Compton Scattering Driven by Quantum Light

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Abstract: We solve the non-perturbative interaction of free electrons with arbitrary intense quantum electromagnetic fields and exemplify it by predicting extended spectra in Compton scattering driven by non-classical photon statistics. © 2023 The Authors

Compton scattering is one of the cornerstones of quantum physics, describing the fundamental interaction of a charged particle with photons. The Compton effect and its inverse are utilized in experiments driving free electrons by high intensity lasers to create high-energy photons. When increasing the intensity of its driving field, the Compton effect transitions into its non-linear regime, dubbed non-linear Compton scattering (NCS) [1], wherein multiple photons are absorbed and converted into higher energy photons.

So far, all theory and experiments of the Compton effect and NCS have relied on electromagnetic fields that can be described classically. The reason that light with non-classical photon-statistics has not been considered is the long-held conception seeing intense (many-photon) light as necessarily classical, whereas quantum states of light are seen as relevant only for a small number of photons. Importantly, recent experimental advances have started to break this conception by generating light states of non-classical photon statistics that have ever-increasing intensities. For example, bright squeezed vacuum (BSV) picosecond pulses with an energy of 10 μJ have been generated through the process of parametric down conversion [2]. Such advances motivate revisiting the Compton effects and its variants using a quantum-optical modelling of the driving field. However, no work so far found the theory required to describe such effects, or more generally any non-perturbative interaction of free electrons with intense non-classical light.

Here we develop the framework for describing the non-perturbative interactions of matter with driving fields ascribed with arbitrary light states. As our primary example, we employ this framework to describe Compton scattering and the general NCS, when driven by intense light of an arbitrary quantum state. Though the framework applies to any fermionic charged particle, we focus on the example of Compton scattering off free electrons. We calculate the NCS emission spectrum and angular distribution, identifying the unique aspects of non-classical driving light states. Specifically, we obtain analytical formulas for driving by intense thermal and BSV states. We find that the resulting spectra are broadened relative to the usual spectrum of classically (coherent-state) driven NCS of the same intensity (see Fig. 1), reaching far higher frequencies in some regimes of parameters. Moreover, in sharp deviation from the conventional Compton scattering, the spectrum per solid angle can become continuous.

![Image](image_url)

**Fig. 1 | Non-linear Compton scattering driven by light with quantum photon statistics: spectrum broadening.** Consider a free electron that scatters off different intense light states described by the Husimi functions in (a1-a3). We consider the cases of a coherent state (a1, red color), a thermal state (a2, blue color), and a bright squeezed vacuum (BSV) state (a3, magenta color). The resulting NCS spectrum per solid angle and angular distribution appear in (b) and (b1), respectively. Panel (c) defines the notation for the polar angle $\theta'$ of photon emission. The angular distribution of emission in (b1) was obtained after integrating over the frequency range that falls in the blue shaded region marked in (b). The spectrum of (b) is plotted at $\theta' = 159.9^\circ$, chosen at the peak of emission in (b1). $\beta = 0.99$ is the velocity of the electron normalized by the velocity of light. Altogether, panel (b) shows how manipulating photon statistics can result in a noticeable spectrum broadening and its extension to higher frequency radiation.

The outline of our framework is as follows. We consider a generic light-matter system (e.g., atoms and free electrons) interacting with a driving field ascribed with an arbitrary single-mode density matrix $\rho_{\text{light}}$. Using the Husimi Q function,
defined as $Q(\alpha) = \langle \alpha | \rho_{\text{light}} | \alpha \rangle / \pi$ where $| \alpha \rangle$ is a coherent state with complex parameter $\alpha$, it is possible to express $\rho_{\text{light}}$ using coherent states as [3]

$$\rho_{\text{light}} = \int d^2\alpha \, d^2\beta \, Q \left( \frac{\alpha + \beta^*}{2} \right) \exp \left( -\frac{|\alpha - \beta^*|^2}{4} \right) |\alpha\rangle \langle \beta^*| .$$  \hspace{1cm} (1)

Let $E = \alpha \sqrt{2\hbar \omega / \epsilon_0} V$ denote the electric field amplitude of a coherent state $|\alpha\rangle$, where $\hbar$ is Planck’s reduced constant, $\epsilon_0$ is the vacuum permittivity, and $V$ is the quantization volume. By exploiting the linearity of the time evolution equation, using Eq. (1), and ensuring that the photon density, rather than the photon number, is well defined as $V \rightarrow \infty$, we obtain that the spectrum for a given solid angle $\Omega$ of the emitted photon is

$$\frac{d\epsilon}{d\omega d\Omega} = \int d^2\varepsilon \, Q(\varepsilon) \left( \frac{d\epsilon}{d\omega d\Omega} \right)_\varepsilon .$$  \hspace{1cm} (2)

Here, $\omega$ is the frequency of the emitted photon, $Q(\varepsilon)$ the limit of $V E_\varepsilon Q(\sqrt{V E_\varepsilon / 2 \hbar \omega}) / 2 \hbar \omega$ as $V \rightarrow \infty$, $E$ is a complex parameter with electric field units, and $(d\epsilon/d\omega d\Omega)_\varepsilon$ is the conventional spectrum per unit solid angle emitted from an interaction with a classical driving field with electric field $E$. Eq. (2) neatly shows that the emission spectrum is only affected by the photon statistics of the driving field (i.e., the diagonal of the density matrix), which is captured via $Q$. Unlike the spectrum, higher order correlations in the emission depend on additional properties of the driving light state besides photon statistics, such as higher order coherence and off-diagonal density matrix elements.

We now employ Eq. (2) in the context of Compton scattering and NCS. Luckily, $(d\epsilon/d\omega d\Omega)_\varepsilon$ has an analytical expression in NCS (found in, e.g., [4]) so that a fully analytical formula can be obtained by performing the integration in Eq. (2). We employ the resulting formula in the case of a coherent-, thermal-, and BSV-driven NCS and plot it in Fig. 2.

Panels (a1-a4) of Fig. 2 show how the energy spectrum in Compton scattering changes when increasing the driving intensity (tenfold between sequential panels). We see that the spectra of BSV- and thermal-driven Compton scattering extend to higher frequencies relative to the coherent-driven case. Moreover, the width of individual spectral peaks are also broader. i.e., BSV-driven and thermal-driven Compton scattering reach higher harmonics of the driving field frequency than conventional Compton scattering driven by the same intensity. This spectral broadening is most notably seen in Fig. 2d, where for the BSV case the overall spectrum extends more than three times farther than for the coherent case. For the thermal case, the spectrum extends roughly two times farther than for the coherent case. The corresponding angular distributions in panels (b1-b4) of Fig. 2 show that the emission becomes more directional for higher driving intensities, being the largest for the BSV case, followed by the thermal case, and then the coherent case. Similar features are found in inverse-Compton scattering, where the non-classical driving fields interact head-on with a relativistic electron.

Looking at the bigger picture, the ideas presented here are not limited to Compton scattering, and can be carried over to other phenomena involving intense driving light fields. Essentially, the entirety of the field of nonlinear optics and its many discoveries over the 20th century can now be revisited when driven by non-classical light. Even more than nonlinear optics, a wide range of fundamental effects in physics such as Rabi oscillations and the photoelectric effect can now be revisited when driven by light with non-classical photon statistics. These novel possibilities hint at a new research field to be explored, at the borderline between quantum optics and strong-field nonlinear optics.

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