Investigating the dynamics of a buck converter with time-delay feedback control

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Abstract. The paper deals with the dynamics features of a closed automatic control system based on a buck converter with a function to control nonlinear dynamic processes. A comprehensive analysis of the system under study is carried out using both small-signal average dynamic models and nonlinear dynamic models. For the first time, a small-signal average model of a system with a control function for nonlinear dynamic processes based on a time-delay feedback control has been obtained. The influence of this control method on Bode plots of an open system is shown.

It is noted that the use of the time-delay feedback control ensures the stability of both the equilibrium point of the small-signal average model and the fixed point of the periodic mode of the nonlinear dynamic model of the system under consideration. This enables to achieve high quality output voltage. The application of the time-delay feedback control does not require to correct the system parameters, which ensures a given speed and low losses in the power switch of the converter. In addition, the quality of the transition process in the system is improved, namely, oscillation is eliminated and overshoot is reduced. Using the time-delay feedback control, it is possible to provide the desired dynamic mode in the areas of instability of the initial system without controlling nonlinear dynamic processes, which makes it possible to increase the speed due to the small time constant of the regulator. These results were obtained for the first time and can be used in the design of pulse power supply systems of a wide variety.

1. Introduction
Pulsed-width converters of electric power [1, 2] are actively used in various fields of activity. They are closed-loop automatic control systems that require stability at the design stage. This is done by choosing the optimal parameters of the regulator based on the required stability margin in phase and/or amplitude when considering the system as linear.

Small-signal average models of pulse voltage converters are known to describe electromagnetic processes only approximately, since these converters belong to the class of nonlinear dynamic systems. Dynamic nonlinearities lead to the fact that undesirable oscillations without loss of stability of the equilibrium point of a small-signal average model of the system can appear at the output of the converter as a result of bifurcations with variations of one or more of its parameters [3]. A complex analysis of such phenomena is possible only with the use of nonlinear dynamic models of systems of the class under consideration, the theory of bifurcations and chaos, as well as the theory of Poincare maps [4-6].

When analyzing pulse converters as nonlinear dynamic systems, the concept of the desired dynamic mode is used. The desired dynamic mode is understood as a mode in which the frequency of oscillations of the output voltage is equal to the frequency of pulse-width modulation (PWM) (1-cycle) [4]. Also, as
a result of bifurcations, \( m \)-cycles can be realized, where \( m \) is the cycle ratio, which shows how many times the frequency of fluctuations in the output voltage is less than the frequency of pulse-width modulation. In addition, chaotic oscillations \( (m = \infty) \) may be observed during the operation of pulse converters. The operation of pulse converters in undesirable modes is usually accompanied by a large amplitude of output voltage fluctuations [3-6].

Elimination of undesirable dynamic modes is currently possible both with the help of parametric synthesis and with the help of structural and algorithmic synthesis. Parametric synthesis is associated with the correction of the controller parameters calculated when considering the system as linear, which can lead to a decrease in its performance. Increasing the switching frequency is an effective method of controlling nonlinear phenomena [7], but it is accompanied by an increase in dynamic losses in power switches, which makes it impossible to use it when using low speed switches. Also, increasing the frequency of power switches may not be possible due to the limited speed of the microcontroller.

The most promising approach to eliminating undesirable dynamic modes is structural and algorithmic synthesis. In this case, it is assumed to make a specific structure of the control system, which eliminates unwanted fluctuations in the output voltage. At the same time, the correction of the parameters of the controller or the power section is not carried out, which gives the desired speed, as well as acceptable dynamic losses.

The most promising and simple method of controlling nonlinear dynamic processes of pulse power supply systems is the time-delay feedback control (TDFC) [8-10]. Here, an additional control system for nonlinear dynamic processes (NDPCS) is introduced into the control system with a standard structure, which supplies a corrective effect to the control circuit of the main control system at the beginning of each clock interval [11]. This allows to change the properties of a nonlinear dynamic system and ensure the stability of the desired dynamic mode. The periodic mode is stable when its largest multiplier lies within the unit circle on the complex plane. The analysis of the local stability of periodic modes in this case is carried out on the basis of the first Lyapunov method [4].

Existing works devoted to the application of the time-delay feedback control, as a rule, show the possibility of stabilizing the desired dynamic mode, but at the same time a number of issues are overlooked. For example, how does the application of the time-delay feedback control affect the transient response of the system or how does Bode plot of an open small-signal average model of the system change? Also, in paper [12] the Bode diagrams of the continuous control system based on TDFC were considered. At the same time, when considering microprocessor control systems, it is necessary to consider a discrete control system.

In this paper, a small-signal average dynamic model of a buck converter with a discrete control system based on the time-delay feedback control is proposed for the first time, and Bode plots of an open-loop system are built. The influence of the time-delay feedback control on the dynamics of the system from various points of view is also studied.

2. Description of a closed automatic control system based on a buck converter with a control function for nonlinear dynamic processes

The functional diagram of the automatic control system (ACS) based on a buck converter with a proportional-integral regulator (PI-regulator) is shown in figure 1, a. The following designations are used in the figure: GCS is the general control system, NDPCS is the nonlinear dynamic processes control system, \( VT \) is a power transistor, \( VD \) is a power diode, \( R_{LD} \) is the load resistance, \( L \) is the filter choke, \( R_{s} \) is the active resistance of the filter choke, \( C \) is the filter capacitor, \( R_{p} \) is the active resistance of the capacitor, \( \beta \) is the feedback amplifier with \( \beta \) coefficient, \( K_{1} \) is the gain coefficient of the proportional part of PI-regulator, \( SH \) is a sample-and-hold device, \( MC \) is a master clock, \( RG \) is a ramp generator, \( e = \Rightarrow \) is a comparator, \( SH_{i} \) are sample-and-hold devices of the scaled value of \( i \)-phase variable with \( \beta \) coefficient, \( CB_{i} \) is the control block for \( i \)-phase variable, \( K_{i} \) is the gain coefficient of \( CB_{i} \), output signal, \( U_{in} \) is an input voltage, \( U_{out} \) is the output signal of the master clock, \( U_{imp} \) is the output signal of the ramp generator, \( U_{ref} \) is the reference signal, \( U_{err} \) is the error signal, \( U_{con} \) is the control signal, \( U_{p} \) are power switch control pulses, \( W_{int} \) is the integrator transfer function.
\[ W_i(s) = \frac{1}{T_i s + K_{r/2}} , \]

where \( T_i \) is the integrator time constant, \( K_{r/2} \) is the coefficient of stray parameters of the integrator circuit [13].

**Figure 1.** Description of ACS operation: a - ACS block diagram, b - CB block diagram.

The green frame in figure 1, a outlines the general control system which aims at stabilizing the average value of the output voltage of the converter. The red frame outlines NDPCS, which is discrete and gives corrective actions with the frequency equal to the pulse-width modulation frequency, which allows to stabilize the desired 1-cycle.

Figure 1, b presents a block diagram of \( CB_i \) correcting block. As the figure shows, this block finds the difference between \( i \)-phase variable at the beginning of the current \( k \)-clock interval \( (x_{k,i}) \) and the previous \( k-1 \) clock interval \( (x_{k-1,i}) \)

\[
\Delta x_{k,i} = x_{k-1,i} - x_{k,i}, \quad \text{where} \quad i = 1, 2, 3.
\]

Then the residual for each phase variable is scaled with \( K_i \) coefficient. The corrective effect of NDPCS for the system in question is defined by the expression

\[
u_k = \sum_{i=1}^{3} K_i \Delta x_{k,i} .
\]

It is known that the main task of controlling a nonlinear dynamic process is to ensure \( u_i = 0 \) in steady-state mode when the system operates in the desired 1-cycle \( x_{k-1,i} = x_{k,i} \).

3. Small-signal linearized dynamic model of the system

The structure of the small-signal dynamic model of the system (figure 1) is shown in figure 2, where the following designations are used: \( W_{pm}(s) \) is the transfer function of the pulse modulator, \( W_{de}(s) \) is the transfer function of the duty cycle–throttle current, \( W_{cl}(s) \) is the transfer function of the \( CB_i \) correcting blocks, \( u_{out} = u_{c} \) is the output voltage, \( u_i \) is the voltage on the capacitor, \( i_L \) is the throttle current, \( u_i \) is the output voltage of the integrator as part of PI-regulator. The rest of the designations correspond to figure 1.
The transfer function of PI-regulator, taking into account its imperfection, is determined by the expression [13]

\[ W_i(s) = K_{r1} + W_j(s) = K_{r1} + \frac{1}{T_s + K_{r2}}. \]

The transfer function of the duty cycle-output is defined as [1]

\[ W_{du}(s) = \frac{\Delta u_{out}(s)}{d(s)} = \frac{DR_{LD} + CDR_{LD}R_Cs}{LC(R_{LD} + R_C)s^2 + (L + R_{LD}R_LC + R_{LD}R_LC + R_CR_LC)s + R_{LD} + R_L}, \]

where \( D \) is the duty cycle, \( \Delta d \) is the small perturbation of the duty cycle, \( \Delta u_{out} \) is the small perturbation of the output voltage.

The transfer function of the duty cycle–choke current is defined as

\[ W_{di}(s) = \frac{\Delta i_t(s)}{d} = \frac{U_{in} + CU_{in}s(R_L + R_C)}{LC(R_{LD} + R_C)s^2 + (L + R_{LD}R_LC + R_{LD}R_LC + R_CR_LC)s + R_{LD} + R_L}. \]

The transfer function of the pulse modulator is as follows

\[ W_{di}(s) = \frac{1}{U_{sweep}}, \]

where \( U_{sweep} \) is the amplitude of the sweeping voltage.

Let us get the transfer function \( W_{cb}(s) \) (figure 1, b). As it is seen from the figure, \( CB \) block is described by a difference equation

\[ \Delta x_{k,i} = x_{k-1,i} - x_{k,i}. \]

Using \( z \)-transformation, we have

\[ \Delta x_{k,i} = x_{k,i}(z^{-1} - 1). \]

The discrete transfer function of \( CB \) block is the following

**Figure 2.** Small-signal average model of the system with controlling nonlinear dynamic processes.
Using a Tustin’s method, we proceed to a continuous transfer function. To do this, we put the following expression in (1) [14, 15]

$$
W_{ch}(s) = -1 + z^{-1},
$$

where $T$ is the duration of the clock interval PWM.

Thus, the continuous transfer function of $CB_i$ blocks looks as follows

$$
W_{ch}(s) = \frac{2Ts}{Ts + 2}.
$$

4. Simulation of system dynamics

This section gives the results of modeling a closed automatic control system based a buck converter. The simulation was carried out with the following system parameters: input voltage $U_{in}=800$ V, choke inductance $L=0.009$ H, active resistance of the choke $R_L=10$ Ohm, capacitor capacity $C=10$ uF, load resistance $R_{LD}=250$ Ohm, coefficient taking into account the imperfection of PI-regulator $K_2=0.001$, feedback factor $\beta=0.01$, amplitude of the sawtooth voltage $U_{rmp,m}=10$ V, PWM frequency $f_{sw}=20$ kHz, $U_{ref}=5$ V.

The experiment is carried out with a system of PI-regulator without NDPCS as well as a system adjusted using NPDCS. When modeling, we used: a nonlinear dynamic model [4] and a small-signal, average model.

Figure 3 presents the results obtained using a nonlinear dynamic model of a system without NPDCS, in the form of a map of dynamic modes and a corresponding diagram of the range of output voltage fluctuations. These diagrams are built in the range of two parameters of PI-regulator: $K_i$ and $K_0$, where $K_i=1/T_i$. On the map of dynamic modes, $D_{i,j}$ symbols mark the areas of different modes, where $i$ symbol is the multiplicity of $m$-cycle, and $j$ symbol is the number of the periodic mode with $i$-multiplicity on the map. So, for example, $D_{1,1}$ is the first area of the desired 1-cycle. $D_{ch,j}$ areas correspond to the chaotic modes of the converter operation ($m \rightarrow \infty$).

$AB$ boundary is a bifurcation boundary, which crossing leads to undesirable dynamic modes (in this case, mainly chaotic). As it can be seen from figure 3, b, undesirable modes are accompanied by a large amplitude of output voltage fluctuations. $CD$ boundary highlighted in red represents the stability boundary of a small-signal average model of a system without NPDCS with a phase stability margin $\varphi=10^\circ$. This boundary was calculated according to the method [16]. It is obvious that the stability boundary of the desired 1-cycle ($AB$) and the stability boundary of the small-signal average $CD$ model do not coincide, which indicates the need to use not only small-signal average models at the design stage, but also nonlinear dynamic models that allow taking into account the possibility of nonlinear oscillations.
Figure 3. Two-parameter diagrams: a - the map of dynamic modes, b - the diagram of the range of output voltage fluctuations.

Figure 3 shows that the desired area of the regulator parameters is the area bounded by AED curve, which is determined by the intersection of the stability area of the 1-cycle and the stability area of the linearized system model. In the area bounded by DEB curve, as follows from figure 3, a, the desired 1-cycle is observed, but the small-signal average model of the system in this case is stable with a stability margin of $\varphi <10^\circ$, so we consider the operation in this area unacceptable.

Using the time-delay feedback control, it is possible to ensure the operation of the system in the desired dynamic mode over the entire range of the regulator parameters (figure 3, a) [7]. It is obvious that NDPCS use changes Bode plot of a small-signal average system model and, accordingly, affects transients. Let us evaluate these indicators with the following NDPCS parameters: $K_1=9.62334$, $K_2=4.74367$, $K_3=0.190714$, $\beta_1=-0.0214938$, $\beta_2=0.0117646$, $\beta_3=-0.836231$.

The calculation results are shown in figure 4 as Bode plots and their corresponding time diagrams. Here, diagrams built for the system without NDPCS are highlighted in red, and for the system using NDPCS in blue. Bode plots are built using a small-signal average model, and time diagrams using a nonlinear dynamic model. As we can see from the figure, the use of NDPCS results in the transformation of diagrams. So, in particular, at $P_1$ point, where the small-signal average model of the system without NDPCS is stable, the use of NDPCS allowed to increase the stability margin from $22^\circ$ to $67^\circ$ (figure 4, a), overshoot was also excluded (the time diagram of the system with NDPCS is highlighted in blue in figure 4, c).

At $P_2$ point, the small-signal average model of the system without NDPCS is unstable, which is accompanied by fluctuations in the output voltage (highlighted in red in figure 4, d). The phase at the edge frequency of the open system in this case is $184^\circ$, i.e. more than $180^\circ$. The use of NDPCS at $P_2$ point made it possible to provide a stability margin of $33^\circ$, which eliminated fluctuations in the output voltage (highlighted in blue in figure 4, d).

The analysis of diagrams in figure 4 shows that the use of NDPCS does not lead to their significant transformation, which provides a speed capacity close to what was defined when choosing the parameters of PI-regulator (figure 4, c). This ensures the stability of both the equilibrium point of the linear system and the fixed point of the desired dynamic mode of the nonlinear dynamic system. It is also obvious that when selecting the controller parameters corresponding to $P_2$ point, the system speed capacity increases significantly (figure 4, d), while taking into account the use of NDPCS, it remains stable.
Figure 4. Diagrams for ACS with buck converter: a - Bode plots at $P_1$ point, b - time diagrams at $P_1$ point, c - Bode plots at $P_2$ point, d - time diagrams at $P_2$ point.

5. Conclusion

According to the research results, the following conclusions can be drawn.

1. For the first time a small-signal linearized model of a system with a control function for nonlinear dynamic processes based on TDFC was obtained, which allows analyzing frequency characteristics.

2. It is shown that the use of TDFC even at the point of stability of an uncorrected system makes it possible to improve the quality of the transition process, in particular, overshoot was completely eliminated while maintaining the duration of the transition process.

3. At the point of instability of an uncorrected system, the use of TFDC ensures its stability, which provides an acceptable quality of the transition characteristic.

4. The presented results were obtained for the first time and they allow deeper understanding of the effect of the time-delay feedback control on the dynamics of the system as a whole.

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