Higher Order Invariants in Supergravity

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Abstract

On a historical note, we first describe the early superspace construction of counterterms in supergravity and then move on to a brief discussion of selected areas in string theory where higher order supergravity invariants enter the effective theories. Motivated by this description we argue that it is important to understand $p$-brane actions with $\kappa$-invariant higher order terms, thus re-opening the question of $\kappa$-invariant “rigidity” terms. Finally we describe a recent construction of such an action using the superembedding formalism.

\small
\begin{itemize}
  \item[1] Invited lecture at “Supergravity at 25”, Stony Brook, December 2001.
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\end{itemize}
1 Introduction

Twenty-two years ago, at the Supergravity workshop in Stony Brook 1979 \cite{1}, I gave one of my very first talks in front of an international audience. A few minutes into that talk the chairman had to break up a heated discussion between some members of the audience, using the phrase “Gentlemen, gentlemen, duels will have to wait until after the talk. Pistols will be provided at the back of the room”. A nerve-racking first encounter with the supergravity community.

So what was I talking about and why did it stir up the emotions? The title of my talk was “Use of dimensional reduction in the search for supergravity invariants” \cite{2}, and it was concerned with finding on-shell invariants that might serve as counter terms in a quantization of supergravity. The reason that this was a “hot” subject, you recall, was that supergravity, or at least its maximally extended version, was believed to be finite. If one could prove the absence of counterterms that would be a very important result.

At the time, three loop counterterms had been shown to exist in supergravity at the linearized level \cite{3} \((N = 1)\), \cite{4} \((N = 2)\), and at the full non-linear level \cite{5} \cite{6} \((N = 1)\). I was hoping to adress the all-important \(N = 8\) case by studying the \(N = 1\) theory in 11 dimensions and dimensionally reduce to four. As a warm-up I had looked at six-dimensional supersymmetric Yang-Mills theory with action

\[
\int d^6x \left[ -\frac{1}{4} F_{mn} F^{mn} + i \bar{\lambda} \gamma^m \partial_m \lambda \right], \tag{1}
\]

and constructed the following on-shell higher order invariant \cite{7}:

\[
\int d^6x \left[ T_{mn} T^{mn} + D_{mn} D^{mn} + i \bar{\mathcal{J}}_m \gamma^m \partial_m \mathcal{J} - \frac{3}{4} C_m \partial^2 C^m \right], \tag{2}
\]

where the stress energy tensor \(T_{mn}\), the supercurrent \(\mathcal{J}_m\), and the spin density \(C_m\) are the currents expected from knowledge of the fourdimensional invariant. \(D_{mn}\) however contains a new identically conserved two-form \(* (F \wedge F)\). This was a first sign of a feature that arises abundantly when trying to construct on-shell invariants in 11 dimensional supergravity. This complicates the calculations a lot, and one needs a good way of organizing the calculation, i.e., one should go to superspace.

2 Supergravity Counterterms

At the Stony Brook conference I met Paul Howe who, with Lars Brink, had just constructed the \(N = 8\) on-shell superspace supergravity in four dimensions \cite{8}. We gradually began looking at the problem of constructing higher order invariants and presented the results at the first Nuffield meeting the following summer (1980) \cite{9}. Here is a brief summary of our results \cite{10}:

Extended supergravity in superspace is described by a superfield \(W\), which is a scalar for \(N \geq 4\) and has spin \(2 - N/2\) for \(N < 4\). The superspace tangent space group is
$SL(2,\mathbb{C}) \otimes G$, where $G$ is (a subgroup of) $U(N)$. The superfield $W$ also carries internal indices corresponding to $G$, and may further transform in a representation of some global symmetry group $G'$ ($=E_7$ for $N = 8$). This description is on-shell.

The linearized form of the $n$-loop counterterm Lagrangians we consider is

$$\mathcal{L}^{(n)} = \kappa^{2(n-1)}\{R^{n+1} + \text{susy completion}\},$$

where $R^{n+1}$ is shorthand for a $n$-fold product of the Riemann tensor with suitably contracted indices. In superspace this corresponds to

$$S^{(n)} = \int d^4xd^{4N}\theta \mathcal{L}^{(n)}(z),$$

where we restrict ourselves to full superspace integrals (over all $z = x, \theta$). For $N \leq 3$, there is a full non-linear 3-loop counterterm respecting all symmetries:

$$\mathcal{L}^{(3)}(z) = \kappa^4 EW^2\bar{W}^2,$$

with $E$ the super-determinant of the super-vielbein and summation of indices is suppressed. At the linearized level Renata Kallosh constructed a 3-loop counterterm also for $N = 8$, although neither the supersymmetry nor the $SU(8)$ is manifest [13]. Its manifest version was given in superspace in [14], and in harmonic superspace in [15].

For $N \leq 4$ there are $N-1$ loop invariants similar to (4), constructed from products of the fundamental superfield to the appropriate power, but they do not respect the global $G'$ invariance of the field equations. To find fully invariant actions we had to go to $N$-loops.

The Lagrangian is

$$\mathcal{L}^{(N)} = \kappa^{2(N-1)}E \epsilon^{\alpha\beta} \epsilon^{\dot{\alpha}\dot{\beta}} \chi_{aabc} \chi_{\beta def} \bar{\chi}_{\dot{a}\dot{b}\dot{c}} \bar{\chi}_{\dot{\beta} \dot{d} \dot{e} \dot{f}},$$

where the torsion $T^{\dot{\gamma}}_{\alpha\beta} = 2\epsilon^{\alpha\beta}\chi^{abc\dot{\gamma}}$, and $\chi$ occurs as first spinor derivative of the fundamental superfield ($a,b,..$ are $G$-indices). At the linearized level, the spin-two content of ($\mathcal{L}$) is a D'Alembertian to the $N - 3$ power sandwiched between the square of the Weyl spinor and the square of its complex conjugate.

In particular our results show that there are possible counterterms even in the maximally extended supergravity, and point towards the rebirth of string theory and the modern interpretation of supergravity as an effective theory.

### 3 String Theory Effective Actions

In string theory, supergravities have their role as effective theories for describing the massless sector of the field theory limit to lowest order in $\alpha'$, as well as, in 11 dimensions, the

\footnote{At the linearized level, this term had been constructed for $N = 1$ [11]. The corresponding non-linear off-shell $N = 1$ term was only recently constructed [12].}

\footnote{An underlined index denotes a pair of tangent space indices, e.g., $\gamma \equiv (c,\gamma)$. This notation is used only here, and in later sections underlining will denote ambient space time indices.}

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lower energy limit of $\mathcal{M}$-theory. As effective theories, the supergravity actions are expected to receive higher order derivative corrections. In the $\mathcal{M}$-theory context in particular, there is very active research aimed at finding the correct modification of supergravity\cite{16, 17, 18, 19}.

Further, the lowest order dynamics of $D$-branes in such a background is given by an effective action which is a sum of the Dirac-Born-Infeld (DBI) action and a Wess-Zumino-Witten (WZW) action $\mathcal{S} = \mathcal{S}_{DBI} + \mathcal{S}_{WZW}$. The higher order corrections to these actions are partly known. E.g., the WZW action, which describes the coupling of $Dp$-branes to the bulk Ramond-Ramond fields\cite{20, 21, 22}:

$$\mathcal{S}_{WZW} = T_p \int_{M^{p+1}} C \wedge tr_N \left( e^{2\pi \alpha' F} \right) \wedge \left( \frac{\hat{A}(4\pi^2 \alpha' R_T)}{\hat{A}(4\pi^2 \alpha' R_N)} \right)^{1/2}, \quad (7)$$

where $T_p$ is the tension of the $Dp$-brane and the trace is in the fundamental representation. The square root of the Dirac ‘roof’ genus has an expansion in even powers of the curvature two-form,

$$\sqrt{\hat{A}} = 1 - \frac{1}{48} p_1(R) + \frac{1}{2560} p_2^2(R) - \frac{1}{2880} p_2(R) + ... , \quad (8)$$

with $p_1$ and $p_2$ the first two Pontryagin classes. The tangential and normal curvatures that occur in (7) are given by the Gauss-Codazzi relations according to

$$(R_T)_{mnr} = R_{mnr} + \delta_{p'}(\Omega_{p'}^r)_{m[r} \Omega_{s]}^t) \quad (9)$$

$$(R_N)_{mn} = -R_{p'}^r m[n + g^r_s (\Omega_{p'}^r)_{m[n} \Omega_{s]}^t) \quad (10)$$

where $\Omega$ is the second fundamental form, unprimed indices are worldvolume indices and primed indices refer to the normal bundle.

Similarly, higher order corrections to the DBI action,

$$\mathcal{S}_{DBI} = T_p \int d^{p+1}x e^\Phi \sqrt{-det[(G_{mn} + B_{mn})\partial_m X^n \partial_n X + F_{mn}]} , \quad (11)$$

have been calculated. (Here $\Phi$ is the dilaton and $B_{mn}$ the antisymmetric NS-NS field, and underlined indices refer to the ambient space-time.)

To second order in $\alpha'$, $\partial F$-corrections were calculated in\cite{23}, and higher order such corrections were computed for the combined DBI-WZW action\cite{24} in a constant background.

The higher $\partial^2 X$ derivative corrections were treated to to second order in $\alpha'$ in\cite{25, 26}. In a background where the $B$ and $F$ fields vanish, the corrections are given by

$$\mathcal{S}_{DBI}^{(2)} = cT_p \int d^{p+1}x e^{-\Phi} \sqrt{-g(\alpha')^2[(R_T)_{mnr}(R_T)^{mnr} - 2(R_T)_{mn}(R_T)^{mn}}$$

$$- (R_N)_{mnpq}(R_N)^{mnpq} + 2\bar{R}_{p'q'} \bar{R}_{p'q'}] , \quad (12)$$

$^5$Here $C = C^{(0)} + C^{(1)} + ... + C^{(9)}$ is a formal sum of the Ramond $n$-forms, odd forms contributing in the IIA and even in the IIB theory.

$^6$In a constant background.
where $g$ is the determinant of the induced metric, $c$ is a numerical constant and $\bar{R}$ is obtained by contracting tangent indices only on the curvature tensor and adding a term quadratic in the second fundamental form. The result in (13) is unique up to an ambiguity involving the trace of the second fundamental form $\Omega_{m}^{p'} m$.

Now, all the higher order terms for the $Dp$-branes described above concern the purely bosonic theory. At the lowest order in $\alpha'$ and in an arbitrary supergravity background, a particular combination $S = S_{DBI} + S_{WZW}$ is needed for $\kappa$ symmetry of the corresponding Green-Schwarz (GS) action [27]. It is thus important for consistency to understand how the above results can be extended in a $\kappa$-symmetric way.

As seen from (10), (7) and (13), this entails finding $\kappa$-symmetric brane actions with terms that involve the second fundamental form. Such terms are often (sloppily) referred to as “extrinsic curvature terms” or “rigidity” terms, and we are thus led to re-open the quest for $\kappa$-symmetric brane actions with rigidity terms.

\section{4 Rigid Branes}

Partly motivated by Polyakov’s suggestion that the QCD string should be viewed as a string with extrinsic curvature terms [28], ”rigid” strings and $p$-branes were quite extensively studied in the 80’s, [29, 30, 31] and early 90’s [32]. The string action that was suggested in [28] is

\begin{equation}
S_{R} = \frac{1}{2\pi\alpha'} \int d^2x \sqrt{-g} [1 + \mu \Omega^2] ,
\end{equation}

where

\begin{equation}
\Omega^2 \equiv \Omega_{m}^{p'}{m} \Omega_{n}^{p'}{n} \sim \eta_{m}^{\bar{n}} \partial_{m} \partial^{m} X^{\bar{n}} \partial_{n} \partial^{n} X^{n} ,
\end{equation}

and $\mu$ is a dimensionful coupling constant. In attempts to include this action, or its generalization, in the bosonic sector of a GS action for a $p$-brane, one has to consider $\kappa$ symmetry, i.e., the generalization to $p$-branes of Siegel’s symmetry for the superparticle [33]. In the simple setting of a flat space background, the action for a the 1-brane (string) reads

\begin{equation}
S_{GS} = T_{p} \int d^2x [\sqrt{-g} + \epsilon^{mn} \bar{\theta} \Pi_{m} \partial_{n} \theta] ,
\end{equation}

where $g_{mn}$ is the globally supersymmetrized induced metric

\begin{equation}
g_{mn} \equiv \eta_{mn} \Pi_{m}^{\bar{n}} \Pi_{n}^{\bar{n}}
\end{equation}

\begin{equation}
\Pi_{m}^{\bar{n}} = \partial_{m} X^{\bar{n}} - i \bar{\theta} \Gamma^{\bar{m}} \partial_{n} \theta
\end{equation}

The action (16) is invariant under the following symmetry with a local parameter $\kappa$:

\begin{equation}
\delta \theta = \kappa \quad \delta X^{\bar{m}} = -i \kappa \Gamma^{\bar{m}} \theta .
\end{equation}

(Apart from $\kappa$ being local, the only formal difference from the global supersymmetry is a sign-change in $\delta X$). The parameter $\kappa$ satisfies a projection relation

\begin{equation}
\kappa = P \kappa ,
\end{equation}
where \( P^2 = P \).

The outcome of the attempts to construct rigid \( p \)-branes may be summarized as follows: A \( \kappa \)-symmetric GS type "rigid" string was proposed in [34], and a generalization to higher \( p \) was suggested in [35]. The \( \kappa \)-symmetry was only shown (for the string) to second order in the spinorial target space coordinate \( \theta \), however, and a fully \( \kappa \)-symmetric formulation was only found for the "rigid" superparticle [36], [37], [38]. (For the 2-brane, spinning, i.e., locally worldvolume supersymmetric, formulations with extrinsic curvature terms were found [39].)

It is interesting to note that the most complete result, the \( \kappa \)-symmetric description of the "rigid" superparticle, is based on a description where \( \kappa \)-symmetry is embedded in a local worldline superconformal symmetry, a fact which points to the way we understand \( \kappa \)-symmetry today, namely as defined in terms of the local supersymmetry of the world-surface in the superembedding approach to \( p \)-branes. This formalism is thus a natural starting point for a renewed attempt at finding a rigid \( \kappa \)-symmetric \( p \)-brane.

## 5 Superembeddings

The superembedding formalism was first used in the context of superparticles [42, 40, 41], and has been applied to various other branes, for a review see [43]. In [43, 46] the formalism is extended to include branes with gauge fields on the world volume and the \( M5 \) brane dynamics derived. Below follows a brief description of the formalism.

We consider an embedding of one superspace \( \underline{M} \) into another \( \underline{\underline{M}} \), given in terms of the coordinates as \( \underline{X}^\underline{M}(X^\underline{\underline{M}}) \). As above, underlined indices refer to the ambient (super-)space and the bare indices to the world volume. Tangent space indices are from the beginning of the alphabet and world indices from the middle. Lower case latin letters denote boson and lower case Greek letters denote fermi indices. Hence, e.g., the ambient superspace has tangent space indices \( A = (a, \alpha) \). Indices for the normal bundle are denoted by primes, as in, e.g., \( (a', \alpha') \).

The embedding matrix is defined as follows

\[
E^A_A \equiv E^M_M \partial_M \underline{X}^\underline{M} E^A_M .
\]

The basic geometric embedding condition which ensures that the odd tangent space of the world volume is a subspace of the odd tangent space of the target superspace is:

\[
E^\alpha_A = 0 .
\]

To explore the consequences of the embedding condition (22) one specifies the geometry of \( \underline{\underline{M}} \), parametrizes the embedding and studies the following torsion equation:

\[
2\nabla_{[A} E^C_{B]} + T^C_{AB} E^C = (\Lambda^{(B+\underline{M})} \Lambda E^A_T)^{\underline{\underline{M}}} .
\]

\[\text{[44] is an earlier attempt to use source and target superspaces.}\]
This relation is the pull-back of the equation defining the target space torsion two-form. (The covariant derivative acts on all types of tensor indices.) The analysis of \((22)\) and \((23)\) yields the induced supergeometry on the embedded brane in terms of multiplets which are typically on-shell for large number of supersymmetries. For \(N \leq 16\) it can also be off-shell. the equations of the component or GS formalism can be obtained by taking the leading \((\theta^\mu = 0)\) components of the equations that describe the brane multiplet. These equations are guaranteed to be \(\kappa\)-symmetric. We summarize the argument:

Let \(v^M\) be a worldvolume vector field generating infinitesimal diffeomorphisms. Then

\[
\delta X^M = v^M \partial_M X^M ,
\]

or

\[
\delta X^\underline{A} \equiv \delta X^M E^\underline{A}_M = v^A E^\underline{A} .
\]

For the special case when \(v^\alpha = 0\), this gives, using \((22)\),

\[
\delta X^\alpha = 0 \\
\delta X^\dot{\alpha} = v^\alpha E^\underline{A}_{\dot{\alpha}} .
\]

We recover the usual \(\kappa\)-symmetry relations by defining

\[
\kappa^\underline{A} \equiv v^\alpha E^\underline{A}_{\dot{\alpha}} ,
\]

and noting that it has to lie in the worldvolume subspace of the odd tangent space of the target space. The latter condition means that there is a projection operator \(P\) such that \(\kappa\) satisfies (c.f. \((20)\)):

\[
\kappa^\underline{A} = \kappa^\underline{B} P^\underline{B}_\underline{D} .
\]

The fact that the embedding formalism yields \(\kappa\)-invariant results makes it eminently suitable when looking for a \(\kappa\)-symmetric rigid \(p\)-brane, provided there is an off-shell description of the embedding.

6 A \(\kappa\)-symmetric Rigid Membrane

The simplest example of the kind of models we are looking for is found in the superembedding of a membrane in flat four-dimensional superspace. In this section we highlight a few of its features, a full description may be found in [47].

For the embedding matrix we can take,

\[
E_{\alpha}^{\underline{a}} = 0 \\
E_{\dot{\alpha}}^{\underline{a}} = u_{\alpha}^{\underline{a}} + i \delta_{\alpha}^{\beta} h u_{\beta}^{\underline{a}} ,
\]

where both \(\alpha\) and \(\alpha'\) are \(d = 3\) spinor indices taking two values. We also have

\[
E_{\underline{a}}^{\underline{a}} = u_{\underline{a}}^{\underline{a}} \\
T_{\alpha \beta}^{c} = -i (\gamma^{c})_{\alpha \beta} \\
E_{a}^{\alpha} = (1 + h^2) u_{a}^{\alpha} ;
\]
where
\[ \text{Spin}(1, 3) \ni u = \left( \begin{array}{c} u_\alpha^\alpha \\ u_\alpha^\alpha' \end{array} \right) \] (35)

and where the corresponding element of \( SO(1, 3) \) is made up by \( u_\alpha^\alpha \) and a normal component \( u_3^\alpha \). The geometry allows us to complement the dimension zero worldvolume torsion in (34) with the rest of the standard off-shell \( N = 1, d = 3 \) supergravity torsion constraints \[ T_{\alpha\beta\gamma} = 0 \quad T_{ab}^c = 0 \] (36)
\[ T_{ab}^c = 0 \quad T_{a\beta\gamma} = i(\gamma_a)_{\beta\gamma} S. \]
The field \( S \) is later determined in terms of the worldvolume multiplet so that the geometry is indeed induced.

From this starting point, we may systematically analyze the consequences of (23), order by order in (mass) dimension. Below only those definitions needed to interpret our final result for the action are given.

One of the fields that enter, \( h \), has already been introduced above. Another, \( X_{ab3} \), is one of the components of the \( \text{spin}(1, 3) \) valued one-form \( X = duu^{-1} \), and \( \Lambda_a^\alpha u_\alpha^\alpha \), finally is defined by
\[ E_a^\alpha = \Lambda_a^\alpha u_\alpha^\alpha + \psi_a^\alpha' u_\alpha'^\alpha. \] (37)

A systematic procedure for constructing actions in the superembedding formalism is presented in [49]. There it was used to rederive the GS action for the our membrane in the form
\[ S_{\text{GS}}^0 = T_2 \int d^3x \left[ \sqrt{-g} \left( \frac{1 - h^2}{1 + h^2} \right) - \frac{1}{6} \epsilon^{mnp} C_{mnp} \right], \] (38)
where \( | \) denotes taking the lowest component of the superfield. Since we want to find an extension of Polyakov’s action (14), the action (38) provides an already \( \kappa \)-symmetric starting point, but we have to provide the higher order terms. From the second term in (14), we see that the higher order Lagrangian has to have dimension 1. Such terms may be found by constructing terms quadratic in \( \Lambda_a^\alpha \) multiplied by arbitrary functions of \( h \) (since \( h \) has dimension zero).

There are two possible quadratic terms one can construct from \( \Lambda \),
\[ L^{(1)} = -\frac{i}{2} \Lambda^{a\alpha} \Lambda_{a\alpha} \] (39)
and
\[ L^{(2)} = -\frac{i}{2} (\gamma^a)^{a\beta} \Lambda_{a\alpha} \Lambda_{b\beta}. \] (40)
The contributions to the GS action from these terms are (neglecting fermions)
\[ -i \nabla^a \nabla_a L^{(1)} = \frac{1}{2} X_{ab3} X^{ab3} + \frac{h^2(2 + 9h^2)}{2(1 + 3h^2)^2} - \frac{2(1 + 2h^2 + 3h^4)}{f^2} (\nabla h)^2 \] (41)
A general linear combination of these two terms with $h$-dependent coefficients will lead to a rather complicated bosonic Lagrangian. Using the freedom to multiply by arbitrary functions of $h$, one may ensure that the $h$-equation remains algebraic (as in (48)) above. It would be very complicated in general, though. A second possibility is to take the linear combination $L^{(1)} + \frac{1}{2} L^{(2)}$. In this case there are still derivative $h$ terms but all the coefficient functions simplify dramatically. We find

$$L_0^{(1)} + \frac{1}{2} L_0^{(2)} = \frac{3}{4} \hat{X}_{ab3} \hat{X}^{ab3} - \frac{2}{f^2} (\nabla h)^2$$

where $\hat{X}_{ab3} := X_{ab3} - \frac{1}{3} \eta_{ab} X_3$ is traceless. Lagrangian for the sum of the GS and higher derivative Lagrangian:

$$L(x) = \sqrt{-\det g} \left( \frac{1}{1 + h^2} \frac{1}{2} \hat{\Omega}^2 - \frac{4}{3 f^2} (\nabla h)^2 \right)$$

where

$$\Omega_{mn}^3 = (\nabla_m \partial_n x^c)(u^{-1})_c^3 \mathcal{R}$$

with $\hat{\Omega}$ being the corresponding traceless tensor. The metric here is

$$g_{mn} = \partial_m x^a \partial_n x^a$$

i.e. the standard induced metric for the associated bosonic embedding. The tensor $\hat{\Omega}_{mn}^3$ is then the traceless second fundamental form for this embedding.

If we now set $h = \tan \phi$ the Lagrangian becomes

$$L(x) = \sqrt{-\det g} (\frac{\beta}{2} \hat{\Omega}^2 - \frac{4}{3} (\nabla \phi)^2 + \cos 2\phi)$$

Hence the equation of motion for the auxiliary field in this case is simply the sine-Gordon equation,

$$\nabla^2 \phi - \frac{3}{4\beta} \sin 2\phi = 0$$

7 Summary

Higher derivative supergravity terms were studied earlier as counter terms for the quantum theory. In that context, the superspace eight-loop counterterm remains the only
example of an higher order invariant in $N = 8$ supergravity which respects all the sym-
metries of that theory. There have been arguments put forward for the existence of a
five-loop counterterm, though \[50\], \[51\].

Nowadays higher derivative supergravity terms arise, e.g., in string theory low energy
effective actions. Investigations of corrections to $D$-brane actions yield “extrinsic curva-
ture” terms in the bosonic sector of the higher order Lagrangians. The full GS actions
must thus contain $\kappa$-symmetric completions of these terms. When so-called “rigid” strings
and branes were studied previously such a $\kappa$-symmetrization proved notoriously difficult.
With the advent of the superembedding formalism we now have a systematic way of
constructing such models, at least for the cases where the multiplets governing the em-
beddings are off-shell. A membrane in flat four-dimensional superspace is such a case and
has been worked out in detail.

An open question is if it is possible to extend the results to $D$-branes and verify that the
known bosonic sectors may be $\kappa$-symmetrized. Perhaps the superembedding formalism
needs to be modified to allow off-shell descriptions of the relevant embeddings.

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References

[1] P. Van Nieuwenhuizen and D. Z. Freedman, “Supergravity. Proceedings, Workshop At Stony Brook, 27-29 September 1979,” Amsterdam, Netherlands: North-holland (1979) 341p.

[2] U. Lindstrom, “Use Of Dimensional Reduction In The Search For Supergravity Invariants. (Talk),” In Stony Brook 1979, Proceedings, Supergravity, 149-154.

[3] S. Deser, J. H. Kay and K. S. Stelle, “Renormalizability Properties Of Supergravity,” Phys. Rev. Lett. 38, 527 (1977).

[4] S. Deser and J. H. Kay, “Three Loop Counterterms For Extended Supergravity,” Phys. Lett. B 76, 400 (1978).

[5] M. Kaku, P. K. Townsend and P. van Nieuwenhuizen, “Superconformal Unified Field Theory,” Phys. Rev. Lett. 39, 1109 (1977).

[6] S. Ferrara and P. Van Nieuwenhuizen, “Structure Of Supergravity,” Phys. Lett. B 78, 573 (1978).

[7] S. Deser and U. Lindstrom, “Extended Supersymmetry Invariants By Dimensional Reduction,” Phys. Lett. B 90, 68 (1980).

[8] L. Brink and P. Howe, “The N=8 Supergravity In Superspace,” Phys. Lett. B 88, 268 (1979).

[9] P. Howe and U. Lindstrom, “Counterterms For Extended Supergravity. (Talk),” In Cambridge 1980, Proceedings, Superspace and Supergravity, 413-422.

[10] P. Howe and U. Lindstrom, “Higher Order Invariants In Extended Supergravity,” Nucl. Phys. B 181, 487 (1981).

[11] S. Ferrara and B. Zumino, “Structure Of Conformal Supergravity,” Nucl. Phys. B 134, 301 (1978).

[12] F. Moura, “Four dimensional N = 1 supersymmetrization of R**4 in superspace,” JHEP 0109, 026 (2001) [arXiv:hep-th/0106028].

[13] R. E. Kallosh, “Counterterms In Extended Supergravities,” Phys. Lett. B 99, 122 (1981): Lebedev Phys.Inst. preprint no 152 (1980).

[14] P. S. Howe, K. S. Stelle and P. K. Townsend, “Superactions,” Nucl. Phys. B 191, 445 (1981).

[15] G. G. Hartwell and P. S. Howe, “(N, p, q) harmonic superspace,” Int. J. Mod. Phys. A 10, 3901 (1995) [arXiv:hep-th/9412147].
\[\text{[16]}\] P. S. Howe, “Weyl superspace,” Phys. Lett. B 415, 149 (1997) [arXiv:hep-th/9707184].

\[\text{[17]}\] M. Cederwall, U. Gran, M. Nielsen and B. E. Nilsson, “Manifestly supersymmetric M-theory,” JHEP 0010, 041 (2000) [arXiv:hep-th/0007033].

\[\text{[18]}\] K. Peeters, P. Vanhove and A. Westerberg, “Supersymmetric higher-derivative actions in ten and eleven dimensions, the associated superalgebras and their formulation in superspace,” Class. Quant. Grav. 18, 843 (2001) [arXiv:hep-th/0010167].

\[\text{[19]}\] K. Peeters, P. Vanhove and A. Westerberg, “Chiral splitting and world-sheet gravitinos in higher-derivative string amplitudes,” [arXiv:hep-th/0112157].

\[\text{[20]}\] M. B. Green, J. A. Harvey and G. W. Moore, “I-brane inflow and anomalous couplings on D-branes,” Class. Quant. Grav. 14, 47 (1997) [arXiv:hep-th/9605033].

\[\text{[21]}\] Y. K. Cheung and Z. Yin, “Anomalies, branes, and currents,” Nucl. Phys. B 517, 69 (1998) [arXiv:hep-th/9710200].

\[\text{[22]}\] R. Minasian and G. W. Moore, “K-theory and Ramond-Ramond charge,” JHEP 9711, 002 (1997) [arXiv:hep-th/9710230].

\[\text{[23]}\] O. D. Andreev and A. A. Tseytlin, “Partition Function Representation For The Open Superstring Effective Action: Cancellation Of Mobius Infinities And Derivative Corrections To Born-Infeld Lagrangian,” Nucl. Phys. B 311, 205 (1988).

\[\text{[24]}\] N. Wyllard, “Derivative corrections to D-brane actions with constant background fields,” Nucl. Phys. B 598, 247 (2001) [arXiv:hep-th/0008125].

\[\text{[25]}\] C. P. Bachas, P. Bain and M. B. Green, “Curvature terms in D-brane actions and their M-theory origin,” JHEP 9905, 011 (1999) [arXiv:hep-th/9903210].

\[\text{[26]}\] A. Fotopoulos, “On \((\alpha')^2\) corrections to the D-brane action for non-geodesic world-volume embeddings,” JHEP 0109, 005 (2001) [arXiv:hep-th/0104146].

\[\text{[27]}\] M. Cederwall, A. von Gussich, B. E. Nilsson, P. Sundell and A. Westerberg, “The Dirichlet super-p-branes in ten-dimensional type IIA and IIB supergravity,” Nucl. Phys. B 490, 179 (1997) [arXiv:hep-th/9611158].

\[\text{[28]}\] A. M. Polyakov, “Fine Structure Of Strings”, Nucl. Phys. B268 (1986) 406.

\[\text{[29]}\] H. Kleinert, “The Membrane Properties Of Condensing Strings” Phys. Lett. B174 (1986) 335.

\[\text{[30]}\] T. Curtright, G. Ghandour and C. Zachos, “Classical Dynamics Of Strings With Rigidity” Phys. Rev. D34 (1986) 3811.

\[\text{[31]}\] U. Lindstrom, M. Rocek and P. van Nieuwenhuizen, “A Weyl Invariant Rigid String”, Phys. Lett. B199 (1987) 219.
[32] J. Polchinski and Z. Yang, “High temperature partition function of the rigid string”, Phys. Rev. D{\textbf{46}} (1992) 3667, hep-th/9205043.

[33] W. Siegel, “Hidden Local Supersymmetry In The Supersymmetric Particle Action,” Phys. Lett. B 128, 397 (1983).

[34] T. Curtright and P. van Nieuwenhuizen, “Supersprings”, Nucl. Phys. B294 (1987) 125.

[35] T. L. Curtright, “Extrinsic Geometry Of Superimmersions”, in Perspectives in String Theory, eds P. Di Vecchia and J.L. Petersen, World Scientific 1988.

[36] E. A. Ivanov and A. A. Kapustnikov, “Towards a tensor calculus for kappa supersymmetry,” Phys. Lett. B 267, 175 (1991).

[37] E. A. Ivanov and A. A. Kapustnikov, “Gauge covariant Wess-Zumino actions for super p-branes in superspace”, Int. J. Mod. Phys. A 7, 2153 (1992).

[38] J. P. Gauntlett, “A kappa symmetry calculus for superparticles”, Phys. Lett. B272 (1991) 25, hep-th/9109039.

[39] U. Lindstrom and M. Rocek, “Bosonic And Spinning Weyl Invariant Rigid Strings”, Phys. Lett. B201 (1988) 63.

[40] D. P. Sorokin, V. I. Tkach and D. V. Volkov, “Superparticles, Twistors And Siegel Symmetry,” Mod. Phys. Lett. A 4, 901 (1989).

[41] D. P. Sorokin, V. I. Tkach, D. V. Volkov and A. A. Zheltukhin, “From The Superparticle Siegel Symmetry To The Spinning Particle Proper Time Supersymmetry,” Phys. Lett. B 216, 302 (1989).

[42] D. V. Volkov and A. A. Zheltukhin, “Extension Of The Penrose Representation And Its Use To Describe Supersymmetric Models,” JETP Lett. 48, 63 (1988) [Pisma Zh. Eksp. Teor. Fiz. 48, 61 (1988)].

[43] D. Sorokin, “Superbranes and superembeddings,” Phys. Rept. 329, 1 (2000) arXiv:hep-th/9906142.

[44] S. J. Gates and H. Nishino, “D = 2 Superfield Supergravity, Local (Supersymmetry)^2 And Nonlinear Sigma Models,” Class. Quant. Grav. 3, 391 (1986).

[45] P. S. Howe and E. Sezgin, “Superbranes,” Phys. Lett. B 390, 133 (1997) arXiv:hep-th/9607227.

[46] P. S. Howe and E. Sezgin, “D = 11, p = 5,” Phys. Lett. B 394, 62 (1997) arXiv:hep-th/9611008.

[47] P. S. Howe and U. Lindstrom, “Kappa-symmetric higher derivative terms in brane actions,” arXiv:hep-th/0111036.
[48] M. Brown and S. J. Gates, “Superspace Bianchi Identities And The Supercovariant Derivative,” Annals Phys. 122, 443 (1979).

[49] P. S. Howe, O. Raetzel and E. Sezgin, “On brane actions and superembeddings,” JHEP 9808, 011 (1998) [arXiv:hep-th/9804051].

[50] Z. Bern et al., “Counterterms in supergravity,” arXiv:hep-th/0012230.

[51] S. Deser and D. Seminara, “Counterterms/M-theory corrections to $D = 11$ supergravity,” Phys. Rev. Lett. 82, 2435 (1999) [arXiv:hep-th/9812136].