A modified method for improving the prediction accuracy of the tunnel shaking table model test based on non-direct similarity technique

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ABSTRACT

The tunnel shaking table model test has many influencing factors, and the test parameters are difficult to meet the strict similarity ratio. There are often large errors in predicting prototypes directly using the similarity ratio derived from the classical similarity theory. In order to improve the prediction accuracy of the tunnel shaking table model test, this article proposes a modified method of the traditional similarity theory. Based on the traditional dimensional analysis method, this method uses a non-direct similarity technique to rebuild the dimensional matrix for the main test parameters, derive a new similarity criterion, and then obtain a new similarity ratio. Different from the traditional similarity ratio which is a certain value, the new similarity ratio varies with dynamic parameters, which is more consistent with the actual situation. The tunnel shaking table model test and numerical simulation are carried out to verify the method. Experiments show that the modified method is superior to the traditional similarity theory in numerical prediction accuracy.

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Tunnel model test; non-direct similarity; prediction accuracy; numerical simulation; modified method

1. Introduction

As the theoretical basis of the model test, the similarity theory is a popular research problem in the model test design. The core of applying similarity theory is to determine the similarity coefficient of external load conditions, so that the model test results are close to the full-scale prototype test results, thus realizes the prediction of prototype by the model (Su & He, 2018). Due to various constraints such as site, equipment, capital, etc., it is difficult to carry out prototype tests of tunnels under the action of seismic wave. However, the limitations of the prototype test can be avoided by carrying out the tunnel shaking table model test according to the similarity theory. The tunnel model test is an important means to study tunnel science (Xia et al., 2010).

In recent years, some scholars have carried out tunnel model tests based on similarity theory and proposed prediction methods. Li et al. (2016) used tunnel diameter, acting head, seepage influence radius, and permeability coefficient of surrounding rock-support system as the main physical quantities, determined the similarity ratio according to the dimensional analysis method and carried out tunnel seepage model tests. Without considering the influence of tunnel excavation disturbance, the water inflow was predicted through model tests, and the numerical modelling analysis was performed with Visual-Modflow software. The comparison and analysis of the two results verified the effectiveness of the prediction method. On the premise of meeting the similarity of main physical quantities, this prediction method has achieved good results in predicting water inflow without considering disturbance. It is a static prediction method. An et al. (2017) carried out a stratigraphic model test based on separate similarity analysis. Through regression analysis of the model test data, a prediction method for vertical displacement of the ground at different depths below the surface of the circular tunnel construction was proposed. This prediction method was proposed under the premise of considering the single similarity of the stratum. The method is simple and effective, and it is also a static prediction method. Wang and Su (2020) designed and carried out indoor model tests based on similarity theory. The experiment considered the influence of the physical and mechanical parameters of rock materials on the range of stratum disturbance caused by shield construction. By revising the Verriuijt formula (Verruijt & Booker, 1998), a method for predicting the settlement value of the ground caused by shield construction in soft rock areas was proposed. The model test was carried out on the basis that the surrounding rock material and the shield
model were both similar. Through the prediction of the model test results, the reliability of the correction method was verified, which belongs to quasi-static prediction on the whole. In summary, the classical similarity theory is relatively mature and effective, and it has been widely verified and applied in static (quasi-static) experiments. However, it is rare to carry out prediction research under dynamic conditions through tunnel shaking table model tests. Compared with the static test, the shaking table test has more influencing factors, and the result analysis is more complicated. Various test parameters, such as the performance of shaking table and model box, the similarity of model materials, the acquisition error of sensor, etc., are difficult to meet the strict similarity ratio. Therefore, in the tunnel shaking table model test, there will be some errors in directly using the similarity ratio derived from the classical similarity theory to predict the prototype, and it is necessary to correct the traditional prediction method.

This article mainly studies the method to improve the prediction accuracy of the tunnel shaking table model test. Firstly, the classical similarity theory is used to establish the model. Then, on this basis, the idea of non-direct similarity is used to re-dimensionally analyse the physical quantities that affect the test results and are difficult to meet the strict similarity, and a new similarity criterion and similarity ratio are obtained to improve the prediction accuracy. The tunnel shaking table model test and numerical simulation are carried out to verify the method. Experiments show that the modified method is superior to the traditional similarity theory in numerical prediction accuracy.

2. Theoretical basis

The early similarity theory believed that the model was equivalent to the micro element separated from the full-scale prototype. Except for the difference in the proportion of appearance size and mass, other physical quantities of the model, such as speed, stress, external load conditions, etc., are the same as the full-scale prototype. Obviously, this theory ignores the similarity coefficient changes due to the size effect, so the experimental results of the model and the full-scale prototype are quite different (Su & He, 2018). In the 1910s, Buckingham (1914) proposed the second theorem of similarity on the basis of previous studies, that is, \( \pi \) theorem. The \( \pi \) theorem is very commonly used in similar model designs (Luo et al., 2016). The application of \( \pi \) theorem in the similarity model design mainly includes three methods, namely dimensional analysis method, equation analysis method, and law analysis method (Schuring, 1977). Among them, the dimensional analysis method is the most widely used in the design of the tunnel model, also known as the MLT method (Birkhoff, 1948). This method is proposed based on the principle of dimensional consistency. After determining the similarity coefficients of the basic physical quantities, the similarity coefficients of the remaining physical quantities can be obtained by the \( \pi \) theorem.

Taking the tunnel shaking table model test as an example, the derivation is as follows. Suppose there are 15 physical quantities are considered, and the expression of dimensionless number \( \pi \) can be listed according to MLT method:

\[
\pi_1 = M X_1 L X_2 T X_3 E X_4 \rho X_5 \sigma X_6 A X_7 K X_8 E X_9 \xi X_{10} \phi X_{11} \sigma X_{12}
\]

The symbols and their meanings mentioned in this article are shown in Table 1. Since \( \mu, \epsilon, \) and \( \phi \) are dimensionless quantities, their similarity ratio is generally set to 1. Therefore, Equation (1) can be changed to the following form:

\[
\pi_1 = M X_1 L X_2 T X_3 E X_4 \rho X_5 \sigma X_6 A X_7 K X_8 E X_9 \xi X_{10} \sigma X_{12}
\]

According to the principle of dimensional consistency, the following equations can be listed:

\[
\begin{align*}
X_1 + X_4 + X_5 + X_8 + X_9 + X_{11} + X_{12} &= 0 \\
X_2 + X_4 - 3X_5 + X_7 - X_9 + X_{10} - X_{11} &= 0 \\
X_3 - 2X_4 - X_5 - 2X_7 - 2X_8 - 2X_9 - X_{10} - 2X_{11} - X_{12} &= 0
\end{align*}
\]

Table 1. Table of notations.

| Notation | Description | Notation description | Description |
|----------|-------------|----------------------|-------------|
| \( M \)  | Mass        | \( c \)              | Cohesion    |
| \( L \)  | Geometric length | \( C \)       | Similarity ratio |
| \( T \)  | Time        | \( P_m \)           | Maximum horizontal seismic dynamic earth pressure at the measuring point of the vault |
| \( F \)  | Load        | \( P_m \)           | Maximum horizontal seismic dynamic earth pressure at the measuring point of the arch foot |
| \( \rho \) | Density     | \( A_m \)          | Maximum acceleration at the measuring point of the arch foot |
| \( f \)  | Frequency   | \( A_m \)          | Maximum acceleration at the measuring point of the arch foot |
| \( A \)  | Acceleration | \( P_{max} \)       | Maximum horizontal seismic dynamic earth pressure |
The above equations are organized into the following dimensional matrix:

\[
\begin{bmatrix}
X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 & X_9 & X_{10} & X_{11} & X_{12} \\
M & L & T & F & \rho & f & A & K & E & \nu & \sigma & \xi
\end{bmatrix}
\]

(4)

According to the \( \pi \) theorem, the number of \( \pi \) criteria are 9. The matrix (4) is transformed into the following \( \pi \) matrix(Huang et al., 2011):

\[
\begin{bmatrix}
X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 & X_9 & X_{10} & X_{11} & X_{12} \\
M & L & T & F & \rho & f & A & K & E & \nu & \sigma & \xi
\end{bmatrix}
\]

(5)

Then, we can obtain the following similarity criterions:

\[
\pi_1 = \left\{ \begin{array}{c}
\frac{L M}{F T^2} \\
\frac{L^3 \rho}{M} \\
\frac{F}{T} \\
\frac{T^2 a}{L} \\
\frac{T^2}{M} \\
\frac{K T^2}{M}
\end{array} \right\}
\]

(6)

The ratio of the physical quantity of the prototype to the corresponding physical quantity of the model is called the similarity ratio, which is represented by the symbol \( C \). From Equation (6), we can get the similar relationship between the physical quantities, as shown in Table 2. We can see that only three of the similarity ratios need to be determined, and the other similarity ratios can be derived from the similarity relationship. Due to the limitation of the test conditions, the acceleration similarity ratio is generally taken as 1. Then, if we assume that the geometric length similarity ratio (\( C_l \)) is 30 and the elastic modulus similarity ratio (\( C_E \)) is 30, the similarity ratios of other physical quantities can be calculated from the similar relationship, as shown in Table 2.

In the MLT method, all similarity ratios are strictly similar. Generally, the prototype test results can be obtained from multiplying the model test results by the corresponding similarity ratio. However, in the shaking table model test, there are many influencing factors and are not easy to control, which makes the test parameters difficult to meet the strict similarity ratio. Therefore, there will be some errors in predicting the prototype with strict similarity ratio.

### 3. Modified method

In order to solve the above problems and improve the prediction accuracy, this article adopts the idea of non-direct similarity (Drazetic et al., 1994; Trímino & Cronin, 2014) to make corrections. Specifically, on the basis of determining the traditional similarity ratio by the MLT method, we first select the main controlling physical quantities that have great influence on the tunnel shake table model test and are difficult to meet the strict similarity ratio, and reestablish the reduced dimensional matrix for these physical quantities. Then, according to the \( \pi \) theorem, we re-deduce the new similarity ratio. Taking the tunnel shaking table model test to predict the maximum earthquake dynamic soil pressure of the tunnel prototype under the same conditions as an example, the derivation process is as follows.

The first step is to establish a similar relationship according to the MLT method, and the results are shown in Table 2.

The second step is to establish a reduced dimensional matrix and solve it to obtain a new similarity criterion. We assume that the main controlling physical quantities considered are acceleration (\( A \)), density (\( \rho \)), buried depth (\( h \)), elastic modulus (\( E \)), earth pressure (\( P \)), and Poisson’s ratio (\( \mu \)). The reduced dimension matrix is established as follows:

\[
\begin{bmatrix}
X_1 & X_2 & X_3 & X_4 & X_5 & X_6 \\
A & \rho & h & E & P & \mu
\end{bmatrix}
\]

(7)

Solve matrix (7) in the same way as matrix (4). Then, we can obtain the following similarity criterions:

\[
\pi_1 = \frac{E}{\rho Ah}; \quad \pi_2 = \frac{P}{\rho Ah}; \quad \pi_3 = \frac{1}{\mu}
\]

(8)
Finally, by $\pi_1 \times \pi_2 \times \pi_3$, we can get that:
\[
C_p = C_\rho^2 C_A^2 C_\mu^2 C_E
\]
(9)

It can be seen from Table 2 that the $C_p$ obtained by the traditional MLT method is equal to $C_E$ and is a certain value. The new similarity criterion $C_p$ from formula (9) is a dynamic value, which takes the factors that are difficult to meet the strict similarity ratio in the shaking table test into account. Compared with the traditional similarity ratio, the revised similarity ratio is more refined. It should be noted that this modified method is not a new theory, but a repeated application of similarity theory. Considering that the traditional similarity criterion is difficult to meet the strict similarity ratio in the case of vibration, this modified method derives a new similarity criterion on the premise of meeting the similarity of the main physical quantities, aiming to improve the prediction accuracy. If each physical quantity can meet the strict similarity ratio under vibration condition, the values of similarity criteria obtained by the modified method and the traditional MLT method are consistent.

It can be seen from the derivation process of Equation (9) that the modified method proposed in this article can select different main controlling physical quantities according to different experimental purposes, establish different dimensional matrices, and thus derive different similarity ratios. Take the test to predict the dynamic strain of the tunnel lining as an example. If the main controlling physical quantities considered are acceleration ($A$), elastic modulus ($E$), mass ($M$), main vibration frequency ($f$), displacement ($\delta$), maximum vibration velocity ($v$), cross-sectional area ($S$), and duration ($T$), then the similarity ratio can be obtained as follows: $C_\delta = C_\rho C_\delta^3 C_\mu^2 C_E C_v^2$. If the acceleration ($A$), elastic modulus($E$), displacement($\delta$), density($\rho$), tunnel area($S$), and vibration frequency($f$) are selected as the main controlling physical quantities, then, $C_\delta = C_\rho C_\delta^3 C_\mu^2 C_f / C_E$. It can be seen that the similarity ratio expression is different when different physical quantities are selected to construct the dimensional matrix. The selection of main controlling physical quantities needs to be determined according to the test conditions, influencing factors and test purpose.

4. Test verification

In order to check the validity of the modified method, the tunnel shaking table model test and numerical simulation were carried out to verify the formula (9). Firstly, the tunnel shaking table model test was carried out to measure the horizontal seismic dynamic earth pressure of vault and arch foot under the action of seismic waves. Then, the similarity ratio obtained by the MLT method and the modified method were used to push back the prototype respectively, and the results were compared with the simulated prototype results to verify the effectiveness of the modified method in the numerical prediction accuracy.

4.1. Model test

The shaking table model test was conducted at the State Key Laboratory of Bridges and Tunnels in Mountain Areas, Chongqing Jiaotong University. The shaking table is 3 m \times 3 m unidirectional seismic simulation shaking table, and the model box size is 190 cm \times 130 cm \times 150 cm, as shown in Figure 1. Through numerous tests for the proportion of similar materials, the ratio of surrounding rock material of tunnel model was finally determined as loess:sand:cement:gypsum:water = 22:52:8:8:10. The mechanical parameters of surrounding rock of tunnel prototype and model are shown in Table 3.

| Table 3. Parameter values of surrounding rock. |
|---------------------------------------------|
| Surrounding rock | $\rho$ (kg/m$^3$) | $E$ (Gpa) | $\mu$ | $\varphi$ | $c$ (Mpa) |
| Prototype | 1800 | 1.3 | 0.30 | 30.3 | 0.3 |
| model | 1787 | 0.044 | 0.28 | 28.3 | 0.0098 |

![Figure 1. The shaking table and the model box.](image)
The prototype material of tunnel lining is C25 concrete with a thickness of 60 cm. The model lining was made of gypsum and water. The water–cement ratio is 1.2. The gypsum used is −200 mesh gypsum powder accounting for 98%. The lining thickness is 2 cm. Figure 2 shows lining fabrication and finished products.

In this experiment, two measuring points were selected to place the accelerometer and the dynamic earth pressure box, which were located at the arch top and arch foot positions, as shown in Figure 3.

This model test used the horizontal unidirectional El-Centro wave. The peak ground acceleration (PGA) was loaded in three conditions, 0.2, 0.4, and 0.6 g, respectively. Figure 4 shows the shaking table model test locale.

According to the test results, the acceleration time histories diagram and the seismic dynamic earth pressure time histories diagram corresponding to the above three conditions are drawn, as shown in Figures 5 and 6.

The maximum acceleration and seismic dynamic earth pressure of each measuring point in the model test are shown in Table 4.

### Table 4. The maximum value of acceleration and horizontal seismic dynamic earth pressure at each measuring point of the model test.

| PGA | 0.2 g | 0.4 g | 0.6 g |
|-----|-------|-------|-------|
| \( P_{v} \) (Mpa) | 0.032 | 0.069 | 0.118 |
| \( P_{a} \) (Mpa) | 0.122 | 0.276 | 0.581 |
| \( A_{v} \) (m/s²) | 2.23 | 5.52 | 8.28 |
| \( A_{a} \) (m/s²) | 2.25 | 5.74 | 8.58 |

### Numerical simulation

The ANSYS software was used to conduct numerical simulation of the above tunnel prototype under the same conditions. The numerical model adopted viscoelasticity artificial boundary (Chen et al., 2018; Tan et al., 2018). The input parameters of the tunnel prototype are shown in Table 3. The time histories data of horizontal seismic dynamic earth pressure at the corresponding measuring points were obtained by numerical simulation and plotted as shown in Figure 7.

The maximum acceleration and seismic dynamic earth pressure of each measuring point of the prototype are obtained by numerical simulation, as shown in Table 5.

### Table 5. The maximum value of acceleration and horizontal seismic dynamic earth pressure at each measuring point of numerical simulation.

| PGA | 0.2 g | 0.4 g | 0.6 g |
|-----|-------|-------|-------|
| \( P_{v} \) (Mpa) | 0.81 | 1.72 | 2.36 |
| \( P_{a} \) (Mpa) | 4.90 | 7.47 | 14.80 |
| \( A_{v} \) (m/s²) | 1.94 | 4.79 | 6.54 |
| \( A_{a} \) (m/s²) | 2.01 | 5.09 | 7.49 |

### Comparative analysis

The maximum seismic dynamic earth pressure of each measuring point in the tunnel prototype is predicted by the MLT method and the modified method. The similarity ratios of the two methods were calculated according to Table 1 and Equation (9), so as to calculate the maximum
Figure 3. Cross-sectional view of measuring point arrangement in the model test.

Figure 4. The shaking table model test locale.
seismic dynamic earth pressure at different measuring points under corresponding working conditions, and the calculation results are compared with the full-scale prototype results. The calculation results are shown in Table 6.

From Table 6, we can see that the similarity ratio calculated by the modified method used in this article is different from the MLT method, and the relative error of this modified method is small, and the prediction accuracy has been improved. The modified method considers
Figure 6. The seismic dynamic earth pressure time histories diagram of the model test.

the actual situation of shaking table test, and incorporates the test factors that are difficult to meet the strict similarity ratio, such as bulk density, acceleration, burial depth, Poisson's ratio, and elastic modulus into the similarity ratio expression, so that the similarity ratio changes dynamically with the input of seismic waves, which is more consistent with the actual situation.

From Table 6, we can see that with the increase of PGA, the deviation of similarity ratio between the modified method and the MLT method gradually increases. This
Figure 7. The time histories data of horizontal seismic dynamic earth pressure obtained by numerical simulation.
is mainly because the acceleration deviation is gradually increasing. The traditional MLT method generally defaults the acceleration similarity ratio to 1. However, in the tunnel shaking table test, it is generally difficult to achieve 1:1 seismic wave loading due to the influence of the performance of the shaking table and model box, boundary conditions, and measurement accuracy. With the increase of PGA, the acceleration amplification coefficient of each measuring point increases nonlinearly, making the acceleration similarity ratio deviate from 1, and the deviation increases gradually.

5. Conclusion

In order to improve the prediction accuracy of the tunnel shaking table model test, this article proposes an improved method of traditional similarity theory by using a non-direct similarity technique, and takes the prediction of the horizontal maximum seismic dynamic earth pressure around the tunnel as an example to carry out the model test and numerical simulation to verify the method. Experiments show that the prediction accuracy of this modified method is better than that of the traditional MLT method. In addition, this article takes the prediction of the maximum dynamic strain of tunnel lining as an example, and derives different similarity ratios by selecting different main controlling physical quantities based on the modified method. The modified method proposed in this article can provide reference for other experiments.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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Data availability statement

I confirm I understand the terms of the share upon reasonable request data policy. The data that support the findings of this study are openly available in Science Data Bank at http://doi.org/10.11922/sciencedb.j00001.00341. The data are reliable and there is no conflict of interest.

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### Table 6. Comparison of prediction prototype results between the MLT method and the modified method.

| Measuring point | Working condition | $P_{\text{max}}$ (Mpa) (model) | $P_{\text{max}}$ (Mpa) (prototype) | C | $P_{\text{max}}$ (predicted value) | Relative error |
|-----------------|------------------|-------------------------------|-------------------------------|----|----------------------------------|----------------|
| 1               | 0.2 g             | 0.032                         | 0.81                          | 30 | 0.96                             | 18.5%          |
| 0.4 g           | 0.069             | 1.72                          |                               | 30 | 2.07                             | 20.3%          |
| 0.6 g           | 0.118             | 2.36                          |                               | 30 | 3.54                             | 50.0%          |
| 2               | 0.2 g             | 0.182                         | 4.90                          | 30 | 3.66                             | 11.4%          |
| 0.4 g           | 0.276             | 7.47                          |                               | 30 | 8.28                             | 10.8%          |
| 0.6 g           | 0.581             | 14.8                          |                               | 30 | 17.43                            | 17.8%          |

MLT method

| C | $P_{\text{max}}$ (predicted value) | Relative error |
|----|----------------------------------|----------------|
| 26.54 | 0.85 | 4.87% |
| 26.41 | 1.82 | 5.95% |
| 21.88 | 2.58 | 9.41% |
| 27.99 | 5.09 | 3.96% |
| 27.58 | 7.62 | 1.90% |
| 26.73 | 15.53 | 4.93% |
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