Proton decay and fermion masses in supersymmetric grand unified theories

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We briefly review the issues of proton decay and fermion masses and mixings in minimal supersymmetric grand unified theories. We argue that minimal SU(5), although tightly constrained by proton decay data, is still not ruled out. However, we outline the advantages of SO(10) unification, in particular in the model with renormalizable see-saw mechanism and its remarkable predictions of (a) exact R-parity at low energies, (b) large atmospheric neutrino angle as a consequence of $b - \tau$ unification and (c) 1-3 leptonic mixing angle close to its upper limit.

1 Introduction

Supersymmetric unification has been the main extension of the Standard Model for about 20 years for three main reasons

1. it provides a natural protection for the weak scale against any large scale as long as supersymmetry breaking is around $\text{TeV}$ or so

2. it predicts correctly the weak mixing angle\textsuperscript{1} (it actually anticipated experiment) and makes proton decay accessible to experiment

3. via the so-called radiative symmetry breaking, it leads naturally to the Higgs mechanism\textsuperscript{2}

Furthermore, in minimal schemes, the enhanced symmetry leads to relations among quark and lepton masses. The most celebrated one is $m_b = m_\tau$ at the unification scale\textsuperscript{3}, which is corroborated by experiment once the running is taken into account. This would have been a great success if not for the fact that the analog relations fail badly for the second and especially for the first family.

Another novel feature of supersymmetric unification is the new source of proton decay through dimension five operators\textsuperscript{4}, generated by the superheavy fermionic fields. This operators originate in Yukawa couplings and are thus intimately correlated with the fermionic mass and mixing angle relations. In this talk, we review in some detail this issue in the context of the minimal SU(5) and SO(10) supersymmetric GUTs. We will argue that the minimal SU(5)

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theory is still in perfect accord with data. However, the non-zero neutrino masses and the almost perfect conservation of R-parity point naturally toward its SO(10) extension. We will briefly recall the essential features of SO(10) grand unification. We emphasize the attractive feature of R-parity remaining an exact symmetry to all orders in perturbation theory in the minimal model with a renormalizable see-saw mechanism. We also discuss some interesting new results on fermion masses such as the deep connection between $b - \tau$ unification and the large atmospheric neutrino mixing. We conclude with SO(10) being a complete and realistic model of both unification and fermion masses and mixings.

2 The minimal supersymmetric SU(5)

We define the minimal SU(5) supersymmetric theory in the conventional manner, that is, with three generations of fermion superfields and a $24$ adjoint ($\Sigma$) and 5 ($\Phi$) and $\bar{5}$ ($\bar{\Phi}$) Higgs superfields. This implies the well-known relations $m_D = m_E$ generation by generation, as we mentioned above. Of course, this must be corrected for the first two generations and in the context of the minimal model this simply implies the existence of higher-dimensional operators cut-off by the Planck scale. Hereafter, by minimal SU(5) we will mean such a theory, where the higher-dimensional operators are taken into account. Notice that this is very similar to the neutrino mass issue in the Standard Model. The nonvanishing neutrino mass does not mean that the Standard Model is ruled out, but simply points out to the existence of higher-dimensional operators.

As mentioned above, supersymmetric GUTs in general, and the SU(5) theory in particular, lead to quite fast proton decay through $d=5$ operators generated by the superheavy colored triplet Higgs supermultiplet ($T$ and $\bar{T}$) with masses $m_T \simeq M_{GUT}$. An important question in recent years was whether the minimal supersymmetric SU(5) theory is already ruled out on this basis, especially after it was found out that the RRRR operators play a crucial role (in the context of SO(10) this was shown before in $^6$), and it was finally argued that this was indeed true$^7$. We will see however that the uncertainty in the masses of the supermultiplets in $\Sigma$, the lack of knowledge of sfermion and fermion masses and mixings, and the presence of higher-dimensional operators is sufficient to keep SU(5) still in accord with experiment.

Let us discuss these points carefully:

2.1 Determination of $M_{GUT}$ and $m_T$

The superpotential for the heavy sector is (up to terms $1/M_{Pl}$)
\[ W = mTrΣ^2 + λTrΣ^3 + a\frac{(TrΣ^2)^2}{M_{Pl}} + b\frac{TrΣ^4}{M_{Pl}}. \]  

(1)

Of course, if \( λ \approx O(1) \), we ignore higher-dimensional terms. However, in supersymmetry \( λ \) is a Yukawa-type coupling, i.e. self-renormalizable. For small \( λ (λ \ll M_{GUT}/M_{Pl}) \), the opposite becomes true and \( a \) and \( b \) terms dominate. In this case, it is a simple exercise to show that

\[ m_3 = 4m_8, \]

(2)

where \( m_3 \) and \( m_8 \) are the masses of the weak triplet and color octet in \( Σ \). In the renormalizable case \( m_3 = m_8 \). The RGE at one loop for the gauge couplings are readily solved to give:

\[ m_T = m_T^0 \left( \frac{m_3}{m_8} \right)^{5/2}, \]

(3)

\[ M_{GUT} = M_{GUT}^0 \left( \frac{M_{GUT}^3}{2m_8} \right)^{1/2}. \]

(4)

where the superscript \(^0\) denotes the values in the case \( m_3 = m_8 \). Since when (2) is valid, \( m_8 \approx M_{GUT}^3/M_{Pl} \), we can also write

\[ M_{GUT} \approx \left[ \left( \frac{M_{GUT}^0}{M_{Pl}} \right)^3 \right]^{1/4}. \]

(5)

From (2) we get

\[ m_T = 32m_T^0, \quad M_{GUT} \approx 10M_{GUT}^0. \]

(6)

Now, \( M_{GUT}^0 \approx 10^{16} \) GeV and it was shown last year\(^7\) that \( m_T > 7 \times 10^{16} \) GeV is sufficiently large to be in accord with the newest data on proton decay. On the other hand, since\(^7\)

\[ m_T^0 < 3 \times 10^{15} \text{GeV}, \]

(7)

from (6) we see that \( m_3 = 4m_8 \) is enough to save the theory. Obviously, an improvement of the measurement of \( τ_p \) is badly needed. It is noteworthy that in this case the usual \( d = 6 \) proton decay becomes out of reach: \( τ_p(d = 6) > 10^{38} \) yrs.
2.2 Higher dimensional operators and fermion masses and mixings

In the minimal SU(5) theory at the renormalizable level we have the Yukawa coupling relations at $M_{GUT}$:

$$Y_U = Y_U^T, \quad Y_E = Y_D,$$

where in the supersymmetric standard model language the Yukawa sector can be written as

$$W_Y = HQ^TY_U u + H Q^T Y_D d + He^T Y_L e,$$

$$+ \frac{1}{2} TQ^T AQ + Tu^T Be^T + T Q^T \bar{c} L + T u^T D d.$$

(9)

Also, in the minimal renormalizable model (at $M_{GUT}$)

$$A = B = Y_U, \quad C = D = Y_D = Y_E.$$

(10)

The fact that $A = B = Y_U, C = Y_E, D = Y_D$, is simply a statement of SU(5) symmetry. On the other hand $Y_U = Y_U^T$ and $Y_D = Y_E$ result from the SU(4)$_c$ Pati-Salam (PS) like symmetry left unbroken by $\langle H \rangle$ and $\langle H \rangle$. Under this symmetry $d^c \leftrightarrow e, u \leftrightarrow u^c, d \leftrightarrow e^c$. It is the above relations that cause the problem, since the color triplet couplings are then well-defined.

Of course, this symmetry is broken by $\langle \Sigma^a \rangle \neq \langle \Sigma^4 \rangle$, where $a = 1, 2, 3$; this becomes relevant when we include higher dimensional operators suppressed by $\langle \Sigma \rangle/M_{Pl}$, such as

$$\frac{1}{M_{Pl}} \Phi \Sigma 10_f 5_f$$

(11)

where $10_f$ and $5_f$ stand for the fermion multiplets. Since there are now neither SU(5) nor SU(4)$_c$ (PS) symmetries, it is a simple exercise to show that the fermion mass relations now become consistent. For a careful study of proton decay in this context see $^9, ^{10}$, where it is shown that this by itself is enough to save the theory.

2.3 Sfermion and fermion masses and mixings and proton lifetime

As we said, d=5 proton decay is generated through Yukawa couplings and thus fermion and sfermion masses and mixings play an important role. We shall not discuss this in detail here, since we know nothing about the sfermion properties; this issue belongs to the realm of supersymmetry breaking and is orthogonal to grand unification. It is highly model-dependent, however the constraints from FCNC can be useful in restricting the proton decay predictions. It has been discussed extensively in $^9, ^{11}$ and we refer the reader to these works.

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In short, the minimal SU(5) supersymmetric theory, although in accord with experiment is highly constrained and thus potentially falsifiable. It is the best, most predictable theory of proton decay, and in our opinion care should be taken before we can proclaim it dead. Certainly, an order of magnitude or two of improvement in proton lifetime measurements is more than welcome.

3 Supersymmetric SO(10) theory

SO(10) grand unified theory is an appealing candidate for the unification of quarks and leptons (family by family) and their interactions for a number of reasons:

1. a family of fermions, included the right handed neutrino, fits into a single spinorial representation

2. it offers a natural explanation of the smallness of neutrino mass through the see-saw mechanism\textsuperscript{12}

3. as any theory of see-saw mechanism, it incorporates leptogenesis, a simple and appealing mechanism for generating the baryon asymmetry of the universe\textsuperscript{13}

4. left-right symmetry in the form of charge conjugation is a finite gauge transformation, which allows for spontaneous parity violation in the context of Left-Right models\textsuperscript{8,14}

5. it contains the Pati-Salam\textsuperscript{8} subgroup which is the prototype of quark-lepton unification; it is this symmetry that leads to the above discussed relation $m_D = m_E$.

6. matter parity, which is equivalent to R parity, is also a finite gauge transformation; furthermore, it remains exact after all the symmetry breaking has taken place\textsuperscript{15,16}

7. in the minimal renormalizable see-saw scenario, it can be shown that SO(10) is a realistic theory of fermion masses and mixings; it correlates $b - \tau$ unification with a large atmospheric neutrino mixing\textsuperscript{19} (see also\textsuperscript{20}) and it predicts the element 1 - 3 of the leptonic mixing matrix to be close to the experimental limit\textsuperscript{21}

We will now discuss the last two issues in some detail.
3.1 SO(10) and R-parity

It is convenient to work with matter parity defined as \( M = (-1)^{3(B-L)} \) instead of its equivalent analog R-parity, \( R = M(-1)^{2S} \), since \( M \) commutes with supersymmetry. In SO(10), under \( M \)

\[
16 \xrightarrow{M} -16 \quad , \quad 10 \xrightarrow{M} 10 ,
\]

which is equal to \( C^2 \), where \( C \) is the center of SO(10). Of course, the crucial thing is the gauging of \( B-L \). Assume next that one utilizes \( 126 \)-dimensional representation in order to break \( B-L \) symmetry and give a mass to the right-handed neutrino, just as in the original proposal. This allows us to stick to the renormalizable theory and simultaneously to keep R-parity intact. The alternative, to use a \( 16 \)-dimensional multiplet in order to break \( B-L \), breaks R-parity at the GUT scale, and the theory must thus be augmented by extra discrete symmetries in order to keep the proton stable. This would take us beyond the SO(10) theory which we pursue here.

This however is not sufficient. Both right-handed and left-handed sneutrino fields can in principle get nonvanishing VEVs and break R-parity. As far as the right-handed sneutrino is concerned, it is a simple exercise to show that the group structure does not allow it. Since its mass is equal to the mass of the right-handed neutrino, and thus very large (close to GUT scale), this fact survives a tiny supersymmetry breaking.

The situation with the left-handed neutrino is far more complex. It belongs to the light sector of the theory and a priori it could be tachyonic to start with, and thus get a VEV. To see that this does not happen requires a subtle interplay between cosmology, phenomenology and the decoupling theorem. First of all, recall that the left-handed sneutrino cannot be a tachyon in the MSSM with R-parity. This we know from the Z-decay width; namely, the opposite would result in the existence of a Goldstone boson, the Majoron \( J \), and the Z-boson would decay into a Majoron and its real counterpart \( R_J \). The mass of \( R_J \) is of the order of the sneutrino VEV, and this VEV must be small in order for the Majoron not to get overproduced in stars and/or for the induced neutrino mass to be small enough. Therefore the left-handed sneutrino VEV would produce a Z decay width incompatible with observations.

A simple application of the decoupling theorem ensures that the same happens in SO(10) theory. In this case a would-have-been Majoron of course gets a mass, but this mass must vanish in the limit of large \( M_R \), the scale of \( B-L \) breaking. More precisely, one shows that

\[
m_J \approx \frac{m^2_{3/2}}{M_R} \quad (13)
\]
where $m_{3/2}$ is the scale of low-energy supersymmetry breaking. Obviously $m_\ell \ll M_Z$, and the above reasoning follows through. Remarkably enough, R-parity can be not broken at all in SO(10), independently of the mechanism of supersymmetry breaking.\textsuperscript{16}

This important fact holds true for any renormalizable theory of the supersymmetric see-saw mechanism based on spontaneous breaking of $B - L$.\textsuperscript{15,16,25} This not only guarantees a stable enough proton, but also a stable LSP (lightest supersymmetric particle), probably the most natural candidate for the dark matter of the Universe.

3.2 SO(10) and fermion masses and mixings

The minimal SO(10) theory, by definition, contains a single 10-dimensional Higgs supermultiplet coupled to fermions. At first glance, in the usual manner, this should imply $m_D = m_E$ as stressed repeatedly. Namely, the Higgs that takes the VEV is a bi-doublet $(2,2,1)$ under the Pati-Salam group $SU(2)_L \times SU(2)_R \times SU(4)_c$. However, the $126$ representation needed for the see-saw mechanism contains also a $(2,2,15)$ field, and in general this field gets a VEV.\textsuperscript{26} In the supersymmetric theory, this amounts to using a $210$ Higgs multiplet at the GUT scale. This has a number of interesting features and should be pursued and studied more carefully.

For us, it is sufficient to have a $(2,2,15)$ with a VEV. This simple fact, surprisingly enough, renders the theory completely realistic as we shall see. On top of that it offers a natural connection between the large $\mu - \tau$ mixing and $b - \tau$ unification. This provides a badly needed answer as to why the small quark mixing angle(s) should be accompanied by large leptonic mixing(s). Let us see how this comes about.

First of all, let us recall an important fact in the SO(10) see-saw mechanism. The Yukawa couplings are given by

$$\mathcal{L}_Y = 10_H \psi Y_{10} \psi + 126_H \psi Y_{126} \psi ,$$

where $\psi$ stands for the 16 dimensional spinors which incorporate a family of fermions, and $Y_{10}$ and $Y_{126}$ are the Yukawa coupling matrices in generation space.

From $126_H = (3,1,10) + (1,3,\overline{10}) + (2,2,15) + (1,1,6)$ one has

$$M_{\nu R} = Y_{126} \langle (1,3,\overline{10})_{126} \rangle ,$$

where $\langle (1,3,\overline{10})_{126} \rangle = M_{R}$, the scale of $SU(2)_R$ gauge symmetry breaking.
It can be shown that, after the SU(2)×U(1) breaking through \( \langle 10_H \rangle = \langle (2, 2, 1) \rangle \approx M_W \), the \((3, 1, 10)\) multiplet from 126_10 gets a small vev \( \langle (2, 2, 15) \rangle \approx M_W \),

\[
\langle (3, 1, 10)_{126} \rangle \propto \frac{M^2_{W}}{M_P},
\]

where \( M_P \) is the scale of the breakdown of parity. In turn, neutrinos pick up small masses

\[
M_{\nu_L} = Y_{126} \langle (3, 1, 10)_{126} \rangle + m_D^T M_D^{-1} m_D ,
\]

where \( m_D \) is the neutrino Dirac mass matrix. It is often assumed, for no reason whatsoever, that the second term dominates. This is the so-called type I (or canonical) see-saw\(^{17}\). In what follows we explore the opposite case, type II (or non-canonical) see-saw. After all, it does not involve Dirac mass terms and so there is no reason a priori in this case to expect quark-lepton analogy of mixing angles. In this sense the type II see-saw is physically more appealing. More than that, we will show that the large leptonic mixing fits perfectly with the small quark mixing, as long as \( m_b = m_\tau \).

To see this, notice that fermion masses take the following form

\[
M_U = Y_{10} v_{10}^u + Y_{126} v_{126}^u ,
\]

\[
M_D = Y_{10} v_{10}^d + Y_{126} v_{126}^d ,
\]

\[
M_E = Y_{10} v_{10}^d - 3 Y_{126} v_{126}^d ,
\]

\[
M_N = Y_{126} \langle (3, 1, 10)_{126} \rangle ,
\]

where \( U, D, E, N \) stand for up quark, down quark, charged lepton and neutrino, respectively, while \( v_{10}^u \) and \( v_{126}^u \) are the two vevs of \((2, 2, 1)\) in \(10_H\) and \((2, 2, 15)\) in \(126_H\), and the last formula is the assumption of the type II see-saw. The result is surprisingly simple. Notice that \(^{18}\)

\[
M_N \propto Y_{126} \propto M_D - M_E .
\]

Now, let us study the 2nd and 3rd generations, and work in the basis of \( M_E \) diagonal. The puzzle then is: why a small mixing in \( M_D \) corresponds to a large mixing in \( M_N \)? For simplicity take the mixing in \( M_D \) to vanish, \( \theta_D = 0 \), and ignore the second generation masses, i.e. take \( m_s = m_\mu = 0 \). Then

\[
M_N \propto \begin{pmatrix}
0 & 0 \\
0 & m_b - m_\tau
\end{pmatrix} .
\]

Obviously, unless \( m_b = m_\tau \), neutrino mixing vanishes. Thus, large mixing in \( M_N \) (the physical leptonic mixing in the above basis) is deeply connected with
the $b - \tau$ unification. Notice that we have done no model building whatsoever; we only assumed a renormalizable SO(10) theory and the type II seesaw.

Of course, this is only very qualitative; for a more careful analysis, one needs to switch on $m_\mu$, $m_s$ and the mixings. This has been done in detail in $^{19}$, and we refer the reader there, suffices it to say that the precise computations confirm the above qualitative analysis.

This is all nice, you may say, but what about three generations? The analysis has just been performed $^{21}$, and it will be reported in the talk by Rabi Mohapatra in this conference. We only cite here the crucial prediction of the theory: the 1-3 leptonic mixing is as large as the experiments allows, making the theory all the more interesting.

4 Conclusions

In these talk, we have focused on some of the central issues in supersymmetric grand unification: the predictions for proton decay and its natural connection with fermion masses and mixings. We started off by showing that the minimal supersymmetric SU(5) theory, although still alive, is tightly constrained and can and should be tested by improved proton decay experiments.

To us, however, SO(10) appears a more complete and more appealing theory. First of all, it succeeds where the MSSM and SU(5) fail, that is, in determining the low energy effective theory. We now know that in the context of SO(10), at least with a renormalizable see-saw mechanism, R-parity can never be broken and neutralinos ought to be some or all of the dark matter of the universe. It is reassuring that the same theory provides a complete and consistent description of all fermion masses and mixings. Most important, if you stick to the type II see-saw the $b - \tau$ unification says simply and clearly that the small quark $u - b$ mixing must be accompanied by a large atmospheric neutrino mixing. Last but not least, it seems that the 1-3 neutrino mixing will provide a crucial test of the theory in the near future.

In spite of the apparent success, a lot remains to be done. The most urgent and most important task is a careful study of proton decay. Namely, the possible existence of intermediate scales in SO(10) lowers the unification scale, and thus the proton lifetime. This is more than welcome for $d=6$ operators mediated proton decay, since it makes it accessible to experiment. However dimension 5 operators run the risk of leading to a too fast decay. We have seen that in SU(5) they are safe, but only marginally.
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