Behaviour of Masonry Walls under Horizontal Shear in Mining Areas

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Abstract. The paper discusses behaviour of masonry walls constructed with small-sized elements under the effects of mining activity. It presents some mechanisms of damage occurring in such structures, its forms in real life and the behaviour of large fragments of masonry walls subjected to specific loads in FEM computational models. It offers a constitutive material model, which enables numerical analyses and monitoring of the behaviour of numerical models as regards elastic-plastic performance of the material, with consideration of its degradation. Results from the numerical analyses are discussed for isolated fragments of the wall subjected to horizontal shear, with consideration of degradation, impact of imposed vertical load as well as the effect of weakening of the wall, which was achieved by introducing openings in it, on the performance and deformation of the wall.

1. Introduction
Mining activity, regardless of its intensity and depth, always has an adverse effect on the surface structures. Loads caused by that effect differ in nature, direction, range and frequency from the loads for which structures are designed in peaceful areas [1, 2, 3].

The intensity of particular actions in engineering practice is evaluated through indices defining the deformation of the free terrain surface. These are: \( w [m] \) – sinking of the ground surface, \( u [m] \) – horizontal displacement of a point on the ground surface, \( \kappa, R [1/km, km] \) – curve and curve radius of the mining subsidence profile, \( e [mm/m] \) – horizontal deformation of the ground surface, \( T [mm/m] \) – inclination of the ground surface and \( a_p [m/s^2] \) – amplitude of acceleration of ground surface vibrations. The values of those indices can be determined based on the geological makeup of the rock mass and the designed activity using one of many known rock mass deformation theories [1, 2, 3].

For the purposes of engineering practice, the full scope of these effects was divided into six parts, and a mining area category was assigned [4-8] to each. Deforming soil typically causes deformation of the structure (figure 1) and additional internal forces [9, 10], resulting mainly from friction between the soil and foundations, influence of various vertical and horizontal displacements of the soil, inclination of the terrain and pressure of the soil on vertical surfaces of walls in it [11]. Disturbances of the ground provoked by mining activity cause horizontal inertia, which loads the structure.
Figure 1. Diagram of wall deformation a) under the effect of vertical displacement of the ground, b) under the influence of mining disturbance.

Figure 2. Deformation of a window opening in a building at ul. Pocztowa in Bytom, Karb, photograph by L. Słowik, ITB, 2013.

Figure 3. Diagonal dissection of a load bearing wall in a building at ul. Pocztowa in Bytom, Karb, photograph by L. Słowik, ITB, 2013.

Figure 4. Diagonal crack in an external wall of a building in Mikołów – outside view, own photograph.

Figure 5. Cracking and separation of a masonry wall between openings, Mikołów, own photograph.
In the case of a masonry structure, its damage results primarily from exceeding the amount of shear strain that the structure is able to withstand [12]. Such deformation appearing while mining activity takes place underneath the building is caused by the nature of ground surface deformation, which is shown in figure 1, presenting the nature of the structure’s deformation, with uplift and under impact load. It can be noted that in both cases the wall structure is subject to intense shear strain. With the case of ground uplift, it is mainly vertical shear, in the case of impact load – horizontal shear. The paper focuses on the behaviour of a masonry structure subjected to shear strains, in particular subjected to horizontal shear.

Figure 6. Diagonal crack, with displacement in an external wall of a building in Mikołów – inside view, own photograph

Figure 7. Diagonal dissection of fragments between windows in a masonry building at ul. Pocztowa in Butom, Karb, photograph by L. Słowik, ITB, 2013

Figures 2-7 show examples of effects of mining activity on masonry structures observed on site, resulting from horizontal shear affecting the masonry elements as a result of static deformations of the mined ground.

According to [5], the risk of cracking of the structure’s walls is assessed by checking condition (1) – chance of not exceeding a certain value of shear strain $\theta_b$:

$$\theta_b \text{ (factored)} \leq \theta_b \text{ (permissible)}$$  \hspace{1cm} (1)

Such methodology may be accepted when damage mechanisms for the wall observed in laboratory tests are consistent with those in real life conditions. Example results of such tests known in subject matter literature, with cyclic horizontal shear on wall fragments, were presented by Lourenço in [13].

Further sections of the paper present attempts to evaluate of the behaviour of large wall fragments of varying proportions in the scenario of modelling their performance in horizontal shear using an elastic-plastic model with degradation. The paper discusses results of calculations – impact of degradation on the failure danger level and deformation of the walls, imposed vertical load, effects of reinforcing the wall by introducing pilasters or reinforced concrete cores, influence of wall weakening which was achieved by introducing openings into the loaded wall.

2. Short description of the elastic-plastic model with degradation used in the paper

In 1989 a team of researchers: Lubliner, Oliver, Oller and Oñate [14, 15] published a paper on the application of a new elastic-plastic-brittle material model accounting for degradation for numerical simulations describing RC concrete structure cracking. In 2002 an expanded version of this model was implemented in the commercial FEM ABAQUS software under the name Damage Concrete Model. This model is later referred to as (e-p-d) model. It is a three-parameter model with unassociated flow rule and with a non-linear isotropic strengthening, two-mechanism type [15-18], based on strain-hardening
hypothesis. The plasticity surface constitutes an extension of the surface in classical Drucker-Prager model in that it accounts for non-circularity of the deviatory section. Plastic strain in (e-p-d) are determined based on plastic potential surface differing from the plasticity surface.

The theoretical basis of the (e-p-d) model is plastic fracture mechanics. Coupling the elastic-plastic characteristics of a material with the description of its fracture is achieved by expressing constitutive equations of plastic theory using effective stresses.

The (e-p-d) model is characterised by isotropic degradation of material, described with two degradation factors: $d_t$ and $d_c$, for tension and compression respectively. These variables, determined based on independent material fracture functions, are connected, and describe both the experimentally evidenced influence of degradation of compressed material ($d_c$) on the degradation of the material in tension ($d_t$) after changing the stress direction, as well as a different phenomenon, which consists in partial restoration of the original rigidity of the material, as a result of closing cracks in a brittle material in transition from tension to compression. For the purposes of the description of the model’s response to cyclic tension-compression load, we defined the fracture variable $d$, according to formula (2),, accounting for degradation of the material in both possible fracture mechanisms - uniaxial compression and uniaxial tension, and additionally two reduction factors corresponding to the transitions: from tension to compression ($\omega_c$) and from compression to tension ($\omega_t$), which are additional material parameters.

$$\left(1 - d\right) = \left(1 - s_t \cdot d_c\right) \cdot \left(1 - s_c \cdot d_t\right), \quad 0 \leq s_t, s_c \leq 1$$

where: $s_t, s_c$ are specific stress functions $\sigma_i, j$ defining reduction of material fracture in the model. The structure and principles of application of the (e-p-d) model is presented in [19].

3. Description of the walls under analysis subjected to horizontal shear

The paper analyses three standalone full brick walls strengthened on the vertical edges with pilasters, marked as wall A, wall B and wall C (figure 8) depending on the dimension ratios. The walls were loaded with: self-weight $\gamma = 20 \text{kN/m}^3$, imposed vertical load $p \text{[kPa]}$, and imposed horizontal displacement of the $\delta$ top edge. At the same time, the possibility of vertical displacement during application of horizontal displacement was limited. The problem was solved with the use of a FEM method, utilising the elastic-plastic model with degradation (e-p-d). The computational model was made up of square, four-node finite elements, with reduction of integration points.

The problem was solved multiple times, for different combinations of load $p$, wall geometry and the fineness of the FEM mesh. Detailed characteristics of parameters adopted for numerical analyses, material and deformation properties of the wall, resulting from the strength of the bricks and mortar, were adopted as for a homogenous material; they are presented in figure 11, and the graphs of stress-strain characteristics, strengthening characteristics and material degradation functions, determined based on tests in [20, 21] and quoted in [22, 23] are shown for compression and tension in figure 9.
Figure 10. Geometry, loading and method of making openings in walls under analysis: a) geometry and loading of wall I, b) making the opening horizontally in layers, c) making the opening vertically in strips d) geometry and loading of wall II

Table 1. Meaning of symbols describing the results of numerical analyses in figures 13, 14, 15 and 16

| No | Description of problem                                                                 | Symbol on the figures |
|----|----------------------------------------------------------------------------------------|-----------------------|
| 1  | wall I full                                                                            | I                     |
| 2  | wall II full                                                                           | II                    |
| 3  | wall I with a 1x1 m opening in the middle of its surface                                | I-o                   |
| 4  | wall I with a large 1.36x1.36 m opening in the middle of its surface                    | I-0.85                |
| 5  | Wall I – δ=1 mm, subsequently a 1.36x1.36 m opening cut into the middle               | I-1mm-o.85            |
| 6  | wall I – δ=1, 1.6, 2, 2.5, 3 mm, subsequently a 1x1 opening cut in the middle         | I-δmm-o              |
| 7  | wall I – δ=1.5, 2, 3 mm, subsequently a 1x1 m opening cut in it – gradually, in horizontal layers | I-δmm-oH              |
| 8  | wall I – δ=2, 3 mm, subsequently a 1x1 m opening cut in the middle, only first layer – gradually element after element | I-δmm-oH-1w          |
| 9  | wall I – δ=2 mm, subsequently a 1x1 m opening cut in the middle, only first vertical strip – gradually element after element | I-2mm-oV-1p          |
| 10 | wall I – δ=2, 3 mm, subsequently a 1x1 m opening cut in the middle – gradually in vertical strips | I-δmm-oV             |
| 11 | wall I – δ=3 mm, subsequently a 1x1 m opening cut in the middle, horizontally, all 6 layers – gradually element after element | I-3mm-oH-6w          |
| 12 | wall II with 1x1 m Left opening (L)                                                   | II-o.L               |
| 13 | wall II with 1x1 m Middle opening (S)                                                  | II-o.S               |
| 14 | wall II with 1x1 m Right opening (P)                                                   | II-o.P               |
| 15 | wall II – δ=0.75, 1.5, 2 mm, subsequently an entire opening cut in it                  | II-δmm-oL            |
| 16 | wall II – δ=1.5 mm, subsequently an entire opening cut in it                           | II-1.5mm-oS          |
| 17 | wall II – δ=1.5 mm, subsequently an entire opening cut in it                           | II-1.5mm-oP          |
| 18 | wall I – δ=3 mm, subsequently a 1x1 m opening cut in it – gradually, in horizontal layers, with a finer element mesh | I-3mm-oH-zs          |
| 19 | wall I – δ=3 mm, subsequently a 1x1 m opening cut in the middle – gradually in vertical strips, with a finer element mesh | I-3mm-oV-zs          |

For two walls, selected from those shown in figure 8 and subsequently presented in figure 10a and figure 10d, analyses were carried out regarding the impact of their geometry on the rigidity and general strength of the wall. The problem was solved multiple times for both cases. Firstly, the full wall was analysed, loaded with self-weight and imposed vertical load, subsequently subjected to horizontal shear.
In the second stage, the analysis was conducted for a wall with an opening, loaded with self-weight and imposed vertical load and subjected to horizontal shear. Next, an opening was cut into the wall loaded with self-weight and vertical $q$ and subjected to horizontal shear. In the case of wall I the opening was located in the middle. Wall II was analysed in different configurations of opening placement: Left, Middle and Right (L, S, P) – figure 10d. Three different methods of cutting the opening were considered: removing elements from the entire opening area evenly, removing elements in horizontal layers (figure 10b) and removing them in vertical strips (figure 10c).

During analysis of the impact of the openings on the structure’s failure danger level, cutting was started at different values of horizontal displacement $\delta$ of the top wall edge – table 1 gives meaning of symbols describing the numerical results. Increasing the opening area by 85% was also considered.

4. Impact of wall dimension ratios on calculation results

Figure 11 compiles graphs of relation between horizontal force $R_{hor}$ and the imposed horizontal displacement $\delta$ – of the top wall edge in relation to the bottom edge. It can be noted that doubling the initial imposed vertical load on the wall from $\sigma_v=100$ kPa to $\sigma_v=200$ kPa, only resulted in a minor improvement in rigidity. The strength of the wall was unaffected, and the peak of $R_{hor}$ force occurred earlier – at a displacement lower by c. 10% $\delta$. The proportions of the wall’s dimensions are of far greater significance as regards its behaviour. Assuming wall A with dimensions ratio of $H/L = 1$ as baseline, then for a wall with $H/L = 1.5$ its rigidity measured as the angle of the first section of the graph decreased by c. 45%, and for a wall with $H/L = 2/3$ it increased by c. 21%.

Accounting for degradation in numerical calculations for the wall (impact of the type of constitutive model) may be observed through comparison of graph 3 with graph 5. It can be noticed that the degradation of rigidity only has a minor influence (c. 2-2.5%) on the wall’s strength in shear. However, the influence is clearly visible in the evaluation of the wall’s deformability outside elastic performance range (difference of c. 40-45%). An increase of the diameter of edge pilasters has a similar effect on the wall’s performance (graphs 4 and 6).

Comparing solutions 1, 2, 3 and 4 it can be concluded that increasing the fineness of the mesh produced visible results mostly in the evaluation of walls’ rigidity outside their elastic performance range. It does not have any significant impact on the calculations of a wall’s strength; the largest elements (0.24 x 0.24 m) proved sufficient.

Figure 11. Graph of the relation between horizontal reaction $R_{hor}$ and horizontal displacement $\delta$ for full walls
Figure 12. Wall I: a) and b) maximum principal plastic strains for $\delta=2.4\,\text{mm}$ and for $\delta=3.6\,\text{mm}$, c) and e) maximum principal stresses $\sigma_1\,[\text{Pa}]$ for $\delta=2.4\,\text{mm}$ and for $\delta=3.6\,\text{mm}$, d) total degradation $d$ for $\delta=6.0\,\text{mm}$

The results, in the form of maps of plastic strains, degradation and distribution of maximum principal stresses, show possible fracture mechanisms for the wall – dependent on their dimension ratios. The results suggest that walls with proportions closest to square experience shear roughly diagonally (the angle of the shear plane is slightly smaller than 45°) – figure 12. In walls extending farther horizontally this plane is strongly inclined towards the horizontal position. On the other hand, high walls primarily get damaged in the middle, in a plane close to vertical. The obtained results show a direct relation with the wall’s proportions; however, the degradation surface (area of the main cracks) is inclined at a larger angle – compared to a square wall – than the wall’s diagonal.

5. Calculation results for walls weakened with openings

Selected calculation results are presented in figures 13-16. They compile graphs of the relation between the horizontal reaction force $R_{\text{hor}}$ and horizontal displacement $\delta$ of the top edge of the wall in relation to the bottom edge. Detailed specification of the problems solved, together with explanation of symbols used in figures 13-16 is presented in table 1. It was assumed that in all cases the basic load of the wall consists of self-weight and imposed vertical load $q=100\,\text{kPa}$ applied to the top horizontal edge. Horizontal shear is executed through imposed displacement of the top horizontal edge in relation to the bottom edge by $\delta$.

Figure 13 contains the most significant results for wall I. As can be noted, the decrease of the horizontal reaction value $R_{\text{hor}}$ on the top edge is greater the closer the opening was cut to the extremum of the curve for the full wall (graphs I-1mm-o, I-1.6mm-o, I-2mm-1, I-2.5mm-o, I-3mm-o). The obtained solutions also indicate that the lower the failure danger level at the moment of beginning the cutting operation, the bigger the strength margin is in the wall after the opening has been completed. Deformability of the wall can be measured e.g. with horizontal displacement of the top horizontal edge in relation to the bottom edge. The graphs in figure 13 show that cutting the opening in a wall close to failure increases its deformability as compared to a wall with an opening which existed before loading, the more so the closer the wall is to failure at the moment of beginning the cut; the increase in deformability goes down as the wall gets closer to failure.
Figure 13. Selected results of wall I analysis – impact of failure danger level of the wall at the moment of cutting the opening on the wall’s strength.

Figure 14. Impact of the increased fineness of the mesh (\(-z_s\)) on the results of gradual cutting of a 1\(^2\)m opening in wall I.

Figure 14 shows, on the example of wall I, influence of increasing the fineness (\(-z_s\)) of the discrete mesh on the obtained results. The graphs reveal that with the fineness of the mesh that was used, this influence is minor and expresses itself only when the wall is close to failure. This confirms the results obtained earlier for other models. The wall strength envelope after openings were made at different stages of wall’s performance traces a curve reminiscent of the curve obtained using the model with an opening made before loading the wall and without taking into consideration its weakening from degradation. This relation is also confirmed in a scenario where the dimensions of the opening were increased (graphs I-o85, I-1mm-o85).

Figure 15a and figure 15b show the results obtained for wall I and different methods of cutting the opening. For example, for a stress level corresponding to displacement \(\delta=3\ mm\) the opening was made using four methods:
- entire opening being cut at once (graphs: I-1.5mm-o, I-2mm-1, I-3mm-o),
- the opening being cut gradually in horizontal layers (graphs: I-1.5mm-oH, I-2mm-oH, I-3mm-oH),
- only the first layer being cut horizontally – element after element (graph: I-3mm-oH-1w),
- entire opening being cut gradually element after element (I-3mm-oH-6w),
- entire opening being cut in vertical strips (I-2mm-oV),
- entire opening being cut gradually in vertical strips (I_2mm_oV-1p).

Figure 15. Selected analysis results for wall I – impact of the opening cutting method.
Figure 16. Selected analysis results for wall II: a) impact of the failure danger level in the wall at the time of cutting the opening, b) impact of the opening location (L, S, P) in the wall

The shape of the graphs shown in figure 15a suggests that enlarging the opening gradually results in a greater drop in strength than when the opening is made at once. That drop is greater, the higher the closer for the wall was to failure. For example, for an opening after forced displacement $\delta = 3\, \text{mm}$, cutting the first layer causes a 50% change in the wall’s strength – as compared to the final result. In the case of openings cut gradually, the size of fragments being cut out is of no consequence.

Figure 15b compares results obtained for different directions of cutting the opening: horizontally or vertically. The horizontal direction of cutting causes a decrease in strength only slightly greater than the vertical method. It can be assumed that in real circumstances, horizontal cutting would connect diagonal cracks, removing the sections of the wall between them from non-linear performance.

Results obtained for wall II are shown in figure 16. They confirm the influence of the wall’s danger level at the moment of starting the cutting on the wall’s strength, which was observed previously in wall I (figure 16a). On the other hand, the impact of the opening’s location on the wall strength is insignificant (figure 16b).

6. Conclusions

The ratio of the wall’s height to its length (H/L) is of primary importance from the point of view of rigidity. The wall’s rigidity decreases almost proportionally to the increase in height, but rises about half as fast as the width of an originally square wall, used as baseline.

The wall fracture (degradation, cracks) sets in its middle section, and propagates in a direction close to the diagonal connecting compressed corners. In particular, for a square, low wall, the shear plane (area of degradation and cracks) is tilted towards the horizontal direction, whereas for a high wall, it is tilted strongly towards the vertical position, from the direction of principal compression.

Degradation of rigidity has a minor influence on the wall’s shear strength, until maximum strength is reached; however, it manifests in a significant manner in performance outside the elastic range and after maximum stress is reached. Doubling the initial imposed vertical load on the wall results in only slight improvement of the wall’s rigidity, without affecting its strength in any significant manner. The fineness of the mesh has an effect mainly on the evaluation of wall rigidity, beyond the elastic performance range. It has no significant impact on the calculation of strength.

In the case where openings in the wall are cut before application of load it can be determined that the lower the failure danger level at the moment of starting the cutting operation, the higher the wall’s strength margin will be. Cutting the opening in a wall when it is close to failure increases its deformability compared to a wall with an opening already existing when the load is applied, the more so the closer the wall is to failure when cutting starts. At the same time, the increase in the wall’s deformability becomes smaller the higher the failure danger level is at the moment of making the
opening. Gradually enlarging the opening effects a higher drop in strength than making the entire opening at once, and that drop is greater the closer the wall was to failure. The size of fragments that were removed from the wall when the opening was made gradually did not matter; however, differences were observed depending on the direction of cutting. The horizontal direction of the process resulted in a slightly greater drop in strength than the vertical direction. The location of the opening in a wide wall was not found to play a role either.

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