ESTIMATING CHROMATIC PROPERTIES OF ELECTRON BEAMS FROM FREQUENCY ANALYSIS OF TURN-BY-TURN BEAM POSITION DATA

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ABSTRACT

A method to estimate linear chromaticity, RMS energy spread, and chromatic beta-beating, directly from turn-by-turn beam position data in a circular electron accelerator, is presented. This technique is based on frequency analysis of a transversely excited beam, in the presence of finite chromaticity. Due to the turn-by-turn chromatic modulation of the beam’s envelope, betatron sidebands appear around the main frequency of the Fourier spectra. By determining the amplitude of both sidebands, chromatic properties of the beam can be estimated. In this paper, analytical derivations justifying the proposed method are given, along with results from tracking simulations. To this end, results from practical applications of this technique at the KARA electron ring are demonstrated.

Keywords chromaticity, RMS energy spread, Fourier Analysis, NAFF, Decoherence, Beam Dynamics

1 Introduction

The measurement of chromaticity [Verdier(1993)] is an indispensable process in a circular accelerator, due to the impact it has on beam lifetime and quality. At the modern high-intensity proton and electron rings, the thresholds of several beam instabilities are controlled through the value of chromaticity.

A plethora of methods for measuring chromaticity, each with different beam parameters as observables, have been developed so far. Some of the existing techniques are mentioned below, however a more detailed review of the corresponding approaches can be found in [Steinhagen(2009), Minty and Zimmermann(2003)]. The basic relationship that defines chromaticity in terms of beam dynamics parameters is presented in Sec. 6.1.1 of the Appendix.

One of the most simple and widely used techniques of chromaticity is the RF-sweep, where the operating frequency of the radio-frequency (RF) system is changed, in order to induce a change in the energy of the beam. At the same time, betatron tune measurements [Serio(1989), Bartolini et al.(1996)] are performed, usually by analysing turn-by-turn (TbT) data from beam position monitors (BPMs) and employing Fourier algorithms. From there, chromaticity can be determined from the correlation of the measured tunes with the varying energy of the beam. This method usually requires dedicated experimental time, thus making it impossible to use during normal operation of the accelerator.

The estimation of chromaticity is also possible by measuring the incoherent power spectrum of the beam with Schottky monitors [Boussard(1989)]. This method is particularly useful for high-intensity, high energy proton machines like the Large Hadron Collider (LHC), however in the case of bunched beams, strong coherent beam modes can pollute the Schottky spectra and thus reduce the efficiency of the method [Betz et al.(2017)].
A very interesting technique to measure the relative chromaticity $\xi = Q^\prime/Q$, where $Q$ is the betatron tune, is by observing the phase shift of the head and the tail of a bunch [Cocq et al.(1998)Cocq, Jones, and Schmücker, Fartoukh and Jones(2002)] during one synchrotron period. This phase-shift is correlated with the value of chromaticity and it can be measured with instruments that can resolve intra-beam movements, like the Head-Tail Monitor [Catalan-Lasheras et al.(2003)Catalan-Lasheras, Fartoukh, and Jones]. This method has been found to be affected by the quality of the TBT signal [Ranjar et al.(2017)Ranjar, Marusis, and Minty], where kinematic decoherence [Meller et al.(1987)Meller, Chao, Peterson, Peggs, and Furman, Lee(1991)] arising from the frequency distribution over the particles in the bunch, can severely diminish the signal-to-noise ratio. As a result, only a few hundreds of turns can be left for meaningful frequency analysis, whereas the synchrotron period of a proton machine can be much larger. Note that this technique cannot be easily applied in a high-energy electron ring, where the bunch lengths are in the range of ps, rendering the phase differences between the head and the tail of the bunches hard to resolve.

Another class of methods that can be employed are the ones that extract information from the chromatic decoherence of the beam [Siemann(1989)]. Due to this mechanism, the envelope of the beam is modulated in amplitude with respect to the number of revolutions. By determining the maximum and the minimum of the coherent TBT signal i.e. the modulation depth of the betatron oscillations, information on chromaticity and the RMS energy spread can be acquired.

In fact, measuring and controlling the RMS energy spread is of paramount importance as well for the modern high-intensity electron accelerators. For example, the modern light sources are designed as to achieve very low emittances, in the regime that intra-beam scattering becomes important [Kubo et al.(2005)Kubo, Mtingwa, and Wolski, Bane et al.(2002)Bane, Hayano, Kubo, Naito, Okugi, and Urakawa, Antoniou(2012)]. At an electron ring, the RMS energy spread is typically measured by evaluating information from the synchrotron light of the electron bunches with specific instrumentation e.g. a streak camera. However, the same goal can be achieved by analysing TBT BPM data in the time domain [Hsu(1990), Bassi et al.(2015)Bassi, Blednykh, Choi, and Smaluk], by performing multi-parametric fits, based on the existing analytical models of the beam centroid motion in the presence of chromaticity. Unfortunately, the unavoidable noise in the TBT data and small values of the chromaticity can drive large measurement errors when these methods are employed [Manukyan et al.(2011), Manukyan, Sargsyan, Amatuni, and Tsakanov].

In this paper, by acknowledging the necessity of an operationally efficient technique, an alternative method is proposed for estimating the chromaticity or the RMS energy spread through the Fourier spectra of a transversely excited beam. The proposed technique is based on the analytical relationships derived in [Rumolo et al.(2004)Rumolo, Schmidt, and Tomas], which govern the Fourier spectra of a longitudinally excited beam, in the presence of chromaticity. The method takes advantage of the fact that chromatic decoherence modulates the envelope of the beam, with the modulation period to be exactly the synchrotron period. As a result, chromatic sidebands appear around the main betatron tune in the transverse spectra of the excited beam, with an amplitude proportional to chromaticity. With the proposed method, the estimation of chromaticity is performed through the measurement of both the chromatic sidebands of the beam, and by using the following simple equation

$$Q^\prime_s = \pm \frac{Q_s}{\sigma_s} \sqrt{\frac{A_1 + A_{-1}}{A_0}},$$

(1)

where $Q^\prime_s$, with $z = x, y$, is the horizontal or vertical chromaticity respectively, $Q_s$ is the synchrotron tune, $\sigma_s$ is the RMS energy spread, $A_1$ and $A_{-1}$ are the amplitudes of the first order chromatic sidebands that appear around the horizontal or vertical betatron frequency which has an an amplitude of $A_0$. Precise measurements of chromaticity with the aforementioned method are possible only if the Fourier amplitudes $A_{\pm 1}$ are known with high certainty. Fortunately, the existence of powerful numerical tools for the Fourier decomposition of the beam’s motion allow for this. The Numerical Analysis of Fundamental Frequencies (NAFF) [Laskar et al.(1992) Laskar, Froeschle, and Celletti, Laskar(1993), Laskar(2003), Papaphilippou(2014)] algorithm has been used with success in the field of accelerator physics, for precise optics and dynamical stability measurements through frequency and amplitude analysis. The first results from the application of this method can be found in [Zisopoulos et al.(2014)Zisopoulos, Antoniou, Papaphilippou, Streun, and Ziemann], where TBT RMS energy spread measurements are performed at the Swiss Light Source (SLS).

It is important to mention that a method which follows a similar approach i.e. it is based on the frequency response of the chromatic motion of the beam, has been developed in [Nakamura(1999)] and employed for measurements in some applications [Nakamura et al.(1999)Nakamura, Takano, Masaki, Soutome, Kumagai, Ohshima, and Tsumaki, Kiselev et al.(2007)Kiselev, Muchnoi, Meshkov, Smaluk, Zhilich, and Zhuravlev]. A limitation of this method is that it assumes equal amplitudes for both chromatic sidebands, and it uses only one sideband for measurements. However, this would be true only in the absence of non-linearities which can cause beating of the optics in the lattice.

In order to by-pass this limitation, the method uses both amplitudes of the chromatic sidebands as it is testified in Eq. (1). The same methodology has been also applied to the Diamond Light Source [Rehm et al.(2010)Rehm, Abbott, Morgan, Rowland, and Uzun], where a piecewise fit of the ratio of the chromatic sidebands, for various chromaticities, has been implemented and used as a reference, in order to estimate...
chromaticity during operation. However, in the present work, the estimation of chromaticity is model independent, by solving analytically the equations of decoherence, and by using the amplitudes of the sidebands as an observable. An advantage of this method requires the same experimental procedure as the typical betatron tune measurements in a ring.

The present paper is organised as follows: In Section 2 the theoretical basis of this method is presented. In Section 3, the method is employed in tracking simulations for estimating chromaticity and chromatic beta-beating from TbT BPM data, by using the accelerator model of the KARA light source and NAFF. In Section 4 the proposed method is employed in experimental measurements at the KARA light source, including an application to characterise the optics response during the commissioning of the Compact Linear Collider (CLIC) Superconducting Wiggler prototype [Bernhard et al. (2016)].

2 Analytical Derivations

Analytical formulas of the Fourier spectra of a Gaussian beam, which is longitudinally excited in the presence of chromaticity, have been derived in [Rumolo et al. (2004)]. These expressions take into account non-linearities, which arise from the distortion of the beam’s motion due to finite dispersion at the sextupoles. From these relationships, the Fourier amplitudes $A_q$ of the synchrotron sidebands of order $q$ are given from

$$A_q = e^{-s^2} \left| I_q(s^2 - isk) + \frac{\Delta \beta_z}{4l \beta_z} \sigma_\delta (k + is) \right|,$$

where $s = Q'_z \sigma_\delta / Q_s$, with $Q'_z$ the chromaticity for the $z = x, y$ planes, $\sigma_\delta$ is the RMS energy spread of the bunch, $Q_s$ the synchrotron tune defined from the RF cavities, $\tilde{a}$ is the initial transverse amplitude of the kicked beam, $I_q(x)$ is the $q$-order modified Bessel function of the first kind for argument $x$, $k$ is the longitudinal kick in units of the RMS bunch length $\sigma_t$ of the beam, and $\Delta \beta_z$ is the chromatic beta beating. Analytical derivations of the chromatic beta-beating for circular accelerators can be found in [Fartoukh (1999)], which includes the effect of linear and non-linear magnetic fields. The appearance of the $I_q(s^2 - isk)$ term is a signature of amplitude modulation, where the argument $s^2 - isk$ represents the modulation index [Oppenheim et al. (1997)]. The same modulation occurs in the case of a pure transverse excitation instead of a longitudinal one. A numerical example is shown in Fig. 1, where TbT pseudo-data are produced, based on the kinematic decoherence relationships in [Rumolo et al. (2004)]. The envelope (red curve) of the oscillation (blue curve) exhibits a modulation period of $N_s = 100$ turns i.e. the inverse of the synchrotron tune $Q_s = 0.01$. Every $N_s/2$ turns the envelope passes a trough, thus exhibiting a minimum projection of the synchrotron oscillations in the transverse betatron motion. The carrier frequency is the betatron frequency, which is excited with a transverse impulse at $N = 0$ turns.

Concerning the application which is presented in this paper, the longitudinal excitation is dropped in favour of a transverse excitation. The motivation behind this choice is that the goal of this analysis is to develop a method for chromaticity or RMS energy spread measurements, based on the same procedure of transverse betatron tune measurements. In order to do so, the parameter of the longitudinal excitation $k$, is set to $k = 0$ and Eq. (2) now becomes
\[ A_q = e^{-s^2|\tilde{a}|} \left| I_q(s^2) + s \frac{\Delta \beta_z}{4 \beta_z} \sigma_\delta [I_{q-1}(s^2) - I_{q+1}(s^2)] \right|, \]  
(3)

where now the Fourier amplitudes of the beam, \( A_q \), are determined from the transverse impulse given to the beam. As a matter of fact, one of the assumptions of the current study is that the transverse kick given to the beam is adequately small, such as not to induce rapid decrease of the beam’s envelope due to decoherence, as a result of tune-shift with amplitude.

The previous expression can be written in a simpler form by using the following relationships [Abramowitz and Stegun(1965)]:

\[ I_{q-1}(x) - I_{q+1}(x) = \frac{2q}{x} I_q(x) \]  
(4)

\[ I_q(x) = I_{-q}(x) \]  
(5)

\[ I_q(x) \approx \frac{1}{\Gamma(q+1)} \left( \frac{x}{2} \right)^q, \quad 0 < |x| \ll \sqrt{q+1}. \]  
(6)

Indeed, by using Eq. (4) in Eq. (3) and computing the absolute value of the result, the expression

\[ A_q = |\tilde{a}| e^{-s^2 I_q(s^2)} \left( 1 + q \frac{Q_s \Delta \beta_z}{Q'_s 2 \beta_z} \right), \]  
(7)

is obtained.

It is evident from Eq. (7) that chromatic beta-beating becomes significant for higher-order modes i.e. it is linear with \( q \). In addition, it is clear that by using only one synchrotron sideband for the estimation of the chromatic properties induces an uncertainty due to the beta-beating. However, by forming the sum of both symmetric sidebands \( A_q + A_{-q} \) with the help of Eq. (5), and normalising it with the main betatron amplitude \( A_0 \) for \( q = 0 \), results in the elimination of the perturbation term \( \frac{\Delta \beta_z}{2 \beta_z} \). Moreover, by normalising the difference \( \pm |A_q - A_{-q}| \) by \( A_q + A_{-q} \), an analytical relationship for the estimation of the chromatic beta-beating \( \frac{\Delta \beta_z}{\beta_z} \) is obtained. The aforementioned operations result in the following equations:

\[ \frac{A_q + A_{-q}}{A_0} = \frac{2I_q(s^2)}{I_0(s^2)} \]  
(8)

\[ \left| \frac{A_q - A_{-q}}{A_q + A_{-q}} \right| = \pm \frac{q Q_s \Delta \beta_z}{2 Q'_s \beta_z}. \]  
(9)

The expression in Eq. (8) can be numerically solved in order to estimate the \( s = \frac{Q'_s \sigma_\delta}{Q_s} \) parameter. The synchrotron tune \( Q_s \) is usually known with fair accuracy from the properties of the RF system, but it can also be inferred from the frequency offset of the chromatic sidebands, with respect to the main frequency line. Then, by knowing the RMS momentum spread \( \sigma_\delta \), one can estimate the chromaticity \( Q'_s \) of the machine or vice-versa.

On the other hand, simple analytical relationships that determine the chromaticity or the RMS energy spread can be developed by combining the approximation in Eq. (6) with Eq. (8).

Assuming that \( s^2 \ll \sqrt{q+1} \), for the \( q \) order chromatic sideband under consideration, the analytical form of Eq. (8) becomes

\[ \frac{A_q + A_{-q}}{A_0} = \frac{2}{\Gamma(q+1)} \left( \frac{s^2}{2} \right)^q \]  
(10)
Solving Eq. (10) with respect to $s$, and further with respect to chromaticity $Q'_z$, yields

$$Q'_z = \pm \frac{Q_s}{\sigma_\delta} \sqrt{2^{q-1} \Gamma(q+1) \left[ \frac{A_q + A_{-q}}{A_0} \right]}$$

(11)

The previous expression allows for the estimation of chromaticity $Q'_z$ or the RMS energy spread $\sigma_\delta$ based on the amplitudes of the chromatic sidebands $A_q$ and $A_{-q}$, for $q \geq 1$. Since in a real Fourier spectrum of TbT data, assuming that the second order chromaticity $Q''_z$ is not large, the beam is longitudinally matched to the RF bucket, and that collective effects are not important, the first order chromatic sidebands $A_1$ and $A_{-1}$ are most easily resolved. Thus, by setting $q = 1$ in Eq. (11) and in Eq. (9), the following expressions are obtained:

$$Q'_z = \pm \frac{Q_s}{\sigma_\delta} \sqrt{A_1 + A_{-1}}$$

(12)

$$\frac{\Delta \beta_z}{\beta_z} = \pm \frac{2Q'_z}{Q_s} \frac{A_1 - A_{-1}}{A_1 + A_{-1}}$$

(13)

These relationships constitute of a method, which is independent from the calibration of the BPMs due to normalization to the main amplitude $A_0$, for estimating the chromaticity or the RMS energy spread of the beam. As a by-product of the method, a relationship which determines the chromatic beta-beating at the position of the BPMs is recovered. Note that the previous expressions do not differentiate between positive and negative solutions, since they depend on the Fourier amplitudes of the beams. Thus, the proposed method cannot recover the sign of the chromaticity $Q'_z$ and of the chromatic beta beating $\frac{\Delta \beta_z}{\beta_z}$. Analytical expressions for the measurement errors of chromaticity and chromatic beta-beating through Eq. (12) and Eq. (13), can be found in Sec. 6.1.2 of the Appendix.

3 Simulations

Tracking simulations with the model of the KARA electron accelerator are undertaken using MADX-PTC [Schmidt et al. (2002) Schmidt, Forest, and McIntosh], in order to numerically investigate the proposed method and to identify the dependence of the proposed method on the initial excitation of the beam. The fundamental parameters of the simulations can be found in Table 1. During the simulations, the particles are tracked around the KARA lattice for different cases of initial excitation, and for $N = 1200$ turns. The lattice of the KARA accelerator consists of Double Bend Achromat (DBA) cells and exhibits a four-fold symmetry. The number of BPMs is $M = 35$ and the optics of at the position of the BPMs are shown in Fig. 2, where the maximum horizontal beta function (top) is at around $\beta_x = 19$ m at the position of the arcs of the ring, the maximum vertical beta function is approximately $\beta_y = 30$ m and the horizontal dispersion (bottom) exhibits an average value of around $\langle D_x \rangle = 0.25$ m. The simulations employ an error-free lattice, with no radiation effects taken into consideration, which result in a vanishing vertical dispersion $D_y$ around the accelerator.

Table 1: Parameters of the tracking simulations with the KARA model. All the position dependent parameters are measured at the injection point of the ring.

| Parameter | Value |
|-----------|-------|
| Energy | 2.50 [GeV] |
| Circumference | 110.40 [m] |
| Tune $Q_x$, $Q_y$, $Q_z$ | 6.79, 2.70, 0.016 [2π] |
| Chromaticity $Q'_x$, $Q'_y$ | 2.57, 6.90 [2π] |
| RMS beam size $\sigma_x$, $\sigma_y$, at injection | 0.94, 0.080 [mm] |
| Beta function $\beta_x$, $\beta_y$, at injection | 18.89, 1.67 [m] |
| RMS bunch length $\sigma_z$ | 10.30 [mm] |
| RMS energy spread $\sigma_\delta$ | 9.80 $[10^{-4}]$ |
| Initial excitation at injection | $1 - 7 | \sigma_x |$, 20 – 80 $| \sigma_y |$ |
| Dimensionality | 6-D |
| Distribution | Gaussian |
| Number of particles $N_p$ | $10^4$ |

The TbT evolution of the centroid of the beam, is inferred from the arithmetic mean of all the particles for each turn $N$. The beam is transversely excited in a range of initial amplitudes, which are referred in this paper in units of the RMS...
transverse beam size $\sigma_z$, where $z = x, y$ for the horizontal and vertical planes respectively. In Fig. 3a the horizontal TbT are shown for initial excitations of $1 \sigma_x$ to $4 \sigma_x$, where for that particular position in the ring, the initial amplitude ranges from 0.5 mm to 2 mm. Kinematic decoherence is responsible for the simultaneous chromatic oscillations and the TbT damping of the beam’s envelope due to non-linear tune-shift. The TbT simulated data for the vertical plane, are presented in Fig. 3b after initial excitations of 20 to 50 $\sigma_y$ and initial amplitudes similar to the horizontal case, Fig. 3a. The trend of the centroid’s evolution in the vertical plane exhibits a smaller non-linear detuning and larger chromatic oscillations, compared to the horizontal plane, due to the higher vertical chromaticity.

(a) Horizontal TbT data for excitations of $1 \sigma_x$ (top left), $2 \sigma_x$ (top right), $3 \sigma_x$ (bottom left) and $4 \sigma_x$ (bottom right).

(b) Vertical TbT data for excitations of $20 \sigma_y$ (top left), $30 \sigma_y$ (top right), $40 \sigma_y$ (bottom left) and $50 \sigma_y$ (bottom right).

Figure 3: The simulated TbT tracking data of the beam’s centroid for the KARA accelerator model, with respect to the number of turns $N$. The initial amplitude of the beam, in terms of the transverse beam sizes $\sigma_x, \sigma_y$, is indicated in the legend.
3.1 Envelope evolution

![Graph](image)

(a) Horizontal normalized envelopes of the centroid for the case of $1 \sigma_x$ (blue line), $2 \sigma_x$ (orange line), $3 \sigma_x$ (green line), and $4 \sigma_x$ (red line).

(b) Vertical normalized envelopes of the centroid for the case of $20 \sigma_y$ (blue line), $30 \sigma_y$ (orange line), $40 \sigma_y$ (green line), and $50 \sigma_y$ (red line).

Figure 4: The upper envelopes for the horizontal (a) and vertical (b) oscillations of the centroid of the beam for the KARA model, with respect to the number of turns $N$. The envelopes are normalised to the oscillation value at the first turn $N = 1$. The initial excitation for each simulation is shown in the legend.

As it has been discussed already, chromatic decoherence results in the periodic modulation of the beam’s envelope with a period $\tau_s$ equal to the inverse of the synchrotron tune $\tau_s = \frac{1}{Q_s}$. For the present simulations, the TbT evolution of the horizontal envelopes, normalised to their maximum value at $N = 1$ turns, is shown in Fig. 4a for a range of initial excitations. The synchrotron period is about $\tau_s = 63$ turns and during the first synchrotron period, the centroid of the beam oscillates with the same phase, regardless of the initial excitation. However, during and after the second synchrotron period, a distinct amplitude-dependent damping of the synchrotron oscillations appears. This behaviour can be attributed to the non-linear detuning of the centroid, which becomes dominant after around $N = 300$ turns i.e. almost 5 synchrotron periods $\tau_s$. In addition, the phases of the oscillations for all cases appear to become different after the first two synchrotron periods $\tau_s$. This indicates that the synchrotron oscillations exhibit a frequency shift, which depends to the initial excitation amplitude. For the $4 \sigma_x$ case and for that particular BPM, the value of the envelope is at 50 % of the initial amplitude at around $N = 300$ turns already, indicating a relatively small decoherence time. The same value is at around 70 % for the excitation of $1 \sigma_x$.

The vertical envelopes are shown in Fig. 4b. The amplitude modulation is visible for every $N = \tau_s$ turns, with the modulation occurring around the high-frequency component i.e. the betatron oscillations. Note that the chromatic oscillations are visible also in the vertical plane, even if there is no vertical dispersion. Due to the small vertical emittance, the TbT evolution of the vertical envelopes exhibit more coherent behaviour with respect to the horizontal plane, and the de-phasing of the oscillations due to decoherence begins at around five synchrotron periods. As a result, estimating the vertical chromaticity with the proposed method, is expected to be less dependent on the initial excitation. In addition, the damping of the chromatic oscillations appears to be very slow, which results in a wider range of number of turns $N$ for the survival of the synchrotron oscillations.
An immediate conclusion of the analysis the TbT evolution of the envelopes, is that the applicability of Eq. (12) and Eq. (13) is expected to depend on the number of turns $N$, and the initial transverse excitation of the beam. For this reason, the number of turns $N$ should be restricted to as small values as possible. At the same time, that particular number of turns $N$ should allow for enough resolution for the estimation of the Fourier spectra.

### 3.2 Fourier spectra

The Fourier spectra of the centroid of the beam are determined with Py-NAFF [Asvesta et al. (2017)], the adaptation of the NAFF [Laskar (2003)] algorithm in Python programming language, which offers an increased precision in the estimation of the spectral components of a quasi-periodic signal. The appearance of the chromatic sidebands around the main betatron frequency line is shown in Fig. 5a for the horizontal plane, and in Fig. 5b for the vertical plane. The amplitudes are normalised to the maximum of the main Fourier amplitude $A_0$ for both cases of initial excitation, and the measurements are performed with $N = 300$ turns or almost five synchrotron periods $\tau_s$.

Concerning the horizontal case, a slight increase of the horizontal tune $Q_x$ is observed for the $4 \sigma_x$ case, with respect to the $2 \sigma_x$ case, due to tune-shift with amplitude. Note that the tune-shift appears as a decrease of the horizontal betatron tune $Q_x$ in the Fourier spectra, since the fractional part of the tune $Q_x$ is larger than 0.5.

For the vertical case, the chromatic sidebands are found to be almost 14 times greater in amplitude, than the horizontal sidebands, due to the difference in the magnitude of the two transverse chromaticities. Furthermore, there is a negligible shift of the frequencies for the $50 \sigma_y$ case. The frequency spectra of all the simulated cases can be similarly produced, and chromaticity can be inferred from Eq. (12).

![Figure 5a](image1.png)

(a) Horizontal Fourier spectra, where the $2 \sigma_x$ case is shown in blue and the $4 \sigma_x$ case in orange.

![Figure 5b](image2.png)

(b) Vertical Fourier spectra, the $30 \sigma_y$ case is shown in blue and the $50 \sigma_y$ case in orange.

Figure 5: The frequency spectra of the beam’s centroid for the horizontal (a) and vertical (b) planes, normalised to the amplitude of the main peak $A_0$. The chromatic sidebands $A_1$ and $A_{-1}$ are observed at a distance equal to the synchrotron tune $Q_s$, marked with dashed lines (green for $A_{-1}$ and black for $A_1$). The initial excitation amplitude of the beam, in terms of the transverse beam sizes $\sigma_x$ and $\sigma_y$, is indicated in the legend.
3.2.1 Synchrotron tune

The synchrotron tune $Q_s$, is usually well known from the parameters of the RF system, however it can be also estimated from the distance between the main betatron frequency line $A_0$ and the first order chromatic sidebands $A_1$ and $A_{-1}$. Although for the present study the value of the synchrotron tune $Q_s$ is obtained from MAD-X, the synchrotron tune $Q_s$ is also estimated from the response of the transverse Fourier spectra of the beam. Such an analysis can provide useful insights on the behaviour of the TbT data. The estimation of the average synchrotron tune $Q_s$, where the average is performed on the $M = 35$ BPMs of the KARA model, is graphically presented in Fig. 6 against the value from MAD-X (solid black line) for the horizontal (top) and vertical (bottom) planes, and for various initial excitations.

Figure 6: The synchrotron tune $Q_s$ measured from the Fourier spectra for each case of initial excitation, and averaged over all the BPMs of the KARA model. The standard deviation $\sigma_{Q_s}$ of the tunes for all cases is at the order of $\sigma_{Q_s} = 10^{-7}$. The tunes are shown with respect to the number of turns $N$ for the horizontal (top) and vertical (bottom) planes. The legend indicates the initial excitation amplitude, while the black solid line shows the theoretical synchrotron tune $Q_s$, which is obtained in MAD-X.

For the horizontal plane, it is evident that amplitude dependent effects can impact the survival of the chromatic sidebands. For example, while for the $1 \sigma_x$ case the synchrotron tune can be recovered for up to $N = 800$ turns, the $4 \sigma_x$ case exhibits synchrotron sidebands up to $N = 500$ turns. In addition, a longitudinal tune-shift with amplitude is evident during the first six synchrotron periods i.e. up to $N = 300$ turns, which is highlighted at $N = 180$ turns, and converges to a constant value after the characteristic decoherence time [Meller et al. (1987)] of each initial excitation. The nature of this effect stems from the injection of the beam in the RF bucket. A larger transverse excitation results in a beam which covers more space in the longitudinal phase space, resulting in larger longitudinal emittance and synchrotron tune $Q_s$.

The synchrotron tune inferred from the vertical TbT data exhibits no amplitude-dependent effects, and the extracted value remains constant with respect to the number of turns, and close to the expected synchrotron tune.

The TbT dependency of the synchrotron tune $Q_s$ on the initial excitation can introduce systematic errors in the estimation of chromaticity with Eq. (12), which is more pronounced for stronger initial excitations. On the other hand, larger initial excitations might be favorable for resolving the chromatic sidebands. In order to correct for this source of systematic error in the synchrotron tune measurement, analytical models can be employed which describe the relationship of the synchrotron tune and amplitude. However, in order to avoid this error, the value of the theoretical synchrotron tune, or the value estimated with the parameters of the RF system, could be used for chromaticity measurements.

3.2.2 Chromatic sidebands

From Eq. (12), the inferred chromaticity is given as the ratio of the sum of the chromatic sidebands $A_{\pm 1}$, to the amplitude of the main frequency line $A_0$. Due to the effect of decoherence, the amplitudes of the spectral lines decrease with respect to the number of turns $N$, with a decay rate that is similar for every chromatic sideband. As a consequence, systematic errors inhibit the estimation of any parameters from the spectral lines of the beam, with the magnitude of the error depending on the optics and the TbT evolution of beam dynamics.

It is worth mentioning that combined decoherence models, which describe the motion of the centroid of the beam in the presence of finite chromaticity and tune-shift with amplitude, do exist and could be “fed-back” to the data, in order to correct for the aforementioned turn-by-turn error. This would require a proper tuning of the model and a good knowledge of the optics, as to fit the experimental data as better as possible. Another alternative would be to estimate the frequency response of the combined decoherence, and correct the amplitude of the spectral lines of the centroid of the beam.
For the sake of estimating the effect of amplitude-dependent decoherence, an analysis of the spectral lines of the beam’s centroid for various initial excitations is employed in these simulations. First the chromatic ratios

\[ R_x = \frac{A_{1x} + A_{-1x}}{A_{0x}} \]  
\[ R_y = \frac{A_{1y} + A_{-1y}}{A_{0y}} \]  

are defined, where \( A_{1x}, A_{-1x}, A_{1y}, A_{-1y} \) are the first order chromatic sidebands of the main betatron amplitudes \( A_{0x}, A_{0y} \) of the horizontal and vertical planes respectively.

The dependence of \( \sqrt{R_x} \) on the number of turns \( N \) and on the initial excitation amplitude, can be visualized in Fig. 7a while the same dependence for the vertical chromatic ratio \( \sqrt{R_y} \) are shown in Fig. 7b. The values represent the average across all the BPMs of the KARA model, and the error-bars represent one standard deviation from the average. Both figures contain the theoretical chromatic ratio values that are expected from the model, by taking into account the statistical error in the estimation of the RMS energy spread, arising from the usage of different particle distributions for each initial excitation. This spread, which is taken as the standard deviation, \( SD(\sigma_\delta) \), of the RMS energy spread \( \sigma_\delta \) of all the simulated distributions, is at the range of \( 10^{-6} \) for both transverse planes, and it results in an uncertainty \( \sigma_\sqrt{R_x} \) of the chromatic ratios, for \( z=x,y \), which by simple error propagation is estimated to be:

\[ \sigma_\sqrt{R_x} = \frac{Q_x}{Q_s} SD(\sigma_\delta) \]  

where the \( SD(\sigma_\delta) \) is the standard deviation of the RMS energy spread \( \sigma_\delta \).

The horizontal ratio \( \sqrt{R_x} \) exhibits a significant TbT spread between the different excitations of the beam, which cannot be explained by the uncertainty in the estimation of the RMS energy spread \( \sigma_\delta \). More specifically, while the cases of 2, 3, and 4 \( \sigma_x \) excitations produce the same \( \sqrt{R_x} \) ratio at \( N = 180 \) turns, the 1 \( \sigma_x \) case exhibits an offset of around 0.01 from that value. At the same time, the 1 \( \sigma_x \) case exhibits the slowest damping due to decoherence. This suggests that the for the current simulations, the 1 \( \sigma_x \) excitation is not enough for precise chromaticity estimations. The curves of 2 \( \sigma_x \) and 3 \( \sigma_x \) initial excitation, exhibit similar trends, albeit a small separation is visible after \( N = 360 \) turns, or after about six synchrotron periods. The 4 \( \sigma_x \) excitation undergoes a steep decrease in the very first turns, due to non-linearities, and then quickly decreases in magnitude. This behaviour points to the fact that the available number of turns for precise chromaticity measurements, is limited for this case. Note that for the 4 \( \sigma_x \) excitation, the systematic errors are almost 10 times larger than the excitations of smaller magnitude.

Concerning the vertical plane, the \( \sqrt{R_y} \) curves can be visualized in Fig. 7b and suggest an insignificant effect of decoherence on the chromatic oscillations of the beam’s centroid. This is expected from the small vertical beam size \( \sigma_y \). All the simulated cases demonstrate a uniform damping of the vertical chromatic ratio \( \sqrt{R_y} \), with a total drop of 0.05 from \( N = 180 \) to \( N = 480 \) turns. The distribution of the curves falls inside the uncertainty interval of the theoretical value, however the case of 40 \( \sigma_y \) exhibits a damping rate which is larger the rate of the 50 \( \sigma_y \). Although this effect is counterintuitive, it can be explained by statistical fluctuations.

Overall, the measurements of the vertical chromatic ratio \( \sqrt{R_y} \) fall inside the expected range, and since the impact of non-linear dynamics is not significant for the vertical plane, accurate measurements of the vertical chromaticity are possible.

### 3.3 Chromaticity Estimations

The findings of the previous section lead to the conclusion that the chromaticity estimations, via the proposed method, depend on the number of turns \( N \) of the TbT BPM signal. An immediate conclusion is that one should use the least number of turns \( N \) for the estimation of the chromatic properties of the beam, in order to avoid the effects of decoherence. At the same time, the number of turns \( N \) must allow for the observation of chromatic sidebands with enough precision i.e. the BPM signal should contain several synchrotron periods. Depending on the exact value of the synchrotron tune \( Q_s \) and the characteristic decoherence time, this is typically achieved at four to six synchrotron periods \( \tau_s = \frac{1}{Q_s} \), where \( \tau_s = 63 \) turns. In this section, the goal is to investigate the output of the proposed method with respect to different initial excitations.
3.3.1 Accuracy

For the current simulations, the chromaticity is estimated by direct application of Eq. 12 and by using a fixed range of number of turns $N$, for the frequency analysis of the TbT data. The value of the synchrotron tune $Q_s$ used in the analysis, corresponds to the reference value i.e. the value which is estimated by the simulation program MAD-X. The number of turns $N$ correspond to a range of three to six synchrotron oscillations $\tau_s$. By using the chromaticities $Q'_x$ calculated from MAD-X as reference values, the absolute normalized error $|\Delta Q'/Q'|_x = |Q'_x - Q'_{x0}|/Q'_{x0}$ of the estimated horizontal chromaticity $Q'_x$ is presented in Fig. 8a, with respect to the initial excitation amplitude $x_0$, in units of the RMS horizontal width $\sigma_x$. The absolute error of the vertical chromaticity $|\Delta Q'/Q'|_y$ with respect to the vertical excitation in units of RMS vertical width $\sigma_y$ are shown in Fig. 8b. For both planes, the measurements are given as a percentage of the reference value, and they refer to the average values from all the BPMs, with the error-bars representing one standard deviation from the average value.

As it has been already discussed, the $1\sigma_x$ case exhibits the least accurate chromaticity estimations, with a normalized error of just below 10%. Integrating the analysis over a longer number of turns seems to improve the error, but not adequately. However, if the initial excitation amplitude $x_0$ increases, the value of the chromaticities converge around the expected value with a normalized error $|\Delta Q'/Q'|_x = |Q'_x - Q'_{x0}|/Q'_{x0}$ of below 5% for the $2\sigma_x$ excitation and a number of turns $N$ of six synchrotron periods. As a matter of fact, further increase of the excitation amplitude and appropriate choice of the number of turns $N$ leads to even smaller errors, which correspond to below 1% of the reference value of the horizontal chromaticity $Q'_x$. Such a case is presented for the $4\sigma_x$ excitation, and for a number of

Figure 7: The TbT evolution of the chromatic ratios $\sqrt{R_x}$ (a) and $\sqrt{R_y}$ (b) for the horizontal and vertical planes respectively, averaged across all the BPMs of the KARA model, for different initial excitations. The error-bars represent one standard deviation from the average values. The theoretical values are shown in black, and the uncertainty due to statistical errors, is shown in light green.
(a) Horizontal chromaticity error with respect to the initial horizontal excitation of the beam. The error-bars indicate one standard deviation from the average value of all the BPMs.

(b) Vertical chromaticity error with respect to the initial vertical excitation of the beam. The error-bars indicate one standard deviation from the average value of all the BPMs.

Figure 8: Normalized absolute error of the horizontal (a) and vertical (b) chromaticity measurements for the simulations with the KARA accelerator model. Each chromaticity measurement is the average from all BPMs, and it is shown as a percentage of the reference value against the initial excitation amplitudes of the simulation. The error-bars measure one standard deviation from the average value. The different colours refer to the number of turns \( N \) that were used for the measurements, which are multiples of the synchrotron period \( \tau_s = 63 \) turns.

For both transverse planes, the spread of the chromaticity estimations across the BPMs of the KARA model is observed to be below the order of 0.1 \%, with respect to the average value. This observation confirms one of the assumptions of the method that the chromaticity measurement should be independent of the location in the ring. An immediate conclusion from the results of these simulations is that accurate chromaticity estimations, by using the Fourier sidebands on the main betatron lines, are possible, if the RMS energy spread of the beam and the synchrotron tune are known. However, care has to be taken for the choice of initial excitation of the beam, as it is found to play a role in the estimations of chromaticity. Concerning the current simulations, the dependence is more pronounced for the horizontal plane, where a minimum of the error in the chromaticity estimations can be achieved by a proper choice of the number of turns \( N \), and the initial amplitude of the beam.
3.4 Chromatic beta-beating estimations

The chromatic beta-beating is defined as the perturbation of the beta function $\beta$, with respect to the momentum offset $\delta$. Expanding the beta function up to first order gives

$$\beta(\delta) = \beta_0 + \frac{\partial \beta}{\partial \delta} \delta,$$  \hspace{1cm} (17)

where $\beta_0$ is the unperturbed beta function. The chromatic beta-beating $\frac{\Delta \beta}{\beta}$ is defined as

$$\frac{\Delta \beta}{\beta} = \frac{1}{\beta_0} \frac{\partial \beta}{\partial \delta},$$  \hspace{1cm} (18)

and it can be calculated by integrating along the whole circular accelerator with the relationship [Fartoukh(1999)]

$$\frac{\Delta \beta}{\beta}(s) = \pm \frac{1}{2 \sin(2\pi Q)} \int \beta(s')[\mp K_1(s')]$$

$$+ K_2(s')D_x(s') \cos(2(\phi(s) - \phi(s')) - 2\pi Q) ds',$$  \hspace{1cm} (19)

where $s$ is the path length, $Q$ is the betatron tune, $K_1(s)$ and $K_2(s)$ are the quadrupolar and sextupolar integrated magnetic strengths, $D_x(s)$ is the horizontal dispersion and $\phi(s)$ is the phase advance of the betatron oscillations.

The experimental measurements of chromatic beta-beating are similar to the measurements of chromaticity [Calaga et al.(2010)Calaga, Aiba, Tomas, and Vanbavinckhove]: The RF system is ramped up and down to induce deviations of the energy of the particles from the reference value, and the response of the beta function is then measured by other experimental methods. A fit of the beta function measurements to the known RF energy deviations estimates the chromatic beta-beating of the lattice.

The proposed method in this paper, simply utilizes an initial transverse excitation of the beam, in order to produce coherent betatron oscillations. With knowledge of the chromaticity $Q'_x$ and the synchrotron tune $Q'_s$, the chromatic beta-beating can be inferred from Eq. (13). Note that, if chromaticity is not known, then the knowledge of the RMS energy spread $\sigma_x$ can be used instead. One arrives to this conclusion by substituting Eq. (12) in Eq. (13).

Concerning the current simulations, Eq. (12) is used to estimate the chromatic beta-beating, by measuring the chromatic sidebands with PyNAFF, and by using the model values of the synchrotron tune $Q_s$ and the chromaticity $Q'_z$, where $z = x, y$. By utilizing the conclusions of Sections 3.2.2 a number of turns $N$ equal to 5 synchrotron periods is used for the analysis. It is found that, for both planes, there is no significant change in the estimated chromatic beta-beating with respect to the initial excitation, however for very large excitations (above $5 \sigma_x$ for the horizontal plane, and above $70 \sigma_y$ for the vertical plane) accurate chromatic beta-beating measurements were not possible due to the impact of non-linearities. The weak dependence on the initial excitation can be explained by the fact that in the chromatic beta-beating estimations, only the chromatic sidebands $A_{\pm 1}$ are used, while for the chromaticity measurements, the main amplitude $A_0$ of the beam is needed, which depends strongly on the initial excitation.

The estimated horizontal chromatic beta-beating averaged across $1 \sigma_x, 2 \sigma_x, 3 \sigma_x$ and $4 \sigma_x$ initial excitations is shown in Fig. 9a, while the vertical chromatic beta-beating, averaged across $20 \sigma_y, 30 \sigma_y, 40 \sigma_y$ and $50 \sigma_y$ initial excitations, is shown in Fig. 9b. The light green band around the measurements defines one standard deviation from the average. The response of the model beta functions are obtained by using the PTC module of MAD-X, and it is superimposed on the results.

It is important to note that the proposed method does not differentiate between negative and positive solutions i.e. the sign of the chromatic beta-beating is ambiguous. However, for the current simulations, the phase of the beta-beating wave of the model can be introduced in the estimations of the proposed method, by first estimating absolute chromatic beta-beating with Eq. (13), and then multiplying them with the sign of chromatic beta-beating of the model. This results in two chromatic beta-beating waves, one wave from the response of the model and one from the estimations of the proposed method, that can be easily understood and compared for the current simulations, since they demonstrate similar phases. In the absence of reference measurements from the model, this operation would be of course impossible, and only absolute values of the chromatic beta-beating could be used.

The results from the horizontal plane exhibit a good agreement of with the expected chromatic beta-beating. The maximum chromatic beta-beating from the simulations is around $\frac{\Delta \beta}{\beta} = 10$ for the current simulated KARA optics. The good agreement in the results for the horizontal plane is also explained by the presence of horizontal dispersion $D_x$,
as it is testified from Eq. (19). For the same reason, the results from the vertical plane do not agree as well with the model, since vertical dispersion $D_y$ is zero everywhere. For the vertical chromatic beta-beating, the maximum beating is found at a value of $\Delta \beta_y = 15$. As it can been for the uncertainty of the measurements for both planes, there is quite a good agreement in the results from the current simulated cases. Note that, as expected from theory, the results from the proposed method return a well defined beta-beating wave, with a frequency of almost twice the corresponding betatron tunes.

4 Experimental measurements at KARA

4.1 Proof of concept

The proposed method for estimation of chromaticity through TbT data, is tested experimentally at the KARA light source. The KARA ring is equipped with $M = 35$ BPMs, and a total of $n_b = 110$ bunches can be injected in the ring, with a range of nominal beam currents at $I_b = 90 - 120$ mA. At flat-top energy of operation $E = 2.5$ GeV and for the KARA optics, the synchrotron tune is around $Q_s = 0.012$ and the measured bunch length for the nominal machine settings at KARA is $\sigma_z = 1.3 \cdot 10^{-2}$ m, which can be used to estimate the RMS energy spread at $\sigma_\delta = 8.9 \cdot 10^{-4}$.

The TbT data are generated by the use of the injection kicker, which can induce horizontal betatron oscillations. The vertical excitation is transferred to the beam by virtue of betatron coupling, which will unfortunately result in TbT signals with a much lower Signal-to-Noise Ratio (SNR). The data are gathered from the $M = 35$ BPMs for around $N = 2000$ turns. The kick signal from the magnet lasts for about $N = 8$ turns or about 2.5 ms, and for the current experiment, a range of kicker currents from 100 A to 800 A is used, in order to generate initial excitations of different
amplitudes. A sample of the experimental horizontal TbT data can be visualised in Fig. 10a, for different magnitudes of the kicker current, which give a maximum RMS oscillation amplitude of $\sqrt{\langle x^2 \rangle} = 0.2\,\text{m}$ for kicker current of $I_k = 750\,\text{A}$. The vertical TbT data for the same kicker configurations are shown in Fig. 10b, where the loss of signal purity is evident from the random spikes arising from the low oscillation amplitudes. At a kicker current of $I_k = 750\,\text{A}$, the RMS vertical oscillation amplitude $\sqrt{\langle y^2 \rangle}$ is 10 times smaller than the horizontal one. As a result, larger systematic errors are expected in the vertical chromaticity measurements.

![Figure 10: Experimental TbT data at KARA light source. The horizontal oscillations are presented in (a) and the vertical oscillations in (b). The TbT data are generated by the KARA injection kicker and the current at the kicker for each case is shown in the legend of the graphs.](image)

(a) Horizontal TbT data, after excitation of the injection kicker in a range of currents, which is shown in the legend.

(b) Vertical TbT data, generated from betatron coupling, following the horizontal excitation of the beam.

4.2 Chromaticity measurements

The first chromaticity estimations from TbT data, based on the application of the proposed method for a number of turns equal to 5 synchrotron periods, is shown in Fig. 11, for the horizontal chromaticity $Q'_x$ (top) and vertical chromaticity $Q'_y$ (bottom), with respect to each KARA BPM. The chromaticity estimations from the raw experimental data are shown with blue markers, while the estimations from the same TbT data, but after post-processing with Singular Value Decomposition (SVD) [Wang(1999)] analysis in order to increase SNR, are shown in orange. Both markers correspond to the average value from 10 injections of bunches, and the error-bars represent one standard deviation from the average. In addition, the method is benchmarked against the traditional method for chromaticity measurements that is used at KARA ring i.e. with the RF sweep. More specifically, for the generation of the coherent betatron oscillations, the current at the horizontal injection kicker is set to the nominal value ($I_k = 450\,\text{A}$), the RF frequency is changed over a range of values, and the betatron tune measurements are performed with the bunch-by-bunch (BBB) feed-back system [Hertle et al.(2014)Hertle et al.] feedback system.

For the horizontal chromaticity measurements, it is evident that the proposed method can be used for on-line chromaticity measurements, as the estimations agree very well with the RF sweep method. Even for the raw data the agreement is impressive, while the outliers are absent in the same measurements with the SVD filtered data. The statistical error from 10 consecutive shots is at around 3% while the agreement with the RF sweep method is below 1% for the SVD measurements.
Concerning the vertical chromaticity measurements, it is clear that not all BPMs could yield Fourier spectra with detectable chromatic sidebands. However, for the SVD filtered data, the population of the successful BPMs increases due to the improvement of SNR, yielding a relatively good agreement with the RF sweep results. A systematic error is also present in the measurements, probably coming from the betatron coupling mechanism, which results in the overestimation of the vertical chromaticity for all the BPMs. In any case, the vertical chromaticity is estimated with an accuracy of around 9% for the SVD measurements.

Similar measurements can be employed, by computing the BPM average with respect to the excitation kicker current, in order to quantify the effect of the initial excitation in the TbT chromaticity estimation.

Such a measurement is graphically presented in Fig. 12a, where in the top plot the average horizontal chromaticity from all BPMs is plotted with respect to the kicker current \( I_k \), for a number of turns equal to three (blue), four (orange) and five (green) synchrotron periods. The error-bars correspond to one standard deviation from the average value. The vertical amplitude-dependent chromaticity measurements are shown at the bottom plot for the same number of turns as before and with the same colour code as before.

Clearly, as the kicker current increases, the horizontal chromaticity estimations converge to a value which agrees very well with the measurement provided by the RF sweep method, with five synchrotron periods yielding the most accurate and precise results. More specifically, for the aforementioned number of turns, the horizontal chromaticity measurements converge to the expected value at around \( I_k = 650 \) A, while at \( I_k = 100 \) A the error in accuracy is estimated at around 5%. Note that, when the least number of turns is used, the error between the two methods increases after the current of the kicker is set to \( I_k = 420 \) A.

The impact of the SNR of TbT beam position signal is visible at the bottom plot, where the vertical chromaticity is only reproducible for an excitation kick of \( I_k \geq 425 \) A. The choice of a large number of turns seems to improve the measurement error, which reduces significantly at \( I_k = 800 \) A. The reproducibility of the vertical chromaticity measurements is lower that the horizontal chromaticity measurements, due to the non-existence of a pure vertical kick.

The particular dependence of the accuracy of the proposed method, for an increasing excitation amplitude, is also observed in the simulations of Sec. 3, Fig. 8a and Fig. 8b. As an immediate conclusion, the excitation amplitude should be large enough as to induce chromatic sidebands which can be accurately resolved. Empirical studies show that such a configuration would be an excitation amplitude that allows for around 4 – 5 synchrotron periods, before it reaches 50% of the initial amplitude of the centroid at the BPMs. On the other hand, a limit to the maximum excitation should also exist due to non-linearities, but it was not observed until the maximum current of the horizontal kicker.

4.3 The CLIC Superconducting Wiggler

The KARA light source has been recently selected to commission the new prototype of the Compact Linear Collider (CLIC) [Aicheler et al. (2012)] and the new prototype of the Compact Linear Collider (CLIC) [Bernhard et al. (2016)], which will be used at the Damping Rings [Papaphilippou et al. (2012)] in order to cool-down the electron and positron beams, i.e. reduce the initially large transverse emittances. The basic parameters of the CLIC SC wiggler, which define the linear dynamics, are summarised in Table 2.
Figure 12: Estimation of chromaticity at KARA by using the proposed method, with respect to the current of the excitation kicker. The synchrotron period is $\tau_s = \frac{1}{Q_s} = 90$ turns. The measurements are produced from analysis of several synchrotron periods, which correspond to $3\tau_s$ (blue), $4\tau_s$ (blue) and $5\tau_s$ (green). The error-bars indicate one standard deviation from the average value all the BPMs at KARA. The measurements from the traditional RF sweep method are superimposed with black lines.

Table 2: Basic parameters of the CLIC Superconducting Damping Wiggler prototype, commissioned at the KARA light-source

| Parameter                        | Value |
|----------------------------------|-------|
| Max. on-Axis Magnetic field $B_w$ [T] | 2.90  |
| Period length $\lambda_w$ [mm]   | 51.40 |
| Total length $L$ [m]             | 1.85  |
| Horizontal beta function $\beta_x$ [m] | 18.96 |
| Vertical beta function $\beta_y$ [m] | 2.17  |

Since an insertion device, as powerful as the CLIC SC wiggler, can potentially influence the beam dynamics of the KARA ring, a series of measurement campaigns have been deployed for the characterisation of the linear and non-linear beam dynamics’ response in the presence of the wiggler [Bernhard et al. (2013)Bernhard, Huttel, Peiffer, Bragin, Mezentsev, Syrovatin, Zolotarev, Ferracin, and Schoerling, Gethmann et al. (2017)Gethmann et al., Papash et al. (2018)Papash et al.].

The first set of measurements during the commissioning of the CLIC SC wiggler at KARA, focuses on the evaluation of the transverse betatron tunes and chromaticities for various magnetic fields of the wiggler.

Due to the symmetries of the magnetic field components at the wiggler, it acts as a focusing quadrupole in both planes [Venturini (2003)]. For the present study, the following points from theory are taken into consideration:

1. A vertical focusing component is expected, due to the existence of a non-vanishing longitudinal field component at the wiggler, coupled to the trajectory of the beam (“wiggling”) along the insertion device. The focusing results in vertical betatron tune-shift which depends on the period and the peak field of the wiggler.

2. The horizontal focusing in the wiggler is compensated by feed-downs of the sextupolar components. However, a slight horizontal defocusing might be observed, in the case that the feed-downs are larger than the linear
focusing of the wiggler. A beam which enters the wiggler with a non-vanishing horizontal position, experiences a gradient in the distribution of the magnetic field at the successive poles of the wiggler. The net effect is a slight horizontal deflection, which can be observed as a horizontal tune-shift. This defocusing depends on the period and the peak field of the wiggler.

iii) The previous horizontal defocusing highlights the existence of sextupolar components at the wiggler magnetic field. In addition, the vertical focusing of a wiggler can have contributions from higher-order multipoles as well. As a result, the wiggler is expected to alter the chromaticity of the ring in both planes, albeit the effect will be smaller in the vertical plane. Therefore, chromaticity measurements are also important in the commissioning of a wiggler.

The measurement campaign at KARA consisted of gathering and analysing TbT data, while ramping up the magnetic field of the CLIC SC wiggler, in steps of 0.5 T until the maximum field of 2.9 T is encountered. The analysis is performed with the PyNAFF software [Asvesta et al.(2017)Asvesta, Karastathis, and Zisopoulos], for each of the M = 39 BPMs available at KARA, during the measurements. The stored current during the measurement procedure is around $I_B = 100$ mA. The horizontal excitation is delivered through the horizontal injection kicker, while the vertical excitation is induced by virtue of betatron coupling. The linear chromaticities are set to low enough values as to assert the observation of any sextupolar components due to the ramp-up of the wiggler. In order to assess the contribution of the CLIC SC wiggler on the linear beam dynamics, experimental measurements of the CLIC SC wiggler tune-shift and beta-beating are performed, and the results can be inspected in sections 6.1.3 of the Appendix.

### 4.3.1 RMS Energy spread-shift due to the CLIC Wiggler

The dependence of the RMS energy spread $\sigma_\delta$ in the magnetic field of a wiggler can be expressed as [Wiedemann(1979)]

$$\sigma_\delta^2 = \sigma_0^2 + \frac{L_w}{2\pi\rho_0} \left( \frac{\rho_w}{\rho_0} \right)^3$$

(20)

where $\sigma_0$ is the value of the natural RMS energy spread without the wiggler, $L_w$ is the length of the wiggler, $\rho_0$ is the bending radius of the main dipoles, and $\rho_w$ is the bending radius of the wiggler magnet. The previous relationship depends on the wiggler magnetic field $B_w$ through the conservation of the beam rigidity

$$\rho_w = \frac{B_0\rho_0}{B_w}$$

(21)

where $B_0$ is the magnetic field of main dipoles.

The RMS energy spread $\sigma_\delta$ of the beam is experimentally measured for each step of the CLIC SC wiggler. The synchrotron light diagnostics at KARA allow for the measurement of the RMS bunch length $\sigma_z$ by using a Hamamatsu streak camera [Kehrer et al.(2013)Kehrer et al.]. The RMS energy spread $\sigma_\delta$ is inferred by

$$\sigma_\delta = \frac{Q_s}{a_p R} \sigma_z$$

(22)

where $Q_s$ is the synchrotron tune, $a_p$ is the momentum compaction factor, and $R$ is the radius of the KARA ring. Note that the momentum compaction factor $a_p$ depends also on the magnetic field of the wiggler $B_w$, however the contribution for the CLIC SC wiggler is negligible. The resolution of the streak camera is $\Delta \sigma_z = 1.5$ ps, which defines the error of a single RMS energy spread measurement to be $\Delta \sigma_\delta = 1.4 \cdot 10^{-5}$. For the current value of the RMS energy spread $\sigma_\delta$, the normalised uncertainty of the measurement is around $\Delta \sigma_\delta / \sigma_\delta = 1.6\%$.

For the current experimental measurements at KARA, the ratio $(\sigma_\delta / \sigma_0)^2$ is estimated for wiggler fields of $B_w = 0.0$ T, 1.5 T, 2.0 T and 2.9 T, due to the unavailability of the streak camera for the intermediate steps of $B_w = 0.5$ T, 1.0 T and 2.5 T. In order to estimate the RMS energy spread in the missing steps, a non-linear fit of the available RMS energy spread measurements to Eq. (20) is performed. The results are graphically presented in Fig. [13] where the ratio $(\sigma_\delta / \sigma_0)^2$ is plotted with respect to the CLIC SC wiggler field $B_w$. The experimental measurements are marked with blue, with the error-bars corresponding to the uncertainty of the streak camera measurements, while the aforementioned fit is shown in orange.

From the trend of the measurements, the total increase of the initial RMS energy spread $\sigma_0$ at $B_w = 0.0$ T, is around 20% for the CLIC SC wiggler operating at $B_w = 2.9$ T.
4.3.2 Chromaticity-shift due to the CLIC Wiggler

The first order chromaticity $Q'_{x,y}$ of a circular accelerator with respect to the optics, is given by

$$Q'_{x,y} = \frac{1}{4\pi} \oint_C \beta_{x,y}(s) \left( K_{x,y}(s) \mp S(s) D_x(s) \right) ds,$$

where the integration is performed around the circumference of the ring $C$, $\beta_{x,y}(s)$ is the transverse beta function, $K_{x,y}(s)$ is the quadrupolar focusing gradient, $S(s)$ is the sextupolar focusing gradient, and $D_x(s)$ is the dispersion generated by horizontal bending in the main dipoles. In the case of a wiggler with peak magnetic field of $B_w$, additional quadrupolar and sextupolar gradients are superimposed to the magnetic fields of the ring. Furthermore, the wiggler creates additional dispersion, whose average value $\langle D_w \rangle$ scales linearly with the magnetic field of the wiggler $B_w$ as $\langle D_w \rangle \propto B_w$.

In order to establish the presence of sextupolar components in the location of the CLIC SC wiggler, dedicated chromaticity measurements are performed by recording TbT data for around $N = 2000$ turns, while ramping up the CLIC SC wiggler from 0 T to 2.9 T in steps of 0.5 T. The analysis is performed with PyNAFF by using two methods:

i) **RF-sweep:** During each step, the RF frequency is modulated in order to induce a change in the relative momentum offset of the beam $\delta = \Delta p/p_0$ in the range of $|\delta| \leq 0.5\%$. Chromaticity is estimated from the chromatic response of the betatron tunes for each value of the wiggler field $B_w$.

ii) **Chromatic sidebands:** The suggested method for chromaticity measurements is benchmarked against the RF-sweep method. For this method, chromaticity is inferred by inspecting the Fourier spectra of the beam and applying Eq. (12), where the RMS energy spread $\sigma_\delta$ estimations are known from previous analysis, see Sec. 4.3.1. The measurements of the chromatic sidebands are performed for around 4 to 5 synchrotron periods. It should be mentioned that the analysis of the chromatic sidebands is performed on TbT data that are gathered while the beam is on the nominal chromatic orbit i.e. for $\delta = 0$. For each step of CLIC SC wiggler, three sets of data, where the beam follows the nominal chromatic orbit, are available. This results in fewer statistics than the RF-sweep method, which uses all the available data.

During the initial set-up of the experiment, transverse chromaticity is trimmed to low-enough values ($Q'_{x} \approx 1$, $Q'_{y} \approx 4$) by reducing the strength of the lattice sextupoles, in order to observe the effect of the CLIC wiggler. These values are the lower limit that ensure beam stability, since the BBB feedback is not employed during the measurements to avoid distortion of the Fourier spectra.

The results for the chromaticity measurements with respect to the field of the CLIC SC wiggler $B_w$ are shown in Fig. 14a for the horizontal chromaticity $Q'_x$ and in Fig. 14b for the vertical chromaticity $Q'_y$. The estimations with the

Figure 13: RMS Energy spread measurements (blue markers) from the KARA streak camera, with respect to the magnetic field of CLIC SC wiggler. The error-bars represent one standard deviation from the average. The fit to the theoretical model is shown in orange.
traditional RF-sweep method are shown in orange and they correspond to the average chromaticity, from all the \( M = 39 \) KARA BPMs. The standard error of the mean is below \( 10^{-4} \) for both planes, due to the very good Signal-to-Noise Ratio (SNR) and the excellent reproducibility of the betatron tunes for each bunch injection. The output from the analysis of the chromatic sidebands with the suggested method are shown in blue, where each marker corresponds to the average value from all the BPMs, and for the three available data-sets, while the error-bars correspond to the standard error of the mean, whereas the uncertainty of the RMS energy spread \( \sigma_{\delta} \) measurements has been added in quadrature.

![Figure 14: Experimental measurements of the horizontal (a) and vertical (b) chromaticities by using the proposed method (blue markers) and the standard method of RF sweeping (orange markers). The values from the RF-sweep method correspond to the average from all the 39 BPMs at KARA. The respective measurements of the proposed method quote the average value from all the 39 BPMs and the three available data sets. The error-bars indicate the standard error of the average, where the uncertainty in the RMS energy spread is added in quadrature.](image)

Concerning the horizontal chromaticity, both methods agree very well, within the margin of error, and both signify the presence of sextupolar components in the CLIC SC wiggler, revealing a positive correlation with the magnetic field of the wiggler. The error-bars for the suggested method is of the order of \( 10^{-2} \). The trend of the measurements suggests two different regions of horizontal chromaticity increase: a small increase of about 5\% from 0 T to 1 T, and a more steep, almost linear increase of about 12\% from 1 T to 2.5 T, which slightly drops at the final step of \( B_{w} = 2.9 \) T. The drop is reported from both methods, and it is possible that ramping the wiggler at the maximum field of \( B_{w} = 2.9 \) T, can generate additional non-linearities which can slightly perturb the optics. A similar behaviour, albeit of less significance, is observed in the horizontal betatron tune measurements at the top of Fig. 16, where the measurement at \( B_{w} = 2.9 \) T exhibits a slight focusing with respect to the tunes measurements at lower fields. Note that magnetic quenches for the CLIC SC wiggler prototype have been reported in [Bernhard et al. (2016)] for higher values of the magnetic field \( B_{w} \), limiting the stable operating region to \( 0 \leq B_{w} \leq 2.9 \) T.

As for the vertical plane, there is also a good agreement between the two methods, by considering also the technique for generating the vertical Tbt through coupling, which results in a weak vertical Tbt signal at the BPMs. Note that in general the uncertainty of the proposed method is smaller than the uncertainty in the horizontal plane, due to the larger vertical chromaticity. However, for some steps of the wiggler field \( B_{w} \), the beam measurements were observed to be less reproducible in the vertical plane, probably due to the simultaneous ramp up and down of the RF system, during the continuous ramping of the wiggler.

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In any case, the vertical chromaticity measurements reveal a pattern similar to the one observed in the horizontal chromaticity measurements. A pronounced difference is that from 0 T to 0.5 T both methods report a drop in the chromaticity, which would be expected from the simultaneous increase of the horizontal chromaticity. However, from 1 T to 2.5 T the vertical chromaticity increases, leading to the conclusion that the distribution of the sextupolar components of the CLIC SC wiggler changes with respect the magnetic field. Nevertheless, the effect is very small, for the current optics, and it should not pose a problem for the operation of the KARA ring with the CLIC SC wiggler at maximum field.

4.3.3 Second Order Effects

During the experimental measurements at the KARA light source, the influence of the CLICK SC wiggler on second order chromaticity \( Q''_x \) and chromatic beta-beating \( \Delta \beta_y \) is determined, by employing the proposed method, shown in Eq. 13, for the latter, and by using the RF-sweep method for the former. Similar to the chromaticity measurements in Sec. 4.3.2 the field of the wiggler is ramped up from 0 T to 2.9 T, in steps of 0.5 T. Second order chromaticity is measured by the response of the centroid of the beam to ramping up and down the frequency of the RF cavities, while the chromatic beta-beating is measured after analyzing for the first 5 synchrotron periods the TbT BPM data, for each value of the magnetic field of the CLIC SC wiggler, in order to estimate the amplitude of the chromatic sidebands, \( A_{\pm 1} \).

The values of chromaticity \( Q''_z \), measured with the suggested method, are used in Eq. 13.

Excluding second order dispersion effects, the chromatic beta-beating \( \Delta \beta_y \) and the second order chromaticity \( Q''_x \) are coupled together through the relationship [Luo et al.(2010)Luo, Fischer, Robert-Demolaize, Tepikian, and Trbojevic]:

\[
Q''_x = -\frac{1}{2} Q''_z + \frac{1}{4\pi} \int ds \left[ K_1 + K_2 D_z \right] \Delta \beta_{x,y},
\]

where \( \Delta \beta_{x,y} = \frac{\partial \beta_{x,y}}{\partial B} \) is the chromatic dependence of the horizontal and vertical beta functions, \( K_1, K_2 \) are the integrated magnetic strength of the quadrupoles and sextupoles respectively, and \( D_z \) is the horizontal dispersion along the ring. Assuming that the magnetic strengths are static, one concludes that an increase of the horizontal beta-beating translates in a increase or decrease of the second order chromaticity, according to the plane of reference.

The measurements for the horizontal plane are presented in Fig. 15a, while the measurements for the vertical plane are shown in Fig 15b. Both figures illustrate the evolution of the chromatic beta-beating \( \Delta \beta_y \), measured with the suggested method, on the left axis (green color), while the dependence of the second order chromaticity \( Q''_x \) on the field of the wiggler is shown on the right axis (red). For both measurements and for both planes, the points are the average measurements, across 3 data sets and 39 BPMs of KARA, and the error-bars represent the standard error of the mean.

The normalized error for the horizontal chromatic beta-beating is at the range of \( \sigma_{\Delta \beta_y}/\Delta \beta_y \approx 10 \% \), while for the vertical plane, the same parameter is \( \sigma_{\Delta \beta_y}/\Delta \beta_y \approx 0.1 \% \). The small error in the vertical plane is explained by the fact that the vertical chromaticity is on average 3 times larger than the horizontal one, which results in more resolvable chromatic sidebands. In addition, the vertical data are produced through the betatron coupling mechanism, which means that finite vertical dispersion is generated in the sextupoles and thus chromatic beta-beating is estimated with more confidence. The errors of the second chromaticity measurements are negligible in both planes, due to the very good reproducibility of the TbT data.

The trend of the curves for the horizontal plane, exhibits an increase of the chromatic beta-beating of around 100 % with respect to the nominal value at \( B_w = 0 \) T, followed by an increase of the second order chromaticity for around 4 %. Note that from \( B_w = 1 \) T to \( B_w = 2.5 \) T, chromatic beta-beating appears to slow down in growth, while the last point at \( B_w = 3 \) T can be explained by non-linearities, as explained in Sec. 4.3.2.

The results of the vertical indicate that, vertical chromatic beta-beating is resolved with higher confidence by using the proposed method, and it is found to be also increasing with respect to the CLIC SC magnetic field \( B_w \), with a total change of around 50 \%, which is half the increase of the horizontal chromatic beta-beating \( \Delta \beta_y \). However, the growth in the vertical chromatic beta-beating is followed by a decrease in the vertical second order chromaticity \( Q''_y \) of about 8 \%, two times larger than the change in the horizontal second order chromaticity \( Q''_x \).

Note that the behaviour of second order chromaticity and chromatic beta-beating is fully expected from theory, as it is justified from Eq. 25. An immediate conclusion is that, during experimental measurement, where the reproducibility of the beam is at an adequate level, the chromatic beta-beating can be estimated with the proposed method. Unfortunately, there are no reference measurements in order to compare the results, neither from experiment nor from the model. In any case, the increase of the chromatic beta-beating is another indication of the existence of additional sextupolar fields from the CLIC SC wiggler, which however do not seem to complicate the operation of the KARA ring.
Two simple equations are proved and proposed, which can be used to estimate linear chromaticity and chromatic beta-beating, directly from the Fourier spectra of the TbT beam position of an electron beam, in a similar manner to the procedure of measuring the betatron tunes i.e. with a transverse excitation. Such a possibility would add further flexibility to the effort for continuous on-line measurements and control of the beam optics. By using the same method, the RMS energy spread can be also estimated in a TbT manner.

One of the most important assumptions of the methods is that the distribution of the electron beam is Gaussian, which is almost always the case for electron beams in high-energy circular accelerators, and that the initial excitation of the beam is not too strong as to generate non-linearities which can possibly affect the efficiency of the proposed methods.

Tracking simulations are deployed with the KARA model, where the chromaticity is estimated with the proposed technique. The efficiency of the method is demonstrated, as it shown that chromaticity can indeed be recovered from TbT data. More specifically, the results indicate that two parameters can be used to fine-tune the estimations: the initial excitation amplitude and the number of turns that is used for the frequency analysis. Since decoherence is found to play an important role in the final result, the number of turns should be as small as to allow the generation of synchrotron sidebands in the transverse Fourier spectra. For a powerful frequency analysis tool like NAFF, this is usually achieved in four to six synchrotron periods.

Similar behaviour is found for the chromatic beta-beating measurements, as it is found that it can be fully recovered in the simulations, via the proposed method. For these particular simulations, it is found that the presence of dispersion can
increase the confidence in the measurements. This is to be expected, since dispersion at the location of the sextupoles is one of the main contributors to the generation of chromatic beta-beating.

The method is also deployed in experimental measurements at the KARA light source, where horizontal excitations are applied to the beam, and the vertical TbT data are generated from betatron coupling. The concept of using the chromatic sidebands for chromaticity estimations is demonstrated with success, and the importance of the signal-to-noise ratio in the beam position signal is highlighted. This is also reflected from the fact that the vertical chromaticity measurements, which are not produced by direct excitation, are less precise and accurate than the horizontal ones. Nevertheless, an important outcome is that the reduction of noise with numerical methods such as the SVD, leads to more precise results.

In another series of experiments, the method is utilized for the measurement of chromaticity under the influence of the CLIC Superconducting Damping Wiggler. In addition, the impact of the CLIC wiggler on the KARA beam dynamics is characterised qualitatively and quantitatively. The most important outcome of these measurements, is the demonstration of the existence of non-linear fields at the position of the wiggler. The influence of these fields on chromaticity is presented, by using the traditional RF-sweep method and measurements with the proposed methodology. In both transverse planes, the suggested method is benchmarked with success against the traditional technique, and report a simultaneous increase of both transverse chromaticities. The magnitude of the increase however is not severe, and as a result, it is concluded that the operation of the CLIC wiggler at maximum, does not have an important effect on the KARA beam dynamics.

Moreover, the proposed method is employed in order to estimate chromatic beta-beating and demonstrate its relationship to the wiggler field. At the same time, measurements of the second order chromaticity are performed by using the RF-sweep method. The increase in horizontal chromatic beta-beating is followed by an increase of second order chromaticity. Vertical chromatic beta-beating is found also to be increasing, which leads to the decrease of vertical second order chromaticity. Both chromaticity and chromatic beta-beating measurements with the proposed method, suggest that additional sextupolar fields are generated during operation of the CLIC SC wiggler, which however do not pose a problem for the operation of the KARA ring.

Finally, as a future work, a similar methodology could be also developed for proton machines, where the on-line monitoring and control of chromaticity and/or RMS energy spread is of high importance as well.

6 Appendix A

6.1 Insights from theory

6.1.1 Chromaticity

The linear $Q'$ and non-linear $Q''$ chromaticities are defined as the betatron tune-shift of a single particle, due to a change of the particle’s energy. The energy dependent betatron tune-shift is defined from the well known formula

$$Q(\delta) = Q_0 + Q' \delta + \frac{1}{2} Q'' \delta^2 + O(\delta^3) , \quad (26)$$

where $\delta = \frac{\Delta p}{p_0}$ with $\Delta p$ the deviation from the reference momentum $p_0$, $Q(\delta)$ is the energy dependent betatron tune, and $Q_0$ is the unperturbed betatron tune, defined by the lattice.

6.1.2 Error propagation

By using the formulas:

$$Q'_z = \pm \frac{Q_s}{\sigma_\delta} \sqrt{A_1 + A_{-1}} \quad A_0 \quad A_1 \quad A_{-1} \quad (27)$$

$$\frac{\Delta \beta_z}{\beta_z} = \pm \frac{2 Q'_z}{Q_s} \left| \frac{A_1 - A_{-1}}{A_1 + A_{-1}} \right| \quad (28)$$

where $z=x, y$ the horizontal and vertical planes respectively, $Q'_z$ the chromaticity, $Q_s$ the synchrotron tune, $\sigma_\delta$ the RMS energy spread, $A_0$ the amplitude of the main betatron line, and $A_{\pm 1}$ the chromatic sidebands that appear around $A_0$, one can estimate the chromaticity $Q'_z$ and chromatic beta-beating $\frac{\Delta \beta_z}{\beta_z}$, as it is suggested in this paper. In the following analysis, new symbols are introduced for the chromatic beta-beating and the chromatic ratios as:
\[ \frac{\Delta \beta_z}{\beta_z} = \Delta \beta \]  
\[ \sqrt{\frac{A_1 + A_{-1}}{A_0}} = R_0 \]  
\[ \left| \frac{A_1 - A_{-1}}{A_1 + A_{-1}} \right| = R_1 \]

for reasons of convenience. In addition the index of the chromaticity is dropped and it is thus symbolized as \( Q' \), since the expressions are valid for both planes.

The measurement errors of chromaticity \( \sigma_{Q_s} \) and chromatic beta-beating \( \sigma_{\Delta \beta} \) are simply given by:

\[ \sigma_{Q'}^2 = \left( \frac{\partial Q'}{\partial Q_s} \sigma_{Q_s} \right)^2 + \left( \frac{\partial Q'}{\partial \sigma_\delta} \sigma_{\sigma_\delta} \right)^2 + \left( \frac{\partial Q'}{\partial R_0} \sigma_{R_0} \right)^2 \]  
\[ \sigma_{\Delta \beta}^2 = \left( \frac{\partial \Delta \beta}{\partial Q_s} \sigma_{Q_s} \right)^2 + \left( \frac{\partial \Delta \beta}{\partial \sigma_\delta} \sigma_{\sigma_\delta} \right)^2 + \left( \frac{\partial \Delta \beta}{\partial R_1} \sigma_{R_1} \right)^2 \]  

where \( \sigma_{Q_s} \) is the error in the synchrotron tune \( Q_s \) measurement, \( \sigma_{\sigma_\delta} \) the error in the RMS energy spread \( \sigma_\delta \) measurement, \( \sigma_{R_0} \) the error of the chromatic ratio \( R_0 \), and \( \sigma_{R_1} \) the error of the chromatic ratio \( R_1 \).

By computing the partial derivatives that are present in Eq. (32) as:

\[ \frac{\partial Q'}{\partial Q_s} = \frac{Q'}{Q_s} \]  
\[ \frac{\partial Q'}{\partial \sigma_\delta} = -\frac{Q_s}{\sigma_\delta} \]  
\[ \frac{\partial Q'}{\partial R_0} = \frac{Q_s}{R_0} \]

and continuing with the derivatives in Eq. (33) as:

\[ \frac{\partial \Delta \beta}{\partial Q_s} = -\frac{Q'_1 R_1}{Q_s^2} \]  
\[ \frac{\partial \Delta \beta}{\partial Q'} = \frac{R_1}{Q_s} = \frac{\Delta \beta}{Q_s} \]  
\[ \frac{\partial \Delta \beta}{\partial R_1} = \frac{Q'_1}{Q_s} = \frac{\Delta \beta}{R_1} \]

results in the necessary expressions to calculate Eq. (32) and Eq. (33).

However, an intermediate step is the calculation of the errors of the chromatic sidebands \( \sigma_{R_0} \) and \( \sigma_{R_1} \) which is given by the expressions:

\[ \sigma_{R_0}^2 = \left( \frac{\partial R_0}{\partial A_0} \sigma_{A_0} \right)^2 + \left( \frac{\partial R_0}{\partial A_1} \sigma_{A_1} \right)^2 + \left( \frac{\partial R_0}{\partial A_{-1}} \sigma_{A_{-1}} \right)^2 \]  
\[ \sigma_{R_1}^2 = \left( \frac{\partial R_1}{\partial A_0} \sigma_{A_0} \right)^2 + \left( \frac{\partial R_1}{\partial A_1} \sigma_{A_1} \right)^2 + \left( \frac{\partial R_1}{\partial A_{-1}} \sigma_{A_{-1}} \right)^2 \]

where \( \sigma_{A_0}, \sigma_{A_1} \) and \( \sigma_{A_{-1}} \) are the measurement errors of the amplitudes \( A_0, A_1 \) and \( A_{-1} \) respectively. With no loss of generality, the errors in measuring \( A_1 \) and \( A_{-1} \), which mostly come from the limitations of the BPM system and the noise in the TbT signal, can be considered similar i.e. \( \sigma_{A_{-1}} \approx \sigma_{A_1} \approx \sigma_{A_{\pm 1}} \). By computing the partial derivatives:
\[
\frac{\partial R_0}{A_0} = -\frac{R_0}{2A_0} \\
\frac{\partial R_0}{A_1} = \frac{\partial R_1}{A_{-1}} = \frac{1}{2A_0R_0} \\
\frac{\partial R_1}{A_1} = \frac{2A_{-1}}{(A_1 + A_{-1})^2} \\
\frac{\partial R_1}{A_{-1}} = \frac{2A_1}{(A_1 + A_{-1})^2},
\]

the errors \(\sigma_{R_0}\) and \(\sigma_{R_1}\) in Eq. (40) and Eq. (41) are calculated as:

\[
\sigma_{R_0}^2 = R_0^2 \frac{\sigma_{A_0}^2}{4A_0^2} + \frac{1}{2A_0^2R_0^2} \sigma_{A_{\pm 1}}^2
\]

\[
\sigma_{R_1}^2 = 4 \frac{A_1^2 + A_{-1}^2}{(A_1 + A_{-1})^2} \sigma_{A_{\pm 1}}^2
\]

\[
= 4 \left( \frac{\sigma_{A_{\pm 1}}}{A_1 + A_{-1}} \right)^2 \left[ 1 - \frac{2A_1A_{-1}}{(A_1 + A_{-1})^2} \right].
\]

Concretely, the normalized errors in the chromaticity \(\sigma_{Q'}\), and chromatic beta-beating \(\sigma_{\Delta \beta}\) measurements are found to be:

\[
\left( \frac{\sigma_{Q'}}{Q'} \right)^2 = \left( \frac{\sigma_{Q_s}}{Q_s} \right)^2 + \left( \frac{\sigma_{A_0}}{A_0} \right)^2 + \left( \frac{\sigma_{A_{\pm 1}}}{A_1 + A_{-1}} \right)^2
\]

\[
\left( \frac{\sigma_{\Delta \beta}}{\Delta \beta} \right)^2 = \left( \frac{\sigma_{Q_s}}{Q_s} \right)^2 + \left( \frac{\sigma_{Q'}}{Q'} \right)^2 + 4 \left[ 1 - \frac{2A_1A_{-1}}{(A_1 + A_{-1})^2} \right] \left( \frac{\sigma_{A_{\pm 1}}}{A_1 + A_{-1}} \right)^2.
\]

The quadratic terms in the previous expressions testify that, in the presence of measurement errors of the Fourier amplitudes \(A_0, A_1\) and \(A_{-1}\), the chromaticity error \(\sigma_{Q'}\) increases for a vanishing betatron amplitude \(A_0\) and a small sum of the synchrotron sidebands \(A_1\) and \(A_{-1}\). Since the amplitude of the synchrotron sidebands depends on the quantity \(s = \frac{Q'}{Q_s}\), as it is shown in Sec. 2, therefore it cannot be altered except by changing the beam dynamics parameters, one could experimentally use a sufficiently large excitation in order to increase the \(A_0\) term. However, care has to be taken as to be influenced by non-linearities as less as possible, since they might perplex the results.

Moreover, it is evident that similar in amplitude synchrotron sidebands, \(A_1\) and \(A_{-1}\), penalize the error \(\sigma_{\Delta \beta}\) of the chromatic beta-beating \(\Delta \beta\) measurement, since this would mean that the chromatic beta-beating is itself very small. Finally, as it is also found for the chromaticity measurement error, the error \(\sigma_{\Delta \beta}\) of the chromatic beta-beating increases for a small sum of amplitudes \(A_1\) and \(A_{-1}\).

### 6.1.3 Impact of a wiggler on optics

The 3-D Hamiltonian that describes the motion of an on-momentum particle in the field of a wiggler, with sinusoidal field variation, and expanded up to fourth order is [Gao(2003)].
The presence of a wiggler in a circular accelerator induces perturbation to the linear optics in the form of beta-beating due to the wiggler generated quadrupolar error, Eq. \((52)\). In theory, only vertical perturbations are allowed in the ideal case. The theoretical average vertical beta-beating \(\Delta \beta_y^W\) due to the presence of the wiggler is \([\text{Walker}(1994)]\)

\[
\frac{\langle \Delta \beta_y \rangle}{\beta_y} = \frac{2\pi \Delta Q_y}{\sin(2\pi Q_y)} ,
\]

where \(Q_y\) is the unperturbed betatron tune, and \(\Delta Q_y\) is the tune shift due to the finite wiggler field, given in Eq. \((54)\).

7 Appendix B

7.1 Optics measurements at KARA

In the following sections, the results from optics measurements at the KARA ring, under the influence of the CLIC SC wiggler are presented in order to establish quantitatively and qualitatively the impact of the wiggler on the linear beam.
dynamics at KARA. The agreement between experiment and theory is also examined, in order to validate the efficiency of the CLIC SC wiggler models that are currently used.

7.1.1 Tune-shift due to the CLIC Wiggler

For the KARA ring and the CLIC SC wiggler, the expected vertical betatron tune-shift $\Delta Q_y(B_w)$, by virtue of Eq. (55) is calculated to be

$$\Delta Q_y(B_w) = 2.6 \cdot 10^{-3} B_w^2,$$  \hspace{1cm} (56)

which is indeed small due to the small vertical beta function at the position of the CLIC SC wiggler. In order to experimentally measure the betatron tune-shift, tune measurements are performed with the mixed BPM method [Zisopoulos et al. (2019) Zisopoulos, Papaphilippou, and Laskar] for around $N = 50$ turns. The results are presented in Fig. 16, for the horizontal (top) and vertical (bottom) tunes with respect to the increasing magnetic field $B_w$ of the CLIC SC wiggler.

![Figure 16: Betatron tune-shift in the horizontal (top) and vertical (bottom) planes at KARA, as a function of the magnetic field of the CLIC SC wiggler. The fits are performed with quadratic models. For the vertical plane, the theoretical tune-shift is shown in black.](image)

For the evolution of the horizontal tune, a slight shift is observed, with a magnitude that is estimated with a fit of the measured tunes (blue curve) to Eq. (53), (orange curve). Although the observed tune-shift, is not dangerous for the operation and beam quality at KARA, it proves the existence of non-linear fields at the location of the CLIC SC wiggler.

Concerning the vertical tune-shift, a similar fit is performed (black curve) in order to estimate its magnitude, which is found similar to the theoretical expectations (blue line), quoted in Eq. (56).

The tune measurements reveal a total normalised tune-shift of around $\Delta Q_x/Q_x = 0.5\%$ for the horizontal plane, and roughly $\Delta Q_y/Q_y = 2\%$, for the wiggler at maximum field i.e. $B_w = 2.9$ T.

7.1.2 Beta-beating due to the CLIC Wiggler

For the current experimental measurements with the operation of the CLIC SC wiggler, TbT data are recorded at the $M = 39$ BPMs, while ramping up the CLIC SC wiggler in a series of steps, from $0$ T to $2.9$ T. The estimations of the beta-beating are performed by using information on the Fourier amplitudes of the oscillations [Zisopoulos et al. (2013) Zisopoulos, Papaphilippou, Streun, and Ziemann], measured with Pynaff, for no wiggler field ($B_w = 0$ T) and maximum wiggler field ($B_w = 2.9$ T).

In order to estimate the beta-beating at the maximum field, first the models are set-up. The CLIC SC wiggler is inserted in the KARA model in ELEGANT [Borland (2000)] tracking code, and the response of the beta function is recorded for both values of the field. The beta-beating of the model due to the maximum field of the wiggler from the model can be calculated as

$$\frac{\Delta \beta^\text{model}}{\beta} = \frac{\beta^m W - \beta^m_0}{\beta^m_0} \bigg|_{W=2.9 \ T},$$  \hspace{1cm} (57)

27
where $\beta_m^{0\psi}$ is the value of the model beta-function for $B_w = 0$ T, and $\beta_m^{W\psi}$ is the value of the model beta-function for $B_w = 2.9$ T.

In order to compare with the experimental estimation of the beta-beating generated purely by the wiggler, one needs first to disentangle the baseline beta-beating, which originates from the quadrupolar errors in the lattice. The amplitudes of the beam for both cases of $B_w = 0$ T and $B_w = 2.9$ T are fitted with the model’s beta functions at $B_w = 0$ T. From this operation, two sets of model dependent, but experimentally measured, beta functions become available, at both fields. Then the experimental beta-beating is found as the difference of the two sets of beta functions, in a manner similar to Eq. (57). The difference cancels out the term of the beta-beating due to the quadrupolar errors of the lattice.

The previous considerations can be visualized in Fig. 17a for the horizontal plane, and in Fig. 17b for the vertical plane, where the estimated wiggler dependent beta-beating is plotted with respect to the azimuthal position of the BPMs. In both plots, the position of the CLIC SC wiggler is marked with a black line.

Concerning the vertical beta-beating, the agreement between the model (green curve) and the experimental measurements (red curve) is very good. Some deviations are present but they are attributed to the calibration of the BPMs. The experimentally measured vertical beta-beating is less than 15%, with an average value of 12% in agreement to the theoretical predictions of Eq. (55). As expected, due to the small value of the vertical beta-function at the position of the CLIC SC wiggler constraints the vertical beta-beating in relatively low values, which are of no concern for beam stability at KARA.
References

[Verdier(1993)] A. Verdier, Chromaticity, in Advanced accelerator physics. Proceedings, 5th Course of the CERN Accelerator School, Rhodes, Greece, September 20-October 1, 1993. Vol. 1, 2 (1993) pp. 77–100.

[Steinhagen(2009)] R. J. Steinhagen, Tune and chromaticity diagnostics 10.5170/CERN-2009-005.317 (2009).

[Minty and Zimmermann(2003)] M. G. Minty and F. Zimmermann, Measurement and control of charged particle beams. Particle acceleration and detection (Berlin, Springer, 2003).

[Serio(1989)] M. Serio, Transverse betatron tune measurements, in Frontiers of Particle Beams: Observation, Diagnosis and Correction, edited by M. Month and S. Turner (Springer Berlin Heidelberg, Berlin, Heidelberg, 1989) pp. 65–93.

[Bartolini et al.(1996)] Bartolini, Giovannozzi, Scandale, Bazzani, and Todesco R. Bartolini, M. Giovannozzi, W. Scandale, A. Bazzani, and E. Todesco, Precise measurement of the betatron tune, Single Particle Effects in Large Hadron Colliders. Proceedings, 2nd International Workshop (LHC95): Montreux, Switzerland, October 15-21, 1995, Part. Accel. 55, 1 (1996).

[Boussard(1989)] D. Boussard, Schottky noise and beam transfer function diagnostics 10.5170/CERN-1989-001.90 (1989).

[Betz et al.(2017)] Betz, Jones, Lefevre, and Wendt M. Betz, O. R. Jones, T. Lefevre, and M. Wendt, Bunched-beam Schottky monitoring in the LHC. Nucl. Instrum. Methods Phys. Res., A 874, 113 (2017)

[Cocq et al.(1998)] Cocq, Jones, and Schmickler D. Cocq, O. R. Jones, and H. Schmickler, The Measurement of chromaticity via a head - tail phase shift, Beam instrumentation. Proceedings, 8th Workshop, BIW’98, Stanford, USA, May 4-7, 1998, AIP Conf. Proc. 451, 281 (1998).

[Fartoukh and Jones(2002)] S. Fartoukh and R. Jones, Determination of chromaticity by the measurement of head-tail phase shifts: Simulations, results from the SPS and a robustness study for the LHC, (2002).

[Catalan-Lasheras et al.(2003)] Catalan-Lasheras, Fartoukh, and Jones N. Catalan-Lasheras, S. D. Fartoukh, and R. Jones, Recent Advances in the Measurement of Chromaticity via Head-Tail Phase Shift Analysis, , 4 p (2003).

[Ranjbar et al.(2017)] Ranjbar, Marusic, and Minty V. Ranjbar, A. Marusic, and M. Minty, Review of Chromaticity Measurement Approaches Using Head-Tail Phase Shift Method at RHIC, in Proceedings, 5th International Beam Instrumentation Conference (IBIC 2016): Barcelona, Spain, September 11-15, 2016 (2017) p. TUPG49.

[Meller et al.(1987)] Meller, Chao, Peterson, Peggs, and Furman R. E. Meller, A. W. Chao, J. M. Peterson, S. G. Peggs, and M. Furman, Decoherence of Kicked Beams, Tech. Rep. (1987).

[Lee(1991)] S. Y. Lee, Decoherence of the Kicked Beams II (1991).

[Siemann(1989)] R. H. Siemann, Bunched beam diagnostics, Physics of particle accelerators. Proceedings, Fermilab Summer School, Batavia, USA, July 20-August 14, 1987. Proceedings, Cornell Summer School, Ithaca, USA, August 1-12, 1988. Vol. 1, 2, AIP Conf. Proc. 184, 430 (1989)

[Kubo et al.(2005)] Kubo, Mtingwa, and Wolski K. Kubo, S. K. Mtingwa, and A. Wolski, Intra-beam scattering formula for high energy beams, Phys. Rev. ST Accel. Beams 8, 081001 (2005)

[Bane et al.(2002)] Bane, Hayano, Kubo, Naito, Okugi, and Urakawa [K. L. F. Bane, H. Hayano, K. Kubo, T. Naito, T. Okugi, and J. Urakawa, Intra-beam scattering analysis of measurements at KEK’s ATF damping ring, Phys. Rev. ST Accel. Beams 5, 084403 (2002) arXiv:physics/0206003 [physics]].

[Antoniou(2012)] F. Antoniou, Optics design of Intra-beam Scattering dominated damping rings, Ph.D. thesis, CERN (2012).

[Hsu(1990)] I. C. Hsu, The Decoherence and recoherence of the betatron oscillation signal and an application, Part. Accel. 34, 43 (1990).

[Bassi et al.(2015)] Bassi, Blednykh, Choi, and Smaluk G. Bassi, A. Blednykh, J. Choi, and V. Smaluk, Decoherence due to Second Order Chromaticity in the NSLS-II Storage Ring, in Proceedings, 6th International Particle Accelerator Conference (IPAC 2015): Richmond, Virginia, USA, May 3-8, 2015 (2015) p. MOPMN016.

[Manukyan et al.(2011)] Manukyan, Sargsyan, Amatuni, and Tsakanov K. Manukyan, A. Sargsyan, G. Amatuni, and V. Tsakanov, Energy spread extraction and processing from the kicked beam centroid motion in electron storage rings, JINST 6, T10003.

[Rumolo et al.(2004)] Rumolo, Schmidt, and Tomas G. Rumolo, F. Schmidt, and R. Tomas, Decoherence of a longitudinally kicked beam with chromaticity, Nucl. Instrum. Meth. A 528, 670 (2004).
[Laskar et al. (1992)] J. Laskar, C. Froeschle, and A. Celletti, The measure of chaos by the numerical analysis of the fundamental frequencies. Application to the standard mapping. *Physica D*, **56**, 253 (1992).

[Laskar (1993)] J. Laskar, Frequency analysis for multi-dimensional systems. Global dynamics and diffusion. *Physica D: Nonlinear Phenomena*, **67**, 257 (1993).

[Laskar (2003)] J. Laskar, Frequency map analysis and quasiperiodic decompositions, ArXiv Mathematics e-prints (2003), arXiv:math/0305364.

[Papaphilippou (2014)] Y. Papaphilippou, Detecting chaos in particle accelerators through the frequency map analysis method. *Chaos*, **24**, 024412 (2014), arXiv:1406.1545 [nlin.CD].

[Zisopoulos et al. (2014)] P. Zisopoulos, F. Antoniou, Y. Papaphilippou, A. Streun, and V. Ziemann, Frequency Maps Analysis of Tracking and Experimental Data for the SLS Storage Ring, in *Proc. 5th International Particle Accelerator Conference (IPAC'14), Dresden, Germany, June 15-20, 2014* International Particle Accelerator Conference No. 5 (JACoW, Geneva, Switzerland, 2014) pp. 3056–3058, https://doi.org/10.18429/JACoW-IPAC2014-THPR0076.

[Nakamura (1999)] T. Nakamura, Excitation of betatron oscillation under finite chromaticity, (1999).

[Nakamura et al. (1999)] T. Nakamura, S. Takano, M. Masaki, K. Soutome, K. Kumagai, T. Ohshima, and K. Tsumaki, Energy spread measurement of the SPring-8 storage ring with chromatic sideband peak height of betatron oscillation spectrum, in *Accelerator science and technology. 12th Symposium, SAST'99, Wako, Japan, October 27-29, 1999* pp. 537–539.

[Kiselev et al. (2007)] V. A. Kiselev, N. Y. Muchnoi, O. I. Meshkov, V. V. Smaluk, V. N. Zhilich, and A. N. Zhuravlev, Beam energy spread measurement at the VEPP-4M electron-positron collider, *JINST*, **2**, P06001.

[Rehm et al. (2010)] G. Rehm, M. Abbott, A. Morgan, J. Rowland, and I. Uzun, Measurement of Lattice Parameters Without Visible Disturbance to User Beam at Diamond Light Source, in *Proceedings, 14th Beam Instrumentation Workshop (BIW 2010): Santa Fe, New Mexico, USA, May 2-6, 2010* (2010) p. MOCNB01.

[Bernhard et al. (2016)] A. Bernhard et al., A CLIC Damping Wiggler Prototype at ANKA: Commissioning and Preparations for a Beam Dynamics Experimental Program, in *Proceedings, 7th International Particle Accelerator Conference (IPAC 2016): Busan, Korea, May 8-13, 2016* (2016) p. WEPMW002.

[Fartoukh (1999)] S. Fartoukh, Second order chromaticity correction of LHC V6.0 at collision, (1999).

[Oppenheim et al. (1997)] A. V. Oppenheim, S. H. Nawab, and A. S. Willsky, *Signals and systems; 2nd ed.*, Prentice-Hall signal processing (Prentice-Hall, London, 1997) with exercises.

[Abramowitz and Stegun (1965)] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions* (Dover Publications, 1965).

[Schmidt et al. (2002)] F. Schmidt, E. Forest, and E. McIntosh, Introduction to the polymorphic tracking code: Fibre bundles, polymorphic taylor types and exact tracking, (2002).

[Asvesta et al. (2017)] F. Asvesta, N. Karastathis, and P. Zisopoulos, PyNAFF: A (C)Python module that implements NAFF, https://github.com/nkarast/PyNAFF (2017).

[Laskar (2003)] J. Laskar, Frequency Map Analysis and Particle Accelerators, *Particle accelerator. Proceedings, Conference, PAC 2003, Portland, USA, May 12-16, 2003*, Conf. Proc. *C030512*, 378 (2003).

[Calaga et al. (2010)] R. Calaga, M. Aiba, R. Tomas, and G. Vanbavinckhove, Linear and Chromatic Optics Measurements at RHIC, Conf. Proc. *C100523*, THPE053 (2010).

[Wang (1999)] C.-x. Wang, *Model independent analysis of beam centroid dynamics in accelerators*, Ph.D. thesis, Stanford U., Phys. Dept. (1999).

[Hertle et al. (2014)] E. Hertle et al., First Results of the New Bunch-by-bunch Feedback System at ANKA, in *Proc. 5th International Particle Accelerator Conference (IPAC’14), Dresden, Germany, June 15-20, 2014* International Particle Accelerator Conference No. 5 (JACoW, Geneva, Switzerland, 2014) pp. 1739–1741, https://doi.org/10.18429/JACoW-IPAC2014-TUPRI074.

[Aicheler et al. (2012)] M. Aicheler, P. Burrows, M. Draper, T. Garvey, K. Lebrun, N. Phinney, H. Schmickler, D. Schulte, and N. Toge, *A Multi-TeV Linear Collider Based on CLIC Technology: CLIC Conceptual Design Report*, CERN Yellow Reports: Monographs (CERN, Geneva, 2012).
[Papaphilippou et al. (2012)] Papaphilippou, Antoniou, Barnes, Calatroni, Chiggiato, Corsini, Grudiev, Koukovini, Lefevre, Martini, Modena, Mounet, Perin, Renier, Russenschuck, Rumolo, Schoerling, Schulte, Schmickler, Taborelli, Vandoni, Zimmermann, Zisopoulos, Boland, Palmer, Bragin, Levichev, Syniatkin, Zolotarev, Vobly, Korostelev, Vivoli, Belver-Aguilar, Faus-Golfe, Rinolfi, Bernhard, Pivi, Smith, Rassool, and Wootton, Conceptual Design of the CLIC Damping Rings, Conf. Proc. C1205201, TUPPC086. 3 p (2012).

[Bernhard et al. (2013)] Bernhard, Huttel, Peiffer, Bragin, Mezentsev, Syrovatin, Zolotarev, Ferracin, and Schoerling, Preparations for Beam Tests of a CLIC Damping Wiggler Prototype at ANKA, in Proceedings, 4th International Particle Accelerator Conference (IPAC 2013): Shanghai, China, May 12-17, 2013 (2013) p. TUPMOD05.

[Gethmann et al. (2017)] J. Gethmann et al., Non-Linear Beam Dynamics Studies of the CLIC Damping Wiggler Prototype, in Proc. of International Particle Accelerator Conference (IPAC’17), Copenhagen, Denmark, 14-19 May, 2017, International Particle Accelerator Conference No. 8 (JACoW, Geneva, Switzerland, 2017) pp. 3087–3090, https://doi.org/10.18429/JACoW-IPAC2017-WEPIK068.

[Papash et al. (2018)] A. Papash et al., Non-Linear Optics and Low Alpha Operation at the Storage Ring KARA at KIT, in Proc. 9th International Particle Accelerator Conference (IPAC’18), Vancouver, BC, Canada, April 29-May 4, 2018, International Particle Accelerator Conference No. 9 (JACoW Publishing, Geneva, Switzerland, 2018) pp. 4235–4238, https://doi.org/10.18429/JACoW-IPAC2018-THPMF070.

[Venturini (2003)] M. Venturini, Effect of wiggler insertions on the single-particle dynamics of the NLC Main Damping Rings, 10.2172/827950 (2003).

[Wiedemann (1979)] H. Wiedemann, WIGGLERS, BEAM EMITTANCE AND ENERGY SPREAD, SLAC-PEP-0319, (1979).

[Kehrer et al. (2015)] B. Kehrer et al., Visible Light Diagnostics at the ANKA Storage Ring, in Proc. 6th International Particle Accelerator Conference (IPAC’15), Richmond, VA, USA, May 3-8, 2015, International Particle Accelerator Conference No. 6 (JACoW, Geneva, Switzerland, 2015) pp. 866–868, https://doi.org/10.18429/JACoW-IPAC2015-MOPHA037.

[Walker (1994)] R. P. Walker, Wugglers, in Advanced accelerator physics. Proceedings, 5th Course of the CERN Accelerator School, Rhodes, Greece, September 20-October 1, 1993. Vol. 1, 2 (1994) pp. 807–835.

[Luo et al. (2010)] Y. Luo, W. Fischer, G. Robert-Demolaize, S. Tepikian, and D. Trbojevic, Sorting Chromatic Sectupoles for Second Order Chromaticity Correction in the RHIC, Proceedings, 1st International Particle Accelerator Conference (IPAC’10): Kyoto, Japan, May 23-28, 2010, Conf. Proc. C100525, THPE103 (2010).

[Gao (2003)] J. Gao, Analytical estimation of dynamic apertures limited by the wigglers in storage rings, Particle accelerator. Proceedings, Conference, PAC 2003, Portland, USA, May 12-16, 2003, Conf. Proc. C030512, 3267 (2003) [Nucl. Instrum. Meth.A516,243(2004)].

[Zisopoulos et al. (2019)] P. Zisopoulos, Y. Papaphilippou, and J. Laskar, Refined betatron tune measurements by mixing beam position data, Phys. Rev. Accel. Beams 22, 071002 (2019).

[Zisopoulos et al. (2013)] P. Zisopoulos, Y. Papaphilippou, A. Streun, and V. Ziemann, Beam Optics Measurements through Turn by turn Beam Position Data in the SLS, in Proceedings, 4th International Particle Accelerator Conference (IPAC 2013): Shanghai, China, May 12-17, 2013 (2013) p. WEPEA067.

[Borland (2000)] M. Borland, elegant: A Flexible SDDS-Compliant Code for Accelerator Simulation, in 6th International Computational Accelerator Physics Conference (ICAP 2000) Darmstadt, Germany, September 11-14, 2000 (2000).