A new approach solution for fuzzy assignment problem using the development Zimmermann method

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Abstract. Transportation problems and assignment problems are special cases of linear programming problems. Puspita, Lukman and Agustina [1] studied about the development of Zimmermann method to solve fuzzy linear programming problems was known as the Zimmerman method of development. The Zimmerman method of development was carried out using the concept of Zimmermann method and Thorani’s ranking function. This paper discusses about solution for fuzzy assignment problems with parameter costs being fuzzy number using the development Zimmermann method that is new method that introduced by Puspita, Lukman, and Agustina [1]. In this article also presents an illustration of a numerical example about the development Zimmermann method for solving fuzzy assignment problems.

1. Introduction
Assignment problems are one special case of linear programming problems. Assignment problems have an important role in industry and other application fields. This is because the assignment problem is a special case of linear programming problems relating to the distribution of tasks or responsibilities of a number of jobs to a number of workers or employees who are considered effective and efficient to do the job. The assignment problem assumes that a worker or an employee can only be assigned to one type of work, and vice versa. The purpose of the assignment problem is to obtain an optimal assignment so that the total cost of production is as small as possible or the total profit is as large as possible. The parameter of the assignment problem is the cost of assigning a job i by a worker j.

There is an effective algorithm for solving Crips assignment problems that is when the values of the parameters are known with certainty. But in reality it rarely happens, usually the parameter values cannot be known with certainty. Uncertainty in the value of this parameter can be due to data uncertainty as a result of company policy or lack of information. If this happens, then the procedure of solving Crips assignment problems cannot be used. One method that can be used to overcome uncertainties that arise in parameter values is the use of fuzzy set operations. Therefore, the fuzzy assignment problem (fuzzy assignment model) is a model that can represent assignment problems in the real world. The fuzzy assignment problem is an assignment problem where the cost of assigning a job i by worker j is in a fuzzy number.

Bellman and Zadeh [2], and Zimmermann [3] were the first scientists that introduce about the solution of the fuzzy linear programming by transforming the fuzzy linear programming into the Crips linear programming problem. Furthermore, developing studies on fuzzy linear programming problems along with their solutions, especially fuzzy assignment problems. Some researchers who examine the problem of fuzzy assignment problems include: Lin and Wen [4] solve the fuzzy assignment problem with parameter cost is a fuzzy number using a labelling algorithm. Mukherjee and Basu [5] discussed the...
settlement of fuzzy assignment problems with parameter cost is fuzzy number using the Yager Ranking Index [6] by transforming the problem of fuzzy assignment problems into crips assignment problems. Kumar and Gupta [7] discussed about the fuzzy assignment problem solving and traveling salesman problem using the fuzzy membership functions are different and the index ranking Yager [6].

On further developments, Thorani, Phani, and Ravi [8] introduced the index ranking Thorani to solve the linear fuzzy programming problems. The advantages of the index ranking Thorani are compared with other index ranking methods, namely Thorani’s index ranking is considered more accurate in comparing several fuzzy numbers. In 2014, Thorani and Shankar introduced methods to find optimal solutions for fuzzy transportation problems. The principle is to convert fuzzy transportation problems into Crips transportation problems by using the index ranking Thorani. Furthermore, the optimal solution is determined by using the Crips transportation problems. Subsequent developments, Puspita, Lukman, and Agustina [1] proposed a new method to solve fuzzy linear problems by using the concept of the Zimmermann method and the index ranking Thorani. This new method is known as the development Zimmermann method. The development Zimmermann method was carried out to overcome the fuzzy linear programming problem with an unlimited solution area. The result was that the development Zimmermann method was seen as effective in solving the fuzzy linear programming problems with unlimited solutions. Because of that, in 2016, Agustina and Puspita [9] discussed the use of the development Zimmermann method to solve fuzzy transportation problems. So, in this paper will discuss about the use of the development Zimmermann method for solving fuzzy assignment problems.

2. Methods

Definition 1

General Fuzzy Trapezoid Numbers \( \tilde{A} = (a, b, c, d; w) \) called the generalized fuzzy numbers where \( w \in [0,1] \) with the membership function as follows [1,9].

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{w x - a}{b - a}, & a \leq x \leq b \\
 \frac{w}{2}, & b \leq x \leq c \\
\frac{w d - c}{d - c}, & c \leq x \leq d 
\end{cases}
\]  

(1)

Arithmetic Operation Fuzzy Trapezoid Numbers [1,9]

Suppose \( \tilde{A}_1 = (a_1, b_1, c_1, d_1; w_1) \) and \( \tilde{A}_2 = (a_2, b_2, c_2, d_2; w_2) \) then the arithmetic operations of the trapezoidal fuzzy numbers are:

(i) \( \tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min (w_1; w_2)) \)

(ii) \( \tilde{A}_1 \otimes \tilde{A}_2 = (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2; \min (w_1; w_2)) \)

(iii) \( k\tilde{A}_1 = (ka_1, kb_1, kc_1, kd_1; w_1), k > 0 \)

\( k\tilde{A}_1 = (kd_1, kc_1, kb_1, ka_1; w_1), k < 0 \)

Definition 2

Suppose \( \tilde{A} = (a, b, c, d; w) \) is a trapezoidal fuzzy number, then the \( \tilde{A} \) rank is

\[
R(\tilde{A}) = \left[ x^{\frac{a+b+2b}{3}} \frac{y b + x^2 + z^2 + z^2}{y + z} \right] \times \frac{x^\frac{w}{3} + y^\frac{w}{2} + z^\frac{w}{2}}{x + y + z}
\]  

(2)

where, \( x = \sqrt{\frac{(2d-2b)^2 + 4w^2}{6}} \), \( y = \sqrt{\frac{(d-a)^2}{3}} \), \( z = \sqrt{\frac{(2b-2a)^2 + 4w^2}{6}} \).

Definition 3

Suppose that \( b \) and \( p \) are positive real numbers that are obtained from decision maker as stated by Bellman and Zadeh [2]. Based on the form of symmetric trapezoidal fuzzy numbers along with the membership function of the Zimmermann fuzzy number, the spread \( \text{spread}(\tilde{A}) = wp_0 \) is obtained [1, 9].
Suppose that $\tilde{A}$ is a symmetric trapezoidal fuzzy number formed from the coefficient of real numbers $c$, then:

$$
\tilde{A}(x) = \begin{cases} 
0, & x \in (-\infty, c - \frac{1}{2}p_0] \cup [c + \frac{1}{2}p_0, +\infty) \\
3w - w \frac{(c-x)}{p_0}, & x \in [c - \frac{1}{2}p_0, c - \frac{1}{2}p_0] \\
w, & x \in [c - \frac{1}{2}p_0, c + \frac{1}{2}p_0] \\
4w - w \frac{(x-c)}{p_0}, & x \in [c + \frac{1}{2}p_0, c + \frac{1}{2}p_0] 
\end{cases}
$$

(3)

where $w \in [0,1]$.

2.1. The development Zimmermann method

Suppose that it is known the Fuzzy Linear Programming (PLF) Zimmermann model [9]

Objective Function
Max $z = CX$

Subject to,
$AX \leq b$
$X \geq 0$

(4)

and

Objective Function
Max $z = CX$

Subject to,
$AX \leq b + p$
$X \geq 0$

(5)

where $p = (p_1, p_2, ...)$ is chosen by the decision maker. Suppose the optimal solution from (4) and (5) are $z_1$ and $z_2$, then choose $b_0 = z_2$ and $p_0 = b_0 - z_1$.

The PMZ method is divided into two phases, namely:

**PHASE I:**

In the first phase the thing that is done is by using definition 3 fuzzy linear programming Zimmermann model is converted into fuzzy linear programming Thorani model.

The fuzzy linear programming Zimmermann model (4) and model (5) are known. Then, select $b_0 = z_2$ and $p_0 = b_0 - z_1$, where $z_1$ and $z_2$ are the optimal solution model (4) and model (5) respectively. Then make a fuzzy linear programming model which is made from the real number coefficient crip linear programming model (4) and obtained,

$$
\tilde{c}_j = (c_j - \frac{1}{2}p_0, c_j - \frac{1}{2}p_0, c_j + \frac{1}{2}p_0, c_j + \frac{1}{2}p_0)
$$

(6)

$$
\tilde{a}_{ij} = (a_{ij} - \frac{1}{2}p_0, a_{ij} - \frac{1}{2}p_0, a_{ij} + \frac{1}{2}p_0, a_{ij} + \frac{1}{2}p_0)
$$

(7)

$$
\tilde{b}_i = (b_i - \frac{1}{2}p_0, b_i - \frac{1}{2}p_0, b_i + \frac{1}{2}p_0, b_i + \frac{1}{2}p_0)
$$

(8)

so, the fuzzy linear programming Thorani model is

Objective Function
Max $Z \cong \tilde{c}_1x_1 \oplus \tilde{c}_2x_2 \oplus ... \oplus \tilde{c}_nx_n$

Subject to,

$\tilde{a}_{11}x_1 + \tilde{a}_{12}x_2 + \ldots + \tilde{a}_{1n}x_n \leq \tilde{b}_1$

$\tilde{a}_{21}x_1 + \tilde{a}_{22}x_2 + \ldots + \tilde{a}_{2n}x_n \leq \tilde{b}_2$

$\vdots$

$\vdots$

$\tilde{a}_{m1}x_1 + \tilde{a}_{m2}x_2 + \ldots + \tilde{a}_{mn}x_n \leq \tilde{b}_m$

$X \geq 0$

(9)
PHASE 2
The completion process in phase 2 consists of several steps, as follows:

- First step is to convert fuzzy linear programming Thorani model (9) into Crips linear programming model,

\[
\text{Objective Function} \quad \text{Max} \quad Z \equiv R(\tilde{c}_1)x_1 \oplus R(\tilde{c}_2)x_2 \oplus \cdots \oplus R(\tilde{c}_n)x_n
\]

\[
\text{Subject to,} \quad R(\tilde{a}_{11})x_1 + R(\tilde{a}_{12})x_2 + \cdots + R(\tilde{a}_{1n})x_n \leq R(\tilde{b}_1)
\]

\[
R(\tilde{a}_{21})x_1 + R(\tilde{a}_{22})x_2 + \cdots + R(\tilde{a}_{2n})x_n \leq R(\tilde{b}_2)
\]

\[
\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots
\]

\[
R(\tilde{a}_{m1})x_1 + R(\tilde{a}_{m2})x_2 + \cdots + R(\tilde{a}_{mn})x_n \leq R(\tilde{b}_m)
\]

\[X \geq 0\]

The conversion is done using a fuzzy number sequencing algorithm. If the model (10) does not have a feasible solution, then the process stops. This means that fuzzy linear programming model Thorani (9) does not have a feasible solution. If it does not have a feasible solution, the process does not continue to the next step.

- The second step is to determine the optimal solution for Crips linear programming model (10). Suppose that \(Z_{\text{max}} = z^*\) is obtained with the decision variable \(X^*\).

- The third step is to substitute \(X^*\) that obtained in second step into fuzzy linear programming Thorani model (9) so that an optimal solution of fuzzy linear programming can be obtained, that is

\[
Z_{\text{max}} = \tilde{c}_1x_1^* \oplus \tilde{c}_2x_2^* \oplus \cdots \oplus \tilde{c}_n x_n^*
\]

with decision variables \(X^* = (x_1^*, x_2^*, \ldots, x_n^*)\)

3. Results and Discussion

3.1. Fuzzy assignment problem

As previously stated that the assignment problem is a special case of linear programming problems. Based on this, the use of Zimmermann method of development to solve fuzzy assignment problems with parameter cost coefficients in the form of fuzzy numbers and decision variables in the form of an explicit number (Crips).

Suppose that it is known the fuzzy assignment Zimmermann model

**Objective Function**

\[
\text{Min} \quad z = C X
\]

**Subject to,**

\[
\sum_{j=1}^{n} x_{ij} = 1; \quad \sum_{i=1}^{m} x_{ij} = 1
\]

\[
x_{ij} = 0 \text{ atau } 1
\]

and

**Objective Function**

\[
\text{Min} \quad z = (C + p) X
\]

**Subject to,**

\[
\sum_{j=1}^{n} x_{ij} = 1; \quad \sum_{i=1}^{m} x_{ij} = 1
\]

\[
x_{ij} = 0 \text{ atau } 1
\]

where \(p = (p_1, p_2, \ldots)\) is chosen by the decision maker. Suppose the optimal solution from (5) and (6) are \(z_1\) and \(z_2\), then choose \(b_0 = z_2\) and \(p_0 = b_0 - z_1\).

where: \(c_{ij}\), called the objective function coefficient or cost / time coefficient which states the cost or time to assign the worker \(i\) to do the work \(j\), \(x_{ij}\) value 1 if the worker \(i\) is assigned to do the work \(j\) with \(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n\) and value 0 for the opposite.
3.2. Illustration

Given the following fuzzy assignment problems:

\[
\min z = 15x_{11} + 20x_{12} + 18x_{13} + 22x_{14} + 14x_{21} + 16x_{22} + 21x_{23} + 17x_{24} + 25x_{31} + 20x_{32} + 23x_{33} + 20x_{34} + 17x_{41} + 18x_{42} + 18x_{43} + 16x_{44}
\]

\[
x_{ij} = 0 \text{ atau } 1; \quad i = 1, 2, 3, 4 \quad \text{&} \quad j = 1, 2, 3, 4
\]

**PHASE 1**

Based on the value of \( p_i \) for \( i = 1, 2, 3, 4 \) which is determined by the decision maker for that problem obtained \( b_0 = 82 \) and \( p_0 = 14 \). Besides that the decision maker also determines the value of \( w \), for example selected \( w = 3/4 \). By using definition 3, obtained the following fuzzy assignment Thorani model:

\[
\min z = (8,10^{\frac{1}{3}}219^{\frac{2}{3}}22;\frac{3}{4})x_{11} \oplus (13,15^{\frac{1}{3}}24^{\frac{2}{3}}27;\frac{3}{4})x_{12} \oplus (11,13^{\frac{1}{3}}22^{\frac{2}{3}}25;\frac{3}{4})x_{13}
\]

\[
\oplus (15,17^{\frac{1}{3}}26^{\frac{2}{3}}29;\frac{3}{4})x_{14} \oplus (7,9^{\frac{1}{3}}18^{\frac{2}{3}}21;\frac{3}{4})x_{21} \oplus (9,11^{\frac{1}{3}}20^{\frac{2}{3}}23;\frac{3}{4})x_{22}
\]

\[
\oplus (14,16^{\frac{1}{3}}25^{\frac{2}{3}}28;\frac{3}{4})x_{23} \oplus (10,12^{\frac{1}{3}}21^{\frac{2}{3}}24;\frac{3}{4})x_{24} \oplus (18,20^{\frac{1}{3}}29^{\frac{2}{3}}32;\frac{3}{4})x_{31}
\]

\[
\oplus (13,15^{\frac{1}{3}}24^{\frac{2}{3}}27;\frac{3}{4})x_{32} \oplus (16,18^{\frac{1}{3}}27^{\frac{2}{3}}30;\frac{3}{4})x_{33} \oplus (13,15^{\frac{1}{3}}24^{\frac{2}{3}}27;\frac{3}{4})x_{34}
\]

\[
\oplus (10,12^{\frac{1}{3}}21^{\frac{2}{3}}24;\frac{3}{4})x_{41} \oplus (11,13^{\frac{1}{3}}22^{\frac{2}{3}}25;\frac{3}{4})x_{42} \oplus (11,13^{\frac{1}{3}}22^{\frac{2}{3}}25;\frac{3}{4})x_{43}
\]

\[
\oplus (9,11^{\frac{1}{3}}20^{\frac{2}{3}}23;\frac{3}{4})x_{44}
\]

\[
x_{ij} = 0 \text{ atau } 1; \quad i = 1, 2, 3, 4 \quad \text{&} \quad j = 1, 2, 3, 4
\]

**PHASE 2**

In phase 2 it was done to change the fuzzy assignment Thorani model to the Crisp assignment model using index ranking Thorani, obtained:

\[
\text{Min } z = 3^{\frac{2}{3}}x_{11} + 4^{\frac{1}{3}}x_{12} + 4^{\frac{1}{6}}x_{13} + 5^{\frac{2}{5}}x_{14} + 3x_{21} + 3^{\frac{1}{2}}x_{22} + 5^{\frac{1}{9}}x_{23} + 3^{\frac{6}{7}}x_{24} + 6^{\frac{1}{3}}x_{31} + 4^{\frac{3}{5}}x_{32} + 5^{\frac{3}{4}}x_{33} + 4^{\frac{4}{5}}x_{34} + 3^{\frac{6}{7}}x_{41} + 4^{\frac{4}{9}}x_{42} + 4^{\frac{1}{6}}x_{43} + 3^{\frac{1}{2}}x_{44}
\]

\[
x_{ij} = 0 \text{ atau } 1; \quad i = 1, 2, 3, 4 \quad \text{&} \quad j = 1, 2, 3, 4
\]
\[ \begin{align*}
    x_{14} + x_{24} + x_{34} + x_{44} &= 1 \\
    x_{ij} &= 0 \text{ atau } 1; \quad i = 1, 2, 3, 4 \quad \& \quad j = 1, 2, 3, 4
\end{align*} \]

The minimum optimal solution is:

\[ x_{13} = 4 \frac{1}{6}; \quad x_{21} = 3; \quad x_{32} = 4 \frac{1}{5}; \quad x_{44} = 3 \frac{1}{2} \quad \text{with } z = 15 \frac{3}{7} \]

\[ z_{\text{min}} = \left( 8 \frac{3}{7}, 10 \frac{3}{4}, 20, 22 \frac{3}{7} \right) \]

This means that the minimum cost is \( 8 \frac{3}{7} \), the average minimum cost is in the range of \( 10 \frac{3}{4} \) to 20, and the lowest minimum cost is \( 22 \frac{3}{7} \).

4. Conclusion

Based on the numerical test results which are limited by Crips number variables, it can be concluded that the Zimmerman method of development can be used to solve fuzzy assignment problems with parameter cost coefficients in the form of fuzzy numbers and decision variables in the form of Crips.

5. References

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