Non-Local Ductile Damage Formulations for Sheet Bulk Metal Forming

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Abstract. A ductile damage model for sheet bulk metal forming processes and its efficient and accurate treatment in the context of the Finite Element Method is presented. The damage is introduced as a non-local field to overcome pathological mesh dependency. Since standard elements tend to show volumetric locking in the bulk forming process a mixed formulation is implemented in the commercial software simufact.forming to obtain better results.

1. Introduction
The sheet bulk metal forming process [4] is characterized by large plastic bulk deformation of a metal sheet. This way it is possible to create functional parts, such as gears or drivers just by forming and without additional cutting processes. In order to simulate such challenging processes an accurate material model and non standard numerical techniques are required. To model the plastic behaviour the framework of elastic isomorphism, see e.g. [1, 10] is used and coupled to a continuum damage model. To circumvent the mesh depended localisation of such models a gradient enhanced non-local theory [5, 2] is used. Since the plastic deformation is nearly incompressible, standard finite elements will show volumetric locking behaviour. Therefore a strategy based on a mixed formulation is presented and implemented in the simulation environment simufact.forming.

2. Elasto-Plastic Material Model
Because of its general applicability and its straight forward derivation the concept of elastic isomorphism as e.g. proposed by Bertram [1] or Svendsen [10] is used for the formulation of the elasto-plastic constitutive laws. This concept states, that for every plastic deformation the elastic material response remains unchanged. If the stored elastic energy for an elastically isotropic material $\psi = \psi(C, P)$ is a function of the right CAUCHY-GREEN Tensor $C$ and the plastic Transformation $P$, then this concept necessarily leads to a functional relationship of the form:

$$\psi(C, P) = \psi(P^T C P) =: \psi(C^E) \quad \text{with} \quad C^E := P^T C P. \quad (1)$$

By introducing a yield function $\Phi(M^E, \sigma_y)$, where $M^E$ is the elastic MANDEL stress, the elastic range is defined for all stress states by a to negative value of $\Phi$. For $\Phi = 0$ plastic yielding takes place and $P$ is assumed to evolve by means of the principle of maximum dissipation. Because of isotropic hardening the elastic range will grow during yielding. To model this effect in a physical meaningful way, we introduce an internal variable $\xi$ which is proportional to the square root of...
the dislocation density on the micro scale of the considered steel. The evolution of this internal variable is motivated by elementary processes on the microlevel, see e.g. Stainier et al. [9]:

\[
\dot{\xi} = H(\xi_\infty - \xi)\lambda. \tag{2}
\]

As last ingredient of our model the dissipative effect of damage is considered. Here the anisotropically extended LEMAITRE damage model of Soyarslan et al. [8] is used. Despite their original idea of splitting the stresses in positive and negative parts by means of an eigenvalue decomposition and using the complementary energy function, we use a split of the free energy function \(\psi = \psi^+ + \psi^-\) depending on either the positive or negative eigenvalues of the strain like quantity \((C_E - 1)\). The damage potential then reads:

\[
\Phi_D = \frac{S}{s + 1} \left( -\frac{(\psi^+ + h \cdot \psi^-) - Y_0}{S} \right)^{s+1} \frac{1}{(1 - \tilde{\phi})^\beta}, \tag{3}
\]

where \(h \in [0,1]\) weights the compression part \(\psi^-\). In equation (3) \(\tilde{\phi} \in [0,1]\) is the local damage factor and \(\{S, s, \beta\}\) are material parameters for this model. The evolution of the scalar damage variable \(\tilde{\phi}\) is then obtained by an associative evolution law.

3. Non-Local Damage Model

It is well known that local softening materials exhibit a strong mesh dependency [6]. The smaller the elements become the earlier a fully degraded element will cause a singular tangent stiffness matrix. One efficient way to circumvent these issues is the application of a non-local theory. This is done by the introduction of a new field variable, the non-local damage \(\phi\) and by coupling the local damage variable \(\tilde{\phi}\) to its non-local counterpart \(\phi\) and its gradient \(\nabla \phi\). Computationally this results in solving an additional equation with homogeneous NEUMANN boundary conditions in the entire domain \(\Omega\):

\[
\phi - \ell^2 \nabla \cdot \nabla \phi = \tilde{\phi} \quad \forall x \in \Omega \quad \text{with:} \quad \nabla \phi \cdot n = 0 \quad \forall x \in \partial \Omega. \tag{4}
\]

This form is either known as implicit gradient formulation [5] or as micromorphic damage model [2]. The constant \(\ell\) represents an internal length scale which in general is determined by experiments for the considered material. In the limiting case of \(\ell \to 0\), the purely local model is obtained. The benefit of additional stability has to be paid by an increase of computational costs by a factor of about 1.3, assuming the same mesh is used as for the standard local damage simulation.

The functionality of the model is best explained by the one dimensional example shown in Fig. 1. There, a clamped beam is exposed to a displacement boundary condition and the reaction
forces are recorded depending on the number of elements used for the discretisation. The yield strength is reduced by a factor of 0.99 for the two elements in the middle of the beam. While the reaction force for the non-local model converges to a stable solution the local model tends to diverge. The reason is the small domain (one integration point) where the damage is localising. In the non-local model the damage evolution is spread also to some neighbouring elements. In this example the ratio \( \frac{\ell}{a} \) is 0.1.

4. Finite Element Formulation and Implementation

The balance equation for the non-local damage (4) and the linear momentum balance is solved by the finite element method. However, standard low order displacement finite elements tend to show volumetric locking behaviour when plastically incompressible materials are involved. Especially in sheet bulk metal forming processes this is often observed when functional elements are formed by large bulk deformations. There is a huge variety of element technologies to treat this issue and a very simply and quite effective technique was first reported by Simo et.al. [7]. This is based on the HU-WASHIZU variational formulation, where the additional fields \( \bar{p} \) and \( \bar{J} \) are introduced and the free energy function is additively split into an isochoric part \( \psi_{iso}(C_{iso}) \) and a volumetric part \( \psi_{vol}(\bar{J} \cdot \det(P)) \). The HU-WASHIZU potential then reads:

\[
\Pi^{HW}(u, \bar{p}, \bar{J}) = \int_{\Omega_0} \psi_{iso}(C_{iso}) + \psi_{vol}(\bar{J} \cdot \det(P)) + \bar{p}(\det(F) - \bar{J}) \, dV , 
\]

where the last term penalises the deviation of \( \bar{J} \) from the determinant of the deformation gradient \( \det(F) \). By computing the variation of eq. (5) the field \( \bar{p} \) is identified as the hydrostatic pressure. In order to avoid additional equations in the global equation system piecewise constant shape functions are used for the discretisation of \( \{\bar{p}, \bar{J}\} \) and a static condensation is performed. The displacement field \( u \) and the non-local damage \( \phi \) are discretised by standard low order shape function of LAGRANGE type. To show the performance of the presented mean dilatation (MD) element the classical COOK’s membrane problem is solved using the hexahedral 8 node MD element and compared to the results of a higher order tetrahedral element. In figure 2 the problem setup is shown as well as the convergence behaviour and the resulting pressure distribution. Especially for coarse discretisations the presented element performs good and the convergence rate for finer meshes is comparable to the 10 node tetrahedral element.

The implementation is done in the commercial FE software simufact.forming as a new user element in subroutine USELEM. To circumvent difficulties of the built in contact formulation regarding the additional degrees of freedom needed for the field \( \phi \), just a weak coupling of damage the equation and the linear momentum is realized. Therefore in every load step first the
damage equation is solved and afterwards the mechanical equilibrium is calculated by means of a
NEWTON-RHAPSON iteration. For post processing purposes an interface to the visualisation
software ParaView is implemented.

5. Testcases
Two examples are considered to reflect the main load cases of sheet bulk metal forming. The
first is a demonstrator problem which was first considered in [3]. In this example eight punches
are placed in an axisymmetric way and they coin a round sheet of metal. In figure 3 on the
left the simufact.forming model is shown and on the right the deformed configuration and the
resulting damage distribution are shown. Compared to the local model the damage field is
more smooth and the absolute value is identical at each indent, so damage is not localising
in this example. The other main load case is a bending dominated problem. In figure 4 the
demonstration problem of simufact.forming for deep drawing is shown. As expected the main
region for damage accumulation is the rounding between bottom an wall of the cup. To validate
the numerical model experimental tests for the two examples are in preparation.

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