Testing of car speed approximation equation from yaw mark using radio control car simulator

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Abstract. In forensic science, yaw mark is one of the interesting evidences from the traffic accident. It caused by a vehicle wheel slides on the surface of the road when the speed before braking of the vehicle is high enough. Therefore, prediction of the vehicle speed is important to compare with the legal maximum speed. This research focused on the simulation of a car's braking on a curve using the radio control car. High-speed video and Tracker Video Analysis (Tracker) software were used to investigate the positions and speeds of the radio control car. The braking path from Tracker software was assumed as the yaw mark in the real accident scene. The equation of the braking path was investigated using the total least-squares spiral curve fitting. The speeds of the car for each position of the braking path were calculated by the method from Daily’s research and the new method modified from Daily’s research. The predicted speed was compared with the one measured from Tracker. The results were similar at the beginning of the braking path, but increasingly different when the speed of the radio control car decreased. Moreover, the modified Daily’s method can investigate the speed better than the Daily’s method for the speed based on a clothoid spiral fitting and Archimedean spiral fitting. Findings from this study can be used to develop instructional materials for intermediate mechanics to demonstrate a real application.

1. Introduction

Driving over the speed limit is a major cause of the traffic accident. However, one of the problems from the traffic accident is investigating the speed of car before the accident to compare with the legal maximum speed. Since some accident scenes and some cars do not have the surveillance camera to record the video when the accident happens. Yaw mark is the interesting evidence from the traffic accident scene especially the curved yaw mark from the accident in road curve because we can approximate the maximum speed of car from the radius of curvature.

An interesting example of the research was performed in 2012 by Daily [1]. He studied the real curved yaw mark from breaking and proposed the method to investigate the speed of car and used the clothoid spiral, logarithmic spiral and Archimedean spiral to fit the equation of the braking path. In this study, the radio control car was used to simulate a car's braking on a curve and the speeds of the radio control car were calculated using Daily’s method and the new method modified from Daily’s research.
2. Methodology

The radio control car was used to test a car’s braking on a curve. The high-speed video was used to record the video from the radio control car's braking and Tracker Video Analysis (Tracker) software [2] were used to investigate the positions and speeds of the radio control car as shown in figure 1. The braking path from this software was assumed as the yaw mark in the real accident scene.

Figure 1. Investigation of the positions and speeds of the radio control car using Tracker software.

After that, the data of positions from Tracker software were used to calculate the speed for each position of the braking path using the Daily’s method. The steps are as follows:

1. Find the equation of the braking path using the total least-squares spiral curve fitting in the Solver Add-in for Microsoft Excel program. The spiral curves used in this study are the clothoid spiral, logarithmic spiral and Archimedean spiral.
2. Find the radii of curvature $r_n$ and the distances $s_n$ when starting break in all data points.
3. Calculate the maximum speed $v_{\text{max}}$ in all points from the radius of curvature $r_n$ in step 2 using the equation

$$v_{\text{max}} = \sqrt{\mu_s r_n g},$$

where $\mu_s$ is the coefficient of static friction and in this study $\mu_s = 0.472$.
4. Calculate the tangential accelerations $a_{n \rightarrow n+1}$ between two data points ($n$ and $n+1$) using the equation

$$a_{n \rightarrow n+1} = \frac{v_{n+1(\text{max})}^2 - v_{n(\text{max})}^2}{2(s_{n+1} - s_n)},$$

where $v_{n(\text{max})}$ and $v_{n+1(\text{max})}$ are the maximum speed in step 3.
5. Calculated the average tangential acceleration $a_{\text{avg}}$ from the average all $a_{n \rightarrow n+1}$ data in step 4.
6. Calculate the speeds at the starting break point $v_0$ from the other data points using the equation

$$v_0 = \sqrt{r_n \sqrt{\left(\mu_s g\right)^2 - a_{\text{avg}}^2} - 2a_{\text{avg}} s_n}.$$
Equation (3) is derived from considering both tangent and normal acceleration, assuming that the tangential acceleration along the breaking path is constant, and use the radius of curvature \( r_n \) and the distance \( s_n \) from step 2.

7. Calculate the average speed at the starting break point from the average all \( v_0 \) data in step 6.

8. Calculate the speeds \( v_n \) for each position of the braking path using the equation

\[
 v_n = \sqrt{v_0^2 + 2a_{\text{avg}}s_n},
\]

assuming that the tangential acceleration \( a_{\text{avg}} \) along the breaking path is also constant.

Since the Daily’s method assumes that the tangential acceleration along the breaking path is constant, we will propose the new method modified from Daily’s research to investigate the speed of car. The difference between the new method and the Daily’s method is in step 8. The tangential accelerations between two data points in step 4 are used to calculate the speeds for each position instead of the average tangential acceleration in step 5 as shown in the equation

\[
 v_{n+1} = \sqrt{v_n^2 + 2a_{n-n+1}(s_{n+1} - s_n)}.
\]

Finally, the speeds from all methods were compared with the speed from Tracker software.

3. Results and discussion

From the total least-squares spiral curve fitting of the braking path, the result from clothoid spiral, logarithmic spiral and Archimedean spiral are shown in figure 2. The spiral curve fitting of the braking path using clothoid spiral is quite different from Tracker data points compared with the other spirals especially at the first and the end of considered data points.

Figure 2. Spiral curve fitting of the braking path using (a) clothoid spiral (b) logarithmic spiral and (c) Archimedean spiral.

From the Daily’s method and the modified Daily’s method, the speeds of the radio control car based on each spiral fitting are shown in figures 3-5. The modified Daily’s method can calculate the speed better than the Daily’s method for the speed based on all spiral fitting except logarithmic spiral has the same results. The speed at the starting break point based on clothoid spiral fitting is quite different from the Tracker software. This corresponds to the spiral curve fitting results in figure 2. For the speed based on Archimedean spiral fitting, the speed at the beginning of the braking path are nearest to the speed from Tracker software compared with the other spirals. However, the speed based on both three spirals fitting is increasingly different from Tracker when the speed of the car decreased.
Figure 3. Speed of the radio control car for each position of the braking based on a clothoid spiral fitting compared with the Tracker software data.

Clothoid spiral equations

\[ x = x_0 + a \left( C(t) \cos \theta_0 - S(t) \sin \theta_0 \right) \]
\[ y = y_0 + a \left( S(t) \cos \theta_0 + C(t) \sin \theta_0 \right) \]

Figure 4. Speed of the radio control car for each position of the braking based on a logarithmic spiral fitting compared with the Tracker software data.

Logarithmic spiral equations

\[ x(\theta) = x_0 + ae^{\beta \theta} \cos \theta_0 \]
\[ y(\theta) = y_0 + ae^{\beta \theta} \sin \theta_0 \]

Figure 5. Speed of the radio control car for each position of the braking based on an Archimedean spiral fitting compared with the Tracker software data.

Archimedean spiral equations

\[ x(\theta) = x_0 + c\theta^{\beta} \left( \cos \theta \cos \theta_0 - \sin \theta \sin \theta_0 \right) \]
\[ y(\theta) = y_0 + c\theta^{\beta} \left( \sin \theta \cos \theta_0 + \cos \theta \sin \theta_0 \right) \]

4. Conclusion
The radio control car can be used to simulate a car’s braking situation on a curve. The braking path assumed as the yaw mark can be calculated using Tracker software. From the results, the modified Daily’s method can improve the accuracy of speed from the Daily’s method especially the speed based on Archimedean spiral fitting at the beginning of the braking path although the speed is more different from Tracker software when the speed of the radio control car decreased.

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