LYAPUNOV FUNCTION CONSISTENT ADAPTIVE NETWORK SIGNAL CONTROL
WITH BACK PRESSURE AND REINFORCEMENT LEARNING

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ABSTRACT
This research studies the network traffic signal control problem. It uses the Lyapunov control function to derive the back pressure method, which is equal to differential queue lengths weighted by intersection lane flows. Lyapunov control theory is a platform that unifies several current theories for intersection signal control. We further use the theorem to derive the flow-based and other pressure-based signal control algorithms. For example, the Dynamic, Optimal, Real-time Algorithm for Signals (DORAS) algorithm may be derived by defining the Lyapunov function as the sum of queue length. The study then utilizes the back pressure as a reward in the reinforcement learning (RL) based network signal control, whose agent is trained with double Deep Q-Network (Double-DQN). The proposed algorithm is compared with several traditional and RL-based methods under passenger traffic flow and mixed flow with freight traffic, respectively. The numerical tests are conducted on a single corridor and on a local grid network under three traffic demand scenarios of low, medium, and heavy traffic, respectively. The numerical simulation demonstrates that the proposed algorithm outperforms the others in terms of the average vehicle waiting time on the network.

Keywords: Signal control, Network signal, Lyapunov control, Reinforcement learning, Back pressure
INTRODUCTION

Millions of travelers’ experience delay at signalized intersections daily (1). Intersection Signal control plays a vital role in urban life since it provides orderly movement and increases intersection traffic throughput (2). Quality signal control has been an intensively studied topic for long in the literature of traffic engineering. It remains to be one of the most fundamental traffic control problems. This paper specially examines the network signal control problem. It is taken as evident that efficient algorithms for traffic signal control shall be capable of efficiently dealing with various mix of traffic such as that with heavy freight traffic and also with various network topological characteristics. Therefore, in this paper, the traffic and the network are both assumed to be general when the control algorithm is derived.

This research assumes an information transparent environment for signal intersections in which traffic information such as queues and traffic progression are assumed known, at least for the purpose of derivation. The data rich environment is emerging today with the wide application of sensor and other information technologies. Technically, it has been possible and feasible to make the entire network traffic transparent when cost is not a major factor. Rapid progress in computational power has further made it feasible to conduct area wide or network wide signal control without having to worry about computational time. All of these has contributed to the so called information-driven intersection traffic control regime in real time. Of course, control regimes that relieve the computational burden by decomposing the complicated system into independent sub-systems are still desirable.

Traffic signal control is a distinct phenomenon in transportation engineering. Signals are designed to allocate the right-of-way in an efficient way at level crossings among traffic coming from different approaches. Based on The Manual on Uniform Traffic Control Devices (MUTCD), approach refers to a lane or lane groups that accommodates all left-turn, through, and right-turn movements from a given direction, such as north, south, west, and east. Phase is defined as "the right-of-way, yellow change, and red clearance intervals in a cycle that are assigned to an independent traffic movement or combination of traffic movements" (3). Signal timing is the outcome of a control policy that determines duration and sequences of phases at the intersection. The signal control is realized by signal timing. The signal timing can take the form of a fixed-time control, actuated control and adaptive control. Fixed timing has a fixed sequence of phases within the cycle and fixed duration for each phase. It use historical data and does not respond to real-time traffic. In contrast to fixed-timed control, actuated control has varying green time intervals in response to vehicle detectors. Detectors are used to collect arrival information under actuated control. Under actuated control, the duration of each phase is determined by detector input and corresponding controller parameters. Adaptive signal control refers to theories that use the current traffic data and historical performances to adjust traffic signal timing to optimize traffic flow in traffic signal systems. To design a signal control policy, we need to collect information through all the devices used in an intelligent system, such as sensors and inter-vehicle communication. The introduction of the Internet of Things (IoT) makes this large data collection possible, and the computation advantage in this era enables scholars to use highly complex optimization and decision-making methods. Reinforcement Learning (RL) is one of the most recent methods used in designing efficient and responsive decision-making tools in transportation networks (4).

RL is the method that can assist intelligent decision-making. It takes sequential actions similar to Markov Decision Process (MDP) with a rewarding criterion (5). It interacts with the environment and try to learn how to behave to maximize rewards, which can be used in complex
systems such as a transportation network. Using the RL, the agent in state $s_t$ takes an action of $a_t \in A$, at each time step $t$, where $A$ is the set of all actions. The actions are taken according to a policy $\pi = \{a_1, a_2, \ldots, a_k\}$. Taking an action results in a new state defined as $s_{t+1}$, and receive a reward (or penalty) $r(s_t, a_t, s_{t+1})$, which is defined as the feedback it gets from the environment based on its performance, typically the incremental performance. The goal of the agent is to maximize the expected cumulative reward $r(s_t, a_t, s_{t+1})$ and maintain a desirable state. Based on state and reward obtained from the environment, the RL agent updates policy to adjust the output action and finally achieves the near optimal policy (6).

RL has been widely applied in solving complex problems that involve a sequence of actions or decisions (7, 8). Our study aims to design a reinforcement learning based method to improve control policies from the observed traffic data, and in a larger aspect, define an optimum state and reward functions to improve mobility along corridors and grid networks. However, RL is usually computationally heavy when dealing with huge state spaces. To reduce and decouple the input state space to RL, we look into the control theory and address the scalability. We borrow some ideas from the max-weight policy in the stochastic queue network control and utilize the Lyapunov control theory, which provides a solid theoretical basis for the control policy and reward function design, to design a reliable and stable traffic signal control policy. Moreover, this study aims at providing a new objective to be used in network signal control to mitigate the overall delay of the network. Additionally, this study looks at a large network to design highly efficient network control algorithms by combining transport theory with reinforcement learning methods. The developed method can deal with traffic with uncertainty or randomness. The performance of the developed method is measured by using a back pressure metric, and the evaluation metric with other methods is the average delay of vehicles pass the network.

LITERATURE REVIEW

In literature, researchers typically classify along two categories: isolated and coordinated signals. Our study falls into the later one. Traffic signal timing comes to intense study in the late 1950s with deteriorating roadway traffic. Green bandwidth maximization algorithm was among the early methods to generate timing plans. The method tries to max the green time intervals along a corridor in which vehicles can progress without stop (9, 10). Morgan and Little first formulate the bandwidth maximization optimization problem as a mixed integer linear programming (MIP) problem and develop MAXBAND algorithm on arterial and network case (11–13). Gartner et al. develops MULTIBAND, which optimizes all the signal control variables and bandwidth progressions on individual link. (14, 15). The second algorithm to develop timing plans for an arterial street is based on minimizing the delay and number of stops, called flow profile methods. The methods have been applied in commercial software such as SYNCHRO, TRANSYT-7F (16), and PASSER V. PASSER V uses both algorithms to maximize progression or minimize total delay and works smoothly under both under-saturated and over-saturated traffic conditions over a set of possible phase sequences (17).

The literature behind adaptive control has been enriched over time. One of the first methods introduced in 1981 is SCOOT, a centralized system using data collected from upstream detectors for each intersection. SCOOT produces cycles, offsets, and splits the phases to maintain the saturation rate of an intersection around the "ideal" value. The method needs to get calibrated each time before it is used (18, 19). OPAC eliminates these three constraints and maximizes the number of vehicles passing through an intersection. It maximizes performance by continually optimizing
the system rather than periodically updating local controller settings \((20, 21)\). RHODES is another method that uses upstream vehicle data and stop-line detectors for each of the approaches to calculate loads on links and predict future platoon size and route choice \((22, 23)\). Varaiya introduced the max pressure (MP) concept to reduce the risk of over-saturation and maximize the network’s throughput. He defined the "pressure" of a phase as the difference between the total queue length of incoming and outgoing approaches and proposed a formulation to minimize the pressure for a signalized network with multiple intersections. The green time is given to phases with the most pressure. This concept was borrowed from in hydraulic when fluid flow is driven by differential pressure between upstream and downstream locations \((24)\). The MP algorithm is widely tested in simulation under various cases and needs only queue information at the intersection, however, it does not guarantee optimal results \((25, 26)\). DORAS-Q is a most recently developed traffic responsive control method that can be used for an isolated intersection in real-time. This method changes phases by measuring its efficiency, which is a function of the existing queues, short-term predictions for the current approach arrivals rates, and average historical arrival rates for other phases. The controller chooses the phase with the highest efficiency at the time of making a switch decision. This method is less data demanding and requires knowledge of the existing queues and queue discharge rate\((27)\).

Most adaptive methods are essentially modeling of queues or delays at intersections and then seeking, by acceptable approximation, control methods that achieve nearly optimal objective in the state transfer equation. They mainly develop algorithms to optimize a mathematically formulated objective approximation to facilitate the efficient movement of vehicles at an intersection. Different functions have been proposed to quantify efficiency, such as maximizing the green bandwidth on major arteries, minimizing the average travel time of vehicles, and maximizing the total number of vehicles through the network. These functions highly rely on travel time, delay, queue length, number of stops, and throughput \((28)\). Also, these formulations depend on certain assumptions about the traffic, such as assuming a uniform and constant rate for the vehicles’ movements.

Technologies today allow more granular details about traffic to become available such as real time traffic location and speed, real time queue length, headway, vehicle itineraries, etc. These additional information available about traffic enables development and application of more sophisticated methodologies and algorithms for traffic signal control. RL is one of them. With the computational advantages in today’s technology, new methods based on reinforcement learning can be an effective tool for optimizing traffic signal control. There are extensive attempts to improve traffic signal control performance by reinforcement learning to outperform the traditional transportation method \((29–33)\). However, the key question is how to define the state, action and reward functions in the RL’s framework. Different elements have been proposed to describe an environment state in the traffic signal control problem, such as queue length, waiting time, volume, speed, the position of vehicles, delay, phase, and duration \((28)\). The criterion for a good design of states and rewards is to enable the agent to extract useful information to be used in the optimization. There is a rich literature dedicated finding the optimal reward function \((34–37)\). The common idea to define the reward function is using a weighted sum of state components such as queue length, waiting time, and delay \((38)\), which could not always reach the objective of minimizing the delay or travel time.

There is a challenge in using the delay or travel time as an effective reward function in RL, although these two features seem to be aligned with the ultimate goal of the problem, which is to minimize the delay or travel time of all vehicles trespassing an intersection or intersections.
However, these two features may neither be instantly observable nor be directly obtainable at low cost for real time control. The delay can be influenced by several other factors such as free-flow speed, platoon dispersion, travel patterns, and vehicle components. This dependency can cause randomness in modeling a real-time signal control policy and map the states to unrealistic rewards, thus failing the model from convergence. Moreover, there is a gap in connecting the reward with measurement endorsed by traffic flow theories that can be effectively observed after an action. So, there is a need to properly justify the heuristics developed in RL literature based on traffic flow theories. Those reward functions include link congestion (critical roads’ density) (39) and difference between upstream and downstream flows (26).

Moreover, most current traffic signal control treats all vehicles the same when designing signal control strategies in the intersection (40). However, freight traffic is more likely to cause a longer delay due to vehicle dimension and slow dynamics (41). Representing the volume of mixed traffic is through converting the traffic to the passenger car equivalents (PCE) by using adjustment factors. PCE was first introduced in the US Highway Capacity Manual (HCM) to illustrate the effect of the truck on traffic stream according to headway ratio (42). HCM 2010 (43) assumes a PCE value of 2.0 for heavy vehicles approaching signalized intersections. Hereafter, researchers developed several methods and techniques, including saturation flow ratio method (44) and regression technique (45), to obtain static or dynamic PCE of the truck to consider freight traffic in signal control. Recently, freight traffic has been treated as noise or perturbation into state-space to examine the robustness of Reinforcement Learning methods on multi-modal traffic signal control (46).

This study proposes a new RL framework to design a traffic control policy considering freight traffic in major corridors. It considers traffic variations instead of using an idealistic traffic flow to bridge the gap in this research topic. Besides, the model aims to utilize myopic and easy-measured information to optimize signal control. Moreover, it looks at the network implication rather than focusing on an isolated intersection. The following section discusses the methodology of our algorithm by explaining the reinforcement learning method called Double-DQN, the stochastic optimization and reward design for the reinforcement learning algorithm, and the rationale behind designing a highly efficient network control algorithm. Then, the simulation framework to test the algorithm on arterial and grid network cases is explained, followed by the comparison method with well-established algorithms. Lastly, the results and conclusions are provided.

**NOTATIONS**

We first introduce the notations.

- \( N \): Total number of intersections in the network.
- \( l_m \): Lane \( m \). There are \( M \) lanes in the network in total.
- \( v_f \): Free-flow speed on the lane, different lanes may have different free-flow speed.
- \( M_i \): Set of all possible movements through intersection \( i \). A movement \((l_a, l_b) \in M_i\) represents the movement from lane \( l_a \) through intersection \( i \) to lane \( l_b \).
- \( \Phi_i \): Set of all the possible phases of intersection \( i, i \in \{1, 2, \ldots, N\} \).
\( \phi_k \)  
\( \phi_k \in \Phi_i \) are the phase controlling single or multiple movements at intersection \( i \). If \( (l_a, l_b) \in \phi_k \), phase \( \phi_k \) give the right of way to the movement from lane \( l_a \) through intersection \( i \) to lane \( l_b \).

\( d(t) \)  
Density on the lane at time step \( t \), \( d_j \) is the jam density on the lane.

\( \Theta(t) \)  
Queue vector, \( \Theta(t) = (\Theta_1^i(t), \ldots, \Theta_N^i(t)) \), where \( \Theta_1^i(t) \) denotes the queue length in lane \( l \) at intersection \( i \).

\( L(t) \)  
Lyapunov function, a non-negative scalar measure defined as the sum of squares of stochastic variables on slot \( t \).

\( \Delta(\cdot(t)) \)  
Lyapunov drift, defined as the difference in the Lyapunov function from time slot \( t \) and \( t + 1 \).

\( W_{ab} \)  
Differential queue backlog between lane \( l_a \) and lane \( l_b \). \( W_{ab} = \Theta_{l_a}^i(t) - \Theta_{l_b}^j(t) \).

\( f_i(\phi_{ij}, l_a, l_b, s) \)  
Flow going from \( l_a \) to \( l_b \) through intersection \( i \) under state \( s \) when phase \( \phi_{ij} \) is activated.

\( z_{im}^{\text{out}} \)  
Flow moves from intersection \( i \) to lane \( l_m \), \( z_{\max}^{\text{out}} = \max_{i,m} z_{im}^{\text{out}} \) represents the max outflow of a lane across the network.

\( z_{mi}^{\text{in}} \)  
Flow moves from lane \( l_m \) to intersection \( i \). \( z_{\max}^{\text{in}} = \max_{m,i} \sum_m z_{mi}^{\text{in}} \) represents the max inflow of a lane across the network.

\( A_{ci}(t) \)  
Process of exogenous vehicles arrival from lane \( c \) into the intersection \( i \) during slot \( \tau \). \( \mathbb{E}[A_{ci}(t)] = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} A_{ci}(\tau) = \lambda_{ci} \). \( A_{\max} \) is the max number of arrival vehicles into an intersection during a time slot across the network. \( \mathbb{E}[\sum_c A_{ci}(t)] \leq A_{\max} \).

\( s \)  
State, for example, queue length. State \( s \in S \), where \( S \) is the set of states and \( s_t \) represents the state at time step \( t \).

\( a \)  
Action, the decision made upon a state. Action \( a \in A \), where \( A \) is the set of all actions, and specifically, \( a_t \) represents the action at time step \( t \).

\( p(s_{t+1}|a_t, s_t) \)  
Transition probability function from state \( s_t \) to the next state \( s_{t+1} \) due to action \( a_t \).

\( r(s_t, a_t, s_{t+1}) \)  
Reward obtained after performing the action \( a_t \).

\( R_t \)  
Total discounted reward from time step \( t \).

\( \gamma \)  
Discount factor.

\( \alpha \)  
Learning rate, a hyper-parameter that controls how much to adjust the weights of the neutral network with respect to the loss gradient.

\( V(s) \)  
State value function under optimal policy \( \pi \) of state \( s \).

\( Q(s, a) \)  
State-action value function under policy \( \pi \) of the state-action pair \( (s, a) \).

Lyapunov control refers to the use of a Lyapunov function to adaptively control a dynamical system. A typical goal of Lyapunov control is to stabilize network queues while improving on some performances, such as average energy consumption. If a function \( L \) is a non-negative function on \( \mathbb{R} \), it is called Lyapunov function when two conditions are met. One is that it is a non-negative scalar measure of multidimensional state vectors reflecting on the system performance, and the second is that the time derivative of \( L \) along trajectories of the system is always negative. Alternatively, the function has its value becoming larger as the system moves to an more undesired
state. The change of the Lyapunov function from one time \( t \) to the next time \( t + \Delta t \) is called a drift. When the drift is controlled with a tendency to move in the negative direction, gradually it tends to change towards zero, and one can expect the system to behave stably over the long run. The Lyapunov function is a user defined function that desirably reflects the system state. There may be multiple forms of functions defined for control of the same system, each may well result in a different control policy. In particular, we are going to examine two different definitions of the Lyapunov function, one being the sum of squared queues and the other being the sheer sum of queue lengths over the network intersections. We will find that they each lead to a different control policy and different performance, and that they each imply a different logic of control.

Usually the control is conducted by discretizing the continuous time into time slices. In each slice, a control action is taken according to the policy. The basic idea of Lyapunov control is to decompose a long-term consolable index with long-term constraints into sub-control problems according to time slices. The action in each slice is considered independent of each other, therefore the long term control is approximated by a set of independent ones, greatly simplifying the problem. In the case of network signal control, Lyapuniv control would make a decision in a time slice based on the queueing situation and direct connectivity between intersections, disregarding the impact of the control action on future queues and future control. Tassiulas and Ephremides first bring the Lyapunov function into queuing network control problems and propose stable routing and scheduling policies, back pressure routing, and max-weight scheduling algorithms, which stabilize the network over a long run and are throughput optimal (48, 49). It is worth noting that the policy only requires knowledge of the current network states, and they do not require knowledge of the probabilities associated with future random events (50).

![FIGURE 1: Illustrative flow movement for left-turn phase on major arterial](image)

In order to facilitate modeling, we reiterate the study problem here. Consider a roadway network with \( N \) signalized intersections and time varying traffic flows. Each intersection \( i \in \{1, 2, \ldots, N\} \) is connected to a set of \( L(i) \) lanes (for inbound or outbound flows), where \( L(i) = \{l_1, l_2, \ldots, l_m\} \). The pair \((l_a, l_b)\) denotes the movement from lane \( l_a \) through intersection \( i \) to lane
Each intersection $i$ can be described by a tuple $(\mathcal{M}_i, \Omega_i, S_i)$, where $\mathcal{M}_i \subseteq \mathcal{L}^2(i)$ is the set of all possible movements through intersection $i$. Each flow movement through the intersection $i$ is defined by a pair $(l_a, l_b)$, where $(l_a, l_b) \in \mathcal{M}_i$. $\Omega_i$ is the set of all phases of signal at intersection $i$, $\phi_{ij} \in \Omega_i$ is phase $j$ controlling single or multiple movements at intersection $i$. We use the notion $(l_a, l_b) \in \phi_{ij}$ to indicate that phase $\phi_{ij}$ gives the right of way to, but not limited to, the movement from lane $l_a$ through intersection $i$ to lane $l_b$. As in the Manual on Uniform Traffic Control Devices (MUTCD), a phase refers to "the right-of-way, yellow change, and red clearance intervals in a cycle that are assigned to an independent traffic movement or combination of movements" (3). $S_i$ is the set of the traffic state at intersection $i$. Clearly, a traffic state shall be identified easily by observable attributes preferably through detectors around the intersection. In this study, a state at an intersection is represented by the current queues at the intersection.

The following queue dynamics is used for each lane across this paper.

$$\Theta_i(t + 1) = \max[\Theta_i(t) - \sum z_{ib}^{\text{out}}(t) + \sum z_{ai}^{\text{in}}(t) + A_i(t), 0]$$

(1)

**LYAPUNOV CONTROL AND WEIGHTED BACK PRESSURE**

We first show that Lyapunov function may give rise to a popular back pressure method. Back pressure is a measure of differential pressure in a fluid system especially around control valves, where the fluid is primarily driven by the differential pressure in flowing through the system. This concept is borrowed by the general network control, which was first seen in the electrical engineering, in the area of communication and grid (48, 50–52). Back pressure routing is a routing algorithm that uses queue differentials between sequential nodes to dynamically route traffic over a network. The back pressure algorithm can operate in slotted time, and it seeks to route data in directions that maximize the weighted differential backlog between neighboring nodes at every slot it. This is similar to fluid flow through a network of pipes via pressure gradients. The pressure is due to differential queue between upstream lanes and downstream lanes. The algorithm can be implemented in a distributed manner. Such a distributed system reduces the computational difficulties and avoids the curse of dimensionality.

The time horizon $[0, T]$ is descretized into a set of time points $\{t_0, t_1, t_2, ..., t_k\}$. At each time step $t \in \{t_i\}$, vehicles enter and exit the lanes. For each lane $l_a \in \mathcal{L}(i)$, $f_{ab}(\phi_{ij}, l_a, l_b, s_i)$ gives the flow rate that going from $l_a$ to $l_b$ through intersection $i$ under state $s_i$ and phase $\phi_{ij}$. Note that $l_a$ is the lane through which traffic arrive at intersection $i$ while $l_b$ is the lane through which the traffic is discharged to a downstream intersection $j$. Note that lane $l_b$ is an exit lane from intersection $i$ but is an entry lane to a downstream intersection $j$. $f_{ab}$ is the flow rate (measure by time slot) that goes from $l_a$ through intersection $i$ out to lane $l_b$ in the current time slot and under the phase of signal. Therefore, $f_{ab}$ leave the queue (if there is one) in lane $l_b$ before $i$ will arrive at intersection $j$ through lane $l_b$ to possibly join the queue in lane $l_b$ at intersection $j$.

$f_{ab}$ has its value determined depending on the traffic situation. When there is a queue in lane $l_a$ before intersection $i$, $f_{ab}$ is equal to the saturation flow rate of the move through the intersection; otherwise, it is equal to the arrival traffic flow rate from upstream intersection(s). Again, when we say 'under a phase', we mean that the phase has the right of way in the form of green signal.

Next, we define our Lyapunov function and show an implied, weighted max pressure method. Let the Lyapunov function be as follows.

$$L(\Theta(t)) = (1/2) \sum_{n,l} [\Theta_n^l(t)]^2$$

(2)
Note that the Lyapunov function defined in Equation (2) has its inherent connection to the intersection waiting time. In a deterministic queue for an approach $l$, one may roughly estimate the total vehicle waiting time to be $\frac{[\Theta^l(t)]^2}{4\lambda_l^\theta}$, where $\lambda_l^\theta$ is the discharge rate under policy $\theta$. Note that the discharge rate $\lambda$ here for a queue in the lane may be equal to sum of several $z$ values depending on the signal phase that permits the discharging operations. To formally align the Lyapunov function with the vehicle queueing time minimization, one might define the Lyapunov function to be as follows.

$$L(\Theta(t)) = \left(\frac{1}{2}\right) \sum_{n,l} \frac{[\Theta^l_n(t)]^2}{4\lambda_l^\theta}.$$  \hspace{1cm} (3)

Be aware that the definition in Equation (3) has its squared queue term normalized by the according discharge rate. Different lane queue has different discharge rate $\lambda$. For the ease of presentation, we choose to continue to use Equation (2) for technical derivation while we keep in mind that $\lambda_l^\theta$ is possible to use in order to differentiate the queues. For simplicity, we implicitly treat all the discharge rate with saturation flow rate be equation. Therefore Equation (2) is proportional to the RHS of Equation (3), both of which may serve for the purpose of vehicle waiting time minimization. In summary, the proposed Lyapunov function is inherently related to (or, proportional to, to be precise) to the intersection vehicle waiting time. This roughly explains why our proposed control algorithm outperforms other algorithms.

its drift function is therefore may be derived as follows.

$$\Delta(\Theta(t)) = E[L(\Theta(t + 1)) - L(\Theta(t))].$$  \hspace{1cm} (4)

$$= \sum_{1 \leq i \leq N} \sum_{l \in M_i} \left[\frac{1}{2} \left( \sum_a z_{ai}^{in}(t) + A_i(t) - \sum_b z_{ib}^{out}(t) \right)^2 \right.$$  \hspace{1cm} (5)

$$\left. + \Theta_i(t) \left( \sum_a z_{ai}^{in}(t) + A_i(t) - \sum_b z_{ib}^{out}(t) \right) \right]$$  \hspace{1cm} (6)

Square at both sides of 1 and summing all intersection $i$ and all movements $l$, assuming queue is always exist and deterministic at the intersection, we have

We start by a simple derivative to simplify the technical presentation. Conduct the first order partial derivative on the RHS wrt $z_{ab}$. $z_{ab}$ is a flow from lane $l_a$ to lane $l_b$ through the intersection under a control $\theta$. Here, For illustrative purpose, we tentatively only consider a flow under control of a phase. Note that when the drift function is minimized in the discrete case, the original Lyapunov function is minimized (Technical references for this point).

Consider the partial derivative on the Lyapunov drift with respect to a flow $z_{ab}$ through the
intersection

\[
\frac{\partial \Delta(\Theta(t))}{\partial z_{ab}} = \sum_{i,j} \sum_{l \in M_{i}} \frac{\partial}{\partial z_{ab}} \left[ \frac{1}{2} \left( \sum_{a} z_{ai}^{in}(t) + A_{i}(t) - \sum_{b} z_{ib}^{out}(t) \right)^{2} \right] + \Theta_{i}(t) \left( \sum_{a} z_{ai}^{in}(t) + A_{i}(t) - \sum_{b} z_{ib}^{out}(t) \right) 
\]

\[= \Theta_{i}(t) \left( \sum_{a} z_{ai}^{in}(t) + A_{i}(t) - \sum_{b} z_{ib}^{out}(t) \right) - (\sum_{a} z_{ai}^{in}(t) + A_{i}(t) - \sum_{b} z_{ib}^{out}(t)) \Theta_{i}(t) + (\sum_{a} z_{aj}^{in}(t)) \]

\[+ A_{j}(t) - \sum_{b} z_{jb}^{out}(t) + \Theta_{j}(t) \]

\[= \Theta_{i}(t + 1) - \Theta_{b}(t + 1) \]

(7)

Note that Equation (8) has \(\partial z_{ab}\) indicating the flow rate through intersection. Equation (8) gives rise to the result in its general form:

\[
\frac{\partial \Delta^{\theta}(\Theta(t))}{\partial z_{ab}} = \sum_{\forall(a,b)} \partial z_{ab}^{\theta} \times \left( \Theta_{a}(t + 1) - \Theta_{b}(t + 1) \right). 
\]

(12)

where \(\partial z_{ab}^{\theta}\) in Equation (12) represents the incremental flow during the current time epoch under policy \(\theta\). Note that the policy \(\theta\) here may be interpreted as a sequence of a particular signal plan that comprises of the set of phases, sequence and timing. In particular, for a particular phase \(\phi_k \in \Phi_i\), the resulting drift value associated with the intersection \(i\) may be expressed as follows.

\[
\frac{\partial \Delta^{\theta}(\Theta(t))}{\partial z_{ab}}|_{\phi_k} = \sum_{(a,b) \in \phi_k} \partial z_{ab}^{\theta} \times \left( \Theta_{a}(t + 1) - \Theta_{b}(t + 1) \right). 
\]

(13)

Note that phases in \(\Phi_i\) are exclusive of each other. When one phase gets the right of way, all other phases in \(\Phi_i\) get red signal. Equation (13) indicates that each phase \(\phi_k \in \Phi_i\) at the particular intersection state \(s\) has its resulting drift value. In order to minimize the drift function, the set of phases for the next time epoch over the intersections shall be chosen so that the drift value is minimized over all the intersections. There are two cases that govern the sequence of phases at intersections. One is a fixed sequence of phases, in which case, the control decision at any time epoch would be whether it leads to a smaller drift value if the signal switches to the next phase; if the answer is yes, the signal is switched to the next phase; otherwise, the current phase continues. The other case does not have a fixed sequence of phases, in which case, all the phases in \(\Phi_i\) for intersection \(i\) are examined in light of Equation (13) in terms of the drift value. The phase with the smallest drift value is chosen to be the next phase to switch to.

Lyapunov function here does not anticipate ripple effect beyond the next time epoch and beyond the adjacent intersections, Therefore it is a myopic policy along time dimension. As a benefit, it allows decomposition of the network intersections into a set of independent intersections, to each of which the control process is implemented independently. Because the policy is implemented in a distributed way, the result indicates that the expected differences of queue represents the gradient along which the downward drift to be minimized. This comes to the connection between Lyapunov drift and the pressure series algorithm. The difference between queue is the pressure. Back pressure can be interpreted as the flow weighted pressure. note that the queue
length at the next step \( t + 1 \) may not be convenient or easy to estimate accurately within a short period of time because both the upstream and downstream queues may have flows fed from other entry/exit approaches. However, technically, the estimate shall be feasible and shall not be too challenging. Because the time epoch is usually short, which would not cause dramatic changes to the queues, we hereby propose to use the existing queue length to estimate such value, and push the Lyapunov function to smaller value and stabilize the system at each time step.

**Network Stability with Back pressure**

The above sections assume the case that queue is deterministic and always exist, we will use drift method to show more general case, and the method fit for the stochastic situation. Consider the quadratic Lyapunov function

\[
L(\Theta(t)) = \sum_{l \in M} \sum_{i \in N} \Theta_{li}(t)^2
\]

and assume \( \mathbb{E}[L(0)] < \infty \). There are constants \( B > 0, \epsilon \geq 0 \), such that the following drift condition holds for all slots \( \tau \in T \) and all possible \( \Theta(\tau) \):

\[
\Delta(\Theta(t)) \leq B - \epsilon \sum_{n, l} \Theta_{nl}(t)
\]  

(14)

If \( \epsilon \geq 0 \), all queues are mean rate stable, more strictly, if \( \epsilon > 0 \), all queues are strongly stable and all queues \( \Theta_{nl}(t) \) have

\[
\lim_{t \to \infty} \sup_{\tau=0} \frac{1}{t} \sum_{\tau=\theta}^{t-1} \sum_{1 \leq i \leq N} \sum_{l \in M_i} \mathbb{E}[\Theta_{nl}(\tau)] \leq \frac{B}{\epsilon}
\]  

(15)

For any control policy, the Lyapunov drift at any time step \( t \) satisfies:

\[
\Delta(\Theta(t)) \leq B - [\Phi(\Theta(t)) - \Gamma(\Theta(t))]
\]  

(16)

where,

\[
B = (z_{\text{out}}^{\text{max}})^2 + (z_{\text{in}}^{\text{max}} + A_{\text{max}})^2
\]  

(17)

\[
\Phi(\Theta(t)) = \sum_{i, l} \Theta_{il}(t) \mathbb{E}[\sum_{b} z_{\text{out}}(t) - \sum_{a} z_{\text{in}}(t) | \Theta(t)]
\]  

(18)

\[
\Gamma(\Theta(t)) = \sum_{i, l} \Theta_{il}(t) \mathbb{E}[\sum_{c} A_{ic}(t) | \Theta(t)]
\]  

(19)

\( z_{\text{out}}^{\text{max}} \) and \( z_{\text{in}}^{\text{max}} \) are the max input and output flow, respectively.

The network capacity region \( \Lambda \) is the closure of the set of all matrices \( \lambda \) that can be stably supported over the network, considering all possible algorithms, it has proven that no control algorithm can achieve stability beyond the set \( \Lambda \), even if the entire set of future events is known in advance (48). The signal control problem assumes that the road network is not beyond its capacity, which is a reasonable assumption. If it exceeds its capacity, an overpass should be considered at the intersection. The capacity region of the network is given by the set \( \Lambda \), such that there exists a policy that makes the network stable and has

\[
\lambda_{ci} + \epsilon \leq \sum_{b} z_{\text{out}}^{b} - \sum_{a} z_{\text{in}}^{a}
\]  

(20)

Also notice in equation 18, we have

\[
\Phi(\Theta(t)) - \Gamma(\Theta(t)) = \sum_{i, l} \Theta_{il}(t) \mathbb{E}[\sum_{b} z_{\text{out}}(t) - \sum_{a} z_{\text{in}}(t) - A_{ci}(t)] \geq \epsilon \sum_{i, l} \Theta_{il}(t)
\]  

(21)
In equation 16, B is the network property, which out of our control. Therefore, equation 16 satisfies the Lyapunov control theory in equation 14 and the network is stable.

\[ \Delta(\Theta(t)) \leq B - 2\epsilon \sum_{i,l} E[\Theta_i^l(t)] \]  

(22)

Summing the equation 22 from \( t = 0 \) to \( t = T \) and notice the definition of Lyapunov drift in 36

\[ \Delta(\Theta(t)) = L(\Theta(t+1)) - L(\Theta(t)) \] assuming \( \Theta(0) = 0 \), we have:

\[ 0 \leq E[[\Theta_i^l(t)]^2] \leq tB - \epsilon \sum_{t=1}^{T} \sum_{i,l} E[\Theta_i^l(t)] \]  

(23)

\[ \frac{1}{t} \sum_{\tau=0}^{t-1} \sum_{i,l} E[\Theta_i^l(\tau)] \leq \frac{B}{\epsilon} \]  

(24)

Notice that \( B \) is defined in equation 17. Little’s theorem provides a base for analyzing the queuing delay. The theorem states that when a network reaches a steady state, the average number of jobs in a queue is equal to the product of the average arrival rate of the jobs and the average time a job is kept in the queue. Thus, the max back pressure algorithm stabilize the system and thereby achieve maximum throughput and maintain acceptably low network delay.

To make a clarification here, if the objective is to minimize the drift-plus-penalty, equation 16 becomes:

\[ \Delta(\Theta(t)) + \mu f(t) \leq B - 2[\Phi(\Theta(t)) - \Gamma(\Theta(t))] + \mu f(t) \]  

(25)

When \( \mu \) is not zero, after we go through the same logic of the proof, finally we have

\[ E[\Delta(\Theta(t)) + \mu f(t)] \leq tB - 2\epsilon \sum_{i,l} E[\Theta_i^l(t)] + \mu \sum_{t=1}^{T} f(t) \]  

(26)

If we consider multiple task trade-off in the future work, the overall logic does not change. And it is easy to see that the long term average queue of the whole network with other consideration \( f(t) \) must be greater than that without the trade-off term.

**Connection with Reinforcement learning**

Over The dynamic equation in DORAS basically represents the logic that the total intersection vehicle waiting time from time \( T \) equals the waiting time between \( t \) and \( T \) plus the waiting time at time \( t \). When we review DORAS, we find that the overall objective is to find some switching time point to minimize the waiting time of an intersection based on a waiting time dynamics with arrival and departure flow known ahead. The whole dynamic equation is built from deterministic perspective, and only consider one possible realization over a time horizon, which are very strong assumptions. Actually, based on the same logic, we can give up such assumptions and construct the Markov Decision Process (MDP). The Bellman equation of the process is:

\[ w(s_t) = \min_a E[r(s_t, a_t, s_{t+1}) + \gamma w(s_{t+1})] \]  

(27)

or

\[ w(s_t) = \min_a \sum_{s_{t+1}, \tau} p(s_{t+1}|s, a) [r(s_t, a_t, s_{t+1}) + \gamma w(s_{t+1})] \]  

(28)
Here, $s_t$ is the state, based on DORAS’s framework, it can be the queue length of on a lane. $a_t$ is the action. The selection of switching point is equivalent to make a decision of whether give release the flow or not within a time interval. $s_{t+1}$ is the state at the next time step. The Bellman equation can be solved by reinforcement learning. Also, the asymptotic optimal policy can be obtained by reinforcement learning. Therefore, the RL-waiting time in the simulation is the asymptotic optimal policy under such framework in theory. However, through the simulation, we find that it lacks the connection between intersections. There are three core terms to formulate a fundamental RL problem: environment, state, and reward. The environment in this study is SUMO, in which the agent operates. State described the agent’s current situation, including queuing length and signal phases. The reward is the feedback from the environment. The states and reward function are intimately tied up, and defining them is essential for RL because they incentive the agent to do “right” actions and to learn suitable policy. However, specifying a reward function can be one of the trickiest parts if applying RL to a real-world problem (53). Designing a reward function is a debatable question. Typically, the reward function should provide more frequent feedback on actions and have practical significance (54). Back pressure is a good fit for the reward design.

**Q-learning**

The Q-learning approach to solving the Bellman equation:

$$V(s_t) = \sum_{a \in A} \sum_{s_{t+1} \in S} p(s_{t+1}|s_t, a)[r(s_t, a, s_{t+1}) + \gamma V(s_{t+1})]$$  \hspace{1cm} (29)

$$Q(s_t, a_t) = \sum_{s_{t+1} \in S} p(s_{t+1}|s_t, a)[r(s_t, a_t, s_{t+1}) + \gamma \sum_{a \in A} \pi(a|s_{t+1})Q(s_{t+1}, a)]$$ \hspace{1cm} (30)

Q-Learning poses an idea of assessing the quality of an action that is taken to move to a state rather than determining the possible value of the state being moved to. The Q-learning algorithm makes the following update:

$$Q^{new}(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[ r(s_t, a_t, s_{t+1}) + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t) \right]$$ \hspace{1cm} (31)

The quantity in square brackets in equation 31 is exactly zero when $a$ is the optimal action to take under states $s_{t+1}$. In other word, $Q(s_t, a_t)$ is the optimal action-state value pair. The quantity in the square brackets can be interpreted as the "Bellman error", the error term describes how far off the target quantity $r(s_t, a_t, s_{t+1}) + \gamma \max_a Q(s_{t+1}, a)$ is from the estimates $Q(s_t, a_t)$ in the current step. Q-learning algorithm iteratively updates $Q(s_t, a_t)$ by equation 31 to reduce the Bellman error until reach a converged solution.

To store all the action-state pair value $s$, Q-learning requires a finite state and action space where it is possible to maintain a table lookup the estimated Q-value. However, it is not always the case where we have finite states and/or actions. When we have an infinite state space and/or action space, specifically in developing the traffic signal control policy, then it becomes impossible to store all the value pairs. An elegant way is to use function approximation to generalize across states and store the approximation function, which is typically done using a deep neural network due to their expressive power. And thus, we will introduce Deep Q-Network (DQN) in the next subsection.
Reinforcement Learning and Double DQN

The basic idea behind many reinforcement learning algorithms is to estimate the action-value function, by using the Bellman equation as an iterative update, e.g. equation 31. Such value iteration algorithms converge to the optimal action-value function as \( t \to \infty \) (8, 55).

In DQN, the experience replay memory and the target network were decisive in allowing the neural network to learn the tasks through RL. Their drawback is that they drastically increase the sample complexity and overestimate the target Q value. This over-estimation is inevitable in regular Q-learning, and therefore the double DQN is proposed (56). Applying double learning to DQN is straightforward: there are already two value networks: the trained and target networks. Instead of using the target network to both select the greedy action in the next state and estimate its Q-value, here the trained network weights \( \theta \) is used to select the greedy action \( a^* = \arg\max_{a'} Q_\theta(s', a') \) while the target network only estimates its Q-value. This induces only a minor modification of the DQN algorithm and significantly improves its performance and stability. The main idea of double DQN is to train independently two value networks: one will be used to find the action with the max Q-value and estimate the Q-value itself. Even if the first network chooses an over-estimated action as the greedy action, the other might provide a less over-estimated value for it, resulting in a better solution.

Back pressure with Reinforcement Learning

One major issue of current RL-based traffic signal control approaches is that the setting is often heuristic and lacks proper theoretical justification from transportation literature. Common goals are either to minimize the average travel time of vehicles or delay. However, these goals are either "delayed" or heavily rely on estimation, resulting in a mismatch between state and reward, which leads to poor performance. The algorithm put forward in this section is based on RL but theoretically grounded by the back pressure method mentioned above. Back pressure has the property of being based on real-time observable quantities, a perfect fit for the state-reward pair design in reinforcement learning. The only information required is the queue backlog on each lane of the intersection in the roadway network, the lane density, and lane length. Detectors can acquire the first two. Greenshields model can calculate the flow with a known density. The RL formulation is demonstrated as the following:

**State:** current phase \( \phi_{ij} \), the total number of moving vehicles and stopped vehicles (speed < 0.1 m/s, or 0.22 mph) on each incoming lanes \( (l_a) \) and outgoing lanes \( (l_b) \).

**Action:** at each time \( t \), each agent chooses a phase as its action \( a_t \) from action set \( A \), indicating the traffic signal should be set as current phase \( \phi_{ij} \). Each action candidate \( a_i \) is represented as a one-hot vector.

**Reward:** the reward for an intersection is the back pressure defined as

\[
R_i(t) = - \sum_{\phi_{ij} \in P_i} |D_{\phi_{ij}}(t)|
\] (32)

In RL, the long-term reward is the objective for optimization, and the solution is derived from the trial-and-error search. We adopt Double-DQN as a function approximator. To stabilize the training process, we maintain an experience replay memory by adding the new data samples and removing the old samples occasionally. Periodically, the agent will take samples from memory and use them to update the network.
Stability of Lyapunov Control

The following is a synopsis of it. A discrete time process $\Theta(t)$ is mean rate stable if

$$\lim_{t \to \infty} \frac{\mathbb{E}[\Theta(t)]}{t} = 0$$

and is strongly stable if

$$\lim_{t \to \infty} \sup \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[\Theta(\tau)] < \infty$$

Consider a network of $N$ queues, and let $\Theta(t) = (\Theta_1^l(t), \ldots, \Theta_N^l(t))$ be the queue vector. Assume the $\Theta(t)$ vector evolves over $t \in T$. As a scalar measure of the “size” of the vector $\Theta(t)$, a Lyapunov function $L(\Theta(t))$ is defined as follows:

$$L(\Theta(t)) = \sum_{1 \leq i \leq N} \sum_{l \in M_i} L[\Theta_i^l(t)]$$

(35)

Given the current state $\Theta(t)$, the expected change in the Lyapunov function over one slot, Lyapunov drift $\Delta(\Theta(t))$ is defined as (47):

$$\Delta(\Theta(t)) = \mathbb{E}[L(\Theta(t+1)) - L(\Theta(t))]$$

(36)

LYAPUNOV FUNCTION AND OTHER SIGNAL CONTROL

Lyapunov control represents an umbrella that may unite different control policies that have been proposed in the transportation literature. We have also found the popular Lyapunov theories connected to other traffic signal control literature. One of them is the intersection efficiency based signal policy, called DORAS. DORAS is a policy that switch the right of way to the phases with most discharge efficiency. Details of it see in (27). Here we show how Lyapunov control implies the DORAS policy.

Define $L(\Theta(t)) = \sum_{1 \leq i \leq N} \sum_{l \in M_i} |\Theta_i^l(t)|$ as the Lyapunov function, representing a scalar measure of the network congestion. For a given control policy and network at time $t$, the Lyapunov drift is:

$$\Delta(\Theta(t)) = \mathbb{E}[|\Theta(t+1)| - |\Theta(t)|]$$

(37)

Because the queue $\Theta_i^l(t)$ is never negative $\forall l, t$, we simplify the equation into the following form, $L(\Theta(t)) = \sum_{1 \leq i \leq N} \sum_{l \in M_i} |\Theta_i^l(t)|$. Summing all intersection $i$ and all phase $l$, assuming queue is always exist at the intersection, we have

$$\Delta(\Theta(t)) = \sum_{1 \leq i \leq N} \sum_{l \in M_i} [\Theta_i^l(t+1) - \Theta_i^l(t)] = \sum_{1 \leq i \leq N} \sum_{l \in M_i} \left[ \sum_a z_{ai}^{in}(t) + A_i(t) - \sum_b z_{ib}^{out}(t) \right]$$

(38)

Conduct the partial derivative on the right hand side of Equation (38) with respect to the flow through the intersection $z_{ab}$, which represent the control at a phase. For a single intersection:

$$\frac{\partial \Delta(\Theta(t))}{\partial z_{ab}} = \frac{\partial}{\partial z_{ab}} \sum_{l \in M_i} \left[ \sum_a z_{ai}^{in}(t) + A_i(t) - \sum_b z_{ib}^{out}(t) \right]$$

$$= -\delta$$

(40)
The net change of drift, or the control-drift under control $\theta$ is $\{-\delta\} \cdot \{z_{ab}^{\theta}\}$. The derivative equals $-1$ when the movement leads to the vehicle leaving the network and $\delta$ for the intersections within the network, where $0 \leq \delta \leq -1$. Here $z_{ab}^{\theta}$ represents the flow of a move through an intersection under control $\theta$. Control $\theta$ may be taken as a signal plan. The value of $z_{ab}^{\theta}$ is equal to the saturation flow rate of the move $(l_a, l_b)$ when the green signal is given to the approach with queued vehicles; it is equal to the arrival flow rate when the green signal is given to an approach without queued vehicles. The value of $z_{ab}^{\theta}$ is zero when a red signal is given to the move $(l_a, l_b)$.

Note that the derivative of the RHS of Equation (39) means that the gradient of the drift function decreases in the fastest rate along the approaches that discharge at the most flow rates. If we treat each intersection individually, the control-drift shows that a policy $\theta$ (or simply take as a control) shall be adopted when it leads to the minimum drift (or maximum negative drift). This is achieved when the phase that gives the maximum flow rate in the time interval gets the green signal. In other words, the signal phase that discharges at the most total rate would decrease the drift function in the fastest rate, meaning that that these approaches shall be given the right of way. Of course, this is still a myopic view of control as it does not consider the causal effect of it along the time and down to other related intersections. If taking such constrain into consideration, more accurate measure of the dispersion effect is needed to get $\delta$ estimation. The policy under this case should be designed as maximizing the output flow of each intersection if each one is considered individually, which naturally leads to the concept of throughput maximization in DORAS. However, the definition of $L(\Theta(t)) = \sum_{1 \leq i \leq N} \sum_{l \in M_i} |\Theta_l^n(t)|$ may not be appropriate under a network of signal because it lack the consideration of interaction between the control of each intersections. It shows that the Lyapunov function should be chosen wisely, although there are no excessive selection restrictions.

Define $W_{ab}^{\phi} = \Theta_i^l(t) - \Theta_j^l(t)$ as the differential queue backlog of the movement $(l_a, l_b)$, assuming that the queue in $l_a$ is only discharged to the downstream queue in lane $l_b$. In the case that vehicles from $l_a$ is discharged to multiple queues in downstream intersection(s), the back pressure is defined in a weighted manner over individual queue differentials as defined above according to the traffic split among them. A general definition for $W_{ab}^{\phi}$ is as follows:

Under each phase $\phi_{ij}$ at intersection $i$, likely there are multiple concurrent moves of traffic, each having a back pressure due to the queue differentials. A phase differential is therefore defined by including differentials for all the moves at time step $t$ as follows:

\[
D_{\phi_{ij}}(t) = \sum_{(l_a, l_b) \in \phi_{ij}} W_{ab} f_{ab}(\phi_{ij}, l_a, l_b, s_t) \tag{41}
\]

Unlike the max pressure policy using saturation flow, with traffic density known, we use the Greensheild model to estimate the max flow rate through the intersection of each movement controlled by a phase at each time step:

\[
z(t) = v_f d(t) - \left[\frac{v_f}{d_{jam}}\right] d^2(t) \tag{42}
\]

The back pressure policy gives the green time to the phase with max back pressure and thus uses back pressure in an effort to equalize differential queue backlog. Instead of making prediction on the arrivals, it use flow info on the lanes to incorporate the arrival information. The algorithm is similar in nature to back pressure routing for a communication network. In previous literature \cite{48, 50–52}, it has been shown that back pressure routing leads to maximum network throughput.
SIMULATION

Numerical tests are conducted on two types of networks: a single corridor and a grid network. The reason for separately testing on a single corridor is that single corridors are often the major means of dealing with urban traffic. We have utilized the SUMO 1.8 micro-simulator in conjunction with TraCI (Traffic Control Interface) 1.8 for modeling the case. SUMO (Simulation of Urban MOBility) is an open-source, microscopic and continuous traffic simulation package designed to handle large traffic networks simulation with a large set of tools for scenario creation (57). SUMO allows us to create a traffic simulation environment and track every vehicle. TraCI implements RL-based real-time signal control possible. The RL agent is built and trained in Pytorch 1.7.1 and Python 3.8.

(a) Intersection of major roads and minor roads without truck
(b) Intersection of major roads and major roads without truck
(c) Intersection of major roads and minor roads with truck
(d) Intersection of major roads and major roads with truck

FIGURE 2: Intersection environment

Figure 2(a) and Figure 2(c) presents the layout of an individual intersection of major and minor arterial, without and with truck volume, respectively, in the simulation. Figure 2(b) and Figure 2(d) presents the layout of an individual intersection of two major arterials, without and with truck volume, respectively, in the simulation. The link on minor arterial at each intersection consists of two through lanes. Each of the through lanes also servers the turning traffic. The link on major arterial at each intersection consists of one left-turn lane and two through lanes. One
of the through lanes also servers the right-turning traffic. The setting is more general than, for example, a separated lane for the left turn, through, and right turn traffic. In reality, there are singular dominating corridors that can be easily identified. For those cases where it is hard to find the dominating singular corridors or several crossing traffic, all being likely major ones.

Figure 3 presents the available phase timing plan for the arterial, which also contains the available action for selecting the RL agent. The ring-and-barrier diagram is for illustrative purposes and presents the phase plan for the arterial simulation. The amber interval is set as 5 seconds and represents the time between two consecutive phases to clear the intersection, consisting of 3 seconds yellow and 2 seconds all-red interval. The min green time is 5 seconds, and the max green time is 30 seconds. The ring-and-barrier diagram is for illustrative purposes and presents the phase plan for the simulation. In reality, the RL algorithm neither requires four phases nor a fixed sequence.

| Action1 | Action2 | Action3 | Action4 |
|---------|---------|---------|---------|
| \(\phi_1\) | \(\phi_2\) | \(\phi_4\) | \(\phi_8\) |
| (a) Phase plan for intersection of major roads and major roads |

![Phase plan for intersection of major roads and major roads](image)

| Action1 | Action2 | Action3 | Action4 |
|---------|---------|---------|---------|
| \(\phi_1\) | \(\phi_2\) | \(\phi_3\) | \(\phi_4\) |
| \(\phi_5\) | \(\phi_6\) | \(\phi_7\) | \(\phi_8\) |
| (b) Phase plan for intersection of major roads and minor roads |

![Phase plan for intersection of major roads and minor roads](image)

**FIGURE 3:** Available phases in traffic signal control of the simulation

Figure 4 illustrates the arterial (a) and grid network (b) used in the simulation. The test arterial consists of five intersections and the grid network contains \(4 \times 4 = 16\) intersections. The arterial in the numerical test consists of one major arterial road (North-south direction) with higher traffic volumes and five minor roads with lower volumes. The minor street crossings spaced 1640 ft (500 m) along the major arterial with free-flow speed 50 mph. The normal travel times on the major road traversing the intersection are approximately 20s. The grid network in the simulation makes of two major arterial roads with higher volumes and three minor roads with lower volumes in each direction. The distance between roads, free-flow speed, and the normal travel times are the same with the arterial.
Three traffic situations are used for the test as indicated in Table 2. They represent high, medium, and low traffic scenarios, respectively.

**TABLE 2: Traffic Volume in the Simulation**

| Traffic Scenario | Major Roads Traffic (veh/h) | Minor Roads Traffic (veh/h) |
|------------------|------------------------------|-----------------------------|
| Low              | 500                          | 200                         |
| Medium           | 900                          | 300                         |
| High             | 1300                         | 400                         |

Also, in each scenario, the effect of different truck ratios (0 %, 10 %, 25 %, and 40 %) on each control algorithm is tested simultaneously for the same major and minor traffic volumes scenarios. We will first present the performance of each algorithm under uniform passenger vehicle flow (0 % truck) and later present the effect of truck flow on the algorithm performance. For simplicity without loss of generality, the research will convert truck to two passenger vehicle in the simulation (58). The vehicle type defaults are shown in Table 3 (59). The dissertation will focus on addressing the near-optimal control mechanism and evaluating the effectiveness and robustness of the designed algorithm.
TABLE 3: Vehicle Type parameter defaults in the simulation

| Vehicle Type | Length, Width, Height | MinGap | Acceleration | Deceleration | Emergency Deceleration |
|--------------|-----------------------|--------|--------------|--------------|------------------------|
| Passenger    | 5 m, 1.8 m, 1.5 m     | 2.5 m  | 2.6 m/s²     | 4.5 m/s²     | 9 m/s²                 |
| Truck        | 16.5 m, 2.55 m, 4 m   | 2.5 m  | 1.1 m/s²     | 4 m/s²       | 7 m/s²                 |

TESTED ALGORITHMS IN SIMULATION

We compare our model with the following two categories of methods: non-machine learning transportation methods and RL-based methods. Non-machine learning methods include Fixed timing plan with coordination, DORAS-Q, Max Pressure and Back pressure. We directly optimize the waiting time and average queue length by double DQN and use them as baselines of the reinforcement learning based algorithms. All algorithms are fine-tuned. And all RL-based algorithms are trained by Double DQN.

- Fixed-time: Fixed-timing plan and offsets optimized with PASSER V. Fixed timing plan with green wave progression is the most classical approach achieving coordination on arterial in practise.
- DORAS-Q (27): DORAS-Q is designed for isolated intersection control, and may be applied to the network as a distributed control system in which each intersection only optimizes its control and the entire system adapts gradually, it requires the existing queue length, short-term (usually 5 seconds) and the average historical arrival rates for each phase to estimate the switch-to efficiency and phase efficiency. Then decide on changing or keeping the current phase based on the discharge efficiency.
- Max Pressure (24): Max pressure defines the differences of queue between the current and the downstream intersection as pressure of the phase, and greedily chooses the phase with the maximum pressure.
- Back Pressure: Consider the flow on the lanes, greedily chooses the phase with the maximum back pressure.
- RL-WaitingTime: Directly define the waiting time as the reward function. Train RL agent to minimize the waiting time of all vehicles in the network.
- RL-Queue: Directly define the average queue length as the reward function. Train RL agent to minimize the average queue length of each lane in the network.
- RL-MaxPressure: Directly define the total network Max Pressure as the reward function. Train RL agent to maximize the negative of absolute Max Pressure.
- RL-BackPressure: Directly define the total network back pressure as the reward function. Train RL agent to maximize the negative of absolute back pressure defined in equation 32.

Agent Performance on Scenarios with Uniform Passenger Vehicle Flow

Figure 5 shows the agent’s performance and the fast convergence in the arterial environment during the training process. The horizontal axis of the figures reflects the episodes. The top to the bottom of the figure, corresponds to low, medium, and high scenarios, respectively. The vertical axis of Figure 5a reflects the average back pressure of an intersection, while the vertical axis of Figure 5b reflects the average waiting time of each vehicle. The figures show that the training agent in the arterial environment converges after 100 episodes. Figure 5 illustrates the convergence curve of
our agents’ learning process with respect to the average waiting time of each episode. Compared with the back pressure curves, we can see that the travel time is closely correlated with pressure. Convergence curve of average duration and our reward design (back pressure).

![FIGURE 5: Average back pressure and waiting time during training process over 300 episodes in arterial case](image)

Figure 6 is the coverage curve of average back pressure and waiting time in the grid network case. The convergence trend is similar to that of the arterial case. The training processing converges around 100 episodes. Because the structure of the network is more complex than arterial so that the fluctuation after convergence is more unstable and violent. The correlation between back pressure and travel time can also be found in the grid network case.
We compare our model with the following two categories of methods: non-machine learning methods and RL-based methods. Non-machine learning method includes Fixed timing plan with coordination, DORAS-Q and Max Pressure. Fixed timing plan with green wave progression is the most classical approach to achieving coordination on the arterial in practice. Fixed-timing plans and offsets are optimized with PASSER V. DORAS-Q (27) is designed for isolated intersection control, and may be applied to the network as a distributed control system in which each intersection only optimizes its control and the entire system adapts gradually, it requires the existing queue length, short-term (usually 5 seconds) and the average historical arrival rates for each phase to estimate the switch-to efficiency and phase efficiency. Then decide on changing or keeping the current phase based on the discharge efficiency. Max Pressure (24) defines the differences of the queue between the current and the downstream intersection as the pressure of the phase and greedily chooses the phase with the maximum pressure. All algorithms are fine-tuned. Table 4 illustrates the results of the simulation.
TABLE 4: Average vehicle delay in arterial and grid network case with uniform passenger vehicle flow (in seconds)

|                  | Low Volume Arterial | Low Volume Network | Medium Volume Arterial | Medium Volume Network | High Volume Arterial | High Volume Network |
|------------------|---------------------|--------------------|------------------------|-----------------------|----------------------|---------------------|
| Fixed-time       | 30.93               | 97.97              | 38.06                  | 139.87                | 89.82                | 191.16              |
| DORAS-Q          | 23.25               | 72.77              | 36.64                  | 84.82                 | 76.64                | 148.92              |
| Max Pressure     | 19.88               | 54.17              | 31.59                  | 73.26                 | 71.62                | 157.61              |
| Back Pressure    | 18.99               | 44.75              | 26.92                  | 69.56                 | 56.86                | 137.82              |
| RL-WaitingTime   | 15.61               | **39.73**          | 34.16                  | 69.27                 | 80.13                | 140.84              |
| RL-Queue         | 15.38               | 40.67              | 26.47                  | 60.88                 | 66.47                | 140.87              |
| RL-MaxPressure   | 16.36               | 44.56              | 25.57                  | 51.43                 | 51.56                | 107.43              |
| RL-BackPressure  | **13.46**           | 42.95              | **22.64**              | **47.75**             | **46.07**            | **92.08**           |

FIGURE 7: Average vehicle delay in arterial and grid network simulation environment with uniform passenger vehicle flow (in second)

Figure 7 presents RL-Backpressure outperforms the other four signal control algorithms in both arterial and grid network cases. Not surprisingly, fixed-time control performs at the bottom, but it does not stop using it as a benchmark. Under all scenarios, DORAS-Q and Max Pressure outperform the fixed time control with coordination. RL baseline have satisfied performance in low volume scenarios, are exceeded by the RL-MaxPressure and RL-BackPressure under medium and high-volume cases.

Agent Performance on Scenarios with Truck Flow

The performance under different truck ratios (0%, 10%, 25%, and 40%) on each signal control algorithm is investigated. All settings are the same as the uniform passenger vehicle flow case except for the various truck ratio. The same as uniform passenger vehicle flow cases, the simulations with the truck flow are conducted under low, medium and high traffic flow scenarios. Specifically, the high traffic volume of 25% means that the traffic volume on the major/minor arterial remains at 1300/400 vehicles/hour, with 975/300 trucks/hour and 325/100 vehicles/hour passenger vehicles, respectively, on the major/minor arterial. When calculating the queue, we assume a truck equals two passenger vehicles.
10% truck volume

**TABLE 5:** Average vehicle delay in arterial and grid network case with 10% truck volume (in seconds)

|                     | Low Volume     | Medium Volume   | High Volume    |
|---------------------|----------------|-----------------|----------------|
|                     | Arterial       | Network         | Arterial       | Network         | Arterial       | Network         |
| Fixed-time          | 36.96          | 99.26           | 52.72          | 153.79          | 132.38         | 226.39          |
| DORAS-Q             | 28.87          | 74.82           | 49.76          | 96.09           | 130.47         | 184.25          |
| Max Pressure        | 20.96          | 50.20           | 41.50          | 85.99           | 125.14         | 184.25          |
| Back Pressure       | 19.87          | 46.69           | 37.14          | 73.74           | 113.65         | 148.83          |
| RL-WaitingTime      | **15.35**      | 45.99           | 40.76          | 89.58           | 108.69         | 143.35          |
| RL-Queue            | 15.45          | **43.56**       | 32.21          | 88.17           | 106.03         | 147.22          |
| RL-MaxPressure      | 18.47          | 46.12           | 34.87          | 79.54           | 91.72          | 129.86          |
| RL-BackPressure     | 17.82          | 44.73           | **30.73**      | **70.61**       | **90.55**      | **123.57**      |

**FIGURE 8:** Average vehicle delay in arterial and grid network simulation environment with 10% truck volume (in second)

Table 5 and Figure 8 illustrates the results of the simulation under all scenarios with 10% of truck volume. RL-BackPressure outperforms the other four signal control algorithms in both arterial and grid network cases. Not surprisingly, fixed-time control performs at the bottom, but it does not stop using it as a benchmark. Under all scenarios, DORAS-Q and Max Pressure outperform the fixed time control with coordination. RL baseline have satisfied performance in low volume scenarios, are exceeded by the RL-MaxPressure and RL-BackPressure under medium and high-volume cases.
25% truck flow

TABLE 6: Average vehicle delay in arterial and grid network case with 25% truck volume (in seconds)

|                     | Low Volume | Medium Volume | High Volume |
|---------------------|------------|---------------|-------------|
|                     | Arterial   | Arterial      | Arterial    |
|                     | Network    | Network       | Network     |
| Fixed-time          | 42.13      | 84.94         | 148.03      |
|                     | 100.62     | 181.88        | 258.91      |
| DORAS-Q             | 31.14      | 57.52         | 118.36      |
|                     | 75.38      | 128.95        | 227.39      |
| Max Pressure        | 23.18      | 41.92         | 122.27      |
|                     | 51.76      | 100.83        | 203.20      |
| Back Pressure       | 21.95      | 36.58         | 90.15       |
|                     | 50.27      | 90.15         | 191.65      |
| RL-WaitingTime      | 19.29      | 47.26         | 120.31      |
|                     | **35.61**  | **89.97**     | **178.33**  |
| RL-Queue            | **17.78**  | 35.39         | 119.78      |
|                     | 36.23      | 87.11         | 176.39      |
| RL-MaxPressure      | 19.39      | 39.01         | 115.39      |
|                     | 49.38      | 93.28         | 170.49      |
| RL-BackPressure     | 18.50      | **34.58**     | **113.48**  |
|                     | 46.63      | **87.26**     | **167.27**  |

FIGURE 9: Average vehicle delay in arterial and grid network simulation environment with 25% truck volume (in second)

Table 6 and Figure 9 illustrates the results of the simulation under all scenarios with 25% of truck volume. RL-BackPressure outperforms the other four signal control algorithms in both arterial and grid network cases. Similar to the 10% truck volume case, DORAS-Q and Max Pressure outperform the fixed time control with coordination. RL baseline have outperformed other algorithms under low volume scenarios. RL-MaxPressure still have the best performance under medium and high-volume cases.

40% truck flow

Table 7 and Figure 10 illustrates the results of the simulation under all scenarios with 40% of truck volume.
### TABLE 7: Average vehicle delay in arterial and grid network case with 40% truck volume (in seconds)

|                      | Low Volume | Medium Volume | High Volume |
|----------------------|------------|---------------|-------------|
|                      | Arterial   | Network       | Arterial    | Network    | Arterial   | Network    |
| Fixed-time           | 69.04      | 106.25        | 129.64      | 192.63     | 267.68     | 274.39     |
| DORAS-Q              | 34.02      | 76.68         | 120.33      | 146.14     | 225.39     | 259.32     |
| Max Pressure         | 27.93      | 65.68         | 127.96      | 110.89     | 239.37     | 259.45     |
| Back Pressure        | 23.25      | 63.38         | 117.83      | 98.46      | 216.21     | 240.86     |
| RL-WaitingTime       | 25.11      | 52.94         | 119.72      | 162.89     | 232.39     | 263.18     |
| RL-Queue             | 23.74      | 51.14         | 114.71      | 165.79     | 232.79     | 247.28     |
| RL-MaxPressure       | 22.92      | 52.78         | 120.05      | 106.46     | 216.39     | 207.91     |
| RL-BackPressure      | **22.18**  | 51.49         | **111.05**  | **95.87**  | **208.72** | **187.45** |

### FIGURE 10: Average vehicle delay in arterial and grid network simulation environment with 40% truck volume (in second)

### DISCUSSION

In general, The RL-based algorithm has overall good performance in terms of total delay among all scenarios. Besides, traditional signal control methods perform much better with the addition of reinforcement learning algorithms. Under the med and high-volume scenarios, the RL-BackPressure algorithm outperforms all other comparable algorithms. Under the low-volume scenario, the advantage of RL-BackPressure is not clear enough, but the RL baselines have satisfied performance.

In the fixed timing design, the green wave undoubtedly facilitates the vehicle’s movement on the arterial and grid network at signalized intersections. Although the green wave is added to signal control, the fixed timing design does not inherently consider the dynamic of traffic flow or responsiveness to the current situation at the intersection. That is why fixed-time control performs worse among candidates.

DORAS-Q estimates the intersections’ efficiency within a certain period and predicts future arrivals. However, inaccurate estimation or prediction inevitably leads to flawed decisions. Even if the controller can receive accurate information (e.g., during a fully connected vehicle), DORAS-Q optimizes based only on the predicted arrival stream information, and there is no correction mechanism for the decision due to limited information. Minor errors will gradually accumulate so that the decision does not match the future situation. Further, the vehicle will still interact
Max Pressure utilizes the current information at the intersection, which overcomes the shortcoming of prediction. However, the algorithm greedily chooses the phase with the maximum pressure without using historical knowledge so that the solution may be the local optimum rather than the global optimum. The more complex the road network, the more likely it converges to the local optimum. That is why the Max Pressure surprisingly underperforms DORAS-Q in the grid network case. In the simulation, either pure Max Pressure or pure back pressure algorithm is susceptible to the flow pattern. The difference in realizations is significant with the same initial conditions, especially under high-volume traffic. This inspires us to apply the back pressure, whose discharge flow rate changes with the link’s current flow. After the RL is added to it, the deficiency of RL-MaxPressure is compensated by learning iterations, obtaining an approximate optimal solution in the same state. Back pressure considers the potential flow through the node and refines the control theory in queuing networks. Neither of the pressure-like control requires input flow prediction or forecast. The only info required is the current state. A slight delay in the information collected will not affect the performance.

In general, considering the truck flow may increase the total delay at the intersection. The higher the truck volume percentage, the more delay driver may experience passing the intersections. However, the higher the truck percentage in the network is, the better the performance of the RL-based algorithm, especially RL-BackPressure. The truck volume may bring more impact or turbulence on downstream traffic, which may be why the pressure-like algorithm relatively underperforms in the low volume and low truck percentage scenarios. However, the impact is more consistent in the medium and high volume cases, which can be captured by RL algorithms. The addition of truck volume increases the overall delay compared with the pure passenger flow case. There are two reasons to explain the phenomenon. First, a truck usually has a larger dimension than a passenger vehicle. Passenger car equivalent of trucks could not sufficiently represent the impact of the truck on traffic flow. Second, trucks are slower to accelerate and decelerate than passenger vehicles. When trucks approach or leave the intersection, the inhomogeneous of vehicles’ dynamic may heterogeneity the traffic flow. The passenger car equivalent of trucks does not consider the problem, particularly when the controller decides the phase. Also, the truck volume brings more disturbance to the coverage of the RL baseline algorithms. The results variation is more significant than that of pure passenger vehicle flow case. And it takes more time to converge in each RL enhanced algorithm.

Considering that the traffic movement process at each intersection is stable, the system is
also stable accordingly. In the road environment without turnarounds, the actions taken by the RL agents do not create loops or block the network, decreasing the imbalance between the intersections and thus allowing efficient use of green light time. A common pseudo-refutation scenario is that if each intersection has the same long queue in each direction, then the pressure at each intersection is 0. Wouldn’t the agent exacerbate the congestion? It is worth noting that the so-called refutation scenario can only exist in a single moment and is very unstable. Under such a case, once a phase turns green, the back pressure in that direction will gradually decrease. After a certain point, the RL agent will choose to end the green light in that direction and give green time to the next phase to earn a higher “reward.” Then, the whole network system will evolve again toward the lowest overall flow and pressure. Therefore, for a given period $T$, our RL agent can provide the maximum throughput, thus minimizing the travel time of all vehicles in the system.

**CONCLUSION**

Traffic signal control is installed to manage traffic flow and allocate the right of way. An effective and well-performed signal control algorithm is needed to alleviate traffic congestion and increase the efficiency of the road network in the urban. Deep reinforcement learning algorithms have recently become increasingly popular and have been widely recognized as an effective tool to solve the traffic control problem.

Currently, several measurements have been used for evaluating network performance, including but not limited to the total travel time of vehicles in the network, average vehicle delay and the number of stops, and average travel speed. Evaluating the performance of signal control usually needs more than one metric to validate each other. Thus, the fundamental problem of signal control is how to confidently translate performance metrics (i.e., minimum delay, minimum stops) into timely, observable, explicitly controllable variables.

This study proposes a weighted max pressure method (back pressure) for network signal control, which is implied in the Lyapunov control function. This method builds on the differential queues between queues before and after an intersection connected by vehicle flow. The weight is the traffic discharge rate. Lyapunov control is a general umbrella, which is also shown to imply a method called DORAS in the literature that maximizes the intersection discharge efficiency. Lyapunov control is a dynamic control framework whose objective is to minimize the drift function, which is usually not the objective function in the usual intersection signal control area. A significant advantage of Lyapunov control is that it maintains network stability under mild conditions, which is often most desired in the general network control area. Surprisingly, the derived weighted max pressure method proves to perform well, and when enhanced with the Deep Reinforcement Learning (DRL) method, it outperforms all other control methods that exist in the literature.

The designed signal control algorithm combines traffic theory and control theory with the reinforcement learning method. The Lyapunov control is introduced in queuing control problems to reduce the impact of inaccurate prediction and increase the robustness to queuing estimation. The state of the RL algorithm is the total number of vehicles and stopped vehicles on each incoming and outgoing lane. The action is phase to allocate right-out-way. The reward for the intersection is the back pressure to reduce the imbalance across the network queues. The Double DQN was adopted as a function approximator for higher accuracy. The proposed signal control framework has solid theoretical support from control theory and is robust to distribution and minor errors. Moreover, the result shows that RL could improve the performance of traditional signal control algorithms. RL-based algorithms have relatively better performance than fixed-time, DORAS-Q,
Max Pressure, and Back Pressure in terms of total delay under varying volumes and in consideration of freight traffic. Among the RL-Based models, the proposed RL-BackPressure always has lower average vehicle delay in arterial and the grid network, especially under high volumes. In addition, the RL-BackPressure relies on the current state rather than predicting future states and can be distributed implemented, so that the framework is more robust and accurate than current models.

Compared with the uniform passenger flow case, the additional truck volume increases the overall delay at the intersection. The higher the truck volume percentage, the more travel delay drivers suffer. Besides, the travel time and back pressure are closely correlated. As the back pressure gradually coverage to zero, the travel time also decreases step by step. Usually, freight traffic is more likely to disturb the downstream traffic, which may decrease the pressure series algorithm’s performance in the low volume and low truck percentage scenarios. However, RL-BackPressure is still robust in the high volume and high truck volume percentage case.

There are a few limitations to this study. First, the RL-BackPressure’s performance is not outstanding for light traffic loads in the case of both arterial and gird networks. It is worth investigating how to improve the algorithm when little network effect exists. Second, a conversion factor may not fully represent trucks’ characteristics. Considering detailed vehicle-specific characteristics and performance may facilitate a better comprehensive study and robust traffic signal control algorithm in future studies. It may be necessary to incorporate Monte Carlo tree search or embed spatial information into the state, such as the state of adjacent intersection, into the algorithm to increase its robustness under various scenarios.
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