Hadron spectroscopy

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1. Introduction

2. Hadron resonances discovered since 2003
   - Open-flavor heavy mesons
   - $XYZ$ states
   - Pentaquark candidates

3. Theory ideas and applications
   - Symmetries of QCD: chiral and heavy quark
     - Applications to new heavy hadrons
   - Threshold cusps and triangle singularities
   - Compositeness and hadronic molecules
Two recent reviews:

- S. L. Olsen, T. Skwarnicki, *Nonstandard heavy mesons and baryons: Experimental evidence*, Rev. Mod. Phys. 90 (2018) 015003 [arXiv:1708.04012] experimental facts and interpretations

- F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, B.-S. Zou, *Hadronic molecules*, Rev. Mod. Phys. 90 (2018) 015004 [arXiv:1705.00141] theoretical formalisms
Two facets of QCD

- Running of the coupling constant \( \alpha_s = g_s^2/(4\pi) \)

- High energies
  - asymptotic freedom, perturbative
  - degrees of freedom: quarks and gluons

- Low energies
  - nonperturbative, \( \Lambda_{QCD} \approx 300 \text{ MeV} = \mathcal{O}(1 \text{ fm}^{-1}) \)
  - color confinement, detected particles: mesons and baryons

⇒ challenge: how do hadrons emerge/how is QCD spectrum organized?

lectures by Gavin Salam
Theoretical tools for studying nonperturbative QCD

- Lattice QCD: numerical simulation in discretized Euclidean space-time
  - finite volume ($L$ should be large)
  - finite lattice spacing ($a$ should be small)
  - often using $m_{u,d}$ larger than the physical values $\Rightarrow$ chiral extrapolation

- Phenomenological models, such as quark model, QCD sum rules, ...

- Low-energy EFT:
  mesons and baryons as effective degrees of freedom
Quarks and hadrons

Mesons and baryons in quark model
Light flavor symmetry

Light meson SU(3) \([u, d, s]\) multiplets (octet + singlet):

- **Vector mesons**

| meson          | quark content       | mass (MeV) |
|----------------|---------------------|------------|
| \(\rho^+ / \rho^-\) | \(ud / \bar{d}\bar{u}\) | 775        |
| \(\rho^0\)     | \((u\bar{u} - d\bar{d}) / \sqrt{2}\) | 775        |
| \(K^{*+} / K^{*-}\) | \(u\bar{s} / s\bar{u}\) | 892        |
| \(K^{*0} / \bar{K}^{*0}\) | \(d\bar{s} / s\bar{d}\) | 896        |
| \(\omega\)     | \((u\bar{u} + d\bar{d}) / \sqrt{2}\) | 783        |
| \(\phi\)       | \(s\bar{s}\)        | 1019       |

- approximate SU(3) symmetry

\[ m_\rho \simeq m_\omega, \quad m_\phi - m_{K^*} \simeq m_{K^*} - m_\rho \]

- very good isospin SU(2) symmetry

\[ m_{\rho^0} - m_{\rho^\pm} = (-0.7 \pm 0.8) \text{ MeV}, \quad m_{K^{*0}} - m_{K^{*\pm}} = (6.7 \pm 1.2) \text{ MeV} \]
Light flavor symmetry

Light meson SU(3) \([u, d, s]\) multiplets (octet + singlet):

- **Pseudoscalar mesons**

| meson     | quark content          | mass (MeV) |
|-----------|------------------------|------------|
| \(\pi^+ / \pi^-\) | \(ud\bar{d} / d\bar{u}\) | 140        |
| \(\pi^0\)   | \( (u\bar{u} - d\bar{d})/\sqrt{2} \) | 135        |
| \(K^+ / K^-\) | \(u\bar{s} / s\bar{u}\) | 494        |
| \(K^0 / \bar{K}^0\) | \(d\bar{s} / s\bar{d}\) | 498        |
| \(\eta\)    | \( (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6} \) | 548        |
| \(\eta'\)   | \( (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3} \) | 958        |

\(\text{very good isospin } SU(2)\) symmetry

\[m_{\pi^\pm} - m_{\pi^0} = (4.5936 \pm 0.0005) \text{ MeV}, \quad m_{K^0} - m_{K^\pm} = (3.937 \pm 0.028) \text{ MeV}\]

**Q:** Why are the pions so light?
What are exotic hadrons?

- **Quark model notation:**
  any hadron resonances beyond picture of $q\bar{q}$ for a meson and $qqq$ for a baryon

  - Gluonic excitations: hybrids and glueballs

- **Multiquark states**

- **Hadronic molecules:**
  bound states of two or more hadrons, analogues of nuclei

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We then refer to the members $u^{\frac{1}{2}}$, $d^{-\frac{1}{2}}$, and $s^{-\frac{1}{2}}$ of the triplet as "quarks" and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(qqq)$, $(qqq\bar{q}\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qqq\bar{q})$, etc. It is assuming that the lowest baryon configuration $(qqq)$ gives just the representations 1, 8, and 10 that have been observed, while the lowest meson configuration $(q\bar{q})$ similarly gives just 1 and 8.
\( J^{PC} \) and exotic quantum numbers

- \( J^{PC} \) of regular \( q\bar{q} \) mesons
  \[ P = (-1)^{L+1} \]
  \[ C = (-1)^{L+S} \] for flavor-neutral mesons
  
  \( L \): orbital angular momentum
  \( S = (0, 1) \): total spin of \( q \) and \( \bar{q} \)

\[ \text{For } S = 0, \text{ the meson spin } J = L, \text{ one has } P = (-1)^{J+1} \text{ and } C = (-1)^J. \] Hence,

\[ J^{PC} = \text{even}^{--} \text{ and odd}^{+-} \]

\[ \text{For } S = 1, \text{ one has } P = C = (-1)^{L+1}. \] Hence,

\[ J^{PC} = 1^{--}, \{0, 1, 2\}^{++}, \{1, 2, 3\}^{--}, \ldots \]

- Exotic \( J^{PC} \) for mesons:

\[ J^{PC} = 0^{--}, \text{even}^{+-} \text{ and odd}^{--} \]
Some trivial facts about additive quantum numbers of regular mesons

- Light-flavor mesons (here $S =$ strangeness)
  - Nonstrange mesons: $S = 0, I = 0, 1$
  - Strange mesons: $S = \pm 1, I = \frac{1}{2}$

- Open-flavor heavy mesons
  - $Q\bar{q}(q = u, d)$: $S = 0, I = 1/2$
  - $Q\bar{s}$: $S = 1, I = 0$

- Heavy quarkonia ($Q\bar{Q}$): $S = 0, I = 0$, neutral

Charge, isospin, strangeness etc. which cannot be achieved in the $q\bar{q}$ and $qqq$ scheme would be a smoking gun for an exotic nature

more subtlety later...
Decay patterns: regular hadrons

- SU(3) flavor symmetry is usually satisfied to 30%

  \[ \frac{\Gamma(K^{*-+})}{\Gamma(\rho^+)} = \frac{51 \text{ MeV}}{149 \text{ MeV}} = 0.34 \text{ [exp]}, \quad \frac{3}{4} \left( \frac{M_{\rho}}{M_{K^*}} \right)^2 \left( \frac{q_{K\pi}}{q_{\pi\pi}} \right)^3 = 0.29 \text{ [SU(3)]} \]

- Okubo–Zweig–Iizuka (OZI) rule:

  Drawing only quark lines, the disconnected diagrams are strongly suppressed relative to the connected ones

  \[ \psi(3770) : \sim 40 \text{ MeV above the } D\bar{D} \text{ threshold} \]

  \[ B(D\bar{D}) = (93^{+8}_{-9})\% \gg B(\text{sum of any other modes}) \]
Godfrey–Isgur quark model

\[
\left( \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2} + V \right) |\Psi\rangle = E |\Psi\rangle
\]

Potential $V$: One-gluon exchange + linear confinement + relativistic effects

Isospin-1 light meson spectroscopy
Many new hadron resonances observed in experiments since 2003

- Inactive: BaBar, Belle, CDF, CLEO-c, D0, ...
- Running: Belle-II, BESIII, COMPASS, LHCb, ...
- Under construction/discussion: PANDA, EIC, EicC, ...

Common strategy: search for peaks, fit with Breit–Wigner

\[
\alpha \frac{1}{(s - M^2)^2 + s \Gamma^2(s)}
\]

Lots of mysteries right now ...
Open-flavor heavy mesons

Most quark-model predicted states were still missing before 2003
Charm-strange mesons (1)

Discoveries in 2003 (both Belle and BaBar started data taking in 1999):

- $D_{s0}^*(2317)$: discovered in $e^+e^- \rightarrow D_s^+\pi^0X$
  
  \[ J^P = 0^+, M = (2317.7 \pm 0.6) \text{ MeV}, \Gamma < 3.8 \text{ MeV} \]
  
  $I = 0, \rightarrow D_s\pi^0$: breaks isospin symmetry

- $D_{s1}(2460)$: discovered in $e^+e^- \rightarrow D_{s1}^*\pi^0X$
  
  \[ J^P = 1^+, M = (2459.5 \pm 0.6) \text{ MeV}, \Gamma < 3.5 \text{ MeV} \]
  
  $I = 0, \rightarrow D_{s1}^*\pi^0$: breaks isospin symmetry

other decays: $D_s^+\gamma, D_s^+\pi^+\pi^-, D_{s0}^*(2317)\gamma$
**Charm-strange mesons (2)**

**$D_{s0}^*(2317)$ and $D_{s1}(2460)$**: the first established new hadrons

- **Puzzle 1**: Why are $D_{s0}^*(2317)$ and $D_{s1}(2460)$ so light?
- **Puzzle 2**: Why $M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)} \simeq M_{D^*\pm} - M_{D^\pm}$?

\[
\begin{align*}
M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)} &= (141.8 \pm 0.8) \text{ MeV} \\
M_{D^*\pm} - M_{D^\pm} &= (140.67 \pm 0.08) \text{ MeV}
\end{align*}
\]
Observations of charm-nonstrange excited mesons in 2003

\[ B^- \rightarrow D^{(*)+} \pi^- \pi^- \]

- **\( D^*_0(2400) \):** \( J^P = 0^+ \)
  - \( \Gamma = (267 \pm 40) \text{ MeV} \)
  - Mass (MeV):
    - PDG18: 2318 ± 29
    - BaBar: 2297 ± 22 \( B \) decays
    - Belle: 2308 ± 36 \( B \) decays
    - FOCUS: 2401 ± 41 \( \gamma A \)
    - LHCb: 2360 ± 34 \( B \) decays

- **\( D^*_1(2430) \):** \( J^P = 1^+ \)
  - \( \Gamma = 384^{+130}_{-110} \text{ MeV} \)
  - \( M = (2427 \pm 36) \text{ MeV} \)
Puzzle 3: Why $M_{D_0^* (2400)} \gtrsim M_{D_s^* (2317)}$ and $M_{D_1 (2430)} \sim M_{D_{s1} (2460)}$?
Most exotic and newest observation: \(X(5568)\)

- \(X(5568)\) by D0 Collaboration (\(p\bar{p}\) collisions)

\[M = (5567.8 \pm 2.9^{+0.9}_{-1.9}) \text{ MeV}\]
\[\Gamma = (21.9 \pm 6.4^{+5.0}_{-2.5}) \text{ MeV}\]

- Observed in \(B_s^{(*)}\)\(0\)\(\pi^+\), sizeable width
  \[\Rightarrow I = 1:\]
  minimal quark contents is \(\bar{b}s\bar{d}u\)!

- a favorite multiquark candidate:
  explicitly flavor exotic, minimal number of quarks \(\geq 4\)

\[\Gamma_I \sim \left(\frac{m_d-m_u}{\Lambda_{QCD}}\right)^2 \times O(100 \text{ MeV})\]
\[= O(10 \text{ keV})\]

PRL117(2016)022003; PRD97(2018)092004
$XYZ$ states

Charmonium spectrum in Godfrey-Isgur quark model

$J^P C = 0^+ \quad 1^- \quad 1^+ \quad 0^{++} \quad 1^{++} \quad 2^{++} \quad 2^{--}$

$3^3S_1, 2^3D_1, 1^3D_0, 2^3S_1, 1^3P_1, 1^3P_0, 1^3P_1, 1^3P_2, 2^3P_2, 2^3D_2$
$XYZ$ states

Experimental status of charmonium spectrum
(before 2003)

- $\psi(415)$
- $\psi(4160)$
- $\psi(4040)$
- $\eta_c(2S)$
- $\psi(2S)$
- $h_c(1P)$
- $\chi_{c0}(1P)$
- $\chi_{c1}(1P)$
- $\chi_{c2}(1P)$

$J^{PC} = 0^+ , 1^- , 1^+ , 0^{++} , 1^{++} , 2^{++} , 2^{--}$

- Black: Godfrey-Isgur quark model
- Red: discovered before 2003
Naming convention

For states with properties in conflict with naive quark model (normally):

- **X**: $I = 0$, $J^{PC}$ other than $1^{--}$ or unknown
- **Y**: $I = 0$, $J^{PC} = 1^{--}$
- **Z**: $I = 1$

PDG2018 naming scheme:

\[
J^{PC} = \begin{cases} 
0^{--} & 1^{+-} & 1^{--} & 0^{++} \\
2^{--} & 3^{+-} & 2^{--} & 1^{++} \\
& & \vdots & \vdots \\
& & \vdots & \vdots
\end{cases}
\]

Minimal quark content

\[
\begin{aligned}
\text{ud, u}\bar{u} &- d\bar{d}, d\bar{u} \quad (I = 1) \\
\text{d}\bar{d} + u\bar{u} \quad \{ \text{I = 0} \} \\
\text{and/or s}\bar{s}
\end{aligned}
\]

- \(\pi\)
- \(b\)
- \(\rho\)
- \(a\)
- \(\eta, \eta'\)
- \(h, h'\)
- \(\omega, \phi\)
- \(f, f'\)
- \(\eta_c\)
- \(h_c\)
- \(\psi^\dagger\)
- \(\chi_c\)
- \(\eta_b\)
- \(h_b\)
- \(\Upsilon\)
- \(\chi_b\)
- \(\Pi_c\)
- \(Z_c\)
- \(R_c\)
- \(W_c\)
- \(\Pi_b\)
- \(Z_b\)
- \(R_b\)
- \(W_b\)

\(^\dagger\)The $J/\psi$ remains the $J/\psi$.

"Young man, if I could remember the names of these particles, I would have been a botanist." — Enrico Fermi
The beginning of the $XYZ$ story, discovered in $B^\pm \to K^\pm J/\psi\pi\pi$

$M_X = (3871.69 \pm 0.17) \text{ MeV}$

- $\Gamma < 1.2 \text{ MeV}$
- Confirmed in many experiments: Belle, BaBar, BESIII, CDF, CMS, D0, LHCb, ...
- 10 years later, $J^{PC} = 1^{++}$

$\Rightarrow S$-wave coupling to $D\bar{D}^*$

Mysterious properties:

- Mass coincides with the $D^0\bar{D}^{*0}$ threshold:
  \[ M_{D^0} + M_{D^{*0}} - M_X = (0.00 \pm 0.18) \text{ MeV} \]
Mysterious properties (cont.):

- Large coupling to $D^0 \bar{D}^{*0}$:
  \[ \mathcal{B}(X \rightarrow D^0 \bar{D}^{*0}) > 30\% \]  
  \[ \mathcal{B}(X \rightarrow D^0 \bar{D}^{0}\pi^0) > 40\% \]  
  Belle, PRD81(2010)031103  
  Belle, PRL97(2006)162002

- No isospin partner observed $\Rightarrow I = 0$ 
  but, large isospin breaking:
  \[ \frac{\mathcal{B}(X \rightarrow \omega J/\psi)}{\mathcal{B}(X \rightarrow \pi^+\pi^- J/\psi)} = 0.8 \pm 0.3 \]
  BaBar, PRD77(2008)011102

- Radiative decays:
  \[ \frac{\mathcal{B}(X \rightarrow \gamma \psi')}{\mathcal{B}(X \rightarrow \gamma J/\psi)} = 2.6 \pm 0.6 \]  
  PDG18 average of BaBar(2009) and LHCb(2014) measurements

Exercise:

1) Why is the isospin of the negative $C$-parity $\pi^+\pi^-$ system equal to 1? 
2) Is $\Upsilon \pi^+\pi^-$ a good choice of final states for the search of $X_b$, the $J^{PC} = 1^{++}$ bottom analogue of the $X(3872)$?
Y(4260)

- Discovered by BaBar in 2005

\[ J^{PC} = 1^{--} \], confirmed by Belle, CLEO, BESIII

\[ M = (4230 \pm 8) \text{ MeV}, \Gamma = (55 \pm 19) \text{ MeV} \]

- Puzzles:
  - no slot in quark model
  - well above \( D\bar{D} \) threshold, but not seen in \( D\bar{D} \) (recall the OZI rule)

PRL95(2005)142001

BESIII, PRL118(2017)092001

PDG2018
$Z_c^\pm$ and $Z_b^\pm$ (1)

- $Z_c^\pm$, $Z_b^\pm$: charged structures in heavy quarkonium mass region, excellent tetraquark candidates: $Q\bar{Q}\bar{d}u$, $Q\bar{Q}\bar{u}d$

- $Z_b(10610)^\pm$ and $Z_b(10650)^\pm$: observed in $\Upsilon(10860) \rightarrow \pi^\mp [\pi^\pm \Upsilon(1S, 2S, 3S)/h_b(1P, 2P)]$

$$Z_b(10610)^\pm$$ and $$Z_b(10650)^\pm$$ very close to $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds

Belle, arXiv:1105.4583; PRL108(2012)122001
$Z_c^\pm$ and $Z_b^\pm$ (2)

- $Z_c(3900)^\pm$: structure around 3.9 GeV seen in $J/\psi\pi^\pm$ by BESIII and Belle in $Y(4260) \rightarrow J/\psi\pi^+\pi^-$, and in $D\bar{D}^*$ by BESIII in $Y(4260) \rightarrow \pi^\pm(D\bar{D}^*)^\mp$
  
  BESIII, PRL110(2013)252001; Belle, PRL110(2013)252002

- $Z_c(4020)^\pm$ observed in $h_c\pi^\pm$ and $(\bar{D}^*D^*)^\pm$ distributions

  BESIII, PRL111(2013)242001; PRL112(2014)132001

- $Z_c(3900)^\pm$ and $Z_c(4020)^\pm$ very close to $D\bar{D}^*$ and $D^*\bar{D}^*$ thresholds
Charmonium spectrum: current status

Note: $J^{PC}$ of $X(3915)$ might also be $2^{++}$
Pentaquark candidates
Two Breit–Wigner resonances needed:

\[ M_1 = (4380 \pm 8 \pm 29) \text{ MeV}, \]
\[ M_2 = (4449.8 \pm 1.7 \pm 2.5) \text{ MeV}, \]
\[ \Gamma_1 = (205 \pm 18 \pm 86) \text{ MeV}, \]
\[ \Gamma_2 = (39 \pm 5 \pm 19) \text{ MeV}. \]
LHCb’s $P_c$ (2)

- In $J/\psi p$ invariant mass distribution, with hidden charm
  \[ \Rightarrow \text{pentaquarks if they are hadron resonances} \]
- Quantum numbers not fully determined, for $(P_c(4380), P_c(4450))$: $(3/2^-, 5/2^+), (3/2^+, 5/2^-), (5/2^+, 3/2^-), \ldots$

From a reanalysis using an extended $\Lambda^*$ model: N. Jurik, CERN-THESIS-2016-086

| $J^p(4380, 4450)$ | $(\sqrt{\Delta(-2 \ln L)})^2$ | $P_c(4380)$ | $P_c(4450)$ |
|------------------|-------------------------------|-------------|-------------|
|                  |                               | $M_0$  | $\Gamma_0$ | $M_0$  | $\Gamma_0$ |
| $(3/2^-, 5/2^+)$ | solution                      | 4359   | 151        | 4450.1 | 49        |
| $5/2^+, 3/2^-$   | $-3.6^2$                      | 10     | -7         | -1.6   | -6        |
| $5/2^-, 3/2^+$   | $-2.7^2$                      | -4     | -9         | -3.6   | -2        |
| $3/2^-, 5/2^+$   |                               |        |            |         |           |

- Early prediction:
  *Prediction of narrow $N^*$ and $\Lambda^*$ resonances with hidden charm above 4 GeV*,
  J.-J. Wu, R. Molina, E. Oset, B.-S. Zou, PRL105(2010)232001
Recent reviews on new hadrons

- H.-X. Chen et al., *The hidden-charm pentaquark and tetraquark states*, Phys. Rept. 639 (2016) 1 [arXiv:1601.02092]
- A. Hosaka et al., *Exotic hadrons with heavy flavors — X, Y, Z and related states*, Prog. Theor. Exp. Phys. 2016, 062C01 [arXiv:1603.09229]
- R. F. Lebed, R. E. Mitchell, E. Swanson, *Heavy-quark QCD exotica*, Prog. Part. Nucl. Phys. 93 (2017) 143, arXiv:1610.04528 [hep-ph]
- A. Esposito, A. Pilloni, A. D. Polosa, *Multiquark resonances*, Phys. Rept. 668 (2017) 1 [arXiv:1611.07920]
- F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, B.-S. Zou, *Hadronic molecules*, Rev. Mod. Phys. 90 (2018) 015004 [arXiv:1705.00141]
- S. L. Olsen, T. Skwarnicki, *Nonstandard heavy mesons and baryons: Experimental evidence*, Rev. Mod. Phys. 90 (2018) 015003 [arXiv:1708.04012]
- M. Karliner, J. L. Rosner, T. Skwarnicki, *Multiquark states*, Ann. Rev. Nucl. Part. Sci. 68 (2018) 17 [arXiv:1711.10626]
- C.-Z. Yuan, *The XYZ states revisited*, Int. J. Mod. Phys. A 33 (2018) 1830018 [arXiv:1808.01570]
Symmetries of QCD: chiral and heavy quark

Useful monographs:

- H. Georgi, *Weak Interactions and Modern Particle Physics* (2009)
- J.F. Donoghue, E. Golowich, B.R. Holstein, *Dynamics of the Standard Model* (1992)
- S. Scherer, M.R. Schindler, *A Primer for Chiral Perturbation Theory* (2012)
- A.V. Manohar, M.B. Wise, *Heavy Quark Physics* (2000)
Symmetries for different sectors

- Different quark flavors:
  - Light quarks: u (~2 MeV), d (~5 MeV), s (~100 MeV)
  - Heavy quarks: c (~1.3 GeV), b (~4.2 GeV), t (~173 GeV)

- Spontaneously broken chiral symmetry: \( \pi, K \) and \( \eta \) as the pseudo-Goldstone bosons

- Heavy quark spin symmetry
- Heavy quark flavor symmetry
- Heavy antiquark-diquark symmetry
Chiral symmetry in a nutshell
Chiral symmetry (1)

- QCD Lagrangian

\[ \mathcal{L}_{\text{QCD}} = i \bar{q}_L \not{D} q_L + i \bar{q}_R \not{D} q_R - (\bar{q}_L \mathcal{M} q_R + \bar{q}_R \mathcal{M} q_L) + \ldots \]

\[ q = \frac{1}{2} (1 - \gamma_5) q + \frac{1}{2} (1 + \gamma_5) q \equiv P_L q + P_R q = q_L + q_R \]

- For \( m_{u,d,s} = 0 \), invariant under \( U(3)_L \times U(3)_R \) transformations:

\[ \mathcal{L}_{\text{QCD}}^0 (G_{\mu \nu}, q', D_\mu q') = \mathcal{L}_{\text{QCD}}^0 (G_{\mu \nu}, q, D_\mu q) \]

\[ q' = R P_R q + L P_L q = R q_R + L q_L \]

\[ R \in U(3)_R, \quad L \in U(3)_L \]

- Parity:

\[ q(t, \vec{x}) \xrightarrow{P} \gamma^0 q(t, -\vec{x}) \]

\[ q_R(t, \vec{x}) \xrightarrow{P} P_R \gamma^0 q(t, -\vec{x}) = \gamma^0 P_L q(t, -\vec{x}) = \gamma^0 q_L (t, -\vec{x}) \]

\[ q_L(t, \vec{x}) \xrightarrow{P} \gamma^0 q_R(t, -\vec{x}) \]

- \( U(3)_L \times U(3)_R = SU(3)_L \times SU(3)_R \times U(1)_L \times U(1)_R \)
Chiral symmetry (2)

\[ q_{y_{LR}} = u^{\dagger}_{y_{LR}} q_{y_{LR}} = e^{-i d_{L}^{a} T_{a}} e^{-i d_{R}^{a} T_{a}}. \]

\[ q = q_{L} + q_{R} = \frac{1 + \gamma_{5}}{2} q + \frac{1 - \gamma_{5}}{2} q \]

\[ \rightarrow \frac{1}{2} \left( e^{-i d_{L}^{a} T_{a}} e^{-i d_{L}^{a} T_{a}} e^{-i d_{R}^{a} T_{a}} e^{-i d_{R}^{a} T_{a}} \right) q + \frac{\gamma_{5}}{2} \left( -e^{-i d_{L}^{a} T_{a}} e^{-i d_{L}^{a} T_{a}} e^{-i d_{R}^{a} T_{a}} e^{-i d_{R}^{a} T_{a}} \right) q \]

\[ = \frac{1}{2} \left( 2 - i d_{L}^{a} T_{a} - i d_{L}^{a} T_{a} - i d_{R}^{a} T_{a} - i d_{R}^{a} T_{a} \right) q + \frac{\gamma_{5}}{2} \left( i d_{L}^{a} T_{a} + i d_{L}^{a} T_{a} - i d_{R}^{a} T_{a} - i d_{R}^{a} T_{a} \right) q \]

\[ = \left( 1 - i d_{V}^{a} T_{a} - i d_{V}^{a} T_{a} \right) q + \left( -i d_{A}^{a} T_{a} - i d_{A}^{a} T_{a} \right) \gamma_{5} q + \ldots \]

\[ = \left( \frac{1}{2} (d_{L}^{a} + d_{R}^{a}) \right) \left( \frac{1}{2} (d_{L}^{a} + d_{R}^{a}) \right) \gamma_{5} \gamma_{5} q \]

\[ = e^{-i d_{V}^{a} T_{a}} e^{-i d_{V}^{a} T_{a}} e^{-i d_{A}^{a} T_{a}} \gamma_{5} e^{-i d_{A}^{a} T_{a}} \gamma_{5} q \]

\[ \text{Note: } “SU(3)_{A}” \text{ not a group} \]

\[ U(3)_{L} \times U(3)_{R} = SU(3)_{L} \times SU(3)_{R} \times U(1)_{V} \times U(1)_{A} \]

baryon number cons. broken by quantum anomaly
Chiral symmetry (3): Wigner–Weyl v.s. Nambu–Goldstone

- **Noether’s theorem:** *continuous* symmetry $\Rightarrow$ conserved currents
  
  Let $Q^a$ be *symmetry charges*: 
  
  $$ Q^a = \int d^3 \vec{x} \ J^a,0(t, \vec{x}), \quad \partial_\mu J^{a,\mu} = 0 $$

- $Q^a$ is the *symmetry generator*: $g = e^{i\alpha^a Q^a}$, $H$: Hamiltonian, thus
  
  $$ gHg^{-1} = H \Rightarrow [Q^a, H] = 0, $$

  $$ [Q^a, H]|0\rangle = Q^a H|0\rangle - H Q^a |0\rangle = 0 $$

- **Wigner–Weyl mode:** $Q^a |0\rangle = 0$ or equivalently $g |0\rangle = |0\rangle$
  
  *degeneracy in mass spectrum*

- **Nambu–Goldstone mode:** $g |0\rangle \neq |0\rangle$, spontaneously broken (hidden)
  
  $Q^a |0\rangle \neq |0\rangle$: *new states* degenerate with vacuum, massless *Goldstone bosons*

  - spontaneously broken continuous global symmetry $\Rightarrow$ *massless GBs*
  - the same quantum numbers as $Q^a |0\rangle \Rightarrow$ *spinless*
  - $(\#(\text{GBs})) = (\#(\text{broken generators}))$

A. Zee: “*If you want to show off your mastery of mathematical jargon you can say that the Nambu–Goldstone bosons live in the coset space $G/H$.***
Chiral symmetry (4)

- $P Q_A^a P^{-1} = -Q_A^a$, if $G = \text{SU}(N_f)_L \times \text{SU}(N_f)_R$ realized in Wigner-Weyl mode
  
  $\Rightarrow$ parity doubling: hadrons have degenerate partners with opposite parity, but

  $m_{\text{Nucleon}}, P=+ = 939$ MeV $\ll m_{N^*(1535)}, P=- = 1535$ MeV,
  
  $m_\pi, P=- = 139$ MeV $\ll m_{a_0(980)}, P=+ = 980$ MeV

- vacuum invariant under $H = \text{SU}(N_f)_V$: $Q_V^a |0\rangle = 0$, $Q_A^a |0\rangle \neq 0$

- SSB $\Rightarrow$ massless pseudoscalar Goldstone bosons

  $\#(\text{GBs}) = \dim(G) - \dim(H) = N_f^2 - 1$

  for $N_f = 3$, 8 GBs: $\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta$

  for $N_f = 2$, 3 GBs: $\pi^\pm, \pi^0$

- Pions get a small mass due to explicit symmetry breaking by tiny $m_u, d$ (a few MeV)

  pseudo-Goldstone bosons, Gell-Mann–Oakes–Renner: $M_\pi^2 \propto (m_u + m_d)$

  $M_\pi \ll M_{\text{other hadron}}$, also, $m_s \gg m_u, d$ $\Rightarrow$ $M_K \gg M_\pi$

- Mechanism for $\text{SU}(N_f)_L \times \text{SU}(N_f)_R \rightarrow \text{SU}(N_f)_V$ in QCD not understood
Heavy quark symmetries
Heavy quark symmetries (1)

- For heavy quarks (charm, bottom) in a hadron, typical momentum transfer $\Lambda_{QCD}$.

Heavy quark spin symmetry (HQSS):

- Chromomagnetic interaction $\propto \sigma \cdot B/m_Q$
- Spin of the heavy quark decouples

Let total angular momentum $J = s_Q + s_\ell$,

- $s_Q$: heavy quark spin,
- $s_\ell$: spin of the light degrees of freedom (including orbital angular momentum)

- Angular momentum conservation $m_Q \to +\infty$ $s_\ell$ is conserved
- Spin multiplets:
  - For singly-heavy mesons, e.g., $\{D, D^*\}$ with $s_\ell^P = \frac{1}{2}^-$,
    $M_{D^*} - M_D \simeq 140$ MeV,
    $M_{B^*} - M_B \simeq 46$ MeV
  - For $Q\bar{Q}$, e.g., $\{\eta_c, J/\psi\}$, $\{\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c\}$, $\{\eta_b, \Upsilon\}$
For heavy quarks (charm, bottom) in a hadron, typical momentum transfer $\Lambda_{QCD}$

- **Heavy quark flavor symmetry (HQFS):**
  - for any hadron containing one heavy quark:
  - velocity remains unchanged in the limit $m_Q \to \infty$:
    \[
    \Delta v = \frac{\Delta p}{m_Q} = \frac{\Lambda_{QCD}}{m_Q}
    \]
  - $\Rightarrow$ heavy quark is like a static color triplet source, $m_Q$ is irrelevant

- **Heavy anti-quark–diquark symmetry**
  - if $m_Q v \gg \Lambda_{QCD}$, the diquark serves as a point-like color-$\bar{3}$ source, like a heavy anti-quark.
  - It relates doubly-heavy baryons to anti-heavy mesons

M. Savage, M. Wise, PLB248(1990)177
Intermediate summary

- Many new hadrons observed (in particular in the charm sector), lots of mysteries
- Symmetries of QCD:
  - spontaneously broken chiral symmetry for light flavors
  - heavy quark spin and flavor symmetry for heavy flavors

⇒ next, applications of symmetries to the new hadrons
HQS for open-flavor heavy hadrons
Applications of HQS: $P$-wave heavy mesons

Examples of HQSS phenomenology:

- Widths of the two $D_1 (J^P = 1^+)$ mesons
  \[ \Gamma[D_1(2420)] = (27.4 \pm 2.5) \text{ MeV} \ll \Gamma[D_1(2430)] = (384^{+130}_{-110}) \text{ MeV} \]

- $s_\ell = s_q + L \Rightarrow$ for $P$-wave charmed mesons: $s_\ell^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$ for decays $D_1 \rightarrow D^*\pi$:
  \[ \frac{1}{2}^+ \rightarrow \frac{1}{2}^- + 0^- \text{ in } S\text{-wave} \Rightarrow \text{large width} \]
  \[ \frac{3}{2}^+ \rightarrow \frac{1}{2}^- + 0^- \text{ in } D\text{-wave} \Rightarrow \text{small width} \]

- Thus, $D_1(2420)$: $s_\ell^P = \frac{3}{2}^+$, $D_1(2430)$: $s_\ell^P = \frac{1}{2}^+$

- Suppression of the $S$-wave production of $\frac{3}{2}^+ + \frac{1}{2}^-$ heavy meson pairs in $e^+e^-$ annihilation
  \[ \text{Table VI in E.Eichten et al., PRD17(1978)3090; X. Li, M. Voloshin, PRD88(2013)034012} \]

**Exercise**: Try to understand this statement as a consequence of HQSS.
Applications of HQS: $D_{s0}^*(2317)$ and $D_{s1}(2460)$ (1)

- HQFS: for a singly-heavy hadron,

$$M_{HQ} = m_Q + A + O\left(\frac{\Lambda_{QCD}^2}{m_Q}\right) \text{ with } A \text{ independent of } m_Q$$

- rough estimates of bottom analogues whatever the $D_{sJ}$ states are

$$M_{B_{s0}^*} = M_{D_{s0}^*(2317)} + \Delta_{b-c} + O\left(\Lambda_{QCD}^2 \left(\frac{1}{m_c} - \frac{1}{m_b}\right)\right) \simeq (5.65 \pm 0.15) \text{ GeV}$$

$$M_{B_{s1}} = M_{D_{s1}(2460)} + \Delta_{b-c} + O\left(\Lambda_{QCD}^2 \left(\frac{1}{m_c} - \frac{1}{m_b}\right)\right) \simeq (5.79 \pm 0.15) \text{ GeV}$$

here $\Delta_{b-c} \equiv m_b - m_c \simeq \overline{M}_{B_s} - \overline{M}_{D_s} \simeq 3.33 \text{ GeV}$, where

$\overline{M}_{B_s} = 5.403 \text{ GeV}, \overline{M}_{D_s} = 2.076 \text{ GeV}$: spin-averaged g.s. $Q\bar{s}$ meson masses

both to be discovered $^1$

- Lattice QCD results: Lang, Mohler, Prelovsek, Woloshyn, PLB750(2015)17

$$M_{B_{s0}^*}^{\text{lat.}} = (5.711 \pm 0.013 \pm 0.019) \text{ GeV}$$

$$M_{B_{s1}}^{\text{lat.}} = (5.750 \pm 0.017 \pm 0.019) \text{ GeV}$$

$^1$The established meson $B_{s1}(5830)$ is probably the bottom partner of $D_{s1}(2536)$.  

13–14.09.2018 46/72
Applications of HQS: $D_{s0}^*(2317)$ and $D_{s1}(2460)$ (2)

- in hadronic molecular model: $D_{s0}^*(2317)[\simeq DK], D_{s1}(2460)[\simeq D^*K]$

  Barnes, Close, Lipkin (2003); van Beveren, Rupp (2003); Kolomeitsev, Lutz (2004); FKG et al. (2006); ...

\[ T = V + V \otimes G \otimes V + \ldots \]

$D^{(*)}K$ bound states: poles of the $T$-matrix

- HQSS $\Rightarrow$ similar binding energies $M_D + M_K - M_{D_{s0}^*} \simeq 45$ MeV

  $M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)} \simeq M_{D^*} - M_D$ is natural

- HQFS $\Rightarrow$ predicting the $0^+$ and $1^+$ bottom-partner masses

  $M_{B_{s0}^*} \simeq M_B + M_K - 45$ MeV $\simeq 5.730$ GeV

  $M_{B_{s1}} \simeq M_{B^*} + M_K - 45$ MeV $\simeq 5.776$ GeV
Applications of HQS: $X(5568)$

FKG, Meißner, Zou, *How the X(5568) challenges our understanding of QCD*, Commun.Theor.Phys. 65 (2016) 593

- **mass too low** for $X(5568)$ to be a $\bar{b}s\bar{u}d$: $M \simeq M_{B_s} + 200$ MeV
  - $M_\pi \simeq 140$ MeV because pions are pseudo-Goldstone bosons
  - Gell-Mann–Oakes–Renner: $M_\pi^2 \propto m_q$
  - For any matter field: $M_R \gg M_\pi$; one expects $M_{\bar{u}d} \sim M_R \gtrsim M_\sigma$
    
    $$M_{bs\bar{u}d} \gtrsim M_{B_s} + 500 \text{ MeV} \sim 5.9 \text{ GeV}$$

- HQFS predicts an isovector $X_c$:
    
    $$M_{X_c} = M_{X(5568)} - \Delta_{b-c} + \mathcal{O} \left( \Lambda_{\text{QCD}}^2 \left( \frac{1}{m_c} - \frac{1}{m_b} \right) \right) \simeq (2.24 \pm 0.15) \text{ GeV}$$

  but in $D_s \pi$, only the isoscalar $D^*_{s0}(2317)$ was observed!  

- properties of $X(5568)$ hard to understand

- negative results reported by LHCb,
  by CMS,
  by CDF,
  by ATLAS

LHCb, PRL117(2016)152003
CMS, PRL120(2018)202005
CDF, PRL120(2018)202006
ATLAS, PRL120(2018)202007
From heavy baryons to doubly-heavy tetraquarks

Development inspired by the LHCb discovery of the $\Xi_{cc}^{++}(3620)$

- **Heavy antiquark-diquark symmetry (HADS):**

  - replacing $\bar{Q}$ in $\bar{Q}q$ by $QQ$ $\Rightarrow$ $QQq$;
  - replacing $\bar{Q}$ in $\bar{Q}q\bar{q}$ by $QQ$ $\Rightarrow$ $QQ\bar{q}\bar{q}$;

  \[
  \bar{Q}q \Rightarrow QQq, \quad \bar{Q}q\bar{q} \Rightarrow QQ\bar{q}\bar{q}
  \]

  \[
  M : \quad m_Q + A \Rightarrow m_{QQ} + A, \quad m_Q + B \Rightarrow m_{QQ} + B
  \]

  **Prediction:**

  \[
  M_{QQq\bar{q}\bar{q}} - M_{\bar{Q}q\bar{q}} \simeq M_{QQq} - M_{\bar{Q}q}
  \]

- **Doubly-charmed baryon discovered by LHCb**

  $M_{\Xi_{cc}^{++}} = (3621.40 \pm 0.78)$ MeV can be used as input
HADS $\Rightarrow$ stable doubly-bottom tetraquarks (only decay weakly) are likely to exist

see also Carlson, Heller, Tjon, PRD37(1988)744; Manohar, Wise, NPB399(1993)17; Karliner, Rosner, PRL119(2017)202001; Czarnecki, Leng, Voloshin, PLB778(2018)233; ...

support from lattice QCD

Francis, Hudspith, Lewis, Maltman, PRL118(2017)142001

\[ \text{Eichten, Quigg, PRL119(2017)202002} \]
HQS for $XYZ$ states
• Assuming the $X(3872)$ to be a $D\bar{D}^*$ molecule

• Consider $S$-wave interaction between a pair of $s_\ell^P = \frac{1}{2}^-$ (anti-)heavy mesons:

\[0^{++} : \quad D\bar{D}, \quad D^*\bar{D}^*\]
\[1^{+-} : \quad \frac{1}{\sqrt{2}} (D\bar{D}^* + D^*\bar{D}), \quad D^*\bar{D}^*\]
\[1^{++} : \quad \frac{1}{\sqrt{2}} (D\bar{D}^* - D^*\bar{D})\]
\[2^{++} : \quad D^*\bar{D}^*\]

Here, charge conjugation phase convention: $D \xrightarrow{C} +\bar{D}, \quad D^* \xrightarrow{C} -\bar{D}^*$

• Heavy quark spin irrelevant $\Rightarrow$ interaction matrix elements:

\[\left\langle s_1 \ell, s_2 \ell, s_L \left| \hat{H} \right| s'_1 \ell, s'_2 \ell, s_L \right\rangle\]

For each isospin, 2 independent terms

\[\left\langle \frac{1}{2}, \frac{1}{2}, 0 \left| \hat{H} \right| \frac{1}{2}, \frac{1}{2}, 0 \right\rangle, \quad \left\langle \frac{1}{2}, \frac{1}{2}, 1 \left| \hat{H} \right| \frac{1}{2}, \frac{1}{2}, 1 \right\rangle\]

$\Rightarrow$ 6 pairs grouped in 2 multiplets with $s_L = 0$ and 1, respectively
For the HQSS consequences, convenient to use the basis of states: $s_{PC}^L \otimes s_{PC}^{c\bar{c}}$

- $S$-wave: $s_{PC}^L$, $s_{PC}^{c\bar{c}} = 0^{--}$ or $1^{--}$
- Multiplet with $s_L = 0$:
  
  $0_L^{-+} \otimes 0_{c\bar{c}}^{-+} = 0^{++}$, $0_L^{-+} \otimes 1_{c\bar{c}}^{--} = 1^{+-}$

- Multiplet with $s_L = 1$:
  
  $1_L^{--} \otimes 0_{c\bar{c}}^{-+} = 1^{+-}$, $1_L^{--} \otimes 1_{c\bar{c}}^{--} = 0^{++} \oplus 1^{++} \oplus 2^{++}$

- Multiplets in strict heavy quark limit:
  - $X(3872)$ has three partners with $0^{++}$, $2^{++}$ and $1^{+-}$

  Hidalgo-Duque et al., PLB727(2013)432; Baru et al., PLB763(2016)20

  - Might be 6 $I = 1$ molecules:

    $Z_b[1^{+-}]$, $Z_b'[1^{+-}]$ and $W_{b0}[0^{++}]$, $W_{b0}'[0^{++}]$, $W_{b1}[1^{++}]$ and $W_{b2}[2^{++}]$

    Bondar et al., PRD84(2011)054010; Voloshin, PRD84(2011)031502; Mehent, Powell, PRD84(2011)114013
• Recall the exercise in Lecture-1:

Is $\Upsilon \pi^+ \pi^-$ a good choice of final states for the search of $X_b$, the $J^{PC} = 1^{++}$ bottom analogue of the $X(3872)$?

Answer: No. $X_b \rightarrow \Upsilon \pi \pi$ breaks isospin symmetry

FKG, Hidalgo-Duque, Nieves, Valderrama, PRD88(2013)054007; Karliner, Rosner, PRD91(2015)014014

$$M_{B^0} - M_{B^\pm} = (0.31 \pm 0.06) \text{ MeV} \quad [M_{D^\pm} - M_{D^0} = (4.822 \pm 0.015) \text{ MeV}]$$

• Negative results:

CMS, Search for a new bottomonium state decaying to $\Upsilon(1S)\pi^+ \pi^-$ in $pp$ collisions at $\sqrt{s} = 8$ TeV, PLB727(2013)57;

ATLAS, Search for the $X_b$ and other hidden-beauty states in the $\pi^+ \pi^- \Upsilon(1S)$ channel at ATLAS, PLB740(2015)199

• The results can be reinterpreted as for the search of $W_{bJ}$ ($I = 1, J^{++}$)
HQSS for $XYZ$ (4)

\[ 1_{L}^{-} \otimes 1_{c\bar{c}}^{-} = 0^{++} \oplus 1^{++} \oplus 2^{++} \]

- Heavy quark spin selection rule for $X(3872)$:
  for $X(3872)$ being a $1^{++}$ $D\bar{D}^*$ molecule, $s_{L} = 1$, $s_{c\bar{c}} = 1$

- Spin structure of $Q\bar{Q}$:

|       | $s_{L}$ | $s_{c\bar{c}}$ | $J^{PC}$   | $c\bar{c}$ |
|-------|--------|----------------|------------|------------|
| $S$-wave | 0      | 0              | 0$^{--}$   | $\eta_{c}$ |
|        | 0      | 1              | 1$^{--}$   | $J/\psi$   |
| $P$-wave | 1      | 0              | 1$^{+-}$   | $h_{c}$    |
|        | 1      | 1              | $(0,1,2)^{++}$ | $\chi_{c0}, \chi_{c1}, \chi_{c2}$ |

- Allowed: $X(3872) \rightarrow J/\psi \pi \pi$, $X(3872) \rightarrow \chi_{cJ} \pi$, $X(3872) \rightarrow \chi_{cJ} \pi \pi$

- Suppressed: $X(3872) \rightarrow \eta_{c} \pi \pi$, $X(3872) \rightarrow h_{c} \pi \pi$

- Interesting feature of $Z_{b}^{'(\prime)}$: observed with similar rates in both $\Upsilon \pi \pi [s_{b\bar{b}} = 1]$ and $h_{b} \pi \pi [s_{b\bar{b}} = 0]$

$$Z_{b} \sim B \bar{B}^{*} \sim 0_{bb}^{--} \otimes 1_{q\bar{q}}^{--} - 1_{bb}^{--} \otimes 0_{q\bar{q}}^{--}, \quad Z_{b}^{'} \sim B^{*} \bar{B}^{*} \sim 0_{bb}^{--} \otimes 1_{q\bar{q}}^{--} + 1_{bb}^{--} \otimes 0_{q\bar{q}}^{--}$$

Voloshin, PLB604(2004)69

Bondar, Garmash, Milstein, Mizuk, Voloshin, PRD84(2011)054010
Unitary transformation from two-meson basis to $|s_{1c}, s_{2c}, s_{c\bar{c}}; s_{1\ell}, s_{2\ell}, s_L; J\rangle$:

$$|s_{1c}, s_{1\ell}, j_1; s_{2c}, s_{2\ell}, j_2; J\rangle = \sum_{s_{c\bar{c}}, s_L} \sqrt{(2j_1 + 1)(2j_2 + 1)(2s_{c\bar{c}} + 1)(2s_L + 1)}$$

$$\times \begin{pmatrix}
  s_{1c} & s_{2c} & s_{c\bar{c}} \\
  s_{1\ell} & s_{2\ell} & s_L \\
  j_1 & j_2 & J
\end{pmatrix}
\begin{pmatrix}
  s_{1c}, s_{2c}, s_{c\bar{c}}; s_{1\ell}, s_{2\ell}, s_L; J
\end{pmatrix}$$

$j_{1,2}$: meson spins;
$J$: the total angular momentum of the whole system

$s_{1c}(2c) = \frac{1}{2}$: spin of the heavy quark in meson 1 (2)

$s_{1\ell}(2\ell) = \frac{1}{2}$: angular momentum of the light quarks in meson 1 (2)

- $s_{c\bar{c}} = 0, 1$: total spin of $c\bar{c}$, conserved but decoupled
- $s_L = 0, 1$: total angular momentum of the light-quark system, conserved
- only two independent $\langle s_{\ell 1}, s_{\ell 2}, s_L | \hat{H} | s'_{\ell 1}, s'_{\ell 2}, s_L \rangle_I$ terms for each isospin $I$:

$$F_{I0} = \langle \frac{1}{2}, \frac{1}{2}, 0 | \hat{H} | \frac{1}{2}, \frac{1}{2}, 0 \rangle_I, \quad F_{I1} = \langle \frac{1}{2}, \frac{1}{2}, 1 | \hat{H} | \frac{1}{2}, \frac{1}{2}, 1 \rangle_I$$
\[
\begin{pmatrix}
D \bar{D} \\
D^* \bar{D}^*
\end{pmatrix}: \quad V^{(0^{++})} = \begin{pmatrix}
C_{IA} \\
\sqrt{3}C_{IB}
\end{pmatrix},
\]
\[
\begin{pmatrix}
D \bar{D}^* \\
D^* \bar{D}^*
\end{pmatrix}: \quad V^{(1^{-+})} = \begin{pmatrix}
C_{IA} - C_{IB} \\
2C_{IB}
\end{pmatrix},
\]
\[
D \bar{D}^*: \quad V^{(1^{++})} = C_{IA} + C_{IB},
\]
\[
D^* \bar{D}^*: \quad V^{(2^{++})} = C_{IA} + C_{IB},
\]

here, \(C_{IA} = \frac{1}{4}(3F_{I1} + F_{I0})\), \(C_{IB} = \frac{1}{4}(F_{I1} - F_{I0})\)

- This predicts a spin partner for \(X(3872)\): \(Nieves, Valderrama, PRD86(2012)056004; \ldots\)

\[
M_{X_2(4013)} - M_{X(3872)} \approx M_{D^*} - M_D
\]

ongoing efforts searching for \(X_2\), not found yet
Threshold cusps and triangle singularities
Peaks and resonances

Resonances do not always appear as peaks:

![Graph showing cross section and energy vs. resonance and poles.]

J. R. Taylor, *Scattering Theory: The Quantum Theory on Nonrelativistic Collisions*

Peaks are not always due to resonances:

- **Dynamics** $\Rightarrow$ poles in the $S$-matrix (resonances): genuine physical states.
- **Kinematic effects** $\Rightarrow$ branching points of $S$-matrix
  - normal two-body threshold cusp
  - triangle singularity
  - ...

tools/traps in hadron spectroscopy
Unitarity of the $S$-matrix: $SS^\dagger = S^\dagger S = 1$, $S_{fi} = \delta_{fi} - i(2\pi)^4\delta^4(p_f - p_i)T_{fi}$

$T$-matrix: $T_{fi} - T_{fi}^\dagger = -i(2\pi)^4 \sum_n \delta(p_n - p_i)T_{fn}^\dagger T_{ni}$

assuming all intermediate states are two-body, partial-wave unitarity relation:

$$\text{Im} T_{L,fi}(s) = -\sum_n T_{L,fn}^\ast \rho_n(s) T_{L,ni}$$

2-body phase space factor: $\rho_n(s) = q_{cm,n}(s)/(2\sqrt{s})\theta(\sqrt{s} - m_{n1} - m_{n2})$, $q_{cm,n}(s) = \sqrt{s - (m_{n1} + m_{n2})^2}[s - (m_{n1} - m_{n2})^2]/(2\sqrt{s})$

- There is always a cusp at an $S$-wave threshold
Threshold cusp: a well-known example

- Cusp effect as a useful tool for precise measurement:
  - example of the cusp in $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$
  - strength of the cusp measures the interaction strength!

  Meißner, Müller, Steininger (1997); Cabibbo (2004); Colangelo, Gasser, Kubis, Rusetsky (2006); ...

\[ \sim \text{threshold, only sensitive to scattering length, } (a_0 - a_2)M_{\pi^+} = 0.2571 \pm 0.0056 \]

- Very prominent cusp $\Rightarrow$ large scattering length $\Rightarrow$ likely a nearby pole

effective range expansion (ERE): $f(k) = \frac{1}{1/a + rk^2/2 - ik}$
Triangle singularity (TS)

\[ \frac{1}{2m_A} \sqrt{\lambda(m_A^2, m_1^2, m_2^2)} \equiv p_{2,\text{left}} = p_{2,\text{right}} \equiv \gamma (\beta E_2^* - p_2^*) \]

on-shell momentum of \( m_2 \) at the left and right cuts in the \( A \) rest frame

\[ \beta = |\vec{p}_{23}|/E_{23}, \quad \gamma = 1/\sqrt{1-\beta^2} \]

- \( p_2 > 0, \quad p_3 = \gamma (\beta E_3^* + p_2^*) > 0 \Rightarrow m_2 \) and \( m_3 \) move in the same direction
- velocities in the \( A \) rest frame: \( v_3 > \beta > v_2 \)

\[ v_2 = \beta \frac{E_2^* - p_2^*}{E_2^* - \beta p_2^*} < \beta, \quad v_3 = \beta \frac{E_3^* + p_2^*}{E_3^* + \beta p_2^*} > \beta \]

- Conditions (Coleman–Norton theorem):
  - all three intermediate particles can go on shell simultaneously
  - \( \vec{p}_2 \parallel \vec{p}_3 \), particle-3 can catch up with particle-2 (as a classical process)
  - needs very special kinematics \( \Rightarrow \) process dependent! (contrary to pole position)

Bayar et al., PRD94(2016)074039

Coleman, Norton (1965); Bronzan (1964)
Coincidence of $P_c(4450)$ with kinematic singularities

- **Mass:** $M_{P_c(4450)} = (4449.8 \pm 1.7 \pm 2.5) \text{ MeV}$

- **Our trivial observation:** $P_c(4450)$ coincides with the $\chi_{c1}p$ threshold:

  $$M_{P_c(4450)} - M_{\chi_{c1}} - M_p = (0.9 \pm 3.1) \text{ MeV}$$

- **Our non-trivial observation:** there is a triangle singularity at the same time!

  Solving the equation $p_{2, \text{left}} = p_{2, \text{right}}$

  $$\Rightarrow$$

  to have a TS at $M_{J/\psi p} = M_{\chi_{c1}} + M_p$, we need $M_{\Lambda^*} \simeq 1.89 \text{ GeV}$

  On shell $\Rightarrow \Lambda^*$ must be unstable, the TS is then a finite peak

More possible relevant TSs, see X.-H. Liu, Q. Wang, Q. Zhao, PLB757(2015)231
Compositeness and hadronic molecules

FKG, Hanhart, Meißner, Wang, Zhao, Zou, *Hadronic molecules*, Rev. Mod. Phys. **90** (2018) 015004
• Hadronic molecule: dominant component is a composite state of 2 or more hadrons

• Concept at large distances, so that can be approximated by system of multi-hadrons at low energies

Consider a 2-body bound state with a mass $M = m_1 + m_2 - E_B$

size: $R \sim \frac{1}{\sqrt{2\mu E_B}} \gg r_{\text{hadron}}$

• scale separation $\Rightarrow$ power expansion in $p/\Lambda$, (nonrelativistic) EFT applicable!

• Only narrow hadrons can be considered as components of hadronic molecules, $\Gamma_h \ll 1/r$, $r$: range of forces

Filin et al., PRL105(2010)019101; FKG, Meißner, PRD84(2011)014013
Hadronic molecules (2)

- Why are hadronic molecules interesting?
  - one realization of color-neutral objects, analogue of light nuclei
  - important information for hadron-hadron interaction
  - understanding the $XYZ$ states
  - EFT applicable; model-independent statements can be made
    for $S$-wave, compositeness $(1-Z)$ related to measurable quantities

$|g_{NR}|^2 \approx (1-Z)\frac{2\pi}{\mu^2}\sqrt{2\mu E_B} \leq \frac{2\pi}{\mu^2}\sqrt{2\mu E_B}$

$a \approx -\frac{2(1-Z)}{(2-Z)\sqrt{2\mu E_B}}$, $r_e \approx \frac{Z}{(1-Z)\sqrt{2\mu E_B}}$

Weinberg, PR137(1965); Baru et al., PLB586(2004); Hyodo, IJMPA28(2013)1330045; …

see also, e.g., Weinberg’s books: QFT Vol.I, Lectures on QM
Compositeness (1)

Model-independent result for $S$-wave loosely bound composite states:

Consider a system with Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + V$$

$\mathcal{H}_0$: free Hamiltonian, $V$: interaction potential

- **Compositeness:** the probability of finding the physical state $|B\rangle$ in the 2-body continuum $|q\rangle$

$$1 - Z = \int \frac{d^3q}{(2\pi)^3} |\langle q|B\rangle|^2$$

- $Z = |\langle B_0|B\rangle|^2$, $0 \leq (1 - Z) \leq 1$
  - $Z = 0$: pure bound (composite) state
  - $Z = 1$: pure elementary state
Compositeness (2)

**Compositeness:**

\[ 1 - Z = \int \frac{d^3q}{(2\pi)^3} \left| \langle q|B\rangle \right|^2 \]

- **Schrödinger equation**

\[
(\mathcal{H}_0 + V)|B\rangle = -E_B|B\rangle
\]

multiplying by \( \langle q| \) and using \( \mathcal{H}_0|q\rangle = \frac{q^2}{2\mu}|q\rangle \):

\[ \Rightarrow \text{momentum-space wave function:} \]

\[ \langle q|B\rangle = -\frac{\langle q|V|B\rangle}{E_B + q^2/(2\mu)} \]

- **S-wave, small binding energy** so that \( R = 1/\sqrt{2\mu E_B} \gg r, r: \text{range of forces} \)

\[ \langle q|V|B\rangle = g_{NR} \left[ 1 + \mathcal{O}(r/R) \right] \]

- **Compositeness:**

\[
1 - Z = \int \frac{d^3q}{(2\pi)^3} \frac{|g_{NR}|^2}{[E_B + q^2/(2\mu)]^2} \left[ 1 + \mathcal{O}\left(\frac{r}{R}\right) \right] = \frac{\mu^2|g_{NR}|^2}{2\pi \sqrt{2\mu E_B}} \left[ 1 + \mathcal{O}\left(\frac{r}{R}\right) \right]
\]
Compositeness (3)

- Coupling constant measures the compositeness for an $S$-wave shallow bound state

\[
|g_{NR}|^2 \approx \left(1 - Z\right)^{2\pi} \frac{\mu^2 \sqrt{2\mu E_B}}{2\pi} \leq \frac{2\pi}{\mu^2} \sqrt{2\mu E_B}
\]

bounded from the above

**Exercise:**

Show that $|g_{NR}|^2$ is the residue of the $T$-matrix element at the pole $E = -E_B$:

\[
|g_{NR}|^2 = \lim_{E \to -E_B} (E + E_B) \langle k | T | k \rangle
\]

Hint: use the Lippmann–Schwinger equation $T = V + V \frac{1}{E - H_0 + i\epsilon} T$ and the completeness relation $|B\rangle \langle B| + \int \frac{d^3q}{(2\pi)^3} |q(+)\rangle \langle q(+) | = 1$ to derive the Low equation (noticing $T |q\rangle = V |q(+)\rangle$):

\[
\langle k'|T|k\rangle = \langle k'|V|k\rangle + \frac{\langle k'|V|B\rangle \langle B|V|k\rangle}{E + E_B + i\epsilon} + \int \frac{d^3q}{(2\pi)^3} \frac{\langle k'|T|q\rangle \langle q|T^\dagger|k\rangle}{E - q^2/(2\mu) + i\epsilon}
\]
- Z can be related to scattering length $a$ and effective range $r_e$ [Weinberg (1965)]

\[
a = - \frac{2R(1-Z)}{2-Z} \left[ 1 + \mathcal{O}\left(\frac{r}{R}\right) \right], \quad r_e = \frac{RZ}{1-Z} \left[ 1 + \mathcal{O}\left(\frac{r}{R}\right) \right]
\]

Effective range expansion:

\[
f^{-1}(k) = \frac{1}{a} + r_e k^2/2 - i k + \mathcal{O}\left(k^4\right)
\]

**Derivation:**

\[
T(E) \equiv \langle k|T|k \rangle = -\frac{2\pi}{\mu} f(k) \quad \Rightarrow \quad \text{Im} T^{-1}(E) = \frac{\mu}{2\pi} \sqrt{2\mu E} \theta(E)
\]

Twice-subtracted dispersion relation for $T^{-1}(E)$

\[
T^{-1}(E) = \frac{E + E_B}{|g_{\text{NR}}|^2} + \frac{(E + E_B)^2}{\pi} \int_0^{+\infty} dw \frac{\text{Im} T^{-1}(w)}{(w - E - i\epsilon)(w + E_B)^2}
\]

\[
= \frac{E + E_B}{|g_{\text{NR}}|^2} + \frac{\mu R}{4\pi} \left( \frac{1}{R} - \sqrt{-2\mu E - i\epsilon} \right)^2
\]

- **Example:** deuteron as $pn$ bound state. Exp.: $E_B = 2.2$ MeV, $a_{3S_1} = -5.4$ fm

\[
a_{Z=1} = 0 \text{ fm}, \quad a_{Z=0} = (-4.3 \pm 1.4) \text{ fm}
\]
Applications

- **Coupling constant fixed by binding energy**, long-distance processes such as \( X(3872) \rightarrow D^0 \overline{D}^0 \pi^0, D^0 \overline{D}^0 \gamma \) calculable
  
  E.g., XEFT prediction of \( \Gamma(X \rightarrow D^0 \overline{D}^0 \pi^0) \)

- Compositeness from scattering length:
  
  scattering lengths calculable using the Lüscher formalism in lattice QCD

  E.g., from \( DK I = 0 \) scattering length \( \Rightarrow D^*_{s0}(2317) \) contains \( \gtrsim 70\% DK \)

  Liu et al., PRD86(2013)014508; Martínez Torres et al., JHEP1505,053; Bali et al., PRD96(2017)074501
Summary

- Lots of resonances or resonance-like structures observed in recent years, many puzzles
- QCD symmetries (chiral, heavy quark) prove to be useful tools
- Many more data needed, lots of work needs to be done

Thank you for your attention!
Backup slides
Symmetry implies a derivative coupling for GBs, i.e.,

GBs do not interact at vanishing momenta

- Consider GB \( \pi^a \): \[ \langle \pi^a | Q_A^a | 0 \rangle = \int d^3 x \langle \pi^a | A_0^a(x) | 0 \rangle \neq 0 \]
  Lorentz invariance \( \Rightarrow \) \[ \langle \pi^a(q) | A_\mu^a(0) | 0 \rangle = -i q_\mu F_\pi \]

- Consider the matrix element

\[
\langle \psi_1 | A_\mu^a(0) | \psi_2 \rangle = R_\mu^a + q_\mu F_\pi \frac{1}{q^2} T^a
\]

Current conservation \( \Rightarrow q_\mu A_\mu^a = 0 \), thus

\[
q_\mu R_\mu^a + F_\pi T^a = 0 \Rightarrow \lim_{q_\mu \to 0} T^a = 0
\]

- \( \Rightarrow \) GBs couple in a derivative form!!
Consider the scalar three-point loop integral

\[ I = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{[(P - q)^2 - m_1^2 + i\epsilon] (q^2 - m_2^2 + i\epsilon) [(p_{23} - q)^2 - m_3^2 + i\epsilon]} \]

Rewriting a propagator into two poles:

\[ \frac{1}{q^2 - m_2^2 + i\epsilon} = \frac{1}{(q^0 - \omega_2 + i\epsilon)(q^0 + \omega_2 - i\epsilon)} \quad \text{with} \quad \omega_2 = \sqrt{m_2^2 + \vec{q}^2} \]

focus on the positive-energy poles

\[ I \simeq \frac{i}{8m_1 m_2 m_3} \int \frac{dq^0 d^3 \vec{q}}{(2\pi)^4} \frac{1}{(P^0 - q^0 - \omega_1 + i\epsilon)(q^0 - \omega_2 + i\epsilon)(p_{23}^0 - q^0 - \omega_3 + i\epsilon)} \]
Contour integral over $q^0 \Rightarrow$

\[
I \propto \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{1}{[P^0 - \omega_1(q) - \omega_2(q) + i\epsilon][p_{23}^0 - \omega_2(q) - \omega_3(p_{23} - q^*) + i\epsilon]}
\]

\[
\propto \int_0^\infty dq \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\epsilon} f(q)
\]

The second cut:

\[
f(q) = \int_{-1}^1 dz \frac{1}{p_{23}^0 - \omega_2(q) - \sqrt{m_3^2 + q^2 + p_{23}^2 - 2p_{23}qz + i\epsilon}}
\]
Relation between singularities of integrand and integral

- singularity of integrand does not necessarily give a singularity of integral: integral contour may be deformed to avoid the singularity
- Two cases that a singularity cannot be avoided:
  - endpoint singularity
  - pinch singularity
\[ I \propto \int_0^\infty dq \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i \epsilon} f(q) \]

\[ f(q) = \int_{-1}^1 dz \frac{1}{A(q, z)} = \int_{-1}^1 dz \frac{1}{p_{23}^0 - \omega_2(q) - \sqrt{m_3^2 + q^2 + p_{23}^2 - 2p_{23}qz + i \epsilon}} \]

Singularity of the integrand of \( I \) in the rest frame of initial particle \((P^0 = M)\):

- **1st cut**: \( M - \omega_1(l) - \omega_2(l) + i \epsilon = 0 \) \( \Rightarrow \)
  \[ q_{on} \equiv \pm \left( \frac{1}{2M} \sqrt{\lambda(M^2, m_1^2, m_2^2)} + i \epsilon \right) \]

- **2nd cut**: \( A(q, \pm 1) = 0 \) \( \Rightarrow \) endpoint singularities of \( f(q) \)

\[ z = +1 : \quad q_{a+} = \gamma (\beta E_2^* + p_2^*) + i \epsilon, \quad q_{a-} = \gamma (\beta E_2^* - p_2^*) - i \epsilon, \]

\[ z = -1 : \quad q_{b+} = \gamma (-\beta E_2^* + p_2^*) + i \epsilon, \quad q_{b-} = -\gamma (\beta E_2^* + p_2^*) - i \epsilon \]

\[ \beta = \frac{|\vec{p}_{23}|}{E_{23}}, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{E_{23}}{m_{23}} \]

\( E_2^*(p_2^*) \): energy (momentum) of particle-2 in the cmf of the (2,3) system
All singularities of the integrand of $I$:

\[ q_{on^+}, \quad q_{a^+} = \gamma (\beta E_2^* + p_2^*) + i \epsilon, \]

\[ q_{on^-} < 0, \quad q_{b^-} = -q_{a^+} < 0 \quad \text{(for } \epsilon = 0), \]

\[ q_{a^-} = \gamma (\beta E_2^* - p_2^*) - i \epsilon, \]

\[ q_{b^+} = -q_{a^-}, \]

2-body threshold singularity at

\[ m_{23} = m_2 + m_3 \]

here $p_{2,\text{left}} = q_{on^+}$, $p_{2,\text{right}} = q_{a^-}$
We may also start from a QFT (for very small $E_B$, nonrelativistic)

\[
\begin{align*}
\text{Free field} & \quad \text{Interacting field} \\
\frac{i}{E - E_0} & + \quad \frac{i}{E - E_0 - \Sigma(E)} \\
\end{align*}
\]

Here $E_0 = M_0 - m_1 - m_2$ with $M_0$ the bare mass, $\Sigma(E)$ is the self-energy ($g_0$: bare coupling constant)

\[
\begin{align*}
\Sigma(E) &= ig_0^2 \int \frac{d^4k}{(2\pi)^4} \left[ \left( \frac{k^0}{2m_1} + i\epsilon \right) \left( E - k^0 - \frac{k^2}{2m_2} + i\epsilon \right) \right]^{-1} \\
&= -ig_0^2 \frac{\mu}{2\pi} \sqrt{-2\mu E - i\epsilon} + \text{constant} \\
&= g_0^2 \frac{\mu}{2\pi} \left[ \sqrt{-2\mu E \theta(-E)} - i\sqrt{2\mu E \theta(E)} \right] + \text{constant}
\end{align*}
\]
The physical mass is $M = m_1 + m_2 + E_B (E_B \geq 0)$ with $-E_B$ the solution of $E - E_0 - \Sigma(E) = 0$, i.e.

$$E_B = -E_0 - \Sigma(-E_B)$$

Expanding the self-energy around the pole, we rewrite the propagator

$$\frac{i}{E - E_0 - \Sigma(E)} = \frac{i}{E - E_0 - \left[\Sigma(-E_B) + (E + E_B)\Sigma'(-E_B) + \tilde{\Sigma}(E)\right]}$$

$$= \frac{i}{E + E_B - (E + E_B)\Sigma'(-E_B) - \tilde{\Sigma}(E)}$$

$$= \frac{iZ}{E + E_B - Z\tilde{\Sigma}(E)}$$

$Z$ is the wave function renormalization constant

$$Z = \frac{1}{1 - \Sigma'(-E_B)} = \left[1 + \frac{g_0^2 \mu^2}{2\pi \sqrt{2\mu E_B}}\right]^{-1}$$
Compositeness (7)

The physical coupling constant

\[ \tilde{g}^2 = Z g_0^2 = \frac{1}{g_0^2} + \frac{\mu^2}{2 \pi \sqrt{2 \mu E_B}} = (1 - Z) \frac{2 \pi}{\mu^2} \sqrt{2 \mu E_B} \]

Taking into account the nonrel. normalization, we get the one in rel. QFT

\[ g^2 = 8 m_1 m_2 (m_1 + m_2) \tilde{g}^2 = 16 \pi (1 - Z) (m_1 + m_2)^2 \sqrt{\frac{2 E_B}{\mu}} \]

If the ERE is dominated by the scattering length (when the pole is extremely close to threshold),

\[ T(E) = \frac{2 \pi / \mu}{-1/a - \sqrt{-2 \mu E - i \epsilon}} \]

At LO, effective coupling strength for bound state

\[ |g_{NR}|^2 = \lim_{E \to -E_B} (E + E_B) T(E) = -\frac{2 \pi}{\mu} \left( \frac{d}{dE} \sqrt{-2 \mu E - i \epsilon} \right)^{-1}_{E = -E_B} \]

\[ = \frac{2 \pi}{\mu^2} \sqrt{2 \mu E_B} \quad \Rightarrow \quad Z = 0 \text{ at this leading order approximation} \]