Building Model of Behaviour of Concrete Under Load

Varlamov A. A.¹, Rimshin V. I ², Norec A. I. ¹, Davydova A. M. ¹

¹ Magnitogorsk state technical University G. I. Nosov, Lenin Ave, 27, City of Magnitogorsk, 455000, Russian Federation
² Scientific-research Institute of building physics (NIISF RAASN), Locomotive travel, 21, City of Moscow, 127238, Russian Federation

Abstract. The proposed new theory "theory of degradation". The theory allows to define a curve of the behaviour of concrete. Conducted experimental research with the aim of charting the concrete for comparison with a new theory. The paper presents analysis of experimental data. Diagram of concrete is built according to \( \varepsilon_{b.short} = \varepsilon_{b.el} + \varepsilon_{b.pl} + \varepsilon_{cl} \). Experiments for each part of the equations has been treated and constructed statistical dependence. The dependences obtained were the basis for building graphs and comparing them with charts of the theory of degradation.

1. Introduction
Diagram of concrete are of great value in the calculation of the structures [1-5]. Diagram of concrete get in the design and inspection of structures [6-9]. For building diagrams, developed the theory of the degradation of concrete [10-14]. With the aim of obtaining energy characteristics of the theory of degradation were tested concrete prisms.

The results of short-term tests of concrete prisms was received the dependence of stress-strain in four samples of concrete with strength within 12 … 41 MPa. In this case the obtained graphs were isolated deformation quickly impingement creep (\( \varepsilon_{cl} \)), instantaneous inelastic deformation (\( \varepsilon_{b.pl} \)) and a deformation of the elastic aftereffect (\( \varepsilon_{b.aft} \)). The analysis was performed in relative units. The analysis was performed in relative units. For the base prism strength \( R_{b0} \) for data processing was adopted the strength of the concrete composition of the fourth – of 34.4 MPa.

Data processing was performed on the program STATUS independently on the parameters «\( \varepsilon - \sigma/R_{b0} \)» and «\( \varepsilon - R_{bi} / R_{b0} \)». As a result, choosing those dependencies, which had the highest correlation coefficient with the experimental data (not less than 0.99). General view of the final dependency is obtained in the product of two independent parameters[15].

2. Build simple models
The total relative deformation (\( 10^{-5} \)) chart «\( \sigma_b - \varepsilon_b \)» written in the following form[15]:

\[
\varepsilon_{b.short} = \varepsilon_{b.el} + \varepsilon_{b.pl} + \varepsilon_{cl}
\]  

(1)
Deformation quickly impingement creep recorded in accordance with the proposed form of the expression

\[ \varepsilon_{cl} = 0.366 \times \left(1 + \frac{1.014R_{b0}}{R_{bi}} + \frac{0.007R_{b0}^2}{R_{bi}^2}\right) \times \left(1 + \frac{124\sigma^3}{R_{bi}^4}\right) \times 10^{-5} \]  

(2)

Inelastic instantaneous strain recorded in the form

\[ \varepsilon_{b.pl} = 2.593 \times \left(1 - \frac{1.040R_{b0}}{R_{bi}} + \frac{0.369R_{b0}^2}{R_{bi}^2}\right) \times \left(1 + \frac{54.560\sigma^3}{R_{bi}^4}\right) \times 10^{-5} \]  

(3)

The elastic deformation is represented by the expression

\[ \varepsilon_{b.el} = \frac{\sigma}{E_0} = \frac{\sigma}{R_{bi}} \times \frac{0.552R_{b0}+R_{bi}}{1130R_{b0}} \]  

(4)

Deformation of elastic aftereffect in the form of

\[ R_{b.aft} = 2.122 \times \left(1 - \frac{1.368R_{b0}}{R_{bi}} + \frac{0.878R_{b0}^2}{R_{bi}^2}\right) \times \left(1 + \frac{15.833\sigma^3}{R_{b0}^4}\right) \times 10^{-5} \]  

(5)

The obtained model adequately describes the source data at relative stress equal to 0.4...0.6. In other ranges the model does not adequately described the data. To clarify the reasons for this were additional statistical research.

Using factor analysis was allocated to two significant factors (strength and volume of the mortar part) source characteristics describing deformation of the concrete and an output parameter.

To build the model of behavior of the concrete and finding the dependencies between the input \((x_1, x_2, \ldots, x_p)\) and output \((y)\) parameters are further used regression analysis \([16-18]\) for each deformation.

To build the model, you can offer countless functions in the form of regression equations describing the experiment. However, the analysis of primary experience in building models (see functions above) showed that the most suitable for our experimental data power-law function. In addition, one of the most fundamental theorems of mathematical analysis, which is called the Weierstrass theorem, the approximation (approximation) of continuous functions by polynomials reads as follows: no matter how difficult it was arranged continuous function, and whatever little \(\delta\) we may choose, there is a polynomial which cannot be distinguished with accuracy to \(\delta\) from this specific continuous function. Of course, the higher the required accuracy of approximation of a function by a polynomial (i.e., the less \(\delta\)), the higher will be the degree of the approximating polynomial (polynomial).

3. Build models in form of polynomials

The structure of the polynomial regression are the most common type of the model \([15]\). The General form of the polynomial represented by the equation:

\[ Y = \beta_0 + \sum_{j=1}^{k} \beta_j x_j + \sum_{u=1}^{k} \sum_{j=1}^{k} \beta_{uj} x_u x_j + \sum_{j=1}^{k} \beta_{jj} x_j^2 + \ldots \]  

(6)

In addition to pairwise interaction effects \(\beta_{uj}\), a polynomial regression may contain triple \(\beta_{ujp}\) and higher order interactions.

To find the regression equation describing adequately obtained our experimental data, analyzed the polynomial of the fifth degree with all possible interactions and combinations. Of all the possible permutations of choosing a model according to the following principles: - the minimum number of members...
of the models most adequately describes the source data and the lowest value of the free term of the equation

Estimates the desired regression coefficients were determined by least-squares method with the use of STATISTIKA in a Windows environment (manufacturer StatSoft Inc., USA) where one of the modules of this system Reg. Analysis.

Baseline data are presented in tables 1, 2, 3.

| Load | Strength (Rb) | The volume of mortar part ,V mortar | Deformation (ε) |
|------|---------------|-----------------------------------|-----------------|
| 1    | 0.299         | 1                                 | 0.59            |
| 2    | 0.299         | 0.765                             | 0.6             |
| 40   | 0.897         | 0.389                             | 0.569           |
|      |               |                                   | 63.51           |

### 4. Results of mathematical processing of initial data

The results of processing the source data selected regression equation best meets the original principles

\[ Y = \beta_0 + \beta_1 \cdot x_1^3 + \beta_2 \cdot x_2^3 + \beta_3 \cdot x_1 \cdot x_2 \cdot x_3^2 \]

where \( \beta_0 = 4.691; \beta_1 = 74.773; \beta_2 = 19.781; \beta_3 = -151.991. \)

The coefficient of determination obtained by the model is equal to \( R^2 = 0.831. \) The value of \( R^2 \) reflects the contribution of regression to the explanation of the phenomenon under study or that the same measure of the usefulness of the parameters \( \beta_j \) (except \( \beta_0 \)).

To test the null hypothesis about the significance of the coefficients of regression-term model used the sampling distribution of \( t \) - statistics (student distribution). Tabular value of \( t \) criterion was determined according to the table of Annex VI [19], for the selected models when the number of degrees of freedom \( \nu \) for the significance level \( \alpha = 0.05 \) got \( t(\nu;\alpha) = 1.96. \) Calculated \( t \) values for the coefficients of the model \( t(\beta_0) = 2.602; \beta_1 = 9.816; \beta_2 = 3.571 \) and \( \beta_3 = -4.46 \) greater than the critical value therefore the null hypothesis \( H_0 \) is considered valid, and the evaluation of the coefficients \( \beta \) recognizes the importance.

To assess the adequacy of the model used \( F \) – Fisher criterion. Tabular value of \( F \) – Fisher criterion for the significance level \( \alpha \) and numbers of degrees of freedom \( \nu_1 \) and \( \nu_2 \), in which the determined variance \( S^2_Y \) and \( S^2_{residual} \) accordingly, for the check of the model is equal to \( F(\nu_1, \nu_2) = 2.86. \) The calculated value of \( F \) – test was equal \( F_{calc} = 59.06. \) As \( F_{calc} > F(\nu_1, \nu_2) \) - the developed model adequately describes the phenomenon being studied.

| Load | Strength (Rb) | The volume of mortar part ,V mortar | Deformation (ε) |
|------|---------------|-----------------------------------|-----------------|
| 1    | 0.299         | 1                                 | 0.59            |
| 2    | 0.299         | 0.765                             | 0.6             |
| 40   | 0.906         | 0.389                             | 0.569           |
|      |               |                                   | 36.316          |

\[ Y = \beta_0 + \beta_1 \cdot x_1^3 \cdot x_2 + \beta_2 \cdot x_2 \cdot x_3 \]
The coefficients of the model | The value of the coefficient | The calculated value of Student's criterion, $t_{\text{est}}$ | The table value of Student's criterion are, $t_{\text{table}}$
---|---|---|---
$\beta_0$ | 14.1329 | 8.717 | 1.96
$\beta_1$ | 59.7722 | 17.354 | 1.96
$\beta_2$ | 30.3164 | -7.398 | 1.96

Since the calculated values of Student test and Fisher more table, all the coefficients are significant and the model is adequate.

**Table 3.** Source data for regression analysis of deformations of elastic aftereffect under short term testing

| Load | Strength (Rb) | The volume of mortar part, $V$ | Mortar | Deformation ($\varepsilon$) |
|---|---|---|---|---|
| $X_1$ | $X_2$ | $X_3$ | $Y$ |
| 1 | 0.299 | 1 | 0.59 | 1.535 |
| 2 | 0.299 | 0.765 | 0.6 | 0.999 |
| 40 | 0.904 | 0.389 | 0.569 | 4.592 |

$$Y = \beta_0 + \beta_1 \cdot x_1 + \beta_2 \cdot x_2$$

Since the calculated values of Student test and Fisher more table, all the coefficients are significant and the model is adequate.

5. Conclusions
1. Building a model based on two factors - stress and prismatic strength of concrete adequately describes the deformation only at the level of the voltage equal to 0.4...0.6

2. The introduction of an additional factor in the model (this need is shown by factor analysis) allowed us to extend the range of validity of the model from 0.3 to 0.9 of the stress level, but it complicated the model.

3. Further increase in the number of factors in the model is impractical because it can improve the convergence of the data obtained using the model with the experimental data by no more than 5...7 %, but much more difficult or impossible to identify these factors in full-scale structures under examination. Additionally, the determination of additional characteristics of the material leads to additional errors.

References
[1] E. Kuzina, and V. Rimshin, Strengthening of Concrete Beams with the Use of Carbon Fiber, *Advances in Intelligent Systems and Computing*, 2019.
[2] V. Rimshin, B. Labudin, V. Morozov, A. Kazarian and V. Kazaryan, Calculation of Shear Stability of Conjugation of the Main Pillars with the Foundation in Wooden Frame Buildings, *Advances in Intelligent Systems and Computing*, 2019.

[3] N. I. Karpenko, V. A. Eryshev, and V. I. Rimshin, The Limiting Values of Moments and Deformations Ratio in Strength Calculations Using Specified Material Diagrams, *IOP Conference Series: Materials Science and Engineering*, 2018.

[4] A. Varlamov, and Yu. M. Krutsilyak, Evaluation of variations of structural and deformation characteristics of concrete during its operation: *Beton i Zhelezobeton* (5), pp. 14-16, 2003.

[5] A. Varlamov, and Yu. M. Krutsilyak, Method of evaluating the stressed-strained state of operated reinforced concrete: *Beton i Zhelezobeton* (6), pp. 18-20, 2005.

[6] A.A. Varlamov, V.I. Rimshin, and S.Y. Tverskoi, The modulus of elasticity in the theory of degradation, *IOP Conference Series: Materials Science and Engineering*, 2018.

[7] E. Kuzina, V. Rimshin, and V. Kurbatov, The Reliability of Building Structures Against Power and Environmental Degradation Effects IOP Conference Series: Materials Science and Engineering, 2018.

[8] A. A. Varlamov, and S. Y. Tverskoi, Experimental selection of young's modulus according the structure of concrete, *IOP Conference Series: Materials Science and Engineering* 451(1), 2018.

[9] A. Varlamov, S.Tverskoi, and V. Gavrilov, Samples of concrete of small sizes, *E3S Web of Conferences* 91, 2019.

[10] A. A. Varlamov, V. I. Rimshin, and S. Y. Tverskoi, The General theory of degradation, *IOP Conference Series: Materials Science and Engineering*, 2018.

[11] A. A.Varlamov, V. I. Rimshin, and S. Y. Tverskoi, The modulus of elasticity in the theory of degradation, *IOP Conference Series: Materials Science and Engineering*, 463(2), 2018.

[12] A. A.Varlamov, E. L. Shapovalov, V. B. Gavrilov, Estimating Durability of Reinforced Concrete, *IOP Conference Series: Materials Science and Engineering*, 262(1), 2017.

[13] A. A. Varlamov, S. Y. Tverskoi, V. B. Gavrilov, Charting standard concrete based on the theory of degradation, *IOP Conference Series: Materials Science and Engineering*, 463(2), 2018.

[14] A. A. Varlamov, V. I. Rimshin, and S. Y. Tverskoi, Security and destruction of technical systems, *IFAC-PapersOnLine*, 1(30), pp. 808-811, 2018.

[15] Behaviors of concrete. The General theory of degradation / Varlamov A. A., Rimshin, V. I. – Moscow: INFRA-M, 436p., 2019.

[16] N. V. Smirnov, and I. V. Dunin-Barkovsky, Course of probability theory and mathematical statistics. Moscow, *Nauka*, - 512 p., 1969.

[17] N. Draper, and G. Smith, Applied regression analysis. Moscow, 391 p., 1973

[18] L. D. Dewatchenko, L. E. Valeeva, and D. Kamardina, Matrix of multidimensional observations in linear regression analysis, *Magnitogorsk*, - 36p., 1988.

[19] L. N. Bol'shev, and N.V. Smirnov, Tables of mathematical statistics, Moscow, *Nauka*, - 464 p., 1965