Quark and Gluon Propagators in Covariant Gauges

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We present data for the gluon and quark propagators computed in the standard lattice Landau’s gauge and for three values of the covariant gauge-fixing parameter ($\lambda = 0, 8, 16$). Our results are obtained using the $SU(3)$ Wilson action in the quenched approximation at $\beta = 6.0$ and $V = 16^3 \times 32$.

In the last few years we have proposed a new lattice gauge-fixing algorithm \cite{1}-\cite{6} to study in a non perturbative framework the gauge dependence of the fundamental QCD correlators, i.e. gluon and quark propagators. Since a na"ive generalization of the standard Landau gauge-fixing functional is not possible \cite{1}, we propose to fix a generic covariant gauge by minimizing a discretized version of the functional

$$ H_A[G] \equiv \int d^4x \text{Tr} \left[ (\partial_\mu A_\mu^G - \Lambda)(\partial_\nu A_\nu^G - \Lambda) \right] $$

where the $\Lambda$’s belong to the Lie algebra of the group and are generated with a Gaussian distribution $\exp[-1/\lambda \int d^4x \text{Tr}(\Lambda^2)]$ (for all notations and conventions see \cite{2},\cite{6}). We have implemented a discretization of the functional (1) which is linear in any single local gauge transformation. This can be achieved with a careful manipulation of $O(a)$ terms in the lattice functional definition (1).

We retrieved 221 $SU(3)$ gauge configurations at $\beta = 6.0$ and $V = 16^3 \times 32$ available from the repository at the “Gauge Connection” (http://qcd.nersc.gov). Covariant gauges corresponding to $\lambda = 0, 8$ have been fixed for all of them, while the gauge corresponding to $\lambda = 16$ has been imposed on 80 configurations only. We have required always a gauge fixing quality factor $\theta < 10^{-6}$. The calculation of the gauge-fixing rotation has been performed on the Boston University’s Origin2000.

Last year we presented in Bangalore \cite{8} some preliminary results on the gluon propagator in the $SU(3)$ quenched theory computed in the standard Landau and in the covariant gauge at two values of the parameter $\lambda = 0, 8$. Here we present another set of data ($\lambda = 16$) for the gluon propagator and the quark propagator, with the Wilson fermionic action, for three values of the gauge parameter $\lambda = 0, 8, 16$.

In Fig. 1 we show the behaviour of the gluon propagator at zero spatial momentum

$$ \langle A_i A_i \rangle(t) \equiv \frac{1}{3V^2} \sum_i \sum_{x,y} \text{Tr} \langle A_i(x,t) A_i(y,0) \rangle $$

as a function of $t$. While the data for the standard Landau gauge and for $\lambda = 0$ are compatible within errors, the results for $\lambda = 8, 16$ show signals of a gauge dependence of the gluon propagator.

We have analyzed the data of the gluon propagator also in the Fourier space by using the usual separation of transverse and longitudinal components, i.e.

$$ D_{\mu\nu}(q) = (\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) D_T(q) + \frac{q_\mu q_\nu}{q^2} \frac{D_L(q)}{q^2}. $$

In Fig. 2 we present the behaviour of the transverse part $q^2 D_T(q)$ as a function of $q = \sqrt{q^2}$. The data have been averaged over the $Z_3$ group.

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and only values corresponding to \( k_0 = \frac{2\pi}{32}(0 \div 12) \) and \( k_i = \frac{2\pi}{16}(0 \div 6) \) are reported. We see that at large \( q^2 \) the data at the three values of the gauge parameter tend to coincide, while at low \( q^2 \) they exhibit a clear gauge dependence. The data with higher \( \lambda \) show a lower peak than those with \( \lambda = 0 \), but the peak position remains fixed. Note that in this region physical effects caused by confinement are expected, see for example [10].

The inverse of the quark propagator \( S^{-1}(q) \), at \( K = 0.1515 \), has been decomposed in the standard way

\[
S(q)^{-1} = i \, q\Sigma_1(q^2, m) + m\Sigma_2(q^2, m).
\]  

In Fig. 3 we report \( \Sigma_1 \) as a function of \( q^2 \). At large \( q^2 \) in the Landau gauge, \( \Sigma_1 \) is expected to be a constant at leading order in perturbation theory. On the contrary, in the covariant gauge it should decrease logarithmically as a function of \( q^2 \) with a slope which depends on \( \lambda \). The data can be fitted using the perturbation expansion in order to extract the gauge fixing parameter renormalization constant \( Z_\lambda \). The data favor a small value of the gauge parameter renormalization constant: \( Z_\lambda \approx .35 \) for \( \lambda = 8 \) and \( Z_\lambda \approx .20 \) for \( \lambda = 16 \). In order to complete the presentation of our results, we show in Fig. 4 the behaviour of \( \Sigma_2 \). It is well known that in the Wilson fermionic regularization this function is affected from large \( O(a) \) effects and our data confirm this pathology in the covariant gauge too. In conclusion, our data show signals of gauge dependence in the gluon and quark propagator. We observe a small value of \( Z_\lambda \) with a large dependence on \( \lambda \) which seems to reduce the separation among gauges with different \( \lambda \).

Our study implicitly indicates that lattice Gribov copies do not have a statistical significance in the quark and gluon propagators in the Landau gauge. In fact Gribov copies are averaged with different weights in the standard Landau gauge-fixing and in the covariant gauge at \( \lambda = 0 \). Nevertheless anything different turns out to be within our statistical errors.

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