Quarks play an active role in shaping the QCD vacuum structure. Being dual carriers of both ‘color’ and ‘electric’ charges they also respond to externally applied electromagnetic fields. Thus, in principle, the vacuum of strong interactions influences higher order QED processes such as photon-photon scattering. We survey here the current status of the understanding of the vacuum structure of strong interactions, and take a fresh look at its electromagnetic properties.

Dedicated to the memory of Peter A. Carruthers

1 QCD Vacuum

1.1 Gluon condensate

Due to attractive glue-glue interaction inherent in the non-Abelian nature of color charges, the naive, i.e., non-interacting product wave function of the vacuum state is known to be unstable. It is generally believed that the QCD-originating structures are the source of the confinement effect which restricts quarks to colorless bound states. Many features of the structured vacuum have been studied in past 20 years with a wealth of methods, but one aspect, the appearance of a glue ‘condensate’ field, i.e, vacuum expectation value (VEV) of the gluon field-correlator in the vacuum state, is of particular relevance in our study. Its value obtained from QCD sum-rules is today not much different from values first noted nearly 20 years ago:

\[ \langle V | \frac{\alpha_s}{\pi} G^2 | V \rangle \simeq (2.3 \pm 0.3) \times 10^{-2} \text{GeV}^4 = [390 \pm 12 \text{MeV}]^4, \]  

(1)

where \( \alpha_s = g^2 / 4\pi \) is the strong interaction (running) coupling constant, and

\[ \frac{1}{2} G^2 \equiv \sum_a \frac{1}{2} G_{\mu\nu}^a G^{\mu\nu}_a = \sum_a |B_a^2 - E_a^2|, \]  

(2)

with \( a = 1, \ldots, N^2 - 1 \) gauge field components for the SU(N) color charge group. This value of the glue condensate is in agreement with the results
obtained numerically using lattice gauge theory (LGT) methods, which in addition provide the shape of the condensate fluctuations in Euclidean time.

What is the meaning of the vacuum condensate field? The vacuum must be field free, so that the appearance of a field correlator has no classical analog, it expresses a Bogoliubov-type rotation away from the trivial Fock space state, induced by the interactions. The effect is often compared to the ferromagnetism since one can prove that one of the QCD instabilities is the magnetic gluon spin-spin interaction. On the other hand, the confinement effect of color charged quarks is best understood invoking an anomalous dielectric property. Both these classical analogs are probably applicable; namely, Lorentz and gauge invariance property of the vacuum state dictates that the VEV of a product of two field operators satisfies:

$$\langle G^a_{\mu\nu}(x)G^b_{\rho\sigma}(x) \rangle = (g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})\delta^{ab}\langle G^2(x) \rangle /96.$$  \hspace{1cm} (3)

Taking the required contractions and using Eq. (2) one finds:

$$\langle \sum_a \vec{B}_a^2 \rangle = - \langle \sum_a \vec{E}_a^2 \rangle.$$  \hspace{1cm} (4)

We find that, in the vacuum $|V\rangle$, we have $\langle \vec{B}_a^2 \rangle$ positive, and $\langle \vec{E}_a^2 \rangle$ negative. The signs of the matrix elements arise with reference to the perturbative non-interacting Fock state, i.e., with respect to products of field operators which are normal-ordered with respect to the unstructured ‘free’ state $|0\rangle$, the so called ‘perturbative vacuum’ — this use of language is an oxymoron since there can be only one ‘true’ vacuum $|V\rangle$. In the perturbative state $|0\rangle$ the VEV of the gluon field vanishes by definition because of normal ordering. Without normal ordering there are (infinite) zero-point fluctuations of the field. The interpretation of Eq. (4) is that the B-field fluctuates in the true QCD vacuum $|V\rangle$ with a bigger amplitude than in the perturbative ‘vacuum’ $|0\rangle$, while the E-field fluctuates with a smaller amplitude than in the perturbative ‘vacuum’ state. This combination of effects is a necessary consequence of the symmetries, the primary effect can be seen as being due to the magnetic gluon spin-spin interaction. There are obviously many different and equivalent ways to model and understand the glue condensate, and we shall not pursue this here in greater detail, though we shall mention some of the models as needed. Our primary interest is in quark fluctuations (condensates as well) to which we turn our attention now.

2
1.2 Chiral symmetry and quark vacuum structure

Dirac spinor operators for up and down quarks satisfy the two identities:

\[ \partial_\mu j_5^u \equiv \partial_\mu (\bar{u} \gamma_\mu d) = i(m_u + m_d)\bar{u}d, \]
\[ \partial_\mu j_5^d \equiv \partial_\mu (\bar{u} \gamma_\mu \gamma_5 d) = i(m_u - m_d)\bar{u}\gamma_5 d, \]

(5)

(6)

Here \( u, d \) are the spinor field operators representing the two light quark flavor fields of mass \( m_u \) and \( m_d \) respectively. The subscript ‘+’ reminds us that these currents ‘lift’ the ‘down’ quark to ‘up’ quark, it is an iso-raising operator which increases the electrical charge by \( +|e| \).

We see that when the quark masses are equal the isospin-quark-current Eq. (5) is conserved, which implies that the Hamiltonian is symmetric under rotations which mix ‘u’ with ‘d’ quarks; this is an expression of the isospin-SU(2) symmetry of strong interactions. In case that the quark masses would vanish, by virtue of Eq. (6) the pseudo-vector isospin-quark-current would be conserved. This implies that the Hamiltonian would be even more symmetric, specifically it would be invariant under two chiral symmetries \( SU(2)_L \times SU(2)_R \). While \( m_q = u, d = 5–15 \text{ MeV} \approx 0 \) to a good approximation on hadronic scale of 1 GeV, there is no sign of the corresponding symmetry, which would be represented in the hadronic spectrum by doublets of hadronic parity states. For example, we find only one isospin doublet of nucleons, not two. On the other hand, the [Adler-Weisberger] sum rules which relate weak and strong sectors confirm the presence of the intrinsic \( SU(2)_L \times SU(2)_R \) symmetry in the elementary Hamiltonian. Nambu pointed to this symmetry-breaking in which the ground state breaks the intrinsic (almost) chiral symmetry of the Hamiltonian.

The way we look at this issue (symmetry breaking by the ground state) today is as follows: by virtue of the Goldstone theorem, in the event that the quark masses vanish exactly, there should be an exactly massless Goldstone boson with quantum numbers of the broken symmetry, thus spin zero, negative parity and \( I = 1 \). However, since the chiral symmetry of the Hamiltonian was not exact, the low mass pion state expresses the massless Goldstone meson of strong interactions. In a way one can then see the parity doublets of all strongly interacting particles as being substituted for by a ‘direct product’ of the Goldstone boson (pion) with the elementary hadron states. This, in turn, means that many features of the hadronic spectrum and possibly of the vacuum structure should strongly depend on the small and seemingly irrelevant quark masses.

A nice illustration of this phenomenon is the observation that in the limit of vanishing quark masses, the pion mass also vanishes. We follow the standard approach and consider the two matrix elements of the pseudoscalar and the
pseudo-vector between the vacuum state and one pion state:

\[
\langle \pi^+ (p) | \bar{u}(x) \gamma^\mu \gamma_5 d(x) | V \rangle = -i \sqrt{2} p^\mu f_\pi e^{ip_{\mu}x^\mu},
\]

(7)

\[
\langle \pi^+ (p) | \bar{u}(x) \gamma_5 d(x) | V \rangle = i \sqrt{2} g_\pi e^{ip_{\mu}x^\mu}.
\]

(8)

The right hand side arises by the Lorentz symmetry properties of the (true) vacuum state \( | V \rangle \) and the \( \pi^+ \)-state — hence \( p^\mu p^\mu = m_{\pi}^2 = 139.6 \) MeV. The matrix element \( f_\pi = 93.3 \) MeV is known in magnitude since it governs the weak interaction decay of pions, and \( g_\pi \simeq (350 \) MeV\( )^2 \) follows indirectly from the sum-rules (see below).

Taking the divergence of Eq. (7) and recalling relation Eq. (6) we obtain:

\[
m_{\pi}^2 f_\pi = (m_u + m_d) g_\pi.
\]

(9)

A remarkable consequence of this relation is \((m_u + m_d) \simeq 0.1 m_\pi\), an unexpected result, since the current quark mass comes out to be smaller than the ‘massless’ pion. Indeed, one finds a quite memorable comment to this point in Weinberg’s treatise (Volume II, p190, bottom) on Quantum Theory of Fields: ‘... One of the reasons for the rapid acceptance of quantum chromodynamics in 1973 as the correct theory of strong interactions was that it explained the \( SU(2) \times SU(2) \) symmetry [inherent in Adler-Weisberger sum rule of 1965] as a simple consequence of the smallness of the \( u \) and \( d \) quark masses.’

Some of us, who have studied in depth the quark-bag model, will wonder how the \( q \bar{q} \) structure of the pion, that is so evident in this approach, can be made compatible with its Goldstone nature. In our opinion the matter is very simple: there is already a big cancellation of different contributions which leads in a comprehensive fit to a pion of mass \( \mathcal{O}(100) \) MeV. The theoretically, not fully understood, but critical component in the pion mass is the so called zero-point energy \( E_0 \simeq -(1.8 \hbar c)/R \), where \( R \) is the hadron radius. A small change in \( E_0 \) suffices to render pion massless, yet the origin of the sign and magnitude of \( E_0 \) remains a theoretical challenge; it can not be understood as the Casimir energy of the cavity as it has been originally conceived. It is easy to imagine that \( E_0 \) expresses aside of the center of momentum projection correction also the structure of the vacuum, and by virtue of the Goldstone theorem, it has to be self-consistently-fine-tuned so that for \( m_q \to 0 \) the mass of the pion vanishes.

The Nambu-Goldstone structure of the vacuum was explored intensely in terms of symmetry relations between current matrix elements (current algebra), even before QCD was discovered — for example the Adler-Weisberger sum rule we mentioned above was a stepping stone to the understanding of the underlying symmetry. Many rather general vacuum matrix element relations
were obtained, of which, in our context, the most important is the GOR (Gell-Mann-Oakes-Renner) relation, which adopted to the quark language reads:

\[ m_π^2 f_π^2 = 0.17 \text{GeV}^4 \simeq -\frac{1}{2} (m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle + \ldots \]

(10)

Sum-rules have been developed which allow to estimate the condensate of the Fermi fields from the particle spectra and cross sections, and the current best value is:

\[ \frac{1}{2} \langle \bar{u}u + \bar{d}d \rangle|_{1 \text{ GeV}} \equiv \frac{1}{2} \langle \bar{q}q \rangle|_{1 \text{ GeV}} = -[(225 \pm 9) \text{ MeV}]^3. \]

(11)

When combined with Eq. (10), one can estimate the magnitude of running QCD-current quark masses, which at 1 GeV scale are:

\[ (m_u + m_d)|_{1 \text{ GeV}} \simeq 15 \text{ MeV}, \quad m_s|_{1 \text{ GeV}} \simeq 182 \text{ MeV}. \]

(12)

### 1.3 Quark-gluon relation in the vacuum state

We have thus established that the vacuum state has a complex structure in which significant fluctuations of the quantum gluon and quark fields occur. Is there a relation between these two different phenomena: i.e., the chiral structure, and gluon instability? One would be tempted to infer that the chiral symmetry-breaking features in QCD and quark condensation have little if anything to do with gluon condensation we described above. However, studies of symmetry restoration at high temperature have yielded contrary evidence: at high temperature the vacuum structure of QCD melts, as expressed by Eq. (1) in terms of the glue condensate, and one reaches the perturbative vacuum. This confinement to deconfinement transformation and the chiral symmetry restoration, as expressed by the melting of the quark condensate, are seen exactly at the same temperature.

Furthermore, model calculations, employing mean field configurations of gauge fields in the QCD vacuum, invariably suggest that it is the presence of the glue field condensate which is the driving force behind the appearance of the quark condensate. For example, a model which employed a self-dual covariantly constant field for the non-perturbative gauge field configurations in the structured QCD vacuum finds that the quark condensation is a minor and stabilizing contributor (6%) to the vacuum energy due to its bulk part to the glue degrees of freedom.

The mechanism how this can happen is, in principle, not very difficult to understand. Schwinger in his seminal paper on gauge invariance and vacuum
fluctuations [see Eq. (5.2)] shows already that:

$$\langle \bar{\psi}(x)\psi(x) \rangle \equiv \frac{1}{2} \langle [\bar{\psi}(x),\psi(x)] \rangle \equiv \Sigma = -\frac{\partial \Gamma}{\partial m}.$$  \hspace{1cm} (13)

The left-hand side of Eq. (13) defines more precisely the meaning of the quark condensate in terms of the Fermi field operators at equal space-time points. The right-hand side refers to the effective action density $\Gamma[A_\mu]$ of fermions in presence of gauge potentials $A_\mu$.

For the case of constant gauge fields, the specific relation Schwinger obtained for the fermion condensate in the true vacuum, with operators normal ordered with reference to perturbative vacuum, to first order in coupling strength but all orders in the gauge field strength is:

$$-\frac{m^2}{2} \langle [\bar{\psi}(x),\psi(x)] \rangle^{(1)} = \frac{m^2}{4\pi^2} \int_0^\infty ds \frac{m^2 - s^2}{s^2 e^{-m^2 s} \tan sE \tanh sB}.$$  \hspace{1cm} (14)

where the gauge field configurations are paralleling the Maxwellian electric $\vec{E}$ and magnetic $\vec{B}$ fields:

$$B^2 = \frac{e^2}{2} \sqrt{(\vec{E}^2 - \vec{B}^2)^2 + 4(\vec{E} \cdot \vec{B})^2} - \frac{e^2}{2} (\vec{E}^2 - \vec{B}^2)$$

$$\rightarrow |e\vec{B}|^2, \quad \text{for} \ |\vec{E}| \rightarrow 0; \hspace{1cm} (15)$$

$$E^2 = \frac{e^2}{2} \sqrt{(\vec{E}^2 - \vec{B}^2)^2 + 4(\vec{E} \cdot \vec{B})^2} + \frac{e^2}{2} (\vec{E}^2 - \vec{B}^2)$$

$$\rightarrow |e\vec{E}|^2, \quad \text{for} \ |\vec{B}| \rightarrow 0.$$  \hspace{1cm} (16)

The standard relations of Abelian gauge fields

$$\vec{E}^2 - \vec{B}^2 = -\frac{1}{2} F_{\mu\nu} F^{\mu\nu}, \quad \vec{E} \cdot \vec{B} = -\frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} \equiv -\frac{1}{4} F \tilde{F},$$  \hspace{1cm} (17)

where:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu, \quad \tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}. \hspace{1cm} (18)$$

apply in suitable generalization for non-Abelian gauge fields. We note that the effective action is greatly sensitive to details of the gauge field configuration, and thus its derivative which is the Fermi field condensate, is probably even more unpredictable as the assumption of the constant (on scale of the fermion considered, $1/m$) gauge fields is relaxed. Thus the precise relationship here given is not likely to apply to any realistic study of QCD structure.

However, the result inherent in Eq. (14) is proving that the Fermi condensate is driven by the presence of the gauge field fluctuations, which as we
alluded to above, are in turn just little influenced by the quark condensation. Thus in a self-consistent description of both condensates the exact solution should be very close to the gauge field vacuum configuration found without Fermi fields, while quark field vacuum configuration should be strongly dependent on the presence of the gauge field fluctuations, which are the key force driving quark condensation.

The Nambu-Goldstone mechanism assures that the quark vacuum must have a profoundly non-trivial structure which ‘remembers’ even the values of small quark masses. The Nambu-Jona-Lasinio model of strong interactions incorporates in the relevant (spontaneous) symmetry-breaking features and has become a frequently studied model of strong interactions. It also offers an opportunity to explore the interference between electromagnetic and strong forces, which has been considered with an eye for the possible chiral symmetry restoration in fields of extreme strength. However, what interests us here is a small distortion of the vacuum in consequence of a relatively weak Maxwell field being applied. It remains to be seen if this question, also subtly dependent on the exact wave function of the vacuum state, can be studied within the Nambu-Jona-Lasinio approach, or if we need to address these issues within a new scheme of approach to QCD.

2 Electromagnetic Properties of the Vacuum

2.1 Precision vacuum structure experiments

The presence of the quark vacuum structure invites naturally an experimental study of the QCD vacuum employing the electromagnetic quark interactions. When assessing this option we have to consider that QCD-QED effects are constrained by the well known QED precision results. Consider as example the QED-vacuum polarization (VP) effect, which on distance scale of the electron Compton wavelength, alters the \(1/r\)-nature of Coulomb’s law by a 0.1%-relative-strength correction. The VP effect arises as the response of the vacuum to a highly inhomogeneous field-strength, and is often interpreted as a dielectric particle-hole (electron-positron) photon polarizibility. Because of its range, the VP-potential can only be detected in the vicinity of the atomic nucleus. VP-effect does not violate the superposition principle of electromagnetic fields, thus there is no possibility for an effective new interaction, an ‘anomaly’ such as photon-photon scattering, or photon-electromagnetic field interaction. VP has been explored to considerable precision and its agreement with theoretical expectations speaks for many as evidence against other measurable electromagnetic vacuum structure.

We need to debunk this myth: the predictability of the usual vacuum
polarization effects in QED arises because of gauge invariance related to charge conservation, and the process of charge renormalization. These symmetry effects combine to ‘protect’ the electromagnetic interactions in strength and shape. These effects are not present in the nonlinear higher order terms which generate otherwise absent interactions, i.e. ‘anomalies’. We specifically note the photon-photon scattering process which is forbidden in classical electromagnetism, and which is the key new feature of QED noted quite early in its development by Euler, Heisenberg, Kockel 22 60 years ago. This effect is not ‘protected’ by symmetries and can be greatly influenced by the QCD vacuum structure, as we shall discuss below.

The theoretical question is here in what physical environment we can best explore such effective interactions. One particularly interesting environment involves macroscopic Maxwell EM-fields (quasi-constant magnetic fields and laser fields), fields which even on atomic scale are extremely homogenous. In such situation renormalization absorbs all effects due to VP and the only effect that remains is the effective higher order interaction, specifically the effect of light-light scattering. Much recent effort has addressed the possibility to study QCD-QED vacuum structure using precision laser-optical QED probes in strong magnetic fields, an approach which promises to test vacuum structure effects we develop below.

In order to understand the question how quarks could contribute to the photon-photon scattering in the presence of electromagnetic fields we first need to remind ourselves how this process works in QED. We than show that the lowest order Feynman diagrams of QCD in which a virtual up-quark is immersed in the glue vacuum fluctuations and is polarized by the applied Maxwell fields, contribute 1000 times the strength of the usual Euler-Heisenberg term to the light-light scattering process. We then discuss several different sub-resummation of higher-order Feynman diagrams and obtain quite different results. This suggests that the non-perturbative approach, based on partial resummation, has limited validity for understanding how glue vacuum fluctuations impact the quark fields.

2.2 EH-QED-effective action: constant Abelian gauge field

The effective ‘one-loop’ action $\Gamma^{(1)}$ that is to first order in the coupling constant $\alpha$ in the Abelian (QED) theory, and evaluated in the limit that the (Maxwell) field is constant on the scale of electron’s Compton wave length (which is the situation for all externally applied macroscopic fields) was studied and its analytical structure and particle production instability was fully understood in the seminal paper of Schwinger 17; the non-perturbative aspects of pair production...
described in that work provide today basis for particle production dynamics by gauge fields. The form of (renormalized) $\Gamma^{(1)}$ is:

$$\Gamma^{(1)} = -\frac{1}{8\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-m^2 s} \left[ \frac{sE}{\tan sE} \tanh sB - 1 + \frac{1}{3} (E^2 - B^2) s^2 \right].$$

(19)

We note here two subtractions. The first accounts for the field independent, zero-point action of the perturbative vacuum, and corresponds to normal-ordering of the field operators with respect to the no-field non-interacting vacuum. The second subtraction is introduced in order to implement charge renormalization. It assures that for weak fields the perturbative asymptotic series begins $O(E^4, B^4, E^2 B^2)$. This subtraction is accounted for by the subscript ‘r’ in $\Gamma$ in Eq. (19).

The analytical structure of the highly non-linear effective action comprises poles, and the action comprises an imaginary component akin to the situation one finds for unstable decaying states when interactions are turned on. Schwinger identified the singularities along the real $s$-axis of the proper-time integral in Eq. (19) with the pair production instability of the vacuum, a process in principle possible when potentials are present that can rise more than $2m$, which is of course the case in presence of constant, infinite range, electrical fields. This requirement is consequence of the properties of QED which remains for time independent fields a stable theory, i.e. there is no spontaneous particle production unless a potential difference (which cannot be gauged away) of more than $2m$ arises. An interesting point in our current discussion is that if we arrange for an electrical field which is quasi constant on Compton wavelength scale of the electron, but for which the potential never exceeds this spontaneous pair production threshold $|V| < 2m$, the Schwinger pair production singularities must vanish, no matter how the potential varies (and fields are time independent). How this can mathematically occur remains a mystery of higher order quantum electrodynamic processes, but this observation shows the sensitivity of the QED to subtle changes of the physical constraints which seemingly only little impact the mathematical structures. Similar situation arises in QCD: Matinyan and Savvidy identified the magnetic QCD instability, which was stabilized by slight inhomogeneity of the vacuum fields in subsequent work, some mentioned above.

We will need both the leading and next to leading terms in the (asymptotic) expansion of Eq. (19) for small fields:

$$\Gamma^{(1)}_2 = -\frac{1}{90 m^4} \left[ (\frac{\alpha}{\pi} F^2)^2 + \frac{7}{4} (\frac{\alpha}{\pi} F\bar{F})^2 \right] +$$
$$+ \frac{1}{315 m^8} \left[ 4 (\frac{\alpha}{\pi} F^2)^3 + \frac{13}{2} \frac{\alpha}{\pi} F^2 (\frac{\alpha}{\pi} F\bar{F})^2 \right] + \ldots .$$

(20)
In pure QED, the greatest contribution arises from the smallest mass charged fermion, the electron. But even for $m_e = 0.511 \text{ MeV}$, the nonlinearity arising from electron fluctuations are extremely small compared to the laboratory fields that can be established. The (nearly) observable macroscopic effect of the QED-fermion vacuum fluctuation is the effect of laser-external field interaction inherent in the nonlinearity of effective action expansion, Eq. (20). In such experiments, the external homogeneous magnetic field is crossed by polarized laser light beams. The laser field corresponds to the visible wavelengths, and thus millions of electron Compton wavelengths.

One should expect that this field can be understood as being homogeneous. But given the proverbial sensitivity of the effective action, it is not quite certain that the wave character of the laser beam can be fully ignored. However, the perturbative expansion which can only have asymptotic meaning, since the imaginary part of the action is non-analytical for $E \to 0$, promises to be non-sensitive, and thus as long as the perturbative expansion remains an asymptotic approximation and can be used given the magnitude of fields used, the consequences of the laser-field scattering experiment are predictable in the framework of this approximate action.

The EM-field driven fermion condensate is now obtained according to Eq. (13):

$$ -m\langle \bar{\psi}(x)\psi(x) \rangle^{(1)} = \frac{2}{3} \frac{\alpha}{\pi} F^2 - \frac{1}{90} \frac{\pi^2}{m^4} \left[ 4 \left( \frac{\alpha}{\pi} F^2 \right)^2 + \frac{7}{\alpha^2} (F \tilde{F})^2 \right] + \ldots . \quad (21) $$

The first term is not dependent on the scale of the Fermi field, as can be verified on dimensional grounds. For situations where perturbative expansion is applicable, it provides an interesting relationship between the Fermi and gauge field condensate, which has been also proposed for the QCD vacuum.

### 2.3 Magnitude of QCD Vacuum influence on light-light scattering

The effective action example derived for non-varying background fields presented in Eq. (19) allows to obtain the quark condensate, Eq. (14), allowing, in principle, to understand how it can be driven by the gauge field (glue) condensate. In this simple non-varying background field case, we can fully explore the quark vacuum, including both Abelian Maxwell (QED) fields and non-Abelian (strong in comparison) QCD-fields. We can assume that the quark structure in the true vacuum is perturbed by Maxwell-EM-fields, without impacting significantly other properties of the true vacuum state which are mostly in the glue sector. In other words, we neglect the feed-back of the quark condensate polarization by Maxwell fields into the gauge field QCD vacuum structure.
We now consider how quarks contribute to the effective action, remembering to include the quark charge \( q \), in units of \( e \) and quark mass \( m_q \), and we subject quarks also to the glue condensate along with the relatively small Maxwell field. Up to factors of order one, we ought to substitute, in the effective action Eq. (19), the pure U(1) Maxwell gauge field by the invariant combination of U(1) and the condensed SU(3) gauge fields:

\[
\frac{\alpha}{\pi} F^2 \to \langle \frac{\alpha_s}{\pi} G^2 \rangle + \frac{\alpha}{\pi} q^2 F^2 .
\]  

(22)

Here \( q = 2/3, -1/3 \) are the fractional quark charges or \( q = \pm 1 \) for leptons. We are presently rederiving the effective action, incorporating the U(1) and SU(3) gauge fields from the outset, and consider Eq. (22) as an ansatz for now.

As next step, we now adopt the result Eq. (20), employing Eq. (22). We are interested in light-light scattering to lowest order, thus in the terms comprising four Maxwell fields. In addition to the first term in Eq. (20), we obtain the term comprising one gluon condensate component from the second next to leading order expansion term given in Eq. (20) substituting there Eq. (22). Specifically we obtain:

\[
\Gamma^{(1)}_{4,2} = -\frac{1}{90 m^4} \left[ \frac{\alpha}{\pi} F^2 \right]^2 \left( \frac{7 \alpha_s}{4} F \tilde{F} \right)^2 + \sum_{i=u,d} \frac{1}{315} \frac{q_i^2 \pi^4}{m_i^4} \left[ 12 \left( \frac{\alpha}{\pi} F^2 \right)^2 \left( \frac{\alpha_s}{\pi} G^2 \right) + \frac{13}{2} \left( \frac{\alpha}{\pi} F \tilde{F} \right)^2 \left( \frac{\alpha_s}{\pi} G^2 \right) \right] + \ldots .
\]  

(23)

The relevant numerical value determining the relative magnitude of the usual EH-term and the QCD-vacuum driven term can be now easily obtained evaluating the ratio of the coefficients of the second and the first term in Eq. (23),

\[
f = \sum_{i=u,d} \frac{24 q_i^2 \pi^4}{7 m_i^4} m_e \left( \frac{\alpha_s}{\pi} G^2 \right) \simeq e^{9\pm 2.5},
\]  

(24)

a large number indeed in favor of the QCD driven effects, even if the magnitude of the effects still remains as we believe just below detection sensitivity of past experiments! It is interesting to note that the up-quark dominates the contribution in Eq. (23) by as much as a factor 1000 over the down quark, since \( q_u^2 = \frac{4}{9} = 4 q_d^2 \) and \( m_u \simeq \frac{1}{2} m_d \simeq 5 \pm 1.5 \) MeV.

It is quite rare that the perturbative relative strength is completely erased by non-perturbative effects, and thus we hope and expect that the light-light EH-anomalous effective interaction could be indeed governed by QCD-vacuum
fluctuations, rather than QED. On the other hand, the fact that the up and
down quark asymmetry introduces a difference by a factor 1000 suggests con-
siderable opportunity for a change of the magnitude of this result when the
non-perturbative effects are considered.

2.4 Non-perturbative treatment of glue condensate

The effective expansion parameter of Schwinger action is \( \frac{\langle \alpha_s \pi G^2 \rangle}{m^4_q} \), which
is much greater than unity. Thus we consider the full EH-effective action of
quarks in presence of gluon condensate and of small Maxwell fields. The prob-
lem with this approach is that the condensate fluctuates on a scale which is 100
times shorter than the Compton wavelength of the quark, and in fact therefore
the EH-action is quite inappropriate for this limit. Yet, let us see what hap-
pens using the non-perturbative fermion action \( \Gamma^{(1)} \), evaluated substituting
with relation Eq. (22). For very large (constant) fields, here taken simply as
magnetic fields it is well known that:

\[
\Gamma^{(1)} \rightarrow \propto B^2 \ln B^2 .
\]  

(25)

Performing the substitution Eq. (22) and expanding through quadratic terms
in \( B^2 \) we obtain:

\[
\Gamma^{(1)} \rightarrow \propto \frac{B^4}{\langle \alpha_s \pi G^2 \rangle} .
\]  

(26)

We see that the scale of the gluon condensate replaces fermion mass as the
characteristic dimension in the non-perturbative treatment of the light quark
sector. Thus the perturbative results hold but with the substitution \( m_q \rightarrow
390 \text{ MeV} \).

However, this result cannot be correct since, as noted above, it is mathem-
atically inconsistent, relying on the validity of constant gauge field approxi-
mation, where we know that glue fluctuations are short ranged, and moreover,
we have argued that even subtle deviations from ‘constancy’ can lead to a
qualitative change of the behavior of effective action. Moreover, the approach
proposed above is ignoring the physics of the chiral symmetry-breaking and the
Goldstone vacuum structure. These two crucial aspects of the QCD vacuum
have now been completely lost.

Thus we turn around to see how we can explore the massless (Goldstone)
quark limit within the effective action. We will only indicate the steps which
are pointing to new and interesting physics. We follow here the suggestion
that the main effect of vacuum response to the Maxwell field is confined to the
action of (massless?) Goldstone bosons (pions). Then:

$$\Gamma_{\text{QCD}} \equiv \frac{1}{16\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-m^2 \pi s} \left[ \frac{eBs}{\sinh eBs} - 1 \right], \quad (27)$$

with the Goldstone mass parameter $m_{\pi}$ comprising all dependences on vacuum structure. Therefore

$$\Sigma(B) - \Sigma(B = 0) = -\frac{\partial \Gamma_{\text{QCD}}}{\partial m_q} = -\frac{\partial \Gamma_{\text{QCD}}}{\partial m^2_{\pi}} \frac{\partial \Gamma_{\text{QCD}}}{\partial m_q} \frac{\partial \Sigma(B = 0, m_q = 0)}{f^2_{\pi}}, \quad (28)$$

where we used the GOR relation (10) in the last equality. Moreover using residuum expansion one easily shows that:

$$\lim_{m_q \to 0} \frac{\partial \Gamma_{\text{QCD}}}{\partial m^2_{\pi}} = \frac{1}{16\pi^2} \int_0^\infty \frac{ds}{s^2} \left[ \frac{eBs}{\sinh eBs} - 1 \right] = -eB \ln 2 \frac{1}{16\pi^2}, \quad (29)$$

which leads to the relation:

$$\lim_{m_q \to 0} [\Sigma(B) - \Sigma(B = 0)] = -eB \ln 2 \frac{\Sigma(B = 0, m_q = 0)}{16\pi^2 f^2_{\pi}(m_q = 0)}. \quad (30)$$

This result suggests that the mass-scale for the QCD vacuum effect is provided by $4\pi f_{\pi}(m_q = 0)$. More importantly, it also implies a very different behavior of the magnetic field dependence of the vacuum action — even though relation Eq. (30) is, strictly speaking, valid in the Goldstone limit only, which here means that the dominant infrared scale is the magnetic field and not the mass. An estimate of the two loop corrections [13] shows that these contribute $\propto \Sigma_0(eB)^2/(2\pi f_{\pi})^4$ to the quark condensate. One can anticipate a new n-loop series in powers of $eB/(2\pi f_{\pi})^2$. Clearly the full action is in the limit $m_q \to 0$ completely different from the full EH-from, and one can only wonder about resummation approaches based on ‘constant field’ approximations.

3 Discussion

We have shown that we cannot, at present, predict the magnitude of the QCD-vacuum deformation by an applied Maxwell magnetic field. We have pursued several approaches and have seen that the results differ greatly. A few interesting insights are worth remembering. The perturbative analysis suggests that the up-quark fluctuations in the structured QCD vacuum dominate, and dominate the light-light scattering over the pure QED Euler-Heisenberg effect. We have also seen, assuming that the glue condensate field is constant
on scale of the quark Compton wavelength, that full re-summation of all possible diagrams including n-loop series is required for the proper evaluation of the effective action. We also were able to confirm that non-perturbative but ‘constant’ condensate assumption implies that the QED-QCD interference effect is too small to be observed, see section 2.4. The assumptions entering the derivation of this result are, however, gravely inconsistent with e.g. latest QCD-lattice simulations. These show that glue condensate fluctuations are occurring on a scale well below hadronic size.

This means that the variation of the condensate is an essential and yet unaccounted factor in non-perturbative evaluation of the QED-QCD effective action. We also recall that the QCD-glue condensate field is nearly 100 times stronger than the so-called critical field strength at which the perturbative expansion parameter in Eq. (19) is unity, e.g. $gE/m^2, gB/m^2$, but that it is believed to be a stochastic, fluctuating vacuum field. Given these observations, we firmly believe that in fact the physics issue we have addressed remains unresolved and, moreover, one may completely ignore the popular assumption that the glue condensate provides the scale in the QCD based light-light scattering effect, and thus renders it unobservable.

In fact, the first term in lowest order in the glue condensate field, as here presented, is perhaps the only term that will remain unchanged in the full treatment of the QCD fluctuations: the perturbative expansion does not suffer from averaging introduced by the realistic fluctuation properties of the gauge field condensate, this term is already result of quantum state averaging. Indeed, one could argue that the fast short-wavelength fluctuations average out the contributions of the higher order terms to the action, and thus, in such a case, the perturbative result we presented could even turn out to experience only minor corrections. However, extensive work on stochastic vacuum models suggest just the contrary, namely that there are reasons to believe

$$\langle G^{2n} \rangle = (?)\langle \langle G^2 \rangle \rangle^n,$$  \hspace{1cm} (31)

which would mean that all higher order terms are significant. This conjecture is confirmed by recent LGT results.

In summary, the theory of the QED-QCD vacuum structure interference is not settled at this time. The perturbative result is most encouraging as point of departure for more theoretical studies, which may be done using lattice gauge theory, or stochastic field methods with QED drift fields. A fully satisfactory treatment of this effect requires a totally novel description in which in our opinion an ultra-strong, but stochastic non-Abelian gauge field with short correlation length which is driving the effective quark action is superposed with a weak Maxwell ‘drift’ field. We hope to revisit this problem in the near future.
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