Fault detection in uncertain systems via proportional integral observer

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ABSTRACT
This article focuses on the design of a proportional integral observer (PIO) for fault detection in uncertain systems. For multi-output linear systems with actuator fault and disturbances, an approach using two PIOs is designed to estimate both the system states and disturbances simultaneously when the actuator fault is free. Then a residual generator based on the output function is used to identify whether the fault is occurring. For a class of nonlinear systems with constant disturbances, a full-order PIO is designed to estimate the disturbances. Similarly, the residual generator is used to identify whether the fault is occurring. Numerical examples are given to show the efficiency of the proposed approach.

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Proportional integral observer; fault detection; uncertain systems; disturbance estimation

1. Introduction

Observers play an important role in control because many control algorithms, such as proportional integral differential controller or sliding controller, require the accurate estimation of system states to realize the close loop control. To realize accurate tracking of system states, integral terms are usually added to the control design (Anderson & Moore, 1990). Wojciechowski (1978) first proposed the PIO for Single Input Single Output (SISO) linear time-invariant systems. Then Shafai and Beale (1989) proposed PIO to make the designed controller less sensitive to parameter variation of the system. This type of observer has been studied by a number of researchers (Chen et al., 2021; Li et al., 2018; Othman et al., 2018; Piltan & Kim, 2019; Tang, 2022; Wang et al., 2019). A comparison of classical PO to PIO was given via a simulation example (Busawon & Kabore, 2000). It was shown that the PIO allowed to completely decouple the modelling uncertainties when modelling errors and sensor noise were presented simultaneously. An approach based on complete parametric eigenstructure assignment for PIOs of multi-variable linear systems was proposed by Duan et al. (2001). An observer called Proportional Integral Adaptive Observer (PIAO) was introduced in Shafai et al. (2002), by adding a control law to the PIO, which had the ability to estimate constant input disturbances, states and parameters of the system. The conclusion was brought to a class of time-varying systems. A robust fractional-order PIO for nonlinear fractional-order chaotic systems was proposed by Ibrahimia et al. (2019), which could achieve a well estimation of modelling errors and external disturbance. To attenuate the influence on filtering performance caused by the event-triggering mechanism, an algorithm with time-update and event-update recursions was proposed by Zhong et al. (2022).

At present, there are many achievements in fault detection. A new active fault diagnosis (AFD) framework was proposed by Z. Zhang and He (2022), which consisted of the geometric-approach-based fault diagnosis observer and the signal processing method. To tackle the state estimation problem for delayed impulsive neural networks, a delay-range-dependent approach was proposed by Luo, Wang, Sheng et al. (2021). An overview of state-of-the-art methods for input design for active fault diagnosis and a discussion of the primary considerations in the formulation and solution of the input-design problem were introduced in Heirung and Mesbah (2019). An $H_{\infty}$ estimation approach was proposed by Luo, Wang, Chen et al. (2021) for an array of coupled stochastic complex networks with intermittent nonlinearity switching. A brief survey of linear and nonlinear PIO design and their applications was given in Bakhshande and Sffker (2015): (1) State estimation for better control; (2) Identification and estimation of the system nonlinearities and model uncertainties; (3) Improving the stability margin in Loop Transfer Recovery (LTR) design; (4) Elimination or attenuation of disturbance in the system; and (5) Fault Detection and Isolation (FDI). Especially in FDI, PIO could be designed to achieve disturbance attenuation and fault detection (Bakhshande & Sffker, 2017;
Considering the following multi-output linear system

\[ \begin{align*}
\dot{x} &= Ax + Bu + Dd + Mf \\
y &= Cx + Ev
\end{align*} \tag{1} \]

where \( x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^p, d, f \) and \( v \) denote the state vector, inputs, outputs, independent disturbance, actuator fault and sensor measurement noise, respectively. The matrices \( A, B, D, M, C \) and \( E \) are of appropriate dimensions satisfying the following assumptions

**Assumption 2.1:** The matrix \( C \) is of full-row rank.

**Assumption 2.2:**

\[
\text{rank}\left[ \begin{array}{c}
sI - A \\ C \\
C A \\
\vdots \\
C A^{N-1}
\end{array} \right] = n \tag{2}
\]

The sufficient and necessary conditions for observability are (Kalman observability condition):

\[
\text{rank}\left[ \begin{array}{c}
C \\ CA \\
\vdots \\
CA^{N-1}
\end{array} \right] = n \tag{3}
\]

According to linear system theory, the condition (3) is equivalent to Assumption 2.2.

**Assumption 2.3:** The disturbance \( d \) and sensor measurement noise \( v \) are slowly varying signals and satisfying

\[
\dot{d} = 0 \quad \text{and} \quad \dot{v} = 0 \tag{4}
\]

For system (1), a full-order PIO is in the following form:

\[
\begin{align*}
\dot{x} &= Ax + Bu + Dd + K_p(y - \hat{y}) \\
\dot{y} &= Cx + Ev \\
\dot{d} &= K_i d(y - \hat{y}) \\
\dot{\hat{y}} &= K_v(y - \hat{y})
\end{align*} \tag{5}
\]

where \( \hat{x} \in \mathbb{R}^n \) is the estimated state vector, \( \hat{y} \in \mathbb{R}^p \) is the estimated output. The matrices \( K_i \) and \( K_v \) are the integral gain matrix, and the matrix \( K_p \) is the proportional gain matrix.

**Theorem 2.1:** If \( f = 0 \), Assumptions 2.1, 2.2 and 2.3 hold, for arbitrary initial condition \( x(0) \) and \( \dot{x}(0) \), the following relations hold:

\[
\lim_{t \to \infty} e_d(t) = 0, \quad \lim_{t \to \infty} e_v(t) = 0, \quad \lim_{t \to \infty} e(t) = 0 \tag{6}
\]

where \( e_d(t) = d(t) - \dot{d}(t), e_v(t) = v(t) - \dot{v}(t) \) and \( e(t) = x(t) - \hat{x}(t) \). Then the system (5) is said to be a PIO for the system (1).

**Proof:** The derivative of \( e(t) \) is

\[
\begin{align*}
\dot{e} &= \dot{x} - \dot{\hat{x}} \\
&= Ax + Bu + Dd + Mf - (A\dot{x} + Bu + D\dot{d} + K_p(y - \hat{y})) \\
&= (A - K_p C)(x - \hat{x}) + D(d - \dot{d}) - K_p E(v - \dot{v}) + Mf \\
&= (A - K_p C)e(t) + D e_d(t) - K_p E e_v(t) + Mf. \tag{7}
\end{align*}
\]

Since Assumption 2.3 holds, then

\[
\begin{align*}
\dot{e}_d &= d - \dot{d} = -d = -K_i d(y - \hat{y}) = -K_i Ce - K_i E e_v \tag{8}
\end{align*}
\]

and

\[
\dot{e}_v = v - \dot{v} = -\dot{v} = -K_v(y - \hat{y}) = -K_v Ce - K_v E e_v. \tag{9}
\]

From (7), (8) and (9), one can obtain that

\[
\begin{align*}
\begin{bmatrix}
\dot{e} \\
\dot{e}_d \\
\dot{e}_v
\end{bmatrix} &= \begin{bmatrix}
(A - K_p C)e + D e_d - K_p E e_v + Mf \\
-K_i Ce - K_i E e_v \\
-K_v Ce - K_v E e_v
\end{bmatrix} \\
&= \begin{bmatrix}
(A - K_p C) D - K_p E \\
-K_i C 0 - K_i E \\
-K_v C 0 - K_v E
\end{bmatrix} \begin{bmatrix}
e \\
e_d \\
e_v
\end{bmatrix} + \begin{bmatrix}
Mf \\
0 \\
0
\end{bmatrix}. \tag{10}
\end{align*}
\]

Considering the candidate Lyapunov function

\[
V = \frac{1}{2} e^2 + \frac{1}{2} e_d^2 + \frac{1}{2} e_v^2,
\]
the derivative of the candidate Lyapunov function is
\[
\dot{V} = e \dot{e} + e \dot{e}_d + e \dot{e}_v = \begin{bmatrix} e & \dot{e}_d & e_v \end{bmatrix}
\begin{bmatrix}
  A - K_p C & D & -K_p E \\
- K_id C & 0 & - K_id E \\
- K_i\nu C & 0 & - K_i\nu E
\end{bmatrix}
\begin{bmatrix} e \\ \dot{e}_d \\ e_v \end{bmatrix}
\]
\[
+ [e \ e_d \ e_v] \begin{bmatrix} M_f \\ 0 \\ 0 \end{bmatrix}.
\]
If \( f = 0 \), one can obtain that
\[
\dot{V} = [e \ e_d \ e_v] \begin{bmatrix}
  A - K_p C & D & -K_p E \\
- K_id C & 0 & - K_id E \\
- K_i\nu C & 0 & - K_i\nu E
\end{bmatrix}
\begin{bmatrix} e \\ \dot{e}_d \\ e_v \end{bmatrix}.
\]

According to the asymptotically stability Theorem 4.1 in Khalil (2011), if the eigenvalue of matrix
\[
\begin{bmatrix}
  A - K_p C & D & -K_p E \\
- K_id C & 0 & - K_id E \\
- K_i\nu C & 0 & - K_i\nu E
\end{bmatrix}
\]
has negative real parts, then the system (10) is asymptotically stable. The parameters of the feedback gain matrices \( K_p, K_id \) and \( K_i\nu \) can be selected by pole assignment technology. Then the PIO (5) can decouple the effect of constant plant modelling disturbance and sensor measurement noise from the state estimation when \( f = 0 \). The proof is completed.

\[
\lim_{t \to \infty} e_{d}(t) = 0, \quad \lim_{t \to \infty} e(t) = 0 \tag{18}
\]

3. Nonlinear system case

Considering the following nonlinear system
\[
\begin{align*}
\dot{x} &= Ax + Bf(x) \\
y &= Cx
\end{align*} \tag{14}
\]
where \( C \) is a unit matrix, and \( y \) stands for the measured values of the sensor. Since \( C \) is a unit matrix, it means that all states of the system (14) are measurable. Therefore, the output function can be reconstructed as
\[
y^c = f(x) + Kx \tag{15}
\]
where \( K \) is the feedback gain matrix, which will be designed later. Assuming that disturbance and actuator fault affect the system (14) linearly
\[
\begin{align*}
\dot{x} &= A\hat{x} + Bf(\hat{x}) + Dd + Mf \\
y^c &= f(x) + Kx.
\end{align*} \tag{16}
\]

A full-order PIO is in the following form
\[
\begin{align*}
\dot{x} &= A\hat{x} + Bf(\hat{x}) + D\hat{d} + B(y^c - \hat{y}^c) \\
\dot{\hat{d}} &= K_i(y - \hat{y}) \\
\hat{y}^c &= f(\hat{x}) + K\hat{x}.
\end{align*} \tag{17}
\]

Theorem 3.1: If \( f = 0 \), Assumption 2.3 and formula (3) hold, for arbitrary initial condition \( x(0) \) and \( \dot{x}(0) \), the following relations hold:
\[
\lim_{t \to \infty} e_{d}(t) = 0, \quad \lim_{t \to \infty} e(t) = 0 \tag{18}
\]
where \( e_{d}(t) = d(t) - \hat{d}(t) \) and \( e(t) = x(t) - \hat{x}(t) \). Then system (17) is said to be a PIO for the system (16).
Proof: The derivative of $e(t)$ is
\[
\dot{e} = \dot{x} - \dot{\hat{x}} = A\dot{x} + Bf(x) + Dd + Mf - (A\dot{\hat{x}} + Bf(\hat{x}) + D\dot{\hat{d}} + M\dot{\hat{f}})
\]
\[
= A(x - \hat{x}) + B(f(x) - f(\hat{x})) + D(d - \hat{d}) + Mf - B(f(x) + Kx - f(\hat{x}) - K\hat{x})
\]
\[
= (A - BK)(x - \hat{x}) + D(d - \hat{d}) + Mf
\]
\[
= (A - BK)e + De_d + Mf. \tag{19}
\]
Since Assumption 2.3 holds, then
\[
\dot{e}_d = d - \dot{\hat{d}} = -K(y - \hat{y}) = -K_ke. \tag{20}
\]
Combining formula (19) and (20), one can obtain that
\[
\begin{bmatrix}
\dot{e} \\
\dot{e}_d
\end{bmatrix}
= \begin{bmatrix}
(A - BK)e + De_d + Mf \\
-K_Ke_e
\end{bmatrix}
\]
\[
= \begin{bmatrix}
(A - BK) \\
-K_K
\end{bmatrix}
\begin{bmatrix}
e \\
e_d
\end{bmatrix}
+ \begin{bmatrix}
e \\
e_d
\end{bmatrix}
+ \begin{bmatrix}
Mf
\end{bmatrix}. \tag{21}
\]
Considering the candidate Lyapunov function
\[
V = \frac{1}{2}e^2 + \frac{1}{2}e_d^2,
\]
the derivative of the candidate Lyapunov function is
\[
\dot{V} = [e \\
e_d]
\begin{bmatrix}
(A - BK) \\
-K_K
\end{bmatrix}
\begin{bmatrix}
e \\
e_d
\end{bmatrix}.
\]
According to the asymptotically stability Theorem (Khalil, 2011), if the eigenvalue of matrix
\[
\begin{bmatrix}
(A - BK) \\
-K_K
\end{bmatrix}
\]
has negative real parts, then the system (21) is asymptotically stable. The parameters of the feedback gain matrices $K$ and $K_e$ can be selected by pole assignment technology. The proof is completed. \hfill \blacksquare

The residual is
\[
r = y - \hat{y} = Cx - C\hat{x} = Ce. \tag{22}
\]
According to formula (19), the following frequency domain relationship can be obtained
\[
e = (sI - A - BK)^{-1}De_d + (sI - A - BK)^{-1}Mf,
\]
then
\[
r = C(sI - A - BK)^{-1}De_d + C(sI - A - BK)^{-1}Mf. \tag{23}
\]
From formula (23), one can find out that when fault $f$ does not exist and the error system (19) is asymptotically stable, then $e_d \to 0$ and $r \to 0$. Therefore, fault detection can be carried out through $r$.

4. Simulation

4.1. Linear system case

To verify the efficiency of the proposed fault detection scheme, consider the following example:
\[
\begin{align*}
x_1 &= -10x_1 + 10x_2 + u_1 + 6f \\
x_2 &= -9x_1 - 11x_2 - u_1 + 2d + 4f \\
x_3 &= -2x_3 + 6f \\
y_1 &= x_1 + 3v \\
y_2 &= x_2 \\
y_3 &= x_3
\end{align*}
\tag{24}
\]
where $x = [x_1 \quad x_2 \quad x_3]^T$. System (24) is in the form of (1) with
\[
A = \begin{bmatrix}
-10 & 10 & 0 \\
9 & -11 & 0 \\
0 & 0 & -2
\end{bmatrix}, \quad B = \begin{bmatrix}
1 \\
-1 \\
0
\end{bmatrix},
\]
\[
C = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad D = \begin{bmatrix}
0 \\
2 \\
0
\end{bmatrix}, \quad E = \begin{bmatrix}
3 \\
0 \\
0
\end{bmatrix}, \quad M = \begin{bmatrix}
6 \\
6 \\
6
\end{bmatrix}. \tag{25}
\]
Assuming $d = 0$, $v = 0$ and $f = 0$, then system (24) can be written as:
\[
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= \begin{bmatrix}
-10 & 10 & 0 \\
9 & -11 & 0 \\
0 & 0 & -2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
+ \begin{bmatrix}
1 \\
-1 \\
0
\end{bmatrix}u_1
\]
\[
\begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}. \tag{26}
\]
Assumptions 2.1 and 2.2 are met. At present, there are many fault-tolerant control methods, such as SFDCC (simultaneous fault detection and consensus control) (Davood et al., 2016), COD (collaborative design) framework (Jia et al., 2022), etc.

This article is mainly to verify the feasibility of PIO. Then the control law is designed by using pole assignment technology. Take the control law $u$ as a state feedback control law
\[
u_1 = K_U \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}. \tag{27}
\]
The feedback gain matrix $K_U$ is designed as $K_U = [9 \quad -10 \quad 0]$. One can obtain that
\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix}
= \begin{bmatrix}
-1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
\]
\[
\begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}. \tag{28}
\]
The eigenvalue of the system (28) is $s = \text{diag}[-1 - 1 - 2]$, which satisfies the requirement of global asymptotic stability. A PIO is designed as

$$
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
-10 & 10 & 0 \\
9 & -11 & 0 \\
0 & 0 & -2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} +
\begin{bmatrix}
-1 \\
0 \\
0
\end{bmatrix} u_1
$$

+ $2 \hat{d} + K_p \begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix} - \begin{bmatrix}
\hat{y}_1 \\
\hat{y}_2 \\
\hat{y}_3
\end{bmatrix}$

$$
\begin{bmatrix}
\dot{\hat{y}}_1 \\
\dot{\hat{y}}_2 \\
\dot{\hat{y}}_3
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\hat{x}_1 \\
\hat{x}_2 \\
\hat{x}_3
\end{bmatrix} +
\begin{bmatrix}
3 \\
0 \\
0
\end{bmatrix} \dot{\nu}
$$

\begin{align*}
\hat{d} &= K_{id} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} - \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \end{bmatrix}, \\
\dot{\nu} &= K_{iv} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} - \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \end{bmatrix}.
\end{align*} \tag{29}

By selecting $K_{id}, K_{iv}$ and $K_p$ as

$$
K_{id} = \begin{bmatrix} 4 \\ 5 \\ -6 \end{bmatrix}, \quad K_{iv} = \begin{bmatrix} 3 \\ -8 \\ -1.5 \end{bmatrix},
$$

$$
K_p = \begin{bmatrix} 1.3 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & -0.5 \end{bmatrix}. \tag{30}
$$

So that all the eigenvalues of the error system (10) have negative real parts. The error system (10) is asymptotically stable in the case of $f = 0$. The residual generator is designed as

$$
r = y - \hat{y} = Cx - \hat{C} \hat{x} = Ce = C \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}. \tag{31}
$$

where $e_1 = x_1 - \hat{x}_1, e_2 = x_2 - \hat{x}_2, e_3 = x_3 - \hat{x}_3$. The disturbance $d$ and the sensor measurement noise $\nu$ are given as $d = 6$ and $\nu = 4$. The actuator fault is given as $f = 0.5 \sin(5t)$. The disturbance $d$ is added to the system (26) at 10s and $\nu$ is added to the output function at 20s. Simulation results are shown in Figures 1–4. Figure 1 shows the tracking trajectories of the system states. In Figure 1(a,b), the trajectories of $x_1$ and $x_2$ have changed due to the addition of $d$ and $\nu$. Under the influence of $d$ and $\nu$, $\hat{x}_1$ and $\hat{x}_2$ can still quickly track the changes of $x_1$ and $x_2$. From system (24), $x_3$ is not affected by $d$ and $\nu$, and its trajectory does not change, as shown in Figure 1(c). The control law $u_1$ is added at 30s. Due to the action of $u_1, x_1, x_2$ and $x_3$ are rapidly globally stable at zero, and $\hat{x}_1, \hat{x}_2$ and $\hat{x}_3$ also realize zero-difference tracking. In Figure 2, $\dot{\hat{d}}$ can achieve an accurate estimation of the disturbance $d$. In Figure 3, $\dot{\nu}$ can achieve an accurate estimation of the disturbance $\nu$. The residual signal is also affected by $d$ and $\nu$. Due to the action of the PIOs, the residual signal quickly tends to zero, as shown in Figure 4. Actuator fault $f$ occurs at 50s. At this time, the PIO (29) does not keep a zero-difference tracking, and the residual signal is no longer maintained at zero point, as shown in Figure 4. This simulation example shows that the proposed PIO can reject the effects of system uncertainties successfully and accurately estimate the disturbance. The real-time fault detection can be realized.

### 4.2. Nonlinear system case

To verify the efficiency of the proposed fault detection scheme for nonlinear system, considering the following nonlinear system

$$
\begin{align*}
x_1 &= 5x_2 - 10x_1 + d + f \\
x_2 &= 2x_1 - x_2 - x_1x_3 + 0.5f \\
x_3 &= -8x_3 + x_1x_2 + 3f \\
y_1 &= x_1 \\
y_2 &= x_2 \\
y_3 &= x_3.
\end{align*} \tag{32}
$$

Assuming $d = 0$ and $f = 0$, then system (32) can be written as

$$
\begin{align*}
x_1 &= 5x_2 - 10x_1 \\
x_2 &= 2x_1 - x_2 - x_1x_3 \\
x_3 &= -8x_3 + x_1x_2 \\
y_1 &= x_1 \\
y_2 &= x_2 \\
y_3 &= x_3.
\end{align*} \tag{33}
$$

where $x = [x_1 \ x_2 \ x_3]^T, y = [y_1 \ y_2 \ y_3]^T$. System (32) is in the form of (14) with

$$
A = \begin{bmatrix} -10 & 5 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -8 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},
$$

$$
f(x) = \begin{bmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \end{bmatrix}, \quad D = \begin{bmatrix} 1 \ 0 \ 0 \end{bmatrix}, \quad M = \begin{bmatrix} 1 \\ 0.5 \ 3 \end{bmatrix}. \tag{34}
$$

Reconstruct the output function $y$ as

$$
y^c = f(x) + K_p \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}. \tag{35}
$$
Figure 1. Tracking trajectories. (a) $x_1$ and $\hat{x}_1$, (b) $x_2$ and $\hat{x}_2$, (c) $x_3$ and $\hat{x}_3$.

Figure 2. Estimation of the disturbance $d$. (a) $\hat{d}$, (b) $e_d$.

Figure 3. Estimation of the sensor measurement noise $v$. (a) $\hat{v}$, (b) $e_v$. 
The disturbance is given as $d = 6$. A PIO is designed in the form of (17)

$$
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
-10 & 5 & 0 \\
2 & -1 & 0 \\
0 & 0 & -8
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
-1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\hat{x}_1 \\
\hat{x}_2 \\
\hat{x}_3
\end{bmatrix} +
\begin{bmatrix}
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
(\hat{y} - \hat{\hat{y}}) +
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
d,
$$

Figure 4. Residual signal trajectories. (a) $e_1$, (b) $e_2$, (c) $e_3$.

Figure 5. Tracking trajectories. (a) $x_1$ and $\hat{x}_1$, (b) $x_2$ and $\hat{x}_2$, (c) $x_3$ and $\hat{x}_3$. 
Figure 6. Estimation of the disturbance $d$. (a) $d$, (b) $e_d$.

Figure 7. Residual signal trajectories. (a) $e_1$, (b) $e_2$, (c) $e_3$.

\[ \dot{\hat{d}} = K_i \begin{bmatrix} x_1 - \hat{x}_1 \\ x_2 - \hat{x}_2 \\ x_3 - \hat{x}_3 \end{bmatrix}, \]

\[ \dot{\hat{y}} = \begin{bmatrix} 0 \\ -\hat{x}_1 \hat{x}_3 \\ \hat{x}_1 \hat{x}_2 \end{bmatrix} + K \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix}. \]  

(36)

Using pole assignment techniques, the gain matrix $K_p$ of the output function can be designed as:

\[ K = \begin{bmatrix} -9 & 5 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & -7 \end{bmatrix}. \]  

(37)

The gain matrix $K_i$ of the observer (36) can be designed as $K_i = [3 \ 0 \ 0]^T$. The residual generator is designed as:

\[ r = y - \hat{y} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}. \]  

(38)

Simulation results are shown in Figures 5–7. Figure 5 shows the tracking trajectories of the system (33)’ states. The disturbance $d$ is added to the system (33) at 30s. In Figure 5, the trajectories of $x_1$, $x_2$ and $x_3$ have changed due to the addition of $d$. Under the influence of $d$, $\hat{x}_1$, $\hat{x}_2$ and $\hat{x}_3$ can still quickly track the changes of $x_1$, $x_2$ and $x_3$. In Figure 6, $\hat{d}$ can achieve an accurate estimation of the disturbance $d$. The residual signal is also affected by $d$. Due to the action of the PIO, the residual signal quickly tends to zero, as shown in Figure 7. Actuator fault $f$ occurs at 70s.
At this time, the PIO (36) does not keep a zero-difference tracking, and the residual signal is no longer maintained at zero point, as shown in Figure 7. The above simulation results show the effectiveness of the full-order PIO designed in Section 3 for fault detection.

5. Conclusion

A new approach to design PIO for multi-output systems with actuator faults and disturbances has been proposed in this article. Compared with Busawon and Kabore (2000); Tang (2022), this article considers both the plant modeling of unknown boundary disturbance and the sensor measurement noise, and designs two PIOs for them to achieve disturbance suppression, and realizes the fault detection through the residual generator. Compared with Mohamed et al. (2018), the fault detection of a class of nonlinear systems is realized by reconstructing the output function and designing PIO. The feasibility of the proposed method is verified by simulation. This article hopes to judge whether the fault occurs through the residual signal, but it does not involve the residual evaluation part.

Data availability statement

The data that support the findings of this study are available on request from the corresponding author.

Disclosure statement

No potential conflict of interest was reported by the authors.

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