Optomechanical Sensing in the Nonlinear Saturation Limit

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Abstract: The dynamic range of optomechanical-displacement measurement is limited by the nonlinearity of the cavity transmission. We demonstrate that the dynamic range can be made arbitrarily large within a small detection bandwidth in the resolved-sideband regime.

Radiation-pressure interactions of light in optical cavities have given rise to many interesting phenomena including cooling and mechanical lasing, quantum state control of light and mechanical motion, nonlinear and chaotic dynamics and so on [1]. These cavities have also proven to be excellent transducers for displacement and force sensing [2, 3]. These applications rely on the shift of the optical resonance with the displacement of the cavity walls. For a given resolution, determined by the linewidth of the cavity, there is an upper limit on the dynamic range of the displacement measurement. This is due to the inherent nonlinearity of the Lorentzian lineshape of the resonance as shown in Fig. 1(b) and (c). When the mechanical motion is large, the transmission signal from the cavity saturates, preventing any measurement. In this summary, we propose and experimentally demonstrate a new technique that can break the saturation limit using the second and third harmonic of the modulation signal.

For a cavity mode under optomechanical modulation in the resolved sideband regime, the transmitted optical power can be shown to have the form:

\[ P_t = P_0 + c_1(\Delta) \sum_{n=0}^{\infty} J_{2n+1}(\eta) \sin[(2n+1)\Omega_m t] + c_2(\Delta) \sum_{n=1}^{\infty} J_{2n}(\eta) \sin[2n\Omega_m t], \quad \eta = g_{om}x_0/\Omega_m, \tag{1} \]

where \( g_{om} \) is the optomechanical coupling strength, \( \Omega_m \) is the mechanical resonance frequency, \( x_0 \) is the displacement amplitude, and \( c_1 \) and \( c_2 \) are constants that depend on laser-cavity detuning \( \Delta \). A simple recursion relation of the Bessel function \( J_n(x) \): \( 2nJ_n(x) = x[J_{n+1}(x) + J_{n-1}(x)] \) gives measurement of \( x_0 \) as

\[ x_0 = \frac{4\Omega_m c_1(\Delta)}{g_{om} c_2(\Delta)} \frac{P(2\Omega_m)}{P(\Omega_m) + P(3\Omega_m)}. \tag{2} \]

Fig. 1. (a) Illustration of the mechanical radial breathing mode of a microdisk resonator. (b) In the linear regime, the cavity’s response is linear with the modulation amplitude with the transmitted mechanical noise spectrum shown in (e) identifying the mechanical resonance frequency. (c) When the modulation amplitude increases further, the response saturates preventing a correct measurement of the modulation. The corresponding noise spectrum is shown in (f) obtained at a much higher input power where the higher harmonics are now visible. (d) SEM image of the Silicon microdisk resonator used in the experiment.
Therefore, the first, second and third harmonic of the modulation signal are sufficient to measure arbitrarily large \(x_0\), even when the first harmonic at \(\Omega_m\) saturates.

In order to verify this result, we perform phonon lasing experiment using a Silicon whispering-gallery-mode (WGM) microdisk resonator with a 4 \(\mu\)m radius and a radial breathing mechanical mode with a 644 MHz resonance frequency and a quality factor of 8000 in vacuum. The resonator has a quasi-transverse-electric (TE) mode at a wavelength of 1551 nm with an intrinsic quality factor of \(1.4 \times 10^6\) and a loaded cavity linewidth of 225 MHz. After the lasing threshold is reached, the cavity oscillates continuously with an energy that increases with the driving optical power. We can use the transmitted noise spectrum to measure the harmonics of the modulation signal using homodyne detection with a fast photodetector. The laser is tuned to the edge of the cavity on the blue side where \(\Delta \approx \Omega_m\). Figure 2(a) shows the peak optical power at the first harmonic with increasing input power. The peak power increases initially and after the lasing threshold is reached, becomes linear with the driving power. The linear growth, however, does not persist and the signal eventually saturates at a higher power. This behavior has been observed in previous phonon lasing experiments [4–6], but has not been investigated in detail. After the power at frequency \(\Omega_m\) saturates the energy shifts to the second harmonic at \(2\Omega_m\) and subsequently to the third harmonic at \(3\Omega_m\) and so on. Figure 2(e) shows the peak power at the three harmonics as the driving optical power increases. Incorporating these values into Eq. (2) yields the measured mechanical energy (square of the displacement) shown in Fig. 2(b). The plot shows a clear linear trend indicating that the modulation amplitude is still increasing with the optical power.

Fig. 2. (a) Peak optical power at \(\Omega_m\) showing phonon lasing with a threshold of approximately 18 \(\mu\)W. The power is normalized to the saturation level. (b) Linearized mechanical response after taking into account the powers at \(2\Omega_m\) and \(3\Omega_m\). The solid line shows a linear fit of the data verifying Eq. (2). (c) and (d) Power spectrum at the three harmonics at the two points identified in (b). (e) Peak power at the three harmonics in the transmitted noise spectrum.

In conclusion, we have theoretically and experimentally demonstrated a method to measure mechanical motion of a cavity beyond its saturation limit. Our analysis has shown that a highly nonlinear process, i.e. modulation nonlinearity, can be rendered linear due to a simple property of the Bessel function. We envision that this will enable optomechanical sensing with potentially unlimited dynamic range within a small measurement bandwidth provided the measurement is done in the resolved-sideband regime.

References
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