Damping of the baryon acoustic oscillations in the matter power spectrum as a probe of the growth factor

Hidenori Nomura$^1$, Kazuhiro Yamamoto$^1$ and Takahiro Nishimichi$^2$

$^1$ Graduate School of Sciences, Hiroshima University, Higashi-Hiroshima 735-8526, Japan
$^2$ Department of Physics, School of Science, The University of Tokyo, Tokyo 113-0033, Japan
E-mail: hide@theo.phys.sci.hiroshima-u.ac.jp, kazuhiro@hiroshima-u.ac.jp and nishimichi@utap.phys.s.u-tokyo.ac.jp

Received 3 February 2008
Accepted 19 September 2008
Published 17 October 2008

Online at stacks.iop.org/JCAP/2008/i=10/a=031
doi:10.1088/1475-7516/2008/10/031

Abstract. We investigate the damping of the baryon acoustic oscillation (BAO) signature in the matter power spectrum due to the quasi-non-linear clustering of density perturbations. On the basis of the third-order perturbation theory, we construct a fitting formula for the damping in an analytic way. This demonstrates that the damping is closely related to the growth factor and the amplitude of the matter power spectrum. Then, we investigate the feasibility of constraining the growth factor through a measurement of the damping of the BAO signature. An extension of our formula including higher order corrections of density perturbations is also discussed.

Keywords: surveys of galaxies, power spectrum
Contents

1. Introduction 2
2. Quasi-non-linear evolution of the matter power spectrum 3
3. Damping of the BAO signature 5
   3.1. Analytic approach 6
   3.2. Fitting formula 8
   3.3. Possible extension and discussion 12
4. Feasibility of constraining $\sigma_8D_1(z)$ 16
5. Summary and conclusions 21
Acknowledgments 22
References 22

1. Introduction

The baryon acoustic oscillations (BAO) imprinted in the galaxy power spectrum have recently attracted remarkable attention, as a useful probe for exploring the origin of dark energy [1, 2]. The BAO signature has been clearly detected in the 2dFGRS and the SDSS galaxy samples [3]–[7]. The feasibility of constraining the equation of state parameter of dark energy $w$ is demonstrated [8]–[10], where $w$ is defined by $w = p_{de}/\rho_{de}$, where $p_{de}$ and $\rho_{de}$ are the pressure and the energy density of the dark energy component, respectively. Furthermore, a lot of BAO survey projects are in progress, or planned [11]–[16].

Though these BAO surveys will precisely measure the galaxy power spectra, we also need precise theoretical templates, in order to obtain a useful cosmological constraint from observational data. In practice, we need to resolve uncertainties about the gravitational non-linear clustering of the density perturbations, the redshift-space distortions, and the galaxy clustering bias, because these uncertainties might yield some systematic effects on the BAO signature. Then, a lot of works related to these topics were done recently, which are very important and challenging for future research into dark energy [17]–[27].

In the present paper, we focus on the damping of the BAO signature in the matter power spectrum due to the non-linear gravitational clustering. We investigate this damping in a semi-analytic way on the basis of the third-order perturbation theory of density fluctuations. The non-linear clustering is a consequence of mode couplings of the density fluctuations and the peculiar velocity divergence in Fourier space. In particular, we demonstrate how the damping of the BAO signature can be expressed with/without $P_{22}(k)$ and $P_{13}(k)$, which describe the mode coupling (see sections 2 and 3). Then, we develop a simple fitting formula relevant to the damping, which demonstrates the fact that the damping of the BAO signature is closely related to the growth factor and the amplitude of the matter power spectrum. This suggests that a measurement of the damping of the BAO signature might be a probe of the growth factor of the density fluctuations, though
we need to further investigate whether the effects of the redshift-space distortion or the galaxy clustering bias are influential in the damping.

This paper is organized as follows. In section 2, we briefly review the third-order perturbation theory, and derive the second-order power spectrum. In this formalism, the non-linear gravitational clustering is described by the coupling of the Fourier modes of the density fluctuations and the peculiar velocity divergences. In section 3, we investigate the damping of the BAO signature due to the coupling of the Fourier modes in an analytic way. And a fitting formula of the damping, applicable in weakly non-linear regimes, is developed. The formula is compared with a result from N-body simulation. Extensions of the formula are also discussed. In section 4, with the use of the fitting formula, we demonstrate the future feasibility of constraining the growth factor through a measurement of the damping of the BAO signature. The last section is devoted to a summary and conclusions. Throughout this paper, we use units in which the velocity of light equals 1, and adopt the Hubble parameter \( H_0 = 100h \) km s\(^{-1}\) Mpc\(^{-1}\) with \( h = 0.7\), unless explicitly stated otherwise.

2. Quasi-non-linear evolution of the matter power spectrum

To investigate the effect of the non-linear gravitational clustering in the present work, we employ the standard perturbation theory (SPT) of the density fluctuations and the peculiar velocity divergences [28]–[36]. Numerical simulation is a rigorous approach to the gravitational clustering, but the perturbative approach has an advantage of providing an understanding of which factors are relevant to the damping of the BAO signature, in an analytic way. Recently, several authors have developed new formalism beyond the SPT [37]–[46]. In the later section 3.3, we discuss a possible extension of our main result to include these formalism. However, let us start with briefly reviewing the derivation of the second-order power spectrum on the basis of the SPT up to the third order of perturbations.

We consider the matter fluctuations after the recombination whose wavelength of interest is smaller than the horizon size; then the evolution of the matter fluctuations can be analyzed via pressureless non-relativistic fluid, with the Newtonian gravity. Denoting the comoving coordinates by \( x \), and the conformal time by \( \eta \), the evolution equations are

\[
\dot{\delta}(x, \eta) + \nabla \cdot [(1 + \delta(x, \eta)) \mathbf{v}(x, \eta)] = 0, \\
\dot{\mathbf{v}}(x, \eta) + (\mathbf{v}(x, \eta) \cdot \nabla) \mathbf{v}(x, \eta) + \mathcal{H}(\eta) \mathbf{v}(x, \eta) = -\nabla \phi(x, \eta), \\
\nabla^2 \phi(x, \eta) = \frac{3}{2} \mathcal{H}^2(\eta) \delta(x, \eta),
\]

where \( \delta \) is the density contrast, \( \mathbf{v} \) is the peculiar velocity, \( \phi \) is the gravitational potential, \( a \) is the scale factor, a dot denotes the derivative with respect to the conformal time, and \( \mathcal{H} = \dot{a}/a \). Here, the Einstein–de Sitter universe is assumed.

We ignore the rotational mode of the velocity, since our interest is only in the growing solution, and the rotational mode is the decaying solution in the expanding universe. Then, we introduce the velocity divergence,

\[
\theta(x, \eta) \equiv \nabla \cdot \mathbf{v}(x, \eta).
\]
It is convenient to analyze the perturbations in the Fourier space, and we define the Fourier coefficients as

$$\delta(x, \eta) = \int \frac{d^3k}{(2\pi)^3} \delta(k, \eta)e^{-ik \cdot x},$$

$$\theta(x, \eta) = \int \frac{d^3k}{(2\pi)^3} \theta(k, \eta)e^{-ik \cdot x}.$$  \hspace{1cm} (5)

Then, equations (1)–(3) yield

$$\dot{\delta}(k, \eta) + \dot{\theta}(k, \eta) = -\int d^3k_1 \int d^3k_2 \delta^{(3)}(k_1 + k_2 - k) \frac{k \cdot k_1}{k_1^2} \theta(k_1, \eta) \delta(k_2, \eta),$$

$$\dot{\theta}(k, \eta) + \mathcal{H}(\eta) \theta(k, \eta) = \frac{2}{3} \mathcal{H}^2(\eta) \delta(k, \eta)$$

$$= -\int d^3k_1 \int d^3k_2 \delta^{(3)}(k_1 + k_2 - k) \frac{k^2 (k_1 \cdot k_2)}{2k_1^2 k_2^2} \theta(k_1, \eta) \theta(k_2, \eta),$$

where \(\delta^{(3)}(k)\) denotes the Dirac delta function. The right-hand side of these equations describes the mode couplings which govern the non-linear evolution of the matter fluctuations.

To solve these coupled equations, we adopt the perturbative expansion as

$$\delta(k, \eta) = \sum_{n=1}^{\infty} a^n(\eta) \delta_n(k), \quad \theta(k, \eta) = \mathcal{H}(\eta) \sum_{n=1}^{\infty} a^n(\eta) \theta_n(k).$$  \hspace{1cm} (9)

In general, the \(n\)th-order solution can be written as

$$\delta_n(k) = \int d^3q_1 \cdots \int d^3q_n \delta^{(3)} \left( \sum_{i=1}^{n} q_i - k \right) F_n(q_1, \ldots, q_n) \prod_{i=1}^{n} \delta(q_i),$$

$$\theta_n(k) = -\int d^3q_1 \cdots \int d^3q_n \delta^{(3)} \left( \sum_{i=1}^{n} q_i - k \right) G_n(q_1, \ldots, q_n) \prod_{i=1}^{n} \delta(q_i),$$

where \(F_n(q_1, \ldots, q_n)\) and \(G_n(q_1, \ldots, q_n)\) are defined as

$$F_n(q_1, \ldots, q_n) = \sum_{m=1}^{n-1} \frac{G_m(q_1, \ldots, q_m)}{(2n + 3)(n - 1)} \left[ (1 + 2n) \frac{k \cdot k_1}{k_1^2} F_{n-m}(q_{m+1}, \ldots, q_n) \right.$$

$$+ \left. \frac{k^2 (k_1 \cdot k_2)}{k_1^2 k_2^2} G_{n-m}(q_{m+1}, \ldots, q_n) \right],$$

$$G_n(q_1, \ldots, q_n) = \sum_{m=1}^{n-1} \frac{G_m(q_1, \ldots, q_m)}{(2n + 3)(n - 1)} \left[ 3 \frac{k \cdot k_1}{k_1^2} F_{n-m}(q_{m+1}, \ldots, q_n) \right.$$

$$+ \left. \frac{k^2 (k_1 \cdot k_2)}{k_1^2 k_2^2} G_{n-m}(q_{m+1}, \ldots, q_n) \right].$$

\(\n\)
Damping of baryon acoustic oscillations

Assuming that the first-order density perturbation described by $\delta_1(k)$ is a Gaussian random field, we obtain the second-order matter power spectrum

$$P_{SPT}(k, z) = D_1^2(z) P_{\text{lin}}(k) + D_1^4(z) P_2(k),$$

(14)

where $P_{\text{lin}}(k)$ is the linear power spectrum given by

$$(2\pi)^3 \delta^{(3)}(k+k') P_{\text{lin}}(k) = \langle \delta_1(k) \delta_1(k') \rangle,$$

(15)

$D_1(z)$ is the linear growth factor, and $P_2(k)$ is the second-order contribution to the power spectrum, which is conventionally expressed as follows:

$$P_2(k) = P_{22}(k) + 2P_{13}(k).$$

(16)

Taking the four-point correlations of $\delta_1(k)$ into consideration, we obtain the explicit form of $P_{22}(k)$ expressed as the integral of the square of the linear power spectrum,

$$P_{22}(k) = 2 \int d^3q \ P_{\text{lin}}(q) P_{\text{lin}}(|k-q|) [F_2^3(q, k-q)]^2,$$

(17)

where $F_2^3(q_1, q_2)$ is symmetrized over its arguments. On the other hand, $P_{13}(k)$ has a slightly different form to $P_{22}(k)$:

$$2P_{13}(k) = 6P_{\text{lin}}(k) \int d^3q \ P_{\text{lin}}(q) F_3^3(q, -q, k).$$

(18)

The solid curve in figure 1 shows typical behavior of $P_{22}(k)$ and $2P_{13}(k)$, where the cosmological parameters are $h = 0.7$, $\Omega_m = 0.28$, $\Omega_b = 0.046$, $n_s = 0.96$ and $\sigma_8 = 0.82$. We discuss further details of the typical behavior of $P_{22}(k)$ and $2P_{13}(k)$ in the next section.

Originally, this perturbation expansion was consistently formulated in the Einstein–de Sitter universe. The result is extensively used for the other general cosmological model, with the corresponding growth factor normalized as $D_1(z) = a(z)$ at $a(z) \ll 1$. We follow this prescription because the validity of the extensive use of the result is known from, for example, [36,47].

3. Damping of the BAO signature

In this section, we will examine the damping of the BAO signature due to the non-linear gravitational clustering, employing the third-order perturbation theory. The BAO signature in the matter power spectrum can be extracted as follows:

$$B(k, z) = \frac{P(k, z)}{\tilde{P}(k, z)} - 1,$$

(19)

where $P(k, z)$ is the matter power spectrum including the BAO signature, but $\tilde{P}(k, z)$ is the matter power spectrum without the BAO, which is calculated using the no-wiggle transfer function in [48]. Hereafter, a quantity having a tilde implies that the quantity was computed using the no-wiggle transfer function. Within the third-order perturbation theory, we may write

$$\tilde{P}_{SPT}(k, z) = D_1^2(z) \tilde{P}_{\text{lin}}(k) + D_1^4(z) \tilde{P}_2(k),$$

(20)
Damping of baryon acoustic oscillations

Figure 1. Typical behavior of $P_{22}(k)$ (upper solid curve) and $2P_{13}(k)$ (lower solid curve), which are calculated using the transfer function suggested by [48] with the BAO. The cosmological parameters are $h = 0.7$, $\Omega_m = 0.28$, $\Omega_b = 0.046$, $n_s = 0.96$ and $\sigma_8 = 0.82$. The dotted curves show $\tilde{P}_{22}(k)$ (upper curve) and $2\tilde{P}_{13}(k)$ (lower curve).

where $\tilde{P}_2(k)$ is

$$\tilde{P}_2(k) = \tilde{P}_{22}(k) + 2\tilde{P}_{13}(k),$$

and $\tilde{P}_{22}(k)$ and $\tilde{P}_{13}(k)$ are, respectively, defined by (17) and (18), but with the no-wiggle transfer function.

Figure 2 shows $B_{\text{SPT}}^{\text{exact}}(k, z)$ as a function of the wavenumber $k$ for several redshifts, which is obtained using the third-order perturbation theory (see also below). Here, a spatially flat universe with cold dark matter (CDM) and the cosmological constant $\Lambda$ is assumed, where the cosmological parameters are the same as those of figure 1. The oscillating behavior of the curves is the BAO signature. As the redshift becomes small, one can see that the amplitude of the oscillations decreases. This damping of the oscillations becomes more significant as $k$ becomes larger.

3.1. Analytic approach

The aim of this paper is to understand the nature of the damping of the BAO signature in detail. To this end, the analytic approach based on the third-order perturbation theory is useful. With the use of the formula of the second-order power spectra, (14) and (20), we have

$$B_{\text{SPT}}^{\text{exact}}(k, z) = \frac{P_{\text{SPT}}(k, z)}{P_{\text{SPT}}(k, z)} - 1 = \frac{P_{\text{lin}}(k) + D_{\text{lin}}^2(z)P_2(k)}{P_{\text{lin}}(k) + D_{\text{lin}}^2(z)\tilde{P}_2(k)} - 1.$$
Figure 2. $B_{\text{SPT}}^{\text{exact}}(k, z)$ as a function of $k$ for several redshifts, $z = 2$, 1, 0.5, which are derived from the matter power spectrum including the second-order contributions. The solid curve is the linear theory. The cosmological parameters are the same as those of figure 1. One can see the damping of the amplitude of the BAO as the redshift becomes small. In addition, this damping becomes more significant as the wavenumber $k$ becomes larger.

First, we adopt an approximation, 

$$P_2(k) \simeq \tilde{P}_{22}(k) + 2P_{13}(k).$$

(23)

In brief, $P_{22}(k)$ in (16) is replaced with $\tilde{P}_{22}(k)$. The validity of this approximation is demonstrated in figure 1, where the upper solid curve is $P_{22}(k)$, and the upper dotted curve is $\tilde{P}_{22}(k)$. The validity of this approximation comes from the fact that the mode coupling of different Fourier modes decreases the coherent BAO signature. As shown in this figure, the tiny oscillatory feature remains. This may induce to some extent the small shift of the peaks (or troughs) of the BAO, as mentioned in [41]. However, this shift of peak location would not be problematic, as long as we are interested in the damping of the BAO signature.

On the other hand, $P_{13}(k)$ cannot be simply replaced with $\tilde{P}_{13}(k)$. However, careful consideration leads to an expression for $\tilde{P}_{13}(k)$ in terms of the linear power spectrum multiplied by a monotonically decreasing function, as follows. First, we define

$$B_{\text{lin}}(k) \equiv \frac{P_{\text{lin}}(k)}{\tilde{P}_{\text{lin}}(k)} - 1,$$

(24)

which corresponds to (22), within the linear theory of density fluctuations. Note that $B_{\text{lin}}(k)$ is not time dependent. With this definition, we obtain

$$2P_{13}(k) = 6\tilde{P}_{\text{lin}}(k) \left[ 1 + B_{\text{lin}}(k) \right] \int d^3q \ P_{\text{lin}}(q) F^3_3(q, -q, k).$$

(25)
Damping of baryon acoustic oscillations

Here, we apply the following approximation to the linear power spectrum of the integrand:
\[
2P_{13}(k) \simeq 6\tilde{P}_{\text{lin}}(k) [1 + B_{\text{lin}}(k)] \int d^3q \tilde{P}_{\text{lin}}(q) F_3^*(q, -q, k)
\]
\[
= 2 [1 + B_{\text{lin}}(k)] \tilde{P}_{13}(k).
\]  
(26)

Substituting (26) into (23), \( P_2(k) \) is written as
\[
P_2(k) = \tilde{P}_2(k) + 2B_{\text{lin}}(k)\tilde{P}_{13}(k).
\]  
(27)

Then, from (22), we obtain
\[
B_{\text{SPT}}^{\text{exact}}(k, z) \simeq 1 + D_1^3(z) \left( \frac{2\tilde{P}_{13}(k)}{\tilde{P}_{\text{lin}}(k)} \right) \frac{\tilde{P}_2(k)}{\tilde{P}_{\text{lin}}(k)} B_{\text{lin}}(k).
\]  
(28)

This formula indicates how the BAO signature is modified as the gravitational clustering evolves, which is expressed by the BAO signature in the linear theory multiplied by the correction determined by the no-wiggle quantities and the growth factor. The second term of the denominator in (28) is small within the perturbation scheme, we may expand it as
\[
B_{\text{SPT}}^{\text{exact}}(k, z) \simeq \left[ 1 - D_1^3(z) \left( \frac{\tilde{P}_{22}(k)}{\tilde{P}_{\text{lin}}(k)} \right) \left( 1 - D_1^3(z) \left( \frac{\tilde{P}_2(k)}{\tilde{P}_{\text{lin}}(k)} \right) \right)^2 \cdots \right] B_{\text{lin}}(k),
\]  
(29)

though the higher order terms make no sense because we are working in the second-order theory of the power spectrum. The expression (29) indicates that the leading effect of the non-linear mode coupling on the damping is described by the factor \(-D_1^3(z)\tilde{P}_{22}(k)/\tilde{P}_{\text{lin}}(k)\), and that the sign of the term clearly shows that this effect is a damping.

### 3.2. Fitting formula

As we are working within the third-order perturbation theory, its prediction does not perfectly coincide with the result of full order computation, which can be obtained via \(N\)-body simulations. However, we believe that the prediction of the third-order perturbation theory is useful in constructing a semi-analytic formula which reproduces the result of \(N\)-body simulations. Then, we find here a simple fitting formula which reproduces the prediction of the third-order perturbation theory. We discuss the validity of the formula in comparison with an \(N\)-body simulation below.

Up to the second order of \(D_1(z)\), (29) yields
\[
B_{\text{SPT}}^{\text{exact}}(k, z) \simeq \left[ 1 - D_1^3(z) \left( \frac{\tilde{P}_{22}(k)}{\tilde{P}_{\text{lin}}(k)} \right) \left( 1 - D_1^3(z) \left( \frac{\tilde{P}_2(k)}{\tilde{P}_{\text{lin}}(k)} \right) \right)^2 \cdots \right] B_{\text{lin}}(k).
\]  
(30)

From a detailed analysis of \(\tilde{P}_{22}(k)/\tilde{P}_{\text{lin}}(k)\) as a function of \(k\), we find that the following fitting formula works well:
\[
\frac{\tilde{P}_{22}(k)}{\tilde{P}_{\text{lin}}(k)} = \sigma_s^2 \left( \frac{k}{k_n} \right)^2 \left( 1 - \gamma \right),
\]  
(31)
where $\sigma_8$ is the rms matter density fluctuations averaged over the sphere with the radius of $8h^{-1}\text{Mpc}$, $k_n$ and $\gamma$ are the constant parameters which depend on $\Omega_m h^2$, $\Omega_b h^2$ and $n_s$. Figures 3 and 4 show the best-fit value of $k_n$ and $\gamma$ as a function of $\Omega_m h^2$ and $\Omega_b h^2$, where we fixed $n_s = 0.96$. This shows that these two parameters depend on $\Omega_m h^2$ and $\Omega_b h^2$ linearly. We can show a similar dependence on $n_s$. Then, we found the following fitting formula:

$$k_n = -1.03(\Omega_m h^2 + 0.777)(\Omega_b h^2 - 0.24)(n_s + 0.92)\, h\, \text{Mpc}^{-1},$$

(32)

$$\gamma = -11.4(\Omega_m h^2 - 0.50)(\Omega_b h^2 - 0.076)(n_s - 0.34)\, h\, \text{Mpc}^{-1}.\quad (33)$$

Though these dependences on the cosmological parameter might have to be investigated more carefully, the validity is guaranteed to be in the following narrow range:

$$0.13 \lesssim \Omega_m h^2 \lesssim 0.15, \quad (34)$$

$$0.022 \lesssim \Omega_b h^2 \lesssim 0.024, \quad (35)$$

$$0.94 \lesssim n_s \lesssim 0.98. \quad (36)$$

In the present paper, we simply assume that the dependence on the other cosmological parameters, except for $\Omega_m h^2$, $\Omega_b h^2$ and $n_s$, can be ignored.

Finally, we obtain the heuristic expression for the leading correction for the BAO signature:

$$B_{\text{SPT}}^\text{fit}(k, z) = \left[ 1 - f(\Omega_m h^2, \Omega_b h^2, n_s, \sigma_8, k, z) \right] B_{\text{lin}}(k),$$

(37)
with
\[ f(\Omega_m h^2, \Omega_b h^2, n_s, \sigma_8, k, z) = \sigma_8^2 D_1^2(z) \left( \frac{k}{k_n} \right)^2 \left( 1 - \frac{\gamma}{k} \right). \]
(38)

Figure 4. The best-fit \( \gamma \) as a function of \( \Omega_m h^2 \) and \( \Omega_b h^2 \). The other cosmological parameters are the same as those of figure 1.

Figure 5 demonstrates the agreement between this fitting formula, \( B_{SPT}^{\text{fit}} \), and the prediction of the third-order perturbation theory, \( B_{SPT}^{\text{exact}} \), (22). As one can see from this figure, the agreement becomes worse as the redshift becomes lower and the wavenumber larger. Figure 6 shows the relative error of the fitting formula, \( \left| B_{SPT}^{\text{fit}} - B_{SPT}^{\text{exact}} \right| / B_{SPT}^{\text{exact}} \), as a function of the redshift at the wavenumbers of P1, P2, P3, T1, T2 and T3, which are defined in figure 5. The relative error is less than about 10% at worst in the range of the wavenumber \( k \lesssim 0.2 h \, \text{Mpc}^{-1} \) until \( z \sim 1 \). The sign of \( B_{SPT}^{\text{fit}} - B_{SPT}^{\text{exact}} \) for the third peak (trough) changes around the redshift 2.7 (1.4), where the relative error becomes zero. For redshift less than 1, the agreement becomes worse, especially at large wavenumber \( k \gtrsim 0.2 h \, \text{Mpc}^{-1} \). However, we should also note that the third-order perturbation theory becomes worse for reproducing \( N \)-body simulations for lower redshift and for larger wavenumbers.

Figure 7 shows a comparison of our fitting formula (\( B_{SPT}^{\text{fit}} \), solid curve) and a result of \( N \)-body simulation (squares with error bar) [49]. This demonstrates that our fitting formula reproduces the result of \( N \)-body simulation for \( k \lesssim 0.2 h \, \text{Mpc}^{-1} \) until \( z \sim 1 \) within error bars, roughly. In the conclusion, note that we focus on the damping of the BAO signature, not on the amplitude of the power spectrum itself.

In the \( N \)-body simulation, we adopt a \( \Lambda \)CDM cosmology (\( \Omega_m = 0.279 \), \( \Omega_\Lambda = 0.721 \), \( \Omega_b/\Omega_m = 0.165 \), \( h = 0.701 \), \( n_s = 0.96 \), \( \sigma_8 = 0.817 \); WMAP5 best-fit values [50]), and calculate the linear matter power spectrum using CAMB [51]. We adopt 512\(^3\) particles in periodic cubes with each side 1000\(h^{-1}\) Mpc, and displace \( N \)-body particles using the second-order Lagrangian perturbation theory (e.g., [52]) from uniform grid positions at
Damping of baryon acoustic oscillations

Figure 5. This figure compares our fitting formula ($B_{\text{fit}}^{\text{SPT}}$, dashed curve), (37), and the formula for the third-order perturbation ($B_{\text{exact}}^{\text{SPT}}$, solid curve), (22), for various redshifts ($z = 3, 2, 1, 0.5$). The dotted curve shows the linear theory. $\delta B$ is the relative difference between the solid curve and the dotted curve ($\delta B = B_{\text{fit}}^{\text{SPT}} - B_{\text{exact}}^{\text{SPT}}$). Here, the cosmological parameters are the same as those of figure 1. P1, P2 and P3 (T1, T2 and T3) denote the first, the second and the third peak (trough), respectively, used in figure 6.

$z = 31$. The simulations are carried out using the Gadget2 code [53] to output data for four redshifts ($z = 3, 2, 1, 0.5$). Our total simulation volume is $8h^{-3}\text{Gpc}^3$, which might be small for investigating our scales of interest (first a few BAO peaks). Then we correct the deviations from the ideal case of infinite volume, as follows.

In addition to the time integration using Gadget2, we also calculate the time evolution of the density contrast using the second-order perturbation theory starting from the same initial condition as was used for the $N$-body simulation. We then calculate the power spectrum from the result from the perturbation theory, and measure the deviation from the linear power spectrum at each output time. The deviation of our finite volume simulation is well explained by the mode couplings predicted by the second-order perturbation theory. Then, we can obtain a corrected power spectrum by multiplying the measured power
spectrum from $N$-body simulations by the ratio of the linear power spectrum to the power spectrum predicted by the perturbation theory. Thus the error bars in the figure stand for the remaining standard errors after this correction, which are very small compared with the usual cases in the region of the small wavenumbers. This is because the perturbation theory used for the correction is more accurate at small wavenumbers, which can remove the deviations from the linear power spectrum better. This correction does not improve the errors at large wavenumbers (see [24], [49] for more details). Moreover, in order to construct the no-wiggle spectrum required for computing the ratio $B$, we use a cubic basis-spline fitting with the same break points as in [27] (see also [8]).

3.3. Possible extension and discussion

There are other possible effects which may affect the damping of the BAO signature: the higher order non-linear effect, the redshift-space distortions and the clustering bias. These effects may be influential for the damping of the BAO signature; however, there remain a lot of uncertainties concerning these effects, at present. Here, let us consider a possible extension of our work to include the higher order non-linear corrections in real space.

The standard perturbation theory is useful for analyzing the damping of the BAO signature in an analytic way; however, it is not enough for precise predictions that match with the result of $N$-body simulations in the regime where the non-linear effect becomes significant. Recently, several authors have developed a non-perturbative approach to the non-linear density clustering, as mentioned above [37]–[46].

As an alternative to the SPT, we here consider the work proposed by [46], which uses the technique of resumming infinite series of higher order perturbations on the basis of
the Lagrangian perturbation theory (LPT). One of the advantages of this approach is the simplicity of the resulting expression for the non-linear power spectrum, which enables us to incorporate the result into our formula. In the framework of the LPT [46], the matter power spectrum can be given by

\[
P_{\text{LPT}}(k, z) = e^{-D_1(z)^2g(k)}[D_1(z)^2P_{\text{lin}}(k) + D_1(z)^4P_2(k) + D_1(z)^4P_{\text{lin}}(k)g(k)],
\]

(39)

where

\[
g(k) = \frac{k^2}{6\pi^2} \int dq \, P_{\text{lin}}(q).
\]

(40)
We define, corresponding to (22), the BAO signature of the matter power spectrum based on the LPT:

\[ B_{\text{LPT}}^{\text{exact}}(k, z) = \frac{P_{\text{LPT}}(k, z)}{P_{\text{LPT}}(k, z)} - 1, \]  

(41)

where \( P_{\text{LPT}}(k, z) \) is defined by (39) but with the no-wiggle transfer function. Adopting an approximation, \( g(k) \approx \tilde{g}(k) \), we can obtain the following expression:

\[ B_{\text{LPT}}^{\text{exact}}(k, z) \approx \frac{1 + (D_2^l(z)/(1 + D_2^l(z)\tilde{g}(k)))(2\tilde{P}_{33}(k)/\tilde{P}_{\text{lin}}(k))}{1 + D_2^l(z)/(1 + D_2^l(z)\tilde{g}(k))}(\tilde{P}_2(k)/\tilde{P}_{\text{lin}}(k))B_{\text{lin}}(k). \]  

(42)

Repeating the same procedure from (28) to (30), one can obtain the leading correction to the BAO:

\[ B_{\text{LPT}}^{\text{exact}}(k, z) \approx \left[ 1 - \frac{D_2^l(z)}{1 + D_2^l(z)\tilde{g}(k)} \frac{\tilde{P}_{22}(k)}{\tilde{P}_{\text{lin}}(k)} \right] B_{\text{lin}}(k), \]  

(43)

and then we obtain

\[ B_{\text{LPT}}^{\text{fit}}(k, z) = \left[ 1 - \frac{f(\Omega_m h^2, \Omega_b h^2, n_s, \sigma_8, k, z)}{1 + D_2^l(z)\tilde{g}(k)} \right] B_{\text{lin}}(k), \]  

(44)

as an extended fitting formula. Note that this reduces to the expression (30) in the limit of \( \tilde{g}(k) \rightarrow 0 \). Comparing (44) and (37), the difference is the contribution from \( D_2^l(a)\tilde{g}(k) \) in the denominator in front of \( \tilde{P}_{22}(k)/\tilde{P}_{\text{lin}}(k) \). Since \( \tilde{g}(k) \) is positive, this correction makes the damping of the BAO signature weaker compared with (37). In figure 7, the dashed curve plots \( B_{\text{LPT}}^{\text{fit}}(k, z), \) (44).

In order to show the validity of the fitting formula, figure 8 plots the relative errors \( \delta B/B \) at the wavenumbers of the peaks and troughs as a function of the redshift; it is the same as figure 6 but with the LPT instead of the SPT. As one can see from this figure, the fitting function \( B_{\text{LPT}}^{\text{fit}} \) reproduces the damping of the BAO signature \( B_{\text{LPT}}^{\text{exact}} \) better, compared with the case of the SPT, especially for larger wavenumber, even at lower redshift. The relative error is less than 10% in the range of the wavenumber \( k \lesssim 0.2h \text{ Mpc}^{-1} \) (near the third peak) until the present epoch, \( z = 0 \).

Below, we investigate other possible ways of describing the damping of the BAO signature due to the non-linear effect.

One of the other possible formulas for the fitting function is an exponential function, as has been discussed in references (e.g., [7, 20, 22, 54]). Angulo et al [22] discussed the following fitting formula with the Gaussian damping function:

\[ B(k, z) = \exp \left[ -\left( \frac{k}{\sqrt{2}k_*(z)} \right)^2 \right] B_{\text{lin}}(k, z), \]  

(45)

where \( k_*(z) \) is a time-dependent free parameter, which should be calibrated with measurements from N-body simulations. This free parameter describes the time dependence of the BAO damping, and Sanchez et al [54] obtained the best-fit value \( k_*(z) = 0.172h \text{ Mpc}^{-1} \) at \( z = 1 \), from their result from N-body simulations, in [21]. Their model includes another parameter, \( \alpha \), which describes the shift of the BAO scale in the wavenumber. However, in the present paper, we adopt the case of no shift of the BAO.
Damping of baryon acoustic oscillations

Figure 8. Same as figure 6 but with the LPT instead of the SPT. The relative error is $|\delta B/B| = |B_{\text{fit}}^\text{LPT} - B_{\text{exact}}^\text{LPT}|/|B_{\text{LPT}}^\text{exact}|$.

scale ($\alpha = 1$) because our interest is focused on the damping of the BAO signature not on the shift of the BAO scale. It is also shown that $\alpha$ almost equals 1 [21].

Figure 9 compares our fitting functions and the Gaussian damping function. The vertical axis stands for $B_{\text{fit}}^\text{SPT}(k,z=1)$ divided by $B_{\text{lin}}^\text{SPT}(k,z=1)$, which means the damping function. The dotted curve and the solid curve are $B_{\text{fit}}^\text{SPT}$ and $B_{\text{fit}}^\text{LPT}$, respectively. The dashed curve is the Gaussian damping function, (45), with $k_*(z) = 0.172 h$ Mpc$^{-1}$. $B_{\text{fit}}^\text{SPT}$ becomes negative as the wavenumber becomes larger, $k \gtrsim 0.25 h$ Mpc$^{-1}$. This is because the phase inversion of the BAO signature appears in the SPT, as shown in the panel for $z = 1$ in figure 5, where the SPT breaks down in this regime as mentioned in section 3.2. On the other hand, the damping function based on the LPT, $B_{\text{fit}}^\text{LPT}/B_{\text{lin}}$, shows similar behavior to the Gaussian damping function, which approaches zero in the limit of $k \to 0.7$. This is due to the modification by the factor $1/[1 + D_1^2(z) g(k)]$ in (44). $B_{\text{fit}}^\text{LPT}/B_{\text{lin}}$ becomes negative for $k \gtrsim 0.7 h$ Mpc$^{-1}$ for the same reason as $B_{\text{fit}}^\text{SPT}/B_{\text{lin}}$.

In figure 9, we also plot the following damping function (dot–dashed curve):

$$B_{\text{exp}}^\text{SPT}(k,z) = \exp \left[-f(\Omega_m h^2, \Omega_b h^2, n_s, \sigma_8, k, z)\right] B_{\text{lin}}(k).$$

The leading term of the expansion of this damping function with respect to $D_1^2(z)$ leads to $B_{\text{fit}}^\text{SPT}$, (37). As one can see, this damping function $B_{\text{exp}}^\text{SPT}$ agrees with the Gaussian damping function, (45). This means that the higher order perturbations is important in describing the BAO signature for the regime of the wavenumber, $k \gtrsim 0.2 h$ Mpc$^{-1}$ at redshift 1.

The result of this section gives us a clue as regards how to describe the BAO damping due to the non-linear gravitational clustering. The BAO damping is determined by the normalization $\sigma_8$ and the growth factor $D_1(z)$. This suggests that a measurement of the BAO damping might be useful for estimating the growth factor of the amplitude of the
Figure 9. Comparison of fitting functions for the damping. The dotted curve is $B_{\text{SPT}}^{\text{fit}}$, (37), while the solid curve is $B_{\text{LPT}}^{\text{fit}}$, (44). The dashed curve is the Gaussian damping function, (45), with $k_\ast(z) = 0.172 h \text{ Mpc}^{-1}$. The dot–dashed curve is $B_{\text{SPT}}^{\text{exp}}$, (46). In this plot, we adopted the same cosmological parameters as are used in [54].

density perturbations, $\sigma_8 D_1(z)$. In section 4, we demonstrate the feasibility of constraining $\sigma_8 D_1(z)$ by measuring the damping of the BAO signature.

4. Feasibility of constraining $\sigma_8 D_1(z)$

As shown in the previous section, the damping of the BAO signature is closely related to the amplitude of the power spectrum, which is determined by $\sigma_8 D_1(z)$, while the BAO signature within the linear theory is determined by the density parameters $\Omega_m h^2$ and $\Omega_b h^2$. In this section, we discuss the feasibility of constraining $\sigma_8 D_1(z)$ by measuring the damping of the BAO in the power spectrum in the quasi-non-linear regime. To this end, the formula developed in the previous section is useful.

As mentioned in sections 1 and 3.3, the redshift-space distortions and the clustering bias might be influential, additionally to the damping of the BAO signature. However, we here assume an optimistic case where the damping is determined by the quasi-non-linear clustering effect and neglect the effects on the damping of the BAO signature from the redshift-space distortions and the clustering bias. Then, we study how a measurement of the damping is useful for determining $\sigma_8 D_1(z)$. Very recently, it has been recognized that a measurement of the growth factor of the density perturbations is a key to distinguishing between the dark energy model and modified gravity model for the cosmic accelerated expansion (e.g., [55, 56], and references therein). Our investigation is the first step towards
investigating whether a measurement of the BAO damping is useful for measuring the growth factor.

In our investigation, we adopt a simple Monte Carlo simulation of the galaxy power spectrum, assuming the ΛCDM model. The error of the galaxy power spectrum depends on its amplitude. For definiteness, we assume that the galaxy power spectrum \( P_{\text{gal}}(k, z) \) can be modeled with the no-wiggle linear power spectrum in real space, \( \tilde{P}_{\text{lin}}(k, z) \), as

\[
P_{\text{gal}}(k, z) = [1 + B(k, z)] \tilde{P}_{\text{gal}}(k, z),
\]

with

\[
\tilde{P}_{\text{gal}}(k, z) = b^2 \frac{1 + Qk^2}{1 + A_1 k + A_2 k^2} \tilde{P}_{\text{lin}}(k, z),
\]

where we use (44) as \( B(k, z) \), \( b \) is a constant bias factor, and \( A_1, A_2 \) and \( Q \) are the parameters which describe the correction of the non-linear clustering, the redshift-space distortions and the scale-dependent bias to the no-wiggle power spectrum. This model is based on the \( Q \)-model of Cole et al [57], and was elaborated by Sanchez et al [54] by adding the new parameter \( A_2(=Q/10) \) which better reproduces the non-linear power spectrum at large wavenumber. Cole et al [57] showed that, from numerical study, the value of \( A_1 = 1.4 \) is adequate for reconstructing the galaxy power spectrum in redshift space, though \( Q \) strongly depends on the galaxy type [57]–[59]. Then, for simplicity, we here adopt \( A_1 = 1.4 \) and consider the cases \( Q = 4, 8, 16, 32 \), to estimate the error for the galaxy power spectrum. We assume the bias parameter \( b = 2 \), for simplicity.

To estimate the constraint on the growth factor, we perform a simple Monte Carlo simulation. We assume that the BAO signature, \( B(k, z) \), can be extracted from a galaxy power spectrum by a method like that in [5]. The variance of the error in measuring the BAO signature can be estimated as

\[
\Delta B^2(k) = \frac{\Delta P_{\text{gal}}^2(k, z)}{[P_{\text{gal}}(k, z)]^2},
\]

with

\[
\Delta P_{\text{gal}}^2(k, z) = 2 \frac{(2\pi)^3}{\Delta V_k} Q^2(k, z),
\]

where \( \Delta V_k = 4\pi k^2 \Delta k \) is the volume of the shell in the Fourier space, and \( Q^2(k, z) \) is defined as

\[
Q^{-2}(k, z) = \Delta A \int dz \frac{d^2s[z]}{dz} s^2[z] \frac{\bar{n}(s[z])}{[1 + \bar{n}(s[z])P_{\text{gal}}(k, s[z])]^2},
\]

where \( \bar{n}(s) \) is the comoving mean number density, \( s = s[z] \) is the comoving distance-redshift relation, and \( \Delta A \) is the survey area.

With the use of the above formulas, we assess \( \chi^2 \) defined by

\[
\chi^2 = \sum_i \frac{[B(k_i, z)^{\text{th}} - B(k_i, z)^{\text{obs}}]^2}{\Delta B^2(k_i, z)},
\]

where \( B(k_i, z)^{\text{th}} \) is the theoretical form of a fiducial target model at the wavenumber \( k_i \), while \( B(k_i, z)^{\text{obs}} \) is the corresponding observational one. The fiducial target model is \( \Omega_m = 0.28 \) and \( \sigma_8 = 0.82 \), \( \Omega_b = 0.046 \), \( h = 0.7 \), and \( n_s = 0.96 \), which are the same as
Figure 10. The solid curve is the prediction of the fiducial model, whose parameters are the same as those of figure 1, while the square with the error bar is an example of $B^{\text{obs}}(k_i, z = 0.9)$, obtained through our Monte Carlo simulation. For the galaxy sample, we assumed the WFMOS-like sample of comoving mean number density $\bar{n} = 5.0 \times 10^{-4} \, [h^{-1} \text{Mpc}]^{-3}$ and the survey area 2000 deg$^2$ in the range of redshift $0.5 < z < 1.3$. The bin size of the Fourier space is $\Delta k = 0.01 h \text{Mpc}^{-1}$.

The values adopted in figure 1. $B(k_i, z)^{\text{obs}}$ is obtained through a Monte Carlo simulation, following the steps below.

(i) On the basis of the fiducial model, compute $B(k_i, z)$ and the variance $\Delta B(k_i, z)$ at $k_i = \Delta k(i - 0.5)$, for $i = 4, 5, \ldots, 19$. Here we specify a bin of the Fourier space, $\Delta k = 0.01 h \text{Mpc}^{-1}$, and consider the range of $0.03 < k < 0.19$, where the validity of our formula in the previous section is guaranteed. We assume two galaxy redshift samples as typical future surveys. One is a WFMOS-like sample, $\Delta A = 2000 \text{deg}^2$ in the range of redshift $0.5 < z < 1.3$, and $\bar{n} = 5.0 \times 10^{-4} \, [h^{-1} \text{Mpc}]^{-3}$, which contains $2.1 \times 10^8$ galaxies. The other sample has the same number density and the range of the redshift, but a larger survey area, $\Delta A = 4\pi \text{sr}$.

(ii) Each $B(k_i, z)^{\text{obs}}$ is obtained through a random process assuming the Gaussian distribution function with the variance $\Delta B^2(k_i, z)$. The data points in figure 10 show an example of a set of $B(k_i, z)^{\text{obs}}$ generated through the random process.

(iii) We assess the values of $\chi^2$ with this sample obtained by the steps (i) and (ii).

We iterate these steps and compute 1000 sets of $\chi^2$, then obtain the average of $\chi^2$. For the theoretical model, $B(k_i, z)^{\text{th}}$, we fixed $\Omega_m h^2$, $h$, and $n_s$ as those of the fiducial model, but took $\Omega_m h^2$ and $\sigma_8 D_1(z = 0.9)$ as the variable parameters. Solid curves in figure 11 show the contours of $\Delta \chi^2 = 2.3$ (inner curve) and $\Delta \chi^2 = 6.17$ (outer
Figure 11. Contours of $\Delta \chi^2$ in the $\Omega_m h^2$ and $\sigma_8 D_1(z = 0.9)$ plane. The solid curves assume the WFMOS-like sample with the survey area $\Delta A = 2000$ deg$^2$. The inner (outer) curve is the contour of $\Delta \chi^2 = 2.3 (\Delta \chi^2 = 6.17)$, which corresponds to the 1$\sigma$ (2$\sigma$) confidence level. The dotted curve is the same as the solid curve, but assumes the survey area $\Delta A = 4\pi$ sr.

From figure 11, one can read that the 1$\sigma$ error of $\sigma_8 D_1(z = 0.9)$ is about 0.2 for the sample with $\Delta A = 2000$ deg$^2$, and is about 0.05 for the sample with $\Delta A = 4\pi$ sr. Thus, the constraint on $\sigma_8 D_1(z = 0.9)$ for the WFMOS-like sample ($\Delta A = 2000$ deg$^2$) is not very stringent. We already have the small uncertainty of $\sigma_8 D_1(z = 0.9)$ at the level of a few $\times$ 0.01 [36], from the result from the WMAP observations, which is obtained on the basis of the flat CDM cosmological model with the cosmological constant. However, we note that the BAO damping is unique and independent for obtaining a constraint on $\sigma_8 D_1(z)$ around the redshift 1.

It would be useful to discuss the origin of the error. To this end, we adopt the Fisher matrix approach. As we are considering the constraint from the damping of the BAO
Damping of baryon acoustic oscillations

Figure 12. The 1σ-level statistical errors of $\Omega_m h^2$ (solid curve) and $\sigma_8 D_1(z = 0.9)$ (dotted line) as a function of the number density $\bar{n}(b/2)^2$. The thick curves assume $\Delta A = 2000$ deg$^2$, and the thin curves assume $\Delta = 4\pi$ sr. In this figure, we fixed $Q = 16$.

signature, we may work with the formula for the Fisher matrix

$$F_{ij} = \frac{1}{4\pi^2} \int_{k_{\min}}^{k_{\max}} dk k^2 \frac{\partial B(k, z)}{\partial \theta_i} \frac{\partial B(k, z)}{\partial \theta_j} \frac{\tilde{P}_{\text{gal}}^2(k, z)}{Q^2(k, z)}$$

where $\theta_i$ denotes cosmological parameters, for which we focus on $\sigma_8 D_1(z)$ and $\Omega_m h^2$, $\tilde{P}_{\text{gal}}(k, z)$ is defined by (48), $Q^2(k, z)$ is defined by (51), we adopt (44) as $B(k, z)$, and $k_{\min} = 0.02h$ Mpc$^{-1}$ and $k_{\max} = 0.2h$ Mpc$^{-1}$. The minimum error attainable on $\theta_i$ is expressed by the diagonal part of the inverse Fisher matrix, $\Delta \theta_i = F^{-1/2}_{ii}$, if the other parameters are known. Figure 12 plots the errors $\Delta(\sigma_8 D_1(z = 0.9))$ and $\Delta(\Omega_m h^2)$, obtained from the Fisher matrix, as a function of the number density $\bar{n}(b/2)^2$. The thick (thin) solid curve is $\Delta(\sigma_8 D_1(z = 0.9))$ for the sample with $\Delta A = 2000$ deg$^2$ ($\Delta A = 4\pi$ rad). The dotted curve is $\Delta(\Omega_m h^2)$. Here we adopted $Q = 16$, but the result is not sensitive to this choice. One can read that the value of the error at the point $\bar{n}(b/2)^2 = 5 \times 10^{-4} [h^{-1} \text{Mpc}]^{-3}$ is consistent with the result of figure 11.

Figure 12 demonstrates that the constraint can be improved by increasing the number density of the galaxy sample, but cannot be improved for $\bar{n}(b/2)^2 > 10^{-3} [h^{-1} \text{Mpc}]^{-3}$. The reason can be explained using figure 13, which plots $\bar{n} P_{\text{gal}}(k, z = 0.9)$ as a function of the wavenumber, where we fixed $\bar{n}(b/2)^2 = 5 \times 10^{-4} [h^{-1} \text{Mpc}]^{-3}$. In the region $k < 0.2h$ Mpc$^{-1}$, we have $\bar{n} P_{\text{gal}}(k, z = 0.9) > 1$, which means that the shot noise is not the dominant component of the error. However, the constraint on the parameter is slightly improved by increasing the number density $\bar{n}$, depending on the bias $b$.  

Journal of Cosmology and Astroparticle Physics 10 (2008) 031 (stacks.iop.org/JCAP/2008/i=10/a=031)
Figure 13. The galaxy power spectrum multiplied by the mean number density, $\tilde{n}P_{\text{gal}}(k, z = 0.9)$, as a function of $k$. Here, we fixed $\bar{n}(b/2)^2 = 5 \times 10^{-4} \, [\text{h}^{-1} \text{Mpc}]^{-3}$. We have $\tilde{n}P_{\text{gal}}(k, z = 0.9) > 1$ for $k < 0.2h \, \text{Mpc}^{-1}$, where the shot noise is subdominant.

Finally in this section, we mention the effect of the redshift-space distortions and the clustering bias on the BAO damping, which might be influential in measuring $\sigma_8 D_1(z)$. It would be true that there still remains room to investigate how the redshift-space distortions and the clustering bias affect the BAO damping. If the effect of the redshift-space distortions and the clustering bias on the BAO damping could not be clarified, it would make it difficult to conclude that the BAO damping is useful. However, very recently, several authors have discussed the issue [54, 60, 61]. If the effect on the BAO damping is well understood, it can be a signal, and would be useful in measuring the growth factor.

5. Summary and conclusions

In the present paper, we examined the effect of the non-linear gravitational clustering on the BAO signature in the matter power spectrum. In particular, we focused on the damping of the BAO signature in the quasi-non-linear regime. Our approach is based on the third-order perturbation theory of the matter fluctuations, which enables us to investigate the damping in an analytic way. We found a simple analytic expression that describes the damping of the BAO signature, which clarifies what the important factor is for the damping. We showed that the leading correction for the damping is in proportion to the combination of $(\sigma_8 D_1(z))^2$. On the basis of the result, we constructed a fitting formula for the correction of the damping of the BAO signature in the weakly non-linear

\[^3\] Note that the Fisher matrix depends on the parameter $\bar{n}b^2$. 

regime, which is expressed as a function of $k$, $\Omega_m h^2$, $\Omega_b h^2$ and $n_s$. This fitting formula reproduces the damping of the BAO signature of the second-order power spectrum within the standard perturbation theory at 10% level for $k \lesssim 0.2 h \, \text{Mpc}^{-1}$ until $z \sim 1$, though the formula is only guaranteed in a narrow range of the parameters $\Omega_m h^2$, $\Omega_b h^2$ and $n_s$.

We also discussed a possible extension of our formula for elaborating higher order non-linear corrections using a technique of resumming infinite series of higher order perturbations on the basis of the Lagrangian perturbation theory. This extended formula reproduces the damping of the BAO signature of the second-order power spectrum based on the Lagrangian perturbation theory at 10% level for $k \lesssim 0.2 h \, \text{Mpc}^{-1}$ until $z \sim 0$. This formula was compared with a result of $N$-body simulation, which showed the validity of the extended formula.

A measurement of the damping of the BAO signature might be useful as a probe of the growth factor of the density fluctuations. As a first step towards investigating such the possibility, we assessed the feasibility of constraining $\sigma_8 D_\| (z)$ by measuring the damping of the BAO in the power spectrum. For a useful constraint, we need a very wide survey area of the sky. For a definite conclusion, however, we must include the effect of the redshift-space distortions and the galaxy clustering bias on the damping of the BAO signature. Thus, a more sophisticated formula including the effect of the redshift-space distortions and the galaxy clustering bias at the same time is required. If this effect on the BAO damping was well understood, it could be useful in measuring the growth factor.

We plan to revisit this issue in future work.

Acknowledgments

We thank A Taruya, T Matsubara, Y Suto and G Hütsi for useful discussions and comments. We are also grateful to an anonymous referee for useful comments which helped to improve the earlier version of the manuscript. TN is supported by a Grant-in-Aid from Japan Society for the Promotion of Science (JSPS) Fellows (DC1: 19-7066). This work was supported by a Grant-in-Aid for Scientific Research from the Japanese Ministry of Education, Culture, Sports and Technology (Nos. 18540277, 19035007).

References

[1] Albrecht A et al, 2006 arXiv:astro-ph/0609591
[2] Peacock J A et al, 2006 arXiv:astro-ph/0610906
[3] Eisenstein D J et al, 2005 Astrophys. J. 633 560 [SPIRES]
[4] Hüetsi G, 2006 Astron. Astrophys. 449 891 [SPIRES]
[5] Percival W J et al, 2007 Astrophys. J. 657 645 [SPIRES]
[6] Percival W J et al, 2007 Astrophys. J. 657 51 [SPIRES]
[7] Tegmark M et al, 2006 Phys. Rev. D 74 123507 [SPIRES]
[8] Percival W J et al, 2007 Mon. Not. R. Astron. Soc. 381 1053
[9] Okumura T et al, 2008 Astrophys. J. 677 889 [SPIRES]
[10] Hüetsi G, 2006 Astron. Astrophys. 459 375 [SPIRES]
[11] http://www.sdss3.org/
[12] Totani T, 2006 private communication
[13] Bassett B A, Nichol R C and Eisenstein D J (the WFMOS Feasibility Study Dark Energy Team), 2005 arXiv:astro-ph/0510272
[14] http://www.lsst.org/
[15] http://www.skatelescope.org/
[16] Robberto M et al, 2007 arXiv:0710.3970
[17] Seo H-J and Eisenstein D J, 2003 Astrophys. J. 598 720 [SPIRES]
Damping of baryon acoustic oscillations

[18] Seo H-J and Eisenstein D J, 2005 Astrophys. J. 633 575 [SPIRES]
[19] Seo H-J and Eisenstein D J, 2007 arXiv:astro-ph/0701079
[20] Seo H-J et al., 2008 arXiv:0805.0117
[21] Angulo R et al., 2005 Mon. Not. R. Astron. Soc. Lett. 362 L25
[22] Angulo R et al., 2008 Mon. Not. R. Astron. Soc. 383 755
[23] Shoji M et al., 2008 arXiv:0805.4238
[24] Takahashi R et al., 2008 arXiv:0802.1808
[25] Wang Y, 2006 Astrophys. J. 647 1 [SPIRES]
[26] McDonald P and Eisenstein D, 2007 Phys. Rev. D 76 063009 [SPIRES]
[27] Nishimichi T et al., 2007 Publ. Astron. Soc. Japan 59 1049
[28] Juszkiewicz R, 1981 Mon. Not. R. Astron. Soc. 197 931
[29] Vlahic E T, 1983 Mon. Not. R. Astron. Soc. 203 345
[30] Fry J N, 1984 Astrophys. J. 279 499 [SPIRES]
[31] Goroff M H et al., 1986 Astrophys. J. 311 6 [SPIRES]
[32] Madau P, Sasaki M and Suto Y, 1992 Phys. Rev. D 46 585 [SPIRES]
[33] Jain B and Bertschinger E, 1994 Astrophys. J. 431 495 [SPIRES]
[34] Bernardeau F, Colombi S, Gaztanaga E and Scoccimarro R, 2002 Phys. Rep. 367 1 [SPIRES]
[35] Saito S, Takada M and Taruya A, 2008 Phys. Rev. Lett. 100 191301 [SPIRES]
[36] Jeong D and Komatsu E, 2006 Astrophys. J. 651 619 [SPIRES]
[37] Valageas P, 2004 Astron. Astrophys. 421 23 [SPIRES]
[38] Valageas P., 2007 Astron. Astrophys. 465 725 [SPIRES]
[39] Crocce M and Scoccimarro R, 2006 Phys. Rev. D 73 063519 [SPIRES]
[40] Croce M and Scoccimarro R, 2006 Phys. Rev. D 73 063520 [SPIRES]
[41] Croce M and Scoccimarro R, 2008 Phys. Rev. D 77 023533 [SPIRES]
[42] McDonald P, 2007 Phys. Rev. D 75 043514 [SPIRES]
[43] Matarrese S and Pietroni M., 2007 J. Cosmol. Astropart. Phys. JCAP06(2007)026 [SPIRES]
[44] Izumi K and Soda J, 2007 Phys. Rev. D 76 083517 [SPIRES]
[45] Taruya A and Hiramatsu T, 2007 arXiv:0708.1367
[46] Matsubara T, 2008 Phys. Rev. D 77 063530 [SPIRES]
[47] Bernardeau F., 1994 Astrophys. J. 433 1 [SPIRES]
[48] Eisenstein D J and Hu W, 1998 Astrophys. J. 496 605 [SPIRES]
[49] Nishimichi T et al., 2008 arXiv:0810.0813
[50] Komatsu E et al., 2008 arXiv:0803.0547
[51] Lewis A et al., 2000 Astrophys. J. 538 473 [SPIRES]
[52] Croce M et al., 2006 Mon. Not. R. Astron. Soc. 373 369
[53] Springel V, 2005 Mon. Not. R. Astron. Soc. 364 1105
[54] Sanchez A G et al., 2008 arXiv:0804.0233
[55] Yamamoto K et al., 2007 Phys. Rev. D 76 023504 [SPIRES]
[56] Yamamoto K, Sato T and H{"u}ttsi G, 2008 Prog. Theor. Phys. 120 609 [SPIRES]
[57] Cole S et al., 2005 Mon. Not. R. Astron. Soc. 362 505
[58] Sanchez A G and Cole S, 2008 Mon. Not. R. Astron. Soc. 385 830
[59] Hamann J et al., 2008 J. Cosmol. Astropart. Phys. JCAP07(2008)017 [SPIRES]
[60] Matsubara T, 2008 arXiv:0807.1733
[61] Jeong D and Komatsu E., 2008 arXiv:0805.2632