Morphological comparison of signals and its application for estimating signal lag time

A I Chulichkov1,2, S N Kulichkov1,2, N D Tsybulskaya2 and G A Bush3

1Moscow State University, Faculty of Physics, Moscow, 119991 Russia;
2Obukhov Institute of Atmospheric Physics, Russian Academy of Sciences, Moscow, 119017 Russia

achulichkov@gmail.com

Abstract. Mathematical methods are proposed for comparing the shape of signals with amplitudes distorted by unknown monotonic transformations. These methods are based on the solution of the problems of the best approximation of the signal under analysis to signals of a specified class, and used in estimating the lag time of one fragment of signal with respect to another. Such problems arise, for example, in determining the direction of propagation of a sound wave in the atmosphere. The solution to this problem is based on the assumption that the conditions of the signal detection are different at different spatial points; as a result, the measured signals differ, not only in their lag time but also in nonlinear distortions, such that only the general shape of the signal is preserved: if the amplitude of one signal is the result of a strictly monotonic transformation of the amplitude of another signal, then their shapes are equivalent. In addition, the measurements are accompanied by an additive noise with an unknown dispersion.

1. Introduction

Problems, in which the signal from its source propagates along different paths to receivers, at which its amplitude is recorded at discrete times, are not uncommon in applications. In such problems, it is necessary to determine the lag time of one recorded signal with respect to another. At the output of different receivers, signals differ not only by time lags, but also by unknown nonlinear distortions of their amplitudes. In this case, such distorted signals are perceived by a human as similar signals in shape; however, their absolute values may significantly differ. One can solve the problem of estimating lag time, if the formal rule, which allows one to identify signals similar in shape, is observed. The methods, which allow one to compare signals or images in form, are called morphological.

As an example, figure 1 shows two infrasonic signals from industrial explosions. Different paths of sound propagation result in distortions that make signals similar but not identical.

The range of application of morphological methods is wide, but different approaches are used. The common feature of these approaches is that they all are based on mathematical models of amplitude-geometric signal structures. These methods are most extensively used in analyzing images, however, they may also be used in analyzing one-dimensional signals.

One of the most consistent approaches to morphological analysis is contained in Yu.P. Pyt’ev’s works. A review and a comparative analysis of the morphological methods of solving these and many
other problems of analyzing and interpreting signals and images are given in [1-7]. These methods are based on a mathematical model of recording signal from some source. It is assumed that this signal is recorded under some (unknown) conditions; however, a class of probable recording conditions is specified, and some signal corresponds to each of these conditions. Thus, a set of signals, which may be recorded under all possible conditions, are related to the concrete source. Such set of images is called the signal shape.

Figure 1. Acoustic signals (from industrial explosions) recorded by receivers located at different points on the land surface. The signals are similar in shape, but not identical as the time functions of signal amplitudes.

This approach makes it possible to solve a number of problems of analyzing and interpreting data, such as the problems of recognizing and identifying signal fragments (for this purpose, it is necessary to determine whether the signal fragment under analysis belongs to the given shape or to find the shape closest to the fragment under analysis), the problems of estimating the parameters of shapes, and others. Moreover, this approach allows researchers to subjectively formulate the model of forming and recording signals and to obtain optimal solutions of morphological problems.

In this work, the morphological analysis methods are used in solving the problems of estimating lag times of infrasonic signals propagating throughout the Earth’s atmosphere.
2. Signal shape. Standard shape

A human perceives the signal shapes in Figure 1 as invariable and conserved, even if the amplitudes and phase shifts of the signals vary within a wide range. From this follows the conclusion that, for the solution of the problems of recognizing and classifying signal fragments, estimating their features, etc., it is important to know some structures (in this case, the intervals of amplitude increasing and decreasing, which do not change under changes in the conditions of signal transmission and recording) rather than exact values of the amplitude of a signal fragment at every point of the domain of its definition. In other words, the shape of signal fragment is determined from the location and sequence order of local maxima and minima.

On the other hand, under real conditions, data on channel properties and sensor characteristics, which make it possible to unambiguously relate the amplitude of a sensor output signal with the initial signal of source, are, as a rule, not available. A set of the mathematical characteristics of signals, which (independently of the conditions of their transmission and recording) makes it possible to isolate and recognize initial signals and to estimate their characteristics (for example, lag time) under arbitrarily varying recording conditions, is called the signal shape, and the mathematical methods of analyzing and interpreting signals are called morphological. The morphological methods of analyzing signals are based on mathematical models and a specially developed mathematical formalism, which allow one to characterize signals in terms of the transformation invariants of their amplitudes according to the domain of their definition, which conserve information necessary for the solution of the problem.

Mathematically, the signal fragment is a set of signal amplitude readings $S_i$, $i=1,...,N$. These readings will be considered as the vector coordinates $S \in R^N$, here $R^N$ is the Euclidean space. The point $i$ of signal fragment is the local minimum, if $S_{i-1} \geq S_i$, $S_i \leq S_{i+1}$, and local maximum, if $S_{i-1} \leq S_i$, $S_i \geq S_{i+1}$. Since both local minima and maxima are conserved under the strictly monotonic transformation of the amplitude of signal fragment $S$, the form of the fragment $S$ is defined as the closure $\overline{V_S}$ into $R^N$ of the subset

$$V_S = \{ \bar{f} \in R^N : f_i = F(S_i), i = 1,...,N, \quad F \in F \},$$

where $F: R^1 \to R^1$ is a class of monotonically increasing functions determined and taking on values on the number axis. If $f \in \overline{V_S}$, for any $k > 0$, $kS \in \overline{V_S}$ is satisfied and, then $\overline{V_S}$ is a convex closed cone in $R^N$. Therefore, $\overline{V_S}$ is biuniquely related with the operator of projection $P_3 : R^N \to R^N$ onto $\overline{V_S}$, whose effect on any element on $f \in R^N$ is determined as the solution of the problem of the best approximation $f$ by the elements from $\overline{V_S}$:

$$\|f - P_3f\| = \inf_{g \in \overline{V_S}} \|f - g\|, \quad f \in R^N;$$

the $P_3f$ vector is the projection of $f$ onto $\overline{V_S}$, and $P_3f$ is the projector onto $\overline{V_S}$. The operator $P_3$ is called the image form $S$ or, more exactly, the (operator) representation of the form $\overline{V_S}$ of the fragment $S \in \overline{V_S}$.

The problems of comparing signal fragments in shape, determining differences in signal shapes, detecting sources of signals according to signal shapes, estimating the parameters of signal shapes, and others are solved in terms of projector. The morphological analysis methods are also successfully used in studying infrasonic signals propagating throughout the atmosphere [8, 9].

In this section, the signal fragment $S$ specifies the standard shape. However, it is not always possible to indicate the standard shape. The problem of comparing signals in shape in the absence of the standard shape is considered in the following section.
3. Comparing signals in shape in the absence of its standard

In practice, errors can never be completely avoided in recording signals, therefore, it is not always possible to specify signal fragment \( S \), according to which one can construct form \( \overline{V}_S \) and (or) projector \( P_S \).

Let the results \( \xi \) and \( \eta \) obtained from recording two signal fragments be known. In this case, the recording is accompanied by some random error so that \( \xi = g + v \), \( \eta = q + \mu \). Here, the fragments of signals \( g \) and \( q \) are unobservable and \( v \) and \( \mu \) are the \( \mathbb{R}^N \) vectors simulating an error in recording signals \( g \) and \( q \). The problem is to find out whether the unobservable signals \( g \) and \( q \) belong to the same form \( \overline{V}_S \) at some (unknown) \( S \).

Let us formulate this question more specifically: let us determine the class of monotonic images, which includes only the images within which the interval \( F(x) = [F_-(x), F_+(x)] \subset \mathbb{R}^1 \) (possibly, of zero length, i.e., when \( F_-(x) = F_+(x) \)) corresponds to every \( x \in \mathbb{R}^1 \); in this case, if \( x \leq y \), \( F_+(x) \leq F_+(y) \). If \( g_i \in F(q_i), i = 1, ..., N \), then, such a signal fragment \( S \) will be found so that \( q \in \overline{V}_S \) and \( g \in \overline{V}_S \). The geometric image \( F() \) is a point set \( (x, y(x)) \) of the plane \( \mathbb{R}^2 \), where \( x \) runs through the entire domain of definition \( F() \), and \( y(x) \) runs through the entire interval \( F(x) \). This point set differs from the graph of monotonic function by the presence of vertical segments. Let us call this point set of \( \mathbb{R}^2 \) a monotonic curve. It is necessary to determine whether it is possible to find such \( F \in \mathbb{F} \) transformation, at which the undistorted components \( g \) and \( q \) of signals \( \xi \) and \( \eta \) are connected by the relation \( g_i \in F(q_i), i = 1, ..., N \), for some monotonic image of \( F \in \mathbb{F} \).

The problem of comparing (in shape) signals \( g \) and \( q \) related by the monotonic image on the basis of observational data distorted by noise reduces to answering the question whether it is possible to draw a monotonic curve closely enough to all points \( (\xi_i, \eta_i) \), \( i = 1, ..., N \), on the plane so that the deviation of this curve from observational data could be explained (with sufficiently high probability) by noise. In fact, if larger value of the \( g_i \) coordinate corresponds to that of the corresponding \( q_i \) coordinate, then the point set \( (g_i, q_i) \) is bound to fall on the «monotonically nondecreasing curve», and conversely if these points fall on such a curve, larger value of \( q_i \) corresponds to that of \( g_i \), and the signal \( q \) may be considered as a monotonic transformation of the signal \( g \). If random noise vectors \( v \) and \( \mu \) with sufficiently high probability take on values in some bounded sets \( \mathcal{N} \) and \( \mathcal{M} \), the problem reduces to answering the question whether it is possible to find such \( F \in \mathbb{F} \) transformation and such vectors \( v \in \mathcal{N} \) and \( \mu \in \mathcal{M} \) so that the relations \( \xi - v \in F(\eta - \mu) \) are satisfied. Geometrically, this implies the following. For every \( i = 1, ..., N \), let us form the two-dimensional vector with coordinates \( (\xi_i, \eta_i) \), \( i = 1, ..., N \), and plot these points on the number plane \( \mathbb{R}^2 \). The answer to the question will be positive, if it is possible to draw (on this plane) a monotonically nondecreasing curve that lags behind the points \( (\xi_i, \eta_i) \), \( i = 1, ..., N \), by the vector \( v \in \mathcal{N} \) along the abscissa axis and by the vector \( \mu \in \mathcal{M} \) along the ordinate axis.

The formal statement of the problem is the following. Let us denote the bijection of a set of natural numbers from 1 to \( N \) onto themselves by \( j(\cdot) \). It is necessary to find points \( (g_i, q_i) \) that are closest to some corresponding points \( (\xi_j, \eta_j) \), where \( j = j(i) \), \( i = 1, ..., N \). In this case, the following inequalities are bound to be satisfied:

\[
g_1 \leq g_2 \leq \ldots \leq g_N \ , \ q_1 \leq q_2 \leq \ldots \leq q_N .
\]

(1)

Thus, we come now to the problem of approximation:
The decision rule may be the following: if the minimum value of functional (2) may be explained by the presence of an error in recording the signal amplitude, the fragments of $g$ and $q$ may be considered to belong to the same shape.

Under the given ordering of points $(\xi_j, \eta_j)$, the «inner» problem of calculating the greatest lower bound in (2) reduces to calculating the projection onto a convex closed cone within the space $\mathbb{R}^{2N}$, which is specified by inequalities (1). The solution algorithm for such a problem is known [7]. The calculation of the greatest lower bound using all bijections is a cumbersome procedure, therefore, a more rational solution of (2) should be found. Moreover, geometrically, problem (2) reduces to finding the vector projection $(\xi, \eta) \in \mathbb{R}^{2N}$ onto the union of convex closed cones, and such a set remains to be a cone but may be non-convex. This implies that the solution of problem (2) is not always unique. However, in the morphological analysis methods, of interest is not the projection, but the estimation of the distance from the point $(\xi, \eta) \in \mathbb{R}^{2N}$ to the cone, because the probability of noise realization, which determines whether it is possible to consider the signals $g$ and $q$ as signals belonging to the same shape, depends on this distance.

4. Method of comparing signals in shape in estimating the lag time of signal fragments

Let us consider some approximate methods of solving problem (2) and their use in estimating the lag time of acoustic-signal fragments. We will illustrate the idea of these methods. Figure 2 gives the two acoustic-signal fragments $\xi$ and $\eta$ obtained from the same source, but by different receivers. Figure 3 shows the distribution of points $(\xi_j, \eta_j)$, $j=1,\ldots,N$, on the two-dimensional plane for the signals given in figure 3. The points clustered around point $(0,0)$ correspond to the signal fragments without useful signal. The diameter of the dense spot with its center at zero evaluates the error in recording the signal amplitude. It is seen that most of these points are in the neighborhood of the monotonic curve passing from the left lower angle of this two-dimensional plane to its right upper angle, which implies that the two signal fragments shown in figure 2 are sufficiently close in shape. However, there are points that are away from this monotonic curve; this may be due to a short lag time of one signal with respect to the other. Shifting one signal fragment in figure 2 with respect to the other and analyzing the obtained distribution of points $(\xi_j, \eta_j)$, $j=1,\ldots,N$, one can achieve such a point spacing, when the minimum number of points fall outside the neighborhood of the monotonic curve. This procedure makes it possible to obtain a sufficiently reliable estimate of the lag time of one signal with respect to the other.

Let us give one more example. Figure 4 shows the three signal fragments recorded by receivers in the form of the time dependence of the three signal amplitudes (green, blue, and red). Let us compare (in shape) the signals represented by blue and red colors.

To this end, let us plot the point set $(\xi_j, \eta_j)$ on the number plane (see figure 5); here, $\xi$ and $\eta$ are the signals represented by the blue and red colors in figure 4, respectively. In figure 5, let us draw broken lines, whose rectangular segments are parallel to the coordinate axes. The broken dashed blue line bounds the point cloud from above and from the left, and the broken dashed red line bounds the point cloud from below and from the right.

Let us assume that maximum errors in recording signals do not depend on receivers. Then half the length of the side of the square of maximum size, which fits between these lines, is the estimate of the width $\delta$ of the point cloud around the optimal monotonic curve.

For the situation shown in figure 5, this estimate amounts to 0.46 (conventional units), which is approximately half the maximum signal amplitude, therefore, it is impossible to consider these signals comparable in form.
However, shifting the graph of one of these signals along the time axis, one can obtain the situation shown in figure 6. After superposing one signal on the other, one can obtain the reasonable estimate $\hat{\delta} = 0.05$ (conventional units) corresponding to the amplitude signal/noise ratio equal to 9.2, which is completely realistic for such measurements. The superimposed signals are shown in figure 7.

Figure 2. Infrasonic signals $\xi$ (green) and $\eta$ (blue) compared in shape, conventional units.

Figure 3. Set of points $(\xi_j, \eta_j)$, $j = 1, \ldots, N$, for the signals shown in figure 2.
Figure 4. Output signals of microphones recording infrasounds.

Figure 5. Spacing of the points $(\xi_j, \eta_j)$, $j = 1, ..., N$, the upper and lower envelope curves.
Figure 6. Spacing of the points $(\xi_j, \eta_j), \; j = 1, \ldots, N$, and the upper and lower envelope curves after reciprocal signal shifting.

Figure 7. Superimposed (due to shifting) output signals of the microphones recording infrasounds.

5. Application of the method to determine the difference of signals in shape segments
The method of determine the difference of signals in shape is best demonstrated by the example of images.

Consider the two images $\xi$ and $\eta$ of scenes shown in figure 8 at the top. These images are distinguished by changing lighting. In addition, there is an object in the image $\eta$ (on the right) that is absent on $\xi$ (left).
Figure 8. Scene images (top), highlighted differences in the form of images (bottom left), diagram of points \((\xi_j, \eta_j)\), \(j = 1, ..., N \times M\), the boundary of domain \(D\) is shown by dotted lines, (bottom right).

Below to the right is a diagram of the location of points with coordinates \((\xi_j, \eta_j)\), \(j = 1, ..., N \times M\), where \(N, M\) is the number of rows and columns of the presented images, \(\xi_j, \eta_j\) is the brightness of the \(j\)-th point of the left and right images. The diagram is presented in the form of an image, the brightness of which displays the number of points with given coordinates \((\xi_j, \eta_j)\).

It is seen that on the diagram there is a domain \(D\) around a sufficiently smooth curve in which the main number of points of the diagram is concentrated. This curve can be estimated as a second-degree polynomial connecting the brightness of the image points corresponding to the objects available in both image \(\xi\) and image \(\eta\):

\[
\eta_j - \mu_j = f(\xi_j - \nu_j)
\]

for some \(\mu_j\) and \(\nu_j, |\mu_j| \leq \delta, |\nu_j| \leq \delta\). The width of this domain corresponds to the error in registering the images \(\xi\) and \(\eta\). Points with number \(j\) that do not belong to domain \(D\) correspond to image points whose brightness does not satisfy relation (3). The points of the field of view of the images \(\xi\) and \(\eta\) not related by relation (3) are shown in figure 8 below to the right. They demonstrate the objects present in one of the images, and are absent on the other.
6. Conclusions
The method of comparing signals in shape is proposed. It is assumed that two signal fragments from the same source are similar in waveform, if such a standard prototype is found, from which each of these two fragments may be obtained through the monotonic amplitude transformation conserving the position and order of the local extrema of the signal fragments. The method of estimating the similarity between the waveforms of two signals recorded with an error, when the standard is unknown, is described. It is shown that this method may successfully be used in estimating the lag time of one signal with respect to another and for determine the difference of signals in shape.

Acknowledgments
This work was partially supported by the Russian Foundation for Basic Research projects №№ 17-07-00832 (parts 4-6);18-05-00576 (part 3) and the program of the Presidium of Russian Academy of Science № 56 (parts 1-2).

References
[1] Pyt’ev Yu P 1993 Morphological image analysis Pattern Recognition and Image Analysis (Advances in Mathematical Theory and Applications) 3 19–28
[2] Pyt’ev Yu P 1997 The morphology of color (multispectral) images, Pattern Recognition and Image Analysis (Advances in Mathematical Theory and Applications) 7 467–73
[3] Pyt’ev Yu P 1998 Methods of morphological analysis of color images Pattern Recognition and Image Analysis (Advances in Mathematical Theory and Applications) 8 517–31
[4] Pyt’ev Yu P, Kalinin A V, Loginov E O and Smolovik V V 1998 On the problem of object detection by black-and-white and color morphologies Pattern Recognition and Image Analysis (Advances in Mathematical Theory and Applications) 8 532–6
[5] Pyt’ev Yu P and Zhivotnikov G S 2004 On the methods of possibility theory for morphological image analysis Pattern Recognition and Image Analysis (Advances in Mathematical Theory and Applications) 14 60–71
[6] Zhivotnikov G S, Pyt’ev Yu P and Falomkin I I. 2007 On the filtering algorithm for images Pattern Recognition and Image Analysis (Advances in Mathematical Theory and Applications) 17 408–20
[7] Pyt’ev Yu P and Chulichkov A I 2010 Methods of Morphological Image Analysis (Moscow: Fizmatlit) [in Russian].
[8] Kulichkov S N, Chulichkov A I and Demin D S 2011 Izv. Atmos. Ocean. Phys. 47 154
[9] Kulichkov S N, Tsybulskaya N D, Chunchuzov I P, Gordin V A, Bykov Ph L, Chulichkov A I, Perepelkin V G , Bush G A and Golikova E V 2017 Izv. Atmos. Ocean. Phys. 53 402