Constraining the curvaton scenario

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We analyse the curvaton scenario in the context of supersymmetry. Supersymmetric theories contain many scalars, and therefore many curvaton candidates. To obtain a scale invariant perturbation spectrum, the curvaton mass should be small during inflation $m \ll H$. This can be achieved by invoking symmetries, which suppress the soft masses and non-renormalizable terms in the potential. Other model-independent constraints on the curvaton model come from nucleosynthesis, gravitino overproduction, and thermal damping. The curvaton can work for masses $m \gtrsim 10^4$ GeV, and very small couplings (e.g. $h \lesssim 10^{-6}$ for $m \lesssim 10^8$ GeV).

1 The curvaton scenario

It is now widely believed that the early universe went through a period of rapid expansion, called inflation. In addition to explaining the homogeneity and isotropy of the observable universe, inflation can provide the seeds for structure formation. In the usual picture quantum fluctuations of the slowly rolling inflaton field “freeze in” soon after horizon exit, and become essentially a classical perturbation which remains constant until the moment of horizon re-entry. The resultant perturbations are adiabatic and Gaussian, in agreement with observations. Moreover, they solely depend on the form of the inflaton potential, and are independent of what goes on between horizon exit and re-entry. This makes models of inflation predictive, but also restrictive. The observed, nearly scale-invariant perturbation spectrum requires very small coupling constants and/or masses, which renders many models unnatural. For this reason it is worthwhile to explore alternative ways of producing density perturbations.

In the curvaton scenario, the adiabatic perturbations are not generated by the inflaton field, but instead result from isocurvature perturbations of some other field — the curvaton field. Adiabatic or curvature perturbations are local perturbations of the curvature of space-time; isocurvature perturbations on the other hand do not perturb space-time but correspond to a local perturbation in the equation of state. After inflation the isocurvature perturbations have to be converted into adiabatic ones. Such a conversion takes place with the growth of the curvaton
energy density compared to the total energy density in the universe. This alternative method of producing adiabatic perturbations was first noted years ago\textsuperscript{2}, but it did not attract much attention until recently\textsuperscript{3}.

The usual implementation of the curvaton scenario is the following. If the curvaton is light with respect to the Hubble constant during inflation, it will fluctuate freely, leading to condensate formation. In the post-inflationary epoch the expansion of the universe acts as a friction term in the equations of motion, and the field remains effectively frozen at large field value. This stage ends when the Hubble constant becomes of the order of the curvaton mass, $H \sim m_\phi$, at which point the curvaton starts oscillating in the potential well. During oscillations, the curvaton acts as non-relativistic matter, and its energy density red shifts as $\rho_\phi \propto a^{-3}$ with $a$ the scale factor of the universe. After inflaton decay, the universe becomes radiation dominated, with the energy density in radiation red shifting as $\rho_\gamma \propto a^{-4}$. Hence, the ratio of curvaton energy density to radiation energy density grows $\rho_\phi/\rho_\gamma \propto a$, and isocurvature perturbations are transformed into curvature perturbations. This conversion halts when the curvaton comes to dominate the energy density, or if this never happens, when it decays.

2 Supersymmetry

It seems natural to try to embed the curvaton scenario within supersymmetric (SUSY) theories. SUSY theories contain many flat directions, i.e., directions in field space along which the scalar potential vanishes in the supersymmetric limit. The fields parametrising these directions can condense during inflation, and are therefore possible curvaton candidates. The problem, however, is that inflation is driven by the non-zero energy density stored in the inflaton field, and necessarily SUSY is broken during inflation. One way to see this, is to note that the inflaton potential is a sum of $F$ and $D$ terms, and that a non-zero $F$ ($D$)-term does not leave the SUSY transformation of the quarks (gauginos) invariant. As a result, soft mass terms are generated, which are typically of the order of the Hubble constant. But this is no good: $m_\phi \sim H$ during inflation leads to a large scale dependence of the produced perturbations, in conflict with observations.

A way out of this is to invoke symmetries. If inflation is driven by $D$-terms, soft mass terms are forbidden by gauge symmetries, and the problem does not arise. In no-scale type supergravity, a so-called Heisenberg symmetry forbids mass terms at tree level, and soft masses are suppressed by loop factors. Another possibility is to consider pseudo-Goldstone bosons, whose mass is protected by approximate global symmetries.

3 constraints

There are several model independent constraints on the curvaton scenario. We will discuss them briefly here; see the original paper for more details\textsuperscript{4}.

First of all, the curvaton scenario should give rise to the observed spectrum of density perturbations. Curvature perturbations $R$ of the correct magnitude are obtained for\textsuperscript{11,13}

$$R \approx \frac{f H_*}{3\pi \phi_*} \approx 5 \times 10^{-5}. \tag{1}$$

Here the subscript $*$ denotes the quantity at the time observable scales leave the horizon, some 60 $e$-folds before the end of inflation. Further, $f = \rho_\phi/\rho_{\text{tot}}$ evaluated at the time of curvaton decay. If the curvaton contributes less than 1\% to the total energy density, i.e., $f < 0.01$, then the perturbations have an unacceptable large non-Gaussianity. If during inflation $m_\phi \ll H$ — which is required to get a nearly scale invariant perturbation spectrum — quantum fluctuations of the curvaton grow until $m_\phi^2 \langle \phi^2 \rangle \sim H^4$, with an exponentially large coherence length. We
will assume that this sets the initial curvaton amplitude $\phi_* \sim \sqrt{\left\langle \phi^2 \right\rangle}$. The non-detection of tensor perturbations puts an upper bound on the Hubble scale during inflation $H_* \lesssim 10^{14}$ GeV. Finally, in the curvaton scenario the adiabatic density perturbations can be accompanied by isocurvature perturbations in the densities of the various components of the cosmic fluid. There are particularly strong bounds on the isocurvature perturbations in cold dark matter.

In the absence of non-renormalizable terms in the potential the initial curvaton amplitude can be arbitrarily large, as long as the curvaton energy density is sub-dominant during inflation. However, to avoid a period of inflation driven by the curvaton field, the curvaton energy density should be still sub-dominant at the onset of curvaton oscillations. This restricts the amplitude $\phi_0 \lesssim M_P$. The constraints are stronger if non-renormalizable terms are taken into account:

$$V_{NR} = \frac{|\lambda|^2}{M_P^n} \phi^{4+n}. \quad (2)$$

Non-renormalizable terms are unimportant for small enough masses, $m_\phi \lesssim m_{\text{eff}} = V_{\text{eff}}''$. For larger masses, the curvaton slow-rolls in the non-renormalizable potential during and after inflation. In the post-inflationary epoch this leads to a huge damping of the fluctuations, making it is impossible to obtain the observed density contrast within the context of the curvaton scenario.

The curvaton scenario should not alter the successful predictions of big bang nucleosynthesis (BBN). This implies that the curvaton should decay before the temperature drops below MeV, and its coupling to other fields cannot be arbitrarily small. Gravitinos have only Planck suppressed couplings and generically decay after BBN, thereby spoiling BBN predictions if their number density is large. To avoid gravitino overproduction requires a reheat temperature $T_R \lesssim 10^9$ GeV: the inflaton should decay sufficiently late. This also constrains the curvaton scenario, since isocurvature perturbations are converted in adiabatic perturbations only after inflaton decay. Note that the entropy production at curvaton decay dilutes the gravitino density, thereby ameliorating the gravitino problem. In no-scale type supergravity the gravitino mass is undetermined at tree level; the gravitino problem is solved if the gravitino is heavy, $m_{3/2} \gtrsim 100$ TeV, and decays before BBN.

Finally, one should take into account various thermal effects. A large thermal mass may be induced when the condensate is submerged in a thermal bath. The heat bath is in thermal equilibrium for temperatures $T \gtrsim h\phi$, where $h\phi$ is the effective mass of the particles the curvaton couples to, and $h$ is the coupling constant. Large thermal masses, $m_{\text{th}} \gtrsim m_\phi$, induce early oscillations. The curvaton energy density not only decreases due to the expansion of the universe, but also due to the decreasing mass: $\rho_\phi(T) = \frac{m(T)}{m(T_0)} \frac{a(T)}{a(T_0)} \rho_\phi(T_0)$. Further, it should be demanded that the curvaton does not decay too early, through either thermal (through scattering) or resonant decay. These constraints turn out to be less stringent.

### 4 Results

The parameter space for a successful curvaton scenario is shown in Fig. 1 and Fig. 2. In all plots $|\lambda| = 1$ and $M_P = 1/\sqrt{8\pi G}$. We have assumed that the perturbations generated by the inflaton are negligible small.

Fig. 1 shows the parameter space for curvaton domination; the curvaton dominates the energy density for $f \gtrsim 0.5$. In the figure on the left the reheating temperature is arbitrary high, whereas in the figure on the right the gravitino constraint is taken into account and $T_R \lesssim 10^9$ GeV. In all parameter space $m_\phi \sim 10^{-4} H_*$. Models with $V_{NR} \sim \phi^{4+n}/M_P^n$ and $n \leq 2$ are ruled out. For higher values of $n$, the curvaton scenario can be successful for curvaton masses
Figure 1: Parameter space for curvaton domination ($f \gtrsim 0.5$). In the plot on the left the reheating temperature is arbitrary high, whereas in the plot on the right $T_R \lesssim 10^9$ GeV. The constraints from BBN, domination, non-renormalizable terms, $\phi$-dominated inflation, and thermal damping are shown.

Figure 2: Parameter space for curvaton non-domination ($10^{-2} \lesssim f \lesssim 0.5$). In the plot on the left the reheating temperature is arbitrary high, whereas in the plot on the right $T_R \lesssim 10^9$ GeV. The constraints from BBN, non-renormalizable terms, $\phi$-dominated inflation, non-Gaussianity, and thermal damping are shown.
in the range $10^4 \text{GeV} \lesssim m_\phi \lesssim 10^9 \text{GeV}$. Couplings have to be small $h \lesssim 10^{-6}$, even $h \lesssim 10^{-10}$ if the gravitino constraint is taken into account, unless renormalizable terms are absent to a very high order. In all plots we assumed that the curvaton has a typical initial value $\phi_0 \sim H_s^2/m_\phi$. Dropping this assumption allows for larger coupling constants; however, thermal damping should be taken into account for $h \gtrsim 10^{-5}$.

Fig. 2 shows the results for non-domination, $10^{-2} \lesssim f \lesssim 0.5$; smaller values of $f$ lead to perturbations with unacceptable large non-Gaussianity. In all parameter space $m_\phi \sim 10^{-4} H_s/f$. The results are similar to the domination case: a succesfull curvaton scenario needs small couplings, especially for a low reheat temperature, and masses $m_\phi \gtrsim 10^4 \text{GeV}$. For very large masses $m_\phi \to 10^{12} \text{GeV} \approx 10^{-2} H_{\text{max}}$, larger couplings are possible.

The only constraints not considered yet pertain to residual isocurvature perturbations. Isocurvature perturbations are defined (on unperturbed hypersurfaces) as $S_i = 3(R_i - R)$, with $i = \text{CDM, B}$ for cold dark matter (CDM) and baryons respectively. Further, $R_i = -3H(\delta \rho_i / \rho_i) \propto \delta \rho_i / \rho_i$ is the curvature perturbation of fluid $i$, and $R$ the total curvature of the universe. The constraints are strongest for CDM. There are three possibilities:

- CDM number is created after curvaton decay. The epoch of creation is defined as the epoch after which the comoving CDM particle number is conserved. CDM and radiation have the same curvature perturbation, and there are no residual isocurvature perturbations, $S_{\text{CDM}} = 0$.

- CDM is created before curvaton decay. If at creation the curvaton energy density is still negligible small, $f \ll 1$, CDM has a negligible curvature perturbation. The isocurvature perturbation at the epoch of last scattering then is $S_{\text{CDM}} = -3R$. The isocurvature and curvature perturbations are correlated. The bound from cosmic microwave background (CMB) measurements is $|S_{\text{CDM}}/R| < 0.1$. The constraint is weaker by a factor $(\Omega_{\text{CDM}}/\Omega_B) \sim 10$ for baryons. Creation of dark matter well before curvaton decay is in conflict with experiment. The same holds true for baryons, unless there is a cancelling CDM isocurvature perturbation created by curvaton decay.

- CDM is created by curvaton decay. Then $S_{\text{CDM}} = 3(1/f^\star) R$. The isocurvature and curvature perturbations now are anti-correlated. In this case the bound from CMB measurements is $|S_{\text{CDM}}/R| < 0.2$. CDM and baryon isocurvature perturbations are unobservable small for $f > 0.9$ and $f > 0.6$ respectively.

5 Conclusions

In the curvaton scenario, the adiabatic perturbations are not generated by the inflaton field, but instead result from isocurvature perturbations of some other field — the curvaton field. We have analyzed various model independent constraints on such a scenario. The curvaton scenario can work for small couplings $h \lesssim 10^{-6}$ and large masses $m_\phi \gtrsim 10^4 \text{GeV}$. Strong bounds come from non-renormalizable operators in the potential, and from gravitino overproduction.

One can ask whether there are any natural candidates for the curvaton. Moduli and other fields with only Planck suppressed couplings generically decay after big bang nucleosynthesis, thereby spoiling its succesful predictions. This problem is avoided for moduli with large soft masses $m_{3/2} \gtrsim 100 \text{TeV}$. Fields parametrizing flat directions in the potential of the supersymmetric standard model typically have too small masses and too large couplings to play the rôle of the curvaton. Better curvaton candidates are the right-handed sneutrino and the Peccei-Quinn axion, which can have large masses and small couplings. In all cases though, considerable tuning of parameters is needed. For example, for the right-handed sneutrino curvaton, one needs to explain why the sneutrino mass is much smaller than the grand unification scale.
Acknowledgments

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