Sivers and Collins Effects: from SIDIS to Proton-Proton Inclusive Pion Production

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We consider the Sivers, Collins and transversity functions as extracted from SIDIS and $e^+e^-$ experimental data and investigate to what extent they might explain the large Single Spin Asymmetries (SSA) observed in proton-proton inclusive processes. This phenomenological study is performed within the TMD factorization scheme. As the SIDIS data cover only a limited range of $x$ values ($x \sim < 0.3$), we allow for different large $x$ behaviours of the SIDIS Sivers functions and transversity distributions. We conclude that, within the available experimental constraints, one cannot observe any clear universality breaking effect for the Sivers functions.

We report \cite{1} on some work in progress, exploring the simple phenomenological idea of adopting the same Sivers, Collins and transversity functions, as extracted from SIDIS and $e^+e^-$ experimental data \cite{2 3}, to evaluate the corresponding Sivers and Collins effects in proton-proton scattering, assuming a TMD factorized scheme. These effects are then compared to the RHIC proton-proton data on SSAs at $\sqrt{s} = 200$ GeV \cite{4 5}.

The $A_{UT}^{Sivers}$ transverse single spin asymmetry, measured by the HERMES \cite{6 7} and COMPASS \cite{8-10 11} collaborations in $\ell N \rightarrow \ell h X$ SIDIS processes, has been analyzed according to the expression:

$$A_{UT}^{Sivers} \propto \frac{\sum q e_q^2 \Delta \hat{f}_{q/p}(x, k_\perp) \otimes \frac{d^2 \sigma_{\ell q \rightarrow \ell q}}{dq^2} \otimes D_{h/q}(z, p_\perp)}{2 \sum q e_q^2 f_{q/p}(x, k_\perp) \otimes \frac{d^2 \sigma_{\ell q \rightarrow \ell q}}{dq^2} \otimes D_{h/q}(z, p_\perp)}, \quad (1)$$

where $f_{q/p}(x, k_\perp)$ and $D_{h/q}(z, p_\perp)$ are the unpolarized distribution and fragmentation functions, with $k_\perp$ and $p_\perp$ being, respectively, the transverse momentum of the quark in the proton and of the final hadron $h$ with respect to the fragmenting quark $q$; $\frac{d^2 \sigma_{\ell q \rightarrow \ell q}}{dq^2}$ is the partonic cross section corresponding to the underlying elementary process $\ell q \rightarrow \ell q$. The numerator of this azimuthal asymmetry contains the Sivers distribution function \cite{12}, $\Delta \hat{f}_{q/p}(x, k_\perp)$, related to the number density of unpolarized quarks inside a transversely polarized proton

$$\Delta \hat{f}_{q/p}(x, k_\perp) = \hat{f}_{q/p}(x, k_\perp) - \hat{f}_{q/p}(x, \bar{k}_\perp) \equiv \Delta^N f_{q/p}(x, k_\perp) \mathbf{S}_T \cdot (\hat{p} \times \hat{k}_\perp). \quad (2)$$
For the purpose of our fit, we parametrize this function by factorizing the $x$ and $k_\perp$ dependences, as follows

$$
\Delta f_{q/p}^q (x, k_\perp) \propto x^{\alpha_q} (1 - x)^{\beta_q} f_{q/p} (x) h(k_\perp),
$$

where $\alpha_q$ and $\beta_q$ are free parameters which control the details of the low-$x$ and large-$x$ behaviour of the Sivers function, for each given flavour $q$.

A fit of the available experimental data allowed us the extraction of the Sivers functions, shown in the first panel of Fig. 1 by using the unpolarized distribution and fragmentation function sets (with their appropriate $Q^2$ dependence) as given in Refs. \cite{13} and \cite{14} respectively.

The transversity and the Collins functions, which, being chirally even, can only contribute in pairs to physical observables, were determined in Ref. \cite{15} and updated in Ref. \cite{3} by performing a simultaneous fit of the Collins azimuthal asymmetry, $A_{UT}^{\text{Collins}}$, measured in SIDIS by HERMES and COMPASS,

$$
A_{UT}^{\text{Collins}} \propto \frac{\sum_q e_q^2 h_{1q}(x, k_\perp) \otimes d\sigma_{\text{SIDIS}}^{x_{\text{coll},-\text{coll}}}}{2 \sum_q e_q^2 f_{q/p}(x, k_\perp) \otimes d\sigma_{\text{SIDIS}}^{x_{\text{coll},-\text{coll}}}} \otimes \Delta h_{q/p}^q (z, p_\perp),
$$

and of the azimuthal correlations $A_{12}$ measured in $e^+e^- \rightarrow h_1h_2X$ by the BELLE collaboration \cite{13} \cite{17}. $h_{1q}(x, k_\perp)$ and $\Delta h_{q/p}^q (z, p_\perp)$ are the transversity and Collins functions and $d\sigma_{\text{SIDIS}}^{x_{\text{coll},-\text{coll}}}$ is the partonic spin transfer cross section, while $A_{12}$ contains the product of two Collins functions.

The Collins function is related to the number density of unpolarized hadrons $h$ resulting from the fragmentation of a transversely polarized quark:

$$
\Delta \tilde{D}_{h/q}^q (z, p_\perp) = \tilde{D}_{h/q}^q (z, p_\perp) - \tilde{D}_{h/q}^q (z, p_\perp) \equiv \Delta f_{h/q}^q (q, p_\perp) S_q \cdot (\hat{p}_q \times \hat{p}_\perp),
$$

where $S_q$ and $p_q$ are, respectively, the polarization vector and the momentum vector of the fragmenting quark $q$, while $p_\perp$ is the intrinsic transverse momentum of the produced hadron with respect to the $\hat{p}_q$ direction.

As in the Sivers fit, the transversity (Collins) functions were parametrized so that their $x$ ($z$) and $k_\perp$ ($p_\perp$) dependences were factorized and their low-$x$ (low-$z$) and large-$x$ (large-$z$) behaviour controlled by the appropriate $\alpha$ and $\beta$ parameters. The transversity and the Collins functions as determined using this strategy are shown in Fig. 1. One can observe that while the Collins fragmentation functions are rather well constrained thanks to the high statistics of the BELLE data \cite{3}, the Sivers and the transversity distributions are affected by much higher uncertainties. Moreover, it is important to notice that the available SIDIS experimental data span a relatively limited range of $x$ values ($x \lesssim 0.3$): therefore, the SIDIS data are unable to fix the parameters $\beta$ which control the large-$x$ behaviour of the

DIS 2009
transversity and of the Sivers distribution functions. As a consequence, in our fits the $\beta$
parameters were chosen to be flavor independent. This observation is relevant when turning
to polarized proton-proton SSAs \[18\]: in fact, $pp$ experimental data from RHIC cover a
range of much larger $x$ values as compared to SIDIS data. When exploring the Sivers and
the Collins effects induced in $pp$ processes by the SIDIS extracted functions, this large $x$
uncertainty should be taken into account.

The actual consensus and understanding about the Collins and Sivers functions is that while
the former are expected to be universal \[19\] \[20\] \[21\], the latter can be process dependent.
In particular the Sivers functions in SIDIS and Drell-Yan processes are expected to be
opposite. The situation is much less clear concerning SSAs in $AB \rightarrow hX$ processes in
which the only large scale measured is the $P_T$ of the final hadron $h$ \[22\] \[23\] \[24\]: even the
factorization scheme with transverse momentum dependent distribution and fragmentation
functions (TMD factorization) has not been proven in such cases.

We adopt here a pragmatic attitude and explore the possible values of SSAs in $p^+p \rightarrow \pi X$
processes at moderately large $P_T$, assuming TMD factorization and universality, that is using
the same Sivers and Collins functions as extracted from SIDIS data. A failure to reproduce
the experimental results would be a clear indication that these assumptions cannot be valid.

The expression of $A_N$ in TMD factorization is given by \[18\]:

$$A_N \sim A_N^{\text{Sivers}} + A_N^{\text{Collins}}$$

$$\propto \sum_{a,b,c,d} \Delta f_{a/p}(x_a, k_{\perp a}) \otimes f_{h/p}(x_b, k_{\perp b}) \otimes \frac{d\hat{\sigma}^{ab\rightarrow cd}}{dt} \otimes D_{h/c}(z, p_{\perp})$$

$$+ \sum_{a,b,c,d} h_1(x_a, k_{\perp a}) \otimes f_{h/p}(x_b, k_{\perp b}) \otimes \frac{d\Delta \hat{\sigma}^{ab\rightarrow cd}}{dt} \otimes \Delta \hat{D}_{h/c}(z, p_{\perp})$$

$$+ \sum_{a,b,c,d} 2 \Delta f_{a/p}(x_a, k_{\perp a}) \otimes f_{h/p}(x_b, k_{\perp b}) \otimes \frac{d\hat{\sigma}^{ab\rightarrow cd}}{dt} \otimes D_{h/c}(z, p_{\perp})$$

where $a$, $b$, $c$ and $d$ can be either quarks $q$ or gluons $g$, and all possible pQCD elementary
interactions at lowest order contribute. Notice that the Sivers and Collins effects add up in
$A_N$, and cannot be separated as it is done in SIDIS. Further contributions, proportional to
the Boer-Mulders and other TMDs, are negligible, as we have checked numerically \[18\].

We now apply the quark Sivers, transversity and Collins functions extracted by fitting SIDIS
and $e^+e^-$ experimental data to compute $A_N$ for $p^+p \rightarrow \pi X$ processes \[18\] and compare with
data \[4\] \[5\]. As mentioned above, the available SIDIS experimental measurements refer to
a limited range of $x$ values ($x \lesssim 0.3$): therefore, the SIDIS data are unable to fix precisely
the parameters $\beta$ which control the large-$x$ behaviour of the transversity and of the Sivers
distribution functions, while the RHIC $pp$ experimental data cover a range of much larger
$x$ values. We need to take this large-$x$ behaviour uncertainty into account. We do so by
letting the $\beta$ parameters vary; we consider a grid of configurations in which $\beta_a$ and $\beta_d$, both
for the transversity and the Sivers distributions, range from 0 to 4 in steps of 0.5. For each of
these configurations, we re-run the SIDIS best fit. Then we select out only the parameter
configurations that correspond to a $\chi^2_{dof}$ not larger than about 20% more of the minimum
original value \[2\] \[3\]. Finally, we construct a variation band, for both the Sivers and Collins
contributions to $A_N$, given by the convolution of all the curves obtained from the parameter sets we have selected.

Figures 2 and 3 show the results we obtain, from which we can draw a few conclusions. We do not find any clear strong indication of universality breaking effects; on the contrary, given the constraints offered by the presently available SIDIS and $e^+e^-$ experimental data, our results show that there might exist a set of SIDIS extracted Sivers functions which can account for the transverse single spin asymmetry $A_N$ for neutral and charged pion production in polarized proton-proton scattering measured by RHIC [4, 5]. Instead, the Collins effect alone, for which universality is usually accepted, only contributes a fraction of the whole $pp$ asymmetries.

The study of the dependence of these results on the choice of the fragmentation function set is currently under way. There is evidence that this dependence, which is quite mild in SIDIS processes, can be much more pronounced in the case of $pp$ scattering where the cross sections become much more sensitive to the details of the gluon distribution function (recall that there is no glue contribution to SIDIS processes at LO). Moreover the $Q^2$ evolution of the Sivers and of the Collins functions are yet unknown. In our fit we assume the same evolution as that of the corresponding unpolarized density functions: the consequences of this simplification have to be analyzed in more details.

These results are entirely phenomenological and preliminary: fur-
ther studies and data are obviously necessary before one can definitely conclude whether or not the same sets of Sivers and Collins distributions, within a TMD factorized scheme, can explain the SSAs measured in SIDIS and hadronic processes. At the moment we can only conclude that, within the large variation bands, the sum of the (SIDIS extracted) Sivers and Collins contributions could fit the RHIC data on $A_N$ at $\sqrt{s} = 200$ GeV.

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