Configuration dependent reflection induced by dissipated localized modes

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We study the one photon scattering problem for a super cavity (SC) coupling with two two-level atoms. With atomic decay, we find a sudden drop in reflection at $\Delta = 0$ for the two atoms in the node-antinode configuration but not in the antinode-node one. The underlying mechanism is due to the scatterer has a configuration dependent localized dissipated eigen-mode at $\Delta = 0$. In the node-antinode configuration, the eigen-mode localized near the input side, which can transport the photon into the SC. By exciting the atom at the node, the photon can leak into the reservoir due to atomic decay, which causes the sudden drop at $\Delta = 0$ in reflection. In the antinode-node case, however, the eigen-mode is localized near the output side, no photon can be transported into the SC and leads to completely reflection at $\Delta = 0$. A similar phenomenon has been observed in a recent experiment of X-ray quantum optics [Nature 482, 199 (2012)] but with a much more complicated explanation due to electromagnetically induced transparency.

I. INTRODUCTION

The studies on photon scattering play an important role in quantum optics. Various significant phenomena in quantum optics are related to scattering, such as electromagnetically induced transparency (EIT) and resonance fluorescence [1, 2]. Photon scattering for models like single or multi atoms coupling with optical cavity [3–6] or waveguide [7–13] have long long been active research area both in theory and experiment. In recent years, a theoretical model based on coupling cavity array (CCA) is proposed and quickly attracts a great deal of attention [14]. CCA is a perfect platform for scattering study, important achievements have been made both on the one dimensional [14–17] and the two dimensional [18] CCA platform.

In nature, the spontaneous emission of the atoms are inevitable due to their coupling with the surrounding electromagnetic environment [19, 20], and the atomic decay has long been included in the scattering research through master equation [11, 21]. But except the intuitive results as the decrease and expansion of the reflection (transmission) peaks, no qualitative difference has been found in the past researches for atoms coupling with waveguide or CCA.

In this paper, we revisit the photon scattering problem for two two-level atoms coupling with the super cavity [22] but include atomic decay. A super cavity (SC) is first present in our former work to help study the photon scattering for a two-level atom coupling with a multi-mode cavity on the CCA platform [23]. In the two atoms case, whether including the atomic decay or not leads to a fundamental difference in the scattering results. Without atomic decay, the scattering results for the two atoms are very similar with the one atom case, configuration of the atoms has no relevant effect [22]. When including the atomic decay, for reflection a dip appears around the resonant energy ($\Delta = 0$) only for the atoms arranged in node-antinode configuration corresponding to the resonant mode. A very similar phenomenon has been observed in Ref. [5] and a complicated theory which attributes it to EIT is given. In this paper, we reveals the physical mechanism behind this phenomenon is much simpler and has nothing to do with EIT.

The rest of the paper is organized as follows. In Sec. [1], we introduce our model and briefly give the results for the condition without atomic decay. In Sec. [11], we use the master equation to handle the photon scattering problem with atomic decay. We reveal the physical mechanism behind the configuration dependent phenomenon. In Sec. [11], single-mode approximation is introduced to support the analysis we give above. Finally a brief conclusion is given.

II. MODEL: TWO TWO-LEVEL ATOMS IN A SUPER-CAVITY

The system we consider consists of three parts, see Fig. 1. The central part contains a SC with two two-level atoms embedded in, where the SC is formed by a 1D single-mode cavity array with $N$ cavities, and these two atoms interacts with two cavities of the SC respectively. The second (third) part is the left (right) photon channel, formed by a semi-infinite 1D cavity array connected to our central part from its left (right) side.

We will study single-photon scattering problem on this system. One photon with wave vector $k$ from the left photon channel is scattered by the SC system. Our aim is to figure out how the reflection and transmission coefficients depend on the position of the two atoms in the SC. More precisely, we will study the cases when one atom is at the node of the resonant mode of the SC while the other is at the antinode. In particular, we are interested
in whether the position order of the two atoms, i.e., the node-antinode configuration or the antinode-node configuration for the resonant mode, is related.

![Diagram](image_url)

**FIG. 1.** (Color online). Schematic set of the single-photon scattering problem for the 1D CCA model. One photon (filled red circle) with the wave vector \( k \) injects from the left side of the SC composed of \( N \) cavities. The SC is formed by a relatively small coupling strength \( \eta \) with the outside cavities. Two two-level atoms (filled blue circle) are in the \( n_1 \)-th and \( n_2 \)-th cavities of the SC respectively. Here we take \( N = 7 \), \( n_1 = 3 \) and \( n_2 = 5 \).

Under the rotating wave approximation, the Hamiltonian of our system is given by

\[
H = H_S + H_L + H_R + H_{SL} + H_{SR},
\]

where

\[
H_S = \sum_{j=1}^{N} \omega_c a_{j}^\dag a_j - \sum_{j=2}^{N} \xi (a_{j-1}^\dag a_j + a_j^\dag a_{j-1}) + \sum_{i=1}^{2} [\omega_e |e_i\rangle \langle e_i| + \Omega (a_{n_i}^\dag \sigma_i^- + H.c.)],
\]

\[
H_L = \sum_{j=-\infty}^{0} [\omega_c a_{j}^\dag a_j - \xi (a_{j-1}^\dag a_j + a_j^\dag a_{j-1})],
\]

\[
H_R = \sum_{j=N+1}^{\infty} [\omega_c a_{j}^\dag a_j - \xi (a_{j-1}^\dag a_j + a_j^\dag a_{j-1})],
\]

\[
H_{SL} = -\eta (a_{n_1}^\dag a_1 + H.c.),
\]

\[
H_{SR} = -\eta (a_{n_2}^\dag a_{N+1} + H.c.).
\]

Here \( H_S \) is the free Hamiltonian for the central part, and \( H_L, H_R \) is the Hamiltonian for the left (right) photon channel. \( a_j (a_j^\dag) \) is the annihilation (creation) operator of the photon in the \( j \)-th cavity, \( |e_i\rangle \) \((i = 1, 2)\) is the excited state of the atom \( i \), \( n_i \) is the label of the cavity that the atom \( i \) interacts with, \( \Omega \) the coupling strength between each atom and its cavity, \( \omega_c \) is the mode frequency for each cavity, \( \omega_e \) is the eigen-energy of the atomic excited state, \( \xi \) is the hopping strength between nearest neighbor cavities within the three parts, and \( \eta \) is the hopping strength between nearest neighbor cavities between the SC system and the left or right photon channel.

Without atomic decay, due to the excitation number is conserved in this model, the scattering state can be expanded as

\[
|\psi_k^{(+)}\rangle = \sum_{l} C_l |l; g_1, g_2\rangle + \alpha_1 |\text{vac}; e_1, g_2\rangle + \alpha_2 |\text{vac}; g_1, e_2\rangle,
\]

with

\[
C_l = \begin{cases} 
  e^{ikl} + r e^{-ikl}, & l < 0, \\
  c_1 e^{ikl} + \rho_l e^{-ikl}, & 0 < l < n_1, \\
  c_2 e^{ikl} + d_2 e^{-ikl}, & n_1 < l < n_2, \\
  c_3 e^{ikl} + d_3 e^{-ikl}, & n_2 < l < N, \\
  t e^{ikl}, & l > N.
\end{cases}
\]

Here the parameters \( t \) and \( r \) are the single-photon transmission and reflection amplitudes, respectively. Meanwhile the wave function must be continuous at nodes 0, \( n_1, n_2 \) and \( N \). According to the scattering theory [24], the scattering state \( |\psi_k^{(+)}\rangle \) is an eigen-state of the Hamiltonian \( H \) with eigen-energy \( E_k \), i.e., we have

\[
H |\psi_k^{(+)}\rangle = E_k |\psi_k^{(+)}\rangle.
\]

Numerically solving Eq. (9) we can obtain the transmission \( (T = |t|^2) \) and reflection \( (R = |r|^2) \) coefficients. The case without decay has been thoroughly investigated in our precious work [22]. As Fig. 2 shows, no qualitative difference appears in transmission or reflection between arranging the atoms in node-antinode and antinode-node configurations. Actually the result is quite similar with the one-atom situation [23].

![Graph](image_url)

**FIG. 2.** (Color online). (a) Single-photon reflection for system without decay. (b) Single-photon transmission for system without decay. Blue solid line represents the atoms in node-antinode (8-12) configuration while the green solid line is for the antinode-node (12-16) configuration. Here, \( N = 31 \), \( \eta = 0.01 \) and \( \Omega = 0.1 \).

### III. SCATTERING WITH DECAY

Now we consider the situation with the atomic decay. Since the two atoms locate in two distant cavities, it is reasonable to assume that each atom is coupling with an independent reservoir. Then the dynamics of our system is controlled by the master equation [2]

\[
\frac{d \rho(t)}{dt} = -i[H, \rho(t)] + \sum_{l=1}^{2} \frac{\gamma}{2} (2\sigma_i^- \rho(t) \sigma_i^+ - |e_i\rangle \langle e_i| \rho(t) - \rho(t) |e_i\rangle \langle e_i|),
\]

where \( \gamma \) is the spontaneous decay rate of the atomic excited state. Here we assume the decay rate for each atom is the same.
The steady state for our scattering problem is

$$
\rho = |\Psi\rangle\langle\Psi| + \kappa |G\rangle\langle G|,
$$

with $|G\rangle = |\text{vac}; g_1, g_2\rangle$ being the ground state of our system, and $|\Psi\rangle$ being the scattering state

$$
|\Psi\rangle = |k\rangle_L + r| - k\rangle_L + \sum_{j=1}^{N} c_j |j\rangle + d_1 |e_1\rangle + d_2 |e_2\rangle + t |k\rangle_R,
$$

where

$$
|k\rangle_L = \sum_{j=-\infty}^{0} e^{ikj} |j\rangle,
$$

$$
|k\rangle_R = \sum_{j=N+1}^{\infty} e^{ikj} |j\rangle,
$$

with $|j\rangle = \sigma^j_\downarrow |G\rangle$ and $|e_1\rangle = \sigma^+ \downarrow |G\rangle$. Then the time independent master equation implies that the scattering state $|\Psi\rangle$ satisfies

$$
(H - i\frac{\gamma}{2} \sum_{l=1}^{2} |e_l\rangle\langle e_l|) |\Psi\rangle = E_k |\Psi\rangle,
$$

where $E_k = \omega_k - 2\eta \cos k$. So we can use the effective Hamiltonian $H_{\text{eff}} = H - i\frac{\gamma}{2} \sum_{l=1}^{2} |e_l\rangle\langle e_l|$ to describe this decay system [10, 21]. Note that Eq. (15) is sufficient to determine the transmission coefficient $T = |r|^2$ and the reflection coefficient $R = |r|^2$.

In Fig. 3, we give the numerical results for single-photon transmission and reflection coefficients varying with $\Delta$. Here, $\Delta = E_k - E_n$ is the energy difference between the incoming photon and the resonant mode of the SC. For transmission, we see two peaks in our selected region of $\Delta$ and their space between is configuration-dependent. This result is quite similar with the case without atomic decay (see Fig. 2), and due to the atomic decay the transmission has a dramatic decline. As for reflection, despite the peaks we expect to appear at the same positions corresponding to the transmission, one more peak emerges or in another word the reflection experience a sudden drop at $\Delta = 0$ only for the node-antinode configuration. This is a major difference comparing with the situation without the atomic decay. A qualitatively similar phenomenon has been observed in Ref. [2], which is explained with a much complicated model. Moreover, the influence of the atomic decay on reflection is much smaller. Next we will try to figure out the physical mechanism underlying this interesting configuration-dependent phenomenon.

Similarly as that discussed in Ref. [2], the reflection coefficient $R$ at $E_k$ for our setting is essentially determined by the eigen-modes of $H_S$ near resonant with $E_k$. The obvious qualitative difference in $R$ between the two cases for the two atoms locating in the node-antinode configuration or in the antinode-node configuration occurs at $\Delta = 0$, i.e., the input photon with energy resonant with the resonant eigen-mode of the empty SC. In our model the scatterer consists of SC and two atoms resonant coupling with SC’s $n$-th eigen-mode (with eigenvalue $E_n$), we can prove that $E_n$ is also an eigen-value of the scatterer ($H_S$) only if either atom is located at the node of the mode. Thus, the condition is met for the above two configurations, and $E_n$ will still be an eigen-value of the scatterer in both situations. But no peak appears at $\Delta = 0$ for the reflection or transmission without the atomic decay (see Fig. 2) which violates the resonant tunneling assumption. Therefore we need to analyze the eigen-mode of the SC system when $\Delta = 0$.

In order to clarify the confusion, Fig. 3(a) shows this special mode with atoms arranged in two configurations. This mode is localized and its localization condition is configuration dependent. As for the node-antinode configuration the mode localized between the left wall of the SC and the atom at node while for the antinode-node configuration it localized between the atom at node and the right wall. The appearance of this localized mode is due to coupling between the node atom and the non-resonance modes [23], and the antinode atom should not be excited and there are no photons around it.

Due to the mode is localized, no photon can be transported through the SC under this incoming energy. So the reflection (transmission) shows no peak at $\Delta = 0$ without the atomic decay, no matter atoms arranged in which configuration. When the atoms coupling with the reservoir, new photon leakage way is introduced due to spontaneous emission. Thus mode localized in different way can lead to fundamental difference. For antinode-node configuration, the mode is localized at the output (right) side of the SC so the incoming photon from left to the SC is totally reflected by the left wall. Then with or without the atomic decay makes no difference since no photon goes into SC at all, so $R = 1$ ($T = 0$) at $\Delta = 0$. For the node-antinode configuration, the photon
has probability to go into the SC as the mode localized at the input (left) side of the SC. The incoming photon will excite the atom at node and then leak into the reservoir due to spontaneous emission. This leads to the appearance of the sudden drop of reflection at $\Delta = 0$.

Next we calculate the photon flow inside the SC at $\Delta = 0$. The photon flow for the $l$-th cavity is defined as

$$J_l = -i[C_l(C_{l+1}^* - C_l^*) + C_l^*(C_{l+1} - C_l)].$$

Fig. 4(b) shows no photon flow within the SC for the atoms arranged in antinode-node configuration, meanwhile for the node-antinode configuration there is a steady photon flow name $J_s$ before the node atom. This confirms the analysis we give above. Further, the steady incoming photon is $J_i = 2 \sin k$, the leaking rate $L$ of the photon into the reservoir is defined as $L = J_s/J_i$. Then we numerically obtain $L + R + T = 1$ as expected. Another major consequence by introducing the decay is the decline of the transmission. Under the above mentioned parameter condition $\gamma = 10^{-5}$, the transmission is so small that can be ignored (see Fig. 3). Thus closing the transmission channel by setting $\eta = 0$ for the right wall of the SC will cause little change to the above results but making our model much similar with the experiment condition [3].

The localized eigen-mode can be analytically solved. When the wave vector of the injection photon is $k = \frac{l\pi}{N+1}$, the atom $n_l$ in the node implies that $\sin(kn_l) = 0$, and the atom in the antinode implies that $|\sin(kn_j)| = 1$. For example, $\sin(kn_1) = 0$ and $|\sin(kn_2)| = 1$ in the case of node-antinode configuration. In this case, we find an analytical solution of the eigen-mode of $H_s$ with eigenvalue $E_i = \omega_e - 2 \cos k$:

$$|\psi_i\rangle = \sum_{j=1}^{N} b_j |l_j; g_1, g_2\rangle + \alpha |\text{vac}; e_1, g_2\rangle,$$  

where

$$b_j = \begin{cases} \sin(kj)\sqrt{\frac{\eta}{2} + \frac{\sin^2 k}{g^2}}, & 1 \leq j \leq n_1, \\ 0, & n_1 + 1 \leq j \leq N, \end{cases}$$  

$$\alpha = -\frac{b_{n_1+1}}{g}.$$  

The localized mode for antinode-node configuration is almost the same only with the state localized to the right.

IV. SINGLE MODE APPROXIMATION

For $\Delta = 0$, we believe only the localized mode is important and this is the starting point of our analysis above. To prove the validness of this assumption, we introduce the single mode approximation Hamiltonian of the scatterer

$$H_S = E_i |\psi_i\rangle \langle \psi_i|$$

while keeping $H_L, H_R, H_{SL}$ and $H_{SR}$ the same. Now the scattering state can be expanded as

$$|\Psi_k^{(+)}\rangle = |\varphi_k\rangle + r|\varphi_k\rangle + \mu |\psi_i\rangle + t|\phi_k\rangle$$  

with

$$\begin{cases} |\varphi_k\rangle = \sum_{j=-\infty}^{0} e^{ikj}|j\rangle, \\ |\phi_k\rangle = \sum_{j=N+1}^{\infty} e^{ikj}|j\rangle. \end{cases}$$

Without the atomic decay, through Eq. (9) we have

$$\Delta' \mu + \eta b_1 (1 + r) + t \eta b_N e^{ik(N+1)} = 0,$$  

$$\eta b_1 - (e^{ik} + re^{-ik}) = 0,$$  

$$\eta b_N - t e^{ikN} = 0$$

with $\Delta' = E_k - E_i$. For the antinode-node configuration, $b_1 = 0$ leads to $R = |r|^2 = 1$. While for the node-antinode case, $b_N = 0$ leads to $t = 0$ and we can analytically solve $r$ as

$$r = -\frac{\Delta' e^{ik} + b_1 \eta^2}{b_1 \eta^2 + \Delta' e^{-ik}}$$

leading to $R = |r|^2 = 1$.

For the condition with atomic decay, we use the standard master equation to handle. The master equation for steady state is

$$-i[H, \rho] = i\Gamma (\rho \sigma^+_1 \sigma^-_1 + \sigma^+_2 \sigma^-_2 + \Gamma \sigma^-_1 \rho \sigma^+_1) = 0$$

with

$$\rho = \kappa |\psi_i\rangle \langle \psi_i| + (1 - \kappa)|\text{vac}\rangle \langle \text{vac}|.$$
Projecting the master equation to various bases, we obtain the following independent equations
\[
(\Delta' + i\Gamma|\alpha|^2/2)\mu + i\eta b_1(1 + r) + t\eta b_N e^{ik(N+1)} = 0,
\]
\[
\eta b_1\mu - (e^{ik} + re^{-ik}) = 0,
\]
\[
\eta b_N\mu - te^{ikN} = 0,
\]
which can be analytically solved:
\[
r = \frac{e^{ik} - \eta b_1\beta}{\eta b_1\beta - e^{-ik}},
\]
where \(\beta = \frac{\eta b_1}{1 + r e^{2i\Delta'}}\). For antinode-node configuration, \(b_1 = 0\) and we obtain the same result \(R = 1\) as the condition without the atomic decay. Meanwhile for the node-antinode configuration, Eq. \(32\) can perfectly give the reflection coefficient around \(\Delta = 0\) as show in Fig. 5. Here, the parameters are the same as in Fig. 4.

**V. CONCLUSION**

In conclusion, based on the 1D CCA platform we investigate the single-photon scattering problem with a SC coupling with two atoms under the condition with decay. Compared with the condition without atomic decay, the reflection with decay shows a significant difference, it will drop suddenly (peak) at \(\Delta = 0\) only for the node-antinode configuration. We propose the EIT like phenomenon is actually not determined by EIT mechanism. It is due to the special eigen-mode condition for the scatterer at \(\Delta = 0\). This eigen-mode is localized and its localization condition is configuration dependent, so photon can not be transported through this mode. This explains why without the atomic decay, \(R = 1\) at \(\Delta = 0\) in any of the two configurations. With atomic decay, the eigen-mode for node-antinode configuration can transport photon into the SC, and the photon can leak into the reservoir through exciting the atom at the node. So the reflection shows a sudden drop at \(\Delta = 0\). Meanwhile, no photon can be transported into the SC through the eigen-mode for antinode-node configuration which leads to \(R = 1\) at \(\Delta = 0\). We calculate the photon flow and use the results of the single-mode approximation to support our analysis. We hope our analysis can help understand the experiment results and enlighten the study for using 1D CCA to simulate real experiment settings.

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**FIG. 5.** (Color online). Reflection vs \(\Delta\) around \(\Delta = 0\). The blue solid line represents the exact numerical result, the yellow dashed line is obtained through Eq. \(32\). Here, the parameters are the same as in Fig. 4.
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