Implications of the Muon Anomalous Magnetic Moment for Supersymmetry

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Abstract

We re-examine the bounds on supersymmetric particle masses in light of the E821 data on the muon anomalous magnetic moment, $a_\mu$. We confirm, extend and supersede previous bounds. In particular we find (at 1σ) no lower limit on tan $\beta$ or upper limit on the chargino mass implied by the data at present, but at least 4 sparticles must be lighter than 700 to 820 GeV and at least one sparticle must be lighter than 345 to 440 GeV. However, the E821 central value bounds tan $\beta > 4.7$ and $m_{\tilde{\chi}_1^\pm} < 690$ GeV. For tan $\beta \lesssim 10$, the data indicates a high probability for direct discovery of SUSY at Run II or III of the Tevatron.
Recently, the Brookhaven E821 experiment [1] has reported evidence for a deviation of the muon magnetic moment from the Standard Model expectation. Immediately following that announcement appeared a number of papers analyzing the reported excess in terms of various forms of new physics, including supersymmetry (SUSY) [2, 3, 4, 5, 6]. The SUSY explanation is, for many of us, the most exciting of the various proposals since it implies SUSY at a mass scale not far above the weak scale. In particular, it implies a light slepton and a light gaugino, though “light” can still be as heavy as several hundred GeV.

Previous analyses have generally concentrated on bounding the masses of the chargino and the sneutrino, or on the lightest of the sparticles. In this paper, we will re-examine the SUSY calculation, both in very general scenarios and in well-motivated, simplified SUSY models. We will show that there exist mass bounds on more than two SUSY particles, though which sparticles are bounded changes with \( \tan \beta \). In particular, we will show that throughout parameter space, one may actually bound at least 4 sparticle masses if we are to use SUSY to explain the E821 data. The existence of these bounds will rely on very simple and clearly stated assumptions about the SUSY particle spectrum; these assumptions will not include a fine-tuning constraint.

1 SUSY and \( a_\mu \)

The actual measurement performed by the E821 collaboration is of the muon’s anomalous magnetic moment, which is to say, the coefficient \( a_\mu \) of the non-renormalizable operator

\[
\frac{a_\mu}{2m_\mu} \bar{\psi} \sigma^{\alpha\beta} \psi F_{\alpha\beta}.
\]

Within the Standard Model, \( a_\mu \) is predicted to be [7]

\[
a^{SM}_\mu = 11 659 159.6(6.7) \times 10^{-10}.
\]

It has been claimed that the quoted uncertainty is actually too small [8]; the dominant uncertainty is the hadronic contribution to the photon polarization diagrams which is extracted from experimental measurements of \( R(e^+e^- \rightarrow \text{hadrons}) \) in the vicinity of the low-energy meson resonances. However, strong arguments have been made to reinforce the quoted values [9], and we will accept them here as given.

The measurement made by E821 is [1]:

\[
a^{E821}_\mu = 11 659 202(14)(6) \times 10^{-10}
\]

from which one deduces a discrepancy between the experiment and the Standard Model of

\[
\delta a_\mu = 43(16) \times 10^{-10},
\]

that is, the measured value is larger than the prediction by \( 2.7\sigma \).
The SUSY contributions to $a_\mu$ have been known since the early days of SUSY and have become more complete with time\cite{10}. In this paper we will follow the notation of Ref. \cite{5} which has the advantage of using the standard conventions of Haber and Kane \cite{11}; any convention which we do not define here can be found in either of these two papers.

Before we begin the discussion of our work, let us review briefly a few of the analyses to date. The first paper to use the new E821 data in the context of SUSY was that of Czarnecki and Marciano \cite{2} who only attempted to approximately bound the sparticle spectrum. More complete analyses followed quickly thereafter by Everett et al. \cite{3} and Feng and Matchev \cite{4}. The former \cite{3} argued that only very large values of $\tan \beta$ were generically consistent with the data; given those, a $1.5 \sigma$ upper bound of 450 GeV could be placed on some sparticle (gaugino or slepton) for $\tan \beta = 35$, and 900 GeV for both a chargino and the muon sneutrino. The latter analysis \cite{4} found a model-independent $1 \sigma$ limit on the mass of the lightest “observable” sparticle of 490 GeV for $\tan \beta < 50$. Martin and Wells did a more complete analysis, but focussed on the lighter chargino and charged smuon. Their model-independent analysis found no upper bound on the chargino mass at $1 \sigma$ but found $m_{\tilde{\mu}} < 500$ GeV. With the added assumption of gaugino mass unification (i.e., $M_2 \simeq 2M_1$), the lighter chargino was bounded by about 700 GeV at $\tan \beta = 30$. The lack of a strong upper bound on the chargino was due to the neutralino contributions at small $\tan \beta$ (all the way down to $\tan \beta = 3$) which can explain the E821 data without any chargino piece at all. Finally, there have been a number of other papers \cite{6} which have studied the SUSY contributions within specialized scenarios which we will not discuss here.

1.1 The diagrams

In the mass eigenbasis, there are only two one-loop SUSY diagrams which contribute to $a_\mu$, shown in Figure 1. The first has an internal loop of smuons and neutralinos, the second a loop of sneutrinos and charginos. But the charginos, neutralinos and even the smuons are themselves admixtures of various interaction eigenstates and we can better understand the physics involved by working in terms of these interaction diagrams, of which there are many more than two. We can easily separate the leading and subleading diagrams in the interaction eigenbasis by a few simple observations.

First, the magnetic moment operator is a helicity-flipping interaction. Thus any
diagram which contributes to $a_\mu$ must involve a helicity flip somewhere along the fermion current. This automatically divides the diagrams into two classes: those with helicity flips on the external legs and those with flips on an internal line. For those in the first class, the amplitude must scale as $m_\mu$; for those in the second, the amplitudes can scale instead by $m_{\text{SUSY}}$, where $m_{\text{SUSY}}$ represents the mass of the internal SUSY fermion (a chargino or neutralino). Since $m_{\text{SUSY}} \gg m_\mu$, it is the latter class that will typically dominate the SUSY contribution to $a_\mu$. Therefore we will restrict further discussion to this latter class of diagrams alone.

Secondly, the interaction of the neutralinos and charginos with the (s)muons and sneutrinos occurs either through their higgsino or gaugino components. Thus each vertex implies a factor of either $y_\mu$ (the muon Yukawa coupling) or $g$ (the weak and/or hypercharge gauge coupling). Given two vertices, the diagrams therefore scale as $y_\mu^2$, $gy_\mu$, or $g^2$. In the Standard Model, $y_\mu$ is smaller than $g$ by roughly $10^{-3}$. In the minimal SUSY standard model (MSSM) at low $\tan\beta$, this ratio is essentially unchanged, but because $y_\mu$ scales as $1/\cos\beta$, at large $\tan\beta$ ($\sim 60$) the ratio can be reduced to roughly $10^{-1}$. Thus we can safely drop the $y_\mu^2$ contributions from our discussions, but at large $\tan\beta$ we must preserve the $gy_\mu$ pieces as well as the $g^2$ pieces.

The pieces that we will keep are therefore shown in Figures 2. In Fig. 2(a)-(e) are shown the five neutralino contributions which scale as $g^2$ or $gy_\mu$; in Fig. 2(f) is the only chargino contribution, scaling as $gy_\mu$. The contributions to $a_\mu$ from the $i$th neutralino and the $m$th smuon due to each of these component diagrams are found to be:

$$\delta a_\mu = \frac{1}{48\pi^2} \frac{m_\mu m_{\text{SUSY}}}{m_\mu^2} F_2^N(x_{im}) \times \left\{ \begin{array}{l}
g_1^2 N_{i1}^2 X_{m1} X_{m2} \quad (\tilde{B}\tilde{B}) \\
g_1 g_2 N_{i1} N_{i2} X_{m1} X_{m2} \quad (\tilde{W}\tilde{B}) \\
-\sqrt{2} g_1 y_\mu N_{i1} N_{i3} X_{m2}^2 \quad (H\tilde{B}) \\
\frac{1}{\sqrt{2}} g_2 y_\mu N_{i1} N_{i3} X_{m1}^2 \quad (\tilde{B}\tilde{H}) \\
\frac{1}{\sqrt{2}} g_2 y_\mu N_{i2} N_{i3} X_{m1}^2 \quad (\tilde{W}\tilde{H}) \end{array} \right\}$$

and for the $k$th chargino and the sneutrino:

$$\delta a_\mu = -\frac{1}{24\pi^2} \frac{m_\mu m_{\text{SUSY}}}{m_\mu^2} F_2^C(x_k) g_2 y_\mu U_{k2} V_{k1}.$$ 

The matrices $N$, $U$ and $V$ are defined in the appendix along with the functions $F_2^{N,C}$. A careful comparison to the equations in the appendix will reveal that we have dropped a number of complex conjugations in the above expressions; it has been shown previously \cite{3, 4} that the SUSY contributions to $a_\mu$ are maximized for real entries in the mass matrices and so we will not retain phases in our discussion.

In many of the previous analyses of the MSSM parameter space, it was found that it is the chargino-sneutrino diagram at large $\tan\beta$ that can most easily generate values of $\delta a_\mu$ large enough to explain the observed discrepancy. From this observation, one can obtain an upper mass bound on the lightest chargino and the muon sneutrino. However, this

\footnote{Pieces which are dropped from our discussion are still retained in the full numerical calculation.}
behavior is not completely generic. For example, Martin and Wells have emphasized that the \( \tilde{B}\tilde{B} \) neutralino contribution can by itself be large enough to generate the observed excess in \( a_\mu \), and since it has no intrinsic \( \tan \beta \) dependence, they could explain the data with \( \tan \beta \) as low as 3. We can reproduce their result in a simple way because the \( \tilde{B}\tilde{B} \) contribution has a calculable upper bound at which the smuons mix at \( 45^\circ \), \( m_{\tilde{\mu}_1} \ll m_{\tilde{\mu}_2} \), and \( m_{\tilde{N}_1} \ll m_{\tilde{N}_{2,3,4}} \) with \( \tilde{N}_1 = \tilde{B} \). Then

\[
|\delta a_\mu|_{(\tilde{B}\tilde{B})} \leq \frac{g^2}{32\pi^2} \frac{m_{\mu} m_{\tilde{N}_1}}{m_{\tilde{\mu}_1}^2} \approx 3800 \times 10^{-10} \times \left( \frac{m_{\tilde{N}_1}}{100 \text{ GeV}} \right) \left( \frac{100 \text{ GeV}}{m_{\tilde{\mu}_1}} \right)^2
\]

where we have used the fact that \( (X_{11} X_{12}) \leq \frac{1}{2} \) and \( F^N_2 \leq 3 \) and have included a 7\% two-loop suppression factor. Though any real model will clearly suppress this contribution somewhat, this is still \( 10^2 \) times larger than needed experimentally.

This pure \( \tilde{B}\tilde{B} \) scenario is actually an experimental worst-case, particularly for hadron colliders. The only sparticles that are required to be light are a single neutralino (which is probably \( \tilde{B} \)-like) and a single \( \tilde{\mu} \). The neutralino is difficult to produce, and if stable, impossible to detect directly. The neutralino could be indirectly observed in the decay of the \( \tilde{\mu} \) as missing energy, but production of a \( \tilde{\mu} \) at a hadron machine is highly suppressed. In the worst of all possible worlds, E821 could be explained by only these two light sparticles, with the rest of the SUSY spectrum hiding above a TeV. Further, even the “light” sparticles can be too heavy to produce at a 500 GeV linear collider. While this

\[ (a) \quad \mu_L \quad \mu_R \quad \tilde{\mu}_L \quad \tilde{\mu}_R \quad \tilde{B} \quad \tilde{B} \]

\[ (b) \quad \mu_L \quad \mu_R \quad \tilde{\mu}_L \quad \tilde{\mu}_R \quad \tilde{B} \]

\[ (c) \quad \mu_L \quad \mu_R \quad \tilde{\mu}_L \quad \tilde{\mu}_R \quad \tilde{B} \quad \tilde{H}_D \]

\[ (d) \quad \mu_L \quad \mu_R \quad \tilde{\mu}_L \quad \tilde{\mu}_R \quad \tilde{B} \quad \tilde{H}_D \]

\[ (e) \quad \mu_L \quad \mu_R \quad \tilde{\mu}_L \quad \tilde{\mu}_R \quad \tilde{W} \quad \tilde{H}_D \]

\[ (f) \quad \mu_L \quad \mu_R \quad \tilde{\mu}_L \quad \tilde{\mu}_R \quad \tilde{W} \quad \tilde{H}_U \quad \tilde{H}_D \]
case is in no way generic, it demonstrates that the E821 data by itself does not provide any sort of no-lose theorem for Run II of the Tevatron or even for the LHC and NLC.

This raises an important experimental question: how many of the MSSM states must be “light” in order to explain the E821 data? In the worst-case, it would appear to be only two. Even in the more optimistic scenario in which the chargino diagram dominates $\delta a_\mu$, the answer naively appears to be two: a single chargino and a single sneutrino. In this limit,

$$|\delta a_\mu|_{(\tilde{C}\tilde{\nu})} \lesssim 2600 \times 10^{-10} \times \left(\frac{m_{\tilde{C}_i}}{100 \text{ GeV}}\right) \left(\frac{100 \text{ GeV}}{m_{\tilde{\nu}}^2}\right) \left(\frac{\tan \beta}{30}\right)$$

where we have bounded $|F_2^C|$ by 10 by assuming $m_{\tilde{\nu}} \lesssim 1 \text{ TeV}$. But this discussion is overly simplistic and we can do much better, as we will see.

### 1.2 Mass correlations

There are a total of 9 separate sparticles which can enter the loops in Fig. 1: 1 sneutrino, 2 smuons, 2 charginos and 4 neutralinos. The mass spectrum of these 9 sparticles is determined entirely by 7 parameters in the MSSM: 2 soft slepton masses ($m_L, m_R$), 2 gaugino masses ($M_1, M_2$), the $\mu$-term, a soft trilinear slepton coupling ($A_{\tilde{\mu}}$) and finally $\tan \beta$. Of these, $A_{\tilde{\mu}}$ plays almost no role at all and so we leave it out of our discussions (see the appendix). And in some well-motivated SUSY-breaking scenarios, $M_1$ and $M_2$ are also correlated. Thus there are either 5 or 6 parameters responsible for setting 9 sparticle masses. There are clearly non-trivial correlations among the masses which can be exploited in setting mass limits on the sparticles.

First, there are well-known correlations between the chargino and neutralino masses; for example, a light charged $\tilde{C}_i \sim \tilde{W}$ implies a light neutral $\tilde{N}_j \sim \tilde{W}$ and vice-versa.

There are also correlations in mixed systems (i.e., the neutralinos, charginos and smuons) between the masses of the eigenstates and the size of their mixings. Consider the case of the smuons in particular; their mass matrix is given in the appendix. On diagonalizing, the left-right smuon mixing angle is given simply by:

$$\tan 2\theta_{\tilde{\mu}} \simeq \frac{2m_{\mu}\mu \tan \beta}{M_L^2 - M_R^2}.$$  

The chargino contribution is maximized for large smuon mixing and large mixing occurs when the numerator is of order or greater than the denominator; since the former is suppressed by $m_\mu$, one must compensate by having either a very large $\mu$-term in the numerator or nearly equal $M_L$ and $M_R$ in the denominator, both of which have profound impacts on the spectrum.

There is one more correlation/constraint that we feel is natural to impose on the MSSM spectrum: slepton mass universality. It is well-known that the most general version of the MSSM produces huge flavor-changing neutral currents (FCNCs) unless some external order is placed on the MSSM spectrum. By far, the simplest such order is
the demand that sparticles with the same gauge quantum numbers be degenerate. Thus we expect \( m_{\tilde{\tau}_L} = m_{\tilde{\mu}_L} \equiv m_L \) and \( m_{\tilde{\tau}_R} = m_{\tilde{\mu}_R} \equiv m_R \). Then the mass matrix for the stau sector is identical to that of the smuons with the replacement \( m_\mu \to m_\tau \) in the off-diagonal elements. This enhancement of the mixing in the stau sector by \( m_\tau/m_\mu \simeq 17 \) implies that \( m_{\tilde{\tau}_1} < m_{\tilde{\mu}_1} \). In particular, if

\[ M_L^2 M_R^2 < m_\tau^2 \mu^2 \tan^2 \beta \]

then \( m_{\tilde{\tau}_1}^2 < 0 \) and QED will be broken by a stau vev. Given slepton universality, this imposes a constraint on the smuon mass matrix:

\[ M_L^2 M_R^2 > \left( \frac{m_\tau}{m_\mu} \right)^2 m_\mu^2 \mu^2 \tan^2 \beta \]

or on the smuon mixing angle:

\[ \tan 2\theta_\tilde{\mu} < \left( \frac{m_\mu}{m_\tau} \right) \frac{2M_L M_R}{M_L^2 - M_R^2} \]

where \( M_{L,R} \) are the positive roots of \( M_{L,R}^2 \). While not eliminating the possibility of \( \theta_\tilde{\mu} \simeq 45^\circ \), this formula shows that a fine-tuning of at least 1 part in 17 is needed to obtain \( \mathcal{O}(1) \) mixing. We will not apply any kind of fine-tuning criterion to our analysis, yet we will find that this slepton mass universality constraint sharply reduces the upper bounds on slepton masses which we are able to find in our study of points in MSSM parameter space.

(As an aside, if one assumes slepton mass universality at some SUSY-breaking messenger scale above the weak scale, Yukawa-induced corrections will break universality by driving the stau masses down. This effect would further tighten our bounds on smuon masses and mixings.)

The above discussion has an especially large impact on the worst-case scenario in which the \( \bar{B}\bar{B} \) contributions dominates \( \delta a_\mu \). For generic points in MSSM parameter space, one expects that \( \tan 2\theta \lesssim 1/17 \) which reduces the size of the \( \bar{B}\bar{B} \) contribution by a factor of 17. As a byproduct, the masses required for explaining the E821 anomaly are pushed back towards the range that can be studied by a 500 GeV linear collider.

## 2 Numerical results

Now that we have established the basic principle of our analysis, let us carry it out in detail. We will concentrate on three basic cases. The first case is the one most often considered in the literature: gaugino mass unification. Here one assumes that the weak-scale gaugino mass parameters (\( M_1 \) and \( M_2 \)) are equal at the same scale at which the gauge couplings unify. This implies that at the weak scale \( M_1 = (5/3)(\alpha_1/\alpha_2)M_2 \). The second case we consider is identical to the first with the added requirement that the
lightest SUSY sparticle (LSP) be a neutralino. This requirement is motivated by the desire to explain astrophysical dark matter by a stable LSP. Finally we will also consider the most general case in which all relevant SUSY parameters are left free independent of each other; we will refer to this as the “general MSSM” case.

The basic methodology is simple: we put down a logarithmic grid on the space of MSSM parameters ($M_1$, $M_2$, $m_L$, $m_R$ and $\mu$) for several choices of tan $\beta$. The grid extends from 10 GeV for $M_1$, $M_2$ and $\mu$, and from 50 GeV for $m_{L,R}$, up to 2 TeV for all mass parameters. For the case in which gaugino unification is imposed $M_2$ is no longer a free parameter and our grid contains $10^8$ points. For the general MSSM case our grid contains $3 \times 10^9$ points. Only $\mu > 0$ is considered since that maximizes the value of $\Delta a_{\mu}$. Finally, for our limits on tan $\beta$ we used an adaptive mesh routine which did a better job of maximizing $\Delta a_{\mu}$ over the space of MSSM inputs. By running with grids of varying resolutions and offsets we estimate the error on our mass bounds to be less than $\pm 5\%$.

2.1 Bounds on the lightest sparticles

Perhaps the most important information that can be garnered from the E821 data is an upper bound on the scale of sparticle masses. In particular, one can place upper bounds on the masses of the lightest sparticle(s) as a function of $\Delta a_{\mu}$. Previous analyses have often followed this approach, deriving upper bounds on the lightest sparticle from among the gauginos and sleptons, or even more specifically, from among the charginos and smuons. We too derive bounds on the lightest sparticles, but as we have argued in the previous section, we can also derive bounds on the second, third, and even fourth lightest sparticles through the mass correlations.

These bounds on additional light sparticles provide an important lesson. Without them there remains the very real possibility that the E821 data is explained by a pair of light sparticles and that the remaining SUSY spectrum is out of reach experimentally. But our additional bounds will give us some indication not only of where we can find SUSY, but also of how much information we might be able to extract about the fundamental parameters of SUSY — the more sparticles we detect and measure, the more information we will have for disentangling the soft-breaking sector of the MSSM.

In Fig. 3 we have shown the upper mass bounds for the lightest four sparticles assuming gaugino mass unification. These bounds are not bounds on individual species of sparticles (which will come in the next section and always be larger than these bounds) but simply bounds on whatever sparticle happens to be lightest. The important points to note are: (i) the maximum values of the mass correspond to the largest value of tan $\beta$, which is to be expected given dominance of the chargino diagram at large tan $\beta$; (ii) the 1$\sigma$ limit (central value) of the E821 data requires at least 4 sparticles to lie below roughly 700 (500) GeV; and (iii) for low values of tan $\beta$ a maximum value of $\Delta a_{\mu}$ is reached (we will return to this later).

The same plots could be produced with the additional assumption that the LSP be a neutralino, but we will only show the case for the LSP bound, in Fig. 4. In this
Figure 3: Bounds on the masses of the four lightest sparticles as a function of $\delta a_\mu$ for $\tan \beta = 3, 5, 10, 30$ and $50$. These figures assume gaugino unification only.
Figure 4: Bound on the mass of the LSP as a function of $\delta a_\mu$ for $\tan \beta = 3, 5, 10, 30$ and 50. The dotted lines assume gaugino unification only while the solid lines require additionally that the LSP is a neutralino. The dashed line is the bound in the general MSSM, calculated at $\tan \beta = 50$.

In the figure, the solid lines correspond to a neutralino LSP, while the dotted lines are for the more general case discussed above (i.e., they match the lines in Fig. 3(a)). Notice that for $\delta a_\mu \gtrsim 40 \times 10^{-10}$ there is little difference between the cases with and without a neutralino LSP. Furthermore, at the extreme upper and lower values of $\tan \beta$ there is little difference. It is only for the intermediate values of $\tan \beta$ that the mass bound shifts appreciably; for $\tan \beta = 10$ it comes down by as much as 50 GeV compared to the more general case.

Finally, we consider the most general MSSM case, i.e., without gaugino unification. Here the correlations are much less pronounced, but interesting bounds still exist. For example the central value of the E821 data still demands at least 3 sparticles below 500 GeV (rather than four for the previous cases). In figure 4 we demonstrate this explicitly by plotting the masses of the four lightest sparticles for $\tan \beta = 50$ and a wide range of $\delta a_\mu$. We see that dropping the gaugino unification requirement has one primary effect: the mass of the LSP is significantly increased. This is because the LSP in the unified case is usually $\tilde{N}_1 \sim \tilde{B}$ but isn’t itself responsible for generating $\delta a_\mu$. In the general case, the LSP must participate in $\delta a_\mu$ (otherwise its mass could be arbitrarily large) and so is roughly the mass of the second lightest sparticle in the unified case, whether that be a $\tilde{\mu}$ or $\tilde{C}$. Otherwise the differences between the more general MSSM and the gaugino unified MSSM are small. In particular we still find that at least 4 sparticles must be light, though the $1\sigma$ bound of 700 GeV for the unified case extends now slightly to 820 GeV.
Figure 5: Bounds on the masses of the four lightest sparticles as a function of $\delta a_\mu$ for $\tan \beta = 50$. The dotted lines assume gaugino unification only while the solid lines are for the general MSSM.

We have summarized all this data on the LSP in Table 1 where we have shown the mass bounds (using both the $1\sigma$ limit and the central value of the E821 data) on the LSP for various $\tan \beta$ values and with our various assumptions. The last line in the table represents an upper bound for any model with $\tan \beta \leq 50$: $m_{\text{LSP}} < 440 \, (345) \, \text{GeV}$ for the E821 $1\sigma$ lower bound (central value) of $\delta a_\mu$. But perhaps of equal importance are the bounds on the next 3 lightest sparticles (the “2LSP,” “3LSP” and “4LSP”): $m_{2\text{LSP}} < 460 \, (355) \, \text{GeV}$, $m_{3\text{LSP}} < 600 \, (465) \, \text{GeV}$ and $m_{4\text{LSP}} < 820 \, (580) \, \text{GeV}$. Thus there must be at least 4 sparticles below 820 GeV even in the most general MSSM scenario, and at least two sparticles accessible to a $\sqrt{s} = 1 \, \text{TeV}$ linear collider.

2.2 Bounds on the sparticle species

In the previous subsection, we derived bounds on the lightest sparticles, independent of the identity of those sparticles. Another important piece of information that can be provided by this analysis is bounds on individual species of sparticles, for example, on the charginos or on the smuons. These bounds will of mathematical necessity be higher than those derived in the previous section, but still provide important information about how and where to look for SUSY. In particular, they can help us gauge the likelihood of finding SUSY at Run II of the Tevatron or at the LHC.

There is one complication in obtaining these bounds. At low $\tan \beta$ the data is most easily explained by the neutralino diagrams and as such there must be at least one light smuon and one light neutralino. At larger $\tan \beta$ values of $\delta a_\mu$ as large as the E821 central value generally require contributions from the chargino diagrams, so there must
Table 1: Upper bounds on the mass (in GeV) of the lightest sparticle for the general MSSM, the MSSM assuming gaugino mass unification, and the MSSM with gaugino mass unification plus a neutralino LSP. The entries represent the bound for the 1σ limit (central value) of the E821 data. The boldfaced tan β = 50 entries represent upper bounds over all tan β ≤ 50.

| tan β | General MSSM | Gaugino Unification | + Dark Matter |
|-------|--------------|---------------------|--------------|
| 3     | 140 ( )      | 135 ( )             | 105 ( )      |
| 5     | 165 (75)     | 160 (65)            | 125 (65)     |
| 10    | 215 (135)    | 210 (105)           | 140 (105)    |
| 30    | 335 (255)    | 285 (205)           | 260 (205)    |
| 50    | **440 (345)**| **345 (265)**       | **345 (265)**|

Figure 6: Bounds on the masses of $\tilde{N}_1$ and $\tilde{\mu}_1$ as a function of $\delta a_{\mu}$ for various tan β with gaugino unification assumed.

be a light chargino and a light sneutrino. However the correlations already discussed preserve the bounds on the various species over the whole range of tan β. A bound on $m_{\tilde{\nu}}$ implies a bound on $m_{\tilde{\mu}_1}$, and a bound on $m_{\tilde{C}_1}$ implies a bound on at least one of the $m_{\tilde{N}_i}$, and in certain cases (such as gaugino unification), the converses may be true as well.

We have shown in Fig. 6 the mass bounds on $\tilde{\mu}_1$ and $\tilde{N}_1$ under the assumption of gaugino unification; a plot for $\tilde{C}_1/\tilde{N}_2$ will appear later in our discussion of Tevatron physics. Note that $\tilde{N}_1$ must be relatively light, even for large tan β, thanks to the gaugino unification condition, while $\tilde{\mu}_1$ can be heavier but must still lie below 820 GeV at 1σ.

Finally, we can consider the general MSSM without gaugino unification. The results can best be summarized by showing the mass bounds on the various sparticles at tan β = 50 in relation to their bounds in the unified case. This is done in Fig. 7. We can see from the figure that the bound on the $\tilde{\mu}_1$ is essentially identical to that in the gaugino
unification picture. However the gaugino masses have shifted, and the reason is no mystery. Once again, as discussed in the previous section, the lightest neutralino is no longer a $\tilde{B}$-like spectator to the magnetic moment, but is a $\tilde{W}$-like partner of a participating $\tilde{W}$-like chargino.

We summarize our results for the various sparticle species in Table 2. There we have shown the upper bounds on several sparticles in the general MSSM, the MSSM with gaugino unification, and the previous case with the additional requirement of a neutralino LSP ("dark matter"). The bounds represents those obtained using the $1\sigma$ limit (central value) of the E821 data.

### 2.3 Bounds on $\tan \beta$

The final bound we will derive using the E821 data is on $\tan \beta$. There has been some discussion in the literature about which values of $\tan \beta$ are capable of explaining the data. And in fact, we concur that at lower $\tan \beta$, there is a real suppression in the maximum size of $\delta a_{\mu}$. So in Figure 8 we have shown the maximum attainable value of $\delta a_{\mu}$ as a function of $\tan \beta$ with and without the added assumption of gaugino mass unification.

The limit in Fig. 8 clearly divides into two regions. At $\delta a_{\mu} > 36 \times 10^{-10}$ the chargino contribution dominates and thus $\delta a_{\mu} \propto y_{\mu}$, scaling linearly with $\tan \beta$. At lower $\delta a_{\mu}$, however, both neutralino and chargino contributions can be important so it becomes
Table 2: Upper bounds on the mass (in GeV) of various sparticles for the general MSSM, the MSSM assuming gaugino mass unification, and the MSSM with gaugino mass unification plus a neutralino LSP. The entries represent the bound for the 1σ limit (central value) of the E821 data. These bounds are for all tan β ≤ 50.

| Mass Bound | General MSSM | Gaugino Unification | + Dark Matter |
|------------|--------------|---------------------|---------------|
| \( \tilde{N}_1 \) | 610 (455) | 470 (350) | 340 (265) |
| \( \tilde{N}_2 \) | none (none) | 830 (565) | 575 (440) |
| \( \tilde{C}_1 \) | none (690) | 830 (565) | 575 (440) |
| \( \tilde{\mu}_1 \) | 870 (680) | 825 (665) | 825 (665) |
| \( \tilde{\nu} \) | 865 (675) | 820 (660) | 820 (660) |

Figure 8: Bounds on tan β as a function of \( \delta a_\mu \) for the general MSSM. The dotted lines represent the E821 central value and ±1σ bounds on \( \delta a_\mu \).

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possible to generate $\delta a_\mu$ with much smaller values of $\tan \beta$ than would be possible from the charginos alone. The central value of the E821 data implies $\tan \beta > 4.7$; but already at 1$\sigma$ all values of $\tan \beta$ down to 1 are allowed. This result contradicts most statements made in the literature to date about limiting $\tan \beta$ using $a_\mu$: at present we are not able to place any 1$\sigma$ bound on $\tan \beta$ larger than 1 due to the very real possibility of a neutralino-dominated $\delta a_\mu$. Future reduction in the error bars on $a_\mu$ will provide an important opportunity for placing a meaningful lower bound on $\tan \beta$.

2.4 Implications for the Tevatron

At its simplest level, the measurement of $\delta a_\mu$, an anomaly in the lepton sector, has little impact on the Tevatron, a hadron machine. In particular, the light smuons associated with $\delta a_\mu$ cannot be directly produced at the Tevatron, occurring only if heavier (non-leptonic) states are produced which then decay to sleptons. In the calculation of $a_\mu$, the only such sparticles are the neutralinos and charginos. These states can be copiously produced and in fact form the initial state for the “gold-plated” SUSY trilepton signature.

Of particular interest for the trilepton signature are the masses of the lighter chargino ($\tilde{C}_1$) and 2nd lightest neutralino ($\tilde{N}_2$). Studies of mSUGRA parameter space indicate that the sensitivity to the trilepton signature at Run II/III of the Tevatron depends strongly on the mass of sleptons which can appear in the gaugino decay chains. For heavy sleptons, the Tevatron is only sensitive to gaugino masses in the range $m_{\tilde{C}_1, \tilde{N}_2} \lesssim 130$ to 140 GeV for 10 fb$^{-1}$ of luminosity and 145 to 155 GeV for 30 fb$^{-1}$, with smaller values for smaller $\tan \beta$. However, for light sleptons (below about 200 GeV) the range is considerably extended, up to gaugino masses around 190 to 210 GeV. Of course, this latter range is exactly the one most relevant for understanding $a_\mu$.

It is impossible in the kind of analysis presented here to comment on the expected cross-sections for the neutralino-chargino production (there is no information in $a_\mu$ on the masses of the $t$-channel squarks, for example) but we can examine the mass bounds on $\tilde{C}_1$ and $\tilde{N}_2$. In Fig. 9 we have shown just that: the upper bound on the heavier of either $\tilde{C}_1$ or $\tilde{N}_2$ as a function of $\delta a_\mu$ for several values of $\tan \beta$.

A few comments are in order on the plot. First, the plot assumes gaugino unification; dropping that assumption can lead to significantly heavier masses for the $\tilde{N}_2$ though not for the $\tilde{C}_1$; this is because the light $\tilde{C}_1$ is needed to participate in the magnetic moment, while $\tilde{N}_2$ can decouple. Second, we have also assumed a neutralino LSP; this is to be expected since the event topology for the trilepton signal assumes a stable, neutralino LSP. Finally, on the y-axis is actually plotted $m_{\tilde{C}_1}$, but in every case we examined with gaugino unification, the difference in the maximum masses of $\tilde{C}_1$ and $\tilde{N}_2$ differed by at most a few GeV. This is because they are both dominantly wino-like in the unified case and thus have masses $\simeq M_2$.

From the figure it is clear that a no-lose theorem for the Tevatron is not lurking in the current E821 data. However, if the central value reported by E821 holds up and
Figure 9: Mass bounds on $\tilde{C}_1$ and $\tilde{N}_2$ (where $m_{\tilde{C}_1} \simeq m_{\tilde{N}_2}$) as a function of $\delta a_\mu$ for $\tan \beta = 3, 5, 10, 30$ and 50. The dotted lines represent the E82I central value and 1σ bounds on $\delta a_\mu$. This figure assumes gaugino unification and a neutralino LSP.

$\tan \beta \lesssim 10$ then one should expect the Tevatron to find a trilepton signal for SUSY. So there is actually significant hope for a positive result. We cannot emphasize enough too that these are upper bounds on the sparticle masses and in no way represent best fits or preferred values. Thus even for larger $\tan \beta$ or smaller $\delta a_\mu$, there is good reason to hope that the Tevatron will be able to probe the gaugino sector in Run II or III.

2.5 Implications for a Linear Collider

A consensus is currently forming in favor of building a $\sqrt{s} = 500$ GeV linear collider, presumably a factory for sparticles with masses below 250 GeV. What does the measurement of $a_\mu$ tell us with regards to our chances for seeing SUSY at $\sqrt{s} = 500$ GeV? And how many sparticles will be actually accessible to such a collider?

The analysis of the previous section can put a lower bound on the number of observable sparticles at a linear collider as a function of $\delta a_\mu$ and $\tan \beta$ and we show those numbers as a histogram in Fig. [11]. In this figure, we have shown the minimum number of sparticles with mass below 250 GeV for $\tan \beta = 5, 10, 30$ and 50, assuming gaugino unification. In the graph, the thinner bars represent smaller $\tan \beta$. As is to be expected, the number of light states increases with increasing $\delta a_\mu$ and with decreasing $\tan \beta$. However note that there are no $\tan \beta = 5$ lines for $\delta a_\mu > 40 \times 10^{-10}$ since there is no way to explain such large $\delta a_\mu$ values at low $\tan \beta$.

We see from the figure also that for $\tan \beta \gtrsim 30$ there is no guarantee that a 500 GeV machine would produce on-shell sparticles; this is not to be taken to mean that one
should not expect their production, simply that $a_\mu$ cannot guarantee it. However for $\tan \beta \lesssim 10$, the 1σ limit on $a_\mu$ from E821 indicates that at least 2 to 3 sparticles will be accessible to a 500 GeV machine. This counting does not include extra sleptons due to slepton mass universality; for example, a light muon sneutrino also implies light tau and electron sneutrinos, and likewise for the charged smuon.

A similar bar graph can also be made for a 1 TeV machine, though we do not show it here. However the relevant numbers can be inferred from Fig. 3; we see that such a machine is guaranteed to produce at least three sparticles for $\tan \beta \leq 50$ at the E821 1σ limit.

3 Conclusions

Deviations in the muon anomalous magnetic moment have long been advertised as a key hunting ground for indirect signatures of SUSY. And it may be that we finally have the evidence we need in the 2.7σ deviation reported by E821. If this signal is real and if SUSY is the correct explanation for it, then light sparticles are requisite. In this analysis, we have confirmed, extended and overridden some of the bounds on these light sparticles that have appeared previously. In particular, for the 1σ lower bound on $\delta a_\mu$ derived from the E821 data, we obtain the following for the most general MSSM parametrization:

- there must be at least 4 sparticles with masses below 820 GeV (700 GeV for unified
• the lightest sparticle must lie below 440 GeV (345 GeV for unified gauginos);
• there is no lower bound on $\tan \beta$ (the central value of the E821 data requires $\tan \beta > 4.7$);
• the $\tilde{\mu}_1$ and the $\tilde{\nu}$ must lie below 870 GeV while one neutralino must fall below 610 GeV (those bounds become 680 and 455 GeV respectively for unified gauginos);
• there is no upper bound on $m_{\tilde{\nu}}$ (the data’s central value requires $m_{\tilde{\nu}} < 690$ GeV);
• if $\tan \beta < \sim 10$, then a 500 GeV linear collider is guaranteed to produce at least 2 sparticles, and Run II/III of the Tevatron is likely to see a trilepton signature (assuming a stable neutralino LSP); for $\tan \beta \gtrsim 10$, neither of these can be guaranteed.

In the most interesting theoretical scenario, i.e. gaugino unification with a neutralino LSP, the bounds on SUSY masses can be much lower, with the neutralino LSP below 340 GeV, $\tilde{C}_1$ and $\tilde{N}_2$ below 575 GeV and sleptons below 825 GeV. Our only important simplifications were to drop the dependence on $A_\mu$ from the smuon mixing matrices, and to assume slepton mass universality.

If E821 has discovered evidence for SUSY, then it is clear that we have many more discoveries awaiting us. The current data implies that the LHC will find SUSY directly; such a discovery for the Tevatron or the NLC is not guaranteed, though the data on $a_\mu$ greatly enhances the likelihood that these programs will be successful.

Acknowledgments

We are grateful to S. Martin for discussions during the early parts of this project. CK would also like to thank the Aspen Center for Physics where parts of this work were completed, and the Notre Dame High-Performance Computing Cluster for much-needed computing resources. This work was supported in part by the National Science Foundation under grant NSF-0098791. The work of MB was supported in part by a Notre Dame Center for Applied Mathematics Graduate Summer Fellowship.

Appendix

The supersymmetric contributions to $a_\mu$ are generated by diagrams involving charged smuons with neutralinos, and sneutrinos with charginos. The most general form of the calculation, including phases, takes the form (we follow Ref. [4]):

$$
\delta a^{(N)}_\mu = \frac{m_\mu}{16\pi^2} \sum_{i,m} \left\{ -\frac{m_\mu}{12 m_\mu^2} (|n_{\mu i}^L|^2 + |n_{\mu i}^R|^2) F_1^N(x_{\mu i}) + \frac{m_{N_i}}{3 m_\mu^2} \text{Re}[n_{\mu i}^L n_{\mu i}^R] F_2^N(x_{\mu i}) \right\}
$$
\[
\delta \theta_{\mu}^{(C)} = \frac{m_\mu}{16\pi^2} \sum_k \left\{ \frac{m_\mu}{12m_\tilde{\nu}} (|c_k^L|^2 + |c_k^R|^2) F_1^C(x_k) + \frac{2m_\tilde{\nu}}{3m_\tilde{\nu}} \text{Re}[c_k^L c_k^R] F_2^C(x_k) \right\}
\]

where \( i = 1, 2, 3, 4 \), \( m = 1, 2 \), and \( k = 1, 2 \) label the neutralino, smuon and chargino mass eigenstates respectively, and

\[
\begin{align*}
n_{im}^R &= \sqrt{2} g_1 N_{i1} X_{m2} + y_\mu N_{i3} X_{m1}, \\
n_{im}^L &= \frac{1}{\sqrt{2}} (g_2 N_{i2} + g_1 N_{i1}) X_{m1}^* - y_\mu N_{i3} X_{m2}^*, \\
c_k^R &= y_\mu U_{k2}, \\
c_k^L &= -g_2 V_{k1},
\end{align*}
\]

\( y_\mu = g_2 m_\mu/\sqrt{2}m_W \cos \beta \) is the muon Yukawa coupling, and \( g_{1,2} \) are the U(1) hypercharge and SU(2) gauge couplings. The loop functions depend on the variables \( x_{im} = m_{N_i}^2/m_{\tilde{\nu}_m}^2 \), \( x_k = m_{\tilde{\nu}_k}/m_{\tilde{\nu}_m} \) and are given by

\[
\begin{align*}
F_1^N(x) &= \frac{2}{(1-x)^3} \left[ 1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x \right] \\
F_2^N(x) &= \frac{3}{(1-x)^3} \left[ 1 - x^2 + 2x \ln x \right] \\
F_1^C(x) &= \frac{2}{(1-x)^3} \left[ 2 + 3x - 6x^2 + x^3 + 6x \ln x \right] \\
F_2^C(x) &= -\frac{3}{2(1-x)^3} \left[ 3 - 4x + x^2 + 2 \ln x \right]
\end{align*}
\]

For degenerate sparticles \( x = 1 \) the functions are normalized so that \( F_1^N(1) = F_2^N(1) = F_1^C(1) = F_2^C(1) = 1 \). We can also bound the magnitude of some of these functions; in particular \( |F_2^N(x)| \leq 3 \) while \( |F_2^C(x)| \) is unbounded as \( x \to 0 \).

The neutralino and chargino mass matrices are given by

\[
M_{\tilde{N}} = \begin{pmatrix}
M_1 & 0 & -m_Z s W c_\beta & m_Z s W s_\beta \\
0 & M_2 & m_Z c W c_\beta & -m_Z c W s_\beta \\
-m_Z s W c_\beta & m_Z c W c_\beta & 0 & -\mu \\
m_Z s W s_\beta & -m_Z c W s_\beta & -\mu & 0
\end{pmatrix}
\]

and

\[
M_{\tilde{C}} = \begin{pmatrix}
M_2 \\
\sqrt{2} m_W c_\beta \\
m_\mu
\end{pmatrix}
\]

where \( s_\beta = \sin \beta \), \( c_\beta = \cos \beta \) and likewise for \( \theta_W \). The neutralino mixing matrix \( N_{ij} \) and the chargino mixing matrices \( U_{kl} \) and \( V_{kl} \) satisfy

\[
\begin{align*}
N^* M_{\tilde{N}} N^{\dagger} &= \text{diag}(m_{\tilde{N}_1}, m_{\tilde{N}_2}, m_{\tilde{N}_3}, m_{\tilde{N}_4}) \\
U^* M_{\tilde{C}} V^{\dagger} &= \text{diag}(m_{\tilde{C}_1}, m_{\tilde{C}_2}).
\end{align*}
\]
The smuon mass matrix is given in the \{\tilde{\mu}_L, \tilde{\mu}_R\} basis as:
\[
M^2 = \begin{pmatrix}
M^2_L & m_\mu (A_\tilde{\mu} - \mu^* \tan \beta) \\
M^2_R & m_\mu (A_\tilde{\mu} - \mu^* \tan \beta)
\end{pmatrix}
\]
(7)
where
\[
M^2_L = m^2_L + (s^2_W - 1/2)m^2_Z \cos 2\beta \\
M^2_R = m^2_R - s^2_W m^2_Z \cos 2\beta
\]
(8)
for soft masses \(m^2_L\) and \(m^2_R\); the unitary smuon mixing matrix \(X_{mn}\) is defined by
\[
XM^2_\tilde{\mu} X^\dagger = \text{diag}(m^2_{\tilde{\mu}_1}, m^2_{\tilde{\mu}_2}).
\]
(9)
We will define a smuon mixing angle \(\theta_\tilde{\mu}\) such that \(X_{11} = \cos \theta_\tilde{\mu}\) and \(X_{12} = \sin \theta_\tilde{\mu}\). In our numerical calculations we will set \(A_\tilde{\mu} = 0\). At low \(\tan \beta\) we have checked that varying \(A_\tilde{\mu}\) makes only slight numerical difference, while at large \(\tan \beta\) it has no observable effect whatsoever. Finally, the muon sneutrino mass is related to the left-handed smuon mass parameter by
\[
m^2_{\tilde{\nu}} = m^2_L + 1/2m^2_Z \cos 2\beta.
\]
(10)

The leading 2-loop contributions to \(\delta a_\mu\) have been calculated \cite{13} and have been found to suppress the SUSY contribution by a factor \((4\alpha/\pi) \log(m_{\text{SUSY}}/m_\mu) \approx 0.07\); we will include this 7\% suppression in all our numerical results.

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