Aggregative simulation method for implementing mathematical models for gas transmission systems

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Abstract. The method of aggregative analysis makes it possible to simulate various technological situations arising during the operation of a gas pipeline. The method makes it possible to determine the deviation of the operating mode of the gas pipeline from the operating mode. In this case, the curves for the temperature and pressure changes of the transported product along the length of the pipeline are calculated.

1. Purpose of study

The operation of a main pipeline is accompanied by a change in the properties of its elements. Linear aggregates are used to describe such systems. The linear unit contains the movements and jumps necessary for the calculation of the operation modes of gas pipeline. The process of operation of any unit (element) of a gas pipeline system is determined by a set of characteristics [1-3]:

- equation of the boundary of state space;
- equations of motion of the point of the vector of additional coordinates in state space;
- ratio for calculating a new state as a result of a jump when it reaches an acceptable boundary and when an input signal is received;
- ratio for calculating coordinates of the original signals.

2. Carrying out the research

The aggregate is determined by sets T, X, Y, Z and transition operators H and outputs G.

T – the multiplier of the considered instants of time \( \tau \); \( \tau \in T \); T – the set of input signals \( x(\tau) \); \( x(\tau) \in X \) – the set of output signals \( y(\tau) \); \( y(\tau) \in Y \). Z – the state space \( z(\tau) \); \( z(\tau) \in Z \).

The plural number of states of a linear aggregate is a finite set of subsets that do not intersect, where \( z_\nu \) – a polytope in \( n(\nu) \) dimensional Euclidean space [4].

State \( z \in Z \) is defined as:

\[
(1) \quad z = (v, z_\nu),
\]

where \( v(\tau) \) – the discrete component or the time interval number;
\( z_\nu \) – the vector of additional coordinates.

Analogously, input and output signals of aggregates are described.

The input signal is:
\[ x = (\mu, x_\mu), \]

where \( \mu \) – the discrete component of the input signal,
\( x_\mu \) – the vector of additional coordinates.

The output signal is:
\[ y = (\lambda, y_\lambda), \]

where \( \lambda \) - the discrete component of the output signal,
\( y_\lambda \) - the vector of additional coordinates.

At the initial time \((\tau = \tau_0)\), the aggregate is in state \( z_0 \), which is defined as:
\[ z_0 = (v_0, z_{v0}), \]

where \( z_{v0} \) – the interior point of polyhedron.

For \( \tau > \tau_0 \), point \( z_{v0}(\tau) \) moves through the interspace of \( z_{v0} \) until it extends beyond boundaries to point \( z^*_{v0} \) at time \( \tau^* \). The state inside the polyhedron varies linearly and is described by relations [5]:
\[ v(\tau) = \text{const}; \]
\[ z_{v1}(\tau) = z_{v1}^0 + \alpha_{v1}(\tau - \tau_{v1}), i = 1, 2, ..., n(v); \]
\[ \alpha_{v1} = \alpha_{v1}, \alpha_{v2}, ..., \alpha_{n(v)} = \text{const}. \]

After the state of the linear aggregate goes over to point \((v, z^*_v)\) to \( \tau = \tau^* + 0 \), a jump from one state to another occurs. In this case, discrete component \( v \) goes to \( v' \) (a jump to a new polyhedron), and the vector of additional coordinates \( z_{v^*} \) into a vector characterizing the internal starting point of new polyhedron \( z_{v1}^0 \).

The time of the exit of coordinate \( z_{v1} \) to boundary \( z_v \) is calculated by the expression:
\[ \tau^* = \tau^0 + z_{v1}. \]

The new state of the system is described as follows:
\[ z = (v', z_{v1}'), z_{v1}' = \left\{ z_{v1}', z_{v2}', ..., z_{vN_1}' \right\}, \]

where \( v' = v_0 + 1 \) – the discrete component;
\( z_{v1}' \) - a vector of additional coordinates;
\( h \)– the integration step in time.

Other coordinates of the spatial state vector are calculated using equation [6]:
\[ z_{v1} = z_{v1}^0 + \alpha_{v1}(\tau^* - \tau^0), i = 2, 4N_x + 4. \]

At the moment of reaching the boundary or at the moment of input of the input signal, the linear aggregate produces an output signal of the form: \( y = (\lambda, y_\lambda) \).

The discrete component of this signal, which appeared as a result of the exit of point \( z_v \) to the boundary, has the form:
When the aggregate receives input signal $x$ at the instant of time $\tau = \hat{\tau}$, $z_\nu(\tau)$ inside the polytope stops, and the state of aggregate $z_\nu(\hat{\tau})$ passes abruptly from point $(\nu, \hat{Z}_\nu)$ to new interior point $(\nu', \hat{Z}_\nu')$ of new polyhedron $z_\nu^*$. The new state is described as:

$$ z^* = (\nu', z_{\nu'}) ,$$

(10)

where

$$ v^* = v, \quad z_{\nu'} = \{ z_{\nu'}^1, ..., z_{\nu'}^{N_x+4} \} .$$

(11)

In this way, the elements of a complex system are described. It is worth noting that the main gas pipeline is divided into linear sections and compressor stations and an external environment is also allocated by a separate element.

Regression models are used to describe the external environment with further correction of the provided value using a first-order autoregression model [7].

Equations describing the motion of gas are transition functions and they are also calculated by grids with time integration step $h$. The total number of grid nodes is $2N_x$.

The state of the unit "linear section" is defined as:

$$ z^* = (v, z_\nu) , \quad z_\nu = \{ z_{\nu}^1, ..., z_{\nu}^{N_x+4} \} ,$$

(12)

where

$\nu$ - the distance from point $h$;

$z_{\nu_1} = \tau$ - time remaining until the end of the interval;

$z_{\nu+i} = P_i$ - the pressure at the $i$ node of the grid, $i = 1 - N_x$;

$z_{\nu+i+N_x+1} = M_i$ - the mass flow in the $i$ node of the grid, $i = 1 - N_x$;

$z_{\nu+i+N_x+1} = T_i$ - the gas temperature at the $i$ node of the grid, $i = 1 - N_x$;

$z_{\nu+i+3N_x+1} = T_{cm_i}$ - the wall temperature of the pipeline in the $i$ node of the grid, $i = 1 - N_x$.

For the combination of aggregates, it is necessary to know the temperature at the beginning of the next aggregate and the length of its breakdown along the $X$ [8].

Let us call such aggregate $j+1$. Then for the $j$ aggregate:

$z_{\nu4N_x+2}$ - gas temperature in second node $j + 1$ of the unit;

$z_{\nu4N_x+4}$ - the wall temperature in $N_x + 2$ at the node + 1 aggregate;

$z_{\nu4N_x+4}$ - the integration step along the length for the $j + 1$ aggregate.

Output signals are:

$$ y = (\lambda, y_\lambda) , \quad y_\lambda = \{ y_\lambda_1, ..., y_\lambda_i \} ,$$

(13)

where

$\lambda = v(\tau)$ - the time interval after which the output signal is output;

$y_\lambda_1 = P_N$ - pressure at the end of the linear section of the pipeline;

$y_\lambda_3 = z_{\nu4N_x+4}$ - the wall temperature of the pipeline at the end of the section;

$y_\lambda_3 = z_{\nu4N_x+4} = M_1$ - the mass flow at the beginning of the section.

If before the unit selection or pumping is performed, then:
\[ y_{\lambda_3} = y_{\lambda_3} \pm M(k), \]  

where \( M(k) \) – the mass flow of selection or pumping ("the plus" sign is taken in the case of selection, and "the minus" sign in the case of pumping);

\[ y_{\lambda_4} = T_2 = z_{v_2N_x+3} \quad \text{temperature in the second node of the site}; \]

\[ y_{\lambda_5} = T_{cm N_x+2} = z_{v_3N_x+3} \quad \text{the wall temperature in the } N_x+2 \text{ node}; \]

\[ y_{\lambda_6} = \Delta x \quad \text{the integration length.} \]

The input signal is:

\[ x = (\mu, x_{\mu}); \]

\[ \mu = \lambda; \]

\[ x_{\mu} = \{x_{\mu_1}, \ldots, x_{\mu_6}\}; \]

\[ x_{\mu_i} = y_{\lambda_4}, i = 1 - 7, i = 4. \]

If the aggregate outputs the output signal to only one aggregate, then \( x_{\mu_4} = y_{\lambda_4} \); if the linear part is ramified, then:

\[ x_{\mu_4} = y_{\lambda_4} \pm \sum_1^N M(k) = y_{\lambda_4} \pm \sum_1^N \sqrt{\frac{(P_{L-\Delta x,k(j)} - P_1)D_{k(j)}^2 P_{k(j)}\pi^2}{\xi_{k(j)}N_{x(j)}}}; \]

where \( k(j) \) – the numbers of the aggregates to which the output signal is output;

\( \xi_{k(j)}, D_{k(j)} \) – coefficients of hydraulic resistance and diameters;

\( P_{L-\Delta x,k(j)} \) – the pressure at section of the pipeline with number \( k \) at a distance of step integration over length \( \Delta x_{k(j)} \) from the \( j \) aggregate.

The signals received by the unit as input, coming from previous unit \( j-1 \) (pressure, temperature) and unit \( j + 1 \) (mass flow) following the same scheme. The operation of the unit "linear section" is determined by the behavior of the system when the input signal arrives [9, 10, 11].

If the signal arrives at the input of the aggregate of the \( c_{j-1} \) aggregate and this signal is determined by parameter \( \mu > v \) in the model, the system goes to state \( z' \) :

\[ z^* = (\nu^*, z_{\nu}); \]

\[ \nu^* = v, \quad z_{\nu} = \{z_{\nu}, ..., z_{\nu}\}; \]

\[ z_{\nu} = 10^{-6}, \]

\[ z_{\nu} = P_1 = x_{\mu_4}, \]

\[ z_{v_2N_x+2} = T_1 = x_{\mu_4}, \]

\[ z_{v_3N_x+2} = x_{\mu_3}. \]

Other additional coordinates of the state remain the same \( (z_{\nu} = z_{\nu}^* = \text{numbers indicated}) \). When the input signal arrives from aggregate \( j+1(\mu = v) \), aggregate state \( x' \) is formed as follows:

\[ z_{v_4N_x+3} = x_{\mu_5}, \]

\[ z_{v_Nx+4} = \Delta x = x_{\mu_7}, \nu^* = v, z_{\nu} = 10^{-6}, z_{v_4N_x+2} = x_{\mu_5}, z_{v_2N_x+1} = M_{N_x} = x_{\mu_4}. \]

The coordinates of vector \( z_{\nu} \) vary according to corresponding laws, which are described by differential equations.
\[
\frac{dz_{vi}}{d\tau} = \alpha_{vi}, \quad i = 1 - 4N_x + 4;
\]
\[
z_{vi} = z_{0vi} + \alpha_{vi}(\tau - \tau_{0vi}).
\]  

Time coordinate \(z_{vi}\) decreases with unit velocity, that is \(\alpha_{vi} = -1\). The pressure and temperature of gas flow, the temperature of the wall at the beginning of the pipeline during the time interval \(h\) does not change and is determined by the corresponding incoming signals. In this case, \(i = 2, \alpha_{v2} = 0, \alpha_{v3N_x+1} = 0\). The mass flow of gas at the end of the pipeline is specified with the input signal and \(\alpha_{v2N_x+1} = 0\).

After simple transformations, let us obtain a system of equations

\[
\frac{dx}{d\tau} = f_i(\tau, z);
\]
\[
z = \{z_i, ..., z_{3N_x}\};
\]
\[
z_i = P_i, i = 1 - N_x, z_{N_x+i} = M_i, i = 1 - N_x
\]
\[
z_{v2N_x+i} = P_i, i = 1 - N_x, z_{v3N_x+i} = T_{cm+iN_x}, i = 1 - N_x.
\]
\[
f_i(\tau, z) = f_{2N_x}(\tau, z) = f_{2N_x+1}(\tau, z) = 0;
\]
\[
f_{2N_x+i}(\tau, z) = \alpha_2 \left(\frac{\partial P}{\partial x}\right) + \frac{\alpha_3}{\alpha_2} M_i;
\]
\[
f_{3N_x+i}(\tau, z) = \alpha_8 \left(\frac{\partial^2 T_{cm}}{\partial x^2}\right) + \alpha_6 \left(T_i - T_{cm+iN_x}\right) + \alpha_7 \left(T_i - T_{cm+iN_x}\right).
\]
\[
f_i(\tau, z) = \frac{1}{\alpha_2} \left(\frac{\partial M}{\partial x}\right), i = 1 - 4N_x;
\]
\[
\alpha_{v_i+1} = 0.5(K^i_1 + K^i_2), i = 1 - 4N_x 4.
\]

For each time interval with step \(h\):
\[
K^i_1 = f_i(\tau, z), K^i_2 = f_i \left(\tau \pm \frac{h}{2}, z \pm \frac{h}{2} K_1\right).
\]  

To find second-order derivatives, equations \(18\) use the values of the corresponding temperatures and incremental integration of the real. These values are the states of the \(j\) aggregate: \(z v_i, i = 4N_x + 2 - 4N_x + 4, \alpha v_i = 0\).

The values of \(\alpha_{v_i}\) found in this way allow us to determine the motion of a point in the state space. The equation of the state space boundary is relation \(z v_i = 0\), which is determined by technological limitations. When the unit leaves the boundary, a new state forms:

\[
z' = (v', z') \quad (21)
\]

The time of the exit of coordinate \(z_{v_i}\) beyond boundary \(z_{v_i}\) is determined from the relation:

\[
\tau^* = \tau^0 + z_{v_i}^0 22\)
\]

where \(\tau^0, z_{v_i}^0\) – the initial time and state of the aggregate.

State \(z'\) of the aggregate is defined as follows:

- discrete component \(v' = v + 1\),
• vector of additional coordinates \( z_{v1} = h \).

Time integration step \( h \) for different aggregates may be different. Other coordinates of the state space vector are determined from equation (18) with \( r = \tau^* \), \( i = 2 - 4N_x + 4 \).

After the formation of new state \( z \), there are: \( v = v', z_y = z_{v'} \). Output signal \( y \) is formed. When the signal is issued, the unit jumps to a new state. And in order to avoid looping, the output signal is delayed. Thus the whole set of characteristics of the unit "linear site ".

The state of the "compressor station" unit, as well as the state of the linear part, is described by the formula (1), and also:

\[
z_y = \{z_{v1}, \ldots, z_{v7}\}, \quad (23)
\]

where \( z_y \) - the number of the time interval of length \( h \);
\( z_{v1} \) - the time remaining until the end of the interval of length \( h \);
\( z_{v2} \) - the pressure at the inlet of the compressor station;
\( z_{v3} \) - the pressure at the station outlet;
\( z_{v4} \) - mass flow of gas at the entrance to the station;
\( z_{v5} \) - mass flow of gas at the outlet of the station;
\( z_{v6} \) - the gas temperature at the entrance to the station;
\( z_{v7} \) - the gas temperature at the station outlet.

The outgoing signal is:

\[
y = (\lambda, y_\lambda), \quad (24)
\]

where \( \lambda = v(\zeta) \) — the number of the time interval after which the output signal is output.

\[
y_\lambda = \{y_{\lambda1}, y_{\lambda2}, y_{\lambda3}\}, \quad (25)
\]

where \( y_{\lambda1}, y_{\lambda2} \) — the pressure and temperature at the outlet of the compressor station;
\( y_{\lambda3} \) — the mass flow of gas at the entrance to the station.

The outgoing signal is:

\[
x = (\mu, x_\mu), \quad x_\mu = \{x_{\mu1}, x_{\mu2}, x_{\mu3}\}; \quad (26)
\]

where \( \mu \) — the number of the time interval of length;
\( x_{\mu1}, x_{\mu2} \) — pressure and temperature at the entrance to the station;
\( x_{\mu3} \) — the mass flow at the output from the station.

The main difference between the description of the compressor station and the linear part is that the station state variables do not change in coordinate, that is the compressor station unit moves only one coordinate \( z_{v1} \).

Time and trajectory of motion are described by equation \( \frac{dz_{v1}}{dt} = -1 \).

Other coordinates \( z_{v1}, i = 2 \div 7 \) are determined by the relation:

\[
\frac{dz_{v1}}{dt} = 0, \quad i = 2 \div 7. \quad (27)
\]
When the incoming signal arrives from the previous aggregate ($\mu > \nu$), state $z^*$ is determined as follows: $z_{v_i} = z_{v_{i'}}$, $z_{v} = \{z_{v_1}, z_{v_2}\}$, where $z_{v_1} = 10^{-6}, z_{v_2} = x_{\mu_1}, z_{v_3} = x_{\mu_4}, z_{v_4} = z_{v_6}, i = 3, 2, 5, 7$.

When the outgoing signal comes from the next aggregate ($\mu = \nu$), state $z^*$ is determined as follows:

$\nu^* = \nu, z_{\nu^*} = \{z_{v_1}' , ..., z_{v_7}'\}, z_{v_8} = x_{\mu_3}, z_{v_5} = z_{v_6} , i = 1 - 4, 5, 7$. (28)

When incoming signals arrive, outgoing signals are not output since when the output to the face of the polyhedron the state of the aggregate executes a jump to state $z' = \{v', z_v\}, \nu' = v + 1$.

$z_{v_4} = \{z_{v_1}, ... , z_{v_7}\}$, (29)

where $z_{v_1} = h$ – the step of integration over time, $z_{v_2} = z_{v_2}', z_{v_4} = z_{v_4}', z_{v_5} = z_{v_5}', z_{v_6} = z_{v_6}'$.

Then the pressure at the outlet from the compressor station is calculated after expression:

$P_n = z_{v_3}(DN^2 - e^n)\rho g + P_1, B = \alpha_0 + \alpha_1X_1 + \alpha_2X_2 + \alpha_3X_3 + \alpha_4X_4$. (30)

In this case:

$X_1 = \ln N, X_2 = \ln v, X_3 = \ln M, X_4 = \ln N \ln v + \ln M \ln N - \ln v \ln M$. (31)

The outgoing temperature is determined by formulas:

$T_v = T_0 + A \frac{\rho h^3}{c_p m}, z_{v_9} = z_{v_0} + A \frac{\rho h^3}{c_p c_v}$. (32)

The outgoing signal from the compressor station:

$y = (\lambda, y_\lambda), y_\lambda = \{y_{\lambda_1}, ..., y_{\lambda_3}\}, y_{\lambda_1} = z_{v_3}, y_{\lambda_2} = z_{v_7}, y_{\lambda_3} = z_{v_4}, \lambda = v$. (33)

When the state of the unit changes:

$\nu = \nu', z_{v_4} = z_{v_6}', i = 1 - 7$.

### 3. Conclusions

Thus a complete description of the compressor station and the linear section of the gas pipeline in the form of a piecewise linear unit is given. To describe the functioning of gas pipeline’s system as a whole, it is necessary to determine the connections between the units to develop a scheme for connecting the elements of the system.

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