On convergence of the HFF expansion for meson production in NN collisions

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Abstract

We consider the application of heavy fermion formalism based $\chi$PT to meson production in nucleon-nucleon collisions. It is shown that to each lower chiral order irreducible diagram there corresponds an infinite sequence of loop diagrams which are of the same momentum power order. This destroys the one-to-one correspondence between the loop and small momentum expansion and thus rules out the application of any finite order HFF $\chi$PT to the $NN \to NN\pi$ reactions.

Key Words: hadroproduction, chiral perturbation theory, heavy fermion formalism.

13.75Cs, 14.40Aq, 25.40Ep

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There have been several attempts recently, to apply the heavy fermion formalism (HFF) based chiral perturbation theory (χPT) \[1,2\], to calculate meson production rate in nucleon-nucleon collisions \[3–8\]. It is well known that in a fully relativistic χPT there is no one-to-one correspondence between the loop and small momentum expansion. Such a correspondence is believed to be restored in the extremely non-relativistic approach of the HFF \[1,2\]. An essential and most important clue to assess the validity of these calculations resides on how rapid the HFF expansion converges. Detailed χPT calculations which account for all contributions from tree and one loop diagrams to chiral order D=2, show that within the framework of the HFF, one loop contributions are sizably bigger than the lowest-order impulse and rescattering terms, indicating that the HFF power series expansion does not converge fast enough and therefore may not be suitable to calculate pion production rate in NN collisions \[3\]. More recently Bernard et al. \[3\] and Gedalin et al. \[4\] have shown that the HFF power series expansion of the nucleon propagator is on the border of its convergence circle. Consequently, a finite order HFF can not possibly predict nucleon pole terms correctly and should not be applied to meson production. It is the purpose of the present comment to call attention to the fact that within the framework of the HFF χPT, the one-to-one correspondence between the loop and small momentum expansion is badly destroyed for processes of sufficiently large momentum transfer. Particularly, for each low chiral order D diagram there corresponds an infinite sequence of n-loop diagrams, \(n = 1, 2, ...\), of chiral order \(D_n = D + 2n\), which have the same low momentum power as the original diagram. Therefore, any finite chiral order HFF based χPT calculations, can not possibly explain meson production in NN collisions.

To be specific we consider pion production via the \(NN \rightarrow NN\pi\) reaction. Such a process necessarily involves large momentum transfer. The characteristic four momentum transferred at threshold is \(Q \approx (-m_\pi/2, \sqrt{Mm_\pi})\), where \(M\) and \(m_\pi\) are masses of the nucleon and meson produced. This stands in marked difference with the Weinberg’s standard power counting \[11\], where presumably the momentum transferred is considerably smaller of the order \(Q^2 \approx m^2_\pi\). Clearly, one can not use directly the original power counting scheme of
Weinberg [11]. However, we may apply the modified Weinberg’s power counting, a scheme tailored to deal specifically with the production process [4]. This scheme includes the following rules:

(i) a $\pi NN$ vertex of zero chiral order $D=0$, $V^{(0)}_{\pi NN}$, contributes a factor $Q/F$,

(ii) a pion propagator contributes a factor $Q^{-2}$,

(iii) a nucleon propagator $(v \cdot Q)^{-1} \approx m^{-1}_\pi$,

(iv) a $\pi NN$ vertex of chiral order $D=1$, $V^{(1)}_{\pi NN}$ contributes a factor $(k_0 Q/F M) \approx m^{3/2}_\pi/(FM^{1/2})$,

(v) a $2\pi NN$ $D=1$ vertex, $V^{(1)}_{\pi\pi NN}$, contributes a factor $k_0 Q^0/(F^2 M)$.

In our notations (see Fig. 1) we refer to the four-momentum squared $Q^2 = (p_1 - p_2)^2 = (vQ)^2 - \vec{Q}^2 \approx -m_\pi M$, with $v$, $|\vec{Q}| \sim \sqrt{m_\pi M}$, $Q^0 = vQ \sim m_\pi$, $k_0$, being the nucleon four-velocity, the transferred three-momentum, the transferred energy and the pion total energy, respectively. The radiative pion decay constant is denoted by $F$. In terms of $Q = \sqrt{m_\pi M}$ and $vQ = m_\pi$, the rules listed above are exactly the ones quoted by Cohen et al. [4]. To calculate loop contributions one has to add three more rules:

(vi) a loop integration contributes a factor $(Q^2/4\pi)^2$,

(vii) a four pion vertex of zero order, $V^{(0)}_{\pi\pi\pi\pi}$, contributes a factor $Q^2/F^2 \sim (m_\pi M)/F^2$,

(viii) a $3\pi NN$ zero order vertex, $V^{(0)}_{\pi\pi\pi NN}$, contributes a factor $Q/F^3 \sim \sqrt{m_\pi M}/F^3$.

The last two factors originate, respectively, from the terms $\pi^2 (\partial_\mu \pi)^2/6F^2$ and $S^\mu_\pi \pi^2 \partial_\mu \pi/6F^3$ in the lowest-order Lagrangian. (see for example Eqn. 2 of Ref. [6]).

We now turn to demonstrate that for each low chiral order $D$ diagram there exist infinite sequence of loop diagrams of higher chiral order, which have the same low momentum power as the original diagram. Consider for example the diagrams shown in Fig. 1. The simplest irreducible diagram is that of the so called impulse term, (graph 1a), corresponding to a chiral order $D=1$. As shown in Ref. [4], by using the rules quoted above, the low momentum power order of this term is, $\Theta_0 \sim F^{-3}(m_\pi/M)^{1/2}$. Next, by adding two zero chiral order $\pi NN$ vertices, two lowest order nucleon propagators and one meson propagator to the diagram 1a, one obtains the irreducible one loop diagram 1b. We recall that a zero chiral
order $\pi NN$ vertex is proportional to the meson three momentum, i.e.,

$$V_{\pi NN} = \frac{g_A}{F} S \cdot Q\tau,$$

where $S$ is the nucleon spin-operator and contributes a factor $QF^{-1}$ (rule (i) above). Thus the two added nucleon vertices give a factor $Q^2F^{-2}$. Likewise, a meson propagator contributes a factor $Q^{-2}$, the two nucleon propagators give $(v \cdot Q)^{-2}$ and the loop integral contributes a factor of $Q^4(4\pi)^{-2}$. Altogether the diagram 1b has an additional factor $Q^4(4\pi F v \cdot Q)^{-2}$ with respect to original diagram 1a. The power factor of diagram 1b is therefore,

$$\Theta_3 = \Theta_1 \frac{Q^4}{(4\pi F)^2(v \cdot Q)^2}.\quad (2)$$

With $4\pi F \sim M, v \cdot Q \sim m_\pi$, $\Theta_3 = \Theta_1$, so that although higher in chiral order, the diagram 1b is of the same order as the diagram 1a. Similarly, by adding progressively, two zero order $\pi NN$ vertices, a pion propagator and two nucleon propagators, as mentioned above, one obtains the other irreducible n-loop diagrams shown in Fig. 1.

By making use of the same power counting rules as above, the momentum power of n-loop diagram would be,

$$\Theta_{2n+1} = \Theta_1 \left(\frac{Q^4}{(4\pi F)^2(v \cdot Q)^2}\right)^n = \Theta_1.\quad (3)$$

Thus for the impulse diagram 1a there exists an infinite sequence of n-loop diagrams, $n = 1, 2, ...$ of chiral order $2n + 1$ all having the same characteristic momentum power as the lowest chiral order tree diagram. Quite obviously, such a sequence of loop diagrams can be constructed in a similar manner for any irreducible diagram that may contribute to the production process.

Thus the basic principle of the HFF $\chi$PT of one-to-one correspondence between the loop and small momentum expansion is badly destroyed. The primary production amplitude becomes the sum over infinite sequences of loop diagrams all having the power order, and thus excluding the possibility that a finite chiral order HFF based $\chi$PT calculations can explain meson production in NN collisions. This result, along with the observation made in
Refs. [9,10], that the HFF series of the nucleon propagator is on the border of its convergence circle, lead us to conclude that the $NN \rightarrow NN\pi$ production process falls outside the HFF validity domain.

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FIG. 1. An infinite sequence of diagrams contributing to the production process (see text). A solid line stands for a nucleon and a dashed line represents a meson. The black dot are zero chiral order $\pi NN$ vertex, an open circle denotes a $\pi NN$ vertex of chiral order 1. Shown in the figure are: the irreducible impulse diagram (1a), a one-loop diagram (1b), two-loop diagrams (1c, 1d) and three-loop diagram (1e). The ellipsis denote all other loop diagrams of the sequence.