Scotogenic dark matter and single-zero textures of the neutrino mass matrix

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Abstract. We consider the phenomenology of the scotogenic model with the one-zero-textures of the neutrino flavor mass matrix. One of the six patterns of the neutrino mass matrix is favorable for the real Yukawa matrix. On the other hand, for the complex Yukawa matrix, five of the six patterns are compatible with observations of the neutrino oscillations, dark matter relic abundance and branching ratio of the $\mu \to e\gamma$ process.

1. Introduction
The nature of the dark matter and neutrinos cannot be explained within the standard model of particle physics. The scotogenic model can simultaneously account for dark matter candidates and the origin of tiny masses of neutrinos [1]. In this model, neutrino masses are generated by one-loop interactions mediated by a dark matter candidate. On the other hand, there have been various discussions on flavor neutrino mass matrices with zero elements [2]. In particular, textures of the flavor neutrino mass matrix with single-zero element is called one-zero-textures.

In this study, we show a new parametrization of the complex Yukawa matrix in the scotogenic model [3]. For the phenomenology of the scotogenic model with the PDG parametrization [4], the parametrization of the Yukawa matrix which is shown in this paper may be useful. Moreover, we consider the phenomenology of the scotogenic model with the one-zero-textures of the neutrino flavor mass matrix [3, 5]. We show that one of the six patterns of the neutrino mass matrix is favorable for the real Yukawa matrix. On the other hand, for the complex Yukawa matrix, five of the six patterns are compatible with observations of the neutrino oscillations, dark matter relic abundance and branching ratio of the $\mu \to e\gamma$ process.

2. Scotogenic model
The scotogenic model has three extra Majorana $SU(2)_L$ singlets $N_k$ ($k = 1, 2, 3$) and one new scalar $SU(2)_L$ doublet $\eta = (\eta^+, \eta^0)$. $N_k$ and $\eta$ are odd under $Z_2$ symmetry while other fields are even under $Z_2$ symmetry. The Lagrangian of the scotogenic model contains new terms for the new fields,

$$L \supset Y_{ak} (\bar{\nu}_{aL}\eta^0 - \bar{\ell}_{aL}\eta^+) N_k + \frac{1}{2} M_k N_k^C N_C^k + H.c.,$$ (1)
and the scalar potential of the model contains the quartic scalar interaction

\[ V \supseteq \frac{1}{2} \lambda (\Phi^\dagger \eta)^2 + H.c., \]

(2)

where \( L_\alpha = (\nu_\alpha, \ell_\alpha) \) is the left-handed lepton doublet and \( \Phi = (\phi^+, \phi^0) \) is the Higgs doublet in the standard model. The elements of the flavor neutrino mass matrix are obtained as

\[ M_{\alpha\beta} = \sum_{k=1}^3 Y_{\alpha k} Y_{\beta k} \Lambda_k, \]

(3)

where

\[ \Lambda_k = \frac{\lambda M_k}{16\pi^2} \frac{M_k}{m_0^2 - M_k^2} \left( 1 - \frac{M_k^2}{m_0^2 - M_k^2} \ln \frac{m_0^2}{M_k^2} \right), \]

(4)

\[ m_0^2 = \frac{1}{2}(m_R^2 + m_I^2), \text{ and } v, m_R, \text{ and } m_I \text{ denote the vacuum expectation value of the Higgs field, and the masses of } \sqrt{2}\text{Re}[\eta^0] \text{ and } \sqrt{2}\text{Im}[\eta^0], \text{ respectively.} \]

The lightest \( Z_2 \) odd particle is stable and becomes a dark matter candidate. We assume that the lightest Majorana singlet fermion, \( N_1 \), becomes the dark matter and \( N_1 \) is considered to be almost degenerate with the next to lightest Majorana singlet fermion \( N_2 \), \( M_1 \leq M_2 < M_3 \). In this case, the relic abundance of the dark matter is given by \([6, 7, 8, 9, 10]\). Moreover, in the scotogenic model, flavor-violating processes such as \( \mu \to e\gamma \) are induced at the one-loop level \([6]\).

3. Parametrization of Yukawa matrix

In order to obtain any phenomenological prediction in the scotogenic model, the elements of the Yukawa matrix should be determined. A Yukawa matrix with PDG parametrization have been proposed in terms of \( Y_{e1}, Y_{e2} \) and \( Y_{e3} \) by Ho and Tandean with \( P = \text{diag.}(e^{i\alpha_2/2}, e^{i\alpha_3/2}, 1) \) \([11]\). In this study, we use the following new PDG compatible Yukawa matrix parametrization in terms of \( Y_{e1}, Y_{e2} \) and \( Y_{e3} \) with \( P = \text{diag.}(1, e^{i\alpha_2/2}, e^{i\alpha_3/2})[3]\):

\[ Y = \begin{pmatrix} Y_{e1} & Y_{e2} & Y_{e3} \\ a_1 Y_{e1} & a_2 Y_{e2} & a_3 Y_{e3} \\ a_4 Y_{e1} & a_5 Y_{e2} & a_6 Y_{e3} \end{pmatrix}, \]

(5)

where

\[ a_1 = -\frac{s_{23} t_{12}}{c_{13}} - e^{-i\delta} s_{23} t_{13}, \quad a_2 = \frac{s_{23} t_{12}}{c_{13}} - e^{-i\delta} c_{23} t_{13}, \]

\[ a_3 = \frac{c_{23}}{t_{12} c_{13}} - e^{-i\delta} s_{23} t_{13}, \quad a_4 = -\frac{s_{23}}{t_{12} c_{13}} - e^{-i\delta} c_{23} t_{13}, \]

\[ a_5 = e^{-i\delta} s_{23}, \quad a_6 = e^{-i\delta} c_{23}/t_{13}, \]

(6)

and \( c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij} \) and \( t_{ij} = \tan \theta_{ij} \) \((i, j=1,2,3)\). The neutrino mass eigenvalues are obtained by

\[ m_i = b_i A_i y_e^2, \]

(7)

where

\[ b_1 = \frac{1}{c_{12} c_{13}}, \quad b_2 = \frac{e^{i\alpha_2}}{s_{12} c_{13}}, \quad b_3 = \frac{e^{i(\alpha_3-2\delta)}}{s_{13}}. \]

(8)
A global analysis shows that the preference for the normal mass ordering is mostly due to neutrino oscillation measurements [12]. We assume the normal mass ordering (NO) for the neutrinos. In this case, the squared mass differences of the neutrinos are given by
\[ \Delta m^2_{21} = m_2^2 - m_1^2, \quad \Delta m^2_{31} = m_3^2 - m_1^2, \] (9)
and we obtain the relations
\[ Y^2_{e2} = \frac{\sigma_2}{b_2 A_2} \sqrt{\Delta m^2_{21} + b_1^2 A_1^2 Y^4_{e1}}, \quad Y^2_{e3} = \frac{\sigma_3}{b_3 A_3} \sqrt{\Delta m^2_{31} + b_1^2 A_1^2 Y^4_{e1}}, \] (10)
where \(\sigma_{2,3} = \pm 1\).

4. One-zero-textures

As we addressed in introduction, we assume that the flavor neutrino mass matrix \(M_\nu\) has one zero element. There are the following six patterns for the flavor neutrino mass matrix \(M_\nu\) in the one-zero-textures:

\[ G_1 : \begin{pmatrix} 0 & \times & \times \\ - & \times & \times \\ - & - & - \end{pmatrix}, \quad G_2 : \begin{pmatrix} \times & 0 & \times \\ - & \times & \times \\ - & - & - \end{pmatrix}, \quad G_3 : \begin{pmatrix} \times & \times & 0 \\ - & \times & \times \\ - & - & - \end{pmatrix}, \]
\[ G_4 : \begin{pmatrix} \times & \times & \times \\ - & 0 & \times \\ - & - & - \end{pmatrix}, \quad G_5 : \begin{pmatrix} \times & \times & \times \\ - & \times & 0 \\ - & - & - \end{pmatrix}, \quad G_6 : \begin{pmatrix} \times & \times & \times \\ - & - & 0 \\ - & - & - \end{pmatrix}. \] (11)

The assumption of a real Yukawa coupling matrix is not realistic; however, we start our study without CP violation just as a simple case. For the \(G_1\) pattern, the relation
\[ M_{ee} = Y^2_{e1} A_1 + Y^2_{e2} A_2 + Y^2_{e3} A_3 = 0 \] (12)
is required by Eq.(3). Since \(A_k > 0\) and \(Y_{ak}\) is real, Eq.(12) yields \(Y_{ek} = 0\). However, the vanishing \(Y_{ek}\) yields
\[ M_{e\mu} = \sum_{k=1}^{3} Y_{ek} Y_{\mu k} A_k = 0, \quad M_{e\tau} = \sum_{k=1}^{3} Y_{ek} Y_{\tau k} A_k = 0, \] (13)
as well as
\[ \begin{pmatrix} 0 & 0 & 0 \\ - & \times & \times \\ - & - & - \end{pmatrix}, \] (14)
and the one-zero-texture assumption should be violated. The \(G_1\) pattern is excluded in the scotogenic model. Similarly, the \(G_4\) and \(G_6\) patterns are also excluded. Thus, three patterns (\(G_1, G_4\) and \(G_6\) ) are impossible in the scotogenic model with the real Yukawa matrix.

To see that whether the \(G_2, G_3\) and \(G_5\) are consistent with observation or not, we performed numerical calculations with the following input values: (1) For neutrino sector, we fix the masses of the neutrinos with the best-fit values of the squared mass differences and vary the mixing angles in the 3σ region [13]. (2) For dark sector, we adopt the standard criteria [6, 14, 15] and we take \(1 \times 10^{-10} \leq \lambda \leq 5 \times 10^{-9}, 0.5 \leq r_1 \leq 0.9, 1.5 \leq r_3 \leq 3.0, 2\text{TeV} \leq m_0 \leq 4\text{TeV},\) where \(r_k = \frac{M_k}{m_0}\).

It turned out that the \(G_3\) pattern is consistent with the observed energy density of the cold dark matter component in the \(\Lambda\)CDM cosmological model by the Plank Collaboration \(\Omega h^2 = \)
0.120 ± 0.001 [16] and the measured upper limit of the branching ratio \( \text{Br}(\mu \rightarrow e\gamma) \leq 4.2 \times 10^{-13} \) [17]. On the other hand, the predicted values of \( \Omega h^2 \) and \( \text{Br}(\mu \rightarrow e\gamma) \) for G\(_2\) seems to be unlikely with the observed data. Moreover, G\(_5\) pattern is excluded from observation. The G\(_2\) and G\(_5\) patterns are not favorable for the scotogenic model with the real Yukawa matrix elements. For more detail, see Ref.[5].

If we include CP-violating phases, the results to be different. We performed numerical calculations with the following input values: (1) For neutrino sector, we fix the masses and mixing of the neutrinos with the best-fit values. The Dirac CP phase is fixed in the best-fit values \( \delta = 261^\circ \) [13] and Majorana CP phases are varied as \( 0^\circ \leq \alpha_2, \alpha_3 \leq 360^\circ \). (2) For dark sector, we adopt the same criteria in the case of the real Yukawa couplings. From our parameter search, it turned out that G\(_1\) pattern is unfavorable from observation. On the other hand, remaining five patterns of G\(_2\), G\(_3\), · · · , G\(_6\) are consistent with observation.

5. Summary

We have shown a new parametrization of the complex Yukawa matrix in the scotogenic model [Eq.(5) with Eq.(6)]. Although there are many other ways to parametrize the Yukawa matrix, the way in this paper may be one of the useful method to consider the phenomenology of the scotogenic model with the standard PDG parametrization.

Moreover, we have considered some phenomenology of the scotogenic model with the one-zero-textures of neutrino flavor mass matrix. If the elements of the Yukawa matrix are real, one of the six patterns of the flavor neutrino mass matrix with single-zero element is favorable [5]. On the other hand, if the Yukawa matrix is complex, the five patterns G\(_2\), G\(_3\), G\(_4\), G\(_5\) and G\(_6\) are compatible with observations of the neutrino oscillations, dark matter relic abundance and branching ratio of the \( \mu \rightarrow e\gamma \) process; however, G\(_1\) is unfavorable with observations [3].

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