Random matrix theory filters and currency portfolio optimisation

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Abstract. Random matrix theory (RMT) filters have recently been shown to improve the optimisation of financial portfolios. This paper studies the effect of three RMT filters on realised portfolio risk, using bootstrap analysis and out-of-sample testing. We considered the case of a foreign exchange and commodity portfolio, weighted towards foreign exchange, and consisting of 39 assets. This was intended to test the limits of RMT filtering, which is more obviously applicable to portfolios with larger numbers of assets. We considered both equally and exponentially weighted covariance matrices, and observed that, despite the small number of assets involved, RMT filters reduced risk in a way that was consistent with a much larger S&P 500 portfolio. The exponential weightings indicated showed good consistency with the value suggested by Riskmetrics, in contrast to previous results involving stocks. This decay factor, along with the low number of past moves preferred in the filtered, equally weighted case, displayed a trend towards models which were reactive to recent market changes. On testing portfolios with fewer assets, RMT filtering provided less or no overall risk reduction. In particular, no long term out-of-sample risk reduction was observed for a portfolio consisting of 15 major currencies and commodities.

1. Introduction

Markowitz portfolio theory [1], an intrinsic part of modern financial analysis, relies on the covariance matrix of returns and this can be difficult to estimate. For example, for a time series of length $T$, a portfolio of $N$ assets requires $(N^2 + N)/2$ covariances to be estimated from $NT$ returns. This results in estimation noise, since the availability of historical information is limited. Moreover, it is commonly accepted that financial covariances are not fixed over time (e.g. [2–4]) and thus older historical data, even if available, can lead to cumulative noise effects.

Random matrix theory (RMT), first developed by authors such as Dyson and Mehta [5–8], to explain the energy levels of complex nuclei [9], has recently been applied (by several authors including Plerou et al. [9–13] and Laloux et al. [14, 15]) to noise filtering in financial time series, particularly in large dimensional systems such as stock markets. Both groups have analysed US stock markets and have found that the eigenvalues, of the correlation matrix of returns, were consistent with those calculated using random returns, with the exception of a few large eigenvalues. Moreover, their findings indicated that these large eigenvalues, which did not conform to random returns, had eigenvectors that were more stable over time $^1$. Of

$^1$ and thus forecasts made here with matrices which have been filtered (i.e. by keeping the stable eigenvalues and smoothing the unstable ones) are expected to show greater reliability over time.
particular interest was the demonstration [9, 15], that filtering techniques, based on RMT, could be beneficial in portfolio optimisation, both reducing the realised risk of optimised portfolios, and improving the forecast of this realised risk.

More recently, Pafka et al. [16] extended RMT to provide Riskmetrics type [4] covariance forecasts. Riskmetrics, dating from the 1990’s and considered a benchmark in Risk management [16], uses an exponential weighting scheme to model the heteroskedasticity of financial returns. Pafka et al. [16] showed that RMT-based eigenvalue filters can improve the optimisation of minimum risk portfolios, generated using exponentially weighted forecasts. However, these authors found that the decay factors, which produced the least risky portfolios, were higher than the range suggested by Riskmetrics and further concluded that unfiltered Riskmetrics-recommended forecasts were unsuitable for their portfolio optimisation problem.

A recent paper by Sharifi et al. [17], using equally weighted, high frequency returns for estimating covariances, proposed an alternative eigenvalue-filtering method, based on a principal components technique developed by Krzanowski [18] for measuring the stability of eigenvectors, in relation to small perturbations in the corresponding eigenvalues. Sharifi et al. [17] concluded that filtering correlation matrices according to the method outlined in Laloux et al. [15] had a negative effect on this stability. Following this, Daly et al. [19], extended the filtering method of Sharifi et al. [17], and showed that the extended filter offered improvements in terms of risk and stability compared to other RMT filters tested. Indeed, all three RMT filters examined were found, overall, to reduce mean realised risk in all out-of-sample tests. However, in some individual years this was not the case, while on specific days, and for all types of filters, RMT filtering was found to be capable of increasing realised risk, substantially in some cases. In agreement with Pafka et al. [16], the optimal out-of-sample decay factors, for both filtered and unfiltered forecasts, were found to be higher in all cases than those suggested by Riskmetrics [4].

While RMT has been extensively applied to large portfolios, such as the S&P 500, some authors such as Conlon et al. [20], have examined its application to smaller portfolios. Conlon et al. [20] studied a portfolio consisting of 49 hedge funds, with limited historical data, and found that the eigenvalues deviating from RMT contained distinct strategy clustering, while RMT filtering of the correlation matrix resulted in a 35% improvement between the risk of the predicted and realised portfolios.

Our objective in this paper is to evaluate the three RMT filters of Laloux et al. [15], Plerou et al. [9], and Daly et al. [19], applied to both equally and exponentially weighted covariance forecasts, of a trading portfolio consisting of 39 currency pairs and commodities. The effect of filtering on the realised risk of minimum risk portfolios, constructed from these assets, is examined. The filters are studied both in-sample using bootstrapping, and out-of-sample using forward validation, following Daly et al. [19], as described in Sections 3 and 4. Finally, the impact of a further reduction in asset numbers is also assessed.

The paper is organised as follows. Section 2 reviews the theoretical background for the three RMT filters. Section 3 contains an in-sample analysis of the filters, including an analysis of reducing the number of assets from 432, as studied in Daly et al. [19], to 39, focusing on S&P data. Section 4 contains the results, of the out-of-sample tests, on effectiveness of the filters in reducing realised risk. In Section 5, the impact of further asset number reduction is studied, and Section 6 contains the concluding remarks.

2. Background

2.1. Random Matrix Theory and Historical Covariance

As described by Laloux et al. [14], Plerou et al. [9], Sharifi et al. [17] and others, in the context of correlation matrices of financial returns, if $R$ is any matrix defined by

$$ R = \frac{1}{T} A A' $$

(1)
where $A$ is an $N \times T$ matrix whose elements are i.i.d. random variables with zero mean, then it has been shown [21] that, in the limit $N \to \infty, T \to \infty$ such that $Q = T/N \geq 1$ is fixed, the distribution $P(\lambda)$ of the eigenvalues of $R$ is self-averaging, and is given by

$$P(\lambda) = \begin{cases} \frac{Q}{2\pi\sigma^2} \sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)} \quad & \text{if } \lambda_- \leq \lambda \leq \lambda_+ \\ 0 & \text{otherwise} \end{cases}$$

(2)

where $\sigma^2$ is the variance of the elements of $A$ and

$$\lambda_\pm = \sigma^2 \left(1 + 1/Q \pm 2\sqrt{1/Q}\right).$$

(3)

Financial correlation and covariance matrices can be expressed, in general, in the form given by Equation (1), so matrices for historical data can be compared to those generated from random returns. Here we define the covariance matrix $\mathbf{V} = \{\sigma_{ij}\}_{i,j=1}^N$ of returns $^3$ by

$$\sigma_{ij} = \langle G_i(t)G_j(t) \rangle - \langle G_i(t) \rangle \langle G_j(t) \rangle$$

(4)

where $\langle \cdot \rangle$ refers to the mean over time, and the correlation matrix $\mathbf{C} = \{\rho_{ij}\}_{i,j=1}^N$ is given by

$$\rho_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}\sigma_{jj}}}$$

(5)

where $\{G_i(t)\}_{t=1,...,T}^N$ are the returns

$$G_i(t) = \ln(S_i(t)/S_i(t-1))$$

(6)

and where $S_i(t)$ is the spot price of asset $i$ at time $t$.

2.2. Random Matrix Theory and Exponentially Weighted Covariance

In extending RMT filtering to exponentially weighted matrices, Pafka et al. [16] have analysed matrices of the form $\mathbf{M} = \{m_{ij}\}_{i,j=1}^N$ with

$$m_{ij} = \sum_{k=0}^{\infty} (1 - \alpha)\alpha^k x_{ik}x_{jk}$$

(7)

and where $\{x_{ik}\}_{k=0,...,\infty}^N$ are assumed to be $\text{N.I.D.} (0, \sigma^2)$. $^4$ They have shown that, in the special case $N \to \infty, \alpha \to 1$ with $Q \equiv 1/(N(1-\alpha))$ fixed, the density, $\rho(\lambda)$, of the eigenvalues of $\mathbf{M}$ is given by $\rho(\lambda) = Qv/\pi$ where $v$ is the root of

$$F(v) = \frac{\lambda}{\sigma^2} - \frac{v\lambda}{\tan(v\lambda)} + \ln(v\sigma^2) - \ln(\sin(v\lambda)) - \frac{1}{Q}$$

(8)

$F(v)$ is well defined on the open interval $(0, \pi/\lambda)$. If a root does not exist on this interval for a given value of $\lambda$ we define $\rho(\lambda) = 0$ for that $\lambda$. The family of matrices, defined by Equation (7), includes the Riskmetrics [4] covariance and correlation matrices. Following this, we define the exponentially weighted covariance matrix $\mathbf{V}^* = \{\sigma_{ij}^*\}_{i,j=1}^N$ by

$^2$ i.i.d. $\equiv$ independent and identically distributed

$^3$ throughout this paper the following notation is used: $\{x_i\}_{i=1}^N = \{x_i : i = 1, \ldots, N\}, \{x_{ij}\}_{i,j=1}^N = \{x_{ij} : i = 1, \ldots, N; j = 1, \ldots, N\}, \{x_{ij}\}_{i=1,...,T}^N = \{x_{it} : i = 1, \ldots, N; t = 1, \ldots, T\}$ etc.

$^4$ $\text{N.I.D.} (\mu, \sigma^2) \equiv$ Normally and identically distributed (with mean $\mu$ and variance $\sigma^2$)
\[
\sigma^*_ij = \frac{1 - \alpha}{1 - \alpha T} \sum_{t=0}^{T-1} \alpha^t (G_i(T-t) - \langle G_i \rangle)(G_j(T-t) - \langle G_j \rangle) \tag{9}
\]

and define the corresponding, exponentially weighted, correlation matrix \( C^* = \{ \rho^*_ij \}_{i,j=1}^N \) by

\[
\rho^*_ij = \frac{\sigma^*_ij}{\sqrt{\sigma^*ii \sigma^*jj}} \tag{10}
\]

Here, \( \alpha \) is commonly called the decay factor.

The maximum eigenvalue of an exponentially weighted random matrix can be found using Equation (8). Equivalently [19], the maximum eigenvalue is the solution of

\[
\frac{\lambda}{\sigma^2} - \ln \left( \frac{\lambda}{\sigma^2} \right) = 1 + \frac{1}{Q}, \quad \lambda > \sigma^2 \tag{11}
\]

2.3. Krzanowski Stability

One of the filtering methods discussed, Sharifi et al. [17], and considered also here, is based on the stability, as described by Krzanowski [18], of the filtered matrix. Krzanowski [18] measured eigenvector stability, specifically the effect on each eigenvector of a perturbation in the corresponding eigenvalue. This is in contrast to stability over time, as analysed by many other authors, e.g. [9, 15]. Krzanowski [18] considered the angle, \( \theta_i \), between an eigenvector \( v_i \) and \( v^p_i \), where \( v^p_i \) is the maximum perturbation that can be applied to \( v_i \) while ensuring that the eigenvalue, \( \lambda^p_i \), corresponding to \( v^p_i \) is within \( \epsilon \) of the eigenvalue, \( \lambda_i \), corresponding to \( v_i \), and showed that \( \theta_i \) is given by:

\[
\cos \theta_i = \begin{cases} 
\left( 1 + \frac{\epsilon}{\lambda_i - \lambda_{i-1}} \right)^{-\frac{1}{2}} & \text{for } \lambda^p_i < \lambda_i \\
\left( 1 + \frac{\epsilon}{\lambda_{i+1} - \lambda_i} \right)^{-\frac{1}{2}} & \text{for } \lambda_i < \lambda^p_i 
\end{cases} \tag{12}
\]

where \( \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_L \) are the eigenvalues.

2.4. Filtering Methods

All three filtering methods compared here are based on replacing the “noisy” eigenvalues of the correlation matrix, while maintaining its trace. The noisy eigenvalues are taken to be those that are less than, or equal to, the maximum possible eigenvalue of the corresponding random matrix (as defined in Sections 2.1 and 2.2). The correlation matrix being filtered is first deconstructed via the eigen decomposition theorem. The noisy eigenvalues are subsequently replaced using one of the three methods outlined here, and the matrix is rebuilt, again referring to the eigen decomposition theorem, resulting in the filtered matrix, \( M_{\text{filtered}} \).

The first filtering method examined is that of Laloux et al. [15] (referred to here as LCPB), which replaces the noisy eigenvalues with their mean, thus maintaining the trace. The second filtering method is that described by Plerou et al. [9] (and referred to here as PG+). This method replaces the noisy eigenvalues by zeroes and, after \( M_{\text{filtered}} \) is built, replaces its main diagonal with that of the original matrix \( M \), again preserving the trace. The third filtering method, Daly et al. [19], is adapted from that of Sharifi et al. [17]. To maximise the Krzanowski stability of the filtered matrix, while also maintaining its trace, the method of Sharifi et al. [17] replaces

\[ 5 \text{ Let } M \text{ be a square matrix and let } E \text{ be the matrix of its eigenvectors. If } E \text{ is a square matrix then } M = E D E^{-1} \text{ where } D \text{ is a diagonal matrix containing the corresponding eigenvalues on the main diagonal. [22]} \]
the noisy eigenvalues with ones that are equally and maximally spaced, are positive, and have sum equal to the sum of those replaced. To achieve maximal spacing, it is assumed that the smallest replacement eigenvalue should be very close to zero. The method described in Daly et al. [19], referred to here as the KR method, adapts this by making the smallest replacement eigenvalue a parameter of the filter. This parameter can then be adjusted to maximise the forecasting potential of the filter. It was found, in Daly et al. [19], that the optimal parameter value for reducing realised portfolio risk involved some reduction in stability. However, the KR method was found to improve stability, compared to both the LCPB and PG+ methods. The non-limiting case KR methods considered here, and their defining minimum replacement eigenvalues, were: KR2 (1/2Λ noisy), KR4 (1/4Λ noisy), KR8 (1/8Λ noisy), KR16 (1/16Λ noisy), KR64 (1/64Λ noisy), KR100 (1/100Λ noisy) and KR1000 (1/1000Λ noisy), where Λ noisy is the mean of the noisy eigenvalues. The limiting minimum replacement eigenvalues, Λ noisy and 10^{-8}, were also considered.

2.5. Data

In order to examine the effects on RMT filtering of reducing the number of assets, from that of the S&P portfolio studied in Daly et al. [19], to that of the currency and commodity portfolio analysed here, daily closing price data for the S&P 500 index stocks were used. The index composition was taken as of 1st February 2006. The dataset runs from 1st June 1995 to 1st February 2006, and any series not covering the entire period were discarded, leaving a total of 432 stocks. To examine the application of RMT filters to the currency and commodity portfolio, and for all the remaining analysis, daily currency spot prices vs. the US Dollar (USD) were used, along with equivalent rates for Silver, Gold, Platinum and Oil. This data, provided by Pacific Exchange Rate Service, covered the period from 4th January 1999 to 31st December 2007. The currencies and commodities selected for this analysis are outlined in Table 1. This group of assets is referred to as the “Fx portfolio” throughout this paper.

3. In-Sample Analysis

For the in-sample analysis, and following [16], bootstrapped samples were taken, together with the mean across these samples. For a given value of N (the number of assets), we randomly selected N assets from the data set, and a random test date. Everything up to and including the test date was taken as historical information and everything afterwards as realised, future information. For each N, we repeated this random selection 1000 times, with replacement, and calculated the mean, across all bootstrapped samples, of the realised risk of the forecast minimum risk portfolio [16] (calculated using our forecast covariance).

A covariance forecast in this context consisted of a raw forecast, which was either exponentially or equally weighted, and could be unfiltered, or filtered by one of the LCPB, PG+ or KR methods applied to the correlation matrix. Filtering the covariance matrix directly, along the lines of Daly et al. [19], was found to be unsuitable when applied to the Fx portfolio, increasing out-of-sample mean realised risk, in most cases, compared to the correlation filter.

On each test date, we calculated the forecast minimum risk portfolio, optimised as follows [16]. Choose a portfolio weighting \( \{w_i\}_{i=1}^N \) that minimises

\[
\sum_{i,j=1}^{N} w_i w_j \tilde{\sigma}_{ij}
\]

6 from www.standardandpoors.com
7 http://fx.sauder.ubc.ca/data.html
Table 1. List of currencies and commodities used. The base currency was the U.S. Dollar.

| Name            | Code | Name            | Code |
|-----------------|------|-----------------|------|
| Australian Dollar | AUD  | Peruvian Nuevo Sol  | PEN  |
| Brazilian Real  | BRL  | Philippine Peso  | PHP  |
| Canadian Dollar  | CAD  | Platinum Ounce   | XPT  |
| Chilean Peso     | CLP  | Polish Zloty     | PLN  |
| Colombian Peso   | COP  | Romanian New Leu | RON  |
| Czech Koruna     | CZK  | Russian Rouble   | RUB  |
| Euro            | EUR  | Silver Ounce     | XAG  |
| Fijian Dollar    | FJD  | Singapore Dollar | SGD  |
| Gold Ounce       | XAU  | Slovak Koruna    | SKK  |
| Hungarian Forint | HUF  | South African Rand | ZAR |
| Icelandic Krona   | ISK  | South Korean Won | KRW  |
| Indian Rupee     | INR  | Sri Lankan Rupee | LKR  |
| Indonesian Rupiah | IDR  | Pound Sterling   | GBP  |
| Israeli New Sheqel | ILS | Swedish Krona    | SEK  |
| Japanese Yen     | JPY  | Swiss Franc      | CHF  |
| Mexican Peso     | MXN  | Thai Baht        | THB  |
| Moroccan Dirham  | MAD  | Thaiwanese Dollar | TWD |
| New Zealand Dollar | NZD  | Tunisian Dinar  | TND  |
| Norwegian Krone  | NOK  | Turkish New Lira | TRY  |
| Oil, Brent Crude, Barrel | XCB |

while satisfying the budget constraint

$$\sum_{i=1}^{N} w_i = 1$$  \hspace{1cm} (14)

where $\hat{\mathbf{V}} = \{\hat{\sigma}_{ij}\}_{i,j=1}^{N}$ is the forecast covariance matrix. The solution, $\{\hat{w}_i\}_{i=1}^{N}$, of this problem is:

$$\hat{w}_i = \frac{\hat{\sigma}_{i1}^{-1}}{\sum_{j=1}^{N} \hat{\sigma}_{ij}^{-1}} \quad \forall i$$  \hspace{1cm} (15)

where $\hat{\mathbf{V}}^{-1} = \{\hat{\sigma}_{ij}^{-1}\}_{i,j=1}^{N}$ is the matrix inverse of $\hat{\mathbf{V}}$. The realised risk of the optimal portfolio is defined by

$$\sqrt{\sum_{i,j=1}^{N} \hat{w}_i \hat{w}_j \tilde{\sigma}_{ij}}$$  \hspace{1cm} (16)

Here, $\tilde{\mathbf{V}} = \{\tilde{\sigma}_{ij}\}_{i,j=1}^{N}$ is the realised covariance matrix, and is the (equally weighted) covariance matrix of the realised future returns over the investment period. We used a forecasting period of 20 days throughout. The forecast risk is calculated analogously, using the forecast covariance matrix, $\hat{\mathbf{V}}$. 
By comparing the covariance forecasts in this way, we measured their effect on realised risk without using forecast returns, which would have introduced additional noise into the results. Further, we have not used any knowledge of future returns in our tests, since we wished to evaluate both forecasting methods (equal vs. exponential weighting) as well as filtering methods. This is in contrast to some previous studies (e.g. [15]), that have isolated the effect of the filtering method on the correlation matrix, by using future knowledge of realised returns to estimate the variance of each individual asset.

3.1. Reduction of S&P Assets
We first analysed the effect on risk of reducing the number of assets in the S&P 500 portfolio [19], from 432 to 39. As a preamble, Figure 1 shows a sample comparison, between the eigenvalue distribution of 1000 sampled random matrices with 39 assets, and the corresponding RMT approximation, for some equally and exponentially weighted random matrices. Here we saw close agreement between the sampled distributions and their RMT approximations. However, the RMT distributions, which are approximations as \( N \to \infty \), underestimated the maximum random eigenvalue in both sample cases. Despite this, subsequent results showed that RMT filters could provide risk reduction for portfolios of 39 assets.

Figures 2 and 3 show the effect of RMT filtering on in-sample mean realised risk, for different sizes of S&P portfolio, in the equally and exponentially weighted cases respectively. Here we saw that filtering can be effective for low asset numbers. This was observed in the equally weighted case when the amount of historical data used was low (equivalent to a low value of \( Q \)), and in the exponentially weighted case across a wide range of decay factors. The effect of filtering was however seen to be reduced, in general, as the number of assets was decreased. The best unfiltered result also approached the best filtered for lower values of \( N \).

These results also confirmed previous work [23, 24], which showed via simulations that as the value of \( Q \) increased, for example through the use of more data, the effect of noise reduced. We also noted another effect here. For the higher values of \( N \), the best overall risk in the equally weighted case was achieved by reducing the amount of data and filtering, thereby uncovering flavours of models that were previously masked by noise.
3.2. In-Sample Results for the Fx Portfolio

Figure 4 shows the corresponding effect of RMT filtering on the in-sample mean realised risk of the Fx portfolio, with 39 assets. As in the S&P case [19], the in-sample results showed, in general, the potential of RMT filters to reduce realised risk, and the KR2, KR4 and KR8 methods were found to be amongst the best performing of all filters, and were reasonably consistent with each other, while optimisation performance of the KR filters disimproved, in general, as the minimum replacement eigenvalue approached zero. In the equally weighted case, a preference was again shown here for the lowest available number of past moves to be used (equivalent to the lowest $Q$ value), in conjunction with RMT filtering. In the exponentially weighted case, RMT filtering was again preferred overall, and the optimal in-sample decay factor coincided with the Riskmetrics [4] recommendation of 0.97, in contrast to the S&P case [19]. For both weightings, we observed that the use of filtering allowed for a more reactive model to be used. In the equally weighted case in particular, good agreement was found with the S&P portfolio. However, the closest agreement was clearly between the Fx portfolio with 39 assets, and the S&P portfolio with 432, and not 39 as might be expected.
Figure 3. Mean bootstrapped (in-sample) realised risk, for selected filters, applied to exponentially weighted volatility forecasts of S&P stocks, and for unfiltered volatility (“ORIG”), for $N =$ (a) 39, (b) 100, (c) 250 and (d) 432 assets.

Figure 4. Mean bootstrapped (in-sample) realised risk, for selected filters, applied to (a) equally weighted and (b) exponentially weighted volatility forecasts, and for unfiltered volatility (“ORIG”), for the Fx portfolio, with 39 assets.
4. Out-of-Sample Analysis of the Fx Portfolio

For comparing the models out-of-sample we used forward validation. This method considers every available test date and for each one uses data prior to the test date to optimise any model parameters. This allows the comparison of filtering methods with different numbers of parameters and also gives some insight into the stability of the models over time. The value of the weighting parameter ($\alpha$ or $T$) and the choice of KR model were determined out-of-sample. Possible KR models were all those defined in Section 2.4, as well as the LCPB model (for completeness). The forward validation was performed over a period of 1837 days, 129 of which were used as the initial training period. Subsequent retraining was done daily. We again used the realised risk of the forecast minimum risk portfolio as our metric and all 39 assets were used to eliminate the need to arbitrarily choose assets each day.

Table 2 shows the performance of the covariance forecasting and filtering combinations. The figures shown are mean realised risk as a percentage of the result for unfiltered equally weighted covariance. RMT filtering was seen on average to reduce realised risk in all cases, compared to the unfiltered portfolio. The best overall performance was seen when applying the PG+ filter to the equally weighted case. In the exponentially weighted case, KR filtering was found to be best, which was also the case for the larger S&P 500 portfolio [19], and at similar levels of risk reduction. We further observed that RMT filtering reduced mean realised risk in the majority of individual years. However, while realised risk was reduced on the majority (62.6%) of individual days, the filters were also found capable of increasing realised risk substantially on any one day. These results were consistent with recent results for a much larger S&P 500 portfolio [19].

Table 2. Mean out-of-sample realised risk as a percentage of that for unfiltered equally weighted covariance, for the full Fx portfolio, with 39 assets. Filtering was seen to reduce mean realised risk in all cases.

| Model         | Unfiltered | LCPB  | PG+  | KR  |
|---------------|------------|-------|------|-----|
| Equal Weights | 100        | 87    | 85.8 | 86.6|
| Exponential   | 98.1       | 91.8  | 92.7 | 87.4|

The optimal number of past moves (equally weighted), and decay factors, as selected through time by the forward validation, are shown in Figure 5. In this case the decay factors chosen were quite consistent with the value of 0.97 suggested by Riskmetrics [4], unlike in the S&P case [16,19]. In the equally weighted case, low numbers of past moves were ultimately preferred on average, in agreement with the in-sample results. Here, this resulted in equally weighted models outperforming exponential, when filtered. A trend towards models which were reactive to recent market changes can be seen in both cases.

5. Further Asset Reduction

Finally, we examined the effect of filtering an Fx portfolio with a reduced number of assets. Figures 6 and 7 show the effect of RMT filtering on in-sample mean realised risk, for portfolios with $N = 10, 15, 20$ and $30$ assets, in the equally and exponentially weighted cases respectively. RMT filtering was seen to be effective for a small $Q$ value, at all four values of $N$, and so, in an environment where the number of past data points, $T$, is limited, either by choice or necessity, RMT filtering may be of some benefit. However, without any restrictions on $T$, the effectiveness of RMT filtering was seen to be reduced as $N$ is reduced. In the case of exponential weights, RMT filtering was seen to provide most benefit for lower decay factors. However, in the presence of a free choice of decay factor, filtering provided little or no overall risk reduction for $N \leq 20$. 

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Figure 5. Optimal number of (a) past moves and (b) decay factor values, as selected by forward validation, for the Fx portfolio. In the unfiltered cases (not shown), the number of past moves ranged between 200 and 240, and the decay factors were usually equal to 0.98, and sometimes in the range 0.99 - 0.995.

These results for the Fx portfolio showed agreement with those for the S&P 500, although in the two markets, similar effects of asset reduction occurred at different values of $N$. In both cases, filtering was most effective for larger portfolios.

These conclusions were also reflected in the out-of-sample results for a portfolio consisting of 15 major currencies and commodities, selected from the main portfolio. In this case, RMT filtering was found to provide no benefit in the long run, as shown in Table 3, where filtering was seen to increase risk in most cases.

Table 3. Mean out-of-sample realised risk as a percentage of that for unfiltered equally weighted covariance, for the Fx portfolio with 15 major assets. Filtering was seen to increase mean realised risk in most cases.

| Model                  | Unfiltered | LCPB | PG+  | KR  |
|------------------------|------------|------|------|------|
| Equal Weights          | 100        | 103.8| 103.1| 99.1 |
| Exponential Weights    | 97.1       | 103.1| 102.6| 98.2 |

6. Conclusions
We have studied the application of RMT filters to a currency and commodity portfolio consisting of just 39 assets and found that, broadly, our results were in agreement with those reported [16, 19], that RMT-based filtering can improve the realised risk of minimum risk portfolios, despite the low number of assets involved in this case.

Using forward validation, RMT filters were found, overall, to reduce mean realised risk in all cases tested, and the majority of years and days. However, they were also found capable of increasing realised risk substantially on some individual days. These results were consistent with the S&P 500 case previously examined [19]. In contrast to these previous results, the decay factors chosen here showed good consistency with the value of 0.97 suggested by Riskmetrics [4]. In the equally weighted case, the value of Q ultimately preferred, when filtering, involved the
Figure 6. Mean bootstrapped (in-sample) realised risk, for selected filters, applied to equally weighted volatility forecasts of the Fx portfolio, and for unfiltered volatility (“ORIG”), for (a) 10, (b) 15, (c) 20, and (d) 30 assets.

least amount of historical data possible, indicating very reactive models. These low values of Q would have been unsuitable in the absence of filtering, and were also seen in the S&P case when the maximum amount of stocks was used.

The observed behaviour of the stability-based filter was in agreement with that of the S&P case [19], namely that the KR2, KR4 and KR8 methods were amongst the best performing of all filters, and reasonably consistent with each other, while optimisation performance disimproved, in general, as the minimum replacement eigenvalue approached zero. In the exponentially weighted case, the KR filter outperformed the others by a factor of 70%.

When RMT filtering was applied to Fx portfolios with fewer asset numbers it was observed, in general, that the benefit of filtering was reduced as asset numbers (N) decreased. Similar reduction was noted for the S&P portfolio, but at different values of N. This effect was also reflected in the out-of-sample filter performance, for a portfolio consisting of 15 major currencies and commodities, where RMT filtering provided no long term risk reduction, and was more likely to increase realised risk.

Taken as a whole, these results suggested that RMT filtering can provide strong risk reduction for foreign exchange and commodity portfolios involving sufficient numbers of assets. RMT filters also uncovered different styles of models than were possible with unfiltered analysis, namely ones
Figure 7. Mean bootstrapped (in-sample) realised risk, for selected filters, applied to exponentially weighted volatility forecasts of the Fx portfolio, and for unfiltered volatility (“ORIG”), for (a) 10, (b) 15, (c) 20, and (d) 30 assets.

that reacted quickly to market events. Without filtering these models utilising very recent data were found to be hidden by noise.

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