The Suitable Index of Flow and Density in the Mixed Traffic

Fadly Arirja Gani¹, Toshio Yoshii² and Shinya Kurauchi²

¹ Graduate School of Science and Engineering, Ehime University, Bunkyo-cho, Matsuyama 790-8577, Japan.
² Department of Civil and Environmental Engineering, Ehime University, Bunkyo-cho, Matsuyama 790-8577, Japan.
Corresponding Author: fadly_agani@yahoo.com

Abstract. Some traffic indices were developed to consider the existence of vehicle variance either in the homogeneous or mixed traffic. This study investigates the proper index of flow and density in the mixed traffic. There are eight pairs of indices of traffic flow and density that will be compared, in which it was varied by the concept and the measurement interval. The concept was distinguished according to the respond to the variance of vehicles in the traffic analysis, and the measurement interval was differentiated over time and time-space. The traffic was observed over a short section of a road which set as the detection zone. All pairs of indices will be applied and evaluated on the Fundamental Diagram to describe the actual traffic state in the density-flow plane. Then, the R-square of each density-flow curve will be compared. The result shows that the existence of vehicle variance in the share of traffic can be described more properly by considering the projected area of vehicles. The index of flow and density which considers the projected area of vehicles can explain the actual traffic state more appropriately by increasing the R-square of the model. In addition, the result does not provide a significant difference between the measurement over time and time-space in a short section of the road.

Keywords: Vehicle Variance; Mixed Traffic; Flow; Density; Occupancy; Area Occupancy

1. Introduction

According to the existence of vehicle variance, the condition of traffic is distinguished in two terms, which known as homogeneous and heterogeneous or mixed traffic. In each condition, traffic has a different characteristic. The terminology of homogeneous traffic refers to the condition which follows the lane rule, and the composed vehicle has a similar characteristic. This condition was addressed to the traffic in the developed countries. The traffic condition will be different in the developing countries, which associated by another terminology namely heterogeneous or mixed traffic. The composition of traffic in this condition is mixed, with a variety of vehicles, motorized and non-motorized, using the same right of way [1]. In addition, it was also classified as a non-lane based traffic due to the poor lane-discipline of many road users [2]. The term of lane discipline refers to the driver behavior in the share of traffic: changing and following the lane rule of a road.

In the reality, it shows that the traffic entity is composed by vehicles, which varies in terms of physical and behavior. Even in the homogeneous traffic, the presence of vehicle variance cannot be ignored. The term of physical in the vehicle variance refers to the different size or vehicle dimensions, and the behavior variance refers to the different of vehicle speed. There is a little different effect of the vehicle variance between the homogeneous traffic and the mixed traffic. The existence of vehicle variance will be responded differently in both conditions of traffic due to the characteristic. Therefore,
the vehicle variance should be considered in the analysis either in the homogeneous or heterogeneous traffic.

Generally, there are two approaches that can be used to analyze the traffic such as microscopic and macroscopic perspective. However, it is not known yet, which one the approach is superior for analyzing the traffic in both homogeneous and heterogeneous or mixed condition. In this study, the index of macroscopic traffic flow variable will be investigated. In the macroscopic perspective, traffic state can be described by the relationship between two variables such as flow, density and speed. The proper relation can be obtained by the suitable index of each variable. In the analysis, the index of macroscopic traffic flow variables should be able to consider the different effect due to the vehicle variance.

In homogeneous condition with lane based traffic, the length of vehicles was considered to affect the traffic. So, the index of macroscopic traffic flow variable was developed to consider the effect of variance of vehicle length. It was different in the mixed condition, with non-lane based traffic. In addition to the length, the width of vehicles was also considered to influence the traffic. So, over this study, the projected area of vehicles will be considered to develop the index of macroscopic traffic flow variable in the mixed traffic. There are some improvements related to the development of the index of traffic flow and density. In the mixed condition, mostly the index of traffic flow and density was adopted from the homogeneous condition. So, the proposed index of traffic flow and density was developed based on the existing index in homogeneous condition. However, it was not clarified yet whether all the vehicle variance has the significant effect to the traffic analysis or vice versa. This point is important to be known that the variance of vehicles needs to be considered or can be ignored. This knowledge can be used to validate the accuracy of the proposed index on the mixed condition.

Specifically, this study focuses to investigate the proper index of macroscopic flow variable in the mixed traffic. It was needed to describe the flow rate and the density of traffic which able to obtain the effect of the vehicle variance. To establish the proper index of these variables, the specific criteria of vehicle variance should be identified. So, the index of flow rate and density will be varied, which considers and not considers the difference of traffic entity. Then, the effect can be shown by comparing all the indices to describe the traffic state on the fundamental diagram (density-flow plane). The analysis result will show whether the variance needs to be considered or not in the traffic flow and density. Finally, it will lead to the suitable index of flow and density in the mixed traffic.

2. Literature Review
The first mathematic model was developed to estimate the actual traffic state by considering all the vehicles are similar, which expressed in vehicle unit. In the reality, vehicles have a different characteristic. So, in the estimation of actual traffic state, the characteristic of vehicles was important to be considered. It may produce a better model and possible to explain the variation in the share of traffic. Since the first traffic flow model was introduced, several concepts have been developed at the macroscopic perspective. These developments are the effort to consider the characteristic of vehicles in the traffic analysis appropriately. The characteristic of vehicle can be considered as a group or individually. In the heterogeneous or mixed traffic, the concept of PCU was widely developed to obtain the effect of different characteristic of vehicles. The concept of PCU classified the entity of traffic into several categories which has a similar characteristic. In the PCU concept, several approaches are developed in the mixed traffic to derive an appropriate result in the traffic analysis [3–6]. By using numerical approaches, the others studies have been conducted to incorporate the different characteristics of vehicles in macroscopic models [7–9]. Both concepts show that the characteristic of vehicle should be considered in the traffic model. In the other concept, the characteristic of vehicles was considered individually. The concept of occupancy and area occupancy [10–12] are proposed to consider the different of vehicle in length and the projected area respectively. The concept was developed to provide the proper density variable. In addition, an extensive review of the heterogeneous traffic flow model has been conducted [1]. It was concluded that linear density measurements are inadequate in the mixed traffic and suggest adopting the areal density concept,
which consider the variation of the projected area of vehicles. All the concept which provided from several literatures are related one another. By understanding the measurement interval of macroscopic variable, it can provide an illustration of the relation and the proper concept. Then, the appropriate index can be developed and summarized by showing the relation with each other.

2.1. The Measurement Interval of Macroscopic Variable

At the macroscopic perspective, three main variables are defined: density $k$, flow rate $q$ and mean speed $u$. In this perspective, traffic state can be described by the relation between these variables. The three aggregate variables are always connected by the fundamental relation or identity relation in which it can be expressed by the Equation (1).

$$q = k \cdot u$$  \hspace{1cm} (1)

This fundamental relation considers the traffic flow under stationary and homogeneous condition. So, by this equation, knowing two variables automatically leads to the third. The definitions of macroscopic variables can be explained in the measurement interval by using trajectory data in the time-space diagram, which illustrates in the Figure 1. In this illustration, macroscopic variables can be derived by three measurement intervals: measurement over time (local measurement), measurement over space (Instantaneous measurement), and measurement over time and space.

![Figure 1. Measurement of macroscopic variables.](image)

In the Figure 1, density was defined on the $S_1$ measurement interval. This rectangular measurement interval covers a road section of length $\Delta X$ during a slight time interval $dt$. Density $k$ reflects the number of vehicles ($m$) per kilometer of the road, in which it was measured over space with $\Delta X$ length at any time instant ($t_1$). In this measurement interval, density $k$ is formulated as the Equation (2):

$$k = \frac{m}{\Delta X}$$  \hspace{1cm} (2)

The definition of density can be expanded into time-space $S$ measurement intervals. In the rectangular measurement interval over time $\Delta T$ and space $\Delta X$, the generalized density was defined as the Equation (3):

$$k = \frac{m \cdot dt}{\Delta X \cdot dt} = \frac{Total \ time \ spent \ by \ all \ vehicles \ in \ S}{Area(S)} = \frac{\sum t_i}{\Delta X \cdot \Delta T}$$  \hspace{1cm} (3)
Where, \( t_i \) – occupied time or time spent by vehicle \( i \) on the measurement interval.

In \( S_2 \) measurement interval, the definition of flow can be obtained. This rectangular measurement interval represents a slight road length \( dx \) during a time interval \( \Delta T \). Flow rate \( q \) represents the number of vehicles (\( n \)) that pass a certain cross-section per time unit. Flow was measured over time with time interval \( \Delta T \) at any location \( (x_2) \). In this measurement interval, flow rate \( q \) is formulated as the Equation (4):

\[
q = \frac{n}{\Delta T}
\]

By expanding the meaning into time-space \( S \) measurement intervals, the generalized flow was obtained as the Equation (5):

\[
q = \frac{n \cdot dx}{\Delta T \cdot dx} = \frac{\text{Total distance covered by all vehicles in } S}{\text{Area}(S)} = \frac{\sum_i d_i}{\Delta T \cdot \Delta X}
\]

Where, \( d_i \) - the distance covered by vehicle \( i \) on the measurement interval.

Then, having known two previous variables, the third variable (speed, \( u \)) can be derived by using the identity relation. In this perspective, the speed \( u \) was defined as the quotient of the flow rate \( q \) and the density \( k \). So, it was clear that the speed variable should comply with the identity relation.

2.2. The Index of Traffic Flow
According to the previous definition, the flow was measured and represented in the homogeneous vehicle unit index (vehicle unit/time unit) either measurement over time or time-space. In this flow variable, the traffic was considered under homogeneous condition. It means the entity was composed by the similar vehicles. However, the real traffic will never be in such a condition. Realizing the existence of vehicle variance in the traffic entity, a new index was developed to understand and deal with this condition. In Highway Capacity Manual 1965 [13], concept of passenger car equivalent (PCE) was first specifically discussed significantly as a measure index to represent the intensity of traffic. At the first, this concept classified the traffic entity into some types, which has a similar characteristic on speed. The term of “PCE” or also known as “PCU” was defined as the number of passenger cars displaced in the traffic flow by a truck or a bus under the prevailing roadway and traffic condition. To express the flow rate, all the vehicle in the traffic will be converted into a homogeneous equivalent number of passenger car by the Equation (6).

\[
q = \frac{\sum_i (a_j \cdot n_j)}{\Delta T}
\]

Where, \( a_j \) - passenger car equivalent of a vehicle type \( j \); \( n_j \) - number of vehicle category \( j \) that pass a cross section during time interval \( \Delta T \).

PCU method is widely used for modeling the heterogeneous traffic in developing countries. However, this method has a significant limitation. In Indonesian Highway Capacity Manual 1997 [14], the traffic entity was simplified into three types such as motorcycle, light vehicle and heavy vehicle. In this concept, this simplification is the advantage and the weakness. The analysis result by simplifying the traffic will deviate from the true condition. Number of vehicle category and equivalent value for each category will determine the accuracy of the method. So, it makes the method cannot apply in general due to the problem of setting PCE value in which it is possible to take the different values in each region/country. This method cannot cover the development of the kind of vehicle without improvement of PCE value, so that the improvement of PCE value is required to obtain a better result. By considering the limitation of PCU method, the other concept was adopted to capture the existence of the vehicle variance by treating the traffic entity as an individual vehicle. In the previous study [15], this concept measures the flow rate by adopting the dimension of vehicle or vehicle size, which obtained by the projected area of the vehicle in \( \text{m}^2/\text{time unit} / \text{road} \). In this study,
adopting “vehicle size” as an index of flow variable will be measured per whole width of the road by following the expression as the Equation (7).

\[ q = \frac{\sum a_i}{\Delta T}, a_i = w_i \cdot l_i \]  

(7)

Where, \( a_i \) - projected area of vehicle \( i \) that passes a certain cross-section during time interval \( \Delta T \) (m²); \( w_i, l_i \) - width and length of the vehicle \( i \).

Adopting “vehicle size” to describe the flow variable in traffic flow analysis can distinguish the effect each kind of vehicle based on the vehicle size and can be applied generally. This index can distinguish the effect of the vehicle variance in dimension that would be different. So, flow variable will describe a different value for each kind of vehicle based on the projected area by measuring dimension each vehicle on the road. The term of “vehicle size” refers to the projected area of each vehicle that obtained by multiplying the dimension (width and length) of the vehicle.

2.3. The Index of Traffic Density

Some index of density variable was developed to capture and deal with the existence of vehicle variance in share of traffic. Density is one of the macroscopic traffic flow variables, which explains the extent of usage of road space by vehicles. Density is useful in getting an idea about how crowded a certain section of the road is. Since density is essentially defined as a spatial measurement, it is one of the most difficult quantities to obtain. So, it can be derived simply from the identity relation by analyzing the flow and speed variable in advance. Basically, density can be defined as vehicles per length unit of the road. By adopting PCU concept, density of a certain section of the road also can be defined over time and space. It was defined as pcu per length unit of the road by the Equation (8), where the notation of the space interval \( \Delta X \) was replaced by the length of the detection zone \( L \).

\[ k = \frac{\sum_i (a_i \cdot \sum_{t_i \in t_i} t_i)}{L \cdot \Delta T} \]  

(8)

Then, the new index to represent the density was developed in the measurement over time. The term of occupancy was introduced. It was developed to capture the length difference of the vehicle and make it easier to be obtained. The concept of occupancy is a function of speed and length of a vehicle. In this concept, occupied time was measured over a certain loop or a certain cross-section by using the additional device. The occupancy can be measured by using ultrasonic sensors, induction loops and image-processing techniques. Occupancy can be distinguished into two types. The first type, occupancy was defined as a non-dimensional variable that represents the percentage time of the road section (detection zone) which occupied by a vehicle over a given period. This occupancy was measured by using induction loops and can be formulated as shown in the Equation (9). The second type, occupancy was defined as a non-dimensional variable that represents the percentage time of the cross-section of the road which occupied by a vehicle over a given period. In this term of occupancy, the length of the detection zone \( L \) was ignored, and it was measured by ultrasonic sensors at certain cross-section and can be formulated in the Equation (10).

\[ \rho_1 = \frac{\sum_i O_{1,i}}{\Delta T}, O_{1,i} = \frac{l_i + L}{v_i} \]  

(9)

\[ \rho_2 = \frac{\sum_i O_{2,i}}{\Delta T}, O_{2,i} = \frac{l_i}{v_i} \]  

(10)

Where, \( \rho_{1,2} \) - occupancy; \( O_{1,i} \) - the time the vehicle \( i \) occupied the detection zone or road section; \( O_{2,i} \) - the time the vehicle \( i \) occupied the detection line or cross-section of road; \( l_i \) - the length of the vehicle \( i \); \( v_i \) - speed of vehicle \( i \); \( L \) - length of the detection zone; \( \Delta T \) - time interval.

The first type of method was not proper to be said as a measurement over time because it considers a space interval. In the equation, it seems to overestimate the occupied time in the detection zone by
considering the length of the vehicle. So, its value will be changed by varying the length of the detection zone. However, in the traffic with different vehicle lengths, it is apparent that both occupancy methods are more meaningful than density (in homogenous traffic). Even though, the concept of occupancy is considered not proper for mixed or heterogeneous traffic as no-lane based traffic flow. The main reason is both density and occupancy developed for lane based traffic to represent the traffic density variable in homogeneous traffic.

In heterogeneous or mixed condition, the width of the vehicle was considered to affect the traffic. Instead of length difference, the variation of the projected area of a vehicle was more meaningful to be considered in this condition. As no-lane based traffic, the width of the vehicle can affect the performance of traffic. The index of traffic density variable was developed to capture the variance of the vehicle projected area (length and width). The term of “area occupancy” was proposed in the earlier study [11,12]. It was considered more suitable to represent the density variable in the mixed traffic condition. Area occupancy considers the horizontal projected area of the vehicle as the basis without any restriction on the length of detection zone and width of the road [16]. It was treating the detection zone which analyzed per length and width unit of the road. Area occupancy can be defined as a ratio of the detection zone area (in terms of time) that is occupied by vehicles. The concept of area occupancy as an index of traffic density variable in time-space measurement can be seen in the Equation (11) and (12).

\[
\rho_a = \frac{\sum_{i=1}^{N} \frac{L - x_i}{v_i} \times w_i \times l_i}{T \times W \times L} \tag{11}
\]

\[
\rho_a = \frac{\sum_i (a_i \cdot t_i)}{A \cdot \Delta T} \tag{12}
\]

Where, \(\rho_a\) - area occupancy on the observed road over time interval \(\Delta T\); \(A, W, L\) – area, width and length of the road section under consideration; \(x_i\) - denotes the distance between the front-end of the vehicle \(i\) and any of the two ends of the detection zone; \((L - x_i)\) - denotes the distance traveled by the vehicle \(i\) over the road section under consideration; \(a_i, w_i, l_i\) - projected area, width and length of vehicle \(i\); \(t_i\) - occupied time of vehicle \(i\) in the detection zone during the time interval \(\Delta T\); \(\Delta T\) - time interval.

In the Equation (11), \((L - x_i)/v_i\) represents the time spent by each vehicle \(t_i\) over the road section under consideration. Since the difficulties to consider the variable of \((L - x_i)\) in the measurement, the area occupancy was measured by the temporal measurement [11] as seen in the Equation (13).

\[
\rho_a = \frac{\sum_{i=1}^{N} l_i}{T \times W} \tag{13}
\]

In the Equation (12), the concept of area occupancy was formulated over time and space by the following definition [12]:

1. Area occupancy will be unchanged for any length of the detection zone. This result can be obtained by two conditions. The first condition: modeling the vehicle under stationary and homogeneous condition where the vehicles on this road share the same speed and traveling with constant speeds over the detection zone. The second, all the vehicles have traversed the entire detection zone during the observation period. So, at the beginning and at the end of the observation period there are no vehicles located in the detection zone.

2. Area occupancy considers the horizontal projected area of the vehicle as the basis without any restriction on the length of detection zone and width of the road (treating the whole of the width of the road as a single unit without consideration of traffic lanes).
3. Methodology
In this study, the proper index of flow and density will be investigated in the mixed traffic. The indices of traffic flow and density will be varied by the concept and the measurement interval. The concept was distinguished according to the respond to the variance of vehicles in the traffic analysis which measure over time and time-space. At the first, flow was measured over time, and density was measured over space. Measuring traffic over time is easier to be conducted rather than the measurement which considers a space interval, mainly over a long section of the road. So, traffic flow is relatively easier to be measured than density. By the difficulties, the density was proposed to be measured over time or time-space with a short space interval. The traffic flow and density will be analyzed on the observed traffic data in the mixed condition. The flow and density are measured over time and time-space in the detection zone which considers a short section of a road as seen in the Figure 2.

![Figure 2. Detection zone of observation area.](image)

The flow and density variables which measured over time are based on the second line of the detection zone. In order to define the flow and density, the front end of the vehicle is used as a reference point to establish the trajectory which will be analyzed within each time interval. Taking the measurement of density and flow over time and space requires defining the occupied time $t_i$ and the distance covered by vehicle $d_i$. Both variables in the time-space interval can be described in Figure 3.

![Figure 3. Occupied time over time and space.](image)
According to the previous illustration in the Figure 3, the occupied time $t_i$ and the distance covered by vehicle $d_i$ relate each other. Between these variables, the occupied time $t_i$ was easier to be defined than the distance $d_i$ which also denoted by $(L - x_i)$ as seen in the Equation (11). The occupied time $t_i$ will be used to obtain the density variable for all indices in the time-space measurement. It will be defined by knowing the period when the vehicle was at the first and the second line by the Equation (14):

$$t_i = t_{out} - t_{in}; \quad t_{in} = \max(t_{i,1}, T - \Delta T); \quad t_{out} = \min(t_{i,2}, T)$$  \hspace{1cm} (14)

Where, $t_i$ - occupied time of vehicle $i$ in the detection zone during the time interval $\Delta T$; $t_{i,1}, t_{i,2}$ - time when the trajectory line of vehicle $i$ get the 1st and the 2nd line of detection zone.

Then, the variable of the distance covered by vehicle $d_i$ can be defined. In time-space measurement, the distance covered by vehicle $d_i$ was based on the occupied time $t_i$ in the detection zone, which used to obtain the flow for all indices. By assuming the speed of the vehicle $v_i$ is constant over the space interval, the distance covered by vehicle $d_i$ can be formulated as the Equation (15), in which occupied time $t_i$ refers to the Equation (14).

$$d_i = v_i \cdot t_i$$  \hspace{1cm} (15)

The index of flow and density was developed to capture the existence of vehicle variance in share of traffic. In the development of this index, the important thing which should be noticed is showing whether this variance needs to be considered or can be ignored in the traffic analysis. The effect of vehicle variance on the traffic analysis at macroscopic perspective can be captured by treating the vehicle as an individual or simplifying them into several groups by using PCU concept in the index flow and density. By direct observation, the existence of vehicle variance was easier to be identified. The difference of the vehicle in the properties such as width and length is the clear evidence to show the existence of vehicle variance. It becomes a basis to develop the index of flow and density which able to show the respond of traffic. These indices will be used to identify the existence of vehicle variance and its effect analytically. Responding to the existence of vehicle variance can be obtained by comparing using the index of flow and density, which considers and not considers the variance of the vehicle in the traffic analysis.

In the basic relation, traffic flow and density are described in the Fundamental Diagram by the term of the density-flow plane. Generally, traffic flow variable can be explained by using different index of density. Flow was expressed by the index of vehicles per time unit, and density was expressed by the index of vehicles per length unit. In the mixed traffic, both flow and density were adjusting to being measured per whole road instead of per lane. In two-dimensional diagram, traffic flow and density are described on the y-axis and x-axis respectively. The relations of both variables are required to comply with the identity relation of Fundamental Diagram. In such case, the gradient of each point on the diagram has to show the speed of traffic. By keeping this rule, density also can be transformed into the other term which measured over time. It can be derived from the generalized density definitions by ignoring the length of the detection zone $L$ as a space interval $\Delta X$. By assuming the length of section was very small, the distance covered for each vehicle will be equal $d_i \approx L$. So, density over time can be expressed as the Equation (16).

$$k = \sum_i \frac{d_i}{\Delta X \cdot \Delta T} = \frac{\sum_i \left( \frac{1}{v_i} \right)}{L \cdot \Delta T} = \frac{\sum_i \frac{1}{v_i}}{\Delta T}$$  \hspace{1cm} (16)

Where, $k$ - density; $t_i$ - occupied time of vehicle $i$ in the detection zone during the time interval $\Delta T$; $v_i$ - speed of vehicle $i$.

The principal to derive the Equation (16) shows the relation between the Equation (11) and (13) which shown in the previous discussion. Basically, all developed indices which mention on the
previous discussion can be used to describe the flow and density variable. However, the combination of traffic flow and density should be selected carefully to comply with the fundamental relation in Equation (1). At the first, both of flow and density were developed by adopting the passenger car unit (PCU) concept. It was also measured per whole road. On this concept, the relation between flow and density still complies with the fundamental relation. Traffic flow in this concept was expressed by the index of PCU per time unit, and density was expressed by the index of PCU per length unit. In this study, the PCU concept refers to the Indonesian Highway Capacity Manual 1997 [14] by the PCE value and number of vehicle category in the Table 1.

| No. | Type of Vehicle | Passenger Car Equivalent Value |
|-----|-----------------|--------------------------------|
| 1   | Motorcycle      | 0.4                            |
| 2   | Light Vehicle   | 1                              |
| 3   | Heavy Vehicle   | 1.3                            |

By adopting PCU, flow can be expanded into the measurement over time and space by determining a length of the detection zone $L$ as the Equation (17).

$$ q = \frac{\sum_j (a_j \cdot \sum_{i=j}^j d_i)}{L \cdot \Delta T} $$  \hspace{1cm} (17)

Where, $d_i$ – the distance covered by vehicle $i$ in the detection zone during the time interval $\Delta T$.

In addition, adopting PCU in density can be expressed on the measurement over time. It can be derived from the Equation (8) by eliminating the length of the detection zone $L$ as the Equation (16), in which it can be formulated as the Equation (18).

$$ k = \frac{\sum_j (a_j \cdot \sum_{i=j}^j (1/v_i))}{\Delta T} $$  \hspace{1cm} (18)

On the further development, traffic density was measured over time by considering the variance of vehicle length, which known by the term of occupancy. In this study, the second occupancy was more appropriate to be used. It can be explained by expanding the meaning into time-space intervals. The definition can be derived by considering the length of the detection zone $L$ as a space interval into the Equation (10) as the Equation (5), which can be expressed as seen in the Equation (19).

$$ \rho = \frac{dx \cdot \sum_i \left( l_i/v_i \right)}{dx \cdot \Delta T} = \frac{\sum_i \left( l_i \cdot d_i/v_i \right)}{L \cdot \Delta T} = \frac{\sum_i (l_i \cdot t_i)}{L \cdot \Delta T} $$  \hspace{1cm} (19)

Where, $\rho$ - occupancy; $l_i$ - the length of the vehicle $i$.

The definition of occupancy in time-space is similar with density by considering the difference of vehicle length. To adopt this occupancy either over time or time-space in the Fundamental Diagram, the flow also should be expressed by considering the difference of vehicle length as shown in the Equation (20) and (21).

$$ q = \frac{\sum_i l_i}{\Delta T} $$  \hspace{1cm} (20)

$$ q = \frac{\sum_i (l_i \cdot d_i)}{L \cdot \Delta T} $$  \hspace{1cm} (21)
In the mixed condition which no-lane based traffic, the index of density was developed by considering the projected area of the vehicle. This measure refers to the two terms namely area occupancy and areal density. Understanding this concept can be explained by the illustration in the Figure 4.

In the Figure 4, the occupied time $t_i$ of each vehicle can be measured by two reference points of the trajectory: the front end and the rear end of the vehicle. So, this occupied time can be defined as the time difference to get the exit and the entry point or the time difference to leave the exit and the entry point. Area occupancy can be measured over time by local measurement, over space by instantaneous measurement, and measured over time and space. In the homogeneous and stationary condition, the concept of area occupancy is not restricted by the length of the detection zone, which mean the area occupancy will not differ by the change of the detection length. In the local or over time measurement, the length of detection zone $L$ as a space interval will be ignored. It can be assumed that the length of section was very small. So, area occupancy over time as seen in the Equation (22) can be derived from Equation (12) by the same process to obtain the Equation (16).

$$\rho_a = \frac{\sum_i (a_i \cdot t_i)}{A \cdot \Delta T} = \frac{\sum_i (a_i \cdot d_i / v_i)}{W \cdot L \cdot \Delta T} = \frac{\sum_i (a_i \cdot L / v_i)}{W \cdot L \cdot \Delta T} = \frac{\sum_i (a_i / v_i)}{W \cdot \Delta T}$$ \hspace{1cm} (22)

Where, $\rho_a$ - area occupancy on the observed road over time interval $\Delta T$; $W, L$ - width and length of detection zone; $w_i, l_i$ - width and length of the vehicle $i$; $v_i$ - speed of vehicle $i$; $\Delta T$ - time interval.

In the instantaneous measurement, area occupancy was measured over space at a time instant, in which time interval will be ignored. It also can be assumed that the time interval was very small. So, the occupied time for each vehicle will be equal $t_i \approx \Delta T$. In the measurement over space, area occupancy can be derived from Equation (12) as seen in the Equation (23).

$$\rho_a = \frac{\sum_i (a_i \cdot t_i)}{A \cdot \Delta T} = \frac{\sum_i a_i}{A}$$ \hspace{1cm} (23)

By this equation, area occupancy over space can be defined as the ratio of the total projected area of vehicles to the area of roadway. This definition was similar with the other term of index traffic density variable namely areal density [10]. So basically, the term of area occupancy leads to the extended definition of areal density, which measured over time and space. Area occupancy over space
or time-space was easier to be measured on a short section of the road as the detection zone by knowing the time at the entry point and the exit point. However, it was difficult and needs more effort to measure area occupancy over the long section of the road.

To adopt this measure either over time or time-space in the Fundamental Diagram, the flow should be expressed by considering the projected area of the vehicle as shown in the Equation (7). This definition of flow can be extended into the time-space measurement by the Equation (24).

$$q = \frac{\sum_i (a_i \cdot d_i)}{L \cdot \Delta T}$$  \hspace{1cm} (24)

By the equation, area occupancy or areal density are measured per width unit of the road and flow is measured per whole road for each direction. The combination of this index can be used by transforming one of them into the same treatment of the width of the road. In this study, area occupancy or areal density either over time or time-space will be transformed. By considering the width of the road \( W \) as one, Equation (12) and (22) can be rewritten as seen in the Equation (25) and (26).

$$\rho_a = \frac{\sum_i (a_i \cdot t_i)}{A \cdot \Delta T} = \frac{\sum_i (a_i \cdot t_i)}{W \cdot L \cdot \Delta T} = \frac{\sum_i (a_i \cdot t_i)}{L \cdot \Delta T}$$  \hspace{1cm} (25)

$$\rho_a = \frac{\sum_i (\frac{a_i}{v_i})}{W \cdot \Delta T} = \frac{\sum_i (\frac{a_i}{v_i})}{\Delta T}$$  \hspace{1cm} (26)

Where, \( \rho_a \) - area occupancy on the observed road over time interval \( \Delta T \); \( W, L, A \) - width, length and area of detection zone.

In this study, all the combination of traffic flow and density will be applied and evaluated on the Fundamental Diagram to describe the actual traffic state in the density-flow plane. The combinations that will be analyzed are summarized in the Table 2.

### Table 2. The index of traffic flow and density.

| No. | Concept and Measurement Interval | Flow Variable | Density Variable |
|-----|---------------------------------|---------------|-----------------|
| 1   | Vehicle Unit - Over Time        | \( q_{1a} = \frac{n}{\Delta T} \) | \( k_{1a} = \frac{\sum_i (\frac{1}{v_i})}{\Delta T} \) |
| 2   | Vehicle Unit - Over Time and Space | \( q_{1b} = \frac{\sum_i d_i}{L \cdot \Delta T} \) | \( k_{1b} = \frac{\sum_i t_i}{L \cdot \Delta T} \) |
| 3   | Passenger Car Unit - Over Time  | \( q_{2a} = \frac{\sum_j (a_j \cdot n_j)}{\Delta T} \) | \( k_{2a} = \frac{\sum_j (a_j \cdot \sum_{i\neq j} (\frac{1}{v_i}))}{\Delta T} \) |
| 4   | Passenger Car Unit - Over Time and Space | \( q_{2b} = \frac{\sum_j (a_j \cdot \sum_{i\neq j} d_i)}{L \cdot \Delta T} \) | \( k_{2b} = \frac{\sum_j (a_j \cdot \sum_{i\neq j} t_i)}{L \cdot \Delta T} \) |
| 5   | Vehicle Length Unit - Over Time | \( q_{3a} = \frac{\sum_i l_i}{\Delta T} \) | \( k_{3a} = \frac{\sum_i \left( \frac{l_i}{v_i} \right)}{\Delta T} \) |
| 6   | Vehicle Length Unit - Over Time and Space | \( q_{3b} = \frac{\sum_i (l_i \cdot d_i)}{L \cdot \Delta T} \) | \( k_{3b} = \frac{\sum_i (l_i \cdot t_i)}{L \cdot \Delta T} \) |
Table 1. The combination of the index of traffic flow and density can be distinguished into four different concept as follows:

- The first concept is the combination which not considers the existence of vehicle variance, so all vehicles are considered similar and it will be indexed by vehicle unit.
- The second concept is the combination which considers the existence of vehicle variance by simplifying the traffic entity into some categories. This combination adopts the concept of the passenger car unit.
- The third concept is the combination which considers the variance of vehicle length, so the vehicle will be indexed by vehicle length unit (m$^1$).
- The fourth concept is the combination which considers the variance of the projected area of the vehicle, so the vehicle will be indexed by vehicle area unit (m$^2$).

All the combination will be plotted in a two-dimensional diagram which describes the relation between flow and density. In addition, the combination of index of flow and density will be modeled by a polynomial of degree two equations to describe the actual traffic state. The goodness of fit on this model is measured based on the R-square value. These values will be a basis to evaluate the model, which represent each combination. The R-square value of each model will be compared to obtain the effect of vehicle variance in the estimation of actual traffic state. Then, the proper index of flow and density will be established based on the highest R-square of the model.

4. Observation Data

In the present study, the traffic in Makassar will be observed. It was selected due to the characteristic of the traffic in Makassar classified as the mixed traffic. The traffic was observed at the morning peak hours. The observed location was a main road, unaffected by major intersection, has a separator direction and having a good view for video recording. The observation was conducted on 20$^{th}$ September 2013 at 08.30-11.30 a.m. in sunny conditions. The traffic was observed by video recording at the pedestrian bridge of Urip Sumoharjo Street. Data for each vehicle will be collected during the observation period. These individual data consist of the projected area as representation of the vehicle dimension (width and length), the period when the vehicle at the first line $t_{1,1}$ and at the second line $t_{1,2}$. The video data will be extracted into image data for 10 frames per second. By using a computer programming which has developed, all these image data will be analyzed. The required time data will be tracked for each vehicle. In addition, the dimension for each vehicle can be obtained by collecting the image coordinates for the specific part of each vehicle. At this stage, the human resources as the operator still require for tracking each vehicle by clicking the pointer manually. Then, the coordinate of image will be converted to get the real coordinate by projective transformation. The method which called a numerical rectification will be used to transform the coordinate. This approach was also used in several studies to transform the screen coordinates to be a real coordinates [17–20].

5. Result and Analysis
In this study, all the raw data of each vehicle will be analyzed to derive the variable of traffic flow and density. The aggregate data of traffic flow and density will be analyzed for every minute. By this time interval, there are 148 observed data of density and flow, which have been collected during three-hour observation. The remaining data were not used due to the error at image processing stage. Briefly, the actual traffic state can be known by plotting the flow and density in the two-dimensional diagram. By using the first index in the Table 2, the relation between flow and density can be shown in the Figure 5. The first index of flow and density is expressed by vehicle unit, which measured over time.

![Figure 5. Density-flow plane.](image)

In the Figure 5, the actual traffic state during the observation period shows that almost the observed traffic was on the uncongested phase. There is just a few sample on the congested phase. Therefore, the model should be applied only on the uncongested phase. The present study emphasizes to the comparison of several indices of flow and density to estimate the actual traffic state. The relation between flow and density was modeled on parabolic equation by using regression analysis method. By using different index of traffic flow and density, it will produce the different regression model. There are eight combinations of the index of traffic flow and density by variation on the concept and the measurement interval. All the indices will be used to model the actual traffic state. The analyzed regression model for each combination can be seen in the Table 3.

| No. | Combination of $q - k$ | Measurement Interval | Coefficient $k$ | $k^2$ | R | R-Square |
|-----|------------------------|----------------------|----------------|-------|---|----------|
| 1   | $q_{1a} - k_{1a}$      | Over Time            | 527.75         | -432.13 | 0.62 | 0.39     |
| 2   | $q_{1b} - k_{1b}$      | Over Time and Space  | 527.77         | -432.40 | 0.62 | 0.39     |
| 3   | $q_{2a} - k_{2a}$      | Over Time            | 541.89         | -957.03 | 0.66 | 0.44     |
| 4   | $q_{2b} - k_{2b}$      | Over Time and Space  | 542.29         | -960.25 | 0.66 | 0.44     |
| 5   | $q_{3a} - k_{3a}$      | Over Time            | 525.74         | -228.02 | 0.72 | 0.52     |
| 6   | $q_{3b} - k_{3b}$      | Over Time and Space  | 526.12         | -228.79 | 0.72 | 0.52     |
| 7   | $q_{4a} - k_{4a}$      | Over Time            | 517.49         | -220.93 | 0.79 | 0.62     |
| 8   | $q_{4b} - k_{4b}$      | Over Time and Space  | 518.40         | -222.55 | 0.79 | 0.63     |

Number of Observation: 148
In the Table 3, the comparison between the concepts which adopts different combination can be evaluated. This table shows the equation and the goodness of fit each model to estimate the actual traffic state. The purposes of this study can be obtained by comparing the goodness of fit all the models, which represent its own concept. The 1st and 2nd model describe the index of flow and density, which not consider the existence of vehicle variance. On this concept, all vehicles are considered equal. It will be set as a basis for comparison to the other concept. Both models as the representation of the first concept show the similar R-square: 0.39. The second concept which represented on the 3rd and the 4th model adopts the passenger car unit concept as an index of the traffic flow and density. This concept simplifies the entity of traffic into some categories based on the physical and the difference of speed. In this concept, the simplification makes the result deviates from the true condition. The accuracy of methods will depend on how well this simplification to represent the variance of the vehicle that exists in the share of traffic. Both models which consider this variance show a better R-square value: 0.44. It was increased as compared with the first concept. In this analytical method, it is quite enough to show that the variance of the vehicle in share of traffic exists. So, the entity of traffic should be differentiated either individually or as a group on any basis which appropriate. The proper index of flow and density should cover the existence of vehicle variance, which possible to explain the traffic behavior well.

By using the individual characteristic as a basis to cover the variation in the share of traffic, two concepts are introduced: occupancy and area occupancy. In the concept of occupancy, the length of vehicles becomes a basis to distinguish the variance of traffic entity as an individual vehicle. This concept was represented by the 5th and the 6th model. The models on this concept show R-square value: 0.52. It shows the better value as compared with the first and the second concept. This result describes that the variation of traffic behavior can be explained more properly by the difference of vehicle length. It also shows that the PCU method which adopted from IHC, 1997 [14] was not enough to represent the variance of vehicles in the share of traffic. In the concept of area occupancy, the projected area of vehicles become a basis to explain the variance of the vehicle in share of traffic. The projected area is the function of width and length of the vehicle. This concept was represented by the 7th and the 8th model. The models on this concept show the improvement by the R-square value: 0.62 and 0.63 respectively. This result shows that the index of flow and density, which considers the projected area of vehicles can explain the actual traffic state more properly by obtaining the highest R-square values. So, instead of the vehicle length, projected area of the vehicle can be considered to explain the variation in the share of traffic flow by individual data.

In addition, the result in the Table 3 also shows the comparison between the measurements over time and time-space, which differentiated for each concept. This comparison was used to verify the result on both density and flow, which measured over time. Taking the measurement over time is more convenient than the measurement which involves the space interval. By considering this reason, variable density and flow are preferred to be measured over time. The result shows a similar result between measurement over time and time-space on short section of the road. It can be obtained by comparing the R-square models which measure over time and time-space for each concept. The comparison for each concept shows the similar of R-square value. The similarity of the result can be explained by two reasons. The length trap which considers as a detection zone for the measurement over time and space is too short. The other reason is the speed variable of each vehicle, which used for the measurement over time. This speed variable was measured on the same longitudinal trap or detection zone for the measurement over time and space. Finally, by the similarity on the result, analyzing the actual traffic state can rely on the both variable density and flow, which measured over time.

6. Conclusion
This study investigates the existence of vehicle variance and the proper index of traffic flow and density to capture its effect in the mixed condition. It was needed to consider any specific criteria to distinguish the vehicle in the share of traffic either as a group or as an individual. As the result of this
study, there were three different concept or criteria, which used to distinguish the vehicle: based on the group using PCU, the length of vehicle and projected area of the vehicle. By adopting these concepts as the index of flow and density, the model which obtained can estimate the actual traffic state well. It was analytically proof that the vehicle variance in the mixed traffic needs to be considered in the analysis. Among the three concept, the projected area of the vehicle, which used as a criterion to distinguish the vehicle can explain the variation of traffic behavior in the share of the road more properly. It also describes that the motions of the vehicle are affected by the dimension or the projected area of vehicles: length and width. Therefore, considering the projected area of the vehicle as the index of density and flow variable can produce a better model to estimate the actual traffic state on the Fundamental Diagram. Moreover, taking the measurement over time, and the measurement over time and space do not provide a significant difference on the traffic observation over a short section of a road.

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