New cutoff frequency for torsional Alfvén waves propagating along wide solar magnetic flux tubes

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Abstract An isolated, isothermal, and wide magnetic flux tube embedded either in the solar chromosphere or in the lower solar corona is considered, and the propagation of linear torsional Alfvén waves is investigated. It is shown that the wideness of the tube leads to a new cutoff frequency, which is a local quantity that gives the conditions for the wave propagation at different atmospheric heights. The cutoff is used to establish the ranges of frequencies for the propagating and reflected waves in the solar chromosphere and lower solar corona. The obtained results are compared to those previously obtained for thin magnetic flux tubes and the differences are discussed. Moreover, the results are also compared to some current observational data, and used to establish the presence of propagating waves in the data at different atmospheric heights; this has profound implications on the energy and momentum transfer by the waves in the solar atmosphere, and the role of linear torsional Alfvén waves in the atmospheric heating and wind acceleration.

Keywords Magnetohydrodynamics · Wide solar magnetic flux tubes · Torsional Alfvén waves · Cutoff frequency

1 Introduction

The presence of magnetic flux tubes in the solar atmosphere is well-established observationally. In the solar photosphere and lower chromosphere, these tubes can be approximated as thin, however, they must be considered as wide tubes in the upper chromosphere, transition region and solar corona (e.g., Priest (1982), Solanki (1993)). The fundamental difference between the thin and wide magnetic flux tubes is that for the former the field remains the same for all magnetic field lines, however, it changes from one magnetic line to another in wide flux tubes (e.g., Roberts and Ulmschneider 1997). Thin solar magnetic tubes can support longitudinal (sausage), transverse (kink) and torsional (Alfvén) waves and also fluting modes (e.g., Spruit 1982, Hollweg 1990, Roberts 1991, Roberts and Ulmschneider 1997, Stix 2004). Similar types of waves also exist in wide solar magnetic flux tubes but their propagation conditions are different (e.g., Roberts and Ulmschneider 1997). The fact that different tube waves may significantly contribute to the local heating required in different layers of the solar atmosphere has been extensively investigated in the literature (e.g., Narain and Ulmschneider 1996, Roberts and Ulmschneider 1997, Ulmschneider and Musielak 2003, Musielak 2004).

A significant amount of work was done on torsional Alfvén waves as they may carry enough wave energy and momentum to heat the upper solar chromosphere, transition region and corona (e.g., Parker 1979, Heinemann and Olbert 1980, Priest 1982, Hollweg 1985, Poedts et al. 1985, Ferriz-Mas et al. 1989, Krogulec et al. 1994, Ferriz-Mas and Schuessler 1994, Roberts and Ulmschneider 1997, Musielak et al. 2007, Routh et al. 2007, Murawski and Musielak 2010, Routh et al. 2010, Webb et al. 2012, Chmielewski et al. 2013, Murawski et al. 2014, Perera et al. 2015). In the cited papers, thin solar magnetic flux tubes were mainly considered and the propagation conditions for torsional Alfvén
waves were investigated. Our approach presented in this paper is different as we deal primarily with wide solar magnetic flux tubes located either in the solar chromosphere or in the lower part of the solar corona; however, we validate our results by taking the limit of thin magnetic flux tubes and comparing the results in this limit with those presented in some of the above cited papers.

In general, the problem of propagation of torsional Alfvén waves along thin solar magnetic flux tubes was formulated and studied in Hollweg (1978, 1981, 1982). The fundamental physical concept in this problem is a cutoff frequency, which exists because of the presence of gradients of physical parameters in the solar atmosphere. According to Musielak et al. (2007), the propagation of torsional Alfvén waves along thin and isothermal solar magnetic flux tubes is cutoff free. However, the cutoff frequency also appears as a result of solar temperature gradients Routh et al. (2007). With the presence of inhomogeneities in the solar atmosphere, a new concept of cutoff frequencies is required because the cutoff originally introduced by Lamb (1911) can only be applied to a homogeneous atmosphere, which is unrealistic for the Sun as shown by Routh et al. (2010), Murawski and Musielak (2010), and Perera et al. (2015).

There are basic differences in settings between the papers by Musielak et al. (2007) and Routh et al. (2007, 2010), who considered the propagation of linear torsional Alfvén waves along thin solar magnetic flux tubes, and the approach presented in this paper, which concentrates on linear torsional Alfvén waves and their propagation along wide solar magnetic flux tubes. It must be also pointed out that in the work by Murawski and Musielak (2010) and Perera et al. (2015), linear transverse Alfvén waves were considered and their propagation along uniform magnetic field lines was investigated. In all these papers, the cutoff frequencies were obtained, Thus, our aim is to compare their results to ours presented in this paper, which would allow us to determine how much the tube wideness affects the wave cutoff frequencies.

We also compare our results to those previously obtained in numerical studies of dissipation and momentum deposition by torsional Alfvén waves propagating along thin solar magnetic flux tubes; the governing equations for those studies were derived by Hollweg (1981, 1982) and then extended to more than one dimension with nonlinear terms included (e.g., Kudoh and Shibata 1999, Saito et al. 2001, Chmielewski et al. 2013, Murawski et al. 2015, Wójcik et al. 2017). Since most of this cited work concerns nonlinear waves, the comparison to our linear results is limited to only several special cases. We also plan to compare our results to some observational data, so let us now briefly review these observations.

Observational evidence for the presence of torsional Alfvén waves in the solar atmosphere was given by Jess et al. (2009), who analyzed high spatial resolution of Hα observations by the Swedish Solar Telescope (SST). They identified Alfvén waves with periods from 12 min to 2 min and with the maximum power near 6-7 min. According to these authors, the waves carry enough energy to heat the solar corona, which is still questionable (Dwivedi and Srivastava 2010). Pure Alfvén motions were also observed by Bonet et al. (2008) who showed evidence for vortex motions of G band bright points around downflow zones in the photosphere; the lifetimes of these motions is around 5 min. In addition, Wedemeyer-Böhm and Rouppe van der Voort (2009) demonstrated the presence of some disorganized relative motions of photospheric bright points and identified them as swirl-like motions in the solar chromosphere. Finally, McIntosh et al. (2011) reported indirect evidence for Alfvén waves found in observations by Solar Dynamic Observatory (SDO), see also Cargill and de Moortel (2011).

Observations of atmospheric oscillations were made by SOHO and TRACE. Periods of these oscillations range from 6 to 10 min, from 3 to 7 min and from 1 to 10 min respectively in the solar chromosphere, transition region and corona, respectively (e.g., De Pontieu et al. (2003, 2005), McIntosh et al. (2004), McAteer et al. (2004), Hasan (2008)). More recently, the GREGOR Infrared Spectrograph measured variations of the magneto-acoustic cutoff frequency in a sunspot umbra as reported by Felipe et al. (2018) and found this cutoff to be of the order of 3.1 mHz, which is in agreement with theoretical predictions. The magneto-acoustic cutoff represents typical frequency of solar chromospheric oscillations and confirms that magnetoacoustic waves are responsible for the origin of these oscillations. Since linear torsional Alfvén waves cannot excite these oscillations, the results obtained in this paper can only be compared to the available solar observations of Alfvén waves.

It was suggested that some transverse oscillations observed in the solar photosphere and chromosphere could be driven by Alfvén or kink waves (Van Doorsselaere et al. (2008)). Solar p-mode oscillations were also detected in sunspots (Zirin and Stein 1972), whose atmosphere with large and strong magnetic fields maybe suitable for the propagation of Alfvén waves. A mechanism of mode conversion from fast to slow modes in the lower chromosphere may also generate Alfvén waves (Cally and Hansen (2011)). Observations also show that the magnetic field over the umbral region of a sunspot reduces the amplitude of oscillations in the solar photosphere. Lites (1992) classified the waves in the sunspots into the following 3 categories: 5 minute oscillations in the sunspot photosphere, 3 minute oscillations and umbral flashes in the umbral chromosphere, and running penumbral waves in the penumbral chromosphere. It was found that the umbral waves and flashes travel upward along the vertical magnetic field in the umbral region of sunspots. Optical observations of the solar chromosphere
demonstrated that there is a connection between the umbral disturbances and penumbral waves (Lites et al. (1982)). Finally, observations and polarimetric studies of the solar corona (De Pontieu et al. (2007), Tomczyk et al. (2007)) showed the presence of Alfvén waves.

The main purpose of this paper is to determine analytically a new cutoff frequency and used it to determine the conditions for propagation of linear torsional Alfvén waves along isolated and isothermal magnetic flux tubes that are considered here to be wide. As the obtained results show the derived cutoff is different than others previously obtained for thin magnetic flux tubes; the main reason is that for thin tubes there is no structure in the horizontal direction as all magnetic-field lines have the same physical properties across these tube. Since at each given height of a wide flux tube, the magnetic field line is characterized by different physical parameters, different Alfvén velocity are obtained (e.g., Hollweg (1981)). In addition, there is a gradient of Alfvén velocity along each field line. Both effects lead to a new cutoff frequency that is determined here by using a method previously developed by Musielak et al. (2007).

The results obtained in this paper significantly generalize the previous work on cutoff frequencies in thin magnetic flux tubes (see citations above). The theoretically established cutoff frequencies are compared to some available observational solar data as well as to some previously found cut-offs for Alfvén waves propagating in the solar atmosphere. We use the comparison to discuss the role of the new cutoff in transporting energy and momentum by torsional Alfvén waves to different layers of the solar atmosphere.

The outline of our paper is as follows: in Sect. 2, we describe our flux tube model and present the basic equations. The cutoff frequency for torsional waves propagating along wide and isothermal magnetic flux tubes is derived in Sect. 3. The conditions for the wave propagation in the solar atmosphere are presented and discussed in Sect. 4. Comparison of our theoretical results to the observational data is given in Sect. 5, and our conclusions are given in Sect. 6.

2 Governing and wave equations

Following Routh et al. (2007), we consider an isolated wide magnetic flux tube that is embedded either in the solar chromosphere or in the lower solar corona. In our model, the solar atmosphere is approximated by an incompressible and isothermal medium that has fixed density stratification. The tube is untwisted, has a circular cross-section and is in temperature equilibrium with the external medium. According to Hollweg (1978, 1981) (also Kudoh and Shibata 1999, Saito et al. 2001), the propagation of linear torsional Alfvén waves along such a flux tube can be described by using an orthogonal curvilinear coordinate system \((\xi, \theta, s)\), where \(s\) is a parameter along a given magnetic field line, \(\theta\) is the azimuthal angle about the axis of symmetry, and \(\xi\) is a coordinate in the direction \(\hat{\xi} = \hat{\theta} \times \hat{s}\).

The background magnetic field becomes \(B_{0\xi} = B_{0\theta}(s)\hat{s}\) with \(B_{0\xi} = 0\) and \(B_{0\theta} = 0\). Since only linear torsional waves are considered, the pressure and density variations associated with the waves are neglected, and the waves are described by \(v = v_\theta(x, t)\hat{\theta}\) and \(b = b_\theta(x, t)\hat{\theta}\).

From the work of Hollweg (1982), the curvilinear scale factors, \(h_\theta = R\), are introduced, where \(R = R(s)\) represents the distance from the magnetic-field line to the tube axis, \(h_\xi = 1\), and \(h_\theta = R\), with the latter being determined by the conservation of magnetic flux. Using these scale factors, we obtain the following momentum and induction equations

\[
\frac{\partial}{\partial t} \left( \frac{v_\theta}{R} \right) - \frac{B_{0\xi}}{4\pi\rho_0 R^2} \frac{\partial}{\partial t} \left( R b_\theta \right) = 0 , \quad (1)
\]

\[
(R b_\theta) - R^2 B \frac{\partial}{\partial s} \left( \frac{v_\theta}{R} \right) = 0 . \quad (2)
\]

These are the basic equations that describe the propagation of torsional waves along the magnetic flux tube embedded in the solar atmosphere.

We combine (1) and (2) and derive the wave equations for torsional tube waves. For this we consider the following two sets of the wave variables: \(v_\theta\) and \(b_\theta\), and \(x \equiv \frac{R_0}{R}\) and \(y \equiv R b_\theta\), which are used here to study the wave behavior in the solar atmosphere Hollweg (1982).

To derive the wave equations for the variables \(v_\theta\) and \(b_\theta\), we use the conservation of magnetic flux \(\pi R^2(s)B_0 = \text{constant}\) to express \(R(s)\) in terms of \(B_{0\xi}\), and applying them in MHD equations (1, 2) gives the wave equations as:

\[
\frac{\partial^2 v_\theta}{\partial t^2} - C_A^2 \frac{\partial^2 v_\theta}{\partial s^2} - \frac{C_A^2}{B_{0\xi}^2} \frac{d B_{0\xi}}{d s} \frac{\partial v_\theta}{\partial s} + C_A^2 \left[ \frac{1}{4B_{0\xi}^2} \left( \frac{d B_{0\xi}}{d s} \right)^2 \right] v_\theta = 0
\]

\[
\frac{\partial^2 b_\theta}{\partial t^2} - C_A^2 \frac{\partial^2 b_\theta}{\partial s^2} + \frac{C_A^2}{B_{0\xi}^2} \left[ \frac{1}{B_{0\xi}^2} \left( \frac{d B_{0\xi}}{d s} \right)^2 - \frac{3}{C_A} \left( \frac{d C_A}{d s} \right) \right] \times \frac{\partial b_\theta}{\partial s} + C_A^2 \left[ \frac{1}{C_A} \frac{d B_{0\xi}}{d s} - \frac{3}{4B_{0\xi}^2} \left( \frac{d B_{0\xi}}{d s} \right)^2 \right] + \frac{1}{2B_{0\xi}^2} \left( \frac{d B_{0\xi}}{d s} \right)^2 b_\theta = 0
\]

where \(C_A(s) = \frac{B_{0\xi}(s)}{\sqrt{4\pi\rho_0(s)}}\) is the Alfvén velocity along a given magnetic-field line. Note that the derived wave equations have different forms, which means that the behavior of the wave variables \(v_\theta\) and \(b_\theta\) is not the same. Similarly, using the Hollweg variables \(x\) and \(y\), Eqs. (1) and (2) give also
two different wave equations
\[
\frac{\partial^2 x}{\partial t^2} - C_A \frac{\partial^2 x}{\partial s^2} = 0 ,
\]
(3)
and
\[
\frac{\partial^2 y}{\partial t^2} - \frac{\partial}{\partial s} \left[ C_A'(s) \frac{\partial y}{\partial s} \right] = 0 ,
\]
(4)
where the condition \( B_0 R^2(s) = \text{constant} \) is satisfied.

The difference in the derived wave equations reflect the fact that \( x \) and \( y \) behave differently. Comparison of these wave equations to those derived for the wave variables \( v_\theta \) and \( b_\theta \) clearly shows that each wave variable has different behavior. The comparison also shows that there is an advantage in using Hollweg’s variables because the wave equations for these variables are much simpler than those obtained for the variables \( \nu \) and \( b \). An interesting result is that the first set of wave equations derived above, is of the same form as that obtained by Musielak et al. (2006) in their studies of acoustic waves propagating in a non-isothermal medium.

3 Cutoff frequency: theory

Musielak et al. (2006) developed a method to determine a cutoff frequency from given wave equations. The first step of this method is to transform the wave equations into the corresponding Klien-Gordon equations and then the cutoff frequency is derived by using the oscillation theorem (e.g., Kahn 1990). Let us introduce the new variable \( d\tau = \frac{dr}{C_A} \) and following Routh et al. (2007), we write the critical frequencies as
\[
\Omega_{x,v\theta}^2(\tau) = \frac{3}{4} \left( \frac{C_A'}{C_A} \right)^2 - \frac{1}{2} \left( \frac{C_A''}{C_A} \right) ,
\]
(5)
and
\[
\Omega_{y,b\theta}^2(\tau) = \frac{1}{2} \left( \frac{C_A''}{C_A} \right) - \frac{1}{2} \left( \frac{C_A'}{C_A} \right)^2 ,
\]
(6)
where \( C_A' = \frac{dC_A}{dr} \) and \( C_A'' = \frac{d^2C_A}{dr^2} \). Using these frequencies, we define their corresponding turning-point frequencies \( \Omega_{tp} \) as
\[
\Omega_{tp}^2 = \Omega_{crit}^2 + 1/4\tau^2 ,
\]
(7)
where \( \Omega_{crit} \) is either \( \Omega_{x,v\theta} \) or \( \Omega_{y,b\theta} \).

These turning-point frequencies separate the solutions into propagating and non-propagating (evanescent) waves. Since there is the turning-point frequency for each wave variable, only one of them can be the cutoff frequency. We follow Musielak et al. (2007) and identify the largest turning-point frequency as the cutoff frequency. The choice is physically justified by the fact that in order to have propagating torsional tube waves, the wave frequency \( \omega \) must always be higher than any turning-point frequency. To determine which turning-point frequency is larger, we need to specify a model of magnetic flux tubes embedded in the solar atmosphere.

A single and isothermal magnetic-flux tube is considered to be wide when its horizontal magnetic field is nonuniform, which means that each magnetic-field line has different physical properties in the horizontal direction. Let us assume that this tube is approximated by a simple model in which the Alfvén velocity varies exponentially along a given field line; the model was originally considered by Hollweg (1981), and it shall be used here to determine the cutoff frequency for torsional tube waves propagating in this model.

For the exponential model, the Alfvén velocity is given by
\[
C_A = C_{A0} e^{(mH/\pi)} ,
\]
(8)
where \( C_{A0} = C_A \) at \( s = 0 \), \( m \) is a positive scaling factor, and \( h \) is the characteristic scale height; we take \( h = H \), with \( H \) being the pressure (density) scale height. The reason for choosing different values of \( m \) is that, in general, \( C_A(s) \) must be different for each magnetic field line. To calculate \( \tau(s) \) we evaluate the following integral
\[
\int_{\tau_0}^{\tau_1} d\tau = \int_0^1 \frac{1}{C_A} d\tau ,
\]
(9)
which, after substitution and integration, becomes
\[
\tau = -mH \left[ \frac{1}{C_A} - \frac{1}{C_{A0}} \right] + \tau_0 .
\]
(10)
Thus, the characteristic wave velocity \( C_A \) is expressed as,
\[
C_A = \frac{mHC_{A0}}{mH - (\tau - \tau_0)C_{A0}}
\]
(11)
Differentiating \( C_A \) with respect to \( \tau \) twice we respectively obtain
\[
\frac{dC_A}{d\tau} = \frac{mHC_{A0}^2}{[mH - (\tau - \tau_0)C_{A0}]^2} ,
\]
(12)
and
\[
\frac{d^2C_A}{d\tau^2} = \frac{2mHC_{A0}^3}{(mH - (\tau - \tau_0)C_{A0})^3} .
\]
(13)
Substituting equations (11), (12) and (13) in (5) and (6), the obtained equations are,
\[
\Omega_x^2(\tau) = -\frac{1}{4} \frac{C_{A0}^2}{(mH - (\tau - \tau_0)C_{A0})^2} ,
\]
(14)
and

\[ \Omega_1^2(\tau) = \frac{3}{4} \frac{C_{A0}^2}{(mH - (\tau - \tau_0)C_{A0})^2}. \]  

(15)

The cutoff frequency is given by

\[ \Omega_{\text{cut}}^2(\tau) = \text{Max} \left[ \Omega_1^2(\tau), \Omega_2^2(\tau) \right] + \frac{1}{4\tau^2}, \]  

(16)

which becomes

\[ \Omega_{\text{cut}}^2(\tau) = \frac{3}{4} \frac{C_{A0}^2}{(mH - (\tau - \tau_0)C_{A0})^2} + \frac{1}{4\tau^2}. \]  

(17)

The cutoff frequency \( \Omega_{\text{cut}}^2(\tau) \) in terms of \( s \) is given by

\[ \Omega_{\text{cut}}^2(s) = \frac{3}{4} \frac{C_{A0}^2 e^{\left(\frac{2s}{mH}\right)}}{m^2 H^2} + \frac{C_{A0}^2}{4 \left[ \tau_0 C_{A0} - mH \left( e^{\left(\frac{s}{mH}\right)} - 1 \right) \right]^2}. \]  

(18)

Clearly, the cutoff frequency is a local quantity that varies with \( s \) in the same way as \( C_A \) does. Because \( \Omega_{\text{cut}} \) depends on height, its physical meaning is different than the global cutoff frequencies for longitudinal and transverse tube waves obtained by Defouw (1976) and Spruit (1982), respectively. From a physical point of view, \( \Omega_{\text{cut}}(s) \) represents locally the cutoff in the solar atmosphere, and torsional tube waves must have their frequency (\( \omega \)) higher than \( \Omega_{\text{cut}} \) at a given height to reach this height and be propagating waves at this height. In other words, the cutoff allows us to determine the height \( s \) in the model at which torsional Alfvén waves of a given frequency become non-propagating (evanescent) waves.

### 4 Cutoff frequency: results

The formulas for the cutoff frequencies derived in the previous section are now used to calculate these cutoffs in the models of magnetic flux tubes considered in this paper. The calculations are performed for a magnetic flux tube with \( B_0 = 1500 \) G at the atmospheric height \( s = 0 \), which corresponds to the location in the solar atmosphere where the flux tubes have widened enough so that \( C_A \) is no longer constant but increases exponentially with height. This height depends on the local magnetic filling factor and typically it should lie at the solar temperature minimum level or in the lower chromosphere.

As the characteristic temperature for our chromosphere model, we choose the effective temperature of the Sun \( (T_{\text{eff}} = 5770 \) K). The considered chromosphere model isothermal with \( T_{\text{eff}} \), which gives \( C_{A0} = 11.0 \) km/s and \( H = 135 \) km; moreover, \( \tau_0 = 50 \) s, and the height \( s = 0 \) is located at the temperature minimum, more precisely 500 m above it. The calculated cutoff frequencies are local quantities and their variations with height in the solar chromosphere are shown in Figs. 1 and 2; in both figures the values of \( \Omega_{\text{cut}}/2\pi \) are plotted. As expected, the cutoff is much steeper for low values of \( m \). It is also seen that the effect of the cutoff on the wave propagation becomes important at atmospheric heights higher than 100 km above the base of the model.

The solar corona model considered in this paper is also isothermal with the temperature \( 200 \times T_{\text{eff}} \), which gives \( C_{A0} = 1.05 \) Mm/s and \( H = 60 \) Mm; these values are the same as those used by Murawski and Musielak (2010). Moreover, \( \tau_0 = 200 \) s and for the coronal models the height \( s = 0 \) is located above the solar chromosphere and corresponds 10 Mm. The variations of the cutoff frequencies with height in the solar corona are shown in Figs. 3 and 4 for different values of \( m \); again, in both figures the values of \( \Omega_{\text{cut}}/2\pi \) are plotted. A steep increase of the cutoff frequency with height is seen for the models with \( m = 1 \) and \( m = 2 \).
For the torsional Alfvén waves to be propagating along a wide magnetic flux tube embedded in the solar atmosphere, it is required that the wave frequency $\omega$ exceeds the cutoff frequency $\Omega_{\text{cut}}$. Specific values of the cutoff frequencies for different flux tube models are given in Table 1 and 2 for the solar chromosphere and corona, respectively; in both tables the values of $\Omega_{\text{cut}}/2\pi$ are plotted. It must be noted that these values are given only for the bottom and top of each flux tube model label by different $m$. The values of the cutoff frequencies range from 0.011 Hz ($m=1$) to 0.002 Hz ($m=5$) at the bottom, and 0.23 Hz ($m=2$) to 0.01 Hz ($m=5$) at the top of the solar chromosphere (see Table 1). However, according to Table 2, the cutoff period ranges from 0.0024 Hz ($m=1$) to 0.0007 Hz ($m=4$) at the bottom, and 0.0043 Hz ($m=1$) to 0.0008 Hz ($m=4$) at the top of the solar corona.

The above cutoff frequencies for the solar chromosphere are consistent with the values of the cutoff frequencies for Alfvén waves propagating in the non-isothermal solar chromosphere with a uniform magnetic field studied by Murawski and Musielak (2010), whose cutoff frequencies range from 0.01 Hz at the bottom of their model to 0.25 Hz at the top of their model. Moreover, their value of the cutoff frequency in the solar corona is around 0.01 Hz, which show significant differences with the results presented in Figs. 3 and 4, and in Table 2. These differences are likely to be the effects of magnetic fields, which in this paper is non-uniform and diverges rapidly with height but in Murawski and Musielak (2010)’s paper the field is uniform. The values of cutoff periods obtained in this paper are also consistent with studies performed by Perera et al. (2015), whose cutoff frequency ranges from 0.022 Hz to 0.005 Hz for propagating linear Alfvén waves in an isothermal solar atmosphere with a uniform magnetic field.

In our previous work on cutoff frequencies for transverse Alfvén waves in a thin non-isothermal magnetic flux tube (e.g., Routh et al. 2010, Hammer et al. 2010), the ranges of cutoff periods for different regions of the solar chromosphere are: 0.003-0.008 Hz for the lower chromosphere, 0.003-0.008 Hz for the middle chromosphere, and 0.003-0.005 Hz for the upper chromosphere. Again, these results are consistent with those obtained in this paper, and some discrepancies between the results reflect the differences in the models of considered flux tubes, thin (previous work) and wide (this paper).

### Table 1

| Model | Bottom (Hz) | Top (Hz) |
|-------|-------------|----------|
| $m=1$ | 0.011       | 18.51    |
| $m=2$ | 0.006       | 0.228    |
| $m=3$ | 0.004       | 0.044    |
| $m=4$ | 0.003       | 0.018    |
| $m=5$ | 0.002       | 0.010    |

### Table 2

| Model | Bottom (Hz) | Top (Hz) |
|-------|-------------|----------|
| $m=1$ | 0.0024      | 0.03     |
| $m=2$ | 0.0013      | 0.0043   |
| $m=3$ | 0.0008      | 0.0017   |
| $m=4$ | 0.0007      | 0.0009   |
| $m=5$ | 0.0006      | 0.0008   |
5 Comparison to observations

There is observational evidence for Alfvénic-like fluctuations in the solar corona and chromosphere (De Pontieu et al. (2007), Tomczyk et al. (2007), Bonet et al. (2008), Fujimura and Tsuneta (2009), Jess et al. (2009), Wedemeyer-Böhm and Rouppe van der Voort (2009)), and the observations show wave periods ranging from 3 to 9 min, which is consistent with the theoretical predictions of this paper. Moreover, Fujimura and Tsuneta (2009) reported a detection of torsional tube waves with periods ranging from 0.001 Hz to 0.008 Hz, up to the top of the solar chromosphere in the expanding wave-guide rooted in the strongly magnetized MBP (magnetic bright point). Their observational results match our theoretical results in the solar chromosphere (especially for $m = 5$, which gives period range as 0.002 Hz to 0.01 Hz) and corona (for $m = 1$ and 2). In general, our results predict low frequency waves to be present in the solar corona while higher frequency waves are propagating in solar chromosphere, which may have implications on the energy and momentum transfer by linear torsional Alfvén waves in the solar chromosphere and corona. Another important aspect of our results is that they can be used to determine whether the observed waves are propagating or evanescent (e.g., Hammer, Musielak and Routh 2010).

6 Conclusions

We determined the conditions for propagation of linear (fully incompressible) torsional Alfvén waves along an isolated and isothermal magnetic flux tube embedded in the solar chromosphere and corona. The main difference between the results of this paper and those considered previously (e.g., Hollweg (1981, 1982), Musielak et al. (2007), Routh et al. (2007, 2010), Hammer et al. (2010)) is the fact that in the previous work thin magnetic flux tubes were studies but this paper investigates the effects caused by wide magnetic flux tubes on the torsional Alfvén wave propagation. It must be noted that in wide magnetic flux tubes the horizontal magnetic field is non uniform, and that the Alfvén velocity varies with height along the tubes.

Our main theoretical result is derivation of the cutoff frequencies for linear torsional Alfvén waves propagating along wide magnetic flux tubes embedded in the solar chromosphere and corona. The cutoff frequency is a local quantity and its variation with height is used to identify regions in the solar atmosphere where strong wave reflection occurs. Using the condition $P_w = P_{cut}$, where $P_w$ is the wave period, the atmospheric height at which Alfvén waves of a given period are reflected can be determined. Wave reflection and the resulting constructive interference between the propagating and reflected Alfvén waves can form standing wave patterns as also seen in numerical studies performed by Murawski and Musielak (2010).

The theoretically predicted cutoff frequency for torsional Alfvén waves propagating along wide magnetic flux tubes is consistent with the previous studies performed by Murawski and Musielak (2010), Routh et al. (2010), Hammer et al. (2010), and Perera et al. (2015) in the solar chromosphere. However, the novelty of this work is that the computed cutoff frequencies in the solar corona are different than those found previously because in the wide magnetic flux tubes considered here, the magnetic field alters greatly with the atmospheric height. The derived chromospheric and coronal cutoffs are in agreement with the currently available observational data; actually our results can be used to determine whether the observed Alfvén waves are propagating or not in the solar chromosphere and corona.

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