Membrane Solitons as Solitary Waves of Non-Linear Strings Dynamics

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Abstract

Families of solutions to the field equations of the covariant BRST invariant effective action of the membrane theory are constructed. The equations are discussed in a double dimensional reduction, they lead to a nonlinear equation for a one dimensional extended object. One family of solutions of these equations are solitary waves with several properties of solitonic solutions in integrable systems, giving evidence that in this double dimensional reduction the nonlinear equations are an integrable system. The other family of solutions found, exploits the property that the non linear system under some assumptions is equivalent to a non linear Schrödinger equation.
1 Introduction

In recent years there has been a renewed interest in the theories of Supermembranes and Dirichlet branes, specially due to the duality relations between many Super-D brane, Super-p branes and Superstring theories in several dimensions [1]. These duality relations has been stablished at the solitonic sectors [2] of the low energy phenomenological actions. Presumably there exists a fundamental 11 dimensional supersymmetric theory, from which all brane theories and its duality relations could be obtained. In particular this theory must contain the solitonic sector of both Superstrings and its dual membranes. Although, a lot of progress in the understanding at the spectral [3][4][5] level from the nonperturbative point of view has been reached, there is still not satisfactory answer to fundamental questions as its geometrical principle, dynamics and quantum consistency. One of the most striking difficulties of the bosonic p-brane theory is that the field equations are intrinsically non linear in all known gauges, so it is not know how to solve the field equations at the classical level neither how to perform its non perturbative quantization. This is the reason that almost all quantization aproach to p-branes or Super-p-brane are semiclassical [6]. We study in this paper some aspects of the non linear structure of the field equations of the covariant, BRST invariant, effective action of the Membrane Theory. One way of having some insight into this nonlinear structure is to study sectors of the system which have integrability properties. To do so we will consider a double dimensional reduction of the membrane theory. We will then analyse the existence of solitary waves to this non linear sector. The properties of the solitary waves give in general direct evidence of the existence or not of solitonic solutions to the non-linear system. In fact, the properties of the solitary waves in systems like KdV, Sine Gordon, non-linear Schrödinger equation and other members of their hierarchy give direct evidence of their solitonic behaviour. These properties usually point out the way to perform the analysis of the non linear integrable equations by the inverse scattering method, which in most cases completely resolves the evolution problem of the non-linear system. In section 2 we briefly discuss the construction of the BRST invariant effective action of membrane theories, including the boundary conditions on ghost fields [7][8]. In section 3 we perform a double dimensional reduction leading to non linear field equations for a 1-dim extended object. In section 4 we find solitary wave solutions to it and discuss
their properties. In section 5, we consider the particular case of 4 dimensions and relate the problem to the static non linear Schrödinger equation.

# 2 Off-shell BRST invariant effective action

The off-shell BRST invariant effective action for the membrane can be obtained from the general modified BFV approach \[8\]. \( x^\mu, p_\mu \) denote the canonical conjugate bosonic variables, \( c^a, \mu_a \) are the conjugate canonical ghost variables, \( \lambda \) are the Lagrange multipliers, \( \xi_a \) are auxiliary ghost fields and \( \chi^a \) are the gauge fixing conditions. The effective action is then

\[
S_{\text{eff}} = \int dt \{ p_\dot{x} - \mu_a \dot{c}^a - \mathcal{H}_0^{(\text{brst})} - \delta_{\text{brst}}(\chi^a \mu_a) + \delta_{\text{brst}}(\xi_a \chi^a) \},
\]

where \( \delta_{\text{brst}}(\cdot) = [\cdot, \Omega] \) and \( \Omega \) is the BRST generator \[10\].

\[
\Omega = c^a \phi_a + \mu_a (1) U^a + \mu_b \mu_a (1) U^{a,b}
\]

\[
(1) U^i = c^j \partial_j c^i - \gamma \gamma^{ij} c^3 \partial_j c^3,
\]

\[
(1) U^3 = c^3 \partial_a c^a - \partial_a c^3 c^a,
\]

\[
(2) U^{ij} = \frac{3}{2} c^3 \partial_j c^3 \partial_i c^3
\]

\[
(2) U^{3a} = 0.
\]

The functional integral obtained from (1) reduces correctly to the canonical functional integral in terms of the physical modes with the correct functional measure, and is independent of the gauge fixing conditions \[10\].

The action (1) may be rewritten as

\[
S_{\text{eff}} = \int d\sigma^3 p_\dot{x} - \mu_a \dot{c}^a - \chi^a \phi_a^{(\text{brst})} + B^a \chi^a - \xi_a \delta_{\text{brst}}(\chi^a) - \theta^a \mu_a,
\]

where we have introduced the non canonical auxiliary fields \( B^a \) and the ghost auxiliary fields \( \theta^a, \phi_a^{(\text{brst})} = \{ \mu_a, \Omega \} \) are the BRST extension of the first class constraints

\[
\phi_i = p_\mu x_\mu^i, \quad \phi_i = \frac{1}{2} (p^2 + \gamma).
\]
From (3) we obtain the Hamiltonian density as

$$H^{eff} = \lambda_a \phi_a^{(brst)} - B^a \chi_a + \mathcal{L}_a \delta_{brst}(\chi_a) + \theta^a \mu_a,$$

(6)

and the effective BRST Lagrangean

$$L^{eff} = p\dot{x} - \mu_a c^a - H^{eff}. $$

(7)

Now we consider the covariant gauge fixing conditions

$$\chi^1 := \lambda^1 = 0, \quad \chi^2 := \lambda^2 = 0, \quad \chi^3 := \lambda^3 - 1 = 0. $$

(8)

after performing the functional integration on $B_a$, $\theta$, and $\lambda^a$ in (3) we obtain

$$H^{eff} = \phi_3^{(brst)}, $$

(9)

$$S_{eff} = \int d\sigma^3 p\dot{x} - \mu_a c^a - \phi_3^{(brst)}, $$

(10)

$$\phi_j^{(brst)} = \dot{\phi}_j + (\partial_j c^a)\mu_a + \partial_i (c^i \mu_j) + \partial_j (c^3 \mu_3), $$

(11)

$$\phi_3^{(brst)} = \frac{1}{2}(p^2 + \gamma) + \gamma \gamma^{ij}[\partial_i c^3 \mu_j] + (\partial_i c^i)\mu_3 + \frac{3}{2} \partial_i c^3 \partial_j c^3 \mu_j \mu_i + \partial_i (c^i \mu_3). $$

(12)

with periodic boundary conditions for the closed membrane theory and the following boundary conditions for the open membrane [8]:

$$c^3|_B = 0, \quad \text{and} \quad n_i c^i|_B = 0, $$

(13)

(9,10,11,12) agree with the results in [11] which were obtained for the closed membrane only under several assumptions.

(13) are the generalization of the boundary conditions for the string [12]. Additionally the ghost fields must satisfy initial and final time conditions, in order to obtain the BRST invariance of the action, they are:

$$c^3|_{t_i} = 0, \quad \text{and} \quad c^3|_{t_f} = 0, $$

(14)

which are associated to the constraint quadratic in the momenta, and are equivalent to the usual restriction on the gauge parameters [13].
From the effective Hamiltonian we get the field equations for the ghost sector:

\[ \dot{c}^i = \gamma^{ij} \partial_j c^3 + 3 \partial^i c^3 \partial_j c^3 \mu_j \]
\[ \dot{c}^3 = \partial_j c^j \]
\[ \dot{\mu}_3 = \partial_i (\gamma^{ij} \mu_i) + 3 \partial_j (\partial_i c^3 \mu_j \mu_i) \]
\[ \dot{\mu}_i = \partial_i \mu_3 \]

We will consider the class of solutions satisfying the BRST invariant condition \( c_3 = 0 \) for all \( \sigma_1, \sigma_2, \tau \). It is consistent with the previous boundary conditions. The field equations for the ghosts reduce then to:

\[ \dot{c}^i = 0, \text{ and } \partial_i c^i = 0. \] (15)

The field equations for the antighost are:

\[ \dot{\mu}_3 = \partial_j (\gamma^{ij} \mu_i) \] (16)
\[ \dot{\mu}_i = \partial_i \mu_3 \] (17)

from which we obtain:

\[ \ddot{\mu}_3 = \partial_j (\gamma^{ij} \partial_i \mu_3). \] (18)

Finally the field equations for the bosonic coordinates become under our assumption:

\[ \ddot{x}^\mu = \partial_j (\gamma^{ij} \partial_i x^\mu). \] (19)

Once (19) is solved (18) is a linear equation in \( \mu_3 \). Consequently it is enough to discuss the solutions of (19). We will construct in the next sections solitonic solutions to the covariant equation (19). We will also consider solutions in the LCG. We notice that \( X^+, X^- \) as in the LCG are solutions to (19).

3 Non Linear String Equation from D=11 Membrane Theory

One way of having some insight on the structure of a nonlinear dynamical system is through the study of the solitary waves solutions of the system. It
usually happens that the existence of solitary waves leads to the existence of solitonic solutions of the system, which in turn allow through the inverse scattering approach a complete understanding of at least an important sector of the full space of solutions of the nonlinear system.

The structure of the solitary wave solutions we propose is closely related to the decomposition of the 11 dimensional spacetime as the product $S^1 \times M_{10}$ and in this sense we believe that our solutions should be related to the IIA Dirichlet Supermembranes. It is well known that the $D = 11$ supermembrane with one coordinate of the target space compactified on $S^1$ is equivalent as a quantum field theory to the $D = 10$ IIA Dirichlet supermembrane [9]. We will consider the particular limit in which the radius of compactification of the membrane together with one of the world volume coordinates, also taken as an angular coordinate, are contracted to zero.

We start analysing solutions with the following structure

$$x^\mu = y(\sigma_2) \ f^\mu(\sigma_1, \tau). \quad (20)$$

In this ansatz we separate the $\sigma_2$ dependence in order to study the a dimensional reduction on one of the target space time coordinates, which we will assume to have a $S^1 \times M^{10}$ topology. It is well known that separation of variables do not lead to a complete set of solutions of non linear equations as (19), but this ansatz is well suited for the double dimensional reduction we will shortly consider. Our argument is based on the remark that the field equations (19) allow $y(\sigma_2)$ to be identified as a local coordinate over $S^1$.

Equation (19) reduces after using (20) to

$$y \ddot{f}^\mu - \dot{y}^2 [f^\mu_1 (f^2)],_1 - \dot{y} f^\mu y^2 f^\mu_1 \quad (21)$$

$$+ [y(\dot{y})^2 (f.f_1) f^\mu],_1 + [y^2 y'(f.f_1) f^\mu_1],_2 = 0$$

which after some manipulations yields

$$\ddot{f}^\mu = (y')^2 ([f^\mu_1 f^2],_1 + 2 f^\mu f^2_1 - [(f.f_1) f^\mu],_1 - 2 (f.f_1) f^\mu_1)$$

$$+ y y''[f^\mu f^2_1 - (f.f_1) f^\mu_1] \quad (22)$$

In standard Kaluza Klein compactifications it is assumed a Fourier expansion for the local coordinate on a compact manifold, but in this case
\[ y(\sigma_2) = \kappa^{(n)} e^{in/R\sigma_2} \] leads to different orders in the \((y')^2\) and \((yy'')\) terms, it may also be meaningless to take linear combinations to study nonlinear equations, so it is better to consider each of the n-th modes by taking a polynomial dependence in \(\sigma_2\)

\[ y(\sigma_2) = \kappa^{(n)} \sigma_2^n, \tag{23} \]

then \((y_n)'^2 = n^2 \kappa_n^2 \sigma_2^{2n-2}\) and \(y_n'' = n(n-1) \kappa_n^2 \sigma_2^{2n-2}\) that have the same order.

So, to factorize the \(\sigma_2\) dependence in (22) we must have

\[ n^2 = n(n-1) \]

this implies that \(n = 0\) and \(n = 1\). Then it is natural to take \(y(\sigma_2)\) of the form

\[ y(\sigma_2) = \kappa \sigma_2 + c \tag{24} \]

where \(\kappa\) and \(c\) are constants.

Using (24) equation, (22) reduces to the nonlinear equation

\[ \ddot{f}^\mu = \kappa^2 (f_{11}^\mu f^2 + f^\mu f_1^2 - (f_{11} \cdot f) f^\mu - (f_1 \cdot f) f_1^\mu) \tag{25} \]

Let us take \(\theta\) on the interval \([0, 2\pi]\) as the angular coordinate on \(S^1\), and let \(\sigma_2\) be a world sheet coordinate wrapped around a circle of radius \(r\), so that \(\sigma_2 = r\theta\). We will consider \(x^{11}\) to be wrapped around a circle of radius \(R = O(r)r\kappa\) with \(O(r) \to 0\) when \(r \to 0\), and \(c\) as the initial length on the circle: \(c = 2\pi NR\). Now we perform a double dimensional reduction on the target and the worldsheet spaces, we take \(N \to \infty\) keeping \(c\) constant. Then \(R \to 0\) which implies \(r \to 0\). We thus obtain

\[ x^{11} \to 0, \tag{26} \]

\[ x^u \to cf^u(\sigma_1, \tau), \text{ where } u=1,...,10 \tag{27} \]

So, now \(x^u\) are the coordinates of a 10 dimensional string that obeys the nonlinear equation (25).

The double dimensional reduction we have performed is different from the one in [4]. However the nonlinear string equation (25) is clearly related to the field equations of the Dirichlet branes in ten dimensions. The membrane theory, in this limit, may then be interpreted as the interaction of 10-dim strings.
In the next section we will study the solitary wave solutions of the nonlinear equation (25). We will perform the analysis in $D$ dimension and discuss afterward the particular case $D = 4$.

4 Solitary wave solutions

Now we try a solitary wave solution

$$f^\mu = f^\mu (\sigma_1 - v \tau), \quad (28)$$

then (24) may be rewritten as

$$(v^2 - f^2) f_{111}^\mu - f^\mu (f_1^2 - f_{11} f) + (f_1 f) f^\mu_1 = 0. \quad (29)$$

where we have taken $\kappa = 1$.

After multiplying by $f^\mu_1$, and some rearrangements we obtain

$$\frac{1}{2} [(v^2 - f^2) f_1^2]_1 + \frac{1}{8} [(f^2)_1^2]_1 = 0. \quad (30)$$

From which

$$(v^2 - f^2) f_1^2 + \frac{1}{4} [(f^2)_1^2] = C \quad (31)$$

so we can solve for $f_1^2$ and introduce back it into (29), and after some calculations we get

$$(v^2 - f^2) [-(f_1^2) - \frac{1}{2} (f^2)_1^1] - 2 f^2 \frac{C - \frac{1}{4} [(f^2)_1^2]}{v^2 - f^2} + \frac{1}{2} f^2 (f^2)_1^1 + \frac{1}{4} [(f^2)_1^2] = 0. \quad (32)$$

Defining $u = v^2 - f^2$, it is straightforward to obtain

$$-2C + u \frac{C}{v^2} + \frac{1}{2} u_1^2 - \frac{1}{2} uu_{11} = 0. \quad (33)$$

When $C = 0$ (33) may be integrated giving

$$Be^{k(\sigma - v \tau)} = f^2 - v^2. \quad (34)$$

Introducing the latest equation into (29) we obtain
\[- f^\mu_{11} + \frac{1}{2} k f^\mu_1 = 0, \quad (35)\]
hence the general solitary wave solution with \( C = 0 \) is
\[ f^\mu = B^\mu e^{\frac{k}{2} (\sigma - vt)} + A^\mu \quad (36) \]
where \((B^\mu)^2 = B\), \((A^\mu)^2 = v^2\), and \( B^\mu A^\mu = 0 \). When \( C \) is not null, we can obtain the solution to (33) by considering
\[ u_c = B e^{k(\sigma - vt)} + 2v^2 \quad (37) \]
where \( k = \frac{\sqrt{C}}{v^2} \). Using \( u_c = v^2 - f^2 \) we obtain \( f^2 \) and its derivatives, and introducing these into (32) we get a linear ordinary differential equation for the coordinates \( f^\mu(\sigma) \)
\[ u_c f^\mu_{11} - \frac{1}{2} (u_c)_1 f^\mu_1 - k^2 v^2 f^\mu = 0, \quad (38) \]
where \( \mu = 1, \ldots, 9 \) and \( u_c \) is given by (37). This implies that one may find all the solutions of the type (20) which are solitary waves by solving the linear differential equation (38), together with the consistency condition \( u_c = v^2 - f^2 \).

Note that the amplitude of this solitary wave solution is velocity dependent, this is a strong evidence that this solution must be a soliton for the classical bosonic membrane.

It is also interesting to see that all the solutions with \( c = 0 \) and \( c \neq 0 \) are non degenerate, in the sense that its area at constant time folding of the space-time is not zero (\( \gamma \neq 0 \)). The solutions (36)-(38) have several properties in common with the solitonic solutions of integrable systems like KdV, Sine-Gordon, Non Linear Schrödinger Equation. The amplitude, \( A \) and \( B \) in (36), depends on the velocity of propagation \( v \). The wave number is \( v \) dependent (\( C \neq 0 \)). There are solitary waves solutions for any value of \( v \) while the non linear equation (19) is independent of \( v \). There is a balance between the dispersion relation and the nonlinearity of the system allowing solitary wave solutions.

For a particular value of \( v \) (36) represents a solution of the Nambu Goto string fields equations. This string solution represents a “Soliton” of the membrane equations.
We can also discuss following the same lines solutions to the field equations of the Fujikawa [11] effective action. In that case there are different boundary conditions and the following gauge conditions have to be imposed:

\[
\begin{align*}
g^{oo} + \gamma &= 0 \\
g_{oi} &= x_o . x_i = 0
\end{align*}
\] (39)

then in order to have a solution, (39) has to satisfy further restrictions which lead to

\[
\begin{align*}
-\frac{1}{4}k^2 v(\kappa\sigma_2 - c)^2 B e^{k(\sigma_1 - v\tau)} &= 0 \\
-\frac{1}{2} kv(\kappa\sigma_2 - c) B e^{k(\sigma_1 - v\tau)} &= 0.
\end{align*}
\]

Since \( k \) and \( v \) are not zero, then \( B = 0 \), and

\[
\begin{align*}
\gamma &= \det x_i x_j \\
\gamma &= \frac{1}{4} B k^2 v^2 (\kappa\sigma_2 - c) e^{k(\sigma_1 - v\tau)} = 0
\end{align*}
\] (40)

This implies then, that for the Fujikawa effective theory we obtain solutions describing a degenerated membrane with no area. These degenerate non linear string solitonic solutions are then solutions of the bosonic membrane theory even without performing any dimensional reduction. We will obtain now solitary wave solutions of equations (19) satisfying also Fujikawa conditions with non zero area. We do so by working in the light cone gauge (LCG). As explained in [14] we take

\[
x^+ = c^+ \tau
\] (41)

then the gauge fixing condition (39) allows us to solve for the minus coordinate

\[
x^-_i = \frac{1}{c^+} \hat{x} \cdot \hat{x}_i
\] (42)
in term of the LCG tranverse sector. The effective action and the field equations are the same as before (see (10)-(12) and (19)) upon replacement of $x^\mu$ by $\vec{x}$ and the area expanded by the tranverse solitary wave solution is not zero because now (39) implies (42) and then neither $B$ not $\gamma$ are null.

If we perform a double dimensional reduction as explained in section (3), we end up with a non linear string solution $x^u = cf^u(\sigma_1, \tau)$ where $u = 1, ..., 8$ and with the usual LCG prescription for $x^+$ and $x^-$. 

5 New membrane solutions in 4 dimensions

Very interesting solutions for the bosonic membrane arise in four dimension. We again choose the conformal (8) and LCG (41) fixing. Denoting

$$x^\mu = (x^+, x^-, x^1, x^2)$$

we will use complex variable to represent the LCG tranverse sector

$$Z = x^1 + ix^2$$

and look for solutions of the type

$$Z = y(\sigma_2)F(\sigma_1, \tau).$$

Equation (25) may be written as

$$\ddot{F} = \left(\partial_2 y\right)^2\{(F_1|F|^2)_1 + 2F|F_1|^2 - \frac{1}{2}[(F_1 \dot{F} + \dot{F}_1 F)F]_1 -(F_1 \dot{F} + \dot{F}_1 F)F_1\}$$

$$-\frac{1}{2} y(\partial_{22} y) \left[ F_1^2 \dot{F} - |F_1|^2 F \right],$$

This equation allows as before, the solution if $y = k\sigma_2$ and from (40) we get

$$k^2 = \frac{\ddot{F}}{(F_1|F|^2)_1 + 2F|F_1|^2 - \frac{1}{2}[(F_1 \dot{F} + \dot{F}_1 F)F]_1 -(F_1 \dot{F} + \dot{F}_1 F)F_1}.$$
The $\sigma_1$ depending equation is

$$AX - X_{11}|X|^2 - |X_1|^2 X + X^2 \bar{X}_{11} + (X_1)^2 \bar{X} = 0$$

(49)

and may be solved introducing polar coordinates $X = r(\sigma_1)e^{i\theta(\sigma_1)}$ that reduce the equation to:

$$\frac{A}{2} = i r (r\theta')' + (r\theta')^2$$

(50)

As $A$ is a constant, this implies that $r\theta' = 0$ then $Im[A] = 0$ so $A$ must be a real nonnegative number and $\theta$ can be directly integrated

$$\theta = \sqrt{\frac{A}{2}} \int \frac{d\sigma_1}{r(\sigma_1)}$$

(51)

where $r(\sigma_1)$ is an arbitrary real function.

The time depending equation is a non linear static Schrodinger equation [15] upon interchange of $\tau$ and $\sigma_1$.

$$\ddot{T} = \frac{A}{2} T|T|^2,$$

(52)

this equation can be solved, as usual, trying with a function that is product of a phase function by an envelope function

$$T = h(t)e^{i\phi(t)}$$

(53)

spliting (52) into real and imaginary parts, we get

$$\ddot{h} - h\dot{\phi}^2 = \frac{A}{2} h^3$$

(54)

$$h\ddot{\phi} + 2h\dot{\phi} = 0$$

(55)

Equation (54) may be integrated to give

$$\phi = \int \frac{C}{h^2(t)} dt$$

(56)
if $C = 0$, then $T$ is real and the (52) reduces to the well known case of a particle submitted to a cubic force. This is easily solved by a quadrature that leads to first class elliptic Jacobi functions [16]. Also when $C$ is not null the (54) (real part of (52)) can be solved by more complicated elliptic Jacobi functions.

Using (56) into (54) we obtain a quadrature from which

$$\dot{h}^2 = \frac{1}{4}(Bh^2 + Ah^6 - C^2h^{-2})$$

(57)

making the variable change $u = h^2$ the latest equation yields

$$t = \int \frac{du}{\sqrt{Au^3 + Bu - C^2}}$$

(58)

that is one Weirstrass elliptic function, where $A$, $B$, and $C$ are constants and $u(t)$ could be obtained by inverting $t(u)$ as an Jacobi elliptic function [16].

6 Conclusions

Starting for the BRST invariant effective action for the membrane theory we found new solutions to the non-linear field equation. The analysis was performed in a double dimensional reduction leading to a nonlinear field equation for a 1-dim extended object. In this reduction the membrane represents then an interacting theory of 10-dim strings, in agreement with [17].

We constructed the general solitary wave satisfying that non-linear differential system. The solitary waves present similar properties to the solitonic solutions on integrable systems like $KdV$, Sine Gordon, and other members of their hierarchy. It is natural to conjecture that this solutions are true solitons of the non linear field equations giving then strong evidence that the system is integrable in the sense that can be completely resolved by using the inverse scattering method. Pointing in that direction, a particular construction was presented in 4-dim relating the non-linear system to the non-linear Schrödinger equation.

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