Higher Dimensional Charged Rotating Dilaton Black Holes

A. Sheykhi\textsuperscript{1,2\ast} and M. Allahverdizadeh\textsuperscript{1}

\textsuperscript{1}Department of Physics, Shahid Bahonar University, P.O. Box 76175, Kerman, Iran
\textsuperscript{2}Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), Maragha, Iran

Abstract

In this paper, we present the metric for the $n$-dimensional charged slowly rotating dilaton black hole with $N = [(n - 1)/2]$ independent rotation parameters, associated with $N$ orthogonal planes of rotation in the background of asymptotically flat and asymptotically (anti)-de Sitter spacetime. The mass, angular momentum and the gyromagnetic ratio of such a black hole are determined for the arbitrary values of the dilaton coupling constant. We find that the gyromagnetic ratio crucially depends on the dilaton coupling constant, $\alpha$, and decreases with increasing $\alpha$ in any dimension.

\* sheykhi@mail.uk.ac.ir
I. INTRODUCTION

There has been a considerable attention in general relativity in more than four space-time dimensions in recent years. There are several reasons why it should be interesting to study this extension of Einstein’s theory, and in particular its black hole solutions (for a recent review on higher dimensional black holes see [1]). The first reason originates from developments in string theory, which is believed to be the most promising approach to quantum theory of gravity in higher dimensions. In fact, the first successful statistical counting of black hole entropy in string theory was performed for a five-dimensional black hole [2]. This example provides the best laboratory for the microscopic string theory of black holes. Second, the AdS/CFT correspondence relates the properties of an $n$-dimensional black hole with those of a quantum field theory in $(n-1)$-dimensions [3]. According to the AdS/CFT correspondence, the rotating black holes in AdS space are dual to certain CFTs in a rotating space [4], while charged ones are dual to CFTs with chemical potential [5]. Furthermore, as mathematical objects, black hole spacetimes are among the most important Lorentzian Ricci-flat manifolds in any dimension. One striking feature of the Einstein equations in more than four dimensions is that many uniqueness properties holding in four dimensions are lost. For instance, four-dimensional black holes are known to possess a number of remarkable features, such as uniqueness, spherical topology, dynamical stability, and the laws of black hole mechanics. One would like to know which of these are peculiar to four-dimensions, and which hold more generally. At least, this study will lead to a deeper understanding of classical black holes and of what spacetime can do at its most extreme. Besides, it was confirmed by recent investigations that the gravity in higher dimensions exhibits much richer dynamics than in four dimensions. For example, the discovery of dynamical instabilities of extended black hole horizons [6], and the discovery of black ring solutions with horizons of non-spherical topology and not fully characterized by their conserved charges [7]. While the nonrotating black hole solution to the higher-dimensional Einstein-Maxwell gravity was found several decades ago [8], the counterpart of the Kerr-Newman solution in higher dimensions, that is the charged generalization of the Myers-Perry solution [9] in higher dimensional Einstein-Maxwell theory, still remains to be found analytically. The most general higher dimensional uncharged rotating black holes in anti-de Sitter space with all rotation parameters have been recently found [10]. As far as we know, rotating black holes for the Maxwell field minimally coupled
to Einstein gravity in higher dimensions, do not exist in a closed form and one has to rely on
perturbative or numerical methods to construct them in the background of asymptotically
flat \cite{11, 12} and AdS \cite{13} spacetime. There has also been recent interest in constructing
the analogous charged rotating solutions in the framework of gauged supergravity in various
dimensions \cite{14, 15, 16}.

There has been a renewed interest in studying scalar coupled solutions of general relativity
ever since new black hole solutions have been found in the context of string theory. The
low energy effective action of string theory contains two massless scalars namely dilaton and
axion. The dilaton field couples in a nontrivial way to other fields such as gauge fields and
results into interesting solutions for the background spacetime \cite{17, 18, 19}. These solutions
\cite{17, 18, 19}, however, are all asymptotically flat. In the presence of one or two Liouville-type
dilaton potentials, black hole spacetimes which are neither asymptotically flat nor (anti)-de
Sitter [(A)dS] have been explored by many authors (see e.g \cite{20, 21, 22, 23, 24, 25}).
Recently, by using the combination of three Liouville type dilaton potentials, static charged
dilaton black hole solutions in four \cite{26} and higher dimensions \cite{27} have been found in the
background of (A)dS spacetime. Such potential may arise from the compactification of a
higher dimensional supergravity model \cite{28} which originates from the low energy limit of a
background string theory.

In the backdrop of the scenarios described so far it is therefore worthwhile to study
higher dimensional rotating black holes in a spacetime with nonzero cosmological constant
in the presence of dilaton-electromagnetic coupling. For some limited values of the dilaton
coupling constant, $\alpha$, exact rotating black hole solutions have been obtained in \cite{29, 30, 31, 32}.
For general dilaton coupling constant, the properties of charged rotating dilaton
black holes in four \cite{33, 34, 35, 36} and five \cite{37} dimensions have been studied in the small
angular momentum limit. The generalization of these slowly rotating solutions in all higher
dimensions with a single rotation parameter have also been done \cite{38, 39, 40}. However, in
more than three spatial dimensions, black holes can rotate in different orthogonal planes, so
the general solution has several angular momentum parameters. Indeed, an $n$-dimensional
black hole can have $N = [(n - 1)/2]$ independent rotation parameters, associated with $N$
orthogonal planes of rotation where $[x]$ denotes the integer part of $x$. In this paper we would
like to extend our former works \cite{39, 40} to the higher dimensional charged rotating dilaton
black holes with all rotation parameters. We then determine the angular momentum and
the gyromagnetic ratio of such a black hole for the arbitrary values of the dilaton coupling constant.

This paper is organized as follows. In the next section we introduce the action and the field equations. In section [III] we present the metric of the higher dimensional charged slowly rotating dilaton black hole in asymptotically flat spacetime with all rotation parameters. We also compute the angular momentum and the gyromagnetic ratio of the solution. In section [IV] we extend our solution to asymptotically (A)dS spacetimes. The last section is devoted to summary and conclusions.

II. FIELD EQUATIONS AND METRIC

Our starting point is the $n$-dimensional theory in which gravity is coupled to dilaton and Maxwell field with an action

$$
S = -\frac{1}{16\pi} \int d^n x \sqrt{-g} \left( R - \frac{4}{n-2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) - e^{-\frac{4\alpha\Phi}{n-2}} F_{\mu\nu} F^{\mu\nu} \right) - \frac{1}{8\pi} \int_{\partial M} d^{n-1} x \sqrt{-h} \Theta(h),
$$

where $R$ is the scalar curvature, $\Phi$ is the dilaton field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor, and $A_\mu$ is the electromagnetic potential. $\alpha$ is an arbitrary constant governing the strength of the coupling between the dilaton and the Maxwell field and $V(\Phi)$ is the potential for $\Phi$. The last term in Eq. (1) is the Gibbons-Hawking surface term. It is required for the variational principle to be well-defined. The factor $\Theta$ represents the trace of the extrinsic curvature for the boundary $\partial M$ and $h$ is the induced metric on the boundary. While $\alpha = 0$ corresponds to the usual Einstein-Maxwell-scalar theory, $\alpha = 1$ indicates the dilaton-electromagnetic coupling that appears in the low energy string action in Einstein’s frame.

Varying the action (1) with respect to the gravitational field $g_{\mu\nu}$, the dilaton field $\Phi$ and the gauge field $A_\mu$, we obtain the field equations

$$
R_{\mu\nu} = \frac{4}{n-2} \left( \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{4} g_{\mu\nu} V(\Phi) \right) + 2 e^{-\frac{4\alpha\Phi}{n-2}} \left( F_{\mu\eta} F^{\eta\nu} - \frac{1}{2(n-2)} g_{\mu\nu} F_{\lambda\eta} F^{\lambda\eta} \right),
$$

$$
\nabla^2 \Phi = \frac{n-2}{8} \frac{\partial V}{\partial \Phi} - \frac{\alpha}{2} e^{-\frac{4\alpha\Phi}{n-2}} F_{\lambda\eta} F^{\lambda\eta}, \tag{3}
$$

$$
\partial_\mu \left( \sqrt{-g} e^{-\frac{4\alpha\Phi}{n-2}} F^{\mu\nu} \right) = 0. \tag{4}
$$
We would like to find the rotating solutions of the above field equations with all rotation parameters in all higher dimensions in the limit of slow rotation. The rotation group in \( n \)-dimensions is \( SO(n-1) \) and therefore the number of independent rotation parameters for a localized object is equal to the number of Casimir operators, which is \( N = [(n-1)/2] \), where \([x]\) is the integer part of \( x \). For small rotation parameters, we can solve Eqs. (2)-(4) to first order in the angular momentum parameters \( a_i \). Inspection of the Myers-Perry solution \([9]\) shows that the terms in the metric that change to the first order of the rotational parameters \( a_i \)'s are \( g_{t\phi_i} \)'s \( (i = 1 \ldots N) \). Similarly, the dilaton field does not change to \( O(a_i) \) and \( A_{\phi_i} \)'s are the components of the vector potential that change. Therefore, inspired by the Myers-Perry metric \([9]\), for infinitesimal angular momentum, we assume the metric being of the following form

\[
\begin{align*}
    ds^2 &= -U(r)dt^2 + \frac{dy^2}{W(r)} - 2f(r)dt\sum_{i=1}^{N} a_i\mu_i^2d\phi_i \\
    &\quad +r^2R^2(r)\sum_{i=1}^{N} (d\mu_i^2 + \mu_i^2d\phi_i^2) + \varepsilon r^2R^2(r)d\nu^2,
\end{align*}
\]

Where for even \( n \) we have \( N = \frac{n-2}{2} \) and \( \varepsilon = 1 \), whereas for odd \( n \), \( N = \frac{n-1}{2} \) and \( \varepsilon = 0 \). The \( \mu_i \) (and \( \nu \), for even dimensional cases) coordinates are not independent but have to obey the constraint

\[
\sum_{i=1}^{N} \mu_i^2 + \varepsilon \nu^2 = 1.
\]

The functions \( U(r) \), \( W(r) \), \( R(r) \) and \( f(r) \) should be determined. For small \( a_i \) \( (i = 1 \ldots N) \), we can expect to have solutions with \( U(r) \) and \( W(r) \) still functions of \( r \) alone. The \( t \) component of the Maxwell equations can be integrated immediately to give

\[
F_{tr} = \sqrt{\frac{U(r)}{W(r)}} Q e^{4\phi_{n-2}} (rR)^{n-2},
\]

where \( Q \), an integration constant, is the electric charge of the black hole. In general, in the presence of rotation, we have \( A_{\phi_i} \neq 0 \). In addition we can write

\[
A_{\phi_i} = -QC(r)a_i\mu_i^2 \quad \text{(no sum on } i\text{)}.
\]

### III. Rotating Dilaton Black Holes in Flat Spacetime

We begin by looking for the asymptotically flat solutions, therefore we set \( V(\Phi) = 0 \) in the field equations. In a recent work \([39]\), we found a class of asymptotically flat slowly rotating...
charged dilaton black hole solution in higher dimensions with single rotation parameter.
Here we are looking for the asymptotically flat slowly rotating dilaton black hole solutions
with all rotation parameters in all higher dimensions. Inserting the metric (5), the Maxwell
fields (7) and (8) into the field equations (2)-(4), one can show that the static part of the
metric leads to the following solutions [19]

\[ U(r) = \left[ 1 - \left( \frac{r_+}{r} \right)^{n-3} \right] \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{1-\gamma(n-3)}, \]  

(9)

\[ W(r) = \left[ 1 - \left( \frac{r_+}{r} \right)^{n-3} \right] \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{1-\gamma}, \]  

(10)

\[ R(r) = \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{\gamma/2}, \]  

(11)

\[ \Phi(r) = \frac{n-2}{4} \sqrt{\gamma(2+3\gamma-n\gamma)} \ln \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right], \]  

(12)

while the rotating part of the metric admits a solution

\[ f(r) = (n-3) \left( \frac{r_+}{r} \right)^{n-3} \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{\frac{n-3-\alpha^2}{n-3+\alpha^2}} \]

\[ + \frac{(\alpha^2 - n + 1)(n-3)^2}{\alpha^2 + n - 3} r_-^{n-3} r_+^{n-3+\alpha^2} \left( 1 - \left( \frac{r_-}{r} \right)^{n-3} \right)^\gamma \]

\[ \times \int \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{\gamma(2-n)} \frac{dr}{r^n}, \]  

(13)

\[ C(r) = \frac{1}{r^{n-3}}. \]  

(14)

We can also perform the integration and express the solution in terms of hypergeometric
function

\[ f(r) = (n-3) \left( \frac{r_+}{r} \right)^{n-3} \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{\frac{n-3-\alpha^2}{n-3+\alpha^2}} \]

\[ + \frac{(\alpha^2 - n + 1)(n-3)^2}{(1-n)(\alpha^2 + n - 3)} \left( \frac{r_-}{r} \right)^{n-3} \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{\gamma} \]

\[ \times _2 F_1 \left( \left[ (n-2)\gamma, 1, \frac{n-1}{n-3} \right], \left[ \frac{2n-4}{n-3} \right] \left( \frac{r_-}{r} \right)^{n-3} \right). \]  

(15)
Here $r_+$ and $r_-$ are the event horizon and Cauchy horizon of the black hole, respectively. The constant $\gamma$ is

$$\gamma = \frac{2\alpha^2}{(n-3)(n-3+\alpha^2)}. \tag{16}$$

The charge $Q$ is related to $r_+$ and $r_-$ by

$$Q^2 = \frac{(n-2)(n-3)^2}{2(n-3+\alpha^2)}r_+^{n-3}r_-^{n-3}, \tag{17}$$

and the physical mass of the black hole is obtained as follows [41]

$$M = \frac{\Omega_{n-2}}{16\pi} \left[ (n-2)r_+^{n-3} + \frac{n-2-p(n-4)}{p+1}r_-^{n-3} \right], \tag{18}$$

where $\Omega_{n-2}$ denotes the area of the unit $(n-2)$-sphere and the constant $p$ is

$$p = \frac{(2-n)\gamma}{(n-2)\gamma - 2}. \tag{19}$$

The metric corresponding to (9)-(15) is asymptotically flat. In the special case $n = 4$, the static part of our solution reduces to

$$U(r) = W(r) = \left( 1 - \frac{r_+}{r} \right) \left( 1 - \frac{r_-}{r} \right)^{\frac{1-\alpha^2}{1+\alpha^2}}, \tag{20}$$

$$R(r) = \left( 1 - \frac{r_-}{r} \right)^{\frac{\alpha^2}{1+\alpha^2}}, \tag{21}$$

$$\Phi(r) = \frac{\alpha}{(1+\alpha^2)} \ln \left( 1 - \frac{r_-}{r} \right), \tag{22}$$

while the rotating part reduces to

$$f(r) = \frac{r^2(1+\alpha^2)^2(1-r_-)}{(1-\alpha^2)(1-3\alpha^2)r_-^2} - \left( 1 - \frac{r_-}{r} \right)^{\frac{1-\alpha^2}{1+\alpha^2}} \left( 1 + \frac{(1+\alpha^2)^2r_-^2}{(1-\alpha^2)(1-3\alpha^2)r_-^2} + \frac{(1+\alpha^2)r_-}{(1-\alpha^2)r_-} - \frac{r_+}{r} \right), \tag{23}$$

which is the four-dimensional charged slowly rotating dilaton black hole solution of Horne and Horowitz [33]. One may also note that in the absence of a non-trivial dilaton ($\alpha = 0 = \gamma$), our solutions reduce to

$$U(r) = W(r) = \left[ 1 - \left( \frac{r_+}{r} \right)^{n-3} \right] \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right], \tag{24}$$

$$f(r) = (n-3) \left[ \frac{r_+^{n-3} + r_-^{n-3}}{r^{n-3}} - \left( \frac{r_+ + r_-}{r^2} \right)^{n-3} \right], \tag{25}$$

which describe $n$-dimensional Kerr-Newman black hole in the limit of slow rotation [12]. Next, we calculate the angular momentum and the gyromagnetic ratio of these rotating
dilaton black holes which appear in the limit of slow rotation parameters. The angular momentum of the dilaton black hole can be calculated through the use of the quasi-local formalism of the Brown and York [42]. According to the quasilocal formalism, the quantities can be constructed from the information that exists on the boundary of a gravitating system alone. Such quasilocal quantities will represent information about the spacetime contained within the system boundary, just like the Gauss’s law. In our case the finite stress-energy tensor can be written as

\[ T^{ab} = \frac{1}{8\pi} (\Theta^{ab} - \Theta h^{ab}) \],

which is obtained by variation of the action (1) with respect to the boundary metric \( h_{ab} \). To compute the angular momentum of the spacetime, one should choose a spacelike surface \( B \) in \( \partial M \) with metric \( \sigma_{ij} \), and write the boundary metric in ADM form

\[ \gamma_{ab} dx^a dx^b = -N^2 dt^2 + \sigma_{ij} (d\phi^i + V^i dt) (d\phi^j + V^j dt) , \]

where the coordinates \( \phi^i \) are the angular variables parameterizing the hypersurface of constant \( r \) around the origin, and \( N \) and \( V^i \) are the lapse and shift functions respectively. When there is a Killing vector field \( \xi \) on the boundary, then the quasilocal conserved quantities associated with the stress tensors of Eq. (26) can be written as

\[ Q(\xi) = \int_B d^{n-2}\varphi \sqrt{\sigma} T_{ab} n^a \xi^b , \]

where \( \sigma \) is the determinant of the metric \( \sigma_{ij} \), \( \xi \) and \( n^a \) are, respectively, the Killing vector field and the unit normal vector on the boundary \( B \). For boundaries with rotational \( (\varsigma = \partial/\partial \phi) \) Killing vector field, we can write the corresponding quasilocal angular momentum as follows

\[ J = \int_B d^{n-2}\varphi \sqrt{\sigma} T_{ab} h^a \varsigma^b , \]

provided the surface \( B \) contains the orbits of \( \varsigma \). Finally, the angular momentum of the black holes can be calculated by using Eq. (28). We find

\[ J_i = \frac{a_i \Omega_{n-2}}{8\pi} \left( r_+^{n-3} + \frac{(n-3)(n-1-\alpha^2)r_+^{n-3}}{(n-3+\alpha^2)(n-1)} \right) . \]

For \( a_i = 0 \) the angular momentum vanishes, and therefore \( a_i \)'s are the rotational parameters of the dilaton black hole. For \( n = 4 \), we have only one rotation parameter, \( a \). In this case, the corresponding angular momentum reduces to

\[ J = \frac{a}{2} \left( r_+ + \frac{3-\alpha^2}{3(1+\alpha^2)} r_- \right) . \]
which restores the angular momentum of the four-dimensional Horne and Horowitz solution [33], while in the absence of dilaton field \((\alpha = 0)\), the angular momentum reduces to

\[
J_i = \frac{a_i \Omega a^2}{8\pi} (r_+^{n-3} + r_-^{n-3}),
\]

which is the angular momentum of the \(n\)-dimensional Kerr-Newman black hole. Next, we are going to calculate the gyromagnetic ratio of this rotating dilaton black holes. As we know, the gyromagnetic ratio is an important characteristic of the Kerr-Newman-AdS black hole. Indeed, one of the remarkable facts about a Kerr-Newman black hole in asymptotically flat spacetime is that it can be assigned a gyromagnetic ratio \(g = 2\), just as an electron in the Dirac theory. It should be noted that, unlike four dimensions, the value of the gyromagnetic ratio is not universal in higher dimensions [43]. Besides, scalar fields such as the dilaton, modify the value of the gyromagnetic ratio of the black hole and consequently it does not possess the gyromagnetic ratio \(g = 2\) of the Kerr-Newman black hole [33].

The magnetic dipole moments for this asymptotically flat slowly rotating dilaton black hole can be defined as

\[
\mu_i = Qa_i.
\]

The gyromagnetic ratio is defined as a constant of proportionality in the equation for the magnetic dipole moments

\[
\mu_i = g \frac{QJ_i}{2M}.
\]

Substituting \(M\) and \(J_i\) from Eqs. (18) and (29), the gyromagnetic ratio \(g\) can be obtained as

\[
g = \frac{(n-1)(n-2)[(n-3+\alpha^2)r_+^{n-3} + (n-3-\alpha^2)r_-^{n-3}]}{(n-1)(n-3+\alpha^2)r_+^{n-3} + (n-3)(n-1-\alpha^2)r_-^{n-3}}.
\]

The above expression shows that the value of the gyromagnetic ratio \(g\) is the same as in the case of single rotation parameter [39]. We can also see that the dilaton field modifies the value of the gyromagnetic ratio through the dilaton coupling constant \(\alpha\) which measures the strength of the dilaton-electromagnetic coupling. This is in agreement with the arguments in [33, 39]. We have shown the behaviour of the gyromagnetic ratio \(g\) of the dilatonic black hole versus \(\alpha\) in Fig. 1. From this figure we find out that the gyromagnetic ratio decreases with increasing \(\alpha\) in any dimension. In the absence of a non-trivial dilaton \((\alpha = 0)\), the gyromagnetic ratio reduces to

\[
g = n - 2,
\]
which is the gyromagnetic ratio of the $n$-dimensional Kerr-Newman black hole in the slow rotation limit \[12\]. When $n = 4$, Eq. \[34\] reduces to

$$g = 2 - \frac{4\alpha^2 r_-}{(3 - \alpha^2)r_- + 3(1 + \alpha^2)r_+},$$

(36)

which is the gyromagnetic ratio of the four-dimensional Horne and Horowitz dilaton black hole \[33\].

**IV. ROTATING DILATON BLACK HOLES IN ADS SPACETIME**

Now we consider the solutions of Eqs. \(2\)-\(4\) with Liouville-type dilaton potential. For arbitrary value of $\alpha$ in (A)dS space the form of the dilaton potential in arbitrary dimensions is chosen as \[27\]

$$V(\Phi) = \frac{\Lambda}{3(n - 3 + \alpha^2)^2} \left[ -\alpha^2(n - 2) \left( n^2 - n\alpha^2 - 6n + \alpha^2 + 9 \right) e^{-\frac{4(n-3)\Phi}{(n-2)\alpha}} ight. \\
+ (n - 2)(n - 3)^2(n - 1 - \alpha^2)e^{\frac{4\Phi}{n-2}} + 4\alpha^2(n - 3)(n - 2)^2 e^{-\frac{2\Phi(n-3-\alpha^2)}{(n-2)\alpha}} \right].$$

(37)

Here $\Lambda$ is the cosmological constant. It is clear the cosmological constant is coupled to the dilaton in a very nontrivial way. This type of the dilaton potential was introduced for the first time by Gao and Zhang \[26, 27\]. They derived, by applying a coordinates transformation which recast the solution in the Schwarzchild coordinates system, the static dilaton black hole solutions in the background of (A)dS universe. For this purpose, they required the existence of the (A)dS dilaton black hole solutions and extracted successfully the form of
the dilaton potential leading to (A)dS-like solutions. They also argued that this type of derived potential can be obtained when a higher dimensional theory is compactified to four dimensions, including various supergravity models [28]. In the absence of the dilaton field the action (1) reduces to the action of Einstein-Maxwell gravity with cosmological constant. Inserting the metric (5), the Maxwell fields (7) and (8), and the dilaton potential (37) into the field equations (2)-(4), one can show that the static part of the metric leads to the following solutions [27]

\[ U(r) = \left[ 1 - \left( \frac{r_+}{r} \right)^{n-3} \right] \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{1-\gamma(n-3)} - \frac{1}{3} \Lambda r^2 \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^\gamma, \]

\[ W(r) = \left\{ \left[ 1 - \left( \frac{r_+}{r} \right)^{n-3} \right] \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{1-\gamma(n-3)} - \frac{1}{3} \Lambda r^2 \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^\gamma \right\} \times \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{\gamma(n-4)}, \]

\[ \Phi(r) = \frac{n-2}{4} \sqrt{\gamma(2+3\gamma-n\gamma)} \ln \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right], \]

\[ R(r) = \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{\gamma/2}, \]

while we obtain the following solution for the rotating part of the metric

\[ f(r) = \frac{2\Lambda r^2}{(n-1)(n-2)} \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{\gamma} + (n-3) \left( \frac{r_+}{r} \right)^{n-3} \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right] \frac{n-3-\alpha^2}{n-3+\alpha^2} \]

\[ + \frac{(\alpha^2-n+1)(n-3)^2 \alpha^2 + n-3}{r^{n-3} r_-^{n-3} r_-^{n-3}} \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{\gamma} \times \int \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{\gamma(2-n)} \frac{dr}{r^n}. \]

\[ C(r) = \frac{1}{r^{n-3}}. \]

We can also perform the integration and express the solution in terms of hypergeometric function

\[ f(r) = \frac{2\Lambda r^2}{(n-1)(n-2)} \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right]^{\gamma} + (n-3) \left( \frac{r_+}{r} \right)^{n-3} \left[ 1 - \left( \frac{r_-}{r} \right)^{n-3} \right] \frac{n-3-\alpha^2}{n-3+\alpha^2} \]

\[ + \frac{(\alpha^2-n+1)(n-3)^2 \alpha^2 + n-3}{(1-n)(\alpha^2 + n-3)} \left( \frac{r_-}{r} \right)^{n-3} \left[ 1 - \left( \frac{r_-}{r} \right) \right]^{\gamma} \times \, _2F_1 \left( \left[ (n-2)\gamma, \frac{n-1}{n-3} \right], \left[ \frac{2n-4}{n-3}, \left( \frac{r_-}{r} \right)^{n-3} \right] \right). \]
It is apparent that the metric corresponding to (38)-(44) is asymptotically (A)dS. For $\Lambda = 0$, the above solutions recover our previous results for asymptotically flat rotating dilaton black holes. It is worth noting that in the absence of a non-trivial dilaton ($\alpha = 0 = \gamma$), our solutions reduce to

$$U(r) = W(r) = \left[1 - \left(\frac{r_+}{r}\right)^{n-3}\right]\left[1 - \left(\frac{r_-}{r}\right)^{n-3}\right] - \frac{1}{3} \Lambda r^2$$

which describe $n$-dimensional charged Kerr-(A)dS black hole in the limit of slow rotation. We can also calculate the angular momentum and the gyromagnetic ratio of these asymptotically (A)dS rotating dilaton black holes. We find

$$J_i = \frac{a_i \Omega_{n-2}}{8\pi} \left( r_+^{n-3} + \frac{(n-3)(n-1-\alpha^2)r_+^{n-3}}{(n-3+\alpha^2)(n-1)} \right).$$

$$g = \frac{(n-1)(n-2)[(n-3+\alpha^2)r_+^{n-3} + (n-3-\alpha^2)r_-^{n-3}]}{(n-1)(n-3+\alpha^2)r_+^{n-3} + (n-3)(n-1-\alpha^2)r_-^{n-3}}.$$
which appear up to the linear order of the rotational parameters $a_i$'s. We have shown that the gyromagnetic ratio crucially depends on the dilaton coupling constant, $\alpha$, and decreases with increasing $\alpha$ in any dimension.

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