Influence of finite baryon density on hadronization in nucleus-nucleus collisions via recombination

C.B. Yang and H. Zheng
Institute of Particle Physics, Central China Normal University, Wuhan 430079, P.R. China

Abstract. In this paper is investigated the influence of net baryon density on baryon and meson yields in relativistic nucleus-nucleus collisions, based on the recombination model for hadronization. Unitarity condition is used as a constraint on the model. Three cases with different assumptions on the expansion of partonic system are considered and the baryon to meson ratio is calculated for those situations.

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§ To whom correspondence should be addressed (cbyang@mail.ccut.edu.cn)
1. Introduction

Partons, both soft and hard, are produced in high energy nucleus-nucleus collisions. Because of color confinement colored objects (quarks and gluons) must convert to some hadrons which can be detected experimentally. The problem we concern in this paper is how those partons convert into the observed final state hadrons and the relation of hadron yield with the parton density. On the topics of hadron production, there are two widely used traditional models: the string model for soft hadron production and the Feynman and Field’s independent fragmentation model for hard hadron production. The string model worked well for elementary collisions, such as $e^+e^-$ annihilations, where strings can be formed among a few initial partons and break up to form the final state hadrons. In relativistic heavy ion collisions, there are thousands initial partons. It is very hard to pair partons and have a string for each pair. Even if strings are formed, their properties must be modified by the presence of many other color charges. So the soft hadron production must be different from that in elementary collisions. In some researches such a modification is treated by the fusion of strings in terms of percolation model [1]. The fragmentation approach, on the other hand, is based on the factorization theorem which is true only for processes with large momentum transfer and predicts a small baryon over meson ratio. This prediction is in contradiction with experimental observations at RHIC in Au+Au collisions, where the $p/\pi$ ratio can exceed one at $p_T \sim 3 \text{ GeV}/c$ [2], showing strong influence of the hot medium produced in the collisions. RHIC also observed the constituent quark number scaling for the elliptic flow [3], species dependence of the Cronin effect [4], and jet shape modifications [5]. All these observations cannot be explained with the two traditional hadronization models.

Last a few years witnessed the rapid development of the quark recombination models [6, 7, 8, 9] as a new approach for the problem of hadronization in relativistic heavy ion collisions at RHIC. Viewed in these models, the colliding system generates partons which evolve in phase space according to the principles of quantum chromodynamics. In this process, quarks get dressed gradually. Finally, when the energy density (or temperature) reaches the critical value, the parton system hadronizes into the final state hadrons. The quark recombination model deals with only the final stage, i.e. the process from dressed quarks to the hadrons. A basic assumption in all the recombination models is that the final state hadrons are formed by recombining two (or three) dressed (anti)quarks. An important feature for the models is the efficiency in producing hadrons with intermediate transverse momentum from thermal (or soft) parton system. The efficiency comes from the fact that the momentum of a hadron is the sum of that of the constituent quarks forming the hadron. Because of the abundance of soft quarks, the recombination approach ensures high yield of mesons with intermediate transverse momentum. More importantly, the interactions among soft quarks and shower partons in jets are the key in explaining experimental observations such as the constituent quark number scaling of the elliptic flow, the modification of the jet structure, the species dependence of the Cronin effect etc., because of the colliding system dependence of the
produced (hot) soft medium.

There are, however, some difficulties with this model. The yield of meson (baryon) is proportional to the square (cubic) of quark density or constituent quark number. This dependence violates unitarity, because the number of mesons (baryons) produced will increase by a factor of four (eight) if the quark density increases by a factor of two. As a result the numbers of dressed constituent quarks can not be conserved and unitarity is violated. A way to overcome this difficulty is to reformulate the quark recombination as a dynamical process with the introduction of hadronization time \[10\]. At each moment, the production rates for mesons and baryons are proportional to square and cubic of the quark density, so the basic idea in the recombination model retains. The total yields are, however, constrained by the conservation of numbers of constituent quarks, and the unitarity condition is also satisfied in this approach.

In \[10\] only the simplest case with zero net baryon density was considered. In that case, the number of baryons produced is equal to that of anti-baryons. So one can basically consider only the production of mesons and baryons. In real Au+Au collisions the net baryon density in the central rapidity region is not zero because of the nuclear stopping effect. The net baryon density enables more baryons produced than anti-baryons. It would be important to know how the ratio of baryon to meson depends on the net baryon density. This is the issue we want to discuss in this paper. We only investigate the yields of mesons and baryons but not the spectra for the final state hadrons, as done in earlier work of the coalescence model \[11\].

2. Basic formulism

In every implementation of the quark recombination models, the transverse momentum spectrum of mesons at mid-rapidity can be written, after some algebras, as

\[
\frac{dN^M}{p_T dp_T} = \int dp_1 dp_2 F(p_1, p_2) R^M(p_1, p_2, p_T),
\]

where a factor \(\delta(p_T - p_1 - p_2)\) is included in the recombination function \(R^M(p_1, p_2, p_T)\) to ensure momentum conservation and \(F(p_1, p_2)\) is associated with the joint quark-antiquark momentum distribution. For baryon production a similar equation can be written out. When we are interested in the total multiplicity of a kind of hadron, we can consider the contribution from recombination of pure thermal partons only, because most produced hadrons are in the low transverse momentum region where pure thermal recombination dominates. In the region of low transverse momentum, the joint distribution for quark-antiquark pair can be written as \(F(p_1, p_2) = V \rho^2 f_1(p_1) f_2(p_2)\) with \(V\) the spatial volume of the partonic system and \(\rho\) the thermal parton density just before hadronization when the (anti)quark transverse momentum distributions \(f_{1,2}\) are normalized to some fixed constant. Then one can carry out the integration over \(p_T\) and the result shows that the yield of meson (baryon) is proportional to the square (cubic) of quark density at hadronization. This shows a violation of unitarity, since the total number of constituent quarks should be conserved because of their low virtuality.
and energy and the total number of hadrons produced must be about proportional to the total quark number. In [10] it was suggested that the production rate for meson (baryon) is proportional to the square (cubic) of the quark density, but the hadronization process conserves the total constituent quark numbers. In this way, the main features of the quark recombination models retain and unitarity is also conserved. Because we are going to consider the case with nonzero net baryon density, the yields of baryon and anti-baryon are different. Thus different from consideration in [10] one should have anti-baryon yield as an additional variable being taken into account. For simplicity we consider hadronization of a partonic system with light quarks \((u, d, \bar{u}, \bar{d})\) and catalog final state hadrons by their baryon numbers only. Extension to include strange quarks is straightforward. The rate equations for the yields of baryons and mesons can be expressed as

\[
\frac{dN_M}{dt} = A_M V \rho_q(t) \rho_q(t), \\
\frac{dN_B}{dt} = A_B V \rho_q^3(t), \\
\frac{dN_{\bar{B}}}{dt} = A_B V \rho_{\bar{q}}^3(t),
\]

with \(V\) the volume of partonic system, \(\rho_q\) and \(\rho_{\bar{q}}\) densities for quarks and antiquarks. In last equations, \(A_B\) and \(A_M\) are determined mainly by the hadronic structures of meson and baryon. The information of both the shape of quark distributions and the recombination functions is encoded in \(A\)'s. From the reaction-rate theory [12] \(A_M\) and \(A_B\) are also proportional to the corresponding cross-sections and the degeneracy factors. Normally \(A_B/A_M \ll 1\), and densities of quarks and antiquarks are a few times higher than that in normal nuclear matter. In quark recombination models gluons are assumed to have turned into \(q\bar{q}\) before hadronization. So there is no term for gluon contribution in last equations. Because hadronization takes place at low temperature, collisions between low momentum quarks and formed hadrons will not likely break the hadrons into quarks. In addition most of the produced hadrons will fly out of the reaction zone and will not collide with quarks left. As a reflection of this fact, there is no reverse term in last equations for the process \(q + \text{hadrons} \rightarrow \text{quarks}\). The same assumptions are adopted in [10]. The conservation conditions for the numbers of constituent quarks read

\[
\frac{d(V \rho_q)}{dt} = -3 \frac{dN_B}{dt} - \frac{dN_M}{dt}, \\
\frac{d(V \rho_{\bar{q}})}{dt} = -3 \frac{dN_{\bar{B}}}{dt} - \frac{dN_M}{dt}.
\]

The last two expressions in last equations ensure the unitarity in hadronization process. The initial conditions are: \(V(t = 0) = V_0, \rho_q(t = 0) = \rho_0, \rho_{\bar{q}}(t = 0) = \kappa \rho_0\), with \(V_0, \rho_0, \kappa\) parameters to be input from other models. Eqs. (2) and (3) form the fundamental formulas for the yields of mesons and baryons in hadronization from quark recombination model with unitarity constraint.
One can see easily that the above equations are not closed. To solve these equations additional input on the relation of $\rho_q, \rho_{\bar{q}}$ and $V$ must be introduced from elsewhere. In ultra-relativistic heavy ion collisions the produced partonic system expands. If the expansion of the parton system retains to the last stage of its evolution and the transition from quarks to hadrons is of first-order, there are two competing trends in hadronization on the change of the volume. One trend is the system’s hydrodynamical expansion which will make the partonic volume larger, and the other is hadronization process which happens on the surface of the system and tries to shrink the system. Thus the hadronization dynamics should, in general, be connected with the hydrodynamical calculations. Such combination is beyond the scope of this paper. Even without hydrodynamical input, qualitative behaviors of $N_B$ etc can be made from Eq. (2). The behaviors of the density $\rho$’s for all the situations discussed below are very similar to those shown in [10] for the corresponding cases, and we will not show them in this paper because they are not observable. Since $\rho_q$’s decrease with time in hadronization, production rates for baryons and mesons also decrease with time, more obvious for baryons than for mesons. So $N_B, N_M$ etc increase with time rapidly at first, then slow down, and finally saturate at the end of hadronization.

In this paper we only discuss the situations we can do numerically without involving hydrodynamical calculations. We will show some results under simplest assumptions. This is enough for illustrating the net baryon density dependence in hadronization from the quark recombination model.

3. Main results

3.1. For fixed volume of the system

The first case one can investigate is with fixed $V = V_0$. For later convenience, we define

$$n_B \equiv N_B/(\rho_0 V_0), n_M \equiv N_M/(\rho_0 V_0), n_{\bar{B}} \equiv N_{\bar{B}}/(\rho_0 V_0).$$

They are the average hadron multiplicities produced from per constituent quark in the state just before hadronization. After integrating Eq. (3) over $t$ one gets

$$3n_B + n_M = 1,$$
$$3n_{\bar{B}} + n_M = \kappa.$$

So $n_B - n_{\bar{B}} = (1 - \kappa)/3$, independent of model assumptions in this paper. In fact this expression is nothing more than the conservation of net baryon number in hadronization. We also define $r = \rho_{\bar{q}}/\rho_q, u = A_B\rho_0/A_M, \rho = \rho_q/\rho_0$ and $\tau = A_M\rho_0 t$, then Eqs. (2) and (3) can be rewritten as

$$\frac{d\rho}{d\tau} = -3u\rho^3 - r\rho^2,$$
$$\frac{dr}{d\tau} = 3u\rho^2(r - r^3) - \rho(r - r^2),$$
$$\frac{dn_B}{d\tau} = u\rho^3,$$

(4)
\[ \frac{dn_M}{d\tau} = r \rho^2 , \]
\[ \frac{dn_B}{d\tau} = ur^3 \rho^3 . \]

Initial conditions for last equations are \( \rho(0) = 1, r(0) = \kappa \). The obtained yields \( n_B \) etc depend on values of parameters \( u \) and \( \kappa \). We try to input two typical values for \( \kappa = 0.8 \) and 0.6 and investigate the yields as functions of \( u \) which depends on the initial quark density \( \rho_0 \) and the competition factor \( A_B/A_M \) of baryon production relative to that of meson. A large value of \( u \) may be caused due to a higher initial quark density or larger probability for baryon production relative to that for mesons. The obtained results for the yields are shown in Fig. 1. From the figure one can see that with the increase of \( u \)

more baryons but less mesons can be produced. This is from the different dependence of their production rates on the quark density. The enhancement of baryon production can be seen more clearly from the ratio \( R_{B/M} = n_B/n_M \) as a function of \( u \), as shown in Fig. 2. The baryon to meson ratio increases almost linearly with \( u \) and can be larger than 0.8 at \( u = 2 \) for \( \kappa = 0.6 \). One may have noticed that the ratio is larger at the same \( u \) when the parameter \( \kappa \) is smaller. This is not surprising, because if \( \kappa \) is smaller there are less anti-quarks and thus less mesons can be produced from the system while most of the quarks can only recombine to form baryons.

### 3.2. For fixed quark density

The second case we can consider is when the quark density is assumed to remain unchanged in hadronization. Then the volume of the system and the anti-quark density can change in the process. We define a new variable instead of the volume \( \mu = \ln(V/V_0) \), and the equations governing the process are

\[ \frac{dr}{d\tau} = 3ur(1 - r^2) - r(1 - r) , \]
Figure 2. Ratio of baryon yield to that of meson as a function of $u$ for given $\kappa = 0.8$ and 0.6 for the first case.

$$
\frac{d\mu}{d\tau} = -(3u + r),
\frac{dn_B}{d\tau} = u \exp(\mu),
\frac{dn_M}{d\tau} = r \exp(\mu),
\frac{dn_{\bar{B}}}{d\tau} = ur^3 \exp(\mu).
$$

In this case the system shrinks almost exponentially, when $u$ is large, with time and hadronization process finishes very quickly. So one can take, as an approximation, $r$ in last two equations as a constant. Then one can see that with increase of $u$ the baryon yield can be larger than that for meson at $u \sim \kappa$. When $u$ is large enough the anti-baryon yield can also be larger than that of meson. The calculated baryon and meson yields are shown in Fig. 3 for this case. Numerical calculations confirm above estimate, as shown in Fig. 3. The baryon to meson ratio, $R_{B/M}$, is much larger than for the first case, as shown in Fig. 4 for both $\kappa = 0.8$ and 0.6. The smaller $\kappa$ the larger baryon to meson ratio, as argued for the first case.

3.3. For the case with hydrodynamic flow

Now we consider a more realistic case when the partonic system is assumed to expand according to some rules from the hydrodynamics. The true velocity profile of the produced partonic system from the study of hydrodynamics for the evolution is quite complicated and is not suitable for the study in this paper. We now assume that the expansion rate is proportional to the distance from the center, as for the galaxies [13]. If the partonic system is regarded as an expanding ellipsoid with principal axes $a$ and $b$ then one roughly has $da/d\tau = va$ and $db/d\tau = vb$ with $v$ a constant. In this case, one
can have approximately
\[ \frac{dV}{d\tau} = \nu V, \]  
and the densities of quarks and anti-quarks decrease rapidly with \( \tau \), because of the decrease of quark number from hadronization and the increase of system’s volume. Then we define \( \rho_1 = \rho_q/\rho_0, \rho_2 = \rho_{\bar{q}}/\rho_0 \), and get
\[ \frac{d\rho_1}{d\tau} = -3u\rho_1^3 - \rho_1\rho_2 - \nu\rho_1, \]
\[ \frac{d\rho_2}{d\tau} = -3u\rho_2^3 - \rho_1\rho_2 - \nu\rho_2, \]
\[ \frac{dn_B}{d\tau} = u\rho_1^4 \exp(\nu\tau), \]
\[ \frac{dn_M}{d\tau} = \rho_1\rho_2 \exp(\nu\tau), \]  
\[ (7) \]

**Figure 3.** Yields for baryon, meson, and anti-baryon for the case with fixed quark density as functions of \( u \) for given \( \kappa = 0.8 \).

**Figure 4.** Ratio of baryon yield to that of meson as a function of \( u \) for given \( \kappa = 0.8 \) and 0.6 for the second case.
\[ \frac{dn_B}{d\tau} = u\rho_2^3 \exp(\nu \tau) , \]

with \( \rho_1(0) = 1, \rho_2(0) = \kappa \). We choose the parameter for system’s expansion \( \nu = 0.1 \) in our calculation as an example. In this case the hadronization process is similar to the first case in the beginning of hadronization with the decrease of parton densities a little faster than in the first case. Because of the \( \rho^3 \) dependence of baryon production rates, most baryons are produced when the parton density is high. So the expansion rate \( \nu \) has smaller influence on baryon production than for mesons. Consequently the meson yield is a little bit less than in the first case. The results for the yields are shown in Fig. 5 for \( \kappa = 0.8 \) and the corresponding baryon to meson ratio is given in Fig. 6 for \( \kappa = 0.8 \) and 0.6.

**Figure 5.** Yields for baryon, meson, and anti-baryon as functions of \( u \) for the case with hydrodynamical expansion for the partonic system for given \( \kappa = 0.8 \).

**Figure 6.** Ratio of baryon yield to that of meson as a function of \( u \) for given \( \kappa = 0.8 \) and 0.6 for the third case.
4. Discussions

From the results obtained for the three cases discussed above one can see that the
yields of baryons and mesons depend on the details of the evolution of the system.
From experimentally measured yields it is possible to constrain the dynamics in the
hadronization process. The net baryon density influence strongly on baryon and meson
yields and their ratio. More realistic investigations on this subject, with the physical
expansion of the partonic system taken into account, are needed to draw more reliable
conclusions.

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