Method for numerical optimization of the parameters of regulator from unstable condition

A A Voevoda, D O Romannikov
Automation Department, Novosibirsk State Technical University, Novosibirsk, Russia
E-mail: dmitry.romannikov@gmail.com

Abstract. The task of selecting the values of coefficients for a given controller structure from the initial unstable position is considered using the methods of numerical optimization. A method is proposed which consists in correcting the error function which is used for optimization. The main idea is multiplication by the exponent in the degree of the simulation step which the convergence of the transition process is achieved. After reaching convergence, you can perform numerical optimization. It is recommended to reduce the value of the exponent so that the figure of the transition process remains convergent. Further, when a stable transition process is reached, the value of the exponent should be zeroed out and then the optimization procedure for the error of the transition process should be performed without adjusting it until the change in the value of the objective function changes less than the specified threshold value. An example of such optimization is considered on the example of the transfer function of a second order oscillator.

1. Introduction

At present, the task of developing regulators for closed systems is relevant and is used in a variety of automatic and automated industrial systems. During the development and research of the theory of automation, a number of methods have been created that allow the structure and values of the controller coefficients to be obtained in [1, 2] proposed. All the above methods allow to obtain a regulator for the linearized model.

Another approach to the creation of control systems is the direction of learning with the confirmation of [3, 4], which assumes the absence of detailed information about the object and the formation of the control law on the basis of the obtained statistics of the object's responses to specified control actions. But reconciliation of the teaching direction methods with confirmation faces the problem that for some objects the formation of the control law takes quite a long time (there is a theoretical justification for the convergence of training with confirmation for time tending to infinity, which is hardly applicable in practice).

On the other hand, numerical optimization [3, 5] methods are often used for a stable closed system, which allow adjusting the controller coefficients depending on the optimization criterion used. At the same time, the authors did not encounter papers that propose the use of numerical methods for obtaining controller coefficients for an initially unstable system.

A closed system is considered with the object \( W(s) \) and the regulator \( W_r(s) \). The structure of the regulator is obtained by one of the existing methods for the synthesis of regulators, for example [1, 2]. In the problem under consideration, according to the known structure of the regulator, the values of the initial coefficients are arbitrarily chosen in such a way that the closed system is in an unstable state. In this case, the task is to apply numerical optimization methods in order to obtain controller values to ensure the minimum error of the optimization criterion function.
The above problem was considered for the transfer function of the object \(W(s) = \frac{1}{s^2}\).

In conclusion, we propose the idea of how to switch to neural networks from the obtained regulator structure. The resulting structure was also applied to the proposed method of stabilizing a closed system.

2. An optimization method with known regulator structure

In the article, for convenience of simulation, discrete models of transfer functions are used with a sampling time of 0.1 s, which have the following form:

\[
W(z) = \frac{0.005z + 0.005}{z^2 - 2z + 1},
\]

\[
W_r(z) = \frac{az + b}{z + c}
\]

The optimization criterion is \(Q = |v - y|\), where the value of the setpoint \(v\) takes the value of one. The initial values of parameters \(a, b, c\) are chosen in such a way that a closed system is unstable, for example, \(a = -5, b = 10, c = -10\). With the selected initial conditions, the simulation time of 3 seconds, optimization did not produce any results, and the optimization criterion did not change its value. Thus, we can conclude that the optimization by numerical methods of an unstable system is a rather complicated task. Figure 1 shows a graph of the divergent transition process, which is obtained as a result of performing an optimization procedure with an error module as an optimization criterion (similar results were obtained for the sum of squares of error as an optimization criterion).

To optimize the unstable closed dynamic system, the authors propose to additionally multiply the error by the factor \(e^{-ki}\), where \(i\) is the iteration number in the simulation. The above multiplier allows you to make a graph of errors converging and allows you to perform optimization. The purpose of this multiplication is to obtain a converging error graph and, as a consequence, the possibility of optimizing it. At the same time, in the course of research, it was found that for the convergence of the optimization process it is necessary to adjust the value of the coefficient \(k\) so that the error graph remains convergent, but not too close to zero.

Thus, for it is possible to formulate an algorithm for the method of finding parameters of a regulator according to a known structure.

1. For the object \(W\), specify the structure of the regulator \(W\) with unknown values of the coefficients.
2. To obtain the value of the transient error during the simulation time \(T\).
3. Select the value of the coefficient \(k\) so that the error graph, corrected by the factor \(e^{-ki}\), is convergent.
4. Perform the optimization procedure until the value of the optimization coefficient changes by less than e.

5. If the graph of the transition process has become stable, then set the value of the coefficient k to zero. Otherwise, go to step 3.

6. Perform the optimization procedure without considering the correction of the error value (k = 0) until the change in optimization criterion will be less than e.

The authors used the simulation to optimize the stochastic optimization Adam [6, 7] with a learning parameter value of 0.05, without regularization, with exponential decay rates of the first and second moments of 0.9 and 0.999, respectively.

3. An example of applying an optimization method with a known regulator structure

Consider the application of the optimization method on the example of the transfer function of the object $W(s) = \frac{1}{s^2}$. In the discrete representation, it has the form: $W(z) = 0.005z + 0.005z^2 - 2z + 1$ with a sampling time of 0.1 seconds. According to the above method, it is necessary to specify the structure of the regulator, which can be selected as $(as + b) / (s + c)$. Under the initial conditions of -5, 10, -10 for the coefficients a, b, c, respectively, the graphs of the error are presented in Figure 2.

![Figure 2](image-url)

**Figure 2.** The initial figure of the transition process error of the studied closed system, corrected by the factor $e^{-kc}$ with $k = 2.4$.

Further, according to the method proposed above, the value of the coefficient k was corrected several times and consistently took the values 0.4, 0. In this case, the graph of changes in the values of the coefficients has the following form (see Figure 3).

![Figure 3](image-url)

**Figure 3.** Changes in the values of the coefficients a, b, c.

In Figure 3, the number of simulations is shown on the abscissa axis, and the values of the coefficients a, b, c are plotted on the ordinate axis. The change of the coefficients occurred at 2300 and 2900 iterations.

The graph of changes in the values of coefficients in simulations from other initial conditions (the values of the coefficients a, b, c are equal to -40) is shown in Figure 4.
Figure 4. Figures of changes in the values of the coefficients during optimization starting from -40.

From Figure 4 it is clear that the simulation required a greater number of iterations. In this case, abrupt changes in the values of the coefficients on the graph also correspond to differences and coincide with the moments of the change in the value of the coefficient k.

A further area of research for this task is to replace the controller coefficients with neural networks while maintaining the controller structure.

4. Using Neural Networks with a Known Regulator Structure

Using the structure of neurons, which allows to divide the input signal into ranges (it was originally used for static tasks [8–10], for which the values of the coefficients of neurons could be calculated) did not give a positive result due to too sharp transition between the shared ranges. The values of the coefficients of the above structure for the replaced coefficient c from the structure of the regulator are shown in Figure 5.

Figure 5. Figures of the neural structure used instead of the coefficient with: a) after 100 iterations; b) after 1000 iterations; c) after 10,000 iterations.
To prevent significant differences in values, it is proposed to use an approximation of the value between two neurons instead of “hard” partitioning. For example, when dividing a range from minus one to three using four neurons, as shown in Figure 6.

![Figure 6. The structure of the neural network for the range from -1 to 3.](image)

In Figure 6, when the value of the input value falls into one of the ranges, the interpolation of a straight line between neighboring neurons is taken to form the output value of the considered neural structure, which allows optimization of the resulting structure.

5. Conclusion

The article proposes a method for optimizing the controller values by its given structure from the initial unstable position, which is based on stabilizing the error graph by multiplying it by $e^{-ki}$, where $i$ is the iteration number of the closed system simulation, $k$ is a coefficient to ensure the convergence of the graph mistakes. With a stable graph of errors, it is proposed to perform the optimization procedure gradually reducing the value of the coefficient $k$. An example of the application of the proposed method on the object $W(s) = \frac{1}{s^2}$ and the structure of the regulator $W(s) = \frac{as + b}{s + c}$ is considered.

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