Pair production of neutral Higgs bosons from the left-right twin Higgs model at the ILC and LHC

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Abstract

In the framework of the left-right twin Higgs model, we study pair production of the neutral Higgs bosons at the International Linear Collider (ILC) and the CERN LHC. We find that the production cross section of the process $e^+e^- \rightarrow \phi^0h$ are at the level of several tens fb at the ILC, the production cross section of the $\phi^0\phi^0$ pair and $\phi^0h$ pair are at the level of several hundreds fb at the LHC. As long as the neutral Higgs boson $\phi^0$ is not too heavy, we conclude that its pair production might be used to test for the left-right twin Higgs model at the LHC experiment or in the future ILC experiment.

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I. Introduction

The Higgs mechanism is the heart of the standard model (SM) providing masses to gauge bosons via electroweak symmetry breaking (EWSB). However, the SM fails to explain the origin of the fermion mass and has naturalness problems. Many alternative new physics models with extended Higgs sectors are free from the above difficulties. The hunt for the Higgs bosons came to be one of the most important goals for present and future high energy collider experiments. Apart from the SM, neutral Higgs bosons appear in almost every scenario exploring new phenomena [1]. Pair production of neutral Higgs bosons at the CERN LHC, which will provide a way to test the Higgs boson self-coupling, may be sensitive to new physics [2, 3]. Many works have contributed to studies of the neutral Higgs pair production at the hadron collider in model independent [4], in SM [5-8], and in new physics models beyond the SM, such as little Higgs models [9], Randall-Sundrum-like models [10], top condensation models [11], supersymmetric models (SU SY) [12, 13] and models of universal extra dimensions (UED) [14].

The SM has been proved by all existing precise experimental data with its theoretical predictions beyond one-loop level being coincident with experimental observations. But in the SM the Higgs boson mass suffers from an instability under radiative corrections, which is called ”hierarchy problem” [15]. Recently, the twin Higgs mechanism has been proposed as a solution to the little hierarchy problem. The Higgs bosons emerge as pseudo-Goldstone bosons once the global symmetry is spontaneously broken. Gauge and Yukawa interactions that break the global symmetry give masses to the Higgses. The twin Higgs mechanism can be implemented in left-right models with the additional discrete symmetry being identified with left-right symmetry [16, 17]. The left-right twin Higgs (LRTH) model contains the $U(4)_1 \times U(4)_2$ global symmetry as well as the gauged symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. After Higgs obtained vacuum expectation values $(f, \hat{f})$, the global symmetry $U(4)_1 \times U(4)_2$ breaks down to $U(3)_1 \times U(3)_2$, and the gauge group $SU(2)_R \times U(1)_{B-L}$ breaks down to the SM $U(1)_Y$. Thus, the LRTH model predicts the existence of the new particles, such as heavy gauge bosons, heavy scalars, and the
top partner $T$, which can generate rich phenomenology at present and in future collider experiments [17-21].

In the context of the $LRTH$ model, pair production of the charged Higgs bosons ($\phi^+$, $\phi^-$) in the $LRTH$ model at the ILC and LHC are studied in Ref.[21], but they did not consider production of the neutral Higgs bosons ($\phi^0$, $h$). As we know, so far production of the neutral Higgs pair at the LHC and the ILC in the $LRTH$ model has not been considered, which is the main aim of this paper.

Besides the SM-like Higgs boson $h$, there are two additional neutral Higgs bosons in the $LRTH$ model, which are $\hat{h}_2^0$ and $\phi^0$. The neutral Higgs boson $\hat{h}_2^0$ is a possible dark matter candidate that only couples to the gauge bosons (including the SM gauge bosons $\gamma$, $Z$, $W$, and the new gauge boson $Z_H$). The production cross section of $\hat{h}_2^0$ at the collider is very small and escapes the detector. Therefore, in this paper, we will not discuss the production of $\hat{h}_2^0$ at the ILC or LHC. The neutral Higgs boson $\phi^0$ is a pseudoscalar that couples to both the SM fermions and gauge bosons. The neutral Higgs boson pair $\phi^0 h$ can be produced via the processes $e^+e^- \rightarrow Z(Z_H) \rightarrow \phi^0 h$ at the ILC, and via the partonic processes $q\bar{q} \rightarrow \phi^0 h (q = u, c, d, s, b)$, $gg \rightarrow \phi^0 h$ at the LHC, respectively. While the neutral Higgs pair $\phi^0 \phi^0$ can only be produced via the partonic process $gg \rightarrow \phi^0 \phi^0$ and the t-channel partonic process $b\bar{b} \rightarrow \phi^0 \phi^0$ at the LHC. We calculate all above these processes. Our numerical results denote that, for $m_h = 120 GeV$, $120 GeV \leq m_{\phi^0} \leq 180 GeV$ and $500 GeV \leq f \leq 1500 GeV$: (i) the production cross section of $\phi^0 h$ at the ILC with the center-of-mass (c.m.) energy $\sqrt{s} = 500 GeV$ is in the range of $0.92 fb \sim 20 fb$; (ii) the production cross section of $\phi^0 h$ at the LHC with the c.m. energy $\sqrt{s} = 14 TeV$ is in the range of $34 fb - 306 fb$, and the main contribution comes from light quarks; (iii) the production cross section of $\phi^0 \phi^0$ at the LHC is in the range of $4 fb - 122 fb$, and the main contribution comes from the top quark loop.

This paper is organized as follows. In Sec. II, we briefly review the essential features of the $LRTH$ model. The relevant couplings of the neutral Higgs bosons to other particles and the feature of the decay for the neutral Higgs bosons $\phi^0$ are also discussed in this section. In Secs. III and IV, we give our numerical results for pair production of neutral
Higgs bosons predicted by the LRTH model at the ILC and LHC, respectively. Our conclusions are given in Sec. V.

II. The LRTH Model

The LRTH model was first proposed in Ref.[16] and the details of the model as well as the particle spectrum, Feynman rules, and some phenomenology analysis have been studied in Ref.[17]. Here we will briefly review the essential features of the model and focus our attention on the neutral Higgs bosons.

The LRTH model is based on the global $U(4)_1 \times U(4)_2$ symmetry with a locally gauged subgroup $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Two Higgs fields, $H = (H_L, H_R)$ and $\hat{H} = (\hat{H}_L, \hat{H}_R)$, are introduced and each transforms as $(4, 1)$ and $(1, 4)$, respectively, under the global symmetry. $H_{L,R} (\hat{H}_{L,R})$ are two component objects which are charged under $SU(2)_L$ and $SU(2)_R$, respectively. For the gauge couplings $g_{2L}$ and $g_{2R}$ of $SU(2)_L$ and $SU(2)_R$, the left-right symmetry implies that $g_{2L} = g_{2R} = g_2$.

The $U(4)_1 [U(4)_2]$ group is spontaneously broken down to its subgroup $U(3)_1 [U(3)_2]$ with nonzero vacuum expectation value ($VEV$) $< H > = (0, 0, 0, f)$ [$< \hat{H} > = (0, 0, 0, \hat{f})$]. The Higgs VEVs also break $SU(2)_R \times U(1)_{B-L}$ down to the SM $U(1)_Y$. After spontaneous global symmetry breaking by $f$ and $\hat{f}$, three Goldstone bosons are eaten by the new gauge bosons $W^\pm_H$ and $Z_H$. After the SM electroweak symmetry breaking, the three additional Goldstone bosons are eaten by the SM gauge bosons $W^\pm$ and $Z$.

The fermion sector of the LRTH model is similar to that of the SM, with the right-handed quarks ($u_R, d_R$) and leptons ($l_R, \nu_R$) form fundamental representations of $SU(2)_R$. In order to give the top-quark mass of the order of the electroweak scale, a pair of vectorlike quarks $Q_L$ and $Q_R$ are introduced. The mass eigenstates, which contain one the SM top quark $t$ and a heavy top partner $T$, are mixtures of the gauge eigenstates. Their masses are given by

$$m_t^2 = \frac{1}{2}(M^2 + y^2 f^2 - N_t), \quad M_T^2 = \frac{1}{2}(M^2 + y^2 f^2 + N_t).$$

(1)

where $N_t = \sqrt{(y^2 f^2 + M^2)^2 - y^4 f^4 \sin^2 2x}$ with $x = \nu / \sqrt{2}f$, in which $\nu = 246 GeV$ is the
scale of the EWSB. Provided $M_T \leq f$ and that the parameter $y$ is of order one, the top Yukawa coupling will also be of order one. The parameter $M$ is essential to the mixing between the SM top quark and its partner $T$.

According the symmetry-breaking pattern discussed above, with certain reparametrizations of the fields, there are left with four Higgs bosons in the $LRTH$ spectrum that couple to both the fermion sector and the gauge boson sector. They are one neutral Higgs bosons $\phi^0$, a pair of charged Higgs bosons $\phi^\pm$, and the SM-like physical Higgs $h$. In addition, there is an $SU(2)_L$ doublet $\hat{h} = (\hat{h}_1^+, \hat{h}_0^0)$ that couples to the gauge boson sector only (including the SM gauge bosons $\gamma$, $Z$, $W$, and the new gauge boson $Z_H$). The lightest particle in $\hat{h}$, typically one of the neutral components, is stable, and therefore constitutes a good dark matter candidate.

These neutral Higgs bosons can couple to each others, and also can couple to the ordinary fermions, ordinary gauge bosons, new top quark $T$, and new gauge boson $Z_H$. The couplings expression forms which are related our calculation, are shown as [17]

$$
\begin{align*}
\phi^0 \bar{d}_{1,2,3}d_{1,2,3} & : im_d\gamma_5/(\sqrt{2} f); \\
\phi^0 \bar{u}_{1,2}u_{1,2} & : -im_u\gamma_5/(\sqrt{2} f); \\
ht & : -em_tC_LC_R/(2m_WS_W); \\
h\bar{t}t & : iyS_RS_L\gamma_5/\sqrt{2}; \\
h\bar{T}T & : -y(S_RS_L - C_LC_R x)/\sqrt{2}; \\
h\phi^0 & : x(30p_2p_3 + 11p_1p_0)/(27\sqrt{2} f); \\
h\phi^0 & : iexp_{3\mu}/(6C_WS_W); \\
Z_{H\mu} & : e\gamma_\mu(2S_W^2P_L + (1 - 7\cos2\theta_W))P_R)/(12C_WS_W\sqrt{\cos2\theta_W}); \\
Z_{H\mu} & : e\gamma_\mu(S_W^2P_L + (3 - 5S_W^2)P_R)/(6C_WS_W\sqrt{\cos2\theta_W}); \\
h\phi^0 & : iexp((14 - 17S_W^2)p_2\mu - (4 - 5S_W^2)p_1\mu)/(18S_WC_W\sqrt{\cos2\theta_W}). \quad (2)
\end{align*}
$$

Where $p_1$, $p_2$, and $p_3$ refer to the incoming momentum of the first, second and third particles, respectively. $u_i$ and $d_i$ represent the upper- and down-type fermions, respectively. $S_W = \sin\theta_W$, $C_W = \cos\theta_W$, and $\theta_W$ is the Weinberg angle. At the leading order of $1/f$, the sine values of the mixing angles $\alpha_L$ and $\alpha_R$ can be written as

$$
S_L = \sin\alpha_L \approx \frac{M}{M_T}\sin x, \quad S_R = \sin\alpha_R \approx \frac{M}{M_T}(1 + \sin^2 x). \quad (3)
$$
$C_L$ and $C_R$ are the cosine values of the mixing angles $\alpha_L$ and $\alpha_R$, respectively. $P_{L(R)} = (1 \mp \gamma_5)/2$ is the left (right)-handed projection operator.

In the framework of the LRTH model, the mass of the neutral Higgs boson $\phi^0$ can be anything below $f$ here we consider another possibility, in which the mass is around 150GeV [17]. Similar to the SM Higgs boson, $\phi^0$ can decay to $\gamma\gamma$ through the top-quark loop and heavy top-quark loop. But unlike the SM Higgs boson, in the LRTH model, the light neutral Higgs boson $\phi^0$ is a pseudoscalar boson, due to its pseudoscalar nature, there is no $\phi^0 W W$ and $\phi^0 Z Z$ couplings at tree level. So, the one-loop SM gauge boson contribution to $\phi^0 \gamma\gamma$ is zero. In general, the light neutral Higgs boson $\phi^0$ decays into $b\bar{b}$, $c\bar{c}$, $\tau^+\tau^-$, $gg$ and $\gamma\gamma$. Now we discuss the branching ratios for the possible decay modes of $\phi^0$. The decay width of $\phi^0 \rightarrow f\bar{f}$ is proportional to the square of the corresponding Yukawa coupling, with an additional suppression factor of $\nu^2/(2f^2)$ comparing to that of the SM Higgs boson. The concrete expressions of the decay widths for the different decay channels are given as follows:

$$\Gamma(\phi^0 \rightarrow b\bar{b}) = \frac{3G_F m_{\phi^0} \nu^2 m_b^2}{8\sqrt{2}\pi f^2} (1 - 4m_b^2/m_{\phi^0}^2)^{3/2},$$

$$\Gamma(\phi^0 \rightarrow c\bar{c}) = \frac{3G_F m_{\phi^0} \nu^2 m_c^2}{8\sqrt{2}\pi f^2},$$

$$\Gamma(\phi^0 \rightarrow \tau^+\tau^-) = \frac{G_F m_{\phi^0} \nu^2 m_{\tau}^2}{8\sqrt{2}\pi f^2},$$

$$\Gamma(\phi^0 \rightarrow \gamma\gamma) = \frac{G_F \alpha^2 m_{\phi^0}^3}{128\sqrt{2}\pi^3} |\sum_f N_f^l Q_f^2 A_{f}^{\phi^0}(\tau_f)|^2,$$

$$\Gamma(\phi^0 \rightarrow gg) = \frac{G_F^2 \alpha^2 m_{\phi^0}^3}{48\sqrt{2}\pi^3} |\sum_q A_q^{\phi^0}(\tau_q)|^2.$$ (4)

Where $m_b$, $m_c$, and $m_{\tau}$ are the masses of the SM fermions $b$, $c$ and $\tau$, respectively. The index $f$ corresponds to $q$ and $l$ ($q = \text{quark}, \ l = \text{lepton}$). Where $N_f^l = 1, 3$ for $f = l, q$, respectively. $Q_f$ is the charge of the fermion $f$. Similar with Ref.[22], the function $A_f^{\phi^0}$ can be written as:

$$A_f^{\phi^0} = 2\tau_f [1 + (1 - \tau_f)f(\tau_f)].$$ (5)

where $\tau_f = 4m_f^2/m_{\phi^0}^2$. In Ref.[22], the function $f(\tau_f)$ has two parts corresponding to the $\tau_f \geq 1$ and $\tau_f < 1$ two conditions. In our numerical estimation, we have neglected
the contributions of the light fermions. Therefore, in the \textit{LRT H} model, there is \( \tau_{t(T)} = 4m_{t(T)}^2/m_{\phi^0}^2 \geq 1 \), and the function \( f(\tau_f) \) is given by

\[
f(\tau) = \arcsin^2 \frac{1}{\sqrt{\tau}}.
\]

where \( A_q^0, \tau_q \), and \( f(\tau_q) \) in Eq.(4) are defined the same as \( A_f^0, \tau_f \) and \( f(\tau_f) \), but only for quarks.

Using above partial widths of the neutral Higgs boson \( \phi^0 \), its total width \( \Gamma \) can be approximately written as

\[
\Gamma = \Gamma_{bb} + \Gamma_{cc} + \Gamma_{\tau^+\tau^-} + \Gamma_{\gamma\gamma} + \Gamma_{gg}.
\]

Figure 1: The branching ratios of the neutral Higgs boson \( \phi^0 \) for different decay modes as functions of the free parameter \( f \) for \( M = 150 GeV \), \( m_{\phi^0} = 120 GeV \). In order to see the trend clearly, we have multiplied \( Br(\phi^0 \rightarrow cc) \), \( Br(\phi^0 \rightarrow \tau^+\tau^-) \), and \( Br(\phi^0 \rightarrow \gamma\gamma) \) by the factors 10, 20, and 300, respectively.

We summed up our numerical results of the branching ratios of the neutral Higgs boson \( \phi^0 \) for different decay modes \( Br(\phi^0) \) in Fig.1. To get the numerical results, the \textit{SM} parameters involved are taken as \( m_b = 4.8 GeV \), \( m_c = 1.25 GeV \) and \( m_\tau = 1.78 GeV \) \cite{23}. In Fig.1, we plot \( Br(\phi^0) \) as a function of free parameter \( f \) for \( M = 150 GeV \)
and $m_{\phi} = 120\text{GeV}$. One can see from Fig.1 that the decay branching ratios of $\phi^0$ are sensitive to the parameter $f$. If we assume that the parameter $f$ is in the range of $500\text{GeV} \sim 1500\text{GeV}$, the value of the branching ratio $Br(\phi^0 \rightarrow b\bar{b})$ is in the range of $14\% - 55\%$, and the branching ratio $Br(\phi^0 \rightarrow gg)$ is in the range of $38\% - 85\%$. The values of $Br(\phi^0 \rightarrow c\bar{c})$, $Br(\phi^0 \rightarrow \tau^+\tau^-)$ and $Br(\phi^0 \rightarrow \gamma\gamma)$ are much smaller than those of $Br(\phi^0 \rightarrow b\bar{b})$ and $Br(\phi^0 \rightarrow gg)$. Therefore, in order to see the trend clearly, in Fig.1 we have multiplied them by 10, 20 and 300, respectively. The real numerical results are $Br(\phi^0 \rightarrow c\bar{c})= 0.9\% - 3.8\%$, $Br(\phi^0 \rightarrow \tau^+\tau^-)= 0.6\% - 2.6\%$, and $Br(\phi^0 \rightarrow \gamma\gamma)= 0.09\% - 0.2\%$. Our numerical results agree quite well with Ref.[17], in that the branching ratio $Br(\phi^0 \rightarrow \gamma\gamma)$ is roughly same as $Br(h \rightarrow \gamma\gamma)$ for $m_h = m_{\phi^0}$.

III. Pair production of neutral Higgs bosons at the ILC

In many cases, the ILC can significantly improve the LHC measurements. If a Higgs boson is discovered, it will be crucial to determine its couplings with high accuracy, to understand the so-called mechanism of EWsb [24]. The high resolution profile determination of a light Higgs boson (mass, couplings, self-couplings, etc.) can be carried out at the ILC, where clear signals of Higgs events are expected with backgrounds that can be reduced to a magnitude level. With the LHC guidance, the ILC, which is currently being designed, will further improve our knowledge of the Higgs sector if that is how nature decided to create mass [24]. It was demonstrated in Ref.[25] that physics at the LHC and at the ILC will be complementary to each other in many respects. So far, many works have been contributed to studies of the neutral Higgs boson pair production at the ILC, in the SM [26-28] and in new physics beyond the SM [29-32].

From the discussions given in Sec. II, we can see that the neutral Higgs boson pair $\phi^0\phi^0$ cannot be produced exclusively at the ILC because $\phi^0\phi^0$ cannot couple with gauge boson $Z$ or $Z_H$. However, the neutral Higgs boson pair $\phi^0 h$ can be produced via the processes $e^+e^- \rightarrow Z(Z_H) \rightarrow \phi^0 h$ at the ILC. The Feynman diagrams of the process $e^+(p_1)e^-(p_2) \rightarrow \phi^0(p_3)h(p_4)$ are shown in Fig.2.
Figure 2: Feynman diagrams for the process $e^+e^- \rightarrow \phi^0h$.

At the leading order, the production amplitude of the process can be written as

$$M_1 = M_Z + M_{ZH}$$

with

$$M_Z = \frac{e^2x(-1 + 4S_W^2)}{24C_W^2S_W^2} \bar{v}_e(p_2) \frac{\gamma_{i2}u_e(p_1)}{p_{12}^2 - m_Z^2}$$

$$+ \frac{e^2x}{24C_W^2S_W^2} \bar{v}_e(p_2) \frac{\gamma_{i2}u_e(p_1)}{p_{12}^2 - m_Z^2} \gamma_5u_e(p_1),$$

$$M_{ZH} = -\frac{e^2x(14 - 17S_W^2)}{36C_W^2S_W^2} \bar{v}_e(p_2) \frac{\gamma_{5}u_e(p_1)}{p_{12}^2 - m_{ZH}^2}$$

$$+ \frac{e^2x(14 - 17S_W^2)}{72C_W^2S_W^2} \bar{v}_e(p_2) \frac{P_Lu_e(p_1)}{p_{12}^2 - m_{ZH}^2},$$

$$+ \frac{e^2x(4 - S_W^2)}{36C_W^2S_W^2} \bar{v}_e(p_2) \frac{\gamma_{i4}u_e(p_1)}{p_{12}^2 - m_{ZH}^2}$$

$$+ \frac{e^2x(4 - S_W^2)}{72C_W^2S_W^2} \bar{v}_e(p_2) \frac{P_Ru_e(p_1)}{p_{12}^2 - m_{ZH}^2}.$$
parameter \( m_{\phi^0} \). For \( 500 GeV \leq f \leq 1500 GeV \) and \( 120 GeV \leq m_{\phi^0} \leq 180 GeV \), its value is in the range of \( 0.92 fb - 20 fb \). According to an update of parameter for ILC at 2006 \[33\], one can see that, an integrated luminosity of \( 500 fb^{-1} \) should be achieved in the first four years of running after one year of commissioning. Therefore, if we assume the integrated luminosity for the ILC is \( 500 fb^{-1} \), there will be \( 10^2 - 10^4 \phi^0h \) events to be generated at the ILC.

![Production cross section](image)

**Figure 3:** The production cross section \( \sigma \) of \( e^+e^- \to \phi^0h \) as a function of the parameter \( f \) for three values of \( m_{\phi^0} \), \( m_h = 120 GeV \), and the c.m. energy \( \sqrt{s} = 500 GeV \).

From the discussions given in Sec. II, we can see that the possible decay modes of the neutral Higgs boson \( \phi^0 \) are \( bb, c\bar{c}, \tau^+\tau^- \), \( gg \) and \( \gamma\gamma \). The SM-like neutral Higgs boson \( h \) has similar decay features with those of \( \phi^0 \). Therefore, the signatures of neutral Higgs boson pair \( \phi^0h \) is similar to those of the neutral Higgs boson pair \( \phi^0\phi^0 \) at the high energy colliders. From the numerical results given in Sec. II, one can see that, for the masses \( m_{\phi^0} \leq 180 GeV \), the possible signals of \( \phi^0h \) can be seen as four \( b \) quarks,

\[
e^+e^- \to \phi^0h \to b\bar{b}b\bar{b}. \tag{9}
\]

The production rate of the \( b\bar{b}b\bar{b} \) final state in the \( LRT H \) model can be easily estimated

\[1\] Thanks to the referees for offering this reference to us.
using the formula \(\sigma_s = \sigma \times Br(\phi^0 \rightarrow b\bar{b}) \times Br(h \rightarrow b\bar{b})\). If we assume the integrated luminosity \(\mathcal{L}_{int} = 500 fb^{-1}\) for the ILC with the c.m. energy \(\sqrt{s} = 500 GeV\), then there will be \(9 - 3.0 \times 10^3\) \(b\bar{b}b\bar{b}\) events to be generated at the ILC, which is significantly larger than that for the SM Higgs boson pair production process \(e^+e^- \rightarrow hh \rightarrow b\bar{b}b\bar{b}\) [26-28]. Therefore, we hope that by using very efficient \(\mu\)-vertex detectors to tag the \(b\) quark jets, we might detect the possible signatures of the neutral Higgs boson \(\phi^0\) via the process \(e^+e^- \rightarrow \phi^0h\) in the future ILC experiments. Certainly, detailed confirmation of the observability of the signals generated by the process \(e^+e^- \rightarrow Z(Z_H) \rightarrow \phi^0h\) would require Monte-Carlo simulations of the signals and backgrounds, which is beyond the scope of this paper.

**IV. Pair production of neutral Higgs bosons at the LHC**

The LHC has a good potential for discovery of a neutral Higgs boson. Now we look at pair production of the neutral Higgs bosons predicted by the LRT H model at the LHC. From the above discussions, we can see that both the \(\phi^0\phi^0\) pair and \(\phi^0h\) pair can be produced at the LHC. In this section, we will consider both of these cases.

**A. \(\phi^0\phi^0\) pair production**

First, we study production of the neutral Higgs boson pair \(\phi^0\phi^0\) at the LHC. At the LHC, the neutral Higgs boson pair \(\phi^0\phi^0\) can be produced through two mechanisms. One is loop-induced production via gluon fusion \((gg \rightarrow \phi^0\phi^0)\) and the other is from the t-channel quark-antiquark annihilation \((q\bar{q} \rightarrow \phi^0\phi^0)\). The relevant Feynman diagrams are shown in Fig.4. Considering the couplings of the neutral Higgs boson \(\phi^0\) to the SM fermions are proportional to the factor of \(m_q/f\) and the smallness masses of the quarks \(q = u, c, d,\) and \(s\), we have neglected their contributions to production of the neutral Higgs boson pair \(\phi^0\phi^0\).

In this paper, we calculate all production channels for the neutral Higgs boson pair \(\phi^0\phi^0\) at the LHC, as shown in Fig.4, including triangle diagrams, box diagrams and tree-level diagram. Each loop diagram is composed of some scalar loop functions, which are
calculated by using *LoopTools* [34]. The hadronic cross section at the *LHC* is obtained by convoluting the partonic cross sections with the parton distribution functions (PDFs). In our numerical calculation, we will use CTEQ6L PDFs for the gluon and quark PDFs [35]. The renormalization scale \( \mu_R \) and the factorization scale \( \mu_F \) are chosen to be \( \mu_R = \mu_F = 2m_\phi^0 \). Because the calculation of the loop diagrams are too tedious and the analytical expression are lengthy, we will not present those here.

![Feynman diagrams](image)

**Figure 4:** One-loop *Feynman* diagrams for the subprocess \( gg \rightarrow \phi^0\phi^0(a,b) \) and tree-level *Feynman* diagram for the subprocess \( b\bar{b} \rightarrow \phi^0\phi^0(c) \) in the LRT H model. The diagrams obtained by exchanging the two gluons or exchanging the two Higgs bosons are not shown here.

It is obvious that the production cross section \( \sigma \) of the neutral Higgs boson pair \( \phi^0\phi^0 \) at the *LHC* are dependent on the model dependent parameters \( f, m_\phi, \) and \( M \). Similar to the calculation at the *ILC*, we assume that the values of the free parameters \( f \) and \( m_\phi \) are in the ranges of \( 500GeV - 1500GeV \) and \( 100GeV - 180GeV \), respectively. Besides, we assume the mixing parameter \( M \) is in the range of \( 100GeV - 200GeV \). Our numerical results are summarized in *Figs.5 and 6.*

To see contributions of the different partonic processes to the total hadronic cross section, we plot the total and partial hadronic cross sections for different partonic processes as functions of the scale parameter \( f \) for the parameters \( M = 100GeV \) and \( m_\phi = 120GeV \) in *Fig.5.* We see from *Fig.5* that production of the neutral Higgs boson pair \( \phi^0\phi^0 \) is
dominated by the partonic process $gg \rightarrow \phi^0\phi^0$ induced by the top-quark loop diagrams. For $M = 100\text{GeV}$, $m_{\phi^0} = 120\text{GeV}$, and $500\text{GeV} \leq f \leq 150\text{GeV}$, the value of the total production cross section is in the range of $4\text{fb} \sim 122\text{fb}$, and the value of the production cross section coming from the top-quark loop diagrams is in the range of $1.5\text{fb} - 105\text{fb}$. This is because the contributions of the box diagrams are generally much smaller than those of the triangle diagrams, and furthermore the coupling $ht\bar{t}$ is much larger than the coupling $hTT$ or the coupling $\phi^0b\bar{b}$. If we assume the integrated luminosity $\mathcal{L}_{\text{int}} = 100\text{fb}^{-1}$ for the LHC with the c.m. energy $\sqrt{s} = 14\text{TeV}$, then there will be $4 \times 10^2 - 1.22 \times 10^4$ events to be generated at the LHC.

![Figure 5: The total and partial hadronic cross sections for different partonic processes as functions of the free parameter $f$ for the parameters $M = 100\text{GeV}$ and $m_{\phi^0} = 120\text{GeV}$.](image)

In order to see the effects of the mass parameter $m_{\phi^0}$ on the total cross section $\sigma$, we plot $\sigma$ as a function of $m_{\phi^0}$ for $f = 500\text{GeV}$ and three values of the mixing parameter $M$ in Fig. 6. One can see from Fig. 6 that the total cross section $\sigma$ is sensitive to the mass parameter $m_{\phi^0}$, while is not sensitive to the mixing parameter $M$. This is because $M$ is introduced to generate the mass mixing term $Mq_Lq_R$, which is included in the gauge
invariant top *Yukawa* terms allowed by gauge invariance. From the relevant *Feynman* rules we can see that, the mixing parameter $M$ does not influence the production cross section $\sigma$ of the neutral Higgs boson $\phi^0$ too much. For $f = 500$ GeV, $M = 200$ GeV, and $m_{\phi^0} = 100$ GeV − $180$ GeV, the total cross section $\sigma$ is in the range of $16 fb$ − $253 fb$.

![Figure 6: The total production cross section $\sigma$ as a function of free parameter $m_{\phi^0}$ for three values of mixing parameter $M$.](image)

**Figure 6**: The total production cross section $\sigma$ as a function of free parameter $m_{\phi^0}$ for three values of mixing parameter $M$.

**B. $\phi^0h$ pair production**

Now we consider production of the neutral Higgs boson pair $\phi^0h$ at the *LHC*. At the *LHC*, the neutral Higgs boson pair $\phi^0h$ can be mainly produced through two mechanisms: (i) $q\bar{q} \rightarrow \phi^0h$, where $q = u, d, c, s, b$; (ii) the loop-induced gluon fusion process $gg \rightarrow \phi^0h$. The relevant *Feynman* diagrams are shown in *Fig. 7*.

Using the relevant *Feynman* rules, we can write the invariant amplitude for the partonic process $q(p_1)\bar{q}(p_2) \rightarrow \phi^0(p_3)h(p_4)$ as

$$
M_2(q) = M_{21}(q), \quad \text{for } q = u, c
$$

$$
M_2(q) = M_{22}(q), \quad \text{for } q = d, s
$$

$$
M_2(q) = M_{22}(q) + M_{23}(q), \quad \text{for } q = b
$$

(10)
For the s-channel partonic processes $q\bar{q} \rightarrow Z(Z_H) \rightarrow \phi^0 h (q = u, d, c, s, b)$, the invariant amplitude can be written

$$M_{21}(q) = \frac{-e^2 x}{24S_W^2 C_W^2} + \frac{e^2 x}{18C_W^2} \bar{v}(p_2) \frac{\psi_{12}}{p_{12} - m_Z^2} P_L u(p_1)$$

$$+ \frac{e^2 x}{18C_W^2} \bar{v}(p_2) \frac{\psi_{12}}{p_{12} - m_Z^2} P_R u(p_1)$$

$$+ \frac{e^2 x (14 - 17S_W^2)}{108C_W^2 \cos 2\theta_W} \bar{v}(p_2) \frac{\psi_{3}}{p_{12}^2 - m_{Z_H}^2} P_L u(p_1)$$

$$+ \frac{-e^2 x (4 - S_W^2)}{108C_W^2 \cos 2\theta_W} \bar{v}(p_2) \frac{\psi_{3}}{p_{12}^2 - m_{Z_H}^2} P_R u(p_1)$$

$$+ \frac{e^2 x (1 - 3S_W^2)(14 - 17S_W^2)}{216S_W^2 C_W^2 \cos 2\theta_W} \bar{v}(p_2) \frac{\psi_{4}}{p_{12}^2 - m_{Z_H}^2} P_L u(p_1)$$

$$+ \frac{-e^2 x (4 - S_W^2)(1 - 3S_W^2)}{216S_W^2 C_W^2 \cos 2\theta_W} \bar{v}(p_2) \frac{\psi_{4}}{p_{12}^2 - m_{Z_H}^2} P_R u(p_1).$$

For the s-channel partonic processes $q\bar{q} \rightarrow Z(Z_H) \rightarrow \phi^0 h (q = d, s$ and $b)$, the invariant amplitude can be written

$$M_{22}(q) = \left( \frac{e^2 x}{24S_W^2 C_W^2} - \frac{e^2 x}{18C_W^2} \right) \bar{v}(p_2) \frac{\psi_{12}}{p_{12}^2 - m_Z^2} P_L u(p_1)$$

15
\[ + \frac{e^2 x}{36 C_w^2} \tilde{v}(p_2) \frac{\bar{q}_{12}}{p_{12}^2 - m_2^2} \mu_R u(p_1) \]
\[ + \frac{e^2 x (14 - 17 S_W^2)}{108 C_w^2 \cos 2 \theta_W} \tilde{v}(p_2) \frac{\bar{q}_3}{p_{12}^2 - m_2^2} \mu_L u(p_1) \]
\[ + \frac{e^2 x (4 - S_W^2)(3 - 5 S_W^2)}{108 S_W^2 C_w^2 \cos 2 \theta_W} \tilde{v}(p_2) \frac{\bar{q}_4}{p_{12}^2 - m_2^2} \mu_R u(p_1) \]
\[ - \frac{e^2 x (4 - 3 S_W^2)}{108 C_w^2 \cos 2 \theta_W} \tilde{v}(p_2) \frac{\bar{q}_4}{p_{12}^2 - m_2^2} \mu_L u(p_1) \]
\[ + \frac{e^2 x (4 - S_W^2)(3 - 5 S_W^2)}{108 S_W^2 C_w^2 \cos 2 \theta_W} \tilde{v}(p_2) \frac{\bar{q}_4}{p_{12}^2 - m_2^2} \mu_R u(p_1). \]

For the t-channel partonic process \( b \bar{b} \rightarrow \phi^0 h \) as shown in Fig.7b, the invariant amplitude can be written

\[ M_{23}(q) = m_\phi^2 \tilde{v}(p_2) \frac{\bar{q}_{13} + m_b}{p_{12}^2 - m_\phi^2} \gamma_5 u(p_1). \]

Where \( p_{13} = p_1 - p_3 \). Considering the couplings of the neutral Higgs boson \( \phi^0 \) to the SM fermions are proportional to the factor of \( m_q/f \) and the smallness masses of the quark \( q = u, c, d, s \), we have neglected their contributions to production cross section of the neutral Higgs boson pair \( \phi^0 h \) via the t-channel process in our calculations. When we calculate the loop diagrams Figs.7(c)-7(d), and Fig.7e, we will use the same method with Figs.4(a) and 4(b).

To see contributions of the different partonic processes to the total hadronic cross section, we plot the total and partial hadronic cross sections for different partonic processes as functions of the parameter \( f \) for \( m_{\phi} = m_h = 120 \text{GeV} \) and \( M = 150 \text{GeV} \) in Fig.8. We see that the production cross sections of the neutral Higgs bosons \( \phi^0 h \) mainly come from the contributions of the light quarks \( (u, d, c, s) \) through the s-channel \( Z \) exchange and \( Z_H \) exchange. Our numerical results show that, the contributions coming from the partonic processes \( gg \rightarrow \phi^0 h \) [including Figs.7(e)-7(e)] to total production cross section are at the orders of \( 10^{-5} \text{fb} - 10^{-1} \text{fb} \), which are much smaller than those of the tree-level processes. This is because the Yukawa couplings depend sensitively on the free parameters \( M \) and \( f \). The parameter \( M \) is very smaller than the scale parameter \( f \). So, although the gluon fusion get an enhancement due to large parton distribution functions, the contribution
of the gluon fusion process is suppressed by the order of \((M/f)^4\) [21]. Thus, in Fig.8, we did not show the line corresponding to the value of the production cross section contributed by the \(gg\) fusion. The value of the production cross section of the neutral Higgs bosons \(\phi^0 h\) is insensitive to the mixing parameter \(M\). For \(m_{\phi^0} = m_h = 120\text{GeV}\) and \(500\text{GeV} \leq f \leq 150\text{GeV}\), its value is in the range of \(34\text{fb} - 306\text{fb}\), the partial value of the total production cross section coming from light quarks contributions is in the range of \(31\text{fb} - 281\text{fb}\). If we assume the integrated luminosity \(\mathcal{L}_{int} = 100\text{fb}^{-1}\) for the LHC with the c.m. energy \(\sqrt{s} = 14\text{TeV}\), then there will be \(3.4 \times 10^3 - 3.1 \times 10^4 \phi^0 h\) events generated at the LHC.

![Figure 8: The total and partial hadronic cross sections for different partonic processes as function of the parameter \(f\) for \(m_{\phi^0} = m_h = 120\text{GeV}\) and \(M = 150\text{GeV}\).](image)

Similar to those of the discussions for neutral Higgs boson pair \(\phi^0\phi^0\) production, we plot \(\sigma\) as a function of free parameter \(f\) for \(m_h = 120\text{GeV}\), \(M = 150\text{GeV}\) and three values of \(m_{\phi^0}\) in Fig.9. One can see from Fig.9 that the total cross section \(\sigma\) is sensitive to mass parameter \(m_{\phi^0}\). For \(f = 500\text{GeV}\) and \(120\text{GeV} \leq m_{\phi^0} \leq 180\text{GeV}\), its value is in the range of \(101\text{fb} - 306\text{fb}\).

From the above discussions, we can see that the decay features of \(\phi^0\) are similar to
those of the $SM$-like neutral Higgs boson $h$, as far as decays into $b\bar{b}$ and $\gamma\gamma$ are concerned. Therefore, when we analyze the signatures of the neutral Higgs boson pairs from the $LRT H$ model at the colliders, we will take the $\phi^0\phi^0$ pair, for example.

![Figure 9](image_url)

Figure 9: The total production cross section as a function of free parameter $f$ for $m_h = 120GeV$, $M = 150GeV$ and three values of $m_{\phi^0}$.

In most of the parameter space of the $LRT H$ model, the main decay modes of $\phi^0$ are $gg$ and $b\bar{b}$. However, the final states $gggg$ and $b\bar{b}b\bar{b}$ induced by pair production of the neutral Higgs boson $\phi^0$ at the $LHC$ have large $QCD$ backgrounds and thus are insignificant for $\phi^0$ discovery. If we assume that one of the neutral Higgs boson $\phi^0$ decays to $b\bar{b}$ and the other decays to $\gamma\gamma$, then pair production of the neutral Higgs boson $\phi^0$ at the $LHC$ can give rise to the $b\bar{b}\gamma\gamma$ final state, and the production rate of the $b\bar{b}\gamma\gamma$ final state can be easily estimated using the formula $\sigma_s = \sigma \times Br(\phi^0 \rightarrow b\bar{b}) \times Br(\phi^0 \rightarrow \gamma\gamma)$. If we assume the integrated luminosity $L_{int} = 100fb^{-1}$ for the $LHC$ with the c.m. energy $\sqrt{s} = 14TeV$, then there will be several hundreds of $b\bar{b}\gamma\gamma$ events to be generated at the $LHC$. Furthermore, the narrow $\gamma\gamma$ peak can be reconstructed to distinguish the signal from the backgrounds. Detailed analysis of the signals and the relevant backgrounds about this kind of the final state has been given in Ref.[36].
V. Conclusions

The twin Higgs mechanism provides an alternative method to solve the little hierarchy problem. The $LRT_H$ model is a concrete realization of the twin Higgs mechanism. In this paper, we discuss the possible decay modes of the neutral Higgs boson $\phi^0$ predicted by the $LRT_H$ model and consider its pair production at the $ILC$ and $LHC$ via suitable mechanisms.

At the $ILC$, we study production of the neutral Higgs boson pair $\phi^0h$ via the processes $e^+e^- \rightarrow Z(Z_H) \rightarrow \phi^0h$. Our numerical results show that, for $m_{\phi^0} = m_h = 120\text{GeV}$ and $500\text{GeV} \leq f \leq 1500\text{GeV}$, the total production cross section of neutral Higgs boson pair $\phi^0h$ at $ILC$ is in the range of $0.92\text{fb} - 20\text{fb}$. If we assume the integrated luminosity $\mathcal{L}_{\text{int}} = 500\text{fb}^{-1}$ for the $ILC$ with the c.m. energy $\sqrt{s} = 500\text{GeV}$, there will be $10^2 - 10^4 \phi^0h$ events to be generated at the $ILC$. If we assume that the neutral Higgs bosons $\phi^0$ and $h$ both decay to $b\bar{b}$, then the process $e^+e^- \rightarrow \phi^0h$ can give rise to the $b\bar{b}bb$ final state. There will be $9 - 3.0 \times 10^3 \times 10^4 \times 10^3 \phi^0h$ events to be generated at the $ILC$. Owing to the $b\bar{b}bb$ events, we might detect the possible signatures of the neutral Higgs boson $\phi^0$ via the processes $e^+e^- \rightarrow Z(Z_H) \rightarrow \phi^0h$ in the future $ILC$ experiments.

At the $LHC$, we study production of the neutral Higgs boson pairs $\phi^0\phi^0$ and $\phi^0h$. First, we study production of the neutral Higgs boson pair $\phi^0\phi^0$ via the processes $gg \rightarrow \phi^0\phi^0$ and $q\bar{q} \rightarrow \phi^0\phi^0$. Our numerical results show that, for $M = 100\text{GeV}$, $m_{\phi^0} = 120\text{GeV}$ and $500\text{GeV} \leq f \leq 1500\text{GeV}$, the value of the hadronic cross section $\sigma_{\phi^0\phi^0}$ is in the range of $4\text{fb} - 122\text{fb}$, which mainly comes from the contributions of the top-quark loop. Then we study production of the neutral Higgs boson pair $\phi^0h$ via the processes $q\bar{q} \rightarrow \phi^0h (q = u, c, d, s, b)$ and $gg \rightarrow \phi^0h$. Our numerical results show that, for $M = 150\text{GeV}$, $m_{\phi^0} = m_h = 120\text{GeV}$ and $500\text{GeV} \leq f \leq 1500\text{GeV}$, the value of $\sigma_{\phi^0h}$ is in the range of $34\text{fb} - 306\text{fb}$, of which about 91% of the contributions comes from light quarks $u, d, c, s$. If we assume the integrated luminosity $\mathcal{L}_{\text{int}} = 100\text{fb}^{-1}$ for the $LHC$ with the c.m. energy $\sqrt{s} = 14\text{TeV}$, then there will be $3.4 \times 10^3 - 3.1 \times 10^4 \phi^0h$ events to be generated at the $LHC$. If we assume that one of the neutral Higgs bosons $\phi^0$ and $h$ decays to $b\bar{b}$ and the
other decays to $\gamma\gamma$, then the processes $pp \rightarrow \phi^0 \phi^0 + X$ and $pp \rightarrow \phi^0 h + X$ all can give rise to the $b\bar{b}\gamma\gamma$ final state. There will be several hundreds and up to thousands of $b\bar{b}\gamma\gamma$ events to be generated at the LHC with the c.m. energy $\sqrt{s} = 14TeV$ and $\mathcal{L}_{int} = 100fb^{-1}$.

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References

[1] For example see: N. E. Adam et al., arXiv: 0803.1154[hep-ph].

[2] U. Baur, T. Plehn, and D. L. Rainwater, Phys. Rev. Lett. 89, 151801(2002).

[3] M. Moretti, S. Moretti, F. Piccinini, R. Pittau, A. D. Polosa, JHEP 0502, 024(2005); T. Binoth, S. Karg, N. Kauer, R. Ruckl, Phys. Rev. D74, 113008(2006).

[4] A. Pierce, J. Thaler, Lian-Tao Wang, JHEP 0705, 070(2007); S. Kanemura, K. Tsumura, arXiv: 0810.0433[hep-ph].

[5] E. W. N. Glover and J. J. van der Bij, Nucl. Phys. B309, 282(1988).

[6] U. Baur, T. Plehn, David L. Rainwater, Phys. Rev. D67, 033003(2003).

[7] S. Dawson, S. Dittmaier, M. Spira, Phys. Rev. D58, 115012(1998).

[8] S. Dawson, C. Kao, Yili Wang, P. Williams, Phys. Rev. D75, 013007(2007).
[9] J. J. Liu, W. G. Ma, G. Li, R. Y. Zhang and H. -S. Hou, *Phys. Rev. D* **70**, 015001(2004); C. O. Dib, R. Rosenfeld and A. Zerwekh, *JHEP* **0605**, 074(2006); L. Wang, W. Y. Wang, J. M. Yang, H. J. Zhang, *Phys. Rev. D* **76**, 017702(2007).

[10] P. K. Das and B. Mukhopadhyaya, *hep-ph/0303135*.

[11] M. Spira and J. D. Wells, *Nucl. Phys. B* **523**, 3(1998).

[12] A. A. Barrientos Bendezu, Bernd A. Kniehl, *Phys. Rev. D* **64**, 035006(2001).

[13] T. Plehn, M. Spira and P. M. Zerwas, *Nucl. Phys. B* **479**, 46(1996); A. Djouadi, W. Kilian, M. Muhlleitner and P. M. Zerwas, *Eur. Phys. J. C* **10**, 45(1999); A. Belyaev, Manuel Drees, Oscar J. P. Eboli, J. K. Mizukoshi, S. F. Novaes, *Phys. Rev. D* **60**, 075008(1999); A. Belyaev, M. Drees and J. K. Mizukoshi, *Eur. Phys. J. C* **17**, 337(2000); R. Lafaye, D. J. Miller, M. Muhlleitner and S. Moretti, *hep-ph/0002238*; M. Moretti, S. Moretti, F. Piccinini, R. Pittau, *JHEP* **0502**, 024(2005).

[14] H. de Sandes, R. Rosenfeld, *Phys. Lett. B* **659**, 323(2008).

[15] R. Barbieri and A. Strumia, *Phys. Lett. B* **462**, 144(1999); A. Falkowski, S. Pokorski, M. Schmaltz, *Phys. Rev. D* **74**, 035003(2006); Z. Chacko, H. -S. Goh, R. Harnik, *Phys. Rev. Lett.* **96**, 231802(2006).

[16] Z. Chacko, H. -S. Goh and R. Harnik, *JHEP* **0601**, 108(2006).

[17] H. -S. Goh and S. Su, *Phys. Rev. D* **75**, 075010(2007).

[18] A. Abada, I. Hidalgo, *Phys. Rev. D* **77**, 113013(2008).

[19] D. -W. Jung and J. Y. Lee, *arXiv: 0710.2589*[hep-ph].

[20] E. M. Dolle, S. F. Su, *Phys. Rev. D* **77**, 075013(2008).

[21] Y. B. Liu , H. M. Han , X. L. Wang, *Eur. Phys. J. C* **53**, 615(2008).
[22] J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, "The Higgs Hunter’s Guide", Addison-Wesley, Reading, MA(1990); L. Reina, hep-ph/0512377.

[23] W. -M. Yao et al. [Particle Data Group], J. Phys. G33, 1(2006) and partial update for the 2008 edition.

[24] P. W. Higgs, Phys. Rev. Lett. 13, 508(1964); G. S. Guralnik, C. R. Hagen and T. W. B. Kibble, Phys. Rev. Lett. 13, 585(1964); F. Englert, R. Brout, Phys. Rev. Lett. 13, 321(1964).

[25] G. Weiglein et al. [ILC/LC Study Group], Phys. Rept. 426, 47(2006); A. Arhrib, R. Benbrik, C. -H. Chen, Rui Santos, arXiv: 0901.3380[hep-ph].

[26] J. J. Lopez-Villarejo, J. A. M. Vermassen, arXiv: 0812.3750[hep-ph].

[27] A. Djouadi, V. Driesen, C. Junger, Phys. Rev. D54, 759(1996).

[28] A. Gutierrez-Rodriguez, M. A. Hernandez-Ruiz, O. A. Sampayo, Phys. Rev. D67, 074018(2003).

[29] H. Grosse, Yi Liao, Phys. Rev. D64, 115007(2001).

[30] J. L. Feng, T. Moroi, Phys. Rev. D56, 5962(1997).

[31] A. Djouadi, H. E. Haber, P. M. Zerwas, Phys. Lett. B375, 203(1996).

[32] R. N. Hodgkinson, D. Lopez-Val, Joan Sola, Phys. Lett. B673, 47(2009); A. Arhrib, R. Benbrik, C. W. Chiang, Phys. Rev. D77, 115013(2008).

[33] http://www.linearcollider.org/newsline/pdfs/20061207_LC_Parameters_Novfinal.pdf

[34] T. Hahn, M. Perez-Victoria, Computl. Phys. Commun. 118, 153(1999); T. Hahn, Nucl. Phys. Proc. Suppl. 135, 333(2004).

[35] J. Pumplin et al. (CTEQ Collaboration), JHEP 0602, 032(2006).

[36] U. Baur, T. Plehn, David L. Rainwater, Phys. Rev. D69, 053004(2004).