Analysis for hysteresis of piezoelectric actuator based on microscopic mechanism

H Li¹, Y Xu³, M Shao³, L Guo¹ and D An²,³*

¹ School of Mechanical Engineering, Shenyang Jianzhu University, Shenyang 110168, China
² Research Center for Analysis and Detection Technology, Shenyang Jianzhu University, Shenyang 110168, China
³ Research Institute for Micronano Detection and Motion Control, Shenyang Jianzhu University, Shenyang 110168, China

* E-mail: andong@sjzu.edu.cn

Abstract. The hysteresis widely existing in piezoelectric ceramic actuators seriously affects the positioning accuracy in practical applications. Based on the microscopic displacement mechanism of piezoelectric actuators, this paper studied the displacement mechanism of the electrostrictive effect and the inverse piezoelectric effect from the microscopic polarization mechanism and, at the same time, the displacement mechanism of electric domain reversal in ferroelectric effect. This paper clarified that the inverse piezoelectric effect and the ferroelectric effect are the main causes for the displacement of the piezoelectric actuator. The contribution of the electrostrictive effect microscopically is extremely weak and can be ignored. This paper shows that the relationship between the voltage and displacement of the piezoelectric actuator in the inverse piezoelectric effect is linear, and the hysteresis characteristic mainly exists in the ferroelectric effect. In this paper, we pointed out that there is energy loss during the piezoelectric domain transition of piezoelectric ceramics, and some irreversible non-180° domain steering is the root cause of the hysteresis of the piezoelectric actuator. This paper provides a scientific basis for further hysteresis curves modelling or correcting hysteresis modelling errors of piezoelectric ceramic and for improving the control accuracy of piezoelectric ceramic actuators in practical applications.

1. Introduction
With the rapid development of nano and micro-level electronic technology, people's requirements for electronic devices are gradually turning to the microscopic field. The research and application of micro-displacement technology has received more and more attention, such as the case of piezoelectric ceramics as a nanometer-level driving element, due to its high displacement resolution, high electromechanical coupling efficiency, large output force, small size, fast response, etc. In the field of micron and nanometer, drive has an excellent performance, thus, it is widely used in precision positioning, precision machining, biomedical, robotics, aerospace and other micro-displacement technologies [1, 2].

However, in the practical application of piezoelectric ceramics, the widely-occurring hysteresis characteristics in piezoelectric materials seriously affect the control accuracy of piezoelectric ceramic
actuators, thus greatly limiting its application in micro-displacement technology [3-5]. The hysteresis characteristic of piezoelectric ceramics means that the corresponding displacement curves of piezoelectric ceramics in the voltage lift and return intervals do not overlap, and there is a displacement difference between this two curves. The hysteresis characteristic of piezoelectric ceramics is the non-local storage type inherent characteristic of piezoelectric ceramics [6]. Figure 1 shows the relationship between the displacement and the voltage across the piezoelectric actuator. It can be seen from the figure that when the voltage is applied to the piezoelectric ceramic, the voltage rising displacement curve and the voltage falling displacement curve are inconsistent, and the displacement does not return to zero after the applied voltage is reduced. This property is called the hysteresis characteristic of the piezoelectric ceramic. This indicates that the output displacement of the piezoelectric actuator is determined not only by the current input or applied voltage, but also by the historical input voltage [7].

![Figure 1. Hysteresis characteristic of a piezoelectric ceramic.](image)

As more and more researchers focus on piezoelectric ceramic actuators, many workers have tried to compensate for hysteresis by using modelling methods. They can be roughly divided into two parts: physics-based models and phenomenological-based models [8]. The physics-based model refers to the scientific concept abstracted from a large number of experiments for the purpose of facilitating research. It is based on the physical meaning of hysteresis characteristics and can be strictly verified. One of the advantages of physics-based models is that their physical meaning is clear. However, due to the complexity of the form, physics-based models are not often used for the control of piezoelectric ceramic actuators.

The commonly used physics-based hysteresis modelling methods for piezoelectric ceramics include the Jiles–Atherton (J–A) model [9] and the Maxwell model [10].

Robert Malczyk et al proposed an extension of the Jiles–Atherton (J–A) magnetic hysteresis model to describe the hysteresis curve narrowing phenomenon in ferrite ZnMn material. Their new model permits the inclusion of a wide variety of additional effects observed for ferromagnetic materials without invalidating the well-known and broadly used J–A model parameters. The experiment proves the feasibility of this method [11].

Yanfang Liu et al. presented a Maxwell model to describe the hysteresis in a piezoelectric actuator. They studied the effect of the number of elements and presented both the forward and inverse
algorithms. Moreover, they used the inverse Maxwell model obtained an almost linear performance of hysteresis compensate. The result of their experimental validates the effectiveness of the proposed algorithm and show that hysteresis nonlinearity reduces from 13.8 to 0.4% [12]. The Phenomenon-based models refers to the models that the researchers obtain by generalizing and summarizing the input and output data as well as the actual experimental phenomena. These models use mathematical methods which directly establish mathematical models to satisfy the experimental rules, regardless of physical meanings, such as Preisach model [13], Prandtle-Ishlinskii (PI) model [14], Duhem model [15] and Bouc-Wen model [16].

Song et al. proposed a novel modified Preisach model to identify and simulate the hysteresis phenomenon observed in a piezoelectric stack actuator. Their approach can handle a varying-frequency dependence by employing a time-derivative correction technique. Parameter estimation and model verification demonstrate high accuracy of the derived model, keeping the deviation in a low percentage range (about 2–3%) [8].

Wang et al. proposed a novel modified Bouc–Wen (MBW) model to describe the asymmetric hysteresis of a piezoelectric actuator. They used a polynomial-based non-lag component to realize the asymmetric hysteresis property. The results demonstrate that their model is superior to its competitors’ models in describing the asymmetric hysteresis of a piezoelectric actuator [17]. However, the lack of physical meaning makes the above models difficult to understand. At the same time, neither of these two types of models reveals the cause of hysteresis characteristics from a microscopic perspective, so the modelling errors in these modelling methods are inevitable.

Conducting a research about the hysteresis characteristics of piezoelectric ceramics is the basis for hysteresis modelling of piezoelectric ceramics and for improving the control accuracy of piezoelectric ceramic actuators.

Based on the microscopic displacement mechanism of piezoelectric actuators, this paper analysed the electrostrictive effect and the inverse piezoelectric effect displacement mechanism from the microscopic polarization mechanism and analysed the ferroelectric’s effect domain reversal mechanism.

We explained that the inverse piezoelectric effect and ferroelectric effect are the main causes of the displacement of the piezoelectric actuator. The energy loss during the micro domain reversal of piezoelectric ceramics was analysed. The results show that some irreversible non-180° domain steering is the root cause of the hysteresis in the piezoelectric actuator.

This paper provides a scientific basis for further hysteresis curves modelling or correcting hysteresis modelling errors of piezoelectric ceramic and improving the control precision of piezoelectric ceramic actuators in the practical application process.

2. Micro-mechanism for displacement of piezoelectric actuator
Piezoelectric ceramic is a kind of ferroelectric material. In the absence of an external electric field, its internal structure is the same as that of a ferroelectric material, and it is in a highly polarized state, therefore, the piezoelectric ceramic is consistent with ferroelectric material in performance. When an external electric field is applied across the piezoelectric ceramic, the internal structure of the piezoelectric ceramic will follow the change of the applied electric field to generate the electrostrictive effect, the inverse piezoelectric effect and the ferroelectric effect [18].

Piezoelectric ceramics are obtained after ferroelectric ceramic materials pre-polarization, generally piezoelectric ceramic working area used for precision positioning as shown in figure 2.
Figure 2. Curves for the electric polarization with electric field.

For piezoelectric ceramics, after pre-polarization, the residual polarization $P_r$ is fixed. The mechanism of displacement deformation of piezoelectric ceramics can be understood as occurring on the basis of residual polarization strain. Without applying an external electric field, the total polarization strain of the piezoelectric ceramic is equal to its residual polarization strain, i.e. $X = X_r$. The total polarization of piezoelectric ceramics is mainly composed of remnant polarization $P_r$ and irreversible polarization $P_\nu$. According to the ferroelectric phenomenological theory of ferroelectric crystals, the Gibbs free energy power series expansion is:

$$G_i = \frac{T - T_0}{2C}P^2 + \frac{1}{4}\beta P^4 + \frac{1}{6}\gamma P^6 - \frac{1}{2}sY^2 - GP^2Y$$  \hspace{1cm} (1)$$

In the formula: $T$ is temperature, $T_0$ is Curie-Weiss temperature, $s$ is elastic modulus, $C$ is Curie constant, $G$ is electrostriction coefficient, $Y$ is external stress of piezoelectric ceramics, $\beta$, $\gamma$ is irrelevant coefficient.

The strain of piezoelectric ceramic is:

$$X = -\frac{\partial G_i}{\partial X} = sX + GP^2$$  \hspace{1cm} (2)$$

When there is no external stress apply to the piezoelectric ceramic, its strain is:

$$X = GP^2 = G(P_r + P_\nu) = GP_r^2 + gP_r + GP_\nu^2$$  \hspace{1cm} (3)$$

In the formula: $g$ is piezoelectric coefficient, $g = 2GP_r$. The formula (3) shows that the strain of the piezoelectric ceramic is determined by the strength of the polarization. When the ferroelectric material is pre-polarized, its internal structure has changed and will exist in the form of a piezoelectric ceramic. The change of the strain displacement is determined by the following three:

- Residual polarization strain:
  $$X_r = GP_r^2$$  \hspace{1cm} (4)$$
When the external electric field is zero, the remanent polarization $P_r$ is considered to be reversible, so the relationship between the residual polarization strain $X_r$ and the applied electric field is also reversible, and there is no hysteresis. In this paper we defined that the strain of the piezoelectric ceramic produces a certain amount of elongation and contraction based on the residual polarization strain.

Electrostrictive effect strain:

$$X_k = GP_{ir}$$

For a piezoelectric ceramic, when the inverse piezoelectric effect and the electrostrictive effect exist simultaneously, because the magnitude of the electrostriction coefficient is much smaller than that of the piezoelectric coefficient, the contribution of electrostrictive effect to the macroscopic displacement of the piezoelectric ceramic is very weak and often ignored.

Inverse piezoelectric effect strain:

$$X_p = gP_e = dE$$

In the formula, $d$ is the piezoelectric strain coefficient.

The inverse piezoelectric effect was discovered when the Curie brothers studied quartz crystals in 1880 [19]. In this paper, we give the piezoelectric effect and inverse piezoelectric effect strain mechanism: When the mechanical force is applied to the outer surface of the piezoelectric ceramic, the surface of the crystal will generate a polarization charge. This phenomenon of the piezoelectricity of piezoelectric ceramics caused by mechanical forces is called the direct piezoelectric effect, as shown in figure 3a. In contrast, if an external voltage is applied across the piezoelectric crystal, the electric dipole inside the crystal is polarized, causing the piezoelectric ceramic to deform macroscopically. This phenomenon of piezoelectric ceramic deformation caused by an applied electric field is called inverse piezoelectric effect, as shown in figure 3b. From the derivation of the above equation, it can be seen that the relation of output displacement in the inverse piezoelectric effect with the strength of the applied electric field is linear, and there is no hysteresis characteristic.

Figure 3. Piezoelectric effect diagram (Red dashed lines indicate after deformation): (a) Direct piezoelectric effect diagram; (b) Inverse piezoelectric effect diagram. The black rectangle represents the original shape of the piezoelectric ceramic block, and the red dashed rectangle represents the deformed shape.
Piezoelectric ceramic is a kind of ferroelectric material. In the case of no external electric field inside the piezoelectric ceramic, the dipole distance between the internal crystal molecules is fixed and aligned in a certain direction. At this time, the piezoelectric ceramic crystal is in the high polarization state. In a highly-polarized ferroelectric material, it is always spontaneously split into a series of small regions with different polarization directions, so that the electric field intensity generated by the spontaneous polarization inside the crystal and the electric field applied by the external space cancel each other out. Therefore, there is no electricity in the piezoelectric ceramic. These spontaneously polarized microdomains with uniform orientation are called domains. The internal domains of piezoelectric ceramics usually have four directions: 71° domains, 90° domains (as shown in figure 4), 109° domains, and 180° domains. It should be noted that for the piezoceramic crystal strain, only the non-180° domain steering contributes to the displacement of the piezoceramic actuator, while the steering of other electric domains has no effect on the displacement. The spontaneous polarization direction of the domains will be redirected under the influence of an external electric field. In the presence of an external electric field, the phenomenon that the spontaneous domain in the piezoelectric ceramic reorientation is called ferroelectric effect.

![Figure 4. Piezoelectric crystal domain diagram.](image)

From the above analysis, we summarized as follows: The displacement of piezoelectric ceramic is caused by the electrostrictive effect, the inverse piezoelectric effect and the ferroelectric effect. The strain mechanisms of the electrostrictive effect and the inverse piezoelectric effect are the electric dipole polarization in the piezoelectric ceramic based on the residual polarization strain, but the contribution of the electrostrictive effect to the macroscopic displacement of the piezoelectric ceramic is very weak, and often negligible. The relation between the output displacement of the inverse piezoelectric effect and the applied electric field is linearly, there is no hysteresis. The ferroelectric effect displacement mechanism is due to the internal domain reversal of the piezoelectric ceramic. When a fixed electric field is applied across the piezoelectric ceramic, the electric domains inside the piezoelectric ceramic will generate a certain degree of turn and elongation along the direction of the electric field, and the boundaries of the domains will also generate elongational deformation. Therefore, the piezoelectric ceramic will generate elongational deformation along the direction of the electric field (as shown in figure 5).

![Figure 5. Schematic diagram of the spontaneous polarization alignment: (a) before; (b) during; (c) after presence of an electric field.](image)
3. Analysis for hysteresis cause

In the above discussion, we know that the displacement of the piezoelectric ceramic is caused by the electrostrictive effect, the inverse piezoelectric effect, and the ferroelectric effect. Theoretically, the three effects of piezoelectric ceramics exist more or less at the same time in practical applications, but the microscopic performance of the electrostrictive effect has a very weak influence on the displacement of the piezoelectric actuator, so it is negligible. We can know from equation (6): in the inverse piezoelectric effect, the relation between the piezoelectric ceramic output displacement and the electric field is linear, and there is no hysteresis, so the ferroelectric effect is the root cause of piezoelectric ceramic hysteresis.

In this paper, we analyse the causes of piezoelectric ceramic hysteresis based on the energy loss model during microscopic domain reversal. In 2007, Labey et al. proposed that when the domains are rotated under the influence of an electric field, the entire domain is not oriented like a dipole. On the contrary, there will be the following four stages: the nucleation of new domains, the vertical growth of new domains, the lateral expansion of new domains and the consolidation of new domains [20]. Piezoelectric deformation is due to the internal domain transition. Previous experiments have confirmed that the physical mechanism of domain reversal is a nucleation process, and the nucleation rate of domains is a function of the applied electric field. Therefore, through the change of the nucleation rate, the volume change rate of the domain reversal can be obtained, and the deformation rate of the piezoelectric ceramic can be obtained. It is generally believed that in the low electric field range, the nucleation rate has the following relationship with the electric field:

$$n_1 = k \exp \left( -\frac{\delta}{E} \right)$$

(7)

Where the $n$ is the number of nucleation inside the piezoelectric ceramic per unit time, $\delta$ is the activation field strength, $k$ is a constant. In the high electric field, the nucleation rate has the following relationship with the electric field:

$$n_2 = h E^{1.4}$$

(8)

Where $h$ is a constant. Here we assume that the saturation field strength of the ferroelectric ceramic is, the electric field changes uniformly, the total number of domains contained in the piezoelectric ceramic crystal is:

$$N = \int_0^{E_1} k \exp \left( -\frac{\delta}{E} \right) dE + \int_{E_1}^{2E_1} h E^{1.4} dE$$

(9)

Theoretically, the total number of domains before and after the deformation of the piezoelectric ceramic should be the same, i.e. $N_1 = N_2$ ( $N_1$ is the number of non-180° domain steering in the piezoelectric ceramic when the applied field strength crosses the piezoelectric ceramic increases, $N_2$ is the number of non-180° domain steering in the piezoelectric ceramic when applied field strength across the piezoelectric ceramic decreases). When the applied electric field strength exceeds a certain critical field strength (when the direction of the electric field begins to change), the piezoelectric ceramic strain besides the contribution of the inverse piezoelectric effect. Non-180° domain steering began to dominate. During the turning of the domain, there are some obstacles inside the crystal that prevent the domains from turning, which causes energy loss in the domain turning process. According to the method proposed by Jiles [21], it is assumed that the surface area of the domains is $A$, the movement distance of the domains is $x$, the energy consumed by the domain crossing the obstacle is:

$$u = -\int_0^x \frac{n <u_x^2>}{2}(1 - \cos \theta)Adx$$

(10)
Here, $n$ is the average number of obstacles, $u_x$ is the average energy required for the non-180° domain to cross the obstacle.

Since this part of the energy loss is not recoverable, when the field strength decreases, some non-180° domains cannot recover to the same level as when the field strength is increased and resulting in the hysteresis of the piezoelectric ceramic. From the above analysis, it can be concluded that the hysteresis of the piezoelectric ceramic is due to the irreversible energy loss in the domain turning process, resulting in the number of the non-180° domain rotation during the process of the applied electric field increases and decreases is different, i.e. $N_1 > N_2$, then, resulting in the hysteresis in the process of piezoelectric ceramic deformation. In addition, the greater the field strength, the greater the irreversibility of the non-180° domain steering and the larger the hysteresis displacement of the piezoelectric ceramic actuators.

4. Conclusion
In this paper, we analysed the micro displacement mechanism of piezoelectric actuators systematically. The electrostrictive effect displacement mechanism and inverse piezoelectric effect displacement mechanism of piezoelectric ceramic is analysed in detail based on the microscopic polarization mechanism. The displacement mechanism of the ferroelectric effect is analysed from a microscopic view based on the domain reversal theory. The energy loss model of piezoelectric ceramics during domain reversal is given, the energy loss in the domain reversal process of piezoelectric ceramics is explained and then we propose that the partially irreversible non-180° domain steering is the root cause of piezoelectric ceramic hysteresis. This paper provides a scientific basis for further hysteresis curves modelling or correcting hysteresis modelling errors of piezoelectric ceramic and improving the control accuracy of piezoelectric ceramic actuators in practical applications.

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References
[1] Fan W, Lin Y Y and Zhong-Shen L I 2016 Hysteresis characteristics of piezoelectric ceramic actuators Optics & Precision Engineering 24 1112-1117
[2] Bahrami A, Tafaoli-Masoule M and Bahrami M N 2013 Active Vibration Control of Piezoelectric Stewart Platform Based on Fuzzy Control. International Journal of Material & Mechanical Engineering 2 17-22
[3] Qin Y, Tian Y, Zhang D, Shirinzadeh B and Fatikow S 2013 A Novel Direct Inverse Modelling Approach for Hysteresis Compensation of Piezoelectric Actuator in Feedforward Applications IEEE/ASME Transactions on Mechatronics 18 981-989
[4] Rakotondrabe M 2011 Bouc–Wen Modelling and Inverse Multiplicative Structure to Compensate Hysteresis Nonlinearity in Piezoelectric Actuators IEEE Transactions on Automation Science & Engineering 8 428-431
[5] Chang-Hai R U, Wang ZH, Chen L G and Sun L N 2010 A Hysteresis Control Model of Piezoelectric Actuator Based on Microscopic Polarization Mechanisms Control Engineering of China 17 1107-1114
[6] Gu G Y, Zhu L M, Su C Y, Ding H and Fatikow S 2016 Modeling and Control of Piezo-Actuated Nanopositioning Stages: A Survey IEEE Transactions on Automation Science & Engineering 13 313-332
[7] Al Janaideh M F 2009 Generalized Prandtl-Ishlinskii hysteresis model and its analytical inverse for compensation of hysteresis in smart actuators. *Mechanical & Industrial Engineering* **09** 307-312

[8] Song X, Duggen L, Lassen B and Mangeot C 2017 Modeling and Identification of Hysteresis with Modified Preisach Model in Piezoelectric Actuator *IEEE International Conference on Advanced Intelligent Mechatronics* **2017** pp 1538-1543

[9] Gao X, Liu Y and Pei Z 2016 Minor loop dynamic Jiles-Atherton model in giant magnetostrictive actuator *Journal of Beijing University of Aeronautics & Astronautics* **42** 2648-2653

[10] Wang B S, An WG and Zhang D 2008 Electrical breakdown mechanics and reliability analysis for PZT piezoelectric ceramics *Journal of Harbin Engineering University* **29** 129-337

[11] Malczyk R and Izydorczyk J 2015 The frequency-dependent Jiles–Atherton hysteresis model *Physica B Condensed Matter* **463** 68-75

[12] Liu Y, Liu H, Wu H and Zou D 2015 Modelling and compensation of hysteresis in piezoelectric actuators based on Maxwell approach *ELECTRON LETT* **52** 188-190

[13] Liu L, Tan K K, Chen S L, Huang S and Lee TH 2012 SVD-based Preisach hysteresis identification and composite control of piezo actuators *ISA T* **51** 430-438

[14] Hassani V, Tjahjowidodo T and Do T N 2014 A survey on hysteresis modeling, identification and control *MECH SYST SIGNAL PR* **49** 209-233

[15] Chen H, Tan Y, Zhou X, Dong R and Zhang Y 2011 Identification of Dynamic Hysteresis Based on Duhem Model *Fourth International Conference on Intelligent Computation Technology and Automation* **2011** 810-814

[16] Zhu W and Rai X T 2015 Online parameter identification of Bouc-Wen model for piezoelectric actuators *Optics & Precision Engineering* **38** 921-927

[17] Wang G, Chen G and Bai F 2015 Modeling and identification of asymmetric Bouc–Wen hysteresis for piezoelectric actuator via a novel differential evolution algorithm *Sensors and Actuators A: Physical* **235** 105-118

[18] Huang X, Zeng J, Ruan X, Zheng L and Li G 2017 Structure, electrical and thermal expansion properties of PZnTe-PZT ternary system piezoelectric ceramics *J AM CERAM SOC* **101** 274-282

[19] Weihua X U, Hai B, Yang Y and Wei X 2010 Voltage Signal Transmission Principle Based on the Inverse Piezoelectric Effect of Piezoelectric Ceramic *Automation of Electric Power Systems* **34** 80-83

[20] Rabe K M, Ahn CH and Triscone J M 2007 *Physics of Ferroelectrics* Springer Berlin Heidelberg p 203-234

[21] Jiles DC and Thoelke J B 1989 Theory of ferromagnetic hysteresis: determination of model parameters from experimental hysteresis loops *Magnetics IEEE Transactions on* **25** 3928-3930