Numerical simulation of large elastoplastic deformation of shells of revolution in the “spacesuit” under the influence of pulse overload

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Abstract. An axisymmetric problem of high strains in a spherical lead shell enclosed into an aluminum “spacesuit” under the influence of pulse overload is considered. The shell straining is described with the use of equations of mechanics of elastoviscoplastic media in Lagrangian variables. Kinematic relations are determined in the current state metrics. Equations of state are taken in the form of equations of the flow theory with isotropic hardening. Contact interaction of a shell and a “spacesuit” is modeled by conditions of non-penetration with friction. Numerical solution under the given boundary and initial conditions is based on the moment schema of the finite element method and the explicit time integration scheme of the “cross”. For sampling the space variables 4-node isoparametric finite elements with multilinear forms features are used. As it was shown by the results of calculations, spherical shell suffers significant local forming, characterized by large displacements and rotation angles of finite elements as a rigid body in the process of loading. The calculation results of the residual form are in good agreement with the experimental data.

1. Introduction
Thin-walled spherical shells are widely used in modern technology. Experimental and theoretical techniques of studies of their stability are presented in [1-5]. The analysis of computer modeling of deformation processes and the loss of stability of spherical shells under the quasi-static and impact loading, taking into account the effects of geometrical and physical nonlinearities are published in [6-15]. The results of numerical analysis of large elastic-plastic deformation of lead spherical shell in an aluminum “spacesuit” under explosive loading and contact with friction are shown below.

2. Constitutive system of equations and problem solution method
Deformation of the shell is analyzed in axisymmetric formulation and is described in Lagrangian variables using equations of mechanics of continuous media in a cylindrical coordinate system $X_1, X_2$ ($X_1$ – axis of rotation, $X_2$ – radius). The equations of motion are derived from the balance of virtual capacity [17, 18]:

\[ \dot{\mathbf{u}} = \mathbf{S} \mathbf{F} \mathbf{n} \]

\[ \mathbf{S} = \frac{1}{E} \begin{bmatrix} 3 \nu & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix} \]

\[ \mathbf{F} = \begin{bmatrix} 1 & -u_1 & -u_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ \mathbf{n} = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} \]

where $\mathbf{S}$ is the compliance matrix, $\mathbf{F}$ is the deformation gradient, $\mathbf{n}$ is the surface traction vector, and $E$ is the Young's modulus. The boundary conditions are applied to the contact surfaces, and the frictional forces are calculated using the Coulomb friction model.
\[
\int_\Omega \left( \sigma_{ij} \delta_{ij} \dot{e}_{11} + \sigma_{23} \delta_{ij} \dot{e}_{22} + \sigma_{33} \delta_{ij} \dot{e}_{33} + 2\sigma_{12} \delta_{ij} \dot{e}_{12} \right) X_2 d\Omega + \int_\Omega \left( \rho \dot{U}_i \delta_{ij} \dot{U}_1 + \rho \dot{U}_2 \delta_{ij} \dot{U}_2 \right) X_2 d\Omega
- \int_{\partial_\rho} \left( p_i \delta_{ij} \dot{U}_1 + p_2 \delta_{ij} \dot{U}_2 \right) X_2 dG - \int_{\partial_\epsilon} \left( q_i \delta_{ij} \dot{U}_1 + q_2 \delta_{ij} \dot{U}_2 \right) X_2 dG = 0
\]

where \( \sigma_{ij}, \dot{\epsilon}_{ij} \) are the components of the Cauchy stress tensor and the strain rate tensor (symmetric part of the displacement velocity gradient), \( U_i \) are displacement in the coordinate system \( X \), \( \rho \) is density, \( p_\alpha, q_\alpha \) are the components of the surface and contact loads (\( \alpha = 1, 2 \)), \( \Omega \) is the domain occupied by the construction, \( G_p \) is the scope of the external pressure, \( G_q \) is the part of the surface subjected to contact pressures determined in the course of solving the problem. Full stop above the symbol indicates the partial derivative over time \( t \). The summation is over repeated indices.

Strain rates are determined in the current state metrics as:

\[
\dot{\epsilon}_{11} = \dot{U}_{1,1}, \quad \dot{\epsilon}_{22} = \dot{U}_{2,2}, \quad \dot{\epsilon}_{33} = X_2^{-1} \dot{U}_2, \quad \dot{\epsilon}_{12} = \frac{1}{2} \left( \dot{U}_{1,2} + \dot{U}_{2,1} \right),
\]

\[
\dot{\omega} = \frac{1}{2} \left( \dot{U}_{1,2} - \dot{U}_{2,1} \right), \quad \dot{U}_{i,j} = \partial_\Gamma \dot{U}_i / \partial X_j, \quad X_j = X_j \big|_{t=0} + \int_0^t \dot{U}_j dt
\]

Equations of state are set separately for spherical \( \dot{\epsilon}^S, \sigma^S \) and deviatoric components of the velocties of deformations and stresses \( \dot{\epsilon}^D, \sigma^D \). The dependence of the spherical components of the strain rate and stress are assumed to be linear. Deviatoric strain rate components \( \dot{\epsilon}^D_{ij} \) are decomposed on plastic \( \dot{\epsilon}^p_{ij} \) and elastic components \( \dot{\epsilon}^e_{ij} \). Deviatoric stress tensor components are calculated using the flow theory of relations with isotropic hardening [19]:

\[
D_j \sigma^D_{ij} = 2G \dot{\epsilon}^e_{ij}, \quad \sigma^D_{ij} \leq 2/3 \sigma^D_{yield} \left( \epsilon, I_{2e} \right) \quad I_{2e} = \sqrt{2/3} \sqrt{\dot{\epsilon}^p_{ij} \dot{\epsilon}^p_{ij}}, \quad \epsilon = \int_0^{I_{2e}} dt,
\]

\[
\dot{\epsilon}^p_{ij} = \dot{\lambda} \sigma^D_{ij};
\]

Here \( D_j \) is the Jaumann derivative [20] (\( D_j \sigma^D_{ij} = \sigma^D_{ij} - \dot{\omega}_k \sigma^D_{kj} - \dot{\omega}_j \sigma^D_{ik} \)), \( G \) – shear modulus. \( \sigma^D_{yield} \) is the yield limit, \( \epsilon \) is the Odquist parameter, \( I_{2e} \) is the strain rate intensity, \( \dot{\lambda} \) is a parameter, which is identically equal to zero in the case of elastic straining and \( \dot{\lambda} > 0 \) determined from the condition of landing on the surface of the flow under elastoplastic deformation.

The components of the contact force \( q_\alpha \) (\( \alpha = 1, 2 \)) are calculated in the course of solving the problem in a local coordinate basis (\( s, \xi \) are the tangential and normal directions to the contact surface). The contact pressure along the normal is determined from the non-penetration condition:

\[
\begin{align*}
\dot{u}_s &= \dot{u}_s^*, \\
q_\xi &= 0, & q_\xi &\geq 0 \\
q'_\xi &= q'_\xi, & q_\xi &< 0
\end{align*}
\]

The tangential pressure is first determined from the condition of rigid bonding and then, if the static friction force is exceeded - by the Coulomb law:
\[
\begin{aligned}
\begin{cases}
\dot{u}_s' = \dot{u}_s'^{\ast} \\
q_s' = -q_s''
\end{cases} \\
q_s' = q_s' = \begin{cases}
q_s, & |q_s| \leq k_z|q_{\xi}|
\\
k_z|q_{\xi}|\text{sign}(q_{\xi}), & |q_s| > k_z|q_{\xi}|
\end{cases}
\end{aligned}
\]  

(5)

where \( k_z \) is the friction coefficient.

The decision of the constitutive system of equations (1) - (5) is based on the finite element method and finite difference explicit scheme of integration over time, such as "cross" [16,21,22]. Contact nodal forces are determined based on the algorithm [22]. To reduce high-frequency nonphysical oscillations of the numerical solution, the procedure of conservative smoothing is applied [23]. The described finite element method for solving two-dimensional (axisymmetric) problems of elastoplastic deformation of structural elements is implemented as separate software modules in the computer system "Dynamics-3" [24].

3. Results of studying dynamic buckling of the lead shell in a “spacesuit” under explosive loading

The results of the experimental investigation of this problem were reported in [25].

\[ \sigma_{yield} /10, \text{MPa} \]

*Figure 1.* Experimental assembly

*Figure 2.* The dependence of the yield stress of the Odquist parameter

The experimental assembly consisting of the lead shell (1), the monolithic half-casing of the "spacesuit" (3) and the cover attached to the half-casing by screws (4) is schematically shown in figure 1. The ratio of the spherical shell thickness to its radius is \( h/R = 0.054 \). The shell mass is 2.95 kg, and the total mass of the assembly is 21 kg. The shell material is S1 lead (\( E = 1.8 \times 10^4 \text{ MPa}, \nu = 0.45, \rho = 11.34 \text{ g/sm}^3 \)). According to [26] the stress-deformation diagram of lead (figure 2) obtained from the experimental data [27] is used in the calculation. The "spacesuit" material is the D16 aluminum alloy (\( E = 7.1 \times 10^4 \text{ MPa}, \nu = 0.3, \rho = 2.78 \text{ g/sm}^3 \)). The friction coefficient \( k_z \) between the lead shell and the "spacesuit" is set equal to 0.2 [28].

The "spacesuit" is loaded from below by an explosive device [29] consisting of the high explosive (HE), flyer plate, and damper converting the action of the explosion on the "spacesuit" into a trapezoidal loading pulse \( n \) (figure 3).

For calculations the shell was divided into finite elements (6 finite elements over the shell thickness and 200 finite elements over the generatrix). As it was shown by preliminary calculations, the deformation of the "spacesuit" can be neglected. For this reason the "spacesuit" is modeled on the equivalent size and weight elastic prototype.
The calculation results are shown in figures 4-6. In figure 4 the shapes of the shell in a residual state derived from natural experiment [25], and the numerical solution using the finite element method [16], are compared. The graphs of time dependence of upsetting of the shell (changes its diameter in the vertical direction) and the thickness at several locations of the lower half of the shell, spaced 30 degrees from the equator to the poles are shown in figures 5,6.

The elastoplastic shell buckling is continued throughout 2.5ms under the influence of inertial load and contact with “spacesuit”. Subsequently, the velocities of the shell and the “spacesuit” are aligned. After unloading, the change of shape of the shell is substantially completed.

As a result of intensive loading, the shape of the shell and the thickness are being changed (figure 6). Increase of the thickness of the shell is irregular along its generatrix. Before buckling maximum
deformations are developed in the lower hemisphere of the shell. The shell thickness is increased by 76% on rotation axis. With the removal from the lower pole shell thickness is reduced to 36% at the equator. Estimated values of the change in thickness along the generatrix are in good agreement with the experimental data in figure 5. Change in thickness of the shell stops after the formation of buckle for t > 1 ms.

Variation of friction coefficient leads to a change in the character of the distribution of shell thickening by generatrix during deformation. The change is barely noticeable, when $k_f \leq 0.35$. With further increase of the friction coefficient the highest thickening shell is moved toward the equator, that contradicts the experimental data.

The shape of the buckles and its location data of experiment and calculation are in good agreement. The difference between the residual vertical shell size (upsetting) based on the experimental value is 3%.

According to the calculated data, strain rate of the shell is changed from a minimum of 60 s$^{-1}$ in the upper shell pole to the maximum value of 1200 s$^{-1}$ in the buckle. Application of static stress-deformation diagram [27] does not allow to obtain quantitatively and qualitatively correct results at high strain rates. For example, the upsetting shell in residual state is greater than the experimental value of more than 2 times in this case.

Moment stress-strain state is realized in the zone of the ring buckles. The longitudinal strains are 70%, shear strain are 5%, the maximum rotation of the finite element as a rigid body in the area in the ring buckles is approximately 120 degrees. In these conditions, the application of Jaumann derivative provides acceptable accuracy of the numerical solution of the problem.

Computational experiments showed that the numerical schemes, based on the strain and stress velocity definition in the local basis [8,19,22], corresponding to the model shell theory of Timoshenko type based compression, allow to solve our problem on the grid with 1 layer of finite elements in thickness, which reduces computational resources on an order of magnitude or more, without loss of accuracy.

4. Conclusion
Elastoplastic straining, buckling, and further deformation of a spherical shell in a “spacesuit” under pulsed loading are numerically analyzed. There are large deformation angles of rotation of the shell elements as a rigid body and the change in thickness in the process of buckling of the lead shell. The dependence of viscosity characteristics of lead on the strain rate and the value of the friction coefficient have significant influence on forming and residual shell thickness. It is shown that the use of Jaumann derivative for small steps of integration over time provides sufficient accuracy of numerical solution of the axisymmetric problem for large deformations and rotation angles.

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