Stress Intensity Factors of Interfacial Crack with Arbitrary Crack Tractions

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Abstract. Interfacial crack is much more complicated than a crack in homogeneous material. The crack is always in mixed-mode even for symmetrical geometry and symmetrical load, and the stress intensity factor has a complex value, which poses great challenges to numerical modelling. In some cases, there is surface traction on the crack faces, which can be crucial to the stability of the crack. In the presented research, a procedure is brought forward for analysis of interfacial crack with arbitrary surface traction based on the Scaled boundary finite element method (SBFEM). The surface traction on the crack is firstly divided into two parts, one in the direction normal to the crack face and the other in the tangential direction. Then each part is further divided into a series of power function of the radial coordinate. The solution for each can be solved semi-analytically by the SBFEM and is then superimposed to get the total solution. With this procedure, the stress and displacement are solved analytically in the radial direction, and the stress singularity at crack tip is obtained with high precision without refined mesh. Both anisotropic and isotropic materials can be considered. For validation of the procedure, a plate containing an interfacial crack and assuming different material properties and different surface traction is investigated. Sensitivity analysis is also performed concerning the plate geometry and material properties. Finally the model is applied to solving the stress intensity factors of an interfacial crack of a gravity dam filled with water.

1. Introduction

Interfaces between different materials are common in engineering structures. Taking a concrete dam constructed on rock foundation as an example, the interface between the concrete and the rock is a weak link of the dam-foundation system and the dam heel is prone to cracking. Once water comes into the crack, it exerts high water pressure on the crack faces, which further promotes the crack propagation and decreases the stability of the dam. Engineering structures facing similar problems also include wharfs and underground geological repositories for nuclear waste storage built on rock foundations. And with the rapid development of science and technology, problems involving interfaces have become common, both on the macroscopic level and microscopic level. Since the interfaces, no matter introduced by welding or bonding or other methods, are the weak links in the system and influence the performance of the whole system, the study on interfacial structure has gained more and more popularity.

As it’s known that the interfacial cracks have peculiar features as compared to the cracks in homogeneous materials. Williams[1] analysed the singular stress field of interfacial cracks and found that the singularity index is \((0.5 \pm i\varepsilon)\), which is a complex value. This results in the oscillatory...
singularity of stress near the crack tip and overlapping of crack faces. In addition to that, the mode I fracture and mode II fracture are coupled, which means that even under symmetric loading, the crack is still a mixed mode one. These features of interfacial cracks expose great challenges to numerical models since standard Finite element method (FEM) [2][3], Boundary element method (BEM) and Extended finite element method (XFEM) all fall short due to the employment of smooth interpolation functions unless very fine mesh is used [4]. To avoid local mesh refinement at the crack tip, special singular finite elements or local enrichment functions that incorporate the asymptotic expansion at a bi-material crack formulated have to be used [5][6][7][8][9].

The surface tractions on the crack faces have crucial effect on the stability of the interfacial crack. In this case, the singular stress field and stress intensity factors are changed as compared with the case of free crack faces, and there is only limited study devoted to this topic. Among them, Hu et al [10] presented an analytical singular finite element based on the symplectic space. Liu et al [11] employed the Scaled boundary finite element method to investigate the singular stress field of the crack subjected to a certain type of load on the crack faces. Tu et al studied the crack with uniform load on the crack faces based on the Boundary element method. It’s worth noting that the distribution and direction of the loads considered in the research mentioned above are relatively simple.

The Scaled boundary finite element method (SBFEM), a semi-analytical method, is a highly competitive candidate for analysis of fracture problems [12][13][14]. It can be employed to compute the singular stress field at the interface of multiple materials. Various loads can be considered including temperature change and dynamic loads. Material nonlinearities can also be modelled. In the presented research, by employing the SBFEM and based on a decomposition of the load on the crack faces and linear superimposition of their effect, a model is brought forward to study the interfacial crack subjected to arbitrary surface traction on the crack faces. The model is verified by the cracking of a plate assuming isotropic material and orthotropic material. Based on that, a parametric analysis is carried out. Finally, the application to a gravity dam containing a preset crack at the dam heel is presented.

2. Fundamentals of the Scaled boundary finite element method

Detailed derivation of the Scaled boundary finite element method (SBFEM) can be found in the literature [15][16][17]. In the presented paper only the details concerning the treatment of arbitrary surface tractions on the crack faces and analysis of the singular stress field are presented. Shown in Fig.1 is a plate with a V-notch analyzed by the SBFEM. O is the crack tip and also the scaling centre for the analysis. A radial coordinate \( \xi (0 \leq \xi \leq 1) \) and a circumferential coordinate \( \eta (0 \leq \eta \leq 1) \) are defined. The boundary of the plate is discretized by one-dimensional element, and \( O - \xi - \eta \) forms the Scaled boundary finite element coordinate system.

![Figure 1. A plate modelled by the SBFEM](image-url)
The Scaled boundary finite element coordinate of a point whose coordinate in the Cartesian coordinate system is \((x,y)\) can be expressed as

\[
\begin{align*}
\xi(x,\eta) &= \xi[N(\eta)] x \\
\eta(x,\eta) &= \eta[N(\eta)] y
\end{align*}
\]

where \(\{N(\eta)\}\) is the shape function. \(\{u(\xi)\}\) denotes the nodal displacement function, so \(\{u\} = \{u(\xi = 1)\}\) represents the displacement of nodes at the boundary. The displacement of any point in the plate can be expressed as

\[
\{u(\xi,\eta)\} = [N(\eta)] \{u(\xi)\}
\]

And the stress is

\[
\{\sigma(\xi,\eta)\} = [D][\{B(\eta)\][u(\xi)\}_{\xi_1} + \frac{1}{\xi}[B^2(\eta)] [u(\xi)]\]
\]

where \([D]\) is the elasticity matrix of the material, \([B(\eta)\) and \([B^2(\eta)\) are the strain displacement matrices[15].

The fundamental equation of the SBFEM with the displacement function as the unknown variables is expressed as[15]

\[
\left[\begin{array}{c}
\left[E^0\right]\xi^2 \{u(\xi)\}_{\xi_1} + \left[E^1\right] - \left[E^2\right]\end{array}\right] \xi \{u(\xi)\}_{\xi_1} - \left[E^2\right]\{u(\xi)\} + \{F(\xi)\} = 0
\]

in which \([E^1]\), \([E^2]\) and \([E^3]\) are coefficient matrices[15], which are dependent on the geometry and material properties of the domain only. The matrices are assembled to form that corresponding to the whole plate in a similar way as in the Finite element method. \(\{F(\xi)\}\) is the vector of external loads, including those acting on the crack faces.

For the loads acting on the crack faces, the loads are approximated as the sum of a series of power functions as

\[
\{F(\xi)\} = \sum_{i=1}^{\mu} \{F_i\}
\]

\[
t_i = i + 1 + \mu (1\cdots M)
\]

where \(\mu\) has a very small value and 0.0001 is used in the presented research. It’s to be noted that both the surface traction normal and tangential to the crack face can be treated in the similar way.

The internal nodal load in the radial direction can be expressed as[17]

\[
\{q(\xi)\} = \left[E^0\right] \xi \{u(\xi)\}_{\xi_1} + \left[E^1\right] \{u(\xi)\}
\]

Eqs. (4) and (7) can be rewritten as a first-order ordinary differential equation as

\[
\begin{align*}
\xi \{u(\xi)\}_{\xi_1} &= \left(-[Z]\right) \{u(\xi)\} - \left\{0\right\} \\
\xi \{q(\xi)\}_{\xi} &= \left(-[Z]\right) \{q(\xi)\} - \left\{F(\xi)\right\}
\end{align*}
\]

Here \([Z]\) is a Hamiltonian matrix[17] whose eigenvalues are denoted as \(\lambda_i\) and \(-\lambda_i\). Eq. (8) is solved through a block-diagonal Schur decomposition of the matrix \([Z]\) as[17]

\[
[Z][\Psi] = [\Psi][S]
\]

The nodal displacement mode corresponding to the load on the crack faces is

\[
\{\psi_i\} = \left[t + 1\right]\left[E^0\right] + \left(t + 1\right)\left[E^1\right] - \left[E^2\right]^{-1} \{-F_i\}
\]

with the equivalent nodal force correspondingly.
\{q_i\} = \left[(t+1)[E^0] \right] + \left[E^1\right]^T \{\psi_i\}

(11)

So the displacement of the domain is obtained as

\{u(\xi,\eta)\} = \left[N(\eta)\right] \sum_{i=1}^{n} \xi_{i+1} \{\psi_i\} + \sum_{i=1}^{n} c_i \xi^{i-1} \{\psi_i\}

(12)

For given integration constants \{c\}, the displacement of nodes on the boundary \{u_b\} = \{u(\xi = 1)\} is

\{u_b\} = \{\psi_i\} + \{\Psi\} \{c\}

(13)

And the corresponding equivalent nodal load vector is

\{P\} = \{q_i\} + \{Q\} \{c\}

(14)

From Eq.(13) the integration constants can be solved

\{c\} = \{\Psi\}^T (\{u_b\} - \{\psi_i\})

(15)

By substituting Eq. (15) into (14) the following relation can be obtained

\[K]\{u_b\} = \{P\} - \{q_i\} + \{K\} \{\psi_i\}

(16)

with \([K]\) denoting the stiffness matrix of the domain. By using the boundary condition, the nodal displacement \{u_b\} can be solved from Eq.(16), which is then substituted into Eq.(15) to obtain the integration constants \{c\}. The displacement of any point in the domain is solved from Eq. (12), which is then substituted into Eq. (3) and the stress field can be obtained as follows

\{\sigma(\xi,\eta)\} = \left[D\sum_{i=1}^{n} c_i \xi^{i-1}\right] + \left[B^1(\eta)\right] + \left[B^2(\eta)\right] \{\psi_i\} + [D] \sum_{i=1}^{n} c_i \xi^{i-1} \left[-\lambda_i \left[B^1(\eta)\right] + \left[B^2(\eta)\right]\right] \{\psi_i\}

(17)

The equation above can be rewritten as

\sigma(\xi,\eta) = \sum_{i=1}^{n+m} c_i \xi^{i-1} \psi_{\alpha i}(\eta)

(18)

with

\lambda_i = -\lambda_i \quad i = 1, \ldots, n;

\lambda_i = t_i + 1 \quad i = n+1, \ldots, n+m

(19)

\psi_{\alpha i}(\eta) represents the stress modes and can be expressed as

\begin{bmatrix}
\psi_{\alpha x}(\eta) \\
\psi_{\alpha y}(\eta) \\
\end{bmatrix}
= \begin{bmatrix}
D \left[\lambda_i \left[B^1(\eta)\right] + \left[B^2(\eta)\right]\right]
\end{bmatrix}

(20)

3. Computation of stress intensity factors

The stress intensity factors are defined through the singular stress \(\sigma_{\alpha 0}^{(i)}(r,\theta)\) and \(\sigma_{\alpha 0}^{(i)}(r,\theta)\) in the polar coordinate system. For the case of interfacial cracks, which is characterized by oscillatory singularity, the singularity index is a pair of conjugate complex numbers \(0.5 \pm i\varepsilon\), where \(\varepsilon\) is the oscillatory index depending on the mismatch of the two adjoining materials.

\varepsilon = \frac{1}{2\pi} \ln \left(\frac{\kappa_{1}/\mu_1 + 1/\mu_2}{\kappa_{2}/\mu_2 + 1/\mu_1}\right)

(21)

where \(\mu_i\) is the shear modulus

\kappa_i = \begin{cases}
3 - 4v_i & \text{plane strain} \\
(3-v_i)/(1+v_i) & \text{plane stress}
\end{cases}

(22)
For a crack lying between two distinct isotropic materials, the stress intensity factors are defined in
\[ \sigma_{\infty}^{(i)} (\hat{r},0) + i r_{\infty}^{(i)} (\hat{r},0) = \frac{1}{2\pi\hat{r}} \left( \hat{r} \right)^{\nu} (K_1 + iK_\eta) \]  
(23)
where \( L \) is a characteristic length. Eq.(23) can be written in a matrix form as
\[ \begin{bmatrix} \sigma_{\infty}^{(i)} (\hat{r},0) \\ r_{\infty}^{(i)} (\hat{r},0) \end{bmatrix} = \frac{1}{\sqrt{2\pi\hat{r}}} \begin{bmatrix} c(\hat{r}) & -s(\hat{r}) \\ s(\hat{r}) & c(\hat{r}) \end{bmatrix} \begin{bmatrix} K_1 \\ K_\eta \end{bmatrix} \]  
(24)
with
\[ c(\hat{r}) = \cos (\varepsilon \ln(\hat{r}/L)); \quad s(\hat{r}) = \sin (\varepsilon \ln(\hat{r}/L)) \]  
(25)
For a crack lying between two distinct anisotropic materials, the stress intensity factor is defined in
\[ \begin{bmatrix} \sigma_{\infty}^{(i)} (\hat{r},0) \\ r_{\infty}^{(i)} (\hat{r},0) \end{bmatrix} = \frac{1}{\sqrt{2\pi\hat{r}}} \begin{bmatrix} W_1 & W_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c(\hat{r}) - s(\hat{r}) \\ s(\hat{r}) & c(\hat{r}) \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \]  
(26)
in which \( W_1, W_2 \) can be computed from the elastic constants of the orthotropic materials.[18] The generalized stress intensity factors can be written as[18]
\[ \{ \psi^{(i)} (\theta) \} = \sqrt{2\pi L} \{ \psi^{(i)} (\theta) \} \{ c^{(i)} \} \]  
(27)
where \( \{ K(\theta) \} = [K_1 (\theta), K_\eta (\theta)] \), \( \psi^{(i)} (\theta) \) is the singular stress mode and \( \{ c^{(i)} \} \) is the corresponding integration constant.

4. Numerical examples
Sections 4.1 to 4.4 deal with a plate containing a crack lying between two distinct materials. Isotropic and anisotropic materials are modelled, normal and tangential surface traction on the crack faces are considered. In Section 4.5, the procedure is applied to the analysis of a gravity dam.

4.1. Single-cracked homogeneous plate subjected to normal load on the crack faces
A single-cracked plate as shown in Fig.2 is considered. The size of the plate is \( W \times 2W \), and the crack length is \( a \). The crack faces are subjected to a distributed load in the direction normal to the crack face, which can be expressed as \( \sigma_\infty = \lambda (r/a)^n \) with \( n=1,2,...,n \) and \( n \) is an arbitrary index. \( N \) is the number of line elements on the boundary with a length of \( W \). The SBFEM discretization of the plate for the case \( N=6 \) is shown. Each element is modelled by an 11-node element with Gauss-Lobatto-Legendre functions.

Difference sizes of the plate with \( W=(5, 10, 20, 30)a \) and different meshing schemes with \( N=2,6,10 \) are considered. The dimensionless stress intensity factors \( K_1, K_\eta \sqrt{\pi a} \) are given in Table 1. The analytical solution for a single-cracked semi-infinite plate subjected to crack face loads is also listed in the table. As can be seen with the increase of index \( n \), the stress intensity factor decreases. The increase in the size of the plate also lead to the decrease of the stress intensity factors. For \( Wa=30 \) and \( N=10 \), the presented result is close to the analytical solution, with an error less than 3%.

shown in Table 1 is the comparison of the results obtained by the presented model and the analytical solution[19] for different \( n \). It can be seen that for different meshing schemes similar results are obtained by the presented model, favourable accuracy can be obtained even with a small number of elements.

| \( Wa \) | Meshing scheme | \( n=0 \) | \( n=1 \) | \( n=2 \) | \( n=3 \) |
|---|---|---|---|---|---|
| 5 | \( N=2 \) | 1.367882 | 0.587560 | 0.388117 | 0.291725 |
| 5 | \( N=6 \) | 1.367883 | 0.587560 | 0.388100 | 0.291690 |
| 5 | \( N=10 \) | 1.367882 | 0.587558 | 0.388105 | 0.291812 |
### Table 2.

| N  | 1.189265 | 0.479268 | 0.310218 | 0.232504 |
|----|---------|----------|----------|----------|
| 20 | 1.189329 | 0.479292 | 0.310272 | 0.231388 |
| 30 | 1.189328 | 0.479289 | 0.310225 | 0.230874 |

### 4.2. Single-cracked bi-material plate subjected to normal load on the crack faces

A single-cracked bi-material plate as shown in Fig.3 is considered. The size of the plate is \( W \times 2W \), and the crack length is \( a \). The crack faces are subjected to a distributed load in the direction normal to the crack face, which can be expressed as \( \sigma_n = \lambda (r/a)^n \) with \( n=1,2,\ldots,n \) and \( n \) is an arbitrary constant.

Although the geometry and the load of the plate is symmetrical, the crack is under mix-mode due to asymmetry of the material. Two dimensionless stress intensity factors are investigated, i.e., \( K_i = K_i / \lambda \sqrt{\pi a} \), \( K_{ii} = K_{ii} / \lambda \sqrt{\pi a} \). \( W/a=30 \), \( \eta=E_2/E_1 \), \( \nu=0.3 \) are chosen, and \( n \) is assumed to be 1 and 3 respectively. The 11-node elements with Gauss-Lobatto-Legendre functions are used to discretize the boundary.

The results obtained are shown in Table 2. The results from Hu et al.[10] who constructed singular finite element based on the symplectic space are used for comparison. It can be seen that the results are consistent. And with the increase of \( \eta = E_2/E_1 \), \( K_{ii} \) increases correspondingly, which is due to the intensification of the mixmatch of the two adjoining materials.
Table 2. Comparison with reference results

| η    | Presented model | Reference results | Presented model | Reference results | Presented model | Reference results |
|------|----------------|------------------|----------------|------------------|----------------|------------------|
| 1    | 0.443476       | 0.000000         | 0.195010       | 0.000000         | 0.436264       | 0.000000         |
| 2    | 0.443854       | -0.006482        | 0.217686       | -0.007837        | 0.438484       | -0.004901        |
| 5    | 0.444680       | -0.015739        | 0.206461       | -0.022349        | 0.438604       | -0.012702        |
| 10   | 0.443746       | -0.018292        | 0.209939       | -0.024628        | 0.444920       | -0.021941        |

4.3. Single-cracked bi-material orthotropic plate subjected to normal load on the crack faces

A single-cracked bi-material plate as shown in Fig. 4 is considered. The size of the plate is $W \times 2W$, and the crack length is $a=W/5$. The crack faces are subjected to a distributed load in the direction normal to the crack face, which can be expressed as $\sigma_0 = \lambda (r/a)^\eta$ with $n=1,2,\ldots,n$ and $n$ is an arbitrary constant. The elastic constants of material 1 are $E_{11}=200\text{MPa}$, $E_{22}=200\text{MPa}$, $\nu_{12}=0.4$, $G_{12}=29.41\text{MPa}$. The elastic constants of material 2 are $E_{11}=10\text{MPa}$, $E_{22}=100\text{MPa}$, $\nu_{12}=0.02$, $G_{12}=28.07\text{MPa}$. $\varphi_1$ and $\varphi_2$ are the angles between the material axes and the coordinate axis $\hat{x}$ for the two materials respectively. Line elements are used to discretize the boundary and for each section of the boundary with a length of $W$, four element are used. The stress intensity factors for different $\varphi_i$ and assuming $\varphi_2 = 0$ are computed. Dimensionless stress intensity factors $K_1^* = K_1 / \lambda \sqrt{\pi a}$ and $K_\Pi^* = K_\Pi / \lambda \sqrt{\pi a}$ are provided in Table 3.

Although the geometry and the load of the plate is symmetrical, the crack is under mix-mode due to asymmetry of the material. With the increase of the index $n$, both stress intensity factors show a trend of decreasing. For the case of $\varphi_1 = 0, 90^\circ$, materials 1 is orthotropic, the absolute value of the mode I and mode II stress intensity factors are small than those corresponding to $\varphi_1 = 30, 60^\circ$. This happens because for orthotropic materials the $W_2$ equals 0 in Eq. (26).

Table 3. Dimensionless stress intensity factors for different $\varphi_i$

| $\varphi_i$ | SIF | $n=0$       | $n=1$       | $n=2$       | $n=3$       |
|------------|-----|------------|------------|------------|------------|
| 0°         | $K_1^*$ | 1.376708  | 0.623168  | 0.414962  | 0.312437  |
|            | $K_\Pi^*$ | -0.219304 | -0.044082 | -0.021666 | -0.013602 |
| 30°        | $K_1^*$ | 1.529958  | 0.664131  | 0.438071  | 0.330323  |
|            | $K_\Pi^*$ | -0.417664 | -0.147609 | -0.095482 | -0.071517 |
| 60°        | $K_1^*$ | 1.601302  | 0.691245  | 0.455605  | 0.342079  |
|            | $K_\Pi^*$ | -0.654783 | -0.244728 | -0.160870 | -0.121811 |
| 90°        | $K_1^*$ | 1.489204  | 0.651089  | 0.429986  | 0.322961  |
|            | $K_\Pi^*$ | -0.723042 | -0.240713 | -0.152680 | -0.112912 |

4.4 Single-cracked bi-material orthotropic plate subjected to normal and tangential load on the crack faces

A single-cracked bi-material plate as shown in Fig. 5 is considered. The size of the plate is $W \times 2W$, and the crack length is $a=W/5$. Aside from the distributed load $\sigma_0 = \lambda (r/a)^\eta$ being normal to the crack faces, distributed load $\tau_n (r) = \lambda (r/a)^\eta$ in the tangential direction is also considered. $\varphi_1$ and $\varphi_2$ are the
angles between the material axes and the coordinate axis $\hat{x}$ for the two materials respectively. The elastic constants of material 1 are $E_{11}=100\text{GPa}$, $E_{22}=10\text{GPa}$, $\nu_{12}=0.3$, $G_{12}=27.03\text{GPa}$, $\varphi_1 = 0$. The elastic constants of material 2 are $E_{11}=100\text{GPa}$, $G_{12}=27.03\text{GPa}$, $\varphi_2 = 0$. Its $E_{22}$ is set as multiples (1, 0.5, 0.3, 0.1) of the $E_{11}$ of material 1 in order to consider the mismatch of the two materials. Line elements are used to discretize the boundary and for each section of the boundary with a length of $W$, four element are used.

The stress intensity factors of the crack are related to both the normal load and the tangential load. And the mismatch of the two materials also influences the results. As can be seen from the dimensionless stress intensity factors in Table 4, for fixed elastic constants of material 1, with the decrease of the elastic modulus of material 2 in the $y$ direction, the mode I stress intensity factors increase while the mode II counterpart decrease.

![Figure 4. Single-cracked bi-material orthotropic plate 1](image1)

![Figure 5. Single-cracked bi-material orthotropic plate 2](image2)

Table 4. Dimensionless stress intensity factors for different $E_{22}/E_{11}$

| $E_{22}/E_{11}$ | SIF  | $n = 0$    | $n = 1$    | $n = 2$    | $n = 3$    |
|-----------------|------|------------|------------|------------|------------|
| 1               | $K_1^*$ | 1.125992   | 0.520949   | 0.347632   | 0.262732   |
|                 | $K_1^n$ | 1.474827   | 0.517280   | 0.321445   | 0.234461   |
|                 | $K_1^*$ | 1.196175   | 0.539462   | 0.358774   | 0.270020   |
| 0.5             | $K_1^n$ | 1.409067   | 0.504930   | 0.316323   | 0.231479   |
|                 | $K_1^*$ | 1.254433   | 0.556023   | 0.369132   | 0.278416   |
| 0.3             | $K_1^n$ | 1.345349   | 0.492732   | 0.310875   | 0.228502   |
| 0.1             | $K_1^*$ | 1.386411   | 0.598864   | 0.395958   | 0.297984   |
|                 | $K_1^n$ | 1.148765   | 0.454600   | 0.292409   | 0.216945   |

4.5 Fracture of a gravity dam

Taking the Koyna gravity dam (Fig.6) as an example, the stress intensity factors of the crack at the dam heel subjected to arbitrarily distributed water pressure is investigated. The crack has a length of 1.93m and lies on the interface between the dam and the foundation. Loadings including the water pressure corresponding to a full reservoir and self weight of the dam are considered. The dam and the
foundation materials are both isotropic and have a Poisson’s ratio equal to 0.25. The density for the two materials are 2450kg/m$^3$ and 0 respectively. Their elastic moduli are labelled as $E_1$ and $E_2$ respectively, the ratio $E_1/E_2$ are changed in order to investigate the influence of the mismatch of material constants. A foundation extending by twice the dam height from the dam towards the upstream, downstream directions and downward is modelled. The dam-foundation system is divided into seven subdomains, and the boundary of each subdomain is discretized by the 11-node line elements with Gauss-Lobatto-Legendre shape functions. 108 line elements are used for the whole system (Fig.7). By placing the origin of the coordinate system at the crack tip, the crack faces are subjected to water pressure $\sigma_0(r) = \lambda (r/a)^n$ $(n=0,1,2)$, in which $r$ is the distance between the nodes on the crack face and the crack tip while $\lambda$ is the water pressure at the crack mouth. $\lambda = 0$ stands for a crack free from water pressure, while $n=0,1,2$ stands for water pressure with a uniform distribution, linear distribution and a quadratic distribution respectively.

The stress intensity factors for different $E_1/E_2$ and different water distribution are listed in Table 5. It can be seen that for a given distribution pattern of the water pressure, an obvious decrease in $K_I$ can be observed. While for given elastic moduli, $K_I$ drops to the minimum when $n$ equals 2. With the decrease of $n$, the water pressure in the crack increases and $K_I$ increases accordingly. When the elastic modulus of the foundation is relatively small, the effect of the water pressure on $K_I$ is extraordinarily remarkable. With the increase in the ratio of the elastic modulus, the coupling effect of the two materials also increases. $K_{II}$ is mainly dependent on the water pressure in the reservoir and the water pressure in the crack has little effect. But as the ratio of the elastic modulus increases, $K_{II}$ also increases due to the coupling effect of interfacial fracture. Since in this case $K_I$ decreases and $K_{II}/K_I$ increases, the mode II component becomes more distinct.

| $E_1/E_2$ | SIF | $\lambda = 0$ | $n = 0$ | $n = 1$ | $n = 2$ |
|-----------|-----|---------------|---------|---------|---------|
| 1         | $K_I$ | 1.296381      | 1.659483| 1.383028| 1.335719|
|           | $K_{II}$ | 1.407656     | 1.461453| 1.439359| 1.407656|
| 2         | $K_I$ | 1.010123      | 1.504426| 1.135653| 1.071478|
|           | $K_{II}$ | 1.422328     | 1.393004| 1.426053| 1.427230|
| 5         | $K_I$ | 0.611296      | 1.253552| 0.795935| 0.712791|
\[ K_{II} \quad 1.510472 \quad 1.349369 \quad 1.473253 \quad 1.485265 \\
K_{I} \quad 0.347189 \quad 1.066808 \quad 0.569772 \quad 0.476663 \\
K_{II} \quad 1.603841 \quad 1.360383 \quad 1.540006 \quad 1.559327 \\
\]

5. Conclusion
A procedure for the interfacial crack with the crack faces subjected to distributed loads of arbitrary direction and arbitrary magnitude is presented. The fundamental equations based on the SBFEM are presented. The arbitrary load is decomposed to two components first, one in the normal direction and the other in the tangential direction. Each component is subdivided into the sum of a series of power functions, the solution for each of which is obtained semi-analytically and then superimposed to arrive at the overall solution. The procedure is verified by a plate with load on the crack faces. The cases considered include isotropic and orthotropic materials, homogeneous and bi-material cracks, normal and tangential loads. Based on that, a gravity dam with a preset crack at the dam heel is studied. It’s found that the water pressure inside the crack is significant to the mode I stress intensity factor of the crack. With the increase of the ratio between the elastic modulus of the dam and that of the foundation, \( K_I \) decreases remarkably while \( K_{II} \) only decreases slightly, leading to the raise of the mode II component in the mixed mode fracture.

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