Perturbations in Quintessential Inflation

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Recently Peebles and Vilenkin proposed and quantitatively analyzed the fascinating idea that a substantial fraction of the present cosmic energy density could reside in the vacuum potential energy of the scalar field responsible for inflation (quintessential inflation). Here we compute the signature of this model in the cosmic microwave background polarization and temperature anisotropies and in the large scale structure.

I. INTRODUCTION

In the last decade the connection between cosmology and particle physics has become more and more interesting. We have several sectors in which a comparison between the theoretical high energy physics processes of the very early universe and their observable traces is possible. These research frontiers include the exploration of the anisotropies in the cosmic microwave background (CMB) and of the large scale structure in the present matter distribution (LSS); wide and high resolution experiments designed to gain observationally insight into these topics are currently in preparation.

Let us approach the subject of this work. Very recently the emissions from deep type Ia supernova have become observable with unprecedented high resolution. The surprising news arising from these observations suggest that we are currently living in a universe that is accelerating its expansion. As it is well known, this could be the observable effect of a vacuum energy density comparable with the critical one. A possible explanation could be the existence of a cosmological constant, much smaller than the characteristic energy scales of quantum gravity effects but non-zero. This \textit{ad hoc} possibility is unlikely for theorists, tending to believe that some unknown process set the cosmological constant to zero in the very early universe.

Recently the idea that the vacuum energy density could be mimicked by a dynamical scalar field $\phi$, named \textit{quintessence}, has been considered with more and more interest since it provide several nice features in the CMB power spectrum and LSS. More precisely, this occurs in the two most popular candidates proposed so far for this field, a cosine potential for the Pseudo Nambu Goldstone Boson and an exponential potential.

However, since most of the inflationary phenomenology is based on the dynamics of a scalar field, the inflaton, it is tempting to relate quintessence to inflaton. A first proposal from the scientific community in this sense was made very recently. The authors suggested that quintessence and inflaton are the same field seen at different times. They proposed a detailed model, providing an appropriate set of values for the physical constants in order to realize this fascinating possibility. The key feature is the occurring of a kinetic energy dominated phase that connects inflation and radiation era. Although the model has to be further investigated, especially for what concerns the spectrum of gravitational waves and the possible production of topological defects, it is interesting to look at the perturbations dynamics induced by its basic features. A quintessence model with similar characteristics is being considered by other authors, with emphasis on arguments regarding the viability of general quintessence models.

In this work we concentrate on the power spectra of CMB polarization and temperature anisotropies as well as on the LSS perturbations produced in this scenario. In section II we recall the background dynamics in quintessential inflation and in section III we describe its perturbations; finally, in section IV we numerically compute and discuss the signature of this scenario on the CMB polarization and temperature power spectra and on the LSS.

II. QUINTESSENTIAL INFLATION

In this section we briefly review the basic features of quintessential inflation; the reader is advised to look at the original work \textsuperscript{7} for a complete exposition and references.

The model involves a minimally coupled scalar field with potential

$$V(\phi) = \lambda \cdot (M^4 + \phi^4) \quad \text{for} \quad \phi < 0,$$

$$V(\phi) = \frac{\lambda M^8}{(M^4 + \phi^4)} \quad \text{for} \quad \phi \geq 0.$$  \hfill (1)

Inflation occurs for $\phi \ll -M$; radiation and matter dominated eras for $\phi \gg M$. In order to make particles and perturbations production just as in chaotic inflationary models \textsuperscript{9}, and to have about 70% of the critical energy today in quintessence, the following physical constants, in $\hbar = 1, c = 1$ units, have been chosen:

$$\lambda = 10^{-14}, \quad M = 8 \cdot 10^5 \text{ GeV}. \quad \hfill (2)$$
The cosmic trajectory is assumed to begin at $\phi \ll -M_{PL}$, with an era of chaotic inflation. At $\phi \simeq -M_{PL}$ a kinetic energy dominated era begins; in this epoch the (kinetic) quintessence energy density decreases very rapidly, $\rho_\phi \sim a^{-6}$ where $a$ is the scale factor. As in the ordinary scenario, particles are produced in the curved space-time from an initial quantum vacuum state [3]. In order to have a workable model, it is necessary that the kinetic era ends when the total field energy is negligible with respect to the radiation one; if this is the case, the radiation era begins and $\phi$ starts its slow rolling toward the present state. It is interesting to note that this model gives a reheating temperature curiously comparable with the supposed electroweak symmetry breaking scale, $T_{rh} \simeq 10^3 N_\psi^{3/4}$ GeV, where $N_\psi$ is the number of scalar fields involved in the process.

This is the general phenomenology imposed by the constants [3]. At the present time, in the matter dominated era, the inflaton is totally equivalent to a quintessence field, rolling on the potential $V(\phi) \simeq \lambda M^4/\phi^4$ with a simple time evolution, $\phi \simeq 2^{1/3} \lambda^{1/6} M^{4/3} H_0^{-1/3}/\sqrt{1+z}$.

III. PERTURBATIONS

Perturbations in models with a dynamical scalar field together with the other ordinary matter and radiation particles require a generalization [1,11] of earlier works [10]; a complete treatment of this subject can be found in the cited works and here we report only the relevant issues for the present problem.

Even if the reheating temperature in the present scenario is much smaller than in chaotic inflation, radiation dominates well before nucleosynthesis. In the most simple view, the cosmic fluid can be thought composed by photons ($\gamma$), baryons ($b$), cold dark matter ($cdm$) and three families of massless neutrinos ($\nu$). As we briefly exposed in the previous section, Gaussian perturbations arise adiabatically from the inflaton dynamics at the end of inflation [3]. They involve matter and radiation as well as fluctuations $\delta \phi$ of the scalar field around its background value $\phi$.

The initial conditions for the perturbations are posed at early conformal time $\tau = \int_0^t dt/a(t)$ when essentially all the perturbation wavenumber $k$ interesting for structure formation are well outside the effective horizon, $k\tau \ll 1$. Adiabatic conditions are posed initially by requiring that no gauge invariant entropy perturbation difference exists between any pair of components [3], in the conformal Newtonian gauge, the leading order early time behaviour for the scalar quantities evolving from adiabatic initial conditions is

$$\begin{align*}
\delta_\gamma &= \frac{4}{3} \delta_b = \frac{4}{3} \delta_{cdm} = \delta_\nu \propto \text{constant} , \\
v_\gamma &= v_b = v_{cdm} = v_\nu \propto k^2 \tau , \\
\sigma_\nu &\propto k^2 \tau^2 , \\
\delta \phi &\propto \left( \frac{d\phi}{dt} \right)_{t=0} \cdot \tau^2 ,
\end{align*}$$

(3)

where $\delta$, $v$, $\sigma$ means density, velocity and shear perturbations respectively. Note that $\delta \phi$ is initially linked to the kinetic field energy. A multipole expansion accounts for temperature as well as polarization perturbations of the Planckian black body spectrum arising mainly from Thomson scattering at the energies relevant in the present problem; neutrinos also are treated similarly without the Thomson scattering terms [4].

From this initial regime, perturbations evolve according the linearized Einstein and Boltzmann equations in a flat Friedmann Robertson Walker background; the latter involves quintessence, scale factor and unperturbed density of all the fluid species, being driven by the Klein Gordon and unperturbed Einstein equations respectively (see [11] and references therein).

IV. RESULTS AND DISCUSSION

We require that at the present about 70% of the critical energy density resides in quintessence. This energy comes essentially from the potential component, since $\phi$ is rolling very slowly making the kinetic energy negligible, as we show below; the baryon abundance respects the nucleosynthesis constraint:

$$\begin{align*}
\Omega_\phi &= .7 , \\
\Omega_b &= .05 , \\
\Omega_{cdm} &= 1 - \Omega_\phi - \Omega_b .
\end{align*}$$

(4)

Also we adopt $H_0 = 70 \text{ km/s/Mpc}$ consistently with some present measurements [12] and assume an initial power spectrum exactly scale-invariant.

The request that the present amount of quintessence energy is $\Omega_\phi$ does not fix completely its dynamics; since it obeys the Klein Gordon equation we need to specify its time derivative. At the present this is very low since it has
been redshifted away during the expansion occurred in the radiation and matter eras (assuming that it is not too large with respect to the potential energy at the beginning of the radiation dominated era). However, particularly in this scenario where the kinetic energy plays a fundamental role during the cosmic evolution, it is important to take into account the field time derivative. This is realized in the following way: after fixing $\Omega$, this scenario where the kinetic energy plays a fundamental role during the cosmic evolution, it is important to take large with respect to the potential energy at the beginning of the radiation dominated era. However, particularly in this model has to be further investigated, and hopefully its predictions will be enriched by a more deep understanding of its phenomenology at the transition between inflation and radiation era; ultimately this scenario will be further constrained by experimental enterprises of the next generation, beginning from the primordial gravitational waves spectrum.

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FIG. 1. Polarization power spectra in quintessential inflation (solid lines) compared with the CDM model (thin dashed line). As in the following figure, solid curves with increasing amplitude represent $\Omega_\phi = 60\%, 70\%, 80\%$ respectively.
FIG. 2. COBE normalized CMB power spectra in quintessential inflation and CDM.
FIG. 3. Large scale structure in quintessential inflation and CDM models.