What is the optimal way to measure the galaxy power spectrum?

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Measurements of the galaxy power spectrum contain a wealth of information about the Universe. Its optimal extraction is vital if we are to truly understand the micro-physical nature of dark matter and dark energy. In Smith & Marian (2015) we generalized the power spectrum methodology of Feldman et al. (1994) to take into account the key tenets of galaxy formation: galaxies form and reside exclusively in dark matter haloes; a given dark matter halo may host galaxies of various luminosities; galaxies inherit the large-scale bias associated with their host halo. In this paradigm we derived the optimal weighting and reconstruction scheme for maximizing the signal-to-noise on a given band power estimate. For a future all-sky flux-limited galaxy redshift survey of depth $b_1 > 22$, we now demonstrate that the optimal weighting scheme does indeed provide improved $S/N$ at the level of $\sim 20\%$ when compared to Feldman et al. (1994) and $\sim 60\%$ relative to Percival et al. (2003), for scales of order $k \sim 0.5 \text{ h Mpc}^{-1}$. Using a Fisher matrix approach, we show that the cosmological information yield is also increased relative to these alternate methods – especially the primordial power spectrum amplitude and dark energy equation of state.

**Introduction** — The matter power spectrum is a fundamental tool for constraining the cosmological parameters. It contains detailed information about the large-scale geometrical structure of space-time, as well as the phenomenological properties of dark energy and dark matter. Given a galaxy redshift survey two things are crucial: how to obtain an unbiased and optimal estimate of the information in the matter fluctuations.

State-of-the-art galaxy redshift surveys, such as the Baryon Oscillation Spectroscopic Survey [BOSS] hereafter BOSS, Galaxy And Mass Assembly [GAMA], and WiggleZ [WZ], have all used the approach of Feldman et al. [FKP] hereafter FKP to estimate the power spectrum. This assumes that galaxies are a Poisson sampling of the underlying density field. Hence, provided one subtracts an appropriate shot-noise term, and deconvolves for the survey window function, one should obtain an unbiased estimate of the matter power spectrum.

In the last two decades, our understanding of galaxy formation has made rapid progress since the work of FKP and our current best models strongly suggest that galaxies are not related to matter in the way they envisioned. Furthermore, observational studies have discovered that galaxy clustering depends on various physical properties: e.g., luminosity [12–18], colour [17, 19–22], morphology [14, 23, 24], and stellar mass [25] etc.

Percival et al. hereafter PVP attempted to correct the FKP framework to take into account the effects of luminosity-dependent clustering. In a recent paper hereafter SM15, we argued that the approach of PVP, whilst appearing qualitatively reasonable, is in fact at odds with our current understanding of galaxy formation, and so non-optimal. More recent studies by Seljak et al. [29], Hamaus et al. [29] and Cai et al. [30] suggested that weighting the galaxy density field by a linear function of halo mass would reduce stochasticity.

In SM15 we developed a new scheme incorporating a number of the key ideas from galaxy formation: galaxies only form in dark matter haloes [27]; haloes can host galaxies of various luminosities; the large-scale bias associated with a given galaxy is largely inherited from the bias of the host dark matter halo.

In this work we demonstrate that our new optimal estimator indeed provides both improved signal-to-noise (hereafter $S/N$) estimates of the galaxy power spectrum and boosted cosmological information content, when compared with the FKP and PVP approaches.

This letter is broken down as follows: First, we provide a brief overview of the results from SM15. Next, we evaluate the $S/N$ expressions for the various weighting schemes, followed by the cosmological information from a putative all-sky galaxy redshift survey. Finally, we conclude.

**Optimal power spectrum estimation** — In the original work of FKP, the starting concept is that galaxies are an independent point sampling of the underlying galaxy field. Hence,

$$n_g(r) = \sum_{i=1}^{N_g} \delta^D(r - r_i),$$

(1)

where $N_g$ is the number of galaxies, and $r_i$ is the position of the $i$th galaxy in the survey. From this field one then may construct an effective galaxy overdensity field:

$$F_{FKP}^g(r) = \Theta(r) w(r) \left[n_g(r) - \alpha n_s(r)\right],$$

(2)

where $\Theta(r)$ is a survey mask function, which is 1 if the galaxy lies inside the survey volume and 0 otherwise, $\alpha$ is a scaling factor for the spatially random galaxy field $n_s(r)$ and $w(r)$ is an optimal weight function that depends on $r$. If we now follow the FKP logic and compute the power spectrum of the $F_{FKP}^g$ field, one finds that it is related to the galaxy power spectrum $P_g(k)$ through the relation:

$$\langle |F_{FKP}^g(k)|^2 \rangle = \int \frac{d^3k}{(2\pi)^3} P_g(k') \left|\tilde{F}_{FKP}^g(k-k')\right|^2 + \rho_{shot},$$

(3)
where \( \hat{G}_{FKP}(k) \) is the weighted version of the Fourier transform of the survey mask function \( \Theta(r) \), and \( P_{\text{PKP}}^\text{shot} \) is an effective shot noise correction. If one subtracts the shot noise and deconvolves for the survey window function, then one may obtain an estimate of \( P_g(k) \). It is important to realise that the above procedure is only correct under the assumption that the galaxy power spectrum does not depend on any observable, e.g. galaxy luminosity, colour, spectral type, host halo mass, etc. and that shot noise is as was given by FKP. If these assumptions are wrong, then the functions \( P_g(k), \hat{G}_{FKP}(k), \) and \( P_{\text{PKP}}^\text{shot} \) will all pick up these dependencies, resulting in a biased and sub-optimal reconstruction of the ‘true’ power spectrum [31]. As noted earlier, current observational evidence indicates that clustering strength does depend on the sample selection. Hence, FKP must be biased and sub-optimal (PVP also came to a similar conclusions).

We now summarise the SM15 formalism, designed to account for a number of these effects. Consider a large survey volume containing \( N_g \) dark matter haloes. Thus the \( j \)th dark matter halo of mass \( M_j \) and position of the centre of mass \( \mathbf{x}_j \), will have \( N_g(M_j) \) galaxies. The \( j \)th galaxy will have a position vector \( \mathbf{r}_j \) relative to the centre of the halo and a luminosity \( L_j \). For this more complicated distribution, Eq. (1) can be generalized to:

\[
n_g(\mathbf{r}, L, x, M) = \sum_{i=1}^{N_g(M_i)} N_g(M_i) \times \sum_{j=1}^{N_g(M_j)} \delta^D(\mathbf{r} - \mathbf{r}_j - \mathbf{x}_i) \delta^D(M - M_j).
\]

where the four Dirac delta functions, going from right to left, are sampling: the luminosity of each galaxy in a given halo; the spatial location of a given galaxy relative to the halo centre; the halo mass from the distribution of masses; and the halo centre in the survey volume.

In direct analogy with FKP’s Eq. (2), we define an effective galaxy overdensity field:

\[
F_g(\mathbf{r}) = \int dL \int d^3x \int dM \Theta(r/L) \frac{w(\mathbf{r}, L, x, M)}{\sqrt{A}} \times \left[ n_g(\mathbf{r}, L, x, M) - \alpha n_g(\mathbf{r}, L, x, M) \right],
\]

where \( \Theta(r/L) \) is the luminosity-dependent survey geometry function; \( A \) is a normalisation constant; \( \alpha \) is a scaling factor for the random halo catalogue; \( w \) is a general weight function. The function \( n_g \) is the same as \( n_g \), except the spatial locations of the halo centres have been randomized and the number density has been scaled up by a factor of \( 1/\alpha \). As shown by SM15, in the large-scale limit, when the distribution of galaxies in haloes adopts a Dirac-delta-function-like behaviour, the \( F_g \) power spectrum can be written:

\[
\left\langle |F_g(\mathbf{k})|^2 \right\rangle \approx \int \frac{d^3q}{(2\pi)^3} P(q) \left| \hat{g}^{(1)}_{(1,1)}(\mathbf{k} - q) \right|^2 + P_{\text{shot}},
\]

where \( P(q) \) is the true matter power spectrum, which is convolved with the effective survey window function \( \hat{g}^{(1)}_{(1,1)}(\mathbf{k}), \) and \( P_{\text{shot}} \) is a new effective shot noise. The set of survey window functions that are required to evaluate these expressions can be written in general:

\[
g^{(n)}_{(l,m)}(\mathbf{r}) \equiv A^{-nl/2} \int dM \bar{n}(M, \chi) b^n(M, \chi) N_g^{(n)}(M) \times \left[ \int dL \Theta(\mathbf{r}/L) \Phi(L|M) w^l(\mathbf{r}, L, x, M) \right]^n,
\]

where in the above \( \bar{n}(M, \chi) \) and \( b(M, \chi) \) are the mass function and large-scale bias of haloes of mass \( M \) at radial position \( \chi \) from the observer (\( \chi \) here is also acting as coordinate time); \( N_g^{(n)}(M) \) gives the \( n \)th factorial moment of the halo occupation distribution (hereafter HOD); \( \Phi(L|M) \) gives the conditional probability density that a galaxy hosted in a halo of mass \( M \) has a luminosity \( L \). Using these functions the effective shot noise term can be written:

\[
P_{\text{shot}} \equiv (1 + \alpha) \left[ \int \frac{d^3q}{(2\pi)^3} \hat{g}^{(2)}_{(1,0)}(\mathbf{q}) + \hat{g}^{(1)}_{(2,0)}(\mathbf{0}) \right],
\]

We also introduce the normalisation-free window functions \( \tilde{g}^{(n)}_{(l,m)} = A^{nl/2} g^{(n)}_{(l,m)}, \) which enables us to write:

\[
A \equiv \int d^3r \left| \hat{g}^{(1)}_{(1,1)}(\mathbf{r}) \right|^2.
\]

We thus see that, similar to the FKP approach, in order to recover the matter power spectrum, one must subtract the effective shot-noise term and deconvolve for the square of the effective survey window function \( \hat{g}^{(1)}_{(1,1)}(\mathbf{k}) \).

In the large-scale limit and under the assumption that the matter density field is Gaussianly distributed, SM15 also showed that the \( S/N \) can, for an arbitrary weight \( w \), be written in general as:

\[
\left( \frac{S}{N} \right)^2 \approx \frac{V(k)}{2(2\pi)^3} \left[ \int d^3r \left| \hat{g}^{(1)}_{(1,1)}(\mathbf{r}) \right|^2 \right]^2 \left\{ \int d^3r \left[ \left| \hat{g}^{(1)}_{(1,1)}(\mathbf{r}) \right|^2 + \frac{(1 + \alpha)}{P(k)} \left| \hat{g}^{(2)}_{(1,0)}(\mathbf{r}) + \hat{g}^{(1)}_{(2,0)}(\mathbf{r}) \right|^2 \right]^{-1} \right\},
\]

where \( V(k) = 4\pi k^2 \Delta k \left[ 1 + (\Delta k/k_0)^2 / 12 \right] \) is the volume of the \( i \)th \( k \)-space shell in which \( P(k) \) is estimated.
Comparison of weighting schemes — The failure of the FKP scheme to characterise the true clustering strengths of galaxies means that it is a biased and sub-optimal estimator. We will now show explicitly, under the assumption that the SM15 description of the galaxy population is the correct one, that both the FKP and PVP weighting schemes do indeed lead to sub-optimal measurements of \( P(k) \). The weighting schemes are:

- **The FKP weights:** These depend only on the position of the galaxy in the survey:
  \[
  w_{\text{FKP}}(r) = 1/[1 + \pi_g(r)P(k)],
  \tag{10}
  \]
  where \( \pi_g(r) \) is the mean number density of galaxies.

- **The PVP weights:** These depend explicitly on the luminosity dependence of the galaxy bias and also the position in the survey:
  \[
  w_{\text{PVP}}(r, L) = b(L)/\left[1 + \pi_g(r)\beta_L^2(r)P(k)\right],
  \tag{11}
  \]
  where the luminosity-dependent galaxy bias is \( b(L) \equiv \int dM\tilde{n}(M)b(M)N_g^{(1)}(M)\Phi(L|M)/\Phi(L) \), \( \beta_L^2(r) \equiv \int L_{\text{min}}(r) dL\beta^2(L)\Phi(L)/\pi_g(r) \) is the average square of the luminosity bias. The galaxy luminosity function is given by \( \Phi(L) \equiv \int dM\tilde{n}(M)N_g^{(1)}(M)\Phi(L|M) \), and \( N_g^{(1)}(M) \) was introduced after Eq. (7).

- **Optimal weights:** In the large-scale limit, these weights depend only on the galaxy’s spatial position and its host halo mass, and not explicitly on its luminosity. The weights are:
  \[
  w_{\text{OPT}}(r, M) = b(M)/\left[1 + R(M)S(r, M)\right]\left[1 + \tilde{n}_{\text{eff}}(r)P(k)\right],
  \tag{12}
  \]
  where \( R(M) \equiv N_g^{(2)}(M)/N_g^{(1)}(M) \) is the ratio of the second and first factorial moments of the halo occupation distribution and we introduced the effective number density of galaxies: \( \tilde{n}_{\text{eff}}(r) \equiv \int dM\tilde{n}(M)b^2(M)N_g^{(1)}(M)S(r, M)/\left[1 + R(M)S(r, M)\right] \).

We defined \( S(r, M) \equiv \int L_{\text{min}}(r) dL\Phi(L|M) \) as the fraction of galaxies hosted by haloes of mass \( M \) that are observable at a spatial position \( r \), with \( L_{\text{min}}(r) \) the minimum luminosity that a galaxy could have and still be observable given the survey flux-limit. Explicitly, \( L_{\text{min}}(r) = 10^{-\frac{1}{2}\frac{m_{\text{lim}}-25-\log(L_{\odot})}{h^2}}[d_L(r)/h^{-1}\text{Mpc}]^{-2}L_{\odot} \), where \( m_{\text{lim}} \) is the apparent magnitude limit of the survey, \( L_{\odot} \) is the absolute magnitude of the sun, \( h \) is the dimensionless Hubble parameter and \( d_L(r) = (1+z)\chi(z) \) is the luminosity distance in flat cosmological models. Note that \( S(0, M) = 1 \) and \( S(\infty, M) = 0 \). For more details on the \( S/N \) expressions for the three weights considered, we refer the interested reader to SM15.

We now show the \( S/N \) on the galaxy power spectrum corresponding to the FKP, PVP and SM15 methods for weighting the galaxy distribution. As a concrete example we consider a flux-limited, full-sky galaxy redshift survey spanning the redshift range \( z = 0.3-0.9 \). In order to evaluate the above expressions we need to specify several model ingredients. For the evolution of \( \tilde{n}(M) \) and \( b(M) \) we use the models of Sheth and Tormen [32]. For the conditional probability distribution \( \Phi(L|M) \) and the first factorial moment of the HOD \( N_g^{(1)}(M) \), we use the Conditional Luminosity Function (CLF) model of Yang et al. [33]. For the second factorial moment we use the model: \( N_g^{(2)}(M) = \beta(M)\left[N_g^{(1)}(M)\right]^2 \), where from fitting to semi-analytic models of galaxy formation \( \beta^{1/2}(M) = \frac{1}{2}\log_{10}(M/10^{11}h^{-1}\text{M}_\odot) \) for the case that \( M < 10^{13}h^{-1}\text{M}_\odot \) and unity otherwise [34]. From these ingredients all required variables may be computed.

Figure 1 shows the \( S/N \) for the SM15 (blue lines) and PVP (red lines) schemes ratioed with the \( S/N \) for the FKP scheme, respectively. The results are presented as a function of limiting \( b_1 \) magnitude and for various \( k \)-mode bins. Clearly, the optimal scheme of SM15 does indeed lead to the largest \( S/N \), \( \gtrsim 5\% \) improvement over FKP at \( k \sim 0.2\text{ h Mpc}^{-1} \), and \( \gtrsim 20\% \) improvement at \( k \sim 0.5\text{ h Mpc}^{-1} \) for surveys with depth \( b_1 \gtrsim 22 \). Interestingly, the scheme of PVP leads to the least optimal set of estimates, being \( \sim 20\% \) lower than FKP at \( k = 0.2\text{ h Mpc}^{-1} \) and \( \sim 40\% \) lower by \( k = 0.5\text{ h Mpc}^{-1} \), again for surveys with \( b_1 \gtrsim 22 \).

**Forecasting cosmological information** — The ability of a set of power spectrum band-power estimates to con-
FIG. 2. Forecasted 1D marginalized errors and relative errors on cosmological parameters as a function of the maximum wavenumber considered in the power spectrum estimates from a full-sky galaxy clustering survey of depth $b_1 \sim 22$. The solid red, dashed blue and black dotted lines represent the SM15, FKP and PVP weighting schemes, respectively. The panels, going clockwise from the top-left show the results for the eight cosmological parameters considered. The largest potential information gains to be had from optimal weighting are in the measurements of $\{w_0, w_1, A_s\}$. Note that we have not properly taken into account the growth evolution of structure, and used power spectrum derivatives suitable for only a single redshift.

strain the cosmological parameters $\theta_\alpha$, can be forecasted through construction of the Fisher information matrix [35]. For a continuum limit of Fourier modes the Fisher matrix can be expressed as [36]:

$$
\mathcal{F}_{\alpha\beta} = \int \frac{d^3k}{V(k)} \frac{\partial \log P(k)}{\partial \theta_\alpha} \frac{\partial \log P(k)}{\partial \theta_\beta} \left( \frac{S}{N} \right)^2(k).
$$

(13)

Thus, in order to compute the Fisher matrix, one needs to specify the $S/N$, and the derivatives of the power spectra with respect to the cosmological parameters. The former were computed in the previous section, and we estimate the latter at a single redshift. Therefore our forecasts will be pessimistic, since we do not fully take into account the information in the growth of structure, but here we are only interested in the relative differences between the three weighting schemes.

For our fiducial model we adopt a flat, dark-energy dominated cosmological model, characterised by eight parameters: $\theta_\alpha \in \{w_0, w_1, \Omega_{DE}, \Omega_c h^2, \Omega_b h^2, A_s, n_s, \alpha_s\}$. The first two characterise the equation of state for dark energy: $w(a) = p_w/\rho_w = w_0 + (1 - a)w_1$; $\Omega_{DE}$ is the dark energy density parameter; $\Omega_c h^2$ and $\Omega_b h^2$ are the physical densities in CDM and baryons, respectively; and $A_s$, $n_s$, and $\alpha_s$ denote the amplitude, spectral index, and running of the primordial scalar power spectrum, respectively. We adopt the values $\theta_\alpha = \{-1, -0.69, 0.12, 0.02, 2.15 \times 10^{-9}, 0.96, 0\}$, consistent with Planck data [37]. The power spectrum derivatives we compute through finite differencing matter power spectra from CAMB [38].

Figure 2 shows the forecasted 1D marginalized errors on the parameters, as a function of the maximum wavenumber $k_{\text{max}}$ entering the integral of Eq. (13). The panels show the fractional error, or if the fiducial value is zero, the error. Clearly, the smallest errors are obtained when one implements the optimal weighting scheme of SM15 (red solid lines), followed by FKP (blue dashed line) and then PVP (black dotted lines). We notice that the constraints on ($A_s, w_0, w_1$) show the most significant improvements from the optimal weighting.

Figure 3 shows the forecasted 2D marginalized errors on various parameter combinations. The line styles are the same as in Figure 2. Again, the optimal weighting of SM15 performs best and the parameters ($A_s, w_0, w_1$) appear to be the most affected by the new scheme.

Conclusions — In this letter we presented an overview of the optimal power spectrum estimation scheme of SM15. We argued that the FKP scheme was biased and sub-optimal since it does not take into account variations of clustering with the galaxy sample. We argued that the SM15 framework, which encodes several key concepts from the theory of galaxy formation, is able to describe these variations. We evaluated the $S/N$ resulting from
FIG. 3. Forecasted 2D marginalized errors on cosmological parameters for the eight cosmological parameters considered. The maximum wavenumber was set to $k = 0.5h\,\text{Mpc}^{-1}$ and the flux-limit was taken to be $b_J = 22$. Note that we have not properly taken into account the growth evolution of structure, and have used only power spectrum derivatives suitable for a single redshift. Nevertheless, it can be clearly seen that the optimal weighting scheme provides the tightest constraints on parameters.

In the previous studies of [28, 29] it was claimed that weighting galaxy groups by some linear function of the halo mass would lead to reduced shot-noise and hence boosted signal-to-noise estimates. In the limit of large number of galaxies per halo, the mass dependence of the SM15 weights is $w \propto b(M)/N_h^{(1)}(M)$, whereas in the limit of small numbers $w \propto b(M)$. Clearly, the SM15 weighting scheme does not follow this mass scaling. The calculation of SM15 has maximised the $S/N$ on power spectrum estimates, albeit under certain assumptions, whereas the calculations of Seljak et al. [28] and later Hamaus et al. [29] have minimised the stochasticity on the halo density field. These two things are not obviously the same. We argue that our approach is the correct path to follow since, by design, it minimises directly errors in the power spectrum and it has clearly built into its framework the corner stones of galaxy formation theory.

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