The Symmetry and the Problem of Mass Generation

Harald Fritzsch
Sektion Physik, Ludwig–Maximilians–Universität München,
Theresienstrasse 37, 80333 München, Germany
(E–Mail: bm@hep.physik.uni–muenchen.de)

Abstract

The mass problem in particle physics and its impact for other fields is discussed. While the problem of the nuclear masses has been resolved within the QCD framework, many parameters of the “Standard Model” are related to the fermion sector. The origin of the fermion masses remains unresolved. We discuss attempts to explain the observed hierarchical features of the mass spectrum by a symmetry, relating the mass eigenvalues to the flavor mixing angles.

Invited plenary talk given at the XXI International Colloquium on Group Theoretical Methods in Physics (Goslar, Germany, July 1996)

Towards the end of the last century the electron was discovered. In retrospect this discovery marked the beginning of a remarkable development, which eventually led to the emergence of the “Standard Model of Fundamental Particles and Forces” in the 70ies. According to the latter all the visible matter in the universe is composed of fundamental objects of two different categories – leptons (among them the electron) and quarks. The latter do not exist as free particles, but are bound among each other to form the protons and neutrons, the building blocks of the atomic nuclei.

Symmetries have played a decisive role in this development, especially at the beginning, and group theory became since the sixties an essential tool of the particle physicists. They learned more than any other scientists that symmetry is the poetry of nature, and group theory its language.

The dynamics of matter in our universe can be traced back to the action of four types of fundamental forces: the strong forces among the quarks, the electromagnetic forces among charged particles, the weak interactions responsible for the phenomenon of radioactivity,
and gravity.

The “Standard Model” constitutes a consistent theory of the fundamental forces, based on quantum field theory and the concepts of non–Abelian gauge theories. Since about 1980 the quantitative predictions of the “Standard Model” have been subjected to severe experimental tests, with the result that no departures from the theoretical expectations have been found. The “Standard Model” provides us not only with a fairly good description of the fundamental particles and forces, but gives an excellent picture of reality, both qualitatively and quantitatively. Not a single confirmed result from the particle physics experiments is in conflict with it.

With the help of the Large Electron Positron collider (LEP) at CERN one was able in the recent years to test the predictions of the “Standard Model”, especially its electroweak sector, with a high order of precision. Furthermore the study of the collisions at LEP has helped to unify particle physics, astrophysics and cosmology to a coherent picture of the cosmic evolution. The collisions at LEP recreate the conditions which were present in the universe about $10^{-10}$ seconds after the Big Bang.

According to the “Standard Model” there exist two different categories of fundamental particles: the matter particles (quark, leptons) carrying spin 1/2 and the force particles (photons, $W$, $Z$, gluons) carrying spin 1. The latter are gauge bosons, and their interactions with the matter particles and with themselves are dictated by the principles of non–Abelian gauge symmetry. The symmetry group is given by the direct product of three simple groups: $SU(3)_c \times SU(2)_w \times U(1)$ (c: color, w: weak). The gauge bosons are given in the following table:

| Gauge Bosons (spin 1) | Mass | Electric Charge | Color |
|----------------------|------|-----------------|-------|
| $\gamma$            | 0    | 0               | 0     |
| $W^-$                | 80.35 GeV | $-1$         | 0     |
| $W^+$                | 80.35 GeV | $+1$         | 0     |
| $Z$                  | 91.19 GeV | 0             | 0     |
| g                    | 0    | 0               | 8     |

The visible matter in the universe is composed of the elements of the first lepton–quark family:

$$
\begin{pmatrix}
\nu_e & u_r & u_g & u_b \\
e^- & d_r & d_g & d_b
\end{pmatrix}
$$

($r$, g, b: color index).

The fact that the electric charges of these eight objects add to zero indicates that the
two leptons and six quarks are related to each other in a way which cannot be described within the “Standard Model”, but is subject to the yet hypothetical physics beyond the “Standard Model”, perhaps described by a grand unification of all interactions.

All known particles can be described in terms of three lepton–quark families:

\[
\begin{pmatrix}
\nu_e & u \\
e^- & d
\end{pmatrix}
\begin{pmatrix}
\nu_\mu & c \\
\mu^- & s
\end{pmatrix}
\begin{pmatrix}
\nu_\tau & t \\
\tau^- & b
\end{pmatrix},
\]

It should be stressed that several algebraic properties of the observed pattern cannot be deduced from the underlying symmetry of the “Standard Model”, e.g. the charge and spin assignments of the leptons and quarks, or the number of families. It is remarkable that the number “three” plays a three–fold significant role:

a) Quarks come in three colors, hence the nucleon consist of three quarks.

b) There exist three families of leptons and quarks.

c) There are three different gauge interactions, based on the three different gauge groups (gravity is not considered here).

The question arises whether there are connections between the three different “three–nesses” of the Standard Model. For example, the numbers of families and of colors might be identical due to an underlying, yet unknown symmetry principle.

The most unsatisfactory feature of the “Standard Model” is the fairly large number of free parameters which need to be adjusted. First of all, the values of the three gauge coupling constants \( g_1, g_2, g_3 \) have to be taken from experiment. All other parameters are related to the masses of the fermions or gauge bosons. They are: the mass of the \( W \)–boson, the masses of the three charged leptons \( m_e, m_\mu, m_\tau \) (the neutrinos will be kept massless in our discussion), the masses of the six quark flavors \( m_u, m_d, m_c, m_s, m_t, m_b \) and the four mixing angles of the weak interaction sector \( \Theta_{sd}, \Theta_{sb}, \Theta_{bd} \) and \( \delta \) (\( \Theta_{sd} \) denotes the angle, which describes primarily the s – d mixing etc., \( \delta \): phase angle, relevant for \( CP \)–violation).

Thus 17 parameters are needed to describe the observed particle physics phenomena.

We point out that 13 free parameters of the “Standard Model” are associated with the masses of the matter fermions (nine masses, four mixing parameters, which arise due to a mismatch between the mass matrices of the \( u \)–type quarks and the \( d \)–type quarks). Thus a deeper insight into the dynamics of the mass generation for the leptons and quarks is needed in order to reduce the number of free parameters – an insight which would definitely carry us beyond the physics of the “Standard Model”. Certainly the problem of
mass and mass generation are on the top of the priority list of problems in fundamental physics for the immediate future.

In all physics phenomena the different interactions and mass scales enter in a variety of ways, thus producing the multitude of phenomena we observe. Before discussing the problem of the lepton and quark masses, let me stress that the problem of the nucleon mass (and therefore of the masses of the atomic nuclei, which represent more than 99% of the mass of the visible matter in the universe) has found an interesting solution within the framework of QCD. Due to the renormalization effects the QCD gauge coupling constant \( \alpha_s \) depends on the scale at which the interaction is studied. It decreases logarithmically at high energies. At a scale \( \mu \) its decrease is given by \( \alpha_s(\mu^2) = \text{const.}/\ln(\mu^2/\Lambda^2) \), where \( \Lambda \) is a scale parameter, which serves as the fundamental mass scale of the theory, i.e. a mass scale which fixes all other mass scales in strong interaction physics (the effects of the quark masses are neglected). Phenomenologically \( \Lambda \) is about 150 . . . 200 MeV, i.e. \( \Lambda^{-1} \) corresponds to the typical extension of a hadron.

Using powerful computers and sophisticated nonperturbative methods one has been able to calculate the masses of the lowest–lying hadrons (nucleon, \( \Delta \)–resonance, \( \rho \)–meson . . .) in terms of \( \Lambda \), with impressive results. Especially the mass ratios \( (m_\Delta/m_p) \), \( (m_\rho/m_p) \) etc. can be calculated with high precision. Thus we can say that the problem of the nuclear masses, especially of the proton mass, has found a solution. The mass of a proton (about 940 MeV) is nothing but the field energy of the quarks and gluons, which are confined within a radius of the order of \( 10^{-13} \) cm (of order \( \Lambda^{-1} \)). Therefore, a direct link exists between the nucleon mass and the size of the nucleon. Both have to be of the same order of magnitude – more specifically the nucleon mass is expected to be of the order of \( 3 \cdot \Lambda \), where 3 denotes the number of the constituent quarks. Furthermore the nucleon mass is a truly nonperturbative phenomenon, directly related to the confinement aspect of QCD. The QCD gauge interaction itself creates its own mass scale – the nuclear mass is generated dynamically.

This phenomenon of dynamical mass generation is not primarily related to the quark substructure of the hadrons, but rather to the gluonic degrees of freedom. This can be seen by studying the mass spectrum of “pure QCD”, i.e. QCD without quarks. This theory of eight interacting gluons displays at low energies a discrete mass spectrum, which starts at the mass of the lowest lying glue meson. Thus unlike “pure QED” the theory displays a mass gap which is generated by nonperturbative effects.

The theory of the nucleon mass described above is remarkable in the sense that the mass of an elementary particle can in principle be calculated. Note that the mass is directly related to the field energy density inside the nucleon. It would be of high interest to know whether the masses of the leptons, the \( W \)–, \( Z \)–particles and the quarks are due to a similar mechanism, or are generated by a qualitatively different mechanism.
In the standard electroweak theory these masses are due to a spontaneous breaking of the
electroweak gauge symmetry caused by an elementary scalar field $\varphi$. The order parameter
of the symmetry breaking is given by the vacuum expectation value $v$ of the field $\varphi$ which
in turn is related to the Fermi constant $G$ and the $W$–mass:

$$
\frac{G}{\sqrt{2}} = \frac{g_{w}^2}{8M_{w}^2} = \frac{1}{2v^2}
$$

($v = 246.2$ GeV)

($g_{w}$: gauge coupling constant).

The main consequence of this mechanism is a relation between the $Z$–mass and the $W$–
mass in terms of the electroweak mixing angle $\Theta_{w}$: $M_{z} = M_{w} / \cos \Theta_{w}$. Since the mixing
angle $\Theta_{w}$ can be determined independently by studying the neutral current interaction of
leptons and quarks, this mass relation is a nontrivial constraint, in excellent agreement
with the experimental data.

The success of the electroweak mass relation does not necessarily imply that the mechanism
of the spontaneous symmetry breaking is realized in the real world, but it implies that an
alternative mechanism must lead to the same mass relation. This is the case, for example,
in technicolor models in which the scalar field $\varphi$ is replaced by a field composed of new
fermions which are tightly bound by the new technicolor interaction.

If the standard electroweak model is correct, it implies the existence of a particle, the
“Higgs” particle, whose couplings are given by the observed particle masses. The mass
of this particle is unknown, but it can hardly be larger than about 1000 GeV. The LEP
experiments exclude the mass region lower than about 60 GeV.

Certainly the most interesting question is the one about the origin of the lepton and quark
masses. The spectrum extends over five orders of magnitude, starting with the electron,
and ending with the $t$–quark with a mass of about 180 GeV. Thirteen free parameters are
needed to describe the properties of the lepton and quark mass spectrum: the three lepton
masses, the six quark masses, and the four parameters describing the mixing of the quark
flavors. Unlike the masses of the leptons, the quark masses cannot be determined directly,
but have to be inferred from the properties of the hadronic spectrum. Furthermore they
are scale dependent, i. e. they vary logarithmically, if the corresponding renormalization
point is shifted. In lowest order this change is given by $m_{q}(\mu) = m_{q}(\mu_{0})(1 - \frac{\alpha_{s}(\mu)}{\pi} \ldots)$, where
$\alpha_{s}(\mu)$ is the QCD coupling constant. A suitable renormalization point for the quark masses
is the mass of the $Z$–boson, which is known with a high precision: $M_{z} = 91.1884 \pm 0.0022$
GeV.
One finds:

\[
\begin{align*}
m_u(M_z) &= 3.4 \pm 0.6 \text{ MeV}, & m_d(M_z) &= 6.3 \pm 0.9 \text{ MeV} \\
m_c(M_z) &= 880.0 \pm 48.0 \text{ MeV}, & m_s(M_z) &= 118.0 \pm 17.0 \text{ MeV} \\
m_t(M_z) &= 172.0 \pm 6.0 \text{ GeV}, & m_b(M_z) &= 3.31 \pm 0.11 \text{ GeV}
\end{align*}
\]

A closer inspection of the mass spectrum tells us:

a) The mass spectra of the three flavor channels (charged leptons, \(u\)-type quarks, \(d\)-type quarks) are almost entirely dominated by the mass of the member of the third generation.

b) The relative importance of the second generation decreases as we proceed upwards in the charge (\(\mu \rightarrow s \rightarrow c\)). In the lepton case the muon contributes about 5.6% to the sum of the masses, while in the charge \((-1/3)\)–channel the \(s\)-quark contributes only 3.2%, and in the charge \((+2/3)\)–channel the \(c\)-quark contributes only 0.5%.

c) The relative importance of the masses of the members of the first generation is essentially negligible.

d) The entire mass spectrum of the leptons and quarks is dominated fairly well by the \(t\)-quark alone. For example, in the case \(m_t = 100\) GeV the \(t\)-quark contributes 97.5% to the sum of all fermion masses. All other quarks, mostly the \(b\)-quark, contribute only 2.5%.

The spectrum exhibits clearly a hierarchical pattern: The masses of a particular generation of leptons or quarks are small compared to the masses of the following generation, if there is any, and large compared to the previous one if there is any. Furthermore another hierarchical pattern emerges if we consider the weak interaction mixing parameters. The mixing matrix, if written in terms of quark mass eigenstates, is not far from the diagonal matrix (no mixing). The mixing angles are typically rather small; the Cabibbo angle being the largest of all, is about 13°, while \(\Theta_{sb}\) is about 2.2° and \(\Theta_{bd}\) about 0.2°.

What kind of symmetry could one discuss in view of the observed lepton–quark mass spectrum? The observed two different hierarchies suggest that we are very close to a limit, which I like to call the “rank 1”–limit, in which both the \(u\)-type and \(d\)-type mass matrix can be diagonalized at the same time and in which they both take the diagonal form \((0, 0, 1)\), multiplied by \(m_t\) or \(m_b\) respectively. Thus the masses of the first two generations vanish (the mass matrix has rank one), likewise all mixing angles.

Of course, it depends on yet unknown details of the mass generation mechanism whether such a limit can be achieved in a consistent way. We simply assume that this is the case. In this limit there exists a mass gap: The third generation is split from the massless first two generations. Obviously nature is not far away from this limit, and therefore one is invited
to speculate about the dynamical origin of such a situation. A mass matrix proportional to the matrix
\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
can always be obtained from another matrix, namely the one in which all elements are equal:
\[
\begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{bmatrix}
\]
by a suitable unitary transformation. Matrices of this type, which might be called “democratic mass matrices” have been considered recently by a number of authors. They can be used as a starting point to construct the full mass matrices of the quarks, including the weak interaction mixing terms. We note that such a matrix plays an important role in other fields of physics, where mass gap phenomena are observed:

a) In the BCS theory of superconductivity the energy gap is related to a “democratic” matrix in the Hilbert space of the Cooper pairs.

b) The pairing force in nuclear physics which is introduced in order to explain large mass gaps in nuclear energy levels has the property that the associated Hamiltonian in the space of nucleon pairs has equal matrix elements, i.e. it has a structure of the type given above.

c) The mass pattern of the pseudoscalar mesons in QCD in the chiral limit $m_u = m_d = 0$. In this limit the $\pi^0$ and the $\eta$ are massless Goldstone bosons, while the $\eta'$ acquires a mass due to the gluon anomaly.

Once we write the mass matrices of the leptons and quarks in their “democratic” form, it is obvious that there exists a symmetry, namely the symmetry $S_3$ of permutations among the three different flavors$^1)$. This symmetry suggests that one should consider the eigenstates of the quarks and leptons in this basis as the fundamental dynamical entities. Let us denote them as $(l_1, l_2, l_3)$ and $(q_1, q_2, q_3)$ respectively.

The heaviest lepton and quark, i.e. the $\tau$–lepton, the $t$ and $b$ quarks, would be coherent states of the type:

$$\tau = \frac{1}{\sqrt{3}} (l_1 + l_2 + l_3) \quad etc.$$  

In view of the scarce information we have at present about the internal dynamics of the leptons and quarks we do not know, whether this description of the fermions in terms
of coherent states is more than a specific mathematical representation. In a composite model, for example, the fermion states $f_1, f_2, f_3$ would be those states which are “pure” in a dynamical sense, e. g. they have simple unmixed wave functions.

We remind the reader that also in the case of superconductivity, of the nuclear pairing force and of the lightest pseudoscalar mesons the mass eigenstates are coherent superpositions of “physical” states which are described by simple wave functions (e. g. the Cooper pairs in superconductivity).

Within our approach we see a solution to a problem, which has plagued many models of the physics beyond the standard model, the problem of the near masslessness of the first and to some extent also of the second generation. In the coherent state basis this is easily understood. For example, the electron state $e = 1/\sqrt{2}(l_1 - l_2)$ is nearly massless, since there is a nearly complete cancellation of the $l_1$– and $l_2$–mass terms, as a consequence of the rank one structure of the dominant lepton–quark mass term.

It would be interesting to see whether a simple breaking of the democratic symmetry leads to a satisfactory understanding of the masses of the second generation and the associated mixing. Indeed a simple breaking of the symmetry leads to a relation between the mixing angle and the mass eigenvalues:

$$\Theta_{sb} \cong 1/\sqrt{2} \cdot (m_s/m_b + m_c/m_t).$$

Using the observed eigenvalues, one finds $\Theta_{sb} \cong 0.036$, in reasonable agreement with the observed value $0.032 \ldots 0.048$.

A further breaking of the remaining $S(2) \times S(2)$ symmetry leads to the generation of the masses of the first generation, and the associated mixing angles $\Theta_{ds}$ and $\Theta_{bd}$. Details can be found in ref. [4].

Although this is not the place to discuss dynamical details of the mass generation it is important to note that in various dynamical schemes, in particular in one based on a composite structure of the leptons and quarks, the introduction of the masses for the second and third generations leads to slight breakings of the flavor conservation, especially in reactions associated with large momentum transfers. For example, decays like $t \rightarrow c + \text{gluon}$ or $\tau \rightarrow \mu + \gamma$ will occur with rates accessible in future experiments.

In this talk I have described a number of ideas which one might consider after looking at the pattern of masses exhibited in the lepton–quark mass spectrum. I have emphasized the role of symmetries in the space of the generations of the quarks in providing relations between the various mass eigenvalues and the mixing angles. An approach to the flavor problem and to the hierarchical mass spectrum of the leptons and quarks, based on the introduction of coherent states, was discussed. It was argued that the mass generation for
the third lepton–quark generation is nothing but a gap phenomenon and is rather similar
to the mass generation for the pseudoscalar mesons in QCD. Thus the third lepton–quark
generation is somewhat distinct from the other ones. The same mechanism which leads to
the mass generation causes the appearance of flavor changing effects; only in the absence
of the lepton and quark masses of the first and second generation the various quark and
lepton flavors are conserved.

If our interpretation of the mass gap seen in the lepton–quark spectrum is correct, it
would mean that all mass gap phenomena seen in physics – superconductivity, nuclear
pairing forces, QCD mass gap, lepton–quark mas spectrum – are due to an analogous
underlying dynamical mechanism. The exploration of further details of this mechanism
could lead soon to a deeper understanding of the physics beyond the standard model.

Literatur

[1] See also: H. Fritzsch, in: Proc. Europhysics Conf. on Flavor Mixing, Erice, Italy
(1984). L. Chau, editor.

[2] H. Harari, Haut and J. Wyers, Phys. Lett. 78B (1978) 459;
Y. Chikashige, G. Gelmini, R. P. Peccei and M. Roncadelli, Phys. Lett. 94B (1980)
499;
C. Jarlskog, in: Proc. of the Int. Symp. on Production and Decay of Heavy Flavors,
Heidelberg, Germany, 1986;
P. Kaus and S. Meshkov, Mod. Phys. Lett. A3 (1988) 1251; A4 (1989) 603;
G. C. Branco, J. I. Silva–Marcos, N. M. Rebelo, Phys. Lett. B237 (1990) 446;
H. Fritzsch and J. Plankl, Phys. Lett. B237 (1990) 451;

[3] H. Fritzsch and D. Holtmannspötter, Phys. Lett. B338 290 (1994).

[4] M. Aguilar–Benitez et al., Particle Data Group, Phys. Lett. D50, 1173 (1994).

[5] H. Fritzsch and Z. Xing, Phys. Lett. B328 (1994) 411