Higher-shell corrections for systems with one and two valence nucleons: A spin-orbit and tensor interaction analysis

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Abstract

It is shown that when higher-shell admixtures are included for systems with two valence particles or holes, there are effects which are quite different from those for one-valence-nucleon systems. For example, for nuclei with one valence particle or hole, there is no first-order correction for the magnetic dipole moment or the Gamow-Teller transition amplitude. However for nuclei with two valence particles or holes, one can get substantial corrections. The effects of the tensor and spin-orbit interactions in core renormalization are emphasized. We find that in $^6\text{Li}$, the spin-orbit interaction causes the quadrupole moment of the $J^\pi=1^+$ ground state to be positive, but the tensor interaction causes it to be negative. The G-matrices derived from realistic interactions are employed in these calculations.
1 Introduction

There has been considerable study of core renormalization for a system of a closed \(LS\) shell plus or minus one nucleon. The most studied problem is probably the electric quadrupole (E2) effective charge. When doing calculations of E2 properties, i.e., quadrupole moments or \(B(E2)\)'s in a single major shell, it is necessary to use effective charges for the valence protons and neutrons. The quadrupole moment for the ground state of \(^{17}\text{O}\) is about \(-3e\text{ fm}^2\), suggesting that the effective charge for the valence \(d_{5/2}\)-neutron is about \(-3/(-6)=0.5\), where the quadrupole moment for a \(d_{5/2}\)-proton is taken to be \(-6e\text{ fm}^2\). The same analysis would lead to an effective charge of 1.2 for a valence proton, although more careful considerations may lead to somewhat different values. The use of effective charges can be partly justified in first-order perturbation theory by the core-polarization process shown in Fig.1(a). Larger effective charges can be obtained in R.P.A. calculations \([1]\). A typical RPA diagram is shown in Fig.1(b). Although there have been calculations with certain interactions which show quantitative agreement with empirical values, there have been complete second-order perturbation calculations by Siegel and Zamick \([1]\) and Ellis and Siegel \([2]\) which yielded very small polarization charges. Evidently the non-RPA diagrams cancel with the RPA diagrams. More recently, Skouras and M"uther \([3]\) also obtained small polarization charges in higher-order calculations with G-matrices evaluated from Bonn potentials \([4]\).

At the extreme it is known that for a closed \(LS\) shell plus or minus one particle, the first-order perturbation-theory corrections for the magnetic dipole (M1) operator and to the Gamow-Teller (GT) operator \[\sqrt{\frac{3}{4\pi}}(g_l\vec{l}+g_s\vec{s})\] and \(\sigma t_\pm\) respectively] vanish. One has to go to higher-order perturbation theory and invoke exchange currents in order to explain deviations from the lowest order.

In this work, we will study higher-shell corrections to properties of sys-
tems involving one and two valence nucleons or holes. Although the first-order perturbation-theory correction vanishes for systems with one valence particle or hole, we may get some effects for systems with two valence particles or holes. For example, the effective interaction between the two valence nucleons or holes can be renormalized (see, for example, Fig. 2) and hence the two-body wave function can change.

Rather than perform perturbative calculations, we will perform matrix diagonalizations using the OXBASH shell-model (SM) code [5]. The model space will consist of one or two valence particles or holes, plus excitations of the core through “$2\hbar\omega$”. By this we mean that either two core-nucleons will be excited through one major shell or one core-nucleon will be excited through two major shells.

There have been previous SM calculations with “$2\hbar\omega$” corrections. For example, van Hees et al [6] and Wolters et al [7] have extended the classic “$0\hbar\omega$” calculations of Cohen and Kurath [8] by including higher shells. At the time of this writing, another paper has appeared [9]. Whereas the above works have used a phenomenological interaction and focused on getting a global fit to experiment — certainly a worthwhile quest — we are more concerned with seeing how well the G matrices, derived from modern realistic nucleon-nucleon (NN) potentials, are able to account for the measured properties. Furthermore, we want to see how the various parts of the NN interaction, especially the tensor and spin-orbit components, affect the physical observables.

The calculations are performed with Brueckner G-matrices [10] calculated from a new Nijmegen potential (NijmII) [11]. Comparisons with other potentials, including the Hamada-Johnson hard-core [12], the original Reid-soft-core [13] and a new Reid-soft-core (Reid93) [14] potential, will be made on a selected basis. It should be emphasized that our calculations will not assume an inert core. The single-particle energies are implicitly generated in OXBASH [3] from the two-body G-matrix elements as well as the one-body
kinetic energy. More explicitly, the matrix diagonalization will be performed for the SM Hamiltonian

\[ H_{SM} = \left( \sum_{i=1}^{A} t_i - T_{CM} \right) + \sum_{i<j}^{A} G_{ij} + \lambda (H_{CM} - \frac{3}{2} \hbar \omega), \] (1)

where \( t \) and \( T_{CM} \) are the one-body and center-of-mass (CM) kinetic energies respectively. The G matrix is calculated according to

\[ G(E_s) = v_{12} + v_{12} \frac{Q}{E_s - (h_1 + h_2 + v_{12})}, \] (2)

with \( E_s \) the starting energy and \( h_i = t_i + u_i = \frac{p_i^2}{2m} + \frac{1}{2} m \omega^2 r_i^2 \) the one-body harmonic-oscillator Hamiltonian. We choose \( \hbar \omega = 16 \text{MeV} \) for \( A=5 \) and \( 6 \) and \( \hbar \omega = 14 \text{MeV} \) for \( A=14, 15, 17 \) and \( 18 \). The Pauli operator \( Q \) in Eq.(2) is defined to exclude the scattering of the two particles in the ladder diagrams into the occupied states as well as the states that will be included in the model space. For example, in the ladder diagrams, the scattering into an intermediate state with two nucleons in the 1s-0d major shells or a state with one nucleon in the 0p shell and the other in the 0f-1p shell is forbidden \((Q=0)\), since these states will be included in our \((0+2)\hbar \omega\) shell model diagonalization. The starting energy \( E_s \) in the G-matrix (2) is taken to be

\[ E_s = \epsilon_a + \epsilon_b + \Delta, \] (3)

where \((\epsilon_a + \epsilon_b)\) is the unperturbed energy for the initial two-particle state in the ladder diagrams \([\epsilon_i = (2n_i + l_i + \frac{3}{2}) \hbar \omega] \) with \( i = a, b \) and \( \Delta \), roughly speaking, represents the interaction energy which in this work is treated as an adjustable parameter whose value is chosen to yield a reasonable nuclear binding energy.

Note that the G-matrix elements used in the \( 0 \hbar \omega \) calculation will be the same as those used in the \((0+2)\hbar \omega \) calculation so that the difference in the results is solely due to the core-renormalization effects.
We force, for the lower-lying states, the spurious CM motion to be in its lowest $0\hbar\omega$ configuration by adding to the SM Hamiltonian the last term in Eq. (1) with $\lambda \gg 1$.

2 Results for systems with one valence particle or hole

We give results using the G-matrices from the Nijmegen NN potential \[1\] for $A=5, 6, 14, 15, 17$ and $18$ in Table I. The data in moments is taken from the compilations of Ajzenberg-Selove \[14\] and Raghavan \[15\].

We emphasize again that the single-particle energies in our SM studies are not taken as parameters, rather, they are implicitly generated by the kinetic energy and the G-matrix. In a $(0 + 2)\hbar\omega$ space, the calculated “single-particle” splitting between the $1/2^-_1$ and $3/2^-_1$ states in $^5\text{He}$ is 3.633 MeV. In $^15\text{O}$, the $3/2^-_1 – 1/2^-_1$ splitting is 5.479 MeV. The splitting between the $3/2^+_1$ and $5/2^+_1$ states in $^17\text{O}$ is 6.260 MeV.

Focusing first on the one-valence-nucleon system $A=17$, we see that indeed the values of $B(\text{GT})$ and $B(\text{M1})$ are nearly the same in the $0\hbar\omega$ space, in which the configuration is a $d_{5/2}$ nucleon outside a closed $^{16}\text{O}$ core, as in the larger $(0 + 2)\hbar\omega$ space. The value of $B(\text{GT})$ [our definition is such that the factor $(1.251)^2$ is not included; the operator is $\sum_i \sigma(i) t_\pm(i)$] in the small $0\hbar\omega$ space is 1.400 and in the large $(0 + 2)\hbar\omega$ space 1.381. For $A=15$, the story is the same. The value of $B(\text{GT})$ changes very little (from 0.333 to 0.326) in going from the $0\hbar\omega$ to the $(0 + 2)\hbar\omega$ space. Underlying this behavior for $A=15$ and $A=17$ is the previously mentioned theorem that there are no first-order corrections to the GT or M1 moment for a closed $LS$ shell plus or minus one particle. The small deviations are due to the fact that the shell model goes beyond first order.

We next consider the E2 properties for $A=17$. In the simple SM picture, the quadrupole moment of $^{17}\text{O}$ is zero because the valence nucleon is a
neutron. In first-order perturbation theory, there is a contribution due to the excitation of a core proton through two major shells, e.g. from 0s to 1s-0d or from 0p to 0f-1p. In the SM calculation that we do here, that effect is implicitly taken into account, but also one has the excitation of two core nucleons through one major shell, i.e., \((0p)^{-2}(1s0d)^2\). These configurations contribute only in second-order perturbation theory.

The calculated polarization charge for the \(d_{5/2}\)-neutron is the ratio of the quadrupole moment of \(^{17}\text{O}\) to that of a \(d_{5/2}\)-proton which can be read off from Table I:

\[
\epsilon_\nu = \frac{-0.720}{-5.924} = 0.123. \tag{4}
\]

Similarly, for the valence proton,

\[
\epsilon_\pi = \frac{-6.183 - (-5.924)}{-5.924} = 0.044. \tag{5}
\]

These are much smaller than the phenomenological values of 0.5 and 0.2. These results agree qualitatively with the higher-order perturbation theory results in Ref.\[8\] where small polarization charges are also obtained.

To check if the second-order contributions are canceling the first-order ones, we consider the first-order perturbation theory. If we use an energy denominator of \((-2\hbar\omega)\) for Fig.1(a), the polarization charge for the neutron is \(\epsilon_\nu = 0.385\) and that for the proton is \(\epsilon_\pi = 0.131\). However, when the calculated single-particle energy splittings are used, the values are \(\epsilon_\nu = 0.263\) and \(\epsilon_\pi = 0.087\). These are still larger than the SM values of 0.123 and 0.044.

These discrepancies are partly due to the fact that in the above first-order perturbation-theory approach, the wave function is not normalized. The ground-state wave function of \(^6\text{Li}\) in the \((0 + 2)\hbar\omega\) SM calculation is

\[
|\text{g.s.}\rangle = 68.446\%|1p-0h\rangle + 27.727\%|2p-1h\rangle + 3.827\%|3p-2h\rangle. \tag{6}
\]

A properly normalized wave function in the perturbation theory would therefore lead to an approximately 32%’s reduction (from 0.263 to 0.18) in the
first-order contribution to the effective charge. Further reductions are caused by the higher-order effects that are included in the \((0+2)\hbar\omega\) SM calculation.

We thus see that one must exhibit care when employing the perturbation-theory approach to calculate effective operators. The first-order perturbation theory first appears to be accountable for the large empirical polarization charge of nearly 0.5 for the \(d_{5/2}\)-neutron in \(^{17}\text{O}\), a more careful analysis involving the use of the self-consistent single-particle energies and the normalized wave function results in a factor of two’s reduction in \(\epsilon_\nu\), leading to a much smaller result (0.385 to 0.18). The \((0 + 2)\hbar\omega\) SM value of 0.123 is more consistent with the refined first-order perturbation-theory result of 0.18, both are smaller than the empirical value. The large quadrupole moment observed for \(^{17}\text{O}\) is due to the fact the “spherical core” is actually deformed and carries a non-zero angular momentum. To give sufficiently large polarization charges, bigger than \((0 + 2)\hbar\omega\) calculations are needed which will simulate highly deformed admixtures into the ground state.

3 Higher-shell effects for systems with two valence nucleons (or holes)

For \(A=14\), we have a striking example of a case where there is a large effect due to higher-shell admixtures, while there is almost none for a one-valence-nucleon system. The value of \(B(\text{GT})\) for the decay \(^{14}\text{C}(J = 0^+, T = 1) \rightarrow ^{14}\text{N}(J = 1^+, T = 0)\) changes from 2.627 to 1.305. The value of \(B(\text{M1})\) changes by almost the same ratio; indeed from isospin consideration the spin part of \(B(\text{M1})\) and \(B(\text{GT})\) must change by exactly the same ratio. The large change has been previously noted by us \([16]\).

What must be happening is that the wave function of the two valence nucleons is changing due to a renormalization of the effective interactions from higher shells. A typical second-order diagram is shown in Fig.2. We recall that this weak beta decay is famous because, although it is \textit{a priori} an
allowed GT transition, the life-time for the transition is very long, indicating a near vanishing GT matrix element.

Furthermore, it was noted by Inglis [17] that in a two-hole calculation one needs a tensor interaction to get a vanishing GT matrix element. In our calculation we obtain a $B(\text{GT})$ value of 1.305. We believe that the bare tensor interaction is too strong so if we imagine turning on the tensor interaction from zero to its full (bare) value, we get an overshoot. The matrix element first gets smaller in magnitude, then goes through zero, changes sign and gets large in magnitude again. The effect of the higher-shell admixtures is to effectively weaken the tensor interaction in the valence space, thus causing the GT matrix element to decrease in magnitude.

For $A=6$, there are some encouraging results from higher-shell effects. The experimental magnetic dipole moment and electric quadrupole moment of the $1^+$ ground state of $^6\text{Li}$ are respectively $0.822\mu_N$ and $-0.082e\text{fm}^2$ [18]. The sign of the $^6\text{Li}$ quadrupole moment is opposite to that of the deuteron ($Q = +0.28e\text{fm}^2$). In the $LS$ limit, the ground state of $^6\text{Li}$ has a $p^2$ configuration with $L=0$, $S=1$, $J=1$. In that limit, the M1 moment is $\mu = \mu_n + \mu_p = (-1.913 + 2.793)\mu_N = 0.880\mu_N$ and the quadrupole moment is zero.

In the small space, the results for $\mu$ and $Q$ are respectively $0.866\mu_N$ and $-0.237e\text{fm}^2$. In the large space, the results are $0.849\mu_N$ and $-0.163e\text{fm}^2$ in both cases closer to experiment.

In Table II we give the results of the $jj$ and $LS$ limits for the M1 and E2 moments of the $1^+$ ground state of $^6\text{Li}$. For the M1 moment, these two limits yield respectively $\mu = (\mu_n + \mu_p + 1)/3 = 0.627\mu_N$ and $\mu = \mu_p + \mu_n = 0.880\mu_N$. The experimental value $0.822\mu_N$ lies between these two extremes. The quadrupole moment $Q$ is zero in the $LS$ limit ($L=0$, $S=1$, $J=1$) and is positive in the $jj$ limit. The expression for $Q$ in the $jj$ limit can be related to that of a $p_{3/2}$ proton via the Wigner-Eckart theorem in terms of

\[ \frac{1}{3} \]
Clebsh-Gordan and unitary Racah coefficients:

\[
Q((p_{3/2}^2)^{J=1,T=0}) = (e_\nu + e_\pi)\left(\begin{array}{c|c}
2 & 0 \\
3/2 & 1
\end{array}\right) U(2,3/2; 1,3/2; 3/2,1) Q(p_{3/2,\pi})
\]

\[
= -0.4 Q(p_{3/2,\pi}).
\]

The quadrupole moment of a \( j = l + 1/2 \) proton is \(-\hbar/(m\omega)\). Hence the quadrupole moment for a \( p_{3/2} \)-proton is negative: \( Q(p_{3/2,\pi}) = -\hbar/(m\omega) \).

The \((p_{3/2}^2)^{J=1,T=0}\) quadrupole moment is therefore positive (1.04 \( e\,fm^2 \) when \( \hbar\omega = 16 \) MeV).

To get further insight into the \( A=6 \) behavior, we use a schematic interaction

\[
V_{\text{sche}} = V_c + xV_{\text{so}} + yV_t.
\]

With \( x=y=1 \), this interaction gives relative matrix elements similar to those of Bonn A [4] for the region of \( ^{16}O \) [15]. Only partial waves with a relative angular momentum \( l=0 \) and \( l=1 \) for the diagonal and \( l=0 \) to \( l=2 \) for the off-diagonal are included [16]. It is constructed so that it is easy to control the strengths of the spin-orbit and tensor interactions by adjusting \( x \) and \( y \), e.g., for \( x=0 \), we turn off the spin-orbit interaction and for \( y=0 \), we turn off the tensor interaction. The results for the schematic interaction are also presented in Table II. The matrix elements of the interaction have been multiplied by 1.15 since they were fitted to \( ^{16}O \) but are being applied to \( A=6 \).

We verify that for \( x=y=0 \), one gets the \( LS \) wave function with \( L=0, S=1 \) for the ground state of \( ^6\)Li. The magnetic moment in both the \( 0\hbar\omega \) and \( (0+2)\hbar\omega \) calculations is \( \mu_n + \mu_p = 0.880\mu_N \) and the quadrupole moment is zero. These results transcend the truncated shell model.

Note that the spin-orbit interaction and the tensor interaction have opposite effects on the quadrupole moment. In the small space, when the spin-orbit interaction is turned on at full strength and the tensor interaction is turned off (i.e., \( x=1, y=0 \)), the quadrupole moment \( Q \) is 0.121 \( e\,fm^2 \). This
is consistent with the fact that making the spin-orbit interaction stronger takes one towards the $jj$ limit of Table II.

In the opposite extreme with the tensor interaction on and the spin-orbit interaction off (i.e., $x=0, y=1$), the value of $Q$ is $-0.608 \text{ e fm}^2$. For $x=y=1$, the value of $Q$ is $-0.281 \text{ e fm}^2$. In a previous work [20], we argued that the effective spin-orbit interaction in the $p$ shell should be even stronger than that given by the relativistic Bonn A with $m^*/m = 1$ [4]. A reasonable set of parameters is $x=1.5, y=1$ with which the small-space value of $Q$ is $-0.114 \text{ e fm}^2$, in reasonable agreement with experiment. The main point here, however, is that the quadrupole moment of $^6\text{Li}$ is extremely sensitive to the relative strengths of the spin-orbit and tensor interactions in the nucleus.

As seen in Table I, the $2\hbar \omega$ admixtures enhance the quadrupole properties of $A=5$, as expected. But but for $A=6$, the magnitude of the quadrupole moment decreases from $-0.255$ to $-0.175 \text{ e fm}^2$ when $2\hbar \omega$ configurations are included. We believe that for $A=6$ there are two opposing tendencies. The building up of the effective charge would cause the magnitude of $Q$ to increase. But the “self weakening” of the tensor interaction due to higher-order admixtures will cause $Q$ to go from negative towards positive values.

A cluster approach for $^6\text{Li}$, i.e., an alpha-deuteron cluster model calculation has been carried out by Eskondarian et al [21]. Without going into too much detail we focus on their results. Whereas we get the magnetic moment of $^6\text{Li}$ larger than experiment, they get it smaller. Whereas we get the quadrupole moment of $^6\text{Li}$ to be negative, they get it positive. It is not clear if the differences are due to the differences between the SM and the cluster model, or in the differences between the interactions that are used. The SM will also yield a positive quadrupole moment if the spin-orbit interaction is stronger and/or the tensor interaction is weaker than what we have. Another possibility to be explored is that the cluster model has correlations which require even larger SM calculations than the “$2\hbar \omega$” ones performed here.
4 Closing remarks

We have shown not only that higher-shell effects are important for nuclear properties but that they enter in a nonuniform way as we add nucleons to the nucleus. For example, higher-shell effects are much larger for the spin properties of a two-valence-particle (or hole) system than they are for a one-valence-particle (or hole) system. As we go from a small space \((0\hbar \omega)\) to a large space \([(0+2)\hbar \omega]\), the value of \(B(GT)\) for \(A=15\) experiences little change (from 0.333 to 0.326), but for \(A=14\) the change is much bigger (from 2.627 to 1.305). This can be understood by the fact that the effective interaction between the two valence holes gets renormalized. In this renormalization, the effective tensor interaction (acting in the valence space) becomes weaker.

The \(A=6\) and \(A=14\) cases offer good tests of the effective interaction. For \(^6\text{Li}\), which has often been proposed as an ideal isoscalar target (see Ref. [13] for more details), the results for the \(J=1^+\) static quadrupole moment and magnetic moment are very sensitive to the details of the spin-orbit and tensor interactions that are used. There is a surprisingly large scatter of theoretical results — both positive and negative values for the quadrupole moment of the \(J=1^+\) ground state of \(^6\text{Li}\) have been obtained. We note that the tensor interaction gives a negative value and the spin-orbit interaction a positive value for this quantity. Our results with the effective interactions calculated from the Nijmegen potential NijmII as applied to “two-particle” systems are encouraging in the sense that higher-shell effects consistently go in the right direction. There is still a problem, however, in that the one-body quadrupole polarization charges come out too small.

If we want to use the nucleus as a laboratory for fundamental studies, e.g., of fundamental symmetries, applications to astrophysics, or, as we have emphasized here, the effective interaction between two nucleons in a nucleus, we must include higher-shell effects. Indeed, our work here represents only a step in the right direction. In the future, excitations beyond \(2\hbar \omega\) will
have to be included. Still, it is encouraging that we are able to get a better systematic feeling for how higher-shell admixtures affect nuclear properties in a nuclear medium.

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Table I. The static and transitional properties in selected nuclei with one or two valence nucleons (or holes) in the $0\hbar\omega$ and $(0 + 2)\hbar\omega$ shell-model calculations using G-matrices calculated from a new Nijmegen local NN potential \[\text{[11]}\]. $B$(M1) is in units of $\mu^2_N$, $\mu$ in units of $\mu_N$ and $Q$ in units of $e\text{fm}^2$. Experimental data are taken from Refs.\[\text{[14, 15]}\].

| Nucleus | Quantity | $0\hbar\omega$ | $(0 + 2)\hbar\omega$ | Expt |
|---------|----------|-----------------|----------------------|------|
| $A=5$   | $B$(GT)$(3/2^- \rightarrow 3/2^-)$ | 1.666 | 1.632 | |
|         | $\mu(5\text{He})$ | -1.913 | -1.864 | |
|         | $\mu(5\text{Li})$ | 3.793 | 3.741 | |
|         | $Q(5\text{He})$ | -2.592 | -2.722 | |
|         | $Q(5\text{Li})$ | 0.000 | -0.303 | |
| $A=6$   | $B$(GT)$(0^+1 \rightarrow 1^+0)$ | 5.626 | 5.397 | 4.847 |
|         | $B$(M1)$(0^+1 \rightarrow 1^+0)$ | 16.29 | 15.57 | |
|         | $\mu(1^+0)$ | 0.866 | 0.849 | 0.822 |
|         | $Q(1^+0)$ | -0.255 | -0.175 | -0.082 |
| $A=14$  | $B$(GT)$(0^+1 \rightarrow 1^+0)$ | 2.627 | 1.305 | 0.000 |
|         | $B$(M1)$(0^+1 \rightarrow 1^+0)$ | 6.916 | 3.842 | |
|         | $\mu(1^+0)$ | 0.634 | 0.499 | 0.404 |
|         | $Q(1^+0)$ | 1.761 | 2.164 | 1.56 |
| $A=15$  | $B$(GT)$(1/2^- \rightarrow 1/2^-)$ | 0.333 | 0.326 | 0.270 |
|         | $\mu(15\text{N})$ | -0.264 | -0.276 | -0.283 |
|         | $\mu(15\text{O})$ | 0.638 | 0.654 | 0.719 |
| $A=17$  | $B$(GT)$(5/2^+ \rightarrow 5/2^+)$ | 1.400 | 1.381 | 1.089 |
|         | $\mu(17\text{O})$ | -1.913 | -1.869 | -1.894 |
|         | $\mu(17\text{F})$ | 4.793 | 4.748 | 4.722 |
|         | $Q(17\text{O})$ | 0.000 | -0.720 | -2.578 |
|         | $Q(17\text{F})$ | -5.924 | -6.183 | -5.8 |
| $A=18$  | $B$(GT)$(0^+1 \rightarrow 1^+0)$ | 5.312 | 5.436 | 3.301 |
|         | $B$(M1)$(0^+1 \rightarrow 1^+0)$ | 16.75 | 16.19 | |
|         | $\mu(1^+0)$ | 0.858 | — | |
|         | $Q(1^+0)$ | -0.514 | — | |
Table II. The magnetic dipole moment $\mu$ (in units of $\mu_N$) and quadrupole moment $Q$ (in units of $e\text{fm}^2$) for the ground state of $^6\text{Li}$ in the $0\hbar\omega$ and $(0 + 2)\hbar\omega$ shell-model calculations using (1) the schematic interaction with varying spin-orbit ($x$) and tensor ($y$) strengths, and (2) G-matrices calculated from different realistic NN potentials: Hamada-Johnson [12], Reid-soft-core [13], new Reid-soft-core (Reid93) [11], and new Nijmegen [11].

| Interaction | $\mu$ | $Q$ |
|-------------|-------|-----|
|             | $0\hbar\omega$ | $(0 + 2)\hbar\omega$ | $0\hbar\omega$ | $(0 + 2)\hbar\omega$ |
| $x$ | $y$ |
| 0.0 | 0.0 | 0.880 | 0.880 | 0.000 | 0.000 |
| 0.0 | 0.5 | 0.867 | 0.859 | -0.345 | -0.415 |
| 0.0 | 1.0 | 0.840 | 0.816 | -0.608 | -0.711 |
| 0.5 | 0.0 | 0.877 | 0.878 | +0.038 | +0.033 |
| 0.5 | 0.5 | 0.872 | 0.866 | -0.221 | -0.276 |
| 0.5 | 1.0 | 0.856 | 0.835 | -0.446 | -0.533 |
| 1.0 | 0.0 | 0.869 | 0.871 | +0.121 | +0.106 |
| 1.0 | 0.5 | 0.870 | 0.866 | -0.085 | -0.135 |
| 1.0 | 1.0 | 0.863 | 0.845 | -0.281 | -0.358 |
| 1.5 | 0.0 | 0.854 | 0.859 | +0.221 | +0.195 |
| 1.5 | 0.5 | 0.861 | 0.860 | +0.055 | +0.006 |
| 1.5 | 1.0 | 0.860 | 0.848 | -0.114 | -0.185 |
| $LS$ limit | | 0.880 | | 0.000 |
| $jj$ limit | | 0.627 | | 1.037 |

Hamada-Johnson | 0.861 | 0.840 | -0.364 | -0.285 |
Reid-soft-core | 0.862 | 0.842 | -0.307 | -0.224 |
Reid93 | 0.866 | 0.848 | -0.255 | -0.181 |
NijmII | 0.866 | 0.849 | -0.255 | -0.175 |
**Figure caption**

**Fig.1** Core-polarization diagrams for E2 effective charge. (a) First-order diagram; (b) Typical RPA graph.

**Fig.2** Core renormalization of the effective interaction. This changes the two-particle wavefunction and hence static properties like M1 and E2 moments.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9311016v1