2016.1272.1276

Petrescu, F.I.T., A. Apicella, R. Aversa, R.V. Petrescu and J.K. Calautit et al., 2016c. Something about the mechanical moment of inertia. Am. J. Applied Sci., 13: 1085-1090. DOI: 10.3844/ajassp.2016.1085.1090
Petrescu, R.V., R. Aversa, A. Apicella, F. Berto and S. Li et al., 2016d. Ecosphere protection through green energy. Am. J. Applied Sci., 13: 1027-1032. DOI: 10.3844/ajassp.2016.1027.1032

Petrescu, F.I.T., A. Apicella, R.V. Petrescu, S.P. Kozaitis and R.B. Bucinell et al., 2016e. Environmental protection through nuclear energy. Am. J. Applied Sci., 13: 941-946. DOI: 10.3844/ajassp.2016.941.946

Petrescu, F.I.T. and J.K. Calautit, 2016a. About nano fusion and dynamic fusion. Am. J. Applied Sci., 13: 261-266. DOI: 10.3844/ajassp.2016.261.266

Petrescu, F.I.T. and J.K. Calautit, 2016b. About the light dimensions. Am. J. Applied Sci., 13: 321-325. DOI: 10.3844/ajassp.2016.321.325

Petrescu, R.V., R. Aversa, B. Akash, R. Bucinell and J. Corchado et al., 2017a. Modern propulsions for aerospace-a short review. J. Aircraft Spacecraft Technol., 1: 1-8. DOI: 10.3844/jastsp.2017.1.8

Petrescu, R.V., R. Aversa, B. Akash, R. Bucinell and J. Corchado et al., 2017b. Modern propulsions for aerospace-part II. J. Aircraft Spacecraft Technol., 1: 9-17. DOI: 10.3844/jastsp.2017.9.17

Petrescu, R.V., R. Aversa, B. Akash, R. Bucinell and J. Corchado et al., 2017c. History of aviation-a short review. J. Aircraft Spacecraft Technol., 1: 30-49. DOI: 10.3844/jastsp.2017.30.49

Petrescu, R.V., R. Aversa, B. Akash, R. Bucinell and J. Corchado et al., 2017d. Lockheed martin-a short review. J. Aircraft Spacecraft Technol., 1: 50-68. DOI: 10.3844/jastsp.2017.50.68

Petrescu, R.V., R. Aversa, B. Akash, J. Corchado and F. Berto et al., 2017e. Our universe. J. Aircraft Spacecraft Technol., 1: 69-79. DOI: 10.3844/jastsp.2017.69.79

Petrescu, R.V., R. Aversa, B. Akash, J. Corchado and F. Berto et al., 2017f. What is a UFO? J. Aircraft Spacecraft Technol., 1: 80-90. DOI: 10.3844/jastsp.2017.80.90

Petrescu, R.V., R. Aversa, B. Akash, J. Corchado and F. Berto et al., 2017g. About bell helicopter FCX-001 concept aircraft-a short review. J. Aircraft Spacecraft Technol., 1: 91-96. DOI: 10.3844/jastsp.2017.91.96

Petrescu, R.V., R. Aversa, B. Akash, J. Corchado and F. Berto et al., 2017h. Home at airbus. J. Aircraft Spacecraft Technol., 1: 97-118. DOI: 10.3844/jastsp.2017.97.118

Petrescu, R.V., R. Aversa, B. Akash, J. Corchado and F. Berto et al., 2017i. Airlander. J. Aircraft Spacecraft Technol., 1: 119-148. DOI: 10.3844/jastsp.2017.119.148

Petrescu, R.V., R. Aversa, B. Akash, J. Corchado and F. Berto et al., 2017j. When boeing is dreaming-a review. J. Aircraft Spacecraft Technol., 1: 149-161. DOI: 10.3844/jastsp.2017.149.161

Petrescu, R.V., R. Aversa, B. Akash, J. Corchado and F. Berto et al., 2017k. About Northrop Gruman. J. Aircraft Spacecraft Technol., 1: 162-185. DOI: 10.3844/jastsp.2017.162.185

Petrescu, R.V., R. Aversa, B. Akash, J. Corchado and F. Berto et al., 2017l. Some special aircraft. J. Aircraft Spacecraft Technol., 1: 186-203. DOI: 10.3844/jastsp.2017.186.203

Petrescu, R.V., R. Aversa, B. Akash, J. Corchado and F. Berto et al., 2017m. About helicopters. J. Aircraft Spacecraft Technol., 1: 204-223. DOI: 10.3844/jastsp.2017.204.223

Petrescu, R.V., R. Aversa, B. Akash, F. Berto and A. Apicella et al., 2017n. The modern flight. J. Aircraft Spacecraft Technol., 1: 224-233. DOI: 10.3844/jastsp.2017.224.233

Petrescu, R.V., R. Aversa, B. Akash, F. Berto and A. Apicella et al., 2017o. Sustainable energy for aerospace vessels. J. Aircraft Spacecraft Technol., 1: 234-240. DOI: 10.3844/jastsp.2017.234.240

Petrescu, R.V., R. Aversa, B. Akash, F. Berto and A. Apicella et al., 2017p. Unmanned helicopters. J. Aircraft Spacecraft Technol., 1: 241-248. DOI: 10.3844/jastsp.2017.241.248

Petrescu, R.V., R. Aversa, B. Akash, F. Berto and A. Apicella et al., 2017q. Project HARP. J. Aircraft Spacecraft Technol., 1: 249-257. DOI: 10.3844/jastsp.2017.249.257

Petrescu, R.V., R. Aversa, B. Akash, F. Berto and A. Apicella et al., 2017r. Presentation of romanian engineers who contributed to the development of global aeronautics-part I. J. Aircraft Spacecraft Technol., 1: 258-271. DOI: 10.3844/jastsp.2017.258.271

Petrescu, R.V., R. Aversa, B. Akash, F. Berto and A. Apicella et al., 2017s. A first-class ticket to the planet mars, please. J. Aircraft Spacecraft Technol., 1: 272-281. DOI: 10.3844/jastsp.2017.272.281

Petrescu, R.V., R. Aversa, B. Akash, F. Berto and A. Apicella et al., 2017t. Forces of a 3R robot. J. Mechatronics Robotics, 1: 15-23. DOI: 10.3844/jmrrsp.2017.15.23

Petrescu, R.V., R. Aversa, B. Akash, J. Corchado and F. Berto et al., 2017u. Direct geometry and cinematic to the MP-3R systems. J. Mechatronics Robotics, 1: 1-14. DOI: 10.3844/jmrrsp.2017.1-14

Electronic copy available at: https://ssrn.com/abstract=3306815
Petrescu, R.V., R. Aversa, B. Akash, F. Berto and A. Apicella et al., 2017. Dynamic elements at MP3R. J. Mechatronics Robotics, 1: 24-37. DOI: 10.3844/jmrsp.2017.24.37

Petrescu, R.V., R. Aversa, B. Akash, F. Berto and A. Apicella et al., 2017w. Geometry and direct kinematics to MP3R with 4×4 operators. J. Mechatronics Robotics, 1: 38-46. DOI: 10.3844/jmrsp.2017.38.46

Petrescu, R.V., R. Aversa, B. Akash, F. Berto and A. Apicella et al., 2017x. Geometry and direct kinematics to MP3R with 4×4 operators. J. Mechatronics Robotics, 1: 38-46. DOI: 10.3844/jmrsp.2017.38.46

Petrescu, R.V., R. Aversa, A. Apicella, M.M. Mirsayar and S. Kozaitis et al., 2017y. Geometry and inverse kinematics at the MP3R mobile systems. J. Mechatronics Robotics, 1: 58-65. DOI: 10.3844/jmrsp.2017.58.65

Petrescu, R.V., R. Aversa, A. Apicella, M.M. Mirsayar and S. Kozaitis et al., 2017aa. The inverse kinematics of the plane system 2-3 in a mechatronic MP2R system, by a trigonometric method. J. Mechatronics Robotics, 1: 75-87. DOI: 10.3844/jmrsp.2017.75.87

Petrescu, R.V., R. Aversa, A. Apicella, M.M. Mirsayar and S. Kozaitis et al., 2017ab. Serial, anthropomorphic, spatial, mechatronic systems can be studied more simply in a plan. J. Mechatronics Robotics, 1: 88-97. DOI: 10.3844/jmrsp.2017.88.97

Petrescu, R.V., R. Aversa, A. Apicella, M.M. Mirsayar and S. Kozaitis et al., 2017ac. Analysis and synthesis of mechanisms with bars and gears used in robots and manipulators. J. Mechatronics Robotics, 1: 98-108. DOI: 10.3844/jmrsp.2017.98.108

Petrescu, R.V., R. Aversa, A. Apicella, M.M. Mirsayar and S. Kozaitis et al., 2017ad. Speeds and accelerations in direct kinematics to the MP3R systems. J. Mechatronics Robotics, 1: 109-117. DOI: 10.3844/jmrsp.2017.109.117

Petrescu, R.V., R. Aversa, A. Apicella, M.M. Mirsayar and S. Kozaitis et al., 2017ae. Geometry and determining the positions of a plan transporter manipulator. J. Mechatronics Robotics, 1: 118-126. DOI: 10.3844/jmrsp.2017.118.126

Reddy, P., K.V. Shihabudeen and J. Jacob, 2012. Precise non linear modeling of flexible link flexible joint manipulator. IReMoS, 5: 1368-1374.

Sevil, H.E. and A. Dogan, 2015. Fault diagnosis in air data sensors for receiver aircraft in aerial refueling. J. Guidance Control Dynam., 38: 1959-1975. DOI: 10.2514/1.G000527

Sherson, J.F., H. Krauter, RK. Olsson, B. Julsgaard and K. Hammerer et al., 2006. Quantum teleportation between light and matter. Nature, 443: 557-560. DOI: 10.1038/nature05136

Sun, J.Z. and S.M. Joshi, 2009. An indirect adaptive control scheme in the presence of actuator and sensor failures. Proceedings of the AIAA Guidance, Navigation and Control Conference, Aug. 10-13, Chicago, Illinois. DOI: 10.2514/6.2009-5740

Tabaković, S., M. Zeljković, R. Gatalo and A. Zivković, 2013. Program suite for conceptual designing of parallel mechanism-based robots and machine tools. Int. J. Adv. Robot Sys. DOI: 10.5772/55936

Tang, X., D. Sun and Z. Shao, 2013. The structure and dimensional design of a reconfigurable PKM. IJARS. DOI: 10.5772/54201

Venkataraman, G., 1992. Chandrasekhar and his Limit. 1st Edn., Universities Press, ISBN-10: 817371035X, pp: 89.

Wang, K., M. Luo, T. Mei, J. Zhao and Y. Cao, 2013. Dynamics analysis of a three-DOF planar serial-parallel mechanism for active dynamic balancing with respect to a given trajectory. Int. J. Adv. Robotic Sys. DOI: 10.5772/54201

Williams, D.R., 1995. Saturnian satellite fact sheet. NASA. https://nssdc.gsfc.nasa.gov/planetary/factsheet/saturniansatfact.html

Wen, S., J. Zhu, X. Li, A. Rad and X. Chen, 2012. End-point contact force control with quantitative feedback theory for mobile robots. IJARS. DOI: 10.5772/53742
Appendix, Figure 12 with 15 Tables

Figure 12, Table 1: $\alpha_0 = 35$ [deg], $\beta = 0$ [deg]

| $\alpha_0$ [deg] | $\beta$ [deg] | $Z_{\text{min}}$ |
|------------------|----------------|------------------|
| 35               | 0              | 4.8828 5.0325 5.1986 5.3204 5.4386 5.5467 5.6431 5.7198 5.7867 5.8439 |
| i 1              | 1.25 1.6 2 2.5 3.15 4 5 6.3 8 |
| i 10             | 12.5 16 20 25 31.5 40 50 63 80 |
| $Z_{\text{min}}$| 5.8880 5.9243 5.9568 5.9805 5.9997 6.0158 6.0290 6.0389 6.0471 6.0539 |

Figure 12, Table 2: $\alpha_0 = 35$ [deg], $\beta = 10$ [deg]

| $\alpha_0$ [deg] | $\beta$ [deg] | $Z_{\text{min}}$ |
|------------------|----------------|------------------|
| 35               | 10             | 4.7254 4.8679 5.0174 5.1419 5.2545 5.3576 5.4495 5.5226 5.5865 5.6411 |
| i 1              | 1.25 1.6 2 2.5 3.15 4 5 6.3 8 |
| i 10             | 12.5 16 20 25 31.5 40 50 63 80 |
| $Z_{\text{min}}$| 5.6832 5.7178 5.7498 5.7715 5.7898 5.8052 5.8178 5.8273 5.8351 5.8416 |

Figure 12, Table 3: $\alpha_0 = 35$ [deg], $\beta = 20$ [deg]

| $\alpha_0$ [deg] | $\beta$ [deg] | $Z_{\text{min}}$ |
|------------------|----------------|------------------|
| 35               | 20             | 4.2799 4.4022 4.5307 4.6379 4.7354 4.8240 4.9035 4.9667 5.0219 5.0693 |
| i 1              | 1.25 1.6 2 2.5 3.15 4 5 6.3 8 |
| i 10             | 12.5 16 20 25 31.5 40 50 63 80 |
| $Z_{\text{min}}$| 5.1058 5.1358 5.1627 5.1824 5.1983 5.2116 5.2226 5.2308 5.2376 5.2432 |

Figure 12, Table 4: $\alpha_0 = 35$ [deg], $\beta = 30$ [deg]

| $\alpha_0$ [deg] | $\beta$ [deg] | $Z_{\text{min}}$ |
|------------------|----------------|------------------|
| 35               | 30             | 3.6198 3.7136 3.8123 3.8949 3.9699 4.0388 4.1004 4.1495 4.1925 4.2294 |
| i 1              | 1.25 1.6 2 2.5 3.15 4 5 6.3 8 |
| i 10             | 12.5 16 20 25 31.5 40 50 63 80 |
| $Z_{\text{min}}$| 4.2578 4.2812 4.3022 4.3176 4.3300 4.3404 4.3490 4.3554 4.3608 4.3651 |

Electronic copy available at: https://ssrn.com/abstract=3306815
**Figure 12, Table 5:** $\alpha_0 = 35$ [deg], $\beta = 40$ [deg]

| $\alpha_0$ [deg] | 35 | $\beta$ [deg] | 40 |
|------------------|----|----------------|----|
| $Z_{\min}$      |    |                |    |
| i 1              | 1.25 | 1.6            | 2  |
|                 | 2.5  | 3.15           | 4  |
|                 | 5    | 6.3            | 8  |
| $Z_{\min}$      | 2.8475 | 2.9104        | 2.9769 |
|                 | 3.0327 | 3.0836         | 3.1304 |
|                 | 3.1724 | 3.2060         | 3.2355 |
|                 | 3.2608 |                |      |

| i 10             | 12.5 | 16             | 20 |
|                 | 25   | 31.5           | 40 |
|                 | 50   | 63             | 80 |
| $Z_{\min}$      | 3.2804 | 3.2965        | 3.3110 |
|                 | 3.3216 | 3.3302         | 3.3374 |
|                 | 3.3433 | 3.3477         | 3.3514 |
|                 | 3.3545 |                |      |

**Figure 12, Table 6:** $\alpha_0 = 20$ [deg], $\beta = 0$ [deg]

| $\alpha_0$ [deg] | 20 | $\beta$ [deg] | 0 |
|------------------|----|----------------|---|
| $Z_{\min}$      |    |                |    |
| i 1              | 1.25 | 1.6            | 2  |
|                 | 2.5  | 3.15           | 4  |
|                 | 5    | 6.3            | 8  |
| $Z_{\min}$      | 12.323 | 12.966        | 13.624 |
|                 | 14.161 | 14.637         | 15.066 |
|                 | 15.443 | 15.740         | 15.997 |
|                 | 16.215 |                |      |

| i 10             | 12.5 | 16             | 20 |
|                 | 25   | 31.5           | 40 |
|                 | 50   | 63             | 80 |
| $Z_{\min}$      | 16.382 | 16.519        | 16.641 |
|                 | 16.730 | 16.802         | 16.911 |
|                 | 16.978 | 17.003        |      |

**Figure 12, Table 7:** $\alpha_0 = 20$ [deg], $\beta = 10$ [deg]

| $\alpha_0$ [deg] | 20 | $\beta$ [deg] | 10 |
|------------------|----|----------------|----|
| $Z_{\min}$      |    |                |    |
| i 1              | 1.25 | 1.6            | 2  |
|                 | 2.5  | 3.15           | 4  |
|                 | 5    | 6.3            | 8  |
| $Z_{\min}$      | 11.835 | 12.447        | 13.075 |
|                 | 13.586 | 14.041         | 14.450 |
|                 | 14.810 | 15.094         | 15.339 |
|                 | 15.547 |                |      |

| i 10             | 12.5 | 16             | 20 |
|                 | 25   | 31.5           | 40 |
|                 | 50   | 63             | 80 |
| $Z_{\min}$      | 15.706 | 15.837        | 15.954 |
|                 | 16.039 | 16.107         | 16.164 |
|                 | 16.211 | 16.246         | 16.275 |
|                 | 16.299 |                |      |

**Figure 12, Table 8:** $\alpha_0 = 20$ [deg], $\beta = 20$ [deg]

| $\alpha_0$ [deg] | 20 | $\beta$ [deg] | 20 |
|------------------|----|----------------|----|
| $Z_{\min}$      |    |                |    |
| i 1              | 1.25 | 1.6            | 2  |
|                 | 2.5  | 3.15           | 4  |
|                 | 5    | 6.3            | 8  |
| $Z_{\min}$      | 10.446 | 10.994        | 11.535 |
|                 | 11.977 | 12.370         | 12.725 |
|                 | 13.036 | 13.282         | 13.495 |
|                 | 13.676 |                |      |

| i 10             | 12.5 | 16             | 20 |
|                 | 25   | 31.5           | 40 |
|                 | 50   | 63             | 80 |
| $Z_{\min}$      | 13.814 | 13.927        | 14.028 |
|                 | 14.102 | 14.161         | 14.211 |
|                 | 14.252 | 14.282         | 14.308 |
|                 | 14.328 |                |      |
**Figure 12, Table 9:** $\alpha_0 = 20 \text{ [deg]}, \beta = 30 \text{ [deg]}

| $i$ | $\alpha$ [deg] | $\beta$ [deg] | $Z_{\text{min}}$ |
|-----|----------------|----------------|-----------------|
| 2   | 1.25           | 1.6            | 8.4777 9.8841   |
| 2   | 2.5            | 3.15           | 10.225 10.468 10.659 10.825 10.966 |
| 4   | 5              | 6.3            | 10.966 |
| 5   | 8              | 11.074 11.163 11.242 11.299 11.346 11.385 11.417 11.441 11.461 11.477 |

**Figure 12, Table 10:** $\alpha_0 = 20 \text{ [deg]}, \beta = 40 \text{ [deg]}

| $i$ | $\alpha$ [deg] | $\beta$ [deg] | $Z_{\text{min}}$ |
|-----|----------------|----------------|-----------------|
| 2   | 1.25           | 1.6            | 6.2280 6.5020 6.7854 7.0182 7.2263 7.4147 7.5813 7.7128 7.8269 7.9241 |
| 2   | 2.5            | 3.15           | 8.1862 8.2352 8.2517 8.2654 8.2767 |
| 4   | 5              | 6.3            | 8.2767 |
| 5   | 8              | 11.9985 8.0597 8.1143 8.1540 8.1862 8.2351 8.2517 8.2654 8.2767 |

**Figure 12, Table 11:** $\alpha_0 = 5 \text{ [deg]}, \beta = 0 \text{ [deg]}

| $i$ | $\alpha$ [deg] | $\beta$ [deg] | $Z_{\text{min}}$ |
|-----|----------------|----------------|-----------------|
| 2   | 1.25           | 1.6            | 176.52 188.86 201.22 211.13 219.81 227.54 234.28 239.55 244.09 247.93 |
| 2   | 2.5            | 3.15           | 250.85 253.24 255.37 256.92 258.17 259.21 260.06 260.70 261.23 261.67 |
| 4   | 5              | 6.3            | 261.67 |
| 5   | 8              | 250.85 253.24 255.37 256.92 258.17 259.21 260.06 260.70 261.23 261.67 |

**Figure 12, Table 12:** $\alpha_0 = 5 \text{ [deg]}, \beta = 10 \text{ [deg]}

| $i$ | $\alpha$ [deg] | $\beta$ [deg] | $Z_{\text{min}}$ |
|-----|----------------|----------------|-----------------|
| 2   | 1.25           | 1.6            | 168.66 180.45 192.25 201.72 210.01 217.38 223.83 228.86 233.19 236.86 |
| 2   | 2.5            | 3.15           | 239.65 241.94 243.97 245.45 246.64 247.63 248.45 249.06 249.57 249.98 |
| 4   | 5              | 6.3            | 249.98 |
| 5   | 8              | 168.66 180.45 192.25 201.72 210.01 217.38 223.83 228.86 233.19 236.86 |

Electronic copy available at: https://ssrn.com/abstract=3306815
Figure 12, Table 13: $\alpha_0 = 5$ [deg], $\beta = 20$ [deg]

| $\alpha_0$ [deg] | 5 | $\beta$ [deg] | 20 |
|------------------|---|----------------|----|
| i                | 1 | 1.25           | 1.62 |
| $Z_{min}$        | 146.73 | 156.95 | 167.20 | 175.42 | 182.62 | 189.03 | 194.63 | 198.99 | 202.76 | 205.94 |
| i                | 10 | 12.5           | 16   | 20   | 25   | 31.5 | 40   | 50   | 63   | 80   |
| $Z_{min}$        | 208.37 | 210.35 | 212.12 | 213.40 | 214.44 | 215.30 | 216.01 | 216.54 | 216.98 | 217.34 |

Figure 12, Table 14: $\alpha_0 = 5$ [deg], $\beta = 30$ [deg]

| $\alpha_0$ [deg] | 5 | $\beta$ [deg] | 30 |
|------------------|---|----------------|----|
| i                | 1 | 1.25           | 1.62 |
| $Z_{min}$        | 115.16 | 123.15 | 131.16 | 137.58 | 143.22 | 148.23 | 152.61 | 156.03 | 158.97 | 161.47 |
| i                | 10 | 12.5           | 16   | 20   | 25   | 31.5 | 40   | 50   | 63   | 80   |
| $Z_{min}$        | 163.37 | 164.92 | 166.30 | 167.31 | 168.12 | 168.79 | 169.35 | 169.76 | 170.11 | 170.39 |

Figure 12, Table 15: $\alpha_0 = 5$ [deg], $\beta = 40$ [deg]

| $\alpha_0$ [deg] | 5 | $\beta$ [deg] | 40 |
|------------------|---|----------------|----|
| i                | 1 | 1.25           | 1.62 |
| $Z_{min}$        | 80.086 | 85.602 | 91.136 | 95.575 | 99.465 | 102.93 | 105.96 | 108.33 | 110.36 | 112.09 |
| i                | 10 | 12.5           | 16   | 20   | 25   | 31.5 | 40   | 50   | 63   | 80   |
| $Z_{min}$        | 113.40 | 114.48 | 115.43 | 116.13 | 116.69 | 117.16 | 117.54 | 117.83 | 118.07 | 118.26 |