Analysis of the strong decay $X(5568) \to B^0_s \pi^+$ with QCD sum rules

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Abstract

In this article, we take the $X(5568)$ to be the scalar diquark-antidiquark type tetraquark state, study the hadronic coupling constant $g_{XB,s\pi}$ with the three-point QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension-6 and including both the connected and disconnected Feynman diagrams, then calculate the partial decay width of the strong decay $X(5568) \to B^0_s \pi^+$ and obtain the value $\Gamma_X = (20.5 \pm 8.1)$ MeV, which is consistent with the experimental data $\Gamma_X = (21.9 \pm 6.4^{+5.0}_{-2.5})$ MeV from the D0 collaboration.

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1 Introduction

Recently, the D0 collaboration observed a narrow structure, $X(5568)$, in the decay $X(5568) \to B^0_s \pi^+$ with significance of $5.1\sigma$ [1]. The measured mass and width are $m_X = (5567.8 \pm 2.9^{+0.9}_{-1.9})$ MeV and $\Gamma_X = (21.9 \pm 6.4^{+5.0}_{-2.5})$ MeV, respectively. The D0 collaboration fitted the $B^0_s \pi^+$ systems with the Breit-Wigner parameters in relative S-wave, the favored quantum numbers are $J^P = 0^+$. However, the quantum numbers $J^P = 1^+$ cannot be excluded according to decays $X(5568) \to B^+_s \pi^+ \to B^0_s \pi^+ \gamma$, where the low-energy photon is not detected. There have been several possible assignments, such as the scalar-diquark-scalar-antidiquark type tetraquark state [2, 3, 4, 5, 6, 7], axialvector-diquark-axialvector-antidiquark type tetraquark state [3, 8, 9], $B^{(*)}K$ hadronic molecule state [10], threshold effect [11].

The calculations based on the QCD sum rules indicate that both the scalar-diquark-scalar-antidiquark type and axialvector-diquark-axialvector-antidiquark type interpolating currents can give satisfactory mass $m_X$ to reproduce the experimental data [2, 3, 4, 5, 8]. In Ref.[7], Agaev, Azizi and Sundu choose the axialvector-diquark-axialvector-antidiquark type interpolating current, calculate the hadronic coupling constant $g_{XB,s\pi}$ with the light-cone QCD sum rules in conjunction with the soft-$\pi$ approximation and other approximations, and obtain the partial decay width for the process $X(5568) \to B^0_s \pi^+$. In Ref.[7], Dias et al choose the scalar-diquark-scalar-antidiquark type interpolating current, calculate the hadronic coupling constant $g_{XB,s\pi}$ with the three-point QCD sum rules in the soft-$\pi$ limit by taking into account only the connected Feynman diagrams in the leading order approximation, and obtain the partial decay width for the decay $X(5568) \to B^0_s \pi^+$. In previous work [2], we choose the scalar-diquark-antidiquark type interpolating current to study the mass of the $X(5568)$ with the QCD sum rules. In this article, we extend our previous work to study the hadronic coupling constant $g_{XB,s\pi}$ with the three-point QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension-6 and including both the connected and disconnected Feynman diagrams, then calculate the partial decay width of the strong decay $X(5568) \to B^0_s \pi^+$.

The article is arranged as follows: we derive the QCD sum rule for the hadronic coupling constant $g_{XB,s\pi}$ in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusion.

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2 QCD sum rule for the hadronic coupling constant $g_{XB_s \pi}$

We can study the strong decay $X(5568) \rightarrow B_s^0 \pi^+$ with the three-point correlation function $\Pi(p, q)$,

$$\Pi(p, q) = i^2 \int d^4 xd^4 ye^{ip \cdot x} e^{iq \cdot y} \langle 0 | T \{ J_{B_s}(x) J_\pi(y) J_X(0) \} | 0 \rangle,$$  \hspace{1cm} (1)

where the currents

$$J_{B_s}(x) = \bar{s}(x)i\gamma_5 b(x),$$  

$$J_\pi(y) = \bar{u}(y)i\gamma_5 d(y),$$  

$$J_X(0) = \epsilon^{ijk}t^{imn}u^j(0)C\gamma_5 s^k(0)d^m(0)\gamma_5 \bar{b}^n(0),$$  \hspace{1cm} (2)

interpolate the mesons $B_s$, $\pi$ and $X(5568)$, respectively, the $i, j, k, m, n$ are color indexes, the $C$ is the charge conjugation matrix. In Ref.[7], the axialvector current is used to interpolate the $\pi$ meson.

At the hadron side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_{B_s}(x), J_\pi(y)$ and $J_X(0)$ into the three-point correlation function $\Pi(p, q)$ and isolate the ground state contributions to obtain the following result,

$$\Pi(p, q) = \frac{f_\pi m^2 f_{B_s} m^2_{B_s} \lambda_X g_{XB_s \pi}}{(m_u + m_d)(m_b + m_s)} \frac{1}{(m_X^2 - p^2) (m^2_{B_s} - p^2) (m^2_s - q^2)}$$  

$$+ \frac{1}{(m_X^2 - p^2) (m^2_{B_s} - p^2)} \int_{s^2_0}^{\infty} dt \frac{\rho_{X\pi}(p^2, t, p^2)}{t - q^2}$$  

$$+ \frac{1}{(m_X^2 - p^2) (m^2_s - q^2)} \int_{s^2_0}^{\infty} dt \frac{\rho_{XB_s}(t, q^2, p^2)}{t - p^2} + \cdots,$$  \hspace{1cm} (3)

where $p' = p + q$, the $f_{B_s}, f_\pi$ and $\lambda_X$ are the decay constants of the mesons $B_s, \pi$ and $X(5568)$, respectively, the $g_{XB_s \pi}$ is the hadronic coupling constant.

In the following, we write down the definitions,

$$\langle 0 | J_X(0) | X(p') \rangle = \lambda_X,$$  

$$\langle 0 | J_{B_s}(0) | B_s(p) \rangle = \frac{f_{B_s} m^2_{B_s}}{m_b + m_s},$$  

$$\langle 0 | J_\pi(0) | \pi(q) \rangle = \frac{f_\pi m^2_\pi}{m_u + m_d},$$  \hspace{1cm} (4)

$$\langle B_s(p) \pi(q) | X(p') \rangle = ig_{XB_s \pi}.$$  \hspace{1cm} (5)

The two unknown functions $\rho_{X\pi}(p^2, t, p^2)$ and $\rho_{XB_s}(t, q^2, p^2)$ have complex dependence on the transitions between the ground state $X(5568)$ and the excited states of the $\pi$ and $B_s$ mesons, respectively. We introduce the parameters $C_{X_X}$ and $C_{X_{B_s}}$ to parameterize the net effects,

$$C_{X_X} = \int_{s^2_0}^{\infty} dt \frac{\rho_{X\pi}(p^2, t, p^2)}{t - q^2},$$  

$$C_{X_{B_s}} = \int_{s^2_0}^{\infty} dt \frac{\rho_{XB_s}(t, q^2, p^2)}{t - p^2},$$  \hspace{1cm} (6)

and rewrite the correlation function $\Pi(p, q)$ into the following form,

$$\Pi(p, q) = \frac{f_\pi m^2 f_{B_s} m^2_{B_s} \lambda_X g_{XB_s \pi}}{(m_u + m_d)(m_b + m_s)} \frac{1}{(m_X^2 - p^2) (m^2_{B_s} - p^2) (m^2_s - q^2)}$$  

$$+ \frac{C_{X_X}}{(m_X^2 - p^2)(m^2_{B_s} - p^2)} + \frac{C_{X_{B_s}}}{(m^2_s - q^2)(m^2_{B_s} - p^2)} + \cdots.$$  \hspace{1cm} (7)
We set $p'^2 = p^2$ and take the double Borel transform with respect to the variable $P^2 = -p^2$ and $Q^2 = -q^2$ respectively to obtain the QCD sum rule at the left side (LS),

$$\text{LS} = \frac{f_\pi m_\pi^2 f_B m_B^2 \lambda_X g_{X_B,\pi}}{(m_u + m_d)(m_b + m_s)} \frac{1}{m_X^2 - m_B^2} \{ \exp \left( \frac{m_B^2}{M_1^2} \right) - \exp \left( \frac{-m_X^2}{M_1^2} \right) \} \exp \left( \frac{-m_s^2}{M_2^2} \right)$$

\[ + C_{XB, s} \exp \left( -\frac{m_s^2}{M_1^2} \right) \exp \left( -\frac{m_s^2}{M_2^2} \right). \] (8)

In calculations, we neglect the dependencies of the $C_{X\pi}$ and $C_{XB, s}$ on the variables $p^2$, $p'^2$, $q^2$ therefore the dependencies of the $C_{X\pi}$ and $C_{XB, s}$ on the variables $M_1^2$ and $M_2^2$, take the $C_{X\pi}$ and $C_{XB, s}$ as free parameters, and choose the suitable values to eliminate the contaminations so as to obtain the stable sum rules with the variations of the Borel parameters [12][13].

Now we carry out the operator product expansion at the large Euclidean space-time region $-p^2 \to \infty$ and $-q^2 \to \infty$, take into account the vacuum condensates up to dimension 6 and...
neglect the contribution of the three-gluon condensate, as the three-gluon condensate is the vacuum expectation of the operator of the order $O(\alpha_s^{3/2})$. In other words, we calculate the Feynman diagrams shown in Fig.1. For example, the first diagram is calculated in the following ways,

$$
\Pi(p, q) = -\frac{6}{(2\pi)^6} \int d^4k d^4l \text{Tr} \left\{ \gamma_5 (k + p_\nu) \gamma_5 (k + p_\nu + m_b) \gamma_5 (\gamma^\mu q) \gamma_5 \gamma^\mu \right\} 
$$

$$
= -\frac{6}{(2\pi)^6} \frac{(-2\pi)^2}{2\pi i} \int_{m_b^2}^{\infty} ds \frac{1}{s - p^2} \int d^4k \delta \left[ k^2 \right] \delta \left[ (k + p)^2 - m_b^2 \right] \frac{(-2\pi)^2}{2\pi i} \int_0^\infty du \frac{1}{u - q^2} 
$$

$$
\int d^4l \delta \left[ l^2 \right] \delta \left[ (l + q)^2 \right] \text{Tr} \left\{ \gamma_5 (k + p_\nu) \gamma_5 (k + p_\nu + m_b) \gamma_5 (\gamma^\mu q) \gamma_5 \gamma^\mu \right\} 
$$

$$
= \frac{3}{128\pi^4} \int_{m_b^2}^{\infty} \frac{ds}{s - p^2} \frac{1}{s} \int_0^\infty du \frac{u}{u - q^2} 
$$

$$
+ \frac{3m_sm_b}{64\pi^4} \int_{m_b^2}^{\infty} \frac{ds}{s - p^2} \frac{1}{s} \int_0^\infty du \frac{u}{u - q^2}.
$$

The operator product expansion converges for large $-p^2$ and $-q^2$; it is odd to take the limit $q^2 \to 0$.

Then we set $p^2 = 0$, take the quark-hadron duality below the continuum thresholds, and perform the double Borel transform with respect to the variables $P^2 = -p^2$ and $Q^2 = -q^2$ respectively to obtain the perturbative term,

$$
B_{M_1^2, M_2^2} \Pi(p, q) = \frac{3}{128\pi^4} \int_m^{s_0} ds \int_0^{u_0} du \frac{(s - m_b^2)^2}{s} u \exp \left( -\frac{s}{M_1^2} - \frac{u}{M_2^2} \right) 
$$

$$
+ \frac{3m_sm_b}{64\pi^4} \int_{m_b^2}^{s_0} ds \int_0^{u_0} du \frac{s - m_b^2}{s} u \exp \left( -\frac{s}{M_1^2} - \frac{u}{M_2^2} \right) 
$$

$$
+ \frac{1}{192\pi^4} \left( \frac{\alpha_s GG}{\pi} \right) \int_{m_b^2}^{s_0} ds \int_0^{u_0} du \frac{2 - m_b^2}{s} \exp \left( -\frac{s}{M_1^2} - \frac{u}{M_2^2} \right) 
$$

$$
+ \frac{m_b(\bar{s}s)}{16\pi^2} \int_0^{u_0} duu \exp \left( -\frac{m_b^2}{M_1^2} - \frac{u}{M_2^2} \right) 
$$

$$
+ \frac{m_s(\bar{s}s)}{32\pi^2} \left( 1 + \frac{m_b^2}{M_1^2} \right) \int_0^{u_0} duu \exp \left( -\frac{m_b^2}{M_1^2} - \frac{u}{M_2^2} \right) 
$$

$$
+ \frac{1}{192\pi^2} \left( \frac{\alpha_s GG}{\pi} \right) \int_0^{u_0} duu \exp \left( -\frac{m_s^2}{M_1^2} - \frac{u}{M_2^2} \right) 
$$

$$
+ \frac{1}{128\pi^4} \left( \frac{\alpha_s GG}{\pi} \right) \int_{m_b^2}^{s_0} ds \frac{(s - m_b^2)^2}{s} \exp \left( -\frac{s}{M_1^2} \right) 
$$

$$
+ \frac{m_b(\bar{s}g_s\sigma G s)}{32\pi^2} \int_0^{u_0} du \left( 1 + \frac{u}{M_1^2} - \frac{um_b^2}{2M_1^2} - \frac{um_s^2}{6M_1^2} \right) \exp \left( -\frac{m_b^2}{M_1^2} - \frac{u}{M_2^2} \right).
$$

The terms $\langle \bar{q}q \rangle \langle \bar{s}s \rangle$ disappear after performing the double Borel transform, the last Feynman diagram in Fig.1 have no contribution.
In Refs. [13, 14], the width of the $Z_c(4200)$ is studied with the three-point QCD sum rules by including both the connected and disconnected Feynman diagrams, which is contrary to Ref. [15], where only the connected Feynman diagrams are taken into account to study the width of the $Z_c(3900)$. In this article, the contributions come from the connected diagrams can be written as

$$R_{S_c} = \frac{1}{192\pi^2} \frac{\alpha_s G_F}{8\pi} \int_0^{s_0} ds \int_0^{u_0} du \left( 2 - \frac{m_b^2}{s} \right) \exp \left( -\frac{s}{M_{s}^2} - \frac{u}{M_b^2} \right)$$

$$- \frac{m_b(\bar{s}g_s \sigma G_s)}{32\pi^2} \int_0^{u_0} du \exp \left( -\frac{m_b^2}{M_b^2} - \frac{u}{M_b^2} \right),$$

(12)

which is too small to account for the experimental data [1].

Finally, we obtain the QCD sum rule,

$$L_S = R_S.$$  

(13)

There appear some energy scale dependence at the hadron side (or $L_S$) of the QCD sum rule according to the factors $m_u + m_d$ and $m_b + m_s$, we can eliminate the energy scale dependence by using the currents $\tilde{J}_{B_s}(x)$ and $\tilde{J}_\pi(y)$,

$$\tilde{J}_{B_s}(x) = (m_b + m_s) \bar{s}(x) i \gamma_5 b(x),$$

$$\tilde{J}_\pi(y) = (m_u + m_d) \bar{u}(y) i \gamma_5 d(y),$$

(14)

then

$$\langle 0 | \tilde{J}_{B_s}(0) | B_s(p) \rangle = f_{B_s} m_{B_s}^2,$$

$$\langle 0 | \tilde{J}_\pi(0) | \pi(q) \rangle = f_\pi m_\pi^2,$$

(15)

and

$$C_{X\pi} \rightarrow C_{X\pi} (m_b + m_s) (m_u + m_d),$$

$$C_{X_{B_s}} \rightarrow C_{X_{B_s}} (m_b + m_s) (m_u + m_d),$$

(16)

the resulting QCD sum rule at the right side also acquires a factor $(m_b + m_s) (m_u + m_d)$, an equivalent QCD sum rule is obtained, the predicted hadronic coupling constant $g_{X_{B_s}\pi}$ is not changed.

We can also study the strong decay $X(5568) \rightarrow B_0^0 \pi^+$ with the three-point correlation function $\Pi_{\mu\nu}(p, q)$,

$$\Pi_{\mu\nu}(p, q) = i^2 \int d^4 x d^4 y e^{ipx} e^{iqy} \langle 0 | T \left\{ \tilde{\eta}_\mu^{\nu b}(x) \eta_\nu^{\tilde{d}}(y) J_X(0) \right\} | 0 \rangle,$$

(17)

where the currents

$$\eta_\mu^{\nu b}(x) = \bar{s}(x) \gamma_\mu \gamma_5 b(x),$$

$$\eta_\nu^{\tilde{d}}(y) = \bar{u}(y) \gamma_\nu \gamma_5 d(y),$$

(18)

interpolate the mesons $B_s$ and $\pi$, respectively. At the hadron side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $\eta_\mu^{\nu b}(x)$ and $\eta_\nu^{\tilde{d}}(y)$ into the three-point correlation function $\Pi_{\mu\nu}(p, q)$ and isolate the ground state contributions.
to obtain the following result,

\[
\Pi_{\mu\nu}(p, q) = \frac{f_{B_\pi} \lambda X g_{X B_\pi}}{(m_X^2 - p^2)} \left( -p_\mu q_\nu \right)
+ \frac{f_{B_\pi} m_{B_1} f_{\pi} \lambda X g_{X B_1 \pi}}{(m_X^2 - p^2)} \left( -q_\mu q_\nu + \frac{p \cdot q}{p^2} p_\mu q_\nu \right)
+ \frac{f_{B_1} m_{B_1} f_{a_1} m_{a_1} \lambda X g_{X B_{1a_1}}}{(m_X^2 - p^2)} \left( g_{\mu\nu} - \frac{1}{p^2} p_\mu p_\nu - \frac{1}{q^2} q_\mu q_\nu + \frac{p \cdot q}{p^2 q^2} p_\mu q_\nu \right) + \cdots ,
\]

where \( p' = p + q \), the \( f_{B_1}, f_{B_\pi}, f_{a_1} \), and \( f_\pi \) are the decay constants of the mesons \( B_{1}(5830) \), \( B_\pi \), \( a_1(1260) \) and \( \pi \), respectively, the \( g_{X B_\pi \pi} \) and \( g_{X B_{1a_1}} \) are the hadronic coupling constants.

In the following, we write down the definitions,

\[
\langle 0 | J^{\mu}_B (0) | B_s (p) \rangle = i f_{B_s} p_\mu ,
\]

\[
\langle 0 | J^{\mu}_\pi (0) | \pi (q) \rangle = i f_{\pi} q_\mu ,
\]

\[
\langle 0 | J^{\mu}_{a_1} (0) | B_{1a_1} (p) \rangle = f_{B_{1a_1}} m_{B_{1a_1}} \epsilon_{\mu} ,
\]

\[
\langle 0 | J^{\mu}_{a} (0) | a_1 (q) \rangle = f_{a_1} m_{a_1} \epsilon_{\mu} ,
\]

\[
\langle B_{1a_1} (p) | \pi (q) | X (p') \rangle = \epsilon^* \cdot q g_{X B_{1a_1} \pi} ,
\]

\[
\langle B_1 (p) | a_1 (q) | X (p') \rangle = i \epsilon^* \cdot \epsilon^* g_{X B_{1a_1} a_1} ,
\]

where the \( \epsilon_{\mu} \) and \( \epsilon_{\nu} \) are the polarization vectors of the axialvector mesons \( B_{1}(5830) \) and \( a_1 (1260) \), respectively. From the values \( m_X = (5567.8 \pm 2.9^{+1.9}_{-1.3}) \) MeV \([1] \), \( m_{B_{1a_1}} = (5828.40 \pm 0.04 \pm 0.41) \) MeV, \( m_{B_\pi} = (5366.7 \pm 0.4) \) MeV \([10] \), we can obtain \( m_{B_{1a_1}} - m_{B_\pi} \approx 462 \) MeV and \( m_{B_1} - m_X \approx 261 \) MeV. If we take the interpolating currents \( \eta^{ab}_\mu (x) \) and \( \eta^{\alpha \beta}_\mu (y) \), there are contaminations from the axialvector mesons \( B_{1}(5830) \) and \( a_1 (1260) \). We should multiply both sides of Eq.(19) by \( p^\mu q' \) to eliminate the contaminations of the axialvector mesons \( B_{1}(5830) \) and \( a_1 (1260) \),

\[
p^\mu q' \Pi_{\mu\nu}(p, q) = \frac{f_{B_{1a_1}} m_{a_1} \lambda X g_{X B_{1a_1}}}{(m_X^2 - p^2)} \left( -p_\mu q_\nu \right) + \cdots ,
\]

which corresponds to taking the pseudoscalar currents \( \widehat{J}_{B_{1}} (x) \) and \( \widehat{J}_{\pi} (y) \) according to the following identities,

\[
\partial^\mu \eta^{ab}_\mu (x) = (m_b + m_s) s(x) i\gamma_5 b(x) = \widehat{J}_{B_{1}} (x) ,
\]

\[
\partial^\mu \eta^{\alpha \beta}_\mu (y) = (m_u + m_d) u(y) i\gamma_5 d(y) = \widehat{J}_{\pi} (y) .
\]

The axialvector currents \( \eta^{ab}_\mu (x) \) and \( \eta^{\alpha \beta}_\mu (y) \) can also be chosen to study the strong decay \( X (5568) \to B_{1}^{0} \pi^{+} \).

We also expect to study the strong decay \( X (5568) \to B_{1}^{0} \pi^{+} \) with the light-cone QCD sum rules using the two-point correlation function \( \Pi(p, q) \),

\[
\Pi(p, q) = i \int d^4 x e^{ip \cdot x} \langle \pi (q) | T \{ J_{B_{1}} (x) J_{X} (0) \} | 0 \rangle ,
\]

where the \( \langle \pi (q) \rangle \) is an external \( \pi \) state.

At the QCD side, we obtain the following result after performing the wick’s contraction,

\[
\Pi(p, q) = i \int d^4 x e^{ip \cdot x} \langle \pi (q) | e^{ijm} u_5^m (0) C_{\gamma_5 S^{bl}} (-x) i\gamma_5 S^{ln}_b (x) (x) c^{T} d_m^T (0) | 0 \rangle ,
\]

where \( S^{bl}_b (-x) \) and \( S^{ln}_b (x) \) are the full \( s \) and \( b \) quark propagators, respectively. The \( u \) and \( d \) quarks stay at the same point \( x = 0 \), the light-cone distribution amplitudes of the \( \pi \) meson are
almost useless, the integrals over the \( \pi \) meson’s light-cone distribution amplitudes reduce to overall normalization factors. In the light-cone QCD sum rules, such a situation is possible only in the soft pion limit \( q \to 0 \), and the light-cone expansion reduces to the short-distance expansion \(^{17}\). In Ref.\(^{9}\), Agaev, Azizi and Sundu take the soft pion limit \( q \to 0 \), and choose the \( C\gamma_\mu \otimes \gamma^i \mathcal{C} \) type current to interpolate the \( X(5568) \), and use the light-cone QCD sum rules to study the strong decay \( X(5568) \to B_s^0 \pi^+ \). The light-cone QCD sum rules are reasonable only in the soft pion approximation.

### 3 Numerical results and discussions

The hadronic parameters are taken as \( m_X = 5.5678 \text{ GeV} \) \(^{11}\), \( \lambda_X = 6.7 \times 10^{-3} \text{ GeV}^5 \), \( \sqrt{s_0} = (6.1 \pm 0.1) \text{ GeV} \) \(^{2}\), \( m_\pi = 0.13957 \text{ GeV} \), \( m_{Bs} = 5.3667 \text{ GeV} \) \(^{10}\), \( f_\pi = 0.130 \text{ GeV} \), \( \sqrt{m_\pi} = (0.85 \pm 0.05) \text{ GeV} \) \(^{18}\), \( f_{Bs} = 0.231 \text{ GeV} \) \(^{19}\), \( f_\pi m_\pi^2/(m_u + m_d) = -2\langle q\bar{q}/f_\pi \rangle \) from the Gell-Mann-Oakes-Renner relation, and \( M_2^2 = (0.8 - 1.2) \text{ GeV}^2 \) from the QCD sum rules \(^{18}\). At the QCD side, the vacuum condensates are taken to be standard values, \( \langle q\bar{q} \rangle = -(0.24 \pm 0.01) \text{ GeV}^3 \), \( \langle \bar{s}s \rangle = (0.8 \pm 0.1)\langle q\bar{q} \rangle \), \( \langle q\bar{q}, \sigma Gq \rangle = m_0^2\langle q\bar{q} \rangle \), \( m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2 \) and \( 2\langle \sigma Gq \rangle = (0.33 \text{ GeV})^4 \) at the energy scale \( \mu = 1 \text{ GeV} \) \(^{20} \text{21}\). The quark condensates and mixed quark condensates evolve with the renormalization group equation, \( \langle q\bar{q} \rangle(\mu) = \langle q\bar{q} \rangle(Q) \left[ \frac{\alpha_s(\mu)}{\alpha_s(\mu)} \right]^\frac{4}{9} \) and \( \langle q\bar{q}, \sigma Gq \rangle(\mu) = \langle q\bar{q}, \sigma Gq \rangle(Q) \left[ \frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^\frac{4}{9} \), where \( q = u, d, s \).

In the article, we take the \( \overline{\text{MS}} \) masses \( m_b(0) = (4.18 \pm 0.03) \text{ GeV} \), \( m_s(0) = (0.095 \pm 0.005) \text{ GeV} \) from the Particle Data Group \(^{16}\), and take into account the energy-scale dependence of the \( \overline{\text{MS}} \) masses from the renormalization group equation,

\[
m_b(\mu) = m_b(0) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_b(0))} \right]^\frac{4}{9},
\]

\[
m_s(\mu) = m_s(2 \text{ GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2 \text{ GeV})} \right]^\frac{4}{9},
\]

\[
\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{1}{b_1} \log \frac{b_2}{t} + \frac{b_3}{b_0^2} \left( \log^2 \frac{t}{b_0^2} - \log t - 1 \right) + b_4 b_5 \right], \tag{26}
\]

where \( t = \log \frac{\mu^2}{\Lambda^2} \), \( b_0 = \frac{33 - 2n_f}{12\pi} \), \( b_1 = \frac{153 - 19n_f}{24\pi^2} \), \( b_2 = \frac{2857 - 5033n_f + 225n_f^2}{128\pi^4} \), \( \Lambda = 213 \text{ MeV} \), and \( 339 \text{ MeV} \) for the flavors \( n_f = 5, 4 \) and 3, respectively \(^{16}\). Furthermore, we set the \( u \) and \( d \) quark masses to be zero. In the heavy quark limit, the \( b \)-quark can be taken as a static potential well, and unchanged in the decay \( X(5568) \to B_s^0 \pi^+ \). In this article, we take the typical energy scale \( \mu = m_b \).

The unknown parameter is chosen as \( C_{XB_s} = -0.00059 \text{ GeV}^8 \). There appears a platform in the region \( M_2^2 = (4.5 - 5.5) \text{ GeV}^2 \). Now we take into account the uncertainties of the input parameters and obtain the value of the hadronic coupling constant \( g_{XB_s\pi} \), which is shown explicitly in Fig.2,

\[
g_{XB_s\pi} = (10.6 \pm 2.1) \text{ GeV}. \tag{27}
\]

Now we obtain the partial decay width,

\[
\Gamma (X(5568) \to B_s^0 \pi^+) = \frac{g_{XB_s\pi}^2}{16\pi M_X} \sqrt{\left[ m_X^2 - (m_{Bs} + m_\pi)^2 \right] \left[ m_X^2 - (m_{Bs} - m_\pi)^2 \right]} = (20.5 \pm 8.1) \text{ MeV}. \tag{28}
\]

The decays \( X(5568) \to B^+ K^0 \) are kinematically forbidden, so the width \( \Gamma_X \) can be saturated by the partial decay width \( \Gamma (X(5568) \to B_s^0 \pi^+) \), which is consistent with the experimental value
Dirac-spinor spaces to obtain the result, $\mathcal{X} \left( 5568 \right) \rightarrow B_s \pi^+$, while the decays $X(5568) \rightarrow B^+ K^0$, which is consistent with the observation of the D0 collaboration [1]. In previous works, we observed that the $C \gamma_5 \otimes \gamma_5 C$ type hidden-charm tetraquark states have slight smaller masses than that of the $C \gamma_5 \otimes \gamma^\mu C$ type hidden-charm tetraquark states, the predicted lowest masses are $m_{C \gamma_5 \otimes \gamma_5 C} = (3.82^{+0.05}_{-0.10})$ GeV and $m_{C \gamma_5 \otimes \gamma^\mu C} = (3.85^{+0.15}_{-0.09})$ GeV [22]. We expect that a $C \gamma_5 \otimes \gamma^\mu C$ type current can also reproduce the experimental value $m_X = (5567.8 \pm 2.9^{+0.9}_{-1.1})$ MeV approximately [3][8].

Now we construct the current $\eta_X$ and perform Fierz re-arrangement both in the color and Dirac-spinor spaces to obtain the following result,

$$\eta_X = \epsilon_{ijk} \epsilon^{lmn} u^{2} C_{\gamma_{5}} s^{k} d^{m} \gamma_{\mu} C \bar{b}^{\alpha},$$

$$\mathcal{X} \left( 5568 \right) \rightarrow B_s \pi^+,$$

the components $b i \gamma_5 s d i \gamma_5 u$ and $b \gamma_{\mu} \gamma_5 s d \gamma_{\mu} \gamma_5 u$ couple potentially to the meson pair $B_s \pi^+$, while the decays $X(5568) \rightarrow B^+ K^0$, which is consistent with the observation of the D0 collaboration [1]. In previous works, we observed that the $C \gamma_5 \otimes \gamma_5 C$ type hidden-charm tetraquark states have slight smaller masses than that of the $C \gamma_5 \otimes \gamma^\mu C$ type hidden-charm tetraquark states, the predicted lowest masses are $m_{C \gamma_5 \otimes \gamma_5 C} = (3.82^{+0.05}_{-0.10})$ GeV and $m_{C \gamma_5 \otimes \gamma^\mu C} = (3.85^{+0.15}_{-0.09})$ GeV [22]. We expect that a $C \gamma_5 \otimes \gamma^\mu C$ type current can also reproduce the experimental value $m_X = (5567.8 \pm 2.9^{+0.9}_{-1.1})$ MeV approximately [3][8].

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Now we construct the current $\eta_X$ and perform Fierz re-arrangement both in the color and Dirac-spinor spaces to obtain the following result,
analogous to the current $J_X$. It is also sensible to assign the $X(5568)$ to be an axialvector-diquark-axialvector-antidiquark type tetraquark state or the $X(5568)$ has some axialvector-diquark-axialvector-antidiquark type tetraquark components.

The $C \otimes C$ type current $\tilde{J}_X$ and $C \gamma_5 \gamma_5 \otimes \gamma_5 \gamma_5 \mu C$ type current $\tilde{\eta}_X$ are expected to couple potentially to the scalar tetraquark with much larger masses,

$$\tilde{J}_X = \epsilon^{ijk} \epsilon^{imn} u^j C s^k \bar{d}^m \bar{C} b^n,$$

$$\tilde{\eta}_X = \epsilon^{ijk} \epsilon^{imn} u^j C \gamma_\mu \gamma_5 s^k \bar{d}^m \gamma_5 \gamma_\mu \bar{C} b^n,$$

as the favored configurations are the scalar diquarks ($C \gamma_5$-type) and axialvector diquarks ($C \gamma_\mu$-type) from the QCD sum rules [23, 24].

4 Conclusion

In this article, we take the $X(5568)$ to be the scalar diquark-antidiquark type tetraquark state, study the hadronic coupling constant $g_{X B_s \pi}$ with the three-point QCD sum rules, then calculate the partial decay width of the strong decay $X(5568) \rightarrow B_s^0 \pi^+$ and obtain the value $\Gamma_X = (20.5 \pm 8.1) \text{ MeV}$, which is consistent with the experimental data $\Gamma_X = (21.9 \pm 6.4 \pm 5.0) \text{ MeV}$ from the D0 collaboration. In calculation, we carry out the operator product expansion up to the vacuum condensates of dimension-6, and take into account both the connected and disconnected Feynman diagrams. The present prediction favors assigning the $X(5568)$ to be the diquark-antidiquark type tetraquark state with $J^P = 0^+$. However, the quantum numbers $J^P = 1^+$ cannot be excluded according to decays $X(5568) \rightarrow B_s^* \pi^+ \rightarrow B_s^0 \pi^+ \gamma$, where the low-energy photon is not detected.

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