Thermal hadron production in high energy collisions

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**Abstract**

It is shown that hadron abundances in high energy $e^+e^-$, pp and $p\bar{p}$ collisions, calculated by assuming that particles originate in hadron gas fireballs at thermal and partial chemical equilibrium, are in very good agreement with the data. The freeze-out temperature of the hadron gas fireballs turns out to be nearly constant over a large center of mass energy range and not dependent on the initial colliding system. The only deviation from chemical equilibrium resides in the incomplete strangeness phase space saturation. Preliminary results of an analysis of hadron abundances in S+S and S+Ag heavy ion collisions are presented.

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1 Introduction

The average multiplicities of particles produced in high energy collisions are very useful tools to investigate the process of hadron production in virtue of some peculiar features. Unlike momentum spectra, hadron abundances (and correlations) are Lorentz-invariant quantities; hence they do not depend on complicated collective motions possibly present in the system and may be calculated in the local comoving frames. In elementary collisions, such as $e^+ e^-$, they are a direct and unique probe of the hadronization process since they are independent of the perturbative parton dynamics which is inherited by hadrons mainly in their momentum spectrum. Therefore, it is very important to study hadron abundances in order to reveal the basic mechanisms governing hadron production in all kinds of collisions.

In the following sections we will sum up briefly the statistical-thermodynamical approach to the problem of hadron production and we will show its stunning capability of fitting all existing hadron average multiplicities data in $e^+ e^-$, pp and p$\bar{p}$ collisions by using only three free parameters. A preliminary analysis of hadron abundances measured in heavy ion collisions in full phase space within the same model will be discussed in Sect. 4.

2 The model

The thermodynamical model of hadron production in $e^+ e^-$, pp, p$\bar{p}$ has been described in detail elsewhere [1–3]; in this section it is briefly summarized.

The basic assumption of the model is the formation of an arbitrary number of hadron gas fireballs moving away from the primary interaction region each with its own collective momentum. The parameters describing the $i^{th}$ hadron gas fireball at thermal and chemical equilibrium are the temperature $T_i$ and the volume $V_i$ in its rest frame as well as its quantum numbers electric charge $Q$, baryon number $N$, strangeness $S$, charm $C$ and beauty $B$. The partition function of this system is calculated in the framework of the canonical formalism of statistical mechanics, namely by summing only over the multi-hadronic states having the same quantum numbers of the fireball. Therefore, if $Q^0_i = (Q, N, S, C, B)$ is the vector of fireballs quantum numbers and $Q$ is the vector of quantum numbers of a particular multi-hadronic state, the partition function of the fireball reads:

$$Z(Q^0_i) = \sum_{\text{states}} e^{-E/T_i} \delta_Q Q^0_i.$$  \hspace{1cm} (1)

A parameter $\gamma_s$ accounting for a possibly incomplete strangeness chemical equilibrium is introduced in the partition function by multiplying by $\gamma_s$ the Boltzmann factors $e^{-\epsilon_j/T}$ associated to the $j^{th}$ hadron where $s$ is the number of its valence strange quarks and anti-quarks.

The average multiplicity of any hadron species in the $i^{th}$ fireball can be derived from the partition function (1). As this quantity depends on the quantum vector $Q^0_i$, the overall average multiplicity depends on the number of fireballs $N$ and on their quantum configuration \{\(Q^0_1, \ldots, Q^0_N\)}.

In principle any configuration may occur provided that $\sum_{i=1}^{N} Q^0_i = Q^0$, where $Q^0$ is the quantum vector fixed by the initial state. However, it can be shown [3] that the overall average multiplicity of any hadron indeed depends only on the global quantities $Q^0$ and $V = \sum_{i=1}^{N} V_i$ (namely the sum of all fireball volumes in their rest frames), provided that the temperatures and the strangeness suppression factors $\gamma_s$ are the same for all fireballs and the probabilities $w(Q^0_1, \ldots, Q^0_N)$ of occurrence of a given quantum configuration are chosen to be:
\[ w(Q_1^0, \ldots, Q_N^0) = \frac{\delta_{\Sigma_i Q_i^0 \Pi_i^N Z_i^i(Q_i^0)}}{\sum_{Q_1^0, \ldots, Q_N^0 \Sigma_i Q_i^0 \Pi_i^N Z_i^i(Q_i^0)}.} \]  

It can be proved that this choice corresponds to the minimal deviation of the system from global (i.e. thermal, chemical and mechanical) equilibrium. After making use of the probabilities (2) to average the hadron production over all possible quantum configurations, the overall average multiplicity of the \( j^{th} \) hadron turns out to be:

\[ \langle \langle n_j \rangle \rangle = \frac{1}{(2\pi)^5} \int d^5\phi \ e^{iQ_0 \cdot \phi} \exp[V \sum_j F_j(T, \gamma_s, \phi)] \times \frac{(2J_j + 1)V}{(2\pi)^3} \int \frac{d^3p}{\gamma_s^{-\gamma_j}} \exp (\sqrt{p^2 + m_j^2/T + iQ_j \cdot \phi}) \pm 1, \]  

where the upper sign is for fermions, the lower for bosons and:

\[ F_j(T, \gamma_s, \phi) = \sum_j \frac{(2J_j + 1)V}{(2\pi)^3} \int \frac{d^3p}{\gamma_s^{-\gamma_j}} \log (1 \pm \gamma_s \ e^{-\sqrt{p^2 + m_j^2/T - iQ_j \cdot \phi}}) \pm 1. \]  

Thus, under the previous assumptions, the hadron yields (3) depend only on three unknown parameters \( T, \gamma_s \) and \( V \); the latter re-absorbs the dependence on the number of fireballs. These unknown parameters have to be determined by fitting the calculated multiplicities to the measured ones at each center of mass energy.

### 3 Results in \( e^+e^-, pp \) and \( p\bar{p} \) collisions

In order to calculate hadron abundances to be compared with experimental data the primary yield of each hadron species calculated with eq. (4) is added to the contribution stemming from the decay of heavier hadrons, which is calculated by using experimentally known decay modes and branching ratios [4, 5]. All light-flavored hadrons up to a mass of 1.7 GeV and all heavy-flavored states inserted in the JETSET tables [5] have been used as primary species. The effect of this cut-off of hadron mass spectrum on final results has been shown to be negligible [1, 3].

The primary yield of resonances has been determined by convoluting the eq. (4) with a relativistic Breit-Wigner function within \( 2\Gamma \) from the central mass value.

The measurements from different experiments have been averaged according to a procedure described in ref. [6] taking into account \textit{a posteriori} disagreements and correlations.

Since the temperature is expected to be \( \mathcal{O}(100) \) MeV the thermal production of heavy flavored hadrons can be neglected while the perturbative production is significant only in \( e^+e^- \) collisions, where c and b quarks are created in the primary interaction and do not re-annihilate. In this case the presence of one charmed (bottomed) flavored hadron-anti-hadron pair is demanded in a fraction of events \( \sigma(e^+e^- \rightarrow c\bar{c}(b\bar{b})) / \sigma(e^+e^- \rightarrow \text{hadrons}) \).

The fit is performed by minimizing the \( \chi^2 \):

\[ \chi^2 = \sum_i \frac{(n_i^{[\text{theo}] - n_i^{[\text{expe}]})^2}{\sigma_i^2} \]  

as a function of \( T, V \) and \( \gamma_s \). The errors \( \sigma_i \) include contributions from uncertainties on masses, widths and branching ratios of various hadrons involved in the decay chain process; they have
Table 1: Values of fitted parameters. The parameter $VT^3$ has been used instead of $V$ in hadronic collisions because less correlated to the temperature. The additional errors within brackets have been estimated by excluding data points deviating the most from fitted values and repeating the fit.

| $\sqrt{s}$ (GeV) | Temp. (MeV) | Volume (Fm$^3$) | $\gamma_s$ | $\chi^2$/dof |
|------------------|-------------|----------------|-----------|--------------|
| 29               | 163.6 ± 3.6 | 26.7 ± 4.1     | 0.724 ± 0.045 | 24.7/13     |
| 35               | 165.2 ± 4.4 | 24.9 ± 4.7     | 0.788 ± 0.045 | 10.5/8      |
| 44               | 169.6 ± 9.5 | 23.2 ± 8.7     | 0.730 ± 0.060 | 4.9/4       |
| 91               | 160.3 ± 1.7 (3.3) | 50.0 ± 3.9 | 0.673 ± 0.020 (0.028) | 70.1/22     |

| $\sqrt{s}$ (GeV) | Temp. (MeV) | $VT^3$ | $\gamma_s$ | $\chi^2$/dof |
|------------------|-------------|--------|-----------|--------------|
| 19               | 190.8 ± 27.4 | 5.8 ± 3.1 | 0.463 ± 0.037 | 6.4/4       |
| 23.6             | 194.4 ± 17.3 | 6.3 ± 2.5 | 0.460 ± 0.067 | 2.4/2       |
| 26.0             | 159.0 ± 9.5  | 13.4 ± 2.7 | 0.570 ± 0.030 | 1.9/2       |
| 27.4             | 169.0 ± 2.1 (3.4) | 11.0 ± 0.69 | 0.510 ± 0.011 (0.025) | 136.4/27    |

| $\sqrt{s}$ (GeV) | Temp. (MeV) | $VT^3$ | $\gamma_s$ | $\chi^2$/dof |
|------------------|-------------|--------|-----------|--------------|
| 200              | 175.0 ± 14.8 | 24.3 ± 7.9 | 0.537 ± 0.066 | 0.70/2      |
| 546              | 181.7 ± 17.7 | 28.5 ± 10.4 | 0.557 ± 0.051 | 3.78/1      |
| 900              | 170.2 ± 11.8 | 43.2 ± 11.8 | 0.578 ± 0.063 | 1.8/2       |

been determined with an iterative fit procedure [1,3]. The results of the fit are shown in table 1. The quoted numbers are the same as in refs. [2,3] except at $\sqrt{s} = 91.2$ GeV where the fit has been repeated with new LEP measurements [7] (see fig. 1). The fit quality is remarkably good at all center of mass energy points. The most interesting result is undoubtedly the uniformity, within the fit errors, of the freeze-out temperature values independently of kind of reaction and center of mass energy. The fact that $\gamma_s$ is always less than 1 demonstrates that strangeness chemical equilibrium is not reached in any of the examined collisions. Nevertheless, it is worth noticing that $\gamma_s$ is higher in e$^+$e$^-$-collisions than in hadronic collisions at the same center of mass energy. The use of the canonical formalism is essential since the system turns out to be small enough to generate charged hadron ($q_j \neq 0$) suppression with respect to neutral ones ($q_j = 0$) even in an initially neutral system ($Q^0 = 0$); for a more detailed discussion see refs. [2,3]. All the fits have been performed by using as experimental input the measured yields of light-flavored hadrons. Once the parameters of the model are determined it is possible to predict the heavy-flavored hadrons abundances provided that the production rate of c and b quark pairs is known. In table 2 predictions for $\sqrt{s} = 91.2$ GeV are compared to actual LEP experiments measurements [8] averaged according to the procedure mentioned above; the agreement is indeed very good.
Table 2: Predictions of heavy flavored hadron abundances at $\sqrt{s} = 91.2$ GeV obtained by using $T$, $V$ and $\gamma_s$ parameters quoted in table 1 and $R_c = 0.17$, $R_b = 0.22$ according to LEP measurements [9]. The $B_s^{*\ast}$ prediction is affected by the interpretation of the observed peaks as four different states or two different states (within brackets).

| Hadron               | Prediction | Measured   | Residual |
|----------------------|------------|------------|----------|
| $D^+$                | 0.0926     | 0.087±0.008| -0.67    |
| $D^0$                | 0.233      | 0.227±0.012| -0.50    |
| $D_s$                | 0.0579     | 0.066±0.010| +0.81    |
| $D^{*+}$             | 0.108      | 0.0880±0.0054| -3.7     |
| $D_s^{+}/c$-jet      | 0.103      | 0.128±0.027| +0.92    |
| $D_1^{0}/c$-jet      | 0.0347     | 0.038±0.009| +0.37    |
| $D_2^{*}/c$-jet      | 0.0471     | 0.135±0.052| +1.7     |
| $D_{s1}^{0}/c$-jet   | 0.00536    | 0.016±0.0058| +1.8     |
| $B^0/b$-jet          | 0.412      | 0.384±0.026| -1.1     |
| $B^{*}/B$            | 0.692      | 0.747±0.067| +0.82    |
| $B^{*}/b$-jet        | 0.642      | 0.65 ±0.06 | +0.13    |
| $B_s/b$-jet          | 0.106      | 0.122±0.031| +0.52    |
| $B_s^{*}/b$-jet      | 0.206      | 0.26 ±0.05 | +1.0     |
| $B_s^{*}/B$          | 0.251      | 0.27 ±0.06 | +0.32    |
| $B_s^{*}/b$-jet      | 0.021(0.011)| 0.048±0.017| +1.6     |
| $B_s^{*0}/B^+$       | 0.026(0.013)| 0.052±0.016| +1.6     |
| $\Lambda_c^+$        | 0.0248     | 0.0395±0.0084| +1.7    |
| b-baryon/b-jet       | 0.0717     | 0.115±0.040| +1.1     |
| $(\Sigma_b + \Sigma_b^*)/b$-jet | 0.0404 | 0.048±0.016| +0.48    |
| $\Sigma_b/(\Sigma_b^* + \Sigma_b)$ | 0.411 | 0.24±0.12 | -1.4     |

4 Thermal fits in heavy ion collisions

The model described in Sect. 2 may be used to fit hadron abundances measured in heavy ion collisions, provided that the same assumptions still hold. Comparison of thermal calculations with experimental data have been done recently by several authors with a grand-canonical, rather than canonical, approach and by using multiplicities measured either in a restricted rapidity range or in full phase space [10–13].

In principle the canonical formalism is the only correct one in that it ensures the exact conservation of initial quantum numbers. However, if the volume $V$ is very large, it can be shown (see ref. [3]) that the formula (3) giving the average primary $j^{th}$ hadron multiplicity in the canonical formalism reduces to:

$$
\langle \langle n_j \rangle \rangle = (2J_j + 1) \frac{V}{(2\pi)^3} \sum_{n=1}^{\infty} (\pm 1)^{n+1} \gamma_s^{n s_j} \frac{1}{\sqrt{n^2 + m_j^2}} \int d^3 p \ e^{-n\sqrt{p^2 + m_j^2}/T} e^{n Q A^{-1} q_j/2} e^{-n^2 q_j A^{-1} q_j/4},
$$

by using a saddle-point approximation of the $\phi$-integrals in eq. (3). $A$ is a $N \times N$ matrix, where $N$ is the dimension of the quantum vectors $Q, q_j$, whose elements are proportional to $V$. Hence,
in the limit $V \to \infty$, the second exponential factor in the above equation goes to 1 as the $q_j$ terms are finite (i.e. the hadrons quantum numbers). On the other hand, the first exponential factor can be written $\exp[n\mu \cdot q_j]$ where $\mu$ is a set of $N$ traditional chemical potentials; the grand-canonical formalism is recovered in the large volume limit. In heavy ion collisions one expects the canonical factor $\exp[-n^2 q_j A^{-1} q_j / 4]$ to be a small correction of the grand canonical formulae as the particle multiplicities, hence the volume, are very large compared to pp or $e^+e^-$-collisions.

We fitted hadron abundances measured in SS \cite{15} and SAg \cite{16} collisions in full phase space by using four free parameters: $T$, $V$, $\gamma_s$ and $\mu_b$, the baryochemical potential. The strangeness and electric chemical potential $\mu_s$ and $\mu_q$ have been determined with the constraints of strangeness neutrality and conservation of the initial electric charge/baryon number initial ratio:

\begin{align*}
\sum_j S_j \langle n_j \rangle &= 0 \\
\sum_j Q_j \langle n_j \rangle &= \frac{Z}{A} \sum_j N_j \langle n_j \rangle.
\end{align*}

Table 3: Values of fitted parameters in SS and SAg collisions. Also quoted the calculated chemical potentials $\mu_s$ and $\mu_q$.

| Parameter       | SS            | SAg           |
|-----------------|---------------|---------------|
| $T$ (MeV)       | 182.1$\pm$9.0 | 180.0$\pm$3.2 |
| $VT^3 \exp[-0.7\text{GeV}/T]$ | 3.51$\pm$0.14 | 5.43$\pm$0.35 |
| $\gamma_s$     | 0.732$\pm$0.037 | 0.830$\pm$0.061 |
| $\mu_b/T$      | 1.243$\pm$0.071 | 1.323$\pm$0.069 |
| $\chi^2$/dof   | 17.2/5       | 5.5/3         |
| $\mu_s/T$      | -0.332       | -0.364        |
| $\mu_q/T$      | -0.0222      | -0.00316      |

The results of the fit are shown in table 3 while the comparison between fitted and experimental average multiplicities are shown in table 4. Due to the strong correlation between $T$ and $V$ we chose to fit the parameter $VT^3 \exp[-0.7\text{GeV}/T]$ instead of $V$. It should be mentioned that these results have been obtained by using only the experimental errors without taking into account the uncertainties arising from hadron parameters like masses, widths and branching ratios.

The resulting elements of the $A$ matrix range between -0.02 and 0.06 in SS collisions and between -0.012 and 0.039 in SAg, confirming the proximity to the grand-canonical regime.

The fitted temperature is compatible with that found in $e^+e^-$, pp, and p$\bar{p}$ collisions and the quality of the fit is good as well. Strangeness chemical equilibrium is not reached as demonstrated by the $\gamma_s$ values $< 1$ although there is a clear increase with respect to pp and p$\bar{p}$ collisions. Our results differ from those obtained in ref. \cite{10} mainly because of the available larger number of data points and the use of updated hadron parameters in the decay chain. A new fit performed by one of the authors of ref. \cite{10} shows a clear consistency with our results \cite{14}.
Table 4: Comparison between fitted and measured multiplicities in SS and SAg collisions.

| Particles SS | Fitted | Measured | Residual |
|--------------|--------|----------|----------|
| Baryons-Antibaryons | 54.57 | 54±3 | -0.19 |
| \( h^- \) | 93.41 | 98±3 | +1.53 |
| \( K^+ \) | 12.61 | 12.5±0.4 | -0.28 |
| \( K^- \) | 7.456 | 6.9±0.4 | -1.39 |
| \( K^0 \) | 9.834 | 10.5±1.7 | +0.39 |
| \( \Lambda \) | 7.798 | 9.4±1.0 | +1.60 |
| \( \bar{\Lambda} \) | 1.425 | 2.2±0.4 | +1.94 |
| \( p - \bar{p} \) | 22.59 | 21.2±1.3 | -1.07 |
| \( \bar{p} \) | 2.094 | 1.15±0.4 | -2.36 |

| Particles SAg | Fitted | Measured | Residual |
|---------------|--------|----------|----------|
| Baryons-Antibaryons | 92.02 | 90±9 | -0.22 |
| \( h^- \) | 152.04 | 160±8 | +1.00 |
| \( K^0 \) | 17.49 | 15.5±1.5 | -1.33 |
| \( \Lambda \) | 14.39 | 15.2±1.2 | +0.68 |
| \( \bar{\Lambda} \) | 2.440 | 2.6±0.3 | +0.53 |
| \( p - \bar{p} \) | 36.76 | 34±4 | -0.68 |
| \( \bar{p} \) | 3.043 | 2.0±0.8 | -1.31 |

5 Conclusions

The analysis of hadron abundances in \( e^+e^- \), pp and p\( \bar{p} \) collisions performed in a suitable canonical formalism is in very good agreement with the hypothesis of local thermal and chemical equilibrium. The most interesting results of the thermal fits to experimental data is the constant value of freeze-out temperature in all three kinds of collisions independently of center of mass energy. This fact indicates that the transition from quarks-gluons to hadrons occurs in a purely statistical fashion at critical values of pre-hadronic matter parameters (such as energy density or pressure) corresponding to a (partially) equilibrated hadron gas at \( T_c \approx 170 \text{ MeV} \). Furthermore, evidence is found for an incomplete strangeness phase space saturation.

The preliminary analysis of hadron abundances in full phase space in SS and SAg heavy ion collisions resulted in a good agreement with the data as well and a temperature value consistent with that found in elementary collisions.

The strangeness enhancement going from pp to heavy ion collisions is explained by two different effects: the increase of the extension of the system reduces the suppression due to strangeness conservation (canonical suppression) whilst the increase of \( \gamma_s \) further raises the yield of particles containing strange quarks.

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Figure 1: Fit of hadron average multiplicities at $\sqrt{s} = 91.2$ GeV measured by LEP experiments. Above: black dots are the experimental data, the solid line connects fitted values. Below: residuals distribution.