Family symmetries and the SUSY flavour problem

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Abstract

We re-examine the constraints on continuous family symmetries coming from flavour changing neutral
current limits on the $D$–term contributions to squark and slepton masses. We show that, for a restricted
choice of the familon sector, continuous family symmetries are consistent with even the most conserva-
tive limits both for the case of gauge mediated supersymmetry breaking and the case of gravity mediated
supersymmetry breaking.

1 Introduction

The SUSY flavour problem is a long-standing problem for SUSY theories. Due to the introduction of additional
flavoured particles, the sfermions, there are extra contributions to 4-fermion flavour changing interactions,
leading to flavour changing neutral currents (FCNC) that are potentially too large. For example, FCNC
processes can occur for quarks through box diagrams involving gluino and squarks in the internal lines, with
the contribution depending on the masses of the squarks that mediate the flavour change. Considering such
processes, experiments have placed rather stringent constraints on the mass matrices of the sfermions [1]. While
there are alternatives (e.g. alignment of the sfermions with the fermions) [2], the simplest way to satisfy the
constraints is by requiring the sfermions of each family to have nearly degenerate masses.

There are three established ways of obtaining the required sfermion mass degeneracy: gauge mediated
models, in which case the SUSY breaking mechanism giving rise to the soft masses is generation blind; SUGRA
models, where universal soft masses are generated by gravitational interactions; and family symmetries, where
the added symmetry explaining the fermion mass structure requires the sfermions of each family to have nearly
degenerate masses.

Although these mechanisms are necessary to avoid large FCNC they may not be sufficient if there are
further sources of family dependent masses. This is the case if there is a continuous family symmetry because
the associated $D$-term spoils the desired degeneracy of sfermion masses [3] and is commonly thought to rule
out symmetries which differentiate between the first two families. In this paper we show how this problem can
readily be avoided. The way this works depends on the origin of the soft masses and we discuss the cases of
both gravity and gauge mediation.

In section 2 we review how to express the sfermion mass matrices in the “Super-CKM” (SCKM) basis [5]
and compare with the experimental bounds presented in [1]. Section 3 shows how the $D$-term gives rise to
family dependent contributions which potentially violate the FCNC bounds. We also present a method for
obtaining an upper bound on the model predictions for the FCNC effects and give a discussion of the energy
scales relevant to the analysis. In section 4 we present ways of solving the SUSY flavour problem associated
with continuous family symmetries. We conclude with a summary in Section 5.

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If the family symmetry is discrete there are no $D$-terms associated with it e.g. [3]. In this case there may be light pseudo
Goldstone bosons associated with the breaking of the family symmetry.
2 Super-CKM basis and the experimental bounds

In SUSY models we need to specify not just the basis chosen for the fermion states, as in the Standard Model (SM), but also the basis chosen for the sfermion states. In the SM it is often convenient to use either the "Mass basis", where each fermion state has a well defined mass (the mass matrices are diagonal) and the "Weak basis", where each fermion state has a well defined flavour (the weak interaction matrix is diagonal). Similarly, it is convenient to use a specific SUSY basis, and it is particularly useful to use the SCKM basis [5]. We turn to a brief review of the SCKM basis, illustrating it by the down quark and squark sector. We generalize the mass matrices to a $6 \times 6$ notation, distinguishing left-handed (LH) and right-handed (RH). In an arbitrary basis for the quarks, we have:

\[
\begin{pmatrix}
\bar{d}_L, \bar{d}_R
\end{pmatrix}
\begin{bmatrix}
0 & M_d^D \\
M_d^{D\dagger} & 0
\end{bmatrix}
\begin{pmatrix}
d_L' \\
d_R'
\end{pmatrix}
\]

(1)

where $d_L'$, $d_R'$ are 3 component columns containing the 3 down-type quarks ($d$, $s$, $b$), and the $6 \times 6$ matrix is represented in four $3 \times 3$ blocks. The prime denotes that the states are taken in the arbitrary basis. $M_d^D$ is the Dirac mass matrix for the down quarks.

The associated scalar partners have a squared mass matrix containing their squared masses:

\[
M_d^2 = \begin{bmatrix}
M_{LL}^2 & M_{LR}^2 \\
M_{LR}^2 & M_{RR}^2
\end{bmatrix}
\]

(2)

where we expressed the full $6 \times 6$ matrix in terms of $3 \times 3$ block matrices. In the LR quadrant, $M_{LR}^2$ is equal to the down quark Dirac mass matrix $M_d^D$ times a generation independent mass factor, and in the RL quadrant, $M_{LR}^2$ is similarly proportional to the hermitian conjugate, $M_d^{D\dagger}$.

In the LL and RR quadrants we have squark masses for the LH and RH squarks respectively. We start in a basis where these are diagonal, and parametrize the matrix with an explicit universal contribution $m_0^2$ that is generation blind, plus deviations from the degenerate spectrum which are parametrized as $\Delta m_j^2$. It is useful to use such a parametrization as we will be considering models where the common $m_0$ comes from a specific SUSY breaking messenger mechanism (e.g. SUGRA), and deviations arise from $D$-term contributions associated with a continuous family symmetry. For the down-type squarks we have:

\[
M_{LL:RR}^2 = \begin{bmatrix}
m_0^2 + \Delta m^2_{d_{L,R}} & 0 & 0 \\
0 & m_0^2 + \Delta m^2_{s_{L,R}} & 0 \\
0 & 0 & m_0^2 + \Delta m^2_{b_{L,R}}
\end{bmatrix}
\]

(3)

It is convenient to re-express this matrix in the SCKM basis. The SCKM basis consists of having the fermion states in the basis where their Dirac mass matrices are diagonalised, and the sfermion states in the basis that has the neutral gauginos couplings flavour diagonal. This requires changing the quark and squark states by the same transformations. To do this we use the $6 \times 6$ mixing matrix that diagonalises the quark Dirac masses:

\[
\begin{pmatrix}
\bar{d}_L, \bar{d}_R
\end{pmatrix}
\begin{bmatrix}
V_L & 0 \\
0 & V_R
\end{bmatrix}
\begin{pmatrix}
d_L' \\
d_R'
\end{pmatrix}
\]

(4)

where the unprimed quark states are the mass eigenstates. From eq. (1), we get:

\[
\begin{pmatrix}
\bar{d}_L, \bar{d}_R
\end{pmatrix}
\begin{bmatrix}
V_L & 0 \\
0 & V_R
\end{bmatrix}
\begin{bmatrix}
0 & M_d^D \\
M_d^{D\dagger} & 0
\end{bmatrix}
\begin{pmatrix}
V_L^\dagger & 0 \\
0 & V_R^\dagger
\end{bmatrix}
\begin{pmatrix}
d_L' \\
d_R'
\end{pmatrix}
\]

(5)

where the product of the three $6 \times 6$ matrices will result in diagonalised LR and RL quadrants (the diagonal down quark Dirac mass matrix and its hermitian conjugate, respectively). In the SCKM basis the down squark mass matrix has the form:

\[
M_d^2 = \begin{bmatrix}
V_L M_{LL}^2 V_L^\dagger & V_L M_{LR}^2 V_R^\dagger \\
V_R M_{LR}^2 V_L^\dagger & V_R M_{RR}^2 V_R^\dagger
\end{bmatrix}
\]

(6)
Note that the LR and RL blocks are now diagonal, but (due to the $\Delta m^2_f$) the LL and RR blocks need not be.

The mass insertions $\Delta \tilde{f}$ are defined as the components of the sfermion mass matrix in the SCKM basis. For example, $\Delta^\tilde{d}_{12LL}$ (the $\tilde{d}_L \tilde{s}_L$ component) is given by:

$$
\Delta^\tilde{d}_{12LL} \equiv (m^2_0 + \Delta m^2_{\tilde{d}_L}) V_{L11} V^*_{L21} + (m^2_0 + \Delta m^2_{\tilde{s}_L}) V_{L12} V^*_{L22} + (m^2_0 + \Delta m^2_{\tilde{b}_L}) V_{L13} V^*_{L23}
$$

(7)

Since $V_L$ is unitary:

$$
V_{L11} V^*_{L21} + V_{L12} V^*_{L22} + V_{L13} V^*_{L23} = 0
$$

(8)

Using this immediately shows that the terms proportional to $m^2_0$ contribution in eq.(7) vanish as expected (as would any generation independent contribution).

In [1] the experimental constraints are presented in terms of quantities $\delta$, which are obtained by dividing the mass insertions $\Delta$ by the average sfermion mass. To illustrate, with down squarks in the LL block, we have (from eq.(6), eq.(7), eq.(8)):

$$
\delta^\tilde{d}_{12LL} \equiv \left( \frac{V_L M^{LL}_d V^*_L}{\langle m^2_{\tilde{q}} \rangle} \right)_{12}
$$

$$
= \frac{\Delta m^2_{\tilde{d}_L} V_{L11} V^*_{L21} + \Delta m^2_{\tilde{s}_L} V_{L12} V^*_{L22} + \Delta m^2_{\tilde{b}_L} V_{L13} V^*_{L23}}{\langle m^2_{\tilde{q}} \rangle}
$$

(9)

where $\langle m^2_{\tilde{q}} \rangle$ is the geometrical average for the squark mass [1].

The most stringent experimental upper bounds for the $\delta$ from [1] are shown in Table 1 (for quarks) and Table 2 (for leptons) [2].

3 The family symmetry flavour problem

3.1 $D$-term contributions

We now illustrate how $D$-terms associated with continuous family symmetry groups can generate family dependant contributions to the sfermion masses. We consider a simple example of $U(1)_{\text{family}}$ symmetry, with the field content extended to include two familons, $\phi$ and $\bar{\phi}$. We define the coupling constant so that the charge of $\phi$ is +1, meaning that the other family charges are defined relative to the charge of this familon. The $D$-term associated with the continuous family symmetry is then:

$$
(D - \text{term})^2 = g_f^2 \left( |\phi|^2 + |\bar{\phi}|^2 + c_{\tilde{d}_L} |\tilde{d}_L|^2 + c_{\tilde{d}_R} |\tilde{d}_R|^2 + \ldots \right)^2
$$

(10)

where $g_f$ is the family coupling constant, $c$ is the family charge of $\bar{\phi}$, $c_{\tilde{d}_L,R}$ are the family charges of the down squark with left and right handedness respectively, and the (...) stands for similar terms for all the other sfermions.

Expanding the $D$-term we can identify terms quadratic in the down squarks, i.e. potential contributions for their masses. Using the notation of eq.(6):

$$
\Delta m^2_{f_{L,R}} = 2 c_{f_{L,R}} \langle D^2 \rangle
$$

(11)

where

$$
\langle D^2 \rangle = g_f^2 \langle |\phi|^2 + |\bar{\phi}|^2 \rangle
$$

is the magnitude of the $D$–term. The contributions shown in eq.(11) are generation dependant, and give rise to the family symmetry flavour problem. We now quantify the problem by calculating the $\delta$ predicted by the

Making some assumptions about the underlying physics (e.g. GUTs) one may obtain stronger bounds [1] but we do not consider these here.
\[
\begin{array}{|c|c|c|}
\hline
\frac{m^2}{m_q^2} & \sqrt{\text{Re} \left( \delta_{12}^d \right)^2} & \sqrt{\text{Re} \delta_{12}^d \delta_{12}^d} \\
\hline
0.3 & 1.9 \times 10^{-2} & 2.5 \times 10^{-3} \\
1.0 & 4.0 \times 10^{-2} & 2.8 \times 10^{-3} \\
4.0 & 9.3 \times 10^{-2} & 4.0 \times 10^{-3} \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\frac{m^2}{m_q^2} & \sqrt{\text{Re} \left( \delta_{13}^d \right)^2} & \sqrt{\text{Re}\delta_{13}\delta_{13}^d} \\
\hline
0.3 & 4.6 \times 10^{-2} & 1.6 \times 10^{-2} \\
1.0 & 9.8 \times 10^{-2} & 1.8 \times 10^{-2} \\
4.0 & 2.3 \times 10^{-1} & 2.5 \times 10^{-2} \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\frac{m^2}{m_q^2} & \sqrt{\text{Im} \left( \delta_{12}^u \right)^2} & \sqrt{\text{Im}\delta_{12}\delta_{12}^u} \\
\hline
0.3 & 4.7 \times 10^{-2} & 1.6 \times 10^{-2} \\
1.0 & 1.0 \times 10^{-1} & 1.7 \times 10^{-2} \\
4.0 & 2.4 \times 10^{-1} & 2.5 \times 10^{-2} \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\frac{m^2}{m_q^2} & |\delta_{23}^d| \\
\hline
0.3 & 4.4 \\
1.0 & 8.2 \\
4.0 & 26 \\
\hline
\end{array}
\]

Table 1: Bounds for $\delta$, assuming $m_q = 500$ GeV [1]

\[
\begin{array}{|c|c|c|}
\hline
\frac{m^2}{m_f^2} & |\delta_{12}^d| \\
\hline
0.3 & 4.1 \times 10^{-3} \\
1.0 & 7.7 \times 10^{-3} \\
5.0 & 3.2 \times 10^{-2} \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\frac{m^2}{m_f^2} & |\delta_{13}^d| \\
\hline
0.3 & 15 \\
1.0 & 29 \\
5.0 & 1.2 \times 10^2 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\frac{m^2}{m_f^2} & |\delta_{23}^d| \\
\hline
0.3 & 2.8 \\
1.0 & 5.3 \\
5.0 & 22 \\
\hline
\end{array}
\]

Table 2: Bounds for $\delta$, assuming $m_f = 100$ GeV [1]
model, allowing for a direct comparison with the experimental bounds. For example, we substitute eq. (11) into eq. (9):

$$\delta_{12}^{LL} \simeq \frac{2 \langle D^2 \rangle (c_{d_L} V_{L_{11}} V_{L_{21}} + c_{s_L} V_{L_{12}} V_{L_{22}} + c_{b_L} V_{L_{13}} V_{L_{23}})}{\langle m_q^2 \rangle}$$ (12)

Similar expressions are obtained for the other $\delta$.

### 3.2 Upper bounds on the theoretical predictions

Since the mixing matrices are a priori unknown, it is useful to derive an upper bound on the $\delta$ that is independent of them. Consider just the part of eq. (12) dependent on the charges and on the mixing matrix entries:

$$c_{d_L} V_{L_{11}} V_{L_{21}} + c_{s_L} V_{L_{12}} V_{L_{22}} + c_{b_L} V_{L_{13}} V_{L_{23}}$$ (13)

Each of the terms in eq. (13) contains two elements of the mixing matrix that share a common column (e.g. $V_{L_{11}}, V_{L_{21}}$). We designate these as “mixing pairs”. The unitarity of the mixing matrix imposes restrictions on the mixing pairs: for example eq. (8) has three such pairs summing up to zero (a unitarity triangle). Also because of unitarity, a mixing pair can be written in the form $\frac{1}{2} \sin(2\theta) \cos(\phi)$, where $\theta$ and $\phi$ are mixing angles, thus the maximum magnitude of any of these pairs is $\frac{1}{2}$. Furthermore eq. (8) shows that if one of the pairs has the maximum magnitude of $\frac{1}{2}$, the other two pairs have to point opposite in relation to the maximum magnitude pair (thus closing the respective unitarity triangle). Because of this, in order to maximize eq. (13), we identify which two of the three family charges produce Max $|c_i - c_j|$ (the dominant charge difference). To obtain the maximum value the mixing pair corresponding to the other charge must vanish, and the mixing pairs corresponding to the other two charges must take the maximum magnitude of $\frac{1}{2}$.

Thus, for example, from eq. (12):

$$|\delta_{12}^{LL}| < \left| \frac{\langle D^2 \rangle}{\langle m_q^2 \rangle} \right| \text{Max} |c_i - c_j|$$ (14)

A specific model can saturate the upper bounds given in eq. (14) if there is maximal mixing in two families with no mixing from the other family and the families that mix correspond to those that maximize $|c_i - c_j|$. Further, when comparing with the experimental bounds, the most stringent constraints arise if it is the $(1, 2)$ families that mix corresponding to the most stringent experimental upper bounds. Finally the phase should also be in the direction that yields the strictest experimental bound. Given this it is quite unlikely that a specific model will indeed saturate the bound so the bounds should be considered as very conservative.

### 3.3 Running effects

Before comparing theory to experiment it is important to discuss the energy scales at which the comparison should be made. The soft SUSY breaking masses are generated at a scale corresponding to the mediator scale $M_X$ communicating SUSY breaking from the hidden to the visible sector and radiative corrections to the mass will be cutoff at this scale. For the case of SUGRA this is the Planck scale and there are substantial radiative corrections in continuing to the electroweak scale where the experimental bounds are obtained. For the case of gauge mediation the gauge messenger scale can be much lower than the Planck scale and so the radiative corrections may be much smaller. The dominant radiative corrections are due to the gauge interactions which are flavour blind. They have the effect of increasing $\langle m_q^2 \rangle$ while leaving $\Delta m_{fL,R}^2$ unchanged. As a result they systematically reduce the FCNC effects [6].

Actually it is more convenient, when comparing with the theoretical expectation, to make the comparison at the messenger scale by continuing the experimental bounds up in energy. Due to the radiative corrections just discussed $\delta$ will depend on the scale $\mu$ at which the comparison is to be made, $\delta = \delta(\mu^2)$. We have:

$$\delta(M_X^2) = \delta(M_W^2) \frac{m_f^2(M_W^2)}{m_f^2(M_X^2)}$$ (15)
In the gravity mediation scenario, taking the value of 1100 GeV and running to 1000 GeV at the messenger scale significantly up to that energy range; the low scale average squark mass however is considerably higher than in the messenger scale to be run to a unified value.

To solve the problem requires alignment [2], which is only natural if explained by some specific mechanism. In this paper we eschew this explanation and look for a more general explanation for the suppression of the FCNC.

Table 3: Experimental upper bounds evaluated at the mediator scale relevant to gauge and gravity mediation.

| $\delta(M_X^2)$ | Gauge mediation ($M_X = 200$ TeV) | Gravity mediation ($M_X = M_P$) |
|-----------------|----------------------------------|-------------------------------|
| $|\delta_{12 LL}^{dd}|$ | $5.9 \times 10^{-4}$ | $8.6 \times 10^{-3}$ |
| $|\delta_{12 RR}^{dd}|$ | $5.9 \times 10^{-4}$ | $8.6 \times 10^{-3}$ |
| $|\delta_{13 LL}^{dd}|$ | $4.8 \times 10^{-2}$ | $7.0 \times 10^{-1}$ |
| $|\delta_{13 RR}^{dd}|$ | $4.8 \times 10^{-2}$ | $7.0 \times 10^{-1}$ |
| $|\delta_{12 LL}^{tu}|$ | $4.5 \times 10^{-2}$ | $6.6 \times 10^{-1}$ |
| $|\delta_{12 RR}^{tu}|$ | $4.5 \times 10^{-2}$ | $6.6 \times 10^{-1}$ |
| $|\delta_{23 LL}^{dd}|$ | $22$ | $3.2 \times 10^2$ |
| $|\delta_{12 LL}^{tc}|$ | $7.7 \times 10^{-3}$ | $1.2 \times 10^{-2}$ |
| $|\delta_{13 LL}^{tc}|$ | $29$ | $45$ |
| $|\delta_{23 LL}^{tc}|$ | $5.3$ | $8.3$ |

To evaluate the size of the effect, one can use the renormalisation group equations [7]. In Table 3 we display sample values of $\delta(M_X)$ for gauge and gravity mediation. For gravity mediation, we considered the low energy squark masses of $m_{\tilde{q}} = 500$ GeV and slepton masses of $m_{\tilde{l}} = 100$ GeV (as used in the bounds of [1]) and running effects corresponding to a common unified gaugino mass $m_{1/2} \sim 250$ GeV lead the sfermions masses to run to a unified value $m_0 = 80$ GeV at the Planck scale ($M_X = M_P$). For gauge mediation, we considered the messenger scale to be $M_X = 200$ TeV (we use the SPS 8 scenario in [7]). The slepton masses don’t run significantly up to that energy range; the low scale average squark mass however is considerably higher than in the gravity mediation scenario, taking the value of 1100 GeV and running to 1000 GeV at the messenger scale $M_X = 200$ TeV. $\delta(M_W^2)$ needs to be scaled with respect to the higher squark mass [1] before applying eq. (15).

In obtaining the values, we use as starting point the $\delta(M_W^2)$ corresponding to the mass ratios $\frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2}$ and $\frac{m_{\tilde{l}}^2}{m_{\tilde{l}}^2}$ of 1.0 in Tables 1 and 2 (i.e. the middle rows in each sector). The bounds for $|\delta|$ shown in Table 3 are obtained under the most conservative assumption about the phases to make the bound as strong as possible. When two different $\delta$ are present in the original experimental bound (as in the 2nd column of Table 1), we took the value for $|\delta_{LL}|$ to be the same as $|\delta_{RR}|$ (this leads to the same upper bound for $\delta_{LL}$ and $\delta_{RR}$ in consecutive rows of Table 3).

It may be seen from Table 3 that the most stringent limits apply to the first two generations. For them it is necessary that there should be a strong suppression of the flavour changing SUSY mass squared difference compared to the the average squark or slepton mass squared. This is the family symmetry flavour problem. One should note however that we are considering the most pessimistic case (as discussed subsection 3.2). For example, it is quite conceivable that the mixing matrices feature small mixing, which implies the mixing pairs in e.g. eq. (12) can readily take values $O(10^{-1})$ or even smaller, rather than $\frac{1}{2}$. How much suppression one allows from the mixing depends on what is considered natural - requiring them to be very small in order to completely solve the problem requires alignment [2], which is only natural if explained by some specific mechanism. In this paper we eschew this explanation and look for a more general explanation for the suppression of the FCNC.
4 Solving the family symmetry flavour problem

In this section we discuss the conditions for the $D$–term in eq.(14) to be anomalously small. The $D$–term is fixed when minimising the familon potential and this relates it to the familon masses. We illustrate the general expectation for this in the context of a $U(1)$ family symmetry with a simple familon sector. As we shall discuss, the important aspects of the form of the $D$–term are common to more complicated familon sectors and to non-Abelian family symmetries.

4.1 $F$-term breaking

We first consider a case where the familons acquire non-vanishingVEVs due to an $F$-term. We take the real potential to be:

$$V = g_f^2 (|\phi|^2 + c|\bar{\phi}|^2)^2 + m^2|\phi|^2 + \bar{m}^2|\bar{\phi}|^2 + g (\phi\bar{\phi} - M^2)^2$$

where we have the $D$-term, familon soft mass terms and also an $F$-term coming from a super-potential term $g (\phi\bar{\phi} - M^2) \bar{\psi}$. By minimising the potential we find:

$$\langle D^2 \rangle \equiv \langle g_f^2 (|\phi|^2 + c|\bar{\phi}|^2) \rangle \simeq \langle -m^2 - \bar{m}^2/c \rangle$$

$$\langle |\phi|^2 \rangle \simeq -c \langle |\bar{\phi}|^2 \rangle$$

where $c < 0$ and we have assumed the family symmetry breaking scale is much greater than the SUSY soft masses.

4.2 Radiative breaking

As a second example, we consider a case where the VEVs are driven radiatively. We take $V$ to have the form:

$$V = g_f^2 (|\phi|^2 + c|\bar{\phi}|^2)^2 + \alpha_\phi|\phi|^2m^2\ln \left(\frac{|\phi|^2}{|\Lambda|^2}\right) + \alpha_{\bar{\phi}}|\bar{\phi}|^2\bar{m}^2\ln \left(\frac{|\bar{\phi}|^2}{|\bar{\Lambda}|^2}\right)$$

where the two last terms include the effects of radiative corrections, $\alpha_\phi$ and $\alpha_{\bar{\phi}}$ are the fine structure constants associated with the interactions of $\phi$ and $\bar{\phi}$ and the tree level contributions have been absorbed in $\Lambda$ and $\bar{\Lambda}$. This gives:

$$\langle D^2 \rangle \equiv \langle g_f^2 (|\phi|^2 + c|\bar{\phi}|^2) \rangle \simeq \langle -\alpha_\phi m^2\ln \left(\frac{|\phi|^2}{|\Lambda|^2}\right) - \alpha_{\bar{\phi}}\bar{m}^2\ln \left(\frac{|\bar{\phi}|^2}{|\bar{\Lambda}|^2}\right) / c \rangle$$

which has the form of eq.(17), where $m$ and $\bar{m}$ are now interpreted as running masses:

$$m^2 \equiv \alpha_\phi m'^2\ln \left(\frac{|\phi|^2}{|\Lambda|^2}\right)$$

$$\bar{m}^2 \equiv \alpha_{\bar{\phi}}\bar{m}'^2\ln \left(\frac{|\bar{\phi}|^2}{|\bar{\Lambda}|^2}\right)$$

(21)

With this form of the $D$–term we have from eq.(11):

$$\Delta m^2_{f_{L,R}} \simeq \frac{c_{f_{L,R}}}{2} (-m^2 - \bar{m}^2/c)$$

(22)

which immediately allows us to determine $\delta$ from eq.(14). For example the $(1, 2)$ element is
where \( \langle m_q^2 \rangle \) is to be evaluated at the mediator scale. To estimate how large the factor involving the familon masses is we must consider the origin of the soft masses \( m \) and \( \bar{m} \). To do this we consider separately the case of gravity and of gauge mediation.

### 4.3 Gravity mediated SUSY breaking

We first consider SUGRA as the origin of the familon and sfermion masses. The soft masses are generated at the Planck scale, so we use the estimated values in the third column of Table 3. It may be seen that the bounds are only significant for the mixing between the first two generations. Are there ways these bounds can naturally be satisfied without appealing to small mixing angles?

As we have stressed, SUGRA models solve the FCNC problem by taking all the soft masses to have a common value at the Planck scale, arguing that gravity is family and flavour blind. This naturally extends to the familon sector too so we expect \( m^2 = \bar{m}^2 \) at the Planck scale. It is immediately obvious that, provided there are not large radiative corrections, this offers an elegant solution to the family symmetry flavour problem too if the familons have equal but opposite charges, i.e. \( c = -1 \). In this case, eq(17), the \( D \)-term vanishes and the FCNC bounds are satisfied. The underlying reason for this is because the familon potential is symmetric under the interchange of \( \phi \) and \( \bar{\phi} \). Of course radiative corrections involving Yukawa couplings may spoil this symmetry but these radiative corrections are suppressed by the loop correction factor and so can satisfy the bounds even making the very conservative assumptions about mixing angles and phases discussed above. Although we illustrated the generation of a \( D \)-term by a very simple familon sector the solution applies too in the case where several familons contribute significantly to the \( D \)-term, provided all their charges have equal magnitude. In this case one may readily check that the \( D \)-term still vanishes.

The case of radiative family symmetry breaking is perhaps more interesting as it does not require the introduction of the mass scale \( \mu \). For the case \( c = -1, m^2 = \bar{m}^2 \) the initial tree level contributions cancel and we have:

\[
\langle D^2 \rangle = (\alpha_\phi - \bar{\alpha}_\bar{\phi}) m^2 \ln\left(\frac{\langle |\phi|^2 \rangle}{M_P^2}\right)
\]

Since the radiative breaking mechanism requires that the radiative corrections to the soft masses are of the same order as the tree level contributions each of the two terms is of \( O(m^2) \). Thus, if the \( D \)-term is to vanish, it is necessary for \( \alpha_\phi = \bar{\alpha}_\bar{\phi} \) corresponding to the couplings driving the radiative breaking being symmetric under the interchange of \( \phi \) and \( \bar{\phi} \). The possibility of such a symmetry is not unnatural for the class of family models discussed in [8] in which the fermion mass hierarchy is generated through the Froggatt-Nielsen mechanism through the coupling of \( \phi \) to heavy supermultiplets which come in vectorlike pairs, \( X, \bar{X} \) and \( Y, \bar{Y} \) say. Being vectorlike if the coupling \( \phi X \bar{Y} \) is allowed then so too is the coupling \( \bar{\phi} \bar{X} Y \) so it is easy to implement a symmetry connecting these terms.

The case of anomaly mediated SUSY breaking offers another way of suppressing the FCNC effects because the soft masses are given in terms of the anomalous dimensions of the fields. If \( c = -1 \) the gauge contributions to \( m \) and \( \bar{m} \) are equal. The family dependent non-gauge contributions are expected to be small leading to a suppression of the \( D \)-term as discussed above. The important point is that UV effects decouple in anomaly mediation meaning that the anomalous dimension has only contributions from fields light at the relevant scale, which here is the scale of family symmetry breaking. Provided the family dependent couplings of the familons involve only states heavier than this scale they will not split the degeneracy driven by the gauge coupling. This is the case in the class of family models discussed in [8] because there the vectorlike supermultiplet mass, \( M \), is necessarily heavier than the familon VEV to generate the small expansion parameter \( \langle \phi \rangle / M \) which drives the fermion mass hierarchy.

Yet another way of suppressing the \( D \)-term is provided by orbifold compactification of string models where the soft masses depend on the the modular weights of the superfields [9] and can be anomalously small if their modular weights are -3. Thus if the familons have this modular weight and the squarks and sleptons do not the factor \( (m^2 + \bar{m}^2/c) / \langle m_q^2 \rangle \) appearing in eq.(20) may be very small leading to the required \( D \)-term suppression.
Our discussion so far has dealt with the form of the $D$–term coming from an Abelian family symmetry. However it also applies to non-Abelian family symmetries. This may readily be seen from the fact that it is always possible to choose a basis in which the dominant $D$–term contribution to the mixing between two particular generations corresponds to a diagonal generator and thus has the same form as the Abelian case.

4.4 Gauge mediated SUSY breaking

We turn now to the case where the soft masses are due to gauge mediated SUSY breaking. Since the mediator mass is low the radiative corrections discussed in subsection 3.3 are small. This may be seen by the values in the second column of Table 3 where the bounds are close to the experimental values obtained at the electroweak scale. To be consistent with these bounds requires a larger suppression to come from the $(m^2 + \tilde{m}^2/c)/\langle m^2 \rangle$ factor than in the gravity mediated case.

Fortunately, gauge mediated models naturally provide such a suppression provided the familons have no direct coupling to the SUSY breaking sector. This follows because the gauginos do not couple directly to the familons (the familons are not charged under the SM gauge group) and so the contributions to the familon masses occur at one loop order higher than the contributions to the sfermion masses. To see this explicitly, note that the gaugino masses are generated as a one loop effect, with the heavy messenger(s) of SUSY breaking coupling directly to the gaugino. The (generation blind) contributions to sfermion masses are two loop effects through their coupling to gauginos. The only way for the gauginos to communicate the SUSY breaking to the familon sector is through the familon coupling to the sfermions making it a three loop effect with an additional loop suppression which depends on the family symmetry gauge coupling strength.

For low gauge mediation scale, $(m^2 + \tilde{m}^2/c)/\langle m^2 \rangle$ needs to take values as small as $6 \times 10^{-4}$. This is possible if the family gauge symmetry coupling is very small, $\alpha_f/4\pi < 10^{-3}$. In practice one might expect a combination of the loop factor and a mixing angle suppression below the maximum used in deriving the upper bounds will allow for a solution with a larger gauge coupling. Alternatively it may be that the gauge mediation scale is higher than 200 TeV leading to a further suppression of the bound compared to that shown in Table 3. For the case $c = -1$ the suppression is complete for the family symmetry gauge contribution because it is proportional to the square of the family charge (i.e. it generates $m^2 = \tilde{m}^2$). In this case the bounds are satisfied for any value of the gauge coupling, as the non-gauge interactions are very heavily suppressed.

5 Summary and conclusions

To summarize, we have re-examined the bounds on continuous family symmetries coming from the need to suppress the associated $D$–term contributions to sfermion masses below the experimental bounds coming from FCNC processes. The FCNC effects coming from the $D$–terms depend on unknown mixing angles and we first derived upper bounds on these effects which are independent of the mixing angles. We then compared these upper bounds with the experimental bounds in each sector, accounting for the weakening of the constraints at higher energy scales. For the case of gravity mediation the constraints are only significant for the mixing of the first two generations. We identified several ways in which these constraints are automatically satisfied without appealing to a suppression involving alignment between the fermions and sfermions. In the SUGRA and anomaly mediated cases, if the familon fields spontaneously breaking the family symmetry relating the first two generations have the same magnitude of family charge, the $D$–terms vanish up to radiative corrections which may readily be within the constraints. Even if the radiative corrections are large the $D$–terms may still be within the limits if there is an underlying symmetry relating the couplings of the familon fields and this may happen quite readily in family schemes relying on the Froggatt-Nielsen mechanism to generate the fermion mass hierarchy. Yet another possibility, motivated by orbifold string compactified models, is that the familons have modular weights such that they are anomalously light. This mechanism works irrespective of the family charge carried by the familons. For the case of gauge mediated models the lower mediation scale leads to stronger constraints which are non-trivial for all the mixings between the three generations. For arbitrary familon charges these bounds can be satisfied for small family gauge coupling if, as is generally the case, the familons couple to the messenger sector only via the quark and lepton sector. For the case that the magnitude of familon charges are equal the bounds are satisfied for arbitrary gauge coupling.

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In conclusion, the $D$–terms associated with continuous family symmetries may be consistent with the experimental bounds on FCNC for a large class of family models and SUSY breaking schemes. However in almost all cases the present bounds on the mixing between the first two families are quite close to the expected signals in these models demonstrating, yet again, the importance of improving the experimental searches for FCNC effects.

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