F-theory and 2d (0,2) Theories

Sakura Schäfer-Nameki

Dave Day, Burke Institute, Caltech, February 25, 2016

1601.02015 in collaboration with Timo Weigand
F-theory at (2,0)

Sakura Schäfer-Nameki

Dave Day, Burke Institute, Caltech, February 25, 2016

1601.02015 in collaboration with Timo Weigand
Setup

Setup: F-theory on elliptically fibered Calabi-Yau five-fold $Y_5$  
$\Rightarrow$ 2d $N = (0, 2)$ gauge theory, coupled to gravity.

Questions:

1. Why is this interesting?
2. What is the geometry-gauge theory dictionary?
3. Does this allow classification of 2d SCFTs?
Motivation

★ 2d \( N = (0, 2) \) theories provide a rich and complex class of susy gauge theories. Despite long history, much remains to be understood.

★ Well-studied as heterotic worldsheat theories

[Witten et al; Candelas, de la Ossa,...]

★ Recent resurgence in the context of M5-branes: In particular
M5-branes on \( X_4 \) (embedded as co-associatives in \( G_2 \))
[Assel, SSN, Wong; to appear]
Computation of elliptic genera, localization, etc have become available
[Benini, Bobev][Benini, Eager, Hori, Tachikawa], [Closset, Sharpe...]

Goal: Develop a setup where 2d (0,2) theories can be constructed systematically and potentially comprehensively.

F-theory on elliptic Calabi-Yau five-folds provide such a framework
2d Vacua: Brief History

2d Superstring vacua: very sparse history.

★ Type II on CY4 and $T^8/\Gamma$  
  [Gates, Gukov, Witten][Font]

★ 1998 and again since 2015: D1s probing CY4 singularities  
  [Hanany, Uranga] [Franco, Kim, Seong, Yokoyama]

★ 1/2016: F-theory on elliptic CY5  
  1601.02015 [SSN, Weigand]

★ 2/2016: heterotic/Type I/F-theory compactified to 2d  
  1602.04221 [Auzzi, Heckman, Hassler, Melnikov]
Plan

I. 2d (0,2) Supersymmetry
II. F-theory on CY5: Gauge Theory
III. F-theory on CY5: Geometry
IV. F-theory on CY5: Effective Theory and Anomalies
V. F-theory on CY5: Spacetime-Worldsheet Correspondence
I. 2d (0,2) Supersymmetry
2d (0, 2) Supersymmetry

# $\mathbb{R}^{1,1}$ with coordinates $y^\pm = y^0 \pm y^1$

# $SO(1, 1)_L \equiv U(1)_L$: ±1 charge $\leftrightarrow$ ±2d chirality

# Negative chirality susy parameters: $\epsilon_-$ and $\bar{\epsilon}_-$.

# Spectrum:
   Vector Multiplet $(v_0 - v_1, \eta, \bar{\eta}; \mathcal{D})$ and matter multiplets:

| Multiplet  | Content          | SUSY                                      |
|------------|------------------|-------------------------------------------|
| Chiral $\Phi$ | $(\varphi, \chi_+)$ | $\delta \varphi = -\sqrt{2} \epsilon_- \chi_+$ |
|            |                  | $\delta \chi_+ = i\sqrt{2}(D_0 + D_1)\varphi \bar{\epsilon}_-$ |
| Fermi $P$  | $\rho_-$         | $\delta \rho_- = \sqrt{2} \epsilon_- G - i \bar{\epsilon}_- E$ |

with $\bar{\mathcal{D}}_+ \Phi = 0$ but $\mathcal{D}_+ P = \sqrt{2} E$:

$$P = \rho_- - \sqrt{2} \theta^+ G - i \theta^+ \bar{\theta}^+ (D_0 + D_1) \rho_- - \sqrt{2} \bar{\theta}^+ E$$
$$\bar{P} = \bar{\rho}_- - \sqrt{2} \bar{\theta}^+ \bar{G} + i \theta^+ \bar{\bar{\theta}}^+ (D_0 + D_1) \bar{\rho}_- - \sqrt{2} \theta^+ \bar{E}.$$
Interactions

Interacting theory with $i = 1, \cdots, \#\Phi$ Chirals and $a = 1, \cdots, \#P$ Fermis:

# $J$-term (superpotentials) for $J^a = J^a(\Phi_i)$:

$$
L^J = -\frac{1}{\sqrt{2}} \int d^2 y d\theta^+ P_a J^a(\Phi_i)|_{\bar{\theta}^+ = 0} - \text{c.c.}
= -\int d^2 y \left( G_a J^a + \rho_{-a} \chi_{+,i} \frac{\partial J^a}{\partial \varphi_i} \right) - \text{c.c.}
$$

# $E$-term with $E_a = E_a(\Phi_i)$:

$$
L^E = -\int d^2 y \left( \bar{\rho}_{-a} \chi_{+,i} \frac{\partial E_a}{\partial \varphi_i} + \rho_{-a} \bar{\chi}_{+,i} \frac{\partial \bar{E}_a}{\partial \bar{\varphi}_i} \right) \subset -\frac{1}{2} \int d^2 y d^2 \theta P \bar{P}.
$$

Supersymmetry:

$$
\text{Tr} J^a E_a = 0.
$$
II. F-theory on CY5: Gauge Theory
F-theory on CY5

Elliptic CY5 with Kähler 4-fold base $B_4$ and (wtmlog) a section:

$$y^2 = x^3 + fx + g$$

# Singular loci: $\{ \Delta = 4f^3 + 27g^2 = 0 \} \supset M_G$.

7-branes wrap $M_G \leftrightarrow$ gauge algebra $g \leftrightarrow$ Singular fiber type

# In gauge-theory limit: effective theory on 7-branes, i.e.

8d SYM on $M_G \times \mathbb{R}^{1,1}$. 1601.02015 [SSN, Weigand]

1602.04221 [Apruzzi, Heckman, Hassler, Melnikov]

# $M_G =$ Kähler 3-fold: $SO(6)_L \rightarrow U(3)_L \equiv U(1)_L \times SU(3)_L$

Scalar supercharges on $M_G$: topological twist along $M_G$ with $U(1)_R$:

$$J_{\text{twist}} = \frac{1}{2} (J_L + 3J_R) .$$

Two susy parameters, with $q_{\text{twist}} = 0$ and are $SO(1,1)_L$ left-chiral spinors:

$$\bar{\epsilon}_- = 1_{-1}, \quad \epsilon_- = 1_{-1}$$
Twisted 8d SYM

Remaining symmetries after the partial twist:

\[ SO(1, 7)_L \times U(1)_R \rightarrow SU(3)_L \times SO(1, 1)_L \times U(1)_{\text{twist}} \]

8d SYM spectrum: gauge fields \(8^0\), fermions \(8^c_{-1}\) and \(8^s_{+1}\) and scalars \(1 \pm 2\).

After topological twisting: fields along \(M_G\) become forms:

\[
\begin{align*}
U(1)_{\text{twist}} \text{ charge } q &\geq 0 \ (q \leq 0) \\
&\uparrow \\
\text{field is section of } &\Omega^{(0,q)}(M_G) \ (\Omega^{(q,0)}(M_G))
\end{align*}
\]

\([U(1)_L \text{ twisting corresponds to twisting with } K_{M_G} = \Omega^{(0,3)}]\).
'Bulk Spectrum'

$\pm$ = chirality in 2d and $L_R$ = line bundle breaking $\text{Ad}(G) \supset \mathbf{R} \oplus \bar{\mathbf{R}}$

| Cohomology                                      | Bosons   | Fermions in $\mathbf{R}$, $\bar{\mathbf{R}}$ | Multiplet               |
|------------------------------------------------|----------|-----------------------------------------------|-------------------------|
| $H^0_0(M_G, L_R) \oplus H^0_\partial(M_G, L_R)^*$ | $\nu_\mu, \mu = 0, 1$ | $\text{dblue}\bar{\eta}_-, \bar{\eta}_-$ | Vector                  |
| $H^1_\partial(M_G, L_R) \oplus H^1_\partial(M_G, L_R)^*$ | $a_m, \bar{a}_m$ | $\bar{\psi}_+ \bar{\psi} + m$ | Chiral and Chiral       |
| $H^2_0(M_G, L_R) \oplus H^2_\partial(M_G, L_R)^*$ | $-$      | $\bar{\rho} - m \bar{n}, \rho - mn$ | Fermi and Fermi         |
| $H^3_0(M_G, L_R) \oplus H^3_\partial(M_G, L_R)^*$ | $\bar{\varphi}_{k\bar{m}\bar{n}}, \varphi_{kmn}$ | $\bar{\chi} + k\bar{m}\bar{n}, \chi + kmn$ | Chiral and Chiral       |
Supersymmetry and Hitchin Equations

Dim redux and twist of 10d SYM supersymmetry results in $(0, 2)$ in 2d:

* Gaugino variation:

\[
\mathcal{D} = \frac{i}{2} (J \wedge J \wedge F_{MG} + [\varphi, \bar{\varphi}])
\]

* Variation of Fermi

\[
\delta \rho_\pm = \sqrt{2} \epsilon_\pm G - i \bar{\epsilon}_\pm E \Rightarrow \begin{cases} G_{mn} = \overline{F}_{mn} \\ E_{mn} = (\bar{\partial}_a \varphi)_{mn} \end{cases}
\]

* BPS equations: Higgs bundle \((a, \varphi)\) over \(M_G\)

\[
D_+ \varphi = D_+ \bar{\varphi} = 0, \quad F^{(0, 2)} = F^{(2, 0)} = 0, \quad J \wedge J \wedge F + [\varphi, \bar{\varphi}] = 0
\]

→ cue Spectral covers, T-branes/Gluing data
‘Bulk’ Interactions

Interactions arise from overlaps of internal wave-functions:

Superpotential (\(J\)-term):

\[
S^{(J)}_{\text{bulk}} = g_{\alpha\beta\gamma} \int d^2 y \, \rho_{-}^{\alpha} a^{\beta} \psi_{+}^{\gamma} + \text{c.c.}
\]

with internal overlap

\[
g_{\alpha\beta\gamma} = \int_{M_G} \tilde{\rho}_{kmn\bar{n},\alpha} \wedge \hat{a}_{\bar{k},\beta} \wedge \hat{\psi}_{\bar{m},\gamma}, \quad \tilde{\rho}_{kmn\bar{n},\alpha} = (\Omega \cdot \hat{\rho}_{\alpha})_{kmn\bar{n}}
\]

\(E\)-term:

\[
S^{(E)}_{\text{bulk}} = f_{\alpha\mu\epsilon} \int d^2 y \, \bar{\rho}_{-}^{\alpha} \left( \varphi_{\mu} \psi_{+}^{\epsilon} + \chi_{+}^{\mu} a^{\epsilon} \right) + \text{c.c.}
\]

with internal overlap

\[
f_{\alpha\mu\epsilon} = \int_{M_G} \hat{\tilde{\rho}}_{k\bar{m},\alpha} \wedge \left( \hat{\varphi}_{kmn,\mu} \wedge \hat{\psi}_{\bar{n},\epsilon} \right)
\]
Matter from Defects

Matter from interacting 7-branes or codim 2 singularity enhancements in the elliptic CY5 along $S_R =$ matter surface.

Twisted SYM: 6d defect theory, with bulk compatible twist to 2d (0,2).

Spectrum:

Chirals: $S = (\bar{S}, \bar{\sigma}_+)$ and $T = (T, \tau_+)$

Fermi: $\bar{\mu}_-$

$$
\psi_+ \in H^1_\partial (M_G, L_R) \rightarrow \tau_+ \in H^0_\partial (S_R, L_R \otimes K_{S_R}^{1/2})
$$

$$
\bar{\rho}_- \in H^2_\partial (M_G, L_R) \rightarrow \bar{\mu}_- \in H^1_\partial (S_R, L_R \otimes K_{S_R}^{1/2})
$$

$$
\bar{\chi}_+ \in H^3_\partial (M_G, L_R) \rightarrow \bar{\sigma}_+ \in H^2_\partial (S_R, L_R \otimes K_{S_R}^{1/2}),
$$

$\rightarrow$ cue study of wave-function profiles.
Interactions

See also (for description in terms of $W^{\text{top}}$) [Apruzzi, Heckman, Hassler, Melnikov]

⋆ ‘Bulk’-Matter-surface interactions: (codim 2)

\[
J_{(\mu^\delta_\_)} = -c_{\delta\beta\epsilon} \mathcal{T}^\beta A^\epsilon \\
E^{(\mu^\alpha_\_)} = -f_{\alpha\mu\epsilon} \Phi^\mu A^\epsilon - b_{\alpha\beta\gamma} \mathcal{T}^\beta S^\gamma \\
E^{(\mu^\delta_\_)} = -e_{\delta\gamma\epsilon} S^\gamma A^\epsilon
\]

⋆ Cubic Matter-surface interactions for any $\mathcal{Z} = S, \mathcal{T}$ and gauge invariant triplet of representations $R_i$ (codim 3)

\[
J_{(\mu_{R_{b_1}^\_}^\delta)} = -h_{\delta\epsilon\gamma}(R_{b_1} R_{b_2} R_{b_3}) \left( Z_{b_2}^{R_{b_2},\epsilon} Z_{b_3}^{R_{b_3},\gamma} \right) \\
E_{(\mu_{R_{a_1}^\_}^\delta)} = -d_{\delta\epsilon\gamma}(R_{a_1} R_{a_2} R_{a_3}) \left( Z_{a_2}^{R_{a_2},\epsilon} Z_{a_3}^{R_{a_3},\gamma} \right)
\]

⋆ Quartic Matter-surface interactions: (codim 4) → see example
III. F-theory on CY5: Geometry
F-theory and Singular Fibers

Numerous F-theory@20 Talks: Above codim 1: Singular fibers are trees of \( \mathbb{P}^1 \)'s associated to simple roots. Codim 2: these can split into weights:

How exactly this happens: see Box Graph paper

[Hayashi, Lawrie, Dave Morrison, SSN]
F-theory on elliptic CY5

Much as in higher-dimensions: Singular fibers above discriminant loci determine gauge algebra and higher codim give rise to matter (codim 2) and interactions (codim 3+).

| Codim | $\mathbb{P}^1$s in Fiber | Gauge Theory |
|-------|--------------------------|--------------|
| 1: $M_G$ | Simple roots | Gauge algebra $\mathfrak{g}$ |
| 2: $S_R$ | Weights for Reps $\mathbb{R}$ | Matter in $\mathbb{R}$ |
| 3: $\Sigma$ | Splitting gauge invariantly | Cubic interactions |
| 4: $p$ | Further gauge invariant splitting | Quartic interactions |

Note:

- Rational sections: $U(1)$ gauge factors ⇒ engineer known and unknown (0,2) GLSM

- Non-abelian gauge groups occur quite naturally ⇒ non-abelian generalizations easily accessible (e.g. GLSM into Grassmanians)
An Example

An old friend: \( SU(5) \) with \( 10 \) and \( 5 \) matter. Almost as old, but some interesting new effects in fiber: non-Kodaira \( I_{n}^{\ast} \) fibers from monodromy

\[
\begin{array}{ccc}
\text{codim 1} & \text{codim 2} & \text{codim 3} \\
\end{array}
\]

\( F_{i} = \mathbb{P}^{1}s \) associated to simple roots \( C = \mathbb{P}^{1}s \) associated to weights of \( 5 \) \( (C_{i}) \) and \( 10 \) \( (C_{i,j}) \) with sign specifying whether \( \pm \) the curve is effective.
Quartic Couplings from Codim 4

Codim 3:
$C_4^{-}$ participates in $10 \times 10 \times 5$.

Codim 4:
$C_4^{-}$ splits further into $C_{34}^{-} + \tilde{C}_3^+$
$\Rightarrow$ quartic coupling over codim 4 point

$10 \times 10 \times 10 \times 5$

Likewise:
$5 \times 5 \times 5 \times 10$
IV. F-theory on CY5: Effective Theory and Anomalies
LEEA by M/F duality

Comparison of the 1d Super-QM, which describes the effective theory of M-theory on resolved (not necessarily elliptic) CY5 with the circle-reduction of the 2d F-theory compactification:

\[
\begin{align*}
\text{M-theory on } Y_5 & \xrightarrow{\text{Vol}(E_r) \to 0} \text{F-theory on } Y_5 \\
\text{1d Super-Mechanics} & \xrightarrow{R_A \sim \frac{1}{R_B} \to 0} \text{2d } (0,2) \text{ Gauge Theory}
\end{align*}
\]

[Haupt, Lukas, Stelle] \downarrow
$G_4$-flux

Fluxes are vital to generate chirality of the spectrum. Study via M/F [lessons from CY4: [Grimm, Hayashi]. Here: Dual M-theory was analyzed in [Haupt, Lukas, Stelle].

- Flux quantization + susy: $G_4 + \frac{1}{2}c_2(Y_5) \in H^4(Y_5, \mathbb{Z}) \cap H^{(2,2)}(Y_5)$

- Transversality constraints to ensure gauge fluxes:

$$\int_{Y_5} G_4 \wedge S_0 \wedge \omega_4 = 0 \quad \text{and} \quad \int_{Y_5} G_4 \wedge \omega_6 = 0, \quad \forall \omega_4 \in H^4(B_4), \ \omega_6 \in H^6(B_4)$$

$S_0 =$ zero-section

Induced gauge flux: $\int_{C_{\lambda}} G_4 = c_1(L_R)$

Chirality contribution: $\chi(S_R) = \frac{1}{2} \int_{S_R} c_1^2(L_R)$
Direct relation of chirality to the intersections of $G_4$ and Cartans $D_i$:

E.g. in the situation $F_i \rightarrow C^+ + C^-$

$$\chi(S_R) = \frac{1}{2} \int_{S_R} c_1^2(L_R) = -\frac{1}{2} G_4 \wedge G_4 \cdot Y_5 \cdot D_i$$

In general: using box graphs, can determine the chiralities in terms of these fiber intersections

$$D_i \cdot Y_5 \left( \frac{1}{24} [c_4(Y_5)] - \frac{1}{2} G_4 \wedge G_4 \right)$$

$$= -\frac{1}{2} \sum_{R} (n_R^+ - n_R^-) \left( \sum_{a=1}^{\text{dim}(R)} \varepsilon(\lambda^R_a) \lambda^R_{ai} \right)$$

$$= -\frac{1}{2} \sum_{R} (n_R^+ - n_R^-) \left( \sum_{a=1}^{\text{dim}(R)} D_i \cdot Y_5 \cdot C^\varepsilon(\lambda^R_a) \right) = (\star)$$

Will see: this follows from 1-loop CS terms.
Wrapped M2/D3-branes

Additional sectors of chiral matter: D3-branes wrapping curves $C$ in the base $B$ that intersect $M_G$: chiral 3-7 strings.

In M-theory: M2-brane states.

Chiralities: $\#$ intersection points $= [M_G] \cdot_{B_4} [C^B_{M2}]$

Key in anomaly cancellation.

First principle description in F-theory: from D3s wrapping curves (→ in progress)
Anomalies and Tadpoles

Global consistency of the compactification: tadpole cancellation and anomaly cancellation. Again, consider M-theory effective action:

Two topological terms: $C_{M2} = \text{wrapped M2 curve class}$

$$S_{M2} + S_{\text{curv}} = -2\pi \int_{\mathbb{R} \times Y_5} C_3 \wedge \delta([C_{M2}]) + 2\pi \int_{\mathbb{R} \times Y_5} C_3 \wedge \left( \frac{1}{24} c_4(Y_5) - \frac{1}{6} G_4 \wedge G_4 \right)$$

Via reduction of $C_3$ along $\omega^{(1,1)}_\alpha$ forms in $Y_5$: CS-terms:

$$S_{\text{top}} = 2\pi \sum_\alpha \int_{\mathbb{R}} A_\alpha \wedge (k^\alpha_{\text{M2}} + k^\alpha_{\text{curv}}) \begin{cases} k^\alpha_{\text{M2}} = -\int_{Y_5} \omega_\alpha \wedge \delta([C_{M2}]) \\ k^\alpha_{\text{curv}} = \int_{Y_5} \omega_\alpha \wedge \left( \frac{1}{24} [c_4(Y_5)] - \frac{1}{2} G_4 \wedge G_4 \right) \end{cases}$$

$A_\alpha$-tadpole: $\delta([C_{M2}]) = \frac{1}{24} c_4(Y_5) - \frac{1}{2} G_4 \wedge G_4$

F-theory: 1-loop CS term

$$k^i_{\text{curv}} \equiv k^i_{1-\text{loop}} = -\frac{1}{2} \sum_R \left( n^+_R - n^-_R \right) \sum_{\alpha = 1}^{\dim(R)} q_{ai} \text{sign}(m_0(\lambda^R_\alpha)) = (*)$$
Anomalies

Chiral fermions ⇒ require gauge anomalies to cancel.
For non-abelian gauge anomaly:

- Bulk matter: $A_{\text{bulk}}(R) = -C(R)\chi(M_G, L_R)$
- Surface matter $R$: $A_{\text{surface}}(R) = C(R)\chi(S_R, L_R)$
- 3-7 sector: $A_{3-7} = -C(R)\int_{B_4} [M_G] \wedge [C_{M2}^B]$

$$A_{\text{bulk}} + A_{\text{surface}} + A_{3-7} = 0$$

Note: tadpole cancellation implies anomaly cancellation via anomaly inflow (at least for perturbative vacua).

Abelian gauge anomalies: rich structure of GS/Stückelberg couplings.
V. F-theory on CY5:  
Spacetime-Worldsheet Correspondence
2d F-theory vacua = heterotic ws theories

Proposed correspondence:

A 2d $N = (0, 2)$ F-theory compactification can be viewed as (the UV completion of a) heterotic worldsheet theory

Example:

Heterotic on Quintic hypersurface in $\mathbb{P}^4 + \text{rk 3 vector bundle}$
$\leftrightarrow$ F on CY5 with rank 1 Mordell-Weil ($\to U(1)$) + $G_4$ flux.

In this case: phases of GLSM have interpretation in terms of topological transition in CY5: FI: $r \simeq G_4 \cdot S_1 \cdot J_B \cdot J_B$ ($S_1 = \text{Shioda of the section}$)

$r \gg 0$: Non-linear sigma-model phase (no $U(1)$ gauge symmetry)
$r \ll 0$: Landau-Ginzburg phase ($\mathbb{Z}_5$)
| NLSM – phase | GLSM | LG – phase |
|--------------|------|-----------|
| $G = \emptyset$ | $G = U(1)$ | $G = \mathbb{Z}_5$ |
| $\tilde{Y}_5$ | $Y_5$ | $\hat{Y}_5$ |
| MW($\tilde{Y}_5$) = 0 | MW($Y_5$) = $\mathbb{Z}$ | MW($\hat{Y}_5$) = 0 |
| TS($\tilde{Y}_5$) = 0 | TS($Y_5$) = 0 | TS($\hat{Y}_5$) = $\mathbb{Z}_5$ |
Remarks on 2d F-theory vacua

• The proposed correspondence can be useful in various ways:
  – Not necessarily critical string worldsheat theories, nevertheless F-theory provides framework to study them coupled also to gravity, see also [Apruzzi, Heckman, Hassler, Melnikov]
  – Generically, non-abelian gauge symmetries are present ⇒ interesting models to study as GLSMs.

• Test whether a 2d F-theory vacuum flows to interesting SCFTs by computing elliptic genus [Benini, Eager, Hori, Tachikawa]

• Classification of 2d SCFTs → cue NHC

⇒ Lots of things to explore.
Epilogue

F-theory certainly has something going for itself when it comes to even dimensions:

\[
\begin{align*}
12d & \leftarrow \text{F-theory@20?} \\
10d & \leftarrow \text{[Morrison 2015]} \\
8d & \leftarrow \text{[Vafa], [Morrison Vafa]} \\
6d & \leftarrow \text{[Vafa], [Morrison Vafa]} \\
4d & \leftarrow \text{F-theory@20} \\
2d & \leftarrow \text{F-theory@20} \\
0d & \leftarrow \text{?}
\end{align*}
\]

Happy \(n \times 20\)th Birthdays and many happy returns!