The effect of 3D wear state of wheel polygon on wheel–rail system dynamics

Dabin Cui\textsuperscript{a,b}, Xing Zhang\textsuperscript{a}, Ruichen Wang\textsuperscript{c}, Boyang An\textsuperscript{b} and Li Li\textsuperscript{a}

\textsuperscript{a}School of Mechanical Engineering, Southwest Jiaotong University, Chengdu, People’s Republic of China; \textsuperscript{b}Key Laboratory of High-speed Railway Engineering, Ministry of Education, Chengdu, People’s Republic of China; \textsuperscript{c}Institute of Railway Research, University of Huddersfield, Huddersfield, UK

\textbf{ABSTRACT}

Wheel wear is a natural phenomenon of wheel–rail rolling contact during vehicle operation. The non-uniform wear in lateral and circumferential directions is normally occurred on wheels. The lateral non-uniform wear will lead to the unexpected hollow wear which can cause flange to flange vibration at low-frequency band, and the circumferential non-uniform wear will result in the vertical vibration which is the originate from the polygon phenomenon. These two forms of wear are coupled to each other on wheels and affect the dynamic behaviours of wheel–rail system. In this paper, a wheel–rail contact dynamics model which includes the 3D geometric shape of wheel profile is established and employed to vehicle system dynamics. The results show that the contact mechanical properties of wheel–rail are greatly affected by the 3D wheel profile. The low-frequency vibration of wheelset caused by hollow wear and the high-frequency vibration by polygonised wear are coupled, which increase the vibrational acceleration of wheel–rail system and even made for the main problem of the periodic separation of wheel and rail and aggravate the wear of wheel polygon.

\section{1. Introduction}

The wheel polygon is an intractable problem for running safety and reliability in railway engineering, which has an insufficient way to solve it completely \cite{1}. The vibration and noise of wheel–rail system caused by the wheel polygon seriously reduce ride comfort of the vehicle and the service life of components of the vehicle–track system \cite{2}. According to the large-scale operation of high-speed trains in China, the wheel polygon of high-speed wheel gradually appears and causes serious wheel–rail damage and additional vibration of vehicle system.

Wheel–rail impact caused by wheel polygon and its development mechanism has been attracted massive attention in the research of wheel–rail interaction. Polygon phenomenon of wheels was first found on the ICE high-speed train in Germany and the metro train...
in Stockholm, Sweden [3]. Meinke et al. [4] established a rigid and flexible coupling model, flexible axle connecting with rigid wheel and brake disc, studied the wheel polygon problem by numerical simulation method, and pointed out that wheel axle torsion has a great influence on the development of wheel polygon. Through simulation analysis, Morys [5] suggested that the wheel polygon will arouse the bending resonance mode of the wheelset when the train is running, so the lateral force of wheel–rail will change periodically, and finally lead to the wheel polygon wear. Johansson et al. [6] analysed the causes of out-of-round of different types of wheels by means of field test and theoretical analysis and pointed out the mechanism of fixed wavelength polygon related to track properties. Zhai [7] proposed the expressions of impulse and harmonic wheel–rail excitation in vehicle/track coupling dynamics model in order to study the influence of flat scar and polygon wear on wheel–rail forces. Cui et al. [8] studied the effect of measured out-of-round wheels on system dynamics and discussed the effect of turning characteristics of wheel lathes on the evolution of wheel polygonisation [9]. Yuan et al. [10] discussed the influences of wheelset lateral vibration on wheel polygon based on elastic vehicle system dynamics model and pointed out that the order of wheel polygon is determined by the phase angle difference of hunting motion. Yin [11] and Song [12] established multi-rigid body dynamics model and rigid-flexible coupling dynamics model respectively, and found out the limit value of wheel polygon from the Angle of vertical force of wheel–rail. Wu et al. [13] studied the vibration response of turnout when the polygonised wheels running through. Nielsen et al. [15] studied the wear problem of wheel polygons in detail and summarised the numerical method of wheel wear prediction. Jin et al. [16] reviewed the research history and current research condition of the wheel polygon wear, and discussed from various aspects such as wheel polygon, development mechanism and detection method. All of the above studies are based on certain wheel–rail contact surfaces, and reasonable modelling methods are very important to the research of related problems.

In recent years, with the remarkable improvement of train running speed and the wide range of the use of large stiffness of the integrated bed, the wheel polygon wear and the secondary damages have become increasingly prominent, and the research on wheel polygon has gradually become a hot issue in the railway engineering. At present, in the study of system dynamics, the wheel polygon is considered as the vertical harmonic excitation of the wheel–rail interface, and the coupling effect of cross section wear and polygon wear on the system dynamics has not been fully considered. In fact, the polygonised wear of the wheel is a kind of 3D wear, not a simple periodic change of the wheel radius. In this paper, the 3D wear of the wheel polygon is discussed, and a vehicle system dynamics model with the 3D wheel polygon is established. Considering the third-order wheel polygon wear as an example, the importance of the 3D wheel polygon wear is simulated and analysed in detail.

2. 3D wheel profile of polygon wear

Wheel polygonisation is the phenomenon of non-uniform wear on wheels along the circumference, which can be found in metro and passenger train. The wheel polygon can
Figure 1. Wheel polygons of different orders.

be described by order in terms of the number of wheel–rail impacts caused by the wheel polygon during one circle rotation of the wheel, as shown in Figure 1.

The wheel polygon excitation is normally described by Fourier series with $1 \sim N$ order harmonics [17]. The amplitude of the wheel polygon is

$$Z_0(t) = \sum_{i=1}^{N} A_i \sin \left[ i \left( \frac{\nu}{r} \right) t + \varphi_i \right]$$

(1)

where $A_i$ is the amplitude of the $i$th-order wheel polygon, $\nu$ is the vehicle running speed, $r$ is the nominal rolling circle radius of the wheel, $\varphi_i$ is the phase Angle of the $i$th-order wheel polygon.

When the wheel is dominated by the $n$th order polygon, periodic wheel–rail excitation comes out during the train running, and the excitation frequency is

$$f_n = \frac{\nu n}{2\pi R}$$

(2)

The wheel polygon causes the high frequency wheel–rail impact, increases the risk of wheel–rail damage and the fatigue failure of the key components on the bogies.

In the traditional wheel–rail force analysis, Hertz nonlinear elastic contact theory is used to calculate the normal force between wheels and rails. Since Hertz contact, the normal force can be expressed as:

$$F_N = \begin{cases} K_{Hz} \delta^{3/2} \left[ 1 + \frac{3(1 - \varepsilon^2)}{4} \frac{\delta}{\delta^{(-)}} \right], & \delta > 0 \\ 0, & \delta \leq 0 \end{cases}$$

(3)

Where $K_{Hz}$ is the Hertz contact stiffness, $\delta$ is the contact penetration amount, $\varepsilon$ is the collision recovery coefficient, $\dot{\delta}$ is the approach speed of the wheel/rail at any time, and $\dot{\delta}^{(-)}$ represents the relative approach speed of the two objects when they start to contact.
The contact penetration amount of wheel–rail is determined by the surface roughness of wheel–rail, which can be expressed as

$$\delta = Z_w - Z_r - Z_0$$  \hspace{1cm} (4)

Where $Z_w$ is the vertical displacement of the wheel at the current moment; $Z_r$ is the vertical displacement of the rail at the current moment. $Z_0$ is the polygon amplitude of the wheel.

From Equations (1–4), the lateral wear state of the wheel is not considered when studying the wheel–rail force and system vibration caused by the wheel polygon. Figure 2 shows the picture of the polygonised wheel. It can be seen that wheel–rail wear occurs in a certain area near the nominal rolling circle of the wheel, but have less impact on the root of wheel flange and the field side of the tread.

Without considering the 3D wear state of wheel polygon in wheel–rail contact analysis, the actual input wheel shape which is expanded in Cartesian coordinates is shown in Figure 3(a). This method ignores the influence of wheel cross section changes at different phases on wheel–rail contact, which is not accordance with the actual wheel–rail contact state. The actual profile of wheel with polygon wear is shown in Figure 3(b). To analyse the wheel–rail contact behaviours in the actual wear state, Equation (1) is modified as follows:

$$Z_0(t, y) = \sum_{i=1}^{N} \xi(y, \theta) A_i \sin \left[ i \left( \frac{y}{r} \right) t + \varphi_i \right]$$  \hspace{1cm} (5)

Where $\xi(y, \theta)$ is the amplitude correction coefficient of the polygon; $y$ is the transverse position of the contact point on the wheel; $\theta$ is the phase angle at the contact point of the wheel, which can be obtained by calculating the residual of the total rotation angle of the wheel by Equation (6)

$$\theta = \text{mod} \left( \frac{\omega t}{2\pi} \right)$$  \hspace{1cm} (6)

where $\omega$ is the wheel rotation angular velocity.
Due to the light wear of wheel polygons, the 3D wheel profile will not affect the applicability of the traditional theory. There is no need to improve the wheel–rail contact model. Considering the computational efficiency, Hertzian contact theory is used to solve the normal contact problem and FastSim is employed to solve the tangential contact problem.

In the wheel–rail contact theoretical model, the contact spot size can be calculated as follows:

\[
\begin{align*}
a &= C_a \sqrt{\frac{3F_N}{2K_1K_2}}, \\
b &= C_b \sqrt{\frac{3F_N}{2K_1K_2}}, \\
K_1 &= \frac{1}{R_1} + \frac{1}{R_1'} + \frac{1}{R_2} + \frac{1}{R_2'}
\end{align*}
\]

Where \( a \) is the long half axis of the ellipse contact spot; \( b \) is the short half axis of the ellipse contact spot; \( R_1 \) and \( R_1' \) are respectively the longitudinal and transverse curvature radii at the wheel contact points; \( R_2 \) and \( R_2' \) are, respectively, the longitudinal and transverse curvature radii at the contact point of the rail; \( E \) is the elastic modulus of the material, \( \nu \) is the Poisson’s ratio.

From Equation (7), the transverse wear of the wheel tread directly affect the size of wheel–rail contact spot. Therefore, when analysing the wheel–rail contact state, the 3D abrasion profile of the wheel cannot be ignored.
3. Vehicle system dynamics modelling

To study the influence of wheel polygon on wheel–rail dynamics, a vehicle system dynamics model was established according to the actual structural parameters of high-speed trains, as shown in Figure 4. In the model, the positioning mode of high-speed train’s revolving arm and the joint stiffness of shock absorber were considered [18]. The normal force of wheel and rail is considered as the Hertz spring, which is solved according to formula (3). Fastsim was used to solve the tangential force of wheel–rail. When solving the transient contact of wheel–rail, the difference of contact geometry caused by the 3D wear profile is considered, and the change of the creep rate/force of wheel–rail is also considered. Due to the main purpose of this paper is to discuss the effects of the 3D wear profile of the polygonised wheel on the dynamic behaviours of wheel–rail system, the rail and the structure under the rail are regarded as rigid bodies to simplify the calculation.

In the model, the equivalent force method is used to solve the mechanical model of the stiffness of the shock absorber joints [18]. The longitudinal force between carbody and bogies is mainly provided by air spring and anti-yaw dampers. The nonlinear characteristic of the anti-yaw damper is considered in the model as shown in Figure 5. Where $V_L$ is the unloading velocity, $F_L$ is the unloading force, and $c_1$, $c_2$ is the equivalent damping coefficient before and after unloading.

![Figure 4. The model of passenger car.](image-url)
Figure 5. Characteristic curve of anti-yaw damper.

The longitudinal velocity at end near the carbody of the anti-yaw damper can be expressed as

\[ V_{csx(L,R)i} = H_{cBsc} \dot{\beta}_c \pm d_{sc} \dot{\varphi}_c \mp (-1)^{i-1} d_{sc} \frac{d}{dt} \left( \frac{l_c}{R_c} \right) \] (8)

The longitudinal displacement at the end point near the bogie is

\[ X_{tsx(L,R)i} = H_{Bts} \dot{\beta}_ti \mp d_{sc} \dot{\varphi}_ti \] (9)

Where \( i = 1, 2 \) respectively represent front and rear bogie. In the calculation process, when the damping force of the anti-yaw damper is less than the unloading force, \( c_1 \) should be selected as the damping coefficient. Otherwise, \( c_2 \) should be selected.

\[ C_{sx}(\dot{Y}_{sx} - V_{csx}) + K_{dx}(Y_{sx} - X_{tsx}) + \frac{|F_{csx}|}{F_{csx}} F_{tl} = 0 \] (10)

The equivalent damping force of the damper can be obtained as

\[ F_{xt(L,R)i} = K_{sx} \left[ H_{cB} \dot{\beta}_c + H_{Bt} \dot{\beta}_ti \pm d_s \dot{\varphi}_c \mp d_s \dot{\varphi}_ti \mp (-1)^{i-1} d_s \left( \frac{l_c}{R_c} \right) \right] + F_{csx(L,R)i} \] (11)

Using the same method, the lateral force between carbody and bogie can be solved by

\[ F_{yt(L,R)i} = K_{sy} \left[ Y_{ti} - Y_c + H_{Bt} \varphi_c + (-1)^i l_c \dot{\varphi}_c + \frac{l_c^2}{2R_c} \right] + F_{csy(L,R)i} \] (12)

In Equations (8–12), \( H_{cBsc} \) is the height of the carriage centre from anti-yaw damper location; \( H_{Bts} \) is the height of the anti-hunting damper from the bogie centre; \( d_{sc} \) is the half distance between the anti-hunting damper of the two sides of the bogie; \( l_c \) is half of the distance between the bogie centre; \( H_{cB} \) is the height of the carriage centre from the secondary suspension location; \( H_{Bt} \) is the height of the secondary suspension from the bogie centre; \( d_s \) is the half distance between the secondary suspension systems positioned on each side of
the bogie frame; $H_{B_{ld}}$ is the height of the lateral damper from the bogie centre; $H_{c_{Bld}}$ is the height of carriage centre from the lateral damper; $\beta_c$ and $\beta_t$ are pitch degree of freedom of car body and frame, respectively; $\varphi_c$ and $\varphi_t$ are body and frame shaking angle respectively; $Y_{sx}$ is the displacement of the node of the anti-snake shock absorber.

To accurately express the 3D profile of the wheel, it is necessary to test the wheel profiles at different phases to obtain the envelope shape of the wheel, which is expressed in the form of Equation (5). The wheel radius at any time and lateral displacements can be expressed as

$$R(t, y) = r + Z_0(t, y)$$  \hspace{1cm} (13)

Where $r$ is the average radius at the nominal rolling circle of the wheel.

Since the polygon amplitude of the wheel is much smaller than the nominal rolling circle radius of the wheel, the longitudinal curvature radius of the wheel is directly taken as $r$.

In this paper, the influence of 3D wear state on wheel–rail dynamic performance is calculated and analysed by taking two 3rd order polygon wheels with different amplitude as an example. The radial run-out of the two selected polygonised wheels are both 0.3 mm. According to the wear at the crest of the polygonised wheel is small and at the trough is large, the two selected wheels have the similar profiles at crest and there has a great difference at the trough, as shown in Figure 6. The two selected wheels have hollow wear at the trough point, and hollow wear values are 0.16 and 0.2 mm, respectively.

The value of $\zeta$ at any phase is obtained by linear interpolation between the crest profile and trough profile of the measured wheels. Ignored the polygon phase $\varphi_i$, the 3rd order polygonised wheel profile can be written as

$$Z_0(t, y) = \zeta(y, t)A \sin \left[ 3 \left( \frac{V}{r} \right) t \right]$$  \hspace{1cm} (14)

Where $A = 0.15$ mm, $v = 50$ m/s, $/r = 0.46$ m.

![Figure 6. The crest and trough profiles of the selected polygonised wheels.](image-url)
4. Dynamics of wheel–rail system

By using the above model, the dynamics of wheel–rail system with and without the 3D wear surface are compared and analysed. To clearly reveal the influence of the 3D wear profiles, the track irregularity excitation is not applied in the simulation process. 1.6 mm lateral displacement of the wheelset was applied at the initial position. For convenience,

| Item                  | Case 1 | Case 2 | Case 3 |
|-----------------------|--------|--------|--------|
| Memory consumption    | 3.5G   | 4.8G   | 5.1G   |
| Calculation time      | 246 s  | 845 s  | 881 s  |

Figure 7. (a) Lateral displacement of wheelset. (b) Spectrum characteristics of lateral displacement of wheelset.
the Case1, Case2 and Case3 are employed to represent three different analytical conditions. Case1 represents the condition in which wear of polygons is described by the wheel radius changing along the wheel circumference. Case2 represents the condition with 0.16 mm hollow wear wheels at the trough, and Case3 represents the case with 0.2 mm hollow wear wheels at the trough.

After considering the 3D wear state of the wheel, the wheel has different geometric parameters at each calculation step, which will increase the memory consumption of the computer and increase the calculation time. The information is given in Table 1. In simulation, the length of rail line is 1000 m. Although statistics are influenced by computer configuration and programming software, it can be clearly seen that the memory consumption of the computer after considering the 3D wear (Case2 and Case3) increases.

![Graph](image-url)

**Figure 8.** (a) Vertical displacement of wheelsets. (b) Spectrum characteristics of vertical displacement of wheelset.
significantly, and the calculation time becomes about four times that of the tradition (Case1).

4.1. Vibration characteristics of wheelset

To clearly express the differences among the three cases, Figures 7 and 8 show the response of wheelset displacement and its spectrum characteristics. From Figure 7(a), the vibration of wheelset under different cases has a significant different after being excited by the initial lateral displacement. The primary performance of Case 1 obtains a small fluctuation of 52 Hz, which is caused by 3rd polygon, while the wheelset hunting frequency 1.7 Hz is not significant. When considering the 3D wear profile of the wheels (Case2 and Case3), under the influence of periodic hollow wear on the wheel, the wheelset shows obvious

![Figure 9](image-url)

**Figure 9.** (a) Lateral acceleration of wheelsets. (b) Frequency spectrum characteristics of lateral acceleration of wheelsets.
hunting motion of about 2.0 Hz, and the hunting amplitude increases with the increase of wheel hollow wear value. It can be seen from Figure 7 that the vibration caused by wheel polygon is coupled with the vibration caused by tread depression, which affects the motion behaviours of wheelset together. The motion behaviours of the wheelset can be regarded as a kind of motion with hunting frequency as group frequency and polygonal vibration frequency as phase frequency.

Figures 9 and 10 show the lateral and vertical accelerations of wheelsets and their spectral characteristics. Without considering lateral wear, in Case 1, the wheelset acceleration is mainly the vibration of 52 Hz dominated by the wheel polygon and its multiple frequencies. Affected by lateral wear, Case 2 and Case 3 gradually show low-frequency hunting vibration, 2 Hz. The wheel polygon in case 3 excites the double-frequencies 4 Hz vibration of wheelset.

![Figure 10](image-url)

**Figure 10.** (a) Vertical vibration acceleration of wheelsets. (b) Frequency spectrum of vertical acceleration of wheelsets.
This phenomenon is manifested in the vertical displacement and acceleration of the wheelset, just the acceleration shows stronger. Under the coupling of polygon wear and lateral wear of the wheel, the maximum acceleration of wheelset is significantly increased.

4.2. Wheel–rail contact characteristics

The contact characteristics of wheel–rail mainly include the geometric information of wheel–rail contact patch and the creep properties. Figure 11 shows the wheel–rail contact position on the wheel during wheel running. The vertical coordinate 0 corresponds to the nominal rolling circle of the wheel. The negative direction is the flange side, and the positive direction is the field side. The contact region is mainly affected by three parameters: the lateral displacement of wheelset shown in Figure 7(a), the profiles of the wheel shown in Figure 6, and the normal pressure of the wheel and rail. The lateral displacement and the profiles determine the centre of wheel/rail contact region, and the normal pressure influences the contact area. It can be seen from Figure 11 that the wheel–rail contact position in Case 2 and 3 is significantly shifted to the flange side compared with that in Case 1. It is noted that, under Case 3, the low-frequency hunting coupled with the high-frequency vibration, which cause periodic wheel–rail separation. The frequency of wheel–rail separation is identical to the group frequency of wheelset.

Figure 12 shows the area of wheel–rail contact patch with vehicle running. The area of wheel–rail contact patch under the three conditions presents periodic fluctuations, which is related to the periodic vibration of wheelset. The fluctuation amplitude of Case 2 is less than that of Case 1, and the area of contact patch is also significantly less than that of Case 1. This is because the consideration of the lateral wear of the tread, the wheel and rail contact points are mainly distributed on both sides of the hollow wear, and the wheel profile is not well matched with the rail. When the hollow wear of wheel is further increased, it causes impact vibration of wheelset [17], which is coupled with the high-frequency vibration caused by the polygon, resulting in the violent shaking of wheelset. Under the influence
of the unexpected periodic vibration, the area of contact patch can also fluctuate greatly. When periodic separation occurs, the area of contact patch is zero.

Therefore, the 3D wear state of the polygonised wheel has a great influence on the geometric information of the contact patch of the wheel–rail, which will inevitably lead to the difference of the mechanical characteristics of the wheel–rail in the contact patch.

Figures 13 and 14, respectively, show the vertical and normal forces of wheel–rail. According to wheel hollow wear is not serious in Case2, the pit has little influence on the movement of the wheel pair, see as Figures 8 and 9, so the wheel–rail forces in Case2 are similar with that in Case1. When the hollow wear increased as Case3, the coupling vibration caused by two kinds of wear significantly increases the vertical force and lateral force of wheel–rail.

Figures 15 and 16 show the response of longitudinal and lateral creep force of wheel–rail. The wheel–rail creep force has some difference between Case1 and Case2. The maximum
Figure 14. Lateral forces of wheel–rail.

Figure 15. Longitudinal creep force of wheel–rail.

difference of longitudinal creep is 75.9% and lateral creep force is 11.6%. Since the impact of periodic wheel–rail separation, the peak value of wheel–rail creep force is increased significantly. The fluctuation of creep force will affect the development of non-uniform wear of wheel. Figure 17 shows the wear number of the wheel. The wear number in Case1 and Case2 varies periodically with wheel polygon. The maximum wear number in Case1 and Case2 respectively is 1.52 and 2.09. The sharp fluctuation of wheel–rail creep force under Case3 results in a significant growth in wear number. The maximum wear number in Case3 is 33.8, which is 22.2 times than the value of Case1. It is worth noting that the wheel–rail separation occurs once in every 9 circles of wheels. In other words, each wheel–rail separation occurs in the same phase of the wheel. This means that the maximum wear number only appears in the same position of the wheel, where the wear must be the maximum. At
the same time, Figure 17 shows that in the interval between the two wheel–rail separations, the frequency of the wheel and rail wear number fluctuation caused by the wheel polygon is consistent with the vibration frequency of the original wheel polygon, which will accelerate the wear rate of the polygon.

5. Conclusions

In the study of the vibration characteristics of the polygonised wheel, the wheel polygon wear is generally regarded as a 2D problem, which is quite different from the real wheel polygon and not appropriate to describe the influence of the real wheel polygon on the dynamic behaviours of wheel–rail system. The function of wheel polygon is modified in this paper according to the measured 3D wear state and applied to the vehicle system...
dynamic theory. Then the vehicle system dynamics model which can consider the 3D wheel polygonised wear state is completed.

The influence of three polygonised wear patterns on the dynamics of wheel–rail system is simulated by the new model. The outputs show that when considering the 3D wheel polygonised wear surface, the low-frequency hunting motion caused by lateral wear couples with the high-frequency vertical vibration caused by wheel polygon, the wheelset appears a kind of motion with hunting frequency as group frequency and polygonised vibration frequency as phase frequency. When the serious hollow wear occurs on the wheel surface, the separation of wheel and rail can be brought in, which is dominated by group frequency. Because of the wheel–rail separation occurs in the same position of wheel regularly, it seriously aggravates the development of wheel polygonised wear.

**Disclosure statement**

No potential conflict of interest was reported by the author(s).

**Funding**

This work was supported by Fundamental Research Funds for the Central Universities: [Grant Number 2682021CX016]; Sichuan Province Science and Technology Support Program: [Grant Number 2021YJ0026].

**References**

[1] Zhu H, Hu H, Yin B, et al. Research progress on wheel polygons of rail vehicles. J Traffic Transp Eng. 2020;20(01):102–119.

[2] Han T, Jia S, Wu Y, et al. Effect of high-order wheel polygonal wear on the noise of bogie area of high speed trains. Noise Vib Control. 2019;39(3):88–91. 127.

[3] Kaper H. Wheel corrugation on Netherlands rail-ways (NS): origin and effects of “polygonization” in particular. J Sound Vib. 1988;120(2):267–274.

[4] Meinke P, Meinke S. Polygonalization of wheel treads caused by static and dynamic imbalances. J Sound Vib. 1999;227(5):979–986.

[5] Morys B. Enlargement of out-of-round wheel profiles on high speed trains. J Sound Vib. 1999;227(5):965–978.

[6] Johansson A, Andersson C. Out-of-round rail-way wheels—a study of wheel polygonalization through simulation of three-dimensional wheel-rail interaction and wear. Veh Syst Dyn. 2005;43(8):539–559.

[7] Zhai W. Vehicle-track coupled dynamics. 4thed. Beijing: Science Press; 2016.

[8] Cui D, Liang S, Song C, et al. Out of round high-speed wheel and its influence on wheel/rail behavior. J Mech Eng. 2013;49(18):8–16.

[9] Su J, Li L, Cui D. Study on influence of turning repair operations on wheels with initial polygonal state. J China Railw Soc. 2017;39(05):57–61.

[10] Yuan Y. Study on the mechanism and influence of the wheel out-of-round of high speed train. Beijing: Beijing Jiaotong University; 2016.

[11] Yin Z, Wu Y, Han J. Effect of polygonal wear of high-speed train wheels on vertical force between wheel and rail. J China Railw Soc. 2017;39(10):26–32.

[12] Song Z, Hou Y, Hu X, et al. Research on vibration characteristics of wheel-rail corrugation under flexible wheel and rail. J China Railw Soc. 2018;40(11):33–40.

[13] Wu Y, Han J, Liu J, et al. Effect of high-speed train polygonal wheels on wheel/rail contact force and bogie vibration. J Mech Eng. 2018;54(4):37–46.

[14] Wang P, Zhang R, Chen J, et al. Influence of polygonised wheels in high-speed trains on dynamic performance of turnout. J Mech Eng. 2018;54(4):47–56.
[15] Nielsen J, Johansson A. Out-of-round railway wheels-A literature survey. Proc Inst Mech Eng F J Rail Rapid Transit. 2000;214(2):79–91.

[16] Jin X, Wu Y, Liang S, et al. Mechanisms and countermeasures of out-of-roundness wear on railway vehicle wheels. J Southwest Jiaotong Univ. 2018;53(1):1–14.

[17] Jin X, Wu L, Fang J, et al. An investigation into the mechanism of the polygonal wear of metro train wheels and its effect on the dynamic behaviour of a wheel/rail system. Veh Syst Dyn. 2012;50(12):1–18.

[18] Cui D, Li L, Wang H, et al. High-speed EMU wheel re-profiling threshold for complex wear forms from dynamics viewpoint. Wear. 2015;338:307–315.