The handedness of a supramolecular chiral aggregates is often assigned based on the sign of circular dichroism spectra, adopting the exciton chirality method. However, the method does not properly account for the nature of intermolecular interactions. We introduce a generalized picture on the use of the sign of chiral signals in determining the helicity of chiral aggregates, rooted in the exciton model, supported by TD-DFT results.

Chirality remains a fascinating area of research, encompassing all branches of science since the first observation of optical activity by François Arago in 1811. Most of the initial research activities in the field were focused on molecular chirality with a significant impact on drug design. With the advent of supramolecular chemistry, the research activity in the area boosted, extending to macroscopic chirality and opening up promising applications in drug delivery, catalysis, nonlinear optics, chiral sensing, etc. The emergence of optical properties in supramolecular assemblies is governed by excitonic interactions. When two molecules are in close proximity, the electrostatic interaction between their transition dipole moments leads to the splitting of the excited state energy levels (exciton model). As a typical example, in aligned dimers the exciton coupling leads to the emergence of new spectral features, specifically a blue-shifted absorption band compared to the monomer, denoted as H-band (H for hypsochromic) and a red-shifted absorption band for J-aggregate denoted as J-band (J for Jelly, one of the researchers who first observed J-aggregates). In a classic work, Nakanishi and Harada brought out the implications of exciton coupling in chiral aggregates, providing the basis of the exciton chirality (EC) rule. It states “if the exciton circular dichroism (CD) shows a positive (negative) first and a second negative (positive) cotton effect, then the two electric transition dipole moments constitute a clockwise (anticlockwise) screw sense”. Despite caution that the rule does not have a general validity, EC rule became the method of choice for chemists and biologists to experimentally determine the absolute handedness of chiral supramolecular and nanoscopic structures. Recently, a few exceptions to the EC rule were highlighted: they were ascribed to conformational disorder or strong transition magnetic dipoles. The exciton chirality rule for vibrational circular dichroism has been recently discussed to highlight its limitations and potentials. Herein, we re-examine the EC rule for electronic circular dichroism, focusing on the relation between the sign of CD spectra and the absolute chirality of the supramolecular system. Moreover, we address the sum rule for CD spectra, underlying the limits of the exciton model. Results are supported by a detailed comparison with state of the art quantum chemical calculations.

To begin with, we considered a dimeric structure, built-up of equivalent molecules, as in Fig. 1. In the spirit of exciton
model, each molecule is described in terms of just two electronic states: the ground g and the excited state e, so that four states should be considered for a dimer: \( gAg_b, eAg_b, gBe_b, eBe_b \), where A and B indices denote the two molecules. In the exciton model, only intermolecular interactions between the two degenerate states, \( eAg_b \) and \( gBe_b \), are considered. As schematically shown in Fig. 2 (see also ESI), this leads to the in-phase (E\(_i\)) and out-of-phase (E\(_o\)) combination of the \( eAg_b \) and \( gBe_b \) states. The relative energy of the two states depends on the sign of the electrostatic interaction between the transition dipole moments on the two molecules, with the in-phase state lying lower (higher) in energy for the attractive (repulsive) interactions. The rotational strength associated with the two states reads:  

\[
\mathcal{R}_\pm = \frac{-\omega}{2} \mathcal{R} \cdot \left( \mu^g_{\pm} \times \mu^e_{\pm} \right),
\]

where \( \omega \) is the frequency of the \( G \rightarrow E_\pm \) transition and \( \mu^A/B_\pm \) is the contribution of the A/B molecules to the corresponding transition dipole moment. \( R_i \) and \( R_o \) have opposite sign, justifying the appearance of an exciton doublet (bisignation) whose sign changes for the left and right helical arrangements. The absolute sign of the doublet, however, depends on the relative energy of the \( E_\pm \) states (see Fig. 2). For repulsive interactions (H-type), \( E_- \) is the lowest excited state so that the right-handed structure gives rise to a positive CD signal in the low-energy (higher wavelength) side of the doublet, in line with the exciton chirality rule. \(^3\) However, the opposite occurs for attractive interactions (J-type) with \( E_- \) lying lower than \( E_+ \), so that for the right-handed structure a negative signal is expected in the low-energy side of the doublet, breaking the generality of exciton chirality rule.

The exciton model is very useful to understand the basic features of CD spectra. However, it suffers from a fundamental problem that is well evident from the above equation. A general and fundamental sum rule states that the sum of the rotational strengths for all the transitions must vanish. \(^11\) In the exciton model, however, this rule is not obeyed: the geometrical part of the rotational strength (the term \( \mathcal{R} \cdot (\mu^g_+ \times \mu^e_+ \) in Equation 1) is indeed equal and opposite for the two transitions, but their (slightly) different frequencies lead to (slightly) different rotational strengths. As detailed in the ESI, this fundamental inconsistency is related to the approximation used in the exciton model and is solved by accounting for the intermolecular interactions that mix \( gAg_b, gBe_b \) and \( eAg_b, eBe_b \) states.

To validate our results, we compare CD spectra calculated in the exciton approximation with TD-DFT results. We select a linear phenyleneethynylene dye (PE, Fig. 1A), a rigid-rod like molecule characterized by a single low-lying excited state, as the molecular building block for the supramolecular structure. \(^12\) The ground state geometry of the PE is optimized at the DFT level using the CAM-B3LYP functional and 6-31G(d) as basis set in the gas phase. The lowest energy transition is calculated at 313.13 nm with a transition dipole moment of 11.28 Debye, aligned along the main molecular axis. PE does not have a chiral centre but can be designed to form chiral supramolecular assemblies. \(^13\)

In the exciton model, with repulsive (V>0; top panel) and attractive (V<0; bottom panel) interactions.

![Figure 2](https://via.placeholder.com/150)

**Figure 2**: Schematic representation of the energy diagram for dimers with repulsive and attractive interactions using exciton model and corresponding bisignated circular dichroic (CD) spectrum for right- and left-handed dimers. The shaded left and right columns show the bisignated CD of the dimeric structures, organized in left- and right-handed fashion, respectively. The central column shows the energy level diagram for dimers with repulsive (V>0; top panel) and attractive (V<0; bottom panel) interactions.

Table 1 shows the transition energy, oscillator strength (f) and rotational strength (R) calculated using TD-DFT for helical and creeper dimers with various twist angles. For small twist angles in the helix arrangement, the oscillator strength (roughly twice the oscillator strength of the monomer) is mainly concentrated in the high energy state, while the opposite occurs for the creeper dimer. This is in line with the exciton model for aligned molecules (θ=0°), which predicts that only a transition from the ground state to the \( E_\pm \) state is allowed. Upon increasing θ, the oscillator strength redistributes between \( E_\pm \) and \( E_- \) states. CD spectra in Fig. 3 and 4 are calculated assigning a Gaussian band shape to the two transitions (with standard deviation \( \sigma = 0.35 \) eV) using equation 2,\(^2\)

\[
\Delta \alpha(\tilde{\nu}) = \sum_{i=1}^{n} \left( \frac{R_i}{2.296 \times 10^{-35} \sqrt{\sigma}} e^{-\left( \frac{\tilde{\nu} - \nu_i}{\sigma} \right)^2} \right),
\]

where \( i \) counts the electronic excitations, \( R_i \) is the corresponding rotational strength and \( \tilde{\nu}_i \) is the transition wavenumber (nm\(^{-1}\)).
The intensity of calculated CD signals increases from 10° to 40° (at 40° (A
Figure 3 - B)) in 10° increments. Panels (A,C) and (E,G) show TD-DFT while panels (B,D) and (F,H) show exciton model results, respectively. Results for left-handed systems are provided in ESI (Fig. S1 and S2).

Table 1: Oscillator strength (f) and rotational strength (R) of electronic transition in dimer at angles 10° - 80° in helical and creeper-helical geometry

| Angle | Right-handed helix (dimer): TD-DFT calculations | Creep-helical geometry |
|-------|--------------------------------------------------|------------------------|
|       | Simple-helical geometry                          | Creep-helical geometry |
|       | Trans. Energy eV                                  | f \( \times 10^{-54} \) | Cm\(^2\)s\(^{-1}\) | Trans. Energy eV | f \( \times 10^{-54} \) | Cm\(^2\)s\(^{-1}\) |
| 10°   | 3.93 0.03 359.85                                 | 3.93 0.35 -158.05                           | 3.93 0.35 -185.02|
| 20°   | 3.99 0.03 350.05                                 | 3.99 0.03 174.31                           | 3.99 0.03 174.31|
| 30°   | 3.99 0.03 275.38                                 | 3.99 0.03 275.38                           | 3.99 0.03 275.38|
| 40°   | 3.99 0.03 131.98                                 | 3.99 0.03 675.78                           | 3.99 0.03 675.78|
| 50°   | 3.99 0.03 158.88                                 | 3.99 0.03 801.50                           | 3.99 0.03 801.50|
| 60°   | 3.99 0.03 193.22                                 | 3.99 0.03 901.09                           | 3.99 0.03 901.09|
| 70°   | 3.99 0.03 194.50                                 | 3.99 0.03 986.97                           | 3.99 0.03 986.97|
| 80°   | 3.99 0.03 203.46                                 | 3.99 0.03 980.73                           | 3.99 0.03 980.73|

where \( \varepsilon_0 \) is the vacuum permittivity, n is the medium refractive index and \( \vec{r} \) is the vector that connects the two molecular centres. Finally, \( \vec{\mu}_{AB} \) is the transition dipole moment for the two molecules, which is set to the value calculated in TD-DFT for the isolated PE molecule. We fix the squared refractive index (\( n^2 \)) to reproduce the exciton splitting calculated in TD-DFT for the aligned dimer. The resulting values (2.48 and 2.31 for helix and creeper, respectively) are in line with typical values for organic materials. Results of the exciton model in Fig. 3 B,D,F,H confirm that the chirality rule as commonly defined only applies to chiral aggregates with repulsive interactions and cannot be generalized. We further investigated the CD response of a trimeric assembly in helical and creeper geometry (Fig. 4 and Table S2, S3). These results are again in line with the previous discussions, proving that the exciton model is fully consistent with TD-DFT results.

Having validated the exciton model against TD-DFT results for small aggregates, we exploit the exciton model to study the chiroptical response of larger aggregates. Fig. 4 reports the results for an aggregate constituted by ten molecules arranged in helix and creeper structures using same geometrical parameters as defined for the dimer. Once again, the exciton chirality rule is obeyed in the helix geometry and is broken in the creeper geometry. Of course, when ten molecules are put together, the exciton model predicts ten excited states. Nevertheless, Fig. 4 confirm that only two transitions mainly contribute to the CD spectrum, while the remaining transitions have negligible rotational strengths. In the helical geometry, the main contribution to the CD spectra is associated with states lying at higher energies (shorter wavelength). In contrast, states lying at lower energies (longer wavelength) are the dominant contributors for the creeper geometry. Thus, for a supramolecular assembly even if there are ‘n’ number of molecules and consequently ‘n’ number of higher energy states, the CD spectrum gives only a single CD couplet (provided there is single transition dipole moment, and exciton coupling is possible). We repeated the calculations for different twist angles like 45°, and 80°; the results are consistent, and the
Conflicts of interest

“There are no conflicts to declare”.

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