A Condition-based maintenance and spare parts provisioning based on markov chains

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Abstract. Machine is a vital tool of the company in helping the production process. Every company expects the production to run smoothly, but sometimes it is hampered by damage that happened to the machine, so that the production process is disrupted and causes losses to the company. Engine damage can be minimized by regularly evaluating the condition of the spare parts. In practice, if the spare parts inventory policy is not accurate it will cause stock outs or overstocks, which can lead to more costs for the company. The worst case is if there is no spare part stored in the warehouse when it is needed, it can make the production floor stopped, which in the end makes the company can’t fulfil their production target. This research aims to obtain an optimal preventive maintenance schedule by calculating the machine’s reliability and inventory provisioning policy for the spare parts according to predicted amount that will be needed in the future calculated using Markov chains so the company can determine the reorder point (r) and the economic order quantity (EOQ).

1. Introduction

Along with the continued development of the world economy and the strict competition in the industrial world, a company must be able to become a superior one in order to survive. A few things that can be done by a company are increasing the smoothness of the products movement in production floor, the effectiveness, and the efficiency of the production process. In order to maintain and repair good machinery, the number of spare parts stored by the company must be optimized. So when spare parts are needed either for preventive or corrective maintenance it is available, not less, and also not excessive. This is important because if there is no spare part stored in the warehouse when it is needed, it can cause a lot of loss for the company. These loss means if there is any machine that is broken, it can’t be fix, so it can’t be running as it should be, the production floor will be stopped, which in the end makes the company can’t fulfil their production target or their customer’s demand. By using inventory control, company can improve service level and reduce the total inventory costs [1]. Also inventory planning needs to be done to reduce the influence of nonconformity between demand and inventory due to fluctuations of inventory amount [2].

The results of this research are the reliability of spare part and predicts amount needed in the future that calculated by using the Markov chains. The reliability result can help the company to arrange preventive maintenance schedules and the predict can be useful to determine the amount of spare parts provisioning that should be stored by the company within a certain period. This paper does not use real case and only provide numerical example.
2. Literature Review

Spare parts are all types of goods or parts used for running a company or factory operations and also for maintaining any equipment that used. This type of inventory often called as maintenance, repair and operation, or MRO materials (maintenance, repair, and operation) [3]. The spare part is usually stocked because it is difficult to obtain the part from the supplier on short notice and it is used in equipment for which prolonged downtime would be very expensive or unsafe [4]. But overstock condition can further cause of loss to the company because it can make the holding cost higher [5].

2.1. Inventory

Inventory is an idle resources whose existence awaits further processing. The advanced process can be in the form of production activities such as those found in manufacturing systems, marketing activities in distribution system, of consumption activities in household systems, offices, etc. [6]. In general, inventory function is to manage any kind of inventory goods so that the company can face the uncertainty of demand [7].

This model is known as economic order quantity (EOQ) model, because it established the most economic size of order to place [8].

\[
Q = \left(\frac{2 \times A \times D}{l \times p}\right)^{1/2} \quad (1)
\]

Reorder point is a method for determining the quantity of inventory that will trigger re-ordering to add units. The formula for calculating ROP is as follows [9].

\[
ROP = 2 \times \frac{D}{\text{working days in one year}} \times L \quad (2)
\]

Safety stock (SS) is the amount of inventory stored in case of unexpected things such as delays in lead time or unexpected requests. The formula for calculating SS is as follows [9].

\[
SS = \frac{D}{\text{working days in one year}} \times L \quad (3)
\]

2.2. Markov Chains

Markov chains is a technique that deals with the probability of future state by analyzing current probabilities [10]. Markov process is a stochastic system which for the emergence of a situation in the future depends on the condition that immediately precedes it and only depends on it.

3. Methodology

3.1. Discrete markov chains

In this research, a study of ergodic Markov chains was carried out, as a requirement to obtain long-term engine requirements and determine the reliability value of a machine using the Markov chains. For example \{X(n), n = 0,1,2, \ldots\} a stochastic process with a discrete T parameter index and S state space meets

\[
P = \{X(n+1) = j | X(0) = i_0, X(1) = i_1, X(2) = i_2, \ldots, X(n-1) = i_{n-1}, X(n) = i\} = P[X(n+1) = j|X(n) = 1] = p_{ij} \quad (4)
\]

For all times \(n \in T\) and conditions \(i_0, i_1, \ldots, i, j \in S\), then the process is called discrete time Markov chains, and \(p_{ij}\) is called transition probability [11]. The one step opportunity transition matrix \(P\) is defined as follows:

\[
P = [p_{ij}] = \begin{bmatrix}
p_{00} & p_{01} & p_{02} & \cdots \\
p_{10} & p_{11} & p_{12} & \cdots \\
p_{20} & p_{21} & p_{22} & \cdots \\
\vdots & \vdots & \vdots & \ddots 
\end{bmatrix} \quad (5)
\]
With \( p_{ij} \geq 0 \) and \( \sum p_{ij} = 1(i, j = 0,1,2, ...); \) [1]. A state \( j \) is said to be achieved from state \( i \), if for \( n \geq 0 \) then \( p_{ij} \geq 0 \), denoted \( i \rightarrow j \), and each condition can be achieved by itself, denoted \( i \rightarrow i \) because \( p_{ii} \geq 0 \). Circumstances \( i \) and \( j \) are said to communicate with each other if \( i \rightarrow j \) and \( j \rightarrow i \), denoted by \( i \leftrightarrow j \), meaning for integers \( m \geq 0 \) and \( n \geq 0 \) then \( p_{ij}^m \geq 0 \) and \( p_{ji}^n \geq 0 \).

- **Definition 1.** For a Markov chains, if there is only one class of communication it is called the irreducible Markov chains. This means that for the Markov irreducible chain, all circumstances communicate with each other [11].
- **Theorem 2.** The state \( i \) said to be recurrent if and only if \( \sum p_{ij}^n = \infty \). The condition \( i \) is said to be transient if and only if \( \sum p_{ij}^n < \infty \) [11].

A state is said to be recurrent if when entering a certain state, the process will definitely return to that state again. Therefore, the state is recurrent if and only if the state is not transient. A state is said to be transient if when entering a certain state, the process will never return to that state again [12].
- **Theorem 3.** If the condition \( i \) is recurrent and \( i \leftrightarrow j \), then the condition \( j \) is recurrent; [11].
- **Definition 4.** Circumstance \( i \) have a period \( d(i) \), if \( d(i) \) is the largest alliance factor (FPB) of \( n \), for all integers \( n \geq 1 \) where \( p_{ii}^n > 0 \).

\[ d(i) = \text{FPB}[n \geq 1|p_{ii}^n > 0] \]  \hspace{1cm} (6)

If \( d(i) = 1 \), state \( i \) is aperiodic, and if \( d(i) > 1 \), state \( i \) is periodic [11].
- **Theorem 5.** If \( i \leftrightarrow j \), then if \( (i) = \text{d}(j) \) [11].
- **Definition 6.** For a Markov chains all recurrent conditions are classified as positive (non-null) recurrent or null recurrent seen from \( (i) = \text{d}(j) \) or \( (i) = \text{d}(j) \) where

\[ \mu_j = \sum_{n=1}^{\infty} n p_{jj}^n \]  \hspace{1cm} (7)

States the average time recurrent for the situation \( j \) [11]. The Markov chains is said to be ergodic if the Markov chains is positive recurrent and aperiodic.
- **Theorem 7.** If the condition \( j \) is recurrent and aperiodic, then \( p_{jj}^n \rightarrow \frac{1}{\mu_j} \), for \( n \rightarrow \infty \) interpreted \( \frac{1}{\mu_j} = 0 \) if \( \mu_j = \infty \); [11]
- **Theorem 8.** If a Markov chains is irreducible and ergodic, then there is a limit of opportunities.

\[ \lim_{n \to \infty} p_{jj}^n = \pi_j, i, j = 0,1,2, \ldots \]  \hspace{1cm} (8)

Which does not depend on the initial state \( i \), where \( \{\pi_j, j = 0,1,2, \ldots \} \) is the stationary distribution of the Markov chains a unique solution and positive from \( \pi_j = \sum_{l=0}^{\infty} \pi_l p_{lj} \), \( j = 0,1,2, \ldots \) with \( \sum_{j=0}^{\infty} \pi_j = 1 \) [11]

### 3.2. Reliability using markov chains

Reliability is the probability that a part or system will perform a function that is needed for a certain period of time when used in operating conditions. The reliability of spare part can decrease according to time [13].

- **Definition 9.** Given a limited state spaces \( S = \{0,1, \ldots, s\} \), partitioned into two parts, \( U = \{0,1, \ldots, r\} \) for working conditions (up state) and \( D = \{r + 1, \ldots, s\} \) for a failed state (down state). Meaning \( S = U \cup D, U \cap D = \emptyset \) and \( U \neq \emptyset, D \neq \emptyset \). Given \( \{X_n\}, n = 0,1, \ldots, m \) Markov chains is ergodic with limited states \( S = \{0,1, \ldots, s\} \), then reliability based on Markov chains \( \{X_n\} \) at time \( n \geq 0 \) is defined

\[ R(n) = P(\forall \nu \in [0,n] \cap \mathbf{X}_n \in U) \]  \hspace{1cm} (9)

Where \( \nu \) is a time index for a certain period; [11]. For \( n = 0 \), the machine is in very good condition and has never experienced damage at all, meaning that the level of reliability or reliability of a machine is very good so that \( R(0) = 1 \). By looking at the state partition \( U \) and \( D \), the transition probability matrix
$P$ and the initial distribution $\pi(0)$ become $P = \begin{bmatrix} P_{UU} & P_{UD} \\ P_{DU} & P_{DD} \end{bmatrix}$ and $\pi(0) = [\pi_u(0), \pi_D(0)]$. Then the reliability for the Markov chain is defined as follows \[11].

\[ R(n) = \pi_u(0)P_n U U 1_r \quad for \quad n \geq 0 \quad (10) \]

3.3. Time dependent probability evaluation

If the initial state of the system is represented by a probability matrix $P(0)$ which states the probability of each state at the beginning of the system mission, then after $n$ the probability interval of the system can be written into an equation.

\[ P(n) = P(0)P^n \quad (11) \]

$P(n)$ = probability matrix which states the probability of each state after $n$ time intervals

$P(0)$ = probability matrix which states the probability of each state at the beginning of the system mission

$P$ = the STP (state transition probability) matrix that represent the system

4. Numerical Example

4.1. Reliability

From the history data of machine A’s total damage time, grouped to define the following conditions.

(i) If $tdt = 0$, the machine is categorized in good condition, given the symbol number 0.

(ii) If $0 < tdt < 4$, the machine is categorized as lightly damaged, given the symbol number 1.

(iii) If $tdt > 4$, the machine is severely damaged and some parts need to be replace, given the symbol number 2.

**Table 1.** Transitions number of machine A’s conditions.

| $i,j$ | 0 (Good) | 1 (Minor Damage) | 2 (Severely Damage) | Total |
|------|---------|-----------------|-------------------|-------|
| 0 (Good) | 102 | 75 | 85 | 262 |
| 1 (Minor Damage) | 95 | 54 | 55 | 204 |
| 2 (Severely Damage) | 0 | 98 | 0 | 98 |
| Total | 197 | 227 | 140 | 564 |

\[ P = \begin{bmatrix} 0.389313 & 0.286260 & 0.324427 \\ 0.465886 & 0.264706 & 0.269608 \\ 0 & 1 & 0 \end{bmatrix} \quad (12) \]

It is assumed that the initial state of the machine is in good condition. Then the state space S is partitioned into two parts, namely the working state U and the failure state D. First thing that need to be calculated is making sure that the transition probability matrix is ergodic. Next is the calculation of the reliability value.

\[ R(n) = [\pi_0(0) \quad \pi_1(0)] \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (13) \]

Reliability value is calculated for $n = 1, 2… 24$.

**Table 2.** Reliability value machine A.

| $n$ | $R(n)$ | $n$ | $R(n)$ | $n$ | $R(n)$ |
|-----|--------|-----|--------|-----|--------|
| 1   | 0.645573 | 9   | 0.028887 | 17  | 0.001290 |
| 2   | 0.438500 | 10  | 0.019586 | 18  | 0.000875 |
| 3   | 0.297299 | 11  | 0.013280 | 19  | 0.000593 |
| 4   | 0.201579 | 12  | 0.009005 | 20  | 0.000402 |
| 5   | 0.136678 | 13  | 0.006105 | 21  | 0.000273 |
| 6   | 0.092672 | 14  | 0.004139 | 22  | 0.000185 |
| 7   | 0.062835 | 15  | 0.002807 | 23  | 0.000125 |
| 8   | 0.042604 | 16  | 0.001903 | 24  | 0.000085 |
Viewed from the calculation, the reliability value of machine A decreases every day. At the 24th day, the reliability of machine A is 0.0085%, which is very low. So it is recommended that company can carry out preventive maintenance or even replacement by controlling the machine’s condition at least once every 24 days.

4.2. Predicting spare parts needs

The first step is to decide which spare parts are critical and need to be prioritized for spare provisioning. For example, it is decided that spare part B are chosen. In the company there are 70 machine A, each machine requires one spare part B, it means that the company need 70 spare part B in good condition to keep the machines running as it should.

(i) If there is no damage to the spare parts then the condition of the spare parts is categorized into good condition, given the symbol number 0, which states the condition 0.

(ii) If there is damage to spare parts then the condition of the spare parts is categorized into bad condition, given the symbol 1, which states condition 1.

| Table 3. Amount of transition from state i to state j each day for a year. |
|---------------------------------------------------------------|
|                      Condition 0 (Good) | Condition 1 (Bad) | Total   |
| Condition 0 (Good)  | 25.833            | 137     | 25.970  |
| Condition 1 (Bad)   | 0                 | 137     | 137     |
| Total               | 25.833            | 274     | 26.107  |

Then table above is made into the transition opportunity matrix, so that it is obtained:

\[ P = \begin{bmatrix} 0.99472 & 0.00528 \\ 0 & 1 \end{bmatrix} \] (14)

Furthermore, the probability value of demand for spare part B is sought in the following year. First, look for the possible value for \( n = 1 \).

\[ P(1) = P(0)P^1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.99472 & 0.00528 \\ 0 & 1 \end{bmatrix}^1 = \begin{bmatrix} 0.99472 & 0.00528 \\ 0 & 1 \end{bmatrix} \] (15)

Based on the Markov chain, look for the possible system values for \( n = 2, 3, 4 \ldots 365 \).

| Table 4. Possible values of spare parts system. |
|-----------------------------------------------|
| Month      | Day | State 1 Probability | State 2 Probability | Estimated Amount of Damage |
|------------|-----|---------------------|---------------------|---------------------------|
| January    | 31  | 0.84865             | 0.15135             | 11                        |
| February   | 59  | 0.73173             | 0.26827             | 19                        |
| March      | 90  | 0.62098             | 0.37902             | 27                        |
| April      | 120 | 0.52979             | 0.47021             | 33                        |
| May        | 151 | 0.44960             | 0.55040             | 39                        |
| June       | 181 | 0.38358             | 0.61642             | 44                        |
| July       | 212 | 0.32525             | 0.67448             | 48                        |
| August     | 243 | 0.27625             | 0.72375             | 51                        |
| September  | 273 | 0.23569             | 0.76431             | 54                        |
| October    | 304 | 0.20001             | 0.79999             | 56                        |
| November   | 334 | 0.17064             | 0.82936             | 59                        |
| December   | 365 | 0.14481             | 0.85519             | 60                        |

The spare part system’s probability value in state 2 (bad or damaged) in the next year is 0.85519. If the probability value is multiplied by the total number of the spare parts in the system totalling 70 units, then there are 60 units damaged in the next year. The company needs to prepare 60 units of spare part B for the next one year.
4.3. Spare parts inventory

It is known that the demand for spare part B is 60 units, assume the spare part B price is $5,000, the ordering fees for each order is $10,000, the annual holding cost is 10.5%, the lead time is 30 days, and the working days in one year is 365 days because the manufacture works 24/7 non-stop with three shifts.

- **Economic Order Quantity (EOQ)**
  
  \[
  Q = \left( \frac{2 \times A \times D}{I \times p} \right)^{\frac{1}{2}} = \left( \frac{2 \times 10,000 \times 60}{10.5\% \times 5,000} \right)^{\frac{1}{2}} = 47.8 \approx 48 \text{ units} \tag{16}
  \]

- **Reorder Point (ROP)**
  
  \[
  ROP = 2 \times \frac{D}{\text{working days in one year}} \times L = 2 \times \frac{60}{365} \times 30 = 9.9 \approx 10 \text{ units} \tag{17}
  \]

- **Safety Stock (SS)**
  
  \[
  SS = \frac{D}{\text{working days in one year}} \times L = \frac{60}{365} \times 30 = 4.9 \approx 5 \text{ units} \tag{18}
  \]

5. Conclusion

A new condition-based maintenance and spare parts provisioning has been proposed based on Markov chain. Whether it’s a preventive or corrective maintenance will be decided by the condition of the machine that were calculated using its reliability value. The procedure of the Markov chain, inventory and cost calculation has been described in this paper. The presented numerical example shows exactly how to do the calculation. The authors believe that the condition-based maintenance and spare parts provisioning may be beneficial from company to estimating the future. Considering the importance of the availability of spares for maintenance decisions. Further work is required to develop a better method.

6. References

[1] Fauziah S, Ridwan, A Y and Santosa B 2016 *Jurnal Rekayasa Sistem & Industri* **3** 4 66-71
[2] Wahyuni N K C, Ridwan A Y and Santosa B 2018 *J. Ind. Serv.* **3** 2 115-151
[3] Indrajit, Eko R and Djokopranoto R 2003 *Manajemen Persediaan* (Jakarta: PT Grasindo)
[4] Kennedy W J, Patterson J W and Fredendall L D 2001 *Int. J. Prod. Ec.* **76** 201-15
[5] Permatasari P M, Ridwan A Y and Santosa B 2017 *MATEC Web of Conf.* **135** 00056
[6] Syamil R A, Ridwan A Y and Santosa B 2018 *Jurnal Integrasi Sistem Industri* **5** 1 49-55
[7] Nurrhma D A, Ridwan A Y and Santosa B 2016 *Jurnal Rekayasa Sistem & Industri* **3** 2 47-51
[8] Kumar R 2016 *Glob. J. of F. and E. M.* **5** 1 1-5
[9] Bahagia S N 2006 *Sistem Inventori* (Bandung: Penerbit ITB)
[10] Render B and Heizer J 2006 *Manajemen Operasi, Edisi Ketujuh* (Jakarta: Salemba Empat)
[11] Andriani M N, Firdaniza and Irianingsih I 2017 *Jurnal Matematika Integratif* **13** 1 41-7
[12] Langi Y A R 2011 *Jurnal Ilmiah Sains* **1** 1 124-30
[13] Ebeling C E 1997 *An Introduction to Reliability and Maintainability* (Singapore: McGraw Hill Book Co.)