SARSA(0) Reinforcement Learning over Fully Homomorphic Encryption

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Abstract: We consider a cloud-based control architecture in which the local plants outsource the control synthesis task to the cloud. In particular, we consider a cloud-based reinforcement learning (RL), where updating the value function is outsourced to the cloud. To achieve confidentiality, we implement computations over Fully Homomorphic Encryption (FHE). We use a CKKS encryption scheme and a modified SARSA(0) reinforcement learning to incorporate the encryption-induced delays. We then give a convergence result for the delayed updated rule of SARSA(0) with a blocking mechanism. We finally present a numerical demonstration via implementing on a classical pole-balancing problem.

Keywords: Homomorphic Encryption, Reinforcement Learning, Privacy, Cloud-based Control

1. INTRODUCTION

Certain algorithms such as Model Predictive Control (MPC) and visual servoing require heavy real-time computational operations while on-site controllers in industrial control systems are often resource-constrained. The cloud-based control may resolve this issue by allowing on-site controllers to outsource their computations to cloud computers. Many such cloud-based control schemes have been proposed recently in the literature. For instance, Hegazy and Hefeeda (2014) considered a cloud-based MPC architecture with an application to large-scale power plant operations. Wu et al. (2012) considered a cloud-based visual servoing architecture, where an UDP-based communication protocol was developed for latency reduction.

Despite the computational advantages, a naive implementation of cloud-based control can leak operational records of clients control systems, which often contain sensitive information. Since private information can be a valuable asset in our modern society, an appropriate privacy protecting mechanism is a critical requirement for cloud-based control services in practice. Both non-cryptographic (e.g., differential privacy) and cryptographic (e.g., homomorphic encryption (HE)), the focus of this paper) approaches have been considered in the literature.

Cloud-based control over HE is pioneered by Kogiso and Fujita (2015), Farokhi et al. (2016), Kim et al. (2016), followed by rapid developments of related technologies in more recent works. The focus of the literature to date has been mainly on the cloud-based implementations of control policies (Fig. 1 (a)). In such scenarios, the role of the cloud is to construct a control policy $\pi_t$ while the synthesized policy may be implemented locally by the client. Control synthesis problems are classified into model-based and data-driven approaches. In model-based approaches, a mathematical model of the system to be controlled is explicitly used for the policy construction (e.g., synthesis of explicit MPC laws; Bemporad et al. (2002)), while data-driven approaches construct a policy $\pi_t$ from the historical input-output data without using system models (e.g., reinforcement learning). Since there are scenarios in which control synthesis involves heavy computations (e.g., explicit MPC for time-varying systems, reinforcement learning by deep Q-network; Mnih et al. (2015)), it will be beneficial to consider cloud-based implementations of such computational procedures. To the best of our knowledge, cloud-based control synthesis over HE has not been investigated in the literature yet.

Although a large portion of the existing work on control over HE is restricted to partially homomorphic encryption (PHE) schemes (notable exceptions include Kim et al. (2019)), this paper adopts a framework of FHE schemes. This is because it is hard to find a useful application of PHE schemes (which support only addition or multiplication in ciphertext domain) in control synthesis problems, where more advanced algebraic operations are typically required. As a step forward to general control synthesis over FHE, this paper focuses on implementing one of the most elementary reinforcement learning (RL) algorithms, namely the SARSA(0) algorithm. We note that RL over FHE is largely unexplored, while supervised learning over FHE has been studied in recent years, e.g., Dowlin et al. (2016). To deal with delays due to computational overhead by FHE, we consider a modified SARSA(0) and discuss its convergence properties. We implement the modified SARSA(0) over Cheon-Kim-Kim-Song (CKKS) encryption scheme and apply it to a benchmark RL problem (pole balancing) to demonstrate the feasibility of our study.
Fig. 1. Cloud-based implementation of control policies vs. cloud-based control synthesis. Control synthesis can be based on plant models (model-based approach) or operational data (data-driven approach).

In the remainder of this paper, we first summarize the overview of HE schemes in Section 2. A modified SARSA(0) algorithm is described in Section 3, and the results of numerical experiments are shown in Section 4. We conclude in Section 5 with future research directions.

2. PRELIMINARIES

**Homomorphic Encryption** (HE) is an encryption method that possesses homomorphic properties with respect to addition and/or multiplication operations. Modern cryptography can be classified into two categories, namely symmetric key encryption and asymmetric key encryption. Symmetric key encryption model assumes that the encryption and decryption keys are interchangeable. For example, Advanced Encryption Standard (AES) is a widely known symmetric encryption algorithm. On the other hand, asymmetric key encryption, often known as public-key encryption, requires the decryption key to be different from the encryption key. Thus, a public-key homomorphic encryption method can enable cloud outsourced computations with data protection. A comprehensive introduction to this technology can be found otherwise in Yi et al. (2014).

2.1 Fully Homomorphic Encryption

Homomorphic Encryption is *Partially Homomorphic* if only one operation (either an addition or a multiplication) is preserved. Some popular Partially Homomorphic Encryption schemes include ElGamal Cryptosystem and Paillier Cryptosystem. The former is homomorphic with respect to multiplication and the latter with addition. Recently, both schemes have been applied in control literatures, Kogiso and Fujita (2015), Farokhi et al. (2016), Alexandru et al. (2018), and Schulze Darup et al. (2018).

If both additions and multiplications are preserved but for a limited number of operations, then the encryption scheme is called *Somewhat (Levelled) Homomorphic*. Gentry (2009) proved that any Somewhat Homomorphic scheme with a bootstrapping procedure can be promoted to *Fully Homomorphic Encryption* by controlling the noise growth in ciphertexts. Since then, many more efficient schemes emerged. It is also worth mentioning that many of recent generation FHE schemes are among a few candidates for *Quantum-resistant cryptosystem*, Lange (2015).

A more in-depth coverage on FHE can be found in Halevi (2017).

2.2 CKKS Encryption scheme

Cheon et al. (2016) developed a CKKS scheme. A bootstrapping procedure for CKKS was proposed by Cheon et al. (2018), thus elevating it to a fully homomorphic encryption scheme. CKKS is unique in that it can encrypt a vector of complex numbers and can perform approximate arithmetic as opposed to other schemes, which encrypt integers and perform exact arithmetic. Note that a special encoding structure is necessary as the plaintext space of CKKS scheme is an integer-coefficient polynomial ring modulo cyclotomic polynomial. A detailed review of CKKS scheme is out of scope of this paper and we refer Cheon et al. (2016).

CKKS scheme’s precision loss is only slightly more than a precision loss due to the unencrypted floating point arithmetic. Therefore, this scheme is very convenient for control applications. A small precision loss can be modeled as an existing disturbance and applications involving large data can reap benefits via batched operations. Moreover, Microsoft SEAL provides an open-source library that supports CKKS with a simple tutorial for the use.

2.3 Cloud-based control over homomorphic encryption

More recently, the secure evaluation of affine control law for explicit MPC using Paillier Cryptosystem (PCS) is shown in Schulze Darup et al. (2018). In Alexandru et al. (2018), an implicit MPC control evaluation using PCS was shown to be possible through the projected Fast Gradient Method and the use of a communication protocol. Also, in Darup et al. (2018), a PCS encrypted implicit MPC was shown via the use of real-time proximal gradient method. But due to the nature of PCS, or any other partially homomorphic schemes, some parameters are assumed to be public. This is a valid assumption in some cases but may not fit some other scenarios.

On the other hand, FHE is not as adopted as PHE in control systems because it is still far more computationally demanding compared to PHE. Nonetheless, the feasibility of using the FHE for a cloud-based control system was first shown in Kim et al. (2016). Subsequently, in Kim et al. (2019), a secure dynamic control was proposed using LWE-based FHE and a critical observation was that the noise growth of FHE is bounded by the stability of the closed-loop system under some conditions, eliminating the need for bootstrapping, which is one of the most computationally involved procedures.

Previous attempts to apply homomorphic encryption to cloud-based control mostly focused on cloud-based implementations of control laws. In many cases, implementations of control laws imposes stringent real-timeness requirements (10 – 100 milliseconds), which significantly restricts the use of homomorphic encryption, which introduces delay. Instead of focusing on the control implementations, we apply homomorphic encryption to cloud-based control synthesis (computation of feedback control policies to be implemented) as seen in Figure 1(b). Compared to Figure 1(a), we assume that a feedback control policy...
for each (s, a) ∈ S × A is implemented by the client. Since sensor data s and control command a are not encrypted, fast implementation of the control policy is possible.

The role of the cloud in our model is to update the control policy π in a way that the cloud synthesizes a policy π every time a new observation s and reward signal r are received. Consequently, an optimal policy of the form π can be assumed without loss of performance. The value π satisfies the Bellman’s equation

\[ V^\pi(s) = E^\pi \left[ \sum_{t=1}^{\infty} \gamma^t r(s_t, a_t) | s_0 = s \right] , \]

where 0 ≤ γ < 1 is a predefined discount factor. We say that a policy π is optimal if it maximizes the value of each state simultaneously. The existence of time-invariant, Markov, and deterministic optimal policy under the present setup is well-known, (e.g., Bertsekas (2011)). Consequently, an optimal policy of the form \( a_t = \pi^*(s_t) \) can be assumed without loss of performance. The value function \( V^\pi \) under an optimal policy \( \pi^* \) satisfies the Bellman’s equation

\[ V^*(s) = \max_{a \in A} \left[ r(s, a) + \gamma \sum_{s' \in S} P(s'|s, a)V^*(s') \right] . \]  

3. REINFORCEMENT LEARNING OVER FULLY HOMOMORPHIC ENCRYPTION

Consider a Markov decision process (MDP) defined by a tuple \((S, A, P, r)\), where \( S \) is a finite state space, \( A \) is a finite action space, \( P(s_{t+1}|s_t, a_t) \) is the state transition probability and \( r(s, a) \) is the reward. A policy is a sequence of stochastic kernels \( \pi = (\pi_1, \pi_2, \ldots) \), where \( \pi_t(a_t|s_{1:t}, a_{1:t-1}) \) assigns the probability of selecting the next action \( a_t \in A \) given the history \((s_{1:t}, a_{1:t-1})\). For a fixed policy \( \pi \), the value of each state \( s \in S \) is defined by

\[ V^\pi(s) = E^\pi \left[ \sum_{t=1}^{\infty} \gamma^t r(s_t, a_t) | s_0 = s \right] , \]

where \( 0 \leq \gamma < 1 \) is a predefined discount factor. We say that a policy \( \pi \) is optimal if it maximizes the value of each state simultaneously. The existence of time-invariant, Markov, and deterministic optimal policy under the present setup is well-known, (e.g., Bertsekas (2011)). Consequently, an optimal policy of the form \( a_t = \pi^*(s_t) \) can be assumed without loss of performance. The value function \( V^\pi \) under an optimal policy \( \pi^* \) satisfies the Bellman’s equation

\[ V^*(s) = \max_{a \in A} \left[ r(s, a) + \gamma \sum_{s' \in S} P(s'|s, a)V^*(s') \right] . \]  

3.1 Q-learning

The focus of reinforcement learning algorithms in general is to construct an optimal policy \( \pi^* \) based on the observed history of the states, actions, and reward signals. The Q-learning, Watkins and Dayan (1992), achieves this by attempting the construction of the optimal Q-function defined by

\[ Q^*(s, a) := r(s, a) + \gamma \sum_{s' \in S} P(s'|s, a)V^*(s') \]  

for each \((s, a) \in S \times A\). It follows from the Bellman’s equation (1) that

\[ Q^*(s, a) = r(s, a) + \gamma \sum_{s' \in S} P(s'|s, a)Q^*(s', a') \]

for each \((s, a) \in S \times A\). Once \( Q^* \) is obtained, an optimal policy can also be obtained by \( \pi^*(s) = \arg \max_{a \in A} Q^*(s, a) \).

Let \( \{\alpha_n\}_{n=0, 1, 2, \ldots} \) be a predefined sequence such that \( 0 \leq \alpha_n \leq 1 \) \( \forall n \), \( \lim_{n \to \infty} \alpha_n = \infty \) and \( \sum_{n=0}^{\infty} \alpha_n^2 < \infty \). Denote by \( n_t(s, a) \) the number of times that the state-action pair \((s, a)\) has been visited prior to the time step \( t \). Upon the observation of \((s_t, a_t, r_t, s_{t+1})\), the Q-learning updates the \((s_t, a_t)\) entry of the Q-function by

\[ Q_t(s_t, a_t) = (1 - \alpha_{n_t(s_t, a_t)})Q_{t-1}(s_t, a_t) + \alpha_{n_t(s_t, a_t)} (r_t + \gamma \max_{a' \in A} Q_{t-1}(s_{t+1}, a')) . \]  

No update is made to the \((s, a)\) entry if \((s, a) \neq (s_t, a_t)\). The following result is standard in the literature:

**Theorem 1.** For an arbitrarily chosen initial Q-function \( Q_0 \), the convergence \( Q_t(s, a) \to Q^*(s, a) \) as \( t \to \infty \) holds under the update rule (3) provided each state-action pair \((s, a) \in S \times A\) is visited infinitely often.

In what follows, we assume that the underlying MDP is communicating, i.e., every state can be reached from every other state under certain policies.

3.2 SARSA(0)

The Q-learning method described above is considered an off-policy reinforcement learning algorithm in the sense that the hypothetical action \( a' \) in the update rule (3) need not be actually selected as \( a_{t+1} \). The SARSA(0) algorithm, on the other hand, is an on-policy counterpart that performs the Q-function update based on the experienced trajectory \((s_t, a_t, r_t, s_{t+1}, a_{t+1})\):

\[ Q_t(s_t, a_t) = (1 - \alpha_{n_t(s_t, a_t)})Q_{t-1}(s_t, a_t) + \alpha_{n_t(s_t, a_t)} (r_t + \gamma Q_{t-1}(s_{t+1}, a_{t+1})) . \]  

Here, we remark that the absence of the max operation in (4) is a significant advantage for the implementation over FHE because the current polynomial approximations of comparison operations are highly inefficient, Cheon et al. (2019).

The convergence of SARSA(0) can be guaranteed under certain conditions. Although a complete discussion must be differed to Singh et al. (2000), roughly speaking it requires that (i) the learning policy is infinitely exploring, i.e., each state-action pair \((s, a) \in S \times A\) is visited infinitely often; and (ii) the learning policy is greedy in the limit, i.e., the probability that \( a_{t+1} \neq \arg \max_{a' \in A} Q_{t-1}(s_{t+1}, a') \) tends to zero. Condition (i) is required for the convergence of Q-learning (off-policy counterpart of SARSA). The additional condition (ii) is needed due to the on-policy nature of SARSA(0). The following is a simple example of learning policies satisfying (i) and (ii):

**Definition 2.** (Decreasing \( \epsilon \) policy) Let \( n_t(s) \) be the number of times that the state \( s \) has been visited prior to time step \( t \) and define \( \epsilon(s) = c/n_t(s) \) for some \( 0 < c < 1 \). We say that \( \pi \) is a decreasing \( \epsilon \) policy if it selects an action \( a_{t+1} \) randomly with the uniform distribution over \( A \) with probability \( \epsilon(s_{t+1}) \) and the greedy action \( \arg \max_{a' \in A} Q_{t-1}(s_{t+1}, a') \) with probability \( 1 - \epsilon(s_{t+1}) \).

The following result is a consequence of Theorem 1 in Singh et al. (2000).
Theorem 3. For an arbitrarily chosen \( Q_0 \), the convergence \( Q_t(s,a) \rightarrow Q^*(s,a) \) as \( t \rightarrow \infty \) for each state-action pair \( (s,a) \in S \times A \), occurs with probability one under the SARSA(0) update rule (4) and the decreasing \( \epsilon \) policy.

Proof. (Outline only) The result relies on the convergence of stochastic approximation (Singh et al., 2000, Lemma 1), whose premises are satisfied if the learning policy is greedy in the limit with infinite exploration (GLIE):

\[ T(s) \leq T(s) + 1 \]

(i) infinitely exploring, and (ii) greedy in the limit in that \( E[Q_{t+1}(s_{t+1}, a_{t+1}) - \max_{a \in A} Q_{t+1}(s_{t+1}, a)] \rightarrow 0 \) as \( t \rightarrow \infty \) with probability one. To verify (i), we note that, under the assumption of communicating MDP, performing each action infinitely often in each state is sufficient to guarantee the infinite exploration of states. Let \( t_x(i) \) be the time step that the state \( s \) is visited the \( i \)-th time. Since we are adopting the decreasing \( \epsilon \) policy, e.g., \( \epsilon_t(s) = c/n_x(s) \) with \( 0 < c < 1 \), \( \Pr(a_t(i) = a) \) is dependent on \( \Pr(a_t(i) = a) \). But by the extended Borel-Cantelli lemma, we have \( \sum_{i=1}^{\infty} \Pr(a_t(i) = a) = \infty \) for each \( a \in A \). Thus, by Lemma 4 (Singh, 2000), we have \( n_t(s,a) \rightarrow \infty \) a.s., where \( n_t(s,a) \) denotes the number of actions performed at state \( s \) at time \( t \). The condition (ii) holds by construction of the decreasing \( \epsilon \)-greedy policy. The conditions 1, 2, and 3 of the Theorem 1 in Singh et al. (2000) are satisfied by construction.

3.3 SARSA(0) with blocking states

Now, consider a cloud-based implementation of SARSA(0). We assume that the Q-function update (4) is performed by the cloud while the decreasing \( \epsilon \) policy is implemented by the client. As shown in Fig. 1(b), at each time step \( t \), the client can encrypt and upload a new data set \( z_t = (s_t, a_t, r_t, s_{t+1}, a_{t+1}) \) and, upon the completion of the Q-function update (4) on the cloud, the updated entry of the Q-function is downloaded and decrypted. If the computation of Q-update takes less than a unit time interval, then SARSA(0) together with the decreasing \( \epsilon \) policy as described above can be implemented in the considered cloud-based architecture without any modifications.

However, encrypted Q-update can take up to some \( L \geq 2 \) unit time intervals. In such scenarios, a new data set may arrive before the previous Q-update is complete. For simplicity, we assume there is no communication delay between the cloud and the client. In what follows, we propose a modified SARSA, which we call SARSA(0) with blocking states. The proposed Q-update rule is depicted in Fig. 2, where a special case with \( |S| = |A| = 2 \) and \( L = 3 \) is shown.

First, encrypted values of the initial Q-function

\[ Q_0 = (Q_0^{[0]}(1,1), Q_0^{[0]}(1,2), Q_0^{[0]}(2,1), Q_0^{[0]}(2,2)) \]

are recorded on the cloud’s memory. At time step \( t = 1 \), the state-action pair \( (s_1, a_1) = (1,1) \) is visited, so the encrypted data set \( z_1 = (s_1, a_1, r_1, s_2, a_2) \) is uploaded to the cloud. The computation (4) for \( Q(1,1)-update \) is initiated, which will take three time steps to complete. While the \( Q(1,1) \) is being updated on the next \( L - 1 \) time steps (i.e., \( t = 2,3) \), the corresponding state \( s = 1 \) is added to the list of blocking states. When the state \( s \) is blocking, no updates are allowed to the entries of \( Q(s, \cdot) \).

For instance, at \( t = 2 \), the state-action pair \( (s_2, a_2) = (1,2) \) is visited. However, since the state \( s = 1 \) is in a blocking state list, the update request is rejected and the data set \( z_2 = (s_2, a_2, r_2, s_3, a_3) \) is discarded. At \( t = 3 \), the state-action pair \( (s_3, a_3) = (1,2) \) is visited when the state \( s = 1 \) is removed from the blocking list, a new data set \( z_3 \) is accepted and the computation for \( Q(1,2)-update \) is initiated.

Under SARSA(0) with blocking states described above, we denote by \( Q_t \) the most updated version of the Q-function recorded on the memory as of time step \( t \). For instance, Fig. 2 reads

\[ Q_{10} = (Q_1^{[1]}(1,1), Q_1^{[1]}(1,2), Q_0^{[0]}(2,1), Q_0^{[0]}(2,2)). \]

Let \( m_t = m_t(s_t, a_t) \) be the number of times that the update has been accepted at the state-action pair \( (s, a) \) prior to time step \( t \). Whenever the update with data \( z_{t-L} = (s_{t-L}, a_{t-L}, r_{t-L}, s_{t-L+1}, a_{t-L+1}) \) is accepted at time step \( t-L \), the following update is made at time step \( t \):

\[ Q_t(s_{t-L}, a_{t-L}) = (1 - \alpha_{m_{t-L}})Q_{t-L}(s_{t-L}, a_{t-L}) + \alpha_{m_{t-L}}(r_{t-L} + \gamma Q_{t-L}(s_{t-L+1}, a_{t-L+1})). \]

Since the blocking mechanism ensures that the entries \( Q_k(s_{t-L}, \cdot) \) are unchanged over the time steps \( t-L \leq k \leq t-1 \), the above update rule can also be written as

\[ Q_t(s_{t-L}, a_{t-L}) = (1 - \alpha_{m_{t-L}})Q_{t-1}(s_{t-L}, a_{t-L}) + \alpha_{m_{t-L}}(r_{t-L} + \gamma Q_{t-1}(s_{t-L+1}, a_{t-L+1})). \]

Definition 4. We define the decreasing \( \epsilon \) policy with delay similarly to the decreasing \( \epsilon \) policy (Definition 2) except that greedy actions are selected by \( a_{t+1} = \arg \max_{a \in A} Q_{t-1}(s_{t+1}, a) \), where \( Q_{t-1} \) is the most updated version of the Q-function available at \( t-1 \).

We have the following convergence result.

Theorem 5. For an arbitrarily chosen \( Q_0 \), the convergence \( Q_t(s,a) \rightarrow Q^*(s,a) \) as \( t \rightarrow \infty \) for each state-action pair \( (s,a) \in S \times A \), occurs with probability one under SARSA(0) with delayed update and blocking if the decreasing \( \epsilon \) policy with delay is adopted.
Proof. We will prove that, under delayed update and blocking mechanism, (i) every state is explored infinitely often, and (ii) the update is still executed infinitely often, and (iii) the policy is greedy in the limit, again satisfying the convergence conditions.

(i) Since the decreasing \( \epsilon \) policies with and without delay (Definitions 2 and 4) share the same probability \( \epsilon_i(s) \) for random exploration, the inequality \( \sum_{i=1}^{\infty} \Pr(\epsilon_{t_i}(i) = a) \geq \sum_{i=1}^{\infty} \epsilon_i/s = \infty \) remains valid under the decreasing \( \epsilon \) policy with delay. As in the proof of Theorem 3, this implies that each state-action pair is visited infinitely often.

(ii) Assume that update occurs only \( K(\infty) \) times for some state-action pair \((s, a)\). Since \( L < \infty \), the Q-update can be blocked at most \( K(L-1) < \infty \) times. This means that updates are accepted infinitely many times, and this contradicts the assumption that updates occur only \( K \) times.

(iii) The policy is greedy in the limit in that

\[
\mathbb{E}[Q_{t-1}(s_t-L+1, a_t-L+1) - \max_{a \in \mathcal{A}} Q_{t-1}(s_t-L+1, a)] \to 0 \quad (5)
\]

as \( t \to \infty \) with probability one. To see this, suppose that \( a_{t-L+1} \) was a greedy action selected at time step \( t - L \), i.e.,

\[
a_{t-L+1} = \arg \max_{a \in \mathcal{A}} Q_{t-L}(s_{t-L+1}, a).
\]

Due to the blocking mechanism, \( Q_k(s_{t-L+1}, \cdot) \) are unchanged over the time steps \( t - L \leq k \leq t - 1 \). This means that

\[
a_{t-L+1} = \arg \max_{a \in \mathcal{A}} Q_{t-L}(s_{t-L+1}, a).
\]

Thus, whenever \( a_{t-L+1} \) was selected greedily at \( t - L \), we have

\[
Q_{t-1}(s_{t-L+1}, a_{t-L+1}) - \max_{a \in \mathcal{A}} Q_{t-1}(s_{t-L+1}, a) = 0.
\]

Since the delayed \( \epsilon \)-greedy policy is greedy in the limit, the convergence (5) holds with probability one.

4. NUMERICAL DEMONSTRATION

We used a classical pole-balancing problem from RL. The setup can be found from Sutton and Barto (2018). With slight modifications, we demonstrate our encrypted SARSA(0) updates. For encryption, we used Microsoft SEAL to set up the CKKS scheme. Algorithm 1 is the pseudocode of Encrypted SARSA(0) with delayed updates. We assume that the client transmitted Q-values directly corresponding to the state-action pair it visited. We fixed a step size \( \alpha \) and we queued the data sets at the client’s memory to take advantage of a batch encryption. We let the data set be \( z_i = (Q_i(s_t, a_t), r_t, Q_i(s_{t+1}, a_{t+1}), \alpha, \gamma) \) and we simply use \( Q' \) for \( Q'_{t}(s_{t+1}, a_{t+1}) \). Instead of attempting to encrypt and upload the data \( z_t \) every time step, we assumed the situation where the agent can explore longer and collect the larger data sets. The agent then uploads the batch data and the cloud computes back the batch update. We use a tilde to denote the encoded version (e.g., \( \tilde{z} \) to be the encoded version of \( L \) data sets, \( \tilde{z} = Encode([z_{t-L}, \ldots, z_{t-1}]]) \) and a symbol \# in front to denote the encrypted version (e.g., \#\( \tilde{z} \) is the encrypted version, \#\( \tilde{z} \) = Encrypted(\( \tilde{z} \)).

Table 1 lists the CKKS Parameters used for the demonstration. Table 2 shows the delay introduced by operations involved in Homomorphic Encryption. The maximum precision error by encrypted updates was only 0.0063%. We used \( L = 1000 \) to generate the particular result below and accordingly only 1000 slots were used out of 4096 available slots (encryption parameter-specific). But the time taken by each operation of homomorphic encryption listed in Table 2 is fixed for all \( L \leq 4096 \), and thus it can allow larger batch operations if necessary. We verified that, given enough trials, learning outcomes remain successful for large \( L \) and can learn much faster than small \( L \) by performing more compact and less frequent encrypted operations.

We note that the client is still burdened with non-trivial computing tasks, most notably CKKS encoding and homomorphic encryption, which takes up more than a half of the homomorphic operations involved. On the other hand, decryption and decoding tasks are less strenuous. This is a prevalent issue in encrypted control as seen in the results of Darup et al. (2018) and many others, dwindling the appeal of cloud-based control. This result is therefore encouraging for a further investigation in possibility of outsourcing the encryption itself. A novel concept in this direction was first suggested in Nachrig et al. (2011), where one can combine the relatively light AES encryption to offload the homomorphic encryption process all together to the cloud. But even without such future improvements, in certain control synthesis problems, the homomorphic encryption-induced computing requirements and delays may be well compensated with the privacy guarantee acquired. Another significant task we note is Relinearization. But this task is for the cloud during the homomorphic operations.

| Algorithm 1 Encrypted SARSA(0) |
|----------------------------------|
| **Client (Start)**               |
| 1: Perform actions and state transitions \( L \) times according to the up-to-date Q-table values |
| 2: Store data \( z_i \), where \( i = t - L, \ldots, t - 1 \) |
| 3: Encode \( [z_{t-L}, \ldots, z_{t-1}] \) and get \( \tilde{z} \) |
| 4: Encrypt and get \#\( \tilde{z} \) and upload to the cloud |
| **Cloud**                        |
| 1: Update: \#\( \tilde{Q} \) ← \((-1 - \#\tilde{\alpha})\#\tilde{Q} + \#\tilde{\alpha}(\#\tilde{\beta} + \#\tilde{\gamma})\) |
| 2: Upload the updated \#\( \tilde{Q} \) back to the Client |
| **Client**                       |
| 1: Decrypt \#\( \tilde{Q} \) to get \( \tilde{Q} \) |
| 2: Decode \( \tilde{Q} \) to get the updated vector of \( Q_i \)'s |
| 3: go to **Start**               |

| Table 1. Encryption Parameters |
|---------------------------------|
| CKKS Parameters                | Chosen |
| \( N \) (Poly. Modulus Deg.)   | 8192   |
| \( q \) (Cipher. Coeff. Modulus)| \((2^{50}, 2^{30}, 2^{30}, 2^{30}, 2^{50})\) |
| Scale Factor                   | \(2^{30}\) |
| Available Slots                | 4096   |

5. SUMMARY AND FUTURE WORK

We tackled a problem of privately outsourced control. In this work, we provide a general framework for control synthesis over fully homomorphic encryption. We
showed a convergence result for the SARSAR(0) with delayed updates. We then demonstrated solving a classical reinforcement learning problem with a privacy objective and privacy-induced delayed updates. Numerical results showed that the homomorphic encryption via CKKS scheme could successfully complete the private learning with the precision loss being minimal. We saw that a batch operation can be of our advantage.

Many challenges remain for encrypted control. Obviously, the delays introduced may limit the area of applications. However, as we considered in this paper, using homomorphic encryption for control synthesis may be feasible. The other critical challenge is the difficulty of implementing certain critical operations such as comparison and sorting over FHE. This makes the execution of Bellman operations or optimizations on ciphertext domain challenging. As our future work seeks to extend our framework to more advanced evaluation tasks such as training and execution of artificial neural network or control synthesis, (e.g., model predictive control) over fully homomorphic encryption, an efficient polynomial comparison function will be of significant value. We will also continue to investigate the idea of outsourcing the homomorphic encryption task as discussed to improve our framework.

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