The Uniqueness of the $\Theta^+$ Pentaquark$^1$

P.J. Mulders$^*$ and P. Jiménez Delgado$^*$

$^*$Department of Physics and Astronomy, Vrije Universiteit Amsterdam, De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands

Abstract. The existence of the $\Theta^+$ pentaquark requires a peculiar mechanism to explain its stability. Looking at quark clusters, notably diquark and triquark configurations, such a mechanism may be found in the color-magnetic interaction between quarks. It is possible to understand why the $\Theta^+$ is unique. Chiral dynamics, in particular the ease of pion emission, will render other members of the same flavor antidecuplet, such as the $\Xi^{--}$ very unstable.

INTRODUCTION

The existence of a $\Theta^+(1540)$ baryon with $I = 0$ and $Y = 2$ decaying into a kaon and a nucleon with a small width of less than 10 MeV, has been confirmed by several experiments [1, 2, 3, 4, 5, 6, 7, 8]. A problem is the distribution of masses, which spans the region from 1520 to 1540 MeV, but not in a way consistent with one object [9].

Baryon and meson resonances in hadron physics generally appear in SU(3) flavor multiplets. The lowest-dimensional possibility for the $\Theta^+(1540)$ is a flavor antidecuplet ($10^*$). Weak evidence exists for another exotic member in such an antidecuplet in the form of a $\Xi^{--}(1862)$ resonance [10], although there are by now many experiments that don’t see this state.

Interesting fact about the $\Theta^+(1540)$ was the accurate prediction made in the chiral soliton model [11], including the prediction of weak coupling to the $KN$-channel. The presence of anti-strangeness in the system, however, calls for an understanding in terms of quark models.

Baryons with $Y = 2$ require a minimal configuration containing four quarks and one antiquark. This opens up a multitude of possibilities and it is therefore surprising that there does not seem to be a rich spectrum of pentaquark states. The general qualitative understanding of the absence of multi-quark states (color singlet combinations other than $q\bar{q}$ and $qqq$ or $q^3$) is the possibility of fission of such configurations into simpler color singlets, $q^4\bar{q} \rightarrow (q^3)(q\bar{q})$ or $q^6 \rightarrow (q^3)(q^3)$. For such a mechanism one naturally expects widths larger than those for decays of baryons or mesons via $q\bar{q}$-creation, thus several hundreds MeV, unless the state is less than a few tens of MeV above threshold.

Actually, multiquark configurations may not be visible at all. The possibility of fission spoils the confinement, which is the necessary ingredient to produce hadrons. A way to deal with the ’artificial’ confinement in the case of multiquark states is the P-matrix

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formalism [12, 13] in which the ‘calculated’ configurations appear as the poles in the P-matrix. This quantity merely describes the boundary condition needed to match a short-range quark description onto hadronic decay channels and the poles do not (necessarily) show up as poles in the (physical) S-matrix.

Of course, the mechanism of fission is also impossible if the mass is below the threshold of the fission products. An example is the deuteron containing six quarks, which is stable against decay into two nucleons. The example of the deuteron also shows that a multi-quark configuration still allows many classes of configurations (see Fig. 1) including bound states of color singlet subsystems. This has been studied for the $KN$ system already in the sixties (see e.g. [14]) The small width of the $\Theta^+(1540)$ points to a specific internal structure that evades the fission procedure, such as was for instance suggested in the work of Jaffe and Wilczek [15]. This still asks for an explanation of the absence of rich spectra of multi-quark states such as studied already decades ago [16, 17].

Two possible ingredients that are relevant in a qualitative analysis of the stability of multi-quark states, are

1. The first is the observation that hadron spectra follow a constituent quark behavior with superimposed on that a color-magnetic spin-spin interaction.
2. The second ingredient is the chiral symmetry implementation in the form of the light pions, of which the masses are lighter than expected on the basis of (1).

### STABILITY OF MULTIQUARK CONFIGURATIONS

I will combine results of previous systematic studies of the spin-spin interaction in multi-quark clusters [16, 17] to look for exceptional configurations that experience a strong attractive color-magnetic interaction and that cannot be lowered by fissioning the system, adding (anti-)quarks, or emitting pions. The hyperfine structure in the spectrum of baryons and mesons can be understood in terms of the color spin-spin interaction.
arising from lowest-order one gluon exchange between pairs of quarks [18]

\[
\Delta E_{cm} = - \sum_{i > j} M_{ij} (F^a_i F^a_j) (\sigma_i \cdot \sigma_j)
\]

where \(\sigma_i\) are the spin matrices and \(F^a_i\) are the color charges of the \(i\)th constituent quark and \(M_{ij}\) measures the interaction strength, which in a specific model can be calculated and depends then e.g. on quark masses and/or size parameters. We will simply use an average strength in our estimates, in which case the color-magnetic interactions becomes proportional to a group-theoretical factor,

\[
\Delta = - \sum_{i > j} (F^a_i F^a_j) (\sigma_i \cdot \sigma_j).
\]

This expression will also be used for colored quark-clusters in which all quarks are assumed to be in relative s-waves.

The color-magnetic interaction in colorless \((q\bar{q})_1\) and \((q^3)_1\) situations (mesons and baryons) is given in Fig. 2. It explains e.g. the splitting between the baryon octet and decuplet, such as the \(N - \Delta\) splitting of about 300 MeV. Hence the proportionality constant relating the hyperfine structure and the factor \(\Delta\) is about 75 MeV. This also works fine for mesons like \(K\) and \(K^*\). With a flavor independent strength one should not expect extreme accuracy as is clear from the fact that \(\Lambda\) and \(\Sigma\) baryons would remain degenerate instead of the observed 70 MeV splitting, but this can be cured by using flavor dependence, in particular a slight decrease of the strength for strange quarks. At this stage we will not worry about these details. A notable exceptional hadron is the pion. The pion is much lighter (and \(\pi - \rho\) splitting is much larger) than expected on grounds of the naïve use of the color-magnetic interaction.

The calculation of color-magnetic hyperfine structure for s-wave \((q^4\bar{q})_1\) baryons was studied by Jaffe in the bag model [19]. The results for \(\Delta\) are shown in Fig. 3 and show the lowest configuration with \(\Delta = -9.045\). This, however, has flavor structure \(3 \times 3^* = 1 + 8\).
FIGURE 3. Color magnetic hyperfine structure for multiquark baryons. The configurations are labeled by flavor and spin (f,s)

FIGURE 4. Color magnetic hyperfine structure for two-quark and four-quark configurations, the latter only for nonstrange quarks. The configurations are labeled by flavor and spin (f,s) or isospin and spin (i,s)

hence non-exotic. It has been suggested that the $q^4\bar{q}$-configuration may be an important component of the $\Lambda(1405)$ hyperon near the $KN$-threshold. The configuration with flavor $6^* \times 3^* = 8 + 10^*$ does contain a $\Theta^+\bar{\Theta}^+$ candidate with spin-parity $1/2^-$, but it is unstable against fission into $KN$ (indicated as BP = Baryon-Pseudoscalar meson threshold in Fig. 3).

Turning attention to colored clusters, it is long known that the color magnetic interaction for $(q^2)_2^*$ prefers spin 0 diquarks (with antisymmetric flavor $3^*$ or for nonstrange quarks isospin $0$) over spin 1 diquarks (Fig. 4), which has been the basis of the expectation of a special role for spin 0 over spin 1 diquarks in hadron spectroscopy or in color superconductivity [15, 20]. Actually, having more quarks with a limited number of
flavors, in general, leads to color-magnetic repulsion. This phenomenon is for instance visible for \((q^0)\) (six quark) clusters with for only nonstrange quarks, for which the lowest possibility is \(\Delta = +2/3\), far above the \(\Delta = -4\) of the (open) baryon-baryon channel, which is the qualitative explanation of the repulsive core in the nucleon-nucleon interaction at the quark level. Also for \((q^4)\) the lowest possibility \((\Delta = -4/3)\) allows fission \((q^4) \rightarrow (q^3)\) (see Fig. 4).

Looking at \((q^2)^*\) configurations one of course can obtain a lower configuration with \(\Delta = -4\). For this one needs a spatial wave function that reduces the overlap between the clusters, turning off the short-range color-magnetic interaction between quarks belonging to the different clusters. Given the quantum numbers of the bosonic diquarks, one needs in that case an antisymmetric wave function, \([((q^2)^*)^*]_A\), realized e.g. through an orbital angular momentum \(\ell = 1\).

Actually, adding a strange antiquark to the nonstrange diquark leads to a strong enhancement of the color-magnetic interaction (from \(\Delta = -2\) to \(\Delta = -5.42\)), as shown in Fig. 5. This has led to suggestions for triquark-quark configurations [21, 22]. Actually, it is important to note that because of the identical structure of the two diquarks involved, the strange quark will oscillate between the diquark configurations, a mechanism proposed already in Ref. [23]. The situation resembles systems like the \(H_2^+\)-ion \((p - e - p)\) or \(^9\text{Be} (\alpha - n - \alpha)\) [24].

**CONCLUSIONS**

Quark clustering may provide an answer to the exotic structure of the \(\Theta^+\) pentaquark. The particular combination of two nonstrange diquarks and a strange antiquark may be a rather unique one. For instance the \([ud]\pi\)-configuration is stable against fission into \(u + K^-\) or \(d + K^0\) (with \(\Delta = -4\)). Although flavor symmetry is usually a good guidance in hadron spectroscopy, it may fail because the structure of \([ds]\pi\) that would be the one in the exotic \(\Xi^-\) pentaquark would most likely because of the exceptional role played
by pions be very unstable against fission into $s + \pi^-$. Hence the uniqueness could even prevail over flavor symmetry. If established beyond any doubt one shouldn’t expect a proliferation of such states. The mechanism to understand the qualitative clustering picture alluded to in this talk, however, would be a challenge in the study of nonperturbative QCD.

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