Modelling strain processes in earth dams with account for inhomogeneous features of a structure

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Abstract. A calculation procedure, results of the stress-strain and strength state studies of earth dams under the effect of their own weight and hydrostatic pressure of water are given in the paper in a generalized plane-deformed state, taking into account real geometry of a dam, design features and inhomogeneous properties of the structure materials. Three different earth dams with their design features are considered in the paper. Potential dangerous zones (in the dam body) with reduced strength characteristics of soil are investigated and revealed at various schemes of dam construction and the level of reservoir filling.

The reliability of the methods, algorithm and the accuracy of the results obtained are verified by solving test problems.

1. Introduction.

To date, a great number of earth dams have been built around the world, most of which are low dams with a height of no more than 30 meters. However, the lives of millions of people depend on the reliability of these structures, so, a high level of their safety is a necessity. The operation of earth dams during their long-term exploitation is complicated by such natural processes as filtration into soil medium of the dam itself, into the dam base, the creep phenomena of soil medium, soil buckling under freezing and soil settlement under wetting, soil weathering and suffusion phenomena. Different (in properties) soils, used in dam construction, complicate the work of the structures due to inhomogeneities in earth embankments, different nature of the water-material interaction. The features of the interaction of various parts of the dam become even more complicated under additional dynamic loads, including seismic ones. Certain success has been achieved in the study of these processes; a special attention in the design of dams is given to the development of new effective methods for predicting the stress-strain state, assessing the strength and ensuring the earthquake resistance of earth dams.

Ensuring the strength of the dam material, stability of slopes and the failure-free operation of anti-filtration elements, drains, piezometers, parapets and other elements of earth dams is one of the urgent tasks at present. To ensure these requirements in the design and calculation of earth dams, it is necessary to assess their strain and strength state under static and dynamic effects, taking into account the specific features of dams design and their interaction with water medium of the reservoir.

This, first of all, requires the development of adequate mathematical models and methods for calculating the stress-strain state (SSS) of dams, taking into account inhomogeneous properties of their material, design features and operation conditions in order to ensure the reliability and impermeability
to water of anti-filtration elements of dams; to predict virtual strains and the sediments during long-term operation of these structures.

2. State of the problem and literature review

The state of the problem and the review of publications show that a number of questions on assessing the strength of earth dams material and the stability of their slopes are currently being solved using various scientifically based theories of strength and models of soil strain under load. The review of publications shows that the issues of determining the stability and strength of the material of earth structures today are relevant and cover a wide range of unresolved problems.

Various theories and methods for solving the problems of the stress-strain state of earth dams are considered in [1-9,11,12,24,29,30,35], taking into account the hydrostatic and hydrodynamic pressure of water, filtration and pore pressure of water under dynamic loads, nonlinear elastic-plastic strain of soil, structural features of the body and core of the dam. In [11-12,29-32,35], on the basis of the variation principle, the methods and algorithms are proposed for calculating the earth dams under the effect of their own weight and hydrostatic pressure of water in conditions of plane and spatial stress-strain state.

Studies in [10,13-15,16] are devoted to the problems of reliability and safety of dams, the construction of dams in unfavorable engineering and geological conditions, and the long-term stability of slopes. The analysis of Russian regulatory documents on the reliability and safety of earth dams, the criteria for their reliability and safety, the safety monitoring systems in hydrotechnical structures in Russia, methods for risk assessment of earth dams accidents and the state of earth dams based on field observations data, as well as the safety assessment of earth dams under seismic effect are given in these papers.

In [7,8,16,24,28] it was determined that the state of earth structures during a certain period changes, i.e. dams settle, move horizontally, embankments are compacted, a change in the stress state occurs due to the strain in the structure. It is noted that due to imperfection of existing calculation methods, the incompleteness of survey materials and construction defects, the estimated predictions often do not coincide with reality.

The study in [17] presents the results of experiment son the properties of soil samples taken from the dam base; it was established that the parameters of soil strength depend on the magnitude of the current normal pressure. Recommendations are given to increase the stability of the left-bank dam, to load the downstream slope in a curvilinear section (PK9 and PK11) to a mark of 276 m in the center of the section above the drainage prism, lowering to the edges of the section.

Besides, there is a number of papers in which the study of the stress-strain state of dams is carried out taking into account dynamic loads in a linear and nonlinear elastic-plastic statement, the features of dynamic behavior of structures during earthquakes [11-12,18-20,25-35].

The fundamental studies by leading Russian and Uzbek scientists in the field of mathematical simulation of soil behavior, strength and stability calculations of earth dams became the basis for the foundation of the school of "ground behavior researchers" in modern hydro-engineering. The authors of these studies are M.M.Mirsaidov, Yu.K.Zaretsky and V.N.Lombardo, Z.G.Ter-Martirosyan, A.M.Belostotsky, V.M.Lyatcher, N.D.Krasnikov, G.S.Shulman, M.M. Grishin, N.P.Rozanov, L.N.Rasskazov and many other scientists. Investigations carried out by N.A.Aniskin, A.S.Bestuzheva, T.V.Matrosilina, M.T.Urazbaev, T.Sh.Shirinkulov, V.T.Rasskazovskiy and others are devoted to certain issues in filtration strength of dam elements, pore pressure in dams and dam bases, issues of soil liquefaction under dynamic loads, seismic resistance of earth dams in spatial conditions. Along with this, the latest published works by a number of authors [1-6,11,12,35,36] provide theoretical substantiation and methods for calculating the SSS of earth dams for various strain and strength models of soil, calculations for static and dynamic loads in plane and spatial statements. However, all known soil models and methods for solving static and dynamic problems of the stress-strain state of dams have their own assumptions and applicability areas, which is especially pronounced in a large number of indices for the physic-mechanical characteristics of soils in problem solution.
Therefore, at present, the development of new effective calculation methods to predict the stress-strain state, to assess the strength and to provide the earthquake resistance of earth dams is an important and urgent task. In addition, the development of intrinsic computing software is a powerful tool in scientific and research work in dam design.

3. Mathematical models and solution methods

The earth dam is considered as a substantially inhomogeneous structure, since its individual parts \( S_n (S=S_1+S_2+S_3) \) have different physical-mechanical and strain properties (Figure 1). When modeling the processes of earth dams strain in an extended section with constant base properties along the length, for example, on a rock base, it is possible to simulate the problem according to the scheme of plane-deformable state. In this case, the upstream and one of lateral faces - “the downstream slope of the dam” - \( \Sigma_1, \Sigma_2 \) are stress-free, and the lower part – “the base” - \( \Sigma_\text{u} \) is rigidly fixed.

![Figure 1. Plane-deformed model of a structure](image)

To describe the equilibrium state of the structure (Figure 1) under various static loads, the principle of possible displacements for a generalized plane-deformed state is used, according to which the sum of the work of all active forces at possible displacements is zero, i.e. [6]:

\[
- \int_{\Sigma_1} \sigma_{ij} \delta \varepsilon_{ij} ds - \int_{\Sigma_2} \sigma_{ij} \delta \varepsilon_{ij} ds - \int_{\Sigma_3} \sigma_{ij} \delta \varepsilon_{ij} ds + \int_{\Sigma_\text{u}} \rho \delta \vec{u} d\Sigma = 0
\]  

(1)

Kinematic boundary conditions are:

\[
\vec{x} \in \Sigma_\text{u} : \vec{u} = 0; \ \vec{\xi} \vec{u} = \vec{0}.
\]  

(2)

Here \( \vec{u} = \{u_1, u_2\} \) is the displacement vector; \( \varepsilon_{ij}, \sigma_{ij} \) are the components of the strain and stress tensor; \( \delta \vec{u}, \delta \varepsilon_{ij} \) are the isochronous variations of the displacement vector and the strain tensor; \( \rho \) is the hydrostatic pressure of water; \( \vec{f} \) is a vector of mass forces.

The hydrostatic pressure of water on the upstream face of the dam \( \Sigma_\rho \) is determined by the formula

\[
\rho = \rho_0 g (h - y)
\]  

(3)
Where $\rho_0$ is the water density, $g$-acceleration of gravity, $(h-y)$ - the depth of the point on the upstream face of the dam.

To describe physical properties of the material in each section of the dam body $\{S_1, S_2, S_3\}$ the Hooke’s law for the generalized plane-deformed state is used; it connects the components of the stress tensor $\sigma_{ij}$ with strains $\varepsilon_{ij}$ (at $\varepsilon_{33}=0$)

$$\sigma_{11} = \frac{E_n(1-v_n)}{(1+v_n)(1-2v_n)} \varepsilon_{11} + \frac{v_n E_n}{(1+v_n)(1-2v_n)} \varepsilon_{22};$$

$$\sigma_{22} = \frac{E_n(1-v_n)}{(1+v_n)(1-2v_n)} \varepsilon_{22} + \frac{v_n E_n}{(1+v_n)(1-2v_n)} \varepsilon_{11};$$

$$\sigma_{12} = \mu_n \varepsilon_{12}$$

$$\sigma_{33} = -\mu_n (\sigma_{11} + \sigma_{22})$$

Where $\mu_n$ is the shear modulus, $E_n$ is the elastic modulus, and $v_n$ is the Poisson’s ratio of the material (index $n$ shows the corresponding section of the body $\{S_1, S_2, S_3\}$, to which this mechanical characteristic belongs).

This problem is considered within the frame of a geometrically linear statement. So, the Cauchy relation is used to describe the relationship between the components of the strain tensor and the displacement vector

$$\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

$$\{i, j = 1, 2\}$$

$$\vec{x} = \{x_1, x_2\} = \{x, y\}$$

(5)

To simulate the strain process in earth dams under static loads, the general problem can be formulated as follows: it is necessary to find the components of displacements $\{u, v\} = \{u_1, u_2\}$, the components of the strain $\varepsilon_{ij}$ and stress $\sigma_{ij}$ tensors (arising in the dam body under mass forces and hydrostatic pressure of water) at each point of the structure, satisfying the variation equations (1), Cauchy relations (5) and Hooke’s law (4), with kinematic boundary conditions (2), at any possible displacements $\delta \vec{u}$.

The problem under consideration is solved by the finite element method (FEM) using a triangular finite element with a linear approximation of the displacements field inside the elements [6,21]. The division of a given domain into finite elements is carried out taking into account the design features and physico-mechanical properties of the dam material.

After finite-element discretization [6,21] the problem is reduced to a resolving system of algebraic equations of the $n$-th order

$$[K]\{u\} = \{F\}$$

(6)

where $[K]$ is the general matrix of structure rigidity; $\{F\}$ is the vector of external load (i.e., mass forces and hydrostatic pressure of water) applied to the nodal points of structures; $\{u\}$ is the sought for vector of nodal displacements of structures.

The obtained algebraic equations (6) are solved by the Gauss method.

4. Test problems. To substantiate the adequacy of the model, the reliability of the developed algorithms and the accuracy of the computer program, the following test problem was solved. In [22],
there is an exact analytical solution to the plane problem of the theory of elasticity for a long rectangular parallelepiped in a plane-deformed state (i.e., $u_z = 0$) under uniform pressure $P$ and supported on absolutely rigid ($u = 0$) smooth base. This problem is very convenient for checking the accuracy of the above methods, algorithm and calculation program. Therefore, this problem was solved using the methods for determining the displacement fields and the stress-strain state of the parallelepiped. Solving this problem, the following dimensionless characteristics of the structure, material and load were taken: $P=1$, $b=h=2$, $E=1$, $\mu = 0.3$ and a triangular finite element was used to discretize the rectangular parallelepiped.

A comparison of the obtained results shows high accuracy of the methods, algorithm and calculation program and their effectiveness for further use when predicting the stress-strain state of earth dams.

5. Assessment of the stress-strain state of earth dams under own weight and hydrostatic pressure of water.

Three different earth dams with design features, built in various areas of Central Asia are considered in the paper; the dams heights are: $H = 73$ m (with an earth screen); $H = 70$ m and $H = 138.5$ m (with an earth core). Structurally, the difference between the first and second dams lies in the presence of a reflecting screen of loamy soil located parallel to the upstream pressure face.

Slope ratios are: upper slope $m_1 = 2.5$ and lower slope $m_2 = 2$. The characteristics of the dam material have the following values: loamy soil: specific weight $\gamma = 1.7$ tf/m$^3$, internal friction angle for dry soil $\varphi = 24^\circ$ for water-saturated soil $\varphi = 19^\circ$; gravel-pebble soil: specific weight $\gamma = 2.0 - 2.1$ tf/m$^3$, angle of internal friction for dry soil $\varphi = 38^0 - 40^0$, for water-saturated soil $\varphi = 36^0 - 38^0$. The optimum soil moisture content is $W = 17\%$. The average coefficient of soil cohesion is $C = 1.9$ tf/m$^2$. At the project stage, the estimated seismicity was taken to be 8 points. The second earth dam has a height of $H = 70$ m, and the third one $H = 138.5$ m (with a core of loamy soil).

The main task here is to determine the components of the stress tensors $\sigma_i(\sigma_{11}, \sigma_{22}, \sigma_{12})$ and the stress intensity $\sigma_i$ at various points of the dam cross section. Along with the components of the stress tensor $\sigma_{ij}$, when determining the strength reliability of a structure, the main role is played by the intensity of normal stresses $\sigma_i$ [9], therefore, we will pay a special attention to the definition of $\sigma_i$.

The intensity of normal stresses for the generalized plane-deformed state is determined by the formula:

$$\sigma_i = \frac{1}{\sqrt{2}} \sqrt{\left(\sigma_{11} - \sigma_{22}\right)^2 + \left(\sigma_{22} - \sigma_{11}\right)^2 + \left(\sigma_{12} - \sigma_{12}\right)^2 + 6\sigma_{12}^2} \quad (7)$$

The study of the stress-strain state of earth dam with a screen of loamy soil (Figure 2) under its own weight (nor accounting for hydrostatic pressure) leads to a symmetric distribution of the intensity pattern of stresses $\sigma_i$ and stress components $\sigma_{11}, \sigma_{22}, \sigma_{12}$ in the dam body. When filled with water to half of the dam height, i.e. an account for hydrostatic pressure of water leads to a change in the SSS of the dam in the upstream slopes zones only. In the active part of hydrostatic pressure, depending on the zone, the stress components increase: $\sigma_{11}$ by 5-10%, $\sigma_{22}$ by 10-20%, $\sigma_{12}$ by 10-30% and stress intensity $\sigma_i$ by 5-15%.

When the reservoir is fully filled with water, hydrostatic pressure dramatically changes the pattern of stress components $\sigma_{11}$ and $\sigma_{22}$ (Figure 2). Due to hydrostatic pressure, the stress components in the upstream slope zones increase: $\sigma_{11}$ increases by 10-60%, $\sigma_{22}$ by 5-45%, $\sigma_{12}$ by 10-50% and the
stress intensity $\sigma_i$ by 10-25%. In areas free of surface forces (hydrostatic pressures), the pattern of the dam SSS almost does not change.

The study of the SSS of the high earth dam (H = 138.5 m) showed that the results qualitatively repeat the pattern of the stress-strain state of H = 70 m dam.

The difference lies mainly in the quantitative pattern of stress changes. When the reservoir is filled with water to half the height of the dam (as compared to the empty reservoir), the stresses change in the upper retaining prism only. In a fully filled reservoir, in the upper retaining prism, the stress intensity $\sigma_i$ and stress $\sigma_{22}$ increase by 10-50%, and the increase in stress $\sigma_1$ in the dam is 20-70%.

The research results confirm the different pattern of the stress state distribution (both qualitatively and quantitatively) for each dam. The effect of hydrostatic water pressure significantly changes the SSS of the upstream slope of the dam. Therefore, when designing hydrotechnical structures from earth materials in earthquake-prone areas, a serious attention should be paid to ensuring the strength of upstream slopes [3]. According to the results of observation carried out by Japanese scientists, the destruction of the upstream slopes occurs almost 3 times more often than of the downstream ones [3].

This is explained by the fact that the upstream slope of the dam experiences additional hydrostatic and hydrodynamic pressure of water and is affected by the inertial load not only of soil mass, but also of the mass of water that fills the soil pores, which can cause the development of shear strains. Here, the slightest additional force effect or a decrease in the soil strength can lead to a violation of the existing equilibrium.

The study of the stress state features of structures makes it possible to more fully reveal the essence of mechanical processes occurring both in the dam body and in sloping zones. By comparing the values of normal stresses $\sigma_{11}$ and $\sigma_{22}$, it is possible to establish the zones where horizontal stresses are greater than vertical ones; this can lead to undesirable phenomena in these zones - to a possible shift of one part of soil relative to another. To ensure the strength of soil structure, both normal stresses $\sigma_{11}$, $\sigma_{22}$, and principal stresses $\sigma_1$, $\sigma_2$ must be compressive ones, i.e. with a minus sign (-).

Along with the intensity of normal stresses $\sigma_i$, a reliable determination of the structure strength [9] requires an accurate determination of principal stresses $\sigma_1$, $\sigma_2$ and maximum tangential stresses $\tau_{\text{max}}$, since they play a decisive role in strength assessing by the Coulomb-Moore theory.

In addition, when assessing the structure strength, it is necessary to define the stress concentration zones; this is especially true for maximum shear stresses $\tau_{\text{max}}$ that are very dangerous, especially near the slopes. They may lead to local instability of certain sections of structures.

Principal normal stresses and maximum tangential stresses are determined using the well-known formulas

$$
\sigma_{12} = \frac{\sigma_{11} + \sigma_{22}}{2} \pm \frac{1}{2} \sqrt{\left(\sigma_{11} - \sigma_{22}\right)^2 + 4\sigma_{12}^2}
$$

$$
\tau_{\text{max}} = \frac{1}{2} \sqrt{\left(\sigma_{11} - \sigma_{22}\right)^2 + 4\sigma_{12}^2}
$$

Based on calculation results, principal stresses $\sigma_1$, $\sigma_2$, $\tau_{\text{max}}$ were determined in the body of each of the dams under consideration.

Figure 3 shows isolines of equal levels of principal stresses in a high dam; it shows the change in principal stresses obtained without/with taking into account the hydrostatic pressure when the reservoir is fully filled with water.
Figure 2. Distribution lines of equal levels of normal component $\sigma_{11}$, tangent component $\sigma_{12}$ and stress intensity $\sigma_i$ for earth dam (H=73 m) with a screen of loamy soil when the reservoir is fully filled:------ - an account for hydrostatic pressure of water; -----not accounting for hydrostatic pressure.

Figure 3. Isolines of principal stresses distribution $\sigma_1$, $\sigma_2$ and $\tau_{\text{max}}$ for a high (H=138.5 m) earth dam with a core of loamy soil at fully filled reservoir: ------ - with account for hydrostatic pressure of water; ----- - without account for hydrostatic pressure.

An analysis of the results obtained of principal stresses $\sigma_1, \sigma_2, \tau_{\text{max}}$ shows that an account for hydrostatic pressure of water significantly changes the distribution of the SSS in the upstream retaining prism of the dams under consideration. The greatest values of $\sigma_1$ and $\sigma_2$ occur in the lower part of the core and in the lower part of the zone of the upstream and downstream slopes of the retaining prism adjacent to the dam core. Here, the values of $\sigma_2$ in all sections are greater than $\sigma_1$. The greatest value of $\tau_{\text{max}}$ arises in the lower part of the dam, where the retaining prism joins the dam core.

The study of the stress state of the dam without consideration of hydrostatic pressure shows that small values of $\tau_{\text{max}}$ appear near the slopes, and an account for hydrostatic pressure at partially and fully filled reservoir leads to significant values of $\tau_{\text{max}}$ near the upstream slope. Large values of $\tau_{\text{max}}$ arise at the upper point of the dam, where the structure-water contact ends. The second such zone occurs in the lower part of the upstream slope. This phenomenon is undesirable for earth dams [23], since there is a possibility of lateral shear and the risers in such zones.

Conclusions

1. To predict the stress-strain state and assess the strength of earth dams under various static effects, a mathematical model is proposed that adequately describes the dam operation in a generalized plane-deformed state. A computer technique, algorithm and computing program are developed,
covered by a number of patent certificates of the Intellectual Property Agency of the Republic of Uzbekistan.

2. The reliability and accuracy of the results obtained, both in displacements and in stresses, are verified by solving test problems. Comparison with the well-known analytical solutions obtained by other authors shows high accuracy of results.

3. The stress-strain state of earth dams under the influence of their own weight and hydrostatic pressure was studied at various levels of reservoir filling with water to predict the structure strength, taking into account their construction features and in homogeneity of soil mechanical characteristics.

4. The strength of three different earth dams under their own weight and hydrostatic pressure was studied at various levels of reservoir filling with water and it was found that the strength of the slope zones of an empty reservoir is less than that of the filled one; this is apparently due to the retention of stresses arising in the dam body under its own weight and under acting hydrostatic pressure of water.

5. It was found that large values of $\tau_{\text{max}}$ occur at the upper point of the dam, where the structure-water contact ends, which is undesirable for earth dams, since lateral shear and the risers can occur in such zones under dynamic effect.

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