The gravitational collapse of a dust ball
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Abstract It is shown that the description of collapse given by the classic model of Oppenheimer and Snyder fails to satisfy a crucial matching condition at the surface of the ball. After correcting the model so that the interior and exterior metrics match correctly, it is established that the contraction process stops at the Schwarzschild radius, that there is an accumulation of particles at the surface of the ball, and that in the limit of infinite time lapse the density of particles at the surface becomes infinite. A black hole cannot form. This result confirms the judgements of both Einstein and Eddington about gravitational collapse when the collapse velocity approaches that of light.

1 Introduction

The Relativistic Theory of Gravitation (RTG)[1] may be considered an example of a class of theories introduced by Rosen[2], known as bimetric theories. Coexisting with the field, or Riemann metric
\[ ds^2 = g_{\mu\nu}dx^\mu dx^\nu, \]  
there is a space, or Minkowski metric
\[ d\sigma^2 = \gamma_{\mu\nu}dx^\mu dx^\nu. \]  
Unlike in General Relativity (GR), where the field is synonymous with the geometry, we recognize that the field, as in the electromagnetic case, is propagated through an underlying Minkowski space. In RTG there is a preferred coordinate system, namely that for which the space metric is Galilean. It may be called the inertial system or frame, and its corresponding field metric \( g_{\mu\nu} \) gives rise to a gravitational potential \( \Phi_{\mu\nu} = g^{\mu\nu}\sqrt{-g} \) whose divergence is zero. The latter condition may be made covariant by requiring the covariant divergence of the gravitational field in the Minkowski metric to be zero. This is the coordinate system which Einstein[3] used in order to derive his formula for the gravitational radiation emitted from a time-varying quadrupole source. In this article I shall show how the comoving coordinate frame introduced by Tolman[4], and developed by Oppenheimer and Snyder[5][6] (OS) to describe gravitational collapse, may be transformed to the inertial, or harmonic frame. It is then possible to track the trajectories of individual particles in the collapse of a dust ball, for which the equation of state is simply \( p = 0 \). We establish that the collapse can go no further than the Schwarzschild radius, that the collapse to this state takes an infinite time, and that the density of particles in the limit becomes infinite at the surface of the ball. This confirms the description given by Logunov[1], in which gravity changes from being attractive to repulsive for certain high-density conditions. The description of OS, namely "continued
gravitational contraction”, which may be considered the ancestor of the contemporary "black hole", is now seen, in the light of RTG, to be incorrect. It is more appropriate to call the process one of gravitational compression; the combination of an attraction of the surface particles with a repulsion of the particles beneath the surface produces an infinite density at the surface; presumably this balance of purely gravitational forces will be modified by the action of nuclear forces in an actual neutron star, so the state of infinite density is never reached. But, in any case, contraction does not go beyond the Schwarzschild radius, as I shall be able to show by tracking the trajectories of individual particles. The results obtained here confirm the intuitions of both Eddington[7] and Einstein[8], as expressed in the early period of the black-hole hypothesis. Particles which crossed the "event horizon" would acquire velocities exceeding that of light, which runs contrary to the prescription of Einstein’s Special Theory of 1905; so black holes are ruled out by that theory.

It may be the case that a modified Einstein-Hilbert equation incorporating a nonzero cosmological constant, a natural and possibly even necessary feature of RTG, will, as suggested in a recent article of Gerstein, Logunov and Mestvirishvili[9], stop the compression process after a certain time and convert it into a cyclical, or pulsating, process. However, it is not clear what will happen if, as seems plausible, nuclear forces intervene before these extra gravitational terms can take effect.

Although RTG is a distinct theory, constructed in opposition to GR and following ideas planted by earlier workers like Rosen[2] and Fock[10], it is possible to view the results obtained in the present article as simply a way of transforming a long established solution of the GR field equations, namely that of Tolman, into a more realistic coordinate system. Previous attempts of this nature, starting from the Schwarzschild solution, are those of Eddington and Finkelstein and of Kruskal, described in the book of Landau and Lifshitz[6] (see also Hartle[11]), but these made no attempt to penetrate into the interior of the collapsing object.

2 The Oppenheimer-Snyder metric

The Oppenheimer-Snyder (OS) field metric for a dust ball may be written

\[ ds^2 = d\tau^2 - V^2 dR^2 - W^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]

(3)

where

\[ \frac{W}{2m} = \left( \sqrt{R^3 - \frac{3\tau}{4m}} F(R) \right)^{2/3}, \]

\[ \frac{V}{2m} = \frac{2m\sqrt{R} - \tau F'(R)}{\sqrt{2mW}}. \]

(4)

In this comoving metric the free-fall radial geodesics are simply \( R = \)constant, and the coordinate \( \tau \) is the particle’s proper time; in particular the surface of the ball is specified by a fixed \( R \), which will be put as 1. The function \( F(R) \), or
rather the product $F F'$, gives the mass distribution of the dust particles. For
$R > 1$ (the external region), we put
\[ F(R) = 1 \quad (R > 1) \quad , \]
giving zero density there, and for $R < 1$ (the internal region), $F$ is left arbitrary
for the moment, but we note that, for the metric to be continuous it has to satisfy
\[ F(1-) = 1, \quad F'(1-) = 0 \quad . \]
At this point OS made a fatal error by choosing an $F$ which fails to satisfy the
second of these, so my choice of $F$ will constitute a corrected version of OS. The
cumulative mass distribution is given by the function
\[ M(R) = \int_0^R 4\pi T^{00}(R') \sqrt{-g(R')dR'} = m F^2(R) \quad . \]
The harmonic coordinates[10][12][14] $(t, r, \theta, \phi)$ for this system are given by
the solutions of
\[ \Box t = 0, \quad \Box r = -\frac{2r}{W^2} \quad , \]
where the spherical d’Alembertian operator is given by
\[ \Box = \partial^2_\tau - \frac{1}{V^2} \partial^2_R + \left( \frac{\dot{V}}{V} + \frac{2W}{V} \right) \partial_\tau + \frac{1}{V^2} \left( \frac{V'}{V} - \frac{2W'}{W} \right) \partial_R \quad , \]
and we use dot and prime to signify differentiation with respect to $\tau$ and $R$
respectively. The solution will be chosen to satisfy the asymptotic conditions
\[ t \sim \tau, \quad r \sim W \quad (\tau \to -\infty) \quad , \]
and in that case the evolution $R =$constant describes, in the asymptotic regi-
on, the Newtonian collapse of a dust ball of uniform density. This may be
demonstrated from the relation between $r$ and $t$, which is
\[ r(R, t) \sim \left[ \frac{9m|t|^2 F^2(R)}{2} \right]^{1/3} \quad (t \to -\infty) \quad , \]
which, combined with (7), gives
\[ M(R) = m \left[ \frac{r(R, t)}{r(1, t)} \right]^3 \quad , \]
and shows that the mass contained in a ball of radius $r$ is proportional to the
ball’s volume. The $t$-dependence of $r$ has, of course, been known since Michell
discovered it in the eighteenth century.
In the exterior region the solutions of (8) are[14]
\[ t = \tau - 2\sqrt{2mW} + 2m \ln \frac{\sqrt{W} + \sqrt{2m}}{\sqrt{W} - \sqrt{2m}}, \quad r = W - m \quad (R > 0) \quad , \]
and the OS metric becomes, in this region,

\[ ds^2 = \frac{r - m}{r + m} dt^2 - \frac{r + m}{r - m} dr^2 - (r + m)^2 \left( d\theta^2 + \sin^2\theta d\phi^2 \right), \tag{14} \]

which is the Schwarzschild metric, except that the Schwarzschild ”radius” \( r \) has been replaced by \( r + m \). We shall see that the solution of (8) may be continued into the interior region all the way to \( R = 0 \), and that this corresponds to \( r = 0 \) for all \( t \). I deduce that it is this \( r \), rather than Schwarzschild’s, which should be regarded as the true radius, a conclusion which will be reinforced by making a closer examination of the internal solutions of (8). It is clear that the above solution (13) gives, as \( t \) goes to plus infinity, that \( W \) approaches \( 2m \) and \( r \) approaches \( m \), and that these limits are reached at a finite value of proper time

\[ \tau_f (R) = \frac{4m}{3} \left( \sqrt{R^3} - 1 \right) \quad (R > 1) \quad . \tag{15} \]

The fact that this is finite has led to the widespread, indeed almost universally held, conclusion that a falling particle goes on to cross the ”event horizon” at \( r = m \) for proper times greater than \( \tau_f \), and therefore to be swallowed by the black hole at \( r = 0 \). I shall show that this conclusion is incorrect.

All I have to do is demonstrate that the interior solutions of (8) give a limiting value \( r_f (R) \) in \( 0 < R < 1 \), with \( r_f (0) = 0 \) and \( r_f (1) = m \), together with a corresponding \( \tau_f (R) \), for which \( t \) goes to plus infinity. I have been able to make the demonstration numerically for \( r_f (R) \) with a particular choice of the function \( F(R) \). However, for general \( F \), the values of \( \tau_f (R) \) may be obtained simply by examining the characteristics of (8). These satisfy, for both of these partial differential equations, the same pair of ordinary differential equations

\[ \frac{d\tau}{dR} = \pm V(\tau, R) = \pm \left( 2m \sqrt{R} - \tau F' \right) \sqrt{\frac{2m}{W}} \quad . \tag{16} \]

It is a simple matter to verify that the above value of \( \tau_f (R) \) satisfies this with the upper sign in \( R > 1 \), where \( F = 1 \) and \( F' = 0 \). In the interior region the upper-sign characteristic through the point

\[ \tau_f (1) = 0 \quad , \tag{17} \]

is the first one to be met by a geodesic \( R = \text{constant} \) coming from \( \tau = -\infty \). The situation is illustrated in Figure 1, where I have plotted this characteristic, in both exterior and interior regions, for the case \( 2m = 1 \), using in the interior the function \( F \) of my previous article\[13\], namely

\[ F(R) = R^{3/2} e^{3X/2}, \quad X = 1 - R \quad (R < 1) \quad . \tag{18} \]

The figure represents the whole of space, that is \( 0 < r < +\infty \), and the whole of time, that is \( -\infty < t < +\infty \); there are no singularities and no trapped surfaces of the kind supposed by Penrose and Hawking\[15\]. Such strange objects are to
Figure 1: The limit of physical space-time with $2m = 1$. $\tau$ is the proper time of a dust particle and $R$ is its comoving coordinate, so that $R = 1$ indicates a surface particle. A given dust particle, or in the exterior region a test particle, moves along the abcissa $R =$constant, arriving at the boundary curve after an infinite time $t$. 


be found in the region beyond \( t = +\infty \), and belong in the realm of science fiction. The finite value of the proper time \( \tau_f \) is an indication that a falling particle, as it approaches the boundary characteristic passing through the point \( R = 1, \tau = 0 \), suffers an infinite gravitational red shift. We shall see a natural explanation for this in the infinite surface density of the dust ball in the limit \( t \to +\infty \).

The integration of (8) in the interior region may be best achieved by changing to the characteristic coordinates \((R, S)\), defined by

\[
\tau = \tau(R, S), \quad \tau(1, S) = -2mS, \quad D_R\tau = V = \left(2m\sqrt{R} - \tau F'\right)\sqrt{\frac{2m}{W}}.
\]

The family of curves given by \( S = \text{constant} \) are the upper characteristics of (8), and in particular the boundary characteristic is \( S = 0 \). In terms of these coordinates the d’Alembertian (9) is transformed by putting

\[
D_R = \partial_R + V\partial_\tau,
\]

where \( D_R \) is the derivative with respect to \( R \) along the characteristic, giving

\[
\Box = -\frac{1}{V^2}D_R^2 + \frac{2}{V}\partial_\tau D_R + \frac{1}{V^2}\left(V' - \frac{2V}{W}\right)D_R + \frac{2}{W}\left(1 + \dot{W}\right)\partial_\tau.
\]

Note that \( \partial_\tau \) does not commute with \( D_R \), but that if we write it as

\[
\partial_\tau = \Psi^{-1}D_S, \quad \Psi = \partial_S\tau(R, S),
\]

then \( D_R \) and \( D_S \) commute. In the numerical procedure, described in the next Section, there is no need to find the function \( \Psi \) explicitly, because we work with the differential operator \( \partial_\tau \) rather than \( D_S \). Although this integration is rather formidable, it may be guaranteed that no singularity occurs in the whole physical region, because the coefficients of the PDEs are nonzero and nonsingular, and the determinant of the second-order coefficients is negative throughout, thereby preserving their hyperbolic character.

Note that the characteristic curves we just introduced are also the null geodesics of the (corrected) OS metric. They are the paths taken by outward going light signals, and since the local light velocity is \( V \), their form underlines the importance of the correction I made to the original OS metric, insisting on \( V \) being continuous at \( R = 1 \). The time required for such a signal to go from an internal point \((R_1, S)\) to an external point \((R_2, S)\) may be written as

\[
t(R_2, S) - t(R_1, S) = t(1, S) - t(R_1, S) + G(R_2, S),
\]

where \( G \) is the travelling time in the exterior region, that is

\[
G(R_2, S) = t(R_2, S) - t(1, S).
\]

In view of the simple form (14) of the exterior metric, this latter integral may be simplified to give

\[
G(R_2, S) = r(R_2, S) - r(1, S) + 2m \ln \frac{r(R_2, S) - m}{r(1, S) - m}.
\]
Now, since $r(1,S) > m$ for all $t$, this result shows that any light signal emitted from inside the ball eventually reaches the exterior region, and this should lay to rest all preexisting ideas regarding trapped surfaces. But note that I said "eventually"; owing to the infinite red shift suffered at the boundary $R = 1$, this travel time becomes infinite as $r(1,S)$ approaches $m$, that is in the closing stages of the compression process.

3 Numerical integration

It is convenient to define the operator

$$P = -V^2\Box = D_R^2 - 2\xi W \partial_r D_R - 2\xi^2 W \left(1 + \dot{W}\right) \partial_r + \left(\xi - \frac{\xi'}{\xi}\right) D_R,$$

where $\xi = V/W$, that is, for the choice we made in (18) for $F$,

$$\xi = R^{-1} \left(1 + \frac{3\tau}{4m} X e^{3X/2}\right) \left(1 - \frac{3\tau}{4m} e^{3X/2}\right)^{-1},$$

and the harmonic coordinates then satisfy

$$Pt = 0, \quad (P - 2\xi^2) r = 0.$$  (28)

These PDEs must be integrated in $0 < R < 1, S > 0$ with surface boundary conditions at $R = 1$

$$t(1,S) = -2mS - 2\sqrt{2mW_0} + \ln \frac{\sqrt{W_0} + \sqrt{2m}}{\sqrt{W_0} - \sqrt{2m}}, \quad r(1,S) = W_0 - m,$$  (29)

where

$$W_0(S) = W(1,S) = 2m \left(1 + \frac{3S}{2}\right)^{2/3},$$  (30)

and also with the asymptotic condition (10) for $S \to \infty$ and with finite values at $R = 0$.

Because $t(1,S)$ becomes infinite at $S = 0$, the integration must be over a range $S \geq S_1 > 0$, and we may then convert each PDE into a set of coupled ODEs in $R$, starting from $t(1,S)$ and $r(1,S)$, for a discrete set of $N$ values of $S$ between $S_1$ and an upper limit $S_2$ for which the asymptotic values may be used. Because $S$ is itself defined by the first-order ODE (19), we retain $\tau$ as the dependent variable, so there are a total of $3N$ coupled ODEs for $t, Dt$ and $\tau$. Fuller details, including the asymptotic matching procedure, are given in the Appendix.

From the solutions $r(R,S)$ and $t(R,S)$ I interpolated to obtain $r(R,t)$. Putting $r_0(t) = r(1,t)$, the relative position of a dust particle whose position
Figure 2: Evolution of the particle distribution $u(R)$. $u$ is the position of a given particle relative to the surface, indexed by its comoving coordinate $R$. The lower bold curve gives $u(R)$ in the limit $r_0 \to \infty$, that is $t \to -\infty$, while the upper bold curve gives $u(R)$ for $r_0 = m$, that is $t \to +\infty$. The lighter curves give $u(R)$ for the values (i) $r_0 = 2m$ (ii) $r_0 = 1.4m$ (iii) $r_0 = 1.1m$.

in the ball is indexed by $(R, \theta, \phi)$, with $0 < R < 1$, may then be described by the coordinate

$$u(R, r_0) = \frac{r[R, t(r_0)]}{r_0} \quad (0 < R < 1), \quad (31)$$

I have plotted, in Figure 2, $u(R, r_0)$ against $R$ for various values of $r_0$. In the early stage of collapse the slopes at both $R = 0$ and $1$ increase as $r_0$ decreases, indicating that particles near the centre move towards the surface and particles near the surface move towards the centre. However, towards the end of the process the curve $u(R, r_0)$ almost immediately crosses the curve $u(R, \infty)$ (the lower bold curve in Figure 2) near $R = 1$, and in the limit $r_0 \to m$ approaches the upper bold curve. The latter has both zero slope and zero curvature at $R = 1$, indicating that the particle distribution in the final state has an infinite density at the surface.
4 Conclusion

A surprising feature of Figure 2 is that the density of dust particles, which started off uniform in the Newtonian region of $t$, becomes infinite at the surface of the ball as $t \to +\infty$; this may be considered as confirming Eddington’s intuition\cite{7} that something intervenes to prevent the “absurdity” (to quote Eddington) of the ball collapsing to a point. We see that the finite radius of the end state is a consequence of a purely gravitational field, without the need for any other forces to counter the gravitational attraction. Indeed, if we commit the heresy of speaking of gravitational forces, we may say that they turn from being attractive to repulsive in the high-density region; it is the accumulation of gravitational energy inside the ball which prevents it from collapsing to a point.

Such heresy is at least hinted at by the favoured status afforded here to the harmonic coordinate frame, often referred to as a ”gauge”. But is it possible to put the results I have established into a coordinate-free form? My central result is the form of the boundary characteristic given by (16) and (17). This makes no reference to the harmonic frame, even though I went via that frame to establish it, so a coordinate-free derivation may be possible.

I would, nevertheless, argue that use of the harmonic frame advances our understanding of the collapse, and especially of the stable final state, which was declared nonexistent in the OS analysis. There was a mystery about the infinite red shift suffered by a free-falling exterior particle as it approaches $r = m$. Now, in the light of the discovery that particles inside the ball behave in a similar manner, such behaviour becomes less mysterious. We merely have to find an explanation for the build up of density at the surface, and that, expressed in suitable coordinates, is to be found in the Einstein-Hilbert field equations themselves. It may be that some change of frame could transform away such an infinite density so that it becomes finite, but the theory of gravity which motivated the analysis of this article\cite{1}, and also the earlier statement of opposition to Strong Equivalence by Fock\cite{10}, seem to offer substantial support to the assertion that the harmonic, or inertial, coordinate frame should have a privileged status.

Finally I remark that the harmonic frame was first accorded such a status in Einstein’s pioneer article on gravitational waves\cite{3}. The subsequent history\cite{15} of gravitational waves is a process of repeated assertion and denial of the Strong Equivalence Principle that all frames are equivalent, beginning perhaps with Eddington’s observation\cite{17} that, if we allow arbitrary coordinate transformations, the waves may be made to appear and disappear at will; gravitational waves travel at the speed of thought. The converse of this statement, of course, is that if we do not allow such transformations, but instead insist on a privileged frame, such waves do exist and are worth searching for.
5 Appendix

The values of \( t \) and \( r \) in the asymptotic region may be obtained in terms of the variables \((R, v)\), where

\[
v^3 = e^{-3X/2} - \frac{3r}{4m}, \quad X = 1 - R.
\]

(32)

Then the operator \( P \), defined by (26), becomes

\[
P = \partial_R^2 + \frac{\eta}{v^2} \partial_R \partial_v + \left( \frac{\eta^2}{4v^6} - \frac{\xi^2 R^2 e^{2X}}{4v^2} \right) (v \partial_v)^2 - \left( \frac{3\xi R X e^{2X}}{4v} + \frac{\eta X}{4R v^2} - \frac{\eta^2}{2v^3} \right) \partial_v
\]

\[
+ \left( \frac{X^2 + 1}{RX} + \frac{5\eta}{2v^3} \right) \partial_R - \frac{\eta X}{R v^3 \xi} \partial_R \left( \frac{\eta}{X} \right) \left( \partial_R - \frac{v X}{2R} \partial_v \right) ,
\]

(33)

where

\[
\xi = \frac{X}{R} + \frac{\eta}{v^3}, \quad \eta = e^{-3X/2} .
\]

(34)

By expanding this in inverse powers of \( v \) and substituting in (28), one obtains the first few terms in the asymptotic series of \( t \), with the first term given by the Newtonian limit, that is

\[
\frac{t}{2m} \sim -\frac{2}{3} v^3 + \sum_{n=0}^{\infty} t_n (R) v^{1-n} ,
\]

(35)

with

\[
t_0 = -\frac{1}{2} R e^{-2X} - \frac{3}{2}, \quad t_1 = \frac{2}{3} e^{-3X/2} ,
\]

\[
t_2 = \frac{91}{40} R e^{-2X} - \frac{1}{40} R^2 e^{4X} ,
\]

\[
t_3 = \frac{1}{2} e^{-3X/2} + \frac{3}{2} (13 - 6X + X^2) e^{X/2} - 20 ,
\]

\[
t_4 = \frac{571}{672} R e^{-2X} + \frac{1}{100} R^2 e^{4X} + \frac{1}{3360} R^6 e^{6X} ,
\]

\[
t_5 = \frac{5}{6} t_3 + \frac{41}{120} \left( e^{-3X/2} - 1 \right) + t_{51} - t_{52} + \frac{e^{-X}}{R} t_{53} + t_{52} (0) - t_{53} (0) ,
\]

\[
t_6 = \frac{1}{10} + \frac{3}{2} (9 - 4X + X^2) e^{-X} - \frac{40}{3} e^{-3X/2} + \frac{1}{6} e^{-3X} .
\]

(36)

where

\[
t_{51} = \frac{1}{6} \int_X^{2X} \frac{XU^2}{R} t_3 ,
\]

\[
t_{52} = \frac{5}{16} \int_X^{1} \frac{\eta U^4}{R} ,
\]

(37)

and

\[
t_{53} = \frac{1}{5} \int_X^{1} \frac{\eta U^5} .
\]

(38)
Note that the latter three integrals may all be given in terms of elementary functions, though they are rather complicated.

The corresponding series for $r$ is

$$\frac{r}{2m} = e^X R \left[ v^2 + \sum_{n=0}^{\infty} r_n(R) v^{-n} \right].$$

(39)

with the coefficients

$$r_0 = \frac{1}{4} R^2 e^{2X} - \frac{3}{4}, \quad r_1 = r_2 = r_4 = r_5 = r_7 = 0,$$

$$r_3 = 20 \left( e^{-3X/2} - \frac{3}{2} \right) (13 - 6X + X^2) e^{X/2} ,$$

$$r_6 = \frac{5}{6} \left( 9 - 4X + X^2 \right) e^{-X} + \frac{100}{3} e^{-3X/2} - \frac{5}{12} e^{-3X} ,$$

$$r_8 = \frac{2403}{8} - \frac{27}{8} e^X \left( X^4 - 9X^3 + 40X^2 - 89X + 89 \right) + \frac{9}{8} e^{-X} \left( X^2 + X + 1 \right) + \frac{3}{8} e^{2X} (X-1)^2 + 60 e^{X/2} \left( (X^2 - 5X + 10) \right)$$

$$r_9 = \frac{16}{675} + \frac{20}{9} e^{-3X/2} - \left( 6X^2 - \frac{96}{5} X + \frac{1158}{25} \right) e^{-5X/2}$$

$$+ \frac{400}{9} e^{-3X} - \frac{10}{27} e^{-3X/2}.$$

(40)

In each coefficient of both expansions two constants of integration had to be specified. They were determined from the continuity condition at $R = 1$ and the finiteness condition at $R = 0$. The derivatives of $t_n$ at $R = 1$ are fixed by this procedure, and we find that

$$t_0' = t_2' = t_3' = t_4' = t_6' = 0, \quad t_1' = 1, \quad t_5' = \frac{2}{5} \quad (R = 1).$$

(41)

Rather remarkably, although the asymptotic series breaks down for $X = O(v^{-3})$, a match with the solution in this boundary layer confirms the above values, and gives the additional ones

$$t_6' = 0, \quad t_7' = \frac{68}{105}, \quad t_8' = -0.0616 \ldots, \quad t_9' = 1,$$

$$t_{10}' = 0.6314 \ldots, \quad t_{11}' = -2.3253 \ldots.$$

(42)

The corresponding procedure for the $r$ coefficients gives $r_n' = 0$ for $n \leq 9$, and the boundary layer matching not only confirms these, but shows that $r_{10}', r_{11}', r_{12}'$ are also zero. I have been emboldened to conjecture that the partial derivative of $r$ with respect to $R$ is zero for all $v > 1$.

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1Note that this result is at variance with the assumption of continuity for $t_R$ made by me in Ref[13]. Essentially, we must replace this condition by one of finiteness at $R = 0$. But I stress that the condition imposed by Ref[5] on the interior metric in the entire region $R < 0$ cannot be satisfied by any solution with an acceptable degree of continuity at the surface.
The numerical integration procedure I have described requires a knowledge of not only $t$ and $r$ at $R = 1$, but also their $R$-derivatives there. For $r$ the conjecture I just described may be tested by moving the lower limit of the region of integration close to $S = 0$, and when this was done there was no noticeable tendency for the solution to diverge as the limit $R = 0$ was approached; of course rounding errors always allow the divergent solution to enter at some stage, but this tendency seems to be constant over the whole range of $S$. In the case of $t$, all we can do is include all the asymptotic coefficients we have, in order to approximate the initial value of its $R$-derivative. Such a procedure gives a stable solution down to a dustball radius $r_0$ of around $1.1m$, and thanks to our rather precise $r$-solution we already know what happens for infinite $t$, that is $r_0 = m$.

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