CP violations in the $K$ and $B$ meson systems in the SUSY models with $(S_3)^3$ flavor symmetry

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Abstract

Rare decays of $B$ and $K$ mesons and CP violations thereof are considered in SUSY models with $S_3^3$ flavor symmetry. SUSY contributions to $\epsilon_K$ and $\epsilon'/\epsilon_K$ can be large, but not to $B^0 - \bar{B}^0$ mixing because of strong constraint from $B \to X_d\gamma$ for $(\delta_{13}^d)_{LR}$. Still large deviations from the SM predictions are possible in the branching ratio for $B \to X_d\gamma$ and direct CP violations therein, even if the KM angle $\gamma$ is in the range preferred by the SM CKM fit. Experimental study of $B \to X_d\gamma$ is strongly recommended as a probe for new physics with nontrivial flavor structure in the $(13)$ mixing.

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By recent observations of $\epsilon'/\epsilon_K$ and CP asymmetry in $B^0(\bar{B}^0) \rightarrow J/\psi K_S$, we now have a solid evidence that CP is indeed violated both in the $K$ and $B$ meson sectors, both in the mixing and in the decay. These observations are in accord with the SM predictions with a single CP violating KM phase. The current world averages of the relevant observables are summarized as follows [1]:

$$\epsilon_K = e^{i\pi/4} (2.280 \pm 0.013) \times 10^{-3},$$
$$\text{Re}(\epsilon'/\epsilon_K) = (19.2 \pm 2.4) \times 10^{-3},$$
$$\sin 2\beta_{\psi K} = (0.79 \pm 0.10).$$

The nonzero $\epsilon_K$ implies the CP violation in the $K^0 - \bar{K}^0$ mixing, and provides a useful constraint on the CKM elements, albeit with some theoretical uncertainties related with nonperturbative QCD effects. The nonvanishing $\epsilon'/\epsilon_K$ excludes the original form of superweak model put forth by L. Wolfenstein [2]. Also relatively large hadronic uncertainties in the kaon physics forbid us from drawing definite conclusions on the validity of the SM or necessity for new physics to understand the observed $\epsilon'/\epsilon_K$ [3]. The current world average of $\sin 2\beta_{\psi K}$ indicates that the KM paradigm for CP violations in the SM is in a good shape, and it starts to provide a meaningful constraint on various new physics scenarios. For example, some SUSY models that were invented to explain the low value of $\sin 2\beta_{\psi K}$ reported by BaBar at the early stage are now being challenged by the updated data. We also include the dilepton charge asymmetry $A_{ll!}$ and the $B_d \rightarrow X_d\gamma$ branching ratio constraint extracted from the recent experimental upper limit on the $B \rightarrow \rho\gamma$ branching ratio [4]

$$B(B \rightarrow \rho\gamma) < 2.3 \times 10^{-6}.$$  

We will take $B(B \rightarrow X_d\gamma) < 1 \times 10^{-5}$ as a representative value.

In the modern language of effective field theory, it would be unnatural to have a CP violation entirely from $|\Delta S| = 2$ four-fermion operators. Any flavor changing ($\Delta F = 1, 2 \text{ with } F = S \text{ or } B$) four-fermion operators can have CP violating phases in principle. This has an important implication for the current construction of the CKM unitarity triangle, since two informations come from the $\Delta F = 2$ (with $F = S \text{ or } B$) processes, namely $\epsilon_K$ and $\Delta M_{B^0}$. In the presence of new physics that could give substantial contributions to the $\Delta F = 2$ mixing, there may be additional contributions to these quantities so that the current CKM global analysis could change. Of course, in the SM, three vastly different constraints from $b \rightarrow ul\nu$, $\epsilon_K$ and $\Delta M_{B^0}$ give a single overlapping region in the $(\rho, \eta)$ plane due to the heaviness of top quark. This is nontrivial at all, and could be regarded as a strong evidence that the CKM picture is really a correct description for the CP violations (at least within the flavor changing sector). However it is still premature to conclude that there are no new flavor or CP violations beyond those residing in the CKM matrix.

In particular, if we consider weak scale supersymmetric (SUSY) models as the new physics beyond the SM, one would expect generically large flavor and CP violations from the soft SUSY breaking terms. These SUSY flavor and SUSY CP problems put serious constraints on realistic model buildings. There are different avenues for SUSY flavor problem : universality, decoupling and alignment using flavor symmetry. The universality assumption is usually taken in the minimal supergravity model, and can be justified in the string models with dilaton dominated SUSY breaking or in the gauge mediation scenarios and some other scenarios. The phenomenology of this class of models are well studied. The general conclusions on the flavor physics are that the deviations from the SM predictions are rather
small and will be hard to be observed considering experimental errors and theoretical uncertainties (mainly from QCD). Another way to solve the SUSY flavor problem is the so-called decoupling solution or effective SUSY models, in which the 1st/2nd generation squarks are very heavy (> $O(10)$ TeV) and almost degenerate, whereas third family squarks and gauginos are relatively light ($\lesssim 1$ TeV). The sleptons can be either heavy or light. In this case, the flavor structures of the sfermion masses and trilinear couplings are not much constrained, and there is enough room for large deviations in the $B$ meson sector from gluino mediated $b \rightarrow s(d)$ transitions. However, it is fair to say that there is no well defined effective SUSY models in which various soft parameters appear with definite relations. This makes it hard to make definite predictions in effective SUSY models, unlike the minimal SUGRA models or GMSB, etc.. Finally, the third way to solve SUSY flavor problem is to invoke the so-called alignment mechanism using some flavor symmetries which could be either abelian or nonabelian. In this approach, flavor structures in soft terms and Yukawa couplings are treated on the equal footing from the outset. Therefore, one can understand both large hierarchies in the fermion masses and their mixings, and suppressions of SUSY induced FCNC amplitudes. This may be aesthetically more attractive than assuming the universality of sfermion mass terms or decoupling solution, since the latter approach cannot shed light on the flavor problem in the Yukawa sectors. There are already many works in the literatures on the alignment, depending on the flavor symmetry groups that are chosen. Generic predictions within this approach are larger effects on flavor physics without leading too much $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$ mixings and $\mu \rightarrow e\gamma$, etc. compared to the solution based on the universality assumption. Since there are no unique flavor models however, one has to rely on theoretical arguments and one’s tastes to choose particular flavor groups and study their phenomenology.

In alignment mechanisms, there are still small residual misalignments between the Yukawa couplings and sfermion mass matrices in the flavor space. When the Yukawa coupling is diagonalized, this misalignment leads to the flavor changing interactions involving a gluino. The gluino mediated $b \rightarrow d$ transitions can be handled in the mass insertion approximation, in which the slight misalignment between fermion and sfermion mass matrices is characterized by parameters $(\delta_{ij}^d)_{AB}$ where $i, j = 1, 2, 3$ are flavor indices and $A, B = L, R$ represent the chirality of their superpartner fermions. In order to study any nontrivial flavor physics from SUSY sector, it is essential to estimate typical sizes of $(\delta_{ij}^d)_{AB}$ with $AB = LL, LR, RR$ and $RL$. The answer will depend on how to solve the SUSY flavor/CP problems.

In this work, we consider a flavor group $S_3^Q \times S_3^U \times S_3^D \equiv S_3^3$, which was considered by Hall and Murayama sometime ago. In SUSY models with the $S_3^3$ flavor group, flavor structures of the Yukawa couplings in the superpotential and sfermion mass matrices in the soft SUSY breaking terms are both controlled by the same flavor symmetry group. The $S_3$ group acts on three elements $(1, 2, 3)$, and has six elements, $S_3 = e, (12), (13), (23), (123), (132)$, with $e$ being the identity element. The group $S_3$ has the following irreducible representations: $1_A$, $2$ and $1_S$. $1_A$ keeps (flips) its sign under even (odd) permutations of $(1, 2, 3)$, and $1_S$ is a trivial representation of $e$. It is assumed that this flavor symmetry is a local symmetry, in order that quantum gravity effects do not spoil it. Furthermore, the flavor symmetry is taken to be a local discrete symmetry. Otherwise, there would appear dangerous family dependent $D$ term contributions to sfermion masses after gauged flavor symmetry is broken to generate Yukawa couplings.

In this model, the 1st/2nd generations are taken to transform as the $2$, whereas the 3rd generation transforms as $1_A$ in order to make discrete flavor gauge symmetry anomaly
It is well known that generic SUSY models will have too large FCNC amplitudes, unless free. Finally one has to introduce three independent $S_3$ groups for $Q,U,D$ superfields in the MSSM in order to have a top quark the only massive particle in the flavor symmetry limit. Both Higgs doublets $H_u, H_d$ transform as $(1_A, 1_A, 1_S)$ under $S_3^Q \times S_3^U \times S_3^D$. Based on these assignments, one can construct the Yukawa and sfermion mass matrices. After diagonalizing the Yukawa matrices, one finds that typical sizes of mass insertion parameters $(\delta_{ij})_{AB}$ are given by (see also Table I)

$$
(\delta_{12}^3)_{LL}, \ (\delta_{13}^3)_{LL} \sim O(\lambda^3) \sim O(10^{-2} - 10^{-3}),
$$

whereas the typical sizes of $(\delta_{ij}^d)_{LR}$ insertions are

$$
(\delta_{12}^d)_{LR} \sim \lambda^5, \quad (\delta_{13}^d)_{LR} \sim O(\lambda^3).
$$

Other parameters are typically much smaller $\lesssim O(\lambda^4)$. Then, $(\delta_{12}^d)_{LL} \sim O(10^{-2} - 10^{-3})$ can generate both $\epsilon_K$, $\text{Re}(\epsilon'/\epsilon_K)$ as discussed by us [9, 10]. On the other hand, for $B^0 - \bar{B}^0$ mixing, both $(\delta_{12}^d)_{LL}$ and $(\delta_{13}^d)_{LR}$ should be considered together, since they are of the same sizes of $O(\lambda^3)$ for large $\tan \beta$ [20]. Each element could have a phase of $\sim O(1)$, leading to new CP violations on top of the KM phase. Therefore, it is important to keep both $(\delta_{13}^d)_{LL}$ and $(\delta_{13}^d)_{LR}$, when we consider the $B^0 - \bar{B}^0$ mixing, $\sin 2\beta \psi_K$, the dilepton CP asymmetry $A_{ll}$ and $B \to X_{d}\gamma$. Since $B^0 - \bar{B}^0$ mixing can be saturated by gluino mediated amplitude only if $(\delta_{13}^d)_{LL} \sim O(10^{-1}) \sim O(\lambda)$ or $(\delta_{13}^d)_{LR} \sim 10^{-2}$, it is not possible to have significant contributions to $B^0 - \bar{B}^0$ mixing from $(\delta_{13}^d)_{LL} \sim O(\lambda^3)$ only in $S_3^3$ model. The LR insertion parameter $(\delta_{13}^d)_{LR} \sim 10^{-2}$ is at the right range to saturate $B^0 - \bar{B}^0$ mixing in the $S_3^3$ model. But it turns out that such a large $(\delta_{13}^d)_{LR}$ may overproduce $B_d \to X_{d}\gamma$, thus being strongly constrained [27]. In short, there could be large SUSY contributions to the $\epsilon_K$ and $\epsilon'/\epsilon_K$, but not in the $B^0 - \bar{B}^0$ mixing, if we consider $B \to X_{d}\gamma$ constraint. Still we find that there could be large deviations in the $B_d \to X_{d}\gamma$ branching ratio and the direct CP violation therein compared to the SM predictions. Thus the detailed experimental study of $B_d \to X_{d}\gamma$ can provide us with the flavor structure of the sfermion mass matrices, thereby a hint for a possible solution to the SUSY flavor problem.

With these comments in mind, let us first consider the CP violation in the kaon sector. It is well known that generic SUSY models will have too large FCNC amplitudes, unless

| (12) | (13) |
|------|------|
| $(LL)$ | $h_t A \lambda^3$ |
| | (0.05) |
| $(RR)$ | $h_2^2 \lambda$ |
| | (0.05) |
| $(LR)$ | $h_2^2 \lambda$ |
| | (0.008) |

TABLE I: Typical sizes of $(\delta_{ij}^d)_{AB}$’s in the $(S_3)^3$ model. The row and the column denote the chiralities $AB$ and the family indices $ij$, respectively. $h$’s are Yukawa couplings, $m_t = h_t(H_u)$ and $m_b = h_b(H_d)$, etc., and $\lambda = 0.22$ is the sine of Cabbibo angle. The numbers in the parentheses are the limits on the quantities (for $\hat{m} = m_\tilde{\chi} = 500$ GeV) derived from various low energy data such as neutral meson mixings, $\epsilon'/\epsilon_K$ and $b \to s\gamma$, etc. [13].
\((\delta^d_{ij})_{AB}\)'s are small enough \([13, 14]\). This is nothing but the SUSY flavor/\(\epsilon_K\) problem \([4]\). A simple twist of this observation is that the SUSY contribution can be saturated by \((\delta^d_{ij})_{AB}\). In particular, the observed \(\epsilon_K\) can be saturated by \((\delta^d_{12})_{LL} \sim O(10^{-3})\). Masiero and Murayama showed that \((\delta^d_{12})_{LR} \sim O(10^{-5})\) is possible in general SUSY models, and can explain the observed \(\epsilon'/\epsilon_K\) \([15]\). Also, flavor nonuniversal trilinear couplings can lead to large \(\epsilon'/\epsilon_K\) in these cases, \(\epsilon_K\) and \(\epsilon'/\epsilon_K\) are generated by two different parameters. On the other hand, the present authors showed that the parameter \((\delta^d_{12})_{LL}\) can also saturate and \(\epsilon'/\epsilon_K\) in the double mass insertion approximation, if \(|\mu| \tan \beta \sim O(10)\) TeV \([8]\). Thus a single SUSY parameter \((\delta^d_{12})_{LL} \sim 10^{-2} - 10^{-3}\) with a phase \(\sim O(1)\) can generate (dominant portions of) both \(\epsilon_K\) and \(\epsilon'/\epsilon_K\). Since there could be large SUSY contributions in the \(S_3^3\) model, the usual CKM phenomenology should be altered and the KM angle \(\gamma\) would not be strongly constrained by \(\epsilon_K\) any longer. The implications of this scenario for other rare kaon decays such as \(K \rightarrow \pi \nu \bar{\nu}\) and \(K_L \rightarrow \pi^0 e^+ e^-\) were studied in detail in Ref. \([10]\).

The most general effective Hamiltonian for \(B^0 - \overline{B^0}\) mixing (\(\Delta B = 2\)) can be written as the following form:

\[
H^B_{\text{eff}}^{\Delta B = 2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i, \tag{4}
\]

where the operators \(Q_i\)'s are defined as

\[
Q_1 = \bar{d}_L^\beta \gamma_\mu b^\beta_L \, d_L^\gamma \mu b^\beta_L \\
Q_2 = \bar{d}_L^\beta b^\beta_R \, d_L^\beta b^\beta_R \\
Q_3 = \bar{d}_L^\beta b^\beta_R \, d_L^\beta b^\beta_R \tag{5}
\]

\(\alpha, \beta\) are color indices, and \(q_{L,R} \equiv (1 \mp \gamma_5)q/2\).

The Wilson coefficients \(C_i\)'s in Eq. (4) are obtained by calculating the \(t-W\) in the SM and \(\tilde{g} - \tilde{q}\) box diagrams in the mass insertion approximations in general SUSY models:

\[
C_1 = -\frac{\alpha_s^2}{216 m^2} \left( 24 x f_6(x) + 66 \tilde{f}_6(x) \right) \quad (\delta^d_{13})_{LL}^2 \\
\tilde{C}_2 = -\frac{\alpha_s^2}{216 m^2} \left( 204 x f_6(x) \right) \quad (\delta^d_{13})_{LR}^2 \\
\tilde{C}_3 = \frac{a_s^2}{216 m^2} \left( 36 x f_6(x) \right) \quad (\delta^d_{13})_{LR}^2 \tag{6}
\]

Other Wilson coefficients are zero in the \(S_3^3\) models ignoring \((\delta^d_{13})_{RR}\) and \((\delta^d_{13})_{RL}\). The SM contribution generate only operator \(Q_1\), and the corresponding Wilson coefficient \(C_1^{SM}\) is given by \([17]\):

\[
C_1^{SM} = \frac{G_F^2}{4 \pi^2} M_W^2 (V_{td}^* V_{tb})^2 S_0(x_t), \tag{7}
\]

where

\[
S_0(x_t) = 4x_t - 11x_t^2 + x_t^3 \quad \frac{3x_t^3 \ln x_t}{2(1-x_t)^3}, \tag{8}
\]
with $x_t \equiv m_t^2/m_W^2$. These Wilson coefficients for SUSY and SM are calculated at $\mu \sim m_t \sim \tilde{m}$ and $m_t$, respectively, and the RG running between these two scales will be ignored. Here $\tilde{m}$ is the common squark mass used in the mass insertion approximation, and $x \equiv m_g^2/\tilde{m}^2$.

The loop functions $f_0(x)$ and $\tilde{f}_0(x)$ are given by

$$f_0(x) = \frac{6(1 + 3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(x - 1)^5},$$

$$\tilde{f}_0(x) = \frac{6x(1 + x)\ln x - x^3 - 9x^2 + 9x + 1}{3(x - 1)^5}. \quad (9)$$

The above $\Delta B = 2$ effective Hamiltonian will contribute to $\Delta m_B$, the dilepton CP asymmetry and time dependent CP asymmetry in the decay $B \to J/\psi K_s$ via the phase of the $B^0 - \bar{B}^0$ mixing. Defining the mixing matrix element by

$$M_{12}(B^0) \equiv \frac{1}{2m_B} \langle B^0|H_{\text{eff}}|B^0\rangle \quad (10)$$

one has $\Delta m_{B_d} = 2|M_{12}(B_d^0)|$ since this quantity is dominated by the short distance contributions, unlike the $\Delta m_K$ for which long distance contributions would be significant. Therefore the data on $\Delta m_{B_d}^{\text{exp}}$ will constrain the modulus of $M_{12}(B_d^0)$.

When we take matrix elements for four quark operators between $B^0$ and $\bar{B}^0$ states, we use the lattice improved calculations for the bag parameters:

$$\langle B_d|Q_1(\mu)|\bar{B}^0\rangle = \frac{2}{3} m_{B_d}^2 f_{B_d}^2 B_1(\mu),$$

$$\langle B_d|\bar{Q}_2(\mu)|\bar{B}^0\rangle = -\frac{5}{12} \left( \frac{m_{B_d}}{m_b(\mu) + m_d(\mu)} \right)^2 m_{B_d}^2 f_{B_d}^2 B_2(\mu),$$

$$\langle B_d|\bar{Q}_3(\mu)|\bar{B}^0\rangle = \frac{1}{12} \left( \frac{m_{B_d}}{m_b(\mu) + m_d(\mu)} \right)^2 m_{B_d}^2 f_{B_d}^2 B_3(\mu), \quad (11)$$

where the $B$ parameters $B_i(\mu)$’s are given by [13]

$$B_1(m_b) = 0.87(4)^{+5}_{-4}, \quad B_2(m_b) = 0.82(3)(4), \quad B_3(m_b) = 1.02(6)(9). \quad (12)$$

Also, we use the following running quark masses in the RI-MOM scheme:

$$m_b(m_b) = 4.6 \text{ GeV}, \quad m_d(m_b) = 5.4 \text{ GeV}. \quad (13)$$

The bottom quark mass is obtained from the $\overline{MS}$ mass $m_b^{\overline{MS}}(m_b^{\overline{MS}}) = 4.23$ GeV. For the $B_d$ meson decay constant, we assume $f_{B_d} = 200 \pm 30$ MeV. We set the common squark mass to $\tilde{m} = 500$ GeV and $x = 1$, and $|V_{cb}| = (40.7 \pm 1.9) \times 10^{-3}$, $|V_{ub}| = (3.61 \pm 0.46) \times 10^{-3}$ with 2σ variation. There are also contributions from $(\delta_{13}^{d})^{\text{ind}}$, which is the induced $LR$ mixing due to the double mass insertion:

$$\frac{(\delta_{13}^{d})^{\text{ind}}}{(\delta_{13}^{d})_{LR}} = (\delta_{13}^{d})_{LL} \times \frac{m_b(A_b - \mu \tan \beta)}{m^2}, \quad (14)$$

which could be important for large $|\mu \tan \beta| \sim O(5-10)$ TeV [3, 10]. We numerically checked that this contribution of $(\delta_{13}^{d})^{\text{ind}}$ to $M_{12}$ is smaller than the SUSY contribution we showed
above, and we do not show the explicit form here. However its effect can be important for the radiative decay \(B \to X_d \gamma\), for which we will keep the induced LR mixing.

Since \(\Delta M_B = (0.472 \pm 0.017) \text{ ps}^{-1}\) is dominated by the short distance physics, it can be reliably calculated in the perturbation theory and is equal to \(2|M_{12}^{\text{full}}|\). The argument of \(M_{12}^{\text{full}}\) (denoted by \(2\beta_{\psi K}\)) is determined by the time dependent CP asymmetry in \(B^0(\overline{B}^0) \to J/\psi K_S\) as long as this decay is dominated by the SM tree amplitude, which is still a good assumption in our model. In the SM, one has \(2\beta_{\psi K} = 2\beta\). Since \(M_{12}^{\text{SUSY}}\) can carry additional phases due to complex parameters \((\delta_{13}^d)_{LL}\) and \((\delta_{13}^d)_{LR}\), these will interfere with \(M_{12}^{\text{SM}}\), and its effect will appear in the net \(B^0 - \overline{B}^0\) mixing. If we fix the KM angle \(\gamma\) to a certain value, the \(\Delta M_B\) will give a relation between the modulus and the phases of \((\delta_{13}^d)_{LL}\) and \((\delta_{13}^d)_{LR}\). We varied their phases from 0 to 2\(\pi\), and their moduli from 0 to 0.06, since their sizes have the order of magnitude \(\sim \lambda^3\) with a fuzzy factor of \(\sim 1/5 - 5\) in the SUSY models with \(S_3^3\) flavor symmetry. The procedure of parameter space searching is as follows. For a particular set of moduli of \((\delta_{13}^d)_{LL}\) and \((\delta_{13}^d)_{LR}\), their phases are varied between 0 and 2\(\pi\), \(\sqrt{\rho^2 + \eta^2}\) within 2\(\sigma\) range, and \(f_{B_d}\) between 200 \pm 30 MeV. If there exists a parameter set that gives \(\Delta M_B\) and \(\sin 2\beta_{\psi K}\) within 2\(\sigma\) from the central values of the measurements, this set of moduli is considered to be consistent with the experiments, and those parameters are used to compute other observables such as \(B(B \to X_d \gamma)\), \(A_\eta\), and \(A_{CP}^{b \to d \gamma}\) discussed below. Throughout the presentation, a parameter set that gives \(B(B \to X_d \gamma) > 1 \times 10^{-5}\) is marked by a light gray (magenta) point, and \(B(B \to X_d \gamma) < 1 \times 10^{-5}\) by dark gray (blue).

In Figs. (a) and (b), we show the allowed region in the \(|(\delta_{13}^d)_{LL}|,|(\delta_{13}^d)_{LR}|\) plane, which is consistent with the measured values of \(\Delta M_B\) and \(\sin 2\beta_{\psi K}\), for (a) \(\gamma = 0^\circ\) and (b) \(\gamma = 50^\circ\), respectively. In both cases, there are allowed regions of \(|(\delta_{13}^d)_{LL}|\) and \(|(\delta_{13}^d)_{LR}|\) with their
magnitudes about $O(\lambda^3)$, as predicted in the $S_3^3$ flavor symmetry. We also note that the observed $\sin 2\beta_{\psi K}$ can be explained entirely by the SUSY effect (namely for $\gamma = 0^\circ$) for $(\delta_{13})_{LR} \sim 10^{-2}$ which can be accommodated in our model. In other words, the real CKM is still consistent with the observed large CP asymmetry $a_{\psi K}$ in our model. [Note that the SM prediction for this asymmetry is $\sin 2\beta_{\psi K} = (0.698 \pm 0.066).$] For this to be true, it is crucial to have both LL and LR mixings in the (13) sector of similar sizes $\sim \lambda$. If we neglected LR mixing, then the size of the (LL) mixing should be $\sim O(\lambda^2 - 10^{-1})$ in order to saturate the measured $\Delta M_B$ and $a_{\psi K}$, which is too large to be accommodated within the $S_3^3$ flavor symmetry group. However, this is not the whole story, since the $(\delta_{d_{13}})^{LR}$ parameter is strongly constrained by $B \to X_d\gamma$, just as the $(\delta_{d_{23}})^{LR}$ parameter is strongly constrained by $B \to X_s\gamma$. Relegating the discussion on this radiative decay to the later part of this work, we simply represent the parameter space with $B(B \to X_d\gamma) < (>) 1 \times 10^{-5}$ by the dark (light) area. Then $\gamma = 0^\circ$ is no longer possible. Namely fully SUSY CP violations in the $B$ meson sector is not possible in the $S_3^3$ model, unlike the $K$ meson sector. For $\gamma = 55^\circ$, there exists some parameter space where SUSY contributions to $B^0 - \bar{B}^0$ mixing is consistent with the current data and also with the SM case.

The new parameters $(\delta_{13})_{LL,LR}$ would affect the dilepton asymmetry $A_l$ through $B^0 - \bar{B}^0$ mixing [19]:

$$A_l \equiv \frac{N(BB) - N(\bar{B}B)}{N(BB) + N(\bar{B}B)} \approx \text{Im}(\Gamma_{12}/M_{12}).$$

Here $M_{12}, \Gamma_{12}$ are the matrix element of $B - \bar{B}$ mixing

$$\langle \bar{B}|H|B \rangle = M_{12} - i\frac{\Gamma_{12}}{2}.$$ 

In the SM, the phases of $M_{12}$ and $\Gamma_{12}$ are approximately equal and

$$\Delta M_{\text{SM}} \approx 2|M_{12}\text{SM}|, \quad \Delta \Gamma_{\text{SM}} \approx 2|\Gamma_{12}\text{SM}|.$$ 

Using $42.4^\circ \leq \gamma \leq 67.2^\circ$ [20] and the other parameters in the same range as is used in the figures, we get

$$-1.98 \times 10^{-3} \leq A_l^{\text{SM}} \leq -0.16 \times 10^{-3},$$

whereas the current world average is

$$A_l^{\text{exp}} \approx (0.2 \pm 1.4) \times 10^{-2}.$$ 

In the presence of SUSY, the phases of $M_{12}$ and $\Gamma_{12}$ may be no longer the same, and potentially larger dilepton asymmetry may be possible. In particular, $M_{12}$ could be affected a lot by SUSY particles, whereas $\Gamma_{12}$ is not (since it would be at higher order) : $M_{12}^{\text{FULL}} = M_{12}^{\text{SM}} + M_{12}^{\text{SUSY}}$ whereas $\Gamma_{12}^{\text{FULL}} \approx \Gamma_{12}^{\text{SM}}$. In this case, the dilepton asymmetry could be approximated as

$$A_l = \text{Im} \left( \frac{\Gamma_{12}^{\text{SM}}}{M_{12}^{\text{SM}} + M_{12}^{\text{SUSY}}} \right).$$ (15)
FIG. 2: \(A_{ll}\) in our model as a function of \(\gamma\). The vertical lines are the data. The black rectangle around \(\gamma \approx 55^\circ\) is the SM prediction. Those parameters which lead to \(B(B \rightarrow X_d\gamma) > 1 \times 10^{-5}\) are represented by light gray (magenta), and \(B(B \rightarrow X_d\gamma) < 1 \times 10^{-5}\) by dark gray (blue).

In the SM, \(\Gamma_{12}\) is given by

\[
\Gamma_{12}^{\text{SM}} = (-1) \frac{G_F m_b^2 M_{B_d} B_{B_d} f_{B_d}^2}{8\pi} \left[ v_t^2 + \frac{8}{3} v_c v_t \left( z_c + \frac{1}{4} z_c^2 - \frac{1}{2} z_c^3 \right) + \right.
\]

\[
\left. v_c^2 \left( \sqrt{1 - 4z_c \left(1 - \frac{2}{3} z_c\right)} + \frac{8}{3} z_c + \frac{2}{3} z_c^2 - \frac{4}{3} z_c^3 - 1 \right) \right],
\]

(16)

where \(v_i \equiv V_{ib} V_{id}^*\) and \(z_c \equiv m_c^2/m_b^2\). The minus sign comes from the convention of CP operation on neutral \(B\) mesons: \(\text{CP}|B_0^d\rangle = -|B_0^d\rangle\). In Fig. 2, we show the dilepton asymmetry \(A_{ll}\) as functions of \(\gamma\), after scanning over the allowed region of complex parameters, \((\delta_{13}^d)_{LL}\) and \((\delta_{13}^d)_{LR}\). The SM case is represented by the filled black rectangle. The region leading to too large \(B(B \rightarrow X_d\gamma)\) is denoted by the light gray region, and the dark gray region is with \(B(B \rightarrow X_d\gamma) < 1 \times 10^{-5}\). Note that the KM angle \(\gamma\) cannot be arbitrary, but should be somewhere between \(\sim 20^\circ\) and \(80^\circ\) because of the \(B(B \rightarrow X_d\gamma)\) constraint. The resulting \(A_{ll}\) is essentially the same as the SM predictions. Thus, one cannot expect a large deviation in \(A_{ll}\) from the SM prediction in SUSY models with \(S_3^3\) flavor symmetry group.

Now let us consider the branching ratio for a radiative decay \(B \rightarrow X_d\gamma\) and direct CP violation therein. The relevant operators for this decay are the current-current \(b \rightarrow du\bar{u}\) and (chromo)magnetic dipole operators:

\[
\mathcal{H}_{\text{eff}}(b \rightarrow d\gamma(\pm g)) = -\frac{4G_F}{\sqrt{2}} V_{td}^* V_{tb} \sum_{i=1,2,7,8} C_i(\mu_b) O_{ic}(\mu_b) + \frac{4G_F}{\sqrt{2}} V_{ud}^* V_{ub} \sum_{i=1,2} C_i(\mu_b) \left[ O_{1u}(\mu_b) - O_{1c}(\mu_b) \right]
\]

(17)

with

\[
O_{1c} = \bar{d}_L \gamma^\mu c_L \bar{c}_L \gamma_\mu b_L, \quad O_{1u} = \bar{d}_L \gamma^\mu u_L \bar{u}_L \gamma_\mu b_L, \quad O_{2c} = \bar{d}_L \gamma^\mu c_L \bar{c}_L \gamma_\mu b_L, \quad O_{2u} = \bar{d}_L \gamma^\mu u_L \bar{u}_L \gamma_\mu b_L, \quad O_{7\gamma} = \frac{e}{16\pi^2} m_b \bar{d}_L \sigma^{\mu\nu} F_{\mu\nu} b_R, \quad O_{8g} = \frac{g_s}{16\pi^2} m_b \bar{d}_L \sigma^{\mu\nu t^a} G_{\mu\nu}^a b_R.
\]

(18)
Here the renormalization scale $\mu_b$ is of the order of $m_b$, and we have used the unitarity of the CKM matrix elements

$$V_{cd}^*V_{cb} = -(V_{ud}^*V_{ub} + V_{td}^*V_{tb}),$$

which should be valid even in the presence of SUSY flavor violations.

In the SM, all the three up-type quarks are relevant to this decay, since all the relevant CKM factors are of the same order of magnitude. The strong phases are provided by the imaginary parts of one loop diagrams at the $O(\alpha_s)$ order by the usual unitarity argument. The resulting branching ratio for this decay is $7.5 \times 10^{-6} - 1.3 \times 10^{-5}$ in the SM depending on the CKM elements, and the direct CP asymmetry is about $-18\% \sim -8\%$ in the SM.

The CP averaged branching ratio for $B \to X_d \gamma$ in the leading log approximation is given by

$$B(B \to X_d \gamma) = \left| \frac{V_{td}^*V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha}{\pi f(z)} |C_{7}(m_b)|^2. \quad (19)$$

where $f(z) = 1 - 8z + 8z^3 - z^4 - 12z^2 \ln z$ is the phase space factor for the $b \to c$ semileptonic decays and $\alpha^{-1} = 137.036$. Neglecting the RG running between heavy SUSY particles and top quark mass scale, we get the following relations:

$$C_{7}(m_b) \approx -0.31 + 0.67 C_{7}^{\text{new}}(m_W) + 0.09 C_{8}^{\text{new}}(m_W),$$

$$C_{8}(m_b) \approx -0.15 + 0.70 C_{8}^{\text{new}}(m_W). \quad (20)$$

The new physics contributions to $C_{7}$ are negligible so that we use $C_{2}(m_b) = C_{2}^{\text{SM}}(m_b) \approx 1.11$. The direct CP asymmetry can be written as

$$A_{CP}^{b\to d\gamma} (\text{in}\%) = \frac{1}{|C_{7}|^2} \left[ 10.57 \text{ Im} (C_{2}C_{7}^*) - 9.40 \text{ Im} ((1 + \epsilon_{d})C_{2}C_{7}^*) \right. \left. - 9.51 \text{ Im} (C_{8}C_{8}^*) + 0.12 \text{ Im} ((1 + \epsilon_{d})C_{2}C_{8}^*) \right] \quad (21)$$

where

$$\epsilon_{d} \equiv \frac{V_{ud}^*V_{ub}}{V_{td}^*V_{tb}} \approx \frac{(\rho - i\eta)}{(1 - \rho + i\eta)}.$$

A remark is in order for the above CP asymmetry in $B \to X_d \gamma$. Unlike the $B \to X_s \gamma$ case for which the $|C_{7}\gamma|$ is constrained by the observed $B \to X_s \gamma$ branching ratio, the $B \to X_d \gamma$ decay has not been observed yet, and its branching ratio can be vanishingly small in the presence of new physics. In that case, $|C_{7}\gamma| \approx 0$ so that the denominator of $A_{CP}^{b\to d\gamma}$ becomes zero and the CP asymmetry blows up. This could be partly cured by replacing the denominator $|C_{7}|^2$ by $K_{\text{NLO}}(\delta)$ defined in Ref. [24]:

$$K_{\text{NLO}}(\delta)(\text{in}\%) = 0.11|C_{2}|^2 + 68.13|C_{7}|^2 + 0.53|C_{8}|^2 - 16.55 \text{Re}(C_{2}C_{7}^*)$$

$$- 0.01 \text{Re}(C_{2}C_{8}^*) + 8.85 \text{Re}(C_{7}C_{8}^*) + 3.86 \text{Re}(C_{7}^{(1)}C_{7}^*) \quad (22)$$

for the photon energy cutoff factor $\delta = 0.3$. Here $C_{7}^{(1)}$ is the next-to-leading order contribution to $C_{7}(m_b)$:

$$C_{7}^{(1)} \approx 0.48 - 2.29 C_{7}^{\text{new}}(m_W) - 0.12 C_{8}^{\text{new}}(m_W). \quad (23)$$
FIG. 3: (a) Branching ratio and (b) direct CP asymmetry of $B \to X_d \gamma$ in our model as functions of the KM angle $\gamma$. The SUSY contributions are added to the SM contributions, and the induced LR mixing is ignored. The black rectangle around $\gamma \simeq 55^\circ$ is the SM prediction. Those parameters which lead to $B(B \to X_d \gamma) > 1 \times 10^{-4}$ are represented by light gray (magenta), and $B(B \to X_d \gamma) < 1 \times 10^{-4}$ by dark gray (blue).

In our model, the Wilson coefficients $C_{7\gamma}$ and $C_{8g}$ are modified in the double mass insertion approximation as follows [10, 13, 25]:

$$C_{7\gamma}^{\text{SUSY}}(m_W) = \frac{8\pi Q_b \alpha_s}{3\sqrt{2} G_F \tilde{m}^2 V_{ts} V_{tb}} \left[ (\delta_{13}^{d})_{LL} M_4(x) - (\delta_{13}^{d})_{LR} \left( \frac{\tilde{m} \sqrt{x}}{m_b} \right) M_2(x) \right] ,$$  \hspace{1cm} (24)

$$C_{8g}^{\text{SUSY}}(m_W) = \frac{2\pi \alpha_s}{\sqrt{2} G_F \tilde{m}^2 V_{td} V_{tb}} \left[ (\delta_{13}^{d})_{LL} \left( \frac{3}{2} M_3(x) - \frac{1}{6} M_4(x) \right) + (\delta_{13}^{d})_{LR} \left( \frac{\tilde{m} \sqrt{x}}{m_b} \right) \frac{1}{6} (4B_1(x) - 9x^{-1}B_2(x)) - (\delta_{13}^{d})_{\text{ind}} \left( \frac{\tilde{m} \sqrt{x}}{m_b} \right) \left( \frac{3}{2} M_1(x) - \frac{1}{6} M_2(x) \right) \right] .$$  \hspace{1cm} (25)

Note that both $(\delta_{13}^{d})_{LR}$ and $(\delta_{13}^{d})_{\text{ind}}$ are enhanced by $m_\tilde{g}/m_b$ due to the chirality flip from the internal gluino propagator in the loop. Explicit expressions for the loop functions $B_i$’s and $M_i$’s can be found in Ref. [10]. In Figs. 3(a) and (b), we show the branching ratio and the direct asymmetry of $B \to X_d \gamma$ as functions of $\gamma$ when the induced LR mixing can be neglected. In Fig. 3(a) and (b), we show the same plots when the induced LR mixing is important by fixing $\mu \tan \beta = +5$ TeV. First of all, the branching ratio for $B \to X_d \gamma$ comes out too large for $(\delta_{13}^{d})_{LR} \sim (\text{a few}) \times 10^{-2}$, compared to the well measured $B \to X_s \gamma$: $B(B \to X_s \gamma) = (3.21 \pm 0.43_{\text{stat}} \pm 0.27_{(s_{90})}^{+0.18}_{-0.10(\text{th})}) \times 10^{-4}$. Although there is no reported upper limit on $B \to X_d \gamma$ at the moment, it would be reasonable to assume $B(B \to X_d \gamma) \lesssim 1 \times 10^{-5}$, as discussed in the introduction. Imposing this condition, substantial parameter space in
FIG. 4: (a) Branching ratio and (b) direct CP asymmetry of $B \to X_d \gamma$ in our model as functions of the KM angle $\gamma$. The SUSY contributions are added to the SM contributions, and the induced LR mixing is included by setting $\mu \tan \beta = +5$ TeV. The black rectangle around $\gamma \simeq 55^\circ$ is the SM prediction. Those parameters which lead to $B(B \to X_d \gamma) > 1 \times 10^{-5}$ are represented by light gray (magenta), and $B(B \to X_d \gamma) < 1 \times 10^{-5}$ by dark gray (blue).

the $(LL, LR)$ plane is excluded, and the KM angle $\gamma$ cannot be too much different from $\gamma \sim \gamma_{SM} \sim 55^\circ$. This would imply that the kaon physics in this model would not be too different from the SM case. Still, there is some room that $B(B \to X_d \gamma)$ can differ from the SM case. Especially, the direct CP asymmetry in $B \to X_d \gamma$ can be significantly different from the SM prediction. Even its sign can change from the SM case. Therefore the branching ratio of $B \to X_d \gamma$ and direct CP asymmetry therein could be a sensitive probe for gluino mediated $b \to d \gamma$ transitions in $B^0 - \bar{B}^0$ mixing and $b \to d \gamma$, and will provide an important information for the flavor structure of down squark mass matrices.

In conclusion, we presented a simple SUSY model where SUSY flavor problem is solved by $S^3$ flavor symmetry group, in which the mass insertion parameters have typical sizes shown in Table 1. In the kaon sector $(\delta_{13}^d)_{LL}$ insertion is dominant over other parameters, and it is possible to have large SUSY contributions to $\epsilon_K$ and $\epsilon'/\epsilon_K$ as discussed in Refs. [8, 11] with unconstrained KM angle $\gamma$. However, this picture cannot be retained when CP violations in the $B$ meson sector is considered. Since $(\delta_{13}^d)_{LL} \sim (\delta_{13}^d)_{LR} \sim O(\lambda^3)$, only LR mixing can be important for $B^0 - \bar{B}^0$ mixing. However the LR mixing is strongly constrained by $B \to X_d \gamma$ which has not been observed yet. By imposing a reasonable limit on $B(B \to X_d \gamma) < 1 \times 10^{-5}$, we find that $B^0 - \bar{B}^0$ mixing is dominated by the SM contributions, and not by SUSY contributions. The KM angle $\gamma$ must lie between $\sim 20^\circ$ and $\sim 80^\circ$. Still a small amount of $(\delta_{13}^d)_{LR}$ can give a significant contribution to $B \to X_d \gamma$ and direct CP asymmetry therein. One can still expect large deviations in these observables in SUSY models with flavor $S^3$ symmetry. Since the KM angle $\gamma$ is not free, the resulting predictions for $K \to \pi \nu \bar{\nu}$ and $K_L \to \pi^0 e^+e^-$ will be constrained as well within the predictions given in Ref. [10]. If the low energy SUSY is relevant to the nature, some mechanism is needed to solve the SUSY flavor/CP problems. If the approximate alignment based on $S^3$ is such a solution, one expects generically large modifications in $K^0 - \bar{K}^0$, $K \to \pi \nu \bar{\nu}$, $K_L \to \pi^0 e^+e^-$ and $B \to X_d \gamma$, but
not in $B^0 - \bar{B}^0$ mixing. Detailed experimental search for these decays at $K$ and $B$ factories will shed light on the flavor physics in SUSY models.

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Another motivation for choosing the $S^3_3$ flavor group was the claim that $(\delta_{13})_{LL} \sim O(\lambda^2)$, which could lead to large SUSY CP violations in the $B$ meson sector. If their claim were true, SUSY contribution from $(\delta_{13})_{LL}$ could saturate the $B^0 - \bar{B}^0$ mixing and CP asymmetry in $B^0 \rightarrow J/\psi K_S$. Then CP violations in both $K$ and $B$ meson sectors could be dominated by SUSY contributions. Unfortunately, we could not confirm their claim. Instead, we found $(\delta_{13})_{LL} \sim O(\lambda^3)$, which is too small to saturate CP violations in the $B$ meson sector. Also it has a similar size as the $LR$ insertion, which is strongly constrained by $B \rightarrow X_d \gamma$ as discussed in the following.

This point was overlooked in the recent analysis by Bećirević et al. [11] (see Ref. [12] for detailed model independent analysis.)