We discuss various aspects of parity, CP, and time reversal invariances in QCD. In particular, we focus attention on the previously proposed possibility that these experimentally established symmetries of strong interactions may be broken at finite temperature and/or density. This would have dramatic signatures in relativistic heavy ion collisions; we describe some of the most promising signals.

1. Introduction

It is commonly accepted that strong interactions do not break parity or CP. This is because in QCD, CP violation can arise only through a non-zero value of the $\theta$ angle, and the experimental limit on the neutron dipole electric moment imposes a stringent limit $\theta \lesssim 3 \times 10^{-10}$. Why $\theta$ is so close to zero in Nature is a fundamental question, known as the “Strong CP Problem”. There are various proposed solutions to this problem, the most popular being the axion hypothesis (see $\cite{2}$), but the discussion of these solutions is not the topic of this talk. Instead we shall assume that, for whatever reason, $\theta = 0$ precisely, and discuss the behavior of the theory at finite temperature and/or density.

If $\theta = 0$, a theorem by Vafa and Witten states that the vacuum is unique and conserves CP. There are circumstances under which the Vafa-Witten theorem does not apply, however. For instance, no conclusion can be drawn in the presence of a finite net density of quarks. Indeed, the existence of degenerate vacuum states with opposite parity in the superconducting phase of QCD has been discussed by Pisarski and Rischke and the (pseudoscalar) pion condensate at finite isospin density has been found by Son and Stephanov. If $\theta = \pi$, the theorem also does not apply. A priori, this corresponds to a CP conserving case because $\theta \rightarrow -\theta$, $\pi$...
and $-\pi$ are equivalent modulo $2\pi$. However, CP can still be spontaneously broken, a phenomenon discovered by Dashen before the advent of QCD. Finally, the theorem is only a statement about the ground state of the theory. In particular, it does not preclude the existence of metastable vacua within which CP is broken. In this talk we gather some of the evidence for the existence of extra vacua in hot QCD, and discuss its possible relevance to the ongoing relativistic heavy ion program.

2. $\Theta$ vacuum in gluodynamics

Consider the Euclidean Yang-Mills action with gauge group $SU(N_c)$,

$$S = \frac{1}{4g^2} \int \text{Tr} F^2 + \frac{\theta}{32\pi^2} \int d^4x \text{Tr} F \tilde{F}$$

(1)

where $\tilde{F}_{\mu\nu} = \frac{i}{2} \epsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}$. Because the integrand in the second term is a total derivative $q(x) = \frac{1}{32\pi^2} \int d^4x \text{Tr} F \tilde{F} = \partial_\mu K_\mu$, it cannot influence equations of motion on the classical level, and all $\theta$ dependence is through the quantum anomaly. In the dilute instanton gas approximation, the ground state energy is

$$E(\theta) \propto e^{-8\pi^2/g^2} \cos \theta,$$

(2)

which illustrates that $\theta$ is an angle.

While the dilute instanton gas should be an adequate concept at high temperature, when the instantons are suppressed by screening, we would like to rely on another approximation in the confined phase. Unfortunately, lattice simulations are inefficient for $\theta \neq 0$, and the only reliable analytical limit of the theory we know of is the case of large $N_c$. The large $N_c$ limit is reached by rescaling the gauge coupling $g^2 \rightarrow \lambda = g^2 N_c$, with $\lambda$ kept fixed as $N_c \rightarrow \infty$. Similarly, one must rescale $\theta$ to $\theta/N_c$. The latter implies that the low energy effective action for YM theory in the confined phase takes the form

$$L_{\text{eff}} = \frac{1}{N_c^2} L[\Phi, \partial \Phi, \ldots, \theta/N_c]$$

(3)

where $\Phi$ denotes some generic colorless (glueball) state, with mass $M_{\Phi} = \mathcal{O}(N_0^2)$. The $N_c^2$ factor comes from counting the number of degrees of freedom. In particular the vacuum energy as function of $\theta$ is

$$E[\theta] = N_c^2 F[\theta/N_c]$$

(4)

To leading order at large $N_c$,

$$E[\theta] \approx N_c^2 F[0] + \frac{1}{2} \chi \theta^2$$

(5)

where $\chi = \int d^4x \langle q(x)q(0) \rangle \sim \mathcal{O}(N_c^2)$ is the topological susceptibility. There is a little problem however: this expression is manifestly incompatible with the requirement of $E[\theta] \equiv E[\theta + 2\pi]$. The remedy has been proposed by Witten. If one takes

$$E[\theta] = N_c^2 \min_k F\left(\frac{\theta + 2k\pi}{N_c}\right)$$

(6)
the vacuum energy becomes a multivalued function, with \( k \) labelling the different branches. A similar construction is known to arise, for instance, in the Schwinger model. The expression (6) is quite remarkable: it predicts that for each fixed \( \theta \),

![Vacuum energy in gluodynamics as a function of \( \theta \). At fixed \( \theta \), there is a ground state and a tower of extra vacua.](image)

there is a tower of extra vacua (see Fig.1).

A corollary is that at \( \theta = \pi \mod 2\pi \) there are two degenerate vacua and CP is spontaneously broken. Also, there is a stable domain wall in the spectrum, very much like in the Schwinger model. If \( \theta \) departs from \( \pi \), one of the vacua becomes unstable. The decay rate of this false vacuum has been estimated to scale with the number of colors as \( \Gamma \sim \exp(-N_c^4) \) – as \( N_c \to \infty \), the metastable (lowest lying) vacua become stable.

3. Vacuum in QCD with \( N_f \) light quark flavors

The low energy phenomenological Lagrangian that incorporates both leading order quark mass and large \( N_c \) effects is

\[
L = \frac{f^2}{4} \text{tr}(\partial_\mu U^\dagger \partial^\mu U) + \Sigma \text{Re}[\text{tr}(MU^\dagger)] - \frac{\chi}{2}(\theta + i \log \det U)^2
\]

where \( M = \text{diag}(m_u, m_d, m_s) \) is the quark mass matrix, \( \Sigma = |\langle \bar{q}q \rangle| \), \( U = \exp i\phi^a \lambda^a / f_\pi \) is a \( N_f \times N_f \) unitary matrix and the \( \phi^a \) are the \( a = 1, \ldots, N_c^2 \) Goldstone modes. Specialising to the neutral modes, the potential term in (7) gives

\[
V = - \sum_{i=u,d,s} m_i \Sigma \cos \frac{\phi_i}{f_\pi} + \frac{\chi}{2}(\theta - \sum_{i=u,d,s} \phi_i / f_\pi)^2
\]

A non-vanishing topological susceptibility explicitly breaks the \( U(1)_A \) symmetry and gives a mass to the \( \eta' \) meson, \( m_{\eta'}^2 \approx \chi / f_\pi^2 \). In Nature, the value of the susceptibility in the vacuum is \( \chi \approx (180 \text{ MeV})^4 \). In one plots the potential for realistic values of the quark mass, the result is a set of almost perfect parabolic curves: this is because the \( \theta \) term dominates over the terms containing the quark
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In the large $N_c$ limit, because $\Sigma \propto N_c$, things become more interesting. To simplify the discussion, let us concentrate on the case of one light quark flavor, with pseudo-Goldstone field $\eta$. For $\theta = 0$, the potential energy at large $N_c$ is as in Fig.2. There is an absolute minimum at $\eta = 0$ and extra local minima at $\eta \approx 2\pi k$. These extra minima violate $CP$ because

$$\langle q \rangle = \frac{\partial V}{\partial \theta} \approx \chi \langle \eta \rangle \quad \text{CP} \rightarrow -\chi \langle \eta \rangle$$

(9)

A technical remark is in order here. The potential (8) is not periodic in the flavor singlet phase. On the other hand it is periodic in $\theta$. Specialising again to one light quark flavor, we see that a shift $\theta \rightarrow \theta + 2\pi$ is compensated by $\eta \rightarrow \eta - 2\pi$. Incidentally, this remark provides us with a way to derive the pure Yang-Mills potential introduced in the previous section. In the large $N_c$ limit, one can integrate out the $\eta$ field to the

$$E[\theta] \approx \frac{1}{2} \chi \min(k + \langle \eta \rangle)^2 \approx \frac{1}{2} \chi \min(k + 2\pi)^2$$

(10)

This construction is ad hoc as the $\eta$ field is essentially made of quark-antiquark pairs while the pure YM potential should be made of gluons. This is remedied if we postulate the existence of a heavy pseudoscalar “glueball” field $\Omega$ and consider instead the following modified $N_f = 1$ potential

$$V' = -\Lambda^4 \cos \Omega - m \Sigma \cos \eta + \frac{1}{2} \chi (\theta + \Omega + \eta)^2$$

(11)

This potential is now manifestly $2\pi$-periodic in $\theta$ and $\eta$ and $\Omega$. The fact that understanding the $\theta$ dependence requires the introduction of a gluonic field is perhaps not surprising - after all, the physical $\eta'$ field must contain a significant gluonic component. Similar potentials, with extra heavy degrees of freedom, have been suggested to arise in some supersymmetric YM theories, like $\mathcal{N} = 1$ SYM.

\footnote{Some interesting features arise at $\theta \sim \pi$; these include Dashen’s phenomenon and the appearance of stable domain walls, but unfortunately this is beyond the scope of this talk.}
Various arguments indicate that these degree of freedom should become very heavy in the large $N_c$ limit, $M_\Omega \propto N_c^{2/3}$. This is unlike the usual glueball states, whose mass is $O(N_c)$. If we integrate out the heavy $\Omega$ field, the effective potential becomes multivalued, pretty much like the pure Yang-Mills potential of Eq. (6). Such a potential has been constructed in [26], starting from the QCD partition function.

Fig. 3. Amended potential for $\eta$ at $\theta = 0$. The relative scales have been chosen so as to give a pleasant picture.

4. $U(1)_{A}$ symmetry at finite temperature and density

The considerations of the last section should apply in the limit of very large number of colors, $N_c \to \infty$. Inversely, for fixed $N_c$, the same effects would arise if the topological susceptibility $\chi$ drops with respect to its vacuum expectation value, $\chi \sim (180 \text{ MeV})^2$ or, in other words, if there is an approximate restoration of $U(1)_{A}$ symmetry. This could arise at finite temperature and/or density. Let us consider again the limit of large number of colors. At very high temperature, because of Debye screening in the deconfined phase of QCD, the instanton calculus is reliable. Contributions to the topological susceptibility are then exponentially small $\chi \sim \exp(-N_c/\lambda) \sim 0$. On the other hand, in the confined phase, $\chi \sim O(1)$. The transition between these two behaviors is not well understood. Toy model calculations in $D = 1 + 1$ show that the instanton picture can be extended all the way down to the critical temperature, but not beyond $\Theta = 0$. In $D = 3 + 1$, lattice simulations of the topological susceptibility show a similar behavior: the topological susceptibility is essentially constant at low temperature, and then drops sharply near $T_c$. Whether this effect permits the appearance of metastable vacua is hard to
answer. If we nevertheless naively extrapolate the range of validity of the effective potential (7) to higher temperature, we see that $\chi$ must drop dramatically because the existence of the extra vacua is controlled by the mass of the lightest quarks, $\chi(T_c)/\chi(0) \sim 5\%$. While the current lattice simulations do not seem to see such a large suppression, they are not far from it. Also, things could improve at finite $T$ and $\mu$, where $\mu$ is the quark chemical potential. The argument is as follows.

If the $U(1)_A$ symmetry is effectively restored at $T_c$, then universality arguments suggest that the chiral phase transition should be first order, driven by fluctuations. Consider now the phase diagram for QCD with two light quark flavors in the plane of temperature and chemical potential, $T-\mu$. Even though there is no phase transition along the $\mu = 0$ axis, various arguments suggest that there is a line of first order phase transitions, terminated by a critical point $(T_*,\mu_*)$. The position of the point is uncertain, since it is influenced by various effects. For instance if the mass of the strange quark were below some critical value $m_*$, the critical point would move toward the temperature axis and the chiral symmetry breaking phase transition would be first order everywhere. A similar conclusion would be reached if the $U(1)_A$ symmetry was restored. This could be controlled by changing the number of colors, for instance. The fact that $N_c$ is a discrete variable is not really an issue here. What we want to emphasize is that, perhaps, $N_c = 3$ is close to a critical value, $N^*_c$, at which the $U(1)_A$ symmetry is restored, which would “explain” why lattice simulations show a sharp drop near $T_c$ even though $N_c$ is only $N_c = 3$. By the same token, the restoration of $U(1)_A$ could become more pronounced closer to the critical point.

These arguments suggest that CP-odd false vacua might exist in hot QCD. While our assumptions might be wrong, they aren’t completely crazy, at least to us. If anything, we have exhibited an explicit mechanism through which CP and P-odd effects could exist in QCD, despite the fact that $\theta \approx 0$ to a very high precision in Nature. Whether these effects states could have observable consequences is another question. Metastable vacua such as those discussed here could be formed in the early universe, near the QCD phase transition, or in heavy ion collisions. The latter possibility will be the topic of the last section. Of course, the outcome will depend very much on the probability of formation of these metastable states, of their size and number density and, finally, on their lifetime. Once they are allowed to exist, a metastable region is easily formed assuming that the phase of the singlet Goldstone mode, i.e. $\theta_{eff}$, is initially random, as the root mean square of $\theta_{eff}$ is quite large, $\langle \theta_{eff} \rangle_{rms} = 2\pi/\sqrt{3}$. The size $l_B$ of a CP-odd bubble is harder to estimate given the uncertainties of our model, but it could be as large as a few fermi. Finally, the lifetime will depend of the relative size of the false and true vacua region, the difference in pressure between these regions (which could be rather small if $U(1)_A$ is very nearly restored), and other factors, like the viscosity of the medium. All

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\(^{c}\text{This is one of the manifestations of the fact there can be no CP violation if any of the quark is massless.}\)

\(^{d}\text{Ideally one would prefer a dynamical explanation, such as the one provided by the instanton liquid picture.}\)
together, there are many uncertainties. The most favorable case is obviously if very large bubbles, of about the size of the hot region, can be formed. On the opposite, if many tiny CP-odd regions form and quickly collapse, all CP violating effect would average to zero. Whether this or the former (or none of the above) is realized is a question which, at this stage, can only answered by the experiments.

5. Parity odd bubbles in heavy ion collisions: the observables

Hot QCD matter can be produced and studied in ongoing experiments with relativistic heavy ion collisions. It is therefore important to investigate the possible signatures associated with the excitation of metastable states discussed above. From [8], [11] it is clear that these states will act like regions of nonzero $\theta$. Parity, CP, and time invariance will be violated spontaneously in such a region.

The first signature of such states stems from the observation that when the anomaly term $a$ becomes small, there is maximal violation of isospin [39], [40], [41], [42]. At zero temperature, the nonet of pseudo-Goldstone bosons — the $\pi$’s, $K$’s, $\eta$, and $\eta'$, are, to a good approximation, eigenstates of $SU(3)$ flavor. It is not often appreciated, but this is really due to the fact that the anomaly term is large, splitting off the $\eta'$ to be entirely an $SU(3)$ singlet. When the anomaly term becomes small, however, while the charged pseudo-Goldstone bosons remain approximate eigenstates of flavor, the neutral ones do not. Without the anomaly, the $\pi^0$ becomes pure $uu$, the $\eta$ pure $\bar{d}d$, and the $\eta'$ pure $\bar{s}s$. Consequently, these three mesons become light. This is especially pronounced for the $\eta$, as it sheds all of its strangeness, to become purely $\bar{d}d$. Thus the $\eta$ and $\eta'$ would be produced copiously, and would manifest itself in at least two ways. First, light $\eta$’s and $\eta'$’s decay into two photons, and so produce an excess at low momentum. Secondly, these mesons decay into pions, which would be seen in Bose-Einstein correlations [43]. Further, through Dalitz decays, the enhanced production of $\eta$’s and $\eta'$’s will enhance the yield of low mass dileptons [42].

Recently, a detailed study of the $\eta$ and $\eta'$ enhancement was carried out by Ahrensmeier, Baier and Dirks [44], who considered the dynamical evolution of these fields in the decay of the $P$-odd bubbles. Their result is a dramatic enhancement in the production of flavor-singlet mesons, leading to about a hundred particles in a single event. Similar studies, were recently presented by Buckley, Fugleberg, and Zhitnitsky [45].

The maximal violation of isospin is true whenever the anomaly term becomes small. There are other signals which only appear when parity odd bubbles are produced. Since parity is spontaneously violated in such a bubble, various decays, not allowed in the parity symmetric vacuum, are possible. Most notably, the $\eta$ can decay not just to three pions, as at zero temperature, but to two pions. Because of the kinematics, in a parity odd bubble, $\eta \to \pi^0\pi^0$ is allowed, but $\eta \to \pi^+\pi^-$ is not. Unfortunately, the search for the parity violating decays will be made difficult by the fact that the masses of the $\eta$ and $\eta'$ in the bubble are different from their canonical values, and depend on the temperature of the system.

There is another measure of how parity may be violated [4], which we will now
discuss following the paper. We first argue by analogy. Consider propagation in
a background magnetic field. As charged particles propagate in the magnetic field,
those with positive charge are bent one way, and those with negative charge, the
other. This could be observed by measuring the following variable globally, on an
event–by–event basis:
\[
J = \sum_{\pi^+ \pi^-} \frac{(\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}) \cdot \hat{z}}{|\vec{p}_{\pi^+}| |\vec{p}_{\pi^-}|};
\] (12)
here \(\hat{z}\) is the beam axis, and \(\vec{p}\) are the three momenta of the pions.

If the quarks were propagating through a background chromo-magnetic field,
then \(J\), which is like handedness in jet physics, is precisely the right quantity.
However, a parity odd bubble is not directly analogous to a background chromo-
magnetic field: \(\pi^+\)'s and \(\pi^-\)'s propagate in a region with constant but nonzero \(\phi\)
in the same fashion. Consider, however, the edge of the parity odd bubble: in such a
region, \(U^\dagger \partial_\mu U\) is nonzero, and does rotate \(\pi^+\) and \(\pi^-\) in opposite directions. Thus
it is the edges of parity odd bubbles which contribute to the parity odd asymmetry
of (12). Purely on geometric grounds, this suggests that a reasonable estimate for
the maximal value of \(P\) is on the order of a few percent.

Let us now construct global observables which are odd under the discrete sym-
metries of \(P, C,\) and/or \(CP\) using only the momenta of charged pions in an event.
Our discussion will be general, independent of whatever detailed mechanism might
produce nonzero values for these variables. For the collisions of nuclei with equal
atomic number, as the initial state is even under \(P\), the observation of a \(P\)-odd final
state must be due to parity violation, such as by a \(P\)-odd bubble. Based upon our
specific model, we will then give a rough estimate of the magnitude of the \(P\)-odd
and \(CP\)-odd effects; we find that the asymmetries can be relatively large, at least
\(\sim 10^{-3}\).

At high energy, nucleus-nucleus collisions produce many pions, on the order of
\(\sim 1000\) per unit rapidity at RHIC energies. Experimentally, it is probably easiest to
detect charged pions and their three-momenta. (All of our comments apply equally
well to charged kaons.) Thus we are led to consider constructing observables only
from the three-momenta of \(\pi^+\)'s, \(\vec{p}_+\), and \(\pi^-\)'s, \(\vec{p}_-\). As vectors, under parity the
three-momenta transform as
\[
P : \quad \vec{p}_+ \rightarrow -\vec{p}_+, \quad \vec{p}_- \rightarrow -\vec{p}_-.
\] (13)
Charge conjugation switches \(\pi^+\) and \(\pi^-\),
\[
C : \quad \vec{p}_+ \leftrightarrow \vec{p}_-.
\] (14)
Lastly, time reversal, \(T\), acts like parity on the pion momenta, switching their sign:
\(\vec{p}_+ \rightarrow -\vec{p}_+\) and \(\vec{p}_- \rightarrow -\vec{p}_-\) under \(T\).

It is a theorem that any \(P\)-odd invariant formed from three-vectors can be
represented as a sum of terms, each of which involves one antisymmetric epsilon
tensor. The variable which we considered previously (12) is of this type. In order
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to form $J$ we have introduced an arbitrary, fixed vector of unit norm, $\hat{z}$. If $\hat{z} \to -\hat{z}$ under parity, then $J$ is odd under $P$. In $J$ we use the unit vectors $\hat{p}_\pm = \hat{p}_\pm / |\hat{p}_\pm|$ so that it is a pure, dimensionless number. The variable $J$ is separately odd under $P$ and $C$, and so is even under $CP$. It is even under $T$ and $CPT$.

The variable $J$ is closely analogous to “handedness”, originally introduced to study spin dependent effects in jet fragmentation. There the axis $\hat{z}$ is usually defined by the thrust of the jet, with $\hat{p}_+$ and $\hat{p}_-$ representing the directions of pions formed in the fragmentation of the jet. Correlations between the handedness of different jets produced in a given event are sensitive to $CP$-violating effects.

It is not difficult to construct other invariants with different transformation properties. We introduce $\vec{k}_\pm$ as

$$\vec{k}_\pm = \sum_{\pi^\pm} \vec{p}_\pm \pm \sum_{\pi^\mp} \vec{p}_\mp , \quad \hat{k}_\pm = \vec{k}_\pm / |k_\pm| ,$$

and then form

$$K_\pm = \sum_{\pi^+,\pi^-} (\hat{p}_+ \times \hat{p}_-) \cdot \hat{k}_\pm .$$

The variables $K_\pm$ are $P$-odd; $K_+$ is $C$-odd, and so $CP$-even, while $K_-$ is $C$-even, and so $CP$-odd. Both $K_\pm$ are $T$-odd, so that $K_+$ is $CPT$-odd, and $K_-$ is $CPT$-even. The vector $\vec{k}_+$ measures the net flow of the charged pion momentum, while $\vec{k}_-$ measures the net flow of charge from pions.

We can also form

$$L = \sum_{\pi^+,\pi^-} (\hat{p}_+ \times \hat{p}_-) \cdot \hat{z} \left( \frac{p_+^2 - p_\mp^2}{p_+^2 + p_\mp^2} \right) .$$

The variable $L$ is $P$-odd, $C$-even, and $T$-even, so it is odd under $CP$ and $CPT$. It does not require a net flow of momentum or charge to be nonzero, although as for $J$, we do need to introduce an arbitrary unit vector $\hat{z}$.

Similar to $L$, we can form

$$M = \sum_{\pi^+,\pi^-} \left( \frac{p_+^2 - p_\mp^2}{p_+^2 + p_\mp^2} \right) .$$

This variable is $P$-even, $C$-odd, $T$-even, and so odd under $CP$ and $CPT$.

Besides using the vectors $\vec{k}_\pm$, another way of avoiding the introduction of an arbitrary unit vector $\hat{z}$ is to use ordered pairs of pion momenta; this procedure is also used in the studies of spin-dependent effects in jet fragmentation. For any given pair of like sign pions, let $\vec{p}_\pm'$ denote the $\pi^+$ with largest momentum, so $|\vec{p}_\pm'| > |\vec{p}_\pm|$. This ordering is done in the frame in which the observable is measured. Then we can form a triple product as

$$T_\pm = \sum_{\pi^+,\pi^-} (\hat{p}_+ \times \hat{p}_-) \cdot (\hat{p}_+ \pm \hat{p}_-') , \quad |\vec{p}_\pm'| > |\vec{p}_\pm|$$
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The variables $T_\pm$ are odd under $P$ and $T$; $T_+$ is $C$-odd, $CP$-even, and $CPT$-odd, while $T_-$ is $C$-even, $CP$-odd, and $CPT$-even. Besides the variables $T_\pm$, one can clearly construct other $P$- and $C$-odd observables from triplets, or higher numbers, of pions.

On general grounds, any scalar observable should be invariant under the combined operation of $CPT$. Consequently, the $CP$-odd variables $J$, $K_-$, and $T_-$ are allowed under $CPT$, while $K_+$, $L$, $M$, and $T_+$ must vanish.

We have previously derived the metastable $P$-odd bubbles within the context of a nonlinear sigma model, with a $U(3)$ matrix $U$, $U^\dagger U = 1$. The metastable vacua are stationary points with respect to the nonlinear sigma model action, including the terms with two derivatives, a mass term, and an anomaly term (8). In terms of the underlying gluonic fields, the $P$-odd bubbles arise from fluctuations in the topological charge density, $G_{\mu\nu}\tilde{G}^{\mu\nu}$. It is easy to understand how a region in which $G_{\mu\nu}\tilde{G}^{\mu\nu} \neq 0$ can produce a $P$-odd effect. Consider the propagation of a quark anti-quark pair through a region in which $G_{\mu\nu}\tilde{G}^{\mu\nu} \neq 0$; in terms of the color electric, $\vec{E}$, and color magnetic, $\vec{B}$, fields, $G_{\mu\nu}\tilde{G}^{\mu\nu} \sim \vec{E} \cdot \vec{B}$. If $\vec{E}$ and $\vec{B}$ both lie along the $\hat{z}$ direction, then a quark is bent one way, the anti-quark the other, so that $(\vec{p}_q \times \vec{p}_{\bar{q}}) \cdot \hat{z} \neq 0$, where $\vec{p}_q$ and $\vec{p}_{\bar{q}}$ are the three-momenta of the quark and anti-quark, respectively. While physically intuitive, this picture does not allow us to directly relate this bending in the momenta of the quark and anti-quark to an asymmetry for charged pions.

To do so, we again resort to using an effective lagrangian. It is known that the effects of the axial anomaly show up in the effective lagrangians of Goldstone bosons in two, and only two, ways. One is through the anomaly term, $\sim a$, which we have already included. Besides that, however, the axial anomaly also manifests itself in the interactions of Goldstone bosons through the Wess-Zumino-Witten term $\frac{2}{5\pi^2} \int d^4x \varepsilon^{\alpha\beta\gamma\delta} tr (u \partial_\alpha u \partial_\beta u \partial_\gamma u \partial_\delta u)$. This term is nonzero only when the fields are time dependent, which is why we could ignore it in discussing the static properties of $P$-odd bubbles. It cannot be ignored, however, in computing the dynamical properties, and in particular the decay, of $P$-odd bubbles. The Wess-Zumino-Witten term is manifestly chirally symmetric when written as an integral over five dimensions,

$$S_{wzw} = -i \frac{1}{80\pi^2} \int d^5x \varepsilon^{\alpha\beta\gamma\delta\sigma} tr (R_\alpha R_\beta R_\gamma R_\delta) ,$$

where $R_\alpha = U^\dagger \partial_\alpha U$, but reduces to a boundary term in four space-time dimensions. For $U = \exp(iu)$, when $\partial_\alpha u \ll 1$,

$$S_{wzw} \approx \frac{2}{5\pi^2} \int d^4x \varepsilon^{\alpha\beta\gamma\delta} tr (u \partial_\alpha u \partial_\beta u \partial_\gamma u \partial_\delta u) .$$

As discussed by Witten, the coefficient of the Wess-Zumino-Witten term is fixed by topological considerations, and is proportional to the number of colors, which equals three.

In a collision, we envision that the trivial vacuum heats up, a $P$-odd bubble forms, and then decays as the vacuum cools. Since this represents bubble formation...
and decay, there is no net change in any topological number. Therefore, it is possible for a given event to contain an excess of bubbles over anti-bubbles (or vice versa), and thus to manifest true parity violation on an event by event basis.

To estimate the magnitude of the Wess-Zumino-Witten term for a $\mathcal{P}$-odd bubble, and to understand its effect on pion production, in (21) we can take three $u$’s to be condensate fields, $u \sim \phi_{u,d,s}$, and two to be charged pion fields, $u \sim \pi_{\pm}/f_\pi$. Suppose that the $\mathcal{P}$-odd bubble is of size $R$, with unit normal $\hat{r}$ to the surface, and lasts for some period of time. Because of the antisymmetric tensor in (21), all three components of the condensate field must enter. Schematically, we obtain

$$S_{wzw} \approx \frac{2}{5\pi^2} \int dt \int d^3 r \phi_u \partial_t \phi_d \partial_\phi_s (\vec{p}_\pi^+ \times \vec{p}_\pi^-) \cdot \hat{r}$$

(22)

The time integral is $\int dt \partial_\phi_s \sim \delta \phi_s = \phi_s$, since $\phi_s = 0$ in the normal vacuum. Similarly, the spatial integral is $\int d^3 r \partial_r \phi_d (\vec{p}_\pi^+ \times \vec{p}_\pi^-) \cdot \hat{r} \sim \int d\Omega \int R^2 \phi_d (\vec{p}_\pi^+ \times \vec{p}_\pi^-) \cdot \hat{r}$, where $d\Omega$ represents an integral over the direction of the normal, $\hat{r}$. Further, as the average momentum within the condensate is of order $p_\pi \sim 1/R$, the size of the bubble drops out as well. We thus obtain a final result which is independent of the size of the bubble, its lifetime, and its width:

$$S_{wzw} \approx \frac{2\phi_u \phi_d \phi_s}{5\pi^2} \int d\Omega (\hat{p}_\pi^+ \times \hat{p}_\pi^-) \cdot \hat{r}$$

(23)

Since the Wess-Zumino-Witten term vanishes for a static field, an asymmetry is only obtained from the decay of a $\mathcal{P}$-odd bubble. In addition, scattering of pions off the $\mathcal{P}$-odd bubble will also produce an asymmetry, although we do not include this at present. Lastly, note that there is only an observable asymmetry when $\phi_s \neq 0$; this is because in the absence of external gauge fields, there is only a Wess-Zumino-Witten term for three, and not for two, flavors. Within this model, $S_{wzw}$ is of similar form for two charged kaons.

Using our previous estimates for the $\phi$’s, $\phi_u \sim \phi_d \sim 1$ and $\phi_s \sim 10^{-2}$, we obtain an effect of order $\sim 10^{-3}$. At the point where the $\mathcal{P}$-odd bubble first appears, $(a/c)_{cr}$, one can estimate that the energy density within the bubble, relative to the ordinary vacuum, is $\sim 25 n^2 \text{MeV}/\text{fm}^3$, where $n$ is the winding number of the bubble, $n = 1, 2, 3...$ For a bubble $\sim 5\text{fm}$ in radius, there are about $\sim 100 n^2$ pions produced in the decay of the bubble. If a fraction of the produced pions are observed within a given kinematical window, and we assume that all observed pions come from a portion of the total bubble, then we recover the variable $J$, introduced before in (12), and find an estimate of $J \sim 10^{-3}$. Moreover, we find a natural interpretation of the direction $\hat{z}$, which was needed to define $J$, as the normal to the bubble’s surface. One might wonder if the effect is diluted by the necessity to average over uncorrelated pairs. This does not happen, however, because the pion field within the bubble is a classical field, so that all charged pions are affected similarly.

Naively, one might expect that $J$ would average to zero over a single bubble. As the bubble is topological, though, the direction in which charged pions are swept
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is correlated with the sign of the condensate, so that a single $\mathcal{P}$-odd bubble can produce an effect in $J \sim 10^{-3}$. Thus it is possible to distinguish between events in which bubbles are produced, and those in which bubbles are not, by measuring $J$.

At first it may seem surprising that our $\mathcal{P}$-odd, $\mathcal{C}$-even, and $\mathcal{CP}$-odd bubble produces a signal in $J$, which by previous analysis is $\mathcal{P}$-odd, $\mathcal{C}$-odd, and $\mathcal{CP}$-even. The Wess-Zumino-Witten term is even under parity, which is $\mathcal{P}_0(-1)^{N_B}$, where $\mathcal{P}_0$ is the operation of space reflection, and $N_B$ counts the number of Goldstone bosons $\phi$. By scattering off a $\mathcal{P}$-odd bubble, we bring in an odd number of condensate fields, $J \sim \phi_u\phi_d\phi_s$, and so change the (apparent) quantum numbers to be $\mathcal{P}$-odd and $\mathcal{CP}$-even. This is only apparent, as scattering off an anti-bubble will give the opposite sign of $J$.

We expect that bubbles will generate signals for the other variables presented of similar magnitude. For example, a single bubble will induce a net flow of pion charge, and so contribute to $K_- \sim 10^{-3}$, (10b). Through coherent scattering in a bubble, we would also expect the variables $K_+$, $L$, and $T_\pm$ to develop signals $\sim 10^{-3}$. Further, hot gauge theories can also exhibit metastable states which are $\mathcal{P}$-even and $\mathcal{C}$-odd; these generate signals for $M$.

The idea of exciting metastable vacua in hadronic reactions is an old one, as is the idea that a collective pion field can produce large fluctuations in heavy ion collisions on an event by event basis. We wish to emphasize that the $\theta$ dependence in QCD, and associated with it complicated vacuum structure, may lead to the possibility of exciting the metastable vacuum states with broken discrete symmetries of $\mathcal{P}$, $\mathcal{CP}$, and $\mathcal{T}$ in experiments with relativistic heavy ions. The search for these phenomena at RHIC is underway, and we are anxiously awaiting the experimental verdict.

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