Heat Losses of Window Compact Shutters

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Abstract. Window shutters are usually made of wood. They protect glazed windows at their external sides from mechanical damages and inconvenient weather conditions. A typical window compact shutter is described and its thermal properties are discussed. In winter season, the heat transfers inside the window shutters represent unwanted heat losses. These transfers of heat are caused especially by convection and radiation running between the warm glazed side of the window to the cold external side of the shutter and through this external side into outdoor space. The heat transfers are computed on the basis of physical relations. The computation of convective heat transfer is based on the correlation relations utilizing Rayleigh’s and Prandtl’s numbers. The radiative transfer is computed by means of the Stefan-Boltzmann law and the Kirchhoff law. The thermal resistance of the typical shutter is derived. It is shown that this resistance is temperature dependent. It is the radiation that is responsible for the temperature dependency of the thermal resistance of compact window shutters. The formula for thermal resistance derived in this contribution may assist the building technologists to evaluate insulation capabilities of similar compact shutters possessing comparable geometric dimensions and similar thermal emissivities.

1. Introduction
The window shutters protect the glazed windows at their external sides from mechanical damages and inconvenient weather conditions. They also partially serve as simple thermal insulators. However, a simple plate-like shutter may provide the window with only small thermal protection. A better thermal protection may be expected when the compact box shutters are used. One type of box compact window shutters is depicted in figure 1. If closed and attached to windows, the compact box shutter forms a closed cavity that is filled with air. The air inside a closed cavity is known to be a good thermal insulator [1-2]. Yet, the compact shutters do not represent quite perfect thermal insulators since some portion of heat may be transferred by convection and radiation from the warm glazed side to the cold external side of the shutter and through this external side into outdoor space. These transfers cause unwanted heat losses. A question concerning the thermal resistance of the shutter may arise. This conference contribution attempts to answer such a question.

Section 2 of this contribution presents the basic physical relations concerning convective and radiative heat transfers occurring in closed air cavities. The convective heat transfer will be described by correlation relations utilizing Rayleigh’s and Prandtl’s numbers whereas the description of the radiative heat transfer is based on the Stefan-Boltzmann law and the Kirchhoff law.

Section 3 contains numerical computations of convective and radiative heat transfers in order to compare their quantitative relation. The computations use the relations introduced in Section 2.

Section 4 is devoted to derivation of thermal resistance of the studied compact window shutter. The temperature dependence of this thermal resistance is highlighted.
Section 5 is a concluding part of this contribution and contains a brief overview of attained results.

Figure 1. A scheme of the compact box window shutter with a triple glazed window.

2. Basic relations
We are going to analyze the air cavity inside the compact box window shutter (figure 1) that has the following dimensions: \( H = 1.2 \text{ m} \), \( L = 0.06 \text{ m} \), \( w = 0.8 \text{ m} \). Let us suppose the temperature of the glazing of the window to be \( 20 \degree C \) and the temperature of the shutter \(-10 \degree C \). Natural convection in closed cavities has been experimentally studied by various authors [3-5]. A comprehensive series of correlation functions for Nusselt number associated with the natural convection running in vertical cavities has been published by Catton [6] and his results have also been included into some monographs [7] [8]. These correlations are dependent on the values of the aspect ratio \( H / L \), Prandtl’s number \( Pr \) and Rayleigh’s number \( Ra_L \)

\[
Ra_L = \frac{g \beta (T_1 - T_2) L^3}{\nu \alpha} 
\]

where \( \nu \) is kinematic viscosity of the fluid inside the cavity (i.e. air in our case), \( \alpha \) thermal diffusivity of the fluid, \( g = 9.81 \text{ ms}^{-2} \), \( \beta = 2 / (T_1 + T_2) \) (in Kelvins). For the average temperature of the convective flow \( T_f = (T_1 + T_2) / 2 = 278 \text{ K} \), the values of \( \nu = 13.932 \cdot 10^{-6} \text{ m}^2 / \text{s} \), \( \alpha = 19.596 \cdot 10^{-6} \text{ m}^2 / \text{s} \), \( Pr = 0.7127 \) and thermal conductivity of air \( \lambda = 24.54 \cdot 10^{-3} \text{ W m}^{-1} \text{ K}^{-1} \) can be found e.g. in the thermodynamic tables published in [7] or [8].

\[
Ra_L = \frac{g \beta (T_1 - T_2) L^3}{\nu \alpha} = \frac{9.81 \cdot \frac{1}{278} \cdot (293.15 - 263.15) \cdot 0.06^3}{13.932 \cdot 10^{-6} \cdot 19.596 \cdot 10^{-6}} = 8.3756 \cdot 10^5
\]

As seen, the chosen window shutter may be characterized by the following parameters \( H / L = 20 \), \( Pr = 0.7127 \) and \( Ra_L = 8.3756 \cdot 10^5 \) that enable us to choose the most convenient correlation function for the Nusselt number \( \overline{Nu} \) by means of the published Catton series [6]:
\[ \bar{N}u = 0.42 \cdot Ra^{0.25} \cdot Pr^{0.012} \left( \frac{H}{L} \right)^{-0.3} \]  
(3)

\[
\begin{cases}
10 \leq \frac{H}{L} \leq 40 \\
1 \leq Pr \leq 2 \cdot 10^5 \\
10^4 \leq Ra_L \leq 10^7
\end{cases}
\]

The Nusselt number is defined as the following ratio

\[ \bar{N}u = \frac{h \cdot L}{\lambda} \]  
(4)

where \( h \) is the coefficient of heat transfer and \( \lambda \) is thermal conductivity of the fluid (in our case it is air). By combining Eqs. (1), (3) and (4), the coefficient of heat transfer \( h \) for natural convection inside the closed cavity emerges:

\[ h = \frac{\lambda}{L} \cdot 0.42 \cdot \left( \frac{g \beta (T_1 - T_2)}{v \alpha} \right)^{0.25} \cdot Pr^{0.012} \left( \frac{H}{L} \right)^{-0.3} \]  
(5)

\[ h = 0.42 \cdot \lambda \cdot \left( \frac{g \beta}{v \alpha} \right)^{0.25} \cdot (T_1 - T_2)^{0.25} \cdot Pr^{0.012} \cdot L^{0.05} \cdot H^{-0.3} \quad \text{Wm}^2 \text{K}^{-1} \]  
(6)

The coefficient of heat transfer \( h \) enables to compute the convective density of heat flow \( q_{conv} \) directed from the warm side to the cold one

\[ q_{conv} = h \cdot (T_1 - T_2) \quad \text{Wm}^{-2} \]  
(7)

\[ q_{conv} = 0.42 \cdot \lambda \cdot \left( \frac{g \beta}{v \alpha} \right)^{0.25} \cdot (T_1 - T_2)^{1.25} \cdot Pr^{0.012} \cdot L^{0.05} \cdot H^{-0.3} \]  
(8)

As known, the convective heat transport mechanism is supported by the conduction transfer of heat that takes place between the solid surfaces of the cavity and the first immobilized layer of the fluid. This conduction energy contribution is distributed over the cavity by other circulating layers of fluid. Consequently, in the case of the fully developed convective mechanism it is not necessary to add an extra conductive energy contribution.

Besides the natural convective heat transfer, the closed cavity of the compact shutter experiences the radiative heat transfer directed in the same way as the convective heat flow. This is in fact a heat transfer between two parallel plates of the same large areas. For such a simple configuration, the density of radiative heat flow \( q_{rad} \) may be expressed by means of the Stefan-Boltzmann law and the Kirchhoff law [1], [2]:

\[ q_{rad} = \frac{C_b}{\varepsilon_1 + \frac{1}{\varepsilon_2} - 1} \cdot \left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right] \quad \text{Wm}^{-2} \]  
(9)

where \( C_b = 5.67 \ \text{Wm}^2 \text{K}^{-4} \), \( \varepsilon_1 = 0.92 \) is the thermal emissivity of window glazing and \( \varepsilon_2 = 0.87 \) is the thermal emissivity of the wooden shutter.

The total heat transfer \( q_{tot} \) in the closed air cavity is given by the sum of radiative and convective heat transfers.
\[ q_{\text{tot}} = q_{\text{conv}} + q_{\text{rad}} \] (10)

3. Numerical computations

As soon as the computations of convective and radiative heat transfers are accomplished, their values lead us to the conclusion that it is the radiative heat transfer that dominates in the investigated air cavity of the window shutter:

\[ q_{\text{conv}} = 63.2 \text{ Wm}^{-2}, \quad q_{\text{rad}} = 118.6 \text{ Wm}^{-2} \] (11)

The density of radiative heat flow is almost two times larger as compared with the density of convective heat flow, which clearly indicates importance of radiative heat transfer within the air cavity of the compact window shutter

\[ q_{\text{rad}} \approx 2 \cdot q_{\text{conv}} \] (12)

The same conclusion may be drown if the energy powers of both the transfers are estimated

\[ \Phi_{\text{conv}} = S \cdot q_{\text{conv}} = 60.7 \text{ W}, \quad \Phi_{\text{rad}} = S \cdot q_{\text{rad}} = 113.9 \text{ W} \] (13)

\[ \Phi_{\text{rad}} \approx 2 \cdot \Phi_{\text{conv}} \] (14)

4. Thermal resistance of the cavity

By inserting Eqs. (8) and (9) into Eq. (10), we obtain the density of heat flow \( q_{\text{tot}} \) in a detailed form

\[
q_{\text{tot}} = 0.42 \cdot \lambda \cdot \left( \frac{g \beta}{v \alpha} \right)^{0.25} \cdot \left( T_1 - T_2 \right)^{1.25} \cdot Pr^{0.012} \cdot L^{0.05} \cdot H^{-0.3} + \frac{C_p}{1 + \frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_2}} \cdot \frac{\left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right]}{\left( T_1 - T_2 \right)}
\] (15)

\[ h_{\text{tot}} = h_{\text{conv}} \cdot (T_1 - T_2) \] (16)

\[
h_{\text{tot}} = 0.42 \cdot \lambda \cdot \left( \frac{g \beta}{v \alpha} \right)^{0.25} \cdot \left( T_1 - T_2 \right)^{2.25} \cdot Pr^{0.012} \cdot L^{0.05} \cdot H^{-0.3} + \frac{C_p}{1 + \frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_2}} \cdot \frac{\left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right]}{\left( T_1 - T_2 \right)}
\] (17)

\[ h_{\text{tot}} = h_{\text{conv}} + h_{\text{rad}} \quad \text{Wm}^{-2} \cdot K^{-1} \] (18)

The total thermal resistance of the air cavity \( R_{\text{tot}} = 1/ h_{\text{tot}} \) is given as follow

\[
R_{\text{tot}} = 0.42 \cdot \lambda \cdot \left( \frac{g \beta}{v \alpha} \right)^{0.25} \cdot \left( T_1 - T_2 \right)^{2.25} \cdot Pr^{0.012} \cdot L^{0.05} \cdot H^{-0.3} + \frac{C_p}{1 + \frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_2}} \cdot \frac{\left[ \left( \frac{T_1}{100} \right)^4 - \left( \frac{T_2}{100} \right)^4 \right]}{\left( T_1 - T_2 \right)} \right]^{-1}
\] (19)

As seen, the total thermal resistance of the closed air cavity is non-linearly dependent on temperature since both the convective and radiative heat transfers are also non-linearly dependent on temperature. After the numerical evaluation of formula (19), the value of the total thermal resistance of the air cavity of the investigated compact window shutter may be presented. The value of this thermal resistance amounts \( R_{\text{tot}} = 0.165 \text{ m}^2 \cdot \text{KW}^{-1} \).
5. Conclusions
In this contribution, the air cavity of window box-like compact shutter has been studied from the viewpoint of thermal properties. This study has revealed several facts that may be summarized as follows:

a. From the three possible kinds of heat transfer mechanisms, it is the radiative heat transfer that dominates over the remaining two processes provided the geometrical dimensions and thermal basic parameters of the window shutter show common values usually implemented in building practice.

b. The coefficient of heat transfer $h_{\text{conv}}$ that characterizes convective transfer processes inside the cavity shows weak non-linear temperature dependence. It can hardly be considered as a constant quantity but it has to be computed from the geometrical and thermal parameters of the cavity.

c. The coefficient of heat transfer $h_{\text{rad}}$ that characterizes radiant transfer processes inside the cavity shows strong non-linear temperature dependence. It is the main factor causing the temperature non-linearity of the total coefficient of heat transfer $h_{\text{tot}}$.

d. When the total thermal resistance of the air cavity of window compact shutter $R_{\text{tot}}$ is to be computed, the influences of both the heat convective and radiative transfer processes must be considered. If only the convective process were considered, the thermal resistance would be strongly overestimated.

e. The total thermal resistance of the air cavity of window compact shutter $R_{\text{tot}}$ shows strong thermal dependence and has to be computed from the geometrical and thermal parameters of the cavity under investigation.

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