Analysis of Traffic Processing Efficiency Based on Cellular Automaton Model

Jianjun Wang¹,a, Gaozhan Wang²,b, Xingqian Li³,c, Hongru Zhao⁴,d and Hongwei Zhao⁵,e

¹,³,⁵College of Computer Science and Technology, Jilin University, Changchun 130012, China
²,⁴Mathematics School & Institute of Jilin University, Changchun, Jilin University, Changchun 130012, China

a616925652@qq.com, b531857050@qq.com, cSisyphusli@foxmail.com, d591236801@qq.com, ezhaohw@jlu.edu.cn

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Abstract. To evaluate the traffic on a straight road, we analyze the driving behavior of self-driving cars and human-driving cars from the micro perspective. Based on cellular automaton model, we simulate two sorts of cellular by using two kinds of cars. Moreover, basic driving rules and peculiar lane-changing rules of these cars are designed. According to a one-day count of traffic on one route in Washington, the mixed β distribution function with the method of least square estimate can be obtained. We simulate the incoming of vehicles then the cellular generation is acquired. Similarly, we simulate the outgoing of vehicles and gain the cellular output. Output-to-generation ratio is defined as the traffic processing efficiency, which is the most essential index in evaluating the traffic capacity. Two traffic count’s tipping points (high, low) merely related to the number of lanes could be calculated. The traffic processing efficiency is deteriorated sharply when the traffic count outnumbers the high tipping point. On the contrary, the traffic processing efficiency is improved to 100% when the traffic count is inferior to the low tipping point. The equilibria for the percentage of self-driving cars exists when the traffic count ranges between the two tipping points. Then we draw a conclusion that there should not be lanes dedicated to self-driving cars after analyzing the traffic processing efficiency and the critical value of different number of lanes.

1. Introduction

In America, the proportion of people with driver license is relatively high[1]. There were about two hundred million drivers in 2007, which constitute 68 percent of the overall population[2]. The United States owns the most vehicles in the world[3]. About fifteen million cars have been produced since 1998. Besides, the usage of the vehicles is quite common now[4]. More than one billion people commute by car each day. And the overall vehicle-miles of travel are as high as four thousand billion kilometers. The following is the bar graph about the motorization in American highways.

Such high rate of vehicle usage has inevitably caused huge pressure on the traffic safety and traffic capacity[5]. In particular, the commuters in Seattle spend 49 percent longer time on worst driving hours on Thursday than on normal driving hours[6]. The I-405 southbound between Exit 14 and Exit 10 witness the slowest traffic flow[7]. It takes 13 minutes during rush hour while it only takes 5 minutes during light traffic to complete the 4.5 mile trip.

First, we want to build a micro model with two driving rules for human-driving cars and self-driving cars[8]. Second, we want to obtain an index evaluating the traffic capacity by simulating the incoming and outgoing vehicles[9]. Then get the solutions to the problems mentioned.

The actual operating process could be described as follows in Figure 1.
2. The Construction of Traffic Model Based on Cellular Automaton

2.1 The Analysis of Traffic Flow and Its Parameter

HDDL rules, Common lane-changing rules, SCDL rules, The cooperative lane-changing performance occurs to two self-driving, cooperating cars, based on the fact that its basic lane-changing rules are the same as those common lane-changing rules.

Following the NS rule and cellular automaton, we choose Volume of traffic, Average car speed, Vehicle density as the most suitable describing parameters.

- **Volume of traffic**: the number of vehicles traversing the cross section per unit time.
  \[ V = \frac{\sum V_i}{n} \]  
  \( n \) denotes the number of vehicles and \( V_i \) denotes the actual instantaneous speed of the \( i \)th vehicle.

- **Average car speed**: 
  \[ \bar{V} = \frac{\sum V_i}{n} \]

- **Vehicle density**: the number of vehicles per unit length.
  \[ \rho = \frac{n}{L} \]
  \( L \) denotes the length of the road

2.2 The Construction of Vehicle's Driving Model

As is described in NS rule, \( V_j \) and \( X_j \) denote the location and speed of the \( j \)th vehicle respectively. \( V_{max} \) denotes the maximum speed. \( Acc \) denotes the amount of speed increase per unit time. \( Dec \) denotes the amount of speed decrease per unit time. \( d \) denotes the distance between two positions.

- **Acceleration rules**: 
  \[ V_j(t+1) = \min(v_j(t) + acc_j, V_{max}) \]  
  In formula (3), we suppose that the car always tries to arrive at the destination as soon as possible.

- **Deceleration rules**: 
  \[ V_j(t+1) = \min(v_j(t+1), d_{j-1,j}) \]  
  The car need to keep a certain distance from the neighboring cars for safety concern.

Randomization rules:
\[ V_j(t+1) = \max(v_j(t) - dec_j, 0) \]  

(5)

The formula (5) depicts the random deceleration of a certain car. This is usually not caused by the blocking of the cars ahead but by the nonproficiency of the driver, the fatigue of the driver or the disturbing illumination effect. \( P_{dec} \) showsthe probability of such situation, which is defined as randomization probability. Rand is the random number, ranging from 0 to 1.

Position updating rules:
\[ X_j(t+1) = X_j(t) + V_j(t+1) \]  

(6)

The position in the next moment is affected by its present position and speed. Then we describe the common lane-changing rules:

To describe lane-changing process, we analyze the positional relationship among cars in multiple-lane highways. The results are as follows: Based on the position of car \( C_{ij} \) at moment.

\( L_i \) denotes lane \( i \), and \( L_{i+1}, L_{i-1} \) mean the right-hand and left-hand lane respectively.

\[ X_{(i+1),j}(t) \] denotes car \( C_{ij} \)’s position on right-hand or left-hand lane.

\[ X_{(i+1),(j-1)}(t), X_{(i+1),(j+1)}(t) \] denote the cars that are ahead of or behind the neighboring car \( X_{(i+1),j}(t) \).

Double-lane’s lane-changing rules: Double-lane rules introduce the state changing of vehicles, such as overtaking and lane-changing. The identification of traffic conditions is depicted at formula (6).

\[
\begin{align*}
    & \text{if } d_{ij,x(j-1)} < \min(V_y + acc_j, V_{\max}) \\
    & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad d_{i(\pm1)(j)(j-1)} > d_{ij,x(j-1)} \\
    & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad d_{i(\pm1)(j)(j+1)} > 1 - \min\{V_y + acc_j, V_{\max}\} + \min\{V_{(i+1)(j+1)} + acc_j, V_{\max}\} \\
    & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad L_i \rightarrow L_{i-1} \\
    & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \ quad
The cooperative lane-changing rules are depicted as follows:

When there are needs to change the lane, namely, when the condition of inequation (8) is satisfied,

\[
\begin{align*}
X_g(t) + V_g(t+1) & \geq X_{g-1}(t+1) - 1 \\
X_g(t) + V_g(t+1) & \leq X_{g+1}(t+1) - 1
\end{align*}
\]

The lane-changing model is the same as the common rules if inequation (9) applies here.

\[
X_{i\pm1,j+1}(t) + V_{i\pm1,j+1}(t+1) \leq X_{ij}(t+1) - 1
\]

If the inequation (10) does not apply but the cooperative conditions are satisfied:

\[
\begin{align*}
X_{i\pm1,j+1}(t) + V_{i\pm1,j+1}(t+1) & \geq X_{ij}(t+1) - 1 \\
X_{i\pm1,j+1}(t) + V_{i\pm1,j+1}(t+1) & \leq X_{i\pm1,j-1}(t+1) - 1
\end{align*}
\]

So if the car behind on the neighboring lane changes its lane simultaneously, neither of them needs to decelerate. They conduct cross cooperative lane-changing here, just as is depicted in formula (11).

\[
\begin{align*}
C_{ij} & \rightarrow \text{lane}(i \pm 1) \\
C_{i\pm1,j+1} & \rightarrow \text{lane}(i \pm 1) + 1
\end{align*}
\]
We shorten common lane-changing rules as HDDL rules. Similarly, we shorten cooperative lane-changing rules as SCDL rules. As is shown in the diagram, C32 has to brake at t moment as it does not satisfy the conditions of lane-changing according to the HDDL rules. While in self-driving, cooperating model, C22 and C32 could exchange data. C22 could implement cooperative lane-changing at t+1 moment if it meets conditions of changing the present lane to lane 1, which helps to maintain the good motion state.

3. Simulation Results and Discussion

As to three-lane highways, based on the model built earlier, we simulate different traffic flow and compare the clear images of road’s traffic efficiency by using cellular automaton. Then we obtain the tipping point of traffic flow during evening rush hours under the condition that there are only self-driving cars (w=1) on the road. Moreover, we obtain another tipping point under the condition of non-self-driving cars (w=0) on the road. The tipping points are 19375 and 15625 respectively, as are shown in the follow diagram as follows in Figure 4:

![Figure 4. Tipping points are 19375 and 15625 respectively.](image)

By applying the same rule, we figure out that the tipping points during the morning rush hours are 30000 and 24750 under the condition of all self-driving cars and no self-driving cars respectively. The following is the diagram as follows in Figure 5:

![Figure 5. Morning rush hours are 30000 and 24750.](image)
The traffic count is 65000 in this unidirectional three-lane highway (data in the second row). Similarly, we obtain the images of traffic efficiency via simulation when the self-driving cars account for 10%, 50% and 90% as follows in Figure 6.

![Figure 6. Self-driving cars account for 10%, 50% and 90%.

Because the traffic count is 32500 on unidirectional highway and it does not fit in any of tipping point intervals (morning, evening), there is no equilibrium point on this section.

The traffic count is 17500 on unidirectional highway. And we figure out the equilibrium point of the percentage of self-driving cars on this section in the tipping point intervals between morning and evening. And self-driving cars account for 45%, as is described in the following diagram.

![Figure 7. Self-driving cars account for 45%.

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