SCALING IN MANY-BODY SYSTEMS AND PROTON STRUCTURE FUNCTION

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The observation of scaling in processes in which a weakly interacting probe delivers large momentum $q$ to a many-body system simply reflects the dominance of incoherent scattering off target constituents. While a suitably defined scaling function may provide rich information on the internal dynamics of the target, in general its extraction from the measured cross section requires careful consideration of the nature of the interaction driving the scattering process. The analysis of deep inelastic electron-proton scattering in the target rest frame within standard many-body theory naturally leads to the emergence of a scaling function that, unlike the commonly used structure functions $F_1$ and $F_2$, can be directly identified with the intrinsic proton response.

1 Introduction

Scaling is observed in a variety of scattering processes involving many-body systems. For example, at large momentum transfer $|q|$ the response of liquid helium measured by inclusive scattering of thermal neutrons, which in general depends upon both $q$ and the energy transfer $\nu$, exhibits a striking scaling behavior, i.e. it becomes a function of the single variable $y = (m/|q|)(\nu - q^2/2m)$, $m$ being the mass of the helium atom. Scaling in a similar variable occurs in inclusive electron-nucleus scattering at $|q| > 500$ MeV and electron energy loss $\nu < Q^2/2M$, where $Q^2 = |q|^2 - \nu^2$ and $M$ is the nucleon mass. Another most celebrated example is scaling of the deep inelastic proton structure functions measured by lepton scattering at large $Q^2$, in the Bjorken variable $x = Q^2/2M\nu$.

The relation between Bjorken scaling and scaling in the variable $y$, whose definition and interpretation emerge in a most natural fashion from the treatment of the scattering process within many-body theory in the target rest frame, has been discussed by many authors (see, e.g., ref.\[6\]). Recently, deep inelastic data have been also shown to scale in the variable $\bar{y} = \nu - |q|$, related to both $y$ and the Nachtman variable $\xi$ which in turn coincides with $x$ in the $Q^2 \to \infty$ limit.

The fact that scaling is observed in processes driven by different interactions clearly indicates that its occurrence reflects the dominance of a common reaction mechanism, independent of the underlying dynamics. In all instances, scaling is indeed a consequence of the onset of the impulse approximation (IA) regime, in which scattering of a weakly interacting probe by a composite target reduces to the incoherent sum of elementary scattering processes involving its constituents.

While the primary goal of scaling analysis is the identification of the dominant reaction mechanism, it has to be pointed out that the scaling variable has a straightforward physical interpretation, and a suitably defined scaling function, being directly related to the target response, contains a great deal of dynamical information.

In general, extracting the target response from the measured cross section re-
quires careful consideration of the nature of the interaction driving the scattering process. In neutron-liquid helium scattering, as the probe-constituent coupling is purely scalar, the cross section coincides with the response up to a kinematical factor. On the other hand, to obtain the target response in the case of electron-nucleus scattering one has to divide out the elementary electron-nucleon cross section. In this paper we will discuss the application of the latter procedure to the analysis of deep inelastic scattering (DIS) of electrons by protons.

The theoretical treatment of scattering off a many-body system and the assumptions involved in the impulse approximation (IA) are described in Section 2, where the case of neutron-liquid helium scattering is considered as a pedagogical example. Section 3 is devoted to the application of the many-body approach to the more complex case of electron-proton scattering, as well as to the derivation of the appropriate scaling variable and scaling function. The relation between the analysis of DIS proposed in this paper and the standard Bjorken scaling analysis is discussed in Section 4, where the implications of the differences between the two approaches are emphasized. Finally, Section 5 contains a summary and the conclusions.

2 Scattering off many-body systems in the IA regime and $y$-scaling

Let us consider scattering off a nonrelativistic bound system consisting of $N$ pointlike scalar particles of mass $m$, and assume that the probe-target interaction be weak, so that Born approximation can be safely used. The differential cross section of the process in which a beam particle, carrying momentum $k$ and energy $E$, is scattered into the solid angle $d\Omega$ with energy $E' = E - \nu$ and momentum $k'$ can then be written

$$\frac{d\sigma}{d\Omega dE'} = \frac{\sigma}{4\pi} \left| \frac{k'}{k} \right| S(q, \nu) ,$$

(1)

where $\sigma$ is the probe-constituent total cross section and the response function $S(q, \nu)$, containing all the information on the structure of the target, is defined as

$$S(q, \nu) = \sum_n |\langle n|\rho_q|0\rangle|^2 \delta(\nu + E_0 + E_n) = \int \frac{dt}{2\pi} e^{iqt} \langle 0|\rho_q^\dagger(t)\rho_q(0)|0\rangle .$$

(2)

In the above equations, $|0\rangle$ and $|n\rangle$ are the target ground and final state, satisfying $H|0\rangle = E_0|0\rangle$ and $H|n\rangle = E_n|n\rangle$, $H$ being the hamiltonian describing the internal target dynamics, $\rho_q(t) = e^{iHt}\rho_q e^{-iHt}$, with $\rho_q = \sum_k a_{k+q}^\dagger a_k$, and $a_{k+q}^\dagger$ and $a_k$ are constituent creation and annihilation operators, respectively.

Rewriting Eq.(2) in coordinate space leads to the expression

$$S(q, \nu) = \sum_n \left| \int dR \langle n|R \rangle \sum_{i=1}^N e^{i\mathbf{q}\cdot\mathbf{r}_i} \langle R|0 \rangle \right|^2 \delta(\nu + E_0 + E_n) ,$$

(3)

where $R \equiv (r_1, \ldots, r_N)$ specifies the target configuration, while $\langle R|0 \rangle$ and $\langle R|n \rangle$ denote its initial and final state wave functions.

The main assumption underlying IA is that, as the space resolution of a probe delivering momentum $q$ is $\sim 1/|q|$, at large enough $|q|$ (typically $|q| \gg 2\pi/d$, $d$
being the average separation between target constituents) the target is seen by the probe as a collection of individual particles. In addition, final state interactions (FSI) between the hit constituent, carrying a large momentum $\sim q$, and the residual (N-1)-particle system are assumed to be negligibly small.

In the IA regime the scattering process reduces to the incoherent sum of elementary processes involving only one constituent, the remaining (N-1) particles acting as spectators, and Eq.(3) simplifies to

$$S(q, \nu) = \sum_i \sum_n \left| \int dR \langle n|R \rangle e^{iq \cdot r_i} \langle R|0 \rangle \right|^2 \delta(\nu + E_0 + E_n) .$$

(4)

The IA final state $|n\rangle$, carrying total momentum $q$, has the structure

$$|n\rangle = |i, R\rangle = |i(p')\rangle \otimes |R(q - p')\rangle ,$$

(5)

its energy being given by $E_n = E_{p'} + E_R$, where $E_{p'} = |p'|^2/2m$ and $E_R$ denote the energies of the free struck constituent, with momentum $p'$, and the spectator (N-1)-particle system, with momentum $q - p'$, respectively. As a consequence, the sum over final states can be carried out replacing

$$\sum_n |n\rangle\langle n| \to \int d^3 p' |i(p')\rangle\langle i(p')| \sum_R |R(q - p')\rangle\langle R(q - p')| .$$

(6)

Substitution of Eqs.(5) and (6) into Eq.(4) yields (see, e.g., ref.8)

$$S(q, \nu) = \sum_i \int d^3 p \int dp_0 P_i(p, p_0) \delta(\nu + p_0 - E_{p+q}) ,$$

(7)

where the function

$$P_i(p, p_0) = \sum_R |\langle 0|i, R\rangle|^2 \delta(p_0 + E_R - E_0)$$

(8)

gives the probability of finding the $i$-th constituent with momentum $p$ and energy $p_0$ in the target ground state.

Eqs.(5) and (8) show that the IA response only depends upon $q$ and $\nu$ through the energy-conserving $\delta$-function, requiring

$$\nu + E_0 - E_R + E_{p+q} = 0 .$$

(9)

The occurrence of scaling, i.e. the fact that, up to a kinematical factor $K(|q|, \nu)$, $S(q, \nu)$ becomes a function of a single variable, simply reflects the fact that in the IA regime, in which energy conservation is expressed by Eq.(5), $q$ and $\nu$ are no longer independent variables. One can then define a new variable $y = y(q, \nu)$ such that, as $|q| \to \infty$, $K(|q|, \nu)S(q, \nu) \to F(y)$.

In the simple case of neutron scattering off liquid helium, in which the cross section can be written exactly as in Eq.(1) and the energy dependence of $P_i(p, p_0)$ can be safely neglected[6], Eq.(8) takes the form

$$\nu + \frac{p^2}{2m} - \frac{|p + q|^2}{2m} = 0 ,$$

(10)

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$m$ being the mass of the helium atom. It follows that, defining

$$y = \frac{m}{|q|} \left( \nu - \frac{|q|^2}{2m} \right), \tag{11}$$

in the $|q| \to \infty$ limit

$$\frac{|q|}{m} S(q, \nu) \to F(y). \tag{12}$$

Fig.1 shows the behavior of $F(y)$, defined by the above equation, measured in neutron scattering off superfluid $^4$He at $T = 1.6 \, ^\circ\text{K}$. The curves corresponding to $F(y)$ at $|q| \geq 15 \, \text{Å}^{-1}$ lie on top of one another, clearly indicating the onset of the scaling regime.

![Figure 1. Scaling functions $F(y)$, defined as in Eq.(12), measured by neutron scattering off superfluid $^4$He at $T = 1.6 \, ^\circ\text{K}$](image)

The variable $y$ defined by Eq.(11) does have a straightforward physical interpretation, and so does the scaling function of Eq.(12). The scaling variable can be identified with the initial longitudinal momentum of the struck atom, $p_\parallel$, while $F(y)$ can be related to the momentum distribution, $n(|p|)$. In fact, the connection linking the response in the scaling regime to $n(|p|)$ has been extensively exploited to obtain momentum distributions of normal and superfluid $^4$He from neutron scattering data.

3 Scaling in deep inelastic scattering and proton response

The unpolarized electron-proron ($ep$) scattering cross section is usually written in the form (see, e.g., ref.11)

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{Q^4} \frac{E'}{E} L_{\mu\nu} W^{\mu\nu}, \tag{13}$$
where $E$ and $E'$ denote the initial and final electron energy, respectively, $Q^2 = -q^2$, $q = k - k' \equiv (\nu, q)$ being the four momentum transfer, $\alpha$ is the fine structure constant and $L_{\mu\nu}(k, k')$ is the tensor associated with the electron, fully specified by the measured kinematical variables.

The information on the structure of the target is contained in the tensor $W_{\mu\nu}$. It can be written in a form reminiscent of Eq.(2), in which the density fluctuation operator $\rho_q$ is replaced by the proton electromagnetic current $J_{\mu}$ and the relativistic nature of the process is properly taken into account:

$$W_{\mu\nu} = \sum_n \langle 0| J_{\mu}^n|n\rangle \langle n| J_{\nu}(0) \rangle \delta^{(4)}(P_0 + q - P_n) = \int \frac{d^4x}{(2\pi)^4} e^{iqx} \langle 0| J_{\mu}^n(x) J_{\nu}(0)|0\rangle,$$

where $P_0 \equiv (M, 0)$, $M$ being the proton mass, and $P_n$ are the four-momenta of the initial and final hadronic states.

We will now make the assumption that the proton can be viewed as a many-body system, consisting of bound pointlike Dirac particles of mass $m$, and proceed to apply the analysis described in Section 2 to $ep$ scattering. This will of course amount to assume, as in the standard parton model of DIS, that over the short spacetime scale relevant to the scattering process confinement does not play a role, so that proton constituents can be described in terms of physical states. It has to be emphasized, however, that, unlike the parton model, the present approach does not involve the additional requirement that proton constituents be on mass shell.

In the IA scheme the target tensor given by Eq.(14) is replaced by a weighted sum of tensors describing the electromagnetic structure of proton constituents (compare to Eq.(7)):

$$W_{\mu\nu} \rightarrow \sum_i q_i^2 \int d^4p P_i(p) \frac{1}{2E_p} \frac{1}{2E_{p+q}} w_{\mu\nu}(p, q) \delta(\nu + p_0 - E_{p+q}),$$

where $E_p = \sqrt{|p|^2 + m^2}$, $E_{p+q} = \sqrt{|p + q|^2 + m^2}$ and $P_i(p)$ is the probability that the $i$-th constituent, whose charge in units of the electron charge is denoted $q_i$, carry four-momentum $p \equiv (p_0, p)$. As a consequence, the cross section of Eq.(13) can be cast in the form

$$\frac{d\sigma}{d\Omega dE'} = \sum_i q_i^2 \int d^4p P_i(p) \left( \frac{d\sigma_c}{d\Omega dE'} \right) \delta(\nu + p_0 - E_{p+q}),$$

$(d\sigma_c/d\Omega dE')$ being the elementary electron-constituent cross section.

Substitution of Eq.(15) into Eq.(13) and comparison with Eq.(16) leads to

$$\left( \frac{d\sigma_c}{d\Omega dE'} \right) = \frac{\alpha^2}{Q^4} \frac{E'}{E} \frac{1}{2E_p} \frac{1}{2E_{p+q}} L_{\mu\nu} w^{\mu\nu},$$

where the tensor $w^{\mu\nu}$ is defined as

$$w^{\mu\nu} = 2 \left\{ \tilde{\rho}^{\mu}(\tilde{\nu} + \tilde{q})^\nu + \tilde{\nu}^{\mu}(\tilde{p} + \tilde{q})^\nu - g^{\mu\nu} \left[ (\tilde{p}(\tilde{p} + \tilde{q})) - m^2 \right] \right\}.$$

with $\tilde{p} \equiv (E_p, p)$, $\tilde{q} \equiv (\nu, q)$ and $\tilde{\nu} = E_{p+q} - E_p = \nu + p_0 - E_p$. The above equations show that the formalism of IA allows one to describe scattering off a bound constituent in terms of the electromagnetic tensor associated with a free Dirac particle.
of initial four-momentum $\tilde{p}$ and final four-momentum $p' = \tilde{p} + \hat{q} = p + q$. It has to be mentioned, however, that, on account of the replacement $q \rightarrow \tilde{q}$, $w^{\mu\nu}$ defined by Eq. (18) is manifestly non gauge-invariant, i.e. $q_{\mu}w^{\mu\nu} \neq 0$. Gauge invariance can be restored using a somewhat ad hoc prescription, originally proposed by De Forest and widely used in the theoretical analysis of electron-nucleus scattering processes. Although violation of gauge invariance is in general an unpleasant feature inherent in the IA scheme, it does not play a quantitatively significant role in the context of DIS, as the non gauge-invariant contributions to the elementary cross section become vanishingly small in the $|q| \rightarrow \infty$ limit.

Using Eqs. (17) and (18) and following ref. 12 one gets

$$\left(\frac{d\sigma}{d\Omega dE'}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{M} \left[\sigma_{2} + 2\sigma_{1} \tan^{2} \frac{\theta}{2}\right],$$

(19)

where $(d\sigma/d\Omega)_{M}$ denotes the Mott cross section, $\theta$ is the electron scattering angle,

$$\sigma_{1} = \frac{1}{E_{p}E_{p+q}} \left( -\frac{q^{2}}{4} + \frac{p_{\perp}^{2}}{2} \right),$$

(20)

and

$$\sigma_{2} = \frac{1}{E_{p}E_{p+q}} \left( \frac{q^{2}}{|q|^{2}} \right) \left[ -\frac{q^{2}}{4} \left( \frac{q^{2}}{|q|^{2}} - 1 \right) + \left( \frac{q^{2}}{|q|^{2}} \right) \left( \frac{E_{p} + E_{p+q}}{2} \right)^{2} - \frac{p_{\perp}^{2}}{2} \right],$$

(21)

$p_{\perp}$ being the component of the constituent momentum perpendicular to the momentum transfer.

Substitution Eqs. (19)-(21) into Eq. (13) leads to the familiar expression of the $ep$ cross section in terms of the two structure functions $W_{1}$ and $W_{2}$:

$$\frac{d^{2}\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{M} \left[ W_{2}(|q|, \nu) + 2W_{1}(|q|, \nu) \tan^{2} \frac{\theta}{2} \right],$$

(22)

with

$$W_{1,2} = \sum_{i} q_{i}^{2} \int d^{4}p \; P_{i}(p) \; \sigma_{1,2}(p, q) \; \delta \left( \nu + p_{0} - \sqrt{|p + q|^{2} + m^{2}} \right).$$

(23)

Comparison between Eq. (13) and Eq. (1) shows that, due to the complexity of the electromagnetic interaction, in the case of electron scattering the quantity carrying the information on the structure of the target, i.e. its intrinsic response, no longer appears as a multiplicative factor in the cross section. The electron scattering cross section is indeed not trivially related to the target response. For example, while the response of a many-body system to a probe delivering momentum $q$ is in general nonzero in the region $\nu \geq |q|$, inaccessible to electron scattering, at $\nu = |q|$ the structure function $W_{2}$ of Eq. (22) vanishes, as required by gauge invariance, while $W_{1}$, proportional to the photoabsorption cross section, does not contribute to the cross section.

The above problem can be circumvented if, as in the case of electron-nucleus scattering\[3\] the momentum and energy dependence of the electron-constituent cross section is much weaker than that exhibited by the distribution function $P_{i}(p)$, which is a rapidly decreasing function of $|p|$ and the (positive) constituent binding energy.
\[ B = -p_0. \] Under this assumption, the elementary cross section, evaluated at a constant \( p = \overline{p} \) corresponding to the maximum of \( P_i(p) \), can be moved out of the integral in Eq. (16) and the proton response can be obtained from

\[ S(q, \nu) = \frac{d\sigma}{d\Omega dE'} / \left( \frac{d\sigma_c}{d\Omega dE'} \right)_{p = \overline{p}}. \] (24)

Having identified the target response, we can now proceed to define the scaling variable exploiting energy conservation. The requirement

\[ \nu + p_0 - \sqrt{|p + q|^2 + m^2} = \nu + p_0 - |q| + O \left( \frac{1}{|q|} \right) = 0, \] (25)

where \( p_0 = M - E_R \) and \( E_R = \sqrt{|p|^2 + M^2} \), \( M \) being the mass of the spectator system, implies that, as \( |q| \to \infty \), the quantity

\[ \tilde{y} = \nu - |q| = p_\parallel + p_0 \] (26)

becomes independent of \( q \). Hence, in this limit \( S(q, \nu) \) exhibits scaling in the variable \( \tilde{y} \), i.e.

\[ S(q, \nu) \to F(\tilde{y}). \] (27)

It has to be pointed out that \( \tilde{y} \) does not have the same physical interpretation as the variable \( y \) defined in Section 2, as it does not coincide with the constituent longitudinal momentum. Obviously, as \( p_0 \) is independent of \( q \), scaling in \( y \) necessarily implies scaling in \( \tilde{y} \), and vice versa. The choice of \( \tilde{y} \) as scaling variable in DIS is motivated by the fact that \( \tilde{y} \) is trivially related to another variable commonly used in the same context, the Nachtman variable \( \xi \) through

\[ -\frac{\tilde{y}}{M} = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}} = \xi, \] (28)

where \( x \) is the Bjorken variable. The above equation shows that in the \( Q^2 \to \infty \) limit \( \tilde{y} \) coincides with \( x \). It has to be noticed that the IA scheme provides a simple physical interpretation of Nachtman’s variable, whose definition was originally obtained in a totally different fashion.

4 \( \tilde{y} \)-scaling analysis of DIS data

According to the IA picture, the \( \tilde{y} \)-scaling function can be obtained dividing either structure function by the appropriate contribution to the elementary cross section. In fact, from Eqs. (22)-(24) and (27) it follows that, in the \( |q| \to \infty \) limit,

\[ S(q, \nu) = \frac{W_1}{\overline{\sigma}_1} = \frac{W_2}{\overline{\sigma}_2} \to F(\tilde{y}), \] (29)

where \( \overline{\sigma}_{1,2} = (\sigma_{1,2})_{p = \overline{p}} \), \( \overline{p} \equiv (B_0, p_{\text{min}}) \), \( B_0 \) is the minimum constituent binding energy and the magnitude of the minimum momentum allowed in the kinematics specified by \( q \) and \( \nu \) is

\[ |p_{\text{min}}| = \frac{1}{2} \left| \frac{M_R^2 - (\tilde{y} + M)^2}{\tilde{y} + M} \right|. \] (30)
with $M_R = M - m + B_0$.

The scaling analysis in terms of $\tilde{y}$ involves two parameters: the constituent mass, $m$, and binding energy, $B_0$, or equivalently $m$ and the mass of the spectator system, $M_R$. Fig. 2 shows the quantities $W_1/\sigma_1$ (upper panel) and $W_2/\sigma_2$ (lower panel), obtained from data taken at SLAC and CERN and rearranged in bins of constant $|q|$ centered at 11, 19 and 27 GeV, plotted as a function of $\tilde{y}$. The structure functions $W_1$ have been obtained using the parametrization of $R = \sigma_L/\sigma_T$ of ref. 14 while the elementary cross sections have been evaluated from Eqs. (20) and (21), with $m = 300$ MeV and $B_0 = 200$ MeV. It clearly appears that in both cases scaling sets in at $|q| > 10$ GeV.

Figure 2. Scaling functions obtained from $W_1/\sigma_1$ (upper panel) and $W_2/\sigma_2$ (lower panel). The experimental structure functions are from refs. 14, 15, 16 whereas the elementary cross sections have been evaluated using Eqs. (20) and (21), with $m = 300$ MeV and $B_0 = 200$ MeV.

The second feature predicted by the IA analysis, i.e. that $W_1/\sigma_1$ and $W_2/\sigma_2$ scale to the same function $F(\tilde{y})$ is illustrated in fig. 3.

It has to be pointed out that the occurrence of $\tilde{y}$ scaling does not depend upon the values of either $m$ or $B_0$. In particular, it does not require that the constituent mass be vanishingly small, nor that the constituent be on mass shell. Varying $m$ and $B_0$ in the range 10-300 MeV does not affect the scaling behavior displayed in figs. 2 and 3. However, it does affect the scaling function extracted from the data.
Figure 3. Comparison between $W_1/\sigma_1$ (diamonds) and $W_2/\sigma_2$ (crosses) of fig. 2 at $10 \leq |q| \leq 12$ GeV.

While in general different combinations of $m$ and $B_0$ correspond to different $F(\tilde{y})$’s, the scaling function turns out to be only sensitive to the difference $m - B_0$, i.e. to the mass of the spectator system $M_R$. Fig. 3 shows that increasing $M_R$ results in a shift of $F(\tilde{y})$ towards larger values of $\tilde{y}$ at $\tilde{y} > -0.5$ GeV.

Figure 4. Dependence of the $\tilde{y}$-scaling function upon the mass of the spectator system $M_R$. Crosses and diamonds correspond to $M_R = 638$ and 838 MeV, respectively. The constituent mass is $m = 300$ MeV.

DIS data are usually analyzed in terms of the two dimensionless structure functions $F_1 = MW_1$ and $F_2 = \nu W_2$. In the Bjorken limit $Q^2, \nu \to \infty$, with $Q^2/\nu$ finite
and $\nu/|q| \rightarrow 1$, both $F_1$ and $F_2$ exhibit scaling in the variable $x$, whose definition in the target rest frame is $x = Q^2/2M\nu$.

The discussion of the previous Sections shows that, while $\tilde{y}$ coincides with $x$ in the Bjorken limit (see eq. (28)), the $\tilde{y}$-scaling function cannot be identified with either $F_1$ or $F_2$. In fact, the structure of $F(\tilde{y})$ is dictated by target dynamics only, whereas both $F_1$ and $F_2$ depend upon the electron-constituent coupling as well. In general the scaling behavior displayed by $F_1$ and $F_2$ is a consequence of the fact that, in addition to the proton response, the quantities $\sigma_1$ and $\nu(\sigma_2)$ also scale.

$x$- and $\tilde{y}$-scaling analyses can be reconciled making the standard assumption of parton model that the constituent mass be vanishingly small. In fact, in the $m \rightarrow 0$ limit $\sigma_1 = 1$ and $\sigma_2 = Q^2/|q|^2$, implying in turn $F_1 = MF(\tilde{y})$ and $F_2 = \nu Q^2 F(\tilde{y})/|q|^2 = 2xF_1$. However, it has to be emphasized that, in spite of the fact that in textbook derivations the requirement $m \sim 0$ is often introduced as a necessary condition for Bjorken scaling (see, e.g., ref. [11]), the present analysis shows that scaling occurs irrespective of the constituent mass.

5 Conclusions

The results described in this paper show that the IA analysis of scattering processes off many-body targets, successfully employed to describe neutron-liquid helium and electron-nucleus data, can be extended to the case of DIS of electrons by protons.

While the scaling variable of many-body theory, whose definition and interpretation naturally emerge in the target rest frame, is trivially related to the Bjorken variable $x$, $\tilde{y}$-scaling analysis differs from the standard parton model of DIS in that it does not make any assumptions on the mass and binding energy of proton constituents. The occurrence of $\tilde{y}$-scaling turns out to be a mere consequence of the onset of the IA regime, and cannot be taken as unambiguous evidence of scattering off massless, on shell constituents.

The $\tilde{y}$-scaling function also differs from the commonly used structure functions $F_1$ and $F_2$, and can be directly identified with the intrinsic proton response. The dependence of $F(\tilde{y})$ upon the constituent binding energy implies that, if the interactions between proton constituents are strong enough to shift a fraction of the strength into the timelike region $\nu > |q|$, the proton response extracted from electron scattering data does not fulfill the constituent number sum rule. The occurrence of strength located in the region unaccessible to electron scattering is a well known feature of the response of interacting many-body systems. One that has long been recognized as the reason why the integral of the charge response measured by electron-nucleus scattering does not yield the total nuclear charge $Z$. Recently, it has been also shown that about 10% of the response of a scalar relativistic particle bound in a linear confining potential resides in the timelike region.

As a final remark, it has to be mentioned that the approach discussed in this paper, showing that scaling is by no means incompatible with massive constituents, suggests that the ability of the constituent quark model of the proton to describe DIS should be reconsidered, carrying out a $\tilde{y}$-scaling analysis in which finite mass effects are consistently taken into account in both the proton response and the electron-constituent interaction vertex.
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