EINSTEINIAN MANIFOLDS
AND GRAVITATIONAL WAVES

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Abstract. The full relativity of the concepts of motion and rest, which is characteristic of the Einsteinian general relativity (GR), does not allow the generation of physical gravitational waves (GW’s). The undulatory nature of a metric tensor is not an invariant property, but depends on the coordinate frame. An undulation of a metric tensor is propagated with a speed that can have any value between zero and infinite.

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1. – The exact (non-approximate) formulation of general relativity (GR) does not allow the existence of physical gravitational waves (GW’s). I have given several proofs of this fact [1]. Quite simply, we can observe, e.g., that bodies which interact only gravitationally describe geodesic lines, and therefore – as it is very easy to see – they do not generate any GW. If we add non-gravitational forces, the conclusion remains the same, because the new trajectories do not possess kinematical elements (velocity, acceleration, time derivative of the acceleration, etc.) different from those of the geodesic motions.

Another plain proof of the unreality of the GW’s runs as follows. Remember in primis that, contrary to what happens in Maxwell theory, for which the class of the inertial systems has a physical privilege, in GR all coordinate systems are on the same footing, none of them is physically privileged. Let us consider, to be determinate, the Schwarzschild solution to Einstein equations which gives the field generated by a gravitating spherically symmetrical body $B$ – as it is seen by an observer $\Omega$ at rest together with $B$. Now, the two instances: $i)$ body $B$ at rest and observer $\Omega$ in any whatever motion $W$, and $ii)$ $\Omega$ at rest and $B$ $W$-moving, are indistinguishable, because in GR we can only speak of relative motions. But $B$ at rest cannot emit GW’s. We can also say: the gravitational potential of $B$ at rest is characterized by the static $g_{jk}(x)$ of Schwarzschild solution. The gravitational potential of $B$ in motion with respect to $\Omega$ is characterized by a given time-dependent $g^*_{jk}(x^*, t^*)$. Of course, the curvature tensor remains intrinsically unaltered by the transition $(x, t) \rightarrow (x^*, t^*)$. Assume now that the motion of $B$ happens in a limited spatial region $L$. Then, it is possible that at a great distance from $L$ the tensor $g^*_{jk}$ has a wavy form, which however would represent a wave due to the starred coordinates $x^*, t^*$, i.e. an illusive undulation.
**Conclusion:** no motion of $B$ generates physical GW’s. This argument is more straightforward than the reasoning developed in a previous paper [2], in which however it is also considered the case of the relative motion of two bodies that can be both reduced to rest, with respect to an observer $\Omega$, by a convenient reference system (Weyl).

2. – A diffuse, wrong belief affirms: “A bar turning round its midpoint has a quadrupole moment, and therefore it generates GW’s.” Two errors: $i$) the quadrupole formula has been derived from the linear version of GR, which is inadequate to treat the question of the GW’s (Weyl 1944, see [3]); $ii$) a convenient coordinate change reduces the bar to rest; further, in the exact GR there are no kinematical elements of any whatever motion that are responsible for the emission of GW’s. (The “bar argument” has many affectionate supporters; thus, in order to legitimate the use of linearized approximation, it has been affirmed that for an observer not very distant from the bar the fact that somewhere far the metric tensor cannot be represented by small corrections to Minkowskian tensor is irrelevant. Now, Weyl [3] has demonstrated that the gravitational field of the linear version exerts no force on matter, i.e. is a “powerless shadow”!).

Analogous considerations can be made for the instance of two masses in relative oscillation, linked together with a spring.

The mere fact that the linearized approximation of GR has a covariant character only with respect to Lorentz transformations ought to be sufficient for not giving it an unconditioned credit. Moreover, we can also remark that from a mathematical standpoint the splitting of metric tensor $g_{jk}$ in a Minkowskian part $\eta_{jk}$ plus another (“small”) part $h_{jk}$ has in general a dubious legitimacy.

3. – The traditional way of investigating the question of the GW’s consists in attempts to derive from Einstein field equations a solution provided with an undulatory character. The conceptual scheme can be sketched as follows [4].

Consider in Minkowski spacetime a hypothetical scalar field $S(r, t)$, which satisfies d’Alembert equation:

$$\nabla^2 S - \frac{1}{c^2} \frac{\partial^2 S}{\partial t^2} = -4\pi \rho$$

a particular solution, as it is known, is given by the retarded field:

$$S_{ret}(r, t) = \int \frac{\rho(r', t')}{|r' - r|} dV'$$

where $ct' = ct - |r' - r|$, and the integration extends over the volume $V'$ in which $\rho$ is different from zero. At large distances from $V'$ the field $S_{ret}$, which satisfies the homogeneous d’Alembert equation in the region where $\rho = 0$, has the asymptotic form:
\[ S^{(a)}_{\text{rel}}(\mathbf{r}, t) = \frac{1}{r} \mu \left( t - \frac{\mathbf{r}}{c}, \frac{\mathbf{r}}{r} \right), \]

the function \( \mu \) being given by:

\[ \mu \left( t - \frac{\mathbf{r}}{c}, \frac{\mathbf{r}}{r} \right) = \int \rho \left( \mathbf{r}', t - \frac{\mathbf{r}}{c} + \frac{\mathbf{r}' \cdot \mathbf{r}}{c} \right) dV'. \]

The general solution of eq. (1) can be written:

\[ S = S_{\text{rel}} + S_{\text{in}}, \]

or

\[ S = S_{\text{adv}} + S_{\text{out}}, \]

where: \( S_{\text{adv}} \) is the advanced field, \( S_{\text{in}} \) and \( S_{\text{out}} \) are the ingoing and outgoing fields, solutions of the homogeneous d’Alembert equation (\( \rho = 0 \)).

The above equations have the \textit{same} formal structure for \textit{all} the inertial frames of reference. Consequently, they assure us of the \textit{real} existence of the waves of our field \( S \). This scheme – as it is well known – works very well for Maxwell electromagnetism, but \textit{in GR matters stand otherwise}. First of all, as I have previously emphasized, the \textit{linearized} approach must be discarded, owing to its inadequacy to investigate the GW’s [3]. (And the inadequacy of quadrupole formula remains also for perturbative refinements of third order in \( G \) and fifth order in \( v/c \)). On the other hand, the application of the above scheme to \textit{exact} GR – apart from the analytical difficulties due to the nonlinearity of Einstein equations – is doomed to a failure, as we shall see presently.

The relevant computations are executed, of course, in a given reference frame, usually in a \textit{harmonic} system of coordinates, and people have succeeded, in particular, in deriving perturbative expansions of Einstein equations of various kinds, mainly described with the adjectives “post-Newtonian” and “post-Minkowskian” [5]. However, as we know, there exist in GR \textit{no} coordinate frames that are \textit{physically} privileged – not even the harmonic ones (contrary to a firm conviction of Fock [6]). Now, a property is invariant, and has consequently a physical meaning, \textit{only} if it holds for \textit{any} system of coordinates. But the wavy character of a gravitational field is \textit{not} a property independent of the coordinate system: a field, which is undulatory in a given reference frame \( F \) \textit{loses} this property with the passage to a suitable frame \( F' \) belonging to an infinite class of similar systems, that are on the same conceptual footing of \( F \). This simple consideration proves the futility of the traditional attempts to demonstrate the existence of physical GW’s. It is also useful to remark that, dependently on the reference frame, the speed of an undulation of metric tensor can have \textit{any} value between zero and infinite, contrary to an old conviction (derived from the linearized version of GR), which attributes to the GW’s the speed of light \( c \) \textit{in vacuo}. 
APPENDIX A

The Einsteinian thesis that in GR there are no coordinate systems which are physically privileged gave origin in past times to a lively debate, provoked by the publication in 1917 of an important article by Kretschmann [7]. A point on the question was made by Pauli, who showed convincingly the correctness of Einstein’s thesis [8]. Fock was not persuaded, and always affirmed that in GR too (as in SR) there is a class of physically privileged frames: the harmonic ones [9]; but the proof of this assertion is defective. 

Et pour cause.

Back to Kretschmann [7]. His paper was admirably summarized by Ph. Frank in the following terms [10]: “Einstein versteht unter seinem allgemeinen Relativitätsprinzip die Forderung, daß die Naturgesetze durch Gleichungen ausgedrückt werden sollen, die gegenüber beliebigen Koordinatentransformationen kovariant sind. Der Verf. zeigt nun, daß jede beliebigen Gesetzen gehorhende Naturerscheinung durch allgemeine kovariante Gleichungen beschrieben werden kann, daß also das Bestehen solcher Gleichungen keine physikalische Eigenschaft aussagt. Zum Beispiel kann die gleichmäßige Ausbreitung des Lichtes im schwerelosen Raum auch kovariant dargestellt werden. Es ergibt aber dann eine Darstellung derselben Erscheinungen, die nur eine engere Gruppe (die Lorentz-Transformationen) zulässt. Diese Gruppe, die durch keine Darstellung der Erscheinung mehr eingeengt werden kann, ist für das betreffende System charakteristisch. Die Invarianz ihr gegenüber ist eine physikalische Eigenschaft des Systems und stellt im Sinne des Verf. das Relativitätspostulat für das betreffende Erscheinungsbereich dar. In der Einsteinschen allgemeinen Relativitätstheorie können nun durch geeignete Wahl der Koordinaten die Feldgleichungen auf eine Gestalt gebracht werden, die nicht mehr gegenüber der Gruppe der Koordinatentransformationen kovariant ist. Der Verf. gibt eine Reihe von Beispielen solcher Umformungen. Die so umgeformten Gleichungen gestatten aber überhaupt keine Gruppe mehr, und in diesem Sinne ist die Einsteinsche allgemeine Relativitätstheorie eine “Absoluttheorie”, während die spezielle Relativitätstheorie auch im Sinne des Verf. dem Relativitätspostulat für die Lorentz-Transformationen genügt.”

However, Kretschmann’s interpretation of GR, according to which GR would be in reality an “Absoluttheorie”, revealed itself as sterile, and Pauli [8] emphasized that the attempts by Kretschmann and Mie to “normalize” with suitable criteria the coordinate system resulted fruitful only in special instances. In the general case, and in fundamental questions, the general covariance is indispensable. Indeed, the general covariance is an essential and characterizing property of GR, while for other physical theories (Maxwell electrodynamics, special relativity, etc.) the formulation in terms of general coordinates is only an additional possibility, which leaves unchanged their physical characteristics.

In a sense, the astrophysical community considers GR as an “Absoluttheorie” in regard to the class of the cosmological models, with their “cosmic times”. The instance of Friedmann models – which however are isomorphic to corresponding Newtonian models – is very significative.
In reality, we should bear in mind that not all the properties attributed to a given cosmological model have an invariant character, and are therefore real properties. In particular, the age of the universe depends on the coordinate system that was chosen for the considered model. Further, the durations of the temporal stages in which the above age is usually divided in the instance of a Friedmann model, have only a conventional value from the general-relativistic standpoint. Only if we consider the isomorphic Newtonian model, all time periods have an absolute meaning.

**APPENDIX B**

In April, in May, and in June 2007 three teams of astrophysicists published the following three papers:

- “Maximum Entropy for Gravitational Waves Data Analysis: Inferring the Physical Parameters of Core-Collapse Supernovae” [11];
- “Rates and Characteristics of Intermediate Mass-Ratio Inspirals Detectable by Advanced LIGO” [12];
- “Host Galaxies Catalog Used in LIGO Searches for Compact Binary Coalescence Events” [13].

The authors describe various astrophysical phenomena that, in their opinion, should generate GW’s, and could be detectable by LIGO interferometers. It is easy to foresee that no GW will be registered by the apparatuses.

**References**

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[2] A. Loinger, arXiv:0709.0490 v1 [physics.gen-ph] 4 Sep 2007. And references therein.
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[4] See also A. Loinger, arXiv:gr-qc/9909091 (September 30th, 1999).
[5] See the review article by D. Kennefick, arXiv:gr-qc/0704002 v1 (April 1st, 1997), which contains a vast bibliography.
[6] V. Fock, *The Theory of Space, Time and Gravitation* (Pergamon Press, Oxford, etc.) 1964, passim.
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[9] Cf. [6].
[10] Ph. Frank, JFM 46.1292.01 (*Jahrbuch für Mathematik*).
[11] T.Z. Summerscales et al., arXiv:0704.2157 v1 [astro-ph] (17 April 2007).
[12] J. Mandel et al., arXiv:0705.0285 v1 [astro-ph] (2 May 2007).
[13] R.K. Kopparapu et al., arXiv:0706.1283 v1 [astro-ph] (9 Jun 2007).

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