Rolling of Modulated Tachyon with Gauge Flux and Emergent Fundamental String

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abstract

We investigate real-time tachyon dynamics of unstable D-brane carrying fundamental string charge. We construct the boundary state relevant for rolling of modulated tachyon with gauge fields excited on the world-volume, and study spatial distribution of the fundamental string charge and current as the D-brane decays. We find that, in contrast to homogeneous tachyon rolling, spatial modulation of the tachyon field triggers density wave of strings when electric field is turned on, and of string anti-string pairs when magnetic field is turned on. We show that the energy density and the fundamental string charge density are locked together, and evolve into a localized delta-function array (instead of evolving into a string fluid) until a critical time set by rolling tachyon’s initial condition. When the gauge fields approach the critical limit, the fundamental strings produced become BPS-like. We also study the dynamics via effective field theory, and find agreement.

\textsuperscript{*}Work supported in part by the BK-21 Initiative in Physics (SNU Project-2), the KRF Overseas Research Grant, the KOSEF Interdisciplinary Research Grant 98-07-02-07-01-5, and the KOSEF Leading Scientist Grant.
1 Introduction

Recently, there has been considerable progress in understanding decay of unstable D-brane in string theory. The decay proceeds, as analyzed first by Sen [1], via rolling of the tachyon living on the D-brane world-volume from the top of the tachyon potential to the bottom (closed string vacuum). It was found that, in the weak coupling limit $g_{st} = 0$, as the tachyon rolls down the potential hill, the unstable D-brane is converted to a pressureless gas, referred as “tachyon matter”. Details of the decay process is of considerable physical interest, as it would shed light on semi-classical and non-perturbative aspects of string dynamics.

An interesting physics question is, as the D-brane decays, how various constituents bound inside an unstable D-brane are liberated. To answer the question, we will consider a bound-state of unstable D-brane with fundamental string, and study dynamical evolution of energy and charge distributions. In the previous works [10, 15], this problem was studied for homogeneous rolling of the tachyon, and it was found that the constituents form a uniformly distributed fundamental string fluid on the hyper-surface of the D-brane world-volume, behaving BPS-like in the limit we take the world-volume gauge fields to a critical value. In this work, we shall examine whether and, if it does, how spatial modulation of the rolling tachyon field would affect fate of constituent’s distribution in the ambient space-time. In approaching the problem, we shall continue utilizing the approach set forth in the previous work [15].

We begin with salient features concerning homogeneous rolling of the open string tachyon. The real-time dynamics is described [1], in the boundary state approach, by turning on an appropriate boundary interaction to the $c = 1$ conformal field theory of the $X^0$-field:

$$
\Delta S = \tilde{\lambda} \int dt \oint d\sigma \delta(t) \cosh X^0(t, \sigma).
$$

Boundary state corresponding to the above interaction was constructed in [1], and takes the form

$$
|D25\rangle_{T(x^0)} = |B\rangle_{X^0} \otimes_{i=1}^{25} |N\rangle_{X^i} \otimes |\text{ghost}\rangle,
$$

where

$$
\begin{align*}
|B\rangle_{X^0} &= f(\bar{x}^0)|0\rangle + g(\bar{x}^0)\alpha^0_{-1}\bar{\alpha}^0_{-1}|0\rangle \\
&\quad + \left[h_1(\bar{x}^0)(\alpha^0_{-2}\bar{\alpha}^0_{-2} + h_2(\bar{x}^0)(\alpha^0_{-1})^2(\bar{\alpha}^0_{-1})^2 + h_3(\bar{x}^0) \left((\alpha^0_{-1})^2\alpha^0_{-2} + \alpha^0_{-2}(\bar{\alpha}^0_{-1})^2\right)|0\rangleight] + \ldots \\
|N\rangle_{X^i} &= \exp\left(-\sum_{n=1}^{\infty} \frac{1}{n} \alpha^i_{-n}\bar{\alpha}^i_{-n}\right)|0\rangle \quad (i = 1, \ldots, 25)
\end{align*}
$$

†Among various works contributed to the progress, a list of those more relevant to the present work includes [1]–[18].
\[ |\text{ghost} \rangle = \exp \left( -\sum_{n=1}^{\infty} (\bar{b}_{-n}c_{-n} + b_{-n}\bar{c}_{-n}) \right) (c_0 + \bar{c}_0)c_1|0\rangle. \]

The coefficient functions \( f(x^0), g(x^0), h_1(x^0), h_2(x^0), h_3(x^0), \cdots \) describe time evolution of the rolling tachyon in the closed string channel, and are computable explicitly, as was done in [1, 10, 11]. Expanding the boundary-state Eq.(1.1) in oscillator levels, one can obtain explicit form of the linear coupling of the tachyon-rolling D25-brane to each closed string mode. It was found that, at late time, the rolling tachyon evolves into a pressureless “tachyon matter”. It was further found that the coupling to higher-level closed string states is hierarchically pronounced [11], a feature escalated even further in a situation the world-volume electric and magnetic fields are turned on [10, 15]. It also entailed an issue on the stability of the “tachyon matter” once a non-zero \( g_{\text{st}} \) is turned on.

The rolling of spatially modulated tachyon field can also be studied using the boundary state approach [7]. In [7], Sen considered the inhomogeneous tachyon corresponding to the following boundary interaction:

\[ \Delta S = \tilde{\lambda} \int d\sigma \cosh \frac{X^0}{\sqrt{2}} \cos \frac{X^1}{\sqrt{2}}. \]

It was found that half of the total energy start to accumulate at localized points and form an array of delta-function-like singularity at finite time. Furthermore, after the system hits the singularity, the total energy suddenly reduces to zero and the energy density vanishes eventually everywhere.

In this paper, we turn on gauge fields along with the inhomogeneous rolling tachyon, and study time evolution of the decaying D-brane. Surprisingly, we find that, in the presence of spatial modulation of the tachyon field, magnetic field (as well as electric field) induces fundamental string constituents on the D-brane world-volume. We also find that time evolution develops a singularity not only for the energy density but also for the fundamental string charge density. Consequently, our study indicates the following picture of the D-brane decay. Initially, driven by the tachyon field modulation, the fundamental string energy and charge densities evolve into free streaming of thin flux-tubes, whose thickness approaches zero up to a critical time set by the initial condition of the tachyon and gauge fields. The critical time is the moment a singularity develops to the boundary state — in particular, energy and charge densities are discontinuous across the critical time, leading allegedly to energy and charge non-conservation. Physically, as the energy and the string charge ought to be conserved, a possibility is that macroscopic fundamental strings of zero thickness are built up until the critical time and then are decoupled from the system right after the critical time (recall that we are working with \( g_{\text{st}} \) set to zero.).

We study couplings to the massless as well as some of the lower-level massive closed string modes, which can be extracted from the boundary state. Through these couplings, the bound-
ary state acts as a space-time varying source for the closed string field in the linearized equation of motion

\[(Q_B + \overline{Q}_B) |\Phi\rangle = |B\rangle,\]  

where \(Q_B\) is the BRST operator and \(|\Phi\rangle\) is the closed string field. We find that the coupling to the closed strings generally blows up sharply as the system evolves across the critical time, suggesting that the source \(|B\rangle\) of the closed string field equation in Eq.(1.3) may be approximated as a sort of impulse. Then, once \(g_{st}\) is turned on, the equations of motion for both open and closed strings will be altered by nonlinear interactions, and our analysis may cease to hold close to the critical time. The situation is analogous to the formation of a heavy star in general relativity: once the energy density becomes huge compared to the Planck scale, one cannot ignore the nonlinear terms in the Einstein field equation.

Several previous works [20, 21, 22] examined fate of the fundamental string constituents using the effective field theory approximation to an unstable D-brane, and claimed that a diffusive string fluid of arbitrary density profile is formed at the tachyon potential minimum, at least at the classical level. Our boundary state (as well as effective field theory) analysis shows the contrary that thin string-like flux tubes can be formed via real-time tachyon rolling ‡.

This paper is organized as follows. In section 2, we begin with recapitulating Sen’s construction of the boundary state representing rolling of modulated tachyon in bosonic string theory, and examine couplings to massless and massive closed string modes. We present detailed analysis of the time evolution of these couplings, and indicate that the couplings to both massless and massive closed string modes exhibit a singularity at the critical time. In section 3, adopting the recipe prescribed in [15], we construct the boundary state describing rolling of modulated tachyon with constant electric and magnetic fields. We extract time evolution of energy and charge density associated with fundamental string and analyze the behavior. In section 4, we study the tachyon rolling dynamics again, but now in the effective field theory approach. We find similar but somewhat differing result compared to those obtained from the boundary state approach. We contrast our results to earlier works [20, 22], and assert that stability criterion used in these works needs to be reconsidered in our case. Section 5 is devoted to discussion including some speculation concerning the physics around the singularity.

2 Rolling of Modulated Tachyon

2.1 Boundary State Construction

Following [7], consider the boundary-state description of the rolling of inhomogeneous tachyon on a D25-brane in the bosonic string theory. In the boundary state approach, it amounts to

‡This might be related to the result of [13].
turning on the following boundary action:

\[ \Delta S = \tilde{\lambda} \int d\sigma \cosh \frac{X^0}{\sqrt{2}} \cos \frac{X^1}{\sqrt{2}} \]  \hspace{1cm} (2.4) 

to the \( c = 2 \) conformal field theory (CFT) associated with \( X^0, X^1 \) of the bosonic D25-brane [23, 24]. The corresponding boundary state describes a D25-brane whose world-volume tachyon field is modulated sinusoidally in space \( x^1 \), and rolling in time \( x^0 \). Schematically, it takes the form:

\[ |D25\rangle_{T(x^0, x^1)} = |B\rangle_{X^0, X^1} \otimes i^{35 \sum_{i=2}^5} |N\rangle_{X_i} \otimes |\text{ghost}\rangle. \]  \hspace{1cm} (2.5) 

The boundary action Eq.(2.4) turns out to define an exactly solvable perturbation, as the \( c = 2 \) CFT boundary state \( |B\rangle_{X^0, X^1} \) is obtainable in a factorized form:

\[ |B\rangle_{X^0, X^1} = |B\rangle_+ \otimes |B\rangle_- , \]  \hspace{1cm} (2.6) 

where \( |B\rangle_\pm \) are the \( c = 1 \) boundary state Eq.(1.2) with the replacement of \( X^0 \rightarrow (X^0 \pm iX^1)/\sqrt{2} \) and \( \tilde{\lambda} \rightarrow \tilde{\lambda}/2 \) [7]. Explicitly, we find that

\[ |B\rangle_\pm = f^{\pm}(\tilde{x}^0, \tilde{x}^1)|0\rangle + \frac{1}{2} g^{\pm}(\tilde{x}^0, \tilde{x}^1)(\alpha^0_\pm + i\alpha^1_\pm)(\bar{\alpha}^0_\pm + i\bar{\alpha}^1_\pm)|0\rangle + \frac{1}{4} h^{\pm}_1(\tilde{x}^0, \tilde{x}^1)(\bar{\alpha}^0_\pm + i\alpha^1_\pm)|0\rangle + \frac{1}{8} h^{\pm}_2(\tilde{x}^0, \tilde{x}^1)(\alpha^0_\pm + i\alpha^1_\pm)^2|0\rangle + \frac{i}{4} h^{\pm}_3(\tilde{x}^0, \tilde{x}^1) \left[ (\alpha^0_\pm + i\alpha^1_\pm)(\bar{\alpha}^0_\pm + i\bar{\alpha}^1_\pm) + (\alpha^0_\pm + i\alpha^1_\pm)(\bar{\alpha}^0_\pm + i\bar{\alpha}^1_\pm)^2 \right]|0\rangle + \ldots \]

where the coefficient functions are defined by

\[ f^{\pm}(x^0, x^1) = \left( 1 + e^{x^0 \cos(\tilde{\lambda} \pi/2)} - 1 \right)^{-1} + \left( 1 + e^{-x^0 \cos(\tilde{\lambda} \pi/2)} - 1 \right)^{-1} - 1, \]  \hspace{1cm} (2.7) 

\[ g^{\pm}(x^0, x^1) = 2 \cos^2(\tilde{\lambda} \pi/2) - f^{\pm}(x^0, x^1), \]  \hspace{1cm} (2.8) 

\[ h^{\pm}_1(x^0, x^1) = 2 \cos(\tilde{\lambda} \pi/2) - 2 \sin(\tilde{\lambda} \pi/2) \cos^2(\tilde{\lambda} \pi/2) \cosh \left( \frac{x^0 \pm ix^1}{\sqrt{2}} \right) - f^{\pm}(x^0, x^1), \]  \hspace{1cm} (2.9) 

Note that the functions labeled with + superscript are complex conjugate of those labeled with − superscript. Note also that, except \( h^{\pm}_3(x^0, x^1) \), all coefficient functions are related to \( f^{\pm}(x^0, x^1) \).
Expanding the matter part of the boundary state Eq.(2.5) in powers of the oscillators and coupling functions associated with them, we obtain

\[
|D25⟩_T = F(x^0, x^1)|0⟩ + G_{ab}(x^0, x^1)\alpha_a^\alpha \bar{\alpha}_b^\beta|0⟩ + \frac{1}{2}H_{ab}(x^0, x^1)\alpha_a^\alpha \bar{\alpha}_b^\beta|0⟩ + \frac{1}{4}I_{abcd}(x^0, x^1)\alpha_a^\alpha \bar{\alpha}_b^\beta \bar{\alpha}_d^\delta \bar{\alpha}_c^\gamma|0⟩ + \frac{i}{2}J_{abc}(x^0, x^1)(\alpha_a^\alpha \bar{\alpha}_b^\beta + \bar{\alpha}_c^\gamma \bar{\alpha}_a^\alpha)|0⟩ + \cdots ,
\]  

(2.10)

where the coupling functions are given by

\[
F = ||f^+||^2, \quad G_{00} = -G_{11} = \text{Re}(f^+ g^-), \quad G_{01} = +G_{10} = \text{Im}(f^+ g^-), \\
G_{ij} = -||f^+||^2 \delta_{ij} \quad (\text{for } i, j = 2, 3, \cdots, 25),
\]  

(2.11)

for the tachyon and massless level modes, and

\[
H_{00} = -H_{11} = \text{Re}(f^+ h^-_1), \quad H_{10} = +H_{01} = \text{Im}(f^+ h^-_1), \\
H_{ij} = -||f^+||^2 \delta_{ij} \quad (\text{for } i, j = 2, 3, \cdots, 25), \\
I_{0000} = I_{1111} = +\text{Re}(f^+ h^-_2) + ||g^+||^2, \\
I_{0011} = I_{1100} = -\text{Re}(f^+ h^-_2) + ||g^+||^2, \\
I_{0101} = +I_{1010} = +I_{0101} = -\text{Re}(f^+ h^-_2), \\
I_{0100} = -I_{0011} = +I_{0001} = -I_{1101} = +\text{Im}(f^+ h^-_2), \\
I_{ijkl} = ||f^+||^2(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \quad (\text{for } i, j, k, l = 2, 3, \cdots, 25), \\
J_{001} = -J_{111} = \text{Re}(f^+ h^-_3), \quad J_{000} = -J_{110} = -\text{Im}(f^+ h^-_3), \\
J_{010} = +J_{100} = \text{Re}(f^+ h^-_3), \quad J_{011} = +J_{101} = +\text{Im}(f^+ h^-_3),
\]  

(2.12)

etc. for the first massive level modes. Various relations among the coupling functions are attributable to the specific factorized form of the boundary state, as given in Eq.(2.6).

### 2.2 Time Evolution of Coefficient Functions

As alluded above, the D25-brane boundary state \( |D25⟩_T \) acts as a source to the closed string field equation Eq.(1.3). The source being space-time varying, we thus need to examine its time evolution. In Eq.(2.10), the source is decomposed into closed string mass-Levels, and we shall be interested in the evolution of the coupling functions Eqs.(2.11, 2.12).

As \( f^\pm(x^0, x^1) \) are the building blocks for most of these coefficients, we will analyze their behavior first. From Eq.(2.7), we see that they are singular at \( x^0 \pm ix^1 = \mp\sqrt{2} \log \left( -\sin(\lambda \pi /2) \right) \).
To be specific, consider the case $0 < \sin(\tilde{\lambda} \pi / 2) < 1$ in the following. Flipping the sign of $\tilde{\lambda}$, interpreted as flipping the sign of the initial value of the tachyon field, amounts to relocating the $x^1$-locus of the singularities by $\sqrt{2}\pi$. When $0 < \sin(\tilde{\lambda} \pi / 2) < 1$, the singularities are located at $x^1 = \sqrt{2}(2n + 1)\pi$ ($n \in \mathbb{Z}$) at the critical time $x^0 = \pm x^0_c$, defined by

$$x^0_c = \sqrt{2} \log \left( \frac{1}{|\sin(\lambda \pi / 2)|} \right).$$

(2.13)

Let us examine closely the behavior of $f^\pm(x^0, x^1)$ around the critical time. Useful formulae for our analysis are:

$$\lim_{\beta \to 1 \mp 0} \left( \frac{1}{1 + \beta e^{ix/\sqrt{2}}} + \frac{1}{1 + \beta e^{-ix/\sqrt{2}}} \right) = 1 \mp 2\sqrt{2}\pi \sum_{n \in \mathbb{Z}} \delta \left( x - \sqrt{2}(2n + 1)\pi \right),$$

and

$$\int_0^{2\sqrt{2}\pi} dx^1 \frac{1}{1 + \alpha e^{ix/\sqrt{2}}} = \begin{cases} 0 & \text{for } |\alpha| > 1 \\ 2\sqrt{2}\pi & \text{for } |\alpha| < 1 \end{cases}.$$

(2.14)

(2.15)

The behavior of the real part of $f^\pm(x^0, x^1)$ close to the critical time $x^0 = x^0_c$ is obtainable by making use of Eq.(2.14):

$$\lim_{x^0 \to x^0_c \mp 0} \left( f^+(x^0, x^1) + f^-(x^0, x^1) \right) = \pm 2\sqrt{2}\pi \sum_{n \in \mathbb{Z}} \delta \left( x^1 - \sqrt{2}(2n + 1)\pi \right)$$

$$+ \left[ \left( 1 + \sin^2(\tilde{\lambda} \pi / 2) e^{ix^1/\sqrt{2}} \right)^{-1} + \left( 1 + \sin^2(\tilde{\lambda} \pi / 2) e^{-ix^1/\sqrt{2}} \right)^{-1} \right].$$

(2.16)

Note that the sign of the $\delta$-function part (the first line) flips as $x^0$ passes through the critical time $x^0_c$, whereas the regular part (the second line) remains the same. Note also that, for a fixed $x^0 \sim x^0_c$, the real part Eq.(2.16) is an even function of $x^1$ across each singularity located at $x^0 = \sqrt{2}(2n + 1)\pi$. The imaginary part of $f^\pm(x^0, x^1)$ is also singular at the critical time $x^0_c$:

$$\lim_{x^0 \to x^0_c} \left( f^+(x^0, x^1) - f^-(x^0, x^1) \right)$$

$$= -i \tan \left( \frac{x^1}{2\sqrt{2}} \right)$$

$$+ \left[ \left( 1 + \sin^2(\tilde{\lambda} \pi / 2) e^{ix^1/\sqrt{2}} \right)^{-1} - \left( 1 + \sin^2(\tilde{\lambda} \pi / 2) e^{-ix^1/\sqrt{2}} \right)^{-1} \right].$$

(2.17)

In this case, there is no discontinuity in $x^0$ for both the singular part (the first line) and the regular part (the second line). For a fixed $x^0 \sim x^0_c$, the imaginary part Eq.(2.17) is an odd function of $x^1$ across each singularity at $x^0 = \sqrt{2}(2n + 1)\pi$. 

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Figure 1: Behavior of the energy density ($\propto \text{Re} f^+(x^0, x^1)$) (vertical axis in arbitrary unit) for $\lambda = 0.3$ in $x^0 = [0, 2]$ and $x^1 = [0, 2\sqrt{2}\pi]$ in the left, and its section just before the critical time $x^0_c \sim 1.116 \cdots$ in the right.

It is also of interest to examine the spatial averages of the functions $f^\pm(x^0, x^1)$. We obtain them from Eq.(2.15) as

$$\langle f^\pm \rangle (x^0) := \frac{1}{2\sqrt{2}\pi} \int_0^{2\sqrt{2}\pi} dx^1 f^\pm(x^0, x^1) = \begin{cases} 0 & \text{for } |x^0| > x^0_c, \\ 1 & \text{for } |x^0| < x^0_c. \end{cases} \quad (2.18)$$

Generically, functions associated with massive modes, such as $h_1^\pm, h_2^\pm$, depend on $f^\pm$ (see Eq.(2.9)), and hence develop the same singularity as $f^\pm$ at the critical time $x^0_c$. There are, however, some exceptions and some other functions, such as $h_3^\pm$, do not depend on $f^\pm$, and evolve in time in a regular manner.

### 2.3 Coupling to Closed String Modes

Having obtained the coupling functions Eqs.(2.11, 2.12), let us now examine time evolution of each of them.

- **graviton coupling**

The energy-momentum tensor of the decaying D25-brane can be read from the boundary state Eq.(2.10) as [1]

$$T_{ab}(x^0, x^1) = \frac{T_{25}}{2} \left( G_{ab}(x^0, x^1) - \eta_{ab} F(x^0, x^1) \right).$$

Utilizing the expressions Eq.(2.11), we obtain [7]

$$T_{00} = -T_{11} = T_{25} \cos^2(\lambda \pi/2) \text{Re} f^+(x^0, x^1),$$

$$T_{01} = +T_{10} = T_{25} \cos^2(\lambda \pi/2) \text{Im} f^+(x^0, x^1),$$

$$T_{ii} = -T_{25} |f^+(x^0, x^1)|^2 \quad \text{(for } i = 2, 3, \cdots, 25). \quad (2.19)$$
Figure 2: Behavior of the momentum density ($\propto \text{Im} f^+(x^0, x^1)$) (vertical axis in arbitrary unit) for $\tilde{\lambda} = 0.3$ in $x^0 = [0, 2]$ and $x^1 = [0, 2\sqrt{2}\pi]$ in the left, and its section at the critical time $x^0_c \sim 1.116 \cdots$ in the right.

One readily sees that the energy-momentum tensor behaves as a sort of impulse source to the closed string field equation. From Eq.(2.18), the average energy is

$$\langle E \rangle (x^0) := \frac{1}{2\sqrt{2\pi}} \int_0^{2\sqrt{2}\pi} dx^1 T_{00}(x^0, x^1) = \begin{cases} 0 & \text{for } |x^0| > x^0_c \\ \mathcal{T}_{25} \cos^2(\tilde{\lambda}\pi/2) & \text{for } |x^0| < x^0_c. \end{cases}$$

It indicates that the energy suddenly disappears after the critical time! This comes about as follows. As can be seen from Eq.(2.16) and Eq.(2.18), half of the total energy is squeezed at $x^1 = \sqrt{2}(2n + 1)\pi$ ($n \in \mathbb{Z}$) and form the $\delta$-function singularities as the critical time $x^0_c$ is approached, while the other half is smoothly spread out along the $x^1$-direction (See Fig.1). After the critical time $x^0_c$, the sign of the $\delta$-function flips (see Eq.(2.16)) and cancels out contribution of the other half that are smoothly spread out. The momentum density is zero on the average, but develops a singular profile which may be approximated by $\sim \tan(x^1/2\sqrt{2})$ (See Fig.2).

To gain a better picture concerning the origin of the singularity, let us examine the energy-momentum conservation carefully. Introduce for convenience complex coordinates $z = (x^0 + ix^1)$, $\overline{z} = (x^0 - ix^1)$ so that $\partial_0 = (\partial_z + \partial_{\overline{z}})$, $\partial_1 = i(\partial_z - \partial_{\overline{z}})$. The coefficient functions $f^\pm$ are analytic functions of $z, \overline{z}$, respectively. From Eq.(2.19), we then find that

$$\partial^a T_{a0} = -\mathcal{T}_{25} \cos^2(\tilde{\lambda}\pi/2) \left[ \partial_z f^-(\overline{z}) + \partial_{\overline{z}} f^+(z) \right]$$

$$\partial^a T_{a1} = i\mathcal{T}_{25} \cos^2(\tilde{\lambda}\pi/2) \left[ \partial_z f^-(\overline{z}) - \partial_{\overline{z}} f^+(z) \right].$$

(2.20)

As alluded in the previous section, $f^+(z)$, $f^-(\overline{z})$ have simple poles at $z = -\sqrt{2}\log(-\sin(\tilde{\lambda}\pi/2))$ and $\overline{z} = +\sqrt{2}\log(-\sin(\tilde{\lambda}\pi/2))$, respectively. Thus, the first relation in Eq.(2.20) is proportional to $\sum_{n \in \mathbb{Z}} \delta(x^0 - x^0_c) \delta(x^1 - \sqrt{2}(2n + 1)\pi)$, invalidating the energy-momentum conservation. On the other hand, the second relation in Eq.(2.20) vanishes, yielding the correct force law.
Physical interpretation of such singular behavior is not clear to us, so we shall primarily concentrate our consideration on the early evolution before hitting the critical time, i.e. $|x^0| < x^0_c$. As the total energy ought to be conserved, a plausible possibility suggested in [7] is that the missing energy has escaped through the singularity to form an array of codimension-one D-branes. We leave viability of this scenario aside for future study. (See section 5 for further discussion.)

- **tachyon coupling**
  The tachyon coupling density is given by
  $$\rho_{\text{tachyon}}(x^0, x^1) = T_{25} ||f^+(x^0, x^1)||^2,$$
  and its behavior is plotted in Fig.3. The coupling again develops a singularity at the critical time $x^0 = x^0_c$. From Eqs.(2.11,2.12), we also find that couplings to all higher closed string modes with polarization along $i = 2, 3, \cdots, 25$ directions behave the same way as the tachyon coupling.

- **dilaton coupling**
  As the part of the boundary state that couples to the sigma model dilaton is
  $$\langle c_0 + \bar{c}_0 | c_{-1}c_1 + \bar{c}_{-1}\bar{c}_1 | 0 \rangle_{\text{gh}},$$
  the dilaton coupling of the decaying D25-brane is given by
  $$\rho_{\text{dilaton}}(x^0, x^1) = T_{25} ||f^+(x^0, x^1)||^2.$$
  Its behavior is the same as that of the tachyon coupling and the pressure $T_{ii}$ ($i = 2, 3, \cdots, 25$).

- **massive string mode coupling**
  Coupling to massive closed string modes, as compared to tachyon and massless string modes,
entails certain new features. As is seen from Eq.(2.12), these couplings are distinguished from those for tachyon and massless modes in that, in addition to $f(x^0, x^1)$, new coefficient functions $h^{\pm}_1(x^0, x^1), h^{\pm}_2(x^0, x^1), h^{\pm}_3(x^0, x^1), \cdots$ are involved. We thus expect that time evolution of the massive mode couplings would display qualitatively significant departure from those of the tachyon and the massless state couplings. Generally, they do not vanish at late time, though the energy density vanishes soon after the system hits the singularity at the critical time. As an example, let us consider $H_{ab}(x^0, x^1)$ couplings. They are governed by the real and the imaginary parts of the coefficient functions $f^\pm$ and $h^{\pm}_1$:

\[ H_{00} + iH_{01} = -H_{11} + iH_{10} = f^+ h^-_1. \]

We plot the real and the imaginary parts of the function

\[ f^+ h^-_1 = 2 \cos^2(\tilde{\lambda}\pi/2) \left[ 1 - 2 \sin(\tilde{\lambda}\pi/2) \cosh \frac{x^0 - ix^1}{\sqrt{2}} \right] f^+ - ||f^+||^2 \]

in Fig.4. The plots show that a singularity develops precisely at the critical time $x^0 = x^0_c$, where the couplings to tachyon and massless modes diverged. This is again attributable to the fact that all these couplings depend on the function $f^\pm$. On the other hand, for the asymptotic behavior, the $H_{ab}$ coupling differs from the couplings to tachyon and massless modes. At $x^0 \gg x^0_c$, we find that

\[ f^+ h^-_1 \sim -2 \cos^2(\tilde{\lambda}\pi/2) e^{-\sqrt{2}x^1}, \]

exhibiting persistent modulation along the $x^1$-direction. Note that, as compared to the early epoch $x^0 \ll x^0_c$, the modulation wave-number has increased twice asymptotically.

The coupling functions at higher massive levels would be governed by two qualitatively different contributions. These coupling functions are extractable from expanding the product.
of two $c = 1$ boundary states Eq.(2.6) in powers of closed string oscillator levels. For the first massive level modes, we have seen in Eq.(2.12) that the couplings are quadratic products among the coefficient functions ($f^\pm, g^\pm, h^\pm_1, h^\pm_2, h^\pm_3$) up to level-2. Likewise, for a generic massive level mode, the coupling would be quadratic products among the coefficient functions, now involving higher-level ones. Expanding the products, we see that the coupling function comprises of two kind of contributions: the one depending on $f^\pm$ (e.g. most of Eq.(2.12) in the case of the first massive level modes) and the one independent of $f^\pm$ (e.g. products among the first parts of $g^\pm, h^\pm_1, h^\pm_2, h^\pm_3$ in the case of the first massive level modes). As the first type of contributions depends on $f^\pm$, there will always be a singularity at $x^0 = x^0_c$. The second type of contributions is non-singular, generally increasing as $x^0 \to \infty$, and hence is qualitatively analogous to the behavior of the couplings for homogeneous tachyon rolling.

A remark is in order. So far, we have focused primarily on constructing the boundary state Eq.(2.10) itself. It might well be that some components of the boundary state Eq.(2.10) are BRST exact, viz. couple only to off-shell or unphysical closed string states. In that case, recalling that the boundary state Eq.(2.10) acts as a source to the closed string field equation Eq.(1.3), the BRST exact components would not be contributing to on-shell closed string processes, such as closed string radiation out of decaying D-brane. Whether there indeed exists BRST exact components and, if so, which modes (and their time-dependent coefficient functions) belong to them are pertinent questions to be examined in case one needs to study on-shell processes. In this work, we will leave the issue aside for future study.

3 Turing on Electric and Magnetic Fields

With the physics motivation addressed in section 1, we extend the analysis of section 2 and construct a boundary state for an unstable D-brane with electric and magnetic fields turned on.

3.1 Rolling of Modulated Tachyon under Electric Flux

We shall now consider turning on a constant electric field on the D25-brane world-volume, and study rolling of the spatially modulated tachyon field. As is well-known, the electric field induces fundamental string constituents on the D25-brane world-volume. If the tachyon field were spatially homogeneous, the induced string charge density (which is given by the electric displacement field) would be homogeneous over the D25-brane world-volume as well. On the other hand, if the tachyon field were spatially modulated, the induced string charge density would be modulated accordingly and are driven to evolve as the tachyon rolls down to the potential minimum. In such a circumstance, an interesting and potentially important question
is whether the displacement field flux can possibly get squeezed into (an array of) thin flux tubes. If so, adding spatial modulation on a decaying D-brane would be a new way of manufacturing a macroscopically large, fundamental closed string!

With such motivation, consider turning on the world-volume electric field along 2-direction \( e = F_{02} \). With tachyon modulation along 1-direction, this amounts to a fluid of fundamental strings, whose macroscopic configuration is stretched along 2-direction and charge density varies across 1-direction. As prescribed in the work \[15\], the corresponding boundary state is obtainable by a sequence of T-dual map, boost by \( e \), and inverse T-dual map, all along the 2-direction. Denote the new coordinates as \( y^\alpha \)'s and the new string oscillators as \( \beta_n^a, \bar{\beta}_n^a \)'s. Starting from Eq.(2.10), the new boundary state would again be expressible as:

\[
|B\rangle_{T,e} = F_e(y^0, y^1)|0\rangle + G^{ae}_{ab}(y^0, y^1)\beta_{-1}^a \beta_{-1}^b|0\rangle + \frac{1}{2} H^{ae}_{ab}(y^0, y^1)\beta_{-2}^a \beta_{-2}^b|0\rangle + \frac{1}{4} I^{ae}_{abcd}(y^0, y^1)\beta_{-1}^a \beta_{-1}^b \beta_{-2}^c \beta_{-2}^d|0\rangle + \frac{i}{2} J^{ae}_{abcd}(y^0, y^1)\left(\beta_{-1}^a \beta_{-1}^b \beta_{-2}^c + \beta_{-2}^a \beta_{-1}^b \beta_{-1}^d\right)|0\rangle + \cdots,
\]

where

\[
F_e(y^0, y^1) = \gamma^{-1} F(\gamma^{-1} y^0, y^1),
\]
\[
G^{ae}_{ab}(y^0, y^1) = \gamma^{-1} \left(t^\Lambda^{-1} G \Lambda\right)_{ab} (\gamma^{-1} y^0, y^1),
\]
\[
H^{ae}_{ab}(y^0, y^1) = \gamma^{-1} \left(t^\Lambda^{-1} H \Lambda\right)_{ab} (\gamma^{-1} y^0, y^1),
\]
\[
I^{ae}_{abcd}(y^0, y^1) = \gamma^{-1} \left(t^\Lambda^{-1} (t^\Lambda^{-1} I \Lambda)_{ac} \Lambda\right)_{bd} (\gamma^{-1} y^0, y^1),
\]

etc., and

\[
\Lambda = \begin{pmatrix}
\gamma & 0 & \gamma e \\
0 & 1 & 0 \\
\gamma e & 0 & \gamma \\
& & \\
& & 1 \\
& & \\
& & . . .
\end{pmatrix}, \quad \gamma = \frac{1}{\sqrt{1 - e^2}}.
\]

Explicitly, the new coefficient functions are given as

\[
F_e(y^0, y^1) = \gamma^{-1} F(\gamma^{-1} y^0, y^1) = \gamma^{-1} |f^+(\gamma^{-1} y^0, y^1)|^2,
\]
\[
G^{00}_{00}(y^0, y^1) = \gamma \left[ G^{00}_{00}(\gamma^{-1} y^0, y^1) - e^2 G_{22}(\gamma^{-1} y^0, y^1) \right] = -\gamma^{-1} |f^+(\gamma^{-1} y^0, y^1)|^2 + 2 \gamma \cos^2(\lambda \pi/2) \text{Re} f^+(\gamma^{-1} y^0, y^1),
\]
\[
G^{11}_{11}(y^0, y^1) = \gamma^{-1} G_{11}(\gamma^{-1} y^0, y^1)
\]

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The energy-momentum tensor is given in terms of the coupling functions by

\[ G_{22}(y^0, y^1) = 2\gamma^{-1}||f^+(\gamma^{-1}y^0, y^1)||^2 - 2\gamma^{-1} \cos^2(\lambda \pi/2) \Re f^+(\gamma^{-1}y^0, y^1), \]

\[ G_{ii}^e(y^0, y^1) = 2\gamma^{-1}||f^+(\gamma^{-1}y^0, y^1)||^2 \]

for the tachyon and the massless level modes, and

\[ G_{02}^e(y^0, y^1) = -G_{20}^e(y^0, y^1) = 2\gamma e [G_{00}(\gamma^{-1}y^0, y^1) - G_{22}(\gamma^{-1}y^0, y^1)] \]

\[ G_{01}^e(y^0, y^1) = +G_{10}^e(y^0, y^1) = 2\gamma e \cos^2(\lambda \pi/2) \Im f^+(\gamma^{-1}y^0, y^1), \]

\[ G_{12}^e(y^0, y^1) = -G_{21}^e(y^0, y^1) = 2\gamma e \cos^2(\lambda \pi/2) \Im f^+(\gamma^{-1}y^0, y^1) \]

for the coupling to the first massive level modes.

\[ H_{00}^e(y^0, y^1) = \gamma [H_{00} - e^2 H_{22}] = 2\gamma [\Re(f^+ h_{i1}^-) + e^2 ||f^+||^2] (\gamma^{-1}y^0, y^1), \]

\[ H_{11}^e(y^0, y^1) = -\gamma^{-1}H_{11} = -\gamma^{-1} \Re(f^+ h_{i1}^-)(\gamma^{-1}y^0, y^1), \]

\[ H_{22}^e(y^0, y^1) = \gamma [H_{22} - e^2 H_{00}] = -\gamma \left[ e^2 \Re(f^+ h_{i1}^-) + ||f^+||^2 \right] (\gamma^{-1}y^0, y^1), \]

\[ H_{ii}^e(y^0, y^1) = -\gamma^{-1}H_{ii} = -\gamma^{-1}||f^+(\gamma^{-1}y^0, y^1)||^2 \]

for the coupling to the first massive level modes.

### 3.2 Coupling to Closed String Modes

Coupling to the closed string modes are readily obtained from the boundary state, as we did in section 2.

- **Tachyon and dilaton couplings**

  For tachyon and dilaton fields, the couplings are given by

  \[ \rho_{\text{tachyon}}^e(y^0, y^1) = \rho_{\text{dilaton}}^e(y^0, y^1) = \mathcal{T} \gamma^{-1}||f^+(\gamma^{-1}y^0, y^1)||^2. \]

We see that, as the electric field is turned on along \( x^2 \)-direction, the rolling is time-dilated and the spatial modulation along \( x^1 \)-direction remains unaffected. In the critical limit, \( e \to 1 \), these couplings vanish because the overall Born-Infeld factor \( \gamma^{-1} \) dilutes the coupling density.

- **Graviton coupling**

  The energy-momentum tensor is given in terms of the coupling functions by

  \[ T_{ab}^e(y^0, y^1) = \frac{\mathcal{T}}{2} \left( G_{(ab)}^e(y^0, y^1) - \eta_{ab} F^e(y^0, y^1) \right), \]
so we obtain

\[
T_{00}(y^0, y^1) = + T_{25} \gamma \cos^2(\tilde{\lambda} \pi/2) \Re f^+(\gamma^{-1}y^0, y^1),
\]

\[
T_{01}(y^0, y^1) = + T_{25} \cos(\tilde{\lambda} \pi/2) \Im f^+(\gamma^{-1}y^0, y^1),
\]

\[
T_{11}(y^0, y^1) = - T_{25} \gamma^{-1} \cos^2(\tilde{\lambda} \pi/2) \Re f^+(\gamma^{-1}y^0, y^1),
\]

\[
T_{22}(y^0, y^1) = - T_{25} e^2 \gamma \cos^2(\tilde{\lambda} \pi/2) \Re f^+(\gamma^{-1}y^0, y^1) - T_{25} \gamma^{-1} ||f^+(\gamma^{-1}y^0, y^1)||^2,
\]

\[
T_{ii}(y^0, y^1) = - T_{25} \gamma^{-1} ||f^+(\gamma^{-1}y^0, y^1)||^2 \quad \text{(for } i = 3, \cdots, 25). \tag{3.21}
\]

Let us compare Eq. (3.21) with the energy-momentum tensor Eq. (2.19) at zero electric flux. Apart from the expected Born-Infeld time dilation and Lorentz contraction, we see that the pressure along the electric field direction receives a new contribution, proportional to the energy density. This part is attributed to the fundamental string constituents.

- **Kalb-Ramond coupling**

At nonzero electric flux, coupling to the Kalb-Ramond field

\[
Q_{ab}^e(y^0, y^1) := \frac{T_{25}}{2} G_{[ab]}^e(y^0, y^1),
\]

is newly induced, whose non-zero components are

\[
Q_{02}(y^0, y^1) = T_{25} \gamma e \cos^2(\tilde{\lambda} \pi/2) \Re f^+(\gamma^{-1}y^0, y^1),
\]

\[
Q_{12}(y^0, y^1) = T_{25} e \cos(\tilde{\lambda} \pi/2) \Im f^+(\gamma^{-1}y^0, y^1). \tag{3.22}
\]

As anticipated, the induced Kalb-Ramond coupling is proportional to the nonzero electric field, thus making up the decaying D25-brane to carry a fundamental string charge. Note that the charge density of the fundamental string stretched along 2-direction is given by \(Q_{02}\), and it is proportional to the energy density in Eq. (3.21) as

\[
Q_{02}(y^0, y^1) = \epsilon \ T_{00}^e(y^0, y^1). \tag{3.23}
\]

Likewise, the current density of the fundamental string \(Q_{12}\) is proportional to the momentum density in Eq. (3.21) as

\[
Q_{12}(y^0, y^1) = \epsilon \ T_{01}^e(y^0, y^1). \tag{3.24}
\]

Therefore, just as the behavior of energy density explained in section 2.3, half of the fundamental string charge density would squeeze to form (an array of) localized \(\delta\)-function profile as the time \(y^0 \) approaches the critical time \(y^0_c\). The off-diagonal (01)-component of the energy-momentum tensor and (12)-component of the Kalb-Ramond coupling imply that the fundamental string density is actually flowing along the 1-direction! This is to be contrasted with the situation of homogeneous tachyon rolling, where turning on electric field \(e\) induces a fundamental string...
fluid on the decaying D-brane world-volume, but the fluid is at rest. As shown in [15, 16],
turning on magnetic field \( b \) perpendicular to the electric field lets the fluid to flow rigidly. We
now find that modulated tachyon field can also trigger the fundamental string gas to flow.
Moreover, the flow velocity varies spatially, albeit it is determined by the modulation of the
rolling tachyon itself.

- **BPS Limit**

Now that the electric field \( e \) is restricted to \( |e| \leq 1 \), Eqs.(3.23, 3.24) imply the following BPS-like
inequalities

\[
T^e_{00} \geq |Q^e_{02}| \quad \text{and} \quad |T^e_{01}| \geq |Q^e_{12}|
\]

which are saturated precisely at the limit \( |e| \rightarrow 1 \).

One important effect of turning on the electric field is the time dilation by the \( \gamma^{-1} \) factor:
\( x^0 \rightarrow \gamma^{-1} y^0 \). Therefore, the critical time is now dilated to

\[
y^0_c = \gamma \sqrt{2 \log \left( \frac{1}{|\sin(\tilde{\lambda} \pi/2)|} \right)} = \gamma x^0_c.
\]

If we take the extremal limit

\[
e \rightarrow 1 \quad \text{and} \quad \tilde{\lambda} \rightarrow 1
\]

while holding the average energy \( \langle E \rangle = \gamma T_{25} \cos^2(\tilde{\lambda} \pi/2) \) fixed, we find that the critical time is
situated at a finite value:

\[
y^0_c \rightarrow \frac{E}{\sqrt{2} T_{25}},
\]

indicating that the relaxation time scale grows with the total energy one starts with.

In the extremal limit, the real part of \( f^\pm \) becomes

\[
\text{Re} f^\pm(\gamma^{-1} y^0, y^1) \rightarrow \left\{ \begin{array}{ll}
2 \sqrt{2} \pi \sum_{n \in \mathbb{Z}} \delta \left( y^1 - \sqrt{2}(2n + 1) \pi \right) & \text{for } |y^0| < y^0_c, \\
0 & \text{for } |y^0| > y^0_c.
\end{array} \right.
\]

Thus, prior to reaching the critical time \( y^0_c \), non-vanishing components of the energy-momentum
tensor and the Kalb-Ramond tensor current density become

\[
T^e_{00} = -T^e_{22} = |Q_{02}| = 2 \sqrt{2} \pi E \sum_{n \in \mathbb{Z}} \delta \left( y^1 - \sqrt{2}(2n + 1) \pi \right).
\]

This is the same relation as that saturated by an array of BPS fundamental strings stretched
along the \( y^2 \)-direction, except that we are now considering homogeneous distribution of the
string fluid along the transverse \( y^3, \cdots, y^{25} \)-directions. It thus indicates that the fundamental
string constituents are confined to co-dimension one hypersurfaces located at \( y^1 = \sqrt{2}(2n + 1) \pi \).
Note, however, that this extremal limit is not a smooth limit for the full boundary state. As pointed out in [10, 15] for the homogeneous case, the coefficients in the boundary state for higher-level massive modes with more than three oscillators for 0- or 2-directions are weighted with extra $\gamma$ factors, and hence will diverge in the extremal limit. In light of the remark at the end of section 2, a possible way out is that (part of) the non-smooth modes belong to the BRST exact class.

3.3 The Effect of Magnetic Field

Physics-wise, it is also of interest to turn on magnetic field on the decaying D-brane world-volume. The world-volume gauge field is combined with pull-back of the Kalb-Ramond field $X^*B_{ab}$ into a gauge-invariant combination $(F_{ab} + X^*B_{ab})$. It thus implies that nonzero magnetic field induces current density of the fundamental string on the decaying D-brane world-volume. Here, we examine how the fundamental string constituents are affected by the magnetic field. We also consider the case with pure magnetic field. In this case, the total fundamental string charge is zero, but, nevertheless we find that polarized fundamental string charge density is induced, and the system evolves to a configuration with an array of fundamental string - anti-fundamental string pairs (i.e. an array of fundamental strings of alternating orientations). We will see that, again, spatial modulation of the rolling tachyon field plays a prominent role.

Following the prescription given in the previous work [15], we find that the boundary state is

$$|B\rangle_{T,e+b} = F^{e+b}(y^0, y^1)|0\rangle + G^{e+b}_{ab}(y^0, y^1) \beta_a \beta_b + \frac{1}{2} H^{e+b}_{ab}(y^0, y^1) \beta_a \beta_b |0\rangle + \frac{1}{4} I^{e+b}_{abcd}(y^0, y^1) \beta_a \beta_b \beta_c \beta_d |0\rangle + \cdots,$$

where the coefficient functions are obtained as

$$F^{e+b}(y^0, y^1) = \gamma^{-1}\bar{\gamma}^{-1} F(\gamma^{-1}y^0, \bar{\gamma}^{-1}y^1 - be\bar{\gamma}y^0),$$
$$G^{e+b}_{ab}(y^0, y^1) = \gamma^{-1}\bar{\gamma}^{-1} \left(t(\Omega)\right)^{-1} F_G(\gamma^{-1}y^0, \bar{\gamma}^{-1}y^1 - be\bar{\gamma}y^0),$$
$$H^{e+b}_{ab}(y^0, y^1) = \gamma^{-1}\bar{\gamma}^{-1} \left(t(\Omega)\right)^{-1} F_H(\gamma^{-1}y^0, \bar{\gamma}^{-1}y^1 - be\bar{\gamma}y^0),$$
$$I^{e+b}_{abcd}(y^0, y^1) = \gamma^{-1}\bar{\gamma}^{-1} \left(t(\Omega)\right)^{-1} F_I(\gamma^{-1}y^0, \bar{\gamma}^{-1}y^1 - be\bar{\gamma}y^0),$$
etc., in terms of transformation matrices

$$\begin{pmatrix} \gamma & 0 & \gamma e' \\ 0 & 1 & 0 \\ \gamma e' & 0 & \gamma \end{pmatrix}$$

and

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \bar{\gamma} & -\bar{\gamma}b \\ 0 & +\bar{\gamma}b & \bar{\gamma} \end{pmatrix}$$

and

$$\gamma = \frac{1}{\sqrt{1-e'^2}} \text{ with } e' = \frac{e}{\sqrt{1+b^2}} \text{ and } \bar{\gamma} = \frac{1}{\sqrt{1+b^2}}.$$ 

We then find that the nonzero components of the energy-momentum tensor are

$$T^{e+b}_{00} = +T_{25} \gamma \bar{\gamma} (1 + b^2) \cos^2(\bar{\lambda}\pi/2) \text{Re} f^+,$$

$$T^{e+b}_{01} = +T_{25} \cos^2(\bar{\lambda}\pi/2) \left[ \bar{\gamma} \gamma e \text{Re} f^+ + \bar{\gamma} \gamma eb \text{Re} f^+ \right],$$

$$T^{e+b}_{11} = -T_{25} \cos^2(\bar{\lambda}\pi/2) \left[ \bar{\gamma} \gamma (1 - e^2) \text{Re} f^+ - \frac{2eb}{(1+b^2)} \text{Im} f^+ \right],$$

$$T^{e+b}_{22} = +T_{25} \cos^2(\bar{\lambda}\pi/2) \left[ \bar{\gamma} \gamma (b^2 - e^2) \text{Re} f^+ + \frac{2eb}{(1+b^2)} \text{Im} f^+ \right] - T_{25} \gamma^{-1} \bar{\gamma}^{-1} ||f^+||^2,$$

$$T^{e+b}_{ii} = -T_{25} \gamma^{-1} \bar{\gamma}^{-1} ||f^+||^2 \quad \text{(for } i = 3, \ldots, 25),$$

while the nonzero components of the Kalb-Ramond coupling tensor are

$$Q^{e+b}_{[02]} = T_{25} \cos^2(\bar{\lambda}\pi/2) \left[ \gamma \bar{\gamma} e \text{Re} f^+ - b \text{Im} f^+ \right],$$

$$Q^{e+b}_{[12]} = T_{25} \cos^2(\bar{\lambda}\pi/2) \left[ e \left( \frac{1 - b^2}{1+b^2} \right) \text{Im} f^+ + \bar{\gamma} \bar{\gamma} b \text{Re} f^+ \right],$$

where $f^\pm := f^\pm(\gamma^{-1}y^0, \bar{\gamma}^{-1}y^1 + eb\bar{\gamma}y^0)$. One can check that the conservation equations $\partial^a T^{e+b}_{ab} = 0$ and $\partial^a Q^{e+b}_{ab} = 0$ are satisfied. Couplings to other closed string modes are also obtainable analogously from the prescription given in [15]. Before proceeding further, we will contrast the above results against those obtained in the previous sections.

First, the spatial coordinate $x^1$ is now replaced by $(\bar{\gamma}^{-1}y^1 + eb\bar{\gamma}y^0)$ in the functions $f^\pm$, so the whole system would be moving in the $y^1$-direction with a constant velocity

$$V := \frac{eb}{1+b^2} \leq 1 \text{.}$$

Indeed, prior to the critical time $y^0_c$, the average momentum

$$\langle P \rangle := \frac{1}{2\sqrt{2\pi}} \int_0^{2\sqrt{2\pi}} dy^1 T^{e+b}_{01}(y^0, y^1) = \gamma \bar{\gamma} eb T_{25} \cos^2(\bar{\lambda}\pi/2)$$

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and the average energy
\[ \langle E \rangle := \frac{1}{2\sqrt{2\pi}} \int_{0}^{2\sqrt{2\pi}} dy^1 T_{00}^{e+b}(y^0, y^1) = \gamma \bar{\gamma} (1 + b^2) \mathcal{T}_{25} \cos^2(\lambda \pi/2) \]
are nonzero, and the velocity of the energy flow is computed as \( \langle P \rangle / \langle E \rangle = eb/(1 + b^2) \). This is consistent with Eq.(3.26).

Second, we note that the (01)-, (11)-, (22)-components of the energy-momentum tensor are now composed of both real and imaginary parts of \( f^\pm \). Recall that real and imaginary parts of \( f^\pm \) are even and odd functions, respectively, of the comoving spatial coordinate \( (y^1 + V y^0) \), and develop a singularity at the critical time \( y^c_0 \). This gives rise to several effects. For (01)-component of the energy-momentum tensor, Poynting field momentum of the electromagnetic fields (proportional to \( eb \) in magnitude) adds up into an inhomogeneous distribution along \( y^1 \)-direction, and accumulates to an array of delta functions as the critical time \( y^c_0 \) is approached. For the (11)- and (22)-components of the energy-momentum tensor, the electromagnetic fields induce additional contribution proportional to \( \text{Im} f^+ \) whose magnitude is determined by the velocity \( V \).

Third, [02]- and [12]-components of the Kalb-Ramond coupling tensor indicate that the fundamental string charge and current densities are induced not only by the electric field \( e \) but also separately by the magnetic field \( b \). We are thus led to a surprising observation that, even without the electric field, a decaying D25-brane can carry fundamental string constituents induced by the modulated tachyon and the magnetic flux! In this case, as real and imaginary parts of \( f^\pm \) are even and odd functions of \( (y^1 + V y^0) \), the net charge would be zero while the net current is nonzero — the system is a sort of plasma of the fundamental strings (consisting of strings of alternating orientations).

Of particular interest is the extremal limit. Take
\[ e \to \sqrt{1 + b^2} \quad \text{and} \quad \bar{\lambda} \to 1, \]
while keeping the average energy \( \langle E \rangle \) held finite. Then, the terms which survive in this limit are
\[
\begin{align*}
T_{00}^{e+b}(y^0, y^1) &= \Gamma^2 \left( 2\sqrt{2\pi} \bar{E} \delta(y^0, y^1) \right) = -\Gamma^2 T_{22}^{e+b}(y^0, y^1) \\
T_{01}^{e+b}(y^0, y^1) &= \Gamma^2 V \left( 2\sqrt{2\pi} \bar{E} \delta(y^0, y^1) \right) = V T_{00}^{e+b}(y^0, y^1) \\
T_{11}^{e+b}(y^0, y^1) &= \Gamma^2 V^2 \left( 2\sqrt{2\pi} \bar{E} \delta(y^0, y^1) \right) = V^2 T_{00}^{e+b}(y^0, y^1) \\
T_{22}^{e+b}(y^0, y^1) &= -2\sqrt{2\pi} \bar{E} \delta(y^0, y^1) \\
Q_{[02]}^{e+b}(y^0, y^1) &= 2\sqrt{2\pi} \bar{E} \Gamma \delta(y^0, y^1) \\
Q_{[12]}^{e+b}(y^0, y^1) &= 2\sqrt{2\pi} \bar{E} V \Gamma \delta(y^0, y^1) = V Q_{[02]}^{e+b}(y^0, y^1). 
\end{align*}
\]
Here, \( V = b/\sqrt{1 + b^2} \) is the velocity Eq.(3.26) in the extremal limit, \( \Gamma = 1/\sqrt{1 - V^2} = \sqrt{1 + b^2} \) is the corresponding gamma factor, \( \hat{E} = \langle E \rangle / \Gamma^2 \) is the average energy in the rest frame, and

\[
\delta(y^0, y^1) := \sum_{n \in \mathbb{Z}} \delta\left(\Gamma(y^1 + V y^0) - \sqrt{2}(2n + 1)\pi\right)
\]

is an array of the delta functions. As such, each component of the energy-momentum and the fundamental string current tensors have a constant magnitude. Evidently, if we observe the system in the rest frame, the results are in agreement with the BPS equality, Eq.(3.25). Therefore, the system is viewed most transparently as an array of BPS fundamental strings stretched along the \( y_2 \)-direction, but now moving in \( y_1 \)-direction uniformly with a constant velocity \( V \).

It would also be of interest to consider the case with large magnetic field to learn the effect of pure magnetic field. If we set \( e = 0 \) and take \( b \to \infty \), the leading terms for the energy-momentum tensor and the Kalb-Ramond coupling tensor are

\[
T^{b}_{00}(y^0, y^1) \simeq +\mathcal{T}_{25} \cos^2(\tilde{\lambda} \pi/2) b \text{Re} f^+(y^0, y^1) \\
T^{b}_{01}(y^0, y^1) \simeq +\mathcal{T}_{25} \cos^2(\tilde{\lambda} \pi/2) \text{Im} f^+(y^0, y^1) \\
T^{b}_{11}(y^0, y^1) \simeq \mathcal{O}(b^{-1}) \\
T^{b}_{22}(y^0, y^1) \simeq +\mathcal{T}_{25} \cos^2(\tilde{\lambda} \pi/2) b \text{Re} f^+(y^0, y^1) - \mathcal{T}_{25} b \|f^+\|(y^0, y^1) \\
T^{b}_{i(i}(y^0, y^1) \simeq -\mathcal{T}_{25} b \|f^+\|(y^0, y^1) \\
Q^{b}_{(02]}(y^0, y^1) \simeq -\mathcal{T}_{25} \cos^2(\tilde{\lambda} \pi/2) b \text{Im} f^+(y^0, y^1) \\
Q^{b}_{(12]}(y^0, y^1) \simeq +\mathcal{T}_{25} \cos^2(\tilde{\lambda} \pi/2) \text{Re} f^+(y^0, y^1),
\]

(3.27)

with \( f^\pm(y^0, y^1) = f^\pm(y^0, by^1) \). Now, the period in the \( y_1 \)-direction is shortened to \( 2\sqrt{2}\pi/b \). The energy and momentum densities are proportional to \( \text{Re} f^+ \) and \( \text{Im} f^+ \), respectively, which is qualitatively similar to the previous results Eq.(2.19) or Eq.(3.21). On the other hand, the fundamental string charge and current densities are now proportional to \( \text{Im} f^+ \) and \( \text{Re} f^+ \), respectively, which is opposite to what we have found for the case with pure electric field Eq.(3.22). The behavior of the fundamental string charge and current densities can be read from Fig.2 and Fig.1, respectively. Since \( \text{Im} f^+ = 0 \) at \( y^0 = 0 \), the charge density is initially zero everywhere, but then starts to get polarized as the tachyon rolls down the potential hill.

Intuitively, such a behavior can be understood as follows. As one can readily see from the gauge invariance \( X^* B \to X^* B + d\Lambda_1, F \to F - d\Lambda_1 \), the magnetic field on a D-brane world-volume induces fundamental string current density. However, when the tachyon field is at the bottom of its potential and the D-brane disappears, there is nowhere for the string current to flow. Therefore, the region of the D-brane world-volume where the tachyon is near the bottom of the potential acts just like an insulator for an electric current. In our situation, as the
modulated tachyon evolves, the regions around \( y^1 = 2\sqrt{2}n\pi/b \ (n \in \mathbb{Z}) \) behave as insulators while those near \( y^1 = \sqrt{2}(2n + 1)\pi/b \ (n \in \mathbb{Z}) \) behave as conductors for the string current. Thus, our system is analogous to a series of capacitors injected with a non-zero electric current, in which electric charge starts to accumulate at the edges of the conductors. The polarization of the fundamental string charge density is induced in this way. When the system reaches the critical time, the current density develops a delta-function singularity, leading to a configuration with a polarized fundamental string - anti-fundamental string pair near the critical time.

4 Effective Field Theory Approach

It is also of interest to compare the results obtained from the boundary-state approach with those from other approaches. In this section, we analyze the D-brane decay in the effective field theory approach, extending previous results [15] to the case with inhomogeneous tachyon rolling. Consider the world-volume Lagrangian of an unstable D\( p \)-brane:

\[
\mathcal{L}_{\text{DBI}} = -T_p V(T) \sqrt{- \det (\eta + F)} F(z),
\]

where \( V(T) \) is a tachyon potential and \( F(z) \) is a function of

\[
z := \left( \frac{1}{\eta + F} \right)^{(ab)} \partial_a T \partial_b T,
\]

normalized such that \( F(0) = 1 \). The energy-momentum tensor \( T_{ab} \) and the Kalb-Ramond current tensor \( Q_{ab} \) are obtainable by replacing \( \eta_{ab} \to \eta_{ab} + h_{ab} \), \( F_{ab} \to F_{ab} + b_{ab} \) in the Lagrangian Eq.(4.28) and expanding in powers of \( h_{ab}, b_{ab} \):

\[
\delta \mathcal{L}_{\text{DBI}} = \frac{T_p}{2} V(T) \sqrt{- \det (\eta + F)} D^{ab} (h_{(ab)} + b_{[ab]}) + \cdots
\]

\[
\equiv \frac{1}{2} (T^{(ab)} h_{(ab)} + Q^{(ab)} b_{[ab]}) + \cdots,
\]

where

\[
D^{ab} = F(z) \left( \frac{-1}{\eta + F} \right)^{ab} + 2 F'(z) \left( \frac{1}{\eta + F} \right)^{ac} \partial_c T \partial_d T \left( \frac{1}{\eta + F} \right)^{db}.
\]

Here, we consider rolling tachyon field with spatial modulation in \( x^1 \)-direction,

\[
\partial_0 T \equiv \dot{T}, \quad \partial_1 T \equiv T', \quad \partial_2 T = \cdots = \partial_p T = 0,
\]

and, for simplicity, turn off all other components of the gauge fields but \( F_{02} = e \). Then, Eq.(4.29) and Eq.(4.31) imply that nonzero components of the energy-momentum tensor \( T_{ab} \)
are

\begin{align}
T_{00} &= +\mathcal{T}_p \gamma V(T) \left( F(z) + 2\gamma^2 \dot{T}^2 F'(z) \right), \\
T_{01} &= +\mathcal{T}_p \gamma V(T) 2\dot{T}^T F'(z), \\
T_{11} &= -\mathcal{T}_p \gamma^{-1} V(T) \left( F(z) - 2T^2 F'(z) \right), \\
T_{22} &= -\mathcal{T}_p \gamma e^2 \gamma V(T) \left( F(z) + 2\gamma^2 \dot{T}^2 F'(z) \right) - \mathcal{T}_p \gamma^{-1} V(T) F(z), \\
T_{ii} &= -\mathcal{T}_p \gamma^{-1} V(T) F(z), \quad \text{for } i = 3, 4, \ldots, p, \\
\end{align}

and nonzero components of the Kalb-Ramond current tensor \( Q_{\langle ab \rangle} \) are

\begin{align}
Q_{02} &= \mathcal{E} \mathcal{T}_p \gamma V(T) \left( F(z) + 2\gamma^2 \dot{T}^2 F'(z) \right), \\
Q_{12} &= \mathcal{E} \mathcal{T}_p \gamma V(T) 2\dot{T}^T F'(z), \\
\end{align}

with \( z = (-\gamma^2 \dot{T}^2 + T^2) \) and \( \gamma = 1/\sqrt{1 - e^2} \).

In the effective field theory approach to unstable D-brane based on the Lagrangian Eq.(4.28), second and higher derivatives of the tachyon and gauge potential are neglected. As such, in case the tachyon evolves sharply and rapidly, we would not expect a good agreement with the corresponding results of the previous section, where the boundary state approach surely indicated rapid and sharp evolution of the tachyon field. Despite such shortcomings anticipated, we see in the above result a behavior observed in the boundary-state approach in Eq.(3.21) and Eq.(3.22). For example, relations

\begin{align}
Q_{02} &= eT_{00}, \quad Q_{12} = eT_{01}, \quad T_{22} = -e^2 T_{00} + T_{ii}, \\
\end{align}

featured by Eqs.(3.21, 3.22) also follow from Eqs.(4.32, 4.33). In particular, we again obtain from the first equation in Eq.(4.34) that the energy density is proportional to the fundamental string charge density.

One might think from the above observation Eq.(4.34) that, in so far as the electric field \( e \) is kept fixed, any configuration is classically stable since energetics does not tell which one is favored among those with a fixed total fundamental string charge. However, this is not correct. As we have seen explicitly in the previous section, the system does evolve to thin electric flux tubes without changing total energy and fundamental string charge during the conversion process. The reason is as follows. In the conventional stability analysis, one compares the energy of static configurations with a fixed total charge. If one finds a minimum energy configuration among them, this configuration is considered stable. This is because the kinetic energy is always positive-definite and the minimum energy configuration cannot evolve dynamically to other ones without increasing the total energy. Note that, in this argument, one has implicitly assumed that variation of the fields does not affect the total charge. In our situation, however,
it is not enough to consider the energy of static configurations with a fixed fundamental string charge, as the kinetic energy § is also seen to contribute to the fundamental string charge. Even if one finds a configuration of a minimum energy among the static ones with a fixed charge, it is always possible for the configuration to evolve to some other configuration with “less” fundamental string charge, (here we mean “less” provided we set $\dot{T} = 0$ by hand), as the rest of the fundamental string charge can be carried off by the kinetic energy. This is to be contrasted against the reasoning of [20, 22], where the conventional stability analysis was taken for granted.

If we allow the electric field $e$ to vary, the minimum energy configuration with a fixed total fundamental string charge is realized in the extremal limit $|e| \to 1$. Note that, in this limit, one also needs to set $V(T) \to 0$ from the outset to keep the total energy finite ¶. This is a special limit in which the time evolution is frozen because of the (by now well-understood) time-dilation effect of the electric field, and can be easily seen from the fact that this system is T-dual to a codimension-one unstable D-brane moving with the speed of light [15]. In this limit, it is claimed that, since the system does not evolve at all, one can engineer a string fluid of an arbitrary energy density distribution [20]. On the other hand, if one starts from a generic point of the potential hill and consider the dynamical decay of the unstable D-brane, there is no reason that the system would evolve to a string fluid of arbitrary energy distribution advocated in [20]. Indeed, we shall now show that the system evolves to the contrary.

Suppose we start with an inhomogeneous tachyon configuration with $V(T) \neq 0$ and $|e| < 1$, as considered above. The pressure along the $x^1$-direction, $p_1 \equiv T_{11}$, is, in general, non-zero and gets modulated along the $x^1$-direction. Then, the momentum density along the $x^1$-direction, $T_{01}$, cannot remain constant, so the energy (and hence fundamental string charge density) distribution would change every moment during the evolution. In order to see whether the flux will be squeezed and form an arbitrarily thin string-like flux tube or diffuse to a uniform distribution, it is imperative to follow the dynamics of the system carefully.

As an example, let us examine the background-independent string field theory action for non-BPS D-branes in superstring theory. We have [25]

$$V(T) = e^{-\frac{1}{2} T^2} \quad \text{and} \quad \mathcal{F}(z) = \frac{z 4^z \Gamma^2(z)}{2 \Gamma'(2z)}.$$  \hspace{1cm} (4.35)

If we take a linear-gradient profile of the tachyon field $T = u x^1$ and $\dot{T} = 0$ as an initial tachyon configuration, the energy density $\rho$ and the pressure $p$ along the $x^1$-direction are

$$\rho = T_{00} = + \mathcal{T}_p \gamma V(u x^1) \mathcal{F}(u^2),$$

$$p_1 = T_{11} = - \mathcal{T}_p \gamma^{-1} V(u x^1) \mathcal{D}(u^2),$$

§more generally, contribution of time-dependence of the tachyon to the total energy

¶Here, we have assumed that the kinetic energy is positive-definite, i.e. $\mathcal{F}(z) + 2 \gamma^2 \dot{T}^2 \mathcal{F}'(z) \geq \mathcal{F}(0) = 1.$
where $D(z) := F(z) - 2zF'(z)$. In this case, we see that the $x^1$-coordinate dependence is carried solely by the tachyon potential, $V(ux^1)$. One can show that the function $F(u^2)$ is always positive for any value of $u^2 > 0$, so the energy density $\rho$ is manifestly positive-definite. The energy density $\rho$ is accumulated around the origin, $x^1 = 0$, with a typical width of order $O(u^{-1})$. It was shown in [25] that the total energy decreases monotonically with $u$. The energy density becomes delta-function distribution $\rho(x^1) \propto \delta(x^1)$ in the $u \to \infty$ limit, representing a BPS D$(p - 1)$-brane with the electric field. The function $D(u^2)$ is also a positive function with $\lim_{u \to \infty} D(u^2) = 0$. Therefore, in so far as $u^2$ is finite, the pressure $p_1$ is negative-definite with the minimum located at the maximum of the energy density. From the continuity equation $\partial_0 T_{01} + \partial_1 p_1 = 0$, we see that the momentum density $T_{01}$ is turned on, and the energy starts to accumulate to the center. This clearly suggest that the system will evolve to a thin string configuration. In order to learn detailed time evolution of the system, one will need to solve the equations of motion and follow the evolution explicitly.

In the above analysis, we considered the evolution under the initial condition of a kink-like tachyon profile. It carries a D$(p - 1)$-brane charge, and hence the decay product would be a bound state of the D$(p - 1)$-brane and fundamental string constituents. In order to obtain purely fundamental string constituents in the final state, we will need to consider a configuration with $T'' \neq 0$. In this case, it is difficult to analyze the system based on the effective action Eq.(4.28), as the second and higher derivatives of the tachyon field are assumed small and thus truncated. Qualitatively, however, until the tachyon modulation has not grown up large enough, the energy density and the pressure of the system are well approximated by Eq.(4.32). Recalling that the qualitative behavior is governed primarily by the tachyon potential $V$, we expect that the pressure $p$ reaches its minima at locations where the energy density reaches its maxima. As such, again, we expect that the inhomogeneity tend to grow as above. (See [8, 9, 17, 18] for related arguments.)

5 Discussion

In this paper, to gain deeper understanding concerning dynamical fate of the fundamental string constituents, we have studied rolling of the inhomogeneous tachyon in the presence of constant gauge fields on the world-volume of the decaying D-brane. Quite interestingly, we have found that the fundamental string charge density is induced not only by the electric field but also by the magnetic field. Although the gauge fields turned on are uniform, distribution of the fundamental string constituents is spatially inhomogeneous because of modulation of the rolling tachyon field. We have shown that the tachyon rolling defines an exactly marginal boundary perturbation to the boundary state and have solved the system exactly. It represents an exact classical solution of string theory, at least prior to reaching the critical time, where
the system develops a singularity.

The most pressing point for further study is a physical interpretation of the critical time and singularity therein. Notice that our consideration corresponds to the weak coupling perturbation theory at the lowest order, and we ignored radiation of massless and massive closed string modes, triggered by the rolling tachyon, as well as back-reaction to the decaying D-brane and background geometry. Indeed, we have seen that the total energy is conserved and stored in the open string sector until the critical time. The singularity seems to suggest that the weak coupling limit is not a smooth limit to take. Once we turn on $g_{st}$, no matter how tiny it is, the coupling to closed string modes blow up at the critical time. So, the physics with $g_{st} = 0$ and that with tiny $g_{st}$ could be completely different at the critical time.

In the more realistic set-up with small but non-zero $g_{st}$, inferring from Eq.(1.3), it is plausible to suppose that the energy will be transferred to a collection of closed strings near the critical time. After some energy is converted to closed string modes, it would appear as if the energy is lost for the observer looking only at the open string modes. It is not clear whether the energy is radiated away or stored at the singularity to form a macroscopic fundamental string or a D-brane. Once we turn on non-zero $g_{st}$, since the coupling to closed string becomes impulsive and huge near the singularity, there ought to be a large back reaction. In particular, it is not hard to imagine that the space-time around the energy core will be severely curved to form an event horizon. If it is the case, the energy cannot escape through it, at least classically. This could be a possible scenario for the formation of a macroscopic fundamental string or a D-brane. If the string coupling $g_{st}$ is nonzero (no matter how small), nonlinear coupling of the tachyon to other open string and closed string states becomes important near the singularity. It would be certainly of interest to take these string states into consideration, and obtain a more complete picture of the rolling tachyon dynamics.

In [19], it was claimed that the non-perturbative confinement mechanism is responsible for the formation of string-like object in unstable D-brane system. It was further argued in [20, 21] that classical dynamics has no mechanism to make the electric flux tube thin. In [22], classical stability analysis for the unstable D-brane effective action was made, and it was claimed that the flux tube is unstable against uniform spread-out. However, our result suggest that the flux can squeeze dynamically to a thin flux tube even at the classical level, in contrast to the conclusion drawn in [20, 21, 22]. Note that [20, 21] considered the situation that the tachyon potential is set to zero from the outset (instead of scaling limit), so it is delicate to make a direct comparison of their results with ours in which we considered the tachyon rolling down from a generic point of the potential hill.

Furthermore, in the works [20, 22], (in)stability of the flux tube was analyzed by looking

\[\text{See [11] for related issues for the late time behavior of the homogeneous rolling tachyon.}\]
for a static configuration minimizing the total energy among those with a fixed fundamental string charge. With the identification of the correct electric displacement field, one readily sees that all configurations with the same fixed charge have the same total energy, so it appears superficially that any of these configurations is classically stable. However, as we have argued thoroughly in section 4, this stability criterion actually does not tell us anything about the stability of the flux distribution in our case. This is because, for the decaying D-brane, it should be noted that the kinetic energy (in addition to the gradient energy) of the tachyon field carries the fundamental string charge as well. Consequently, in general, the system does evolve even for the total charge held fixed, and this is the physical reason behind the conclusion drawn in this work differently from [20, 21, 22].

How generic is the conclusion we have drawn? We have analyzed in detail the tachyon rolling of the special configuration given in Eq.(2.4), and observed the formation of the string-like thin electric flux tube. We also pointed out that modulation of the tachyon field tends to grow by using both the boundary-state and the effective field theory descriptions. On the other hand, not every inhomogeneous tachyon configuration will eventually evolve to a configuration with thin flux tubes like this. In fact, we can easily construct an example with static inhomogeneous energy density by turning on the tachyon profile of the form:

\[ T(x^0, x^1) = \lambda \cosh(x^0) + \lambda' \cos(x^1). \]

This profile also induces an exactly solvable boundary perturbation. Thus, in order to see the fate of unstable D-branes with generic tachyon configuration, a more general analysis is desirable, which we leave for future study.

Finally, [13] has proposed an another approach in which the dynamical formation of fundamental string in unstable D-brane is argued within the context of classical effective field theory. It would also be interesting to elaborate relation of theirs to our approach.

Acknowledgement

We thank J. Ambjorn, K. Hashimoto, R.A. Janik, H. Kawai, O. Lunin, J. Maldacena, F. Sugino, and S. Terashima for discussions. This work was carried out while SJR was a member of the Institute for Advanced Study. SJR thanks the School of Natural Sciences for the hospitality and the Fellowship. SS is supported in part by Danish Natural Science Research Council.

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