MODEL ATMOSPHERES AND X-RAY SPECTRA OF BURSTING NEUTRON STARS:
HYDROGEN-HELium COMPTONIZED SPECTRA

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ABSTRACT

Compton scattering plays a crucial role in determining the structure of the atmosphere of an X-ray burster and its theoretical spectrum. Our paper presents a description of the plane-parallel model atmosphere of a very hot neutron star and its theoretical flux spectrum of outgoing radiation. Our model equations take into account all bound-free and free-free monochromatic opacities relevant to hydrogen-helium chemical composition and take into account the effects of Compton scattering of radiation in thermal plasma with fully relativistic thermal velocities. We use Compton scattering terms in the equation of transfer, which precisely describe photon-electron energy and momentum exchange for photons with initial energies exceeding the electron rest mass of 511 keV. Model atmosphere equations are solved with the variable Eddington factors technique. The grid of H-He model atmospheres and flux spectra is computed on a dense mesh of 10^7 K and a surface gravity of log g. In many cases, the assumed log g approached the critical gravity log g_{cr}, i.e., the Eddington limit. We confirm that H-He spectra of X-ray bursters deviate from blackbody spectra and discuss their shapes. The table of color to effective temperature ratios shows that theoretical values of T_c/T_{eff} do not exceed 1.9 in H-He atmospheres in hydrostatic and radiative equilibrium.

Subject headings: radiative transfer — scattering — stars: atmospheres — stars: neutron — X-rays: bursts

1. INTRODUCTION

The scattering of photons by free electrons is a very important opacity and emission process in stellar atmospheres and accretion disks. In most existing research papers and numerical methods, the process is routinely approximated by coherent and isotropic photon-electron collision. A scattered photon can change the direction of propagation, whereas its frequency remains unaltered (Thomson approximation), and differential cross section for scattering only weakly depends on the scattering angle and is considered as an isotropic process. Such an approximation causes the scattering emission j_{sc} = \sigma_T j_{\nu}, where \sigma_T denotes the monochromatic mean intensity of radiation. The monochromatic source function in the equation of transfer exhibits a very simple form, S^\nu = J_{\nu}, and involves only the angular integration of specific intensity I_{\nu}.

The assumption of coherent Thomson electron scattering causes decoupling between the radiation field and local temperature, particularly in hot stellar atmospheres on the main sequence, where scattering exceeds thermal opacity by a few orders of magnitude (Mihalas 1978). This is because temperature (or Planck function B_{\nu}) does not appear in the scattering source function S^\nu.

The realistic scattering process is referred to as Compton scattering. Scattering of photons by electrons at rest always causes electron recoil and, simultaneously, a decrease of photon energy (redshift). The increase of photon wavelength is equal to a fraction of \Delta \lambda_C = 0.026 \AA (Compton wavelength), depending on the cosine of the scattering angle (Rybicki & Lightman 1979). Therefore, the relative change of photon energy and energy transfer are most significant in X-rays, where initial wavelengths are comparable to the Compton wavelength. Both changes are negligible for visual or even infrared photons.

Compton scattering of photons by electrons in thermal motion, and the corresponding redistribution of final photon energy, is much more complicated. Principles of energy and momentum conservation imply that photons and free electrons in thermal motion change their momenta and energies at the time of collision. The photon redistribution function has to account for three physical effects: (1) reddening of scattered photons due to electron recoil, (2) broadening of the initial spectrum due to chaotic thermal motion of electrons, and (3) the blueshift of incident photons, i.e., inverse Compton scattering (Pomraning 1973). Display of all three effects makes it possible to maintain detailed balance between free electrons and photons in thermodynamic equilibrium.

Rybicki & Lightman (1979) showed that, in the case of diffusion approximation in energy space (\nu \ll mc^2) and nonrelativistic Maxwellian electron velocity distribution, the average change of photon energy per scattering is given by

\frac{\Delta \nu}{\nu} = \frac{4kT - \hbar \nu}{mc^2}, \tag{1}

and that energy change is negative for photons of energy significantly exceeding the local gas temperature T.

In a stellar atmosphere with no external irradiation, temperature usually decreases outward, if we neglect non-LTE
(NLTE) effects, which dominate mostly in the highest line-forming regions. Electron gas in or above the photosphere is then illuminated by radiation flux originating in deeper and hotter layers. The difference between gas and radiation temperatures causes hot photons to typically lose energy at the time of scattering and electrons to heat up. There are two consequences of that situation. First, the whole hard energy branch of the spectrum gets steeper than in the Thomson scattering model atmosphere and exhibits deficiency of photons. In addition, the spectrum around the peak flux is distorted in a characteristic way (Rybicki & Lightman 1979). Second, the temperature in the uppermost layers of the atmosphere rises, which has a direct impact on line profiles and their equivalent widths. Both effects are closely related.

Many papers have derived various types of the equation of transfer with Compton scattering terms, starting with Dirac (1925), who studied the importance of Compton scattering in line wings emitted from the solar limb. Kompaneets (1957) proposed an approximation to Compton scattering, which was frequently used for the study of scattering, mostly in simplified cases in which semianalytic considerations were applicable. Model atmospheres of X-ray bursters were treated in such a way by Ebisuzaki, Hanawa, & Sugimoto (1984), Ebisuzaki (1987), and Babul & Paczyński (1987), for example. First, a useful set of model atmospheres of X-ray bursters Madej & Rożanska (2000a, 2000b), and Joss & Madej (2001) abstract (No. 214: “Theoretical Spectra of Unmagnetized Neutron Stars”) is available under the category: “Supernovae, Supernova Remnants and Isolated Neutron Stars” at the Two Years of Science with Chandra Web site: http://cxc.harvard.edu/symposium_2001/abstracts_list.html. A critical review of various techniques was recently given by London, Taam, & Howard (1986), who computed NLTE model atmospheres subject to hydrostatic and radiative equilibrium. In the latter paper and in, e.g., Ross, Fabian, & Mineshige (1992), Nobili, Turolla, & Zampieri (1993), Titarchuk (1994), Madej (1994), and Hubeny et al. (2001), authors use the Kompaneets equation or its approximate forms, both in stellar atmospheres and in accretion disk atmospheres.

There are a few important approximations connected with the Kompaneets equation, which are removed in our paper. First, our equation of transfer allows for a nonisotropic radiation field; nevertheless, in the present stage of computations, we assume isotropy of radiation in the Compton scattering terms. Second, our model equations do not make any assumptions about gas temperature, allowing electrons to have fully relativistic thermal (isotropic) velocities. Third, in our treatment initial energies of photons can be much larger than the electron rest mass, whereas the Kompaneets equation is only valid in the low-energy limit. Eventually, the Compton scattering energy redistribution function is carefully computed here, following the method by Guilbert (1981). The redistribution function in the Kompaneets approximation is infinitely narrow, and therefore, some physical implications can be lost there.

Other papers on the transfer problem with realistic Compton scattering terms include Sampson (1959), Madej (1989, 1991a, 1998), van Teeseling, Heise, & Paerels (1994), and Zane et al. (1996). A critical review of various techniques was recently given by Psaltis & Lamb (1997).

The equations, Compton redistribution functions, and the method of solution used in §§ 2–5 are improved versions of those published in Pomraning (1973), Madej (1989, 1991a), Madej & Rożanska (2000a, 2000b), and Joss & Madej (2001). A useful set of model atmospheres of X-ray bursters was previously published by Madej (1991b).

In §§ 2–5 we discuss scattering emission, the source function, and the equation of transfer for unpolarized radiation and solve model equations of a very hot neutron star atmosphere. We also show that noncoherent Compton scattering closely relates properties of the radiation field and the temperature of the gas and even dominates heating effects in the uppermost layers of a neutron star atmosphere.

Section 2 presents a brief description of equations that define a model atmosphere in radiative and hydrostatic equilibrium. The equations of radiative transfer and radiative equilibrium take into account Compton scattering terms, which influence both the run of temperature and the outgoing spectrum of the model.

The Compton scattering energy redistribution function was computed allowing for a large energy change at the time of single scattering of electrons with a relativistic thermal velocity distribution (Guilbert 1981). We also consider the decrease of the total Compton scattering cross section with initial photon energy, and we interpolate precomputed tables of this quantity after each temperature iteration, which allow for electron thermal motion. Therefore, our cross sections are superior to the Klein-Nishina expression, which is valid only for electrons at rest. The following equations were used for the computation of radiative transfer with Compton scattering and the variable Eddington factors, for photons with initial energy exceeding the electron rest mass in some models.

### 2. MODEL ATMOSPHERE EQUATIONS

The equation of radiative transfer in the static atmosphere of a nonrotating neutron star (planar geometry) can be written in the following form:

\[
\frac{\partial I(z, \mu)}{\partial z} = \kappa_0 \left(1 - e^{-h/\delta}\right) (B_\nu - I_\nu) + \int_0^\infty \frac{d\nu'}{4\pi} \left(\frac{1 + e^2}{2h}\right)^\nu I_\nu(z, \nu') \frac{d\nu'}{\nu'} \sigma(\nu' - \nu, \nu') I_\nu(z, \nu') d\nu'
\]

Second, the temperature in the uppermost layers of the atmosphere rises, which has a direct impact on line profiles and their equivalent widths. Both effects are closely related.

\[
\left. I_\nu(z, \mu) \right|_{0}^{\infty} = \kappa_0 \left(1 - e^{-h/\delta}\right) (B_\nu - I_\nu) - I_\nu(z, \mu) + j_\nu^\infty, \tag{2}
\]

assumed that sources of true absorption (\(\kappa_0\)) and thermal emission (\(j_\nu = \kappa_0 B_\nu\)) are in LTE. Compton scattering monochromatic emission, \(j_\nu^\infty\), fully includes stimulated scattering corrections. Equation (2) does not include relativistic corrections on the neutron star surface. In the case of a nonrotating neutron star, relativistic corrections would simply cause a shortening of the independent variable \(dz\). The above equation of transfer was a starting point in earlier investigations (Sampson 1959; Madej 1989, 1991a, 1991b).

Equation (2) also includes NLTE noncoherent Compton scattering terms. The variable \(\sigma_0\) denotes the coefficient of Compton scattering at a given frequency \(\nu\) integrated over all incoming frequencies \(\nu'\). The variable \(\sigma(\nu' - \nu, \nu')\) denotes the differential Compton scattering cross section, and \(\kappa_0\) is the absorption coefficient uncorrected for stimulated emission. All the opacity coefficients are given for 1 g of matter (units of \(\text{cm}^2\,\text{g}^{-1}\)). The relation between the differential Compton scattering cross section and the integrated Compton scattering coefficient is given by the following equation:

\[
\sigma_\nu = \int_0^\nu \frac{d\nu'}{4\pi} \int_0^{\nu'} \sigma(\nu' - \nu', \nu') d\nu'. \tag{3}
\]
We stress here that, although thermal absorption and emission remains in LTE, the presence of Compton scattering integrals implies strong deviations from LTE in the corresponding part of equation (2). In the case of atmospheres of hot neutron stars composed of hydrogen and helium alone, both species are practically completely ionized. Therefore, the only source of equation (2). In the case of atmospheres of hot neutron stars implies strong deviations from LTE in the corresponding part remains in LTE, the presence of Compton scattering integrals.

We know that the Compton scattering differential cross section must fulfill the relation

$$\sigma(\nu \to \nu', \mathbf{n} \cdot \mathbf{n}') \nu^2 e^{-h\nu/kT} = \sigma(\nu' \to \nu, \mathbf{n}' \cdot \mathbf{n}) \nu'^2 e^{-h\nu'/kT},$$

which results from the detailed balancing of this process in global thermodynamic equilibrium (Pomraning 1973). Moreover, we define new variables,

$$\kappa_{\nu} = \kappa_{\nu}'(1 - e^{-h\nu/kT}),$$

and

$$\sigma(\nu \to \nu', \mathbf{n} \cdot \mathbf{n}') = \sigma_{\nu}(\nu, \nu', \mathbf{n} \cdot \mathbf{n}),$$

where the Compton scattering redistribution function $\phi$ is normalized to unity,

$$\int_{\omega} \frac{d\omega'}{4\pi} \int_0^\infty \phi(\nu, \nu', \mathbf{n} \cdot \mathbf{n}) d\nu' = 1.$$  

Then we write the equation of transfer (eq. [2]) on the monochromatic optical depth scale $d\tau_{\nu} = -\kappa_{\nu}(1 + \sigma_{\nu}) \rho dz$,

$$\mu \frac{dI_{\nu}}{d\tau_{\nu}} = I_{\nu} - \epsilon_{\nu} B_{\nu} - (1 - \epsilon_{\nu}) J_{\nu} + (1 - \epsilon_{\nu}) J_{\nu},$$

$$\times \int_0^\infty \Phi(\nu, \nu') \left( 1 + \frac{e^2}{2h\nu^2} I_{\nu} \right) d\nu'$$

$$- (1 - \epsilon_{\nu}) \left( 1 + \frac{e^2}{2h\nu^2} J_{\nu} \right)$$

$$\times \int_0^\infty \Phi(\nu, \nu') J_{\nu} \left( \frac{\nu}{\nu'} \right)^3 \exp \left[ - \frac{h(\nu - \nu')}{kT} \right] d\nu',$$

where the angle-dependent Compton scattering redistribution function $\phi$ was approximated by its zeroth angular moment,

$$\Phi(\nu, \nu') = \int_{\omega} \phi(\nu, \nu', \mathbf{n} \cdot \mathbf{n}) \frac{d\omega'}{4\pi}.$$  

The dimensionless monochromatic absorption is defined as $\epsilon_{\nu} = \kappa_{\nu}/(\kappa_{\nu} + \sigma_{\nu})$. We point out here that the equation of transfer (eq. [8]) is not linear and is of quadratic form with respect to the unknown $J_{\nu}$. In our computational algorithm we substituted the mean intensities $J_{\nu}$ or $J_{\nu}'$ with values from the previous temperature iteration in stimulated scattering terms, $1 + e^2/2h\nu^2 J_{\nu}$. In such a way, the equation of transfer (eq. [8]) became linear and therefore can be solved by the Feautrier method.

The exact derivation of equation (8) and the numerical method of solution has been given in earlier papers (Madej 1989, 1991a; Madej & Różańska 2000a, 2000b). Here, we stress that the above equation assumes that the Compton scattering coefficient is isotropic. This is also not true in the absence of the magnetic field. Pomraning (1973) and Psaltis (2001) have formulated the corresponding equations of transfer in the case of nonisotropic Compton scattering, which included higher moments of angular distributions of $\phi_{\nu}$. However, we solve the model atmosphere problem assuming that Compton scattering is isotropic. In practical computations, we always determine the differential Compton scattering cross section for eight discrete cosines of scattering angle and then averaged them over the whole sphere.

Pomraning (1973) noted that estimated errors introduced in model atmosphere computations by the assumption of isotropic electron scattering are similar in value to errors caused by neglecting the linear polarization of radiation. We do not solve here the equation of transfer for polarized radiation; therefore, we assumed that the inclusion of nonisotropy of the Compton scattering process is useless in our approximation.

We did not mention here recent papers on Compton scattering treated in the diffusion approximation (with the Kompaneets equation). Our equations and the Compton redistribution functions (Guilbert 1981) work correctly in cases of both large and small energy exchanges between an X-ray photon and a free electron at the time of scattering. Our model atmosphere and theoretical spectra represent the most accurate quantum mechanical solution of the Compton scattering problem, which is available in the literature. The above equations also produce accurate solutions in cases in which the initial photon energy before or after scattering exceeds electron rest mass ($m_e c^2 = 511$ keV).

We transform the equation of transfer (eq. [8]) in the standard manner. Zeroth- and first-order moments of the equation yield the ordinary differential equation of the second order,

$$\frac{d^2}{d\tau_{\nu}^2} \left( f_{\nu} J_{\nu} \right) = \epsilon_{\nu} \left( J_{\nu} - B_{\nu} \right) + (1 - \epsilon_{\nu}) J_{\nu} \int_0^\infty \Phi_1(\nu, \nu') d\nu'$$

$$- (1 - \epsilon_{\nu}) \int_0^\infty J_{\nu} \Phi_2(\nu, \nu') d\nu'.$$

The variable $f_{\nu}$ denotes the usual variable Eddington factor, $K_{\nu} = f_{\nu} J_{\nu}$, and the auxiliary functions $\Phi_1$ and $\Phi_2$ are defined as

$$\Phi_1(\nu, \nu') = \left( 1 + \frac{e^2}{2h\nu^2} J_{\nu} \right) \Phi(\nu, \nu'),$$

$$\Phi_2(\nu, \nu') = \left( 1 + \frac{e^2}{2h\nu^2} J_{\nu} \right) \left( \frac{\nu}{\nu'} \right)^3 \exp \left[ - \frac{h(\nu - \nu')}{kT} \right] \Phi(\nu, \nu').$$

Both functions $\Phi_1$ and $\Phi_2$ contain stimulated scattering correction factors and strictly fulfill the detailed balancing condition in thermodynamic equilibrium.

In global thermodynamic equilibrium we have $J_{\nu} = B_{\nu}$. Therefore, the equation of transfer reduces to the identity,

$$B_{\nu} \int_0^\infty \Phi_1(\nu, \nu') d\nu' - \int_0^\infty B_{\nu} \Phi_2(\nu, \nu') d\nu' = 0.$$
In practical computations we express the scattering integrals in equation (10) by quadrature sums, therefore introducing some error. We can also compute the left-hand side of the above identity by the same quadrature sums and also obtain some nonzero error. To minimize errors in our calculations, we always subtract discretized integrals of equation (13) from the right-hand side of the equation of transfer (eq. [10]).

\[
\frac{d^2}{d\tau^2} (f_\nu I_\nu) = \epsilon_\nu (J_\nu - B_\nu)
+ (1 - \epsilon_\nu) (J_\nu - B_\nu) \int_0^\infty \Phi_1(\nu, \nu') d\nu'
- (1 - \epsilon_\nu) \int_0^\infty (J_\nu' - B_\nu') \Phi_2(\nu, \nu') d\nu'.
\] (14)

Such a strategy was absolutely necessary to obtain a working code converging to a model atmosphere in radiative equilibrium.

The above equation of transfer is accompanied by two boundary conditions, the first being

\[
\frac{d}{d\tau} (f_\nu I_\nu) = H_\nu(0)
\] (15)

at the top of the model atmosphere (\(\tau_\nu = 0\)). At the bottom, we require \(J_\nu = B_\nu\) according to the usual thermalization condition.

In this paper we solve the above equation precisely iterating Eddington factors \(f_\nu\) at all frequencies and optical depth points. In such a way, we exactly reproduce the angular behavior of the radiation field and determine angular stratification of the specific intensity \(I_\nu(\tau, \mu)\). Note that the angular distribution of \(I_\nu(\tau, \mu)\) and its nonisotropy can be determined in the frame of the Eddington approximation \(f_\nu = 1/3\) as well, but with lower accuracy.

3. LINEARIZATION AND DISCRETIZATION

Static model atmospheres of neutron stars are subject to the constraint of radiative equilibrium, and the constraint of hydrostatic equilibrium, simultaneously. The condition of radiative equilibrium implies, on each depth level \(\tau_\nu\), that

\[
\int_0^\infty H_\nu(\tau_\nu) d\nu = \frac{\sigma_R T_\nu^4}{4\pi},
\] (16)

where \(\sigma_R = 5.66961 \times 10^{-5}\) (in cgs units). Taking the derivative \(d/d\tau = d/d\tau_\nu (d\tau_\nu/d\tau)\) of the above equation, we obtain the alternative form of the equation of radiative equilibrium,

\[
\int_0^\infty \eta_\nu \epsilon_\nu (J_\nu - B_\nu') d\nu + \int_0^\infty \eta_\nu (1 - \epsilon_\nu) J_\nu' d\nu
\times \int_0^\infty \Phi_1(\nu, \nu') d\nu'
- \int_0^\infty \eta_\nu (1 - \epsilon_\nu) J_\nu' \Phi_2(\nu, \nu') d\nu' = 0,
\] (17)

where \(\eta_\nu = (\kappa_\nu + \sigma_\nu)/(\kappa + \sigma)_{\rm std}\). The functions \(B_\nu', \Phi_1', \Phi_2'\) are computed at the unknown temperature \(T^*\), at which point they strictly fulfill the constraint of radiative equilibrium.

The second condition of hydrostatic equilibrium can be expressed in the usual form,

\[
\frac{dP_g}{dT} = \frac{g}{(\kappa + \sigma)_{\rm std}} \quad \frac{dP_r}{dT} = \frac{g}{(\k + \sigma)_{\rm std}} - \frac{4\pi}{c} \int_0^\infty \eta_\nu H_\nu d\nu.
\] (18)

The constraint of radiative equilibrium (eq. [17]) is linearized with respect to the unknown temperature stratification in three different variables (Madej 1991a, 1998; Madej & Różańska 2000b),

\[
B_\nu'(\tau) = B_\nu(\tau) + \left( \frac{\partial B_\nu}{\partial T} \right)_\tau \Delta T(\tau),
\] (19)

\[
\Phi_1'(\nu, \nu') = \Phi_1(\nu, \nu') + \left( \frac{\partial \Phi_1}{\partial T} \right)_\tau \Delta T(\tau),
\] (20)

\[
\Phi_2'(\nu, \nu') = \Phi_2(\nu, \nu') + \left( \frac{\partial \Phi_2}{\partial T} \right)_\tau \Delta T(\tau),
\] (21)

where \(\Delta T = T^* - T\), with variables \(T\) and \(T^*\) denoting the actual and final values of temperature, respectively. Introducing the above linearized perturbations into the equation of radiative equilibrium (eq. [17]) and solving for \(\Delta T\), we obtain

\[
\Delta T(\tau) = \frac{\int_0^\infty \eta_\nu \epsilon_\nu (J_\nu - B_\nu') d\nu + L(\tau)}{\int_0^\infty \eta_\nu \epsilon_\nu (\partial B_\nu/\partial T)_\tau d\nu - L'(\tau)}.
\] (22)

The new functions are defined as

\[
L(\tau) = \int_0^\infty \eta_\nu (1 - \epsilon_\nu) J_\nu' d\nu \int_0^\infty \Phi_1(\nu, \nu') d\nu' - \int_0^\infty \eta_\nu (1 - \epsilon_\nu) J_\nu' \Phi_2(\nu, \nu') d\nu',
\] (23)

\[
L'(\tau) = \int_0^\infty \eta_\nu (1 - \epsilon_\nu) J_\nu' d\nu \int_0^\infty \Phi_1(\nu, \nu') d\nu' - \int_0^\infty \eta_\nu (1 - \epsilon_\nu) J_\nu' \Phi_2(\nu, \nu') d\nu'.
\] (24)

The prime denotes a partial derivative of the corresponding functions with respect to temperature \(T\). The final linearized form of the useful equation of transfer (eq. [14]) can be expressed as

\[
\frac{d^2}{d\tau^2} (f_\nu I_\nu) = \epsilon_\nu (J_\nu - B_\nu) + (1 - \epsilon_\nu) J_\nu
\times \left[ \int_0^\infty \Phi_1(\nu, \nu') d\nu' - (1 - \epsilon_\nu) J_\nu \Phi_2(\nu, \nu') d\nu' \right]
- \left[ \epsilon_\nu \frac{\partial B_\nu}{\partial T} - (1 - \epsilon_\nu) J_\nu \frac{\partial \Phi_1}{\partial T} d\nu' + (1 - \epsilon_\nu) J_\nu \frac{\partial \Phi_2}{\partial T} d\nu' \right]
\times \left( \int_0^\infty \eta_\nu \epsilon_\nu (J_\nu - B_\nu) d\nu + L(\tau) \right)
\int_0^\infty \eta_\nu \epsilon_\nu (\partial B_\nu/\partial T)_\tau d\nu - L'(\tau).
\] (25)

The useful form of the inner boundary condition has been directly taken from Madej & Różańska (2000b), neglecting external illumination, and its derivation is not presented here.
We consider the above linearization as an extension of the partial linearization scheme outlined by Mihalas (1978, p. 189).

The above equations of transfer were written on the discrete grid of standard optical depth points \( \tau_d, d = 1, \ldots, D \) and the discrete set of frequencies \( \nu_i, i = 1, \ldots, I \) using discrete values of unknown monochromatic mean intensities \( J_\nu \). Therefore, the full problem of an unknown radiation field and temperature corrections was reduced to \( D \times I \) algebraic equations, which were solved by the Feautrier technique and elimination scheme outlined in detail by Madej & Różańska (2000a).

4. NUMERICAL RESULTS

We have computed a total of 47 hydrogen-helium model atmospheres of X-ray burst sources for effective temperatures \( T_{\text{eff}} = 10^7, 1.5 \times 10^7, 2 \times 10^7, 2.5 \times 10^7, \) and \( 3 \times 10^7 \) K, and surface gravities from \( \log g = 15.0 \) in cgs units down to the critical gravity \( g_{\text{cr}} \), at which true gravity forces are overcome by the gradient of radiation pressure from the bottom of an atmosphere. The number of standard optical depth points was equal to 139 in all models. The standard optical depth \( \tau \) is defined as the monochromatic optical depth computed at the fixed wavelength \( \lambda_{\text{std}} = 3.5 \) Å. We always choose a grid of standard optical depths equidistant in logarithmic scale, starting from the uppermost \( \tau_1 = 10^{-7} \). The number of discrete frequencies varied between 901 and 951, depending on the effective temperature. The above sizes of both the optical depth and frequency grids are very large and certainly allow for the precise reproduction of both model atmospheres and theoretical spectra of X-ray burst sources.

The model atmospheres of our grid fulfill the basic condition of radiative equilibrium very well. We were able to ensure constancy of the radiation flux to better than 0.5% for each zone, and in only several intermediate discrete standard optical depths did this error exceed 2.5%.

We have computed grids of models with \( \log g \) spaced by 0.1 in order to produce equidistant grids of models up to the critical gravity \( g_{\text{cr}} \). For each \( T_{\text{eff}} \), the minimum value of \( g_{\text{cr}} \) is not the critical gravity. The latter is presented and discussed in § 4.2

4.1. Temperature Structure of Model Atmospheres

The run of temperature in our models is determined by the two most important monochromatic opacities. Free-free absorption of hydrogen and helium ions usually causes a temperature drop in the outer layers of model atmospheres (Figs. 1, 2, and 3, solid lines labeled “Thom.”). It is always the case that highly nongray true absorption causes efficient cooling of external atmospheric layers, as long as this particular opacity source remains in LTE (Mihalas 1978). On the other hand, Compton scattering of hard X-ray photons from below the photosphere by initially cooler electron gas increases its temperature several times. Therefore, runs of temperature in atmospheres of X-ray bursters exhibit a distinct minimum above the photosphere and also a distinct rise in the uppermost layers. Both effects have also been presented in previous papers (London et al. 1986; Ebisuzaki 1987; Madej 1991a, 1991b); however, our paper discusses this effect in a much more exact way.

In models of the lowest \( \log g \), which closely approach the critical gravity \( g_{\text{cr}} \), the distinct minimum of \( T \) vanishes and the atmosphere becomes almost isothermal. This effect has also been discussed by Illarionov & Sunyaev (1975) in layers with efficient Compton scattering. Unfortunately, we cannot determine precisely the value of \( g_{\text{cr}} \) for a given \( T_{\text{eff}} \). In each temperature iteration, we compute values of \( C_7 \) for each frequency and optical depth, and this parameter decreases below the Thomson scattering cross section in a very complex way. We cannot present any simple prescription that determines a realistic value of \( g_{\text{cr}} \) for models discussed here.

Figures 1–3 also demonstrate that for a given \( T_{\text{eff}} \), temperature distributions in series of models with different \( \log g \) converge at large depth, \( \tau_{\text{Ross}} \to \infty \). This is because in each atmosphere,

\[
T^4(\tau_{\text{Ross}}) = \frac{1}{4} T^4(\tau_{\text{Ross}} + 0.710446),
\]

independent of the details of the opacity and temperature stratification in higher layers (Mihalas 1978).

4.2. Critical Gravity of X-Ray Burst Sources

The critical gravity of a stellar atmosphere is defined as the gravity for which the acceleration exerted by the radiation
field equals the true gravity. In realistic model atmospheres, stratification of monochromatic opacities is usually very complex. Therefore, in a series of models with fixed $T_{\text{eff}}$ and decreasing surface gravity $\log g$, the balance between true gravity and radiative acceleration appears in just one standard optical depth point and not in the whole atmosphere at once.

In the rather schematic picture at the critical gravity $g_{\text{crit}}$, the atmosphere expands and the density arbitrarily decreases; therefore, all opacities vanish except Thomson scattering on free electrons. The critical gravity is then determined by the well-known equation,

$$
g_{\text{crit}} = \frac{0.2(1 + X)\sigma_{\text{T}} T_{\text{eff}}^4}{c}.
$$

In this equation, the term $0.2(1 + X)$ denotes the Thomson scattering coefficient for 1 g of matter, and $X$ is the mass abundance of hydrogen.

In our model atmosphere of X-ray bursters, we compute Compton scattering coefficients, which depend on temperature and photon frequency and are usually lower than the Thomson scattering coefficient. Consequently, in our models the critical gravity is slightly lower than that predicted by equation (27).

Table 1 presents a comparison of critical gravity extrapolated from our models $\log g_{\text{crit}}^{\text{model}}$ to the critical gravity computed for the Thomson scattering opacity $\log g_{\text{crit}}^{\text{Thom}}$ (see eq. [27]).

### Table 1

| $T_{\text{eff}}$ (K) | $\log g_{\text{crit}}^{\text{model}}$ | $\log g_{\text{crit}}^{\text{Thom}}$ |
|---------------------|-------------------------------|-------------------------------|
| $1 \times 10^7$ | 12.804 | 12.808 |
| $1.5 \times 10^7$ | 13.507 | 13.513 |
| $2 \times 10^7$ | 13.995 | 14.012 |
| $2.5 \times 10^7$ | 14.386 | 14.400 |
| $3 \times 10^7$ | 14.697 | 14.717 |

Note.—In col. (2), gravity is estimated from our models, whereas col. (3) uses eq. (27) with Thomson electron scattering.

The differences of $\log g_{\text{crit}}$ derived in both ways are very small and amount to 0.02 at most.

The model atmospheres presented in our grid do not approach the Eddington limit very closely. In all cases, the geometric thickness of our models is small compared with the neutron star radius. We have chosen two sample model atmospheres closely approaching the critical gravity. In the case of a model of $T_{\text{eff}} = 1.5 \times 10^7$ K and $\log g = 13.6$, the total geometric thickness of the model is about $1.35 \times 10^3$ cm, whereas for $T_{\text{eff}} = 2 \times 10^7$ K and $\log g = 14.1$, the thickness is about $5.1 \times 10^3$ cm (uncorrected for general gravity effects).

In both cases, the thickness of the model atmosphere is much smaller than the neutron star radius, typically of the order of $10^6$ cm. For higher surface gravities, the geometric thickness decreases and scales approximately as $(g - g_{\text{crit}})^{-1}$.

### 4.3. Emergent Flux Spectra

Theoretical spectra of X-ray burst sources differ significantly from the blackbody spectra for a given effective temperature. Figures 4, 5, and 6 present some of our model spectra compared with the blackbody spectrum. For such a high $T_{\text{eff}}$, our spectra of hydrogen-helium atmospheres are featureless. Their shape differs from the blackbody only slightly, while for the largest $\log g$, the shapes of blackbody and Comptonized spectra are almost identical; for the lowest $\log g$, Comptonized spectra get flattened from the low-energy side of the top and develop a distinct soft X-ray excess. What is most important is that for all $\log g$, there exists a shift on the energy scale between the peak fluxes of Comptonized spectra and the blackbody spectrum.

Such a shift is always present in scattering atmospheres, in the cases of both Thomson and Compton electron scattering. Madej (1974) has studied the influence of coherent Thomson scattering on the evolution of the spectrum of a gray atmosphere. The shift of peak fluxes between the scattering atmosphere and the blackbody measured on the horizontal photon energy scale is usually expressed as the ratio of color to effective temperature, $T_c/T_{\text{eff}}$. In the case of a Thomson scattering atmosphere, the ratio $T_c/T_{\text{eff}}$ reaches arbitrarily large values, if the ratio of averaged absorption to scattering coefficients goes to zero. Therefore, van Paradijs (1982) pointed out that the presence of scattering in atmospheres of X-ray bursters should cause a distinct shift between $T_c$ and $T_{\text{eff}}$.

In the case of a Compton scattering atmosphere, the expected shift between $T_c$ and $T_{\text{eff}}$ is smaller than that in a Thomson scattering atmosphere, which is exhibited by Figure 7. In the other words, effects of Comptonization diminish the ratio $T_c/T_{\text{eff}}$ expected in a Thomson scattering atmosphere. Precise values of these ratios for our models are listed in Table 1.

Note that $T_c/T_{\text{eff}}$ reaches an exceptionally large value of 1.93 in the model of $T_{\text{eff}} = 2 \times 10^7$ K and $\log g = 14.1$. This is simply because in our grid of models separated by a fixed difference of $\Delta \log g = 0.1$, this model accidently approaches the critical gravity $g_{\text{crit}}$ particularly closely.

Table 2 clearly demonstrates that in the reasonable range of $T_{\text{eff}}$ and $\log g$ there are no hydrogen-helium models that exhibit $T_c/T_{\text{eff}} > 2$. One can expect a slight increase of $T_c/T_{\text{eff}}$ for $\log g$ slightly smaller than those presented in Table 2; however, we expect that models with still lower $\log g$ would quickly lose hydrostatic equilibrium because of huge radiation forces. In such a case, our models in radiative and hydrostatic equilibrium would become irrelevant, and then the stellar wind models by Titarchuk (1994) would become valid. In the
latter case, however, expected values of $T_c/T_e$ are smaller than 1 (Shaposhnikov & Titarchuk 2002).

Our models of a neutron star atmosphere in hydrostatic and radiative equilibrium can most likely be applied to an analysis of X-ray burst spectra with small or zero radius expansion. Models by Shaposhnikov & Titarchuk (2002) are applicable to phases of extremely large radius expansion from an initial $\approx 10$ km to more than 10,000 km (see Table 2 in their paper). We believe that such large radius expansion can occur very seldom.

5. GENERAL RELATIVISTIC EFFECTS

In our treatment we have not explicitly included the effects of general relativity. However, as long as the neutron star atmosphere is geometrically thin, the general relativistic corrections to the relevant physics are negligible for our purposes (Ayasli & Joss 1982). The properties of the emitted radiation as seen by an observer on the neutron star surface (as we have calculated), including the bolometric luminosity and the effective and color temperatures, will be different from those measured by a distant observer. However, the properties seen by a distant observer can be readily determined from our results for any given value of the gravitational redshift at the neutron star surface (i.e., any value of $M/R$, where $M$ and $R$ are the gravitational mass and metric radius, respectively, of the star) by use of the prescription given by Ayasli & Joss (1982).

As shown below, general relativistic effects, combined with the equation of state for matter at extremely high densities, place an extreme upper limit on $\log g$ of $\sim 15.0$ in cgs units. We have therefore placed an upper limit of $\log g = 15.0$ on our grid of model atmospheres.

In general relativity, the gravitational acceleration at the surface of a neutron star, as measured by a local observer on the surface, is given by

$$
g = \frac{GM(1+z)}{R^2},$$

(28)
where $z$ is the gravitational redshift from the surface to infinity,

$$z = \left(1 - \frac{R_S}{R}\right)^{-1/2} - 1,$$

with $R_S$ being the Schwarzschild radius, $R_S = 2GM/c^2$. Equations (28) and (29) can be solved to obtain $M$ as a function of $R$ and $g$,

$$M = \frac{g^2 R^3}{c^2 G} \left[ \left(1 + \frac{c^4}{g^2 R^2} \right)^{1/2} - 1 \right].$$

For a given neutron star mass, the minimum physically plausible radius is given by the “causality limit” (Lattimer et al. 1990), which can be expressed almost exactly as

$$R = 1.5 R_S = \frac{3GM}{c^2}$$

(eq. [31] also represents the “photon recapture limit”; see Joss & Rappaport 1984, Fig. 17). In fact, the maximum value of $R_S/R$ for many physically realistic neutron star equations of state corresponds closely to the locus of values given by this equation (Lattimer & Prakash 2001 and references therein; see Joss & Rappaport 1984 for earlier references).

For $\log g = 15$, the loci of equations (30) and (31) intersect at $M = 1.02 M_\odot$ and $R = 4.5$ km. Hence, neutron stars with $\log g \geq 15$ and $M \geq 1 M_\odot$ cannot exist. Since the empirical evidence strongly suggests that many, if not all, neutron stars found in nature have masses close to $1.4 M_\odot$ (Joss & Rappaport 1976, 1984; Thorsett & Chakrabarty 1999), it is likely that the equation of state for matter at neutron star densities actually constrains the maximum possible $\log g$ to a value slightly smaller than 15.0. Nevertheless, for completeness and to facilitate interpolation between our models, we have included atmospheres with $\log g = 15.0$ in our model grid.

6. SUMMARY AND CONCLUSIONS

This paper presents model atmosphere equations and theoretical Comptonized spectra of X-ray burst sources. Computational results are obtained for five effective temperatures $T_{\text{eff}}$ in the range $10^7-3 \times 10^7$ K, and surface gravities from $\log g = 15.0$ down to the critical gravity $g_{\text{crit}}$, in steps of $\Delta \log g = 0.1$. We take into account only hydrogen and helium in the chemical composition. Model atmosphere equations assume LTE in the equation of state of perfect gas, radiative and hydrostatic equilibrium, and include precise formulation of Compton scattering on free electrons in thermal relativistic motion. Compton scattering in our equations was described by precise integral expressions that allows us to consider scattering of very energetic X-rays (up to 511 keV), for which diffusion approximation fails.

We stress here that our computations of the theoretical spectra were done on a very extensive grid of the photon energy points from 10 eV up to 430 keV. This was possible only because of our integral formulation of the Compton scattering terms (cf. eq. [14]), in which we differ from other papers that are based on the diffusion approximation (Kompaneets equation).

Theoretical spectra obtained in our research confirm previous predictions that the peak frequency of X-ray burst spectrum is always higher than the peak frequency of the blackbody with the same $T_{\text{eff}}$; therefore, $T_e/T_{\text{eff}} > 1$. We did not find very large values of $T_e/T_{\text{eff}} > 2$, at least for mixed hydrogen and helium models with zero abundance of heavier elements (cf. discussion by Kuulkers et al. 2002). Our grid of models is a substantial revision and extension of earlier grids (London et al. 1986; Madej 1991a, 1991b). We have obtained a set of $T_e/T_{\text{eff}}$ values located in a range similar to that presented in London et al. (1986) and Madej (1991a). However, these values are substantially lower for specific $T_{\text{eff}}$ and $\log g$.
X-ray spectra computed at the highest gravities are similar to a shifted blackbody. X-ray spectra approaching the critical gravity develop distinct low-energy excess and get flattened on the low-energy side of the peak.

Our grid of model atmospheres extends up to \( \log g = 15.0 \) for each effective temperature \( T_{\text{eff}} \). Such a high surface gravity probably does not occur in real neutron stars. We attempted to search for the highest gravities, which can be predicted by the existing equations of state (EOS) for neutron stars. We have selected a few EOSs presented by Haensel (2001) and tentatively determined minimum radii and maximum masses allowed by the corresponding EOSs.

For example, the EOS (eq. [1]) of this paper (taken from Bombaci 1995) predicts that for the maximum mass of neutron star \( M = 1.47 \, M_\odot \), the radius \( R = 8.9 \, \text{km} \), where \( R \) is measured on the neutron star surface. With the help of the equation

\[
g = \frac{GM}{R^2} \left( 1 - \frac{R_S}{R} \right)^{-1/2}
\]

we have determined the maximum \( \log g = 14.53 \) (cgs units). Similarly, the EOS (No. 6) in Haensel (2001) given by Balberg, Lichtenstadt, & Cook (1999) implies that the maximum mass \( M = 2.18 \, M_\odot \), which yields \( R = 10.8 \, \text{km} \), and therefore \( \log g = 14.60 \).

It seems to us that the maximum surface gravity can be obtained with the EOS corresponding to dot-dashed line in Figure 1 of Haensel (2001) (see EOS of Kubis 2001). The maximum possible mass in this case \( M = 2.01 \, M_\odot \); hence, \( R = 8.8 \, \text{km} \) and \( \log g = 14.78 \). We see that, extending our grid to \( \log g = 15.0 \), we slightly exceed the range of \( \log g \) actually expected by theory and \( M-R \) relations for neutron stars.

Model atmospheres presented in this paper do not include relativistic correction factors (general relativistic formulation was presented by Zane et al. 1996 and Psaltis & Lamb 1997). The surface redshift factor does not enter the equation of transfer (eq. [14]) since it is written on the optical depth scale (Madej 1991a). However, true effective temperature \( T_{\text{eff}} \) measured on the neutron star surface differs from \( T_{\text{eff}} \) seen by a distant observer by the redshift factor. Our theoretical spectra can be regarded as reddened spectra from the neutron star surface, in which case there are simply shifted long horizontal axes in Figures 4–6.

Unfortunately, we cannot propose here any prescription to determine \( T_{\text{eff}} / T_{\text{d}} \) from the observed continuum spectra, because their shapes are similar to a blackbody and no spectral features are present in our theoretical spectra. We plan to compute iron-rich model atmospheres in a forthcoming paper, in which iron spectral line series are also computed. Identification of individual spectral lines were recently done by Cottam, Paerels, & Mendez (2002), who determined that the redshift factor, and therefore \( T_{\text{eff}} / T_{\text{d}} = 1.35 \) in the low-mass X-ray binary EXO 0748–676.

We stress here that our rather dense grid of theoretical model atmospheres of neutron stars and their spectra can be used to fit the observed X-ray burst spectra. We plan to extend our research in near future.

Note added in manuscript.—Extensive reviews of properties of X-ray burst sources were presented by Lewin, van Paradijs, & Taam (1993) and by Lewin, van Paradijs, & Taam (1995, 1998).

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