TABULATED EQUATION OF STATE FOR SUPERNOVA MATTER INCLUDING FULL NUCLEAR ENSEMBLE

N. BUYUKCIZMECI1,2, A. S. BOTVINA1,3,4, AND I. N. MISHUSTIN1,5

1 Frankfurt Institute for Advanced Studies, J.W. Goethe University, D-60438 Frankfurt am Main, Germany
2 Department of Physics, Selcuk University, 42079 Kampus, Konya, Turkey
3 Institute for Nuclear Research, Russian Academy of Sciences, 117312 Moscow, Russia
4 Institut für Kernphysik, J. Gutenberg Universität, D-55099 Mainz, Germany
5 Kurchatov Institute, Russian Research Center, 123182 Moscow, Russia

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ABSTRACT

This is an introduction to the tabulated database of stellar matter properties calculated within the framework of the Statistical Model for Supernova Matter (SMSM). The tables present thermodynamical characteristics and nuclear abundances for 31 values of baryon density ($10^{-8} < \rho / \rho_0 < 0.32$, $\rho_0 = 0.15$ fm$^{-3}$ is the normal nuclear matter density), 35 values of temperature ($0.2$ MeV $< T < 25$ MeV), and 28 values of electron-to-baryon ratio ($0.02 < Y_e < 0.56$). The properties of stellar matter in $\beta$ equilibrium are also considered. The main ingredients of the SMSM are briefly outlined, and the data structure and content of the tables are explained.

Key words: dense matter – equation of state – nuclear reactions, nucleosynthesis, abundances – stars: neutron – supernovae: general

Online-only material: color figures, machine-readable tables

1. INTRODUCTION

As established in intensive experimental studies of last 20 yr, many nuclear reactions lead to the formation of thermalized nuclear systems characterized by subnuclear densities and temperatures of 3–8 MeV. De-excitation of such systems goes through nuclear multifragmentation, i.e., break up into many excited fragments and nucleons. As it is generally accepted, thermal and chemical equilibrium can be established in such multifragmentation reactions. Transport theoretical calculations of both central heavy-ion collisions around the Fermi energy and peripheral heavy-ion collisions predict momentum distributions of nucleons that are similar to equilibrium ones after $\approx 100$ fm $c^{-1}$ (see references in the topical issue of Chomaz et al. 2006). In the early 1980s, several versions of the statistical approach were proposed to describe multifragmentation of highly excited equilibrated sources (see, e.g., Randrup and Koonin 1981; Gross 1982; Bondorf et al. 1985). These models were able to describe many characteristics of nuclear fragments observed in nuclear experiments: multiplicities of intermediate-mass fragments, charge and isotope distributions, event-by-event correlations of fragments (including fragments of different sizes), their angular and velocity correlations, and other observables (Gross 1990; Bondorf et al. 1995; Souliotis et al. 2007; Oglu et al. 2011; Botvina et al. 1990, 1995; Scharenberg et al. 2001; D’Agostino et al. 1996, 1999; Bellaize et al. 2002; Avdeyev et al. 2002; Hauger et al. 2000; Iglío et al. 2006; Hudan et al. 2009; Wang et al. 1999; Viola et al. 2001; Rodionov et al. 2002; Pienkowski et al. 2002). The temperature and density of nuclear matter at the stage of fragment formation can also be established reliably in experiment by measuring relative velocities of fragments and ratios of isotope yields.

A nuclear matter of a similar type is expected to be formed in astrophysical processes, such as the collapse of massive stars and supernova explosions. To compare thermodynamical conditions obtained in nuclear reactions and in astrophysics, in Figure 1 (Buyukcizmeci et al. 2013), we show the phase diagram for symmetric and asymmetric nuclear matter for a corresponding range of densities and temperatures. Typical conditions associated with multifragmentation reactions are indicated by the shaded area in Figure 1. These reactions give us a chance to study hot nuclei in the environment of other nuclear species in thermodynamical equilibrium, as we expect in hot stellar matter at subnuclear densities. The properties of such nuclei can be directly extracted from experimental data, and then this information can be used for more realistic calculations of nuclear composition and equation of state (EOS) of stellar matter. This is a new opportunity to study stellar matter in the laboratory, besides the theoretical approaches based on phenomenological nuclear forces. As one can see from Figure 1, in the course of massive star collapse the stellar nuclear matter passes exactly through the multifragmentation region with typical entropy per baryon $S/B = 1–4$. The electron fraction $Y_e$, which is equal to the total proton fraction, in the supernova core varies from 0.1 to 0.5. In the two-phase coexistence region (below the solid and dot-dashed lines) at densities $\rho \approx 0.3–0.8\rho_0$ the matter should be in a mixed phase, which is a strongly inhomogeneous state with intermittent dense and dilute regions. In the coexistence region at lower densities, $\rho < 0.3\rho_0$, the nuclear matter breaks up into compact nuclear droplets surrounded by nucleons. These relatively low densities dominate during the main stages of stellar collapse and explosion. Under such conditions, one can expect a mixture of different species, including nucleons and light and heavy nuclei. In Figure 1, we also show isentropic trajectories of nuclear matter (dashed curves) as well as dynamical trajectories of density and temperature inside the supernova core (red dotted curves), taken from the supernova simulation of a 15 solar mass progenitor (Sumiyoshi et al. 2005). The snapshots in the supernova dynamics are selected for three stages: before bounce (BB), at core bounce (CB), and after bounce (PB). At the gravitational collapse of the iron core just BB, the density and temperature roughly follow the isentropic curves with $S/B = 1–2$. At the CB, the central density increases just above the nuclear matter density $\rho_0 = 0.15$ fm$^{-3}$. After that, the temperature of the inner core becomes higher than 10 MeV.
Figure 1. Nuclear phase diagram in the “temperature–baryon density” plane. Solid and dash-dotted blue lines indicate boundaries of the liquid-gas coexistence region for symmetric ($Y_e = 0.5$) and asymmetric matter ($Y_e = 0.2$) calculated with TMI interactions (Sugahara & Toki 1994). The shaded area corresponds to typical conditions for nuclear multifragmentation reactions. The dashed black lines are isentropic trajectories characterized by constant entropy per baryon, $S/B = 1, 2, 4, 6$ calculated with SMSM (Botvina & Mishustin 2010). The dotted red lines show model results of Sumiyoshi et al. (2005) for just before the bounce (BB), at the core bounce (CB), and the post bounce (PB) in a core-collapse supernova. This figure is taken from Buyukcizmeci et al. (2013).

(A color version of this figure is available in the online journal.)

due to the passage of the shock wave. The temperature of the whole supernova core is still high at 150 ms after the bounce, when the shock is stalled around 130 km. This one-dimensional simulation does not lead to a successful explosion.

One can see that thermodynamic conditions inside the supernova core cover interesting regions of the phase diagram. The BB and CB trajectories pass through the multifragmentation region, which is explored in heavy-ion collisions at intermediate energies. The trajectories CB and PB traverse the phase boundary between the mixture of nuclei and the nuclear gas. This region is dominated by nucleons and light nuclei ($n, p, d, ^3\text{He}, ^4\text{He}$). It is well known (see, e.g., Arcones et al. 2008) that dynamics of the shock wave are strongly affected by the neutrino-induced reactions on nucleons and nuclei. Therefore, it is very important to determine the realistic composition of hot stellar matter in this region. This can be done only by considering the full ensemble of nuclear species without artificial constraints (Ishizuka et al. 2003; Botvina & Mishustin 2004). The nuclear composition and thermodynamic properties of nuclear matter under supernova conditions were studied recently within different approaches (Botvina & Mishustin 2010; Hempel & Schaffner-Bielich 2010; Hempel et al. 2012; Typel et al. 2010; Hempel et al. 2011; Sumiyoshi & Röpke 2008; Buyukcizmeci et al. 2013).

2. STATISTICAL MODEL FOR SUPERNOVA MATTER

The Statistical Model for Supernova Matter (SMSM) was developed in (Botvina & Mishustin 2004, 2010) as a direct generalization of the Statistical Multifragmentation Model (SMM; Bondorf et al. 1995). The SMM was successfully used for description of nuclear multifragmentation reactions (Ogul et al. 2011; Botvina et al. 1995; Scharenberg et al. 2001; D’Agostino et al. 1996, 1999; Bellaize et al. 2002; Avdeyev et al. 2002; Hauger et al. 2000; Iglio et al. 2006; Hudan et al. 2009; Wang et al. 1999; Viola et al. 2001). This gives us confidence that this model can realize a realistic approach to clustered nuclear matter under astrophysical conditions, as discussed in Section 1. We treat supernova matter as a mixture of nuclear species, electrons, and photons in statistical equilibrium. Below, we briefly present the model with the latest modifications.

2.1. Equilibrium Conditions

For describing stellar matter, we adopt the grand canonical approximation where macroscopic states are characterized by temperature $T$ and chemical potentials of the constituents. The chemical potentials for nuclear species $(A, Z)$ are expressed as

$$\mu_{AZ} = A\mu_B + Z\mu_Q. \quad (1)$$

For protons, $\mu_p = \mu_p + \mu_Q$, and for neutrons, $\mu_n = \mu_B$. The chemical potentials $\mu_B$ and $\mu_Q$ are found from the conservation laws for baryon number $B$ and electric charge $Q$ in the normalization volume $V$:

$$\rho = \frac{B}{V} = \sum_{AZ} A\rho_{AZ},$$

$$\rho_Q = \frac{Q}{V} = \sum_{AZ} Z\rho_{AZ} - \rho_e = 0. \quad (2)$$

Here, $\rho_{AZ}$ is the average density of a nuclear species with mass $A$ and charge $Z$ (see below); $\rho_e = \rho_e - \rho_e$ is the net electron density. The second equation requires any macroscopic volume of the star $(V)$ to be electrically neutral.

The lepton number conservation is a valid concept only if $\nu$ and $\bar{\nu}$ are trapped in the matter within the neutrino-sphere (Prakash et al. 1997). If they escape freely from the star, the lepton number conservation is irrelevant and $\mu_L = 0$. In this case, the two remaining chemical potentials are determined by Equations (2). In the $\beta$-equilibrated matter the electron chemical potential $\mu_e$ is found from the condition $\mu_e = \mu_p - \mu_n$. However, $\beta$ equilibrium may not be achieved in a fast explosive process. In this case, it is more appropriate to perform calculations for fixed values of electron fraction $Y_e$. Then, $\mu_e$ is determined from the given electron density $\rho_e = Y_e\rho$.

2.2. Ensemble of Nuclear Species

The nuclear component of stellar matter is represented as a mixture of gases of different species $(A, Z)$, including nuclei and nucleons. It is convenient to introduce the numbers of particles of different kind $N_{AZ}$ in a normalization volume $V$. In SMSM, we use the grand canonical version of the SMM formulated in Botvina et al. (1985), and developed further in Botvina et al. (2002). The corresponding thermodynamic potential of the system can be expressed as

$$\Omega(T, \mu_B, \mu_Q, [N_{AZ}], V) = E_{\nu}(T, [N_{AZ}], V) + \sum_{AZ} N_{AZ}F_{AZ}(T, \rho) - \mu_B \sum_{AZ} AN_{AZ} - \mu_Q \sum_{AZ} ZN_{AZ}, \quad (3)$$

where the first term accounts for the translational degrees of freedom of nuclear fragments and the second term
is associated with their internal degrees of freedom. Assuming the Maxwell–Boltzmann statistics for all nuclear species, including nucleons, the translational free energy can be explicitly written as

$$F^T(T, \{N_{AZ}\}, V) = -T \sum_{AZ} N_{AZ} \left\{ \ln \left( \frac{g_{AZ} V_f A^{3/2}}{N_{AZ} \lambda_T^3} \right) + 1 \right\},$$

where $g_{AZ}$ is the ground-state degeneracy factor for species $(A, Z)$, $\lambda_T = (2\pi/m_N T)^{1/2}$ is the nuclear thermal wavelength, $m_N \approx 939$ MeV is the average nucleon mass, and $V_f$ is so-called free volume of the system, which accounts for the finite size of nuclear species and is only a fraction of the total volume $V$. We assume that all nuclei with $A > 4$ have normal nucleon density $\rho_0 \approx 0.15$ fm$^{-3}$ so that the proper volume of a nucleus with mass $A$ is $A/\rho_0$. At the relatively low densities considered here, one can adopt the excluded volume approximation, $V_f/V \approx (1 - \rho/\rho_0)$. This approximation is commonly accepted in statistical models (see the extended discussion of this problem in Sagun et al. 2014). Certain information about the free volume in multifragmentation reactions has been extracted from analysis of experimental data (Scharenberg et al. 2001).

### 2.3. The Internal Free Energy of Fragments

The internal excitations of nuclei play an important role in regulating their abundances since they significantly increase their entropy. Some authors (see, e.g., Ishizuka et al. 2003) limit the excitation spectrum by particle-stable levels known for low excited nuclei. Within the SMM, we follow quite different philosophy motivated by experimental investigations of nuclear disintegration reactions. Namely, we assume that excited states are populated according the internal temperature of nuclei, which is assumed to be the same as the temperature of the surrounding medium. In this case, not only particle-stable states but also particle-unstable states will contribute to the excitation energy and entropy. This assumption can be justified by the dynamical equilibrium between emission and absorption processes in the hot medium. Moreover, in the supernova environment, both the excited states and the binding energies of nuclei may be strongly affected by the surrounding matter. For this reason, we find it more appropriate to use an approach which can easily be generalized to include in-medium modifications. Namely, the internal free energy of species $(A, Z)$ with $A > 4$ is parameterized in the spirit of the liquid drop model, which has been proven to be very successful in nuclear physics:

$$F_{AZ}(T, \rho) = F^B_{AZ} + F^S_{AZ} + F^{sym}_{AZ} + F^{C}_{AZ}. \quad (5)$$

Here, the right-hand side contains, respectively, the bulk, the surface, the symmetry, and the Coulomb terms. The first three terms are taken in the standard form (Bondorf et al. 1995),

$$F^B_{AZ}(T) = -\frac{w_0 - T^2}{\epsilon_0} A, \quad (6)$$

$$F^S_{AZ}(T) = \beta_0 \frac{\left( T_0^2 - T^2 \right)}{T_0^2 + T^2} A^{5/4}, \quad (7)$$

$$F^{sym}_{AZ} = \gamma \frac{(A - 2Z)^2}{A}. \quad (8)$$

where $w_0 = 16$ MeV, $\epsilon_0 = 16$ MeV, $\beta_0 = 18$ MeV, $T_0 = 18$ MeV, and $\gamma = 25$ MeV are the model parameters which are extracted from nuclear phenomenology and provide a good description of multifragmentation data (Bondorf et al. 1995; Botvina et al. 1995; Scharenberg et al. 2001; D’Agostino et al. 1996, 1999; Bellaize et al. 2002; Avdeyev et al. 2002). However, these parameters, especially the symmetry coefficient $\gamma$, may be different in hot nuclei, and therefore they should be determined from corresponding experimental data (see the discussions in Botvina et al. 2006, 2002; Buyukcizmeci et al. 2008; Ogul et al. 2011).

In the electrically neutral environment, the nuclear Coulomb term should be modified to include the screening effect of electrons. This can be done, e.g., within the Wigner–Seitz approximation as was proposed in Lamb et al. (1981) and Lattimer et al. (1985). One should imagine that the whole system is divided into spherical cells each containing one nucleus. The radius of the cell is determined by the condition that it contains the same number of electrons as the number of protons in the nucleus. The interaction between the cells is neglected. Then, assuming a constant electron density, one obtains

$$F^C_{AZ}(\rho) = \frac{3}{5} \epsilon Z^2 \rho_0 A^{2/3},$$

$$\epsilon = 1 - \frac{3}{2} \frac{\rho_e}{\rho_0} \frac{\rho_e}{\rho_0}^{1/3} + \frac{1}{2} \left( \frac{\rho_e}{\rho_0} \right), \quad (9)$$

where $\rho_0 = 1.17$ fm, $\rho_e = Y_e \rho$ is the average electron density, and $\rho_0 = (Z/A) \rho_0$ is the proton density inside the nucleus. The screening function $\epsilon$ is 1 at $\rho_e = 0$ and at $\rho_e = \rho_0$. In fact, in this work, all presented results are obtained with the approximation $\rho_e/\rho_0 \approx \rho_0/\rho_0$, as in Lattimer et al. (1985), which works well when neutrons are mostly bound in nuclei so that $\rho_0 \approx Y_e \rho_0$. Although this approximation gives satisfactory results in many cases, it may deviate at very high temperatures and very low baryon densities. However, under these conditions, the nuclei are getting smaller and the Coulomb interaction effects become less important. Generally, the reduction of the Coulomb energy due to electron screening favors the formation of heavy nuclei.

In the SMM nucleons and light nuclei ($A \leq 4$) are considered as structureless particles characterized only by exact masses, proper volumes, and spin degeneracy factors $g_{AZ}$ (Bondorf et al. 1995): for nucleons, $g_A = g_p = 2$; for deuterons, $g_{21} = 3$; for $^3$H, $g_{31} = 2$; for $^3$He, $g_{32} = 2$; for $^4$He, $g_{41} = 1$. Their Coulomb interaction is taken into account within the same Wigner–Seitz approximation. For all nuclear species with $A > 4$, we use $g_{AZ} = 1$, but include internal excitations.

### 2.4. Thermodynamical Quantities

Within the grand canonical approximation, the conservation law Equations (2) are fulfilled only for the mean densities of nuclear species $\rho_{AZ} = \langle N_{AZ}/V \rangle$. The mean fragment numbers $\langle N_{AZ} \rangle$ in volume $V$ are found by minimizing the thermodynamic potential Equation (3) with respect to fragment multiplicities $\{N_{AZ}\}$ at fixed $T$ and $V$. In the lowest-order approximation, this gives

$$\langle N_{AZ} \rangle = g_{AZ} \frac{V_f}{\lambda_T^3} A^{3/2} \exp \left[ -\frac{1}{T} (F_{AZ} - \mu_{AZ}) \right]. \quad (10)$$

where $\mu_{AZ}$ is defined by Equation (1). The corrections to this expression become significant at higher densities and low temperatures when heavy nuclei are abundant.

\[\text{Here and below, we use units with } \hbar = c = 1.\]
The pressure associated with nuclear species can be calculated by differentiating Equation (3) with respect to V at fixed T and \(|N_{AZ}|\), which gives

\[
P_{\text{nuc}} = P_u + P_C,
\]

where first term comes from the translational motion of fragments and the second term from the density-dependent Coulomb interaction. Their explicit expressions are

\[
P_u = T \sum_{AZ} \rho_{AZ} \, T, \quad P_C = \rho \sum_{AZ} \rho_{AZ} \frac{\partial F_{AZ}}{\partial \rho}.
\]

As one can see from Equation (9), the Coulomb term gives a negative contribution to pressure (Baym et al. 1971).

In Figure 2, we present the translational pressure \(P_u\), Equation (12), caused by single nucleons and nuclei. The total nuclear pressure \(P_{\text{nuc}}\) is also shown in Figure 2 as a function of temperature at \(T = 5 \text{ MeV}\), when matter nearly completely dissociates into nucleons and lightest clusters, \(P_C\) is close to zero. In this region, the total nuclear pressure coincides with the pure nuclear pressure. The Coulomb pressure becomes very important when heavy clusters dominate in the system. One can see in Figure 2 that at low temperatures and high density, the total nuclear pressure may be negative. In this case, the nuclear clusterization is favorable for the collapse. However, the positive pressure of the relativistic electron Fermi gas is considerably (more than an order of magnitude) larger.

Finally, the entropy of nuclear species is calculated by differentiating Equation (3) with respect to T at fixed V and \(|N_{AZ}|\), which gives

\[
S_{\text{nuc}} = \sum_{AZ} N_{AZ} \left[ \ln \left( \frac{g_{AZ} V_f A^{3/2}}{\gamma_{AZ}^{3/2} T} \right) + \frac{5}{2} \right] - \sum_{AZ} N_{AZ} \frac{\partial F_{AZ}}{\partial T}.
\]

Here, the first term comes from the translational motion, and the second term is from internal excitations of fragments.

As follows from Equation (10), the fate of heavy nuclei depends strongly on the relationship between \(F_{AZ}\) and \(\mu_{AZ}\). In order to avoid an exponentially divergent contribution to the baryon density, at least in the thermodynamic limit (\(A \to \infty\)), inequality \(F_{AZ} \geq \mu_{AZ}\) must hold. The equality sign here corresponds to the situation when a large (infinite) nuclear fragment coexists with the gas of smaller clusters (Bugaev et al. 2001). When \(F_{AZ} > \mu_{AZ}\), only small clusters with a nearly exponential mass spectrum are present. However, there exists a region of thermodynamic quantities corresponding to \(F_{AZ} \approx \mu_{AZ}\) when the mass distribution of nuclear species is close to a power-law \(A^{-\tau}\) with \(\tau \approx 2\). This is a characteristic feature of the liquid-gas phase transition. The advantage of our approach is that we consider all of the fragments present in this transition region and, therefore, can study this phase of nuclear matter in full detail.

2.5. Leptons and Photons

We assume that besides nuclear species, the supernova matter also contains electrons, positrons, and photons.\(^{7}\) At \(T, \mu_e > m_e\), the pressure of the relativistic electron–positron gas can be written as

\[
P_e = \frac{g_e \mu_e^4}{24 \pi^2} \left[ 1 + \frac{2}{\pi} \left\{ \left( \frac{\pi T}{\mu_e} \right)^2 + \frac{7}{15} \left( \frac{\pi T}{\mu_e} \right)^4 - \frac{m_e^2}{\mu_e^2} \left( 3 + \left( \frac{\pi T}{\mu_e} \right)^2 \right)^2 \right\} \right],
\]

where first-order corrections (~\(m_e^2\)) due to the finite electron mass are included and \(g_e = 2\) is the spin degeneracy factor for electrons. Due to the correction terms of order \((m_e/\pi T)^2\) and \((m_e/\mu_e)^2\), these expressions can be used even at \(T, \mu_e\) of the order of \(m_e\). At these condition, the contributions of electrons to the pressure and entropy become negligible.

The corresponding expressions for net number density \(\rho_e\) and entropy density \(s_e\) are obtained from standard thermodynamic relations, \(\rho_e = \partial P_e/\partial \mu_e\) and \(s_e = \partial P_e/\partial T\), which give

\[
\rho_e = \frac{g_e \mu_e^3}{6 \pi^2} \left[ 1 + \frac{1}{\mu_e} \left( \frac{\pi^2 T^2}{2} - \frac{3}{2} m_e^2 \right) \right],
\]

\[
s_e = \frac{g_e T \mu_e^2}{6} \left[ 1 + \frac{7}{15} \left( \frac{\pi T}{\mu_e} \right)^2 - \frac{m_e^2}{2 \mu_e^2} \right].
\]

The photons are always close to the thermal equilibrium, and they are treated as massless Bose gas with zero chemical potential. The corresponding density \(\rho_\gamma\), energy density \(e_\gamma\), pressure \(P_\gamma\), and entropy density \(s_\gamma\) of photons are given by the standard formulae

\[
\rho_\gamma = \frac{g_\gamma \xi(3) T^3}{\pi^2}, \quad e_\gamma = \frac{g_\gamma \pi^2 T^4}{30}, \quad P_\gamma = \frac{e_\gamma}{3}, \quad s_\gamma = \frac{4 e_\gamma}{3 T},
\]

\(^{7}\) As has been already mentioned in Section 2.1, here, we do not consider the situation when neutrinos also participate in statistical equilibrium.
where $g_\nu = 2$. A possible participation in the statistical ensemble of $\nu_e, \bar{\nu}_e, \mu, \nu_\mu$, and $\bar{\nu}_\mu$ is not considered in this work, so the current SMSM tables do not include these particles. However, the computer SMSM code includes the options of $\beta$ equilibrium, as well as the full lepton (including neutrino) conservation; therefore, such calculations are possible (see Botvina & Mishustin 2004, 2010).

The SMSM was realized with a generalization of the special computer code developed previously for the grand canonical calculations of the full ensemble of nuclear species, as performed in Botvina et al. (2002). All kinds of particles contribute to the free energy, pressure, and other thermodynamical characteristics of the system, and we sum up all these contributions. The densities of all particles are calculated self-consistently by taking into account the relations between their chemical potentials. In the solution of the coupled equations, we used the step-by-step approximation method and control the found potentials with the relative precision of 0.001. We have checked that it is sufficient for our purposes. In this procedure, we directly observe how the successive steps approach the correct values during the simulations. We take baryon number $B = 1000$ and perform calculations for all fragments with $1 \leq A \leq 1000$ and $0 \leq Z \leq A$ in the ensemble. This restriction on the size of nuclear fragments is fully justified in our case since fragments with larger masses ($A > 1000$) can be produced only at very high densities $\rho \gtrsim 0.3\rho_0$ (Lamb et al. 1981; Lattimer et al. 1985), which are appropriate for the regions deep inside the protoneutron star. Such nuclei, as well as the “pasta” phase at the high densities, are not considered here.

3. THE SMSM EOS TABLES

We have constructed the SMSM EOS tables, which are available at http://fias.uni-frankfurt.de/en/physics/mishus/research/smssm/. Below, we give detailed information about the format of these tables and their possible applications.

3.1. Description of the SMSM EOS Tables

The SMSM EOS tables cover the following ranges of control parameters:

1. Temperature: $T = 0.2–25$ MeV; for 35 $T$ values.
2. Electron fraction $Y_e$: 0.02–0.56; linear mesh of $Y_e = 0.02$, giving 28 $Y_e$ values. It is equal to the total proton fraction $X_p$, due to charge neutrality.
3. Baryon number density fraction $\rho/\rho_0 = (10^{-8}–0.32)$, giving 31 $\rho/\rho_0$ values.

In our calculations, we consider all nuclear species with $1 \leq A \leq 1000$ and $0 \leq Z \leq A$. These restrictions on fragment size are justified within the above-defined intervals of control parameters. The different $T$ (35 points), $\rho/\rho_0$ (31 point), and $Y_e$ (28 points) values are listed in Table 1. They sum up to 30,380 different grid points. We believe that physical conditions for the SMSM EOS are sufficiently well represented by this grid. Running time for calculations was approximately 7.8 days (Work station: 4x Dual core AMD 2.2 GHz processor). We have obtained the file SMSM-EOS-Tables.txt (datafile2.txt, size 6.8 MB).

We can certainly say that our model is not applicable at densities above $0.3\rho_0$ and temperatures above 25 MeV. At higher densities, the matter will be rearranged into more complicated geometrical structures known as “pasta” phases (Lamb et al. 1981; Newton & Stone 2009). We are not considering them in our present work. The assumption of statistical equilibrium also requires that the temperature is high enough to allow nuclear transformations leading to this equilibrium. As is commonly believed, this could be a good approximation at $T > 1$ MeV. Nevertheless, for completeness, we extend our calculations to $T = 0.2$ MeV, when the statistical equilibrium could be problematic.

The information is stored in a format that is very similar to the tables of Shen et al. (1998) or Hempel & Schaffner-Bielich (2010) or Furusawa et al. (2011) so that it can easily be implemented in running codes. The SMSM EOS tables (“SMSM-EOS-Tables.txt”) is the main file which is available in the online journal.

In the beginning of file, we write the general parameters of the liquid-drop description: $w_0 = 16$ MeV, $\beta_0 = 18$ MeV, $T_c = 18$ MeV, $\gamma = 25$ MeV, $\epsilon_0 = 16$ MeV, $r_0 = 1.17$ fm. The tables are written in the order of increasing $T$. For each $T$, we present the results in the order of increasing $Y_e$ and $\rho/\rho_0$ (see Table 1). The first three lines in the SMSM EOS tables are shown in Table 2 for guidance regarding its form and content. Each line of the table corresponds to one density grid point.

### Table 1

| $T$ (MeV) | $Y_e$ | $\rho/\rho_0$ |
|-----------|-------|----------------|
| 0.20      | 0.02  | 0.00–0.56      |
| 1.75      | 0.04  | 0.00–0.56      |
| 3.50      | 0.06  | 0.00–0.56      |
| 5.25      | 0.08  | 0.00–0.56      |
| 8.00      | 0.10  | 0.00–0.56      |
It contains 21 different thermodynamic quantities which are defined as follows.

1. \( T \), temperature (MeV),
2. \( Y_e \), electron fraction: \( Y_e = \rho_e / \rho \),
3. \( \rho / \rho_0 \), density fraction,
4. \( X_n \), the mass fraction of free neutrons is given by \( X_n = \rho_n / \rho \),
5. \( X_p \), the mass fraction of free protons: \( X_p = \rho_p / \rho \),
6. \( X_d \), the mass fraction of deuterons \((A = 2, Z = 1)\) is given by \( X_d = 2 \rho_d / \rho \),
7. \( X_t \), the mass fraction of tritons \((A = 3, Z = 1)\) is given by \( X_t = 3 \rho_t / \rho \),
8. \( X_{He} \), the mass fraction of heliums \((A = 4, Z = 2)\) is given by \( X_{He} = 3 \rho_{He} / \rho \),
9. \( X_{alpha} \), the mass fraction of alpha particles \((A = 4, Z = 2)\) is given by \( X_{alpha} = 4 \rho_{alpha} / \rho \),
10. \( X_{heavy} \), the mass fraction of heavy nuclei \((A > 4, Z > 2)\) is defined by 
    \[ X_{heavy} = \frac{\sum_{A>4,Z>2} A \rho_{AZ}}{\rho} \],
11. \( \langle A_{heavy} \rangle \), the average mass number of heavy nuclei \((A > 4, Z > 2)\) is defined by 
    \[ \langle A_{heavy} \rangle = \frac{\sum_{A>4,Z>2} A \rho_{AZ}}{\sum_{A>4,Z>2} \rho_{AZ}} \],
    \( \langle A_{heavy} \rangle \) is set to zero if \( X_{heavy} = 0 \).
12. \( \langle Z_{heavy} \rangle \), the average charge number of heavy nuclei \((A > 4, Z > 2)\) is defined by 
    \[ \langle Z_{heavy} \rangle = \frac{\sum_{A>4,Z>2} Z \rho_{AZ}}{\sum_{A>4,Z>2} \rho_{AZ}} \],
13. \( E_{tot} \), total energy, can be written as the sum of nuclear \( E_{nuc} \), electron \( E_e \), and photon \( E_{\gamma} \) energy contributions (MeV nucleon\(^{-1}\)), 
    \[ E_{tot} = E_{nuc} + E_e + E_{\gamma} \],
14. \( E_{nuc} \), the energy of nuclear species (MeV nucleon\(^{-1}\)), is calculated as \( F + TS \), using Equations (4), (5), and (13).
15. \( S_{tot} \), total entropy, is the sum of nuclear \( S_{nuc} \), electron \( S_e \), and photon \( S_{\gamma} \) entropy contributions (1/nucleon), 
    \[ S_{tot} = S_{nuc} + S_e + S_{\gamma} \].
16. \( S_{nuc} \), nuclear entropy (1/nucleon) is given by Equation (13).
17. \( P_{tot} \), total pressure, is the sum of the pressure of nuclear species \( P_{nuc} \) (Equation (11)), electrons \( P_e \) (Equation (14)), and photons \( P_{\gamma} \) (Equation (17); MeV fm\(^{-3}\)), 
    \[ P_{tot} = P_{nuc} + P_e + P_{\gamma} \],
18. \( P_{nuc} \), total nuclear pressure, is the sum of translational pressure and Coulomb pressure (Equation (12); MeV fm\(^{-3}\)),
19. \( \mu_e \), chemical potential of electrons (MeV),
20. \( \mu_B \), chemical potential of baryons (MeV),
21. \( \mu_Q \), chemical potential of charged particles (MeV).

The thermodynamic consistency of the SMSM EOS table is guaranteed by using the grand canonical ensemble with the thermodynamical potential (Equation (3)). This approximation should be good for the large volumes of matter considered here. The numerical accuracy of our calculations is typically better than 1%. The mass conservation and electrical neutrality conditions are especially checked. For example, the mass fractions of the different particle species sum up to unity: 
\[ \delta_X = 1 - X_n + X_p + X_d + X_t + X_{He} + X_{alpha} + X_{heavy} \].
\( \delta_X \) is found around 0.1%.

### 3.2. Application of the EOS Tables and Illustrative Results

In addition to standard output, i.e., the 21 different thermodynamic quantities listed above, such as the pressure, entropy, energy, fractions of light particles, and heavy nuclei (actually, in Shen et al.’s (1998) case, a single heavy nucleus), it is possible to determine the ensemble-averaged densities of all heavy nuclei, \( \rho_{AZ} \). This is achieved by first finding the chemical potentials \( \mu_B \) and \( \mu_Q \) and then using Equation (10). This formula is valid for all \((T, \rho, Y_e)\) values and can be applied for nuclei with all possible mass numbers and charges. We believe that the knowledge about full nuclear ensemble could be quite helpful for accurate calculations of weak reactions with neutrinos and electrons in supernova simulations.

The mass distributions contain important information about the composition of nuclear matter in the nuclear liquid-gas phase transition (coexistence region). The concept of statistical equilibrium assumes an intensive interaction between the fragments.
via specific microscopic processes, such as absorption and emission of neutrons, which provide equilibration (see, e.g., the discussion in Botvina & Mishustin 2010). In this situation, the nuclei may remain hot and have modified properties, such as binding energies, excited states, etc., which differ from those in cold, isolated nuclei. We note that the data file for mass distributions “SMSM-EOS-Mass-Distribution-Tables.txt” (datafile3.txt) is also stored for each calculation. This file is also available in a machine-readable form on the online journal and on the Web page of SMSM. The file size is 319 MB, it contains 30,380 calculations, and every line has 1003 columns. The first three columns show \( T \), \( Y_e \), and \( \rho/\rho_0 \), and columns 4–1003 show the number of mass yields per nucleon (relative yield) for \( A = 1–1000 \). Data are presented for \( 35T \), 28 \( Y_e \), and 31 \( \rho/\rho_0 \) values with the same order as in Table 1. The first 3 lines and the first 10 columns in “SMSM-EOS-Mass-Distribution-Tables” are shown in Table 3.

Since we have \( 35(T) \times 28(Y_e) = 980 \) values for 31 \( \rho/\rho_0 \) values, we grouped four density intervals from lower to higher as

1. Density-1 \( = \rho/\rho_0 = 10^{-8}–5.62 \times 10^{-7} \) (eight values),
2. Density-2 \( = \rho/\rho_0 = 10^{-10}–5.62 \times 10^{-9} \) (eight values),
3. Density-3 \( = \rho/\rho_0 = 10^{-12}–5.62 \times 10^{-11} \) (eight values),
4. Density-4 \( = \rho/\rho_0 = 10^{-14}–3.17 \times 10^{-14} \) (seven values).

For these intervals, we have plotted 3920 figures as EPS files containing information about mass distributions for each grid point. We have presented example figures for \( T = 1, 3 \) MeV, \( Y_e = 0.2–0.4 \), and different densities in Figures 3–6. As seen in these figures, we also put \( X_n \), \( X_p \), \( X_{\alpha} \), \( X_{\text{heavy}} \), \( \langle A_{\text{heavy}} \rangle \), \( \mu_B \), and \( \mu_Q \) values from tables with the same order of \( \rho/\rho_0 \) as the control parameters inside of figures. These figures are presented on the SMSM Web page with files

1. SMSM-Massdist-Figs-Density-1.zip,
2. SMSM-Massdist-Figs-Density-2.zip,
3. SMSM-Massdist-Figs-Density-3.zip,
4. SMSM-Massdist-Figs-Density-4.zip.

Each file contains 980 EPS figures. For example, inside SMSM-Massdist-Figs-Density-4.zip, one can find the mass distribution figure for \( T = 1 \) MeV, \( Y_e = 0.20 \), and \( \rho/\rho_0 = 10^{-2}–3.17 \times 10^{-1} \) in T-1-Ye-020-DENSITY-4.eps, as shown in Figure 3.

At very low temperatures, the SMSM predicts a Gaussian-like distribution for heavy nuclei as seen in Figures 3 and 4. In this case, an approximation of a single heavy nucleus adopted in the EOS by Lattimer & Swesty (1991) and Shen et al. (1998) may work reasonably well for calculations of thermodynamical characteristics of matter. However, at \( T \geq 1 \) MeV, the gap between the Gaussian peak and light clusters and nucleons is essentially filled by nuclei of intermediate masses.

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**Table 3**

SMSM-EOS-mass-distribution Tables

| Column 1 | Column 2 | Column 3 | Column 4 | Column 5 |
|----------|----------|----------|----------|----------|
| \( T(\text{MeV}) \) | \( Y_e \) | \( \rho/\rho_0 \) | \( Y_1 \) | \( Y_2 \) |
| 0.20     | 0.02     | 0.100E-07| 0.9447E+00| 0.1248E-35 |
| 0.20     | 0.02     | 0.178E-07| 0.9447E+00| 0.4504E-36 |
| 0.20     | 0.02     | 0.316E-07| 0.9442E+00| 0.1609E-36 |

| Column 6 | Column 7 | Column 8 | Column 9 | Column 10 |
|----------|----------|----------|----------|-----------|
| \( Y_3 \) | \( Y_4 \) | \( Y_5 \) | \( Y_6 \) | \( Y_7 \) |
| 0.2067E-26| 0.7178E-25| 0.0000E+00| 0.0000E+00| 0.0000E+00 |
| 0.1327E-26| 0.1667E-25| 0.0000E+00| 0.0000E+00| 0.0000E+00 |
| 0.8426E-27| 0.3797E-26| 0.0000E+00| 0.0000E+00| 0.0000E+00 |

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**Figure 3.** Mass distributions of nuclear fragments (yields per nucleon or relative yields) produced in stellar matter with temperature \( T = 1 \) MeV, electron fraction \( Y_e = 0.2 \), and at several densities \( \rho/\rho_0 = 10^{-2}–3.17 \times 10^{-1} \). (A color version of this figure is available in the online journal.)
leading to characteristic U-shaped distributions. The examples of such distributions are shown in Figures 5 and 6. They give a typical evolution of mass distributions of nuclei in the coexistence region of the liquid-gas phase transition. Previously, it was demonstrated for disintegration of finite nuclei (Bondorf et al. 1995). These distributions continuously transform with decreasing density as well as with increasing temperatures into exponential falloff of fragment yields as functions of $A$.

Using SMSM EOS tables and Equation (10), it is also possible to obtain isotopic distributions of stellar matter. Examples of isotopic distributions for $Z = 8$ and $Z = 26$ are shown in Figure 7. As seen from the figure, the SMSM predicts Gaussian-type distributions, which is a consequence of the liquid-drop description of fragments. Isotopic yields help to understand the trends observed for summed quantities such as mass yields or mass fractions. They are also needed...
to calculate average $\langle Z \rangle$ values, which are listed in the tables. This information is important for realistic calculations of the weak reactions with electrons and neutrinos.

Recently, in Buyukcizmeci et al. (2013), we have compared mass and isotope distributions of the SMSM (Botvina & Mishustin 2010) with predictions of two other models by Hempel & Schaffner-Bielich (2010) and Furusawa et al. (2011), under conditions expected during the collapse of massive stars and supernova explosions. Recently, we have found significant differences between mass distributions predicted by these three models, which use different assumptions on properties of hot fragments in a dense environment, especially at low electron fractions, low temperatures, and high densities. We note that the differences are also present in the behavior of nuclear pressure (see Figure 22 in Buyukcizmeci et al. 2013).

Indeed, the main assumption of the SMSM is that most interaction effects are included into the internal binding energy of the fragments. From that, we also take into account the Coulomb energy and excluded volume effects. As one can see from the tables, the mass fraction of single nucleons is rather small (less than 20%) at baryon densities around $0.1\rho_0$ and electron fractions around 0.2. Their actual density is therefore only $0.02\rho_0$. The interaction effects are rather weak at such a low density (see, e.g., Talahmeh & Jaqaman 2013). At higher densities and lower electron fractions, the share of unbound nucleons becomes even larger and their interaction may become more important (Typel et al. 2010). But even at baryon densities around $0.3\rho_0$ and temperatures $T \sim 1$ MeV, most nucleons are bound in clusters, and most interaction effects come from the nuclear binding energy. Exactly in this domain of parameters, different models predict very different nuclear ensembles (see Buyukcizmeci et al. 2013). Certainly, improving the EOS calculations in this region is an important issue for future studies. Recently, Furusawa et al. (2013) have made an attempt to improve the description of nucleons and light nuclei in supernova matter.

### 3.3. Nuclear Composition Under Condition of $\beta$ Equilibrium

An important special case when full $\beta$ equilibrium is reached in stellar matter corresponds to the condition $\mu_\beta = \mu_\beta - \mu_p$. It is expected to be fulfilled in certain situations, such as the slow collapse of a massive star, late stages of a supernova explosion, or in crusts of neutron stars. On the other hand, the $\beta$ equilibrium can be a useful physical limit for theoretical estimates of the nuclear composition without full knowledge of weak reactions. The SMSM allows for this kind of calculations (Botvina & Mishustin 2004, 2010) and the corresponding tables of matter properties at baryon densities $\rho = (10^{-5} - 0.3)\rho_0$ and temperatures $T = 0.2 - 25$ MeV are under construction.

At a given $\rho$ and $T$, the electron fraction $Y_e$ can be evaluated iteratively to fulfill the above-mentioned condition.

In Figure 8, we show the calculated electron fraction, the chemical potential of electrons $\mu_e$, and the average mass number of heavy nuclei $\langle A_{\text{heavy}} \rangle$ ($A > 4$) under the $\beta$-equilibrium condition as a function of temperature for various densities. Figure 8 gives valuable information about the composition of stellar matter under the condition of $\beta$ equilibrium. In Figure 8(a), one can see an interesting behavior, i.e., a non-monotonic change of $Y_e$ with increasing temperature, which has a simple explanation. As follows from Figure 8(b), electrons have larger chemical potentials at higher densities ($\rho = 0.1 - 0.3\rho_0$; Figure 8(b)), where heavy nuclei (Figure 8(c)) can survive even at high temperatures $T = 6 - 8$ MeV. That is why the fraction of electrons $Y_e$ increases slowly with temperature. On the other hand, at lower densities ($\rho \lesssim 0.01\rho_0$), the heavy nuclei can exist only at low temperatures, and they disintegrate into free nucleons and light clusters at temperatures $T > 3 - 5$ MeV.
Figure 7. Isotopic distributions of \( Z = 8 \) and \( Z = 26 \) fragments produced in matter with temperatures \( T = 1, 2, 3, \) and 5 MeV; electron fractions \( Y_e = 0.2 \) and 0.4; and densities \( \rho / \rho_0 = 10^{-3} - 10^{-1} \).
However, in Figure 8(a), one can see that $Y_e$ exhibits a minimum at $T \approx 2$ MeV for $10^{-3}/10^{-2} \rho_0$ and $T = 3/4$ MeV for $(10^{-3}/10^{-2}) \rho_0$. As follows from Figure 8(c), these temperatures correspond to the transition from heavy nuclei to free nucleons and light clusters. At higher densities ($\rho = (0.3/0.1) \rho_0$), when heavy nuclei are present in the ensemble even at higher temperatures, the electron fraction grows slowly with the temperature, reaching $(10/12)\%$ at $T = 10$ Mev. These results reveal an interesting correlation between the nuclear composition and electron fraction in $\beta$-equilibrated stellar matter.

4. CONCLUSIONS

We have presented the SMSM EOS tables that give comprehensive information on physical properties of stellar matter at subnuclear densities, temperatures $T < 25$ MeV, and electron fractions between 0.02 and 0.56. The SMSM EOS can be used for hydrodynamical simulations of massive star collapse and supernova explosion processes. We consider the whole ensemble of nuclear species, without any artificial constraints on their masses and charges, as well as thermodynamical characteristics of matter, such as temperature and chemical potentials. The nuclear mass and isotope distributions are needed for realistic calculations of electron capture and neutrino scattering processes in supernova environments. The SMSM is directly linked to nuclear multifragmentation reactions, which allow one to study in a laboratory the properties of nuclei embedded in hot and dense surrounding. The present tabulated calculations are performed at the standard parameters of nuclei. However, as was demonstrated previously (see Botvina & Mishustin 2004, 2010), modifications of these parameters, in particular, the symmetry energy of nuclei extracted from multifragmentation reactions, can essentially change the nuclear composition and influence the rate of weak reactions. Our calculations clearly demonstrate that the nuclear composition is very sensitive to temperature, density, and electron fraction of stellar matter. A special case of $\beta$-equilibrated nuclear matter is also considered. In this case, we have found non-monotonous behavior of the electron fraction as a function of temperature at $\rho < 0.2 \rho_0$. This effect is connected with the rearrangement of the fragment mass distribution from U shape to exponential fall-off. These results can be used for modeling the outer layers of proto-neutron stars and crusts of neutron stars. We believe that the comparison of our results with predictions of other models, as was done in Buyukcizmeci et al. (2013), will help to better understand properties of hot and dense stellar matter.

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