Stable and mobile excited two-dimensional dipolar Bose–Einstein condensate solitons

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Abstract

We demonstrate robust, stable, mobile excited states of quasi-two-dimensional (quasi-2D) dipolar Bose–Einstein condensate (BEC) solitons for repulsive contact interaction with a harmonic trap along the \(x\) direction perpendicular to the polarization direction \(z\). Such a soliton can freely move in the \(y–z\) plane. A rich variety of such excitations is considered: one quanta of excitation for movement along (i) \(y\) axis or (ii) \(z\) axis or (iii) both. A proposal for creating these excited solitonic states in a laboratory by phase imprinting is also discussed. We also consider excited states of quasi-2D dipolar BEC soliton where the sign of the dipolar interaction is reversed by a rotating orienting field. In this sign-changed case the soliton moves freely in the \(x–y\) plane under the action of a harmonic trap in the \(z\) direction. At medium velocity the head-on collision of two such solitons is found to be quasi elastic with practically no deformation. The findings are illustrated using numerical simulation in three and two spatial dimensions employing realistic interaction parameters for a dipolar \(^{164}\text{Dy}\) BEC.

Keywords: Bose–Einstein condensate, dipolar atoms, two-dimensional soliton, excited state

(Some figures may appear in colour only in the online journal)

1. Introduction

A bright soliton is a localized intensity peak that maintains its shape, while traveling at a constant velocity in one dimension (1D), due to a cancellation of nonlinear attraction and dispersive effects [1]. Solitons have been observed in water waves, nonlinear optics, and Bose–Einstein condensate (BEC) etc among others. In physical three-dimensional (3D) world, quasi-one-dimensional (quasi-1D) solitons are observed where a reduced (integrated) 1D density exhibit soliton-like property [2]. Experimentally, bright matter-wave solitons and soliton trains were created in a BEC of \(^7\text{Li}\) [3] and \(^{85}\text{Rb}\) atoms [4]. However, due to collapse instability, 3D BEC bright solitons are fragile and can accommodate only a small number of atoms [2].

Recent observation of BECs of \(^{164}\text{Dy}\) [5, 6], \(^{168}\text{Er}\) [7] and \(^{52}\text{Cr}\) [8, 9] atoms with large magnetic dipole moments has opened new directions of research in BEC solitons. In a dipolar BEC, in addition to the quasi-1D solitons [10] it is possible to have a quasi-two-dimensional (quasi-2D) soliton [11] free to move in a plane with a constant velocity and trapped in the perpendicular direction. Although, such a quasi-2D dipolar BEC soliton has not been experimentally observed, this seems to be the only simplest example of an experimentally realizable quasi-2D soliton. Dipolar BEC solitons can be stabilized either by a harmonic or an optical-lattice trap in quasi-1D [12] and quasi-2D [13] configurations. More interestingly, one can have dipolar BEC solitons for fully repulsive contact interaction [10]. Hence these dipolar solitons bound by long-range dipolar interaction could be robust and less vulnerable to collapse for a large number of atoms due to the repulsive contact interaction. The contact repulsion gives stability against collapse and dipolar attraction prevents the escape of the atoms from the soliton.

The possibility of realizing a trapped BEC in an excited (non-ground) state has been a topic of intense research [14]. Here we demonstrate the existence of stable excited quasi-2D dipolar BEC solitons harmonically trapped in the \(x\) direction capable of moving in the \(y–z\) plane with a constant velocity with \(z\) the polarization direction. They are stable and
stationary excitations of the quasi-2D bright solitons. We consider three types of excitations of the solitons: lowest excitation for dynamics along (i) y axis, (ii) z axis, and (iii) both y and z axis. The wave function for the lowest y (z) excitation is antisymmetric in y (z) with a zero in the y = 0 (z = 0) plane. The symmetries of these states are quite similar to the excited states of a 3D trapped harmonic oscillator, the only difference is that in the case of the quasi-2D dipolar solitons there is no harmonic trap along y and z axis and the confinement in these directions is achieved solely by dipolar attraction.

In addition to the normal excited quasi-2D dipolar BEC soliton, we also considered a set-up where the sign of the dipolar interaction is reversed by a rotating orienting field [15]. In this sign changed configuration, the dipolar interaction is repulsive along z axis and attractive in the x–y plane and an excited quasi-2D dipolar BEC soliton can be realized by applying a harmonic trap along the z axis. The stationary excited quasi-2D bright solitons were obtained by a imaginary-time propagation of the mean-field Gross–Pitaevskii (GP) equation in 3D. These stationary excited solitons can also be termed quasi-2D dipolar dark-in-bright solitons bearing resemblance with the quasi-1D dipolar dark-in-bright solitons studied recently [16]. The quasi-1D dipolar dark-in-bright solitons were also established to be excitations of dipolar bright solitons [16, 17].

We also studied the dynamics of these quasi-2D solitons by real-time simulation using an effective 2D mean-field model. The stability of the excited quasi-2D dipolar solitons was established by studying the breathing oscillation of the system upon small perturbation over long time in real-time propagation. The head-on collision between two excited quasi-2D solitons is found to be quasi elastic at medium velocities of few mm s\(^{-1}\). In such a collision, two excited solitons pass through each other without significant deformation. However, at lower velocities the collision becomes inelastic, as only the strictly 1D integrable solitons can have elastic collision at all velocities [1]. We also demonstrate by real-time simulation in 2D the possibility of the creation of the excited quasi-2D solitons by phase imprinting [18, 19] a normal quasi-2D bright soliton with identical parameters. For example, a phase difference of \(\pi\) between the two parts of the BEC wave function situated at \(y > 0\) and \(y < 0\).

In section 2 the time-dependent 3D mean-field model for a quasi-2D dipolar BEC soliton is presented. A reduced 2D model appropriate for the present study is also considered. The numerical investigation of the quasi-2D solitons is considered in section 3. The domain of the appearance of the ground and excited states of the quasi-2D solitons is illustrated in a phase plot with realistic values of contact and dipolar interactions of \(^{164}\)Dy atoms exhibiting the maximum number of atoms in the quasi-2D soliton versus the scattering length. The evolution of collision between two excited quasi-2D solitons is considered by real-time evolution. The dynamical simulation of the creation of an excited quasi-2D soliton from a phase-imprinted ground state of a quasi-2D soliton is also demonstrated. The possibility of creating a dark soliton by phase imprinting a BEC by means of a detuned laser has been illustrated experimentally [18]. Finally, in section 4 we present a brief summary and concluding remarks.

2. **Mean-field GP equations**

We consider a dipolar BEC soliton, with the mass, number of atoms, magnetic dipole moment, and scattering length given by \(m, N, \mu, a\). The interaction between two atoms at \(\mathbf{r}\) and \(\mathbf{r}'\) is [20]

\[
V_{ij}(\mathbf{R}) = 3\alpha a_{ij} V_{dd}(\mathbf{R}) + 4\pi a_\delta(\mathbf{R}), \quad \mathbf{R} = (\mathbf{r} - \mathbf{r'}),
\]

with

\[
a_{dd} = \frac{\mu_0 \mu^2 m}{12\pi \hbar^2}, \quad V_{dd}(\mathbf{R}) = \frac{1 - 3 \cos^2 \theta}{\mathbf{R}^3},
\]

where \(\mu_0\) is the permeability of free space, \(\theta\) is the angle made by the vector \(\mathbf{R}\) with the polarization direction. The parameter \(\alpha\) is tuned by a rapidly rotating magnetic field allowing the change of the sign of dipolar interaction [15]. We will consider two cases in this study: (i) \(\alpha = 1\) corresponding to the normal dipolar interaction and (ii) \(\alpha = -1/2\) corresponding to a sign-changed dipolar interaction.

The first possibility (i) leads to a *fully asymmetric* quasi-2D excited solitonic state in the presence of a harmonic trap along z axis. The dimensionless GP equation in this case can be written as [20]

\[
i \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = \left[ -\frac{\nabla^2}{2} + \frac{1}{2} \mathbf{r}^2 \right]
\]

\[
+ g |\psi|^2 + g_{dd} \int V_{dd}(\mathbf{R})
\]

\[
\times |\psi(\mathbf{r'}, t)|^2 d\mathbf{r'}, \quad (3)
\]

where \(g = 4\pi a N, g_{dd} = 3N a_{dd}\). In (3), length is expressed in units of oscillator length \(l = \sqrt{\hbar/(ma)}\), where \(\omega\) is the circular frequency of the harmonic trap acting along x axis. The energy is in units of oscillator energy \(\hbar \omega\), probability density \(|\psi|^2\) in units of \(l^{-3}\), and time in units of \(t_0 = 1/\hbar \omega\).

The second possibility (ii) leads to an *axially-symmetric* excited quasi-2D dipolar soliton in the presence of a harmonic trap along z axis. The dimensionless GP equation in this case is [11]

\[
i \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = \left[ -\frac{\nabla^2}{2} + \frac{1}{2} \mathbf{r}^2 \right]
\]

\[
+ g |\psi|^2 - \frac{1}{2} g_{dd} \int V_{dd}(\mathbf{R})
\]

\[
\times |\psi(\mathbf{r'}, t)|^2 d\mathbf{r'}, \quad (4)
\]

where now \(\omega\) is the circular frequency of the harmonic trap along z axis. In the case of (3) the dipolar interaction is attractive along z axis and repulsive in the x–y plane; the opposite is true for (4). The anisotropic dipolar interaction is circularly symmetric in the x–y plane in both cases. Hence, (3) is fully anisotropic and leads to fully-anisotropic quasi-2D
solitons, whereas (4) is axially symmetric and hence leads to quasi-2D solitons with this symmetry. In place of (4), a quasi-2D model appropriate for the quasi-2D excited solitons of large spatial extension is very economic and convenient from a computational point of view, specially for real-time dynamics. The system is assumed to be in the ground state \( \phi(z) = \exp(-z^2/2)/(\pi)^{1/4} \) of the axial trap and the wave function can be written as 
\[ \phi(r, t) = \phi(z)\phi_{2D}(\bar{\rho}, t), \]
where \( \phi_{2D}(\bar{\rho}, t) \) is an effective wave function in the \( x-y \) plane. Using this ansatz in (4), the \( z \) dependence can be integrated out to obtain the following effective 2D equation

\[ i\frac{\partial \phi_{2D}}{\partial t} = \left[ -\frac{\nabla^2}{2} + \frac{g}{\sqrt{2\pi}}|\phi_{2D}(\bar{\rho}, t)|^2 \right] \phi_{2D}(\bar{\rho}, t) \]

For this study we consider \( \mu_B \) atoms of magnetic moment \( \mu = 10 \mu_B \) [6] with \( \mu_B \) the Bohr magneton leading to the dipolar length \( a_{dd}(164\text{Dy}) \approx 132.7a_0 \), with \( a_0 \) the Bohr radius. We consider here \( l = 1 \) \( \mu m \) corresponding to a radial angular trap frequency \( \omega \approx 2\pi \times 61.6 \) Hz corresponding to \( t_0 \approx 2.6 \) ms.

3. Numerical results

We solve the 3D equations (3) and (4) or the 2D equation (5) by the split-step Crank–Nicolson discretization scheme using both real- and imaginary-time propagation in 3D or 2D Cartesian coordinates, respectively, using a space step of 0.1 \( \sim 0.2 \) and a time step of 0.001 \( \sim 0.005 \) in the imaginary-time simulation, and of 0.004 \( \sim 0.002 \) in real-time simulation [22]. A smaller time step is employed in the real-time propagation to obtain reliable and accurate results. The dipolar potential term is treated by a Fourier transformation to the momentum space using a convolution rule [23]. To obtain good convergence in the 3D calculation, a cut-off was introduced in the dipolar potential for \( r > R \), where \( R \) is greater than the size of the condensate, as suggested in [24].

The stationary profile of the excited quasi-2D solitons can be obtained by imaginary-time simulation. A symmetric initial state in the harmonic oscillator problem converges to the ground state in imaginary-time propagation, whereas an antisymmetric initial state converges to the lowest excited state. The imaginary-time routine preserves the symmetry of the initial state. The anisotropic quasi-2D soliton of (3) lies in the \( y-z \) plane and we consider three types of excited solitons: excitation in (i) \( y \), (ii) \( z \), and in both (iii) \( y \) and \( z \). In
The results for the normal dipolar $^{164}$Dy atoms trapped along the $x$ axis ($a = 1$, $a_{dd} = 132.7a_0$) as well as for the $^{164}$Dy atoms with the dipole interaction reversed by a rotating orienting field ($a = -\frac{1}{2}$, $a_{dd} = -66.35a_0$) and trapped along the $z$ axis are shown in addition to the critical number of atoms in quasi-1D solitons of [16]. Stable quasi-2D solitons appear for the number of atoms $N$ below the critical number $N_{\text{crit}}$. The width of the lines gives the error in the numerical calculation. The oscillator length $l = 1 \text{ \mu m}$.

The excited quasi-2D solitons to be considered next have a larger spatial extension and can accommodate a larger number of atoms. However, due to a larger spatial extension, an accurate calculation of the critical number of atoms in excited quasi-2D solitons necessitates a huge amount of RAM and CPU time and the critical number of these excited solitons will not be considered here. The plots of figure 1 represent a lower limit for the critical numbers for the excited quasi-2D solitons. From figure 1 we see that the number of atoms in the quasi-2D solitons in ground and excited states could be quite large and will be of experimental interest. The size of quasi-1D nondipolar solitons is usually quite small and these solitons can accommodate only a small number of atoms. A quasi-2D soliton cannot be realized in a nondipolar BEC as the long-range dipolar interaction plays a crucial role in their formation and stability.

Next we present the profile of the quasi-2D solitons as obtained from (3) by imaginary-time propagation for 1000 $^{164}$Dy atoms with the scattering length adjusted to $a = 80a_0$ by the Feshbach resonance technique [26] and with the dipolar length $a_{dd} = 132.7a_0$. A 3D Gaussian input wave function converges to the unexcited quasi-2D soliton illustrated in figure 2(a). An input of 3D Gaussian times $z$, with a node at $z=0$ representing an excitation in $z$, converges to the excited quasi-2D soliton of figure 2(b). Similarly figure 2(c) shows a quasi-2D soliton with the lowest excitation in $y$ and figure 2(d) shows the same with an excitation in both $y$ and $z$. From figure 2 we find that the excitation of the quasi-2D soliton increases the spatial extension significantly for the same number (1000) of $^{164}$Dy atoms. Moreover, the excited quasi-2D solitons can accommodate a larger number of atoms compared to the unexcited quasi-2D solitons implying a larger critical number $N_{\text{crit}}$ shown in figure 1. The quasi-2D solitons of figures 2(b) and (c) with one quantum of excitation in $y$ or $z$ are larger in size compared to the ground state shown in figure 2 (a) and the quasi-2D soliton of figure 2(d) with two quanta of excitation is larger those of figures (b) and (c) with one quantum of excitation each. The size of the excited quasi-2D solitons of figure 2 is about $120 \text{ \mu m}$ and it increases rapidly with the scattering length $a$. The size tends to $\infty$ as $a \rightarrow a_{dd} = 132.7a_0$.

Similar to the Feshbach resonance technique for manipulating the scattering length by magnetic [26] and optical [27] means, it is possible to manipulate the dipolar interaction by a rotating orienting field [15]. Now we present results for quasi-2D solitons with a sign-changed dipolar length of $a_{dd} = -67.35a_0$ for $^{164}$Dy atoms with an axial trap along the polarization $z$ direction satisfying (4). Here we consider the lowest-energy quasi-2D soliton of 1000 $^{164}$Dy atoms for the scattering length $a = 80a_0$. The 3D isodensity contour is illustrated in figure 3(a) as obtained by imaginary-time propagation of (4). As (4) is axially symmetric, profile of the quasi-2D soliton with one quantum of excitation in $x$ is the same as the one with one quantum of excitation in $y$ and we present the one with one quantum of excitation in $x$ in figure 3(b). Finally, in figure 3(c) we present a quasi-2D soliton with one quantum of excitation each in $x$ and $y$. As in figure 2, the excited quasi-2D solitons of figure 3 containing the same number of atoms have larger spatial extension. The size of the quasi-2D soliton of figure 3(c) with two quanta of excitation is larger than that of figure 3(b) with one quantum of excitation, which is larger than that of the ground state shown in figure 3(a).

The excited quasi-2D solitons presented in figures 2 and 3 are stable and robust as tested under real-time propagation in 3D with a reasonable perturbation in the parameters. The robustness comes from the large contact repulsion for a
reasonably large scattering length which strongly inhibits collapse. Also, an appropriate combination of the harmonic trap and long-range dipolar interaction provides confinement of the quasi-2D soliton and prevents leakage of the atoms to infinity. In a nondipolar BEC soliton, the contact attraction alone provides the binding and there is no repulsion to stop the collapse. Consequently, a nondipolar BEC soliton is usually fragile against collapse and auto-destruction. A stringent test of the robustness of these excited solitons is provided in their behavior under head-on collision. Like the quasi-1D dipolar solitons [10], the collision of quasi-2D solitons are expected to be quasi elastic with the solitons emerging with little deformation at medium velocities. However, at low velocities the collision is expected to be inelastic. Only the collision between two integrable 1D solitons is known to be perfectly elastic at all velocities.

To demonstrate the robustness of the solitons we consider a head-on collision between two excited quasi-2D solitons moving in opposite directions. However, because of the large spatial extension of the excited solitons, from a consideration of the required RAM and CPU time, it is prohibitive to carry on these studies in 3D. Hence for the study of the dynamics of the excited quasi-2D solitons we consider the reduced 2D equation (5) with the sign changed dipolar interaction. The reduced 2D equation (5) should give a reasonable account of the quasi-2D solitons for medium values of contact and dipolar nonlinearity parameters. Before the study of this collision dynamics, we
first compare the densities of a quasi-2D soliton calculated using imaginary-time propagation of the 3D and 2D equations (4) and (5), respectively. In figures 4(a)–(c) we plot the effective 2D density \( |\psi(x, y)|^2 = \int dx |\psi(x, y)|^2 \) in the \( x-y \) plane of the quasi-2D excited solitons shown in figures 3(a)–(c), respectively, from a numerical solution of the 3D GP equation (4). In figures 4(d)–(f) we plot the same as obtained from the imaginary-time solution of the reduced 2D (5). The agreement between the two sets of densities presented in figure 4 justifies the use of (5) for the study of the dynamics. However, it should be noted that nonetheless the collision dynamics can differ slightly in the 2D and 3D descriptions, because, in the 3D case, energy can be transferred in the additional degree of freedom in the \( x \) direction which is eliminated in the 2D case [28].

We consider the collision dynamics between two identical excited quasi-2D solitons each of 1000 \(^{164}\text{Dy} \) atoms as illustrated in figure 3(b). The collision dynamics of two such solitons as generated from a real-time simulation of (5) is shown in figure 5. The initial profiles of the two colliding solitons are obtained by a imaginary-time simulation of the same. The initial velocities of the two solitons placed at \( y = \pm y_0 \) are attributed by multiplying the initial wave functions of the two solitons by phase factors \( \exp(\pm iv_y y) \). The parameter \( v_y \) is chosen by trial. The two excited quasi-2D solitons are initially placed at \( y = \pm 80 \) and the real-time simulation started. The two solitons move in opposite directions and suffer a head-on collision. The collision dynamics is best illustrated by plotting effective linear density along \( y \) direction \( |\psi(y, t)|^2 = \int dx |\psi_{2D}(x, t)|^2 \) versus \( y \) and \( t \) as shown in figure 5. The velocity of each soliton is about 2.67 mm s\(^{-1}\) using \( l_0 = 1 \) \( \mu \)m and \( t_0 = 2.6 \) ms. The smooth density profiles of the dynamics presented in figure 5 illustrates the quasi elastic nature of the dynamics.

As the excited quasi-2D solitons are stable and robust, they can be prepared by phase imprinting [19] a bright soliton. In experiment a homogeneous potential generated by the dipole potential of a far detuned laser beam is applied on one half of the bright soliton (\( y < 0 \)) for an interval of time so as to imprint an extra phase of \( \pi \) on the wave function for \( y < 0 \) [18]. The thus phase-imprinted quasi-2D bright soliton is propagated in real-time, while it slowly transforms into a excited quasi-2D soliton. This simulation is done with no axial trap. In actual experiment a very weak axial trap can be kept during generating the excited quasi-2D soliton, which can be eventually removed. The simulation is illustrated in figure 6, where we plot the linear axial density versus time. It is demonstrated that at large times the linear density tends towards that of the stable excited quasi-2D soliton.

4. Summary

We demonstrated the possibility of creating mobile, stable, excited quasi-2D solitons in dipolar BEC with a notch in the central plane and capable of moving in a plane with a constant velocity. These solitons are stationary solutions of the mean-field GP equation. The head-on collision between two such solitons with a relative velocity of about 5 mm s\(^{-1}\) is quasi elastic with the solitons passing through each other with practically no deformation. A possible way of preparing these excited quasi-2D solitons by phase imprinting a bright soliton is demonstrated using real-time propagation in a mean-field model. The results and conclusions of this paper can be tested in experiments with present-day know-how and technology and should lead to interesting future investigations.

![Figure 5](image-url)  Linear density \( |\psi(y, t)|^2 \) of two colliding excited quasi-2D solitons of 1000 \(^{164}\text{Dy} \) atoms each of figure 3 (b) as calculated using the reduced 2D GP equation (5). The excited quasi-2D solitons free to move in the \( x-y \) plane are traveling with constant speed along the \( y \) axis.

![Figure 6](image-url)  Creating the excited quasi-2D soliton of figure 3(b) by real-time simulation of the reduced 2D equation (5) with the phase imprinted quasi-2D soliton of figure 3(a). Linear density of the excited quasi-2D soliton (blue) for (a) small and (b) large times. In (b) the constant density of the stationary excited state (red) is shown for comparison.
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