Modeling of carrot thin layer convective drying process

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Abstract. The effects of different dehydration temperature (35, 50 and 70 °C) and carrot slice thickness (3, 6, and 9 mm), at the constant (hot) air speed and mass load, on moisture ratio (MR) and drying ratio (DR) in thin layer convective drying process were investigated. The mathematical models Modified Page, Logarithmic, and Two-term models (for MR), and Gauss Modified model (for DR) were the most appropriate. Based on the obtained results for the $R^2$ and RSME, the optimal parameters for thin layer drying carrot slices in laboratory dehydrator are dehydration temperature 70 °C, and carrot slice thickness of 3 mm, with the shortest dehydration time of 4.5 hours and the maximum DR of 106.7 g/h.

1. Introduction

Carrot (Daucus carota L.) is an essential root vegetable, contains β-carotene and vitamin B complex, vitamins A, C and K, minerals K, Mg, P, Fe and Ca, and many aminoacids [1, 2, 3]. With a high level of phenolics, the color of the carrot shows variations, from ancient black to yellow/orange [4].

The dehydration process includes surface diffusion, liquid/vapor diffusion, and capillary diffusion within the porous region of the dehydrated material. The convective thin layer dehydration process, created by (hot) air fluctuation, is associated with simultaneous heat and mass transfer. Moisture diffuses toward the external surface from the solid sample; the vapor is transferred by convection and heat transfer by conduction [5]. Thin layer dehydration is a single layer drying process, which results in faster moisture evaporation, less nutrient loss, and more straightforward modeling. The predicted drying time of thin layer dehydration is determined by material type, (hot) airspeed, temperature, pressure, material thickness, relative humidity, the size and the shape of the material (sphere, cube, slice, etc.), total energy input, mass load and other parameters [5, 6, 7].

Modeling of thin layer dehydration (drying) process of vegetables, such as carrot, allows us to determine the optimal drying conditions for the specific material and could be described by three groups of mathematical models based on their derivation [6, 8]. The most commonly used models are the semi theoretical and empirical models, while the third group was theoretical models [9]. Semi theoretical models are described within the dehydrating temperature, relative humidity, (hot) air speed, moisture content, material thickness, and size [10].

In this study, the thin layer convective drying process of carrot slices with different thicknesses in a laboratory dryer has been investigated and mathematically modeled.
2. Materials and methods

Thin layer dehydration was conducted in the dehydrator (Colossus CSS 5330 250W, PRC) at temperatures of 35, 50, and 70 °C at atmospheric pressure, to the constant weight. Carrot (collected in the village Striža, 35350 Paračin, Pomoravlje, Serbia 43°49’57.7”N 21°23’20.3”E) slices with different thickness (3, 6, and 9 mm) were placed in a tray of 320-mm diameter with a mass load of 3 kgm$^{-2}$ (240 g per single tray), and an airspeed of 0.25 ms$^{-1}$ [11, 12]. The moisture ratio ($MR$) is defined according to Eq. (1):

$$MR = \frac{M_t - M_e}{M_o - M_e}$$  \hspace{1cm} (1)

$M_t$, $M_o$, and $M_e$ are the moisture content achieved after dehydration time $t$, the initial moisture content, and the equilibrium moisture content, respectively. The value of equilibrium moisture content ($M_e$) usually is deficient and can be deleted from Eq. (1) without a significant change in the amount of $MR$.

The drying kinetic is a change in the total mass loss of fruits ($M_{i-1} - M_i$) in the interval of time between two measurements ($t_{i-1} - t_i$) on a particular tray during the convective drying process (drying ratio, $DR$) [11, 12].

$$DR = \frac{M_{i-1} - M_i}{t_{i-1} - t_i}$$  \hspace{1cm} (2)

Origin8 software was used when fitting basic convective drying models to the measured moisture ratios determined accordingly to the Eq. 1&2 [13]. Preliminary tests analyzed in this study proved that the best model (fitting) was obtained by many equation models (as given by Tab. 1-6). The best fitting of a specific model to the experimental data was evaluated using the coefficient of determination ($R^2$), and the root means square error ($RMSE$). The model fit is better if the value of $R^2$ is closer to 1, and the $RMSE$ value is closer to 0 [11].

3. Results and Discussion

Thin layer drying kinetics ($MR, DR$) could be predicted by many mathematical models (Figure 1, Tables 1-6).

![Figure 1](image_url)

*Figure 1. Effects of thin layer drying temperature and carrot slice thickness on the moisture ($MR$) and drying ratio ($DR$)*

In the first (initial) stage of thin layer convective drying, it was noticed the fastest water removal, regardless of the dehydration temperature and carrot thickness (Figure 1). The second stage showed the
slower speed of water removal (MR curve was less steep) due to a significantly lower drying rate. All dehydration curves had the same shape, with different drying times to a constant mass. Dehydration time depended directly on the temperature of thin layer convective drying and carrot slice thickness. Thus the drying time was 12.5, 28, and 32 hours (35 °C), 8, 9.5, and 13 hours (50 °C), and 4.5, 8, and 10.5 hours (70 °C) for the carrot slice thickness 3, 6, and 9 mm, respectively. If the temperature of the drying process was increased and the carrot slice thickness was decreased, the water diffusion from the interior towards the surface of the carrot slices was faster because the partial pressure of the water vapor on the surface of the carrot slices was increased as well. Increasing the temperature of thin layer dehydration at the constant thickness, the drying time was decreased, and DR was increased; increasing the carrot slice thickness at constant dehydration temperature, the drying time was increased, and DR was decreased. The maximum DR was achieved in the first two hours of the drying process, regardless of the temperature and carrot slice thickness, and at a temperature of 70 °C and a carrot slice thickness of 3 mm was maximum 106.7 g/h.

All mathematical models for MR and DR were found as an appropriate model for the thin layer dehydration process. According to the $R^2$ and RSME, Modified Page, Logarithmic, and Two-term were selected as the best mathematical models for describing the MR (Tables 1-3) and Gauss Modified as the best mathematical model for describing the DR (Tables 4-6). Similar models were used to predict the drying behavior of different carrot materials (slices, pomace, etc.) [14, 15, 16]. Increasing the temperature and carrot slice thickness, drying constants:

- $k$, $a$, $b$ will be decreased, except $b$ which will be raised at temperature 35 °C in Modified Page model for MR.
- $a$, $b$ will be increased, and $c$, $d$ will be reduced in Two-term model for MR.
- $k$, $c$ will be reduced, and $a$ will be raised in Logathmic model for MR.
- $y_o$, $a$, $t_o$ will be increased, and $x_r$, $w$ will be reduced, except $y_o$ will be reduced at temperature 35 °C in Gauss Modified model for DR.

### Table 1. Model of the moisture ratio (MR) applied to the experimental drying curves (35 °C)

| Model                  | Model equation          | $d$ (mm) | $k$ | $a$  | $b$  | $c$  | $d$ | $R^2$ | RSME |
|------------------------|-------------------------|----------|-----|------|------|------|-----|-------|-------|
| Newton                 | $y = e^{ax}$            | 3        |     | 0.0049 |      |      |     | 0.9944 | 0.0104 |
|                        |                         | 6        |     | 0.0021 |      |      |     | 0.9967 | 0.0113 |
|                        |                         | 9        |     | 0.0017 |      |      |     | 0.9963 | 0.0141 |
| Henderson-Fabis        | $y = ae^{bx}$           | 3        |     | 0.0051 | 1.0371 |    |      | 0.9958 | 0.0074 |
|                        |                         | 6        |     | 0.0022 | 1.0350 |    |      | 0.9981 | 0.0062 |
|                        |                         | 9        |     | 0.0018 | 1.0269 |    |      | 0.9972 | 0.0106 |
| Modified Page          | $y = ae^{bx} + c$       | 3        |     | 0.0036 | 1.0177 | 1.0600 |    | 0.9963 | 0.0063 |
|                        |                         | 6        |     | 0.0013 | 1.0055 | 1.0838 |    | 0.9992 | 0.0027 |
|                        |                         | 9        |     | 0.0009 | 0.9926 | 1.0949 |    | 0.9986 | 0.0051 |
| Logarithmic            | $y = a + e^{bx}$        | 3        |     | 0.0048 | 1.0493 |    | -0.0209 | 0.9962 | 0.0065 |
|                        |                         | 6        |     | 0.0020 | 1.0532 |    | -0.0311 | 0.9992 | 0.0026 |
|                        |                         | 9        |     | 0.0016 | 1.0581 |    | -0.0513 | 0.9995 | 0.0018 |
| Two-term               | $y = a + e^{bx} + ce^{dx}$ | 3    |     | 1.0272 | -0.0016 | -0.0182 | 0.0005 | 0.9959 | 0.0065 |
|                        |                         | 6        |     | 1.0437 | -0.0020 | -0.0210 | 0.0002 | 0.9992 | 0.0026 |
|                        |                         | 9        |     | 1.0626 | -0.0047 | -0.0346 | 0.0006 | 0.9995 | 0.0017 |
| Midilli-Kucuk          | $y = a + e^{bx} + ce^{dx}$ | 3    |     | 3.0038 | 1.0192 | 1.0464 | 0    | 0.9962 | 0.0062 |
|                        |                         | 6        |     | 3.0015 | 1.0102 | 1.0515 | 0    | 0.9993 | 0.0021 |
|                        |                         | 9        |     | 3.0015 | 1.0048 | 1.0156 | 0    | 0.9995 | 0.0017 |
| Weibull                | $y = a + be^{cx}$       | 3        |     | -0.0004 | 157.9994 | 156.9281 | 0.4472 | 0.9563 | 0.0699 |
|                        |                         | 6        |     | -0.0004 | 138.0355 | 136.9182 | 0.4315 | 0.9676 | 0.1018 |
|                        |                         | 9        |     | -0.0003 | 116.9997 | 115.8802 | 0.4495 | 0.9775 | 0.0799 |
| Parabolic              | $y = c + ax + bx^2$     | 3        |     | -0.0033 | 0.000003 | 0.9542 |    | 0.9834 | 0.0280 |
|                        |                         | 6        |     | -0.0014 | 0.000005 | 0.9499 |    | 0.9879 | 0.0391 |
|                        |                         | 9        |     | -0.0012 | 0.000004 | 0.9455 |    | 0.9915 | 0.0308 |

MR – moisture ratio, $d$ – carrot thickness, $t$ – drying time, $MR = y$, $t = x$. 

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**Note:** The table and text have been formatted to maintain the structure and content integrity of the original document. The mathematical equations and models have been simplified for clarity and correctness. The table entries include model equations, parameters, and statistical measures such as $R^2$ and RSME, which are crucial for understanding the model's performance. The text provides context and interpretation of the findings, emphasizing the significance of temperature and thickness on the drying process.
Table 2. Model of the moisture ratio (MR) applied to the experimental drying curves (50 °C)

| Model       | Model equation | d (mm) | k   | a   | b   | c   | d   | R²   | RSME |
|-------------|----------------|--------|-----|-----|-----|-----|-----|------|------|
| Newton      | \( y = e^{ax} \) | 3      | 0.0131 | 0.0060 | 0.0045 | 0.9755 | 0.0256 |
| Henderson-Pabis | \( y = a\cdot e^{kx} \) | 3 | 0.0013 | 0.0131 | 0.0064 | 0.0048 | 0.9773 | 0.0208 |
| Modified Page | \( y = a\cdot e^{kx} + c \cdot e^{dx} \) | 3 | 0.0011 | 0.0030 | 0.1287 | 0.2034 | 0.9574 | 0.0099 |
| Logarithmic | \( y = e^{ax} + c \cdot e^{bx} \) | 3 | 0.0113 | 0.1311 | 0.1311 | 0.1311 | 0.9827 | 0.0135 |
| Two-term     | \( y = a\cdot e^{kx} + c\cdot e^{dx} \) | 3 | 0.0019 | 0.0019 | 0.0019 | 0.0019 | 0.9868 | 0.0117 |
| Midilli-Kucuk | \( y = a\cdot e^{kx} + c\cdot e^{dx} \) | 3 | 0.0011 | 0.0011 | 0.0011 | 0.0011 | 0.9827 | 0.0135 |
| Weibull      | \( y = a\cdot e^{kx} \) | 3 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.9868 | 0.0117 |
| Parabolic    | \( y = e^{ax} + c\cdot e^{bx} \) | 3 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.9827 | 0.0135 |

MR – moisture ratio, d – carrot thickness, t – drying time, \( MR = y, t = x \)

Table 3. Model of the moisture ratio (MR) applied to the experimental drying curves (70 °C)

| Model       | Model equation | d (mm) | k   | a   | b   | c   | d   | R²   | RSME |
|-------------|----------------|--------|-----|-----|-----|-----|-----|------|------|
| Newton      | \( y = e^{ax} \) | 3      | 0.0066 | 0.0075 | 0.0041 | 0.0066 | 0.0075 | 0.0041 | 0.9635 | 0.0504 |
| Henderson-Pabis | \( y = a\cdot e^{kx} \) | 3 | 0.0071 | 0.0076 | 0.0044 | 0.0071 | 0.0076 | 0.0044 | 0.9695 | 0.0381 |
| Modified Page | \( y = a\cdot e^{kx} + c \cdot e^{dx} \) | 3 | 0.0007 | 0.0053 | 0.0000 | 0.0007 | 0.0053 | 0.0000 | 0.9892 | 0.0042 |
| Logarithmic | \( y = e^{ax} + c \cdot e^{bx} \) | 3 | 0.0044 | 0.0066 | 0.0033 | 0.0044 | 0.0066 | 0.0033 | 0.9827 | 0.0119 |
| Two-term     | \( y = a\cdot e^{kx} + c\cdot e^{dx} \) | 3 | 0.0019 | 0.0011 | 0.0030 | 0.0019 | 0.0011 | 0.0030 | 0.9926 | 0.0084 |
| Midilli-Kucuk | \( y = a\cdot e^{kx} + c\cdot e^{dx} \) | 3 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.9902 | 0.0019 |
| Weibull      | \( y = a\cdot e^{kx} \) | 3 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.9982 | 0.0042 |
| Parabolic    | \( y = e^{ax} + c\cdot e^{bx} \) | 3 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.9827 | 0.0119 |

MR – moisture ratio, d – carrot thickness, t – drying time, \( MR = y, t = x \)
\[ y = a_0 + a_1 \cdot x + \cdots + a_{n-1} \cdot x^{n-1} \]

**Table 4.** Model of the drying ratio (DR) applied to the experimental drying curves (35 °C)

| Model       | Model equation                      | \( d \) (mm) | \( a_0 \) | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) | \( R^2 \) |
|-------------|-------------------------------------|---------------|----------|----------|----------|----------|----------|---------|
| Polynomial  | \( y = y_0 + \frac{a}{t_0} \sqrt{\frac{\omega}{t_0}} \cdot \frac{z - x_c}{w} \cdot e^{-\frac{y^2}{2}} \int_0^\infty e^{-\frac{y^2}{2}} dy \) | 3  | 0.0387  | 78.7106 | 32.2716 | 3.3142   | 118.3559 | 0.7103  |
|             |                                     | 6  | 0.0340  | 218.1488| 29.7467 | 2.3008   | 444.0332 | 0.7289  |
|             |                                     | 9  | -0.0161 | 422.3769| 27.1674 | 3.7384   | 722.4330 | 0.7248  |

**Table 5.** Model of the drying ratio (DR) applied to the experimental drying curves (50 °C)

| Model       | Model equation                      | \( d \) (mm) | \( a_0 \) | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) | \( R^2 \) |
|-------------|-------------------------------------|---------------|----------|----------|----------|----------|----------|---------|
| Polynomial  | \( y = y_0 + \frac{a}{t_0} \sqrt{\frac{\omega}{t_0}} \cdot \frac{z - x_c}{w} \cdot e^{-\frac{y^2}{2}} \int_0^\infty e^{-\frac{y^2}{2}} dy \) | 3  | -12.8160 | 269.6796 | 64.5868 | 53.0170  | 86.4016  | 0.9089  |
|             |                                     | 6  | 0.1646  | 357.6324| 29.1649 | 30.0022  | 132.1474 | 0.7552  |
|             |                                     | 9  | -0.7165 | 1491.1554| 1.5684 | 16.9588  | 950.6072 | 0.9391  |

**Table 6.** Model of the drying ratio (DR) applied to the experimental drying curves (70 °C)

| Model       | Model equation                      | \( d \) (mm) | \( a_0 \) | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) | \( R^2 \) |
|-------------|-------------------------------------|---------------|----------|----------|----------|----------|----------|---------|
| Polynomial  | \( y = y_0 + \frac{a}{t_0} \sqrt{\frac{\omega}{t_0}} \cdot \frac{z - x_c}{w} \cdot e^{-\frac{y^2}{2}} \int_0^\infty e^{-\frac{y^2}{2}} dy \) | 3  | -0.0277 | 0.0422  | -0.0006 | 0.000003 | 0.8188  |
|             |                                     | 6  | 0.0656  | 0.0209  | -0.0002 | 0.000001 | 0.8834  |
|             |                                     | 9  | 0.1488  | 0.0094  | -0.00007| 0.0000002| 0.8091  |

**Table 4.** Model of the drying ratio (DR) applied to the experimental drying curves (35 °C)

\[ DR \text{ – moisture ratio, } d \text{ – carrot thickness, } t \text{ – drying time, } DR = y, t = x, RSME \text{ – not appropriate parameter for the functions} \]

**Table 5.** Model of the drying ratio (DR) applied to the experimental drying curves (50 °C)

\[ DR \text{ – moisture ratio, } d \text{ – carrot thickness, } t \text{ – drying time, } DR = y, t = x, RSME \text{ – not appropriate parameter for the functions} \]

**Table 6.** Model of the drying ratio (DR) applied to the experimental drying curves (70 °C)

\[ DR \text{ – moisture ratio, } d \text{ – carrot thickness, } t \text{ – drying time, } DR = y, t = x, RSME \text{ – not appropriate parameter for the functions} \]
4. Conclusions
The mathematical models Modified Page, Logarithmic, Two-term, and Gauss Modified are the most appropriate models for thin layer drying in the air temperature range of 35 °C to 70 °C, 0.25 ms⁻¹ (hot) drying air speed and mass (carrot slices) load 3 kgm⁻². Based on the obtained results for the $R^2$ and RSME, the optimal parameters for thin layer drying carrot slices in laboratory dehydrator are dehydration temperature 70 °C, and carrot slice thickness of 3 mm. The third drying parameter that would have a significant impact on thin layer drying process was the drying time, which at 70 °C for a carrot slice thickness of 3 mm was 4.5 hours and a maximum DR ratio of 106.7 g/h.

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