Cosmological Unification, Dark Energy and the Origin of Neutrino Mass

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We suggest that quintessential vacuum energy could be the source of right handed neutrino masses that feed the seesaw mechanism, which may provide observed small masses to light standard neutrinos. This idea is naturally implemented in the Cosmological Unification model based on the global $SO(1,1)$ symmetry, where early inflation and late accelerated expansion of the Universe are driven by the degrees of freedom of a doublet scalar field. In this model, the $SO(1,1)$ custodial symmetry naturally provides the quintessence to standard model singlet fermion couplings that sources neutrino masses. We also show that the model predicts a highly suppressed contribution to relativistic degrees of freedom from quintessential quanta at any late Universe epoch, ensuring the consistency of the model.

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I. INTRODUCTION

Contemporary cosmological surveys\textsuperscript{[1–6]} have shown that the energy density of our Universe, in the framework of General Relativity, consists mainly of an unknown substance having the exotic property of overcoming the pull of gravity compelling our Universe to a stage of accelerated expansion. Whatever this component is made of it is known as dark energy (DE). The simplest candidate for DE is the cosmological constant ($\Lambda$)\textsuperscript{[7, 8]}, other more elaborated proposals invoke the existence of scalar fields\textsuperscript{[9–11]} which near its vacuum state behave like $\Lambda$ and additionally have the advantage to allow a dynamics which could alleviate the problems of smallness and fine-tuning that $\Lambda$ has to deal with\textsuperscript{[12–15]}.

Since it was proposed, the feasibility of dynamic DE has been checked, as it can be seen for instance in early works like\textsuperscript{[16]} To name just a more recent one see\textsuperscript{[17]}. It is also expected to be checked in the near future through scheduled high precision probes like DESI\textsuperscript{[18]}. Several scalar fields have been proposed as DE, for instance Kessence\textsuperscript{[19, 20]}, Chaplygin Gas\textsuperscript{[21, 22]}, Phantom\textsuperscript{[23, 24]}, Hessian\textsuperscript{[25, 26]}, but among them likely the most known and studied is Quintessence (Q)\textsuperscript{[27–29]}, which is thought as a canonical scalar field minimally coupled to gravity, its potential being flat enough to guarantee the slow-rolling evolution of the field, which in turn is necessary to violate the strong energy condition and so to realize the accelerated cosmic expansion.

The cosmological evolution of Q has been studied widely regardless of its origin or the phenomenology of the high energy theory it could come from, to name only a few references see\textsuperscript{[30–33]}. It is possible to do this because to realize Q as DE it is only required the existence of a vacuum state that can be used as a classical source in Einstein’s equations.

On the other hand, an underlying theory has to be considered when interactions between DE and other fields are taken into account, see for instance\textsuperscript{[34, 35]} for DE and Dark Matter (DM) interactions, (for a review about DE and DM see\textsuperscript{[36]}). Other examples are the effects of coupling Q with ordinary matter, as it was revised in\textsuperscript{[37]}. The first mention and a posterior study on the possible connection among active neutrinos and Q, grounding the mass-varying neutrinos models, can be consulted in\textsuperscript{[38, 39]}.

A series of related studies can be found for instance in\textsuperscript{[40] and [41, 42]}. The study of Yukawa couplings between DE and fermionic DM and the effects of radiative corrections on the mass of Q, as well as the proposal of multi-axion DE/DM models and their cosmological evolution, were addressed in\textsuperscript{[43]}. Early ideas regarding a possible connection among sterile Majorana neutrino masses and ultra-light bosons that could be Q were presented in\textsuperscript{[44]}, although no reference to any governing principle for that was given there. To name only a few, studies on Q as an axionic particle or its connection with higher energy theories like string, superstring or M theory can be found in\textsuperscript{[45–47]}.

The dynamics of Q resembles that of the inflaton, which is the scalar field hypothesized in order to solve, among others, the horizon and flatness problems that non-inflationary (Friedmann) cosmologies suffered\textsuperscript{[48–52]}. Inflation assumes the early universe underwent an exponential expansion phase driven by a state of almost pure vacuum energy, that behaves like a cosmological constant, generated through the slow-rolling evolution of the inflaton, but unlike Q, at a higher energy scale and totally dominating the content of the universe. Despite these facts, both dynamics are evidently similar to each other, and it seems reasonable to assume that Q and the inflaton may be deeply interrelated.

Such is the line of thought of the cosmological unification idea presented in Ref.\textsuperscript{[53]}. According to that, one can unify, in the field theory sense, using symmetries, both stages of accelerated expansion by relating inflation and quintessence fields with the degrees of freedom of a

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unique scalar field representation. In such an approach, DE would be just the remnant of the very early stages of cosmological evolution (see also Ref. [54]). Although the original model, based on the $SO(1,1)$ global symmetry, as discussed in [53], was intended for phantom instead of Q as DE, on the basis of the same symmetry the Unification of inflation and Q is very well possible, as we will show below, by describing both the fields as associated to the components of a doublet scalar representation.

The interesting aftermath of this symmetry guided cosmological unification model, is that all possible interactions become very well defined at the Lagrangian level solely by the symmetry, in terms of a few field invariant couplings. That is the case of both scalar self-interactions as well as scalar to fermionic matter couplings. Hence, to the extent of a few fundamental parameters, all possible physics derived from the model becomes mostly determined. Exploring these and probe up to what extent the $SO(1,1)$ cosmological unification model can provide acceptable physical consequences is the main goal of the present paper.

Interestingly enough, as we will discuss later on, $SO(1,1)$ symmetry does provide a set of bilinear field invariants that allow accommodating inflation and quintessence dynamics from the most generic quadratic scalar potential. As explained briefly latter, more general potentials can be built by choosing higher-order invariants, nonetheless, we study the simplest one as the first approximation to the phenomenology of our model, despite the fact that the quadratic potential, in the inflation sector, is disfavoured by the Planck data [55].

To allow for a fermion to scalar coupling the symmetry enforces the introduction of a fermion doublet and a singlet. We assume these fermions to be right handed and singlets under the Standard Model (SM) of particle physics symmetries and naturally identify them as neutrinos. As expected, such Yukawa couplings would provide an inflaton decay channel for the reheating after inflation.

However, as we shall discuss, due to the symmetry, the same set of couplings would keep right-handed neutrinos couple to the quintessence field. The last would remain trapped in a false vacuum configuration along the evolution of the observed Universe. According to quintessence model, such a false vacuum is the actual source of the observed DE, yet, what becomes even more interesting is the observation that in the context of our model, this explanation of DE would also introduce a natural way to generate large masses to right handed neutrinos, which would become connected to the cosmological accelerated expansion.

Right handed neutrino masses are the main known ingredient of the seesaw mechanism [56–62], which provides a natural explanation to the tiny standard neutrino masses observed in neutrino oscillation experiments [63], which are so far bounded to be in the sub eV scale. (See also [64, 65] for very strong constraints on the sum of neutrino masses from cosmological data in the context of both, constant and dynamical DE.)

In its simplest one family formulation a right handed singlet neutrino, $N$ is added to the SM particle content, and the most general Lagrangian terms that contribute to neutrino masses are then written as $y \bar{L}_a H N + (h.c) + M_R \bar{N}^2 N$, where $L$ stands for the SM lepton doublet, $H$ for the Higgs and $y$ for the Yukawa couplings. By introducing the Higgs vacuum, $\langle H \rangle$, the first term becomes a Dirac mass term for the neutrino, $m_\nu \bar{N} N$, where $m = y \langle H \rangle$, which jointly to the Majorana mass term, provides a small effective mass for the standard neutrino that goes as $m_\nu \approx m^2 / M_R$. Assuming an order one Yukawa, the only way to understand a sub eV $m_\nu$ is to have $M_R$ as large as $10^{13}$ GeV or so. Smaller values are yet possible if smaller Yukawa couplings are considered. Nevertheless, notice that Majorana mass enters as a free parameter in the theory, with no connection to the Higgs mechanism whatsoever. Therefore, understanding neutrino masses with the seesaw mechanism becomes the search for an understanding of the origin of $M_R$. Here is where the outcome of the $SO(1,1)$ model becomes of relevance by suggesting that such mass could actually have a cosmological origin, associated with the source of DE. This is a striking observation that deserves to be closely analyzed in order to establish its consistency in the cosmological setup and doing so is the main goal of this paper.

To this end, we have organized our discussion as follows. In the next section, we introduce the Cosmological Unification model based in the $SO(1,1)$ symmetry. There we present the Lagrangian of the model, which is based on the most general bilinear invariants built upon a dimension two fundamental representation to which cosmological scalar fields are assigned. We then discuss how inflation and quintessence emerge in the model. Right handed neutrinos are introduced to the model in section three. Yukawa couplings to the cosmological scalars are explored and the conditions upon which these get masses from the cosmic vacuum energy is discussed. As this mechanism also implies that quintessence quanta, $\chi$, can be excited in the primordial plasma from out of equilibrium right handed neutrino interactions, due to the same couplings that provide neutrino masses, in section four we explore the consequences of it, by studying the production of relativistic $\chi$ fields through Boltzmann equations, which shows the consistency of the scenario with Big Bang Nucleosynthesis requirements. Section five contains a short discussion and some final remarks about our proposal. Finally, two appendices containing some technical details and relevant calculations are also included.

\section{II. THE $SO(1,1)$ COSMOLOGICAL MODEL}

Following the motivations of the $SO(1,1)$ model as presented in Ref [53], we consider the scalar doublet

\begin{equation}
\Phi = \begin{pmatrix}
\phi \\
\varphi
\end{pmatrix},
\end{equation}

where $\phi$ and $\varphi$ are real fields, and $\phi = \langle \phi \rangle$ is the Higgs vacuum. The field $\phi$ is responsible for breaking the $SO(1,1)$ global symmetry to the SM with the Yukawa couplings,

\begin{equation}
y \bar{L}_a H N + (h.c). \end{equation}
with $\phi$ and $\varphi$ complex scalar fields, which for convenience can be written in terms of four real fields as

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \quad \varphi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2).$$

This representation transforms under the global $SO(1,1)$ group as $\Phi \to g_a \Phi$, where $g_a$ stands for an arbitrary element in the corresponding $SO(1,1)$ matrix representation, whose exponential mapping is in general given by

$$g_a = e^{i\alpha_a \sigma_1}, \quad \alpha \in \mathbb{R},$$

with $\sigma_1$ the first Pauli matrix.

There are four bilinear invariants formed with this doublet [53]:

$$\Phi^†\Phi = |\phi|^2 + |\varphi|^2, \quad \Phi^†\sigma_1\Phi = \phi^*\varphi + \varphi^*\phi,$$

$$\Phi^†i\sigma_2\Phi = \phi\varphi - \varphi\phi, \quad \Phi^†\sigma_3\Phi = \varphi^2 - \varphi^2,$$

with $\sigma_2$ and $\sigma_3$, the other two Pauli matrices. Clearly, the kinetic term $\partial^\mu\Phi^†\partial_\mu\Phi$ belongs to the first class of invariants in the above equation. The potential of the model, on the other hand, is restricted to be built out of these invariants in order to keep the symmetry.

It is worth noticing that these terms still allow for some diversity on the possible cosmological potentials one may consider. In the case of real field representations, for instance, first and third invariants can be added together to provide for a whole class of systems where the fields have an independent evolution, simply because one can write $\phi^2 = \Phi^†\Phi + \Phi^†\sigma_3\Phi$, and $\varphi^2 = \Phi^†\Phi - \Phi^†\sigma_3\Phi$. In such a case, the potentials $U(\phi^2)$ and $V(\varphi^2)$ written on the terms of such combinations would always have a quadratic dependence on the fields. Of course, such a scenario implies the removal of the $\Phi^†\sigma_3\Phi$ term from the theory, but as stated in Ref. [53] this could be done by noticing that such a term is actually a pseudo-scalar bilinear under the parity transformation defined as $\Phi \to \sigma_3\Phi$, which can easily be added to the model. Such a construction, however, ignores the most general complex nature of the cosmological field $\Phi$ and we will avoid it.

Next, for our model, we consider the most general theory we can build out of the invariant terms in Eq. (3), but considering for simplicity only mass like terms in the potential. As it should be clear, more general potentials based on these same bilinears are also possible, but considering this simplest form, although disfavoured by Planck data, will suffice for our propose. Therefore, the Lagrangian we consider would be

$$\mathcal{L}_\Phi = \partial^\mu\Phi^†\partial_\mu\Phi - V(\Phi),$$

where the potential is formed from the most general linear combination of the non trivial invariants,

$$V(\Phi) = \Phi^†(\alpha_0 \mathbb{1} + \alpha_1 \sigma_1)\Phi + \alpha_3 \Phi^†\sigma_3\Phi + h.c.$$  

Here $\alpha_i=0,1,3$ are mass dimension two quantities which in general can be complex. As the model intends to incorporate inflation, we should assume that the involved scales are naturally large, perhaps as few orders below the Planck scale, $M_{pl}$. Also, a contraction with the background Friedmann-Robertson-Walker metric should be understood in the kinetic terms. In order to identify the dynamics of the so constructed cosmological model, we need to explore the potential in detail and identify the proper set of initial conditions that should give rise to inflation and DE.

As explained in detail in Appendix A, the above generic potential can be diagonalized using an orthogonal rotation, $S'$, on the four dimensional field space of initial real field components, such that we can use the new fields defined as $(Q_1, Q_2, \xi_1, \xi_2)^T = S(\phi_1, \phi_2, \varphi_1, \varphi_2)^T$, to build the mass eigenstate complex scalars

$$Q = \frac{1}{\sqrt{2}}(Q_1 + iQ_2), \quad \xi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2),$$

out of which the Lagrangian simply becomes [see Eq. (A6)]

$$\mathcal{L}_\Phi = \partial^\mu\mathcal{Q}^†\partial_\mu\mathcal{Q} - \mathcal{M}\mathcal{Q}^†\mathcal{M},$$

$$= \partial^\mu Q^†\partial_\mu Q + \partial^\mu \xi^†\partial_\mu \xi - m^2|Q|^2 - M^2|\xi|^2,$$

where

$$\mathcal{Q} = \begin{pmatrix} Q \\ \xi \end{pmatrix} \quad \text{and} \quad \mathcal{M} = \begin{pmatrix} m^2 & 0 \\ 0 & M^2 \end{pmatrix}.$$  

As stated in Appendix A, above masses, written in terms of $\alpha_i$, are expressed as

$$M^2 = \mu_0^2 + \mu^2; \quad \text{whereas} \quad m^2 = \mu_0^2 - \mu^2;$$

where $\mu_0^2 = 2Re\alpha_0$ and $\mu^2 = 2\sqrt{(Re\,\alpha_1^2 + |\alpha_3|^2)}$.

Notice that even though in Eq. (4) we started with a coupled system of complex fields, after field rotation we have ended with a new description where $Q$ and $\xi$ degrees of freedom had been decoupled. However, we should also notice that, even though this is a more suitable way of writing the potential, it is on the cost of hiding the $SO(1,1)$ symmetry, which now is not explicit in the Lagrangian.

Furthermore, the potential in Eq. (7) shows no explicit dependence on the phase fields which suggests that they should not play any fundamental role in the slow-roll evolution phase of the background cosmological system. In accordance with this, for purposes of simplicity, we shall proceed with the analysis of the cosmological model by only considering the modular field components as a good first approximation, fixing the phases to zero. However, as one may still consider worth asking about the role played by these field phases on other effects of cosmological interest, particularly as in the DE sector where this phase could play a regulatory role, as it is done, for instance, in spintessence models [66, 67], we are addressing the issue in some detail in Appendix B, where it is shown that the system dynamics of the background universe does indicate that it is indeed consistent to choose
the initial phase value being zero, such that the phase does not evolve.

The supplementary condition $\mu_0^2 \approx \mu^2$, which is consistent with the assumption that all involved scales were naturally about the same order, allows incorporating a fine-tuning in the masses just to have $M_2^2 \gg m^2$, which permits to identify $\xi$ as the inflaton field, and Q as the quintessence source of DE. As a matter of fact, in such a case the cosmological system involves the independent evolution of two fields that fall on a par aboloidal potential from some given initial condition towards the absolute minimum located in $\xi = Q = 0$. Clearly, for $M_2^2 \gg m^2$, the potential is steeper along $\xi$ direction with $Q$ behaving almost like a flat direction. Assuming that the initial condition is such that $(\xi) \sim (Q) \sim M_p$, in the slow-roll regime the source for inflation in the model would then be proportional to the squared modulus of the inflaton, as it is done in chaotic inflation. Similarly, the source for DE is proportional to the squared modulus of $(Q)$. According to the standard dynamics, $\xi$ should slow-roll down the potential towards the local minimum at $\xi = 0$ but where $Q$ is frozen at its initial value, $(Q)$ due to its small mass since $m \ll H \approx M$, with $H$ the Hubble parameter, and thus $Q/\xi \approx m^2/H \approx 0$, until $H$ catches with $m$ scale. Effectively, $Q$ would behave most of the time as a perfect fluid with an equation of state $p = \rho\omega$ with $\omega = -1$. Eventually, $\xi$ would exit inflation and suddenly evaporates and reheats the Universe. As usual for chaotic inflation, the observed amount of density perturbations in the cosmic microwave background would require $M \approx 10^{-5} M_p$. Q, on the other hand, should stay fixed at its initial value along most eras of evolution, until matter density, $\rho_m$, catches with quintessence false vacuum energy density,

$$\rho_{(Q)} = \frac{1}{2} m^2 (Q)^2,$$

near the coincidence era. After that, Q gets released and starts slow-rolling down towards its true minimum at zero. Most of the Q-models use this expression to rewrite the observed DE density, $\rho_{DE} = M^2_{pl} \Lambda \approx 10^{-47} \text{GeV}^4$, such that

$$(Q)^2 = 2 \frac{M^2_{pl}}{m^2} \Lambda.$$

Therefore, the mass of Q should be as small as $m \sim 10^{-23} \text{ eV}$, to provide a successful scenario. The smallness of this parameter indicates the need for a fine-tuning as large as in the cosmological constant problem.

### III. ADDING FERMIONS: REHEATING AND NEUTRINO MASS

Reheating after inflation in the usual approach uses the sudden decay of the inflaton into other particles in order to inject matter in an otherwise empty Universe. In accordance with the global $SO(1, 1)$ symmetry we adopted as the protective one for our cosmological model, the minimal fermionic matter content is accounted by introducing a total of three spinorial fields, $\Psi^a_{i=0,1,2}$, two of them arranged into a doublet

$$\psi = \begin{pmatrix} N^a_1 \\ N^a_2 \end{pmatrix},$$

and the remaining one treated as a singlet. We choose fermions to be (two-component) right handed Weyl fields, such that they can be identified with those usually introduced in extensions of the standard model of particle physics in order to have massive neutrinos through the seesaw mechanism. Thus, the two-component spinorial index $\hat{a} = 1, 2$ and Dirac matrices are written as

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu_{\hat{a}c} \\ \sigma^{\mu c}_{\hat{a}} & 0 \end{pmatrix},$$

with

$$\sigma^\mu_{\hat{a}c} = (\bar{l}, \sigma), \quad \sigma^{\mu c}_{\hat{a}} = (\bar{l}, -\sigma), \quad \bar{\sigma}^{\mu c}_{\hat{a}} = \epsilon^{\hat{a}b} \epsilon^{\sigma b} \sigma^\mu_c,$$

and $\sigma = (\sigma_1, \sigma_2, \sigma_3)$. In this notation, the charge conjugation matrix and the $\beta$ matrix (which is numerically equal to $\gamma^0$ but carrying different index structure), are respectively given by

$$C = \begin{pmatrix} 0 & \delta_c^\alpha \\ \bar{\delta}_c^\alpha & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & \delta^\alpha_c \\ \bar{\delta}^\alpha_c & 0 \end{pmatrix}. \quad (13)$$

From each Weyl field, a four-component $(a, \hat{a} = 1, 2)$ sterile Majorana neutrino is built by writing

$$\psi_i = \begin{pmatrix} N^a_{i\hat{a}} \\ N^a_i \end{pmatrix}, \quad (14)$$

where $N^a_{i\hat{a}}$ is the charge conjugate of the right handed Weyl field, given by $N^a_{i\hat{a}} = (N^a_{\hat{a}i})^C$. The previous can be seen from Eq. (14) and $\psi^C = C \psi^T$ with the application of Eq. (13). The doublet in Eq. (11) transforms under $g_{a} \in SO(1, 1)$ as

$$\begin{pmatrix} N^a_1 \\ N^a_2 \end{pmatrix} \rightarrow g_{a} \epsilon^a_{\alpha \sigma i} \begin{pmatrix} N^a_1 \\ N^a_2 \end{pmatrix} = \begin{pmatrix} N^a_{i\hat{a}} \\ N^a_i \end{pmatrix}, \quad (15)$$

with the new Weyl fields arising from combinations and global phase changes of the previous ones. It is important to note that since the Weyl fields admit global phase transformations, it will be always possible to build a new four-component sterile Majorana neutrino

$$\psi_i' = \begin{pmatrix} N^a_{i\hat{a}} \\ N^a_i \end{pmatrix}, \quad \text{such that} \quad \psi_i \rightarrow \psi_i^C \rightarrow \psi_i' = \psi_i'^C,$$

therefore the transformation of the field $\psi_i$ induced by the $SO(1, 1)$ rotation in Eq. (15) does not violate the Majorana condition.
With these conventions, the general fermion kinetic terms for the Majorana fields become \( \frac{1}{2} \psi_i \gamma^\mu \partial_\mu \psi_i = N_i^\dagger i \sigma_\mu \partial_\mu N_i \), where a background metric contraction should be understood as before. Next, it is easy to see that one can write the kinetic terms in a clearly \( SO(1,1) \) and Lorentz invariant form, as

\[
L_\Psi = N_i^\dagger i \sigma_\mu \partial_\mu N_i + \Psi^\dagger i \sigma_\mu \partial_\mu \Psi .
\]  

(16)

On the other hand, by taking the Hermitian conjugate of \( N_0^\dagger \) and the fermion and scalar doublets, the most general Yukawa interaction terms from the linear combination of the invariants one can build are

\[
-L_I = N_{0\bar{a}} \left\{ a_0 \Phi \bar{\psi} + a_1 \Phi \bar{\sigma}_1 \Psi \right. \\
\left. + a_2 \Phi^\dagger i \sigma_2 \Psi + a_3 \Phi^\dagger i \sigma_3 \Psi \right\} + h.c.,
\]  

(17)

where \( a_i = 0, \ldots, 3 \) are complex dimensionless couplings.

Notice that analogous to the invariant terms which appear in Eq. (3), there exist bilinear \( SO(1,1) \) invariants that are formed from the fermion doublet taken with itself, \( \Phi \bar{\psi}, \Phi \bar{\sigma}_1 \Psi, \Phi^\dagger i \sigma_2 \Psi \) and \( \Phi^\dagger i \sigma_3 \Psi \), which, however, are not Lorentz invariant objects and therefore we take them off from the Lagrangian.

It is also worth asking if there are allowed mass terms for the fermions. We note that such terms can be built by defining an additional doublet formed from the charge conjugate fields of \( N_{i=1,2}^\dagger \), as

\[
\Psi^C = \begin{pmatrix} N_{1a}^\dagger \\ N_{2a}^\dagger \end{pmatrix} .
\]  

(18)

The following product, which is a Lorentz-invariant scalar

\[
\Psi^C \bar{\psi} + h.c. = N_{1a} N_{1b} + N_{2a} N_{2b} + h.c.,
\]  

(19)

clearly produces Majorana mass terms \((\bar{\psi}^C \cdot C \psi_i)\) for the fields \( \psi_i = 1,2 \), however, in order to get a consistent transformation of \( \Psi^C \) under the symmetry, it is necessary to impose the condition that

\[
N_{1a}^\dagger = (N_{1a}^\dagger)^C ,
\]  

which means that the components of the charge conjugate rotated doublet \( \Psi^C \) must be equal to the charge conjugate components of the rotated doublet \( \Psi^\prime \). In order to achieve this, the doublet in Eq. (18) has to transform with the Hermitian conjugate matrix \( g_{ai}^1 \), as can be checked by means of the two-dimensional matrix representations. Consequently, the term in Eq. (19) is not invariant under \( SO(1,1) \) rotations and we must remove it from the Lagrangian. The same occurs for all the terms formed from Eq. (18) and (11). On the other hand, a mass term for \( N_0^\dagger \) does is allowed by the \( SO(1,1) \) symmetry because it transforms as a singlet, however, we note that the interaction sector in Eq. (17) is invariant under the following \( U(1) \) transformation

\[
\Psi \rightarrow e^{i q} \Psi, \quad N_0^\dagger \rightarrow e^{-i q} N_0^\dagger,
\]  

(20)

as long as \( q = -g_0 \). So, the fields \( N_{i=1,2}^\dagger \) transform with the same charge and \( N_0^\dagger \) does it with the opposite. Thereby, by imposing invariance under \( U(1) \) in the fermion sector, which implies lepton number conservation, we remove the singlet’s mass term. We note that the same argument can be invoked in order to forbid mass terms for the fermions \( \psi_i = 1,2 \), but this only confirms what the \( SO(1,1) \) symmetry suggests.

Finally, the complete Lagrangian we are left with, is

\[
\mathcal{L} = \mathcal{L}_\Psi + \mathcal{L}_\Psi + L_I ,
\]  

(21)

where the three sectors are respectively given by Eq. (4), Eq. (16) and Eq. (17). The above Lagrangian is the most general one that can be written with \( SO(1,1) \) bilinear invariant terms, it is also Lorentz invariant, \( P \) (as long as both scalar fields transform with the same parity phase) and \( CP \) invariant. As mentioned above the fermionic sector is \( U(1) \) invariant, similarly, there is \( U(1) \) invariance in the scalar sector, as long as both \( \phi \) and \( \varphi \) transform with the same charge.

\[ \textbf{A. Reheating} \]

By performing the rotation in field space that diagonalizes the scalar sector and allows to identify the inflaton and quintessence fields, one has also to redefine the general Yukawa couplings introduced in Eq. (17). After some algebra, as explained in detail in Appendix A, scalar to fermion couplings [see Eq. (A16)] can be put into the following simple expression

\[
-L_I = N_{0\bar{a}} \{ \phi \Gamma_1 F + \varphi \Gamma_2 G \} + h.c.,
\]  

(22)

where the new coupling constants, which are just simple linear combinations of the original \( a_i \) constants written in Eq. (17), are contained in the matrices [see Eq. (A17)]

\[
\Gamma_1 = \begin{pmatrix} 0 & g_2 \\ h_1 & 0 \end{pmatrix} \quad \Gamma_2 = \begin{pmatrix} g_1 & 0 \\ 0 & -h_2 \end{pmatrix} .
\]

Here, the Weyl fields \( F_{i=1,2} \) are the components of the doublet

\[
F = \begin{pmatrix} F_1^\dagger \\ F_2^\dagger \end{pmatrix} ,
\]  

(23)

which arises from Eq. (11) after performing a \( SO(2) \) rotation, \( e^{-i \sigma_2 \pi/4} \Psi = F \), as can be seen in equation (A15). Notice that this rotation also transforms the spinor kinetic terms, which remain diagonal [see equation (A18)]. Clearly, as for the scalar sector, after the transformations the \( SO(1,1) \) symmetry is not explicit in the Yukawa Lagrangian anymore. The assumed \( U(1) \) symmetry imposed in the fermion sector remains explicit, on the other hand.
Former couplings can be written in a more useful way, as

\[ -\mathcal{L}_I = N_{0i} \{ g_1 Q F_1^i + g_2 Q^* F_2^i \\
+ h_1 \xi^* F_1^{i*} - h_2 \xi F_2^{i*} \} + h.c. \]  

(24)

The last two terms of Eq. (24) provide the inflaton decay channels, \( \xi \to N_0 F_i \), that are required for reheating after inflation. The sudden evaporation of inflaton energy would inject entropy to the emptied Universe by inflation. Since the fermions on final states are assumed to be right handed neutrinos they should provide the portal, through the standard couplings \( \bar{L} H N_0 \) and \( \bar{L} H F_i \), to produce all types of SM fields, which in turn should thermalize producing the primordial plasma. Assuming that such a process is efficient enough, the reheating temperature of the plasma should be \( T_r \sim 6 \times 10^{-3} \) max\( \{|h_1|, |h_2|\} \) \( M_{pl} \).

**B. Sourcing neutrino mass with DE**

At the end of inflation the \( \xi \) field evaporates completely, such that its energy density becomes null, sitting the inflaton field at its zero value which makes its couplings of no further relevance for thermal history. On the other hand, as we have already discussed in the previous section, the \( Q \) field would remain trapped on its initial homogeneous configuration all along the Universe evolution, perhaps changing quite slowly until recent times, when it is still slow-rolling down its almost flat potential while causing the Universe accelerated expansion.

By inserting the \( Q \) false vacuum, conveniently defined as \( \langle Q \rangle / \sqrt{2} \), back in Eq. (24), one immediately realizes that due to the couplings provided by the \( SO(1,1) \) model, DE naturally generates masses for the right handed neutrinos, given as

\[ \mathcal{L}_m = m_1 N_{00} F_1^i + m_2 N_{00} F_2^i + h.c. \]  

(25)

where \( m_i = g_i \langle Q \rangle / \sqrt{2} \). These mass terms, as discussed in detail in Appendix B, give rise to two degenerate massive Majorana neutrinos, \( \nu_{1,2} \), for which one can write

\[ -\mathcal{L}_m = \frac{1}{2} m_k (\bar{\nu}_1 \nu_1 + \bar{\nu}_2 \nu_2) \]  

(26)

This is a striking result, which connects the seesaw mechanism, and thus the origin of standard neutrino mass, to the origin of DE. Here, we have implicitly written the Majorana condition, namely \( \bar{\nu} = \nu^c C \), with \( C \) the charge conjugation matrix [see equation (13)]. Likewise, the mass \( m_k \) appearing in Eq. (26), as defined in (B23), is given by

\[ m_k = \frac{a_c \langle Q \rangle}{\sqrt{2}} \]  

(27)

where the effective coupling \( a_c = \sqrt{|g_1|^2 + |g_2|^2} \). We note that by choosing \( a_c \) in the interval \( 10^{-3} \lesssim a_c \lesssim 10^{-5} \), which seems reasonable, we can get right handed neutrino masses in the range of \( 10^{13} \) GeV \( \lesssim m_k \lesssim 10^{15} \) GeV, which are values around those needed to implement the standard seesaw mechanism.

Another immediate outcome of the present model is the alignment to mass terms of couplings among quintessence quantum excitations, \( \chi \), and neutrinos. Setting in the excitations over the false vacuum, by redefining \( Q = (\langle Q \rangle + \chi)/\sqrt{2} \), it is clear that after diagonalizing fermion masses, one gets

\[ -\mathcal{L}_{I\chi} = \frac{a_c}{2\sqrt{2}} \chi (\bar{\nu}_1 \nu_1 + \bar{\nu}_2 \nu_2) \]  

(28)

This coupling has relevance for thermal history. Equations (26) and (28) show that only two neutrinos are massive and interact with the DE field. The third neutrino remains massless and decoupled. Heavy neutrinos will eventually become non-relativistic in the very early stages of the Universe and decay. Main decay process would go into SM particles as \( \nu_i \to LH \), injecting entropy to the primordial plasma. However, there could be an increase in the relativistic energy density due to out-of-equilibrium processes allowed by (28), since quintessence is a rather ultralight field, and the co-annihilation process \( \nu \nu \to \chi \chi \) will populate this degree of freedom as we will examine in the next section.

It is worth mentioning that although the above analysis assumed neglecting phases for the fields, their inclusion has little impact on our main conclusions. To state our point we are including in Appendix B a detailed discussion of the changes and effects that are involved when the phase of the scalar fields are considered. In particular, we notice that the phase of the scalar DE field does not take part in the interaction sector beyond the term that involves the inflaton (see equation (B31)), where the value of the phase \( \vartheta \) can change the rate of the decay of the complex inflaton into neutrinos. Both mass and \( \chi \) interaction terms, as expressed by Eqs. (26) and (28) remain unchanged [see Eqs. (B29) and (B30)]. On the other hand, this phase could impact the evolution of the homogeneous background universe, since it appears as part of the total DE density, as it is shown in equation (B38). However, as it can be seen from the first slow-roll condition, which in the polar base, where we define

\[ Q = \frac{(\langle Q \rangle + \chi)}{\sqrt{2}} e^{i \vartheta}, \]  

(29)

takes the form [see equation (B42)],

\[ \frac{1}{2} \dot{\chi}^2 + \frac{1}{2} \left( 1 + \frac{\chi}{\langle Q \rangle} \right)^2 \dot{\vartheta}^2 \ll \frac{1}{2} m^2 (\langle Q \rangle + \chi)^2, \]  

(30)

the phase does not contribute effectively to the DE density, but controls it indirectly, because the fulfillment of the condition depends on the initial values of the phase and its velocity. Condition (30) is fulfilled during the DE dominated age for most of the initial values of the phase.
and its velocity, as can be checked by the evolution of the dynamic system (B43).

In particular, for the simplest $\dot{\phi}_{\text{ini}} = \dot{\phi}_{\text{ini}} = 0$, the phase $\dot{\phi}$ remains null during all the history of the universe, therefore, for these values, the condition (B42) is simplified to the expected one for the usual case of a real scalar field.

Since in the rest of the present work we will only focus on (28), the value of the phase will not play a crucial role, then we can choose the simplest initial condition without losing generality. Our model, nonetheless, is completely compatible with different values, as shown in Appendices A and B, and although it is not developed here, we believe that a deeper analysis of the initial conditions could be related to the studies on the problem of coincidence, as well as to effects beyond the homogeneous limit.

IV. QUINTESSENCE QUANTA PRODUCTION

Because $X$-particles have the same mass associated with Q they are ultra-relativistic, and thus, right handed pair annihilation constitutes a source that can inject an extra degree of freedom during the radiation dominated age. Hence, it is necessary to check whether the presence of such radiation is compatible or not with the predictions of Big Bang Nucleosynthesis (BBN).

In order to do that, we consider standard BBN (SBBN) [68–70] (for a recent review see [71]), in which all of the input parameters, namely, the number of relativistic degrees of freedom in equilibrium ($g_*$), the neutron lifetime, the cross-sections of the involved nuclear processes, the mass difference between neutrons and protons and the strength of both the weak force and gravity, are in accordance with the Standard Model of Particle Physics and Einstein gravity. In SBBN all of those parameters are well determined. The unique input free parameter is the baryon to photon ratio, which determines the primordial abundances of the four light nuclei, namely $^4\text{He}, ^3\text{He}, H$ or D and $^7\text{Li}$. None of them is modified directly in our model, apart, perhaps, from $g_*$. Since SBBN assumes a Friedmann-Lemaître-Robertson-Walker (FLRW) universe and it occurs during the radiation domination age, any increment on $g_*$ increases the value of the Hubble parameter, $H$, consequently, the value of the freeze-out temperature of the neutron-to-proton ratio also increases, which in turn implies an increment on the final primordial helium abundance. The same is accomplished if there is some net increase in the total radiation energy density due to any process beyond thermal equilibrium. That is just the kind of process of neutrino pair annihilation.

Once the system formed by the neutrinos and $X$-particles goes out of equilibrium, the energy density of the latter becomes relevant, otherwise, the pair annihilation can be reversed yielding to a net increment of zero in the total radiation energy density. Therefore, to evaluate the total impact on the Hubble parameter, it is necessary to determine the out-of-equilibrium radiation production along with the one in equilibrium, by evolving the Boltzmann equation for the radiation number density, $n_X$, as a function of the temperature in an FLRW Universe. As the whole process is controlled solely by the coupling $a_e$, and thus by the scale of right handed neutrino masses, the analysis of such a process should constrain this parameter in order to avoid perturbing the predictions of SBBN through an excess of injected $X$. Nevertheless, as we will show hereafter, the process is already so inefficient, that no additional constraints are needed on $a_e$, in such a way that our model appears as consistent with SBBN. Let us proceed next with the detailed analysis.

In order to write the Boltzmann equation, we have to explicitly calculate the collision term, which in turn involves the thermally averaged cross-section for the pair annihilation. (For the last calculation we follow [72, 73]). We start by calculating the total cross-section for the part of the Lagrangian (28) that corresponds to only one of the neutrinos, namely

$$-\mathcal{L}_{\nu_i} = \frac{a_e}{2\sqrt{2}} X \bar{\nu}_i \nu_i.$$  

For this Lagrangian, the total annihilation cross-section of neutrino pairs going to a pair of $X$-particles, calculated in the center of mass frame (CM), is

$$\sigma \equiv \sigma_{\bar{\nu}_i \nu_i \to X X} = \frac{1}{2048\pi s} \frac{a_e^4}{v_r(s) \sqrt{\lambda(s, m_X^2)}} F(s),$$  

where $v_r(s)$ is the relative velocity between the neutrinos and

$$F(s) = \left[ s + 16m_X^2 \left( 1 - \frac{2m^2_X}{s} \right) \log \left[ \frac{s + \sqrt{\lambda(s, m_X^2)}}{s - \sqrt{\lambda(s, m_X^2)}} \right] - 2 \left( 1 + \frac{8m^2_X}{s} \right) \sqrt{\lambda(s, m_X^2)} \right].$$  

where we have neglected the ultra-relativistic mass $m$ respect to the non-relativistic neutrino mass $m_k$. In the previous equations $s$ is a Mandelstam variable, which in the CM corresponds to $s = 4E^2$, with $E$ the energy of each incoming neutrino and $\lambda(s, m_X^2)$ is the Mandelstam triangular function, which is given by

$$\lambda(s, m_X^2) = s(s - 4m_X^2).$$

Next, the thermally averaged cross-section becomes

$$\langle \sigma v_r \rangle = \frac{a_e^4}{4096 \pi m_k T K_2(m_k/T)} I(m_k/T),$$  

where $K_2$ is the modified Bessel function of the second kind of order 2, and where we have defined the integral

$$I(m_k/T) \equiv \int_0^1 dx \frac{g(x)}{x \sqrt{x}} K_1 \left( \frac{2m_k}{T \sqrt{x}} \right),$$
with $K_1$ the modified Bessel function of the second kind of order 1, and $g(x)$ the function coming from (32) after the change of integration variable

$$s \rightarrow 4m_k^2/x, \quad F(s) \rightarrow 4m_k^2g(x).$$

On the other hand, the out-of-equilibrium number density for the $X$-particles, $n_X$, by means of the Boltzmann equation in an FLRW Universe, is given as

$$\frac{1}{a^3} \frac{d}{dt} (a^3 n_X) = (\sigma v_r)(n_\nu)_{eq},$$

where $a = a(t)$ is the universal scale factor and $(n_\nu)_{eq}$ the neutrino number density in equilibrium, which is given by

$$(n_\nu)_{eq} = 4\pi m_k^2 TK_2(m_k/T).$$

It is important to note that $n_\nu$ will go out of equilibrium through processes allowed by the coupling of $\nu_i$ with the SM lepton doublets above mentioned. Such processes can be either, decaying of $\nu_i$ into Higgs and leptons or its co-annihilation into Higgs pairs. From these, the former occurs with a decay rate $\Gamma_d \sim y^2 m_k$, where $y$ is the Yukawa coupling.

It turns out that $y$ is always greater than the coupling $a_c$, as can be checked by considering $\langle Q \rangle \sim M_{pl}$, and the mass of the light neutrinos $m_\nu = m_\nu^2/m_k \sim 10^{-2}$ eV, where $m \sim y(H)$, all of these together with equation (27) leads to $a_c \sim 10^{-3}y^2$, which means that both, the decomposition channel and the co-annihilation channel, dominate over that of $X$-quanta production.

By taking for instance, $y \sim 1$, it is clear that it is in accordance with our assumption of $a_c \sim 10^{-3}$. For this value of $y$, the decay of $\nu_i$ occurs around $T \sim m_k$, such that $n_\nu = (n_\nu)_{eq}$ as far as $T > m_k$, and thus $n_\nu < (n_\nu)_{eq}$ for $T < m_k$, leading to a strong suppression on the resulting number density $n_X$.

Nevertheless, for our calculations we will not involve such transitions, because we are interested in maximizing the production of $X$-quanta, which states the worst possible scenario for the model. Thus, we overestimate it by choosing $(n_\nu)_{eq}$ as the source of the Boltzmann equation (35). This, in turn, simplifies its numerical evolution. In the event that the result conflicts with the requirements of Big Bang Nucleosynthesis, we would refine the calculation by taking into account the extra suppression due to the Higgs channels, but as we will shown next, that will not be neecesary to stablize the cosmological consistency of the model.

Turning back to the equations (33) and (36) notice that these are valid only in the non-relativistic limit $T \ll m_k$. By using them, the Boltzmann equation (35) becomes

$$\frac{1}{a^3} \frac{d}{dt} (a^3 n_X) = \frac{\pi}{256} m_k^3 a_c^4 T I(m_k ; T).$$

After changing time evolution in favor of the temperature, which is possible to do during the radiation dominated age, the Boltzmann equation (37) becomes

$$\frac{d}{dT} (a^3 n_X) = -\frac{M_{pl}}{256} \left( \frac{90}{g_*(T)} \right)^{1/2} m_k^3 a_c^4 T a^3(I(m_k ; T),$$

where $g_*(T)$ is the number of relativistic degrees of freedom in energy density in equilibrium.

Since the Universe is cooling, we perform the integration at both sides backward in $T$, from $T_{out}$ to a certain temperature $T' < T_{out}$, so, we have

$$\int_{(a^3 n_X)(T_{out})}^{(a^3 n_X)(T')} d(a^3 n_X) = -\frac{M_{pl}}{256} \sqrt{90} m_k^3 a_c^4 \int_{T_{out}}^{T'} dT \frac{a^3(T)}{\sqrt{g_*(T)} T^2},$$

where in the RHS, we have written explicitly the universal scale factor dependence on $T$, such a dependence, during the radiation dominated age, is given by

$$a(T) = \frac{a_0}{g_{*s}(T)} t^{1/3},$$

where $a_0$ is a constant and $g_{*s}(T)$ is the number of relativistic degrees of freedom in entropy density in equilibrium.

When the cooling Universe reaches the temperature $T_{out}$ the density $n_X$ starts to increase, i.e. the system goes out of equilibrium, which is true whenever

$$\Gamma \equiv (n_\nu)_{eq}(\sigma v_r) \lesssim H,$$

where $\Gamma$ is the neutrino interaction rate, which can be estimated by using (33) and (36). It turns out that for any value of $T \lesssim m_k$, the integral (34) is very suppressed and so is the rate $\Gamma$ as is shown in the figure 1. Then the inequality (41) is always fulfilled and we can use the temperature $T_{out} \sim m_k$ as the lower limit to obtain a good estimate of the integral that appears in the RHS of Eq. (39).

Furthermore, as the initial state of the $X$-field is one of pure vacuum, and this is not coupled to the inflaton, there are not initial quanta, consequently, we can impose the condition

$$(a^3 n_X)(T_{out}) = 0,$$

which jointly to Eq. (40) allows expressing the integral in Eq. (39) as

$$n_X(T') \cong N a_c^2 g_{*s}(T') T^3 \int_{T_{out}}^{m_k} dT \frac{T}{T^5} \frac{I(m_k ; T)}{g_{*s}(T)} \sqrt{g_*(T)},$$

where $N$ is a constant factor given by

$$N = 2 \times \frac{M_{pl}}{512} \sqrt{\frac{135}{m_k^3}}(Q)^3,$$

and where we have multiplied it by 2 because there are two Majorana neutrinos involved [see equation (28)].
By considering $g_{sn} \sim g_{sn}$, whith $g_{sn}$ the relativistic degrees of freedom in number density in equilibrium, the integral (42) can be written as

$$n_X(T') = n_r(T') \times f(T'),$$

(44)

where $n_r(T')$ is the relativistic number density in equilibrium, given by

$$n_r(T') = \frac{\zeta(3)}{\pi^2} g_{sn}(T') T'^3,$$

(45)

with $\zeta(3)$ the Apéry’s constant, and where

$$f(T') = N\alpha c^7 \frac{\pi^2}{\zeta(3)} \int_{m_r}^{\infty} \frac{dT}{T^5} \frac{I(m_k; T)}{g_{sn}(T) \sqrt{g_{sn}(T)}}.$$  

(46)

By means of equation (44) we write the total relativistic number density of our model $n_{TOT}$, in terms of (46) as

$$n_{TOT}(T') = n_r(T')(1 + f(T')).$$

(47)

The integral (46) can be calculated numerically for different values of the $\alpha_c$ parameter, with the result that for each value of the latter, the integral depends smoothly on the temperature and it is easy to maximize.

Since $n_r(T)$ is a growing monotonic function, it is enough to know whether, for certain $\alpha_c$, the value of $f(T_{mx})$ exceeds that of $n_r(T_{mx})$, where $T_{mx}$ is the temperature that maximizes the integral (46). What we found is that $f(T_{mx})$ is always several orders of magnitude below one for any value of $\alpha_c < 1$, as shown in figure 2, so the increase in the total relativistic number density of $X$ particles due to the co-annihilation of right handed neutrinos is of no cosmological consequences. Clearly, once neutrino decay into SM fields is switched on, the actual $X$ would be much smaller than the value we have just calculated. The model, to this extent, appears consistent with the cosmological constraints.

V. SUMMARY AND CONCLUDING REMARKS

We have presented a cosmological model that unifies early inflation and late accelerated expansion, driven by a quintessence field, where both cosmological scalar fields belong to the degrees of freedom of the same fundamental field representation, $\Phi$, of the $SO(1,1)$ symmetry. This symmetry, as it is usual in particle physics model building, in particular in the construction of the Standard Model, is the guiding principle that dictates and governs the dynamics of the system. It is really interesting that such a simple principle allows reproducing chaotic type potentials for both inflation and DE, which are derived from considering all possible bilinear field operators based on $\Phi$ that are invariant under the symmetry. As a matter of fact, the field system of the model can be rewritten in terms of two scalar fields with and independent evolution, which in the cosmological setup will fall down on simple mass type potential. Upon fine-tuning, one can easily understand the reason why one of such fields breaks down the slow-roll condition at large scale, ending inflation, whereas the other stays trapped in a false vacuum configuration that we see as a cosmological constant nowadays.

The need for reheating after inflation, which requires the coupling of the inflaton to matter fields, is fulfilled by introducing a set of fermions which, in order to be consistent with the symmetry, belong to a doublet and singlet of $SO(1,1)$. Enforcing the symmetry to build the Yukawa couplings as also invariant terms has two out-

![Figure 1](image1.png)

Figure 1: The out-of-equilibrium condition given in Eq. (41) for some values of the parameter $\alpha_c$. As stated in the text, the integral (34) is very suppressed, hence the system is always out of equilibrium, even for temperatures as high as of the one for reheating.

![Figure 2](image2.png)

Figure 2: The maximums of the function $f(T)$ given in (46) for different values of the parameter $\alpha_c$. Notice that the integral is always less than the unit and so the increase in $n_{TOT}$ given in Eq. (47) is negligible.
standing implications. First, since the cosmological field does not belong to the Standard Model particle sector, neither the new fermions will, and thus they are naturally identified as right handed neutrinos. Second, the invariant couplings among $\Phi$ and the fermions do provide the appropriate inflaton couplings to allow inflaton decay and reheating, but furthermore, they also mean that right handed neutrinos would couple to the cosmological DE field. Without any further assumption, beyond the use of symmetries, our model introduces a way to naturally understand the existence of large sterile Majorana neutrino masses as sourced by DE, which, on the other hand, is a need for the standard seesaw mechanism to work. The last is the simplest known mechanism that provides very small masses to the standard neutrinos, required to explain neutrino oscillation phenomena.

Here, we have studied in some detail the mechanism contained in the $SO(1,1)$ cosmological Unification model that is beneath the generation of neutrino masses. Our analysis shows that the origin of the mass is independent of the field phases and their dynamics. However, it may not be the only possible mechanism in nature, as the $SO(1,1)$ symmetry does not prohibit to write an independent mass associated to any singlet fermion. Such a mass seems unnatural since there is no a priory mass scale associated to it, an issue already present in the seesaw. Nevertheless, as we have argued, such a mass can easily be removed if additional global symmetries are involved in the fermion sector. In such a scenario, DE arises as the natural source of such a neutrino mass, through its false vacuum energy that supports current accelerated expansion of the Universe. As this last has a large scale, then it comes naturally that the right handed neutrinos would have masses in the $10^{13}$ GeV, scale, or so.

Our study has also looked upon the possible impact that the model and in particular quintessence quanta, $\chi$ may have in the thermal history of the Universe. The inflaton in the model does not couple to quintessence field, and thus, it does not inject entropy through that channel upon decay. As a matter of fact, the only allowed decay channel for the inflaton is for its decay into the heavy right handed neutrinos, which, eventually would create the primordial plasma through Higgs and Standard Model lepton couplings, of the form $\nu\nu\to H\nu$. After this we expect Standard thermal history to proceed as usual, but for the possible contributions to entropy that the right handed neutrinos would inject back into the Universe, in the form of quintessence quanta through out-of-equilibrium co-annihilation processes, $\nu\nu\to 2\chi$. To further estimate this effect, we have calculated the thermally averaged cross section for the process, which depends on the same Yukawa coupling that provides neutrino masses, $a_c$. As discussed in the paper, the numerical integration of the Boltzmann equations with $a_c$ varying on a wide range of values shows that the process is so suppressed that the total amount of injected quintessence quanta number density is negligible. This clearly indicates that the model, without any further constraints or assumptions, remains consistent with the conditions required for a successful Big Bang Nucleosynthesis.

The present model uses complex scalars to realize the symmetry, and thus it involves dynamical phases for which we have not explored yet their possible role in the Universe evolution. Our analysis does show that they are not potentially relevant for the after-inflation evolution, provided the initial conditions fix them to zero, at least for the mechanism that generates neutrino masses and the production of quintessence quanta. However, other roles may be possible that would be interesting to look at.

As final comments. One of the issues that remain to be explored in detail to make for a more realistic model is the connection with the standard particle physics. In particular, we have not yet explored whether standard model particles should be assigned into a set of singlets under $SO(1,1)$, or if this last could play the role of a flavor symmetry, at least for the lepton sector, to which our heavy neutrinos would couple. If so, it would be interesting to explore if such symmetry may account for masses and mixings of the light neutrinos as well.

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Appendix A: Diagonalization of the Lagrangian

In this appendix, we present in detail the diagonalization analysis of our model Lagrangian whose results are used along the discussion in the main text. First, we consider the scalar sector, whose Lagrangian (4) in terms of the doublet complex field components becomes

$$\mathcal{L}_\Phi = \partial^\mu \phi^* \partial_\mu \phi + \partial^\mu \varphi^* \partial_\mu \varphi - V(\phi, \varphi),$$

with the potential

$$V(\phi, \varphi) = a_0(|\phi|^2 + |\varphi|^2) + a_1 (\phi^* \varphi + \varphi^* \phi) + a_3 (\varphi^2 - \phi^2) + \text{c.c.} \quad (A1)$$

Next, we rewrite the Lagrangian in terms of the hermitian base

$$\phi = \frac{1}{\sqrt{2}} (\phi_1 + i \phi_2), \quad \varphi = \frac{1}{\sqrt{2}} (\varphi_1 + i \varphi_2),$$

where $\phi_i$, $\varphi_i$, $i = 1, 2$ are real scalar fields. This lets us put the potential in a matrix form which we will diagonalize in order to identify physical fields having separated dynamics. The potential (A1) becomes

$$V = \frac{1}{2} \Phi_R^\dagger A \Phi_R,$$
with $\Phi_R$ being the vector formed from above real scalar fields components of $\phi$ and $\varphi$, given by $\Phi_R = (\phi_1, \phi_2, \varphi_1, \varphi_2)$, and $A$ is the $4 \times 4$ mass coupling matrix

$$A = \begin{pmatrix}
m_{1}^2 & \lambda^2 & \mu_{1}^2 & 0 \\
\lambda^2 & m_{2}^2 & 0 & \mu_{2}^2 \\
\mu_{1}^2 & 0 & m_{2}^2 - \lambda^2 & 0 \\
0 & \mu_{1}^2 & 0 & m_{2}^2
\end{pmatrix},$$

where we have defined

$$m_1^2 = \mu_0^2 + \mu_3^2, \quad m_2^2 = \mu_0^2 - \mu_3^2, \quad \lambda^2 = 2Re(\alpha_3),$$

and

$$\mu_0^2 \equiv 2Re(\alpha_0), \quad \mu_1^2 \equiv 2Re(\alpha_1), \quad \mu_2^2 \equiv 2Re(\alpha_3).$$

Notice that by definition all the involved mass terms, $m_1^2$, $m_2^2$, $\lambda^2$, $\mu_0^2$, $\mu_1^2$ and $\mu_2^2$ are real and by construction, we have chosen them to be positive.

Since the $A$ matrix is real and symmetric, by mean of the proper orthogonal rotation of the field base, $S$, through which we redefine

$$\Phi_D = S\Phi_R, \quad A_D = S\lambda S^T,$$

we should get a diagonal mass sector. It is not difficult to check that such a matrix can be expressed as

$$S = (I_{2\times2} \otimes B - i\sigma_2 \otimes H) \cos(\omega),$$

where

$$B = \begin{pmatrix}
\cos(\rho) & 0 \\
0 & \cos(\rho)
\end{pmatrix}, \quad H = \begin{pmatrix}
\tan(\omega) & \sin(\rho) \\
\sin(\rho) & -\tan(\omega)
\end{pmatrix}. $$

In the above, we have made use of the shorthand notation where

$$\cos(\rho) = \frac{\mu_1^2}{\sqrt{\mu_1^4 + \lambda^4}}, \quad \sin(\rho) = \frac{\lambda^2}{\sqrt{\mu_1^4 + \lambda^4}},$$

$$\cos(\omega) = \frac{\alpha^2}{\sqrt{2h^2(h^2 + \Delta^2)}}, \quad \sin(\omega) = \frac{\alpha^2}{\sqrt{2h^2(h^2 - \Delta^2)}},$$

and

$$\alpha^4 = 4(\mu_1^4 + \lambda^4), \quad \Delta^2 = m_1^2 - m_2^2, \quad \hbar^4 = \Delta^4 + \alpha^4. $$

After performing the $S$ rotation, the potential becomes

$$V = \frac{1}{2} \Phi_D^T A_D \Phi_D,$$

with $\Phi^T_D = (Q_1, \xi_1, \xi_2, Q_2)^T$ and

$$A_D = \text{diag}\left(m^2, M^2, M^2, m^2\right),$$

where the eigenvalues $m^2$ and $M^2$ are given by

$$m^2 = \mu_0^2 - \mu^2 \quad \text{and} \quad M^2 = \mu_0^2 + \mu^2, \quad (A2)$$

where $\mu^2 = \sqrt{\mu_1^4 + \mu_2^4 + \lambda^4}$. In terms of the $\alpha$ couplings, we get $\mu_0^2 = 2\text{Re} \alpha_0$ and $\mu^2 = 2\sqrt{\text{Re} \alpha_1^2 + |\alpha_2|^2}$.

The requirement that $M^2, m^2 > 0$, which guarantees that the potential is bounded from below, is fulfilled if $\mu_0^2 > \mu^2 > 0$. If both parameters were of the same order, $\mu_0^2 \approx \mu^2 > 0$, we would naturally get $M^2 \gg m^2 \approx 0$. In such a scenario it becomes natural to identify $\xi$ with the inflaton and $Q$ with the DE field, provided $M$ is as large as the inflation scale.

Notice that the mass eigenstates in $\Phi_D$ can be rearranged in a more natural ordering by the permutation matrix

$$P = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix},$$

such that $(Q_1, Q_2, \xi_1, \xi_2)^T = S^T \Phi_R$ with $S^T = P S$.

In terms of the diagonal base and given that there are two degenerated scalar degrees of freedom for each mass, the potential finally can be expressed as

$$V = m^2|Q|^2 + M^2|\xi|^2, \quad (A3)$$

where we have introduced the new complex scalar fields

$$Q = \frac{1}{\sqrt{2}}(Q_1 + iQ_2), \quad \text{and} \quad \xi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2). \quad (A4)$$

Analogously, the scalar kinetic term can be easily put in terms of the new fields after the $S'$ rotation on $\Phi_R$, to get the also diagonal terms $\partial^\mu Q^* \partial_\mu Q + \partial^\mu \xi^* \partial_\mu \xi$.

Finally, by introducing the doublet

$$\varphi = \begin{pmatrix}Q \\ \xi\end{pmatrix}, \quad (A5)$$

the whole Lagrangian of the scalar sector becomes

$$\mathcal{L}_\varphi = \partial^\mu \varphi^* \partial_\mu \varphi - \varphi^* M \varphi, \quad (A6)$$

where $M$ is the diagonal mass matrix

$$M = \begin{pmatrix}m^2 & 0 \\
0 & M^2\end{pmatrix}. \quad (A7)$$

We should emphasize that this new doublet notation is not a faithful representation of $SO(1,1)$, since the $SO(4)$ rotation, $S'$, and the $SO(1,1)$ transformations do not commute. Therefore, the diagonal Lagrangian $(A7)$, which provides the decoupled field system which evolves explaining inflation and the late accelerated expansion of the Universe, is not explicitly invariant under $SO(1,1)$, even though the original model does so.

Let us now move into analyzing the fermion sector of the theory, for which the corresponding kinetic terms, as given in Eq. (16), are

$$\mathcal{L}_{N_i} = \sum_{i=0}^{2} N_i^{\alpha \bar{\alpha}} i \sigma_{\alpha \bar{\alpha}}^\mu \partial_\mu N_i^{\bar{\alpha}}, \quad (A8)$$
and the interaction terms (17) which takes the form
\[-\mathcal{L}_I = N_{0a} \left\{ a_0 (\phi^* N_1^a + \varphi^* N_2^a) + a_1 (\phi^* N_1^a + \varphi^* N_1^a) + a_2 (\phi N_2^a - \varphi N_1^a) + a_3 (\phi N_1^a - \varphi N_2^a) \right\} + h.c. \]
\[(A9)\]

Last, written in terms of the real field components in \( \Phi_R \), we finally rewrite the interaction terms as
\[-\mathcal{L}_I = \frac{1}{\sqrt{2}} N_{0a} \Phi_R^\dagger \left\{ \mathcal{V} N_1^a + \Gamma \mathcal{V} N_2^a \right\} + h.c., \]
\[(A10)\]

where \( \mathcal{V} \) is the vector formed from the complex couplings \( a_i \), given by
\[\mathcal{V} = \begin{pmatrix} a_3 + a_0 \\ i(a_3 - a_0) \\ a_1 - a_2 \\ -i(a_1 + a_2) \end{pmatrix},\]
and \( \Gamma \) is a \( 4 \times 4 \) matrix given by \( \Gamma = -\sigma_1 \otimes \sigma_2 \). After the \( S \) rotation in the scalar sector is set in, and noticing that \( \Gamma \) is actually an invariant matrix, since \( \Gamma = S \Gamma S^\dagger \), the interaction Lagrangian becomes
\[-\mathcal{L}_I = \frac{1}{\sqrt{2}} N_{0a} \Phi_R^\dagger \left\{ \mathcal{V}' N_1^a + \Gamma' \mathcal{V}' N_2^a \right\} + h.c., \]
\[(A11)\]

where \( \mathcal{V}' = SV \).

It is important to note that \( \mathcal{V}' \) just corresponds to a redefinition of the Yukawa couplings, for which one can always assume a convenient parameterization, implicitly defined in terms of the initial \( a_i = 0, \ldots, 3 \) couplings. Hence, using this freedom we choose the following combinations to define the couplings in the rotated scalar base:
\[\mathcal{V}' = \frac{1}{\sqrt{2}} \begin{pmatrix} g_1 + g_2 \\ h_1 - h_2 \\ -i(h_1 + h_2) \\ i(g_1 - g_2) \end{pmatrix}, \]
\[(A12)\]

where \( g_i = 1, 2 \) and \( h_i = 1, 2 \) are complex numbers. Substituting the last expression and the redefinition of the scalar fields given in Eq. (A4) into Eq. (A11), after some simple algebra, we finally rewrite the interaction terms as
\[-\mathcal{L}_I = N_{0a} \left\{ g_1 Q F_1^a + g_2 Q^* F_2^a \right\} + h_1 \xi^* F_1^a - h_2 \xi F_2^a \right\} + h.c., \]
\[(A13)\]

where the new Weyl fields \( F_i^a \) (i = 1, 2) are the components of the doublet
\[F = \begin{pmatrix} F_1^a \\ F_2^a \end{pmatrix}, \]
\[(A14)\]

which in turn comes from the transformation
\[e^{-i\sigma_2 \pi/4} \Psi = F, \]
\[(A15)\]

i.e., the diagonalization of the scalar potential through \( S \), induces an \( \text{SO}(2) \) rotation over the doublet Eq. (11), by an angle of \( \pi/4 \). Note that we still can define the \( U(1) \) global transformation used in (20) with the same charge for the new Weyl fields as \( F \rightarrow e^{i\alpha} F \), and so this convenient transformation does not alter the argument used to remove the mass of \( N_0 \) in the main text. Nevertheless, as for the scalar sector, the transformations used to rewrite the interactions hide the \( \text{SO}(1, 1) \) of the theory, but on the other hand, allows to write down Eq. (A13) in a simple and compact way, as
\[-\mathcal{L}_I = N_{0a} \left\{ \varphi^* G_1 F + \varphi^* G_2 F \right\} + h.c., \]
\[(A16)\]

where we have defined the coupling matrices as
\[G_1 = \begin{pmatrix} 0 & g_2 \\ h_1 & 0 \end{pmatrix} \quad G_2 = \begin{pmatrix} g_1 & 0 \\ 0 & -h_2 \end{pmatrix}. \]
\[(A17)\]

Finally, notice that the transformation given in Eq. (A15) keeps the diagonal form of fermion kinetic terms, as expected, which can now be expressed as
\[-\mathcal{L}_F = N_0 [\iota_\alpha \iota_\beta \partial_\mu N_0^\dagger + F^\dagger \iota_\mu \partial_\mu F]. \]
\[(A18)\]

Appendix B: Including phase fields on the \( \text{SO}(1, 1) \) model

Here we explore some of the possible effects that considering dynamical phase fields for the cosmological scalars may have in the model outcomes discussed in the main text, as well as other interesting aspects that we believe might be of further interest for field dynamics. For this, we assume that after reheating, the Q field remains dynamically trapped in a homogeneous and isotropic false vacuum configuration, which sources DE and breaks the \( U(1) \) global symmetry in the neutrino sector, whereas the inflaton field \( \varphi \) has already settled on its null value, and thus, quantum perturbation for our cosmological scalar fields can be conveniently introduced in a polar base as
\[Q = \frac{(\langle Q \rangle + \tilde{X})}{\sqrt{2}} e^{i\theta/(Q)}, \quad \xi = \frac{1}{\sqrt{2}} |\xi| e^{i\theta/(Q)}, \]
\[(B1)\]

where the degrees of freedom of the complex scalar field \( Q \) are now given by the real scalar field \( \tilde{X} \), and the dynamical phase \( \theta \). Similarly, for \( \xi \), its degrees of freedom are given by its modulus and its own dynamical phase \( \theta \).

Next, we proceed to rewrite the Lagrangian of our model in terms of the above parameterization, for this we first notice that the doublet (A5) can be written as
\[\varphi = \mathbb{P}\varphi_R, \]
\[(B2)\]

where we have defined the radial field part as
\[\varphi_R = \frac{1}{\sqrt{2}} \begin{pmatrix} (Q) + \tilde{X} \\ |\xi| \end{pmatrix}, \]
\[(B3)\]
and the field phase matrix given by

$$\mathcal{P} = \begin{pmatrix} e^{i\theta/(Q)} & 0 \\ 0 & e^{i\theta/(Q)} \end{pmatrix}. \quad (B4)$$

By substituting Eq. (B2) into the scalar sector of the theory, it is straightforward to see that the Lagrangian (A6) simply becomes

$$\mathcal{L}_\varphi = \partial^\mu \varphi^T \eta \partial_\mu \varphi + \varphi^T i \mu \varphi + \mathcal{T}(\varphi, \mathcal{P}), \quad (B5)$$

where \(\mathcal{T}(\varphi, \mathcal{P}) = \varphi^T (\partial^\mu \mathcal{P}) (\partial_\mu \mathcal{P}) \varphi\), is a dimension six and highly suppressed operator. Thus we do not expect it to be relevant for the later dynamics of DE.

Explicitly, in terms of inflaton and DE fields, the above Lagrangian reads

$$\begin{align*}
\mathcal{L}_\varphi &= \frac{1}{2} \partial^\mu |\xi| \partial_\mu |\xi| + \frac{1}{2} \partial^\mu X \partial_\mu X \\
&\quad + \frac{1}{2} m^2 (|\xi| + X)^2 + \frac{1}{2} M^2 |\xi|^2 + \mathcal{T}(\xi, x, \theta, \vartheta),
\end{align*} \quad (B6)$$

where the last term on the RHS is given by

$$\mathcal{T}(\xi, x, \theta, \vartheta) = \frac{|\xi|^2}{2Q^2} \partial^\mu \theta \partial_\mu \theta + \frac{1}{2} \left(1 + \frac{X}{\langle Q \rangle}\right)^2 \partial^\mu \vartheta \partial_\mu \vartheta. \quad (B7)$$

As for the interaction with fermions given by Eq. (A16), this is now written as

$$\begin{align*}
-\mathcal{L}_I &= N_{0a} \varphi_n^T \{\bar{\psi}^\tau G_1 + \bar{\psi}^\tau G_2\} F + h.c., \\
&= N_{0a} \varphi_n^T \xi G F' + h.c.,
\end{align*} \quad (B8)$$

where the new coupling matrix is given by

$$\xi = \left( \begin{array}{cc} g_1 & g_2 \\ h_1 e^{-i(\theta + \vartheta)/(Q)} & -h_2 e^{i(\theta + \vartheta)/(Q)} \end{array} \right), \quad (B9)$$

and where we have performed a local phase transformation over the fermions in the doublet to introduce

$$F' = \begin{pmatrix} F_1^\dagger \\ F_2^\dagger \end{pmatrix}, \quad (B11)$$

with \(F_1^\dagger = e^{i\theta/(Q)} F_1^\dagger\) and \(F_2^\dagger = e^{-i\theta/(Q)} F_2^\dagger\). This redefinition of the fermion fields removes the dynamical phases on the X-sector, as can be seen from (B10). Nonetheless, they will reappear as currents coming from the transformation of the kinetic terms (A18), which now read as

$$\mathcal{L}_F = N_0^{\dagger} i \sigma_\mu \partial_\mu N_0^{\dagger} + F_1^\dagger i \sigma_\mu \partial_\mu F+ \frac{\partial_\mu \vartheta}{\langle Q \rangle} F_1^\dagger \sigma_\mu \sigma_3 F',$$

where in the last term the effect of \(\sigma_3\) is to switch the sign of the lower entry of the doublet. Notice that once again the phase field enters in a suppressed way. Apart from these new terms where the phase fields are explicit, the part of the Lagrangian that matters for the model remains the same.

1. Revisiting massive neutrino base

Let us now execute a new transformation with the aim to remove the constant phases of the couplings \(g_1\) and \(g_2\) appearing in (B10), by means of a SU(2) rotation on the doublet fermion sector

$$\eta = \mathcal{R} F' = \begin{pmatrix} \eta_1^2 \\ \eta_2^2 \end{pmatrix}, \quad (B12)$$

with

$$\mathcal{R} = \frac{1}{a_c} \left( \begin{array}{cc} g_1 & g_2 \\ -g_2^* & g_1^* \end{array} \right), \quad (B13)$$

where \(a_c = \sqrt{|g_1|^2 + |g_2|^2}\). After this rotation, the interaction term (B9) becomes

$$-\mathcal{L}_I = N_{0a} \varphi_n^T \xi G' \eta + h.c., \quad (B14)$$

where now, the coupling matrix is

$$\xi' = \mathcal{G} \mathcal{R}^\dagger = \left( \begin{array}{cc} a_c & 0 \\ C_1(\theta, \vartheta) & C_2(\theta, \vartheta) \end{array} \right). \quad (B15)$$

In above we have used for a shorthand notation

$$\begin{align*}
C_1(\theta, \vartheta) &= (g_{11} e^{-i(\theta + \vartheta)/(Q)} - g_{22} e^{i(\theta + \vartheta)/(Q)})/a_c, \\
C_2(\theta, \vartheta) &= -(g_{12} e^{i(\theta + \vartheta)/(Q)} + g_{21} e^{-i(\theta + \vartheta)/(Q)})/a_c,
\end{align*}$$

where \(g_{11} = g_1 h_1, g_{22} = g_2^* h_2, g_{12} = g_1 h_2,\) and \(g_{21} = g_2 h_1\). On the other hand, upon the same rotation, the fermion kinetic terms are now written as

$$\mathcal{L}_F = N_0^{\dagger} i \sigma_\mu \partial_\mu N_0^{\dagger} + \eta^\dagger i \sigma_\mu \partial_\mu \eta + \frac{\partial_\mu \vartheta}{\nu} \eta^\dagger \sigma_\mu \nu \eta, \quad (B16)$$

where \(\nu\) is a coupling matrix, that comes from the transformation of \(\sigma_3\) under (B13), given by

$$\nu = \left( \begin{array}{cc} y_1 & -y_2 \\ -y_2^* & -y_1 \end{array} \right),$$

where \(y_1 = (|g_1|^2 - |g_2|^2)/a_c^2,\) and \(y_2 = 2 g_1 g_2/a_c^2,\) i.e., \(y_1 \in \mathbb{R}\) and \(y_2 \in \mathbb{C}\). (Notice that \(y_1^2 + |y_2|^2 = 1\).)

Let us now concentrate our analysis towards the interaction among neutrinos and the DE field, which after above mathematical manipulations has gotten the simple expression

$$-\mathcal{L}_{\nu X} = \frac{a_c}{\sqrt{2}} \langle Q \rangle \left\{ N_{0a} \eta_1^{\dagger} + h.c. \right\}. \quad (B17)$$

The part between braces can be expressed also as

$$N_{0a} \eta_1^{\dagger} + h.c. = N_{0a} \eta_1^{\dagger} + \eta_1^{\dagger} N_0^{\dagger} \quad (B18)$$

$$= \frac{1}{2} \left\{ N_{0a} \eta_1^{\dagger} + N_{0a} \eta_1^{\dagger} + \eta_1^{\dagger} N_0^{\dagger} + \eta_1^{\dagger} N_0^{\dagger} \right\}$$

$$= \frac{1}{2} \left\{ N_{0a} \eta_1^{\dagger} + \eta_1^{\dagger} N_0^{\dagger} + \eta_1^{\dagger} N_0^{\dagger} + N_0^{\dagger} \right\},$$
wherein both, the second and the fourth terms in the last line, we have used the anti-commutation properties plus an extra minus sign coming from the change from $\tilde{a}$ to $\tilde{\bar{a}}$ (and similarly for the undotted indices). Now, we define two four-component Dirac neutrinos as

$$u_1 = \left( \begin{array}{c} N_{1a}^\dagger \\ \eta_1^\dagger \end{array} \right), \quad u_2 = \left( \begin{array}{c} \eta_1^a \\ N_0^a \end{array} \right),$$

(B19)

in terms of which the last line in Eq. (B18) can be written as

$$N_{0a} \nu_1^a + h.c. = \frac{1}{2} (\bar{\nu}_1 u_1 + \bar{\nu}_2 u_2).$$

(B20)

As it can be seen from (B19), the neutrinos $u_1$ and $u_2$ are conjugates of charge of each other, this let us put them in terms of two Majorana neutrinos $\nu_1$ and $\nu_2$, through of another rotation, which is given by

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}.$$  \[ \text{(B21)} \]

Therefore, Eq. (B20) directly becomes

$$N_{0a} \nu_1^a + h.c. = \frac{1}{2} (\bar{\nu}_1 \nu_1 + \bar{\nu}_2 \nu_2),$$

(B22)

which explicitly provide the neutrino mass eigenstates, with a mass given by

$$m_k = \frac{a_c(Q)}{\sqrt{2}}.$$  \[ \text{(B23)} \]

Notice that this same rearrangement of the neutrinos provide the interaction Lagrangian with $\chi$ fields,

$$-\mathcal{L}_{IX} = \frac{a_c}{2\sqrt{2}} \chi (\bar{\nu}_1 \nu_1 + \bar{\nu}_2 \nu_2),$$  \[ \text{(B24)} \]

that we use on our discussions along the paper. We stress that these results are independent of the phase fields and link the origin of the heavy right handed neutrino masses to DE, as already argued in the main text.

As a final note on this regard, notice that the Majorana neutrinos, in four-component notation, can be expressed as

$$\nu_i = \begin{pmatrix} k_{1a}^i \\ k_2^a \end{pmatrix}, \quad i = 1, 2.$$  \[ \text{(B25)} \]

In the last equation, we have introduced the new right-handed Weyl field in two-component notation: $k_{1a}^i$, $k_2^a$. Note that the transformation (B21) together with (B19) are equivalent to the transformations

$$\begin{pmatrix} k_{1a}^i \\ k_2^a \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} N_0^a \nu_1^a \\ \eta_1^a \end{pmatrix},$$  \[ \text{(B26)} \]

and

$$\begin{pmatrix} k_{1a}^i \\ k_2^a \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} N_0^a \nu_1^a \\ \eta_1^a \end{pmatrix}.$$  \[ \text{(B27)} \]

It is important to remark that these transformations do not respect the $U(1)$ invariance of the fermionic sector since it mixes fields with different global charges.

Summarizing, we can either, substitute (B25) into (B22) or directly operate over (B16) through of (B26) and (B27) to get

$$N_{0a} \eta_1^a + h.c. = \frac{1}{2} \{ k_{1a}^1 k_1^1 + k_{2a}^1 k_2^1 \} + h.c.$$  \[ \text{(B28)} \]

By substituting equation (B28) into equation (B17), one gets the mass terms

$$-\mathcal{L}_m = \frac{1}{2} m_k (k_{1a}^1 k_1^1 + k_{2a}^1 k_2^1) + h.c.,$$  \[ \text{(B29)} \]

with the mass given as before and the interaction term

$$-\mathcal{L}_{IX} = \frac{a_c}{2\sqrt{2}} \chi (k_{1a}^1 k_1^1 + k_{2a}^1 k_2^1) + h.c.$$  \[ \text{(B30)} \]

In the same footing, and for future use, we also write the inflaton to neutrino interactions, as derived from Eq. (B14), for which we also rename $k_3^a \equiv \eta_2^a$, to write

$$-\mathcal{L}_g = \frac{1}{4} C_1(\theta, \vartheta)|\xi | (k_{1a}^1 k_1^1 + k_{2a}^1 k_2^1)$$

$$+ \frac{1}{2\sqrt{2}} C_2(\theta, \vartheta)|\xi | (k_{1a}^1 - i k_{2a}^1) \eta_2^a + h.c.$$  \[ \text{(B31)} \]

Similarly, by expanding Eq. (B16) and by transformation (B26), whereas the kinetic terms for $k_{1a}^i = 1, 2, 3$ remain as usual,

$$\mathcal{L}_K = \sum_{i=1}^3 k_{1a}^i i\sigma_{\mu}^a \partial_{\mu} k_i^c,$$  \[ \text{(B32)} \]

the current-couplings among the phase scalar $\partial_{\mu} \vartheta$ and the neutrinos go as

$$\mathcal{L}_c = \mathcal{L}_{c_1} + \mathcal{L}_{c_2},$$  \[ \text{(B33)} \]

where

$$\mathcal{L}_{c_1} = y_1 \frac{\partial_{\nu} \vartheta}{\langle Q \rangle} \left\{ \frac{1}{2} \left( k_{1a}^1 \sigma_{\alpha}^a k_1^1 + k_{2a}^1 \sigma_{\alpha}^a k_2^1 \right)$$

$$+ \frac{i}{2} \left( k_{1a}^1 \sigma_{\alpha}^a k_1^2 - k_{2a}^1 \sigma_{\alpha}^a k_2^1 \right) - k_{3a}^1 \sigma_{\alpha}^a k_3^1 \right\},$$  \[ \text{(B34)} \]

and

$$\mathcal{L}_{c_2} = - \frac{\partial_{\nu} \vartheta}{\langle Q \rangle} \left\{ y_2 \frac{1}{\sqrt{2}} (k_{1a}^1 - i k_{2a}^1) \sigma_{\alpha}^a k_3^1 + h.c. \right\}.$$  \[ \text{(B35)} \]

2. Energy density and equations of motion for the DE sector

We close this appendix by presenting the results of the calculation of the equation of state parameter for DE in the present model. For this purpose, we made explicit use
of the model Lagrangian, as defined in Eq. (B6), where the DE part is written as

\[ \mathcal{L}_{X,\vartheta} = \frac{1}{2} \partial^\mu X \partial_\mu X + \frac{1}{2} \left( 1 + \frac{\chi}{\langle Q \rangle} \right)^2 \partial^\vartheta \partial_\vartheta \vartheta + V(X), \]  

(B36)

where the potential is defined as

\[ V(X) = \frac{1}{2} m^2 (\langle Q \rangle + X)^2. \]  

(B37)

From equation (B36) and by calculation of the energy-momentum tensor in an FLRW Universe, we obtain both, the energy density and the pressure in terms of \(X\) and the phase \(\vartheta\). These are given by

\[ \rho_{DE} = \frac{1}{2} \dot{X}^2 + \frac{1}{2} \left( 1 + \frac{\chi}{\langle Q \rangle} \right)^2 \dot{\vartheta}^2 + \frac{1}{2 a^2} (\nabla X)^2 \]  

+ \[ V(X) + \frac{1}{2a^2} \left( 1 + \frac{\chi}{\langle Q \rangle} \right)^2 (\nabla \vartheta)^2, \]  

(B38)

and

\[ P_{DE} = \frac{1}{2} \dot{X}^2 + \frac{1}{2} \left( 1 + \frac{\chi}{\langle Q \rangle} \right)^2 \dot{\vartheta}^2 - \frac{1}{6a^2} (\nabla X)^2 \]  

\[ - V(X) - \frac{1}{6a^2} \left( 1 + \frac{\chi}{\langle Q \rangle} \right)^2 (\nabla \vartheta)^2. \]  

(B39)

In the homogeneous case, the previous equations are reduced to

\[ \rho_{DE} = \frac{1}{2} \dot{X}^2 + \frac{1}{2} \left( 1 + \frac{\chi}{\langle Q \rangle} \right)^2 \dot{\vartheta}^2 + V(X), \]  

(B40)

and

\[ P_{DE} = \frac{1}{2} \dot{X}^2 + \frac{1}{2} \left( 1 + \frac{\chi}{\langle Q \rangle} \right)^2 \dot{\vartheta}^2 - V(X). \]  

(B41)

In order to realize the accelerated expansion, the DE field has to accomplish an equation of state such that

\[ \omega = \frac{P_{DE}}{\rho_{DE}} \approx -1, \]

which means, according to (B40) and (B41), that the first slow-roll condition is of the form

\[ \frac{1}{2} \dot{X}^2 + \frac{1}{2} \left( 1 + \frac{\chi}{\langle Q \rangle} \right)^2 \dot{\vartheta}^2 \ll \frac{1}{2} m^2 (\langle Q \rangle + X)^2. \]  

(B42)

The dynamics for the homogeneous background involving both, \(X\) and \(\vartheta\), is given by substitution of the equation (B40) into the first Friedman equation, after application of the first slow-roll condition, together with those coming from application of the Euler-Lagrange equations to (B36). For completeness, we also included DM, baryons (\(b\)), photons (\(\gamma\)) and active neutrinos (\(\nu\)). Taking into account the first slow-roll condition, the whole system is:

\[ H^2 = \frac{1}{3M_{pl}^2} V(X), \]  

\[ \ddot{X} + 3H \dot{X} + V(X) = 0, \]  

\[ \ddot{\vartheta} + 3H \dot{\vartheta} = 0, \]  

\[ \dot{H} = -\frac{1}{2M^2_{pl}} \left( \dot{\rho}_{DM} + \dot{\rho}_b + \frac{4}{3} \dot{\rho}_\gamma + \frac{4}{3} \dot{\rho}_\nu \right), \]  

\[ \dot{\rho}_{DM,b} + 3H \rho_{DM,b} = 0, \]  

\[ \dot{\rho}_{\gamma,n} + 4H \rho_{\gamma,n} = 0. \]  

(B43)

\[ H^2 = \frac{1}{3M_{pl}^2} V(X), \]  

\[ \ddot{X} + 3H \dot{X} + V(X) = 0, \]  

\[ \ddot{\vartheta} + 3H \dot{\vartheta} = 0, \]  

\[ \dot{H} = -\frac{1}{2M^2_{pl}} \left( \dot{\rho}_{DM} + \dot{\rho}_b + \frac{4}{3} \dot{\rho}_\gamma + \frac{4}{3} \dot{\rho}_\nu \right), \]  

\[ \dot{\rho}_{DM,b} + 3H \rho_{DM,b} = 0, \]  

\[ \dot{\rho}_{\gamma,n} + 4H \rho_{\gamma,n} = 0. \]  

(B43)
