Evolution of mechanical parameters of deep sandstone and its constitutive model under the condition of different stress paths

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Abstract: Using a multifunctional true triaxial fluid–solid coupling system, the mechanical properties of deep sandstone and its constitutive model were studied under the simulated depths 1000, 1500, and 2000 m and different stress paths. In stress path 1, \( \sigma_z \) is increased while unloading along the x-axis (\( \sigma_x \) decreased). Path 2 involves an increase in \( \sigma_z \) with two-sided unloading. Path 3 involves an increase in \( \sigma_z \) with one-sided unloading \( \sigma_x \) and \( \sigma_y \) true triaxial loading. In addition, the evolution law of the Poisson's ratio \( \mu \) and deformation modulus \( E \) of the deep sandstone under different stress paths and simulated depths were studied. The experimental results show that the following: (1) The \( \mu–\sigma_z \) curves of the sandstone at different simulated depths and the same stress path are similar. As \( \sigma_z \) increases, the Poisson's ratio first decreases and then increases to a maximum value, and subsequently, it abruptly decreases. On the other hand, the initial value of the deformation modulus \( E \) increases with an increase in the simulated depth. (2) Under the same simulated depth and different stress paths, the deformation trends of the \( E–\sigma_z \) curves of the deep sandstone are quite different. Under the stress paths 1 and 2, the deformation modulus \( E \) of the deep sandstone first increases and then sharply decreases as \( \sigma_z \) increases. On the other hand, it first decreases and then increases under the condition of stress path 3. (3) A constitutive model for the deep sandstone is proposed herein. The research results can provide some important references for deep rock mechanics.
1. Introduction

With the rapid development in the global economy, shallow mineral resources have remarkably depleted; thus, deep underground projects have received increasing attention. During the mining process of deep underground resources, in situ stress is an important force that causes mass destruction of the geological rocks. Meanwhile, the mechanical properties of rocks vary under different buried depths and stress paths. In this study, the evolution of the mechanical parameters and a constitutive model of a deep sandstone under the condition of different simulated depth and stress paths based on the reduction of the initial in situ stresses are studied, which can provide some guidance for deep underground engineering.

At present, the research on the mechanical properties of coal and rocks under true triaxial forces has globally made certain progress. Feng et al. [1] proposed a three-dimensional strength criterion for hard rocks in combination with true triaxial test data on different types of hard rocks, aiming at the problem that the failure modes of hard rocks are quite different under true triaxial compression. Wu et al. [2] conducted directional hydraulic fracturing tests on a fine sandstone and obtained the hydraulic pressure–time and the fracturing volume–strain curves of the sandstone. They concluded that the crack growth in sandstone can be divided into two stages: the gentle wave shape and the fault-rock drop shape stages. Jia et al. [3] simulated the crack growth model of a fractured sandstone using PFC3D and established the relation that low medium principal stresses significantly affect the peak strength of sandstone. Li et al. [4] conducted true triaxial loading and unloading tests on sandstone with different stress paths using the autonomous true triaxial electro-hydraulic servo system. They revealed the effect of intermediate principal stress and established a three-dimensional mechanical model for the rock. Vachaparampil et al. [5] obtained the strength characteristic curves and test-fracture photographs of three kinds of shale by true triaxial compression tests. They concluded that medium principal stresses significantly affect the strength and failure characteristics of the three kinds of shale. Furthermore, Lu et al. [6] conducted true triaxial loading tests on raw coal in two different modes and thoroughly studied the strength and failure characteristics of raw coal under true triaxial ground pressure as well as the coupling effect of coal and gas outburst on the raw coal to establish the fracture mechanism of raw coal under composite dynamic disaster. Lu [7] conducted uniaxial, conventional triaxial, and true triaxial tests on the marble from the Jinping Hydropower Station. The failure modes of the marble under the three tests were compared and analyzed. Furthermore, the Mohr–Coulomb criterion was modified, and the corresponding model was established. Wu et al. [8] adopted a true triaxial loading test system to conduct laboratory tests on the deep-buried sandstone of the Jinping II hydropower station under different high-stress conditions. They obtained that the plate fracture failure phenomenon of rocks under different high-stress conditions can better represent the rule of the plate fracture failure phenomenon in the practice of the project. Yin et al [9–11] conducted true triaxial loading tests on a sandstone under different stress paths and at different loading and unloading rates. They investigated their effects on the failure mode, strength characteristics, deformation characteristics, and permeability evolution law of the sandstone. Du [12] conducted true triaxial compression tests on sandstone, marble, and granite and discovered that the lithology, $\sigma_2$, and $\sigma_3$ all affect the strength, fracture inclination, failure mode, and nonlinear mechanical behavior of rocks.

In the course of studying the mechanical properties of coal and rocks under true triaxial
loading conditions, most studies discovered the effects of principal stress on the strength, deformation, failure mode, permeability, and energy evolution of rocks. However, only few reports exist on the evolution law and constitutive model of the mechanical parameters of rocks under true triaxial loading conditions. As a result, this paper conducts true triaxial loading and unloading tests on deep sandstone with different initial stress paths and simulation depths. It further studies the evolution law of the mechanical parameters of the deep sandstone under the different stress paths of reducing the initial in situ stress and the structure model. The results can provide some theoretical basis for the construction of deep underground projects.

2 Materials and method
The test employed the multifunctional true triaxial fluid–solid coupling system developed by the Chongqing University, which comprises a frame rack, true triaxial pressure chamber, loading system, internal sealed seepage system, control and data measurement devices, acquisition system, and acoustic emission monitoring system. The maximum pressure obtainable in the three directions are 6000, 6000, and 4000 KN. A two-way rigidity, always rigid and flexible loading method, can be performed in the true triaxial loading under different stress paths for the mechanical properties of deep sandstone. The test instrument is shown in Figure 1.

Figure 1. Multifunction true triaxial fluid–solid coupling system [13]

2.1 Extraction of sandstone
A typical sandstone was collected from the Huafeng Coal Mine of Shandong Xinwen Group, as shown in Figure 2. The surface of the sandstone was devoid of macro cracks and flaws, which made it suitable for the test. The sandstone was cut into cubic specimens of 100 mm × 100 mm × 100 mm using a cutter. The physical and mechanical properties of the deep sandstone are listed in Table 1. To avoid the damage of the instrument, the specimens were completely wrapped with thin plastic films after cutting and grinding, and they were then placed in the multifunctional true triaxial fluid–solid coupling system.

Table 1. Physical and mechanical parameters of the deep sandstone under different
2.2 Initial in situ stress determination at different simulated depths

From the results of the initial in situ stress measurements, as shown in Figure 3, the axial stresses of the sandstone sample at simulated depths of 1000, 1500, and 2000 m were calculated using the following empirical formula:

$$\sigma_z = 0.027H$$

(1)
The horizontal stress was calculated using the statistical formula developed by O. Stephansson [15].

$$\begin{align*}
\sigma_y &= \sigma_{h_{\text{max}}} = 6.7 + 0.0444H \\
\sigma_x &= \sigma_{h_{\text{min}}} = 0.8 + 0.0329H
\end{align*}$$

where $\sigma_z$ is the axial stress, $\sigma_y$ the maximum horizontal stress, and $\sigma_x$ the minimum horizontal stress. To facilitate the experiment, the calculated value of the initial in situ stress state was reduced. The initial in situ stress states of the deep sandstone at simulated depths of 1000, 1500, and 2000 m are tabulated in Table 2.

### 2.3 Specific test design

**Table 2.** Simulated depths and initial in situ stress levels of deep sandstone

| The test path                     | simulated depths (m) | $\sigma_z$ (MPa) | $\sigma_y$ (MPa) | $\sigma_x$ (MPa) |
|-----------------------------------|----------------------|------------------|------------------|------------------|
| increasing $\sigma_z$ while       | 1000                 | 27               | 51               | 33               |
| one-sided unloading $\sigma_x$    | 1500                 | 40               | 73               | 50               |
|                                   | 2000                 | 54               | 95               | 66               |
| increasing $\sigma_z$ while       | 1000                 | 27               | 51               | 33               |
| two-sided unloading $\sigma_x$    | 1500                 | 40               | 73               | 50               |
|                                   | 2000                 | 54               | 95               | 66               |
| increasing $\sigma_z$ while       | 1000                 | 27               | 51               | 33               |
| one-sided unloading $\sigma_x$    | 1500                 | 40               | 73               | 50               |
|                                   | 2000                 | 54               | 95               | 66               |

Table 2 shows the simulated depths and initial in situ stress levels of the deep sandstone. The true triaxial test stress paths 1, 2, and 3 on the deep sandstone can be divided into two steps. The first step is the initial in situ stress reduction stage. In this stage, the deep sandstone is synchronously subjected to three-directional stresses. This stage is completed when the three-directional stresses at the different simulated depths of 1000, 1500, and 2000 m reach the preset values. For example, the three-directional stress preset values for the deep sandstone at the simulated depth of 1000 m are $\sigma_z = 27$ MPa, $\sigma_y = 51$ MPa, and $\sigma_x = 33$ Mpa. In the true triaxial loading process, the detailed steps of the stress paths 1, 2, and 3 are as follows:
Path 1: Keep $\sigma_z$ constant, unload $\sigma_x$ under one-sided unloading at an unloading rate 2 KN/s and the other sides maintain the displacement with the displacement control mode while loading the slug $\sigma_z$ at a rate of 0.003 mm/s until $\sigma_x$ unloads to 0.

Path 2: This path is similar to path 1. The difference is that path 2 involves two-sided unloading $\sigma_x$ at an unloading rate of 2 KN/s.

Path 3: To unload the rate 2 KN/s one - sided unloading $\sigma_x$, the corresponding side with displacement control to keep the displacement constant, while unloading rate 2 KN/s one - sided unloading $\sigma_x$, the corresponding side with displacement control to keep the displacement constant, and then in the displacement control way to load the slug $\sigma_z$, rate 0.003 mm/s, until the $\sigma_x$ unload to 0. The corresponding diagrams of the stress paths are shown in Figure 4.

![Diagram of test stress paths](image)

**Figure 4.** The diagram of test stress path

### 3. Strength of deep sandstone under different stress paths and the same simulated depth

**Table 3.** Strength of the deep sandstone under the different stress paths and the same simulated depth

| simulated depth (m) | The test path | peak strength $\sigma_{cf}$ (MPa) | residual strength $\sigma_d$ (MPa) |
|---------------------|---------------|----------------------------------|---------------------------------|
| 1000                | Path 1        | 121.8                            | 38.1                            |
|                     | Path 2        | 112.1                            | 40.2                            |
|                     | Path 3        | 104.1                            | 94.5                            |
Table 3 lists the strengths of the deep sandstone under the same simulated depth and different stress paths. Figure 5 shows the strength-characteristic curve of the sandstone under the same simulated depth and different stress paths; $\sigma_{cf}$ and $\sigma_d$ are the peak and residual strengths of deep sandstone, respectively. As shown in the figure, the peak strength of the deep sandstone under the same simulation depth and different stress paths follows the trend: $\sigma_{cf\text{ path1}} > \sigma_{cf\text{ path2}} > \sigma_{cf\text{ path3}}$. For example, under the conditions of paths 1, 2, and 3 with the simulated depth of 1000 m, the peak strength was 121.8, 112.1, and 104.1 MPa, respectively. The peak strength under paths 2 and 3 showed a similar trend. This shows that the stress path not only affects the deformation characteristics of deep sandstones but also has a significant effect on its intensity evolution features. This is because the deep sandstone under path 1 at the same simulated depth was subjected to the minimum horizontal stress single one-sided unloading, whereas that under path 2 was under the minimum horizontal stress two-sided unloading, and that under path 3 was simultaneously unloaded with the maximum and minimum horizontal stresses, which resulted in the gradual reduction of the circumference of the deep sandstone in the three paths. Based on the effect of the decrease in circumference, the peak strength of the deep sandstone was gradually reduced. Meanwhile, the stress path has a significant effect on the residual strength of the deep sandstone. The trend of the residual strength of the sandstone under paths 1, 2, and 3 at simulated depths of 1500 and 2000 m is $\sigma_{d\text{ path1}} > \sigma_{d\text{ path2}} > \sigma_{d\text{ path3}}$. At simulated depth of 1000 m, the trend is $\sigma_{d\text{ path3}} > \sigma_{d\text{ path2}} > \sigma_{d\text{ path1}}$. For example, the residual strength at the simulation depth of 1500 m was 73.2, 51.2, and 38.7 MPa for paths 1, 2, and 3, respectively, and that under the simulated depth of 2000 m was 64.5, 45.8, and 25.2 MPa, respectively. At the simulated depth of 1000 m, it was 38.1, 40.2, and 94.5 MPa for paths 1, 2, and 3, respectively. We infer that due to the three stress paths, the confining pressure of the deep sandstone was significantly different, resulting in a large difference in the residual strength. The residual strength evolution trend of deep sandstone under paths 1, 2, and 3 at the simulated depth of 1000 m is different from that under 1500 and 2000 m simulated depths, which is attributed to the difference in the opposite characteristics of the deep sandstone.

|     | Path 1 | Path 2 | Path 3 |
|-----|-------|-------|-------|
| 1500 | 161.1 | 131.4 | 127.2 |
| 2000 | 178.4 | 167.9 | 159.8 |
Figure 5. Strength-characteristic curve of deep sandstone under different stress paths and the same simulated depth

4 The strength characteristics of the deep sandstone under different simulation depths and the same stress path

Table 4. Strength of deep sandstone under different simulated depths and the same stress path

| The test path | simulated depth (m) | peak strength $\sigma_{cf}$ (MPa) | residual strength $\sigma_d$ (MPa) |
|---------------|---------------------|----------------------------------|----------------------------------|
| Path 1        | 1000                | 121.8                            | 38.1                             |
|               | 1500                | 161.1                            | 73.2                             |
|               | 2000                | 178.4                            | 64.5                             |
| Path 2        | 1000                | 112.1                            | 40.2                             |
|               | 1500                | 131.4                            | 51.2                             |
|               | 2000                | 167.9                            | 45.8                             |
| Path 3        | 1000                | 104.1                            | 94.5                             |
|               | 1500                | 127.2                            | 38.7                             |
|               | 2000                | 159.8                            | 25.2                             |

Table 4 lists the strengths of the deep sandstone under the same stress path, and the
strength-characteristic curves are shown in Figure 6. As shown in the figure, under the same path, the peak strength of the sandstone increased with depth. For example, the peak strengths at the simulated depth of 1000, 1500, and 2000 m under stress path 1 were 121.39, 160.82, and 178.41 MPa, respectively. A similar trend was obtained under stress paths 2 and 3. We infer that as the depth increases, the initial in situ stress of the sandstone increases, resulting in different degrees of compaction on the deep sandstone. Meanwhile, the greater the degree of compaction, the greater the limit carrying capacity of the deep sandstone in the three directions. In addition, as shown in Figure 6, under the same stress path, with an increase in the simulation depth, the residual strength of the deep sandstone under path 1 and path 2 first increased and then decreased. Under stress path 1, the residual strengths of the sandstone were 38.1, 73.2, and 64.5 MPa at depths of 1000, 1500, and 2000 m, respectively. However, under stress path 3, with an increase in the simulated depth, the residual strength gradually decreased (94.5, 38.7, and 25.2 MPa for depth of 1000, 1500, and 2000, respectively). This shows that the depth of deep sandstone also affects its residual strength.

Figure 6. Strength characteristic curve of the sandstone under different simulated depths and the same stress path. 
5. Evolution analysis of rock deformation parameters under different stress paths and the same simulation depth

Poisson’s ratio and deformation modulus $E$ are the two most important parameters in the study of the evolution law of rock deformation. The Poisson’s ratio of a rock is the ratio of the radial strain to the axial strain when subjected to a load. It is expressed as follows:

$$\mu = \frac{\varepsilon_x}{\varepsilon_y}$$  \hspace{1cm} (3)

On the other hand, the deformation modulus $E$ of a rock is the ratio of the applied stress to the strain when subjected to a load:

$$E = \frac{\sigma}{\varepsilon}$$  \hspace{1cm} (4)

In rock mechanics, $\mu$ and $E$ are usually constant in the elastic region, and in this stage, $E$ is known as the elastic modulus. In the plastic region, both $\mu$ and $E$ vary with the applied stress. Under three-dimensional stresses, $\mu$ and $E$ of a rock cannot be calculated using the formula for the single-axis stresses. Therefore, to obtain $\mu$ and $E$ of a rock under the triaxial stress, we employed the formula adapted from the literature [16]. We assumed that the rock obeys Hook’s law under the three-direction stress state, and the stress–strain relationship of the deep sandstone is as follows:

$$\begin{align*}
\varepsilon_z &= \frac{1}{E} \left[ \sigma_z - \mu \left( \sigma_x + \sigma_y \right) \right] \\
\varepsilon_y &= \frac{1}{E} \left[ \sigma_y - \mu \left( \sigma_x + \sigma_z \right) \right] \\
\varepsilon_x &= \frac{1}{E} \left[ \sigma_x - \mu \left( \sigma_z + \sigma_y \right) \right]
\end{align*}$$  \hspace{1cm} (5)

where $\varepsilon_z$ is the axial strain, $\varepsilon_y$ is the maximum horizontal strain, and $\varepsilon_x$ is the minimum horizontal strain.

Through simple mathematical transformation, the expressions for $E$ and $\mu$ under true triaxial stress are obtained as follows:

$$\begin{align*}
E &= \frac{(\sigma_z + 2\mu\sigma_y)}{\varepsilon_z} \\
\mu &= \left( B\sigma_y - \sigma_z \right) / \left( \sigma_z (2B - 1) - \sigma_z \right)
\end{align*}$$  \hspace{1cm} (6)

Under the same simulated depth, the stress paths of deep sandstone vary, which results in deep sandstones having different stress state under true triaxial loading and unloading. This difference in the stress state leads to a difference in the evolution law of the mechanical parameters of the sandstone. According to Eq. (5), the $E-\sigma_z$ and $\mu-\sigma_z$ curves of deep sandstone at the depth of 1500 m under different stress paths were obtained. In addition, the effect of the stress paths on the mechanical parameters of the deep sandstone was analyzed. As shown in Figure 7, the $E-\sigma_z$ and $\mu-\sigma_z$ curves of the deep sandstone at 1500 m simulated depth under different stress paths are divided into two stages: the AB and BC stages. The AB stage is the pre-peak stage, whereas the BC stage is the post-peak stage. Figure 7 shows that with axial stress loading, the $E$ of the deep sandstone under the same depth and different stress paths
greatly varies. Under the conditions of 1500 m of simulated depth and stress path 1, stage AB shows a slight increase in $E$ as the axial stress increases. When the axial stress reaches point B (peak stress), $E$ also reaches its peak value. This is because, after the initial in situ stress reduction process of deep sandstone and with the increase in the axial and maximum horizontal unloading stresses, the stiffness of deep sandstones increases, leading to the increase in the deformation modulus. In the BC stage, due to the strain loading, the rear axle that decreases the stress in deep sandstones ruptures; thus, $E$ rapidly reduces. The formation of the inflection point (the turn-back phenomenon) is due to the outburst of the deep sandstone. Macroscopic cracks in the rock quickly reduce the bearing capacity and the stiffness of the rock, resulting in a decrease in the deformation modulus of the rock. Herein, the variation in $E$ for the deep sandstone at the simulated depth of 1500 m and under stress path 2 is also established. At the simulated depth of 1500 m and under stress path 3, as the axial stress increased in stage AB, $E$ first rapidly decreased and then slowly increased. As shown in Figure 7, with the axial stress loading, the evolutions of $\mu$ under the same simulated depth and different stress paths slightly differ. In stage AB, under the conditions of paths 1, 2, and 3 and the simulated depth of 1500 m, the Poisson's ratio first decreased with the axial stress loading and then rapidly increased to a maximum value after which it slowly decreased. In the BC stage, under the conditions of paths 1, 2, and 3 at the depth of 1500 m, the axial stress decreased when the strength of the deep sandstone reached its peak value and the Poisson's ratio rapidly decreased. This indicates that the stress path has little influence on the evolution of the Poisson's ratio of deep sandstones.

6 Evolution analysis of deep sandstone deformation parameters under different simulation depths and the same stress path

Under the same stress path and different simulated depth, due to the difference in the initial simulated depth, the initial in situ stress varies. The initial in situ stress affects the stiffness of deep sandstone. With different stiffness, the evolution of the deformation parameters of deep sandstone is significantly different. Therefore, it is necessary to perform an in-depth study on the evolution of deep sandstone deformation parameters under different depths and the same stress path.
sandstones varies. Therefore, studying the evolution of the deformation parameters of the deep sandstone at different simulated depths is necessary. Figure 8 shows the $\mu$–$\sigma_z$ and $E$–$\sigma_z$ curves under different simulated depths and stress path 1. It is divided into two stages. The AB stage is the pre-peak stage, and the BC stage is the post-peak stage. Figure 8 shows that with the axial stress loading, there is little difference in the evolution of the deformation modulus of deep sandstone under the same stress path and different simulated depths. The deformation modulus $E$ increases with an increase in the simulated depth. For example, under the simulated depth of 1000 m in the AB stage, $E$ slowly increased with an increase in the axial stress. However, in the BC stage, it rapidly decreased with a decrease in the axial stress, and the inflection-point phenomenon was observed. The evolutionary trends of the deformation modulus of deep sandstone under path 1 at depths of 1500 and 2000 m are similar. It can be observed that with axial loading, the evolution of the Poisson's ratio of deep sandstone under different simulation depths and the same stress path differs. In the AB stage, at the simulated depth of 1000 m, with an increase in the axial stress, the Poisson ratio first rapidly decreased first and then slowly decreased. At 1500 m simulated depth, as the axial stress increased, the Poisson ratio first decreased and then rapidly increased, whereas at the depth of 2000 MPa, it first rapidly decreased and subsequently slowly increased. In the BC stage, the Poisson's ratio evolution of deep sandstone at the simulated depths of 1000, 1500, and 2000 m show that after the rupture of the deep sandstone, with the axial loading and lateral unloading, the change in the axial strain was much less than that of the lateral strain; hence, the Poisson's ratio rapidly decreased.

![Figure 8. $\mu$–$\sigma_z$ and $E$–$\sigma_z$ curves under stress path 1 and different simulated depths](image)

7. Constitutive analysis of rock mass
The constitutive model of the rock mass represents an essential attribute of the rock mass. The rock-mass structure model can be divided into elastic plastic and nonlinear elastic models. The elastic model is less than the nonlinear elastic model needs to determine the parameters, so it is widely used in rock mechanics. At present, most research on the structure models of a rock mass is based on uniaxial and triaxial stresses. However, the rock mass in the geological environment exhibits the true triaxial stress state. Therefore, the structure model of rock under the minimum horizontal stress under axial loading and single surface unloading is deduced herein by analyzing the true triaxial test results of deep sandstone under different stress paths.
Based on the previous analysis of the rock strength guidelines, the following assumptions are made with regards to the structural model of the deep sandstone:

1. Stages OA1 and A1A involve the initial stress reduction process, and stage AB is the secondary pressure stage; all belong to the elastic stage. In this stage, the stress–strain relationship of deep sandstones obeys Hooke's law.

2. In the BC stage, the strength criterion of the deep sandstone applies to the Mogi–Coulomb strength criterion.

3. In the CD stage, the residual strength of the deep sandstone conforms to the revised Hoek–Brown strength criterion.

4. In the BC stage, the stress–strain line for deep sandstones is a curve, whereas other stages show linear relationships.

Figure 9 shows the stress–strain curve of the deep sandstone of the entire process under the simulated depth of 1500 m and stress path 1. The OA1 and A1A stages represent the initial in situ stress reduction stages, which are considered to be in the elastic region. This is because of the small load applied to the deep sandstone. The OB stage is also an elastic stage, the BC stage is the plastic stage, the CD stage is the strain -softening stage, and the DE stage is the ideal plastic stage.

1. Elastic stage (OB)
In the elastic stage of rocks, with an increase in the three-direction stress, the strain can be determined using Hooke's law:

\[ \{d\varepsilon\} = \{C\} \{\varepsilon\} \]

where \( \{C\} \) is the softness matrix expressed as follows:
where $\mu_e$ is the Poisson's ratio in the elastic stage and $E_e$ is the elasticity modulus.

(2) Plastic yield stage (BC)

At the BC stage, the stress–strain constitutive relation is obtained by linear fitting as follows:

$$\sigma = 36.6718 + 177.7434 \varepsilon$$

(8)

where $\sigma$ is the directional stress and $\varepsilon$ the directional strain in the BC stage.

(3) Strain softening stage CD

From the analysis of the strength criterion, the deep sandstone conforms to the Mogi–Coulomb strength criterion at the maximum load strength and the residual strength conforms to the revised Hoek–Brown strength guideline.

The peak yield function $f_F$ can be expressed as

$$f_F = \tau_{\text{cr}} - \frac{1}{3}\sqrt{(\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2}$$

(9)

On the other hand, the residual yield function $f_c$ can be expressed as

$$f_c = \sigma_1 - \sigma_3 - \sigma_c (m \frac{\sigma_3}{\sigma_c})$$

(10)

In the strain softening stage, the yield function and the volume strain vary between the peak yield function $f_F$ and the residual yield function $f_c$. $F$ is expressed as

$$F = \frac{\varepsilon_1 - \varepsilon_1^c}{\varepsilon_1^c - \varepsilon_1^p} f_F + \frac{\varepsilon_3^c - \varepsilon_3}{\varepsilon_3^c - \varepsilon_3^p} f_c = 0$$

(11)

where $\varepsilon_1^F$ is the volume strain at the peak strength and $\varepsilon_1^C$ is the volume strain at the residual strength.

According to the plasticity theory, the structural equation of the strain softening phase can be expressed as

$$\{d\sigma\} = [D_p] - [D_p] [d\varepsilon]$$

(12)

$$[D_p] = \frac{1}{A} \left[ \begin{array}{c} \frac{\partial F}{\partial \sigma} \\ \frac{\partial F}{\partial \sigma} \\ \frac{\partial F}{\partial \sigma} \\ \frac{\partial F}{\partial \sigma} \end{array} \right] \left[ \begin{array}{c} \frac{\partial F}{\partial \sigma} \\ \frac{\partial F}{\partial \sigma} \\ \frac{\partial F}{\partial \sigma} \\ \frac{\partial F}{\partial \sigma} \end{array} \right]$$

(13)

where $A$ is the hardening modulus and $[D_p]$ the elastic stiffness matrix.

In the strain-softening stage, $A$ is negative and it is expressed as follows:
where \( E_h \) is the softening coefficient.

According to the plasticity theory, the equation for the CD strain-softening phase is as follows:

\[
A = \frac{E_g}{1 - \frac{E_g}{E}}
\]  

(14)

(4) Ideal plasticity (DE) stage: In this stage, \( A = 0 \), the constitutive equation can be written as follows:

\[
\{d\sigma\} = ([D_e] - [D_p])\{d\varepsilon\}
\]  

(15)

\[
[D_e] = \left[ \frac{\partial F}{\partial \sigma} \right] \left[ \frac{\partial F}{\partial \varepsilon} \right]^T [D_e] + \left[ \frac{\partial F}{\partial \varepsilon} \right] \left[ \frac{\partial F}{\partial \sigma} \right]^T
\]  

(16)

Conclusion

From the results obtained in this study, the following conclusions are drawn:

1. Under the same stress path, as the depth increases, the peak strength of deep sandstones increases.

2. The deformation trends of deep sandstone under different simulated depths and the same stress path are similar, as revealed by the \( \mu-\sigma \) curves. As \( \sigma \) increases, the Poisson's ratio first decreases and then increases to a peak value beyond which a sharp decrease is recorded. On the other hand, the initial value of the deformation modulus \( E \) increases with an increase in the simulated depth.

3. The deformation trend of the \( E-\sigma \) curves for the deep sandstone differs under the same simulated depth and different stress paths. Under stress paths 1 and 2 in this study, \( E \) first increased and subsequently sharply decreased as \( \sigma \) increased. On the other hand, under the path 3 condition, it first decreased, then increased, and finally sharply decreased as \( \sigma \) increased.

Acknowledgments

The study was financially supported by the National Key R&D Program of China [2018YFC0604703, 2016YFC0600901], National Natural Science Foundation of China [51974319], and research fund of the State Key Laboratory for Geomechanics and Deep Underground Engineering, CUMTB [SKLGDEUK1828].
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