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Damage localization and quantification of composite stratified beam Structures using residual force method

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Abstract. In this paper, the problem of using measured modal parameters to detect and locate damage in beam composite stratified structures with four layers of graphite/epoxy [0°/90/0°] is investigated. A technique based on the residual force method is applied to composite stratified structure with different boundary conditions, the results of damage detection for several damage cases demonstrate that using residual force method as damage index, the damage location can be identified correctly and the damage extents can be estimated as well.

1. Introduction

The damage assessment of structure using vibration analysis based data has been receiving significant attention for the last three decades. A large number of proposed methods for detecting damage based on vibration characteristics by Doebling et al. [1, 2]. The most existing damage detection study was based on modal curvature and investigated the changes of an indicator between the damaged and undamaged structures [3]. Rytter [4] proposed a comparison between different techniques, which consists of four levels. The first level is based on detection, the second level is based on localization, the third level on assessment, and finally the fourth level, which is the consequence of damage, predicts the remaining life of the structure in a certain state of damage. The mode shape expansion method based on the best eigenvector concept to solve the incomplete measurement problem was presented by Yang and Liu [5]. Recently, transmissibility functions, which are based on using only output data, have been proposed to detect damage in structures [6-9]. Numerical techniques such as Finite Element Analysis [10-22] and isogeometric analysis [23-31] have been used along with experimental data to detect and quantify damage in structures. Khatir and al. [32] used finite element method to build snapshot matrix based on proper orthogonal decomposition with radial basis function (POD-RBF) combined with optimization method for damage identification of composite beam structure using vibration data. The finite element method combined with optimization methods for detecting and locating damage of composite structure based on objective function using several objective functions was introduced by Khatir and al. [33-35].

The modal data in damage detection in the form of the response showed that the use of vibration at higher frequencies is better that the identification of delamination in the cantilever composite beams introduced by Valdes and Souts [36]. In the procedure for detecting and locating the variability in structural stiffness using the data obtained by damaged structure, the mode shapes of undamaged structures were smoothed without irregularity and a curve fitting technique was proposed. The first few mode shapes were approximated using this procedure by Yoon et al. [37]. Chiang and Lai [38] combined the residual forces method with the method of simulated evolution for damage identification. The damage detection and localization approach based on the residual forces method was used successfully to locate structural damage based on analytical model for damage identification. Kahl and Sirkis [39] proposed to use the residual forces concept along with the subspace rotation damage identification approach.
This work presents a force residual method for damage detection and localization of stratified beam structure with four layers of graphite/epoxy [0°/90°/90°/0°] discretised in 20 elements with different boundary conditions using FEM based on Matlab programming.

2. The residual forces method

The damage index of the $j^{th}$ element is here expressed as the change of the rigidity of a finite element:

$$\Delta[K]_j^e = ([K]^e_j - [K]^u_j) = \alpha [K]^e_j$$  \hspace{1cm} (1)

Where $[K]^e_j$ and $[K]^u_j$ are the $j^{th}$ element of the elementary matrix of the damaged and undamaged structure, respectively. $\Delta[K]^e_j$ represents the variation of stiffness, $\alpha \in [0,1]$ indicates a loss of rigidity of $j^{th}$ element; i.e. for undamaged element $\alpha = 0$ and for damaged element $\alpha = 1$. We consider that the mass matrix of the damaged structure is not affected by damage, and the rigidity matrix of the damaged element change as given below:

$$[\Delta M] = 0$$  \hspace{1cm} (2)

$$\Delta[K]^e_j = \alpha [K]^e_j$$  \hspace{1cm} (3)

The modal residual force vector can be written as:

$$\{R\}_j = [\Delta K] \{\phi\}_j = \{\Delta f\}_j = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_m \end{bmatrix} = [F] \{\alpha\}$$  \hspace{1cm} (4)

Equation (4) can be written in matrix form as:

$$[F]\{\alpha\} = \{R\}$$  \hspace{1cm} (5)

Where matrix $[F]$ is:

$$\{F\}_j = [K]^e_j \{\phi\}^u_j$$  \hspace{1cm} (6)

The modal residual force vector can be written as:

$$\{R\}_j = ([K] - \lambda \{M\})\{\phi\}_j$$  \hspace{1cm} (7)

Equation (5) can be rewritten as:
Where $n$ is the number of modes, while $m$ is the number of elements. The resolution of equation (8) allows us to determine the values of the damage indicators as:

$$
\begin{bmatrix}
F_{11} & F_{12} & \ldots & F_{1m} \\
F_{21} & F_{22} & \ldots & F_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
F_{n1} & F_{n2} & \ldots & F_{nm}
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_m
\end{bmatrix}
=
\begin{bmatrix}
R_1 \\
R_2 \\
\vdots \\
R_n
\end{bmatrix}
$$

(8)

The methodological approach used in this paper is illustrated in Figure 1.
3. Numerical applications

To evaluate the performance of the proposed residual force method, we consider a stratified composite beam Graphite/epoxy \([0^\circ/90^\circ_2/0^\circ]\) discretized into 20 finite elements SI12 as shown in Figure 2 [40]. The mechanical and geometrical properties are presented in Table 1.

![Figure 2. SI12 FEM.](image)

### Table 1. The material properties and beam structure composite Graphite / epoxy \([0^\circ/90^\circ_2/0^\circ]\).

| Length (m) | Width (m) | Thickness (m) | Young modulus \(E_x\) (GPa) | Young modulus \(E_y\) (GPa) | \(G_{yz}\) (GPa) | \(G_{xy} = G_{xz}\) (GPa) | \(\rho\) (Kg/m\(^3\)) | \(v_{xy}\) |
|------------|-----------|---------------|-----------------------------|-----------------------------|----------------|----------------------------|----------------|----------|
| 1          | 0.05      | 0.04          | 14.48                       | 0.965                       | 0.345         | 0.414                      | 1390           | 0.25     |

3.1 Case 1. Clamped free beam

In first section, we consider clamped – free a stratified beam structure with four layers of graphite/epoxy \([0^\circ/90^\circ_2/0^\circ]\).

![Figure 3. Clamped free beam stratified graphite/epoxy \([0^\circ/90^\circ_2/0^\circ]\).](image)

3.1.1 Single damage

The results of single damaged element 2, 7 and 13, having 20% loss of rigidity, are presented in Figure 4 and the results of a single damaged element 10 with different loss of rigidity 30%, 40% and 50% are presented in the Figure 5.
According to the results sited in the figures 4-5 the value of the damage indicator of the damaged element affected is largest and that whatever the position and level of damage.

3.1.2 Multiple damage
The results of multiple damaged elements (3, 12), (5, 16) and (10, 15) with loss of rigidity 30% are presented in Figure 6.

The results of multiple damaged elements (2, 11, 17), (3, 9, 11, 17) and (2, 6, 11, 13, 16) with loss of rigidity 30%, 35% and 40%, respectively, are presented in Figure 7.
Figure 7. The variation of the damage indicator of clamped–free stratified beam structure with multiple damaged elements (2, 11, 17), (3, 9, 11, 17) and (2, 6, 11, 13, 16) with loss of rigidity 20%, 30% and 35% Successively.

From the results obtained in this application, we can conclude that the method based on the modal residual force method is possible to locate the damage in case of a beam with multiple damages. The frequencies of damaged and undamaged beam stratified structure are presented in the Table 2.

|    | Frequency (Hz)         |  |  |  |  |
|----|----------------------|---|---|---|---|
|    | $f_1$                | $f_2$ | $f_3$  | $f_4$ | $f_5$  |
| FEM | 62.20                | 357.70 | 911.80  | 1599.90 | 1863.40 |
| 2 (20%) | 60.00               | 354.90 | 910.00  | 1599.50 | 1840.80 |
| 7 (20%) | 60.70               | 356.30 | 906.00  | 1599.60 | 1845.90 |
| 13 (20%) | 61.10               | 354.50 | 904.30  | 1599.60 | 1856.20 |
| 3 and 12 (30%) | 59.40               | 349.60 | 904.50  | 1578.30 | 1811.90 |
| 5 and 16 (30%) | 60.00               | 355.90 | 891.90  | 1563.40 | 1824.70 |
| 10 and 15 (30%) | 60.10               | 348.70 | 895.50  | 1563.20 | 1835.30 |
| 2, 11 and 17 (20%) | 59.90               | 350.60 | 904.50  | 1578.30 | 1828.90 |
| 3, 9, 11 and 17 (30%) | 58.90               | 343.50 | 900.00  | 1547.40 | 1784.30 |
| 2, 6, 11, 13, and 16 (35%) | 57.30               | 344.90 | 863.60  | 1541.50 | 1741.10 |

3.2 Case 2. Clamped – clamped beam
In second section, we consider clamped-clamped stratified beam structure with four layers of graphite/epoxy [0°/90°/0°] as shown in Figure 8.
3.2.1 Single damage

The results of single damaged elements 2, 7 and 13, with 20% loss of rigidity clamped–clamped stratified beam structure are presented in Figure 9, and the results of single damaged element 10 with different loss of rigidity 30%, 40% and 50% are presented in Figure 10.

**Figure 8.** Clamped – clamped a stratified beam structure with four layers of graphite/epoxy [$0^\circ$/90°$/0^\circ$].

**Figure 9.** The variation of the damage indicator of clamped – clamped stratified beam structure with single damaged elements 2, 7 and 13 by 20% loss of rigidity.

**Figure 10.** The variation of the damage indicator of clamped – clamped stratified beam structure with single damaged elements 10 with different loss of rigidity 30%, 40% and 50%.
3.2.2 Multiple damage
The results of multiple damaged elements (3, 12), (5, 16) and (10, 15) with loss of rigidity 30% of clamped–clamped stratified beam structure are presented in Figure 11.

![Figure 11](image1.png)

Figure 11. The variation of the damage indicator of clamped – clamped stratified beam structure with multiple damaged elements (3, 12), (5, 16) and (10, 15) with loss of rigidity 30%.

The results of multiple damaged elements (2, 11, 17), (3, 9, 11, 17) and (2, 6, 11, 13, 16) with loss of rigidity 20%, 30% and 35%, respectively, of clamped – clamped stratified beam structure are presented in Figure 12.

![Figure 12](image2.png)

Figure 12. The variation of the damage indicator of clamped – clamped stratified beam structure with multiple damaged elements (2, 11, 17), (3, 9, 11, 17) and (2, 6, 11, 13, 16) with loss of rigidity 20%, 30% and 35% Successively.
From the results obtained in Figures 9 to 12, we can conclude that the method based on the modal residual force method is successful to locate the damage in the case with single and multiple damages. The frequencies of damaged and undamaged beam stratified structure are presented in the Table 3.

| FEM            | Frequency (Hz) |
|----------------|----------------|
|                | $f_1$       | $f_2$       | $f_3$       | $f_4$       | $f_5$       |
| 2 (20%)        | 350.10      | 860.60      | 1502.50     | 2217.90     | 2976.30     |
| 7 (20%)        | 347.20      | 858.80      | 1502.10     | 2216.50     | 2972.20     |
| 13 (20%)       | 349.20      | 854.80      | 1502.00     | 2210.40     | 2963.50     |
| 3 and 12 (30%) | 348.60      | 856.40      | 1501.30     | 2205.10     | 2972.90     |
| 5 and 16 (30%) | 350.00      | 847.70      | 1477.80     | 2209.90     | 2969.70     |
| 10 and 15 (30%)| 345.60      | 850.40      | 1478.40     | 2213.20     | 2939.30     |
| 2, 11 and 17 (20%) | 344.50   | 857.10      | 1485.00     | 2207.10     | 2958.00     |
| 3, 9, 11 and 17 (30%) | 340.00  | 854.20      | 1461.80     | 2174.40     | 2955.30     |
| 2, 6, 11, 13, and 16 (35%) | 335.90 | 828.20      | 1453.00     | 2179.00     | 2909.40     |

4. Conclusion
The case study demonstrates a methodology for damage detection and localization in beam composite stratified structures with four layers of graphite/epoxy [0°/90°/0°/90°]. Several boundary conditions discussed for the robustness of this method for detecting and locating damage in a structure. The presence of damage in a composite structure based on changes in the dynamic properties of the structure. The results show that the residual force method can be detecting single and multiple damage scenarios.

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