THE DYNAMICAL MECHANISM OF JETS FOR AGN

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Abstract. The black hole core of a galaxy attracts a large amounts of gases around it, forming an active galactic nucleus (AGN). An AGN emits huge quantities of energy, leading to AGN jets. In [16], Ma and Wang established a model governing the AGN, in which they obtain the driving force of AGN jets. In this paper, we generalize their model to couple magnetic fields describing the AGN plasma, and derive the huge explosive electromagnetic energy as proposed in (1.13) of [16].

1. Introduction. An active galactic nucleus (AGN) is a compact region near the center of a galaxy that has a much higher than normal luminosity over at least some portion, and possibly all, of the electromagnetic spectrum. AGN emits huge quantities of energy, leading to AGN jets. AGN jet is one of the important discoveries in the twentieth century, and the research about AGN can date back to early 20th century.

In 1918, American astronomer Heber Curtis of Lick Observatory observed that there was no spiral structure in Messier 87 and he noticed a “curious straight ray... apparently connected with the nucleus by a thin line of matter.” The ray appeared brightest at the inner end. However, in the next 30 years, there is no further research on AGN.

In 1954, it is the American astronomers Baade and Minkowski who firstly called the astronomical phenomena as jet. In the same year, they completed the optical certification of the first radio source outside river. In 1963, Hazard, Mackay and Shimmins observed the jets of 3C273[6].

In the 1970s, with the application (Very Large Array) and VLBI (Very Long Baseline Interferometry), astronomers discovered that there are many galaxies which have jets. The jets are either on one side of the galaxies, or both sides.

At the end of the 20th century, with the help of the Hubble telescope and other new astronomical equipments, astronomers can show more details of jets through the high resolution images. In general, the characteristics of jets are as follows:

1. On the basis of the astronomical observation, there are two kinds of jets: unilateral and bilateral;
2. The AGN jets are collimating and universal; and
3. There exists strong radiation near the core of an AGN.

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With more and more observations for AGN, some astronomers want to find the mechanism of jets by establishing some theoretical models, such as the model of radiation pressure to accelerate gases around AGN\cite{9, 10}. Blandford and Payne established a B-P mechanism\cite{2}, Blandford-Znajek mechanism\cite{1}, and more\cite{3, 4}. But these models can't give the driving force producing jets. Ma and Wang given a new model and derived the driving force\cite{16}, but they ignored the influence of magnetic fields to AGN jets.

Galactic nucleus consist of plasma. The precise description of AGN jets needs to take magnetic effect into account. It is necessary to generalize the model\cite{16} coupling magnetic fields.

The main objectives of this paper are two-fold. The first is to establish the governing equations of AGN jets coupling magnetic fields, and second objective is to derive the magnetic power for the AGN jets.

The paper is based on the principle of symmetry-breaking\cite{16}, which was proposed by Ma and Wang. It is know that different physical systems obey different physical principles. One common view of modeling a multilevel large system is a unification based on larger symmetry. However, Ma and Wang think that the essence of a physical theory for such multilevel system is through coupling different physical laws in different levels by a symmetry-breaking principle mechanism. In other words, the principle of symmetry-breaking is a general principle when we deal with a physical system coupling different subsystems in different levels.

In \cite{13, 15}, Ma and Wang developed a unified field theory coupling four interactions based on a few first principles. The theory, together with the principle of symmetry-breaking, provides a solid theoretic foundation for astrophysics and cosmology.

In astrophysics, the Newton second law for plasma motion and the diffusion law for heat conductivity are not compatible with the principle of general relativity. The reason is that time and space are independent for the Newton Second Law and the diffusion law, but dependent for a relativistic system. To established astrophysical dynamics, we have to couple the Newton second law, heat conductivity, and the general theory of relativity. In this paper, we follow the spirit.

The paper consists of two sections. In Section 1, the model of AGN jets is given, and the driving force of AGN jets in Section 2.

2. Model for AGN governing plasma.
2.1. The incompressible MHD equations on Riemannian manifold. The dynamical equations of incompressible plasma are as follows

\[
\begin{cases}
\frac{\partial u}{\partial t} + (u \cdot \nabla)u = \mu \Delta u - \frac{1}{\rho} \nabla p + \frac{\mu_0}{\rho} (\nabla \times H) \times H, \\
\frac{\partial H}{\partial t} = \frac{1}{\sigma \mu_0} (\nabla \times (\nabla \times H)) + \nabla \times (u \times H), \\
\text{div} u = 0,
\end{cases}
\]

(1)

where \(u = (u^1, u^2, u^3)\) is the velocity field, \(p\) is the pressure, \(\rho\) is the mass density, \(\mu\) is the dynamical viscosity, \(H\) is the magnetic field of plasma, \(\sigma\) is electrical conductivity and \(\mu_0\) is vacuum magnetic permeability.

Let \((M, g_{ij})\) be a 3D Riemannian manifold. The metric of \(M\) is as follows

\[
ds^2 = g_{ij} dx^i dx^j.
\]

(2)

We need to know the governing equations of plasma on \(M\). In other words, we want to know the expressions of plasma under the metric (2).
By (1), in order to get the dynamical equations of plasma in $M$, the key is to get the expressions of $\nabla \times A$ and $A \times B$ on the Riemannian manifold $M$. Hereafter, $A$ and $B$ always represent two smooth vector fields on $M$. For simplicity, let the metric of $M$ be diagonal and be given as follows

$$ds^2 = g_{ij}dx^i dx^j.$$ (3)

Let $A$ and $B$ be

$$A = (A^1, A^2, A^3), B = (B^1, B^2, B^3)$$

Note

$$(A_1, A_2, A_3) = (g_{11}A^1, g_{22}A^2, g_{33}A^3),$$

$$(B_1, B_2, B_3) = (g_{11}B^1, g_{22}B^2, g_{33}B^3).$$

We define $\nabla \times A$ and $A \times B$ as

$$\nabla \times A = (g^{11}(*d\omega_A)_1, g^{22}(*d\omega_A)_2, g^{33}(*d\omega_A)_3),$$ (4)

$$A \times B = (g^{11}C_1, g^{22}C_2, g^{33}C_3),$$ (5)

where

$$\omega_A = A_i dx^i, \quad \omega_B = B_j dx^j, \quad d\omega_A = \frac{\partial A_i}{\partial x^j} dx^i \wedge dx^j,$$

$$*d\omega_A = * \left( \frac{\partial A_i}{\partial x^j} \right) (dx^i \wedge dx^j), C_i = (*) (\omega_A \wedge \omega_B)_{i}, 1 \leq i \leq 3,$$

$$* \left( \frac{\partial A_i}{\partial x^j} \right) = \sqrt{g}g^{ij}g^{kl} \left( \frac{\partial A_i}{\partial x^l} \right), \quad g = \det(g_{ij}),$$

$$* (dx^i \wedge dx^j) = - (* (dx^j \wedge dx^i)),$$

$$\omega_A \wedge \omega_B = A_i B_j dx^i \wedge dx^j,$$

$$* (A_i B_j) = \sqrt{g}g^{ij}g^{kl} (A_i B_j),$$

$$* (dx^2 \wedge dx^3) = dx^1, * (dx^3 \wedge dx^1) = dx^2, * (dx^1 \wedge dx^2) = dx^3.$$

Then,

$$\nabla \times A = \frac{1}{\sqrt{g}} \left( \frac{\partial A_3}{\partial x^2} - \frac{\partial A_2}{\partial x^3}, \frac{\partial A_1}{\partial x^3} - \frac{\partial A_3}{\partial x^1}, \frac{\partial A_2}{\partial x^1} - \frac{\partial A_1}{\partial x^2} \right),$$ (6)

$$g^{11}C_1 = \sqrt{\frac{g_{22}g_{33}}{g_{11}}}(A^2 B^3 - A^3 B^2),$$ (7)

$$g^{22}C_2 = \sqrt{\frac{g_{11}g_{33}}{g_{22}}}(A^3 B^1 - A^1 B^3),$$ (8)

$$g^{33}C_3 = \sqrt{\frac{g_{11}g_{22}}{g_{33}}}(A^1 B^2 - A^2 B^1).$$ (9)

From (3) and (6)-(9), we can get that

$$\nabla \times (\nabla \times H) = \overline{\Delta} H = (\overline{\Delta} H^1, \overline{\Delta} H^2, \overline{\Delta} H^3),$$ (10)

$$(\nabla \times H) \times H = (H \cdot \nabla) H - \frac{1}{2} \nabla(||H||^2),$$ (11)

$$\nabla \times (u \times H) = (H \cdot \nabla) u - (u \cdot \nabla) H.$$ (12)
Hence, the governing equations of plasma motion on $M$ are

$$\begin{align*}
\frac{\partial u}{\partial t} + (u \cdot D)u &= \mu \Delta u - \frac{1}{p} \nabla p^* + \frac{\mu_0}{p} ((H \cdot D)H) \\
\frac{\partial H}{\partial t} + (u \cdot D)H &= \frac{1}{\mu_0} \nabla \Delta H + (H \cdot D)u \\
\text{div}u &= 0
\end{align*}$$

(13)

Where $\Delta$ is the Laplace-Beltrami operator, $p^* = p + \frac{\mu_0}{2}(|H|^2)$. And for any smooth vector field $A$ on $M$, $\Delta A$ is defined as follows

$$\Delta A^i = \nabla^k \left( \nabla_k A^i + \Gamma^i_{kj} A^j \right) + \Gamma^i_{kj} \left( \nabla^j A^j + \Gamma^j_{ks} A^s \right).$$

(15)

Here $R_{ij}$ is the Ricci curvature tensor and $\Gamma^i_{kj}$ the Levi-Civita connection:

$$\begin{align*}
R_{ij} &= \frac{1}{2} g^{kl} \left( \nabla^2 g_{kl} - \nabla^2 g_{lk} \right) + g^{kl} \left( \nabla_k \Gamma^i_{lj} - \nabla_l \Gamma^i_{kj} \right) \\
\Gamma^i_{kj} &= \frac{1}{2} g^{lm} \left( \nabla_k \nabla^j A^l - \nabla_l \nabla^j A^k \right).
\end{align*}$$

(16)

The nonlinear convection term $(A \cdot D)B$ in (13) is defined by

$$(A \cdot D)B = (A^i D_i B^1, A^i D_i B^2, A^i D_i B^3),$$

(18)

$$A^i D_i B^k = A^i \frac{\partial B^k}{\partial x^i} + \Gamma^i_{kj} A^j B^j.$$  

(19)

The pressure term is

$$\nabla p = (g^{1k} \frac{\partial p}{\partial x^k}, \cdots, g^{nk} \frac{\partial p}{\partial x^k}).$$

(20)

The divergence of $A$ is

$$\text{div}A = \frac{\partial A^k}{\partial x^k} + \Gamma^k_{kj} A^j = \frac{1}{\sqrt{g}} \frac{\partial (\sqrt{g} A^k)}{\partial x^k}.$$  

(21)

2.2. PID and symmetry-breaking principle. We now recall the principle of interaction dynamics and the principle of symmetry-breaking introduced by Ma and Wang [13, 16].

**Principle 2.1. Principle of Interaction Dynamics[13]:**

1. For all physical interactions there are Lagrangian actions

$$L(g, A, \psi) = \int_M \mathcal{L}(g_{\mu\nu}, A, \psi) \sqrt{-g} dx,$$

(22)

where $g = \{g_{\mu\nu}\}$ is the Riemannian metric representing the gravitational potential, $A$ is a set of vector fields representing the gauge potentials, and $\psi$ are the wave functions of particles.

2. The action(22) satisfy the invariance of general relativity, Lorentz invariance, gauge invariance and the gauge representation invariance.
3. The states \((g, A, \psi)\) are the extremum points of (22) with the \(\text{div}_A\)-free constraint.

The mass of an AGN is usually in the range
\[
10^5 M_\odot \sim 10^9 M_\odot.
\]

The space-time structure of AGN is a Riemannian manifold, and the metric is
\[
d s^2 = g_{\alpha\beta} d x^\alpha d x^\beta.
\] (23)

Based on PID, \(g_{\alpha\beta}\) satisfy the Einstein’s equations as follows
\[
R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = - \frac{8\pi G}{c^2} T_{\alpha\beta} + D_\alpha \Phi_\beta.
\] (24)

More details about PID and (24) can be found in [15].

There is still a fact that an AGN is made up of plasma gases, and the Newton Second Law for plasma motion and the diffusion law for heat conduction are not compatible with the principle of general relativity. The reason is that time and space are independent for the Newton Second Law and the diffusion law, but dependent for a relativistic system. The symmetry-breaking principle firstly proposed by Ma and Wang in [16] can solve this problem.

**Principle 2.2. Symmetry-breaking principle[16]:**

1. The three sets of symmetries,
   - the general relativistic invariance,
   - the Lorentz and gauge invariances, and
   - the Galileo invariance,

   are mutually independent and dictate in part the physical laws in different levels of the physical world.

2. for a system coupling different levels of physical laws, part of these symmetries must be broken.

The symmetry-breaking mechanism and the coupling in this case are achieved by prescribing the coordinate system:
\[
x^\mu = (x^0, x), \quad x^0 = c t, \quad x = (x^1, x^2, x^3),
\]
such that the metric (23) is given by
\[
d s^2 = -(1 + \frac{2}{c^2} \psi) c^2 d t^2 + g_{ij} d x^i d x^j.
\] (25)

Here \(g_{ij}(1 \leq i, j \leq 3)\) are the spatial metric, \(\psi\) is gravitational potential. Obviously, the system with metric as (25) breaks the symmetry of general coordinate transformations. Ma and Wang called such symmetric-breaking principle as relativistic symmetric breaking.

In fact, the space time metric of the AGN is the Schwarzschild metric as follows
\[
d s^2 = -c^2(1 + \frac{2}{c^2} \psi) d t^2 + (1 + \frac{2}{c^2} \psi)^{-1} d r^2 + r^2 d \theta^2 + r^2 \sin^2 \theta d \varphi^2,
\] (26)

where \(\psi = \frac{GM}{r}\), and \(M\) is the mass of the black hole core.

The domain of an AGN is a spherical annulus
\[
B = \{ x \in \mathbb{R}^3 | r_s \leq |x| \leq r_0 \},
\] (27)
and the metric for the domain is the spatial metric of (26), which is
\[ ds^2 = (1 + \frac{2}{c^2} \psi)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2. \] (28)

To study the plasma in \( B \), we need to know the governing equations of plasma in \( B \) under the metric (28).

The temperature of plasma gases near the core of AGN is so high that we cannot ignore the heat diffusion. Here, we suppose that there is no direct relationship between the temperature and magnetic field of plasma. Based on the relativistic symmetric breaking and coupling equations (13) with heat diffusion equation, we get the incompressible heat plasma equations in \( B \) as follows
\[
\begin{align*}
\frac{\partial u}{\partial t} + (u \cdot \nabla)u &= \mu \Delta u - \frac{1}{\rho} \nabla p^* + \frac{e^2}{r^2} (1 - \beta T) \nabla \psi + \frac{\mu_0}{\rho} (H \cdot \nabla)H, \\
\frac{\partial T}{\partial t} + (u \cdot \nabla)T &= \kappa \Delta T + Q \\
\text{div} u &= 0
\end{align*}
\] (29)

where \( \kappa \) is heat diffusion coefficient, \( Q \) is heat source, and
\[
\tilde{\Delta} T = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^k} (\sqrt{g} g^{ks} \partial \Delta T \partial x^s).
\] (30)

2.3. Differential operators in spherical coordinates. In Sections 2.1 and 2.2, we give the heat plasma equation on general Riemannian manifold. In the following section, we always take 3D Riemannian manifold \( M \) with the metric
\[ ds^2 = \alpha(r) dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2, \] (31)
where \( \alpha(r) = (1 + \frac{2}{c^2} \psi)^{-1} \), and \( \psi \) is the gravitational potential.

In (31) we have
\[ g_{11} = \alpha(r), \quad g_{22} = r^2, \quad g_{33} = r^2 \sin^2 \theta, \quad g_{ij} = 0 \text{ for } i \neq j. \] (32)

By (17) and (32), we can get the Levi-Civita connection as
\[
\begin{align*}
\Gamma^1_{21} &= \frac{1}{r}, \quad \Gamma^2_{33} = -\sin \theta \cos \theta, \quad \Gamma^3_{11} = \frac{1}{\alpha}, \\
\Gamma^3_{32} &= \frac{\cos \theta}{\sin \theta}, \quad \Gamma^1_{32} = -\frac{\alpha}{r}, \quad \Gamma^1_{33} = -\frac{\alpha}{\sin ^2 \theta}, \\
\Gamma^1_{11} &= \frac{1}{2} \frac{\partial \alpha}{\partial r}, \quad \Gamma^k_{ij} = 0, \text{ for others.}
\end{align*}
\] (33)

and the Ricci curvature tensors:
\[
\begin{align*}
R_{11} &= -\frac{\alpha}{r^2} \frac{\partial \alpha}{\partial r}, \quad R_{22} = \frac{1}{\alpha} - \frac{\alpha}{r^2} \frac{\partial \alpha}{\partial r} - 1, \\
R_{33} &= R_{22} \sin^2 \theta, \quad R_{ij} = 0, \quad i \neq j.
\end{align*}
\] (34)

Hence, we can obtain the expressions of the differential operators (14) and (15) as follows:

1. The Laplace-Beltrami operator \( \Delta u = (\Delta u_r, \Delta u_\theta, \Delta u_\varphi) \)
\[
\Delta u_r = \tilde{\Delta} u_r - \frac{2}{\alpha^2 r^2} \left[ u_r + r \frac{\cos \theta}{\sin \theta} u_\theta + r \frac{\partial u_\theta}{\partial \theta} + r \frac{\partial u_\varphi}{\partial \varphi} - \frac{\alpha'}{2 \alpha} u_r - \frac{r \alpha'}{4 \alpha} \frac{\partial u_r}{\partial r} \right],
\] (35)
\[
\Delta u_\theta = \tilde{\Delta} u_\theta + \frac{1}{r^2} \left[ \frac{2}{r} \frac{\partial u_r}{\partial \theta} - \frac{2 \cos \theta}{\sin \theta} \frac{\partial u_\varphi}{\partial \varphi} - \frac{1}{\sin \theta} u_\theta \right] + \frac{1}{\alpha r^2} \left[ \frac{\alpha'}{2 \alpha} \frac{\partial u_\theta}{\partial r} + 2 \frac{\partial}{\partial r} (ru_\theta) - \frac{\alpha'}{2 \alpha} \frac{\partial}{\partial r} (r^2 u_\theta) \right], \quad (36)
\]

\[
\Delta u_\varphi = \tilde{\Delta} u_\varphi + \frac{1}{r^2} \left[ \frac{2 \cos \theta}{\sin \theta} \frac{\partial u_\varphi}{\partial \theta} + \frac{2 \cos \theta}{\sin \theta} \frac{\partial u_\theta}{\partial \varphi} + \frac{2}{r \sin^2 \theta} \frac{\partial u_r}{\partial \varphi} \right] + \frac{1}{\alpha r^2} \left[ \frac{\alpha'}{2 \alpha} \frac{\partial u_\varphi}{\partial r} + \frac{\partial}{\partial r} (ru_\varphi) - \frac{\alpha'}{2 \alpha} \frac{\partial}{\partial r} (r^2 u_\varphi) - 2 \alpha u_\varphi \right]. \quad (37)
\]

2. The Laplace operator \( \tilde{\Delta} H = (\tilde{\Delta} H_r, \tilde{\Delta} H_\theta, \tilde{\Delta} H_\varphi) \):

\[
\tilde{\Delta} H_r = \tilde{\Delta} H_r - \frac{2}{\alpha r^2} \left[ H_r + r \frac{\cos \theta}{\sin \theta} H_\theta + r \frac{\partial H_\theta}{\partial \theta} + r \frac{\partial H_\varphi}{\partial \varphi} - \frac{\alpha'}{2 \alpha} \frac{r}{r} H_r \right], \quad (38)
\]

\[
\tilde{\Delta} H_\theta = \tilde{\Delta} H_\theta + \frac{1}{r^2} \left( \frac{2}{r^2} \frac{\partial H_r}{\partial \theta} - \frac{2 \cos \theta}{\sin \theta} \frac{\partial H_\varphi}{\partial \varphi} - \frac{1}{\sin^2 \theta} H_\theta + H_\theta \right) + \frac{1}{\alpha r^2} \left( H_\theta + 2r \frac{\partial H_\theta}{\partial r} + \frac{\alpha'}{2 \alpha} \frac{\partial H_\varphi}{\partial r} - \frac{\alpha'}{2 \alpha} \frac{r}{r} H_\theta \right), \quad (39)
\]

\[
\tilde{\Delta} H_\varphi = \tilde{\Delta} H_\varphi + \frac{1}{r^2} \left( \frac{2 \cos \theta}{\sin \theta} \frac{\partial H_\varphi}{\partial \theta} - H_\varphi + \frac{2 \cos \theta}{\sin \theta} \frac{\partial H_\theta}{\partial \varphi} + \frac{2}{r \sin^2 \theta} \frac{\partial H_r}{\partial \varphi} \right) + \frac{1}{\alpha r^2} \left( H_\varphi + 2r \frac{\partial H_\varphi}{\partial r} + \frac{\alpha'}{2 \alpha} \frac{\partial H_\theta}{\partial r} - \frac{\alpha'}{2 \alpha} \frac{r}{r} H_\varphi \right) \quad (40)
\]

3. \( \tilde{\Delta} \) is the Laplace operator for scalar fields given by

\[
\tilde{\Delta} T = \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \varphi^2} + \frac{1}{r^2 \alpha} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) - \frac{\alpha'}{2 \alpha} \frac{\partial T}{\partial r}, \quad (41)
\]

24. Equations of AGN governing plasma. We need to note that the two components \( u_\theta, u_\varphi \) are angular velocities in the \( \theta \) and \( \varphi \) directions:

\[
u_\theta = \frac{d \theta}{dt}, \quad u_\varphi = \frac{d \varphi}{dt}. \quad (42)
\]

In classical fluid dynamics, the velocity field \( v = (v_\theta, v_\varphi, v_r) \) is the line velocity. The relation of \( u \) and \( v \) is given by

\[
u_r = v_r, \quad u_\theta = \frac{1}{r} v_\theta, \quad u_\varphi = \frac{1}{r \sin \theta} v_\varphi. \quad (43)
\]

Similarly, replace \( H \) by \( \Pi \) as follows:

\[
H_r = \Pi_r, \quad H_\theta = \frac{1}{r} \Pi_\theta, \quad H_\varphi = \frac{1}{r \sin \theta} \Pi_\varphi. \quad (44)
\]
Inserting (42) and (43) into equation (29) and by the expressions (14)-(20), we get the model of AGN governing plasma as follows:

\[
\begin{align*}
\frac{\partial v_r}{\partial t} + (v \cdot \dot{D}) v_r &= \mu \Delta v_r - \frac{1}{\rho_0} \frac{\partial p^*}{\partial r} + (1 - \beta T) \frac{d\psi}{dr} + \frac{\mu_0}{\rho} (\mathcal{H} \cdot \dot{D}) \mathcal{P}_r, \\
\frac{\partial v_\theta}{\partial t} + (v \cdot \dot{D}) v_\theta &= \mu \Delta v_\theta - \frac{1}{\rho_\theta} \frac{\partial p^*}{\partial \theta} + \frac{\mu_0}{\rho} (\mathcal{H} \cdot \dot{D}) \mathcal{P}_\theta, \\
\frac{\partial v_\varphi}{\partial t} + (v \cdot \dot{D}) v_\varphi &= \mu \Delta v_\varphi - \frac{1}{\rho r \sin \theta} \frac{\partial p^*}{\partial \varphi} + \frac{\mu_0}{\rho} (\mathcal{H} \cdot \dot{D}) \mathcal{P}_\varphi, \\
\frac{\partial \mathcal{P}_r}{\partial t} + (v \cdot \dot{D}) \mathcal{P}_r &= \frac{1}{\sigma_0} \Delta \mathcal{P}_r + (\mathcal{H} \cdot \dot{D}) v_r, \\
\frac{\partial \mathcal{P}_\theta}{\partial t} + (v \cdot \dot{D}) \mathcal{P}_\theta &= \frac{1}{\sigma_\theta} \Delta \mathcal{P}_\theta + (\mathcal{H} \cdot \dot{D}) v_\theta, \\
\frac{\partial \mathcal{P}_\varphi}{\partial t} + (v \cdot \dot{D}) \mathcal{P}_\varphi &= \frac{1}{\sigma_\varphi} \Delta \mathcal{P}_\varphi + (\mathcal{H} \cdot \dot{D}) v_\varphi, \\
\frac{\partial T}{\partial t} + (v \cdot D) T &= \kappa \tilde{T} + Q.
\end{align*}
\]

Here

\[
\begin{align*}
\Delta v_r &= \frac{2}{\alpha r^2} \left( v_r + \frac{\partial v_\theta}{\partial \theta} + \frac{\cos \theta}{\sin \theta} v_\theta + \frac{1}{\sin \theta} \frac{\partial v_\varphi}{\partial \varphi} \right) \\
&\quad + \frac{1}{\alpha r^2} \frac{\partial}{\partial r} \left( \frac{\alpha' v_r}{\alpha} \right) + \frac{\alpha'}{2 \alpha^2 r} \frac{\partial v_r}{\partial r}, \\
\Delta v_\theta &= \frac{2}{\alpha r^2} \left( v_\theta + \frac{\partial v_r}{\partial r} + \frac{2 \cos \theta}{\sin \theta} \frac{\partial v_\varphi}{\partial \varphi} - \frac{v_\theta}{r^2 \sin^2 \theta} \right) \\
&\quad - \frac{\alpha'}{2 \alpha^2 r} \frac{\partial}{\partial r} \left( rv_\theta \right) + \frac{\alpha'}{2 \alpha^2 r} \frac{\partial v_\theta}{\partial r}, \\
\Delta v_\varphi &= \frac{2}{\alpha r^2} \left( v_\varphi + \frac{\partial v_r}{\partial r} + \frac{2 \cos \theta}{\sin \theta} \frac{\partial v_\theta}{\partial \varphi} - \frac{v_\varphi}{r^2 \sin^2 \theta} \right) \\
&\quad - \frac{\alpha'}{2 \alpha^2 r} \frac{\partial}{\partial r} \left( rv_\varphi \right) + \frac{\alpha'}{2 \alpha^2 r} \frac{\partial v_\varphi}{\partial r}, \\
\tilde{T} &= \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \varphi^2} \\
&\quad + \frac{1}{\alpha r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) - \frac{\alpha'}{2 \alpha^2} \frac{\partial T}{\partial r}, \\
\Delta \mathcal{P}_r &= \Delta \mathcal{P}_r - \frac{2}{\alpha^2} \left( \mathcal{P}_r + \frac{\partial \mathcal{P}_\theta}{\partial \theta} + \frac{\cos \theta}{\sin \theta} \mathcal{P}_\theta + \frac{1}{\sin \theta} \frac{\partial \mathcal{P}_\varphi}{\partial \varphi} \right) \\
&\quad + \frac{\alpha'}{2 \alpha^2 r} \frac{\partial \mathcal{P}_r}{\partial r} + \frac{1}{\alpha r} \frac{\partial}{\partial r} \left( \alpha' v_r \right) + \frac{\alpha'}{\alpha^2} \mathcal{P}_r, \\
\Delta \mathcal{P}_\theta &= \Delta \mathcal{P}_\theta + \frac{2}{\alpha^2} \left( \frac{\partial \mathcal{P}_r}{\partial r} + \frac{2 \cos \theta}{\sin \theta} \frac{\partial \mathcal{P}_\varphi}{\partial \varphi} - \frac{1}{r^2 \sin^2 \theta} \mathcal{P}_\theta \right) \\
&\quad - \frac{1 - \alpha}{\alpha^2} \mathcal{P}_\theta - \frac{\alpha'}{2 \alpha^2 r} \frac{\partial}{\partial r} \left( r \mathcal{P}_\theta \right) + \frac{\alpha'}{2 \alpha^2} \frac{\partial \mathcal{P}_\theta}{\partial r} + \frac{\alpha'}{\alpha^2} \mathcal{P}_\theta, \\
\Delta \mathcal{P}_\varphi &= \Delta \mathcal{P}_\varphi + \frac{2}{\alpha^2} \left( \frac{\partial \mathcal{P}_r}{\partial r} + \frac{2 \cos \theta}{\sin \theta} \frac{\partial \mathcal{P}_\theta}{\partial \varphi} - \frac{1}{r^2 \sin^2 \theta} \mathcal{P}_\varphi \right)
\end{align*}
\]
\[-\frac{1}{\alpha v^2}H_\varphi - \frac{\alpha'}{2\alpha^2 r} \frac{\partial}{\partial r}(rH_\varphi) + \frac{\alpha'}{2\alpha^2 r^2} \frac{\partial H_\varphi}{\partial r} + \frac{\alpha'}{2\alpha^2 r}H_\varphi, \]

and

\[(\vec{H} \cdot D)H_r = (\vec{H} \cdot \tilde{\nabla})H_r + \frac{\alpha'}{\alpha r} \frac{\partial H_\varphi}{\partial r}, \]

\[(\vec{H} \cdot D)H_\theta = (\vec{H} \cdot \tilde{\nabla})H_\theta + \frac{H_r H_\theta}{r} - \cos \theta r \sin \theta H_\varphi, \]

\[(\vec{H} \cdot D)H_\varphi = (\vec{H} \cdot \tilde{\nabla})H_\varphi + \frac{H_r H_\varphi}{r} + \cos \theta r \sin \theta H_\theta H_\varphi, \]

\[(\vec{H} \cdot \tilde{\nabla})\vec{H} = \frac{\partial \vec{H}}{\partial r} + \frac{\vec{H}_\theta}{r} \frac{\partial \vec{H}}{\partial \theta} + \frac{\vec{H}_\varphi}{r \sin \theta} \frac{\partial \vec{H}}{\partial \varphi}. \]

(v \cdot \dot{D})v as in (59)-(62),

\[(\vec{H} \cdot \dot{D})v_r = (\vec{H} \cdot \tilde{\nabla})v_r, \]

\[(\vec{H} \cdot \dot{D})v_\theta = (\vec{H} \cdot \tilde{\nabla})v_\theta - \frac{\partial v_\varphi}{\partial r}, \]

\[(\vec{H} \cdot \dot{D})v_\varphi = (\vec{H} \cdot \tilde{\nabla})v_\varphi - \frac{\cos \theta r \sin \theta}{r} v_\varphi - \frac{H_\theta v_r}{r}, \]

\[(v \cdot \dot{D})\vec{H}_r = (v \cdot \tilde{\nabla})\vec{H}_r, \]

\[(v \cdot \dot{D})\vec{H}_\theta = (v \cdot \tilde{\nabla})\vec{H}_\theta - \frac{\partial v_r}{\partial r}, \]

\[(v \cdot \dot{D})\vec{H}_\varphi = (v \cdot \tilde{\nabla})\vec{H}_\varphi - \frac{\cos \theta r \sin \theta}{r} v_\varphi - \frac{H_\theta v_r}{r}. \]

2.5. Momentum representation. In the universe, galaxies and galactic clusters are composed of stars and interstellar nebulae. Then the velocity fields are not continuous. Hence, it is not appropriate that we study the dynamical behavior of jets by the model (45)–(51). In [16], the authors propose a new idea that we should replace the velocity field \(v(x, t)\) by the momentum density field \(P(x, t)\). The main reason is that the momentum density field \(P\) is the energy flux containing the mass, the heat, and all interaction energy flux, and can be regarded as a continuous field.

In fact, the equations (45)–(47) are the mathematical expressions of the Newton Second Law expressed as

\[\frac{dP}{dt} = F, \]

where \(F\) is total force that contains the frictional force \(\nu \Delta P\), the pressure gradient \(\nabla p\), the gravitational force \((1 - \beta T) \frac{dx}{dt}\) and the Lorentz force \((\vec{H} \cdot \tilde{\nabla})\vec{H} - \frac{1}{2} \nabla (|\vec{H}|^2)\). \(P\) is the momentum density field, formally defined by

\[v = \frac{dx}{dt} = \frac{P}{\rho} \]

with \(\rho\) being the energy density. The reason that we replace \(v\) by \(\frac{P}{\rho}\) is that \(\frac{P}{\rho}\) is more general than \(\frac{dx}{dt}\), and (70) holds under the condition that \(x(t)\) is differential.

Replace \(\vec{H}\) by \(H\)
By (70), the equations (45)–(51) can be written as
\[
\frac{\partial P_r}{\partial t} + \frac{1}{\rho} (P \cdot \dot{D}) P_r = \mu \Delta P_r - \frac{1}{\alpha} \frac{\partial p^*}{\partial r} + \rho(1 - \beta T) \frac{d\psi}{dt}
+ \mu_0 (H \cdot \dot{D}) H_r,
\]
\[
\frac{\partial P_\theta}{\partial t} + \frac{1}{\rho} (P \cdot \dot{D}) P_\theta = \mu \Delta P_\theta - \frac{1}{r} \frac{\partial p^*}{\partial \theta} + \mu_0 (H \cdot \dot{D}) H_\theta,
\]
\[
\frac{\partial P_\varphi}{\partial t} + \frac{1}{\rho} (P \cdot \dot{D}) P_\varphi = \mu \Delta P_\varphi - \frac{1}{r \sin \theta} \frac{\partial p^*}{\partial \varphi} + \mu_0 (H \cdot \dot{D}) H_\varphi,
\]
\[
\frac{\partial H_r}{\partial t} + \frac{1}{\rho} (P \cdot \dot{D}) H_r = \frac{1}{\sigma \mu_0} \Delta H_r + (H \cdot \dot{D}) \frac{P_r}{\rho},
\]
\[
\frac{\partial H_\theta}{\partial t} + \frac{1}{\rho} (P \cdot \dot{D}) H_\theta = \frac{1}{\sigma \mu_0} \Delta H_\theta + (H \cdot \dot{D}) \frac{P_\theta}{\rho},
\]
\[
\frac{\partial H_\varphi}{\partial t} + \frac{1}{\rho} (P \cdot \dot{D}) H_\varphi = \frac{1}{\sigma \mu_0} \Delta H_\varphi + (H \cdot \dot{D}) \frac{P_\varphi}{\rho},
\]
\[
\frac{\partial T}{\partial t} + \frac{1}{\rho} (P \cdot \nabla) T = \kappa \Delta T + Q.
\]
Here the differential operators are all with respect to the spatial metric \(g_{ij}(1 \leq i, j \leq 3)\) determined by (28), as defined in (52)-(68).

Suppose the momentum \(P\) is incompressible, i.e.
\[
div P = 0.
\]

Then the model for AGN can be given by
\[
\begin{align*}
\frac{\partial P_r}{\partial t} + \frac{1}{\rho} (P \cdot \dot{D}) P_r &= \nu \Delta P_r - \nabla p^* + \rho(1 - \beta T) \frac{\partial p^*}{\partial r} \vec{k} + \mu_0 (H \cdot \dot{D}) H_r, \\
\frac{\partial P_\theta}{\partial t} + \frac{1}{\rho} (P \cdot \dot{D}) P_\theta &= \nu \Delta P_\theta - \frac{1}{r} \frac{\partial p^*}{\partial \theta} + \mu_0 (H \cdot \dot{D}) H_\theta, \\
\frac{\partial P_\varphi}{\partial t} + \frac{1}{\rho} (P \cdot \dot{D}) P_\varphi &= \nu \Delta P_\varphi - \frac{1}{r \sin \theta} \frac{\partial p^*}{\partial \varphi} + \mu_0 (H \cdot \dot{D}) H_\varphi, \\
\frac{\partial H_r}{\partial t} + \frac{1}{\rho} (P \cdot \dot{D}) H_r &= \frac{1}{\sigma \mu_0} \Delta H_r + (H \cdot \dot{D}) \frac{P_r}{\rho}, \\
\frac{\partial H_\theta}{\partial t} + \frac{1}{\rho} (P \cdot \dot{D}) H_\theta &= \frac{1}{\sigma \mu_0} \Delta H_\theta + (H \cdot \dot{D}) \frac{P_\theta}{\rho}, \\
\frac{\partial H_\varphi}{\partial t} + \frac{1}{\rho} (P \cdot \dot{D}) H_\varphi &= \frac{1}{\sigma \mu_0} \Delta H_\varphi + (H \cdot \dot{D}) \frac{P_\varphi}{\rho}, \\
\frac{\partial T}{\partial t} + \frac{1}{\rho} (P \cdot \nabla) T &= \kappa \Delta T + Q,
\end{align*}
\]
where \(\vec{k} = (1, 0, 0)\).

3. The dynamical process of AGN jets. Physical reality tells us that there is one kind of force providing energy for jets. The force drives the gases away from AGN. Jets straightly shot into space. In this section, we want to explain the physical observation with equations (80).

3.1. The driving force of AGN jets. The domain of an active galactic nucleus is spherical annulus.
\[
B = \{ x \in \mathbb{R}^3 \mid r_s \leq |x| \leq r_0 \},
\]
where \(r_s\) is the Schwarzschild radius of the black hole core, and \(r_0\) is the radius of the galaxy nucleus.

The model governing AGN is given by (80), defined in the domain (81) with boundary conditions.
\[
\begin{align*}
P_r &= 0, \quad \frac{\partial P_r}{\partial r} = 0, \quad P_\varphi = P_0, \\
H_r &= 0, \quad \frac{\partial H_r}{\partial r} = H_0, \quad H_\varphi = 0, \quad \text{for } r = r_s, \\
T &= T_0,
\end{align*}
\]
\[
\begin{aligned}
\left\{
\begin{array}{ll}
 P_r = 0, & \frac{\partial P_r}{\partial r} = 0, P_\varphi = P_1, \\
 H_r = 0, & \frac{\partial H_r}{\partial r} = H_1, H_\varphi = 0,
\end{array}
\right. \\
T = T_1,
\end{aligned}
\]

for \( r = r_0 \), \( r \to \infty \).

Note that
\[
\Delta P = \Delta \tilde{P} + F_1(P) + F_2(P),
\]
\[
\Delta H = \Delta \tilde{H} + G_1(H) + G_2(H).
\]

By (52)-(58),
\[
F_1(P) = \left( -\frac{2}{\alpha_1} \left( P_r + \frac{\partial P_r}{\partial \varphi} + \frac{\cos \theta}{\sin \theta} P_\theta + \frac{1}{\sin \theta} \frac{\partial P_\varphi}{\partial \varphi} \right) \right) \left( \frac{1}{r^2 \sin \theta} \frac{\partial P_r}{\partial r} - \frac{1}{r \sin \theta} \frac{\partial P_\varphi}{\partial r} \right),
\]
\[
F_2(P) = \left( \frac{1}{\alpha_1} \frac{\partial \left( \alpha_1 P_r \right)}{\partial r} + \frac{\alpha_1}{\alpha_2} \frac{\partial P_\varphi}{\partial r} \right) - \frac{\alpha_1}{\alpha_2} \frac{\partial \left( \alpha_1 P_r \right)}{\partial \varphi},
\]
\[
G_1(H) = \left( -\frac{2}{\alpha_1} \left( v_r + \frac{\partial H_r}{\partial \varphi} + \frac{\cos \theta}{\sin \theta} H_\theta + \frac{1}{\sin \theta} \frac{\partial H_\varphi}{\partial \varphi} \right) \right) \left( \frac{1}{r^2 \sin \theta} \frac{\partial H_r}{\partial r} - \frac{1}{r \sin \theta} \frac{\partial H_\varphi}{\partial r} \right),
\]
\[
G_2(H) = \left( -\frac{1}{\alpha_1} \frac{\partial H_r}{\partial r} + \frac{1}{\alpha_2} \frac{\partial \left( \alpha_1 v_r \right)}{\partial r} + \frac{\alpha_1}{\alpha_3} H_r \right) - \frac{\alpha_1}{\alpha_3} \frac{\partial \left( \alpha_1 v_r \right)}{\partial \varphi}.
\]

Based on (28), \( F_2(P) \) and \( G_2(H) \) can be written as
\[
F_2(P) = \left( \frac{-x}{r^2} \frac{\partial P_r}{\partial r} - \frac{r_x}{r^3} \frac{\partial P_r}{\partial \varphi} + \left( 1 - \frac{x}{r} \right) \frac{1}{2 \alpha_1} \frac{1}{r^2} \frac{\partial P_r}{\partial \varphi} + \frac{x}{r_3} P_r \right),
\]
\[
G_2(H) = \left( \frac{-x}{r^2} \frac{\partial H_r}{\partial r} - \frac{r_x}{r^3} \frac{\partial H_r}{\partial \varphi} + \left( 1 - \frac{x}{r} \right) \frac{1}{2 \alpha_1} \frac{1}{r^2} \frac{\partial H_r}{\partial \varphi} + \frac{x}{r_3} H_r \right).
\]

By (89) and (90),
\[
F_2(P)_r = -\frac{r_x}{r^4} \frac{\partial P_r}{\partial r} - \frac{r_x}{r^4} \frac{\partial P_r}{\partial \varphi} + \left( 1 - \frac{x}{r} \right) \frac{1}{2 \alpha_1} \frac{1}{r^2} \frac{\partial P_r}{\partial \varphi} + \frac{x}{r_3} P_r,
\]
\[
G_2(H)_r = -\frac{r_x}{r^4} \frac{\partial H_r}{\partial r} - \frac{r_x}{r^4} \frac{\partial H_r}{\partial \varphi} + \left( 1 - \frac{x}{r} \right) \frac{1}{2 \alpha_1} \frac{1}{r^2} \frac{\partial H_r}{\partial \varphi} + \frac{x}{r_3} H_r.
\]

Obviously,
\[
F_3(P)_r \to \infty \quad r \to r_{s_+},
\]
\[
G_2(H)_r \to \infty \quad r \to r_{s_+}.
\]

The huge driving force \( F_2(P)_r \) in (93) is derived in [16]. Here, we obtain the following result.

**Physical Conclusion 1.** \( G_2(H)_r \) is source of electromagnetic radiation. It has the properties that
\[
\begin{aligned}
G_2(H)_r &\to \infty \quad \text{for} \quad H_r > 0, \\
G_2(H)_r &\to -\infty \quad \text{for} \quad H_r < 0, \quad \text{as} \quad r \to r_s
\end{aligned}
\]
3.2. Stationary equations. Let the stationary solutions of the equations (80) only depend on \((r, \theta)\) and

\[
\begin{align*}
\bar{P}_r &= 0, \bar{P}_\theta = 0, \bar{P}_\varphi = \bar{P}_\varphi(r, \theta), \\
\bar{H} &= (\bar{H}_r(r, \theta), \bar{H}_\theta(r, \theta), \bar{H}_\varphi(r, \theta)), \\
\bar{p} &= \bar{p}(r, \theta), \bar{T} = \bar{T}(r), \rho_0 = \rho_0(r, \theta).
\end{align*}
\]

Then, the stationary equations for the seven unknown functions \(\bar{P}_\varphi, \bar{H}_r, \bar{H}_\theta, \bar{H}_\varphi, \bar{p}, \bar{T}\) and \(\rho_0(r, \theta)\) are in the form

\[
\frac{1}{\rho_0 r} \frac{\bar{P}_{\varphi}^2}{r} = \frac{1}{\alpha r} \frac{\partial \bar{p}}{\partial r} + \mu_0 \left( \frac{\bar{H}_\theta}{r} \frac{\partial \bar{H}_r}{\partial \theta} - \frac{1}{\alpha r} H_\varphi \frac{\partial \bar{H}_\theta}{\partial r} \right.
\]
\[
- \frac{1}{\alpha r} \bar{H}_\varphi^2 - \frac{1}{\alpha} \bar{H}_r \frac{\partial \bar{H}_\varphi}{\partial r} \left. - \rho_0 \left( 1 - \frac{\rho_0}{\rho_0} \right) \frac{d \psi}{d r} \right), \quad (95)
\]
\[
\begin{align*}
- \frac{\cos \theta}{\rho_0 \sin \theta} \frac{\bar{P}_{\varphi}^2}{r^2} &= \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial r} + \mu_0 \left( \bar{H}_r \frac{\partial \bar{H}_\varphi}{\partial r} + \frac{\bar{H}_\theta}{r} \frac{\partial \bar{H}_\varphi}{\partial \theta} \right. \\
&\quad \left. - \frac{1}{\rho_0} \bar{H}_\varphi \frac{\partial \bar{H}_\varphi}{\partial \theta} \right) \quad (96)
\end{align*}
\]
\[
\bar{\Delta} P_\varphi = \frac{1}{r^2 \sin^2 \theta} P_\varphi + \frac{\alpha'}{2 \alpha r} P_\varphi + \mu_0 \left( \bar{H}_r \frac{\partial \bar{H}_\varphi}{\partial r} + \frac{\bar{H}_\theta}{r} \frac{\partial \bar{H}_\varphi}{\partial \theta} + \frac{\bar{H}_\varphi}{r} \frac{\partial \bar{H}_\varphi}{\partial \theta} \right)
\]
\[
+ \frac{\bar{H}_r \bar{H}_\varphi}{r} + \frac{\cos \theta}{\rho_0} \frac{\bar{H}_\theta}{r} \bar{H}_\varphi \quad (97)
\]
\[
\bar{\Delta} \bar{H}_r = 0, \bar{\Delta} \bar{H}_\theta = 0, \quad (98)
\]
\[
\frac{1}{\sigma \mu_0} \frac{\bar{\Delta} \bar{H}_r}{r} = - \bar{H}_r \frac{\partial \bar{P}_\varphi}{\partial r} - \frac{\bar{H}_\theta}{r} \frac{\partial \bar{P}_\varphi}{\partial \theta} + \frac{\cos \theta}{\rho_0} \frac{\bar{H}_\theta}{r} \bar{P}_\varphi
\]
\[
+ \frac{1}{\rho_0} \left( (\bar{H} \cdot \bar{\nabla}) \rho \right) \bar{P}_\varphi, \quad (99)
\]
\[
\kappa \left( \frac{1}{\alpha r^2} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \right. - \frac{\alpha'}{2 \alpha^2} \frac{\partial T}{\partial r} \left. \right) = Q(r) \quad (100)
\]

3.3. Perturbed equations. Making the translation:

\[
P \to \bar{P} + \bar{P}, \quad H \to \bar{H} + \bar{H}, \quad p \to p + \bar{p}, \quad T \to T + \bar{T}.
\]

Then the perturbed equations of (80) can be written as

\[
\begin{align*}
\frac{\partial P}{\partial t} + \frac{1}{\rho} (P \cdot \bar{D}) P &= \nu \Delta P - \nabla p + \rho \beta T \frac{d \psi}{d r} \bar{k} + \mu_0 (H \cdot \bar{D}) H \\
&\quad - \frac{\mu_0}{2} \nabla \left( |H|^2 \right) - \frac{1}{\rho} (\bar{P} \cdot \bar{D}) P - \frac{1}{\rho} (P \cdot \bar{D}) \bar{P} \\
&\quad + \mu_0 [ (H \cdot D) H + (H \cdot \bar{D}) \bar{H} - \nabla (H \cdot \bar{H})],
\end{align*}
\]
\[
\begin{align*}
\frac{\partial H}{\partial t} + \frac{1}{\rho} (P \cdot \bar{D}) H &= \frac{1}{\sigma \mu_0} \Delta H + \frac{1}{\rho} (H \cdot \bar{D}) P + \frac{1}{\rho} |(H \cdot \bar{D})| P \\
&\quad + (H \cdot \bar{D}) \bar{P} - (P \cdot \bar{D}) \bar{H} - (\bar{P} \cdot \bar{D}) H
\end{align*}
\]
and the perturbed equations (106)-(109) can be written as

\[
\frac{\partial P}{\partial t} + \frac{1}{\rho_0} (P \cdot \hat{D}) P = \nu \Delta P - \nabla p + \rho_0 \beta T \frac{d\psi}{dr} k + \mu_0 (H \cdot \hat{D}) H - \frac{\mu_0}{2} \nabla |H|^2.
\]
\[
\frac{\partial H}{\partial t} + \frac{1}{\rho_0} (P \cdot \dot{D}) H = \delta \Delta H + \frac{1}{\rho_0} (H \cdot \dot{D}) P, \quad (113)
\]
\[
\frac{\partial T}{\partial t} + \frac{1}{\rho_0} (P \cdot \nabla) T = \kappa \Delta T - \frac{1}{\rho_0} (P \cdot \nabla) \widetilde{T}, \quad (114)
\]
\[
div P = 0. \quad (115)
\]
Choose the following dimensionless forms:
\[
(r,t) = \left( r, \frac{r^2 \dot{t}}{\kappa} \right), \quad (116)
\]
\[
(P,H,T,p) = \left( \rho_0 \kappa \dot{P}/r, \frac{\kappa}{r} \sqrt{\frac{\rho_0}{\mu_0}} \dot{H}, -\frac{d \widetilde{T}}{dr} r \dot{T}, \rho_0 \kappa^2 \dot{P}/r^2 \right). \quad (117)
\]
The equations (112)-(115) are given by
\[
\frac{\partial P}{\partial t} + (P \cdot \dot{D}) P = \frac{\nu}{\kappa} \Delta P - \nabla p - \beta \frac{r^4}{\kappa^2} \frac{d \psi}{dr} \frac{d \widetilde{T}}{dr} T \frac{\vec{k}}{r} + (H \cdot \dot{D}) H - \frac{1}{2} \nabla (|H|^2), \quad (118)
\]
\[
\frac{\partial H}{\partial t} + (P \cdot \dot{D}) H = \frac{\delta}{\kappa} \Delta H + (H \cdot \dot{D}) P, \quad (119)
\]
\[
\frac{\partial T}{\partial t} + (P \cdot \nabla) T = \kappa \Delta T - (P \cdot \nabla) \widetilde{T}, \quad (120)
\]
\[
div P = 0 \quad (121)
\]
with the boundary conditions
\[
\begin{cases}
  P_r = 0, \quad \frac{\partial P_\theta}{\partial r} = 0, \quad P_\phi = 0, \\
  H_r = 0, \quad \frac{\partial H_\theta}{\partial r} = 0, \quad H_\phi = 0, \quad \text{for } r = r_s, r_0, \\
  T = 0,
\end{cases} \quad (122)
\]
The eigenvalue equation of (118)-(121) is given by
\[
\frac{\nu}{\kappa} \Delta P - \nabla p - \beta \frac{r^4}{\kappa^2} \frac{d \psi}{dr} \frac{d \widetilde{T}}{dr} T \frac{\vec{k}}{r} = \lambda P \\
\frac{\delta}{\kappa} \Delta H = \lambda H, \quad (123)
\]
\[
\kappa \Delta T - \frac{d \widetilde{T}}{dr} P_r = \lambda T, \\
div P = 0,
\]
with the boundary conditions (122).

The eigenvalues of (123) are discrete (not counting multiplicity):
\[
\lambda_1 > \lambda_2 > \cdots > \lambda_k > \cdots \to -\infty.
\]
The dimensionless parameter \( \beta \frac{r^4}{\kappa^2} \frac{d \psi}{dr} \frac{d \widetilde{T}}{dr} \) in (118) and (123) is called Rayleigh number, which is proportional to the temperature gradient
\[
1 \ll DT = T_0 - T_1. \quad (124)
\]

Based on the dynamic transition theory in [11] and [12], equations (118)-(121) have non zero solutions \((P,H)\) for \(DT \geq \text{critical } \tilde{T}\). Likely, equations (106)-(109) also have non zero solutions, but the critical \( \tilde{T} \) for the case of \( \tilde{P} \neq 0, \tilde{H} \neq 0 \) is different to \( \tilde{P} = \tilde{H} = 0 \). It means the essence of bifurcation for equations (106)-(109) and (118)-(121) is same, but different \( \tilde{P} \) and \( \tilde{H} \) corresponding to different critical \( \tilde{T} \).
3.5. **Mechanism of radio jets.** We can deduce from (94) that the first equation of (106) equivalents to

$$\frac{\partial H_r}{\partial t} = (1 - \frac{r_s}{r})^{-1} \frac{r_s^2}{2r^4} H_r \rightarrow \infty \text{ for } H_r > 0 \text{ and } r \rightarrow r_s$$

(125)

near the black hole core of an AGN.

Equations (106)-(109) also have non zero solutions. This means that $H_r \neq 0$ and the huge change of the radial magnetic fields $H_r$ generates strong electromagnetic radiation spreading out along $r$-direction.

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