SECOND OVERTONE PULSATORS AMONG δ SCUTI STARS

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ABSTRACT

We investigate the modal stability of stellar models at masses and luminosity levels corresponding to post–main-sequence luminous δ Scuti pulsators. The envelope models have been computed at fixed mass value ($M/M_\odot = 2.0$), luminosity level [$\log (L/L_\odot) = 1.7$], and chemical composition ($Y = 0.28$, $Z = 0.02$). According to a nonlinear approach to radial oscillations, the present investigation for the first time predicts the occurrence of stable second overtone pulsators. More generally, we found that when moving inside the instability strip from lower to higher effective temperatures, the models show a stable limit cycle in three different pulsation modes: fundamental and first and second overtones. The shapes of both light and velocity curves are presented and discussed, providing a useful tool for the identification of second overtone pulsators among the known groups of radially pulsating stars.

Comparison with observations shows that our nonlinear, nonlocal, and time-dependent convective models provide light curves in agreement with observed values, suggesting that second overtone pulsators have already been observed, though misclassified as fundamental pulsators. In a limited region of the instability strip we also found some models presenting mixed-mode features, i.e., radial pulsators that show a stable limit cycle in more than one pulsational mode.

The period ratios of mixed-mode pulsators obtained by perturbing the first and the second overtone radial eigenfunctions are in agreement with observed values. This result is a crucial point for understanding the pulsation properties of δ Scuti stars, since it provides sound evidence that these variables during their evolution off the main sequence are pure or mixed-mode radial pulsators. Finally, the physical structure and the dynamical properties of second overtone pulsators are discussed in detail. The role played by the nodal lines in the destabilization of second overtone pulsators is also pointed out.

Subject headings: stars: interiors — stars: oscillations — stars: variables: other (δ Scuti)

1. INTRODUCTION

The occurrence of variable stars radially pulsating in the fundamental and/or the first overtone modes is a well-known and well-established piece of observational evidence based on both Population I and Population II variables. The possible occurrence of stars pulsating in the second overtone (SO) is still a much-debated argument. For a long time the observational scenario concerning such an occurrence has been limited to the suggestions of several authors (van Albada & Baker 1973; Demers & Wehlau 1977; Clement, Dickens, & Bingham 1979; Nemec, Wehlau, & Mendes de Oliveira 1988 and references therein) that some globular cluster RRc variables (first overtone) characterized by very short periods ($P \sim 0.3$ days) and small pulsational amplitudes could be good SO candidates. Only in recent times, the large and homogeneous photometric databases collected by the MACHO project for radial pulsators in the Large Magellanic Cloud (LMC) have brought out for the first time sound evidence of the occurrence of SO pulsators in classical Cepheids (Alcock et al. 1995). On the basis of the same extensive photometric survey, from the period distribution of about 8000 RR Lyrae variables, it has been also suggested (Alcock et al. 1996) that a peculiar peak located at $P = 0.28$ days could be due to low-mass SO variables.

From a theoretical standpoint, the limit cycle stability of second overtone pulsators in Population I and Population II variable stars has been an odd problem for a long time. As a matter of fact, at present we still lack either firm theoretical predictions concerning the existence of SO pulsators, or convincing insights on the physical mechanisms which could govern their approach to mode stability. Classical nonlinear analyses of pulsation properties and modal stability of Population II low-mass variables (Christy 1966; Stellingwerf 1975) never produced any firm evidence of unstable SO pulsators. By using a simple nonlinear and nonadiabatic one-zone model, Stellingwerf, Gautschy, & Dickens (1987, hereafter SGD) succeeded in producing an unstable SO pulsator. However, even though one-zone pul-
sating models can be of some help in disclosing the main processes that constrain the physical structure of stellar envelopes, unfortunately this approach cannot supply definitive conclusions on the existence of these variables, since these models do not take into account the dynamical effects of the regions that operate the damping of the pulsation. An unstable SO model at limiting amplitude was constructed for RR variables by Stothers (1987) on the basis of a nonlinear radiative approach. However, the light curve of his model shows a peculiar feature, i.e., a deep splitting just after the maximum in luminosity, which is not observed in any known group of radial pulsators and in the light curve predicted by SGD. Moreover, no evidence of unstable SOs was found in the homogeneous and systematic survey of nonlinear, nonlocal, and time-dependent convective models provided by Bono & Stellingwerf (1994, hereafter BS) and by Bono et al. (1996a, hereafter BCCM).

According to the previous discussion, no SO pulsators appear to be foreseen in low, low-mass variable stars. In this context, it is worth emphasizing that periods of RRc variables in globular clusters are not in conflict with first overtone expectations, and their small amplitudes could be due to the sudden decrease of the first overtone amplitudes near the first overtone blue boundary (BCCM). Moreover, the peak in the period distribution at $P = 0.28$ days of RR Lyrae stars in the LMC can be taken as compelling evidence of a moderately metal-rich population, with periods that overlap the characteristic short periods already found both in the Galactic bulge and in the metal-rich field RR Lyrae stars. The reader interested in a detailed discussion on the evolutionary and pulsational properties of metal-rich RR Lyrae variables is referred to Bono et al. (1997).

As far as more massive stars are concerned, it has often been suggested that the Hertzsprung progression of the bump in the light curve of classical Cepheids could be closely correlated with a spatial resonance between the SO and the fundamental mode (Simon & Schmidt 1976; Buchler et al. 1996). However, present nonlinear calculations devoted to the modal stability of these variables did not supply any firm conclusion on the existence of SO pulsators. Although further nonlinear investigations have not been so far undertaken to properly tackle the problem, it is worth mentioning that linear nonadiabatic models provided by Baker (1965) for the mass value $M/M_\odot = 1.0$ disclosed a SO instability over a wide range of luminosities and temperatures. Moreover, Deupree & Hodson (1977), in their study on the instability strip of anomalous Cepheids, gave an evaluation of the SO linear blue edge for even larger masses ($1.0 \leq M/M_\odot \leq 2.0$). However, the cited authors did not come to a sound conclusion, since the numerical difficulties caused by the increased efficiency of the convective overshooting prevented them from evaluating the nonlinear modal stability of these pulsators.

An extensive grid of SO pulsation properties has recently been provided by Milligan & Carson (1992), taking into account a combination of stellar evolution models and both linear and nonlinear pulsation models. Although in this investigation they carried out a detailed analysis of the linear and nonlinear properties of SO stars, only a few models have been followed at limiting amplitude, owing to the small value of the growth rates involved. Therefore, the ultimate modal stability of the wide range of nonlinear models could not be firmly established. Let us note that SO variables are most attractive objects among the several groups of variables presently known, since they also show simultaneous excitation of both radial and nonradial modes. This peculiar feature is of the utmost importance for properly addressing several astrophysical questions concerning the physical mechanisms that rule the driving and the quenching of stellar oscillations.

The observational panorama of SO stars is rather complex, since both the rapidly oscillating roAp magnetic stars (Shibahashi 1987; Martínez & Kurtz 1995) and the large-amplitude metal-poor SX Phoenicis stars (Nemec, Linnell Nemec, & Lutz 1994; Nemec et al. 1995) belong to this group of variable stars as well. However, new high-quality photometric data of SO variables in the Galactic field (Rodriguez et al. 1995), in Baade’s window (Udalski et al. 1995), in open clusters (Frandsen et al. 1995), and in globular clusters (Nemec et al. 1994) have recently been provided, and therefore a proper classification of these objects based on their evolutionary and pulsational properties will soon be available. Another interesting aspect that makes SO variables worth investigating is that during their evolution intermediate-mass stars cross the instability strip in different evolutionary phases. In fact, in contrast to canonical Cepheid-like variables that are connected with helium-burning evolutionary phases, in this region of the H-R diagram the current evolutionary theory foresees stars in pre-main-sequence, main-sequence, and post-main-sequence phases. As a consequence, their pulsational properties can supply useful constraints on their evolutionary properties and at the same time an independent check of stellar models (Breger 1993, 1995).

Linear adiabatic and nonadiabatic models of these objects have been computed by Stellingwerf (1978), Andersen, Hejlesen, & Petersen (1983), and Andersen (1983). Even though these investigations have significantly improved our knowledge of the location of the linear blue boundaries of the instability strip, the theoretical scenario of SO stars presents several unsettled problems. According to this evidence, we decided to extend the investigation on modal stability and pulsational properties already provided for low-mass He-burning stars to larger mass values. Canonical evolutionary prescriptions indicate that stars with masses of the order of 1.5–2.0 $M_\odot$ spread a nonnegligible portion of their lifetime inside the instability strip during their evolution off the main sequence. Therefore, relatively massive stars are expected to generate, during this phase, short-period radial pulsators.

In this paper we investigate the nonlinear pulsational properties of this kind of variable star (Eggen 1994; McNamara 1995) by exploring the pulsational behavior of a sequence of models as a function of the surface effective temperature. For the first time we show that the adopted theoretical framework accounts for the modal stability of SO pulsators and allows some predictions concerning the full-amplitude behavior of SO light and velocity curves. In § 2 we describe the numerical and physical assumptions adopted for constructing both linear and nonlinear pulsation models, whereas in § 3 we discuss the full-amplitude pulsation behavior for the first three modes. The shape and the secondary features of both light and velocity curves are presented in § 4, together with a plain comparison with available photometric data. The physical parameter that rules the SO limit cycle stability and its dynamical properties are discussed in § 5. Our conclusions are presented in § 6. In this section the consequences arising from these
new findings and the observed features worth being investigated are outlined as well.

2. STANDARD PULSATION MODELS

The theoretical framework adopted for investigating linear and nonlinear pulsation characteristics of radial pulsators have been previously described in a series of papers (Stellingwerf 1982; BS and references therein). The sequence of models presented in this paper was computed at fixed mass value \( M/M_\odot = 2.0 \), chemical composition \( Y = 0.28 \), \( Z = 0.02 \) and luminosity level \( \log \left( L/L_\odot \right) = 1.7 \), and by exploring a wide range of effective temperatures \( 7500 \lesssim T_\text{e} \lesssim 6000 \) K. The static envelope models were constructed by adopting an optical depth of the outermost zone of the order of 0.001, and the inner boundary was assumed fixed so that possible destabilization effects due to variations in the efficiency of the H shell burning are ignored. Moreover, we adopted the OP radiative opacities provided by Seaton et al. (1994) for temperatures higher than 10,000 K, whereas for lower temperatures we adopted molecular opacities provided by Alexander & Ferguson (1994). The method adopted for handling the opacity tables has already been described in Bono, Incerpi, & Marconi (1996).

Each envelope model extends from the surface to 20%–10% of the stellar radius, and the zone closest to the hydrogen ionization region (HIR) was constrained to \( T_{\text{HIR}} \approx 1.3 \times 10^4 \) K. Between the HIR and the surface we inserted 20 zones to ensure a good spatial resolution of the outermost regions throughout the pulsation cycle. Because of the key role played by spatial resolution for firmly estimating both linear and nonlinear modal stability of higher modes, the envelopes were discretized by adopting a detailed zoning in mass. The mass ratio \( \theta \) between consecutive zones has been assumed equal to 1.1 for temperatures lower than \( 6 \times 10^5 \) K, whereas for higher temperatures it has been set equal to 1.2. By adopting this type of zoning, the ionization regions and the opacity bump due to iron are covered with a number of zones lying between 100 and 150. The fine spatial resolution of the ionization fronts provides, in turn, an accurate treatment of both the formation and the propagation of shock fronts during the phase interval between the phase of minimum radius and the phase of maximum luminosity (for complete details see BS). On the basis of these assumptions a typical envelope model is characterized by roughly 20%–30% of the total stellar mass and by 150–250 zones.

To clarify matters concerning the dependence of the pulsation behavior on the spatial resolution, we have constructed a much finer envelope model. This model is located at \( T_\text{e} = 7000 \) K, and, in contrast to the standard sequence of linear nonadiabatic models, it was constructed by adopting a smaller mass ratio \( \theta = 1.08 \) for the regions located at temperatures lower than \( 6 \times 10^5 \) K. Moreover, the inner boundary condition for this model was chosen in such a way that the base of the envelope was located below 0.1 of the photospheric radius and its temperature was of the order of (5–6) \( \times 10^6 \) K.

As a consequence of these assumptions, the detailed model presents an increase in both the envelope mass \( M_{\text{env}} = 0.46M_\star \) against \( M_{\text{env}} = 0.21M_\star \) and the number of zones that cover the ionization regions (180 versus 150). We eventually found that for the first three modes the differences between the pulsation characteristics of both detailed and standard models are quite negligible. In fact, the discrepancies range from \( 10^{-4} \) to \( 10^{-3} \) for the periods and from \( 10^{-6} \) to \( 10^{-4} \) for the growth rates. As a result we can assume that the spatial resolution of the standard sequence allows a proper treatment of the radial pulsation of \( \delta \) Scuti stars during their off–main-sequence evolution.

3. APPROACH TO LIMIT CYCLE STABILITY

According to the usual approach, a sequence of linear nonadiabatic models was first constructed for supplying the static structure of the envelope to the nonlinear stability analysis. Then the equations governing both the dynamical behavior of a case that presents a permanent mixture of modes, we evaluated the asymptotic behavior of each mode by performing very long runs. This approach leads to an integration of the governing equations for a number of periods that ranges from 5000 to 50,000 for some peculiar cases. The integration is generally stopped when the nonlinear total work is vanishing and the pulsation amplitudes present over two consecutive periods a periodic similarity of the order of or lower than \( 10^{-3} \) to \( 10^{-2} \).

Since this is the first time that hydrodynamic calculations are performed over such a long time interval, Figures 1, 2, and 3 show the time behavior of period, velocity, and magnitude for three cases characterized by a different approach to nonlinear limit cycle stability. In particular, Figure 1 shows the variation of the quoted quantities for a single pure SO model, whereas Figure 2 shows that radial motions at \( t \approx 7 \) yr experience a mode switching from the first overtone to the SO. Figure 3 finally presents the limiting amplitude behavior of a case that presents a permanent mixture of different radial modes.

As a result of the modal stability analysis, we found stable nonlinear limit cycles in the fundamental, in the first overtone, and, for the first time, in the second overtone when the effective temperature is increased. Even though so far the location inside the instability strip of \( \delta \) Scuti stars characterized by different pulsation modes has not been firmly established, the previous finding confirms the distribution originally suggested by Breger & Bregman (1975). In fact, by assuming that \( \delta \) Scuti variables are radial pulsators, these authors found that observed second and first overtone variables were located at effective temperatures higher than the fundamental ones.

In Table 1 are listed selected observational parameters for the sequence of \( \delta \) Scuti models. As a first result, data in
Table 1 show that theoretical periods appear in general agreement with the observed range of δ Scuti values. It is worth emphasizing that the effective temperature of the fundamental red edge should be considered an upper limit. As a matter of fact, even though the model located at 6300 K after 23,000 periods presents both a constant negative value in the total work term and a very low pulsational amplitude \( |\Delta M_{\text{bol}}| \approx (4-6) \times 10^{-3} \text{ mag} \), this region of the instability strip, due to the slow approach to limit cycle stability, should be investigated in more detail before firmly constraining the location of the red edge.

In order to disclose the main features of the modal behavior in δ Scuti stars, Figure 4 shows the bolometric light curves and the surface radial velocities of SO pulsators; Figure 5 shows the same quantities, but they are referred to selected first overtone (solid lines) and fundamental (dashed lines) pulsators. Figure 6 shows the light and radial velocity curves of mixed-mode pulsators, i.e., of models that present a permanent mixture of different radial modes at limiting amplitude.

Inspection of light curves discloses the surprising evidence that the shape of SO light curves—sudden increase in the rising branch and slow decrease in the decreasing branch—closely resembles canonical fundamental mode rather than first overtone RR Lyrae pulsators. This finding confirms the original prediction concerning the shape of SO light curves made by SGD. Moreover, we find that in moving from SO to lower pulsational modes the amplitudes progressively decrease and the shape of the light curves
becomes more sinusoidal. It is worth noting that the pulsation amplitudes of RR Lyrae variables, which are located in the same region of the instability strip, present an opposite trend. In fact, for this group of pulsators the RRab variables (fundamental) show the largest amplitudes. These theoretical prescriptions can be usefully compared with the observational scenario recently discussed by McNamara (1995 and references therein).

According to this author, δ Scuti stars on the basis of their luminosity amplitude can be empirically divided into two groups. The light curves of stars with larger amplitudes appear to be asymmetrical, whereas the light curves for lower amplitudes tend to be much more symmetrical. However, McNamara (1995) also suggests that for low-amplitude variables, which are poorly sampled, it is often difficult to determine whether the light curves are symmetric or asymmetric. Therefore, for light curves that are only partially covered by photometric data, several cases of probable asymmetry are brought forward. By analogy with the behavior of RR Lyrae variables, McNamara (1995) assumes that stars with asymmetric, large-amplitude light curves are fundamental pulsators, whereas symmetric, low-amplitude light curves belong to first overtone pulsators.

The comparison of similar empirical prescriptions with the current theoretical scenario shows a convincing degree of agreement. However, theory now tells us that asymmetric, large-amplitude pulsators could be good SO candidates, whereas low-amplitude pulsators could be a mixture of fundamental and first overtones.

To go further with this comparison, let us refer to the sample of variable stars recently collected by the OGLE collaboration (Udalski et al. 1995 and references therein) as the result of their search for evidence of microlensing in the bulge of the Galaxy. Inspection of photometric data connected with short-period variables discloses that the observed light curves can be arranged in three typical classes, as shown in Figure 7, with class “A” representing McNamara large-amplitude pulsators and classes “B” and “C” the small-amplitude ones. For a meaningful comparison between theory and observation, the bolometric light curves have been transformed into the I band according to Kurucz’s (1992) atmosphere models. Figures 8 and 9 show the light curves for single-mode pulsators. Because of the magnitude scale adopted for plotting data in Figure 9, the light curves of fundamental pulsators (dashed lines) seem almost perfectly sinusoidal. However, even though the luminosity variations throughout the cycle are quite smooth, a bump appears before the phase of minimum radius. Taking into account that we explored only one mass value and only one luminosity level, the comparison should be considered more than satisfactory.

**4. LIGHT-CURVE MORPHOLOGY**

Available observational data hardly allow the detection of minor details in the light curves. However, the quality of both spectroscopic and photometric data is rapidly improving (see, e.g., Milone, Wilson, & Fry 1994; Breger et al. 1995 and references therein), and therefore in this section we...
discuss even minor features of theoretical light curves in order to underline the theoretical predictions worth being investigated with the required accuracy. As a first point, let us notice that the light and velocity curves of SO pulsators present two further relevant distinctive features:

1. As in the case of canonical first overtone RR Lyrae variables, the bump does not appear along the decreasing branch of the light curves, and, moving from higher to lower effective temperatures, the dip becomes more and more evident along the rising branch.

2. Moving from the blue to the red boundary of the SO instability region, the velocity curves show smooth variations, but at phases 0.2–0.3 a bump appears due to the propagation of an outgoing shock.

As a second point, we find that the shape of first overtone light curves presents some features that allow a careful distinction between different radial modes. In fact, for these models the dip is the main maximum, whereas the “true” maximum takes place along the decreasing branch. Moreover, the first overtone light curves show that the bump appears along the increasing branch and that just before the phase of minimum radius they also display a short stillstand phase ($\phi \approx 0.45$).

The scenario concerning the pulsation characteristics of $\delta$ Scuti stars can now be nicely completed by the models that
FIG. 7.—Light curves of δ Scuti variables selected from the OGLE photometric database. The variables were chosen in such a way that the top curve can be considered a template for second overtone pulsators (class "A"), the middle curve for first overtone pulsators (class "B"), and the bottom one for fundamental pulsators (class "C").

Fig. 8.—Second overtone light curves for two consecutive periods. The luminosity curves have been transformed into the $I$ band by adopting the static atmosphere models provided by Kurucz (1992). The magnitude scale refers to the top light curve; the other curves are artificially shifted by 1 mag. Diamonds mark the phase of minimum radius.

Fig. 9.—Same as Fig. 8, but the solid lines are referred to pure first overtone pulsators, whereas the dashed lines are referred to pure fundamental pulsators.

are simultaneously excited in two or more radial modes. Figure 6 shows a collection of light and velocity curves of mixed-mode pulsators; a glance at these curves brings out both the expected fluctuation of the pulsation amplitudes between consecutive cycles and the appearance of the secondary features that characterize the first three lower single-mode pulsators.

A thorough comparison with observed period ratios of δ Scuti variables is beyond the scope of the present study, since the “true” period ratios should be evaluated through a Fourier decomposition of the theoretical light curves. Moreover, for properly constraining the stellar masses and the luminosity levels of these objects by means of the Petersen diagram ($P_1/P_0$ versus $P_0$), an extensive set of nonlinear models computed for different assumptions on astrophysical parameters is necessary (Bono et al. 1996b).

Nevertheless, since period ratios of double mode pulsators can provide valuable clues on several astrophysical problems involving both the evolutionary and the pulsational properties of these objects, we constructed a new sequence of detailed, linear, radiative, nonadiabatic models by adopting the assumptions already discussed at the end of § 2. This analysis has been undertaken only to supply a preliminary but meaningful theoretical guess concerning the location of this group of variables inside the instability strip.

At first it is important to note that the linear periods and the related period ratios listed in Table 2 are, within the estimated uncertainties, in agreement with observed values (see, for example, data on double-mode δ Scuti stars collected by Andreasen 1983 and by Petersen 1990). Indeed, the observed period ratio between first overtone and fundamental pulsators ranges, for large-amplitude δ Scuti stars, from 0.760 to 0.780, whereas the period ratio between second overtone and first overtone is approximately of the
order of 0.800. This agreement is a remarkable result, since the period ratios predicted by pulsation models are the most important observable adopted for finding out whether the pulsation is ‘‘driven’’ by radial or by nonradial modes. As a consequence, this finding provides sound evidence that these variables are mixed-mode radial pulsators (Breger 1979).

Moreover, previous linear and nonlinear results suggest that, owing to the appearance of three different modal stabilities inside the instability strip, double-mode pulsators belonging to this group of variables are located close to the fundamental blue edge when they are a mixture of fundamental and first overtone, and close to the first overtone blue edge when they are a mixture of first and second overtones. A detailed investigation of the dependence of this peculiar occurrence among radial pulsators, together with a straightforward analysis of the envelope structure, will be discussed in a forthcoming paper et al. (Bono 1996c).

5. SECOND OVERTONE INSTABILITY

In order to identify properly the regions of the stellar envelope which drive or damp the pulsation instability of SO pulsators, shows the nonlinear differential work Figure 10 integrals versus the logarithm of the external mass for a model located close to the SO blue edge. In this plane the positive areas denote driving regions (growing oscillations), whereas the negative areas denote damping regions (quenching oscillations). The total work curve shows quite clearly the two driving sources due to the hydrogen and helium ionization zones as well as the radiative damping due to the inner regions. In contrast to the situation for canonical cluster variables, the second helium ionization zone provides a stronger destabilization if compared with the HIR. This effect is mainly due to the increase in effective temperature, which causes a shift of the HIR toward the surface and therefore a decrease of the mass that lies above these layers. However, the total work plotted in Figure 10 clearly shows that this element, in contrast to previous qualitative arguments, provides a substantial amount of driving to the pulsation instability of $\delta$ Scuti stars. For the reasons previously discussed, all other nonlinear work terms supply a negligible damping effect on the pulsation.

Nevertheless, the physical parameter that rules the SO instability is the location of the nodes inside the envelope. In fact, the nodes of temperature, luminosity, and radius fall within the region of radiative damping. As a consequence, the amount of damping is strongly reduced, and the desta-

![Figure 10](image-url)

**Figure 10.**—Nonlinear work integrals for a second overtone vs. the logarithm of the external mass, surface at right. The solid line shows the total work, the dashed line the total turbulent work (i.e., turbulent pressure plus eddy viscosity works), and the dashed-dotted line the artificial viscosity work. The different symbols mark, according to linear eigenfunctions, the location of the nodes in radius, temperature, and luminosity. The hydrogen and helium ionization regions are also indicated.
In this paper we report the first theoretical evidence for the occurrence of pulsators that, when moving inside the instability strip from lower to higher effective temperatures, show three different stable pulsation modes, namely, fundamental and first and second overtone. We found that the predicted features of the light curves appear in general agreement with observational constraints concerning δ Scuti variables, suggesting that SO pulsators have already been observed and pointing out, at the same time, a revised approach to observational evidence for different pulsation modes. Moreover, we show that the location of radius, luminosity, and temperature nodes in the damping region of the stellar envelope is the main physical parameter that governs the limit cycle stability of SO pulsators. This result strongly supports the discussion given in the Introduction to this paper about the absence of SO pulsators in low-mass variable stars.

The satisfactory agreement between the computed period ratios and the observed ones casts new light on the problem of limit cycle stability of mixed-mode variables belonging to Population II stars and provides a valuable piece of information for accounting for the pulsation properties of δ Scuti stars. At the same time, there is growing strong evidence that these stars during their evolution off the main sequence are pure or mixed-mode radial pulsators.

Before it is possible to provide a sound comparison with available photometric data on δ Scuti variables, the present sequence of nonlinear models should be extended to both lower luminosity levels and smaller mass values. However, even though the computed models cover a restricted range of effective temperatures, the theoretical framework currently adopted accounts for both pure and mixed-mode radial pulsators. This new scenario presents several interesting features, since both the modal stability and the pulsational behavior have been investigated in a homogeneous physical context without invoking unpleasant ad hoc physical mechanisms and/or peculiar characteristics.

Plenty of new high-quality photometric data on δ Scuti stars will soon be available as a by-product of the international projects involved in the search for microlensing events, and therefore new sequences of nonlinear δ Scuti models at full amplitude, although such analysis requires nontrivial computational efforts, are necessary to firmly establish the pulsation properties of these objects.

Finally, we suggest that short-period RR Lyrae–like pulsators found in the Galactic bulge as well as in dwarf spheroids like Carina and Sagittarius should be regarded as evidence of relatively massive stars, a witness to the efficiency of star formation until relatively recent times. On the other hand, this finding stresses once again the key role played by variable stars as tracers of stellar populations which experienced different dynamical and/or chemical evolutions.

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