DISTANCES FROM THE CORRELATION BETWEEN GALAXY LUMINOSITIES AND ROTATION RATES

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Abstract

A large luminosity–linewidth template sample is now available, improved absorption corrections have been derived, and there are a statistically significant number of galaxies with well determined distances to supply the zero point. A revised estimate of the Hubble Constant is $H_0 = 77 \pm 4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ where the error is the 95% probable statistical error. Systematic uncertainties are potentially twice as large.

1. Introduction

More massive galaxies have more stars than less massive galaxies and more massive galaxies rotate faster. This simple reasoning leads to the expectation of a correlation between the light of the stars and a measure of the rotation rate (Tully & Fisher 1977) though the small scatter of the correlation is not trivially explained. The measurement of the rotation rate, say from the width of a global neutral hydrogen profile, is independent of distance. Hence, if the relationship is calibrated in terms of the intrinsic luminosity dependence with linewidth then the modulus between the apparent and absolute magnitude at a given linewidth gives a distance.

Nowadays there are more accurate methods for measuring the distance to an individual galaxy. However, the great virtue of the luminosity–linewidth method is that it can be used to get distances for thousands of galaxies all over the sky. Roughly 40% of galaxies with $M_B < -16^m$ are potential targets. Current techniques allow candidates to be accessed out to $V \sim 10,000 \text{ km s}^{-1}$. Consequently, a determination can be made of the Hubble Constant based on distance measurements dispersed on the sky and in a regime with modest consequences due to peculiar velocities.
It is a good moment to review the results obtained with this method. Lately there has been a big input of high quality data. Coupled with this happy situation is the dramatic improvement in the absolute calibration as a result of the determination of distances by the cepheid pulsation method with Hubble Space Telescope. These two improvements address what were the greatest deficiencies in the path to the Hubble Constant based on the luminosity–linewidth method. It will be shown during the ensuing discussion that today there are sufficiently large numbers of both calibrators and targets that the statistical accuracy of the method is very good. Uncertainties are now dominated by potential systematic effects.

Here is an outline of the structure of the article. First there will be a discussion of the raw data: luminosities, axial ratios, and linewidths. The information comes from many sources. The observed parameters require adjustments for modifying effects like extinction and projection. Then the methodology of the construction and application of the luminosity–linewidth correlation will be described. The potential problems of Malmquist bias must be confronted. It will be described how a template luminosity–linewidth correlation is constructed and then transformed to an absolute magnitude scale. Then the template relation is imposed upon data in clusters that for the most part are beyond the Local Supercluster, at distances where peculiar velocities should be only a modest fraction of expansion velocities. That exercise results in distance estimates for the clusters and a determination of the Hubble Constant.

2. Data

Three parameters must be measured: an apparent magnitude, a characterization of the rotation rate, and an estimator of the inclination needed to compensate for projection effects. Each of these components will be considered in turn. Then, there will be a discussion of the adjustments to be made to get to the parameters that are used in the correlations.

2.1. LUMINOSITIES

Area photometry with optical and near-infrared imagers has come of age. Large format detectors on modest sized telescopes provide fields of view that encompass the entire target galaxies. The author has had an on-going program of both optical and infrared photometry (Pierce & Tully 1988, 1992; Tully et al. 1996, 1998). For the purposes of the present discussion, the other important sources of luminosities are Mathewson, Ford, & Buchhorn (1992), Han (1992) and Giovanelli et al. (1997b). The latter three sources provide $I$ band magnitudes for galaxies in clusters at intermediate to large distances. The collaboration involving Pierce and Tully is producing $B, R, I$
magnitudes for many nearby galaxies, including the calibrators, and for
galaxies in a couple of distant clusters that overlap with the other sources.
In addition, data in the $K'$ band is provided in Tully et al. (1996, 1998) for
two of the clusters.

At the moment, there is a lot more data available at $I$ than at other
bands so most of the analysis presented in this paper will be based on
this material. There is interest in the other bands, though, because of the
insidious effects of obscuration. There will be some comfort that there is
proper compensation for these effects if relative distances are the same at
different passbands. The $K'$ material is of particular interest in this regard
since obscuration should be very small at 2 microns.

The issue of adjustments to magnitudes because of obscuration and
spectral shifting will be discussed in a later section. The concern at this
point is the homogeneity of the raw magnitudes from various sources. Dif-
ferent authors measure magnitudes to slightly different isophotal levels then
usually extrapolate to total magnitudes: Han extrapolates from $23.5^m$, Gio-
vanelli et al. extrapolate from $\sim 24^m$, Mathewson et al. extrapolate from
$25.0^m$, Tully et al. (1996) extrapolate from $25.5^m$, and Pierce & Tully give
a total magnitude to sky at $\sim 26^m$. The added light at the faintest levels is
small for the high surface brightness galaxies that are relevant for the deter-
mination of $H_0$. Typical extrapolations from $25.5^m$ to infinity add $\sim 0.02^m$
and always less than $0.1^m$ for galaxies in the appropriate magnitude range
(Tully et al. 1996). Magnitudes measurements are more vulnerable to the
detailed fitting of the sky level. Variations at the level of $\sim 0.05^m$ can arise
with systematic differences in sky fitting procedures.

Inter-comparisons between sources indicate that everybody is working
on the same system and that systematics are almost negligible. Some offsets
have been reported, for example Giovanelli et al (1997b) adjust Mathewson
et al. data (1992) to match their own. However, globally the data sets are
consistent with each other at a level of 2% in effect on distances. Object by
object, rms differences between any pair of observers is at or below $\pm 0.1^m$.
In the present analysis all sources are given equal weight and luminosi-
ties are averaged if there are multiple observations. Overlap measurements
reveal spurious results in a few percent of cases. If a difference between
sources is big enough it is usually evident which measurement is wrong.

2.2. INCLINATIONS

Projection corrections are required to recover true disk rotation rates and
to compensate for differential obscuration. Uncertainties in inclination es-
specially affect de-projected velocities as one approaches face-on. With rare
exception, inclinations are derived from a characteristic axial ratio of the
main or outer body of a galaxy. From experience, it is found that such inclination measurements are reproducible at the level of $\pm 3^\circ$ rms. However, errors are non-gaussian. From the radial variations in axial ratios and from such independent considerations as inclination estimates from velocity fields it is suspected that errors of $\sim 10^\circ$ are not uncommon. The $1/\sin i$ deprojection correction becomes grossly uncertain toward face-on. A sample cut-off at $i = 45^\circ$ is invoked to avoid large errors.

The derivation of an inclination from an axial ratio requires an assumption about the intrinsic thickness of the system. The standard formulation for the derivation is $\cos i = \sqrt{(q^2 - q_0^2)/(1 - q_0^2)}$ where $q = b/a$ is the observed ratio of the minor to major axes and $q_0$ is the intrinsic axial ratio. The thinnest systems are spirals of type Sc. Earlier types have bulges and later types are puffed up. For simplicity, $q_0 = 0.20$ is often used. A more elaborate specification of $q_0$ that depends on type could be justified. Giovanelli et al. (1997b) provide an extreme example with their choice $q_0 = 0.13$ for type Sc. A smaller $q_0$ value results in derived inclinations that are more face-on. Fortunately the choice of $q_0$ has negligible effect on the measurement of distances as long as one is consistent. The difference $q_0 = 0.13$ or 0.20 gives a difference in inclination, for an observed $q = 0.20$, of $81^\circ$ or $90^\circ$ respectively. However the $1/\sin i$ difference on the corrected linewidth is only $1.2\%$. As one progresses toward larger $q$ the difference in assigned inclination is reduced but the $1/\sin i$ amplifier is growing. The product of the two effects is a roughly constant shift of $1.2\%$ in the corrected linewidth at all inclinations $i > 45^\circ$. If both calibrators and distance targets are handled in the same manner there will be no effect on measured distances.

Difficulties with projection enter luminosities in the opposite regime, as galaxies are presented toward edge-on. It has become popular to formulate extinction corrections directly in terms of the observed $q$ value which avoids a dependence on the parameter $q_0$.

Inter-comparisons between the sources of photometry used in this study fail to reveal any systematic differences in $q$ measurements between authors, though big individual differences are not uncommon. Big differences raise flags that prompt special attention.

2.3. LINEWIDTHS

It has become popular to measure rotation parameters via both optical and radio techniques. The original radio methods are simpler but are constrained by detector sensitivity to modest redshifts. The methods that involve optical spectra require more work but can be used to larger distances. With care, the two techniques can be reconciled in a common characterization of the projected rotation. However that synthesis will not be attempted
here. There are plentiful and sufficiently distant observations of profiles in
the 21 cm neutral hydrogen line for the purpose of determining $H_0$. The
complexity of intermingling radio and optical data can be avoided.

Even restricted to the HI data, there is a bit of a mess. For once in
astronomy, resolution is not an unmitigated advantage. In this case it is
desired that the beam project onto an area larger than the galaxy in order
to enclose most of the emission. As a result, the data on nearby, large
galaxies is handed down from observations on old telescopes from the days
of paper strips. The parameterizations are still quite ‘personalized’. Worse
than with magnitudes, one has to be careful that one is using a consistent set
of linewidth information from the near field to far. In this study, HI profile
linewidths defined at the level of 20% of the peak flux are used (called
$W_{20}$). These linewidths are only adequately measured if the line signal-to-
noise is greater than 7. A decent signal typically provides a measurement
with an accuracy of better than 10 km s$^{-1}$. The 20% linewidths are then
transformed into the parameter $W_R$ defined by Tully & Fouquè (1985). This
parameter approximates twice the maximum rotation velocity of a galaxy.

The other linewidth characterization in common use is the width at 50%
of peak flux in each horn of the profile ($W_{50}$: Haynes et al. 1997) which is
then adjusted to account for instrumental and thermal broadening (Gio-
vanelli et al. 1997b). The advantages and disadvantages of the alternative
systems are technical and not important. The key concern is that the in-
formation available over both north and south hemispheres and for both
nearby large galaxies and those distant and small be brought to a common
system. The current analysis draws on a large database of $W_{20}$ measure-
ments and a supplement of $W_{50}$ values in clusters well beyond the Local
Supercluster. The important new contributions in the $W_{50}$ system are only
partially within the public domain so it has not yet been possible to make a
detailed comparison based on the $\sim 10^3$ galaxies that must be mutually ob-
served in the two systems. The present study relys on an inter-comparison
of only 66 galaxies in 3 clusters. It is found that $< W_{20} - W_{50} > = 25$ km s$^{-1}$
with 12 km s$^{-1}$ rms scatter and a standard deviation of 2 km s$^{-1}$. There is
no apparent trend with $W$ or between the three clusters. While acceptable
for a preliminary foray, this inter-comparison of the $W_{20}$ and $W_{50}$ systems
must, and can easily, be improved upon.

2.4. EXTINCTION CORRECTIONS

Along with the Hubble Space Telescope contribution to the zero-point cal-
ibration and the abundance of new material, the third significant improve-
ment of late has been in the compensation for extinction. Giovanelli et al.
(1995) made a convincing case for a strong luminosity dependence in the
Figure 1. Dependence of the extinction amplitude parameter $\gamma_\lambda$ on absolute magnitude. Data are presented for $B, R, I$ bands in the three separate panels. The filled squares correspond to values of $\gamma_\lambda$ derived from deviations from mean color relations as a function of $b/a$. The filled triangles correspond to equivalent information derived from deviations from luminosity–line profile width correlations as a function of $b/a$. The small circles in the $I$ panel are data taken from Giovanelli et al. (1995). The dashed straight line in the $I$ panel is a least squares fit to the Giovanelli et al. data (errors in $\gamma$). The solid straight lines in this and the other panels are least squares fits to the data by Tully et al. (1998). The dotted straight line in the $I$ panel gives equal weight to the two sources of data.

Obscuration properties of galaxies and Tully et al. (1998) have further quantified the effect. The latter work has profited from the leverage provided by information in passbands from $B$ to $K'$. A giant galaxy can be dimmed by 75% at $B$ if it is viewed edge-on rather than face-on, although a dwarf galaxy of the luminosity of the Small Magellanic Cloud statistically has not enough extinction at $B$ to measure. At $K'$ the most luminous galaxy is dimmed by a maximum of 20%.

The extinction can be described by the expression $A_\lambda = \gamma_\lambda \log(a/b)$ where $a/b$ is the major to minor axis ratio and $\lambda$ is the passband. The correction is to face-on orientation but does not account for the residual absorption in a face-on system. Figure 1 provides a plot from Tully et al. (1998) that shows the dependence on luminosity of $\gamma_\lambda$ for $\lambda = B, R, I$. Given this strong luminosity dependence, there is a problem because absolute magnitudes are not known a priori. Absolute magnitudes are to be an output of the distance estimation process so they cannot also be an input. Both Giovanelli et al. (1997b) and Tully et al. (1998) recast the corrections for magnitudes so the dependency is on the distance independent linewidth parameter. This conversion is provided through the luminosity–linewidth
calibrators. The formulations presented by Tully et al. (1998) are:

\[
\begin{align*}
\gamma_B &= 1.57 + 2.75(\log W_R^i - 2.5) \\
\gamma_R &= 1.15 + 1.88(\log W_R^i - 2.5) \\
\gamma_I &= 0.92 + 1.63(\log W_R^i - 2.5) \\
\gamma_{K'} &= 0.22 + 0.40(\log W_R^i - 2.5)
\end{align*}
\]

There is a fortunate interplay that minimizes the effect of uncertain inclination on \(A_\lambda\). If the inclination is taken too face-on because of an spuriously large \(b/a\) then \(W_R^i\) is overestimated, which drives up \(\gamma_\lambda\), but is offset by a low \(\log(a/b)\) in the product that gives \(A_\lambda\).

The other corrections to be made are modest and non-controversial. Absorption at \(I\) due to obscuration in our own Galaxy is taken to be 41\% of the \(B\) band value given by Burstein & Heiles (1984). There is a small ‘k-correction’ of 1.27\%.

3. Methodology

Over the years many people have used luminosity–linewidth relations to measure distances and there has been controversy. An extreme view has been presented by Sandage (1994b). According to him, there can be large biases that distort distance measurements and limit the usefulness of the procedure. In this section there will be a description of a way of conducting the analysis that results in unbiased distance estimates and, hopefully, accurate results. The reader interested in making a comparison will find that the method to be described is not the method used by Sandage.

3.1. BIASES

Malmquist (1920) discussed a bias that might create a problem with measurements of distances to objects selected by apparent magnitude. Teerikorpi (1984) and Willick (1994) have discussed the problem in the present context. Schechter (1980) and Tully (1988a) have described a procedure that is expected to nullify the bias. That procedure will be summarized.

An example of when the bias arises is provided by considering the description of the luminosity–linewidth correlation given by the regression with errors taken in magnitudes – sometimes called the ‘direct’ relation. Use the ‘direct’ relation to determine distances to objects in the field. By the construction of the regression, the brightest galaxies will tend to lie above the correlation line. Suppose one considers a group. The brightest galaxies, drawn from above the mean correlation but assigned the absolute magnitude of the mean correlation, will be given a closer distance than
is correct. As fainter galaxies in the group are sampled, they progressively sample the true distribution around the mean correlation, so that the mean distances of the fainter galaxies are larger. Kraan-Korteweg, Cameron, & Tammann (1988) have shown that the measured mean distance of a group increases as fainter objects are included. For the same reason, as one probes in the field to larger redshifts one samples progressively only the brightest galaxies, those that tend to be drawn from above the mean correlation. Hence one progressively assigns erroneously low distances. Low distances give a high $H_0$.

In an analysis made this way it is imperative that a correction be made for the bias. However, to make the correction it is necessary to have detailed information on the form of the luminosity–linewidth correlation and the nature of the scatter. With adequate information, it is possible to correct statistically for the bias, though the trend of deviations with magnitude would persist in the individual measurements. It is submitted that Sandage (1994b) provides an example where the characteristics of the correlation and scatter are not understood and the corrections are erroneous.

Variations on the procedures that require bias corrections are pervasive (eg, Willick et al. 1997). For example, a maximum likelihood description of the relationship (Giovanelli et al. 1997b) still retains the bias and requires corrections. The corrections might be done properly. However, these procedures require (1) that the calibrators and targets have the same statistical properties, and (2) detailed specification of the sources of scatter and of properties of the luminosity function from which the sample is drawn. As an alternative, the method to be described nulls the bias rather than corrects for it. Consequently, there is no requirement to specify the sources of scatter or the properties of the sample. One is relying only on the assumption that calibrators and targets have the same properties.

The magic description that nulls the bias is given by the regression with errors in linewidth (Schechter 1980; Tully, 1988a, b) – the ‘inverse’ relation. Two qualitative comments might crystallize the merits of the procedure. The first point to appreciate is that the amplitude of the bias depends on the assumed slope of the correlation. The flatter the dependence of magnitude with linewidth the greater the bias. Conversely, if the slope is taken steep enough the sign of the bias can be reversed. Hence it can be understood that there is a slope that nulls the bias. That slope is given by the regression on linewidth if the sample is only limited in magnitude. The second key point is made by a consideration of the regressions on the separate axes of a luminosity–linewidth plot. Suppose one considers successively brighter magnitude cuts on an intrinsic distribution. As one progressively limits the magnitude range, the correlation coefficient of the fit will degrade. Presented graphically, the correlations on the two axes will progressively
diverge as the fitting range is reduced. Here is the critical point. As the truncation is progressively advanced in magnitude the slope of the correlation with errors in linewidths is always the same but the slope with errors in magnitudes is progressively splayed to shallower values.

Since the amplitude of the bias depends on the slope of the correlation, it should be seen that an analysis based on the direct relation is on slippery ground because the value of the slope depends on the magnitude limit of the sample. One needs a lot of information for an internally consistent application. The maximum likelihood approach raises the same qualitative concerns although, because it involves a slope intermediate between the direct and inverse correlations, the quantitative problem is also intermediate.

It has been pointed out by Willick (1994) that a bias can enter the inverse correlation in practical applications. The bias can be introduced because the cutoff may not be strictly in magnitude. For example, the sample might be chosen at $B$ band but applied at a more redward band such as $I$. A correlation between color and linewidth generates a slope to the magnitude cutoff at a band other than $B$. Or suppose the sample is selected by apparent diameter. A correlation between surface brightness and linewidth can again give a slope to the magnitude cutoff. A slope in the magnitude cutoff is equivalent to the introduction of a linewidth stricture. Any restriction in linewidths brings the problem of bias over to the orthogonal axis. Two things can be said of this problem. First it is a small effect, down by a factor of five in amplitude in Willick’s analysis. Second the problem is partially avoided by building the calibration out of only galaxies that satisfy a completion limit at the band to be considered; i.e., a stricter limit is taken than the one that provided the initial sample.

Most important: to achieve the correlation that nulls the bias one wants a complete magnitude limited calibration sample. In the population of the luminosity–linewidth diagram with the calibration sample there should not be any discrimination against candidates in any particular part of the diagram above the magnitude limit. Selection based on inclination is inevitable but that restriction should be distributed across the diagram. Other potential restrictions must be considered in a similar light.

The good news is that, with due care to the calibration, then the method can be applied to give unbiased distances to individual galaxies in the field as long as the inclusion of those galaxies is not restricted in linewidth. In other words, there will not be a correlation between luminosity and distance within a group as found by Kraan-Korteweg et al. (1988) nor a correlation between $H_0$ and redshift as found by Sandage (1994a). The method will break down if the target galaxy is a dwarf intrinsically fainter than the limit of the calibration. The latter issue is only a concern in our immediate neighborhood, not for the $H_0$ problem.
3.2. THE TEMPLATE RELATION AT I BAND

The creation of the template relation is a critical step. In the section on biases it has been described how important it is to have a sample that only suffers magnitude constraints. Often the calibration relationship is formed out of the ensemble of a field sample (Willick et al. 1996) but the constraints on such samples are usually ambiguous. Also, the calibration relationship is inevitably broadened and distorted by non-Hubble expansion motions.

Cluster samples have evident advantages. It is possible to be complete to a magnitude limit and it can be assumed that the galaxies are all at the same relative distance. The biggest concern with cluster samples is whether there are intrinsic differences between galaxies in a cluster environment and those that are more isolated. An operational disadvantage of cluster samples is that an individual cluster does not provide enough systems to provide good statistics. These two disadvantages can be addressed simultaneously by building a template relation out of several cluster samples. The ‘clusters’ can have a sufficient range in their properties that one can begin to evaluate the issue of environmental dependence. The combination of several cluster samples takes care of the problem of poor statistics.

This study uses samples drawn from five clusters with reasonable completion characteristics. There is best control with the nearby Ursa Major and Fornax clusters. The completeness limits in Ursa Major are discussed by Tully et al. (1996) and in Fornax by Bureau, Mould, and Staveley-Smith (1996). After corrections for obscuration, and translation to I magnitudes, the completion limit for both clusters is $I = 13.4^m$. There are 38 galaxies in Ursa Major with type Sa or later and $i \geq 45^\circ$ above this limit. There are 16 galaxies in Fornax satisfying these constraints. It was appreciated in advance that Ursa Major and Fornax are at similar distances. Hence the apparent magnitude limits conform to about the same absolute magnitude limits. Fornax is indicated by these data to be $0.10^m$ closer.

Already a diverse environmental range has been explored between the Ursa Major and Fornax cases. Tully at al. (1996) have labored the point that the Ursa Major Cluster environment is more similar to that of low density spiral groups than to what is generally considered a cluster. The structure must be dynamically young. By contrast, Fornax has a dense core of early type systems, evidence of a dynamically evolved structure. Granted, the spirals in the Fornax sample are more widely distributed than the central core and may represent recent arrivals.

The next component to be added to the template is drawn from the filament that passes through what has been called the Pisces Cluster. Aaronson et al. (1986) and Han & Mould (1992) have included the region in their distance studies but Sakai, Giovanelli, & Wegner (1994) have shown that one
is dealing with an extended structure with separate sub-condensations. It is unlikely that the region as a whole is collapsed. Indeed, what will be considered here is a length of $\sim 20^\circ$ along the Pisces filament, which corresponds to an end-to-end distance of $\sim 20$ Mpc. The mean redshift is constant to $\sim 4\%$ along the filament though individual redshifts scatter over a range of $\pm 20\%$ relative to the mean. It can be asked if the full length of the filament is at a common distance or if variations in distance can be identified. A luminosity–linewidth correlation is constructed for the ensemble, then inter-compared by parts to determine if components deviate significantly from the mean. There is not the slightest hint of deviations from the mean. Six sub-components along the $20^\circ$ filament have consistent distances within a few percent. To within measurement errors, the filament is tangent to the plane of the sky in both real space and velocity space. Given this circumstance, all the galaxies with $3700 < V_{\text{cmb}} < 5800$ km s$^{-1}$ along the $20^\circ$ segment of the Pisces filament $00^h 44^m < \alpha < 02^h 13^m$ will be taken to be at the same distance. Failures of this assumption will act to increase the scatter of the luminosity–linewidth relationship but the scatter is found to be only 0.31$m$, as small as for any sub-component of the template. This scatter is with 46 galaxies, after rejection of one object that deviates by $\sim 4\sigma$. There is reasonable completion brighter than $I = 13.8^m$ which is taken as the magnitude limit for the present sample. The Pisces filament data is added to the Ursa Major/Fornax template by (1) calculating the offset from the slope of the 2 cluster template, (2) redetermining a new slope now with 3 clusters, (3) iterating the distance offset with the new slope, and (4) iterating the new 3 cluster template slope. The distance shift at step 3 is of order 1% and the slope shift at step 4 is $\sim 1\%$. The final step in the development of the template is the addition of the Coma and Abell 1367 clusters. These clusters are at the same distance to within a few percent so they are treated together until the final iteration, at which point they are considered separately against the mean relation. Only galaxies within 4.3$^\circ$ of the cluster centers are accepted and the velocity constraints described by Giovanelli et al. (1997b) are accepted. As with Pisces, there is substantial but not full completion to $I = 13.8^m$. Iterations like those described with the Pisces filament converge to provide the five cluster template. There could have been a problem if there is curvature in the template, as might be indicated if, say, the slope flattened for samples with more luminous cutoffs (more distant clusters). However there is no suggestion of such a flattening. The Coma sample provides 28 galaxies. A1367 adds 23, after one 5$\sigma$ rejection. In total, there are 151 galaxies in the 5 cluster template. There are three distinct absolute magnitude cutoffs (UMa/Fornax; Pisces; Coma/A1367) but, to the degree that the slopes are indeed constant between components,
the calibration slope does not depend on the galaxy luminosity function. If there was evidence of a slope change, a slightly more complicated analysis involving a non-linear relationship would have been necessary.

3.3. ABSOLUTE CALIBRATION AT I BAND

Currently there are 17 galaxies with distances determined through observations of cepheid variable stars, mostly from observations with the Hubble Space Telescope (Freedman et al. 1997; Sandage et al. 1996; Tanvir et al. 1995). Two of the systems, NGC 2366 and NGC 3109, are fainter than the
template cutoff so will be ignored. One more galaxy is added as a calibrator, NGC 4258, which has a distance from the geometry inferred for the circum-nuclear masers (Miyoshi et al. 1995). Hence 16 calibrators are used.

It would be improper to do a regression on the calibrator relationship because in no way do they provide a complete sample. It can only be assumed that the calibrators are drawn from a similar distribution as the template objects, perhaps with magnitude as a selection criterion but not
linewidth. Effectively, each of the 16 calibrators provides a separate zero-point offset. The least-squares average provides the optimum fit. The final result is shown in Figure 2 where the $I$ band luminosity–linewidth relation is shown for the 16 calibrators and the 151 cluster template galaxies shifted to the absolute magnitude scale of the calibrators (the 2 rejected galaxies are also plotted). Figures 3-8 present the same material but separated to distinguish the fits to the calibrators and the individual clusters.
3.4. THE B, R AND K’ RELATIONS

Less complete information is available at other bands than I. However, inter-comparisons are valuable because of the potential problem with obscuration. Information is available at B and R for the calibrators, all the galaxies in the Ursa Major sample, most of those in Coma, and for most in the part of the Pisces region at $00^h49^m < \alpha < 01^h32^m$ (Pierce & Tully 1998). Material is available at $K'$ for the same Ursa Major and Pisces
Figure 9. Luminosity–linewidth relations in $B, R, I, K'$ after corrections for inclination. Filled circles: Ursa Major; open circles: Pisces. Straight lines are regressions with errors in linewidths.
galaxies (Tully et al. 1996, 1998). The luminosity–linewidth relations are illustrated in Figure 9.

The magnitude scatter is essentially the same at $R, I, K'$ and $\sim 20\%$ worse at $B$. Obscuration corrections diminish toward the infrared until they are tiny at $K'$. However, sky background contamination causes degradation toward the infrared from the favorable situation at $R$ to the poor situation at $K'$ where one loses almost 2 scalelengths to the sky compared with an $R$ exposure of the same duration. The correlations are seen to steepen toward the infrared. However, this steepening is less extreme than had been seen in the past because of the strong luminosity dependence of the reddening corrections that are now applied. The biggest corrections are made to the most luminous galaxies in the bluest bands. Hence the corrected relations at shorter wavelengths are steepened toward the slopes of the almost-reddening-free infrared relations. As shown in Tully et al. (1998), only a weak color dependency on luminosity remains after reddening is taken into account. Slopes at $B, R, I, K'$ are -7.8, -8.0, -8.2, and -8.7, respectively, with the correlation against the same linewidth information. These slopes are based on the linewidth regression which is appropriate for bias-free distance determinations but not the slopes that one wants to give a physical interpretation. The true nature of the correlation is characterized better by a maximum likelihood fit. An approximation to that is a double regression, which at $I$ gives the slope -7.9. These fits indicate infrared convergence toward $L \propto W^n$ where $n = 3.4 \pm 0.1$.

4. Summary of Results

The template relation with the zero-point given by 16 galaxies can be used to determine distances to any other galaxy, with the proviso that the procedure must fail if the target is intrinsically less luminous than $M^*_I = -18m$, the faintness limit of the calibration. If the problem is to measure $H_0$, this faint limit is of no concern because targets of interest are beyond the Local Supercluster where peculiar velocities are expected to be a small fraction of expansion velocities.

It would be possible to apply the calibration to measure distances to hundreds of galaxies. For the moment, with the interest of maintaining as homogeneous a set of measurements as possible, the $H_0$ determination will be based on the 5 clusters that went into the template plus 7 other clusters each with of order a dozen measures. The results are presented in Table 1 and Figure 10. The table provides (col. 2) the number of measures in the cluster, (col. 3) the rms scatter about the template relation, (col. 4/5) the distance modulus/distance of the cluster, (col. 6) the velocity of the cluster in the CMB frame as given by Giovanelli et al. (1997b), and (col. 7) the
measure of $H_0$ from the cluster. The velocity given to the Pisces filament is the average of the values for the three main sub-condensations.

TABLE 1. Five Template Clusters and Seven More

| Cluster        | No. | RMS Modulus | Distance V$_{cmb}$ | $H_0$ |
|----------------|-----|-------------|-------------------|-------|
| Fornax         | 16  | 0.50        | 31.21             | 17.5  | 1321  | 76   |
| Ursa Major     | 38  | 0.41        | 31.34             | 18.5  | 1101  | 59   |
| Pisces Filament| 46  | 0.31        | 33.90             | 60.3  | (4779)| 79   |
| Abell 1367     | 23  | 0.41        | 34.64             | 84.8  | 6735  | 79   |
| Coma           | 28  | 0.33        | 34.67             | 85.9  | 7185  | 84   |
| Antlia         | 11  | 0.27        | 32.78             | 35.9  | 3120  | 87   |
| Centaurus 30   | 13  | 0.52        | 32.94             | 38.9  | 3322  | 86   |
| Pegasus        | 12  | 0.37        | 33.36             | 46.9  | 3519  | 75   |
| Hydra I        | 11  | 0.35        | 33.84             | 58.6  | 4075  | 70   |
| Cancer         | 16  | 0.34        | 33.89             | 60.0  | 4939  | 83   |
| Abell 400      | 9   | 0.24        | 34.91             | 96.1  | 6934  | 72   |
| Abell 2634     | 16  | 0.32        | 35.23             | 111.0 | 7776  | 70   |

Weighted average 77

The error bars in Fig. 10 contain distance and velocity components. The errors associated with distance depend directly on the rms dispersion in a cluster and inversely with the square root of the number of galaxies in the cluster sample. The error associated with velocity streaming is taken to be 300 km s$^{-1}$. The velocity component to the error is totally dominant inside 2000 km s$^{-1}$. The statistical errors in distance become the dominant factor beyond $\sim 6000$ km s$^{-1}$. The symbols in Fig. 10 differ for different regions of the sky. There is a hint of systematics; for example the filled circles lie above the open circles. For the present purposes, the best estimate of $H_0$ is derived by taking an average of log $H_0$ values with weights proportional to the inverse square of the error bars that are plotted. The result is $H_0 = 77 \pm 4$ km s$^{-1}$ Mpc$^{-1}$. The error is the 95% probability statistical uncertainty. It is small because it is based on a template of 151 galaxies, a zero-point fixed by 16 galaxies, and application to 12 clusters distributed around the sky and out to 8,000 km s$^{-1}$.

This result is somewhat higher than ‘interm’ value of $H_0 \simeq 73$ reported by the HST Key Project team (Mould et al. 1997) or the value of $H_0 = 69 \pm 5$ found by Giovanelli et al. 1997$^a$ based on similar applications of luminosity–linewidth correlations. The latter value is outside the bound of
Figure 10. Individual estimates of $H_0$ as a function of systemic velocity. Errors are a convolution of the statistical errors in distance and an uncertainty of 300 km s$^{-1}$ in velocities. Symbols vary with location on the sky: filled circles: north celestial and north galactic; open circles: north celestial and south galactic; filled squares: south celestial and north galactic; open squares: south celestial and south galactic.

The statistical error found in this paper. It must be attributed to a small systematic difference that remains to be identified.

The value of $H_0 = 77$ found here is about 13% lower than the value found by the same author and method in the past. There has been a 17% decrease due to the revision of the luminosity–linewidth zero-point as a consequence of the increase from 4 to 16 in the number of calibrator galaxies with distances determined through the cepheid period–luminosity relation. There has been an 8% increase as a consequence of the revised reddening corrections now being applied. A 5% decrease has come about with the introduction of material on more clusters around the sky at distances well
beyond the Local Supercluster. These changes are a sobering illustration of random and systematic errors. The shift associated with the improved cepheid calibration is comparable to the rms dispersion of the luminosity–linewidth relations at $R, I, K'$ bands. Either by statistical fluke or an unknown systematic, the original 4 calibrators are drawn from the faint side of the intrinsic correlation.

In conclusion, there has been a decrease of $\sim$ one standard deviation from the value of $H_0$ measured previously by this author with the luminosity–linewidth method, to $H_0 = 77 \pm 4 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The greatest perceived problems of old have been addressed: there are now many more zero-point calibrators, the template is much more extensive and complete, reddening corrections are under better control, and the method is applied to more targets distributed around the sky. Formal statistical errors are cut in half. Uncertainties are now dominated by potential, unidentified systematics.

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