Optical trapping performance of dielectric-metallic patchy particles

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Abstract: We demonstrate a series of simulation experiments examining the optical trapping behavior of composite micro-particles consisting of a small metallic patch on a spherical dielectric bead. A full parameter space of patch shapes, based on current state of the art manufacturing techniques, and optical properties of the metallic film stack is examined. Stable trapping locations and optical trap stiffness of these particles are determined based on the particle design and potential particle design optimizations are discussed. A final test is performed examining the ability to incorporate these composite particles with standard optical trap metrology technologies.

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1. Introduction

Optical trapping (OT) is a prevalent technology in the realm of micro-manipulation. It is routinely used to apply forces on the order of 0.1 to 100 pN to particles spanning a few nanometers all the way up to tens of microns in diameter [1]. Translational force control with OT has been incorporated into a wide variety of applications including micro-robotics [2], micro-scaled manufacturing [3, 4] and single molecule force spectroscopy [5]. Optical trapping experiments typically rely on pure dielectric micro-particles to serve as handles for interfacing with these systems, since the low absorption and relatively high index of refraction of the dielectric material are ideal for generating stable optical traps. As the state of the art advances, researchers often look to incorporate novel anisotropic composite particles within OT experiments. These composite patchy particles can provide additional degrees of control using magnetic actuation for biomedical [6–8], advanced manufacturing [9, 10], or metrology purposes [11–14]. Similar particles have also been implemented, without the use of OT, to enhance imaging capabilities of integrated sensors [15].

Our group has previously described two unique methods of fabricating dielectric-metallic patchy particles for simultaneous optical and magnetic actuation. The first method developed particles with small spherical caps of metallic films, which we have designated dot-Janus parti-

Fig. 1. A simplified ray-optics model showing the marginal rays incident upon two patchy dielectric-metallic particles with an optical trap. a) When the patch surface area is large, some of the rays will strike the surface of the patch, leading to scattering, absorption and non-ideal trapping behavior and eventual particle rejection. b) Smaller patches allow optical momentum to pass through the particle unimpeded, enabling stable OT.
Fig. 2. a) Optical trapping paths for a dielectric and dot-Janus particle from twelve unique starting locations. The dot-Janus particle modeled has a $0.15a$ cap-height, where $a$ is the radius of the trapped particle. The particle is initially located in the xz-plane at the locations marked with black circles. In each test, the patch’s normal was initially oriented parallel with the positive x-axis. b) Shows this orientation with the particle centered at the origin of the coordinate system for reference. Two tests in which the dot-Janus particle failed to converge are omitted for clarity.

cles due to their appearance [16]. The second fabrication method implemented a glancing angle deposition (GLAD) process, leading to a range of patch geometries [17]. These particles contained only a small patches covering less than 20% of the total surface area in every test case, enabling optical trapping on axis much like a pure dielectric, as shown in Fig. 1.

While these dielectric-metallic patchy particles are observed to trap stably on-axis using optical microscopy, some deviations from a standard dielectric-only particle is expected. Examples of potential deviations include non-axial stable trapping locations [18] and non-standard behavior of OT stiffness [19]. For example, Fig. 2 shows how the inclusion of a metallic patch may adversely affect the path of an optically trapped particle. Near the trap center, these variations may be less than the diffraction limit of an optical system, but still be large enough to affect experiments requiring the precision normally associated with OT. The small size of the patch prevents in situ identification and characterization of the patch’s shape parameters for performance optimization experimentation. Additionally, it is not apparent whether the presence of the metallic cap will permit the use of quadrant photodiode (QPD) position sensing for tracking the OT performance of these particles empirically. For each of these reasons, a model of the OT behavior of these dielectric-metallic patchy particles is necessary to evaluate their efficacy as OT handle particles and to identify any limitations or design optimizations that might be inherent with this class of particle.

Custom Matlab based software capable of estimating the optical force and torque on a dielectric-metallic patchy particle was developed to identify the OT performance characteristics of dielectric-metallic patchy particles. Simulation experiments were conducted on both dot-Janus and GLAD particles to determine the location of stable OT positions and trap stiffness across a full parameter space of particle designs. The influence of scattering and absorption from the particles was also examined to demonstrate the limitations of QPD sensing techniques for these classes of particles. The results from these analyses can be used to design patchy particles with OT performance tailored to a specific application while optimizing the magnetic rotational response.
2. Optical force calculation theory

The optical force acting upon a dielectric-metallic particle may be approximated using geometric optics analysis [20]. Such approximations are valid for particles in the Mie optical trapping regime where the radius of the particle, \( a \), is much larger than the wavelength of the trapping laser, \( \lambda \). Detailed explanations of optical force and torque estimation using ray tracing analysis on dielectric particles has previously been examined at length, so only a brief overview is presented here for continuity [21]. In this approach, the electric field is modeled as a collection of rays, each imparted with some optical power proportional to the magnitude of the local optical field. The power distribution was modeled as an ideal Gaussian beam with a wavelength of 1064 nm overfilling the system’s entrance pupil such that 90% of the total optical power passed into the trapping region. The collection of rays were focused by an ideal objective with a 1.4 numerical aperture. The forces and torques produced by OTs can be described by the transfer of optical momentum of each ray. The optical momentum flux for a ray traveling through a material with index of refraction \( n \) is given by the equation \( n P/c \), where \( P \) is the power of the ray and \( c \) is the speed of light in vacuum [22]. The force produced by a single ray at a single surface can be calculated from the optical momentum balance given in Eq. (1). Subscripts \( i \), \( t \), and \( r \) represent the incident, transmitted, and reflected rays associated with the unit vectors \( \hat{u}_i \), \( \hat{u}_t \), \( \hat{u}_r \) respectively.

\[
F_{ray,\text{surface}} = \frac{n_i P_i}{c} \hat{u}_i - \frac{n_t P_t}{c} \hat{u}_t - \frac{n_r P_r}{c} \hat{u}_r
\]  
(1)

The optically generated forces and torques can be calculated by balancing all of the incident optical momentum with the optical momentum exiting the particle. Since a ray will reflect off of the internal surface several times before the power is fully dissipated, the total force produced by the ray must include the momentum transfer from each surface reflection. The effect of the multiple reflections is accounted for by the summation in Eq. (2). Assuming that the particle is located at the origin of the coordinate system, an optically induced torque may also be estimated using Eq. (3).

\[
F_{ray} = \frac{n_i P_i}{c} \hat{u}_i - \frac{n_r P_r}{c} \hat{u}_r - \sum_{m=2}^{\infty} \frac{n_{t,m} P_{t,m}}{c} \hat{u}_{t,m}
\]  
(2)

\[
T_{ray} = P_1 \times \frac{n_i P_i}{c} \hat{u}_i - P_1 \times \frac{n_r P_r}{c} \hat{u}_r - \sum_{m=2}^{\infty} P_m \times \frac{n_{t,m} P_{t,m}}{c} \hat{u}_{t,m}
\]  
(3)

From the optical force and torque equations in Eqs. (2)–(3), the optically induced forces are not dependent upon the size of the particle. The reported results are therefore normalized to a unit bead radius wherever appropriate to provide generality. In order to correctly predict the optical forces, the distribution of power from incident to reflected and refracted rays must be calculated for each ray and surface intersection. The power distribution is a function of the incident angle, optical material properties, and the polarization state of the incoming ray. The program created for this analysis identified whether an incoming ray would strike a metallic patch for any arbitrary particle position, rotation, or patch geometry. The polarization state of the incoming rays with respect to the surface was identified, and the distribution of power from incident to transmitted and reflected rays was calculated from Fresnel’s equations. When the rays intersect the metallic patch, the recursive analysis technique, assuming isotropic multilayers, was employed to capture the effects of multiple reflections between laminates and identify the total reflectance and transmittance [23].

3. Test particle parameterization

A parameter space analysis was performed with both dot-Janus and GLAD dielectric-metallic patchy particles. All test particles are based off of 3 \( \mu \text{m} \) diameter (\( a=1.5\mu \text{m} \)) polystyrene (PS).
beads with evaporated metallic patches suspended in water. This diameter was selected as a realistically sized particle for typical OMT applications. Each metallic patch may be defined using a number of shape parameters to control the surface area and profile of the metallic patch. A dot-Janus particle’s only shape parameter is the cap-height ($h$) of the metallic spherical cap shown in Fig. 3(b). This parameter was varied over a range of $0.1a \leq h \leq 0.5a$ corresponding to total surface area coverage from 5% to 25% of the particle’s total surface area. A GLAD particle may be described by two surface patch parameters: substrate tilt and substrate rotation. Detailed discussion of these parameters may be found in [17]. Briefly, the substrate rotation controls the shape profile of the metallic patch while the substrate tilt controls the overall area of the patch. In these analyses, the substrate tilt was held constant at $82^\circ$ while the rotation was varied from $0^\circ$ to $30^\circ$; yielding a surface area coverage of $10\% \pm 1\%$ and encompasses the entire range of GLAD shape parameters. Examples of typical GLAD shape parameters are shown in Fig. 3(a).

Reflectivity of the metal patch, determined by composition of deposited films, was also included as a test parameter. All test particles contained a 50 nm thick film of Co along with a variable inner and outer film stack to control the reflectivity from both directions. The highly absorptive properties of metallic materials makes the transmission for any film design negligible, therefore only the reflectivity constants are parameterized. Three film stacks (labeled in Fig. 4(a)) were designed to control the reflectivity off of the patch from both the polystyrene (inner) and fluid (outer) surfaces giving a total of nine test film stacks. The film stacks were designed using realistic film combinations that could be reproduced in future tests, if desired. While the reflection coefficients vary considerably at higher incidence angles, these higher angle coefficients are not typically observed within OT experiments. Most incident angles near the stable trapping locations are less than $20^\circ$, as is shown in Fig. 4(b), where the differences in reflectivity due to polarization are less pronounced.

4. Results and discussion

4.1. Dynamic particle tracking

The first OT performance test sought to determine stable trapping locations and orientations of dielectric-metallic patchy particles as a function of the defined particle parameters. A stable location is defined by the position in Cartesian space as well as the rotational orientation of the particle where the net force on the particle is zero and small perturbations produce a restoring force. Given the computational requirements for ray tracing analyses, a direct full factorial analysis of all six positional degrees of freedom across the entire trapping region is computationally unreasonable. Additionally, standard gradient search methods are unsuitable given the discontinuity of the particle’s optical properties around the surface. Instead, a dynamic model was developed to identify stable OT locations. Particles were first placed with some initial location
and orientation in the presence of an optical field. The optical force and torque was calculated according to the previously outlined procedure. These forces, balanced against the viscous drag forces, determined a new testing position and rotation. This dynamic modeling process was continued until the particle converged to a stable location, determined by a root mean square error of the position less than $10^{-6}$, or was expelled from the trapping region. A convergent test would therefore indicate a location and orientation with a local potential minimum that acts as a stable optical trapping position.

A large number of starting locations and orientations were selected in order to test the ability of a particle to locate a stable trapping position and to identify the possibility of multiple stable trapping arrangements. Particles were initially placed at locations on a sphere centered on the focal point with a radius slightly smaller than the radius of the test particle. A full factorial examination of the particle shape parameters, film compositions, polarization orientation, and optical power were then computed for each initial location. In total, 324,900 and 167,580 unique dynamic modeling tests were initiated with 21,322 and 20,017 convergent results for dot-Janus and GLAD particles respectively. The large number of dynamic tracking tests allows the determined output locations to represent the statistical trapping performance of these particles in a Monte Carlo fashion.

The dynamic tracking results indicate that the OT has minimal impact on the rotational alignment of the particles. Figure 5 shows the results of initial azimuthal rotations ($\gamma$) and also describes the coordinate system used for describing the particle rotation. In every test case with an initial polar displacement ($\beta$), the patch acted like an optical sail, catching the ray’s momentum, and was driven out of the trapping region. Particles without initial polar angle displacements did converge to non-zero polar angle orientations, but these were less than 6° in all cases. When particles were given an initial azimuthal angle displacement ($\gamma$) the particles were not rotated, but translated into a new position that oriented their patch normal in plane with and oriented away from the optical axis, as described by Fig. 5(a). The largest observed displacement in the $\gamma$ direction was less than 2°. The location of the trap was not effected by the direction of the

Fig. 4. a) Fresnel power reflection coefficients for all tested film designs as a function of the incident angle. Films were designed to achieve normal incident reflection coefficients of 0.86, 0.69, and 0.52 for rays propagating from the fluid medium and 0.87, 0.74, and 0.61 for rays propagating from the internal particle. b) Histogram of optical power with respect to the incident angle for a dielectric particle located in the trap center versus one at an extreme location for OT modeling. The concentration of optical power at lower incidence, where polarization is less influential to reflectivity constants, enables a parameterized study of film properties to be explored.
Fig. 5. a) Locations of stable optical traps, reduced to their x and y coordinates for dot-Janus particles. In each test, the particle was initially located on the positive half of the xz-plane but was rotated around the z-axis ($\gamma$) by one of five discrete angles. The optical forces did not produce a relative rotation of the particles, but the particle instead translated into a plane rotated an equal distance from the xz-plane. b) Relative Coordinate system of discussed results. All rotations can be defined by subsequent rotations $\alpha$, $\beta$, and $\gamma$.

polarization, as expected since the Fresnel reflection coefficients are similar for incidence angles in the trapping region. Since polarization is not a significant factor of trapping location, the stable position for all of the dynamic modeling tests is rotationally symmetric and the results may be reduced to their radial and axial displacement coordinates.

Stable trapping locations of all converged dot-Janus and GLAD particles are shown in Figs. 6(a)-6(c) and 6(d)-6(f) for dot-Janus and GLAD particles respectively. In each sub-figure, the data points are identical but the colorization is altered to highlight different testing parameters. It is apparent that the location of stable traps does not exist at single stationary positions as it does with pure dielectric particles, but is instead defined as a stable trapping region. At each power level, the trapping region is defined as a nearly linear path beginning at the origin and extending away from the optical axis and above the focal plane. These stable trapping locations above the focal plane are due to momentum transfer from the incident beam into the metallic patch. Given the rotational symmetry described earlier, this path represents a surface of stable trapping positions in the shape of a truncated cone. These surfaces are unique between the different power levels, although the difference is less pronounced at higher power settings indicating an asymptotic limit.

The location of a particle within each truncated cone surface is also dependent upon the surface area of the patch. For dot-Janus particles in Fig. 6, this surface is composed of sub groupings of particles grouped by cap-heights. While particles having cap-heights up to $0.5a$ radii were tested, this procedure did not locate any stable trapping locations for particles with cap-heights larger than $0.22a$. As the size of the cap height is increased, the particle tends to move up and away from the axial focal position.

The GLAD particles also trapped on separate surfaces dictated by the optical power and the substrate rotation parameter, which also defines the total surface coverage. Some deviations from the linear path existed with the GLAD particles due to the non-linearity of the patch. This was most apparent at the lower OT power parameters. Most of the GLAD particles that successfully converged were oriented such that the final rotation around the patch’s normal, $\alpha$, was either $0^\circ$ or $180^\circ$, placing the long axis of the patch parallel to the focal plane. All GLAD particles with this orientation observed a linear power trend similar to dot-Janus particles. At the lowest power setting, some GLAD particles trapped with the long axis oriented approximately
Fig. 6. Radial and axial components of each stable trapping position in the dynamic tracking study for both dot-Janus (top row) and GLAD (bottom row) particles due to the optical power and particle shape parameters. Figures (b) and (e) show subsets of Figs. (a) and (d) to highlight the effect of the optical power at the higher power settings.

45° from the optical focal plane and were observed to trap within the clusters seen at radial displacements 0.015a, 0.035a, and 0.125a. When the power was increased above 5 mW, all of these rotated particles were ejected.

Reflectivity off of the inner film layer also contributed to the regions of stable trapping. Figure 7 shows the same trap locations highlighting the inner film parameter. Inner films that were more reflective, and therefore less absorptive, could generate traps further out along the power-trapping path. Having an inner film with higher reflectivity also allowed particles with larger surface coverage to stably trap since the larger reflectivity minimized the absorption inherent with the larger patches. Surprisingly, no significant correlation between the trapping locations or the ability for a particle to generate a trap was observed with the choice of outer film properties. Our reasoning for this is that at the stable trapping orientation for these small patched particles, none of the rays are incident onto the outer surface of the metallic patch. This result is promising for two reasons. First, if maintaining a stable trap near the optical axis is desired, the inner film stack can be designed in such a way to impose such a constraint or a more reflective film stack may be used to provide additional tolerance to the metallic cap synthesis process. Secondly, the lack of influence on the outer film stack allows the outer surface to be tailored for specific chemical or functional purposes, such as biocompatibility or presenting a desired surface for binding.

In the case of dot-Janus particles, which tested a greater range of surface area coverages, the radial location along the stable trapping surface was heavily influenced by the absorbed power
and rotational alignment. Figure 8(a) shows the measured absorbed power for each converged testing location. When the particles absorb additional power, they are forced away from the optical axis and were rotated so that the scattering force from the absorbed energy balanced the new restoring gradient force. Figure 8(b) shows the stable trapping locations for all dot-Janus tests involving a 0.13μm cap-height with 5 mW of optical power. While this figure shows only one combination of cap size and power level for clarity, similar relationships were found for every other test combination. The amount of absorbed power, and therefore the position of the particle along the trapping region, were linearly related to the rotation of the particle above focal plane indicating the potential to limit radial displacement or absorbed optical energy by controlling the polar angle displacement of the metallic patch.

The range of the optical trapping locations is also important to note. For the lowest optical power, stable trapping locations were observed for radial displacements ranging from 0.0μm to 0.178μm and axial displacements ranging from 0.170μm to 0.488μm. For a 3 μm diameter bead this corresponds to a total potential shift of 267 nm radially and 477 nm axially; a significant source of energy.

Fig. 7. Converged test locations for all tests separated by the inner film reflectivity parameter. Each inner film variety was capable of trapping particles equally at locations with smaller radial displacements. However, when the reflectivity of the inner film was increased, which simultaneously decreases the optical absorption, stable trapping locations were generated at larger radial and axial coordinates along the stable trapping surface.

Fig. 8. (a) Absorbed optical power for dot-Janus particles based on the particle’s location along the truncated cone surface. When particles absorbed additional power, they were pushed up and away from the optical axis causing the particle to trap at a different position. (b) Polar angle rotation at stable trapping locations for one of the dot-Janus test particles. As the particles were pushed further out along the stable trapping surface, the particle’s rotated down to balance the increase in absorbed optical power with the restoring gradient force.
of error for many OT applications. This range can be greatly reduced simply by increasing the optical power. For all tests with 20 mW or more optical power, the range of observed trapping locations was reduced to 0.020\(a\) radially and 0.027\(a\) axially, corresponding to 30 nm and 41 nm for a 3 \(\mu m\) diameter bead. These ranges can be further reduced by careful control of the metallic patch compositions and surface profile.

4.2. Optical trap stiffness

Using the same set of functions for calculating optical force and torque, a second analysis was performed investigating OT stiffness. The stable OT locations determined in the previous section were used as starting locations for each stiffness test. From these positions, the force on the particle was measured at discrete displacements along the axial, radial, and transverse directions. Additionally, the optical torque was measured at discrete rotational displacements around the same orthogonal axes to estimate any rotational stiffness produced by the beam-patch interaction. The difference in force, or torque, along each orthogonal direction was fit to a line in a least squares sense and the slope was used to determine the optical stiffness.

The calculated translational stiffness constants for dot-Janus and GLAD particles are presented in Fig. 9. The stiffness of the OT was observed to increase linearly with increasing optical power. This observation is consistent with OT theory for dielectric particles as well as empirical data for GLAD particle trapping [17]. Variations in the radial and axial stiffnesses occur based on the size of the patch for a tested particle allowing some degree of particle stiffness design. The orientation of polarization with respect to the trapped particle is statistically significant to the translational stiffness in both the radial and transverse directions. However, the difference in translational stiffness observed between the trapping directions was extremely small: 0.46 pN/\(\mu m\) in the most extreme case which worked out to slightly more than 2% of the mean stiffness for the 30 mW power level. Therefore, the variation due to polarization is safely neglected. The effect of the inner film stack on the OT stiffness is omitted in these results because, as described in the previous section, this parameter is confounded with the size of the metallic patch.

No significant rotational stiffness was discovered due to the presence of the optical beam, which is consistent with the lack of rotational motion in the dynamic tracking test and previous empirical results. Given the near incident angles of the rays near the focus position, rotating the particle away from its stable location resulted in optical forces that drove the particle to a new location along the defined stable trapping surface instead of producing a restoring torque.

4.3. Predicted QPD response

Knowing the OT performance for a specifically designed, dielectric-metallic patchy particle, the question remains about how effectively the positions of these particles may be tracked experimentally. One of the most common techniques to monitor position is to collect the scattered light off of the trapped particle and image it onto a quadrant photodiode (QPD). As the trapped particle is displaced from its stable trapping positions, the scattered light is deflected causing more energy to strike one half of the QPD that the other. For standard dielectric particles, the deflection signal is linear with respect to the particle displacement. The QPD detection process was simulated with the same ray tracing techniques discussed previously. The rays reflected or transmitted away from the particle were traced to a reference sensor surface some arbitrary distance along the optical axis. Assuming that the numerical aperture of the QPD imaging system matched the trapping system, an appropriate entrance pupil diameter could be determined for this sensor plane and all rays intersecting outside of this diameter were discarded. The optical power of the remaining rays were binned based on the quadrant of the intersection and the total intersected power was summed. Displacement signals were then calculated in the same manner...
Fig. 9. Calculated optical trap stiffness for tested dielectric-metallic patchy particles. a-b) Dot Janus particles behaved similarly to pure dielectric particles as the size of the metallic patch decreased towards zero. These particles showed a decrease in axial stiffness as the size of the metallic patch increased while the radial stiffness showed a similar increase. The stiffness in the transverse direction remained nearly constant across the range of cap sizes. c-d) GLAD particles exhibited similar decreases and increases to their axial and radial stiffness respectively. These variations are due to the shape of the patch induced by the GLAD manufacturing process.

as a physical QPD system.

Figure 10 shows the typical QPD response for dot-Janus particles of varying metallic patch size. These responses are also typical of GLAD particles. A common DC offset was applied to all signals so that the standard dielectric particle has null signal at zero displacement. Near the trap center, each response behaves linearly and is parallel to the expected response of a pure dielectric particle. As the particle is displaced from its trap center, the interaction of the beam with the metallic patch causes nonlinearities in the response signal. The larger the size of the patch, the smaller the region of linearity. This deviation is far more influential when translated in the radial direction than in the transverse direction. Additionally, particles with larger patches tended to have a positive DC shift near zero displacement compared to a standard dielectric. This bias is not due to increased reflections off of the metallic patch; the increased absorption should cause the total transmitted power to drop, not increase. Instead, this bias is proportional to the radial displacement of the stable trapping position of the larger patched particles compared to dielectrics which trap on axis. Such a bias should not be a concern since a simple DC offset of the signal may be applied to zero the QPD.

Determining whether a dielectric-metallic patch particle is viable for QPD position detection is a matter of several testing parameters including the desired OT stiffness and the expected load on the particle. While no single answer is applicable for all tests, this study can provide some useful guidelines for selecting an appropriate particle. The linear range of the QPD signal
Fig. 10. Predicted responses of a QPD signal for displacements in the radial and transverse directions, centered at the stable trapping position. The y-axis shows the measured power difference between the halves of the QPD in units of mW. A uniform DC bias was added to all signals in order to center the response of a dielectric particle at zero. All signals behave linearly near the center of the trapping region but begin to deviate after some displacement due to the interaction of the optical beam and the metallic patch. The magnitude of this displacement and the linear range are proportional to the size of the patch. Additionally, a DC shift is observed in the linear response region, also proportional to the size of the metallic patch.

Fig. 11. Mean squared error of the measured position of a dot-Janus particle measured using a QPD technique. All four plots show the error as a function of displacement for the same particle at four different power settings. Increasing the optical power, leads to a larger error in the signal away from the trapping center.

is a function of the total optical power of the test. Figure 11 shows the mean square error (MSE) of each test, neglecting DC offsets, with respect to the expected response from a pure dielectric particle. Increasing the power will force the particle into a new stable trapping position which affects ray scattering. Increasing the compliance of the trap, by decreasing the optical power, will result in a greater range of linearity at the cost of decreased position resolution. For dot-Janus test cases, the MSE remained less than 1% of the standard signal within 0.1a, except for cap-heights greater than 0.18a. When the desired testing range is increased to 0.25a, linearity remained than 1% MSE for cap-heights of 0.15a, 0.13a and 0.12a for optical powers of 5 mW, 10 mW, and both 20 and 30 mW respectively.
4.4. Optical heating

The absorbed optical energy measurements discussed in section 4.1 can also be used to gain insight into potential fluid heating effects. Heating the surrounding fluid will not have any effect on the location of the stable optical positions, stiffness of the OT, or on the sensitivity of position sensors based on scattered light. These values and properties were determined solely from the transfer of optical momentum and, assuming negligible thermo-optic material effects, are not influenced by temperature of the surrounding medium. Heating of the surrounding fluid will lead to an increase in magnitude of thermally driven random motion which can affect the performance of an OT due to changes in the trapped particle’s dynamics.

If the absorbed optical power is assumed to radiate uniformly from the particle’s surface, the steady state temperature of the fluid at the particle’s surface can be determined by solving the heat equation in spherical coordinates giving a value of:

\[
T_{\text{particle}} = T_{\infty} + \frac{P}{4\pi a \kappa}
\]

where \(T_{\infty}\) is the bulk temperature of the fluid, \(P\) is the absorbed power and \(\kappa\) is the thermal conductivity of the fluid. Assuming again that the particle has a radius of 1.5 \(\mu\)m and the surrounding fluid is water with thermal conductivity \(\kappa = 0.6 \text{ Wm}^{-1}\text{K}^{-1}\), the surrounding fluid will experience heating of approximately 0.088 K \(\mu\text{W}^{-1}\). The largest absorption for the dot-Janus particles was only 17 \(\mu\text{W}\) (Fig. 8(a)) resulting in a temperature rise of approximately 1.5 K.

The thermally driven motion of the trapped particle is typically modeled as a zero mean white noise positional disturbance source. The variance of the positional disturbance is determined from the equipartition theorem as

\[
\langle \chi^2 \rangle = \frac{k_B T}{k}
\]

where the constants \(k_B\) and \(T\) are Boltzmann’s constant and the fluid temperature respectively. The variable \(k\) is the effective spring constant of the OT and is typically on the order of pN/nm. The effect of an increase in the particle’s random motion will limit the resolution of the particle’s position within an OT. This spatial resolution, \(\delta x\), can be determined from Eq. (5).

\[
\delta x = \sqrt{\frac{k_B T}{k}}
\]

A temperature rise of 1.5 K for the particle described earlier with a moderate OT stiffness of 10 pN \(\mu\text{m}^{-1}\) would lead to an increase of the resolvable position from 6.14 nm to 6.16 nm.

In actuality, the heating profile of the surrounding medium will vary from a uniform radiator described here because the absorbing metal patch is only located on a small portion of the surface. However, these calculations show that the heating effects due to this level of absorption will be relatively negligible and can be easily accounted for through system calibrations. Particles that absorb larger amounts of optical power, and therefore experience greater heating effects, are ejected during the dynamic particle modeling test in section 4.1 and are not examined further in this study.

5. Conclusion

We have demonstrated a series of simulation experiments to characterize the OT response of dielectric-metallic patchy particles. A parameter space analysis was conducted on the shape, surface area, and optical properties of the metallic patch. We showed that these particles optically trap not at a single stationary point, but along a rotationally symmetrical truncated cone
surface. The position within this surface may be adjusted based on the particle parameters and control of the polar angle rotation of the particle. These results indicate that tight control of a particle’s position on the order of 10 nm is still achievable with optical trapping of metallic patchy particles. Effects due to fluid heating from optical absorption of the metal patches was negligible due to the minimal optical-patch interactions needed to generate stable traps. Further studies were performed to characterize the optical trap stiffness of these particles and indicated that the overall stiffness behaved linearly with optical power, as expected, but was also affected by the surface area of the metallic patch. A final study was performed to model the response of a QPD position tracking system when trapping such patchy particles. Interference with the optical beam and the metal patch does occur after a certain displacement, however these particles are expected to display a linear response region around the trap center enabling such particles to be integrated into existing OT systems.

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