Alleviating the tension in CMB using Planck-scale Physics

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Certain anomalies in the CMB bring out a tension between the six-parameter flat ΛCDM model and the CMB data. We revisit the PLANCK analysis with loop quantum cosmology (LQC) predictions and show that LQC alleviates both the large-scale power anomaly and the tension in the lensing amplitude. These differences arise because, in LQC, the primordial power spectrum is scale dependent for small k, with a specific power suppression. We conclude with a prediction of larger optical depth and power suppression in the B-mode polarization power spectrum on large scales.

Introduction: The ΛCDM model selected by the PLANCK satellite data has had impressive success in explaining all major features in the temperature anisotropies and polarizations of the cosmic microwave background (CMB), using only six parameters [1]. Let us begin by recalling the procedure that is used to determine the model from the CMB data. Inspired by inflationary models, one assumes that the primordial power spectrum is nearly scale invariant with a specific form,

$$P(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1},$$

characterized by two parameters $A_s$, the amplitude of the scalar mode, and $n_s$, its spectral index. (Here, $k_\star = 0.05$ Mpc$^{-1}$ is the pivot mode.) We will refer to this form as the standard ansatz (SA). To determine a specific ΛCDM model, one requires four additional parameters: $\Omega_b h^2$, $\Omega_c h^2$, and the cold matter density; and $100 \theta_L$, $\tau$ that determine the observed angular scale associated with acoustic oscillations, and the optical depth that characterizes the reionization epoch, respectively. Given the SA and the six parameters, the Boltzmann codes [2–4] that incorporate subsequent astrophysics provides us with four power spectra $C^{TT}_\ell$, $C^{TE}_\ell$, $C^{EE}_\ell$, $C^{\phi \phi}_\ell$, where $T$, $E$, $\phi$ stand for temperature, E-mode (even-parity) polarization, and lensing potential [5–6]. One compares these theoretical predictions with the observed power spectra and finds the best-fitting values (together with uncertainties) for the six parameters. This fixes the ΛCDM model. One can then work out predictions for other observables, which can be measured independently. For example, the four-point correlation function of the CMB measures the gravitational lensing amplitude $A_L$ [7], and the B-mode (odd-parity) polarization power spectrum $C^{BB}_\ell$ measures the amplitude of tensor perturbation in the early Universe [8–9].

At the same time, the CMB data exhibits some anomalies that bring out tensions between the best-fit ΛCDM model and observation. We will ignore the tension between the CMB and low-$z$ observations, and focus instead on two anomalies in the CMB. The first is large-scale power anomaly related to $S_{1/2} \equiv \int_{-1}^{1/2} |C(\theta)|^2 d(\cos \theta)$, obtained by integrating the two-point correlation function $C(\theta)$ of the CMB temperature anisotropies over large angular scales ($\theta > 60^\circ$). The WMAP [10, 11] and PLANCK [12, 13] measured values of $S_{1/2}$ are much smaller than the expectation from the SA+ΛCDM cosmology. The second is the anomaly associated with the lensing amplitude $A_L$. When it is allowed to vary, $A_L$ prefers a value larger than unity, hinting at an internal inconsistency in the ΛCDM cosmology [6–14] based on the SA. In particular, it was recently suggested [14] that this anomaly gives rise to a “possible crisis in cosmology” because the positive spatial curvature one can introduce to alleviate this tension makes CMB analysis inconsistent with low-$z$ cosmological measurements.

In this letter, we present an intriguing possibility of alleviating both anomalies within a well-motivated theoretical framework of loop quantum cosmology (LQC). First, the LQC prediction modifies the SA for the primordial power spectrum by suppressing its large-scale amplitude, which naturally leads to lower $S_{1/2}$. The scale-dependent primordial power spectrum, in turn, prefers a higher amplitude $A_s$ that pushes lensing amplitude $A_L$ toward unity (making it consistent with flat ΛCDM), and higher optical depth $\tau$ decreasing the discrepancy between the WMAP and PLANCK results [15]. Finally we show that, due to modified primordial power spectrum and higher $\tau$, LQC leaves a specific signature in the B-mode (odd-parity) polarization power spectrum.

Modified primordial power spectrum: In LQC, the big bang singularity is naturally resolved and replaced by a big bounce (see, e.g., [17,18]). Therefore, one can systematically investigate the dynamics of cosmological perturbations in the pre-inflationary epoch starting from the Planck regime (see, e.g., [19,20]). Since the quantum corrected Einstein’s equations never break down, all physical quantities remain finite. In particular, while the scalar curvature $R$ of space-time diverges at the big-bang, it remains finite at the bounce, achieving its universal maximum value $R_{\text{max}} \approx 62$ in Planck
units. Now, curvature –more precisely $R/6$– provides a natural scale in the dynamics of the gauge invariant perturbations (which in de Sitter space-time coincides with $2H^2$). Fourier modes with physical wavenumbers $k_{\text{phys}} \equiv k/a(\eta) \gg (R/6)^{1/2}$ are essentially unaffected by curvature while those with $k_{\text{phys}} \lesssim (R/6)^{1/2}$ get excited. Therefore the evolution during the pre-inflationary epoch of LQC is subject to a new scale: $k_{LQC} = (R_{\text{max}}/6)^{1/2} \approx 3.21$ in Planck units. Modes with $k_{\text{phys}} \lesssim k_{LQC}$ at the bounce are excited during their pre-inflationary evolution. Therefore they are not in the Bunch Davies vacuum at the onset of the relevant slow roll phase of inflation –i.e. a couple of e-folds before the time at which the mode with the largest observable wavelength crosses the Hubble horizon during inflation. (For details, see [19, 20]).

Now, one’s first reaction may be that these excitations are observationally irrelevant because they would be simply diluted away by the end of inflation. However, this is not the case: because of stimulated emission, the number density of these excitations remains constant during inflation [18, 31, 32]. Therefore the primordial LQC power density of these excitations remains constant during inflation. (For details, see [19, 20]).

The key question then is whether these long wavelength modes are in the observable range. The answer depends on the initial conditions both for the background metric that satisfies the quantum corrected Einstein’s equations given by LQC, and for the cosmological perturbations. In standard inflation, one cannot specify initial conditions at the big bang. Instead, one specifies them, so to say, in the ‘middle of the evolution’ by positing that cosmological perturbations are in the BD vacuum at the start of the relevant slow roll. In LQC one can impose them at (or near) the bounce. But then the BD ansatz has to be replaced because the space-time geometry there is very different from de Sitter. A specific proposal has been put forward [20, 27], motivated by features of quantum geometry in LQC and a ‘quantum generalization’ of Penrose’s Weyl curvature hypothesis [33]. Given these initial conditions, the quantum corrected LQC dynamics leads to unique predictions for the primordial power spectrum for any given inflationary potential: there are no parameters to adjust. The viewpoint is to use the proposal as a working hypothesis, analyze the consequences, and use the CMB observations to test its admissibility.

The initial conditions for the background metric imply that the $\Lambda$CDM universe selected by this LQC model has undergone $\simeq 141$ e-folds since the bounce (irrespective of the choice of inflationary potential) [20]. Therefore, the mode with $k_{\text{phys}} = k_{LQC}$ at the bounce corresponds to the co-moving wavenumber $k_0 \simeq 3.6 \times 10^{-4}\text{Mpc}^{-1}$. The primordial power spectrum of LQC is nearly scale invariant for $k \gg k_0$ but power is suppressed for $k \lesssim 10k_0$.

![FIG. 1: Ratio of the primordial TT-power spectrum for LQC and SA. Power is suppressed in LQC for $k \lesssim 10k_0 \simeq 3.6 \times 10^{-4}\text{Mpc}^{-1}$. Plots for the Starobinsky and quadratic potentials are essentially indistinguishable.](image)

\[
P_R^{\text{LQC}} = f(k) A_s \left( \frac{k}{k_s} \right)^{n_s-1},
\]

where the form of the suppression factor $f(k)$ can be seen in Fig. 1 ($f(k) \approx 1$ for $k \gg k_0$). This difference from the standard ansatz can be traced back directly to the modes not being in the BD vacuum at the onset of inflation, which in turn is a consequence of the initial conditions and pre-inflationary dynamics. Now, if the total energy in the scalar field is dominated by the kinetic contribution at the bounce, details of the potential do not affect the pre-inflationary dynamics, and the suppression factor $f(k)$ is also the same. We expect that there is a large class of potentials for which our initial condition for the background geometry will lead to a kinetic energy dominated bounce. This is in particular the case for the Starobinsky inflation [34] and the quadratic potential, as shown in Fig. 1.

Results: All results are based on the PLANCK-2018 data [1] using the observed TT, TE, EE, and $\phi-\phi$ power spectra (including the $\ell < 30$ modes for EE correlations which were excluded in the PLANCK-2015 release).

Fig. 2 shows the observed TT-power spectrum together with the $1-$\(\sigma\) (68% confidence level) error-bars, and the LQC and the SA predictions for the respective best-fit cosmological parameters. Clearly, LQC power is suppressed at $\ell \lesssim 30$ relative to the SA. This is also true for the EE power spectrum (as already noted in [20], using the then available PLANCK-2015 data). Note that the difference between LQC and SA best-fitting curves shown in Fig. 2 underestimates the difference in the predicted primordial spectra, for the best-fitting cosmological parameters are different. Also, had LQC+$\Lambda$CDM model been used for their analysis, the cosmic-variance uncertainties on large-scales may have been smaller than the
To quantify this difference, we computed \( S \) for the SA. It is clear by inspection that the LQC prediction values is < the difference between the SA+\( \Lambda \)CDM and LQC+\( \Lambda \)CDM parameters together with their 1\( \sigma \) probability distributions of the six cosmological parameters, and values of \( S_{1/2} \) calculated using \( C_{\ell}^{TT} \).

![TT power spectra](image1)

**FIG. 2:** TT power spectra. The 2018 PLANCK spectrum (black dots with error bars), the LQC (solid (blue) line) and the standard ansatz (SA) predictions(dashed (red) line).

![Angular power spectrum C(\theta)](image2)

**FIG. 3:** The angular power spectrum \( C(\theta) \). The 2018 PLANCK spectrum (thick black dots), the LQC (solid (blue) line), and the standard ansatz (dashed (red) line) predictions.

The mean values of marginalized PDF for the six cosmological parameters, and values of \( S_{1/2} \) calculated using \( C_{\ell}^{TT} \).

| Parameter        | SA            | LQC           |
|------------------|---------------|---------------|
| \( \Omega_M h^2 \) | 0.02238 ± 0.00014 | 0.02239 ± 0.00015 |
| \( \Omega_b h^2 \)  | 0.1200 ± 0.0012 | 0.1200 ± 0.0012  |
| \( 10^3 P(M) \)  | 1.04091 ± 0.00031 | 1.04093 ± 0.00031 |
| \( \tau \) | 0.0542 ± 0.0074 | 0.0595 ± 0.0079 |
| \( \ln(10^{10} A_s) \) | 3.044 ± 0.014 | 3.054 ± 0.015 |
| \( n_s \) | 0.9651 ± 0.0041 | 0.9643 ± 0.0042 |
| \( S_{1/2} \) | 42496.5 | 14308.05 |

![1\sigma and 2\sigma probability distributions in the \( \tau-A_L \) plane](image3)

**FIG. 4:** 1\( \sigma \) and 2\( \sigma \) probability distributions in the \( \tau-A_L \) plane. Predictions of the standard ansatz (shown in red) and LQC (shown in blue). Vertical lines represent the respective mean values of \( \tau \).

Table I also shows the mean values of the marginalized probability distributions of the six cosmological parameters together with their 1\( \sigma \) ranges. For five of these, the difference between the SA+\( \Lambda \)CDM and LQC+\( \Lambda \)CDM values is < 0.4%. However, the value of the sixth –the optical depth \( \tau \)– has increased in LQC by ~ 9.8%! As we discuss below, this significant change is a direct consequence of the scale-dependent initial power spectrum of LQC, which also leads to a 2.2% decrease in the lensing amplitude \( A_L \). As Fig. 4 shows, the value \( A_L = 1 \) lies outside of the 68% confidence level for the SA+\( \Lambda \)CDM model (red contours). A natural way to alleviate this tension within the SA+\( \Lambda \)CDM is to consider a closed universe. However, then other disagreements with observations arise that prompted the authors of [15] to raise the possibility of a “crisis in cosmology”. What is the situation with the altered values of \( \tau \) and \( A_L \) in LQC? We see from Fig. 4 that now the tension is naturally alleviated because the value \( A_L = 1 \) is within 68% confidence level (blue contours). Therefore, the primary motivation for introducing spatial curvature no longer exists in LQC.

**General implications of power suppression at large angles:** In LQC, the mechanism for departure from the nearly scale invariant ansatz ([1] is rooted in fundamental considerations in the Planck regime. Nonetheless, it is natural to ask if the qualitative features of some of our results will carry over if there were other mecha-
nisms that led to primordial spectrum of the from given in Eq. (2). We now show that this is indeed the case.

Let us then suppose that there is some mechanism that provides a primordial power spectrum of the form (2) for some $k_s$. Let us compare and contrast the resulting best fit $\Lambda$CDM model with that given by the SA of Eq. (1). As a first step, let us restrict the analysis only to smaller angular scales ($k \gg k_s$). Then, the primordial spectrum in both schemes is the same, whence we will obtain the same best fit values of the six cosmological parameters. Denote by $A_s$ the best fit value of the scalar amplitude $A_s$. In the second step, let us bring in the full range of observable modes including $k \leq k_s$. Now, given the observed large-scale suppression in the TT power spectrum, for SA+$\Lambda$CDM model the best-fit value $A_s^{(1)}$ for the entire $k$ range will be lower than $A_s$. By contrast if the primordial power spectrum is of the form of Eq. (2), $A_s$ will not have to be lowered as much to obtain the best fit $A_s^{(0)}$ since the initial power is already suppressed by $f(k)$.

Thus, we have $A_s > A_s^{(2)} > A_s^{(1)}$. (For the $f(k)$ in LQC, we have $\ln[10^{10} A_s] = 3.089$ and $\ln[10^{10} A_s^{(2)}] = 3.054$ and $\ln[10^{10} A_s^{(1)}] = 3.044$.) The key point is the last inequality: $A_s^{(2)} > A_s^{(1)}$. Now, we know that for large $k$, the product $A_s e^{-2\tau}$ is fixed by observations. Hence, it follows that the best fit values of the optical depth in the two scheme must satisfy $\tau^{(2)} > \tau^{(1)}$. Finally, from the very definition of lensing amplitude, the value of $A_L$ is anti-correlated to the value of $A_s$. Therefore, we will have $A_s^{(2)} < A_s^{(1)}$. Thus in any theory that has primordial spectrum of the form (2), $A_s, \tau$ and $A_L$ will have the same qualitative behavior as in LQC, and hence the tension with observations would be reduced. This general implication appears not to have been noticed before. What LQC provides is a precise form of the suppression factor $f(k)$ from ‘first principles’, and hence specific quantitative predictions. The LQC $f(k)$ also leads to other predictions – e.g., for the BB power spectrum discussed below– that need not be shared by other mechanisms.

**Summary and Discussion:** In LQC, curvature never diverges and reaches its maximum value at the bounce. As a result, pre-inflationary dynamics naturally inherits a new scale, $k_{LQC}$, such that modes with $k_{\text{phys}} \lesssim k_{LQC}$ at the bounce are not in the BD vacuum at the start of the slow roll phase of inflation [19, 20], whence the primordial power spectrum is no longer nearly scale invariant, but of the form (2). The LQC dynamics and initial conditions then imply [20] that there is power suppression in CMB at the largest angular scales $\ell \lesssim 30$. As a result, there is an enhancement of optical depth $\tau$ and suppression of the lensing potential $A_L$. The two together bring the value $A_L = 1$ within 1$\sigma$ of the LQC $\tau - A_L$ probability distribution, thereby removing the primary motivation for considering closed universe and the subsequent “potential crisis” [15]. In addition, the anomaly in $C^{TT}(\theta)$ at large angles [10, 13] is significantly reduced; the LQC value of $S_{1/2}$ is $\sim 0.34$ of that predicted by standard inflation. The PLANCK collaboration had suggested [1] that “...if any of the anomalies have primordial origin, then their large scale nature would suggest an explanation rooted in fundamental physics. Thus it is worth exploring any models that might explain an anomaly (even better, multiple anomalies) naturally, or with very few parameters”. In this letter we presented a concrete realization of this idea. (For an alternate proposal within LQC see [35]).

Our results provide support in favor of the specific initial conditions that were proposed in [20, 27]. Therefore, it is worthwhile to explore other consequences of the quantum corrected LQC dynamics using these initial conditions. In particular, this model also leads to other specific predictions. First, as Table I shows, the reionization optical depth $\tau$ is predicted to be $\sim 9.8\%$ higher. This prediction can be tested by the future observation of global 21cm evolution at high redshifts that can reach a percent level accuracy in the measurement of $\tau$ [36]. Second, LQC predicts the same value of $r$ – the tensor to scalar ratio – as the SA, but a specific scale dependence in the large-scale B-mode (odd-parity) polarization power spectrum, as shown in Fig. 5. The difference is driven by the LQC suppression of the primordial tensor amplitude combined with the larger reionization contribution due to higher $\tau$. Provided that $r$ is sufficiently large, for example, $r \gtrsim 0.001$, we may be able to test this prediction against the data from the future B-mode missions such as CLASS [37], LiteBIRD [38], the Simons Observatory [39], CMB-S4 [40], or Probe Inflation and Cosmic Origins (PICO [41]). Again, LQC modifies $C_{\ell}^{BB}$ on large scales where the cosmic variance limits its detectability. However, in light of results presented in this paper, we hope that the LQC primordial power spectrum will be included in the future cosmological analysis.
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