Effective light front quantization of scalar field theories and two-dimensional electrodynamics

E.V. Prokhvatilov,* H.W.L. Naus† and H.–J. Pirner

Institut für Theoretische Physik
Universität Heidelberg

Abstract

We introduce a new method to include condensates in the light-cone Hamiltonian. By using a Gaussian approximation to the ordinary vacuum in a theory close to the light front, we derive an effective Hamiltonian on the light cone, which has new terms reflecting the nontriviality of the vacuum. We demonstrate our method for scalar $\phi^4$-theory and the massive Schwinger model.

*Permanent address: Department of Theoretical Physics, Institute of Physics, St. Petersburg University, Ulyanovskaya 1, St. Petersburg 198904, Russia
†Supported in part by the Federal Ministry of Research and Technology (BMFT) under contract number 06 HD 729
1 Introduction

The idea of quantizing field theories on the light front (i.e. on the hyperplane tangent to the light-cone) was put forward by Dirac [1]. He pointed out that in such a formulation, the part of the Lorentz symmetry described kinematically is maximal. In other words, the number of generators of the Poincaré group, which depend on the dynamics, is minimal. Instead of Lorentz coordinates \( x^\mu (\mu = 0, 1, 2, 3) \) Dirac used the light-like coordinates:

\[
x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3), \\
x^\perp = x^{1,2}.
\]

(1)

The coordinate \( x^+ \) plays the role of the time. The subgroup of the Poincaré group consisting of the generators \( M_{12}, M_{+-}, M_{-\perp} \) and \( P^+, P_\perp \) is dynamically independent. This maximal amount of kinematical symmetry is related to the trivial structure of the vacuum in this formulation [2]. Indeed the vacuum is identified with the lowest eigenstate of the momentum \( P^+ \geq 0 \). The Fock space constructed over this vacuum [2] can be used to solve the eigenvalue problem for the mass (squared) operator:

\[
m^2 = 2P^+P^- - P_\perp^2.
\]

For states with fixed \( P^+ \) and \( P_\perp = 0 \), one has to solve the Schrödinger equation:

\[
P^-|m^2, P^+, P_\perp = 0 > = \frac{m^2}{2P^+}m^2, P^+, P_\perp = 0 >.
\]

(2)

This approach appears promising in non-perturbative studies of gauge theories, in particular QCD [3, 4, 5, 6].

The quantization surface \( x^+ = 0 \), however, is a characteristic surface of the field equations. This peculiarity is reflected in infrared singularities, \( P^+ \to 0 \), in such formulations. Consequently, one is forced to use some regularization. Usually the most simple regularization is chosen: \( P^+ \geq \epsilon > 0 \), where \( \epsilon \) is a cutoff parameter. The simplicity of the vacuum and of the physical Fock space is related to this choice of the regularization.

The question about the equivalence of such a light-front formulation to the usual one arises. To answer this question, results for various two-dimensional models have been considered: Sine Gordon [7, 8], \( \varphi^4 \) model [9, 10], QED [11, 12], QCD [13], etc. The results for the mass spectra agree rather well with the results of the usual approaches, except for some ‘vacuum effects’. These are usually connected with condensates which are zero in the light-front formalism. In four-dimensional space-time the spectrum of positronium in QED was considered with similar results [14].

To gain understanding about the equivalence of light-front formulation to the ordinary one, it is useful to consider the theory again on a space-like plane, close to the light front [15, 16], and investigate the limiting transition to the latter. This can be done by introducing the following coordinates [15]:

\[
y^0 = x^+ + \frac{1}{2}\eta^2 x^- ,
\]

\[
y^3 = x^- ,
\]

\[
y^\perp = x^\perp ,
\]

(3)

with the metric \( g_{\mu\nu}(\eta) (g_{00} = 0, g_{03} = g_{30} = 1, g_{33} = -\eta^2) \). The quantization plane is defined by \( y^0 = 0 \). The parameter \( \eta \) is small and in the limit \( \eta \to 0 \) the exact light front is approached.

In the studies [13, 14] of two-dimensional gauge theories formulated on a finite \( y^3 \) interval with periodic boundary conditions, it was explicitly shown that one obtains
equivalent results only, when the continuum limit \( L \to \infty \) is made first and then followed by the transition to the exact light cone \( \eta \to 0 \) (or \( L \eta \to \infty \), \( \eta \to 0 \)). Taking the limit \( \eta \to 0 \) at fixed \( L \) \((L \eta \to 0)\) yields the usual light-front formulation (with \(|x^-| \leq L\) with zero condensates.

Attempts have been made to take into account vacuum effects by considering zero \((P^+ = 0)\) Fourier modes of the fields \([10, 15, 17]\). However, in the light-front formulation these zero modes have peculiar dynamics \([4, 10, 17]\). For example, they depend on nonzero modes through some specific canonical constraints related to the choice of the boundary conditions for \(|x^-| \leq L\) \([4, 10]\). This means that the physics at low momenta (zero modes) can depend on high momentum modes in a complicated fashion.

In this paper another, more efficient approach to light-front quantization is proposed. It is based on approximations for the vacuum in the ordinary formulation \([18]\) and on the appropriate choice of canonical variables reflecting the non-triviality of the vacuum in the given approximation. In terms of these new variables we then take the naive light front limit \((\eta \to 0 \text{ at fixed } \varepsilon)\); the resulting theory will include information on the approximate, non-trivial, vacuum.

This approach is demonstrated by two simple examples: scalar field theory in two dimensions (next section) and the massive Schwinger model (section 3).

## 2 Scalar field theory in 1+1 dimensions

For scalar field theory, we define the Lagrangian density as

\[
\mathcal{L}(\varphi) = \frac{1}{2} g^{\mu\nu} \partial_{\mu} \varphi(y) \partial_{\nu} \varphi(y) - \frac{1}{2} m_0^2 \varphi^2(y) - \lambda U(\varphi),
\]

where \( U(\varphi) \) is an interaction term. The theory is formulated using the \( y^\mu \) coordinates, eq. (3); here, in the two-dimensional case, the space coordinate is denoted by \( y^1 \). Consequently, we can write

\[
\mathcal{L}(\varphi) = \partial_0 \varphi(y) \partial_1 \varphi(y) + \frac{1}{2} \eta^2 (\partial_0 \varphi(y))^2 - \frac{1}{2} m_0^2 \varphi^2(y) - \lambda U(\varphi).
\]

After introducing the canonical variable \( \Pi(y) \), the conjugate momentum of \( \varphi(y) \),

\[
\Pi(y) = \frac{\partial \mathcal{L}}{\partial (\partial_0 \varphi(y))} = \eta^2 \partial_0 \varphi(y) + \partial_1 \varphi(y),
\]

the Hamiltonian reads

\[
H = \int dy^1 \left\{ \left( \frac{(\Pi - \partial_1 \varphi)^2}{2\eta^2} + \frac{1}{2} m_0^2 \varphi^2 + \lambda U(\varphi) \right) \right\}
\]

\[
= : H_0 + \lambda U.
\]

The usual (equal \( y^0 \)) commutation relations are imposed:

\[
[\varphi(y^1), \Pi(y^{1'})] = i\delta(y^1 - y^{1'}). \]

We make a Fourier-decomposition of the canonical fields \( \varphi \) and \( \Pi \) in terms of the “bare” operators \( b \) and \( b^+ \) \((b|0_b >= 0, \text{ with } |0_b > \text{ as the free field vacuum})\):

\[
\varphi(y) = \frac{1}{\sqrt{4\pi}} \int_{-\infty}^{+\infty} \frac{dk_1}{\sqrt{E_0(k_1)}} (b(k_1) + b^+(-k_1)) e^{-ik_1 y^1}
\]

\[
\Pi(y) = -\frac{i}{\sqrt{4\pi}} \int_{-\infty}^{+\infty} dk_1 \sqrt{E_0(k_1)} (b(k_1) - b^+(-k_1)) e^{-ik_1 y^1},
\]

\((13)\)
where \( E_0(k_1) = \sqrt{k_1^2 + \eta^2 m_0^2} \). For the operators \( b \) and \( b^\dagger \) we have standard commutation relations

\[
[b(k_1), b^\dagger(k'_1)] = \delta(k_1 - k'_1) \\
[b(k_1), b(k'_1)] = 0 = [b^\dagger(k_1), b^\dagger(k'_1)].
\]

In terms of \( b \) and \( b^\dagger \) \( H_0 \) is diagonal by construction:

\[
H_0 = \int_{-\infty}^{+\infty} dk_1 \frac{E_0(k_1) - k_1}{\eta^2} b^\dagger(k_1) b(k_1).
\]  

(8)

Since \( \eta \) appears also in the energy \( E_0 \), only terms in \( H_0 \) with \( k_1 \leq 0 \) are singular in the limit \( \eta \to 0 \). In order to make the energy finite in this limit we consider the restricted Fock space \( \mathcal{F}_{(\varepsilon)} \)

\[
\mathcal{F}_{(\varepsilon)} =: \{ \prod_i b^\dagger(k_i)|0_b >, \ k_i \geq \varepsilon > 0 \}.
\]

where \( \varepsilon \) is the cutoff parameter. If we now take \( \eta \to 0 \) at fixed \( \varepsilon > 0 \), we obtain a finite result for the energy, because \( \frac{E_0(k_1) - k_1}{\eta^2} \to \frac{m_0^2}{2k_1} \), for \( k_1 \geq \varepsilon > 0 \) and \( \eta \to 0 \).

The limiting form of the Hamiltonian on the subspace \( \mathcal{F}_{(\varepsilon)} \) reproduces the usual light-cone Hamiltonian \( P^- \)

\[
P^- = \lim_{\eta \to 0} H_\eta \quad \text{(acting on } \mathcal{F}_{(\varepsilon)} \text{)}
\]

\[
= \int_{-\infty}^{+\infty} dx^- \left\{ \frac{1}{2} m_0^2 \varphi_\varepsilon^2(x) + \lambda U(\varphi_\varepsilon) \right\},
\]

(9)

where \( \varphi_\varepsilon(x) \) is the parametrization of the field in light-front coordinates

\[
\varphi_\varepsilon(x^-, x^+) = 0 = \frac{1}{\sqrt{4\pi}} \int_\varepsilon^\infty \frac{dp^+}{\sqrt{p^+}} \left[ b(p^+) e^{-ip^+x^-} + b^\dagger(p^+) e^{ip^+x^-} \right].
\]

(10)

Note that we would get the same result in the theory formulated on a finite interval, \(-L \leq y^t \leq L\), with periodic boundary conditions in \( y^t \). In this case the role of the cut-off parameter \( \varepsilon \) would be taken over by the parameter \( \pi/L \). Furthermore, the result (9) can be obtained via time-independent perturbation theory in \( \eta \) [15, 16].

At this point we want to introduce a better way to formulate the light-cone limit \( \eta \to 0 \). Before the limiting transition, i.e. still for finite \( \eta \), we approximate the vacuum by a Gaussian trial state [18, 19] using the limit \( \varepsilon \to 0 \). This trial state is parametrized by a Bogoljubow type transformation:

\[
|0_a > = \exp \left[ -\frac{1}{2} \int dk f(k)(b^\dagger(k)b^\dagger(-k) - b(k)b(-k)) + f_0(b^\dagger(0) - b(0)) \right] |0_b >,
\]

(11)

where \( f(k) \) and \( f_0 \) are real, and \( f(k) = f(-k) \). The trial vacuum can be easily defined with new operators \( a(k_1), a^\dagger(k_1) \) such that \( a(k_1)|0_a > = 0 \). As follows from eq. (11) these new operators \( a(k_1) \) and \( a^\dagger(k_1) \) are linear combinations of the old operators \( b^\dagger(k_1) \) and \( b(k_1) \). Therefore one can rewrite the Fourier-decompositions of \( \varphi \) and \( \Pi \) in terms of \( a, a^\dagger \):

\[
\varphi(y) = \varphi_0 + \frac{1}{\sqrt{4\pi}} \int \frac{dk_1}{\sqrt{E(k_1)}} (a(k_1) + a^\dagger(-k_1)) e^{-ik_1y^t},
\]

\[
\Pi(y) = -\frac{i}{\sqrt{4\pi}} \int dk_1 \sqrt{E(k_1)} (a(k_1) - a^\dagger(-k_1)) e^{-ik_1y^t}.
\]

(12)
Identifying these expressions with the corresponding ones in terms of the $b$ and $b^+$ operators (eq. (7)) yields the linear transformations between the sets $(a, a^+)$ and $(b, b^+)$ in terms of $E(k_1), E_0(k_1)$ and $\varphi$. Then the condition $a(k_1)|0_a >= 0$ determines the relation between $(E(k_1), \varphi_0)$ and $(f(k), f_0)$ to be:

$$E(k_1) = E_0(k_1) \exp(2f(k))$$
$$\varphi_0 = \frac{f_0}{\sqrt{\pi \eta m_0}} \frac{1 - \exp(-f(0))}{f(0)}.$$ (13)

In the following we will consider $E(k_1)$ and $\varphi_0$ as parameters of the transformation (or, equivalently, of the trial state).

From now on we specify the interaction as $U(\varphi) = \varphi^4$. We proceed by rewriting the Hamiltonian $H$ in the normal ordered form with respect to the $a, a^+$ operators; :: denotes this normal ordering. The result is:

$$H = \int dy \left[ \frac{(\Pi - \partial_1 \varphi)^2}{2\eta^2} + \frac{1}{2}(m_0^2 + 12\lambda \varphi \varphi') \varphi^2 + \lambda \varphi^4 \right. + \frac{1}{2m_0^2} \varphi \varphi' + \left. \frac{1}{2}m_0^2 \varphi \varphi' + 3\lambda (\varphi \varphi')^2 \right],$$ (14)

where

$$\varphi \varphi := \int \frac{dk_1}{4\pi E(k_1)} \quad \Pi \Pi := \int \frac{dk_1}{4\pi E(k_1)}$$

and

$$\partial_1 \varphi \partial_1 \varphi = \int \frac{dk_1(k_1)^2}{4\pi E(k_1)}.$$

These integrals are understood to be regularized by a cut-off parameter $\Lambda$, $|k_1| < \Lambda$.

In order to fix the parameters $E(k_1)$ and $\varphi_0$ we minimize the expectation value of the Hamiltonian density $\mathcal{H}$ in the trial vacuum $|0_a >$. This expectation value is given by

$$< 0_a | \mathcal{H} | 0_a > = \frac{1}{2\eta^2} (\Pi \Pi + \partial_1 \varphi \partial_1 \varphi) + \frac{1}{2}(m_0^2 + 12\lambda \varphi_0^2) \varphi \varphi'$$

$$+ 3\lambda (\varphi \varphi')^2 + \frac{1}{2}m_0^2 \varphi_0^2 + \lambda \varphi_0^4$$

$$= \frac{1}{8\pi \eta^2} \int dk_1(E(k_1) + k_1^2 + \eta^2 m_0^2 + 12\eta^2 \lambda \varphi_0^2)$$

$$+ 3\lambda \left( \int \frac{dk_1}{4\pi E(k_1)} \right)^2 + \frac{1}{2}m_0^2 \varphi_0^2 + \lambda \varphi_0^4.$$

At the extremum, $\delta < 0_a | \mathcal{H} | 0_a > / \delta E(k_1) = 0$, we obtain:

$$E^2(k_1) = k_1^2 + \eta^2 \left( m_0^2 + 12\lambda \varphi_0^2 + \frac{3\lambda}{\pi} \int_{|q_1| \leq \Lambda} dq_1 E^{-1}(q_1) \right)$$

$$: = k_1^2 + \eta^2 m^2,$$

and

$$\varphi_0(m^2 - 8\lambda \varphi_0^2) = 0.$$ (16)

5
Using the equality (15) we get for a large cut-off $\Lambda$:

$$
\int_{|q_1| \leq \Lambda} dq_1 E^{-1}(q_1) \approx \left( \frac{4\Lambda^2}{\eta^2 m^2} \right) \ln \frac{4\Lambda^2}{\eta^2 \lambda} + \ln \frac{\lambda}{m^2} \approx \ln \frac{4\Lambda^2}{\eta^2 \lambda}.
$$

Since in this limit $m^2 \simeq m_0^2 + 12\lambda \varphi_0^2 + \frac{3\lambda}{\pi} \ln \frac{4\Lambda^2}{\eta^2 \lambda}$, we can renormalize the theory by choosing:

$$
m_0^2 = \frac{3\lambda}{\pi} \ln \frac{\eta^2 \lambda}{4\Lambda^2} + \xi,
$$

with a parameter $\xi$, $-\infty < \xi < \infty$. Then we can convert equation (15) into a nonlinear equation for $m$:

$$
y + \frac{3}{\pi} \ln y = \xi + 12 \varphi_0^2,
$$

with $y = m^2/\lambda$. (17)

This equation should be solved together with eq. (16), which obviously has the solutions:

(1) $\varphi_0 = 0$

(2) $\varphi_0^2 = m^2/8\lambda = \frac{1}{8} y$. (18)

Therefore, there are two different cases:

(1) $y + \frac{3}{\pi} \ln y = \xi$

and

(2) $-\frac{1}{2} y + \frac{3}{\pi} \ln y = \xi$. (19)

The solutions $y_1(\xi)$ (full curve), $y_2(\xi)$ (dashed curve) are shown in Fig. 1. Of course, one needs to choose the solution which corresponds to a minimum of the (trial) vacuum energy. The difference of this energy in the cases (1) and (2) can be calculated straightforwardly; the result is

$$
< 0_a |\mathcal{H}|0_a >_{(1)} - < 0_a |\mathcal{H}|0_a >_{(2)} = \frac{\lambda}{8\pi} (y_1 - y_2) + \frac{\lambda}{48} \left( y_1^2 + \frac{1}{2} y_2^2 \right).
$$

At the critical point $\xi_c = -0.503...$, the sign of the energy difference changes and, consequently, the favoured solution switches from $y_2$ to $y_1$ (for increasing $x$). In other words, we obtain the well-known phase transition in this approximation $\text{[18, 19]}$. Moreover, the exact location of the phase transition, i.e. the critical point, agrees with earlier results $\text{[18]}$.

For the minimal energy solution of eqs. (17) and (18) we introduce the notation

$$
F(\xi) := \begin{cases} 
\left( \frac{m^2}{\lambda} \right)_1, & \xi > \xi_c \\
\left( \frac{m^2}{\lambda} \right)_{up}, & \xi < \xi_c,
\end{cases}
$$

6
where \( \left( \frac{m^2}{\lambda} \right)_{\mu} \) denotes the upper branch of the curve \( \left( \frac{m^2}{\lambda} \right) \). Now we are in the position to present a renormalized Hamiltonian, which is obtained by subtracting the trial-vacuum energy,

\[
H_{\text{ren}} = \int dy \left[ \frac{\Pi^2}{2\eta^2} + \frac{1}{2} \lambda F(\xi) \varphi^2 + \lambda \sqrt{2} F(\xi) \theta(\xi_c - \xi) \varphi^3 + \lambda \varphi^4 \right],
\]

with \( \varphi := \varphi - \varphi_0 \).

We can use \( H_{\text{ren}} \) as the starting Hamiltonian for the limiting transition to the light-cone. Repeating the steps following eq. (8), we obtain the effective light-front Hamiltonian

\[
P^- = \int dx \left[ \frac{\lambda}{2} F(\xi) \varphi_c^2(x) + \lambda \sqrt{2} F(\xi) \theta(\xi_c - \xi) \varphi_c^3(x) + \lambda \varphi_c^4(x) \right],
\]

This expression differs from the usual one by the presence of the function \( F \) describing vacuum effects. In the quadratic term, we see that the effective theory has a renormalized mass term. The cubic term was even completely absent in the usual approach. For \( \xi > \xi_c \), i.e. in the phase without zero mode (\( \varphi_0 = 0 \)), the cubic term vanishes identically and the mass renormalization is all that remains. For \( \xi < \xi_c \), the reflection symmetry \( \varphi \rightarrow -\varphi \) is spontaneously broken and a zero mode \( \varphi_0 \neq 0 \) is present. This zero mode produces in the effective light-cone Hamiltonian an additional interaction term, which explicitly breaks the reflection symmetry. In other words, this formulation converts a spontaneous symmetry breaking into an explicit symmetry breaking in the effective light-cone Hamiltonian. In this way, a rather long-standing defect of light-cone quantization, namely the triviality of the vacuum, can be handled in an approximative way. We emphasize that the proposed approach is very reasonable. The zero modes carry infinite light-cone energy. The strategy to remove high-energy degrees of freedom by effective interactions is the usual strategy of renormalization in equal time field theory.

The effective light-cone Hamiltonian, eq. (21), can be used for explicit calculation using standard light-cone techniques. We note that this approach can easily be generalized to other scalar field theories in two or more dimensions.

## 3 Massive Schwinger Model

The massive Schwinger has also been formulated in the \( y^\mu \) coordinates, i.e. for \( \eta \neq 0 \). The Lagrangian density reads

\[
\mathcal{L}(A_\mu, \psi) = -\frac{1}{4} g^{\mu\rho} g^{\nu\lambda} F_{\mu\nu}(y) F_{\rho\lambda}(y)
+ \bar{\psi}(x(y)) \left[ i \left( \frac{\partial y^\lambda}{\partial x^\mu} \right) \gamma^\mu D_\lambda - M \right] \psi(x(y)),
\]

where the covariant derivative,

\[
D_\mu = \partial_\mu - ie A_\mu(y),
\]

and the field strength tensor, \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \), are expressed in terms of the vector potential \( A_\mu \). The fermion field contains two spinor components, \( \psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \), and
$M$ is the fermion mass. With the definition (3) of the coordinates and the $\gamma$-matrices,
\[
\gamma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix},
\]
we obtain for the Lagrangian density
\[
\mathcal{L}(y) = \frac{1}{2} F_{01}^2(y) + i \sqrt{2} \psi^\dagger D_0 \psi^+ + \frac{1}{2} \sqrt{2} i \eta^2 \psi^\dagger D_0 \psi^- \\
+ i \sqrt{2} \psi^\dagger D_1 \psi^- - i M (\psi^\dagger \psi^+ - \psi^\dagger \psi^-).
\]
(23)

Note that only the mass term couples the two fermion components $\psi^-$ and $\psi^+$. Let us consider the theory on a finite $y^1$ interval: $-L \leq y^1 \leq L$ and impose periodic boundary conditions on the fields $A_\mu$ and $\psi$. We fix the gauge by imposing
\[
\partial_1 A_1 = 0,
\]
(24)
i.e. the ‘Coulomb gauge’.

It has been shown earlier [15, 16] that the zero mode of $A_1$ cannot be gauged away. In the Coulomb gauge the only constraint is Gauss’s law
\[
\partial_1 F_{01} + e \sqrt{2} \psi^\dagger \psi^+ + \frac{1}{2} \sqrt{2} e \eta^2 \psi^\dagger \psi^- = 0.
\]
It can be solved with respect to the nonzero modes of $F_{01}$:
\[
\langle F_{01} \rangle = -\partial_1^{-1} (e \sqrt{2} \psi^\dagger \psi^+ + \frac{1}{2} \sqrt{2} e \eta^2 \psi^\dagger \psi^-),
\]
(25)
where the special brackets $\langle \rangle$ define the non-zero modes $\langle f \rangle = f(y^1) - \frac{1}{2L} \int_{-L}^L f(y^1)dy^1$ and $\partial_1^{-1}$ is the periodic Greens function of the differential operator $\partial_1$ (see e.g. [20]),
\[
\partial_1^{-1}(x) = \sum_{n \neq 0} \frac{1}{2i \pi n} \exp(2i \pi n \frac{x}{L}).
\]
(26)
Substituting eq. (25) into the Lagrangian and performing the Legendre transformation yields the Hamiltonian in terms of the canonical variables $\chi^+ = \eta^{1/4} \psi^+$, $\chi^- = \eta^{-1/4} \eta \psi^-$, $\Pi_1 = \int_{-L}^L dy^1 F_{01}(y^1)$ and $A_1$
\[
H = \int_{-L}^L dy^1 \left\{ \frac{\Pi_1^2}{8 L^2} + \frac{1}{2} e^2 \left( \partial_1^{-1}(\chi^\dagger \chi^+ + \chi^\dagger \chi^-) \right)^2 \\
-2i/\eta^2 \chi^\dagger D_1 \chi^- + i \frac{M}{\eta} (\chi^\dagger \chi^+ - \chi^\dagger \chi^-) \right\}.
\]
(27)
Moreover, integration of Gauss’s law gives a residual constraint, which is to be imposed on the physical states
\[
\int_{-L}^L dy^1 (\chi^\dagger \chi^+ + \chi^\dagger \chi^-)|\text{phys} > = 0.
\]
(28)
Notice that our canonical variables satisfy the following commutation relations:
\[
\{ \chi^\dagger (y^1), \chi^\pm (y^1') \}_{y^1 = y^1'} = \delta(y^1 - y^1'),
\]
\[
[A_1(y^0), \pi_1(y^0)] = i.
\]
(29)
The regularized charge densities of the right and left movers are obtained via point splitting the two densities and connecting the two centers with a string.

\[
I_\pm(y^1) = \lim_{\varepsilon \to 0} \left( \chi_\pm \left( y^1 \mp \frac{i\varepsilon}{2} \right) \chi_\pm \left( y^1 \pm \frac{i\varepsilon}{2} \right) \exp(\pm\varepsilon eA_1) - \frac{1}{2\pi\varepsilon} \right)
\]

\[
:= \lim_{\varepsilon \to 0} \left( I_\pm(y^1, \varepsilon) - \frac{1}{2\pi\varepsilon} \right).
\]

These chiral charge densities have Fourier expansions ($r = \pm$, $p_n = \pi n/L$):

\[
I_r(y) = \frac{1}{2L} \left( Q_r + \sum_{n \neq 0} \sqrt{|n|} I_{n,r}(y^0) \exp(-ip_n y^1) \right).
\]

The zero mode part of the Fourier expansion is defined by the total chiral charges

\[
Q_r = \int_{-L}^{+L} I_r(y^1) dy^1,
\]

which can be calculated via the \(\varepsilon\) prescription by inserting the \(\varepsilon\)-regularized charge density \(I_r(y^1, \varepsilon)\) into eq. (32)

\[
Q_r = \lim_{\varepsilon \to 0} (Q_r(\varepsilon) - L/\pi \varepsilon).
\]

The coefficients \(I_{n,r}\) obey commutation relations, which are a consequence of the commutation relations of the Fourier coefficients \(\chi_{n,r}\) of the fermion fields and of the regularization, eq. (30):

\[
\left\{ \chi_n(y^0), \chi_n(y^0)' \right\} = \delta_{nn'}
\]

\[
[I_{n,r}, I_{n',r}^+] = \frac{n}{|n|} \delta_{rr'} \delta_{nn'}; \quad n \neq 0.
\]

As usually \[21, 22\], we define subspaces \(|\ell>\) of the total Hilbert space, which correspond to sectors \((\ell = (\ell_+, \ell_-))\) with given edges of occupied energy levels for the right and left movers as follows:

\[
\chi_{n,r}|\ell> = \theta(\ell \ell_r - rn)|\ell>,
\]

with \(\theta(\ell) = \begin{cases} 1 & \ell > 0 \\ 0 & \ell \leq 0 \end{cases}\)

Consequently, the operators \(I_{n,+}, I_{n,-}, n > 0\), annihilate the states \(|\ell>\):

\[
I_{n,+}|\ell> = I_{n,-}^+|\ell> = 0.
\]

The charge eigenvalues in sectors \(|\ell>\) depend on the zero mode gauge field \[21, 22\]

\[
Q_r|\ell> = (r \ell_r + r \frac{eLA_1}{\pi} + 1/2)|\ell>, \quad [Q_r, \Pi_1] = \frac{rieL}{\pi}.
\]

We introduce \[22\] the variables \(\omega_r\) canonically conjugated to the \(Q_r\) such that

\[
[\omega_r, Q_{r'}] = i\delta_{rr'}.
\]

Then we can represent the fermion fields with the help of the bosonic operators \(I_r, \omega_r, A_1\):
\[ \chi_r(y^1) = \frac{1}{\sqrt{2L}} \exp \left(-i\omega_r\right) \cdot \exp \left(\frac{ir\pi}{2}(Q_+ + Q_- - 1) - \frac{r\pi y^1}{L} \left(Q_r - \frac{r e L A_1}{\pi} - \frac{1}{2}\right)\right) \cdot \exp \left(-\sum_{n>0} \sqrt{n} I_{n,r}^+ e^{i r n y^1}\right) \cdot \exp \left(\sum_{n>0} \sqrt{n} I_{n,r} e^{-i r n y^1}\right). \quad (41) \]

These operators satisfy the commutation relations eq. (29) and reproduce the regularized charge densities, eq. (29). The necessary explanations can be found in the Appendix. The operators \( I_r \) link the fermionic to the bosonic description: In the Hamiltonian of eq. (27) we recognize four terms. The first three terms can be rewritten as a free boson Hamiltonian in terms of bosonic variables \((\phi, \Pi_{\phi})\) constructed from the charge densities \(I_r\).

\[
\Pi_{\phi} = \sqrt{\pi}(I_+ - I_-), \quad \phi = -\frac{1}{m} \left(\frac{\Pi_1}{2L} - \partial^1 e(I_+ + I_-)\right), \quad m^2 = \frac{e^2}{\pi}. \quad (42)
\]

With the help of the commutation relations (36) one can verify that \(\Pi_{\phi}\) and \(\phi\) are canonically conjugate variables. The mass term of the bosonic Hamiltonian is easily calculable using the fact that the zero mode is subtracted in \(\langle e(I_+ + I_-)\rangle\)

\[
\int_L^{+L} dy^1 \frac{1}{8L^2} m^2 \phi^2 = \int_{-L}^{L} dy^1 \left[\frac{1}{8L^2} \Pi_1^2 + \frac{e^2}{2} (\partial_1^{-1}(I_+ + I_-))^2 \right]. \quad (43)
\]

The momentum term can be expressed with the help of eq. (41) in terms of the chiral charges. The space integral of the square of the zero mode free chiral charge density \(\langle I_r \rangle^2\) is related to the fermionic momentum

\[
\pi \int_{-L}^{L} dy^1 \langle I_r \rangle^2 = r \int_{-L}^{L} dy^1 \chi_r^\dagger(y^1)(iD_1)\chi_r(y^1). \quad (44)
\]

This relation is derived in the Appendix. On the physical subspace defined by \(Q|\text{phys} > = 0, \; Q = Q_+ + Q_-\), we obtain with eqs. (12, 14):

\[
\int_{-L}^{L} dy^1 \frac{(\Pi_{\phi} - \partial_1 \phi)^2}{2\eta^2} = \frac{2}{\eta^2} \int_{-L}^{L} dy^1 \chi_r^\dagger(y^1)(-iD_1)\chi_r(y^1). \quad (46)
\]

The mass term remains as last term in the fermionic Hamiltonian of eq. (27). It is given by direct insertion of the boson representation of the fermion fields eq. (11) into eq. (27). After simplifying this expression with the help of normal ordering with respect to \(I_{n,r}^+\) and \(I_{n,r}\) (cf. Appendix) we obtain:

\[
\int_{-L}^{L} dy^1 \left[\chi_r^\dagger(y^1)\chi_r(y^1) - \chi_r^\dagger(y^1)\chi_r(y^1)\right] \quad (47)
\]

\[
= -\frac{M}{\eta L} : \sin(\omega_+ - \omega_- + \sqrt{4\pi}\langle\phi\rangle) : .
\]
In a similar way to the treatment of the scalar field theory in \((1 + 1)\) dimensions we approximate the vacuum by a trial state \(|0_a\rangle\) which is defined as

\[
a_n|0_a\rangle = 0,
\]

where \(a_n\) and \(a_n^+\) are the normal modes of the boson variables \(\phi(y^1), \Pi_\phi(y^1)\)

\[
\phi(y^1) = \frac{1}{\sqrt{2L}} \sum_{n=-\infty}^{\infty} \frac{1}{\sqrt{2E_n}} (a_n + a_n^+) e^{-ip_n y^1},
\]

\[
\Pi_\phi(y^1) = \frac{-i}{\sqrt{2L}} \sum_{n=-\infty}^{\infty} \sqrt{\frac{E_n}{2}} (a_n - a_n^+) e^{-ip_n y^1}.
\]

The weights \(E_n\) are variational parameters, which then also enter the Fourier coefficients of the chiral charges (cf. eq. (42))

\[
I_{n,r} = -ri\sqrt{\frac{4}{E_n}} |p_n| \left[ \left( E_n + rp_n \right) a_n - \left( E_n - rp_n \right) a_n^+ \right].
\]

Inserting these expressions into eq. (47) and normal ordering with respect to the trial vacuum eq. (48) we obtain on the physical subspace, eq. (45), the effective Hamiltonian

\[
H = \int_{-L}^{L} dy^1 \left[ \frac{\left( \Pi_\phi - \partial_1 \phi \right)^2}{2\eta^2} + \frac{1}{2} m^2 \phi^2 \right.
\]

\[
- \frac{M}{\eta L} \exp \left\{ \frac{\pi}{L} \sum_{n>0} \left( \frac{1}{p_n} - \frac{1}{E_n} \right) \right\} \cos(\omega + 4\pi \langle \phi \rangle) : \cos(\omega + \sqrt{4\pi} \langle \phi \rangle) :,
\]

with \(\omega = \omega_+ - \omega_- - \frac{\pi}{2}\). Note that the normal ordering symbol \(:\) means here to order the operators \(a_n, a_n^+\).

In order to fix the variational parameters we look for a minimum of the vacuum energy density using the trial vacuum state \(|0_a\rangle\). The calculation proceeds in an analogous way to the calculation of the \(\phi^4\) scalar theory. Note that the minimum corresponds to \(\omega = 0\). Using condition (49) we obtain the following expression for the vacuum energy density of the Hamiltonian, eq. (52), at \(\omega = 0\):

\[
<0_a|H(y)|0_a> = \frac{1}{4L\eta^2} \sum_{n>0} \left( E_n + \frac{p_n^2 + \eta^2 m^2}{E_n} \right) - \frac{M}{\eta L} \exp \left\{ \frac{\pi}{L} \sum_{n>0} \left( \frac{1}{p_n} - \frac{1}{E_n} \right) \right\}.
\]

At the minimum of this expression we have \((n > 0)\):

\[
E_n^2 = p_n^2 + \eta^2 m^2 + \frac{4\pi M\eta}{L} \exp \left\{ \frac{\pi}{L} \sum_{n>0} \left( \frac{1}{p_n} - \frac{1}{E_n} \right) \right\} = p_n^2 + \eta^2 \mu^2,
\]

with \(\mu^2 = m^2 + \frac{4\pi M}{\eta L} \exp \left\{ \frac{\pi}{L} \sum_{n>0} \left( \frac{1}{p_n} - \frac{1}{E_n} \right) \right\} \).

From this equation \(\mu\) is to be determined. In order to do that we rewrite the infinite sum in the exponent \((23)\) as

\[
\frac{\pi}{L} \sum_{n>0} \left( \frac{1}{p_n} - \frac{1}{E_n} \right) = -2 \sum_{k=1}^{\infty} K_0(2\pi ak) + \gamma + \ln \frac{1}{2} a + \frac{1}{2} a,
\]
where we introduced $a = \frac{\eta m}{\pi}; K_0$ is the modified Bessel function and $\gamma = 0.5772...$ (Euler’s constant). In the limit $\eta mL \gg 1, \ a \gg 1$, the sum gives $\gamma + \ln \frac{1}{2}a$ and one readily obtains
\[
\mu^2 = m^2 + \frac{4\pi M}{\eta L} \left( \frac{1}{2}ae^\gamma \right) = m^2 + 2e^\gamma M \mu, \quad (54)
\]
which gives
\[
\mu = e^\gamma (M + \sqrt{M + e^{-2\gamma}m^2}) = (M_\gamma + \sqrt{M_\gamma^2 + \frac{e^2}{\pi}}), \quad M_\gamma \equiv e^\gamma M \quad (55)
\]
This value of $\mu$ corresponds to the effective boson mass parameter in the Hamiltonian (52). Some remarks are in order. Taking the $L \to \infty$ corresponds to $\epsilon \to 0, (|k_1| \geq \epsilon)$ in the scalar field theory (eqs. (15) and (16)). For obtaining the effects of the non-trivial vacuum one needs to take these limits in the relevant equations (cf. eqs. (15), (16) and (17)). Indeed, immediately approaching the light front, $\eta \to 0$ at finite $L$ would not reproduce the boson mass, eq. (55). This can easily be seen from eq. (54) in combination with the small $a$ limit of the infinite sum:
\[
\sum_{n > 0} \left( \frac{1}{p_n} - \frac{1}{E_n} \right) = \frac{1}{2}a^2 \xi(3) + O(a^3), \quad \text{where} \quad \xi(3) = 1.201056903... \quad \text{Actually, in this limiting case $\mu$ diverges as} \quad \frac{1}{\sqrt{\eta}}.
\]
The next step is to take this Hamiltonian, eq. (52), with $E_n$ and $\mu$ fixed by eqs. (54) and (55), as the starting one for the transition to the light-cone formulation. Repeating the procedure as outlined in section 2 for the scalar field theory, we obtain the following effective light-cone Hamiltonian for the massive Schwinger model
\[
P^- = : \int_{-L}^{L} dx^- \left\{ \frac{e^2}{\pi} \varphi_L^2 + \frac{M_\gamma}{2\pi} (M_\gamma + \sqrt{M_\gamma^2 + \frac{e^2}{\pi}}) [1 - \cos \sqrt{4\pi} \varphi_L] \right\}, \quad (56)
\]
where $\varphi_L(x)$ is the analog of light-cone field, defined by the Fourier decomposition
\[
\varphi_L(x) = \sum_{n > 0} \frac{1}{\sqrt{4Lp_n}} (a_n e^{-ip_n x^-} + a_n^+ e^{ip_n x^-}). \quad (57)
\]
Note that the operators $a_n, a_n^+, n > 0$ form the light-cone Fock basis. The field $\varphi_L(x)$ can be expressed in terms of light front fermionic variables:
\[
\varphi_L(x) = -\sqrt{\pi} \partial_{-1} (\chi_+^\dagger \chi_+).
\]
This means that the expression (57) can be written on the light cone also in the fermionic basis.

It should be emphasized that the result, eq. (57), indeed yields a correction to the naive light-cone approach [27]. In the future we hope to address the interesting question how this affects the mass spectrum, in particular for small fermion mass.

The generalization of this approach to gauge theories in higher dimensions may be attempted with the help of Hamiltonians where the dependent degrees of freedom have been eliminated after gauge fixing [20, 28].

\section{Acknowledgments}

This work was supported by the exchange program between the University of Heidelberg and the University of St. Petersburg. We would like to thank Profs. F. Lenz and H.C. Pauli, as well as Th. Gasenzer for useful discussions.
A Appendix

In this appendix we give clarifications of eqs. (41), (44) and (47), based on the more general considerations of [22, 23]. Let us demonstrate that the expression in eq. (41) satisfies canonical anticommutation relations. First of all, notice that the representation (41) acts in the Hilbert space spanned by vectors of the form \( \Pi_{i,j} \{ I_{n_i+} \} \{ I_{n_j} \} | \ell \rangle \), where \( n_i > 0, n_j > 0 \), where the \( I_{n_i+} \) and \( I_{n_i-} \) act like annihilation and creation operators with respect to “vacuum” states \( | \ell \rangle \) according to eqs. (37), (38) at \( n > 0 \). Using for these operators the normal ordering symbol :: we can rewrite eq. (41) in more compact form:

\[
\chi_r(y) = \frac{1}{\sqrt{2L}} \exp(-i\omega_r) \exp \left\{ i\pi \left( \frac{1}{2}Q_r \right) \right\} : \exp\{-r2i\partial_1^{-1}(I_r)y\} :,
\]

(A.1)

where we denote by \( Q_r \) and \( Q \) the integer valued parts of the charges \( Q_r \) and \( Q \) (\( Q_r = Q_r - \frac{rELA_1}{\pi} - \frac{1}{2} \)).

Let us consider the products \( \chi_r(y')\chi_r(y') \) and \( \chi_r(y')\chi_r(y') \) as a function of \( z' = \exp(ri\pi y'L^{-1}), z'' = \exp(r'i\pi y'L^{-1}) \), taking the operator products in normal ordered form. We get

\[
\chi_r(y')\chi_r(y') = F_{rr'}(z, z') \exp \left\{ i\pi \left( \frac{1}{2}(r - r') \right) \right\} \left( z' \right) Q_{r'+1}(z') - Q_r - \delta_{r,r'}
\]

\[
\exp \left\{ \sum_{n > 0} \frac{1}{n} \left( \frac{z'}{z} \right)^n \right\} ,
\]

(A.2)

and

\[
\chi_r(y')\chi_r(y') = F_{rr'}(z, z') Q_{r'+1}(z') - Q_r - \delta_{r,r'}
\]

\[
\exp \left\{ \sum_{n > 0} \frac{1}{n} \left( \frac{z}{z'} \right)^n \right\} ,
\]

(A.3)

with the \( F_{rr'}(z, z') = \frac{1}{2L} e^{-i(\omega_r - \omega_r')} e^{\frac{i\pi}{2}(r - r')Q} : \exp\{2\pi i(r'\partial_1^{-1}(I_r)y' - r\partial_1^{-1}(I_r)y')\} : .
\]

Notice that \( F_{rr}(z, z) = \frac{1}{2L} \).

We see that for \( r \neq r' \) the expressions (A.2) and (A.3) differ only by a sign (due to \( \exp(i\pi/2)(r - r') = -1 \)). Hence, \( \{ \chi_r(y'), \chi_r(y') \} = 0 \). For \( r = r' \), we use the analytical regularization of the type used in [22]. This yields

\[
\chi_r(y')\chi_r(y') = \lim_{\epsilon \to 0} \frac{1}{2\pi i} \oint_{|z''| = 1 - \epsilon} \frac{dz''}{z''} \sum_{n} \left( \frac{z''}{z'} \right)^n \left( \frac{z'}{z} \right)^{Q_r + 1} \times
\]

\[
\left( 1 - \frac{z''}{z} \right)^{-1} F_{rr}(z, z''),
\]

(A.4)

and

\[
\chi_r(y')\chi_r(y') = \lim_{\epsilon \to 0} \frac{1}{2\pi i} \oint_{|z''| = 1 + \epsilon} \frac{dz''}{z''} \sum_{n} \left( \frac{z''}{z'} \right)^n \left( \frac{z'}{z} \right)^{Q_r} \times
\]

\[
\left( 1 - \frac{z}{z''} \right)^{-1} F_{rr}(z, z'').
\]

(A.5)
Adding (A.4) and (A.5), we get

\[ \left\{ \chi_r(y^1), \chi_r^\dagger(y^1) \right\} = \lim_{\varepsilon \to 0} \frac{1}{2\pi i} \left( \oint_{|z''|=1+\varepsilon} - \oint_{|z''|=1-\varepsilon} \right) \frac{dz''}{z'' - z} \times \]

\[ \sum_n \left( \frac{z''}{z'} \right)^n \left( \frac{z''}{z} \right)^{Q_r} F_{rr}(z, z'') = \]

\[ \frac{1}{2L} \sum_n \exp \left( \frac{r\pi n}{L} (y^1 - y^1') \right) = \delta(y^1 - y^1'). \quad (A.6) \]

Analogously, one obtains \( \left\{ \chi_r(y^1), \chi_r(y^1) \right\} = 0. \)

To explain eq. (44) let us consider the \( \varepsilon \)-regularized charge densities, eq. (30), using eq. (A.3) of the Appendix with the substitutions: \( y^1 \to y^1 + \frac{ri\varepsilon}{2} \), \( y^1' \to y^1 - \frac{ri\varepsilon}{2} \), and expanding in \( \varepsilon \) up to \( O(\varepsilon^2) \). We get

\[ I_r(y^1, \varepsilon) = \chi_r^+ \left( y^1 - \frac{ri\varepsilon}{2} \right) \chi_r \left( y^1 + \frac{ri\varepsilon}{2} \right) \exp(r\varepsilon eA_1) \]

\[ = \frac{1}{2\pi \varepsilon} + I_r(y^1) + \pi\varepsilon(I_r(y^1))^2 - \frac{\pi\varepsilon}{48L^2} + O(\varepsilon^2), \quad (A.7) \]

in agreement with eq. (30). Differentiating eq. (A.7) with respect to \( \varepsilon \), we obtain

\[ \int_{-L}^{L} dy^1 \chi_r^+ \left( y^1 - \frac{ri\varepsilon}{2} \right) iD_1 \chi_r \left( y^1 + \frac{ri\varepsilon}{2} \right) \]

\[ = -r \left( \frac{L}{\pi \varepsilon^2} + \frac{\pi}{12} \right) + r\pi \int_{-L}^{L} dy^1 (I_r(y^1))^2 + O(\varepsilon), \quad (A.8) \]

that coincides with eq. (44) after subtracting the constant and taking the limit \( \varepsilon \to 0. \)

Eq. (47) is a direct consequence of eq. (A.3) and eq. (41) if it is considered on the physical subspace (\( Q = 0 \)). Indeed, from eq. (A.3) we get

\[ -\frac{iM}{\eta} (\chi_r^+\chi_r - \chi_r^\dagger\chi_r) = \frac{iM}{2L\eta} (-1)^Q [e^{i(\omega_+ - \omega_-)} e^{2y_1^+ L} Q e^{2\pi i(\partial_t^{-1}(I_++I_-))} \]

\[ - e^{-i(\omega_+ - \omega_-)} e^{-i\frac{2y_1^+}{L} Q e^{-2\pi i(\partial_t^{-1}(I_++I_-))}}] \]

which indeed coincides with eq. (47) at \( Q = 0. \)

References

[1] P.A.M. Dirac, Rev. Mod. Phys. 21, 392 (1949).
[2] J.R. Klauder, H. Leutwyler and L. Streit, Nuovo Cim. 66A, 536 (1970).
[3] A. Casher, Phys. Rev. D14, 452 (1976).
[4] V.A. Franke, Yu.V. Novozhilov and E.V. Prokhwatilov, Lett. Math. Phys. 5, 239, 437 (1981).
[5] R.J. Perry, A. Harindranath and K.G. Wilson, Phys. Rev. Lett. 65, 2959 (1990).
[6] A.M. Annenkova, E.V. Prokhvatilov and V.A. Franke, Phys. At. Nucl. 56 (6), 813 (1993).

[7] A.M. Annenkova, E.V. Prokhvatilov and V.A. Franke, Vestn. Leningr. Univ., Ser. 4, No. 4, 80 (1985) [in Russian].

[8] M. Burkardt, Phys. Rev D47, 4628 (1993).

[9] A. Harindranath and J.P. Vary, Phys. Rev. D36, 1141 (1987).

[10] T. Heinzl, S. Krusche, S. Simburger and E. Werner, Z. Phys.C 56, 415, (1992); B. van de Sande and S. Pinsky, Phys. Rev. D49, 2001 (1994).

[11] A.B. Bylev, E.V. Prokhvatilov and V.A. Franke, Vestn. Leningr. Univ., Ser. 4, No. 11, 8 (1986) [in Russian].

[12] T. Eller, H.-C. Pauli and S.J. Brodsky, Phys. Rev. D35, 1493 (1987).

[13] K. Hornbostel, S.J. Brodsky and H.-C. Pauli, Phys. Rev. D41, 3814 (1990).

[14] A. Tang, H.-C. Pauli and S.J. Brodsky, Phys. Rev. D44, 1842 (1991).

[15] E.V. Prokhvatilov and V.A. Franke, Sov. J. Nucl. Phys. 49, 688 (1989).

[16] F. Lenz, M. Thies, S. Levit and K. Yazaki, Ann. Phys. 208, 1 (1991).

[17] E.V. Prokhvatilov and V.A. Franke, Sov. J. Nucl. Phys. 47, 559 (1988).

[18] P.M. Stevenson, Phys. Rev. D30, 1712 (1984); Phys. Rev. D32, 1389 (1985).

[19] L. Polley and V. Ritschel, Phys. Lett. B221, 44 (1989); Phys. Rev. D45, No. 8 (1992).

[20] F. Lenz, H.W.L. Naus, K. Ohta and M. Thies, Quantum Mechanics of Gauge Fixing, to be published in Ann. Phys. (N.Y.).

[21] N.S. Manton, Ann. Phys. 159, 220 (1985).

[22] E.V. Prokhvatilov, Theor. Math. Phys. 88, No. 1 (1991) (Transl. from Russian).

[23] A. Bylev, E.V. Prokhvatilov and V.A. Franke, Vestn. Leningr. Univ., Ser. 4, No. 11, 66 (1989) [in Russian].

[24] S. Iso and H. Murayama, Prog. Theor. Phys. 84, 142 (1990).

[25] I.S. Gradshteyn and I.M. Ryzhik, Tables of Integrals, Series, and Products, Academic Press, New York and London, 1965.

[26] I.B. Frenkel and V.N. Kac, Inv. Math. 62, 23 (1980).

[27] S. Coleman, Ann. Phys. (N.Y.) 101, 239 (1976).

[28] F. Lenz, H.W.L. Naus and M. Thies, QCD in the Axial Gauge Representation, to be published in Ann. Phys. (N.Y.).