Fermion dark matter in gauge-Higgs unification

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ABSTRACT: We propose a Majorana fermion dark matter in the context of a simple gauge-Higgs Unification (GHU) scenario based on the gauge group SU(3) \times U(1)' in 5-dimensional Minkowski space with a compactification of the 5th dimension on $S^1/Z_2$ orbifold. The dark matter particle is identified with the lightest mode in SU(3) triplet fermions additionally introduced in the 5-dimensional bulk. We find an allowed parameter region for the dark matter mass around a half of the Standard Model Higgs boson mass, which is consistent with the observed dark matter density and the constraint from the LUX 2016 result for the direct dark matter search. The entire allowed region will be covered by, for example, the LUX-ZEPLIN dark matter experiment in the near future. We also show that in the presence of the bulk SU(3) triplet fermions the 125 GeV Higgs boson mass is reproduced through the renormalization group evolution of Higgs quartic coupling with the compactification scale of around $10^8$ GeV.

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1 Introduction

The existence of dark matter (DM) is promising from the various cosmological observations and one of the keys for exploring physics beyond the Standard Model (SM). It is still a mystery in particle physics and cosmology to clarify the identities of the dark matter particle. Among various possibilities, the so-called Weakly Interacting Massive Particle is a prime candidate for the DM particle, which is the thermal relic from the early Universe and whose relic abundance is calculable independently of the history of the Universe before the DM has gotten in thermal equilibrium. A variety of experiments aiming for directly/indirectly detecting DM particles is ongoing and planned, and the discovery of the dark matter may be around the corner. In this paper we consider a fermion DM in the context of a simple gauge-Higgs Unification (GHU) scenario in 5-dimensions and identify a model-parameter region which is consistent with the current experimental constraints.

The GHU scenario [1–6] is a unique candidate for new physics beyond the SM, which offers a solution to the gauge hierarchy problem without invoking supersymmetry. An essential property of the GHU scenario is that the SM Higgs doublet is identified with an extra spatial component of the gauge field in higher dimensions. Associated with the higher-dimensional gauge symmetry, the GHU scenario predicts various finite physical observables, irrespective of the non-renormalizability of the scenario, such as the effective Higgs potential [7–12], the effective Higgs coupling with digluon/diphoton [13–15], the anomalous magnetic moment $g - 2$ [16, 17], and the electric dipole moment [18].

In the previous paper by some of the present authors [15], the one-loop contributions of Kaluza-Klein (KK) modes to the Higgs-to-digluon and Higgs-to-diphoton couplings were calculated in a 5-dimensional GHU model by introducing color-singlet bulk fermions with a half-periodic boundary condition, in addition to the SM fermions. It was shown that the color-singlet bulk fermions play a crucial role not only to explain the observed Higgs-to-digluon and Higgs-to-diphoton couplings, but also to achieve the 125 GeV Higgs boson
mass. See also refs. [19, 20] for extended analysis including color-triplet bulk fermions. As a bonus, it was pointed out that the lightest KK mode of the bulk fermions can be a DM candidate by choosing their hypercharges appropriately. The main purpose of this paper is to pursue this possibility and investigate the DM physics in the context of the GHU scenario. For related works on the DM physics in GHU scenarios, see refs. [21–25].

Towards the completion of the GHU scenario as new physics beyond the SM, we need to supplement a DM candidate to the scenario. In order to keep the original motivation of the GHU scenario to solve the gauge hierarchy problem, the DM candidate to be introduced must be a fermion. Since the GHU scenario is defined in a higher dimensional space-time with a gauge group into which the SM gauge group is embedded, it would be the most natural/general to introduce a DM candidate as a bulk fermion of a certain representation under the gauge group of the GHU scenario. Hence, the DM candidate is accompanied by its partners in decomposition of the SM gauge group and has a Yukawa coupling with the SM Higgs doublet, which originates from the higher-dimensional gauge interaction. Thanks to the structure of the GHU scenario, once the representation of the bulk fermion is defined, the Yukawa coupling is predicted. This is in a sharp contrast with 4-dimensional DM models, where Yukawa couplings are generally undetermined. In addition, as we will show in section 4, the fermion DM multiplet in the bulk plays a crucial role to lower the compactification scale of the (5-dimensional) GHU scenario while reproducing the observed Higgs boson mass of 125 GeV.

The plan of this paper is as follows. In the next section, we consider a 5-dimensional GHU model based on the gauge group SU(3) × U(1)′ with an orbifold $S^1/Z_2$ compactification. In this context, we propose a Majorana fermion DM scenario, where a DM particle is provided as the lightest mass eigenstate in a pair of bulk SU(3) triplet fermions introduced in the bulk along with a bulk mass term and a periodic boundary condition. In section 3, we focus on the case that the DM particle communicates with the SM particles through the Higgs boson. Solving the Boltzmann equation, we identify an allowed parameter region of the model to reproduce the observed DM density. In section 4, we further constrain the allowed parameter region by considering the upper limit of the elastic scattering cross section of the DM particle off with nuclei from the current DM direct detection experiments. An effective field theoretical approach of the GHU scenario will be discussed in section 5, and the 125 GeV Higgs boson mass is reproduced in the presence of the bulk SU(3) triplet fermions with certain boundary conditions. The compactification scale is determined in order to reproduce the Higgs boson mass of 125 GeV. The last section is devoted to conclusions.

2 Fermion DM in GHU

We consider a GHU model based on the gauge group SU(3) × U(1)′ [26] in a 5-dimensional flat space-time with orbifolding on $S^1/Z_2$ with radius $R$ of $S^1$. In our setup of bulk fermions including the SM fermions, we follow ref. [27]: the up-type quarks except for the top quark, the down-type quarks and the leptons are embedded, respectively, into $3, \bar{5}$, and $10$ representations of SU(3). In order to realize the large top Yukawa coupling, the
The top quark is embedded into a rank 4 representation of SU(3), namely $\mathbf{15}$. The extra U(1)$'$ symmetry works to yield the correct weak mixing angle, and the SM U(1)$_Y$ gauge boson is realized by a linear combination between the gauge bosons of the U(1)$'$ and the U(1) subgroup in SU(3) [26]. Appropriate U(1)$'$ charges for bulk fermions are assigned to yield the correct hypercharges for the SM fermions.

The boundary conditions should be suitably assigned to reproduce the SM fields as the zero modes. While a periodic boundary condition corresponding to $S^1$ is taken for all of the bulk SM fields, the $Z_2$ parity is assigned for gauge fields and fermions in the representation $\mathcal{R}$ by using the parity matrix $P = \text{diag}(-, -, +)$ as

$$A_{\mu}(-y) = P^\dagger A_{\mu}(y)P, \quad A_y(-y) = -P^\dagger A_y(y)P, \quad \psi(-y) = \mathcal{R}(P)\gamma^5\psi(y) \quad (2.1)$$

where the subscripts $\mu$ ($y$) denotes the four (the fifth) dimensional component. With this choice of parities, the SU(3) gauge symmetry is explicitly broken down to SU(2) $\times$ U(1). A hypercharge is a linear combination of U(1) and U(1)$'$ in this setup. One may think that the U(1)$_X$ gauge boson which is orthogonal to the hypercharge U(1)$_Y$ also has a zero mode. However, the U(1)$_X$ symmetry is anomalous in general and broken at the cutoff scale and hence, the U(1)$_X$ gauge boson has a mass of order of the cutoff scale [26]. As a result, zero-mode vector bosons in the model are only the SM gauge fields.

Off-diagonal blocks in $A_y$ have zero modes because of the overall sign in eq. (2.1), which corresponds to an SU(2) doublet. In fact, the SM Higgs doublet ($H$) is identified with

$$A_y^{(0)} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & H \\ H^\dagger & 0 \end{pmatrix}. \quad (2.2)$$

The KK modes of $A_y$ are eaten by KK modes of the SM gauge bosons and enjoy their longitudinal degrees of freedom like the usual Higgs mechanism.

The parity assignment also provides the SM fermions as massless modes, but it also leaves exotic fermions massless. Such exotic fermions are made massive by introducing brane localized fermions with conjugate SU(2) $\times$ U(1) charges and an opposite chirality to the exotic fermions, allowing us to write mass terms on the orbifold fixed points. In the GHU scenario, the Yukawa interaction is unified with the gauge interaction, so that the SM fermions obtain the mass of the order of the $W$-boson mass after the electroweak symmetry breaking. To realize light SM fermion masses, one may introduce $Z_2$-parity odd bulk mass terms for the SM fermions, except for the top quark. Then, zero mode fermion wave functions with opposite chirality are localized towards the opposite orbifold fixed points and as a result, their Yukawa couplings are exponentially suppressed by the overlap integral of the wave functions. In this way, all exotic fermion zero modes can be heavy and the small Yukawa couplings for the light SM fermions can be realized by adjusting the bulk mass parameters. In order to realize the top quark Yukawa coupling, we introduce a rank 4 tensor representation, namely, a symmetric $\mathbf{15}$ without a bulk mass [27]. This leads to a group theoretical factor 2 enhancement of the top quark mass as $m_t = 2m_W$ at the compactification scale [26]. Note that this mass relation is desirable since the top quark pole mass receives QCD threshold corrections which push up the mass about 10 GeV.
Now we discuss the DM sector in our model. In addition to the bulk fermions corresponding to the SM quarks and leptons, we introduce a pair of extra bulk fermions $\psi, \tilde{\psi}$ which are triplet representations under the bulk SU(3) and have a $U(1)'$ charge 1/3. With this choice of the $U(1)'$ charge, the triplet bulk fermions include electric-charge neutral components and a linear combination among the charge neutral components serves as the DM particle. Associated with $S^1$ we impose the periodic boundary condition in the fifth dimension, while the $Z_2$ parity assignments are chosen as

$$\psi(-y) = P\gamma^5\psi(y), \quad \tilde{\psi} = -P\gamma^5\tilde{\psi}(y).$$

After the electroweak symmetry breaking, the lightest mass eigenstate among the bulk fermions is identified with the DM particle. As we will discuss in section 5, these bulk fermions also play a crucial role to reproduce the observed Higgs boson mass of 125 GeV.

The Lagrangian relevant to our DM physics discussion is given by

$$\mathcal{L}_{\text{DM}} = \bar{\psi}i\slashed{D}\psi + \bar{\tilde{\psi}}i\slashed{D}\tilde{\psi} - M \left( \bar{\psi}\tilde{\psi} + \bar{\tilde{\psi}}\psi \right) + \delta(y) \left[ \frac{m}{2} \bar{\psi}_{3R}^{(0)}\psi_{3R}^{(0)} + \frac{\tilde{m}}{2} \bar{\psi}_{3L}^{(0)}\psi_{3L}^{(0)} + \text{h.c.} \right],$$

where the covariant derivative and a pair of the bulk SU(3) triplets are given by

$$\slashed{D} = \Gamma^M \left( \partial_M - igA_M - ig' A'_M \right),$$

$$\psi = (\psi_1, \psi_2, \psi_3)^T, \quad \tilde{\psi} = (\tilde{\psi}_1, \tilde{\psi}_2, \tilde{\psi}_3)^T.$$  (2.5)

(2.6)

With the non-trivial orbifold boundary conditions, the bulk SU(3) triplet fermions are decomposed into the SM SU(2) doublet and singlet fermions. As we will see later, the DM particle is provided as a linear combination of the second and third components of the triplet fermions. In eq. (2.4) we have introduced a bulk mass ($M$) to avoid exotic massless fermions. Here we have also introduced Majorana mass terms on the brane at $y = 0$ for the zero-modes of the third components of the triplets ($\psi_{3R}^{(0)}$ and $\psi_{3L}^{(0)}$), which are singlet under the SM gauge group. The superscript “c” denotes the charge conjugation. With the Majorana masses on the brane, the DM particle in 4-dimensional effective theory is a Majorana fermion, and hence its spin-independent cross section with nuclei through the Z-boson exchange vanishes in the non-relativistic limit.

Let us focus on the following terms in eq. (2.4), which are relevant to the mass terms in 4-dimensional effective theory:

$$\mathcal{L}_{\text{mass}} = \bar{\psi}i\Gamma^5 (\partial_y - ig\langle A_y \rangle) \psi + \bar{\tilde{\psi}}i\Gamma^5 (\partial_y - ig\langle A_y \rangle) \tilde{\psi} - M \left( \bar{\psi}\tilde{\psi} + \bar{\tilde{\psi}}\psi \right) + \delta(y) \left[ \frac{m}{2} \bar{\psi}_{3R}^{(0)}\psi_{3R}^{(0)} + \frac{\tilde{m}}{2} \bar{\psi}_{3L}^{(0)}\psi_{3L}^{(0)} + \text{h.c.} \right],$$

where $\Gamma^5 = i\gamma^5$. Expanding the bulk fermions in terms of KK modes as

$$\psi(x, y) = \frac{1}{\sqrt{2\pi R}} \psi^{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \psi^{(n)}(x) \cos \left( \frac{n}{R} y \right) \left( \text{for } \psi_{1L,2L,3R}, \tilde{\psi}_{1R,2R,3L} \right),$$

$$\tilde{\psi}(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \psi^{(n)}(x) \sin \left( \frac{n}{R} y \right) \left( \text{for } \psi_{1R,2R,3L}, \tilde{\psi}_{1L,2L,3R} \right),$$

(2.8)

(2.9)
and integrating out the fifth coordinate $y$, we obtain the expression in 4-dimensional effective theory. The zero-mode parts for the electric-charge neutral fermions are found to be

\[
\mathcal{L}_{\text{zero\@mode\ mass}} = i m_W \left( \bar{\psi}_L^{(0)} \gamma^2 \psi_{3R}^{(0)} + \bar{\psi}_L^{(0)} \gamma^3 \psi_{3L}^{(0)} \right) - M \left( \bar{\psi}_{2L}^{(0)} \gamma^0 \psi_{2R}^{(0)} + \bar{\psi}_{3L}^{(0)} \gamma^3 \psi_{3R}^{(0)} \right) + \text{h.c.}
\]

\[
\rightarrow - m_W \left( \bar{\psi}_{2L}^{(0)} \gamma^0 \psi_{2R}^{(0)} - \bar{\psi}_{3L}^{(0)} \gamma^3 \psi_{3R}^{(0)} \right) - M \left( \bar{\psi}_{2L}^{(0)} \gamma^0 \psi_{2R}^{(0)} + \bar{\psi}_{3L}^{(0)} \gamma^3 \psi_{3R}^{(0)} \right) + \text{h.c.}
\]

\[
\frac{1}{2} \left( \begin{array}{cc}
\chi & \tilde{\chi} \\
\omega & \tilde{\omega}
\end{array} \right) M_N \left( \begin{array}{c}
\chi \\
\tilde{\chi} \\
\omega \\
\tilde{\omega}
\end{array} \right) = 0.
\]

where $m_W = g v / 2$ is the $W$-boson mass, and the arrow means the phase rotations $\psi_{3R}^{(0)} \rightarrow i \psi_{3R}^{(0)}$ and $\tilde{\psi}_{3L}^{(0)} \rightarrow i \tilde{\psi}_{3L}^{(0)}$. It is useful to rewrite these mass terms in a Majorana basis defined as

\[
\chi \equiv \psi_{3R}^{(0)} + i \psi_{3L}^{(0)}, \quad \tilde{\chi} \equiv \gamma^3 \psi_{3L}^{(0)} + \gamma^0 \psi_{3R}^{(0)},
\]

\[
\omega \equiv \psi_{2L}^{(0)} + \gamma^0 \psi_{2R}^{(0)}, \quad \tilde{\omega} \equiv \gamma^3 \psi_{2R}^{(0)} + \gamma^0 \psi_{2L}^{(0)}.
\]

and we then express the mass matrix $(M_N)$ as

\[
\mathcal{L}_{\text{zero\@mode\ mass}} = - \frac{1}{2} \left( \begin{array}{cccc}
\chi & \tilde{\chi} & \omega & \tilde{\omega}
\end{array} \right) M_N \left( \begin{array}{cccc}
m & 0 & m_W & 0 \\
0 & m & -m_W & 0 \\
m_W & 0 & 0 & M \\
0 & -m_W & M & 0
\end{array} \right) \left( \begin{array}{c}
\chi \\
\tilde{\chi} \\
\omega \\
\tilde{\omega}
\end{array} \right).
\]

The zero-modes of the charged fermions, $\psi_{1L}^{(0)}$ and $\tilde{\psi}_{1R}^{(0)}$, have a Dirac mass of $M$.

To simplify our analysis, we set $m = \tilde{m}$, and in this case we find a simple expression for the mass eigenvalues of $M_N$ as

\[
m_1 = \frac{1}{2} \left( m - \sqrt{4m_W^2 + (m - 2M)^2} \right),
\]

\[
m_2 = \frac{1}{2} \left( m + \sqrt{4m_W^2 + (m - 2M)^2} \right),
\]

\[
m_3 = \frac{1}{2} \left( m - \sqrt{4m_W^2 + (m + 2M)^2} \right),
\]

\[
m_4 = \frac{1}{2} \left( m + \sqrt{4m_W^2 + (m + 2M)^2} \right).
\]
for the mass eigenstates defined as $(\chi \tilde{\chi} \omega \tilde{\omega})^T = U_M (\eta_1 \eta_2 \eta_3 \eta_4)^T$ with a unitary matrix

$$U_M = \begin{pmatrix} u_1 & u_2 & u_3 & u_4 \\ -u_1 & -u_2 & u_3 & u_4 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{c_1} & 0 & 0 & 0 \\ 0 & \frac{1}{c_2} & 0 & 0 \\ 0 & 0 & \frac{1}{c_3} & 0 \\ 0 & 0 & 0 & \frac{1}{c_4} \end{pmatrix}, \quad (2.14)$$

where

$$u_1 = \frac{m_1 - M}{m_W}, \quad u_2 = \frac{m_2 - M}{m_W}, \quad u_3 = \frac{m_3 + M}{m_W}, \quad u_4 = \frac{m_4 + M}{m_W}, \quad (2.15)$$

$$c_1 = \sqrt{2(u_1^2 + 1)}, \quad c_2 = \sqrt{2(u_2^2 + 1)}, \quad c_3 = \sqrt{2(u_3^2 + 1)}, \quad c_4 = \sqrt{2(u_4^2 + 1)}. \quad (2.16)$$

Note that without loss of generality we can take $M, m \geq 0$. Considering the current experimental constraints from the search for an exotic charged fermion, we may take $M \gtrsim 1 \text{ TeV} \gg m_W$ [28]. In this case, the lowest mass eigenvalue (dark matter mass $m_{\text{DM}}$) is given by $|m_1|$. From the explicit form of the mass matrix $M_N$ in eq. (2.12) and $M \gg m_W$, we notice two typical cases for the constituent of the DM particle: (i) the DM particle is mostly an SM singlet when $m = \tilde{m} \lesssim M$, or (ii) the DM particle is mostly a component in the SM SU(2) doublets when $m = \tilde{m} \gtrsim M$. In the case (i), the DM particle communicates with the SM particle essentially through the SM Higgs boson. On the other hand, the DM particle is quite similar to the so-called Higgsino-like neutralino DM in the minimal supersymmetric SM (MSSM) for the case (ii). Since the Higgsino-like neutralino DM has been very well-studied in many literatures,\(^1\) we focus on the case (i) in this paper. Note that the case (i) is a realization of the so-called Higgs-portal DM from the GHU scenario.

We emphasize that in our scenario, the Yukawa couplings in the original Lagrangian are not free parameters, but are the SM SU(2) gauge coupling, thanks to the structure of the GHU scenario.

Now we describe the coupling between the DM particle and the Higgs boson. In the original basis, the interaction can be read off from eq. (2.12) by $v \rightarrow v + h$ as

$$\mathcal{L}_{\text{Higgs-coupling}} = -\frac{1}{2} \left( \frac{m_W}{v} \right) h \left( \begin{array}{c} \chi \\ \tilde{\chi} \\ \omega \\ \tilde{\omega} \end{array} \right) \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \chi \\ \tilde{\chi} \\ \omega \\ \tilde{\omega} \end{pmatrix} = -\frac{1}{2} \left( \frac{m_W}{v} \right) h \left( \begin{array}{c} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{array} \right) C_h \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{pmatrix}, \quad (2.17)$$

\(^1\)In this case, a pair of the DM particles mainly annihilates into the weak gauge bosons through the SM SU(2) gauge coupling, and the observed DM relic abundance can be reproduced with the DM mass of around 1 TeV [29].
where $h$ is the physical Higgs boson, and the explicit form of the matrix $C_h$ is given by

$$C_h \equiv \begin{pmatrix} C_1 & C_5 & 0 & 0 \\ C_5 & C_2 & 0 & 0 \\ 0 & 0 & C_3 & C_6 \\ 0 & 0 & C_6 & C_4 \end{pmatrix}, \quad (2.18)$$

where

$$C_1 = \frac{4u_1}{c_1^2}, \quad C_2 = \frac{4u_2}{c_2^2}, \quad C_3 = \frac{4u_3}{c_3^2}, \quad C_4 = \frac{4u_4}{c_4^2}, \quad C_5 = \frac{2(u_1 + u_2)}{c_1 c_2}, \quad C_6 = \frac{2(u_3 + u_4)}{c_3 c_4}. \quad (2.19)$$

The interaction Lagrangian relevant to the DM physics is given by

$$L_{DM-H} = -\frac{1}{2} \left( \frac{m_W}{v} \right) C_1 h \bar{\psi}_{DM} \psi_{DM} - \frac{1}{2} \left( \frac{m_W}{v} \right) C_5 h (\eta_2 + h.c.), \quad (2.20)$$

where we have identified the lightest mass eigenstate $\eta_1$ as the DM particle ($\psi_{DM}$).

3 Dark matter relic abundance

In this section, we evaluate the DM relic abundance and identify an allowed parameter region to be consistent with the Planck 2015 measurement of the DM relic density [30] (68 % confidence level):

$$\Omega_{DM} h^2 = 0.1198 \pm 0.0015. \quad (3.1)$$

In our model, the DM physics is controlled by only two free parameters, namely, $m$ and $M$. As we discussed in the previous section, we focus on the Higgs-portal DM case with $0 \leq m \lesssim M$. Using $M \gg m_W$, we can easily derive approximate formulas for parameters involved in our DM analysis. For the mass eigenvalues listed in eq. (2.13), we find

$$m_1 \simeq -M + m - \frac{m_W^2}{2M - m}, \quad m_2 \simeq M + \frac{m_W^2}{2M - m}. \quad (3.2)$$

By using these formulas, we express $u_{1,2}$ and $c_{1,2}$ in eqs. (2.15) and (2.16) as

$$u_1 \simeq -\frac{2M - m}{m_W}, \quad u_2 \simeq \frac{m_W}{2M - m}, \quad c_1 \simeq \sqrt{2} \left( \frac{2M - m}{m_W} \right), \quad c_2 \simeq \sqrt{2}, \quad (3.3)$$

which lead to

$$C_1 \simeq -\frac{2m_W}{2M - m}, \quad C_5 \simeq 1. \quad (3.4)$$

For $m \lesssim M$ and a fixed value of $M \gg m_W$, the DM particle can be light when $m \simeq M$, otherwise $m_{DM} \simeq M$ while $m_2 \simeq M$ for any values of $m \lesssim M$. The coupling of a DM particle pair with the Higgs boson is always suppressed by $|C_1| \ll 1$ while $C_5 \simeq 1$.

According to the interaction Lagrangian in eq. (2.20), we consider two main annihilation processes of a pair of DM particles. One is through the $s$-channel Higgs boson exchange, and the other is the process $\psi_{DM} \bar{\psi}_{DM} \rightarrow hh$ through the exchange of $\eta_2$ in the $t/u$-channel. Since $|C_1| \ll 1$ and $C_5 \simeq 1$, the $t/u$-channel processes dominate for the DM
pair annihilations when the DM particle is heavier than the Higgs boson. In evaluating this process, we may use an effective Lagrangian of the form,

$$\mathcal{L}_{\text{DM-H}}^{\text{eff}} = \frac{1}{2} \left( \frac{m_W}{v} \right)^2 \frac{\mathcal{C}_5^2}{m_2} \, h \, h \, \bar{\psi}_{\text{DM}} \psi_{\text{DM}},$$

(3.5)

which is obtained by integrating $\eta_2$ out, and calculate the DM pair annihilation cross section times relative velocity ($v_{\text{rel}}$) as

$$\sigma_{v_{\text{rel}}} = \frac{1}{64\pi} \left( \frac{m_W}{v} \right)^4 \left( \frac{\mathcal{C}_5^2}{m_2} \right)^2 v_{\text{rel}}^2 \equiv \sigma_0 v_{\text{rel}}^2.$$

(3.6)

It is well-known that the observed DM relic density is reproduced by $\sigma_0 \sim 1$ pb. Since we find $\sigma_0 \sim 0.02$ pb for $\mathcal{C}_5 \simeq 1$ and $m_2 \simeq M = 1 \text{ TeV}$, we conclude that the observed relic density is not reproduced by the process $\psi_{\text{DM}} \psi_{\text{DM}} \rightarrow hh$.

Next we consider the DM pair annihilation through the $s$-channel Higgs boson exchange when the DM particle is lighter than the Higgs boson. Since the coupling between the pair of DM particles and the Higgs boson is suppressed by $|\mathcal{C}_1| \ll 1$, an enhancement of the DM annihilation cross section through the Higgs boson resonance is necessary to reproduce the observed relic DM density. We evaluate the DM relic abundance by integrating the Boltzmann equation

$$\frac{dY}{dx} = -\frac{x s \langle \sigma v \rangle}{H(m_{\text{DM}})} \left( Y^2 - Y_{\text{EQ}}^2 \right),$$

(3.7)

where the temperature of the Universe is normalized by the DM mass as $x = m_{\text{DM}}/T$, $H(m_{\text{DM}})$ is the Hubble parameter as $T = m_{\text{DM}}$, $Y$ is the yield (the ratio of the DM number density to the entropy density $s$) of the DM particle, $Y_{\text{EQ}}$ is the yield of the DM in thermal equilibrium, and $\langle \sigma v_{\text{rel}} \rangle$ is the thermal average of the DM annihilation cross section times relative velocity for a pair of the DM particles. Various quantities in the Boltzmann equation are given as follows:

$$s = \frac{2\pi^2}{45} g_* m_{\text{DM}}^3 x^3, \quad H(m_{\text{DM}}) = \sqrt{\frac{\pi^2}{90} g_*} \frac{m_{\text{DM}}^2}{M_P}, \quad s Y_{\text{EQ}} = \frac{g_\text{DM}}{2\pi^2} \frac{m_{\text{DM}}^3}{x} K_2(x),$$

(3.8)

where $M_P = 2.44 \times 10^{18}$ GeV is the reduced Planck mass, $g_\text{DM} = 2$ is the number of degrees of freedom for the DM particle, $g_*$ is the effective total number of degrees of freedom for the particles in thermal equilibrium (in our analysis, we use $g_* = 86.25$ corresponding to $m_{\text{DM}} \simeq m_h/2$ with the Higgs boson mass of 125 GeV), and $K_2$ is the modified Bessel function of the second kind. For $m_{\text{DM}} \simeq m_h/2 = 62.5$ GeV, a DM pair annihilates into a pair of the SM fermions as $\psi_{\text{DM}} \psi_{\text{DM}} \rightarrow h \rightarrow f \bar{f}$, where $f$ denotes the SM fermions. We calculate the cross section for the annihilation process as

$$\sigma(s) = \frac{y_{\text{DM}}^2}{16\pi} \left[ 3 \left( \frac{m_b}{v} \right)^2 + 3 \left( \frac{m_c}{v} \right)^2 + \left( \frac{m_\tau}{v} \right)^2 \right] \frac{\sqrt{s} \left( s - 4 m_{\text{DM}}^2 \right)}{(s - m_h^2)^2 + m_h^2 \Gamma_h^2},$$

(3.9)

where $y_{\text{DM}} = (m_W/v)|\mathcal{C}_1|$ (see eq. (2.20)), and we have only considered pairs of bottom, charm and tau for the final states, neglecting the other lighter quarks, and used the following
values for the fermion masses at the Z-boson mass scale \cite{31}: $m_b = 2.82 \text{ GeV}$, $m_c = 685 \text{ MeV}$ and $m_\tau = 1.75 \text{ GeV}$. The total Higgs boson decay width $\Gamma_h$ is given by $\Gamma_h = \Gamma_h^{\text{SM}} + \Gamma_h^{\text{new}}$, where $\Gamma_h^{\text{SM}} = 4.07 \text{ MeV}$ \cite{32} is the total Higgs boson decay width in the SM and

$$\Gamma_h^{\text{new}} = \begin{cases} \frac{m_h}{16\pi} \left(1 - \frac{4m_{\text{DM}}^2}{m_h^2}\right)^{3/2} \frac{m_h^2}{y_{\text{DM}}^2} m_h < 2m_{\text{DM}} \\ \frac{m_h}{16\pi} \frac{16m_{\text{DM}}^2}{m_h^2} y_{\text{DM}}^2 m_h > 2m_{\text{DM}} \end{cases}, \quad (3.10)$$

is the partial decay width of the Higgs boson to a DM pair. The thermal average of the annihilation cross section is given by

$$\langle \sigma v \rangle = (sY_{\text{EQ}})^{-2} \frac{m_{\text{DM}}}{64\pi^4 x} \int_{4m_{\text{DM}}}^{\infty} ds \hat{\sigma}(s) \sqrt{s} K_1 \left(\frac{x \sqrt{s}}{m_{\text{DM}}}\right), \quad (3.11)$$

where $\hat{\sigma}(s) = 2(s - 4m_{\text{DM}}^2)\sigma(s)$ is the reduced cross section with the total annihilation cross section $\sigma(s)$, and $K_1$ is the modified Bessel function of the first kind. We solve the Boltzmann equation numerically and find an asymptotic value of the yield $Y(\infty)$ to obtain the present DM relic density as

$$\Omega h^2 = \frac{m_{\text{DM}} s_0 Y(\infty)}{\rho_c/h^2}, \quad (3.12)$$

where $s_0 = 2890 \text{ cm}^{-3}$ is the entropy density of the present universe, and $\rho_c/h^2 = 1.05 \times 10^{-5} \text{ GeV/cm}^3$ is the critical density.

In figure 1 we show the resultant DM relic density as a function of the DM mass for various values of $y_{\text{DM}}$. The solid lines from top to bottom correspond to $y_{\text{DM}} = 0.005$, 0.00692 and 0.01, respectively, while the dashed line denotes the observed DM density $\Omega_{\text{DM}} h^2 = 0.1198$ from the Planck 2015 result. For a fixed $y_{\text{DM}}$ value, intersections of the solid and the dashed lines denote the DM mass to reproduce the observed DM density. We can see that there is a lower bound on $y_{\text{DM}} \geq 0.00692$ in order to reproduce the observed DM density.

In the left panel of figure 2, we show $y_{\text{DM}}$ as a function of $m_{\text{DM}}$ (solid line) along which the observed DM density $\Omega_{\text{DM}} h^2 = 0.1198$ is reproduced. Here, the current experimental upper bound from the LUX 2016 result \cite{33} and the prospective reach in the future LUX-ZEPLIN DM experiment \cite{34} are also shown as the dashed and the dotted lines, respectively, which will be derived in section 4. In order to satisfy the LUX 2016 constraint, we find the parameter regions such as $58.0 \leq m_{\text{DM}}[\text{GeV}] \leq 62.4$ and $(0.00692 \leq) y_{\text{DM}} \leq 0.0164$. Recall that the Yukawa coupling between the DM particle and the Higgs boson, $y_{\text{DM}} = (m_W/v)|C_1|$, and the DM mass are determined by the two parameters, $M$ and $m$, from eqs. (2.13), (2.15) and (2.16). Using these formulas, we can express $M$ as a function of $m_{\text{DM}}$ along the solid line in the left panel. Our result is shown in the right panel. Since $M \gg m_W$, the parameter $m$ as a function of $m_{\text{DM}}$ is approximately given by $m \simeq M - m_{\text{DM}}$. Corresponding to the parameter regions of $58.0 \leq m_{\text{DM}}[\text{GeV}] \leq 62.4$ and $(0.00692 \leq) y_{\text{DM}} \leq 0.0164$, we find $3.14 \leq M[\text{TeV}] \leq 7.51$. 
Figure 1. The DM relic density as a function of the DM mass for various $y_{\text{DM}}$ values (solid lines), along with the observed DM density $\Omega_{\text{DM}} h^2 = 0.1198$ (horizontal dashed line). The three solid lines form top to bottom correspond to $y_{\text{DM}} = 0.005$, 0.00692, and 0.01, respectively.

Figure 2. Left panel: $y_{\text{DM}}$ as a function of $m_{\text{DM}}$ (solid line) along which the observed DM density $\Omega_{\text{DM}} h^2 = 0.1198$ is reproduced. Here, the current experimental upper bound from the LUX 2016 result [33] and the prospective reach in the future LUX-ZEPLIN DM experiment [34] are also shown as the dashed and the dotted lines, respectively. Right panel: $M$ as a function of $m_{\text{DM}}$, along the solid line in the left panel.

4 Direct dark matter detection

A variety of experiments are underway and also planned for directly detecting a dark matter particle through its elastic scattering off with nuclei. In this section, we calculate the spin-independent elastic scattering cross section of the DM particle via the Higgs boson exchange to lead to the constraint on the model parameters from the current experimental results.
The spin-independent elastic scattering cross section with nucleon is given by

$$\sigma_{SI} = \frac{1}{\pi} \left( \frac{y_{DM}}{v} \right)^2 \left( \frac{\mu_{\psi_{DM}N}}{m_N^2} \right)^2 f_N^2$$  \hspace{1cm} (4.1)$$

where $\mu_{\psi_{DM}N} = m_N m_{DM}/(m_N + m_{DM})$ is the reduced mass of the DM-nucleon system with the nucleon mass $m_N = 0.939$ GeV, and

$$f_N = \left( \sum_{q=u,d,s} f_{T_q} + \frac{2}{9} f_{TG} \right) m_N$$  \hspace{1cm} (4.2)$$
is the nuclear matrix element accounting for the quark and gluon contents of the nucleon. In evaluating $f_{T_q}$, we use the results from the lattice QCD simulation [35]: $f_{T_u} + f_{T_d} \simeq 0.056$ and $|f_{T_s}| \leq 0.08$. For conservative analysis, we take $f_{T_s} = 0$ in the following. Using the trace anomaly formula, $\sum_{q=u,d,s} f_{T_q} + f_{TG} = 1$ [36–40], we obtain $f_N^2 \simeq 0.0706 m_N^2$ and hence

$$\sigma_{SI} \simeq 4.47 \times 10^{-7} \text{ pb} \times y_{DM}^2$$  \hspace{1cm} (4.3)$$

for $m_{DM} = m_h/2 = 62.5$ GeV.

The LUX 2016 result [33] currently provides us with the most severe upper bound on the spin-independent cross section, from which we read $\sigma_{SI} \leq 1.2 \times 10^{-10}$ pb for $m_{DM} \simeq 62.5$ GeV. From eq. (4.3), we find $y_{DM} \leq 0.0164$, which is depicted as the horizontal dashed line in the left panel of figure 2. The next-generation successor of the LUX experiment, the LUX-ZEPLIN experiment [34], plans to achieve an improvement for the upper bound on the spin-independent cross section by about two orders of magnitude. When we apply a conservative search reach to the LUX-ZEPLIN experiment as $\sigma_{SI} \leq 1.2 \times 10^{-11}$ pb (just an order of magnitude improvement from the current LUX bound), we obtain $y_{DM} \leq 0.00518$. This prospective upper bound is shown as the dotted line in the left panel of figure 2. We can see that the present allowed parameter region all covered by the future LUX-ZEPLIN experiment.

5 Higgs boson mass in effective theory approach

In this section, we calculate the Higgs boson mass by using a 4-dimensional effective theory approach of the GHU scenario in 5-dimensional Minkowski space, which is developed in refs. [41, 42]. In this paper, it has been shown that an effective Higgs quartic coupling derived from the 1-loop effective Higgs potential after integrating out all KK modes coincides with a running Higgs quartic coupling at low energies obtained from the renormalization group (RG) evolution with a vanishing Higgs quartic coupling at the compactification scale (“gauge-Higgs condition” [41, 42]). This vanishing Higgs quartic coupling indicates a restoration of the 5-dimensional gauge invariance at the compactification scale. With this approach, we can easily calculate the Higgs quartic coupling at low energies by solving the RG equations, once the particle contents and the mass spectrum of the model below the compactification are defined. Assuming that the electroweak symmetry breaking is
correctly achieved, the Higgs boson mass is calculated by the Higgs quartic coupling value at the electroweak scale.

There are two scales involved in our RG analysis, namely, the bulk mass $M \approx m$ and the compactification scale $M_{\text{KK}} = 1/R$. In the following analysis, we ignore the mass splitting among the bulk fermion zero modes and set all of their masses as $M$. As we will show in the following, a hierarchy $M \ll M_{\text{KK}}$ is necessary to reproduce the 125 GeV Higgs boson mass, and hence this treatment is justified.

For the renormalization scale smaller than the bulk mass $\mu < M$, all bulk fermions are decoupled and we employ the SM RG equations at two loop level [43–49]. For the three SM gauge couplings $g_i$ $(i = 1, 2, 3)$, we have

$$\frac{dg_i}{d\ln \mu} = \frac{b_i}{16\pi^2} g_i^3 + \frac{g_i^3}{(16\pi^2)^2} \left( \sum_{j=1}^3 B_{ij} g_j^2 - C_i y_t^2 \right), \quad (5.1)$$

where

$$b_i = \left( \frac{41}{10} - \frac{19}{6} - 7 \right), \quad B_{ij} = \begin{pmatrix} 199 & 27 & 44 \\ 9 & 10 & 6 \\ 11 & 9 & -26 \end{pmatrix}, \quad C_i = \left( 17, \frac{3}{2}, 2 \right). \quad (5.2)$$

Here, among the SM Yukawa couplings, we have taken only the top Yukawa coupling $(y_t)$ into account. The RG equation for the top Yukawa coupling is given by

$$\frac{dy_t}{d\ln \mu} = y_t \left( \frac{1}{16\pi^2} \beta_t^{(1)} + \frac{1}{(16\pi^2)^2} \beta_t^{(2)} \right), \quad (5.3)$$

where the one-loop contribution is

$$\beta_t^{(1)} = \frac{9}{2} y_t^2 - \left( \frac{17}{20} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2 \right), \quad (5.4)$$

while the two-loop contribution is given by

$$\beta_t^{(2)} = -12 y_t^4 + \left( \frac{393}{80} g_1^4 + \frac{225}{16} g_2^4 + 36 g_3^4 \right) y_t^2$$

$$+ \frac{1187}{600} g_1^4 - \frac{9}{20} g_1^2 g_2^2 + \frac{19}{15} g_1^2 g_3^2 - \frac{23}{4} g_2^4 + 9 g_2^2 g_3^2 - 108 g_3^4$$

$$+ \frac{3}{2} \lambda^2 - 6 y_t^2. \quad (5.5)$$

The RG equation for the Higgs quartic coupling is given by

$$\frac{d\lambda}{d\ln \mu} = \frac{1}{16\pi^2} \beta_{\lambda}^{(1)} + \frac{1}{(16\pi^2)^2} \beta_{\lambda}^{(2)}, \quad (5.6)$$

with

$$\beta_{\lambda}^{(1)} = 12 \lambda^2 - \left( \frac{9}{5} g_1^4 + 9 g_2^2 \right) \lambda + \frac{9}{4} \left( \frac{3}{25} g_1^4 + \frac{2}{5} g_2^2 g_2^2 + g_2^2 \right) + 12 y_t^2 \lambda - 12 y_t^4, \quad (5.7)$$
and

$$\beta^{(2)}_{\lambda} = -78\lambda^2 + 18\left(\frac{3}{5}g_1^2 + 3g_2^2\right)\lambda^2 - \left(\frac{73}{8}g_1^2 - \frac{117}{20}g_1^2g_2^2 - \frac{1887}{200}g_1^4\right)\lambda - 3\lambda y_t^4 + 305\frac{g_2^6}{8} - \frac{289}{40}g_1^2g_2^4 + \frac{1677}{200}g_1^4g_2^2 + \frac{3411}{1000}g_1^6 - 64g_3^2y_t^4 - \frac{16}{5}g_1^2y_t^4 - \frac{9}{2}g_2^4y_t^2 + 10\lambda\left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right)g_1^2 - \frac{3}{5}g_1^2\left(\frac{57}{10}g_1^2 - 21g_2^2\right)y_t^2 - 72\lambda^2 y_t^2 + 60y_t^6. \tag{5.8}$$

In solving these RGEs, we use the boundary conditions at the top quark pole mass \(M_t\) given in [50]:

$$g_1(M_t) = \sqrt{\frac{5}{3}}\left(0.35761 + 0.00011(M_t - 173.10) - 0.00021\left(\frac{M_W - 80.384}{0.014}\right)\right),$$

$$g_2(M_t) = 0.64822 + 0.00004(M_t - 173.10) + 0.00011\left(\frac{M_W - 80.384}{0.014}\right),$$

$$g_3(M_t) = 1.1666 + 0.00314\left(\frac{\alpha_s - 0.1184}{0.0007}\right),$$

$$y_t(M_t) = 0.93558 + 0.0055(M_t - 173.10) - 0.00042\left(\frac{\alpha_s - 0.1184}{0.0007}\right) - 0.00042\left(\frac{M_W - 80.384}{0.014}\right),$$

$$\lambda(M_t) = 2(0.12711 + 0.00206(m_h - 125.66) - 0.00004(M_t - 173.10)). \tag{5.9}$$

We employ \(M_W = 80.384\) GeV, \(\alpha_s = 0.1184\), the central value of the combination of Tevatron and LHC measurements of top quark mass \(M_t = 173.34\) GeV [51], and the central value of the updated Higgs boson mass measurement, \(m_h = 125.09\) GeV from the combined analysis by the ATLAS and the CMS collaborations [52].

For the renormalization scale \(\mu \geq M\), the SM RG equations are modified in the presence of the bulk fermions. In this paper, we take only one-loop corrections from the bulk fermions into account. In the presence of a pair of the SU(3) triplet bulk fermions, the beta functions of the SU(2) and U(1)\(_Y\) gauge couplings receive new contributions as

$$\Delta b_1 = \Delta b_2 = \frac{2}{3}. \tag{5.10}$$

The beta functions of the top Yukawa and Higgs quartic couplings are modified as

$$\beta^{(1)}_t \to \beta^{(1)}_t + 2y_t|Y_S|^2, \quad \beta^{(1)}_\lambda \to \beta^{(1)}_\lambda + 8\lambda|Y_S|^2 - 8|Y_S|^4, \tag{5.11}$$

where \(Y_S\) is the universal Yukawa coupling of \(\psi\) and \(\bar{\psi}\) with the Higgs doublet in eq. (2.7), which obeys the RG equation,

$$16\pi^2 \frac{dY_S}{d\ln \mu} = Y_S \left[3|y_t|^2 - \left(\frac{9}{20}g_1^2 + \frac{9}{4}g_2^2\right) + \frac{7}{2}|Y_S|^2\right]. \tag{5.12}$$

In our RG analysis, we numerically solve the SM RG equations from \(M_t\) to \(M\), at which the solutions connect with the solutions of the RG equations with the bulk triplet
Figure 3. Left panel: RG evolution of the Higgs quartic coupling with the bulk mass $M = 1$ TeV (solid line), along with the result in the SM (dashed line). The compactification scale is found to be $M_{KK} = 1.9 \times 10^8$ GeV, where the gauge-Higgs condition $\lambda(M_{KK}) = 0$ and $|Y_S(M_{KK})| = g_2(M_{KK})/\sqrt{2}$ are satisfied. Right panel: the relation between the bulk mass ($M$) and the compactification scale ($M_{KK}$) so as to reproduce the 125 GeV Higgs boson mass.

For a fixed $M$ values, we arrange an input $|Y_S(M)|$ value so as to find numerical solutions which satisfy the gauge-Higgs condition and the unification condition between the gauge and Yukawa couplings at the compactification scale, such that

$$\lambda(M_{KK}) = 0, \quad |Y_S(M_{KK})| = \frac{g_2(M_{KK})}{\sqrt{2}}. \quad (5.13)$$

In the left panel of figure 3 we show the RG evolution of the Higgs quartic coupling (solid line) for $M = 1$ TeV, along with the one in the SM (dashed line). At $M_{KK} = 1.9 \times 10^8$ GeV, the boundary condition in eq. (5.13) is satisfied. For a fixed $M$ value, we numerically find a $M_{KK}$ value. Our result for the relation between $M$ and $M_{KK}$ is shown in the right panel of figure 3. For $M_{KK} \simeq 10^8$ GeV, the 125 GeV Higgs boson mass is reproduced. We obtained the hierarchy $M \ll M_{KK}$ mentioned above. Note that in the absence of the bulk SU(3) triplet fermions, the RG evolution of the Higgs quartic coupling follows the SM one and the compactification scale, at which the quartic coupling becomes zero, is found to be $M_{KK} \simeq 10^{10}$ GeV [43–49]. In the presence of the bulk fermions, the compactification scale is lowered from $M_{KK} \simeq 10^{10}$ GeV to $10^8$ GeV.

6 Conclusions and discussions

In this paper, we have proposed a Majorana fermion DM scenario in the context of a 5-dimensional GHU model based on the gauge group SU(3) $\times$ U(1)$_Y'$ with a compactification of the 5th dimension on $S^3/Z_2$ orbifold. A pair of bulk SU(3) triplet fermions is introduced along with a bulk mass term and a periodic boundary condition. The bulk fermions are decomposed into a pair of the SU(2) doublets and a pair of the electric-charge neutral singlets under the SM gauge group of SU(2) $\times$ U(1)$_Y$. With Majorana mass terms for the singlets, which are introduced on a brane at an orbifold fixed point in general, the lightest mass eigenstate among the doublet and singlet components serves as a DM candidate.
We have focused on the case that the DM particle is mostly composed of the SM singlet fermions, and have investigated the DM physics. In this case, the DM particle communicates with the SM particles through the Higgs boson. We have found that an allowed parameter region to reproduce the observed DM density is quite limited and the DM particle mass is to be a vicinity of a half of the Higgs boson mass. The allowed region has been found to be further constrained when we take into account the upper limit of the elastic scattering of the DM particle off with the nuclei by the LUX 2016 result. We have found that the entire allowed region will be covered by the LUX-ZEPLIN experiment in the near future.

Note that even if the parameter region shown in the left-pane of figure 2 is entirely excluded in the future, our DM scenario can be still viable for the case where the DM particle is mostly a component in the SM SU(2) doublets. As mentioned in section 2, the DM particle property in this case is very similar to the Higgsino-like neutralino DM in the MSSM and the observed DM relic abundance is reproduced with the DM mass of around 1 TeV [29]. Since the reduced mass is $\mu_{\psi_{DM} N} \simeq m_N$ for $m_{DM} \gg m_N$, we apply eq. (4.3) for the spin-independent elastic scattering cross section also for the present case. However, the limit on $y_{DM}$ is weaker since the experimental upper bound on $\sigma_{SI}$ for $m_{DM} \sim 1$ TeV is about an order of magnitude higher than the one for $m_{DM} \approx 62.5$ TeV [33]. Furthermore, we can estimate $y_{DM} = (m_W/v)|C_1|$ with eqs. (2.15) and (2.16) as

$$y_{DM} \simeq 2 \left( \frac{m_W}{v} \right) \left( \frac{m_W}{m} \right)$$ (6.1)

for $m \gtrsim M \simeq m_{DM} \sim 1$ TeV. Therefore, in the decoupling limit of the SM singlet components, namely $m \gg M$, the spin-independent elastic scattering cross section is highly suppressed, and therefore the DM particle escapes detection. This limit is analogous to the pure Higgsino dark matter in the MSSM.

Employing the effective theoretical approach with the gauge-Higgs condition, we have also studied the RG evolution of Higgs quartic coupling and shown that the observed Higgs mass of 125 GeV is achieved with the compactification scale of around $10^{8}$ GeV. In the presence of the bulk DM multiplets, the compactification scale to reproduce the 125 GeV Higgs boson mass is reduced by about two orders of magnitude from $M_{KK} \simeq 10^{10}$ GeV. However, in terms of providing a solution to the gauge hierarchy problem by the GHU scenario, $M_{KK} \approx 10^{8}$ GeV is too high for the scenario to be natural. In fact, as has been shown in refs. [15, 19, 20], when we introduce a pair of bulk fermions in higher dimensional SU(3) representations such as 10-plet and 15-plet, the compactification scale can be as low as $\mathcal{O}(1 \text{ TeV})$, while reproducing the 125 GeV Higgs boson mass. Hence, toward a natural GHU scenario with a fermion DM, it is worth extending our present model and introducing the bulk DM multiplets in such a higher dimensional representation. In this case, we will see that the DM physics investigated in this paper remains almost the same while the Higgs boson mass of 125 GeV can be reproduced with the compactification scale of order 1 TeV [53].
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