Quasinormal modes and strong cosmic censorship in near-extremal Kerr-Newman-de Sitter black-hole spacetimes

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The quasinormal resonant modes of massless neutral fields in near-extremal Kerr-Newman-de Sitter black-hole spacetimes are calculated in the eikonal regime. It is explicitly proved that, in the angular momentum regime $\bar{a} > \sqrt{1 - 2\bar{\Lambda}/3}$, the black-hole spacetimes are characterized by slowly decaying resonant modes which are described by the compact formula $\Im\omega(n) = \kappa_+ \cdot (n + \frac{1}{2})$ [here the physical parameters $\{\bar{a}, \kappa_+, \bar{\Lambda}, n\}$ are respectively the dimensionless angular momentum of the black hole, its characteristic surface gravity, the dimensionless cosmological constant of the spacetime, and the integer resonance parameter]. Our results support the validity of the Penrose strong cosmic censorship conjecture in these black-hole spacetimes.

I. INTRODUCTION

The dynamics of linearized matter and radiation fields in black-hole spacetimes are characterized by a discrete family of complex (decaying in time) oscillation modes. These exponentially damped quasinormal resonances, which dominate the late-time relaxation dynamics of composed black-hole-field systems, have attracted the attention of physicists and mathematicians during the last five decades (see [1–3] for excellent reviews and detailed lists of references).

The characteristic quasinormal resonance spectrum $\{\omega(l, m; n)\}_{n=\infty}^n$ (here the dimensionless angular parameters $\{l, m\}$ are respectively the spheroidal harmonic index and the azimuthal harmonic index which characterize the linearized perturbation modes of the composed black-hole-field systems) of an asymptotically flat composed black-hole-field system is determined by the linearized Einstein-matter field equations with the physically motivated boundary conditions of purely ingoing waves at the absorbing black-hole event horizon and purely outgoing waves at spatial infinity [4, 5]. The fundamental black-hole-field quasinormal resonant mode (the mode with the smallest value of $\Im\omega$) determines the characteristic timescale

$$\tau_{\text{relax}} \equiv 1/\Im\omega(n = 0)$$

for the decay (relaxation) of linearized perturbation fields in the exterior regions of the curved black-hole spacetime.

Due to the mathematical complexity of the linearized Einstein-matter field equations, the complex resonant spectra of most black-hole spacetimes are not known in a closed (and compact) analytical form. Instead, one is usually forced to use numerical techniques in order to solve the linearized Einstein-matter field equations with the appropriate physically motivated boundary conditions which determine the composed black-hole-field quasinormal resonance spectra.

Near-extremal (rapidly-spinning) Kerr black holes are unique in this respect. In particular, solving analytically the linearized Einstein-matter field equations, it has been explicitly proved that the fundamental (least damped) quasinormal resonant frequencies of equatorial massless perturbation modes of near-extremal Kerr black holes are characterized by the remarkably compact analytical relation [6–10]:

$$\Im\omega(n) = 2\pi T_{\text{BH}} \cdot (n + \frac{1}{2}) ; \quad n = 0, 1, 2, \ldots ,$$

where $T_{\text{BH}}$ is the semi-classical Bekenstein-Hawking temperature [11, 12] of the black-hole spacetime, which is related to the classical surface gravity $\kappa_+$ of its outer (event) horizon by the simple relation [11, 12]

$$T_{\text{BH}} = \frac{\kappa_+}{2\pi} .$$

In a very interesting work [13], it has recently been demonstrated, using semi-analytical techniques, that near-extremal Kerr-de Sitter black holes are also characterized by the simple functional relation (2).

Interestingly, defining the dimensionless black-hole rotation parameter

$$\bar{a} \equiv \frac{a}{r_+}$$
(here \( r_+ \) is the radius of the black-hole outer (event) horizon [see Eq. (13) below]), one finds \cite{7} that scalar perturbation modes of near-extremal charged and spinning Kerr-Newman black-hole spacetimes with large enough angular momenta,  
\[
\bar{a} > \bar{a}_c(l),
\]
are also characterized by the compact analytical relation \cite{2, 14, 16}. Here the critical (minimal) black-hole rotation parameter \( \bar{a}_c(l) \), above which the quasinormal resonant modes of the near-extremal Kerr-Newman black-hole spacetimes are characterized by the compact analytical relation \cite{2}, depends on the angular harmonic index \( l \) of the linearized perturbation modes. In particular, it has been proved analytically that \cite{7}
\[
\bar{a}_c^{\text{KN}}(l \gg 1) = \frac{1}{2}
\]
for near-extremal Kerr-Newman black holes in the eikonal (geometric-optics) \( l = m \gg 1 \) regime.

The main goal of the present paper is to study analytically the quasinormal resonance spectra of massless neutral perturbation fields in non-asymptotically flat charged and rotating Kerr-Newman-de Sitter (KNdS) black-hole spacetimes. It is important to note that black holes in asymptotically de Sitter spacetimes have recently attracted much attention in the context of the intriguing Penrose strong cosmic censorship (SCC) conjecture \cite{17, 18}. In particular, it has been shown (see \cite{13, 19–21} and references therein) that the validity of the fundamental SCC conjecture in asymptotically de Sitter spacetimes depends on the existence of (at least) one black-hole-field perturbation mode with the property
\[
\Im \omega \leq \frac{1}{2} \kappa_-, \tag{7}
\]
where \( \kappa_- \) is the surface gravity which characterizes the inner (Cauchy) horizon of the black-hole spacetime \cite{13, 19–21}.

Using analytical techniques, we shall explicitly prove below that, in the eikonal (geometric-optics) regime \( l \gg 1 \), the quasinormal resonant modes of the near-extremal KNdS black-hole spacetimes with large enough angular momenta, \( \bar{a} > \bar{a}_c^{\text{KNdS}}(\bar{\Lambda}) \) (here \( \bar{\Lambda} \equiv \Lambda r_+^2 > 0 \) is the dimensionless cosmological constant of the black-hole spacetime), are characterized by the compact functional relation \cite{2}. In particular, we shall determine the functional dependence \( \bar{a}_c^{\text{KNdS}} = \bar{a}_c^{\text{KNdS}}(\bar{\Lambda}; l \gg 1) \) of the critical black-hole rotation parameter above which the neutral large-\( l \) fundamental perturbation modes of the near-extremal KNdS black holes conform to the important inequality \cite{7, 22}.

\section{II. DESCRIPTION OF THE SYSTEM}

We analyze the quasinormal resonance spectra of massless neutral fields which are linearly coupled to a non-asymptotically flat Kerr-Newman-de Sitter black-hole spacetime of mass \( M \), angular momentum \( J \equiv Ma \), electric charge \( Q \), and cosmological constant \( \Lambda > 0 \). The line element of the curved black-hole spacetime can be expressed in the form \cite{23, 24}
\[
ds^2 = -\frac{\Delta_r}{\rho^2} \left( \frac{dt}{I} - a^2 \sin^2 \theta \frac{dL_z}{I} \right)^2 + \frac{\Delta_\theta}{\rho^2} \left[ \frac{adt}{I} - (r^2 + a^2) \frac{dL_z}{I} \right]^2 + \rho^2 \left( \frac{dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_\theta} \right), \tag{8}
\]
where the metric functions are given by \cite{23, 24}
\[
\Delta_r \equiv r^2 - 2Mr + Q^2 + a^2 - \frac{1}{3} \Lambda r^2 (r^2 + a^2), \tag{9}
\]
\[
\Delta_\theta \equiv 1 + \frac{1}{3} \Lambda a^2 \cos^2 \theta, \tag{10}
\]
\[
\rho^2 \equiv r^2 + a^2 \cos^2 \theta, \tag{11}
\]
and
\[
I \equiv 1 + \frac{1}{3} \Lambda a^2. \tag{12}
\]
The zeroes of the radial metric function \[ \Delta_r(r_*) = 0 \quad \text{with} \quad * \in \{-, +, c\} \] determine the horizon radii which characterize the KNdS black-hole spacetime \[8\]. For generic KNdS black holes, there are four distinct (non-degenerate) roots \( r_0 < 0 < r_- \leq r_+ \leq r_c \) to the characteristic equation \[13\], where \( r_- \) is the inner (Cauchy) horizon, \( r_+ \) is the outer (event) horizon of the black hole, and \( r_c \) is the radius of the cosmological horizon.

In the next section we shall explicitly prove that the quasinormal resonance spectra of near-extremal KNdS black holes can be determined analytically. These black-hole spacetimes are characterized by the dimensionless relation \((r_+ - r_+)/r_+ \ll 1\), or equivalently \[20\]

\[
r_+^{-1} \Delta_r'(r_+) \ll 1 .
\]

It is worth noting that, taking cognizance of Eqs. \[9\], \[13\], and \[14\], one finds that near-extremal Kerr-Newman-de Sitter (NEKNdS) black-hole spacetimes are characterized by the dimensionless relation \( \bar{a}_{\text{NEKNdS}} \simeq \sqrt{1 - \bar{\Lambda} Q^2} \), where the dimensionless black-hole physical parameters \( \{\bar{a}, \bar{Q}, \bar{\Lambda}\} \) stand respectively for \( \{a/r_+, Q/r_+, \bar{\Lambda} r_+^2\} \). Note that, in the \( \bar{\Lambda} \rightarrow 0 \) limit, this relation reduces to the familiar dimensionless relation \( \bar{a}_{\text{NEKN}} \simeq \sqrt{1 - Q^2} \) which characterizes near-extremal Kerr-Newman (NEKN) black-hole spacetimes.

### III. THE QUASINORMAL RESONANCE SPECTRA OF NEAR-EXTREMAL KERR-NEWMAN-DE SITTER BLACK-HOLE SPACETIMES

In the present section we shall use analytical techniques in order to calculate the quasinormal resonance spectra of near-extremal charged and rotating KNdS black-hole spacetimes. In particular, we shall use the well established \[27, 29\] relation between the black-hole quasinormal resonant frequencies in the eikonal large-\( l \) regime and the unstable null circular geodesics which characterize the corresponding black-hole spacetimes.

The quasinormal resonant modes which dominate the linearized relaxation dynamics of neutral perturbation fields in asymptotically de Sitter back-hole spacetimes are characterized by the physically motivated boundary conditions of purely ingoing waves at the outer (event) horizon of the black hole and purely outgoing waves at the cosmological horizon of the spacetime \[30\]:

\[
\psi \sim \begin{cases} 
  e^{-i\omega y} & \text{for } r \rightarrow r_+ \quad (y \rightarrow -\infty) ; \\
  e^{i\omega y} & \text{for } r \rightarrow r_c \quad (y \rightarrow \infty) ,
\end{cases}
\]

where the tortoise radial coordinate \( y \) is defined by the differential relation \( dy = [(r^2 + a^2)/\Delta_r]dr \).

As explicitly proved in \[27, 29\], in the eikonal (geometric-optics) \( l \gg 1 \) regime, the real parts of the black-hole quasinormal resonant frequencies are directly related (proportional) to the characteristic angular velocity \( \Omega_c \) of null particles which are trapped at the unstable null circular geodesic of the black-hole spacetime. Likewise, the imaginary parts of the complex resonant frequencies are given, in the eikonal large-\( l \) regime, by the remarkably compact relation \[29\]

\[
\Im \omega(n) = -i(n + \frac{1}{2}) \cdot |\gamma| \quad ; \quad n = 0, 1, 2, ... ,
\]

where the integer \( n \) is the resonance parameter of the composed black-hole-field perturbation mode, and \[29\]

\[
\gamma = \sqrt{\frac{V''}{r^2}}
\]

is the Lyapunov exponent which characterizes the instability timescale \( \tau = \gamma^{-1} \) of the null circular orbit \[31\] (the dot symbol `` denotes a derivative with respect to the proper time \( \tau \)). Here \( V'(r) \) [see Eq. \[21\] below] is an effective radial potential which determines the geodesic motions of test particles in the black-hole spacetime. It is worth emphasizing again that, as shown in \[29\], Eq. \[17\] is valid in the asymptotic large-\( l \) regime.

The geodesic motions of a test particle of proper mass \( m \) in the KNdS black-hole spacetime are characterized by three conserved quantities \( \{E, L_z, K\} \) which are respectively related to the stationarity property of the spacetime geometry, to its axial symmetry, and to the hidden symmetry of the black-hole geometry \[22, 32, 34\]. In particular,
the equatorial motions of test particles in the KNdS black-hole spacetime are governed by the geodesic equations \[ \frac{dr}{d\lambda} = \pm \frac{V_r^{1/2}(r)}{r}, \] \[ r^2 \frac{dL_z}{d\lambda} = -IP_\theta + \frac{aIP_r}{\Delta_r}, \] and \[ r^2 \frac{dt}{md\lambda} = -aIP_\theta + \frac{(r^2 + a^2)IP_r}{\Delta_r}, \] where \[ V_r(r) = r^{-4}[P_r^2 - \Delta_r(m^2r^2 + K)], \] \[ P_r = I(Er^2 + a^2) - aIL_z, \] \[ P_\theta = I(aE - L_z), \] and \[ K = I^2(aE - L_z)^2. \] Here the affine parameter \( \lambda \) is related to the proper time \( \tau \) by the simple relation \( \tau = m\lambda \). The characteristic equatorial circular geodesics of the black-hole spacetime are determined by the relations \[ V_r(r = r_c) = 0 \quad \text{and} \quad V_r'(r = r_c) = 0, \] which, taking cognizance of Eqs. \( (9), (12), (21), (22), (24) \) and defining the angular velocity parameter \( \Omega_c = \frac{E}{L_z} \), yield the coupled algebraic equations \[ [(r_c^2 + a^2)\Omega_c - a]^2 = \Delta_r(r = r_c) \cdot (a\Omega_c - 1)^2 \] \[ 4r_c\Omega_c \cdot [(r_c^2 + a^2)\Omega_c - a] = \Delta'_r(r = r_c) \cdot (a\Omega_c - 1)^2 \] for the null circular geodesics of the KNdS black-hole spacetimes.

We shall now use analytical techniques in order to determine the physical and mathematical properties which characterize the equatorial null circular geodesics of near-extremal [see Eq. \( (14) \)] KNdS black-hole spacetimes. To this end, it proves useful to define the dimensionless small physical parameters \( x \equiv \frac{r_c - r_+}{r_+} \); \( y \equiv \Omega_c \cdot \frac{r_+^2 + a^2}{a} - 1 \). Substituting Eq. \( (29) \) into Eqs. \( (27) \) and \( (28) \), and using the near-horizon expansions \[ \Delta_r(r = r_c) = r_+\Delta'_r(r = r_+) \cdot x + \frac{1}{2} r_+^2 \Delta''_r(r = r_+) \cdot x^2 + O(x^3) \] and \[ \Delta'_r(r = r_c) = \Delta'_r(r = r_+) + r_+ \Delta''_r(r = r_+) \cdot x + O(x^2), \]
one obtains the leading-order (with $x \ll 1$ and $y \ll 1$) equations
\[
\left(\frac{a}{r_+^2 + a^2}\right)^2 \cdot (2r_+^2 x + a^2 y)^2 = \left[r_+ \Delta'(r = r_+) \cdot x + \frac{1}{2} r_+^2 \Delta''(r = r_+) \cdot x^2 \right] \cdot (a\Omega_c - 1)^2 \cdot [1 + O(x, y)]
\] (32)
and
\[
\frac{a}{r_+^2 + a^2} \cdot (2r_+^2 x + a^2 y) = \left[\Delta'(r = r_+) + r_+ \Delta''(r = r_+) \cdot x \right] \cdot \frac{(a\Omega_c - 1)^2}{4 r_+ \Omega_c} \cdot [1 + O(x, y)]
\] (33)
for the equatorial null circular geodesics of the near-extremal KNdS black-hole spacetimes.

From the two coupled equations (32) and (33) one finds the two dimensionless physical parameters
\[
x = \frac{\Delta'_c(r = r_+)}{r_+ \Delta''(r = r_+)} \cdot \left[\frac{1}{\sqrt{1 - \frac{r_+^2}{8 a^2} \Delta''(r = r_+)}} - 1\right]
\] (34)
and
\[
y = \frac{2 r_+ \Delta'_c(r = r_+)}{a^2 \Delta''(r = r_+)} \cdot \left[1 - \sqrt{1 - \frac{r_+^2}{8 a^2} \Delta''(r = r_+)}\right],
\] (35)
which characterize the near-horizon ($x \ll 1$) null circular geodesics of the near-extremal $[\Delta'_c(r_+)/r_+ \ll 1$, see Eq. (13)] KNdS black-hole spacetimes.

It is important to stress the fact that the physical requirement $x, y \in \mathbb{R}$ implies that the analytically derived relations (34) and (35) are valid for near-extremal charged and spinning KNdS black holes in the dimensionless angular momentum regime [see Eqs. (14), (20), (24), and (25)].

As a consistency check, we note that the critical rotation parameter $\tilde{a}^{\text{KNdS}}(\tilde{\Lambda})$ of the Kerr-Newman-de Sitter black-hole spacetimes [see Eq. (30)] reduces, in the $\tilde{\Lambda} \to 0$ limit, to the critical rotation parameter $\tilde{a}^{\text{KN}}$ [see Eq. (3)] which characterizes the Kerr-Newman black-hole spacetimes. It is important to note that, for KNdS black-hole spacetimes, the value of the maximally allowed dimensionless cosmological constant $\tilde{\Lambda}_{\text{max}}$ is a monotonically increasing function of the charge parameter $Q$ from $\tilde{\Lambda}_{\text{max}} = \sqrt{3}(2 - \sqrt{3})$ for neutral Kerr-de Sitter black holes [37] to $\tilde{\Lambda}_{\text{max}} = 1/2$ for maximally charged Reissner-Nordström-de Sitter black holes. Thus, the critical physical parameter $\tilde{a}^{\text{KNdS}}(\tilde{\Lambda})$ as given by Eq. (30) is real.

From Eqs. (21), (22), (24), and (26), one finds the relation
\[
V_r = \frac{f^2 L^2}{r^4} \left\{\left[\Omega_c (r^2 + a^2) - a\right]^2 - \Delta_a(a\Omega_c - 1)^2\right\}.
\] (37)

From Eqs. (25) and (37) one deduces the relations
\[
\left[\Omega_c (r^2 + a^2) - a\right]^2 - \Delta_a(a\Omega_c - 1)^2 = \left\{\left[\Omega_c (r^2 + a^2) - a\right]^2 - \Delta_a(a\Omega_c - 1)^2\right\}' = 0,
\] (38)
which imply [see Eq. (37)]
\[
V_r'' = \frac{f^2 L^2}{r^4} \left\{\left[\Omega_c (r^2 + a^2) - a\right]^2 - \Delta_a(a\Omega_c - 1)^2\right\}''.
\] (39)

From Eq. (39) one finds
\[
V_r''(r = r_c) = \frac{f^2 L^2}{r_c^4} \left\{8 (\Omega_c r_c)^2 + 4 \Omega_c \left[\Omega_c (r_c^2 + a^2) - a\right] - \Delta''(r = r_c) (a\Omega_c - 1)^2\right\}.
\] (40)

Substituting Eq. (29) into Eq. (41), one obtains
\[
V_r''(r = r_c) = \left(r_c^2 + a^2\right)^2 \cdot \frac{8 a^2}{r_c^4} - \Delta''(r = r_+) \cdot [1 + O(x, y)]
\] (41)
for the near-horizon equatorial null circular geodesics of the near-extremal KNdS black holes. In addition, from Eqs. (20), (22), (23), and (26), one finds the relation

$$i = \frac{r^2 L_x}{r^2} \left\{ -a(a \Omega_c - 1) + \frac{r^2 + a^2}{\Delta} \Omega_c (r_c^2 + a^2) - a \right\}.$$  (42)

Substituting Eq. (24) into Eq. (42), one obtains

$$i(r = r_c) = \frac{r^2 L_x (1 - a \Omega_c)}{r_c^2} \left[ -a + \frac{r^2 + a^2}{\Delta}(r = r_c) \right].$$  (43)

Substituting Eq. (30) into Eq. (43), one finds the near-horizon ($x \ll 1$) relation

$$i^{-1}(r = r_c) = \frac{r^2 + a^2}{I^2 L_x (1 - a \Omega_c)(r_c^2 + a^2)} \sqrt{r + \Delta_r'(r = r_+) \cdot x + \frac{1}{2} r^2 + \Delta_r''(r = r_+) \cdot x^2 \cdot [1 + O(x)]}.$$  (44)

Substituting Eqs. (29) and (34) into Eq. (44), one obtains

$$i^{-1}(r = r_c) = \frac{r + \Delta_r'(r = r_+)}{I^2 L_x \sqrt{16a^2 - 2r^2 + \Delta_r''}} \cdot [1 + O(x, y)].$$  (45)

Substituting Eqs. (41) and (45) into the characteristic eikonal relation (17), and using the expression (37)

$$k_+ = \frac{\Delta_r'(r = r_+)}{2I(r_c^2 + a^2)}$$  (46)

for the surface gravity of the outer black-hole horizon, one obtains the simple equality [see Eq. (14)]

$$\gamma = k_+ [1 + O(r_+ k_+)] \quad ; \quad r_+ k_+ \ll 1,$$  (47)

which, taking cognizance of Eq. (16), yields the remarkably compact functional relation

$$\Im \omega(n) = -i(n + \frac{1}{2}) \cdot k_+ \quad ; \quad n = 0, 1, 2, ...$$  (48)

for the fundamental quasinormal resonant frequencies which characterize the near-extremal ($r_+ k_+ \ll 1$) charged and rotating KNdS black-hole spacetimes in the eikonal (geometric-optics) regime.

IV. BLACK-HOLE QUASINORMAL RESONANT FREQUENCIES AND THE PENROSE STRONG COSMIC CENSORSHIP CONJECTURE

The strong cosmic censorship conjecture, introduced by Penrose almost five decades ago [18], asserts that, starting with physically reasonable (spatially regular) initial conditions, the dynamics of self-gravitating matter and radiation fields, which are governed by the Einstein field equations, will always produce globally hyperbolic spacetimes. If true, this physically important conjecture guarantees that classical general relativity is a deterministic theory.

It is well known that eternal black-hole spacetimes which possess regular inner Cauchy horizons are not globally hyperbolic [38-40]. In particular, for eternal charged and rotating black holes, the inner spacetime regions which are located beyond the black-hole Cauchy horizons are characterized by the presence of past directed null geodesics that terminate on the inner timelike singularities of the eternal black-hole spacetimes [38-40]. Thus, the Einstein field equations may fail to determine uniquely the future dynamics of physical observers who fall into eternal black holes which contain regular inner Cauchy horizons [38-40].

Intriguingly, however, as originally discussed by Penrose [18], physically realistic (dynamically formed) black-hole spacetimes are probably globally hyperbolic. In particular, according to the mass-inflation scenario [39-49], remnant perturbation fields that fall into dynamically formed black holes are infinitely blue-shifted as they approach the inner Cauchy horizons. This blue-shift mechanism may turn the pathological inner Cauchy horizons of eternal black-hole spacetimes into singular hypersurfaces that provide natural non-extendable boundaries to the inner spacetime regions of dynamically formed black holes [39-41].

As discussed in [18, 20] (see also [13, 21] and references therein), in asymptotically de Sitter black-hole spacetimes, the final fate of the fundamental Penrose SCC conjecture [18] is determined by the dimensionless ratio between the
physical parameters $\Im \omega_0$ and $\kappa_-$, which respectively characterize the decay rate of the linearized perturbation modes in the exterior regions of the dynamically formed black-hole spacetimes and the amplification rate of the infalling fields as they approach the inner Cauchy horizons of the corresponding black-hole spacetimes.

In particular, the validity of the SCC conjecture in dynamically formed non-asymptotically flat charged and spinning KNdS black-hole spacetimes depends on the existence of (at least) one black-hole-field perturbation mode which is characterized by the property \[ \frac{\Im \omega}{\kappa_-} \leq \frac{1}{2} \, . \] \tag{49}

Taking cognizance of the analytically derived functional relation \[ \text{(48)}, \] and using the inequality $\kappa_+ \leq \kappa_- \leq \kappa_{\text{KNdS}}$ which characterizes the surface gravities of the black-hole outer (event) and inner (Cauchy) horizons, one deduces that the fundamental (least damped, $n = 0$) neutral perturbation mode of the KNdS black-hole spacetimes conforms to the dimensionless relation \[ \text{(49)}. \] We therefore conclude that near-extremal charged and rotating KNdS black-hole spacetimes in the dimensionless physical regime respect the fundamental Penrose strong cosmic censorship conjecture \[ \text{(18, 51)}. \]

V. SUMMARY

The quasinormal resonance spectra which characterize the relaxation dynamics of linearized neutral fields in non-asymptotically flat near-extremal Kerr-Newman-de Sitter black-hole spacetimes have been studied analytically in the eikonal (geometric-optics) regime. We have proved that the characteristic relaxation rates of the composed black-hole-field systems are determined by the surface gravities [see Eqs. \[ \text{(17), (46), and (47)}. \] of the black-hole outer (event) horizons.

Interestingly, we have revealed the existence of a critical dimensionless black-hole rotation parameter $a_{\text{KNdS}} = a_{\text{KNdS}}(\Lambda)$, above which the eikonal neutral perturbation modes of the near-extremal black holes become long lived. In particular, using the well established relation between the black-hole quasinormal resonant frequencies in the eikonal large-$l$ regime and the physical properties of the unstable null circular geodesics which characterize the corresponding black-hole spacetimes \[ \text{(27, 29)}, \] we have derived the remarkably compact analytical formula [see Eqs. \[ \text{(36) and (48)\]} for the quasinormal resonant modes of the composed near-extremal-Kerr-Newman-de-Sitter-black-hole-linearized-neutral-field systems.

Finally, we have pointed out that the fundamental (least damped, $n = 0$) resonant mode of the near-extremal black-hole spacetimes conforms to the inequality $\Im \omega / \kappa_- \leq 1/2$ [see Eq. \[ \text{(19)}\] which is imposed by the fundamental SCC conjecture \[ \text{(18)}. \] The compact analysis presented in this paper therefore supports the validity of the Penrose strong cosmic censorship conjecture in these non-asymptotically flat charged and spinning Kerr-Newman-de Sitter black-hole spacetimes.

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We use natural units in which $G = c = h = k_{B} = 1$.

Here we have taken $m < l$ quasinormal resonant modes of near-extremal black holes.

It is worth noting that, as explicitly proved in [6–8], the simple analytical relation (2) also characterizes part of the non-equatorial ($m < l$) quasinormal resonant modes of near-extremal black holes.

We use natural units in which $G = c = h = k_{B} = 1$.

We would like to emphasize that this is not to say that the Penrose SCC conjecture [17, 18] is violated by highly charged KN dS black holes in asymptotically de Sitter spacetimes.
black-hole spacetimes outside the regime (32). In particular, it is worth mentioning that it has been explicitly proved in [21] that charged matter perturbation fields may also conform to the dimensionless inequality (49) which, as discussed in [19, 20] (see also [13, 21] and references therein), provides a necessary condition for the validity of the SCC conjecture [18] in asymptotically de Sitter black-hole spacetimes.