Lack of correlation-length scaling for an array of boxes

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Abstract. Finite-size scaling theory predicts that uniformly small critical systems that have the same dimensionality and belong to the same universality class will scale as a function of the ratio of the spatial length $L$ to the correlation length $\xi$. This should occur for all temperatures within the critical region. Measurements of the heat capacity of liquid $^4$He confined to a two-dimensional (2D) planar geometry agree well with this prediction when the $^4$He is normal but disagree near the specific heat maximum where the confined $^4$He becomes superfluid. Data for $^4$He confined to 1D structures show a similar behavior (however the lack of data collapse is not as dramatic). Recent measurements of the heat capacity from two 0D confinements, which differ by a factor of two in size, fail to scale at any temperature within the critical region. This lack of scaling may be due to the interaction of neighboring boxes through the shallow channels used to fill them. This is quite surprising since the liquid in the channels is not superfluid at the temperatures of interest for the helium in the boxes. Furthermore, measurements of the superfluid density of the helium within the channels reveal a critical temperature that is higher than expected suggesting that the normal fluid is affected by the already superfluid regions at each end of these channels. Both of these anomalies might be explained by a proximity effect analogous to what is seen when normal metals are sandwiched between two superconductors.

Several measurements have been made of the heat capacity of liquid $^4$He confined to small geometries. When confined to uniform planar (2D) geometries these measurements show remarkable agreement with finite-size scaling theory above the superfluid transition temperature $T_\lambda$, and above the temperature of the specific heat maximum $T_{max}$ [1]. This is also true for $^4$He confined to channel or pore geometries (1D) [2; 3]. However, measurements on $^4$He confined to small boxes (0D) disagree with scaling theory throughout the critical region. Figure 1 shows specific heat data from two 0D confinements. These data cross at a reduced temperature $t = |T - T_\lambda|/T_\lambda = 3 \times 10^{-5}$ below the transition, a clear indication that when cast in scaling form, following a prescription discussed in [4], these data will not collapse. This is shown explicitly in the plots of figure 2.

Further, comparison of the 0D data with the 1D and 2D data shows other systematic differences. When the magnitude of the specific heat maximum $C_{max}$ is plotted vs. confinement size $L$ (see figure 3), the 0D data are shown to deviate from trends seen in the other two data sets. It would seem that either the $C_{max}$ of the $(1 \mu\text{m})^3$ confinement is inflated, the $C_{max}$ of the $(2 \mu\text{m})^3$ confinement is lowered, or some combination thereof.

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Figure 1. Specific heat of $^4$He confined in all three dimensions plotted vs. reduced temperature $t = |T - T_\lambda|/T_\lambda$. Two data sets are shown, one of $^4$He confined to (1 $\mu$m)$^3$ boxes, and the other in (2 $\mu$m)$^3$ boxes.

Figure 2. Scaling plots of the data from figure 1 both above and below $T_\lambda$. Clearly the data do not collapse on either side of the transition.

To determine possible causes for the lack of scaling, one might reconsider the assumption that these measurements represent $^4$He in isolated boxes. This assumption seems reasonable when comparing these data with those for 2D and 1D crossover [6]. This also seems appropriate on the basis that the $^4$He confined to the connecting channels in the 0D cells remains normal throughout the temperature range of interest [7; 8]. Also, all of these data were taken using the same procedure as outlined in [9]. The only difference in the two 0D cells, apart from their dimensions, is in the details of the filling channels. The (1 $\mu$m)$^3$ boxes are connected with 1 $\mu$m long, 18.5 nm tall channels, while the (2 $\mu$m)$^3$ boxes are connected with 1 $\mu$m long, 10 nm tall channels. If we re-evaluate the assumption of isolated boxes and consider the possibility of neighboring boxes coupling through these filling channels, one would expect this coupling to be stronger for the (1 $\mu$m)$^3$ boxes since these are twice as close and connected by channels twice as tall as the (2 $\mu$m)$^3$ boxes. In the limit of strong coupling, where an array of boxes would behave as a 2D system, one would expect an increase in $C_{\text{max}}$ over the value for truly isolated boxes. This then could be an explanation for the behavior of $C_{\text{max}}$ seen in figure 3. Also, if the (1 $\mu$m)$^3$ data are indeed influenced more strongly by coupling then the (2 $\mu$m)$^3$ data, then the data for
Figure 3. $C_{\text{max}}$ for various confinement sizes in all three confinement geometries. The solid lines are fits of the 2D and 1D data to the power law $L^{\alpha/\nu}$. The 0D data deviate from trends seen in the other two geometries, showing less variation in $C_{\text{max}}$ with varying $L$. Plot taken from [5], see also references therein, and discussion of the solid lines.

truly isolated $(1 \, \mu m)^3$ boxes in the entire region of the maximum would be lowered relative to the data for truly isolated $(2 \, \mu m)^3$ boxes. This would shift the data in the right direction to recover scaling.

One could also argue that the lack of scaling for the 0D data is caused by a contribution to the specific heat from the filling channels rather than a coupling between boxes. If one approximates the $^4$He in the filling channels as a 2D film\(^2\), the specific heat can be determined from the scaled planar data in [9]. This specific heat is then used to calculate the heat capacity of the $^4$He in the filling channels. When this heat capacity is subtracted from the measured heat capacity of the 0D confinement the resultant change in specific heat is on the order of 1% (compare figures 1 and 4). This is not enough to explain the behavior seen in figure 3 or, more importantly, the overall lack of scaling shown in figure 2.

Evidence of coupling can also be seen in data on the superfluid fraction of $^4$He confined in the filling channels. This has been obtained in a separate measurement on the same cell as the heat capacity [8]. The superfluid onset in the short filling channels that link the $(2 \, \mu m)^3$ boxes can be compared with that of the very long 1 mm channels studied in [10]. One can see that the onset in the filling channels takes place at a much higher temperature than expected on the basis of the very long channel data [5]. The same is also true for the onset in the filling channels of the $(1 \, \mu m)^3$ cell [7].

In summary, the increased magnitude of $C_{\text{max}}$ in the $(1 \, \mu m)^3$ 0D confinement and the increased $T_c$ in the filling channels both suggest some type of proximity effect existing between connected regions of superfluid and normal liquid $^4$He. Thus, properties of superfluid $^4$He are altered when in contact with regions of normal fluid and vice versa. A systematic study of these effects is in progress.

Acknowledgments

We want to thank the National Science Foundation for its support, DMR-0605716, and the Cornell NanoScale Science and Technology Facility, project 526-94.

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\(^2\) We note that this ‘film’ in the channels has an aspect ratio, length\times width\times height, of $54\times 54\times 1$ and $200\times 200\times 1$ for the $(1 \, \mu m)^3$ and $(2 \, \mu m)^3$ respectively.
Figure 4. The specific heat data from the (1 $\mu$m)$^3$ and (2 $\mu$m)$^3$ confinements minus the estimated contribution from the filling channels. The data still cross and will not scale anywhere in the critical region.

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