Comparison Study on Four Types Mathematical Model for Sand-Casting Process

Rahaini Mohd Said1,2, Roselina Sallehuddin1, Nor Haizan Mohd Radzi1, Mohamad Ridzuan Mohamad Kamal3, Nor Erne Nazira Bazin1

1School of computing, Faculty of Engineering, Universiti Teknologi Malaysia
2Department of Electronic and Computing, Faculty of Electric and Electronic Engineering Technology, Universiti Teknikal Malaysia Melaka.
3Department of Manufacturing, Faculty of Mechanical and Manufacturing Engineering Technology, Universiti Teknikal Malaysia Melaka.

E-mail: rahaini@utem.edu.my

Abstract. Modelling and optimisation of sand-casting process have been widely studied in casting research. The present study was developed to compare the four types of mathematical model and find the best model used in the optimised process. The four types of mathematical model discussed in this paper were multiple linear regression model, full factorial model, fractional factorial model and response surface methodology model. The best model will used for the optimized process using soft computing technique. The input parameters of these models were silica sand, bentonite, water and coal dust. Meanwhile, permeability was the model output. The four types of mathematical model produced were statistically validated to test for model adequacy by using ANOVA and coefficient of determination. Each model was finally, validated by using mean square error (MSE) and root square error (RMSE). The research findings proved that respond surface methodology model was the best model as compared the other models, which was significant with p-values of less than 0.05. The model also produced the smallest MSE.

1. Introduction

In sand casting process, proper and complete process of silica sand, clay powder, coal dust, and water combination is known as green sand. A typical green sand mixture contains about silica sand (85%), clay (9%), water (3%), and organic additives (3%) [1]. Then, the sand will undergo a “mulling” process, whereby clay acts as a binder in the sand which will produce a suitable composition for the sand moulding process. The sand mixt ure is first compressed around the pattern at specific pressures and temperatures to ensure that its shape is maintained in the required casting process. The sand is compacted around the pattern which followings the desired casting. The sand mould plays a vital role in the sand-casting process to help remove gases in the moulded part during process. Therefore, maximum permeability must be achieved to remove gases from the mould through the sand grains. Permeability is the flow capacity measure of a porous media to emit gases from an object [2]. So far optimisation of the casting process has been the subject of different conducted studies. The studies had many approaches which were aimed to improve the casting process. They differ in parameters to be optimised and applied optimization techniques. According to Ganesh et al., there were two issues in casting, modelling, and optimisation. Modelling is a process in determining the minimum or maximum value of casting...
performances while optimisation is a selection process of optimal semi-casting process parameters that lead to the minimum or maximum value of casting performances [3].

The modelling process plays an important role, whereby the mathematical model is developed to act as an objective function for the optimisation process. In modelling the casting process, several approaches were widely considered by previous researchers, such as multiple regression approaches, response surface methodology (RSM), and artificial neural network (ANN) [4-6]. On the other hand, there are two types of optimisation, namely conventional approaches and nonconventional approaches. Examples of conventional approaches are Taguchi Method, Design of Experiment (DOE), and iterative mathematical search. Meanwhile examples of nonconventional approaches are genetic algorithm (GA), and backpropagation (BP) and Neural network [7-9].

The modelling process was purposely used to define the experimental data in terms of the composition relation of casting parameters and moulding performance to assist the moulding optimisation process. Due to the stochastic and complex nature of casting process, development of the convincing theoretical model has become more challenging. Therefore, many previous researchers had adopted various modelling methods. This paper is aimed to propose four regression technique approaches to develop the mathematical models for casting performance, which were factorial design, Taguchi method, respond surface method, and multi regression method. All these models will be compared to their performance. Finally, these models will be used in the next computational optimisation process by using the soft-computing technique.

2. Methodology: Experimentation, Modelling and Statistical Analysis

The present research methodology is discussed in this section. Experiment parameter, modelling, and statistical analysis used of the sand-casting system that were carried out are as shown in Figure 1.

### 2.1. Selection of Input Variables and Their Operating Levels

Determination of proper combination input variables selection and their experiment range affects the modelling results accuracy. Different combination amounts of composition parameter influence the casting performance. Therefore, selecting the parameter with too wide levels or too narrow levels will result in a possible or inaccurate input-output relation. Table 1 shows the study input variables, propose level, and operating range.

| Input variable | Range | Coded Unit | Low level (-) | High level (+) |
|----------------|-------|------------|----------------|----------------|
| Silica sand    | A     | g          | 95             | 100            |
| Bentonite      | B     | g          | 21             | 24             |
| Water          | C     | g          | 6.5            | 8              |
| Coal dust      | D     | g          | 6              | 8              |

### 2.2. Design Modelling

The next crucial step is to design an experiment with the selection of points, whereby the response should be estimated. Several design methods were applied for the permeability optimisation model. The most popular were full factorial design, fractional factorial design, and central composite design (CCD) for optimisation with many variables. Several types of software are easily accessible to researchers and they can be used for these methods. The most popular program for casting process studies is Design Expert (Stat-Ease, Inc), Minitab (Minitab Inc.), and MATLAB (MathWorks).

### 2.3. Conducting Experiment

Greensand is a combination of silica sand, coal dust, bentonite, and water. For optimum compositions of green sand, the ratio aspect of these components were readjusted; hence the defects in a cast product
were reduced. The sand specimen size is a cylinder of dimension 50mm x 50mm. The specimen tube was placed with its base under the ram punch. The ram punch was slowly lowered onto the sand specimen by lifting the cam plate, and three ram blows were simultaneously executed by turning the working cam plate.

2.4. Developing the Regression Model and Conducting Statistical Analysis

The experiment was conducted by using three types of design models. The experiment was used to develop a nonlinear regression model and explain the respond (permeability) as a nonlinear function of moulding parameters (silica sand, bentonite, water, and coal dust). Four types of mathematical model were produced, and every design was statistically validated to test the model adequacy by using ANOVA and coefficient of determination. Finally, each model was validated by using mean square error (MSE) and root square error (RMSE).

\[ y(\text{output}) = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ij} x_i^2 + \sum_{i<j}^{k} \beta_{ij} x_i x_j \]

**Figure 1.** Step in modelling of sand-casting process.
3. Results and Discussion

In this study, four types of mathematical regression modelling were used to seek the best model for the casting process of independent parameter. The significant model and coefficient determination values for all types of model are presented in Table 2. From the Analysis of Variance (ANOVA) analysis, all the types of models were significant since the p-values were less than 0.05. A perfect fit model was observed and sustained by the coefficient of determination ($R^2$). The fractional factorial design and central composite showed good models since the coefficient value was more than 90%, where the coefficient of determination is $R^2 = 0.97$ and $R^2=0.92$ respectively. Furthermore, 0.92 indicated that the polynomial regression model explained almost 92% of variability in the casting process.

Table 2. Mathematical models for four types of models.

| Type               | Mathematical Model                                                                 | P-values | $R^2$ |
|--------------------|------------------------------------------------------------------------------------|----------|-------|
| Full factorial     | $P = 338.5267 - 0.9404*Silica Sand - 0.79218*Bentonite - 68685*Coal dust + 0.13365*Silica Sand*Water + 0.2408*Silica sand*Coal dust$ | 0.04     | 0.76  |
| Multiple regression Model | $P = 163.1-0.874*Silica sand - 1.93*Water -2.478*Coal dust$ | 0.02     | 0.74  |
| Central Composite Design | $P = 101.422 + 6.24718 *Silica sand-0.8607*Bentonite - 30.7222*Water – 0.1135*Silica sand*Bentonite - 0.1336*Silica sand*water + 0.24075*Silica sand*Coal dust +1.444*Bentonite*water + 0.5195*Bentonite*coal dust- 0.0238*(Silica sand)^2-0.066*(Bentonite)^2 - 0.7921*(water)^2 - 0.1485*(Coal dust)^2$ | 0.00     | 0.92  |
| Fractional Factorial Design | $P = 681.4 – 4.850 *Silica sand – 2.083*Bentonite -13.14*water – 74.50*Coal dust +0.5680*Silica sand*Coal dust +0.2083*Bentonite*coal dust + 1.650*water*Coal dust$ | 0.00     | 0.97  |
The graphical representations in Figure 2 shows four different types of mathematical model with the actual experimental data. All types of model showed that they were close to each other, but the RSM model showed that it was slightly close with the actual experimental data, which means that the RSM model was the significant model among the other models. However, to prove that the model was accurate and the best, all models were validated based on insertion value to the mathematical model. Next, the value of mean square error and roof mean square error were calculated, as shown in table 3. The smallest error produced the accurate model between experimental data and mathematical model. The table 3 show the RSM model produced the smallest MSE and RMSE.

Table 3. Mean square error (MSE) and root mean square error (RMSE).

| Model                          | MSE   | RMSE |
|-------------------------------|-------|------|
| Full Factorial Design         | 4.94  | 2.22 |
| Response Surface Methodology  | 2.05  | 1.43 |
| Fractional Factorial Design   | 52.32 | 7.23 |
| Multi regression              | 15.01 | 3.87 |

4. Conclusions
Modelling and analysis of four types of mathematical model for sand moulding composition are presented in the research work by utilising DOE software, Minitab software, and ANOVA. This study aims to present the four types of mathematical model for the optimisation of sand-casting mould performance using the soft-computing technique. Response surface methodology model shows the best mathematical model for the sand moulding composition process. The present research work also helps the foundrymen to predict the mould parameters by using the developed models. Further, these models can be used to optimise the composition of parameters for sand mould, whereby the study is currently working for the optimisation process by using the soft-computing technique.

Acknowledgement
Special appreciative to reviewers for the useful advices and comments. The authors greatly acknowledge the Research Management Centre, UTM for financial support through the Fundamental Research Grant Scheme (FRGS) Vot. No. R.J130000.7851.5F154.

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