Light hadrons in 2+1 flavor lattice QCD

Urs M. Heller (For the MILC Collaboration)
American Physical Society, One Research Road, P.O. Box 9000, Ridge, NY 11961-9000, USA
E-mail: heller@aps.org

Abstract. This talk will focus on recent results by the MILC collaboration from simulations of light hadrons in 2+1 flavor lattice QCD. We have achieved high precision results in the pseudoscalar sector, including masses and decay constants, plus quark masses and Gasser-Leutwyler parameters from well controlled chiral perturbation theory fits to our data. We also show spectroscopy results for vector mesons and baryons.

1. Introduction
The MILC Collaboration has been engaged in the study of QCD with three flavors of improved staggered sea quarks [1] for some time. Our goals are to determine the hadron mass spectrum, the properties of light pseudoscalar mesons, the topological susceptibility, the decay properties of B and D mesons, and the behavior of strongly interacting matter at high temperatures, all in as realistic simulations as possible. In this talk I will concentrate on the light pseudoscalar sector and the light hadron spectroscopy [2, 3].

While we have made simulations with lighter u and d quark masses (with $m_u = m_d = m_l$) than have been reached before, to simulate at the physical mass value has not been possible so far even with the improved “asqtad” quarks [1]. We have created gauge field ensembles at two lattice spacings, $a \sim 0.125$ and $a \sim 0.09$ fm, with the strange quark mass $m_s$ fixed near its physical value and several light quark masses, as low as $m_l = 0.1 m_s$. This allows controlled chiral extrapolations to the physical light quark mass, and from the two lattice spacings, extrapolations to the continuum limit, $a \to 0$. The physical spatial size of our gauge field ensembles is $L \geq 2.5$ fm. At one set of simulation parameters we created ensembles with two physical sizes, $L = 2.5$ and $3.5$ fm, to check for finite volume effects. These were found to be small.

All gauge field ensembles created by the MILC collaboration are made available to other researchers at the “NERSC Gauge Connection”. They have been used, for example, by the Fermilab, HPQCD and UKQCD collaborations for the study of heavy quarkonia and heavy–light mesons. A comparison of quantities that can be computed on the lattice with small systematic uncertainties and that are well known experimentally shows agreement within errors of 1–3% [4].

2. The light pseudoscalar sector
In the pseudoscalar sector we have the most accurate data with many partially quenched measurements, i.e., measurements of pseudoscalar masses and decay constants with valence quark masses different from the sea quark masses. Fits, even at a single lattice spacing, to continuum partially quenched chiral perturbation theory (XPT) do not work [2]. Lattice effects,
in particular the $\mathcal{O}(\alpha_s^2 a^2)$ taste breaking effects of the improved staggered fermions, need to be taken into account.

The staggered $\chi$PT (S$\chi$PT) formalism for our case of 2+1 sea quark flavors with partially quenched measurements has been worked out in detail in [5]. All our data for $f_\pi$ and $m_\pi^2$ are fit to the S$\chi$PT form simultaneously, taking account of the taste violations. The continuum $\chi$PT parameters were allowed to have $\mathcal{O}(\alpha_s a^2)$ terms. After the fit, all the terms taking account of the finite lattice spacing effects were set to zero to take the continuum limit. Small finite volume errors ($< 1.5\%$) were corrected within the $\chi$PT framework. Details can be found in [2]. Sample plots for “full QCD”, i.e. with $m_{val} = m_{sea}$ are shown in Fig. 1.

**Figure 1.** Illustration of S$\chi$PT fit showing full QCD points for $f_\pi$ (left) and $m_\pi^2/(m_x + m_y)$ (right). Also shown are the fits extrapolated to the continuum limit, and after correcting from the simulation strange quark mass to its physical value.

The strange quark mass in the simulations, our best guess beforehand and denoted by $m'_s$ in Fig. 1, turned out a little larger than the physical strange quark mass, $m_s$. We can correct for this within the chiral fits. The corresponding curves are also shown in Fig. 1.

To find the quark masses, we must extrapolate to the physical meson masses. Electromagnetic and isospin-violating effects ($m_u \neq m_d$) are important. Denoting by $m_\pi$ and $m_K$ the masses with EM effects turned off and $m_u = m_d = \bar{m}$, i.e. what is done in our simulations, we have

\[ m_\pi^2 \approx \left(m_{\pi}^{\text{QCD}}\right)^2 \approx \left(m_{\pi}^{\text{expt}}\right)^2 \]
\[ m_K^2 \approx \left(m_{K}^{\text{QCD}}\right)^2 + \left(m_{K}^{\text{QCD}}\right)^2 \]

\[ (m_{K_0}^{\text{QCD}})^2 \approx (m_{K_0}^{\text{QCD}})^2 \]
\[ (m_{K_+}^{\text{QCD}})^2 \approx (m_{K_+}^{\text{QCD}})^2 - (1 + \Delta_E) \left(m_{\pi}^{\text{expt}} - m_{\pi}^{\text{expt}}\right)^2 \]

where the superscript QCD indicates the masses with EM effects turned off. $\Delta_E = 0$ is “Dashen’s theorem.” Continuum considerations suggest that $\Delta_E \approx 1$.

With the considerations in eq. (1) we found, in collaboration with the HPQCD and UKQCD groups, using a one-loop mass renormalization constant, [6]

\[ m_s^{\text{MS}}(2 \text{ GeV}) = 76(3)(7)(0) \text{ MeV} \]
Here, the errors are from statistics, simulation, perturbation theory, and EM effects (obtained from varying $\Delta E$ between 0 and 2), respectively.

For the pseudoscalar decay constants we obtain

$$f_{\pi} = 129.5 \pm 0.9 \pm 3.5 \text{ MeV}, \quad f_K = 156.6 \pm 1.0 \pm 3.6 \text{ MeV}, \quad f_K/f_{\pi} = 1.210(4)(13),$$

where the first error is statistical and the second is systematic. More details, including our results for the Gasser–Leutwyler chiral parameters and the ratio $m_u/m_d$ can be found in [2].

Marciano has pointed out [7] that an accurate determination of $f_K/f_{\pi}$ can be used to calculate the CKM matrix element $V_{us}$. With the above result we obtain $V_{us} = 0.2219(26)$. The error, already comparable to that from the standard method using $K_{\ell 3}$ decays, is dominated by the lattice error on $f_K/f_{\pi}$. With the coming simulations, $V_{us}$ will be known more precisely.

3. Light hadron spectroscopy

In the pseudoscalar sector the statistical errors on the correlation functions were small enough that we could get some signal for excited states, as shown in Fig. 2. It is noteworthy that in kaon correlation functions we could observe an opposite parity state – correlation functions with staggered fermions generically contain opposite parity states – with energy close to $m_\pi + m_K$, i.e. the expected decay products of the opposite parity excited meson. No such state was found in matched quenched simulations – absence of virtual quark loops in quenched QCD prevents emergence of the “$\pi + K$” intermediate state.

Our vector meson and nucleon masses are shown in Figs. 3 and 4. In the latter we also show some possible chiral extrapolations, after taking the continuum limit at fixed $m_l/m_s$. For further details and more spectroscopy results see [3].

We end by showing a summary of our current spectroscopy results in Fig. 5.
Figure 3. Vector meson masses.

Figure 4. Nucleon masses.

Figure 5. Comparison of our hadron spectroscopy, from a crude chiral and continuum extrapolation, with experiment. $\pi$ and $K$ were used to set the light and strange quark masses. The upsilon and charmonium columns are differences from the ground state (1S) masses, from work of the HPQCD and Fermilab groups [4, 8]. The $T$ 1P-1S splitting was used to fix the lattice spacing.

References

[1] Kostas Orginos and Doug Toussaint, Phys. Rev. D 59, 014501, (1999); Kostas Orginos, Doug Toussaint and R.L. Sugar, Phys. Rev. D 60, 054503 (1999); G.P. Lepage, Phys. Rev. D 59, 074502 (1999).

[2] The MILC Collaboration: C. Aubin et al., Phys. Rev. D 70 114501 (2004).

[3] The MILC Collaboration: C. Bernard et al., Phys. Rev. D 64, 054506 (2001); Phys. Rev. D 70 094505 (2004).

[4] The Fermilab, HPQCD, MILC, and UKQCD Collaborations: C.T.H. Davies et al., Phys. Rev. Lett. 92, 022001 (2004).

[5] C. Bernard, Phys. Rev. D 65, 054031 (2002); C. Aubin and C. Bernard, Phys. Rev. D 68, 034014 (2003); Phys. Rev. D 68, 074011 (2003).

[6] The HPQCD, MILC and UKQCD Collaborations: C. Aubin, et al., Phys. Rev. D 70 031504(R) (2004).

[7] W. J. Marciano, Phys. Rev. Lett. 93, 231803 (2004).

[8] C.T.H. Davies et al., private communication; M. Wingate et al., Phys. Rev. Lett. 92, 162001 (2004).