Bianchi - III inflationary universe for perfect fluid distribution using Cosmological constant in general relativity

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Abstract: In this study Bianchi- III universe for perfect fluid under effect of the cosmological constant is investigated. The complicated system of field’s equations are resolved in the existence of conservation condition of energy-momentum and a relativistic relation among metric potential is assumed i.e. $b = e^k$ here $k \neq 1$ is a constant. The dynamical features of the constructed model are also pointed

Keywords: Bianchi - III, Inflationary Universe, Perfect Fluid, Cosmological Constant

1. Introduction

The study of anisotropic and homogenous model of the universe has scientific importance in astrophysics to search out the realistic behavior of current scenario in cosmology. Bianchi-I models are widely used to investigate the accelerated phase of the physical cosmos. and the large scale of evolution. but we have considered the Bianchi –III because it is more general. Our universe has covered a long period of expansion in exponentially way from the initial stage of evolution. An anisotropic approach toward the cosmological model helps to understand the structure of present cosmology more generally. The set of Einstein field equations are highly complicated non-linear differential equations coupled with a scalar region which provides the analytical description of applications in astrophysics. After an inflationary epoch, the scale factor of the space-time increase exponentially allowed a small casualty; articulate region to grow large enough to resemble the currently observed universe. In this way, the inflationary nature of the universe provides relativistic solution to some of the conceptual problems in cosmology which was unexplained in the case of the big-bang theory. The recent scenario in inflation explains most of the mysteries related to modern cosmology like homogeneity, monopole, isotropy, and the flatness concept in the observed universe. Guth[1] provides the introductory idea about the
phase of inflation in the context of GUT. Author [2-6] discussed the accelerated expansion under various circumstances. The term Λ used in the QF theory and considered as the energy density in case of the vacuum. The model containing the cosmological term is very popular in recent research. The astrophysical evidence also detects the cosmological constant or a form of matter with the content of the universe. Authors [7-15] have developed significant results in their research on effects of the cosmological term at cosmology in various contexts.

Poonia and Sharma [16-18] also studied the inflationary model of the universe in a different context. In this work, we have obtained the solution of a complicated set of non-linear differential eqs. under consideration of Bianchi – III space metric with a framework of a perfect fluid and cosmological constant. To get the relativistic model, we have assumed that is proportionality relation between shear and expansion which leads to $b = c^k$ where $b$, $c$ are metric functions and $k$ is a constant. Structural features of the model are pointed.

2. Metric and field equations

Bianchi – III line elements can be defined as

$$ds^2 = -dt^2 + a^2 dx^2 + b^2 e^{-2t} dy^2 + c^2 dz^2$$

(2.1)

here $a$, $b$ and $c$ are metric functions of cosmic time.

Energy-momentum tensor for the perfect fluid is

$$T_{uv} = (\rho + p) V_u V_v + pg_{uv}$$

(2.2)

Here $P$ and $\rho$ indicates thermodynamic pressure and energy density. $V_u$ indicates the four velocity tensor for the fluid obeys the criteria

$$V_u V^u = -1$$

Equation of state is satisfied as

$$p = \omega \rho, \ 0 \leq \omega \leq 1$$

(2.3)

The Einstein field eq. in terms is of Λ given by

$$R_u^r - \frac{1}{2} R g_u^r = -T_u^r + \Lambda g_u^r$$

(2.4)

Proper volume $V$ for the model (2.1) may be obtained as

$$V = R^3 = abc$$

(2.5)

Hubble parameter ($H$) for this model defined as
\[ H = \frac{R_a}{R} = \frac{1}{3} \left( \frac{a}{a} + \frac{b}{b} + \frac{c}{c} \right) \]  
\ \ \ \ \ (2.6)

Where symbol 4 indicates time derivative

\[ H = \frac{1}{3}(H_1 + H_2 + H_3) \]  
\ \ \ \ \ (2.7)

Where \( H_1 = \frac{a}{a}, H_2 = \frac{b}{b} \) and \( H_3 = \frac{c}{c} \) are directional components of the Hubble factors in the co-ordinates directions.

The field equations (2.4) for the model (2.1) reduces to

\[ \left[ \frac{b_{44}}{b} + \frac{b_{24}}{bc} + \frac{c_{44}}{c} \right] = -p + \Lambda \]  
\ \ \ \ \ (2.8)

\[ \left[ \frac{a_{44}}{a} + \frac{a_{24}}{ac} + \frac{c_{44}}{c} \right] = -p + \Lambda \]  
\ \ \ \ \ (2.9)

\[ \left[ \frac{a_{44}}{a} + \frac{b_{44}}{ba} + \frac{c_{44}}{c} - \frac{a^2}{a} \right] = -p + \Lambda \]  
\ \ \ \ \ (2.10)

\[ \left[ \frac{b_{44}}{ab} + \frac{a_{44}}{ca} + \frac{b_{44}}{bc} - \frac{a^2}{a} \right] = p + \Lambda \]  
\ \ \ \ \ (2.11)

\[ \alpha \left( \frac{b_4}{b} - \frac{a_4}{a} \right) = 0 \]

\[ \frac{b_4}{b} - \frac{a_4}{a} = 0 \]  
\ \ \ \ \ (2.12)

3. Field equation with solution

Equation (2.12) provides

\[ \frac{b_4}{b} = \frac{a_4}{a} , \]  
\ \ \ \ \ (3.1)

on solving

\[ a = lb \]  
\ \ \ \ \ (3.2)

\( l \) is the integrating constant

From equation (2.5) and (2.6) with (3.1) we have

\[ \frac{b_{44}}{b} - \frac{c_{44}}{c} + \frac{b_4}{b} \left[ \frac{b_4}{c} - \frac{c_4}{c} \right] = \frac{1}{l^2b^2} \]  
\ \ \ \ \ (3.3)
To solve the system of six independent equations containing seven unknown quantities $a$, $b$, $c$, $\rho$, $p$ and $\Lambda$ an extra relation among expansion scalar and shear term is assumed which provides an appropriate relation between metric scalars such as

$$b = c^k$$

(3.4)

where ‘k’ is the constant

By equation (3.4) and (3.3), we get

$$\frac{c_4}{c} + 2k \left( \frac{c_4}{c} \right)^2 = \frac{1}{T^2 (k-1)c^{2k}}$$

($k \neq 1 \& l \neq 0$)

(3.5)

Further from the equation (3.5), we have

$$c_4 = \sqrt{\frac{1}{T^2 (k-1)c^{2k-1} + \beta}}$$

(3.6)

here $\beta$ has constant value

Equation (3.2-3.6) with equation (2.1) reduces in to form

$$ds^2 = -\left[T^2 (k^2 - 1)c^{2k-1} + \beta\right]dc^2 + T^2 c^{2k}dx^2 + C^{2k}e^{-2s}dy^2 + c^2 dz^2$$

(3.7)

using suitable transformation

$$ds^2 = -\left[T^2 (k^2 - 1)T^{2k-1} + \beta\right]dT^2 + T^2 T^{2k}dx^2 + T^{2k}e^{-2s}dy^2 + T^2 dz^2$$

(3.8)

4. Physical and structural features

The expression for the energy density $\rho$ and cosmological constant $\Lambda$ is given by

$$\rho = \frac{\gamma}{T^{(2k+1)\alpha+1}}$$

(4.1)

where $\gamma$ is constant

$$\Lambda = \frac{1}{(\omega+1)T^2} \left[ \frac{T^2 (k^2 - 1)(4k - 3 + 2\omega)}{2k\beta(\omega + 1) + \frac{k^2}{\gamma^2} [T^2 (k^2 - 1)T^{2k-1} + \beta]} \right] - \frac{1}{T^{2k}}$$

(4.2)

$$\theta = \frac{(2k+1)\left[ T^2 (k^2 - 1)T^{2k-1} + \beta \right]^{1/2}}{T}$$

(4.3)

$$\sigma = \frac{(k-1)\left[ T^2 (k^2 - 1)T^{2k-1} + \beta \right]^{1/2}}{\sqrt{3}T}$$

(4.4)
Case – I

For \( \rho > 0, \gamma > 0 \), the model is singular at initial stage. The terms \( \rho, \Lambda, \sigma \) and \( \theta \) approaches to infinite initially and continuously diminishes towards \( T = \infty \). Since the ratio of shear and expansion scalar is non-zero, i.e. the model is anisotropic. Finally it concludes the space-time describes non-rotating, shearing, accelerated fate of the cosmos and begin with Big-Bang. Cosmological term \( \Lambda \) tends to infinite at initial conditions and decreases slowly tend zero to a large value to \( T \).

Case – II

For \( \beta = 0 \), the model reduces to

\[
\frac{ds^2}{dt^2} + \frac{k^2T^2}{(k^2-1)} dx^2 + \frac{k^2T^2}{a^2(k^2-1)} e^{-2T} dy^2 + \left( \frac{kT}{k_T \sqrt{k^2-1}} \right)^{2k} dz^2
\]

(4.5)

using appropriate transformation equation (4.5) give

\[
\frac{ds^2}{dt^2} = -dT^2 + T^2 dx^2 + T^2 e^{-2x} dy^2 + T^{2k} dz^2
\]

(4.6)

The parameter \( \sigma \) and \( \theta \) are for the model (4.6) is

\[
\sigma = \frac{k-1}{\sqrt{3kT}} \quad (4.7)
\]

\[
\theta = \frac{2k+1}{kT} \quad (4.8)
\]

\[
q = -1 + \frac{H}{H^2} = \frac{RR_{\mu}}{(R_{\mu})^2} \quad (4.9)
\]

\[
q = \frac{k-1}{\sqrt{2k+1}} \quad (4.10)
\]

Since the fraction of shear and expansion is constant, the model maintained anisotropy and deceleration scalar \( q \) is also constant the proper volume becomes zero as \( T \) tends to zero, Thus the singularity exists at the initial frame. The cosmological term inversely proportional to the age of the cosmos and approaches to zero as \( T \to \infty \). The deceleration factor is constant for all time.

5. Discussion

In this study, we obtained the solution of a complicated set of nonlinear differential eqs. under consideration of Bianchi – III space metric with a framework of the perfect fluid and cosmological constant. The constructed model leads to point type singularity at the initial epoch and cosmological term varies inversely as the life span of the universe. The cosmological term
becomes infinitely large at the initial stage, it tends to zero at late times. The parameter of deceleration is constant at all

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