xloops

A Program Package calculating

One- and Two-Loop Feynman Diagrams

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1 Introduction

1.1 Intention

The aim of xloops is to calculate one-particle irreducible Feynman diagrams with one or two closed loops for arbitrary processes in the Standard model of particles and related theories. Up to now this aim is realized for all one-loop diagrams with at most three external lines and for two-loop diagrams with two external lines.

The results can be returned both algebraically and numerically. All necessary tensor integrals are treated for arbitrary masses and momenta. Those two-loop two-point functions for which no analytic result is known are integrated numerically by xloops.

The calculations are performed in Maple V – a language for symbolic computations [1,2]. Numerical integrations are done using VEGAS – a procedure for adaptive Monte Carlo integration [3,4]. For convenient input xloops has an Xwindows interface based on Tcl/Tk [5].

The package consists of the following parts:

▷ Input via Xwindows interface:
  In a window the user selects the topology which shall be calculated. A Feynman diagram pops up in which the particle names have to be inserted.

▷ Processing with Maple:
  The selected diagram is evaluated. The necessary steps for reducing the numerator (SU(N) algebra, Dirac matrices) are performed. The result is expressed in terms of one- and two-loop integrals.

▷ Evaluation of one-loop integrals:
  One-loop one-, two- and three-point integrals are calculated analytically or numerically to any tensor rank using Maple. This part was already released separately [6,7].

▷ Evaluation of two-loop integrals:
  All two-loop two-point topologies including tensor integrals are supported by the Maple routines. For those topologies where no analytic result is known, xloops creates either an analytic two-fold integral representation or integrates numerically with the help of VEGAS using C++ [8]. A parallelized implementation of the VEGAS algorithm invented by R. Kreckel [9,10] is part of this distribution.

  The other topologies can be calculated analytically or numerically like one-loop integrals.

In the second chapter we briefly denote how to install and start the program. Chapter three gives several examples for quick and easy usage. A more detailed description of the user-interface can be found in chapter four of this manual.
1.2 Copyright

The copyright of this program package is owned by the Johannes Gutenberg-Universität Mainz, Germany.

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Any further program development which intends to use the name \textit{xloops} or the incorporation of \textit{xloops} or a part of the \textit{xloops} code into any other program needs the permission of the authors.

With the receipt of \textit{xloops} the customer accepts that he uses the program entirely on his own risk. Neither the authors nor the distributors of \textit{xloops} are liable for any loss or damage as a result of any use which is made of this program.

1.3 Comments and bug reports

\textit{xloops} is still under development. So if you have questions, comments or if you have found any bug – please don’t hesitate to contact

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or one of the authors.
2 Preliminaries

2.1 Requirements

\texttt{xloops} can be used on any computer where \texttt{Maple V} is working. If \texttt{Maple} is not yet installed on your computer system you have to do that first. For numerical integration of two-loop integrals a compiler for \texttt{C++} is necessary. The \texttt{Xwindows} interface needs \texttt{Tcl/Tk} and is at present only available for Unix. On other platforms or if \texttt{Tcl/Tk} is not installed the \texttt{Xwindows} interface will not work. In this case you can use the \texttt{Maple} part directly (cf. sect. 2.4 and 2.5). The \texttt{Maple} part covers most features of the \texttt{xloops} package but the input may be found not as user-friendly as if the interface is used.

2.2 Installation

The distribution of the \texttt{xloops} package is accessible via WWW at

\begin{verbatim}
http://wwwthep.physik.uni-mainz.de/~xloops/
\end{verbatim}

It consists of one packed (tared, gzipped) file

\texttt{xloops-1.0.tgz}

This file contains all files needed for \texttt{xloops}. To install the package unpack \texttt{xloops-1.0.tgz}:

\begin{verbatim}
tar -xzvf xloops-1.0.tgz
\end{verbatim}

\texttt{xloops} then automatically creates a subdirectory of the current directory which is called \texttt{xloops}. The package is then deposited there. In addition there will occur several subdirectories of \texttt{xloops}. A complete list of all files in the distribution is given in appendix B.4. Moreover, there exists an additional (tared, gzipped) file

\texttt{xloops-lib.tgz}

in the same directory as \texttt{xloops-1.0.tgz} which contains a library of one-loop diagrams (cf. sect. 4.5). For convenience one should unpack this file in the same directory where \texttt{xloops-1.0.tgz} was unpacked before. \texttt{xloops} will read from this library whenever a one- or two-loop integral has to be evaluated. If this library is missing \texttt{xloops} will create the necessary files automatically, but this will usually take some time. If you have the \texttt{bash} shell type

\begin{verbatim}
  cd xloops/
  configure
\end{verbatim}
This will settle the configuration of the Xwindows interface. configure will request

▷ the release of Maple V (1, 2, 3 or 4) you are using. There were subtle changes from release 2 to 3 in handling local and global variables, therefore it is important to select the correct option. Since we tested only Maple V release 1 and 3, for release 2 and 4 configure internally does the following replacement:

release 2: use the same files as for release 1
release 4: use the same files as for release 3 (with some patches)

which is expected to be consistent.

▷ the WWW browser which should be used for the on-line html version of this manual. The name of the executable (cf. netscape, mosaic, ...) must be inserted.

▷ the character encoding of your terminal (Isolatin 1 or standard ascii).

▷ whether multiprocessing is enabled. This question is necessary to choose between normal VEGAS or the parallelized version of VEGAS running on multiprocessor computers only. If multiprocessing is enabled the number of processors is requested.

▷ the platform. Several frequently used systems (Linux, Digital Unix, SUN Solaris, IBM AIX) are distinguished from other Unix to install the right Makefile for the numerical integration.

▷ whether a library of one-loop integrals should be build. It can take a lot of time (up to an hour) on slower computers to build the whole library. Therefore it is recommended not to do that, but to load a pre-built library xloops-lib.tgz from our WWW server instead.

After that several checks are performed. First an embarrassing bug occurring in Maple releases up to release 3 is tested: If a file is read in a procedure, no output of the procedure will appear. Therefore we have written a little workaround for this bug, which can be installed by the user.

configure continues with a test run of the Makefile which is needed for numerical integrations. This Makefile depends on the platform. The run will take some time.

Finally you are asked to agree that automatically a mail is sent to the developer team of the xloops package. This mail is understood as registration of your xloops installation.

A typical run of configure is given in appendix A.1. configure automatically creates the xloops executable file.

If you have root access you can make it executable for all users. You then can set a symbolic link to /usr/bin, i.e.

    cd /usr/bin
    ln -s ⟨directory⟩/xloops/xloops .

where ⟨directory⟩ is the directory where xloops was installed.

If you don’t have bash you can find the necessary instructions in the file README which is part of the xloops distribution.

### 2.3 Getting started

You can now type
./xloops

from the directory where \texttt{xloops} was installed before. This starts your \texttt{xloops} session. The main window of \texttt{xloops} (cf. fig. 3.1) will appear. If \texttt{xloops} was made executable for all users (cf. previous section) it can be started with

\texttt{xloops}

from any directory.

There is on principle no restriction in the number of \texttt{xloops} sessions which can be used simultaneously – by the same user or by different users. On the other hand the numerical integration with \texttt{VEGAS} should be used only once by each user (cf. sect. 4.6).

\section*{2.4 Installing the Maple part only}

If you are only interested in the Maple routines you have to start the installation as in sect. 2.2, but you then skip \texttt{configure} and just copy all files with extension \texttt{.ma}

\begin{itemize}
  \item loops.ma
  \item fmrules.ma
  \item fmuser.ma
  \item evalproc.ma
  \item oneloop.ma
  \item cfcn.ma
  \item pv.ma
  \item r.ma
  \item simple.ma
  \item twoloop.ma
  \item numint.ma
  \item values.ma
  \item unvalue.ma
  \item mess.ma
\end{itemize}

in one directory instead and delete the rest of the distribution. The correct \texttt{.ma} files you can find in the subdirectory \texttt{xloops/MapleVR1} – if you use release 1 or 2 – or in the subdirectory \texttt{xloops/MapleVR3} – if you have release 3 or higher.
2.5 **Usage within a Maple session**

You can use $xloops$ within an ordinary Maple session as well: After having started Maple just type the following lines:

```maple
LoopPath:=⟨path⟩;
read′⟨path⟩loops.ma′;
```

$⟨path⟩$ describes the directory where the .ma files are located. If the $xloops$ package is installed properly this should be the path of the $xloops$ directory to which you have to add MapleVR1 – if you are using release 1 or 2 of Maple V – or MapleVR3 – if you are using release 3 or higher. If the .ma files were copied to the current directory before you can skip the assignment of LoopPath and only type

```maple
read′loops.ma′;
```

Now you can use the functions described in chapter 4.
3 Quick reference

3.1 Main window

After having typed the command xloops the main window (cf. fig. 3.1) appears on the screen. It consists of three parts:

1. The menu bar which contains several menus for xloops commands (for details cf. sect. 4.1.1).

2. The topology bar which displays all possible topologies for a definite \(m\)-loop \(n\)-point function. The selection of \(m\) closed loops and \(n\) external lines is done with the help of the menus Loops and Ext. Lines.

3. The text window with scroll bar which contains the results of calculations.

Figure 3.1: Main window
Before we will explain the functions and their meanings in detail in the next chapter, we demonstrate here as an example how to use $\chi^3$ for the calculation of a two-loop self-energy diagram and a one-loop vertex correction.

### 3.2 Two-loop self-energy

We now perform the calculation of a diagram which contributes to the Higgs self-energy $\Sigma$ at the two-loop level:

The underlying topology is the master topology, which is located in the fourth place in the topology bar (cf. fig. 3.1). In this case the number of external lines and the number of closed loops is already fixed correctly, so one now has just to click on the topology.

Then the master topology appears in a separate window, the diagram window (cf. fig. 3.2). In this window for each internal and external particle the corresponding particle name has to be inserted. Only particles can occur here, no antiparticles. The allowed terms for particle names are listed in sect. 4.2.4. They can be obtained alternatively with the menu Help. The flow of charges and fermion or ghost numbers is determined by the arrows of the lines. The direction of an arrow is reverted with a click on it.

Figure 3.2: The diagram for the two-loop Higgs self-energy
Since in our example we have to deal only with chargeless scalars, the direction of arrows is meaningless and we have to insert the name $H$ or $higgs$ for each particle only.

For the sake of clearness, the flow of external momenta in our conventions is displayed in blue. In this example there is only one momentum

$$q_1 = (q_{10}, 0, 0, 0) = (q_{10}, 0).$$

Vanishing components are put together to a common zero-vector. In our case there exists only one parallel space component $q_{10}$. The orthogonal space is represented through a common 0.

When all particles are inserted, $\chi$loops is ready for calculation. The diagram window gives you the possibility to select different modes of evaluation.

- Click on the $\mathbf{Evaluate}$ button: The output – for simplicity – is returned in terms of OneLoop and TwoLoop functions which correspond directly to one-loop and two-loop functions explained in sect. 4.3 and 4.4. In our example the result consists only of the scalar master two-loop two-point function which $\chi$loops calls TwoLoop2Pt2:

$$> \text{G1 := EvalGraph2 (7, [higgs,higgs,higgs,higgs,higgs,higgs,higgs,higgs,higgs,}$$
$$> \text{higgs,higgs,higgs]);}$$

$$\text{G1 := \{ [81 8 4 2 2 2 2 2 2 2,}$$
$$\text{C1 = ---- I Mhiggs e (1 + 2 eps Ln(4 Pi MU ) + 2 eps Ln(4 Pi MU ) ) }$$
$$\text{8192}$$

$$\text{TwoLoop2Pt2(0, 0, 0, 0, q10, Mhiggs, Mhiggs, Mhiggs, Mhiggs, Mhiggs, Mhiggs)}$$

$$\text{/ 4 4 8}$$

$$\text{/ (sin(tw) Mw Pi ),}$$

$$\text{/ C1}$$

Generally, the output is decomposed in one or several form factors $C_1$, $C_2$, ... to reflect the Dirac $\gamma$ or Lorentz structure of the result. Therefore the result is a list of form factors. The last entry in this list is the defining equation for the form factors. In our example of a scalar self-energy there appears only one form factor $C_1$ and the defining equation is just the scalar $C_1$. A more complicated situation can be found in the next section (for details cf. sect 4.2.10).

Moreover $\chi$loops assigned the whole result to a variable G1. Within an $\chi$loops session each calculation will get such a name in an unique way. This gives the user the opportunity to perform further calculations with the results of diagrams, for instance to sum up all evaluated graphs which contribute to the same process.

- Click on the $\mathbf{Eval. Full}$ button: $\chi$loops evaluates directly the OneLoop and TwoLoop integrals. Now each form factor $C_1$, ... is a Laurent expansion in terms of the ultraviolet regulator $\varepsilon$, where the significant coefficients of this expansion – $O(\varepsilon^{-1}), O(\varepsilon^0)$ in the one-loop case, $O(\varepsilon^{-2}), O(\varepsilon^{-1}), O(\varepsilon^0)$ in the two-loop case – are denoted in form of a list (cf. sect 4.2.10).

For those two-loop topologies where no analytic result is known – like in the case of our example, the finite part contributes two entries to the output list. The first
of them is a list itself which denotes an integral representation, the second contains
the part which is analytically calculable:

\[
GF1 := [ \\
C1 = [0, 0,
\begin{align*}
&81 \quad 4 \\
&\text{Mhiggs} e \\
&8192 \quad 4 \quad 4 \\
&\sin(tw) \quad \text{Mw} \quad \text{Pi} \\
&2 \\
&\text{Mhiggs} \\
&\text{I} \sqrt{(x - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4}) + \sqrt{(y - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4})}} \\
&\frac{2}{\text{q10}} \\
&\text{q10} (-2x - 1) (-2y + 1) \\
&4 \\
&\text{Mhiggs} \\
&-4 \sqrt{(x + 2x + 1 - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4})} + \frac{2}{\text{q10}} \\
&\text{q10} \\
&\text{q10} (-2x - 1) (-2y + 1) \\
&\text{Mhiggs} \\
&\text{I} \sqrt{(x - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4}) + \sqrt{(y - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4})}} \\
&\frac{2}{\text{q10}} \\
&\text{q10} (-2x - 1) (-2y + 1) \\
&\text{Mhiggs} \\
&\text{I} \sqrt{(x + 2x + 1 - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4}) + \sqrt{(y - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4})}} \\
&\frac{2}{\text{q10}} \\
&\text{q10} (-2x - 1) (-2y + 1) \\
&\text{Mhiggs} \\
&\text{I} \sqrt{(x - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4}) + \sqrt{(y - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4})}} \\
&\frac{2}{\text{q10}} \\
&\text{q10} (-2x - 1) (-2y + 1) \\
&\text{Mhiggs} \\
&\text{I} \sqrt{(x + 2x + 1 - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4}) + \sqrt{(y - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4})}} \\
&\frac{2}{\text{q10}} \\
&\text{q10} (-2x - 1) (-2y + 1) \\
&\text{Mhiggs} \\
&\text{I} \sqrt{(x - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4}) + \sqrt{(y - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4})}} \\
&\frac{2}{\text{q10}} \\
&\text{q10} (-2x - 1) (-2y + 1) \\
&\text{Mhiggs} \\
&\text{I} \sqrt{(x + 2x + 1 - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4}) + \sqrt{(y - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4})}} \\
&\frac{2}{\text{q10}} \\
&\text{q10} (-2x - 1) (-2y + 1) \\
&\text{Mhiggs} \\
&\text{I} \sqrt{(x - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4}) + \sqrt{(y - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4})}} \\
&\frac{2}{\text{q10}} \\
&\text{q10} (-2x - 1) (-2y + 1) \\
&\text{Mhiggs} \\
&\text{I} \sqrt{(x + 2x + 1 - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4}) + \sqrt{(y - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4})}} \\
&\frac{2}{\text{q10}} \\
&\text{q10} (-2x - 1) (-2y + 1) \\
&\text{Mhiggs} \\
&\text{I} \sqrt{(x - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4}) + \sqrt{(y - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4})}} \\
&\frac{2}{\text{q10}} \\
&\text{q10} (-2x - 1) (-2y + 1) \\
&\text{Mhiggs} \\
&\text{I} \sqrt{(x + 2x + 1 - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4}) + \sqrt{(y - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4})}} \\
&\frac{2}{\text{q10}} \\
&\text{q10} (-2x - 1) (-2y + 1) \\
&\text{Mhiggs} \\
&\text{I} \sqrt{(x - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4}) + \sqrt{(y - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4})}} \\
&\frac{2}{\text{q10}} \\
&\text{q10} (-2x - 1) (-2y + 1) \\
&\text{Mhiggs} \\
&\text{I} \sqrt{(x + 2x + 1 - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4}) + \sqrt{(y - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4})}} \\
&\frac{2}{\text{q10}} \\
&\text{q10} (-2x - 1) (-2y + 1) \\
&\text{Mhiggs} \\
&\text{I} \sqrt{(x - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4}) + \sqrt{(y - \frac{M_\text{higgs}}{2} + \frac{\text{I} \rho}{4})}} \\
&\frac{2}{\text{q10}} \\
&\text{q10} (-2x - 1) (-2y + 1) \\
$, 
\(x = -\text{infinity .. infinity}, \ y = -\text{infinity .. infinity}\),
0],
C1]
In our example the result is finite, therefore the first two entries in the list are zero. The only nonvanishing contribution comes from the integral representation. A more detailed description of the integral representation is given in sect. 4.4.3.

The result is a function of the Higgs mass $M_{\text{higgs}}$ and the external momentum $q_{10}$. The coupling of the vertices involve the $W$ mass $M_w$, the Weinberg angle $\theta_W$ and the electromagnetic coupling constant $e$. $\rho$ denotes the infinitesimal imaginary part of the masses in the denominator (cf. sect. 4.2.10), $x$ and $y$ are the variables which have to be integrated out numerically.

Click on the [Eval. Numeric] button: The integral representation described above is evaluated directly with the help of VEGAS. This option exists only for two-loop integrals, because all one-loop integrals are analytically calculable.

Please be aware that xloops can only perform the numerical integration if all variables – $M_{\text{higgs}}$ and $q_{10}$ in our example – are assigned with numerical values. For that purpose the menu File provides the Insert Maple Command entry. If this option is selected, a window (cf. fig. 3.3) appears, where any command can be passed directly to Maple. In our example we insert

$$q_{10} := 91.17; \quad M_{\text{higgs}} := 300;$$

to evaluate the integral on the $Z$ resonance for an assumed value of 300 GeV for the Higgs mass. After clicking on [Eval. Numeric] xloops starts the numerical integration.

![Figure 3.3: Direct Maple input of masses and momenta](image)

The result including the numerical uncertainty appears in the main window and reads:

$$GM1 := [\begin{array}{c} 2 \cr 2 \cr \frac{2}{\sqrt{x + 2xy + y - \frac{x}{2} + \frac{y}{2} + I\rho}} \cr q_{10} \end{array}$$

$$C1 = [0, 0,$$

$$\frac{4}{16} e^{\frac{4}{e}} \left[.66598680119771701375*10^{-4} \right],$$

$$\sin(\theta_W) M_w$$
In the result – compared to the \textbf{Eval. Full} option – the integral representation is now replaced by the numerical result of the integral. This result consists of a list: a pre-factor and another list. This list contains two entries: first the result of the \textsc{vegas} calculation, then the uncertainty \textsc{vegas} detected for this result (for details cf. sect. 4.2.10).

In our example the real part of the diagram is less than the numerical uncertainty, whereas the imaginary part is significantly larger. This means that the real part in fact is zero. Since we calculated the entire diagram which is not precisely $\Sigma$ but $-i\Sigma$, this is what one expects below threshold.

Alternatively to the direct input of \textsc{maple} commands described above it is possible to assign all known masses of particles, couplings and elements of the CKM matrix with the option Insert Particle Properties of the \textbf{Options} menu.

The input information for \textsc{vegas} can also be determined with the menu \textbf{Options}: Click on Numeric and an additional window will appear where the input parameters for \textsc{vegas} can be fixed (for details cf. sect. 4.1.4).

Finally the result can be saved in text mode to a file. Use in the menu \textbf{File} the entry \textbf{Save Maple Output}.

$\triangleright$ Click on the \textbf{Close} button: The \textit{diagram window} disappears.

The four buttons described above are completely independent. You can for example select \textbf{Eval. Numeric} without having clicked on \textbf{Evaluate} or \textbf{Eval. Full} before.

### 3.3 One-loop vertex correction

What follows is the calculation of a one-loop contribution of the correction to the $eeZ$ vertex:

First one has to select the correct values in the menus Loops and Ext. Lines of the \textit{main window} to get access to the one-loop three-point topologies. A click on the correct topology opens the \textit{diagram window} (cf. fig. 3.3) where the particle names have to be inserted. Again the correct particle names can be found in sect. 4.2.4 or in the menu \textbf{Help}. The direction of the fermion line must be adjusted by clicking on the arrows. The flow of external momenta...
Figure 3.4: The diagram for the $eeZ$ one-loop vertex correction

is displayed in blue again. The three momenta are

$$\mathbf{q}_1 = (q_{10}, 0, 0, 0) = (q_{10}, 0)$$

$$\mathbf{q}_2 = (q_{20}, q_{21}, 0, 0) = (q_{20}, q_{21})$$

$$\mathbf{q}_1 - \mathbf{q}_2 = (q_{10} - q_{20}, -q_{21}, 0, 0) = (q_{10} - q_{20}, -q_{21})$$

where the vanishing orthogonal space components are not displayed in the window. $q_{10}, q_{20}, q_{21}$ are the kinematical variables xloops uses for three-point functions. To get a numerical result, we assign these variables with

$q_{10} := M_{\text{Z}}; q_{20} := M_{\text{Z}}/2; q_{21} := \text{evalf}(\sqrt{M_{\text{Z}}^2/4 - M_{\text{Melec}}^2});$

using the option Insert Maple Command of the menu File so that all external particles are on-shell. Moreover the masses and couplings are assigned by the option Insert Particle Properties of the Options menu. The renormalization scale $\mu$ and the unit GeV can be omitted by setting

$\mu := 1; \text{GeV} := 1;$

Clicking on the [E. Full] button gives the following result:

$$GF2 := \{$$

\[
\begin{align*}
C_1 &= [0.000010058632263504308717 \, \text{i}, \\
&\quad -0.00862764964328278638 - 0.0072236143340212361270 \, \text{i}], \\
C_2 &= [0.00010510130634321490041 \, \text{i}, \\
&\quad -0.090149159887518402249 - 0.075478582279918574719 \, \text{i}], \\
C_3 &= [0, 0.0083116483915048522171 + 0.0071524795241752712917 \, \text{i}],
\end{align*}
\]

$$\}$$
The output is decomposed in several form factors $C_1, C_2, \ldots$ to reflect the Dirac $\gamma$ and Lorentz structure of the result. The last entry in this list is the defining equation for the form factors (cf. sect. 4.2.10).
4 Detailed description

There are two intentions covered by \texttt{xloops}:

▷ Either the calculation of complete Feynman diagrams. Here you need the \texttt{Evaluate Graph} window (see sect. 4.1) where you can insert particles into a Feynman diagram. Alternatively you can use the \texttt{EvalGraph} procedures in \texttt{Maple} described in sect. 4.2.

▷ Or the solution of single integrals occurring in the calculation of one- and two-loop diagrams. Then you can make use of the \texttt{OneLoop} and \texttt{TwoLoop} procedures explained in sect. 4.3 and 4.4.

The creation of a library for the integrals follows in sect. 4.5. The numerical integration of two-loop integrals is explained in sect. 4.6.

4.1 The \textit{X}windows interface

4.1.1 The functions of the menu bar

The menu bar provides a variety of different functions for the user. These functions are combined to groups which appear as menus if the user clicks on it. The list below gives a short overview. Additional descriptions can be found in the following sections.

▷ The menu \texttt{File} (cf. fig. 4.1a) has the following entries:

  ▷ \texttt{Insert Maple Command} allows the insertion of an arbitrary command which is passed directly to \texttt{Maple} (cf. fig. 3.3). Especially the numerical input of masses, momenta and renormalization conditions can be done easily this way.

  ▷ \texttt{Save Maple Output} saves the output as it is denoted in the \texttt{Maple text window} to an extra file. The name of the file has to be determined in a separate window, which appears if the user clicks on this entry.

  ▷ \texttt{New Process} allows the user to collect a set of graphs and save them with a common process name. Detailed explanation can be found in section 4.1.3.

  ▷ With \texttt{Load Process} processes which were declared with \texttt{New Process} before can be reloaded into the current \texttt{xloops} session (cf. sect. 4.1.3).

  ▷ \texttt{Create Process} will call the graph generator to produce all graphs to a given number of loops and external lines in a pre-defined model (in future versions of \texttt{xloops} only).

  ▷ \texttt{Restart Maple} terminates the current \texttt{Maple} session and removes all assigned variables. \texttt{Maple} and \texttt{xloops} are started again.

  ▷ \texttt{Quit} terminates \texttt{xloops}.
The menu Model (cf. fig. 4.1d) offers several physical models for calculation (for details cf. sect. 4.2.3).

- The menu Loops (cf. fig. 4.1e) selects the number of closed loops for the calculation. The user can choose between 0, 1 and 2. A change of this value results in different topologies displayed in the topology bar.

- The menu Ext. Lines (cf. fig. 4.1f) allows the user to choose the number of external lines. At the moment the upper limit is three. The change of this number results in a different topology bar as well.

- The menu Options (cf. fig. 4.1c) contains the following entries:
  - If History is selected \texttt{xflops} saves all calculations under a common process name. Those diagrams which were already saved before are read from disk. Otherwise, if History is not selected they are recalculated (cf. sect. 4.1.3).
  - With the selection of OneLoop Library \texttt{xflops} takes all one-loop functions from a pre-built library. Otherwise the one-loop functions are calculated every time they are needed (cf. sect. 4.7 and appendix B.2).
  - If Insert Particle Properties is selected the values of masses and couplings reported by the Particle Data Group \cite{PDG} are inserted (cf. sect. 4.2.4).
  - Set Neutrino Masses 0 assigns all neutrino masses to zero (default) or, if not selected, leaves the neutrino masses unassigned. In this case an undefined neutrino mixing matrix exists as well (cf. sect. 4.2.4).
Massless Light Fermions assigns the masses of all fermions except the $b$ and $t$ quark to zero (cf. sect. 4.2.4).

Numeric defines the number of points and the number of iterations for the numerical integration with VEGAS (cf. sect. 4.6).

Window Options allows the user to adjust the appearance of the Xwindow interface, especially the placement of the red arrows of the diagram window (cf. sect. 4.1.4).

Save Options saves all options in the file .xloops in the home directory of the user.

The menu Help (cf. fig. 4.1b) has three entries:

About shows a brief information on xloops.

Help gives a short help where especially the notation of all particle names in xloops is written.

Tutorial calls a WWW browser which displays the on-line html version of this manual.

### 4.1.2 Topology bar and diagram window

According to the pre-defined values for the numbers of closed loops and external lines the topology bar displays all possible topologies. If the user clicks on one of them this topology appears in a separate window, the diagram window. This window consists of three parts: a menu bar, an area for inserting particles (diagram area) and a command bar with different buttons for evaluation (cf. fig. 4.2).

The menu bar provides two options. The right one, Export, gives the user the possibility to export the graph to a Postscript file. Only the diagram area of the diagram window is exported, menu bar and command bar are not included.

The left option, Subtopologies, will only be available in future versions. It will provide the possibility to switch between several subtopologies, which are obtained when external lines are interchanged. The upper topology in fig. 4.2 may serve as an example. This topology can be rotated by $120^\circ$ or $240^\circ$ which corresponds to a cyclic permutation of the external lines. This changes the topology to the same type of topology but with different momentum flow. In contrast, the lower topology in fig. 4.2 does not change if it is rotated by $120^\circ$ or $240^\circ$. So this topology does not exhibit different subtopologies.

In the diagram area the topology is displayed graphically. For every propagator an entry for the insertion of particles is located. The correct terms for particle names are written in section 4.2.4. They can also be obtained from the Help function of the main window. The flow of charge and fermion and ghost numbers are determined by the red arrows on the propagator lines. Clicking on an arrow changes its direction. The correct flow of charge and fermion and ghost numbers can be settled in this way. Only particles, no antiparticles can be inserted. Charge conjugation may be achieved by inversion of arrows. Please keep in mind that the arrows don’t describe the direction of momenta. The convention of momentum flow is shown graphically in the diagram window in blue. It is defined for all diagrams in the same way, to facilitate a final summation of all calculated diagrams.

If the user has inserted all particles and checked the charge flow, the command area gives him several possibilities for the evaluation of the diagram. They correspond to the different modes of evaluation as explained in section 4.2.10.
Figure 4.2: Different diagram windows
4.1.3 Loading and saving of processes

For the evaluation of complete processes it is possible to save the contributing graphs with a common process name. Therefore xloops provides the File menu entries Load Process and New Process.

If Load Process is selected the file window (cf. fig. 4.3) appears on the screen. In this window the processes which were already saved are listed on the left hand side. In addition for every process a user specified description is given on the right hand side. This may help to reidentify a process later. To select a specific process one clicks on a process name on the left. After clicking on the OK button the history table (cf. fig 4.4) of the selected process appears whereas Cancel closes the file window. In the history table all diagrams which were included in the corresponding process occur with a button. A click on such a button opens a diagram window (cf. fig 4.2) with which the Feynman diagram can be evaluated. The history table contains two menus:

▷ In Graphs one has the opportunity – if the number of diagrams exceeds the capacity of the window – to switch to another sheet of diagrams. The same effect have the arrows at the bottom of the history table.

▷ Close lets the history table disappear.

If the entry History of the Options menu is selected all results are saved. With History all calculations which were already saved in this or another xloops session are read in automatically and not calculated again. For each process xloops automatically creates a subdirectory of the directory xloops_user where the results are saved to different files.

If History is selected without having specified a process before xloops assumes that a new process shall be added to the list and opens a similar file window (cf. fig 4.3) – the same window appears if New Process is selected. Two entries are provided for inserting a new process name and description at the bottom of the window. After the user inserted the name and description for his process and clicked on the OK button a new – and therefore empty – history table appears on the screen. All diagrams which are calculated from now are included in the history table as additional entries. The topology of the graph and the list of interacting particles are displayed.
Figure 4.4: The *history table* containing diagrams of the process $H \rightarrow H$. 

- Menu for selecting arbitrary diagrams
- Single diagram
- Topology
- Particle list
- Diagram numbers shown
- Forward/backwards in diagram list
4.1.4 Window options

With the entry Window Options of the Options menu the appearance of some windows of \textit{xloops} can be adjusted to fit the terminal characteristics (cf. fig. 4.6). First, the size of the history table (cf. fig. 4.4) can be changed by decreasing or increasing the numbers of the \textbf{Rows} and \textbf{Columns} entry in fig. 4.6. Then, the position of the arrows of the propagators in the \textit{diagram window} can be corrected. This might be necessary on some terminal types, where the arrows of the propagators are misplaced. This has to do with the fact that the relation between screen dots (as used by \textit{Tk}) and centimeters is different on different \textit{X} terminals. The embarrassing consequence can be that the arrows of the propagators are misplaced. The problem can be avoided by inserting a global \textbf{Scale Factor}. Alternatively, if this factor is not obvious, \textit{xloops} will calculate it. Therefore the user must insert the
position of the arrow on the ruler in the Arrow Scaling field of fig. 4.6.

4.2 Evaluation of Feynman diagrams

4.2.1 Input

To evaluate a particular Feynman diagram \texttt{xloops} needs to know four ingredients:

1. The number of closed loops.
2. The number of external lines.
3. The topology, that means the information how the lines of the diagram are connected.
4. The particles of the diagram.

The first two numbers have to be selected with the help of the \textit{menu bar} entries \texttt{Loops} and \texttt{Ext. Lines} (cf. sect. 4.1.1). Then the topology can be selected on the \textit{topology bar} so that the \textit{diagram window} pops up where the particles can be inserted (cf. sect. 4.1.2).

Internally the input from the Xwindows interface is converted to a procedure which performs the calculations in Maple. It is also possible to start directly from this routine for Maple – either with the \texttt{Insert Maple Command} of the File menu or within an ordinary Maple session where the Maple part of \texttt{xloops} was read in before (cf. sect. 2.5).

For that purpose the internal Maple commands will be mentioned in the following. All commands which follow without reference to the Xwindows interface have to be passed to Maple.

The input is then of the following form

\begin{verbatim}
  ⊲ for tree diagrams: EvalGraph0(n,⟨list⟩);
  ⊲ for one-loop diagrams: EvalGraph1(n,⟨list⟩);
  ⊲ for two-loop diagrams: EvalGraph2(n,⟨list⟩);
\end{verbatim}

The different topologies with the same number of loops are numbered. The number of the desired topology has to be inserted in the function call for \texttt{n}. The correct number for each topology will be given below in sect. 4.2.2. \texttt{⟨list⟩} describes the list of particles involved in the Feynman diagram. The conventions for the ordering of the particles are also explained for every topology in 4.2.2.

4.2.2 Internal Notation

What follows is a list of all topologies which at present can be solved by \texttt{xloops}. This list is only of interest for direct Maple input. To fix the flow of charges and of fermion or ghost numbers in an unique way, each internal propagator is assigned with two particle names. For example \texttt{[up,upbar]} and \texttt{[upbar,up]} describe the up-quark propagator but with different direction of the quark line arrow. External lines are described by only one particle name.

As a convention all particles at each vertex are incoming. The arrows of the diagrams displayed below only describe the flow of momenta. Of course these arrows need not to coincide with the arrows of fermions or other charged particles.

For each topology we give the correct \texttt{EvalGraph} command as well as the command for the corresponding OneLoop or TwoLoop integral.

\footnote{If the scaling factor is already correct, the arrow should point on the 5.}
Tree level

Three-point diagrams

Four-point diagrams
One-loop level

One-point diagrams

\[ \text{EvalGraph1}(1, \{1, 2, 3\}) \]

\[ \text{OneLoop1Pt}(p, m) \]

Two-point diagrams

\[ \text{EvalGraph1}(2, \{1, 2, 3, 4\}) \]

\[ \text{OneLoop1Pt}(p, m) \]
Three-point diagrams

**EvalGraph1(3, [1, 2, 3, 4, 5, 6])**

**OneLoop2Pt(p_0, p_\perp, q_{10}, m_1, m_2)**

**EvalGraph1(4, [1, 2, 3, 4, 5, 6, 7])**

**OneLoop2Pt(p_0, p_\perp, q_{10}, m_1, m_2)**

**EvalGraph1(5, [1, 2, 3, 4, 5, 6, 7, 8, 9])**

**OneLoop3Pt(p_0, p_1, p_\perp, q_{10}, q_{20}, q_{21}, m_1, m_2, m_3)**
Two-loop level

Two-point diagrams

EvalGraph2(6, [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12])

TwoLoop2Pt1(p_0, p_⊥, r_0, r_⊥, s, q_0, m_1, m_2, m_3, m_4, m_5)

EvalGraph2(7, [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12])

TwoLoop2Pt2(p_0, p_⊥, r_0, r_⊥, s, q_0, m_1, m_2, m_3, m_4, m_5)
4 Detailed description

EvalGraph2(8, [1, 2, 3, 4, 5, 6, 7, 8, 9, 10])

TwoLoop2Pt3(p₀, p⊥, r₀, r⊥, s, q₁₀, m₁, m₂, m₃, m₄)

EvalGraph2(9, [1, 2, 3, 4, 5, 6, 7, 8])

TwoLoop2Pt4(p₀, p⊥, r₀, r⊥, s, q₁₀, m₁, m₂, m₃)
For each encircled number one particle has to be inserted. The correct symbols for all allowed particles are listed in the next section. The conventions for the OneLoop and TwoLoop functions are denoted in sect. 4.3 and 4.4.

4.2.3 Models

xloops distinguishes several models. The menu Model of the menu bar serves for the selection of the desired model. Internally the global variable Model selects the model which shall be used. The following models are declared in xloops:

EvalGraph2(10, [1, 2, 3, 4, 5, 6, 7, 8, 9, 10])

TwoLoop2Pt5(\(p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_2, m_3, m_4\))

EvalGraph2(11, [1, 2, 3, 4, 5, 6, 7, 8, 9, 10])

TwoLoop2Pt6(\(p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_2, m_3, m_4\))
Detailed description

▷ **Standard Model** – the (minimal) standard model (electroweak and QCD). This corresponds to Model:=SM; and is the default value

▷ **2 Higgs Doublets Model** – the standard model with two Higgs doublets which corresponds to five physical Higgs particles; Model:=THDM;

▷ **QCD** – QCD with three generations of Quarks; Model:=QCD;

▷ **Electroweak Theory** – the electroweak sector of the standard model; Model:=EW;

▷ **QED** – QED with three generations of Leptons and Quarks; Model:=QED;

▷ **Phi^4 Theory** – \( \phi^4 \)-theory in four dimensions; Model:=phifour;

▷ **Phi^3 Theory** – \( \phi^3 \)-theory in four dimensions; Model:=phithree;

▷ **Goldstone Boson Equiv. Model** – the equivalence model for Goldstone bosons – standard model vector bosons are replaced by scalar Goldstone modes \([12–14]\); Model:=GBEM;

▷ a user-defined model where arbitrary Feynman rules can be declared. To work with this option the user has to set Model:=user; and to read an additional file

\[
\text{read'}(\text{path})\text{fmuser.ma'};
\]

where \(\text{path}\) is the same path used for the assignment of LoopPath at the beginning of the Maple session (cf. sect. 2.5). In the file \(\text{fmuser.ma}\) the user-defined rules have to be inserted. Initially all QCD Feynman rules are denoted there as an example. The conventions of declaration can be taken from this example.

### 4.2.4 Particles and Masses

What follows is a list which contains all particles \(xloops\) knows. The names valid in the Xwindows interface are denoted (second column from the left) as well as the names for internal use within Maple (third column from the left). In addition for each particle the models in which it is declared (right column) and the name which \(xloops\) uses for the corresponding particle mass (second column from the right) are listed.
\[ m_\phi \]  
\[ f, \phi \]  
\[ \begin{align*} 1 &= \phi \\ 2 &= \phi \end{align*} \]  
\[ m_\phi = M_\phi \]  
phithree, phifour

\[ m_{\nu_e} \]  
\[ ve, \nu_e \]  
\[ \begin{align*} 1 &= \nu_e \\ 2 &= \nu_e \bar{\nu} \end{align*} \]  
\[ m_{\nu_e} = M_{\nu_e} \]  
SM, EW, QED, THDM, GBEM

\[ m_e \]  
\[ e, e \]  
\[ \begin{align*} 1 &= e \\ 2 &= e \bar{e} \end{align*} \]  
\[ m_e = M_e \]  
SM, EW, QED, THDM, GBEM

\[ m_{\nu_\mu} \]  
\[ \nu_\mu, \nu_\mu \]  
\[ \begin{align*} 1 &= \nu_\mu \\ 2 &= \nu_\mu \bar{\nu}_\mu \end{align*} \]  
\[ m_{\nu_\mu} = M_{\nu_\mu} \]  
SM, EW, QED, THDM, GBEM

\[ m_\mu \]  
\[ \mu, \mu \]  
\[ \begin{align*} 1 &= \mu \\ 2 &= \mu \bar{\mu} \end{align*} \]  
\[ m_\mu = M_\mu \]  
SM, EW, QED, THDM, GBEM

\[ m_{\nu_\tau} \]  
\[ \nu_\tau, \nu_\tau \]  
\[ \begin{align*} 1 &= \nu_\tau \\ 2 &= \nu_\tau \bar{\nu}_\tau \end{align*} \]  
\[ m_{\nu_\tau} = M_{\nu_\tau} \]  
SM, EW, QED, THDM, GBEM

\[ m_\tau \]  
\[ \tau, \tau \]  
\[ \begin{align*} 1 &= \tau \\ 2 &= \tau \bar{\tau} \end{align*} \]  
\[ m_\tau = M_\tau \]  
SM, EW, QED, THDM, GBEM

\[ m_u \]  
\[ u, u \]  
\[ \begin{align*} 1 &= u \\ 2 &= u \bar{u} \end{align*} \]  
\[ m_u = M_u \]  
SM, EW, QED, QCD, THDM, GBEM
\begin{center}
\begin{tabular}{ccc}
\hline
\textbf{\(m_d\)} & \textbf{d, down} & \(1 = \text{down}\) \\
\hline
\textbf{\(m_c\)} & \textbf{c, charm} & \(1 = \text{charm}\) \\
\textbf{\(m_s\)} & \textbf{s, strange} & \(1 = \text{strange}\) \\
\textbf{\(m_t\)} & \textbf{t, top} & \(1 = \text{top}\) \\
\textbf{\(m_b\)} & \textbf{b, bottom} & \(1 = \text{bottom}\) \\
\textbf{\(m_{\bar{\gamma}}\)} & \textbf{\(\gamma, \gamma\)} & \(1 = \text{\gamma}\) \\
\textbf{\(m_{W}\)} & \textbf{W, wp} & \(1 = \text{wp}\) \\
\textbf{\(m_Z\)} & \textbf{Z, z0} & \(1 = \text{z0}\) \\
\hline
\end{tabular}
\end{center}

\begin{center}
\begin{tabular}{ccc}
\hline
\& \textbf{\(1 = \overline{\text{down}}\)} & \textbf{\(m_d = M_{\text{down}}\)} & \textbf{SM, EW, QED, QCD, THDM, GBEM} \\
\& \textbf{\(1 = \overline{\text{charm}}\)} & \textbf{\(m_c = M_{\text{charm}}\)} & \textbf{SM, EW, QED, QCD, THDM, GBEM} \\
\& \textbf{\(1 = \overline{\text{strange}}\)} & \textbf{\(m_s = M_{\text{strange}}\)} & \textbf{SM, EW, QED, QCD, THDM, GBEM} \\
\& \textbf{\(1 = \overline{\text{top}}\)} & \textbf{\(m_t = M_{\text{top}}\)} & \textbf{SM, EW, QED, QCD, THDM, GBEM} \\
\& \textbf{\(1 = \overline{\text{bottom}}\)} & \textbf{\(m_b = M_{\text{bottom}}\)} & \textbf{SM, EW, QED, QCD, THDM, GBEM} \\
\& \textbf{\(1 = \gamma\)} & \textbf{\(m_{\gamma} = M_{\gamma} = 0\)} & \textbf{SM, EW, QED, THDM, GBEM} \\
\& \textbf{\(1 = \overline{\text{wp}}\)} & \textbf{\(m_{W} = M_{W}\)} & \textbf{SM, EW, THDM, GBEM} \\
\& \textbf{\(1 = \overline{\text{z0}}\)} & \textbf{\(m_{Z} = M_{Z0}\)} & \textbf{SM, EW, THDM, GBEM} \\
\hline
\end{tabular}
\end{center}
\[ m_g \]

\[ m_H \]

\[ m_{H^0} \]

\[ m_{h^0} \]

\[ m_{g^0} \]

\[ m_{h^0} \]

\[ m_{H^0} \]

\[ m_{h^0} \]
\begin{center}
\begin{tabular}{c c c c c c c c c c c}
\hline
& $m_H$ & & $H^+$, Higgsp & $1$ = Higgsp & $2$ = Higgs$^*$ & $m_H = M_{Higgs}$ & THDM \\
& & & & & & & \\
& $m_{A_0}$ & & $A_0$, a0 & $1$ = a0 & $2$ = a0 & $m_{A_0} = M_{a0}$ & THDM \\
& & & & & & & \\
& & & $G^-$, ghostgamma & $1$ = ghostgamma & $2$ = ghostgamma$^*$ & & SM, EW, THDM \\
& & & & & & & \\
& & & $GZ$, ghostz0 & $1$ = ghostz0 & $2$ = ghostz0$^*$ & & SM, EW, THDM \\
& & & & & & & \\
& & & $GW^+$, ghostwp & $1$ = ghostwp & $2$ = ghostwp$^*$ & & SM, EW, THDM \\
& & & & & & & \\
& & & $GW^-$, ghostwm & $1$ = ghostwm & $2$ = ghostwm$^*$ & & SM, EW, THDM \\
& & & & & & & \\
& & & $Gg$, ghostgluon & $1$ = ghostgluon & $2$ = ghostgluon$^*$ & & SM, QCD, THDM \\
\hline
\end{tabular}
\end{center}
Please note that the masses of the neutrinos are set to zero by default. To keep them as
variables, one can switch off the entry Set Neutrino Mass 0 of the Options menu or use

UnsetNeutrinoMass0();

within Maple which removes any assignment of \(M_{\text{nelec}}, M_{\text{numu}}\) and \(M_{\text{nutau}}\). As a consequence now a leptonic CKM matrix exists which behaves analogous to the usual CKM
matrix of quarks (cf. sect. 4.2.6). The reverse command

SetNeutrinoMass0();

or the reselection of Set Neutrino Mass 0 sets all neutrino masses back to zero. The leptonic
CKM matrix equals now the unit matrix.

The masses of photon and gluon are also set to zero by default. To keep them as vari-
ables you have to type

\[\text{Mgamma:='Mgamma'; Mgluon:='Mgluon'};\]

The masses of leptons and light quarks – all standard model fermions except \(b\) and \(t\) quark –
can collectively be set to zero with the entry Massless Light Fermions of the Options menu or with

SetLightMass0();

and be unassigned with

UnsetLightMass0();

or another click on Massless Light Fermions. Apart from this, masses, momenta and cou-
plings – as they are explained in the following sections – can be assigned by the user.

Moreover, if you want to assign all particle masses\(^2\) as well as the couplings, the
Cabibbo-Kobayashi-Maskawa matrix (CKM matrix) and the Weinberg angle with their
experimentally determined values, you can use the entry Insert Particle Properties of the
Options menu or type

\[\text{read'}\langle\text{path}\rangle\text{values.ma'};\]

where \(\langle\text{path}\rangle\) is the same path used for the assignment of LoopPath at the beginning of
the Maple session (cf. sect. 2.3). The file values.ma contains the parameters according to
the currently published data of the Particle Data Group \([1]\). To remove all information
of values.ma in a Maple session you can type

\[\text{read'}\langle\text{path}\rangle\text{unvalue.ma'};\]

or switch off the Insert Particle Properties entry. Instead of the fermion names which were
already introduced one can also use the more general terms

\[\text{up1, up2, up3, down1, down2, down3}.\]

The difference is, that for these terms no electric charge \(Q\) and no weak isospin \(T3\) is

\(^2\)Unfortunately the Higgs mass is not yet included
defined. While \textit{xloops} assumes
\[
Q(\text{elec}) = -1; \quad T_{3l}(\text{elec}) = -\frac{1}{2}; \quad Q(\text{up}) = \frac{2}{3}; \quad T_{3l}(\text{up}) = \frac{1}{2}; \quad ...
\]
it has no values for \(Q(\text{up}_1), T_{3l}(\text{up}_1), ...\) and the user has the opportunity to make use of this freedom.

4.2.5 Feynman rules

Since the conventions for Feynman rules vary in the literature, it is important to explain one’s own rules: We use Feynman gauge, for electroweak processes according to the rules given in [15], but with all three-vertices and all propagators multiplied by \(-i\) and all four-vertices and the integration measure multiplied by \(i\). This convention coincides with the rules of [14] – as far as these are applicable in Feynman gauge. From [16] we took the rules for QCD and the notation of the CKM matrix, too. The rules for the Two Higgs doublet model are from [17].

Anyway, you receive the rule for one particular vertex easily if you just use the tree level topologies or call the \texttt{EvalGraph0} procedure.

In our convention the electromagnetic coupling \(e\) and the strong coupling \(g_s\) are positive (the electric charge of the electron is \(-e\)). \textit{xloops} automatically inserts a factor \((2\pi)^2\varepsilon^4\) for every closed loop and an additional \(-1\) for each fermion and ghost loop. This means that the dimension – which \textit{xloops} calls \(D\) – equals \(4 - 2\varepsilon\). The correct symmetry factor of the diagram is also included. The scale which keeps the coupling constants dimensionless is called \(\mu\).

4.2.6 Mixing angles and CKM matrix

Three mixing angles are known by \textit{xloops}: The usual Weinberg angle \(\theta_w\) and – only relevant in the case of the Two Higgs doublet model – the angles \(\alpha\) and \(\beta\) (cf. [17]). The elements of the CKM matrix are denoted by \(\text{CKM}(\text{up,down})\ldots\), its complex conjugates are \(\text{CKMC}(\text{up,down})\ldots\). If neutrino masses are not set equal to zero there exists also a leptonic CKM matrix with elements like \(\text{CKM}(\text{nuelec,elec})\). If the neutrino masses are set to zero – which is the default (cf. sect. 4.2.4) – this matrix equals the unit matrix.

4.2.7 Momenta and Metric

The external momenta \(q_1(nu_1), q_2(nu_1), \ldots\) may occur in two different contexts,

1. as Lorentz vectors \(q_1(nu_1), \ldots\) with Lorentz indices \(nu_1, \ldots\),

2. contracted with another Lorentz vector.

In the first case the momentum will appear in the output as it is. In the second case the contraction will be written in terms of parallel space components:

\[
q_{10} = \sqrt{q_1^2}; \quad q_{20} = \frac{q_1 \cdot q_2}{\sqrt{q_1^2}}; \quad q_{21} = \sqrt{\frac{(q_1 \cdot q_2)^2}{q_1^2} - q_2^2}; \quad q_{30} = \frac{q_1 \cdot q_3}{\sqrt{q_1^2}}; \quad q_{31} = -\frac{q_2 \cdot q_3}{\sqrt{-q_2^2}}; \quad q_{32} = \sqrt{\frac{(q_1 \cdot q_3)^2}{q_1^2} + \frac{(q_2 \cdot q_3)^2}{q_2^2} - q_3^2},
\]

where the auxiliary property
\[
\hat{q}_2 = q_2 - \frac{q_1 \cdot q_2}{q_1^2} q_1
\]

(4.2)
was used. In appendix A we demonstrate how this works in practice.

The metric tensor of \(xloops\) is \(G(\nu_1,\nu_2)\). It obeys the usual conventions

\[
G(0,0) = 1; \quad G(1,1) = -1 \quad \ldots
\]

The contraction of \(G(\nu_1,\nu_2)\) depends on whether the indices \(\nu_1\) and \(\nu_2\) are orthogonal space indices – then \(G(\nu_1,\nu_1) = D - \text{DimP}\) – or belong to the entire Minkowski space – this means that \(G(\nu_1,\nu_1) = D\).

The totally antisymmetric tensor is \(Eps(\nu_1,\nu_2,\nu_3,\nu_4)\) with \(Eps(0,1,2,3) = 1\).

### 4.2.8 Dirac matrices

Dirac \(\gamma\)-matrices \(xloops\) calls \(Dg(\mu_1), \ldots\). Like the external momenta they can appear either as Lorentz vector as they are, or contracted by another Lorentz vector. In the latter case they are denoted by

\[
Dg_0 = \frac{\hat{q}_1}{\sqrt{q_1^2}}; \quad Dg_1 = -\frac{\hat{q}_2}{\sqrt{-q_2^2}}; \quad Dg_2 = -\frac{\hat{q}_3}{\sqrt{-q_3^2}}
\]

with

\[
\hat{q}_3 = q_3 - \frac{q_1 \cdot q_3}{q_1^2} q_1 - \frac{q_2 \cdot q_3}{q_2^2} q_2
\]

and the abbreviation \((4.2)\). The Dirac structure is written as a string of the non-commutative product \&*. Since the program is also designed for diagrams with more than one fermion line it labels each string of Dirac matrices with a number. For example in the case of fermionic self-energies

\[
\&*(1,\text{ONE}); \quad \&*(1,Dg0); \quad \&*(1,Dg5); \quad \&*(1,Dg5,Dg0)
\]

will occur in the output – the latter two of course only in the case of \(W, Z\) or Goldstone boson exchange. \text{ONE} represents the identity of the Clifford algebra, \(Dg5\) is \(\gamma_5\). The first entry in the non-commutative string gives the number of the fermion line. The 1 means that the string belongs to the first fermion line which in this case is the only fermion line. In appendix A this treatment can be seen in practice.

### 4.2.9 \(SU(N_c)\) algebra

In QCD calculations \(xloops\) keeps the colour number \(N_c\) unassigned by default, but with \(Nc:=3;\)

the number is fixed to its standard model value. The \(SU(N_c)\) algebra needs as ingredients

\(\triangleright\) the identity matrix of the \(SU(N_c)\) algebra \(\text{delta}3(\text{ta1},\text{ta}2)\) which should not be mixed with Kronecker \(\text{delta}8(\text{ta1},\text{ta}2)\) which contracts the Gell-Mann matrices with the labels \(\text{ta}1\) and \(\text{ta}2\). Of course \(\text{delta}3(\text{ta1},\text{ta}2)\) and \(\text{delta}8(\text{ta1},\text{ta}2)\) are both diagonal with entries 1 in the diagonal. This means that

\[
\sum_{\text{ta1}} \text{delta}3(\text{ta1},\text{ta}1) = N_c = 3; \quad \sum_{\text{ta1}} \text{delta}8(\text{ta1},\text{ta}1) = N_c^2 - 1 = 8.
\]
the Gell-Mann matrices \( T_s(t_{a1}, t_{a2}, t_{a3}) \) themselves, where \( t_{a1} \) – which is the number of the matrix – can take values from 1 to \( N_c^2 - 1 \). \( t_{a2} \) and \( t_{a3} \) are the indices of a particular Gell-Mann matrix and therefore have values from 1 to \( N_c \).

The \( T_s(t_{a1}, t_{a2}, t_{a3}) \) are normalized by the following trace condition:

\[
\sum_{t_{a3}, t_{a4}} T_s(t_{a1}, t_{a3}, t_{a4}) T_s(t_{a2}, t_{a4}, t_{a3}) = 2 \delta_8(t_{a1}, t_{a2}).
\]

the usual structure functions \( f_s(t_{a1}, t_{a2}, t_{a3}) \) and \( d_s(t_{a1}, t_{a2}, t_{a3}) \) where all three indices vary from 1 to \( N^2_c - 1 \). They are defined by the Lie algebra relation

\[
\sum_{t_{a5}} T_s(t_{a1}, t_{a3}, t_{a5}) T_s(t_{a2}, t_{a5}, t_{a4})
= \frac{2}{N_c} \delta_8(t_{a1}, t_{a2}) \delta_3(t_{a3}, t_{a4})
+ \sum_{t_{a5}} d_s(t_{a1}, t_{a2}, t_{a5}) T_s(t_{a5}, t_{a3}, t_{a4})
+ i \sum_{t_{a5}} f_s(t_{a1}, t_{a2}, t_{a5}) T_s(t_{a5}, t_{a3}, t_{a4}),
\]

where the \( f_s(t_{a1}, t_{a2}, t_{a5}) \) are completely antisymmetric and the \( d_s(t_{a1}, t_{a2}, t_{a5}) \) are completely symmetric.

### 4.2.10 Output

The output of the diagrams is decomposed in one or several form factors \( C_1, C_2, \ldots \) to reflect the Dirac \( \gamma \) or Lorentz structure of the result. Therefore a list of form factors is returned. The last entry in this list is the defining equation for the form factors. The result is presented in the Maple text window of the main window and has the following structure:

\[
G_1 := [C_1 = \ldots, C_n = \ldots, C_1 \ q_1(nu1) \ q_2(nu2) + \ldots + C_n \ G(nu1, nu2) \ldots]
\]

Let us consider for instance the self-energy of a vector boson. \texttt{loops} finds as Lorentz structure \( C_1 \ G(nu1, nu2) + C_2 \ q_1(nu1) \ q_1(nu2) \) and the output roughly looks like

\[
[C_1 = \text{term1}, C_2 = \text{term2}, C_1 \ G(nu1, nu2) + C_2 \ q_1(nu1) \ q_1(nu2)],
\]

where \( \text{term1} \) and \( \text{term2} \) denote the form factors. In the case of a fermionic self-energy the result carries no Lorentz index. Instead, it is decomposed with respect to \( \&*(1, \text{ONE}), \&*(1, \text{Dg5}), \&*(1, \text{Dg0}) \) and \( \&*(1, \text{Dg5}, \text{Dg0}) \) and in the general case the result is of the form

\[
[C_1 = \text{term1}, C_2 = \text{term2}, C_3 = \text{term3}, C_4 = \text{term4},
C_1 \ (1 \ &* \ \text{ONE}) + C_2 \ (1 \ &* \ \text{Dg5}) + C_3 \ (1 \ &* \ \text{Dg0}) + C_4 \ &*(1, \text{Dg5}, \text{Dg0})].
\]

Here all \( \gamma \)-matrices are contracted. In vertex functions with a fermion line and a vector boson there is also a \( \text{Dg}(nu1) \) appearing. In appendix A several examples are given in more detail. In the case of a scalar self-energy there appears only one form factor and the result reads

\[
[C_1 = \text{term1}, C_1].
\]

The form factors can be written in several modes of evaluation which correspond to the different buttons of the diagram windows:
With \texttt{Evaluate} or if no qualifier is added to the \texttt{EvalGraph} procedures the output – for simplicity – is returned in terms of unevaluated \texttt{Oneloop} and \texttt{Twoloop} functions which correspond directly to the \texttt{OneLoop} and \texttt{TwoLoop} functions explained in sect. 4.3 and 4.4.

To get complete answers, \texttt{Eval. Full} or in the case of \texttt{EvalGraph1} and \texttt{EvalGraph2} the qualifier \texttt{full} forces \texttt{xloops} to evaluate directly the \texttt{Oneloop} and \texttt{Twoloop} integrals. To be precise, you have to type

\texttt{EvalGraph}(m, (n, \langle \text{list} \rangle, \texttt{full});}

where \( m \) and \( n \) are numbers. Now each form factor \( C_1, \ldots \) is a Laurent expansion in terms of the ultraviolet regulator \( \varepsilon = (4 - D)/2 \), where the significant coefficients of this expansion – \( \mathcal{O}(\varepsilon^{-1}), \mathcal{O}(\varepsilon^0) \) in the one-loop case, \( \mathcal{O}(\varepsilon^{-2}), \mathcal{O}(\varepsilon^{-1}), \mathcal{O}(\varepsilon^0) \) in the two-loop case – are denoted in form of a list. The bosonic self-energy we introduced before now has the structure

\[
\begin{align*}
C_1 &= [\varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}], \\
C_2 &= [\varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}], \\
& \quad C_1 \text{ \texttt{G}(nu1,nu2)} + C_2 \text{ \texttt{q1}(nu1) q1(nu2)}
\end{align*}
\]

for one-loop contributions and

\[
\begin{align*}
C_1 &= [\varepsilon^{-2}\text{-term}, \varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}], \\
C_2 &= [\varepsilon^{-2}\text{-term}, \varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}], \\
& \quad C_1 \text{ \texttt{G}(nu1,nu2)} + C_2 \text{ \texttt{q1}(nu1) q1(nu2)}
\end{align*}
\]

for two-loop diagrams. \( \varepsilon^{-2}\text{-term} \) and \( \varepsilon^{-1}\text{-term} \) are meant to represent the divergent part (the coefficients of \( \frac{1}{\varepsilon} \) and \( \frac{1}{\varepsilon^2} \)) whereas \( \varepsilon^0\text{-term} \) is describing the finite part of the corresponding form factor. For those two-loop topologies where no analytic result is known, the finite part contributes two entries to the output list. The first of them is a list itself which denotes an integral representation, the second contains the part which is analytically calculable:

\[
\begin{align*}
C_1 &= [\varepsilon^{-2}\text{-term}, \varepsilon^{-1}\text{-term}, \texttt{numeric}, \varepsilon^0\text{-term}], \\
C_2 &= [\varepsilon^{-2}\text{-term}, \varepsilon^{-1}\text{-term}, \texttt{numeric}, \varepsilon^0\text{-term}], \\
& \quad C_1 \text{ \texttt{G}(nu1,nu2)} + C_2 \text{ \texttt{q1}(nu1) q1(nu2)}
\end{align*}
\]

A more detailed description of the integral representation – which was called here \texttt{numeric} – is given in sect. 4.4.3.

The integral representation is evaluated directly with the help of \texttt{VEGAS} if \texttt{Eval. Numeric} is used or if the qualifier \texttt{full} replaced by two lists which contain the input information for \texttt{VEGAS}:

\texttt{EvalGraph}(n, (\langle \text{list} \rangle, [m_1,m_2],[n_1,n_2]));

Within an ordinary \texttt{Maple} session one has in addition to assign the variable \texttt{NumPath} as described in sect. 4.6. The variables \( m_1, m_2, n_1, n_2 \) represent the grids where the integrand is evaluated: first \( m_1 \) iterations with \( m_2 \) points – \texttt{itmx} = \( m_1 \) and \texttt{ncall} = \( m_2 \) are passed to \texttt{VEGAS} – and then \( n_1 \) iterations with \( n_2 \) points – this corresponds to \texttt{itmx} = \( n_1 \) and \texttt{ncall} = \( n_2 \). In the \texttt{Xwindow} interface these values can be changed with the entry \texttt{Numeric} of the \texttt{Options} menu (cf. sect. 4.6). Real and imaginary part are integrated one after the other.
The output has then the following structure:

\[
C_1 = [\varepsilon^{-2}\text{-term}, \varepsilon^{-1}\text{-term}, [\text{factor}, [\text{result}, \text{error}]], \varepsilon^0\text{-term}], \\
C_2 = [\varepsilon^{-2}\text{-term}, \varepsilon^{-1}\text{-term}, [\text{factor}, [\text{result}, \text{error}]], \varepsilon^0\text{-term}], \\
C_1 \ G(\nu_1,\nu_2) + C_2 \ q_1(\nu_1) \ q_1(\nu_2)]
\]

where \textit{factor} denotes an analytically evaluated pre-factor of the integrand. The result of the integral itself is called \textit{result} here, \textit{error} describes the uncertainty of the numerical integration which is returned by \textsc{VEGAS}. To be specific: the entire \(O(\varepsilon^0)\) contribution is obtained by

\[
\text{factor} \times \text{result} + \varepsilon^0\text{-term},
\]

the complete uncertainty is

\[
\text{factor} \times \text{error},
\]

where the uncertainty of \(\varepsilon^0\text{-term}\) is neglected.

If not all parameters of the integrand carry numerical values the numerical integration cannot be performed. Therefore the same output as for the qualifier \textit{full} is returned.

\(\triangleright\) With \texttt{Eval\ More} or in the case of \texttt{EvalGraph1}, with the qualifier \textit{more} instead of \textit{full}, \texttt{xloops} evaluates even the \(O(\varepsilon)\) of one-loop integrals. This becomes relevant if the one-loop integral is part of a two-loop calculation, because then it may be multiplied with a divergent one-loop \(Z\)-factor. This changes the output of a bosonic self-energy:

\[
[C_1 = [\varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}, \varepsilon^1\text{-term}], \\
C_2 = [\varepsilon^{-1}\text{-term}, \varepsilon^0\text{-term}, \varepsilon^1\text{-term}], \ C_1 \ G(\nu_1,\nu_2) + C_2 \ q_1(\nu_1) \ q_1(\nu_2)].
\]

As far as the masses and momenta which enter the \texttt{EvalGraph} functions are symbols the output is given algebraically, whereas if they are all assigned with numbers the result will be numerical.

Generally, in the numerical case, the program sets the value of the imaginary part of the propagators, five digits higher than the numerical accuracy of the whole calculation, for example to \(10^{-15}\) if one calculates with 20 digits. When reading the program \texttt{Maple} sets the number of digits to 20 – except when the user specified a higher accuracy before, for instance:

\[
\texttt{Digits:=40;}
\]

In the algebraic case just \texttt{rho} is returned. The output is facilitated by the usage of several abbreviations \(4.23, 4.24, 4.25, 4.26\) for certain one-loop contributions which are introduced in detail in the context of the \texttt{OneLoop} procedure in sect. 4.3.3. The abbreviations are substituted if the \texttt{evalRex} command is used (cf. sect. 4.3.3).

\subsection*{4.2.11 Special relations}

If external momenta are zero or parallel, \texttt{xloops} omits those Lorentz vectors which are no longer independent and therefore redundant. The number of form factors – and the dimension of the parallel space – may decrease. If this is the case the form factors will no
longer be expressed in terms of proper \texttt{OneloopPt} and \texttt{TwoloopPt} functions, but with the help of its variants \texttt{Oneloop\_\textit{nPt}} and \texttt{Twoloop\_\textit{nPt}} which are reduced in dimension of parallel space. The \texttt{Oneloop\_\textit{nPt}} functions correspond to the \texttt{OneLoop\_\textit{nPt}} functions from sect. 4.3.

If for instance in the case of the two-point function the external momentum \( q_1 \) shall vanish, one has to make the following assignment before starting the calculation:

\[ q_{10} := 0; \]

Now the bosonic self-energy reduces to

\[ [C_1 = \text{term1}, C_1 G(\text{nu1,\text{nu2}})] \]

which means that one form factor vanished. The only remaining form factor is expressed by a \texttt{Oneloop1\_2Pt} function – this is a two-point function with a parallel space of dimension zero. The notation “\texttt{1\_2Pt}” reflects the fact that this function carries properties of the two-point function as well as of the one-point function.

If in the case of a three-point function both external momenta \( q_1 \) and \( q_2 \) have a common rest frame one has to make the assignment

\[ q_{21} := 0; \]

Now the momentum \( q_2 \) will disappear from the equation which defines the form factors since it is no longer independent of \( q_1 \) (“zero recoil limit”). In addition the proper \texttt{Oneloop3Pt} will switch to \texttt{Oneloop2\_3Pt} functions – a mixing of two- and three-point functions.

It is important here to make the assignment first and then to evaluate the diagram, because otherwise number and shape of form factors may be wrong. Moreover, if special relations exist which fix the variables of the \texttt{EvalGraph} functions or connect them with each other – like for instance the on-shell condition which connects masses and momenta – it is important first to declare these relations and then evaluate the functions.

Otherwise, if the functions are evaluated first analytically and then numbers or relations between variables are substituted in the result, \texttt{Maple} has to do more work than necessary. Moreover, it might happen in some special cases that an error occurs – something like “\texttt{division by 0}”. Usually this sort of divergence is artificial and not an error \texttt{xloops} is responsible for – at the moment of calculation the parameters which cause the error message were general analytical expressions. The problem can be overcome in either expanding in the critical parameters before the insertion of values, or – and this is recommended here – in calling the diagram window or the \texttt{EvalGraph} routines with the final values for the arguments from the very beginning, because the program was taught to detect a lot of this critical points and to avoid such artificial infinities in the result.

\subsection{Fast evaluation}

If the entry \texttt{Oneloop Library} of the \texttt{Options} menu is selected or – within an ordinary \texttt{Maple} session – the variable \texttt{LibPath} is assigned with some path, the procedures assume that a library of \texttt{OneLoop} functions is stored in the assigned directory – as described in sect. 4.3. The necessary integrals are then read from this directory – which of course is much faster (see sect. 3.3). Otherwise, if \texttt{LibPath} is not assigned, the procedures are forced to recalculate the necessary \texttt{OneLoop} functions at any occurrence.
4.3 The OneLoop procedures

4.3.1 Input

There exist nine basic functions for the calculation of one-loop integrals. The first three functions

- OneLoop1Pt\((p, m, t)\)
- OneLoop2Pt\((p_0, p_\perp, q_{10}, m_1, m_2, t_1, t_2)\)
- OneLoop3Pt\((p_0, p_1, p_\perp, q_{10}, q_{20}, m_1, m_2, m_3, t_1, t_2, t_3)\)

 correspond to the notation in parallel and orthogonal space variables. The next three functions

- OneLoop1_2Pt\((p, m_1, m_2, t_1, t_2)\)
- OneLoop1_3Pt\((p, m_1, m_2, m_3, t_1, t_2, t_3)\)
- OneLoop2_3Pt\((p_0, p_\perp, q_{10}, q_{20}, m_1, m_2, m_3, t_1, t_2, t_3)\)

 are used in the case of reduced parallel space. All abovementioned functions require the tensor degree, the external momenta \(q_n\) and the masses \(m_n\) as input. The powers \(t_n\) of the propagator terms in the denominator are optional. The remaining three functions

- OneLoopTens1Pt\((i, m, t)\)
- OneLoopTens2Pt\((i, q_1^2, m_1, m_2, t_1, t_2)\)
- OneLoopTens3Pt\((i, q_1^2, q_2^2, (q_2 - q_1)^2, m_1, m_2, m_3, t_1, t_2, t_3)\)

 require the squared external momenta as input variables and return the complete rank-\(i\) tensor. The powers \(t_n\) of the propagator terms in the denominator again are optional.

4.3.2 Notation

In detail the basic functions directly correspond to the following integrals:

- One-point function:

\[
\text{OneLoop1Pt}(p, m, t) = A^{(p)}(t)(m) = \int d^Dl \frac{(l^2)^\frac{p}{2}}{[l^2 - m^2 + iq]^t} \quad (4.5)
\]

\[
\text{OneLoop1Pt}(p, m) = \text{OneLoop1Pt}(p, m, 1) \quad (4.6)
\]

\[
\text{OneLoopTens1Pt}(i, m, t) = A^{(i)}_{\mu_1 \cdots \mu_i}(m) = \int d^Dl \frac{l_{\mu_1} \cdots l_{\mu_i}}{[l^2 - m^2 + iq]^t} \quad (4.7)
\]

\[
\text{OneLoopTens1Pt}(i, m) = \text{OneLoopTens1Pt}(i, m, 1). \quad (4.8)
\]
Two-point function:

\[
\text{OneLoop2Pt}(p_0, p_\perp, q_{10}, m_1, m_2, t_1, t_2) = B^{(p_0 p_\perp)(t_1 t_2)}(q_{10}, m_1, m_2)
\]

\[
= \int d^Dl \frac{(l_0)^{p_0} (l_\perp)^{p_\perp}}{[(l + q_1)^2 - m_1^2 + i\epsilon][l^2 - m_2^2 + i\epsilon]^{t_2}}
\]

(4.9)

\[
\text{OneLoop2Pt}(p_0, p_\perp, q_{10}, m_1, m_2) = \text{OneLoop2Pt}(p_0, p_\perp, q_{10}, m_1, m_2, 1, 1) \tag{4.10}
\]

\[
\text{OneLoop1_2Pt}(p, m_1, m_2, t_1, t_2)
\]

\[
= \int d^Dl \frac{(l^2)^{\frac{\mu_1 \cdots \mu_i}{2}}}{[l^2 - m_1^2 + i\epsilon][l^2 - m_2^2 + i\epsilon]^{t_2}}
\]

(4.11)

\[
\text{OneLoop1_2Pt}(p, m_1, m_2) = \text{OneLoop1_2Pt}(p, m_1, m_2, 1, 1) \tag{4.12}
\]

\[
\text{OneLoopTens2Pt}(i, q_{10}, m_1, m_2, t_1, t_2) = B^{(t_1, t_2)}(q_{10}^2, m_1, m_2)
\]

\[
= \int d^Dl \frac{l_{\mu_1} \cdots l_{\mu_i}}{[(l + q_1)^2 - m_1^2 + i\epsilon][l^2 - m_2^2 + i\epsilon]^{t_2}}
\]

(4.13)

\[
\text{OneLoopTens2Pt}(i, q_{10}, m_1, m_2) = \text{OneLoopTens2Pt}(i, q_{10}^2, m_1, m_2, 1, 1). \tag{4.14}
\]

Abbreviations:

\[
q_{10} = \sqrt{q_1^2}; \quad l_0 = \frac{l \cdot q_1}{\sqrt{q_1^2}}; \quad l_\perp = \sqrt{l_0^2 - l^2}. \tag{4.15}
\]

Three-point function:

\[
\text{OneLoop3Pt}(p_0, p_\perp, q_{10}, q_{20}, q_{21}, m_1, m_2, m_3, t_1, t_2, t_3)
\]

\[
= C^{(p_0 p_\perp)(t_1, t_2, t_3)}(q_{10}, q_{20}, q_{21}, m_1, m_2, m_3)
\]

(4.16)

\[
= \int d^Dl \frac{(l_0)^{p_0} (l_1)^{p_1} (l_\perp)^{p_\perp}}{[(l + q_1)^2 - m_1^2 + i\epsilon][l + q_2]^2 - m_2^2 + i\epsilon][l^2 - m_3^2 + i\epsilon]^{t_3}}
\]
OneLoop3Pt\( (p_0, p_1, p_{\perp}, q_{10}, q_{20}, q_{21}, m_1, m_2, m_3) \)

\[ = \text{OneLoop3Pt}(p_0, p_1, p_{\perp}, q_{10}, q_{20}, q_{21}, m_1, m_2, m_3, 1, 1, 1) \] (4.17)

OneLoop2_3Pt\( (p_0, p_{\perp}, q_{10}, q_{20}, m_1, m_2, m_3, t_1, t_2, t_3) \)

\[ = \int d^Dl \frac{(l_0)^{p_0} (l_{\perp})^{p_{\perp}}}{[(l + q_1)^2 - m_1^2 + i\epsilon]^{t_1} [(l + q_2)^2 - m_2^2 + i\epsilon]^{t_2} [l^2 - m_3^2 + i\epsilon]^{t_3}} \]

OneLoop2_3Pt\( (p_0, p_{\perp}, q_{10}, q_{20}, m_1, m_2, m_3) \)

\[ = \text{OneLoop2_3Pt}(p_0, p_{\perp}, q_{10}, q_{20}, m_1, m_2, m_3, 1, 1, 1) \] (4.18)

OneLoop1_3Pt\( (p, m_1, m_2, m_3, t_1, t_2, t_3) \)

\[ = \int d^Dl \frac{(l_0)^q}{[l^2 - m_1^2 + i\epsilon]^{t_1} [l^2 - m_2^2 + i\epsilon]^{t_2} [l^2 - m_3^2 + i\epsilon]^{t_3}} \]

OneLoop1_3Pt\( (p, m_1, m_2, m_3) = \text{OneLoop1_3Pt}(p, m_1, m_2, m_3, 1, 1, 1) \) (4.19)

OneLoopTens3Pt\( (i, q_1^2, q_2^2, (q_2 - q_1)^2, m_1, m_2, m_3, t_1, t_2, t_3) \)

\[ = C_{\mu_1 \ldots \mu_3}^{(t_1, t_2, t_3)} (q_1^2, q_2^2, (q_2 - q_1)^2, m_1, m_2, m_3) \] (4.20)

\[ = \int d^Dl \frac{l_{\mu_1} \ldots l_{\mu_3}}{[(l + q_1)^2 - m_1^2 + i\epsilon]^{t_1} [(l + q_2)^2 - m_2^2 + i\epsilon]^{t_2} [l^2 - m_3^2 + i\epsilon]^{t_3}} \]

OneLoopTens3Pt\( (i, q_1^2, q_2^2, (q_2 - q_1)^2, m_1, m_2, m_3) \)

\[ = \text{OneLoopTens3Pt}(i, q_1^2, q_2^2, (q_2 - q_1)^2, m_1, m_2, m_3, 1, 1, 1) \] (4.21)

Abbreviations – using the auxiliary property (4.2):

\[ q_{10} = \sqrt{q_1^2}; \quad q_{20} = \frac{q_1 \cdot q_2}{\sqrt{q_1^2}}; \quad q_{21} = \sqrt{\frac{(q_1 \cdot q_2)^2}{q_1^2} - q_2^2}; \]

\[ l_0 = \frac{l \cdot q_1}{\sqrt{q_1^2}}; \quad l_1 = -\frac{l \cdot q_2}{\sqrt{-q_2^2}}; \quad l_{\perp} = \frac{l_0^2 - l_1^2 - l^2}{2}. \] (4.22)

Our notation for the OneLoop\( n \)Pt functions distinguishes between parallel and orthogonal space which is reflected by the definitions of (4.13) and (4.22) for the momentum components. Please keep in mind the fact that in our notation the indices \( p_0, p_1, \ldots \) represent the powers of the different components of the loop momentum \( l \) which should not be mixed with Lorentz indices \( \mu_1, \mu_2, \ldots \) of the corresponding tensor OneLoopTens\( n \)Pt.

If the powers \( t_n \) of the propagator terms in the denominator are omitted like in (4.6, 4.8, 4.10, 4.14, 4.17, 4.21) the program assumes the value 1 for each \( t_n \).

### 4.3.3 Output of OneLoop\( n \)Pt

The results of the OneLoop\( n \)Pt functions are Laurent expansions in terms of the ultraviolet regulator \( \varepsilon \). Therefore the output of the OneLoop\( n \)Pt procedures consists of a list where
the significant coefficients – \( O(\varepsilon^{-1}) \), \( O(\varepsilon^{0}) \) – of this expansion are denoted. In two-loop calculations the term of \( O(\varepsilon^{1}) \) of the one-loop functions is also of interest. For that purpose this coefficient is also calculated if the qualifier \texttt{more} \ is used in the function call:

\[
\begin{array}{|l|l|}
\hline
\text{Eingabe} & \text{Ausgabe} \\
\hline
\text{OneLoop1Pt}(p, m, t) & \left[\varepsilon^{-1}\text{-term, }\varepsilon^{0}\text{-term}\right] \\
\text{OneLoop1Pt}(p, m, t, \text{more}) & \left[\varepsilon^{-1}\text{-term, }\varepsilon^{0}\text{-term, }\varepsilon^{1}\text{-term}\right] \\
\text{OneLoop1_2Pt}(p, m_m, t_1, t_2) & \left[\varepsilon^{-1}\text{-term, }\varepsilon^{0}\text{-term}\right] \\
\text{OneLoop1_2Pt}(p, m_m, t_1, t_2, \text{more}) & \left[\varepsilon^{-1}\text{-term, }\varepsilon^{0}\text{-term, }\varepsilon^{1}\text{-term}\right] \\
\text{OneLoop2Pt}(p_0, p_\perp, q_{10}, m_1, m_2, t_1, t_2) & \left[\varepsilon^{-1}\text{-term, }\varepsilon^{0}\text{-term}\right] \\
\text{OneLoop2Pt}(p_0, p_\perp, q_{10}, m_1, m_2, t_1, t_2, \text{more}) & \left[\varepsilon^{-1}\text{-term, }\varepsilon^{0}\text{-term, }\varepsilon^{1}\text{-term}\right] \\
\text{OneLoop3Pt}(p, m_1, m_2, t_3, t_1, t_2) & \left[\varepsilon^{-1}\text{-term, }\varepsilon^{0}\text{-term}\right] \\
\text{OneLoop3Pt}(p, m_1, m_2, t_3, t_1, t_2, \text{more}) & \left[\varepsilon^{-1}\text{-term, }\varepsilon^{0}\text{-term, }\varepsilon^{1}\text{-term}\right] \\
\text{OneLoop2_3Pt}(p_0, p_\perp, q_{10}, q_{20} m_1, m_2, m_3, t_1, t_2, t_3) & \left[\varepsilon^{-1}\text{-term, }\varepsilon^{0}\text{-term}\right] \\
\text{OneLoop2_3Pt}(p_0, p_\perp, q_{10}, q_{20} m_1, m_2, m_3, t_1, t_2, t_3, \text{more}) & \left[\varepsilon^{-1}\text{-term, }\varepsilon^{0}\text{-term, }\varepsilon^{1}\text{-term}\right] \\
\text{OneLoop3Pt}(p_0, p_\perp, q_{10}, q_{20}, q_{21} m_1, m_2, m_3, t_1, t_2, t_3) & \left[\varepsilon^{-1}\text{-term, }\varepsilon^{0}\text{-term}\right] \\
\text{OneLoop3Pt}(p_0, p_\perp, q_{10}, q_{20}, q_{21}, m_1, m_2, m_3, t_1, t_2, t_3, \text{more}) & \left[\varepsilon^{-1}\text{-term, }\varepsilon^{0}\text{-term, }\varepsilon^{1}\text{-term}\right] \\
\hline
\end{array}
\]

In the output \( \varepsilon^{-1}\text{-term} \) is meant to represent the divergent part (the coefficient of \( \frac{1}{\varepsilon} \)) whereas \( \varepsilon^{0}\text{-term} \) is describing the finite part and \( \varepsilon^{1}\text{-term} \) the \( O(\varepsilon) \) contribution.

As long as the arguments of the \texttt{OneLoop} functions are symbols the output is given algebraically, whereas of course numbers inserted for the arguments imply a numerical result. Like in the case of the \texttt{EvalGraph} routines, in numerical evaluations the program sets the value of \( \rho \), the imaginary part of the propagators, five digits higher than the numerical accuracy of the whole calculation, for example to \( 10^{-15} \) if one calculates with 20 digits. In the algebraic case just \texttt{rho} is returned. To get a compact result also in algebraic calculations, the output is written by using several abbreviations:

\[
R2ex1(x, y) = \sqrt{1 - \frac{x}{y}} \left[ \ln \left( 1 - \sqrt{1 - \frac{x}{y}} \right) - \ln \left( 1 + \sqrt{1 - \frac{x}{y}} \right) + i\pi \right] - \ln(-x) - i\pi \tag{4.23}
\]

\[
R2ex2(x, y) = \left( 1 + \sqrt{1 - \frac{x}{y}} \right) \text{Li}_2 \left( 1 - \frac{1 - \sqrt{1 - \frac{x}{y}}}{1 + \sqrt{1 - \frac{x}{y}}} \right) + \left( 1 + \sqrt{1 - \frac{x}{y}} \right) \left[ \ln \left( 1 - \sqrt{1 - \frac{x}{y}} \right) \right]^2 + \left( 1 - \sqrt{1 - \frac{x}{y}} \right) \text{Li}_2 \left( 1 - \frac{1 + \sqrt{1 - \frac{x}{y}}}{1 - \sqrt{1 - \frac{x}{y}}} \right) + \left( 1 - \sqrt{1 - \frac{x}{y}} \right) \left[ \ln \left( 1 + \sqrt{1 - \frac{x}{y}} \right) \right]^2 + \frac{1}{2} (\ln y)^2 + 2 (\ln x)^2 - 2 \ln x \ln y \tag{4.24}
\]
Detailed description

\[+ (\ln y - 2 \ln x) \left[ \left(1 + \sqrt{1 - \frac{x}{y}}\right) \ln \left(1 - \sqrt{1 - \frac{x}{y}}\right)\right] + \left(1 - \sqrt{1 - \frac{x}{y}}\right) \ln \left(1 + \sqrt{1 - \frac{x}{y}}\right)\]

\[- i\pi \frac{\sqrt{x-y}}{\sqrt{-y}} \left[2 \ln 2 + \ln(x-y)\right]\]

\[\text{R3ex2}(x, y, z) = 2 \ln \left(1 - \frac{x}{z}\right) \eta(x, z) + 2 \ln \left(1 - \frac{y}{z}\right) \eta(y, z) + 2 \ln z^2\]  
(4.25)

\[\text{R3ex3}(x, y, z) = 4 S_{12} \left(1 - \frac{x}{z}\right) - 4 L_3 \left(1 - \frac{x}{z}\right) + 4 S_{12} \left(1 - \frac{y}{z}\right) - 4 L_3 \left(1 - \frac{y}{z}\right) \]

\[\quad - 4 \eta \left(x, \frac{1}{z}\right) \left\{ L_2 \left(\frac{x}{z}\right) \ln \left(1 - \frac{x}{z}\right) \ln \left(- \frac{x}{z}\right) + \frac{1}{2} \left[\ln \left(1 - \frac{x}{z}\right)\right]^2\right\}\]

\[- 4 \eta \left(y, \frac{1}{z}\right) \left\{ L_2 \left(\frac{y}{z}\right) \ln \left(1 - \frac{y}{z}\right) \ln \left(- \frac{y}{z}\right) + \frac{1}{2} \left[\ln \left(1 - \frac{y}{z}\right)\right]^2\right\}\]

\[\quad - 4 \ln z \left[ L_2 \left(1 - \frac{x}{z}\right) + L_2 \left(1 - \frac{y}{z}\right)\right] \]

\[- 4 \ln z \left[ \eta \left(x, \frac{1}{z}\right) \ln \left(1 - \frac{x}{z}\right) + \eta \left(y, \frac{1}{z}\right) \ln \left(1 - \frac{y}{z}\right)\right]\]

\[+ 2 L_3 \left(\frac{z - y}{z - x}\right) - 2 L_3 \left(\frac{(z - y)x}{(z - x)y}\right) + 2 L_3 \left(\frac{x}{y}\right)\]  
(4.26)

\[+ \left[\ln \left(\frac{z - y}{z - x}\right)\right]^2 \eta \left(y - x, \frac{z}{z - x}\right) + 2 \eta \left(y - x, \frac{1}{z}\right)\]

\[\times \left\{ \frac{1}{2} \left[\ln \left(\frac{z - y}{z - x}\right)\right]^2 - \ln \left(- \frac{z - y}{z - x}\right) \eta \left(z - y, \frac{z}{z - x}\right)\right\}\]

\[+ 2 \left(\ln x - \ln y\right) \left[ L_2 \left(\frac{(z - y)x}{(z - x)y}\right) - L_2 \left(\frac{x}{y}\right)\right]\]

\[- \ln \left(1 - \frac{x}{z}\right) \left(\ln x - \ln y\right)^2 - 2 \zeta(3) - \frac{4}{3} \left(\ln z\right)^3\]

\[+ 2 \left[\ln x - \ln y - \frac{1}{2} \ln \left(\frac{(z - y)x}{(z - x)y}\right)\right] \ln \left(\frac{(z - y)x}{(z - x)y}\right) \eta \left(y - x, \frac{z}{z - x}\right)\] .

These functions are related to the corresponding $R$ functions. They represent the coefficients of the Taylor expansion in $\varepsilon$ making use of (cf. [20].

\[\zeta(n) = \sum_{k=1}^{\infty} \frac{1}{k^n}\]

\[\text{Li}_2(z) = - \int_{0}^{z} \frac{\ln(1 - s)}{s} \, ds\]

\[\text{Li}_3(z) = \int_{0}^{z} \frac{\text{Li}_2(s)}{s} \, ds\]

\footnote{The function $\text{R3ex3}(x, y, z)$ assumes that $x$ and $y$ have an imaginary part of different sign which is always the case.}
\[ S_{12}(z) = \frac{1}{2} \int_0^z \frac{\ln^2(1 - s)}{s} \, ds \]

\[ \eta(a, b) = 2\pi i \left[ \theta(-\text{Im} a)\theta(-\text{Im} b)\theta(\text{Im}(ab)) - \theta(\text{Im} a)\theta(\text{Im} b)\theta(-\text{Im}(ab)) \right]. \]

The function

```
 evalRex((expression));
```

substitutes the \texttt{Rnexm} functions in \texttt{(expression)} by the corresponding Polylogarithms.

### 4.3.4 Output of OneLoopTensnPt

The functions \texttt{OneLoopTensnPt} return a full tensor as output.

The output format looks similar to that of the \texttt{EvalGraph} routines. The following types of arguments – here only demonstrated for the two-point case – are allowed for all \texttt{OneLoopTensnPt} functions:

- **OneLoopTens2Pt\((i)\):**
  - The rank \(i\) tensor decomposition of the two-point function is given. The procedure returns a list consisting in the different coefficients, which are expressed in terms of the \texttt{OneLoop}\(\texttt{nPt}\) tensor integrals, and the defining equation for the coefficients, for instance in the case \(i = 2\):
    \[
    \begin{bmatrix}
    C21 = -\frac{\text{OneLoop2Pt}(0, 2)}{3 - 2\varepsilon}, & C20 = -\frac{\text{OneLoop2Pt}(0, 2)}{q^2 (-3 + 2\varepsilon)} \\
    + \frac{\text{OneLoop2Pt}(2, 0)}{q^2}, & C20 \ q1(nu1) \ q1(nu2) + C21 \ G(nu1, nu2)
    \end{bmatrix}.
    \]

- **OneLoopTens2Pt\((i, \text{full})\):**
  - The function inserts the results of the \texttt{OneLoop}\(\texttt{nPt}\) tensor integrals explicitly. Here again the case \(i = 2\):
    \[
    \begin{bmatrix}
    C21 = [\varepsilon^{-1}-\text{term}, \varepsilon^0-\text{term}], & C20 = [\varepsilon^{-1}-\text{term}, \varepsilon^0-\text{term}], \\
    C20 \ q1(nu1) \ q1(nu2) + C21 \ G(nu1, nu2)
    \end{bmatrix}.
    \]

- **OneLoopTens2Pt\((i, q_1^2, m_1, m_2, t_1, t_2)\):**
  - The function returns the same as \texttt{OneLoopTens2Pt\((i, \text{full})\)}, but expressed in the user-defined terms for \(q_1^2, m_1\) and \(m_2\). Of course numerical values are also allowed. \(t_1\) and \(t_2\) are optional.

- **OneLoopTens2Pt\((i, q_1^2, m_1, m_2, t_1, t_2, \text{more})\):**
  - The function returns also the \(O(\varepsilon)\) contribution. Again, \(t_1\) and \(t_2\) are optional:
    \[
    \begin{bmatrix}
    C21 = [\varepsilon^{-1}-\text{term}, \varepsilon^0-\text{term}, \varepsilon^1-\text{term}], & C20 = [\varepsilon^{-1}-\text{term}, \varepsilon^0-\text{term}, \varepsilon^1-\text{term}], \\
    C20 \ q1(nu1) \ q1(nu2) + C21 \ G(nu1, nu2)
    \end{bmatrix}.
    \]

The \(C20, C21, \ldots\) correspond to the coefficients of the Passarino-Veltman procedure.
4.3.5 **Infrared divergences**

The program is regulating the infrared divergence of the three-point function as well – which may occur for instance if the momenta are on-shell and one mass is set to 0. There is no distinction made between UV and IR divergences – that means that there is no infrared dimension parameter $\varepsilon_{\text{IR}}$. Both kinds of divergences are described by $\varepsilon = (4 - D)/2$.

4.3.6 **Special relations**

If special relations exist which fix the variables of the OneLoop functions or connect them with each other – like for instance the on-shell condition – it is important first to declare these relations and then to call the functions – as already stressed in the case of the EvalGraph routines.

If the functions are evaluated first analytically and then numbers or relations between variables are substituted in the result, it might happen in some special cases that an error occurs – something like “division by 0”. Usually this sort of divergences is artificial due to the way the analytical result was written before the insertion (cf. sect. 4.2.11).

4.4 **The TwoLoop procedures**

4.4.1 **Input**

Unlike for the one-loop functions on the two-loop level for each $n$-point case exist different types of integrals – which correspond to different topologies with the same number of external legs. To distinguish between the different $n$-point functions with the same $n$ the TwoLoop procedures acquire an additional identification number:

- $\triangleright$ TwoLoop2Pt1($p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_2, m_3, m_4, m_5, t_1, t_2, t_3, t_4, t_5$)
- $\triangleright$ TwoLoop2Pt8($p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_2, t_1, t_2$)
- $\triangleright$ TwoLoopTens2Pt1($i_1, i_2, q^2_1, m_1, m_2, m_3, m_4, m_5, t_1, t_2, t_3, t_4, t_5$)
- $\triangleright$ TwoLoopTens2Pt8($i_1, i_2, q^2_1, m_1, m_2, t_1, t_2$).

Like in the one-loop case each function needs the tensor degree, the external momenta $q_n$ and the masses $m_n$ as input. The powers $t_n$ are again optional.

The functions TwoLoopTens$n$Pt$m$ require the squared external momenta as input variables and return the complete tensor, whereas the functions OneLoop$n$Pt$m$ correspond to the notation in parallel and orthogonal space variables.
4.4.2 Notation

The functions correspond to the following integrals:

- Two-point functions:

  Topology 1:

  \[
  \begin{align*}
  \text{TwoLoop2Pt1}(p_0, p_\perp, r_0, r_\perp, s, q_1, m_1, m_2, m_3, m_4, m_5, t_1, t_2, t_3, t_4, t_5) &= \int \frac{(l_0)^{p_0} (l_\perp)^{p_\perp} (k_0)^{r_0} (k_\perp)^{r_\perp} (z)^s}{[(l + q_1)^2 - m_1^2 + i\varrho]^{t_1} [l^2 - m_2^2 + i\varrho]^{t_2} [l^2 - m_3^2 + i\varrho]^{t_3}} \\
  \times \frac{1}{[(l + k)^2 - m_4^2 + i\varrho]^{t_4} [k^2 - m_5^2 + i\varrho]^{t_5}} \tag{4.27}
  \end{align*}
  \]

  Topology 2:

  \[
  \begin{align*}
  \text{TwoLoop2Pt2}(p_0, p_\perp, r_0, r_\perp, s, q_1, m_1, m_2, m_3, m_4, m_5, t_1, t_2, t_3, t_4, t_5) &= \int \frac{l_{\mu_1} \cdots l_{\mu_j} k_{\nu_1} \cdots k_{\nu_2}}{[(l + q_1)^2 - m_1^2 + i\varrho]^{t_1} [l^2 - m_2^2 + i\varrho]^{t_2} [l^2 - m_3^2 + i\varrho]^{t_3}} \\
  \times \frac{1}{[(l + k)^2 - m_4^2 + i\varrho]^{t_4} [k^2 - m_5^2 + i\varrho]^{t_5}} \tag{4.29}
  \end{align*}
  \]
TwoLoopTens2Pt2\((i_1, i_2, q_1^2, m_1, m_2, m_3, m_4, m_5, t_1, t_2, t_3, t_4, t_5)\) (4.30)

\[
\begin{align*}
= \int d^D l \int d^D k \frac{l_{\mu_1} \cdots l_{\mu_i_1} k_{\nu_1} \cdots k_{\nu_{i_2}}}{[(l + q_1)^2 - m_1^2 + i\varrho]^t_1 \ [l^2 - m_2^2 + i\varrho]^t_2 \ [(l + k)^2 - m_3^2 + i\varrho]^t_3} \\
\times \frac{1}{[(k - q_1)^2 - m_4^2 + i\varrho]^t_4 \ [k^2 - m_5^2 + i\varrho]^t_5}.
\end{align*}
\]

Topology 3:

TwoLoop2Pt3\((p_0, p_\perp, r_0, r_\perp, s, q_10, m_1, m_2, m_3, m_4, t_1, t_2, t_3, t_4)\) (4.31)

\[
\begin{align*}
= \int d^D l \int d^D k \frac{(l_0)^{p_0} (l_\perp)^{p_\perp} (k_0)^{r_0} (k_\perp)^{r_\perp} (z)^s}{[(l + q_1)^2 - m_1^2 + i\varrho]^t_1 \ [l^2 - m_2^2 + i\varrho]^t_2} \\
\times \frac{1}{[(l + k)^2 - m_3^2 + i\varrho]^t_3 \ [k^2 - m_4^2 + i\varrho]^t_4}.
\end{align*}
\]

TwoLoopTens2Pt3\((i_1, i_2, q_1^2, m_1, m_2, m_3, m_4, t_1, t_2, t_3, t_4)\) (4.32)

\[
\begin{align*}
= \int d^D l \int d^D k \frac{l_{\mu_1} \cdots l_{\mu_i_1} k_{\nu_1} \cdots k_{\nu_{i_2}}}{[(l + q_1)^2 - m_1^2 + i\varrho]^t_1 \ [l^2 - m_2^2 + i\varrho]^t_2} \\
\times \frac{1}{[(l + k)^2 - m_3^2 + i\varrho]^t_3 \ [k^2 - m_4^2 + i\varrho]^t_4}.
\end{align*}
\]

Topology 4:
\[ \text{TwoLoop2Pt4} (p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_2, m_3, t_1, t_2, t_3) \] (4.33)
\[ \int d^D l \int d^D k \left( \frac{(l_0)^p_0 (l_\perp)^p_\perp (k_0)^r_0 (k_\perp)^r_\perp (z)^s}{[(l + q_1)^2 - m_2^2 + i\epsilon]^t_1 [(l + k)^2 - m_2^2 + i\epsilon]^t_2 [k^2 - m_3^2 + i\epsilon]^t_3} \right) \]

\[ \text{TwoLoopTens2Pt4} (i_1, i_2, q_{1}^2, m_1, m_2, m_3, t_1, t_2, t_3) \] (4.34)
\[ \int d^D l \int d^D k \left( \frac{l_{\mu_1} \cdots l_{\mu_i} k_{\nu_1} \cdots k_{\nu_2}}{[(l + q_1)^2 - m_1^2 + i\epsilon]^t_1 [(l + k)^2 - m_2^2 + i\epsilon]^t_2 [k^2 - m_3^2 + i\epsilon]^t_3} \right) \]

Topology 5:

\[ \text{TwoLoop2Pt5} (p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_2, m_3, m_4, t_1, t_2, t_3, t_4) \] (4.35)
\[ \int d^D l \int d^D k \left( \frac{(l_0)^p_0 (l_\perp)^p_\perp (k_0)^r_0 (k_\perp)^r_\perp (z)^s}{[(l + q_1)^2 - m_1^2 + i\epsilon]^t_1 [l^2 - m_2^2 + i\epsilon]^t_2} \times \frac{1}{[l^2 - m_3^2 + i\epsilon]^t_3 [k^2 - m_4^2 + i\epsilon]^t_4} \right) \]

\[ \text{TwoLoopTens2Pt5} (i_1, i_2, q_{1}^2, m_1, m_2, m_3, m_4, t_1, t_2, t_3, t_4) \] (4.36)
\[ \int d^D l \int d^D k \left( \frac{l_{\mu_1} \cdots l_{\mu_i} k_{\nu_1} \cdots k_{\nu_2}}{[(l + q_1)^2 - m_1^2 + i\epsilon]^t_1 [l^2 - m_2^2 + i\epsilon]^t_2} \times \frac{1}{[l^2 - m_3^2 + i\epsilon]^t_3 [k^2 - m_4^2 + i\epsilon]^t_4} \right) \]

Topology 6:
TwoLoop2Pt6\( (p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_2, m_3, m_4, t_1, t_2, t_3, t_4) \)

\[
\begin{align*}
\text{TwoLoop2Pt6}(p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_2, m_3, m_4, t_1, t_2, t_3, t_4) &= \int d^Dl \int d^Dk \ \frac{(l_0)^{p_0} (l_\perp)^{p_\perp} (k_0)^{r_0} (k_\perp)^{r_\perp}}{[(l + q_1)^2 - m_1^2 + i\varrho]^{t_1} [l^2 - m_2^2 + i\varrho]^{t_2}} (z)^s \\
&\times \frac{1}{[(k - q_1)^2 - m_3^2 + i\varrho]^{t_3} [k^2 - m_4^2 + i\varrho]^{t_4}} \quad (4.37)
\end{align*}
\]

TwoLoopTens2Pt6\( (i_1, i_2, q_{10}, m_1, m_2, m_3, m_4, t_1, t_2, t_3, t_4) \)

\[
\begin{align*}
\text{TwoLoopTens2Pt6}(i_1, i_2, q_{10}, m_1, m_2, m_3, m_4, t_1, t_2, t_3, t_4) &= \int d^Dl \int d^Dk \ \frac{l_{\mu_1} \ldots l_{\mu_i_1} k_{\nu_1} \ldots k_{\nu_2}}{[(l + q_1)^2 - m_1^2 + i\varrho]^{t_1} [l^2 - m_2^2 + i\varrho]^{t_2}} \\
&\times \frac{1}{[(k - q_1)^2 - m_3^2 + i\varrho]^{t_3} [k^2 - m_4^2 + i\varrho]^{t_4}} \quad (4.38)
\end{align*}
\]

Topology 7:

TwoLoop2Pt7\( (p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_2, m_3, m_4, t_1, t_2, t_3) \)

\[
\begin{align*}
\text{TwoLoop2Pt7}(p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_2, m_3, m_4, t_1, t_2, t_3) &= \int d^Dl \int d^Dk \ \frac{(l_0)^{p_0} (l_\perp)^{p_\perp} (k_0)^{r_0} (k_\perp)^{r_\perp}}{[(l + q_1)^2 - m_1^2 + i\varrho]^{t_1} [l^2 - m_2^2 + i\varrho]^{t_2}} (z)^s \\
&\times \frac{1}{[(k - q_1)^2 - m_3^2 + i\varrho]^{t_3} [k^2 - m_4^2 + i\varrho]^{t_4}} \quad (4.39)
\end{align*}
\]

TwoLoopTens2Pt7\( (i_1, i_2, q_{10}, m_1, m_2, m_3, m_4, t_1, t_2, t_3) \)

\[
\begin{align*}
\text{TwoLoopTens2Pt7}(i_1, i_2, q_{10}, m_1, m_2, m_3, m_4, t_1, t_2, t_3) &= \int d^Dl \int d^Dk \ \frac{l_{\mu_1} \ldots l_{\mu_i_1} k_{\nu_1} \ldots k_{\nu_2}}{[(l + q_1)^2 - m_1^2 + i\varrho]^{t_1} [l^2 - m_2^2 + i\varrho]^{t_2}} \\
&\times \frac{1}{[(k - q_1)^2 - m_3^2 + i\varrho]^{t_3} [k^2 - m_4^2 + i\varrho]^{t_4}} \quad (4.40)
\end{align*}
\]
Detailed description

Topology 8:

\[
\text{TwoLoop2Pt}_8(p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_2, t_1, t_2)
\]
\[
= \int d^D l \int d^D k \frac{(l_0)^{p_0} (l_\perp)^{p_\perp} (k_0)^{r_0} (k_\perp)^{r_\perp} (z)^s}{[(l + q_1)^2 - m_1^2 + i\varepsilon_1][k^2 - m_2^2 + i\varepsilon_2]}
\]

\[
\text{TwoLoopTens}_8(i_1, i_2, q_1^2, m_1, m_2, t_1, t_2)
\]
\[
= \int d^D l \int d^D k \frac{l_{\mu_1} \cdots l_{\mu_1'} k_{\nu_1} \cdots k_{\nu_2}}{[(l + q_1)^2 - m_1^2 + i\varepsilon_1][k^2 - m_2^2 + i\varepsilon_2]}.
\]

Like in the one-loop case for each function holds

\[
\text{TwoLoop}_n(p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, \ldots, m_k)
\]
\[
= \text{TwoLoop}_n(p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, \ldots, m_k, 1, \ldots, 1)
\]

\[
\text{TwoLoopTens}_n(i_1, i_2, q_1^2, m_1, \ldots, m_k)
\]
\[
= \text{TwoLoopTens}_n(i_1, i_2, q_1^2, m_1, \ldots, m_k, 1, \ldots, 1).
\]

This means that if the powers \(t_n\) of the propagator terms in the denominator are omitted the program assumes the value 1 for each \(t_n\).

The abbreviations (4.15) we made in the one-loop case for parallel and orthogonal space notation are still valid.

There are several relations between different two-point functions. Some of them we would like to mention here:

\[
\text{TwoLoop2Pt}_1(p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_2, m_3, m_4, m_5)
\]
\[
= \frac{1}{m_4^2 - m_5^2} \left[ \text{TwoLoop}_3(p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_3, m_4, m_5)
\right.
\]
\[
- \text{TwoLoop}_3(p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_2, m_4, m_5) \right]
\]

\[
\text{TwoLoop2Pt}_5(p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_2, m_3, m_4)
\]
\[
= \frac{1}{m_3^2 - m_2^2} \left[ \text{TwoLoop}_7(p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_3, m_4)
\right.
\]
\[
- \text{TwoLoop}_7(p_0, p_\perp, r_0, r_\perp, s, q_{10}, m_1, m_2, m_4) \right].
\]
Both relations are only valid for \( m_2 \neq m_3 \), otherwise one has

\[
\text{TwoLoop2Pt1}(p_0, p_{\perp}, r_{\perp}, s, q_{10}, m_1, m_2, m_2, m_4, m_5, t_1, t_2, t_3, t_4, t_5) \\
= \text{TwoLoop2Pt3}(p_0, p_{\perp}, r_{\perp}, s, q_{10}, m_1, m_2, m_4, m_5, t_1, t_2 + t_3, t_4, t_5) \tag{4.47}
\]

\[
\text{TwoLoop2Pt5}(p_0, p_{\perp}, r_{\perp}, s, q_{10}, m_1, m_2, m_4, t_1, t_2, t_3, t_4) \\
= \text{TwoLoop2Pt7}(p_0, p_{\perp}, r_{\perp}, s, q_{10}, m_1, m_2, m_4, t_1, t_2 + t_3, t_4). \tag{4.48}
\]

The two-point functions numbered with 1-4 are nonfactorizable whereas the functions 5-8 are products of one-loop integrals, for instance

\[
\text{TwoLoop2Pt6}(p_0, p_{\perp}, r_{\perp}, s, q_{10}, m_1, m_2, m_3, m_4, t_1, t_2, t_3, t_4) \\
= \left\{ \begin{array}{ll}
\frac{(1/2, s)}{(D-1, 2)} \text{OneLoop2Pt}(p_0, p_{\perp}, q_{10}, m_1, m_2, t_1, t_2) \\
\times \text{OneLoop2Pt}(r_0, r_{\perp}, -q_{10}, m_3, m_4, t_3, t_4) & \text{if } s \text{ even} \\
0 & \text{if } s \text{ odd}
\end{array} \right. \tag{4.49}
\]

\[
\text{TwoLoop2Pt7}(p_0, p_{\perp}, r_{\perp}, s, q_{10}, m_1, m_2, m_3, t_1, t_2, t_3) \\
= \left\{ \begin{array}{ll}
\frac{(1/2, s)}{(D-1, 2)} \frac{-1}{D} \frac{1}{2} \frac{(D-1) \cdots (D-3 + r_0)}{D \cdots (D-2 + r_0 + r_{\perp})} \frac{r_0!}{(4^2 + r_{\perp})!} \\
\times \text{OneLoop2Pt}(p_0, p_{\perp}, q_{10}, m_1, m_2, t_1, t_2) \\
\times \text{OneLoop1Pt}(r_0 + r_{\perp}, m_3, t_3) & \text{if } s, r_0, r_{\perp} \text{ even} \\
0 & \text{else}
\end{array} \right. \tag{4.50}
\]

4.4.3 Output of TwoLoopnPtM

The output – like in the one-loop case – is a list which contains the relevant coefficients of the Laurent expansion in \( \varepsilon \). Ultraviolet divergences at the two-loop level occur as terms of \( \mathcal{O}(\varepsilon^{-2}) \) and \( \mathcal{O}(\varepsilon^{-1}) \). Therefore the notation of the output starts with two divergent coefficients.

For the two-point functions 1-4 the finite part cannot be given completely analytically for all mass cases. Therefore these functions have two entries in the output list which describe the finite part. The first one is a list itself denoting an integral representation, the second one gives the analytically calculable terms.

The integral representation is evaluated directly with the help of VEGAS if two arguments are added. Both arguments must be lists which contain each two numbers. These numbers carry the input information for VEGAS. Within an ordinary Maple session one has in addition to assign the variable NumPath as described in sect. 4.6.

| input                                                                 | output                        |
|----------------------------------------------------------------------|-------------------------------|
| TwoLoop2PtM\((p_0, p_{\perp}, r_{\perp}, s, q_{10}, m_1, \ldots, m_k,\) \(t_1, \ldots, t_k, [l_1, l_2], [n_1, n_2])\) | \([\varepsilon^{-2}\text{-term}, \varepsilon^{-1}\text{-term,}
\text{numeric, } \varepsilon^0\text{-term}]\) |
| TwoLoop2PtM\((p_0, p_{\perp}, r_{\perp}, s, q_{10}, m_1, \ldots, m_k,\) \(t_1, \ldots, t_k, [l_1, l_2], [n_1, n_2])\) | \([\varepsilon^{-2}\text{-term}, \varepsilon^{-1}\text{-term,}
\text{factor, result, error}], \varepsilon^0\text{-term}]\) |
"numeric" describes the integral representation. This list consists of the following entries

\[ \text{factor, integrand, } x=\text{--infinity..infinity, } y=\text{--infinity..infinity} \]

where \text{integrand} is the integrand of the integral representation which has to be integrated over \( x \) and \( y \) in the interval \((-\infty, \infty)\). \text{factor} is an analytical factor which multiplies the integral representation.

The variables \( l_1, l_2, n_1, n_2 \) represent the grids where the integrand is evaluated: first \( l_1 \) iterations with \( l_2 \) points – \text{itmx} = \( l_1 \) and \text{ncall} = \( l_2 \) are passed to \text{VEGAS} – and then \( n_1 \) iterations with \( n_2 \) points – this corresponds to \text{itmx} = \( n_1 \) and \text{ncall} = \( n_2 \). Real and imaginary part are integrated one after the other.

The other entries of the output list have the same behaviour as in the one-loop case: As long as the arguments of the \text{TwoLoop} functions are symbols the output is given algebraically, whereas numbers inserted for the arguments imply a numerical result. In the numerical case, the program sets the value of \( \rho \), the imaginary part of the propagators, five digits higher than the numerical accuracy of the whole calculation, for example to \( 10^{-15} \) if one calculates with 20 digits. In the algebraic case just \( \rho \) is returned.

In the case where all internal particles are massless the result is of course known analytically. In this case \text{numeric} is just \([0]\).

The two-point functions 5-8 are products of one-loop integrals and therefore evaluated completely analytically. The output list has no entry for numerical evaluation and looks simply like:

| input                                                                 | output                           |
|----------------------------------------------------------------------|----------------------------------|
| TwoLoop2Pt \( m(p_0, p_\perp, r_0, r_\perp, s, q_{10}, \ldots, m_k, \ldots, t_k) \) | \( [\epsilon^{-2}\text{-term}, \epsilon^{-1}\text{-term}, \epsilon^0\text{-term}] \) |
| TwoLoop2Pt \( m(p_0, p_\perp, r_0, r_\perp, s, q_{10}, \ldots, m_k, t_1, \ldots, t_k, [l_1, l_2], [n_1, n_2]) \) | \( [\epsilon^{-2}\text{-term}, \epsilon^{-1}\text{-term}, \epsilon^0\text{-term}] \) |

The optional input of two lists for the \text{VEGAS} input data doesn’t affect the output since no numerical integration is performed here. The \( \mathcal{O}(\epsilon^1) \) is not relevant in two-loop calculations. Therefore the qualifier \text{more} does not exist for two-loop functions.

The divergent part of two-loop integrals is closely related to one-loop integrals. Therefore the same abbreviations (4.23, 4.24) as for one-loop integrals are appearing. They are substituted if the \text{evalRex} command is used (cf. sect. 4.3.3).

### 4.4.4 Output of TwoLoopTens\( n \)Pt\( m \)

The functions \text{TwoLoopTens\( n \)Pt\( m \)} return a full tensor as output. The same types of arguments are allowed as for the corresponding \text{OneLoopTens\( n \)Pt\( m \)} functions described in sect. 4.3.4 – except the qualifier \text{more}.

### 4.5 The OneLoopLib procedures

For different calculations of the same \text{OneLoop} function it is of course rather inconvenient to start the whole program again. Therefore our package includes the functions

\[
\text{OneLoopLib2Pt}(i,j) \\
\text{OneLoopLib3Pt}(i,j)
\]
which generate a library of all OneLoop2Pt and OneLoop3Pt functions respectively up to the tensor rank \(i\). All powers of the denominators from 1 to \(j\) are considered. The second parameter \(j\) is optional. If it is omitted the procedures assume the value 1 for \(j\), so that no powers of denominators higher than one are calculated. These functions are then read by the proper procedures described before, so that they in any case speed up.

If you use the OneLoopLib functions within an ordinary Maple session and want to write this library in any other than the actual directory you have to assign the variable LibPath:

\[ \text{LibPath} := \langle \text{path} \rangle; \]

Now the library will be written to \(\langle \text{path} \rangle\). The procedures will only look for the library if the entry OneLoop Library of the Options menu is selected or if LibPath is assigned. If the procedures search for an integral which is not contained in the library it will be calculated and stored in the library on the fly. Each integral corresponds to one file. In appendix B.2 we list the convention for the file names.

### 4.6 Numerical integration with VEGAS

For numerical integration a parallelized implementation of the VEGAS algorithm for \(C++\) invented by R. Kreckel is provided with \(xloops\). To use the routines within an ordinary Maple session one has to declare the variable NumPath:

\[ \text{NumPath} := \langle \text{path} \rangle; \]

\(\langle \text{path} \rangle\) describes the directory where the integration routines are located. If \(xloops\) is installed properly this is the subdirectory \(cxx\) of the \(xloops\) directory.

The routines are customized for those TwoLoop procedures which cannot be calculated analytically. The most elegant way to perform those numerical integrations is described in sect. 4.2.10 and 4.4.3. In addition there is also the possibility to perform an integration separately. For that purpose there exists the command

\[ \text{NumIntC}((\text{integrand}), [m_1, m_2], [n_1, n_2]); \]

\((\text{integrand})\) denotes the expression which has to be integrated numerically. \(xloops\) expects the integration variables to be named \(x\) and \(y\) – as they are called by the EvalGraph2 and TwoLoop procedures. The variables \(m_1, m_2, n_1, n_2\) represent the grids where the integrand is evaluated: first \(m_1\) iterations with \(m_2\) points – \(\text{itmx} = m_1\) and \(\text{ncall} = m_2\) are passed to VEGAS – and then \(n_1\) iterations with \(n_2\) points – this corresponds to \(\text{itmx} = n_1\) and \(\text{ncall} = n_2\). Real and imaginary part are integrated one after the other. If only the real part is of interest one has to specify

\[ \text{NumIntC}((\text{integrand}), [m_1, m_2], [n_1, n_2], \text{Re}); \]

if only the imaginary part shall be evaluated one writes

\[ \text{NumIntC}((\text{integrand}), [m_1, m_2], [n_1, n_2], \text{Im}); \]

If \text{NumIntC} is called only with the first argument \(xloops\) takes the values \(m_1 = 20, m_2 = 1000, n_1 = 5\) and \(n_2 = 10000\).
The output has the following format:

\[ \langle \text{result} \rangle, \langle \text{error} \rangle \].

\textit{result} describes the – possibly complex – numerical result, \textit{error} is the uncertainty (1\(\sigma\)) of the numerical integration which is returned by VEGAS.

\texttt{NumIntC} performs the necessary internal steps („compile, link“) and starts the \texttt{C++} program as subprocess. \texttt{Maple} waits until the subprocess terminates and finally re-reads the result.

The numerical integration with VEGAS should be used only once by each user. Otherwise there will occur conflicts because several numerical integrations would use the same \texttt{C++} files.

![Vegas Options](image)

Figure 4.7: Options for VEGAS

The entry \texttt{Numeric} of the \texttt{Options} menu enables the user to change the parameters for VEGAS. These parameters are set in a window displayed in fig. 4.7.
5 Conclusion and outlook

Our aim was to introduce a program package that gives any computer user the possibility to calculate particle processes without much effort in manpower and hardware. At this stage of development the program is still understood to be under construction. In fact we are planning to incorporate several additional features:

- One-loop four-, five- and six-point functions.
- Two-loop three- and four-point functions.
- Calculation of complete processes, generation of all diagrams which contribute to a process in a given order.
- Incorporation of other models, for instance supersymmetric theories.
- An interface for adding other models.
- A plotting device for automatical plotting of results.
- Automatical renormalization, determination of $Z$ factors and renormalized functions.

Acknowledgements

We would like to thank R. Stemler for his help in developing several procedures and A. Frink for his contribution and advice in developing the numerics and for valuable tests.
Appendix
A Examples

A.1 Installation

A typical run of the configure script looks like the following:

./configure

This is the XLOOPS installation script!

Which Release of Maple V are you using [1/2/3/4] ? 3

Enter the name of the WWW Browser executable,
which XLOOPS should use! netscape

Do you have Isolatin-1 character encoding [y/n] ? y

If you feel unsure, answer the following question with 'n'.
XLOOPS can work in single processor mode on multi processor machines as well.
Do you have multiprocessing (and POSIX thread library installed) [y/n] ? y

How many processors has your computer? 2

On which platform are you installing xloops?
Linux [l] Digital OSF [d] SUN Solaris [s] IBM AIX [a] Other Unix [o] l

Instead of creating the library you can also get a prebuilt library
from the XLOOPS homepage and unpack it.

Shall I create the library of one-loop functions
(takes approx. 1 hour) [y/n] ? n

Now testing for Maple output bug while reading the library!

==============================================================================
|\^/| Maple V Release 3 (Universitaet Mainz)
\._\_| Copyright (c) 1981-1994 by Waterloo Maple Software and the
 `\ MAPLE / University of Waterloo. All rights reserved. Maple and Maple V
 <_____ ____> are registered trademarks of Waterloo Maple Software.
 | Type ? for help.
> read('loops.ma');
Warning: new definition for &*

==============================================================================

This is XLOOPS Version 1.0

by Lars Brücher, Johannes Franzkowski, Dirk Kreimer

©1993-1997 Johannes Gutenberg Universität Mainz, Germany

bytes used=1001220, alloc=851812, time=0.03
bytes used=2011736, alloc=1310480, time=0.03
> LibPath := '../lib/' ;

LibPath := ../lib/
> EvalGraph1(3, [higgs, higgs, higgs, higgs, higgs, higgs], full);
bytes used = 3011944, alloc = 1834672, time = 0.03
> quit;
bytes used = 3662072, alloc = 1834672, time = 0.03

Did you see any result of the Maple command 'EvalGraph' used above [y/n] ? n

System configuration completed, doing now some checks!

This may take a while!

Creating the index file for tcl!

Now testing the Makefile for numerical integrations generated for your System!

rm -f main funct.o numint.o div0.o vegas.o gauss.o utils.o

g++ -DGNU_COMPLEX -O3 -fomit-frame-pointer -finline-functions -m486 -malign-functions=4 -malign-jumps=4 -malign-loops=4 -fexpensive-optimizations -funroll-loops -ffast-math -c funct.cxx
g++ -DGNU_COMPLEX -O3 -fomit-frame-pointer -finline-functions -m486 -malign-functions=4 -malign-jumps=4 -malign-loops=4 -fexpensive-optimizations -funroll-loops -ffast-math -c numint.cxx
g++ -DGNU_COMPLEX -O3 -fomit-frame-pointer -finline-functions -m486 -malign-functions=4 -malign-jumps=4 -malign-loops=4 -fexpensive-optimizations -funroll-loops -ffast-math -c div0.cxx
g++ -DGNU_COMPLEX -O3 -fomit-frame-pointer -finline-functions -m486 -malign-functions=4 -malign-jumps=4 -malign-loops=4 -fexpensive-optimizations -funroll-loops -ffast-math -c vegas.c
g++ -DGNU_COMPLEX -O3 -fomit-frame-pointer -finline-functions -m486 -malign-functions=4 -malign-jumps=4 -malign-loops=4 -fexpensive-optimizations -funroll-loops -ffast-math -c gauss.cxx

g++ -DGNU_COMPLEX -O3 -fomit-frame-pointer -finline-functions -m486 -malign-functions=4 -malign-jumps=4 -malign-loops=4 -fexpensive-optimizations -funroll-loops -ffast-math -c utils.cxx

g++ -o main funct.o numint.o div0.o vegas.o gauss.o utils.o -lm -lpthread main.cxx

Division by 0 Errors will be ignored

Test: 1/0

Initializing SR-sequences with seed 876920052

Input parameters for vegas: ndim = 2 ncall = 968 2 thread(s)

| iteration no. | integral     | chi**2/IT n |
|---------------|--------------|-------------|
| 1             | -1.077957e-06 +/- 9.2e-07 | 0           |
| 2             | 1.703926e-07 +/- 7.1e-07  | 1.2         |
| 3             | -1.407306e-09 +/- 4.3e-07 | 0.67        |
| 4             | -6.310369e-08 +/- 4e-07   | 0.45        |
| 5             | -2.702396e-07 +/- 3.2e-07 | 0.38        |

Input parameters for vegas: ndim = 2 ncall = 9800 2 thread(s)

| iteration no. | integral     | chi**2/IT n |
|---------------|--------------|-------------|
| 1             | -1.077957e-06 +/- 9.2e-07 | 0           |
| 2             | 1.703926e-07 +/- 7.1e-07  | 1.2         |
| 3             | -1.407306e-09 +/- 4.3e-07 | 0.67        |
| 4             | -6.310369e-08 +/- 4e-07   | 0.45        |
| 5             | -2.702396e-07 +/- 3.2e-07 | 0.38        |
A Examples

iteration no. 1: integral = -4.470697e-08 +/- 6.2e-08
all iterations: integral = -4.470697e-08 +/- 6.2e-08 chi**2/IT n = 0

iteration no. 2: integral = 2.028027e-08 +/- 5.3e-08
all iterations: integral = -6.763662e-09 +/- 4e-08 chi**2/IT n = 0.63

iteration no. 3: integral = -2.663549e-08 +/- 5.1e-08
all iterations: integral = -1.435286e-08 +/- 3.2e-08 chi**2/IT n = 0.36

Total Number of ignored Divisions by 0: 0

Input parameters for vegas: ndim= 2 ncall= 968 2 thread(s)
ittot= 1 itmx= 5 22 * 1 hypercubes
nprn= 0 ALPH= 1.50
mds= 1 nd= 200 npg= 2
xl[ 1]= 0 xu[ 1]= 1
xl[ 2]= 0 xu[ 2]= 1

iteration no. 1: integral = 7.704579e-06 +/- 1.3e-06
all iterations: integral = 7.704579e-06 +/- 1.3e-06 chi**2/IT n = 0

iteration no. 2: integral = 8.286222e-06 +/- 6.8e-07
all iterations: integral = 8.160574e-06 +/- 6.1e-07 chi**2/IT n = 0.16

iteration no. 3: integral = 9.681104e-06 +/- 6.2e-07
all iterations: integral = 8.902334e-06 +/- 4.3e-07 chi**2/IT n = 1.6

iteration no. 4: integral = 8.97195e-06 +/- 5e-07
all iterations: integral = 8.938647e-06 +/- 3.4e-07 chi**2/IT n = 1.1

iteration no. 5: integral = 8.732733e-06 +/- 3.7e-07
all iterations: integral = 8.84922e-06 +/- 2.5e-07 chi**2/IT n = 0.85

Input parameters for vegas: ndim= 2 ncall= 9800 2 thread(s)
ittot= 1 itmx= 3 70 * 1 hypercubes
nprn= 0 ALPH= 1.50
mds= 1 nd= 200 npg= 2
xl[ 1]= 0 xu[ 1]= 1
xl[ 2]= 0 xu[ 2]= 1

iteration no. 1: integral = 8.890832e-06 +/- 1e-07
all iterations: integral = 8.890832e-06 +/- 1e-07 chi**2/IT n = 0

iteration no. 2: integral = 8.87138e-06 +/- 5.3e-08
all iterations: integral = 8.875575e-06 +/- 4.7e-08 chi**2/IT n = 0.029

iteration no. 3: integral = 8.798005e-06 +/- 4.7e-08
all iterations: integral = 8.836898e-06 +/- 3.4e-08 chi**2/IT n = 0.68

Total Number of ignored Divisions by 0: 0

Make seems to work, so Makefile is correct!

If you had problems please edit the Makefile in the subdirectory ./cxx
until it compiles correctly. Please report your system configuration and
Makefile changes to xloops@thep.physik.uni-mainz.de!

Do you agree sending the XLOOPS team a mail [Y/n] ? Y

XLOOPS installed successfully, invoke with ./xloops!
A.2 Examples with the Xwindows frontend

To demonstrate the input and output of \( xloops \) some typical examples are given. The Feynman diagrams in this chapter are Postscript pictures produced with \( xloops \).

A.2.1 Output of one-loop integrals

Analytic examples

At first we want to show the different ways of evaluating the diagrams. As an example we have chosen a simple self-energy diagram. The advantage of this diagram is the relatively short result. The diagram on the right shows the process, a nondiagonal \( Z \) self-energy with a \( W \) bubble.

With [Evaluate] the following result is obtained:

\[
G_1 := \left[ \begin{array}{c}
2 \\
C_1 = \frac{1}{16} I e \cos(tw) \left( 1 + \varepsilon \ln(4 \pi \mu) + \frac{1}{2} \varepsilon \ln(4 \pi \mu) \right) \\
(6 I - 4 I \varepsilon) \text{Oneloop1Pt}(0, M_W) / (\sin(tw) \pi), \\\nC_1 G(nu1, nu2) \end{array} \right]
\]

As expected, the structure of form factors, indicated by the last element of the list above, is just proportional to \( g^{\mu\nu} \). The pre-factor \( C_1 \) is determined by an equation, which shows the corresponding \text{Oneloop} functions. In this example just \text{Oneloop1Pt}(0, M_W) appears. As pre-factor the coupling \( e \), the Weinberg angle \( tw \) and the renormalization parameter \( \mu \) occur.

With [Eval.Full] the \text{OneLoop} function can be evaluated further:

\[
G_{F2} := \left[ \begin{array}{c}
2 \\
I e \cos(tw) M_W \\
C_1 = \left[ - \frac{3}{8} \right. \\
\frac{2}{\sin(tw) \pi} \\
\frac{1}{16} I (- 6 \ln(4 \pi \mu) \pi M_W + 4 \pi M_W) \\
- 6 (- \ln(M_W - I \rho) - \ln(\pi) - \gamma + 1) \pi M_W) e \cos(tw) \\
(\sin(tw) \pi), \\\nC_1 G(nu1, nu2) \end{array} \right]
\]
Now the constant $C_1$ is evaluated to a list, which elements represent the coefficients of the Laurent series in the dimensional regulator $\varepsilon = (4 - D)/2$. The first element of the list shows the divergence of the integral, the second element the convergent part.

Additionally the next order in dimensional regularization can be evaluated. Clicking on \[\text{Eval. More}\] shows the result:

\[
\begin{align*}
\text{GM3} & := \left[ \begin{array}{c}
\frac{1}{2} e^{\cos(tw)} \cot(tw) \mu^2 \\
\frac{1}{2} e^{\cos(tw)} (-6 \ln(4\pi\mu) + 4\pi M_w - 6 \gamma_1) e^{\cos(tw)} \\
\end{array} \right], \\
C_1 & = \left[ \begin{array}{c}
\frac{-3}{8} \\
\frac{1}{16} \\
\end{array} \right], \\
\end{align*}
\]

In the above result the list for $C_1$ was just enhanced by an additional element, the $\mathcal{O}(\varepsilon^1)$.

**Numerical examples**

In the following subsection we want to show some numerical examples, where different kinds of form factors occur. The values for couplings and particle masses were inserted with Insert Particle Properties from the Options menu. The momentum was set to $q_{10} := \mu_0$; and the renormalization parameter was chosen to be $\mu_U := 1$. All results are given in natural units $\hbar = c = 1$ and the energy scale GeV.
The results, when evaluated with Eval. Full, read:
(a)

$$GF4 := [$$
\begin{align*}
C1 &= [ - 2143.7993127296539430 I, \\
&\quad - .33537943559210349312*10^15 + 16785.676157258815796 I], C1]
\end{align*}$$

(b)

$$GF5 := [$$
\begin{align*}
C1 &= [ - .11869586429575443972*10^5 I, \\
&\quad .37289405527131873878*10^5 + .66133100616139736842*10^10 I], \\
C2 &= [.052942866215076463492 I, - .16632491956127147989 - .32145019412334161258 I], \\
C1 \ (1 \ &* \ ONE) \ + \ C2 \ (1 \ &* \ Dg0)\]
\end{align*}$$

(c)

$$GF6 := [$$
\begin{align*}
C1 &= [ - 6.4357348171046949024 I, 20.218457221868161079 + 34.784995718164094540 I], \\
C2 &= [.0007742732020705022066450 I, \\
&\quad - .0024324510191403821401 - .0041849285213475828278 I], \\
C1 \ G(nu1, \ nu2) \ + \ C2 \ q1(nu1) \ q1(nu2)\]
\end{align*}$$

Figure (a) shows a scalar self-energy. So the result just consists of the scalar constant $C1$. Like in the previous example $C1$ is expanded as a series in $\varepsilon$. Figure (b) shows a fermionic self-energy, which has Dirac structure. So the form factors are the unity matrix ONE and Dg0. The pre-factors $C1$ and $C2$ are again represented as lists of coefficients of the Laurent series in $\varepsilon$. Finally figure (c) shows a vector boson self-energy. The result is proportional to the metric tensor $G(nu1, nu2)$ and to the product of the external momenta $q1(nu1) \ q1(nu2)$. Again two constants, $C1$ and $C2$, are needed.

### A.2.2 Two-loop diagrams

Now some two-loop diagrams shall be evaluated numerically. The values are chosen as in the previous section, if no comment is made. First we want to show some two-loop diagrams which factorize, starting with a photon self-energy:
The typical structure of a photon self-energy,

\[ C_1 G(\nu_1, \nu_2) + C_2 q_1(\nu_1) q_1(\nu_2) \]

can be seen. The difference to the one-loop self-energy (c) is just the fact, that this graph is more divergent. So the constants \( C_1 \) and \( C_2 \) have an additional list entry, the order \( \varepsilon^{-2} \).

Next we demonstrate a three-point function. All momenta are on-shell, e.g. \( q_{10}: = Mz0; q_{20}: = Mz0/2; q_{21}: = \sqrt{(Mz0^2 - Melec^2)} \); is passed to Maple with Insert Maple Command.

---

\[
GF7 := [C1 = [\begin{array}{c}
-0.16744033026458920315 I,
0.70833637543010397531*10 + 2.1988161494279602884 I,
-0.42232923827218965144*10 - 14.715169445826507291 I,
\end{array}] \\
C2 = [\begin{array}{c}
0.00020136974316008130172 I,
-0.85187071577025180931*10 - 0.00026443751189864126887 I,
-0.50790828055269934758*10 + 0.0017696990248291598135 I,
\end{array}] \\
C1 G(\nu_1, \nu_2) + C2 q_1(\nu_1) q_1(\nu_2)]
\]
A Examples

C2 = [0, 0.1642868599856020572\times 10^{-7} - 0.5002369242512327568\times 10^{-10}]

C3 = [0, 0.25907616430369099002\times 10^{-9} + 0.5002369242512327568\times 10^{-10}]

C4 = [0, 0.56375581291706017052\times 10^{-9} + 0.5226908872160878944\times 10^{-10}]

C5 = [0, 0.11444962108462501382\times 10^{-26} - 0.1304643398124069502\times 10^{-27}]

C6 = [0, 0.19581694212974079513\times 10^{-27} - 0.2247792468575032187\times 10^{-27}]

C7 = [0, 0.1347698993840451368157\times 10^{-23} - 0.1099856810098265\times 10^{-23}]

C8 = [0, -0.13476989941451368157\times 10^{-22} - 0.11492263298098265\times 10^{-22}]

C9 = [0, 0.31582291657035101225\times 10^{-9} - 0.10838466684439346062\times 10^{-9}]

C10 = [0, -0.98315817920409517794\times 10^{-8} - 0.1132496921500662291\times 10^{-8}]

C11 = [0, 0.76454675644007305666\times 10^{-27} - 0.48786422812941166168\times 10^{-27}]

\[ C12 = [0, -0.206288073409613985 \times 10^{-8}, -0.2142392351100969654 \times 10^{-8}] , \]

\[ C1 (1 \& (Dg(nu1))) + C2 \& (1, Dg5, Dg(nu1)) + C3 (1 \& (Dg0) q2(nu1)) \]
\[ + C4 \& (1, Dg5, Dg0) q2(nu1) + C5 (1 \& (Dg1) q2(nu1)) \]
\[ + C6 \& (1, Dg5, Dg1) q2(nu1) + C7 (1 \& (ONE) q1(nu1)) \]
\[ + C8 (1 \& (Dg5) q1(nu1)) + C9 (1 \& (Dg0) q1(nu1)) \]
\[ + C10 \& (1, Dg5, Dg0) q1(nu1) + C11 (1 \& (Dg1) q1(nu1)) \]
\[ + C12 \& (1, Dg5, Dg1) q1(nu1) \]

Of course the result has more form factors as the diagrams evaluated before. The reason for this is, that as external particles vector bosons as well as fermions occur.
B Technical details

B.1 File structure

To understand which files \( \text{xloops} \) uses for loading and saving of processes, it is necessary to have a short glance on the file structure, which \( \text{xloops} \) writes into every user’s home directory. First of all \( \text{xloops} \) creates the subdirectory \( \text{xloops\_user} \). In this directory \( \text{xloops} \) writes all user-specific data. If the user now starts to input a new process, \( \text{xloops} \) creates in \( \text{xloops\_user} \) a new subdirectory, which has the same name as the process. In this subdirectory \( \text{xloops} \) writes to basic files: the file \( \text{Init} \) with all options entered for this process (e.g. the model), and the file \( \text{History.tab} \), where for every diagram a corresponding number and a particle list is saved. If one graph from this list is calculated, \( \text{xloops} \) creates a new file to save the result. The name of this file includes the number of the diagram in the particle list. If the user now tries to evaluate this graph again, \( \text{xloops} \) looks for the corresponding file and reads the result.

B.2 The one-loop library

To avoid multiple evaluation of one-loop integrals \( \text{xloops} \) can read these integrals from a pre-built library. This library is produced with by the \text{OneLoopLib} procedures and written to the directory, which is indicated by the variable \text{LibPath}. If the \text{Xwindows} interface is used, this variable points to a common directory, where all integrals are saved. Otherwise the user can set this variable to the directory, where he has his own library stored. If some integrals in the library are missing, \( \text{xloops} \) creates them automatically.[1] In practice it is necessary to store not only the general mass case of each integral. Several special cases are also needed. The correspondence is

\[
Bp_0p_1t_1t_2n_1.1\text{LOOP} \iff \text{OneLoop2Pt}(p_0, p_1, q, m_1, m_2, t_1, t_2)
\]

\[
Cp_0p_1p_2t_1t_2t_3n_1n_2i_1i_2i_3i_4i_5i_6.1\text{LOOP} \iff \text{OneLoop3Pt}(p_0, p_1, p_2, q_1, q_20, q_21, m_1, m_2, m_3, t_1, t_2, t_3).
\]

[1]Currently only supported on Unix platforms.
The additional numbers \( n_1, n_2 \) and the optional numbers \( i_1, i_2, i_3, i_4, i_5, i_6 \) specify the different kinematical arrangements which have to be taken into account. In the following tables we list the conditions which belong to each case:

### Two-point functions:

| 1 | conditions | 2 | \( n_1 \) | method of solution | effective \( n \)-point |
|---|-------------|---|------------|-------------------|----------------------|
| \( q = 0 \) | \( m_1 = m_2 \) | triv. | \( D \) | \( P \) | 1 |
| \( m_1 = 0 \) | \( m_2 = 0 \) | \( q^2 + m_1^2 - m_2^2 = 0 \) | \( q^2 - m_1^2 + m_2^2 = 0 \) | \( q + m_1 + m_2 = 0 \) | \( q - m_1 + m_2 = 0 \) | \( q + m_1 - m_2 = 0 \) | \( q - m_1 - m_2 = 0 \) | 1 |

### Three-point functions:

| 1 | 2 | 3 | 4 | \( n_{12} \) | method of solution | effective \( n \)-point |
|---|---|---|---|------------|-------------------|----------------------|
| \( q_{21} = 0 \) | \( q_{20} = 0 \) | \( q_1 = 0 \) | \( m_1 = m_2 = m_3 \) | triv. | \( 2 \times D \) | 1 |
| \( m_1 = m_2 \) | \( m_1 = m_3 \) | \( m_2 = m_3 \) | \( q_1 + m_1 + m_3 = 0 \) | \( q_1 + m_1 - m_3 = 0 \) | \( q_1 - m_1 + m_3 = 0 \) | \( q_1 - m_1 - m_3 = 0 \) | 11 |
| \( m_2 = m_3 \) | \( q_1 + m_1 - m_3 = 0 \) | \( q_1 - m_1 + m_3 = 0 \) | \( q_1 - m_1 - m_3 = 0 \) | \( q_1 + m_1 + m_3 = 0 \) | \( q_1 + m_1 - m_3 = 0 \) | \( q_1 - m_1 + m_3 = 0 \) | \( q_1 - m_1 - m_3 = 0 \) | 12 |
| \( q_20 = 0 \) | \( q_1 = 0 \) | \( m_1 = m_2 \) | \( q_1 + m_1 + m_3 = 0 \) | \( q_1 + m_1 - m_3 = 0 \) | \( q_1 - m_1 + m_3 = 0 \) | \( q_1 - m_1 - m_3 = 0 \) | \( q_1 + m_1 + m_3 = 0 \) | \( q_1 + m_1 - m_3 = 0 \) | \( q_1 - m_1 + m_3 = 0 \) | \( q_1 - m_1 - m_3 = 0 \) | 13 |
| \( q_{20} = 0 \) | \( q_1 = 0 \) | \( m_1 = m_3 \) | \( q_1 + m_1 + m_3 = 0 \) | \( q_1 + m_1 - m_3 = 0 \) | \( q_1 - m_1 + m_3 = 0 \) | \( q_1 - m_1 - m_3 = 0 \) | \( q_1 + m_1 + m_3 = 0 \) | \( q_1 + m_1 - m_3 = 0 \) | \( q_1 - m_1 + m_3 = 0 \) | \( q_1 - m_1 - m_3 = 0 \) | 14 |
| \( (q_{20}^2 - m_2^2)q_1 - (q_1^2 - m_1^2)q_{20} - m_3^2(q_{20} - q_1) = 0 \) | \( q_1 = 0 \) | \( m_1 = m_3 \) | \( q_1 + m_1 + m_3 = 0 \) | \( q_1 + m_1 - m_3 = 0 \) | \( q_1 - m_1 + m_3 = 0 \) | \( q_1 - m_1 - m_3 = 0 \) | \( q_1 + m_1 + m_3 = 0 \) | \( q_1 + m_1 - m_3 = 0 \) | \( q_1 - m_1 + m_3 = 0 \) | \( q_1 - m_1 - m_3 = 0 \) | 15 |
| \( q_{20} = 0 \) | \( q_1 = 0 \) | \( m_1 = m_3 \) | \( q_1 + m_1 + m_3 = 0 \) | \( q_1 + m_1 - m_3 = 0 \) | \( q_1 - m_1 + m_3 = 0 \) | \( q_1 - m_1 - m_3 = 0 \) | \( q_1 + m_1 + m_3 = 0 \) | \( q_1 + m_1 - m_3 = 0 \) | \( q_1 - m_1 + m_3 = 0 \) | \( q_1 - m_1 - m_3 = 0 \) | 16 |
| \( q_{20} = 0 \) | \( q_1 = 0 \) | \( m_1 = m_3 \) | \( q_1 + m_1 + m_3 = 0 \) | \( q_1 + m_1 - m_3 = 0 \) | \( q_1 - m_1 + m_3 = 0 \) | \( q_1 - m_1 - m_3 = 0 \) | \( q_1 + m_1 + m_3 = 0 \) | \( q_1 + m_1 - m_3 = 0 \) | \( q_1 - m_1 + m_3 = 0 \) | \( q_1 - m_1 - m_3 = 0 \) | 17 |

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The table shows in the first columns the conditions to the masses $m_i$ and the external momenta for a special case. Then the number occurring in the name of the file, in which the special case is saved is given. In the following two columns the method of solving this integrals and the effective $n$-point function to solve is given. The abbreviations for the method of solution have the following meaning:

- **D** Differentiation to the squares of masses.
- **P** Solving by partial fraction decomposition.
- **R** Special case of the $\mathcal{R}$ function, either vanishing or equal arguments.
- **L** Solution by Lorentz transformation and reduction of parallel space.
- **S** Summands vanish in the sum of residua, e.g. some residua are 0.

For the tensor integrals of the three point function the following additional cases occur:

| 1 | conditions | 2 | 3 | method of solution | effective $n$-point |
|---|------------|---|---|-------------------|-------------------|
| $q_{20} - q_1 = q_{21}$ | $q_{20} = -q_{21}$ | — | — | — | 3 |
|  | $m_3 \neq 0$ | — | — | C0 | $S$ | 3 |
| $q_{20} - q_1 = -q_{21}$ | $q_{20} = q_{21}$ | — | — | — | 3 |
|  | $m_3 \neq 0$ | — | — | D0 | $S$ | 3 |
| $q_{20} = q_{21}$ | $m_2 \neq 0$ | — | — | — | 3 |
| $q_{20} = -q_{21}$ | $m_2 \neq 0$ | — | — | — | 3 |
| $q_{20}^2 - q_{21}^2 = m_2^2$ | $q_1^2 = m_1^2$ | $m_3 = 0$ | — | G0 | $S$ | 3 |
|  | $q_{20}^2 - q_{21}^2 = m_3^2$ | $m_2 = 0$ | — | H0 | $S$ | 3 |
|  | $q_{20}^2 - q_{21}^2 = m_2^2$ | $q_1^2 = m_2^2$ | $m_1 = 0$ | I0 | $S$ | 3 |
| $q_{20}^2 - q_{21}^2 = m_2^2$ | $q_1^2 = m_1^2$ | $m_3 = 0$ | — | — | — | 3 |
| $q_{20}^2 - q_{21}^2 = m_3^2$ | $m_2 = 0$ | — | — | — | 3 |
| $q_{20}^2 - q_{21}^2 = m_2^2$ | $q_1^2 = m_2^2$ | $m_1 = 0$ | — | — | — | 3 |

The table shows in the first columns the conditions to the masses $m_i$ and the external momenta for a special case. Then the number occurring in the name of the file, in which the special case is saved is given. In the following two columns the method of solving this integrals and the effective $n$-point function to solve is given. The abbreviations for the method of solution have the following meaning:

- **D** Differentiation to the squares of masses.
- **P** Solving by partial fraction decomposition.
- **R** Special case of the $\mathcal{R}$ function, either vanishing or equal arguments.
- **L** Solution by Lorentz transformation and reduction of parallel space.
- **S** Summands vanish in the sum of residua, e.g. some residua are 0.

For the tensor integrals of the three point function the following additional cases occur:
To make the notation clearer, we give the following examples:

\[
\begin{align*}
B_{p_0 p_1 t_1 t_2} & .1\text{LOOP} & \text{OneLoop2Pt}(p_0, p_1, q, m_1, m_2, t_1, t_2) \\
B_{p_0 p_1 t_1 t_2}1.1\text{LOOP} & \text{OneLoop2Pt}(p_0, p_1, 0, m_2, t_1, t_2) \\
B_{p_0 p_1 t_1 t_2}2.1\text{LOOP} & \text{OneLoop2Pt}(p_0, p_1, q, m_1, 0, t_1, t_2) \\
B_{p_0 p_1 t_1 t_2}3.1\text{LOOP} & \text{OneLoop2Pt}(p_0, p_1, \sqrt{m_2^2 - m_1^2}, m_1, m_2, t_1, t_2) \\
B_{p_0 p_1 t_1 t_2}4.1\text{LOOP} & \text{OneLoop2Pt}(p_0, p_1, \sqrt{m_1^2 - m_2^2}, m_1, m_2, t_1, t_2) \\
C_{p_0 p_1 p_2 t_1 t_2 t_3} & .1\text{LOOP} & \text{OneLoop3Pt}(p_0, p_1, p_2, q_1, q_20, q_21, m_1, m_2, m_3, t_1, t_2, t_3) \\
C_{p_0 p_1 p_2 t_1 t_2 t_3}1.1\text{LOOP} & \text{OneLoop3Pt}(p_0, p_1, p_2, 0, m_1, m_2, m_3, t_1, t_2, t_3) \\
C_{p_0 p_1 p_2 t_1 t_2 t_3}2.1\text{LOOP} & \text{OneLoop3Pt}(p_0, p_1, q_1, q_20, q_21, \sqrt{q_20^2 - q_21^2}, 0, t_1, t_2, t_3) \\
C_{p_0 p_1 p_2 t_1 t_2 t_3}3.1\text{LOOP} & \text{OneLoop3Pt}(p_0, p_1, p_2, q_1, q_20, q_21, q_1 - q_20, m_1, m_2, m_3, t_1, t_2, t_3) \\
C_{p_0 p_1 p_2 t_1 t_2 t_3}4.1\text{LOOP} & \text{OneLoop3Pt}(p_0, p_1, p_2, q_1, q_20, -q_20, m_1, m_2, m_3, t_1, t_2, t_3) \\
C_{p_0 p_1 p_2 t_1 t_2 t_3}5.1\text{LOOP} & \text{OneLoop3Pt}(p_0, p_1, p_2, q_1, q_20, q_20 - q_1, m_1, m_2, m_3, t_1, t_2, t_3) \\
C_{p_0 p_1 p_2 t_1 t_2 t_3}6.1\text{LOOP} & \text{OneLoop3Pt}(p_0, p_1, p_2, q_1, q_20, q_20, m_1, m_2, m_3, t_1, t_2, t_3) \\
C_{p_0 p_1 p_2 t_1 t_2 t_3}7.1\text{LOOP} & \text{OneLoop3Pt}(p_0, p_1, p_2, q_1, q_1, 0, m_1, m_2, m_3, t_1, t_2, t_3) \\
C_{p_0 p_1 p_2 t_1 t_2 t_3}8.1\text{LOOP} & \text{OneLoop3Pt}(p_0, p_1, p_2, q_1, q_1, 0, m_1, m_2, m_3, t_1, t_2, t_3) \\
C_{p_0 p_1 p_2 t_1 t_2 t_3}9.1\text{LOOP} & \text{OneLoop3Pt}(p_0, p_1, p_2, 0, q_20, q_21, m_1, m_2, m_3, t_1, t_2, t_3)
\end{align*}
\]

B.3 Run time and numerical stability

Typical run times for the different integrals are displayed in the following table. We used two different systems: (i) a DEC Alpha 8400 workstation (ii) a PC with i586 chip, 32 MB RAM and 133 MHz frequency running Linux. All systems are working with Maple V. Without using the library we found the following run times:

|          | DEC Alpha 8400 | i586 133MHz |
|----------|----------------|-------------|
|          | numerical | algebraical | numerical | algebraical |
| One-loop two-point functions |              |             |              |             |
| 0        | 0.02 s     | 0.04 s      | 0.04 s      | 0.06 s      |
| 1        | 0.04 s     | 0.21 s      | 0.06 s      | 0.26 s      |
| 2        | 0.07 s     | 0.28 s      | 0.07 s      | 0.48 s      |
| 3        | 0.08 s     | 0.41 s      | 0.09 s      | 0.76 s      |
| One-loop three-point functions |              |             |              |             |
| 0        | 2.54 s     | 1.83 s      | 4.83 s      | 3.31 s      |
| 1        | 2.57 s     | 2.23 s      | 5.09 s      | 3.69 s      |
| 2        | 3.01 s     | 3.62 s      | 5.67 s      | 6.14 s      |
| 3        | 3.78 s     | 4.16 s      | 7.61 s      | 34.58s      |

It should be emphasized that these are the times which are necessary to generate the functions once and forever using the OneLoopLibPt routines. If they once are stored in the LibPath directory they may be read in quickly. With assigned LibPath we get:
The accuracy and stability of numerical results may suffer from cancellations of large, approximately equal dilogarithms. Since Maple supports calculations of arbitrary precision, it is always possible to increase the number of digits to improve accuracy — instead of Fortran programs which are limited to Fortran’s restricted accuracy.

Usually we calculated with 20 or 40 digits. For all practical purposes our numerical results were not influenced significantly — only in the last four or five digits — if the number of digits increased.

### B.4 The xloops distribution

The distribution (packed in xloops.tgz) consists of the following files:

- README
- XLwidget.tcl
- configure*
- convert.tcl
- graphen.tcl
- MapleVR1:
  - cfnc.ma
  - evalproc.ma
  - fmrules.ma
  - fmuser.ma
  - loops.ma
- MapleVR3:
  - cfnc.ma
  - evalproc.ma
  - fmrules.ma
  - fmuser.ma
  - loops.ma
- cxx:
  - div0.cxx
  - div0.h
  - dpc1.in
  - dpc2.in
  - dpc3.in
  - dpc4.in
  - funct.cxx
  - funct.h
  - gauss.cxx
lib:
B00110.1loop

manual:
manual.html

xbms:
Hp1l1t0.xbm Hp3l2t9.xbm Hp4l2t9.xbm p310t2.xbm p412t13.xbm
Hp1l2t0.xbm Hp4l0t0.xbm attention.xbm p31l10.xbm p412t14.xbm
Hp1l2t1.xbm Hp4l1t0.xbm file.xbm p31l1t1.xbm p412t15.xbm
Hp1l2t2.xbm Hp4l1t1.xbm help.xbm p31l1t4.xbm p412t16.xbm
Hp2l0t0.xbm Hp4l1t2.xbm inmmath.xbm p31l1t5.xbm p412t17.xbm
Hp2l1t0.xbm Hp4l2t0.xbm lefarrow.xbm p31l2t0.xbm p412t18.xbm
Hp2l1t1.xbm Hp4l2t1.xbm p11l0t0.xbm p31l2t1.xbm p412t19.xbm
Hp2l2t0.xbm Hp4l2t10.xbm p11l1t1.xbm p31l2t10.xbm p412t2.xbm
Hp2l2t1.xbm Hp4l2t11.xbm p11l2t0.xbm p31l2t11.xbm p412t20.xbm
Hp2l2t10.xbm Hp4l2t12.xbm p11l2t1.xbm p31l2t12.xbm p412t21.xbm
Hp2l2t11.xbm Hp4l2t13.xbm p11l2t2.xbm p31l2t13.xbm p412t22.xbm
Hp2l2t2.xbm Hp4l2t14.xbm p11l2t3.xbm p31l2t2.xbm p412t23.xbm
Hp2l2t3.xbm Hp4l2t15.xbm p1proc.xbm p31l2t3.xbm p412t24.xbm
Hp2l2t4.xbm Hp4l2t16.xbm p21l0t0.xbm p31l2t4.xbm p412t25.xbm
Hp2l2t5.xbm Hp4l2t17.xbm p21l0t1.xbm p31l2t5.xbm p412t26.xbm
Hp2l2t6.xbm Hp4l2t18.xbm p21l1t0.xbm p31l2t6.xbm p412t27.xbm
Hp2l2t7.xbm Hp4l2t19.xbm p21l1t1.xbm p31l2t7.xbm p412t28.xbm
Hp3l0t0.xbm Hp4l2t2.xbm p21l1t2.xbm p31l2t8.xbm p412t3.xbm
Hp3l1t0.xbm Hp4l2t20.xbm p21l1t3.xbm p31l2t9.xbm p412t4.xbm
Hp3l1t1.xbm Hp4l2t21.xbm p21l2t0.xbm p3proc.xbm p412t5.xbm
Hp3l2t0.xbm Hp4l2t22.xbm p21l2t1.xbm p40l0t0.xbm p412t6.xbm
Hp3l2t1.xbm Hp4l2t23.xbm p21l2t10.xbm p40l0t3.xbm p412t7.xbm
Hp3l2t10.xbm Hp4l2t24.xbm p21l2t11.xbm p41l0t0.xbm p412t8.xbm
Hp3l2t11.xbm Hp4l2t25.xbm p21l2t2.xbm p41l1t1.xbm p412t9.xbm
Hp3l2t12.xbm Hp4l2t26.xbm p21l2t3.xbm p41l1t2.xbm p4proc.xbm
Hp3l2t13.xbm Hp4l2t27.xbm p21l2t4.xbm p41l1t6.xbm rightarrow.xbm
Hp3l2t2.xbm Hp4l2t28.xbm p21l2t5.xbm p41l1t7.xbm schraff.xbm
Hp3l2t3.xbm Hp4l2t3.xbm p21l2t6.xbm p41l1t8.xbm xloops.gif
Hp3l2t4.xbm Hp4l2t4.xbm p21l2t7.xbm p412t0.xbm xloops.xbm
Hp3l2t5.xbm Hp4l2t5.xbm p21l2t8.xbm p412t1.xbm
Hp3l2t6.xbm Hp4l2t6.xbm p21l2t9.xbm p412t10.xbm
Hp3l2t7.xbm Hp4l2t7.xbm p2proc.xbm p412t11.xbm
Hp3l2t8.xbm Hp4l2t8.xbm p310t0.xbm p412t12.xbm
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