Introduction to Hadronic $B$ Physics

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Abstract

An overview of the theory of $B$ physics is given, with an emphasis on issues in the strong interactions and hadronic physics. This article is taken from an introductory chapter of The BaBar Physics Book – Physics at an Asymmetric $B$ Factory, SLAC Report SLAC-R-504. It is written at the level of a basic survey aimed at the experimental community.

Published as Chapter 2 of
The BaBar Physics Book – Physics at an Asymmetric $B$ Factory
SLAC Report SLAC-R-504
I. INTRODUCTION

This review appeared originally as an introductory chapter of The BaBar Physics Book – Physics at an Asymmetric B Factory, SLAC Report SLAC-R-504. As a whole, the book is a collaborative effort to summarize and review the current (1998) state of the art in our understanding of B physics, especially as the theory will be applied to the analyses to be performed by the BaBar Collaboration. The complete text of The BaBar Physics Book is available on the web site of the Stanford Linear Accelerator Center, at http://www.slac.stanford.edu. However, this self-contained chapter is being made available separately, because it might be of interest to an audience somewhat broader than the experimental B physics community toward which the rest of The BaBar Physics Book is aimed. Its purpose is to provide an overview of, and basic introduction to, theoretical techniques essential to the study of B mesons and their decays. Of course, there is an enormous literature on this subject, and it is not the goal here to review it all. Rather, this introduction provides a general context within which to place the theoretical treatments of specific processes discussed elsewhere in The BaBar Physics Book. The focus here is on general issues of philosophy and approach, and the goal is to be pedagogical. No attempt is made to provide a comprehensive review of the field, nor to include references to the voluminous literature of individual contributions on each topic. Instead, at the end are some suggestions for further reading. For a fully referenced review of theoretical B physics, or at least of those parts of direct relevance to BaBar, the reader should see The BaBar Physics Book itself.

A key factor in the experimental interest in b physics is the potential insight it affords into physics at very short distances. In particular, it is hoped that the high precision study of phenomena such as CP violation, rare decays, and flavor changing processes will provide precious insights into new interactions associated with the flavor sector of whatever theory lies beyond the Standard Model. However, in order for this information to become available, it is necessary to confront the fact that the b quarks, which are the ultimate objects of study, are bound by strong dynamics into color neutral hadrons. While understood in principle, the nonperturbative nature of these bound states makes problematic the extraction of precision information about physics at higher energies from even the most clever and precise experiments on B mesons. To explore new physics effects one faces a daunting theoretical challenge to untangle them from the effects of nonperturbative QCD.

This is not a problem which has been solved in its entirety, nor is it likely ever to be. Rather, what is available is a variety of theoretical approaches and techniques, appropriate to a variety of specific problems and with varying levels of reliability. There are a few situations in which one can do analyses which are rigorous and predictive, and many in which what can be said is more imprecise and model dependent. The result is an interesting interplay between theory and experiment, where one often cannot measure what one can compute reliably, nor compute reliably what one can measure. In the search for quantities which can be both predicted and measured, one must be creative and flexible in the choice of theoretical techniques. While approaches which are based directly on QCD, and which allow for quantitative error estimates, are clearly to be preferred, more model-dependent methods are often all that are available and thus have an important role to play as well.

The theoretical methods discussed here fall roughly into three categories. First, there are effective field theories such as the Heavy Quark Expansion (HQE) and Chiral Perturbation
Theory (ChPT). Effective field theories derive their predictive power by exploiting systematically a small expansion parameter. For nonperturbative QCD, this parameter cannot be the strong coupling constant $\alpha_s$; instead, it is a ratio of mass scales obtained by considering a particular limit or special kinematics. Second, there are the approaches of lattice QCD and QCD sum rules, which are based on QCD but do not exploit a large separation of scales. While in principle these techniques are rigorous, they suffer in their current practical implementations from a degree of uncontrolled model dependence. In the case of the lattice, this problem will improve with the availability of ever more powerful computers. Third, there are quark models, which do not purport to be derived from QCD. Instead, in using models one introduces some new degrees of freedom and interactions which, it is hoped, capture or mimic some behavior of the true theory. The advantage of models is their flexibility, since a model may be tuned to particular processes or hadronic states. The disadvantage is that models are intrinsically ad hoc, and it is difficult to assess their reliability. For this reason, one should use them only when no better options are available.

Effective field theories are based on the idea that in a given process, only certain degrees of freedom may be important for understanding the physics. In particular, it is often the case that kinematical considerations which restrict the momenta of external particles effectively restrict the momenta of virtual particles as well. Thus it is sensible to remove from the theory intermediate states of high virtuality. Their absence may be compensated by introducing new “effective” interactions between the degrees of freedom which remain. Effective field theories are often constructed using the technique of the operator product expansion, which provides an elegant and general conceptual framework.

Both the HQE and ChPT are effective field theories which are derived from formal limits of QCD in which the theory exhibits new and useful symmetries. In the case of the HQE, the limit is $m_b, m_c \to \infty$, where a “spin-flavor” symmetry yields a variety of predictions for heavy hadron spectroscopy and semileptonic decays. For ChPT, the limit is $m_u, m_d, m_s \to 0$, which leads to exact predictions for the emission and absorption of soft pions. In both cases, the quark masses are large or small compared to the scale of nonperturbative QCD, typically hundreds of MeV. What makes an effective field theory powerful is that the deviations from the limiting behavior may be organized in a systematic expansion in a small parameter. Hence one can both improve the accuracy of an analysis and derive quantitative error estimates. An effective field theory is predictive precisely because it is under perturbative control.

While the HQE and ChPT are powerful tools where they may be applied, their use is restricted to a small number of processes involving certain initial and final states. Unfortunately, the HQE and ChPT have nothing to say about the vast majority of processes and quantities available for experimental study at a B Factory. Similar considerations affect lattice QCD. Because of both computational and theoretical limitations, reliable lattice predictions are confined largely to spectroscopy and matrix elements with restricted kinematics. QCD sum rules, also for technical reasons, may only be used in limited circumstances.

Thus a serious problem remains, namely that many quantities of experimental and phenomenological importance cannot be analyzed by methods which are systematic and well understood. For inclusive weak decays, some exclusive semileptonic decays, and some static properties, effective field theories or the lattice give controlled theoretical predictions. But for the description of exclusive hadronic weak decays, most exclusive semileptonic decays,
strong decays, fragmentation, and many other interesting aspects of $B$ physics, only a variety of model dependent approaches are available. While no model is “correct”, some models are better than others. A successful model should be motivated by some physical picture, should reproduce much more data than there are input parameters, and should behave correctly in appropriate limits, such as obeying heavy quark symmetry as $m_b \to \infty$. It will not be possible here to discuss or even enumerate all of the models which are used in $B$ physics, but it is generally true that every model ought to be judged by criteria such as these.

Because there is no single theoretical framework which suffices for all of $B$ physics, it is often necessary to utilize a variety of methods in one theoretical analysis. Usually, this is desirable, as a combination of complementary approaches can lead to conclusions which are much more robust. But at the same time, one must be careful to be consistent in the use and definition of theoretical concepts and quantities, and particularly in their translation from one context to another. Otherwise one is led easily to error and confusion.

An excellent illustration of how problems can arise is given by the definition of the heavy quark mass. Clearly there is something which is meant by “the $b$ mass”, because to say that the $b$ quark is heavy is to say that the parameter $m_b$ is large compared to $\Lambda_{\text{QCD}}$. Whatever the $b$ mass is, it is presumably somewhere close to 4 or 5 GeV. But the situation becomes more complicated when one tries to pin down $m_b$ more precisely than that.

On the one hand, it is known that the $b$ quark acquires its mass from its coupling, of strength $\lambda_b$, to the “Higgs vacuum expectation value” $v$, so $m_b = \lambda_b v$. The quark mass which is directly related to this coupling is known as the “current mass” or “short distance mass.” Its value depends on the renormalization scheme, such as $\overline{\text{MS}}$, which is used to define the theory. In perturbation theory, there is also a pole in the $b$ quark propagator, the position of which corresponds to the rest energy of a freely propagating $b$ quark. This “pole mass” is closer to an intuitive notion of an invariant, relativistic mass. Unfortunately, because of confinement, a freely propagating $b$ quark cannot actually exist, and the pole mass is not defined nonperturbatively. In fact, even within perturbation theory the pole mass is ill behaved and can only be defined to a fixed finite order $\alpha_n^\text{pert}$. Hence there is really a family of pole masses, namely the “1-loop pole mass”, the “2-loop pole mass”, and so on, none more “accurate” or intuitively accessible than another. There are also “Wilsonian” running masses $m_b(\mu)$, which are defined with additional subtractions in the infrared.

An analogous variety of $b$ quark masses is defined in lattice calculations. While it is typically understood how these lattice $b$ masses are related to each other, relating them to pole or current masses defined in continuum QCD can be problematic. For example, lattice field theory, both perturbative and nonperturbative, is regulated and subtracted differently from field theory in the continuum, and the relationship between the various schemes often is not straightforward. Similar ambiguities can affect the $b$ quark masses which appear in QCD sum rules. Finally, there are the many quark masses introduced in models, which are free parameters with no rigorous relationship either to each other or to masses defined in QCD. A typical example is the “constituent quark mass” of the nonrelativistic quark model. No matter how precisely one fits the constituent quark mass to data, it can never be used as an input into a lattice or HQE calculation. The most that can be said is that all of these various masses probably are within several hundred MeV of each other.

It is important to understand that there is no more precise way to unify these many masses into a single universal quantity. The ambiguity in $m_b$ is unimportant, so long as
its definition is consistent within a given analysis, and ultimately one predicts measurable quantities in terms of other measurable quantities. The problem is that it is difficult to make an $m_b$ defined on the lattice consistent with one defined in the continuum, and impossible to make a model dependent $m_b$ consistent with either. Hence there can be limits in principle to the accuracy which one can obtain when a variety of methods are combined in a single analysis.

We now turn to elementary introductions to the most important theoretical techniques in $B$ physics. After a general discussion of operator product expansions and effective field theories, Heavy Quark Effective Theory and Chiral Perturbation Theory are introduced. The next two sections contain discussions of lattice QCD and QCD sum rules, followed by a brief discussion of quark models. None of these ideas will be developed in much depth. Rather, they are intended to serve as a background to the variety of detailed theoretical analyses which are presented in The BaBar Physics Book.

II. THE OPERATOR PRODUCT EXPANSION

A. General Considerations

A central observation which underlies much of the theoretical study of $B$ mesons is that physics at a wide variety of distance (or momentum) scales is typically relevant in a given process. At the same time, the physics at different scales must often be analyzed with different theoretical approaches. Hence it is crucial to have a tool which enables one to identify the physics at a given scale and to separate it out explicitly. Such a tool is the operator product expansion, used in conjunction with the renormalization group. Here a general discussion of its application is given.

Consider the Feynman diagram shown in Fig. 1, in which a $b$ quark decays nonleptonically. The virtual quarks and gauge bosons have virtualities $\mu$ which vary widely, from $\Lambda_{\text{QCD}}$ to $M_W$ and higher. Roughly speaking, these virtualities can be classified into a variety of energy regimes: (i) $\mu \gg M_W$; (ii) $M_W \gg \mu \gg m_b$; (iii) $m_b \gg \mu \gg \Lambda_{\text{QCD}}$; (iv) $\mu \approx \Lambda_{\text{QCD}}$. Each of these momenta corresponds to a different distance scale; by the uncertainty principle, a particle of virtuality $\mu$ can propagate a distance $x \approx 1/\mu$ before being reabsorbed. At a given resolution $\Delta x$, only some of these virtual particles can be distinguished, namely those that propagate a distance $x > \Delta x$. For example, if $\Delta x > 1/M_W$, then the virtual $W$ cannot be seen, and the process whereby it is exchanged would appear as a point interaction. By the same token, as $\Delta x$ increases toward $1/\Lambda_{\text{QCD}}$, fewer and fewer of the virtual gluons can be seen explicitly. Finally, for $\mu \approx \Lambda_{\text{QCD}}$, it is probably not appropriate to speak of virtual gluons at all, because at such low momentum scales QCD becomes strongly interacting and a perturbation series in terms of individual gluons is inadequate.

It is useful to organize the computation of a diagram such as is shown in Fig. 1 in terms of the virtuality of the exchanged particles. This is important both conceptually and practically. First, it is often the case that a distinct set of approximations and approaches is useful at each distance scale, and one would like to be able to apply specific theoretical techniques at the scale at which they are appropriate. Second, Feynman diagrams in which two distinct scales $\mu_1 \gg \mu_2$ appear together can lead to logarithmic corrections of the form
\(\alpha_s \ln(\mu_1/\mu_2)\), which for \(\ln(\mu_1/\mu_2) \sim 1/\alpha_s\) can spoil the perturbative expansion. A proper separation of scales will include a resummation of such terms.

**B. Example I: Weak b Decays**

As an example, consider the weak decay of a \(b\) quark, \(b \to c \bar{u} d\), which is mediated by the decay of a virtual \(W\) boson. Viewed with resolution \(\Delta x < 1/M_W\), the decay amplitude involves an explicit \(W\) propagator and is proportional to

\[
\bar{c}\gamma^\mu (1-\gamma^5)b \bar{d}\gamma^\mu (1-\gamma^5)u \times \frac{(ig_2)^2/4}{p^2 - M_W^2},
\]

where \(p^\mu\) is the momentum of the virtual \(W\). Since \(m_b \ll M_W\), the kinematics constrains \(p^2 \ll M_W^2\), so the virtuality of the \(W\) is of order \(M_W\), and it travels a distance of order \(1/M_W\) before decaying. Viewed with a lower resolution, \(\Delta x > 1/M_W\), the process \(b \to c \bar{u} d\) appears to be a local interaction, with four fermions interacting via a potential which is a \(\delta\) function where the four particles coincide. This can be seen by making a Taylor expansion of the amplitude in powers of \(p^2/M_W^2\),

\[
\bar{c}\gamma^\mu (1-\gamma^5)b \bar{d}\gamma^\mu (1-\gamma^5)u \times \frac{g_2^2}{8M_W^2} \left[ 1 + \frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \ldots \right].
\]

The coefficient of the first term is just the usual Fermi decay constant, \(G_F/\sqrt{2}\). The higher order terms correspond to local operators of higher mass dimension. In the sense of a Taylor expansion, the momentum-dependent matrix element (1), which involves the propagation of a \(W\) boson between two spacetime points, is identical to the matrix element of the following infinite sum of local operators:

\[
\frac{G_F}{\sqrt{2}} \bar{c}\gamma^\mu (1-\gamma^5)b \left[ 1 + \frac{(i\partial)^2}{M_W^2} + \frac{(i\partial)^4}{M_W^4} + \ldots \right] \bar{d}\gamma^\mu (1-\gamma^5)u,
\]

where the derivatives act on the entire current on the right. This expansion of the nonlocal product of currents in terms of local operators, sometimes known as an operator product expansion, is valid so long as \(p^2 \ll M_W^2\). For \(B\) decays, the external kinematics requires \(p^2 \leq m_b^2\), so this condition is well satisfied. In this regime, one may consider a nonrenormalizable

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FIG. 1. The nonleptonic decay of a \(b\) quark.
effective field theory, with interactions of dimension six and above. The construction of such a low energy effective theory is also known as matching. As it is nonrenormalizable, the effective theory is defined (by construction) only up to a cutoff, in this case $M_W$. The cutoff is explicitly the mass of a particle which has been removed from the theory, or integrated out. If one considers processes in which one is restricted kinematically to momenta well below the cutoff, the nonrenormalizability of the theory poses no technical problems. Although the coefficients of operators of dimension greater than six require counterterms in the effective theory (which may be unknown in strongly interacting theories), their matrix elements are suppressed by powers of $p^2/M_W^2$. To a given order in $p^2/M_W^2$, the theory is well-defined and predictive.

From a modern point of view, in fact, such nonrenormalizable effective theories are actually preferable to renormalizable theories, because the nonrenormalizable terms contain information about the energy scale at which the theory ceases to apply. By contrast, renormalizable theories contain no such explicit clues about their region of validity.

In principle, it is possible to include effects beyond leading order in $p^2/M_W^2$ in the effective theory, but in practice, this is usually quite complicated and rarely worth the effort. Almost always, the operator product expansion is truncated at dimension six, leaving only the four-fermion contact term. Corrections to this approximation are of order $m_b^2/M_W^2 \sim 10^{-3}$.

C. Radiative Corrections

At tree level, the effective theory is constructed simply by integrating out the $W$ boson, because this is the only particle in a tree level diagram which is off-shell by order $M_W^2$. When radiative corrections are included, gluons and light quarks can also be off-shell by this order. Consider the one-loop diagram shown in Fig. 2. The components of the loop momentum $k^\mu$ are allowed to take all values in the loop integral. However, the integrand is cut off both in the ultraviolet and in the infrared. For $k > M_W$, it scales as $d^4k/k^6$, which is convergent as $k \to \infty$. For $k < m_b$, it scales as $d^4k/k^3m_bM_W^2$, which is convergent as $k \to 0$. In between, all momenta in the range $m_b < k < M_W$ contribute to the integral with roughly equivalent weight.

As a consequence, there is potentially a radiative correction of order $\alpha_s \ln(M_W/m_b)$. Even if $\alpha_s(\mu)$ is evaluated at the high scale $\mu = M_W$, such a term is not small in the limit $M_W \to \infty$. At $n$ loops, there is potentially a term of order $\alpha_s^n \ln^n(M_W/m_b)$. For
$\alpha_s \ln(M_W/m_b) \sim 1$, these terms need to be resummed for the perturbation series to be predictive. The technique for performing such a resummation is the renormalization group.

The renormalization group exploits the fact that in the effective theory, operators such as

$$O_I = \bar{c}_i \gamma^\mu (1 - \gamma^5) b^j \bar{d}_j \gamma^\mu (1 - \gamma^5) u^i,$$(4)

receive radiative corrections and must be subtracted and renormalized. (Here the color indices $i$ and $j$ are explicit.) In dimensional regularization, this means that they acquire, in general, a dependence on the renormalization scale $\mu$. Because physical predictions are of necessity independent of $\mu$, in the renormalized effective theory it must be the case that the operators are multiplied by coefficients with a dependence on $\mu$ which compensates that of the operators. It is also possible for operators to mix under renormalization, so the set of operators induced at tree level may be enlarged once radiative corrections are included. In the present example, a second operator with different color structure, $O_{II} = \bar{c}_i \gamma^\mu (1 - \gamma^5) b^j \bar{d}_j \gamma^\mu (1 - \gamma^5) u^i,$ (5)

is induced at one loop. The interaction Hamiltonian of the effective theory is then

$$\mathcal{H}_{\text{eff}} = C_I(\mu) O_I(\mu) + C_{II}(\mu) O_{II}(\mu),$$ (6)

and it satisfies the differential equation

$$\mu \frac{d}{d\mu} \mathcal{H}_{\text{eff}} = 0.$$ (7)

By computing the dependence on $\mu$ of the operators $O_i(\mu)$, one can deduce the $\mu$-dependence of the Wilson coefficients $C_i(\mu)$. In this case, a simple calculation yields

$$C_{I,II}(\mu) = \frac{1}{2} \left[ \left( \frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right)^{6/23} \pm \left( \frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right)^{-12/23} \right].$$ (8)

For $\mu = m_b$, these expressions resum all large logarithms proportional to $\alpha_s^n \ln^n(M_W/m_b)$.

The decays which are observed involve physical hadrons, not asymptotic quark states. For example, this nonleptonic $b$ decay can be realized in the channels $B \to D\pi$, $B \to D^*\pi\pi$, and so on. The computation of partial decay rates for such processes requires the analysis of hadronic matrix elements such as

$$\langle D\pi | \bar{c}_i \gamma^\mu (1 - \gamma^5) b \bar{u} \gamma^\mu (1 - \gamma^5) d | \bar{b} \rangle.$$ (9)

Such matrix elements involve nonperturbative QCD and are extremely difficult to compute from first principles. However, they have no intrinsic dependence on large mass scales such as $M_W$. Because of this, they should naturally be evaluated at a renormalization scale $\mu \ll M_W$, in which case large logarithms $\ln(M_W/m_b)$ will not arise in the matrix elements. By choosing such a low scale in the effective theory (6), all such terms are resummed into the coefficient functions $C_i(m_b)$. As promised, the physics at scales near $M_W$ has been separated from the physics at scales near $m_b$, with the renormalization group used to resum the large logarithms which connect them. In fact, nonperturbative hadronic matrix elements are usually evaluated at an even lower scale $\mu \approx \Lambda_{\text{QCD}} \ll m_b$, explicitly resumming all perturbative QCD corrections.
D. Example II: Penguins and Box Diagrams

In the previous example of nonleptonic decays, the operator $O_I$ appeared when the matching at tree level was performed. It is also possible to find new operators in the effective theory which appear only when the matching is performed at one loop. The most common such operators are those which arise from penguin and box diagrams, such as those shown in Fig. 3. These diagrams are important in $b$ physics typically because they lead to flavor-changing interactions at low energies which are suppressed at tree level in the Standard Model. Hence the transitions mediated by these operators are potentially a sensitive probe of new physics.

Both penguins and box diagrams can lead, via the operator product expansion, to four-quark operators with new flavor or color structure, such as

$$\bar{s}\gamma^{\mu} T^{a} b \bar{d}\gamma_{\mu} T^{a} d,$$

which mediates nonleptonic $B$ decay, or

$$\bar{b}\gamma^{\mu} \gamma^{5} d \bar{b}\gamma_{\mu} \gamma^{5} d,$$

which is responsible for $B^0 - \bar{B}^0$ mixing. Penguins can also lead to new flavor-changing magnetic interactions, such as

$$\bar{s}\sigma^{\mu\nu} T^{a} b C_{\mu\nu}^{a},$$

when the $d$ quark line in Fig. 3 is removed. The gluon could also be replaced by a photon or a $Z$ boson. From the point of view of the low energy effective theory, it is unimportant that these operators arise at one loop at high energy. They can mix with four-fermion operators induced at tree level, insofar as such mixing is allowed by the flavor and Lorentz symmetries of the effective theory. In fact, the renormalization of penguin-induced operators can be quite complicated, due to the nature of their flavor structure; two loop calculations may be required to resum the leading logarithms $\alpha_s^n \ln^n(M_W/m_b)$.

E. Summary

Low-energy effective theories are constructed using the operator product expansion and the renormalization group. This procedure implements an important separation of scales,
isolating the physics which involves virtualities $\mu \gg m_b$ and accounting for it systematically in a double expansion in powers of $\alpha_s$ and $m_b/M_W$, where $M_W$ is the matching scale at which heavy particles are integrated out of the theory. This procedure may be generalized to integrate out heavy particles of many different kinds.

This analysis explicitly does not address those parts of a process which are dominated by low momenta, which will typically be more difficult to deal with. By breaking the problem up according to momentum scale, one may compute systematically in perturbation theory where it is possible to do so. However, the accuracy obtained in this part of the calculation is useful only if one can also account for physics at lower energy. The chief limitation on the accuracy of most theoretical calculations in $b$ physics is, in fact, from these lower energy effects.

III. THE HEAVY QUARK EXPANSION

A. Separation of Scales

This section considers physics characterized by virtualities $\mu \approx m_b$ and below. The previous section discussed how physics at higher scales is accounted for in QCD perturbation theory, because at high energies $\alpha_s(\mu)/\pi \ll 1$. Although $m_b \ll M_W$, at this "low" energy it is still the case that $\alpha_s(m_b)/\pi \approx 0.1 \ll 1$, and $\Lambda_{\text{QCD}}/m_b \sim 0.1 \ll 1$. Hence one seeks a technique analogous to the operator product expansion by which to exploit the presence of such small parameters.

The status of the $b$ quark in a $B$ meson is different from that of a virtual $W$ in a weak decay, because the $b$ is real, not virtual, and the $B$ carries nonzero $b$-number which persists in the asymptotic state. Hence it is not appropriate to integrate out the $b$ in the same sense as the $W$ was integrated out, removing it from the theory entirely. Rather, when bound into a hadron with light degrees of freedom of typical energies $E \sim \Lambda_{\text{QCD}}$, the $b$ makes excursions from its mass shell by virtualities only of order $\Lambda_{\text{QCD}}$. What can be integrated out is not the $b$ itself, but rather those parts of the $b$ field which take it far off shell. The result will be an effective theory of a static $b$ quark, in its rest frame.

Processes with hard virtual gluons, which drive the $b$ far off shell, will lead to perturbative corrections in the effective theory of order $\alpha_s(m_b)$. They may be included as before. In addition, power corrections appear, analogous to the higher order operators appear in Eq. (8). In this case, it will be necessary to include the leading higher-dimension operators to achieve results of the desired accuracy. These power corrections will lead to terms of order $(\Lambda_{\text{QCD}}/m_b)^n$. The appearance of the scale $\Lambda_{\text{QCD}}$ serves as a reminder that these corrections involve nonperturbative physics, and will typically not be calculable from first principles. Instead, the inclusion of power corrections will require the introduction of new phenomenological parameters, whose values are controlled by nonperturbative QCD. These parameters have precise field-theoretic definitions, and they will be introduced in a systematic manner. Their appearance will not spoil the inherent predictability of the theory, although in practice they will increase the number of quantities which must be determined from experiment before accurate predictions can be made.
Finally, for some applications (notably the analysis of exclusive semileptonic $B$ decays), it will be useful to treat the $c$ quark as heavy, that is, to perform an expansion also in powers of $\Lambda_{\text{QCD}}/m_c$. In this case, clearly, the leading power corrections will be important and will have to be well understood for the theory to be predictive.

### B. Heavy Quark Symmetry

Let us, for generality, consider a hadron $H_Q$ composed of a heavy quark $Q$ and “light degrees of freedom” consisting of quarks, antiquarks and gluons, in the limit $m_Q \to \infty$. The Compton wavelength of the heavy quark scales as the inverse of the heavy quark mass, $\lambda_Q \sim 1/m_Q$. The light degrees of freedom, by contrast, are characterized by momenta of order $\Lambda_{\text{QCD}}$, corresponding to wavelengths $\lambda_{\ell} \sim 1/\Lambda_{\text{QCD}}$. Since $\lambda_{\ell} \gg \lambda_Q$, the light degrees of freedom cannot resolve features of the heavy quark other than its conserved gauge quantum numbers. In particular, they cannot probe the actual value of $\lambda_Q$. Although the structure of the hadron $H_Q$ is determined by nonperturbative strong interactions, the typical momenta exchanged by the light degrees of freedom with each other and with the heavy quark are of order $\Lambda_{\text{QCD}} \ll m_Q$, against which the heavy quark $Q$ does not recoil. In this limit, $Q$ acts as a static source of electric and chromoelectric field.

There is an immediate implication for the spectroscopy of heavy hadrons. Since the interaction of the light degrees of freedom with the heavy quark is independent of $m_Q$, then so is the spectrum of excitations. It is these excitations which determine the spectrum of heavy hadrons $H_Q$. Since the splittings $\Delta_i \sim \Lambda_{\text{QCD}}$ between the various hadrons $H_Q$ are entirely due to the properties of the light degrees of freedom, they are independent of $Q$ and, in the limit $m_Q \to \infty$, do not scale with $m_Q$. For example, if $m_b, m_c \gg \Lambda_{\text{QCD}}$, then the light degrees of freedom are in exactly the same state in the mesons $B_i$ and $D_i$, for a given $i$. The offset $B_i - D_i = m_b - m_c$ is just the difference between the heavy quark masses. By no means does the relationship between the spectra rely on an approximation $m_b \approx m_c$.

This picture is enriched by recalling that the heavy quarks and light degrees of freedom also carry angular momentum. The heavy quark has spin quantum number $S_Q = \frac{1}{2}$, which leads to a chromomagnetic moment $\mu_Q \propto g/2m_Q$. Note that $\mu_Q \to 0$ as $m_Q \to \infty$, and the interaction between the spin of the heavy quark and the light degrees of freedom is suppressed. Hence the light degrees of freedom are insensitive to $S_Q$; their state is independent of whether $S_Q^z = \frac{1}{2}$ or $S_Q^z = -\frac{1}{2}$. Thus each of the energy levels $B_i$ and $D_i$ is actually doubled, one state for each value of $S_Q^z$. In summary, the light degrees of freedom in a heavy hadron are the same when combined with any of the four heavy quark states:

$$b(\uparrow), \ b(\downarrow); \ c(\uparrow), \ c(\downarrow).$$

The result is an $SU(4)$ symmetry which leads to nonperturbative relations between physical quantities.

Suppose the light degrees of freedom have total angular momentum $J_{\ell}$, which is integral for baryons and half-integral for mesons. When combined with the heavy quark spin $S_Q = \frac{1}{2}$, physical hadron states can be produced with total angular momentum

$$J = \left| J_{\ell} \pm \frac{1}{2} \right|.$$

10
If $J_\ell \neq 0$, then these are two degenerate states. For example, the lightest heavy mesons have $J_\ell = 1/2$, leading to a doublet with $J = 0$ and $J = 1$. When effects of order $1/m_Q$ are included, the chromomagnetic interactions split the states of given $J_\ell$ but different $J$. This “hyperfine” splitting is not calculable perturbatively, but it is proportional to the heavy quark magnetic moment $\mu_Q$. Since $\mu_Q \propto 1/m_Q$, one can construct a relation which is a nonperturbative prediction of heavy quark symmetry,

$$m_{B^*}^2 - m_B^2 = m_{D^*}^2 - m_D^2. \quad (15)$$

Experimentally, $m_{B^*}^2 - m_B^2 = 0.49\,\text{GeV}^2$ and $m_{D^*}^2 - m_D^2 = 0.55\,\text{GeV}^2$. The correction to this prediction is of order $\Lambda_{\text{QCD}}^3(1/m_c - 1/m_b) \sim 0.1\,\text{GeV}^2$, so it works about as well as one should expect. Note that the relation (15) involves not only the heavy quark symmetry, but also the systematic inclusion of the leading symmetry violating effects.

So far, heavy quark symmetry has been formulated for hadrons in their rest frame. One can easily boost to a frame in which the hadrons have arbitrary four-velocity $v^\mu = \gamma(1, \vec{v})$. The symmetry will then relate hadrons $H_b(v)$ and $H_c(v)$ with the same velocity but with different momenta. This distinguishes heavy quark symmetry from ordinary symmetries of QCD, which relate states of the same momentum. It will often be convenient to label heavy hadrons explicitly by their velocity: $B(v), B^{*}(v)$, and so on.

C. Heavy Quark Effective Theory

It is quite useful to make heavy quark symmetry manifest within QCD by taking the limit $m_b \to \infty$ of the QCD Lagrangian. This is done by making the dependence of all quantities on $m_b$ explicit, and then developing the Lagrangian in a series in inverse powers of $m_b$. The idea is to write the Lagrangian in a form in which the action of the heavy quark symmetries is well-defined at each order in the expansion, so the effect of symmetry breaking corrections can be studied in a systematic way. The resulting Lagrangian is known as the Heavy Quark Effective Theory (HQET). The HQET is similar to an effective theory which results from an operator product expansion, in the sense that the only virtualities $p$ which are allowed satisfy $p \ll m_b$, with effects of greater virtuality absorbed into the coefficients of higher dimension operators. The difference is that in this case, the heavy $b$ quark is not explicitly removed from the effective theory.

In the heavy quark limit, the velocity $v^\mu$ of the $b$ quark is conserved. Thus one may write its four-momentum in the form $p_b^\mu = m_b v^\mu + k^\mu$, where $m_b v^\mu$ is the on-shell part and $k^\mu$ is the residual momentum. In this decomposition, $k^\mu \sim \Lambda_{\text{QCD}}$ represents the fluctuations in $p_b^\mu$ due to the exchange of soft gluons with the rest of the $B$ meson. Only the on-shell part of $p_b^\mu$ scales with $m_b$. Also, mixing between the “quark” and “anti-quark” components of the Dirac spinor is suppressed by powers of $2m_b$, the mass gap between the positive and negative energy parts of the wavefunction. Hence an effective heavy quark field $h_v$ can be defined,

$$h_v(x) = \frac{1 + \gamma}{2} e^{im_b v \cdot x} b(x), \quad (16)$$
where the Dirac matrix \((1 + \gamma) / 2\) projects out the “quark” part of the field. Furthermore, since \(i \partial \gamma^\mu h_v(x) = (p^\mu_b - m_b \gamma^\mu)h_v(x) = k^\mu h_v(x)\), derivatives acting on \(h_v\) scale as \(\Lambda_{\text{QCD}}\), rather than as \(m_b\).

The next step is to express the QCD Lagrangian, \(\mathcal{L} = \bar{b}(i\gamma^\mu - m_b)\gamma^\mu b\), in terms of the \(m_b\)-independent field \(h_v\). At lowest order in \(1/m_b\), the result is

\[
\mathcal{L}_{\text{HQET}} = \bar{h}_v i v \cdot D h_v .
\]  

(17)

At leading order, \(\mathcal{L}_{\text{HQET}}\) respects the heavy spin and flavor symmetries explicitly. Both bottom and charm quarks can be treated as heavy by introducing separate effective fields \(h_v(b)\) and \(h_v(c)\) and duplicating \(\mathcal{L}_{\text{HQET}}\). The theory has a simple heavy quark propagator and quark-gluon vertex which are manifestly independent of \(m_b\) and have no Dirac structure.

The effective theory is also expanded perturbatively in \(\alpha_s(m_b)\). In particular, the quark mass \(m_b\) is shifted to \(m_{\text{pole}}\), the pole mass at \(n\) loops. The pole mass is a quantity which makes sense only at finite order in perturbation theory. One must always be careful to be consistent in the convention by which one chooses to define it.

The mass of the \(B\) meson may be expanded in powers of \(m_b\),

\[
m_B = m_b + \Lambda + \mathcal{O}(1/m_b) ,
\]

(18)

where \(\Lambda \sim \Lambda_{\text{QCD}}\) is the energy contributed by the light degrees of freedom. Its precise definition depends on the convention by which one chooses to define the heavy quark pole mass. The parameter \(\Lambda\) depends on the flavor, excitation energy and total angular momentum of the light degrees of freedom.

When one includes the leading \(1/m_{b,c}\) corrections, the heavy spin and flavor symmetries are broken by the subleading terms. The leading Lagrangian [17] is modified by the addition of two terms,

\[
\mathcal{L}^{(1)} = \frac{1}{2m_b} (O_1 + O_2) = \frac{1}{2m_b} (\bar{h}_v (iD)^2 h_v + \bar{h}_v \frac{1}{2} g G^{\mu\nu} \sigma^{\mu\nu} h_v) ,
\]

(19)

neglecting terms which vanish by the classical equations of motion. Note that the “kinetic” operator \(O_1\) violates the heavy flavor symmetry, while the “chromomagnetic” operator \(O_2\) violates both the spin and flavor symmetries. When radiative corrections are included, the operator \(O_2\) is renormalized, and its coefficient develops a logarithmic dependence on \(m_b\).

The subleading operators \(O_1\) and \(O_2\) contribute to the mass of the \(B\) meson through their expectation values,

\[
\lambda_1 = \langle B | \bar{h}_v (iD)^2 h_v | B \rangle / 2m_B ,
\]

\[
\lambda_2 = \langle B | \bar{h}_v \frac{1}{2} g G^{\mu\nu} \sigma^{\mu\nu} h_v | B \rangle / 6m_B .
\]

(20)

(21)

The matrix elements \(\lambda_1\) and \(\lambda_2\) are often referred to by the alternate names \(\mu_{\pi}^2 = -\lambda_1\) and \(\mu_G^2 = 3\lambda_2\). The parameter \(\mu_{\pi}^2\) actually differs from \(\lambda_1\) in that it is defined with an explicit infrared subtraction. Because they are defined in the effective theory, the parameters \(\lambda_1\) and \(\lambda_2\) do not depend on \(m_b\). The expansion of \(m_B\) now takes the form
\[ m_B = m_b + \Lambda - \frac{\lambda_1 + 3\lambda_2}{2m_b} + \ldots, \]
\[ m_{B^*} = m_b + \overline{\Lambda} - \frac{\lambda_1 - \lambda_2}{2m_b} + \ldots. \]

Because \( \mathcal{O}_2 \) violates the heavy spin symmetry, it is the leading contribution to the splitting between \( B \) and \( B^* \). From the measured mass difference, \( \lambda_2 \approx 0.12 \text{GeV}^2 \). On the other hand, the parameters \( \overline{\Lambda} \) and \( \lambda_1 \) must be measured indirectly. Estimates from models yield the ranges \( 200 \text{MeV} < \overline{\Lambda} < 700 \text{MeV} \) and \( -0.5 \text{GeV}^2 < \lambda_1 < 0 \). Measurement of various features of inclusive semileptonic \( B \) decays will provide experimental information on \( \overline{\Lambda} \) and \( \lambda_1 \) in the future.

D. Application of the HQE to \( B \) Decays

There is a wide variety of applications of the HQE to \( B \) decays. Here a few general comments and two illustrative examples are given. In principle, the value of using an effective theory such as the HQE is that there is a framework within which one can estimate the error in a calculation, due to uncomputed terms of a definite size. Even when the accuracy is not so good, it is under control in the sense that one can understand the magnitude of the error to be expected. In any application of the HQE, then, two sorts of questions must be addressed in addition to the computation itself:

1. What are the sizes of the leading uncomputed corrections in the expansion in powers of \( \alpha_s \) and \( 1/m_b \) (or \( 1/m_c \), as appropriate)? With what accuracy are the parameters known which appear in the expansion?

2. What other assumptions or approximations have been made, beyond those that go into the HQE itself?

1. Exclusive semileptonic \( B \) decays

The paradigmatic application of heavy quark symmetry is to semileptonic \( B \) decay in the limit \( m_b, m_c \to \infty \). This decay is mediated by the quark transition \( b \to c \ell \bar{\nu} \). Suppose the weak decay occurs at time \( t = 0 \). What happens to the light degrees of freedom? Since the \( b \) quark does not recoil, for \( t < 0 \) they see simply the color field of a point source moving with velocity \( v \). At \( t = 0 \), this point source changes (almost) instantaneously to a new velocity \( v' \); the color neutral leptons do not interact with the light hadronic degrees of freedom as they fly off. The light quarks and gluons then must reassemble themselves about the recoiling color source. There is some chance that this nonperturbative process will lead the light degrees of freedom to reassemble themselves back into a \( D \) meson. The amplitude for this to happen is a function \( \xi(w) \) of the product \( w = v \cdot v' \) of the initial and final velocities of the heavy color sources.

Clearly, the kinematic point \( v = v' \), or \( w = 1 \), is a special one. In this corner of phase space, where the leptons are emitted back to back, there is no recoil of the source of color field at \( t = 0 \). As far as the light degrees of freedom are concerned, nothing happens! Hence
the amplitude for them to remain in the ground state is exactly unity. This is reflected in a nonperturbative normalization of $\xi(w)$ at zero recoil,

$$\xi(1) = 1.$$  \hspace{1cm} (23)

This normalization condition is of great phenomenological use. There are important corrections to this result for finite heavy quark masses $m_b$ and, especially, $m_c$.

The weak decay $b \to c$ is mediated by a left-handed current $\bar{c}\gamma^\mu (1 - \gamma^5) b$, which can also change the orientation of the spin $S_Q$ of the heavy quark during the decay. Since the only difference between a $D$ and a $D^*$ is the orientation of $S_c$, heavy quark symmetry implies relations between the hadronic matrix elements which describe the semileptonic decays $B \to D\ell\bar{\nu}$ and $B \to D^*\ell\bar{\nu}$. These matrix elements are parameterized by six form factors, which are independent nonperturbative functions of $w$. In the heavy quark limit, they are all proportional to $\xi(w)$, a powerful constraint on the structure of semileptonic decays.

Now consider more closely the structure of the theoretical expansion for the decay $B \to D^*\ell\bar{\nu}$, which may be used to measure the CKM matrix element $|V_{cb}|$. Near the zero-recoil point, the transition is dominated by a single form factor, $h_{A_1}(w)$, with the normalization condition $h_{A_1}(1) = 1$ in the heavy quark limit. For general $m_B$ and $m_{D^*}$, the differential decay rate may be written

$$\frac{d\Gamma}{dw} = G_F^2 |V_{cb}|^2 K(m_B, m_{D^*}, w) F^2(w),$$  \hspace{1cm} (24)

where $K(m_B, m_{D^*}, w)$ is a known kinematic function and $F(w)$ has an expansion at $w = 1$ of the form

$$F(1) = \eta_A(\alpha_s) \left[ 1 + \frac{0}{m_c} + \frac{0}{m_b} + \mathcal{O}\left(\frac{1}{m_{b,c}^2}\right) \right].$$  \hspace{1cm} (25)

The perturbative function $\eta_A(\alpha_s)$ has been computed to two loops, with the result $\eta_A = 0.960$. The leading HQE corrections arise at order $1/m_{b,c}^2$ rather than at order $1/m_{b,c}$, and have been estimated to be approximately 5%. More detailed analysis of this decay exist in the literature. The point here is to note how the double expansion in powers of $\alpha_s$ and $1/m_{b,c}$ appears in a physical quantity. This analysis is also typical because it applies only to a very particular case of enhanced symmetry, namely the decay rate as $w \to 1$. The extrapolation of the data to this limit requires both experimental ingenuity and more theoretical input beyond the HQE.

2. Duality and inclusive semileptonic decays

As a second example, consider the inclusive decay $B \to X_c$, where $X_c$ is any final state containing a charm quark. The analysis of inclusive decays, although it relies on a similar expansion, is different from the treatment of exclusive decays. In this case, it is useful to observe that the energy released into the final state by the decay of the heavy $b$ quark is large compared to the QCD scale. Hence the final hadronic state need not be dominated by
a few sharp resonances. If resonances are indeed unimportant, then there is a factorization between the short distance part of the decay (the disappearance of the $b$ quark) and the long distance part (the eventual hadronization of the decay products). This factorization implies that for sufficiently inclusive quantities it is enough to consider the short distance part of the process, with the subsequent hadronization taking place with unit probability. Note that what is important here is that the $b$ quark is heavy, with no restriction placed on the charm mass. In fact, a smaller charm quark mass is better, because it increases the average kinetic energy of the decay products.

This factorization, known as local parton-hadron duality, is an example of a crucial assumption which lies outside of the HQE itself. What is its status? Clearly, local duality must hold as $m_b \to \infty$ with all other masses held fixed. In this limit, wavelengths associated with the $b$ quark decay are arbitrarily short and cannot interfere coherently with the hadronization process. On the other hand, it is not known how to estimate the size of corrections to local duality for $m_b$ large but finite. There is no analog of the heavy quark expansion appropriate to this question, and no way to estimate systematically deviations from the limit $m_b \to \infty$. Although we will incorporate an expansion in powers $1/m_b$ in the calculation of inclusive quantities, the behavior of this expansion does not address directly the issue of violations of duality. The duality hypothesis, while entirely reasonable for inclusive $B$ decays, is not independently verifiable except by the direct confrontation of theoretical calculations with the data.

For semileptonic $B$ decays, $B \to X_c \ell \bar{\nu}$, the situation has additional interesting features. On the one hand, in the region of phase space where the leptons carry away most of the available energy, the final hadronic state is likely to be dominated by resonances and local duality is likely to fail. (In some $B$ decays, local duality can be shown to hold even in the resonance region; however, this requires a more subtle and less intuitive argument than the one on which this discussion is based.) On the other hand, if one integrates over the lepton phase space to compute an inclusive quantity such as the total semileptonic width, then one needs not local duality but rather the weaker notion of global parton-hadron duality. (The use here and elsewhere in this book of this term, while it reflects current practice, is ahistorical. This notion originally was known simply as duality, while global duality was first introduced to describe the technical assumption that one can neglect distant cuts in computing the semileptonic $B$ decay rate, which is an important and distinct issue. Both terminologies remain in use in the literature.) In essence, the argument is as follows. Let $q$ be the momentum carried away by the leptons. The semileptonic width is an integral of a differential width, written schematically as $d\Gamma/dq$, which must be calculated under the hypothesis of local duality. For certain ranges of $q$ ($q^2$ near its kinematic maximum), local duality clearly fails. However, $d\Gamma/dq$ has a known analytic structure as a function of $q$, with cuts and poles, corresponding to thresholds and resonances, which are confined to the real axis. If the integration contour in $q$ is deformed away from the resonances, into the complex plane, then it may be possible to compute the integral without knowing the integrand everywhere along the original (real) contour of integration. From one point of view, complex $q$ forces the final state away from the mass shell, where long distance effects can become important. From another, the integral over $q$ imposes an average over the invariant mass $s_H$ of the hadrons in the final state, which smears out the effect of resonances when they do contribute. This property, that quantities averaged over $s_H$ may
be computable even when differential ones are not, is global duality. The most important feature is the smearing of the perturbative calculation over the resonance region. Note that global duality does not apply to purely hadronic $B$ decays, for which $s_H = m_B^2$ is fixed.

Once the issue of duality has been addressed, the actual expansions obtained for inclusive decays are very similar to those for exclusive decays. For example, the total charmless semileptonic $B$ decay width takes the form

$$
\Gamma(B \to X_u \ell \bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} m_b^5 \left[ 1 - 2.41 \frac{\alpha_s}{\pi} + \frac{\lambda_1 - 9\lambda_2}{2m_b^2} + \ldots \right].
$$

(26)

The leading corrections to this expression are of order $\alpha_s$, $\alpha_s/m_b$ and $1/m_b^3$. Note that the $1/m_b^2$ corrections are far more tractable than in the exclusive decay: first, because $1/m_c^2$ does not appear, and second, because they may all be written in terms of the two parameters $\lambda_1$ and $\lambda_2$, one of which is already known. Finally, note the strong dependence on the mass $m_b$, which is equivalent via the mass expansion (22) to a dependence on $\Lambda$. This is a significant recent source of uncertainty in the expression for the rate. There have been extensive recent theoretical efforts to reduce this uncertainty. The technically more complicated case of $B \to X_c \ell \bar{\nu}$ is discussed in the literature.

**E. Limitations of the HQE**

While the heavy quark expansion and the HQET are powerful tools in the analysis of many aspects of $B$ spectroscopy and decay, there are important issues into which they provide little direct insight. What the HQE provides is a framework within which the dependence of quantities on the large mass $m_b$ may be extracted systematically. However, once this has been accomplished, the task usually remains of analyzing those parts of the process which are characterized by long distances, small momenta, and nonperturbative dynamics. For a few quantities, such as the exclusive and inclusive decay rates discussed in this section, the calculation can be organized so that such effects appear only at subleading order, with the leading order terms controlled by heavy quark symmetry. But this is not the typical situation in $B$ phenomenology; one is usually required to analyze quantities and processes for which the nonperturbative nature of QCD is a dominant effect.

By necessity, such analyses involve a wide variety of methods, techniques, approximations and ansatzes. Some of the most important approaches are briefly discussed in the rest of this introduction. But even where the heavy quark limit is not itself predictive, it still has an important role to play. Any model or effective theory which purports to describe $B$ mesons must obey the heavy quark limit. By the same token, it is often possible to enhance or extend a model by building heavy quark symmetry in explicitly. The information provided by the heavy quark limit will prove to be very useful in this broader context.
IV. LIGHT FLAVOR SYMMETRY

A. Chiral Lagrangians

Complementary to the heavy quark limit, new symmetries of QCD also arise in the limit of vanishing light quark masses. As $m_u, m_d, m_s \to 0$, the quarks of left and right helicity decouple from each other. In this limit, the invariance of the Lagrangian separately under rotations among $(u_L, d_L, s_L)$ and $(u_R, d_R, s_R)$ gives rise to an $SU(3)_L \times SU(3)_R$ chiral flavor symmetry. In the QCD vacuum, this symmetry is dynamically broken to the diagonal subgroup $SU(3)_V$ by the quark condensate $\langle \bar{q} q \rangle \neq 0$. As a consequence, there are eight Goldstone bosons in the light spectrum, which we identify with the physical $\pi$, $K$, and $\eta$. Since the actual $u$, $d$, and $s$ quark masses are small but nonzero, the $\pi$, $K$, and $\eta$ are light but not exactly massless.

The spontaneous breaking of chiral symmetry is characterized by a scale $\Lambda_\chi \sim 1$ GeV, which is related to the value of the quark condensate. For light masses $m_\pi, m_K, m_\eta \ll \Lambda_\chi$ and small momenta $p \ll \Lambda_\chi$, QCD exhibits a separation of energy scales which can be used as the basis for an effective field theory. This chiral Lagrangian describes low energy interactions in a systematic expansion in powers of $p/\Lambda_\chi$ and $m_q/\Lambda_\chi$. Since the fundamental degrees of freedom, the eight “pions” $\pi, K$ and $\eta$, are the Goldstone bosons associated with the spontaneous symmetry breaking $SU(3)_L \times SU(3)_R \to SU(3)_V$, they transform in a complicated nonlinear way under the full symmetry group. It is convenient to assemble them into a matrix $\Pi_{ab}$, which is in turn exponentiated,

$$\Sigma_{ab} = [\exp (2i\Pi/f_\pi)]_{ab}, \quad (27)$$

where $f_\pi \approx 130$ MeV is the pion decay constant and $a, b$ are flavor indices which take values $u, d$ or $s$. The unusual looking field $\Sigma$ has the property that it transforms simply under $SU(3)_L \times SU(3)_R$.

The chiral Lagrangian is the most general function of $\Sigma$ consistent with the symmetries, constructed order by order in powers of $1/\Lambda_\chi$. At lowest order, the Lagrangian is completely fixed,

$$\mathcal{L} = \frac{f_\pi^2}{4} \partial^\mu \Sigma_{ab} \partial_\mu \Sigma_{ba} + \ldots, \quad (28)$$

where the flavor indices are summed over and the ellipsis indicates operators suppressed by $\Lambda_\chi^a$. The exponential form of the $\Sigma$ field allows this simple Lagrangian to describe interactions between arbitrarily large numbers of pions. Indeed, one of the useful features of Chiral Perturbation Theory (ChPT) is its ability to relate scattering amplitudes involving different numbers of external particles.

All hadrons other than the pions, such as vector mesons or baryons, have masses of order $\Lambda_\chi$. Hence for external momenta $p \ll \Lambda_\chi$ they can only appear as virtual states. Their effect on the effective theory is reproduced by higher dimension operators involving $\Sigma$, such as

$$\frac{f_\pi^2}{\Lambda_\chi^2} \text{Tr} \left[ \partial^\mu \Sigma^\dagger \partial^\nu \Sigma^\dagger \partial_\mu \partial_\nu \Sigma \right], \quad (29)$$
where the trace is over flavor indices. Because one cannot solve QCD, the couplings of these operators are unknown constants which must be determined phenomenologically. In practice, the Lagrangian \((25)\) has been generalized to include operators containing up to four derivatives or one power of the light quark masses, as well as effects from electromagnetic and flavor changing currents. By now, most of the couplings have been extracted from experiment. Typical predictions, such as \(\pi-K\) radiative reactions or \(\pi-\pi\) scattering, are accurate at the 10–30% level, although in some cases, such as the extraction of \(|V_{us}|\) from \(K \to \pi\ell\bar{\nu}\), the uncertainties are much smaller. It is important to keep in mind that these predictions are valid only so long as external momenta are small compared to \(\Lambda_{\chi} \sim 1\) GeV.

### B. Heavy Hadron Chiral Perturbation Theory

Although heavy hadrons have masses much larger than \(\Lambda_{\chi}\), it is still possible to incorporate them into ChPT. This is because it is only the light degrees of freedom in the hadron, whose mass does not scale with the heavy quark, which interact with external pions. This extension of the effective theory, known as Heavy Hadron Chiral Perturbation Theory (HHChPT), incorporates the heavy quark spin-flavor symmetry in an expansion in derivatives, light quark masses, and inverse heavy quark masses. It describes soft pions interacting with a static heavy hadron.

A simple example of where such a formalism is useful is the semileptonic decay \(B \to \pi\ell\bar{\nu}\). Over most of the Dalitz plot, the pion is much too energetic for chiral symmetry to apply. However, in the region where the pion is soft, the form factor \(f_+(q^2)\) which determines the differential rate can be determined reliably. For a sufficiently soft pion, the dominant contribution to \(B \to \pi\ell\bar{\nu}\) comes from the process where \(B \to B^*\pi\), with the virtual \(B^*\) then decaying leptonically. The strength of the \(B \to B^*\pi\) transition is proportional to a universal coupling constant \(g\), which may be determined from the rate for the decay \(D^* \to D\pi\). The amplitude for \(B \to \pi\ell\bar{\nu}\) at lowest order in HHChPT is then

\[
f_+(q^2) = \frac{gM_B^2f_B/f_\pi}{M_{B^*}^2 - q^2},
\]

which is simply a statement of nearest pole dominance, which holds rigorously in the combined heavy quark and chiral limit. Physically, pole dominance holds because in this limit the mass splitting between the \(B\) and \(B^*\) vanishes, whereas the energy gap to the nearest excited resonance remains finite. Thus, for arbitrarily soft pions, the \(B^*\) is the only resonance which can affect the form factor. Note that the heavy and light flavor symmetries relate \(B_s, D\) and \(D_s\) states to the \(B\), so there are analogous form factors in \(B_s \to K\ell\bar{\nu}, D_s \to K\ell\bar{\nu}\), and \(D \to \pi\ell\bar{\nu}\).

As is typical in chiral calculations, the amplitude relations hold only where the pions are soft. There will be corrections to these relations at higher order, when loop graphs and explicit symmetry breaking terms are included. Most calculations within HHChPT are done at leading order, or include only some of the numerically important corrections. Since the number of unknown coefficients tends to proliferate at higher order, such results are usually presented as estimates of the size of symmetry breaking effects. Chiral Lagrangians are particularly useful for exploring the light flavor dependence of quantities arising from pion loops and other infrared physics.
C. Factorization, Color Flow, and Vacuum Saturation

The problem with chiral calculations is that they only apply when the external pions are soft, and for most processes of phenomenological interest, nothing constrains this to be the case. For example, it is not very useful to apply such techniques to exclusive nonleptonic decays such as $B \to D\pi$, since the $\pi$ has momentum $p = 2.3 \text{ GeV} > \Lambda_{\chi}$. If one attempts to use the chiral Lagrangian here, one finds that the effects of higher dimension operators, which scale as $(p/\Lambda_{\chi})^2$, are unsuppressed, and the theory loses its predictive power. The hadrons in the final state continue to interact long after the weak decay, and there is no clean separation of scales. The situation is even more complicated for multiple pion production ($B \to D\pi\pi, \cdots$), which is governed over most of the phase space not by low energy theorems but by the nonperturbative dynamics of QCD fragmentation. In the absence of a solution to QCD, exclusive nonleptonic decays remain one of the most intractable problems in $B$ physics.

All that one has is a variety of models, based on ideas such as light cone wavefunctions or fragmenting strings, which describe the data with varying degrees of success. In the absence of any theory based on first principles, phenomenological approaches are often used instead. The most popular of these is the hypothesis of factorization, which applies to certain two body nonleptonic decays. A simple example is $B \to D\pi$, which is mediated by the quark transition $b \to c\bar{u}d$. Immediately after the weak decay, the quarks typically find themselves with a large momentum and in the middle of a medium of gluons and light quark-antiquark pairs, with which they subsequently interact strongly. However, if the $\bar{u}d$ pair has a small invariant mass, $m(\bar{u}d) \approx m_\pi$, then these two quarks will remain close together as they move through the colored medium. If, in addition, they are initially in a color singlet state, then they will interact with the medium not individually but as a single color dipole. Since the distance between the $\bar{u}$ and the $d$ grows slowly, it is possible that the pair will have left the colored environment completely before its dipole moment is large enough for its interactions to be significant. In this case, the pair will hadronize as a single $\pi$. Such a phenomenon is known as “color transparency”.

If, by contrast, the $\bar{u}d$ pair has a large invariant mass, then the quarks will interact strongly with the medium. In this case, their reassembly into a single $\pi$ is extremely unlikely. As a result, it is reasonable to hypothesize that the decay $B \to D\pi$ is dominated by the former scenario, and that the matrix element actually factorizes,

$$\langle D\pi| \bar{c}\gamma^\mu(1 - \gamma^5)b \bar{d}\gamma_\mu(1 - \gamma^5)u | 0 \rangle = \langle D| \bar{c}\gamma^\mu(1 - \gamma^5)b | \bar{b} \rangle \times \langle \pi| \bar{d}\gamma_\mu(1 - \gamma^5)u | 0 \rangle. \quad (31)$$

The result is something much simpler: $\langle \pi| \bar{d}\gamma_\mu(1 - \gamma^5)u | 0 \rangle$ is related to $f_\pi$, and $\langle D| \bar{c}\gamma^\mu(1 - \gamma^5)b | \bar{b} \rangle$ may be extracted from semileptonic $B$ decays. With this ansatz, it is possible to obtain relations among various two body decays which can then be tested experimentally. A proper analysis is fairly complicated, because it is necessary to take into account short distance perturbative corrections and other formally subleading effects. In particular, when the leading QCD radiative corrections are included, the matrix element (2.31) develops a dependence on the renormalization scale $\mu$ which cannot be compensated within the factorization ansatz. Thus even the question of whether a matrix element factorizes has no scale invariant meaning.

There is a heuristic distinction which is often made in the discussion of nonleptonic $B$ decays, between contributions to decays which are “color allowed” and those which are
“color suppressed”. In the spirit of factorization, it is often convenient to use Fierz identities to rewrite the effective Hamiltonian as a sum of products of quark bilinears which could interpolate certain exclusive final states. For example, if one were interested in the semi-inclusive process $B \to X_s \psi$, it would be useful to re-express the combination

$$C_1 \bar{s}_i \gamma^\mu (1 - \gamma^5) c^j \bar{c}_j \gamma_\mu (1 - \gamma^5) b^j + C_2 \bar{s}_i \gamma^\mu (1 - \gamma^5) c^j \bar{c}_j \gamma_\mu (1 - \gamma^5) b^j,$$

where $i$ and $j$ are color indices, as

$$(C_1 + \frac{1}{3}C_2) \bar{c} \gamma^\mu (1 - \gamma^5) c \bar{s} \gamma_\mu (1 - \gamma^5) b + 2C_2 \bar{c} T^a \gamma^\mu (1 - \gamma^5) c \bar{s} T^a \gamma_\mu (1 - \gamma^5) b.$$

Then the first term can be factorized in the sense of Eq. (31), while the second cannot. If the coefficient $C_1 + \frac{1}{3}C_2$ of the factorizable term is large, that is, if $C_1 + \frac{1}{3}C_2 \gg 2C_2$, then the amplitude is said to be “color allowed”; if the reverse is true, then it is said to be “color suppressed”. It is often supposed that amplitudes which have the wrong color structure to factorize are intrinsically small. Of course, soft gluons can always be exchanged to rearrange the color structure, so this distinction does not survive radiative corrections. However, the neglect of nonfactorizable amplitudes is a common phenomenological starting point for analyses of nonleptonic $B$ decays, where it is often useful to have some guess as to which four-quark operators are the most important for mediating a given transition.

Another common ansatz, which is similar in spirit to factorization, is vacuum saturation. The computation of $B^0 - \bar{B}^0$ mixing requires the hadronic matrix element $\langle \bar{b}^0 | \bar{b} \gamma^\mu \gamma^5 d \bar{b} \gamma^a \gamma^5 d | B^0 \rangle$, where the four quark operator has been induced by an interaction (such as a box diagram) at very short distances. In vacuum saturation, one inserts a complete set of states between the two currents, and then assumes that the sum is dominated by the vacuum. This ansatz is neither stable under radiative corrections, nor really well defined, since $\bar{b} \gamma^\mu \gamma^5 d \bar{b} \gamma^a \gamma^5 d$ is an indivisible local operator. The result is of the form

$$\langle \bar{b}^0 | \bar{b} \gamma^\mu \gamma^5 d \bar{b} \gamma^a \gamma^5 d | B^0 \rangle = Af_B m_B B_B,$$

where $A$ is a known constant and $B_B$ absorbs the error induced by keeping only the vacuum intermediate state. Deviations of $B_B$ from unity parameterize corrections to the ansatz. Vacuum saturation becomes exact in the formal limit $N_c \to \infty$, where $N_c$ is the number of colors, since then the mesons are noninteracting. This limit is often cited as a justification of the ansatz. As it turns out, calculations in lattice QCD do seem to prefer a value for $B_B$ which is close to unity. One may use HHChPT to estimate the uncertainty in the light flavor dependence of the ratio $B_{B_s}/B_{B_d}$.

V. LATTICE GAUGE THEORY

An important alternative to the analytic analyses presented so far is the attempt to solve QCD directly via a numerical simulation. As for any quantum field theory, QCD may be defined by a partition function,

$$Z = \int [dA_\mu] [d\bar{\psi}_i] [d\psi_i] e^{iS(A_\mu, \bar{\psi}_i, \psi_i)},$$
where the functional integral is over all configurations with given gauge potential $A_\mu$ and quarks $\psi_i$. The action,

$$S(A_\mu, \bar{\psi}_i, \psi_i) = \int d^4x \left[ \frac{-1}{4} G^{\mu\nu} G_{\mu\nu} + \bar{\psi}_i (i\partial - gA - m)\psi_i + \ldots \right],$$

is supplemented by sources for the quarks and gluons, and by gauge fixing terms. The functional $Z$ and its derivatives are enough to determine all of the correlation functions of the theory. The program of lattice gauge theory is to perform the integral in the action by discretizing space-time on a grid of spacing $a$, and then to compute $Z$ by summing over a finite but representative set of configurations of $A_\mu$ and $\psi_i$. In principle, given enough configurations and a fine enough grid, such an analysis provides an arbitrarily accurate solution to QCD.

However, there are a number of important practical difficulties with this program, which effectively restrict its accuracy and rigor, and the uses to which it may be put. The first is that any realistic analysis requires an enormous amount of computer power. While such resources continue to improve at a remarkable pace, it will be long in the future before it will be possible to analyze processes in which a wide range of momentum scales is important. Effectively, this limits the use of the lattice for the study of exclusive nonleptonic $B$ decays or $\pi-\pi$ scattering. For the time being, it is the static, rather than the dynamical, properties of QCD which are most amenable to a lattice treatment.

Another practical limitation of lattice QCD is that it is extremely expensive to include quark loops in the computation. It is possible to save a huge factor in computing time by working in the quenched or valence approximation, in which quark loops are neglected entirely. Quenching is really more an ansatz than an approximation, in the sense that it is difficult to estimate reliably the error which it induces. It can be argued that in certain contexts, such as heavy quark-antiquark bound states, the primary effect of quenching is to renormalize the effective coupling of the gluons, which can be compensated by adjusting the coupling $g$ at the lattice scale. But in most cases, quenching is just a necessary simplification of the calculation, with a largely unknown effect on the results. With the emergence of a new generation of computers capable of $\sim 10^{12}$ flops, some unquenched calculations will become feasible. Then it will begin to be possible to study the effects of quenching in more quantitative detail.

Other practical difficulties in lattice QCD are more tractable. Because of the nature of the propagator, massless quarks induce singularities in lattice calculations, so one must work with light quarks of mass $m \sim 100$ MeV or larger and then extrapolate to physical $m_u \sim 5$ MeV and $m_d \sim 10$ MeV. The nature of this extrapolation is strongly affected by quenching. It is also necessary to work at nonzero lattice spacing $a$, and finite overall lattice size $L$, extrapolating to $a \to 0$ and $L \to \infty$ at the end. These extrapolations are believed to be reasonably under control in most calculations. Finally, it turns out to be extremely difficult to incorporate chiral quarks in lattice computations, although this is not an important problem for a vector theory such as QCD.

Even given these limitations, the progress in lattice QCD in the past ten years has been phenomenal. This is due both to advances in computing technology, and perhaps more important, to the development of new theoretical methods particular to the lattice. New techniques which are relevant to the study of heavy quarks include the static approximation,
**nonrelativistic QCD, and improved actions.** The first of these is the analogue of HQET for heavy quarks on the lattice, which actually predates (and inspired) the development of HQET in the continuum. Static techniques are necessary because the Compton wavelength of the $b$ quark scales as $1/m_b$ and is much smaller than any lattice spacing $a$ in use, so fully dynamical $b$ quarks are extremely difficult to simulate. The static limit has proven very useful for the computation of heavy hadron spectra and decay constants. Nonrelativistic QCD, a somewhat different expansion in powers of $1/m$, is relevant to the study of heavy quark-antiquark bound states. Such analyses have become so accurate that lattice determinations of quarkonium splittings, when compared with data, provide a measurement of $\alpha_s$ which may be competitive with precision measurements at the $Z$ pole. Finally, it has been understood how to “improve” the action $S(A_{\mu}, \psi_i, \bar{\psi}_i)$ by including discretization effects order by order in $a$, thereby allowing the same accuracy to be obtained with larger lattice spacing. Since for a lattice of a given size in physical units, the number of points scales as $1/a^4$, the result can be a significant saving in computer resources.

In summary, the lattice will continue to be an important tool for $B$ physics, but it is not a universal approach for the numerous important quantities which cannot as yet be treated analytically. Lattice QCD has been very successful for certain quantities, such as the bag constant $B_B$ and the splittings in the $\Upsilon$ system. For others, such as bottom meson and baryon spectroscopy, decay constants, and semileptonic form factors, the situation continues to improve. At the same time, there are quantities, such as exclusive nonleptonic decays or fragmentation functions, where lattices of impractical size and granularity would be required to obtain useful predictions. For such processes, there is no rigorous theoretical calculation based on first principles.

**VI. QCD SUM RULES**

Another theoretical approach which is based, in principle, on QCD is that of *QCD sum rules*. The idea is to exploit parton-hadron duality as fully as possible, by studying inclusive quantities with kinematic or other restrictions which require them to be dominated by a single exclusive intermediate state. In this way one can learn something about nonperturbative physics, which controls the detailed properties of bound states, within a calculation based on a perturbative expansion.

The construction of QCD sum rules has a number of technical subtleties. This section only outlines the ingredients and the general structure. The method involves the study of correlation functions in QCD, as a function of external momenta. The correlation functions of a quantum field theory contain all the information about the theory, and hence in principle this method has access to every observable of QCD. In practice, it is only feasible to study two-point and some three-point functions, so QCD sum rules are useful primarily for computing spectra, decay constants and form factors.

The main features of the method will be illustrated here with a simple example from $B$ physics. One attractive feature of sum rules is that they can be formulated within HQET, thereby incorporating heavy quark symmetry automatically. In HQET, the current $\bar{h}_c \gamma^5 q$ can create a $B$ meson, or other excited states $B_n$ with the same quantum numbers. Since the $b$ quark is static, the appropriate measure of the mass of a state is the excitation energy
\( \nu_n = (m_{B_n}^2 - m_b^2)/2m_b \), which is independent of \( m_b \) as \( m_b \to \infty \). The object of study is the two point function

\[
\Pi(\omega) = i \int d^4x e^{i k \cdot x} \langle 0 | T\{\bar{q}\gamma^5 h_v(x), \bar{v}\gamma^5 q(0)\} | 0 \rangle,
\]

where \( \omega = v \cdot k \) is the energy injected into the correlator. For general \( \omega \), the correlator \( \Pi(\omega) \) receives contributions from all intermediate states \( B_n \). Hence \( \Pi(\omega) \) may be written in the form

\[
\Pi(\omega) = \sum_n \frac{F_n^2}{\nu_n - \omega - i\epsilon},
\]

where \( F_n \) is the coupling of the current to the excited state \( B_n \). The correlator (37) and sum over states (38) are often referred to, respectively, as the “theoretical” and “phenomenological” sides of the sum rule.

The goal is now two-fold: first, to compute the correlator (37) in QCD, and second, to isolate the contribution of the ground state \( B \) to the sum (38), so that its properties can be extracted. These two goals conflict, as they require different limits of \( \omega \). A perturbative calculation of the correlator is appropriate for \( \omega \) far from resonances, \( \omega \gg \Lambda_{\text{QCD}} \), or even better, in the unphysical region \( \omega \to -\infty \). On the other hand, the ground state will only dominate the sum for \( \omega \) small and near the \( B \) resonance. The compromise is to work in a regime of intermediate \( \omega \), where it is hoped that, with some technical improvements, both the correlator and the sum over states can be treated accurately. These improvements are the source of most of the complications in the method.

The first step is to rewrite \( \Pi(\omega) \) as a dispersion integral over its imaginary part, which receives contributions from real intermediate states,

\[
\Pi(\omega) = \int_0^\infty d\nu \frac{\rho(\nu)}{\nu - \omega - i\epsilon},
\]

where \( \rho(\nu) \propto \text{Im} \Pi(\nu) \). For simplicity, local subtractions, which may be required to make this expression well behaved, have been omitted here. Note the similarity between the theoretical expression (39) and the phenomenological sum over states (38). While it is certainly not true that \( \rho(\nu_n) = F_n^2 \) at each point, global duality allows the two expressions for \( \Pi(\omega) \) to coincide once both sides have been integrated. For \( \omega \) large enough, it suffices to compute the density \( \rho(\nu) \) as a power series in \( \alpha_s \). But for intermediate \( \omega \), it is necessary also to include corrections to \( \Pi(\omega) \) of order \( 1/\omega^n \). These corrections appear in the form of condensates, new nonperturbative quantities characteristic of QCD. It is usually enough to include the condensates of dimension \( \leq 5 \), whose values have been extracted at the \( \sim 30\% \) level from other processes:

\[
\langle \bar{q}q \rangle \approx -0.23 \text{ GeV}^3,
\]

\[
\langle \alpha_s G^{\mu\nu} G_{\mu\nu} \rangle \approx 0.45 \text{ GeV}^4,
\]

\[
\langle g \bar{q} \sigma^{\mu\nu} q G_{\mu\nu} \rangle \approx -0.40 \text{ GeV}^5.
\]

The condensates are universal quantities which, it is hoped, capture the leading nonperturbative effects of the QCD vacuum and allow the correlator to be computed accurately even for \( \omega \) not asymptotically large.
The next step is to focus on the ground state $B$. The excited states $B_n$ are all quite broad and unlikely to induce rapid variations in $\Pi(\omega)$, and it is assumed that above some scale $\omega_0$, the integral over the excited states can accurately be described by parton-hadron duality. Actually, it is hoped that the scale $\omega_0$ may be chosen as a threshold, in the sense that the entire contribution of the excited states (and none of the ground state) may be modeled by the perturbative dispersion integral for $\nu > \omega_0$. The contribution of the excited states is then subtracted from both expressions, leaving an upper cutoff $\omega_0$ on the dispersion integral (38), and only the ground state $n = 0$ in the sum over intermediate states (38).

The object of the analysis is now to equate the theoretical and phenomenological sides of the sum rule and attempt to fit the coupling $F$ and the energy $\nu$ of the ground state $B$. To do so, one must fix values for the threshold $\omega_0$ and the energy $\nu$, neither of which is given a priori. (In Borel sum rules, $\omega$ is exchanged for a “Borel parameter” $T$.) While there exists a prescription for choosing these parameters, it is not based directly on QCD, but rather derives from the requirement that the sum rule be self-consistent, that is, dominated neither by the condensates nor by the excited states. In fact, therein lies a fundamental source of uncertainty in the practical application of QCD sum rules. While it is certainly encouraging that $\omega_0$ and $\nu$ usually may be chosen to make the sum rule consistent and well behaved, there is no way to test whether the consistent choice is, in fact, the correct one. It is not clear, from first principles, how the stability of a sum rule corresponds to its accuracy.

The absence of a reliable estimate of the error from choosing $\omega_0$ and $\nu$, as well as of the error from truncating the sum over intermediate states, leaves QCD sum rule analyses with systematic uncertainties which are difficult to quantify. In this respect, they are a lot like lattice gauge theory calculations in the quenched approximation. Both methods are based, in principle, on QCD, which is their most attractive feature. However, in their practical implementation it is unavoidable that uncontrolled model dependence emerges. The result in each case is a bit of a hybrid, a valuable theoretical tool which one must rely on only with considerable care.

VII. QUARK MODELS AND RELATED METHODS

This section discusses quark models and their relatives. While a QCD analysis is always preferable to a model, there are unfortunately many processes and quantities of interest for which models are the only recourse. The variety of models, even commonly used ones, is indeed enormous, and there is no hope to survey the field here. This brief section explains what is meant by a model, and why a model is distinct from QCD. Many models are invented for a very limited purpose, to capture some particular feature of hadron phenomenology such as spectroscopy, fragmentation or weak decay. Here the focus is on a popular model with more general ambitions, the nonrelativistic quark model. This is probably the most intuitively accessible model, and it serves as an excellent illustrative example.

Consider a $\rho^+$ meson. In QCD, this state is a complicated collection of quarks, antiquarks and gluons, carrying overall flavor quantum numbers. Note that although a $\rho^+$ has the flavor of a $u\bar{d}$ pair, there are in fact many $u$, $\bar{u}$, $d$ and $\bar{d}$ quarks in a $\rho^+$, and it is not correct to assign the flavor of the overall $\rho^+$ to any particular ones. In the nonrelativistic quark model, however, a meson is treated as a bound state of a single quark and antiquark. Entirely
new degrees of freedom have been introduced, since these constituent $u$ and $d$ quarks are only indirectly related to the quarks of QCD. They have large masses of order 300 MeV (in contrast to the QCD current quark masses of $5 - 10$ MeV), they have small magnetic moments, they are nonrelativistic, they are not pair produced, and they interact with each other through an instantaneous potential. This is an ansatz, not an approximation to or a limit of QCD.

Given these new degrees of freedom, one can then guess a potential and solve the Schrödinger equation to find quark wavefunctions. Magnetic interactions and other effects are introduced as necessary, as perturbations to the nonrelativistic potential. The wavefunctions then may be used to fit or predict physical observables such as spectra, decay constants, or transition rates. Note that the very idea of a nonrelativistic wavefunction is foreign to QCD, so there is no meaningful sense in which the solutions which are obtained are “correct”. All that one can ask is that the model be “predictive”, in that it fit many independent pieces of data with few adjustable parameters.

In principle, models should be constrained to reproduce the known behavior of QCD in its various limits, but this is not always possible. The chiral limit is a particular problem: it is difficult to tune the nonrelativistic quark model to obtain a massless pion when the $u$ and $d$ current masses vanish. By contrast, heavy quark symmetry provides useful constraints, and it can be used to tune aspects of the quark model when it is applied to bottom and charmed hadrons.

The central problem with models is that it is difficult to accompany their predictions with meaningful error estimates. Since they do not arise as an expansion of QCD, there is no small parameter and no systematic corrections to a controlled limit. It is very difficult to guess, when a model is extended to a new region, at what level to trust its predictions. For example, the nonrelativistic quark model typically works very well for hadron spectroscopy, but this fact gives little insight into its reliability in predicting form factors. It is common practice, unfortunately, to cite uncertainties due to “model dependence” which are obtained by surveying the predictions of a variety of models. This exercise certainly provides more insight into the tastes of model builders than into the accuracy of their predictions.

Of the wide variety of models currently in use, just a few of the most popular ones for $B$ physics are listed here. The Isgur-Scora-Grinstein-Wise model is a version of the nonrelativistic quark model which is tuned to the study of semileptonic $B$ decays. The Bauer-Stech-Wirbel model is a quark model on the light front, used for weak $B$ and $D$ decays and the exploration of the factorization hypothesis. String fragmentation and flux tube models are used to study heavy quark fragmentation. The Skyrme model, derived from the chiral Lagrangian, is a model of light baryons which has been extended to describe heavy baryons as well. An alternative tool for studying baryons is the nonrelativistic diquark model. The ACCMM (Altarelli et al.) model is used to include initial bound state effects in inclusive $B$ decays. What these models, and others like them, have in common is that they are tuned to specific particles or specific processes, for which they typically work reasonably well. By contrast, their predictivity in new contexts is hard to assess reliably. Since it is unavoidable that models will continue to be an indispensable tool in $B$ phenomenology, it is important always to remain mindful of their limitations.
VIII. FURTHER READING

We have given only the briefest discussion to a few topics in the theory of hadronic $B$ physics. Because of the very general level of the discussion, references to the original literature have not been included. The reader who wishes to explore any of these topics further at an introductory level is invited to consult the many textbooks and reviews which now exist.

A few examples are:

- effective Hamiltonians and operator product expansions
  - *Dynamics of the Standard Model*, John Donoghue, Eugene Golowich and Barry R. Holstein, Cambridge University Press (1992)
  - G. Buchalla, A. J. Buras and M. E. Lautenbacher, *Rev. Mod. Phys.* 68, 1125 (1996)

- the heavy quark expansion
  - M. Neubert, *Phys. Rep.* 245, 259 (1994)
  - M. Shifman, in *QCD and Beyond*, Proceedings of TASI 95, ed. D. Soper, World Scientific (1996)
  - B. Grinstein, in *CP Violation and the Limits of the Standard Model*, Proceedings of TASI 94, ed. J. F. Donoghue, World Scientific (1995)

- chiral perturbation theory
  - J. Gasser and H. Leutwyler, *Phys. Rep.* 87, 77 (1982)

- lattice gauge theory
  - *Phenomenology and Lattice QCD*, Proceedings of the Uehling Summer School on Phenomenology and Lattice QCD, eds. G. Kilcup and S. Sharpe, World Scientific (1995)
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- QCD sum rules
  - Stephan Narison, *QCD Spectral Sum Rules*, World Scientific (1989).

ACKNOWLEDGMENTS

I would like to thank the Stanford Linear Accelerator Center and the BaBar Collaboration for inviting me to participate in the writing and editing of *The BaBar Physics Book*, and for their hospitality and support during this process. I am indebted to the many experts on $B$ physics and others who offered criticism of early drafts, thereby improving the
article and making it more reflective of the diverse points of view of those working in this
field. Thanks go to Jonathan Bagger, Ikaros Bigi, Robert Fleischer, Fred Gilman, Benjamín
Grinstein, Dafne Guetta, Laurent Lellouch, Zoltan Ligeti, Michael Luke, Thomas Mannel,
Matthias Neubert, Yossi Nir, Alexey Petrov, Helenka Przysiezniak, Chris Sachrajda, Nicolai
Uraltsev, Mark Wise and Daniel Wyler for their comments and suggestions. I am especially
grateful to Helen Quinn for her editing of later versions of the manuscript, as well as for
her general wisdom. This work was supported in part by the United States National Sci-
ence Foundation under Grant No. PHY-9404057 and National Young Investigator Award
No. PHY-9457916, by the United States Department of Energy under Outstanding Junior
Investigator Award No. DE-FG02-94ER40869, and by the Alfred P. Sloan Foundation. A. F.
is a Cottrell Scholar of the Research Corporation.