Microwave control of coupling parameters in spinor alkali condensates

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We propose a protocol which utilises radio frequency magnetic pulses in order to tune the effective two-particle scattering amplitudes for alkali atoms in the $F=1$ hyperfine ground state. Unlike the Feshbach resonance method, the proposed protocol preserves with controllable accuracy the global rotational symmetry in the spin space offering access to a broad region of the phase diagram of the rotationally-symmetric spinor Bose condensate. Examples of $^{41}$K and $^7$Li are considered and it is shown that for these atoms sufficient variation in the effective coupling constants can be achieved in order to explore phase transitions between different symmetry-broken phases of the condensate.

Ultracold atomic systems serve as potent quantum laboratories, where an amazing range of many-body hamiltonians can be realised and investigated owing to the precise spacial and temporal control over the interactions. Of particular interest is the possibility to fabricate bespoke strongly interacting many body systems with a high spin symmetry, thus paving a way to the investigation of mechanisms of spontaneous symmetry breaking, symmetry changing quantum phase transitions, spin liquid states and other topological states of matter \cite{11,13}. In order to use the full potential offered by cold atoms it is desirable to have control over effective interaction strength while preserving the high degree of symmetry of the interaction. The most popular way of tuning the interaction, the Feshbach resonance, does not offer such a possibility because of the symmetry breaking Zeeman splitting \cite{6}. In some cases, the optical Feshbach resonance can be used as an alternative way of control over interactions \cite{7,9}, which has been demonstrated for $^{87}$Rb, $^{172}$Yb, and $^{88}$Sr atoms \cite{10,13}.

Here we investigate an alternative way of symmetry-preserving control over the coupling constants utilising radio-frequency (RF) dressing technique. Our interest is inspired by the successful use of the radio-frequency radiation for the manipulation of the hyper-fine Zeeman split levels of the atomic ground state manifold. This technique enabled, in particular, creation of topologically nontrivial structures in the magnetic BEC condensate \cite{11,13,14,15,16}. It was also used for the study the nonequilibrium population dynamics of different spin sates \cite{17,19,20}.

We focus on the spin-1 alkali condensate as a minimal nontrivial example with competing spin-dependent interactions. Spin-symmetric interactions are characterized by two interaction constants, which in turn are determined by the spin-scattering length in the total momentum $F=0$ and $F=2$ channels. Different quantum phases are realized depending on the relation between the two coupling constants \cite{21}. Therefore, experimental exploration of the quantum phase diagram requires their tunability. In this work, we find that a train of radio-frequency pulses can be used to control inter-atomic interactions giving access to a broad range of coupling constants. Moreover, we design a protocol of the time-dependent polarization of RF field that suppresses the quadratic Zeeman coupling, thus leaving the spin-rotational symmetry intact. Extended to higher spin, our approach may prove useful for the implementation of tunable $SU(N)$ symmetric interactions that are necessary for the experimental creation of exotic topologically nontrivial spin liquid phases \cite{3,4}.

We begin by considering the basic microscopic process of low-energy collision of two identical alkali atoms. The direct interaction between nuclear spins in such a process can be neglected \cite{6}, therefore the outcome of the collision is determined by scattering lengths in the singlet and triplet electron spin channels, $a_1$ and $a_3$. Assuming that each atom in the wave zone is in its hyperfine ground state, the inter-atomic collision is characterized by the set of scattering lengths corresponding to the channels of different total angular momentum $F$. The scattering problem is encoded in the radial $l=0$ center-of-mass equation

\[
\frac{1}{2\mu} \frac{d^2}{dr^2} + i V(r)^{1/2} \cdot 3 \not{P} + 3 V(r)^{3/2} \cdot \not{P} + A(\hat{I}_1 \cdot \hat{S}_1 + \hat{I}_2 \cdot \hat{S}_2) \psi = E \psi
\]

Here $\hat{S}_i$ is the spin of the $i$th particle, $\hat{I}_i$ is its nuclear spin and $^{m} \not{P}$ is the projector on the two-atom spin state of multiplicity $m$, and $\mu$ denotes the reduced mass of the two atoms. At low energies, $^{m} V(r)$ can be replaced by the zero-range pseudopotentials that impose the appropriate scattering length in the corresponding channel through the Bethe-Peierls boundary condition \cite{22}. The constant $A$ is the hyperfine constant of an individual atom. The two-atom wave function $\psi$ is a reducible spinor of size $4 \times (2I+1)^2$. Here we are interested in the spin-1 bosons, which are realized by alkali atoms with the nuclear spin $I = 3/2$.

For given kinetic energy $E$ the wave-zone motion can be characterized by a set of four good quantum numbers $F_1, F_2, F, M_F$ that is the total momentum of each atom, the total momentum of the pair and its projection. Introducing the projector $\hat{P}_{F_1F_2}$ onto the subspace of given
values of momenta we write the wave zone dispersion relation
\[
\sum_{F_1=1}^{2} \sum_{F_2=1}^{2} \sum_{F=|F_1-F_2|}^{F_1+F_2} \left( \frac{k^2}{2m} + E_{F_1} - E \right) \hat{P}^F_{F_1F_2} \psi = 0,
\]
where
\[
E_{F_1F_2} = \frac{A}{2} [F_1(F_1 + 1) + F_2(F_2 + 1)] - 2A,
\]
and we chose the energy offset such that \(E_{11} = 0\). We are interested in the low energy limit \(E \ll A\), where both atoms are in the lowest hyperfine state \(F_1 = F_2 = 1\). In this limit the higher energy spin configurations can only contribute to scattering as evanescent modes. We introduce the corresponding values of the momentum
\[
k = \sqrt{2\mu E}, \quad \kappa_{F_1F_2} = \sqrt{2\mu (E_{F_1F_2} - E)}.
\]
Furthermore, we define the following matrix
\[
\hat{T}(r) = (e^{-ikr} - Se^{ikr})\hat{P}^0_{11} + \alpha \hat{P}^0_{22}e^{-\kappa_{22}r}
\]
for the \(F = 0\) scattering channel and and seek a solution to Schroedinger’s equation in the form
\[
\Psi(r) = T(r)v
\]
where \(v\) is some arbitrary spinor with \(F_2 = 0\).

For the \(F = 2\) channel we write
\[
\hat{T}(r) = (e^{-ikr} - Se^{ikr})\hat{P}^0_{11} + \left( \alpha \hat{P}^2_{12} + \beta \hat{P}^2_{21} \right) e^{-\kappa_{12}r} + \gamma \hat{P}^2_{22}e^{-\kappa_{22}r}
\]
and assume that \(v\) is any vector with \(F_2 = -2\). The Bethe-Peierls boundary conditions read
\[
m \hat{P} \left( \Psi'(0) + \frac{1}{a_m} \Psi(0) \right) = 0, \quad m = 1, 3
\]
and are to be used for the determination of the unknown coefficients \(S, \alpha, \beta, \gamma\). Since \(S = e^{2iS}\) the scattering length is found from
\[
i k \frac{S + 1}{S - 1} = -\frac{1}{a} + o(k), \quad k \to 0
\]
The result of the calculation is as follows. In the \(F = 0\) channel
\[
\frac{1}{a_{F=0}} = \frac{10a_1 + 6a_3 - 8\lambda}{16a_3a_1 - 3a_1\lambda - 5a_3\lambda}.
\]
In the \(F = 2\) channel
\[
\frac{1}{a_{F=2}} = \left[ 6\sqrt{3}a^2_3 + 26\sqrt{2}a_3a_1 + 16\lambda^2 - (6\sqrt{2} + 14)a_1\lambda - (10\sqrt{2} + 18)a_3\lambda \right] \left[ 3a_1\lambda^2 + 13a_3\lambda^2 - (7\sqrt{2} + 12)a^2_3\lambda - (9\sqrt{2} + 20)a_1a_3\lambda + 32\sqrt{2}a_1a^2_3 \right],
\]
where
\[
\lambda^2 = \frac{\hbar^2}{2\mu A}.
\]
One can see from Eqs. \((10), (11)\) that the numerator vanishes at certain values of \(\lambda\), signaling the resonantly diverging scattering length. The scattering length varies strongly close to the resonance, and it changes sign on different sides of the resonance. In the rest of the paper we show how one can use rf-pulses in order to vary the effective hyperfine interaction constant \(A\), thus changing the value of \(\lambda\) by virtue of Eq. \((12)\). We propose a protocol which enables an order of magnitude variation of the strength of effective hyperfine interaction while maintaining the spin-rotational symmetry.

The proposed protocol consists of periodic sequence of pulses with magnetic field within one period \(T\) given by
\[
\mathbf{h}(t) = \sum_{n=1}^{4} \mathbf{e}_n(t - nT/4).
\]
Here the four unit vectors \(\mathbf{e}_n\) form the vertices of a perfect tetrahedron, and the function \(\eta(t)\) is defined by
\[
\eta(t) = \begin{cases} 
  h_0 & \text{for } 0 \leq t < T/16, \\
  -h_0 & \text{for } 3T/16 \leq t < T/4, \\
  0 & \text{otherwise.}
\end{cases}
\]
The rf-field introduces the periodic time dependent perturbation to the Hamiltonian \(H_{\text{rf}} = g\mu_B \mathbf{h}(t) \cdot \hat{S}\).

The effective Hamiltonian is calculated as the time-independent Hamiltonian that results in the equivalent quantum evolution operator over the period of the sequence \([23, 24]\)
\[
\hat{U}(T, 0) = \prod_{n=1}^{4} \hat{U}_n = e^{-iH_{\text{eff}}T},
\]
where
\[
\hat{U}_n = \exp \left[ \frac{T}{16} (\Omega_R \mathbf{e}_n - A \hat{I}) \cdot \hat{S} \right] \exp \left[ -iA \hat{I} \cdot \hat{S} \frac{T}{8} \right] \times \\
\exp \left[ -i \frac{T}{16} (\Omega_R \mathbf{e}_n + A \hat{I}) \cdot \hat{S} \right],
\]
where \(\Omega_R = g\mu_B h_0\) denotes the Rabi frequency in the magnetic field of the pulse. In accord with Eq. \((15)\), the effective energies are calculated from the eigenvalues \(u_k\) of the evolution operator by
\[
E_k = (i/T) \log u_k.
\]
The effective hyperfine splitting is defined as the difference between the largest and the smallest eigenenergies, \(A_{\text{eff}} = E_{\text{max}} - E_{\text{min}}\). The proposed protocol introduces a small quadratic Zeeman coupling, leading to a splitting of the energy levels with \(M = 0\) and \(M = \pm 1\). The induced quadratic Zeeman splitting is determined by the
The effective Hamiltonian defined by Eqs. (15), (16) is completely determined by two dimensionless parameters, which can be chosen as the dimensionless period of the pulse sequence \( \tau = TA/(2\pi\hbar) \), and the dimensionless Rabi frequency \( h\Omega_R/A \). The change of the effective hyperfine coupling induced by the dressing with rf-pulses is shown in Fig. 1. One can see the suppression of the hyperfine coupling by an order of magnitude (solid line in Fig. 1). Especially interesting for practical applications are the points \( h\Omega_R/A \approx 16, h\Omega_R/A \approx 21 \), at which the quadratic Zeeman coupling is completely suppressed, hence the full spin rotational symmetry remains intact. The data in Fig. 1 were calculated for the dimensionless period of the pulse sequence \( \tau = 0.5 \). Changing the value of \( \tau \) leads to the rescaling of the curves without changing their qualitative behavior. The quadratic Zeeman splitting for different values of \( \tau \) is shown in Fig. 2.

It is important to note that the proposed protocol does not change the inter-atomic two-body interaction potential in the leading order of the high-frequency expansion in \( 1/\omega \), where \( \omega \) is the fundamental frequency of the driving (in the considered case \( \omega = 2\pi/T \)). Indeed, the Hamiltonian of two atoms in the driving field can be represented as \( H = H_1 + H_2 + H_{\text{int}} + H_{\text{ext}}(t) \). In the proposed protocol, the interaction with external RF field for two atoms is written as \( H_{\text{ext}}(t) = h_i(t)(\hat{S}_{1z} + \hat{S}_{2z}) \), hence it commutes with the interaction part of the Hamiltonian \( H_{\text{int}} \) (see Eq. (4)). Performing the canonical transformation \( \hat{H} = UHU^{-1} \) with \( U = (Te^{i\int H_{\text{ext}}(t)dt}) \) we obtain \( \hat{H} = \hat{H}_1(t) + \hat{H}_2(t) + H_{\text{int}} \). The interaction term remains unchanged, since it commutes with \( H_{\text{ext}}(t) \). In the proposed driving protocol, the transformation operator \( U \) is periodic in time, which results in the periodic time dependence of the effective one-atom Hamiltonians \( \hat{H}_1, \hat{H}_2 \) with the frequency \( \omega \). Therefore, the effective one-particle Hamiltonians \( H_1, H_2 \) allow the Fourier expansion \( H_i(t) = \hat{H}_i + \sum_{n=1}^{\infty} (V_{i,n} e^{i\omega_nt} + V_{i,-n} e^{-i\omega_nt}) \). For \( \hbar\omega \) larger than the hyperfine splitting, the effective time-independent Hamiltonian can be constructed using the \( (1/\omega) \) expansion as described in details in Refs. 23, 24. Thereby, the zero order term reads \( H_1 + H_2 + H_{\text{int}} \), where \( \hat{H}_i \) denote the single atom Hamiltonian with renormalized hyperfine coupling. The corrections to the interaction part of the Hamiltonian appear only in the order of \( 1/\omega^2 \) (see Eq. (C10) in Ref. [24]).

To illustrate the proposed method of control over the inter-atomic scattering length, we calculated the change of the scattering length for two types of spin-1 atoms, that are often used for creation of the BEC. The spin-dependent scattering lengths for \(^{41}\)K atoms are shown in upper panel of Fig. 3. Particularly interesting is the region between the two resonances, where the negative scattering length in the \( F = 0 \) channel is complemented by the positive scattering length in the \( F = 2 \) channel. One can expect appearance of unconventional BEC in this regime. The lower panel in Fig. 3 shows the scattering lengths for \(^{7}\)Li atoms. The BEC of \(^{7}\)Li is known to be unstable against collapse under usual conditions, because of the negative scattering lengths in both \( F = 0 \) and \( F = 2 \) channels. One can see in Fig. 3(lower panel), that the application of the external RF field can drive \(^{7}\)Li atoms in the region between the two resonances, where both scattering length become positive. One can expect a stable BEC of \(^{7}\)Li in that regime. In both considered cases, the necessary RF frequencies and the Rabi frequencies associated with the magnetic field pulse lie within several GHz. Such conditions can be realized in the modern RF pulse generators.

In conclusion, we proposed a method of control over
widely used magnetic and optical Feshbach resonances between the two-particle states in open and closed scattering channels, the proposed protocol is based on the change of the effective hyperfine coupling of a single atom dressed by the external RF field. Explicit calculations for $^{41}$K atoms show, that one can reach the regime, where the scattering lengths in $F = 0$ and $F = 2$ channels have different signs, which would allow experimental investigation of spinor Bose condensates with exotic magnetic orderings. Calculations for $^7$Li atoms demonstrate the regime, where both scattering length can change sign from initially negative without the external radiation to positive in the field, which provides a mechanism for the stabilization of spinor condensate of $^7$Li atoms against collapse. Our findings may prove useful for experimental creation and investigation of spinor Bose condensates that could not be realized using magnetic or optical Feshbach resonances.

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