Triangle singularity in the

\[ J/\psi \rightarrow K^+ K^- f_0(980)/a_0(980) \text{ decays} \]

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From threshold cusp to triangle singularity

Singularities systematized by Landau [L. D. Landau (1959)]:

➢ Two-body threshold cusp,
➢ Triangle singularity...

were fashionable in the sixties, see references:

R. Karplus et al, Phys. Rev. 111, 1187 (1958);
R. F. Peierls, Phys. Rev. Lett. 6, 641 (1961);
I. J. R. Aitchison, Phys. Rev. 133, B1257 (1964);
Y. F. Chang and S. F. Tuan, Phys. Rev. 136, B741 (1964);
J. B. Bronzan, Phys. Rev. 134, B687 (1964);
S. Coleman and R. E. Norton, Nuovo Cim. 38, 438 (1965);
C. Schmid, Phys. Rev. 154, 1363 (1967);
N. E. Booth, Phys. Rev. Lett. 7, 35 (1961);
V. V. Anisovich, Phys. Lett. 10, 221 (1964)…
A well-known example of the threshold cusp: the precise measurement of the $\pi\pi$ S-wave scattering length from the cusp at the $\pi^+\pi^-$ threshold.

[Meißner, Müller, Steininger (1997); Cabibbo (2004); Colangelo, Gasser, Kubis, Rusetsky (2006); . . .]
Anomally large isospin breaking in $\eta(1405) \rightarrow \pi^0 f_0(980)$ reaction was found in [BESIII, Phys. Rev. Lett. 108, 182001 (2012)].

This was explained in terms of the $K\bar{K}K^*$ triangle singularity in [J.-J. Wu et al, PRL108, 081803 (2012)], [F. Aceti et al, PRD86, 114007 (2012)], [X.-G. Wu et al, PRD87, 014023 (2013)], [Achasov et al, PRD92, 036003 (2015)] ...
The Coleman-Norton Theorem [Coleman, Norton (1965); Bronzan (1964)] tells that the triangle singularity is in the physical region only when the process can happen classically:

- All the intermediate states are on shell.
- The particle 3 emitted from the decay of the particle 1 moves along the same direction as the particle 2 with a larger speed than the particle 2 and can catch up with it to rescatter.
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\[ q_{on}^+ = q_{a-} \]

\[
q_{on}^+ = \frac{1}{2m_A} \sqrt{\lambda(m_A^2, m_1^2, m_2^2)} + i\epsilon
\]

\[
q_{a+} = \gamma (v E_2^* + p_2^*) + i\epsilon
\]

\[
q_{a-} = \gamma (v E_2^* - p_2^*) + i\epsilon
\]

Only depends on kinematics

[M. Bayar, F. Aceti, F. K. Guo, and E. Oset, PRD94, 074039 (2016)]
References in recent years

M. Mikhasenko, B. Ketzer and A. Sarantsev, PRD91, 094015 (2015);
A. P. Szczepaniak, PLB747, 410 (2015);
F.-K. Guo, U.-G. Meißner, W. Wang and Z. Yang, PRD92, 071502 (2015);
X. H. Liu, M. Oka and Q. Zhao, PLB753, 297 (2016);
X.-H. Liu, Q. Wang and Q. Zhao, PLB757, 231 (2016);
A. E. Bondar and M. B. Voloshin, PRD93, 094008 (2016);
X. H. Liu and U. G. Meissner, EPJC77, 816 (2017);
R. Pavao, S. Sakai and E. Oset, EPJC77, 599 (2017);
J.-J. Xie, L.-S. Geng and E. Oset, Phys. Rev. D 95, 034004 (2017);
E. Wang, J.-J. Xie, W.-H. Liang, F.-K. Guo and E. Oset, PRC95, 015205 (2017);
J. J. Xie and F. K. Guo, PLB774, 108 (2017);
S. Sakai, E. Oset and A. Ramos, EPJA54, 10 (2018);
Q.-R. Gong, J.-L. Pang, Y.-F. Wang and H.-Q. Zheng, EPJC78, 276 (2018);
W. H. Liang, S. Sakai, J. J. Xie and E. Oset, CPC42, 044101 (2018);
J. J. Xie and E. Oset, PLB792, 450-453 (2019);
L. R. Dai, Q. X. Yu and E. Oset, PRD99, 016021 (2019);
V. R. Debastiani, S. Sakai and E. Oset, EPJC79, 69 (2019);
Z. Cao and Q. Zhao, PRD99, 014016 (2019);
F.-K. Guo, PRL122, 202002 (2019) …
$J/\psi \rightarrow K^+ K^- f_0(980)/a_0(980)$ decays

A triangle singularity can be found at $M_{inv}(K^- f_0/K^- a_0) \approx 1515$ MeV:
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$\triangleright \quad Br(J/\psi \rightarrow K^+K^- \phi) = (8.3 \pm 1.2) \times 10^{-4}$

$t_{J/\psi \rightarrow K^+K^- \phi} = A \varepsilon(J/\psi) \cdot \varepsilon(\phi)$
$J/\psi \rightarrow K^+K^- f_0(980)/a_0(980)$ decays

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- $\text{Br}(J/\psi \rightarrow K^+K^-\phi) = (8.3 \pm 1.2) \times 10^{-4}$
- $\Gamma(\phi) = 4.25$ MeV

$t_{\phi \rightarrow K^+K^-} = g_V (p_{K^+}^\mu - p_{K^-}^\mu) \cdot \epsilon_\mu(\phi)$
$J/\psi \to K^+K^- f_0(980)/a_0(980)$ decays

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- $\text{Br}(J/\psi \to K^+K^- \phi) = (8.3 \pm 1.2) \times 10^{-4}$
- $\Gamma(\phi) = 4.25$ MeV
- Well studied within chiral unitary model, see:
  W. H. Liang and E. Oset, PLB 737, 70 (2014)
  J. J. Xie, L. R. Dai and E. Oset, PLB 742, 363 (2015)
Step 1: tree diagram

\[ t_{J/\psi \rightarrow K^+K^-\phi} = A \varepsilon(J/\psi) \cdot \varepsilon(\phi) \]

\[ \frac{d\Gamma_{J/\psi \rightarrow K^+K^-\phi}}{dM_{inv}(K^-\phi)} = \frac{1}{(2\pi)^3} \frac{1}{4M^2_{J/\psi}} p_{K^+} \tilde{p}_{K^-} \sum |t_{J/\psi \rightarrow K^+K^-\phi}|^2 \]
Step 1: tree diagram

\[
\begin{align*}
\mathcal{A}^2 \frac{\Gamma_{J/\psi}}{\Gamma_J} &= \frac{Br(J/\psi \rightarrow K^+K^-\phi)}{\int \frac{1}{(2\pi)^3} \frac{1}{4M_{J/\psi}^2} \rho_{K^+\phi^-} dM_{inv}(K^-\phi)} \\
&= 0.018 \pm 0.003 \text{ MeV}^{-1}
\end{align*}
\]

\[
t_{J/\psi \rightarrow K^+K^-\phi} = A \varepsilon(J/\psi) \cdot \varepsilon(\phi)
\]

\[
\frac{d\Gamma_{J/\psi \rightarrow K^+K^-\phi}}{dM_{inv}(K^-\phi)} = \frac{1}{(2\pi)^3} \frac{1}{4M_{J/\psi}^2} p_{K^+} \tilde{p}_{K^-} \sum |t_{j/\psi \rightarrow K^+K^-\phi}|^2
\]
\[-it = -iA \int \frac{d^4q}{(2\pi)^4} \frac{i}{(P-q)^2 - m^2_{\phi} + i\varepsilon q^2 - m^2_{K^-} + i\varepsilon (P-q-k)^2 - m^2_{K^+} + i\varepsilon} \times \varepsilon(J/\psi) \cdot \varepsilon(\phi) \varepsilon(\phi) \cdot (2k + q - P) \times (-ig_V) \times (-ig_{f_0,K^+K^-})

= -iA g_V g_{f_0,K^+K^-} \varepsilon(J/\psi) \cdot k t_T\]
triangle amplitude $t_T$ with a peak at $M_{inv}(K^-f_0/K^-a_0) \approx 1515$ MeV

$t = A g_V g_{f_0,K^+K^-} \varepsilon(J/\psi) \cdot k t_T$
**Step 2: triangle diagram**

The triangle amplitude $t_T$ with a peak at $M_{\text{inv}}(K^- f_0/K^- a_0) \approx 1515$ MeV.

**triangle singularity:** $q_{on^+} = q_{a^-}$

Mathematically:

$$t = A g_V g_{f_0,K^+K^-} \varepsilon(J/\psi) \cdot k t_T$$

where $A$ is a constant.
Step 3: final state interaction
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\[-i t' = -iA \int \frac{d^4 q}{(2\pi)^4} \frac{i}{(P - q)^2 - m_\phi^2 + i\epsilon \, q^2 - m_{K^+}^2 + i\epsilon \, (P - q - k)^2 - m_{K^-}^2 + i\epsilon} \times \varepsilon(J/\psi) \cdot \varepsilon(\phi) \varepsilon(\phi) \cdot (2k + q - P) \times (-ig_V) \times (-ig_{f_0,K^+K^-}) \times (-i t_{K^+K^-,\pi^+\pi^-}) \]

\[= -iA \, g_V \, t_{K^+K^-,\pi^+\pi^-} \, \varepsilon(J/\psi) \cdot k' \, t'_T \]
Step 3: final state interaction

\[
\frac{1}{\Gamma_{j/\psi}} \frac{d^2 \Gamma_{j/\psi \rightarrow K^+K^-f_0 \rightarrow K^+K^-\pi^+\pi^-}}{dM_{\text{inv}}(K^-f_0)dM_{\text{inv}}(\pi^+\pi^-)} = \frac{A^2}{\Gamma_{j/\psi}} \frac{1}{(2\pi)^5} \frac{1}{4M_{j/\psi}^2} \frac{1}{3} p''_{K^+} \tilde{p}_{K^-} p_{\pi^+} g_V^2 |t_{j/\psi \rightarrow K^+K^-\phi}|^2 |t_T'|^2
\]
Summary

• The following channels are accessible in present facilities:

\[ Br(J/\psi \to K^+K^- f_0 \to K^+K^+\pi^+\pi^-) = 7.6 \times 10^{-6} \]
\[ Br(J/\psi \to K^+K^- a_0 \to K^+K^-\pi^0\eta) = 5.2 \times 10^{-6} \]

• There is a sharp raise of this magnitude around

\[ M_{inv}(K^- f_0/K^- a_0) \approx 1515 \text{ MeV}, \] where the triangle singularity appears.

• All these features are tied to the nature of the \( f_0(980) \) and \( a_0(980) \) as dynamically generated resonances.
Thank you very much!

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