The (dual) Meissner effect in SU(2) and SU(3) QCD

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After making an abelian projection in the maximally abelian gauge, we measure the distribution of abelian electric flux and monopole currents around an abelian Wilson loop in SU(2) and SU(3) QCD. The (dual) Meissner effect is observed clearly. The vacua in the confinement phases of SU(2) and SU(3) are both at around the border between type-1 and type-2 (dual) superconductor.

1. Introduction

It is important to understand the mechanism of quark confinement in QCD. We expect that color electric flux is squeezed by the dual Meissner effect due to color magnetic monopole condensation\cite{1,2}. It is well known that such a situation occurs in compact U(1) lattice gauge theory (LGT)\cite{3}. In QCD, 'tHooft conjectured that the role is played by the monopoles in abelian projected QCD which can be regarded as an abelian theory with abelian charges and magnetic monopoles\cite{4}. We choose the maximally abelian (MA) gauge\cite{5} in which diagonal components of all link variables are maximized as much as possible by the gauge transformation. Gauge-fixed link variables are decomposed into a product of two matrices \( \hat{U}(s,\mu) = V(s)U(s,\mu)V^\dagger(s+\mu) = c(s,\mu)u(s,\mu) \), where \( V(s), c(s,\mu) \) and \( u(s,\mu) \) are a gauge-fixing matrix, an off-diagonal and a diagonal matrix, respectively.

Monte Carlo simulations in this gauge have shown interesting results, i.e., abelian dominance\cite{6–8} and interesting monopole behaviors\cite{5,7–9}. These results show that the monopoles play an important role in QCD confinement, but evidence of the monopole condensation in the confinement phase is not yet obtained.

In this report, we show the (dual) Meissner effect occurs in the presence of external charges also in QCD\cite{10,11}. In the continuum, the assumptions of the abelian dominance and the monopole condensation lead us to a dual form of the abelian Ginzburg-Landau (G-L) theory\cite{12}. So we study the G-L type equations between the abelian electric field and the magnetic monopole currents in the presence of a static quark-antiquark pair in SU(2) and SU(3), following the method developed by Singh et al.\cite{10}. Our purpose is to apply the method to SU(3) LGT as well as SU(2) and to investigate scaling behaviors of the penetration length \( \lambda \) and the coherence length \( \xi \).

2. Simulations and measurements

We used the Wilson action and simulations are performed on \( 16^4 \) lattice in SU(2) (\( \beta = 2.4, 2.45, 2.5 \) and 2.6) and on \( 10^4 \) lattice in SU(3) (\( \beta = 5.6, 5.7, 5.8 \) and 5.9). Measurements are done every 30 sweeps and we adopted totally 400 configurations in SU(2) and 500 configurations in SU(3). For thermalization, we discarded initial 1500 sweeps. Abelian link variables \( u(s,\mu) \) are extracted after the gauge-fixing is done in the MA gauge. The average of an observable \( O(u) \) is computed as

\[
< O(u) > = \frac{\text{Tr}^{-S} W(I,I) O(u) \text{Tr}^{-S} W(I,I) }{\text{Tr}^{-S} W(I,I) },
\]

where \( W(I,I) \) is an abelian Wilson loop. We used \( 3 \times 3 \) and \( 5 \times 5 \) in SU(2) and \( 3 \times 3 \) in SU(3). The electric field operator is defined by

\[
E_i(s) = \text{Im}(u_p(s,i,4)),
\]

where \( u_p(s,\mu,\nu) \) is an abelian plaquette operator. Monopole currents are defined following

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We investigate the dual G-L equations:
\[
\vec{E}(r) = \nabla \times \vec{A}(r),
\]
\[
\vec{J}_m(r) = f(r)^2(\vec{A}(r) - \Phi \vec{\nabla} \alpha/(2\pi))/\lambda^2,
\]
\[
-\xi^2 \nabla^2 f(r) + \xi^2(\vec{\nabla} \alpha - 2\pi \vec{A}(r)/\Phi)^2 f(r) - f(r) + f(r)^3 = 0.
\]

The order parameter \(f(r)\) is given by the following approximate solution of the G-L equation
\[
f(r) = \tanh(\nu r/\xi),
\]
where we adopted \(\nu = 1.0\). Then we get the following extended London equation:
\[
\vec{E}(r) - \lambda^2 \nabla \times (\vec{J}_m(r)/f(r)^2) = n\Phi \delta(r),
\]
where \(n\) is an integer and \(\Phi\) is the quantized electric flux at the origin \(r = 0\).

3. Analyses

To reproduce the \(-\vec{\nabla} \times \vec{J}_m\) data, we choose the following function for the azimuthal component of the monopole current:
\[
J_m(r) = a_1(r + a_2 r^2 + a_3 r^3) \exp(-a_4 r),
\]
where \(a_1, a_2, a_3, a_4\) are parameters. The form ensures that \(J_m(r)\) vanishes linearly at \(r = 0\) and exponentially as \(r \to \infty\). Then, substituting the current into Eq.7 we examine the electric field data except the \(r = 0\) point and determine both the penetration and the coherence lengths. The typical data of the curl of the monopole currents and the electric field are plotted in Fig.1 (SU(3)).

If the extreme type-2 dual superconductor is the case, \(f = 1\). Then, the data with \(-\vec{\nabla} \times \vec{J}_m > 0\) and \(E > 0\) can not satisfy Eq.7. Hence, there must be a region where \(f \neq 1\), i.e., we need both \(\lambda\) and \(\xi\) also in the SU(3) case. The values of the both lengths obtained are plotted in Fig.2. The fluxoid \(\Phi\) evaluated from the fluxoid relation is consistent with the net flux through the entire lattice in SU(2) and SU(3).

The G-L parameter \(\kappa = \lambda/\xi\) are shown in Fig.3. The values are close to the borderline of the type-1 and type-2 dual superconductor. It seems that the data of the large Wilson loop show the scaling behaviors.

4. Results

We have a direct evidence of the dual Meissner effect in SU(2) and SU(3) QCD.

1. The color electric field and the magnetic monopole current satisfy the extended dual London equation in SU(2) and SU(3) lattice QCD. The monopole current plays the role of the dual "diamagnetic current" squeezing the electric flux.

2. All \(\lambda\) and \(\xi\) data show that the SU(2) and SU(3) vacua are both close to the borderline of the type-1 and type-2 dual superconductor. So, it is expected that the vortex-vortex interactions are small.

3. It seems that the G-L parameter scales near the borderline in the case of the large Wilson loop. Therefore, scalar and axial-vector glueball masses are nearly degenerate:
\[
m(\text{scalar}) = \sqrt{2}/\xi,
m(\text{axial-vector}) = 1/\lambda.
\]

4. It is interesting to extend this analysis to the study of the extended monopoles of
the type-2, since large monopole loops are seen to be important [13,16].

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REFERENCES

1. G. ’tHooft, *High Energy Physics*, ed. A. Zichichi (Editori Compositori, Bologna, 1975).
2. S. Mandelstam, Phys. Rep. 23C (1976) 245.
3. T. Banks et al., Nucl. Phys. B129 (1977) 493.
4. G. ’t Hoot, Nucl. Phys. B190 (1981) 455.
5. A.S. Kronfeld et al., Phys. Lett. 198B (1987) 516.
6. A.S. Kronfeld et al., Nucl. Phys. B293 (1987) 461.
7. T. Suzuki and I. Yotsuyanagi, Phys. Rev. D42 (1990) 4257.
8. S. Hioki et al., Phys. Lett. 272B (1991) 326.
9. T. Suzuki, Nucl. Phys. B(Proc. Suppl.) 30 (1993) 176.
10. H. Monden et al., Phys. Lett. B294 (1992) 100.
11. P. Cea and L. Cosmai, Nucl. Phys. B(Proc. Suppl.) 30 (1993) 572.
12. T. Suzuki, Prog. Theor. Phys. 81 (1988) 929.
13. S. Kamizawa et al., Nucl. Phys. B389 (1993) 563.
14. T. Suzuki, Prog. Theor. Phys. 84 (1990) 130.
15. T. Suzuki, Prog. Theor. Phys. 81 (1989) 752.
16. H. Monden et al., Phys. Lett. B294 (1992) 100.
17. P. Cea and L. Cosmai, Nucl. Phys. B(Proc. Suppl.) 30 (1993) 568.
18. T. Suzuki, Prog. Theor. Phys. 81 (1989) 229.
19. S. Maedan et al., Prog. Theor. Phys. 84 (1990) 130.
20. T. Suzuki, Prog. Theor. Phys. 84 (1990) 752.
21. S. Maedan et al., Prog. Theor. Phys. 84 (1990) 130.