Spинор Bose-Einstein condensate (BEC) has been attracting growing attentions in the last decade since it displays a variety of exotic phenomena associated with its spin degree of freedom. Spin-domain formation and spin dynamics are definitely among such topics of particular interest.

Early in 1998, soon after the experimental realization of the spinor BEC, the MIT group investigated the miscibility of different spin domains in the spinor $^{23}$Na condensate. The spin-dependent interaction between $^{23}$Na atoms is antiferromagnetic (AFM) and domain structures were pre-created by applying a gradient magnetic field. However, the $|m_F = \pm 1\rangle$ domains become almost completely miscible as the gradient field is turned down, indicating that spin domains is hard to be formed spontaneously in the AFM spinor condensate. A new experimental result further rules out spontaneous domain formation in $^{23}$Na condensate. Very recently, a theoretical work demonstrates that the homogeneous magnetic field can lead to spatial modulational instability in AFM condensates, followed by the generation of spin domains. In contrast, the spontaneous domain formation has been observed in the ferromagnetic (FM) $^{87}$Rb condensate using an in-situ phase-contrast imaging. This is a pioneering approach in exploring spin domains in spinor bosons. Theoretically, the domain formation is attributed to the FM interaction between $^{87}$Rb atoms which leads to spontaneous polarization in the ground state.

Spin dynamics in spinor BECs, usually referring to evolutions of spin populations in different spin components, arises directly from spin exchange collisions. Taking the $F = 1$ manifold for example, the collision process can be expressed as $|m_F = 1\rangle \rightarrow 2|m_F = 0\rangle$, which naturally hold the conservation of total spins. Owing to the quantum nature of BECs, the collision process is coherent so that it leads to oscillations of spin populations. Such coherent behaviors have been observed experimentally in $^{87}$Rb condensates of both the $F = 1$ and $2$ manifolds, and very recently in the AFM $^{23}$Na condensate. So far, theoretical explorations for the spin dynamics are mainly based on the assumption that each component shares the same spatial wave function, called the single-mode approximation (SMA). It seems that the SMA works well for the AFM BEC, but becomes invalid for the FM condensate due to its domain structures. We have ever proposed a two-domain model to account for the later case and suggest that domain formations inside FM BECs bring about significant influence on spin dynamics.

In this paper, we propose a scheme for generating spin domains both in FM and AFM condensates, and discuss the exotic spin dynamics caused by domain formations. According to this scheme, domain structures can be created by the spatially modulated phases, which is expected to be realized via phase-imprinting method in experiments. Phase-imprinting, as a versatile tool to manipulate BECs, has already been used to create dark solitons, vortices and vortex rings in scalar or two component condensates. One can further expect that it applies to spinor-1 BECs as well.

We start with the mean-field energy functional for a spinor-1 Bose condensate, which is expressed as:

$$E = \int dr \left[ \frac{\hbar^2}{2m} \nabla \psi_i \nabla \psi_i + V_{\text{ext}}(r) \psi_i^* \psi_i + \frac{1}{2} c_0 \psi_i^* \psi_j \psi_j \psi_i + \frac{1}{2} c_2 \psi_i^* \psi_k \psi_j F_{ij} F_{kl} \psi_l \psi_j \right],$$

where $\psi_a$ denotes the condensate wave function for the atomic BEC in the $a$-th internal state $|m_F = \alpha\rangle$ and repeated indices are assumed to be summed. The $c_0$ and $c_2$ terms describe contributions of the spin-independent and spin-dependent interactions between atoms respectively. The spin-dependent interaction could be FM if $c_2 < 0$ or AFM if $c_2 > 0$. $F$ is the vector of spin matrices and $V_{\text{ext}}(r)$ is the external trap potential. Then equations of motion for the spinor condensate, derived from Eqn. via variational principles, are given by:

$$i\hbar \frac{\partial}{\partial t} \psi_{+1} = [\mathcal{H} + c_2 (n_{+1} + n_0 - n_{-1})] \psi_{+1} + c_2 \psi_0^2 \psi_{-1}^*, \quad i\hbar \frac{\partial}{\partial t} \psi_0 = [\mathcal{H} + c_2 (n_{+1} + n_{-1})] \psi_0 + 2c_2 \psi_{+1} \psi_{-1} \psi_0^*, \quad i\hbar \frac{\partial}{\partial t} \psi_{-1} = [\mathcal{H} + c_2 (n_{-1} + n_0 - n_{+1})] \psi_{-1} + c_2 \psi_0^2 \psi_{+1}^*.$$
where $\mathcal{H} = -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}} + c_0 (n_{+1} + n_0 + n_{-1})$ and $n_\alpha$ represents density of the $m_F = \alpha$ atoms. Equations 2 are just the Gross-Pitaevskii (GP) equations for spinor-1 condensates, with the condensate wave function expressed as $\psi_\alpha(r,t) = \sqrt{n_\alpha(r,t)} e^{i\theta_\alpha(r,t)}$.

The GP equations have been intensively employed to describe spin dynamics and domain instabilities of spinor BECs, wherein the phase $\theta$ acts as a crucial factor since it reflects the quantum characteristics of the condensate \cite{9, 10, 11, 12, 13}. However, $\theta$ could play a more important role than recognized previously. In previous works, $\theta$ is usually considered to be spatially invariant. Hereinafter, we look at the case that the phase $\theta$ can be spatially modulated. Such an extension is by no means trivial. We will show that intriguing domain structures and spin dynamics could be induced by appropriate phase-modulations.

We consider a one-dimensional (1D) BEC as an example, supposing that it is confined in the infinitely deep square well potential, approximately corresponding to the elongated cigar-shaped BEC in experiments \cite{21}. The initial condensate density profile is set to be homogeneous-like except at boundaries where it tends to zero \cite{22}, as plotted in Fig. 1(a). We note that the initial phases as given by Eq. (3), which implies that the located domains are in accord with the specific choice of initial phases as given by Eq. (3), which implies that the domains are closely related to the modulated phases.

Obviously, both the shape and location of domains are varying with the time. A very interesting phenomenon occurs during the period from Fig. 1(d) to (f), where the $\pm 1$ domains have been exchanging their positions. It is worth noting that the two domains exchange their positions periodically with time. This kind of dynamical
behavior of the condensate is somewhat similar to the soliton-like dynamics in FM spinor condensates studied by Zhang et al. recently [24]. In Ref. [24], the soliton-like behaviors are attributed to the exchange interaction $c_2$, while here they are resulted from the phase modulation. As to one domain, the inside magnetization density $M = n_+ - n_-$ exhibits perfect oscillation. Figure 2 plots the local magnetization density $M$ at three different points in the left side of the condensate. $M$ at $r = 0.1$ and 0.3 exhibits a good periodicity with almost the same period within the time of our simulation. The oscillation period is dependent on the dimensionality of the condensate and the parameter $c_2$. Nevertheless, at the boundary between two domains $r = 0.5$, the magnetization density is equal to 0 and remains unchanging, owing to the symmetry of initial density profiles and the symmetric choice of initial phases.

Figure 3 illustrates the evolution of total population of each spin component (spin population) and the total spins. The spin population smears out detailed information of inner structures inside the condensate, but it is an important factor concerning spin dynamics of spinor condensates and has been intensively studied previously [11,12,13,14]. It is already discovered that the spin population exhibits a quantum oscillating feature; this point is also confirmed in our calculations. Moreover, the total spin conserves during the evolution, although the local magnetization is allowed. Comparing Figs. 2 and 3, one can get an interesting point that the period of population oscillations is different from the oscillation period of the local magnetization density. This reveals that the spin population is not sufficient to characterize the whole feature of spin dynamics when the condensate has certain inner structures, e.g., domain structures.

It is important to point out that domain structures can appear spontaneously in the FM condensate even if the initial phases are not modulated at all. This kind of domain formation is attributed to the spontaneous symmetry breaking in the spin space cause by the FM interaction between bosons [7,8,9,10] and has been confirmed experimentally in the $^{87}$Rb condensate [6]. The spontaneous domain formation usually leads to some multi-domain structures and domains seem distributed randomly in the condensate [6,25]. However, the phase-imprinting induced domain structure is strongly dependent on the modulation of initial phases. Regular initial phases can induce regular domain structures. As indicated above, one can even produce a simple two-domain structure so as to facilitate experimental probing. Moreover, if the initial phases are not that regular as above, an irregular multi-domain structure can be produced. To demonstrate this point, we suppose that the initial phases are modulated in a more complicated manner, such as

$$\theta_\alpha(r,0) = \sin ((4 + \alpha) \pi r)$$

(5)

where different components are imprinted with different phases. Figs. 4(a-e) show evolution of the condensate. A multi-domain structure emerges and the condensate displays very complicated dynamical behaviors. For example, the local magnetization density shown in Fig. 4(f) is still oscillating, but not periodically.

Another striking difference between the two cases is the time scale of producing domains. The characteristic time for the spontaneous domain formation strongly depends on the spin-dependant interaction ($t_{\text{unit}} = \hbar / |c_2| n$), whereas that for phase-imprint induced domains is mainly determined by the size of the condensate ($t_{\text{unit}} = 2m^2 / \hbar$). The spontaneous domain formation takes a period of relaxation time of nearly 100ms in $^{87}$Rb condensates [6]. In a similar condensate with $l = 334\mu m$, it takes about $0.12 t_{\text{unit}} \approx 36s$ to form the two domain structure resulted from the phase modulation, as can be seen from Fig. 4. In this case, domains appear much more slowly. However, the domain formation time can be significantly reduced by shortening the conden-
faster than the two-domain case where shows, domains are well developed at is to enrich the phase modulation pattern. As Fig. 4(f)
sate length. When the length is reduced to about 10μm, the formation time can be decreased to within 100ms. An alternative way to reduce the domain formation time is to enrich the phase modulation pattern. As Fig. II(f) shows, domains are well developed at t ≈ 0.01t_{unit}, much faster than the two-domain case where t ≈ 0.12t_{unit}. In some sense, enriching the phase modulation pattern is equivalent to reducing characteristic length of the condensate.

In comparison with the case of FM condensate, domain formations in AFM spinor condensates, such as $^{23}$Na, is more fascinating. A very recent experiment suggests that the spatial domain structure could not be formed spontaneously in the $^{23}$Na condensate [3]. Meanwhile, the spin dynamics measurements show good agreement with predictions made on the base of the SMA. Nevertheless, spin domains may be induced by applying some driving factors. For example, a recent theory predicts that spin domains can be generated by applying an external homogeneous magnetic field to the AFM condensates [3]. So a question arises: whether domain formation could be induced if modulating initial phases? To answer this question, we need to simulate dynamic behaviors of the AFM condensate according to Eqs. 4 and 5, with the parameter $c_2$ set to be positive. We choose the parameter $c_0$ and $c_2$ to be 100 and 10 respectively and the obtained results are shown in Fig. 5. The evolution process of AFM condensates is quite similar to the FM case, except that the density profile of each spin component is different from that in the FM condensate. This indicates that one can really create spin domains in AFM condensates by the phase imprinting method, as does in FM condensates. These simulation results await experimental validation, for example, in the AFM $^{23}$Na condensate.

In summary, we have established that the phase-imprinting could induce spin domains both in FM and AFM spinor condensates. Domains come into being after the initial phases are imprinted and the domain structure is strongly related to the spatial modulation of phases. These characters make the phase-imprint induced domain formation differs from the spontaneous domain formation in the FM condensate. Even more interestingly, phase-imprinting offers the opportunity to study spin domains and related spin dynamics in the AFM condensate where domain structure can not form spontaneously. Our results demonstrate that the phase engineering can play more important roles in manipulating the quantum feature of spinor Bose condensates than previously done.

This work is supported by the National Natural Science Foundation of China (Grant No. 10504002), the Fok Yin-Tung Education Foundation, China (Grant No. 101008), and the Ministry of Education of China (NCET-05-0098).

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