Holographic astigmatic particle tracking velocimetry (HAPTV)

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Abstract
The formation of twin images in digital inline holography (DIH) prevents the placement of the focal plane in the center of a sample volume for DIH-based particle tracking velocimetry (DIH-PTV) with a single camera. As a result, it is challenging to apply DIH-PTV for flow measurements in large-scale laboratory facilities or many field applications where it would otherwise be desirable due to the low cost and compact setup. Here we introduce holographic astigmatic PTV (HAPTV) by inserting a cylindrical lens in the optical setup of DIH-PTV, generating distorted holograms. Such distortion is subsequently utilized in a customized reconstruction algorithm to distinguish tracers positioned on different sides of the focal plane which can in turn be placed in the middle of a sample volume. Our HAPTV approach is calibrated under high (1 µm pixel⁻¹) and low (10 µm pixel⁻¹) magnifications with an error standard deviation of 4.2 µm (one particle diameter) and 120.7 µm (~5 times the particle diameter), respectively. We compare the velocity field of a laminar jet flow obtained using HAPTV and conventional PIV to illustrate the accuracy of the technique when applied to practical flow measurement applications. The work demonstrates that HAPTV improves upon the depth of field of conventional astigmatic PTV and enables the implementation of DIH-based PTV for in situ applications.

Keywords: holography, 3D flow measurements, particle tracking velocimetry and astigmatism

(Some figures may appear in colour only in the online journal)
measurement due to their simple setup and minimal calibration requirements [14, 21–24]. Such advantages allow these techniques to be applied underwater and reduce the intrusiveness to the measurement volume, which is crucial for in situ applications.

In defocusing PTV, a multi-hole aperture is commonly used for generating the defocused image [25–27] where the shape variation of the object (i.e. degree of defocusing) as it moves away from the focal plane encodes the depth position, without any other specialized optical components. Objects that are not located on the focal plane produce multiple images on the sensor, one for each pinhole, and the distance between these images encodes the depth of the object. However, due to its multi-image encoding of each particle, the method is limited to low tracer concentration (e.g. 0.12 particles cm\(^{-3}\)) [14] as particle localization becomes challenging as the number of tracers in the sample volume increases. Furthermore, the use of a pinhole aperture significantly reduces the amount of light collected by the sensor [26], demanding the usage of high-power illumination which increases the cost and lowers the reliability of such systems for in situ measurements [28].

Another way to implement defocusing PTV which overcomes the limitation of the traditional defocusing method employs astigmatism, i.e. non-axisymmetric characteristics of the optical system which cause light rays propagating on different planes to focus at different axial locations. A higher degree of astigmatism separates the focal planes by a greater distance, thereby introducing additional distortion. In astigmatic PTV (APTV), astigmatism can be introduced by mounting a cylindrical lens between the imaging lens and the camera sensor [29, 30], tilting the camera by a small angle to the measurement plane [31], or using a spherical wave illumination source offset from the optical axis [32]. Such astigmatic images of spherical particles appear elliptical, with their axes of ellipticity dependent on the distance of the object from each focal plane [33]. By measuring the major and minor axes of the ellipse, the depth of the particle can be uniquely determined [29, 30]. Apart from the high signal-noise ratio relative to conventional defocusing PTV, APTV can handle higher tracer concentrations (e.g. 640 particles cm\(^{-3}\)) [34] as the signal from each object is confined to a single position on the image. However, APTV suffers from several limitations that prevent its adoption for in situ measurements. First, the technique has only been implemented for spherical particles [28] and extending it to non-spherical particles found in natural environments will be challenging. In addition, limited depth of field (~2 mm) [35] and low depth sensitivity have limited the technique primarily to microscopic applications.

A final single-camera approach is holographic PTV which uses the interference between the light scattered from objects and reference light to encode their depth information into holograms. Such encoding is convolutional and compressive, and the depth information can be extracted through a reconstruction process using a diffraction formulation. Therefore, holographic PTV has a higher depth sensitivity and can deal with higher tracer concentrations compared with defocusing approaches because the compressive nature of holography means that the image contains denser information. In particular, holographic PTV using an inline setup (DIH-PTV)—which includes laser, camera, and lens aligned along a single optical axis—requires very low laser power and is highly compact, suitable for in situ measurements [19, 22]. For typical in situ applications, we must place the laser and imaging systems sufficiently far apart to avoid introducing external disturbances (from the enclosure) to the flow measurements. Under such a measurement scenario, it becomes challenging to capture particles located far from the image plane, due to the drop in the signal strength (i.e. the scattered intensity) with distance from the sensor. Such a limitation can be overcome by simply positioning the imaging plane inside the measurement volume. However, the numerical reconstruction of in-line holograms which are usually based on the recorded intensity distributions does not include any phase information. Although the use of phase information for reconstruction is not impossible [36, 37], the focus of these studies is on increasing the accuracy of particles localization rather than differentiating objects on either side of the hologram. This is known as the twin image problem. To tackle the twin image problem, several methods have been proposed for distinguishing the objects on either side of the imaging plane in DIH. For example, the phase shifting method [38–40] provides a solution to reconstruct the complex amplitude of an object by systematically varying the phase of reference light. The recorded objects must remain stationary while the reference phase is varied. Thus, the phase shifting method is only applicable for static or slowly-moving objects. Phase shifting is also mathematically complex, making this approach unsuitable for in situ measurements. Ling et al [41] employ a DIH setup with two synchronized cameras to record holograms at two planes separated by a short distance and determine the side of an object with respect to the focal plane by comparing the object images in the two holograms. However, such a method increases the complexity of the DIH system in terms of the setup and calibration, limiting its applications for in situ measurements.

In this study, we present a new method, termed holographic astigmatic particle tracking velocimetry (HAPTV), which combines the advantages of both APTV and DIH-PTV to solve the twin image problem and acquire 3D flow fields in a sample volume with an extended depth. Section 2 provides a description of the general methodology of HAPTV, followed by the calibration of our method presented in section 3. In section 4, we apply the calibrated HAPTV to the measurement of a jet flow and compare to a PIV measurement. The conclusion of the work is provided in section 5.

2. Methodology

2.1. General principle of HAPTV

Figure 1 presents a schematic of our system where we introduce a planar cylindrical lens between the camera and imaging lens of a conventional digital in-line holographic setup, as a source of astigmatism. We orient the axis of curvature of the cylindrical lens along the horizontal direction, resulting in the focal length along the y axes being altered while the one along x remains the same (i.e. in the absence of the cylindrical lens).
Figure 1. Schematic of the HAPTV optical setup (upper panel) and images of a 25 µm polystyrene particle at various \( z \) distances from the focal plane (lower panel).

Figure 2. The flow chart of the processing procedure to extract 3D positions of particles recorded using holographic astigmatic imaging setup.

This configuration leads to particle fringe patterns appearing elongated when out of focus. Such an elongation encodes the direction of the particle from the image plane, with the orientation changing from horizontal when the particle is on the laser side of the focal plane (\( z < 0 \)) to vertical when the particle is on the camera side (\( z > 0 \)) as illustrated in figure 1. This direction encoding solves the twin image problem associated with DIH.

2.2. Digital reconstruction and 3D localization of objects

In this study, all recorded astigmatic holograms are processed using minor variations on standard holographic processing routines in order to extract the position information, including steps of image enhancement, reconstruction, and centroid calculation. The first step of the algorithm involves the enhancement of the hologram through an ensemble-averaged background subtraction in order to eliminate stationary artifacts as well as increase the signal-to-noise ratio (SNR) of the fringe patterns. Next, the enhanced holograms (figure 2(a)) are numerically reconstructed through convolution with a Rayleigh-Sommerfeld diffraction kernel (also known as the angular spectrum method \([42]\)) resulting in a 3D intensity field as shown in figure 2(b). The volume illustrates the unique signature of a reconstructed astigmatic hologram of a spherical particle, where the object is refocused along the horizontal and vertical directions at two separate planes, with the relative order of the planes dependent on the initial direction of
elongation, as seen in figure 1. To better capture this signature, we divide the centroid calculation step into in-plane localization and depth localization substeps.

2.2.1. In-plane localization. A maximum intensity projection of the 3D intensity field along the longitudinal (z) direction is first calculated to generate a 2D image with all particles appearing as bright crosses with their xy position at the cross centers (figure 2(c)). Next, we employ cross correlation with a cross-shaped template (figure 2(d)), the shape and value of which has been optimized based on the appearance of recorded particles. This template is effective for identifying true particles because ghost particles do not exhibit the same cross shape and thus do not produce peaks in the cross-correlation map. The peak of the cross-correlation map (figure 2(e)) is extracted as an intensity-weighted centroid through a manual thresholding operation as shown in figure 2(f) and used as the xy position of the particle.

2.2.2. Depth localization. Once the particle position in xy is identified, we then determine the depth using an intensity-based focus metric. The sharp edges of the two focal planes are accentuated by applying a Sobel filter to the reconstructed intensity field. The operation is performed separately in both the x and y directions, producing two filtered volumes. Next, we calculate longitudinal (along z axes) profiles of the squared sum of the filtered intensity within two bounding boxes along each of the axes. Each bounding box has a ~4:1 aspect ratio, matching the elongated shape of the particle when in focus. We also include an inverse distance weighting of the intensity along the shorter axes, centered at the middle of the window to further enhance the signal intensity. The peaks of the longitudinal profiles are identified with sub-pixel accuracy using a spline interpolation. We define the depth of the particle using the position of the x peak (i.e. when the object is most focused as a horizontal line), since the focal plane along this direction is insensitive to the presence of the cylindrical lens as demonstrated in section 2.1. Finally, the direction of the particle (positive or negative) is identified by the relative order of the peaks. If the particle focuses along x first then the particle is located on the laser side of the image plane (z < 0). This case is shown in figure 2(g). Conversely, if the particle focuses first along y, the particle is on the camera side of the focal plane. Both cases can be seen in the scans of the single particle included in figure 1. It is worth noting that since we are using the same reconstruction kernel as standard in-line holography, the depth of field that our method can obtain is not limited by astigmatism. Ideally, we can obtain the same valid volume on both sides as in standard in-line holography.

3. Calibration

In order to accurately position particles imaged under the distortion of the cylindrical lens, we require a calibration relating reconstructed position with a ground truth obtained through scanning of a particle field using the HAPTV system. In this section, we present two experimental scans at different magnifications to demonstrate the calibration process and evaluate the accuracy of our method. For both experimental setups shown in figure 3, we use identical illumination systems consisting of a HeNe laser (REO Inc; 12 mW) which passes through a spatial filter (Edmund Optics) to eliminate high frequency noise in the beam profile. The filtered beam is then collimated to a 50 mm diameter Gaussian beam using a pair of lenses. Additionally, we also include a neutral density filter (Newport Inc) to modulate the intensity of the
laser reaching our imaging system. We image the beam using separate imaging lenses and a long focal length cylindrical lens (Thorlabs; \( f = 1000 \text{ mm} \)) onto a CMOS camera (2048 \( \times \) 1088 pixels Flare 2M360-CL) at a rate of 20 frames s\(^{-1}\). The two magnification cases correspond to imaging resolutions of 1 \( \mu \text{m pixel}^{-1} \) using a 5X objective (EO M Plan Apo) and 10 \( \mu \text{m pixel}^{-1} \), achieved with a Nikon imaging lens (Nikkor 105 mm f/2.8).

We record holograms of 4 \( \mu \text{m} \) silver coated hollow glass particles on a glass slide which is positioned at 30 \( \mu \text{m} \) increments using a micrometer stage for the high magnification experiment. Similarly, for the low magnification case, we capture 25 \( \mu \text{m} \) polystyrene particles across a total depth of 60 mm using a motorized translation stage. The use of the motorized translation stage provides a large range of translation at a uniform speed (177 \( \mu \text{m s}^{-1} \)), within which we can specify a constant displacement (1 mm) through sampling the recorded images in time. It should be noted that in order to avoid the clustering of particles, both the hollow glass and polystyrene particles are first suspended in ethanol, then placed on the glass slide and allowed to evaporate completely. The dry particle slide is then mounted in the imaging system and holograms recorded in areas with minimal clustering.

To determine a calibration curve, we first track the particles in the hologram along the \( z \) direction and label them. Then we use 50\% (11 and 17 particles for high and low magnifications, respectively) of the tracked particles, uniformly distributed in our holograms, to fit a linear equation (see equations (1)–(2)) relating the known and reconstructed positions for both magnifications (figures 4(a) and (b)). Note that such sample sizes though small are sufficient to determine the calibration curve accurately as shown in the following analysis. Both cases show a strong linear trend between the positions with slopes near to one which provides support for our depth definition of the particle using the focus position along the horizontal. As particles approach the focal plane (\( z = 0 \)) the paraxial approximation used in the diffraction kernel becomes invalid, preventing the recovery of the particle depth at such distances. One possible way to improve accuracy near \( z = 0 \) is to perform template matching on a stored set of calibration images, a method previously used in traditional defocus PTV [43]. Furthermore, the slopes of the calibration curves change...
with the image magnification, with the lower magnification being closer to unity compared to the other.

For the high magnification case:

$$z_{\text{real}} = 1.07 \times z_{\text{detect}} + 17.91 \text{ (µm)}.$$ (1)

For the low magnification case:

$$z_{\text{real}} = 0.999 \times z_{\text{detect}} - 388.10 \text{ (µm)}.$$ (2)

where $z_{\text{real}}$ represents the ground truth positions and $z_{\text{detect}}$ represents the detected positions. For the high magnification case, the 95% confidence interval on the slope is $1.066 \pm 0.0025$ and the intercept is $17.91 \pm 0.90$. For the low magnification case, the slope confidence interval is $0.9868 \pm 0.0014$ while that of the intercept is $388.1 \pm 25.50$. The uncertainty of intercepts in both magnifications show that our fitting lines have high accuracy with the maximum variation of less than one particle diameter for high and low magnifications.

To validate the accuracy of our calibration curve, we estimate the error between real and detected positions using the remaining 50% of the recorded tracked particles which were not used to generate the fit. In order to eliminate the bias in the initial position for each individual particle from our measurement, we use the single-step displacement ($\Delta z_{\text{detect}}$) between two neighboring recording positions ($\Delta z_{\text{real}}$) as our evaluation metric. The differences between the detected displacement ($\Delta z_{\text{detect}}$) and real ($\Delta z_{\text{real}}$) displacement between two neighboring recording positions are normalized by the particle diameter ($D$) and presented in figures 4(c) and (d) for each magnification. As illustrated, we are able to obtain an average error, represented by the standard deviation of the distribution, of $\sim 1D$ (4.2 µm) for the high magnification case and $\sim 5D$ (120.7 µm) for the low magnification case, respectively.

4. Validation through measurements of 3D jet flow

Finally, we demonstrate our HAPTV technique by capturing a 3D jet flow and validate its accuracy through comparison to particle image velocimetry (PIV). As illustrated in figure 5, the experimental system consists of the flow chamber, the HAPTV setup, and the PIV imaging setup to capture the flow field in a plane within the sample volume of HAPTV. A 2 mm diameter nozzle placed at the bottom of an acrylic water tank ($9 \times 9 \times 40 \text{ cm}^3$) is used to generate a laminar jet flow by injecting water from a syringe pump (Harvard Apparatus 22) at a constant flow rate of 0.25 ml min$^{-1}$ from a 60 ml syringe (Monoject). We use neutrally buoyant 10 µm hollow glass spheres dispersed uniformly in the water as tracers for both experiments and include a 2 h wait between the dispersal and measurement to ensure identical background flows in both cases. In order to ensure the jet has achieved a stable and repeatable flow profile, the syringe pump is started 1 min prior to the start of the recording for both HAPTV and PIV. The flow is captured for a duration of 30 s with each technique. Furthermore, we perform three repetitions of the flow for each technique independently in order to obtain an accurate average velocity field for comparison and avoid crosstalk between signals.

The HAPTV system consists of a green diode laser (Thorlabs CPS532), a beam expander, and a collimation lens, which together produce a 50 mm Gaussian beam to illuminate the sample volume. The motion of the tracers in the jet is imaged onto a CMOS camera (2048 × 1088 pixels Flare 2M360-CL) using a Nikon imaging lens (Nikkor 105 mm f/2.8) and a cylindrical lens ($f = 1000 \text{ mm}$) at a resolution of 5 µm pixel$^{-1}$ recording at 320 frames s$^{-1}$ with an exposure time of 61 µs. To demonstrate our ability to measure the entire flow field
Figure 6. (a) Mean velocity field of vertical velocity ($v$) measured from the PIV and (b) HAPTV. (c) Comparison of mean velocity profiles from the PIV and HAPTV.

with the image plane located at an arbitrary position within the volume, we place the hologram focal plane with an offset of ~6.5 mm from the jet centerline. The PIV setup uses a green DPSS laser (Optoengine MGL-III-532-100), a turning mirror, and a light sheet generator to create a ~1 mm thick laser sheet which covers the $yz$ plane along the center of the jet. The PIV images are sampled by a separate CMOS camera ($7920 \times 6004$ pixels, Flare 48MP) using a Nikon imaging lens (Nikkor 105 mm f/2.8) at a resolution of 6.5 μm pixel$^{-1}$ recording at 30 frames s$^{-1}$ with an exposure time of 1 ms. A beam splitter is placed in the optical path to position the laser sheet, ensuring that the Gaussian beam and laser sheet are coincident with each other and also ensuring that the two setups can capture the flow field at the same sampling region. In addition, we use a calibration target captured with both setups simultaneously as well as an image of the light sheet on the HAPTV camera to align the interrogation domains from both measurements. The image of the target on the two setups allows us to fix the vertical positions ($y$) within the volume to be identical while the image of the light sheet on the HAPTV sensor identifies the lateral ($x$) position on the hologram that corresponds to the PIV measurement location.

We use a multi-pass cross correlation approach using a $64 \times 64$ pixel interrogation window with 50% overlap to calculate the PIV vector fields (figure 6(a)) using PIVlab [44]. For HAPTV, the detected particles are first tracked over time to obtain an unstructured 3D velocity field. This field is then interpolated onto a 3D grid using inverse distance weighting and a grid resolution similar to PIV. This grid is the mean velocity field (figure 6(b)). We perform a separate ground truth calibration in water, since the resolution of the HAPTV measurement is increased for the current experiment relative to the prior low magnification calibration.

The flow profiles indicated in the PIV and HAPTV vector fields illustrate a typical jet flow profile with a peak velocity of 10 mm s$^{-1}$ at the center that decays to zero velocity on either side. To offer a quantitative comparison between the two, we plot longitudinal ($z$) velocity profiles from both at $y = 10$ mm. For HAPTV, the profile is averaged over the thickness of the light sheet. The profiles are symmetric about the centerline even for HAPTV imaged away from the middle of the jet. The maximum difference between the two is ~5.5% of the centerline velocity and the full width at half maxima for the PIV and HAPTV measurements are 9.05 mm and 9.04 mm, respectively. The small difference between two measurements maybe caused by the misalignment of the jet flow and PIV light sheet.

5. Conclusion
In this study, we introduce a novel flow measurement technique, HAPTV, by inserting a cylindrical lens in the setup of DIH-PTV which enables the image plane to be located in the middle of a sample volume while still differentiating the relative directions of particles from the image plane. Based on the holograms generated in this astigmatic system, we utilize a customized reconstruction algorithm with an intensity-based focus metric to distinguish tracers positioned on different sides of the focal plane. We perform a ground truth calibration of the measurement using a linear fit between the detected and true positions at two different magnifications. Our approach is calibrated under high (1 μm pixel$^{-1}$) and low (10 μm pixel$^{-1}$) magnifications with an error standard deviation of 4.2 μm (one particle size) and 120.7 μm (~5 times the particle size), respectively. As a demonstration, our method is implemented for the measurement of a 3D jet flow. The results show a good agreement when compared with the conventional PIV method. Overall, the present work demonstrates the capability of HAPTV to achieve an improved depth of field compared to
the conventional astigmatic PTV with the potential to perform flow measurement in field applications. However, the single template that we used to identify particles may not be optimal in all situations, such as when there is substantial variation in particle size and shape. Possible solutions include using multiple templates for different particles and using a data-driven approach to determine an optimal template suitable for all particles similar to the recent methods developed for 3D particle field reconstruction [45] and particle sizing [46]. The necessity of such implementation will be evaluated in future studies using natural tracer particles.

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