Possible violation of Newtonian gravitational law at small distances and constraints on it from the Casimir effect

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The recent ideas that the gravitational and gauge interactions become united at the weak scale lead to Yukawa-type corrections to the Newtonian gravitational law at small distances. We briefly summarize the best constraints on these corrections obtained recently from the experiments on the measurement of the Casimir force. The new constraints on the Yukawa-type interaction are derived from the latest Casimir force measurement between a large gold coated sphere and a flat disk using an atomic force microscope. The obtained constraints are stronger up to 19 times comparing the previous experiment with aluminum surfaces and up to 4500 times comparing the Casimir force measurements between dielectrics. The application range of constraints obtained by means of an atomic force microscope is extended.

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I. INTRODUCTION

It is common knowledge that the gravitational interaction holds a unique position with respect to the other fundamental interactions described by the Standard Model. Up to now, there is no renormalized Quantum Gravity and the obstacles placed on the way to such a theory seem to be insurmountable. Gravitational physics suffers also from a poor experimental test. Even the classical Newtonian gravitational law which is reliably proved at large distances is lacking experimental confirmation at the distances of order 1 mm or less. At the same time it is generally believed to be correct up to the Planck distance $\sqrt{\frac{G\hbar}{c^3}} \sim 10^{-35}$ m. Needless to say that it is a far-ranging extrapolation over 33 orders of magnitude.

Corrections to Newtonian gravitational law at small distances are predicted by unified gauge theories, supersymmetry, supergravity, and string theory. They are mediated by light and massless elementary particles like scalar axion, graviphoton, dilaton, arion, and others. The exchange of these particles between atoms leads to an interatomic interaction described by Yukawa- or power-type effective potentials. Constraints for their parameters (interaction constants $\alpha$, $\lambda_n$, and interaction range $\lambda$ in case of a massive particle) is the subject for considerable study (see the monograph [1] and references therein). The gravitational experiments of Cavendish- and Eötvös-type lead to rather strong constraints over a distance range $10^{-2}$ m $< \lambda < 10^6$ km [2]. At submillimeter range the existence of corrections to Newtonian gravity is not excluded experimentally which are in excess of it by many orders.

The only constraints on non-Newtonian gravity at a submillimeter range follow from the measurements of the van der Waals and Casimir force (see, e.g., [3–5]). Until recently, they were not enough restrictive in spite of the quick progress in experimental technique and increased accuracy of force measurements. In [3] the Casimir force between the metallic surfaces of a disk and a spherical lens was measured by the use of torsion pendulum. The obtained experimental results and the extent of their agreement with theory were used in [4] to obtain stronger constraints on the corrections to Newtonian gravity in a submillimeter range. The strengthening up to 30 times comparing the previously known constraints was obtained within the range $2.2 \times 10^{-7}$ m $\leq \lambda \leq 1.6 \times 10^{-4}$ m (see also [5]). In [6] the results of the Casimir force measurements between a metallized disk and a sphere attached to a cantilever of an atomic force microscope were reported and accurately confronted with a theory. They were used in [7,11] to constrain the non-Newtonian gravity. The strengthening of constraints up to 560 times comparing the former Casimir force measurements between dielectrics was obtained in the interaction range $5.9 \times 10^{-9}$ m $\leq \lambda \leq 1.15 \times 10^{-7}$ m.

In the present paper we discuss the possible violation of Newtonian gravitational law at small distances and constraints on it following from the Casimir effect. In Sec. 2 the recent impressive ideas are considered that the gravitational and gauge interactions become united at the weak scale (see, e.g., [14]). These ideas in the context of string

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theory inevitably lead to the existence of Yukawa-type corrections to the Newtonian gravitational law at moderate distances in addition to the well known arguments in support of such corrections presented above. Extra spatial dimensions cause even more drastic change of gravitational law at small distances. In Sec. 3 the constraints on the parameters of Yukawa-type interactions are discussed which were obtained recently from the Casimir force measurements between gold and aluminum surfaces. In Sec. 4 the new constraints on the Yukawa interaction are obtained following from the latest Casimir force measurements between gold surfaces by means of an atomic force microscope (these experimental results can be found in [12]). In Sec. 5 (conclusions and discussion) the importance of the new Casimir force measurements for the elementary particle physics, astrophysics and cosmology is underlined. Also the prospects for further strengthening of constraints on non-Newtonian gravity in the submillimeter range are outlined.

Below we use units in which $\hbar = c = 1$.

II. THE YUKAWA-TYPE CORRECTIONS TO NEWTONIAN GRAVITY

As was told in the introduction, the corrections to Newtonian gravitational law are predicted by the unified gauge theories, supersymmetry, supergravity, and string theory. The effective potential of gravitational interaction between two atoms with account of such corrections can be represented in the form

$$V(r_{12}) = -\frac{GM_1 M_2}{r_{12}} \left(1 + \alpha_G e^{-r_{12}/\lambda}\right),$$  \hspace{1cm} (1)

where $M_1$, $M_2$ are the masses of the atoms, $r_{12}$ is the distance between them, $G$ is Newtonian gravitational constant, $\alpha_G$ is a dimensionless interaction constant, $\lambda$ is the interaction range. In the case that the Yukawa-type interaction is mediated by a light particle of mass $m$ the interaction range is described by the Compton wave length of this particle, so that $\lambda = 1/m$.

According to recent ideas the gravitational and gauge interactions may become united at the weak scale $F \sim 1 \text{ TeV} = 10^3 \text{ GeV}$, and the weakness of gravity at macroscopic distances is explained by the existence of $n \geq 2$ compact (but rather large) extra spatial dimensions [14,16–18]. The consequence of these ideas is that the gravitational interaction is described by Eq. (1) for the distances much larger than the size of characteristic compactification dimension [19–21]. One can arrive to potential (1) as follows.

Let $n$ extra dimensions be compactified by making a periodic identification with a period $R$. If one mass $M_1$ is placed at the origin and the test mass $M_2$ is at the distance $r_{12} \ll R$ from $M_1$ the force law of $N = (4+n)$-dimensional space-time is

$$F_N(r_{12}) = -G_N \frac{M_1 M_2}{r_{12}^{N-2}},$$  \hspace{1cm} (2)

which provides the continuity of force lines in $(N-1)$-dimensional space. If $r_{12} \gg R$ one obtains the usual Newtonian force

$$F(r_{12}) = -G \frac{M_1 M_2}{r_{12}^2},$$  \hspace{1cm} (3)

where the Gauss law can be used to find the connection between the $N$-dimensional and the usual Newtonian gravitational constants [19]

$$G = \frac{\pi^{\frac{N-3}{2}}}{2\Gamma\left(\frac{N-1}{2}\right)} \frac{G_N}{R^{N-4}}.$$  \hspace{1cm} (4)

If there is no extra dimensions ($n = 0$, $N = 4$) one finds from (3) $G = G_N$ as it should be. Taking into account the connection between the gravitational constants $G$, $G_N$ and the respective Planckian energy scales

$$M_{Pl}^2 = \frac{1}{G}, \hspace{1cm} M_{Pl,N}^{N-2} = \frac{2\Gamma\left(\frac{N-1}{2}\right)}{G_N \pi^{\frac{N-3}{2}}},$$  \hspace{1cm} (5)

Eq. (4) turns out to be equivalent to

$$M_{Pl}^2 = \frac{M_{Pl,N}^{N-2} R^{N-4}}{G_N \pi^{\frac{N-3}{2}}}.$$  \hspace{1cm} (6)
This result gives the possibility to estimate the allowed values of the compactification dimension \( R \) and the required number of extra dimensions. Putting \( N \)-dimensional Planckean scale be equal to the weak scale, i.e. \( M_{Pl,N} = F \), one obtains from (\ref{eq:1}) \( \ref{eq:14} \)

\[
R = \frac{1}{F} \left( \frac{M_{Pl,N}}{F} \right)^{\frac{2}{N-2}} \sim 10^{\frac{30}{N-2}} \frac{1}{F} \sim 10^{\frac{30}{17}} \text{cm}
\]

(we remind that \( M_{Pl} \approx 2.4 \times 10^{18} \text{ GeV} \)). Evidently, one extra dimension is impermissible because for \( N = 5 \) it follows \( R \sim 10^{13} \text{ cm} \) which is in contradiction with the confirmed validity of Newtonian gravitational law at the scales of solar system. But already \( n = 2 \) (\( N = 6 \)) leads to \( R \sim 10^{-2} \text{ cm} \) which is permitted by the results of gravitational measurements.

The above considerations show that the gravitational law may vary from (\ref{eq:2}) at small, submillimeter distances to the usual form (\ref{eq:3}) at relatively large distances. The corrections to Eqs. (\ref{eq:2}), (\ref{eq:3}) can be found by considering the Newtonian limit of \( N \)-dimensional Einstein gravity \( \ref{eq:16} \). At the distances \( r_{12} \ll R \) the corrections to Eq. (\ref{eq:2}) are of power type. The corrections at the distances \( r_{12} \gg R \) hold the greatest interest because they can be tested experimentally. It is significant that they are of Yukawa-type, so that remote from the compactification scale the gravitational potential is given by Eq. (\ref{eq:1}). Coefficient \( \alpha_{G} \) of Eq. (\ref{eq:1}) depends on compactification geometry and on the number of extra dimensions. By way of example, if \( n \) extra dimensions have the topology of \( n \)-torus \( \alpha_{G}^{(n)} = 2n \), and if they have the topology of \( n \)-sphere \( \alpha_{G}^{(n)} = n + 1 \) \( \ref{eq:21} \).

The above models with \( n = 6 \) can be formulated within type I or type IIB string theories \( \ref{eq:16} \) (in the case of M theory \( n = 7 \)). In so doing, gravitons are described by closed strings and propagate in \( N \)-dimensional bulk. The particles of the Standard Model are described by open strings living on \( (3+1) \)-dimensional wall. This wall should have a thickness of order \( F^{-1} \sim 10^{-17} \text{ cm} \) in the extra dimensions. Gravity becomes unified with the gauge interactions of Standard Model at the weak scale \( F \). The usual Newtonian gravitational constant \( G \) loses its status of a fundamental constant. It is a multidimensional gravitational constant \( G_{N} \) which acquires a meaning of the fundamental one. Note that the separated character of gravitons which may propagate freely in extra dimensions, while all ordinary particles cannot do so, is in some analogy with the field theory of gravity \( \ref{eq:22} \) where the gravity to gravity interaction is quite distinct from the interaction of matter to gravity.

The possibility of serious variations of the gravitational law in submillimeter range makes much-needed the performance of new experiments. As was stressed, e.g., in \( \ref{eq:19} \), the Casimir effect may well become a new method for experimental verification of fundamental physical theories. In \( \ref{eq:23} \) the measurements of the Casimir force between plane plates were considered in order to restrict the extra dimensions and string-inspired forces. In the next two sections the recent experiments on measuring the Casimir force between a disk and a spherical lens (sphere) are used for the same purpose and new more strong constraints are obtained.

### III. REVIEW OF THE BEST CONSTRAINTS ON YUKAWA-TYPE INTERACTION IN SUBMILLIMETER RANGE

It has been known that the Casimir and van der Waals force measurements between dielectrics (see, e.g., \( \ref{eq:24} \)) lead to the strongest constraints on the constants of Yukawa-type interaction given by the second term of Eq. (\ref{eq:1}) with a range of action \( 10^{-9} \text{ m} < \lambda < 10^{-4} \text{ m} \) \( \ref{eq:21} \). In \( \ref{eq:23} \) the Casimir force between two metallized surfaces of a flat disk and a spherical lens was measured with the use of torsion pendulum. The outer metallic layer of gold, covering the test bodies, had the thickness of 0.5 \( \mu \text{m} \). The absolute error of the force measurements in \( \ref{eq:23} \) was \( \Delta F = 10^{-11} \text{ N} \) for the distances \( a \) between the disk and the lens in the range \( 1 \mu\text{m} \leq a \leq 6 \mu\text{m} \). In the limits of this error the theoretical expression for the Casimir force

\[
F^{(0)}(a) = -\frac{\pi^3}{360} \frac{R}{a^3}
\]

(\ref{eq:8}) was confirmed (where \( R \) is lens curvature radius).

No corrections to Eq. (\ref{eq:5}) due to surface roughness, finite conductivity of the boundary metal or nonzero temperature was recorded. These corrections, however, may not lie in the limits of the absolute error \( \Delta F \). By way of example, at \( a \approx 1 \mu\text{m} \) the roughness correction may be around 12% of \( F^{(0)} \) or even larger \( \ref{eq:1} \), and the finite conductivity correction for the gold surfaces at 1 \( \mu\text{m} \) separation is 10% of \( F^{(0)} \) \( \ref{eq:2} \). (Remind that \( \Delta F \) is around 3% of \( F^{(0)} \) at \( a = 1 \mu\text{m} \).) As to the temperature correction, it achieves 174% of \( F^{(0)} \) at the separation \( a = 6 \mu\text{m} \), where, however, \( \Delta F \) is around 700% of \( F^{(0)} \). By this reason the constraints for Yukawa-type interaction following from the experiment \( \ref{eq:1} \) were found from the inequality \( \ref{eq:3} \).
where $F_{th}$ is the theoretical force value, including $F^{(0)}$, all the corrections to it mentioned above, and also the hypothetical Yukawa-type interaction calculated in Eq. (1) for $\alpha_G > 0$ respectively $\alpha_G < 0$. Curve 3 was obtained earlier (see, e.g., [2]) by the results of Casimir force measurements between dielectrics. The strengthening of constraints given by the curves 2.a and 2.b comparing the curve 3 is up to a factor 30 in the interaction range $2.2 \times 10^{-7} \leq \lambda \leq 1.6 \times 10^{-4}$ m (a bit different result was obtained later in [3] where the corrections to the ideal Casimir force were not taken into account).

In [9,10] the results of the Casimir force measurements between a flat disk and a sphere by means of an atomic force microscope were presented in comparison with the theory taking into account the finite conductivity and roughness corrections. Temperature corrections are not essential in the interaction range $0.1 \mu m < a < 0.9 \mu m$. The test bodies were covered by the aluminum layer of 300 nm thickness and Au/Pd layer of the thickness 20 nm (the latter is transparent for electromagnetic oscillations of characteristic frequency). The absolute error of the force measurements in [9,10] was $\Delta F = 2 \times 10^{-12}$ N. In the limits of this error the theoretical expression for the Casimir force with corrections to it due to both surface roughness and finite conductivity was confirmed. The theoretical expression for the Yukawa-type interaction in experimental configuration of [9,10] was obtained in [12]. The constraints on the parameters $\alpha_G$, $\lambda$ calculated in [12] from the inequality

$$|F_{Ya}(a)| \leq \Delta F,$$

turned out to be the best ones in the interaction range $5.9 \times 10^{-9} \leq \lambda \leq 10^{-7}$ m. They are stronger up to 140 times than the previously known ones from the Casimir force measurements between dielectrics (note that here all the corrections were included into the force under measuring; by this reason the constraints on $|\alpha_G|$ rather than on $\alpha_G$ were obtained).

In [11] the improved precision measurement of the Casimir force was performed by means of an atomic force microscope. The experimental improvements which include vibration isolation, lower systematic errors, and independent measurement of surface separation gave the possibility to decrease the absolute error of force measurement by a factor 2. Also the smoother Al coating with thickness 250 nm was used and thinner external Au/Pd layer of the thickness 7.9 nm. The Yukawa-type hypothetical force in the configuration of [11] was calculated in [12] where the stronger constraints on $\alpha_G$, $\lambda$ were also obtained using the inequality (10). These constraints are presented in Fig. 1 (curve 4). They turned out to be up to four times stronger than the constraints obtained from the previous experiment [9,10] within a bit wider interaction range $5.9 \times 10^{-9} \leq \lambda \leq 1.15 \times 10^{-7}$ m. The total strengthening of constraints on the corrections to Newtonian gravitational law from the measurements of the Casimir force by means of atomic force microscope has reached 560 times within the $\lambda$-interval mentioned above.

IV. NEW CONSTRAINTS ON THE YUKAWA-TYPE INTERACTION FROM THE CASIMIR FORCE MEASUREMENT BETWEEN GOLD SURFACES BY MEANS OF ATOMIC FORCE MICROSCOPE

Recently one more measurement of the Casimir force was performed using the atomic force microscope [13]. The test bodies (sphere and a disk) were coated by gold instead of aluminum which removes some difficulties connected with the additional thin Au/Pd layers used in the previous measurements [9,10] to prevent the oxidation processes on Al surfaces. The used polystyrene sphere coated by gold layer was of diameter $2R = 191.3$ $\mu$m and a sapphire disk had a diameter $2L = 1$ cm, and a thickness $D = 1$ nm. The thickness of the gold coating on both test bodies was $\Delta = 86.6$ nm. This can be considered as an infinitely thick in relation to the Casimir force measurements. The root mean square roughness amplitude of the gold surfaces was decreased until 1 nm which makes roughness corrections negligibly small. The measurements were performed at smaller separations, i.e. $62 \mu m < a < 350 \mu m$. The absolute error of force measurements was, however, $\Delta F = 3.5 \times 10^{-12}$ N, i.e., a bit larger than in the previous experiments. The reason is the thinner gold coating used in [13] which led to poor thermal conductivity of the cantilever. At smaller separations of about 65 nm this error is less than 1% of the measured Casimir force.

Now let us calculate the gravitational force acting in experimental configuration due to the potential (4). The Newtonian contribution is found to be negligible. Actually, due to the inequality $R \ll L$ each atom of the sphere can be considered as if it would be placed above the center of the disk. Then the vertical component of the Newtonian gravitational force acting between the sphere atom of a mass $M_1$ situated at a height $h \ll L$ and the disk is
Thus, it is desirable here not only to increase the strength of constraints but also to move the interaction range under $\alpha$. This is just right to attain the desirable values of $\alpha$ following from the experiment \[6\] can be improved up to four orders of magnitude in the range around weak-scale compactification characterized by the value $\alpha$ obtained from the Casimir force measurement by the use of torsion pendulum \[6\].

The new constraints were obtained also following from the recent Casimir force measurements between dielectrics. Although the interaction range where the new constraints are valid was extended, there is yet a gap (shown by curve 3 in Fig. 1) up to the curves 2,a, 2,b from the Casimir and van der Waals force measurements between dielectrics. Although this does not touch the era of big-bang nucleosynthesis which begins at lower temperatures of about 1 MeV, the strongest constraints follow for the smallest possible values of $\lambda$.

The strengthening up to 4500 was achieved by the Casimir force measurement \[15\] between gold surfaces using the atomic force microscope. They are found to be up to 19 times stronger than the previously reported \[13\] and up to 4500 stronger than the well-known constraints obtained from the Newtonian law. These deviations can be described by the Yukawa-type potential. They were predicted as indicated above, there is abundant evidence that the gravitational interaction at small distances undergo deviations from the Newtonian law. These deviations can be described by the Yukawa-type potential. They were predicted in the theoretical schemes with the quantum gravity scale both of order $10^{18}$ GeV and $10^{9}$ GeV. In the latter case the problem of experimental search for such deviations takes on great significance. The existence of large extra dimensions can radically alter many concepts of space-time, elementary particle physics, astrophysics and cosmology. To cite an example, the highest temperature at which the Universe was born turns out to be $\sim 10^3$ GeV instead of $10^{18}$ GeV. Although this does not touch the era of big-bang nucleosynthesis which begins at lower temperatures of about 1 MeV the theory of the very early Universe including inflation may be significantly changed \[19\].

The Casimir and van der Waals force measurements is the main source of constraints on the Yukawa-type interactions at small distances. In the present paper we have reviewed the latest advances deduced in such a manner in the submillimeter interaction range. The new constraints were obtained also following from the recent Casimir force measurements between gold surfaces by means of an atomic force microscope \[13\]. They are found to be up to 19 times stronger than the previously reported \[13\] and up to 4500 stronger than the well-known constraints obtained from the Casimir and van der Waals force measurements between dielectrics. Although the interaction range where the new constraints are valid was extended, there is yet a gap (shown by curve 3 in Fig. 1) up to the curves 2,a, 2,b obtained from the Casimir force measurement by the use of torsion pendulum \[13\].

As it is seen from Fig. 1, the Casimir force measurement between the gold surfaces by means of an atomic force microscope gives the possibility to strengthen the previously known constraints (curve 4) up to 19 times within a submillimeter interaction range. The new constraints were obtained also following from the recent Casimir force measurements between dielectrics. Although the interaction range where the new constraints are valid was extended, there is yet a gap (shown by curve 3 in Fig. 1) up to the curves 2,a, 2,b obtained from the Casimir force measurement by the use of torsion pendulum \[13\].

As is seen from Fig. 1, the Casimir force measurement between the gold surfaces by means of an atomic force microscope was extended, there is yet a gap (shown by curve 3 in Fig. 1) up to the curves 2,a, 2,b obtained from the Casimir force measurement by the use of torsion pendulum \[13\].

$$f_{N,z}(l) = \frac{\partial}{\partial l} \left[ GM_1 \rho 2\pi \int_0^L r \, dr \int_l^{l+D} \frac{dz}{\sqrt{r^2 + z^2}} \right] \approx -2\pi GM_1 \rho D \left( 1 - \frac{D + 2l}{2L} \right),$$

(11)

where $\rho$ is the disk density, and only the first order terms in $D/L$ and $l/L$ are retained.

The Newtonian gravitational force acting between the disk and the sphere is obtained from \[1\] by integration over the sphere volume

$$F_{N,z} \approx -\frac{8}{3} \pi^2 G \rho' DR^3 \left( 1 - \frac{D}{2L} - \frac{R}{L} \right),$$

(12)

where $\rho'$ is the density of the sphere material.

Even with sphere and disk made of the vacuo-distilled gold as a whole with $\rho = \rho' = 18.88 \times 10^3$ kg/m$^3$ one arrives from \[12\] to the negligibly small value of $F_{N,z} \approx 6 \times 10^{-10}$ N $\ll \Delta F$.

According to \[13\] the theoretical value of the Casimir force was confirmed within the limits of $\Delta F = 3.5 \times 10^{-12}$ N and no hypothetical force was observed. In such a situation, the constraints on $\alpha_G$ can be obtained from the inequality \[11\]. The strongest constraints follow for the smallest possible values of $a \approx 65$ nm. The computational results are presented by curve 5 in Fig. 1.

As is seen from Fig. 1, the Casimir force measurement between the gold surfaces by means of an atomic force microscope gives the possibility to strengthen the previously known constraints (curve 4) up to 19 times within a range $4.3 \times 10^{-9}$ m $\leq \lambda \leq 1.5 \times 10^{-7}$ m. The largest strengthening takes place for $\lambda = (5-10)$ nm. Comparing the constraints obtained from the Casimir and van der Waals force measurements between dielectrics (curves 3 and 6) the strengthening up to 4500 was achieved by the Casimir force measurement \[15\] between gold surfaces using the atomic force microscope.

**V. CONCLUSIONS AND DISCUSSION**

As indicated above, there is abundant evidence that the gravitational interaction at small distances undergo deviations from the Newtonian law. These deviations can be described by the Yukawa-type potential. They were predicted in the theoretical schemes with the quantum gravity scale both of order $10^{18}$ GeV and $10^{9}$ GeV. In the latter case the problem of experimental search for such deviations takes on great significance. The existence of large extra dimensions can radically alter many concepts of space-time, elementary particle physics, astrophysics and cosmology. To cite an example, the highest temperature at which the Universe was born turns out to be $\sim 10^3$ GeV instead of $10^{18}$ GeV. Although this does not touch the era of big-bang nucleosynthesis which begins at lower temperatures of about 1 MeV the theory of the very early Universe including inflation may be significantly changed \[19\].
examination to larger $\lambda$ (e.g., by the increase of a sphere radius and the space separation to the disk). In conclusion it may be said that the compact and relatively cheap laboratory experiments on the measurement of the Casimir force offer important advantages over the commonly used techniques intended for the investigation of fundamental interactions and new elementary particles.

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FIG. 1. Constraints on the Yukawa-type interaction. Curve 1 follows from the Cavendish-type experiment of Ref. [26]. Curves 2,a, 2,b follow from the Casimir force measurement by means of a torsion pendulum [6]. Curves 3 and 6 were obtained from the Casimir and van der Waals force measurements between dielectrics. Curve 4 follows [13] from the Casimir force measurement between aluminum surfaces by means of an atomic force microscope [11], and curve 5 is obtained in this paper from the experiment of Ref. [15] with gold surfaces. The regions below the curves are permitted, and those above the curves are prohibited.