Separating the impact of nuclear skin and nuclear deformation on elliptic flow and its fluctuations in high-energy isobar collisions

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Bulk nuclear structure properties, such as radii and deformations, leave distinct signatures in the final states of relativistic heavy-ion collisions. Collisions of isobars, in particular, offer an easy route to establish clear correspondences between the structure of colliding nuclei and the final state observables. Here we investigate the impact of nuclear skin and nuclear deformations on elliptic flow ($v_2$) and its fluctuations in high-energy $^{96}\text{Ru}+^{96}\text{Ru}$ and $^{96}\text{Zr}+^{96}\text{Zr}$ collisions, for which experimental data is available. We show that the difference in skin thickness between these isobars impacts the intrinsic ellipticity of the collision systems, or reaction-plane flow, $v_2^{\text{rr}}$. In contrast, differences in nuclear deformations impact only the fluctuations of $v_2$ around $v_2^{\text{rr}}$. Through isobar collisions, one can separate the influence of nuclear skin and nuclear deformations in the $v_2$ data. This is a significant step towards assessing the consistency of nuclear phenomena across energy scales.

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The structure of most of atomic nuclei is characterized by prominent bulk properties, reflecting collective correlations in many-body systems held together by the strong force. Mapping out such properties and how they evolve across the Segré chart is one of the main goals of nuclear physics [1, 2]. Information about the collective features of nuclei is typically inferred via spectroscopic and scattering experiments conducted at low energies. In the past five years or so, an expanded list of nuclei utilized in collision experiments at ultra-relativistic energies has lead to the identification of fingerprints of collective nuclear properties also in such processes [3–13]. The angular (azimuthal) particle distributions emitted in high-energy nuclear collisions carry, in particular, direct information about the structure of the colliding ions, to the extent that, for certain collision configurations, the shape of the particle distributions in momentum space can be related directly to the shape of the colliding ions at the time of scattering.

This is made possible by the nearly-ideal fluid nature of the quark-gluon plasma (QGP) created in high-energy collisions. In a hydrodynamic picture, the emergence of final-state anisotropies in the azimuthal particle spectra stems from the presence of spatial anisotropies in the initial conditions of the fluid expansion [14–16]. These anisotropies are in turn sourced by the positions in the initial conditions of the fluid expansion [14–16]. The important question is, then, to which extent established information from low-energy nuclear physics can provide a consistent picture of the phenomena observed at high-energy colliders. In this paper we make significant progress in this direction.

In heavy-ion collisions, the deformation of the colliding ions becomes manifest mainly in the limit of fully-overlapping configurations (central collisions), where the shape of the colliding bodies is fully resolved. However, nuclear structure effects can be cleanly isolated as well over the full centrality range by comparing two isobaric collision systems [8, 11, 13]. Isobar nuclei have the same mass number, therefore, any visible difference in the observables measured in two isobaric systems must originate from differences in their structure, which impacts the initial condition and evolution of the QGP. Collisions of isobars, $^{96}\text{Ru}+^{96}\text{Ru}$ and $^{96}\text{Zr}+^{96}\text{Zr}$, have been performed at the BNL Relativistic Heavy Ion Collider (RHIC). They demonstrate this argument. Ratios of observables taken between $^{96}\text{Ru}+^{96}\text{Ru}$ and $^{96}\text{Zr}+^{96}\text{Zr}$ collisions show significant and centrality-dependent departures from unity [17]. Most models of heavy ion collisions describe the data by parameterizing the nucleon density within the colliding ions according to a Woods-Saxon (WS) profile,

$$
\rho(r, \theta, \phi) \approx \frac{1}{1 + e^{(r - R_0(1 + \beta_2 Y_2^2(\theta, \phi) + \beta_3 Y_3^3(\theta, \phi))) / a_0}},
$$

which contains four structure parameters, nuclear skin $a_0$, half-width radius $R_0$, quadrupole deformation $\beta_2$, and octupole deformation $\beta_3$. Model studies have established that isobar ratios are indeed controlled by parameter differences, $\Delta \beta_2^2 = \beta_2^{\text{rr}} - \beta_2^{\text{Zr}}$, $\Delta \beta_3^3 = \beta_3^{\text{rr}} - \beta_3^{\text{Zr}}$, $\Delta a_0 = a_0^{\text{rr}} - a_0^{\text{Zr}}$ and $\Delta R_0 = R_0^{\text{rr}} - R_0^{\text{Zr}}$ [18].

Many observables have been shown to present a sensitivity to the nuclear profile parameters, such as the mean transverse momentum $p_T$ [19], its fluctuations [13],
the spectator neutron number [20], Fourier plane correlations [21, 22], and shape-size correlations [6, 10, 12]. In the present study, the key observable is the simple elliptical asymmetry of the azimuthal particle distributions, known as elliptic flow $v_2 = v_2 e^{2i\Psi}$, which characterizes the anisotropic flow of particles along the direction $\Psi$ with an amplitude $v_2$ and periodicity of $\pi$. $v_2$ emerges as a hydrodynamic response to the elliptical shape of the region of overlap between two colliding ions. In isobar collisions, the ratio of $v_2$ between $^{96}$Ru+$^{96}$Ru and $^{96}$Zr+$^{96}$Zr collisions shows a complex non-monotonic centrality dependence, which can be explained as a combined effect from differences in the WS parameters between $^{96}$Ru and $^{96}$Zr [23]. In this paper, we show that the impact of the deformations parameters ($\beta_2$ and $\beta_3$) can be fully disentangled from that of the radial profile parameters ($a_0$ and $R_0$), and point out the consequences of such a result.

We start from Fig. 1. We parametrize the plane transverse to the collision axis (transverse plane) with Cartesian coordinates where the $x$ direction is along the impact parameter direction. For events at a given centrality, the distribution of $V_2 \equiv (v_{2x}, v_{2y})$ is approximately a two-dimensional Gaussian [24]

$$p(v_{2x}, v_{2y}) = \frac{1}{\pi \delta^2} \exp\left[-\frac{(v_{2x} - v_{2P}^x)^2 + v_{2y}^2}{\delta^2}\right]. \quad (2)$$

The displacement along $x$, $v_{2P}^x$, is the so-called reaction plane flow, associated with the average elliptic geometry, whereas the fluctuation around the intrinsic geometry, $\delta$, is the variance of elliptic flow due to, e.g., fluctuations in the positions of the colliding nucleons. Our point is that, to leading order, a change in the radial profile of the nucleus, determined by either $a_0$ or $R_0$ [see Fig. 1(a)], modifies the intrinsic ellipticity, $v_{2P}$, with little impact on the flow fluctuation. On the other hand, in the presence of nuclear deformations [see Fig. 1(b)], the random orientation of the colliding nuclei implies an increase in $\delta$, with little impact on $v_{2P}^x$ [25]. Now, if $p(V_2)$ is a Gaussian, the root-mean-squared elliptic flow, experimentally accessible via a two-particle correlation, is $v_2(2) = \sqrt{(v_{2P}^x)^2 + \delta^2}$, while higher-order cumulants of $v_2$, measured via multi-particle correlations, are all identical and equal to $v_{2P}$,

$$v_2(4) = v_2(6) = \ldots = v_2(\infty) = v_{2P}^x. \quad (3)$$

In this limit, the fluctuation of $v_2$ can be measured as

$$\delta^2 = v_2(2^2) - v_2(4^2). \quad (4)$$

In the following, we demonstrate our argument about the sensitivity of $v_{2P}^x$ and $\delta$ to the nuclear structure parameters by means of transport model calculations.

We simulate the dynamics of the QGP using the multi-phase transport model (AMPT) [26]. We use AMPT v2.26t5 in the string-melting mode at $\sqrt{s_{NN}} = 200$ GeV with a partonic cross section of 3.0 mb [27, 28]. This model successfully describes the isobar ratios of $v_2$, $v_3$, and $N_{ch}$ measured by the STAR collaboration [9, 18]. We simulate generic isobar $^{96}$X+$^{96}$X collisions with five choices of $\beta_2$, $\beta_3$, $R_0$ and $a_0$, as listed in Table I. This allows us to define ratios that isolate the influence of the nuclear structure parameters step-by-step, e.g., Case1/Case2 includes the effect of $\beta_2$, Case1/Case3 includes the effect of $\beta_2$ and $\beta_3$, and so on. The cumulants of elliptic flow are calculated within the multi-particle cumulant framework [29, 30], for hadrons with $0.2 < p_T < 2$ GeV. The two-particle cumulant $v_2(2)$ is obtained by correlating particles in $0 < \eta < 2$ with those in $-2 < \eta < 0$ to suppress short-range correlations that do not emerge from the collective expansion of the system [31]. $v_2(4)$ is instead free of such contributions by construction, and is thus obtained from all particles with $|\eta| < 2$. Additionally, we calculate the true $v_{2P}$ from the azimuthal correlation of particles relative to the impact parameter, and the true flow fluctuation as $\delta^2 = v_2(2^2) - (v_{2P}^x)^2$. The simulated events are binned in classes defined by the number of nucleons that participate in the interaction $N_{part}$, which is a good enough proxy for the experimentally-defined collision centrality.

In Fig. 2, we show our results for $v_2(2)$, $v_2(4)$, and $\delta$, averaged between $^{96}$Ru+$^{96}$Ru and $^{96}$Zr+$^{96}$Zr collisions, and how they compare with STAR data. The comparison with data is generally good. In central collisions, AMPT correctly predicts a sign change for $v_2(4)$, qualitatively similar to STAR data but with a smaller magnitude. This sign change can be attributed to volume fluctuations associated with the event-activity variable, such as $N_{part}$ or $N_{ch}$, which could have a strong impact on multi-particle cumulants [10, 32, 33]. Indeed, we observe a stronger sign change pattern when the events are binned in $N_{ch}$ [34],

\[ \text{FIG. 1. (a) Sketch of a collision of spherical nuclei with different skin thickness, } a_0. \text{ The influence of the nuclear skin is primarily on the average ellipticity along the } x \text{ direction, i.e., an increase in the reaction plane flow, } v'_{2x}. \text{ (b) Collisions of deformed nuclei with } \beta_2 = 0.25 \text{ and random orientations (four orientations for each nucleus as shown by solid lines). The random orientations mostly result in an enhanced ellipticity fluctuation, } \delta. \]
The results are shown in the left column of Fig. 3. Figure 3(a) shows the complex centrality dependence of $R_{v_2}(2)$, arising from both deformation and radial profile parameters. In contrast, $R_{v_2(4)}$ [Fig. 3(b)] is sensitive mostly to $a_0$, whereas $R_{\delta}$ [Fig. 3(c)] is sensitive mostly to $\beta_n$. Therefore, the behavior of $R_{v_2(2)}$ can be fully decomposed into a part sensitive to the nuclear skin and a part sensitive to the nuclear deformations. One can establish the following relation,

$$R_{v_2(2)}^2 = R_{\delta}^2 + (R_{v_2(4)}^2 - R_{\delta}^2) r,$$

$$R_{v_2(2)} \approx R_{\delta} + (R_{v_2(4)} - R_{\delta}) r,$$

where the second line is obtained by assuming all ratios are close to unity. In central collisions where $r = 0$, the behavior of $R_{v_2(2)}$ is dominated by $R_{\delta}$. In mid-central collisions, the non-monotonic behavior of $R_{v_2(2)}$ in the top-left panel of Fig. 3 results from the interplay of $R_{v_2(4)}$ and $R_{\delta}$. This is our main result. The right column of Fig. 3 shows, in addition, the ratios of the true intrinsic ellipticities, $R_{v_2}^{\text{true}}$ [Fig. 3(d)], and the true flow fluctuation, $R_{\delta_\text{rep}}$ [Fig. 3(e)], obtained from $\delta_{2\text{rep}}^2 = v_2^2(2)^2 - (v_2^{\text{true}})^2$. We see that, for a difference in skin thickness of $\delta_0 = 0.06$ fm, the value of $v_2^{\text{true}}$ is enhanced by about 10% in $^{96}$Ru+$^{96}$Ru collisions. The impact of $\beta_n$ on $R_{v_2}^{\text{true}}$ is subleading, as expected. In addition, we see that the value of $v_2^{\text{true}}$ varies more strongly when changing nuclear structure parameters than that of $v_2^2(4)$. As a result, the ratio $R_{\delta_\text{rep}}$ shows somewhat larger dependence on nuclear structure than $R_{\delta}$. One further comment is in order. Elliptic flow emerges, event-by-event, as a response to the initial ellipticity of the system. This quantity, denoted by $E_2$, is usually quantified as the second Fourier harmonic of the initial density distribution [38]. With the excellent approximation of a linear scaling, $V_2 \propto E_2$, on an event-by-event basis, the ratios of observables analyzed in Fig. 3 can be estimated starting solely from the knowledge of $E_2$ and its fluctuations. In the supplemental material we show that the behaviors observed in Fig. 3 do indeed originate largely from the initial state.

Before concluding, let us comment on the fact that the STAR collaboration has also measured an approximation of $v_2^{\text{true}}$ by correlating particles with the spectator neutrons.

| Ratios | Case1 | Case2 | Case3 | Case4 | Case5 |
|--------|-------|-------|-------|-------|-------|
| $R_{v_2}$ | $0.50$ | $0.46$ | $0.162$ | $0.06$ | $0.20$ |
| $a_0$ | $0.50$ | $0.46$ | $0.06$ | $0.20$ | $0.20$ |
| $\beta_2$ | $0.50$ | $0.46$ | $0.06$ | $0.20$ | $0.20$ |
| $\beta_3$ | $0.50$ | $0.46$ | $0.06$ | $0.20$ | $0.20$ |

Table I. Nuclear structure parameters used in the simulations of $^{96}$Ru+$^{96}$Ru and $^{96}$Zr+$^{96}$Zr collisions. Case1 and Case5 represent, respectively, our full parameterizations of $^{96}$Ru and $^{96}$Zr.
Wrapping up, we have found that the nuclear radial profile parameters, i.e., nuclear skin thickness, $a_0$, and half-density radius, $R_0$, mostly influence the magnitude of $v_2$ along the impact parameter direction, i.e., the reaction-plane elliptic flow, $v_2^{rp}$. Conversely, the nuclear deformations, $\beta_2$, mostly influence the elliptic flow fluctuation, $\delta$. Based on our simulations, we conclude that the measured isobar ratio of $v_2^{zdc}$ is determined by $a_{0}^{0Ru} - a_{0Zr}$, while the measured isobar ratio of $\delta$ arises from the interplay of $\beta_{2}^{0Ru} - \beta_{2}^{2Zr}$ and $\beta_{3}^{0Ru} - \beta_{3}^{2Zr}$. This new result has now to be combined with the information from previous studies [18, 36], showing that the isobar ratio of triangular flow coefficients, $v_3\{2\}$, is dominated by $\beta_{2}^{0Ru} - \beta_{2}^{2Zr}$. Therefore, We have three ratios, $R_{v_2(2)}$, $R_{v_2(4)}$, and $R_\delta$, that provide separate constraints on three features of the colliding nuclei, namely $\Delta a_0$, $\Delta \beta_2^2$, and $\Delta \beta_3^2$. A pressing question in nuclear phenomenology is whether the structure-induced phenomena observed in high-energy collisions are consistent or not with the expectations from low-energy nuclear experiments and theories. These results represent, then, a stepping stone to the achievement of this goal. Finally, let us briefly comment on the fact that our conclusions do not make any specific use of the Gaussian Ansatz for $v_2$ fluctuations that we have used to motivate our analysis. The fluctuations of $v_2$ are, in fact, non-Gaussian, especially in peripheral collisions, where $v_2^{rp}$ is large and one becomes sensitive to the bound $v_2 < 1$ [41, 42]. It would be interesting to generalize this study to higher-order cumulants, $v_2\{4, 6, 8\}$, and see how nuclear structure impacts these quantities in isobar collisions (see Ref. [43] for a study in collisions of uranium-238 nuclei). In the supplemental material, we show results for $R_{v_3(4)}$, $R_{v_3(6)}$ and $R_{v_3(8)}$, and also study the fine splitting of these cumulants at the level of the eccentricity fluctuations. Our preliminary conclusion is that, for such observables, there is no obvious separation of nuclear structure effects, as in the case of $v_2\{2\}$, although this is work in progress.

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SUPPLEMENTAL MATERIAL

A similar analysis is performed for the eccentricity, $\varepsilon_2$, and its fluctuations, where we decompose $\varepsilon_2\{2\} = \sqrt{\langle \varepsilon_2^2 \rangle}$ into a reaction plane component and a fluctua-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{Isobar ratios $R_{v_2(2)}$ (a), $R_{v_2(4)}$ (b), $R_\delta$ (c), $R_{v_2^{rp}}$ (d), and $R_{\delta_{np}}$ (e) plotted as a function of $N_{part}$. They are compared with STAR data from Fig. 23 of Ref. [17]. These ratios are re-calculated at matching $N_{part}$, which are slightly different from the published ratios at matching centrality as described in detail in Ref. [39].}
\end{figure}
tion component. The results are shown in Fig. 4 with identical layout as in Fig. 3. Specifically, we calculate the four-particle cumulant \( \varepsilon_2(4) \) and the standard reaction-plane eccentricity \( \varepsilon_2^R \), as well as the associated fluctuations defined by \( \delta_{\varepsilon_2} \equiv \sqrt{\varepsilon_2(2)^2 - \varepsilon_2(4)^2} \) and \( \delta_{\varepsilon_2^R} \equiv \sqrt{\varepsilon_2(2)^2 - (\varepsilon_2^R)^2} \) to match the corresponding final state quantities in Fig. 3. We see that the isobar ratios of these initial state estimators already qualitatively or even quantitatively reproduce the AMPT result in Fig. 3, with only a few exceptions. Compared to Fig. 3, we see in particular that the values of \( R_{\varepsilon_2(4)} \) systematically smaller than \( R_{\varepsilon_2(4)} \). However, the values of \( R_{\varepsilon_2^R} \) agree quantitatively with \( R_{\varepsilon_2^R} \) in all four cases. The stronger dependence of \( R_{\delta_{\varepsilon_2}} \) on WS parameters, compared to \( R_{\delta_{\varepsilon_2^R}} \), may then be attributed to the role of non-Gaussianities, which are known to be larger in the distribution of \( \varepsilon_2 \) than in that of \( \varepsilon_2 \), as \( \varepsilon_2 \) fluctuations are largely smeared by the hydrodynamic expansion. We also see that \( R_{\delta_{\varepsilon_2}} \) agrees rather nicely with \( R_{\delta_{\varepsilon_2^R}} \) in Fig. 3, which is expected as both \( \varepsilon_2^R \) and \( \varepsilon_2^R \) are defined relative to the impact parameter, and therefore is less affected by non-Gaussianities. However, both \( R_{\delta_{\varepsilon_2}} \) and \( R_{\delta_{\varepsilon_2^R}} \) show some dependence on \( a_0 \) in peripheral collisions, which are absent in \( R_{\delta_{\varepsilon_2}} \) and \( R_{\delta_{\varepsilon_2^R}} \) in Fig. 3. This is once more likely due to a smearing effect from the hydrodynamic expansion, washing out the primordial non-Gaussianities.

Moving to higher-order fluctuations, we also calculate the ratios of higher-order cumulants, \( R_{\varepsilon_2(6)} \) and \( R_{\varepsilon_2(8)} \), and compare them to \( R_{\varepsilon_2(4)} \). In the Gaussian limit, these ratios should all be identical. However, a characteristic fine splitting is observed experimentally, \( \varepsilon_2(4) \gtrsim \varepsilon_2(6) \gtrsim \varepsilon_2(8) \), reflecting the non-Gaussianity of the distribution of \( \varepsilon_2 \). It is interesting to study whether this fine splitting is affected by the nuclear structure differences. The results are shown in Fig. 5. Unfortunately, the statistical precision of our AMPT results does not allow for a definitive answer to this question. On the other hand, the ratios of higher-order cumulants for \( \varepsilon_2 \) can be calculated with high precision to provide a useful guidance. The results, shown in Fig. 6, bring in a sense both good and bad news. The good news is that the cumulant splittings are impacted by the nuclear structure parameters. The bad news is that both \( \beta_\varepsilon \) and \( a_0 \) seem to affect these ratios at similar level, by reducing the cumulant ratios \( \varepsilon_2(6)/\varepsilon_2(4) \) and \( \varepsilon_2(8)/\varepsilon_2(6) \). We note that the reduction is larger for increasing \( N_{\text{part}} \), and may even change the overall trends of these ratios. While more work may be needed on the conceptual side, it would still be interesting to study whether these effects also survive to the final state and leave similar imprints on \( v_2(4, 6, 8) \), both in simulations and in experiment.

![Figure 4: The isobar ratios](image-url)

**FIG. 4.** The isobar ratios \( R_{\varepsilon_2(2)} \) (a), \( R_{\varepsilon_2(4)} \) (b), \( R_{\varepsilon_2} \) (c), \( R_{\varepsilon_2^R} \) (e), and \( R_{\delta_{\varepsilon_2}} \) (f) plotted as a function of \( N_{\text{part}} \). They are compared with the same STAR data shown in Fig. 3.

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