Current-Driven Dynamics of Magnetic Hopfions

X. S. Wang, A. Qaiumzadeh, and A. Brataas

Center for Quantum Spintronics, Department of Physics,
Norwegian University of Science and Technology, NO-7491 Trondheim, Norway

Topological magnetic textures have attracted considerable interest since they exhibit new properties and might be useful in information technology. Magnetic hopfions are three-dimensional (3D) spatial variations in the magnetization with a non-trivial Hopf index. We find that in ferromagnetic materials, two types of hopfions, Bloch-type and Néel-type hopfions, can be excited as metastable states in the presence of bulk and interfacial Dzyaloshinskii-Moriya interactions, respectively. We further investigate how hopfions can be driven by currents via spin-transfer torques (STTs) and spin-Hall torques (SHTs). Distinct from 2D ferromagnetic skyrmions, hopfions have a vanishing gyrovector. Consequently, there are no undesirable Hall effects. Néel-type hopfions move along the current direction via both STT and SHE, while Bloch-type hopfions can be moved either transverse to the current direction by SHT or parallel to the current direction by STT. Our findings open the door to utilizing hopfions as information carriers.

In this Letter, we show that, in addition to interfacial PMA, a bulk PMA assists in stabilizing a localized hopfion that can exist in nanostrips, in contrast to the boundary-confined hopfions in nanodisks proposed in previous studies. In addition to the Bloch-type hopfions studied previously, which can be stable in the presence of bulk DMI, we identify another type of hopfion, Néel-type hopfions, which can be stable in the presence of interfacial DMI. We also introduce an ansatz that can accurately describe the hopfion profile. We then study the current-driven dynamics of ferromagnetic hopfions in nanostrips. Although the hopfions have non-trivial Hopf indices, their gyrovectors vanish, in contrast to skyrmions. As a result, hopfions move along the current via spin-transfer torques (STTs) also cause Néel-type hopfions to move along the current, while Bloch-type hopfions move transverse to the current. Hopfions may be superior to skyrmions as information carriers in racetrack memories since their current-induced motion is more straightforward.

We consider a magnetic film of thickness $d$ with interfacial PMA at the top and bottom surfaces as well as bulk PMA in the bulk. The zero-temperature micromagnetic
free energy of the system reads

\[ F = \int_V A_{ex} \left[ \nabla \times m + D \left( m, \frac{\partial m}{\partial x_i} \right) + K_b(1 - m_x^2) \right] \cdot \nabla \cdot m + \int_{z=\pm d/2} K_s(1 - m_z^2) dS + E_d, \]

where \( A_{ex} \) is the exchange constant; \( D \) is the DMI energy density functional, which depends on the symmetry of the system; \( K_b \) and \( K_s \) are the bulk PMA and the interfacial PMA, respectively; \( B \) is a perpendicular magnetic field; \( M_s \) is the saturation magnetization; and \( E_d \) is the demagnetization energy. In bulk noncentrosymmetric materials such as FeGe and MnSi, the DMI is bulk-like \( D = D_b \mathbf{m} \cdot (\nabla \times \mathbf{m}) \), where \( D_b \) is the bulk DMI strength in units of J/m² [10]. In inversion-symmetry-broken films such as Pt/Co/AlOₓ, the DMI is interfacial-like \( D = D_i [(\mathbf{n} \cdot \mathbf{m}) \nabla \cdot \mathbf{m} - (\mathbf{m} \cdot \nabla)(\mathbf{n} \cdot \mathbf{m})] \), where \( \mathbf{n} \) is the direction normal to the film and \( D_i \) is the interfacial DMI strength in units of J/m² [9, 14]. Because the hopfions are non-isomorphic maps from \( \mathbb{R}^3 \cup \{ \infty \} \) to \( S^2 \), the topological invariant of hopfions, known as the Hopf index, \( H \), differs from the skyrmion number. This index is defined as

\[ H = \frac{1}{4\pi^2} \int_V \mathbf{F} \cdot \mathbf{A} \, dV, \]

where \( F_i = \varepsilon_{ijk} m_j \cdot (\partial_k m_z \times (\partial_k m_i)/2 \) in which \( i, j, k = \{ x, y, z \} \) and \( \varepsilon \) is Levi-Civita tensor, and \( \mathbf{A} \) is a vector potential, which satisfies \( \nabla \times \mathbf{A} = \mathbf{F} \) [16]. The components of \( \mathbf{F} \) are solid angle densities in different coordinate planes. \( \mathbf{F} \) can be understood as the gyrovector density [17], emergent magnetic field [18], or topological charge [10].

Figures 1(a) and 1(b) show the typical magnetization profiles of Bloch-type and Néel-type hopfions, respectively, obtained by numerical simulations. We consider a 16-nm-thick film with \( A_{ex} = 0.16 \) pJ m⁻¹ and \( M_s = 1.51 \times 10^5 \) A m⁻¹, representing MnSi parameters [38]. No external field is applied. The Bloch-type (Néel-type) hopfions are favorable in bulk (interfacial) DMI systems. In Fig. 1(a), we use \( K_s = 0.5 \) mJ m⁻², \( K_b = 41 \) kJ m⁻³, and \( D_b = 0.115 \) mJ m⁻², while in Fig. 1(b), we use \( K_s = 0.5 \) mJ m⁻², \( K_b = 20 \) kJ m⁻³, and \( D_i = 0.115 \) mJ m⁻² (these parameters are also used in the study of current-driven dynamics below). The simulations are mainly performed using mumax³ [39] at zero temperature (additional details of the simulations can be found in the Supplemental Materials [40]). We compute that the Hopf indices are 0.96 (Bloch) and 0.95 (Néel) by numerical integration of Eq. 2. The two types of hopfions are topologically equivalent but behave differently in the presence of SHT, which we will discuss later. The upper and lower panels are the midplane cross-sections in the \( xy \)-plane and \( xx \)-plane. The magnetization profile in each \( xy \)-midplane cross-sections is Bloch-type (a) or Néel-type (b) skyrmionium or the target skyrmion [51, 52], while the \( zz \)-midplane cross-section shows a pair of vortices with opposite chirality. The right \( (x > 0) \) \( zz \)-midplane contains a vortex with chirality +1 for an \( H = +1 \) hopfion or an antivortex with chirality −1 for an \( H = −1 \) hopfion. Outside the hopfions and at the center of the hopfions, the magnetization is along the \( z \) direction, and the dotus-box transition region is chiral (for Bloch-type hopfions) or hedgehog-like (for Néel-type hopfions). Figure 1(c) and (d) show the corresponding preimages (constant-\( \mathbf{m} \) curves in real space) of Fig. 1(a) and (b). The preimages link with each other once, which is consistent with the Hopf index calculation, justifying the hopfion nature of the textures in (a) and (b).

The hopfions observed in previous studies [38, 40] were extended objects confined by the side edges of small magnetic disks. In contrast, the introduction of a finite bulk PMA causes the hopfions in our work to be metastable, localized objects that can exist in long strips with a hopfion radius \( R \), defined as the radius of the preimage \( \mathbf{m} = (0, 0, −1) \). The ground state is a single-domain state with an out-of-plane magnetization. Thus, these hopfions can be candidates of information carri-
ers, and devices such as hopfion racetrack memories can be designed \[53\] \[54\]. Moreover, although the Hopf index is nontrivial, the gyrovector \( G \) of a hopfion vanishes. Consequently, the main drawback of a FM skyrmion racetrack memory, the skyrmion Hall effect, is absent in the hopfion racetrack memory. In addition to the numerical verification, the vanishing gyrovector of a hopfion can be understood as follows. Consider a film that is isotropic in the \( xy \) plane. The hopfion profile centered at a certain location can be expressed via \( \Theta(r, \phi, z) \), \( \Phi(r, \phi, z) \), where \((r, \phi, z)\) are cylindrical spatial coordinates, and \( \Theta, \Phi \) are the polar and azimuthal angles of the magnetization. Because of the isotropy in the \( xy \) plane, it is natural to assume that \( \Theta \) is independent of \( \phi \), and \( \Phi(r, \phi, z) = \Delta \Phi(r, z) + n \phi \), where \( n \) is an integer and \( \Delta \Phi \) is a function independent of \( \phi \). These assumptions are well justified by our numerical results. Thus, in cylindrical coordinates, \( F_z \) = \( n \frac{\sin \Theta}{r} \frac{\partial \Theta}{\partial \phi} \). We can rewrite \( G_z \) as

\[
G_z = \int F_z dV = -2n \pi \int_{-d/2}^{d/2} (\cos \Theta)_{r=0} \, dz. \tag{3}
\]

Since in a hopfion the magnetization directions are the same at both the periphery (\( r = \infty \)) and the center (\( r = 0 \)), \( G_z \) vanishes. Since the two vortices in any \( xy \) (or \( yz \)) midplane cross-section have opposite chirality, as shown in the lower panels of Fig. 1(a)(b), the integration of \( F_x \) (or \( F_y \)) over the volume gives a vanishing contribution to \( G_x \) (or \( G_y \)). The components of \( G \) are invariant under continuous deformation \[10\]; therefore, \( G = 0 \) applies to all the hopfions.

The magnetic hopfions discussed in previous studies \[38\] \[40\] were Bloch-like. In the following, we mainly focus on Néel-type hopfions. Although the analytical expression of the hopfion profile is unknown, we find an ansatz that describes the \( H = +1 \) Néel-type hopfion profile very well:

\[
m_x = \frac{4r'}{1 + r'^2 + z'^2} \left[ 2z' \sin \phi + \cos \phi \left( r'^2 + z'^2 + 1 \right) \right],
\]

\[
m_y = \frac{4r'}{1 + r'^2 + z'^2} \left[ -2z' \cos \phi + \sin \phi \left( r'^2 + z'^2 + 1 \right) \right],
\]

\[
m_z = 1 - \frac{8r'^2}{1 + r'^2 + z'^2},
\]

where \( r' = \frac{R}{w_R}, z' = \frac{z}{w_h}, R \) is the hopfion radius, and \( w_R \) is a hopfion wall width in the radial direction \[55\]. \( w_h \) is a hopfion height describing the extent of the hopfion in the out-of-plane direction, and \( w_h \) is a hopfion wall width in the out-of-plane direction. The continuity of the above ansatz at \( r = 0 \) and \( z = 0 \) can be verified \[50\]. The ansatz \[4\] is based on the well-known ansatz \[50\] augmented by a non-linear rescaling of \( r \) and \( z \) \[55\] and can also describe Bloch-type hopfions and \( H = -1 \) hopfions after simple transformations \[50\]. Figure 2(a) shows a comparison of \( m_z \) between the above ansatz and the numerical data along the \( x \) direction for \( y = z = 0 \) (bottom axis) and along the \( z \) direction for \( r = R \) (top axis), with \( R = 8.3 \) nm, \( w_R = 5.6 \) nm, \( h = 6.3 \) nm and \( h_w = 1.6 \) nm obtained from fitting. The comparison gives good agreement (more comparisons can be found in the Supplemental Materials \[50\]). The numerical data along the \( z \) direction are slightly asymmetric with respect to \( z = 0 \), which is because of the asymmetric bulk magnetic charge. If the dipolar interaction is turned off, or if the hopfion is a Bloch-type hopfion, this asymmetry will vanish. For larger hopfions, the ansatz agrees with the numerical data even better.

Next, we numerically calculate the Hopf index \( H \) and the layer-averaged gyrovector \( G_z/d \) by integrating over a cylinder of height \( d \) and radius \( R_0 \) (symbols), and we compare the numerical results with the analytical result calculated using the ansatz \[4\] (solid lines), as shown in Fig. 2(b). As \( R_0 \) increases, \( H \) converges toward 1, and \( G_z/d \) converges toward 0. Note that the \( R_0 \) used here is smaller than the sample size of our numerical simulation such that the edge structures are discarded. Below, we use this ansatz to discuss the current-driven dynamics of the hopfions, and we compare the results with numerical simulations.

Disregarding deformations, the motion of a hopfion, as a rigid body, is governed by Thiele’s equation \[57\]:

\[
\gamma \frac{\pounds}{M_s} \mathbf{T} + \mathbf{G} \times (\mathbf{v} - \mathbf{u}) - \mathbf{D} \cdot (\alpha \mathbf{v} - \beta \mathbf{u}) = 0, \tag{5}
\]

where \( \gamma \) is the gyromagnetic ratio; \( \alpha \) is the Gilbert damping; \( \beta \) is the STT non-adiabaticity \[49\]; \( \mathbf{v} \) is the velocity of the hopfion; \( \mathbf{u} = -\mu_B \mathbf{p} / [eM_s (1 + \beta^2)] \) is a
vector with dimension of velocity proportional to the current density $\mathbf{J}$, in which $p$ is the spin polarization and $e$ is the electron charge; $\mathbf{G}$ is the above-mentioned gyrovector; and $\mathcal{D}$ is the dissipation tensor defined as $\mathcal{D}_{ij} = \int \partial_i \mathbf{m} \cdot \partial_j \mathbf{m} dV$. $\mathbf{T}$ is the force on the hopfion, expressed as $T_i = -\frac{\partial}{\partial x} F_{x} - \int \frac{e}{m} \cdot (\mathbf{m} \times \mathbf{J}) dV$, where $F$ is the free-energy functional. $X_i$ is the center position of the hopfion, and $\mathbf{J}$ represents non-conservative torques other than STT such as the SHT. In our model, all the material parameters are spatially homogeneous; therefore, the first term in $\mathbf{T}$ is 0. The STT does not contribute to $\mathbf{T}$. Since $\mathbf{G} = 0$, the hopfions move along the applied current via STT with velocity $\mathbf{v} = \frac{\dot{\theta}}{\alpha} \mathbf{u}$. Figure 3(a) shows the simulated trajectory during a period of 15 ns of the Néel-type hopfion driven by STT under $J = 10^{11}$ A m$^{-2}$, with $\alpha = 0.12$ (a typical value for Co [58]). $\alpha = 0.05$ and $\beta = 0.1$. The strip is 128 nm-wide in the $y$ direction, and periodic boundary conditions are used in the $x$ direction. The trajectory is almost along the $x$ direction after moving for 15 ns. The small deviation may come from the discretization and the deformation of the hopfion. Figure 3(b) shows the longitudinal component of the hopfion velocity $v_x$ versus the applied current density $J$. The numerical data (black squares) are in good agreement with the analytical formula $v = \frac{\dot{\theta}}{\alpha} u$ (black line). Above $J = 2 \times 10^{11}$ A m$^{-2}$, the hopfion becomes distorted, and at even higher currents $J = 5 \times 10^{11}$ A m$^{-2}$, the hopfion is destroyed. In contrast to the threshold current for the annihilation of FM skyrmions, this limitation on the current is not intrinsic and can be improved by material engineering. For the Bloch-type hopfion in Fig. 1(a), similar results are obtained.

Recently, spin-orbit torques have attracted attention for driving magnetic textures because of their possibly higher angular momentum transfer efficiency [59]. Spin-orbit torques arise from a variety of origins such as interfacial Rashba spin-orbit coupling (SOT) [60], spin-Hall-effect-induced spin currents from adjacent heavy metal layers [61], and the intrinsic SOT in magnetic materials [62]. The field-like component of the torque [60, 62] can be regarded as a uniform magnetic field on the system. Since a hopfion is a localized object in a domain, a uniform magnetic field deforms (or even destroys) the hopfion without exerting a net force on it. We consider the antidamping-like SHT [61, 63],

$$\tau = \frac{\gamma h J}{e M_d} \theta_{SH} \mathbf{m} \times (\mathbf{m} \times (\hat{\mathbf{J}} \times \hat{\mathbf{z}})), \quad (6)$$

which is usually the dominant SOT for a heavy metal/magnet system. Here, $\theta_{SH}$ is the spin Hall angle, $\hat{\mathbf{J}}$ is the electron current direction, $J$ is the current density, and $\hat{\mathbf{z}}$ is the direction normal to the sample film. Consider a current applied along the $x$ direction. The SHT is then $\tau = \gamma_0 \mathbf{m} \times (\mathbf{m} \times \hat{\mathbf{y}})$, where $\gamma_0$ denotes the prefactors in [60]. Using the ansatz [4] with collective coordinates

$$R, w_R, h \text{ and } w_h \text{ obtained by fitting the numerical data, we can calculate the force } \mathbf{T} \text{ and dissipation tensor } \mathcal{D}.$$  

According to the polarity of the hopfion profile, the force on a Bloch-type hopfion is along the $y$ direction, while the force on a Néel-type hopfion is along the $x$ direction, similar to the skyrmion or target skyrmion [19, 51]. Thus, only Néel hopfions move along the current under SHT, while the Bloch hopfions move transverse to the current and are blocked by the edge of the racetrack. Because of the isotropy in the $xy$ plane, $\mathcal{D}$ is diagonal, with $D_{xx} = D_{yy} \equiv D$. The trajectory of the Néel-type hopfion during 15 ns driven by SHT under $J = 10^{11}$ A m$^{-2}$ and $\theta_{SH} = 0.05$ (a typical value for Pt [61]) is shown in Fig. 3(c). The damping is assumed to be $\alpha = 0.05$. The Néel hopfion propagates along the wire. The longitudinal velocity component $v_x$ under different current
dients is plotted in Fig. 3(b) by red circles. The analytical formula $v_z = \frac{T}{D}$ (red line) agrees well with the numerical data. Note that the values of $T$ and $D$ depend on the hopfion profile. Since the ansatz introduced gives very good agreement with the numerical results, it may be useful in other investigations on hopfions.

Note that hopfions can also be stabilized in AFM systems, where the staggered Néel field $n$ forms a hopfion profile (see Supplemental Materials [50]).

In previous studies, it has been suggested that Bloch-type hopfions may be realized in thick films of noncentrosymmetric materials with strong surface PMA. The Néel-type hopfions that we find may be realized in stacked magnetic multilayers, where each layer has an inversion-symmetry-broken structure and PMA such as Pt/CoFe/MgO [65]. In this case, the SHT is much larger than that calculated above because the torque is exerted on layers with substantially smaller thickness $d$. Similar to Bloch-type hopfions, strong surface PMA is required. The hopfions that we found remain geometrically confined by the thickness of the film. Indeed, in the presence of DMI, Derrick’s theorem [66], which prohibits the existence of 3D solitons in infinite conventional (non-chiral) magnets, is no longer valid [67]. Whether it is possible to stabilize hopfions in 3D chiral magnets without confinement is still an open question for further investigations. Very recently, hopfions in Heisenberg ferromagnets arising from higher order exchange terms in the micromagnetic energy functional and in the absence of any DMI and confinement have been theoretically reported [68]. The current-driven dynamics of those hopfions should be similar to our case. Analytically, the effects of long-range dipolar interactions are difficult to consider. We find numerically that, ignoring the inhomogeneous part of the dipolar field and only keeping the shape anisotropy contribution, hopfions become more localized [39]. Regarding the current-driven dynamics, there are similarities between hopfions and FM target skyrmions [51]. Both of them are free of the skyrmion Hall effect. We observe that, keeping the other material parameters the same, because of the strong surface anisotropy, hopfions are more rigid than target skyrmions, which might be an advantage for further exploitations. Our study also implies that magnetic systems represent a fertile playground for research on nonlinear 3D topological solitons.

In conclusion, we identified a new type of hopfion, the Néel-type hopfion, and studied the current-driven dynamics of hopfions. In FM systems, despite the nontrivial topology, neither Bloch- nor Néel-type hopfions exhibit Hall effects and propagate along external currents via spin transfer torque. The SHT only drives the Néel-type hopfions to move along the wire. Hopfions have the potential to be efficient information carriers.

The research leading to these results was supported by the European Research Council via Advanced Grant No. 609442, “Insulatronics,” and by the Research Council of Norway through its Centres of Excellence funding scheme, Project No. 262633, “QuSpin.” X.S.W. acknowledges the support from the Natural Science Foundation of China (Grant No. 11804045) and the China Postdoctoral Science Foundation (Grant No. 2017M612932 and 2018T110957).

[1] A. M. Kosevich, B. A. Ivanov, and A. S. Kovalev, Phys. Rep. 194, 117 (1990).
[2] N. Manton and P. Sutcliffe, Topological Solitons (Cambridge University Press, Cambridge, England, 2004).
[3] D. A. Allwood, G. Xiong, C. C. Faulkner, D. Atkinson, D. Petit, and R. P. Cowburn, Science 309, 1688 (2005).
[4] S. S. P. Parkin, M. Hayashi, and L. Thomas, Science 320, 190 (2008).
[5] X. S. Wang, P. Yan, Y. H. Shen, G. E. W. Bauer, and X. R. Wang, Phys. Rev. Lett. 109, 167209 (2012).
[6] A. Qaimzadah, L. A. Kristiansen, and A. Brataas, Phys. Rev. B 97, 020402(R) (2018).
[7] B. Van Waeyenberge, A. Puzic, H. Stoll, K. W. Chou, T. Tyliszczak, R. Hertel, M. Fähnle, H. Brückl, K. Rott, G. Reiss, I. Neudecker, D. Weiss, C. H. Back and G. Schütz, Nature 444, 461 (2006).
[8] H. Y. Yuan and X. R. Wang, AIP Adv. 5, 117104 (2015).
[9] J. Sampaio, V. Cros, S. Rohart, A. Thiaville and A. Fert, Nat. Nanotech. 8, 839 (2013).
[10] N. Nagaosa and Y. Tokura, Nat. Nanotech. 8, 899 (2013).
[11] N. Romming, C. Hanneken, M. Menzel, J. E. Bickel, B. Wolter, K. von Bergmann, A. Kubetzka, and R. Wiesendanger, Science 341, 636 (2013).
[12] W. Jiang, P. Upadhyaya, W. Zhang, G. Yu, M. B. Jungfleisch, F. Y. Fradin, J. E. Pearson, Y. Tserkovnyak, K. L. Wang, O. Heinonen, S. G. E. te Velthuis, and A. Hoffmann, Science 349, 283 (2015).
[13] X. S. Wang, H. Y. Yuan, and X. R. Wang, Commun. Phys. 1, 31 (2018).
[14] V. Flovik, A. Qaimzadah, A. K. Nandy, C. Heo, and T. Rasing, Phys. Rev. B 96, 140411(R) (2017).
[15] R. Khoshlahni, A. Qaimzadah, A. Bergman, and A. Brataas, Phys. Rev. B 99, 054423 (2018).
[16] X. Z. Yu, N. Kanazawa, W. Z. Zhang, T. Nagai, T. Hara, K. Kimoto, Y. Matsui, Y. Onose and Y. Tokura, Nat. Commun. 3, 988 (2012).
[17] N. Romming, A. Kubetzka, C. Hanneken, K. von Bergmann, and R. Wiesendanger, Phys. Rev. Lett. 114, 177203 (2015).
[18] J. Zang, M. Mostovoy, J. H. Han, and N. Nagaosa, Phys. Rev. Lett. 107, 136804 (2011).
[19] W. Jiang, X. Zhang, G. Yu, W. Zhang, X. Wang, M. B. Jungfleisch, J. E. Pearson, X. Cheng, O. Heinonen, K. L. Wang, Y. Zhou, A. Hoffmann, and S. G. E. te Velthuis, Nat. Phys. 13, 162 (2017).
[20] K. Litzius, I. Lemesh, B. Krüger, P. Bassirian, L. Caretta, K. Richter, F. Böttner, K. Sato, O. A. Tretiakov, J. Förster, R. M. Reeve, M. Weigand, I. Bykova, H. Stoll, G. Schütz, G. S. D. Beach, and M. Kläui, Nat. Phys. 13, 170 (2017).
[21] M. W. Yoo, V. Cros, and J. V. Kim, Phys. Rev. B 95, 184423 (2017).
[22] R. Tomasello, E. Martínez, R. Zivieri, L. Torres, M. Car-
penticieri and G. Finocchio, Sci. Rep. 4, 6784 (2014).
[23] X. Zhang, M. Ezawa, and Y. Zhou, Sci. Rep. 5, 9400 (2015).
[24] J. Barker and O. A. Tretiakov, Phys. Rev. Lett. 116, 147203 (2016).
[25] X. Zhang, Y. Zhou, and Motohiko Ezawa, Sci. Rep. 6, 24795 (2016).
[26] L. D. Faddeev, Lett. Math. Phys. 1, 289 (1976).
[27] T.H.R. Skyrme, Proc. Roy. Soc. 260, 127 (1961).
[28] J. Hietarinta and P. Salo, Phys. Rev. Lett. B 451, 60 (1999).
[29] N. R. Cooper, Phys. Rev. Lett. 116, 147203 (2016).
[30] H. Hopf, Math. Ann. 104, 637 (1931).
[31] L. Faddeev and A. J. Niemi, Nature (London) 387, 58 (1997).
[32] L. Faddeev and A. J. Niemi, Phys. Rev. Lett. 82, 1624 (1999).
[33] Y. Kawaguchi, M. Nitta, and M. Ueda, Phys. Rev. Lett. 100, 180403 (2008).
[34] D. Kleckner and W. T. M. Irvine, Nat. Phys. 9, 253 (2013).
[35] P. J. Ackerman and I. I. Smalyukh, Phys. Rev. X 7, 011006 (2017).
[36] P. J. Ackerman and I. I. Smalyukh, Nat. Mater. 16, 426 (2017).
[37] I. Georgescu, Nat. Phys. 13, 208 (2017).
[38] J.-S. B. Tai and I. I. Smalyukh, Phys. Rev. Lett. B 98, 174437 (2018).
[39] P. Sutcliffe J. Phys. A: Math. Theor. 51, 375401 (2018).
[40] I. Dzyaloshinskii, J. Phys. Chem. Solids 4, 241 (1958).
[41] T. Moriya, Phys. Rev. 120, 91 (1960).
[42] Y. Ishikawa, K. Tajima, D. Bloch, and M. Roth, Solid State Commun. 19, 525 (1976).
[43] A. Qaiumzadeh, I. A. Ado, R. A. Duine, M. Titov, and A. Brataas, Phys. Rev. Lett. 120, 197202 (2018); I. A. Ado, A. Qaiumzadeh, R. A. Duine, A. Brataas, and M. Titov, ibid. 121, 086802 (2018).
[44] A. Zhang and Z. Li, Phys. Rev. Lett. 93, 172704 (2004).
[45] J. H. C. Whitehead, Proc. Natl. Acad. Sci. U.S.A. 33, 117 (1947).
[46] A. Thiele, Phys. Rev. Lett. 30, 230 (1973).
[47] S. Zhang and S. S.-L. Zhang, Phys. Rev. Lett. 102, 086601 (2009).
[48] A. Vansteenkiste, J. Leliäert, M. Dvornik, M. Helsen, F. Garcia-Sanchez, and B. Van Waeyenberge, AIP Adv. 4, 107133 (2014).
[49] Supplemental Material
[50] X. Zhang, J. Xia, Y. Zhou, D. Wang, X. Liu, W. Zhao, and M. Ezawa, Phys. Rev. B 94, 094420 (2016).
[51] F. Zheng, H. Li, S. Wang, D. Song, C. Jin, W. Wei, A. Kovács, J. Zang, M. Tian, Y. Zhang, H. Du, and R. E. Dunin-Borkowski Phys. Rev. Lett. 119, 197205 (2017).
[52] S. S. P. Parkin, M. Hayashi, and L. Thomas, Science 320, 190 (2008).
[53] R. Tomasello, E. Martinez, R. Zivieri, L. Torres, M. Carpentieri, and G. Finocchio, Sci. Rep. 4, 6784 (2014).
[54] X. S. Wang, H. Y. Yuan, and X. R. Wang, Commun. Phys. 1, 31 (2018).
[55] J. Hietarinta and P. Salo, Phys. Lett. B 451, 60 (1999).
[56] H. Y. Yuan and X. R. Wang, AIP Adv. 5, 117104 (2015).
[57] E. Villamor, M. Isasa, L. E. Hueso, and F. Casanova Phys. Rev. B 88, 184411 (2013).
[58] A. Brataas, A. D. Kent, and H. Ohno, Nat. Mater. 11, 372 (2012).
[59] A. Manchon and S. Zhang, Phys. Rev. B 79, 094422 (2009).
[60] X. Zhang, M. Ezawa, and Y. Zhou, Sci. Rep. 5, 9400 (2015).
[61] L. Liu, T. Moriyama, D. C. Ralph, and R. A. Buhrman, Phys. Rev. Lett. 106, 036601 (2011).
[62] K. M. D. Hals and A. Brataas, Phys. Rev. B 87, 174409 (2013); K. M. D. Hals and A. Brataas, ibid. 88, 085423 (2013).
[63] Y. Zhang, H. Y. Yuan, X. S. Wang, and X. R. Wang, Phys. Rev. B 97, 144416 (2018).
[64] X. S. Wang, A. Qaiumzadeh, and A. Brataas (unpublished).
[65] W. Legrand, J.-Y. Chauleau, D. Maccariello, N. Reyren, S. Collin, K. Bouzehouane, N. Jaouen, V. Cros, and A. Fert, Sci. Adv. 4, eaat0415 (2018).
[66] G. H. Derrick, J. Math. Phys. 5, 1252 (1964).
[67] A. Bogdanov, JETP Lett. 62, 247 (1995).
[68] F. N. Rybakov, N. S. Kiselev, A. B. Borisov, L. Döring, C. Melcher, S. Blügel, [arXiv:1904.00250]
SUPPLEMENTAL MATERIALS

Simulation Details

Most of the simulations are performed using the mumax\textsuperscript{3} package \textsuperscript{[S1]}. Some of the results of the static hopfion profile are double-checked by the OOMMF package \textsuperscript{[S2]}. All the calculates are performed at zero temperature. The mesh size is 0.5 nm×0.5 nm×0.5 nm. The surface pinning is modeled by imposing a very strong PMA $K = 10^6$ J m\textsuperscript{-3} on two additional layers attached to the top and bottom surfaces. This corresponds to a surface anisotropy $K_s = 0.5$ mJ m\textsuperscript{-2} by multiplying the mesh size.

For the static hopfion profile, the conjugate gradient method is used to minimize the total energy with an error toleration of $10^{-5}$. The sample size is 128 nm×128 nm×16 nm, as shown in Fig. S1. To be consistent with the current-driven dynamical simulations, periodical boundary conditions are imposed along the $x$ direction to mimic a long strip along the $x$ direction. Two sets of initial magnetizations are used. One magnetization is a ring of $\mathbf{m} = (0, 0, -1)$ at 25 nm≤ $r$ ≤ 40 nm and $|z|$ < 5 nm inside a uniform domain of $\mathbf{m} = (0, 0, 1)$. The other magnetization is a profile of a well-known ansatz that will be discussed below. Both sets of initial magnetizations give the same results. For the current-driven dynamics, the RK45 method is used for the temporal integration of the Landau-Lifshitz-Gilbert (LLG) equation \textsuperscript{[S4]}. The spin-transfer torque is in the Zhang-Li form \textsuperscript{[S1, S5]}.

Calculation of Hopf Index

**Analytical discussions**-As mentioned in the main text, for an infinite system, the Hopf index is defined as

$$H = \frac{1}{(4\pi)^2} \int \mathbf{F} \cdot \mathbf{A} \, dV,$$

where $F_i = \frac{1}{2} \varepsilon_{ijk} \mathbf{m} \cdot (\partial_j \mathbf{m} \times \partial_k \mathbf{m})$, in which $i, j, k = \{x, y, z\}$ and $\varepsilon$ is the Levi-Civita tensor, and $\mathbf{A}$ is a vector potential satisfying $\nabla \times \mathbf{A} = \mathbf{F}$.

We now demonstrate that the Hopf index is well-defined for an infinite system. Straightforward derivation shows that $\mathbf{F}$ is divergenceless ($\nabla \cdot \mathbf{F} = 0$) when $|\mathbf{m}| = \text{constant}$ such that the vector potential $\mathbf{A}$ exists. However, obviously, $\mathbf{A}$ is not unique. For any continuous function $\varphi(\mathbf{r})$, $\mathbf{A}' = \mathbf{A} + \nabla \varphi$ is also a vector potential. The corresponding Hopf index is

$$H' = \frac{1}{(4\pi)^2} \int \mathbf{F} \cdot \mathbf{A}' \, dV = H + \frac{1}{(4\pi)^2} \int \mathbf{F} \cdot \nabla \varphi \, dV.$$

The integral in the extra term can be rewritten as

$$\int \mathbf{F} \cdot \nabla \varphi \, dV = \int \nabla \cdot (\varphi \mathbf{F}) \, dV - \int \varphi \nabla \cdot \mathbf{F} \, dV = \oint \varphi \mathbf{F} \cdot d\mathbf{S} - 0 = \oint \varphi \mathbf{F} \cdot d\mathbf{S},$$

where Gauss’s theorem has been used and $\oint$ means the integration over the surface of the volume. In an infinite system, the surface is infinitely far away, and the $\mathbf{m}$ field should be homogenous such that, on the surface, $\mathbf{F}$ is 0, and the integral $\oint \varphi \mathbf{F} \cdot d\mathbf{S}$ vanishes. Thus, we have $H' = H$, meaning that the Hopf index is well-defined independent of the choice of $\mathbf{A}$.

For a rotationally symmetric system, it is natural to assume that the hopfion profile following $\Theta$ is independent of $\phi$, and $\Phi(r, \phi, z) = \Delta \Phi(r, z) + n\phi$, where $n$ is an integer. This form means that the polar angle ($z$ component) of $\mathbf{m}$ is independent of $\phi$, and when transversing a whole circle centered at the origin in real space ($\phi$ changes from 0 to $2\pi$), the azimuthal angle $\Phi$ of $\mathbf{m}$ uniformly rotates by $2n\pi$. With this assumption, we can write the $\mathbf{F}$ field in cylindrical coordinates as

$$F_r = \mathbf{m} \cdot \left( \frac{\partial \mathbf{m}}{r \partial \phi} \times \frac{\partial \mathbf{m}}{\partial z} \right) = -\frac{n}{r} \sin \Theta \frac{\partial \Theta}{\partial z},$$

$$F_\phi = \mathbf{m} \cdot \left( \frac{\partial \mathbf{m}}{\partial z} \times \frac{\partial \mathbf{m}}{\partial r} \right) = \sin \Theta \left( \frac{\partial \Theta}{\partial r} \frac{\partial \Delta \Phi}{\partial z} - \frac{\partial \Theta}{\partial r} \frac{\partial \Delta \Phi}{\partial z} \right),$$

$$F_z = \mathbf{m} \cdot \left( \frac{\partial \mathbf{m}}{\partial z} \times \frac{\partial \mathbf{m}}{r \partial \phi} \right) = \frac{n}{r} \sin \Theta \frac{\partial \Theta}{\partial r}.$$
The vector potential $A$ is

$$A_r = -(1 + \cos \Theta) \frac{\partial \Delta \Phi}{\partial r};$$  \hfill (S7)

$$A_\phi = \frac{n}{r}(1 - \cos \Theta),$$  \hfill (S8)

$$A_z = -(1 + \cos \Theta) \frac{\partial \Delta \Phi}{\partial z}. \hfill (S9)$$

Then, the Hopf index is

$$H = \frac{1}{(4\pi)^2} \int \mathbf{F} \cdot \mathbf{A} dV = \frac{n}{4\pi} \int_{-\infty}^{+\infty} \int_0^{+\infty} \sin \Theta \left( \frac{\partial \Theta}{\partial z} \frac{\partial \Delta \Phi}{\partial r} - \frac{\partial \theta}{\partial r} \frac{\partial \Delta \Phi}{\partial z} \right) dr dz. \hfill (S10)$$

Thus, the Hopf index equals the whirling number $n$ along the $\phi$ direction multiplied by the skyrmion number at the $rz$ half plane [S6].

Numerical evaluation of Hopf index- As discussed above, the Hopf index is well-defined when $\oint \mathbf{F} \cdot d\mathbf{S} = 0$ is satisfied. To numerically evaluate the Hopf index, we first cut off the nonhomogeneous edge such that $\mathbf{m}$ is homogeneous on the surface of the sample to ensure that the Hopf index is well-defined (see Fig. S1). We then employ two methods to calculate the Hopf index: a real space method and a Fourier space method.

In real space, we first numerically calculate $\mathbf{F}$ from $\mathbf{m}$ utilizing the standard central finite difference method. Then, we employ a radial basis function (RBF) interpolation to ensure that $\mathbf{F}$ is divergenceless [S7]. We use a Gaussian function $g(x_i, x_j) = e^{-(|x_i - x_j|^2)}$ as the RBF with control parameter $\epsilon = 1$. Here, $i$ and $j$ label two grid points, and $x_i$ and $x_j$ denote the positions of $i$ and $j$. The interpolated field is

$$\mathbf{F}(x) = \sum_i \nabla_\nabla - \nabla_2 \mathbf{g}(x, x_i) c_i. \hfill (S11)$$

FIG. S1. A sample of the simulation for the static hopfion. One octant is made transparent to visualize the magnetization profile inside. The volume in the Hopf index calculation is indicated by the red box.
After obtaining the RBF coefficients $c_i$, $A$ can be directly calculated:
\[
A(x) = -\sum_i (c_i \times \nabla) g(x, x_i),
\] (S12)
where the Coulomb gauge is used. Then, the standard numerical integration is performed to calculate $H = \frac{1}{(4\pi)^3} \int F \cdot AdV$.

The Fourier space method has been introduced in Ref. [S8]. The relative difference between the two methods is less than 5%. In the main text, we show the Fourier space result.

Construction and Verification of the Ansatz for Hopfion Profile

We start from the well-known ansatz for hopfions [S9]:
\[
m_x = \frac{4 \left[ 2xz - y \left( x^2 + y^2 + z^2 - 1 \right) \right]}{(1 + x^2 + y^2 + z^2)^2},
\] (S13)
\[
m_y = \frac{4 \left[ 2yz + x \left( x^2 + y^2 + z^2 - 1 \right) \right]}{(1 + x^2 + y^2 + z^2)^2},
\] (S14)
\[
m_z = 1 - \frac{8 (x^2 + y^2)}{(1 + x^2 + y^2 + z^2)^2},
\] (S15)
which describes an $H = -1$ Bloch-type hopfion with $m$ upward at the center and infinity. Note that in some references, the definitions of Hopf index differ by a sign. Here, we use the definition mentioned in the main text. The radius is 1 because at $r^2 = x^2 + y^2 = 1$, $m = (0, 0, -1)$. The magnetization rotates counterclockwise (clockwise) at $r > 1$ ($r < 1$) in the top view. An $H = +1$ hopfion ansatz can be obtained simply by inverting the sign of the first term in the numerators of $m_x$ and $m_y$:
\[
m_x = \frac{4 \left[ -2xz - y \left( x^2 + y^2 + z^2 - 1 \right) \right]}{(1 + x^2 + y^2 + z^2)^2},
\] (S16)
\[
m_y = \frac{4 \left[ -2yz + x \left( x^2 + y^2 + z^2 - 1 \right) \right]}{(1 + x^2 + y^2 + z^2)^2},
\] (S17)
\[
m_z = 1 - \frac{8 (x^2 + y^2)}{(1 + x^2 + y^2 + z^2)^2}.
\] (S18)

In cylindrical coordinates:
\[
m_x = \frac{4r \left[ -2 \cos \phi z - \sin \phi (r^2 + z^2 - 1) \right]}{(1 + r^2 + z^2)^2},
\] (S19)
\[
m_y = \frac{4r \left[ -2 \sin \phi z + \cos \phi (r^2 + z^2 - 1) \right]}{(1 + r^2 + z^2)^2},
\] (S20)
\[
m_z = 1 - \frac{8r^2}{(1 + r^2 + z^2)^2}.
\] (S21)

This operation can invert the Hopf index and retain the rotational sense, which is preferred for $D_h > 0$. For $D_h < 0$, we let $m_x \rightarrow -m_x$ and $m_y \rightarrow -m_y$. For a realistic hopfion with a given radius $R$ and height, it is natural to consider a linear rescaling, where $h$ describes the extent of the hopfion in the $z$ direction. This profile is used as the initial condition, with $R = 20$ nm and $h = 10$ nm. However, this ansatz cannot describe the numerical data well. In Fig. S2(a), we show the numerical magnetization profile along $x$ at $y = z = 0$ (symbols), and the above ansatz (dashed blue line) with $R = 7.8$ nm obtained from the numerical data for the Bloch-type hopfion is shown in Fig. 1(a) of the main text. Obviously, the numerical data show a much faster decay away from $x = R$ than the polynomial decay of the ansatz. Inspired by work on skyrmion profiles [S10], we introduce another length scale to describe how fast $m$...
decays to $(0, 0, 1)$. Since the $z$ direction is special because of the surface PMA, we introduce $w_R$ and $w_h$ for the $xy$ plane and $z$ direction, respectively. We try a monotonic nonlinear rescaling

$$
 r \rightarrow r' = \frac{e^{r/w_R} - 1}{e^{r'/w_R} - 1}, \quad z \rightarrow z' = \frac{|z| e^{z'/w_h} - 1}{z e^{z'/w_h} - 1},
$$

and use the resulting ansatz to fit the numerical result to determine the parameters $R, w_R, h, \text{and } w_h$. $R$ and $w_R$ are determined by fitting the radial profile $m_z(r)$ at $z = 0$. $h$ and $w_h$ are determined by fitting the profile $m_z(z)$ along $z$ at $r = R$. The result of this nonlinear rescaling (solid red line) is also compared with the numerical data in Fig. S2(a). The agreement is inversely better. For a larger hopfion (which can be obtained by using a smaller $K_b$), the agreement of our ansatz is even better.

For an $H = +1$ hopfion, with $z \neq 0$, the position of the minima of $m_z(r)$ moves outward as $|z|$ increases according to the ansatz, which is consistent with the numerical result, as shown in Fig. S2(b). In contrast, for an $H = -1$ hopfion, the minima of $m_z(r)$ moves inward, as shown in Fig. S2(c). To describe this, we further invert the rescaling.
of $r$ as

$$ r \rightarrow r' = \left( \frac{e^{r/w_R} - 1}{e^{R/w_R} - 1} \right)^{-1} = \frac{e^{R/w_R} - 1}{e^{r/w_R} - 1}, $$

(S23)

which maps $r = 0$ to $r' = \infty$ and vice versa. In cylindrical coordinates for space and spherical coordinates for $\mathbf{m}$, the ansatz can be written as

$$ \cos \Theta = 1 - \frac{8r'^2}{(1 + r'^2 + z'^2)^2}, $$

(S24)

$$ \Phi = \phi + \arctan \left( -\frac{1 + r'^2 + z'^2}{2z'} \right). $$

(S25)

The Hopf index can be calculated using Eq. (S10). Because of the inverse rescaling of $r$, the Hopf index becomes 1. To retrieve the rotational sense, we further let $m_x \rightarrow -m_x$, $m_y \rightarrow -m_y$, or $\Phi \rightarrow \Phi + \pi$. The resultant ansatz gives good agreement for $z \neq 0$, as shown in Fig. S2(b) by the solid lines. Although the ansatz cannot quantitatively overlap with the numerical data, the tendency of the minima position is correct.

![Comparison of $\mathbf{m}$ between numerical data and ansatz for (a) $H = +1$ Bloch-type hopfion ($K_b = 41 \text{ kJ m}^{-3}$, $D_b = 0.115 \text{ mJ m}^{-2}$), (b) $H = -1$ Bloch-type hopfion ($K_b = 39 \text{ kJ m}^{-3}$, $D_b = 0.115 \text{ mJ m}^{-2}$), (c) $H = +1$ Neél-type hopfion ($K_b = 20 \text{ kJ m}^{-3}$, $D_i = 0.115 \text{ mJ m}^{-2}$), (d) $H = -1$ Neél-type hopfion ($K_b = 20 \text{ kJ m}^{-3}$, $D_i = 0.115 \text{ mJ m}^{-2}$).](image-url)
To verify the continuity of the ansatz, we write it in Cartesian coordinates as

\[
\begin{align*}
m_x &= \frac{4r' \left[ -2z' \frac{x}{\sqrt{x^2+y^2}} - \frac{y}{\sqrt{x^2+y^2}} \left( r'^2 + z'^2 + 1 \right) \right]}{(1 + r'^2 + z'^2)^2}, \\
m_y &= \frac{4r' \left[ -2z' \frac{y}{\sqrt{x^2+y^2}} + \frac{x}{\sqrt{x^2+y^2}} \left( r'^2 + z'^2 + 1 \right) \right]}{(1 + r'^2 + z'^2)^2}, \\
m_z &= 1 - \frac{8r'^2}{(1 + r'^2 + z'^2)^2},
\end{align*}
\]

where \( r' = \left( \frac{\sqrt{x^2+y^2}}{e^{R/w_R-1}} \right)^{\pm 1} \). When \( xyz \neq 0 \), the ansatz has no singularity. At \( z = 0 \), we have \( \lim_{z \to 0} z' = 0 \) and \( \partial_z z'|_{z=0} = \partial_z z'|_{z=0} \). At \( x = y = z = 0 \), it is easy to verify \( \lim_{x,y,z \to 0} \mathbf{m} = (0, 0, 1) \), and all the first-order derivatives \( \partial_i m_j \) \((i, j \in x, y, z)\) are continuous. Since the highest order of derivative in the energy functional is 1, the ansatz is well-defined in the whole space.

To obtain a Néel-type hopfion, we can locally rotate the \( \mathbf{m} \) by 90 degrees, as shown in the main text:

\[
\begin{align*}
m_x &= \frac{4r' \left[ 2z' \sin \phi + \cos \phi \left( r'^2 + z'^2 + 1 \right) \right]}{(1 + r'^2 + z'^2)^2}, \\
m_y &= \frac{4r' \left[ -2z' \cos \phi - \sin \phi \left( r'^2 + z'^2 + 1 \right) \right]}{(1 + r'^2 + z'^2)^2}, \\
m_z &= 1 - \frac{8r'^2}{(1 + r'^2 + z'^2)^2}.
\end{align*}
\]

Because the interfacial DMI only exists in the \( xy \) plane and because there is no chiral interaction in the \( z \) direction, the minima in \( m_3(r) \) remains in the same position. Therefore, either \( r' = frac{R}{w_R} - 1 \) or \( r' = \frac{R}{w_R} - 1 \) can be used as the ansatz. We choose \( r' = frac{R}{w_R} - 1 \) because it gives better results in the calculation of \( D \) and \( T \) [Fig. S2(d)].

The midplane cross-sections in the \( xy \) plane and \( yz \) plane are compared in Fig. S3(a-d) for \( H = +1 \) Bloch-type, \( H = -1 \) Bloch-type, \( H = +1 \) Néel-type, and \( H = -1 \) Néel-type hopfions. The 2D cross-sections also give fairly good agreement with the numerical results. The senses of the rotation of the spins are retrieved in all the cross-sections, while the shapes of the textures in \( yz \) cross-sections are not as good as those in the \( xy \) cross-sections. Nevertheless, the ansatz can well describe the topological properties and current-driven dynamics of the hopfions.

**Antiferromagnetic Hopfions**

We consider that \( A_{xx} = -0.16 \text{ pJ m}^{-1}, D_b = 0.115 \text{ mJ m}^{-2}, K_b = 16 \text{ kJ m}^{-3} \), and the other parameters to be the same as those in Fig. 1(a). The ground state is an out-of-plane AFM Néel state. Because the dipolar field is negligible in an antiferromagnet, to speed up the simulation, we turn off the dipole-dipole interaction. For a numerical cell labelled by \((i,j,k)\), if \( i + j + l \) is even (sublattice 1), we impose the ansatz [Eq. (S19-S22)] with the collective coordinates \( R = 15 \text{ nm}, w_R = 5 \text{ nm}, h = 8 \text{ nm} \) and \( h_w = 5 \text{ nm} \). If \( i + j + k \) is odd (sublattice 2), we impose the opposite direction. After relaxation, we obtain an antiferromagnetic Bloch-type hopfion. In Fig. S4(a), the mid-plane cross-sections in the \( xy \) and \( xz \) planes are shown for each sublattice. If we use \( K_b = 5 \text{ kJ m}^{-3} \) and \( D_l = -0.115 \text{ mJ m}^{-2} \) instead, a Néel-type AFM hopfion is obtained, as shown in Fig. S4(b).

---

S1 A. Vansteenkiste, J. Leliaert, M. Dvornik, M. Helsen, F. Garcia-Sanchez, and B. Van Waeyenberge, AIP Adv. 4, 107133 (2014).
FIG. S4. Mid-plane cross-sections in the $xy$ plane (upper) and $xz$ plane (lower) of sublattices 1 (left) and 2 (right). (a) Bloch-type AFM hopfion. (b) Néel-type AFM hopfion.
S2 M. J. Donahue and D. G. Porter, OOMMF User’s Guide, Interagency Report NISTIR 6376, NIST, Gaithersburg, MD, 1999. [http://math.nist.gov/oommf]

S3 M. R. Hestenes and E. Stiefel, J. Research Natl. Bur. Standards 49, 409 (1952).

S4 T. L. Gilbert, IEEE. Trans. Magn. 40, 3443 (2004).

S5 S. Zhang and Z. Li, Phys. Rev. Lett. 93, 127204 (2004).

S6 J. Gladikowski and M. Helimund, Phys. Rev. D 56, 5194 (1997).

S7 C. P. McNally, Mon. Not. R. Astron. Soc. 413, L76 (2011).

S8 Y. Liu, R. K. Lake, and J. Zang, Phys. Rev. B 98, 174437 (2018).

S9 J. Hietarinta and P. Salo, Phys. Lett. B 451, 60 (1999).

S10 X. S. Wang, H. Y. Yuan, and X. R. Wang, Commun. Phys. 1, 31 (2018).