Establishment of rubber thermo-viscoelastic constitutive model and analysis of temperature field

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Abstract. Rubber has strong nonlinear viscoelastic behavior. Under the periodically changing external forces, it show deformation hysteresis, mechanical loss and heat generation. In this work, a pair of extruding and rotating steel-rubber rollers was researched. Stress relaxation data at different temperatures were obtained through dynamic thermomechanical analysis experiments. A thermo-viscoelastic rubber constitutive model with temperature factor was obtained by combining the generalized Maxwell model where parameters of rubber viscoelastic were fitted with Prony series. Then, the temperature experiment of structure was conducted on the experimental platform. Based on thermo-viscoelastic constitutive model, simulation and analysis of temperature distribution of steel-rubber roller was obtained. The results of experimental data were consistent with simulation results. Further, the rubber constitutive models of pure elasticity, the superposition of elasticity and viscoelasticity with non-temperature factors, the superposition of elasticity and viscoelasticity with temperature factors were simulated respectively. The results show that the third model was more consistent with the actual experimental results, which verifies the accuracy of constitutive model building by this work. Through numerical comparison and analysis, it was proved that viscoelastic hysteretic heat generation is an indispensable source of heat generation in rubber structure movement.

1 Introduction

Rubber, as a common shock absorption and seal material, is widely used in building, vehicle, aerospace and other fields. However, the mechanical properties of rubber are significantly affected by temperature. Under the influence of high temperature, the mechanical properties of rubber will change, which cause the increase of relaxation effect, accelerating aging and even failure. Rubber is a typical viscoelastic material. Viscoelastic hysteresis heating isn’t negligible in the process of rubber extrusion and rotating. In order to study viscoelastic hysteretic heat generation of rubber, it is essential to establish an accurate rubber constitutive model that fits the actual situation.

The constitutive relation is a mathematical model that reflects the properties of material [1]. In order to accurately determine the response of an object under the influence of external factors, it is necessary to know the constitutive equations specific. In order to describe the viscoelastic constitutive model of rubber, many studies have been made by scholars. From the viscoelastic and elastoplastic theories, CHRISTIAN[2] and LIN[3] has given a constitutive model describing the correlation between the excited vibration frequency and the excited vibration amplitude of rubber while there are still many inconveniences and difficulties in practical engineering applications. OLSSON[4] connected several ideal elastoplastic units in parallel to establish a constitutive model of the filled rubber, which was better describe the amplitude correlation of the dynamic properties of the filled rubber. Zhong jianlin[5] proposed a sticky hyperelastic constitutive model to describe the mechanical response of rubber under different strain rates. The Instron machine and SHTB device were used to conduct the uniaxial tensile test of the rubber. The viscoelastic constitutive model of rubber established. Wu jiu[6] used the existing hyperelastic model, generalized Maxwell model and ideal elastoplastic model to obtain the
hyperelastic-viscoelastic-elastoplastic constitutive model which characterizes the correlation between vibration frequency and vibration amplitude of the dynamic characteristics of the filled rubber. Bo Song[7] combined the strain energy function with the relaxation function describing viscoelasticity and proposed a material model related to the rate, which described the stress-strain behavior of rubber under pressure and tension. It was found that there is a strong consistency between the theoretical and experimental results at normal temperature while the influence of temperature was not taken into account. Osterl[8] proposed a three-dimensional viscoplastic constitutive model of finite strain in a co-rotating explicit structure, which was described by the finite element method. Zuo[9] studied the determination method of plastic elements and the distribution of yield points to established a new type of rubber superelastic-viscoelastic-plastic elastoplastic constitutive model. According to the rubber performance test results, the parameters were determined, and the theoretical model was transformed into the rubber parameters to establish a finite element model of the rubber. Li[10] used ABAQUS to fit the parameters of four constitutive models, selected the best constitutive model and established a high-density porous rubber quasi-static constitutive model. Since the rubber has strong nonlinear viscoelasticity and rubber is affected by various factors such as temperature and frequency, there has not been a perfect constitutive model that can describe all the nonlinear viscoelasticity.

In the aspect of temperature analysis of rubber structure, many scholars have used different methods to study the viscoelastic hysteretic heat generation of rubber. Wan zhimin[11] conducted experimental and theoretical research on the heat generation fatigue behavior of rubber composites reinforced with cord. They used a homemade fatigue test system to study rubber composites made by different tire manufacturers. The effects of stress amplitude, average stress and loading frequency on fatigue life were analyzed, and the rise of temperature rubber composite surface during fatigue process was obtained by using infrared thermometer, and the general law of temperature rise of rubber composites surface under cyclic loading was studied. Tian zhenhui[12] studied the fatigue damage of rubber composites under tensile cyclic loading. The results show that the hysteresis loss of the material remains the same during the fatigue process, and the difference of the hysteresis loss and thermal conductivity is the main reason for the diversity of surface temperature rise of discrepancy kinds of specimens. Wu fuqi[13] analyzed the temperature field of tires by using the decoupling idea.

To establish the viscoelastic constitutive model of rubber with temperature factors, this work took a pair of extrusion-rotated steel-rubber roller structures as the research object and the data in the stress relaxation experiment of rubber at different temperatures were superimposed by the ANSYS Prony fitting and the generalized Maxwell model. Based on this constitutive model, the thermal-structure coupling field simulation of the steel-rubber roller was carried out, and the temperature field distribution was obtained. The simulation analysis under different conditions was studied. At the same time, the temperature experiment of extruding and rotating steel-rubber roller on the experimental platform was obtained. Finally, the rubber models of pure elasticity, the superposition of elasticity and viscoelasticity with non-temperature factors, the superposition of elasticity and viscoelasticity with temperature factors were simulated respectively and investigated.

2 Rubber properties

The viscoelastic properties of rubber are particularly pronounced compared to other materials, and their viscoelasticity is more significantly dependent on temperature. Materials that haven’t significantly creep at room temperature produce significant deformation or flow at higher temperatures[14]. The change in temperature is so great that it even changes the mechanical form of matter, which is solid or fluid, depending on the temperature.

From the relaxation property of molecular motion, it can be known that the same mechanical relaxation phenomenon can be observed at a higher temperature in a shorter time, or at a lower temperature in a longer time[15]. Therefore, increasing temperature and prolonging time are equivalent to molecular motion and viscoelasticity. It is the principle called the temperature equivalence.

With the help of a shift factor α, mechanical data measured at one temperature and time can be changed into mechanical data at another. Therefore, the viscoelastic data obtained at different
temperatures can be superimposed along the time axis translation. Set a reference temperature, the curve of the reference temperature will not move, the curve below the reference temperature will move to the left, the curve above the reference temperature will move to the right. Finally, the curves will be superimposed into a smooth combined curve, as shown in figure 1. E is the modulus as a function of time and temperature.

![Figure 1. Time temperature equivalence principle](image)

For most amorphous polymers, the relation between the curve translation $\lg \alpha_T$ and temperature T is Williams-Landel-Ferry (WLF) equation [16]. WLF equation studies the dependence of the viscoelasticity of polymer materials on temperature and time. This equation is about the equivalent temperature relationship between any temperature and a reference temperature, and its expression is as follows:

$$\lg \alpha_T = \frac{-C_1(T - T_0)}{C_2 + T - T_0}$$  \hspace{1cm} (1)

Where,  
$C_1, C_2$—empirical constant, temperature-dependent.
$T_0$—selected the reference temperature.

The stress-strain relationship of rubber under cyclic loading is not only related to time but also temperature. Studies have shown that the generalized Maxwell model, which is composed of N Maxwell bodies in parallel, can well describe the properties of viscoelastic materials. The stress relaxation modulus at the reference temperature T is:

$$G(T_0, t) = \sum_{i=1}^{n} G_i \exp(-t / \tau_i)$$  \hspace{1cm} (2)

Where,  
$G_i$—The spring elastic modulus of the Maxwell unit can be determined by experiments.
$\tau_i$—Relaxation time of the Maxwell unit.
$\eta_i$—Viscosity coefficient, which can be determined by experiment.
$n$—Number of Maxwell models.

The dependence on time and temperature is a remarkable characteristic of the mechanical properties of polymer. If the influence of temperature on the mechanical properties of rubber is considered, according to the time-temperature equivalence principle of rubber within a certain temperature range, the relaxation modulus at any temperature can be obtained by the relaxation modulus at the reference temperature:

$$G(T, t) = G(T_0, t / \alpha_T)$$  \hspace{1cm} (3)

The constitutive model determination scheme is: Rubber relaxation experiment was carried out independently under different temperature. The reference temperature was selected and the data was fitted shift in ANSYS with Prony series, and the constitutive relation under constant temperature was
combined to establish the constitutive model of rubber under variable temperature.

3 Constitutive model

3.1 Stress relaxation test
The rubber samples were subjected to relaxation experiments using a German GAOBA EPLECOR 500N dynamic thermomechanical analyzer. The loading mode is shear loading. A disc type sample of nitrile rubber with hardness of Shore 50 was used in experiment whose specification is: radius 5mm, thickness 2mm. Seven temperature conditions which were 20℃, 25℃, 30℃, 35℃, 40℃, 45℃, 50℃ were set up in experiments. The relaxation modulus of rubber at various temperatures changes with time as shown in table 1, and the stress relaxation curve is shown in figure 2.

| Time (s) | The experimental temperature (℃) |
|----------|----------------------------------|
|          | 20  | 25  | 30  | 35  | 40  | 45  | 50  |
| 0.010    | 1465880 | 1411260 | 1365080 | 1332470 | 1302070 | 1271840 | 1262600 |
| 0.013    | 1439760 | 1388790 | 1344640 | 1313470 | 1284520 | 1256050 | 1247920 |
| 0.016    | 1415170 | 1367180 | 1324890 | 1295100 | 1267620 | 1240920 | 1233860 |
| 0.020    | 1391840 | 1346330 | 1305770 | 1277340 | 1251370 | 1226420 | 1220400 |
| 0.025    | 1369550 | 1326150 | 1287290 | 1260220 | 1235780 | 1212530 | 1207500 |
| 0.032    | 1348140 | 1306590 | 1269440 | 1243750 | 1220830 | 1199210 | 1195130 |
| 0.040    | 1327490 | 1287630 | 1252220 | 1227920 | 1206510 | 1186440 | 1183250 |
| 0.050    | 1307530 | 1269270 | 1235650 | 1212730 | 1192780 | 1174180 | 1171820 |
| 0.063    | 1288190 | 1251510 | 1219720 | 1198180 | 1179620 | 1162380 | 1160810 |
| 0.079    | 1269440 | 1234360 | 1204430 | 1184230 | 1166980 | 1151010 | 1150200 |
| 0.100    | 1251290 | 1217830 | 1189790 | 1170870 | 1154850 | 1140030 | 1139970 |
| 0.126    | 1233720 | 1201920 | 1175760 | 1158060 | 1143160 | 1129420 | 1130110 |
| 0.158    | 1216750 | 1186640 | 1162330 | 1145760 | 1131890 | 1119160 | 1120610 |
| 0.200    | 1200390 | 1171990 | 1149480 | 1133940 | 1121010 | 1109230 | 1111480 |
| 0.251    | 1184640 | 1157940 | 1137160 | 1122550 | 1110490 | 1096610 | 1102730 |
| 0.316    | 1169500 | 1144490 | 1125340 | 1111570 | 1100290 | 1090320 | 1094390 |
| 0.398    | 1154980 | 1131600 | 1113990 | 1100970 | 1090420 | 1081360 | 1086500 |
| 0.501    | 1141070 | 1119230 | 1103070 | 1090700 | 1080860 | 1072760 | 1079100 |
| 0.631    | 1127730 | 1107360 | 1092550 | 1080770 | 1071620 | 1064530 | 1072240 |
| 0.794    | 1114960 | 1095950 | 1082400 | 1071150 | 1062700 | 1056740 | 1065980 |
| 1.000    | 1102710 | 1084960 | 1072590 | 1061830 | 1054120 | 1049410 | 1060380 |
| 1.259    | 1090950 | 1074360 | 1063120 | 1052830 | 1045920 | 1042630 | 1055500 |
| 1.585    | 1079650 | 1064120 | 1053970 | 1044160 | 1038130 | 1036440 | 1051380 |
| 1.995    | 1068780 | 1054210 | 1045140 | 1035830 | 1030820 | 1030920 | 1048030 |
| 2.512    | 1058290 | 1044620 | 1036640 | 1027890 | 1024020 | 1026130 | 1045460 |
| 3.162    | 1048160 | 1035330 | 1028490 | 1020370 | 1017810 | 1022100 | 1043600 |
| 3.981    | 1038360 | 1026340 | 1020720 | 1013330 | 1012260 | 1018870 | 1042370 |
| 5.012    | 1028870 | 1017660 | 1013350 | 1006830 | 1007420 | 1016410 | 1041630 |
| 6.310    | 1019690 | 1009300 | 1006450 | 1000920 | 1003350 | 1004660 | 1041240 |
| 7.943    | 1010790 | 1001270 | 1000060 | 995682 | 1000050 | 1013530 | 1041070 |
| 10.000   | 1002200 | 993625  | 994239 | 991159 | 997529 | 1012870 | 1041010 |
| 12.589   | 993920 | 986392 | 989048  | 987393 | 995723 | 1012530 | 1040990 |
| 15.849   | 985968 | 979625 | 984542  | 984399 | 994536 | 1012390 | 1040990 |
Figure 2. Stress relaxation curves at different temperatures

3.2 Curve fitting
In this work, a generalized Maxwell model is chosen to characterize the viscoelastic properties of rubber. General Maxwell model can describe stress relaxation behavior of materials more intuitively. In general, the stress function is given in integral form. In the case of small strain theory, the viscoelastic constitutive equation of generalized Maxwell can be written as following:

\[
\sigma = \int_0^t 2G(t) \frac{de}{d\tau} d\tau
\]

Where,
- \(\sigma\) —— Cauchy stress;
- \(G(t)\) —— Shear relaxation kernel function;
- \(e\) —— Strain deviator (shear deformation);
- \(t\) —— The current time;
- \(\tau\) —— The past time.

In ANSYS, the viscoelastic integral kernel function in equation (4) can be represented by the Prony series, as shown in the following.

\[
G(t) = G_0 \left[ \alpha_\infty + \sum_{i=1}^n \alpha_i \exp \left( -\frac{t}{\tau_i} \right) \right]
\]

Where,
\( G_0 \) —— Shear relaxation modulus at time 0;
\( n \) —— The number of terms in the Prony series;
\( \alpha_i \) —— The relative modulus;
\( \tau_i \) —— Relaxation time.

If there are more Maxwell element models in parallel, the fitting curve will closer to the test curve. Therefore, the fitting accuracy can be improved by increasing the number of unit models. In this work, a five-unit parallel generalized Maxwell model was chosen. Substituting equation (5) into equation (4), the viscoelastic constitutive equation (6) under fixed temperature can be obtained:

\[
\sigma = \int_0^t 2G_0 \left[ \alpha_\infty + \sum_{i=1}^{5} \alpha_i \exp \left( -\frac{t}{\tau_i} \right) \right] \frac{de}{d\tau}, \quad n = 5
\]

After the time calculation with the shift factor, the relaxation modulus at any temperature can be obtained by the relaxation modulus at the reference temperature, then the viscoelastic constitutive model of the temperature change process can be derived by combining equations (3) and (1):

\[
\sigma = \int_0^t 2G_0 \left[ \alpha_\infty + \sum_{i=1}^{5} \alpha_i \exp \left( \frac{-t}{\tau_i \exp \left( \frac{-C_1(T-T_0)}{C_2+T-T_0} \right)} \right) \right] \frac{de}{d\tau}
\]

Equation (7) is the viscoelastic constitutive model of rubber with temperature factors. The experimental data in table 1 were brought into ANSYS for fitting with Prony. Since the relaxation modulus is related to temperature, the transformation function needs to be selected in the interface. The time temperature equivalent equation, namely WLF, is selected as the conversion function. According to the data of relaxation modulus at different temperatures, the viscoelastic parameters has obtained as shown in table 2. The fitting curve was further drawn in 20°C reference temperature, as shown in Figure.3. The non-red line is the experimental data curve, and the red line is the fitting curve after the experimental data is shifted to the reference temperature. This indicates that the mathematical description of the relaxation modulus by using Prony series has a good agreement with the relaxation modulus curve. The viscoelastic constitutive model of rubber at any temperature can be obtained by applying the formula (7) by fitting the curve.

**Table 2. Fitted viscoelastic parameters**

| Shear response coefficient | Temperature (°C) |
|---------------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
|                           | 20               | 25               | 30               | 35               | 40               | 45               | 50               |                  |
| \( \alpha_i \)            | 0.05256          | 0.09332          | 0.05070          | 0.05363          | 0.04942          | 0.04386          | 0.03908          |                  |
| \( \tau_0 \) (s)           | 27.37400         | 0.48052          | 15.35700         | 38.35200         | 27.35500         | 1.05180          | 0.22624          |                  |
| \( \alpha_2 \)            | 0.05699          | 0.11784          | 0.03844          | 0.05374          | 0.05050          | 0.04387          | 0.03898          |                  |
| \( \tau_1 \) (s)           | 3.32340          | 0.8423           | 85.95500         | 1.58240          | 0.29602          | 3.75600          | 0.24371          |                  |
| \( \alpha_3 \)            | 0.07589          | 0.06961          | 0.08442          | 0.05276          | 0.05067          | 0.04391          | 0.03900          |                  |
| \( \tau_2 \) (s)           | 0.44676          | 2.87520          | 0.46564          | 7.57780          | 0.29256          | 15.56800         | 2.53080          |                  |
| \( \alpha_4 \)            | 0.09849          | 0.04593          | 0.10796          | 0.05497          | 0.05014          | 0.04417          | 0.03897          |                  |
| \( \tau_3 \) (s)           | 0.07532          | 132.08000        | 0.09145          | 0.36588          | 1.34580          | 0.26697          | 8.93230          |                  |
| \( \alpha_5 \)            | 0.13249          | 0.05438          | 0.06293          | 0.05505          | 0.04909          | 0.04406          | 0.03892          |                  |
| \( \tau_4 \) (s)           | 0.01211          | 19.29700         | 2.56400          | 0.32217          | 5.44210          | 0.26499          | 0.77296          |                  |
4 Temperature experiment

4.1 Setting
The main experimental equipment is the platform of steel-rubber roller extrusion-rotating (as shown in figure 4 and the infrared temperature gun. Rubber roller which is covered with a layer of rubber and the steel core is the experimental object. The rubber layer material is nitrile rubber with hardness of Shore 50, which is vulcanized by adding carbon black reinforcing agent. The diameter of the rubber roller is 60mm and the thickness is 10mm.

4.2 Program
An experiment was conducted on a rubber roller having a thickness of 10 mm and a hardness of Shore 50, and the temperature rise of the rubber roller under moving conditions was measured. The rotational speed of the experimental platform was set to 201 r/min, and the pressure between the two extrusion rollers was 0.5 MPa. Record the initial ambient temperature before starting the experiment. After starting the experiment, the data was measured every five minutes: the temperature of the middle extrusion area of the two rollers was measured using an infrared temperature gun, and the experiment was terminated when the temperature of the extrusion area rose to stabilize.

4.3 Analysis
It was found that the temperature at edge of the rubber roller extrusion area was significantly higher than that at the middle area. The temperature between the center area was shown as figure 5. Under the condition of initial temperature which is 20 ℃ , the highest temperature between the center area of
rubber roller was 41.1 °C. During the rotating of the two rollers, the highest temperature appears at edge of the extrusion area and is significantly higher than the middle area. The highest temperature reached 43.0248 °C. It is found that the experimental results of temperature rise were similar to the thermodynamic simulation results which shows the accuracy of above analysis.

**Figure 4.** steel-rubber roller experiment platform  
**Figure 5.** Temperature data graph of roller center

### 5 Simulation and analysis

#### 5.1 Simulation setting

The upper part is the steel roller with a diameter of 65mm and a length of 30mm. The lower part is a rubber covered roller. The central shaft is made of steel and the rubber layer covers it with thickness of 10mm. In order to reduce the time of simulation calculation and the accuracy of calculation, the rubber roller only builds the model of rubber layer, and a mass node replaces the steel shaft inside. The rubber roller model is a hollow cylinder with an internal diameter of 45mm, an external diameter of 65mm and a length of 30mm. The mesh of structure is shown in figure 6.

**Figure 6.** 3D model of extruding and rotating steel-rubber structure

The above thermo-viscoelastic properties and the elastic properties were applied to the rubber. The material properties of rollers were shown in table 3. The hardness of rubber is Shore 50, and the friction coefficient between the steel-rubber roller was set as 0.5.

**Table 3.** Material parameters of steel rubber
| Material | Elasticity modulus (GPa) | Density (kg/m³) | Coefficient of thermal conductivity (W/(m²•℃)) | Specific heat capacity (J/(kg•℃)) | Coefficient of thermal expansion (1/℃) | Poisson's ratio |
|----------|--------------------------|-----------------|-----------------------------------------------|-------------------------------|----------------------------------------|----------------|
| Steel    | 206                      | 7800            | 66.6                                          | 460                           | 1.06×10⁻⁵                              | 0.3            |
| Rubber   | 0.008                    | 1300            | 0.25                                          | 1700                          | 65×10⁻⁵                                | 0.47           |

Ink roller will happen heat transfer with surrounding environment in rotating, that is, the external convective heat transfer between the rubber roller and air. The heat transfer coefficient of 280W/(m²•℃). The convection heat transfer coefficient of steel roller and air is 420W/(m²•℃) and the coefficient of convective heat transfer between the two rollers set to 2000W/(m²•℃).

In order to analyze the heat generation during the extrusion and rotation between two rollers, both the motion and thermal analysis should be simulated simultaneously. Therefore, this chapter adopts the method of direct coupling to conduct the transient thermal-structure simulation analysis of the model, which takes into account both the frictional heat generated during the rotating process and the hysteretic heat generated during the movement of rubber. The unit selects SOLID226 structure thermal coupling module. For steel roller downward pressure 0.5 MPa and angular velocity of the 21 rad/s, both roller around their central axis rotation, the initial temperature is 20℃, steel roller press the 1s and then drive the rubber roller rotation 1s for the transient coupled field analysis.

Based on the above modeling and setting, thermodynamic simulation experiments were carried out in ANSYS.

5.2 Analysis of temperature field

Temperature field nephogram of rubber roller after 1 second rotation was shown in figure 7.

![Figure 7. Nephogram of temperature field](image)

5.2.1 Temperature analysis of steel-rubber rollers structure

It can be seen the temperature distribution of the temperature nephogram that the steel roller was maintained at temperature of 36℃, and the highest temperature rubber roller was 43.0248℃ which occurred significant delamination. The overall temperature of the steel roller was slightly lower than rubber roller because the convective heat transfer coefficient between the steel and the air is higher than that between rubber roller and air, and the rubber may cause hysteresis heat generation. The temperature inside the steel roller wasn’t stratified, while the rubber roller was obviously. It is caused by the difference of materials for two rollers. The thermal conductivity of the steel roller is 66.6
W/(m²•℃), while that of the rubber roller is 0.25 W/(m²•℃). The heat transfer speed inside the steel roller was so much faster than rubber roller that the temperature inside the steel roller didn’t occur significantly stratified.

5.2.2 Temperature analysis of the center node of extrusion area
The temperature data of the central node of extrusion area with time was extracted, as shown in figure 8. It can be seen that in the rotation, the temperature of the central area rises quickly first, then slows ascension, and finally tends to stable. The slope rose reached the highest when the temperature of rubber roller rising from 20°C to 30°C. This is because in the 20-30°C, the rubber has good damping performance and it is in transition which from a glassy state to an elastic state, so the heating rate is fast [17].

![Figure 8. Temperature curve of the center area for two rollers with time](image)

5.2.3 Temperature analysis of rubber roller along the length direction
According to the path in figure 9, the temperature data of nodes along the length direction of the rubber roller extrusion area was extracted as shown in figure 10. From the figure, the highest temperature of rubber roller for extrusion area during rotating occurred at edge, the highest temperature reached 43.0248 °C, and the temperature of middle extrusion area is lower than the edge, which reached 38.541 °C. Due to rubber roller edge by the steel roller extrusion, the dislocation between molecular chains got larger and higher hysteretic heat generation were produced.

![Figure 9. The path of temperature data extraction](image)

![Figure 10. Temperature data of length nodes](image)

5.2.4 Temperature analysis of the rubber roller along the wall direction
As shown in figure 11, taking the wall direction of the central section of the rubber roller as the object,
temperature values of 5 nodes were successively extracted from inside and outside along the radial direction which were shown in figure 12. It can be seen that the temperature of the rubber roller increased gradually from inside to outside along the radial direction, which was caused by the low thermal conductivity of rubber. The heat which generated by the friction can’t be transmitted to the inside in time, only the heat generation of rubber inside produces hysteretic during rotation. The heat generation in the inner area was caused by the hysteretic of the rubber during rotation.

5.2.5 Temperature analysis of the rubber roller along the radial direction
The temperature data of the rubber roller along the radial direction were extracted according to the path in figure 13. From the figure 14, the wall temperature of the rubber roller increased slowly from the outward extrusion area. This is due to the extrusion between the rubber and the steel roller, the deformation of rubber becomes larger when entering the extrusion area, resulting in the large stress and strain. The temperature in the extrusion area was higher than that in the non-extrusion area, which is consistent with the conclusions reached by previous scholars.

5.3 Analysis of three rubber constitutive models
In the extruding and rotating, the rubber heat generation is derived from the viscoelastic hysteresis and friction. In order to study the role of viscoelastic constitutive model of rubber with temperature factor in the heat generation, two different rubber constitutive models were simulated again. The first model was set as pure elastic (PE) rubber constitutive model. The second model was set as the superposition of elasticity and viscoelasticity with non-temperature factors (SEV-NT) rubber constitutive model. The third model was set as the superposition of elasticity and viscoelasticity with temperature factors (SEV-T) rubber constitutive model, which obtained in 5.1. The temperature variation data of extrusion...
center area of the rubber roller under the three models was shown in the figure 15.

![Figure 15](image)

**Figure 15.** The temperature varies with time under three rubber models

The highest temperature of PE model which was shown in the green line in figure 15 achieved 26.0316°C, that is, the rise of temperature caused by the frictional heat generation was 6.0316°C. The highest temperature of SEV-NT model which was shown in the red line in Figure 15 achieved 33.5884°C, that is, the rise of temperature caused by viscoelastic hysteresis heat generation was 7.5568°C. The highest temperature of SEV-NT model which was shown in the red line in Figure 15 achieved 33.5884°C, that is, the rise of temperature caused by viscoelastic hysteresis heat generation was 7.5568°C.

The experiment data show that the rise of temperature in PE and SEV-NT was 6.0316°C and 13.5884°C, where in SEV-NT it reaches 18.541°C. The temperature rise test results show that the highest temperature in the center area of rubber roller with hardness 50 is 41.1°C, which increased by 21.1°C. It has found that the SEV-T model was more consistent with the actual experimental results than the SEV-NT model. And the SEV-T has a higher temperature rise in the transient coupled field simulation because the temperature factor has a certain influence on the constitutive model of the rubber. In a certain temperature range, as the temperature increases, the loss factor will increase, further causing the temperature to rise again with rotation. Compared the experimental results, the temperature rise generated in SEV-T model has shown a fine consistency, which verifies the correctness of the constitutive model.

Under the PE model, the frictional heat generation was 6.0316°C; under the SEV-NT model, the temperature rise caused by frictional heat generation was 6.0316°C, accounting for 44.4% of the total temperature rise, while the temperature rise caused by viscoelastic hysteresis was 7.5568°C, accounting for 55.6%; under the SEV-T model, the temperature rise caused by frictional heat generation was 6.0316°C, accounting for 32.5% of the total temperature rise, while the temperature rise caused by viscoelastic hysteresis was 12.5094°C, accounting for 67.5%. The above data are summarized in Table 4, which shows that viscoelastic hysteretic heat generation accounts for a large proportion in the total temperature rise. It was proved that viscoelastic hysteretic heat generation is an important source of heat generation in the movement of rubber roller and steel roller.

| Model     | Total temperature | Friction temperature | Viscoelastic hysteresis | Frictional temperature | Viscoelastic hysteresis |
|-----------|-------------------|----------------------|-------------------------|------------------------|-------------------------|
| PE Model  |                   |                      |                         |                        |                         |
| SEV-NT Model |                 |                      |                         |                        |                         |
| SEV-T Model |                 |                      |                         |                        |                         |

Table 4. Comparative analysis of temperature rise under different rubber models

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6 Conclusion

In this work, the following conclusions are drawn:

1. A rubber viscoelastic constitutive model with temperature factors was established by combining with rubber relaxation test.

2. In the coupled field analysis of the steel-rubber roller, the temperature field of the rubber roller presents a different distribution due to the inconsistent thermal conductivity of the material itself, the temperature of the steel roller was maintained within a certain range. The final temperature of the steel roller was slightly lower than that of the rubber roller because the heat transfer coefficient between the material and air is different and the rubber produces hysteretic heat generation during movement.

3. The highest temperature of the rubber roller appears at the edge of the compressed area during the rotation process, which is due to the large deformation of both ends during the rotary extrusion process. Compared with the results of temperature experiment, the simulation results of thermal structure coupling field are proved to be correct.

4. The three rubber constitutive models were simulated separately. It was concluded that the temperature rise of PE rubber constitutive model was 6.0316 °C, SEV-NT rubber constitutive model was 13.5884 °C, and SEV-T rubber constitutive model was 18.541 °C. The third model was more consistent with the temperature experiment results, which verifies the accuracy of the constitutive model. Viscoelastic hysteretic heat generation accounts for a large proportion in the total temperature rise. It proved that viscoelastic hysteretic heat generation of rubber is a non-negligible source of heat generation in the process of rubber structure movement, and indicated the importance of this study and its practical application significance.

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