Rigorous analysis of bistable memory in silica toroid microcavity

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We model the nonlinear response of a silica toroid microcavity using coupled mode theory and a finite element method, and successfully obtain Kerr bistable operation that does not suffer from thermo-optic effect by optimizing the fiber-cavity coupling. Our analysis shows it is possible to demonstrate a Kerr bistable memory with a memory holding time of 500 ns at an extremely low energy consumption.

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1. Introduction

Optical bistability is a fundamental physical phenomenon where it is possible for certain devices to have two stable transmission states. It occurs when the refractive index or the absorption of the nonlinear medium in an optical cavity is dependent on the light intensity. Optical bistable devices are considered to be an important building blocks in all-optical signal processing for such components as optical memories and optical flip-flops [1], and this phenomenon was extensively studied in 1970s to 80s [2]. However, at that time, the size of the cavity and the operating energy were too large to be considered for practical applications.

Recent progresses in achieving higher quality factors (Q) in ultrasmall microcavities on-chip [3–5] has refocused attention on optical bistability [6], due to the possibility of achieving denser integration and lower energy consumption. Since the photon density in a cavity scales with Q/V, where V is the mode volume, a high Q cavity with a small V enables us to use various nonlinearities at extremely low input powers; hence it allows optical bistability at an ultralow power.

Soljačić et al. demonstrated numerically that optical bistability based on Kerr nonlinearity is possible by using a two-dimensional photonic crystal nanocavity at an ultralow driving power of 133 mW [6]. Various experiments have already been reported in silica microspheres [3], silicon photonic crystals [4, 10], and silicon microring resonators [11], but by using the thermo-optic (TO) effect. Since the TO effect is accompanied by thermal accumulation, the response is relatively slow. To achieve faster speed, the carrier-plasma effect has been utilized. This effect is the result of carrier generation [12–13]. The latest research has reported on 4-bit optical random-access memory operation based on carrier nonlinearity, at a power consumption of only 30 nW, by using InGaAsP photonic crystal nanocavities [14]. The key to achieving the low power consumption is the smallness of the cavity V.

As introduced above, the required power for the operation of optical bistability has been significantly reduced in recent years due to the high Q/V. However, all of the demonstrations use either TO or carrier-plasma effects to drive the bistability whose nonlinearities are accompanied by photon absorption. Recent advances on linear and nonlinear studies of microcavities and chip-based waveguide devices have opened possibility of their use not only for classical all-optical processing but also for loss sensitive applications such as quantum information processing [14, 15, 16]. With those applications in mind, we need to reduce significantly the power loss (consumption) of the bistable system. The use of the optical Kerr effect is the ultimate goal because it does not absorb photons. In addition, the use of an ultra-high Q allows us to reduce the power scattering loss of the signal light from the system, and this will support all-optical information processing for loss-sensitive applications.

Although there have been several attempts to employ the optical Kerr effect by using large-bandgap materials such as AlGaAs [19] and chalcogenide glasses [20], it still seems to be difficult to obtain Kerr bistability without suffering from the carrier effect or TO effect [3, 21, 22]. On the other hand, silica has been an excellent material with which to study of various aspects of χ(3) based physics [23], because of the large bandgap that can suppress carrier generation. Therefore silica microcavities have the potential to achieve a Kerr bistable memory that consumes very little energy.

Among various silica microcavities, the silica toroid microcavity [3] has an ultra-high Q and is capable of integration on a chip. As discussed above, a high Q cavity is attractive for both achieving optical Kerr bistability and for low-loss applications.

In this paper, we demonstrate numerically that an optical bistable memory based on the optical Kerr effect is possible by controlling the coupling between the cavity and the tapered fiber. Our model consists of coupled mode theory (CMT) and the finite element method (FEM).
The paper is organized as follows. First in §2 we provide a simple picture of the physics that we are going to employ. In §3 we describe a numerical simulation model that combines CMT and FEM. In §4 we show the calculation result and in §5 we discuss the energy consumption. Finally, we finish with a conclusion.

2. Simple model

As described in the previous section, it is still difficult to use the optical Kerr effect in ultrahigh- Q silica microcavities, even though the material has a large bandgap. This is because of the small light absorption at the surface, which is caused by a thin water layer [8, 32]. Therefore, we need to analyze this carefully by using rigorous modeling and applying realistic physical parameters to reveal the conditions required for obtaining Kerr bistability. However, before undertaking a rigorous analysis we start with a simple model to gain an intuitive understanding of the strategy for obtaining Kerr bistability. In this section we pursue an analytical discussion about how we can obtain the Kerr effect without exhibiting considerable TO effect.

First we derive the relationship between the wavelength shift and the input energy required for Kerr nonlinearity. The energy $U$ of an electromagnetic wave in a dielectric medium is given by [24],

$$U = \frac{1}{2} \varepsilon E_0^2 V,$$  

where $\varepsilon$, $E_0$ and $V$ are the dielectric constant, the electric field constant and the mode volume, respectively. Using Eq. (1) and $I = \varepsilon E_0^2 / 2$, where $v$ is the light speed of the medium, we obtain the relation between the power density $I$ and the optical energy $U$ as,

$$I = \frac{2 \varepsilon U}{n_0 V},$$  

where $c$ is the velocity of light and $n_0$ is the refractive index of the cavity medium. Then, the refractive index change caused by the Kerr effect is given by,

$$\Delta n_{\text{Kerr}} = n_2 I = \frac{2 n_2 c U}{n_0 V},$$  

where $n_2$ is the nonlinear refractive index. Equation (3) allows us to calculate the refractive index change as a function of the energy in the cavity.

Next we discuss the TO nonlinearity. The refractive index change caused by the TO effect $\Delta n_{\text{TO}}$ is given by,

$$\Delta n_{\text{TO}} = n_0 \xi T,$$  

where $\xi = (1/n)(\partial n / \partial T)$ is the TO coefficient. The energy required to increase the temperature of 1 K for volume $V$ is given by $C \rho V$, where $C$ is the heat capacity and $\rho$ is the density of the material. Then, using Eq. (4), we obtain

$$\Delta n_{\text{TO}} = n_0 \xi V U_{\text{abs}},$$  

where $U_{\text{abs}}$ is the energy absorbed by the material. Equations (3) and (5) describe the refractive index change at a given energy for the Kerr and TO effects respectively. For the Kerr effect to be larger than the TO effect the following condition is required,

$$\frac{\Delta n_{\text{Kerr}}}{\Delta n_{\text{TO}}} = \frac{2 n_2 c \rho C}{n_0^2 \xi U_{\text{abs}}} > 1$$  

(6)

To gain a simple understanding, we assume a steady-state model. The light energy in the cavity is constant and generates heat at a constant rate. Thus, $U(t)$ and $U_{\text{abs}}(t)$ are expressed as,

$$\begin{align*}
\frac{dU_{\text{abs}}(t)}{dt} &= -\frac{1}{\tau_{\text{diff}}} U_{\text{abs}} + \frac{1}{\tau_{\text{abs}}} U(t),
\end{align*}$$  

(7)

where $U_0$, $\tau_{\text{diff}}$ and $\tau_{\text{abs}}$ are the steady energy in the cavity, the thermal relaxation time and the thermal generation rate caused by photon absorption, respectively. When we neglect the thermal diffusion ($\tau_{\text{diff}} = 0$), Eq. (8) is simply expressed as,

$$t < 2 n_2 c \rho C n_0^2 \xi \tau_{\text{abs}},$$  

(8)

This provides direct view as to how we should design the cavity system in order to obtain the Kerr effect without the TO effect being too great. When we use the following parameters for SiO$_2$, $\xi = 5.2 \times 10^{-6}$ K$^{-1}$, $n_2 = 3.67 \times 10^{-20}$ m$^2$/W $n = 1.47$, $C = 7.41 \times 10^{-3}$ J/(g·K), and $\rho = 2.65$ g/cm$^3$, Eq. (8) gives the following condition,

$$t < 3.84 \tau_{\text{abs}}.$$  

(9)

This equation provides a simple understanding of how we can achieve Kerr nonlinearity in SiO$_2$ microcavities. Although the Kerr effect governs the refractive index change ($\Delta n_{\text{Kerr}} > \Delta n_{\text{TO}}$) at an early stage of the operation, the TO effect becomes dominant after a period of time given by Eq. (9). When we employ a $\tau_{\text{abs}}$ of 329 ns (the selection of this value is discussed in detail in §5), Eq. (9) gives $t < 1.26 \mu$s, which is the duration time, for which $\Delta n_{\text{Kerr}}$ is larger than $\Delta n_{\text{TO}}$. Note that even if we consider the thermal relaxation time in our model as, $\tau_{\text{diff}} = 8 \mu$s (obtained from Fig. 3), this duration time does not change greatly and is about 1.35 $\mu$s.

To achieve Kerr nonlinearity we must complete our operation before this time, namely the time that the light is absorbed by the material. This simple picture is straightforward to understand. If we are to obtain a faster operation speed we require a smaller $\tau_{\text{tot}}$ because it determines the rise and fall time of the light energy in the cavity. $\tau_{\text{tot}}$ is given by $\tau_{\text{tot}} = \tau_{\text{loss}} + \tau_{\text{coup}} + \tau_{\text{abs}}$, where $\tau_{\text{loss}}$ and $\tau_{\text{coup}}$ are the photon lifetimes defined by the loss rate that do not contribute to the generation of heat and coupling to the waveguides. Since $\tau_{\text{abs}}$ and $\tau_{\text{loss}}$ are mainly determined by the material and the structure that we use, the only parameter that we can control is $\tau_{\text{coup}}$. This discussion suggests that optical Kerr operation is possible.
by controlling the coupling between the cavity and the waveguides, because it enables us to release light into the waveguides before it is absorbed by the material.

In the following section, we perform a rigorous analysis showing that Kerr operation is indeed possible by changing \( \tau_{\text{coup}} \).

3. Rigorous modeling of the optical Kerr effect and thermo-optic effect in a toroid microcavity

A. CMT in whispering gallery mode resonator

First we describe our master equation based on the coupled mode theory in a whispering gallery mode (WGM) resonator to obtain the linear and nonlinear transmittance. The structure is shown in Fig. 1(a). Because a two-port system (a side coupled cavity with one waveguide) makes the bistable operation difficult to observe, we focus on a side-coupled four-port system throughout this paper. It consists of a toroid (ring) cavity and two waveguides for input and output light. A detailed discussion on the comparison between side-coupled two- and four-port systems will be provided elsewhere. Briefly, as discussed in [26] we need to make the coupling large in order to prevent heat accumulating in the system. However, the transmittance spectrum of a two-port system is shallower in an over-coupled configuration, which makes the switching contrast of the output light very low and difficult to distinguish. Hence, optical-bistable switching with high contrast is difficult to achieve in a two-port system in the presence of the TO effect.

The mode amplitude \( a \) in the cavity is given as [28],

\[
\frac{da}{dt} = \left[ j\omega_0 - \frac{1}{2} \left( \frac{1}{\tau_{\text{abs}}} + \frac{1}{\tau_{\text{loss}}} + \frac{1}{\tau_{\text{coup1}}} + \frac{1}{\tau_{\text{coup2}}} \right) \right] a \\
+ \sqrt{\frac{1}{\tau_{\text{coup1}}} \exp(j\theta)s_{\text{in}}}
\]

where \( \omega_0 \) is the resonant frequency of the cavity. The input wave \( s_{\text{in}} \) excites the counter clock-wise (CCW) mode in the cavity. We assume an ideal cavity where there is no coupling between the CW and CCW modes. \( \tau_{\text{coup1}} \) and \( \tau_{\text{coup2}} \) are photon lifetimes determined by the coupling with the lower and upper waveguides, respectively. \( \theta \) is the relative phase between the mode amplitude in the cavity and the optical wave in the lower waveguide and is given as,

\[
\theta = 4\pi^2 n_0 (R + r) \left( \frac{1}{\lambda_0} - \frac{1}{\lambda} \right).
\]

Here \( R \), \( r \), \( \lambda \) and \( \lambda_0 \) are the major and minor radiiuses of the cavity (shown in Fig. 1(b)), the input wavelength and the resonant wavelength of the cavity, respectively. Equation (11) shows that the phase between \( a \) and \( s_{\text{in}} \) becomes unmatched on off-resonance. Output waves \( s_{\text{out1}} \) and \( s_{\text{out2}} \) are given as,

\[
s_{\text{out1}} = \exp(-j\beta_1 d) \times \left[ s_{\text{in}} - \sqrt{\frac{1}{\tau_{\text{coup1}}} \exp(-j\beta_1 a)} \right] \\
s_{\text{out2}} = \exp(-j\beta_2 d) \sqrt{\frac{1}{\tau_{\text{coup2}}} a},
\]

where \( \beta_1 \) and \( \beta_2 \) are the propagation constants of the lower and upper waveguides and \( d \) is the waveguide length. Note that the relative phase between the mode amplitude in the cavity \( a \) and the upper waveguide \( s_{\text{in}} \) is always zero because there is no incident wave in the upper waveguide.

When we use a slowly varying envelope approximation, i.e.,

\[
a(t) = A(t) \exp(j\omega t) \\
s_{\text{in}}(t) = S_{\text{in}}(t) \exp(j\omega t),
\]

we can rewrite Eq. (10) as,

\[
\frac{dA(t)}{dt} = \left[ j\frac{2\pi e}{n_0} \left( \frac{1}{\lambda_0} - \frac{1}{\lambda} \right) \\
- \frac{1}{2} \left( \frac{1}{\tau_{\text{abs}}} + \frac{1}{\tau_{\text{loss}}} + \frac{1}{\tau_{\text{coup1}}} + \frac{1}{\tau_{\text{coup2}}} \right) \right] A(t) \\
+ \sqrt{\frac{1}{\tau_{\text{coup1}}} \exp(j\theta)S_{\text{in}}(t)}.
\]

where \( A(t) \), \( S_{\text{in}}(t) \) and \( \omega = 2\pi c/(n_0\lambda) \) are the envelopes of the cavity mode and the waveguide mode and the frequency of the input wave, respectively. Equation (10) is
the master equation of the linear system. By using this equation, we now can calculate the energy in the cavity and the output power at an arbitrary time.

**B. Modeling the nonlinearities**

To describe the nonlinear effects in our model, we take account of the Kerr effect \( \Delta n_{\text{Kerr}} \) and TO effect \( \Delta n_{\text{TO}} \) in the master equation (Note that the carrier-plasma effect is negligible in silica due to its large bandgap). The nonlinearities in an optical cavity result in a shift in the resonant wavelength because the optical path length changes. Thus, the shift of the resonant wavelength of a cavity is given as,

\[
\delta \lambda(t) = \frac{\Delta n(t)}{n_0} \lambda_0,
\]

where \( \Delta n(t) \) is the effective nonlinear refractive index change of the cavity. By substituting Eq. (17) into Eqs. (16) and (11), we obtain

\[
\frac{dA(t)}{dt} = \left[ j \frac{2\pi c}{n_0 + \Delta n(t)} \left( \frac{1}{\lambda_0 + \delta \lambda(t)} - \frac{1}{\lambda} \right) - \frac{1}{2} \left( \frac{1}{\tau_{\text{abs}}} + \frac{1}{\tau_{\text{loss}}} + \frac{1}{\tau_{\text{coup1}}} + \frac{1}{\tau_{\text{coup2}}} \right) \right] A(t) + \sqrt{\frac{1}{\tau_{\text{coup1}}} \exp (j\theta) S_{\text{in}}(t)}.
\]

These are the master equations that we used in our model, which take the nonlinearities into account.

Next, we describe how we calculated the nonlinear refractive index change \( \Delta n_{\text{Kerr}} \) and \( \Delta n_{\text{TO}} \). We can directly calculate \( \Delta n_{\text{Kerr}} \) from Eq. (23). Taking the spatial dependency into account, we obtain,

\[
\Delta n_{\text{Kerr}}(x, y, t) = n_2 I(x, y, t) = \frac{2n_p c}{n_0} \tilde{U}_p(x, y, t),
\]

where \( \tilde{U}_p(x, y, t) \) is the energy density distribution of the cavity mode in \( x, y \) cross-sectional coordinates. It is given as,

\[
\tilde{U}_p(x, y, t) = \frac{U_p(t)}{2\pi R} I(x, y),
\]

where \( U_p = |A(t)|^2 \) is the energy of the light stored in the cavity and \( I(x, y) \) is the normalized cross-sectional power density distribution of the whispering gallery mode obtained by FEM (see Ref. [27]). It is normalized as \( \iint I(x, y)dx dy = 1 \), and the profile is shown in Fig. (1b).

The refractive index change \( \Delta n_{\text{TO}} \), which is induced by the TO effect, is described as,

\[
\Delta n_{\text{TO}}(x, y, t) = n_0 \xi [T(x, y, t) - 300 \text{ [K]}].
\]

The cross-sectional temperature distribution is calculated by using 2D-FEM (COMSOL Multiphysics). By setting the heat source in the dielectric cavity as,

\[
Q'(x, y, t) = \begin{cases} \tau_{\text{abs}}^{-1} \tilde{U}_p(x, y, t), & \text{in cavity}, \\ 0, & \text{in air}, \end{cases}
\]

where \( \tau_{\text{abs}}^{-1} \) is the thermal generation rate caused by the material absorption, we can obtain the temperature \( T(x, y, t) \) at any time and at any position by performing an FEM calculation.

Finally, we obtain the effective nonlinear refractive index change \( \Delta n(t) \) as,

\[
\Delta n(t) = \frac{\iint [\Delta n_{\text{TO}}(x, y, t) + \Delta n_{\text{Kerr}}(x, y, t)] I(x, y)dx dy}{\iint I(x, y)dx dy}.
\]

Now by solving Eq. (18) sequentially using Eqs. (17)–(24), we can obtain the light energy in the cavity \( U_p(t) \) and the output powers \( P_{\text{out1}} = |S_{\text{out1}}|^2 \) and \( P_{\text{out2}} = |S_{\text{out2}}|^2 \).

**C. Determining the absorption and the photon lifetimes**

The total photon lifetime \( \tau_{\text{tot}} \) is defined as,

\[
\tau_{\text{tot}} = \left( \tau_{\text{mat}}^{-1} + \tau_{\text{water}}^{-1} + \tau_{\text{cont}}^{-1} + \tau_{\text{rad}}^{-1} + \tau_{\text{surf}}^{-1} + \tau_{\text{coup1}}^{-1} + \tau_{\text{coup2}}^{-1} \right)^{-1},
\]

where \( \tau_{\text{mat}}, \tau_{\text{water}}, \tau_{\text{cont}}, \tau_{\text{rad}}, \tau_{\text{surf}}, \tau_{\text{coup1}} \) and \( \tau_{\text{coup2}} \) are photon lifetimes determined by the absorption of the material, water absorption that is usually present on the surface of the cavity, absorption caused by surface contamination, radiation loss, scattering loss, coupling to the lower waveguide and coupling to the upper waveguide, respectively. Since this expression is very complicated, we define the photon lifetime that is related to the absorption as,

\[
\tau_{\text{abs}} = \tau_{\text{mat}}^{-1} + \tau_{\text{water}}^{-1} + \tau_{\text{cont}}^{-1} + \tau_{\text{rad}}^{-1} + \tau_{\text{surf}}^{-1} + \tau_{\text{coup1}}^{-1} + \tau_{\text{coup2}}^{-1},
\]

where \( \tau_{\text{mat}}, \tau_{\text{water}}, \tau_{\text{cont}}, \tau_{\text{rad}}, \tau_{\text{surf}}, \tau_{\text{coup1}} \) and \( \tau_{\text{coup2}} \) are photon lifetimes determined by the absorption of the material, water absorption that is usually present on the surface of the cavity, absorption caused by surface contamination, radiation loss, scattering loss, coupling to the lower waveguide and coupling to the upper waveguide, respectively. Since this expression is very complicated, we define the photon lifetime that is related to the absorption as,

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\]

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\]

where \( \tau_{\text{mat}}, \tau_{\text{water}}, \tau_{\text{cont}}, \tau_{\text{rad}}, \tau_{\text{surf}}, \tau_{\text{coup1}} \) and \( \tau_{\text{coup2}} \) are photon lifetimes determined by the absorption of the material, water absorption that is usually present on the surface of the cavity, absorption caused by surface contamination, radiation loss, scattering loss, coupling to the lower waveguide and coupling to the upper waveguide, respectively. Since this expression is very complicated, we define the photon lifetime that is related to the absorption as,

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\]

where \( \tau_{\text{mat}}, \tau_{\text{water}}, \tau_{\text{cont}}, \tau_{\text{rad}}, \tau_{\text{surf}}, \tau_{\text{coup1}} \) and \( \tau_{\text{coup2}} \) are photon lifetimes determined by the absorption of the material, water absorption that is usually present on the surface of the cavity, absorption caused by surface contamination, radiation loss, scattering loss, coupling to the lower waveguide and coupling to the upper waveguide, respectively. Since this expression is very complicated, we define the photon lifetime that is related to the absorption as,

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\]

where \( \tau_{\text{mat}}, \tau_{\text{water}}, \tau_{\text{cont}}, \tau_{\text{rad}}, \tau_{\text{surf}}, \tau_{\text{coup1}} \) and \( \tau_{\text{coup2}} \) are photon lifetimes determined by the absorption of the material, water absorption that is usually present on the surface of the cavity, absorption caused by surface contamination, radiation loss, scattering loss, coupling to the lower waveguide and coupling to the upper waveguide, respectively. Since this expression is very complicated, we define the photon lifetime that is related to the absorption as,

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\tau_{\text{abs}} = \tau_{\text{mat}}^{-1} + \tau_{\text{water}}^{-1} + \tau_{\text{cont}}^{-1} + \tau_{\text{rad}}^{-1} + \tau_{\text{surf}}^{-1} + \tau_{\text{coup1}}^{-1} + \tau_{\text{coup2}}^{-1},
\]
thermal accumulation, because a large part of the incident light eventually turns into heat. Any result should be better than that obtained with this latter condition.

To consider those two cases, first we fix the intrinsic photon lifetime τint = (τ−1 \text{int} + τ−1 \text{loss}) as 329 ns (corresponding to Qint = 4 × 10^8) [31], where Qint is the intrinsic Q. Since this is the record largest experimental τint. In the ideal case we disregard losses than the intrinsic material absorption; so we use τabs = 164 µs (corresponding to Qabs ≃ Qmat = 2 × 10^{11}) [31] and τloss = 330 ns; i.e. τ−1 \text{int} ≃ τ−1 \text{loss} ≫ τ−1 \text{abs}. On the other hand, we use τabs = 329 ns (corresponding to Qabs = 4 × 10^8) [31, 32] and τloss = ∞ for the realistic case. This Qabs value is the same as the highest experimental Qint, for the reason discussed above; i.e. τ−1 \text{int} ≃ τ−1 \text{abs} ≫ τ−1 \text{loss}.

Although τint is determined by the material and structure, we can control τcoup1 and τcoup2 by adjusting the distance between the fiber and the cavity [23]. With this in mind, we conducted numerical simulations for various τcoup2 values. Note that τcoup1 is controlled to satisfy τ−1 \text{coup1} = (τ−1 \text{int} + τ−1 \text{coup2})^{-1} and thus achieve critical coupling [28, 34, 35] between the cavity and the lower waveguide. In this condition, the power transmittance through the lower waveguide T_{\text{out1}} = P_{\text{out1}}/P_{\text{in}} decreases to zero on resonance and high contrast can be obtained between two output states [20].

4. Numerical calculations

A. An ideal case: Small material absorption

In this section, we show that Kerr bistable memory is easily feasible, without careful adjustment of τcoup, if only the inherent material absorption of silica is present.

First, we input a rectangular pulse to investigate the refractive index changes Δn_{\text{Kerr}}, and Δn_{\text{TO}}. The result is shown in Fig. 2(a), where τcoup2 is equal to τint. It shows that Δn_{\text{Kerr}} is always larger than Δn_{\text{TO}}, which tells us that the influence of the absorption induced thermal generation is nearly negligible. Hence the Kerr effect is easily obtained without it suffering from the TO effect.

Next, we input a triangular pulse to investigate the relationship between the input and output of the system. To allow us to charge and discharge the cavity gradually, we set the pulse rising/falling rate of the triangular inputs at dP_{\text{in}}/dt = 62.5 nW/µs. Figure 2(b) and (c) are plotted from the input-output response of a triangular input of different detuning values δ. The lower coupling photon lifetime τcoup1 is set equal to (τ−1 \text{int} + τ−1 \text{coup2})^{-1} to achieve critical coupling. When δ is greater than 20 fm, clear hysteresis is observed, which is direct evidence of optical bistability. Optical bistability is observed in the longer side of the wavelength detuning, because the Kerr effect increases the refractive index (shifts the resonance toward the longer wavelength). Note that we obtained a large contrast between the two bistable states in Fig. 2(b) because the cavity transmittance P_{\text{out1}} falls to zero on resonance in the critical coupling [35].

Finally, we performed optical memory operations as shown in Fig. 2(d). Again, we set τcoup2 equal to τint and the detuning δ equal to 27 fm. The solid line is an input with a drive power P_{\text{drive}} of 3.2 µW. The peak power of the 0.8-µs square set pulse is P_{\text{set}} = 5 µW. To reset the system, we reduce the input power to P_{\text{in}} = 2 µW for a duration of 0.8 µs, which we call a reset pulse. The duration of the negative reset pulse must be longer than the discharging time of the cavity, which is equal to τ_{\text{tot}}; otherwise the cavity does not reset. Set and reset pulses are inputted at t = 8 and 20.8 µs. P_{\text{out2}} (shown as the broken line) rises to high (ON) state when the set pulse is inputted. It keeps the ON state until the reset pulse is injected. After the reset pulse has been entered, P_{\text{out2}} drops to low (OFF) state and holds this state. P_{\text{out1}} (indicated by the dotted line in Fig. 2(d)) shows the inverse behavior of P_{\text{out2}}. Figure 2(d) clearly shows optical memory operation, which is based on Kerr nonlinearity. Although the “memory holding time” demonstrated with this calculation (shown in Fig. 2(d)) is about 30 µs, it can be much larger, since P_{\text{out1}} and P_{\text{out2}} exhibit almost an plateau response due to the small material absorption.

B. A realistic case: Large material absorption

As shown in Fig. 1A, the realization of a Kerr bistable memory is feasible without it suffering from the TO effect, even when the coupling is not large, if only the inherent absorption of silica is present in the cavity. In reality, however, other sources of absorption occur in a microcavity, such as surface absorptions caused by water and contamination. Thus, in this section, we use τabs = 329 ns to simulate a case with faster thermal generation. As discussed above, here we assume that the Qint of the cavity is limited by absorption and not by losses to the outside of the cavity, since it appears to be the worst (but realistic case) for toroid microcavities. We employ τabs, which is derived from the highest experimental Qint (τabs = 329 ns corresponds to Qint = 4 × 10^{8}). If we can clarify the requirements for demonstrating a Kerr bistable memory under this condition, it is a significant step toward the experimental realization of a Kerr bistable memory in silica toroid microcavities.

First, in a similar way to that shown in Fig. 2(a), we employ a rectangular pulse to obtain the refractive index change Δn_{\text{Kerr}} and Δn_{\text{TO}}. The calculation results are shown in Fig. 3 for three different τcoup2 values (τcoup1 is adjusted to satisfy τ−1 \text{coup1} = τ−1 \text{coup2} + τ−1 \text{int}). Figure 3 shows that Δn_{\text{TO}} is larger than Δn_{\text{Kerr}} in all three cases when t is larger than ~ 2.3 µs. This number gives us the upper limit of the Kerr memory holding time without the memory suffering from the TO effect. It also tells us that this number is insensitive to τcoup2. This result is consistent with Eq. 3 obtained from a simple model, where the equation is dependent on τabs but independent of τcoup2. Figure 3 also shows the effect of the different charging speed resulted by different τcoup2 values. The cavity charging time is much faster for τcoup2 = τint/100, which allows the cavity to reach a plateau Δn_{\text{Kerr}}, domain much faster. This enables us to have a longer “Kerr mem-
when we made the coupling strong (i.e. τcoup2 ≥ τint/100), but the loops are deformed when τcoup2 ≥ τint/10. This is because the light cannot charge and discharge the cavity quickly enough before the heat accumulates in the system when τcoup2 is large. Again, we obtained a clear hysteresis loop only when we made the coupling strong (i.e. τcoup2 is small).

Finally, the memory operation for various τcoup2 values is shown in Fig. 5. Since the response speed of Pout1 and Pout2 depend on the total photon lifetime τtot, we normalized the temporal axis t by τtot. Figure 5 shows clearly that the reset pulse does not work when τcoup2 = τint and τint/10, and the memory operation cannot be obtained under this condition. If the system is operating in the Kerr dominant regime, we should be able to reset the state by injecting a negative reset pulse. The pulse width needed for the reset pulse is > τtot, since we can discharge the cavity within this time. However, Fig. 6 shows that significant heat is accumulating in the system, which prevents the system from resetting because ΔnTO cannot be reset by such a short negative pulse due to its much longer relaxation time.

On the other hand, when τcoup2 ≤ τint/100, we can successfully set and reset the system, and use the device as a Kerr bistable memory. However, the TO effect cannot be eliminated completely even in this case, and thus a hold-
We showed in previous sections that Kerr bistable operation is possible by adopting a large coupling constant. The basic idea is to allow the light to charge and discharge quickly by adjusting $\tau_{\text{coup}2}$. A Kerr bistable memory usable time exists. $P_{\text{out}1}$ is automatically switched from ON to OFF, or vice versa for $P_{\text{out}2}$, due to the thermal accumulation. Figure 3 shows that the memory holding time for the realistic case is about 500 ns. This value is inconsistent to the length of “Kerr memory usable” regime shown in Fig. 2 since the thermal nonlinearity depends on the accumulation of $U_{\text{abs}}$ while the Kerr nonlinearity depend on the instantaneous $U_p$.

In this section we achieved Kerr bistable memory operation and obtained a sufficiently long memory holding time of 500 ns by allowing the system to charge and discharge quickly by adjusting $\tau_{\text{coup}2}$.

5. Discussion: Power consumption

We showed in previous sections that Kerr bistable operation is possible by adopting a large coupling constant. The basic idea is to allow the light to charge and discharge before it turns into heat. However, stronger coupling with waveguides (i.e. smaller $\tau_{\text{coup}2}$) results in a lower total $Q$, which decreases the photon density in a cavity and makes the nonlinearity small. Hence, there is a trade-off between operating speed and operating power.

Now we consider two measures for evaluating the loss of our system; $Q_{\text{int}}$ and $U_{\text{cons}}$. $Q_{\text{int}}$ is the figure of merit of a cavity, whose value gives the cavity loss, and $U_{\text{cons}}$ is the energy consumed for the operation. We often want to increase $Q_{\text{int}}$ and reduce $U_{\text{cons}}$ to build a lossless system. Table 1 compares these values for different systems. A bistable memory based on an ultrahigh-$Q$ silica toroid microcavity with Kerr nonlinearity has a clear advantage over other schemes in terms of both losses. This lossless nature of this memory is the advantage of this system.

First, a system with a high $Q_{\text{int}}$ yields a low loss, because $Q_{\text{int}}$ corresponds to the fundamental loss characteristics of a cavity. As shown in Table 1 the $Q_{\text{int}}$ of a toroid cavity is much higher than that of other types of cavities. Furthermore, we would like to note that both linear loss and nonlinear losses are significant in other systems, especially those systems that use TO and carrier-based nonlinearities to achieve bistability. This is because carrier generation is unavoidable.
The energy change caused by the resonant wavelength shift \(\Delta \lambda\) is estimated from the cavity resonance shift of \((1/2)\Delta \lambda_{\text{FWHM}}\). We assume no TO and no carrier diffusion.

As shown in Fig. 5, we need \(P_{\text{drive}}\) to drive the system, which is significantly smaller than the value shown in Ref. [7], which uses the same Kerr nonlinearity. This is due to the high \(Q\) factor of our system. However, this value is still larger than the experimentally demonstrated value in a photonic crystal nanocavity using carrier-based nonlinearity [14]. Since the coefficients of carrier nonlinearities in semiconductor materials are normally larger than the Kerr nonlinearity in SiO\(_2\), it is not an easy task to reduce the driving power of our system to the same level. However, glass performs low propagation loss [37, 38], which is usually difficult to obtain with devices made of semiconductors.

Although the driving power \(P_{\text{drive}}\) of our system is not necessarily the smallest, the linear, nonlinear and consumption losses are significantly smaller than those of other devices. With this in mind, this device is attractive for applications that require high efficiency \(\eta = P_{\text{out}}/P_{\text{in}}\), such as quantum information processing [12].

6. Conclusion

We rigorously modeled the Kerr and the TO effects in a silica toroid microcavity by combining CMT and FEM. We gained a clear understanding of the impact of adjusting the coupling, and showed that Kerr optical bistable memory operation is possible by adjusting the coupling between the cavity and the waveguides. The memory holding time was about 500 ns. Although the driving power was 7.3 mW, the energy consumed by the system was extremely low. This is because unlike other nonlinearities such as carrier of TO effects Kerr nonlinearity does not absorb photons. In addition due to the ultrahigh-\(Q\) of the system, the energy loss outside the system is also low. Our Kerr bistable memory in a silica toroid microcavity exhibits extremely low loss and thus is suitable for applications such as quantum signal processing.

Table 1. Comparison of the power consumption of different systems

| System type       | Material      | Nonlinearity       | \(Q_{\text{int}}\) | \(P_{\text{drive}}\) | \(U_{\text{in}}^{(a)}\) | \(U_{\text{out}}^{(b)}\) | \(P_{\text{out}}/P_{\text{in}}\) | Exp./Cal. | Refs |
|-------------------|---------------|-------------------|---------------------|----------------------|-------------------------|-------------------------|-----------------------------|----------|-----|
| Microring cavity  | Si            | Thermo-optic      | 1.43 x 10\(^5\)    | 800 \(\mu\)W         | 3.7 x 10\(^{-12}\) J | Exp.                     | Ref. [11]                   |          |     |
| Photonic crystal  | InGaAsP       | Carrier-plasma    | 1.3 x 10\(^5\)    | 250 \(\mu\)W         | -                       | Exp.                     | Ref. [13]                   |          |     |
| Photonic crystal  | Semiconductor Kerr |            | 557                 | 133 mW         | 2.0 x 10\(^{-17}\) J | Calc.                    | Ref. [7]                   |          |     |
| Toroid microcavity| SiO\(_2\)     | Kerr             | 4 x 10\(^8\)      | 7.3 mW         | 2.6 x 10\(^{-29}\) J | Calc.                    | This work                  |          |     |

\(^a\) Estimated from the cavity resonance shift of \((1/2)\Delta \lambda_{\text{FWHM}}\). We assume no TO and no carrier diffusion.

\(^b\) The energy change caused by the resonant wavelength shift \(\Delta U = U_{\text{FWHM}}((1/2)\Delta \lambda_{\text{FWHM}}/(\lambda_0 + (1/2)\Delta \lambda_{\text{FWHM}}))\) is regarded as \(U_{\text{cons}}\), where \(U_{\text{FWHM}}\) is the energy that can cause a resonant wavelength shift of \((1/2)\Delta \lambda_{\text{FWHM}}\) by using the Kerr effect.
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