Common field-induced quantum critical point in high-temperature superconductors and heavy-fermion metals

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High-temperature superconductors (HTSC) and heavy-fermion (HF) metals exhibit extraordinary properties. They are so unusual that the traditional Landau paradigm of quasiparticles does not apply. It is widely believed that utterly new concepts are required to describe the underlying physics. There is a fundamental question: how many concepts do we need to describe the above physical mechanisms? This cannot be answered on purely experimental or theoretical grounds. Rather, we have to use both of them. Recently, in HTSC, the new and exciting measurements have been performed, demonstrating a puzzling magnetic field induced transition from non-Fermi liquid to Landau Fermi liquid behavior. We show, that in spite of very different microscopic nature of HTSC and HF metals, the behavior of HTSC is similar to that observed in HF compounds. We employ a theory, based on fermion condensation quantum phase transition which is able to resolve the above puzzles.

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The non-Fermi liquid (NFL) behavior of many classes of strongly correlated fermion systems pose one of the tremendous challenges in modern condensed matter physics. Many puzzling and common experimental features of such seemingly different systems as two-dimensional (2D) electron systems and liquid 3He, heavy-fermion (HF) metals and high-temperature superconductors (HTSC) suggest that there is a hidden fundamental law of nature, which remains to be recognized. The key word here is quantum criticality, taking place in quantum critical point (QCP).

Heavy fermion metals provide important examples of strongly correlated Fermi-systems. The second class of substances to test whether or not the Landau Fermi liquid (LFL) theory is fulfilled in them, are HTSC. In these substances, all quantum critical points are almost inaccessible to experimental observations since they are "hidden in superconductivity" or more precisely, the superconductive gap opened at the Fermi level changes the physical properties of corresponding quantum phase transition.

There is a common wisdom that the physical properties of above systems are related to zero temperature quantum fluctuations, suppressing quasiparticles and thus generating their NFL properties, depending on their initial ground state, either magnetic or superconductive. On the other hand, it was shown that the electronic system of HF metals demonstrates the universal low-temperature behavior irrespectively of their magnetic ground state. Recently, the NFL behavior has been discovered experimentally in 2D 3He, Ref. 6, and the theoretical explanation has been given to it,1 revealing the similarity in physical properties of 2D 3He and HF metals. We note here that 3He consists of neutral atoms interacting via van der Waals forces, while the mass of He atom is 3 orders of magnitude larger than that of an electron, making 3He to have drastically different microscopic properties than those of HF metals. Therefore it is of crucial importance to check whether this behavior can be observed in other Fermi systems like HTSC. Recently, precise measurements on HTSC TlBa2CuO6+δ of magnetic field induced transition from NFL to LFL behavior become available.8 This transition takes place under the application of magnetic field B ≥ Bc, where Bc is the critical field at which the magnetic field induced CQP takes place.

Here we pay attention that to study the aforementioned CQP experimentally, the strong magnetic fields of B ≥ Bc are required so that earlier such investigation was technically inaccessible. Here Bc is the critical magnetic field destroying the superconductivity. We note also that an attempt to study the aforementioned CQP experimentally had been done more then 10 years ago.

In our paper, we show, that in spite of very different microscopic nature of HTSC and HF metals, the behavior of HTSC is similar to that observed in HF compounds. We employ a theory, based on fermion condensation quantum phase transition (FCQPT), which is able to demonstrate that the physics underlying the field-induced reentrance of LFL behavior, is the same for both HTSC and HF metals. We demonstrate that there is at least one quantum phase transition inside the superconducting dome, and this transition is FCQPT. We also show that there is a relationship between the critical fields Bc and Bc so that Bc ≥ Bc.

We have shown earlier (see, e.g., Ref. 13) that without loss of generality, to study the above universal behavior, it is sufficient to use the simplest possible model of a homogeneous heavy-electron (fermion) liquid. This permits not only to better reveal the physical nature of observed effects, but to avoid unnecessary complications.
related to microscopic features (like crystalline structure, defects and impurities etc) of specific substances.

We consider HF liquid at \( T = 0 \) characterized by the effective mass \( M^* \). Upon applying well-known Landau equation (see Appendix sections for details), we can relate \( M^* \) to the bare electron mass \( M \) \(^{14,15}\)

\[
\frac{M^*}{M} = \frac{1}{1 - N_0 F^3(x)/3}.
\]

Here \( N_0 \) is the density of states of a free electron gas, \( x = p_F^3/3\pi^2 \) is a number density, \( p_F \) is Fermi momentum, and \( F^3(x) \) is the \( p \)-wave component of Landau interaction amplitude \( F \). When at some critical point \( x = x_c, F^3(x) \) achieves certain threshold value, the denominator in Eq. 1 tends to zero so that the effective mass diverges at \( T = 0 \) and the system undergoes FCQPT. The leading term of this divergence reads

\[
\frac{M^*(x)}{M} = \alpha_1 + \frac{\alpha_2}{x - x_c},
\]

where \( \alpha_1 \) and \( \alpha_2 \) are constants. At \( x > x_c \), the FC takes place. The essence of this phenomenon is that at \( x > x_c \), the effective mass \( M^* \) becomes negative signifying the physically meaningless state. To avoid this state, the system reconstructs its quasiparticle occupation number \( n(p) \) and topological structure so as to minimize its ground state energy \( E \) \(^{10,11,12,16,17} \)

\[
\frac{\delta E}{\delta n(p)} = \mu,
\]

where \( \mu \) is a chemical potential. The main result of such reconstruction is that instead of Fermi step, we have \( 0 \leq n(p) \leq 1 \) in certain range of momenta \( p_i \leq p \leq p_f \). Accordingly, in the above momenta interval, the spectrum \( \varepsilon(p) = \mu \), see Fig. 1 for details of its modification.

Due to above peculiarities of the \( n(p) \) function, FC state is characterized by the superconducting order parameter \( \kappa(p) = \sqrt{n(p)(1 - n(p))} \). This means that if the electron system with FC has pairing interaction with coupling constant \( \lambda \), it exhibits superconductivity since the superconducting gap \( \Delta \propto \lambda \) in a weak coupling limit. This linear dependence is also a peculiarity of FC state and substitutes well-known BCS relation \( \Delta \propto \exp(-1/\lambda) \), see e.g. Ref \(^{18} \) for the systems with FC \(^{10,11,12,13,15,18} \).

Assume now that \( \lambda \) is infinitely small. In that case, any weak magnetic field \( B \) is critical and destroys both \( \kappa(p) \) and FC state. Simple energy arguments suffice to determine the type of FC state rearrangement. On one hand, since the FC state is destroyed, the gain in energy \( \Delta E_B \propto B^2 \) tends to zero as \( B \to 0 \). On the other hand, the function \( n(p) \), occupying the finite interval \( (p_f - p_i) \) in the momentum space, yields a finite gain in the ground-state energy compared to that of a normal Fermi liquid. Such a state is formed by multiply connected Fermi spheres resembling an onion \(^{20} \), see Appendix section. In this state the system demonstrates

\[
\varepsilon(p) = \mu - \frac{1}{\sqrt{B - B_c}}.
\]

Here \( B_c \) is the critical magnetic field driving corresponding QCP towards \( T = 0 \). In some cases, for example in HF metal CeRu\(_2\)Si\(_2\), \( B_c = 0 \), see e.g. Ref \(^{22} \).

At elevated temperatures, (see Appendix section, eq. \(^{30} \)), the system transits from the LFL to NFL regime exhibiting the low-temperature universal behavior independent of its magnetic ground state, composition, dimensionality (2D or 3D) and even nature of constituent Fermi particles which may be electrons or \(^3\)He atoms \(^{5,7} \). To check, whether the quasiparticles are present in the systems in the transition regime, we use the results of measurements of heat capacity \( C \), entropy \( S \) and magnetic susceptibility \( \chi \). If these results can be fitted by the well-known relations from Fermi liquid theory \( C/T = \gamma_0 \propto S/T \propto \chi \propto M^* \), then quasiparticles define the system properties in the transition regime.

As it follows from equation \(^{30} \), \( M^* \) reaches the maximum \( M^*_M \) at some temperature \( T_M \). Since there is no external physical scales near FCQPT point, the normalization of both \( M^* \) and \( T \) by internal parameters \( M^*_M \) and \( T_M \) immediately reveals the common physical nature of above thermodynamic functions which we use to extract the effective mass. The normalized effective mass extracted from measurements on the HF metals \( \mathrm{YbRh_2(SiO_5Ge_0.5)_3} \), \( \mathrm{CeRu_2Si_2} \), \( \mathrm{CePd_1-Rh_x} \), \( \mathrm{CeNi_2Ge_2} \) and \( \mathrm{^3\text{He}} \) along with our theoretical solid curve (also shown in the inset) is reported in Fig. 2. It is seen that above normalization of experimental data yields the merging of multiple curves into single one, thus demonstrating a universal scaling behavior \(^{5,7,29} \).
outlines the transition regime. Several regions are shown as mass given by our theoretical curve agrees well with experimental data. It is also seen that the universal behavior of the effective mass at $T = T_N$ is in good agreement with experiments. As it is seen from Fig. 3 that the $T_N$ point can be regarded as a crossover between LFL and NFL regimes. Since magnetic field enters the Landau equation as $\mu_B B/T$ (see Appendix section), we have

$$T^* (B) = a_1 + a_2 B \simeq T_M \sim \mu_B (B - B_c_0),$$

where $T^* (B)$ is the crossover temperature, $\mu_B$ is Bohr magneton, $a_1$ and $a_2$ are constants. In our simple model $B_c_0$ is taken as a parameter. The crossover temperature is not really a phase transition. It necessarily is broad, very much depending on the criteria for determination of the point of such a crossover, as it is seen from the inset to Fig. 4. As usually, the temperature $T^* (B)$ is extracted from the field dependence of charge transport, for example from the resistivity $\rho (T) = \rho_0 + A (B) T^2$ with $\rho_0$ is a temperature independent part and $A (B)$ is a LFL coefficient. The crossover takes place at $T^* (B)$ where the resistance starts to deviate from the LFL $T^2$ behavior, see e.g. Ref. 8.

Let us now consider the $B - T$ phase diagram of the HTSC substance $Tl_2 Ba_2 Cu O_{6+\delta}$ shown in Fig. 4. The substance is a superconductor with $T_c$ from 15 K to 93 K, being controlled by oxygen content. In Fig. 4 open squares and solid circles show the experimental values of the crossover temperature from the LFL to NFL regimes. The solid line shows our fit with $B_{c_0} = 6 T$ that is in good agreement with $B_{c_0} = 5.8 T$ obtained from the field dependence of the charge transport. As it is seen from Fig. 5, the linear behavior agrees well with the peak temperatures $T_{max}$ shown in the inset to Fig. 3. The inset reports the peak temperatures $T_{max} (B)$, extracted from measurements of $C/T$ and $\chi_{AC}$ on $YbRh_2 (Si_{0.95} Ge_{0.05})$. It follows from eq. (5), $T_{max}$ shifts to higher values with increase of the applied magnetic field. It is seen that both functions can be represented by straight lines intersecting at $B \simeq 0.03 T$. This observation is in good agreement with experiments.

It is seen from Fig. 3 that critical field $B_{c_2} = 8 T$ destroying the superconductivity is close to $B_{c_0} = 6 T$. Let us show that this is more than a simple coincidence, and $B_{c_2} \geq B_{c_0}$. Indeed, at $B_0 > B_{c_0}$ and low temperatures $T < T^* (B)$, the system is in LFL state. The superconductivity is then destroyed since the superconducting gap is exponentially small as we have seen above. At the same time, there is FC state at $B < B_{c_0}$ and this low-field phase has large prerequisites towards superconductivity as in this case the gap is a linear function of the coupling constant. We note that this is exactly the case in CeCoIn$_5$ where $B_{c_0} \approx B_{c_2} \approx 5 T$ Ref. 27, while the application of pressure makes $B_{c_2} > B_{c_0}$ Ref. 28. On the other hand, if the superconducting coupling constant is rather weak then antiferromagnetic order wins a competition. As a result, $B_{c_2} = 0$, while $B_{c_0}$ can be finite as in $YbRh_2 Si_2$ and $YbRh_2 (Si_{0.95} Ge_{0.05})$. Upon comparing the phase diagram of CeCoIn$_5$ with that of $Tl_2 Ba_2 Cu O_{6+\delta}$, it is possible to conclude that...
they are similar in many respects. Further, we note that the superconducting boundary line \( B_{c2}(T) \) at lowering temperatures acquires a step, i.e. the corresponding phase transition becomes first order\textsuperscript{30,31}. This permits us to speculate that the same may be true for \( \text{Tl}_2\text{Ba}_2\text{CuO}_{6+x} \). We expect that in the NFL state the tunneling conductivity is asymmetrical function of the applied voltage, while it becomes symmetrical at the application of elevated magnetic fields when \( \text{Tl}_2\text{Ba}_2\text{CuO}_{6+x} \) transits to the LFL behavior, as it predicted to be in CeCoIn\(_5\), Ref. 32.

Now we consider the field-induced reentrance of LFL behavior in \( \text{Tl}_2\text{Ba}_2\text{CuO}_{6+x} \) at \( B \geq B_{c0} \). The LFL regime is characterized by the temperature dependence of the resistivity, \( \rho(T) = \rho_0 + A(B)T^2 \), see also above. The \( A \) coefficient, being proportional to the quasiparticle-quasiparticle scattering cross-section, is found to be \( A \propto (M^*(B))^2 \), Ref. 13,29. With respect to eq. 4, this implies that

\[
A(B) \simeq A_0 + \frac{D}{B - B_{c0}}, \tag{6}
\]

where \( A_0 \) and \( D \) are parameters. It is pertinent to note that Kadowaki-Woods ratio\textsuperscript{33}, \( K = A/\gamma_0 \), is constant within the FC theory as it follows from equations 4 and 5.

Figure 4 reports the fit of our theoretical dependence of \( \rho(T) \) to the experimental data for two different classes of substances: HF metal YbRh\(_2\)Si\(_2\) (left panel) and HTSC \( \text{Tl}_2\text{Ba}_2\text{CuO}_{6+x} \) (right panel). The different scale of fields is clearly seen as well as good coincidence with theoretical dependence 4. This means that the physics underlying the field-induced reentrance of LFL behavior, is the same for both classes of substances. To further corroborate this point, we replott both dependencies in reduced variables \( A/A_0 \) and \( B/B_{c0} \) on Fig. 4. Such reploting immediately reveals the universal nature of the behavior of these two substances - both of them are initially in the FC state, which is being destroyed by an external magnetic field. Since close to magnetic QCP there is no external physical

In summary, it follows from our study that there is at least one quantum phase transition inside the superconducting dome, and this transition is FCQPT. Moreover, our consideration of above very different strongly correlated Fermi-systems, with leading family resemblance found between HTSC and HF compounds, shows that numerous QCPs assumed earlier to be responsible for the NFL behavior of above substances can be well reduced to a single QCP related to FCQPT.

Appendix

Consider the temperature and magnetic field dependence of the effective mass \( M^*(T, B) \) as system approaches FCQPT. Landau equation\textsuperscript{14} is of the form

\[
\frac{1}{M^2} = \frac{1}{M} + \int \frac{d^3p_F}{p_F^3} F(p_F, p_1) \frac{\partial \mu(p_1, T, B)}{\partial p_1} \left( \frac{dp_1}{2\pi} \right)^3. \tag{7}
\]

The notations here are similar to those in the main text, we suppress the spin indices for simplicity. Approximate interpolative solution for equation (7) reads\textsuperscript{5,13}

\[
M^*(B, T_N, x) = M_N(T_N) \approx c_0 \frac{1 + c_1 T_N^2}{1 + c_2 T_N^{8/3}}. \tag{8}
\]

Here \( M_N(T_N) \) is the normalized effective mass, \( M_N \) is the maximum value, that it reaches at \( T = T_M \). Normalized temperature \( T_N = T/T_M \), \( c_0 = (1 + c_2)/(1 + c_1) \), \( c_1 \) and \( c_2 \) are fitting parameters, parameterizing Landau amplitude. It follows from Eq. 8 that in contrast to the standard paradigm of quasiparticles the effective mass strongly depends on temperature, revealing three different regimes at growing temperature. At the lowest temperatures we have the LFL regime. Then the system enters the transition regime: \( M_N(T_N) \) grows, reaching its maximum \( M_N^* = 1 \) at \( T = T_M, (T_N = 1) \), with

![Graph](image-url)
subsequent diminishing. Near temperatures $T_N \geq 1$ the last "traces" of LFL regime disappear and the NFL state takes place, manifesting itself in decreasing of $M_N^* \sim T_N^{-2/3}$ and then as $T_N^{-1/2}$. These regimes are reported in the inset to Fig. 6

![Diagram](image)

**FIG. 6:** The function $\nu(p)$ for the multiply connected distribution that replaces the function $n(p)$ in the region $(p_f - p_i)$ occupied by Fermi condensate. The momenta satisfy the inequalities $p_i < p_F < p_f$, where $p_F$ is Fermi momentum of a normal Fermi liquid. The outer Fermi surface at $p \gtrsim p_{2n} \approx p_f$ has the shape of a Fermi step so that the system at $T < T^* (B)$ behaves like LFL.

Now we consider the action of external magnetic field on HF liquid in FC phase. Any infinitesimal magnetic field $B \neq 0$ (better to say, $B \geq B_{c0}$) destroys both superconductivity and FC state, splitting it by Landau levels. The simple qualitative arguments can be used to guess what happens to FC state in this case. On one side, the energy gain from FC state destruction is $\Delta E_B \propto B^2$ (see above) and tends to zero as $B \rightarrow 0$. On the other side, $n(p)$ in the interval $p_i \leq p \leq p_f$ gives a finite energy gain as compared to the ground state energy of a normal Fermi liquid. It turns out that the state with largest possible energy gain is formed by a multiconnected Fermi surface, resembling an onion so that the smooth function $n(p)$ is replaced in the interval $p_i \leq p \leq p_f$ by the set of rectangular blocks of unit height, formed from Heavyside step functions as reported in Fig. 6.

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