Spin precession due to spin-orbit coupling in a two-dimensional electron gas with spin injection via ideal quantum point contact

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We present the analytical result of the expectation value of spin resulting from an injected spin polarized electron into a semi-ininitely extended 2DEG plane with [001] growth geometry via ideal quantum point contact. Both the Rashba and Dresselhaus spin-orbit couplings are taken into account. A pictorial interpretation of the spin precession along certain transport directions is given. The spin precession due to the Rashba term is found to be especially interesting since it behaves simply like a windshield wiper which is very different from the ordinary precession while that due to the Dresselhaus term is shown to be crystallographic-direction-dependent. Some crystallographic directions with interesting and handleable behavior of spin precession are found and may imply certain applicability in spintronic devices.

Recent research on spin-polarized electron transport in semiconductors has attracted a great attention in the emerging field of spintronics. Of particular interests is the manipulation of spin via spin-orbit (SO) coupling in semiconductor nanostructures. In a two-dimensional electron gas (2DEG) confined in a heterostructure quantum well (QW), two basic mechanisms of the SO coupling are often taken into account: (i) structure inversion asymmetry (SIA) mechanism described by the Rashba term

\[ H_R = \frac{\alpha}{\hbar} (p_x \sigma_y - p_y \sigma_x), \]

whose coupling strength \( \alpha \) is gate-voltage-dependent and provides certain contribution to the Dresselhaus term.\(^{10,11} \) When restricted to a two-dimensional semiconductor nanostructure with [001] growth geometry, this term is of the form

\[ H_D = \frac{\beta}{\hbar} (p_x \sigma_z - p_y \sigma_y) \]

where the coupling parameter \( \beta \) is material specific. The interface inversion asymmetry (IIA) term also provides certain contribution to the Dresselhaus term in the SO coupling but is phenomenologically inseparable from BIA.

Whereas the competition between Rashba and Dresselhaus terms was concluded that the former dominates in narrow-gap systems while the latter dominates in wide-gap materials.\(^{12,13} \) Datta and Das proposed an theoretical idea constructing an electronic analog of the optic modulator using ferromagnetic contacts as spin injector and detector with a 2DEG channel confined in a narrow-gap semiconductor with only the Rashba SO coupling taken into account. In their proposal, the spin precession is envisioned due to the interference between the two eigenfunctions superposing the wave function of the injected spin with a gate-voltage-tunable phase difference \( \Delta \theta = 2m^* \alpha L / \hbar^2 \) with \( L \) being the channel length. Therefore, the spin orientation angle for electrons arriving at the end of the 2DEG channel, and hence the resulting current, is theoretically tunable via the applied gate-voltage. Hence the proposed device is expected to serve as a field-effect transistor (FET) based on the electron spin and has been commonly referred to as the Datta-Das spin-FET.

Recently, Winkler has further demonstrated this well-known spin precession described above and also the spin orientation in a quasi-two-dimensional (quasi-2D) electron system by using an \( 8 \times 8 \) Kane model which takes into account both SIA and BIA mechanisms.\(^{14} \) The spin orientation is shown to be sensitively dependent on the crystallographic direction for which the quasi-2D system is grown. This is also consistent with the previous results obtained by Lusakowski et al. showing that the conductance of the Datta-Das spin-FET depends significantly on the crystallographic direction of the channel when the Dresselhaus term is also at present.\(^{15} \) Indeed, the contribution to the SO coupling of Rashba and Dresselhaus terms may be of the same order in some QWs (such as GaAs QWs) and their ratio is even shown to be experimentally determinable very recently.\(^{21} \) Therefore, possible effects caused by the Dresselhaus term on spin-related devices has been an imperative issue in semiconductor spintronics.

In this report, we extend Winkler’s work, who calculated the expectation value of the spin operator (\( \mathbf{S} \)) (referred to as the spin vector in his paper) with respect to the injected spin-polarized electron state superposed by the two eigenstates of the 2DEG in the presence of the SO coupling. Whereas the spin precession is shown by calculating the overlaps between the spin vector and the polarization of the ferromagnetic drain contact numerically, we present the analytical result of the spin vector as a function of the coupling strengths \( \alpha \) and \( \beta \), the orientation angle of the injected spin, and the position of determination. A pictorial interpretation of the spin precession along certain transport directions is given. By analyzing the two extreme cases, pure Rashba and pure Dresselhaus, the spin precession due to SO couplings in inversion-asymmetric 2DEGS can be understood more concretely. Some crystallographic directions with interesting and handleable spin precession behavior are found.
and may imply certain applicability in spintronics.

Consider an electron with definite spin perfectly injected from a spin-polarized needle into an inversion-asymmetric 2DEG where both the Rashba and Dresselhaus SO couplings are present. The spin-injector and the 2DEG is connected with an ideal quantum point contact and the 2DEG is assumed to be semi-infinitely extended so that the boundary effect is out of consideration. Let the electron injected at an angle $\phi$ with spin $\mathbf{S}_0$ orienting toward $\phi_s$ with respect to $x$ axis. Setting the growth direction of the 2DEG layer to be [010], and the $x$ and $y$ axes to be [100] and [010], respectively, the single electron Hamiltonian under the effective mass approximation can be written as

$$
H = \frac{p^2}{2m^*} \sigma_0 + H_R + H_D
$$

where $m^*$ is the electron effective mass in the 2DEG. Defining

$$
\gamma(\phi) \equiv \sqrt{\alpha^2 + \beta^2 + 2\alpha\beta \sin 2\phi}
$$

and

$$
e^{-i\varphi} = \frac{\alpha e^{-i\phi} - i\beta e^{i\phi}}{\gamma(\phi)},
$$

the corresponding eigenenergies and eigenfunctions can be easily obtained as

$$
E_{\pm} = \left( \frac{\hbar k_{\parallel}^\pm}{2m^*} \right)^2 \pm \gamma(\phi) k_{\parallel}^\pm
$$

and

$$
\langle \mathbf{r} | \mathbf{k}^\pm_{\parallel}, \pm \rangle = \frac{1}{\sqrt{2}} e^{i\mathbf{k}_{\parallel} \cdot \mathbf{r}} \left( \frac{e^{-i\alpha}}{\pm 1} \right),
$$

where the inplane wave vector $\mathbf{k}_{\parallel}$ and the position vector $\mathbf{r}$ represent two-dimensional vectors ($k_x, k_y$) and $(x, y)$, respectively. Separating the spin part from the state kets $|\mathbf{k}^\pm_{\parallel}, \pm\rangle$, we denote the eigenspinors as $|\varphi - \pi/2, \pm\rangle$ with the usual definition

$$
|\tilde{\alpha}, \pm\rangle \equiv |\beta = 0, \tilde{\alpha}, \pm\rangle = \frac{1}{\sqrt{2}} \left( e^{-i\tilde{\alpha}} \pm 1 \right)
$$

where $\tilde{\beta}$ and $\tilde{\alpha}$ are the polar and azimuthal angles, respectively. Taking Eq. 7 as the basis, the injected spin $|\phi_s, +\rangle$ can be expanded as

$$
|\phi_s, +\rangle = c_+ |\varphi - \pi/2, +\rangle + c_- |\varphi - \pi/2, -\rangle
$$

where

$$
c_\pm = \langle \varphi - \pi/2, \pm | \phi_s, + \rangle = \frac{1}{2} \left( -ie^{i\varphi} |\phi_s, +\rangle + 1 \right).
$$

Since there is a phase difference $\Delta\theta(\mathbf{r}) = 2m^* \gamma(\phi) r/\hbar^2$ between $|\varphi - \pi/2, +\rangle$ and $|\varphi - \pi/2, -\rangle$, the spin state ket at position $\mathbf{r}$ can be equivalently written as

$$
|\phi_s, +\rangle_{\mathbf{r}}^{RD} = c_+ e^{-i(\Delta\theta(\mathbf{r})/2)} |\varphi - \pi/2, +\rangle + c_- e^{i(\Delta\theta(\mathbf{r})/2)} |\varphi - \pi/2, -\rangle
$$

(11)

where the superscript "RD" denotes that both the Rashba and Dresselhaus terms are nonvanishing. By computing the expectation values of $\mathbf{S}$ with respect to the state kets $|\phi_s, +\rangle_{\mathbf{r}}^{RD}$, regardless of the factor $\hbar/2$, we obtain

$$
\langle \mathbf{S} \rangle_{\mathbf{r}}^{RD} = \left( \begin{array}{c}
\langle S_x \rangle_{\mathbf{r}}^{RD} \\
\langle S_y \rangle_{\mathbf{r}}^{RD}
\end{array} \right)
\equiv
\left( \begin{array}{c}
\cos \phi_s \cos^2 \theta - \sin (2\varphi - \phi_s) \sin^2 \Delta\theta/2 \\
\sin \phi_s \cos^2 \theta - \sin (2\varphi - \phi_s) \sin^2 \Delta\theta/2
\end{array} \right).
$$

Note that the phase difference $\Delta\theta$ is generally a function of $\mathbf{r}$ in the presence of both the Rashba and Dresselhaus terms, and returns to be a constant only when either $\alpha$ or $\beta$ vanishes as will be seen in the later discussion. In the following we discuss the behavior of the spin precession under two extreme cases: $(\alpha \neq 0, \beta = 0)$ and $(\alpha = 0, \beta \neq 0)$.

In the absence of the Dresselhaus term, we return to the familiar results of the Rashba case: $\gamma(\beta = 0) = \alpha$. Here the reason we set Eq. 4 can also be seen since the mathematical forms of the eigenvalues, eigenfunctions, and the expectation values of $\mathbf{S}$ expressed in Eqs. 6, 11, and 12 are maintained. The results are obtained simply by replacing $\varphi$ with $\phi$, e.g., the expectation value of $\mathbf{S}$ is

$$
\langle \mathbf{S} \rangle_{\mathbf{r}}^R = \langle \mathbf{S} \rangle_{\mathbf{r}}^{RD} |_{\varphi = \phi}
$$

(13)

where the superscript "R" denotes, similar to the previous ones, that only the Rashba term is nonvanishing.

As an example illustrating the Rashba spin precession, let $\phi_s = \pi/8$. Spin orientations along 9 straight paths are shown in Fig. 1 where a compact unit defined as

$$
R_0 \equiv 2\pi\hbar^2/(m^* \sqrt{\alpha^2 + \beta^2})
$$

(14)

is used. Note that $R_0$ is essentially the length, within which the spin completes one period of precession on $x$ or $y$ axes, and is typically of the order of or less than $1\mu m$ for the Rashba case. Each pair of adjacent paths includes an angle of $\pi/8$, which divides a half circumferential angle into 8 equal parts. Spin precessions are clearly observed except for the path which is perpendicular to the injected spin (see the $-3\pi/8$ path in Fig. 1). This is reasonably expected since the projection of the injected spin on one of the two eigen spin states, which are always perpendicular to the electron wave vector, vanishes. Thus the fact that only one component of the basis is occupied leads to zero spin precession.

Interestingly, the Rashba spin precession (RSP) behaves simply like a windshield wiper swinging about the
direction perpendicular to the electron wave vector (or the propagation path). This is very different from the ordinary full-circle precession. Another feature of the RSP is that the projection of \( \langle S \rangle^R \) on the direction perpendicular to the path is universally conserved. Mathematically, this can be further demonstrated by calculating \( \langle S \rangle^R \cdot \hat{r} \parallel \) and \( \langle S \rangle^R \cdot \hat{r} \perp \) where \( \hat{r} \parallel \) and \( \hat{r} \perp \) are the unit vectors in the directions parallel and perpendicular to the path, respectively. Let us define these two projection quantities to be \( \langle S \rangle^R_{r,\parallel} \) and \( \langle S \rangle^R_{r,\perp} \). After some straightforward mathematical manipulation, we obtain

\[
\begin{align*}
\langle S \rangle^R_{r,\parallel} &= S_{0,\parallel} \cos \Delta \theta \\
\langle S \rangle^R_{r,\perp} &= S_{0,\perp}
\end{align*}
\]

where \( S_{0,\parallel} = \cos \phi_s \) and \( S_{0,\perp} = S_0 \sin \phi_s \) are the projections of the injected spin with normalized magnitude \( S_0 = 1 \) on \( \hat{r} \parallel \) and \( \hat{r} \perp \), respectively.

Also, the transport directions along which no spin precession occurs can be mathematically testified by calculating the scalar product of \( \langle S \rangle^R \) and \( S_0 \). The result is

\[
\langle S \rangle^R \cdot S_0 = \cos^2 \frac{\Delta \theta}{2} - \cos [2(\phi - \phi_0)] \sin^2 \frac{\Delta \theta}{2}
\]

having the maximum value when \( \phi = \phi_0 + (n + 1/2) \pi \) with \( n \) being an integer. That is, on the directions perpendicular to the injected spin, we always have \( \langle S \rangle^R = S_0 \) (see the \(-3\pi/8\) path in Fig. 1).

In the absence of the Rashba term, we have \( \gamma (\alpha = 0) = \beta \) and \( \varphi = \pi/2 - \phi \) from Eqs. (4) and (5). Thus the expectation value obtained in Eq. (12) is modified as

\[
\langle S \rangle^D = \left( \cos \phi_s \cos^2 \frac{\Delta \theta}{2} + \cos (2\phi + \phi_s) \sin^2 \frac{\Delta \theta}{2} \right) - \sin \phi_s \cos^2 \frac{\Delta \theta}{2} - \sin (2\phi + \phi_s) \sin^2 \frac{\Delta \theta}{2}
\]

where the superscript “D” is, again, for reminding that only the Dresselhaus term is at present. Again, the spin orientations on 9 straight paths are plotted in Fig. 2. The Dresselhaus spin precession (DSP), though appears to be more complicated than the Rashba case, is still analyzable mathematically. We first turn to the projections of \( \langle S \rangle^D \) on \( \hat{r} \parallel \) and \( \hat{r} \perp \). Using Eq. (17), we obtain

\[
\begin{align*}
\langle S \rangle^D_{r,\parallel} &= S_{0,\parallel} \left( \cos^2 \frac{\Delta \theta}{2} + \cos 4\phi \sin^2 \frac{\Delta \theta}{2} \right) - S_{0,\perp} \sin 4\phi \sin^2 \frac{\Delta \theta}{2} \\
\langle S \rangle^D_{r,\perp} &= S_{0,\perp} \left( \cos^2 \frac{\Delta \theta}{2} - \cos 4\phi \sin^2 \frac{\Delta \theta}{2} \right) - S_{0,\parallel} \sin 4\phi \sin^2 \frac{\Delta \theta}{2}
\end{align*}
\]
The projections shown above in general do not exhibit conserved quantities as in the Rashba case except two sets of paths: (i) For \( \phi = n/4 \) with \( n \) being an odd integer, we have

\[
\begin{align*}
\langle S \rangle_{r,||}^D &= S_{0,||} \cos \Delta \theta \\
\langle S \rangle_{r,\perp}^D &= S_{0,\perp} \cos n/4
\end{align*}
\]

which is exactly the same as Eq. (15). Thus on these directions, namely, \([100]\), \([010]\), \([1\bar{1}0]\), \([\bar{1}10]\), and \([\bar{1}0\bar{1}]\) (for which only \([100]\) and \([1\bar{1}0]\) are shown in our case) the spin precession behaves like the RSP (see Fig. 2). (ii) For \( \phi = n\pi/4 \) with \( n \) being an odd integer, we have

\[
\begin{align*}
\langle S \rangle_{r,||}^D &= S_{0,||} \\
\langle S \rangle_{r,\perp}^D &= S_{0,\perp} \cos \Delta \theta
\end{align*}
\]

In conclusion, we have presented the analytical results of the space-dependent expectation values of the spin operator \( \langle S \rangle_r \) with respect to the injected spin-polarized electron state superposed by the two eigenstates of the 2DEG in the presence of both Rashba and Dresselhaus SO coupling. The RSP behaves like a windshield wiper resulting from the conservation of \( \langle S \rangle_{r,\perp}^R \) and the oscillation of \( \langle S \rangle_{r,||}^R \) as shown in Eq. 15 while the DSP owns similar "swinging" behavior stemming from the conservation and oscillation of either \( \langle S \rangle_{r,||}^D \) or \( \langle S \rangle_{r,\perp}^D \) only on certain directions and thus exhibits crystallographic-direction dependence. The general behavior of the spin precession due to Rashba and Dresselhaus terms can therefore be envisioned as a superposition of RSP and DSP. This implies that the RSP behavior always exists on \([100]\) and \([1\bar{1}0]\) directions, and therefor an improvement in the quality of Datta-Das transistor may be achieved by setting the 2DEG channel growing toward \([001]\) with transport direction being either \([100]\) or \([1\bar{1}0]\). Furthermore, in both extreme cases a straight path on which no spin precession occurs can always be found. With proper arrangement, this zero spin precession path may be found to be the same \([100]\), for example, when setting the injected spin pointing to \( \pi/4 \) in both the Rashba and Dresselhaus cases. This implies an interesting direction toward which the spin transport is precessionless even in the presence of both Rashba and Dresselhaus SO couplings with different coupling strengths.

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