Toy Models for Black Hole Evaporation*

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Abstract

These notes first present a brief summary of the puzzle of information loss to black holes, of its proposed resolutions, and of the flaws in the proposed resolutions. There follows a review of recent attempts to attack this problem, and other issues in black hole physics, using two-dimensional dilaton gravity theories as toy models. These toy models contain collapsing black holes and have for the first time enabled an explicit semiclassical treatment of the backreaction of the Hawking radiation on the geometry of an evaporating black hole. However, a complete answer to the information conundrum seems to require physics beyond the semiclassical approximation. Preliminary attempts to make progress in this direction, using connections to conformal field theory, are described.

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Since the discovery of black holes, physicists have been faced with the possibility that they engender a *breakdown of predictability*. At the classical level this breakdown arises at the singularity. Classically we do not know how to evolve past it. Inclusion of quantum effects may serve as a remedy, allowing predictable evolution, by smoothing out the singularities of general relativity. However, as suggested by Hawking [1,2], quantum effects also present another sharp challenge to predictability through the mechanism of Hawking evaporation.

To see this, consider a pure quantum state describing an infalling matter distribution. If this matter collapses to form a black hole, it will subsequently emit Hawking radiation. In Hawking’s approximation where the backreaction of the emitted radiation on the geometry is neglected, the radiation is thermal and is described by a mixed quantum state. This suggests that once the black hole evaporates the initial pure state has been converted to a final mixed state; information has been lost, and unitarity has been violated. Hawking proposed that this represents a new and fundamental type of unpredictability inherent in quantum gravity.

Beyond any prejudice that quantum gravity shouldn’t violate unitarity, there are potential problems with this scenario. In particular, as argued in [3-5], naïve attempts to formulate unitarity violating dynamics typically run afoul of essential principles such as energy conservation. We are therefore motivated to look for other possible resolutions to the problem of what happens to information that falls into a black hole.

There have been various proposals for resolving the black hole information problem, but each of them appears to have flaws. A list of these, in ascending order of speculative content, and together with objections, is as follows:

1. Correcting Hawking’s calculation by including the backreaction renders the final state pure; the information escapes in the Hawking radiation. Objection: this would appear to imply that either all of the information has been extracted from the infalling matter by the time it crosses the horizon, or that information propagates acausally from behind the horizon to outside.

2. The information is released in the last burst of radiation as the black hole evaporates to the Planck scale and quantum-gravitational and backreaction effects dominate. Objection: Since the initial black hole could have been arbitrarily massive, it must be

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1 For more comprehensive reviews see [6-8].
2 This alternative has for example been advocated in [9,10].
possible for the remaining Planck scale energy to carry off an arbitrarily large amount of information corresponding to all of the possible black hole initial states. A large amount of information can be transmitted using a small amount of energy only over a long period of time, e.g., through emission of many very soft photons. This implies the next proposal[2,11,7].

3. The black hole leaves behind a long-lived remnant with Planck-sized mass. This remnant must have infinitely many states to allow it to carry the unbounded amount of information that could have been present in the initial state[9]. Objection: an infinite spectrum of states with Planck-size masses wreaks havoc with loop calculations and with production probabilities both in thermodynamics and in background fields. In particular, such a spectrum implies infinite production of these particles in the Hawking radiation from arbitrarily large black holes, and likewise appears to imply infinite production from a thermal ensemble at any given temperature. The resulting instabilities are disastrous[9].

4. Baby universes form and carry away the information that falls down the black hole, and thus unitarity, while apparently violated in our Universe, is restored for the system including the baby universes[16]. Objection: in a different context, it has been argued[17,18] that wormholes simply shift coupling constants and don’t lead to such apparent violations of unitarity.

5. Information emerges from the black hole via a previously unsuspected mechanism rather than through small corrections to the Hawking radiation. Such a mechanism is suggested both by the failure of other attempts to resolve the information problem and by arguments that there should be upper bounds on information content that arise from Planck scale physics[7]. Objection: this proposal appears to require acausal behavior behind the horizon.

There are other variations on these basic possibilities. The objections are not iron-clad, and may have loopholes. One way (and perhaps the only way) to actually solve the information problem is to gain control of backreaction and quantum gravity effects. This is a difficult task in four dimensions, and it behooves us to search for simple toy models to gain more intuition.

3 In one possible realization of this proposal[12,14] the infinite number of states arises from excitations of an infinite “internal” volume of the black hole.

4 For an attempt to evade these see[15].
One such toy model\cite{12} is two-dimensional dilaton gravity, described by the action

\[ S = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left[ e^{-2\phi} \left( R + 4(\nabla\phi)^2 + 4\lambda^2 \right) - \frac{1}{2} \sum_{i=1}^{N} (\nabla f_i)^2 \right]. \] (1)

Here \( \phi \) is the dilaton, \( 4\lambda^2 \) the cosmological constant, and the \( f_i \) are minimally coupled matter fields. Note for future reference that since the gravitational part of the action is multiplied by \( e^{-2\phi} \), the quantity \( e^\phi \) plays the role of the gravitational coupling constant.

This toy model has several virtues. First, it is perturbatively renormalizable by power counting; the only dimensionful coupling constant is \( \lambda \). Secondly, it is completely soluble at the classical level. Among these solutions are black holes, and these black holes Hawking radiate \( f \)-particles. Finally, this model is the low energy effective theory for certain types of four- and five-dimensional black holes, and this provides a direct application to higher-dimensional physics.

Before pursuing the former points, let us recall the connection to higher-dimensions.\footnote{For a more complete explanation see \cite{14}.} The low energy action for string theory is of the form

\[ S = \int d^{10}x \sqrt{-g} e^{-2\phi} \left[ R + 4(\nabla\phi)^2 - \frac{1}{2} F_{\mu\nu}^2 + \cdots + \mathcal{O}(\alpha') \right] \] (2)

where \( F_{\mu\nu} \) is the electromagnetic field strength and where terms involving other fields are neglected, as are higher-dimension operators. This theory is known \cite{19,20} to have magnetically charged black hole solutions,

\[ ds^2 = -\frac{1 - r_+}{1 - r_-} dt^2 + \frac{dr^2}{(1 - r_+)(1 - r_-)} + r^2 d\Omega_2^2 + ds_6^2 \]

\[ e^{-2\phi} = e^{-2\phi_0} \left( 1 - \frac{r_-}{r} \right) \]

\[ F_{\mu\nu} = Q \epsilon_{\mu\nu}^{(2)}. \] (3)

Here \( r_+, r_- \), and \( \phi_0 \) are constants, \( ds_6^2 \) denotes your favorite 6-dimensional string compactification, \( Q \) is the magnetic charge, and \( d\Omega_2^2, \epsilon_{\mu\nu}^{(2)} \) are the line element and Levi-Civita tensor on the two-sphere.

This solution has a causal structure identical to that of Schwarzschild (see Fig. 1), with a horizon at \( r = r_+ \) and a singularity at \( r = r_- \). However, there is a crucial difference between their geometries. Consider the slice \( S \) shown in Fig. 1; in Schwarzschild, this
Fig. 1: The Penrose diagram for the magnetically charged dilatonic black hole in four dimensions. Also shown is a constant-time slice $S$ through the geometry.

spatial slice gives the Einstein-Rosen bridge. In the present case there is also a throat connecting two asymptotically flat regions (Fig. 2), but in the extremal limit $M \rightarrow Q$ the throat becomes infinitely long. In this limit an observer in the asymptotic region sees the horizon and singularity disappear to infinity; likewise an observer fixed near the horizon sees the asymptotic region recede to infinity. For the latter observer the universe is an infinite tube terminating in a black hole; this solution takes the form

$$ds^2 = ds_{2DBH}^2(r, t) + Q^2 d\Omega_2^2$$
$$e^{2\phi} = e^{2\phi_{2DBH}(r)}$$
$$F_{\mu\nu} = Q \epsilon^{(2)}_{\mu\nu}.$$  

(4)

Here $ds_{2DBH}^2$ and $\phi_{2DBH}$ are the metric and dilaton for the two dimensional black hole in string theory that was found by Mandal, Sengupta, and Wadia [21] and Witten [22]; we will see their explicit forms shortly. The important point is that the solution (4) is a direct product of two two-dimensional solutions. The second of these is the round two-sphere threaded by a magnetic flux. A similar construction holds for five-dimensional black holes.

6 At first sight one might think that the extremal Reissner-Nordstrom black hole has the same property, since its spatial geometry also has an infinite throat. However, the horizon does not become causally disconnected from the rest of spacetime as a result of the rapid falloff of $g_{00}$ along the throat.
Fig. 2: Pictured is the spatial geometry of the right half of the slice $S$ of Fig. 1.

The mass of the two-dimensional black hole in (4) depends on the asymptotic value of the dilaton, $\phi_0$. If the latter is scaled to $-\infty$ as the extremal limit is taken, then the mass can be arranged to be finite. Conversely, if the asymptotic dilaton is fixed, then the mass is zero and the resulting two-dimensional solution is the vacuum.

With the dilaton (i.e. the coupling) fixed at infinity, and near the extremal limit, the solution far down the throat is closely approximated by (4) with a small mass. At extremality, and as seen by the asymptotic observer, the horizon is infinitely far down the throat and the $r,t$ solution far down the throat is the vacuum. Let us now consider low-energy scattering of particles from the extremal black hole. Very low energy excitations that penetrate into the throat will not be able to excite the angular degrees of freedom, which have a threshold $\sim 1/Q$. On the other hand, $s$-wave excitations may penetrate the throat and raise the mass of the black hole above extremality. This corresponds to raising the mass of the two-dimensional black hole above zero. The excess mass will later be emitted in Hawking radiation, corresponding to evaporation of the two-dimensional solution back to zero mass. This provides a direct relationship between low-energy scattering by near-extremal dilatonic black holes and formation and evaporation of two-dimensional black holes. The low-energy effective theory describing such excitations is obtained by dropping the angular dependence in (2), and one obtains the action (1).

To describe excitations that can actually penetrate the throat, one must add matter terms to the action (2). This is further described in [14,23].
To investigate the two-dimensional problem we return to the action (1). Although the general solution is easily found [12], we focus on some special cases; units are chosen so that $\lambda = 1$. First are the vacuum solutions, with $f_i \equiv 0$:

$$ds^2 = \frac{-dx^+ dx^-}{M - x^+ x^-}$$

$$e^{-2\phi} = M - x^+ x^-$$

where $x^\pm = x^0 \pm x^1$ are light-cone coordinates and $M$ is an arbitrary parameter. The change of coordinates $x^+ = e^{\sigma^+}$, $x^- = -e^{-\sigma^-}$ (with $\sigma^\pm = \tau \pm \sigma$) gives

$$ds^2 = \frac{-d\sigma^+ d\sigma^-}{1 + Me^{\sigma^- - \sigma^+}}$$

$$\phi = -\frac{1}{2} \ln \left( M + e^{\sigma^+ - \sigma^-} \right).$$

For $M = 0$ the metric is flat, and this solution is identified as the ground state of the theory. Note that the dilaton is then

$$\phi = -\sigma;$$

the $M = 0$ solution is therefore called the linear dilaton vacuum. For $M > 0$ one recovers the two-dimensional black hole of [21,22]. The causal structure of this black hole is identical to that of Schwarzschild; its Penrose diagram is given by Fig. 3. Solutions with $M < 0$ have naked singularities.

**Fig. 3:** The Penrose diagram for the two-dimensional eternal black hole.
Black hole formation occurs when one allows matter to fall into the linear dilaton vacuum. For example, a general left-moving lump of classical matter is given by

\[ f = F(x^+) \],

for an arbitrary function \( F(x^+) \), and has stress tensor

\[ T_{++} = \frac{1}{2} (\partial_+ F)^2 \].

An \( F(x^+) \) that vanishes outside \( x_j^+ > x^+ > x_i^+ \), or equivalently \( \sigma^+ > \sigma > \sigma_i^+ \), corresponding to a lump of finite width, yields a Penrose diagram as in Fig. 4: the matter “collapses” to form a black hole. The metric for \( \sigma^+ < \sigma_i^+ \) is

\[ ds^2 = -d\sigma^+ d\sigma^- \] (10)

and for \( \sigma^+ > \sigma_j^+ \) is

\[ ds^2 = -\frac{d\sigma^+ d\sigma^-}{1 + M e^{\sigma^- - \sigma^+} - \Delta e^{\sigma^-}} \] (11)

Here the constants \( M \) and \( \Delta \) are moments of the matter distribution,

\[ M = \int_{\sigma_i^+}^{\sigma_j^+} d\sigma^+ T_{++}(\sigma^+) \], \[ \Delta = \int_{\sigma_i^+}^{\sigma_j^+} d\sigma^+ e^{-\sigma^+} T_{++}(\sigma^+) \]. (12)

The change of coordinates

\[ \xi^+ = \sigma^+ \], \[ \xi^- = -\ell n \left[ e^{\sigma^-} - \Delta \right] \] (13)

returns (11) to the asymptotically flat form,

\[ ds^2 = -\frac{d\xi^+ d\xi^-}{1 + M e^{\xi^- - \xi^+}} \]. (14)

Hawking radiation arises from evolution of positive frequency incoming states to both positive and negative frequency outgoing states.\(^8\) In a fixed background the left and right moving \( f \)-quanta are decoupled, and in considering the Hawking radiation we focus on the right-movers. The positive frequency modes in the respective regions are

\[ u_\omega = \frac{1}{\sqrt{2\omega}} e^{-i\omega\sigma^-} \] (in)
\[ v_\omega = \frac{1}{\sqrt{2\omega}} e^{-i\omega\xi^-} \] (out)

\[ ^8\) For more details, see e.g., \([24, 25]\).}
The Penrose diagram for an “infalling” lump of classical matter. Also indicated are the “in” and “out” regions for right movers.

and they are related by the Fourier transform,

\[ v_\omega (\xi^-(\sigma^-)) = \int_0^\infty d\omega' \left[ \alpha_{\omega\omega'} u_{\omega'}(\sigma^-) + \beta_{\omega\omega'} u_{\omega'}^*(\sigma^-) \right] . \] (16)

The Fourier coefficients \( \alpha_{\omega\omega'} , \beta_{\omega\omega'} \) give the Bogoliubov transformation, and determine the spectrum of Hawking radiation. For example, it is easily shown that the expectation value of the out number operator in the in vacuum is given by

\[ \text{in} \langle 0 | N_{\omega}^{\text{out}} | 0 \rangle_{\text{in}} = \int_0^\infty d\omega' |\beta_{\omega\omega'}|^2 ; \] (17)

nonvanishing \( \beta_{\omega\omega'} \) therefore implies Hawking radiation.

In the present case the Bogoliubov coefficients can be evaluated exactly \[25\] (contrast four-dimensional black holes!); for example,

\[ \beta_{\omega\omega'} = \frac{1}{2\pi} \sqrt{\frac{\omega'}{\omega}} \Delta^{i\omega} B \left( -i\omega - i\omega', 1 + i\omega \right) . \] (18)

From this one can derive \[25\] the outgoing stress tensor,

\[ \langle T^{f}_{- -} \rangle = \frac{1}{48} \left[ 1 - \frac{1}{(1 + \Delta e^{\xi^-})^2} \right] \] (19)
which, after an initial transitory period, describes a thermal flux of Hawking radiation.

Next we wish to investigate the backreaction of the Hawking radiation on the geometry. It is most easily discussed by considering the vacuum functional integral,

$$ \int Dg D\phi \, e^{i S \phi, g} \int Dg f \, e^{\frac{-i}{4\pi} \int d^2 x \sqrt{-g} \sum_{i=1}^{N} (\nabla f_i)^2}.$$  \hspace{1cm} (20)

Here we have split the action into the gravitational part and the matter part. The subscript on the matter measure indicates that this functional measure is induced from the functional metric

$$ (\delta f_1, \delta f_2)_g = \int d^2 x \sqrt{-g} \, \delta f_1 \, \delta f_2 $$ \hspace{1cm} (21)

in the usual fashion. The matter functional integral has been extensively studied in the string literature and elsewhere, and gives

$$ \int Dg f \, e^{\frac{-i}{4\pi} \int d^2 x \sqrt{-g} \sum_{i=1}^{N} (\nabla f_i)^2} = e^{iNS_{PL}} $$ \hspace{1cm} (22)

where

$$ S_{PL} = -\frac{1}{96\pi} \int d^2 x_1 \sqrt{-g} \int d^2 x_2 \sqrt{-g} \, R(x_1) \Box^{-1}(x_1, x_2) R(x_2) $$ \hspace{1cm} (23)

is the Polyakov-Liouville action. (Here $\Box^{-1}$ denotes the Green function for the d’Alembertian.) Although this action is nonlocal, it appears local in conformal gauge, $ds^2 = -e^{2\rho} dx^+dx^-:

$$ S_{PL} = \frac{1}{24\pi} \int d^2 x \, (\nabla \rho)^2 $$ \hspace{1cm} (24)

Gravitational dynamics, including quantum effects of the matter, therefore follow from the functional integral

$$ \int Dg D\phi \, e^{i S[g,\phi]+iNS_{PL}[g]}.$$  \hspace{1cm} (25)

One way to see the gravitational effects of the matter is to compute the quantum stress tensor,

$$ N \frac{2\pi}{\sqrt{-g}} \frac{\delta S_{PL}}{\delta g^{\mu\nu}} = \langle T^f_{\mu\nu} \rangle $$ \hspace{1cm} (26)

One finds either directly from (23), or equivalently from the well-known trace anomaly,

$$ \langle T^f_{++} \rangle = -\frac{N}{96} R = -\frac{N}{12} \partial_+ \partial_- \rho $$ \hspace{1cm} (27)
The other components of the stress tensor can also be found by varying (23), although it is simpler to integrate the conservation law \(\nabla^\mu \langle T^f_{\mu\nu} \rangle = 0\) using (27). This gives
\[
\langle T^f_{-} \rangle = -\frac{N}{12} \left[ (\partial_- \rho)^2 - \partial_-^2 \rho + t_-(x^-) \right]
\] (28)
where \(t_-(x^-)\) is an integration function that must be fixed by boundary conditions. (Equivalently it arises from the ambiguity in defining the Green function in (23)).

Computation of \(\langle T^f_{-} \rangle\) from this expression and (11) yields agreement with (19), confirming the relationship [26] between the conformal anomaly and Hawking radiation.

We conclude that \(S_{PL}\) incorporates both the Hawking radiation, and the effects of its backreaction on the geometry.

What, then, are these effects? To begin, let’s determine when the backreaction due to the \(f\)-fields has a substantial effect on the geometry. This can be estimated by asking when the Hawking radiation (uncorrected by backreaction effects) has carried away a substantial fraction of the mass of the black hole,
\[
\int d\xi^- \langle T^f_{-} \rangle \sim M.
\] (29)
It is straightforward to see that for large \(M\) this occurs at the retarded time \(x_{\text{evap}}^- \simeq -\Delta\), near the horizon, and where the dilaton at the trailing edge of the incoming matter distribution satisfies
\[
e^{2\phi} \sim \frac{1}{M}.
\] (30)
Therefore if the mass of the black hole is taken to be large (and also \(x_{\text{f}}^+ \Delta \gg 1\)), the evaporation process takes place at weak coupling. This helps us in separating the backreaction from quantum-gravitational effects.

Indeed, the resulting weak-coupling can be used to justify a semi-classical analysis of the functional integral (25), via the classical equations arising from the action
\[
S_{SC} = \frac{1}{2\pi} \int d^2x \ e^{-2\phi} \left[ -2\Box \rho + 4(\nabla \phi)^2 + 4\lambda^2 \right] + \frac{N}{24\pi} \int d^2x (\nabla \rho)^2
\] (31)
where we use the conformal-gauge result \(R = -2\Box \rho\). In order to do this we must take the number of matter fields \(N\) to be large so that the second term dominates the other quantum corrections to the dynamics, and can be treated on the same footing as the first term.
Because the correction term in (31) is quadratic in $\rho$, and $\rho = 0$ for the linear dilaton vacuum, it remains a solution to the semiclassical theory. However, the theory is no longer exactly soluble and, in fact, other solutions are difficult to find. We have, nonetheless, gained considerable insight into the structure of the solutions from numerical and general arguments [27-30].

As an example, from the resulting time-independent equations one can numerically find static solutions [28-30] that correspond to a black hole in equilibrium with an influx of radiation that precisely balances the outward Hawking flux.

In the dynamic case of a black hole formed from collapsing matter, one can argue that an apparent horizon, determined by the condition $(\nabla \phi)^2 = 0$, forms and recedes as the black hole evaporates. A surprise is that behind this apparent horizon is a singularity of the semiclassical equations [27,13]. The reason this is surprising is that it is distinct in behavior from the original classical singularity; in particular, it occurs where

$$e^{2\phi} = e^{2\phi_{cr}} = \frac{12}{N}.$$  (32)

It is therefore (for large $N$) not at large $e^{\phi}$ as was the classical singularity. Mathematically this singularity arises as a result of the vanishing of an eigenvalue of the kinetic operator in (31); this signals a breakdown of the semiclassical approximation. This breakdown means that solutions of the equations following from (31) should really not be trusted for values of the dilaton $\phi \geq \phi_{cr} - \epsilon$, for some small ($N$-dependent) $\epsilon$.

The resulting geometry is pictured in Fig. 5, Fig. 6. Shown are both the apparent horizon, and the effective horizon. The latter will in general be defined as the boundary of the region from which future-directed causal curves can escape to null infinity without encountering Planck-scale physics; it is thus the boundary of the region where we can make predictions using an effective field theory valid below the Planck scale. In the present context “Planck scale” physics is identified with physics beyond the validity of the semiclassical approximation. Eventually the line with $\phi = \phi_{cr} - \epsilon$ crosses the horizons. We cannot determine the physical behavior in the future light cone (or “shadow”) of this point since it depends on the presently ill-understood physics at $\phi \geq \phi_{cr} - \epsilon$. For $\phi < \phi_{cr} - \epsilon$, the solution asymptotes back to the linear dilaton vacuum.

This semiclassical picture, although incomplete, may nonetheless give some clues about the information problem. In particular, working order-by-order in $1/N$, it is probable that one can construct an argument [25] that information does not come out in the
Fig. 5: A Kruskal diagram for the evaporating two-dimensional black hole. $Q$ denotes the line along which $\phi = \phi_{cr} - \epsilon$; beyond this line a full quantum description of the collapse is presumably needed to make predictions.

Hawking radiation before one reaches the shadow of $\phi = \phi_{cr} - \epsilon$. This is analogous to stating that the information doesn’t come out of four-dimensional black holes until they reach the Planck scale. It may therefore, in the present context, rule out the most conventional proposed resolution, resolution 1), of the black hole information conundrum.

On the other hand, semiclassical predictability has failed and we still can’t say what does happen to information. To proceed we must go beyond the large-$N$ approximation and investigate the full quantum theory.

Quantum dilaton gravity is most easily treated by gauge-fixing the metric to the form

$$g_{\mu \nu} = e^{2\rho} \hat{g}_{\mu \nu}$$

(33)

where $\hat{g}_{\mu \nu}$ is a fixed background metric. Although we have argued that dilaton gravity is renormalizable, the fields $\rho$ and $\phi$ are dimensionless so in fact an infinite number of counterterms can occur. The general parity invariant action is of the form

$$S = -\frac{1}{2\pi} \int d^2x \sqrt{-\hat{g}} \left( G_{MN}(X^P) \nabla X^M \cdot \nabla X^N + \frac{1}{2} \Phi(X^P) \hat{R} + T(X^P) \right)$$

(34)
Fig. 6: A Penrose diagram for the evaporating two-dimensional black hole. Prediction of physics in the causal future of the line $Q$ requires a quantum treatment of dilaton gravity. In particular, one cannot at present determine the final fate of the black hole.

where $X^P = (\rho, \phi)$. Here $G_{MN}(X^P)$, $\Phi(X^P)$, and $T(X^P)$ are arbitrary functions, and we use sigma-model notation.

In quantizing dilaton gravity there are several physical restrictions on the general action (34). The most obvious of these are:

1) The theory should depend on $\hat{g}, \rho$ only through the combination in (33); that is, the theory should be invariant under the background transformation

$$
\hat{g}_{\mu \nu} \to e^{2\omega} \hat{g}_{\mu \nu},
$$

$$
\rho \to \rho - \omega .
$$

This condition is on-shell equivalent to invariance under conformal rescaling of the background metric $\hat{g}$, which implies that the sigma-model $\beta$-functions must vanish. If we momentarily reinstate Planck’s constant, and work to leading order
in $\bar{h}$, this implies
\[
\nabla_M \Phi \nabla^M T - 4T - \frac{\bar{h}}{2} \Box T = 0
\]
\[
\nabla_M \nabla_N \Phi + \frac{\bar{h}}{2} R_{MN} = 0
\]
\[
(\nabla \Phi)^2 - \frac{\bar{h}}{2} \Box \Phi + \frac{(N - 24)\bar{h}}{3} = 0 .
\]  
(36)

Off-shell these must be supplemented by the condition that the tangent vector $V^M$ to the $\rho$ direction satisfy $V_M = \nabla_M \Phi / 2$.

2) The theory must agree with dilaton gravity at weak coupling:

\[
G_{MN} \to G_{\ell MN}^{\phi} = \begin{pmatrix} -4e^{-2\phi} & 2e^{-2\phi} \\ 2e^{-2\phi} & 0 \end{pmatrix}
\]
\[
\Phi \to \Phi^{\ell} = -2e^{-2\phi}
\]
\[
T \to T^{\ell} = -4\lambda^2 e^{-2\phi}
\]  
(37)

Furthermore, outside the large-$N$ approximation we must worry about the role of the ghosts. If the measures for the ghost and $(\rho, \phi)$ functional integrals are defined using the metric $g$, as in (21), then this results in the replacement $N \to N - 24$ in (31). This implies the nonsensical result that for $N < 24$, black holes accrete mass by Hawking radiating negative energy in ghosts! The problem is easily resolved by instead using the metric $e^{2\phi} g$ to regulate the functional measures [31,32]. There follows a third condition:

3) At weak coupling, subleading counterterms of the form

\[
G_{MN} \to G_{\ell MN}^{\phi} + \bar{h} \left( \begin{array}{cc} 2 & -2 \\ -2 & \frac{24 - N}{12} \end{array} \right) + \cdots
\]
\[
\Phi \to \Phi^{\ell} + \bar{h} \frac{24 - N}{6} \rho - 4\bar{h}\Phi + \cdots
\]  
(38)

should appear in $G_{MN}$, $\Phi$.

Finally there may be other physical constraints; one such restriction is

4) The theory should have a sensible ground state.

Writing down the full $\beta$-function equations (39), let alone solving them, is no small task. One promising approach has been advocated in refs. [33-36,32]. As is easily seen, the leading order metric given in (38) is flat, and therefore trivially obeys the leading-order $\beta$-function equations. Furthermore, if $T = 0$ this theory is an exact CFT, that is, an exact solution of the $\beta$-function equations.
One can then identify the tachyon as the operator of conformal dimension (1,1) that agrees with $T^{\text{cf}}$ to leading order in $e^\phi$. The theory with $T \neq 0$ is obtained by perturbing the exact flat theory with this operator. This is similar to steps used to define Liouville theory [37], and should yield an exact solution to the $\beta$-function equations.

Although the resulting theory satisfies criteria 1–3, it does not satisfy criterion 4. It can be shown that there are regular solutions with mass unbounded from below [32]. Hawking radiation in these theories does not shut off [36,35], and black holes appear to radiate to infinite negative mass. The necessary modifications for a stable theory are not obvious; one attempt to stabilize such models is by applying suitable boundary conditions at the line where $\phi = \phi_{cr}$ [38,39].

Despite these facts, the general approach of attempting to identify exact conformal field theories that represent evaporating black holes is worthy of pursuit; perhaps other more realistic examples can be constructed. One is still, however, left with the feeling that uniqueness is lacking. Consideration of supersymmetric theories may provide sufficient uniqueness and solve the problem of negative mass. A different tack is to view the problem of the non-uniqueness of quantum dilaton gravity as similar to that of four-dimensional gravity. In the latter case, we expect string theory to provide an escape from non-renormalizability. Perhaps two-dimensional dilaton gravity is best treated as the low-energy limit of string theory as well.

To conclude, we have succeeded in qualitatively understanding two-dimensional black hole formation and evaporation until quantum effects become strong; this is analogous to understanding four-dimensional black holes up to the Planck regime. Furthermore, we may likely rule out the most conservative proposed resolution to the black hole information problem. This is potentially very interesting. However, a solution to the information conundrum is still beyond the horizon. We haven’t seen quantum restoration of predictability, and probably won’t until we understand quantization of the family of theories of quantum dilaton gravity. Although this is a challenge, it is a good toy problem to develop techniques for higher-dimensions: quantization of dilaton gravity should be an excellent warmup for understanding quantum gravity in four dimensions.

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Linear Infalling matter dilaton vacuum

Effective horizon

Apparent horizon

Infalling matter

Linear
dilaton
vacuum

\( Q \)

\( x_f^+ \)

\( x_i^+ \)
\[ x^+ x^- = M \]

\[ x^- = 0 \]

\[ x^+ = 0 \]

\[ x^+ x^- = M \]