INFLUENCE OF DIFFERENT COLLECTIVE COORDINATES
ON SPONTANEOUS FISSION PROCESS
IN Fm ISOTOPES

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Abstract

We study the role of different collective degrees of freedom in spontaneous fission process of even–even Fm isotopes. To find a proper collective space we examine a reach collection of nuclear shape parameters \(\{\beta_\lambda\}\), with \(\lambda=2,3,4,5,6,\) and \(8;\) as well as the paring degrees of freedom i.e. proton \(\Delta_p\) and neutron \(\Delta_n\) pairing gaps. On the basis of the multidimensional dynamic–programming method (MDPM) the optimal collective space \(\{\beta_2, \beta_4, \beta_6, \Delta_p, \Delta_n\}\) for Fm isotopes was found.

1 Introduction

It is commonly known that experimental values of the spontaneous fission half–lives \((T_{sf})\) of nine even–even Fm isotopes (\(N = 142, 144, \ldots, 158\)) form approximately two sides of an acute–angled triangle with a vertex in \(N = 152\). The rapid changes in \(T_{sf}\) on both sides of \(^{252}\text{Fm}\) are particularly dramatic for the heavier Fm isotopes, where \(T_{sf}\) descends by about ten orders of magnitude when one passes from \(^{254}\text{Fm}\) to \(^{258}\text{Fm}\). This strong nonlinear behaviour of \(T_{sf}\) vs. neutron number \(N\) produces good opportunity for testing theoretical models.

The aim of this paper is to present \(T_{sf}\) of even–even Fm isotopes, obtained on the basis of a wholly dynamical analysis in different multidimensional collective spaces.

The used method is described in sect. 2, the results and discussion are given in sect. 3 and the conclusions are presented in sect. 4.

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2 Formulation of the Method

2.1 Spontaneous Fission Half–Life

The spontaneous fission half–lives $T_{sf}$ for the even-even Fm isotopes were evaluated within the one–dimensional WKB semiclassical approximation

$$T_{sf} [\text{yr}] = 10^{-28.4} [1 + \exp S(L)],$$

where $S(L)$ is the action–integral along a fission path $L(s)$ in the multi–dimensional deformation space $\{X_\lambda\}$

$$S(L) = \int_{s_1}^{s_2} \left\{ \frac{2}{\hbar^2} B_{\text{eff}}(s)[V(s) - E] \right\}^{1/2} ds.$$  \hspace{1cm} (2)

An effective inertia associated with the fission motion along the path $L(s)$ is

$$B_{\text{eff}}(s) = \sum_{\lambda,\mu} B_{X_\lambda X_\mu} \frac{dX_\lambda}{ds} \frac{dX_\mu}{ds}.$$ \hspace{1cm} (3)

In above equations $ds$ defines the element of the path length in the $\{X_\lambda\}$ space. The integration limits $s_1$ and $s_2$ correspond to the entrance and exit points of the barrier $V(s)$, determined by a condition $V(s) = E$, where $E = V(X_\lambda^0) + 0.5 \text{ MeV}$ is defined as a sum of a ground–state energy $V(X_\lambda^0)$ and a zero–point energy in the fission direction at the equilibrium deformation $X_\lambda^0$ and denotes the energy of the fissioning nucleus.

2.2 Potential Energy and Inertia Tensor

The potential energy $V$ is calculated by the macroscopic–microscopic model very similar to that used in [4]. For the macroscopic part we used the Yukawa–plus–exponential finite–range model [2] and for microscopic part the Strutinsky shell correction, based on the Woods–Saxon single–particle potential with “universal” variant of the parameters [3]. The single–particle potential is extended to involve residual pairing interaction, which is treated in the BCS approximation. The inertia tensor $B_{X_\lambda X_\mu}$, which describes the inertia of the nucleus with respect to change of its shape, is calculated in the cranking approximation (cf. e.g. [4]).

2.3 Multidimensional Dynamic–Programming Method (MDPM)

Dynamic calculations of the spontaneous–fission half–lives $T_{sf}$ are understood as a quest for least–action trajectory $L_{\text{min}}$ which fulfills a principle of the
least–action $\delta [S(L)] = 0$. To minimize the action integral (2) we used the
dynamic–programming method [4]. Originally this method was used only for
two–dimensional deformation space. We extended the method up to four di-
mensions.

In opposite to approximation used in [1, 5], where only two of coordinates
($\beta_2$ and $\beta_4$) have been handled dynamically and the remaining degrees of free-
dom have been found only by minimization of the potential energy $V$, in our
multidimensional dynamic–programming method (MDPM) all coordinates are
treated dynamically as independent variables.

3 Results and Discussion

3.1 Effect of Higher Even–Multipolarity $\beta_6$ and $\beta_8$ Parameters on
the Spontaneous Fission Half–Lives

Fig. 1. Logarithms of the calculated in the different collective spaces sponta-
neous fission half–lives of the even–even Fm isotopes $vs.$ neutron number.
The experimental values are shown for comparison.

To find the proper collective space for description of the fission process we
examine three kinds of deformation spaces: \{\$\beta_2, \beta_4, \beta_6, \beta_8\}\,$\{\$\beta_2, \beta_4, \beta_35, \beta_6\}\,$\{\$\beta_2, \beta_4, \Delta_p, \Delta_n\}$, where parameter $\beta_35$ defines an average trajectory in a
($\beta_3, \beta_5$) plane; $\Delta_p$ and $\Delta_n$ denotes proton and neutron pairing gap, respectively.

Fig. 1 shows the logarithm of the spontaneous fission half–lives $T_{sf}$, given
in years, for even–even Fm isotopes; obtained when only deformations of the
even–multipolarities \( \{\beta_\lambda\} \), \( \lambda=2,4,6,8 \) are considered. One can see that effect of deformation \( \beta_6 \) on \( T_{sf} \) is stronger than deformation \( \beta_8 \). This is in agreement with earlier results (see e.g. [1]).

3.2 Role of the Reflection–Asymmetry Shape Parameter \( \beta_{35} \)

To examine the role of the deformations with odd multipolarities on \( T_{sf} \) we collect the \( \beta_3 \) and \( \beta_5 \) parameters in one parameter \( \beta_{35} \equiv (\beta_3, \beta_5=0.8\beta_3) \) and perform the dynamical calculation of \( T_{sf} \) in four–dimensional collective space \( \{\beta_2, \beta_4, \beta_{35}, \beta_6\} \). Results of this study are presented in Fig. 2. It is easy to see that reflection–asymmetry shape parameter \( \beta_{35} \) does not change \( T_{sf} \) in Fm region.

The reason of this lies in the dynamical treatment of fission process. The parameters \( \beta_3 \) and \( \beta_5 \) shorten the static fission barriers (i.e. barriers along static paths), however the effective inertia \( B_{eff} \), eq. (3), along these static paths are larger than along the path with \( \beta_{35}=0 \).

![Fig. 2. The same as in Fig.1, but obtained when reflection–asymmetry \( \beta_{35} \) shape parameter (see text) is included in the collective space.](image)

3.3 Influence of the Pairing Degrees of Freedom

The residual pairing interactions are usually treated in the stationary way (i.e. in BCS approximation). The idea of dynamical calculations of \( T_{sf} \) in multidimensional collective space spanned by the shape parameters as well as the
pairing–field parameters \((\Delta_p, \Delta_n)\) was proposed in ref. [3] and practically applied to the macroscopic–microscopic calculations with Nilsson single–particle potential in [4, 8].

Fig. 3. Comparison between \(T_{sf}\) of even-even Fm isotopes, obtained in collective spaces with and without pairing degrees of freedom i.e. proton \((\Delta_p)\) and neutron \((\Delta_n)\) pairing gaps. The difference between two kinds of \(T_{sf}\) is shadowed.

In Fig. 3 we present \(T_{sf}\) of even–even Fm isotopes obtained in four–dimensional collective space \(\{\beta_2, \beta_4, \Delta_p, \Delta_n\}\) and in two–dimensional space \(\{\beta_2, \beta_4\}\). The difference between \(T_{sf}\) got with and without pairing degrees of freedom is shadowed in the figure.

This difference represents the dynamical effect of the pairing degrees of freedom on the spontaneous fission half–lives \(T_{sf}\).

As it is seen, proton \((\Delta_p)\) and neutron \((\Delta_n)\) pairing gaps reduce \(T_{sf}\) for Fm isotopes with \(N > 152\) for about 3 orders of magnitude.

3.4 Optimal Multidimensional Deformation Space

Fig. 4 presents \(T_{sf}\) of even–even Fm isotopes calculated in \(\{\beta_2, \beta_4, \beta_6\}\) collective space and corrected by the effect of the pairing degrees of freedom from Fig. 3.

This correction is strongly isotopic dependent and considerably improves theoretical prediction of \(T_{sf}\). The dynamical study in different deformation
spaces of various dimensions has shown that optimal collective space for description of spontaneous fission half-lives in even-even Fm isotopes is five-dimensional space \{\beta_2, \beta_4, \beta_6, \Delta_p, \Delta_n\}.

Fig. 4. Logarithms of \(T_{sf}\) of even-even Fm isotopes from Fig. 1 calculated in \{\beta_2, \beta_4, \beta_6\} collective space and corrected by the effect of the pairing degrees of freedom from Fig. 3.

4 Conclusions

The following conclusions may be drawn from the present study.

1. The contribution of parameter \(\beta_8\) to \(T_{sf}\) is negligible.

2. In dynamical calculations the deformations with odd multipolarities \(\beta_3\) and \(\beta_5\) do not change \(T_{sf}\).

3. The proton \(\Delta_p\) and neutron \(\Delta_n\) pairing gaps reduce \(T_{sf}\) for heavy even-even Fm isotopes.

4. The optimal collective space for description of spontaneous fission half-lives in even-even Fm isotopes is \{\beta_2, \beta_4, \beta_6, \Delta_p, \Delta_n\}.

References

[1] R. Smolańczuk, J. Skalski and A. Sobiczewski, GSI Preprint 94–77, November, 1994.
[2] P. Möller and J.R. Nix, Nucl. Phys. A361 (1981) 117; At. Data Nucl. Data Tables 26 (1981) 165.

[3] S. Ćwiok, J. Dudek, W. Nazarewicz, J. Skalski and T. Werner, Comput. Phys. Commun. 46 (1987) 379.

[4] A. Baran, K. Pomorski, A. Lukasiak and A. Sobiczewski, Nucl. Phys. A361 (1981) 83.

[5] R. Smolańczuk, H.V. Klapdor–Kleingrothaus and A. Sobiczewski, Acta Phys. Pol. B24 (1993) 685.

[6] L.G. Moretto and R.B. Babinet, Phys. Lett. 49B (1974) 147.

[7] A. Staszczak, A. Baran, K. Pomorski and K. Böning, Phys. Lett. 161B (1985) 227.

[8] A. Staszczak, S. Pilat and K. Pomorski, Nucl. Phys. A504 (1989) 589.