Core dynamics of a multi-armed spiral pattern in a dielectric barrier discharge

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Abstract. Stable multi-armed spiral patterns have been observed in a gas discharge by using water electrodes. It is found that the multi-armed spirals in our experiments generally result from the interaction between a single-armed spiral pattern and dislocations. The number of spiral arms can be increased or decreased depending on the topological charge of the dislocation when it glides into the spiral core. The complex spatiotemporal dynamics of the multi-armed spiral tips has also been investigated. The spiral tips rotate about a common circle for a two-armed spiral pattern. The core dynamics of a three-armed spiral pattern involves intermittent pairwise collision of tips at or near their tips, and it is more complex for a four-armed spiral pattern.

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1. Introduction

One of the most striking examples of spatiotemporal self-organized phenomena in non-equilibrium systems is the spiral pattern. It can be found in many nonlinear systems such as physical [1], chemical [2] and biological systems [3]. It is believed that spiral appearance is related to tachycardia, and a loss of stability of spirals can cause sudden heart death called ventricular fibrillation [4]. Significant progress has been made in the study of single-armed spiral patterns through detailed numerical, analytical and experimental investigations [5]–[8]. Recently, the formation of multi-armed spiral patterns has attracted more and more attention, especially since they were found in rabbit hearts [9]. Multi-armed spiral patterns are not expected to be observed in both experiments and numerical simulations in the reaction-diffusion system, since it has been proved that only one-armed spirals are stable for certain regions of the complex Ginzburg–Landau equation, whereas multi-armed spirals are unstable everywhere [10]. However, they can be constructed experimentally and numerically [11]–[14]. Vasiev et al [12] have shown that stable multi-armed spirals can arise spontaneously from several single-armed spirals if their tips are less than one wavelength apart. Furthermore, multi-armed spiral patterns can occur in biological excitable systems, Faraday experiments and Rayleigh–Bénard convection (RBC) systems [15]–[17]. In 1991, multi-armed spirals with arm number from 1 to 7 were observed in an experiment on non-Boussinesq convection [16]. It was found that the dynamics of the core (domain of the spiral tips) was much faster than the large-scale spiral rotation [17]. Similar phenomena have also been found in oscillated granular layers [18]. However, to the best of our knowledge, there has been little reported on the core dynamics of the multi-armed spirals. In this paper, we will show the complex dynamics, especially the core dynamics, of stable multi-armed spirals in an ac-driven gas discharge system.

In recent years, gas discharge systems have attracted much attention for their luminous features and suitable time scales of pattern forming [19]–[22]. Breazeal et al [19] observed a target pattern in a dielectric barrier discharge (DBD) system under the condition that pd (the product of pressure p and gas gap width d) was less than 4 Torr cm. Purwins and co-workers [20] found single-armed spirals in a dc-driven gas discharge at low pd value. Recently, stable multi-armed spiral patterns were first observed at high pd value by using a DBD device with water electrodes [23]. Here, we will give the formation mechanism of the multi-armed spiral patterns and their core dynamics in the DBD system.

2. Experimental set-up

The schematic diagram of the experimental device has been given previously [23]. Two cylindrical containers with diameters of 65 mm are filled with water. There is a metallic ring immersed in the water of each container and connected to a power supply. Thus, the water acts as liquid electrodes. The parallel glass with thickness of 1.5 mm serves as a dielectric layer. A circular glass ring with a diameter of 60 mm limits the diffusion of the discharge gas laterally. The gap width between the two parallel electrodes can be adjusted in a range from 1.0 to 2.0 mm. All of the apparatus are enclosed in a big container filled with argon at atmospheric pressure. The pd value can range from 76 to 152 Torr cm. A sinusoidal ac voltage at a frequency of 61 kHz is applied to the electrodes. A high-voltage probe (Tektronix P6015A 1000X) is used to measure the applied voltage. A digital camera (Canon Powershot G1) is used to take the pictures of patterns.
Figure 1. Pattern transition as the applied voltage is increased. (a) $U = 3200 \text{ V}$; (b) $U = 3400 \text{ V}$; (c) $U = 3600 \text{ V}$; (d) $U = 3900 \text{ V}$; (e) $U = 4200 \text{ V}$; and (f) $U = 5300 \text{ V}$. Other parameters are $p = 10^5 \text{ Pa}$, $d = 1.4 \text{ mm}$, $f = 61 \text{ kHz}$, $t_{\text{exp}} = 40 \text{ ms}$. The diameter of the discharge domain is 60 mm.

3. Experimental results

Figure 1 shows the pattern transitions with the increase of the applied voltage. Stochastic filaments initially appear when the gas breakdown occurs. The filaments arrange themselves with increasing voltage, resulting in a self-organized hexagon pattern (figure 1(a)). When the applied voltage increases far from the breakdown voltage, the hexagon pattern loses its stability, and numerous short stripes with different orientations appear, as shown in figures 1(b) and (c). These short stripes become longer and longer, and organize into a spiral pattern ultimately (figure 1(e)). Sufficiently far from the breakdown voltage, a static hexagon and/or stripe pattern forms, as shown in figure 1(f).

The spirals in our experiment are finite spirals terminating in dislocations at some radius $r_d$ (the size of the spiral), as shown in figure 1(e). Such an $m$-armed spiral can be described by a modified Archimedean spiral:

$$I(r, t) = A(r) \cos \left( kr + m\phi + \sum_{i=1}^{n} n_i \phi_{d_i} - \omega_m t \right),$$

where $k$ is the wavenumber of the spiral waves, $A(r)$ is the amplitude, and $r$ is the radial distance from the spiral tip, respectively. The spiral tip is defined as a point with maximum curvature in the spiral pattern. $m = \pm 1, \pm 2, \pm 3, \ldots$ is the topological charge of the spiral pattern, and its absolute value stands for the number of arms of the spiral pattern, while the sign corresponds to the chirality. The topological charge is defined via the winding number of the phase gradient $\nabla \phi$ around a spiral tip or a dislocation. The phase $\phi = \arctan[(y - y_0)/(x - x_0)]$ and $\phi_{d_i} = \arctan[(y - y_{d_i})/(x - x_{d_i})]$ are polar angles centered about the spiral center at $r = (x_0, y_0)$ and the outer defect positions $r_{d_i} = (x_{d_i}, y_{d_i})$, respectively. $n_i = \pm 1$ defines the topological charge of the defects, and $n$ stands for the defect number. $\omega_m$ is the rotation frequency of the $m$-armed spiral pattern. When the outer defect glides into the spiral center, the spiral arms will be increased (decreased) if the topological charges of the defects and the spirals have the same
(a) (b) (c) (d)

Figure 2. Transition from a double-armed spiral pattern to a single-armed spiral pattern. Pictures are separated by $1/15$ s. $U = 4200$ V, $p = 10^5$ Pa, $d = 1.4$ mm, $f = 61$ kHz, $t_{\text{exp}} = 40$ ms. The radius of the discharge domain is 30 mm.

(opposite) sign. The corresponding arm number becomes $|m + n_i|$. For example, a two-armed spiral pattern transits into a single-armed spiral pattern as the dislocation with topological charge $n = 1$ glides into the spiral center with topological charge $m = -2$, as shown in figure 2.

Multi-armed spiral patterns in our experiments generally result from the interaction between single-armed spirals and dislocations [24]. Sidewall forcing makes the rolls parallel to the sidewall resulting in the formation of single-armed spiral patterns or multi-armed spiral patterns. The spirals observed in our experiments are meandering spirals rather than rigid spirals. The meander velocity of the spiral is inverse to the number of spiral arms, i.e. the single-armed spiral meanders with a relatively larger velocity than the multi-armed spiral pattern. Due to the movement, the spiral moves off center, compressing the roll on one side while dilating it on the other side. When the wavenumber of the compressed region increases beyond a critical value, a pair of dislocations with opposite topological charge occurs to decrease the wavenumber. A double-armed spiral pattern with arm number $m = 2$ forms when one of the dislocations with the same topological charge $n = 1$ glides to the spiral center. The movement of the double-armed spiral creates new dislocations (figure 3(a)). The dislocation makes the roll break down into two parts. One of them connects with the dislocation and becomes a new roll of the spiral, while the other part moves towards the center and becomes a new dislocation (figure 3(b)). The new dislocation makes the nearby roll break down. This process will continue until the dislocation moves into the core of the spiral. Then a three-armed spiral forms (figure 3(d)). Sometimes multi-armed spiral patterns can also be formed by a curved stripe pattern directly. No matter what the number of arms of the spiral pattern is, its mean wavelength has a constant value under the given experimental conditions.

The tips and the tails of spirals have different dynamical behaviors though they are dislocations in the spiral pattern. The tips act as sources, while the tails as sinks. There are two different types of motion of the spiral tails: one is climbing, also referred to as the travel of the
tails parallel to the axes of the rolls; the other is gliding, i.e. the travel of the tails perpendicular to the rolls. When the tails are located at the sidewall, the spiral is no longer rotating, apparently pinned by the sidewalls. Examples are given in figure 3(d).

Multi-armed spirals are composed of several spirals with like topological charge that remain within a limited distance from each other. Spiral cores are responsible for the mechanism of the spiral interaction. Two types of multi-armed spiral patterns have been found in our experiments according to their core dynamics. The spiral tips rotate about a common circle in the first type, while they have a complex dynamics near their tips in the second type. The two-armed spirals belong to the first type. The distance between the two spiral tips holds a constant value at all times, as shown in figure 4, and the radius of the spiral core is approximately equal to the spiral wavelength. The multi-armed spirals with \( m \geq 3 \), which belong to the second type, lack rotational symmetry and have interesting core dynamics.

Figure 5 shows the time evolution of a three-armed spiral pattern. The core dynamics involves intermittent pairwise collision of tips at or near their tips. The quantitative relationship among the three spiral tips has been measured in figure 6. The parameter \( r_{ij} \) is the distance between the tip \( i \) and the tip \( j \). It is shown that two of the three arms join together and depart, then reappear between another pair of arms. This process repeats with a period that varied somewhat, but on average was \( 0.3 \pm 0.1 \) s, much smaller than the rotation period of the spiral itself. Similar dynamics of the core was observed in cell-filling spirals in RBC [17] and vibrated granular layers [18]. We evaluate the correlation coefficient \( C \) for any two distances between the spiral tips, defined as

\[
C_{ij} = \frac{\sum (I_i - \bar{I}_i)(I_j - \bar{I}_j)}{\sqrt{\sum (I_i - \bar{I}_i)^2 \sum (I_j - \bar{I}_j)^2}},
\]
Figure 4. Time evolution of distance between the tips of a two-armed spiral pattern.

Figure 5. Time evolution of a three-armed spiral. Pictures are separated by 1/15 s. The parameters are same as those in figure 2.

where $I_1 = r_{12}$, $I_2 = r_{13}$, $I_3 = r_{23}$ for the three-armed spiral and $I_3 = r_{14}$ for the four-armed spiral. The correlation coefficient $C_{12} = -0.3$, $C_{13} = -0.2$ and $C_{23} = 0$. The negative value means the collision of the tips is pairwise, i.e. when two of the three tips collide with each other, the third one keeps away from them. In addition, the size of the spiral core has been measured. In this case, radius of the spiral core is 4.6 mm while the wavelength of the spiral is 3.07 mm, i.e. the ratio $r/\lambda \approx 1.5$. 

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Figure 6. Time evolution of distances among tips of a three-armed spiral pattern.

Figure 7. Time evolution of a four-armed spiral. Pictures are separated by 1/15 s. The parameters are same as those in figure 2.

Multi-armed spiral patterns can keep their arm numbers for several seconds because there are many defects (dislocations) in the spiral pattern, while the spiral states can persist for several tens of minutes. For instance, a dislocation appears when there is a large fluctuation in local wavenumber in our system. Then a four-armed spiral forms as the dislocation moves into the center of the three-armed spiral pattern, as shown in figure 7. The dynamics of the
four-armed spiral is more complex. The average period of the spiral interaction process is about 0.4 ± 0.2 s. Figure 8 gives the distances between the tip 1 and the others. It is shown that the interaction among the tips of the four-armed spiral exhibits complex dynamic behavior, where the correlation coefficients are $C_{12} = 0.3, C_{13} = -0.2$ and $C_{23} = 0$, respectively. The wavelength of the four-armed spiral pattern is still 3.07 mm. In this case, the radius of the spiral core is 6.13 mm, therefore the ratio $r/\lambda \approx 2$.

4. Discussion

A spiral pattern is a spatial-temporal structure which is frequently observed in various non-equilibrium systems with different pattern forming mechanisms. The famous examples for studying spiral patterns are the RBC system and the chemical reaction-diffusion system.

RBC is a well-established model system in which patterns are driven by the temperature difference across the fluid layer. Multi-armed spiral patterns have been studied in RBC by several groups [16, 17], [25]–[28]. It is found that multi-armed spiral patterns are stable at low $\varepsilon$ (reduced control parameter) in the presence of sidewall forcing [17, 27]. The multi-armed spiral patterns with $n$ arms rotate in the winding-up direction. The $n$ arms end at $n$ dislocation defects in the interior of the cell. The whole spirals are surrounded by concentric rings. In RBC, the multi-armed spiral is obtained from a target pattern by increasing $\varepsilon$. The center of the target moves off center of the cell causing local variations in the wavenumber of the pattern. This leads to the appearance of dislocations by skewed varicose instability [28], and eventually to a multi-armed spiral pattern.

The dynamics of multi-armed spirals in DBD seems to be a close analogy to that in RBC. In both cases, the core motion is limited in a central area of radius $r \approx n\lambda/2$, where $n$ is the number of spiral arms ending in the core and $\lambda$ is the wavelength of the pattern [17]. The core dynamics of multi-armed spiral patterns also have similar dynamic behaviors. The tips of multi-armed spiral patterns undergo a process of connecting and disconnecting. For example, there is

**Figure 8.** Time evolution of distances among tips of a four-armed spiral pattern.
an interesting intermittent pairwise collision of tips in a three-armed spiral pattern. In the case of a four-armed spiral pattern, the collision of tips has a more complex behavior.

There are also many differences between the behaviors of multi-armed spiral patterns in the DBD system and those observed in RBC. First of all, the mechanisms of pattern formation in the two systems are essentially different. Multi-armed spiral patterns in a DBD system are driven by a sinusoidal applied voltage. They can form either by evolution of single-armed spiral patterns with dislocations or by the self-organization of the disordered stripes in the central region of the cell [23]. In convection, the multi-armed spiral patterns can only evolve from a single-armed spiral pattern by increasing the parameter $\varepsilon$. Secondly, multi-armed spirals in RBC can rotate in both winding-up and winding-down direction, while those in DBD can only rotate in the winding-up direction. In RBC, the spiral tails always locate in the interior of the cell, but in DBD the tails almost always locate at the sidewalls [24]. Thirdly, whether the multi-armed spiral patterns appear depends strongly on the effect of the boundary conditions at the cell wall in convection, while the spiral pattern in DBD can be observed no matter what boundary condition is used [23]. In convection, the role of the sidewall forcing is to orient the stripes parallel to the cell wall and to stabilize the multi-armed spiral pattern. The origin of the sidewall forcing in convection is the horizontal temperature gradient forcing upwelling hot fluid at the sidewalls. In DBD, it is found that the sidewall forcing also exists although its mechanism is still unclear. In addition, the transition between different $n$-armed spiral patterns is dependent upon the increase of the parameter $\varepsilon$, but in DBD this transition is time-dependent. Two-armed spiral patterns in RBC can only be obtained by increasing the temperature difference abruptly rather than quasi-statically. The two tips connect twice during one period of rotation [17]. In the case of DBD, two-armed spiral patterns can be created by the interaction of a single-armed spiral pattern and a dislocation directly, and have a fixed distance between their tips. There is another important difference to be addressed. In RBC, a single-armed spiral pattern displays a periodic rigid rotation in the presence of sidewall forcing. The spiral tip does not move off center of the cell until the parameter $\varepsilon$ is increased. $n$-armed spiral patterns do not have a characteristic frequency $\omega_n$ or a typical size $R_n$, but their product $\omega_nR_n$ is a constant under given experimental conditions [27]. However, in the case of our experiments, a single-armed spiral tip does not rotate around a simple circle. It meanders in a more complex configuration. Different from the multi-armed spirals in RBC, the tips of multi-armed spirals in DBD also meander, with a velocity that is inversely proportional to the spiral arm numbers.

Phenomenally, spiral patterns in the DBD system seem very different from that in chemical reaction-diffusion systems and the biological systems. Reaction-diffusion systems, in which spiral patterns were first obtained, have been proven to be a paradigm for studying spiral patterns. Many of the reaction-diffusion systems can be classified as excitable media. The spiral core acts as a pacemaker, and selects the spatial and temporal evolution of the outward traveling waves [2]. Usually, spirals in the reaction-diffusion system are infinite. To the best of our knowledge, however, spontaneously self-organized multi-armed spiral patterns have not been reported in experiments and numerical simulations in the reaction-diffusion system.

5. Conclusion

In conclusion, DBD provides a new type of pattern formation system for studying spiral patterns, in which stable multi-armed spirals have been observed. It is found that the multi-armed spirals in our experiments generally result from the interaction between single-armed...
spirals and dislocations. The number of spiral arms can be increased or decreased depending on the topological charge of the dislocation when it glides into the spiral core. The complex spatiotemporal dynamics of the multi-armed spiral tips has also been investigated. The spiral tips rotate about a common circle for the two-armed spiral pattern. The core dynamics of the three-armed spirals involves intermittent pairwise collision of vortices at or near their tips, and it is more complex for the four-armed spiral pattern. Phenomenally, the multi-armed spiral pattern in the DBD system seems similar to that in the RBC system.

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