Macroscopic observables and Lorentz violation in discrete quantum gravity

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December 28, 2021

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Abstract

This article concerns the fate of local Lorentz invariance in quantum gravity, particularly for approaches in which a discrete structure replaces continuum spacetime. Some features of standard quantum mechanics, presented in a sum-over-histories formulation, are reviewed, and their consequences for such theories are discussed. It is argued that, if the individual histories of a theory give bad approximations to macroscopic continuum properties in some frames, then it is inevitable that the theory violates Lorentz symmetry.

1 Introduction

Local Lorentz Invariance, or more specifically the question of whether this physical principle will be maintained or broken at the Planck scale, has become a much debated topic both in the theory and phenomenology of quantum gravity. On the phenomenological side, constraints on violation of Lorentz invariance (termed “Lorentz violation” below) are becoming ever more stringent [1], and more arguments have recently appeared suggesting that Planck scale Lorentz violation is incompatible with current observations, without (at best) additional fine-tuning being introduced [2, 3]. With this progress, it becomes increasingly important for the theoretical side to provide predictions, at least of a heuristic nature: in any particular theoretical framework, what becomes of Lorentz invariance? Is it possible to maintain it, or must it be broken at some scale?

It is well-known that waves travelling on lattices violate Lorentz invariance, and this is often used as an argument against fundamental discreteness. Some Loop Quantum Gravity (LQG) based arguments have been made to support this expectation [4, 5, 6], and some further discussion of the methods used is given in [7]. However, there are conflicting arguments for local Lorentz invariance in LQG [8], and also some [9] for the “third way” of Doubly Special Relativity (DSR) [10] in which the Planck length or Planck energy is introduced as an invariant scale along with the speed of light, deforming the Lorentz transformations (an idea that is criticised in [11]). If the spin-foam quantum gravity program is to provide a path-integral formulation of LQG, then it might be expected that the argument which prevails in the LQG program will also prevail for these models, and vice-versa.

Two interesting strategies have been proposed to evade the conclusion that Lorentz violation follows from discreteness. One is to find a form of discreteness that retains Lorentz symmetry at the level of the continuum approximation. The causal set [12, 13] offers such a discretisation of spacetime, as argued in [14] and proved in a strong sense in [15]. This allows waves to travel on a fixed background causal set without extra Lorentz violating terms appearing in the dispersion relation [16]. This special property of the causal set is often given as a motivation for using these structures as the histories in a quantum gravity sum-over-histories (SOH).

In spin-foam models, this Lorentz invariance of individual histories has not been claimed, and so a different argument must be employed. Since discreteness arises in standard quantum theories without violating continuum symmetries, it has been suggested that the same might be true for Lorentz symmetry in quantum gravity. This line of reasoning has been expanded on in the context of LQG [8]. In that article, an analogy is drawn between the rotation group in standard quantum mechanics, and the Lorentz group in quantum gravity. It is pointed out that, although the components of the angular momentum of, say, an electron, cannot be simultaneously measured, nevertheless any one can be measured and the theory is still rotationally invariant. It is claimed that the same reasoning will prevail in the case of measurements made with respect to different frames in loop quantum gravity.
If this evasion of Lorentz violation were to be successful in sum-over-histories approaches such as spin-foams, we would be able to say that, although the individual histories of our theory (which are configurations in one Hilbert-space basis, given at all times) do violate the symmetry, somehow it can still be true that our measurements do not. The underlying reasoning is that the results of measurements need not all directly correspond to properties of the histories. But in standard quantum theory, approximate, macroscopic measurements (measurements that we can make without significantly altering the state of the system) do correspond to properties of the histories. The main point of this article is that even these macroscopic properties are put in danger if the histories are not Lorentz invariant.

In the angular momentum example, this danger does not exist, since all components of the (approximate) angular momentum of a macroscopic object can be seen as a property of the histories. In other words, measurements of angular momentum components of, say, a baseball, can in principle be made in the same basis, up to an acceptable degree of accuracy. But there is an important difference in the case of Lorentz symmetry, more fully explained later on. If a history in a quantum gravity SOH was like a lattice, then even macroscopic quantities could fail to be properties of the history in highly boosted frames. This situation is qualitatively different from that of rotational invariance. It is argued below that this difference will prevent Lorentz invariance in a class of discrete models.

This article is intended to introduce a different way of looking at the problem of Lorentz violation in quantum gravity, through the lens of the sum-over-histories formalism for quantum mechanics. As such it is does not take the argument to any level of technical sophistication, but only presents a framework that, it is hoped, will facilitate further debate. Roughly, the argument proceeds in the following stages:

1) The outcome of the measurement of a macroscopic quantity corresponds to a property of some subset of the histories in the SOH. It follows that, in our quantum gravity SOH, there must be an approximate correspondence between some of the histories and Lorentzian manifolds (or at least those properties of Lorentzian manifolds that we expect to be measurable in principle). It is further assumed that some of the histories should have Minkowski space as an approximation.

2) A semi-classical state that “tracks” Minkowski space is considered, along with quantities that are defined with respect to some co-ordinate frame on this Minkowski space. Consider a quantity that is macroscopic and measurable in this “fiducial” coordinate frame. For Lorentz invariance to hold, this quantity must be macroscopic and measurable in all frames. Therefore there must be histories, each of which has properties that correspond to the outcomes of measurements of the macroscopic quantity in all frames.

3) There is a condition on the histories of a quantum gravity SOH (arguably true for lattices and similar discretisation schemes) under which no one history can contain such properties in all frames. It is shown that this leads to either Lorentz violation or a lack of macroscopically observable properties.

4) Spin foam quantum gravity models are likely to satisfy this condition if they are discrete at the Planck scale.

The argument relies on the correspondence between the outcomes of measurements and the properties of histories in the SOH. This view, suggested by Feynman in his original paper on the subject [17], is not currently en vogue, and so a justification of the point is necessary. Because of this, the first section below is a detour into sum-over-histories quantum mechanics, which also serves to fix some notation. As this viewpoint might be useful for other problems in quantum gravity, a fuller treatment is given in the appendix. In subsequent sections, we return to the specific case of discrete quantum gravity, covering the other points mentioned above. An outline of the conclusion is this: Lorentz violation at the level of the individual histories of a theory, if it causes macroscopic properties to be badly approximated in some frames, leads to Lorentz violating predictions.
2 Properties of histories, outcomes of measurements

Must there be an approximate correspondence between some of the histories in our quantum SOH, and the histories of the classical theory that we wish to approximately recover? Specifically to quantum gravity, we might ask: should some of our histories have Lorentzian manifolds as approximations at some large scale? The existence of such a discrete/continuum correspondence is supported in some of the literature (e.g. [12], and [18] in which such a correspondence is sought for spatial configurations rather than histories); however, it is not an uncommon opinion that this will not be necessary (see e.g. [19]). The properties of the continuum manifold need only become evident, it is argued, in quantum sums over many of the fundamental histories. This view is no doubt motivated by thinking of properties like the momentum of a quantum particle. This property is certainly not a property of any history (if those histories are written in the position basis)\footnote{A similar intuition might be noted, coming from the expansion of path integrals for interacting quantum field theories written in terms of Feynman diagrams. There, the histories appear merely as abstract graphs, plus some extra information, that do not correspond to properties that we eventually measure. But, again, the intuition that histories do not correspond to measurements does not extend to macroscopic properties.}. From this, it is sometimes concluded that individual histories in the SOH are of no physical significance.

Here it is argued that this reasoning fails to apply for properties that are macroscopic in a suitable sense. The momentum of an electron may not be a property of a history, but in a semi-classical state, the momentum of a baseball (measured with some acceptable degree of accuracy) is. On a suitably coarse-grained scale, the momentum of the baseball can be read off from a typical history in the same way as it would have been in the classical theory. This not only holds in all standard quantum theories, but is a necessary feature of any physically realistic quantum theory, as explained in the appendix.

This SOH view of quantum mechanics is nothing but a re-casting of the standard formalism; it is possible to describe our present, successful theories in this language. The following arguments on Lorentz violation apply only to theories that are compatible with this standard framework, and so this is introduced as a first condition for the main argument to hold.

**Condition 1** The quantum gravity theory is compatible with SOH formalism.

2.1 The sum over histories

First, we recall some features of standard quantum mechanics. In the paper in which he introduced the SOH formalism [17], Feynman also gave some appealing and useful ways of picturing it, allowing an easy connection between the formalism and observations.

The theory has a history space Ω, the elements of which are the histories. A history gives a possible configuration of the system in question (in some special Hilbert-space basis) at all times under consideration. For example, in the case of the Schrödinger particle they are continuous (but not necessarily differentiable) paths \( x(t) \) between some initial and final times. A brief quote from Feynman shows how measurements are dealt with in this formalism:

> The probability that a particle will be found to have a path \( x(t) \) lying somewhere within a region of space-time is the square of a sum of contributions, one from each path in the region. \[17\]

So, in measurement situations, our quantum theory associates probabilities to properties of the histories. When we talk of a property \( X \) of a history, we can associate to it a set of histories that have that property, \( \Gamma(X) \subseteq \Omega \), and a question “Does the system have property \( X \)?”. For example, the property may be that \( a < x(t) < b \) for some particular...
Answering the associated question requires a measurement, in this case a standard measurement. More generally, we might ask “did the particle’s world-line pass through spacetime region \( R \)” for some particular \( R \), which amounts to a non-standard, “continuous” measurement.

If we observe a property \( X \) we condition on the set of histories \( \Gamma(X) \) with that property, throwing away all the histories not in \( \Gamma(X) \) and carrying out the appropriate renormalisation of the probabilities (the equivalent of the “collapse of the wave function” from this perspective). We might want to carry out another measurement, finding that the system has another property, \( Y \), and we would then further condition on the smaller set of histories \( \Gamma(X) \cap \Gamma(Y) \), and so on. We will call this subset of the history space the measured set of histories or measured set. See [20] for an application of the formalism to a familiar experiment.

From the quote, we can see that the individual histories in the measured set do have at least some significant properties. The properties of the system that we observe are those shared by all the histories in the measured set. So far, there is no need to talk about anything but properties of histories in order to describe the observations\(^2\). But all these measurements were in the specially selected basis. So the question becomes: how much of the observed data can be described by measurements in this one basis?

As noted above, not all microscopic variables in a quantum system are directly represented in the histories. But it presents no difficulty to perform measurements of all macroscopic quantities in one basis, when they are suitably defined. Recall, for example, the properties of a semi-classical, coherent state for the Schrödinger quantum theory: the product of uncertainties of position and momentum is minimised, and the quantum state, when peaked on a certain position and momentum, “tracks” the associated classical solution over time. An approximate measurement of position (projecting onto a certain macroscopic range, say) can be made without significantly disturbing the state – indeed, this might be taken as a definition of a macroscopic measurement. By appeal to the classical theory, we can now see that a time-sequence of such position measurements must be enough to approximately determine the conjugate variable, the momentum. Thus the (approximate) momentum of a macroscopic object can be a property of the histories in the measured set, just as the position is – even though the “typical path” can be highly fractal on smaller scales.

The argument that this is the case for standard theories, and moreover must be the case for any realistic quantum theory with a good semi-classical regime, is left for the appendix. Also treated there, in section A.2, is a method sketched by Feynman for indirectly finding the results of all microscopic measurements in one basis. The conclusion is that each individual history in the measured set must have all of the properties that we observe.

### 2.2 Effective descriptions

Sometimes our fundamental histories are not directly described in terms of the properties that we are familiar with at the macroscopic level. But our observations may be formalised in terms of these effective properties.

For example, if our system is a collection of molecules, we might make a hydrodynamical approximation. But this continuum description is of course not perfect. Some of the properties of the hydrodynamical description do not have any physical significance. An example is the property that a density perturbation of amplitude \( A \) and wavelength \( \lambda \) exists in some region of the fluid. This ceases to make sense in the underlying history when \( \lambda \) becomes close to the intermolecular separation.

If an effective property \( X \) does not have any corresponding property in a fundamental history \( h \) (i.e. if it is impossible to tell from \( h \) whether or not \( X \) has happened) then we

\(^2\)Note that we have not stepped outside of the normal interpretation of quantum mechanics. Physical properties can therefore be interpreted here in an operationalist sense. However, if preferred, it is also possible to apply a “decoherent histories” style interpretation (see e.g. [21]) without altering the argument.
must consider $X$ false for that history, and so $h$ would not be included in $\Gamma(X)$. Such properties are termed “undecidable” for that history (the converse being “decidable”).

3 Lorentz invariance

These considerations will now be applied to the problem of Lorentz invariance in quantum gravity. But what would Lorentz symmetry mean in this context?

Here our system is spacetime, and our histories, at least on an effective level, are Lorentzian manifolds. It is simplest to assume that there is a semi-classical state that tracks Minkowski space, in order to talk about global Lorentz invariance. This should not be a controversial assumption, as much effort has gone into attempting to describe such states in the various quantum gravity programs. To say that this state is Lorentz invariant means that there should be no way to pick out a preferred direction in spacetime by any measurement. This immediately tells us that, if we expect Lorentz invariance, then we expect the semi-classical behaviour to persist in all frames. But semi-classical behaviour (and what it means to track a classical solution) should be defined in terms of some class of observable quantities that we expect to be able to measure.

3.1 Macroscopic observables in quantum gravity

Our measurements might be measurements of time and length in one frame, made at scales appropriate to the semi-classical state, e.g. the informal, very approximate measurements of space and time that we make every day. They could also include measurements of other fields. Different observers might set out to measure similar macroscopic quantities in other frames.

Already, the terms “macroscopic”, “semi-classical” and “tracking Minkowski” have been used, but no exact definition has been given. What exactly are these scales at which observables can be considered macroscopic, in quantum gravity? No detailed investigation shall be launched into here; it is only necessary to stress the uncontroversial point that some familiar observables must be considered to be in this class. It is trivial to observe good approximations to classical predictions, for example, in electromagnetic waves moving on an approximately flat background, for certain values of frequency and amplitude. Our theory of quantum gravity must be able to reproduce these results and others like them, and so measurements of such waves must be in the class of macroscopic measurements that can be made in that theory. And in our semi-classical state that tracks Minkowski space, we can add the extra expectation of Lorentz invariance. This has the important consequence that any frame-dependent measurements considered macroscopic with respect to one frame $F$ must be considered macroscopic with respect to frames that are boosted with respect to $F$.

As noted in appendix A.3, no such measurements should alter the state of the system in any detectable way, and so they should not affect each other – there must come a point at which the presence or absence of wave-function collapse ceases to be physically significant, and this point comes after the semi-classical scale of the given system is reached. By analogy, we can define an approximate measurement of one component of the angular momentum of a baseball, that does not significantly alter the state; likewise, the measurement can be made in other frames rotated with respect to the first, as many of them as we choose, without causing detectable changes in the state. (Otherwise, to repeat a point made in the appendix, the question of whether wave function collapse occurred when we photographed a spinning ball, or later when we developed the photograph, would become physically significant. Such theories are physically unacceptable.)

This is perhaps a source of the difference between the arguments presented here, and those of [8]. There, the non-commutativity of observables corresponding to measurements in different Lorentz frames is emphasised. The arguments above contrast with this. They
basically assert that for semi-classical states, there is a sense in which non-commutativity must become practically insignificant for some class of approximate measurements, in order to prevent macroscopic quantum interference effects. This is the situation in standard theories, and there seems to be no reason to change the requirement in the case of quantum gravity.

Following the reasoning of the previous section, all of these macroscopic observations must correspond to properties of the underlying histories. Further investigation shows that only certain theories can maintain Lorentz invariance under this condition.

3.2 A simple example

A light-cone lattice offers a simple discretisation of 2D Minkowski space, with a fairly obvious discrete/continuum correspondence. This example offers some interesting insights.

Let the histories of our theory be examples of such a lattice, with a scalar field living on it. More specifically, the structure is a directed graph on a set of elements \( \{ e(i, j) \} \) with \( i, j \in \mathbb{Z} \). The graph edges run between the pairs \( \{ e(i, j), e(i+1, j) \} \) and \( \{ e(i, j), e(i, j+1) \} \).

To each element is associated a real number \( \phi(i, j) \) to represent the scalar field. Different values of the field give rise to different histories.

These histories are not defined in terms of a manifold at all. They are purely “abstract graphs”. Also, the labels given to the elements need not be considered to be part of the structure; they can be reconstructed from the directed graph edges up to “discrete translations” (i.e. \( i \rightarrow i + x, j \rightarrow j + y, x, y \in \mathbb{Z} \)). The only “intrinsic” properties of an element are the graph edges connected to it and value of the field there.

Despite this, a correspondence to 2D Minkowski space can be made in an obvious way: the elements can be assigned to points in Minkowski with \( u = ai \) and \( v = aj \), where \( a \) is some length (which might be imagined to be the Planck length), and \( (u, v) \) are the standard light cone co-ordinates in Minkowski space, in terms of which the metric is \( ds^2 = 2dudv \). This embedding defines the manifold approximation: a scalar field \( \Phi(u, v) \) on Minkowski space can be considered as an approximation to \( \phi(i, j) \) if a frame can be found such that \( \phi(i, j) = \Phi(ai, aj) \). Realistically, we would only assume this to some degree of approximation, and also require that the field \( \Phi(u, v) \) was not “quickly varying” with respect to the distance \( a \). It can easily be seen that this correspondence principle is consistent with the discrete translation invariance of the fundamental histories.

Note that, as well as defining a field on the Minkowski space, the fundamental history also defines a frame on the Minkowski space (in which the \( (u, v) \) co-ordinates are defined), which we will call the “lattice frame”. For any single fundamental history, the lattice frame in the effective Minkowski description is fixed relative to the field configuration.

How do we begin to talk about Lorentz invariance in this model? Lorentz invariance only makes sense in the continuum, and so any Lorentz transformations must be applied at the level of the effective continuum description. It will be said that one effective property \( X \) is related to another \( X' \) by a Lorentz transformation \( \Lambda \) if every field configuration \( \Phi(u, v) \) satisfies property \( X \) if and only if \( \Lambda \Phi(u, v) \) satisfies \( X' \).

The problem with this kind of discretisation is that it is “not equally good in all frames”. For a given fundamental history, some properties that are \textit{decidable} (in the sense of section 2.2) are related by Lorentz transformations to \textit{undecidable} properties. For example, a plane wave, written \( \Phi(u, v) = \sin(u/\lambda) \) in the lattice frame, can only be an approximation to a discrete field when \( \lambda \gg a \). Therefore the property “there is a plane wave of amplitude \( A \) and wavelength \( \lambda \gg a \) with respect to frame \( F \), in region \( R \)” is only decidable for a particular fundamental history when the frame \( F \) is sufficiently near to the lattice frame (remember that the lattice frame is fixed relative to the field configuration for each history).

\footnote{More properly, the histories should be considered to be equivalence classes of the directed graphs under the above relabellings of the elements.}
Figure 1: The diagram represents two regions, $R_1$ and $R_2$, in 2D Minkowski space, into which is embedded a light-cone lattice as described in section 3.2. $R_1$ is related to $R_2$ by a boost. A scalar field defined on the lattice can be approximated by a field on the Minkowski space. But it is hard to imagine how all the properties of the field decidable in $R_1$ could also be decidable in $R_2$, as $R_2$ contains no embedded lattice points. With sufficient boosts, similar situations can be constructed with any finite lattice density and any size of region, leading to the presence of macroscopic properties of the history that are decidable in one frame but not another.

Crucially, there is nothing to stop these properties from being semi-classical, macroscopic properties. Even on a very fine lattice, an extreme enough boost will take a wave of arbitrarily long wavelength to one with “sub-lattice” wavelength. See figure 1 for a further example.

### 3.3 Lorentz violation, from histories to observations

This is one example from a class of such discretisation schemes. A condition on such schemes is now presented that encapsulates the problem.

**Condition 2** The set of histories of the theory that have Minkowski space (plus matter) as an effective description is non-empty. Consider some region $R_1$ in Minkowski and let $X_1$ be a macroscopic effective property of that region, and also consider another region $R_2$ related by a boost transformation (with some sufficiently large boost factor) to $R_1$. Let $X_2$ be the property of $R_2$ related to property $X_1$ of $R_1$ by that boost. Then for some $X_1$, either $X_1$ or $X_2$ is undecidable, for all histories.

In the lattice example above, this condition is no doubt satisfied. We will now consider a hypothetical realistic model in which the above condition is also satisfied. Does this lead to Lorentz violation? It might be imagined that the “lattice frame” could vary over the histories in the measured set, somehow ameliorating the frame dependence of the individual histories. This is now shown to be impossible: the above condition leads to in-principle observable Lorentz violation.

What if we were to attempt to measure for both properties $X_1$ and $X_2$? We need the probability associated with the measured set $\Gamma(X_1) \cap \Gamma(X_2)$. Unfortunately, either $X_1$ or $X_2$ is undecidable in every history. This means that the set $\Gamma(X_1)$ is disjoint from the set $\Gamma(X_2)$, and so $\Gamma(X_1) \cap \Gamma(X_2) = \emptyset$. This pair of macroscopic properties is undecidable in all histories, and hence always false; it is unphysical. If there are many easily measurable pairs like $X_1$ and $X_2$, this is absurd. The only solution is that only one of the properties can be considered a physical, measurable quantity, and so, since one is related to the other by a boost, Lorentz symmetry is violated. The conclusion is that *This kind of Lorentz violation in the individual histories necessarily leads to Lorentz violation at the level of macroscopic observables.*
In this argument, no scale has been put on the Lorentz violation of the individual histories as expressed in condition 2, hence the vagueness of the phrase “with some sufficiently large boost factor”. But we would expect that Planck scale discreteness implies Lorentz violation at a related scale, as it does in the case of the lattice. A quantification of the effect would depend on the range of boosts over which a certain property was well approximated in the individual histories. No attempt to quantify this violation for any theory is made here.

In the specific semi-classical situation being described, some of the symmetries of GR remain; general covariance has been broken to the Poincaré invariance of Minkowski space. How then do we fix the positions of regions $R_1$ and $R_2$ in all relevant histories? The positions of the regions have no meaning a priori. They must be determined with reference to something else, by the properties of other fields in the theory. Here it is assumed that this can be done, the reason being that, if it could not be, then the theory would have failed in any case. It must be possible to identify in all relevant histories, for example, the world-line of the planet earth. With this done, regions could be identified with reference to it and the above conclusion drawn. In the lattice example, the world-line of the planet earth would be described by the configuration of the fields on the lattice, and would necessarily be identifiable across all histories in the measured set. The details of this scheme to fix the positions of $R_1$ and $R_2$ would not impact the argument that no single lattice-like history can accurately represent our two macroscopic properties $X_1$ and $X_2$ (which, it should be noted, may be defined in overlapping regions, and/or be supplemented with many additional, similar properties).

4 Consequences for discrete quantum gravity

In order to reach the conclusion, we must assume condition 2, a condition arguably true for lattice-like discrete structures. The question now becomes: which discrete structures proposed in the various approaches to quantum gravity satisfy the condition, and which evade it?

A discretisation scheme along the lines of a regular lattice would satisfy condition 2 but that is not a particularly interesting case; this is not the basis of any popular attempt to quantise gravity. In order to fully address the issue in a given approach, a method of assigning continuum approximations to the discrete underlying structure would have to be specified: a map from the individual histories to Lorentzian manifolds.

In causal set theory, the correspondence between the discrete underlying structures and the continuum manifold was early on made explicit, and these discrete structures have been shown to be Lorentz invariant in a strong sense [15], evading condition 2. In particular, in contrast to the light-cone lattice, a scalar field dynamics on a fixed causal set background can be defined which does not violate Lorentz symmetry [16].

The situation for spin-foam models is less clear, as here a correspondence between continuum manifolds and the underlying histories has not been made completely explicit. However, some clues are available. An analogous situation has been considered in loop quantum gravity, where discrete spatial configurations must be approximated by continuous Euclidean manifolds. In [18], a correspondence is developed for the simple but suggestive case of unlabelled graphs.

A graph can be associated to a manifold by a random process. First, a locally finite set of points is randomly selected from the manifold by a Poisson process (the resulting set of points being called a “sprinkling”). To this sprinkling, a graph is associated via a Voronoi procedure [18]. A graph is said to be approximated by a manifold if it could have arisen in this way from a sprinkling of that manifold, with relatively high probability. This in-principle correspondence can be reversed to order to recover geometrical information (e.g. on area, volume and dimension) directly from a given graph. To allow for quantum behaviour at short scales, the discrete/continuum correspondence could be applied only
on large scales, by using some coarse-graining of the fundamental graph \[18\].

In this way many desirable features are recovered, for example a proportionality between the area of a surface of co-dimension 1, and the number of edges crossing it (something desirable from the standpoint of loop quantum gravity), and also a proportionality between the number of vertices in a region and its volume. Thus it is plausible that the scheme could be extended to spin networks. Since these are nothing but spatial slices of spin-foams, a similar scheme might be sought in that case, keeping these necessary proportionalities.

However, a naïve application of such a scheme to spin-foams could not be Lorentz invariant. It has been proven that no finite valency graph can be associated to a sprinkling of Minkowski space in a way that respects Lorentz symmetry \[15\], and each spin-foam contains such a graph. Thus there would always be a preferred frame associated to each history like the lattice frame of section \[5\] and it is reasonable to assume that condition \[2\] would be violated in this case. Without the randomness of sprinkling, the correspondence might have to rely on some more regular embedding of vertices into manifolds, something even more likely to produce Lorentz violations, as it occurs for regular lattices.

It could be argued that the correspondence principle will be more subtle, not relying on attempts to embed vertices into manifolds. But this would seem unlikely in the light of the successes of the scheme for spatial configurations mentioned above. Moreover, it is hard to imagine how to avoid the concept of embedding, at least at a coarse-grained level, if the vertices of the discrete structure are to correspond in any way to spacetime points or “elementary regions”.

But there is another objection to this line of argument. The Lorentz violation on a lattice is a result of the discreteness. The finer the lattice, the higher the boosts at which it can successfully represent macroscopic properties. But for any lattice there will come a boost at which these properties cease to be decidable. Condition \[2\] requires that there is a boost factor sufficient to bring about this situation in all histories that correspond to Minkowski. This clearly implies that there is an upper bound on the boost necessary for each individual history. In the case of spin-foams, this might be questioned. Could there be no upper bound to how “fine” an spin-foam can become, in the appropriate sense?

The form of the sum-over-triangulations in spin-foam quantum gravity has not been fixed, and so only more speculation can be offered on this point. An embedding scheme that put no upper bound on the number of vertices per unit of 4-volume could possibly evade condition \[2\] on these grounds. It has even been suggested that a continuum limit may have been taken to get consistent results from spin-foam models \[22\], in which case the fundamental histories would not be discrete at all. But these alternatives seem unlikely to be consistent with the correspondence of continuum 3-volume and area with properties of the slices of the underlying spin-foam. Only the establishment of a full discrete/continuum correspondence principle could unambiguously answer this question. Even so, it is interesting to note that for such an evasion to be possible, Planck scale discreteness would be sacrificed. The import of this observation depends upon the value put on Planck scale discreteness as a desirable aspect of a candidate quantum gravity theory \[4\].

4.1 Further discussion

Apart from the possible applications of condition \[2\] to spin-foams, there are other possible objections to the argument laid out here that should be mentioned. For example, some theories may not satisfy condition \[1\] certain approaches to quantum gravity might not have any SOH formulation in their final forms. But this seems unlikely to affect the arguments on spin-foams, especially when the goal of the program is expressed as finding a path-integral theory for quantum gravity (as, for example, in \[23\]). Any theory that did not satisfy condition \[1\] would differ considerably from standard quantum mechanics;

\[4\] see e.g. \[13\] for the attitude towards discreteness taken in the causal set program.
in that case many questions would arise concerning the consistency of the theory and the interpretational framework to be used.

Secondly, there is some lack of precision in the idea of “effective descriptions” here, but it would be difficult to argue that a lattice-like structure does in fact have enough structure to represent waves in all frames. Similarly, although all the “macroscopic properties” of quantum gravity are not described, they must include the well-observed macroscopic properties that are present in today’s successful theories.

In the argument, it was assumed that the quantum gravity theory would have semi-classical solutions corresponding to Minkowski space, and this was used in subsequent arguments. This might not be possible in principle, and it is not a realistic scenario. But it should be possible to generalise to a situation in which Minkowski was only a good approximation in a region, without altering the main points of the argument. The failure of lattices, and similar structures, to represent macroscopic properties at very high boosts, is not a subtle effect but a very marked one, and so it is unlikely to be removed by small corrections to the picture given above.

Other objections might be found in the practicality of finding and measuring pairs of events such as $X_1$ and $X_2$, but one would have to explain why “an electromagnetic wave with given frequency and amplitude, travelling through a region $R_1$” is not a good property to use for $X_1$, if it is accepted that properties of this form are measured for in tests of Lorentz invariance. The biggest problem is that there is no concrete prediction for the scales at which Lorentz violation would be seen, and this leaves the door open to Lorentz violation at unmeasurably high boosts. Thus the argument presented here should be seen as only one step in the ongoing discussion of Lorentz violation.

5 Conclusion

The argument set out above works from the sum-over-histories view of quantum mechanics. Firstly, it is argued that all macroscopic observations correspond, in a fairly direct way, to properties of histories in the measured set of histories. If a semi-classical state is invariant under some symmetry, then the class of observables that can be considered to be macroscopic will also be invariant under this symmetry. In a semi-classical state of quantum gravity that tracks Minkowski space, we expect to be able to make macroscopic measurements in any frame. Above, a condition has been set down under which this is impossible, as there are not enough properties in the underlying histories to represent these observations. Some discussion has been given of the relevance to the spin-foam and causal set quantum gravity programs.

One purpose of the article was to call attention to the necessity of an in-principle correspondence between fundamental histories in a quantum gravity theory, and Lorentzian manifolds which approximate to them. In the case of spin-foams, even establishing some broad features of this correspondence would enable some conclusions to be drawn from the arguments above, and would doubtless be useful elsewhere.

Finally, no discussion of doubly special relativity has been made here, and it is possible that this allows some compromise between this line of reasoning and those put forward elsewhere, for example in \cite{9}. In conclusion, the arguments given may put the spin-foam program in the happy situation of predicting effects that are almost within reach of observation, or conversely, the principle of Lorentz invariance could be used to decide amongst various spin-foam models. But these applications are dependent on further developments in the ongoing discussion of Lorentz violation in quantum gravity.

The author is grateful to Jeremy Butterfield, Adrian Kent, Fay Dowker, Rafael Sorkin and Sumati Surya for correspondence and discussions of this work, and to the organisers and participants in the Loops ’05 conference, many of whom contributed to the debate on this issue. The author was supported by DARPA grant F49620-02-C-0010R at UCSD, where some work on the article was carried out.
A Appendix: Properties individual histories in the SOH

A.1 The significance of measurements in one basis

In section 2.1 of the main text, it was shown how measurements in the histories basis correspond to properties of histories in the measured set. It was also briefly argued that all macroscopic observations can be expressed as measurements in this one basis, e.g. the position bases. This discussion is expanded on here.

Above, it was claimed that the approximate momentum of a non-relativistic particle in a semi-classical state can be determined from a series of position measurements. It is interesting to note that this is true even though the sum-over-histories is dominated by nowhere-differentiable paths, and so the classical meaning of momentum is not immediately applicable. It only comes back into relevance at large scales. In other words, at scales above the spread of the wave-function, a typical path approximates to the classical one, and the momentum can be approximately defined at those scales in the a way that corresponds to the classical definition.

This would apply to any system, not just the position and momentum of a particle. In some approaches to quantum gravity, the history is a path in superspace (i.e. a sequence of 3 manifolds). It should in principle be possible to set up experimental apparatuses in order to measure the intrinsic geometry of a spatial hypersurface at different times (time, and the foliation considered, being decided by the experimental apparatus). In an informal sense this happens to us all every day – we are equipped with an obvious time parameter, and we observe approximately flat space. It has been conjectured by Wheeler \cite{24} that a sequence of 3D Riemannian manifolds parameterised by time should be enough to uniquely reconstruct a 4D solution to the equations of motion of GR. Therefore, we are able, by making a series of approximate measurements in one basis (that of the intrinsic geometry of a hypersurface), to reconstruct the conjugate variables (in this case, the extrinsic geometry). Our eventual quantum theory of spacetime should allow this to be possible for semi-classical states, such as the one which we observe. This requirement would not be altered if Lorentzian manifolds were taken to be an approximation to some other (possibly discrete) structure.

A.2 Feynman’s argument

Although it is stronger than necessary for the purposes of the arguments of sections \[3\] and \[4\] for completeness it is interesting to see how measurements in other bases were dealt with by Feynman:

The [SOH measurement] postulate is limited to defining the results of position measurements. It does not say what must be done to define the result of a momentum measurement, for example. This is not a real limitation however, because in principle the measurement of momentum of one particle can be performed in terms of position measurements of other particles, e.g. meter indicators. Thus, an analysis of such an experiment will determine what it is about the first particle which determines its momentum.

Here Feynman is arguing that the amount of information available from measurements in this one basis is good enough to describe the results of all experiments. It is assumed that the correct correlation can indeed be set up between the meter and the quantum system, in a measurement situation. But the possibility of such correlations is indeed necessary if we are to be able to make any consistent connection with experiment, as noted in \[25\].
This is a method of casting all measured properties as properties of histories in the measured set. But this is not directly useful in the present discussion, since the property of the histories corresponding to, say, the momentum of a particle in a measurement situation, will not be immediately recognisable. This method offers no easy connection between simple properties of histories and observations. But with macroscopic observations of systems in semi-classical states we have the more direct given at the end of section A.1 – there is no need to couple a pointer to a baseball in order to know its momentum.

A.3 The necessity of semi-classical behaviour

Here it has been assumed that there exist semi-classical states, with small uncertainty (relative to the required experimental accuracy) in all the macroscopic variables that we might want to measure. In particular, one must be able to approximately measure macroscopic quantities without noticeably affecting the state. This property of standard quantum mechanics must also be true for any future quantum theory of gravity. It is not an incidental property of current quantum theories, but is essential for the theoretical framework to make sense.

Von Neumann stressed the requirement [25] that we have the freedom to change the classical/quantum boundary at which wave-function collapse occurs. In an experimental set-up there is a division of the universe into quantum system, and classical apparatus and environment. This division must be made so that none of the microscopic degrees of freedom of interest are taken to be classical; but any choice of division that accords with this rule-of-thumb is valid, and the choice should not be physically significant. To maintain this principle, it must be possible to (approximately) measure macroscopic properties, without significantly altering the state. Our theory would be faulty if an approximate measurement of the position of a rock necessarily affected the results of subsequent approximate measurements. In such a case of “macroscopic complementarity”, the imposition of wave-function collapse has detectable consequences, opening a Pandora’s box of conceptual problems that are kept at bay in standard quantum theories.

A lack of semi-classical states, with small quantum fluctuations in all the quantities that we expect to behave approximately classically, would create such problems. The allowable fluctuations must be decided with respect to experimental constraints and expectations stemming from these constraints, as noted in [26]. This allows such results as that of [27], in which a symmetry reduced gravitational system is quantised, and relatively large fluctuations in the metric are found to be possible, even in coherent states. The situation described there is far from being one that we have experimental access to, and so “reasonable expectations” for the magnitude of quantum fluctuations should be, in this case, very loose. In the light of this, the result need not be viewed as a failure of semi-classicality, but only as a situation in which the meaning of semi-classicality is unusual. But we are not at all far from experiment when discussing approximately flat spacetime, and so the constraints here are much tighter, especially if one of our expectations is the survival of Lorentz invariance.

A.4 Summary

The conclusion is that each individual history in the measured set must have all of the properties that we observe. Extracting these observations when given such a history may not be easy, but it must in principle be possible. For semi-classical quantities, the relationship to properties of histories should be direct, allowing us to proceed as we do classically, albeit on a suitably course-grained scale at which the classical quantities make sense. This is the case in standard quantum theories. In the original example, a typical path of a Schrödinger particle might be highly fractal on small scales, but in semiclassical situations it will approximate the classical solution with an acceptable degree
of accuracy. Any series of position measurements of a macroscopic object could be said to be an informal verification of this aspect of the theory.

Conversely, there are no measured properties that belong only to sets of histories, and not to its members. Such properties fall into two other categories: (a) Properties in other bases that are never measured, and so are never correlated to a “meter” property in the way that Feynman requires; (b) Mathematical attributes that do not represent the outcomes of any real or imagined experiment, for example, the property of a set of histories decohering with another set of histories (see e.g. [21]).

Feynman’s remark that all observations can be handled in one basis is little known, and sometimes explicitly contradicted, in modern work on theoretical physics. The point seems to be raised only in work on the measurement problem and its various proposed solutions, e.g. [28, 29, 30]. However, this picture of quantum mechanics is not only of interest from this perspective. It impacts on an important current issue in quantum gravity: exploring the properties of semi-classical states.

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