Laser pulse amplification and dispersion compensation in an effectively extended optical cavity containing Bose–Einstein condensates

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Received 31 July 2012, in final form 19 November 2012
Published 10 December 2012
Online at stacks.iop.org/JPhysB/46/015501

Abstract

We review and critically evaluate our proposal of a pulse amplification scheme based on two Bose–Einstein condensates inside the resonator of a mode-locked laser. Two condensates are used for compensating the group velocity dispersion. Ultraslow light propagation through the condensate leads to a considerable increase in the cavity round-trip delay time, lowers the effective repetition rate of the laser, and hence scales up the output pulse energy. It has been argued recently that atom–atom interactions would make our proposal even more efficient. However, neither in our original proposal nor in the case of interactions, were limitations due to heating of the condensates by optical energy absorption taken into account. Our results show that there is a critical time of operation, $\tau_{\text{crit}} \approx 0.3\text{ ms}$, for the optimal amplification factor, which is of the order of $\sim 10^2$ at effective condensate lengths of the order of $\sim 50\mu\text{m}$. The bandwidth limitation of the amplifier on the minimum temporal width of the pulse that can be amplified with this technique is also discussed.

1. Introduction

High peak-power laser pulses are sought in diverse scientific and technological applications such as biomedical imaging [1], ultrafast spectroscopy [2] and high harmonic generation [3]. Ultrashort laser pulses in the picosecond to femtosecond range are routinely produced by the technique of mode locking, where the axial modes of the cavity are locked in phase to produce a train of pulses with a repetition rate $f_{\text{rep}} = c/2L$, where $c$ is the speed of light and $L$ is the effective optical path length of the resonator [4]. Pulse energies directly generated from such lasers are typically in the nJ range and amplification schemes are necessary in order to scale up the peak intensities to levels where nonlinear interactions can be observed. Recently, a simple amplification method was demonstrated, based on the extension of the standard mode-locked resonator length with a compact multipass cavity [5]. If the added insertion loss of the extended cavity is negligible, the average output power of the laser remains nearly the same and extension of the cavity length scales up the energy per pulse by lowering the repetition rate. While this technique has been widely used to amplify femtosecond pulses up to $\mu\text{J}$ energies [6], there is an ever growing need to find amplification methods that will facilitate the construction of even more compact, high-intensity lasers.

About five years ago, we proposed a compact laser pulse amplification scheme using two Bose–Einstein condensates (BECs) introduced in the resonator of a mode-locked laser [7]. The basic idea was to utilize the electromagnetically induced transparency (EIT) scheme [8–11] in a cavity [12] to make the pulse ultraslow [13, 14] such that the effective pulse repetition rate could be drastically reduced due to the increase of the cavity round-trip time, provided that the average output power of the laser remains nearly the same after the introduction of the condensates, this increase in round-trip time then scales up the output energy.

The promise of laser pulse amplification by atomic condensates is further studied by taking into account...
atom–atom interactions in an f-deformed condensate formalism very recently [15]. It is found that interactions make the amplification effect stronger. Moreover, in addition to subluminal propagation, the possibility of superluminal speeds and a high degree of control over the repetition rate are also shown to be possible. On the other hand, neither of these works consider the effects of the heating of the condensates due to optical pulse absorption. The present work is a critical evaluation of the feasibility of the proposal by taking into account the temperature increase of the condensates during pulse propagation.

In the design of a practical BEC pulse amplifier, certain effects need to be further considered. First, because the BEC would introduce an excessive amount of dispersion to short pulses [16], undesirable pulse broadening results. To overcome this limitation, we offer a method for dispersion compensation by using a second intracavity BEC with the proper choice of energy levels to provide the opposite sign of dispersion. A second BEC can, in fact, be prepared identically to the first, and a magnetic field can be utilized to shift the energy levels of the second BEC to match the desired detuning of the probe pulse. Second, the transparency bandwidth of the EIT process puts a limitation on the shortest temporal pulsewidth that can be amplified in the resonator. The resulting bandwidth limitation of the amplifier is further discussed. Our results show that pulse energy amplification factors of $\sim 10^2$ are possible by using a $^{23}$Na BEC with effective lengths of $\sim 10–100\ \mu$m. Finally, heating of the condensate, due to small but nonzero absorption in the EIT scheme, introduces an operational time beyond which the condensate turns into a usual Maxwell–Boltzmann gas. Our calculations reveal a critical time, $0.3\ \text{ms}$, at which the amplification factor is optimum.

The significance of the proposed method of effectively increasing the length of the cavity against the straightforward solution of extended cavities should be clarified. In fact, it is of high demand to have compact laser cavities in practical laser systems. In applications like optical data storage, filtering or other optical logic and signal processing, having long optical paths, for long optical delay, while maintaining a small cavity volume brings tremendous practical advantages. The search for such compact cavities goes back to the mid 1960s. Herriott et al introduced the method of folding long optical paths for compact cavities [17]. Around early 2000, multi-pass cavities were developed by Cho et al [5]. This is now an active modern research field (for a review see [18]). In an ultraslow light scheme with EIT, optical pulses slow down to speeds of about a few metre/second. Thus the effective optical path is indeed too long for considering an equivalent extended cavity. In addition to the typical applications of compact laser cavities in accurate optical loss measurements, stimulated Raman scattering, long-path absorption spectroscopy, high-speed path-length scanning, the present system of ultraslow light has another crucial application area of coherent optical memory. With a modest amplification power and dispersion compensation, the proposed scheme can also be used for that purpose in the context of correcting pulse shape errors in the stored optical information.

This paper is organized as follows. The method of generating ultraslow light via the EIT scheme is introduced briefly in section 2. We describe our proposed scheme in section 3. The results of the dispersion compensation, heating rate, amplification factor and spectral bandwidth calculations are presented in the corresponding subsections of section 4. We conclude in section 5.

2. Ultraslow light by the EIT scheme

In the EIT configuration, a condition of a weak probe is usually assumed such that a strong drive field with Rabi frequency $\Omega_d$ and the circulating resonator field of frequency $\Omega_p$ satisfy $\Omega_d \gg \Omega_p$ [19]. A conventional configuration is the case where the probe field is pulsed while the drive field is a continuous wave (CW) [10]. Studies and experiments in the strong probe regime indicate deterioration of EIT and enhanced absorption of the probe pulse [20]. Requirements for initiation of EIT is formulated in terms of Rabi frequencies [21]. We assume that the Rabi frequency of the amplified pulsed signal remains sufficiently smaller than the Rabi frequency of the CW drive so that the EIT conditions are maintained during the operation. Our results reveal a maximum of the amplification factor and the strength of the CW control field can be chosen accordingly. Although we did not specifically investigate it here, our scheme may also be used for compensating enhanced absorption in the strong probe EIT experiments. This would be particularly advantageous to examine interactions of probe and coupling fields [22], adiabatons [23], as well as to facilitate nonlinear processes demanded for EIT applications [24–27].

Under the weak probe condition, susceptibility $\chi$ for the probe transition can be calculated as a linear response as most of the atoms remain in the lowest state. Assuming a local density approximation, neglecting local field, multiple scattering and quantum corrections and employing steady-state analysis, $\chi$ is found to be [19]

$$\chi = \frac{\rho|\mu_{31}|^2}{\hbar \omega_p} \frac{i(-\Delta + \Gamma_2/2)}{(\Gamma_2/2 - i\Delta)(\Gamma_3/2 - i\Delta) + \Omega_p^2/4},$$  (1)

where $\rho$ is the atomic density of the condensate, $\Delta = \omega_p - \omega_{31}$ is the frequency detuning of the probe field with frequency $\omega_p$ from the resonant electronic transition $\omega_{31}$. $\Gamma_2$ and $\Gamma_3$, respectively, denote the dephasing rates of the atomic coherences of the lower states, $\mu_{31} = 3\hbar|\lambda^3_{31}|^2 g/8\pi^2$ is the dipole matrix element between the upper state $|3\rangle$ and lower state $|1\rangle$, involved in the probe transition with $\lambda_{31}$ being the resonant wavelength of the probe transition and $g$ being the radiation decay rate between $|3\rangle$ and $|1\rangle$. For the cold gases considered in this paper and assuming co-propagating laser beams, the Doppler shift in the detuning is neglected.

At the probe resonance, the imaginary part of $\chi$ becomes negligible and results in turning an optically opaque medium transparent. Furthermore, EIT can be used to achieve ultraslow light velocities, owing to the steep dispersion of the EIT susceptibility $\chi$ about the probe resonance [14].

3. Our proposed scheme of the pulse amplifier

A schematic of the proposed BEC pulse amplifier is shown in figure 1. The short cavity, initially extending from the output
coupler (OC) up to the end high reflector (HR1), contains a gain medium and is first passively mode-locked by using the technique of Kerr lens mode locking or a saturable absorber [4]. The cavity is then extended by removing HR1 and two different BECs are introduced inside the resonator, which now extends from OC up to the end high reflector HR2.

Although there is some interest in examining Josephson coupled BECs in optical cavities [28], here we assume the BECs are spatially disconnected such that the condensates are kept in traps which are sufficiently far apart from each other to avoid spatial overlap of condensate wave functions. As long as there is Josephson coupling then condensate numbers would have dynamics and our treatment with the frozen density profile cannot be applied. In addition to the Josephson effect, one could consider dense condensates with interaction terms and dissipation effects to find equilibrium density profiles so that our treatment can be extended to such cases to determine electric susceptibility for calculating possibility of dispersion compensation and reduction of group velocity.

The BECs are in the EIT configuration used in ultraslow light experiments [13, 29]. The circulating laser pulse acts as the probe field in the BECs and external coupling lasers are used to achieve transparency. BEC1 introduces a delay to lower the repetition rate and scales up the pulse energy. BEC2 acts as a dispersion compensator. An additional delay is also produced by BEC2. Axial sizes of the BECs are assumed to be sufficiently shorter than the cavity length while we have assumed that proper focusing optics is employed to keep the spotsize within the reduced pulse repetition rate (or the reverse of $\Delta t$).

Let us assume that the repetition rate of the short cavity is $f_0 = c/2L_0$, where $L_0$ is the effective optical path length of the short resonator. In the simulations, we assumed that, without the intracavity BECs, $f_0 = 25$ MHz which is easily achievable in standard mode-locked laser configurations. The total round-trip group delay $T_g$ (in other words, the reverse of the reduced pulse repetition rate) will be given by

$$T_g = 2 \left( \frac{L_4}{v_{gi}} + \frac{L_2}{v_{g2}} \right).$$

Here, we neglected the round-trip group delay of the initial short cavity, and denoted the effective length and the group velocity for the condensates $(i = 1, 2)$ as $L_i$ and $v_{gi}$, respectively, such that [16],

$$L_i = \left[ \frac{4\pi}{N} \int_0^\infty rdr \int_0^\infty r^2 \rho_i(r, z) dz \right]^{1/2}$$

and

$$\frac{1}{v_{gi}} = \frac{1}{c} + \frac{\pi}{\lambda} \frac{\partial \chi_i}{\partial \Delta}.$$  

The effective length is the rms width of the density distribution of the condensates along the cavity axis ($z$). Here $r$ is the radial coordinate. In the ideal case, where the inclusion of the BECs add no loss to the resonator, the average output power of the laser remains nearly equal to that of the short cavity. During mode-locked operation of the extended cavity, the amplification factor $A$ for the pulse energy will, therefore, be given by $A = T_g f_0$.

4. Results

We consider two BECs of $^{23}$Na atoms with parameters [13], $M = 23$ amu, $\alpha_i = 2.75$ nm, $N_1 = N_2 = 8.3 \times 10^6$, $\omega_r = 2\pi \times 69$ Hz, $\omega_c = 2\pi \times 21$ Hz, $\Gamma_1 = 0.5\gamma$, $\Gamma_2 = 2\pi \times 10.01$ MHz and $\Gamma_2 = 2\pi \times 10^3$ Hz. The resonance wavelength for the probe laser transition is $\lambda = 589$ nm. We take $\Omega = 2.5\gamma$ and $\Delta_1 = 0.01\gamma$.

4.1. Dispersion compensation

Within the transparency window of EIT, the imaginary part of the susceptibility ($\chi''$) is very small compared to the real part of the susceptibility ($\chi'$). The group velocity dispersion $D_i$ is given by $D_i = \frac{d^2 \phi_i}{d\omega^2}$ [31], where $\phi_i = n_i \omega L_i / c$ is the accumulated phase, with $n_i$ being the refractive index. Using $n_i = \sqrt{1 + \chi_i} \approx 1 + \chi_i / 2$, $D_i$ can be expressed as

$$D_i = \frac{L_i}{c} \left( \frac{d\phi_i}{d\omega} + \frac{\omega d^2 \chi_i}{2 d\omega^2} \right).$$

In the proposed scheme, the second order dispersion of the BEC1 is positive for a probe pulse slightly blue detuned from the resonance, which can be seen in figure 2.

At the same but red detuning, dispersion would be almost the same, but with the opposite sign. For propagation through a single BEC, this sign change is not relevant on pulse broadening. On the other hand, by introducing a second
as $\Delta_t = \gamma/(\Delta_1)$. The operating point of the amplifier, $D_{\text{BEC}}$, plays a role through the pulse chirp. The pulses emerging from BEC1 are broadened and chirped. In other words, the instantaneous carrier frequency differs across the temporal pulse profile from the central carrier frequency. In the presence of second-order group velocity dispersion, this chirp is approximately linear near the pulse centre. If the chirped pulse then enters the second BEC whose group velocity dispersion parameter is adjusted to be equal in magnitude but opposite in sign to that of BEC1, the linear chirp near the pulse centre will be cancelled and the initial pulsewidth will be restored (see [31, 32] and references therein). For that aim, we assume that BEC2 is also in an EIT configuration but energy levels for the probe transition are shifted by an amount $\Delta \gamma$. As the dispersions are of the same magnitude but of opposite sign, this can be used to compensate for the positive dispersion of the delay segment (BEC1), provided that the dispersion introduced in the other intracavity elements are negligible.

To determine the operating point of the amplifier, we adjust the parameters of the BECs so that $D_{\text{BEC}}(x_1) + D_{\text{BEC}}(x_2) = 0$, where $x_1 = \Delta_1/\gamma$, $x_2 = \Delta_2/\gamma$. The operating point would be independent of temperature for identical condensates as $D_{\text{BEC}}$ are proportional to the $\rho_L$ that are the same for both BECs. Dependence of $D$ on $\Delta$ is shown in figure 2 at the chosen temperature of $T = 381$ mK. We find that dispersion compensation condition, $D_1 = -D_2$, is satisfied by $x_1 = -x_2$ at $\Delta_1/\gamma = 0.01$, which will be the operating point of the amplifier.

We assumed identical traps and condensates only for the sake of simplicity. The main formalism and the dispersion compensation condition $D_1 = -D_2$ is generally valid for non-identical traps and condensates. The effect of a different number of atoms in the BECs would be on the operating point (detuning of the probe field), where the dispersion compensation is achieved. We can make a simple estimate. The dispersion coefficient is proportional to the product of BEC density and length, both of which are related to the chemical potential by power laws. Thus, the ratio $D_1/D_2$ is related to the ratio of the chemical potentials of the BECs. Assuming Thomas–Fermi profiles, $D_1/D_2$ has negligibly weak temperature dependence, but strong dependence on the ratio of condensed particle numbers $N_1/N_2$. If we know $N_1$ and $N_2$ then the operating point can be determined by a simple calculation.

Due to number fluctuations, the compensation of the dispersion would be incomplete. However, the amplification factors are modest in our case and the pulse duration is in the order of microseconds. For such pulses the dispersion is not too strong and even partial compensation of dispersion should be sufficient and thus we can say that a fluctuation of particle number would not have crucial effects on our conclusions.

4.2. The heating rate

The heating rate of a gaseous sample due to optical absorption is a standard problem in laser cooling theory (see e.g. [33] (chapter 4)). The rate of change in the average kinetic energy of an atom due to absorption can be evaluated from

$$\frac{1}{2m} \frac{\text{d}p^2}{\text{d}t} = vF_{\text{rad}},$$

where $v = \hbar k_L/m$ is the recoil velocity of the atom with mass $m$ due to the absorption of a laser photon at wave number $k_L$, and $F_{\text{rad}}$ is the radiation force acting on the atom. An impulse–momentum theorem can be written for the absorption as $F_{\text{rad}} = \hbar k_L$, where $F_{\text{rad}}$ is the characteristic time of interaction between the atom and the radiation field. Introducing $\Gamma_{\text{rad}} = 1/\tau_{\text{rad}}$, we rewrite the relation as $F_{\text{rad}} = \hbar k_L \Gamma_{\text{rad}}$. Second-order perturbation theory can be used to find $\Gamma_{\text{rad}}$. Analogous to ac Stark shifts, we write the energy level shift as $\Delta \epsilon = -\frac{1}{\alpha} \langle \epsilon^2 \rangle$, where $\alpha$ is the single-atom complex polarizability and $\epsilon$ is the laser pulse amplitude. The imaginary part of the level shift can be identified by $\Gamma_{\text{rad}} = -(2/\hbar) \text{Im}(\Delta \epsilon) = \alpha \langle \epsilon^2 \rangle/\hbar$. This yields

$$F_{\text{rad}} = \frac{1}{\hbar} \langle \hbar k_L \alpha \epsilon^3 \rangle.$$

In our case, $\alpha = \mu_L^2 L(\Delta)/\hbar$ is the single atom EIT susceptibility, where we introduced a notation

$$L(\Delta) = \frac{i(\Delta + \Gamma_2/2)}{(\Gamma_2/2 - i\Delta)(\Gamma_2/2 - \Delta + \Omega_3^2/4)}.$$

The average rate of change of kinetic energy due to absorption then becomes

$$\frac{1}{2m} \frac{\text{d}p^2}{\text{d}t} = \frac{2}{m} \langle \hbar k_L \rangle^2 \mu_L^2 \langle \epsilon^2 \rangle^2 \frac{L''(\Delta)}{\hbar^2},$$

where $L''(\Delta)$ is the imaginary part of $L(\Delta)$. Identifying the probe Rabi frequency as $\Omega_p^2 = \mu_L^2 \langle \epsilon^2 \rangle/\hbar^2$, and by using the equipartition theorem, $k_B T = \Omega_p^2/2m$, we finally get the heating rate $\kappa$ to be

$$\kappa = \frac{\text{d}T}{\text{d}t} = \frac{4}{mk_B} \langle \hbar k_L \rangle^2 \Omega_p^2 L''(\Delta).$$
An additional factor of 2 is introduced to take into account subsequent emission and absorption processes together. For the parameters we use in our simulations, taking $\Omega_\rho = 0.1\Omega_c$, the heating rate is evaluated to be $\kappa \sim 1.6 \text{ mK s}^{-1}$. We do not assume zero temperature BECs. The heating rate $\kappa$, or the rate of loss of condensed particles, exhibit linear dependence with the temperature and thus one can always choose the initial time ($t_0$) corresponding to the initial temperature ($T_0$) by $t_0 = T_0/\kappa$ so that $T = \kappa t$. We report our results in the figures as functions of temperature which can be translated to time dependence by this scaling transformation. The effect of dynamically changing temperature $T = \kappa t$ is to make the density of the cloud time dependent. We shall use semi-ideal model of the BEC [34] for an analytical expression of the density of the atomic cloud.

Indeed the main dissipation channel is the loss of the condensate atoms. In addition to heating, other mechanisms of condensed particle loss may occur, such as three-body loss [35]. Here we do not take them into account as we assume BECs are sufficiently dilute, and the number of particles are not too large. Thus in our case the only source of particle loss out of the condensate is heating, or the recoil momentum transfer, by pulse absorption. Atoms with sufficient recoil are thermalized and removed from the condensate phase. This in turn reduces the density of the condensate. Once the density becomes lower than the critical density required to maintain the condensate, then the condensate is destroyed and the gas is entirely thermalized. The semi-classical model we employ is, in fact, to treat such a case. It contains both the thermal and condensate phases. Below the critical temperature, the thermal phase serves as the thermal background for the dominant condensate phase. EIT has normally negligible absorption but still is not zero. A single pulse would transfer a little energy to the condensate. Duration of the interaction due to pulse delay is less than the time needed to transfer sufficient energy to destroy the condensate. The recoil energy or the received kinetic energy is translated in our treatment to the temperature variable. Temperature and time has a linear relation. The loss of the atoms out of the condensate phase or the condensate fraction is reduced by the temperature. These arguments can be put into mathematical context using the semi-ideal model as follows.

The total density at a temperature $T(t)$ is then written to be

$$\rho(\vec{r}, t) = \frac{\mu(t) - V(\vec{r})}{U_0} \Theta(\mu(t) - V(\vec{r})) \Theta(T_C - T(t)) + \frac{g_3/2 e^{-\beta V}}{\kappa_f(t)^3},$$

where $U_0 = 4\pi\hbar^2a_s/m$ is atomic mass; $a_s$ is the atomic s-wave scattering length; $\mu$ is the chemical potential; $\Theta(.)$ is the Heaviside step function; $g_3(x) = \Sigma_j x^j/f^j$ is the Bose function; $\kappa_f$ is the thermal de Bröglie wavelength; $\beta = 1/k_BT$; $z = \exp(\beta \mu)$ is the fugacity, and $T_C$ is the critical temperature which is $T_C = 424 \text{ nK}$ in our case. The optical pulse would heat the cloud to $T_C$ in 265 $\mu$s. The maximum pulse delay time $\sim 65 \mu$s is less than the critical time at which the condensate turns into thermal gas. However, as the multiple passes of the pulse train over the condensate continue to heat it we shall consider time, and corresponding temperature range, in our examinations beyond the critical time and temperature. The external trapping potential is $V(\vec{r}) = (m/2)(\omega_0^2 r^2 + \omega_z^2 z^2)$ with $\omega_0$, the radial trap frequency and $\omega_z$, the angular frequency in the $z$ direction. $\mu$ is determined from $N = \int d^3r \rho(\vec{r})$. At temperatures below $T_C$ this yields [34]

$$\mu(t) = \mu_{TF} \left( \frac{N_0}{N(t)} \right)^{2/5},$$

(12)

where $\mu_{TF}$ is the chemical potential evaluated under the Thomas–Fermi approximation and the condensate fraction is given by

$$\frac{N_0}{N(t)} = 1 - x(t)^3 - \frac{\xi(2)}{\xi(3)} x(t)^2 \left( 1 - x(t)^3 \right)^{2/5},$$

(13)

with $x(t) = T(t)/T_C$, and $\xi$ is the Riemann–Zeta function. The Thomas–Fermi profile is valid for small healing length, $\xi = 1/\sqrt{8\pi a_s \rho}$, of the condensate relative to the harmonic trap length. The scaling parameter $s$, characterizing the strength of atomic interactions within the condensate, is calculated to be [34]

$$s = \frac{\mu_{TF}}{k_b T_C} = \frac{1}{2} (\frac{\xi(3)}{N(3/5)} \left( 15 N^{1/6} \frac{a_s}{a_h} \right)^{2/5})^{1/3}.$$  (14)

Here, $a_h = \sqrt{\hbar/m(\omega_0^2 \omega_z^2)^{1/3}}$ denotes the average harmonic oscillator length scale. At temperatures above $T_C$, $\mu$ can be determined from $L_3(z) = \xi(3)/z^3$, where $L_3(\cdot)$ is the third-order polylogarithm function. The semi-ideal model has a wide-range of validity in representing density distribution of a trapped Bose gas at finite temperature provided that $s < 0.4$ [34]. At the same time, the interactions are assumed to be strong enough to ensure $\mu \gg \hbar \omega_{zz}$, so that the kinetic energy of the condensate can be neglected according to the Thomas–Fermi approximation. In typical slow-light experiments in cold atomic gases, $s$ remains within these limits. The time dependence of the density is translated to the condensate expansion and the group velocity increase, which, in turn, affects the amplification factor as shown in the

Figure 3. Dependence of the amplification factor $A$ on temperature, for the same parameters as in figure 2. The temperature is scaled by the critical temperature $T_C = 424 \text{ nK}$. The heating rate $\kappa = 1.6 \text{ mK s}^{-1}$ translates the same behaviour into the time domain via $T = \kappa t$. 

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following section. We note that decoupling of the matter wave dynamics from the optical pulse dynamics is based upon their significantly different time scales. The dynamics of condensate matter wave happens in the ms scale, while an optical pulse evolves in μs [36]. We also ignore higher dimensional propagation effects in the optical pulse dynamics such as multimode waveguiding [37] and diffraction losses [38].

4.3. The amplification factor

The result of the temperature dependent amplification calculations is shown in figure 3. The amplification factor increases with temperature up to a temperature $T \sim 318$ nK, close but less than the critical temperature $T_c = 424$ nK of the BEC. In the semi-ideal picture of the condensate density profile, this is due to the contribution from the increasing effect of the thermal component in the pulse propagation via the group velocity and the length of the condensate.

As can be seen in figure 4, the group velocity weakly depends on the temperature in the condensate phase. Thermal behaviour of the group velocity can be explained by examining the thermal behaviour of the condensate density. In the semi-ideal model that we use, both the condensate and the thermal gas components are considered. Below the critical temperature, the condensate component emerges sharply and dominates over the thermal background gas. Density varies little at low temperatures. Thus the ultraslow propagation speeds weakly change with the temperature. After the critical temperature, there is no more condensate and the thermal gas becomes more rapidly diluted with the temperature, causing the kink in figure 4(a), which is also observed experimentally [13].

The main effect of the temperature is the expansion of the condensate which causes the increase of the delay times of the pulse through the condensate and hence $T_g$ increases. Behaviour of the amplification factor with the temperature follows that of the effective length. Beyond $T_c$, almost linear behaviour of group velocity and the effective length with temperature result in weak dependence of $T_g$ and hence amplification on the temperature. The amplification is larger in the condensation regime due to large group delays. Figure 4(a) shows that group velocity of the pulse in the thermal gas regime rises sharply and it is faster than expansion of the atomic cloud shown in figure 4(b). Accordingly, the amplification factor sharply drops after a critical temperature close to $T_c$ and continues to decrease slowly in the thermal gas phase.

4.4. Spectral bandwidth

Finally, we investigate the spectral bandwidth limitations of the BEC. Ideally, it is desirable to have a system which has a very broad transparency to support the propagation of pulses with very short duration. To provide a feel for how short a pulse the BECs can support, we calculate the spectral bandwidth $\Delta \nu$ that corresponds to the transparency window of the BEC in the EIT scheme. The net bandwidth due to both of the BECs is $2\Delta \nu$ so that the temporal width $\tau_p$ of the pulse that can be supported can be estimated by $\tau_p = 1/2 \Delta \nu$. Using [10]

$$\Delta \nu = \frac{\Omega_c^2}{\gamma} \frac{1}{\sqrt{\kappa}}.$$

with $\kappa = 3\rho L^3(k_1 L)/8\pi^2$ being the opacity of the atomic cloud of length $L$ and density $\rho$, we find $\Delta \nu \sim 0.1\nu$. Note that the operation point $\Delta = 0.01 \gamma$ lies within the transparency window. The pulses that can be supported by the condensates should have widths of the order of $\sim \mu$s. Pulses of shorter widths could be supported by considering larger $\Omega_c$. The cost, however, would be to get lower amplification factors as the group velocity would increase with increasing Rabi frequency of the control field. More ingenious designs that specifically consider transparency window enhancement for broadband pulses are proposed [39], but their integration to the present proposal require further studies.

5. Conclusions

In conclusion, we have investigated the feasibility of using Bose–Einstein condensates for laser pulse amplification. The method involves the introduction of two BECs inside the resonator of a passively mode-locked laser. The large delay
produced by the BECs lowers the pulse repetition rate and scales up the output energy. Our calculations show that pulse amplification factors of the order \(\sim 10^2\) should be possible over a condensate length of \(\sim 50\ \mu\text{m}\). We further showed that a second BEC could be used to provide dispersion compensation. However, the amplification factor decreases with time due to the presence of optical absorption. We have estimated the heating rate to be about 1.6 \(\text{mKs}^{-1}\), which severely limits the operation time of the system in the condensate regime. A critical time of operation for optimum amplification is found to be about 200 \(\mu\text{s}\), which would require an additional optical switching to extract the pulse at the right time out of the cavity. Alternatively, reducing the effects of absorption by considering multi-level EIT systems \([40, 41]\) or designing a compensating simultaneous cooling mechanism on the atomic cloud in the EIT scheme \([42, 43]\) can be considered. This would eliminate the need to reconstruct the BEC to amplify different pulses at optimum conditions. For quick and easy generation of BECs, atom chips can be promising \([44]\). To aim at higher intensities and amplification factors, the nonlinear response of the atomic gas should also be taken into account. Denser condensates, with a nonlinear response and quantum corrections, including atom–atom interactions, but without local field correction, seem to be beneficial for larger delay times as well \([15]\).

Acknowledgments

DT acknowledges support from TUBITAK (The Scientific and Technological Research Council of Turkey) career grant no 109T686. OEM is supported by TUBITAK under project TBAG-109T267. DT thanks G S Agrawal for helpful discussions.

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