\( \Xi_c \rightarrow \Xi \) Form Factors and \( \Xi_c \rightarrow \Xi \ell^+ \nu_\ell \) Decay Rates From Lattice QCD

Qi-An Zhang,1 Jun Hua,2 Fei Huang,2 Renbo Li,3 Yuanyuan Li,3 Cai-Dian Lüt,4,5 Peng Sun,3 Wei Sun,4 Wei Wang,2 and Yi-Bo Yang6,7,8,†

1 Key Laboratory for Particle Astrophysics and Cosmology (MOE), Shanghai Key Laboratory for Particle Physics and Cosmology, Tsung-Dao Lee Institute, Shanghai Jiao Tong University, Shanghai 200240, China
2 INPAC, Key Laboratory for Particle Astrophysics and Cosmology (MOE), Shanghai Key Laboratory for Particle Physics and Cosmology, School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China
3 Nanjing Normal University, Nanjing, Jiangsu, 210023, China
4 Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
5 School of Physics, University of Chinese Academy of Sciences, Beijing 100049, China
6 CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
7 School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China
8 International Centre for Theoretical Physics Asia-Pacific, Beijing/Hangzhou, China

(Dated: March 15, 2021)

We perform the first lattice QCD calculation of the form factors governing the \( \Xi_c \rightarrow \Xi \ell^+ \nu_\ell \) using the 2+1 clover fermion ensembles with two lattice spacings, \( a = (0.108\text{fm}, 0.080\text{fm}) \). After the continuum extrapolation, partial decay widths are calculated. Using the \( \left| V_{cs} \right| \) from global fit and the \( \Xi_c \) lifetimes, we predict branching fractions as \( B(\Xi^0 \rightarrow \Xi^- \ell^+ \nu_\ell) = 2.38(0.30)(0.33)\% \), \( B(\Xi^+ \rightarrow \Xi^0 \ell^+ \nu_\ell) = 2.29(0.29)(0.31)\% \), \( B(\Xi^+ \rightarrow \Xi^- \ell^+ \nu_\ell) = 6.91(0.87)(0.93)\% \), with the two kinds of uncertainties coming from statistical fluctuations in form factors and \( \Xi_c \) lifetimes, and systematic ones from continuum extrapolation and renormalization, respectively. Combing the experimental measurement by Belle, we determine the CKM matrix element as \( \left| V_{cs} \right| = 0.834(0.074)(0.127) \), where the two uncertainties come from theory and data.

Introduction. Weak decays of heavy hadrons provide an ideal platform to test the standard model (SM) of particle physics, especially the Cabibbo-Kobayashi-Maskawa (CKM) paradigm which describe mixing between quark flavors and also strength of CP violations. Any of significant deviation from the SM expectation of the CKM matrix should provide clues to uncover the new physics beyond. Currently most of previous analysis concentrate on weak decays but also heavy baryon ones.

Among the charmed baryons, the \( \Lambda_c \) has been extensively studied especially by BESIII \cite{BESIII} and Belle \cite{Belle1} \cite{Belle2} collaborations, and Lattice simulation of form factors are also available \cite{Lattice1} \cite{Lattice2}. On the contrary, decays of \( \Xi_c \) receive much less attention, even though some early experimental study can date back to 1990s \cite{Belle3}. Very recently, using a data sample of 89.5 fb\(^{-1} \) recorded at the energy \( \sqrt{s} = 10.52 \text{ GeV} \) at the KEKB \( e^+e^- \) collider, Belle collaboration reported the measurement of branching fractions of semi-leptonic \( \Xi_c \) decays \cite{Belle3}:

\[
B_{\text{exp}}(\Xi^0_c \rightarrow \Xi^- \ell^+ \nu_\ell) = (1.72 \pm 0.10 \pm 0.12 \pm 0.50)\%,
\]

\[
B_{\text{exp}}(\Xi^0_c \rightarrow \Xi^+ \mu^+ \nu_\mu) = (1.71 \pm 0.17 \pm 0.13 \pm 0.50)\%,
\]

where the last errors arise from the uncertainties in \( B(\Xi^0 \rightarrow \Xi^- \pi^+) \) \cite{Belle3}.

In this Letter, we will focus on the Lattice QCD study of \( \Xi_c \rightarrow \Xi \ell^+ \nu_\ell (\ell = e, \mu) \), whose partial decay widths are proportional to \( \left| V_{cs} \right|^2 \). This study is of great value in the following aspects. Firstly, the combination of the form factors from lattice QCD and experimental results for branching fractions allows an independent determination of \( \left| V_{cs} \right| \). Secondly, those form factors are also mandatory inputs for the theoretical analyses of the \( \Xi_c \) non-leptonic decays which give an interesting platform for the study of CP violation. Last but not least, recent studies of heavy bottom mesons have revealed some hints for lepton flavor violation in semi-leptonic decay modes, and then it is equally interesting to examine the lepton flavor violation in charmed baryon leptonic decays.

In the SM, the \( \Xi_c \rightarrow \Xi \ell^+ \nu_\ell \) decay rates depend on six form factors which parametrize the matrix elements of vector and axial-vector currents between the \( \Xi_c \) and \( \Xi \) baryons. These form factors have been estimated in some phenomenological models for instance \cite{Yu, Chen}, but vary substantially depending on the explicit model assumptions. In this work, the first lattice QCD determination of \( \Xi_c \rightarrow \Xi \) form factors is reported, and predictions for semi-leptonic decay widths are presented, based on which the \( \left| V_{cs} \right| \) is also extracted.

Lattice Setup. This work is based on the 2+1 flavor dynamical ensembles with the tree level tadpole improved clover fermion action and tadpole improved Symanzik gauge action. One step of the Stout link smear is ap-
plied on the gauge field used by the clover action to improve the stability of the pion mass with given bare quark mass. The tadpole improvement factors used by the quark and gluon actions are tuned to the fourth root of the plaquette using the Stout link smeared and original gauge links, respectively. We start from the ensemble s108 with bare coupling $\beta = \frac{3}{2g^2} = 6.20$ and dimensionless size $24^3 \times 72$, determine the lattice spacing with Wilson flow [23], and tune the bare coupling on the s080 ensemble with smaller lattice spacing to make their physical volume to be roughly the same. The information of two configuration ensembles used in this letter can be found in Tab. I.

| $\beta$ | $L^a \times T$ | $a$ | $c_{sym}$ | $\eta_l$ | $m_c$ | $k_\eta_s$ | $m_{\eta_s}$ |
|---------|----------------|-----|-----------|---------|-------|-----------|-----------|
| s108   | 24 $^3 \times 72$ | 0.108 | 1.161 | -0.2770 | 290 | 0.1330 | 640 |
| s080   | 32 $^3 \times 96$ | 0.080 | 1.141 | -0.2295 | 300 | 0.1318 | 650 |

On these two ensembles, we use the charm quark mass as $m_c^{s108_a} = 0.485$ and $m_c^{s080_a} = 0.235$, respectively, by requiring the corresponding $J/\psi$ mass to be the physical value $m_{J/\psi} = 3.9690(0)(6)\text{GeV}$ [24] within 0.3% accuracy.

The extraction of $\Xi_c \to \Xi$ form factors requires the lattice QCD calculation of both three-point correlation function (3pt) from $\Xi_c$ to $\Xi$, and also the two point correlation functions (2pt) of both $\Xi_c$ and $\Xi$. The 3pt with weak current $J^{\mu} = V^{\mu} - A^{\mu} = \bar{s}\gamma^{\mu}(1 - \gamma_5)c$ is defined by,

$$C_3^{V-A}(q^2, t, t_{\text{sec}}) = \int d^4x d^4t \bar{\psi}(x) e^{- i p \cdot x} e^{- i q \cdot t} T_{\gamma' \gamma}$$

$$\times \langle 0 | \chi(\bar{x}, t_{\text{sec}}) J^{\mu}(\bar{y}, t) \chi(0, 0) | 0 \rangle, (3)$$

where $\chi_{\Xi_c}^{\Xi}, \Xi_c$ is the interpolation field of $\Xi$ and $\Xi_c$, respectively, $T$ is a combination of the Dirac matrix that is chosen to project out the form factor. For the 2pt with $B = \Xi, \Xi_c$,

$$C_{2B}(t) = \int d^4x e^{- i p \cdot x} T_{\gamma' \gamma}(0) \chi(\bar{x}, t) \chi(0, 0) | 0 \rangle, (4)$$

we choose $T'$ as the identity matrix to simplify the formula reduction. Then we can define the following ratios for different projection matrix $T$ and current operator $V^{\mu}/A^{\mu}$,

$$R_{V/A}(T, \mu) = \sqrt{\frac{C_{2A}^{V/A}(q^2, t, t_{\text{sec}})C_{2A}^{V/A}(q^2, t_{\text{sec}} - t, t_{\text{sec}})}{C_{2B}^{V/A}(t_{\text{sec}})C_{2B}^{V/A}(t_{\text{sec}})}}, \quad (5)$$

where the subscript $V$ or $A$ corresponds to the vector or axial-vector current in the 3pt. After making use of the reduction formula, the ratios $R_F$ for the six form factors $F = (f_{+}, f_0, g_{+}, g_0, f_{++}, g_{++})$ can be constructed by different combinations of $R_{V/A}(T, \mu)$. More details can be found in the supplement material. Then we adopt the parameterization,

$$R_F = F \left( \frac{1 + c_1 e^{- \Delta E_1 t} + c_2 e^{- \Delta E_2 (t_{\text{sec}} - t)}}{1 + d_1 e^{- \Delta E_1 t_{\text{sec}}} + d_2 e^{- \Delta E_2 (t_{\text{sec}} - t)}} \right)^{1/2}$$

$$\cong F \left[ 1 + c'_1 (e^{- \Delta E_1 t/2} + e^{- \Delta E_2 (t_{\text{sec}} - t)/2}) \right], \quad (6)$$

to eliminate the excited-state contaminations and obtain the desired form factor $F$, where $\Delta E_1$ and $\Delta E_2$ with the condition $\Delta E_1 < \Delta E_2$ describe the mass differences between the first excited states and ground states in the initial and final interpolation fields. It should be noted that Eq. (6) is employed in the fit for most cases, while Eq. (7) is used for certain small $q^2$ (large $\bar{q}^2$) cases on the ensemble s080 since the lattice results are noisy. We have checked that in the latter cases using Eq. (6) will lead to consistent central values.

\[\text{FIG. 1. Lattice results for } R_{V}^{+\to +}(t) \text{ and } \sqrt{R_{V}^{+\to +} R_{V}^{-\to -}}. \text{ The bands correspond to the ground state contributions } Z_{V}^{+\to +} \text{ and } \sqrt{Z_{V}^{+\to +} Z_{V}^{-\to -}} \text{ on the s080 and s108 ensembles, respectively.}\]

The vector and axial-vector $c \to s$ currents on the lattice suffer from the finite renormalization. Such a renormalization can be defined by the ratio of the conserved vector current $V_c$ and local current $V$ in the hadron matrix element,

$$R_{V_c}^{V \to V}(t) = \frac{(M_1(T/2) \sum_{\bar{q}} V^{q_1 \to q_2}(\bar{x}, t) M_2(0))}{(M_1(T/2) \sum_{\bar{q}} V^{q_1 \to q_2}(\bar{x}, t) M_2(0))}$$

$$= Z_{V_c}^{V \to V} + O(e^{-T/4\Delta E}), \quad (8)$$

where $M_{1,2}$ are arbitrary pseudoscalar states and $\Delta E$ is the mass gap between the ground state and first excited state. For the $c \to s$ current, one can use either the combination $(M_1, M_2) = (\eta_s, D_s)$, or the geometric average on those of the $s \to c$ current and $c \to c$ current using $(M_1, M_2) = (\eta_c, \eta_c)$ and $(\eta_c, \eta_c)$, respectively. We illustrate the $Z_V$ in Fig. 1 in which the crosses and dots correspond to $R_{V_c}^{V \to V}(t)$ and $\sqrt{R_{V_c}^{V \to V}(t) R_{V_c}^{-\to -}(t)}$, respectively. Constant fits can describe the data at median
$t \sim T/4$ well, and the difference between two definitions becomes smaller at the finer s080 ensemble (upper yellow data), and both of them are also closer to one comparing to that on the coarser s108 ensemble (lower blue data). Thus the differences between two strategies arise from discretization effects. In the following discussion, we will use $Z_{\pi}^{c+s}$ to obtain the central values of the final result, then repeat the analysis with $\sqrt{Z_{\pi}^{c+s} Z_{\pi}^{c-s}}$ and treat the differences as a systematic uncertainty. Due to the chiral symmetry breaking of the clover fermion action, the renormalization factor of the axial-vector current is not exactly the same as the vector one. Thus we use the off-shell quark matrix elements to define $Z_A$ as,

$$Z_A^{c+s} = Z_{\pi}^{c+s} \sqrt{\frac{\text{Tr}(c|V^\mu|c) \gamma^\mu \gamma_5 \text{Tr}(s|V^\mu|s) \gamma^\mu \gamma_5}{\text{Tr}(c|A^\mu|c) \gamma^\mu \text{Tr}(s|A^\mu|s) \gamma^\mu}},$$

with multiple off-shell quark momenta $p^2$. With $a^2 p^2$ extrapolation using three values of $p^2$ in the range of $a^2 p^2 \in [4, 8]$, we obtained $Z_A/Z_{\pi} = 1.010231(69)$ and 1.020296(68) on s108 and s080, respectively.

**Numerical Results.** By choosing different reference time slices, we preform $48 \times 393$ measurements on the s080 ensemble, and $72 \times 436$ measurements on the s080 ensemble. The lattice results for the ratios $R_{f_{\perp}}$ with $\vec{p}_\perp = (0, 0, 1) \times \frac{2\pi}{L_a} \simeq 0.48$GeV are shown in Fig. 2. The $\chi^2/d.o.f.$ are below/close to 1 for most fits of the 400 bootstrap samples, which indicates a good fit quantity. This can be verified that the colored bands in the left panel of Fig. 3 predicted by the fit agree with the data points well. To further validate the results, we calculate the differential summed ratio [25],

$$R(t_{\text{seq}}) = \frac{SR(t_{\text{seq}}) - SR(t_{\text{seq}} - \Delta t)}{\Delta t},$$

and show the results in the right panel of Fig. 2, where $SR(t_{\text{seq}}) \equiv \sum_{t_c < t < t_{\text{seq}} - t_c} R_F(t, t_{\text{seq}})$, $t_c = 3a$ is the requirement used in the fits to suppress higher excited states contributions. One can see that $R(t_{\text{seq}})$ agree with the grey band from the two-state fit well when $t_{\text{seq}} > 14a$.

To access the $q^2$ distribution, we employ the $z$-expansion parametrization of form factors that arises from the analyticity and unitarity [26]

$$f(q^2) = \frac{1}{1 - q^2/(m_{pole}^2)^2} \sum_{n=0}^{n_{\text{max}}} (c_n + d_n a^2) [z(q^2)]^n,$$

where

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}},$$

and the fitted parameters in Tab. I and show the $q^2$ dependent form factor in the continuum limit (by eliminating the $d_n^0$ terms) from the fit and also the Lattice results at given $q^2$ in Fig. 3. As shown in the figure, our form factor results on the s080 ensemble are consistent with those on the s080 ensemble, and agree with the fit prediction in the continuum well.

**TABLE II.** Results for the $z$-expansion parameters describing the form factors with statistical errors.

| $f_{\perp}$ | 1.51(0.09) -1.88(0.21) 1.71(0.49) | $f_0$ | 0.64(0.09) -1.83(0.22) 0.56(0.51) | $f_+$ | 0.77(0.07) -4.09(0.18) 0.35(0.49) | $g_{\perp}$ | 0.56(0.07) -0.35(0.26) 0.15(0.29) | $g_0$ | 0.63(0.07) -1.37(0.36) 0.15(0.29) | $g_+$ | 0.56(0.08) 0.00(0.38) 0.14(0.29) |

In Fig. 4, we use the above form factors to predict the differential decay widths (in units of ps$^{-1}$GeV$^{-2}$) for $\Xi^0_c \to \Xi^- e^+ \nu_e$ divided by $|V_{cs}|^2$ as a function of $q^2$. Results for $\Xi_c^+ \to \Xi^0 e^+ \nu_e$ are also similar. Using the lifetime from PDG: $\tau(\Xi^0_c) = (1.53 \pm 0.06) \times 10^{-13}$s and $\tau(\Xi_c^-) = (4.56 \pm 0.05) \times 10^{-13}$s, and $|V_{cs}| = 0.97320 \pm 0.00011$, one can obtain the decay branching fractions:

$$B(\Xi^0_c \to \Xi^- e^+ \nu_e) = 2.38(0.30)(0.33)\%,$$

$$B(\Xi_c^- \to \Xi^0 e^+ \nu_e) = 2.29(0.29)(0.31)\%,$$

$$B(\Xi^0_c \to \Xi^0 e^+ \nu_e) = 7.18(0.90)(0.98)\%,$$

$$B(\Xi_c^+ \to \Xi^0 e^+ \nu_e) = 6.91(0.87)(0.93)\%.\ (13)$$
The two kinds of uncertainties come from the statistical fluctuations of form factors and $\Xi_c$ lifetimes, and the systematic errors, respectively. The latter arises from differences between the continuum-extrapolated results and the ones using the s080 ensemble, and the differences between the results using $Z_c^{\Xi^0\pi^+}$ or $Z_c^{\Xi^-\pi^+}$ in the renormalization. Our predictions for branching fractions are consistent with the global fit [24] within 1σ. It should be noted that the largest error arises from the experimental result on $B(\Xi_c^0 \rightarrow \Xi^- \pi^+)$ [16]. In a conference proceed-

\[ R_{\mu/e} = \frac{B(\Xi_c^0 \rightarrow \Xi^- \mu^+\nu_\mu)}{B(\Xi_c^0 \rightarrow \Xi^- e^+\nu_e)} = 0.962 \pm 0.003 \pm 0.002, \]  

(14) 

where most uncertainties from form factors have cancelled. The deviation from unity arises from the mass differences between muon and electron. This result is consistent with Belle measurement: $R_{\mu/e} = 1.00 \pm 0.11 \pm 0.09$ [15]. 

Using the recent Belle measurement [15], we give an independent determination of $|V_{cs}|$: 

\[ |V_{cs}| = \begin{cases} 
0.830(0.074)(0.128), & l = e \\
0.846(0.076)(0.135), & l = \mu \\
0.834(0.074)(0.127), & l = (e, \mu)_{\text{fit}} 
\end{cases} \]  

(15) 

where the third line is obtained by considering both $\Xi_c^0 \rightarrow \Xi^- e^+\nu_e$ and $\Xi_c^0 \rightarrow \Xi^- \mu^+\nu_\mu$. This result is consistent with the global fit [24] within 1σ. It should be noted that the largest error arises from the experimental result on $B(\Xi_c^0 \rightarrow \Xi^- \pi^+)$ [16]. In a conference proceed-

FIG. 3. The $q^2$ distribution for the $\Xi_c \rightarrow \Xi$ form factors. The z expansion approach has been used to fit the lattice data. An extrapolation to the continuum limit has been made, and the shadowed region corresponds to the final results with $a \rightarrow 0$. 

FIG. 4. Predictions for the differential decay widths of the $\Xi_c^0 \rightarrow \Xi^- e^+\nu_e$ and $\Xi_c^0 \rightarrow \Xi^- \mu^+\nu_\mu$, divided by the $|V_{cs}|^2$ in units of ps$^{-1}$GeV$^{-2}$. 

From the above equation, one can see that both statistical and systematic uncertainties are about 10%. The statistics uncertainties can be reduced by more measurements, while analyses at some finer lattices can suppress systematic uncertainties from both the continuum extrapolation and renormalization scheme. In this calculation, we have ignored the systematic uncertainty from the chiral extrapolation, which can also be controlled by the simulations using smaller pion masses.
ing [32], ALICE collaboration also reported their measurement of the ratio $B(\Xi_c^0 \to \Xi^- e^+ \nu_e)/B(\Xi_c^0 \to \Xi^- \pi^+)$ which corresponds to:

$$B_{\exp}(\Xi_c^0 \to \Xi^- e^+ \nu_e) = 2.43(0.25)(0.35)(0.72)\%,$$  \hspace{1cm} (16)

where the last error is also from the $B(\Xi_c^0 \to \Xi^- \pi^+)$. If this result is used, the $|V_{cs}|$ is determined as:

$$|V_{cs}| = 0.983(0.081)(0.157).$$  \hspace{1cm} (17)

Conclusions. We have reported the first lattice QCD calculation of the form factors governing the $\Xi_c \to \Xi^+ \pi_e$ at two lattice spacing and extrapolated it to the continuum. Using the CKM matrix element $|V_{cs}|$ from the global fit and the $\Xi_c$ lifetimes, we predict the branching fractions as $B(\Xi_c^0 \to \Xi^- e^+ \nu_e) = 2.38(0.30)(0.33)\%$, $B(\Xi_c^0 \to \Xi^- \mu^+ \nu_\mu) = 2.29(0.29)(0.31)\%$, $B(\Xi_c^0 \to \Xi^- \tau^+ \nu_\tau) = 7.18(0.90)(0.98)\%$, and $B(\Xi_c^+ \to \Xi^0 \mu^+ \nu_\mu) = 6.91(0.87)(0.93)\%$, respectively. These predictions are consistent with very recent experimental measurements by Belle and ALICE collaborations. Using the measured branching fraction from Belle together with the lattice calculation, we determine the CKM matrix element $|V_{cs}| = 0.834(0.074)(0.127)$, where the errors come from the uncertainties of theoretical and experimental results, respectively.

Acknowledgment.— We thank Andreas Schäfer for valuable discussions, and Prof. Y.B. Yin, Dr. J. Zhu and T. Cheng for pointing out the ALICE result in the conference proceeding [32]. The gauge configurations are generated on the cluster supported by Southern Nuclear Science Computing Center (SNSC) and also HPC Cluster of ITP-CAS. The LQCD calculations were performed using the Chroma software suite [27] and QUDA [28-30] through HIP programming model [31]. The numerical calculation is supported by Chinese Academy of Science CAS Strategic Priority Research Program of Chinese Academy of Sciences, Grant No. XDC01040100. The setup for numerical simulations was conducted on the $\pi$ 2.0 cluster supported by the Center for High Performance Computing at Shanghai Jiao Tong University. This work is supported in part by Natural Science Foundation of China under grant Nos. 11735010, U2032102, 11653003, 12005130, 11521505, 12070131001, and 11975127, Natural Science Foundation of Shanghai under grant No. 15DZ2272100, the China Postdoctoral Science Foundation and the National Postdoctoral Program for Innovative Talents (Grant No. BX20190207), National Key Research and Development Program of China under Contract No. 2020YFA0406400, Jiangsu Specially Appointed Professor Program, the Strategic Priority Research Program of Chinese Academy of Sciences, Grant No. XDB34030303, and a NSFC-DFG joint grant under grant No. 12061131006 and SCFA 458/22.
[21] C. Q. Geng, C. W. Liu, T. H. Tsai and S. W. Yeh, Phys. Lett. B 792, 214-218 (2019), doi:10.1016/j.physletb.2019.03.056 [arXiv:1901.05610 [hep-ph]].

[22] C. Q. Geng, C. W. Liu and T. H. Tsai, arXiv:2012.04147 [hep-ph].

[23] S. Borsanyi, S. Durr, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, T. Kurth, L. Lellouch, T. Lippert and C. McNeile, et al. JHEP 09, 010 (2012), doi:10.1007/JHEP09(2012)010 [arXiv:1203.4469 [hep-lat]].

[24] P. A. Zyla et al. [Particle Data Group], PTEP 2020, 083C01 (2020), doi:10.1093/ptep/ptaa104

[25] C. C. Chang, A. N. Nicholson, E. Rinaldi, E. Berkowitz, N. Garron, D. A. Brantley, H. Monge-Camacho, C. J. Monahan, C. Bouchard and M. A. Clark, et al. Nature 558, 91-94 (2018), doi:10.1038/s41586-018-0161-8 [arXiv:1805.12130 [hep-lat]].

[26] C. Bourrely, T. Caprini and L. Lellouch, Phys. Rev. D 79, 013008 (2009) [erratum: Phys. Rev. D 82, 099902 (2010)], doi:10.1103/PhysRevD.82.099902 [arXiv:0807.2722 [hep-ph]].

[27] R. G. Edwards et al. [SciDAC, LHPC and UKQCD], Nucl. Phys. B Proc. Suppl. 140, 832 (2005), doi:10.1016/j.nuclphysbps.2004.11.254 [arXiv:hep-lat/0409003 [hep-lat]].

[28] M. A. Clark, R. Babich, K. Barros, R. C. Brower and C. Rebbi, Comput. Phys. Commun. 181, 1517-1528 (2010), doi:10.1016/j.cpc.2010.05.002 [arXiv:0911.3191 [hep-lat]].

[29] R. Babich, M. A. Clark, B. Joo, G. Shi, R. C. Brower and S. Gottlieb, doi:10.1145/2063384.2063478 [arXiv:1109.2935 [hep-lat]].

[30] M. A. Clark, B. Joó, A. Strelchenko, M. Cheng, A. Gambhir and R. Brower, [arXiv:1612.07873 [hep-lat]].

[31] Y. J. Bi, Y. Xiao, W. Y. Guo, M. Gong, P. Sun, S. Xu and Y. B. Yang, PoS LATTICE2019, 286 (2020), doi:10.22323/1.363.0286 [arXiv:2001.05706 [hep-lat]].

[32] J. Zhu on behalf of the ALICE collaboration, PoS ICHEP2020 (2021) 524.
\section*{Supplemental Materials}

\section*{A. Combination of ratios of correlation functions}

As in Eq. (5), one can define the projected ratios of correlation functions:

\begin{align}
R_1 & \equiv \frac{R_V(I,z) + R_V(\gamma^0,z)}{2}, \quad R_2 \equiv R_V(\gamma^z,0), \quad R_3 \equiv R_V(\gamma_5 \gamma^z, y), \\
R_4 & \equiv \frac{R_A(\gamma_5, z) + R_A(\gamma_5^0, z)}{2}, \quad R_5 \equiv R_A(\gamma_5 \gamma^z, 0), \quad R_6 \equiv R_A(\gamma^z, y). 
\end{align} 

We can construct the combined ratio \( R_F \) of the six form factors \( F = (f_-, f_0, g_-, g_+, g_0) \) as

\begin{align}
R_{f-} &= \frac{R_3}{4m_1 N_x \hat{p}}, \\
R_{g-} &= \frac{R_6}{4m_1 N_x \hat{p}}, \\
R_{f_0} &= -\frac{(E_2 - m_1) (m_1^2 - m_2^2) + (E_1 + m_1) q^2}{8m_1^2 (m_1 - m_2) (E_2 + m_2) N_x \hat{p}} R_1 + \frac{m_1^2 - m_2^2 + q^2}{8m_1^2 (m_1 - m_2) N_x \hat{p}} R_2 + \frac{2m_1 E_2 - m_1^2 - m_2^2 + q^2}{8m_1^2 (m_2 + E_2) N_x \hat{p}} R_3, \\
R_{f_+} &= -\frac{(E_2 - m_1) \left[(m_1 + m_2)^2 - q^2\right]}{8m_1^2 (E_2 + m_2) (m_1 + m_2) N_x \hat{p}} R_1 + \frac{(m_1 + m_2)^2 - q^2}{8m_1^2 (m_1 + m_2) N_x \hat{p}} R_2 + \frac{2m_1 E_2 - m_1^2 - m_2^2 + q^2}{8m_1^2 (m_2 + E_2) N_x \hat{p}} R_3, \\
R_{g_0} &= \frac{(E_2 + m_1) (m_1^2 - m_2^2) + (E_2 - m_1) q^2}{8m_1^2 (m_1 + m_2) (m_1 - m_2) N_x \hat{p}} R_4 - \frac{m_1^2 - m_2^2 + q^2}{8m_1^2 (m_1 + m_2) N_x \hat{p}} R_5 + \left[\frac{2m_1 (E_2 - m_2) - m_1^2 + m_2^2}{8m_1^2 (E_2 - m_2) N_x \hat{p}} + \frac{(m_1 - m_2) q^2}{8m_1^2 (E_2 - m_2) (m_1 + m_2) N_x \hat{p}}\right] R_6, \\
R_{g_+} &= \frac{(E_2 + m_1) \left[(m_1 - m_2)^2 - q^2\right]}{8m_1^2 (E_2 - m_2) (m_1 - m_2) N_x \hat{p}} R_4 - \frac{(m_1 - m_2)^2 - q^2}{8m_1^2 (m_1 - m_2) N_x \hat{p}} R_5 + \left[\frac{2m_1 (E_2 - m_2) - m_1^2 + m_2^2}{8m_1^2 (E_2 - m_2) N_x \hat{p}} + \frac{(m_1 + m_2) q^2}{8m_1^2 (m_1 - m_2) (E_2 - m_2) N_x \hat{p}}\right] R_6,
\end{align}

where \( m_1 \) is the mass of \( \Xi_c \), \( m_2 \), \( E_2 \) are the mass and energy of \( \Xi \), respectively, \( \hat{p} = \frac{2 \pi}{L_a} \simeq 0.48 \text{GeV} \) is the unit momentum for both s108 and s080, and \( \hat{p}_z = (N_x, N_y, N_z) \times \hat{p} \).