Forest Management Research Based on Tree Growth Model

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Abstract. At present, the greenhouse effect is becoming more and more serious, and just considering the reduction of greenhouse gas emissions cannot solve most of the problems, so it is important to solve the balance between deforestation and forest carbon sink for the development of the whole earth and human beings. In this paper, we establish a logical growth model of trees and assume that the weight gain of trees is equal to the carbon sink of trees. On the basis of this model, the annual carbon sequestration at different ages is obtained. Then, the multiple linear relationship between forest age ratio initially and growth to maximum carbon sequestration time was found to achieve the same maximum carbon sequestration time and with age ratio originally. In addition to this, the correlation data between forest carbon sink and was found and the maximum value was obtained by TOPSIS algorithm, which is the best way to manage the forest.

Keywords: Logistic growth model, TOPSIS algorithm, principal component analysis.

1. Introduction

Climate change poses a threat to the earth. In order to reduce the impact of climate change, we need to reduce greenhouse gas emissions and prevent climate change. We also need to strive to increase the amount of carbon dioxide storage isolated by biosphere or mechanical means, which is called carbon sequestration [1].

The biosphere stores carbon dioxide in plants, soil and water environment. While the deforested forest stores carbon dioxide in wood products. Therefore, the combination of carbon sequestration in forest products and carbon sequestration due to the regeneration of young forests may allow more carbon sequestration over time [2].

At the global level, appropriate deforestation may be conducive to carbon sequestration, but excessive deforestation will also limit carbon sequestration. Therefore, forest managers must find a balance between the value of forest products produced by deforestation and the value of carbon sequestration as a living tree.

2. Model assumptions and notation

2.1. Assumptions

1. The growth of trees conforms to the basic biological law, and there are no natural factors such as death in the growth process of trees, and the change of tree age ratio is only affected by human felling.
2. The growth of trees is only affected by light factors.
3. Because data is difficult to obtain, only Chinese data represent global data.
4. Deforestation produces wood products. We only consider wood products, and the life cycle of wood products is less than the life cycle of tree cutting.

2.2. Notations

Important notations used in this paper are listed in Table 1.
### Table 1. Notations

| Symbol | Notations |
|--------|-----------|
| t      | time      |
| $W(t)$ | The tree weight as a function of time |
| $W_{\text{max}}$ | The maximum weight of trees |
| r      | The inherent growth rate of trees |
| $\sigma$ | The environmental factor |
| $T$   | The illumination |
| $S$   | The total forest area |
| $h$ (cm) | The thickness of the soil |
| $Y(t/\text{hm}^2)$ | The mean soil erosion modulus |
| $\rho_{\text{soil}}$ (g/cm$^3$) | The average density of sediment |
| Eyuan/hm$^2$ | The woodland is worth per hectare |

### 3. Model construction and solving

#### 3.1. The Tree growth model

#### 3.1.1. Description of a tree weight and time

For forests, the process of carbon sequestration is the photosynthesis of trees. According to the existing research formula:

$$6\text{CO}_2 + 6\text{H}_2\text{O} = \text{C}_6\text{H}_{12}\text{O}_6 + 6\text{O}_2 \quad (1)$$

At the same time, forests also breathe and release carbon dioxide, taking into account the absorption of forest leaves and soil, so

$$\text{C}_6\text{H}_{12}\text{O}_6 + 6\text{O}_2 = 6\text{CO}_2 + 6\text{H}_2\text{O} \quad (2)$$

Ignoring forest respiration. From formula (1), we can see that every 1.63g CO2 absorbed by the forest can increase 1g of plant dry matter.

According to the relevant data, the carbon sequestration capacity of trees is young trees $>$ middle aged trees $>$ near mature trees $>$ mature trees $>$ over mature trees. Assuming that the carbon sequestration capacity of trees is uninterrupted during the development process, and the carbon sequestration capacity of trees does not increase when they reach mature trees, which is in a stable period, we believe that the slowing carbon sequestration capacity of trees is related to environmental carrying factors, gradual decline of function and other factors [3].

Assuming that we only consider that trees are always in their infancy and function well, the carbon sequestration capacity of trees is as follows:

$$\frac{dW(t)}{dt} = rW(t) \quad (3)$$

However, due to the fact that trees are always in the growth period, we can add a limiting factor.

$$\frac{dW(t)}{dt} = rW(t) \left(1 - \frac{W(t)}{W_{\text{max}}}\right) \quad (4)$$

In addition, we also consider that with the continuous growth of trees, the impact of environmental factors is increasing. Due to the gradual prosperity of branches and leaves, the mutual shielding
between trees and trees makes the light gradually reduce, so we only consider the impact of light on trees.

The differential equation of weight and time as follows.

\[
\frac{dW(t)}{dt} = rW(t) \left( 1 - \frac{W(t)}{W_{\text{max}}} - \sigma T \right)
\]  

We assume that the initial weight of saplings is 10 kg, so we can get the first-order nonlinear differential equation of carbon sequestration varying with time.

\[
\frac{dW(t)}{dt} = rW(t) \left( 1 - \frac{W(t)}{W_{\text{max}}} - \sigma T \right) \\
W(0) = 10
\]  

In order to simplify the problem, we \( \alpha = 1 - \sigma T \). Finally, we get the growth weight of trees as a function of time.

\[
W(t) = 10 \times \alpha \times \frac{W_{\text{max}} \times e^{\alpha t}}{\alpha W_{\text{max}} + 10(e^{\alpha t} - 1)}
\]

In this way, we can get the range of growth rate \( r \) between \( 20 \times 10^3 \) kg and \( 30 \times 10^3 \) kg every year. The Parameter value is shown in Table 2.

| \( W(1) \) (kg) | \( R \) (kg/year) | \( T \) (Lux) | \( \sigma \) |
|----------------|----------------|-------------|---------|
| 35             | 20             | 25          | 0.03744 |

Therefore, we draw the image of function (7) as shown in Figure 1.

![Figure 1. Tree weight growth curve](image)

3.1.2. **Analysis based on tree age ratio and carbon sequestration**

In order to simplify the article, we assume that the age of trees in the young stage is all 5 years, the age of trees in the middle stage is all 10 years, the age of trees in the near mature stage is all 15
years, the age of trees in the mature stage is all 25 years, and the age of trees in the over mature stage is all 30 years. It is not difficult for us to figure out carbon sequestration [4].

It is assumed that the age proportion of forest trees is young trees: medium-aged trees: near mature trees: mature trees: over mature trees, and the proportion is \(x_1: x_2: x_3: x_4: x_5\), we can conclude that the total carbon sequestration of a forest in a year is this.

\[
M = (10.2918 \times x_1 + 7.6855 \times x_2 + 0.3146 \times x_3) \times \beta \times S \tag{8}
\]

It can be seen from the data that in order to maintain the sustainable development of the forest, the proportion of forest trees should be young trees: medium aged trees: near mature trees: mature trees: over mature trees = 2:1:1:2:1[5]. From the above formula, it can be seen that the annual carbon sequestration of the forest is only linear with young trees: medium aged trees: near mature trees. It is assumed that the proportion at the initial stage of forest cutting conforms to the sustainable development strategy, that is, the forest age ratio at the initial stage of felling is young trees: medium aged trees: near mature trees = 2:1:1. Optimal proportion of forest is shown in Figure 2.

**Figure 2.** Optimal proportion of forest

Assumed forest density \(\beta\), the forest area \(s\) remains unchanged. Let the current forest get the maximum value of carbon sequestration after \(t\) years, and then the forest grows to young trees: middle-aged trees: near mature trees = 2:1:1 after felling.

\[
T = m_1x_1 + m_2x_2 + m_3x_3 + K \tag{9}
\]

The results obtained by SPSS fitting are as shown in Table 3.

**Table 3.** Fitting table

|               | Non standardized coefficient | Standardization Beta | t   | p    |
|---------------|------------------------------|----------------------|-----|------|
| (Constant)    | -2.974                       | 2.627                | 1   | 0.301|
| \(m1\)        | 1.312                        | 0.543                | 0.400 | 2.414 | 0.052 |
| \(m2\)        | 1.755                        | 0.540                | 0.534 | 3.248 | 0.018 |
| \(m3\)        | 1.645                        | 0.487                | 0.560 | 3.379 | 0.015 |

It can be seen from the above table that the R square is 0.838, indicating that the equation fitting effect is good, and the following functions can be obtained.

\[
T = 1.312x_1 + 1.755x_2 + 1.645x_3 - 2.974 \tag{10}
\]
It can be seen from function (10) that time $t$ is more related to the proportion of middle-aged trees. When the proportion of young trees and near mature trees remains unchanged, the time increases by 1.755 years for each increase in the proportion of middle-aged trees. When the proportion of young trees and medium-aged trees remains unchanged, the time increases by 1.645 years for each increase in the proportion of near mature trees. When the proportion of middle-aged trees remains unchanged, the time increases by 1.312 years for each increase in the proportion of young trees.

Assuming that the age ratio of young trees: middle-aged trees: near mature trees is 3:5:2, the forest area is 8410013 square hectares, and the forest density is 8800 trees / hm$^2$, the maximum carbon sequestration of the forest can be $2.794035 \times 10^{10}$ kg in 15 years. At this time, the proportion of trees is 5.34:3.46:1.2. If the trees are cut down again as young trees: middle-aged trees: nearly mature trees = 2:1:1, the maximum carbon sequestration of the forest will be reached after 12 years [6]. At this time, the carbon sequestration of the forest is $1.4783 \times 10^{10}$ kg. Optimal proportion of forest is shown in Figure 3.

![THE TREE PROPORTION](image)

**Figure 3.** Optimal proportion of forest

### 3.2. Best forest management solutions

It can be seen from the data that when the forest absorbs carbon dioxide, it is related to the amount of water conserved, vegetation density, precipitation, light time, soil pH, soil ammonium nitrogen content in the growing season, soil nitro nitrogen content in the growing season, temperature, and forest area [7]. Since the same type of fertilizer was applied at the beginning, with the same content per square meter, the biological effects of nitrogen production were negligible.

When the forest's carbon dioxide absorption effect is the best, the amount of water conserved, vegetation density, precipitation, light time, the larger these values are better, the closer the soil PH distance is to the appropriate value, the better, the smaller the value of the soil ammonium nitrogen content in the growing season and the nitro nitrogen content in the growing season, the better, indicating that the better the plant growth and development, the more carbon dioxide is absorbed. According to the data found, the forest was divided into 8 kinds of forests according to the amount of water conserved, vegetation density, precipitation, light time, soil PH, and soil ammonium nitrogen content in the growing season, and the optimal solution was obtained using the Topsis algorithm. The forest management scheme is shown in Table 4 and Table 5.
Table 4. Indicators for different forests

| Forest | Water conservation | Vegetation density | precipitation | Light time | Soil PH |
|--------|--------------------|--------------------|---------------|------------|---------|
| Forest 1 | 7300 | 6500 | 500 | 6.84 | 5.87 |
| Forest 2 | 7800 | 9200 | 600 | 6.84 | 5.34 |
| Forest 3 | 6300 | 7400 | 500 | 8.84 | 5.16 |
| Forest 4 | 6400 | 6300 | 500 | 6.84 | 6.52 |
| Forest 5 | 8500 | 8600 | 500 | 6.84 | 5.83 |
| Forest 6 | 7100 | 9600 | 400 | 6.84 | 6.23 |
| Forest 7 | 9300 | 9300 | 550 | 7.84 | 6.29 |
| Forest 8 | 8100 | 8500 | 500 | 6.84 | 5.03 |

Unit: $m^3$, roots/$hm^2$, ml, h, PH

Table 5. Indicators for different Options

| Option | Soil ammonium nitrogen content in peak growth season | Soil nitro nitrogen content in peak growth season | Temperature | Forest coverage |
|--------|---------------------------------------------------|--------------------------------------------------|-------------|-----------------|
| Option 1 | 9.23 | 0.74 | 24.6 | 90 |
| Option 2 | 7.24 | 0.56 | 26.3 | 69 |
| Option 3 | 8.1 | 0.64 | 21.2 | 76 |
| Option 4 | 9.73 | 0.83 | 28.3 | 77 |
| Option 5 | 11.03 | 0.96 | 19.8 | 85 |
| Option 6 | 10.26 | 0.92 | 23.1 | 83 |
| Option 7 | 9.25 | 0.71 | 21.7 | 81 |
| Option 8 | 8.67 | 0.78 | 25.4 | 91 |

Unit: mg/kg, mg/kg, °C, $hm^2$

Step1:

The soil ammonium nitrogen content in the growing season and the soil nitro nitrogen content in the growing season $x_i$ are treated from minimum value to maximum value, and the positive treatment formula is as follows.

$$X_i = \max - X_i$$ (11)

The soil PH is treated from the intermediate type to the maximum value, and the optimal value is set $X_{best}$ to the following treatment formula.

$$M_i = \max \{|X_i - X_{best}|\} - X_i$$ (12)

$$X_i = 1 - \frac{|X_i - X_{best}|}{M}$$ (13)
Step 2: Normalization of positive matrices

There are 8 evaluation objects and 9 evaluation indicators, which constitute the positive matrix as follows.

\[
X = \begin{bmatrix}
  x_{11} & \cdots & x_{19} \\
  \vdots & \ddots & \vdots \\
  x_{81} & \cdots & x_{89}
\end{bmatrix}
\] (14)

Its normalized matrix is \(Z\) denoted. There is \(Z\) each element.

\[
Z_{ij} = x_{ij} / \sqrt{\sum_{i=1}^{8} x_{ij}^2}
\] (15)

Step 3

Scores are calculated and normalized.

\[
Z = \begin{bmatrix}
  Z_{11} & \cdots & Z_{19} \\
  \vdots & \ddots & \vdots \\
  Z_{81} & \cdots & Z_{89}
\end{bmatrix}
\]

Defines the maximum value

\[
Z^* = (Z_{1}^*, Z_{2}^*, \ldots, Z_{9}^*)
\]

Defines the minimum value

\[
Z^- = (Z_{1}^-, Z_{2}^-, \ldots, Z_{9}^-)
\]

Defines the \(i\) distance between the first evaluation object and the maximum value

\[
D^+_i = \sqrt{\sum_{j=1}^{8} (Z_{j}^+ - z_{ij})^2}
\] (16)

Defines the \(i\) distance between the first evaluation object and the maximum value

\[
D^-_i = \sqrt{\sum_{j=1}^{8} (Z_{j}^- - z_{ij})^2}
\] (17)

The unmeritized score of \(i\) the first reviewer was calculated

\[
S_i = \frac{D^+_i}{D^+_i + D^-_i}
\] (18)

Therefore, we can see that 0 < \(S_i\) < 1, that is \(S_i\), the larger, the better the effect of this management.

TOPSIS evaluation calculation results are shown in Table 6.
Table 6. TOPSIS evaluation calculation results

|        | Positive ideal solution distance D+ | Negative ideal solution distance D- | Relative proximity C | Sort the results |
|--------|--------------------------------------|-------------------------------------|----------------------|------------------|
| Forest1| 3690.532                             | 1024.923                           | 0.217                | 7                |
| Forest2| 1552.576                             | 3271.094                           | 0.678                | 3                |
| Forest3| 3721.596                             | 1104.563                           | 0.229                | 6                |
| Forest4| 4394.338                             | 141.908                             | 0.031                | 8                |
| Forest5| 1284.573                             | 3184.377                           | 0.713                | 2                |
| Forest6| 2209.096                             | 3395.616                           | 0.606                | 5                |
| Forest7| 304.383                              | 4245.309                           | 0.933                | 1                |
| Forest8| 1630.955                             | 2844.384                           | 0.636                | 4                |

4. Conclusion

Considering the amount of carbon sequestration:

When forests have been around for a long time, the carbon sequestration capacity is far less than that of forests that are younger than in the previous year, and the ability to deal with carbon dioxide and other harmful gases has greatly declined, resulting in a decline in air quality in the community and bringing health risks to local residents [8].

From an educational point of view:

The popularization of how to promote the exploitation of trees by residents to promote local economic development shows that it is not a simple protection that can provide more benefits to the community, so that residents can understand the relationship between the forest and the long term development of the community, lay the foundation for the reasonable protection of the forest in the community, and let the residents understand that cutting down trees is not blind, it is purposeful and planned [9].

From the perspective of forest compensation:

The principles of science, fairness and reasonableness. The implementation of forest ecological compensation should be comprehensively planned, and the funds for forest ecological compensation should be distributed fairly and reasonably. Mainly including: stakeholders, the distribution of revenue and expenditure should be fair and reasonable. For example, for some people who love aerobic exercise in the forest and real estate agents are the beneficiaries of the forest from being felled, and the timber company is the beneficiary of the felling of trees, at this time, part of the tax revenue of the timber company can be applied to community construction, further balancing the interest gap between the two, and part of the after-tax of the timber company can be applied to the prevention and control of forest diseases and insect pests, and the resistance of the forest can be improved.

The principle of combining flexibility and utility. Because it does not have the attribute of commodity exchange, the ecological value of forests cannot be direct Enter the market to trade, but the service function value of the forest ecosystem does benefit people, so how much people should pay for this ecological value is the core issue, using market substitution value, shadow engineering method, etc. to estimate the ecological value of the forest, comparing the ecological value before the felling with the ecological value after the felling, when the ecological value after the felling is greater than the ecological value before the felling, explain to the community, and reduce the tax on the compensation community for the timber company.

The principle of openness and transparency. The income, expenditure, and implementation of the various funds compensated by the government and timber companies should be earmarked for special purposes, so that they should be open and transparent, and a unified competent department should be announced to the community on a regular basis, so that residents can further understand the collection and utilization of forest ecological compensation funds, and at the same time play a role in publicity and education.
Adapt to local conditions and actively innovate. It is necessary to combine the characteristics of forest ecological compensation areas, give play to their subjective initiative, actively explore forest ecological compensation methods suitable for their own regions, establish a sound policy and regulation guarantee system and a long-term and stable compensation mechanism for the development of forest ecological protection, and lay a solid foundation for improving forest ecological compensation [10].

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