Thermal gluo-magnetic vacuum of $SU(N)$ gauge theory

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Abstract

The magnetic sector of $SU(N)$ Yang-Mills theory at finite temperature is studied. At low temperatures, $T < 2T_c$, the analytic expressions for the temperature dependence of the magnetic correlator, of the magnetic gluon condensate and of the spatial string tension are obtained. Fair agreement with lattice calculations for spatial string tension is obtained for $SU(2)$ and $SU(3)$ gauge theories. The relative contribution given by non-zero Matsubara modes to the spatial string tension is calculated. At $T = 2T_c$ this contribution is of the order of 5%. The behavior of magnetic correlator at high temperatures is investigated and it is shown that gluo-magnetic condensate increases with temperature as $\langle H^2 \rangle(T) = \text{const} \cdot g^8(T)T^4$.

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1 Introduction

It is known that at finite temperature non-Abelian $SU(N)$ gauge theories undergo a deconfining phase transition passing from the low-energy glueball phase to the phase of hot gluon matter. At the critical point $T_c$ where the phase transition occurs the behavior of the thermodynamic properties of the system, such as energy density $\varepsilon$, specific heat, non-ideality $(\varepsilon - 3P)/T^4$, etc., is drastically changed [1,2,3]. More than that, the phase transition in non-Abelian gauge theories is characterized by the radical rearrangement of the non-perturbative (NP) gluon vacuum.

Recently the behavior of the gauge-invariant two-point correlation functions of the gauge fields across the deconfinement phase transition was investigated by numerical simulations on a lattice both for the pure-gauge $SU(3)$ theory and for the full QCD with two flavors [4]. These data clearly demonstrate strong suppression of the electric
component of the correlator above $T_c$ and persistence of the magnetic component. The magnetic correlator and its contribution to the gluon condensate is kept intact across the phase transition temperature, while the confining electric part abruptly disappears above $T_c$, so that the electric gluon condensate drops to zero at the deconfining phase transition point \cite{4}. These results are completely in line with theoretical predictions of deconfining phase transition within the "evaporation model" \cite{5} approach. Within the framework of the effective dilaton Lagrangian at finite temperature the discontinuity of the gluon condensate at $T = T_c$ was studied in \cite{6} (see also \cite{7}).

At $T = T_c$ the physical string tension becomes zero. It was understood however that in non-Abelian gauge theories the Wilson loop for large space-like contours obeys the area law at arbitrary temperature. This phenomenon is known as "magnetic confinement" and yields non-zero spatial string tension $\sigma_s(T)$ \cite{8}. The temperature dependence of $\sigma_s(T)$ was studied on a lattice for the pure-gauge SU(2) \cite{9} and SU(3) \cite{9,10} theories. It was shown there that $\sigma_s(T)$ smoothly passes through the phase transition temperature and increases with temperature. For temperatures larger than $2T_c$ the scaling behavior, $\sqrt{\sigma_s(T)/g^2T} \sim \text{const}$, settles on \cite{9,10}. This regime is imposed by the non-perturbative magnetic scale $\sim g^2T$ \cite{11}. It is also firmly established that the 4d SU(N) gauge theory at finite temperature is well described by an effective 3d gauge theory with an adjoint Higgs field (the so-called dimensional reduction) \cite{12,13}. Moreover, the scalar Higgs fields only weakly influence the physical properties of the gauge theory magnetic sector \cite{9,10,14,15,16}. Thus the behavior of certain gluo-magnetic quantities in 3d are of direct relevance for the behavior of these quantities in the 4d gauge theory at high temperature. In particular, the spatial string tension $\sigma_s(T)$ at high T coincides with good accuracy with lattice results for the 3d string tension $\sigma_3$ \cite{10,17}.

In the present paper we study the magnetic sector of the 4d SU(N) Yang-Mills theory at finite temperature. It is demonstrated that thermal behavior of magnetic properties of the system is qualitatively different in two temperature regions. We will define $T < 2T_c$ as low temperature region, and $T \geq 2T_c$ as high temperature region\textsuperscript{1}. The temperature behavior of the gluo-magnetic correlator, of the magnetic condensate and of the spatial string tension in low temperature region ($T < 2T_c$) are obtained analytically. The relative contribution of non-zero Matsubara modes to $\sigma_s(T)$ is calculated and found to be of order of 5\% for pure SU(3) gauge theory at $T = 2T_c$. The temperature interval corresponding to the onset of the 3d regime of the Yang-Mills theory is investigated. We consider the thermal character of the gluo-magnetic correlator in the scaling region for the spatial string tension, $\sqrt{\sigma_s} \sim g^2T$. It is shown that the non-perturbative gluo-magnetic condensate grows with temperature as $\sim g^8(T)T^4$ at high $T$.

2 Gluo-magnetic correlator and spatial string tension

Gauge-invariant non-local field correlators are of utmost importance for our understanding of the NP QCD dynamics (see a recent review \cite{18} and references therein). The gauge-invariant two-point correlators in the Yang-Mills vacuum are defined as \cite{19,20,21}

$$D_{\mu\nu\sigma\lambda}(x) = \langle g^2\text{tr}G_{\mu\nu}(x)\Phi(x,0)G_{\sigma\lambda}(0)\Phi(0,x)\rangle,$$

\textsuperscript{1}\textsuperscript{1}2\% should not be considered as an exact number. This matter is discussed in the last section.
where $G_{\mu\nu} = t^a G^a_{\mu\nu}$, is the field-strength tensor, $t^a$ are the SU(N) gauge group generators in the fundamental representation, $\text{tr} t^a t^b = \frac{1}{2} \delta^{ab}$, and

$$\Phi(x, y) = P \exp \{ig \int_x^y dx_\mu A_\mu(x)\}, \quad A_\mu = t^a A^a_\mu$$

is the Schwinger parallel transporter with the integration from $x$ to $y$ along a straight-line path. These phase factors $\Phi$ are introduced to take care of the gauge-invariance of the correlators. In general form bilocal correlators (1) are expressed in terms of two independent invariant functions of $x^2$, $D(x^2)$ and $D_1(x^2)$ [19, 20, 21].

At finite temperature the O(4) space-time symmetry is broken down to the spatial O(3) symmetry and bilocal correlators are described by independent electric and magnetic correlation functions. On general grounds one can write for the magnetic correlators the following expression

$$D^H_{ik}(x) = \langle g^2 \text{tr} H_i(x) \Phi(x, 0) H_k(0) \Phi(0, x) \rangle$$

$$= \delta_{ik} (D^H + D^H_1) + (\delta_{ik} - \frac{x_i x_k}{x^2}) x^2 \frac{\partial D^H_1}{\partial x^2},$$

where $H_i = \frac{1}{2} \varepsilon_{ijk} G_{jk}$ are the magnetic field operators. In a well known way [20] one can obtain the area law for large spatial $W$-loops of size $L \gg \xi_m$, where $\xi_m$ is the magnetic correlation length

$$\langle W(C) \rangle_{\text{spatial}} \sim \exp \{-\sigma_s S_{\text{min}}\},$$

where the spatial string tension $\sigma_s$ is

$$\sigma_s = \frac{1}{2} \int d^2x \langle g^2 \text{tr} H_n(x) \Phi(x, 0) H_n(0) \Phi(0, x) \rangle$$

and $H_n$ is the component orthogonal to the space-like plane of the contour.

Substituting (3) into (5) and keeping in mind that the terms with $D^H_1$ form the complete derivative, one can present $\sigma_s$ in the following form [22]

$$\sigma_s = \frac{1}{2} \int d^2x D^H(x).$$

Note that only the term $D^H$ enters in the expression (6) for $\sigma_s$. At $T = 0$ due to the $O(4)$ invariance electric and magnetic correlators coincide and $\sigma_s = \sigma$, where $\sigma = (420\text{MeV})^2$ is a physical (temporal) string tension.

Note that magnetic correlator $D^H_{ik}$ can be expressed through matrix $U_{\text{adj}}^{ab}$ in the adjoint representation. Since integration in (2) goes along straight-line path, connecting points $x$ and $y$, one has

$$D^H_{ik}(x, y) = \langle g^2 H_i^a(x) U_{\text{adj}}^{ab}(x, y) H_k^b(y) \rangle,$$

$$U_{\text{adj}}^{ab}(x, y) = 2\text{tr} \left( t^a \Phi(x, y) t^b \Phi(y, x) \right).$$

The functions $D^H$ and $D^H_1$ contain both perturbative ($\propto 1/x^4$) and nonperturbative ($\propto \exp \{-|x|/\xi_m\}$) terms and only non-perturbative parts of the correlators contribute to
the string tension (for the detailed discussion of these questions see the review paper [18]). The magnetic gluon condensate can be also determined through the NP contributions to the correlator and expressed in terms of the functions \( D_H \) and \( D_H^1 \) at \( x^2 = 0 \). The gluo-magnetic condensate can be written as

\[
D_{ii}^H(0) = \langle g^2 \text{tr} H_i^2 \rangle \equiv \frac{1}{2} \langle H^2 \rangle = 3(D_H^0(0) + D_H^1(0)),
\]

where \( \langle H^2 \rangle \equiv \langle (gH^a_i)^2 \rangle \).

The lattice data [4] demonstrate that at least in the region of "measured" temperatures, \( T \lesssim 1.5 T_c \), the NP functions \( D_H = B_0 \exp\{-|x|/\xi_m\} \) and \( D_H^1 = B_1 \exp\{-|x|/\xi_m\} \) are almost independent of temperature. Correlation length does not change and equals to it's value at \( T = 0 \), and \( B_1 \approx 0.05B_0 \ll B_0 \). Only \( B_0 \) slightly grows with \( T \), which leads to the slow growth of condensate \( \langle H^2 \rangle \) at low temperatures.

Let us see how one can obtain the analytical expressions for the magnetic correlator and correspondingly for the spatial string tension and magnetic gluon condensate as functions of the temperature. It is well known that the introduction of the temperature for the quantum field system in thermodynamic equilibrium is equivalent to compactification along the euclidean "time" component \( x_4 \) with the radius \( \beta = 1/T \) and imposing the periodic boundary conditions (PBC) for boson fields (anti-periodic for fermion ones).

Thermal vacuum averages are defined in a standard way

\[
\langle \ldots \rangle_\beta = \frac{1}{Z_\beta} \int_{\text{PBC}} [DA] \ldots e^{-S_\beta[A]}, \tag{9}
\]

where partition function is

\[
Z_\beta = \int_{\text{PBC}} [DA] e^{-S_\beta[A]}, \quad S_\beta = \int_0^\beta dx_4 \int d^3x\mathcal{L}_{YM}.
\]

As mentioned above, in the low temperature region gluelump mass \( M \) (inverse magnetic correlation length \( 1/\xi_m \)) does not depend on \( T \). Thus we will use zero-temperature expression for \( D_{ik}^H(x, x_4) \) with PBC to obtain thermal correlator \( \mathcal{D}_{ik}^H = \langle g^2 H_i^a U_{\text{adj}}^{ab} H_k^b \rangle_\beta \) built from gauge fields \( H_i(x, x_4) \) and phase factor \( U_{\text{adj}}[A(x, x_4)] \) at a particular "time" coordinate \( x_4 \)

\[
\mathcal{D}_{ik}^H(x, \beta) = \sum_{n=-\infty}^{+\infty} D_{ik}^H(x, n\beta). \tag{11}
\]

Usually, this approximation is valid in the case of free noninteracting fields. For example, thermal Green function of free scalar field is defined as sum over Matsubara modes of propagator at \( T = 0 \). At the same time, interaction changes particle mass and makes it dependent on \( T \). In our case in the region of low \( T \) we take into account only "kinematic" change of correlator, and consider it as zero-temperature correlator with PBC. Physically the expression (11) is valid at \( \beta > 2\xi_m \), i.e. when the two neighboring exponential functions do not overlap. We use the value \( 1/\xi_m = 1.5 \text{ GeV} \) and it is thus required that \( T < 1/2\xi_m = 750 \text{ MeV} \). In the temperature region we consider in the present section, \( T < 2T_c \sim 600 \text{ MeV} \), this condition is fulfilled. Moreover comparison of
calculated below physical quantities with lattice results confirm validity of expression (11) at \( T < 2T_c \).

It should be noted that using Poisson’s summation formula

\[
\frac{1}{\beta} \sum_{n=-\infty}^{+\infty} e^{i\omega_n x} = \sum_{n=-\infty}^{+\infty} \delta(x - n\beta), \quad \omega_n = 2\pi n / \beta
\]  

(12)

one finds, that (11) is a representation of magnetic correlator \( D \) as a Fourier sum over Matsubara frequencies

\[
\sum_n D^H_{ik}(x, n\beta) = \frac{1}{\beta} \sum_n \tilde{D}^H_{ik}(x, \omega_n),
\]  

(13)

where \( \tilde{D}(\omega) = \int d\tau \exp(i\omega\tau)D(\tau) \) is a Fourier image of function \( D \).

Thus consider the function

\[
f(x, \beta) = \sum_{n=-\infty}^{+\infty} e^{-M\sqrt{x^2 + n^2\beta^2}}, \quad M \equiv 1/\xi_m.
\]  

(14)

In order to carry out the Matsubara summation over the frequencies \( \omega_n = 2\pi n T \) in (14) use can be made of the integral representation of the exponent in (14). This yields

\[
f(x, \beta) = \frac{1}{\sqrt{\pi}} \int_0^{+\infty} ds e^{-s-M^2x^2/4s} \sum_{n=-\infty}^{+\infty} e^{-M^2\beta^2n^2/4s}.
\]  

(15)

The following summation equation holds

\[
\sum_{n=-\infty}^{+\infty} e^{-s\frac{x^2}{4s}} = \frac{2\sqrt{\pi}s}{b} \sum_{n=-\infty}^{+\infty} e^{-\frac{bx^2}{4s}}.
\]  

(16)

It enables to write (15) in the form

\[
f(x, \beta) = \frac{2}{M\beta} \sum_{n=-\infty}^{+\infty} \int_0^{+\infty} ds e^{-as-b/s},
\]  

(17)

where \( a = 1 + 4\pi^2n^2/M^2\beta^2 \) and \( b = M^2x^2/4 \).

The integral in the right-hand side of (17) is expressed in terms of the Macdonald function \( K_1 \). Finally we get

\[
f(x, T) = 2MT|x| \sum_{n=-\infty}^{+\infty} \frac{1}{\sqrt{M^2 + \omega_n^2}} K_1(|x|\sqrt{M^2 + \omega_n^2}).
\]  

(18)

From the asymptotic behavior of function \( f \) at short distances we can obtain the temperature dependence of the magnetic gluon condensate, \( \langle H^2 \rangle(T) = \langle H^2 \rangle f(|x| \to 0, T) \).

Using the behavior \( K_1(x \to 0) \to 1/x \) one has

\[
\frac{\langle H^2 \rangle(T)}{\langle H^2 \rangle} = 2MT \sum_{n=-\infty}^{+\infty} \frac{1}{M^2 + \omega_n^2} = \coth \left( \frac{M}{2T} \right)
\]  

(19)
and at $T \ll M$

$$\langle H^2 \rangle(T)/\langle H^2 \rangle = 1 + 2e^{-M/T} + O(e^{-2M/T}). \quad (20)$$

This is completely in line with the lattice result [4] showing the slow increase of the magnetic gluon condensate at low $T \lesssim 1.5T_c$.

Using the expression (6) for $\sigma_s$ and equation (18) we get

$$\frac{\sigma_s(T)}{\sigma} = 4T \sum_{n=-\infty}^{+\infty} \frac{1}{(1 + \omega_n^2/M^2)^2}. \quad (21)$$

In arriving to (21) use was made of the normalization condition, $\sigma_s(0) = \sigma$, and $\int_0^\infty x^2dxK_1(cx) = 2/c^3$. Performing summation in (21) we arrive to the expression for the temperature dependence of the spatial string tension at low temperatures

$$\frac{\sigma_s(T)}{\sigma} = \frac{\sinh(M/T) + M/T}{\cosh(M/T) - 1}, \quad (22)$$

and $\sigma_s(T \ll M)/\sigma = 1 + 2(M/T + 1)e^{-M/T} + O(e^{-2M/T})$.

From (21) we can determine the relative contribution, $\Delta_n(T) \equiv \sigma_{s}^{n\neq 0}(T)/\sigma_{s}(T)$, of the non-zero Matsubara modes to the spatial string tension as a function of $T$. Sorting out from (21) the term with $n = 0$ we get

$$\Delta_n(T) = 1 - \frac{4T}{M} \frac{\cosh(M/T) - 1}{\sinh(M/T) + M/T} \quad (23)$$

It is clear that $\sigma_{s}^{n=0}(T) + \sigma_{s}^{n\neq 0}(T) = \sigma_s(T)$ and $\Delta_n(0) = 1$.

Also one can obtain from (21) contribution of individual $n$-th non-zero Matsubara mode

$$\frac{\sigma_{s}^{n}(T)}{\sigma_{s}^{n=0}(T)} = \frac{M^4}{\omega_n^4} \frac{1}{(1 + M^2/\omega_n^2)^2} \quad (24)$$

Thus, it can be seen that at $T > M/2\pi$ contribution of $n$-th non-zero mode to the spatial string tension as compared to the contribution of zero mode, $n = 0$, very fast ”dies out” with increasing $T$.

$$\frac{\sigma_{s}^{n}(T)}{\sigma_{s}^{n=0}(T)} = \left(\frac{M}{2\pi T}\right)^4 \frac{1}{n^4} - 2\left(\frac{M}{2\pi T}\right)^6 \frac{1}{n^6} + \ldots \quad (25)$$

3 High temperatures

According to its definition the quantity $M = 1/\xi_m$ is the inverse magnetic correlation length. At $T = 0$ the correlation lengths were determined by lattice calculations [23]. On the other hand $M$ can be identified as the mass of the lowest magnetic gluelump with quantum numbers $J^{PC} = 1^{-+}$. Gluelumps [24] are not physical objects and their spectrum cannot be measured in experiment. However, they play fundamental role in the nonperturbative QCD since gluelump masses define the field correlators in the QCD vacuum and in particular the string tension at $T = 0$. The masses of gluelumps were
calculated analytically within the framework of the QCD sum rules \cite{25}, QCD string model \cite{26} and computed numerically on the lattice \cite{27}. From all the calculations listed above one can deduce the value of the correlation length in the interval $\xi \approx 0.1 \div 0.2$ fm.

In what follows we shall use the value $M = 1.5$ GeV for inverse magnetic correlation length at $T = 0$. In Fig. 1 we present the dependence of the function $\Delta_n(T)$ on $T/T_c$ ($T_c = 270$ MeV for pure-gauge SU(3) theory). It follows that $\Delta_n(2T_c) \simeq 0.05$, i.e. the contribution non-zero (non-static) Matsubara modes into $\sigma_s$ is about 5%. This is of the same order as the error of the lattice calculations of the spatial string tension \cite{2, 9, 10}. Thus at temperatures under consideration, $T > 2T_c$, the main contribution into the dynamics of the gauge theory magnetic sector comes from zero (static) Matsubara mode.

![Figure 1: Relative contribution, $\sigma_{s\neq 0}^n/\sigma_s$, of the non-zero Matsubara modes to the spatial string tension for pure-gauge SU(3) theory.](image)

Next we note that at any $T$ the SU(N) gauge theory is in the phase of magnetic confinement. Therefore at high $T$ the exponential form of the magnetic correlator remains unchanged. On the other hand the two-point correlation function is determined by the amplitude and correlation length which are temperature dependent. Hence one can write the magnetic correlator at high temperature $T$ as

$$D^{H}_{ik}(x, T) = \frac{1}{6} \delta_{ik} \langle H^2 \rangle(T) e^{-|x|/\xi_m(T)} + O \left( x^2 \frac{\partial D^{H}_{11}}{\partial x^2} \right). \quad (26)$$

The term $\sim O(...)$, as explained above, contributes neither into gluo-magnetic condensate, nor into spatial string tension. Then from (5) and (26) we get

$$\sigma_s(T) = \frac{\pi}{6} \langle H^2 \rangle(T) \xi_m^2(T). \quad (27)$$

As it was shown in \cite{2, 9, 10} the temperature dependence of the spatial string tension $\sigma_s(T)$ at $T > 2T_c$ with good accuracy coincides with the behavior of the string tension $\sigma_3$. 


in 3d Yang-Mills theory. The three-dimensional Yang-Mills theory is a superrenormalizable theory and all physical quantities are determined by the only dimensionful coupling constant $g_3^2 = g^2 T$. It was shown in [2, 9, 10] that
\begin{equation}
\sqrt{\sigma_s(T)} = c_s g^2(T) T, \tag{28}
\end{equation}
where use was made of the two-loop expression for $g^2(T)$
\begin{equation}
g^{-2}(T) = 2b_0 \ln \frac{T}{\Lambda_\sigma} + \frac{b_1}{b_0} \ln \left(2 \ln \frac{T}{\Lambda_\sigma}\right), \tag{29}
\end{equation}
\begin{equation}
b_0 = \frac{11N}{48\pi^2}, \quad b_1 = \frac{34}{3} \left(\frac{N}{16\pi^2}\right)^2. \tag{30}
\end{equation}
The two constants $c_\sigma$ and $\Lambda_\sigma$ were determined using a two-parameter fit to lattice results. For the SU(2) gauge theory $c_\sigma = 0.369 \pm 0.014$, $\Lambda_\sigma = (0.076 \pm 0.013) T_c$ [9], while for SU(3) gauge theory $c_\sigma = 0.566 \pm 0.013$, $\Lambda_\sigma = (0.104 \pm 0.009) T_c$ [2]. In Figs. 2 and 3 we present the plots $\sigma_s(T)/\sigma$ for SU(2) (Fig.2) and SU(3) (Fig.3) gauge theories as functions of $T/T_c$ ($T_c^{SU(2)} = 290$ MeV, $T_c^{SU(3)} = 270$ MeV). Solid lines correspond to Eq.(22) and the dashed ones to Eqs.(28, 29). The lattice data are from Refs.[2, 9].

Further, from (27) it is possible to find $\langle H^2(T) \rangle$. The inverse magnetic correlation length $1/\xi_m$ at high temperature behaves as
\begin{equation}
1/\xi_m(T) = c_m g^2(T) T, \tag{30}
\end{equation}
where $c_m$ is some constant. Using (27), (28) and (30) we obtain the following expression for the temperature dependence of the gluo-magnetic condensate
\begin{equation}
\langle H^2(T) \rangle = c_H g^8(T) T^4, \tag{31}
\end{equation}
and
\begin{equation}
c_H = \frac{6}{\pi} c_\sigma c_m. \tag{32}
\end{equation}
From (32) one can estimate $c_H$. For instance in $SU(2)$ Yang-Mills theory, $c_\sigma \approx 0.37$ [9], $c_m \approx 0.92$ [28], and one finds $c_H \approx 0.22$. However, the value of $c_H$ should be found from numerical simulations on a lattice for thermal magnetic correlator.

The result (31) seems quite natural from the 3d gauge theory standpoint. As was already indicated, the thermal behavior of the physical quantities in the magnetic sector of 4d $SU(N)$ gauge theory at high $T$ coincides with the behavior of corresponding quantities in the 3d gauge theory. Then the NP gluo-magnetic condensate is determined by the value of the dimensionful coupling constant $g_3$ and $\langle H^2(T) \rangle = \text{const} \cdot g_3^2$. Therefore at high temperatures (in the scaling region for the spatial string tension) the gluo-magnetic correlator has the form
\begin{equation}
\mathcal{D}_{ik}^H(x, T) = \frac{1}{6} \delta_{ik} c_H g^8(T) T^4 e^{-c_m g^2(T)|x|}. \tag{33}
\end{equation}
The amplitude of this correlator rises as $T^4$ while the correlation length drops with growing temperature as $1/T$ in the high temperature region.

\footnote{Strictly speaking, in superrenormalizable 3d Yang-Mills theory vacuum averages either are equal to zero, or are defined through corresponding power of coupling constant $g_3$. Disappearance of magnetic condensate at high temperatures means that $\sigma_s = 0$, and therefore contradicts lattice data on magnetic confinement.}
Figure 2: Spatial string tension $\sigma_s(T)/\sigma$ for SU(2) gauge theory as function of $T/T_c$. Solid lines correspond to Eq. (22) and the dashed ones to Eqs. (28,29). The lattice data are from Refs. [9].

Figure 3: Spatial string tension $\sigma_s(T)/\sigma$ for SU(3) gauge theory as function of $T/T_c$. Solid lines correspond to Eq. (22) and the dashed ones to Eqs. (28,29). The lattice data are from Refs. [2].

4 Discussions and conclusion

The results which were obtained above allow to estimate the temperature region within which the change of the regime in the magnetic sector of SU(N) gauge theory occurs and it becomes possible to describe the dynamics in terms of the static correlation functions, i.e. in terms of the 3d gauge theory. On physical grounds it is clear that when the compactification radius $\beta$ (the inverse temperature $1/T$) becomes much smaller than the typical dimension $l$ of the system (which in turn is determined either by magnetic string thickness, or by the radius of the magnetic gluelump) $l = 1/\sqrt{\sigma_s(T)}$, the correlators
“cease to feel the time coordinate $x_4$” and become pure static.

Thus the condition $T = \sqrt{\sigma_s(T)}$ enables to estimate the temperature at which the change of the regimes occurs. Consider for definiteness the SU(3) gauge theory. If one starts from the low temperature region then from the numerical solution of the equation $T_\pm = \sqrt{\sigma_s(T_\pm)}$ and from Eq. (22) for $\sigma_s(T)$ one finds $T_\pm = 1.42 T_c$. Alternatively if one starts from the high temperature region then the solution of the equation $T_\pm + \sqrt{\sigma_s(T_\pm)}$ with $\sigma_s(T)$ defined by (28, 29) gives $T_\pm = 2.74 T_c$. Thus we may conclude that within the interval $T_\pm < T < T_\pm$ the change of the regimes takes place and transition to the reduced 3d Yang-Mills theory occurs. Therefore the commonly accepted value $2T_c$ seems quite natural from the outlined physical point of view.

The growth of the magnetic condensate, $\langle H^2 \rangle(T) \propto T^4$, with the increase of the temperature is thermodynamically advantageous for the quantum field system. The vacuum energy density is connected via the trace anomaly\(^3\) with the magnetic condensate by the relation $\varepsilon_{vac} = \langle \theta_{00} \rangle = -(b/64\pi^2)\langle H^2 \rangle$, $b = 11N/3$, and the increase of $\langle H^2 \rangle$ lowers the vacuum energy density.

In the present paper we have investigated the magnetic sector of SU(N) Yang-Mills theory at finite temperature. At low temperatures, $T < 2T_c$, the analytic expressions were obtained for the temperature dependence of the magnetic correlator, magnetic gluon condensate and the spatial string tension. Fair agreement with lattice calculations for $\sigma_s(T)$ was obtained for SU(2) and SU(3) gauge theories. It is demonstrated, that the contribution of $n$-th non-zero Matsubara mode to the spatial string tension as compared to the contribution of zero mode, $n = 0$, very fast ”dies out” with increasing temperature $\sim (M/2\pi nT)^4$. The relative contribution given by non-zero Matsubara frequencies to the spatial string tension was calculated. At $T = 2T_c$ this contribution is about 5%. The estimate was presented of the temperature region corresponding to the transition to the description of the nonperturbative dynamics in the magnetic sector in terms of the 3d correlation functions. We have found the behavior of magnetic correlator at high $T > 2T_c$ and have shown that gluo-magnetic condensate increases with temperature as $\langle H^2 \rangle(T) = c_H g^8(T) T^4$.

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