Based on the R-parity violation option of the minimal supersymmetric Standard Model, we examine the scenario where the massive gravitino, relic from the hot big-bang, is the lightest supersymmetric particle and can decay through one or several of the trilinear R-parity violating interactions. We calculate the rates of the gravitino decay via the various three-body decay channels with final states involving three quarks and/or leptons. By taking into account the present constraints on the trilinear R-parity violating coupling constants and assuming the gravitino and scalar superpartner masses do not exceed \( \sim 80 \text{ TeV} \), it turns out that the gravitinos could easily have decayed before the present epoch but not earlier than the big-bang nucleosynthesis one. Therefore, the considered scenario would upset the standard big-bang nucleosynthesis and we conclude that it does not seem to constitute a natural solution for the cosmological gravitino problem.

1 Introduction

In supergravity theories\(^1\), the gravitino, namely the spin-3/2 supersymmetric partner of the graviton, weakly interacts with all the particle species (including itself) due to the small gravitational strength coupling \( \sqrt{G_N} = 1/M_P \), \( G_N \) being the gravitational constant and \( M_P \) the Planck scale. Hence, the gravitino-gravitino annihilation rate is extremely small so that the gravitinos should decouple at an early epoch of the universe history, and moreover at an epoch characterized typically by a temperature \( T_d \) higher than the gravitino mass: \( kT_d > m_{3/2} \) (\( k \) being the Boltzmann constant)\(^2\). Therefore, the relic abundance of the gravitino should be large, which is often denoted in the literature as the cosmological “gravitino problem”. One of the first solutions for the gravitino problem to be envisaged is to compensate the large gravitino relic abundance by a gravitino mass sufficiently small, namely \( m_{3/2} < 1 \text{ keV} \)\(^3\), to respect the limit on the present universe energy density: \( \Omega_0 \lesssim 1 \). For heavier gravitinos, a second type of available solution is by shortening their lifetime so that they do not survive out at the late epochs\(^2\). This can be realized in two characteristic options: either the gravitino is not the Lightest Supersymmetric Particle (LSP) and thus can decay into an odd number of superpartners through gravitational and gauge interactions (both of these ones couple an even number of superpartners), or it is the LSP. A gravitino LSP can be realized within various supersymmetric models, including the gauge mediated supersymmetry breaking models\(^4,5\), the models of low fundamental energy scale\(^6\) and even the conventional gravity mediated supersymmetry breaking models (for some specific set of the supersymmetry breaking parameters). If the gravitino is the

\(^a\)Invited talk given at the XXXVIIth Rencontres de Moriond session devoted to ELECTROWEAK INTERACTIONS AND UNIFIED THEORIES, March 9-16 2002, Les Arcs (France).
LSP, it can decay only into the ordinary particles of the Standard Model. Such a decay channel must involve both gravitational and the so-called R-parity symmetry violating interactions (the latter ones couple an odd number of superpartners). The R-parity violating ($R_p$) interactions are written in the following superpotential, in terms of the left-handed superfields for the leptons ($L$), quarks ($Q$) and Higgs of hypercharge $1/2$ ($H$) and the right-handed superfields for the charged leptons ($E^c$), up and down type quarks ($U^c, D^c$),

$$W_{R_p} = \sum_{i,j,k} \left( \frac{1}{2} \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \frac{1}{2} \lambda''_{ijk} U_i^c D_j^c D_k^c + \mu_i H L_i \right), \quad (1)$$

$i, j, k$ being flavor indices, $\lambda_{ijk}, \lambda'_{ijk}, \lambda''_{ijk}$ dimensionless coupling constants and $\mu_i$ dimension one parameters. Note that in the scenario in which the gravitino is not the LSP, the gravitino preferentially decays into an ordinary Standard Model particle and its superpartner through gravitational interactions, since the $R_p$ coupling constants are severely constrained by the low-energy experimental bounds obtained at colliders.

In the scenario of an unstable gravitino heavier than the LSP, the gravitino decay can produce an unacceptable amount of LSP which conflicts with the observations of the present mass density of the universe. The scenario containing an unstable LSP gravitino, having decay channels which involve $R_p$ coupling constants, is based on the violation of the R-parity symmetry. Now, neither the grand unified theories, the string theories nor the study of the discrete gauge symmetries give a strong theoretical argument in favor of the conservation of the R-parity symmetry in the supersymmetric extension of the Standard Model. Hence, the scenario with an unstable LSP gravitino constitutes an attractive possibility which must be considered as an original potential solution with respect to the cosmological gravitino problem.

Our main purpose in the present work is to determine whether the scenario of an unstable LSP gravitino decaying via $R_p$ interactions constitutes effectively a natural solution to the cosmological gravitino problem. We will concentrate on the trilinear $R_p$ interactions, namely $\lambda_{ijk} L_i L_j E_k^c, \lambda'_{ijk} L_i Q_j D_k^c$ and $\lambda''_{ijk} U_i^c D_j^c D_k^c$. With the goal of enhancing the gravitino instability, we will consider an optimistic type of scenario in which several trilinear $R_p$ coupling constants have simultaneously non-vanishing values. We will not consider the bilinear $R_p$ term $\mu_i H L_i$ of Eq.(1) which can be rotated away by a suitable redefinition of superfields, for some specific choices of the $R_p$ soft supersymmetry breaking terms. The bilinear $R_p$ interactions as well as the possible alternative of a spontaneous breaking of the R-parity symmetry have been considered, within the context of the cosmological gravitino problem, in a recent study which examines the two-body gravitino decay mode into photon and neutrino. We also mention here the works in which have been considered the $R_p$ decay reactions of single nucleon into a gravitino and a strange meson.

2 Constraints in a scenario containing an unstable gravitino

The equilibrium condition,

$$\Gamma \geq H(T), \quad (2)$$

$\Gamma$ being the gravitino decay rate and $H(T)$ the expansion rate of the universe, is reached at an epoch at which the cosmic energy density is dominated by the gravitino energy density $\rho_{3/2}$. Therefore, in determining the temperature $T_{3/2}$ at which the equilibrium condition of Eq.(2) is reached (the gravitino decay temperature) by solving the equation $\Gamma = H(T_{3/2})$, one must take the following expression for the expansion rate,

$$H(T) \approx \sqrt{8\pi G_N \frac{\rho_{3/2}}{3}}. \quad (3)$$
Their decay energy has been thermalized, the temperature rises to the value $\tilde{T}_{3/2}$. At the temperature $T_{3/2}$, the present epoch. This requirement imposes the following bound on the increased gravitino decay temperature $T_{3/2}$.

$$kT_{3/2} > 2.75K \quad (kT_{3/2} > 2.36 \times 10^{-10} \text{MeV}).$$

Furthermore, the decay of the gravitinos causes an increase of the temperature, which leads to an increase of the entropy density $s = (2\pi^2/45)g(T)(kT)^3$ and hence to a decrease of the baryon-to-entropy ratio $B = n_B/s$, $n_B$ being the baryon number density. Therefore, if the gravitinos decay after the nucleosynthesis epoch, the baryon-to-entropy ratio during the nucleosynthesis epoch must be much greater than the present one, leading to the production through nucleosynthesis of too much helium and too little deuterium (compared with the constraints on the primordial abundances derived from observational data) $^2$. In conclusion, if the gravitinos decay before the present epoch, the temperature $T_{3/2}$, reached after the gravitino decay energy has been thermalized, has to be higher than the nucleosynthesis temperature, namely,

$$kT_{3/2} > 0.4\text{MeV}. \quad (7)$$

### 3 Gravitino life time

The gravitino decay processes which involve the trilinear $R_p$ interactions of Eq. (1) and have the dominant rates are obviously the decays through a virtual scalar superpartner into three ordinary fermions involving one gravitational and one $R_p$ coupling. Those processes are represented in Fig. 1 for the $R_p$ interactions of type $\lambda_{ijk}L_iL_jE_k^c$. The reactions involving the $\lambda_{ijk}'$ and $\lambda_{ijk}''$ couplings have similar structures.
As a function of the relevant ratio of squared masses of the gravitino and exchanged sfermions, \( z = m_{3/2}^2/\hat{m}^2 \), we find that the partial gravitino lifetime, associated to the decay into a given flavor configuration of three light fermions final state, decreases monotonically in the interval \( 0 < z < 1 \) as,
\[
\tau_{3/2}(\hat{G} \rightarrow f_i f_j f_k) \approx 10^9 - 10^{11} \left( \frac{1}{\lambda_{ijk}^2} \right) \left( \frac{1 TeV}{m_{3/2}} \right)^3 \text{sec},
\]
where \( \hat{\lambda} \) stands for any one of the trilinear coupling constants. For light gravitinos of mass \( m_{3/2} = \mathcal{O}(GeV) \) and \( \hat{\lambda}_{ijk} = \mathcal{O}(1) \), we see from Eq.(8) that the gravitino lifetime exceeds by a few order of magnitudes the age of the universe \( t_0 \approx 3.2 \times 10^{17} \text{sec} \).

**4 Numerical results and discussion**

In Fig.2 (plain line), we show the values of the gravitino and superpartner masses for which \( kT_{3/2}' > 0.4 MeV \). These values have been obtained by assuming that the single dominant \( R_p \) coupling constant is \( \lambda_{213}' \) and is equal to its present limit \(^{11} \).

In Fig.2, we have set all the scalar superpartner masses to a common value, denoted by \( \hat{m} \). Moreover, the combination of statistical factors \( g(T_d)^{1/4} / g(T_{3/2})^{1/2} \), which enters Eq.(5), has been set to unity \(^b \). Finally, the gravitino decay rate has been multiplied by a factor of 2 in order to count the charge conjugated gravitino decay process and the masses of the final state particles have been taken into account in the computation.

We see in Fig.2 (plain line) that as the superpartner mass increases, larger gravitino masses are needed to have \( kT_{3/2}' > 0.4 MeV \). The reason is that \( T_{3/2}' \) increases with the gravitino mass but is suppressed if the superpartner is getting heavier (since \( T_{3/2}' \) is proportional to the squared root of the gravitino decay rate, as shown in Eq.(5)).

Let us consider now a type of scenario in which several \( R_p \) coupling constants, having some of the weakest present bounds, are simultaneously non-vanishing and are equal to their present limit obtained in the single dominant coupling hypothesis. Based on the strongest constraints on the products of \( R_p \) coupling constants and on the review \(^{11} \) of the present limits on the single \( R_p \) coupling constants, we find that the scenario of this type leading to the highest gravitino decay temperature \( T_{3/2}' \) (for \( m_{3/2} = 1 TeV \) and \( \hat{m} = 1.5 TeV \)) corresponds to the case where the simultaneously dominant \( R_p \) coupling constants are,
\[
\lambda_{132}', \lambda_{211}', \lambda_{223}', \lambda_{311}', \lambda_{121} \text{ and } \lambda_{233},
\]
which have the present bounds (obtained in the single dominant coupling hypothesis) \( \lambda_{132}' < 0.34 \) for \( m_q = 100 GeV \) (and for instance \( \lambda_{132}' \approx 1.2 \) for \( m_q = 1 TeV \)) \(^{11,17} \), \( \lambda_{211}' < 0.06 (m_{\tilde{b}}/100 GeV) \) \(^{18,19} \), \( \lambda_{223}' < 0.18 (m_{\tilde{b}}/100 GeV) \) \(^{20} \), \( \lambda_{311}' < 0.10 (m_{\tilde{b}}/100 GeV) \) \(^{11,20} \), \( \lambda_{121} < 0.05 (m_{\tilde{b}}/100 GeV) \) \(^{10,18} \) and \( \lambda_{233} < 0.06 (m_{\tilde{t}}/100 GeV) \) \(^{10,18} \). We note that no \( \lambda_{ijk}'' \) coupling can be added to the set 9 of simultaneously dominant \( R_p \) couplings, since the experimental constraints on the proton decay rate force any product \( \lambda_{ijk}'' \lambda_{ij'k'}'' \) to be smaller than \( 10^{-9} \), in a conservative way and for squark masses below 1 TeV. This result has been obtained in \(^{22} \) by calculating the proton decay rate at one loop level. In contrast, we have checked that no strong constraints exist on any

\(^b\)Let us assume tentatively that the thermalized degrees of freedom at the gravitino decoupling and decay temperatures are the same as those for the minimal supersymmetric Standard Model and the present epochs, respectively. Using then the values \( g(T_d) \approx 915/4 \approx 228.75 \) and \( g(T_{3/2}) = 43/11 = 3.909 \), one obtains: \( T_{3/2}' \propto g(T_d)^{1/4} / g(T_{3/2})^{1/2} \approx 1.96 \). The actual numerical value of this factor might be larger but not by very much.

\(^c\)In Fig.2, the \( R_p \) coupling constants of set 9 have been set to their perturbativity limit, obtained from the requirement of perturbativity up to the gauge group unification scale \(^{21} \), since, in the whole interval of \( \hat{m} \) covered by the figure, this latter limit is more severe than the corresponding present bound, which is a low-energy experimental constraint.
Figure 2: Domains of the $m_{3/2}(GeV/c^2)$-$\tilde{m}(GeV/c^2)$ plane (gravitino versus superpartner mass) in which the gravitino decay temperature is higher than the nucleosynthesis one, namely $kT'_{3/2} > 0.4 MeV$. The region situated above the plain line corresponds to $kT'_{3/2} > 0.4 MeV$ in case the dominant $R_p$ coupling constant is $\lambda''_{211} = 1.25$. The domain situated above the dashed line corresponds to $kT'_{3/2} > 0.4 MeV$ in case the dominant $R_p$ coupling constants are $\lambda'_{132} = 1.04$, $\lambda'_{211} = 1$, $\lambda'_{223} = 1.12$, $\lambda'_{311} = 1.12$, $\lambda'_{121} = 1$ and $\lambda'_{233} = 1$. Finally, the colored region corresponds to the situation $m_{3/2} > \tilde{m}$ which must be considered within a scenario where the gravitino is not the LSP.

In this optimistic situation where several $R_p$ coupling constants are simultaneously present, the gravitino masses corresponding to $kT'_{3/2} > 0.4 MeV$ are smaller than in the considered case of a single dominant $R_p$ coupling constant (see Fig.2), since $T'_{3/2}$ is typically proportional to the $R_p$ coupling constants and decreases with the gravitino mass.

The conclusion about Fig.2 is that the gravitino mass, and thus the scalar superpartner masses, must exceed values of $O(80 TeV)$, even within the most optimistic scenarios, so that the condition of Eq.(7) can be fulfilled. Nevertheless, this requirement conflicts with the constraint, $\tilde{m} \sim O(TeV)$, coming from the “hierarchy problem”, namely the problem of natural coexistence of the electroweak symmetry breaking scale and the scale of new physics underlying the Standard Model (grand unification scale, string scale . . . ).

5 Conclusion

Along with the existence of the cosmic microwave background, big-bang nucleosynthesis is one of the most important predictions of the big-bang cosmology. Furthermore, if one assumes that the light nuclei (atomic number less than 7) have effectively been produced through the big-bang nucleosynthesis, one finds that the theoretical predictions on the abundances of these light nuclei are in good agreements with the observational data. Now, as we have seen above, if one believes that the light elements have been synthesized through the standard big-bang nucleosynthesis, the temperature $T'_{3/2}$, reached after the thermalization of the gravitino decay energy, must be higher than the nucleosynthesis temperature. Therefore, since we have found that this cannot happen in the scenario characterized by an unstable LSP gravitino having a decay channel which involves trilinear $R_p$ interactions, this scenario does not seem to provide a realistic solution to the large relic abundance of the gravitino.
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References

1. H. P. Nilles, Phys. Rep. 110 (1984) 1.
2. S. Weinberg, Phys. Rev. Lett. 48 (1982) 1303.
3. H. Pagels and J. R. Primack, Phys. Rev. Lett. 48 (1982) 223.
4. M. Dine, A. E. Nelson and Y. Shirman, Phys. Rev. D51 (1995) 1362.
5. M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D53 (1996) 2658.
6. N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429 (1998) 263.
7. G. R. Farrar and S. Weinberg, Phys. Rev. D27 (1983) 2732.
8. A. Salam and J. Strathdee, Nucl. Phys. B87 (1975) 85.
9. P. Fayet, Nucl. Phys. B90 (1975) 104.
10. H. K. Dreiner, published in Perspectives on Supersymmetry, Ed. by G. L. Kane, World Scientific (1998), hep-ph/9707435.
11. G. Bhattacharyya, Nucl. Phys. B (Proc. Suppl.) 52A (1997) 83; Invited talk presented at ‘Beyond the Desert’, Castle Ringberg, Tegernsee, Germany, 8-14 June 1997, hep-ph/9709395.
12. L. M. Krauss, Nucl. Phys. B277 (1983) 556.
13. T. Banks, Y. Grossman, E. Nardi and Y. Nir, Phys. Rev. D52 (1995) 5319.
14. F. Takayama and M. Yamaguchi, Phys. Lett. B485 (2000) 388.
15. K. Choi, E. J. Chun and J. S. Lee, Phys. Rev. D55 (1997) 3924.
16. K. Choi, K. Hwang and J. S. Lee, Phys. Lett. B428 (1998) 129.
17. G. Bhattacharyya, J. Ellis and K. Sridhar, Mod. Phys. Lett. A10 (1995) 1583.
18. V. Barger, G. F. Giudice and T. Han, Phys. Rev. D40 (1989) 2987.
19. F. Ledroit and G. Sajot, GDR-S-008 (1998), see http://qcd.th.u-psud.fr/GDR_SUSY/GDR_SUSY_PUBLIC/entete_note_publique
20. G. Bhattacharyya and D. Choudhury, Mod. Phys. Lett. A10 (1995) 1699.
21. B. C. Allanach, A. Dedes and H. K. Dreiner, Phys. Rev. D60 (1999) 075014.
22. A. Y. Smirnov and F. Vissani, Phys. Lett. B380 (1996) 317.
23. T. P. Walker, G. Steigman, D. N. Schramm, K. A. Olive and H.-S. Kang, Ap. J. 376 (1991) 51.