Calculation of the strength of objects of water management of the environment

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Abstract. The article discusses the objects of water management of the environment. An algorithm for forming a matrix of the stress-strain state of a prismatic finite element with a triangular cross-section in a mixed formulation is proposed to take into account the physical nonlinearity of water management objects in the form of rotation shells. The basic relations of the flow theory for arbitrary loading are used. The desired displacements and stresses of the inner point of the finite element were approximated by linear functions in terms of their nodal values. The stress-strain state matrix was formed using a functional expressing the equality of the actual work of external and internal work of external and internal forces, in which the actual work of internal forces was represented by the difference between the total and additional work of these forces.

1. Introduction

Calculations for the strength of nonlinearly deformable shells of rotation are performed using the finite element method in various formulations [1, 2, 3, 4, 5]. Finite elements in the mixed formulation have a number of advantages due to the continuity of the desired values not only at the nodes of the finite element, but also at its borders [6, 7, 8]. To take into account the physical nonlinearity of rotation shells, the relations of the deformation theory of plasticity or flow theory are used [9]. With the step loading method, the total increments of deformations are the sum of the increments of elastic and plastic deformations.

In this paper, a finite element of a prismatic shape with a triangular cross-section is developed in a mixed formulation.

The dependences of increments of elastic deformations on increments of stresses in the plastic region are expressed by Hooke's law [9].

The increments of plastic deformations were determined on the basis of the hypothesis that the increments of plastic deformations are proportional to the components of the total stress deviator. The proportionality coefficient is a function of the ratio of the increment of stress intensity to stress intensity. After representing the increment of stress intensity in terms of stress increments, matrix dependences of strain increments on stress increments are formed.

The matrix of the stress-strain state of the finite element at the loading step is formed using a
functional based on the equality of possible and actual work of external and internal forces with the replacement of the actual work of stress increments at the loading step by the difference between the total and additional work [10, 11].

2. Materials and methods

2.1. Flow theory relations

At the loading step, the increments of deformations are assumed to be the sum of the increments of elastic and plastic deformations [9]:

$$\Delta \varepsilon_{ij} = \Delta \varepsilon_{ij}^e + \Delta \varepsilon_{ij}^p,$$

where $\Delta \varepsilon_{ij}$ - total increments of deformations; $\Delta \varepsilon_{ij}^e$ - increment of elastic deformations; $\Delta \varepsilon_{ij}^p$ - increments of plastic deformations.

Increments of elastic deformations are related to stress increments by Hooke’s law [9]:

$$\Delta \varepsilon_{rr}^e = \frac{1}{E} \Delta \sigma_{rr} - \frac{\vartheta}{E} (\Delta \sigma_{\theta \theta} + \Delta \sigma_{zz});$$

$$\Delta \varepsilon_{r\theta}^e = \frac{1}{E} \Delta \sigma_{r\theta} - \frac{\vartheta}{E} (\Delta \sigma_{rr} + \Delta \sigma_{zz});$$

$$\Delta \varepsilon_{zz}^e = \frac{1}{E} \Delta \sigma_{zz} - \frac{\vartheta}{E} (\Delta \sigma_{rr} + \Delta \sigma_{\theta \theta});$$

$$\Delta \varepsilon_{r\theta}^e = \frac{1}{E} \Delta \sigma_{r\theta};$$

$$\Delta \varepsilon_{zz}^e = \frac{1}{E} \Delta \sigma_{zz};$$

$$\Delta \varepsilon_{r\theta}^e = \frac{1}{E} \Delta \sigma_{r\theta};$$

where $E$ - is the elastic modulus of the material; $\vartheta$ - is the coefficient of transverse deformation; $\Delta \sigma_{ij}$ - and is the stress increment.

Increments of plastic deformations are determined by the relations [9]:

$$\Delta \varepsilon_{rr}^p = \frac{3}{2} \varepsilon_{ij}^{\lambda p} \Delta \sigma_{rr} - \frac{1}{\sigma_i} (\sigma_{rr} - \sigma_e);$$

$$\Delta \varepsilon_{r\theta}^p = \frac{3}{2} \varepsilon_{ij}^{\lambda p} \Delta \sigma_{r\theta} - \frac{1}{\sigma_i} (\sigma_{r\theta} - \sigma_e);$$

$$\Delta \varepsilon_{zz}^p = \frac{3}{2} \varepsilon_{ij}^{\lambda p} \Delta \sigma_{zz} - \frac{1}{\sigma_i} (\sigma_{zz} - \sigma_e);$$

$$\Delta \varepsilon_{r\theta}^p = \frac{3}{2} \varepsilon_{ij}^{\lambda p} \Delta \sigma_{r\theta} - \frac{1}{\sigma_i} \Delta \sigma_{r\theta};$$

$$\Delta \varepsilon_{zz}^p = \frac{3}{2} \varepsilon_{ij}^{\lambda p} \Delta \sigma_{zz} - \frac{1}{\sigma_i} \Delta \sigma_{zz},$$

where $\Delta \varepsilon_{ij}^{\lambda p}$ -intensity of increments of plastic deformations; $\sigma_i$ - intensity of stresses; $\sigma_{ij}$ - stresses; $\sigma_e = \frac{\sigma_{rr} + \sigma_{r\theta} + \sigma_{zz}}{3}$ - average stress.

For small deformations at the loading step, the following equalities are assumed:

$$\varepsilon_{ij}^{\lambda p} = \varepsilon_{ij}^p;$$

$$\varepsilon_{ij}^e = \varepsilon_{ij}^p + \varepsilon_{ij}^e,$$

where $\varepsilon_{ij}^p$ - increment of the intensity of plastic deformations; $\varepsilon_{ij}^e$ - increment of the intensity of elastic deformations.

At the loading step, the relations take place between increments of strain intensity and increments of stress intensity:
where $E_1$ is the elastic modulus of the initial section of the material deformation diagram; $E_k$ is the tangent modulus of the deformation diagram at the point under consideration; $\Delta \sigma_i$ is the increment of stress intensity at the loading step.

The increment in the intensity of plastic deformations based on the relations (4,5) is written by the expression:

$$\Delta \varepsilon_{ip}^e = \Delta \varepsilon_i - \Delta \varepsilon_e = \frac{\Delta \sigma_i}{E_k} - \frac{\Delta \sigma_i}{E_1} = \Delta \sigma_i \left( \frac{1}{E_k} - \frac{1}{E_1} \right).$$  \quad (6)

The increment of stress intensity can be determined by the following general formula

$$\Delta \sigma_i = \frac{\partial \sigma_i}{\partial \sigma_{rr}} \Delta \sigma_{rr} + \frac{\partial \sigma_i}{\partial \sigma_{\theta \theta}} \Delta \sigma_{\theta \theta} + \frac{\partial \sigma_i}{\partial \sigma_{zz}} \Delta \sigma_{zz} +$$

$$+ \frac{\partial \sigma_i}{\partial \sigma_{r \theta}} \Delta \sigma_{r \theta} + \frac{\partial \sigma_i}{\partial \sigma_{r z}} \Delta \sigma_{r z} + \frac{\partial \sigma_i}{\partial \sigma_{z z}} \Delta \sigma_{z z},$$  \quad (7)

where

$$\sigma_i = (\sigma_{rr}^2 + \sigma_{\theta \theta}^2 + \sigma_{zz}^2 - \sigma_{rr} \sigma_{\theta \theta} - \sigma_{r r} \sigma_{zz} - \sigma_{\theta \theta} \sigma_{zz} +$$

$$+ 3 \sigma_{r \theta}^2 + 3 \sigma_{r z}^2 + 3 \sigma_{z z}^2) \frac{1}{2}$$

- stress intensity;

$$\frac{\partial \sigma_i}{\partial \sigma_{rr}} = \frac{\sigma_{rr} - \frac{1}{2}(\sigma_{\theta \theta} + \sigma_{zz})}{\sigma_i} = s_{11};$$

$$\frac{\partial \sigma_i}{\partial \sigma_{\theta \theta}} = \frac{\sigma_{\theta \theta} - \frac{1}{2}(\sigma_{rr} + \sigma_{zz})}{\sigma_i} = s_{22};$$

$$\frac{\partial \sigma_i}{\partial \sigma_{zz}} = \frac{\sigma_{zz} - \frac{1}{2}(\sigma_{rr} + \sigma_{\theta \theta})}{\sigma_i} = s_{33};$$

$$\frac{\partial \sigma_i}{\partial \sigma_{r \theta}} = \frac{1}{2 \sigma_i} 6 \sigma_{r \theta} = 3 s_{12}; \quad \frac{\partial \sigma_i}{\partial \sigma_{r z}} = \frac{1}{2 \sigma_i} 6 \sigma_{r z} = 3 s_{13};$$

$$\frac{\partial \sigma_i}{\partial \sigma_{z z}} = \frac{1}{2 \sigma_i} 6 \sigma_{z z} = 3 s_{23}, \quad \psi_1 = \frac{3 \Delta \varepsilon_{ip}^p}{2 \Delta \sigma_i}.$$

Increments of plastic deformations (3) at the loading step, taking into account (6) and (7), can be written as:
\[\Delta \varepsilon^p_{rr} = \Delta \sigma_{rr} \frac{3}{2} \frac{s_{11}}{\sigma_i} \varphi(\sigma_{rr} - \sigma_c) + \Delta \sigma_{\theta \theta} \frac{3}{2} \frac{s_{22}}{\sigma_i} \varphi(\sigma_{rr} - \sigma_c) +
\]
\[+ \Delta \sigma_{zz} \frac{3}{2} \frac{s_{33}}{\sigma_i} \varphi(\sigma_{rr} - \sigma_c) + \Delta \sigma_{r \theta} \frac{3}{2} \frac{3s_{12}}{\sigma_i} \varphi(\sigma_{rr} - \sigma_c) +
\]
\[+ \Delta \sigma_{rz} \frac{3}{2} \frac{3s_{13}}{\sigma_i} \varphi(\sigma_{rr} - \sigma_c) + \Delta \sigma_{\theta z} \frac{3}{2} \frac{3s_{23}}{\sigma_i} \varphi(\sigma_{rr} - \sigma_c);
\]
\[\Delta \varepsilon^p_{\theta \theta} = \Delta \sigma_{rr} \frac{3}{2} \frac{s_{11}}{\sigma_i} \varphi(\sigma_{\theta \theta} - \sigma_c) + \Delta \sigma_{\theta \theta} \frac{3}{2} \frac{s_{22}}{\sigma_i} \varphi(\sigma_{\theta \theta} - \sigma_c) +
\]
\[+ \Delta \sigma_{zz} \frac{3}{2} \frac{s_{33}}{\sigma_i} \varphi(\sigma_{\theta \theta} - \sigma_c) + \Delta \sigma_{r \theta} \frac{3}{2} \frac{3s_{12}}{\sigma_i} \varphi(\sigma_{\theta \theta} - \sigma_c) +
\]
\[+ \Delta \sigma_{rz} \frac{3}{2} \frac{3s_{13}}{\sigma_i} \varphi(\sigma_{\theta \theta} - \sigma_c) + \Delta \sigma_{\theta z} \frac{3}{2} \frac{3s_{23}}{\sigma_i} \varphi(\sigma_{\theta \theta} - \sigma_c);
\]
\[\Delta \varepsilon^p_{zz} = \Delta \sigma_{rr} \frac{3}{2} \frac{s_{11}}{\sigma_i} \varphi(\sigma_{zz} - \sigma_c) + \Delta \sigma_{\theta \theta} \frac{3}{2} \frac{s_{22}}{\sigma_i} \varphi(\sigma_{zz} - \sigma_c) +
\]
\[+ \Delta \sigma_{zz} \frac{3}{2} \frac{s_{33}}{\sigma_i} \varphi(\sigma_{zz} - \sigma_c) + \Delta \sigma_{r \theta} \frac{3}{2} \frac{3s_{12}}{\sigma_i} \varphi(\sigma_{zz} - \sigma_c) +
\]
\[+ \Delta \sigma_{rz} \frac{3}{2} \frac{3s_{13}}{\sigma_i} \varphi(\sigma_{zz} - \sigma_c) + \Delta \sigma_{\theta z} \frac{3}{2} \frac{3s_{23}}{\sigma_i} \varphi(\sigma_{zz} - \sigma_c);
\]
\[\Delta \varepsilon^p_{r \theta} = \Delta \sigma_{ss} \frac{3}{2} \frac{s_{11}}{\sigma_i} \varphi(\sigma_{r \theta} + \Delta \sigma_{\theta \theta} \frac{3}{2} \frac{s_{22}}{\sigma_i} \varphi(\sigma_{r \theta} +
\]
\[+ \Delta \sigma_{zz} \frac{3}{2} \frac{s_{33}}{\sigma_i} \varphi(\sigma_{r \theta} + \Delta \sigma_{r \theta} \frac{3}{2} \frac{3s_{12}}{\sigma_i} \varphi(\sigma_{r \theta} +
\]
\[+ \Delta \sigma_{rz} \frac{3}{2} \frac{3s_{13}}{\sigma_i} \varphi(\sigma_{r \theta} + \Delta \sigma_{\theta z} \frac{3}{2} \frac{3s_{23}}{\sigma_i} \varphi(\sigma_{r \theta};
\]
\[\Delta \varepsilon^p_{r z} = \Delta \sigma_{ss} \frac{3}{2} \frac{s_{11}}{\sigma_i} \varphi(\sigma_{r z} + \Delta \sigma_{\theta \theta} \frac{3}{2} \frac{s_{22}}{\sigma_i} \varphi(\sigma_{r z} +
\]
\[+ \Delta \sigma_{zz} \frac{3}{2} \frac{s_{33}}{\sigma_i} \varphi(\sigma_{r z} + \Delta \sigma_{r \theta} \frac{3}{2} \frac{3s_{12}}{\sigma_i} \varphi(\sigma_{r z} +
\]
\[+ \Delta \sigma_{rz} \frac{3}{2} \frac{3s_{13}}{\sigma_i} \varphi(\sigma_{r z} + \Delta \sigma_{\theta z} \frac{3}{2} \frac{3s_{23}}{\sigma_i} \varphi(\sigma_{r z});
\]

\[(8)\]
\[ \Delta \epsilon_{\theta z}^p = \Delta \sigma_{s_z} \frac{3}{2} \frac{s_{11}}{\sigma_i} \varphi \sigma_{\theta z} + \Delta \sigma_{\theta \theta} \frac{3}{2} \frac{s_{22}}{\sigma_i} \varphi \sigma_{\theta z} + \\
+ \Delta \sigma_{rz} \frac{3}{2} \frac{s_{33}}{\sigma_i} \varphi \sigma_{\theta z} + \Delta \sigma_{r \theta} \frac{3}{2} \frac{s_{12}}{\sigma_i} \varphi \sigma_{\theta z} + \\
+ \Delta \sigma_{rz} \frac{3}{2} \frac{s_{13}}{\sigma_i} \varphi \sigma_{\theta z} + \Delta \sigma_{\theta z} \frac{3}{2} \frac{s_{23}}{\sigma_i} \varphi \sigma_{rz}, \]

where \( \varphi = \frac{1}{E_k} - \frac{1}{E_1}. \)

Relations (8) can be written in matrix form:

\[
\{ \Delta \epsilon^p \} = [C] \{ \Delta \sigma^p \},
\]

where \( \{ \Delta \epsilon^p \}^T = \{ \Delta \epsilon_{rr} \Delta \epsilon_{\theta \theta} \Delta \epsilon_{rz} \Delta \epsilon_{\theta z} \Delta \epsilon_{r \theta} \Delta \epsilon_{\theta z} \} \) - matrix-string of strain increments;
\[
[C] - matrix of material compliance; \{ \Delta \sigma^p \}^T = \{ \Delta \sigma_{rr} \Delta \sigma_{\theta \theta} \Delta \sigma_{rz} \Delta \sigma_{r \theta} \Delta \sigma_{rz} \Delta \sigma_{\theta z} \} - matrix-a string of stress increments.

2.2. Displacements and deformations

Under the action of a load, the shell deforms and its arbitrary point receives a displacement described by the vector:

\[ \vec{V} = \nu^1 \begin{bmatrix} 0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} + \nu^2 \begin{bmatrix} 0 \\ 0 \\ g_2 \\ g_3 \end{bmatrix} + \nu^3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ g_3 \end{bmatrix} = \begin{bmatrix} \nu^1 \\ \nu^2 \\ \nu^3 \end{bmatrix}, \]

where \( \{ \nu \}^T = \left[ \begin{array}{c} \nu^1 \\ \nu^2 \\ \nu^3 \end{array} \right] \) - matrix is a string of basis vectors of an arbitrary point of the shell;
\[ \{ \nu \}^T = \left[ \begin{array}{c} \nu^1 \\ \nu^2 \\ \nu^3 \end{array} \right] \] - matrix is a string of point displacements.

Increments of deformations at an arbitrary point of the shell are expressed in terms of its displacements by the Cauchy relations:

\[ \Delta \epsilon_{rr} = \frac{\partial \nu^1}{\partial r} ; \Delta \epsilon_{\theta \theta} = \frac{\partial \nu^2}{\partial \theta} ; \Delta \epsilon_{rz} = \frac{\partial \nu^3}{\partial \zeta} ; \\
\Delta \epsilon_{r \theta} = \frac{1}{2} \left( \frac{\partial \nu^1}{\partial \theta} + \frac{\partial \nu^2}{\partial r} \right) ; \Delta \epsilon_{rz} = \frac{1}{2} \left( \frac{\partial \nu^1}{\partial \zeta} + \frac{\partial \nu^3}{\partial r} \right) ; \Delta \epsilon_{\theta z} = \frac{1}{2} \left( \frac{\partial \nu^2}{\partial \zeta} + \frac{\partial \nu^3}{\partial \theta} \right), \]

which are represented in matrix form:

\[ \{ \Delta \epsilon \} = [L] \{ \nu \}, \]

where \[ [L] \] is the matrix of differential operators.
3. Results and discussion
The finite element is a prism with a cross-section in the form of arbitrary triangles with nodes \( i, j, k, l \). To perform numerical integration, the finite element is mapped to a local prism with a base in the form of a right triangle \( 0 \leq \xi, \eta \leq 1 \) and with a height coordinate that varies within \( -1 \leq \zeta \leq 1 \). The global coordinates of the inner point of the final element are determined in terms of nodal coordinates by linear relations of local coordinates \( \xi, \eta, \zeta \):

\[
\begin{align*}
  r &= \left\{ f(\xi, \eta, \zeta) \right\}_{1 \times 6}^T \{ r_y \}_{6 \times 1}; \quad \theta = \left\{ f(\xi, \eta, \zeta) \right\}_{1 \times 6}^T \{ \theta_y \}_{6 \times 1}; \\
  z &= \left\{ f(\xi, \eta, \zeta) \right\}_{1 \times 6}^T \{ z_y \}_{6 \times 1},
\end{align*}
\]

where \( \left\{ f(\xi, \eta, \zeta) \right\}_{1 \times 6}^T = \left\{ \frac{(1-\xi-\eta)}{2} \xi \frac{1-\xi}{2} \eta \frac{1-\xi}{2}, \frac{(1-\xi-\eta)}{2} \xi \frac{1+\xi}{2} \eta \frac{1+\xi}{2}, \frac{(1-\xi-\eta)}{2} \xi \frac{1+\xi}{2} \eta \frac{1+\zeta}{2} \right\} \) - shape functions;

\[
\left\{ r_y \right\}_{6 \times 1} = \left\{ r^i r^j r^k r^m r^n r^p \right\}, \quad \left\{ \theta_y \right\}_{6 \times 1} = \left\{ \theta^i \ldots \theta^p \right\}, \quad \left\{ z_y \right\}_{6 \times 1} = \left\{ z^i \ldots z^p \right\} \) - matrices-strings of nodal values of global coordinates \( r, \theta, z \).

By differentiating (13) the derivatives of global coordinates in the local system \( r, \xi, r, \eta, r, \zeta, \Theta, \theta, \zeta, z, \eta, z, \zeta \) and the derivatives of local coordinates in the global system are determined \( \xi_r, \eta_r, \zeta_r, \xi_\Theta, \eta_\Theta, \zeta_\zeta, \xi_z, \eta_z, \zeta_z \).

Increments of displacements of the inner point of a finite element are expressed in terms of increments of displacements of its nodal values at the loading step in the same way as (13) and written as

\[
\Delta u^1 = \left\{ f(\xi, \eta, \zeta) \right\}_{1 \times 6}^T \{ \Delta u^1_y \}_{6 \times 1}; \quad \Delta u^2 = \left\{ f(\xi, \eta, \zeta) \right\}_{1 \times 6}^T \{ \Delta u^2_y \}_{6 \times 1};
\]

\[
\Delta u^3 = \left\{ f(\xi, \eta, \zeta) \right\}_{1 \times 6}^T \{ \Delta u^3_y \}_{6 \times 1},
\]

where \( \left\{ \Delta u^1_y \right\}_{6 \times 1} = \left\{ \Delta u^{i1} \Delta u^{i2} \Delta u^{i3} \Delta u^{i4} \Delta u^{i5} \Delta u^{i6} \right\}; \)

\[
\left\{ \Delta u^2_y \right\}_{6 \times 1} = \left\{ \Delta u^{2i} \Delta u^{2j} \Delta u^{2k} \Delta u^{2m} \Delta u^{2n} \Delta u^{2p} \right\}; \)

\[
\left\{ \Delta u^3_y \right\}_{6 \times 1} = \left\{ \Delta u^{3i} \Delta u^{3j} \Delta u^{3k} \Delta u^{3m} \Delta u^{3n} \Delta u^{3p} \right\} \) - matrices-rows of nodal values of displacements.

Stress increments at the inner point of the finite element at the loading step are expressed in terms of stress increments at the nodal points by relations similar to (13) and are written as:
\[ \Delta \sigma_{rr} = \begin{bmatrix} f(\xi, \eta, \zeta) \end{bmatrix}_{1x6}^T \begin{bmatrix} \Delta \sigma_{rr} \end{bmatrix}_{6x1} \]; \quad \Delta \sigma_{\theta\theta} = \begin{bmatrix} f(\xi, \eta, \zeta) \end{bmatrix}_{1x6}^T \begin{bmatrix} \Delta \sigma_{\theta\theta} \end{bmatrix}_{6x1} \];

\[ \Delta \sigma_{zz} = \begin{bmatrix} f(\xi, \eta, \zeta) \end{bmatrix}_{1x6}^T \begin{bmatrix} \Delta \sigma_{zz} \end{bmatrix}_{6x1} \]; \quad \Delta \sigma_{r\theta} = \begin{bmatrix} f(\xi, \eta, \zeta) \end{bmatrix}_{1x6}^T \begin{bmatrix} \Delta \sigma_{r\theta} \end{bmatrix}_{6x1} \];

\[ \Delta \sigma_{rz} = \begin{bmatrix} f(\xi, \eta, \zeta) \end{bmatrix}_{1x6}^T \begin{bmatrix} \Delta \sigma_{rz} \end{bmatrix}_{6x1} \]; \quad \Delta \sigma_{\theta z} = \begin{bmatrix} f(\xi, \eta, \zeta) \end{bmatrix}_{1x6}^T \begin{bmatrix} \Delta \sigma_{\theta z} \end{bmatrix}_{6x1} \] \quad \text{(15)}

where \( \begin{bmatrix} \Delta \sigma_{rr} \end{bmatrix}_{1x6}^T = \begin{bmatrix} \Delta \sigma_{rr}^j \Delta \sigma_{rr}^k \Delta \sigma_{rr}^m \Delta \sigma_{rr}^n \Delta \sigma_{rr}^p \end{bmatrix} \); matrices-rows of nodal stress values.

On the basis of expressions (14) and (15), matrix relations are formed for the inner point of a finite element:

\[ \{\Delta \nu\} = [A] \{\Delta \nu_y\}; \quad \{\Delta \sigma\} = [S] \{\Delta \sigma_y\}, \quad \text{ (16)} \]

where

\[ \{\Delta \nu_y\} = \begin{bmatrix} \Delta \nu_y^1 \Delta \nu_y^2 \Delta \nu_y^3 \Delta \nu_y^4 \Delta \nu_y^5 \Delta \nu_y^6 \end{bmatrix}_{6x1}; \]

\[ \{\Delta \sigma_y\} = \begin{bmatrix} \Delta \sigma_{rr}^j \Delta \sigma_{rr}^k \Delta \sigma_{rr}^m \Delta \sigma_{rr}^n \Delta \sigma_{rr}^p \end{bmatrix}_{1x6}; \quad \text{matrices-strings of nodal unknowns of a finite element.} \]

Taking into account (16), the increments of deformations (12) are represented in matrix form:

\[ \{\Delta \varepsilon\} = [L] \{\Delta \nu\} = [L][A] \{\Delta \nu_y\} = [B] \{\Delta \nu_y\}, \quad \text{ (17)} \]

To form the matrix of the stress-strain state of a finite element at the loading step, the equality of possible and actual work of external and internal forces is used in the form [12]:

\[ \Delta \varepsilon \]
\[
\int_V \left[ \{\sigma\}^T + \frac{1}{2}\{\sigma\}^T \right] \{\Delta \varepsilon\} \, dV = \int_S \left[ \{\Delta u\}^T + \frac{1}{2}\{\Delta u\}^T \right] \{\Delta q\} + \frac{1}{2}\{\Delta q\} \, dS ,
\]

where \(\{\sigma\}^T = \{\sigma_{rr} \sigma_{\theta \theta} \sigma_{zz} \sigma_{r \theta} \sigma_{rz} \sigma_{\theta z}\}\)-matrix-string of stresses per \(j\) loading steps;

\(\{q\}^T = \{q_1 \ q_2 \ q_3\}\)- vector of increments of loads per loading steps; \(V\)- volume of the finite element; \(S\)- surface area of the load application.

The actual work of the stress increments in (18) is replaced by the difference between the total and additional energy at the loading step in the form [12]

\[
\Phi = \int_V \left[ \{\Delta \sigma\}^T \{\Delta \varepsilon\} - \frac{1}{2}\{\Delta \sigma\}^T \{C\}\{\Delta \sigma\} \right] \, dV - \int_S \left[ \{\Delta u\}^T \{\Delta q\} \right] + \frac{1}{2}\{\Delta q\} \, dS .
\]

Equality (18), taking into account (19), can be written as a functional [12]:

\[
\Phi \equiv \int_V \left[ \{\Delta \sigma\}^T \{L\}\{\Delta \varepsilon\} + \frac{1}{2}\{\Delta \sigma\}^T \{C\}\{\Delta \sigma\} \right] \, dV - \int_S \left[ \{\Delta u\}^T \{\Delta q\} \right] + \frac{1}{2}\{\Delta q\} \, dS .
\]

Taking into account expressions (9) and (17), functional (20) for a single finite element at the loading step will take the form:

\[
\Phi \equiv \left[ \{\Delta \sigma\}^T \{S\}^T \{B\} \right] \{\Delta \varepsilon\} \, dV - \frac{1}{2}\left[ \{\Delta \sigma\}^T \{C\}\{\Delta \sigma\} \right] \, dV - \frac{1}{2}\left[ \{\Delta u\}^T \{\Delta q\} \right] + \frac{1}{2}\{\Delta q\} \, dS .
\]

By varying the functional (21) in \(\{\Delta \sigma\}^T\) and \(\{\Delta \varepsilon\}^T\), systems of equations are obtained:

\[
\frac{\partial \Phi}{\partial \{\Delta \sigma\}^T} = \left[ Q \right] \{\Delta \varepsilon\} - \left[ H \right] \{\Delta \sigma\} = 0;
\]

\[
\frac{\partial \Phi}{\partial \{\Delta \varepsilon\}^T} = \left[ Q \right] \{\Delta \sigma\} - \{\Delta f\} - \{R\} = 0 .
\]
where $[Q] = \int \left[ S \right]^T [B] \, dV$; $[H] = \int \left[ S \right]^T \left[ C \right] [S] \, dV$;

$$
\left\{ \Delta f_q \right\} = \int \left[ A \right]^T \left\{ \Delta q \right\} \, dS; \quad \left\{ R \right\} = \int \left[ A \right]^T \left\{ q \right\} \, dS - \int \left[ B \right]^T \left\{ \sigma \right\} \, dV \quad \text{-Raphson's discrepancy.}
$$

Systems (22) can be represented in the traditional finite element formulation [13]:

$$
\begin{bmatrix}
\left\{ k \right\} \\
\left\{ z_y \right\}
\end{bmatrix} = \begin{bmatrix}
\left\{ F_y \right\}
\end{bmatrix},
$$

(23)

where $\begin{bmatrix}
\left\{ k \right\} \\
\left\{ z_y \right\}
\end{bmatrix} = \begin{bmatrix}
\left\{ \Delta \sigma_y \right\} \\
\left\{ \Delta \nu_y \right\}
\end{bmatrix}$ - vector of nodal unknowns;

$$
\begin{bmatrix}
\left\{ F_y \right\}
\end{bmatrix} = \begin{bmatrix}
\left\{ O \right\} \\
\left\{ \Delta f_q \right\} + \left\{ R \right\}
\end{bmatrix},
$$

- vector of nodal loads.

The stress-strain state matrix of the entire structure is formed using the traditional FEM procedure [6, 11].

4. Conclusion
The desired unknown displacement increments and stress increments obtained using the developed finite element in the mixed FEM formulation when using flow theory to take into account the physical nonlinearity of rotation shells are continuous not only in the nodes of the finite element, but also on its faces and boundaries.

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