The radiative return method - a short theory review

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A short review of the status of the theoretical developments concerning the radiative return method is presented. The emphasis is on the construction of the PHOKHARA Monte Carlo event generator and its tests. It is advocated that the radiative return method provides not only with the hadronic cross section extraction competitive with and complementary to the scan method, but also that it is a powerful tool in detailed studies of the hadron interactions.

1. INTRODUCTION

The hadronic cross section measurement is crucial for the accurate evaluation of the hadronic contributions to the muon anomalous magnetic moment ($a_\mu$) \(^1\) and running of the electromagnetic coupling $\alpha_{QED}$ \(^2\). The traditional way of measuring of the hadronic cross section via the energy scan has one disadvantage - it needs dedicated experiments. An alternative way, the radiative return method, was proposed in \(^3\), even if the radiative process was investigated earlier \(^4\). This method, described in the next section, allows for a simultaneous extraction of the hadronic cross section from the nominal energy of the experiment down to the production threshold, and importantly can profit from the data of all high luminosity meson factories.

2. THE HADRONIC CROSS SECTION VIA THE RADIATIVE RETURN METHOD

The radiative return method relies on an observation that the cross section of the reaction $e^+e^- \rightarrow \text{hadrons} + \gamma$, with photons emitted from the initial leptons, factorizes into a function $H$, fully calculable within QED, and the cross section of the reaction $e^+e^- \rightarrow \text{hadrons}$

$$
\frac{d\sigma(e^+e^- \rightarrow \text{hadrons} + \gamma)}{dQ^2}(s, Q^2) = H \cdot \frac{d\sigma(e^+e^- \rightarrow \text{hadrons})}{dQ^2},
$$

where $Q^2$ is the invariant mass of the hadronic system. Thus from the measured differential, in $Q^2$, cross section of the reaction $e^+e^- \rightarrow \text{hadrons} + \gamma$ one can evaluate $\sigma(e^+e^- \rightarrow \text{hadrons})$ once the function $H$ is known. As evident from the Eq.\(^1\), the radiative return method allows for the extraction of the hadronic cross section from the production energy threshold of a given hadronic channel almost to the nominal energy of a given experiment ($\sqrt{s}$). The smaller cross section of the radiative process as compared to the process without photons emission has to be compensated by higher luminosities. That requirement is met by meson factories (DAPHNE, BaBar, BELLE). All of them were built for other purposes then the hadronic cross section measurements, but their huge luminosities provide with data samples large enough for very accurate measurements of interesting hadronic channels and/or give an information on rare channels, which were not accessible in scan experiments. Two representative examples of such measurements are the very accurate pion form factor extraction by KLOE collaboration \(^5\) and $\sigma(e^+e^- \rightarrow 3\pi)$ extraction by BaBar collaboration \(^6\), where it was shown that the old DM2 scan data were wrong at high values of $Q^2$. An extensive review of the recent results of both collaborations concerning the radiative return is presented.

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in [7]. If one likes to use the formula Eq.(11) in a realistic experimental situation, where sophisticated event selections are used, one needs a Monte Carlo event generator of the measured process. To meet that requirement the PHOKHARA [8] event generator was constructed.

3. THE PHOKHARA MONTE CARLO EVENT GENERATOR AND ITS TESTS

The construction of the PHOKHARA event generator started from the EVA generator [9,10], where structure function method was used to model multi-photon emission. The physical accuracy of the program was however far from the demanding experimental accuracy of the KLOE pion form factor measurement and in a series of papers that high expectations were met. The first distributed PHOKHARA version [8] relied on the one loop initial state radiative corrections calculated in [11] and the two hard photon emission was simulated using exact matrix element written within helicity amplitudes method. That version was designed to run with tagged photon configurations and the radiative corrections necessary for a photon emitted at small angles were calculated afterwords in [12] and implemented into the event generator in [13]. The important issue of the final state emission, which will be discussed in details in the next section, was addressed in [14] and subsequently in [15], while in aspects specific for \( \phi \)-factory DAPHNE in [16]. In parallel the generator was being extended to allow for the generation of more hadronic channels and now it allows for generation of \( \pi^+\pi^-\), \( K^+K^-\), \( \bar{K}^0K^0\), \( \bar{p}p\), \( \bar{n}n\), \( \pi^+\pi^0\), \( 2\pi^+2\pi^-\), \( \pi^+\pi^-2\pi^0\) hadronic states and \( \mu^+\mu^-\). The nucleons final states were discussed in [17], while the three pion current was modeled and implemented into the PHOKHARA event generator in [18].

All that allowed for building of the state-of-the-art event generator. The proper implementation of the radiative corrections as well as the hadronic currents is guarantied by extensive tests of the generator discussed below.

At each step of the generator development, the newly implemented matrix element calculated in the program using the helicity amplitude method, squared and summed over all available helicities is compared with the square of the matrix element summed over polarizations calculated by traditional trace method. All numerical calculations in PHOKHARA are performed using double precision, but in some cases, mainly for double photon emission it was necessary to use the quadrupole precision for the matrix element evaluation calculated analytically by the trace method. It was caused by numerical cancellations up to ten significant digits occurring between various terms.

Another type of tests concern the process of the generation. The initial state emission of one photon with one-loop radiative corrections and two hard real photon emission from initial states were compared [13] separately with existing analytical results for fully inclusive phase space configurations [19]. Both results were in perfect agreement up to the numerical precision of the tests limited by the Monte Carlo statistics. The relative difference between the numerical and analytical results was a few times \( 10^{-4} \), that was well within the statistical Monte Carlo error bars. That accuracy is usually called technical precision of the Monte Carlo generator and I will use that name hereafter. That tests are repeated for each newly added hadronic channel, even if the program uses the same building blocks for every channel, to avoid possible bugs in the implementation.

Similar tests were performed for the reaction \( e^+e^- \rightarrow \pi^+\pi^-\gamma(\gamma) \) with one photon emitted from the initial state and one emitted from the final pions [14]. In that case the analytical results of [20] were used together with the analytical results obtained in [14] to test the Monte Carlo generation. Again the same technical precision was achieved and the tests were repeated for the charged kaons in the final states. For the reaction \( e^+e^- \rightarrow \mu^+\mu^-\gamma(\gamma) \) the configurations with one photon emitted from the initial states and one from final muons were tested in [15] with the same technical precision.

It would be useful to make extensive comparisons with independent Monte Carlo generators, however the only existing Monte Carlo code meeting the accuracy requirements is the KKMC [21], which is limited to muons in the final state as
far as its accurate matrix element is concerned. It means that one can test the initial state emission and in fact detailed tests were performed [22] leading to an excellent agreement of the non exponentiated matrix elements of the virtual corrections to single photon emission (a relative difference of a few times $10^{-5}$ was found). The higher order effects, that can be seen as a difference between the exponentiated and the non exponentiated matrix elements reach at most 2 per mile with the exception of the region of the invariant mass of the hadronic system very close to the nominal energy of the experiment. That region, where soft multi-photon emission play an important role and thus the exponentiation is necessary, is however of no interest to the radiative return method. All that results agree very well with the estimated previously in [8], by means of the structure function approach, PHOKHARA physical precision of 0.5%, attributed to the lack of the higher order effects in the ISR matrix element.

4. THE FINAL STATE EMISSION

The final state emission (FSR) forms a potential problem for the application of the radiative return method and it has to be studied carefully to be sure that the required accuracy of the description is met. First of all one has to have in mind that the situation at B- factories is completely different from the one of the $\phi$- factory DAPHNE. In the former case the region of hadronic masses, which is of physical interests, mainly below 4 GeV, lays far from the nominal energy of the experiments, thus an emission of a hard photon is required to reach it. As a result the typical kinematic configuration of an event consists of a photon emitted in one direction and hadrons going opposite to it. That provides a natural suppression of the FSR contributions, which are large for photons emitted parallel to the direction of a charged hadron in the final state, and makes the measurement of the hadronic cross section easier. For the $\phi$- factory, where the physically interesting region is not far from the nominal energy of the experiment, that natural separation between the emitted photon and the hadrons does not exist and one has to suppress FSR by an appropriate event selection. In that case one has to control the uncertainty due to the model dependence of the final state emission. That is a challenge, as the models were not tested with the adequate precision prior to the DAPHNE results. I will discuss that problem on the basis of the $e^+e^- \rightarrow \pi^+\pi^-\gamma(\gamma)$ process, where the accuracy requirements are the most demanding. For that process the solution of the problem was first proposed in [23] and further elaborated in [14]. One can imagine a similar solution for other hadronic final states, but that kind of analysis was never performed.

The main tool in the tests of the model(s) of the photon emission from the final pions is the charge asymmetry. For one real photon emission the two-pion state is produced in C=-1 and with an odd orbital angular momentum for the real photon emitted from the initial state and in C=1 and with an even orbital angular momentum for the real photon emitted from the final state. As a result, the initial-final state interference is odd under $\pi^+ \leftrightarrow \pi^-$ interchange and it integrates to zero for charge blind event selections. In the same time it is the only source of the charge asymmetry and as such allow for tests of the models of the final state emission. The charge asymmetry depends on the invariant mass of the two-pion system and that allow for deeper insight into details of the tested model(s). In short, the tests should be done in the following way: First one compares the experimental data for the asymmetry with the Monte Carlo where the tested model was implemented. That has to be performed for an event selection which enhance the FSR as compared to the ISR. Once the implemented model agrees with the data one chooses an event selection, which suppresses the FSR and performs the radiative cross section measurement. That guaranties that the ISR and the FSR contributions are separately well under control. For the case of untagged photons a specific background, $e^+e^- \rightarrow \pi^+\pi^-e^+\gamma$, has to be also taken into account [23,24] as the final leptons are not vetoed.

The reaction $e^+e^- \rightarrow \pi^+\pi^-\gamma$, with the photon emitted from the pions, does contribute also to dispersion integrals for evaluation of $a_e$ and $\alpha_{QED}$ and in the former case its theoretically es-
estimated value \[14\] is of the size of the theoretical uncertainty and thus numerically important. As its theoretical estimations are not reliable it has to be measured. The sketched program was successfully undertaken by KLOE and resulted in a sound extraction of the \(\sigma(e^+e^- \to \pi^+\pi^-)\) \[5\] together with the mentioned FSR photon corrections.

Another source of complications for using of the radiative return method at DAPHNE are the radiative \(\phi\) decays. That problem was considered for the first time in \[25\] and it is discussed in more details in the next section.

5. THE RADIATIVE RETURN AS A TOOL IN HADRONIC PHYSICS

The reaction \(e^+e^- \to \phi \to \pi^+\pi^-\gamma\) produces the same final state as the one measured for the \(\sigma(e^+e^- \to \pi^+\pi^-)\) extraction. That contribution is sizable for energy close to the \(\phi\) mass and thus important for DAPHNE. As shown in \[10\], the charge asymmetry has large analyzing power and can provide with information allowing for distinguishing between different models of the radiative \(\phi \to \pi\pi\gamma\) decay, even if that is impossible in the analysis of the differential (in \(Q^2\)) cross section. Again by an appropriate event selection one can suppress those contributions or enhance them as for other sources of the FSR emission discussed in the previous section. That example shows that the radiative return method can be used not only for the hadronic cross section measurement, but also for getting detailed information about the models of hadronic interactions. It was partly exploited in the KLOE analysis \[26\], however as the charge asymmetries were not used in the fits, the collected data contain more information on the tested models then actually was used.

An extensive analysis of the FSR contributions is extremely important especially in the ongoing KLOE analysis, both for tagged \[27\] and untagged \[28\] photon(s) as till now the experimental information on the pion-photon interactions is far from being satisfactory and it is not clear if the model used currently in PHOKHARA (sQED + vector dominance + radiative \(\phi\) decays) describes the FSR with the adequate precision in the threshold region, where other contributions might be important \[29\].

Another example of the power of the radiative return method in the hadronic models tests is the separation of the magnetic and the electric nucleon form factors. The method, which was proposed in \[17\], was used by BaBar collaboration \[30\] for separation of the proton form factors. The obtained results show clearly its competitiveness.

6. THE SUMMARY AND NEAR FUTURE DEVELOPMENTS

A short description of the theoretical status of the radiative return method was presented showing its competitiveness in precise measurements of the hadronic cross section and studies of the hadronic interactions. Many interesting problems, for example a proper modeling of the hadronic current of multi-meson final states observed at BaBar \[7\], the FSR simulation for more than the two-pions final states, the modeling of the narrow resonance contributions and many others not mentioned in this paper still await for detailed theoretical investigations. One of that problems, which will be addressed in the near future by the group working on the PHOKHARA event generator developments and updates, is the improvement of the theoretical description of the \(4\pi\) hadronic current \[31\]. Exploiting isospin symmetry and all available experimental data one comes to the predictions for the \(\sigma(e^+e^- \to 2\pi^0\pi^+\pi^-)\) (central dashed line) shown in Fig.1. The lower and upper dashed lines show the error bars of the model predictions.

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Figure 1. The predictions for the \( \sigma(e^+e^- \rightarrow 2\pi^0\pi^+\pi^-) \) (central dashed line). The lower and upper dashed lines show the error bars of the model predictions.