Temperature dependence of spin correlation and charge dynamics in the stripe phase of high-$T_c$ superconductors

Y. Shibata, T. Tohyama, and S. Maekawa
Institute for Materials Research, Tohoku University, Sendai 980-8577, Japan
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We examine the temperature dependence of the electronic states in the stripe phase of high-$T_c$ cuprates by using the $t$-$J$ model with a potential that stabilizes vertical charge stripes. Charge and spin-correlation functions and optical conductivity are calculated by using finite-temperature Lanczos method. At zero temperature, the antiferromagnetic correlation between a spin in a charge stripe and that in a spin domain adjacent to the stripe is weak, since the stripe charge and the spin domain are almost separated. With increasing temperature, the correlation increases and then decreases toward high temperature. This is in contrast to other correlations that decrease monotonically. From the examination of the charge dynamics, we find that this anomalous temperature dependence of the correlation is the consequence of a crossover from one-dimensional electronic states to two-dimensional ones.

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I. INTRODUCTION

Charge stripes and related phenomena are now hot topics in the high-$T_c$ superconductor research field. Neutron-scattering measurements performed on La$_{1.475}$Nd$_{0.4}$ Sr$_{0.125}$CuO$_4$ (LNSC) with the low-temperature tetragonal (LTT) structure have revealed the presence of a charge order and an incommensurate magnetic order. These charge and spin orders have been explained by assuming a stripe structure that consists of vertical charge stripes and antiphase between spin domains. The incommensurate magnetic order has been observed not only in LNSC but also in La$_{2-x}$Sr$_x$CuO$_4$ (LSC) with $x \approx 0.12$ carrier doping. Around this carrier concentration, an incomplete phase transition from the low-temperature orthorhombic (LTO) phase to the LTT phase has been observed. While clear evidence of the charge order has not been reported, it is considered that the electronic states in LSC are similar to those in the LNSC.

Anomalous behaviors that are supposed to be related to the presence of the stripes have been observed in the angle-resolved photoemission (ARPES) spectrum as suppressed spectral weight along the $(0,0)$-$\pi$-$\pi$ direction and in the optical conductivity as the enhancement of intensity in the mid-infrared region. These features have been explained by the present authors, by using the exact diagonalization calculation at zero-temperature for a model that includes both the strong electron correlation and the stripes, i.e., a $t$-$J$ model with an additional potential introduced to stabilize vertical charge stripes. The ground state of the model is characterized by the change stripes along which the charge carriers can move coherently, but perpendicular to which charge carriers show only incoherent motion. This situation is consistent with the behavior of the Hall coefficient at low temperature as well as the ARPES data in LNSC, both of which indicate the one-dimensional motion of the charge carriers. At the same time, the zero temperature calculation has shown that the spin correlation inside the spin domains is as strong as those for the Heisenberg spin system, whereas the correlation between a spin in a spin domain and that in a charge stripe adjacent to the domain is weak since the charge stripe and the spin domain are almost separated.

With the increase of temperature, the one-dimensional electronic states in the stripe phase mentioned above are expected to be destroyed and evolved into the two-dimensional ones. Therefore it is interesting to know how such evolution occurs and how the physical quantities are affected by the evolution. In this study, we clarify the temperature dependence of the electronic state in the stripe phase. By using the finite-temperature Lanczos method developed by Jaklič and P. Prelovšek, we calculate the optical conductivity and spin-correlation functions at finite temperatures in the $t$-$J$ model with a potential that stabilizes the vertical charge stripes. We find that the spin correlation between a spin in the charge stripe and that in the spin domain increases with increasing temperature and then decreases toward higher temperature. This is in contrast to other spin correlations that decrease monotonically. By examining the dependence of charge dynamics on temperature, this anomalous temperature dependence of the spin correlation is concluded to be the consequence of a crossover from one-dimensional electronic states to two-dimensional ones.

We introduce our model, i.e., a $t$-$J$ model with a stripe potential, and show outlines of the finite temperature Lanczos method in Sec. II. In Sec. III, results and discussions on the temperature dependence of spin-and charge-correlation functions and the optical conductivity are presented. A possible method to confirm experimentally the evolution of the electronic states in stripe phase with temperature is also proposed. The summary is given in Sec. IV.
II. MODEL AND NUMERICAL METHOD

We introduce the $t$-$J$ model, which is given by

$$ H_{tJ} = J \sum_{\{i,j\}} S_i \cdot S_j - t \sum_{\{i,j\}\sigma} \left( \hat{c}_{i\sigma} \hat{c}_{j\sigma} + \text{H.c.} \right), $$

where $\hat{c}_{i\sigma} = c_{i\sigma}(1 - n_{i\sigma})$ is the annihilation operator of an electron with spin $\sigma$ at site $i$ with the constraint of no double occupancy, $S_i$ is the spin operator, and the summation $\{i,j\}$ runs over the nearest-neighbor pairs.

It is controversial whether the $t$-$J$ model itself has the stripe-type ground state. A possible origin of the appearance of stable stripe phases is due to the presence of the long-range part of the Coulomb interaction and/or the coupling to lattice distortions. In LSC, the LTT fluctuation seems to help the latter mechanism.

Among four columns in the cluster, the stripe potential is assumed to depend on the number of holes $n$ and/or the coupling to lattice distortions. In LSC, the potential $V_l$ is set to be $V_l = 0$ for $n_h = 0$ and $V_l = -n_h V$ for $n_h \geq 2$ with $V > 0$. $V_S(n_h)$ behaves like an attractive potential for holes independent of distance between holes. In the following, we use a 4$x$4 cluster of the $t$-$J$ model with two holes to simulate the underdoped system. Among four columns in the cluster, the stripe potential $V_S(n_h)$ is introduced into the second column from the left. Periodic and open boundary conditions are imposed along the directions parallel and perpendicular to the column, respectively.

We set parameters $J/t=0.4$ ($t \approx 0.35$ eV), and $V/t=1$ to obtain the ground state with charge stripes. Here, we note that, if $V = 0$, the ground state in small clusters is not the stripe state but a uniform state as discussed by Hellberg and Manousakis, and the energy difference between the two states is of the order of $J^2$. In order for the stripe phase to be in the ground state, it is necessary to introduce $V$ with a magnitude of more than $J$. Therefore $V/t=1$ is chosen to make the stripe state stable enough.

Examining the $V$ dependence of the spin and charge correlation functions discussed in the next section, we have found that their behaviors are not altered qualitatively if $V > J$.

We employ the finite temperature Lanczos method. Outlines of this method are as follows. The statistical expectation value of an operator $\hat{A}$ is given by

$$ \langle \hat{A} \rangle = \frac{1}{Z} \sum_{l=1}^{L} \langle \Psi_l | e^{-\beta H} \hat{A} | \Psi_l \rangle , $$

where $Z$ is the partition function defined as $Z = \sum_{l=1}^{L} \langle \Psi_l | e^{-\beta H} | \Psi_l \rangle$, $\{ \Psi_l \}$ is a complete basis set, $L$ is the dimension of the Hamiltonian, and $\beta = 1/k_B T$, $k_B$ being the Boltzmann factor. Hereafter, $k_B$ is set to be 1. In order to obtain eigenvectors and eigenvalues, we use the Lanczos procedure. We set an arbitrary vector $| \Psi_l \rangle$ as the initial vector of the Lanczos step $| \psi_{l}^{1} \rangle$, i.e., $| \psi_{l}^{1} \rangle = | \Psi_l \rangle$, and calculate $| \psi_{l}^{2} \rangle, | \psi_{l}^{3} \rangle, \ldots, | \psi_{M}^{l} \rangle$ step by step.

$$ H | \psi_{l}^{1} \rangle = a_{l}^{1} | \psi_{l}^{1} \rangle + b_{l}^{1} | \psi_{l}^{2} \rangle, $$

$$ H | \psi_{l}^{2} \rangle = a_{l}^{2} | \psi_{l}^{1} \rangle + a_{l}^{3} | \psi_{l}^{2} \rangle + b_{l}^{3} | \psi_{l}^{3} \rangle, $$

$$ H | \psi_{l}^{M-1} \rangle = b_{l}^{M-1} | \psi_{l}^{M-2} \rangle + a_{l}^{M-1} | \psi_{l}^{M-1} \rangle + b_{l}^{M} | \psi_{l}^{M} \rangle, $$

$$ H | \psi_{l}^{M} \rangle = b_{l}^{M} | \psi_{l}^{M-1} \rangle + a_{l}^{M} | \psi_{l}^{M} \rangle , $$

where $M$ is a given maximum number of Lanczos steps. Then, we obtain a tridiagonal matrix with diagonal elements $a_{l}^{i}$ and off-diagonal ones $b_{l}^{i}$ with $i = 1, \ldots, M$ and off-diagonal ones $b_{l}^{i}$ with $i = 2, \ldots, M$, $a_{l}^{1}$ and $b_{l}^{1}$ being real. After diagonalizing the matrix, we obtain “eigenvalues” $\epsilon_{l}^{m}$ and “eigenvectors” $| \phi_{l}^{m} \rangle$ ($m = 1, \ldots, M$). Since we stop the Lanczos steps at $M$ steps, these “eigenvalues” and “eigenvectors” are approximate ones. After sampling some arbitrary vectors $| \Psi_l \rangle$ ($l = 1, \ldots, L_0$, $L_0 < L$) and repeating the above process Eq. (3), the expectation value is given by

$$ \langle \hat{A} \rangle = \frac{1}{Z} \sum_{l=1}^{L_0} \sum_{m=1}^{M} e^{-\beta \epsilon_{l}^{m}} \langle \Psi_l | \phi_{l}^{m} \rangle \langle \phi_{l}^{m} | \hat{A} | \Psi_l \rangle , $$

where $Z = \sum_{l=1}^{L_0} \sum_{m=1}^{M} e^{-\beta \epsilon_{l}^{m}} | \langle \Psi_l | \phi_{l}^{m} \rangle |^2$. If we chose all vectors of the basis set, i.e., $L_0 = L$ and obtained eigenvectors and eigenvalues exactly, Eq. (4) should be equivalent to Eq. (2). Jaklič and Prelovšek have shown that, by using a random state $| r \rangle = \sum_{l=1}^{L} \beta_{rl} | \Psi_l \rangle$ ($\beta_{rl}$ is randomly distributed) instead of $| \Psi_l \rangle$, the expectation value obtained using Eq. (4) agrees well with the exact one even if $L_0$ and $M$ are much smaller than $L$.

Similar to static quantities Eq. (4), dynamical quantities of the operator $\hat{A}$ are given by

$$ A(\omega) \sim \frac{1}{Z} \sum_{l=1}^{L_0} \sum_{m=1}^{M} e^{-\beta \epsilon_{l}^{m}} \langle \Psi_l | \phi_{l}^{m} \rangle \langle \phi_{l}^{m} | \hat{A} | \phi_{l}^{n} \rangle \delta(\omega - \epsilon_{l}^{m} + \epsilon_{l}^{n}) , $$

where $| \phi_{l}^{n} \rangle$ are approximate eigenstates with approximate eigenvalues $\epsilon_{l}^{n}$ obtained by the Lanczos procedure Eq. (4) starting from the initial vector $| \phi_{l}^{1} \rangle = \hat{A} | \Psi_l \rangle$.

In small clusters, there is a characteristic temperature $T^*$ below which finite-size effects due to the smallness...
of the systems are appreciable. This temperature is approximately proportional to an average level spacing in the low-energy sector. In our 4×4 t-J cluster without the stripe potential, the finite-size effects should appear as anisotropic behaviors of physical quantities along the horizontal and vertical directions reflecting different boundary conditions along the two directions. In order to estimate \( T^* \), we calculate the hole correlation function. Shown in Fig. 1 is the correlation function defined as

\[
C_h(\vec{r}) = \langle n_i^h n_j^h \rangle ,
\]

where \( \vec{r} = \vec{R}_i - \vec{R}_j \), \( \vec{R}_i \) is the position vector for the site \( i \), \( n_i^h = 1 - n_i \) is the hole-number operator at site \( i \). In the figure, the hole correlations with two-lattice spacing labeled \( b \) and \( d \) are plotted, because the two correlations should be equivalent in magnitude in two-dimensional systems. The magnitude of the correlation in the \( b \) configuration is almost identical to that of \( d \) above \( T \geq 0.3t \). However, below \( T \geq 0.3t \), the two correlations show different temperature dependence. This is due to the difference of the boundary conditions imposed on the cluster. Therefore the characteristic temperature \( T^* \) in the t-J cluster without the stripe potential is about 0.3t. Even in the presence of the stripe potential, we assume that \( T^* \) does not change so much from 0.3t. In fact, the average level spacing in the low-energy sector is similar. In the following, we will show the results for \( T/t \geq 0.3 \).

At \( T = 0 \), we employ the standard Lanczos technique to calculate various quantities. In this case, it is necessary to examine the dependence of given quantities on size and boundary condition in order to estimate finite-size effects. We have examined the dependence by using other clusters with two holes (5×4 with the same boundary condition and 4×4 with periodic boundary condition along both the directions) and have found that the hole and spin-correlation functions discussed in the next section depend only weakly on cluster size and boundary condition.

### III. RESULTS AND DISCUSSION

Figure 2 shows the temperature dependence of the hole correlations given by Eq. (6) in the t-J model with the stripe potential of \( V/t = 1 \). At \( T/t = 0 \), the correlations of holes in the vertical directions labeled \( c \) and \( d \) are larger than those in the horizontal direction labeled \( a \) and \( b \). This anisotropic behavior is due to inhomogeneous distribution of holes induced by the stripe potential. With increasing temperature, the correlations in the \( c \) and \( d \) configurations decrease, while those in \( a \) and \( b \) increase. Therefore the anisotropic behavior of the correlations indicating the confinement of holes in the stripe becomes less pronounced at finite temperatures. The evolution of the hole confinement in the stripe is also observed by examining the hole occupation number as a function of temperature (not shown here); the hole number inside (outside) the stripe decreases (increases) monotonically with increasing temperature.

Figure 3 shows the results of the spin-correlation function around a hole, which is defined as

\[
C(\alpha, \alpha') = \frac{1}{N_h} \sum_i \langle n_i^h S_{i+\alpha}^z S_{i+\alpha'}^z \rangle ,
\]

where \( \alpha \) and \( \alpha' \) denote two sites around a hole at site \( i \) in the stripe following the labeling convention of configurations shown in the inset of Fig. 3 and \( N_h \) is the number of holes \( (N_h=2) \). At high temperatures, all of the nearest-neighbor spin correlations in the \( a, c, e, \) and \( f \) configurations are antiferromagnetic and have the same magnitude, because all spins are disordered by the thermal
fluctuation. The spin correlations between third nearest neighbors (the b and d configurations) are the same. With decreasing temperature, holes begin to be confined into the charge stripes and the magnitude of the spin correlations increase. The charge stripes make spin correlation anisotropic below $T/t \simeq 0.8$. The temperature dependence of the spin correlations in the spin domain (the e and f configurations) differs from that of the spin correlations related with a spin in the charge stripe (a and c). At the same time, the spin correlations in the b and d configurations become different. In the low-temperature region ($T/t \leq 0.4$), the nearest-neighbor spin correlations inside the spin domains (e and f) show temperature dependence similar to that in the Heisenberg spin system (not shown here) because of the confinement of holes in the charge stripe.

The correlation between a spin in the charge stripe and that in the spin domain (the a configuration) in Fig. 3 shows interesting temperature dependence below $T/t \leq 0.4$, when the lowest-temperature data are connected to the zero-temperature ones: The correlation is much suppressed at $T/t=0$, while other spin correlations (b to f) are enhanced. This anomalous temperature dependence of the a configuration can be understood in the following way: At $T/t=0$, holes move along the charge stripe coherently and large Drude weight is obtained parallel to the stripe. Spin correlation in the configuration a is thus suppressed in order to stabilize the coherent motion of holes. With increasing temperature, hole carriers enter into the spin domain, and thermally fluctuating spins in the stripe disturb the motion of holes along the stripe. These effects recover interaction between the stripe and the spin domain, and thus the spin correlation in the a configuration is enhanced with the gain of the exchange energy. In other words, although the charge stripe and the spin domain are almost separated at $T/t=0$, the interaction between the charge stripe and the spin domain is recovered and two-dimensional electronic system is restored with increasing temperature. This picture will be confirmed by examining the optical conductivity as shown below.

In Fig. 4, we show the result of the spin-correlation function given by

$$C_S(\vec{r}) = \frac{1}{N_p} \sum_{\langle i,j \rangle} \langle P(i,j) S_i^z S_j^z \rangle,$$

where $\vec{r} = \vec{R}_i - \vec{R}_j$, and $P(i,j)$ is 1 when $i$ and $j$ are in the same sublattice and it is $-1$ otherwise. The summation $\langle i,j \rangle$ runs over all the pairs satisfying a given $\vec{r}$, and $N_p$ is the number of the pairs. As seen in the figure, the stripe potential, which confines holes in the charge stripe, makes the spin correlation anisotropic. For example, the spin correlation parallel to the stripe labeled c (d) is larger than that perpendicular to the stripe labeled a (b) below $T/t \simeq 0.8$. With increasing temperature, the anisotropy decreases.

Next, we examine the temperature dependence of the optical conductivity. The regular part of the optical conductivity $\sigma_{\mu \nu}^{\text{reg}}(\omega > 0)$ ($\mu=x$ or $y$) is given by

$$\sigma_{\mu \nu}^{\text{reg}}(\omega) = \frac{i e}{\hbar} \int_0^\infty \langle \Phi_n | j_\mu | \Phi_m \rangle \langle \Phi_m | j_\nu | \Phi_n \rangle \delta(\omega + \varepsilon_m - \varepsilon_n),$$

with

$$\Omega_{\mu \nu}(\omega) = \frac{\pi}{N^2} \sum_{n \neq m} e^{-\beta \varepsilon_n} |\langle \Phi_n | j_\mu | \Phi_m \rangle|^2 \delta(\omega + \varepsilon_m - \varepsilon_n)$$

and

$$j_\mu = \frac{i e a_0}{\hbar} \sum_i \left( \hat{c}_{i+\mu} - \hat{c}_i \right),$$

where $|\Phi_n\rangle$ is the eigenstate with the eigenvalue $\varepsilon_n$. $e$ is the unit of the electric charge and $a_0$ is the lattice constant.
FIG. 5. Temperature dependence of the regular part of the optical conductivity in the $t$-$J$ model with the stripe potential on a $4\times4$ cluster with two holes. $J/t=0.4$ and $V/t=1.0$. (a) Perpendicular and (b) parallel to the charge stripe. Small broadening of $0.2t$ is used for delta functions.

spacings. Other definitions are standard. Adding singular contribution to $\sigma_{\mu\mu}^{\text{reg}}(\omega)$, the real part of the optical conductivity reads

$$\sigma_{\mu\mu}(\omega) = 2\pi e^2 D_\mu \delta(\omega) + \sigma_{\mu\mu}^{\text{reg}}(\omega), \quad (12)$$

where the so-called Drude weight $D_\mu$ is given by

$$D_\mu = \frac{\langle K_\mu \rangle}{2N} - \frac{1}{\pi e^2} I_{\mu\mu} \quad (13)$$

with

$$I_{\mu\mu} = \int_0^{\infty} \sigma_{\mu\mu}^{\text{reg}}(\omega) d\omega. \quad (14)$$

Here, $\langle K_\mu \rangle$ is the kinetic energy in the $\mu$ direction. We set $\hbar=e=a_0=1$ hereafter. $\Omega_{\mu\mu}(\omega)$ is calculated using Eq. (3).

Figures 5(a) and (b) show the regular part of the optical conductivity perpendicular, $\sigma_{xx}^{\text{reg}}(\omega) = \sigma_{yy}^{\text{reg}}(\omega)$, and parallel, $\sigma_{xx}^{\text{reg}}(\omega) = \sigma_{yy}^{\text{reg}}(\omega)$, to the stripe, respectively. At $T/t=0$, the intensity of $\sigma_{xx}^{\text{reg}}(\omega)$ is larger than that of $\sigma_{xx}^{\text{reg}}(\omega)$ for $\omega/t \leq 1.0$. In contrast, in the temperature region of $T/t \geq 0.3$, the intensity of $\sigma_{xx}^{\text{reg}}(\omega)$ for $\omega/t \leq 1.0$ is much larger than that of $\sigma_{xx}^{\text{reg}}(\omega)$. The dramatic increase of $\sigma_{xx}^{\text{reg}}(\omega)$ from $T/t=0$ to $T/t=0.3$ is caused by the spectral weight transfer from the Drude weight parallel to the stripe $D_{//}$ as discussed below. Another interesting point in Fig. 5(b) is that the intensity of $\sigma_{xx}^{\text{reg}}(\omega)$ increases with increasing temperature from $T/t=0.3$ to $T/t=0.4$, and then decreases with further increase of $T$. The intensity maximum at $T/t=0.4$ is related to the maximum of $I_{//}$ shown in Fig. 6(b). Note that $I_{//}$ is the integrated spectral weight of the regular part of the optical conductivity $\sigma_{xx}^{\text{reg}}(\omega)$ [see Eq. (14)]. Here, let us consider the origin of this maximum. At $T/t=0$, since the motion of holes in the charge stripe is coherent, $D_{//}$ is very large while $D_{\perp}$ is almost zero as shown in Fig. 6(a). On the contrary, $I_{//}$ is suppressed compared with $I_{\perp}$. With increasing temperature, the motion of holes in the stripe becomes incoherent, because spins in the charge stripe fluctuate thermally as is evidenced by the spin correlations in the $c$ and $d$ configurations in Fig. 3. As a result, $D_{//}$ is strongly suppressed, while $I_{//}$ is enhanced with the increase of temperature. The kinetic energy along the charge stripe $\langle K_{//} \rangle = \langle K_\mu \rangle$ exhibits small temperature dependence below $T/t \approx 0.5$. From the sum rule Eq. (3), this means that almost all of the intensity of
$D_{1//}$ transfers to the regular part $\sigma^{\text{reg}}$. With further increase of temperature above $T/t \approx 0.5$, $\langle K_{1//}\rangle$ gradually decreases. Since $D_{1//}$ is almost zero in such high temperature, the decrease of $\langle K_{1//}\rangle$ leads to the decrease of $I_{1//}$. Therefore $I_{1//}$ shows a maximum at the temperature $T_m \approx 0.45t$. We have examined the dependence of $T_m$ on the parameters $V$ and $J$; $T_m/t=0.4, 0.45$, and $0.6$ for $J/t=0.2, 0.4$, and $0.8$, respectively, keeping $V/t=1.0$, and $T_m/t=0.4, 0.45$, and $0.5$ for $V/t=0.5, 1.0$, and $2.0$, respectively, keeping $J/t=0.4$. $T_m$ is found to be dependent not only on $V$ but also strongly on $J$. These data are consistent with a picture that the spin degree of freedom plays an important role in the charge dynamics in the stripe phase.

Moreover, we find an interesting temperature dependence of the kinetic energy in Fig. 6(b). At $T/t=0$, $\langle K_{1//}\rangle$ is larger than $\langle K_{\perp}\rangle$ as expected. With increasing temperature, $\langle K_{1//}\rangle$ gradually decreases. Such a decrease is also seen in the $t$-$J$ model without the stripe potential (not shown here). In contrast, $\langle K_{\perp}\rangle$ at $T/t=0.3$ is larger than that at $T/t=0$. This anomalous increase of $\langle K_{\perp}\rangle$ is consistent with the picture proposed above, i.e., one-dimensional nature of the electronic states due to the charge stripe is destroyed by the thermal fluctuation of spins and thus two-dimensional electronic states are restored.

Finally, we propose a possible method to compare the present results of the temperature dependence of $\sigma(\omega)$ with experimental ones. The frequency dependence of effective carrier number $N_{e\text{ff}}(\omega)$ is experimentally evaluated from the optical conductivity by using a relation that $N_{e\text{ff}}(\omega) = 2mV/(\pi e^2) \int_{0}^{\infty} \sigma(\omega')d\omega'$, where $m$ is the mass of a free electron and $V$ is the volume of the unit cell. From the theoretical side, the effective carrier number in high-frequency limit $\omega \rightarrow \infty$ is proportional to the average of the kinetic energies $\langle K_{\perp}\rangle$ and $\langle K_{1//}\rangle$ because of a relation that $\int_{0}^{\infty} \sigma(\omega)d\omega = -\pi e^2 \langle (K_{\perp})^2 + (K_{1//})^2 \rangle/(4N)$. Since the dominant part of the temperature-induced change of the calculated $\sigma(\omega)$ and $\sigma_{\perp}(\omega)$ is concentrated in the region of $\omega \lesssim 3t$ ($\approx 1$ eV), the change of the averaged kinetic energies can be a measure of the change of the effective carrier number up to around 1 eV. In a realistic temperature region ($T < J=0.4t$), $\langle K_{1//}\rangle$ is almost temperature independent, while $\langle K_{\perp}\rangle$ increases with increasing $T$ because of the destruction of the stripe as mentioned above. Thus the averaged kinetic energy increases. We propose that $N_{e\text{ff}}(\omega)$ at around $\omega \approx 1$ eV in the stripe phase of the high-$T_c$ cuprates increases with increasing temperature. This may be detectable from the detailed analysis of the temperature dependence of experimental data.

**IV. SUMMARY**

In summary, we have examined the temperature dependence of the electronic states with vertical charge stripes. The spin-correlation function, the optical conductivity, and the kinetic energy have been calculated by using the finite temperature Lanczos method. We have found that the motion of holes along the charge stripes, which is coherent at $T/t=0$, becomes incoherent with increasing temperature. We have also found that the spin correlation between a spin in the charge stripe and that in the spin domain is smaller than that for other spin pairs at $T/t=0$. However, the correlation increases with increasing temperature and then decreases toward high temperature. This anomalous increase of the correlation is a manifestation of the evolution to the two-dimensional electronic states. Moreover, we have found an anomalous change of the kinetic energy perpendicular to the stripe. All of these characteristic phenomena of the stripe phase appear in a realistic temperature region, i.e., $T < J$. The fact that characteristic temperatures such as $T_m$ defined in Fig. 6(b) depend not only on the stripe potential $V$ but also on $J$ suggests that the ordering of spins plays an important role in the confinement of holes in the charge stripe. The disordering of spins due to the thermal fluctuation destroys the charge stripe and makes the crossover from the one-dimensional electronic states to the two-dimensional ones.

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We also examined a $4 \times 4$ cluster of the $t$-$J$ model with periodic boundary condition both parallel and perpendicular to the charge stripe. Although the periodic boundary condition perpendicular to the charge stripe causes frustration of spins, our conclusions are not affected by the boundary condition.

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