SCALING EVOLUTION OF UNIVERSAL DARK MATTER HALO DENSITY PROFILES

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Abstract

Dark matter halos show a universal density profile with a scaling such that less massive systems are typically denser. This mass-density relation is well described by a proportionality between the characteristic density of halos and the mean cosmic density at halo formation time. It has recently been shown that this proportionality could be the result of the following simple evolutionary picture. Halos form in major mergers with essentially the same, cosmogony-dependent, dimensionless profile and then grow inside out as a consequence of accretion. Here we verify the consistency of this picture and show that it predicts the correct zero point of the mass-density relation.

Subject headings: cosmology: theory — galaxies: evolution — galaxies: formation — galaxies: structure

1. introduction

High-resolution cosmological N-body simulations show that the spherically averaged density profile of dark matter halos has the following universal form (Navarro, Frenk, & White 1997, hereafter NFW; but Moore et al. 1997):

$$\rho(r) = \frac{\delta_c}{\xi(1+\xi)^2},$$  \hspace{1cm} (1)

where $\xi$ is the radial distance to the halo center in units of the scale radius $r_s$, and $\delta_c$ is the characteristic halo density $\rho_c$ in units of the critical density for closure $\rho_c$. The parameters $r_s$ and $\rho_c$ are linked by the condition that the mean density within the virial radius $R$ of a halo of a given mass is a constant factor $a$ times the cosmic critical density; here we take $a = 200$. Therefore, the density profiles of halos at a given epoch depend on their mass $M$ through one unique parameter, $\delta_c$ or $r_s = r_c/R$.

The universality of this profile is not surprising as far as the gravitational clustering of dark matter is self-similar in scale-free cosmogonies and approximately so, over a certain range of masses and times, in other cosmogonies. However, the specific form of this profile and the scaling functions $\delta_c(M)$ or $r_s(M)$ at a given time $t$ is hard to predict. Much of the research on this issue has focused on the effects of spherical infall (Gunn & Gott 1972; Fillmore & Goldreich 1984; Bertschinger 1985; White & Zaritsky 1992; Avila-Reese, Firmani, & Hernández 1997; Henriksen & Widrow 1998) and/or secular evolution owing to dynamical friction and tidal disruption on small-mass clumps (Syer & White 1997; Nusser & Sheth 1998). Both of these mechanisms require a certain stability of the system and, therefore, presume the growth of halos governed by minor mergers or accretion. The opposite viewpoint that the typical halo density profile is the result of repetitive, asymmetrical, major mergers producing strong violent relaxation has also been investigated (e.g., Duncan, Farouki, & Shapiro 1983). In fact, accretion and major mergers alternate in hierarchical clustering (see, e.g., Tormen, Bouchet, & White 1997 for the cold dark matter [CDM] cosmogony). Hence, one should in principle take into account the combined action of these two extreme processes.

NFW have found that the smaller the mass of halos, the denser they are. This mass-density correlation was interpreted as reflecting the fact that, in hierarchical clustering, less massive halos form earlier when the mean density of the universe is higher. NFW showed, indeed, that the characteristic density of halos with a given mass at a given epoch is proportional to the mean cosmic density at their time of formation. There is, however, a small caveat in this reasoning. According to it, the structural properties of halos would be fixed on formation. Since major mergers yield a substantial rearrangement of the system, while accretion only causes a smooth secular evolution, it seems consistent to define the formation of halos as the last major merger they experience, their identity being kept during accretion. However, the halo formation time used by NFW, drawn from the extended Press-Schechter (1974; hereafter, PS) formalism, does not match this definition.

Recently, Salvador-Solé, Solanes, & Manrique (1998; hereafter, SSM) have repeated the same analysis as NFW but using a better suited formation time estimate drawn from a modified version of the PS theory. This incorporates a phenomenological distinction between accretion and major mergers. For a fractional mass capture threshold for major mergers $\Delta_m$, equal to 0.6, SSM confirm that, in any hierarchical cosmogony, there is a clear proportionality between the characteristic density of halos and the mean density (or still the critical density) of the universe when they formed. Furthermore, SSM have shown that this proportionality is consistent with the following simple evolutionary picture for the structure of dark matter halos. Halos form as the result of major mergers with essentially universal (i.e., independent of mass and time), cosmogony-dependent, scaling parameters $\delta_c$ and $r_s$. Then, until the next major merger, the corresponding dimensional parameters $\rho_c$ and $r_s$ remain fixed, there only being a shift in $\delta_c$ and $r_s$ as the critical density decreases and the virial radius expands accordingly.

The requirement of steady profiles during the accretion phase which follows from the empirical scaling and the PS clustering model agrees with accurate dynamical studies of self-similar infall (Fillmore & Goldreich 1984; Bertschinger 1985; Henriksen & Widrow 1998). This gives strong support to the validity of that evolutionary picture. However, its consistency has yet to be checked. On the other hand, although this picture might explain the observed proportionality between the mass and characteristic density of halos, the zero point of the mass-
density relation in any given cosmogony remains to be explained. In the present Letter, we address these two issues.

2. THE PREDICTED STRUCTURE OF HALOS FORMED IN MAJOR MERGERS

In this section we derive, in an arbitrary hierarchical cosmogony, the values of the scaling parameters $\delta_0$ or $x$, of halos that result, according to the proposed evolutionary picture, from the major merger of their typical progenitors with appropriate structural properties. All elements we need for this calculation are given in SSM except for the typical mass (and number) of progenitors of a halo with mass $M_0$ at $t_s$. This is next derived in the same frame of the modified PS clustering model used in SSM.

The specific capture rate $r'(M' \leftarrow M, t)$ is the rate at which halos of final mass $M'$ capture, at $t$, other halos of mass $M < M'$. Following Manrique & Salvador-Solé (1996), this is related to the specific major merger rate $r''(M \rightarrow M', t)$, given in SSM through

$$r'(M' \leftarrow M, t) = r''(M \rightarrow M', t) \frac{N(M, t)}{N(M', t)},$$

where $N(M, t)$ is the mass function. For halos with mass $M$ larger than $M'/\Delta_m + 1$, the specific capture rate is null. This reflects the fact that such halos evolve by accretion and are, therefore, identified with the final ones with mass $M'$ which cannot capture themselves. On the other hand, the captures of halos with masses $M$ smaller than $M'/\Delta_m + 1$ do not cause, in general, the destruction of the capturing halos and, hence, do not imply the formation of new ones. Only those captures of halos with masses $M$ ranging between $M'/\Delta_m + 1$ and $M'/\Delta_m + 1$ necessarily correspond to major mergers implying the formation of new halos. Hence, were all major mergers binary, the specific capture rate given by equation (2) would be, within that range, a symmetrical function of $M$ around $M'/2$, which would lead to the equality between the two integrals

$$C(M', t) = \int_{M'/2}^{M'/\Delta_m + 1} r''(M' \leftarrow M, t) dM' = \int_{M'/\Delta_m + 1}^{M'/2} r''(M' \leftarrow M, t) dM.'$$

(3)

Moreover, the probability that one of the two progenitors of a halo of mass $M'$ has mass $M$ within the limits of any of the two preceding integrals would then be given by

$$\Phi_x(M) = \frac{r'[M' \leftarrow M, t(t_x)]}{C(M', t)}.$$  

(4)

and the typical masses $M_i$ and $M_f$ of the progenitors of a halo of mass $M'$ would coincide with the median value of this distribution function and its complementary value, respectively. $N$-body simulations show that major mergers are binary, in agreement with the fact that in all cosmogonies analyzed, the capture rate (eq. [2]) is found to be very closely symmetrical around $M'/2$ in the appropriate range of masses, the fractional difference between the two integrals in equation (3) being less than 3% at any redshift. Thus, the use of the probability (4) to derive the typical mass of progenitors is fully justified.

Let us consider now two gravitationally bound halo progenitors with masses $M_i$ and $M_f$ at turnaround. Each mass $M_i$ includes not only the mass of the main relaxed body but also that of the surrounding matter that will have accreted onto it by the time the merger occurs. Assuming radial orbits, the total energy of the system at turnaround is

$$E_{in} = U_i + U_2 + E_{12},$$

(5)

where $U_i$ is the internal energy of each component and $E_{12}$ is the interaction energy. If we apply energy conservation to each progenitor considered in isolation, the internal energy $U_i$ is approximately equal to the internal energy of the corresponding fully relaxed halo prior to merger. Assuming spherical symmetry and virial equilibrium, this is given by

$$U_i = -\frac{1}{2} \frac{G M_i^2}{R_i},$$

(6)

with $R_i$ and $x_i$ the virial and dimensionless scale radii, respectively, of progenitor $i$ at merger time and $F(x_i)$ is a dimensionless function of order unity dependent on the density profile (eq. [1]). The interaction energy is also approximately given by the expression

$$E_{12} = -\frac{G M_1 M_2}{D},$$

(7)

with $D_n$ the turnaround separation between the two centers of mass, equal to

$$D_n = 2GM \left[\frac{t(t(M_0, t_0))^2}{\pi}\right]$$

(8)

for two point masses falling radially toward each other, where we have taken into account that the cosmic time corresponding to zero separation marking the merger of the two progenitors is the formation time $t(M_0, t_0)$ of the final halo with mass $M_0$ at $t_0$. Finally, taking into account that the internal energy $U$ of the final halo at formation, of the form given in equation (6), coincides with the total energy of the system at turnaround (eq. [5]), we arrive to the approximate relation

$$F(x_1) = \left(\frac{M_1}{M}\right)^{5/3} F(x_1) + \left(\frac{M_2}{M}\right)^{5/3} F(x_2) + \frac{\left(\frac{5}{2}\right)^{2/3} M_1 M_2}{M^2},$$

(9)

where $M = M_1 + M_2$, $\tau$ is equal to $t(t_0, t_0)$ in units of the Hubble time at that epoch, and $x_m$ is the value of $x_i$ at halo formation.

In deriving the previous relation, we have neglected the external gravitational torques acting on the system as well as the mutual tidal interaction between the progenitors. External torques make the orbits nonradial which, for a fixed turnaround separation, increases the infall time. To follow nonradial orbits, it is convenient to use the dimensionless variable $S = J^2 I^2(E)$, where $J$ is the specific angular momentum of $M_2$ relative to $M_1$, $E$ is the corresponding orbital energy, and
orbital parameter in steps of 0.25. Solid lines correspond to equal to the empirical value of the corresponding progenitors (and , respectively) for the case of the NFW parameter equal to one orbital period leads to an extra factor in the last term on the right-hand side of equation (for any fixed value of and energy ) universal density law and the standard CDM cosmogony. The values of the generated mass for the merging time which can also be accounted for by further increasing the value of . Note that the importance of external torques is expected to increase with decreasing mass, while the more massive the halos, the more marked their mutual tides. Therefore, both effects can be taken into account by adopting, in a first approximation, a constant positive value of in the whole range of masses.

From the clustering model given in SSM, we can compute the values of and as well as the typical formation time of each progenitor. Then, the evolutionary picture for the structure of halos we wish to check allows us to calculate the values of of the two progenitors prior to merger from the assumed universal value of this parameter on formation. (This can be done by using any of the two independent fitting formulae given in SSM, the two results typically differing by less than 10%.) Therefore, equation (9) can be solved for .

### 3. Comparison with the Empirical Structure of Halos

In Figure 1 we plot, for the NFW density law, the fractional difference between the solution of equation (9), , and the empirical value of of obtained by SSM used as the formation value for the progenitors, , as a function of at the present time. is scaled to defining a unity rms density fluctuation so that similar graphs hold at other epochs. The curves plotted here correspond to the standard CDM cosmogony with 0.59, although any other cosmogony analyzed (in particular those studied in SSM) leads to very similar results. For , there is very good agreement between and at the large mass end. The deviation increases toward small , but, within the mass range of four decades shown in this figure, it stays below 25%. Increasing the value of of course tends to further improve the agreement at small masses without spoiling too much the good predictions at the large-mass end.

Moreover, the evolutionary picture analyzed implies one single, cosmogony-dependent value of , fixing the zero point of the mass-density relation. In Table 1 we quote, for various cosmogonies (with parameters listed in the first four columns), the value of minimizing, for 0.5, the maximal fractional difference between and in the four decades of mass range analyzed (col. [5]) and the empirical value of obtained in

### Table 1: Scaling of Halo Density Profiles

| $P(k)$ | $\Omega_0$ | $\lambda_0$ | $\sigma_8$ | $x_{s_0}$ | $x_{s_0}^b$ | $90\%$ Confidence Interval |
|--------|------------|-------------|-----------|----------|------------|-------------------------|
| Standard CDM | 1.0 | 0.0 | 0.63 | 0.14 | 0.17 | (0.12, 0.25) |
| ACDM | 0.25 | 0.75 | 1.3 | 0.18 | 0.29 | (0.18, 0.52) |
| $n = -1.5$ | 1.0 | 0.0 | 1.0 | 0.09 | 0.20 | (0.12, 0.36) |
| $n = -1.0$ | 1.0 | 0.0 | 1.0 | 0.10 | 0.17 | (0.10, 0.30) |
| $n = -0.5$ | 1.0 | 0.0 | 1.0 | 0.08 | 0.19 | (0.12, 0.31) |
| $n = 0.0$ | 1.0 | 0.0 | 1.0 | 0.09 | 0.14 | (0.08, 0.23) |
| 0.1 | 0.0 | 1.0 | 0.10 | 0.06 | (0.04, 0.12) |

* Best theoretical prediction (for $S = 0.5$).

* Best fit to the mass-density relation.
SSM from the $\chi^2$ fit of the mass-density relation (col. [6]), with the corresponding 90% confidence interval (col. [7]). As can be seen, the theoretical values favored by the evolutionary picture proposed fall inside the 90% confidence interval associated with the empirical values in six of eight cases. Only in the case of the flat ($n = -1.5$) and the open ($n = -1$) cosmogonies is the theoretical prediction in column (5) a little too small. In fact, the trend for the theoretical value to be slightly smaller than the empirical one seems quite general, suggesting the existence of some slight bias of the kind mentioned in SSM. In any event, the general agreement between the theoretical and empirical values of $x_\delta$ in all cosmogonies considered is hardly a coincidence. This strongly supports the validity of the schematic evolutionary picture for dark matter halos proposed in SSM.

4. DISCUSSION

In § 2 we have considered only elliptical orbits with different possible values of $S$ to account for their unknown typical eccentricity. These correspond, indeed, to the most reasonable initial condition for mergers regardless of the cosmology. Mergers from parabolic or hyperbolic orbits are certainly much more improbable because of their non-negative orbital energy. This seems to contradict the idea that in scale-free cosmogonies [i.e., an Einstein–de Sitter universe with power-law spectrum: $P(k) \propto k^n$] with $n = 1$, mergers occur typically between halos describing parabolic orbits, while for $n > 1$ ($n < 1$), they would follow hyperbolic (elliptical) orbits. The reason is that the mean cosmic density, and hence the mean density of halos with characteristic mass $M_\ast$, scales as $M_\ast^{-1/(3+n)}$. Therefore, for $n = 1$ one has $M_\ast \propto R$, implying that the specific binding energy of such halos is constant in time. One must be very cautious with this kind of qualitative argument. In the previous reasoning, the binding energy refers only to a very specific class of halos, those which have the characteristic mass at the given epoch. Furthermore, the typical progenitors of such halos will hardly coincide with the class of objects which have characteristic mass at the adequate epoch.

In this Letter, we have checked the consistency, relative to energy conservation, of the scaling evolution inferred in SSM from the empirical mass-density relation. We must stress, however, that such a relation is not the simple consequence of energy conservation in mergers. Our results show that the mass-density relation depends, in addition, on the assumed evolution of halo structure. Any departure from the concrete evolutionary picture proposed (e.g., other values of parameters $\delta_0$ and $x_\delta$ at halo formation or a nonsteady behavior of halos during accretion) would have led, in general, to a mass-density relation in disagreement with $N$-body simulations.

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