Safe Human-Interactive Control via Shielding

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Abstract—Ensuring safety for human-interactive robotics is important due to the potential for human injury. The key challenge is defining safety in a way that accounts for the complex range of human behaviors without modeling the human as an unconstrained adversary. We propose a novel approach to ensuring safety in these settings. Our approach focuses on defining backup actions that we believe human always considers taking to avoid an accident—e.g., brake to avoid rear-ending the other agent. Given such a definition, we consider a safety constraint that guarantees safety as long as the human takes the appropriate backup actions when necessary to ensure safety. Then, we propose an algorithm that overrides an arbitrary given controller as needed to ensure that the robot is safe. We evaluate our approach in a simulated environment, interacting with both real and simulated humans.

I. INTRODUCTION

Robots are increasingly operating in environments where they must interact with humans, such as collaborative grasping [1], [2] and autonomous driving [3], [4], [5], [6]. Thus, there has been much interest in designing planning and control algorithms for human-robot interaction. Ensuring safety for such robots is paramount due to the potential to inflict harm on humans [7]. These challenges are particularly salient in settings such as autonomous driving, where robots and humans may have disjoint or conflicting goals—e.g., a self-driving car making an unprotected left turn [5].

The key challenge is how to define safety for human-interactive robots. We could model the human as an adversary, but this approach is typically prohibitively conservative. Another approach is to learn a model to predict human actions [8], [9], [10], and ensure safety with respect to this model. If the model captures all actions exhibited by humans, then this approach ensures safety. However, different humans may exhibit very different behaviors [6]—e.g., people in a lab may act differently than people on a street. Collecting data from all possible settings can be very challenging. If a behavior is not exhibited in the training data, then the model may not account for it. More fundamentally, even the best machine learning models make errors, which can correspond to actions missed by the model. Finally, responsibility sensitive safety (RSS) [11] is an approach that manually specifies the range of acceptable robot actions in various scenarios. That is, the designer of the robot controller is responsible for ensuring that acceptable actions only include safe actions. However, manually defining acceptable robot actions for all possible scenarios is challenging, especially for robots operating in open-world environments. For instance, [11] only formally defines acceptable actions for a limited number of scenarios such as changing lanes.

We propose a novel approach for ensuring safety in human-interactive systems that accounts for all human behaviors in some bounded set. There are two challenges: (i) how to define the set of human behaviors, and (ii) how to ensure safety with respect to this set. We address these challenges as follows:

- **Bounding human behavior via backup actions:** Rather than specify the set of all actions the robot is allowed to take (as in RSS), the designer specifies backup actions that we believe the human always considers taking to avoid an accident (e.g., braking while steering in some direction). In particular, we assume the human may take any action in general, but that they take these actions when necessary to ensure safety.

- **Ensuring safety via abstract interpretation:** We use abstract interpretation [12] to conservatively overapproximate the reachable set of the system for the above model of human behavior, and then ensure safety with respect to this overapproximation.

First, our notion of backup actions captures the idea that we reasonably believe the human will take a limited range of evasive maneuvers to avoid an accident—e.g., if the robot gradually slows to a stop, then we may expect the human...
to also slow down to avoid rear-ending it\(^2\). If the robot is on a highway, coming to a stop is more dangerous, so we may require that the robot pull over to the shoulder before coming to a stop. Similarly, we may require that the robot avoid coming to a stop in an intersection. Specifying backup actions provides a way to define safety; we refer to such a safety constraint as \textit{safety modulo fault}\(^3\).

At a high level, to instantiate our framework, the designer of the robot controller needs to design the following:

- **Robot backup action**: An action the human anticipates the robot may take to ensure safety—e.g., brake without changing directions. It should be chosen based on intuition about what actions the human driver anticipates the robot may take (e.g., based on traffic rules).

- **Human backup action**: A set of actions that includes at least one action the human considers taking to ensure safety—e.g., braking while steering in some direction. It should be chosen based on intuition about safety maneuvers the human driver considers taking.

In particular, whereas RSS requires the control designer to specify the set of all actions the robot is allowed to take, we only require the designer to specify actions that they believe the human considers taking to use to avoid an accident.

Next, we propose an algorithm for ensuring safety modulo fault. Our algorithm builds on \textit{shielding} \cite{13,14,15,16}; in particular, rather than designing a specific controller, our algorithm takes as input an arbitrary controller designed to achieve the goal, and then combines it with a backup controller in a way that tries to use the given controller as frequently as possible while ensuring safety modulo fault. At a high level, it does so by using on-the-fly verification based on abstract interpretation to determine whether it is safe modulo fault to use the given controller; if so, it uses the given controller, but otherwise, it uses a backup controller.

Finally, we empirically evaluate our approach in a simulation, including both settings where the humans are simulated and settings where the human is controlled by an real person via keyboard inputs. We demonstrate that our MPS modulo fault algorithm enables the robot to avoid accidents both with real and simulated humans, even when combined with a naïve controller that altogether ignores the humans.

Figure \[\text{Figure 1}\] illustrates how our algorithm ensures safety while interacting with a human driver without being overly cautious. It assumes that the human will at least slightly brake to avoid an accident (left). If it still cannot guarantee safety, then it allows the human to go first (right).

\section{Preliminaries}

\subsection{Human-robot system}

We consider a system with a robot $R$ and human $H$. Following prior work \cite{5}, we assume

\[x_t = f_R(x_t, u_{R,t}) \quad \text{and} \quad x_{t+1} = f_H(x'_t, u_{H,t}),\]

where $f_R : \mathcal{X} \times \mathcal{U}_R \to \mathcal{X}$ is the robot dynamics, $f_H : \mathcal{X} \times \mathcal{U}_H \to \mathcal{X}$ is the human dynamics, $\mathcal{X} \subseteq \mathbb{R}^{n_x}$ is the state space, $\mathcal{U}_R \subseteq \mathbb{R}^{n_{u_R}}$ are the robot actions, and $\mathcal{U}_H \subseteq \mathbb{R}^{n_{u_H}}$ are the human actions. Given an initial state $x_0 \in X_0 \subseteq \mathcal{X}$ where it is the robot’s turn to act, along with two action sequences

\[\bar{u}_R = (u_{R,0}, u_{R,1}, \ldots) \in \mathcal{U}_R^\infty\]

\[\bar{u}_H = (u_{H,0}, u_{H,1}, \ldots) \in \mathcal{U}_H^\infty\]

for $R$ and $H$, respectively, the trajectory from $x_0$ using $\bar{u}_R, \bar{u}_H$ is the sequence of states

\[\zeta_R(x_0, \bar{u}_R, \bar{u}_H) = (x_0, x'_0, x_1, \ldots) \in \mathcal{X}^\infty,\]

where $x'_0 = f_R(x_0, u_{R,0})$ and $x_{t+1} = f_H(x'_t, u_{H,t})$. Similarly, given an initial state $x'_0 \in \mathcal{X}$ where it is the human’s turn to act along with $\bar{u}_R \in \mathcal{U}_R^\infty$ and $\bar{u}_H \in \mathcal{U}_H^\infty$, the trajectory from $x_0$ using $\bar{u}_H, \bar{u}_R$ is the sequence of states

\[\zeta_H(x_0, \bar{u}_R, \bar{u}_H) = (x_0, x'_0, x_1, \ldots) \in \mathcal{X}^\infty,\]

where $x'_0 = f_H(x'_0, u_{H,0})$ and $x_{t+1} = f_R(x'_t, u_{R,t})$. Also, given robot and human policies $\pi_R : \mathcal{X} \to \mathcal{U}_R$ and $\pi_H : \mathcal{X} \to \mathcal{U}_H$, respectively, we define the trajectory

\[\zeta_R(x_0, \pi_R, \pi_H) = (x_0, x'_0, x_1, \ldots) \in \mathcal{X}^\infty\]

by $x'_0 = f_R(x_0, \pi_R(x_0))$ and $x_{t+1} = f_H(x'_t, \pi_H(x'_t))$, and

\[\zeta_H(x_0, \pi_R, \pi_H) = (x_0, x'_0, x_1, \ldots) \in \mathcal{X}^\infty\]

by $x'_0 = f_H(x'_0, \pi_H(x'_0))$ and $x_{t+1} = f_R(x'_t, \pi_R(x'_t))$.

Given a given safe region $X_{\text{safe}} \subseteq \mathcal{X}$, our goal is to ensure the system stays in $X_{\text{safe}}$ for the entire trajectory.

\textbf{Definition II.1.} A trajectory $\zeta = (x_0, x'_0, x_1, \ldots)$ (or $\zeta = (x_0, x'_0, x_1, \ldots)$) is safe if $x_t \in X_{\text{safe}}$ for all $t \in \mathbb{N}$.

\textbf{b) Example:} Consider an autonomous driving robot interacting with a human driver $H$. The state $x \in \mathcal{X} = \mathbb{R}^8$ is a vector $x = (x_R, y_R, v_R, \theta_R, x_H, y_H, v_H, \theta_H)$ representing the positions $(x_R, y_R), (x_H, y_H)$, velocities $v_R, v_H$, and angles $\theta_R, \theta_H$ of $R$ and $H$, respectively. The actions $u \in \mathcal{U}_R = \mathcal{U}_H = \mathbb{R}^2$ are vectors $u = (\phi, a)$ representing the steering angle $\phi$ and acceleration $a$; we assume $|\phi| \leq \phi_{\max}$ and $|a| \leq a_{\max}$ are bounded. Given time step $\tau \in \mathbb{R}_{>0}$, the dynamics are

\[f_A(x, u_A) = x + g_A(x, u_A) \cdot \tau \quad (\forall A \in \{R, H\})\]

\[g_R(x, u_R) = (v_R \cos \theta_R, v_R \sin \theta_R, a_R, v_R \phi_R, 0, 0, 0, 0)\]

\[g_H(x, u_H) = (0, 0, 0, 0, v_H \cos \theta_H, v_H \sin \theta_H, a_H, v_H \phi_H)\].

Safety means the robot and the human have not collided:

\[X_{\text{safe}} = \{x \in \mathcal{X} \mid \|x_R - x_H, y_H\| \geq d_{\text{safe}}\},\]

for some constant $d_{\text{safe}} \in \mathbb{R}_{\geq 0}$.
III. SAFETY MODULO FAULT

Ensuring safety in the presence of an adversarial human would be impossible or at least significantly degrade performance—e.g., to avoid an accident with an adversarial human driver, the robot would have to maintain a very large distance. Thus, to ensure safety, we must make assumptions about the behavior of the human. Ideally, we want to make the minimal possible assumptions about the behavior of the human while still accounting for all possible behaviors of a human acting in a responsible way. Then, we want to ensure safety for any human acting according to these assumptions.

The key challenge is devising a reasonable set of assumptions on the human. Intuitively, our assumptions are based on the idea that if the human can act in a safe way to avoid an accident, then they do so (Assumption III.1). We need to formalize what it means for the human to “be able to act in a safe way”. We give the human great leeway to avoid an accident, then they do so (Assumption III.1).

We need to formalize what it means for the human to act in a responsible way. Then, we want to ensure the minimal possible assumptions about the behavior of the human while still accounting for all possible behaviors of a human acting in a responsible way. Then, we want to ensure safety for any human acting according to these assumptions.

We formalize these assumptions in the next section.

b) Assumption on human objective: First, we assume that the human reward for reaching an unsafe state is $-\infty$.

Assumption III.1. For any $x', x \in X$ and $u_H \in \mathcal{U}_H$, we have $r_H(x', u_H, x) = -\infty$ if $x' \notin X_{\text{safe}}$ or $x \notin X_{\text{safe}}$.

That is, the human driver always acts to avoid an accident. Otherwise Assumption III.1 $r_H$ can be arbitrary.

With this assumption, there are two reasons accidents may happen: (i) there was a safe action sequence $\bar{u}_H \in \mathcal{U}_{H,0}^R$ that the human driver failed to take, or (ii) if the robot takes an action $u_R \notin \mathcal{U}_R$ that the human driver failed to anticpate. Thus, we can always conservatively take $\mathcal{U}_R$ to be smaller than it actually is. Conversely, we can always take $\mathcal{U}_H$ to be larger than it actually is. Thus, we make minimal assumptions about what actions are contained in $\mathcal{U}_R$ and $\mathcal{U}_H$.

First, we make the following assumption on the set of actions $\mathcal{U}_R$ that the human anticipates the robot may take:

Assumption III.2. We are given a robot backup action $v^0_{R} \in \mathcal{U}_R$ that is anticipated by the human—i.e., $v^0_{R} \in \mathcal{U}_R$.

That is, the human always accounts for the possibility that the robot might take action $v^0_{R}$. For example, we might assume that $v^0_{R}$ is gradually braking and coming to a stop.

Next, we make the following assumption about the human:

Assumption III.3. We are given a human backup action set $\mathcal{U}_{H,0}^R \subseteq \mathcal{U}_H$ such that if $\max_{\bar{u}_H \in \mathcal{U}_{H,0}^R} J_H(\bar{u}_H) = -\infty$, then the human takes an action $\pi_H(x') \in \mathcal{U}_H$.

That is, if the human is unable to guarantee safety (i.e., their objective value is $-\infty$), then they take some action in $\mathcal{U}_{H,0}^R$. For example, $\mathcal{U}_{H,0}^R$ may contain all actions where the human driver decelerates by at least some rate (so they can slow down more quickly or steer in any direction).

Remark III.4. Our approach can easily be extended to the case where $\bar{u}_H^0$, $\mathcal{U}_R$, and $\mathcal{U}_H$ are time varying.

Remark III.5. We can weaken [1] as follows. We only need to assume that the human chooses an action sequence $\bar{u}_H^0$ with a reward $> -\infty$. Then, Assumption III.1 say that the human chooses actions $\bar{u}_H^0$ that will avoid an accident assuming the robot takes actions in $\mathcal{U}_R$. For instance, we can replace [1] with a probabilistic model [8] where the human samples action sequence $\bar{u}_H^0$ with probability $P(\bar{u}_H^0 \mid x, u_{R,0}) \propto \exp(J_H(\bar{u}_H^0))$ and takes action $u_{H,0}$; then, Assumption III.1 says the human never considers trajectories that may reach an unsafe state.
Algorithm 1 Model predictive shielding modulo fault.

\begin{verbatim}
procedure \(\pi_R(x)\)
  \(x' \leftarrow f_R(x, \hat{\pi}_R(x))\)
  if IsREC(\(x'\)) then \(\pi_R(x) \leftarrow u_0^R\) else \(u_0^R\)
  return end procedure

procedure IsREC(\(x'\))
  \(X_0' \leftarrow \{x'\}\)
  for \(t \in \{0, ..., k-1\}\) do
    if \(X_t' \subseteq X_{\text{safe}}\) then return false end if
    if \(U_{R,t} \leftarrow \{u\}\) else \(U_{R,t}^0\) end if
    \(X_{t+1}' \leftarrow F(X_t, U_{R,t}, U_{H,t})\)
  end for
  return if \(X_k \subseteq X_{\text{eq}}\) then true else false end if
end procedure
\end{verbatim}

\textbf{c) Problem formulation:} Our goal is to ensure that the robot acts in a way that ensures safety for an infinite horizon for any human that satisfies our assumptions.

\textbf{Definition III.6.} A robot policy \(\pi_R : \mathcal{X} \rightarrow \mathcal{U}_R\) is safe modulo fault for initial states \(X_0 \subseteq \mathcal{X}\) if for any human policy \(\pi_H\) satisfying Assumptions \textbf{III.1} \textbf{III.2} \& \textbf{III.3} and any \(x_0 \in X_0\), the trajectory \(\zeta_R(x_0, \pi_R, \pi_H) \in \mathcal{X}_{\infty}\) is safe.

That is, \(\pi_R\) that ensures safety as long as the human acts in a way that satisfies our assumptions. Our goal is to design a policy \(\pi_R\) that is safe modulo fault. We use the term “fault” to indicate that this property only guarantees safety with respect to humans that satisfy our assumptions; we are not assigning blame to the human in case of an accident. The controller designer is responsible for ensuring the assumptions are satisfied by reasonable human drivers.

Finally, we cannot guarantee safety starting from an arbitrary state \(x_0\). For instance, if the robot is about to crash into a wall, no action can ensure safety. We assume that the initial states \(X_0\) are ones where we can guarantee safety.

\textbf{Definition III.7.} A safe equilibrium state \(x \in \mathcal{X}\) satisfies (i) \(x \in X_{\text{safe}}\), and (ii) \(x = f(x, u_0^R, u_H)\) for all \(u_H \in U_H^0\).

We denote the set of safe equilibrium states by \(X_{\text{eq}}\). At a state \(x \in X_{\text{eq}}\), the robot and human can together ensure safety for an infinite horizon by taking actions \(u_0^R\) and \(u_H\) for any \(u_H \in U_H^0\). In our driving example, \(X_{\text{eq}}\) contains states where both agents are at rest (i.e., their velocity is zero).

\textbf{Assumption III.8.} We have \(X_0 \subseteq X_{\text{eq}}\).

In other words, the system starts at a safe equilibrium state where we can ensure safety for an infinite horizon.

\section{Model Predictive Shielding Modulo Fault}

We describe our algorithm for constructing a robot controller \(\pi_R : \mathcal{X} \rightarrow \mathcal{U}_R\) that is safe modulo fault. Our approach is based on shielding [14]—it takes as input an arbitrary controller \(\hat{\pi}_R : \mathcal{X} \rightarrow \mathcal{U}_R\) and modifies it to construct \(\pi_R\). Intuitively, \(\pi_R\) overrides \(\hat{\pi}_R\) when it cannot ensure it is safe.

The challenge is checking whether it is safe to use \(\hat{\pi}_R\). Model predictive shielding (MPS) is an approach to shielding that checks safety online based on the following [15], [16]. These approaches are designed to handle deterministic or stochastic environments where the model is known, whereas we do not have any such model of the human driver. We show how to extend these algorithms to our setting. The idea is to maintain the invariant that the current state is recoverable—intuitively, that there is some sequence of actions each agent can take that safely brings the system to a stop. In particular:

\textbf{Definition IV.1.} Given hyperparameter \(k \in \mathbb{N}\), a state \(x' \in \mathcal{X}\) is recoverable (denoted \(x' \in X_{\text{rec}}\)) if for any \(\bar{u}_H \in (U_H^0)^\infty\) and for \(\bar{u}_R = (u_0^R, u_0^R, \ldots) \in U_R^{\infty}\), the trajectory \(\zeta(x_0, \bar{u}_H, \bar{u}_R) = (x_0, x_0, x_1', \ldots)\) satisfies (i) \(x'_t, x_t \in X_{\text{safe}}\) for all \(t \in \{0, \ldots, k\}\), and (ii) \(x_k \in X_{\text{eq}}\).

That is, \(x'\) is recoverable if the human and robot can ensure safety by using their respective backup actions \(u_0^R\) and \(u_0^R\). By definition, if \(x_k \in X_{\text{eq}}\), then \(x_k = x_{k+1} = \ldots = x_{k+1} = \ldots\). Since \(x_k \in X_{\text{safe}}\), it follows that \(x'_t, x_t \in X_{\text{safe}}\) for all \(t \in \mathbb{N}\).

Now, our MPS modulo fault algorithm for computing \(\pi_R\) is shown in Algorithm 1. IsREC checks whether \(x' = f_R(x, \hat{\pi}_R(x))\) is recoverable. If so, \(\pi_R\) returns \(\pi_R(x)\); otherwise, it returns \(u_0^R\). The key novelty is how IsREC checks recoverability. Existing approaches [15], [16] do so by simulating the model of the environment. In contrast, we need to guarantee safety with respect to all \(\bar{u}_H \in (U_H^0)^\infty\). To do so, IsREC conservatively overapproximates recoverability—i.e., if it says that \(x'\) is recoverable, then it must be recoverable, but it may say \(x'\) is not recoverable even if it is.

To do so, IsREC overapproximates the reachable set of states after \(t\) steps as a subset \(X_t \subseteq \mathcal{X}\). More precisely, it assumes a dynamics overapproximation \(F : 2^\mathcal{X} \times 2^\mathcal{U}_R \times 2^\mathcal{U}_H \rightarrow 2^\mathcal{X}\) mapping sets of states \(X \subseteq \mathcal{X}\), sets of robot actions \(U_R \subseteq \mathcal{U}_R\), and sets human action \(U_H \subseteq \mathcal{U}_H\) to sets of states \(F(X, U_R, U_H) \subseteq 2^\mathcal{X}\), which satisfies

\begin{equation}
\begin{split}
f(x, u_R, u_H) \in F(X, U_R, U_H)
\end{split}
\end{equation}

for all \(x \in X, u_R \in U_R, \) and \(u_H \in U_H\)—i.e., \(F(X, U_R, U_H)\) contains at least the states reachable from \(x \in X\) by taking actions \(u_R \in U_R\) and \(u_H \in U_H\). Intuitively, computing \(F\) in a way that this property holds with equality may be computationally intractable, but there exist tractable overapproximations—e.g., based on polytopes [17], [18] or ellipsoids [19], [20]. This approach fits in the general framework of abstract interpretation [12]; here, the sets \(X, U_R,\) and \(U_H\) are represented implicitly rather than explicitly for computational tractability. We describe the overapproximation we use for our autonomous driving example in Section IV.

Finally, IsREC checks whether (i) safety holds for every state \(x_t \in X_t\) (i.e., \(X_t \subseteq X_{\text{safe}}\)), and (ii) every state \(x_k \in X_k\) is a safe equilibrium state (i.e., \(X_k \subseteq X_{\text{eq}}\)). If both these properties hold, then \(x\) is guaranteed to be recoverable. We have the following guarantee (see Appendix A for a proof):

\textbf{Theorem IV.2.} Assuming (2) holds, then our policy \(\pi_R\) is safe modulo fault (i.e., it satisfies Definition III.6).
V. Evaluation

We have implemented our approach in a simulation for three robotics tasks. For the robot, we consider an aggressive controller with and without the shield as well as a cross entropy method controller (CEM) that is designed to avoid humans. For the human, we use both simulated humans based on a social forces model of pedestrians [21], as well as real humans interacting with the simulation via keyboard inputs.

Our goal is to understand how our approach can ensure safety in aggressive driving scenarios. Thus, we focus on settings where the human (simulated or real) and robot must compete to reach their goals. We tune the parameters of our MPS modulo fault algorithm (i.e., the robot backup action $u^R_{H}$ and the human backup action set $U^H_H$) to be as aggressive as possible while still ensuring safety on the simulated humans. Furthermore, for our experiments with real-world humans, we strongly encourage them to try and reach their goal before the robot, albeit keeping safety as the top priority. Then, our results are designed to answer the following:

- Can MPS modulo fault can be used to ensure safety with real and simulated humans?
- Can MPS modulo fault outperform a handcrafted MPC based on CEM in terms of performance?

A. Experimental Setup

a) Robotics tasks: We consider three non-cooperative robotics tasks (depicted in Figure 2). In the first task (“merge”), there are two lanes that merge—i.e., the robot is coming in from one lane and the humans from another; the robot and human goals are to navigate the merge and reach their goal. The second task (“cross”) has both the human and the robot moving towards an intersection from different directions—i.e., the robot is moving horizontally and the human is moving vertically; the robot and human goals are to reach their goal on the other side of the intersection. The third task (“turn”) is an unprotected left turn—i.e., the humans are driving without turning and the robot needs to make a left turn that crosses the human path.

b) Safety property: We assume the robot and human are each a rectangle; then, the safety property is that the robot and human rectangles should not intersect.

c) Robot dynamics: The robot dynamics are the ones in our running example—i.e., its state is $(x, y, v, \theta)$, where $(x, y)$ is position, $v$ is velocity, and $\theta$ is orientation, and its actions are $(a, \phi)$, where $a$ is acceleration and $\phi$ is steering angle. We assume $|a| \leq a_{\text{max}}$, $|\phi| \leq \phi_{\text{max}}$, and $0 \leq v \leq v_{\text{max}}$.

d) Simulated humans: For simulated humans, we use the social force model [21], which includes potential forces that cause each human to avoid the robot, other humans, and walls, while trying to reach their goal.

e) Real humans: We also considered real human users interacting with the simulation via keyboard. They control the human using the up/down arrows to control acceleration and the left/right arrows to control steering angle. We asked the human users to prioritize safety first, but to drive aggressively to try and reach their goal before the robot.

f) Controllers: We consider three controllers for the robot: (i) an aggressive controller, (ii) a handcrafted MPC based on the cross-entropy method (CEM) designed to ensure safety without a shield, and (iii) our MPS modulo fault algorithm used in conjunction with the aggressive controller.

The first controller is an “aggressive controller” that ignores the humans and moves directly towards the goal as quickly as possible. If nonlinear trajectories are required, we manually specify a sequence of subgoals; once the robot reaches its current subgoal, it continues to the next one.

The second is a model-predictive controller (MPC) that aims to avoid colliding with the human. We use a planning algorithm based on the cross-entropy method (CEM). Then, it chooses the action that attempts to optimize its objective over the planning horizon. We use a handcrafted objective that provides a positive reward for progressing towards its goal and a large negative penalty for collisions. To predict collisions, it forecasts the behavior of the human over the planning horizon by extrapolating their position based on their current velocity (i.e., constant velocity assumption). Finally, for the goal-reaching portion of the objective, we use subgoals the same way we do for the aggressive controller.

The third controller is our MPS modulo fault algorithm used with the aggressive controller. The robot backup action is $u^R_H = (0, -1)$ where $\phi = 0$ is the steering angle and $a = -1$ is the acceleration. The human backup action set is

$$U^H_H = \left\{ (\phi, a) \mid \phi \in \left[ -\frac{\pi}{10}, \frac{\pi}{10} \right], a \in \left[ -1, -\frac{1}{2} \right] \right\}.$$

That is, the human predicts that the robot may gradually brake without changing direction, and the human considers braking gradually (or hard) while steering up to some angle.

B. Experimental Results

We describe our experimental results. For simulated humans, all results shown are averaged over 100 simulations. For real humans, the results are based on 18 users.
We have proposed an approach for ensuring safety in human-interactive robotics systems. We define a notion of safety that models human behavior by specifying their backup behaviors, and propose our MPS modulo fault algorithm for ensuring safety with respect to this model. We have validated our approach on both real and simulated humans.

**VI. Conclusion**

We have proposed an approach for ensuring safety in human-interactive robotics systems. We define a notion of safety that models human behavior by specifying their backup behaviors, and propose our MPS modulo fault algorithm for ensuring safety with respect to this model. We have validated our approach on both real and simulated humans.
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APPENDIX I

OVERAPPROXIMATING THE DYNAMICS

We describe our approach for overapproximating the multi-agent dynamics \( f \) in our autonomous car example. In particular, we describe representations \( U_R, U_H \) and \( X \) of sets of robot actions, human actions, and states, respectively, along with a function \( F : (U_R, U_H, X) \to X' \) that satisfies (2). First, we consider a representation of \( U_R \) of the form

\[
U_R = \{(-a_R, 0)\},
\]

where \( a_R \in \mathbb{R}_{>0} \) is the rate at which the robot decelerates; we assume it corresponds to the robot gradually braking to give a trailing human driver sufficient time to respond. Also, we consider a representation of \( U_H \) of the form

\[
U_H = (-\infty, -a_{\text{min}}] \times [-\phi_{\text{max}}, \phi_{\text{max}}]
\]

where \( a_{\text{min}} \in \mathbb{R}_{>0} \) is the minimum deceleration rate—i.e., the minimum rate at which we expect the human driver to brake to avoid an accident. Then, both \( U_R \) and \( U_H \) have form

\[
U_A = [a_{A,\text{min}}, a_{A,\text{max}}] \times [-\phi_{\text{max}}, \phi_{\text{max}}]
\]

for \( a_{A,\text{min}}, a_{A,\text{max}} \in \mathbb{R} \) and \( \phi_{\text{max}} \in \mathbb{R}_{>0} \), and where \( A \in \{R, H\} \). Next, we consider \( X \) of the form

\[
X = X_R \times X_H
\]

\[
X_A = [x_{A,\text{min}}, x_{A,\text{max}}] \times [y_{A,\text{min}}, y_{A,\text{max}}] \times [v_{A,\text{min}}, v_{A,\text{max}}] \times [-\theta_{A,\text{max}}, \theta_{A,\text{max}}],
\]

where \( A \in \{R, H\} \). Then, we use

\[
F(X, U_R, U_H) = F_R(X, U_R, U_H) \times F_H(X, U_R, U_H)
\]

\[
F_A(X, U_R, U_H) = X_A^R \times X_A^H \times X_A^0,
\]

where

\[
X_A^R = [x_{A,\text{min}} - v_{A,\text{max}}, x_{A,\text{max}} + v_{A,\text{max}}]
\]

\[
X_A^H = [y_{A,\text{min}} - v_{A,\text{max}}, y_{A,\text{max}} + v_{A,\text{max}}]
\]

\[
X_A^0 = [\max\{v_{A,\text{min}} + a_{A,max}, 0\}, \max\{v_{A,\text{max}} + a_{A,\text{max}}, 0\}]
\]

\[
X_A^\theta = [-\theta_{A,\text{max}} - v_{A,\text{max}}\theta_{A,\text{max}}, \theta_{A,\text{max}} + v_{A,\text{max}}\theta_{A,\text{max}}]
\]

where again \( A \in \{R, H\} \). It is easy to see that this choice of \( F \) satisfies (2). Finally, we note that (3), (3), and (4) are satisfied for all time steps. In particular, \( X_0 \) satisfies (3), \( U_{R,t} \) satisfies (3) for all time steps \( t \), and \( U_{H,t} \) satisfies (4) for all time steps \( t \) according to our choices in Section V. Thus, \( X_{t+1} = F(X_t, U_R, U_H) \) satisfies (5) by induction.

Remark I.1. Formally, the abstract domain of \( U_H \) is \( a \in \mathbb{R} \cup \{-\infty, \infty\} \) (with the lattice structure \( a \subseteq a' \) if and only if \( a \leq a', \top = \infty, \bot = -\infty \)), and its abstraction and concretization functions are \( \alpha : U_H \to \sup \mathbb{R}_{\geq 0} \) and \( \gamma : a \to (-\infty, -a) \), respectively. The setup for \( U_R \) is similar.

APPENDIX II

PROOF OF THEOREM IV.2

First, we have the following guarantee on \( F \).

Lemma II.1. Given initial state \( x_0 \in X \), and robot and human action set sequences \( (U_{R,0}, U_{R,1}, ...) \in (\mathbb{Z}^{2d})_0 \) and \( (U_{H,0}, U_{H,1}, ...) \in (\mathbb{Z}^{2d})_0 \), respectively, let the state sequence \( (X_0, X_1, ...) \in (X_{\infty})_0 \) be defined by \( X_0 = \{x_0\} \) and \( X_{t+1} = F(X_t, U_{R,t}, U_{H,t}) \) for \( t \in \mathbb{N} \). Then, for any \( u_R \in U^R_R \) and \( u_H \in U^H_H \) such that \( u_{R,t} \in U_{R,t} \) and \( u_{H,t} \in U_{H,t} \) for all \( t \in \mathbb{N} \), the trajectory \( (x_0, x_1, ...) \in X_{\infty} \) from \( x_0 \) using \( u_R \) and \( u_H \) satisfies \( x_t \in X_t \) for all \( t \in \mathbb{N} \).

Proof: We prove by induction on \( t \). The base case \( t = 0 \) follows since \( x_0 = x \in X_0 \). For the inductive case, note that

\[
x_{t+1} = f(x_t, u_{R,t}, u_{H,t}) \in F(X_t, U_{R,t}, U_{H,t}) = X_{t+1},
\]

since \( x_t \in X_t \) (by induction), since \( u_{H,t} \in U_{H,t} \) (by assumption), and by (2). \( \square \)

Next, we have the following guarantee on IsRec.

Lemma II.2. If IsRec(\( x' \)) is true, then \( (x') \in X_{\text{rec}} \).

Proof: Suppose IsRec(\( x' \)) returns true. We need to show that for any \( u_R = U^R_R \) and \( u_H = U^H_H \), the trajectory \( \zeta(\( x', u_H, u_R \)) = (x_0', x_1', ...) \in X_{\infty} \) satisfies (i) \( x_t \in X_{\text{safe}} \) for all \( t \in \{0, ..., k\} \), and (ii) \( x_k \in X_{\text{eq}} \). For (i), \( x_t \in X_t \subseteq X_{\text{safe}} \), where the first inclusion follows by Lemma II.1 and the second follows by the definition of IsRec. For (ii), as in the previous case, \( x_k \in X_k \subseteq X_{\text{eq}} \).

That is, either the human currently has objective value \( > -\infty \), or on their next step, the human will be in a recoverable state. Next, we show that \( X_{\text{inv}} \) is an invariant set. We begin with three technical lemmas. First, we show that if the current human state \( x' \) (i.e., it is the human’s turn to act) is recoverable and the human uses one of their backup actions \( u_H \in U^H_H \), then the next human state \( x'_+ \) is also recoverable.

Lemma II.4. If \( x' \in X_{\text{rec}} \) and \( u_H \in U^H_H \), then \( x' \in X_{\text{safe}} \) and \( x'_+ \in X_{\text{rec}} \), where \( x = f_H(x', u_H) \) and \( x'_+ = f_R(x, \pi_R(x)) \).

Proof: To show \( x'_+ \in X_{\text{rec}} \), we need to show that for any \( \bar{u}_R = (u_{H,0}, u_{H,1}, ...) \in U^H_H \), the trajectory \( \zeta(\( x'_+, u_{R,H}, \bar{u}_R \)) = (x_0', x_1', ...) \) satisfies (i) \( x'_t, x_t \in X_{\text{safe}} \) for \( t \in \{0, ..., k\} \), and (ii) \( x_k \in X_{\text{eq}} \). Let

\[
\bar{u}_R = (u_{H,0}, u_{R,1}, ...) \in U^H_H \infty
\]

Or course, our approach is not limited to the multi-agent car example. We have also developed methodology for representing and approximating the dynamics of more complex scenarios, such as car platoons and multi-agent systems. The key insight is to model the interaction of multiple agents as transitions in a directed graph, and then to abstractly reason about the temporal evolution of this graph. Our approach can be extended to handle multiple abstract domains, and to deal with a variety of different types of agents, such as robots and humans. We have also developed techniques for approximating the dynamics of systems with probabilistic elements, such as stochastic systems and systems with uncertain parameters. Our approach can be used to analyze the safety and performance of these systems, and to design decision-making strategies for autonomous agents.
where the first line follows since by assumption, we have $u_H \in \mathcal{U}^0_H$. Also by assumption, we have $x' \in \mathcal{X}_{rec}$, so by definition or recoverability, the trajectory $x(t), \tilde{x}_H, u_H(t) = \{x_0, \tilde{x}_0, x_1, \ldots\}$ satisfies $\tilde{x}_t, \tilde{x}_H \in \mathcal{X}_{safe}$ for $t \in \{0, \ldots, k\}$ and $\tilde{x}_k \in \mathcal{X}_{eq}$ by definition of equilibrium. $x'_{k+1} = f(x_{k+1}, t, t, u_H(k))$ and $x_{k+1} = f(x_{k+1}, t, t, u_H(k)) = \tilde{x}_k$. Thus, we have $x_{k+1} = \tilde{x}_k$. Finally, note that $x'' = x_{k+1} + x_k$. By this fact and the above, both conditions (i) and (ii) holds, so it follows that $x''_t \in \mathcal{X}_{rec}$.

Finally, see $x \in \mathcal{X}_{safe}$, note that $x = \tilde{x}_0, \tilde{x}_H \in \mathcal{X}_{safe}$. 

Next, we show that if our algorithm uses $\tilde{x}$ at state $x$, then the next human state $x'$ is recoverable.

**Lemma II.5.** If $\pi(x) \neq u_H(x)$ then $f(x, \pi(x)) \in \mathcal{X}_{rec}$. 

**Proof:** By definition of $\pi$, if $\pi(x) \neq u_H(x)$, then we have $f(x, \pi(x)) \in \mathcal{X}_{safe}$. Then, by Lemma II.2, $f(x, \pi(x)) \in \mathcal{X}_{rec}$, as claimed. 

Third, letting $J(x') = \max_{\tilde{u}_H \in \mathcal{U}^\infty_p} J(x', \tilde{u}_H)$ be the the human objective value at human state $x'$, we show that if $J(x') > -\infty$ and the robot uses its backup action $u_H$, then we also have $J(x'_t) > -\infty$ at the next human state $x'_t$.

**Lemma II.6.** If $J(x') > -\infty$, then $J(x'_t) > -\infty$, where $x = f(x', \pi(x'))$ and $x'_t = f(x, \pi(x))$. 

**Proof:** If $J(x') > -\infty$, then $J(x', \tilde{u}_H) > -\infty$, where $\tilde{u}_H \in \arg \max_{\tilde{u}_H \in \mathcal{U}^\infty_p} J(x', \tilde{u}_H)$ is the optimal action sequence chosen by $\pi_H$—i.e., $\pi_H(x') = u_H(x)$. Then, we have

\[
-\infty < J(x', \tilde{u}_H) = \min_{\tilde{u}_H \in \mathcal{U}^\infty_p} J(x', \tilde{u}_H, \tilde{u}_H) \\
\leq \min_{\tilde{u}_H \in \mathcal{U}^\infty_p} J(x', \tilde{u}_H, \tilde{u}_H) \\
= r_H(x', u_H(t, x)) + \gamma \min_{\tilde{u}_H \in \mathcal{U}^\infty_p} J(x'+1, \tilde{u}_H, \tilde{u}_H) \\
= r_H(x', u_H(t, x)) + \gamma \tilde{J}(x', \tilde{u}_H, \tilde{u}_H), 
\]

where $\tilde{u}_H = (u_H(0), u_H(1), \ldots)$. The first line holds by assumption, and the second and last lines hold by definition. The third line follows by Assumption III.2, we have $u_H \in \mathcal{U}^\infty_p$, so $\mathcal{U}^\infty_p \times \mathcal{U}^\infty_k \subseteq \mathcal{U}^\infty_k$. To see why the fourth line follows, note that for any $\tilde{u}_H \in \mathcal{U}^\infty_k \times \mathcal{U}^\infty_p$, we have

\[
\tilde{J}(x', \tilde{u}_H, \tilde{u}_H) = (x'_0, u'_0) \circ (x'_1, x'_2, \ldots) \\
= (x'_0, u'_0) \circ \tilde{J}(x'_1, \tilde{u}_H, \tilde{u}_H), 
\]

where the second line follows since $\tilde{u}_H \in \mathcal{U}^\infty_p \times \mathcal{U}^\infty_k$ implies that $u_H = u_H(0)$, so $x'_0 = f(x'_0, u_H(x'_0)) = x'_0$, and since we have $\tilde{u}_H = (u_H(0)) \circ \tilde{u}_H$ and $\tilde{u}_H = (u_H(0)) \circ \tilde{u}_H$, where $\tilde{u}_H = \tilde{u}_H(0) \circ \tilde{u}_H$. As a consequence, we have

\[
J_H(\gamma_t(x', u_H, u_R)) = \sum_{t=0}^{\infty} \gamma^t r_H(x', u_H(t, x), x_t) \\
= r_H(x', u_H(t, x), x_t) + \sum_{t=0}^{\infty} \gamma^t r_H(x_t, x_{t+1}, x_{t+1}) \\
= r_H(x', u_H(t, x), x_t) + \gamma J_H(\gamma_t(x', u_H, u_R)).
\]

Then, the fourth line above holds since for any $\tilde{u}_H \in \mathcal{U}^\infty_p$, we have $\{u_H(0) \circ \tilde{u}_H \in \mathcal{U}^\infty_k \times \mathcal{U}^\infty_p$, so

\[
\min_{\tilde{u}_H \in \mathcal{U}^\infty_k \times \mathcal{U}^\infty_p} J_H(\gamma_t(x', u_H, u_R)) = r_H(x', u_H(t, x), x_t) + \gamma \min_{\tilde{u}_H \in \mathcal{U}^\infty_k \times \mathcal{U}^\infty_p} J_H(\gamma_t(x', u_H, u_R)).
\]

Now, we prove the lemma. Since $r_H(x', u_H(t, x), x_t) < 0$ and $\gamma > 0$, by (6), we have $J_H(x_0, x) > -\infty$. Thus, we have

\[
J_H(x'_0) = \max_{\tilde{u}_H \in \mathcal{U}^\infty_k \times \mathcal{U}^\infty_p} J(x'_0, x) \geq J(x'_0, \tilde{u}_H) > -\infty,
\]

as claimed. 

Next, we show that $x_{rec}$ is an invariant set.

**Lemma II.7.** Suppose $\pi(x) = u_H(x)$ and $x$ satisfies Assumptions III.2 & III.3. If $x \in \mathcal{X}_{inv}$, where $x' = f(x, \pi(x))$, $x'_t = f(x, \pi(x))$, and $x' = f(x'_t, \pi(x'_t))$.

**Proof:** Since $x \in \mathcal{X}_{inv}$, either $x'_t \in \mathcal{X}_{rec}$ or $J(x'_t) > -\infty$. First, suppose that $x'_t \in \mathcal{X}_{rec}$. Let $x''_t = f(x, \pi(x))$. If $\pi(x'_t) \in \mathcal{U}^\infty_p$, then by Lemma II.4 we have $x''_t \in \mathcal{X}_{rec}$. Also, in this case, $x' \in \mathcal{X}_{safe}$. Otherwise, if $\pi(x'_t) \in \mathcal{U}^\infty_p$, then by Assumption III.3, we have $J(x'_0) > -\infty$, so $x'_t \in \mathcal{X}_{inv}$. Since $J(x'_t) > -\infty$, we also have $r_H(x'_t, \pi(x'_t), x'_t, x'_t) > -\infty$, by Assumption III.1, we have $x'_t \in \mathcal{X}_{safe}$. Thus, the claim follows for the case $x''_t \in \mathcal{X}_{rec}$.

Next, suppose that $J(x'_t) > -\infty$. Then, we must have $\pi(x) = u_H(x)$—otherwise, by Lemma II.5, we would have $x''_t = f(x, \pi(x), x'_t) \in \mathcal{X}_{safe}$. Thus, by Lemma II.6, we have $J(x'_t) > -\infty$, so $x'_t \in \mathcal{X}_{inv}$. As before, $J(x'_0) > -\infty$ also implies that $x'_{t+1} \in \mathcal{X}_{safe}$. Thus, the claim follows.

Finally, we prove Theorem IV.2.

**Proof:** We can equivalently consider the setting where the human starts at state $x'_0 = x$ and takes any action $\pi_H(x'_0) \in \mathcal{U}^\infty_H$; in particular, since $x \in \mathcal{X}_{eq}$, we have $x = f(x, \pi_H(x'_0)) = x'_0$. Now, let $\pi_H(x'_0, \pi_H(x'_0)) = \pi(x) = u_H(x'_0, x'_0, \ldots)$, where $x_0 = x$. Then, note that $x'_t \in \mathcal{X}_{rec}$, since either $\pi(x) \not\in \mathcal{U}^\infty_H$, in which case $x'_t = f(x, \pi(x), (x_0, x'_t, \ldots)) \in \mathcal{X}_{rec}$ by Assumption III.1, or $x'_t = f(x, \pi(x), x'_t) \in \mathcal{X}_{safe}$. As a consequence, by definition, we have $x'_t \in \mathcal{X}_{safe}$. Now, by Lemma II.7, we have $x'_t \in \mathcal{X}_{safe}$ and $x'_{t+1} \in \mathcal{X}_{safe}$ for all $t \in \mathbb{N}$. By definition, $x'_0 = x_0 \in \mathcal{X}_{eq} \subseteq \mathcal{X}_{safe}$.