The Exact Solution of Fractional Coupled EW and Coupled MEW Equations Using Sine-Cosine Method

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Abstract. In the current paper, the coupled of space -time fractional of both the equal width wave equation(FCEWE) and the modified equal width wave equation (FCMEWE) are formally solved. For this purpose, the conformable derivative and Sine-Cosine method have been introduced to reduce these equations into the ordinary differential equations then the exact solution was extracted. The search results showed the possibility of using this method successfully for solving the FCEW and FCMEW equations at different time levels.

Keywords: conformable fractional derivative, Sine-Cosine function method

1. Introduction
Several decades ago, more generalized forms of differential equations are described as fractional differential equations. Various phenomena in many natural and social sciences fields like engineering, geology, economics, meteorology, chemistry, and physics are modeled by those equations[1],[2]. Great efforts were made to find powerful ways to obtain exact and numerical solutions to solve equal width wave equation EWE:
\[ U_t + UU_x - \mu U_{xxx} = 0, \ a \leq x \leq b \]
(1)
with the boundary conditions \( U \to 0 \) as \( x \to \pm \infty \), \( x \) is the space coordinate , \( \mu \) is a positive parameter and \( t \) is time, and modified equal width wave equation MEWE:
\[ U_t + U^2 U_x - \mu U_{xxx} = 0, \ a \leq x \leq b \]
(2)
Using various mathematical methods, as these equations play a fundamental role in the study of nonlinear scattered waves because they describe a set of physical behaviors such as shallow water and plasma sound waves ion. The convergence analysis is discussed for nonlinear Klein-Gordon equation and the solutions are stable and convergent when the sine function is used[3]. In recent years, it has been proposed many of the powerful and reliable ways to get the exact solution of fractional differential equations, such as Lie-Trotter and Strang Splitting methods[4], trigonometric cubic B‐spline collocation method [5], finite element method [6], Lie symmetry method[7] have been used to find solutions for (EWE and GWE) , Fourier spectral method[8],method of the dynamical system[9], Optimal system[10], all used to find solutions for (MEWE).Sine-Gordon expansion[11],the tanh-sech method[12] , the simple hyperbolic tangent Ansatz method [13] and sine-cosine method[14] ,the bright soliton solutions and singular solutions are constructed for space-time fractional EW and modified EW equations[1], the Riccati–Bernoulli sub-ODE method[15] ,modified Kudryashov method [16], the generalized \((G'/G)\)-expansion method[17],[18] for the space-time fractional nonlinear partial differential equations .

In this paper, the Sine and Cosine method was used to find the exact solution of both fractional coupled EW and coupled MEW equations defined with conformable derivative. The organization of this paper is as follows: In Section 2 we present a sum summary about the conformable derivative. Section 3, a brief about the method that has been used in the solution, which is Sine and Cosine with some graphical representatives are presented. Finally, the application, results, and conclusions.
2. Conformable fractional derivative and some properties

A definition of derivative of fractional order which is called conformable fractional derivative, due to its simplicity, has got significant attention these days. It depends on the basic limit definition of the derivative, let first recall the basic limit definition [19],[20]

Definition: Let \( u: [0, \infty) \to \mathbb{R} \) be a function. Then the conformable fractional derivative of order \((r)\) is defined as follows:

\[
D_t^r(u(t)) = \lim_{\Delta t \to 0} \frac{u^{(r+\epsilon t^r)}(t) - u(t)}{\epsilon t^r}
\]

Where \( t > 0 \), \( r \in (0,1] \), \( \epsilon > 0 \), \( \epsilon = \Delta t \) (3)

Some useful properties of the conformable fractional derivative are listed below

(a) \( D_t^r(t^n) = rt^{n-r} \), for all \( n \in \mathbb{R} \).

(b) \( D_t^r(au(t) + bv(t)) = a D_t^r(u(t)) + b D_t^r(v(t)), \) for all \( a, b \in \mathbb{R} \).

(c) \( D_t^r(v(t)u(t)) = D_t^r(v(t))(u(t)) + u(t) D_t^r(v(t)). \)

(d) \( D_t^r(u(t)/v(t)) = \frac{v(t) D_t^r(u(t)) - u(t) D_t^r(v(t))}{v(t)^2}, v(t) \neq 0. \)

(e) \( D_t^r(u(t)) = t^{1-r} \left( \frac{du}{dt} \right), \) If \( u(t) \) is differentiable. [21]

3. The method of Sine-Cosine function. [14]

To clarify the basic ideas of the method that has been used in this paper, Consider the form a system of nonlinear partial differential equations in (4):

\[
Q(u, D_t^{r_1}u, D_x^{r_2}u, D_{xt}^{r_3}u, D_{xx}^{r_2}u, D_{xxt}^{r_3}u, \ldots) = 0
\]

\[
S(v, D_t^{r_1}v, D_x^{r_2}v, D_{xt}^{r_3}v, D_{xx}^{r_2}v, D_{xxt}^{r_3}v, \ldots) = 0, \quad 0 < r_1, r_2 \leq 1 \quad (4)
\]

Where \( u(x, t) \) and \( v(x, t) \) are solutions for the system of nonlinear fractional partial differential equations.

By using the wave transformations.

\[
u(x, t) = w(k), \quad \nu(x, t) = g(k) (5)
\]

where \( k = \frac{c x - d t}{r}, c \) and \( d \) are nonzero constants . \( d \) symbolize the velocity of a wave traveling in the positive direction of \( x \)-axis.

Then, from Eq(5), we have:

\[
u_t = -dw'(k), \quad \nu_x = cw'(k), \quad \nu_{xx} = c^2 w''(k), \quad \nu_{xxt} = -dc^2 w'''(k), \quad \nu_t = -dg'(k), \quad \nu_x = cg'(k), \quad \nu_{xx} = c^2 g''(k), \quad \nu_{xxt} = -dc^2 g'''(k). \quad (6)
\]

Substituting Sine-Cosine function in Eq. (5) we get:

\[
w(k) = p \sin^{m}(\varphi_1 k), \quad |k| \leq \frac{\pi}{2\varphi_1} \quad (7)
\]

or

\[
g(k) = q \cos^{m}(\varphi_2 k), \quad |k| \leq \frac{\pi}{2\varphi_2} \quad (7)
\]

\[
w(k) = p \sin^{m}(\varphi_2 k), \quad |k| \leq \frac{\pi}{2\varphi_2} \quad (8)
\]

where \( p, \varphi_1, \varphi_2, q, m \) and \( n \) are parameters that will be determined.

We present the first and second derivatives of functions in the equation (7) with respect to \( k \):

\[
w'(k) = np\varphi_1 \sin^{m-1}(\varphi_2 k) \cos(\varphi_2 k)
\]

\[
w''(k) = -mp\varphi_1^2 \sin^m(\varphi_2 k) + m(m-1)p\varphi_2^2 \sin^{m-2}(\varphi_2 k)
\]

\[
g'(k) = nq\varphi_2 \sin^{m-1}(\varphi_2 k) \cos(\varphi_2 k)
\]

\[
g''(k) = -nq^2 \sin^m(\varphi_2 k) + n(n-1)q\varphi_2^2 \sin^{m-2}(\varphi_2 k) \quad (9)
\]

For equation (8), we get:

\[
w'(k) = -mp\varphi_2 \cos^{m-1}(\varphi_2 k) \sin(\varphi_2 k)
\]

\[
w''(k) = -mp\varphi_2^2 \cos^m(\varphi_2 k) + m(m-1)p\varphi_2^2 \cos^{m-2}(\varphi_2 k)
\]

\[
g'(k) = -nq\varphi_2 \cos^{m-1}(\varphi_2 k) \sin(\varphi_2 k)
\]
\[ g''(k) = -n^2 q \cos n(\phi_k) + n(n-1)q \cos^{n-2}(\phi_k) \]  

Applying Eq. (7) and its derivatives in Eq. (9) to transmit the system of nonlinear fractional partial differential equations (4) into nonlinear ordinary differential equations:

\[ P(w', w'', w''', g', g'', g''', \ldots) = 0 \]  

Where \( P \) is a polynomial in \((\sin m(\phi_k) \text{ and } \cos m(\phi_k))\). By equating the coefficient of each power of \((\sin m(\phi_k) \text{ and } \cos m(\phi_k))\) in Eq.(11) to zero, a system of algebraic equations will be obtained whose solution yields the exact solution of Eq. (4).

4. Application

In this section, the “Sine-Cosine method” has been applied to find the exact solution of the space – time fractional CEW and CMEW equations.

4.1 Solution for conformable fractional CEW equations

Consider space – time conformable fractional CEW of the form:

\[ D_u^{\alpha} u(x, t) + eD_v^{\beta} u^2(x, t) - \mu D_{xt}^{\gamma} u(x, t) + e D_{vt}^{\delta} v^2(x, t) = 0 \]  

Applying the Eq. (5) and Eq. (6) into Eq. (12), the following system of ordinary differential equations will be obtained:

\[ -dU' + ec(U^2)' + \frac{1}{2}dc^2 U'''' + ec(V^2)' = 0 \]  

Integrating Eq. (13) once with respect to \( k \) and taking the constant of integration to be zero, yields:

\[ -dU' + ecU^2 + \frac{1}{2}dc^2 U'''' + ecV^2 = 0 \]  

Substituting Eq. (9) in to (14) gives:

\[ -d(p \sin m(\phi_k)) + ec(p \sin m(\phi_k))^2 + \frac{1}{2}dc^2 (-m^2 p^2 \sin^m(\phi_k) + m(m-1)p^2 \sin^{m-2}(\phi_k)) + ec(q \sin n(\phi_k))^2 = 0 \]  

To calculate the parameters, the exponent of each pair of sines is balanced to obtain \( m \) & \( n \). Then the terms that have the same power in \( \sin m \) are determined and their coefficients are equaled to zero. During this way, the problem is reduced to a system of algebraic equations that can be solved.

System (15) satisfied if System (16) of the algebraic equations is valid:

\[ m-2=2n, \quad mc^2 dn^2 p_1^2 = 0 \]  

\[ ecq^2 + mc^2 m(m-1) q P_2^2 = 0 \]  

Solving the above system, we obtain:

\[ m= -2, \quad \varphi_1 = \pm \frac{1}{2c} \sqrt{-\frac{1}{\mu}} = \varphi_2, \quad q = \frac{3}{2} \frac{d}{ec}, \quad p = \frac{3}{2} \left( -\frac{1}{2c^2} \sqrt{3} \right) \frac{d}{ec} \]  

Case (1)

Using the sine function, and by substituting for the parameter values, the following exact solution was obtained, as explained below:

\[ u(x, t) = \frac{1}{2} \left( \frac{x^2 + \sqrt{3} \sqrt{d}}{ec} \right) \sin^2 \left( \pm \frac{1}{2c} \sqrt{-\frac{1}{\mu}} (c \frac{x^2}{r} - d \frac{r}{x}) \right) \]
\[ v(x, t) = \frac{3}{2} \frac{d}{2 ec} \sin^{-1} \left( \frac{1}{2c} \sqrt{\frac{1}{\mu}} \left( c \frac{x'}{r} - d \frac{f'}{r} \right) \right) \] (18)

which in turn contains a set of the following states:

1-When \( \rho = \frac{3}{2} \frac{(-1 + \sqrt{3})d}{2 ec} \), \( \phi_1 = \frac{1}{2c} \sqrt{\frac{1}{\mu}} \phi_2 \) and \( q = \frac{3}{2 ec} \)

In compensation for the values of \( t \) and the value of the fractional derivative represented by \( r \), a gap appears due to the values of the inverse of the sine function at zero for some values of \( t \) and the results are infinite, as shown in Figure (1-a) for a fixed value of the fractional derivative and different times, While the three-dimensional Figure (1-b) shows the results for different values of \( r = (0.65, 0.3, 0.15) \) at different times.

![Figure 1](image1)

2-When \( \rho = \frac{3}{2} \frac{(-1 + \sqrt{3})d}{2 ec} \), \( \phi_1 = \frac{1}{2c} \sqrt{\frac{1}{\mu}} \phi_2 \) and \( q = \frac{3}{2 ec} \)

After compensation, it was found that the results are proportional in the essential case, yet dissimilar to the sign (negative sign) and in this manner the shapes are topsy turvy as appeared in Figure(2).

![Figure 2](image2)
Figure 2: Exact solution for the space-time FCEWE for $r=0.45$ with different times.

**Case (2):**
In this part, we used the cosine function to get the exact solution as shown in equation (19) for values of $u$ and $v$ respectively and we will study special cases when substituting the various values of parameters $p$, $\varphi_1$, $\varphi_2$ and $q$.

$$u(x, t) = \frac{3}{2} \left( -\frac{1}{2} + \frac{1}{2} \sqrt{3} \right) d \cos^2 \left( \frac{1}{2c} \sqrt{\frac{1}{\mu}} \left( c \frac{r}{r} - d \frac{r}{r} \right) \right)$$

$$v(x, t) = \frac{3}{2} d \cos^2 \left( \frac{1}{2c} \sqrt{\frac{1}{\mu}} \left( c \frac{r}{r} - d \frac{r}{r} \right) \right)$$

(19)

The states for case (2) are:

1. When $p = \frac{3}{2} \left( -\frac{1}{2} + \frac{1}{2} \sqrt{3} \right) d$, $\varphi_1 = \frac{1}{2c} \sqrt{\frac{1}{\mu}} = \varphi_2$ and $q = \frac{3}{2} d$

As can be seen from Figure (3) above, when using the cosine function, no gaps or unknown values appear for some values of $t$, for example $t \epsilon [0,25]$ and $r=1$.

Figure 3: Exact solution for the space-time FCEWE for $r=1$ and $0 \leq t \leq 25$.

When the value of $r$ is fixed at $r=0.45$ with different values of $0 \leq t \leq 45$ we obtained the results represented in Figure (4).

Figure 4: Exact solution for the space-time FCEWE for $r=0.45$ at different time.
The results, in this case, are negative, therefore the shapes are upside down as it is clear in Figure (5) for \( r=0.45 \) with two different values of \( t \).

Figure 5: Exact solution for the space-time FCEWE for \( r=0.45 \) with different time.

It is the same result in section 1 for case (1) when using sine function and case (2) by using cosine function.

4.2 Solution for conformable fractional CMEW equations

Consider Space – Time conformable Fractional CMEW for finding functions \( u(x,t) \) and \( v(x,t) \) in the form:

\[
2 - p = \frac{3}{2} \left( \frac{1}{2} \sqrt[3]{\frac{1}{c}} \right) \frac{d}{d \mu}, \quad \varphi_1 = \frac{1}{2c} \sqrt{\frac{1}{\mu}} \varphi_2, \quad q = \frac{3}{2} \frac{d}{d \mu} 
\]

The system (23) is valid if the following algebraic equations satisfied:
m-2=3m implies that m=-1, \( \varphi_1 = \mp \frac{1}{c \sqrt{\mu}} = \varphi_2, \ p = \mp \sqrt{\frac{d}{ec}} = q \)

It is worth noting, the values \( u(x, t) \) and \( v(x, t) \) are the same when the particular structure has been found since the parameters have equal values, this feature facilitates the process of finding the exact solution and reduces the possibilities. We will briefly discuss the cases, as it is a repetition of the procedures applied in section (4) for finding the exact FCEW solution. Hint(\( \mu=e=c=d=1 \)).

**Case (1)**

This case represents the solutions for using the sine function of the space values as mentioned previously and different values for each of the fractional derivative and the time value and again we notice the emergence of gaps resulting from the reasons mentioned in the first case when finding the exact solution for FCEWE, the values of (Cosecant) function are infinite.

\[
\begin{align*}
u(x, t) & = \mp \frac{1}{3 40} \sin^{-1} \left( \mp \frac{1}{1} \left( c \frac{x^r}{r} - d \frac{t^r}{r} \right) \right) \\
v(x, t) & = \mp \frac{1}{3 40} \sin^{-1} \left( \mp \frac{1}{1} \left( c \frac{x^r}{r} - d \frac{t^r}{r} \right) \right) \quad (24)
\end{align*}
\]

Now, these solutions can be plotted at different time levels the solitary wave motion is shown. To clarify more we show through Figure (6- a & b) the results obtained by applying the sine function and thus the appearance of some infinite value when compensating the different values of time for each derivative value represented by \( r \).

1- \( u(x, t) = \frac{1}{3 40} \sin^{-1} \left( \frac{1}{1} \left( c \frac{x^r}{r} - d \frac{t^r}{r} \right) \right) \)

![Figure 6: Exact solution for space-time FCMEWE for 0≤ t ≤ 45: (a) r=0.85 and (b) r=0.25](image)

2- \( u(x, t) = -\frac{1}{3 40} \sin^{-1} \left( \frac{1}{1} \left( c \frac{x^r}{r} - d \frac{t^r}{r} \right) \right) \)

The results are exactly the same as in section 1 above for positive values.

3- \( u(x, t) = -\frac{1}{3 40} \sin^{-1} \left( \frac{1}{1} \left( c \frac{x^r}{r} - d \frac{t^r}{r} \right) \right) \)

In this case, the results are the same as in section 1 with a negative signal. Thus, the figures appear upside down as shown in Figure (7) for different values of the fractional derivative \( r \) at different times.
Figure 7: Exact solution for space-time FCMEWE for $0 \leq t \leq 45$: (a) $r=0.85$ and (b) $r=0.25$

\[ 4-u(x, t) = \frac{d}{dt} \sin^{-1} \left( -\frac{1}{\mu} \frac{1}{c} \left( \frac{c x^r}{r} - d \frac{t^r}{r} \right) \right) \]

The results were completely identical to the section 3 as in the above.

Case (2)

During this case, solutions can be drawn at different time levels, as shown in Figure (8) and Figure (9). The results obtained by applying the cosine function when substituting different time values for each fractional derivative value represented by $r$, is characterized by the absence of gaps that appeared when using the sine function.

\[ u(x, t) = \mp \frac{d}{dt} \cos^{-1} \left( \mp \frac{1}{\mu} \frac{1}{c} \left( \frac{c x^r}{r} - d \frac{t^r}{r} \right) \right) \]

\[ v(x, t) = \mp \frac{d}{dt} \cos^{-1} \left( \mp \frac{1}{\mu} \frac{1}{c} \left( \frac{c x^r}{r} - d \frac{t^r}{r} \right) \right) \tag{25} \]

Which contains a set of cases that are displayed through the following graphics triple dimension:

1- $u(x, t) = \frac{d}{dt} \cos^{-1} \left( \frac{1}{\mu} \frac{1}{c} \left( \frac{c x^r}{r} - d \frac{t^r}{r} \right) \right)$

The exact solution of $u$ for the values of $x \in [0,100]$, with $r=0.35$ and $r=0.5$ as shown in Figure (8-a&b) with different time.

Figure 8: Exact solution for space-time FCMEWE for (a) $r=0.35$, (b) $r=0.5$ at different time.
2- \( u(x, t) = -\sqrt{\frac{1}{k(c^2 + \mu^2)}} \cos(\frac{1}{c \sqrt{-\frac{1}{\mu}} (c \frac{x^r}{r} - d \frac{t^r}{r}))} \)

The results are exactly the same as in section 1 above for positive values.

3- \( u(x, t) = -\sqrt{\frac{1}{k(c^2 + \mu^2)}} \cos(\frac{1}{c \sqrt{-\frac{1}{\mu}} (c \frac{x^r}{r} - d \frac{t^r}{r}))} \)

The results are the same as in section 1 with a negative signal, the Figures appear upside down as shown in Figure (9) for different values of the fractional derivative at different times.

![Figure 9](image_url)

**Figure 9:** Exact solution for space-time FCMEWE for (a) \( r=0.35 \), (b) \( r=0.5 \) at different times.

4- \( u(x, t) = \sqrt{\frac{1}{k(c^2 + \mu^2)}} \cos(\frac{1}{c \sqrt{-\frac{1}{\mu}} (c \frac{x^r}{r} - d \frac{t^r}{r}))} \)

The results were completely identical to the section 3 as in the above.

5- **Conclusion**

The research presents the exact solution for space-time fractional CEW equation and modified CEW equation by using conformable fractional derivative with the Sine-Cosine method. After transform equations to ordinary differential equations, the solutions of the sine and cosine type are explored. We presented the properties of the exact solution for the model. The performance of this technique demonstrated the efficiency and strength of this method, especially when using the Cosine function.

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