Modelling the radar signature of rotorcraft

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Abstract
The problem of radar helicopter detection and classification requires simple and fast modelling of rotorcraft signature to aid the design of specific waveforms or to provide quality training data to machine learning algorithms. Here, we present a modular parametric model for the baseband radar cross-section of helicopters in the narrowband approximation. Every significant element of the complex rotorcraft signature is identified and its signature is reproduced faithfully. The full model is validated by comparing simulated data with real radar helicopter data. Particularly important features, such as the overall time-frequency response, blade flash shape and blade flash duration, are shown to be reconstructed very accurately.

1 | INTRODUCTION

Rotorcraft are challenging targets to detect by means of radar because of their ability to hover at low speed and low altitude. This enables them to be masked by the clutter response, eventually being filtered out of data by clutter blanking processing [1]. Identification of this type of target is, however, a strategic issue.

Fortunately, as helicopters rely on large rotating parts to fly, their radar return signal is strongly modulated with characteristic frequencies directly related to that of the craft rotors. This takes the form of a strong modulation of the radar cross-section (RCS) in the time domain, with the formation of periodic surges or flashes with a small duration (typically on the order of 100 μs in X-band). This tell-tale response can be used not only to detect the presence of a helicopter, but also to classify this detection, because the micro-Doppler response of rotorcraft is directly related to the physical characteristics of rotors. If these problems used to be addressed by various algorithms aiming at the extraction of specific features of the helicopter signature [2–4], the recent progress made by machine learning-based processing for micro-Doppler feature recognition [5–7], and their superior capabilities, will pave the way for the deployment of this technology in actual radar systems in the coming years.

However, such algorithms need a large amount of data in order to be trained adequately and perform optimally. As real radar data, especially in the case of airborne radar, is costly to obtain and have low availability, mainly due to classification issues, they often cannot be supplied in an adequate amount to that purpose. Instead one has to resort to simulated data, which can be generated in virtually any quantity at almost no cost. The choice of the simulation method is also important because data must be generated in large quantities in a relatively short time. While electromagnetic modelling, based on the resolution of Maxwell's equation using a discrete representation of the target (typically a computer-assisted drawing), is very accurate, it is also cumbersome and difficult to use for the simulation of highly dynamic situations, as in the case of a flying rotorcraft [8]. The complexity of such models can be decreased by reducing the target to a cloud of judiciously positioned point scatterers [9–11] or to a simpler triangular mesh [12]. Still, they remain heavy to use, with one simulation run needed per rotor position and per view angle, while their performance can be deemed inadequate for target recognition purposes [13, 14]. On the other hand, it is well known that the characteristic micro-Doppler response of the rotating parts of helicopters can be phenomenologically reproduced by considering rotor blades and rotor hub parts as simple geometrical elements, for which the analytical expression of their radar response can be computed [15, 16]. Parametric models
based on such representations are very fast because they rely on the use of simple equations, but they are not able to reproduce fundamental elements of the rotorcraft signature like rotor blade shape, as we showed in our previous works [17, 18].

The parametric model we devised is based on the latter representation, but we managed to address the issue of synthetic data representativeness by the careful physical modelling of the main features of the rotorcraft. Two hypotheses were made to make it fast, yet accurate:

- the model is valid in the narrowband approximation only, that is the radar bandwidth must be small enough for the helicopter not to be resolved in range;
- the model does not take multiple reflections occurring on the target into account.

Here, we start by analysing the characteristic radar response of a helicopter. Based on this analysis, we then present in detail the modelling of each identified significant contributor to this response. The model is then confronted to real helicopter data for two different rotorcrafts. The resulting comparison is shown to be very accurate with respect to reality.

2 | ANALYSIS OF A CHARACTERISTIC HELICOPTER RADAR RESPONSE

In this section, we characterize the main constituents of the radar response of rotorcraft. As seen in Figure 1(a), a helicopter is usually made of the following components:

- the rotorcraft body, designed to carry pilots, passengers and/or cargo, depending on the craft destination;
- a main rotor, which consists of a given number of identically built blades. These blades are affixed to the main rotor shaft through a complex mechanical assembly, called the rotor hub, which enables the pilot to control the blades’ inclination and angle of attack. The goal of the main rotor is to provide enough lift to the craft to take off and fly;
- a secondary rotor, or tail rotor, which is a smaller, faster spinning rotor than the main rotor, and of which axis of rotation is orthogonal to that of the main rotor. Its purpose is to counteract the torque imparted by the main rotor on the helicopter, which would otherwise rotate it in the opposite direction of the main rotor and therefore make it impossible to fly in a controlled way.

Other configurations exist, like the well-known CH-47 Chinook with its two juxtaposed, counter-rotating main rotors, or the Kamov Ka-50 with two superimposed, counter-rotating rotors. Yet, our model is able to deal with any of them without restriction.

We now investigate the real data recorded by a forward-looking, airborne X-band radar on a large, four-bladed helicopter. To this purpose, the complex baseband signal of the target range cell is extracted, yielding the slow-time vector, normalized to unit power, displayed in Figure 1(b). As seen in this graph, nothing can be inferred about the presence of a target from the time signal only. A Doppler analysis is necessary to reveal the different components of the radar signal. Figure 1(c) displays the power spectral density (PSD) computed from the signal of Figure 1(b), so that one can then now distinguish the different contributors to this complex spectrum:
• first, the dominating component comes from a strong and thin spectral line around zero Doppler shift. This corresponds to clutter, or unwanted radar echoes from the static environment. This explains why nothing of the helicopter response can be seen in the time signal;

• second, another narrow spectral line (in red) can be seen slightly on the right of the clutter line. This comes from the helicopter body, which does not display any micro-Doppler component and therefore results in a simple Doppler response. Given the Doppler shift associated to the body line, one can conclude the helicopter was moving towards the radar. Had it been in stationary flight, the body line would have been completely masked by the clutter line at zero Doppler shift;

• third, below the body line and more or less symmetrically on each side of it, the PSD decreases in a characteristic triangular way. This comes from the main rotor hub;

• finally, on each side of the rotor hub response, there are two plateaus with a slightly different PSD level. The one on the right, on the positive Doppler side, comes from rotor blades that are rotating towards the radar. The one on the left, on the negative Doppler side, comes from blades that are rotating away from the radar. The different PSD levels are explained by the variability of the RCS between a blade’s leading and trailing edges. Note that here, contributions of both the main and the tail rotor are mixed and cannot be separated.

A time–frequency analysis can then bring even more information about the target dynamics. Looking at the spectrogram displayed in Figure 1(d), one can clearly distinguish the periodic nature of the radar signature. Blade flashes for the main rotor occur with a period of approximately 3000 radar pulses, and the blades’ leading edges have a significantly higher RCS than their trailing edges. The characteristic blade tip signature, with its quasi-sine behaviour, is also clearly visible. Even receding tail rotor blade flashes can be seen on the negative Doppler side.

From this analysis, it appears that the following components are to be modelled with care in order to accurately emulate the radar response of rotorcraft:

• the body;

• all rotors affixed to the craft, made up of:
  ◦ a rotor hub;
  ◦ a given number of blades; and
  ◦ blade tips, that is, strongly reflecting elements at the blades’ extremities.

The following section is dedicated to the detailed description of the individual modelling of each of these components.

### 3 | MODEL DESCRIPTION

The purpose of the model is to compute a complex scalar time function $s(t)$. $s$ is the ‘square root’ of the target RCS $\sigma(t)$, defined as the reflectivity, since the two are linked by the following relation:

$$|s(t)|^2 = \sigma(t).$$  \hspace{1cm} (1)

Since, the model is only valid in the narrowband approximation, as stated in the introduction, $s$ corresponds to the slow-time evolution of the range cell signal containing the whole helicopter response. To remain as general as possible, $t$ can take an arbitrary value and is not limited to fixed-PRF radar bursts, although almost all Doppler radar systems use such pulse trains in order to perform coherent integration on receive.

#### 3.1 | Helicopter body modelling

In order to design an accurate model for the helicopter body, we will start by a thorough study of the real recorded response. To this purpose, we make use of the time signal corresponding to the frequency bin occupied by the rotorcraft body in the spectrogram displayed in Figure 1(d). From this long complex vector, it is possible to estimate the corresponding temporal covariance matrix over a timescale much shorter than the total duration of the initial time series. In our case, we chose to perform our study using training vectors with size 500 for a total recorded frame of over 10,000 pulses. The resulting sample covariance matrix $\hat{R}$ is displayed in Figure 2(a).

As seen in this figure, the matrix $\hat{R}$ is almost real, with its imaginary part being about 20 dB below the level of its real part. Moreover it shows that the body signal is actually time-correlated, with the covariance slowly decaying with time, and that the correlation level is almost independent on the value of $t_1$ or $t_2$, showing that only the relative time difference $|t_1-t_2|$ matters. From this first analysis, a meaningful model for the covariance matrix is an exponential correlation in the form:

$$\hat{R}_\text{sim}(t_1, t_2) = \hat{R}_\text{sim}(\tau = |t_1-t_2|) = \sigma_b^2 e^{-\tau/T_c},$$  \hspace{1cm} (2)

where $\sigma_b^2$ is the variance of the body RCS and $T_c$ the characteristic decorrelation time. Performing a fit over the real part of $\hat{R}$ using this model yields the synthetic covariance matrix displayed in Figure 2(b), which appears to be in excellent agreement with the true estimated covariance matrix, showing the exponential correlation was indeed a good choice.

We thus established the correlation properties of the helicopter body signal, but we still need to find from which distribution it is drawn, though this random variable is usually considered as a Swerling 1 target [20]. Since, we have an estimate of the covariance matrix, we can use it to whiten the helicopter body time signal $s_{\text{body}}(t)$ following:

$$s_{\text{body, white}}(t) = \hat{R}^{-1/2} s_{\text{body}}(t).$$  \hspace{1cm} (3)

Then we can build the histogram for $|s_{\text{body, white}}|$, displayed in Figure 2(c). This yields a shape very close to that of a Rayleigh probability distribution function:

$$f_\sigma(x) = \frac{x}{\sigma^2} e^{-x^2/(2\sigma^2)}.$$  \hspace{1cm} (4)
Performing a fit over the histogram gives the solid red curve in Figure 2(c) and a value of 0.55 for \( \sigma \). This means that \( s_{\text{body, white}} \) is indeed very close to a standard complex normal variable, for which \( \sigma_{\text{theo}} = 1/\sqrt{2} \approx 0.7 \). Note the original helicopter body time signal extracted from the spectrogram in Figure 1(d) is probably corrupted by the low-frequency component of the main rotor hub signal due to the relatively large width of the selected Doppler bin, which might explain
why the body signal distribution is not perfectly complex Gaussian.

Although the fit of the sample covariance matrix $\hat{R}$ enables us to show that an exponential correlation model is valid, the relatively small time window at hand prevents us from correctly evaluate the decorrelation time $T_\text{c}$, which is obviously much longer than 500 pulses. A solution is to make the fit in the spectral domain instead, since the autocorrelation function and the PSD of a given signal are related through the Fourier transform. Starting from Equation (2), the corresponding frequency signal yields:

$$\text{PSD}_\text{body}(f) = \sqrt{\frac{2}{\pi}} \frac{\sigma_\text{c}^2 T_\text{c}}{1 + 4\pi^2 f^2 T_\text{c}^2}.$$ (5)

Fitting the helicopter body line of the full spectrum displayed in Figure 1(c) using this function, we obtain the red solid curve in Figure 2(d) and an accurate estimate of the decorrelation time, which appears to be on the order of 100 ms in our case. In the first approximation, a Swerling 1 target model for the rotorcraft body seems therefore accurate, because this model assumes a Rayleigh-distributed target amplitude that remains constant during a whole radar coherent integration interval, which is usually on the order of a few tens of milliseconds.

From this we can now describe the steps required for the generation of a synthetic helicopter body signal:

1. Generate a white standard complex normal random vector of length $N$, $s_0 \sim \mathcal{CN}(0, 1)$, and define the corresponding time vector $t$.
2. Generate the $N \times N$ covariance matrix $R$ using $t$ and Equation (2).
3. Compute the Cholesky decomposition of $R$, that is the triangular matrix $C$ satisfying $C^H C = R$, where $\cdot^H$ denotes the conjugate transposition operation.
4. Colour the white noise vector $s_0$ to yield the body reflectivity $s_b = C s_0$.

Figure 3 compares an example of a synthetic body signal generated from this procedure with the original body signal extracted from the spectrogram in Figure 1(d). In order to ensure a similar frequency content between the two signals, a spectrum was also generated from the full synthetic body vector, and the corresponding frequency bin was selected. As seen here, the modelled signal behaves quite closely to the original signal, with a similar frequency content.

3.2 | Rotor modelling

This section deals with the modelling of any rotor affixed to a helicopter. Such a rotor is made up of three different objects: the rotor hub, the rotor blades and the wingtips at the edge of each blade. All calculations are here done in the corresponding rotor Cartesian frame $\mathcal{R}_r$, in which the rotation axis corresponds to the z-axis. The $(x, y)$ plane defines the so-called rotor disc (see Figure 4). The considered generic rotor is made up of $N$ blades that are uniformly positioned in azimuth, splitting the disc into $N$ angular sectors of width $2\pi/N$. The whole rotor rotates with a constant angular velocity $\Omega_r$ is considered positive for a counterclockwise motion. The projection of the radar wavevector $\mathbf{k}_r$ on the rotor disc segments it into four parts, referred to as front right/left, and back right/left respectively.

3.2.1 | Rotor hub

The hub of a helicopter rotor is the name given to its central part, close to the rotor shaft. Its role is dual: it not only ensures the fixation of the rotor blades to the shaft, but also enables the pilot to control the pitch angle of each blade by means of dedicated rods and of a specific element called the swashplate. Many kinds of rotor hubs can be encountered on a different helicopter, but they always take the form of a complex mechanical assembly taking a significant volume with respect to the rotor dimensions (see Figure 5(a)). The main rotor hub is among the most important elements of a rotorcraft because it allows to control the flight direction. The tail rotor hub, if present, is usually less complex because it only controls the collective pitch of tail blades [21]. Moreover its smaller size and
First looking at the spectral signature of the main rotor hub as in Figure 1(e), one can see it takes the form of an approximately linear decay in the logarithmic scale, symmetrically with respect to the rotorcraft body line. This means the spectral density decreases in a roughly exponential fashion on a linear scale. This behaviour is not limited to the helicopter model signals from Figure 1 were recorded on, but appears to be quite general and not limited to a particular radio-frequency band, as seen elsewhere in the literature [3, 15, 17, 23, 24]. To reproduce this shape, point scatterers must therefore be placed in the radial direction using an exponential PDF:

$$f_{r, hub}(r) = \frac{1}{1 - c^2} \frac{2}{r_{hub}} e^{-2r/r_{hub}} H(r_{hub} - r), r \in [0, +\infty].$$  

(6)

Here, $r_{hub}$ stands for the physical hub diameter, and $H$ stands for the Heaviside step function, which ensures that all points lie within the cylinder with radius $r_{hub}$.

As for the azimuthal repartition of scatterers, it is based on the observation that hub components can be broadly segmented in two categories, as seen in Figure 5(a): first, elements that have a cylindrical symmetry like, for example, the swashplate, the hub cover and the rotor shaft. Second, those that are concentrated around the azimuthal position of the rotor blades such as control rods or blade roots. If we consider a single rotor sector with an azimuth opening angle $2\pi/N$ centred on the angular position of a blade $\theta_b$, the azimuth angle $\theta$ of scatterers in that sector is considered distributed as:

$$\theta \sim \alpha_b \mathcal{N}(\theta_b, (\Delta \theta_b)^2) + (1 - \alpha_b) \mathcal{U}(\frac{2\pi}{N})$$  

(7)

where $\mathcal{N}(a, b)$ stands for the normal distribution of the mean $a$ and variance $b$, and $\mathcal{U}(I)$ stands for the uniform distribution over the interval $(I)$. The weighting parameter $\alpha_b$ controls the repartition of scatterers between the blade-related category and the isotropic category. The angular width $\Delta \theta_b$ is chosen wide enough to cover both the blade root and its associated control rod. A value of 0.25 for $\alpha_b$ proved to result in an accurate hub signal reconstruction for the helicopters we studied, but should be adapted to a given rotor hub model.

As for the $z$-repartition of scatterers, it is chosen as uniform. If $h_{hub}$ is the hub height, then the $z$-PDF is therefore:

$$f_{z, hub}(z) = \frac{1}{h_{hub}} z, z \in [-h_{hub}/2, h_{hub}/2].$$  

(8)

Finally, the total hub representation using point scatterers can be constructed as follows: first, $M$ values are drawn independently using distributions from Equations (6) (7) (8):

$$\begin{align*}
    r_0 \sim & f_{r, hub}, r_0 \in \mathcal{M}_1, M(0, r_{hub}) \\
    \theta_0 \sim & f_{\theta, hub}, \theta_0 \in \mathcal{M}_1, M(0, 2\pi/N) \\
    z_0 \sim & f_{z, hub}, z_0 \in \mathcal{M}_1, M([-h_{hub}/2, h_{hub}/2])
\end{align*}$$  

(9)
These values correspond to the seed position of the scatterers. The position of scatterers for the azimuthal sector \( j \in [1, N] \) in the rotor frame \( \mathcal{R}_r \) is then computed as:

\[
\begin{align*}
\mathbf{r}_j(t = 0) &= r_0 + n_{r,j}, n_{r,j} \sim \mathcal{N}(0, \sigma^2_r) \\
\mathbf{\theta}_j(t = 0) &= \theta_0 + \frac{2\pi(j - 1)}{N} + n_{\theta,j} + \Phi_0, n_{\theta,j} \sim \mathcal{N}(0, \sigma^2_{\theta}) \\
\mathbf{z}_j(t = 0) &= z_0 + n_{z,j}, n_{z,j} \sim \mathcal{N}(0, \sigma^2_z).
\end{align*}
\]

(10)

Here, the role of additive position noises \( n_r, n_{\theta} \) and \( n_z \) is to make every angular sector of the hub slightly different from the others as to break the \( 2\pi/(N\Omega_t) \) periodicity that naturally results from our model. The angular shift \( \Phi_0 \) defines the initial position of the first angular sector. Figure 5(b) gives an example of the scatterer representation of the main rotor hub for a four-bladed helicopter, showing both diffuse and blade-related contributors.

Finally, the instantaneous position of points for every sector \( j \in [1, N] \) in the rotor frame \( \mathcal{R}_r \) is given by:

\[
\mathbf{r}'_{j}(t) = \begin{bmatrix}
\mathbf{r}_j(t = 0) \odot \cos(\mathbf{\theta}_j(t = 0) + \Omega_t t) \\
\mathbf{r}_j(t = 0) \odot \sin(\mathbf{\theta}_j(t = 0) + \Omega_t t) \\
\mathbf{z}_j(t = 0)
\end{bmatrix},
\]

(11)

where \( \odot \) denotes the Hadamard matrix product. Once the position of contributors has been determined for any time, one can then compute the hub reflectivity \( s_{hub} \) which, under the assumption that the incoming radar signal is a plane wave of wavevector \( \mathbf{k}_0 \), can be written as:

\[
s_{hub}(t) = \sqrt{\frac{\sigma_{hub}}{NM}} \sum_{j=1}^{N} \sum_{k=1}^{M} e^{-2i\mathbf{k}_0 \cdot \mathbf{r}'_{j,k}(t)}
\]

(12)

\( \sigma_{hub} \) being the desired hub RCS level, \( \mathbf{r}'_{j,k} \) the \( k \)-th column vector from the \( \mathcal{T}'_{j} \) matrix in Equation (11), and \( \varphi_{0,r} \) an arbitrary propagation phase shift from the target to the origin of \( \mathcal{R}_r \), \( \Omega_r \).

Figure 6 depicts the results obtained from the rotor hub model compared to the real main rotor hub signals from the data presented in Section 2, from which all other contributions (rotor blades, craft body and clutter) were filtered out. It is hard to conclude anything from the comparison of time signals (Figure 6(a)) because the model is stochastic by essence. However, one can see the overall energy fluctuation is comparable in both cases. Things become more interesting when looking at the autocorrelation of the same signals (see Figure 6(b)): now it becomes clear that both the model and the real data share the same periodic behaviour, with correlation surges at every rotor quarter-period. Finally, if one looks at the superimposed spectra of the real and synthetic data (Figure 6(c)), there is an excellent agreement between the two signals, that share the same spectral width and decay trend.

3.2.2 | Rotor blades

Since rotor blades are objects with a very large aspect ratio, they are modelled as one-dimensional objects with constant length \( L \), which implies they cannot stretch nor shrink. To be as generic as possible, no hypothesis is made about their shape, and they can be represented as a segment of any three-dimensional curve. Their radar reflectivity per unit length is given by the function \( \alpha(r) \). If the blade happens to lie on a plane, \( \alpha \) can be related to the RCS in normal incidence \( \sigma_b \) following:

\[
\sigma_b = \left( \int_0^L \alpha(r) \, dr \right)^2.
\]

Using the parametrization described in Figure 7 in cylindrical coordinates, the blade can be described by the implicit set of equations:

\[
\begin{align*}
\theta(r(t), t) &= f_1(r(t), t), f_1 \in C^\infty(\mathbb{R}^+ \times \mathbb{R}) \\
z(r(t), t) &= f_2(r(t), t), f_2 \in C^\infty(\mathbb{R}^+ \times \mathbb{R})
\end{align*}
\]

(13)

where the maximum radius value at a given time \( r_{\text{max}}(t) \) can be computed using the fact the blade keeps a constant length \( L \):

\[
\int_{r_{\text{hub}}}^{r_{\text{max}}(t)} \sqrt{1 + \left( \frac{\partial f_1}{\partial x}(x, t) \right)^2 + \left( \frac{\partial f_2}{\partial x}(x, t) \right)^2} \, dx = L.
\]

(14)

The return signal from the blade at any time \( s_{b}(t) \), given the description of Equation (13), can be expressed as:

\[
s_{b}(t) = e^{i\varphi_{0,r}} \times \left( \int_{r_{\text{hub}}}^{r_{\text{max}}(t)} \alpha(r) \exp \left[ -2i\mathbf{k}_0 \cdot \mathbf{r}(t) \cos(f_1(r(t), t)) \right] \right) \times \left[ \int_{r_{\text{hub}}}^{r_{\text{max}}(t)} \alpha(r) \exp \left[ -2i\mathbf{k}_0 \cdot \mathbf{r}(t) \sin(f_1(r(t), t)) \right] \right] \times \sqrt{1 + \left( \frac{\partial f_1}{\partial r}(r, t) \right)^2 + \left( \frac{\partial f_2}{\partial r}(r, t) \right)^2} \, dr.
\]

(15)

No analytic solution for this equation exists in general, except for very specific cases. One of them is that of the straight horizontal blade for which:

\[
\begin{align*}
\left\{ \begin{array}{l}
f_1(r(t), t) = f_1(t) = \Omega_t t + \Phi_0, \\
f_2(r(t), t) = 0
\end{array} \right.
\]

(16)

yielding a constant value \( r_{\text{max}}(t) = L \). If the reflectivity per unit length \( \alpha \) is constant, then Equation (15) reduces to:
This straight wire blade model has many advantages: it is quite easy to compute and offers an intuitive explanation to the existence of blade flashes, which occur every time the scalar product $\mathbf{k}_0 \cdot \mathbf{e}_b(t)$ is null, that is when the blade is perpendicular to the radar wavevector. Unfortunately it cannot reproduce adequately the temporal shape and duration of real blade flashes for the main rotor, as we showed in the previous studies [17, 18]. It is, however, sufficient in the case of the smaller tail rotor.

Actually, main rotor blades experience important movements in flight due to the strong forces they are submitted to [21], as illustrated in Figure 7:

- blade flapping in the vertical plane, due to the inhomogeneous lift and centrifugal force repartition along the blade, since both scale with the velocity (and hence radius) squared;
• blade dragging in the horizontal plane, due to the drag component of the aerodynamic forces;
• blade twisting, that is the wrapping of the blade around its longitudinal axis. As in our model blades are monodimensional objects, this effect can be neglected.

These movements come as whole blade movements plus blade deformation when blades are connected to the rotor hub through hinges, or solely as blade deformation when they are rigidly fixed to the hub. Blade flapping is the most important, with the corresponding flapping angle \( \beta \) able to reach in excess of \( 10^\circ \). The blade dragging angle \( \xi \) can also take a similar value [25], but as it remains almost constant during the blade rotation and since we are mainly interested in hinged rotors, its effect is similar to that of the phase shift \( \Phi_0 \), and can therefore be neglected. From these considerations we can refine our blade model from Equation (13):

\[
\begin{align*}
\theta(r,t) &= \theta(t) = \Omega_r t + \Phi_0 \\
\mathbf{z}(r,t) &= \mathbf{z}(r,t) = f(r,t), f \in C^\infty(\mathbb{R}^+ \times \mathbb{R}), \\
\forall t \in \mathbb{R}, r(t) \in [r_{hub}, r_{max}(t)]
\end{align*}
\]

that is the blade always lie in the \((r, z)\) plane. To further simplify, we consider that \( f \) is a polynomial function of the radius \( r \) with a low degree, for we showed previously that a cubic function was sufficient to yield blade flashes very close to those observed in the experiments [18]. Blade flapping can be decomposed into two components: a static one, that is a deformation that does not depend on the blade azimuth angle and, therefore, remains constant during rotation, and a dynamic one, which evolves with the blade azimuth angle.

Static blade deformation can be illustrated in stationary flight (Figure 8(a)). In that case, the pilot imparts a constant angle of attack to all blades. The forces they are experiencing keep constant during rotation, and the resulting deformation is the same at all azimuth angles. As a consequence blade tips describe a circle with a smaller diameter than the initial rotor disc, an effect called coning. Dynamic blade deformation, on the contrary, occurs when the helicopter is in advancing or manoeuvring flight. In that case, the pilot must induce a cyclic variation of the blade angle of attack so the rotor would tilt in the direction of desired flight and turn a portion of the rotor lift into acceleration (Figure 8(b)). Blade deformation also becomes cyclic, with an average intensity depending on the acceleration, and with the retreating blade experiencing a more important flapping than the advancing blade [25, 26]. The function \( f \) in Equation (18) must therefore be periodic, with a period \( 2\pi/\Omega_r \). As a consequence, it can be decomposed as a time Fourier series. Keeping only the lowest order we can finally describe the space-time blade shape evolution:

\[
\begin{align*}
\theta(t) &= \Omega_r t + \Phi_0 \\
\mathbf{z}(r,t) &= a_0 r^3 + b_0 r + (a_1 r^3 + b_1 r) \sin(\theta(t) + \Phi_1), \\
\forall t \in \mathbb{R}, r(t) \in [r_{hub}, r_{max}(t)]
\end{align*}
\]

where \( \Phi_1 \) is a phase shift controlling the azimuth angle at which maximum deformation occurs. Coefficients \( a_0 \) and \( b_0 \) are the static polynomial coefficients describing the blade shape, that is the time average of function \( f \), while \( a_1 \) and \( b_1 \) are the first-order Fourier expansion polynomial coefficients for \( f \). It is worth mentioning that using this blade shape modelling, there is no closed-form solution to Equation (15). Instead, we have to resort to numerical integration, for which a simple trapezoidal rule using a radial sampling of a quarter wavelength proves sufficient.

A last approximation that we make in our model is to consider the linear blade reflectivity \( \alpha(\mathbf{r}) \) as a constant. One can see, however, that blade RCS can be significantly different for the leading edge and for the retreating edge of the same blade (see e.g. Figure 1(d)). To deal with this we attribute two values of \( \alpha \), \( \alpha_{lead} \) and \( \alpha_{ret} \), to a given blade, and use one or the other value depending on the sign of:

\[
\left( \frac{\mathbf{k}_\theta \times \left[ \begin{array}{c} \sin(\theta(t)) \\
-\cos(\theta(t)) \\
0 \end{array} \right]}{0} \right) \cdot \mathbf{e}_{z,r},
\]

\( \mathbf{e}_{z,r} \) being the unit vector along the \( z_r \) axis.
3.2.3 | Blade tip

Given the visibility of blade tips (for instance as seen in Figure 1 (d)), their radiation pattern must have a very large angular width. Consequently, they are modelled as a one-dimensional straight wire with a short length on the order of the wavelength affixed to the end of rotor blades. As seen in Figure 1(d), their RCS can change during rotation, sharing the behaviour of rotor blades. However, in their case, the RCS changes between the front and back half of the rotor disk while the blade RCS varies between the left and right half of the rotor disk, in the sense defined in Figure 4. We, therefore, use a different reflectivity value by making this time dependent on the sign of:

\[
\overrightarrow{k}_0 \cdot \begin{bmatrix} \cos(\theta(t)) \\ \sin(\theta(t)) \\ 0 \end{bmatrix}.
\]  

(21)

3.3 | Full helicopter signal modelling

Now that all significant elements of the helicopter signature have been reproduced accurately, we can finally build the full rotorcraft model. The helicopter response is computed in the frame centred on the radar phase centre at all times, noted \( R_0 \) (Figure 9). In this frame, the helicopter is oriented arbitrarily by means of standard Tait-Bryan yaw, pitch and roll angles, defining the helicopter body frame \( R_b \). The rotorcraft is supposed to have \( N_{\text{rotor}} \) rotors, with each rotor having \( N_{\text{blade}} \) blades with similar characteristics for \( i \in [1, N_{\text{rotor}}] \). Each rotor keeps a fixed orientation in \( R_b \), thus defining its corresponding rotor frame \( R_{r,i} \). \( \overrightarrow{k}_0 \) defines the radar wavevector described in the \( R_0 \) frame. Due to propagation effects (chiefly atmospheric refraction), \( \overrightarrow{k}_0 \) is not perfectly colinear to \( \overrightarrow{OO}_\text{p} \). Noting \( V_0 \) the rotorcraft velocity vector in \( R_0 \), the Doppler shift associated to it will be:

\[
f_D = -\frac{\overrightarrow{k}_0 \cdot V_0}{\pi}.
\]  

(22)

The total rotorcraft baseband signal \( s \) then reads:

\[
s(t) = e^{i(2\pi f_D t + \varphi_0)} \left( s_{\text{body}}(t) + \sum_{i=1}^{N_{\text{rotor}}} s_{\text{rotor},i}(t) \right).
\]  

(23)

Here, \( \varphi_0 \) is an arbitrary phase shift taking propagation into account. Using the detailed model described in this Section, each rotor signal can be written as follows:

\[
s_{\text{rotor},i}(t) = s_{\text{hub},i}(t) + \sum_{j=1}^{N_{\text{blade},i}} s_{b,j,i} \left( t - \frac{2\pi(j - 1)}{N_{\text{blade},i} \Omega_{r,i}} \right) + s_{b,t,i} \left( t - \frac{2\pi(j - 1)}{N_{\text{blade},i} \Omega_{r,i}} \right).
\]  

\[\text{(24)}\]

\( s_{\text{hub},i} \) is the rotor hub signal computed from Equation (12), \( s_{b,j,i} \) is the blade signal computed from Equation (15), and \( s_{b,t,i} \) is the blade tip signal. It is worth mentioning that in order to use these equations the wavevector \( \overrightarrow{k}_0 \) must first be described in the corresponding rotor frame \( R_{r,i} \), following:

\[
\left( \overrightarrow{k}_0 \right)_{R_{r,i}} = R_{b/r_i} R_{0/h} \overrightarrow{k}_0,
\]  

\[\text{(25)}\]

where \( R_{0/h} \) is the rotation matrix from \( R_0 \) to \( R_b \), and \( R_{b/r_i} \) the rotation matrix from \( R_b \) to the rotor frame \( R_{r,i} \).

4 | MODEL VALIDATION

4.1 | Description of the validation data

The model validation is done by comparing the recorded data from two different helicopters using an experimental airborne X-band radar and results from the simulation. The first helicopter is a large, four-bladed model while the second one is a smaller, three-bladed rotorcraft. Both have a tail rotor counting five and two blades, respectively. They performed the same trajectory, that is, a circular motion with a \( \sim 1 \) km radius at moderate velocity and low altitude while the radar carrier was coming at high velocity and high altitude towards them. Synthetic data generation was performed by coupling the helicopter model described in the previous section with a clutter data generator developed at THALES, yielding a simulated signal as close as possible to reality. In all cases, rotor characteristics (rotation velocity, blade and hub RCS levels, blade deformation parameters) were kept constant, with only the view angles being adapted from one case to the other.

To assess the representativity of our model, we compute spectrograms from the helicopter range cell both for the real and synthetic data. We also extract advancing (positive Doppler) and retreating (negative Doppler) blade flashes by suitably filtering unwanted Doppler shifts in order to remove clutter, rotor hub and helicopter body contributions. In all cases, we also plotted the ideal blade flash shape (from the straight wire model, Equation (17)) to show that our model is significantly more representative.
4.2 Validation for the first helicopter

We start our analysis with the first helicopter seen from the front. Looking at the real (Figure 10(a)) and synthetic spectrograms (Figure 10(b)), one can see that our model accurately reproduces all the phenomenology seen in real data.

It proved able to generate advancing and retreating blade flashes with a different level due to the variable RCS between the blade leading and retreating edges, but also a very realistic blade tip behaviour. The helicopter body signal level and time variability are also very close to the real one. Finally, in order to stick to the experimental data, we attributed a null RCS to the rear rotor leading edge since no advancing tail rotor flash can be seen. These good results can also be seen by looking at the main rotor blade flashes (Figure 10(c) and (d)), particularly for the advancing ones, with the model closely reproducing real blade flashes.

Now looking at the same helicopter seen from the side (i.e., a zero Doppler shift), it appears that the closeness of synthetic data to real data is still high (Figure 11(a) and (b)). However, there are small discrepancies. First, we had to adopt a non-null RCS for the leading edge of the tail rotor blades, unlike the previous case. This highlights the first shortcoming of our model; when seen from the front, the helicopter body is probably masking advancing flashes from the tail rotor, a
phenomenon that cannot be reproduced. Second, one can clearly see obvious Doppler lines in real data (for instance the one, which is close to 0.3 relative Doppler shift), whereas they are not present in the synthetic data. These lines come from the compression and/or the exhaust stages of turbine engines. Their presence in the signal is strongly dependent on the view angle (particularly the azimuth) and usually involves a complex interplay of reflection in turbine ducts, making their modelling challenging. Despite these minor discrepancies, our model appears excellent when it comes to blade flashes (Figure 11(c) and (d)), because it results in synthetic flashes having the same duration and shape as the real ones. Please also note in this case, the elevation angle for the wavevector in the rotor frame is estimated at more than 30°. With such a large angle one could expect that blade responses could only be reproduced using a two-dimensional blade model. These results show that a properly deformed one-dimensional object is sufficient.

Finally, looking at results obtained when the helicopter is seen from the rear side (Figure 12), one can see that our model is here too able to compute a signal very close to the experimental one, as evidenced by the spectrograms and the blade flash shapes.

4.3 | Validation for the second helicopter

The second helicopter is first seen from the side, at zero relative velocity. Its response, as seen in Figure 13(a), is
FIGURE 12 First helicopter seen from the rear side: (a) real-data spectrogram, (b) synthetic-data spectrogram, (c) comparison of real and synthetic advancing blade flashes, and (d) comparison of real and synthetic retreating blade flashes

significantly different from that of the first helicopter. Since, its main rotor is odd-bladed, advancing and receding flashes no longer occur at the same time. Moreover, the retreating side of the main rotor (negative Doppler shifts) appears very noisy, with a very peculiar blade tip behaviour. Still, the synthetic signal (Figure 13(b)) appears close to the real one. A slight difference can be seen in the form of a very broad Doppler line around −0.2 relative Doppler shift, which is caused by a non-representative clutter modelling in that configuration. Now looking at the main rotor flashes (Figure 13(a) and (b)), the overall shape and duration are well reproduced, although the non-conventional appearance of real receding flashes makes their modelling challenging.

Now looking at the results when the rotorcraft is seen from its rear side (Figure 14(a) and (b)), we can see the appearance of continuous Doppler lines in the real data. Just like in the case of the first helicopter, these spectral lines come from the turbine engine of the craft that results in a fast modulation of the body return signal. These do not render in the synthetic signal because turbine modelling is not included in our analysis. Except for this, both spectrograms are very close. Please also note the slight upward temporal bending of receding blade
flashes, which is accurately reconstructed by the model. This shows once again the necessity to use deformed blades to correctly simulate the real helicopter data. As for the main rotor blade advancing flashes (Figure 14(c)), they are relatively well reproduced by the model, which a much broader base than the straight wire model predicts. The agreement is worse for receding flashes (Figure 14(d)), with a total flash duration significantly less for the model than for the real data, while still being closer than the ideal blade model. Once more the real helicopter signal on the negative Doppler side appears very complex, with a peculiar blade tip behaviour and a much noisier background than on the positive Doppler side. The presence of such elements can explain the broadening of blade flashes on the retreating side, which is not seen in the model.

The final investigated case is that of the helicopter seen from the rear, at maximum negative Doppler shift. Again the spectrogram computed from real data (see Figure 15(d)) exhibits turbine spectral lines sharing the same frequency as in the previous case. Given the orientation of the craft with respect to the radar, they are undoubtedly generated by the exhaust stage of the turbine. The signal from the compression stage would probably have a different modulation frequency, because it is equal to the product of the turbine shaft rotation frequency by the number of blades for the involved rotor, with
FIGURE 14 Second helicopter seen from the rear side: (a) real-data spectrogram, (b) synthetic-data spectrogram, (c) comparison of real and synthetic advancing blade flashes, and (d) comparison of real and synthetic retreating blade flashes.

the intake and exhaust stages having a different number of blades. When compared to the synthetic spectrogram (Figure 15(b)), both look very close although the retreating side of the main rotor blades (negative Doppler side) here is also much noisier than the advancing side, with the blade tip being notably ‘thicker’ in terms of Doppler shift. Given our blade tip model, we have much difficulty in reproducing this behaviour; we could of course increase their length in order to fit the real signal around receding blade flashes, but this would also decrease the angular opening of the associated main lobe and restrict blade tip energy only around blade flashes. This is clearly not the case in real data, with the blade tip keeping a similar RCS during the blade rotation. Still, in this precise case, the comparison of the shape of blade flashes is excellent. As seen in Figure 15(c) and (d), both the shape and duration of synthetic flash events are very close to the real ones. Once more, the straight wire blade model proves much too simple to accurately fit the experimental data. Both facts confirm the importance of main rotor blade deformation in the radar return signal of rotorcraft.

5 | CONCLUSION

Here, we introduced a parametric model for the radar signature of helicopters in the narrowband approximation. Starting
from the analysis of representative rotorcraft data, we identified the main contributors to this response as being the craft body and, for each rotor, the rotor blades and their blade tip, and the associated rotor hub. Then a specific model was developed for each of these elements. The helicopter body model is built from an exponentially time-correlated Rayleigh random variable, with a decorrelation time on the order of 100 ms. Rotor hubs are described as a collection of point scatterers, the position of which is randomly drawn at the initial time. The radial component is exponentially distributed in order to closely reproduce the spectral shape associated with the main rotor hub. The azimuthal component is made up of two parts: a uniform one that mimics cylindrically symmetric elements of the hub (e.g., the swashplate, the rotor shaft, etc.) and a Gaussian one that simulates discrete hub components linked to the blade roots (e.g., blade control rods). The whole point cloud is then put into rotation at the rotor angular velocity. Finally, rotor blades are modelled as one-dimensional objects with an arbitrary shape. Although the straight wire blade model is sufficient to simulate tail rotor blades we showed previously [17, 18] that helicopter flight dynamics results in important blade deformations which, in turn, yield blade flashes much broader and with a global shape very different from those obtained from the ideal blade model.
By choosing a cubic polynomial function with a sinusoidal time dependence for the main rotor blade shape, we managed to keep the model complexity at a low level. Still, the estimation of accurate blade deformation parameters is not trivial, and must necessarily involve a fit over real radar data, or the use of aerodynamic simulations. Our model also includes a different blade RCS level between the leading and retreating edges, and a specific blade tip simulation.

The model was validated using data recorded from two different helicopters. Spectrograms obtained from real and synthetic data and advancing and receding blade flashes were compared as well. Results are very close in terms of phenomenology and of signature level, with main rotor blade compared as well. Results in a high-frequency modulation of the craft body return signal. This element is challenging to simulate systematically because it is strongly dependent on the view angle on the helicopter but also involves complex multipath propagation in the turbine ducts.

Note that the model presented herein can be easily adapted to other targets displaying a characteristic micro-Doppler response. As it is able to simulate an arbitrary number of rotors, it can be used to generate synthetic drone signals, given blade deformation parameters that are adapted to them. In the case of micro-drones, a null rotor deformation (i.e., a pure straight wire model) would probably be enough. In theory, it could also be used to model aircraft with propellers.

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