Reliability modeling of power transformers with maintenance outage

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(Received 6 November 2013; final version received 4 March 2014)

Power transformers are critical equipment of the electric power transmission system, because they adjust the electric voltage to a suitable level on each stage of the power transmission from generation station to the end user. Reliability modeling of power transformers is very important for the electric grid design and risk assessment. In this paper, we propose a reliability model of power transformers with maintenance outage incorporated. The power transformer is divided into two groups of components described by different Markov state space models. Both types of state space models are combined to implement the whole power transformer modeling. Reliability indices such as probabilities of full, derated, and zero capacity as well as system unavailability are derived, and numerical evaluation of selected indices is presented with respect to different parameters. The developed models and analysis method are expected to serve as a useful component for the electric grid system design, assessment, and operation.

Keywords: reliability; power transformer; maintenance outage; stochastic modeling

1. Introduction

It is well known that power transformers become increasingly imperative for maintaining the resilience of the electric grids. Power transformers are critical equipment in every stage of electricity transmission from a power plant to the residential houses. Electricity generated at the power plant needs large transformers in the transmission substation to convert the generator’s voltage (thousands of volts level) up to extremely high voltages (hundreds of thousands of volts range) for long-distance transmission on the transmission grid (in order to reduce line losses). For electricity to be applicable to end users, it leaves the transmission grid and is stepped down via the distribution substation to the distribution grids. Various types of transformers are needed to step transmission voltages down to distribution voltages at appropriate levels depending on different requirements.

While transformers are significant to electric grids, transformer failures are common and costly. There are many causes of failure: insulation failures, design/manufacturing errors, oil contamination, overloading, line surge, loose connections, moisture, and other man-made or natural causes. Power transformers are expected to last 30–40 years under “ideal conditions.” However, Hartford steam boiler (HSB) studies (Bartley, 2000a, 2000b) have indicated that is not the case. In a 1975 study, the average age at the time of failure was 9.4 years. In HSB’s 1985 study, the average age was 11.4 years. In a later study by HSB, the average age at failure was 14.9 years. These studies about transformer age should justify the time and expense to periodically check the condition of transformers. A good maintenance program can help improve reliability of transformers and achieve maximum service life (IEEE/PES Task Force [Reliability, Risk and Probability Applications Subcommittee], 1999).

Since transformers are the key components in electric grids in terms of both reliability and investment, the reliability of transformers is a primary concern to grid operators. The reliability models of transformers are very important for the electric grid design, assessment, and operation. Much research has been done on the reliability of transformers. A condition-based evaluation method was presented to assess the lifetime status of power equipment (Sundermann, Petersson, & Fantana, 1998). The method was developed by ABB Group and is a unit-oriented approach that identifies the most vulnerable units in a population by ranking them on the basis of some certain rules. This ranking serves as the basic data for decision-making on appropriate actions. Muthanna, Sarkar, Das, and Waldner (2006) presented techniques for life assessment of the insulation of the transformers in power plants by considering the two important factors of load and ambient temperatures. The two factors are inputted to the IEEE life consumption models to assess the consumed life of insulation. Wang, Gong, and Grzybowski (2011) studied the reliability assessment methods for oil-paper insulation to help determine the reliability level of power transformers accurately and ensure their safe and stable operation. The life of oil-paper insulation was proven to obey the Weibull distribution under eight
different temperatures. Van Schijndel, Wouters, and Wetzer (2012) proposed a policy to determine the reliability of a whole transformer population from individual transformer reliabilities by estimating the life expectancies of individual components. A statistical failure model was used to obtain the population reliability figures.

Carer, Bellvis, Bouissou, Domergue, and Pestourie (2002) proposed a reliability assessment method called Boolean-logic-driven Markov processes for electrical systems along with two application examples. The method allows the definition of complex dynamic models while retaining its tractability as easy as fault trees. Seyedi, Fotuhi, and Sanaye-Pasand (2006) proposed a reliability model for power transformer protection, from which the optimum routine test and self-checking intervals of the power transformer protection system could be determined. Based on failure probabilities obtained from various collected data, Sotiropoulos, Alefragis, and Housou (2007) proposed a hidden Markov model tool for estimating the deterioration level of a power transformer. The dissolved gas analysis (DGA) field methodology was utilized for data analysis.

Sefidgaran, Mirzaie, and Ebrahimzadeh (2010) proposed a three-state Markov reliability model of the power transformer with Oil Natural and Air Natural (ONAN) cooling to calculate the probability and frequency of each state. A numerical analysis and sensitivity analysis were presented to evaluate numerical values of the model parameters and the impact of different components on the model. Hamoud (2012) proposed a probabilistic method based on Markov models for assessing the number of spare transformers required for a group of distribution transformers. Two criteria were used in determining the required number of spare transformers. The first criterion utilized the calculated group availability and the pre-determined level of the group availability. The second criterion used a cost/benefit analysis method with a minimum total cost. Bhalla, Bansal, and Gupta (2013) used the DGA and partial discharge level measurements to evaluate the condition of the insulation (as a type of health indicator) of a transformer for preventing power transformer failures.

In this paper, we propose a reliability modeling approach for power transformers by using Markov state space models. Compared with most of the existing works related to Markov modeling techniques such as Carer et al. (2002), Sotiropoulos et al. (2007), Sefidgaran et al. (2010) and Hamoud (2012), our proposed reliability model is incorporated with maintenance outage. In reality, scheduled or non-scheduled maintenance is always performed for power transformers, and maintenance may temporarily keep the components from service and thus affect the system performance. Therefore, the maintenance outage is a critical factor for the performance evaluation of power transformers and should be considered in their reliability modeling. To the best of our knowledge, little existing research has considered the impact of maintenance outage on transformer reliability modeling. Moreover, besides incorporating maintenance outage in our proposed model and deriving analytic results of interest, we provide detailed quantitative computation of the impact of maintenance outage on system performance through numerical evaluation for a better understanding of the analysis.

The main contribution of this paper is summarized as follows: (i) The main components of the power transformer are described and their respective possible causes of failure are analyzed for preparation of the reliability modeling of these components. (ii) The main components of the power transformer are classified into two groups in terms of the obtained failure modes. Separate Markov state space models are then built for individual groups. (iii) The reliability model of the whole transformer is built by combining the Markov state space models of the two groups and incorporating maintenance outage. It is worthy of note that the proposed modeling approach can be extended to more groups, not just two. (iv) The global balance equations of the reliability model of the whole transformer are developed and based on the solved results, a list of reliability indices is derived and numerical evaluation of the indices is provided with respect to different parameters for performance evaluation of the transformer system.

The remainder of the paper is organized as follows. Section 2 describes a typical power transformer structure with its components. Section 3 develops the reliability modeling of the power transformer and its components. Section 4 derives some reliability indices of interest. Section 5 presents detailed numerical evaluation. Finally, the paper is concluded in Section 6.

2. Power transformer structure

The main components of the transformer are the windings, core, insulation, tank, coolant, and bushings, as shown in Figure 1. A brief description of each component is given as follows.

Winding: The windings consist of primary and secondary coils of conducting wire that contribute to the magnetic field and carry current. The windings are arranged as cylindrical shells around the core legs, where each strand is wrapped with insulation paper. Copper or aluminum is the primary winding material. Windings on both primary and secondary sides may have external connections (called taps) to intermediate points on the winding to allow adjustment of the voltage ratio. Taps may be connected to automatic on-load tap changers for voltage regulation.

There are many types of faults in windings; short circuit is one of the most prevalent faults. If the short circuit is severe, the transformer will be damaged and must be immediately removed from service. Some faults do not cause the outage of the transformer and just disturb the normal operation of the system, which can be diagnosed by preventive tests and online monitoring such as DGA technique. When this type of fault occurs, the transformer can remain in service with a derated power load.
3. Reliability model of power transformers

Based on the description in Section 2, we classify the components of the power transformer system into two groups. Group 1 contains windings. Group 2 includes core, insulation, tank, coolant, and bushings. In the following, we propose the modeling of Group 1 and Group 2, as well as the whole transformer with maintenance outage incorporated.

3.1. Markov model of Group 1

For windings in Group 1, some major faults may cause the transformer to completely fail and need immediate removal from service; other faults may just disturb the normal full-rate operation and lead to a derated operation of the system. Thus, we can model the windings with three states. State 1 corresponds to the condition that the windings have no fault (called Up state, denoted by UP). State 2 corresponds to the condition that the windings have fault but remain in operation, and State 3 corresponds to the condition that the windings have severe fault or need immediate removal from service. The transition rates between these states depend on the failure rates of the windings and the maintenance policies.
In Group 2 is shown in Figure 3, where the five components work in series. Thus, the five components operate in series, and the failure of any one component leads to the failure of the whole transformer. The state transition rates of each component in Group 2 are shown in Figure 3. The Markov state transition diagram of each component in Group 2 is shown in Figure 3.

### 3.2. Markov model of Group 2

Group 2 consists of five components: core, insulation, tank, coolant, and bushings. Since the failure of any component will significantly reduce the transformer’s efficiency and cause critical change of output voltage on the secondary, we model the core by two states, Up and Down. Similarly, insulation, tank, coolant, and bushings all have the same result upon failure. Each of them can be modeled by two states with respective state transition rates. In addition, if a failure event occurs on each of the five components, the transformer will fail. Thus, the five components work in series.

The Markov state transition diagram of each component in Group 2 is shown in Figure 3, where $\lambda_x$ and $\mu_x$ represent the component failure rate and repair rate, respectively. The probability that the component is in Up or Down state can be found as

$$
P_U = \frac{\mu_x}{\lambda_x + \mu_x}, \quad P_D = \frac{\lambda_x}{\lambda_x + \mu_x}.
$$

$\lambda_x$ denotes $\lambda_{c1}, \lambda_{i1}, \lambda_{b1}, \lambda_{c2},$ or $\lambda_{b}.$ $\mu_x$ denotes $\mu_{c1}, \mu_{i1}, \mu_{b1}, \mu_{c2},$ or $\mu_{b}.$

### 3.3. Markov model of the transformer with maintenance outage

The power transformer consists of components of both Group 1 and Group 2. The whole transformer model can be obtained by combining the models of Group 1 and Group 2 and incorporating maintenance outage, as shown in Figure 5. State 1 represents that the transformer system operates without any fault, i.e., Winding is Up and Series structure is Up ($W_{UP}, S_{UP}$). State 2 represents that the system operates in derated service ($W_{DR}, S_{DR}$). State 3a (alternately, State 3b) represents that only Group 1 (alternately, Group 2) of the system fails. State 3c represents that the winding is in derated state while the Series structure fails ($W_{DR}, S_{DN}$), where the system does not operate and needs to be repaired. Since in this state Winding is not fully Up, it will be treated as if it were in Down state (although it is in derated state) and

$$
P_{su} = \frac{\mu_{c1}}{\lambda_{c1} + \mu_{c1}} \frac{\mu_{i}}{\lambda_{i} + \mu_{i}} \frac{\mu_{t}}{\lambda_{t} + \mu_{t}} \frac{\mu_{c2}}{\lambda_{c2} + \mu_{c2}} \frac{\mu_{b}}{\lambda_{b} + \mu_{b}}.
$$

It can also be found (Billinton & Allan, 1994) that the equivalent failure rate $\lambda_s$ is the sum of the failure rates of all five components, i.e.

$$
\lambda_s = \lambda_{c1} + \lambda_i + \lambda_t + \lambda_{c2} + \lambda_{b}.
$$

By definition, we have $P_{su} = \mu_s/(\lambda_s + \mu_s),$ thus the equivalent repair rate $\mu_s$ can be derived as

$$
\mu_s = \frac{\lambda_{c1} + \lambda_i + \lambda_t + \lambda_{c2} + \lambda_{b}}{(1 + \lambda_{c1}/\mu_{c1})(1 + \lambda_i/\mu_{i})(1 + \lambda_t/\mu_{t})}

\frac{1}{(1 + \lambda_{c2}/\mu_{c2})(1 + \lambda_{b}/\mu_{b}) - 1}.
$$
triggered a repair process. That is, when the system works in derated state and a failure event happens for Group 2, a repair process will be triggered for both Group 2 and Group 1 (which is in derated service). This is reasonable since the fully Up state is always preferred for the transformer system. Therefore, State 3c has transition paths to State 3a and State 3b with respective rates.

As mentioned above, a good maintenance program can improve reliability of transformers and achieve maximum service life. Even though the transformer system operates without any fault, it is absolutely necessary to schedule preventive maintenance for all the components. This preventive maintenance may temporarily keep the components from service for a short time of duration, but the recompense to the transformer lifetime is immeasurable. In Figure 5, the transition rate λm from State 1 to State 4 (WMT, SDN) represents the maintenance outage rate of the transformer system. The rate μm from State 4 to State 1 represents the maintenance processing rate, which is the reciprocal of the mean time required for maintenance.

Let Ps denote the steady-state probability that the transformer system is in state k, where k ∈ {1, 2, 3a, 3b, 3c, 4}. The global balance equations of the system (Harrison & Patel, 1993) are as follows.

\[
\begin{align*}
(\lambda_w + \lambda_s + \lambda_m)P_1 &= \mu_w P_{3a} + \mu_s P_{3b} + \mu_m P_4, \\
(\lambda_s + \lambda_{dr})P_2 &= \beta \lambda_w P_1, \\
\mu_w P_{3a} &= (1 - \beta)\lambda_w P_1 + \lambda_{dr} P_2 + \mu_s P_{3c}, \\
\mu_s P_{3b} &= \lambda_s P_1 + \mu_w P_{3c}, \\
(\mu_w + \mu_s)P_{3c} &= \lambda_s P_2, \\
\lambda_m P_1 &= \mu_m P_4.
\end{align*}
\]

By applying the normalization condition, we have

\[P_1 + P_2 + P_{3a} + P_{3b} + P_{3c} + P_4 = 1.\]

Solving the above equations, we determine \(P_1\) as

\[P_1^{-1} = 1 + \frac{(1 - \beta)\lambda_w + \lambda_s + \lambda_m + \beta\lambda_w}{\mu_w + \lambda_s + \lambda_{dr}} \left[1 + \frac{\lambda_{dr}}{\mu_w + \mu_s}\left(1 + \frac{\mu_w + \mu_s}{\mu_w}\right)\right],\]

Then, the rest probabilities can be derived as

\[P_2 = \frac{\beta\lambda_w}{\lambda_s + \lambda_{dr}} P_1,\]

\[P_{3a} = \left[\frac{(1 - \beta)\lambda_w + \beta\lambda_w}{\mu_w + \lambda_s + \lambda_{dr}} \left(\frac{\lambda_{dr} + \mu_s}{\mu_w + \mu_s + \mu_s}\right)\right] P_1,\]

\[P_{3b} = \left(\frac{\lambda_s}{\mu_w + \mu_s}\right) P_1,\]

\[P_{3c} = \frac{\lambda_m}{\mu_m} P_1,\]

\[P_4 = \frac{\mu_m}{\mu_m} P_1.\]

**4. Transformer system reliability indices**

After the system steady-state probabilities are obtained, some reliability indices can be derived. Let us define the following four events:

\[\begin{align*}
E_1 &= \{(W_{UP}, S_{UP})\}, \\
E_2 &= \{(W_{DR}, S_{UP})\}, \\
E_3 &= \{(W_{DN}, S_{UP}), (W_{UP}, S_{DN}), (W_{DR}, S_{DN})\}, \\
E_4 &= \{(W_{MT}, S_{MT})\},
\end{align*}\]

where \(E_1\) represents the event that the transformer system is fully up (i.e. the system has full capacity); \(E_2\) represents the event that the system provides derated service (i.e. the system has derated capacity); \(E_3\) represents the event that the system cannot operate (i.e. the system has zero capacity) due to component down; and \(E_4\) represents the event that the system cannot operate due to component maintenance. Thus, the probability that the system has full capacity is

\[P(E_1) = P_1.\]

The probability that the system has derated capacity is

\[P(E_2) = P_2.\]
The probability that the system has zero capacity due to component down is

\[ P(E_3) = P_{3a} + P_{3b} + P_{3c}. \]  

(20)

The probability that the system has zero capacity due to maintenance is

\[ P(E_4) = P_{4}. \]  

(21)

The transformer system unavailability, denoted by \( U \), is the total probability that the system cannot operate.

\[ U = P_{3a} + P_{3b} + P_{3c} + P_{4}. \]  

(22)

The frequency of an event (Billinton & Allan, 1994) is defined by the product of the transition rates departing from the event state and the state probability, or the product of the transition rates arriving at the event and their respective state probabilities where these transitions start. Thus, the frequencies of the events \( E_1, E_2, E_3 \), and \( E_4 \) are derived as follows:

\[ f_{E_1} = (\lambda_w + \lambda_s + \lambda_m)P_1, \]  

(23)

\[ f_{E_2} = \beta \lambda_m P_1, \]  

(24)

\[ f_{E_3} = (\lambda_w + \lambda_s)P_1, \]  

(25)

\[ f_{E_4} = \lambda_m P_1, \]  

(26)

The mean duration of an event is obtained by the ratio of the event probability to the event frequency. The mean durations of events \( E_1, E_2, E_3 \), and \( E_4 \) are

\[ d(E_1) = \frac{P(E_1)}{f_{E_1}} = \frac{1}{\lambda_w + \lambda_s + \lambda_m}, \]  

(27)

\[ d(E_2) = \frac{P(E_2)}{f_{E_2}} = \frac{1}{\lambda_s + \lambda_{dr}}, \]  

(28)

\[ d(E_3) = \frac{P(E_3)}{f_{E_3}} = \frac{1}{\lambda_w + \lambda_s} \left( \frac{(1 - \beta) \lambda_w}{\mu_w} + \frac{\lambda_s}{\mu_s} + \psi \right), \]  

(29)

where

\[ \psi = \frac{\beta \lambda_w}{\lambda_s + \lambda_{dr}} \left[ \frac{\lambda_{dr}}{\mu_w} + \frac{\lambda_s}{\mu_w} \left( 1 + \frac{\mu_w}{\mu_s} + \frac{\mu_s}{\mu_w} \right) \right]. \]

(30)

5. Numerical evaluation

In this section, we present numerical evaluation for the transformer system by studying selected reliability indices with respect to various parameters, particularly the maintenance outage rate and maintenance processing time. The typical parameter settings for numerical evaluation are given in Table 1. Other values of related parameters are set separately in each figure to study the performance of the relevant reliability index.

![Figure 6. Probability of full capacity \( P(E_1) \) vs. windings failure rate multiplier (increased by 10 times).](image)

Table 1. Typical parameter configuration for numerical evaluation.

| Parameters | Value | Unit   | Description   |
|------------|-------|--------|---------------|
| \( \lambda_w \) | 0.005 | Failures/year | Total fault rate |
| \( 1/\mu_w \) | 1000  | Hours   | Repair time   |
| \( \beta \) | 0.5   |         | Proportion    |
| \( \lambda_{dr} \) | 0.1   | Failures/year | Failure rate |
| \( \lambda_m \) | 0.2   | Outages/year | Maintenance rate |
| \( 1/\mu_m \) | 100   | Hours   | Maintenance time |
| \( \lambda_{cl} \) | 0.001 | Failures/year | Failure rate |
| \( 1/\mu_{cl} \) | 480   | Hours   | Repair time   |
| \( \lambda_i \) | 0.004 | Failures/year | Failure rate |
| \( 1/\mu_i \) | 360   | Hours   | Repair time   |
| \( \lambda_{c2} \) | 0.003 | Failures/year | Failure rate |
| \( 1/\mu_{c2} \) | 360   | Hours   | Repair time   |
| \( \lambda_b \) | 0.002 | Failures/year | Failure rate |
| \( 1/\mu_b \) | 72    | Hours   | Repair time   |
| \( \lambda_{w} \) | 0.003 | Failures/year | Failure rate |
| \( 1/\mu_{w} \) | 48    | Hours   | Repair time   |

Figure 6 shows the probability of full capacity \( P(E_1) \) with respect to the change of parameters \( \lambda_m \) and \( \mu_m \). As expected, when the maintenance rate \( \lambda_m \) increases or the maintenance time \( 1/\mu_m \) increases, the probability \( P(E_1) \) will decrease. The impact of the maintenance time on \( P(E_1) \) is minimal as compared to that of the maintenance rate. It can also be observed that when the windings failure rate is increased, the \( P(E_1) \) will decrease, too. The reason is that the failure of windings is one of the factors leading to the transformer system failure.

Figure 7 shows the probability of zero capacity due to maintenance \( P(E_4) \) with respect to the change of parameters \( \lambda_m \) and \( \mu_m \). When the rate \( \lambda_m \) is increased or the maintenance time \( 1/\mu_m \) is increased, the probability \( P(E_4) \) will increase. Similar to Figure 6, the impact of the maintenance time on \( P(E_4) \) is smaller than that of the maintenance rate. We also observe that the change of the windings failure rate
Figure 7. Probability of zero capacity due to maintenance $P(E_4)$ vs. windings failure rate multiplier (increased by 10 times).

Figure 8. System unavailability $U$ vs. windings failure rate multiplier (increased by 10 times).

Figure 9. Probability of full capacity $P(E_1)$ vs. proportion factor $\beta$.

Figure 10. Probability of derated capacity $P(E_2)$ vs. proportion factor $\beta$.

Figure 11. System unavailability $U$ with respect to the change of parameters $\lambda_m$ and $\mu_m$. As expected, when the maintenance rate $\lambda_m$ is increased or the maintenance time $1/\mu_m$ is increased, the system unavailability $U$ will increase. The maintenance program considered here leads to scheduled outages of short periods. Similarly, the impact of the maintenance time on $U$ is smaller than that of the maintenance rate. It can also be observed that when the windings failure rate is increased, the system unavailability $U$ will increase, too. The failure of windings is one of the reasons causing an unavailable system.

Figure 9 shows the probability of full capacity $P(E_1)$ with respect to the change of parameters $\lambda_m$ and $\beta$. As expected, when the maintenance rate $\lambda_m$ is increased, the probability $P(E_1)$ will decrease. It can also be observed that when the factor $\beta$ is increased, $P(E_1)$ will decrease, too. The reason can be explained as follows. As $\beta$ is increased, more transitions will enter State 2 in Figure 5, which will take longer time to return to State 1 (i.e. system of full capacity) via State 3a, compared with the transitions between State 1 and State 3a.

Figure 10 shows the probability of derated capacity $P(E_2)$ with respect to the change of parameters $\lambda_m$ and $\beta$. We observe that $P(E_2)$ will decrease very little (almost no change) when $\lambda_m$ is increased remarkably. This means that the change of maintenance rate does not have a direct impact on the derated state probability. However, the change of $\beta$ will have direct impact on the probability $P(E_2)$. The larger the value of $\beta$, the larger the probability $P(E_2)$ can be achieved. This is also consistent with the analysis shown in Figure 5.

Figure 11 shows the system unavailability $U$ with respect to the change of parameters $\lambda_m$ and $\beta$. We observe that $U$ will increase when $\lambda_m$ is increased. The reason is explained in Figure 8. On the other hand, we observe that in the given parameter configuration, the change of $\beta$ has
little impact on the system unavailability. This is probably due to the relatively larger value of $\lambda_{dr}$ as compared with $\lambda_w$ (Table 1).

Figure 12 shows the probability of full capacity $P(E_1)$ with respect to the change of parameters $\lambda_m$ and $\lambda_w$. When the maintenance rate $\lambda_m$ is increased, the probability $P(E_1)$ will decrease. This decrease becomes more significant when the windings fault rate $\lambda_w$ is small. As $\lambda_w$ becomes larger, it will dominate the change of $P(E_1)$. In Figure 12, we can observe that when $\lambda_w = 0.005$ outages/year, the change of $\lambda_m$ almost does not affect $P(E_1)$. We also observe that when $\lambda_w$ becomes larger, the probability $P(E_1)$ will decrease distinctly.

Figure 13 shows the probability of derated capacity $P(E_2)$ with respect to the change of parameters $\lambda_m$ and $\lambda_w$. When the maintenance rate $\lambda_m$ changes markedly, the probability $P(E_2)$ almost does not change. The reason is explained in Figure 10. We also observe that when $\lambda_w$ is increased, $P(E_2)$ will increase markedly. As $\lambda_w$ becomes larger, more transitions will enter the derated state, leading to a larger value of $P(E_2)$.

Figure 14 shows the probability of zero capacity due to component down $P(E_3)$ with respect to the change of parameters $\lambda_m$ and $\lambda_w$. It is intuitive to observe that the probability $P(E_3)$ decreases very little (almost no change) with the increase of the maintenance rate $\lambda_m$. On the other hand, we observe that $P(E_3)$ increases with the increase of the windings fault rate $\lambda_w$. This is because the windings is a component of the system and the failure of windings contributes to the probability $P(E_3)$.

Figure 15 shows the probability of zero capacity due to maintenance $P(E_4)$ with respect to the change of parameters $\lambda_m$ and $\lambda_w$. As expected, when the maintenance rate $\lambda_m$ is increased, the probability $P(E_4)$ will increase remarkably, especially at the condition of small value of $\lambda_w$. When $\lambda_w$ becomes larger, the impact of $\lambda_m$ on the probability of zero capacity will become smaller, since $\lambda_w$ becomes dominant.
over $\lambda_m$. Due to the same reason, when $\lambda_w$ is increased, $P(E_4)$ will decrease.

6. Conclusions

We proposed a reliability model of power transformers with maintenance outage. The power transformer was divided into two groups of components described by separate Markov state space models. The two separate models were analyzed and combined to implement the modeling of the whole power transformer by incorporating maintenance outage. The whole transformer model was then analyzed by the balance equations method and some reliability indices of interest such as probabilities of full, derated, and zero capacities, as well as system unavailability were derived. Detailed numerical evaluation of selected indices was provided for further understanding and verification of the analytic results. The developed models and analysis method are expected to serve as a useful component for the electric grid system design, assessment, and operation.

Acknowledgements

This work was supported by the PORTAL program in Missouri Western State University, St. Joseph, MO, USA.

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