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The soul conjecture in Alexandrov geometry in dimension 4. (English) Zbl 07537684
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Summary: In this paper, we prove the Soul Conjecture in Alexandrov geometry in dimension 4, i.e. if $X$ is a complete non-compact 4-dimensional Alexandrov space of non-negative curvature and positive curvature around one point, then a soul of $X$ is a point.

MSC:
53C20 Global Riemannian geometry, including pinching
53C23 Global geometric and topological methods (à la Gromov); differential geometric analysis on metric spaces
51F99 Metric geometry

Keywords:
soul conjecture; Alexandrov geometry; Sharafutdinov retraction; submetry; finite quotient of join

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