Stability analysis of intrinsic camera calibration using probability distributions

Kun Zhan¹, Dieter Fritsch ² and Joerg F Wagner ³

¹ Adaptive Structures in Aerospace Engineering, University of Stuttgart, Pfaffenwaldring 31, D-70569 Stuttgart, Germany
² Institute for Photogrammetry, University of Stuttgart, Geschwister-Scholl-Str. 24D, D-70174 Stuttgart, Germany
³ Adaptive Structures in Aerospace Engineering, University of Stuttgart, Pfaffenwaldring 31, D-70569 Stuttgart, Germany

¹ kun.zhan@pas.uni-stuttgart.de
² dieter.fritsch@ifp.uni-stuttgart.de
³ jfw@pas.uni-stuttgart.de

Abstract. The currently most widely used calibration method in geometric computer vision is based on Zhang’s method. This approach is easy to implement and delivers detailed result evaluations. One drawback, however, might be that the unexperienced user may obtain unstable and unrepeatable calibration results despite small re-projection errors. Therefore, the data must be interpreted with caution and there is definitely a need for further consideration beside re-projections to be taken for better estimations. So far, statistical inference has been used for presenting precision measures only. In our work, we extend the statistical approach quantitatively using large bundles of images and get calibrations from randomly chosen subsets. Then a Maximum Likelihood Estimation of intrinsic parameters is implemented and the statistical behaviour is analyzed. In addition, the recovered expected values of parameters are utilized as ground truth to scrutinize the single influencing factors of the imaging configuration. According to the results we found out that the image block plays a significant role for camera calibration with regard to the orientation of the imaging configuration. Including also suitable manufactured calibration boards as well as other operational issues, finally a well-designed image block provides a repeatable and reliable calibration.

1. Introduction

Camera calibration is one main focus of airborne and close-range photogrammetry. Due to image sensor and lens imperfections, camera calibration is a crucial step in many fields for metric scene measurements. The imperfections lead to deviations of the projection relationships between image and object space and must be compensated. For decades, camera systems are calibrated regarding not only the interior parameters, but also lens distortion, by using additional parameters based on physical and mathematical models [1]. It took decades to prove mathematically, that camera calibration belongs to the problems of algebraic function approximation, as given in detail by [2], [3] and [4].

In geometric computer vision a plane point field calibration with Zhang’s method [5] is widely used, due to its mature algorithm. A comparative review of earlier related works can be found in [6]. In [7] the merits and drawbacks of different methods are illustrated. Test field calibration can work with 3D
test-fields as given in [8], which might, however, require high manufacturing expenses. Zhang’s calibration method requires at least 5 views of images of a chess board from different orientations. After the feature extraction, the projective transformation is calculated for every image. A closed-form solution is applied to derive the initial calibration results and then all parameters including additional distortion parameters will be refined by a global optimization method [5]. Although the method is easy to implement and normally can get high accuracy, users without any related background will end up with unsatisfying accuracies or with small re-projection errors and unstable calibration parameters as stated in [9]. Thus, among many factors, the set-up for image data collection is most important for camera calibration. This is confirmed by many references about image block configurations for camera calibration, as presented in [10], [11].

Our work is proposing a method to quantitatively analyze the different factors to give finally specific suggestions on camera calibration even for users without little prior knowledge. It is triggered by finding a simple but rigorous calibration method for the 3D digitization of the gyroscopic instruments collection of the University of Stuttgart – the Gyrolog project [12].

The ground truth of calibration parameters in reality is unknown and can be approximated only. However, the image configuration and feature extraction process can be assumed to have a normal distribution behavior [13]. Therefore, through a normal distribution estimation from a big amount of calibration experiments, it is possible to recover the expected values. Thus, it can be regarded as ground truth estimating the precision of individual calibration [14]. In this paper, the new calibration evaluation method proposed uses the probability distribution. Moreover, quantitative influences are also analyzed to give practical suggestions.

2. Materials and Methods

In computer vision, the calibration precision is not of high demand. Therefore, the widely-used calibration tools do not deliver strategies for image data collection, which actually will cause either unsatisfied calibration accuracies or small re-projection errors. In [15] the camera calibration wizard delivers some guidance on how to compute the next best pose for calibrations. The method is based on a subsequent optimization process to obtain the next best image for a good calibration. This method has not been widely adopted yet. Most of the methods use well-designed 3D test fields, which provide for better ray intersections, but the expensive costs may be a restriction. In our method, different factors are quantitatively experimented and analyzed and more specific suggestions are given.

2.1. Related Theory

2.1.1. Camera Calibration. For the relationships between the 2D and 3D world in Euclidean geometry, different coordinate systems are defined. At the beginning, the camera model is assumed as an ideal camera model without distortion, which complies with a direct linear transformation.

\[
\begin{bmatrix}
\frac{1}{dX} & 0 & u_0 & f & 0 & 0 & 0 & X \\
0 & 1/dY & v_0 & 0 & f & 0 & 0 & Y \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & Z
\end{bmatrix}
\]

(1)

Here \(X, Y, Z\) are the object coordinates in the world coordinate system, \(R\) and \(t = [X_0, Y_0, Z_0]^T\) are representing rotations and translations, respectively, between the world coordinate system and the camera coordinate system, \(f\) stands for the focal length, \(dX, dY\) are scale factors in the image frame, \((u_0, v_0)\) are principal point coordinates and \(u, v\) are the pixel coordinates. When extending the ideal lens characteristics with distortion, it can be written into the collinearity equations.

\[
u = u_0 + f \frac{r_{11}(X - X_0) + r_{12}(Y - Y_0) + r_{13}(Z - Z_0)}{+r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)} + \Delta u
\]

(2)

\[
v = v_0 + f \frac{r_{12}(X - X_0) + r_{33}(Y - Y_0) + r_{32}(Z - Z_0)}{+r_{13}(X - X_0) + r_{23}(Y - Y_0) + r_{33}(Z - Z_0)} + \Delta v
\]

(3)
Here $\Delta u$ and $\Delta v$ are correction terms for image coordinates, $r_{ij}$ are the components of the rotation matrix $R$. With regard to camera distortion, various models are based on either the mathematical principle, the physical principle or the mixed principle. Among many, the classical Brown model and its variants are most widely used [2]. It classifies the distortion into radial distortion $\Delta_r$ and tangential distortion $\Delta_t$.

$$\Delta = \Delta_r + \Delta_t$$  \hspace{1cm} (4)

In the Brown model, the radial distortion is modeled by the three parameters $K_1$, $K_2$, $K_3$, and $P_1$, $P_2$ are tangential distortion parameters. Furthermore, $r = \sqrt{u^2 + v^2}$:

$$\Delta u = u(1 + K_1 r^2 + K_2 r^4 + K_3 r^6) + (2P_1 u v + P_2(r^2 + 2u^2))$$  \hspace{1cm} (5)

$$\Delta v = v(1 + K_1 r^2 + K_2 r^4 + K_3 r^6) + (2P_2 u v + P_1(r^2 + 2v^2))$$  \hspace{1cm} (6)

Various methods based on the geometrical relationship are put forward with regard to calibration scenes, calibration models and estimation processes. In this work, experiments are mainly depending on the Matlab® Calibration Toolbox, which uses a plane chess-board.

First of all, feature points are extracted from the images, taken from different orientations of the chess-board. After the calculation of view homographies using a non-linear optimization method, extrinsic and intrinsic parameters together with distortion parameters are estimated. In the last step, all parameters are refined with a non-linear optimization as shown in (7).

$$E = \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} || \hat{u}_{i,j} - P(\mathbf{a}, \mathbf{w}, \mathbf{X}_j) ||^2$$  \hspace{1cm} (7)

$E$ is the energy function, $M$ is the number of views, $N$ is the number of points in each image, $\mathbf{a}$ is the vector of intrinsic parameters including distortion parameters, $\mathbf{w}$ is the vector of view parameters, $\mathbf{X}_j$ are 3D coordinates of points on the calibration board, $\hat{u}$ stands for the observed image points, $P(\mathbf{a}, \mathbf{w}, \mathbf{X}_j)$ delivers the projected image coordinates [16].

2.1.2. Influencing Factors for Camera Calibration. In the process of calibration determination, the correlation between the calibration parameters needs to be considered. It has been found, that correlations arise between the interior parameters and exterior orientation parameters [17], [18].

In practice, good ray intersections can reduce the correlation between the exterior and interior orientation parameters. However, within the Matlab® Calibration Toolbox, the guidance on taking images is limited, as such avoiding extreme angles, changing distances, which are important while not comprehensive. Therefore, the images obtained following the instructions will not necessarily deliver satisfying calibration results. In this study, more comprehensive suggestions are given, including relative 90-degree rotations, the complete use of image format and using oblique view images.

2.1.3. Central Limit Theorem (CLT). The general idea of the Central Limit Theorem [19] is that regardless of the population distribution model, if the sample size increases, the sample mean tends to be normally distributed around the population mean, and its standard deviation shrinks as the number of samples $n$ increases. The theorem holds in condition that 1) the sample size must be independent and 2) the sample size must be “big enough”. The CLT is stated as follows:

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \rightarrow N(0,1)$$  \hspace{1cm} (8)

where $\bar{X}$ stands for sample mean, $\mu$ and $\sigma$ are the mean and standard deviation of the normal distribution.

Since the extraction uncertainty of each image feature is independent, different image orientations are considered to be independent as well. Therefore, the final calibration result should fulfill the CLT, which means the parameters from succeeding calibrations should fulfill a normal distribution. More specifically, when taking a big number of images following the guidance to assume that all the images provide useful information for the calibration process, a sufficient number of random subsets can be chosen. It is assumed, that also the unstable calibration parameters should present a normal random error.
distribution behavior. By estimating all the calibration parameters from all calibrations, the expectation values of all parameters should be obtained in a statistical sense:

\[ E(\hat{x}) = x \]  

(9)

This proof is given by a simple definition of the Gauss-Markov model for estimating unknown parameters in a least-squares parameter estimation process. Let be given the linear model

\[ \mathbf{l} = \mathbf{A}x; \quad D(\mathbf{l}) = \sigma^2 \mathbf{I} \]  

(10)

with \( \mathbf{l} \) as observation vector of the image feature coordinates, \( \mathbf{A} \) is the design matrix and \( x \) the vector of unknown parameters; \( E \) is the expectation and \( D \) the dispersion operator. (10) can be reformulated by using the vector of inconsistencies \( \mathbf{v} \),

\[ \mathbf{l} + \mathbf{v} = \mathbf{A}x; \quad D(\mathbf{l}) = \sigma^2 \mathbf{I} \]  

(11)

because

\[ E[\mathbf{l} + \mathbf{v}] = E[\mathbf{A}x] = \mathbf{A}E[x] = \mathbf{A}x \]  

(12)

with \( E(\mathbf{l}) + E(\mathbf{v}) = \mathbf{A}x \) if and only if \( E(\mathbf{v}) \equiv 0 \). The parameter estimation of \( \|\mathbf{v}\|^2 = \text{min} \) finally leads to

\[ \hat{x} = (A' A)^{-1} A' l \]  

(13)

\[ E(\hat{x}) = E[(A' A)^{-1} A' l] = (A' A)^{-1} A' E(l) = (A' A)^{-1} A' A x = x \]  

(14)

2.2. Experiments

For calibration setup, a well-designed chess-board rather than a normal printed paper should be used. The feature point number should also be big enough, due to the fact, that insufficient feature points in every image will cause unstable behavior of the parameter estimation. The illumination on the image together with appropriate settings of the camera is also crucial. Those factors should be fulfilled to avoid any possible impacts. Furthermore, efforts should be paid to design a suitable image configuration.

2.2.1. Experiment Settings. Our experiments are accomplished with two mirrorless cameras, a Sony a7R III with Zeiss Loxia 25mm lens, and a Leica Q (Typ 116) camera with Leica Summilux 28mm lens (see Figure 1). In addition, we investigate the potential of two smartphone cameras: a Galaxy Note 8 from Samsung and an iPhone 7 Plus from Apple. The Sony a7R III and the Leica Q (Typ116) are full frame cameras, while the Galaxy Note 8 smartphone has a wide-angle camera with 1/2.55” sensor and an extra telephoto lens with 1/3.6” sensor. The iPhone 7 Plus’ camera is also a wide-angle lens with 1/3” sensor and an extra telephoto lens with 1/3.6” sensor. In this work, all experimental results are based on a special manufactured 18×29 chess-board by the company calib.io I/S (see Figure 2).

Figure. 1 Camera systems used for calibration. From left to right: Sony a7R III, Zeiss Loxia 25mm lens and Leica Q (Typ 116) (pictures are from respective official websites).

Figure. 2 Camera calibration board (calib.io I/S).

Figure. 3 Workflow.
2.2.2. Experiment Workflow. To start with, we would like to find the ground truth estimation of the intrinsic parameters. According to the instruction of OpenCV or Matlab® Calibration Toolbox [20], 10 to 20 images are appropriate for a calibration. However, in reality, images taken without enough considerations will lead to different intrinsic parameters. Therefore, in our experiment, far more than the required images are taken, with different orientations including slightly tilted or highly tilted images, rotated images, and images with different distances. If some images are ill-conditioned, they may severely influence the final calibration results. In case of ground truth estimation of intrinsic parameters, the ill-conditioned images most probably will have an impact on the confidence interval of the Gaussian fitted functions. We use the Maximum Likelihood Estimation method to calculate the expected values of the calibration parameters. The workflow for the calibration is given in Figure 3.

3. Results

3.1. Gaussian fitting for multisets of calibrations

In the experiments, different numbers of images for the subsets are tested. For a certain subset image number, 1000 subsets are chosen and thus 1000 calibrations are implemented. Therefore, every calibration parameter has 1000 results from different subsets. As stated in 2.2, a Maximum Likelihood Estimation is applied to obtain the expected values (in unit pixels). The respective pixel size of each camera is listed in Table 1, which can be used to convert the given results to metric units. Additionally, for non-full-frame sensors, the crop factor related to the sensor size is needed for the conversion to a 35mm equivalent value. The results of the focal length in X direction are displayed in Figures 4-7.

| Camera Type | Pixel size (μm) | Crop factor | Sony α 7R III | Leica Q | Galaxy Note 8 | iPhone 7 Plus |
|-------------|----------------|-------------|---------------|---------|----------------|----------------|
| Pixel size  | 4.5            | 5.97        | 1.4           | 1.2     | 1.2            | 1.2            |
| Crop factor | 1              | 1           | 6             | 7.2     |                |                |

Table 1. Pixel size of the four different cameras.

Figure. 4 Gaussian fitting experiment results. From top left to bottom right: the calibrated focal length (pixel) in x direction for the Sony α 7R III, for the Leica Q, for the Galaxy Note 8 and for the iPhone 7 Plus.
Figure 4 demonstrates that with different image subsets, the natural mean of the focal length from all the calibrations will converge. Moreover, the Gaussian fitting results indicate that more images for a subset will make a more stable calibration, though more than 20 images do not show a significant improvement. From the comparison of the four curves of Figure 5, the standard deviations of all camera calibrations converge to small values, while camera Sony α 7R III and Leica Q (Typ 116) cameras show better stability than the smartphone cameras of the Galaxy Note 8 and iPhone 7 Plus.

Figure. 5 Gaussian fitting standard deviations of the calibration for four types of cameras.

From the above processing, for different image numbers of subsets we obtain different Gaussian fitting curves with different expected values and standard deviations. Examples of the focal length in x direction and y direction for the four different cameras’ calibration are displayed in figures 6.

Figure. 6 Gaussian fitted standard deviations (SD) of the calibrated focal length. From top left to bottom right: SD of the Sony camera, the Leica camera, Galaxy Note 8 and iPhone 7 plus.

The outcome of the experiments above is that the calibrated parameters are stable and within small range for the calibrated focal length and the other calibration parameters. This means that the influence of the image configuration is greatly reduced by using many subsets for the calibration from a huge image bundle and then estimate the natural mean according to the Central Limit Theorem.

A weighted arithmetic mean for the expected values is determined as the final calibration parameter:
with $X$ as ground truth of the calibration parameter, $x_i$ is the expected value of the $i^{th}$ Gaussian fitting curve, and $\sigma_i$ is the corresponding standard deviation of the normal distribution. Table 2 lists the results of the individual calibration parameters.

Table 2. Calibration parameters for Sony α 7R III, Leica Q, Galaxy Note 8 and iPhone 7 Plus

| Calibration parameters | Estimated Values |
|------------------------|------------------|
|                        | Sony             | Leica            | Galaxy Note 8 | iPhone 7 Plus |
| Focal length x (pixels)| 5819.6096        | 4424.5633        | 3164.8576     | 3281.3755     |
| Focal length y (pixels)| 5819.4368        | 4425.4199        | 3165.3254     | 3281.3854     |
| Principal point x (pixels)| -32.4949      | 3.2504           | 27.8190       | -24.6529      |
| Principal point y (pixels)| -1.4404       | 2.5041           | 13.6578       | 22.9677       |
| Radial distortion K1   | -0.0566          | 0.0479           | 0.1566        | 0.1951        |

The values of the calibration parameters from Table 2 may differ from the specifications of the vendor when transferred to mm. A reason can be the difference between the design value and the production tolerance of the product. However, the concept introduced in this paper is not suggesting readers to implement this workflow for every camera calibration, instead, the method is applied here for the estimation of a ground truth information in order to analyse the single factor influence in section 3.2 so that practical suggestions can be given for a stable calibration with about 20 images.

3.2. Image Block Analysis based on Ground Truth Estimation

After having estimated the intrinsic parameters through the Gaussian Fitting method with the whole image dataset, the composition of the images used for calibration is changed with regard to different factors for further comparison. Oblique imaging angles and roll angles (rotations along the optical axis) are important to make a strong image block for the camera calibration. Among the whole image dataset, a group of photos is taken with relatively small oblique angles, which follows the typical description of camera calibration in most open-source calibration toolboxes. Additional images with bigger oblique angles and roll angles are taken, which compose a better image block to optimize the calibration configurations. Thus, a control experiment with and without big oblique view images and roll movement images is designed to validate the suggestion for taking images. In this section, only the result of the Sony camera experiments is presented in Figure 7.

From Figure 7, the following conclusions can be made:

- Big oblique view images are important for a reliable and stable camera calibration;
- Weak image blocks without big oblique images and roll angles images can lead to unstable calibration results and even deviate from estimated ground truth with more images;
- Roll angle images can contribute to more stable calibration results, and
- Roll angle images affects more the focal length estimation than principle point;

Therefore, for non-expert users, it is of vital importance to pay more attention to the image configuration for camera calibration regarding oblique view angles as well as roll angles.

3.3. Temporal Stability Variation

We have learned that the above factors have a strong influence on the calibration. Therefore, we need to pay attention to the obtained results by taking the right images. For validation, the experiments with the Sony camera are shown as an example. Twenty independent sets of images, for which each consist of 15-25 images, are collected within one week at different epochs for camera calibration. Ten sets of images follow the typical description of camera calibration in most open-source calibration toolboxes, while another ten sets of images with the big oblique angle and roll angle images supplement the
validation, so that the temporal stability of the proposed method is finally proven. Part of the resulting calibration parameters including the calibrated focal length and the principal point coordinates are shown in Figure 8.

**Figure. 7** Single factor analysis results of Gaussian fitting using different image compositions. From top left to bottom right: expected value for the calibrated focal length, standard deviations of the calibrated focal length, expected value for the principal point x coordinate, standard deviations of the principle point x coordinate.
According to the figures, it can be seen that for the image datasets with big oblique angle images and roll angle images, the differences of the calibrated focal length or principal point calibration of different calibrations are within 6 pixels. However, image datasets without strong image block configurations indicate high instability with calibrated parameter deviations of up to 40 pixels. In addition, it must be noted that the experimental conditions in this section are deviating from section 3.1 regarding focus setting and other changes which might cause the deviation of the calibrated parameters between section 3.3 and previous experiments.

4. Discussions

Given the complex nature of the image acquisition process and the uncertainties of the target manufacturing, it is often difficult to identify the exact sources of errors and uncertainties and how they are combined. For reliable and repeatable calibrations for a camera with regard to operational issues especially the strategy of taking images, appropriate hardware preparations, and precisions were studied.

First of all, a sufficient number of images has to be collected, more than enough for a single camera calibration. Also the appropriate conditions concerning camera settings, hardware precision, illumination etc. should be fulfilled. Most importantly, the strategy for taking images has to be respected. Secondly, a big number of subsets from images of different configurations should randomly be chosen, and for each subset a camera calibration has to be implemented with the Matlab® Camera Calibration Toolbox. According to the Central Limit Theorem, the uncertainties of images should follow a normal distribution. Therefore, a Maximum Likelihood Estimation will deliver all calibrated camera parameters - the natural mean is regarded as ground truth of the camera. In addition, different factors, including oblique angles and 90-degree roll angles along the camera axis, should be considered by excluding the corresponding images. The results of such a process were compared and analyzed above. With knowing the influence of validated factors, during the time period of two weeks, also a normal camera calibration with 15-20 images, as suggested by most toolboxes, was repeated 10 times. Finally, the results following our guidelines show much better stability compared with calibrations only following the instructions from computer vision.

In contrast to many camera calibration simulations, our work is based on real situation experiments and aiming for a stable and repeatable camera calibration solution. Effective suggestions are given for non-experienced users. Most important for a more robust and practical solution is the study of precision figures rather than to rely just on re-projection errors, to obtain a stable and repeatable camera calibration [21].

5. Acknowledgements

The authors gratefully acknowledge the funding of this research within the Gyrolog project of the German Ministry of Education and Research (BMBF), FKZ 01UG1774X.
References

[1] Fritsch, D. „Photogrammetrische Auswertung digitaler Bilder – Neue Methoden der Kamerakalibrierung, dichten Bildzuordnung und Interpretation von Punktwolken“. In: Photogrammetrie und Fernerkundung, Ed. C. Heipke, Springer, Berlin, pp. 157-196, 2017.

[2] Tang, R. "Mathematical methods for camera self-calibration in photogrammetry and computer vision." Dissertation, University of Stuttgart, 2013.

[3] Tang, R., Fritsch, D. and Cramer, M. "New rigorous and flexible Fourier self-calibration models for airborne camera calibration." ISPRS Journal of Photogrammetry and Remote Sensing, Vol. 71, pp: 76-85, 2012.

[4] Tang, R., Fritsch, D., Cramer, M. and Schneider, W. “A flexible mathematical method for camera calibration in digital aerial photogrammetry”, Photogrammetric Engineering & Remote Sensing, Vol. 78, No. 10, pp. 1069–1077, 2012.

[5] Zhang, Z. “A flexible new technique for camera calibration”, IEEE Transactions on pattern analysis and machine intelligence, Vol. 22, No. 11, pp. 1330-1334, 2000.

[6] Salvi, J., Armangué, X. and Batlle, J. “A comparative review of camera calibrating methods with accuracy evaluation”, Pattern Recognition, Vol. 35, No. 7, pp. 1617–1635, 2002.

[7] Luhmann, T., Fraser, C. and Maas, H.-G. “Sensor modelling and camera calibration for close-range photogrammetry”, ISPRS Journal of Photogrammetry and Remote Sensing, Vol. 115, pp. 37–46, 2016.

[8] Abraham, S. and Hau, T. “Towards autonomous high-precision calibration of digital cameras”, Videometrics V, International Society for Optics and Photonics, Vol. 3174, pp. 82–93, 1997.

[9] Fraser, C. “Automatic camera calibration in close range photogrammetry”, Photogrammetric Engineering & Remote Sensing, Vol. 79, No. 4, pp. 381–388, 2013.

[10] Hastedt, H. and Luhmann, T. “Investigations on the quality of the interior orientation and its impact in object space for UAV photogrammetry”, International Archives of the Photogrammetry, Remote Sensing & Spatial Information Sciences, Vol. XL-1/W4, pp. 321–328, 2015.

[11] Remondino, F. and Fraser, C. “Digital camera calibration methods: Considerations and comparisons”, International Archives of Photogrammetry, Remote Sensing and Spatial Information Sciences, Vol. 36, No. 5, pp. 266–272, 2006.

[12] Fritsch, D., Wagner, J.F., Simon, S., Ceranski, B., Niklaus, M., Zhan, K., Schweizer, T. and Wang, Z. “Gyrolog – Towards VR Preservations of Gyro Instruments for Historical and Didactical Research”. In: Proceedings 2018 Pacific Neighborhood Consortium Annual Conference and Joint Meetings (PNC). Piscataway, NJ, IEEE, pp. 13-19, 2018.

[13] Taylor, J.R. “Error analysis”, Univ. Science Books, Sausalito, CA, 1997.

[14] Semeniuta, O. “Analysis of camera calibration with respect to measurement accuracy”, Procedia Cirp, Vol. 41, pp. 765–770, 2016.

[15] Peng, S. and Sturm, P. “Calibration wizard: A guidance system for camera calibration”, arXiv preprint arXiv:1811.03264, 2018.

[16] Burger, W. “Zhang’s camera calibration algorithm: In-depth tutorial and implementation”, Report, University of Applied Sciences Upper Austria, Hagenberg, 2016.

[17] Luhmann, T., Robson, S., Kyle, S. and Boehm, J. “Close-range photogrammetry and 3D imaging”, Walter de Gruyter, Berlin/Boston, 2013.

[18] Tang, R. and Fritsch, D. “Correlation analysis of camera self-calibration in close range photogrammetry”, The Photogrammetric Record, Vol. 28, no. 141, pp. 86–95, 2013.

[19] Breuer, P. and Major, P. “Central Limit Theorems for non-linear functionals of Gaussian fields”, Journal of Multivariate Analysis, Vol. 13, No. 3, pp. 425–441, 1983.

[20] Bouguyt, J.-Y. "Camera calibration toolbox for Matlab", URL: http://www.vision.caltech.edu/bouguetj/calib_doc/, last access July 24, 2020.

[21] Fritsch, D. „Ausgleichungsrechnung damals und heute“. In: Johann Gottlieb Friedrich Bohnenberger, Ed. E. Baumann, Verlag W. Kohlhammer, Stuttgart. pp. 303–320, 2016.