ON THE COUPLING BETWEEN HELIUM SETTLING AND ROTATION-INDUCED MIXING IN STELLAR RADIATIVE ZONES. II. NUMERICAL APPROACH

SYLVIE THÉADO AND SYLVIE VAUCLAIR
Laboratoire d'Astrophysique, Observatoire Midi-Pyrénées, 14 avenue Edouard Belin, 31400 Toulouse, France
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ABSTRACT

In the first paper of this series, we discussed analytically, in an approximate way, the mixing processes that occur in slowly rotating stars when the feedback effects due to the diffusion-induced μ-gradients are introduced in the computations. We found that the classical scheme of meridional circulation was dramatically modified, with important possible consequences. Here we present a complete two-dimensional numerical simulation of these processes in a static stellar model. The helium microscopic diffusion and the meridional advection are computed simultaneously, so that we can follow the evolution of the abundances and mixing processes in the radiative zone. Most of the effects discussed in Paper I are confirmed. Some consequences will be discussed in Paper III.

Subject headings: diffusion — hydrodynamics — methods: numerical — stars: abundances — stars: interiors — stars: rotation

1. INTRODUCTION

This paper is the second of a series concerning the importance of diffusion-induced μ-gradients in the computations of rotation-induced mixing in stellar radiative zones. In the first paper of this series (Vauclair & Théado 2003, hereafter Paper I), we discussed in detail the orders of magnitude of the various terms that appear in the computation of the meridional circulation velocity. We showed that the term related to the transport of angular momentum may be important in some cases, particularly at the beginning of the main sequence for F- and G-type stars. However, during most of their lifetime, their hydrodynamic history is dominated by the competition between the classical terms (the Eddington-Sweet meridional circulation) and the terms related to the feedback effect induced by μ-gradients.

The situation that appears when these terms become of the same order of magnitude and nearly cancel out is difficult to handle numerically. In Paper I we gave an approximate analytical approach to better understand, in a physical way, the hydrodynamic processes that may happen when this situation takes place. We found that a quasi-stationary stage can settle down, in which case the circulation pattern is strongly modified. The vertical and horizontal μ-gradients adjust themselves to remain nearly constant in the concerned zone. Such a situation must be taken into account in the computations of abundance variations in stellar outer layers.

In the past, various authors performed two-dimensional numerical simulations of meridional circulation (Tassoul & Tassoul 1982; Mestel & Moss 1986; Charbonneau 1992). In the computations of Tassoul & Tassoul (1982) and Mestel & Moss (1986), the importance of the feedback effect due to nuclear-induced μ-gradients was recognized. In particular, Mestel & Moss (1986) showed that the μ-currents could slowly stabilize the circulation near the core. They called this process "creeping paralysis." However, in these computations the feedback effect due to the diffusion-induced μ-gradients was not included, and the horizontal and vertical composition variations induced by the diffusion/circulation coupling were not accurately studied.

In this paper, we present the results of a two-dimensional numerical simulation of meridional circulation in the presence of helium settling, including the feedback effect due to μ-gradients. In order to calculate and visualize in a realistic way the physical processes that take place below the convective zone, we have computed simultaneously the helium abundance variations and the meridional circulation velocity. No free parameter is introduced in these computations, although some simplifying assumptions are made: the rotation velocity is assumed constant during the time of the simulation, and the modifications of the stellar structure induced by diffusion and evolution are neglected (static model). In the simulation, the meridional circulation is treated as an advection process. However, as is discussed in § 2.2, the discretization leads to some numerical diffusion, which we evaluate and find similar to an anisotropic mixing, larger horizontally than vertically, in a realistic way.

In § 2 we give a complete description of the numerical method. The simulation is first tested on the well-known classical circulation (§ 3), and the results obtained including the effects of μ-gradients are given in § 4. A general discussion of these results is proposed in § 5.

2. NUMERICAL ANALYSIS

We describe below the details of our computational method. Throughout this paper, we use the spherical coordinate system, but we assume axial symmetry so that the studied processes are reduced to a two-dimensional simulation.

2.1. Discretization

As an initial model for our simulation, we use a standard homogeneous stellar model (here we choose a 0.75 M☉ halo star, which has no importance for the general process we intend to describe). To obtain the two-dimensional mesh point of the simulation, each shell of the one-dimensional initial model is first discretized in the latitudinal direction into angular sectors of 1° (Fig. 1).
2.2. Computational Method

This simulation aims to understand the coupling between microscopic diffusion and meridional circulation. These two physical processes have quite different natures and behaviors. Microscopic diffusion is a particle process that leads to a selective transport of chemical elements. On the other hand, meridional circulation is a hydrodynamic process that leads to a global transport of matter. We have included in the code two different routines that compute each of the two transport processes in an adequate way:

1. A particle routine that first computes the microscopic diffusion of helium with an Eulerian scheme.
2. A hydrodynamic routine that includes a Lagrangian description of the transport process to compute the global advection by meridional circulation.

At each time step, we first compute the diffusion-induced composition variations by using the particle routine. Then, taking into account the new chemical composition, we determine the circulation-induced abundance variations by computing the advection of matter with the hydrodynamic routine.

The time steps always satisfy the flow condition. Using smaller time steps does not alter the results. We have also checked that reversing the order of the diffusion and circulation computations does not alter the results in a significant way.

2.2.1. The Treatment of Microscopic Diffusion

The diffusion-induced composition variations are computed by solving the helium mass conservation equation

$$\frac{\partial \rho Y}{\partial t} + \text{div}(\rho V Y) = 0,$$

where \(Y\) represents the helium mass fraction and \(V\) the helium diffusion velocity. In the region below the convective zone, the helium production rate by nuclear reactions is negligible.

The diffusion velocity is computed in the test-atom approximation with respect to hydrogen:

$$V = D_\text{He}\left\{ -\nabla \ln c_\text{He} + k_p \nabla \ln p + k_T \nabla \ln T + \frac{m_F}{kT} \right\};$$

\(D_\text{He}\) is the helium diffusion coefficient in a hydrogen gas, which we compute with the Paquette et al. (1986) formalism, and the four terms in the braces represent respectively the concentration gradient, the gravitational settling, and the thermal and radiative diffusions.

The test-atom approximation is rather crude for helium, whose abundance is about 10% that of hydrogen. However, for the purpose of the present paper, treating helium diffusion with a more precise approximation would have led to useless heavier computations in the two-dimensional simulation, with a difference in the results of 10%-15% (Montmerle & Michaud 1976).

In solar-type stars, the radiative acceleration is negligible. In the vertical direction, the diffusion velocity is then due to the concentration gradient, the gravitational settling, and the thermal diffusion. In the horizontal direction the velocity depends only on the concentration gradient.

The horizontal and the vertical components of \(\nabla \ln c_\text{He}\) are respectively equal to \((1/r)(1/c)(\partial c/\partial \theta)\) and \((1/c)(\partial c/\partial r)\). As a first approximation, the helium concentration is a linear function of the inverse of the molecular weight, so \(\partial c/c\) can be approximated by \(|\partial \mu|/\mu\).

In the horizontal direction, we see in § 4 that in the considered stars \(d\mu/\mu\) is typically of the order of \(10^{-6}\). With a radius of about \(5 \times 10^{10}\) cm, this leads to the following approximation:

$$\frac{1}{r} \frac{\partial \mu}{\partial \theta} \simeq 2 \times 10^{-17} \text{ cm}^{-1}.$$

This leads to a horizontal diffusion velocity of nearly \(8 \times 10^{-17}\) cm s\(^{-1}\), while the horizontal meridional velocity is typically of the order of \(10^{-9}\) to \(10^{-8}\) cm s\(^{-1}\); the horizontal diffusion can therefore be neglected in the computations.

In the vertical direction, \(\left|\frac{1}{\mu}(\partial \mu/\partial r)\right|\) can be approximated by \(\left|\frac{1}{\mu}(\partial \mu/\partial \theta)\right|\). We see in § 3.1 that just below the convective zone, where the \(\mu\)-gradients are the steepest,

$$\frac{1}{\mu} \frac{\partial \mu}{\partial r} \simeq 9 \times 10^{-12} \text{ cm}^{-1}.$$

It is interesting to compare the concentration gradient contribution with the other terms, in particular the gravitational settling term. In the case of a totally ionized gas, this term can be written

$$k_p \nabla \ln p = \frac{5 m_p G M_*}{2 kT r^2}.$$
In a 0.75 $M_\odot$ star (i.e., a radius of nearly $5 \times 10^{10}$ cm), this term is approximately $1.7 \times 10^{-9}$ cm s$^{-1}$. Typically, the vertical concentration gradient term is at least 2 orders of magnitude smaller than the other terms.

Two boundary conditions are needed. The upper one is obtained by computing the dilution in the convective zone:

$$\int_{e_z} \frac{\partial \rho X}{\partial t} \, dv + \int_{e_z} \rho X V \cdot dS = 0 \ ,$$

which can be written in the quasi-stationary regime

$$M_i \frac{\partial X}{\partial t} = 2 \pi \int_{0}^{\pi} \rho X V, r^2 \sin \theta \, d\theta \ ,$$

where $M_i$ is the stellar convective mass.

The nuclear reaction rates are neglected, and we assume that the diffusion timescale is much greater than the stellar lifetime; in these conditions, the lower boundary condition is chosen so that $Y = \text{constant below } r \approx 0.4 \ R_\odot$.

### 2.2.2. The Treatment of Meridional Circulation

The vertical component of the meridional circulation is traditionally expanded in spherical functions. When the rotation rate depends only on depth, the radial velocity involves only the second Legendre polynomial:

$$u_r = U_r P_2(\cos \theta) \ .$$

The velocity amplitude, $U_r$, is derived from the heat transfer equation. It can be written (Paper I)

$$U_r = \frac{P}{\rho g T C_p (\nabla \mu - \nabla + \nabla \mu)} \frac{L}{M_i} (E_{\Omega} + E_\mu + E_\zeta + E_b) \ ;$$

$$\nabla \mu \text{ and } \nabla \text{ represent the usual adiabatic and real ratios } [(d \ln T)/(d \ln P)], \text{ and } \nabla \mu \text{ the mean molecular weight contribution } [(d \ln \mu)/(d \ln P)]. \text{ The four terms in the parentheses are as follows:}

1. The classical Eddington-Sweet term $E_{\Omega}$:

$$E_{\Omega} = 2 \left( 1 - \Omega^2 \frac{2 \pi \mu}{G \rho} \right) \frac{\dot{g}}{g} - \left. \frac{\rho m}{\rho} \frac{d}{dr} \left( H_T \frac{d \zeta}{dr} - \chi_T \zeta \right) - \frac{2 H_T}{r} \zeta + 2 \frac{3}{3} \zeta \right] - \Omega^2 \frac{2 \pi \mu}{G \rho} \zeta \ .$$

Note that the last term in this equation was not written in the previous papers; it was neglected by mistake in Maeder & Zahn (1998). While it is indeed negligible in the deep stellar regions where the so-called Gratton-O"{p}ik term is less than unity, it can become important in the outer layers.

2. The $\mu$-gradient term $E_\mu$:

$$E_\mu = \frac{\rho m}{\rho} \left. \frac{d}{dr} \left( H_T \frac{d \Lambda}{dr} - (\chi_\mu + \chi_T + 1) \Lambda \right) - 2 \frac{H_T}{r} \Lambda \right] \ .$$

3. The term related to the differential rotation, $E_\zeta$:

$$E_\zeta = \frac{M_i}{L} \frac{\partial \kappa}{\partial t} \ .$$

4. The term related to horizontal turbulence, $E_b$:

$$E_b = \frac{\rho m}{\rho} \frac{2 H_T D_b}{r} \zeta \ .$$

In these equations, $\zeta$ represents the density fluctuations along a level surface $\rho/\tilde{\rho}$; $\Lambda$ refers to the horizontal $\mu$-fluctuations $\mu/\tilde{\mu}$; $\rho_m$ is the mean density inside the sphere of radius $r$, while $\tilde{\rho}$ represents the density average on the level surface (as well as $T$ for the temperature and $\Omega$ for the angular rotation velocity); $C_p$ is the specific heat; $H_T$ is the temperature scale height; and $D_b$ is the horizontal turbulent diffusion coefficient. The terms $\chi_\mu$ and $\chi_T$ are given by

$$\chi_\mu = \left( \frac{\partial \ln \mu}{\partial \ln \mu} \right)_{P,T} \ , \ \chi_T = \left( \frac{\partial \ln T}{\partial \ln T} \right)_{P,T} \ .$$

In Paper I, we showed that the term $E_b$, due to horizontal turbulence is always negligible compared to the other terms. We also showed that the term $E_\zeta$, which is related to the transport of angular momentum, may be important at the beginning of the stellar lifetime. We neglect it here, however, since our purpose is to study the behavior of the circulation when the two other terms $E_\Omega$ and $E_\mu$ are preponderant.

The horizontal velocity component of the meridional circulation is then deduced from the equation of matter conservation

$$\text{div}(u_r \rho r^2) = -u_\theta \rho r \ ,$$

which gives

$$u_\theta = -\frac{1}{2 \rho r} \frac{d}{dr} (\rho r^2 U_r) \sin \theta \cos \theta \ .$$

### 2.2.3. The Numerical Method

We use the finite difference technique to discretize the equations. The time step of the simulation is computed in order to satisfy the flow condition. At each time step, the microscopic diffusion is first computed by determining the helium diffusion velocity in each shell and solving the helium mass conservation equation for each cell of the mesh point. Once the diffusion-induced abundance variations are computed in the whole star, the laminar meridional circulation is then treated as an advection process. The velocity components are first determined at the center of each cell. Then, in order to treat in a more accurate way the advection by circulation and to reduce the numerical diffusion, a secondary mesh point is introduced in the computations. Each cell of the main mesh point (described in Fig. 1) is discretized into $10^4$ smaller cells. All the secondary cells belonging to the same main cell are supposed to be advected with the same velocity: the one computed at the center of the main cell. This assumption introduced an error lower than 5% on the velocity components. The matter contained in each cell of the secondary mesh point is supposed to be concentrated at its center. The motion of each secondary cell is computed, and the contained matter is distributed into the main cell in an adequate way (see Fig. 2). The main cells are then homogenized to obtain the new chemical composition in the star.

### 2.3. Numerical Diffusion

In spite of the secondary mesh point introduced in the computations of meridional circulation, the numerical
scheme suffers some numerical diffusion. This is basically due to the fact that, at each time step, matter is homogenized inside each cell of the main mesh point. The numerical diffusion cannot be totally suppressed; however, it can be reduced by increasing the number of cells. In practice, increasing the number of cells leads rapidly to extremely large computation times that are unacceptable. The $800 \times 90$ chosen mesh point leads to both reasonable computation times and reasonable numerical diffusion.

It is possible to evaluate this diffusion by defining an anisotropic numerical diffusion coefficient, which we call $D_{\text{num}}^h$ in the horizontal direction and $D_{\text{num}}^v$ in the vertical direction. In both cases, it is defined as:

$$D_{\text{num}} = \frac{dr^2}{dt},$$

where $dr$ is the length scale of the cell and $dt$ the time step of the computation.

In the horizontal direction $dr \approx r/100$, while in the vertical one $dr \approx r/1000$. The time steps of the simulation lie between $10^6$ and $10^7$ yr. With a radius of about $5 \times 10^{10}$ cm, we find the orders of magnitude: $D_{\text{num}}^h \approx 10^3$–$10^4$ cm$^2$ s$^{-1}$ and $D_{\text{num}}^v \approx 10$–$100$ cm$^2$ s$^{-1}$.

It is interesting to compare the value of the numerical horizontal diffusion coefficient with the prescription given by Chaboyer & Zahn (1992) and Maeder & Zahn (1998):

$$D_h = C_h U_r.$$

With $U_r$ of the order of $10^{-7}$ cm s$^{-1}$, we find $C_h \approx 0.2$–$2$, which is quite reasonable. Meanwhile, the numerical vertical diffusion coefficient is 100 times smaller.

We thus find that our numerical scheme leads to some small artificial mixing, which simulates in a realistic way the kind of turbulence that may occur physically in such a situation.

3. TEST OF THE NUMERICAL SCHEME

To test the numerical scheme, we first check that the simulation reproduces correctly the processes we already know, such as microscopic diffusion alone and classical meridional circulation, without the $\mu$-terms. Then we introduce the feedback effect due to the $\mu$-gradients to try to understand more precisely what happens in those circumstances.

3.1. The Classical Meridional Circulation

We first introduce in the simulation the diffusion of helium and the advection by the $\Omega$-currents only; the effects of the $\mu$-gradients are not taken into account, which means that the $E_{\mu}$-term is not introduced in the computations of the meridional velocity.

Here we present the results obtained for a constant rotation velocity of $5$ km s$^{-1}$. The circulation streamlines are represented in Figure 3. As expected, the circulation leads to ascending flows near the rotation axis and to descending flows near the equator. The flow brings $\mu$-enriched matter up in the polar axis and $\mu$-depleted matter down in the equatorial regions. Figure 4 displays the molecular weight profiles at different steps of the simulation. For comparison it also shows the results obtained when helium diffusion is the only transport process introduced in the computations (i.e., without meridional circulation). Microscopic diffusion alone leads to an important vertical $\mu$-gradient below the convective zone. The spread of the molecular weight values obtained in the presence of meridional circulation shows how $\Omega$-currents turn the diffusion-induced vertical $\mu$-gradients into horizontal ones.

3.2. Importance of $\mu$-Currents

In the previous section we have presented the molecular weight variations under the combined effects of helium diffusion and $\Omega$-currents only. We have shown that diffusion builds vertical $\mu$-gradients that are turned into horizontal ones by $\Omega$-currents. Although the $\mu$-currents are not taken into account in the computations of meridional advection, we have computed at each time step the value of the $E_{\mu}$-term.
by determining the horizontal $\mu$-gradients built in the star and using equation (8). We insist on the fact that these $\mu$-gradients are those built by the classical meridional circulation, i.e., when the feedback effect of the $\mu$-currents is not taken into account.

In these conditions, Figure 5 (left) displays the molecular weight versus the colatitude at different depths below the convective zone and at different steps of the computations. This shows the slow construction of the horizontal $\mu$-currents. Figure 5 (right) compares the $E_{\Omega}$- and $E_{\mu}$-terms inside the stars. It shows that the horizontal $\mu$-gradients induced by the $\Omega$-currents lead rapidly to important values of $E_{\mu}$. After 0.5 Gyr, $E_{\mu}$ is already greater than $E_{\Omega}$. The described situation is then unrealistic: if such horizontal $\mu$-gradients were effectively built in the star, the $\mu$-currents should have changed the sense of the circulation. This clearly shows that the feedback effect of the $\mu$-currents cannot be neglected in the computations.

4. MERIDIONAL CIRCULATION INCLUDING THE $\mu$-CURRENTS

4.1. The Creeping Paralysis

We now present the results obtained by introducing in the simulation the diffusion of helium and the two currents of meridional circulation ($\Omega$- and $\mu$-currents). The advection by circulation is then computed by including in the computations of the meridional velocity the classical term $E_{\Omega}$ but also the $E_{\mu}$-term related to the molecular weight gradients.

Figure 6 displays the $E_{\Omega}$- and $E_{\mu}$-profiles inside the stars at different steps of the simulation. The $\mu$-currents become rapidly of the same order of magnitude as the $\Omega$-currents below the convective zone. As soon as the two currents compensate each other, the meridional circulation seems to freeze out in the concerned regions. The “frozen region” first appears below the convective zone and deepens slowly inside the radiative zone. This is a creeping paralysis process similar to that discussed by Mestel & Moss (1986) for nuclear-induced $\mu$-gradients.

Fig. 3.—Meridional circulation streamlines. Here only the $\Omega$-currents are taken into account in the meridional velocity computations. The rotation velocity is chosen to be constant during the whole simulation, and no structural evolutions are introduced in the computations; therefore, the $\Omega$-current streamlines remain the same at each time step.

Fig. 4.—Molecular weight vs. radius in each cell below the convective zone. Left: Results obtained when helium microscopic diffusion is the only transport process taken into account. Under the effects of microscopic diffusion, vertical $\mu$-gradients build below the convective zone. The horizontal microscopic diffusion is negligible, so all the cells of the same shell have the same chemical composition and the same $\mu$-value. Right: Molecular weight in each cell of the two-dimensional mesh point when the diffusion of helium and the classical meridional circulation (i.e., the $\Omega$-currents) are introduced in the simulation. Each cell of the mesh point is represented with a black square. The $\Omega$-currents bring $\mu$-enriched matter up to the rotation axis and down to the equator. This turns the vertical diffusion-induced $\mu$-gradients into horizontal ones, which explains the spread in the value of $\mu$ at a fixed radius.
Fig. 5.—Computations including the helium microscopic diffusion and the classical meridional circulation. Both the $E_l$- and the $E_\mu$-terms are computed, but only the $E_l$-term is taken into account in the meridional circulation velocity computations (eq. [6]). Left: Molecular weight vs. colatitude ($\theta$ in radians) at different depths (between 0.704$R_\odot$ and 0.698$R_\odot$) below the convective zone. Right: The $E_l$- and $E_\mu$-profiles inside the star. The $E_\mu$-term, which is not introduced in the advection computations, is evaluated by computing the horizontal $\mu$-gradients present in the star and using eq. (8). After 0.5 Gyr, the $E_\mu$-term is greater than the $E_l$-term; therefore, it cannot be neglected in the meridional velocity computations.
concerned region. The paralyzed zone then deepens slowly inside the star. Below the convective zone, which freezes out the circulation in the con-

\[ E \] \[ \text{horizontal} \]

\[ \text{state is reached. The}\]

\[ \text{meridional circulation, the}\]

\[ \text{currents:}\]

\[ \text{imbalance, well above the numerical fluctuations, before}\]

\[ \text{stop the meridional circulation and let helium diffusion pro-}\]

\[ \text{what happens when the balance between} \]

\[ \text{is clearly broken below the convective zone. Afterward, we}\]

\[ \text{introduce the circulation again in order to study its behavior.}\]

\[ \text{4.2. Computations}\]

\[ \text{As an initial situation, we use the results obtained with}\]

\[ \text{the complete simulation after 70 Myr (including diffusion}\]

\[ \text{and circulation,} \]

\[ \text{and} \]

\[ \text{currents:}\]

\[ \text{compares them to the case of classical meridional circula-}\]

\[ \text{and circulation,}\]

\[ \text{the horizontal} \]

\[ \text{is equal to the critical value,}\]

\[ \lambda_{\text{crit}}, \] \[ \text{for which}\]

\[ |E_\mu| \text{is equal to}\]

\[ |E_\Omega|,\] \[ \text{the meridional circulation vanishes. However, helium}\]

\[ \text{diffuses out of the convection zone where it is completely}\]

\[ \text{homogeneous (i.e.,} \]

\[ \Lambda = 0).\] \[ \text{Because of this diffusion, the}\]

\[ \text{horizontal} \]

\[ \mu\text{-gradient decreases below the convective zone, and}\]

\[ \text{the circulation is triggered again until a new equilibrium}\]

\[ \text{state is reached. The} \]

\[ \text{and} \]

\[ \text{profiles, which always}\]

\[ \text{remain very close in the frozen region, suggest that the}\]

\[ \text{balance-restoring timescale below the convective zone is}\]

\[ \text{much more rapid than the diffusion timescale. This is the}\]

\[ \text{reason why this problem is difficult to treat numerically.}\]

\[ \text{4.2. Self-regulating Process}\]

\[ \text{The aim of this section is to understand more accurately}\]

\[ \text{what happens when the balance between} \]

\[ \text{is broken because of the helium microscopic diffusion below the}\]

\[ \text{convective zone. Then, for numerical reasons, it is necessary}\]

\[ \text{to separate the computations of diffusion and circulation in}\]

\[ \text{the simulation because the balance-restoring timescale is}\]

\[ \text{small compared to the diffusion timescale. In other words,}\]

\[ \text{we need to let diffusion proceed long enough to create a real}\]

\[ \text{imbalance, well above the numerical fluctuations, before}\]

\[ \text{studying the restoring effect of the circulation.}\]

\[ \text{We start this study from a situation where}\]

\[ |E_\Omega| = |E_\mu|\] \[ \text{below the convective zone in a significant region. We then}\]

\[ \text{stop the meridional circulation and let helium diffusion pro-}\]

\[ \text{ceed further until the equilibrium between the two currents}\]
Fig. 7.—Influence of microscopic diffusion on the creeping paralysis below the convective zone. Left: Results obtained with the simulation (including diffusion + $E_T + E_l$) after 70 Myr. Then we stop the circulation and let the diffusion proceed further for 400 Myr. After this time, we stop the diffusion and introduce the circulation again. Right: Meridional circulation variables after the alteration of the equilibrium by diffusion.
the magnitude of the horizontal velocity is significantly enhanced above and below the frozen region.

Figure 9 presents the resulting circulation streamlines in the deep interior and just below the convective zone. The left panel shows the circulation lines in a meridional sector after 70 Myr of complete simulation (diffusion and circulation). The frozen region clearly appears below the convective zone. In the radiative interior, the circulation goes on in the classical direction but is compressed and annihilated at the bottom of the frozen region. In the right panel the region just below the convective zone is zoomed and expanded after 400 Myr of pure microscopic diffusion inside the frozen region. As expected from Figure 8, two circulation loops arise. In the radiative interior, the circulation remains as seen on the left panel, while the second loop, which appears just below the convective zone, is clearly seen. The two loops are separated by a quiet zone with an extension $r/R_{\ast} \approx 3\%$.

4.2.2. Discussion

We have shown that, when the frozen state is altered by helium microscopic diffusion below the convective zone, the circulation starts again in order to compensate the created imbalance. A new circulation loop appears, opposite to the classical one. We find numerically a similar situation as described analytically in Paper I. In this simulation, the two loops are separated by about half a pressure scale height. We must not forget, however, that this is not the real situation that occurs in stars, since in the present case the circulation has been completely stopped for 400 Myr.

The restoring timescale is very short compared to the diffusion one. If we let the circulation start again after 400 Myr of pure helium settling, the region polluted by diffusion just below the convective zone is mixed up in less than 1 Myr because of the new loop. In consequence, we may deduce that as soon as diffusion begins to modify the equilibrium between the $\Omega$- and $\mu$-currents, the original situation is immediately restored. This corresponds to the situation described analytically in Paper I, in which we showed that a stationary stage could take place where the horizontal $\mu$-gradients should remain constant as well as the vertical ones, since they are consistently linked.

In the presented computations, we let the circulation start again after 400 Myr of pure helium settling. Other
computations have been done with different timescales. In any case, the restoring timescale is always very much smaller than the pure diffusion time, which is the main conclusion of this study.

5. CONCLUSION

After Paper I, where we had discussed in an analytical approximate way the importance of diffusion-induced \( \mu \)-gradients on the meridional circulation in slowly rotating stars, we presented here a complete two-dimensional simulation of the considered processes. We used a stellar model obtained from our stellar evolution code, which we discretized in the latitudinal direction. Then we computed simultaneously the helium diffusion below the convective zone and the advection due to meridional circulation. The influence of helium variations on the stellar structure was neglected (static model), but the feedback effect of these variations on the meridional circulation was thoroughly introduced, in a complete way.

We confirmed that, in a somewhat short timescale compared to the main-sequence lifetime, the so-called \( \mu \)-currents become of the same order of magnitude as the \( \Omega \)-currents, thereby creating a “frozen” region where the circulation does not proceed anymore.

However, when this occurs the equilibrium between the two opposite currents is permanently destabilized by the helium settling below the convective zone. Helium falling from the mixed outer regions induces a decrease of the horizontal \( \mu \)-gradients, which remain below the critical values for which the circulation is completely stopped. As a consequence, a new circulation loop develops, which mixes up into the convective zone the region polluted by diffusion. We derive two important consequences from this study:

1. The rotation-induced mixing that occurs in stellar radiative zones may be dramatically modified by \( \mu \)-gradients, with the occurrence of disconnected loops of circulation.
2. A new mixing process appears, directly induced and modulated by the microscopic diffusion.

The results concerning the rotation-induced mixing may have important consequences for the light elements that are easily destroyed by nuclear reactions (Théado & Vauclair 2003 [Paper III]). On the other hand, the influence of these processes on the abundance variations induced by diffusion is still not completely settled. A more sophisticated simulation would be helpful to derive it more precisely. However, the results of the present two-dimensional simulation are very encouraging. The diffusion-induced mixing that was analytically discussed in Paper I is confirmed. It acts as a restoring system for the element abundance variations and increases in a significant way their settling timescales. We suggest, as in Paper I, that diffusion and mixing react in such a way as to keep both the horizontal and vertical \( \mu \)-gradients constant in the frozen region, while they proceed freely below.

![Diagram showing self-regulating process](image-url)
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