Synchronization State of Chaotic Circuit Containing Time Delay in One Direction

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Abstract
Synchronization state can be observed in coupled circuits. Further, interesting synchronization state was confirmed in coupled time delayed chaotic circuits. In this study, we propose novel coupled systems and investigate synchronization state in coupled time delayed chaotic circuits. The proposed coupling methods of time delayed chaotic circuits depend on attractor types. We focus on the relationships between synchronization state and the coupling methods. Moreover, we investigate the special coupling methods of time delayed circuit in this study.

1. Introduction
There are many nonlinear systems containing time delay, such as neural networks, control systems, meteorological systems, biological systems and so on in the natural world. Namely, it is considered that investigation of stability in such time-delay systems is significant [1]. Generation of chaos of them all is reported self excited oscillation system containing time delay [2]. This chaotic circuit can be easily realized by using simple electric circuit element and analyzed exactly. On the other hand, there are examples of nonlinear phenomena, chaotic synchronization and so on [3]. In particular, many studies on synchronization of coupled chaotic circuits have been reported [4].

In this study, we devise coupled systems that takes advantage of features of the time delayed chaotic circuit. The novel coupled systems are utilizing the characteristics of the circuit having time delayed feedback. This circuit is auto gain controlled chaotic oscillator containing time delay. The circuit has feedback systems which control the gain. We investigate synchronization state in coupled time delayed chaotic circuits. By carrying out computer simulations, time delay of subcircuits effects a change of synchronization state.

2. Time Delayed Chaotic Circuit
Figure 1 shows the time delayed chaotic circuit. This circuit consists of one inductor $L$, one capacitor $C$, one linear negative resistor $-g$ and one linear positive resistor $G$ of which amplitude is controlled by the switch containing time delay. The current flowing through the inductor $L$ is $i$, and the voltage between the capacitor $C$ is $v$. The circuit equations are normalized as Eqs. (1) and (2) by changing the variables as below.

(A) In case of switch connected to $-g$,
\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= 2\alpha y - x,
\end{align*}
\]

(B) In case of switch connected to $G$,
\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= -2\beta y - x.
\end{align*}
\]

By changing the parameters and variable as follow:

\[i = \sqrt{\frac{C}{L}} V_{th} x, \quad v = V_{th} y, \quad t = \sqrt{LC} \tau,\]
\[g \sqrt{\frac{C}{L}} = 2\alpha \quad \text{and} \quad G \sqrt{\frac{C}{L}} = 2\beta.\]

The switching operation is shown in Fig. 2, it controls the amplitude of the oscillator. This switching operation is included time delay. $T_d$ denotes the time delay. First, the switch is connected to a negative resistor. In state of that, the voltage $v$ is amplified up to while $v$ is oscillating, the amplitudes exceed the threshold voltage $V_{th}$ which is the threshold control.
Second, the system memorize the time as \( T_{th} \) while \( v \) is exceeding the threshold voltage \( V_{th} \) and that state is remained for \( T_{th} \). In subsequent the instant of exceeding threshold \( V_{th} \), the switch stays the state for \( T_{th} \). After that switch is connected to positive resistor during \( T_{th} \). The switch does not immediately connect to the positive resistor however the switch is connected after \( T_{th} \). A set of switching operations control the amplitude of \( v \). Figure 3 shows chaotic attractor of time delayed chaotic circuit. By using mapping method to this circuit, we could derive the 1-dimensional Poincard map explicitly from each circuit, and the Poincard map was proved to have a positive Liapunov number with computer assistance [3].

3. Ring Coupled Time Delayed Chaotic Circuit

In this section, we investigate synchronization state of the coupling methods of three coupled time delayed chaotic circuits. Figure 4 shows the schematic of coupled three time delayed chaotic circuits. Two cases of interest are considered: coupling elements are resistors \( R_0 \) or inductors \( L_0 \). By changing the parameters and variables as follows:

\[
\begin{align*}
i_n &= \sqrt{\frac{C}{L}} V_{th} x_n, \quad v_n = V_{th} y_n, \quad t = \sqrt{LC} \tau, \\
g\sqrt{\frac{C}{L}} &= 2\alpha, \quad G\sqrt{\frac{C}{L}} = 2\beta \quad \text{and} \quad \gamma = R_0 \sqrt{\frac{C}{L}}.
\end{align*}
\]

The normalized circuit equations of the system are given as follows:

(A) In case of that switch is connected to \(-g\),

\[
\begin{align*}
\dot{x}_n &= y_n \\
\dot{y}_n &= -x_n + 2\alpha y_n + \gamma(y_{n-1} - 2y_n + y_{n+1}), \quad (3)
\end{align*}
\]

(B) In case of that switch is connected to \( G \),

\[
\begin{align*}
\dot{x}_n &= y_n \\
\dot{y}_n &= -x_n - 2\beta y_n + \gamma(y_{n-1} - 2y_n + y_{n+1}), \quad (4)
\end{align*}
\]

where \( (n = 1, 2, 3) \) and \( x_0 = x_3, \quad x_4 = x_1 \). Figure 5 shows some of simulation results. In calculation result, in-phase synchronization state can be observed. When the coupling strength \( \gamma \) is large, full in-phase synchronization can be observed. However full in-phase synchronization can not be observed or synchronization is lost in case of small coupling strength \( \gamma \).

3.2 Coupled by inductors \( L_0 \)

In case of time delayed chaotic circuits coupled by inductors \( L_0 \), Fig 6 shows some of simulation results. By changing the parameters and variables when ring coupled system is connected by the inductor as follows:

\[
i_n = \sqrt{\frac{C}{L}} V_{th} x_n, \quad v_n = V_{th} y_n, \quad t = \sqrt{LC} \tau, \\
g\sqrt{\frac{C}{L}} = 2\alpha, \quad G\sqrt{\frac{C}{L}} = 2\beta \quad \text{and} \quad \gamma' = \frac{L}{L_0}.
\]

The normalized circuit equations of the system are given as follows:

(A) In case of that switch is connected to \(-g\),

\[
\begin{align*}
\dot{x}_n &= y_n \\
\dot{y}_n &= -x_n + 2\alpha y_n + \gamma'(x_{n-1} - 2x_n + x_{n+1}), \quad (5)
\end{align*}
\]

(B) In case of that switch is connected to \( G \),

\[
\begin{align*}
\dot{x}_n &= y_n \\
\dot{y}_n &= -x_n - 2\beta y_n + \gamma'(x_{n-1} - 2x_n + x_{n+1}), \quad (6)
\end{align*}
\]

where \( (n = 1, 2, 3) \) and \( x_0 = x_3, \quad x_4 = x_1 \). In-phase synchro...
nization and three-phase synchronization can be observed in the ring coupled system by inductors \( L_0 \). When the coupling strength \( \gamma' \) is equal to 0.01, synchronization is lost. Consequently, the certain level of coupling strength is required for synchronization.

4. System Including Time Delay in One Direction

The circuit in this study have characteristic time delays methods. We have devised coupled systems as shown in Fig. 7. This system is coupled by resistors \( R_0 \) or inductors \( L_0 \). It is called coupled systems and “system including time delay in one direction”

4.1 Coupled by resistors \( R_0 \)

Now, we use resistors \( R_0 \) to coupling elements. The normalized circuit equations of this system are same to Eqs.(3) and (4). The result as shown in the Fig. 8 can be obtained by difference of coupling strength \( \gamma \). The time waveform of Fig. 8 (a) is in-phase synchronization and the amplitude of \( y_n \) is switching sequentially. However when the coupling strength \( \gamma \) is bigger than 0.1, switching synchronization state is lost and full in-phase synchronization state can be observed. When the coupling strength \( \gamma \) is bigger then 0.1, switching synchronization state is lost and full in-phase synchronization state can be observed.

4.2 Coupled by inductors \( L_0 \)

when we use inductors \( L_0 \) to coupling elements, the result as shown in the Fig. 9 can be obtained by difference of coupling strength \( \gamma' \). Eqs.(5) and (6) are same to the normalized circuit equations of coupled by inductors \( L_0 \). The time waveform of Fig. 9 (a) has the phase difference and the amplitude of \( y_n \) is switching sequentially. However when the coupling
strength $\gamma'$ is bigger then 0.1, switching synchronization state is lost and synchronization can be observed by initial values. Generally switching synchronization can be observed when system including time delay in one direction is coupled by resistors $R_0$ or inductors $L_0$. The amplitude is going divergence and convergence. Additionally the time of divergence and convergence is different.

5. Conclusion

In this study, we investigated synchronization state of novel coupled systems observed from some coupling methods of ring coupled by time delayed chaotic circuits. All the investigated coupling systems 4 types. In case of ring coupled by resistors, we observed in-phase synchronization state. The other case of ring coupled by inductors, in-phase synchronization and three-phase synchronization state can be observed. We devised coupled systems that takes advantage of features of the time delayed chaotic circuit. As a result, some special synchronization state can be observed. The switching of the amplitude of voltage in addition to the in-phase synchronization state can be observed by difference of coupling strength.

References

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