MIRROR MODEL FOR STERILE NEUTRINOS

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ABSTRACT

Sterile neutrinos are studied as subdominant contribution to solar neutrino physics. The mirror-matter neutrinos are considered as sterile neutrinos. We use the symmetric mirror model with gravitational communication between mirror and visible sectors. This communication term provides mixing between visible and mirror neutrinos with the basic scale $\mu = v_{\text{EW}}^2/M_{\text{Pl}} = 5 \times 10^{-6}$ eV, where $v_{\text{EW}} = 174$ GeV is the vacuum expectation value of the standard electroweak group and $M_{\text{Pl}}$ is the Planckian mass. It is demonstrated that each mass eigenstate of active neutrinos splits into two states separated by small $\Delta m^2$. Unsuppressed oscillations between active and sterile neutrinos ($\nu_a \leftrightarrow \nu_s$) occur only in transitions between each of these close pairs (“windows”). These oscillations are characterized by very small $\Delta m^2$ and can suppress the flux and distort spectrum of $pp$-neutrinos in detectable way. The other observable effect is anomalous seasonal variation of neutrino flux, which appears in LMA solution. The considered subdominant neutrino oscillations $\nu_a \leftrightarrow \nu_s$ can reveal itself as big effects in observations of supernova neutrinos and high energy (HE) neutrinos. In the case of HE neutrinos they can provide a very large diffuse flux of active neutrinos unconstrained by the e-m cascade upper limit.

1 Introduction

1.1 Why light sterile neutrinos?

A sterile neutrino, $\nu_s$, is a neutral, spin 1/2 particle which is a singlet under SU(3)$\times$SU(2)$\times$U(1) (SM model) group. In supersymmetry, grand unified theories (GUT) and superstring models such singlets appear naturally. It is more difficult however to have them light. Sterile neutrino always interacts, sometimes very weakly, with ordinary neutrinos. This interaction naturally results in small neutrino mixing. It is more difficult to build a model with large mixing between sterile and ordinary neutrinos.

However, there are many models of light sterile neutrinos, which effectively mix with the ordinary ones. It is natural to consider the models in which ordinary and sterile neutrinos mix, even if this effect is weak. In our view, an investigation of sterile neutrinos should not necessarily start from this or that observational motivation; it should rather select a well definite theoretical framework and study its implications, with the hope that some of them are observable. This is the strategy of our work.
It must be said that at present there are not many indications for effective oscillations between sterile and active neutrinos. Two flavor oscillation $\nu_e \to \nu_s$ is excluded as solution to the solar neutrino problem, especially after SNO data \cite{sno}. Oscillation $\nu_\mu \to \nu_s$ is disfavored as solution of atmospheric neutrino anomaly \cite{atm}. In recent works \cite{recent1, recent2, recent3, recent4}, stringent upper limits on the contribution of sterile neutrinos to the solar or atmospheric neutrino experiments are obtained. An indirect hint for a sterile neutrino comes only from interpretation of the above-mentioned data when combined with those of LSND \cite{lsnd} (for the difficulties in such interpretation see \cite{difficulties1}, and \cite{difficulties2} for further discussion).

However, a small mixing of sterile neutrinos with active ones and/or very small $\Delta m^2$ remain a viable possibility. This possibility can reveal itself in subdominant processes, or in some new phenomena such as neutrino oscillations at very large distances: neutrinos from supernovae and high energy (HE) neutrinos from the mirror matter.

The models for sterile neutrinos include those based on supersymmetric theory, with or without R-parity violation e.g. \cite{supersymmetry1, supersymmetry2}, on the GUT and string models, e.g. component of 27-plet in E$_6$ model \cite{gut1} or SM singlet with additional U(1) charge \cite{gut2}, on the hidden sector, modulinos in supergravity \cite{supergravity1}, and many other models.

We believe that any theoretical model for sterile neutrinos should explain why the mass of a sterile neutrino is close to that of the ordinary neutrinos. Actually, this is the reason why we select the mirror models \cite{mirror1, mirror2, mirror3} for sterile neutrinos: neutrino masses arise only from operators with dimensions 5 or larger, and therefore are suppressed by inverse power of large masses, exactly as for ordinary neutrinos.

In this paper we discuss the origin of the mixing of ordinary and mirror neutrinos (‘communication’ term) and estimate its impact on oscillations. The most important effect is the splitting of the unperturbed mass eigenvalues of active neutrinos to two close-set states (see Fig. \ref{fig:split}). The transition between these states results in the oscillation to sterile neutrinos with long wavelength. We consider the applications of our model to solar, supernova and high energy neutrinos.

### 1.2 Models with mirror matter

Mirror matter was first suggested by Lee and Yang \cite{mirror} in 1956, who proposed that the transformation in the particle space which corresponds to space inversion $\vec{x} \to -\vec{x}$ should not be the usual parity transformation $P$, but $P \times R$, where $R$ transforms a particle (Lee and Yang considered the proton) into a reflected state in the mirror particle space. This concept was further developed by Salam \cite{salam}, but in fact this idea was clearly formulated only later, in 1966, by Kobzarev, Okun and Pomeranchuk \cite{kobzarev}. In this work it has been proposed that mirror and ordinary matter may communicate only gravitationally, and that the objects from mirror matter (stars and planets) can be present in the universe. Okun \cite{okun} considered also the communication due to new very weak long-range forces and discussed this interaction for celestial bodies from mirror matter. Since that time mirror matter
Figure 1: The double degeneracy between neutrino mass eigenstates of ordinary and mirror world ($\nu_i$ and $\nu_i'$, respectively, with $i = 1, 2, 3$) is lifted when the small mixing (communication) terms are included. The new mass eigenstates, denoted as $\nu_i^+$ and $\nu_i^-$, are maximal superpositions of $\nu_i$ and $\nu_i'$: $\nu_i^+ = (\nu_i + \nu_i')/\sqrt{2}$ and $\nu_i^- = (\nu_i' - \nu_i)/\sqrt{2}$.

has found interesting phenomenological applications and development [23, 24]. It has been boosted in 1980s by superstring theories with $E_8 \times E_8'$ symmetry. The particle content and symmetry of interactions in each of the $E_8$ groups are identical, and thus the mirror world has naturally emerged.

One can describe the motivation for the mirror sector in the following way.

The Hilbert space of the particles is assumed to be a representation of the extended Poincaré group, i.e. one which includes the space coordinate inversion $\vec{x} \rightarrow -\vec{x}$. Since the coordinate operations, inversion and time shift, commute, the corresponding operations in the particle space, $I_r$ and Hamiltonian $H$, must commute, too. It implies that parity defined as eigenvalue of operator $I_r$ must be integral of motion for a closed system. For this it can be suggested that $I_r = P \times R$, where $P$ is the usual operator of reflection and $R$ is the operator which transforms an ordinary particle to the mirror particle. Thus parity is conserved in the total Hilbert space of ordinary and mirror particles. The assumption of Landau [25] was $R = C$, i.e. we can say that he suggested to use antiparticles as the mirror space, but then $CP$ must be conserved which we know is not the case today.

Mirror neutrinos as sterile neutrinos and various applications of mirror matter have been intensively studied during the past several years in the context of explanation of atmospheric and solar neutrino problems [17, 15, 29], cosmological problems, including inflation and nucleosynthesis [20, 27, 28, 29, 18], dark matter and galaxy formation [20, 21, 30, 31], extra dimensions [32] and high energy neutrinos [18].

Mirror matter scenarios have two basic versions. The symmetric version was suggested in
the early works and more recently advocated in [33, 15]. The Lagrangian which describes the particles and their interactions in the visible and mirror sectors, $\mathcal{L}_{vis}$ and $\mathcal{L}_{mirr}$, are perfectly symmetric and transforms into each other when $\vec{x} \rightarrow -\vec{x}$, accompanying by all left states transforming into right and vice versa: $\Psi_L \rightarrow \Psi'_R$ and $\Psi_R \rightarrow \Psi'_L$, where primes denote the mirror states. The vacuum expectation values (VEV’s) of the Higgs fields are also identical in both sectors. Parity is conserved in the enlarged space of ordinary and mirror states. The masses and mixings of neutrinos in the symmetric mirror model is studied by Foot and Volkas in [15]. The two sectors (ordinary and mirror) communicate through the Higgs potential and mixing of neutrinos. This is a phenomenological description, in contrast to dynamical description of Refs. [16, 17], where the two sectors interact gravitationally and neutrino mixing follows from this interaction. Neutrino masses and mixings in each sector are induced by the usual see-saw mechanism, and mixing of neutrinos of different sectors are postulated as e.g. $m_{\nu_L} \nu'_{L}$, where mirrors neutrinos are denoted by primes. As demonstrated in [15] the most general mixing terms compatible with parity conservation results in maximal mixing of ordinary and sterile neutrinos.

The asymmetric version was suggested by Berezhiani and Mohapatra [17]. They assume that while all coupling constants in the two sectors are identical, the VEV’s are different and break the parity. The ratio $\zeta = v'/v$ of electroweak VEV’s ($\langle H \rangle = v$ and $\langle H' \rangle = v'$) gives thus the scaling factor for ratios of masses in the ordinary and mirror worlds, such as masses of gauge bosons, leptons and quarks. The basic communication between the two worlds is gravitational. It is taken in the form of universal dimension 5 operators, suppressed by the Planckian mass $M_{Pl}$. Operating inside each world and between them, these terms give neutrino masses and mixings. However, to describe the desired neutrino masses, the authors assume also additional communication through the singlet superheavy fields, which results in the similar dimension 5 operators suppressed by superheavy mass $\Lambda < M_{Pl}$.

A similar model–with asymmetric hidden sector–was studied in Ref. [18]. The communication of the two sectors is described by a dimension 5 operator with superheavy mass $\Lambda$ in the denominator. The neutrinos in this model are found to be maximally mixed and mass degenerate. The neutrino masses and mixings are obtained with help of dimension 5 operators with one scale $\Lambda$, with two different electroweak VEV’s, $v$ and $v'$, in the visible and mirror sectors, respectively, and using VEV’s of two SU(2) singlets $\langle \Phi \rangle = V$ and $\langle \Phi' \rangle = V'$.

### 1.3 The nucleosynthesis constraints

There are several dangers to be watched for in the models with mirror matter. They are connected with cosmological (big-bang) nucleosynthesis.

In mirror models the number of massless and light particles is doubled, and this case is excluded by cosmological nucleosynthesis if the temperatures of mirror and ordinary
matter is the same. A natural solution might be given by making the temperature of mirror
matter, \( T' \), lower than that of visible matter \( T \). It is rather difficult to implement this in
the symmetric models [15]. In asymmetric models [17] a natural solution is given by the
different couplings of the inflaton to the visible and mirror particles [26]. The inflaton decays
with different rates to visible and mirror matter, producing thus the different temperatures
of the two sectors. In the symmetric models the different temperatures can be obtained in
a two-inflaton model [18]. In this model there are two inflatons, \( \varphi \) and \( \varphi' \), with identical
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a two-inflaton model [18]. In this model there are two inflatons, \( \varphi \) and \( \varphi' \), with identical
couplings to the visible and mirror matter, respectively. The roll of the inflatons towards the
minimum of the potential is not synchronized, and the particles produced by the inflaton
which reaches the minimum earlier will be diluted by the inflation driven by the second
inflaton. By definition, the first inflaton is the mirror one.

Having the temperature \( T' < 0.5T \), does not solve the problem of cosmological nucleo-
synthesis completely. The number of additional effective neutrino flavors is limited by
cosmological nucleosynthesis as \( \Delta N_\nu < 0.2 - 0.3 \). Even if the initial density of mirror neu-
trinos is strongly suppressed, they might reappear again with the equilibrium density due
to oscillation of the visible neutrinos to sterile neutrinos; for a review and references see [34]. The oscillations might bring the sterile neutrinos in equilibrium with the active ones.
(Indeed, while \( \nu_e \)'s oscillate into \( \nu_s \)'s, the missing \( \nu_a \)'s are replenished again by thermal pro-
duction.) In case of small mixing angles, when \( \nu_a \) and \( \nu_s \) can be approximately considered
as mass eigenstates with masses \( m_a \) and \( m_s \), the non-resonant oscillation \( \nu_a \to \nu_s \) occur
when \( \Delta m^2 = m_s^2 - m_a^2 > 0 \). In this case the limit on \( \Delta m^2 \) allowed by nucleosynthesis is
given by [34]:

\[
\Delta m^2 \sin^4 2\theta \leq \xi \cdot 10^{-5}(\Delta N_\nu)^2 \ eV^2,
\]

where \( \theta \) is the vacuum mixing angle for \( \nu_a - \nu_s \) mixing, and \( \xi = 3.16 \) for \( \nu_a = \nu_e \), and
\( \xi = 1.74 \) for \( \nu_a = \nu_\mu/\tau \). Using \( \Delta N_\nu < 0.2 - 0.3 \) one obtains from Eq. (1) the upper limit on
\( \Delta m^2 \) for given mixing angle \( \theta \).

For large (or maximal) mixing, the upper bound on \( \nu_a \) (where \( a \neq e \)) oscillations con-
tinues to roughly satisfy the scaling law previously given, whereas the bound on \( \Delta m^2_{\nu_a\nu'} \) becomes much more stringent. In a number of calculations [38, 40, 41] bounds were obtained
in the range

\[
\Delta m^2_{\nu_a\nu'} \leq 10^{-8} - 10^{-9} \ eV^2,
\]

where the effect of \( \nu_a \to \nu_e \) oscillations are neglected.

The model for neutrino masses presented in this paper has oscillations into sterile neu-
trinos with small \( \Delta m^2 \), that satisfy the bounds cited above (see Section 2.2). However, we
note that, \( a \ priori \), these bounds are not unavoidable. As was first remarked in Ref. [35],
these limits become much weaker in presence of large lepton asymmetry \( L = (n_\nu - n_\bar{\nu})/n_\gamma \).
For example, in case of \( \nu_e \to \nu_s \) oscillation the limit (1) is replaced by

\[
\Delta m^2/eV^2 < 4 \cdot 10^2 |L_e|.
\]

5
The suppression of $\nu_a \to \nu_s$ oscillation is due to the matter effects, which appear because the neutrino potential depends on lepton asymmetry. The scale for large and small lepton asymmetry is given by the baryon asymmetry $B \sim 10^{-10}$. The lepton asymmetry needed for the above-mentioned effect must be larger than $\sim 10^{-7}$ [36]. An application for atmospheric and solar neutrino oscillations with $L \sim 10^{-5}$ is considered in [37, 42]. The lepton asymmetry can be generated by some unspecified mechanism, but to be larger than $B \sim 10^{-10}$, the lepton asymmetry must be generated after electroweak phase transition, otherwise it would be reprocessed by the sphaleron mechanism into too large a baryon asymmetry. A mechanism of generation of the lepton asymmetry is neutrino oscillation itself (see [34] for the status and references). Lepton asymmetry is generated only in case of small neutrino mixing. At maximal mixing, for example, probability of $\nu_s \to \nu_a$ oscillation is large, and scattering of $\nu_a$ provides thus the equilibrium of sterile and active neutrinos. Foot [42] suggested the following model where suppression of oscillation is provided by self-generating lepton asymmetry. There are four neutrinos $\nu_\tau, \nu_\mu, \nu_e$ and $\nu_s$, among them $\nu_\tau$ and $\nu_s$ have small mixing, while the other neutrinos are allowed to have the large mixing. Large lepton asymmetry $L$ is produced by $\nu_\tau \to \nu_s$ oscillation, and $L$ suppresses the oscillations of the other neutrinos. (The oscillation $\nu_a \to \nu'$, where $\nu'$ is the mirror neutrino, is suppressed more strongly than $\nu_a \to \nu_s$, because of self-interaction of $\nu'$ [37].) Thus, even in the case of large $\Delta m^2$, the $\nu_a \to \nu_s$ oscillation and the resulting nucleosynthesis restrictions are suppressed in presence of large lepton asymmetry, either existing or self-produced.

2 The model

2.1 Origin of the mass terms

Our model belongs to the more general framework of the symmetric models of Foot and Volkas [15]. Differently from these authors, we discuss the origin of the mass terms, and focus on the models with gravitational communication terms between ordinary and mirror neutrinos.

2.1.1 Gravitational communication terms

As in the first classical works [19]-[23] on mirror matter we shall assume that that mirror particles communicate with the visible ones only gravitationally. For the description of this interaction we shall use dimension 5 operators [43, 44]. For neutrinos these communication term, obtained from the $SU(2)_L \times U(1) \times SU(2)'_R \times U(1)'$ scalar, reads:

\[ L_{\text{comm}} = \frac{\lambda_{\alpha\beta}}{M_{\text{Pl}}} (\nu_{\alpha L} \phi)(\nu_{\beta R}' \phi'), \tag{4} \]
where $M_{\text{Pl}} = 1.2 \times 10^{19}$ GeV is the Planckian mass, and $\alpha, \beta = e, \mu, \tau$, and $\phi, \phi'$ are the neutral component of the electroweak Higgses from visible and mirror sectors, respectively.\footnote{Here and everywhere below we use the Greek letters $\alpha, \beta, ...$ for flavor states, and the Latin letters $i, j, k, ...$ for the mass states.}

After spontaneous electroweak symmetry breaking the Lagrangian (4) generates the terms, which mix visible and sterile neutrinos,

$$L_{\text{mix}} = \lambda_{\alpha\beta} \frac{v^2}{M_{\text{Pl}}} \nu_\alpha \nu'_\beta,$$

where $v = 174$ GeV is VEV of the Higgses, which is assumed to be the same for $SU(2)$ and $SU(2)'$ groups.

We assume that the coefficients $\lambda_{\alpha\beta}$ are of the order of unity, and this is a natural assumption, once we consider non-perturbative gravitational interaction as origin of the term (4). In this case we have in our model basically only one mass parameter

$$\mu = \frac{v^2}{M_{\text{Pl}}} = 5.0 \times 10^{-6} \text{ eV.}$$

However, it is important to remark that in some models this parameter might be slightly or essentially different:

1) The non-perturbative gravitational mechanism responsible for the communication term (4) could have explicit suppression factors, flavor independent or perhaps dependent. For instance, in the wormhole model there is an exponential suppression factor $\exp(-S)$, where $S$ is the action, which can be approximately expressed through the wormhole throat radius $R$, as $S \sim M_{\text{Pl}}^2 R^2$. This radius is inversely proportional to $M_{\text{Pl}}$ with unknown proportionality coefficient. The suppression can reach many orders of magnitude \cite{45}.

2) Eq. (4) can contain some numerical factors such as $1/(4\pi)^2$, Clebsch-Gordan coefficients, etc.

3) Also, one has to recall that certain parameters of the standard model, such as gauge couplings and top Yukawa coupling, are indeed of the order of unity, but many other parameters are much smaller; e.g., $V_{ub} \approx 3 \times 10^{-3}, m_s/v = 7 \times 10^{-4},$ and $m_e/v = 3 \times 10^{-6}$. This suggests that some flavor selection rule, or other mechanism, provides explicit suppression of Yukawa couplings. A similar (or the same) mechanism could well be acting on the couplings $\lambda$, and result in their suppression or smallness (e.g. note that in the celebrated seesaw model couplings similar to $\lambda$ in Eq. (4) are related to the neutrino Yukawa couplings, that are likely to be smaller than unity).

The most general neutrino mass matrix in the flavor representation can be written as

$$L_{\nu \text{ mass}} = -\frac{1}{2} \langle \nu, \nu' \rangle \left( \begin{array}{cc} M & m \\ m' & M' \end{array} \right) \left( \begin{array}{c} \nu \\ \nu' \end{array} \right) + h.c.$$
where $3 \times 3$ matrix $m$ is given by $\mathcal{L}_{\text{mix}}$ from Eq. (1), and $3 \times 3$ matrixes $M$ and $M'$ are either complex conjugate or identical due to assumed mirror symmetry (see Appendix).

We shall assume that the content of matrix $M$ in the visible sector is determined by interactions inside this sector, e.g. by the see-saw mechanism. When $m = 0$ the mass matrix $M$ in the mass eigenstates basis is $M = \text{diag}(M_1, M_2, M_3)$, where $M_i$ are masses generated by the see-saw mechanism. The numerical values of these masses are outside the scope of our work.

All elements of matrix $m$ are approximately equal and are mainly controlled by the fundamental scale of our model $\mu$. Due to $M' = M$ (or $M' = M^*$) the diagonalization of mass matrix (7) results in maximal mixing of $\nu_\alpha$ and $\nu'_\beta$.

Before proceeding to detailed calculations let us consider an illustrative example of two neutrinos $\nu$ and $\nu'$. The $2 \times 2$ mass matrix in this case is given by

$$M = \begin{pmatrix} M_i & \mu \\ \mu & M_i \end{pmatrix},$$

(8)

where we assume $M_i \gg 4\mu$. When the interaction between the two sectors is switched off, $\mu = 0$ and the neutrinos are mass degenerate. With $\mu$ taken into account, the mixing is maximal $\sin 2\theta = 1$ and the mass eigenvalues split to $m_{1,2} = M_i \pm \mu$, so that $\Delta m^2 = 4M_i\mu$ (more precisely, $\Delta m^2 = 4\text{Re}(M_i m^*)$, since neutrino oscillations depend on the product of neutrino mass matrix and its hermitian conjugate). The transition between the split levels results in $\nu_\alpha \rightarrow \nu_s$ oscillation with small $\Delta m^2$.

This feature survives in the three neutrino case, when each mass eigenvalue, $M_i$, splits into two close ones (see Fig. 1). This provides additional $\nu_\alpha \rightarrow \nu_s$ oscillation with small $\Delta m^2$ between the split levels. We shall prove that unsuppressed oscillations of active to sterile neutrinos exist only between these split states of one level (“window”) with the small $\Delta m^2$, while the short wave oscillations (large $\Delta M^2$) are suppressed.

### 2.2 Mass spectrum and oscillations

In case of $m = 0$ we have the active neutrino oscillations described by the matrix $M$. We introduce its decomposition into mass eigenstate matrix $M_i$ as:

$$M = U^* \text{diag}(M_i) U^\dagger$$

(9)

The precise numerical values of masses $M_i$ and of the mixing angles have to be taken from the “standard” models of active neutrino oscillations and thus they are outside the scope of our work.\(^2\) However, for two masses ($M_3$ and $M_2$) there are the lower bounds from the

\(^2\)We will consider the case of “normal hierarchy”, which—beside being consistent with all we know—arises most commonly in grand unified theories, models with flavor selection rules, etc. See, e.g., [46].
data on atmospheric and solar neutrinos,

\[
M_3 \geq (\Delta M^2_{\text{atm}})^{1/2} \approx 5 \times 10^{-2} \text{ eV}
\]

\[
M_2 \geq (\Delta M^2_{\text{sol}})^{1/2} \approx 7 \times 10^{-3} \text{ eV}
\]

while the lightest mass \( M_1 \) remains unconstrained, and its value can be (very) small (e.g. \( M_1 \ll M_2 \)). Also, we are allowed to borrow the mixing angles suggested by data, without dwelling on questions of how to justify their precise values. In the three neutrino case, it is convenient to rotate away the phases in the masses \( M_i \), since they do not affect the oscillations. We will see below that these phases play a more interesting role in our 6 neutrino case. Note, that with \( M_i \) given above \( \Delta m^2 \) calculated for oscillations \( \nu_\alpha \leftrightarrow \nu' \) \((\alpha = e, \mu, \tau)\) in all the three windows of Fig. 1 respect the upper limits (1) and (2).

### 2.2.1 Oscillations into mirror neutrinos

Let us come back again to Eq. (7), where the matrices \( M \) and \( m \) are written in the flavor representation. But before, we introduce a notation for the expression of the communication term in the special basis where the matrices \( M = M' \) are diagonal:

\[
\tilde{m} = U^t(M)U
\]

The general 6×6 neutrino mass matrix can be brought in exact diagonal form using two unitary matrices, defined by:

\[
U^t_\pm (M \pm m)U_\pm = \text{diag}(M_{\pm i})
\]

with \( i = 1, 2, 3 \) the indices for mass eigenstates. The expression we get for the mixing matrix, that relates the ordinary and mirror neutrinos to the 6 mass eigenstates, \( \nu^\pm_i \), is the following one:

\[
\begin{align*}
\nu &= \frac{1}{\sqrt{2}}(U_+\nu^+ - U_-\nu^-) \\
\nu' &= \frac{1}{\sqrt{2}}(U_+\nu^+ + U_-\nu^-)
\end{align*}
\]

This is the master equation, that now we discuss and analyze. If \( m = 0 \), we get two equalities: (1) \( U_\pm = U \) (namely, the two unitary matrices are equal) and (2) \( M_{\pm i} = M_i \). In this case, ordinary and mirror neutrinos are pairwise degenerate but do not oscillate into each other as can be verified from Eq. (13). Instead, when \( m \neq 0 \), there are deviations from both these two equalities that, in turn, produce different types of oscillations. Their origin can be traced back to different terms of the matrix \( \tilde{m} \) of Eq. (11):

(1) The off-diagonal terms of \( \tilde{m} \) lead at first order to \( U_+ \neq U_- \) and to no-splitting between
Using Eq. (13), we realize that there are oscillations into mirror neutrinos connected with the splittings $M_i^2 - M_j^2$, i.e. with short wavelength:

$$P_{\text{short}}(\nu_\alpha \rightarrow \text{mirror}) = \frac{1}{4} \sum_{\alpha'} \left| \sum_i (U_{+\alpha i} U^{+\alpha'i} - U_{-\alpha i} U^{-\alpha'i}) \exp \left( -\frac{i M_{\pm i} L}{2E} \right) \right|^2$$

(14)

(the sum over $\alpha'$ accounts for disappearance into any of the mirror neutrinos). In this expression we have differences between $U_+$ and $U_-$, therefore the amplitude is linearly small in the parameters of $\tilde{m}$, and this means that the oscillation probability is doubly suppressed. For this reason, these effects are in practice negligible.

(2) The diagonal terms of $\tilde{m}$ remove the double degeneracy of $M_i$-values:

$$M_{\pm i} = M_i \pm \tilde{m}_{ii}$$

(15)

This leads to oscillations into mirror states with long wavelengths, associated with the scale

$$\Delta m_i^2 = 4 \text{Re}(M_i \tilde{m}_{ii}^*)$$

(16)

namely, using again Eq. (13):

$$P_{\text{long}}(\nu_\alpha \rightarrow \text{mirror}) = \sum_i |U_{\alpha i}|^2 \sin^2 \left( \frac{\Delta m_i^2 L}{4E} \right)$$

(17)

It should be noted that, when $L/E$ becomes sufficiently large, this expression averages to $1/2$: the effect is large.

Note that, using the approximation $U_+ \approx U_- \approx U$, which has been motivated above, and plugging in formula (13) the equation $\nu_\alpha = U_{\alpha i} \nu_i$, one immediately obtains that the would-be mass eigenstates $\nu_i$ are actually maximal superpositions of the true mass eigenstates:

$$\nu_i = \frac{1}{\sqrt{2}}(\nu_i^+ - \nu_i^-),$$

(18)

and similarly for $\nu'_i$:

$$\nu'_i = \frac{1}{\sqrt{2}}(\nu_i^+ + \nu_i^-),$$

(19)

With these equations in mind, it is easy to summarize the pattern of oscillations: apart from the ordinary flavor oscillations (‘short wavelength’) we must also take into account that the state $\nu_i$ are not mass eigenstates, but a superposition of maximally mixed mass eigenstates, Eq. (18) (see again Fig. 1). This leads to further oscillations with ‘long wavelength’, associated with the splitting in Eq. (16).
Till now, we have presented formulae for vacuum oscillations (Eqs. 14 and 17). Occurrence of MSW effect \[47\] leads to other cases: for instance, if $\nu_e$ is adiabatically converted to $\nu_2$, after a sufficient distance, this state will further oscillate in vacuum, since $\nu_2 \approx (\nu_2^+ - \nu_2^-)/\sqrt{2}$. At sufficiently long wavelengths, this will produce a disappearance of $1/2$ of neutrinos, that is the same situation that was discussed after Eq. (17).

We can summarize this subsection with the conclusion that oscillations to sterile neutrinos which remain unsuppressed are caused by transitions between the split mass levels with small $\Delta m^2_i$ in one window shown in Fig. 1.

2.2.2 The scales of oscillation into mirror states

The mirror model we are considering has 3 new parameters in comparison with the usual ones: these are the three ‘small’ $\Delta m^2_i$ of Eq. (16). Large values of these parameters, as compared with $\Delta M^2_{\text{sol}}$, are excluded by solar neutrino data. In this respect the most dangerous parameter is $\Delta m^2_2 \propto M_2$. Indeed, due to $U_{e3} \approx 0$ the splitting $\Delta m^2_3$ is decoupled from solar neutrino oscillations, and $\Delta m^2_1 \propto M_1$ can always be made very small because of the arbitrary value of $M_1$. But $M_2 \propto \sqrt{\Delta M^2_{\text{sol}}}$ is generically large and may create the problems with the second “window” in Fig. 1. However, there exist the allowed textures of matrices $m$ ($\bar{m}$) when oscillations in the second window are suppressed, and we shall describe below two such examples.

(1) Let us consider first the very specific case with $\bar{m} = \mu \text{ diag}(1, 0, 0)$. We shall demonstrate, in fact, that this specific case arises from a class of initial textures of matrix $m$ with all elements of order one, as implied by Lagrangian \[5\] with $\lambda_{\alpha\beta} \sim 1$. We perform rotation of $\bar{m}$ to $m$ using the usual mixing matrix $U$ with $U_{e3} = \sin \phi = 0$, with the maximal atmospheric neutrino mixing angle, i.e. $\psi = 45^\circ$, and with a large solar angle $\omega$, namely

$$U = \begin{pmatrix}
\cos \omega & s_\omega & 0 \\
-\frac{s_\omega}{\sqrt{2}} & \frac{c_\omega}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{s_\omega}{\sqrt{2}} & -\frac{c_\omega}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix},$$

(20)

where $s_\omega = \sin \omega$ and $c_\omega = \cos \omega$ and the common notation for the angles is $\omega = \theta_{12}$, $\phi = \theta_{13}$, $\psi = \theta_{23}$. Using Eq. (11), we obtain the communication matrix $m$ which has all elements $\mathcal{O}(1)$, as should be provided by $\lambda_{\alpha\beta} \sim 1$:

$$m = \begin{pmatrix}
\cos^2 \omega & -\frac{1}{\sqrt{8}} s_{2\omega} & \frac{1}{\sqrt{8}} s_{2\omega} \\
-\frac{1}{\sqrt{8}} s_{2\omega} & \frac{1}{2} s^2_{\omega} & \frac{1}{2} s^2_{\omega} \\
\frac{1}{\sqrt{8}} s_{2\omega} & -\frac{1}{2} s^2_{\omega} & \frac{1}{2} s^2_{\omega}
\end{pmatrix},$$

(21)

This property remains true generically, even for other values of the starting matrix, e.g. $\bar{m} = \mu \text{ diag}(1, i, 3)$ ($i$ here is $\sqrt{-1}$), and actually, this happens even when $\bar{m}$ is non-diagonal. In other words, we may have very small or negligible oscillations to the second window,
without violating the condition that the elements of \( m \) are of order unity.

(2) As the second example, we consider the case when all the elements of the communication term \( m \) are exactly equal to one. That is, \( m \) has the following texture:

\[
m = \mu \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix}.
\]  

(22)

Using Eq. (11) and Eq. (20) we get

\[
\bar{m} = \mu \begin{pmatrix}
\cos^2 \omega & * & *
\\
* & \sin^2 \omega & *
\\
* & * & 2
\end{pmatrix}.
\]  

(23)

We have not written the off-diagonal terms of \( \bar{m} \) because as explained above they play no role in oscillations. Note that for this texture all terms of \( \bar{m} \) are of the same order of magnitude. In particular all diagonal terms are of same order of magnitude, i.e. there is no strong hierarchy in the diagonal terms as in the first scheme. Therefore the splitting in the first two windows is

\[
\Delta m_1^2 \sim 4\mu \cos^2 \omega M_1 \cos \xi_1
\]

\[
\Delta m_2^2 \sim 4\mu \sin^2 \omega \sqrt{\Delta M_{\text{sol}}^2} \cos \xi_2
\]

(24)

where \( \Delta M_{\text{sol}}^2 \) is the solar neutrino mass squared difference for the large mixing angle (LMA) solution, and we have introduced two phase factors \( \xi_1 \) and \( \xi_2 \). For typical values of mixing angles in the LMA regime, \( \sin^2 \omega \sim 0.3 \) we get: \( \Delta m_1^2 \sim 1 \times 10^{-5} \text{ eV} \) \( M_1 \) and \( \Delta m_2^2 \sim 5 \times 10^{-8} \text{ eV}^2 \). Since \( M_1 \) is unconstrained, we can always choose a sufficiently small value for it, so that the splitting of the first level, or \( \Delta m_1^2 \), becomes small and irrelevant for solar neutrino oscillations. If taken at face value, the splitting in the second level is too large, and the net result is too much suppression for solar neutrinos. However, as mentioned in Section 2.1.1 there are several ways to escape this conclusion, for instance, with the help of suppression of scale \( \mu \) by \( \exp(-\mathcal{S}) \) from wormholes effects. Or even, it is possible that \( M_2 \) and \( \bar{m}_{22} \), when regarded as complex numbers, lead to a small \( \Delta m_2^2 \) just because of their relative phase (namely, the phase \( \xi_2 \) can be close to \( \pi/2 \)). Note that in this second case, the smallness of \( \Delta m_2^2 \) can be attributed to the phase of \( M_2 \), instead than to a specific property of the matrix \( m \).

We conclude that mirror models with gravitational communication in many cases have a mass matrix \( m \) (or \( \bar{m} \)) which produces too large oscillation effects, but there are also models compatible with present experimental data.
Figure 2: How the LMA survival probability is modified by the oscillation into mirror states. The values of $\Delta m^2$ are indicated at the curves, in units of $10^{-13}$ eV$^2$. Note the sizeable spectral distortion at low energies.

3 Applications

In this section, we will consider the effect of oscillations into mirror neutrinos on solar, supernova, and high energy neutrinos. An important remark is that the atmospheric neutrino phenomenology is not affected by the splitting of the third level $\nu_3$, since for the values of the $L/E$ relevant to the atmospheric neutrino problem, there are no oscillations into mirror neutrinos. The atmospheric neutrino problem remains pure $\nu_\mu \leftrightarrow \nu_\tau$ oscillations to a very good approximation, since CHOOZ constrains the mixing term $U_{e3}$ to be small. The limit of CHOOZ implies also that solar neutrino oscillations are almost decoupled from “atmospheric” frequency $\Delta M^2_{\text{atm}}$ in the usual three-flavor context. The same remains true in our model: it is sufficient to consider oscillations with frequency $\Delta M^2_{\text{sol}}$ or smaller.

3.1 Subdominant solar neutrinos oscillations

To illustrate the role of oscillations into mirror neutrinos, we will discuss now how the LMA solution of the solar neutrino problems is affected by these oscillations. We emphasize here that our aim is not a detailed global fit to the solar neutrino data. Instead we want (1) to demonstrate that solar neutrino data are consistent with the presence of subdominant oscillations into mirror neutrinos in our model and (2) to calculate the effects produced by these oscillations, which occur mostly at low energies.
3.1.1 CASE 1: $\Delta m^2_{1} \neq 0$.

We consider here the case of sterile oscillations associated with the splitting of the first level. The splitting in the second window is assumed to be small as discussed in Sections 2.2.2 and 2.1.1. The splitting of the third level does not affect solar neutrino oscillations.

The electron neutrino survival probability with the MSW effect taken into account is given by

$$P_{ee} = \cos^2 \omega \cos^2 \omega_m + \sin^2 \omega \sin^2 \omega_m - \cos^2 \omega \cos^2 \omega_m \sin^2 \delta$$

Here, $\delta$ is the phase of vacuum oscillations

$$\delta = \frac{\Delta m^2_{1} L}{4 E}$$

where $L$ is the distance between production and detection, $E$ is the neutrino energy, and $\Delta m^2_{1}$ is the mass squared for oscillations into mirror neutrinos. The mixing angle at the core of the sun $\omega_m$ is:

$$\tan 2\omega_m = \frac{\sin 2\omega}{\cos 2\omega - \alpha}, \text{ with } \alpha = \frac{2\sqrt{2}G_F \rho_i E}{M^4_2 - M^4_1}$$

Note that the above expression for the survival probability is true as long as the propagation in the sun is adiabatic. This happens in the case of the LMA solution, on which we elaborate here.

There are two distinct behaviors of $P_{ee}$ at low and high energies:

**Low energy regime**: If the solar neutrino scale is the LMA mass squared difference then at low energies $\omega_m \approx \omega$. This implies that Eq. (25) becomes

$$P_{ee} = 1 - \frac{1}{2} \sin^2 2\omega - \cos^4 \omega \sin^2 \delta.$$

This can be cast in a more transparent manner:

$$P_{ee} = P_{ee}^{LMA} - \cos^4 \omega \sin^2 \delta.$$  \hspace{1cm} (29)

where $P_{ee}^{LMA}$ is the standard survival probability at low energies of the LMA solution.

**High energy regime**: At high energies $\omega_m \approx \frac{\pi}{2}$. This implies that Eq. (25) becomes

$$P_{ee} = \sin^2 \omega.$$  \hspace{1cm} (30)

This is the standard survival probability at high energies of the LMA solution. Therefore the crucial feature of our model is that its predictions coincide with the standard MSW solution at high energies but are affected by the subdominant sterile oscillations at low
energies: The standard MSW solution is modified at low energies, and most noticeably at pp neutrinos energies. This is evident from Fig. 1 which shows the survival probabilities for some values of $\Delta m^2_1$ superimposed on the usual LMA survival probability.

Indeed, there is an upper bound on $\Delta m^2_1$ which follows from gallium data. For the calculation, we took the fluxes from [51] and the cross sections from [52], and used the average gallium rate as obtained by the Gallex/GNO and SAGE experiments [53], 70.8$\pm$4.4 SNU. As unperturbed case we choose the best fit LMA solution $M^2_2 - M^2_1 = 6.2 \times 10^{-5}$ eV$^2$ and $\tan^2\omega = 0.4$ as obtained in [49], which is consistent with the value found in other calculations, see e.g. [50]. As illustrated in Fig. 3, the largest value of $\Delta m^2_1$ allowed by the 3 sigma range is $10^{-12}$ eV$^2$. (It is curious to note that this coincides with the scale of the "just-so" solution [54, 55, 56] to the solar neutrino problem.)

Two remarks are in order:

(i) As can be seen from Fig. 1 there is a large difference between the profiles of the usual LMA solution and our model in the energy range 0.2 to 0.4 MeV. This is precisely the energy range of the pp neutrinos. So future experiments like LENS [57] which will study pp neutrinos in real time should see a large distortion in the pp spectrum. By contrast, the usual LMA solution does not predict any energy dependent distortion in this energy range. When one decreases the subdominant scale, the effect weakens though remaining still appreciable for a reasonable range of $\Delta m^2_1$.

(ii) The spectral distortion somewhat diminishes when the mixing angle $\omega$ increases. This is simply due to the $\cos^4\omega$ factor in front of the oscillatory term. But even for the largest value of the angle allowed by data, 41$^\circ$, there is an appreciable effect for a range of $\Delta m^2_1$.

An important task is to investigate possible signals of seasonal variations in our scheme. Even if the LMA solution will be firmly established by future experiments like KamLAND [58], an interesting signature of our model will be the presence of further seasonal variations in solar neutrino experiments, as we now discuss. We first rewrite our oscillation probability Eq. (25) in a form suitable for studying seasonal variations.

$$P_{ee} = P_{ee}^{LMA} - \cos^2\omega \cos^2 \omega_m \sin^2 \delta,$$

where $\delta$ is defined in Eq. (26) and

$$L(t) = L_o (1 - \varepsilon \cos \Omega t)$$

In the above equation $\varepsilon = 1.675 \times 10^{-2}$ and $L_o = 1.496 \times 10^{11}$ m. $\Omega = \frac{2\pi}{T}$ where $T = 1$ year, and $t$ is the time since the perihelion. Hence we can write the phase of the oscillating term as

$$\delta = \delta_o (1 - \varepsilon \cos \Omega t)$$

where in analogy with previous notation we define $\delta_o = \Delta m^2_1 L_o/4E$. This modulation implies

$$\sin \delta = \sin [\delta_o (1 - \varepsilon \cos \Omega t)] = \sin \delta_o - \varepsilon \delta_o \cos \delta_o \cos \Omega t$$

15
Figure 3: How the gallium rate changes with the new oscillation scale (the parameter $\Delta m_{12}^2$). For comparison, the 3 sigma experimental band is shown by dashed lines.

In writing Eq. (34) we have used the fact that $\delta \varepsilon \cos \Omega t \ll 1$ which is always true for the range of $\Delta m_{12}^2$ we are considering, and for all neutrino energies of interest. Using Eq. (34) we get:

$$P_{ee} = P_{ee}^{LMA} - \cos^2 \omega \cos^2 \omega_m \sin^2 \delta_o + \varepsilon \sin 2\delta_o \cos \Omega t \cos^2 \omega \cos^2 \omega_m \equiv \langle P_{ee} \rangle + P'_{ee} \cos \Omega t \tag{35}$$

Here, $\langle P_{ee} \rangle$ is the time independent part of $P_{ee}$, while $P'_{ee}$ is the amplitude of the time dependent part.

We first apply this formula to Borexino. For an experiment which measures the monoenergetic Beryllium line the rate, as a function of time, has the following form

$$R(t) \propto 1 + a_{Be} \cos (\Omega t) \text{ with } a_{Be} = \frac{P'_{ee}}{\langle P_{ee} \rangle + 1/5} \tag{36}$$

(a factor $\approx 1/5$ comes from the neutral current contribution). The amplitude $a_{Be}$ of the time varying factor is a function of the small scale $\Delta m_{12}^2$, and it is evaluated at the energy of the Beryllium line. We find that even for the maximum value of $\Delta m_{12}^2$ which is $10^{-12}$ eV$^2$ the amplitude is only 0.001. So there is no significant seasonal variations at Borexino.

However, since the subdominant scale in our model is much smaller than the usual “just-so” scale, there may be some seasonal variations at lower energies. With this motivation in mind, we analyze seasonal variations for the $pp$ neutrinos. We will consider two experimental situations, the first one $a la$ LENS, when the $pp$ flux can be measured separately, the second one $a la$ GNO, when there are contributions to the signal also from other neutrino fluxes.
Figure 4: Amplitude $a$ of seasonal variation of gallium counting rate (‘GNO’ case) due to oscillations into mirror neutrinos as percentage of the excentricity $\epsilon$, in a function of the (average) counting rate. We also show the amplitude $a_{pp}$ of seasonal variation for events induced by $pp$ neutrinos only (‘LENS’ case). The curves are obtained varying $\Delta m_1^2$ in the range below $10^{-12}$ eV$^2$ for fixed LMA parameters.

In the LENS case, the modulation of the signal is:

$$R(t) \propto 1 + a_{pp} \cos(\Omega t)$$

where $a_{pp}$ is given by

$$a_{pp} = \frac{\int \Phi_{pp}(E)\sigma(E)P'_{ee}dE}{\int \Phi_{pp}(E)\sigma(E)\langle P_{ee} \rangle dE}.$$  

Here $\Phi_{pp}$ is the $pp$ flux as given in BP2000 \textsuperscript{[51]}, and $\sigma$ is the cross section for neutrino absorption on gallium. The integral goes from the threshold of 0.23 MeV to the endpoint of the $pp$ spectrum. For the maximum value of $\Delta m_1^2 = 10^{-12}$eV$^2$ we obtain that the amplitude is slightly more than 1 %, which is smaller than the term $\epsilon \sim 1.7$ % resulting from geometrical modulation of the distance in Eq. \textsuperscript{[32]}, but is however non-negligible. Two conclusive remarks are in order:

(i) It should be noted that the new modulation is in phase with the geometrical modulation, in other terms the rate is modulated by $(2\epsilon + a_{pp}) \cos \Omega t$.

(ii) As mentioned above, all these calculations are done with a fixed LMA solution, namely
the best fit in absence of mirror neutrino oscillations. If we decrease $\omega$ to the minimum value allowed by the LMA solution, the amplitude increases a bit to 1.23\%.

Let us consider now the case of an experiment like GNO. When we take into account the contribution due to the other neutrino fluxes (which do not receive seasonal variations in our model) the expression for $a_{pp}$ Eq. (38) gets modified to

$$a = \frac{\int \Phi_{pp}(E) \sigma(E) P'_{ee} dE}{\int \Phi_{pp}(E) \sigma(E) \langle P_{ee} \rangle dE + R_{oth}}. \quad (39)$$

where $R_{oth}$ is the contribution to the gallium experiment due to the other neutrino sources. So the effect will be scaled down in the actual case of GNO (see Fig. 4). The message is that an experiment dedicated to measuring the $pp$ flux only will be better suited to look for this kind of effect.

In concluding this section we again stress the novelty of the phenomena: Even after the MSW solution is established, the model we consider can lead to differences in the counting rate, spectral distortions, and/or seasonal variations for $pp$ neutrinos.

### 3.1.2 CASE 2: $\Delta m_2^2 \neq 0$.

Let us assume that $\Delta m_1^2$ is strongly suppressed, but $\Delta m_2^2$ is not, see Sections 2.2.2 and 2.1.1.
The electron neutrino survival probability becomes

\[
P_{ee} = \cos^2 \omega \cos^2 \omega_m + \sin^2 \omega \sin^2 \omega_m - \sin^2 \omega \sin^2 \omega_m \sin^2 \delta
\]

(40)

(we use the notation of Eq. (26) for the phase of oscillation with \(\Delta m^2_1\) replaced with \(\Delta m^2_2\)).

In this scheme at high energies when \(\omega_m \approx \frac{\pi}{2}\) we get

\[
P_{ee} = \sin^2 \omega \cos^2 \delta
\]

(41)

So the usual LMA behavior is modified also at high energies. When we select the range \(\Delta m^2_2 < 10^{-11} \text{ eV}^2\) there is no spectral distortion at boron energies, but just a scaling down of the usual LMA survival probability. This upper bound is obtained by allowing for a 10 % decrease in the survival probability. Note that the suppression of boron neutrino flux will be accompanied by a disappearance of total neutrino flux, which is given by the factor \(\Phi = \cos^2 \delta \Phi_0\), since only half of the \(\nu_2\) neutrinos reaches the detector. For this reason, the ratio of charged-current to neutral-current events yields the same \(\omega\) that will be obtained by terrestrial experiments, say, by KamLAND. For the same reason, one can use the boron flux to extract information on \(\delta\) only if the absolute flux value is known from the theory, and this makes the investigation difficult.

In this scheme, the gallium rate is weakly dependent on the new scale unlike the previous case. In fact, when \(\Delta m^2_2\) changes from \(10^{-12}\text{eV}^2\) to \(10^{-11}\text{eV}^2\) the gallium rate diminishes only by 3 to 7 SNU. We plot in Fig. 5 the modification at low energies of the usual LMA survival probability due to oscillation to mirror states. One sees that even for the largest value of \(\Delta m^2_2\) allowed the effect is weak, unlike the case of the first scheme. This is simply due to the fact that for typical mixing angles in the LMA region \(\sin^4 \omega \ll \cos^4 \omega\).

The analysis of seasonal variations is exactly analogous to the analysis in the first scheme, except that we now use Eq. (40) for the analysis. We find that the coefficient \(a\) of modulation varies between \(\pm 25\%\) in the allowed \(\Delta m^2_2\) range. In conclusion, this scheme offers much less clear “smoking-gun” signals and its experimental investigation is difficult.

### 3.2 Supernova neutrinos

The framework outlined above has a natural application to supernova neutrinos. In fact, the condition that the phase of vacuum oscillation is large, \(\Delta m^2 L / 4E \gg 1\) can be rewritten as

\[
\Delta m^2 \gg 1.3 \times 10^{-19} \text{eV}^2 \left[ \frac{1 \text{kpc}}{L} \right] \left[ \frac{E}{20 \text{MeV}} \right]
\]

(42)

where \(\Delta m^2\) is the mass scale that gives rise to oscillations into mirror neutrinos. We recall that the energy of supernova neutrino events lies certainly in the range \(1 < E < 100 \text{ MeV}\) (lowest bound being mostly due to detector characteristics), and the distance of the galactic
center is $L_{\text{g.c.}} \approx 8$ kpc. We will consider two cases where mixing with mirror neutrinos leads to observable consequences. The first case is based on rather standard astrophysics of core collapse supernovae and will be discussed at length; the second case starts from a bolder speculation, about the existence of mirror supernovae, however it has a nicer signature. In a sense, these two cases correspond to the classification of oscillations into “disappearance” and “appearance” that we follow below.

### 3.2.1 Disappearance of supernova neutrinos

Let us consider neutrinos from a core collapse supernova. Various patterns of oscillations into mirror neutrinos are possible, according to which vacuum oscillation develops (i.e. which $\Delta m^2_i$ is sufficiently large). Here, we will focus on the simplest possibility: all three $\Delta m^2_i$ satisfy the inequality given above and therefore, averaged oscillations into mirror states take place for all states. As shown above, see Eq. (17) and following discussion, this situation leads to the very simple result that half of active neutrinos of any type $\nu_{e,\mu,\tau}$ and their antineutrinos reach the detector. Hence the signature of this scenario is that the energy observed is half the energy emitted.

The theoretical uncertainties are the key issues to verify or contradict the predictions of the model we propose. It is unclear whether the theoretical predictions of the energy emitted in the gravitational collapse $E_{\text{th}}$ will become accurate enough in future to reveal a difference in neutrino energy by a factor of two, say: $E_{\text{th}} = 4 \times 10^{53}$ erg versus $E_{\text{obs}} = 2 \times 10^{53}$ erg (where we assume that oscillations into mirror neutrinos do take place). Indeed, within the existing theoretical uncertainties [60], these two values are only marginally distinguishable, and can be found by varying the parameters of the equation of state of the nuclear matter within the existing uncertainties.

For what regards $E_{\text{obs}}$, we recall that a future galactic supernova exploding at a distance $\sim L_{\text{g.c.}}$ will yield several thousands of neutrino events at the Super-Kamiokande detector; similarly, very significant statistics will be collected at LVD, Baksan, SNO, etc. This should permit a determination of the total energy $E_{\text{obs}}$ at much better than 10 %; possibly, without assuming energy equipartition of the various neutrino fluxes, but testing it by future data (see below).

In this respect, the $\sim 20$ neutrino events collected from SN1987A might seem already a statistically useful indication; let us look at the point more closely. In a recent work [61], Loredo and Lamb find as optimal fit values $E_{\text{obs}} \sim 3 \times 10^{53}$ erg in practically all models with an accretion and a cooling components of the neutrino signal, which are expected in the ‘delayed scenario’ for supernova explosion. On examination of their Fig. 10, one gets convinced that oscillations into mirror neutrinos would not contradict present theoretical expectations, even though this would suggest a stiff equation of state for nuclear matter. Note incidentally that this paper, though being one of the the most thorough existing analyses, assumes strict equipartition, does not include oscillations into active neutrinos,
and considers only $\bar{\nu}_e p \to n e^+$ signal using the ‘leading order’ cross section. Equipartition is particularly important: for example if $\bar{\nu}_e$ carry 1/4 rather than 1/6 of the total energy, which is a reasonable value \[62\], the best estimate of \[61\] reduces to $E_{\text{obs}} \sim 2 \times 10^{53}$ erg.

### 3.2.2 Appearance of supernova neutrinos

Mirror matter and mirror stars are expected to exist \[21, 23, 27\], and this could ultimately result into an explosion of a mirror galactic supernova. The mirror neutrinos will oscillate into the active one, and half of the the original flux will become observable. All other radiation of mirror supernova is undetectable, that gives the basic signature of the event: there will be no optical burst in the direction of neutrino burst, and no radio or infrared radiation will be detected simultaneously with neutrino burst and later. In the case of an asymmetric gravitational collapse, another detectable signal is the gravitational radiation, see e.g. \[63\]. An additional signature can be given by high energy mirror neutrino radiation from the young supernova shell \[64\].

### 3.2.3 Remarks on neutrino spectra and “equipartition”

On the top of the new effects outlined above, and in both cases considered in Sects.\[3.2.1\] and \[3.2.2\], we will have also the (usual) effects related to flavor oscillations. To be specific, the fluxes of $\nu_e$, $\bar{\nu}_e$ and active neutrinos at the detector are:

$$
\begin{align*}
F_e &= F_0^e / 2 \\
F_{\bar{e}} &= (\cos^2 \omega F_0^\bar{e} + \sin^2 \omega F_0^x) / 2 \\
F_{NC} &= (F_0^e + F_0^\bar{e} + 4F_0^x) / 2.
\end{align*}
$$

(43)

where $F_0^e$, $F_0^\bar{e}$, $F_0^\mu = F_0^\bar{\mu} = F_0^\tau = F_0^x \equiv F_0^\nu$ are the fluxes without oscillations. For definiteness, we assumed a normal mass hierarchy of neutrinos, and an angle $\phi > 1^\circ$: These conditions leads to adiabatic MSW conversion of $|\nu_e\rangle$ into $|\nu_3\rangle$ and of $|\bar{\nu}_e\rangle$ into $|\bar{\nu}_1\rangle$ (terms order $\phi^2 \leq \text{few}\%$ are neglected). These effects—without the oscillation into mirror neutrinos—have been discussed previously by a number of authors \[65\] \[66\] \[67\] \[68\] \[69\], with the generic conclusion that flavor mixing might lead to ‘hotter’ $\nu_e$ and $\bar{\nu}_e$ fluxes due to the $F_0^x$ component.\[3\] The flux of active neutrinos $F_{NC}$—from neutral current events—is unchanged; a peculiar signature of mirror oscillations is that even this flux is reduced by a factor of 2.

Note that, by the time when the next galactic supernova will explode, we might already know the usual parameters of oscillations, and therefore the usual type of supernova neutrino oscillations. In this connection, we believe that it is important to stress an important

\[3\] This statement is intentionally vague, in view of the different conclusions reached by those who investigated SN1987A neutrinos including oscillations, see e.g. \[70\], and in view of the fact that a full fledged theory of supernova explosions is still missing \[71\] \[72\] \[73\] \[73\].
consideration (important, even if mirror neutrino oscillations do not take place): Using the large $\bar{\nu}_e$-produced data samples along with the presumably smaller samples produced by neutral currents and $\nu_e$, we will have chance to test the hypothesis of equipartition by the experimental data, or in other words, we will be able to reconstruct the total energy $E_{\text{obs}}$ with a minimum theoretical bias.

Mirror neutrino oscillations for different energy and situations (supernova remnant) has been recently discussed in Ref.[74], stressing in particular the possibility of a distortion of the spectrum. Of course a similar possibility exists for the neutrino spectrum from core collapse, for the particular case when the parameters of oscillations into mirror states satisfy the relation $\Delta m^2 L/E \sim 1$.

### 3.3 High energy neutrinos

High energy (HE) neutrino astronomy includes the wide range of energies from $E \sim 100$ GeV up to $E \sim 10^{13}$ GeV. We will consider here the diffuse neutrino fluxes at very high energies. These fluxes can be produced by three principal sources: accelerator sources, topological defects (TD) and superheavy relic particles.

There is a very general cascade upper limit on HE diffuse neutrino flux [75, 76]. It is based on the e-m cascade which develops due to collisions of cascade electrons and photons with the target photons, e.g. CMB. The cascade is initiated by high energy electrons or photons which always accompany the production of a HE neutrino. The diffuse neutrino flux $I_\nu(E)$ is limited as

$$E^2 I_\nu(E) \leq \frac{c}{4\pi} \omega_{\text{cas}},$$

where $\omega_{\text{cas}}$ is the energy density of the cascade photons left in intergalactic space. These photons have the energies in the range of EGRET observations, which give the upper limit on the cascade energy density $\omega_{\text{cas}} \leq 2 \times 10^{-6}$ eV/cm$^3$. The limit (44) is very general: it is valid for all processes of neutrino generation in extragalactic space (e.g. generation by TD and by decay of superheavy relic particles) and in the galaxies if they are transparent to gamma radiation.

The only class of sources that escape the cascade upper bound (44) is comprised by the so called “hidden sources” [76]. An example of a powerful hidden source of HE neutrinos is given by mirror matter. As demonstrated in Ref.[18] in some models the density of topological defects in the mirror matter can be much higher than in ordinary one. Superheavy particles produced by mirror-sector TD and the products of their decays are sterile in the visible world, but mirror neutrinos can oscillate into the visible ones. The flux of these neutrinos can be higher than what the limit (44) allows.
3.3.1 Z-bursts

Z-burst is a beautiful idea of generation of Ultra High Energy Cosmic Rays (UHECR) through the resonant production of Z-bosons in the collisions of UHE neutrinos with Dark Matter (DM) neutrinos, $\nu + \nu_{DM} \rightarrow Z^0 \rightarrow all$. The resonant energy of UHE neutrino is $E_0 = m_Z^2/2m_\nu = 4.2 \times 10^{12} m_{eV}^{-1}$ GeV, where $m_{eV}$ is the mass of DM neutrino in eV. Following Ref. [13] we calculate the number of Z-bosons produced per unit volume and unit time, integrating over energy the diffuse neutrino flux $F_\nu(E)$ coupled to the Breit-Wigner cross-section $\sigma(E)$:

$$\dot{n}_Z = 4\pi n_\nu \sum_i \int I_\nu(E)\sigma(E)dE = 4\pi n_\nu \sigma_t E_0 I_\nu(E_0), \quad (45)$$

where $I_\nu = \sum I_{\nu_i}$, $n_\nu = 56$ cm$^{-3}$ is the space density of one flavor DM neutrinos, and $\sigma_t = 48\pi f_\nu G_F = 1.29 \times 10^{-32}$ cm$^2$ is the effective cross-section with $G_F$ the Fermi constant and $f_\nu = 0.019$ the relative width of Z-decay to neutrino channel. In Eq. (45) it is assumed that neutrino masses are degenerate, $m_{\nu_i} = m_\nu$. Using the limit on the sum of neutrino masses $\sum m_{\nu_i} < 1.8$ eV from the spectrum fluctuations derived from 2dF galaxy survey [79], we shall assume in the calculations below the neutrino mass $m_\nu = 0.3$ eV as maximally allowed. Formula (45) is exact.

In the case of decaying superheavy particles (TD and superheavy relic particles) UHE photons dominate in UHECR signals at energy $E \geq 1 \times 10^{20}$ eV [78]. Their flux can be calculated as

$$I_\gamma(E) = \frac{1}{4\pi} \dot{n}_Z R_\gamma(E) Q_\gamma(E), \quad (46)$$

where $R_\gamma(E)$ is absorption length of UHE photon and $Q_\gamma(E)$ is the number of photons with energy $E$ produced (via $\pi^0$ decays) per one Z-decay. Using the observed UHECR flux at $E \sim 10^{20}$ eV, one can calculate the flux of resonant neutrinos $I_\nu(E_0)$ from Eqs. (45) and (46) as $4 \times 10^{-36}$ cm$^{-2}$s$^{-1}$sr$^{-1}$eV$^{-1}$, while the cascade limit (44) is 5 orders of magnitude lower.

In Ref. [80] it was suggested that the cascade limit can be evaded, if X-particles decay exclusively to neutrinos, i.e. $X \rightarrow \nu \bar{\nu}$. However, in a recent work [81] it was demonstrated that this decay results in the electroweak cascading in which electrons, photons and pions are efficiently produced and thus the cascade limit (44) is valid for this case too.

3.3.2 High energy neutrinos from oscillations

The oscillations of mirror neutrinos into the visible ones are characterized by oscillation length $L_{osc} \sim E/\Delta m^2$, much shorter than the typical cosmological distance $L \sim 100$ Mpc. The only exceptional case is given by oscillation of the resonant neutrinos with $E_0 \approx 1 \times$
$10^{13}$ GeV in the first “window” (see the Fig. 1), where $\Delta m^2_1$ can be as small as $1 \times 10^{-13}$ eV$^2$. Therefore, the average suppression due to oscillation length is given by factor $\frac{1}{2}$.

The conversion of the sterile neutrinos into visible ones occurs through two stages. Let us consider a sterile neutrino $\nu'_\alpha$ born with a flavor $\alpha$ and energy $E$. On the short length scale $L_{\text{short}} \sim E/\Delta M^2$, where $\Delta M^2 = M^2_i - M^2_k$ is the mass squared difference of the unperturbed states, $\nu'_\alpha$ oscillates into two other sterile flavors, and we have all three sterile neutrinos $\nu'_\beta$ with $\beta = e, \mu, \tau$. On much longer scale $L_{\text{long}} \sim E/\Delta m^2$, where $\Delta m^2$ is a scale of the window splittings, sterile neutrinos oscillate into visible ones. Taking into account that suppression factors due to oscillation length is $1/2$, we can calculate the probabilities $P_{\nu'_\alpha \nu}$ for conversion of mirror neutrino $\nu'_\alpha$ into visible neutrino $\nu_\beta$, using Eqs. (17) and (20).

In particular, for conversion of mirror muon neutrino $\nu'_\mu$ we obtain the probabilities

$$P_{\nu'_\mu \nu_e} = \sin^2 \frac{2\omega}{8}, \quad P_{\nu'_\mu \nu_\mu} = P_{\nu'_\mu \nu_\tau} = \frac{1}{4} - \sin^2 \frac{2\omega}{16},$$

which depend only on the solar mixing angle $\omega$. For conversion of mirror tau neutrino $\nu'_\tau$ one should replace $\nu'_\mu$ by $\nu'_\tau$ in Eq. (47). For completeness we also give the relevant probabilities for the mirror electron neutrino $\nu'_e$ conversion.

$$P_{\nu'_e \nu_\mu} = P_{\nu'_e \nu_\tau} = \frac{1}{8}, \quad P_{\nu'_e \nu_e} = \frac{1}{2} - \sin^2 \frac{2\omega}{4}.$$  (48)

Note, that as follows from Eq. (17) the probability of conversion $P_{\nu'_\alpha \nu_\beta}$ summed over all visible neutrinos $\nu_\beta$ is equal to $\frac{1}{2}$. It means that for $Z$-burst production when all neutrino flavors participate in the resonant reaction, the total oscillation suppression $P(\nu'_\alpha \to \nu) = \frac{1}{2}$.

As it was already mentioned, in some cosmological models the density of mirror-sector TD can be much larger than in the visible sector. The ratio of neutrino fluxes from mirror and visible TD’s can reach $2 \times 10^4$ [18]. In our model this ratio is modified by the probabilities given by Eq. (17), which are different for different modes of oscillations, but after summation over final states of visible neutrinos the ratio remains the same as in Ref. [18].

4 Conclusions

Oscillations between active and sterile neutrinos, being excluded as the main channel of oscillations both in the solar and the atmospheric neutrinos, can be interesting subdominant processes. In this paper we have considered mirror neutrinos as the sterile ones. Our particular model is the “classical” symmetric model of the mirror matter, when apart from $L \leftrightarrow R$ interchange, mirror and ordinary matter have the same interactions, coupling constants and VEV’s. The communication between these two sectors is gravitational, it is described by a dimension-5 operator, suppressed by Planckian mass (see Eq. (4)). This operator can be additionally suppressed by some factors, most strongly by $\exp(-S)$ due to
wormhole transitions. The gravitational interaction between ordinary and mirror particles, being too weak for heavy particles of the Standard Model, can reveal itself in case of neutrinos. When the communication interaction is switched off, the visible and mirror neutrinos have identical “standard” mass spectrum $M_i$ ($i = 1, 2, 3$), being provided e.g. by “internal” see-saw mechanism. The communication term (4) mixes the visible and mirror neutrinos, generating a non-diagonal term $m$ (in the form of $3 \times 3$ matrix) in the neutrino mass matrix (7). The mixing of active and sterile neutrinos is maximal and each unperturbed mass eigenstate with mass $M_i$ ($i = 1, 2, 3$) splits into two close mass eigenstates with masses $M_{\pm i} = M_i \pm \bar{m}_{ii}$ and $\Delta m^2 = 4 \text{Re}(M_i \bar{m}_{ii}^*)$. Thus three split levels (windows) arise. Our basic observation is that unsuppressed oscillations between active and sterile neutrinos occur only in transitions between the split levels of the same window. Thus, these oscillations are the long-wavelength ones, being characterized by large $L_{\text{osc}} \sim E/\Delta m^2$. These oscillations are unobservable in atmospheric neutrinos and produce subdominant effects in solar neutrinos.

An interesting case (CASE 1, described in Section 3.1) is given by the standard LMA solution for active neutrinos with a perturbation term $\mathcal{L}_{\text{comm}}$ (or matrix $m$) taken as in Eq. (21) of Section 2.2.2. In this case, only the splitting in the first window works: $\Delta m^2_2$ is small and $\Delta m^2_3$ is irrelevant for solar neutrino oscillations. In this case, the large energy effects are practically the same as predicted by the standard LMA solution, but the flux and the spectrum of $pp$-neutrinos are distorted as shown in Figs. 2 and 3. A rather unusual prediction is the anomalous seasonal flux variation shown in Fig. 4. The predicted effects, especially distortion of the $pp$-neutrino spectrum, can be detected by the future LENS experiment [57].

The mirror-visible neutrino oscillations ($\nu_{\text{mirr}} \leftrightarrow \nu_{\text{vis}}$) which are predicted to be observed as subdominant effects in solar neutrinos can reveal itself as big effects in observation of supernova and HE neutrinos. In the former case (supernovae, Section 3.2.1), half of the expected neutrinos might actually be missed, though it would be crucial to improve on the theoretical expectation for total energy released in neutrinos to interpret this signal precisely; another possibility is finding a neutrino signal without any optical counterpart (Section 3.2.2). In the latter case (HE neutrinos, Section 3.3), a large flux of mirror neutrinos can be produced by mirror topological defects. Mirror neutrinos can oscillate into the visible ones, while all accompanying mirror particles remain invisible. This allows a large diffuse neutrino flux unconstrained by e-m cascades or any other restrictions. The probability of conversion of mirror neutrino $\nu'_\alpha$ into ordinary neutrino $\nu_\beta$, summed over flavors $\beta$ and averaged over distance is

$$
\sum_\beta P_{\nu'_\alpha \nu_\beta} = \frac{1}{2}, \quad \alpha, \beta = e, \mu, \tau,
$$

due to the fact that oscillation length $L_{\text{osc}} \sim E/\Delta m^2$ is much shorter than $L \sim 100 – 1000$ Mpc relevant for production of high energy diffuse neutrino flux.
A Neutrino mass matrices and transformation properties of ordinary to mirror particles

We shall consider here the restrictions imposed on the texture of neutrino mass matrices by the transformation properties of ordinary to mirror particles.

Let us study the most general case of three neutrino flavors \((\alpha, \beta)\) for ordinary and mirror neutrinos \(\nu_\alpha(x)\) and \(\nu'_\alpha(x)\). We shall limit to the case of neutrinos with “Majorana masses”. For this reason, we adopt the formalism of Majorana spinors, which obey the relation

\[
\nu = C \bar{\nu}'.
\]

The neutrino mass part of the Lagrangian is given by:

\[
\mathcal{L} = -\frac{1}{2} \left[ M_{\alpha\beta} \bar{\nu}_\alpha(x) P_L \nu_\beta(x) + M^*_{\alpha\beta} \bar{\nu}_\alpha(x) P_R \nu_\beta(x) + M'_{\alpha\beta} \bar{\nu}'_\alpha(x) P_L \nu'_\beta(x) + M'^*_{\alpha\beta} \bar{\nu}'_\alpha(x) P_R \nu'_\beta(x) - m_{\alpha\beta} \bar{\nu}_\alpha(x) P_L \nu'_\beta(x) - m^*_{\alpha\beta} \bar{\nu}'_\alpha(x) P_R \nu_\beta(x) \right]
\]

where the star denotes complex conjugation and \(P_L, R\) are the projection operators. It is easy to see that this is the most general case, by recalling that for the Majorana spinors \(\bar{\chi} P_L R = \bar{\chi} P_L R \chi\). The parameters of this Lagrangian (apart from the arbitrary phases of the fields) are those of the two symmetric matrices \(M\) and \(M'\), and of the arbitrary matrix \(m\), and we can construct from them the neutrino mass matrix:

\[
\mathcal{M} = \begin{pmatrix} M & m \\ m^t & M' \end{pmatrix}
\]

In the context of quantum field theory, the mirror symmetry implies that the action \(S = \int d^4x \mathcal{L}\) is invariant under left↔right transformation of the fields (see Lagrangian \[51\] as a particular example).

Two kinds of left↔right transformations of the arbitrary fermionic fields \(\Psi\) and \(\Psi'\), generically denoted as

\[
\Psi_{L,R} \leftrightarrow \Psi'_{R,L}
\]
can provide this invariance.

(1) The first one is given by

\[ \Psi(x) \leftrightarrow C \Psi'(x) . \] (54)

It is easy to convince oneself that this transformation belongs to the class described by Eq. (53): e.g. the ordinary left current is transformed into a mirror right-current. Applying this transformation to the neutrino mass Lagrangian (51), we obtain that the action is invariant if the following conditions hold: \( M_{\alpha\beta} = M'_{\alpha\beta} \), \( m_{\alpha\beta} = m_{\beta\alpha} \). Then, the neutrino mass matrix reads:

\[ \mathcal{M} = \begin{pmatrix} M & m \\ m & M \end{pmatrix} \] (55)

with the conditions that \( M \) and \( m \) are symmetric.

(2) Now let us consider the standard case of mirror transformation

\[ \Psi(t, \vec{x}) \leftrightarrow \gamma^0 \Psi'(t, -\vec{x}) . \] (56)

Again, this transformation converts left- into right-currents, and belongs to the class described by Eq. (53). Applying this to the Lagrangian given by Eq. (51), and noting that the measure of integration \( d^4x \) is invariant under inversion, we obtain that the action is symmetric if the following conditions hold: \( M_{\alpha\beta} = (M'_{\alpha\beta})^* \) and \( m_{\alpha\beta} = (m_{\beta\alpha})^* \). Then, the neutrino mass matrix reads:

\[ \mathcal{M} = \begin{pmatrix} M & m \\ m^t & M^* \end{pmatrix} \] (57)

with the conditions that \( M \) is symmetric and \( m \) is hermitian.

In most applications considered in this paper one can take \( M = M^* \) and \( m = m^* \), and thus the cases (1) and (2) are identical. However, there can be the cases when the two mass matrices in Eqs. (55) and (57) lead to different physical situations. Consider, for example, the one family case, when the element of the matrix \( m \) is a small value of order \( \epsilon \) and the element of \( M \) is order 1 (that is the case we have in mind in our study). Let us further assume that the first parameter is real and the second is a pure phase factor \( e^{i\xi} \) (we use a very similar notation in Eq. (24)). In the first and second case, we have respectively:

\[ \mathcal{M}_1 = \begin{pmatrix} e^{i\xi} & \epsilon \\ \epsilon & e^{i\xi} \end{pmatrix} , \quad \mathcal{M}_2 = \begin{pmatrix} e^{i\xi} & \epsilon \\ \epsilon & e^{-i\xi} \end{pmatrix} \] (58)

Oscillations are described by the combination \( \mathcal{M}\mathcal{M}^\dagger \). Neglecting the terms of order of \( \epsilon^2 \), we obtain:

\[ \mathcal{M}_1\mathcal{M}_1^\dagger = \begin{pmatrix} 1 & 2\epsilon \cos \xi \\ 2\epsilon \cos \xi & 1 \end{pmatrix} . \] (59)
in case (1) and

\[ \mathcal{M}_2 \mathcal{M}_2^\dagger = \begin{pmatrix} 1 & 2\epsilon e^{i\xi} \\ 2\epsilon e^{-i\xi} & 1 \end{pmatrix}, \]

(60)
in case (2). The mixing angle is in both cases maximal, but in the first case \( \Delta m^2 \) can be much smaller than \( \epsilon \) (when the phase \( \xi \) is close to \( \pm \pi/2 \)), while in the second case it is always of order of \( \epsilon \).

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