A Stochastically Perturbed Particle Swarm Optimization for Identical Parallel Machine Scheduling Problems

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1. Introduction

Identical parallel machine scheduling (PMS) problems with the objective of minimizing makespan (C_max) is one of the well known NP-hard [1] combinatorial optimization problems. It is unlikely to obtain optimal schedule through polynomial time-bounded algorithms. Small size instances of PMS problem can be solved with reasonable computational time by exact algorithms such as branch-and-bound [2, 3], and the cutting plane algorithm [4]. However, as the problem size increases, the computation time of exact methods increases exponentially. On the other hand, heuristic algorithms generally have acceptable time and memory requirements, but do not guarantee optimal solution. That is, a feasible solution is obtained which is likely to be either optimal or near optimal. The well-known longest processing time (LPT) rule of Graham [5] is a sort of so called list scheduling algorithm. It is known that the rule works very well when makespan is taken as the single criterion [6]. Later, Coffman et al. [7] proposed MULTIFIT algorithm that considers the relation between bin-packing and maximum completion time problems. Yue [8] showed that the MULTIFIT heuristic is not guaranteed to perform better than LPT for every problem. Gupta and Ruiz-Torres [9] developed a LISTFIT algorithm that combines the bin packing method of the MULTIFIT heuristic with multiple lists of jobs. Min and Cheng [10] introduced a genetic algorithm (GA) that outperformed simulated annealing (SA) algorithm. Lee et al. [11] proposed a SA algorithm for the PMS problems and compared their results with the LISTFIT algorithm. Tang and Luo [12] developed a new iterated local search (ILS) algorithm that is based on varying number of cyclic exchanges.

Particle swarm optimization (PSO) is based on the metaphor of social interaction and communication among different spaces in nature, such as bird flocking and fish schooling. It is different from other evolutionary methods in a way that it does not use the genetic operators (such as crossover and mutation), and the members of the entire population are maintained throughout the search procedure. Thus, information is socially shared among
individuals to direct the search towards the best position in the search space. In a PSO algorithm, each member is called a particle, and each particle moves around in the multi-dimensional search space with a velocity constantly updated by the particle's experience, the experience of the particle's neighbours, and the experience of the whole swarm. PSO was first introduced to optimize various continuous nonlinear functions by Eberhart and Kennedy [13]. PSO has been successfully applied to a wide range of applications such as automated drilling [14], home care worker scheduling [15], neural network training [16], permutation flow shop sequencing problems [17], job shop scheduling problems [18], and task assignment [19]. More information about PSO can be found in Kennedy et al. [20].

The organization of this chapter is as follows: Section II introduces PMS problem, the way how to represent the problem, lower bound of the problem and overview of the classical PSO algorithm. The third section reveals the proposed heuristic algorithm. The computational results are reported and discussed in the fourth section, while the fifth section includes the concluding remarks.

2. Background
2.1 Problem description
The problem of identical parallel machine scheduling is about creating schedules for a set \( J = \{J_1, J_2, J_3, \ldots, J_n\} \) of \( n \) independent jobs to be processed on a set \( M = \{M_1, M_2, M_3, \ldots, M_m\} \) of \( m \) identical machines. Each job should be carried out on one of the machines, where the time required for processing job \( i \) on a machine is denoted by \( p_i \). The subset of jobs assigned to machine \( M_i \) in a schedule is denoted by \( S_{M_i} \). Once a job begins processing, it must be completed without interruption. Furthermore, each machine can process one job at a time, and there is no precedence relation between the jobs. The aim is to find a permutation for the \( n \) jobs to machines from set \( M \) so as to minimize the maximum completion time, in other words the makespan. The problem is denoted as \( P | \ | C_{\text{max}} \), where \( P \) represents identical parallel machines, the jobs are not constrained, and the objective is to obtain the minimum length schedule. An integer programming formulation of the problem that minimize the makespan is as follows: [5]

\[
\min y
\]

subject to:

\[
\sum_{j=1}^{m} x_{ij} = 1, \ 1 \leq i \leq n, \quad (1)
\]

\[
y - \sum_{j=1}^{m} p_j x_{ij} \geq 0, \ 1 \leq j \leq m \quad (2)
\]

where the optimal value of \( y \) is \( C_{\text{max}} \) and \( x_{ij}=1 \) when job \( i \) is assigned to machine \( j \), otherwise \( x_{ij}=0 \).
2.2 Solution representation and lower bound

The solution for the PMS problem is represented as a permutation of integers $\Pi = \{1, ..., n\}$ where $\Pi$ defines the processing order of the jobs. As mentioned in the text above, three versions of the PSO algorithm are compared in terms of solution quality and CPU time.

In continuous based PSO by Tasgetiren et al. [17], PSO$_{spv}$, particles themselves do not present permutations. Instead, the SPV rule is used to derive a permutation from the position values of the particle. In discrete PSO by Pan et al.[21] and the proposed algorithm (SPPSO), on the other hand, the particles present permutations themselves.

| Jobs | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|---|---|---|---|---|---|---|---|---|
| $p_i$ | 7 | 7 | 6 | 6 | 5 | 5 | 4 | 4 | 4 |

Table 1. An example of 9-job × 4-machine PMS problem

For all of the three algorithms, the process of finding makespan value for a particle can be illustrated by an example. Namely, let’s assume a permutation vector of $\Pi = \{1 \ 8 \ 3 \ 4 \ 5 \ 6 \ 7 \ 2 \ 9\}$. By considering 4 parallel machines and 9 jobs, whose processing times are given in Table 1, the makespan value of the given vector is depicted in Figure 1.

![Fig. 1. Schedule generated from random sequence](image)

According to the schedule, each value of the vector is iteratively assigned to the most available machine. First four elements of the permutation vector (1,8,3,4) are assigned to the four machines respectively. The remaining jobs are assigned one by one to the first machine available. For instance, 5 goes to second machine ($M_2$), since it is the first machine released. If there is more than one available machine at the time, the job will be assigned randomly (ties can be broken arbitrarily). The makespan value of the given sequence is $C_{\text{max}}(\Pi) = 14$, as can easily be seen in figure 1.

The lower bound for $P\mid \mid C_{\text{max}}$ is calculated as follows [22]:

$$LB(C_{\text{max}}) = \max\left\{ \frac{1}{m} \sum_{i=1}^{n} p_i, \max\{p_i\} \right\}$$

(3)

It is obtained by assuming that preemption is not allowed. If $C_{\text{max}}(\Pi) = LB(C_{\text{max}})$, the current solution($\Pi$) is optimum. So, lower bound will be used as one of the termination criteria.
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throughout this chapter. The lower bound of the example presented in Table 1 can be calculated as:

\[ \text{LB}(C_{\text{max}}) = \max \left\{ \frac{1}{4} \sum_{i=1}^{2} p_i \right\} ; \ \max \{p_i\} \right\} = \max(12;7) = 12 \]

2.3 Classic Particle Swarm Optimization

In PSO, each single solution, called a particle, is considered as an individual, the group becomes a swarm (population) and the search space is the area to explore. Each particle has a fitness value calculated by a fitness function, and a velocity to fly towards the optimum. All particles fly across the problem space following the particle that is nearest to the optimum. PSO starts with an initial population of solutions, which is updated iteration-by-iteration. The principles that govern PSO algorithm can be stated as follows:

- \( n \) dimensional position (\( X_i = (x_{i1}, x_{i2}, ..., x_{in}) \)) and velocity vector (\( V_i = (v_{i1}, v_{i2}, ..., v_{in}) \)) for \( i^{\text{th}} \) particle starts with a random position and velocity.
- Each particle knows its position and value of the objective function for that position. The best position of \( i^{\text{th}} \) particle is donated as \( P_i = (p_{i1}, p_{i2}, ..., p_{in}) \), and the best position of the whole swarm as, \( G = (g_1, g_2, ..., g_n) \) respectively. The PSO algorithm is governed by the following main equations:

\[
\begin{align*}
\dot{v}_{in}^{t+1} &= wv_{in}^t + c_1 r_1 (p_{in}^t - x_{in}^t) + c_2 r_2 (g_{in}^t - x_{in}^t), \\
\dot{x}_{in}^{t+1} &= v_{in}^{t+1} + x_{in}^t
\end{align*}
\]  

(4)

where \( t \) represents the iteration number, \( w \) is the inertia weight which is a coefficient to control the impact of the previous velocities on the current velocity. \( c_1 \) and \( c_2 \) are called learning factors. \( r_1 \) and \( r_2 \) are uniformly distributed random variables in \([0,1]\).

The original PSO algorithm can optimize problems in which the elements of the solution space are continuous real numbers. The major obstacle for successfully applying PSO to combinatorial problems in the literature is due to its continuous nature. To remedy this drawback, Tasgetiren et al. [17] presented the smallest position value (SPV) rule. Another approach to tackle combinatorial problems with PSO is done by Pan et al. [21]. They generate a similar PSO equation to update the particle’s velocity and position vectors using one and two cut genetic crossover operators.

3. The proposed Stochastically Perturbed Particle Swarm Optimization algorithm

In this chapter, a stochastically perturbed particle swarm optimization algorithm (SPPSO) is proposed for the PMS problems. The initial population is generated randomly. Initially, each individual with its position, and fitness value is assigned to its personal best (i.e., the best value of each individual found so far). The best individual in the whole swarm with its position and fitness value, on the other hand, is assigned to the global best (i.e., the best particle in the whole swarm). Then, the position of each particle is updated based on the personal best and the global best. These operations in SPPSO are similar to classical PSO.
algorithm. However, the search strategy of SPPSO is different. That is, each particle in the
swarm moves based on the following equations.

\[ s_i = w^t \oplus \eta(X^i_t) \]

\[ w^{t+1} = w \cdot \beta \]

\[ s_2 = c_1 \oplus \eta(P^t_i) \]

\[ s_3 = c_2 \oplus \eta(G^t) \]

\[ X^t_i = \text{best}(s_1; s_2; s_3) \]

At each iteration, the position vector of each particle, its personal best and the global best are
considered. First of all, a random number of \( U(0,1) \) is generated to compare with the inertia
weight to decide whether to apply \( \text{Insert function}(\eta) \) to the particle or not.

\( \text{Insert function}(\eta) \) implies the insertion of a randomly chosen job in front (or back
sometimes) of another randomly chosen job. For instance, for the PMS problem, suppose a
sequence of \{3, 5, 6, 7, 8, 9, 1, 2, 4\}. In order to apply \( \text{Insert function} \), we also need to derive
two random numbers; one is for determining the job to change place and the other is for the
job in front of which the former job is to be inserted. Let’s say those numbers are 3 and 5
(that is, the third job will move in front of the fifth. In other words, job no.6 will be inserted
in front of job no.8 \{3, 5, 6, 7, 8, 9, 1, 2, 4\}). The new sequence will be \{3, 5, 7, 8, 6, 9, 1, 2, 4\}.

If the random number chosen is less than the inertia weight, the particle is manipulated with
this \( \text{Insert function} \) function, and the resulting solution, say \( s_1 \), is obtained. Meanwhile, the inertia
weight is discounted by a constant factor at each iteration, in order to tighten the
acceptability of the manipulated particle for the next generation, that is, to diminish the
impact of the randomly operated solutions on the swarm evolution.

The next step is to generate another random number of \( U(0,1) \) to be compared with \( c_1 \),
cognitive parameter, to make a decision whether to apply \( \text{Insert function} \) to personal best of
the particle considered. If the random number is less than \( c_1 \), then the personal best of the
particle undertaken is manipulated and the resulting solution is spared as \( s_2 \). Likewise, a
third random number of \( U(0,1) \) is generated for making a decision whether to manipulate
the global best with the \( \text{Insert function} \). If the random number is less than \( c_2 \), social
parameter, then \( \text{Insert} \) is applied to the global best to obtain a new solution of \( s_3 \). Unlike the
case of inertia weight, the values of \( c_1 \) and \( c_2 \) factors are not increased or decreased
iteratively, but are fixed at 0.5. That means the probability of applying \( \text{Insert function} \) to the
personal and global bests remains the same. The new replacement solution is selected
among \( s_1, s_2 \) and \( s_3 \) based on their fitness values. This solution may not always be better
than the current solution. This is to keep the swarm diverse. The convergence is traced by
checking the personal best of each new particle and the global best. As it is seen, proposed
equations have all major characteristics of the classical PSO equations. The following
pseudo-code describes in detail the steps of the SPPSO algorithm.

It can be seen from the pseudo-code of the algorithm that the algorithm has all major
characteristics of the classical PSO, the search strategy of the algorithm is different in a way
that the new solution is selected among $s_1$, $s_2$, and $s_3$, based on their fitness values. The selected particle may be worse than the current solution that keep the swarm diverse. The convergence is obtained by changing the personal best of each new particle and the global best.

```
Begin
Initialize particles (population) randomly
For each particle
    Calculate fitness value
    Set to position vector and fitness value as personal best ($P_i^t$)
    Select the best particle and its position vector as global best ($G_t$)
End
Do{
    Update inertia weight
    For each particle
        Apply insert with the probability of inertia weight ($s_1$)
        Apply insert to ($P_i^t$) with the probability of $c_1$ ($s_2$)
        Apply insert to ($G_t$) with the probability of $c_2$ ($s_3$)
        Select the best one among the $s_1$, $s_2$, and $s_3$
        Update personal best ($P_i^t$)
    End
    Update global best ($G_t$)
}While (Maximum Iteration is not reached)
End
```

Fig. 2. Pseudo code of the proposed SPPSO algorithm for PMS problem

4. Computational results

In this section, a comparison study is carried out on the effectiveness of the proposed SPPSO algorithm. SPPSO was exclusively tested in comparison with two other recently introduced PSO algorithms: PSO$_{spv}$ algorithm of Tasgetiren et al. [17] and DPSO algorithm of Pan et al. [21]. Two experimental frameworks, namely E1 and E2, are considered implying the type of discrete uniform distribution used to generate job-processing times. That is, the processing time of each job is generated by using uniform distribution of $U[1,100]$ and $U[100,800]$ for experiments E1 and E2 respectively. All SPPSO, PSO$_{spv}$ and DPSO algorithms are coded in C and run on a PC with the configuration of 2.6 GHz CPU and 512MB memory. The size of the population considered by all algorithms is the number of jobs ($n$).

For SPPSO and DPSO, the social and cognitive parameters were taken as $c_1 = c_2 = 0.5$, initial inertia weight is set to 0.9 and never decreased below 0.40, and the decrement factor $\beta$ is fixed at 0.999. For the PSO$_{spv}$ algorithm, the social and cognitive parameters were fixed at $c_1 = c_2 = 2$, initial inertia weight is set to 0.9 and never decreased below 0.40, and the decrement factor $\beta$ is selected as 0.999. The algorithms were run for 20000/$n$ iterations. All the there algorithms were applied without embedding any kind of local search.

The instances of problems were generated for 3, 4, 5, 10, 20, 30, 40, 50 machines and 20, 50, 100, 200, and 500 jobs. In order to allow for the variations, 10 instances are generated for each problem size. Hence, the overall number of instances added up to 350. The measures considered in this chapter are mainly about the solution quality. The performance measure
is a relative quality measure, C/LB, where C is the result achieved (makespan) by the algorithm and LB is the lower bound of the instance which is calculated in Eq. (3). Once C catches LB, the index results 1.0, otherwise remains larger.

| m  | n  | PSO<sub>spv</sub> |         |         |         | DPSO |         |         |         | SPPSO |         |         |
|----|----|-------------------|---------|---------|---------|------|---------|---------|---------|-------|---------|---------|
| 3  | 20 | 1.000 1.000 1.000 | 1.000 1 | 1.000 1 | 1.000 1 | 1.000 1 | 1.000 1 | 1.000 1 | 1.000 1 |
| 5  | 20 | 1.000 1.000 1.000 | 1.000 1 | 1.000 1 | 1.000 1 | 1.000 1 | 1.000 1 | 1.000 1 | 1.000 1 |
| 10 | 20 | 1.050 1.091 1.168 | 1.050 1 | 1.091 1 | 1.168 1 | 1.050 1 | 1.091 1 | 1.168 1 | 1.050 1 |
| 5  | 50 | 1.015 1.026 1.050 | 1.033 1 | 1.043 1 | 1.053 1 | 1.009 1 | 1.024 1 | 1.050 1 | 1.020 1 |
| 10 | 50 | 1.007 1.009 1.013 | 1.025 1 | 1.029 1 | 1.037 1 | 1.004 1 | 1.009 1 | 1.013 1 | 1.007 1 |
| 20 | 50 | 1.006 1.007 1.010 | 1.013 1 | 1.015 1 | 1.018 1 | 1.004 1 | 1.006 1 | 1.008 1 | 1.007 1 |
| 50 | 50 | 1.004 1.006 1.007 | 1.006 1 | 1.007 1 | 1.009 1 | 1.002 1 | 1.003 1 | 1.005 1 | 1.007 1 |
| 30 | 50 | 1.066 1.154 1.266 | 1.076 1 | 1.161 1 | 1.266 1 | 1.066 1 | 1.154 1 | 1.266 1 | 1.066 1 |
| 100| 50 | 1.013 1.022 1.028 | 1.043 1 | 1.061 1 | 1.072 1 | 1.019 1 | 1.029 1 | 1.039 1 | 1.027 1 |
| 200| 50 | 1.009 1.017 1.021 | 1.032 1 | 1.037 1 | 1.043 1 | 1.014 1 | 1.017 1 | 1.020 1 | 1.016 1 |
| 500| 50 | 1.009 1.011 1.015 | 1.011 1 | 1.016 1 | 1.021 1 | 1.008 1 | 1.009 1 | 1.011 1 | 1.007 1 |
| 40 | 100| 1.282 1.538 1.707 | 1.282 1 | 1.538 1 | 1.707 1 | 1.282 1 | 1.538 1 | 1.707 1 | 1.282 1 |
| 100| 100| 1.033 1.047 1.067 | 1.084 1 | 1.115 1 | 1.142 1 | 1.042 1 | 1.055 1 | 1.061 1 | 1.042 1 |
| 200| 100| 1.021 1.028 1.034 | 1.054 1 | 1.067 1 | 1.075 1 | 1.028 1 | 1.035 1 | 1.042 1 | 1.034 1 |
| 500| 100| 1.016 1.019 1.022 | 1.025 1 | 1.030 1 | 1.031 1 | 1.016 1 | 1.020 1 | 1.026 1 | 1.025 1 |
| 500| 100| 1.070 1.088 1.114 | 1.156 1 | 1.184 1 | 1.220 1 | 1.070 1 | 1.097 1 | 1.140 1 | 1.097 1 |
| 200| 100| 1.036 1.044 1.053 | 1.081 1 | 1.096 1 | 1.106 1 | 1.049 1 | 1.057 1 | 1.065 1 | 1.057 1 |
| 500| 100| 1.023 1.027 1.030 | 1.034 1 | 1.043 1 | 1.046 1 | 1.028 1 | 1.032 1 | 1.035 1 | 1.032 1 |

Table 2. Results for experiment E1:p~U(1,100)
| m | n | min | avg | max | min | avg | max | min | avg | max |
|---|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 3 | 20 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 50 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 100 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 200 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 500 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 4 | 20 | 1.000 | 1.001 | 1.001 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 50 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 100 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 200 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 500 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 5 | 20 | 1.001 | 1.002 | 1.003 | 1.001 | 1.002 | 1.003 | 1.001 | 1.001 | 1.002 |
| 50 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 100 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 200 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 500 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 10 | 20 | 1.046 | 1.071 | 1.128 | 1.040 | 1.068 | 1.128 | 1.040 | 1.068 | 1.128 |
| 50 | 1.001 | 1.003 | 1.005 | 1.004 | 1.010 | 1.006 | 1.004 | 1.006 | 1.004 |
| 100 | 1.000 | 1.001 | 1.001 | 1.003 | 1.004 | 1.004 | 1.001 | 1.001 | 1.001 |
| 200 | 1.000 | 1.000 | 1.001 | 1.002 | 1.003 | 1.003 | 1.001 | 1.001 | 1.001 |
| 500 | 1.000 | 1.000 | 1.000 | 1.001 | 1.002 | 1.002 | 1.000 | 1.000 | 1.000 |
| 20 | 50 | 1.022 | 1.067 | 1.113 | 1.026 | 1.037 | 1.054 | 1.011 | 1.019 | 1.025 |
| 100 | 1.012 | 1.016 | 1.021 | 1.012 | 1.023 | 1.029 | 1.006 | 1.006 | 1.007 |
| 200 | 1.002 | 1.005 | 1.010 | 1.011 | 1.014 | 1.017 | 1.003 | 1.003 | 1.004 |
| 500 | 1.000 | 1.001 | 1.002 | 1.005 | 1.007 | 1.009 | 1.001 | 1.002 | 1.003 |
| 30 | 50 | 1.080 | 1.122 | 1.195 | 1.096 | 1.128 | 1.195 | 1.080 | 1.123 | 1.195 |
| 100 | 1.016 | 1.029 | 1.043 | 1.038 | 1.055 | 1.065 | 1.012 | 1.015 | 1.016 |
| 200 | 1.012 | 1.017 | 1.022 | 1.027 | 1.033 | 1.037 | 1.008 | 1.010 | 1.012 |
| 500 | 1.005 | 1.006 | 1.007 | 1.012 | 1.015 | 1.017 | 1.005 | 1.007 | 1.008 |
| 40 | 50 | 1.268 | 1.378 | 1.534 | 1.268 | 1.378 | 1.534 | 1.268 | 1.378 | 1.534 |
| 100 | 1.024 | 1.069 | 1.095 | 1.077 | 1.093 | 1.102 | 1.022 | 1.029 | 1.036 |
| 200 | 1.016 | 1.022 | 1.028 | 1.046 | 1.057 | 1.066 | 1.015 | 1.019 | 1.021 |
| 500 | 1.009 | 1.010 | 1.011 | 1.022 | 1.025 | 1.027 | 1.011 | 1.012 | 1.014 |
| 50 | 100 | 1.034 | 1.052 | 1.084 | 1.121 | 1.154 | 1.166 | 1.047 | 1.060 | 1.084 |
| 200 | 1.007 | 1.011 | 1.022 | 1.076 | 1.086 | 1.099 | 1.026 | 1.032 | 1.035 |
| 500 | 1.001 | 1.003 | 1.007 | 1.034 | 1.039 | 1.044 | 1.015 | 1.019 | 1.022 |
| Average | 1.016 | 1.025 | 1.038 | 1.026 | 1.035 | 1.046 | 1.016 | 1.023 | 1.033 |

Table 3. Results for experiment E2:p~U(100,800)
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| m  | n  | nopt | CPU | m  | n  | nopt | CPU | m  | n  | nopt | CPU | m  | n  | nopt | CPU |
|----|----|------|-----|----|----|------|-----|----|----|------|-----|----|----|------|-----|
| 3  | 20 | 10   | 0.008 | 10  | 0.266 | 10  | 0.014 | 10  | 0.308 | 10  | 0.005 | 10  | 0.241 |
| 50 | 10 | 0.015 | 10  | 0.571 | 10  | 0.008 | 10  | 0.077 | 10  | 0.003 | 10  | 0.029 |
| 100| 10 | 0.038 | 9    | 2.020 | 10  | 0.010 | 10  | 0.091 | 10  | 0.005 | 10  | 0.023 |
| 200| 10 | 0.310 | 9    | 8.054 | 10  | 0.044 | 10  | 0.239 | 10  | 0.019 | 10  | 0.062 |
| 500| 10 | 3.172 | 10   | 57.143| 10  | 0.259 | 10  | 1.437 | 10  | 0.083 | 10  | 0.180 |
| 4  | 20 | 10   | 0.112 | 1    | 1.007 | 10  | 0.201 | 3   | 0.383 | 10  | 0.096 | 4   | 0.406 |
| 50 | 10 | 0.013 | 2    | 0.836 | 10  | 0.055 | 7   | 0.294 | 10  | 0.024 | 10  | 0.202 |
| 100| 10 | 0.027 | 9    | 1.676 | 10  | 0.059 | 8   | 0.355 | 10  | 0.019 | 10  | 0.126 |
| 200| 10 | 0.202 | 9    | 4.391 | 10  | 0.115 | 7   | 0.865 | 10  | 0.053 | 10  | 0.239 |
| 500| 10 | 3.169 | 10   | 11.438| 10  | 1.085 | 10  | 3.635 | 10  | 0.234 | 10  | 0.485 |
| 5  | 20 | 7    | 0.206 | 8    | 0.218 | 0    | 0.363 | 9   | 0.233 | 0   | 0.430 |
| 50 | 9  | 0.084 | 8    | 0.678 | 10  | 0.134 | 1   | 0.274 | 10  | 0.052 | 5   | 0.286 |
| 100| 8  | 0.028 | 5    | 2.308 | 10  | 0.199 | 3   | 0.424 | 10  | 0.072 | 9   | 0.255 |
| 200| 9  | 0.408 | 9    | 4.877 | 10  | 0.397 | 2   | 1.023 | 10  | 0.127 | 9   | 0.357 |
| 500| 6  | 3.177 | 9    | 15.739| 10  | 2.502 | 3   | 4.576 | 10  | 0.453 | 9   | 0.720 |
| 10 | 20 | 0    | 0.414 | 0    | 0.374 | 0    | 0.401 | 0   | 0.559 | 0   | 0.449 |
| 50 | 5  | 0.799 | 0    | 0.922 | 0    | 0.322 | 0   | 0.329 | 8   | 0.344 | 0   | 0.399 |
| 100| 4  | 0.778 | 1    | 2.853 | 0    | 0.512 | 0   | 0.542 | 8   | 0.354 | 0   | 0.435 |
| 200| 0  | 0.208 | 1    | 10.314| 0    | 1.189 | 0   | 1.259 | 5   | 0.630 | 0   | 0.673 |
| 500| 0  | 3.194 | 5    | 52.414| 0    | 4.869 | 0   | 5.207 | 2   | 1.347 | 0   | 1.439 |
| 20 | 50 | 0    | 0.960 | 0    | 1.514 | 0    | 0.438 | 0   | 0.446 | 0   | 0.450 | 0   | 0.471 |
| 100| 0  | 2.840 | 0    | 2.883 | 0    | 0.627 | 0   | 0.650 | 0   | 0.510 | 0   | 0.551 |
| 200| 0  | 10.385| 0    | 10.671| 0    | 1.397 | 0   | 1.451 | 0   | 0.806 | 0   | 0.862 |
| 500| 0  | 52.525| 0    | 67.284| 0    | 5.334 | 0   | 5.643 | 0   | 1.750 | 0   | 1.853 |
| 30 | 50 | 0    | 1.636 | 0    | 1.631 | 0    | 0.459 | 0   | 0.469 | 0   | 0.485 | 0   | 0.504 |
| 100| 0  | 2.842 | 0    | 2.898 | 0    | 0.643 | 0   | 0.674 | 0   | 0.561 | 0   | 0.607 |
| 200| 0  | 10.495| 0    | 11.330| 0    | 1.455 | 0   | 1.532 | 0   | 0.906 | 0   | 0.972 |
| 500| 0  | 59.247| 0    | 66.154| 0    | 5.550 | 0   | 5.940 | 0   | 1.978 | 0   | 2.324 |
| 40 | 50 | 0    | 1.684 | 0    | 1.636 | 0    | 0.497 | 0   | 0.522 | 0   | 0.518 | 0   | 0.590 |
| 100| 0  | 2.984 | 0    | 2.873 | 0    | 0.699 | 0   | 0.742 | 0   | 0.620 | 0   | 0.726 |
| 200| 0  | 10.625| 0    | 10.531| 0    | 1.568 | 0   | 1.667 | 0   | 1.022 | 0   | 1.164 |
| 500| 0  | 59.573| 0    | 65.551| 0    | 5.829 | 0   | 6.292 | 0   | 2.244 | 0   | 2.548 |
| 50 | 100| 0    | 3.658 | 0    | 3.626 | 0    | 0.813 | 0   | 0.861 | 0   | 0.697 | 0   | 0.745 |
| 200| 0  | 10.702| 0    | 10.556| 0    | 1.680 | 0   | 1.763 | 0   | 1.140 | 0   | 1.247 |
| 500| 0  | 65.759| 0    | 65.793| 0    | 6.117 | 0   | 6.465 | 0   | 2.521 | 0   | 2.844 |

| Total | 148 | 117 | 148 | 94 | 172 | 126 |
|-------|-----|-----|-----|----|-----|-----|
| Average | 8.922 | 14.385 | 1.305 | 1.634 | 0.598 | 0.727 |

Table 4. Results for both experiments
The results for the instances with different sizes are shown in Table 3 and Table 4, where the minimum, average and maximum of the \( C/LB \) ratio are presented. Each line summarizes the values for the 10 instances of each problem size, where 10 replications are performed for each instance.

The result for the experiment E1, in which processing times are generated by using \( U(1,100) \) are summarized in Table 2. In this experiment, it is found that the minimum, average and maximum values of the ratios are quite similar for SPPSO and PSO\(_{spv}\). On the other hand, SPPSO and PSO\(_{spv}\) performed better than DPSO.

The result for the experiment E2 in which processing times are generated by using \( U(100,800) \) are summarized in Table 3. In this experiment, there is also no significant difference between SPPSO and PSO\(_{spv}\). However, in terms of max ratio performance SPPSO performed slightly better than PSO\(_{spv}\). In addition, PSO\(_{spv}\) and SPPSO are also better than DPSO for all the three ratios in this experiment.

Table 4 shows the number of times the optimum is reached within the group (nopt) for each algorithm and their average CPU times in seconds for each experiment. Total number of optimum solutions obtained by PSO\(_{spv}\), DPSO and SPPSO for the both experiment are summarized as (148,148,172) and (117, 94,126) respectively. Here, the superiority of SPPSO over PSO\(_{spv}\) and DPSO is more pronounced in terms of number of total optimum solutions obtained.

In terms of the average CPU, SPPSO shows better performance than PSO\(_{spv}\) and DSPO. SPPSO (0.598, 0.727) is about 15 times faster than PSO\(_{spv}\) (8.922, 14,395) and about 2 times faster than DPSO (1.305, 1.634) in both experiments.

5. Conclusion

In this chapter, a stochastically perturbed particle swarm optimization algorithm (SPPSO) is proposed for identical parallel machine scheduling (PMS) problems. The SPPSO has all major characteristics of the classical PSO. However, the search strategy of SPPSO is different. The algorithm is applied to (PMS) problem and compared with two recent PSO algorithms. The algorithms are kept standard and not extended by embedding any local search. It is concluded that SPPSO produced better results than DPSO and PSO\(_{spv}\) in terms of number of optimum solutions obtained. In terms of average relative percent deviation, there is no significant difference between SPPSO and PSO\(_{spv}\). However, they are better than DPSO.

It also should be noted that, since PSO\(_{spv}\) considers each particle based on three key vectors; position \( (X_i) \), velocity \( (V_i) \), and permutation \( (\Pi_i) \), it consumes more memory than SPPSO. In addition, since DPSO uses one and two cut crossover operators in every iteration, implementation of DPSO to combinatorial optimization problems is rather cumbersome. The proposed algorithm can be applied to other combinatorial optimization problems such as flow shop scheduling, job shop scheduling etc. as future work.

6. References

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