Color coherence in multiple antenna medium radiation

Fabio Domínguez, Carlos A Salgado and Víctor Vila
Instituto Galego de Física de Altas Enerxías (IGFAE), Universidade de Santiago de Compostela, E-15706 Santiago de Compostela, Galicia, Spain
E-mail: victor.vila@usc.es

Abstract. We present the configuration in which a quark-antiquark pair with a fixed opening angle emits a hard gluon inside a medium, and an additional very soft emission afterwards (double antenna). We discuss the coherence effects in terms of the survival probability, which describes the interaction of the $q\bar{q}g$ system with the medium. We generalize previous studies of the antenna radiation to the case of more than two emitters and prove that this generalization provides further support to the picture of jet quenching with effective emitters in the parton cascade.

1. Jet quenching
Jet quenching has become an essential signal for the characterization of the medium formed in experiments of heavy ion collisions. Striking effects of energy loss of leading partons and reconstructed jets have been observed in heavy ion collisions at RHIC and LHC, see e.g. Ref. [1]. These results indicate that the spectra of high-$p_T$ hadrons are strongly affected by the matter formed in the collisions as compared to those from proton-proton collisions. Color coherence is a key ingredient in understanding these effects from a theoretical point of view. It implies a reorganization of the cascade in terms of effective emitters for medium-induced radiation [2].

2. Color coherence in vacuum
The concept of color coherence is useful to understand the qualitative and quantitative features of the radiation in QCD, see e.g. Ref. [3]. In a parton shower, when a highly collinear parton is emitted during the propagation, the splitted collinear gluons are difficult to resolve unless they are sufficiently apart, and for large angles, their radiation interfere rather than add incoherently. A simple interpretation of this result is that, at certain angles, it is difficult to resolve the individual radiated partons, and their net behaviour is rather similar to the radiation from the total color charge of the parent parton. This problem becomes simple when the shower is implemented by using the angle of radiation as the ordering variable. The cascading of a jet, initiated by a hard parton, occurs in a coherent manner. In other words, subsequent parton branchings of the shower are not independent but depend on the characteristics of the previous branching.

The singlet antenna spectrum for $q\bar{q}$ production plus soft gluon emission is

$$dN = \frac{d\omega}{\omega} \frac{d\Omega}{2\pi} \alpha_s C_F \left[R_q + R_{\bar{q}} - 2J\right].$$ (1)
where $R_q$ ($R_{\bar{q}}$) is the radiation spectrum off an independent constituent, and $\mathcal{J}$ describes the quark-antiquark interference ($\omega$ is the energy of emitted gluons, $d\Omega$ is the differential solid angle, $\alpha_s$ is the coupling constant in QCD and $C_F^1$ is a color factor).

This spectrum is divergent when the energy of the emitted gluon becomes soft, $\omega \to 0$, and when the gluon is emitted collinearly to the either the quark or the antiquark. The spectrum is suppressed at large angles due to the presence of destructive interferences ($R_q \simeq \mathcal{J}$ at large angles). Thus, in vacuum, large-angle gluon emission is sensitive to the total charge of the system. This property relies on color conservation and leads to angular ordering of the vacuum cascade in QCD.

3. In-medium antenna radiation

Considering the case in which a very soft gluon is emitted from the antenna, the dominant contribution to the medium-induced radiation can be easily computed from the diagrams of Fig. 1. In this simple case, the spectrum of emitted gluons is a modification of eq. (1) [4]

$$dN = \frac{d\omega}{\omega} \frac{d\Omega}{2\pi} \frac{\alpha_s C_F}{2\pi} \left[ R_q + R_{\bar{q}} - (1 - \Delta_{\text{med}}) \right] \mathcal{J}. \quad (2)$$

The interaction of the $q\bar{q}$ pair with the medium is described by the survival probability

$$S \equiv 1 - \Delta_{\text{med}} \equiv \frac{1}{N_c^2 - 1} \text{Tr} \left[ W(\vec{x}_-) W^\dagger(\vec{y}_-) \right], \quad (3)$$

where the Wilson lines describe the propagation of a gluon through a medium field $\mathcal{A}_-(x_+, \vec{x})$,

$$W(\vec{x}) = \mathcal{P} \exp \left[ ig \int dx_+ \mathcal{A}_-(x_+, \vec{x}) \right]. \quad (4)$$

In the resulting spectrum we have introduced the $\Delta_{\text{med}}$ parameter, which is called the medium decoherence parameter. This factor determines a characteristic time scale for decoherence of the $q\bar{q}$ pair, and for the case of a static medium of size $L$,

$$\Delta_{\text{med}} \simeq 1 - \exp \left[ -\frac{1}{4} \hat{q} L (\vec{x}_- - \vec{y}_-)^2 \right], \quad (5)$$

where $\hat{q}$ is the transport coefficient, encoding all the information about the dynamical properties of the medium; this is the main parameter to be determined by fits to experimental data and to be compared with theoretical calculations.

We can consider two limits [2, 4]

$$C_F = \frac{N_c^2 - 1}{2N_c}, \quad \text{with } N_c \text{ the number of colors.} \quad (1)$$
(i) when the color correlation length of the medium ($\simeq \frac{1}{\sqrt{\ell q}}$) is larger than the size of the pairs, $\Delta_{med} \rightarrow 0$ ($dN \sim R_q + R_{\bar{q}} - 2J$), the medium cannot resolve the individual emitters, thus acting like an individual object with the total charge of the pair ($C_F$ for triplet, $C_A$ for octet or 0 for singlet).

(ii) In the opposite case, $\Delta_{med} \rightarrow 1$ ($dN \sim R_q + R_{\bar{q}}$), the medium resolves the antenna (opaque medium: decoherence, two independent emitters) and it breaks the color coherence of the pair, behaving as two independent particles.

A way to estimate the coherence effects is through the definition of a coherence time, $t_{coh} \sim \left[\frac{1}{12} g^2 q \bar{q} \hat{q}\right]^{1/3}$, so that the pair remains coherent (acting as an individual emitter) for path lengths in the medium smaller than $t_{coh}$.

4. More than two emitters
The antenna provides a simple and intuitive picture, but does it hold for more than two emitters? In order to address this question, we repeated the previous calculations now including an additional hard gluon, with the final goal of extrapolating our results to n-gluon emissions. We restrict to the case of very soft gluon radiation ($\omega \rightarrow 0$), where only the out-out component needs to be computed [4].

So, let us consider now the case of three emitters (two hard splittings inside the medium). The direct terms $R_q, R_{\bar{q}}$ and $R_{\bar{q}}$ are proportional to a color factor, i.e., no medium effects appear as expected with the kinematics chosen.

$$|\mathcal{M}_1|^2 \propto C_F^2$$

$$|\mathcal{M}_2|^2 \propto C_F^2$$

$$|\mathcal{M}_3|^2 \propto N_c C_F^2$$

Figure 2. Direct terms for the case of three emitters.

More interesting results are obtained in the interference terms, where the survival probabilites $S$ appear explicitly, providing information about the color coherence of the emitters.

$$\mathcal{M}_1 \otimes \mathcal{M}_3 \propto \frac{1}{2} N_c^2 S(t, L) J$$

$$\mathcal{M}_2 \otimes \mathcal{M}_3 \propto N_c^2 S(0, t) S(t, L) J$$

The interpretation of equations (7) and (8) is very clear in the large-$N_c$ limit (see Figs. 3 and 4). The survival probability, controlling the degree of color coherence in the soft gluon emission, corresponds to that of the dipole from which this gluon is emitted. In Fig. 3 this dipole is formed at time $t$ during the evolution of the system, while it is produced at time 0 in the diagrams of Fig. 4. In this second case, the survival probability is the product of the two survival probabilities, from 0 to $t$ and from $t$ to $L$. If coherence is not preserved after the in-medium splitting, the antenna won’t radiate coherently in the following emission.
5. Discussion and conclusions
Color coherence is essential to understand if multiple gluon emissions are correlated or not. Proceeding as in the vacuum case, in which coherence leads to angular ordering, antenna is a very convenient laboratory. An interesting goal is to incorporate the medium effects: the medium can destroy the coherence of the system. Our results definitely conclude that the results from the antenna setup (two emitters) can be easily generalized to the case of more than two emitters. These computations go a step forward to obtain a complete description of a QCD cascade.

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