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A combinatorial algorithm for computing the rank of a generic partitioned matrix with 2×2 submatrices. (English) Zbl 07606011 Math. Program. 195, No. 1-2 (A), 1-37 (2022)

Summary: In this paper, we consider the problem of computing the rank of a block-structured symbolic matrix (a generic partitioned matrix) $A = (A_{\alpha \beta} x_{\alpha \beta})$, where $A_{\alpha \beta}$ is a $2 \times 2$ matrix over a field $F$ and $x_{\alpha \beta}$ is an indeterminate for $\alpha = 1, 2, \ldots, \mu$ and $\beta = 1, 2, \ldots, \nu$. This problem can be viewed as an algebraic generalization of the bipartite matching problem and was considered by Iwata and Murota (SIAM J Matrix Anal Appl 16(3):719-734, 1995). Recent interests in this problem lie in the connection with non-commutative Edmonds’ problem by Ivanyos et al. (Comput Complex 27:561-593, 2018) and Garg et al. (Found. Comput. Math. 20:223-290, 2020), where a result by Iwata and Murota implicitly states that the rank and non-commutative rank (nc-rank) are the same for this class of symbolic matrices. The main result of this paper is a simple and combinatorial $O((\mu \nu)^2 \min(\mu, \nu))$-time algorithm for computing the symbolic rank of a $(2 \times 2)$-type generic partitioned matrix of size $2\mu \times 2\nu$. Our algorithm is inspired by the Wong sequence algorithm by Ivanyos et al. for the nc-rank of a general symbolic matrix, and requires no blow-up operation, no field extension, and no additional care for bounding the bit-size. Moreover it naturally provides a maximum rank completion of $A$ for an arbitrary field $F$.

MSC:
90Cxx Mathematical programming

Keywords:
generic partitioned matrix; Edmonds’ problem; non-commutative Edmonds’ problem; maximum rank completion problem

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References:
[1] Buss, JF; Frandsen, GS; Shallit, JO, The computational complexity of some problems of linear algebra, J. Comput. Syst. Sci., 58, 572-596 (1999) · Zbl 0941.68059 · doi:10.1006/jcss.1998.1608
[2] Edmonds, J., Systems of distinct representatives and linear algebra, J. Res. Natl. Bur. Stand., 71B, 4, 241-245 (1967) · Zbl 0178.03002 · doi:10.6028/jres.071B.033
[3] Fortin, M.; Reutenauer, C., Commutative/noncommutative rank of linear matrices and subspaces of matrices of low rank, Sém. Lothar. Comb., 52, B52f (2004) · Zbl 1069.15011
[4] Fujishige, S., Király, T., Makino, K., Takazawa, K., Tanigawa, S.: Minimizing submodular functions on diamonds via generalized fractional matroid matchings. EGRES Technical Reports, TR-2014-14, (2014)
[5] Garg, A.; Gurvits, L.; Oliveira, R., Operator scaling: theory and applications, Found. Comput. Math., 20, 223-290 (2020) · Zbl 1432.68617 · doi:10.1007/s10208-019-09417-z
[6] Hamada, M., Hirai, H.: Maximum vanishing subspace problem, CAT(0)-space relaxation, and block-triangularization of partitioned matrix. arXiv:1705.02060 (2017)
[7] Hamada, M., Hirai, H.: Computing the nc-rank via discrete convex optimization on CAT(0) spaces. SIAM J. Appl. Geom. Algebra (to appear)
[8] Hirai, H., Computing the degree of determinants via discrete convex optimization on Euclidean buildings. SIAM J. Appl. Geom. Algebra, 3, 3, 523-557 (2019) · Zbl 1446.90135 · doi:10.1137/17M190823
[9] Ishikawa, T.: Max-rank matrix completion via Wong sequence. Bachelor thesis, The University of Tokyo (2018) (in Japanese)
[10] Ito, H.; Iwata, S.; Murota, K., Block-triangularizations of partitioned matrices under similarity/equivalence transformations, SIAM J. Matrix Anal. Appl., 15, 4, 1226-1255 (1994) · Zbl 0811.15008 · doi:10.1137/S0895479892235599
[11] Ivanyos, G.; Karpinski, M.; Qiao, Y.; Santha, M., Generalized Wong sequences and their applications to Edmonds’ problems, J. Comput. Syst. Sci., 81, 1373-1386 (2015) · Zbl 1320.94102 · doi:10.1016/j.jcss.2015.04.006
[12] Ivanyos, G.; Qiao, Y.; Subrahmanyan, KV, Non-commutative Edmonds’ problem and matrix semi-invariants, Comput. Complex., 26, 717-763 (2017) · Zbl 1421.13002 · doi:10.1007/s00037-016-0143-x
[13] Ivanyos, G.; Qiao, Y.; Subrahmanyan, KV, Constructive non-commutative rank computation is in deterministic polynomial
time, Comput. Complex., 27, 561-593 (2018) · Zbl 1402.68197 · doi:10.1007/s00037-018-0165-7

[14] Iwata, S.; Murota, K., A minimax theorem and a Dulmage-Mendelsohn type decomposition for a class of generic partitioned matrices, SIAM J. Matrix Anal. Appl., 16, 3, 719-734 (1995) · Zbl 0829.15008 · doi:10.1137/S0895479893255901

[15] Kabanets, V.; Impagliazzo, R., Derandomizing polynomial identity tests means proving circuit lower bounds, Comput. Complex., 13, 1-46 (2004) · Zbl 1089.68042 · doi:10.1007/s00037-004-0182-6

[16] Kuivinen, F., On the complexity of submodular function minimisation on diamonds, Discrete Optim., 8, 459-477 (2011) · Zbl 1261.90047 · doi:10.1016/j.disopt.2011.04.001

[17] Lovász, L.: On determinants, matchings, and random algorithms. In: International Symposium on Fundamentals of Computation Theory (FCT’79) (1979)

[18] Lovász, L., Singular spaces of matrices and their application in combinatorics, Bol. Soc. Bras. Mat., 20, 1, 87-99 (1989) · Zbl 0757.05035 · doi:10.1007/BF02585470

[19] Oki, T.: On solving (non)commutative weighted Edmonds’ problem. In: Proceedings of the 47th International Colloquium on Automata, Languages and Programming (ICALP’20), Leibniz International Proceedings in Informatics (LIPIcs), vol. 168, pp. 89:1-89:14 (2020)

[20] Schwartz, JT, Fast probabilistic algorithms for verification of polynomial identities, J. ACM, 27, 4, 701-717 (1980) · Zbl 0452.68050 · doi:10.1145/322217.322225

[21] Tutte, WT, The factorization of linear graphs, J. Lond. Math. Soc., 22, 2, 107-111 (1947) · Zbl 0029.23301 · doi:10.1112/jlms/s1-22.2.107

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