Higher Twist Effects in Parton Fragmentation Functions

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Abstract

We study twist expansions for parton fragmentation functions based on the definition of the twist as an invariant matrix element of a light-cone, bilocal operator. The results are then applied to a method which might be used to extract higher twist effects in the fragmentation sector using both $e^+e^-$, and $e^-p$ collisions. We discuss how to apply the later measurements to experiments at the Jefferson National Acceleration Facility.

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I. INTRODUCTION

Duality in the nucleon structure function, \( W_2(\nu, Q^2) \), was discovered before Quantum Chromodynamics (QCD) \(^1\). Thus the structure function, \( F_2(Q^2 \omega') = \nu W_2/m_N \), in the resonance region \((W < 2\text{GeV})\), is approximately the same as (duals) the deep inelastic region \((W > 2\text{GeV})\), when the functions are expressed in terms of the scaling variable, \( \omega' = 1 + W^2/Q^2 \). Here \( W \) is the final-state hadron mass. Moreover, the occurrence of duality appears to be local in the sense that it exists in each interval of \( \omega' \) over the prominent nucleon resonances.

An explanation of Bloom-Gilman duality was offered by de Rujula, Georgi, and Politzer in 1977 \(^2\). Using the operator product expansion, they represented the scaling function as a sum of various twist operators, and then studied the contributions from individual twists (moments of the scaling functions) in the scaling variable, \( \xi = 2x/(1 + (1 + 4x^2m_N^2/Q^2)^{1/2}) \), where \( x = Q^2/2m_N\nu \). They argue that the \( n \)th moment, \( M_n(Q^2) \), of \( F_2 \) has the following twist expansion,

\[
M_n(Q^2) = \sum_{k=1}^{\infty} \left( \frac{mM_0^2}{Q^2} \right)^{k-1} B_{n,k}(Q^2) .
\]  

(1)

Here, \( M_0^2 \) is a mass scale \((\approx 400 - 500\text{MeV})^2\), and \( B_{n,k}(Q^2) \) depends logarithmically on \( Q^2 \), being roughly on the order of \( B_{n,0} \). According to eq. 1, there exists a region of \( n \) and \( Q^2(n \leq Q^2/M_0^2) \) where the higher twist contributions are neither large nor negligible, and where the dominant contribution to the moments comes from low-lying resonances. In this region for example, the moments defined by eq. 1 would not correspond to positive definite functions. Thus one might expect the structure functions to oscillate when large \( k \) is important, and this would lead to the appearance of local duality. A more recent study of parton-hadron duality by Ji and Unrua \(^3\) gives more quantitative estimations.

There is no doubt that duality is a very interesting phenomenon, and it could allow one, under certain circumstances, to bridge the gap between perturbative predictions and experimentally observed quantities in non-perturbative regions \(^4\). However, duality, as expressed in the above analyses, only reflects properties of the parton structure functions. Appealing to the concept of factorization in the strong interaction, it is natural to ask whether there is duality in the parton fragmentation sector. Although in twist-two, perturbative calcula-
tions, parton structure functions and parton fragmentation functions have some similarities, this does not indicate that they have similar higher twist expansions.

Recently, an experiment at the Jefferson National Acceleration Facility, Jlab E00-108, studied duality in semi-inclusive \( e^+e^- \) reactions. Semi-inclusive reactions can reflect higher twist contributions in the parton fragmentation sector, so it is worthwhile to study these contributions in more detail in order to understand how one might extract these contributions in such reactions.

We investigate these issues in this note. In Section 2, we study the twist expansions in the fragmentation sector, and investigate twist two perturbative calculations of semi-inclusive \( e^+e^- \) reactions. We also discuss a method to extract higher twist effects in these measurements. The paper is summarized in Section 3.

II. TWIST EXPANSION IN THE PARTON FRAGMENTATION SECTOR

In this section, we study the twist expansions of the parton fragmentation functions in \( e^+e^- \) and \( ep \) semi-inclusive reactions.

A. \( e^+e^- \rightarrow h + X \)

The kinematics for \( e^+e^- \rightarrow h + X \) is illustrated in Fig. 1. Two kinematic invariants, \( Q^2 \) and \( \nu = P \cdot q \), define the process. We consider scalar particles, \( h \). The scattering matrix of the semi-inclusive reaction, \( e^+e^- \rightarrow h + X \), when calculated to lowest order in the electromagnetic interaction, is given by;

\[
\begin{align*}
  e^+(p_e) & \quad q \quad h(p_h) \\
  e(p_e) & \quad X(p_X) \\
  \end{align*}
\]

FIG. 1: Kinematics for \( e^+e^- \rightarrow h + X \).
\[ \langle hX|S_{QCD+em}|e^+e^- \rangle = \langle hX|\mathcal{T}\exp[i\int d^4\xi (\mathcal{L}_{em}^{\text{int}} + \mathcal{L}_{QCD}^{\text{int}})]|e^+e^- \rangle \]
\[ = \langle hX|\frac{(ie)^2}{2!} \int d^4\xi_1 \int_{t_0}^t d^4\xi_2 \mathcal{T}[\mathcal{L}_{em}^{\text{int}}(\xi_1)\mathcal{L}_{em}^{\text{int}}(\xi_2)] \mathcal{T}\exp[i\int d^4\xi \mathcal{L}_{QCD}^{\text{int}}]|e^+e^- \rangle + \mathcal{O}(e^2) , \tag{2} \]

Here we have defined:

\[ \mathcal{L} = eJ^\mu A_\mu , \]
\[ J_q^\mu(p_q) = \sum_q Q_q : \bar{\psi}_q(p_{q'}) \gamma^\mu \psi_q(p_q) : , \tag{3} \]

where \( Q_q \) is the charge of quark, \( q \), in units of the proton charge, \( e \).

Using Feynman rules for QED, one then finds for the kinematics of Fig. 1,

\[ \mathcal{M} = \frac{e^2}{q^2} \bar{\psi}(p_{e'}, \sigma_{e'}) \gamma_\mu u(p_e, \sigma_e) \langle hX|J^\mu(0)|0 \rangle . \tag{4} \]

Hence, the cross section can be written as:

\[ d\sigma \sim \hat{l}_{\mu\nu} \hat{W}_{\mu\nu} \frac{d^3p_h}{(2\pi)^3 2E_h} ; \tag{5} \]

with:

\[ \hat{l}_{\mu\nu} = \frac{1}{4} \sum_{\sigma_e, \sigma_{e'}} \bar{\psi}(p_{e'}, \sigma_{e'}) \gamma_\mu u(p_e, \sigma_e) [\bar{\psi}(p_{e'}, \sigma_{e'}) \gamma_\nu u(p_e, \sigma_e)]' \]
\[ - \frac{1}{2} \{ q_\mu q_\nu - q^2 g_{\mu\nu} - (p_e - p_{e'})_\mu (p_e - p_{e'})_\nu \} , \quad \text{and} \]
\[ \hat{W}_{\mu\nu} = \frac{1}{4\pi} \sum_X (2\pi)^4 \delta^4(p_h + p_X - q) \langle 0|J^\mu(0)|hX \rangle \langle hX|J'^\nu(0)|0 \rangle \]
\[ = \frac{1}{4\pi} \int d^4\xi e^{iq\cdot\xi} \sum_X \langle 0|J^\mu(\xi)|hX \rangle \langle hX|J'^\nu(0)|0 \rangle , \tag{6} \]

We have set the electron mass to zero. The sum over unobserved hadrons, \( X \), cannot be complete because the state \( |hX\rangle \) depends non-trivially on the observed hadron. Therefore one does not have \( \sum_X |hX\rangle \langle hX| = 1 \) and so \( \sum_X \langle 0|J_\mu(\xi)|hX \rangle \langle hX|J_\nu(0)|0 \rangle \neq \langle 0|J_\mu(\xi)J_\nu(0)|0 \rangle \).

Thus \( e^+e^- \rightarrow h + X \) is not controlled by the product of two operators, the operator product expansion does not apply, and no short distance analysis can be formulated.

In a reference frame where the produced hadron, \( h \), is fixed, the hadron and photon momenta can be expanded as;
\[ p_h^\mu = p^\mu + \frac{m_h^2}{2} n^\mu, \]
\[ q^\mu = \frac{1}{m_h^2} \left( \nu - \sqrt{\nu^2 - m_h^2 Q^2} \right) p^\mu + \frac{1}{2} \left( \nu + \sqrt{\nu^2 - m_h^2 Q^2} \right) n^\nu, \]

where;
\[ Q^2 \equiv (p_e + p\text{e}')^2, \]
\[ p_h \cdot q \equiv \nu, \]
\[ 0 < z \equiv \frac{2p_h \cdot q}{q^2}, \]
\[ p^\mu = \frac{m_h}{2} (1, 0, 0, 1), \]
\[ n^\mu = \frac{1}{m_h} (1, 0, 0, -1). \]

Writing;
\[ \xi^\mu = \eta p^\mu + \lambda n^\mu + \xi^{\mu \perp}, \]

one finds in the Bjorken limit;
\[ \lim_{Q^2 \to \infty} q \cdot \xi = \eta \nu - \frac{\lambda}{z}. \]

This result implies that as \( \nu \to \infty \), then \( \eta \to 0 \) and \( \lambda \sim z \). Because \( \xi^2 = \xi_0^2 - \xi_3^2 - \xi_{12}^2 \leq \xi_0^2 - \xi_3^2 \), one finds in the Bjorken limit that \( \xi^{\mu \perp} \to 0 \). Therefore in this limit, light-like separation occurs, \( \xi^\mu \xi_\mu \sim 0 \), which dominates the integration region of eq. (6).

Fragmentation is generally a non-perturbative process. We first consider the simplest quark fragmentation function represented diagrammatically in Fig. 2. More complicated fragmentation processes, such as coherent fragmentation of several quarks and gluons, do not contribute until order \( 1/Q^2 \).

In the case of Fig. 2, the diagram of \( e^+ e^- \to h + X \) shown in Fig. 1 can be re-drawn as in Fig. 3.

The fermion field can be written as:
\[ \psi(x) = \sum_s \int \frac{d^3p}{(2\pi)^{3/2}} \frac{1}{2E_p} \left[ a(p, s) u(p, s) e^{-ip \cdot x} + b^\dagger(p, s) v(p, s) e^{ip \cdot x} \right], \]

where \( a(p, s) \) (\( a^\dagger(p, s) \)) and \( b(p, s) \) (\( b^\dagger(p, s) \)) are annihilation (creation) operators for particles and anti-particles. Assuming that there is no final state interaction between \( Z \)
FIG. 2: Quark fragmentation.

FIG. 3: Kinematics for $e^+e^- \rightarrow h + X$ for quark $b(\bar{b})$ with the Fig. 2 fragmentation.

and $hY$, one can rewrite $\hat{W}^{\mu\nu}$ in eq. (6) as: Fig. 2 as:

$$\hat{W}^{\mu\nu} = \frac{1}{4\pi} \int d^4\xi e^{iq\cdot\xi} \sum_{Y,Z\{Y+Z=X\}} \langle 0 | J^\mu(\xi) | h(Y + Z) \rangle \langle h(Y + Z) | J^\nu(0) | 0 \rangle$$

$$= \frac{1}{8\pi} \int d^4\xi e^{iq\cdot\xi} \left\{ \sum_{Y,Z\{Y+Z=X\}} \langle 0 | \bar{\psi}_\alpha(\xi) | hY \rangle \gamma^\mu_{\alpha\beta} \langle 0 | \psi_\beta(\xi) | Z \rangle \langle Z | \bar{\psi}_\delta(0) | 0 \rangle \gamma^\nu_{\delta\lambda} \langle hY | \psi_\lambda(0) | 0 \rangle : + \sum_{Y',Z'\{Y'+Z'=X\}} \langle 0 | \bar{\psi}_\alpha(\xi) | Z' \rangle \gamma^\mu_{\alpha\beta} \langle 0 | \psi_\beta(\xi) | hY' \rangle \langle hY' | \bar{\psi}_\delta(0) | 0 \rangle \gamma^\nu_{\delta\lambda} \langle Z' | \psi_\lambda(0) | 0 \rangle : \right\}$$

$$= \frac{1}{8\pi} \int d^4\xi \left[ \sum_Y \langle 0 | \bar{\psi}_\alpha(\xi) | hY \rangle \gamma^\mu_{\alpha\beta} \langle 0 | \psi_\beta(\xi), \bar{\psi}_\delta(0) \rangle \gamma^\nu_{\delta\lambda} \langle hY | \psi_\lambda(0) | 0 \rangle + \sum_{Y'} \gamma^\mu_{\alpha\beta} \langle 0 | \psi_\beta(\xi) | hY' \rangle \langle 0 | \bar{\psi}_\delta(0) \rangle \gamma^\nu_{\delta\lambda} \langle hY' | \psi_\lambda(0) | 0 \rangle \right]. \quad (12)$$

To obtain this result we have used the fact that:

$$\sum_Z |Z\rangle \langle Z| = 1 \quad \text{and}$$
\[ \sum_{Z'} |Z\rangle \langle Z'| = 1 \]  

and due to the un-physical energy in the \(|Z\rangle\) state, then;

\[
\frac{1}{(2\pi)^4} \int d^4 \xi e^{iq \cdot \xi} \sum_{Z,Y \{ Y + Z = X \}} \langle 0 | \bar{\psi}_\alpha(\xi) | hY \rangle \gamma_\mu^{\alpha \beta}(0) \langle 0 | \bar{\psi}_\beta(0) | Z \rangle \langle Z | \psi_\beta(\xi) | 0 \rangle \gamma_\delta^\nu \langle hY | \psi_\lambda(0) | 0 \rangle \\
= \delta^4(p_Z + q - p_h - p_Y) \langle 0 | \bar{\psi}_\alpha(0) | hY \rangle \gamma_\mu^{\alpha \beta}(0) \langle 0 | \bar{\psi}_\beta(0) | Z \rangle \langle Z | \psi_\beta(0) | 0 \rangle \gamma_\delta^\nu \langle hY | \psi_\lambda(0) | 0 \rangle \\
= 0; \\
\]

and;

\[
\frac{1}{(2\pi)^4} \int d^4 \xi e^{iq \cdot \xi} \sum_{Z,Y \{ Y + Z = X \}} \gamma_\mu^{\alpha \beta}(0) \langle 0 | \psi_\beta(0) | hY \rangle \langle 0 | \psi_\lambda(0) | Z \rangle \langle Z | \bar{\psi}_\alpha(\xi) | 0 \rangle \langle hY | \bar{\psi}_\lambda(0) | 0 \rangle \gamma_\delta^\nu \\
= \delta^4(p_Z + q - p_h - p_Y) \gamma_\mu^{\alpha \beta}(0) \langle 0 | \psi_\beta(0) | hY \rangle \langle 0 | \psi_\lambda(0) | Z \rangle \langle Z | \bar{\psi}_\alpha(0) | 0 \rangle \langle hY | \bar{\psi}_\lambda(0) | 0 \rangle \gamma_\delta^\nu \\
= 0. \tag{14} 
\]

In this expression, summation over color and flavor is assumed.

Due to the strong interaction Lagrangian, \( \mathcal{L}_{QCD}^{int} \), eq. (12) can be modified because of the higher order contributions in perturbative QCD calculations. These interactions cause vertex corrections, gluon polarization, etc, and their contributions to the cross section give logarithmic corrections. These contributions do not change the twists of the terms in the matrix elements. However, there are terms which represent quarks propagating in a gluon background in order to preserve color gauge invariance of the bilocal operator, \( \hat{W}^{\mu \nu} \). These terms are essential to generate higher twist corrections, and can be included by changing the singular function of the free field theory \[ \{ \psi(\xi), \bar{\psi}(0) \} = \frac{1}{2\pi} \beta \epsilon(\xi_0) \delta(\xi^2) , \text{ to;} \]

\[
\{ \psi(\xi), \bar{\psi}(0) \} \rightarrow \frac{1}{2\pi} \beta \epsilon(\xi_0) \delta(\xi^2) \mathcal{P}(\exp [i \int_0^\xi d\zeta^\mu A_\mu(\zeta)]). \tag{15} 
\]

Substituting eq. (15) into eq. (12) and using the relationships;

\[
\gamma^\mu \gamma^\rho \gamma^\nu = S^{\mu \rho \sigma} \gamma_\sigma - i e^{\mu \rho \sigma} \gamma_\sigma \gamma_5 , \\
S_{\mu \rho \sigma} = \frac{1}{4} Tr (\gamma_\mu \gamma_\rho \gamma_\nu \gamma_\sigma) = g_{\mu \rho} g_{\nu \sigma} + g_{\mu \sigma} g_{\nu \rho} - g_{\mu \nu} g_{\rho \sigma}, \\
\langle 0 | \mathcal{P} (\exp [i \int_0^\xi d\zeta^\mu A_\mu(\zeta)]) | 0 \rangle = \sum_Z \langle 0 | \mathcal{P} (\exp [i \int_0^\xi d\zeta^\mu A_\mu(\zeta)]) | Z \rangle \cdot \\
\langle Z | \mathcal{P} (\exp [i \int_0^\infty d\zeta^\mu A_\mu(\zeta)]) | 0 \rangle , \tag{16} 
\]

one has,

\[
\hat{W}^{\mu \nu} = \frac{1}{8\pi} \int d^4 \xi e^{iq \cdot \xi} \sum_X \langle 0 | \bar{\psi}_\alpha(\xi) \mathcal{P} (\exp [-i \int_0^\infty d\zeta^\mu A_\mu(\zeta)]) | hX \rangle \frac{1}{2\pi} \partial_\rho \epsilon(\xi_0) \delta(\xi^2) .
\]
\[
(S^{\mu \nu \sigma} \gamma_\sigma - i e^{\mu \nu \sigma} \gamma_\sigma \gamma_5)_{\alpha \lambda} \langle hX | \mathcal{P}(e^{i \int_0^\infty d\zeta^\mu A_\mu(\zeta)}) | \psi_\lambda(0) | 0 \rangle + \\
\langle 0 | \psi_\beta(\xi) \mathcal{P}(e^{i \int_0^\infty d\zeta^\mu A_\mu(\zeta)}) | hX \rangle \frac{1}{2\pi} [ - \partial_\rho \epsilon(\xi_0) \delta(\xi^2) ] . \\
(S^{\mu \nu \sigma} \gamma_\sigma - i e^{\mu \nu \sigma} \gamma_\sigma \gamma_5)_{\delta \beta} \langle hX | \mathcal{P}(e^{-i \int_0^\infty d\zeta^\mu A_\mu(\zeta)}) | \bar{\psi}_\beta(0) | 0 \rangle \]
\[= \mathcal{P}_{h/b} + \mathcal{P}_{\bar{h}/b}, \tag{17} \]

where \( \mathcal{P}_{h/b}(\mathcal{P}_{\bar{h}/b}) \) is the fragmentation function of a quark \( b(\bar{b}) \) fragmenting into hadron \( h \).

As shown in eq. (10), light-like separation \( \xi^\mu \xi_\mu \sim 0 \) dominates the semi-inclusive process, \( e^+ e^- \rightarrow h + X \). Therefore one can write,
\[
\xi^\mu = \lambda n^\mu + \hat{\xi}^\mu, \tag{18} \]

where \( \hat{\xi}^\mu \) includes contributions from \( \eta p^\mu \) and \( \xi^{\mu \perp} \). Both \( \eta \) and \( \xi^{\mu \perp} \) go to zero \((1/\sqrt{Q^2})\) as \( Q^2 \rightarrow \infty \)

With eq. (8) and \( n^2 = p^2 = 0 \), one can expand eq. (16) in terms of \( \hat{\xi}^\mu \). This gives,
\[
\hat{W}^{\mu \nu} = \frac{1}{4\pi} \int d\lambda e^{-i\lambda z} \sum_X \{ \\
\sum_Y [(0 | \bar{\psi}_\alpha(\lambda n) \mathcal{P}(e^{i \int_\lambda^\infty d\tau n \cdot A(\tau n)}) | hX \rangle \frac{1}{2\pi} \partial_\rho \epsilon(\xi_0) \delta(\xi^2) . \\
(S^{\mu \nu \sigma} \gamma_\sigma - i e^{\mu \nu \sigma} \gamma_\sigma \gamma_5)_{\alpha \lambda} \langle hX | \mathcal{P}(e^{i \int_0^\infty d\tau n \cdot A(\tau n)}) | \psi_\lambda(0) | 0 \rangle + \\
\langle 0 | \psi_\beta(\lambda n) \mathcal{P}(e^{i \int_\lambda^\infty d\tau n \cdot A(\tau n)}) | hX \rangle \frac{1}{2\pi} \partial_\rho \epsilon(\xi_0) \delta(\xi^2) . \\
(S^{\mu \nu \sigma} \gamma_\sigma - i e^{\mu \nu \sigma} \gamma_\sigma \gamma_5)_{\delta \beta} \langle hX | \mathcal{P}(e^{-i \int_0^\infty d\tau n \cdot A(\tau n)}) | \bar{\psi}_\beta(0) | 0 \rangle \\
+ \mathcal{O}(\hat{\xi}) \} . \tag{19} \]

The \( \langle \hat{\xi} \rangle^0 \) term in eq. (19) is the same as the definition of parton fragmentation given in ref. [7]. Working in the light-cone gauge, \( n \cdot A = 0 \), explicit reference to gluons disappears, and using the definition of the twist of an invariant matrix element of a light-cone bilocal operator [6], ref. [8] studied the twist expansion of the possible terms for the production of scalar hadron or hadrons whose spins are not observable,

\[
z \int \frac{d\lambda}{2\pi} e^{-i\lambda z} \langle 0 | \gamma^\mu \psi(0) | hX \rangle \langle hX | \bar{\psi}(\lambda n) | 0 \rangle = 4 [ \hat{f}_1(z) p^\mu + \hat{f}_4(z) M^2 n^\mu ] , \]

and
\[
z \int \frac{d\lambda}{2\pi} e^{-i\lambda z} \langle 0 | \psi(0) | hX \rangle \langle hX | \bar{\psi}(\lambda n) | 0 \rangle = 4M \hat{e}_1(z) . \tag{20} \]
Here $\hat{f}_1(z)$, $\hat{e}_1$, $\hat{f}_4$ have twists 2, 3, 4, respectively; and $M$ is a generic QCD mass scale.

Since the twists for light-cone bilocal operators only represent the leading $Q^2$ dependence, it is possible for $\hat{f}_1(z)$, $\hat{e}_1$ and $\hat{f}_4$ to include multiplicative factors of $M^2/Q^2$. Therefore eq. (20) can be rewritten as:

$$z \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0|\gamma^\mu \psi(0)|hX\rangle \langle hX|\bar{\psi}(\lambda n)|0 \rangle = 4 \sum_{n=0}^\infty [p^\mu(\frac{M}{\sqrt{Q^2}})^n \hat{f}_{1n}(z) + n^\mu(\frac{M}{\sqrt{Q^2}})^{n+2} \hat{f}_{4n}(z)];$$

and:

$$z \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0|\psi(0)|hX\rangle \langle hX|\bar{\psi}(\lambda n)|0 \rangle = 4M \sum_{n=1}^\infty (\frac{M}{\sqrt{Q^2}})^n \hat{e}_{1n}(z). \quad (21)$$

One can see from eq. (17) that each $\hat{\xi}$ factor always has a gauge-covariant derivative as a companion in the $\mathcal{O}(\hat{\xi}^2)$ terms; i.e. $\hat{\xi}^\mu D_\mu$. Using the light-cone, $\hat{\xi} \rightarrow 1/\sqrt{Q^2}$ as $Q \rightarrow \infty$ and the twist analysis of bilocal operators \cite{1}, one concludes that the $\mathcal{O}(\hat{\xi})$ terms have higher twists than their corresponding ($\hat{\xi})^0$ terms. For example, a general $\mathcal{O}(\hat{\xi})$ term of order $n$ has the following twist expansion:

$$z \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \hat{\xi}_{\mu_1} \cdots \hat{\xi}_{\mu_N} \Gamma_{\alpha\beta}^{\mu_\nu} \langle 0|D_{\mu_1} \cdots D_{\mu_N} \psi_\alpha(\lambda n)|h Y\rangle \langle h Y|\bar{\psi}_\beta(0)|0 \rangle$$

$$\sim (\frac{1}{\sqrt{Q^2}})^N \sum_{j=0}^N [p^{\mu_1} \cdots p^{\mu_j} n^{\mu_{j+1}} \cdots n^{\mu_N} M^{2(N-j)} [p^{\mu} p^{\nu} \hat{f}_{N+3}^{N-j}(z) + (p^{\mu} n^{\nu} + n^{\mu} p^{\nu}) M^2 \hat{f}_{N+4}^{N-j}(z) + n^{\mu} n^{\nu} M^4 \hat{f}_{N+4}^{N-j}(z)] + \text{trace terms}. \quad (22)$$

Based on the twist analysis from bilocal operators, the leading twists for $\hat{f}_{N+2}^{N-j}$, $\hat{f}_{N+3}^{N-j}$, and $\hat{f}_{N+4}^{N-j}$ are $N + 2 + 2(N - j)$, $N + 3 + 2(N - j)$, and $N + 4 + 2(N - j)$ respectively.

Since all the trace terms have higher twists, their corresponding diagonal terms, eq. (22), can be written as:

$$z \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \xi_{\mu_1} \cdots \xi_{\mu_N} \Gamma_{\alpha\beta}^{\mu_\nu} \langle 0|D_{\mu_1} \cdots D_{\mu_N} \psi_\alpha(\lambda n)|h Y\rangle \langle h Y|\bar{\psi}_\beta(0)|0 \rangle$$

$$\sim \sum_{j=0}^N [p^{\mu_1} \cdots p^{\mu_j} n^{\mu_{j+1}} \cdots n^{\mu_N} [p^{\mu} p^{\nu} \sum_{i=0}^\infty (\frac{M}{\sqrt{Q^2}})^{N+2(N-j)+i} \hat{f}_{N+2}^{N-j,i}(z) + \sum_{i=2}^\infty (\frac{M}{\sqrt{Q^2}})^{N+2(N-j)+i} \hat{f}_{N+2}^{N-j,i}(z)]. \quad (23)$$

If there is final state interaction, there can be additional terms in $\hat{W}^{\mu_\nu}$. Ref. \cite{8} has demonstrated the existence of such a terms,

$$M_{\rho\sigma}^{\alpha} = \int \frac{d\lambda d\mu}{2\pi} e^{-i\lambda/z} e^{-i\mu(1/2-1/z)}[\langle 0|\bar{\psi}_\rho(\lambda n)|\pi(P)X\rangle \langle \pi(P)X|\bar{\psi}_\sigma(\lambda n)|0 \rangle]$$

$$+ \int \frac{d\lambda d\mu}{2\pi} e^{i\lambda/z} e^{i\mu(1/2-1/z)}[\langle 0|\psi_\rho(\lambda n)|\pi(P)X\rangle \langle \pi(P)X|\bar{\psi}_\sigma(0)iD^\alpha_{\bot}(\mu n)|0 \rangle]. \quad (24)$$
In more complicated fragmentation processes, more quarks and gluons are usually involved in the operators, and these lead to larger dimensions. In any case, twist expansions can still be carried out based on a twist analysis scheme involving bilocal operators.

In conclusion, one can carry out twist expansions for all bilocal terms in eq. (19), to obtain the twist expansion for $\hat{W}^{\mu\nu}$:

$$\hat{W} = \sum_n (\frac{M^2}{Q^2})^n f_n.$$  \hspace{1cm} (25)

Twist two contributions to $|M|^2$ are the major contributions as $Q^2 \to \infty$ or $(\frac{Q^2}{M^2}) \gg n$. As $(\frac{Q^2}{M^2}) \geq n$, higher twist contributions become important. Depending on the sign of the coefficients of $(\frac{Q^2}{M^2})^n$, the duality phenomenon in the fragmentation sector could occur. Although one cannot carry out non-perturbative calculations, the determination of the sign of $\hat{f}_n$ could be measured in duality experiments. Such experiments are straightforward in $e^+e^-$ collisions, where one can compare the cross sections of hadron, $h$, production at fixed $z$ for various $Q^2$. For a given accuracy, $e^+e^-$ collisions at lower $Q^2$ require higher-twist contributions than those at larger $Q^2$. However, it is meaningless to attempt a twist expansion in the non-perturbative region, $n \geq (\frac{Q^2}{M^2})$.

Thus the fragmentation functions can also be expanded in terms of contributions of operators of various twists, and the phenomenon of duality can appear if the signs of the coefficients of the higher twist contributions are not positive definite. Because complete non-perturbative calculations are not presently possible, a comparison of measurements of fragmentation at various $Q^2$, having the same $z$, can provide information on the coefficients of higher twist contributions.

Although we have studied only the production of scalar hadrons, a richer structure occurs if one includes polarization effects. \[6, 8\].

B. The process $e^- p \to h + X$

The scattering matrix of the process $e^- + p \to h + X$ is given by,

$$W = \langle hX | T \exp[i \int d^4x (L^{\text{int}}_{\text{em}} + L^{\text{int}}_{\text{QCD}})] | ep \rangle.$$  \hspace{1cm} (26)

The lowest order of the electromagnetic contribution is shown in Fig. \[\text{Fig. 4}\].
FIG. 4: Semi-inclusive $ep$ scattering.

The scattering matrix can be written as

$$W = \bar{\psi}_e(p_e')\gamma^\mu \psi_e(p_e) \frac{1}{q^2} \langle hX|J_\mu \exp(i \int d^4\xi L_{QCD}^\text{int})|p\rangle + \mathcal{O}(e^2), \quad (27)$$

where $\hat{J}_\mu$ is the proton electromagnetic current, and depends on the proton structure.

From eq. (26), one has,

$$|W|^2 = \bar{\psi}_e(p_e')\gamma^\mu \psi_e(p_e)\bar{\psi}_e(p_e')\gamma^\nu \psi_e(p_e) \frac{1}{q^2} \sum_X \langle p|J_\mu^\dagger|hX\rangle \cdot \langle hX|J_\nu|p\rangle. \quad (28)$$

This is similar to $e^+e^- \rightarrow h + X$, where $\sum_X \langle hX|hX\rangle \neq 1$ due to the non-trivial dependence of $|hX\rangle$ on the observed hadron, $h$. One can work out a similar expansion as was done in the last subsection, to obtain the twist expansion of eq. (28). The major difference between eq. (28) and eq. (5) is that eq. (28) contains information on the parton structure functions in addition to information on parton fragmentation functions.

The phenomenon of duality in parton structure functions has been established through inclusive $ep$ scattering. In order to extract higher twist information in the parton fragmentation sector using semi-inclusive $ep$ scattering, one needs to separate the contributions from the parton fragmentation and structure sectors. If one allows factorization in the strong interaction, one should be able to separate the scale, $Q_s^2$, in the structure sector from the scale, $Q_F^2$, in the fragmentation sector.

In general, it is more advantageous to obtain higher twist information in the fragmentation sector using $e^+e^-$ scattering. In order to extract information from $ep$ scattering, one needs
to subtract the contributions from the structure sector. This can be achieved by suitably choosing $Q^2_S$ and $Q^2_F$ in an experiment. One way to achieve this is to keep the $Q^2_S$ sufficiently high so that the contributions from the parton structure sectors are mainly from twist two operators which can be confidently calculated. Then one can suitably choose the $Q^2_F$ functions in the fragmentation sector to obtain information of higher twist contributions. Therefore to quantitatively study higher-twist contributions in the parton fragmentation sector, it is critical to identify $Q^2_S$ and $Q^2_F$ and separate the parton fragmentation sector from the structure sector.

An experimental effort to detect higher twist effects in the fragmentation sector through semi-inclusive $e^-p \rightarrow h + X$ scattering has been recently undertaken at Jlab. We comment on this process in more detail here with the intent to clarify the experimental results. The object of such an experiment would be to determine higher twist effects by comparing measurements which include effects from all twists to the calculated twist two results. As discussed previously, in order to study higher twist effects in fragmentation sectors through semi-inclusive $e^-p \rightarrow h + X$ scattering, it is important to subtract the contributions from the parton structure functions. This is feasible by choosing high $Q^2_S$ in the structure sector so that these contributions are predominantly from twist two operators which can be calculated perturbatively by using the well-known information on twist two parton structure functions. One must have measurements in a sufficiently high region of $Q^2_S$, with various $Q^2_F$, in order to compare the measurements with calculations of twist two contributions in the same regions. The differences between the measurements and the calculations provide information of higher twist effects in the fragmentation sector. Therefore we need to locate the regions with high $Q^2_S$ and various $Q^2_F$ to carry out the twist-two calculations.

If $Q^2_S$ is large enough so that the contributions from the parton structure sectors are twist two, the scattering process $e^-p \rightarrow h + X$ is illustrated in Fig. 5. Before presenting a calculation of the process shown in Fig. 5 we would like to make several remarks.

1. In a momentum infinitive reference frame where $|\vec{p}| \gg m_p$, parton momentum and energy are usually expressed as a fraction $x$ of the momentum (energy) of the parent proton momentum (energy), where $x = \frac{Q^2}{2p_e}$. In the CMS or Lab frame, a Jlab beam energy ($E_e \sim 5.5 GeV$) is too low to use $x = \frac{Q^2}{2p_e}$ as the momentum fraction since the beam momentum $p_e$ in both CMS and Lab does not satisfy $|\vec{p}_e| \gg m_p$. In our
approach in this section, we work in the target rest frame. While the target mass, $m_p$, is non-negligible when compared to the beam energy, the quark masses are. Therefore, the quark energy and momentum in the target rest frame can be expressed as;

$$E_q = x m_p, \quad |\vec{p}_q| = x m_p. \quad (29)$$

In a finite momentum reference frame, parton transverse momentum cannot be ignored and the parton structure function $f_q$ should depend on $x$, $Q^2$, and $p^T_q$, i.e. $f_q = f_q(x, Q^2, p^T_q)$, where $x = E_q/E_{\text{targ}}$. This structure function should be the same as the one measured in an infinite momentum reference frame, if one boosts the reference frame to a very high momentum so that $p^T_q$ is negligible. In a target-rest reference frame, the probability for a parton to have a fraction $x$ of the parent proton’s energy should be the same as the corresponding parton structure function measured in momentum-infinitive reference frame because $x = E_q/E_{\text{targ}}$. However, the direction of the momentum is completely random and satisfies $\sum_q \vec{p}_q \equiv 0$.

2. In $e^+e^-$ scattering, the $Q_F^2$ is defined as $Q_F^2 = (p_{e^+} + p_{e^-})^2$ and is equal to $4E_{\text{beam}}^2$ in the CMS. If one concentrates on the fragmentation functions given by Fig. 2, the energy of the produced quarks, $E_q$, is equal to $E_{\text{beam}}$ in the CMS. Therefore $Q_F^2$ can be written as;

$$Q_F^2 = 4E_q^2. \quad (30)$$

The fraction of energy carried by the produced hadron $1$ from the parent parton is $z = 2p_h \cdot q / q^2$, which is equal to $E_h/E_q$ in the CMS.
In semi-inclusive $e^-p$ scattering, one usually defines (see Fig. 4)

\[
Q_F^2 \equiv -q_S^2, \\
z \equiv \frac{pp \cdot ph}{pp \cdot q} = \left(\frac{E_h}{q^0}\right)_{\text{lab. frame}}.
\]

(31)

In order to adopt the fragmentation functions obtained from $e^+e^-$ scattering in the $e^-p$ scattering process, i.e.

\[
\mathcal{P}^{ep}_{h/q}(Q_F^2, z) = \mathcal{P}^{ep}_{h/q}(Q_F^2, z') = \mathcal{P}^{e^+e^-}_{h/q}(Q_F^2, z'),
\]

one should use the same definitions used in $e^+e^-$ scattering. Therefore, we correspondingly redefine the $Q_F^2$ and $z$ in $e^-p$ scattering as

\[
z \equiv \frac{pp \cdot ph}{pp \cdot pq'} = \left(\frac{E_h}{E_q'}\right)_{\text{lab. frame}}, \\
Q_F^2 \equiv \frac{4(pp \cdot pq')^2}{p_p^2} = \left(\frac{4E_q'^2}{\text{lab. frame}}\right).
\]

(32)

Compared to $Q_F^2$ and $z$ in $e^-e^+$ scattering, we believe that these definitions are reasonable and the new $Q_F^2$ is at least proportional to $Q_F^2$ in $e^-e^+$.

3. The $Q_S^2$ in $ep$ scattering is defined as usual as;

\[
Q_S^2 = -q_S^2 = -(p_e - p_{e'})^2 \simeq 2E_eE_{e'}(1 - \cos \theta_{ee'}). 
\]

(33)

The differential cross section of the process $e^-p \rightarrow h + X$ is given by,

\[
d\sigma(e^-p \rightarrow h + X) = \sum_q \int dx f_q(x) \frac{dQ_q}{4\pi} \frac{1}{|\vec{v}_e - \vec{v}_q|2E_e2E_q|\mathcal{M}|^2} \frac{1}{(2\pi)^2} \cdot \\
\delta^4(p_{e'} + p_{q'} - p_e - p_q)\frac{d^3p_{e'}}{2E_{e'}}\frac{d^3p_{q'}}{2E_{q'}}\frac{d^3q}{2E_q}D_h(Q_F^2, z)dz, 
\]

(34)

where $p_e, p_{e'}, p_q$, and $p_{q'}$ are four momenta of the incoming electron, scattered electron, quark within a target proton, and scattered quark, respectively. Also $f_q(x)$ is the structure
function of quark $q$ in a target proton with $x$ fraction of the proton energy, $D_h^q(Q_s^2, z)$ is the fragmentation function of quark $q$ fragmenting to hadron $h$ with $z$ fraction of the quark $q$ energy, $\Omega_q$ is the solid angle of quark $q$ within the target in the lab frame, and the $\delta$-function reflects energy-momentum conservations among $p_e$, $p_e'$, $p_q$, and $p_{q'}$.

We define the electron beam direction as the positive $z$ direction and let;

$$W_1 = E_e E_{q'} (1 - \cos \theta_{q'}) + \frac{Q_s^2}{2 E_e} (E_e - E_{q'} \cos \phi_{q'}) ,$$
$$W_2 = W_1 E_{q'} (1 - \cos \theta_{q'}) + \frac{Q_s^2}{2 E_e} E_{q'}^2 \sin^2 \theta_{q'} \cos^2 (\phi_{e'} - \phi_q) ,$$
$$W_3 = W_1^2 + (\frac{Q_s^2}{2 E_e})^2 E_{q'}^2 \sin^2 \theta_{q'} \cos^2 (\phi_{e'} - \phi_q) .$$

Eq. (34) can be rewritten as;

$$\frac{d\sigma(e^- p \rightarrow h + X)}{dQ_s^2 dQ_h dz} = \frac{1}{(16\pi)^3 m_P^2} \sum_q \int d\phi_{e'} d\cos \theta_{q'} d\phi_q \frac{1}{|\vec{v}_e - \vec{v}_h| E_e E_{q'}^2 f_q(x)} .$$

$$\frac{1}{x^2 (1 - \cos \theta_{q'})^2} |\mathcal{M}|^2 D_h^q(Q_s^2, z) ,$$

where

$$E_{e'} = \left\{ \begin{array}{ll}
W_2 + \sqrt{W_2^2 - W_3 E_{q'}^2 (1 - \cos \theta_{q'})^2} & \text{if } \cos(\phi_{e'} - \phi_{q'}) \geq 0 \\
\frac{W_2 - \sqrt{W_2^2 - W_3 E_{q'}^2 (1 - \cos \theta_{q'})^2}}{W_2 - \sqrt{W_2^2 - W_3 E_{q'}^2 (1 - \cos \theta_{q'})^2}} & \text{if } \cos(\phi_{e'} - \phi_{q'}) < 0
\end{array} \right. ,$$

$$x = \frac{E_{e'} + E_{q'} - E_e}{m_P} ,$$
$$\cos \theta_q = \frac{E_{e'} \cos \theta_{e'} + E_{q'} \cos \theta_{q'} - E_e}{x m_P} ,$$
$$\sin \theta_q = \frac{\sqrt{E_{e'}^2 \sin^2 \theta_{e'} + E_{q'}^2 \sin^2 \theta_{q'} + 2 E_{e'} E_{q'} \sin \theta_{e'} \sin \theta_{q'} \cos(\phi_{e'} - \phi_{q'})}}{x^2 m_P^2} ,$$
$$\cos \phi_q = \frac{E_{e'} \sin \theta_{e'} \cos \phi_{e'} + E_{q'} \sin \theta_{q'} \cos \phi_{q'}}{x m_P \sin \theta_q} ,$$
$$\sin \phi_q = \frac{E_{e'} \sin \theta_{e'} \sin \phi_{e'} + E_{q'} \sin \theta_{q'} \sin \phi_{q'}}{x m_P \sin \theta_q} .$$

Using the parton structure functions from ref. \[10\] and the pion fragmentation functions from ref. \[11\], the pion production cross sections versus $z$ at $Q_s^2 = 10.0 \text{GeV}^2$ and $E_{q'} = 5.0 \text{GeV}$ and $4.0 \text{GeV}$ are plotted in Fig. 6. In general, the production cross section is small and the choices of $E_{q'}$ that satisfy all conditions are limited. This can be seen from the very
small cross section at \( E_{q'} = 4.0 \, \text{GeV} \) in Fig. 6 when \( Q_S^2 = 10.0 \, \text{GeV}^2 \). This problem can be avoided by lowering the beam energy, which leads to lower \( Q_S^2 \) and more choices of \( E_{q'} \). However, lower \( Q_S^2 \) brings more contributions from higher twist operators in the structure sector. When the magnitude of \( Q_S^2 \) is sufficient that the parton structure functions are safely represented by twist 2 operators, one can compare the hadron production data at various \( E_{q'} \) to obtain information on higher twist contributions in the fragmentation sector.

![Graph](image)

**FIG. 6:** Calculations of \( d\sigma/dQ_S^2 \, dQ_T^2 \, dz \) versus \( z \) for \( Q_S^2 = 10.0 \, \text{GeV}^2 \) and \( E_{q'} = 5.0 \, \text{GeV} \) and \( 4.0 \, \text{GeV} \).

The expressions given in eq. (34) are given for the purpose of calculating the hadron production cross section versus \( z \) at \( Q_S^2 \) and \( E_{q'} \). In experiments, the quantities that can be directly measured are \( E_{e'}, \theta_{e'}, \phi_{e'}, E_h \), and approximately \( \theta_{q'} \) and \( \phi_{q'} \), if one takes \( \theta_{q'} \approx \theta_h \) and \( \phi_{q'} \approx \phi_h \). The quantities that can not be directly measured are \( x, \theta_q, \phi_q, E_{q'} \), and \( z \). One can determine \( x, \theta_q, \phi_q, \) and \( E_{q'} \) based on energy and momentum conservation and the measurements of \( E_{e'}, \theta_{e'}, \phi_{e'}, \theta_{q'} \), and \( \phi_{q'} \) with the following equations,

\[
Q_S^2 = 2E_e E_{e'}(1 - \cos \theta_{e'}),
\]

\[
E_{q'} = \frac{E_e E_{e'}(1 - \cos \theta_{e'})}{E_{e'}(1 - \cos \theta_{e'} Q_T^2) - E_e(1 - \cos \theta_{q'})},
\]

\[
x = \frac{E_{e'} + E_{q'} - E_e}{m_P},
\]

\[
\cos \theta_e = \frac{E_{e'} \cos \theta_{e'} + E_{q'} \cos \theta_{q'} - E_e}{m_P}.
\]
\[
\sin \theta_e = \frac{(E^2_e \sin^2 \theta_{e'} + E^2_q \sin^2 \theta_{q'} + 2E_{e'}E_{q'} \sin \theta_{e'} \sin \theta_{q'} \cos(\phi_{e'} - \phi_{q'})^{1/2})}{m_P},
\]
\[
\cos \phi_e = \frac{E_{e'} \sin \theta_{e'} \cos \phi_{e'} + E_{q'} \sin \theta_{q'} \cos \phi_{q'}}{m_P \sin \theta_e \cos \phi_e},
\]
\[
\sin \phi_e = \frac{E_{e'} \sin \theta_{e'} \sin \phi_{e'} + E_{q'} \sin \theta_{q'} \sin \phi_{q'}}{m_P \sin \theta_e \sin \phi_e},
\]
\[
\cos \theta_{e'q'} = \cos \theta_{e'} \cos \theta_{q'} + \sin \theta_{e'} \sin \theta_{q'} \cos(\phi_{e'} - \phi_{q'}).
\] (38)

Also \(z\) can be determined through the above determination of \(E_{q'}\) and the measurements of \(E_h\),
\[
\frac{E_h}{E_{q'}}.
\] (39)

With eqs. (38) and (39), one can determine \(Q^2_S\), \(Q^2_F\), and \(z\) of the hadron production processes and then obtain the cross section versus \(z\). Comparing the cross sections at various \(Q^2_F\) but with the same \(Q^2_S\) and \(z\), one can deduce the higher twist contributions from the fragmentation sector.

### III. SUMMARY

We have studied possible higher twist QCD contributions in the fragmentation sector. In summary we find the following.

1. The contributions to \(e^-e^+ \to h + X\) cross section can be expanded as contributions of operators with various twists. If \(Q^2_F \to \infty\), twist-two operators are the dominant contribution. As one decrease \(Q^2_F\), higher-twist contributions become more and more important. If the sign of the coefficients in higher-twist contributions are not positive definite, oscillation similar to duality in \(e^-p\) inclusive scattering could appear. As \(Q^2_F\) becomes low, QCD becomes completely non-perturbative, and the twist expansion becomes meaningless.

2. A study of higher-twist contributions can be carried out through \(e^+e^- \to h + X\) by comparing measurements of cross sections versus \(z\) at various \(Q^2_F\). In order to carry out the same study using \(e^-p \to h + X\), one needs to make sure that the higher twist contributions from the parton structure sector are well determined, so that one can isolate the contributions from the fragmentation sector. One way to achieve this is...
to choose large enough $Q_2^S$ so that the contributions to the cross section from the structure sectors are dominantly from twist-two operators, which can be calculated with well measured structure functions and perturbation QCD. To achieve higher $Q_2^S$ but different $Q_F^2$, one can suitably choose the detector angles carefully in $e^- p \rightarrow h + X$ experiments for various beam energies.

3. We have not attempted to study the evolution of the ratio,$\sigma_{e^-p\rightarrow hX}(Q_2^S)/\sigma_{tot}(Q_2^S)$ as a function of $z$. However we suggest that this ratio contains information of the higher twist effects in the fragmentation sector and should be studied in the future.

4. To adopt the fragmentation functions obtained in $e^- e^+ \rightarrow h + X$ in the process of $e^- p \rightarrow h + X$, redefinitions of $Q_F^2$ and $z$ are necessary. We currently use the definitions in eq. (31).

More studies on this subject, especially considering the case of polarization, are necessary.

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