Students’ procedural and conceptual Fraction knowledge based on cognitive style

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Abstract. Recent research on children’s procedural conceptual knowledge has suggested that there are individual differences. Some children have more conceptual knowledge, some children have more procedural knowledge, and some children have an equal level of both. In this study we compared and investigated individual differences knowledge of procedural and conceptual fraction among cognitive style of seventh (n=40) graders in Surabaya, East Java. Data collection in three sessions; all participants answered the same tasks. In the first session, students completed the Group Embedded Figures Test (GEFT) in 12 minutes to investigate their cognitive style (Field Dependence cognitive (FD) or Field Independence cognitive (FI) styles). In the second session, students completed three written measures (procedural and conceptual fraction knowledge test in 45 minutes) individually in a classroom. In the third session, students were individually interviewed for approximately 30 minutes. They answered questions on how they would solve fraction tasks. Students of the FI group performed significantly better than the FD group, on a measuring fraction and estimating number line. Overall, these results provide a more detailed picture of individual differences in procedural and conceptual knowledge in cognitive style.

1. Introduction
Knowledge concept is critically important to learn mathematics. Indeed, students must possess chances and sufficient support of knowledge to study mathematics in-depth [3]. One of the concepts in mathematics is fractions. Fractions are one of the most challenging cognitive lessons which mostly leads to problems of teaching [7]. Not only are fractions essential for learning, but they also benefit to solve problems in daily lives, such as solving financial issues and doing home repairs [4]. Therefore, students need to have knowledge of fractions correctly.

The abstraction of the concept of fragmentation about the whole relationship [10]. Related to this, Charalombus proposes that a general relationship is a condition in which a constant quantity or set of discrete objects is divided into equal parts [2]. Insufficient knowledge of the concepts of fractions leads to difficulties in doing calculations with fractions, decimals and concepts, the use of fractions in measurements, and the concepts of ratios and proportions [10]. Students who have insufficient knowledge in learning fractions will encounter difficulties in other mathematics concepts such as algebra, measurements, ratios and proportions [8].

An important aspect that distinguishes various types of numerical knowledge lies in procedural and conceptual knowledge [6]. Knowledge of fractional procedures consists of four fractional arithmetic operations such as summing, subtraction, multiplication, and division. Conceptual knowledge in this area involves knowledge of the properties of factions, including size (e.g., 4/5 greater than 1/2), principles related to factions (e.g., an infinite number of factions can be found between two factions etc.), and notations to express fractions (e.g., 3/4 = 6/8 = .75) [1].
Individual knowledge as a cognitive activity is highly dependent on different cognitive styles. One of the most widely studied cognitive styles is the cognitive style that is psychologically divided into two fields: Field of Cognitive Dependence (FD) and Field of Cognitive Independence (FI). Individuals with FD-induced cognitive tendencies may have difficulty separating information that is in their context and may be more influenced by external cues and thus less selective in the absorption of information. While FI individuals have little difficulty in separating the most important information from their context, they are more likely to be influenced by internal cues than external cues, and they are selective in entering information [11].

This study aimed to describe the knowledge of first-year high school students' cognitive skills with Field Independence (FI) and Field Dependence (FD). This research is qualitative research with a qualitative approach to data collection techniques through interviews. The results of this study are expected to contribute to the development of learning in procedural knowledge.

2. Method
2.1. Participants
A qualitative approach applied in this study to analyse students' procedural and conceptual fraction knowledge with different cognitive style. The sample consisted of 30 on 7th grade students of a school in Surabaya. Only students want to as a volunteer were included in the study. All students are given Group Embedded Figures Test (GEFT) for 20 minutes to investigate their cognitive style (field independence or field dependence) and Procedural-Conceptual Fraction Knowledge Test (PCFUT) to know their procedural and conceptual knowledge in fraction. The result shows, there were 7 field independence students and 23 field dependence students. A volunteer, one student chosen as a participant from each group by considering the criteria of equivalent mathematical ability to analyze deeply about their performance on procedural and conceptual test. In this study, field independence students we labelled as FI and field dependence students we labelled as FD.

Time triangulation is used to determine the validity of the data. In such an area, it is a way of testing the validity of data by comparing data collected at two different times. For instance, data on PCFUT-1 results and interviews 1 compared with data from PCFUT-2 and interview 2. The results of this triangulation indicate that the data is consistent, and therefore obtaining valid data, and PCFUT results and interviews are used as substantial aspects to analyze data to answer research questions.

2.2. Measures
Cognitive Style Task
Participants completed the Group Embedded Figures Test (GEFT) for 12 minutes to investigate their cognitive style. There are eighteen complex numbers in GEFT. Based on the correct answer number given by the student, the score on GEFT may be received from 0 (most FD) to 18 (lowest FI). In addition, a preliminary book that contains strict directions together with examples for discussion of subjects, this exam contains three parts: The first section aims to make students able to use the exam, and the other two parts are GEFT. The first section that has a two-minute time limit includes 7 easy problems to practice, and the items in this section are not included in the total score. The real task starts in the second and third sets, where asking for the exam requires making reproach them in every 9 items specified in having a period of five minutes for each one. Those who scored more than 12 out of 18 were labeled as field freedom (FI) and those who had a score of 11 and less than 11 were labeled as field dependency (FD).

Fraction procedural knowledge task
Participants completed eight fraction arithmetic problems (addition, subtraction, multiplication, and division) with equal denominators and unequal denominators. The items were $3/5 + 1/2$, $3/5 + 2/5$, $3/5 - 1/2$, $3/5 - 2/5$, $3/5 * 1/2$, $3/5 * 2/5$, and $3/5 : 2/5$. The correct answered were scored as correct even if they were not in simplest form.
Fraction conceptual knowledge task

0-1 Number line estimation
Participants completed 10 number lines. Each line’s left endpoint was labelled “0”, and the right endpoint was labelled “1”. The fraction to be estimated were 1/19, 1/7, 1/4, 3/8, 1/2, 4/7, 2/3, 7/9, 5/6, and 12/13. Performance was measured from the correct location of the fraction on the number line.

0-5 Number line estimation
This task and the measure were identical with 0-1 number line estimation task except that the right endpoint of the number line was labelled “5” rather than “1” and the items to be estimated were 1/19, 4/7, 7/5, 13/9, 8/3, 11/4, 10/3, 7/2, 17/4, and 9/2.

Fraction magnitude comparison
Participants compared eight fractions magnitudes (3/8, 5/8, 2/9, 4/5, 4/7, 5/9, 8/9, and 2/3) with the fraction 3/5 which one is greater than or less than.

2.3. Procedure
Data collection in three sessions; all participants answered the same tasks. In the first session, students completed GEFT in 12 min. In the second session, students completed three written measures (procedural and conceptual fraction knowledge test in 45 min) individually in a classroom. In the third session, students were individually interviewed for approximately 30 min. They answered questions on how they would solve fraction tasks. These interviews were always completed after first and second session.

3. Result and Discussion
3.1. Cognitive style differences in knowledge of fraction procedures
Based on their result of fraction procedural knowledge are shown in Figure 1 and 2.

![Figure 1. The answer of FI students in fraction procedural knowledge](image)

From the resulting test, he completed all the problems and correct. From the result of the interview, FI students understand the procedure for operating fraction problems. So, we can conclude that he understand how to operating fraction.
From the resulting test, we know that FD students completed addition, subtraction, and multiplication problems and correct. But for division problems are uncompleted. From the result of the interview, FD students understand the procedure for operating fraction problems (addition, subtraction, and multiplication). In division problems, he could not solve it.

Based on the resulting test and interview of FI students and FD students we can compare their fraction procedural knowledge. The results show that FI students were represented a complete fractional concept and FD students almost complete fractional concept excepts addition.

3.2. Cognitive style differences in knowledge of fraction concepts

Fraction conceptual knowledge task measured by three different tasks (0-1 number line estimating, 0-5 number line estimating, and fraction magnitude comparison). Based on their result of fraction procedural knowledge are shown in Figure 3.

Performance of fraction estimating was measured by the percentage accuracy of each estimate from the correct location of the fraction on the number line (0-1 number line estimating task and 0-5 number line estimating task) and fraction magnitude comparison.

In the model estimating 0-1 number line, there was an effect of different cognitive style, indicating that FI students were more accurate than FD students. From the results of the interview, FI students estimating number line using concept of fraction units. He converting number "1" and number "5" into

Figure 2. The answer of FD students in fraction procedural knowledge

Figure 3. The percentage accuracy of FI students and FD in estimating number line and fraction magnitude comparison
a fraction that has the same denominator with the fraction in question. Therefore, he can estimate the position of this fraction in the number line. The above answer was justified with the result of the interview as follows.

Q : Do you know what the instruction to answer the problem?
F1 : Yes, it is to estimate the location of fraction in the number line
Q : How did you do?
F1 : I make the maximum number line to be a fraction with the same denominator with the fraction in the question
Q : Could you explain that?
F1 : For instance, in this number, we must estimate fraction 1/7 in 0-1 number line. I make number 1 into a fraction that has the same denominator with fraction 1/7, number 1 is equal to 7/7. After that I just compare the numerator of 1/7 and 7/7, it is 1 and 7. Therefore, I just estimate where is the position of 1 in number line 0 until 7.
Q : Why you estimate like that?
F1 : I learn that if we want to compare fractions we must make the fraction into has the same denominator.

From the result of the interview, FD students estimating number line-using concept of part-whole. He imagined a whole number and divided it into 4 parts, one part has become 1/4. He understand about concept is used to estimate fraction, but he had difficulty with large numerator and denominator. The FD students have a dependence on external cues and structures, easily disturbed and easily confused as to lack the ability to solve problems [3].

In the fraction magnitude comparison accuracy, FI students and FD students have no differences of accuracy. The results show that FI students using fraction equivalents and FD student using visualization of concrete objects to estimating a fraction in the number line.

The groups of FI students and FD students differentiated not only seem to match their learning profiles. The differences also relate to problem-solving performance for fractions and intensive quantities [5]. Their understanding of the underlying concepts can we know when students practice solving problems.

4. Conclusion

Based on the results of the study, it is shown that Cognitive Field Independence students measuring fraction very well and express how the procedure with their own words. FI students estimating number line using concept of fraction units. He converting number "1" and number "5" into a fraction that has the same denominator with the fraction in question. Therefore, he can estimate the position of this fraction in the number line. While the cognitive-oriented students of Field Dependence could measuring fraction, but difficulty in addition. FD students estimating number line using concept of part-whole. He imagined a whole number and divided it into 4 parts, one part has become \( \frac{1}{4} \). He understand about concept is used to estimate fraction, but he had difficulty with large numerator and denominator. In the fraction magnitude comparison accuracy, FI students and FD students have no differences of accuracy. The results show that FI students using fraction equivalents and FD student using visualization of concrete objects to estimating a fraction in the number line. These findings suggest that, cognitive style differences are one of the decisive factors that can influence students' knowledge of the concept of fractions.
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