Universal spectral correlations in ensembles of random normal matrices

Ravi Prakash(a) and Akhilesh Pandey(b)

School of Physical Sciences, Jawaharlal Nehru University - New Delhi - 110067, India

received 11 November 2014; accepted in final form 20 April 2015
published online 12 May 2015

PACS 05.45.Mt – Quantum chaos; semiclassical methods
PACS 05.40.-a – Fluctuation phenomena, random processes, noise, and Brownian motion
PACS 02.50.Ng – Distribution theory and Monte Carlo studies

Abstract – We consider non-Gaussian ensembles of random normal matrices with the constraint that the ensembles are invariant under unitary transformations. We show that the level density of eigenvalues exhibits disk or ring structure in the complex plane. We also show that the $n$-eigenvalue correlation and the spacing distribution are universal and identical to that of complex (Gaussian) Ginibre ensemble. Our results are confirmed by Monte Carlo calculations. We verify the universality for dissipative quantum kicked rotor systems.

Copyright © EPLA, 2015

Introduction. – Random matrix ensembles play a prominent role in the study of quantum systems which exhibit chaotic behavior in the classical limits. The local spectral and transition width fluctuations as well as scattering cross-section and conductance fluctuations have been extensively studied and are well described by random matrix ensembles [1–7]. Ensembles of Hermitian matrices (e.g. Gaussian orthogonal, unitary or symplectic ensemble) and of unitary matrices (e.g. circular orthogonal, unitary or symplectic ensemble) are widely used.

Ginibre introduced ensemble of general complex matrices with no unitary or Hermitian condition imposed [8]. The matrix elements follow the Gaussian distribution, i.e., the joint probability distribution (jpd) for Ginibre matrices $G$ is given by $P(G) \propto \exp[-\text{Tr} V(G^\dagger G)]$, where $V(x) = x$ with $x > 0$. Eigenvalues lie in the complex plane and are distributed uniformly in a disk [6,8]. The Ginibre ensemble and its extension to non-Gaussian weight functions (viz. nonlinear potentials $V$) have found applications in several fields, e.g., quantum dissipative systems with eigenvalues having imaginary parts due to dissipation [9–12], quantum chromodynamics at finite chemical potential [13,14], transition from symmetric to asymmetric matrices [15,16], neural networks dynamics [17], truncation of scattering matrices [18] and google networks [19].

Ginibre ensembles with non-Gaussian weight are typically difficult to deal with because of lack of symmetry. We consider ensembles of normal matrices with arbitrary weight functions. Normal matrices commute with their Hermitian adjoints. Such ensembles are exactly solvable because the matrices are diagonalizable by unitary transformations [20]. We believe that normal matrix ensembles will capture the behavior of the spectral density qualitatively and the behavior of two-eigenvalue and higher-order correlations exactly. Moreover, normal matrix ensembles are useful in the study of fractional quantum Hall effect where the Laughlin wave function for a non-uniform magnetic [21–23] field is similar to the eigenvalue jpd and also in the study of other problems, where the matrix may not be normal but the eigenvalue jpd is similar [18,24].

Our purpose in this letter is to study ensembles of normal matrices with arbitrary weight. It has been shown in [25] that the eigenvalues lie in a disk in the complex plane and the two-eigenvalue correlation are similar to that of the Ginibre case. The results in [25] are proved for polynomial potentials with non-negative coefficients; see also [26–28]. Higher-order correlations are shown explicitly to be universal for monomial potentials [25,26]. We show that the eigenvalues are scattered in the complex plane such that it can constitute either a disk or a ring structure for all smooth potentials. A similar theorem has been proved in [29] for non-Gaussian Ginibre ensembles. Moreover, disk or ring structures along with δ-function boundaries are encountered for potentials with hard walls. We also prove that after suitable unfolding, not only the two-eigenvalue correlation but also all higher-order correlations as well as spacing distributions are stationary.
and universal. To elucidate the results, we provide detailed verification of the analytic results by comprehensive numerical study.

We undertake the Monte Carlo (MC) simulation [30] of several non-Gaussian ensembles. As an application, we also study the effect of dissipation on the quantum kicked rotor maps [31–33] and an equivalent random matrix model. The eigenvalue density for both shows a ring structure. We obtain the same universal correlations also for these cases.

Joint probability distribution and correlation functions. – We consider complex normal matrices with JPD of matrix elements given by

\[ P(M) \propto e^{-N \text{Tr}[V(M^\dagger M)]}. \]

The eigenvalues of a normal matrix can be obtained by a unitary transformation \( U \), i.e., \( M = U Z U^\dagger \), where \( Z \equiv \text{diag}(z_1, \ldots, z_N) \). The \( z_j \) are the eigenvalues of matrix \( M \) and are in general complex. Integrating over the eigenvector variables we get [6,20]

\[ P(z_1, \ldots, z_N) = C \prod_{j<k} |z_j - z_k|^{2\beta} e^{-N \sum_{i=1}^N V(|z_i|^2)}, \]

where \( C \) is the normalization constant and \( \beta = 2 \). (We mention that \( \beta = 1 \) and 4 correspond to the complex normal matrices which are symmetric and self-dual, respectively.) We remark that the JPD in (2) for \( \beta = 2 \) turns out to be identical to the JPD for non-normal ensembles for the Gaussian weight [6,8] and some logarithmic [18,24] potentials.

Joint probability distribution and correlation functions. – We consider complex normal matrices with JPD of matrix elements given by

\[ P(M) \propto e^{-N \text{Tr}[V(M^\dagger M)]}. \]

The eigenvalues of a normal matrix can be obtained by a unitary transformation \( U \), i.e., \( M = U Z U^\dagger \), where \( Z \equiv \text{diag}(z_1, \ldots, z_N) \). The \( z_j \) are the eigenvalues of matrix \( M \) and are in general complex. Integrating over the eigenvector variables we get [6,20]

\[ P(z_1, \ldots, z_N) = C \prod_{j<k} |z_j - z_k|^{2\beta} e^{-N \sum_{i=1}^N V(|z_i|^2)}, \]

where \( C \) is the normalization constant and \( \beta = 2 \). (We mention that \( \beta = 1 \) and 4 correspond to the complex normal matrices which are symmetric and self-dual, respectively.) We remark that the JPD in (2) for \( \beta = 2 \) turns out to be identical to the JPD for non-normal ensembles for the Gaussian weight [6,8] and some logarithmic [18,24] potentials. The JPD is rotationally invariant in the complex plane. The factor \( N \) in the exponent will ensure that for large \( N \), any disk or ring structure has \( N \)-independent radii. We will be interested in the \( n \)-eigenvalue correlation functions, \( R_n(z_1, \ldots, z_n) \), defined by

\[ R_n = \frac{N!}{(N-n)!} \int \cdots \int P(z_1, \ldots, z_N) \, d^2z_{n+1} \cdots d^2z_N, \]

where \( d^2z \) represents the infinitesimal area element, i.e., \( d^2z = 2dz \, d\bar{z} \) in Cartesian coordinates. Here \( R_1 \) is the spectral density. To derive the \( n \)-eigenvalue correlation functions we use the method of orthogonal polynomials in the complex plane. Following [6,8] in the Gaussian case and [20] for the general potential, we introduce the orthogonal polynomials, \( p_l(z) = z^l \), where

\[ \int z^l (z^*)^m e^{-N V(|z|^2)} \, d^2z = N_l \delta_{lm}, \]

with \( N_l \) the normalization constant. Note that the orthogonality in (4) comes from the angular integration. We define the kernels,

\[ K_N(z_j, z_k) = e^{-\frac{N}{2} [V(|z_j|^2) + V(|z_k|^2)]} \sum_{l=0}^{N-1} \frac{(z_j z_k)^l}{N_l}. \]

The \( n \)-eigenvalue correlation functions are given by [6,20]

\[ R_n(z_1, \ldots, z_n) = \text{det} [K_N(z_j, z_k)]_{j,k=1,\ldots,n}. \]

Thus, for example,

\[ R_1(z) = K_N(z), \]

and since \( K_N(z_j, z_k) = K_N(z_k, z_j) \), we have

\[ R_2(z_1, z_2) = R_1(z_1) R_1(z_2) - |K_N(z_1, z_2)|^2. \]

Large-\( N \) expression for spectral density. – The JPD of the eigenvalues is equivalent to the partition function of \( N \) particles in the complex plane under the interaction of a repulsive two-body log potential and a one-body binding potential \( V \). We have, \( P(z_1, \ldots, z_N) = C \exp(-W) \), where the effective potential \( W \) is given by

\[ W = -\sum_{1 \leq j < k \leq N} \ln |z_j - z_k|^2 + N \sum_i V(|z_i|^2). \]

The JPD will be maximum when \( W \) is minimum, i.e., \( \partial W/\partial z_j = 0 \) and \( \partial W/\partial z_j^* = 0 \). Thus, for large \( N \),

\[ \frac{1}{z_j - z_k} = z_j^N V'(|z_j|^2). \]

Here \( V' \) is the differential with respect to its argument \( |z|^2 \). For large \( N \) the summation in (10) can be replaced by an integral over \( R_1 \). Using the isotropic invariance of \( R_1 \), we find from (10) that, with \( R_1(z) = R_1(|z|) \),

\[ \int \frac{R_1(|z|)}{z - \xi} \, d^2\xi = z^* V'(|z|^2). \]

Defining \( \xi = r \exp(i\theta) \) and integrating over \( \theta \), we get,

\[ \int_0^{2\pi} r R_1(r) \, dr = \frac{|z|^2}{2\pi} NV'(|z|^2). \]

Differentiating with respect to \( |z| \), we have finally [25–28],

\[ R_1(|z|) = \frac{N}{\pi} \left[ V'(|z|^2) + |z|^2 V''(|z|^2) \right]. \]

Note that the left-hand side of (12) is greater than zero and monotonically increasing function of \( |z| \). Therefore, the right-hand side must show the same behavior. Thus, all the eigenvalues must lie in the range where \( |z|^2 V'(|z|^2) \) is positive and monotonically increasing. This implies that whatever the potential, the constraint allows eigenvalues to lie either in a disk shape or in a ring shape only. This is referred to as the single-ring theorem in the context of non-Gaussian Ginibre ensembles [29]. Potentials considered in [25–28] do not admit a ring structure. The inner radius, \( a \), is obtained from

\[ |z|^2 V'(|z|^2) \geq 0 \text{ and monotonic for } |z| \geq a. \]
The outer radius, \( b \), is obtained from the normalization condition,

\[
2\pi \int_a^b r R_1(r) \, dr = N. \tag{15}
\]

As mentioned above, \( a \) and \( b \) are independent of \( N \). If there are several solutions of (14), we choose the largest value of \( a \). See the example of the cosine potential ahead. Our Monte Carlo results in the following result that for finite but large \( N \) the errors are primarily at the spectral boundaries.

**Spectral density for potentials with arbitrary boundaries.** – We have assumed above that the potential is smooth and its first and second derivatives are continuous. We now consider hard walls at \( |z_1| \) and \( |z_2| \), where \( |z_1| < |z_2| \). Then there are several scenarios. If \( |z_1| \leq a < b \leq |z_2| \), the result (13) remains the same. If \( a \leq |z_1| < |z_2| \leq b \), then the density in (13) is modified. In this case the eigenvalues up to \( |z_1| \) accumulate at \( |z_1| \) and eigenvalues beyond \( |z_2| \) accumulate at \( |z_2| \). This behavior is consistent with the condition (12) and is verified for the Gaussian potential ahead. In this case the spectral density is defined for \( |z_1| \leq |z| \leq |z_2| \) and is given by

\[
R_1(|z|) = \frac{N}{\pi} \left[ |z|^2 V''(|z|^2) \frac{\delta(|z| - |z_1|)}{|z_2| - |z_1|} \right. \\
+ \left. |z|^2 V''(|z|^2) + V'(|z|^2) + \left( \frac{1}{|z_2| - |z_2|} \right) \delta(|z| - |z_2|) \right]. \tag{16}
\]

Other cases can be similarly dealt with.

**Universality of two-eigenvalue and higher-order correlations.** – To evaluate \( n \)-eigenvalue correlation functions we require the knowledge of the normalization constant in (5) for arbitrary potential. From (5), (7) along with (13), we find, for large \( N \),

\[
e^{-NV(|z|^2)} \sum_{l=0}^{N-1} \frac{|z_2|^l}{N_l!} = \frac{N}{\pi} \left[ V'(|z|^2) + |z|^2 V''(|z|^2) \right]. \tag{17}
\]

Thus, the normalization constant \( N_l \) is given by

\[
\frac{1}{N_l} = \frac{N}{\pi l!} \int_0^1 \left[ \exp(NV(x)) \right] V'(x) \, dx \bigg|_{x=0} \to \infty. \tag{18}
\]

Substituting (18) in (5) we get for large \( N \)

\[
K_N (z_j, z_k) = \frac{N}{\pi} e^{-\frac{1}{2}(V(|z_j|^2) + V(|z_k|^2))} \\
\times \left[ z_j z_k^* V'(|z_j|^2) z_k z_j^* V'(|z_k|^2) \right] e^{NV(|z_j|^2)}. \tag{19}
\]

Equation (19) gives the analytic expression for the two-point kernel for arbitrary \( V \). For monomial potentials our result agrees exactly with [26] and also with [25] after rescaling of the potential. The proof of (19), and therefore (27) ahead, relies on the assumption that \( N^{-1} \sum_{\ell=0}^{N-1} \xi^l / N_l \) remains an analytic function of \( \xi \) for

\( N \to \infty \). This has been proved in [25] for polynomial potentials. We have assumed that the analyticity holds for other potentials also. Our assumption is justified by the MC calculations ahead.

In order to investigate the universality of correlation functions we have to transform the spectrum so that the transformed spectral density is uniform. This procedure is called unfolding of the spectrum. For the Gaussian Giniibre ensemble the density is uniform and, therefore, unfolding is not required. We choose the new density to be \( 1/\pi \) so that we can compare with the results of [6,8]. The transformation from \( z \) variables to new variables \( \zeta \) should be such that \( (1/\pi) \, d^2 \zeta = R_1(|z|) \, d^2 z \). Instead of unfolding the entire spectrum by one function, we consider local unfolding. We assume that the spectral density is constant locally for large \( N \). The transformation of the line element can be written as \( d\zeta = \sqrt{\pi R_1(|z|)} \, dz \). Thus, \( \zeta \) is related to \( z \) in the neighborhood of \( z \) by the relation

\[
z_j = z + \delta_j; \quad \delta_j = \frac{\zeta_j}{\sqrt{\pi R_1(|z|)}}, \tag{20}
\]

where \( \zeta_j = O(1) \) and \( \delta_j = O(1/\sqrt{N}) \).

The \( n \)-eigenvalue correlation function, \( R_n (\zeta_1, \ldots, \zeta_n) \), in transformed variables can be obtained from

\[
R_n \, d^2 \zeta_1 \ldots d^2 \zeta_n = R_n (z_1, \ldots, z_n) \, d^2 z_1 \ldots d^2 z_n. \tag{21}
\]

It follows that \( R_n \) can be written as

\[
R_n (\zeta_1, \ldots, \zeta_n) = \det \left[ K_N (\zeta_j, \zeta_k) \right]_{j,k=1,\ldots,n}, \tag{22}
\]

where

\[
K_N (\zeta_j, \zeta_k) = \frac{1}{\pi} \frac{K_N (z_j, z_k)}{\sqrt{K_N (z_j, z_j) K_N (z_k, z_k)}}. \tag{23}
\]

Note that \( R_1 = 1/\pi \), as it should be. By using (19) in (23) and keeping the leading terms we get

\[
K_N (\zeta_j, \zeta_k) = \frac{1}{\pi} \exp \left[ -\frac{N}{2} \left( V(|z_j|^2) \right) \right. \\
+ \left. V(|z_k|^2) - 2V(z_j z_k^*) \right]. \tag{24}
\]

Substituting (20) in (24) and evaluating the exponent to order \( N\delta_z \) and \( N\delta_v ^2 \), we get

\[
K_N (\zeta_j, \zeta_k) = \frac{1}{\pi} \exp \left[ \frac{N}{2} R_1(|z|) (|\delta_j - \delta_k|^2) \right] \\
\times \exp \left[ \frac{N}{2} R_1(|z|) (|\delta_j - \delta_k^*|^2 + |\delta_j^* - \delta_k|^2 + (\chi_j - \chi_k)) \right]. \tag{25}
\]

Here \( \chi \) is purely imaginary and is given by

\[
\chi_j = \frac{N}{2} V'z^* \delta_j + \frac{N}{4} V''z^* \delta_j^2 - C.C., \tag{26}
\]

with C.C. denoting the complex conjugate. The \( \chi \) term in (25) does not affect the determinant in (22) and, therefore, we ignore it. The kernel can then be written in terms of \( \zeta \) as

\[
K_N (\zeta_j, \zeta_k) = \frac{1}{\pi} e^{-\frac{1}{2} |\zeta_j - \zeta_k|^2 + \frac{i}{2} (\zeta_j^* \zeta_k - \zeta_j \zeta_k^*)}, \tag{27}
\]
Thus, the two-eigenvalue correlation function is given by
\[ R_2(\zeta_1, \zeta_2) = \frac{1}{\pi^2} \left( 1 - e^{-|\zeta_1 - \zeta_2|^2} \right). \]  
(28)

Note that, unlike (28), the higher-order functions will depend on the phase term of (27).

The result in (28) is known for the Gaussian Ginibre case [6]. Our proof shows that not only (28) but also the more general \( n \)-eigenvalue correlation functions given in (22), (27) are all stationary (i.e., independent of \( z \)) and universal (i.e., independent of \( V \)). Using the rotational invariance of the correlation functions, we introduce
\[ R_2(s) = \int \int R_2(\zeta_1, \zeta_2) \delta(s - |\zeta_1 - \zeta_2|) \, d^2 \zeta_2. \]  
(29)

Substituting eq. (28) in (29), we get, for large \( N \),
\[ R_2(s) = \frac{2}{\pi} s \left( 1 - e^{-s^2} \right). \]  
(30)

Equation (30) for small \( s \) provides another proof of cubic repulsion [11]. \( \mathcal{R}_2 \) can also be written as
\[ \pi \mathcal{R}_2(s) = \sum_{k=0}^{\infty} P_k(s), \]  
(31)

where \( P_k(s) \) is the density of the \( k \)-th nearest-neighbor spacing, \( s \). Note that since the \( \mathcal{R}_n \) are stationary and universal, so will be the \( P_k(s) \). Thus, for example, \( P_0(s) \) given for the Gaussian ensemble [11],
\[ P_0(s) = -\frac{d}{ds} \prod_{n=1}^{\infty} \left[ e_n(s^2) e^{-s^2} \right], \]  
(32)

where \( e_n(x) = \sum_{j=0}^{n} x^j / j! \), will be valid for all potentials.

**Monte Carlo simulations.** – We verify the spectral density and the two-eigenvalues correlation function numerically using Monte Carlo (MC) simulations for various potentials. In order to obtain samples of spectra from the jpd given in (2), we extend the MC method of [30] to equilibrate the particles (i.e. eigenvalues in our case) in the complex plane under the partition function with effective potential given by (9). We obtain spectral density and correlation functions for several non-Gaussian potentials discussed below. For the calculation of \( \mathcal{R}_2 \), we consider a histogram of all spacings suitably normalized as in (31). The spacing between the two eigenvalues \( z_j \) and \( z_k \) is unfolded as \( s = |z_j - z_k| \sqrt{\pi R_1((z_j + z_k)/2)} \).

**Quartic potential and its correlation functions.** – We consider the potential, \( V(|z|^2) = |z|^4 - \alpha |z|^2 \). For \( \alpha < 0 \), this gives a single-well potential whereas for \( \alpha > 0 \) it gives a double-well potential. We find, by using (13),
\[ R_1(|z|) = \frac{N}{\pi} \left( 4|z|^2 - \alpha \right), \]  
(33)

where \( a \) and \( b \) are the inner and outer radii, respectively, which are obtained by conditions (14) and (15). We get

![Fig. 1: (a) Plot of eigenvalues in the complex plane, (b) spectral density, \( R_1(|z|) \), (c) two-eigenvalue correlation, \( \mathcal{R}_2(s) \), for the quartic potential with various values of \( \alpha \) and \( N = 500 \).](image1)

\[ a = (\alpha/2)^{1/2} \alpha \geq 0 \text{ and } 0 \text{ for } \alpha < 0. \] Similarly \( b = (\alpha/4 + (1/2 + \alpha^2/16)^{1/2})^{1/2} \). Note that there is a disk-to-ring transition at \( \alpha = 0 \). The complex eigenvalues obtained by MC simulations and spectral density for various values of \( \alpha \) are shown in (c) and (d), respectively. \( N = 500 \) in both cases.

![Fig. 2: (a) Eigenvalue plot and (b) spectral density for \( V(|z|^2) = |z|^2 - \alpha \log(|z|^2) \), and similar plots for truncated log potential, \( V = -3 \log(1-|z|^2) \), are shown in (c) and (d), respectively.](image2)

\[ a = (\alpha/2)^{1/2} \alpha \geq 0 \text{ and } 0 \text{ for } \alpha < 0. \] Similarly \( b = (\alpha/4 + (1/2 + \alpha^2/16)^{1/2})^{1/2} \). Note that there is a disk-to-ring transition at \( \alpha = 0 \). The complex eigenvalues obtained by MC simulations and spectral density for various values of \( \alpha \) are shown in fig. 1(a), (b). Note that the spectral density is not constant. The two-eigenvalue correlation is shown in fig. 1(c) and is consistent with (30).

**Logarithmic potential.** – The disk-to-ring transition is also obtained for the logarithmic potential, viz., \( V(|z|^2) = |z|^2 - \alpha \log(|z|^2) \), for \( \alpha \geq 0 \) [24]. The eigenvalue density is \( R_1(|z|) = N/\pi \) for \( a \leq |z| \leq b \) with \( a = \sqrt{\alpha} \) and \( b = \sqrt{\alpha + 1} \). See fig. 2 for MC verification of \( R_1 \). \( \mathcal{R}_2(s) \) is similar to the case shown in fig. 1(c).

**Truncated log potential.** – In [18] the jpd, spectral density and the two-eigenvalue correlation function for truncated unitary matrices are studied. The jpd
in fig. 3(a), there are three rings for $\gamma$ Eigenvalue plot in the complex plane is shown in fig. 3. \( N \) \textit{|} of size \( N \) result is shown for $\gamma$ eigenvalues are distributed uniformly (outermost ring. This is seen in fig. 3(b) and supports distribution is the same as in (2) with the potential, $V$ − \( \sum_{i=-N}^{N} \exp \left[ -i (p + \gamma)^2 / 2h \right] \) with \( p, \gamma \) the position and momentum operators, respectively. Here \( \kappa \) is the kicking parameter, \( \phi \) is the parity breaking parameter and $\gamma$ is the time-reversal breaking parameter. For large $\gamma$, the corresponding classical system displays chaos and the quantum system follows the circular ensemble models [31–33].

For dissipation, we consider the evolution operator $F(\alpha) = BGD$, where $D \equiv D(\alpha) = \exp [-\alpha p^2 / 2h]$, and $\alpha > 0$ is the dissipation parameter. The matrix elements of $F(0)$ in the position representation are given in [31–33]. We obtain similarly the matrix elements of $F(\alpha)$. We have

$$ F_{mn}(\alpha) = \frac{1}{N} \exp \left[ -i \frac{\kappa}{\hbar} \cos \left( \frac{2\pi m}{N} + \phi \right) \right] \times \sum_{i=-N}^{N} \exp \left[ -i \left( \frac{\hbar}{2} \alpha t^2 - \gamma l + 2\pi \left( \frac{m - n}{N} \right) \right) \right], \quad (34) $$

where $m, n = -N', -N' + 1, \ldots, N', N' = (N - 1)/2$. We set $\hbar = 1$. If there is no dissipation (i.e. $\alpha = 0$), the evolution operator is unitary (i.e. $F(0) = U$) and eigenvalues lie on the circle of the unit radius centered at the origin. In the presence of dissipation, the operator is no longer unitary and eigenvalues start coming inside the circle to constitute a ring-like structure. We numerically analyze the spectrum. We take $\phi = \pi / 2N$ and $\gamma = 0.7$ to ensure that the parity and time-reversal symmetries are broken and the circular unitary ensemble (CUE) results are obtained for $\alpha = 0$. The eigenvalue plot, two-eigenvalue correlation function and the spacing distribution are shown in fig. 6. The two-eigenvalue correlation function and spacing distribution exhibit the universality of (30) and (32), respectively. This is consistent with the universalities obtained from dissipative quantum kicked top [11].

We propose a random matrix model for the above dissipative system. We define the evolution operator, $F$, as $F(\eta) = F(0) e^{-\eta M^2}$. Here $F(0)$ is a CUE-type matrix and represents, as mentioned above, the matrix model for the unitary evolution operator $F(0)$. $M$ is a Gaussian unitary ensemble (GUE)-type matrix and represents the momentum operator $p$. It is shown in [32,33] that
ensembles. We study the kicked rotor and its correlation functions for such ensembles. We illustrate our used the MC method for generating the eigenvalues and central density for $F$ support.

We have also shown that the $n$-eigenvalue correlation functions are stationary and universal after unfolding. We have used the MC method for generating the eigenvalues and correlation functions for such ensembles. We illustrate our results by MC calculations for the quartic, cosine and logarithmic potentials as well as for the Guassian case with arbitrary boundaries. We study the kicked rotor and its equivalent random matrix model and find the same universal properties that have been observed in the above ensembles.

Summary. – We have considered non-Gaussian random normal matrix ensembles. We have derived analytic expressions for spectral density for arbitrary weights and proved the single-ring theorem which guarantees the existence of a single ring or disk patterns of eigenvalues. We have also shown that the $n$-eigenvalue correlation functions are stationary and universal after unfolding. We have used the MC method for generating the eigenvalues and correlation functions for such ensembles. We illustrate our results by MC calculations for the quartic, cosine and logarithmic potentials as well as for the Guassian case with arbitrary boundaries. We study the kicked rotor and its equivalent random matrix model and find the same universal properties that have been observed in the above ensembles.

**

RP thanks Ashwani Kumar Tripathi and Dushyant Kumar for useful discussion and CSIR, India for financial support.

REFERENCES

[1] Brody T. A., Flores J., French J. B., Mello P. A., Pandey A. and Wong S. S. M., Rev. Mod. Phys., 53 (1981) 385.