Analysis of interval constants of Shoushili-affiliated calendars

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Abstract We study the interval constants that are related to the motions of the Sun and the Moon, i.e., the Qi, Intercalation, Revolution, and Crossing interval constants, in calendars affiliated with the Shoushi calendar (Shoushili), such as Datongli and Chiljeongsannaepyeon. It is known that those interval constants were newly introduced in Shoushili and revised afterward, except for the Qi interval constant, and the revised values were adopted in the Shoushili-affiliated calendars. In this paper, we investigate first the accuracy of the interval constants and then the accuracy of the Shoushili-affiliated calendars in terms of the interval constants by comparing the times of the new moon and the solar eclipse maximum calculated by each calendar with modern calculations. During our study, we found that the Qi and Intercalation interval constants used in the early Shoushili were well determined, whereas the Revolution and Crossing interval constants were relatively poorly measured. We also found that the interval constants used by the early Shoushili were better than those of the later one, and hence than those of Datongli and Chiljeongsannaepyeon. On the other hand, we found that the early Shoushili is, in general, a worse calendar than Datongli for use in China but a better one than Chiljeongsannaepyeon for use in Korea in terms of the new moon and the solar eclipse times, at least for the period 1281 – 1644. Finally, we verified that the sunrise and sunset times recorded in Shoushili-Li-Cheng and Mingshi are those at Beijing and Nanjing, respectively.

Key words: history and philosophy of astronomy: general — celestial mechanics — ephemerides

1 INTRODUCTION

The Shoushi calendar (Shoushili, hereinafter) was developed by Shoujing Guo and his colleagues from the Yuan dynasty (Bed 1997). It is known as one of most the famous calendars in Chinese history (Needham 1959). Its affiliated calendars, the Datongli of the Ming dynasty and the Chiljeongsannaepyeon (Naepyeon, hereinafter) of the Joseon dynasty, are extremely similar. Compared to previous Chinese calendars, the most distinguishing characteristic of the Shoushili is the abolishment of the use of the Super Epoch, an ancient epoch where the starting points of all interval constants are the same (Sivin 2009). Instead, Shoushili adopted the winter solstice of 1280 (i.e., 1280 December 14.06
in Julian date) as the epoch and introduced seven interval constants from observations in order to reduce the large number of accumulated days in the calculations arising from the use of the Super Epoch. Of the seven interval constants, four are related to the motions of the Sun and the Moon: the Qi, Ren (Intercalation), Zhuan (Revolution), and Jiao (Crossing) interval constants. According to the records of Mingshi (History of the Ming Dynasty), the early values of these interval constants, except for the Qi interval constant, were revised later. Currently, it is known that Datongli and Naepyeon adopted the revised values of the early Shoushili; hence, both calendars are essentially identical to the later Shoushili. The main purpose of this study is to evaluate the effect of the values of these four interval constants on the accuracy of calendar calculations. In this paper, therefore, “Shoushili” refers to theearly one unless stated otherwise.

In this study, we first estimate the accuracy of the values of the Qi, Intercalation, Revolution, and Crossing interval constants of the Shoushili-affiliated calendars. We then investigate the accuracy of each calendar in terms of the interval constants by comparing the timings of the new moon and the solar eclipse maximum calculated by each calendar with modern calculations. Because sunrise and sunset times are needed to calculate the timings of the solar eclipse maximum in the Shoushili-affiliated calendars, we verify those times presented in calendar books as well.

This paper is composed as follows. In section 2, we briefly introduce the Shoushili-affiliated calendars and modern astronomical calculations. In section 3, we present the results on the analysis of the four interval constants, the new moon times, the sunrise and sunset times, and the solar eclipse maximum times. Finally, we summarize our findings in section 4.

2 THE SHOUSHILI-AFFILIATED CALENDARS

2.1 Shoushili

In the chapter related to calendars in the Yuanshi (History of the Yuan Dynasty), Shoushili is described as covering two parts: Shoushili-Yi (Discussion) and Shoushili-Jing (Method). It is known that the calendar was used in the Goryeo dynasty of Korea since the reign of King Chungsun (1308 – 1313) (Jeon 1974). For this reason, Shoushili is also recorded in the chapter related to calendars in the Goryeosa (History of the Goryeo Dynasty). However, the Goryeosa does not include the Shoushili-Yi. Conversely, the Yuanshi contains no Li-Cheng (Ready-reference Astronomical Tables), which is presented in the Goryeosa. In addition, both historical books have no tables on the sunrise and sunset times.

A book entitled Shoushili-Li-Cheng (Ready-reference Astronomical Tables for Shoushili) is preserved in the Kyuganggak Institute for Korean Study (Kyuganggak, hereinafter) in Korea. This book contains various tables for the use of calendar calculations by Shoushili along with the times of sunrise and sunset times (for further details, see Lee & Jing 1998). We have to refer to three books (i.e., Yuanshi, Goryeosa, and Shoushili-Li-Cheng) to completely understand Shoushili. Kyuganggak also possesses a book called Shoushili-Jie-Fa-Li-Cheng (Expeditious Ready-reference Astronomical Tables for Shoushili). According to the preface of the Kyuganggak edition, this book was brought from China by Bo Gang (an astronomer in the Goryeo court). It was printed in 1346, and reprinted in 1444, around the publication year of Naepyeon.

2.2 Datongli

There are two versions of Datongli in the Ming dynasty; Wushen-Datongli (Datongli of the Wushen Year) made by Ji Liu in 1368, and Datong-Lifa-Tonggui (Comprehensive Guide to the Calendrical Method by Datongli) made by Tong Yuan in 1384. Although the epochs of the former and the latter calendars are the winter solstices of 1280 and 1383, respectively, both are based on the Shoushili (Lee 1996). In the chapter related to calendars in the Mingshi, Datongli is described as having three parts: the first is on the origin of the techniques, the second on Li-Cheng including the tables on the timings of sunrise and sunset, and the third on calendar methods.
Table 1  Summary of the four interval constants adopted in the Shoushili-affiliated calendars.

| Interval     | Shoushili | Datongli | Goryeosa | Mingshi* | Naepyeon |
|--------------|-----------|----------|----------|----------|----------|
| Qi           | 550,600   | 550,600  | 550,600  | 550,600  | 550,600  |
| Intercalation| 200,850   | 202,050  | 202,050  | 202,050  | 202,050  |
| Revolution   | 131,904   | 131,904  | 130,205  | 130,205  | 130,205  |
| Crossing     | 260,187.86| 260,388  | 260,388  | 260,388  | 260,388  |

Notes: *Values when the epoch is the winter solstice of 1280.

It is widely known that Datongli was used in the Goryeo dynasty since 1370, although this fact is arguable (refer to Lee et al. 2010). Unlike Shoushili, Datongli is not included in the chapter related to calendars in the Goryeosa. Instead, a series with a name similar to Datong-Lifa-Tonggui of Tong Yuan, for example, Datong-Liri-Tonggui (Comprehensive Guide to the Calendar Day by Datongli), is preserved in the Kyuganggak. According to the work of Lee (1988), the series (Tonggui series, hereinafter) is related to the Datongli and was published around 1444 and its main purpose was being a reference during the compilation of the Naepyeon.

2.3 Naepyeon

King Sejong of the Joseon dynasty ordered In-Ji Jeong et al. to compile the Naepyeon in 1433. Although the data on when the compilation was completed is not clear, the oldest extant version is the one published in 1444 by Sun-Ji Yi and Dam Kim. In terms of the contents, each chapter of the Naepyeon corresponds to the Tonggui series and the timings of sunrise and sunset are contained in the Taeum (Moon) chapter. In addition, these times are different in the Shoushili-affiliated calendars. Because the compilation of the Naepyeon was considered to be one of the greatest works of King Sejong, this book is also appended in his Veritable Record, unlike Veritable Records of other king’s (Lee et al. 2008).

A book entitled Jeongmyoyeon-Gyeosik-Garyeong (Example Supplement for the Calculations of the Solar and Lunar Eclipses Occurred in 1447; shortly Garyeong), which is related to the Naepyeon, also remains in the Kyuganggak. This book contains modified interval constants for the epoch of 1442 and is valuable for step-by-step calculations of not only the solar eclipse but also the new moon by the Naepyeon, and hence by the Shoushili or Datongli. In this study, we refer to Garyeong to calculate the new moon and solar eclipse maximum times by the Shoushili-affiliated calendars, and we refer to the work of Lee (1988), which introduced the differences among the calendars.

In Table 1, we summarize the values of the four interval constants in each calendar. All dates are given in the Julian calendar and all values of interval constants are in units of Part; one day is 10,000 Parts. The Qi interval constant is the interval between the epoch and the midnight of the first day in a sexagenary cycle on counting backwards from the epoch. This value is same in all Shoushili-affiliated calendars. Intercalation interval constant is the interval from the epoch to the ‘mean’ new moon of the month belonging to the epoch. The time of a new moon is determined by correcting the slowness or fastness of the solar and lunar motions to the ‘mean’ new moon time, which is obtained by accumulating the Intercalation interval constant. Revolution and Crossing interval constants are the lengths between the epoch, and the times of lunar perigee and descending node passage, respectively (see also Sivin 2009).

As can be seen in Table 1 the values of interval constants in Datongli and Naepyeon are identical to each other. Interestingly, the numbers for the Shoushili of the Goryeosa are the same as those for the Naepyeon and the Datongli, except for the Revolution interval constant.

2.4 Modern calculations

In modern calculations, we use the astronomical algorithms of Meeus (1989, 1998) and the DE406 ephemeris of Standish et al. (1997). In addition, we use Besselian elements to calculate the solar eclipse
Table 2  Summary of the dates in the Shoushili-affiliated calendars and modern calculations relating to the four interval constants.

| Item | Shoushili-affiliated calendars (A) | Modern Calculation (B) | B − A (min) | Calendar | Remark |
|------|-----------------------------------|------------------------|-------------|----------|--------|
|      | Julian Date JD                   | ID = 2188925.56       | ID = 2188925.56 |          |        |
| WS1280$^1$ | Dec. 14.060000 | 0.000000 | 0.011638 | 16.8 | S, D, N | Epoch |
| MFDSC$^2$ | Oct. 20.000000 | −55.060000 | −55.060000 | 0.0 | S, D, N | Qi |
| MNM$^3$ | Nov. 23.875000 | −20.185000 | −20.205000 | −10.0 | S | Intercalation |
| MN$^4$ | Nov. 23.855000 | −20.191856 | −20.205000 | −10.2 | D, N |
| LPP$^5$ | Nov. 30.869600 | −13.190400 | −13.355960 | 238.4 | S | Revolution |
| LDNP$^6$ | Nov. 18.041214 | −26.018786 | −25.863352 | 223.8 | S | Crossing |

Notes: $^1$ Winter solstice of 1280, $^2$ Midnight of the first day in a sexagenary cycle on counting backwards from the epoch, $^3$ Mean new moon, $^4$ new moon, $^5$ Lunar perigee passage, $^6$ Lunar descending node passage, $^7$: Shoushili, D: Datongli, N: Naepyeon

time in a local circumstance. Although Muckes & Meeus (1983) tabulated Besselian elements for the solar eclipses ranging from −2003 to 2526, they presented only the first order coefficients in each element. Hence, we use Besselian elements extracted from DE406 ephemeris to increase our accuracy (e.g., Lee 2008). One of the important parameters to calculate ancient astronomical phenomena is $\Delta T$, difference between the universal time (UT) and the dynamical time (TD). To estimate $\Delta T$ for a given year, we employ the cubic spline interpolation method (see Press et al. 1992) using the data obtained recently by Morrison & Stephenson (2004). To directly compare the results of modern calculations with those by the Shoushili-affiliated calendars, we convert the universal time into the local apparent solar time by correcting the equation of time. Lastly, we assume that the locations of Beijing, Nanjing, and Seoul are at $39^\circ 55^\prime$ N and $116^\circ 25^\prime$ E, $32^\circ 3^\prime$ N and $118^\circ 53^\prime$ E, and $37^\circ 34^\prime$ N and $126^\circ 59^\prime$ E, respectively.

3 RESULTS

3.1 Values of interval constants

A winter solstice was used as the epoch in ancient Chinese calendars. In the Shoushili, the values of the interval constants were based on the winter solstice of 1280, as mentioned earlier. It is known that Shoujing Guo determined the date of the winter solstice from gnomon shadow measurements (Chen 1983; Li 2005). He used a tall gnomon and estimated the moment when the length of the shadow, caused by the Sun, is the longest, based on several days’ observations. In modern times, the winter solstice is calculated as the time when the Sun is passing the ecliptic longitude ($\lambda$) of 270°, using an astronomical ephemeris such as DE406. For the details on how Shoujing Guo determined the times of the winter solstice and of the lunar perigee and descending node passages, refer to Yuanshi.

In Table 2, we present the dates related to four interval constants in the Shoushili-affiliated calendars along with the results from modern calculations. All dates are in the Julian calendar, in units of the apparent solar time at Beijing unless otherwise mentioned. The difference in the equation of time according to the regions (i.e., Beijing, Nanjing, and Seoul), is negligible. Hence, we can easily convert the times at Beijing into times at other regions by correcting only the longitudinal difference. Therefore, the time at Seoul, for example, is obtained by adding the time at Beijing with +42.26 min (i.e., 10.566° of longitudinal difference between Beijing and Seoul).

In the table, the first column contains items related to the interval constants; WS1280 is the winter solstice of 1280, i.e., the epoch of the Shoushili-affiliated calendars. MFDSC is the midnight of the first day in the sexagenary cycle (i.e., Jiazi day) before the epoch. MNM and MN are mean new moon
and new moon, respectively, and LPP and LDNP are lunar perigee and descending node passages, respectively. The second column contains the Julian dates derived from the epoch and the values of interval constants in the Shoushili-affiliated calendars, except for WS1280, the epoch itself, and NM. The third column is the day number obtained by subtracting 2188925.56 d (i.e., the epoch) from the Julian day number (JD) corresponding to the date given in the second column. The fourth and fifth columns are the results of modern calculations and the difference between the modern calculations and the values derived from the Shoushili-affiliated calendars, respectively. The sixth column represent the calendar; S is Shoushili, D Datongli, and N Naepyeon. The last column contains the interval constants related with the items in the first column.

According to modern calculations, $\Delta T$ in 1280 is 532.6 s and JD of the winter solstice of the year at Beijing is 2188925.571638 d (i.e., 1280 December 14.071638), which is obtained by correcting the equation of time by $-0.087$ min. A difference of $+16.8$ min compared to modern calculation shows that Shoujing Guo accurately estimated the epoch in Shoushili (see Table 2). Although the time difference between Shoushili and modern calculations is 16.8 min, there is no change in a date. Hence, the MFDSCs in the Shoushili and modern calculations are the same as the 56th day in the sexagenary cycle (i.e., Jiwei day), October 20.0. Because of this, the Qi interval constant, which is the length between the epoch and the MFDSC, has the same difference in the epoch, i.e., 16.8 min. To verify our calculation, we compute the winter solstice of 2010 using $\Delta T = 65.9$ s (U.S. Nautical Almanac Office 2009), compare the result with the data of the Purple Mountain Observatory Chinese Academy of Science (2009, shortly CAS2009), and find a good agreement in the values, with the JD being 2455552.485037 d (i.e. 2010 December 21.985037) in UT (see also Korea Astronomy and Space Science Institute 2009).

Although it is known that Shoujing Guo also determined the Intercalation interval constant based on observations, there is not much detail on the calculations he used (Li & Zhang 1998a). Hence, we calculated the date of the new moon in 1280 November as an indirect method to verify the Intercalation interval constant. The dates are calculated to be JD 2188905.71966 and JD 2188905.719181 d (or 1280 November 21.344808 in UT) by Shoushili and modern calculations, respectively, which gives just a difference of only $+10.4$ min. To check our calculation, we also compare the time of the new moon with the data provided by NASA and find that the difference is less than 1 min. The exact times at which the astronomers of the Yuan dynasty modified the values of the interval constants of the Shoushili are not known. On using Datongli’s value of the Intercalation interval constant, the difference increases, becoming $\sim 39.3$ min, at least for the new moon time on 1280 November. We discuss the values for other periods in the next subsection.

Unlike the Qi and Intercalation interval constants, we find relatively large differences in the remaining constants. According to modern calculations, the lunar perigee and the descending node passage times at that time are JD 2188912.204040 and JD 2188899.696648 d, respectively. That is, Revolution and Crossing interval constants show the difference of $\sim 238.4$ (cf. $\sim 216$ min; Chen 2006) and $\sim 223.8$ min, respectively, compared with modern calculations. In particular, there is a large change in the Revolution interval constant in the Datongli, i.e., $+1699$ Parts ($\sim 244.7$ min). Hence, the difference is also larger in proportion to the amount, i.e., $\sim 483.1$ ($= 238.4 + 244.7$) min in Datongli (refer to Tables 1 and 2).

### 3.2 New moon time

In the Shoushili-affiliated calendars, the time of any phase of the Moon, for example, new moon, is determined in the following manners. First calculate the mean new moon time using the Intercalation interval constants and the length of the synodic month (i.e., 295305.93 Parts in the Shoushili-affiliated calendars). The new moon time is then determined by using the Revolution interval constant and by considering the motions of the Sun and the Moon. Because there is no Li-Cheng in Yuanshi, we use the values of Shoushili-Li-Cheng preserved in Kyuganggak for the the motions of the Sun and the Moon.

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1. [http://eclipse.gsfc.nasa.gov/phase/phasecat.html](http://eclipse.gsfc.nasa.gov/phase/phasecat.html)
The number distribution showing the differences in the new moon times between the Shoushili-affiliated calendars and modern calculations at (a) Beijing and (b) Seoul. The red-solid and blue-dotted lines represent the results for Shoushili and Datongli (Naepyeon in (b)), respectively.

For the sake of completeness, we check the values of Shoushili-Li-Cheng against Mingshi and Naepyeon and find that all values are identical to each other, except for a few typos in each book.

Comparing the length of the synodic month in the Shoushili with that obtained by modern calculations (i.e., 29.530587 d in 1280; refer to CAS2009), the difference is less than 1 s. Hence, we ignore the error in the length of the synodic month in the Shoushili. Instead, it is worth noting that the motion of the Sun speeds up or slow down before and after the perihelion or aphelion and not the winter or summer solstice as mentioned in the Shoushili. However, it is well known that the time of the winter solstice was very close to the perihelion passage time of the Earth around 1280. According to our investigations, the Earth passed the perihelion on the JD 2188925.078000d resulting in the difference of 0.48200 d compared to the time of the winter solstice in 1280.

To evaluate the effect of the Intercalation and the Revolution interval constants in determining the new moon times, we compute those times for the Shoushili-affiliated calendars for the years ranging from 1280 to 1644, compare them with the results of modern calculations, and present the number distribution of the differences in Figure 1 along with the root-mean-square (RMS) values. In the figure, the panels (a) and (b) show the results at Beijing and Seoul, respectively. The red-solid and blue-dotted lines represent the results for Shoushili and Datongli (Naepyeon in Fig. 1(b)), respectively. In each panel, the horizontal and vertical axes represent the difference in units of minutes with the intervals of 15 min and the number, respectively.

As shown in Figure 1, the new moon times given by the Datongli are on an average more accurate than those given by Shoushili at Beijing, i.e., RMSs are $\sim 23.8$ min (cf. $\sim 21$ min; Li & Zhang (1998a,b)) and $\sim 30.7$ min, respectively. Therefore, it can be said that the Datongli (or the later Shoushili) is one of the better calendars in China. However, Shoushili gives better results than Datongli in Korea. That is, the RMSs by the former and latter calendars are $\sim 27.9$ and $\sim 46.6$ min, respectively, in Korea. Interestingly, the RMS by Shoushili in Korea (i.e., RMS of $\sim 27.9$ min) is even less than that in China (RMS of $\sim 30.7$ min).
3.3 Sunrise and sunset time

One way to assess the overall accuracy of the four interval constants is to check the solar eclipse time. Before that, we verify the timings of sunrise (SR) and sunset (SS) presented in the calendar books because these times are used for calculating the times of the solar eclipses. In Figure 2, we depict the nighttime lengths (i.e., period from SS to SR) from the Shoushili-Li-Cheng and the Mingshi along with the results of modern calculations. In the figure, the blue-dotted lines represent the results (a) from Shoushili-Li-Cheng at Beijing and (b) from Mingshi at Nanjing, while the red-solid lines are the results from modern calculations at each region. In each panel, the upper and bottom panels show the nighttime lengths after winter and summer solstices, respectively. The horizontal and vertical axes represent the day number and the nighttime length in units of hours, respectively. According the Shoushili-affiliated calendars, the summer solstice of 1281 is June 14, i.e., JD = 2189107.5d (Lee et al. 2010). In this study, we define the SR/SS time as the zenith distance ($z$) of 90$^\circ$, which is different from the modern definition of $z = 90^\circ 50'$.

A table on the sunrise and sunset times is also contained in the Taiyin-Tonggui (Comprehensive Guide on the Sun), which is one of Tonggui series. According to our observations, there were some disagreements between Mingshi and Taiyin-Tonggui in the SR and SS times. We can easily figure out which document is incorrect by checking Daybreak (period from midnight to sunrise or from sunset to midnight) and Dusk (period from midnight to sunset) Parts because the sum of both parts should be 10,000 Parts, i.e., one day. In spite of this check, there were two discrepancies. However, these can be ignored because the differences were less than 1 Part. Hence, we can consider that the values of the SR and SS times are identical in both documents. Although there is no statement in the Taiyin-Tonggui, the Mingshi explicitly states that the SR and SS times are those at Nanjing. As shown in Figure 2, the RMSs are less than ∼1 min, compared to modern calculations, making it hard to distinguish from each other. Therefore, we can confirm that the SR and SS times given in the Shoushili-Li-Cheng are those at Beijing and that the statement on the SR and SS times in Mingshi is true.
Fig. 3 Differences in the solar eclipse maximum time between the Shoushili-affiliated calendars and modern calculations according to the region (a) at Beijing and (b) at Seoul. The circles and crosses represent the results for Shoushili and Datongli (Naepyeon in (b)), respectively.

3.4 Solar eclipse time

Based on the four interval constants and the SR/SS times discussed above, we calculate the timings of the solar eclipse maximum according to the calendars and compare these with the results of modern computations. Prior to the comparison, we validate our calculations by the Shoushili-affiliated calendars using the records in the historical literature. In the Garyeong, the procedure for calculating solar and lunar eclipses by Naepyeon is described in great detail. A calendar book entitled Jiaosi-Tonggui (Comprehensive Guide on the Eclipse), which is preserved in Kyuganggak, also lists step-by-step values in the process of calculating several eclipses by Datongli. We find that the results of our calculations of Naepyeon and Datongli show exact agreement with those of the documents. In particular, we find that the solar eclipse times of Jiaosi-Tonggui come from the result that used the SR/SS times at Nanjing and not Beijing. We compare the calculations in the Shoushili with the records in the Mingshi. In the history book, the times for a total of 32 solar eclipses, calculated by Shoushili, are recorded (see also Yabuuchi & Nakayama 2006). We find that all times match well except for four records: 707 June 1, 1059 January 1, 1061 June 1, and 1162 January 1 in the luni-solar calendar. According to our computations based on the Shoushili, there is no solar eclipse in 707, whereas the others show a difference of 1 Mark (∼15 min). However, we think that the actual differences would be smaller than 1 Mark, which is the significant digit in the records. For the hour systems used in ancient China and Korea, (refer to Saito 1995; Lee et al. 2011).

In this study, we restrict ourselves to the cases where the eclipse maximum occurred during daytime, i.e., the Sun’s altitude is greater than zero at the eclipse maximum. Figure 3 shows the difference between the times given by the Shoushili-affiliated calendars (Ts) and by modern calculations (Tm), i.e., Ts − Tm, between 1281 and 1644, along with RMS. We use SR/SS times (a) at Beijing and (b) at Seoul. In each panel, the circles and crosses represent the results for Shoushili and Datongli (Naepyeon in (b)), respectively.

According to this study, the RMSs for Shoushili and Datongli at Beijing are about ∼38.4 and ∼25.8 min, respectively. Meanwhile, Li & Zhang (1997) also studied the difference between the solar eclipse times given by Shoushili (according to this study, the later Shoushili) and by a modern ephemeris, and found that the RMS is ∼24 min. From the figure, we can easily see that the interval constants of the
Datongli are better than those of Shoushili at Beijing, which is similar to the case of the new moon times. The situation is reversed at Seoul (RMSs for Shoushili and Naepyeon are ~50.8 and ~77.1 min, respectively). Thus, the interval constants of Shoushili give better results than those of Naepyeon for the prediction of the solar eclipse time in Korea.

4 SUMMARY

It is known that Shoushili, of the Yuan dynasty, is one of the most accurate calendars in the history of China. The court of the next dynasty, i.e., the Ming dynasty, revised the calendar and titled it as Datongli. An example of the revisions is that the annual precession was abandoned. In the Joseon dynasty of Korea, both calendars were referred to for the compilation of Naepyeon by the Joseon royal astronomers. With regard to the interval constants, the Joseon court adopted the values of Datongli (i.e., the later Shoushili) in Naepyeon. Although there are some differences, particularly in the values of the interval constants, the Datongli and the Naepyeon are basically identical to the (early) Shoushili.

In this paper, we study the four interval constants given in the Shoushili-affiliated calendars, which are related to the motions of the Sun and the Moon: the Qi, Intercalation, Revolution, and Crossing interval constants. We first compared the values of those interval constants with the results of modern calculations, and then investigate on the accuracy of the timings of the new moon and the solar eclipse maximum given by the Shoushili-affiliated calendars using the interval constants values adopted in each calendar, along with timings of sunrise and sunset. The following is the summary of our findings.

(1) In the Shoushili, the Qi and Intercalation interval constants are well determined (errors of ~16.8 and ~10.4 min, respectively) while Revolution and Crossing ones are relatively poorly measured (errors of ~238.4 and ~223.8 min, respectively). The latter two interval constants are worse in Datongli (errors of ~483.1 and ~252.6 min, respectively).

(2) On calculating the new moon times for the period from 1280 to 1644 using Intercalation and Revolution interval constants of the Shoushili-affiliated calendars, the results show that the RMSs by Shoushili and Datongli are 30.7 and 23.8 min, respectively, at Beijing, and 27.9 and 46.6 min, respectively, at Seoul. Therefore, it can be evaluated that Datongli is the better calendar in China but not in Korea. Moreover, Shoushili is, in general, more suitable for use in Korea rather than in China in terms of the new moon time, at least for the period from 1280 to 1644.

(3) Unlike the interval constants, the timings of sunrise and sunset are accurately determined in each calendar book, with errors of less than 1 min. In addition, the times listed in Shoushili-Li-Cheng are at Beijing, as we confirmed, and in Mingshi are at Nanjing, as noted in the book.

(4) The timings of the solar eclipse maximum by Shoushili and Datongli ranging from 1281 to 1644 show RMSs of 38.4 and 25.8 min, respectively, at Beijing, and 50.8 and 77.1 min, respectively, at Seoul. Similarly to the new moon time, Shoushili gives better results than Datongli at Seoul in the calculation of the solar eclipse.

In the future, we think that more studies are needed to explain why the RMSs for the timings for the new moon and the solar eclipse maximum are small, order of ten, compared to large errors in the Revolution and the Crossing interval constants of the Shoushili-affiliated calendars, order of one hundred. One possibility might be the fact that both constants have opposite signs in the differences when compared to modern calculations.

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References

Bo, S. 1997, in International Conference on Oriental Astronomy from Guo Shoujing to King Sejong, edited by I. S. Nha & F. R. Stephenson, 15 (Seoul: Yonsei University Press)

Chen, M. 1983, ChJAA, 7, 69
Chen, M. 2006, A Critical Biography of Guo Shoujing (Nanjing: Nanjing University Press)
Jeon, S.-W. 1974, Science and Technology in Korea: Traditional Instruments and Techniques (Cambridge: MIT Press)
Korea Astronomy and Space Science Institute 2009, Korean Astronomical Almanac for the Year 2010 (Seoul: Namsandang)
Lee, E.-H. 1996, A Study of the Chiljŏnsan Naepiŏn, PhD Thesis (Seoul: Yonsei University)
Lee, E.-H., & Jing, B. 1998, China Historical Materials of Science and Technology, 19, 55
Lee, K.-W. 2008, The Journal of the Korean Earth Science Society, 29, 408
Lee, K.-W., Ahn, Y. S., Mihn, B.-H., & Lim, Y.-R. 2010, Journal of Astronomy and Space Science, 27, 55
Lee, K.-W., Ahn, Y. S., & Yang, H.-J. 2011, Advances in Space Research, 48, 592
Lee, K.-W., Yang, H.-J., & Park, M.-G. 2008, Journal of Astronomy and Space Science, 25, 199
Lee, M.-U. 1988, Journal of Korean History and Science Society, 10, 76
Li, Y. 2005, Progress in Astronomy, 23, 70
Li, Y., & Zhang, C. Z. 1997, Earth, Moon, & Planets, 76, 11
Li, Y., & Zhang, C. Z. 1998a, A&A, 332, 1142
Li, Y., & Zhang, C. Z. 1998b, A&A, 333, L13
Meeus, J. 1989, Elements of Solar Eclipses 1951 – 2200 (Richmond: Willmann-Bell Inc.)
Meeus, J. 1998, Astronomical Algorithms (Richmond: Willmann-Bell Inc.)
Morrison, L.-V., & Stephenson, F. R. 2004, Journal for the History of Astronomy, 35, 327
Muckes, H., & Meeus, J. 1983, Canon of Solar Eclipse –2003 to +2526 (Wien: Astronomisches Biüro)
Needham, J. 1959, Science and Civilisation in China, vol. 3 (Cambridge: Cambridge University Press)
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, Numerical Recipes (Cambridge: Cambridge University Press)
Purple Mountain Observatory Chinese Academy of Science 2009, Chinese Astronomical Almanac for the Year 2010 (Beijing: Science Press)
Saito, K. 1995, System of Hours in Ancient Japan, China, and Korea (Tokyo: Yuzankaku Inc.)
Sivin, N. 2009, Granting the Seasons (New York: Springer)
Standish, E. M., Newhall, X. X., Williams, J. G., & Folkner, W. M. 1997, JPL Planetary and Lunar Ephemeris (CD-ROM) (Richmond: Willmann-Bell Inc.)
U.S. Nautical Almanac Office 2009, The Astronomical Almanac for the Year 2010 and Its Companion (Washington: U.S. Government Printing Office)
Yabuuchi, K., & Nakayama, S. 2006, The Shoushi Calendar - Annotated Translation and Studies (Kawasaki: I. K. Corporation)