Anti-Zeno quantum advantage in fast-driven heat machines

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Developing quantum machines which can outperform their classical counterparts, thereby achieving quantum supremacy or quantum advantage, is a major aim of the current research on quantum thermodynamics and quantum technologies. Here, we show that a fast-modulated cyclic quantum heat machine operating in the non-Markovian regime can lead to significant heat current and power boosts induced by the anti-Zeno effect. Such boosts signify a quantum advantage over almost all heat machines proposed thus far that operate in the conventional Markovian regime, where the quantumness of the system-bath interaction plays no role. The present effect owes its origin to the time-energy uncertainty relation in quantum mechanics, which may result in enhanced system-bath energy exchange for modulation periods shorter than the bath correlation-time.
The non-equilibrium thermodynamic description of heat machines consisting of quantum systems coupled to heat baths is almost exclusively based on the Markovian approximation\(^2\). This approximation allows for monotonic convergence of the system-state to thermal equilibrium with its environment (bath) and yields a universal bound on entropy change (production) in the system\(^3\). Yet, the Markovian approximation is not required for the derivation of the Carnot bound on the efficiency of a cyclic two-bath heat engine (HE): this bound follows from the second law of thermodynamics, under the condition of zero entropy change over a cycle by the working fluid (WF), in both classical and quantum scenarios. In general, the question whether non-Markovianity is an asset remains open, although several works have ventured into the non-Markovian domain\(^4\). This approximation allows for monotonic bound on the efficiency of the machine. The mechanisms that can cause such a raise include either a conversion of atomic coherence and entanglement in the bath into WF heatup\(^10\),\(^14\),\(^15\), or the ability of a squeezed bath to exchange ergotropy\(^11\),\(^13\) (alias non-passivity or work-capacity\(^17\)–\(^19\)) with the WF, which is incompatible with a standard HE. However, neither of these mechanisms is exclusively quantum; both may have classical counterparts\(^20\). Likewise, quantum coherent or squeezed driving of the system acting as a WF or a piston\(^21\) may boost the power output of the machine depending on the ergotropy of the system-state, but not on its non-classicality\(^13\).

Finding quantum advantages in machine performance relative to their classical counterparts has been one of the major aims of research in the field of quantum technology in general\(^22\)–\(^24\), and particularly in thermodynamics of quantum systems\(^25\). Overall, the foregoing research leads to the conclusion that conventional thermodynamic description of cyclic machines based on a (two-level, multilevel or harmonic oscillator) quantum system in arbitrary two-bath settings may not be the arena for a distinct quantum advantage in machine performance\(^20\). An exception should be made for multiple identical machines that exhibit collective, quantum-entangled features\(^26\),\(^27\).

Here, we show that quantum advantage is in fact achievable in a quantum heat machine (QHM), whether a heat engine or a refrigerator, whose energy-level gap is modulated faster than what is allowed by the Markov approximation. To this end, we invoke methods of quantum system-control via frequent coherent (e.g., phase-flipping or level-modulating) operations\(^28\),\(^29\) as well as their incoherent counterparts (e.g., projective measurements or noise-induced dephasing)\(^30\)–\(^34\). Such control has previously been shown, both theoretically\(^30\),\(^31\),\(^35\),\(^36\) and experimentally\(^34\),\(^37\), to yield non-Markovian dynamics that conforms to one of two universal paradigms: (i) quantum Zeno dynamics (QZD) whereby the bath effects on the system are drastically suppressed or slowed down; (ii) anti-Zeno dynamics (AZD) that implies the opposite, i.e., enhancement or speed-up of the system-bath energy exchange\(^30\),\(^31\),\(^38\). It has been previously shown that QZD leads to the heating of both the system and the bath at the expense of the system-bath correlation energy\(^39\), whereas AZD may lead to alternating cooling or heating of the system at the expense of the bath or vice-versa\(^30\),\(^31\). In our present analysis of cyclic heat machines based on quantum systems, we show that analogous effects can drastically modify the power output, without affecting their Carnot efficiency bound. AZD is shown to bring about a drastic power boost, thereby manifesting genuine quantum advantage, as it stems from the time-energy uncertainty relation of quantum mechanics.

### Results

**Model.** We consider a quantum system \(S\) that plays the role of a working fluid (WF) in a quantum thermal machine, wherein it is simultaneously coupled to cold and hot thermal baths. The system is periodically driven or perturbed with time period \(\tau_S = 2\pi/\Delta_S\) by the time-dependent Hamiltonian \(\hat{H}_S(t)\):

\[
\hat{H}_S(t + \tau_S) = \hat{H}_S(t).
\]

In order to have frictionless dynamics at all times, we choose \(\hat{H}_S(t)\) to be diagonal in the energy basis of \(S\), such that:

\[
[\hat{H}_S(t), \hat{H}_S(t')] = 0 \quad \forall t, t'.
\]

The system interacts simultaneously with the independent cold (c) and hot (h) baths via

\[
\hat{H}_I = \sum_{j=c,h} \hat{S} \otimes \hat{B}_j,
\]

where the bath operators \(\hat{B}_c\) and \(\hat{B}_h\) commute: \([\hat{B}_c, \hat{B}_h] = 0\), and \(\hat{S}\) is a system operator. For example, for a two-level system, \(\hat{S} = \sigma_x\), while \(\hat{S} = \hat{X}\) for a harmonic oscillator, in standard notations. We do not invoke the rotating wave approximation in the system-bath interaction Hamiltonian Eq. (3). As in the minimal continuous quantum heat machine\(^40\), or its multilevel extensions\(^42\), we require the two baths to have non-overlapping spectra, e.g., super-Ohmic spectra with distinct upper cutoff frequencies (see Fig. 1). This requirement allows \(\hat{S}\) to effectively couple intermittently to one or the other bath during the modulation period \(\tau_S\), without changing the interaction Hamiltonian to either bath.

**From Markovian to non-Markovian dynamics.** In what follows we assume weak system-bath coupling, consistent with the Born (but not necessarily the Markov) approximation. Our goal is to examine the dynamics as we transit from Markovian to non-Markovian time-scales, and the ensuing change of the QHM performance as the period duration \(\tau_S\) is decreased. To this end, we have adopted the methodology previously derived in refs.\(^28\),\(^29\),\(^43\),\(^44\),\(^45\),\(^46\) to account for the periodicity of \(H_S(t)\), by resorting to a Floquet expansion of the Liouville operator in the harmonics of \(\Delta_S = 2\pi/\tau_S\). As explained below, we focus on system-bath coupling durations \(\tau_C\) as the order of a few modulation periods, where \(n > 1\) denotes the number of periods. The time-scales of importance are the modulation time period \(\tau_S\), the system-bath coupling duration \(\tau_C\), the bath correlation-time \(\tau_B\) and the thermalization time \(\tau_{th} \sim \gamma_0^{-1}\).
where $\gamma_0$ is the system-bath coupling strength. We consider $n \gg 1$ such that $\tau_C \gg \tau_S$, ($\omega + q \Delta_0$)$^{-1}$, where $\omega$ denotes the transition frequencies of the system $S$, and $q$ is an integer (see Methods “Floquet Analysis of the non-Markovian Master Equation”). This allows us to implement the secular approximation, thereby averaging over the fast-rotating terms in the dynamics. In the limit of slow modulation, i.e., $\tau_S \gg \tau_B$, we have $\tau_C \gg \tau_B$, which allows us to perform the Born, Markov and secular approximations, and eventually arrive at a time-independent Markovian master equation for $\tau_C \gg \tau_S$, $\omega^{-1}$, $\tau_B$ (see Methods ”Floquet Analysis of the non-Markovian Master Equation”).

On the other hand, in the regime of fast modulation $\tau_S \ll \tau_B$, the Markov approximation becomes invalid for coupling durations $\tau_C = n \tau_S \ll \tau_B$. This gives rise to the fast-modulation form of the master equation (see Methods “Floquet Analysis of the non-Markovian Master Equation” and “Non-Markovian dynamics of a driven two-level system in a dissipative bath”):

$$\dot{\rho}_j(t) = \sum_{j'k} L_{j'k} \rho_j(t) = \sum_{j'k} \overline{I}_j(\omega,t) D_{j'k} \rho_j(t) + h.c.;$$

$$\overline{I}_j(\omega,t) \equiv \int_{-\infty}^{\infty} d\nu G_j(\nu) \left[ \frac{\sin[(\nu - \omega)t]}{\nu - \omega} \pm i \frac{\cos[(\nu - \omega)t] - 1}{\nu - \omega} \right]$$

(4)

For simplicity, unless otherwise stated, we consider $\hbar = k_B = 1$. Here, for any modulation period $\tau_S$, the generalized Liouville operators $L_j$ of the two baths act additively on the reduced density matrix $\rho_j(t)$ of $S$, generated by the $\omega$-spectral components of the Lindblad dissipators $D_{j'}$ (see below) for the $j = c, h$ bath acting on $\rho_j(t)$. For a two-level system, or an oscillator, $D$ does not depend on $\omega$. For $\rho_j(t)$ that is diagonal in the energy basis, which we consider below, the dynamics is dictated by the coefficients $I_j(\omega,t)$ (that has spectral width $~\tau_B \sim 1/\tau_B$, with the sinc function, imposed by the time-energy uncertainty relation for finite times (see Methods “Non-Markovian dynamics of a driven two-level system in a dissipative bath”).

Our main contention is that overlap between the sinc function and $G_j(\nu)$ at $t \sim \tau_C \lesssim \tau_B$ may lead to the anti-Zeno effect, i.e., to remarkable enhancement in the convolution $I_j(\omega,t)$, and, correspondingly, in the heat currents and power. One can stay in this regime of enhanced performance over many cycles, by running the QHM in the following two-stroke non-Markovian cycles:

i. Stroke 1: we run the QHM by keeping the WF (system) and the baths coupled over $n$ modulation periods, from time $t = 0$ to $t = n \tau_S = \tau_C \ll \tau_B$ ($n \gg 1$, $\tau_S \ll \tau_B$). The $n$ modulation periods of the WF are equivalent to $n$ cycles of continuous heat machines studied earlier, which have been shown to exploit spectral separation of the hot and cold baths for the extraction of work, or refrigeration, in the Markovian regime (see Eq. (8)). By contrast, in the non-Markovian domain a modulation period is not a cycle, since the time-dependent heat currents and the WF state are not necessarily reset to their initial values at the beginning of each modulation period (see below).

ii. Stroke 2: In order to reset the WF state and the heat currents to their initial ($t = 0$) values in the non-Markovian regime, we have to add another stroke: At $t = n \tau_S = \tau_C$, we decouple the WF from the hot and cold baths. One needs to keep the WF and the thermal baths uncoupled (non-interacting) for a time-interval $\tau_S \gg \tau_B$, so as to eliminate all the transient memory effects. After this decoupling period, we recouple the WF to the hot and cold thermal baths and continue to drive the WF with the periodically modulated Hamiltonian Eq. (1). Thus, the set-up is initialized after time $\tau_C + \tau_S$, provided we choose $n$ to be such that $\rho_j(\tau_C + \tau_S) = \rho_j(0)$, so as to close the steady-state cycle after $n$ modulation periods, with the WF returning to its state at start of the cycle (see Fig. 2 and section “A minimal quantum thermal machine beyond Markovianity”). The QHM may then run indefinitely in the non-Markovian cyclic regime.

By contrast, in the limit of long WF-baths coupling duration $\tau_C = n \tau_S \gg \tau_B$, the sinc functions take the form of delta-functions, and therefore, as expected, the integral Eq. (4) reduces to the standard form obtained in the Markovian regime, given by

$$I_j(\omega,t) = \pi G_j(\omega) > 0.$$  

(5)

A minimal quantum thermal machine beyond Markovianity. Here, we consider as the QHM a two-level system (TLS) WF with states $|0\rangle$ and $|1\rangle$, interacting with a hot and a cold thermal bath, described by the Hamiltonian

$$\hat{H}(t) = \hat{H}_S(t) + \hat{\sigma}_x \otimes (\hat{B}_c + \hat{B}_h) + \hat{H}_B.$$  

(6)

The Pauli matrices $\hat{\sigma}_j$ ($j = x, y, z$) act on the TLS, the operator $\hat{B}_c$ ($\hat{B}_h$) acts on the cold (hot) bath, and $\hat{H}_B$ denotes the bath Hamiltonian. The resonance frequency $\omega(t)$ of the TLS is
sinusoidally modulated by the periodic-control Hamiltonian
\[ \hat{H}_S(t) = \frac{1}{2} \omega(t) \hat{\sigma}_z; \quad \sigma_z(1) = |1\rangle, \quad \sigma_z(0) = |0\rangle \]
\[ \omega(t) = \omega_0 + \lambda \Delta \sin(\Delta t), \tag{7} \]
where the relative modulation amplitude is small: \(0 < \lambda \ll 1\). The periodic modulation Eq. (7) gives rise to Floquet sidebands (denoted by the index \(q = 0, \pm 1, \pm 2, \ldots\)) with frequencies \(\omega_q = (\omega_0 + q \Delta)\) and weights \(P_q\), which diminish rapidly with increasing \(|q|\) for small \(\lambda\) (see Methods “Non-Markovian dynamics of a driven two-level system in a dissipative bath”).

A crucial condition of our treatment is the choice of spectral separation of the hot and cold baths, such that the positive sidebands \((q > 0)\) only couple to the hot bath and the negative sidebands \((q < 0)\) only couple to the cold bath. This requirement is satisfied, for example, by the following bath spectral functions:
\[ G_h(\omega) = 0 \quad \text{for} \quad 0 < \omega \leq \omega_0, \]
\[ G_c(\omega) = 0 \quad \text{for} \quad \omega \geq \omega_0, \tag{8} \]
which ensures that for small \(\lambda\), only the \(q = 1\) harmonic exchanges energy with the hot bath at frequencies \(\pm \omega_1 = \pm (\omega_0 + \Delta)\), while the \(q = -1\) harmonic does the same with the cold bath at frequencies \(\pm \omega_1 = \pm (\omega_0 - \Delta)\). We neglect the contribution of the higher order sidebands \((|q| > 1)\) for \(0 < \lambda < 1\), for which \(P_q \approx 0\). Further, we impose the Kubo-Martin-Schwinger (KMS) detailed-balance condition
\[ G_j(-\omega) = G_\omega(\omega) \exp(-\beta \omega), \tag{9} \]
where \(\beta = 1/T\).

For simplicity, in what follows, \(G_h(\omega)\) and \(G_c(\omega)\) are assumed to be mutually symmetric around \(\omega_0\), i.e., they satisfy
\[ G_h(\omega_0 + \nu) = a G_\omega(\omega_0 - \nu), \tag{10} \]
where \(a\) is a real-positive number and \(0 \leq \nu < \omega_0\) (see Methods “steady states in the anti-Zeno dynamics (AZD) regime”).

The WF is first coupled to the thermal baths at an initial time \(t_{in} = 0\). Irrespective of the value of \(\tau_{ph}\), at large times \(t + \tau_{ph} \gg \tau_{ph}\), and under the condition of weak WF-baths coupling, one can arrive at a time-independent non-equilibrium steady-state \(\rho_S \rightarrow \rho_{ss}\) in the energy-diagonal form (see Methods “steady states in the anti-Zeno dynamics (AZD) regime”):
\[ \rho_{ss} = \rho_{ss}(1)\{1\} + P_{0,ss}\{0\}\{0\} \]
\[ P_{0,ss} = w = \frac{a e^{-\beta \omega_0 - \Delta \nu} + e^{-\beta \omega_0 + \Delta \nu}}{1 + a} \tag{11} \]
One can then decouple the WF and the baths, such that they are non-interacting for a time-interval exceeding \(\tau_{in}\) so as to eliminate all memory effects, then recouple them again at \(t = 0\), keeping \(\rho_S = \rho_{ss}\), and run the QHM in a cycle (as described in the Section “From Markovian to non-Markovian dynamics”).

In general, owing to the finite widths \((\sim 1/\tau_c)\) of \(\mathcal{I}_h(\omega_0, t)\) in the frequency domain for short coupling times \((\tau_c \ll \tau_{ph})\), the WF would be driven away from \(\rho_{ss}\), as follows from Eq. (4), causing \(\rho_S(t)\) to evolve with time within the time-interval \(0 < t \leq \tau_c\). However, in order to generate a cyclic QHM operating in the steady-state, we focus on cycles consisting of \(n\) modulation periods that satisfy
\[ \tau_c^{-1} \ll \tau_{ch}; \quad \tau_c^{-1} < \omega_0 - \Delta \nu, \tag{12} \]
so that
\[ e^{-\frac{\Delta \nu \omega_0 - \Delta \nu}{\tau_c}} \approx e^{-\frac{\Delta \nu \omega_0}{\tau_c}}. \tag{13} \]

The above conditions Eq. (12) and (13), along with the KMS condition Eq. (9), imply that
\[ \mathcal{I}_h(\omega_0 - \Delta \nu, t) \approx e^{-\frac{\Delta \nu \omega_0}{\tau_c}} \mathcal{I}_h(\omega_0 + \Delta \nu, t) \]
\[ \mathcal{I}_c(-\omega_0 - \Delta \nu, t) \approx e^{-\frac{\Delta \nu \omega_0}{\tau_c}} \mathcal{I}_c(-\omega_0 + \Delta \nu, t). \tag{14} \]
Equation (14), in turn, guarantees that Eq. (11) yields the steady-state even at short times, and thus eliminates any time dependence in \(\rho_S\) (see Fig. 2). For a QHM operating in the steady-state,
\[ \rho_S(t) = (\mathcal{L}_h + \mathcal{L}_c)[\rho_{ss}] \]
remains zero even during decoupling from, and recoupling with, the hot and cold baths. This ensures that the system remains in its steady-state \(\rho_S\) throughout the cycle.

Equations (12)–(14) can be easily satisfied for experimentally achievable parameters; e.g., \(\Delta \nu \approx \omega_0\) and \(n = 10\) would imply \(\tau_c \gg \tau_{ph}/2 \pi n \Delta \nu \approx 10^{-9}\) K. The number \(n = 10\) was chosen to be around the minimal number \(n\) that allows for the validity of the secular approximation \(\rho = q\) in Eq. (23) and hence for a simplification (Eq. (24)) in the master equation. This number should be made as low as possible, since by decreasing \(n\) we decrease the cycle duration \(\tau_c\) and hence increase the power boost, as explained above. Since this power boost is then maximized without changing the efficiency, as noted above, the performance is optimized for the chosen \(n\).

From the First Law of thermodynamics, the QHM output power \(\mathcal{W}(t)\) is given in terms of the hot and cold heat currents \(J_h(t)\) and \(J_c(t)\), respectively, by\(^{19}\)
\[ \mathcal{W}(t) = -(J_h(t) + J_c(t)). \tag{15} \]
The possible operational regimes of the heat machine, i.e., its being a heat engine or a refrigerator\(^{19,40}\), are determined by the signs of the WF-baths coupling duration-averaged \(\mathcal{I}_h, \mathcal{I}_c\), and \(\mathcal{W}\). One can calculate the steady-state efficiency \(\eta\), average power output \(\mathcal{W}\) and average heat currents \(J_j (j = h, c)\)
\[ \eta = \frac{\int_{t_c} \mathcal{W}(t) dt}{\int_{t_c} J_h(t) \, dt}, \tag{16} \]
\[ \mathcal{W} = \frac{\int_{t_c} \mathcal{W}(t) dt}{\int_{t_c} J_j(t) \, dt}; \quad J_j = \frac{1}{\tau_c} \int_{t_c} J_j(t) \, dt \]
as a function of the modulation speed \(\Delta \nu\), searching for the extrema of the functions in Eq. (16).

The heat currents \(J_h\) and \(J_c\), flowing out of the cold and hot baths, respectively, are obtained consistently with the Second Law\(^{19,40}\) in the form
\[ J_h(t) = \frac{\lambda^2}{4} (\omega_0 + \Delta \nu) \mathcal{I}_h(\omega_0 + \Delta \nu, t), \]
\[ J_c(t) = \frac{\lambda^2}{4} (\omega_0 - \Delta \nu) \mathcal{I}_c(\omega_0 - \Delta \nu, t), \tag{17} \]
where we have used \(P_{ss} = \lambda^2/4\).

In order to study the steady-state QHM performances for different modulation frequencies, we consider the example of two non-overlapping spectral response functions of the two baths displaced by \(\delta\) with respect to \(\omega_0\), i.e., \(G_h(\nu)\) \((G_c(\nu))\) characterized by a quasi-Lorentzian peak of width \(\Gamma_0\) with the peak at \(\nu_h = \omega_0 + \Delta \nu - \delta\) \((\nu_c = \omega_0 - \Delta \nu + \delta\) (see Methods “Quasi-Lorentzian bath spectral functions”). Alternatively, we also consider the example of two non-overlapping super-Ohmic spectral response functions \(G_h(\nu)\) \((G_c(\nu))\) of the two baths, with their origins shifted from \(\nu = 0\) by \(\nu_h = \omega_0 + \Delta \nu - \delta\) and \(\nu_c = \omega_0 - \Delta \nu + \delta\) respectively, for \(0 < \delta < \Delta \nu, \omega_0 - \Delta \nu\) (see
Fig. 3 Quantum advantage with quasi-Lorentzian spectral functions. The quasi-Lorentzian spectral functions of the hot bath \( G_h(\nu) \) (red filled curve) and the cold bath \( G_c(\nu) \) (blue filled curve) (see Eq. (54)), and the sinc functions \( \text{sinc}(\nu - \omega_0 - \Delta_0) t \) (black solid curve) and \( \text{sinc}(\nu - \omega_0 + \Delta_0) t \) (cyan solid curve) for (a) fast modulation \( \Delta_0 = 60\Gamma_B \) and (b) slow modulation \( \Delta_0 = 10\Gamma_B \) at \( t = 10\tau_S \). Fast (slow) modulation results in broadening (narrowing) of the sinc functions, thus leading to enhanced (reduced) overlap with the spectral functions. (c) Power \( \mathcal{W} \) (black lines) and heat currents \( \mathcal{I}_h \) (red lines) and \( \mathcal{I}_c \) (blue lines) averaged over \( n = 10 \) modulation periods (solid lines) and the same obtained under the Markovian approximation for long cycles, i.e., large number of modulation periods \( n \to \infty \) (dashed lines), versus the modulation frequency \( \Delta_0 \). Anti-Zeno dynamics for \( \tau_C \leq \tau_B \) results in output power boost (shown by dotted double-arrowed lines) by up to more than a factor of 2, signifying quantum advantage in the heat-engine regime. The green dotted line corresponds to zero power and currents. Here \( \lambda = 0.2, \omega_0 = 20, \gamma_0 = 1, \Gamma_B = 0.2, N = 1, \delta = 3, \epsilon = 0.01, \alpha = 1, \beta_h = 0.0005, \beta_i = 0.005 \).

Methods “Super-Ohmic bath spectral functions.” The dependence of \( \eta_{h,c} \) on \( \Delta_0 \) amounts to considering baths with different spectral functions for different modulation frequencies, and ensures that any enhancement in heat currents and power under fast driving results from the broadening (rather than the shift) of the sinc functions, which are centered at \( \omega_0 \pm \Delta_0 \).

We plot quasi-Lorentzian bath spectral functions and the sinc functions in Fig. 3a, b, and the corresponding time averaged heat currents and power (see Eq. (17)) for the heat engine regime in Fig. 3c. We do the same for super-Ohmic bath spectral functions in Fig. 4a–c. The corresponding heat currents and powers for the refrigerator regimes are shown in Fig. 5a, b. The Markovian approximation: \( \text{sinc}(x) \propto \delta(x) \) in Eq. (4) reproduces the correct heat currents and power only in the limit of slow modulation \( \tau_C \gg \tau_B \). By contrast, the Markovian approximation reproduces the exact efficiency for both slow and fast-modulation rates (see Fig. 6a). Thus, although
Enhanced overlap resulting from fast modulation (large $\beta$ over the frequency range $\Delta_s$) boosts whenever the sinc functions have sufficiently detuned from $\omega_0 \pm \Delta_s$ (i.e., $\delta > \Gamma_B$) may increase the overlap with the sinc functions appreciably under fast modulation in the anti-Zeno regime, for $\tau_c^{-1}, \delta \gg \Gamma_B$, thus resulting in substantial output power boost. This regime indicates that finite spectral width of the sinc functions may endow a HE with significant quantum advantage, arising from the time-energy uncertainty relation, which is absent in the classical regime, be it Markovian or non-Markovian. In the numerical examples shown here, the quantum advantage in the HEs powered by baths with quasi-Lorentzian spectral functions can increase the power by a factor larger than two (seven) (see Figs. 3a, b, and 5a), for the same efficiency (see Fig. 6a and Methods “Efficiency and coefficient of performance”).

**Quantum refrigeration.** AZD can lead to quantum advantage in the refrigerator regime as well, for modulation rates beyond the quantum speed limit\(^{42,48}\) (see Supplementary Note 1), by enhancing the heat current $J_h$, thus resulting in faster cooling of the cold bath. As for HE, numerical analysis shows that quasi-Lorentzian, as well as super-Ohmic bath spectral functions can lead to significant quantum advantage in the AZD regime (see Fig. 5). On the other hand, as for the efficiency in case of the HE,
the coefficient of performance
\[ \text{COP} = \frac{-\tau}{W} \] (18)
is not significantly affected by the broadening of the sinc function, and on average remains identical to that obtained under slow modulation in the Markovian regime (see Fig. 6b and Methods “Efficiency and coefficient of performance”).

Discussion
We have explored the hitherto uncharted domain of quantum heat engines (QHEs) and refrigerator (QRs) based on quantum working fluids (WFs) intermittently coupled and decoupled from heat baths operating on non-Markovian time-scales. We have shown that for driving (control) faster than the correlation (memory) time of the bath, one may achieve dramatic output power boost in the anti-Zeno dynamics (AZD) regime.

Let us revisit our findings, using as a benchmark the Markovian regime under periodic driving: In the latter regime, detailed-balance of transition rates between the WF levels, as well as the periodic driving (modulation) rate, determine, according to the First and Second Laws of thermodynamics, the heat currents between the (hot and cold) baths, and thereby the power produced or consumed. In our present treatment, the Markovian regime is recovered under slow modulation, such that the WF-baths coupling duration \( \tau_C \) exceeds the bath correlation-time \( \tau_B \). Then, the Markovian approximation is adequate for studying the operation of the QHE or the QR. By contrast, under fast modulations, such that \( \tau_C \approx n\tau_B \ll \tau_B \), the working fluid interacts with the baths over a broad frequency range of the order of \( \sim \tau_C^{-1} \), according to the time-energy uncertainty relation in quantum mechanics. The frequency-width over which system-bath energy exchange takes place can lead to anti-Zeno dynamics (AZD). The resultant quantum advantage is then especially pronounced for bath spectral functions that are appreciably shifted by \( \delta > \tau_B \approx \tau_B^{-1} \) from the centers of the sinc functions that govern the system-bath energy exchange rates.

We have explicitly restricted the results to mutually symmetric bath spectral functions (e.g., the experimentally common Lorentzian or Gaussian spectra), in order to ensure time-independent steady-states of the WF. Yet this requirement is not essential, since the WF steady-state may be time-dependent as long as it is periodic so as to allow for cyclic operation. The AZD28–35 can arise for any bath spectra of finite width \( \sim 1/\tau_B \), as long as \( n\tau_B \ll \tau_B \). One can therefore operate a thermal machine provided stroke 1 of the cycle is in the AZD regime and achieve a quantum advantage without additional restrictions on the bath spectral functions (see Methods “Thermal machines with arbitrary (asymmetric) spectral functions”).

The QHM discussed here is driven by external modulation. As previously shown both theoretically32–35 and experimentally34,37, periodic perturbations of the TLS state can increase its relaxation rate in the non-Markovian anti-Zeno regime. The reason for the power boost is that at the non-Markovian stage of the evolution, which occurs on short time-scales, the sinc factors in the convolutions with \( G(\omega) \), as in Eq. (32), are sufficiently broad so as to modify the convolutions and hence the relaxation rates in Eq. (34) in comparison with the Markovian case, where these sinc functions are spectrally narrow enough to be approximated by delta-functions. Under the conditions chosen in the paper, this modification leads to an increase in the TLS relaxation rates and hence to a power boost. This boost is of quantum nature, since the broadening of the sinc factors is due to the quantum time-energy uncertainty relation that may lead to the violation of energy conservation at short times. The quantum mechanical time-energy uncertainty relation employed here reflects the fact that the Schrödinger equation for a two-level system coupled to a bath renders the energy transfer probability from the two-level system to the bath and back oscillatory in time. Such oscillation leads at short times (comparable to the required cycle period) to sinc-like deviation from delta-function energy conservation. Classical description of analogous processes, even beyond the Markovian approximation, does not involve discrete energy levels and hence no oscillations of the system-bath transfer rate that deviates from energy conservation. Thus, the effects discussed here are inherently quantum mechanical.

The non-Markovian effect in the present context is quantified by the spectral widths of the sinc functions compared to the bath-response \( G(\omega) \) spectral width \( 1/\tau_B \). If the cycle duration is kept fixed, then the non-Markovian effect scales with the spectral width of \( G(\omega) \). Hence, super-Ohmic bath spectra with their salient cutoff provide realistic examples of the non-Markovian effects described here. Such bath spectra should be contrasted with the flatter and broader Ohmic spectra. Yet, non-Markovian dynamics does not necessarily imply a quantum advantage, as discussed in Supplementary Note 2.

The predicted power boost relies on transient dynamics: the heat fluxes change with time within \( t = \tau_C \) in the non-Markovian AZD regime, even when the WF state hardly changes during that time-interval. Yet it is essential that we incorporate this transient dynamics within steady-state cycles by decoupling the WF from the baths, allowing the bath-correlations to vanish within \( \tau_B \) and then recoupling the WF again to the baths when they have all resumed their initial states. These cycles can be repeated without restriction, thereby allowing us to operate the QHM with quantum-enhanced performance even for long times, despite the reliance on transient dynamics within the stroke 1 of each cycle.

The quantum advantage of AZD, at zero energetic cost (see Supplementary Note 3), manifests itself in the form of higher output power, for the same efficiency, in the QHE regime (\( \Delta_G < \Delta_{\text{sl}} \)), as compared to that obtained under Markovian dynamics in the limit of large \( \tau_C \), all other parameters remaining the same. Alternatively, in the QR regime (\( \Delta_G > \Delta_{\text{sl}} \)), AZD may lead to quantum advantage over Markovian dynamics in the form of higher heat current \( \Gamma_r \), or, equivalently, higher cooling rate of the cold bath, for the same coefficient of performance. The latter effect leads to the enticing possibility of quantum-enhanced speed-up of the cooling rate of systems as we approach the absolute zero, and raises questions regarding the validity of the Third Law of Thermodynamics in the quantum non-Markovian regime, if we expect the vanishing of the cooling rate at zero temperature as a manifestation of the Third Law47,49,50.

The QHE power boost in the anti-Zeno regime results from a corresponding increase in the rates of heat-exchange and entropy production, arising from the TLS relaxation by both baths. This is the reason that the efficiency, i.e., the ratio of the work output to the heat input, is unchanged, i.e., is the same as in the standard Markovian regime. Yet, all parameters being equal, the QHM rate of operation (as measured by the power output) speeds up in the anti-Zeno regime, which constitutes a practical quantum advantage.

One can extend the analysis discussed here to Otto cycles42,51. Fast periodic modulation during the non-unitary strokes of an Otto cycle can speed-up the thermalization through AZD, thereby allowing quantum-enhanced performance. Interestingly, fast modulation in the Otto cycle can yield enhanced power or refrigeration rate, even in the Markovian regime52.

Finally, in the regime of ultrafast modulation with \( \tau_C^{-1} \gg \Gamma_B, \delta \), quantum Zeno dynamics sets in, leading to vanishing heat currents and power, thus implying that such a regime is incompatible with thermal machine operation (see Fig. 7, Supplementary Note 2 and Supplementary Fig. 1). While Zeno dynamics has
The novel effects and performance trends of QHE and QR in the non-Markovian time domain, particularly the anti-Zeno induced power boost, open new, dynamically controlled pathways in the quest for genuine quantum features in heat machines, which has been a major motivation of quantum thermodynamics in recent years.\textsuperscript{10–13,21,23,41,64–68}

Methods
Floquet analysis of the non-Markovian master equation. Let us consider the differential non-Markovian master equation for the system density operator \( \rho_S(t) \) in the interaction picture:\textsuperscript{29}

\[
\dot{\rho}_S(t) = -\int d\tau \mathcal{T}_B \left[ \hat{S}(t) \otimes \hat{B}_B(t) + \hat{S}_B(t) \otimes \hat{B}(t), [\hat{S}(t) \otimes \hat{B}(t)] \right].
\]

(19)

Here \( \rho_B = \rho_{B_1} \otimes \rho_{B_2} \), where \( \rho_B \) is the density operator of bath \( B \). In the derivation of Eq. (19) we have assumed that \( \text{Tr}[\hat{B}(t) \rho_B] = 0 \). We consider commuting bath operators \( [\hat{B}(t), \hat{B}_B(t)] = 0 \), such that the two baths act additively in Eq. (19). Below we focus on only one of the baths and omit the labels \( c, h \) for simplicity. We then have

\[
\begin{aligned}
\dot{\bar{S}}(t) &= \bar{S}(t) \\
\dot{\bar{B}}(t) &= \bar{B}(t) \\
\text{Tr}[\hat{B}(t) \rho_B(t)] &= [\bar{B}(t) \bar{S}(t)] = \Phi(t - s) \\
\bar{S}(t) &= \sum_q \int_0^1 \Phi(t - s) e^{i[\omega_q t + \Delta_q s]} ds,
\end{aligned}
\]

(20)

where \( q \) and \( s \) are transition frequencies of the system \( S \). One can use Eq. (20) to write the first term on the r.h.s. of Eq. (19) as

\[
T_1 = -\sum_{\omega, q, s} \int_0^1 \Phi(t - s) e^{i[\omega q + \Delta_q s]/s] ds,}
\]

(21)

In the limit of times of interest, i.e., times larger than the period of driving \( \tau_4 \) and the effective periods of the system, \( t \gg \tau_4 (\omega + \Delta_0)^{-1} \), the terms with the fast oscillating factor before the integral in Eq. (21) become small and can be neglected, i.e., the secular approximation becomes applicable, such that

\[
(q' - q) \Delta_q = \omega - \omega',
\]

(22)

which generally holds only for

\[
u' = \omega; \quad q' = q.
\]

(23)

as long as \( (q' - q) \Delta_q \) is not close to \( (\omega' - \omega) \) for any \( q, q', \omega, \omega' \). Condition (23) gives us

\[
T_1 \approx -\sum_{\omega, q} \int_0^1 \Phi(t - s) e^{i[\omega q + \Delta_q s]/s] ds,}
\]

(24)

In the limit of slow modulation, such that \( t \approx n_\tau \gg \tau_4 \), one can perform the Markovian approximation, thereby extending the upper limit of the integral in time in Eq. (24) to \( t \rightarrow \infty \), which finally results in the time-independent Markovian form\textsuperscript{34}

\[
T_1 \approx -\pi \sum_{\omega, q} \Delta_q \Phi(q) G(\omega, q).
\]

(25)

On the other hand, in the limit of \( t \approx n_\tau \ll \tau_4 \), the Markovian approximation becomes invalid, and one gets

\[
T_1 \approx -\sum_{\omega, q} \Delta_q \Phi(q) \int_{-\infty}^{\infty} d\nu \Phi(\nu) e^{i[\omega q + \Delta_q s]/s] ds,}
\]

(26)

Progressing similarly as above, one can arrive at similar expressions for other terms in Eq. (19) as well.
The imaginary part in Eq. (31) acts on terms of the form
\[ j(\pm \sigma^\dagger \omega s) \]
and experimentally verified in refs. 32–34. These MEs for the populations (probabilities) \( p_\sigma(\tau) \) and \( \rho_\tau(\tau) \) of the TLS levels are not guaranteed to be non-negative, i.e., to satisfy
\[ 0 \leq \rho_\sigma(\tau) \leq 1 \]
Below we show that the inequalities (Eq. (35)) hold, at least, up to second order in the system-bath coupling strength. This means that for a weak coupling, violations of (Eq. (35)) if any, are negligibly small.
First, we note that sufficiently long times, \( t \gg T_e \), the MEs become Markovian and coincide with the Lindblad equation. In this case, the rates are constant and positive, \( R_1, R_2, T_e > 0 \), as follows from Eq. (33). The inequalities Eq. (35) are now known to hold. Generally, the MEs are valid if the couplings of the TLS with the baths are sufficiently weak, so that
\[ R_{1e} \ll 1, \quad R_{2e} \ll 1. \]
Consider now the short times, \( t \lesssim T_e \), where the non-Markovian effects are important. Since \( p_\sigma(\tau) + p_\tau(\tau) = 1 \), we rewrite
\[ p_\sigma(\tau) = \frac{1 - w(\tau)}{2}, \quad p_\tau(\tau) = \frac{1 + w(\tau)}{2} \]
where \( w(\tau) = p_\sigma(\tau) - p_\tau(\tau) \) is the TLS population inversion. In terms of \( w(\tau) \), inequities Eq. (35) are equivalent to
\[ -1 \leq w(\tau) \leq 1, \]
which we now prove.
The condition Eq. (36) implies that at times \( t \lesssim T_e \), the relaxation can be approximated to first order in the relaxation rates. In this approximation, Eq. (34) yields
\[ w(\tau) = w(0)[1 - J_s(\tau)] + J_s(\tau), \]
where
\[ J_s(\tau) = J_s(\tau) \]
and
\[ J_s(\tau) = \int_0^\tau dr R_s(\tau) \]
which we now prove.
From (32) and (34), one can check that
\[ J_s(\tau) \geq 0. \]
From (39) we obtain
\[ |w(\tau)| \leq |w(0)|[1 - J_s(\tau)] + |J_s(\tau)| \]
yielding Eq. (38). The second inequality in Eq. (43) follows from the assumption \( |w(0)| \leq 1 \) and the relation \( |J_s(\tau)| \leq J_s(\tau) \), resulting from Eqs. (40) and (42).

Steady states in the anti-Zeno dynamics regime. Now, we study the regimes that allow us to operate the set-up with a time-independent steady-state \( \rho_\tau \), even inside the AZD regime. We note that for \( t \to \infty, J_s(\omega T_e) \) reduces to the time-independent form \( \pi G_1(\omega) \), thus leading us to Eq. (11). On the other hand, for \( t \lesssim T_e^0, J_s(\omega T_e) \) includes contributions from \( G_1(\omega), \omega \approx \omega \), where
\[ |\omega| \leq 1/t \Rightarrow 1/(\pi T_e). \]
Further, we consider \( \omega_0, T_e, T_{\text{AZ}}, \delta_\omega \), large enough, such that \( 1/t < \omega_0 \delta_\omega \). Therefore, in this limit the KMS condition gives us
\[ G_1\left(-\left(\omega_0 + \nu \right)\right) \approx e^{-\omega_0/\delta_\omega} G_1(\omega_0 + \nu) \]
\[ \approx e^{-\nu/\delta_\omega} G_1(\omega_0 + \nu). \]
This immediately leads us to
\[ I_x(-\omega_c, t) \approx e^{-\alpha \psi(t)} I_x(\omega_c, t), \] (46)
and consequently (see Eq. (11))
\[ w \approx e^{-(\alpha + \beta) \psi(t)} I_x(\omega_c + \Delta_c, t) + e^{-(\alpha - \beta) \psi(t)} I_x(\omega_c - \Delta_c, t), \] (47)
where we have considered the two-sided cases \( q = 1, -1 \) only.

The condition
\[ I_x(\omega_c + \Delta_c, t) \approx e^{-\alpha \psi(t)} I_x(\omega_c - \Delta_c, t), \] (48)
which holds for mutually symmetric bath spectral functions up to a multiple factor \( G_\beta(\omega + \chi) \propto \omega G_\beta(\omega - \chi) \) for any real \( \chi \) and positive \( \alpha \) (see Supplementary Fig. 2), leads to the time-independent steady-state \( \rho_{0s} \) with (see Supplementary (11))
\[ w \approx \frac{a e^{-(\alpha + \beta) \psi(t)} + e^{-(\alpha - \beta) \psi(t)}}{\alpha + 1}. \] (49)

Efficiency and coefficient of performance. The efficiency in the heat engine regime is given by
\[ \eta = \frac{\frac{\int \omega h_x(t) \, dt}{\int h_x(t) \, dt}}{\xi_c} = \frac{\omega h_x(t) \psi(t) + \omega h_x(t) \psi(t)}{\omega h_x(t) \psi(t) + \omega h_x(t) \psi(t)}, \] (50)
where \( \xi_c = \frac{e^{-(\alpha + \beta) \psi(t)} - w}{\alpha + 1} \)
One can get the results of the Markovian (\( \tau_c \rightarrow \infty \)) limit by replacing \( I_x(-\omega_c, t) \) by \( G_x(\omega_c) \).

Let us consider the integral:
\[ \int I_x(\omega_c + \Delta_c, t) \int I_x(\omega_c - \Delta_c, t) \approx e^{-\alpha \psi(t)} I_x(\omega_c - \Delta_c, t) \] (51)
where \( \omega h_x(t) \) defines the variable \( x = \nu - (\omega_c + \Delta_c) \), and taken into account that \( G_\beta(\nu) \approx 0 \) for \( 0 < \nu < \omega_0 \) (see Fig. 8), and \( \sin(\nu x)/\nu \) is small for large \( |x| \).

Similarly, we have
\[ I_x(\omega_c - \Delta_c, t) \approx e^{-\alpha \psi(t)} I_x(\omega_c + \Delta_c, t), \] (52)
where \( y = (\omega_c - \Delta_c - \nu) \), and we have taken into account that \( G_\beta(\nu) \approx 0 \) for \( \nu \geq \omega_0 \) (Fig. 8), and \( \sin(y)/\nu \) is large for small \( |y| \).

Clearly, for bath spectral functions related by Eq. (10), we have
\[ I_x(-\omega_c + \Delta_c, t) \approx e^{-\alpha \psi(t)} I_x(-\omega_c + \Delta_c, t), \] (53)
which in turn results in the efficiency and the coefficient of performance in the non-Markovian anti-Zeno dynamics regime being approximately equal to those in the Markovian dynamics regime (see Fig. 6).

Quasi-Lorentzian bath spectral functions. We focus on baths characterized by the spectral functions:
\[ G_\beta(\nu) \approx \begin{cases} \frac{a}{(\nu - \omega_0 + \Delta_c)^2 + \beta^2} & \nu < \omega_0 + \Delta, \\ \frac{b}{(\nu - \omega_0 - \Delta_c)^2 + \beta^2} & \nu > \omega_0 - \Delta, \end{cases} \] (54)
where \( \Delta \) and \( \beta \) are the characteristic widths of the 1-th peak. \( \delta_\beta \) are the (real) Lamb self energy shifts, such that \( G_\beta(\Omega) \) is peaked at \( \nu - \omega_0 + \Delta + \delta_\beta, (\nu - \omega_0 - \Delta - \delta_\beta) \).

As seen from Eq. (53), we consider bath spectral functions with different resonance frequencies (\( \omega_0 = \omega_0 \pm \Delta_0 \pm \delta_\beta \)) for different modulation rates \( \Delta_0 \).

One can also still operate the set-up as a cyclic thermal machine for a time \( t \leq I \), as long as

\[ \theta(\nu - \omega_0 - \Delta_c) \approx \begin{cases} \frac{a}{(\nu - \omega_0 + \Delta_c)^2 + \beta^2} & \nu < \omega_0 + \Delta, \\ \frac{b}{(\nu - \omega_0 - \Delta_c)^2 + \beta^2} & \nu > \omega_0 - \Delta, \end{cases} \] (55)

where \( \Delta \) and \( \beta \) are the characteristic widths of the 1-th peak. \( \delta_\beta \) are the (real) Lamb self energy shifts, such that \( G_\beta(\Omega) \) is peaked at \( \nu - \omega_0 + \Delta + \delta_\beta, (\nu - \omega_0 - \Delta - \delta_\beta) \).

Similar to Eq. (53), we consider bath spectral functions with different resonance frequencies (\( \omega_0 = \omega_0 \pm \Delta_0 \pm \delta_\beta \)) for different modulation rates \( \Delta_0 \).

For the single-peaked case \( N = 1 \), the above functions Eq. (53) reduce to quasi-Lorentzian spectral functions of the form
\[ G_\beta(\nu) \approx \begin{cases} \frac{a}{(\nu - \omega_0 + \Delta)^2 + \beta^2} & \nu < \omega_0 + \Delta, \\ \frac{b}{(\nu - \omega_0 - \Delta)^2 + \beta^2} & \nu > \omega_0 - \Delta, \end{cases} \] (56)
where \( \theta(\nu - \omega_0 - \Delta) \approx \frac{a}{(\nu - \omega_0 + \Delta)^2 + \beta^2} \) with the origin shifted from \( \nu = 0 \) by \( \nu_0 = \omega_0 + \Delta_c - \delta \) \( \nu_0 = \omega_0 - \Delta_c + \delta \) (57)

Similarly, we have
\[ G_\beta(\nu) \approx \begin{cases} \frac{a}{(\nu - \omega_0 + \Delta)^2 + \beta^2} & \nu < \omega_0 + \Delta, \\ \frac{b}{(\nu - \omega_0 - \Delta)^2 + \beta^2} & \nu > \omega_0 - \Delta, \end{cases} \] (58)
where, as before, \( \alpha > 0 \) and \( \chi(\nu) \) is the ambient real function of \( \nu \). We then have
\[ \int I_x(\omega_c + \Delta_c, t) \approx e^{-\alpha \psi(t)} I_x(\omega_c + \Delta_c, t), \] (59)
and
\[ \int I_x(\omega_c - \Delta_c, t) \approx e^{-\alpha \psi(t)} I_x(\omega_c - \Delta_c, t), \] (60)
In this case, we get a time-dependent steady-state with
\[ w(t) \approx e^{-(\alpha + \beta) \psi(t)} \left[ a I_x(\omega_c - \Delta_c, t) + \frac{\chi(t)}{(\alpha + 1) I_x(\omega_c - \Delta_c, t) + \chi(t)} \right] \] (61)
Therefore, the rate of change of \( w(t) \) with time is given by
\[ w(t) = \frac{\chi(t)I_x(\omega_c - \Delta_c, t) - \chi(t)I_x(\omega_c - \Delta_c, t)}{[\chi(t)(\omega_c - \Delta_c, t) + \chi(t)]} \] (62)
One can still operate the set-up as a cyclic thermal machine for a time \( t \leq I \), as long as

\[ \int I_x(\omega_c + \Delta_c, t) \approx e^{-\alpha \psi(t)} I_x(\omega_c + \Delta_c, t), \] (63)
and
\[ \int I_x(\omega_c - \Delta_c, t) \approx e^{-\alpha \psi(t)} I_x(\omega_c - \Delta_c, t), \] (64)
where \( \theta(\nu - \omega_0 - \Delta) \approx \frac{a}{(\nu - \omega_0 + \Delta)^2 + \beta^2} \) with the origin shifted from \( \nu = 0 \) by \( \nu_0 = \omega_0 + \Delta_c - \delta \) \( \nu_0 = \omega_0 - \Delta_c + \delta \) (57)

Similarly, we have
\[ G_\beta(\nu) \approx \begin{cases} \frac{a}{(\nu - \omega_0 + \Delta)^2 + \beta^2} & \nu < \omega_0 + \Delta, \\ \frac{b}{(\nu - \omega_0 - \Delta)^2 + \beta^2} & \nu > \omega_0 - \Delta, \end{cases} \] (58)
where, as before, \( \alpha > 0 \) and \( \chi(\nu) \) is an ambient real function of \( \nu \). We then have
\[ \int I_x(\omega_c + \Delta_c, t) \approx e^{-\alpha \psi(t)} I_x(\omega_c + \Delta_c, t), \] (59)
and
\[ \int I_x(\omega_c - \Delta_c, t) \approx e^{-\alpha \psi(t)} I_x(\omega_c - \Delta_c, t), \] (60)
In this case, we get a time-dependent steady-state with
\[ w(t) \approx e^{-(\alpha + \beta) \psi(t)} \left[ a I_x(\omega_c - \Delta_c, t) + \frac{\chi(t)}{(\alpha + 1) I_x(\omega_c - \Delta_c, t) + \chi(t)} \right] \] (61)
Therefore, the rate of change of \( w(t) \) with time is given by
\[ w(t) = \frac{\chi(t)I_x(\omega_c - \Delta_c, t) - \chi(t)I_x(\omega_c - \Delta_c, t)}{[\chi(t)(\omega_c - \Delta_c, t) + \chi(t)]} \] (62)
One can still operate the set-up as a cyclic thermal machine for a time \( t \leq I \), as long as
as \( \dot{y}(t) \) and \( \dot{\dot{y}}(t) \) are small enough so as to ensure

\[
\dot{w}_{\text{max}} \ll \bar{I}^{-1}
\]  

where \( \dot{w}_{\text{max}} \) is the maximum value attained by \( |\dot{w}(t)| \) in the time-interval \( 0 \leq t \leq T \).

**Data availability**

All relevant data are available to any reader upon reasonable request.

**Code availability**

All relevant codes are available to any reader upon reasonable request.

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Author contributions
G.K. and V.M. conceived the idea. V.M. and A.G.K. performed the analytical calculations. V.M. did the numerical simulations. All authors contributed to the interpretations of the results and to the writing of the manuscript.

Competing interests
The authors declare no competing interests.

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