Adjudication with Rational Jurors

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Abstract
We analyze a mechanism for adjudication involving majority voting and rational jurors, who might not be willing to exert effort to properly judge a given case. The mechanism rewards jurors who vote in accordance with the final verdict and optionally punishes jurors who do not. We give bounds on the number of jurors and payments that are sufficient to guarantee a bounded error rate of the resulting adjudication. We show that the mechanism results in a non-trivial adjudication for sufficiently large payments provided that sufficiently many jurors are well-informed (on average). We consider different classes of jurors and show how to instantiate the payments to bound the error rate of the resulting system. Our work has applications to decentralized dispute resolution systems like Kleros.

1 Introduction
We consider the problem of incentivizing jurors to properly assess case evidence, such that the resulting adjudication is better than random. The jurors are assumed to be rational and indifferent to the outcome of the case. As such, the naïve solution of simply paying jurors for their participation does not work, as presumably assessing the case evidence requires an effort by the jurors. Hence, it is rational for a juror to vote randomly with zero effort, producing an adjudication that is trivial. Instead to produce a non-trivial adjudication, we have to pay jurors in some way conditioned on their vote. Hopefully, if the jurors are happy with their payments they will make a reasonable effort to assess the case evidence.

We ask the following question.

Can we design a mechanism for adjudication that is better than random using rational jurors?

Concretely, we analyze the mechanism Kleros proposed by Lesaeghe et al. [2019; 2021] where jurors are paid for voting in favor of the majority, and lose for voting otherwise. Intuitively, this is secure because jurors expect other jurors to vote honestly, so that for sufficiently large payments, a juror has an interest to exert the necessary effort. This is known in economics as a focal point, defined by Schelling [1960] as the strategy chosen in absence of coordination; however, it is not obvious that the focal point of adjudication is honesty. Lesaeghe et al. show that Kleros is truthful, though they do not take into account that jurors lose utility by exerting effort to judge the case. They also do not analyze the error rate of the resulting system, and do not give concrete values for parameters such as rewards or the size of the jury. Until now, to the best knowledge of the authors, it was unclear exactly which assumptions are necessary to produce a non-trivial adjudication with rational jurors.

The problem appears naturally in distributed systems where such a system can be used to forward information from the physical world; this is also the motivation for Kleros that is implemented on the Ethereum blockchain. This provides a powerful primitive for the analysis of distributed protocols, offering an explicit means of interacting with the physical world. Successful deployment of such a system would have applications in decentralized solutions to e.g. supply chain management, commerce, and banking.

1.1 Related Work
Incentivizing agents to report their beliefs truthfully using payments is a well-studied problem in mechanism design. A mechanism that has this property is said to be truthful. Proper scoring rules (Selten [1998]; Gneiting and Raftery [2007]) can be used to elicit truthful predictions from parties. An important theorem by Myerson [1981] gives a large class of mechanisms that can be implemented truthfully. Examples of famous truthful mechanisms include Vickrey [1961] (or second-price) auctions, Bayesian truth serum by Prelec [2004] for voting, or the generic Vickrey-Clarke-Groves mechanism [1961; 1971; 1973] for selecting a socially optimal outcome. However, none of these works are directly applicable to our setting where jurors can exert varying effort.

In cryptography, an entity that forwards information from 'off-chain' resources onto a blockchain is known as a blockchain oracle as discussed by Caldarelli and Ellul [2021]. Designing such an oracle has various applications in rational cryptography. Blockchain oracles were used by Schwartzbach [2021] to design a rational smart contract for decentralized commerce of physical goods. It is a program that holds the payment from the buyer in escrow until the trade has been completed, and settles disputes by allowing each party to ‘threaten’ the other to
invoke the oracle if they are unhappy. The contract is shown to be secure in a strong game-theoretic sense iff the oracle is strictly better than random. They suggest using Kleros to implement the oracle, though as mentioned no formal analysis has been made on the error rate of Kleros.

Chainlink by Breidenbach et al. [2021] is a general-purpose cryptographic framework for designing smart contracts that use blockchain oracles. The nodes run a consensus protocol (such as BFT) to forward off-chain data onto the chain. To participate, a node is required to submit a stake that is optionally repaid. This has the effect of ensuring security against an attacker who bribes the nodes. The whitepaper is largely focused on the cryptographic implementation and on applications of the system. Chainlink is modular and thus transparent to the implementation of the actual oracle; they do not propose a concrete implementation. They suggest using a reputation-based approach, though no formal claims on its security are made. Kleros, as mentioned, is a candidate for implementing this oracle. It is deployed on the Ethereum blockchain and uses the mechanism we analyze in this paper. In fact, its mechanism is slightly more involved than in the present work, owing primarily to issues related to the implementation, such as the mitigation of Sybil attacks through staking of money. Kleros is interesting because it is used in practice; at the time of writing, Kleros has settled 1000+ disputes.

1.2 Our Contributions

We analyze a mechanism for adjudication involving rational jurors who can exert varying effort to judge a given case. In the trivial case, parties may choose to exert zero effort, thus effectively flipping a coin to determine the outcome. The mechanism is a simplification of Kleros, based on the idea of rewarding parties who vote in favor of the output of the jury, and optionally punishing parties who do not. We define the payment of a juror as the sum of their rewards and punishment. We show that as long as the payments are sufficiently large, the dominant strategy results in an adjudication that is strictly better than random, provided sufficiently many jurors are well-informed. Such an adjudication is said to be non-trivial. We give conditions for the existence of a non-trivial equilibrium, and show that when it exists it Pareto-dominates the trivial equilibrium.

Theorem 1. The trivial strategy is always an equilibrium. There is a non-trivial equilibrium that Pareto-dominates the trivial equilibrium if, on average, the jurors are well-informed and the payments are sufficiently large.

We consider different classes of jurors and give concrete bounds on the size of these rewards and punishments necessary to ensure the equilibrium strategy results in an adjudication that has a known bound on its error rate. As a corollary, we improve the analysis in the Kleros yellow paper and show that Kleros with the right choice of parameters is secure in a strong sense. The results are stated here as asymptotic results for brevity; in the main body, we give concrete bounds on the rewards and punishments. We consider two types of jurors: the simplest stepwise juror produces a vote that is better than random only above a threshold effort. Formally, such jurors guess randomly below the threshold and guess correctly with probability $p > \frac{1}{2}$ above the threshold. For each such juror, we are forced to use a constant-sized payment; hence the cost of the adjudication asymptotically equals the number of jurors. Caragiannis et al. [2016] show that for majority voting, a total of $\Omega(\ln \varepsilon^{-1})$ votes (jurors) are required to recover the ground truth with probability $\varepsilon$. We show that we can make do with $O(\ln \varepsilon^{-1})$ jurors which is optimal. We also consider exponential jurors for which the quality of their vote is exponential in the effort they exert. This allows us to strike a trade-off between the payments and the size of the jury. However, we show that this does not asymptotically improve the cost of the adjudication. Our main result is the following.

Theorem 2 (Main result). For stepwise and exponential jurors, the mechanism produces an adjudication with error $\varepsilon$ for a total cost of $O(\ln \varepsilon^{-1})$, and this is optimal.

The $O$-notation hides constant factors that depend on the exact distribution of jurors.

2 Preliminaries

We will use the following version of the Hoeffding bound.

Lemma 1 ([Hoeffding, 1994]). Let $x_1, x_2, \ldots, x_n$ be i.i.d. random variables on $[0, 1]$, and let $\overline{X} = \frac{1}{n} \sum_{i=1}^{n} x_i$. Then, $\Pr[|\mathbb{E}[X] - \overline{X}| > \delta] < \exp(-2n\delta^2)$.

We mostly assume familiarity with game theory and refer to Osborne and Rubinstein [1994] for more details. We give a short definition to establish notation. A game $G$ in extensive form on $n$ players consists of a finite tree $T$ with a set $L \subseteq T$ of leaves. The set of branches $T \setminus L$ is partitioned into $n$ sets, one belonging to each player. We say a node belongs to a player if it belongs to their partition. There is a mapping $u : L \rightarrow \mathbb{R}^n$ that for every leaf $\ell \in L$ decides the utility $u_i(\ell)$ for the player $P_i$ when the game terminates in $\ell$. The game is played as follows. We start at the root of the tree, letting the player who owns that node choose a child to recurse into. A strategy $\sigma_i$ for a player is then a set of distributions, one for each branch, over the children of that branch. When the player arrives at a branch, they sample a child from the distribution and recurse. A set of strategies $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$, one for each player, is said to be a strategy profile and determines a distribution on the leaves. We overload notation and let $u_i(\sigma)$ be the expected utility of $P_i$ when playing the strategy profile $\sigma$. If $I \subseteq \{1, n\}$ is a set of indices, we may write $\sigma = (\sigma_I, \sigma_{\bar{I}})$. A strategy profile $\sigma$ is said to be a (Nash) equilibrium if it is robust to unilateral deviations, that is for any $\sigma_i'$, it holds that, $u_i(\sigma) \geq u_i(\sigma_i', \sigma_{-i})$.

In our model, for technical reasons, we will settle for a slight weakening of the definition of an equilibrium, allowing for a small gap $\varepsilon$, i.e. that $u_i(\sigma) \geq u_i(\sigma_i', \sigma_{-i}) - \varepsilon$ for some parameter $\varepsilon > 0$. However, we will require that this gap is exponentially small in some parameter $\kappa$ that we call the security parameter. Formally, the gap must be a function $\varepsilon(\kappa)$ that satisfies $\varepsilon(\kappa) = o(1/p(\kappa))$ for any polynomial $p$. Such a function is said to be negligible in $\kappa$ as per cryptographic convention. The intuition is that the security parameter may be set to a moderate value to make the gap sufficiently small such that we can ignore it. The resulting equilibrium is said
to be a computational equilibrium and is standard in rational cryptography. We give the definition by Halpern et al. [2016] here, modeling such games as infinite sequences of games indexed by a security parameter.

**Definition 1** (Computational Equilibrium). Let $G = \{G(\kappa)\}_{\kappa \in \mathbb{N}}$ be a sequence of games with $n$ players $P_1, P_2, \ldots, P_n$, indexed by a security parameter $\kappa \in \mathbb{N}$, and let $\{\sigma^{(\kappa)}\}_{\kappa \in \mathbb{N}}$ be a non-uniform sequence of strategy profiles for $G$. We say $\{\sigma^{(\kappa)}\}_{\kappa \in \mathbb{N}}$ is a computational equilibrium for $G$ if for every $i \in \{1, n\}$, and every non-uniform sequence of strategies $\{\sigma^i(\kappa)\}_{\kappa \in \mathbb{N}}$ for $P_i$, there is a negligible function $\text{negl}$ such that,

$$u_i(\sigma^i(\kappa), \sigma^{-i}(\kappa)) - u_i(\sigma(\kappa)) < \text{negl}(\kappa)$$

In other words, any deviation from the equilibrium can at most result in negligible gain in utility. By increasing the security parameter slightly, the gain from deviating reduces to essentially zero and can be ignored in practice.

### 3 Adjudication with Rational Jurors

We consider a set of $n$ rational parties $P_1, P_2, \ldots, P_n$ with $n$ odd (to avoid ties), serving as jurors in some binary dispute whose verdict is a single bit. We assume $\pi^* \in \{0, 1\}$ is the "ground truth" of the dispute that we want to uncover through the mechanism. As a simplification, the jurors are assumed to vote independently. We suppose the jurors are indifferent to the outcome of the case such that their utility remains the same, no matter the outcome of the case (excluding payments). As input, each juror is given a string of evidence to be assessed. We assume assessing the evidence requires effort by a juror, such that an equilibrium in a naïve setting is for each party to vote randomly. Specifically, we let $\lambda_i \geq 0$ be the effort spent by party $P_i$ and denote by $f_i(\lambda_i) \in [0, 1]$ the 'quality' of the vote of $P_i$, i.e. the probability that $P_i$ will guess correctly the ground truth by exerting $\lambda_i$ effort. We assume the utility functions are quasilinear in the effort exerted and the payments received. The function $f_i$ is said to be the effort function of the juror $P_i$ and is assumed known by the juror $P_i$. We further assume $f_i(0) = \frac{1}{2}$ such that no effort results in a random choice, and that each $f_i$ is monotone. If $f_i$ is strictly monotonically increasing, we say $P_i$ is a well-informed voter, and otherwise we say $P_i$ is uninformed. For well-informed voters, we require in addition that their effort functions have diminishing returns in the sense that they are concave. This has the effect that the greatest gain is in the beginning and intentionally excludes effort functions that are e.g. sigmoidal.

We now describe how the adjudication mechanism works. Each juror $P_i$ is given evidence and casts a vote $x_i \in \{0, 1\}$ to the mechanism. The mechanism takes as input all votes $\{x_i\}_{i=1}^n$ cast by the parties and outputs $v \leftarrow R(x_1, x_2, \ldots, x_n) \in \{0, 1\}$ for some voting rule $R$. The mechanism then gives an amount $\omega \geq 0$ of money to every $P_i$ with $x_i = v$, and takes $\ell \geq 0$ money from every $P_i$ with $x_i \neq v$. Exerting zero effort always results in an expected payment of $\frac{1}{2}(\omega - \ell)$; hence, we assume that $\omega \geq \ell$ to ensure interim individual rationality. The payments can be implemented by using a deposit scheme, as in Kleros, where each $P_i$ starts the protocol by depositing $\ell$ money that is repaid unless they lose according to $R$. For a fixed adjudication, we denote by $\varepsilon \in [0, 1]$ its error rate.

Our overall goal is to express $\varepsilon$ in terms of $\{f_i\}_{i=1}^n$ and $\omega, \ell$ such that we can characterize how to instantiate $\omega, \ell$ to ensure the adjudication recovers the ground truth with probability significantly higher than $\frac{1}{2}$.

#### 3.1 Asymptotic Considerations

In the following sections, we will make a rather strong assumption that we shall attempt to justify here. Namely, we assume that a single juror cannot effect the outcome of the adjudication. Of course, this is false in general: unless the jurors are deterministic, there is always a non-zero chance that a single juror decides the final outcome. However, by a Hoeffding bound, the probability of this happening for majority voting decreases exponentially fast in the size of the jury. We say that majority voting is robust.

We will now adopt a formalism inspired by the ideal/real world paradigm from cryptography, see e.g. Canetti [2001]. Here, we explicitly consider an ideal world in which the jury is countably infinite, such that the assumption stated is true. We will conduct all our analysis in this ideal world game $G$, hoping to instantiate the payments to ensure the ideal adjudication is good. However, in practice, we run the protocol in the real world where we are forced to choose a concrete size $n \in \mathbb{N}$ of the jury. Following Halpern et al. [2016], we can imagine a countably infinite sequence of games $G = \{G_n\}_{n \in \mathbb{N}}$, one for each possible size of the jury. In principle, the analysis as conducted in $G$ breaks down in $G_n$ for any choice of $n$ as the latter has a larger strategy space. However, we can still say something about $G_n$, as hinted above: by using a robust voting rule, a juror who deviates from any ideal strategy can only benefit a negligible amount (in $n$) in the real world. Note that in general, there need not be a meaningful translation from strategies in $G$ to strategies in $G_n$. However, for the class of games we consider there is such a translation. Namely, the set of strategies in $G$ is $\mathbb{R}^{\mathbb{N}}$, corresponding to the efforts they can exert. In $G_n$, in addition, they might be able to decide the outcome of the adjudication to win a constant payment of size $\omega$, albeit with negligible probability in $n$; if we make $n$ sufficiently large, this strategy gives only a tiny advantage over any corresponding ideal strategy, and can be ignored. Let us make this argument more formal and let $\sigma$ be a strategy profile for $G$. Now, if there were a sequence of strategy profiles $\{\sigma_n\}_{n \in \mathbb{N}}$ that resulted in non-negligible gain in utility in the real world, it could not have been due to strategizing in the vote, thus giving a corresponding strategy in the ideal world, contradicting the assumption that $\sigma$ is an equilibrium. We conclude that any equilibrium in $G$ is a computational equilibrium in $G$ in the sense of Definition 1.
Lemma 2. Any equilibrium in the ideal world is a computational equilibrium in the real world.

For the remainder of the paper, we work in the ideal world, noting that all equilibria, technically speaking, are computational equilibria in the real world. This model is implicitly used in e.g. the Bayesian truth serum (BTS) model by Prelec [2004], and explicitly avoided to create ‘robust BTS’ by Witkowski and Parkes [2012]. In doing so, they effectively increase the variance of the payments; it is unclear if this is better in practice than allowing for these negligible errors.

3.2 Structure of Equilibria

In this section, we analyze the structure of the equilibria in game induced by the adjudication mechanism. We observe that for any payment rule there is a trivial equilibrium where all parties exert zero effort, resulting in a random adjudication. If the jurors are well-informed on average, and the payments are sufficiently large, we show there is an additional non-trivial equilibrium that Pareto-dominates the trivial equilibrium.

Formally, consider the one-shot ideal game that $P_i$ plays by choosing a value of $\lambda_i$, the effort they are willing to exert. For the sake of analysis, let us assume that $f_i$ is continuous and differentiable. Working in the ideal world, we may assume the adjudication recovers the ground truth $\pi^*$ with probability $1-\varepsilon$ for some constant $\varepsilon$, independent of the vote of $P_i$. Now, let us analyze the utility gained by a juror $P_i$ who chooses the strategy $\lambda_i$. They lose $\lambda_i$ utility from choosing the strategy $\lambda_i$. In addition, they gain $\omega$ if they agree with the adjudication, either if they both guess correctly or both guess incorrectly, and lose $\ell$ otherwise. By quasilinearity, the expected utility of $P_i$ is as follows.

$$E[u_i(\lambda_i)] = -\lambda_i + f_i(\lambda_i)(1-\varepsilon)\omega - f_i(\lambda_i)\varepsilon\ell$$
$$+ (1 - f_i(\lambda_i))\varepsilon\omega - (1 - f_i(\lambda_i))(1 - \varepsilon)\ell$$
$$= -\lambda_i + f_i(\lambda_i)(1 - 2\varepsilon)(\omega + \ell) + \varepsilon\omega - (1 - \varepsilon)\ell. \quad (1)$$

For a fixed $\varepsilon$, a juror $P_i$ chooses $\lambda_i^*$ to maximize Eq. (1); however, this assumes $\varepsilon$ is independent of $\lambda^*$ which is not the case in practice. We say a strategy profile $\lambda^*$ is compatible with $\varepsilon$ if it results in an adjudication with an error $\leq \varepsilon$.

Theorem 1. The trivial strategy is always an equilibrium.

There is a non-trivial equilibrium that Pareto-dominates the trivial equilibrium if sufficiently many jurors are well-informed and the payments are sufficiently large.

Proof. To show the mechanism always has a trivial equilibrium, let $\omega, \ell$ be arbitrary. Note that when all jurors play $\lambda_i = 0$, in the ideal world, the resulting adjudication has an error of $\varepsilon = \frac{1}{2}$. As a result, the expected utility for $P_i$ is $-\lambda_i + \frac{1}{2}(\omega - \ell)$ which is clearly maximized for $\lambda_i = 0$ that in particular is compatible with $\varepsilon = \frac{1}{2}$.

For the latter claim, it is easy to see there can only be a non-trivial equilibrium if there are sufficiently many well-informed voters, as we also have the dual case where uninform voters invest some effort to produce the incorrect outcome. Now, since $f_i$ are all concave and continuous, it suffices to show there is an equilibrium with some well-informed jurors having $\lambda_i^* > 0$. Suppose $\lambda^*$ is such an equilibrium and let $\varepsilon < \frac{1}{2}$ be its error rate. Now consider the strategy of a well-informed juror $P_j$. We compute the derivative.

$$\frac{\partial}{\partial \lambda_j} E[u_j(\lambda_j)] = -1 + f'_i(\lambda_i)(1 - 2\varepsilon)(\omega + \ell) \quad (2)$$

For well-informed jurors, the derivative at $\lambda_j = 0$ is strictly positive when $\omega + \ell > \frac{2}{\varepsilon^2}$, meaning $P_j$ benefits by putting in some amount of effort. Note that this is compatible with some $\varepsilon < \frac{1}{2}$. Since $L$ was chosen arbitrarily, this concludes the proof.

When the payments are sufficiently large (i.e. $\omega + \ell > \frac{2}{\varepsilon^2}$ for every $i$), and there are sufficiently many well-informed jurors, we say the mechanism is sound. We observe that when we let $\omega = \ell$, we obtain a desirable property.

Proposition 1. If the mechanism is sound and $\omega = \ell$, it is strictly interim individually rational for $P_i$ iff $P_i$ is well-informed.

Proof. Since the mechanism is sound, we know $\varepsilon < \frac{1}{2}$. Thus, if $P_i$ is uninformed they maximize their utility by playing $\lambda_i = 0$. In this case, when $\ell = \omega - \delta$, Eq. (1) reduces to $\delta/2$ which for $\delta = 0$ gives a utility of $0 \neq 0$. Instead, as the payments are sufficiently large and $P_i$ is well-informed, the gradient $\frac{\partial}{\partial \lambda_i} E[u_i(\lambda_i)]$ is strictly positive, which means the equilibrium has expected utility $> 0$ for $P_i$.

3.3 Security of Adjudication

We now define what it means for an adjudication to be secure.

Definition 2. An adjudication is $\varepsilon$-secure if every non-trivial equilibrium is $\varepsilon$-compatible.

Unfortunately, finding the precise equilibrium turns out to be difficult because each $\lambda_i^*$ and $\varepsilon$ depend on each other: we only have the general Hoeffding bound to relate the two quantities that might leave a gap. Specifically, a particular choice of $\lambda_i^*$ will likely lead to an adjudication error that is lower than the bound claims, which in turn changes the optimal value of $\lambda_i^*$ and once again possibly changes the bound.

If $\lambda$ is a strategy profile, we denote by $\rho(\lambda)$ the resulting strategy profile. Note that $\lambda$ is an equilibrium iff it is a fixed point. We say $\lambda$ has a finite orbit if there is an integer $k$ such that $\rho^k(\lambda)$ is a fixed point.

Lemma 3. $\rho$ is monotonically increasing.

Proof. Let $\lambda^*$ be a strategy profile, and let $\lambda^{*'} = \rho(\lambda^*)$. If $\lambda^*$ is a fixed point, we are done, so assume $\lambda^*$ is not a fixed point. Now let $\varepsilon$ (resp. $\varepsilon'$) be the fixed error rate for which $\lambda^*$ (resp. $\lambda^{*'}$) is an equilibrium. By Eq. (2) the equilibrium condition for $P_i$ is as follows.

$$\frac{\partial}{\partial \lambda_i} E[u_i(\lambda_i)] \bigg|_{\lambda_i = \lambda_i^*} = 0 \iff f'_i(\lambda_i^*) = \frac{1}{(1 - 2\varepsilon)(\omega + \ell)} \quad (3)$$

Since $\lambda^*$ is not a fixed point, we have $\varepsilon > \varepsilon'$, hence the term $(1 - 2\varepsilon)$ is increased by application of $\rho$. It follows that $f'_i$ is made smaller which happens precisely when $\lambda_i^*$ is increased by concavity of $f'_i$. \qed
Lemma 4. If the non-trivial equilibrium against a fixed arbiter with an error of $\varepsilon$ is $\varepsilon$-compatible, then the adjudication is $\varepsilon$-secure.

Proof. Let $\lambda^{(0)}$ be the non-trivial equilibrium against the fixed arbiter, and define $\lambda^{(k+1)} := \rho(\lambda^{(k)})$. If $\lambda^{(0)}$ has finite orbit, we are done by Lemma 3 and monotonicity of $f_i$, so assume $\lambda^{(0)}$ does not have finite orbit. Note that since $\omega + \ell$ is constant, the sequence $L = \{\lambda^{(k)}\}_{k \in \mathbb{N}}$ is bounded, hence it has a limit by the monotone convergence theorem, let $\lambda^* = \lim_{k \to \infty} \lambda^{(k)}$. Since $L$ is bounded and monotone, it is also Cauchy, and hence $\lambda^*$ is uniquely determined and $\varepsilon$-compatible. Finally, it is not hard to see that $\lambda^*$ is a fixed point of $\rho$ since $\rho(\lambda^{(k)}) \to \rho(\lambda^*)$ and $\rho(\lambda^{(k)}) = \lambda^{(k+1)} \to \lambda^*$.

4 Adjudication with Majority Voting

In this section, we take a closer look at the mechanism when instantiated with majority voting. As a warm-up, we consider a simple model where each $f_i$ is a constant function parameterized with majority voting. As a warm-up, we consider the latter bound. Due to the discrete nature of the effort functions, the resulting game is inherently discrete and in particular, can be represented as a finite one-shot game where $P$ can choose to play randomly by exerting $\lambda_i = 0$ effort, or play honestly by exerting $\lambda_i = T_i$ effort. All other values of $\lambda_i$ are strictly dominated by one of these two strategies. By Eq. (1), we can ensure honesty when,

$$-T_i + p_i (1 - 2\varepsilon)(\omega + \ell) + \varepsilon\omega - (1 - \varepsilon)\ell > \frac{1}{2} (\omega - \ell).$$

Substituting and solving for $\varepsilon$ immediately shows the latter bound.

4.2 Majority Voting, exponential $f_i$’s

In this section, we consider jurors with exponentially decaying effort functions. We model this using functions of the form:

$$f_i(\lambda_i) = \beta_i + \left(\frac{1}{2} - \beta_i\right) \exp(-\alpha_i \lambda_i),$$

where $\alpha_i \geq 0$ and $\beta_i \in [0, 1]$, with well-informed voters having $\beta_i > \frac{1}{2}$. The parameter $\alpha_i$ measures how ‘smart’ the juror $P_i$ is, with higher values of $\alpha_i$ corresponding to recovering the ground truth faster. By Proposition 1, an uninformed
Suppose we want an adjudication with a fixed error in \( \varepsilon \). For the stepwise jurors this also fixes the choice of \( n \) as the bound on \( \varepsilon \) depends only on \( n \) and \( \Delta P \) which is fixed. However, for exponential jurors there is a tradeoff between \( n \) and \( \Delta P \). If there are few jurors we need to pay them a lot of money to exert enough effort. If there are too many jurors, we have to pay the reward too many times so we seek a tradeoff. We can find an optimal set of parameters resulting in an \( \varepsilon \)-secure adjudication by minimizing \( n(\omega + \ell) \) subject to having an error of at most \( \varepsilon \). To enforce this constraint, by Lemma 5 we can let \( \delta = \sqrt{\frac{\ln \varepsilon^{-1}}{2n}} \) to ensure \( \lambda^* \) is \( \varepsilon \)-compatible. Suppose for simplicity that all jurors have \( \alpha_i = \alpha \) and \( \beta_i = \beta \).

**Proposition 3.** With exponential effort functions, the majority-vote adjudication is \( \varepsilon \)-secure with minimal total cost when,

1. \( n = \frac{9 \ln \varepsilon^{-1}}{4(2\beta - 1)^4} \); and,
2. \( \omega + \ell = \frac{2\alpha}{(1 - 2e)(2\beta - 1 - 2\beta^2(2\beta - 1))} \).

In this case, the total cost is \( O(\ln \varepsilon^{-1}) \) as \( \varepsilon \ll \frac{1}{2} \).

**Proof.** Note that minimizing the objective function is equivalent to minimizing \( f_{a,b}(n) := \frac{a}{\ln n} \) for suitable \( a, b \). Let \( c := (a/b)^2 \). When \( a > 0, b > 0 \), this function is undefined at \( n = c \); it is negative for \( n < c \), and positive and (almost) convex for \( n > c \), see Fig. 2. Note that we are only interested in the latter domain. We find \( n \) by setting the derivative to zero and solving for \( n \); this gives \( n^* = \sqrt{\frac{a}{c}} \). Substituting \( a = \sqrt{\ln \varepsilon^{-1}} \) and \( b = 2\beta - 1 \) gives 1., while 2. is obtained by substituting \( n \) into Eq. (5).

To show we have indeed found the global minimum, we compare the second-order derivative with zero.

\[
\frac{\partial^2}{\partial n^2} f_{a,b}(n) \geq 0 \iff 3a^2 - ab \sqrt{n} < 0 \iff n < 9c.
\]

This shows the function is convex for \( n \in (c, 9c) \). Finally, note that \( \frac{\partial}{\partial n} f_{a,b}(n) \geq 0 \) whenever \( n \geq n^* = \frac{9}{4}c \).

5 Conclusion and Future Work

In this paper, we analyzed a class of mechanisms for adjudication with rational jurors. We showed that the mechanism produces a non-trivial adjudication whenever the payments are sufficiently large. We considered two classes of jurors and gave bounds on the size of the payments to ensure the adjudication has bounded error rate.

As future work, it would be interesting to instantiate the mechanism using a different voting rule such as surprisingly popular voting by Prelec [2004], as this might enable the mechanism to recover the ground truth even if most jurors are uninformed. Here, voters report both their vote as well as a prediction of how the other votes are distributed. The mechanism then selects the answer that is surprisingly popular (SP). It is shown by Prelec et al. [2017] that under reasonable assumptions, SP voting recovers the ground truth even in situations where the majority of voters are uninformed and majority vote would fail. Prelec et al. also demonstrate that SP voting works well in practice, outperforming majority-voting by as much as 30-40%. Prelec [2004] proposes a truthful mechanism for SP voting based on the logarithmic scoring rule; however, it does not take into account that jurors can exert varying effort to receive a better or worse signal. It would be interesting to replace majority-vote with SP voting to allow the mechanism to recover the ground truth even in complex cases where expert opinion is needed but hard to come by.
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