Thermal Conductivity in Vortex State of Nodal Superconductors

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How to determine the symmetry of the superconducting order parameter is one of the important issues in novel superconductors, which include charge conjugated organic superconductors. We have proposed that the angular dependence of the thermal conductivity in a planar magnetic field provides a new window to look at the symmetry of the order parameter. After a brief summary of the quasiclassical approach we describe how the symmetry of the superconducting order parameter in Sr\textsubscript{2}RuO\textsubscript{4}, CeCoIn\textsubscript{5} and κ-(ET)\textsubscript{2}Cu(NCS)\textsubscript{2} is determined. Also in some of experiments the phononic thermal conductivity plays the crucial role.

Keywords: Unconventional superconductors, Thermal conductivity, Vortex state

1. Introduction

Since the discovery of the first organic superconductor in (TMTSF)\textsubscript{2}PF\textsubscript{6} (or Bechgaard salts) \textsuperscript{1}, the symmetry in superconducting order parameter has been one of the important issues \textsuperscript{2,3}. Indeed the symmetry of the superconducting order parameter becomes one of the central issues after the establishment of d-wave symmetry in both hole and electron doped high T\textsubscript{c} superconductors \textsuperscript{4,5,6,7}. Most likely the superconductivity in (TMTSF)\textsubscript{2}X with X = ClO\textsubscript{4}, PF\textsubscript{6}, etc. is of p-wave \textsuperscript{8}. In particular, a flat Knight shift seen in a recent NMR experiment \textsuperscript{9} is consistent with this picture. The remaining question is whether the p-wave superconductor belongs to 1D representation \textsuperscript{3} or 2D representation \textsuperscript{8}. After a long controversy \textsuperscript{10}, d-wave superconductivity is emerging in κ-(ET)\textsubscript{2} salts \textsuperscript{11,12,13,14,15}. Here now the question is whether the symmetry is of d\textsubscript{xy}-wave as suggested by the theoretical works \textsuperscript{16,17,18} or d\textsubscript{x}2−y\textsuperscript{2}-wave as the recent STM study suggests \textsuperscript{15}. We shall give a somewhat surprising answer on this based on the recent angular dependent thermal conductivity data of κ-(ET)\textsubscript{2}Cu(NCS)\textsubscript{2} by Izawa et al \textsuperscript{19}. The quasiparticle spectrum of all these new superconductors is well described by the BCS theory for nodal (or unconventional) superconductors \textsuperscript{11,12}. In particular, there are nodal excitations (i.e. the quasiparticles which inhabit in the vicinity of the nodal lines) which persist to low temperatures (i.e. T<< Δ where Δ is the superconducting order parameter). In the vortex state the quasiparticle spectrum is modified due to the supercurrent circling around individual vortices. In order to describe the quasiparticle spectrum in the vortex state, Volovik \textsuperscript{21} has introduced a very simple method to evaluate the effect of the supercurrent within the quasiclassical approximation. In particular, he has shown that the specific heat in the vortex state in nodal superconductors...
is proportional to $\sqrt{H}$ for $H/H_c^2 < 1$ where $H$ is the magnetic field strength. This $\sqrt{H}$ dependence has been seen in YBCO [22,23,24], LSCO [25], $\kappa$-(ET)$_2$ salts [12] and Sr$_2$RuO$_4$ [26,27]. We shall show later that in the presence of impurity scattering the above dependence may change as [28].

\[
\frac{C_s}{\gamma_N T} = \frac{N(0)}{N_0} \left\{ 1 + \frac{\Delta}{\pi T} \frac{1}{x^2} \right\} \\
= \frac{N(0)}{N_0} \left\{ 1 + \frac{3 \tilde{v}^2 (eH)}{16\pi \Gamma \Delta} \left[ \ln \left( \frac{3}{2(eH) \tilde{v}} \right) - \frac{1}{27} \right] \right\} \tag{1}
\]

for Sr$_2$RuO$_4$ in $H//a$-$b$. Similarly for $d$+$s$-wave in $H//a$-$b$,

\[
\frac{C_s}{\gamma_N T} = \frac{N(0)}{N_0} \left\{ 1 + \frac{\tilde{v}^2 (eH)}{8\pi \Gamma \Delta} \left[ (1 - \frac{1}{2} r \cos(2\theta)) \times \ln \left( \frac{\Delta}{\sqrt{\tilde{v}^4 eH}} \right) - \frac{1}{16} (1 - \frac{1}{2} (1 - 2r^2 \cos(4\theta))) \right] \right\} \tag{2}
\]

Here $N(0)$ is the residual density of states in the presence of the impurity scattering and $N_0$ is the one for the normal state. Also $\gamma_N$ is the Sommerfeld coefficient, $\tilde{v} = \sqrt{\frac{\Delta}{\epsilon_{	ext{h}} eH}}$ and $v_a$ and $v_c$ are the Fermi velocities in the $a$-$b$ plane and parallel to the $c$-axis, respectively. For $d$+$s$-wave we took $\Delta(k) \propto \cos(2\phi) - r \cos(2\theta)$ and $\epsilon_{	ext{h}} eH$ is the characteristic magnetic energy. Also when we put $r = 0$ in Eq.(2), we obtain the usual expression for $d$-wave superconductors. The specific heat in the clean limit in LSCO in $H || c$ has been reported recently[30]. The quasiclassical approximation is extended to calculate the thermal conductivity in the vortex state [28,31,32,33,34]. Indeed we can now describe the angular dependent thermal conductivity observed in single crystals of YBCO [35,36,37,38] consistently if we assume that the system is in the superclean limit and $T > \epsilon$ [34].

In the following we shall first review the theory limiting ourselves to the clean limit ($\epsilon \ll \sqrt{\Delta T}$).

As to the result for the superclean limit readers may consult [27,34,39]. Also we consider the phononic thermal conductivity both in Sr$_2$RuO$_4$ and in $\kappa$-(ET)$_2$ salts. The $c$-axis thermal conductivity in Sr$_2$RuO$_4$ [40] appears to be described in terms of the phononic thermal conductivity [41]. As to $\kappa$-(ET)$_2$Cu(NCS)$_2$ it appears the phononic thermal conductivity dominates for $T > 0.5\mathrm{K}$ [19,41]. For $T < 0.47\mathrm{K}$ there appears a clear sign of the electronic contribution. From the angle dependence of the thermal conductivity, we can deduce $d$+$s$-wave ($\Delta(k) \propto \cos(2\phi) - 0.067$) for $\kappa$-(ET)$_2$Cu(NCS)$_2$. In Fig.1 we show $|\Delta(k)|$’s for superconductors a) $s$-wave, (b) $d$-wave as in HTSC, CeCoIn$_5$ [12], $\kappa$-(ET)$_2$ salts [19], (c)2D $f$-wave as in Sr$_2$RuO$_4$ [26,27] and (d)2D $f$-wave as in UPt$_3$ [42,43].
2. Thermal conductivity in the vortex state.

In the following we limit ourselves to nodal superconductors; 2D $f$-wave with $\Delta(k) \propto \cos(\theta) e^{\pm i \phi}$ as in Sr$_2$RuO$_4$ and $d + s$-wave with $\Delta(k) \propto \cos(2\phi) - r$ as in $\kappa$-(ET)$_2$Cu(NCS)$_2$. Also we limit ourselves to the clean limit (i.e. $\sqrt{\Delta T} \gg \epsilon = \hat{v} \sqrt{eH}$, where $\hat{v} = \sqrt{v_a v_b}$) [27]. Then the thermal conductivity for $T \ll \Delta$ and in a planar magnetic field is given by

$$\frac{\kappa_{xx}}{\kappa_0} = 1 + \frac{\tilde{v}_0^2(eH)}{4\pi \Gamma \Delta} \ln(4\sqrt{\frac{2\Delta}{\pi \Gamma}})\ln(\frac{\Delta}{\hat{v} \sqrt{3eH}}) \frac{1}{72}$$

for 2D $f$-wave.

$$\frac{\kappa_{xx}}{\kappa_0} = 1 + \frac{\tilde{v}_0^2(eH)}{6\pi \Gamma \Delta} \ln(4\sqrt{\frac{2\Delta}{\pi \Gamma}})((1 - \frac{1}{2}r^2 \cos(2\theta)) \ln(\frac{\Delta}{\hat{v} \sqrt{3eH}}) - \frac{1}{16}(1 - \frac{1}{2}(1 - 2r^2 \cos(2\theta)))$$

for $d + s$-wave.

Similarly the Hall thermal conductivity is given by

$$\frac{\kappa_{xy}}{\kappa_0} = -\frac{\tilde{v}_0^2(eH)}{24\pi \Gamma \Delta} \sin(2\theta) \ln(2\sqrt{\frac{2\Delta}{\pi \Gamma}})\ln(\frac{\Delta}{\hat{v} \sqrt{3eH}})$$

for 2D $f$-wave, and

$$\frac{\kappa_{xy}}{\kappa_0} = -\frac{\tilde{v}_0^2(eH)}{16\pi \Gamma \Delta} (1 - r^2) \sin(2\theta) \ln(4\sqrt{\frac{2\Delta}{\pi \Gamma}}) \ln(\frac{\Delta}{\hat{v} eH})$$

for $d + s$-wave. Here $\kappa_0$ is the thermal conductivity in the absence of the magnetic field.

3. Phononic thermal conductivity

So far we have considered only the electronic thermal conductivity. In general, the thermal conductivity is written as $\kappa = \kappa_e + \kappa_g$, where the second term is due to phonons. The importance of $\kappa_g$ in high $T_c$ cuprate superconductors, YBCO and Bi2212 have been well-documented [30]. In low temperatures (for $T < 5K$) $\kappa_g \sim T^3$ in single crystals of YBCO and Bi2212 [30]. In high quality crystals phonons are mostly scattered by crystalline defects and crystal boundaries. In the vortex state in nodal superconductors the quasiparticles provide another scattering center. When the thermal phonons are ballistic, the phonon scattering due to the quasiparticles is proportional to $[N(0,\theta)]^2$ at low temperatures [41]. Here $N(0,\theta)$ is the residual density of states in the presence of a magnetic field and $\theta$ refers to the field orientation. Therefore the $c$ axis phononic thermal conductivity $\kappa_g$ in Sr$_2$RuO$_4$ is a planar magnetic field is written

$$\frac{\kappa_g}{\kappa_0} = [1 + \frac{T}{T_0}(1 + \frac{3\tilde{v}_0^2(eH)}{8\pi \Gamma \Delta} \ln(\frac{T}{c \sqrt{eH}}))^{-1}$$

for 2D $f$-wave, and

$$\frac{\kappa_g}{\kappa_0} = [1 + \frac{T}{T_0}(1 + \frac{\tilde{v}_0^2(eH)}{4\pi \Gamma \Delta}(1 - \frac{r}{2} \cos(2\theta)) \ln(\frac{T}{c \sqrt{eH}}))^{-1}$$

for $d + s$-wave, respectively. Here $T_0$ is a constant of the dimension of the energy, and $c$ is the phonon velocity. It is noteworthy that Eq.(8) does not contain the fourfold term.

When the phononic thermal conductivity dominates as in Sr$_2$RuO$_4$ [27], it is more convenient...
(W(H) − W(0))T (K m/W)

\( \mu_0 H (T) \)

T=0.30 K (Suzuki et al.)
Fitted with \( a(H-H_{c1}) \)

Figure 3. The interplane field dependence of the thermal resistance Ref.[47] is compared with \( a(H-H_{c1}) \).

The gap symmetry is the central issue of new superconductors. We have shown the angular dependent thermal conductivity in the vortex state provides a new window to look this question. In this way we have succeeded in identifying the gap symmetry of \( \kappa-(ET)_2 \text{Cu(NCS)}_2 \) and \( \kappa-(ET)_2 \text{Cu(NCS)}_2 \).[4] We expect this method will be very useful to identify the gap symmetry of \( \beta-(ET)_2 \) salts, \( \lambda-(ET)_2 \) salts and other organic superconductors and to clarify existing controversy. Also the success of this method testifies the soundness of both the BCS theory of nodal superconductors and the Volovik’s semiclassical approach to handle the vortex state in nodal superconductors.

4. Concluding Remarks

The gap symmetry is the central issue of new superconductors. We have shown the angular dependent thermal conductivity in the vortex state provides a new window to look this question. In this way we have succeeded in identifying the gap symmetry of \( \kappa-(ET)_2 \text{Cu(NCS)}_2 \) and \( \kappa-(ET)_2 \text{Cu(NCS)}_2 \). We expect this method will be very useful to identify the gap symmetry of \( \beta-(ET)_2 \) salts, \( \lambda-(ET)_2 \) salts and other organic superconductors and to clarify existing controversy. Also the success of this method testifies the soundness of both the BCS theory of nodal superconductors and the Volovik’s semiclassical approach to handle the vortex state in nodal superconductors.

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