Approach and Application of Semi-Blind Source Separation for Aero-Engine Vibration Signals Using ICA-R

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Abstract. In this paper, the approach and application of semi-blind source separation (SBSS) in aero-engine vibration signal is studied. Firstly, the features of aero-engine vibration signal and difficulties for blind source separation (BSS) are summarized, and the SBSS incorporated the available prior knowledge is match to the goal of signal processing. Then, the ICA with reference (ICA-R) algorithm based on classical FastICA is introduced, with Newton iteration and gradient descent iteration approach to obtain optimal solution. The unique parameters in ICA-R for aero-engine vibration signal are also provided. Finally, the efficacy and the accuracy of the ICA-R algorithm are verified by numerical simulations and real engine vibration signals. The approach of SBSS in this paper perfectly suited to handle aero-engine vibration source separation and it lead to efficient implementation in fault diagnosis.

1. Introduction

Vibration signal offers important information for aero-engine condition monitoring and fault diagnosis. However, for the vibration signal of the whole engine, how to judge the position of the specific vibration is imperative to fault diagnosis. Blind source separation (BSS) is the issue of recovering the various independent vibration sources, which is of great significance for locating faulty part accurately. Independent Component Analysis (ICA)[1-2] is a statistical technique that are widely used to detect source or independent component for solving BSS problem.

The semi-blind source separation (SBSS) that incorporates the available prior knowledge into the separation process has gradually become a more suitable method for processing complicate signals. Specifically, ICA with References (ICA-R) [3] impose prior knowledge into mathematical process, forces the optimization process converge to the target component by formulating a rough template as reference signals.

In term of SBSS, Lin[4] introduced the pre-processing and separation vector standardization methods to improve the iteration speed of one-unit ICA-R algorithm. Zhang[5] studied reference signal construction method based on the second-order cumulant. De Vos[6] provided spatially constrained ICA(scICA) to deals with the situation when one mixing vector is exactly known. Wang[7] proposed an original CICA (OCICA) algorithm based on fixed-point iterative to simplify the learning rate parameters in the iterative process. Wang[8] extended the CICA method to the complex domain, and proposed cyclic stationary constraint (ICA-CC) and spatial constraint (ICA-SC) algorithms. Yang[9] used the rotation speed information to separate signals in the wavelet domain, and effectively extracted the information of main vibration sources such as high and low-pressure rotor, combustion chambers and accessory casings. Such methods provided good result but suffer from complexity in algorithm and application in aero-engine.
Combined with the engineering background of aero-engine vibration signals, this paper presents an SBSS approach based on ICA-R. Section 2 discusses the exclusive features of aero-engine vibration signal and describes the basic SBSS model. The ICA-R algorithm is presented in Section 3. Finally, the methodology is verified by number simulation and a convincing case in application of engine in Section 4.

2. Blind source separation model for aero-engine vibration signals

2.1. Basic BSS model

Generally, it can be considered as an instantaneous mixing model for independent signals [10]. The basic mixing-demixing model without noise is:

\[ x = As \]  
\[ y = Wx \]

Where \( A \) is mixing matrix, represents the force transmission path; \( W \) is demixing matrix, and \( W = [w_1^T, w_2^T, \ldots, w_n^T] \), while \( w \) is demixing vector. And different components during mixing-demixing process are systematically displayed as following:

1. the source signal denoted by \( s \).
2. the raw observed signal denoted by \( x \).
3. the observed signal after preprocessing denoted by \( z \).
4. the reference signal denoted by \( r \).
5. the demixing signal (output estimation) denoted by \( y \).

2.2. BSS of aero-engine: a list of exclusive features

Due to the complicated structure of the aero-engine, there are difficulties BSS is faced with when dealing with aero-engine vibration signals. These conflicts have never been explicitly recognised before.

2.2.1. Coupling of vibration sources

There are physical relevance of different parts in aero-engine such as inter-shaft bearing and common bearing cavity, whereas the basic assumption of BSS is the sources are irrelevant from each other. It is general that the dynamics relationship between excitation and response for different parts coupled, which caused vibration sources are not independent. Taking dynamic equations of dual-rotor with the inter-shaft bearing as an example:

\[ m_i \ddot{x}_i + d \dot{x}_i + k_2 x_i = F_{\text{bearing}} + F_{\text{fault}} + m_i g \]  
\[ m_h \ddot{x}_h + d \dot{x}_h + k_2 (x_h - x_i) = F_{\text{bearing}} + F_{\text{fault}} + m_h g \]

The fault force in any rotor can transmits to the other rotor via the inter-shaft bearing, which means the vibration signals in low and high-pressure rotor are relevant in statistics.

2.2.2. Nonlinear mixing

In can be seen from equation (1) and (2) that the BSS adopt linear equation to demix signals, while the fact that strong nonlinear mixing in aero-engine are rarely described by simple linear algebraic equations. For instance, a rigid rotor on anisotropic flexible supports including gyroscopic effects [11] is expressed as:

\[
\begin{bmatrix}
\Omega G & M \\
M & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{r}
\end{bmatrix}
+
\begin{bmatrix}
K & 0 \\
0 & -M
\end{bmatrix}
\begin{bmatrix}
x \\
\dot{x}
\end{bmatrix}
=
\begin{bmatrix}
F \\
F
\end{bmatrix}
\]

Ignoring the nonlinear mixing will reduce the accuracy of BSS, or even lead to wrong result.

2.2.3. Overmuch number of sources

BSS algorithm require the number of observed signals to be equal or greater than the number of source, in other words, that \( s \) and \( y \) must have same number of rows. Yet sensibly the vibration sources cannot be all observable from limited sensors (six in test-rig normally). Figure 1 illustrates the infinite vibration source of aero-engine. It is obvious that deficient measurement unable to meet the requirement of BSS model.
2.3. Prior knowledge and SBSS

Actually, the application of BSS wish to extract the desired subset of source, like low and high-pressure rotor and bearing, and automatically discard the rest of uninteresting sources from the observed mixtures.

Also, the engine vibration test is not blind totally. The vibration of aero-engine is not a completely stochastic process while only the statistical properties of the data are utilized in BSS. Some of source signals can be identified by traditional signal processing (such as Fourier Transform). As shown in figure 2, some parameters are already known: (1) the amplitude and frequency of the low and high-pressure rotor, including fundamental frequency and the multiplier frequency; (2) the characteristic frequency main parts like bearing, driving gear, etc.

Figure 1. Typical example where the number of vibration source can hardly be inferred

Figure 2. Some prior knowledge about the source signals are available in aero-engine test.

Figure 3. Framework of SBSS of aero-engine vibration signals

Figure 3 gives the framework of SBSS, the prior knowledge needs to be fully used to avoid three difficulties and improve its feasibility. Based on this goal, the ICA with reference (ICA-R) that incorporates reference signals can extract the interesting source signal accurately.

3. Approach of semi-blind source separation

3.1. Classical FastICA algorithm

FastICA based on maximization of negentropy [12] is classical algorithm to solve BSS problem, which are introduced firstly here.

Before the FastICA, the raw observed signal should be preprocessed including centering step and whitening step to decorrelate observed signals and orthogonalize the demixing vector. The preprossing phase is introduced detailed in reference [1,2,13]. In this context, all observed signal are assumed been preprocessed and donated as $z$.

An reliable approximation of negentropy is defined as:
\[ J(y) \propto \{E[G(y)] - E[G(v)]\}^2 \]  

(6)

Where \( J \) is negentropy; \( y = w^T z \); \( v \) is a Gaussian variable with zero mean and unity variance. There is an inherent constraint condition \( E[(w^T z)^2] = \|w\|^2 = 1 \) in deducing negentropy. So the equation (6) transfer to:

\[
\begin{align*}
\text{max } & E[G(w^T z)] \\
\text{s.t. } & E[(w^T z)^2] = \|w\|^2 = 1
\end{align*}
\]

(7)

The equation (7) can be solved by Lagrange multipliers method, and the Lagrange function and its gradient is given by equation (8) and equation (9).

\[
L(w) = E[zg(w^T z)] + \beta w = 0
\]

(8)

\[
L'[w] = \frac{\partial L}{\partial w} = E[zz^T g'(w^T z)] + \beta I
\]

(9)

Where \( g(y) = G(y) \). The optimal solution of Lagrange function is searched by Newton iteration scheme as equation (10) and equation (11).

\[
w_{k+1} = w_k - \frac{L(w_k)}{L'(w_k)}
\]

(10)

\[
w_{k+1} = w_k - \frac{E[zg[w_k^T z]] + \beta w_k}{E[zz^T g'[w_k^T z]] + \beta}
\]

(11)

The approximate equation (12) is correct because of \( z \) is whitened, by which the equation (11) is simplified to equation (13).

\[
E[zz^T g'(w^T z)] \approx E[zz^T]E[g'(w^T z)]
\]

\[ = E[g'(w^T z)]I \]

(12)

\[
w_{k+1} = w_k - \frac{E[zg[w_k^T z]] + \beta w_k}{E[g'[w_k^T z]] + \beta}
\]

(13)

The both sides of equation (13) multiply the \( E[g'[w_i^T z]] + \beta \), and the final FastICA iterative formula is obtained as:

\[
\begin{align*}
\begin{cases}
w_{k+1} = E[zg[w_k^T z]] - E[g'[w_k^T z]]w_k \\
w_{k+1} = \frac{w_{k+1}}{\|w_{k+1}\|}
\end{cases}
\end{align*}
\]

(14)

In addition, \( g(\cdot) \) is the nonlinear function. The vibration signal of aero-engine, like sine and cosine signal, is sub-Gaussian signal. The references[14-15] tell a better choice of \( g(\cdot) \) and \( g'(\cdot) \) for aero-engine vibration separation is as follows:

\[
g(y) = -y - \tanh(y)
\]

\[
g'(y) = -1 - (1 - \tanh^2(y))
\]

(15)

FastICA provide decent solution to BSS problems due to clear iteration and cubic convergence speed.

3.2. ICA-R algorithm

The ICA-R take into account the prior knowledge as the reference signal into basic ICA paradigm. The ICA-R algorithm based on FastICA in term of aero-engine vibration signal is introduced here.
On one hand, maximization of negentropy can be adopted as the objective function. On the other hand, the reference signals can be created to represent rough templates of specific component vibration, and minimization of closeness measure function\(^{[16]}\) can be adopted as the other objective function:

\[
\begin{align*}
\max J(w) &= \sum E[G(w^T z)] \\
\min f(w) &= \varepsilon(y, r)
\end{align*}
\]

Where \(\varepsilon(y, r)\) is closeness measure function to scale distance between the demixing signal \(y\) and the corresponding reference signal \(r\).

For the first objective function with maximizing negentropy, it can be iterated by equation (14). For the second objective function with minimizing closeness measure function, it can be updated by gradient descent method:

\[
w_{k+1} = w_{k+1} + \nabla f(w)
\]

Where \(\nabla f(w)\) is the gradient of the closeness measure function with respect to the demixing vector \(w\). And the demixing vector must be standardized after each iteration step as equation (14). The final iteration formulation of ICA-R is given as:

\[
\begin{align*}
w_{k+1} &= E[zg[w_k^T z]] - E[g[w_{k+1}^T]]w_k \\
w_{k+1} &= w_{k+1} + \nabla f(w) \\
w_{k+1} &= w_{k+1}/\|w_{k+1}\|
\end{align*}
\]

In summary, the steps to achieve the ICA-R algorithm are shown as figure 4.

**Figure 4.** Flow chart of ICA-R algorithm

3.3. Analyses of parameter selection

Due to the complicated structure of the aero-engine, there are difficulties BSS is faced with when dealing with aero-engine vibration signals. These conflicts have never been explicitly recognised before.
3.3.1. Closeness measure function. The mean square error (MSE) is used as the similarity measure in previous studies because of simple format, easy derivative and rapid iteration. Mean square error is shown in equation (19) and the derivative function to demixing vector is shown in equation (20).

\[ \varepsilon(y, r) = E[(y - r)^2] \]  
\[ f(w_z) = E[(w_z^T Z - r)^2] \]
\[ f'(w_z) = E[(w_z^T Z - r)^2]' = E[2 \times z(w_z^T Z - r)] \]  

3.3.2. The reference signal for aero-engine. Low and high-pressure rotor, whose vibration is close to sine signal, are main aero-engine vibration sources in form of rotor fundamental frequency and its multiple frequency. So standard sine or sine superimpose signal can be selected as reference signal.

\[ r = \sum A \sin(2\pi \times f_i \times t + \phi) \]  

Where \( A \) is amplitude, \( f \) is frequency (Hz), and \( \phi \) is phase (rad). Subscript \( i \) denote different order of fundamental frequency. The fundamental frequency (rotation speed) of the high and low-pressure rotor is known in aero-engine vibration test, and amplitude and other frequency can be obtained easily through the FFT spectrum.

4. Verification and application

4.1. Numerical simulation on synthetic signals

The vibration of the aero-engine includes periodic vibration, modulated vibration, shock vibration, random vibration and so on. In order to verify the effectiveness of the ICA-R algorithm in aero-engine, the sine signal, modulated signal and the pulsed signal are selected as the three source signals for experiment:

\[
\begin{align*}
    s_1 &= 10 \sin(2\pi \times 10 \times t) \\
    s_2 &= 8 \cos(2\pi \times 75 \times t) \times 2 \sin(2\pi \times 0.8 \times t) \\
    s_3 &= 15 \text{sawtooth}(2\pi \times 30 \times t)
\end{align*}
\]  

The mixing matrix is generated as non-linear formation (equation(23)), and the waveform of source signals and mixing signals are as shown in figure 5 and 6.

\[
\begin{bmatrix}
    \sin & 0.22 & 0.16 \\
    0.22 & \cos & 0.33 \\
    0.25 & 0.15 & \sin \\
    2\sin & 2\sin & 2\sin
\end{bmatrix}
\]  

**Figure 5.** Synthetic source signals  
**Figure 6.** Mixing signals
The characteristics of the source signals are substantially invisible from the mixing signals. For \( s_1 \), it can be assumed that amplitude and frequency are known; For \( s_2 \), it can be assumed that higher frequency is known; For \( s_3 \), the reference are formulated by square signal with same amplitude and frequency. The three reference signals are shown in equation (24). The demixing signals calculated by FastICA and ICA-R are shown in figure. 7 and 8.

\[
\begin{align*}
    r_1 &= 10\sin(2\pi \times 10 \times t) \\
    r_2 &= 8\cos(2\pi \times 75 \times t) \\
    r_3 &= 15\text{square}(2\pi \times 30 \times t)
\end{align*}
\]  
\tag{24}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig7}
\includegraphics[width=0.4\textwidth]{fig8}
\caption{Demixing signals by FastICA}
\caption{Demixing signals by ICA-R}
\end{figure}

The correlation distance between the source signals and demixing signals can be calculated to evaluate the quality of two algorithms, which is defined as:

\[
d_y = \frac{\sum_{i=1}^{n} (s_i - \bar{s}_i)(y_j - \bar{y}_j)}{\sqrt{\sum_{i=1}^{n} (s_i - \bar{s}_i)^2 \times \sum_{j=1}^{m} (y_j - \bar{y}_j)^2}}
\]  
\tag{25}

And correlation distance is \([0.1629, 0.8843, 0.8777]\) in FastICA, while \([0.9390, 0.9678, 0.9621]\) in ICA-R. That means the ICA-R can extract source signals yet the FastICA cannot process non-linear mixing problem.

4.2. Case study with real-world turbine engine vibration signals

The application is on an actual dual-rotor turbine test rig wherein the amplitude of vibration was abnormally large during all test. Six acceleration sensors were measured on the front air intake casing, intermediate casing and rear casing, and in vertical and horizontal direction respectively. The sampling method is uniform period sampling tracking rotation speed of high-pressure rotor. A 16 periods raw observed in stationary state are displayed in frequency domain in figure 9. At the time, fundamental frequency of low and high-pressure rotor is \( f_1=149.7\text{Hz} \) and \( f_2=203.9\text{Hz} \) respectively.
A firstly FFT analysis revealed that combined frequency of $f_1+2f_2$ existed throughout the test. The FastICA is unable to separate rotor signals no matter how many source numbers are assumed, and SBSS is applied to further investigate which part produced the combined frequency. According to the figure 9, there are $f_1$, $2f_1$, $f_2$, $2f_2$, $f_1+2f_2$, and the amplitude of each frequency are also known. The reference signals of the low and high-pressure rotors are formulated respectively:

$$
\begin{align*}
\text{r}_1 &= 1\sin(2\pi \times f_1 \times t + \pi) + 0.5\sin(2\pi \times 2f_1 \times t + 9\pi/5) \\
\text{r}_2 &= 1\sin(2\pi \times f_2 \times t + 7\pi/5) + 0.5\sin(2\pi \times 2f_2 \times t + 2\pi/3)
\end{align*}
$$

The separation results are shown in time domain and frequency domain respectively in figure 10.

It can be seen from the figure 10 that, there are $f_1$, $2f_1$, $f_1+2f_2$ in signal $y_1$, which is vibration signal of low-pressure rotor clearly; there are $f_2$, $2f_2$ in signal $y_2$, which is vibration signal of high-pressure rotor clearly. Hence, it can conclude the combined frequency comes from the low-pressure rotor, and can be check to originate from rubbing between low-pressure rotor and stator[17].

Apart from separating rotor signals, the approach is general enough to apply in other mechanical circumstances, such as bearing, gear, stator, any part that have characteristic frequency.

5. Conclusions

In this work, the approach and application of ICA-R algorithm in aero-engine vibration signals are ascertained, and the SBSS provides promising method for fault diagnosis and condition monitoring of aero-engine. This paper followed the approach of not only explaining the algorithm, but also presenting the engineering background. The paper listed three difficulties related with aero-engine vibration signal when it comes to be processed by BSS: (1)un-independent, (2) nonlinear mixing and (3) overmuch sources. Because of effective utilization of the prior knowledge the SBSS perfectly suited to demix aero-engine vibration signal.
The ICA-R algorithm based on FastICA is introduced with detailed explanations. The maximizing negentropy and minimizing closeness between the reference signal and the demixing signal are selected as two objective functions. And the unconstrained optimization problem is solved by Newton method and gradient descent method. This work also contributes to provide the proper closeness measure function and reference parameters for aero-engine vibration signal. The advantage and superiority of this ICA-R algorithm compared to the earlier methods is that simple update formulation and without manual intervention except reference signal. Moreover, the experiment in synthetic signals and real-world signals have illustrated the use of the ICA-R approach, and the approach provide a efficacious way to diagnose fault for aero-engine. A difficulty in ICA-R is phase varies randomly in different acquisition channels and different frequency and cannot be judged in advance for creating reference signal. Unfortunately, the phase affects the similarity measure extremely, and the phase deviation is likely to cause failure of SBSS. Further exploration of how to decide phase in ICA-R is worth pursuing.

6. References
[1] Tharwat A 2019 Independent component analysis: An introduction. Appl. Comput. Inf., (Preprint)
[2] Choi S, Cichocki A, and Park H 2005 Blind source separation and independent component analysis: A review. Neural Inf. Process. 6 pp.1-57
[3] Lu W, and Rajapakse J C 2006 ICA with reference Neurocomputing 69 pp.2244-2257
[4] Lin Q, Zhang Y, Yin F, Liang H, and Calhoun V C 2007 A fast algorithm for one-unit ICA-R. Inf. Sci. 177 pp.1265-1275
[5] Zhang Z 2008 Morphologically constrained ICA for extracting weak temporally correlated signals. Neurocomputing 71 pp.1669-1679
[6] De Vos M, Lathauwer L, De and Van Huffel S 2011Spatially constrained ICA algorithm with an application in EEG processing. Signal Process. 91 pp.1963-1972
[7] Wang Z 2011 Fixed-point algorithms for constrained ICA and their applications in fMRI data analysis. Magn. Reson. Imaging 29 pp.1288-1303
[8] Wang X, Huang Z, Zhou Y and Ren X 2013 Approaches and applications of semi-blind signal extraction for communication signals based on constrained independent component analysis: The complex case. Neurocomputing 101 pp.204-216
[9] Yang G, Jing J, Ming Y, Yan Y and Chen C 2017 Blind vibration source separation method based on signal feature of aircraft engine. J. Xi’an Jiaotong University 51 pp.20-27
[10] Antoni J 2004 Blind separation of vibration components: Principles and demonstrations. Mech. Syst. Sig. Process. 19 pp.1166-1180
[11] Friswell M L, Penny J E T, Garvey S D and Lees A W Dynamics of Rotating Machines Cambridge university press 2010
[12] Hyvarinen A, Karhunen J and Oja E. Independent component analysis (John Wiley & Son, 2001)
[13] Hyvarinen A 1999 Fast and robust fixed-point algorithms for independent component analysis. IEEE Trans. Neural Networks. 10 pp.626 -634
[14] Dermoune A and Wei T 2013 FastICA algorithm: five criteria for the optimal choice of the nonlinearity function. IEEE Trans. Signal Process. 61 pp.2078-2087
[15] Song X and Liao M 2006 Vibration signal separation technical for double-rotor aero-engines. Mech. Sci. 25 pp.487–496
[16] Li C, Liao G and Shen Y 2010 An improved method for independent component analysis with reference. Digital Signal Process. 20 pp.575-580
[17] Wang Y, Wang L, Liao M and Ding X 2014 Exploring vibration characteristics of dual-rotor engine’s rotor-to-case rub-impact. Mech. Sci. Tech. Aero. Eng. 33 pp.614-620

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