Fine Structure Constant and Fractal Potential

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Abstract

The variation of fine structure constant with the variation of velocity of light has been studied considering contribution to the permeability and permittivity of the vacuum and incorporating contribution of fractal potential originated from the non-differentiability of fractal fluctuation. The time variation of $\alpha$ is found to depend on the complex velocity field characteristics of fractal nature of space time.

Keywords: Fractal, fine structure constant, velocity of light

Introduction:

Fine structure constant represents interaction between two charged particle and is expressed as $\alpha = \frac{e^2}{4\pi \epsilon_0 \hbar}$. This coupling constant is responsible for the stable structure of atoms and is supposed to be a fundamental constant of nature. Recent experimental observations hint at the fact that the fine structure constant may not be constant [1] and varies according to the 'large number' hypothesis by Dirac [2]. QED suggests that the fine structure constant is not a constant rather it varies on distance and energy [3,4]. Some experimental results indicate the variation of $\alpha$ from cosmological time to the present. One of the important candidate for variation of $\alpha$ is the variable speed of light ($c$). In cosmology the variable speed of light is required to solve a number of cosmological problems like flatness. In QED, vacuum polarization plays an important role for the concept of variable $\alpha$. Haranas [5] has investigated the variation of $\alpha$ with the change of speed of light. They have argued that the variation of $\alpha$ may be attributed to the variable speed of light from cosmological time to the present value. Considering $c$ as function of time $c(t)$, they have obtained $|\frac{\alpha'}{\alpha}| = 10^{-15} - 10^{-16} yr^{-1}$ at $z=3$. Tobar [6] has argued that the variation of $\alpha$ is manifested due to the differential change in the fraction of quanta of the electric and magnetic flux of force of electron whereas variation of $\hbar c$ has shown to manifest due to common mode change. Leefer et al [7] have investigated the variation of fine structure constant using atomic Dysprosium. The transition between the nearly degenerate excited states with opposite parity is found to depend on $\alpha$. Ranada [8] has proposed a model to explain the observed cosmological variation of fine structure constant as an effect of quantum vacuum. He has pointed out that due to the fourth
Heisenberg relation, the density of the sea of virtual particles in quantum vacuum must change in a gravitational field with the corresponding change in permittivity and permeability. The variation of fine structure constant and its relation with fractal space has been studied by Bhattacharya et al [9]. They have investigated the effect of space-time geometry to the variation of 'α' between matter and radiation dominated era and have suggested that the variation of 'α' may be attributed to the intrinsic scale dependence of fundamental constants. Tedesco [10] has studied the variation of fine structure constant due to generalised uncertainty principle in the space-time domain wall. They have pointed out that the fine structure constant may be different in different epoch considering the generalized Heisenberg uncertainty principle which include gravitational interaction. Moffat [11] has investigated the variation of fine structure constant with speed of light and observed that the varying speed of light reproduces the observational data well in the range.

In the present work we have investigated the variation of fine structure constant considering the contribution of fractal potential arising due to the fractal nature of vacuum. We have studied the time variation of fine structure constant and have observed that α varies with derivative of ∆ν where ν is the velocity field.

**Formalism:**

The fine structure constant in vacuum can be expressed as:

\[
\alpha = \frac{e^2}{4\pi\epsilon_0 c}
\]  

(1)

Now considering the effect of medium we can rewrite the above expression as [8]:

\[
\alpha' = \alpha \sqrt{\frac{\mu_r}{\epsilon_r}} = \alpha \sqrt{\frac{\epsilon_r \mu_r}{\epsilon_r}}
\]  

(2)

where \( e = \frac{\epsilon}{\epsilon_r} \) and \( e = \frac{\mu}{\sqrt{\epsilon_r \mu_r}} \) in medium. It may be mentioned that the permittivity and permeability of the quantum vacuum changes to \( \epsilon_r \epsilon_0 \) and \( \mu_r \mu_0 \) indicating the effect of thickening and lightening of the medium. Recently Ranada [8] has suggested a model to investigate the variation of α where quantum vacuum has been suggested to be a transparent optical medium characterized by its permittivity and permeability and the change of 'α' is suggested to be a consequence of fourth Heisenberg relation applied to the Gravitational interaction of virtual pairs. In such system the observed quantities like permittivity and permeability of quantum vacuum have been expressed as \( \epsilon_r \epsilon_0 \) and \( \mu_r \mu_0 \) respectively. Incorporating the effect of gravitational field \( \phi_0 \) along
with Newtonian potential \( \phi \), the value of \( \epsilon_r \) and \( \mu_r \) at a space time point \( \phi \) have been expressed as:

\[
\epsilon_r = 1 - \frac{\beta(\phi - \phi_0)}{c^2}
\]  

(3)

\[
\mu_r = 1 - \frac{\gamma(\phi - \phi_0)}{c^2}
\]  

(4)

where \( \beta, \gamma \) are certain positive co-efficient, \( \phi \) is potential at a space-time point and \( \phi_0 \) is present gravitational potential of all universe.

In scale relativity the space-time is described as non-differential continuum [12]. We have introduce in effect of fractal potential due to space time non-differentiability or fractality. The fractal fluctuation can be expressed as effective force of the potential energy from which it has been derived. Nottale [12] has pointed out that in the scale relativistic fractal space (time) approach, the force is manifestation of the very structure of the space. The proposition applied to the standard quantum mechanical case of microphysics, requires such force to be universal and independent of the mass of particles. It may be mentioned that the most interesting consequence of non-differentiability of space time is the introduction of two valuedness in velocity. Nottale [12] has pointed out that the breaking of discrete symmetry like reflection in variance on the differential element of time implies two valuedness of velocity which in turn results in the origin of complex velocity and they are fractal function of the resolution such as:

\[
V_+ [x(t, dt), t, dt] = v_+ [x(t), t] + w_+ [x(t, dt), t, dt]
\]  

(5)

\[
V_- [x(t, dt), t, dt] = v_- [x(t), t] + w_- [x(t, dt), t, dt]
\]  

(6)

where the fractal velocity field has been decomposed in terms of their classical and fractal parts. The fractal fluctuation can be expressed in terms of a fractal force and the potential energy which is obtained from its derivative. In scale relativity and in fractal space time approach is termed as manifestation of the structure of the space. In such understanding as pointed out by Nottale [12], the force then must be independent of mass of the particle. The fractal potential energy can be expressed as [12],
\[ \phi_F = -imD \cdot \nabla \nu \]  
(7)

where \( D = \frac{h}{2m} \) and \( \nu \) is complex velocity field expressed as \( \nu = V - iU \) where \( V = \frac{v_+ + v_-}{2} \), \( U = \frac{v_+ - v_-}{2} \).

Including the contribution to potential from fractal space, ie. fractal potential, the expression (3) and (4) can be recast as:

\[ \epsilon_r = 1 - \frac{\beta(\phi - \phi_0 - imD \cdot \nabla \nu)}{c^2} \]  
(8)

\[ \mu_r = 1 - \frac{\gamma(\phi - \phi_0 - imD \cdot \nabla \nu)}{c^2} \]  
(9)

so that,

\[ \sqrt{\epsilon_r \mu_r} \simeq \left[ 1 - \frac{(\beta + \gamma)(X - Y)}{2c^2} \right] \]  
(10)

where \( X = \phi - \phi_0 \), \( Y = \frac{-imD \cdot \nabla \nu}{c^2} \), we have

\[ \mu_r^2 = \left[ 1 - \frac{\beta(X - Y)}{c^2} \right]^2 \]  
(11)

from (13),(14) and (2) we arrive at:

\[ \frac{\alpha'}{\alpha} \simeq \left[ 1 + \frac{Z(3\beta - \gamma)}{2c^2} \right] \]  
(12)

where \( Z = X - Y \). With \( \alpha' = \alpha + \Delta \alpha \), (15) can be rewritten as:

\[ \frac{\alpha + \Delta \alpha}{\alpha} = \left[ 1 + \frac{Z(3\beta - \gamma)}{2c^2} \right] \]  
(13)

so that:

\[ \frac{\Delta \alpha}{\alpha} = \xi(\phi - \phi_0 - imD \cdot \nabla \nu) \]  
(14)

where \( \xi = \frac{(3\beta - \gamma)}{2c^2} \).

The variation of \( \alpha' \) with time is obtained as:

\[ \frac{1}{\alpha} \cdot \frac{d(\Delta \alpha)}{dt} = \xi \frac{d}{dt}(-imD \cdot \nabla \nu) \]
\[ -\xi mD \cdot \frac{d\nabla \nu}{dt} \]  

(15)

where \( \nu' \) is the velocity field as stated earlier.

with \[ \frac{d\nu}{dt} = iD\Delta \nu \] from [12] we come across:

\[ \frac{1}{\alpha} \cdot \frac{d(\Delta \alpha)}{dt} = AmD^2 \nabla \cdot \Delta \nu \]  

(16)

The rate of change of \( \alpha \) depends upon the fractal potential part where \( \nu' \) is the complex velocity field.

**Discussion**

In the present work we have investigated the variation of fine structure constant with variation of velocity of light resulted from the variation of permittivity and permeability due to fractal fluctuation of space-time. The time variation of \( \alpha' \) is found to depend on the derivative of complex velocity field. The contribution to the potential which results from the fractal fluctuation has been incorporated to the vacuum potential. It may be mentioned that Chang et al [13] have investigated the observed Quasar absorption spectra and suggested that they can be interpreted as the effect of space time inhomogeneity whereas Timotte et al [14] have investigated the effect of fractal potential in the system dynamic using fractal character of particle movement. They have pointed out that complex speed field contributed to establish coherence in the system which results type I superconductivity. Recently Naschie [15] has investigated cantorian space-time and high energy particle physics. He has argued that recent results in particle physics suggested the geometry of space-time to be a non-differential contour set rather than flat and smooth. The radiative correction due to vacuum polarization also impose some correction to the fine structure constant substantially below compton wave length of electron. Contribution of fractal fluctuation potential to the vacuum potential is very important consequence in variation of fine structure constant. Quantum phenomenon emerges as natural consequence of non-differentiability of space-time. The fractal space-time introduces a complex velocity field which has contribution to the vacuum potential energy. It is interesting to note that the origin of changing velocity of light has contribution from the nature of space time. Zhang [16] has suggested that the electric charge as a form of imaginary energy where an electric charge is supposed to be a pack of complex energy. The real part is supposed to be proportional to the mass whereas the imaginary part represent
the electric charge. Electric charge is an important component of fine structure constant. It would be interesting to investigate how complex electric charge affects the fine structure and if there is any connection between the complex electric charge and complex velocity field originated from the non-differential space-time structure. We will investigate this problem in our future work.

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