Theory of the NMR relaxation rates in cuprate superconductors with field induced antiferromagnetic order

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Based on a model Hamiltonian with a $d$-wave pairing interaction and a competing antiferromagnetic interaction, we numerically study the site dependence of the nuclear spin resonance (NMR) relaxation rate $T_1^{-1}$ as a function of temperature for a $d$-wave superconductor (DSC) with magnetic field induced spin density wave (SDW) order. In the presence of the induced SDW, we find that there exists no simple direct relationship between NMR signal rate $T_1^{-1}$ and low energy density of states while these two quantities are linearly proportional to each other in a pure DSC. In the vortex core region, $T_1^{-1}$ on $^{17}O$ site may exhibit a double-peak behavior, one sharp and one broad, as the temperature is increased to the superconductivity transition temperature $T_c$, in contrast to a single broad peak for a pure DSC. The existence of the sharp peak corresponds to the disappearance of the induced SDW above a certain temperature $T_{AF}$ which is assumed to be considerably lower than $T_c$. We also show the differences between $T_1^{-1}$ on $^{17}O$ and that on $^{63}Cu$ as a function of lattice site at different temperatures and magnetic fields. Our results obtained from the scenario of the vortex with induced SDW is consistent with recent NMR and scanning tunneling microscopy experiments.

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I. INTRODUCTION

Intensive efforts have been focused recently on the nature of low-lying excitation spectra around a vortex core in high temperature superconductors (HTS). The excitations around the core play a fundamental role in determining physical property of a superconductor. It has now been established both experimentally and theoretically that a spin density wave (SDW) order could be induced and pinned at the vortex lattice by a strong magnetic field for both under- and optimally-doped HTS. In neutron scattering experiments by Lake et al., a remarkable antiferromagnetic (AF)-like SDW was observed in optimally- and under-doped La$_{2-x}$Sr$_x$CuO$_4$ under a strong magnetic field. The muon spin rotation measurement by Miller et al. studied the internal magnetic field distribution in the vortex core of underdoped YBa$_2$Cu$_3$O$_{6+x}$, and it was revealed a feature in the high-field tail which fits well to a model with static alternating magnetization. Recent nuclear magnetic resonance (NMR) measurements by Mitrovic et al. studied the spatially resolved NMR signal in the mixed state of optimally doped YBa$_2$Cu$_3$O$_{6.67}$, and they found strong AF fluctuations outside the cores and rather different electronic states inside the vortex cores. Another NMR experiment by Kakuyanagi et al. investigated the magnetism in and around the vortex core of nearly optimally-doped Tl$_2$Ba$_2$CuO$_{6+8}$. The NMR signal rate $T_1^{-1}$ at Ti site provides a direct evidence that the AF spin correlation is significantly enhanced in the vortex core region. From theoretical point of view, various approaches suggest the existence of an induced AF order inside the core. Some recent studies also reveal that the AF order could propagate outside the vortex cores and form a SDW.

As proposed in several articles, that the spatially imaging NMR experiment can be a powerful tool to investigate the exotic electronic structure around the vortex cores. The frequency dependence of NMR signal rate reflects the internal magnetic field distribution and the electronic and magnetic excitations in the mixed state can be probed through the temperature and site-dependence of $T_1^{-1}$. The spatial variation of the vortex lattice in the NMR experiments can be resolved by the distribution of internal magnetic field. In the present paper, we study the NMR theory for HTS with the effect of the magnetic field induced SDW being taken into account. Our approach is based upon an effective model Hamiltonian with a $d$-wave pairing interaction $V_{DSC}$ between nearest neighboring sites and a competing onsite Coulomb repulsion $U$ which may generate a competing AF order. The parameters are chosen in such a way that only $d$-wave superconductivity (DSC) prevails in the optimally doped HTS samples when the magnetic field $B=0$. Using the Bogoliubov-de Gennes (BdG) equations, the dynamic spin-spin correlation function between site $i$ and site $j$ can be numerically obtained. From this correlation function and the established methods, we are able to derive the NMR relaxation rates $T_1^{-1}$ on $^{17}O$ and on $^{63}Cu$ nuclei, and from which the NMR signals as a function of temperature $T$ and site location can be determined. Our calculation is performed for an optimally doped HTS sample with the chosen $U$ so that the B induced SDW would appear only below a critical temperature $T_{AF}$ which is considerably smaller than the superconductivity transition temperature $T_c$. We find that there exists no simple direct relationship between the NMR signal rate $T_1^{-1}$ and the low energy local density...
of states (LDOS) while these two quantities are linearly proportional to each other in a pure $d$-wave superconductor ($U = 0$). In the core region, we find that $T^{-1}$ exhibits a double-peak behavior, one sharp and one broad, as the temperature is increased to $T_c$, in contrast to a single broad peak for a pure $d$-wave superconductor. It is found that the low temperature peak becomes sharper and moves to lower $T$ when $T_{AF}$ becomes lower. This result is consistent with the NMR experiments.\textsuperscript{4,14,15} We also show the differences between $T^{-1}_1$ on $^{17}O$ and that on $^{63}Cu$ as a function of lattice site at different temperatures and magnetic fields. In general the NMR signal at the $^{63}Cu$ site is larger than that at $^{17}O$ site, and its magnitude can be quite enhanced at higher magnetic field. In section II we will outline our method and the numerical scheme for calculating the NMR relaxation rate. In section III, we will present our numerical results and their comparison with the experimental measurements. In addition we are also going to compare our work with other similar theoretically studies.\textsuperscript{17,30,31} there the authors only consider the case for a pure $d$-wave superconductor. In all these works\textsuperscript{17,30,31}, the NMR signals are calculated for the square lattice sites which may not rigorously related to the $^{17}O$ and $^{63}Cu$ sites as discussed in the present paper. In the last section, we will give a summary and discussion of our results.

II. FORMALISM

Let us begin with a phenomenological model in which interactions describing both DSC and SDW order parameters in a two-dimensional lattice are considered. The effective Hamiltonian can be written as:

$$H = -\sum_{i,\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{i,\sigma} (U n_{i\sigma} - \mu) c_{i\sigma}^\dagger c_{i\sigma} + \sum_{i,j} (\Delta_{ij} c_{i\uparrow}^\dagger c_{j\downarrow} + h.c.) ,$$  
(2.1)

where $c_{i\sigma}^\dagger$ is the electron creation operator, $\mu$ is the chemical potential, and the summation is over the nearest neighboring sites. In the presence of magnetic field $B$ normal to the two dimensional plane, the hopping integral can be expressed as $t_{ij} = t_0 \exp[i\Phi_0 \int_{r_i}^{r_j} A(r) \cdot dr]$ for the nearest neighboring sites $(i,j)$, with $\Phi_0 = h/2e$ as the superconducting flux quantum. We assume that the strength of $B$ is large enough such that it can be regarded as uniform. Here we choose the Landau gauge for the vector potential $A = (-By, 0, 0)$. Since the internal magnetic field induced by the supercurrent around the vortex core, as it will be numerically shown later, is so small as compared with $B$, the above assumption should be justified.

The mean-field Hamiltonian (1) can be diagonalized by solving the resulting BdG equations self-consistently

$$\sum_{j} \left( \mathcal{H}_{ij,\sigma} - \Delta_{ij} \right) (v_{ij}^n) = E_n (v_{ij}^n) ,$$  
(2.2)

where the single particle Hamiltonian $\mathcal{H}_{ij,\sigma} = -t_{ij} + (U n_{i\sigma} - \mu) \delta_{ij}$, and $n_{i\sigma} = \sum_{n} |u_{i\sigma}^n|^2 f(E_n)$, $n_{ij} = \sum_{n} |u_{ij}^n|^2 f(E_n)$, $\Delta_{ij} = \frac{\tanh(E_{\uparrow\rightarrow\downarrow}/2T)}{\tanh(E_{\uparrow\rightarrow\downarrow}/2T)}$ with $f(E)$ as the Fermi distribution function and the electron density $n_{\uparrow} = n_{i\uparrow} + n_{i\downarrow}$. The DSC order parameter at the ith site is $\Delta_{D} = (\Delta_{D,\uparrow} + \Delta_{D,\downarrow})/4$ where $\Delta_{D} = \Delta_{ij} \exp[i\Phi_0 \int_{r_i}^{r_j}(r_i + r_j)/2 \mathbf{A}(r) \cdot dr]$, and $\mathbf{e}_{x,y}$ denotes the unit vector along $(x,y)$ direction. The main procedure of our self-consistent calculation is summarized as follows. For a random set of initial parameters $n_{ij}$ and $\Delta_{ij}$, the Hamiltonian is numerically diagonalized and the obtained quasiparticle wave functions $u_{i\sigma}^n$ and $v_{i\sigma}^n$ are used to calculate the new parameters for the next iteration step. The iteration continues until the relative difference of order parameter between two consecutive iteration steps is less than $10^{-4}$.

In the following section, we need to calculate the local density of states (LDOS) according to the formula:

$$\rho_n(E) = -\sum_n [ |u_{i\uparrow}^n|^2 f'(E_n - E) + |v_{i\downarrow}^n|^2 f'(E_n + E) ] ,$$  
(2.3)

where $f'(E) \equiv df(E)/dE$. The LDOS is proportional to the local differential tunneling conductance at low temperature which can be measured by STM experiments. In addition, we also need to calculate the internal magnetic field distribution of the vortex state. The internal magnetic field is computed through the Maxwell equation, $\nabla \times \mathbf{H}_{int}(r) = \frac{\Phi_0}{c} \mathbf{j}(r)$, where the current $\mathbf{j}(r)$ is calculated as:

$$j_{e_{x,y}}(r_i) = 2|\epsilon|c|\epsilon|t_{i+e_{x,y},i} \int_{n} |u_{i+e_{x,y}}^n|^2 f(E_n) + v_{i+e_{x,y}}^n v_{i+e_{x,y}}^* (1 - f(E_n))| .$$  
(2.4)

The nuclear spin relaxation rate $T_1^{-1}$ can be obtained from the spin-spin correlation function $\chi_{+,-}(i,j;\Omega)$ at zero energy through the following site dependent function:\textsuperscript{17,18,25}

$$R(i,j) = \text{Im} \chi_{+,-}(i,j;\Omega_n = \Omega + i\eta)/(\Omega/T)|_{\Omega \rightarrow 0} = -\sum_{n,n'} u_{i\sigma}^n u_{i\sigma}^{n'} [u_{j\sigma}^n u_{j\sigma}^{n'} + v_{i\sigma}^n v_{i\sigma}^{n'}] \times \pi T f'(E_n) \delta(E_n - E_n') .$$  
(2.5)

Since all of the experiments\textsuperscript{26,27,32} are performed at the oxygen, copper, or other nuclei in HTS, we need the expressions of the NMR relaxation rate at these sites. In this paper we shall mainly consider the contribution from the planar oxygen ($^{17}O$) and copper ($^{63}Cu$) nuclei. The
oxygen nucleus is located between two Cu nuclei along the \(x\) or \(y\) axis. The internal magnetic field \(H_{\text{int}}\) on \(^{17}O\) site is averaged over its two nearest neighboring \(^{63}Cu\) sites. The NMR relaxation rate form factor on \(^{17}O\) and \(^{63}Cu\) in momentum space is: \(F_{O,x} \sim 1 + \cos(q_x)/2\) and \(F_{Cu} \sim (\cos(q_x) + \cos(q_y))^2\). It can be expressed in terms of the real-space spin-spin correlations as follows:\(^{20}\)

\[
\frac{1}{17}T_1 T = C[R(i,i) + 1/2 \sum_{j,x} R(i,j)],
\]

\[
\frac{1}{63}T_1 T = B[R(i,i) + 1/2 \sum_{j} R(i,j) + 1/4 \sum_{j} R(i,j)],
\]

where the notation of \(\sum_{x}^{"}, \sum_{y}^{"}\) and \(\sum_{x}^{"'}\) means summation over two nearest-neighbor sites along the \(x\) axis, four next-neighbor sites, and four next-next-neighbor sites, respectively. Here we calculate \(T_1^{-1}\) in terms of arbitrary unit and choose the ratio of the coefficients\(^{21}\) \(B/C = 1.583\).

Throughout this paper, we use the following parameters: \(U = 2.4, V_{\text{DSC}} = 1.2\), the linear dimension of the unit cell of the vortex lattice is chosen as \(N_x \times N_y = 40 \times 20\) sites and the number of the unit cell \(M_x \times M_y = 20 \times 40\), and the hole doping level \(x = 0.15\). This choice corresponds to a uniform magnetic field \(B \approx 37T\). We set \(t_0 = a = 1\). The standard procedures\(^{22}\) are followed to introduce magnetic unit cells, where each unit cell contains two superconducting flux quanta. By introducing the quasi-momentum of the magnetic Bloch state, we obtain the wave function under the periodic boundary condition whose region covers many unit cells.

III. NUMERICAL RESULTS AND COMPARING WITH EXPERIMENTS

Our previous numerical calculations\(^{12}\) show clearly that the induced AF order exists both inside and outside the vortex cores, and behaves like an inhomogeneous SDW with the same wave length in the \(x\) and \(y\) directions. Fig. 1 depicts the SDW amplitude or the staggered magnetization \(M_i^z = \Delta_i^{\text{SDW}}/U\) distribution in a square lattice. It is seen that the induced SDW order reaches its maximum value at the vortex core center (V), zero value at the saddle-point midway between two nearest neighboring vortices (S), and in between at the center of the squared vortex lattice unit cell (C). Using Eq.(2.4), the internal magnetic field \(H_{\text{int}}\) at different site can be numerically determined. In the upper right inset of Fig. 1 we show the histogram of \(H_{\text{int}}\), where \(P(H_{\text{int}})\) measures the number of lattice sites (or plaquettes) with fixed \(H_{\text{int}}\). We can approximately identify each site (V, S, C) including the sites around it in the vortex lattice to the distribution. The correspondence between the site position and \(H_{\text{int}}\) can roughly be established. It is easy to see that the magnitude of \(H_{\text{int}}\) is decreasing along the path (V→S→C) and its direction is always perpendicular to the lattice plane. In a narrow region, \(H_{\text{int}}\) may even become negative. This is very different from the case for a pure DSC where \(H_{\text{int}}\) is always positive\(^{12}\). Our histogram of \(H_{\text{int}}\) distribution is consistent with the experimental data\(^{17}\) except there the region of negative \(H_{\text{int}}\) is somewhat wider than ours. The experimentally observed negative \(H_{\text{int}}\) may indicate the existence of magnetic field induced SDW around the vortex cores.

The NMR signal at the maximum cutoff of the histogram as a function of internal magnetic field comes from the V-site. The minimum cutoff is from the C-site. The peak value of the internal magnetic field comes from the S-site. Contrary to the case for a pure \(d\)-wave superconductor, the SDW order outside the vortex core may have a remarkable contribution to both the internal magnetic distribution and the NMR signal. It is supposed that \(^{17}O\) NMR experiment can provide direct information on the LDOS as the antiferromagnetism spin fluctuation is cancelled\(^{27,28}\). From our study, we find that in the presence of SDW vortex cores, the internal magnetic field on \(^{17}O\) site can not be cancelled exactly and appreciable residue exists.

Next, we study the relationship between low energy

FIG. 1: The amplitude distribution of the SDW order parameter \(M_i^z\) (a) in one magnetic unit cell. V, S, C points represents the vortex core center, the saddle point midway between two nearest neighboring vortices, and the center of the squared vortex lattice unit cell, respectively. The inset shows the histogram \(P(H_{\text{int}})\) of the induced magnetic field distribution where \(H_{\text{int}}\) is in a unit of \(10^3\) Gauss. The calculation is performed with \(U = 2.4\) and \(V_{\text{DSC}} = 1.2\).
FIG. 2: The spatial distribution images of the LDOS at the energy $E=0$ (a) and NMR relaxation rate $1/T_1(i)T$ (b) in a vortex unit cell for a pure DSC with $U=0$. Other parameter values are the same as those in Fig. 1.

LDOS and NMR signal rate $1/T_1$. In a pure $d$-wave superconductor (DSC), a linear relationship is found between $1/T_1(i) \sim R(i,i)$ and the LDOS $\rho_i(E=0)$ at zero temperature. We have checked our program and reproduced similar results. Figure 2 depicts the spatial distribution images of LDOS $\rho_i(E=0)$ (a), and $1/T_1(i)T$ (b) for a pure DSC with $U=0$ at low temperature. In this case, the core state shows zero-energy peak and nodal directions are along $x=y$ and $x=-y$. The images in Fig. 2(a) and 2(b) are similar to each other and the results can be regarded as linearly proportional to each other. Thus the NMR signal for a pure DSC can be estimated quantitatively from its corresponding zero energy LDOS according to the linear relationship.

From Fig. 2(a), the LDOS of quasiparticles decreases along the path ($V\rightarrow C \rightarrow S$). In the nodal directions, appreciable weight of the low-energy states extending from the vortex core still remains there. Our numerical result indicates that the LDOS of the low-energy states at the C-site is slightly larger than that of the S-site, although C-site is farther away from the vortex center. This is different from the work by Takigawa et al. where the LDOS at the S site is found to be larger than that at the C site. The authors there employed the symmetrical gauge and they found that their unit vector of vortex lattice is oriented along 45° from the Cu-O bond. But our result is consistent with the study of Knapp et al. who used a different approach and calculated the NMR signal at various sites for a pure DSC. Furthermore, a recent small-angle neutron scattering measurement for La$_{1.83}$Sr$_{0.17}$CuO$_{4+\delta}$ indicates that the unit vector of vortex lattice is clearly oriented along the Cu-O bonds, consistent with our result in Fig. 1. The NMR experiments also show that the NMR signal first decreases as one moves away from the V-site to S-site and then increases at the C-site, consistent with the results in Fig. 2. Here we notice that the NMR signal at C site is slightly larger than that at S-site, their difference could be seen in Fig. 6 and Fig. 7. It appears that the above analysis could provide a basic understanding of the spatial distribution of the experimentally observed NMR relaxation rates in HTS in the framework of pure DSC. However, it is well known that the theoretical predicted LDOS, which shows a strong zero energy peak for a pure DSC at the vortex core is in contradictory to the STM measurement on HTS. In order to overcome this difficulty, the magnetic induced AF order has been introduced to account for the experiments. In the following we shall investigate the effect of induced AF or SDW order on the NMR theory in HTS.

The presence of the induced SDW order in the mixed state may lead to violation of the linear relationship between the LDOS at $E=0$ and the NMR relaxation rate $1/T_1(i)$ . This can be seen clearly in Fig. 3, where we show the spatial images of the LDOS (a) and the $1/T_1(i)T$ (b) according to our numerical calculations for $U=2.4$. There is little resemblance between Figs. 3(a) and 3(b), and the linear relationship does not exists here. Fig. 3(a) indicates that the LDOS is smallest at V-site, and largest at S-site, it clearly can not be applied to explain the NMR experiments as in the case for a pure DSC. But the LDOS obtained here exhibits no zero energy peak at the vortex core which is consistent with the STM experiments. The averaged LDOS of quasiparticles in Fig. 3(a)($U=2.4$) is greatly suppressed than that in Fig. 2(a)($U=0$). This is because the presence of the induced SDW enhances the insulating nature of the system and thus reduces the number of the quasiparticles. When $U=2.4$, there are two contributions to the NMR signal $1/T_1(i)T$, at the low temperature, the dominant one is from the induced SDW order and the minor one is from the low energy quasiparticle states. The LDOS is the smallest at the vortex core center but the SDW order is at its maximum as shown in Fig. 1. From Fig. 3(b), it is easy to see that the spatially distributed NMR signal $1/T_1(i)T$ which has the largest value at the vortex center because of the induced AF order there and decreases along the path ($V\rightarrow C \rightarrow S$). This feature is consistent with what has been observed by the experiments and in sharp contrast with Takigawa et al.’s results. They claimed that $T_{1}^{-1}$ at V-site is smaller than the neigh-
boring sites and the NMR signal rate at S-site is higher than that of C-site. The choice of lattice size in their calculations corresponds to a very strong magnetic field at about 51 Tesla. Their results seem to be inconsistent with the measurement of $1/\tau_1$ on YBa$_2$Cu$_3$O$_{7-\delta}$.

The NMR signals also exhibit the temperature $T$ and magnetic field $B$ dependences. Our approach described in Section II is easily extended to study the finite $T$ case, but not the $B$ dependence because weaker $B$ implies larger size calculation which we are not able to do at the present moment. Since the observed NMR signals are coming from the oxygen, copper, or other atoms in HTS, here we shall mainly consider the temperature dependence of the NMR signal rate $1/\tau_1$ from the $^{17}O$ sites using Eq. (2.6). It needs to be point out that the value of $R(i,j \neq i)$ in Eq. (2.6) is of the same order with $R(i,i)$ and its contribution should not be neglected. In Fig. 4, the NMR relaxation rates $1/\tau_1$ are plotted as a function of $T$ for sites $V$, $C$, and $S$. The calculations are performed using $U=2.4$ except the dashed curve associated with the $V$ site where $U=0$ and is obtained for a pure DSC. The inset plots the temperature dependence of the NMR signal rate $1/\tau_1$ at the vortex core center $V$, the evolution of $T_1^{-1}$ with temperature exhibits a double peak structure below $T_c$; the sharp peak occurs at lower $T$ and the broad peak is at higher $T$. This is in contrast to a single peak obtained with $U=0$ for a pure DSC. The minimal value of the staggered magnetization occurs near the saddle point $S$. It reflects the fact that the SDW order at $C$ site is larger than that at $S$-site. Because both NMR signals at $C$ and $S$ sites are very weak as compared to that at $V$ site, their distinction is not reflected in Fig. 4 below $T < 0.4T_c$. The maximum magnitude of $1/\tau_1$ at $V$-site is about two orders of magnitude larger than that far from the vortex core, and the NMR signal rate at C-site is also larger than that at S-site. 

![FIG. 3: The spatial distribution images of the LDOS at the energy $E = 0$ (a) and NMR relaxation rate $1/\tau_1$ (b) in a vortex unit cell for a DSC with induced SDW. All parameter values are the same as those in Fig. 1.](image)

![FIG. 4: Temperature dependences of $^{17}O$ NMR signals $1/\tau_1$ normalized by its value $1/\tau_1^c$ at $T_c$ are plotted as a function of $T/T_c$ at the sites $V$, $S$, $C$. The dashed line associated with the site $V$ is obtained for a pure DSC by using $U=0$. The inset plots the temperature dependence of SDW order parameters at $V$-site and DSC order at $S$-site.](image)
case the value of $T_{AF}$ is slightly higher than the position of the broad peak, only a single peak for the NMR signal is predicted. When $T_{AF}$ is close or larger than $T_c$, the NMR signal would be an increasing function of $T$ and may not show any peak structure. We also have done the same calculation for the NMR relaxation rates on $^{63}Cu$ nuclei on our model lattice sites ($1/T_1(i)$), and found similar $T$ and site dependence as shown in Fig. 4.

In the experimental aspect, a sharp peak has been observed at $T \sim 0.235T_c$ in Tl$_2$Ba$_2$CuO$_6$ while there is no observed peak in YBa$_2$Cu$_3$O$_7$ up to $T \sim 0.272T_c$. Therefore the value $U$ in YBa$_2$Cu$_3$O$_7$ should be larger than that in Tl$_2$Ba$_2$CuO$_6$. This conclusion is opposite to the theoretical prediction made by Takigawa et al.$^{31}$

We also would like to examine the difference in the spatial distributions of the NMR signals from $^{63}Cu$ and $^{17}O$ nuclei. In Fig. 5, The site distribution maps of $T_1^{-1}$ for $^{63}Cu$ and for $^{17}O$ in a pure DSC ($U=0$) are respectively illustrated in (a) and (b). Both of these maps reach a similar peak value at the vortex core and then fall away smoothly from it. In the vortex core, the magnitude of $T_1^{-1}$ at the $^{63}Cu$ site is about three times larger than that at the $^{17}O$ site. In Figs. 5(c) and 5(d), we present the results for $^{63}Cu$ and $^{17}O$ nuclei in a DSC with induced SDW ($U=2.4$). The site-dependent spectra for both nuclei show staggered oscillations in the vortex core, there the NMR signal rate from the $^{17}O$ site is about one fourth of the value from that of $^{63}Cu$ site. This is because the AF order at the $^{17}O$ sites could be dramatically weakened by the partial cancellation of the staggered magnetization between two nearest neighboring sites on the original lattice. It is useful to point out that the NMR signal $^{17}T_1^{-1}$ outside the vortex core region is one order of magnitude smaller than that of $^{63}T_1^{-1}$. In the presence of the induced SDW, the NMR signal rate is enhanced at the core center and suppressed away from the vortex core.

Next, the temperature dependence of NMR signal rates for the case of $U=2.4$ is examined. Here, we plot the variation of $T_1^{-1}$ along the path $V \rightarrow S \rightarrow C$ for $^{63}Cu$ nuclei in Fig. 6(a) and for $^{17}O$ nuclei Fig. 6(b) at two different temperatures. The values of $T_1^{-1}$ in Figs. 6(a) and 6(b) exhibit similar behavior and oscillate from site to site. The amplitude of the oscillation reflects the underlying staggered magnetization and is apparently weakened at higher temperature. In fact, the $T$ dependence of NMR signal rate at three typical points has been seen from Fig. 3. Figure 6 shows more clearly that NMR signal rate at each site increases with temperature and its value at site C is always larger than that at site S. We have reproduced the power law relation $T_1^{-1} \sim T^{3}$ for a pure DSC at zero magnetic field and at low temperature.
In the presence of vortices with induced SDW order, it should not be surprised that $T_1^{-1}$ does not follow this $T^3$ law.

Finally, we show the site dependence of NMR signal rates on $^{63}\text{Cu}$ and on $^{17}\text{O}$ nuclei at two different magnetic field B respectively in Figs. 7(a) and 7(b). The NMR signals $T_1^{-1}$ in Figs. 7(a) and 7(b) are similar to each other. Since the magnitude of the induced SDW order is strongly dependent on the external magnetic field B — higher B leads to more pronounced SDW order which may in turn yield a larger NMR signal. This conclusion is consistent with the NMR experiments. As a result, the amplitude of oscillation at different sites, which measures the strength of the induced staggered magnetization, is dramatically reduced when the magnetic field B is decreased from 37T to 27T as shown in Fig. 7. Here $B = 27T$ corresponds to a vortex lattice unit cell of $48 \times 24$ sites. On the other hand, the NMR signal rate at V-site is suppressed with increasing B for a pure DSC which reflects the density of quasiparticle around the vortex core is reduced due to the enhancement of quasiparticle leakage along the nodal directions. It was argued by Takigawa et al. that when AF order inside the vortex core is large enough, $T_1^{-1}$ becomes the absolute minimum at V-site. This conjecture is based on the assumption that the linear relation between $T_1^{-1}$ and LDOS still exists even in the presence of the SDW order. On the other hand, our results indicate strongly that with the increasing B, the enhancement of AF order inside the vortex core may lead to more pronounced $T_1^{-1}$ value at the V-site. This is very different from what has been predicted in Ref. 31. Finally we point out that existing experimental data show the presence of appreciable NMR signal rate at S-site and its value is insensitive to T. But in our calculations there is a very weak NMR signal at S-site. The discrepancy between experimental data and our results may be due to the reason unclear to us at this moment.

IV. SUMMARY

Based on a model Hamiltonian as described in Sections I and II, we have investigated the site- and temperature-dependence of NMR signal rate $T_1^{-1}$ near the vortex core in an optimally doped HTS. The calculations are carried out with ($U = 2.4$) and without ($U = 0$) the magnetic field induced SDW order. Although the relative values of $T_1^{-1}$ at V, S, and C sites near the vortex core obtained theoretically for a pure DSC are consistent with recent experiments, the temperature dependent of $T_1^{-1}$ thus derived fails to explain the sharp peak observed in NMR experiments for Tl$_2$Ba$_2$CuO$_{6+x}$ at low temperature. Moreover, the LDOS obtained for a pure DSC has a zero energy peak at the core center and this is in contradictory to the STM observations. All these difficulties could somewhat be removed by introducing the field induced SDW pinned at the vortex cores as it has been studied in the present paper. With the induced SDW order, the LDOS exhibits no zero energy peak at the core center and is consistent with experiments. Unlike the case for a pure DSC, there exists no linear relationship between zero energy LDOS and $T_1^{-1}$. We show that SDW order could strongly enhance the NMR signal rate, and as a result, the NMR signal $T_1^{-1}$ from $^{17}\text{O}$ site has its largest value at the core center V, and smallest value at the saddle point S site (see Figs. 6 and 7). The temperature dependent of $T_1^{-1}$ at V site exhibits a sharp peak at low temperature which is in agreement with the NMR experiments. Since the strength of the SDW order depends on the magnitude of B, the value of $T_1^{-1}$ should also be enhanced by increasing B. We also compare $T_1^{-1}$ from $^{63}\text{Cu}$ and $^{17}\text{O}$ nuclei at different T and B, their essential features are similar to each other. The magnitude of $^{17}T_1^{-1}$ outside the vortex core region is about one order of magnitude smaller than that of $^{63}T_1^{-1}$ while these two values are closer to each other inside the vortex core. Finally, we would like to point out that...
the difference between our results and those of a similar work has been discussed in detail in Section III. Another fundamental difference is that their LDOS at the vortex center for a pure DSC ($U=0$) shows double peaks near $E=0$ while only a single peak is obtained in previous and present works. It is our hope that the present calculation may shed more light on the understanding of recent NMR and STM experiments for the mixed state of HTS in a unified picture, and stimulate more theoretical activity in this field.

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