Phenomenological study of two-gluon densities

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Abstract

The decay of colour neutral 2-gluon systems in $q\bar{q}$ and $2q2\bar{q}$ has been simulated with the Monte-Carlo method, taking into account an effective 1-gluon exchange interaction between the emitted quarks, which was folded with a 2-gluon density determined self-consistently. Finite 2-gluon densities were obtained with a mean square radius $<r^2>$ of about 0.5 fm$^2$. These give a significant contribution to the 2-gluon field correlators and the gluon propagator from lattice QCD. By solving a relativistic Schrödinger equation the binding potential of the 2-gluon system is computed. By the relation $2g \rightarrow (q\bar{q})^n$ this corresponds to the confinement potential between quarks, which is in excellent agreement with the confinement potential from lattice QCD. The lowest bound state in this potential allows an interpretation of the 2-gluon mass as binding energy of the two gluons. The lowest glueball with $E_0 = 0.68 \pm 0.10$ GeV is consistent with the scalar $\sigma(550)$, with a mass significantly lower than obtained in lattice QCD simulations. However, the first and second radial excitations agree with the lattice results.

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Quantum chromodynamics (QCD) is not well understood in the infrared region, where the most important properties of the strong interaction are generated, i.e. the hadronic masses and the confinement of quarks and gluons to hadrons. As perturbative methods are generally not applicable, simulations of the QCD equations on the lattice together
with studies of Dyson-Schwinger equations [2, 3] offer the best known methods to study the structure of QCD in the infrared region. However, because of their complexity and severe problems in the lattice simulations (due to discretisation, with small quark masses and the need of a very high computing power) it is very difficult to reveal the underlying structure of QCD. Therefore, alternative approaches are welcome, which can shed further light on the underlying physics.

In the present paper we discuss a phenomenological description of two-gluon systems following the old idea (see e.g. in ref. [4]), that the non-Abelian structure of QCD may allow a coupling of two gluons to bound colour singlets with spin J=0 (and 2). For the description of such a scalar bound state Φ we may write the radial wave functions in the form

\[ \psi_{\Phi}(\vec{r}) = \psi_1(\vec{r}_1) \psi_2(\vec{r}_2), \]

where \( \psi_j(\vec{r}_j) \) are the radial wave functions of the two gluons. To investigate the properties of such a 2-gluon system we studied the decay \( 2g \rightarrow q\bar{q} + 2q2\bar{q} \) with an attractive interaction between the emitted quarks.

There is evidence for the existence of scalar and tensor gluonic fields from the formation of glueballs in pure Yang-Mills theory [5], from the mechanism of Pomeron-exchange [6, 7] in high energy hadron-hadron scattering (which is understood by the exchange of two non-perturbative gluons which couple mainly to spin zero), scalar excitation of baryon resonances [8] and hadron compressibilities [9]. Finally, the effective confining quark-quark potential [10] contains a sizeable scalar contribution.

Assuming an effective 1-gluon exchange interaction \( V_{1g}(R) = -\alpha_s/R \) between the emitted quarks (or antiquarks) with relative distance \( R = |\vec{r}_1 - \vec{r}_2| \), the decay from a 2-gluon system \( 2g \rightarrow (q\bar{q})^n \) requires a modification of the free q-q interaction by the density of the 2-gluon system, which may be expressed by a folding integral

\[ V_{qq}(R) = \int d\vec{r} \rho_{\Phi}(\vec{r}) V_{1g}(\vec{R} - \vec{r}), \]

where \( \rho_{\Phi}(\vec{r}) \) is the 2-gluon density \( \rho_{\Phi}(\vec{r}) = |\psi_{\Phi}(\vec{r})|^2 \).

It is interesting to note, that for a spherical density the Fourier transform of eq. (2) to momentum (Q) space yields

\[ V_{qq}(Q) = -\frac{4\pi\alpha_s}{Q^2} \rho_{\Phi}(Q), \]
where \[ \rho_\Phi(Q) = 4\pi \int r^2 dr \ j_\alpha(Qr) \ \rho_\Phi(r) . \] Comparing this with the standard 1-gluon exchange force yields a Q-dependent strong coupling \[ \alpha_s(Q) = \alpha_s C \rho_\Phi(Q) , \] which is qualitatively consistent with the known fact of a “running” of \( \alpha_s(Q) \) and the condition \( \alpha_s(Q) \to 0 \) for \( Q \to \infty \) (asymptotic freedom)\(^1\). Note, that the 2-gluon densities discussed below yield \( \alpha_s(Q) \) in reasonable agreement with the lattice data of ref. [12] for small momentum transfers.

Further, the interaction has to be modified by the finite size of the decaying 2-gluon system. This is approximated in our calculation by replacing the 1-gluon exchange potential \( V_{1g}(R) \) by

\[ \begin{align*}
V'_{1g}(R) = V_{1g}(R) \ e^{(-aR^2)} \end{align*} \]

with the requirement \( < r_2^2 > \approx < r_\rho^2 \). This effect is shown in the upper part of fig. 1, which gives a 2-gluon density by the upper dot-dashed line and pure 1-gluon and modified 1-gluon exchange potential by the lower dashed and dot-dashed lines, respectively. The resulting interaction is given by

\[ \begin{align*}
V'_{qq}(R) = \int d\vec{r} \ \rho_\Phi(\vec{r}) \ V'_{1g}(\vec{R} - \vec{r}) ,
\end{align*} \]

which has the characteristics of a quite rapid fall-off to zero for large radii (solid line in the upper part of fig. 1).

Finally we have to take into account the fact, that the decay into \( 2q2\bar{q} \) favours relative angular momentum \( L=0 \) between the emitted quarks, whereas for the decay in \( q\bar{q} \) the outgoing quarks are in a relative p-state (\( L=1 \)). For the p-wave density \( \rho_p^2(\vec{r}) = \rho_p^2(r) Y_{1,m}(\theta, \phi) \) we take \( \rho_p^2(r) = (1 + \beta \cdot d/dr) \rho_\Phi(r) \) with the constraint \( < r > = \int d\tau \ r \rho_{vec}(r) = 0 \) (to suppress spurious motion), by which \( \beta \) is fixed. By relativistic Fourier transformation \([13] \) the effective interaction (6) can be transformed to momentum space

\[ \begin{align*}
V'_{qq}(Q') = 4\pi \int R^2 dR \ j_\alpha(QR) \ V'_{qq}(R) ,
\end{align*} \]

with \( Q' = Q \ \sqrt{1 + [Q^2/4m^2]} \) and \( m_\Phi \) being the mass of the 2-gluon field. For \( \rho_\Phi(r) \) and \( \rho_p^2(r) \) the potential in momentum space \( V'_{qq}(Q) \) is given in the lower part of fig. 1 by the

\[ \text{However, it is clear that the 2-gluon densities deduced here fall off much faster than the slow logarithmic decrease found for } \alpha_s(Q) \text{ (see [11]).} \]
solid and dot-dashed line, respectively, which show significant differences. Due to the finite size effects both interactions are finite at vanishing momentum transfer.

Using this effective q-q interaction Monte-Carlo simulations of gluon-gluon scattering have been performed in fully relativistic kinematics using the CERN routine GENBOD \[14\], in which the 2 gluons in the final state can decay in $q \bar{q}$ and $2q2\bar{q}$ (we used massless quarks). Random Q-transfers between 0 and 6 GeV/c have been used, from these the momenta $\vec{p}_i$ of the outgoing quarks or antiquarks were also determined randomly, but restricted by the available phase space. The potential $V_{qq}'(\Delta p)$ \[7\] has been used as a weight function between the outgoing quarks (with the relative momenta $\Delta \vec{p} = \vec{p}_i - \vec{p}_j$) in addition to the normal phase space weight. Resulting gluon momentum distributions $d_{qq}(Q)$ and $d_{2q2\bar{q}}(Q)$ for decay into $q \bar{q}$ and $2q2\bar{q}$ were generated. Their sum $D_\Phi(Q') = d_{qq}(Q') + d_{2q2\bar{q}}(Q')$ can be related to the radial density $\rho_\Phi(r)$ of the 2-gluon system

$$D_\Phi(Q') = 4\pi \int r^2 dr \ j_o(Qr) \ \rho_\Phi(r) , \quad \text{(8)}$$

with $Q'$ as in eq. \[7\]. The mass $m_\Phi$ to determine $Q'$ in the relativistic Fourier transformations \[7\] and \[8\] has been used as a fit parameter, for which a value of 0.68 GeV yields the best results.

The condition, that $\rho_\Phi(r)$ in the interaction \[6\] and in eq. \[8\] should be the same, allows us to determine this density. As the 2-gluon density was not known initially, our simulation procedure had to be iterated: starting with a certain density $\rho_\Phi(r)$ the folding potential \[6\] was calculated. The Fourier transform \[7\] was then inserted in the Monte-Carlo simulations, from which gluon momentum distributions $d_{qq}(Q)$ and $d_{2q2\bar{q}}(Q)$ were generated as shown in the upper part of fig. 2. By inversion of eq. \[8\] a Fourier retransformation of $D_\Phi(Q)$ to r-space had to be made and the resulting density $\rho_\Phi(r)$ compared to the one used in eq. \[6\]. Self-consistency of both densities was required.

Resulting momentum distributions $d_{qq}(Q)$ and $d_{2q2\bar{q}}(Q)$ for a self-consistent solution (discussed below) are given in the upper part of fig. 2. These are very different for the two decays: the decay in $q \bar{q}$ falls off rapidly with increasing Q, whereas a much slower fall-off for $2q2\bar{q}$ decay is observed. This difference can be readily explained by the fact, that 4 quarks can absorb much higher momenta than 2 quarks. We see that the sum $D_\Phi(Q)$ is in reasonable agreement with $\rho_\Phi(Q)$ from the Fourier transformation of $\rho_\Phi(r)$ inserted in
eq. (6) (dot-dashed line), which is quite well approximated by a radial dependence

$$\psi_\Phi(r) = \psi_o \exp[-(r/a)\kappa]$$

(9)

with values of $a$ and $\kappa$ in table 1 and $\psi_o$ taken from the normalisation $4\pi \int r^2 dr \ \psi_\Phi(r)^2 = 1$.

The self-consistency condition for our simulations is far from trivial. The resulting density is given in the lower part of fig. 2, which indicates clearly that a self-stabilized 2-gluon field is generated. In table 1 we give two solutions of about equal quality, which may reflect the minor ambiguities of our method. The dashed area indicates the estimated uncertainties of the deduced density.

The extracted 2-gluon density should give a significant contribution to the gluon 2-point functions extracted from lattice QCD simulations in form of gluon field correlators [15, 16] and the QCD gluon propagator (see e.g. [17]). This is shown in fig. 3. In the upper part we make a comparison of our results with the 2-gluon field correlator $C_\perp(r)$ of Di Giacomo et al. [16]. $C_\perp(r)$ has been weighted exponentially with distance (yielding $<r_\perp^2> \sim 0.01 \text{ fm}^2$); therefore we have to compare the logarithmic values of $C_\perp(r)$ with our 2-gluon density (lower solid line). This leads to a reasonable agreement at large radii. For a comparison with the gluon propagator we have taken the lattice data (obtained in Landau gauge) of Bowman et al. [18]. These are compared with the Fourier transformed 2-gluon density $\rho_\Phi(Q)$ shown by the lower solid line in the lower part of fig. 3.

In spite of a reasonable agreement with our densities at large radii or small $Q$-values, respectively (note that in fig. 3 the gluon propagator is multiplied by $Q^2$), both lattice data show deviations, which may be described by a 2-gluon vector component (where the two gluons couple to angular momentum $L=1$). Such a (current) field is constrained by the condition $<r>_c = 0$ to suppress spurious motion. We assume a form $\rho_\text{vec}(r) = \rho_o(1 - 0.25 r/a') \exp\{-(r/a')\}$ with $a'=0.11 \text{ fm}$, which fulfills the requirement $<r>_c = 0$, yielding a contribution given by the dot-dashed lines in fig. 3. Using for this a strength of about 18 % of the scalar component yields a consistent description of the (gauge independent) 2-gluon correlator and the gluon propagator in Landau gauge. The latter is not very different in Laplacian gauge [19], but a fit to the gluon propagator in Coulomb gauge [20] yields a relative vector component about a factor 2 smaller, but in this gauge the gluon propagator is not covariant.
In various investigations of the gluon propagator and Pomeron-exchange it has been concluded, that 2-gluon fields should be massive. Cornwall \[4\] has discussed the gluon propagator in relation to a (relativistic) generation of mass. In studies of Pomeron-exchange \[6, 7\] an effective gluon mass between 0.3 and 0.8 GeV has been extracted. In our relativistic Fourier transformations we had to assume \( m_\Phi \sim 0.68 \text{ GeV} \). This is consistent with the gluon pole mass of 0.64 ± 0.14 GeV deduced in ref. [21]. We shall see, that the extracted mass can be understood as binding energy of the 2-gluon system including relativistic mass corrections.

A finite 2-gluon density as shown in fig. 2 may be interpreted as a quasi bound state of the two gluons (glueball). Therefore, from the 2-gluon density the binding potential of the 2-gluon system can be obtained by solving a three-dimensional reduction of the Bethe-Salpeter equation in form of a relativistic Schrödinger equation

\[
- \left( \frac{\hbar^2}{2\mu_\Phi} \left[ \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} \right] - V_\Phi(r) \right) \psi_\Phi(r) = E_i \psi_\Phi(r) ,
\]

where \( \psi_\Phi(r) \) is the 2-gluon wavefunction \[11\] and \( \mu_\Phi \) a relativistic mass parameter, which is related to the mass \( m_\Phi \) of the 2-gluon system by \( \mu_\Phi = \frac{1}{4} m_\Phi + \delta m \). In the non-relativistic limit \( \delta m = 0 \) and \( \mu_\Phi \) corresponds to the reduced mass \( \mu = \frac{m_1 m_2}{m_1 + m_2} \) which gives \( \mu = \frac{1}{4} m_\Phi \) for \( m_1 = m_2 = m_\Phi/2 \). Using a 2-gluon wave function of the form of eq. \[9\] yields the explicit form for the binding potential

\[
V_\Phi(r) = \frac{\hbar^2}{2\mu_\Phi} \left[ \frac{\kappa}{a^2} \left( \frac{r}{a} \right)^{\kappa - 2} \left[ \kappa \left( \frac{r}{a} \right)^\kappa - (\kappa + 1) \right] \right] + E_o .
\]

To calculate the shape of the potential the used value of \( E_o \) (which gives only an absolute shift) is not important. With \( \kappa, a, \delta m \) in table 1, slightly different solutions of the binding potential were obtained, which are given by the dot-dashed and dashed lines in fig. 4. Because of the relation \( 2g \rightarrow (q\bar{q})^n \) this potential can also be considered as confinement.

| 2-gluon field | \( \kappa \) | \( a \) | \( \langle r^2 \rangle \) | \( \delta m \) | \( E_o \) | \( E_1 \) | \( E_2 \) |
|---------------|--------|--------|----------------|--------|--------|--------|--------|
| (solution 1)  | 1.50   | 0.66   | 0.48          | 0.04   | 0.68±0.10 | 1.70±0.15 | 2.58±0.20 |
| (solution 2)  | 1.51   | 0.69   | 0.52          | 0.12   | 0.68±0.10 | 1.74±0.15 | 2.68±0.20 |
potential between the emitted quarks. This is in surprising agreement with the $1/r + \text{linear}$ form expected from potential models \cite{10} and consistent with the lattice data of Bali et al. \cite{22}. It is important to note, that our potential \cite{11} reproduces the $1/r + \text{linear}$ form without any assumption on its distance behavior; this is entirely a consequence of the deduced radial form of the 2-gluon wave function, see eq. \cite{9}.

Absolute values of binding energies $E_i$ in the potential \cite{10} can be obtained by fitting the binding potential by a form $V_{\text{fit}}(r) = -\frac{a}{r} + br$, and adding to the eigenvalues $\epsilon_i$ a constant $\delta E$ to give a g.s. energy $E_o = \epsilon_o + \delta E$ consistent with $m_\Phi$. This yields bound state energies for the ground state and the two lowest radial excitations given in table 1.

By computing also eigenstates in the q-q potential \cite{6} we find only one bound state with an energy in the order of -10 MeV. This very low binding energy indicates clearly, that the glueball states must have a large width, since $\Gamma \sim 1/E_o$ should be valid for the decay width. This is consistent with the general expectation of the width of glueball states ($\Gamma \geq 500 \text{ MeV}$, see e.g. ref. \cite{23}). From these results we may conclude, that the glueball ground state with $E_o=0.68 \pm 0.10 \text{ GeV}$ and a large width may be identified with the scalar $\sigma(550)$, which is most likely not a $q\bar{q}$ structure (see e.g. the review by Bugg \cite{24}).

Glueball masses have been deduced also from lattice simulations \cite{5,25}, in which a glueball mass below 1 GeV has not been found. However, in these simulations $0^{++}$ glueball masses have been extracted at about 1.7 and 2.6 GeV, which correspond very nicely to the first and second radial excitation in table 1. Our evidence for a low lying glueball is supported by QCD sum rule estimates \cite{26}, which also require the existence of a low lying gluonium state below 1 GeV.

**Conclusion:** We have presented a phenomenological method which yields a good understanding of important properties of the non-perturbative structure of QCD. The basic assumption, that stable 2-gluon colour singlets exist, which decay into $q\bar{q}$ pairs, has allowed a self-consistent determination of the densities of these 2-gluon systems. These give important contributions to the gluon 2-point functions, field correlators and gluon propagator, and yield a two-gluon binding potential which can be identified with the confinement potential determined from potential models and lattice QCD.

A detailed comparison of our approach with lattice QCD will be important. In the latter two-gluon fields are facilitated by loop integrations. By relating gluon loops to our 2-gluon
fields (recall that Wilson loops are gauge invariant as colour singlet 2-gluon systems) one can imagine, that Wilson’s confinement picture (by evaluating closed gluon loops around separated quarks) has a correspondence to our interpretation of confinement as binding of 2-gluons.

New information is obtained on the mass of glueballs. Consistent with pole analyses of the gluon propagator and QCD sum rules we obtain a low mass of the 2-gluon system (glueball ground state) in the order of 0.7 GeV. This is in the mass region of the scalar $\sigma(550)$. Higher radial excitations are obtained at about 1.7 and 2.7 GeV, consistent with $0^{++}$ glueball masses from lattice QCD.

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References

[1] K.G. Wilson, Phys. Rev. D 10, 2445 (1974); M. Creutz, ”Quantum fields on the computer”, Adv. Series on directions in high energy Phys. 12, World Scientific, Singapure, 1992; J. Greensite, Progr. in Part. and Nucl. Phys. 51, 1 (2003); and refs. therein

[2] C.D. Roberts and S.M. Schmidt, Prog. Part. Nucl. Phys. 45, S1 (2000)

[3] R. Alkofer and L. von Smekal, Phys. Rep. 353, 281 (2001), and refs. therein

[4] J. M. Cornwall, Phys. Rev. D 26, 1453 (1982)

[5] C. Morningstar and M. Peardon, Phys. Rev. D 56, 4043 (1997); Phys. Rev. D 60, 034509 (1999); and refs. therein

[6] A. Donnachie and P.V. Landshoff, Nucl. Phys. B 231 (1984) 189 and B 311 (1989) 509; P.V. Landshoff and O. Nachtmann, Z. Phys. C 35, 405 (1987); and refs. therein

[7] F. Halzen, G. Krein and A.A. Natale, Phys. Rev. D 47, 295 (1993); M.G. Gay Ducati, F. Halzen and A.A. Natale, Phys. Rev. D 48, 2324 (1993); and refs. therein

[8] H.P. Morsch, W. Spang and P. Decowski, Phys. Rev. C 67, 064001 (2003); H.P. Morsch and P. Zupranski, Phys. Rev. C 71, 065203 (2005)
[9] H.P. Morsch, Z. Phys. A 350, 61 (1994)

[10] R. Barbieri, R. Kögerler, Z. Kunszt, and R. Gatto, Nucl. Phys. B 105, 125 (1976);
    S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985); D. Ebert, R.N. Faustov, and
    V.O. Galkin, Phys. Rev. D 67, 014027 (2003); and refs. therein

[11] Review of particle properties, S. Eidelman et al., Phys. Lett B 592, 1 (2004);
    and http://pdg.lbl.gov/

[12] S. Furui and H. Nakajima, Phys. Rev. D 69, 074505 (2004) and Phys. Rev. D 70,
    094504 (2004)

[13] J.J. Kelly, Phys. Rev. C 66, 065203 (2002); and refs. therein

[14] A detailed write up of GENBOD can be found in the CERN library

[15] A.Di Giacomo, H.G. Dosch, V.I. Shevchenko, and Yu.A. Simonov, Phys. Rep. 372,
    319 (2002)

[16] A. Di Giacomo, E. Meggiolaro, and H. Panagopoulos, Nucl. Phys. B 483, 371 (1997)

[17] J.E. Mandula, hep-lat/9907020 and refs. therein

[18] P.O. Bowman, U.M. Heller, D.B. Leinweber, M.B. Parappilly, and A.G. Williams,
    Phys. Rev. D 70, 034509 (2004)

[19] P.O. Bowman, U.M. Heller, D.B. Leinweber, and A.G. Williams, Phys. Rev. D 66,
    074505 (2002);

[20] A. Cucchieri and D. Zwanziger, Phys. Rev. D 65, 014001 (2001) and Phys. Lett. B
    524,123 (2002); L. Moyaerts, doctoral thesis, University of Tübingen (2004);
    K. Langfeld and L. Moyaerts, Phys. Rev. D 70, 074507 (2004)

[21] C. Alexandrou, P. de Forcrand, and E. Follana, Phys. Rev. D 63,094504 (2001)

[22] G.S. Bali, B. Bolder, N. Eicker, T. Lippert, B. Orth, K. Schilling, and T. Struckmann,
    Phys. Rev. D 62, 054503 (2000)

[23] P. Minkowski and W. Ochs, hep-ph/0511126 and refs. therein

[24] D.V. Bugg, hep-ex/0510014 and refs. therein

[25] G.S. Bali, K. Schilling, and A. Wachter, Phys. Rev. D 56, 2566 (1997)

[26] S. Narison, Nucl. Phys. B 509, 312 (1998); and Phys. Rev. D 73, 114024 (2006)
Figure 1: Upper part: Two-gluon density $\rho_{\Phi}(r)$ (upper dot-dashed line) and potentials: 1-gluon exchange potential (dashed line), modified by size effects (dot-dashed line) and folding potential given by solid line. Lower part: Fourier transform of the folding potential using $\rho_{\Phi}(r)$ (solid line) and $\rho_{\Phi}^p(r)$ (dot-dashed line), respectively.
Figure 2: Upper part: Resulting 2-gluon momentum distributions (multiplied by $Q^2$) for decay in $q\bar{q}$ and $2q2\bar{q}$ and sum. Lower part: deduced 2-gluon density with estimated error band. The dot-dashed lines represent solution 1 in table 1.
Figure 3: Gluon field correlator $\log(C_\perp)$ from ref. [16] (upper part) and gluon propagator from ref. [18] (lower part) from lattice QCD simulations in comparison with our results. The lower solid lines correspond to the density $\rho_\Phi(r)$ and its Fourier transform $\rho_\Phi(Q)$, respectively, and the dot-dashed lines to an additional vector component with the same relative normalisation in the upper and lower part. The sum of both contributions yields a good description of both lattice data.
Figure 4: Two-gluon binding potential (11) given by the dot-dashed and dashed lines (with the respective parameters from table 1) in comparison with the confinement potential from lattice QCD [22].