Adequate and fair explanations

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Abstract

Explaining sophisticated machine-learning based systems is an important issue at the foundations of AI. Recent efforts [Ribeiro et al., 2016; Ribeiro et al., 2018; Wachter et al., 2017; Ignatiev et al., 2019; Bachoc et al., 2018] have shown various methods for providing explanations. These approaches can be broadly divided into two schools: those that provide a local and human interpretable approximation of a machine learning algorithm, and logical approaches that exactly characterise one aspect of the decision. In this paper we focus upon the second school of exact explanations with a rigorous logical foundation.

There is an epistemological problem with these exact methods. While they can furnish complete explanations, such explanations may be too complex for humans to understand or even to write down in human readable form. Interpretability requires epistemically accessible explanations, explanations humans can grasp. Yet what is a sufficiently complete epistemically accessible explanation still needs clarification. We do this here in terms of counterfactuals, following [Wachter et al., 2017].

With counterfactual explanations, many of the assumptions needed to provide a complete explanation are left implicit. To do so, counterfactual explanations exploit the properties of a particular data point or sample, and as such are also local as well as partial explanations. We explore how to move from local partial explanations to what we call complete local explanations and then to global ones. But to preserve accessibility we argue for the need for partiality. This partiality makes it possible to hide explicit biases present in the algorithm that may be injurious or unfair. We investigate how easy it is to uncover these biases in providing complete and fair explanations by exploiting the structure of the set of counterfactuals providing a complete local explanation.

1 Introduction

Explaining the predictions of sophisticated machine-learning algorithms is an important issue for the foundations of AI. Recent efforts [Ribeiro et al., 2016; Ribeiro et al., 2018; Wachter et al., 2017; Ignatiev et al., 2019; Bachoc et al., 2018] have shown various methods for providing explanations. These approaches can be broadly divided into two schools: those that provide a local and human interpretable approximation of a machine learning algorithm, and logical approaches that completely characterise one aspect of the decision. In this paper we investigate a comparison between complete explanations and partial, epistemically accessible ones.

There is an epistemological problem with these complete methods. While they can furnish complete explanations, such explanations may be too complex for humans to understand or even to write down in human readable form. Interpretability requires epistemically accessible explanations, explanations humans can grasp. Yet what is a sufficiently complete or adequate epistemically accessible explanation still needs analysis. We provide such an analysis in terms of counterfactuals, following [Wachter et al., 2017].

With counterfactual explanations, many of the assumptions needed to provide a complete explanation are left implicit. To do so, counterfactual explanations exploit the properties of a particular data point or sample, and as such are also local as well as partial explanations. We explore how to move from local partial explanations to what we call complete local explanations and then to global ones. But to preserve accessibility we argue for the need for partiality. This partiality makes it possible to hide explicit biases present in the algorithm that may be injurious or unfair. We investigate how easy it is to uncover these biases in providing complete and fair explanations by exploiting the structure of the set of counterfactuals providing a complete local explanation.

To make the point about biases in counterfactual explanations concrete, consider the following scenario. An ML program judges A’s application for a loan. A is turned down. When A asks the bank for an explanation of the decision, the bank returns with the following.

(1) Your income is 50K euro per year.
(2) If your income had been 100K euro per year, you would have gotten the loan.
The counterfactual in (2) might be true but it also might be misleading, hiding a bias that one might find unfair. Suppose that (1)-(2) is the explanation the bank gives A. But suppose also that there is another more morally repugnant explanation: A black. If A had been white he would have gotten the loan with your current income of 50K per year. There’s also a question of features that indirectly code bias. For example, tying the loan availability to the postal code of A’s residence could be a way of encoding racial bias.

The problem of dissimulating a bias when giving an explanation is a direct outcome of the partiality of epistemically accessible explanations.

2 Background on explanations

Suppose that \( f : X^n \to Y \) is the “ideal” function taking data encoded in an n-dimensional space of features \( X^n \) into the representations in \( Y \) and that \( \hat{f} : X^n \to Y \) is the function that the algorithm has encoded. For this paper, we’ll assume that \( \hat{f} \) is some sort of classifier and thus we can assume \( Y \) to be a set of classes. We want to have an explanation of why \( \hat{f} \) outputs the predictions it does. We might want to know its behavior over the total space \( X^n \); this would be a complete explanation. But for many purposes, we might only need to know how \( \hat{f} \) behaves on a data point of interest or focal point, like A’s profile that was submitted to the loan program. Note that, we are implicitly assuming that \( \hat{f} \) is too complex or opaque for its behaviour to be analyzed statically. For instance, \( \hat{f} \) might be a neural network with multiple layers/parameters/variables etc. Or, it might be be case that we have access only to the binaries of \( \hat{f} \), from which the actual algorithm cannot be reverse-engineered.

There are two sorts of explanations of program behavior. Internal explanations involve the internal states of the program—if these are linked by logic then we can have a deductive explanation for a particular response in \( X^n \). However, there are also external explanations that involve linking features of \( X \) with output \( Y \) (we can also have a deductive link or something else). These are initially attractive because they do not involve unpacking the algorithms’ internal states and assigning them a meaning, which in the case of deep learning networks with multiple hidden layers can be a very complicated affair. Even partial explanations that exploit internal states of a complex neural architecture may be epistemically inaccessible.

[Ignatiev et al., 2019] provide a definition of complete explanations. They assume a classifier \( \hat{f} \) can be represented as a set of logic formulas, which we assume here too. In addition, we will assume that \( \hat{f} \) has a constant set of features with binary values making the encoding into logic transparent. For [Ignatiev et al., 2019], an explanation, or what we call and MS explanation, of a prediction \( \pi \) of a classifier \( \hat{f} \) given a feature space \( X \) is a subset minimal set of literals \( \mathcal{E} \) (each one describing a value of a feature in the problem space) such that

\[ \mathcal{E} \models \land \hat{f} \to \pi \]

where \( \land \) ranges over the set of formulas encoding \( \hat{f} \). \( \mathcal{E} \models \hat{f} \to \pi \) means that we can prove in the logic representation that \( \hat{f} \) predicts \( \pi \) given any instance with features as described in \( \mathcal{E} \). A complete explanation of the behavior of \( \hat{f} \) in our sense would be the set of all possible MS explanations.

An instance in such a set up is a set of literals that assigns values to every feature in the feature space. The underlying logical form of explanations discussed in [Ignatiev et al., 2019] thus exploit universal generalizations and a deductive consequence relation. The explanations in [Ignatiev et al., 2019] thus explain in principle sets of instances, and they are known as global explanations and are a version of a deductive nomological explanation, where a relation of entailment holds between the explanans and the explanandum.

Counterfactuals offer a natural way to provide epistemically accessible, partial explanations geared toward properties of individuals or focal points. Such explanations directed to a particular case are often called local explanations in contrast to global ones [Ribeiro et al., 2016; Ribeiro et al., 2018]. One reason for using counterfactuals in explanations is that counterfactuals lend themselves to an attractive analysis of causation, as [Lewis, 1973] proposed.

The reason why counterfactual explanations furnish natural candidates for partial epistemically accessible explanations is that they single out properties or features that would make a difference to a decision about an individual as in (2), other things being as equal as they can be given that the individual has the property described by the counterfactual’s antecedent. This ceteris paribus property of counterfactuals means that many factors that would be mentioned in a complete explanation can remain implicit. They are thus more partial than MS explanations.

The canonical semantics for counterfactuals symbolised via \( \Box \to \) as outlined in [Lewis, 1973] exploits a possible worlds model for propositional logic and crucially a similarity relation: \( A \Box \to B \) is true at world \( w \) just in case for all worlds \( w' \) in which \( A \) is true and that are the closest worlds to \( w \) in which \( A \) is true, \( B \) is true.

The similarity relation in Lewis’s semantics is used to model the complicated nature of causal laws, which are themselves often formulated with ceteris paribus assumptions. In particular, allows us to have consistent laws with conflicting consequents and antecedents ordered by entailment as in the following set of cascading counterfactuals:

(3) a. If I were making 100K euro or more, I would have gotten the loan.

b. If I were making 100K euro or more but were convicted of a serious financial fraud, I would not

Note however, such global generalizations may not offer a mathematical function or rule based reconstruction of \( \hat{f} \)’s behavior. Thus, global explanations may be extensional as in the case [Ignatiev et al., 2019] or intensional in the form of a rule reconstruction of \( \hat{f} \).
get the loan.

c. If I were making 100K euro or more and were convicted of a serious financial fraud but then the conviction was overturned and I was awarded a medal, I would get the loan.

In a cascading set of counterfactuals we can count how many times the value of the consequent changes as we move from one antecedent to a logically more specific one (e.g., does the prediction flip from $A$ to $A \land C$ or from $A \land C$ to $A \land C \land D$).

We will call the number of flips the degree of the cascading set. The counterfactual semantics with weak centering permits the counterfactuals in (3) to be satisfiable at a world without forcing the antecedents of (3)b or (3)c to be inconsistent. The reason for this is that strengthening of the antecedent fails for counterfactuals; the closest worlds in which I make 100k euro do not include a world $w$ in which I make 100k euro but am also convicted of fraud. Counterfactuals share this property with other conditionals that have been used as the basis for nonmonotonic reasoning [Ginsberg, 1986; Pearl, 1990]. However, if the actual world turns out to be like $w$, then by weak centering (3)a turns out to be false, because the ceteris paribus assumption in (3)a is that the actual world is one in which I’m not convicted of fraud.

In adapting counterfactuals to provide explanations of a learning algorithm’s behavior around a focal point, it is natural to interpret the similarity relation appealed to in the semantics of counterfactuals as a distance function over the feature space $X$ used to describe data points—in effect identifying the latter as the relevant “worlds” for the semantics of the counterfactuals to be defined over. To find the relevant counterfactuals to explain the behavior of $\hat{f}$ around a focal point $x_p \in \mathbb{R}^n$ where $X$ has $n$ dimensions, we exploit a linear map $\Delta_i : \mathbb{R}^n \rightarrow \mathbb{R}^n$ where $\Delta_i$ is defined such that there is a subset $\{i \in \{1,2,\ldots,n\} \}$ such that (i) for any $x \in X$, $\Delta_i(x)$ differs from $x$ only concerning values for dimensions in $i$, (ii) $\hat{f}(x_p) = \eta_i$, (iii) $\hat{f}(\Delta_i(x_p)) = \pi$, with $\eta, \pi$ two incompatible predictions in $Y$ and (iv) $\forall x \in X$ such that $\hat{f}(x) = \pi$ and $x$ differs from $x_p$ only on the values of dimensions $i$, $\|x - x_p\|_i \geq \|\Delta_i(x_p) - x_p\|_i$, where $\|\cdot\|_i$ is a natural norm on $X$ like the Euclidean norm.

Thus, $\Delta_i$ represents a minimal change in the features of $x_p$ (values of $x_p$ in the dimensions $i$) needed to shift the predictions of $\hat{f}$ to $\pi$ from the unwanted prediction $\hat{f}(x_p)$. Exploiting the translation from feature values to literals, the antecedents of our target counterfactuals will express these features as a conjunction of literals.

As discussed in [Kusner et al., 2017], these linear maps can be generated via techniques of adversarial perturbations. A typical definition of an adversarial perturbation of an image $x$, given a classifier, is that it is a smallest change to $x$ such that the classification changes. Essentially, this is a counterfactual by a different name. Finding a closest possible world to $x$ such that the classification changes is, under the right choice of distance function, the same as finding the smallest change to $x$ to get the classifier to make a different prediction. Adversarial perturbation has been the locus of a lot of recent research activity and can be computed quite efficiently [Dube, 2018]. Proofs for minimal perturbations can be found using optimal transport theory [Bachoc et al., 2018].

The fact that counterfactuals are closely tied to adversarial examples relative to some focal point invites a comparison to recent work by [Ignatiev et al., 2019] on explanations and adversarial examples in a deductive framework. The discussion in [Ignatiev et al., 2019] of counterexamples and adversarial examples builds a bridge between deductive nomological explanations and local counterfactual explanations based on a particular focal point. A counterexample to a prediction $\pi$ is a subset minimal set of literals $\mathcal{A}$ such that

$$\mathcal{A} \models \bigwedge \delta \rightarrow \bigvee_{\mathcal{A} \models \delta} \delta.$$  

[Ignatiev et al., 2019] show that counterexamples and explanations are incompatible in that every explanation and every counterexample to a prediction $\pi$ contain literals $e_i$ and $c_f$ such that $e_i$ is inconsistent with $c_f$. An adversarial example to a prediction $\pi$ for $x$ is then an instance that has all the features of a counterexample to $\pi$ and otherwise has the features of the instance $x$. An adversarial example for a learning model $\hat{f}$ thus is a closest element $y$ in $X$ to a focal point $x_p$, in which certain features are shifted so that $\hat{f}(y)$ is a different prediction from $\hat{f}(x)$. An adversarial example then can be defined in terms of a linear map $\Delta_0$ on $X$, and this map links the adversarial example with a counterfactual.

Counterexamples can also serve as the basis of explanations of properties of a focal element.

- Why not $\pi$ for $x$?
- $x$ has features of $\mathcal{A}$ and $\mathcal{A}$ is a counterexample to $\pi$ ($\mathcal{A} \models \hat{f} \rightarrow \neg \pi$).

Note that this deductive explanation is distinct from counterfactual explanations. When a counterfactual is used to give an explanation, the relationship between the explanans and the explanandum is not logical consequence but a more pragmatic relation based on a Lewisian analysis of causation. The counterfactual in (2) gives a sufficient reason for $A$’s getting the loan, all other factors of my situation being equal or being as equal as possible given the assumption of a different salary for me. Deductive explanations specify those ceteris paribus conditions; this makes them more complex but also invariable with respect to the choice of focal point. Counterfactual explanations depend on the nature of the focal point.

This (relative) simplicity comes at a cost. Counterfactuals may offer only a partial explanation in some cases [Wachter et al., 2017]. In fact there are two sorts of partiality in a counterfactual explanation. First a counterfactual explanation doesn’t specify the ceteris paribus conditions and so doesn’t specify what is necessary for the prediction—call this partiality. On the other hand counterfactual explanations are also partial in the sense that they don’t specify all the sufficient conditions for the prediction; they are hence what are called local explanations. A given counterfactual might give

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3Weak centering requires that $w \leq w'$ for all $w'$.
4[Kusner et al., 2017] explore how to uncover these biased explanations and reveal the ones that are damaging. He uses a Pearl causal system to generate counterfactuals, where the causal variables are exogenous descriptions of input data for the ML algorithm.
only a partial picture of the behavior of the program or agent. It might not give a complete local explanation of a decision, as such a decision might be over determined for the given point at hand. A prediction $\pi$ is over determined for a focal point $x_0$ iff the following set of linear maps contains at least two elements

$$\{\Delta_i : i \subseteq n \land f(\Delta_i(x_0)) = \pi \land f(x_0) = \eta \land \eta \neq \pi\}$$

Many real world applications like our bank loan example will have this feature.

3 From partial to complete explanations

In principle, we can move from a partial picture of the behavior of $f$ to a more complete one. The linear maps $\Delta_i$ associated with counterfactuals permit us to plot the local behavior of $f$ around a focal point $x_p$. If we look at all possible $\Delta_i$ using all the possible combinations of dimensions of $X$, we can plot a neighborhood around $x_p$, $N_{f,x_p}$, where for all points $z$ in the interior $N_{f,x_p}$, $f(z) = f(x_p)$ and for points $w$ on the boundary of $N_{f,x_p}$, $f(w) \neq f(x_p)$. We call the collection of such linear maps a complete local explanation of the decision at $x_p$.

$N_{f,x_p}$ captures the fact that there may be several distinct conditions the lack of which would be causally responsible for a particular prediction, like my not getting the loan in our motivating example. Let us call the set of counterfactuals corresponding to $N_{f,x_p}$, $S(N_{f,x_p})$. $S(N_{f,x_p})$ may contain many cascading counterfactuals, and its cascading degree may be high.

There are some cases in which the geometry of the prediction space allow us to move from complete local to global explanations of the behavior of $f$. Suppose $f$ changes values only once for each feature/dimension $d_i$ moving out from a focal point $x_p$. If in such a case $N_{f,x_p}$ forms a convex sub-space of $\pi[N]\}$ and a complete local explanation provides a full global explanation.

There is an important connection between the cascading degree of $S(N_{f,x_p})$ and the geometry of $f$ on the feature space. If $f$ changes values only once for each feature/dimension $d_i$, $S(N_{f,x_p})$, has cascading degree 1. In addition, $S(N_{f,x_p})$, has cascading degree 1 iff the feature/dimensions are pairwise independant of each other with respect to $f$’s predictions.

Proposition 1. Suppose that the feature space is Boolean valued. Then $S(N_{f,x_p})$, has a cascading degree $\leq 2$ iff $N_{f,x_p}$ is convex.

Proof: Note that a cascading set of counterfactuals exhibits an entailment relation between antecedents. Thus, if $\phi \implies \psi$ and $\chi \implies \neg \psi$ are counterfactuals of degrees $n$ and $n-1$, $\phi$ entails $\chi$. This means that if we have a set $S$ of cascading counterfactuals of degree 3 or more, we will have antecedents $\phi, \chi, \delta$ which are conjunctions of feature values, such that if $\phi \models \chi \models \delta$ that in the Manhattan space of Boolean features puts the points in the feature space corresponding to $\phi, \chi, \delta$ are all on the same line. But if $S$ has degree 3 or greater, this forcibly means that one of the points will not be in the space of predictions made $f$ at the other two points. So $N_{f,x_p}$ is non convex. Conversely, suppose $N_{f,x_p}$ is non convex. Using the construction of counterfactuals from $N_{f,x_p}$ will immediately yield a cascading set of degree 3 or higher.

The cascading degree of $S(N_{f,x_p})$ for nonconvex $N_{f,x_p}$ thus gives a measure of the degree of non-convexity of $N_{f,x_p}$, and a measure of the complexity of an explanation.

Remark 1. Suppose $N_{f,x_p}$ gives rise to a set of cascading counterfactuals of degree 2. In this case two complete local explanations can determine a full global explanation (and determine the behavior of $f$).

The problem with complete local explanation is that they may be still too complex for any human to understand. It is not unusual for AI applications to encode data via hundreds even thousands of features. The complete local explanation would involve too many counterfactuals for humans to grasp.

4 Pragmatic constraints on explanations

Because the set of causal factors can be so complex, it becomes apparent that explanations must have an important pragmatic component. All explanations have an “explanation” who asks for the explanation, and an explanation for an explainee must respond to the particular conundrum that brought the explainee to ask for one [Bromberger, 1962; Achinstein, 1980]. For instance, our loan seeker $A$ may wonder why the bank refused her a loan when she has what she thinks is an adequate qualifying income and other qualities. An appropriate explanation for $A$ would then explain which of his assumptions was faulty or incomplete, thus solving the conundrum. In essence, what is going on here is that $A$ has in mind the “ideal” function $f$ and is confused about why the value of $f$ on her data $x_p$ is not that of $f$; i.e. her conundrum is that $f(x_p) \neq f(x_p)$. The conundrum can come about for two reasons: either $0$ is simply mistaken about the nature of $f$ (perhaps she is also mistaken about $f$ or if not, she is mistaken about how $f$ differs from $f$), or her understanding of $f$ is incomplete.

For the incompleteness sense, suppose $x_p$ is decomposed into $(x_{d_1}, x_{d_2})$; for $A f$ only pays attention to the values of dimensions $d_i$ in the sense that for her $f((x_{d_1}, x_{d_2})) = f((x_{d_1}, x_{d_2}))$, for any values $x_{d_2}$. An adequate explanation will then point out that for some $\Delta$ where $\Delta((x_{d_1}, x_{d_2})) = (x_{d_1}, x_{d_2})$, $f(\Delta((x_{d_1}, x_{d_2}))) = \pi$ while $f(x_p) = \eta$.

So we have, simplifying, conundra resulting from incompleteness and conundra resulting from mistaken information. We then stipulate:

CI Suppose we have a conundrum based on incompleteness. An adequate explanation for explainee $A$ who re-

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5By complicating the language of counterfactuals, we can have probability estimates of literals and so approximate continuous feature spaces.

6A more contemporary view aligned with this is [Miller, 2019].
requests an explanation why \( \hat{f}(x_p) = \eta \) must resolve A’s conundrum arising from attending only to some dimensions \( \vec{d}_1 \) of \( x_p = \langle x_{d_1}, \ldots, x_{d_k} \rangle \). More precisely, the explanation must provide a \( \Delta \) such that \( \Delta(x_{d_1}, \ldots, x_{d_k}) = \langle y_{d_1}, \ldots, y_{d_k} \rangle \) and \( \hat{f}(\Delta(x_p)) = \hat{f}(\langle y_{d_1}, \ldots, y_{d_k} \rangle) = \pi \) while \( \hat{f}(x_p) = \eta \).

**CM** Suppose A’s conundrum is based on error. An adequate explanation for explainee A who requests an explanation why \( \hat{f}(x_p) = \eta \) must resolve A’s conundrum by providing the values for the dimensions \( \vec{d}_2 \) of \( x_p \) on which A is mistaken. More precisely, supposing a decomposition of \( x_p = \langle x_{d_1}, \ldots, x_{d_k} \rangle \), the explanation must provide a \( \Delta \) such that \( \Delta(x_{d_1}, \ldots, x_{d_k}) = \langle y_{d_1}, \ldots, y_{d_k} \rangle \) such that \( \hat{f}(\langle y_{d_1}, \ldots, y_{d_k} \rangle) = \hat{f}(\Delta(x_p)) = \pi \) while \( \hat{f}(x_p) = \eta \).

\[ \hat{f}(\Delta(x_p)) = \delta \]  
\[ \hat{f}(P(\Delta(x_p))) = \pi \]

Note that the incompleteness condition for a conundrum mirrors the notion of a biased dependency. As a person with data \( x_p \) might reasonably want to know whether such biases were the result of a particular decision, we put one additional constraint on appropriate explanations:

**CB** an appropriate explanation for explainee A must lay bare any prejudicial factors \( P \) that affect A. That is, where \( \pi, \delta \) and \( P \) are defined as above, the explanation must provide \( \hat{f}(\Delta(x_p)) = \delta \) and \( \hat{f}(P(\Delta(x_p))) = \pi \) in our loan example, explanations that obey (CB) might not be in the interest of the bank that owns the ML algorithm. For instance, the bias of the bank against loans to people of color is unfair to A. But the bank might not want to have this bias exposed. This is the ethical problem for explanations.

To attain an appropriate explanation meeting CB, we cannot simply rely on the bank’s preferred explanation, as the bank may have something to hide. In similar fashion, even if we have access to all of \( \hat{f} \), we cannot rest content with just one counterfactual explanation that satisfies CM and CI. We have to ensure that CB is met as well. We will say that a set of counterfactuals provides an adequate local explanation just in case it obeys CM, CI and CB.

Let us suppose that we do not have access to \( \hat{f} \). To find an appropriate explanation, we imagine a game played between the bank and the would be loan taker, in which the loan taker can ask questions of the bank (or owner/ developer of the algorithm) about the algorithm’s decisions. More particularly, we propose to use a two player game, an explanation game to get at appropriate explanations for A.

To define an explanation game, we first fix a set of two players \( \{0, 1\} \). (1 – i) denotes the opponent of i.

The moves or actions in the game for 0, denoted \( A_0 \), are to request an explanation from 1 about the behavior of \( \hat{f} \) at \( x_p \), which means requesting a particular linear map \( \Delta \). 0 may accept an explanation provided by 1 or reject 1’s explanation and request an explanation different from explanations previously offered by 1; that is request a \( \Delta \) where \( j \) and \( i \) are different sets of dimensions of \( X \) (this move is known as restriction). Alternatively, 0 may force 1 to provide a counterfactual explanation generating linear map for a particular set of dimensions. If 0 is allowed to use forcing, then she can name which dimensions she wants varied to find a boundary point of \( \mathcal{N}_{\hat{f}^{-1}} \).

1’s moves \( A_1 \) consists of the following: 1 may offer a \( \Delta \) generating a counterfactual explanation; 1 may also claim that a particular map \( \Delta_j \) for dimensions \( j \) does generate a counterfactual explanation—that is, for no values in dimensions \( j \) do we get the requested prediction; 1 may also insist that a previously given \( \Delta \) solve 0’s conundrum. The game terminates when (a) 0 is convinced that her conundra are resolved or (b) when all possible \( \Delta \) have been examined or (c) 0 does not want to continue anymore.

We assume that in an explanation game it is common knowledge among the players that one can check the truth of 1’s responses and whether 1’s counterfactuals address the constraints we have mentioned. Thus, we do not address issues of deceit here.

We can now specify an explanation game and its winning condition for player 0.

**Definition 1.** An Explanation game, \( \mathcal{G} \), concerning a polynomially computable function \( \hat{f} : X^n \rightarrow Y \), where \( X^n \) is a space of data and \( Y \) a set of predictions, is a tuple \((\mathcal{A}_0 \cup A_1, \hat{f} : X^n \rightarrow Y, x_p, C)\) where:

i. 1, but not 0 has access to the behavior of \( \hat{f} \).

ii. \( x_p \in X^n \) is the starting position, and 0 opens \( \mathcal{G} \) with a request for an explanation.

iii. 1 responds to 0’s requests (forcing or restriction) or claims an adequate explanation is already provided.

iv. \( C \subseteq X^n \) is a local minimum of \( \hat{f} \) with respect to the neighbourhood \( \mathcal{N}_{\hat{f}} \), where \( x \) is the current position in the game. Every \( c \in C \) resolves one or more of 0’s conundrum CM or CI.

We say that 0 wins \( \mathcal{G} \) just in case in \( \mathcal{G} \) she acquires a set of counterfactuals about the behavior of \( \hat{f} \) concerning features \( C \) that answer her conundrum.
Given that 0 can only ask for a single dimension to be shifted, her strategy is just to go through all the dimensions by continuing to ask for a new explanation. Eventually 0 will have gone through all of the dimensions of $X^n$. As the cascading degree of $S(\mathcal{N}_{f,x_p}) < 2$, she will have determined $S(\mathcal{N}_{f,x_p})$.

Once the cascading degree associated with $\mathcal{N}_{f,x_p} \geq 2$, Player 1 can hide, if he wishes, prejudicial factors for exponential time.

**Proposition 5.** Suppose that for $X \subset R^n$ in an explanation game $G$, with $\hat{f}$ and 0 with focal point $x_p \in X^n$ and the cascading degree of $S(\mathcal{N}_{f,x_p}) \geq 2$. Then 0 has a worst case exponential time with respect to $n$ winning strategy in $G$.

Player 1 in this case is free to provide counterfactuals that satisfy constraint (CI) and (CM) but possibly not (CB) and can provide counterfactuals with arbitrarily complex antecedents. Without forcing, 0 cannot make 1 provide counterfactuals that take satisfy (CB). However, given that the boundary $\mathcal{N}_{f,x_p}$ is computable, 0 still has a winning strategy with restriction in an explanation game. She will eventually compute the boundary of $\mathcal{N}_{f,x_p}$ in exponential time wrt to $n$ in the worst case.

We draw from this the moral that in order to be effective an explanation game must allow 0 to force 1 to vary certain dimensions that 0 picks and $S(\mathcal{N}_{f,x_p})$ must have a fixed cascading degree. We also claim that having such a fixed cascading degree might be an important aspect of good explanations, as there is an important connection between cascading degrees and the generality of laws. A low cascading degree means a general set of laws, which a priori is preferable scientifically.

## 6 Conclusions and outlook

We have used the semantics and logic of counterfactuals to explore epistemically accessible and adequate but partial explanations. Counterfactual explanations are promising vehicles for epistemic accessibility, and we have shown that they can algorithmically provide adequate explanations where all biases are made clear if certain conditions (forcing) obtain and the cascading degree of the set of counterfactuals describing the local neighborhood around the focal point is fixed. However, we have not explored here another important parameter for epistemic adequacy. Adequacy also depends on the characterization of the input data $X$ for a learning algorithm $\hat{f} : X \rightarrow Y$. If the dimensionality of $X$ is too high or if the dimensions don’t correspond to intuitive concepts, then even partial explanations may not be satisfactory. So to get fully explanatory counterfactuals for the behavior of $\hat{f}$, it will be important in such cases to find the right representation of the data for the explainee.

Counterfactual explanations may also express adversarial examples. These typically aren’t good explanations of the phenomenon $\hat{f}$ is trying to model. Nevertheless such counterfactuals explain the behavior of $\hat{f}$ and can be as such very valuable.
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