Nanoflare Theory Revisited

Amir Jafari1, Ethan T. Vishniac2, and Siyao Xu3

1 Department of Applied Mathematics and Statistics, Johns Hopkins University, Baltimore, MD, USA; elenceq@jhu.edu
2 Department of Physics & Astronomy, Johns Hopkins University, Baltimore, MD, USA; evishni1@jhu.edu
3 Institute for Advanced Study, Princeton, NJ, USA; sxu@ias.edu

Received 2020 April 21; revised 2020 November 9; accepted 2020 November 10; published 2021 January 14

Abstract

At any scale $l$ in the turbulent inertial range, the magnetic field can be divided up into a large-scale component and a small-scale, high spatial frequency component which undergoes magnetic reversals. Such local reconnections, i.e., on any inertial scale $l$, seem to be an inseparable part of magnetohydrodynamic (MHD) turbulence, whose collective outcome can lead to global reconnection with a rate independent of the small-scale physics dominant at dissipative scales. We show that this picture, known as stochastic reconnection, is intimately related to nanoflare theory, proposed long ago to explain solar coronal heating. We argue that, due to stochastic flux freezing, a generalized version of magnetic flux freezing in turbulence, the field follows the flow in a statistical sense. Turbulence bends and stretches the field, increasing its spatial complexity. Strong magnetic shears associated with such a highly tangled field can trigger local reversals and field annihilations on a wide range of inertial scales which convert magnetic energy into kinetic and thermal energy. The former may efficiently enhance turbulence and the latter heat generation. We support this theoretical picture using scaling laws of MHD turbulence and also recent analytical and numerical studies which suggest a statistical correlation between magnetic spatial complexity and energy dissipation. Finally, using an MHD numerical simulation, we show that the time evolution of the magnetic complexity is statistically correlated with the rate of kinetic energy injection and/or magnetic-to-thermal energy conversion, in agreement with our proposed theoretical picture.

Unified Astronomy Thesaurus concepts: Astrophysical fluid dynamics (101); Solar magnetic fields (1503); Stellar magnetic fields (1610); Solar magnetic reconnection (1504); Stellar accretion disks (1579); Jets (870)

1. Introduction

Over half a century ago, Grotrian (1939) and Edlén (1946) pointed out that the unexpected emission lines detected in the spectrum of the solar corona indicated a very high ionization, which would require extremely high temperatures of order $10^{6} K$. One implication is that the solar corona is much hotter than the lower layers, which are much closer to the Sun. This phenomenon, known as the coronal heating problem (see, e.g., Einaudi et al. 1996; Califano & Chiuderi 1999; Romeou et al. 2009) is puzzling of course because one expects the temperature to drop off with distance from the solar surface. Several mechanisms have been proposed to resolve this theoretical difficulty; however, it is still the subject of ongoing research. Interestingly, though, almost all of these models rely, in one way or another, on a common phenomenon—magnetic fields.

Alfvén wave dissipation and magnetohydrodynamic (MHD) shocks (Morinay et al. 2004; McIntosh et al. 2011), magnetic reconnection (Roald et al. 2000; Liu et al. 2002; Aulanier et al. 2007; Hood et al. 2009), and MHD turbulence (Matthaeus et al. 1999; Cranmer et al. 2007; Rappazzo et al. 2007) are among the proposed mechanisms to solve the coronal heating problem. Each model is often backed up by a few numerical simulations, as is usual nowadays, yet there is no consensus on which processes play a more fundamental role. A typical model identifies a source of energy, e.g., magnetic energy or energy carried by Alfvén waves, which is finally converted into thermal energy. The second component is a mechanism to do the energy conversion, e.g., MHD turbulence damping Alfvén waves which generates heat. It is highly plausible, on the other hand, to think that several mechanisms work together to give rise to such a bizarre situation. One should keep in mind that real astrophysical systems, including the solar corona, are much more complicated than what a simple theoretical picture may present based on a few physical mechanisms. In any case, magnetic fields seem to play a very important role in theories that attempt to explain the coronal heating problem (Golub et al. 1980; Heyvaerts & Priest 1984).

As one of the mechanisms possibly responsible for, or else at least partly contributing to, the solar coronal heating phenomenon, magnetic reconnection has frequently been invoked through analytical, observational, and numerical studies (see, e.g., Biskamp 2005; Priest & Forbes 2007; Lazarian et al. 2020). Recent advances in both observations, with high spatial and temporal resolution, and numerical simulations in studying the solar coronal heating problem, are summarized by De Moortel & Browning (2015). See also reviews by, e.g., Priest (1999) and Low (2003) on the role of magnetic reconnection in the coronal heating and solar coronal phenomena, and Parnell & De Moortel (2012) for different coronal heating mechanisms. As an example of recent observations, Longcope & Tarr (2015) quantified the contribution of magnetic reconnection to the coronal heating for one special case of an active region (AR 11112) by measuring the rate of magnetic reconnection and the rate of energy dissipation in the solar corona. Extrapolating the result to other regions, they concluded that magnetic reconnection can in fact account for the measured temperatures. More recently, the fragmented and turbulent nature of magnetic reconnection has also been confirmed by observations of a superhot current sheet during the SOL2017-09-10T X8.2-class solar flare (Cheng et al. 2018). Serving as other observational evidence for the role of reconnection in the solar coronal heating, Yang et al. (2018) used extreme-ultraviolet observations to argue that the impulsive reconnection is responsible for...
the active region coronal heating. In another recent work, based on high-cadence observations from the Interface Region Imaging Spectrograph, Chitta & Lazarian (2020) identified the important role of turbulence in triggering fast reconnection and driving solar microflares. Numerical simulations, too, have emphasized the role of reconnection as a major component in the coronal heating process. For instance, Kanella & Gudiksen (2017) identified individual heating events in 3D MHD simulations of the solar corona and the corresponding released energy rate and volume ranges. Their results suggest the stochastic nature of magnetic reconnection (Lazarian & Vishniac 1999; Eyink et al. 2013; Jafari & Vishniac 2019; Lazarian et al. 2020) in releasing a random fraction of the energy stored in the magnetic fields as heat. The kinetic particle-in-cell simulations performed by Shay et al. (2018) to study the heating effects of magnetic reconnection showed that the statistics of the turbulent reconnection is important for determining the ion and electron heating. In a recent review, Vlahos & Isliker (2019) provided evidence from numerical simulations showing that the turbulent reconnection with spontaneous formation of current sheets in the solar corona drives both coronal heating and particle acceleration.

Magnetic reconnection generates fast, explosive motions in magnetized fluids, thereby enhancing particle diffusion at large scales. It may also start a turbulent cascade in an initially quiet medium or else it may help the present turbulence by injecting kinetic energy at large scales, typically of order the scale of the reconnection zone (Kowal et al. 2017; see also Servidio et al. 2009, 2010; Shay et al. 2018). In the stochastic model of reconnection (Lazarian & Vishniac 1999), energy is basically injected on a range of scales, which provides a more efficient way to enhance turbulence as we will show in the present paper. On the other hand, it is well known that resistivity is not enough to generate appreciable heat in typical astrophysical systems. It is true that the Sweet–Parker model (Parker 1957; Sweet 1958) predicts a much faster magnetic energy conversion rate than what is achievable by magnetic dissipation alone; however, it is still very slow (Yamada et al. 2010; Jafari & Vishniac 2018a). Although a small, but of course finite, resistivity is required for stochastic reconnection to start and proceed, neither the reconnection rate nor its underlying mechanism depends on resistivity (Lazarian et al. 2020). The major role, instead, is played by the turbulence. In typical reconnection models, a global reversal converts magnetic energy into kinetic energy and pumps it into the medium at large scales. This will generally enhance the diffusion process in the fluid at large scales, not necessarily generating a fully developed turbulence. In addition, such large-scale motions are generated even in occasional global reconnections. In contrast, local stochastic reconnections over a continuum of scales can generate small-scale motions, efficiently enhancing the local turbulent cascade even in the absence of a global field reversal. This process constitutes a more effective way of generating, enhancing, and maintaining turbulence, as we will argue in the following sections. As discussed before, the topological dissipation of stochastic magnetic fields has been identified as an alternative mechanism of coronal heating (Parker 1972, 1983, 1988; Levine 1974).

The dissipation of magnetic energy via magnetic reconnection may occur at many small-scale tangential discontinuities (current sheets), which are caused by the photospheric footpoint motions (Parker 1987). These ubiquitous impulsive heating events are referred to as nanoflares. Parker (1972, 1983, 1988) also suggested that the initially slow reconnection can be enhanced by hydromagnetic and plasma turbulence followed by an explosive reconnection phase. The nanoflare model (Parker 1988) for coronal heating has been further investigated both analytically and numerically by different authors; see, e.g., Einaudi & Velli (1994, 1999), Cargill & Klimchuk (2004), Rappazzo et al. (2008, 2019), Parnell et al. (2010), Bowness et al. (2013), Rappazzo & Parker (2013), and Jess et al. (2019).

In this paper, using both analytical and numerical considerations, we argue that local reversals in MHD turbulence can be studied in a statistical framework which unifies reconnection and other magnetic phenomena, in particular the magnetic heating process invoked in the nanoflare theory. We illustrate that stochastic reconnection, as the global outcome of many simultaneous local reversals, is more efficient than conventional models in enhancing turbulence and heat generation. Finally, we show that, even in the absence of a global reconnection at larger scales of order than the system’s scale, the local reversals at comparatively smaller scales in the inertial range can maintain and enhance the turbulence and also the process of heat generation at much smaller scales in the dissipative range. The theoretical picture we invoke to advance our arguments can be summarized as follows. Turbulence stretches and bends the magnetic field, which follows the turbulent flow in a statistical sense due to stochastic flux freezing (Eyink 2011), producing magnetic gradients at random regions in the turbulence inertial range. The resultant magnetic shears can give rise to local, small-scale magnetic reconnection events whose collective outcome may lead to a global reconnection event (Lazarian & Vishniac 1999; Lazarian et al. 2020; see also Matthaeus & Lamkin 1986 for reconnection in 2D MHD turbulence). As the magnetic field gets stretched, bent, and tangled by the turbulence, its spatial complexity increases in a geometric sense (Jafari & Vishniac 2019). The field resists tangling because of the magnetic tension forces, which at some point make the field slip through the fluid to relax (Jafari & Vishniac 2019; Jafari et al. 2020b).

The relaxing field may in turn accelerate particles (de Gouveia dal Pino & Lazarian 2005; Kowal et al. 2012a; Khiali et al. 2015; Beresnyak & Li 2016; Lu et al. 2020), producing jets of fluid. Hence, during reconnection, the spatial complexity of the velocity field increases while that of magnetic field decreases after reaching a local maximum. Previous work has in fact quantified the level of spatial complexity associated with magnetic and velocity fields (Jafari & Vishniac 2019; Jafari et al. 2019, 2020b) with the implication that these local reconnection events are ubiquitous in MHD turbulence. Using this novel statistical picture, we show that these local events can generate or enhance an existing turbulent cascade and also boost the magnetic-to-thermal energy conversion down the cascade. A simple numerical study is also presented to illustrate the statistical correlation between the time evolution of magnetic spatial complexity and magnetic energy conversion rate. This picture in fact reformulates the theory of coronal heating by local nanoflares, proposed by Parker (1988) to explain the solar coronal heating, in a statistical picture connecting it to the stochastic model of magnetic reconnection (Lazarian & Vishniac 1999), and also the recent topological and statistical studies of turbulent magnetic fields (Jafari & Vishniac 2019, 2020; Jafari et al. 2020b).
We begin by revisiting dissipative anomalies and stochastic reconnection in Section 2 to illustrate how magnetic flux freezing fails in turbulence, which is intimately related to stochastic reconnection. These considerations are already well established and our emphasis here is due to the fact that they play a major role in the development of the ideas presented in this paper. For a detailed review of stochastic reconnection and stochastic flux freezing, see, e.g., Jafari & Vishniac (2018a) and Lazarian et al. (2019, 2020). In Section 3, which presents the main results of this paper, we use the notion of vector field complexity to argue that local reversals involved in stochastic reconnection are efficient in both maintaining the local turbulent cascade and also heating the fluid. To support these statistical arguments, we also use simple scaling laws of the Goldreich–Sridhar MHD turbulence model (Goldreich & Sridhar 1995, 1997). We also test our main results using an incompressible, homogeneous MHD turbulence numerical simulation. Finally, in Section 4, we summarize and discuss our results.

2. MHD Turbulence

In this section, we present a brief review of the tools required to study stochastic heating in turbulent fluids, including dissipative anomalies in incompressible fluids (Eyink 2018; Jafari & Vishniac 2019), which make the turbulent velocity field Hölder singular (see below), and the failure of flux freezing in MHD turbulence (Eyink 2011; Eyink et al. 2013). We also briefly revisit stochastic magnetic reconnection (Lazarai & Vishniac 1999; Jafari & Vishniac 2018a; Lazarai et al. 2019, 2020).

2.1. Dissipative Anomalies

In a magnetized fluid with a large characteristic length scale \( L \), a large characteristic velocity \( U \), or a tiny viscosity \( \nu \), the Reynolds number \( \text{Re} = LU/\nu \) can be very large. If the magnetic diffusivity \( \eta \) is of the same order as the viscosity \( \nu \), implying a magnetic Prandtl number of order unity \( \text{Pr}_m = \nu/\eta \sim 1 \), the magnetic Reynolds number \( \text{Re}_m = LU/\eta \) will also be large. In order to see what a large kinetic Reynolds number means, we can re-write the momentum equation in the common notation, using the parameters

\[
\mathbf{x} = x/L, \quad t' = t/(L/U), \quad \mathbf{u} = u/U, \quad \rho = p/U^2,
\]

in a dimensionless form as

\[
\frac{\partial \mathbf{u}}{\partial t'} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \rho + \frac{1}{\text{Re}} \nabla^2 \mathbf{u}.
\]

Let us assume an incompressible fluid; \( \nabla \cdot \mathbf{u} = 0 \). Apparently, as \( \text{Re} = LU/\nu \) increases, by either increasing the system’s characteristic size or velocity or decreasing the viscosity, the last term in the momentum equation tends to vanish. This might for example justify ignoring a small viscosity altogether in some cases, but not always. As the Reynolds number increases, i.e., \( \text{Re} \rightarrow \infty \), the flow becomes unstable: like a pen balanced on its tip, any small perturbation would lead to turbulence (Monin & I’Aglom 1971; Stuart 1986). This is why the initially slow and laminar flow coming out of a faucet would become turbulent at some point if we kept increasing the flow velocity \( U \) (i.e., increasing \( \text{Re} \)). Indeed, large Reynolds numbers, frequently encountered in astrophysical fluids, are typically associated with turbulence. On the other hand, numerous numerical simulations and experiments have shown (Sreenivasan 1984, 1998; Eyink & Sreenivasan 2006; Eyink & Eyink 2018) that in turbulence the kinetic energy dissipation rate \( \epsilon_k(t) = \nu |\nabla \mathbf{u}|^2 \) does not approach zero when viscosity tends to vanish; rather it approaches a non-zero constant \( \lim_{\nu \to 0} \nu |\nabla \mathbf{u}|^2 \to \epsilon_k > 0 \)—the phenomenon of dissipation anomaly (Leray 1934; Onsager 1949; Duchon & Robert 1999; Galtier 2018). Thus the velocity gradients should diverge in the limit of vanishing viscosity, \( \lim_{\nu \to 0} |\nabla \mathbf{u}| \to \infty \), to keep \( \nu |\nabla \mathbf{u}|^2 \) constant. With diverging and ill-defined velocity gradients, hydrodynamics equations will consequentially become ill-defined in ideal turbulence; for more details see, e.g., Eyink (2018) and Jafari & Vishniac (2019). Incidentally, we should note that the limit \( \nu \to 0 \) (or equivalently \( \text{Re} \to \infty \)) is just the mathematical translation of the physical statement that one can take an arbitrarily small viscosity (or an arbitrarily large \( \text{Re} \)): viscosity is not required, or assumed, to vanish: viscosity never vanishes in most astrophysical plasmas but it can be taken as small as one wishes.

Similar to the momentum equation, the induction equation governing the evolution of magnetic field \( \mathbf{B} \), with a characteristic strength \( \mathbf{B} \), can be written in a dimensionless form as follows:

\[
\frac{\partial \mathbf{B}}{\partial t'} = \frac{1}{\text{Re}_m} \nabla^2 \mathbf{B} + \nabla \times \left( \mathbf{u} \times \mathbf{B} \right),
\]

where \( \mathbf{B} = \mathbf{B}/B \). In turbulence, the magnetic dissipation rate \( \epsilon_m(t) = \eta |\nabla \mathbf{B}|^2 \) does not approach zero as the diffusivity tends to vanish, i.e., \( \lim_{\eta \to 0} \eta |\nabla \mathbf{B}|^2 \to 0 \) (magnetic dissipation anomaly). Magnetic field gradients diverge, \( |\nabla \mathbf{B}| \to \infty \), and MHD equations become ill-defined as a result; see, e.g., Eyink et al. (2013) and Jafari & Vishniac (2019). It is physically naive and mathematically incorrect, therefore, to ignore viscosity altogether and use ideal fluid equations in real fluids, unless we apply careful measures to keep the Reynolds number small to avoid the development of turbulence. Likewise, a vanishingly small magnetic diffusivity cannot justify ignoring the diffusivity altogether. If turbulence is developed, magnetic and velocity field gradients will typically become ill-defined or singular, i.e., the field gradients will diverge. If we insist on using MHD equations, which we do, we would first have to remove these singularities. One way to do so is to smooth the fields or, in other words, to use the average velocity field \( \mathbf{u}(x, t) \) or magnetic field \( \mathbf{B}(x, t) \) in a parcel of fluid of length scale \( l \) located at the spacetime point \( (x, t) \) instead of using the bare mathematical fields \( \mathbf{u}(x, t) \) and \( \mathbf{B}(x, t) \). This simple coarse-graining methodology, to be revisited in Section 3, can be applied to any scalar or vector field in MHD turbulence (Jafari & Vishniac 2019; see also Eyink 2005; Lee & Sung 2011; Aluie 2017).

\[4\] Mathematically, this means that these vector fields become Hölder singular instead of being Lipschitz continuous. For a Lipschitz function \( f(x) \), the slope (derivative) at any point of the domain has an upper bound, i.e., there is a positive constant \( f_1 \) such that \( |f(x_1) - f(x_2)| < f_1 |x_1 - x_2|^h \) with \( h = 1 \). For Hölder functions \( 0 < h < 1 \), which means that the slope can increase indefinitely. Generalization to vector fields is straightforward: the field \( \mathbf{B}(x) \) satisfying \( |\mathbf{B}(x) - \mathbf{B}(y)| < B_0 |x - y|^h \), with \( B_0 > 0 \), is Lipschitz continuous if \( h = 1 \), and Hölder singular if \( 0 < h < 1 \). In the latter case, \( \nabla \mathbf{B} \) will in general become ill-defined.
2.2. Failure of Flux Freezing

One important implication of the above considerations, as far as the reconnection of turbulent magnetic fields is concerned, is the breakdown of the standard Alfvén flux-freezing law (Alfvén 1942) in turbulent systems. If the flow remains laminar but the diffusivity $\eta$ is very small, under certain conditions the diffusive term $\eta \nabla^2 B$ may be ignored in the bare (i.e., not coarse-grained) induction equation, $D_t B = B \nabla u - B \nabla u + \eta \nabla^2 B$, with Lagrangian derivative $D_t \equiv (\partial_t + u \nabla)$. Thus, using the continuity equation $D_t \rho + \rho \nabla u = 0$, one finds $D_t (B/\rho) = (B/\rho) \nabla u$, which means that the magnetic field is frozen into the fluid, i.e., the integral curves of $B/\rho$ are advected with the fluid and the field follows particle trajectories. However, at least in most astrophysical systems, a vanishingly small diffusivity (i.e., a large $Re_m$) will typically be accompanied with a small viscosity, which translates into large kinetic Reynolds numbers, i.e., turbulence. Hence, the induction equation used in the above derivation of ideal flux freezing will not remain well-defined because of the blow-up of velocity and magnetic gradients. All other derivations of the Alfvén flux-freezing law, in a similar way, assume that the induction equation (and/or other MHD equations) are well-defined, thus neither such derivation guarantees the validity of flux freezing in turbulence. Indeed the Alfvén flux-freezing theorem fails in turbulence. Particle trajectories are random in turbulent flows, therefore the magnetic field which tends to follow these trajectories will become a stochastic (random) field; see, e.g., Jafari & Vishniac (2019) and references therein. It is possible, however, to generalize the standard flux freezing to stochastic fields in turbulence using a little more advanced mathematics. The result, called stochastic flux freezing developed by Eyink (2011), states that magnetic field will follow the random particle trajectories in a statistical sense; see also Eyink et al. (2013) and Eyink (2015).

2.3. Stochastic Reconnection

Reconnection rate in a laminar flow can be estimated, or defined, in terms of normal diffusion of the magnetic field by magnetic diffusivity $\eta$ on large scales or, in other words, in terms of the (root-mean-square, henceforth rms) average distance $\delta(t)$ the field spreads relative to a fixed point. The related random walk of magnetic field lines was discussed by Jokipii (1966); see also Snodin et al. 2013, 2016; Servidio et al. 2014. This is of course the Taylor or normal diffusion (of particles or magnetic field) in which the rms distance between the diffusing material and a fixed point increases as $\delta \sim t^{1/2}$ with time; see, e.g., Eyink et al. (2013) and Jafari et al. (2019). For a diffusing magnetic field, $\delta^2 \approx \eta t$. In the absence of turbulence, in a reconnection zone of width $\delta$ and length $\Delta$ (parallel to the local magnetic field), using the Alfvén timescale $t_A = \Delta/V_A$, and using mass conservation $V_A \delta = V_R \Delta$, we recover the reconnection speed

$$V_R \approx \eta V_A / \Delta^{1/2}. \quad (1)$$

Figure 1. Top: Richardson superdiffusion occurs in the turbulence inertial range. The rms width of a bundle of magnetic field lines increases super-linearly with time $\sim t^{1/2}$ (two-particle diffusion). The magnetic field follows the flow in a statistical sense, hence turbulence creates local current sheets by tangling the magnetic field. The reconnecting small-scale fields at multiple local current sheets diffuse super-linearly to larger scales and may give rise to a global reconnection event—stochastic reconnection. Turbulence also increases the spatial complexity of the field which is statistically frozen into the fluid; see Section 3.1. Reconnection relaxes the field decreasing its complexity level. Bottom: Taylor (normal) diffusion occurs in laminar flows, or at scales much larger than the turbulence inertial range. The rms distance of magnetic field lines from a fixed point increases sublinearly with time $\sim t^{1/2}$ (one-particle diffusion). The Sweet–Parker model corresponds to Taylor diffusion.

This is, of course, the well-known Sweet–Parker reconnection rate (Parker 1957; Sweet 1958). Reconnection, and/or other instabilities such as tearing modes (Furth et al. 1963), will in general generate turbulence (Eastwood et al. 2009; Jafari & Vishniac 2018a), with the implication that the laminar Sweet–Parker model is far from realistic in turbulent systems such as most astrophysical fluids. In the turbulence inertial range, i.e., at scales larger than dissipative scale but smaller than the larger scales where Taylor (normal) diffusion of the magnetic field occurs, diffusing particles will undergo super-linear Richardson diffusion; $d^2 \propto t^3$ which is a two-particle diffusion, i.e., $d$ is the rms separation between any pair of particles undergoing diffusion in the inertial range. If we consider magnetic diffusion in the turbulence inertial range, we have to consider Richardson diffusion of the field, in terms of the rms distance the field spreads during the time $t$; see Figure 1. The eddy turnover time, in the inertial range, is of order $t \sim \epsilon^{-1} d^{2/3}$ with $d$ being the length scale perpendicular to the mean magnetic field. Here, $\epsilon \approx V_T^2 V_A / l_i$ denotes the energy transfer rate, with turbulent velocity $V_T$ and parallel energy injection length scale $l_i$. This corresponds to the Richardson diffusion; $d^2 \approx \epsilon t^3$. The super-linear nature of Richardson diffusion broadens the reconnection zone and thereby enhances the reconnection rate. To see this, using mass conservation $V_A d = V_R \Delta$, and substituting the Alfvén time $t_A = \Delta/V_A$, one arrives at the fast reconnection speed (Lazarian & Vishniac 1999; Jafari et al. 2018; Lazarian et al. 2019, 2020);

$$V_R \sim V_T \min \left[ \left( \frac{\Delta}{l_i} \right)^{1/2}, \left( \frac{l_i}{\Delta} \right)^{1/2} \right]. \quad (2)$$

For instance, in highly ionized accretion disks, in which the magneto-rotational instability is thought to be active, $Pr_m$ is usually assumed to be of order unity (Lesur & Longaretti 2009; Murphy et al. 2009; Jafari & Vishniac 2018b) while it is much smaller in planetary and stellar interiors. In any case, at least in astrophysics, huge kinetic Reynolds numbers are typically accompanied by huge magnetic Reynolds numbers.
Depending on the parallel (with respect to the local field) length scale of the current sheet, i.e., \( \Delta \), and the parallel energy injection scale \( l_i \), the smaller ratio, either \((\Delta/l_i)^{1/2}\) or \((l_i/\Delta)^{1/2}\), should be taken in the above formula. This reconnection speed is of order the large turbulent eddy velocity \( V_T \), independent of diffusivity and is in agreement with numerical simulations to date (Kowal et al. 2009, 2012b). The stochastic model of reconnection was also examined with a large viscosity-to-diffusivity ratio in a recent work (Jafari & Vishniac 2018a; Jafari et al. 2018).

3. Stochastic Heating

In this section, we present the main results of this paper. First, in Section 3.1, employing the recent statistical formalism developed by Jafari & Vishniac (2019), we argue that local magnetic reversals are ubiquitous in MHD turbulence, which continuously convert magnetic energy into kinetic and thermal energy; see also Jafari et al. (2020b). Then, we use simple scaling laws of MHD turbulence in Section 3.2 to support the idea that these local events are efficient in maintaining the turbulent cascade in the inertial range and heat generation at smaller scales down the inertial range. Finally, in Section 3.3, we numerically test our theoretical prediction that magnetic complexity’s rate of change should be statistically correlated with magnetic energy dissipation rate \( \eta |\nabla B|^2 \) and/or the rate of change of the kinetic energy.

3.1. Statistics of Local Reversals

A fluid parcel of an arbitrary size \( l \) located at spacetime point \((x, t)\) has an average velocity

\[
\mathbf{u}_l(x, t) = \int_V G(r/l) \mathbf{u}(x + r, t) \frac{d^3r}{l^3},
\]

where \( V \) is the volume of interest whose length scale is at least of order the largest scales in the inertial range of the turbulence and \( G(r) = G(r) \) is a smooth and rapidly decaying kernel, e.g.,

\[
G(r) = G(r) \sim e^{-(r/\Delta)^2} \text{ for } |r| \leq 1 \text{ and } G = 0 \text{ for } |r| > 1.
\]

The real mathematical field \( \mathbf{u} \) is sometimes called the bare field while \( \mathbf{u}_l \) is called the renormalized, or coarse-grained, field at scale \( l \). Instead of the vector field \( \mathbf{u}(x, t) \) which mathematically assigns a unique velocity to the point \((x, t)\) in space and time, we consider the average velocity of a fluid parcel of size \( l \) located at \((x, t)\). Hence, we ignore the fact that the parcel itself is made of many particles that are below our resolution scale \( l \). In other words, we look at the fluid with our spectacles off in the sense that we cannot observe or resolve scales smaller than \( l \).

The interaction between a turbulent flow and the magnetic field threading the fluid can be understood in terms of stochastic flux freezing and the spatial complexity of the velocity and magnetic fields. A simplified picture can be described as follows (Jafari & Vishniac 2019; Jafari et al. 2019, 2020b).

(i) Stochastic flux freezing. The magnetic field will tend to become increasingly tangled as it statistically follows the turbulent flow. This is an implication of stochastic flux freezing

\[ S_m(t) = \frac{1}{2} \langle \hat{B}_l \hat{B}_l \rangle - 1 \]

(Eyink 2011) on a range of inertial scales \([l, L]\) with \( L > l \). The magnetic spatial complexity, quantified by the function

\[
S_m(t) = \frac{1}{2} \left( \langle \hat{B}_l \hat{B}_l \rangle - 1 \right)^{1/2} \left( \frac{d^3x}{V} \right)^{1/2}
\]

with unit direction vector \( \hat{B}_l \equiv B_l / B_0 \), will increase over time by the turbulent motions until it reaches a maximum, at which point magnetic tension forces are strong enough to resist further tangling of the field; see Figure 2.

(ii) Field–fluid slippage. Magnetic complexity increases until the tension forces associated with large magnetic field curvatures become strong enough to make the field suddenly slip through the fluid (Eyink 2015; Jafari & Vishniac 2019). Such field–fluid slippages should be a ubiquitous phenomenon in astrophysical systems in which magnetic fields, although typically very complex geometrically, could have survived for millions of years without being infinitely tangled. Field–fluid slippage corresponds to a sudden drop in magnetic complexity \( S_m \) after it reaches a maximum. The corresponding magnetic complexity

\[
S_m(t) = \frac{1}{2} \left( \langle \hat{B}_l \hat{B}_l \rangle - 1 \right)^{1/2} \left( \frac{d^3x}{V} \right)^{1/2}
\]

with unit direction vector \( \hat{B}_l \equiv B_l / B_0 \), will increase over time by the turbulent motions until it reaches a maximum, at which point magnetic tension forces are strong enough to resist further tangling of the field; see Figure 2.

8 The motivation behind the definitions (4) and (6) is as follows: the renormalized field \( \mathbf{u}_l(x, t) \) represents the average velocity of a fluid parcel of size \( l \) at \( x \). Because \( G(r/l) \) is a rapidly decaying function (with compact support), the integral \( \mathbf{u}_l(x, t) = \int_V G(r/l) \mathbf{u}(x + r, t) d^3r/l^3 \) gets much smaller contributions from distant points located at \( x \sim l \). The large-scale field \( \mathbf{u}_L \) with \( L > l \) is the average velocity field of a fluid parcel of scale \( L \) at the same point \( x \). In a laminar flow whose velocity field has a large curvature radius \( \gg L \), we expect \( \mathbf{u}_L \sim \mathbf{u} \). For a stochastic velocity field in a fully turbulent medium, however, \(-1 \leq \mathbf{u}_L \leq 1 \) becomes a rapidly varying stochastic variable. This quantity thus measures the spatial complexity (or stochasticity level) of \( \mathbf{u} \) at point \( x \). Its rms value tells us how spatially complex (or stochastic) the velocity field is on average in a given volume \( V \). In order to obtain a non-negative global quantity, we can volume average \( \frac{1}{2} \left( \langle \mathbf{u}_L \mathbf{u}_L \rangle - 1 \right)^{1/2} \left( \frac{d^3x}{V} \right)^{1/2} \). Magnetic complexity is defined similarly.
cross-energy, which is defined as the geometric mean
\[ E_m(t) = \left( \frac{B_t^2 + B_l^2}{2} \right)_{\text{rms}} = \left( \frac{1}{2} B_t B_l \right)_{\text{rms}}, \tag{5} \]
will tend to decrease (increase) as the magnetic complexity \( S_m(t) \) increases (decreases). In passing, note that the magnetic complexity \( S_m \) and cross-energy \( E_m \), respectively given by (4) and (5), are basically obtained from a scalar field \( \psi(x, t) = \frac{1}{2} B_t B_l - \frac{1}{2} B_t^2 \), called scale-split magnetic energy density (Jafari & Vishniac 2019).

Figure 3 plots such a typical relationship between \( S_m \) and \( E_m \) (and also the rms magnetic energy density \( (B^2/2)_{\text{rms}} \)) in a typical subvolume of the simulation box of an incompressible, homogeneous numerical simulation (Jafari et al. 2020b). The anticorrelation between magnetic spatial complexity and magnetic energy density implies, in a fully developed turbulence, the higher the magnetic complexity, the more efficient the magnetic energy conversion.

(iii) Local reversals. If the field–fluid slippage is strong enough such that the relaxing field accelerates fluid elements efficiently, converting magnetic energy into kinetic energy, the resultant eruptive, spontaneous fluid motions will increase the spatial complexity of the velocity field, which is defined by
\[ S_k(t) = \frac{1}{2} (\vec{u}_t \cdot \vec{u}_l - 1)_{\text{rms}}. \tag{6} \]

Thus \( \partial_t S_k(t) \) will take positive values as \( S_m(t) \) reaches its maxima (at times for which \( \partial_t S_m = 0 \) & \( \partial_t^2 S_m < 0 \)). Figure 4 illustrates this typical behavior between \( \partial_t S_m \) and \( \partial_t S_k \) in the same subvolume as in Figure 3 (with size \( 194 \times 42 \times 33 \) in grid units, equivalent to \( 1.2 \times 0.26 \times 0.20 \) in code units). The corresponding cross-energy is defined as
\[ E_k(t) = \left( \sqrt{\frac{u_t^2 + u_l^2}{2}} \right)_{\text{rms}} = \left( \frac{1}{2} u_t u_l \right)_{\text{rms}}. \]

The kinetic cross-energy \( E_k(t) \) and the kinetic complexity \( S_k(t) \) are in fact defined using the scale-split kinetic energy density \( \Psi(x, t) = \frac{1}{2} u_t u_l - \frac{1}{2} u_t^2 \), which is a scalar field (Jafari & Vishniac 2019). Incidentally, note that the acceleration of fluid particles by a slipping magnetic field ultimately results from Lorentz forces \( \mathbf{F} = (j \times \mathbf{B}) - \mathbf{j} \times \mathbf{B} \), with electric current \( j \) acting on the fluid elements at an arbitrary scale \( l \). The reconnection power on an arbitrary range of inertial scales \([l, L]\), defined as \( P = \frac{1}{2} (u_t \mathbf{N}_t + u_l \mathbf{N}_l)_{\text{rms}} \), is therefore expected to be statistically correlated with the rate of change of the kinetic energy \( \partial_t \Psi \), see Figure 5.

In short, on the one hand, the interplay between turbulence and magnetic field results in rapid temporal variations in magnetic complexity \( S_m \) in an arbitrary spatial volume \( V \). A sudden decrease in \( S_m \) indicates the presence of field–fluid slippage and/or local magnetic reversals, which are indeed observed almost everywhere in the inertial range of MHD turbulence (Eyink et al. 2013; Eyink 2015; Jafari et al. 2020b; see also Servidio et al. 2009, 2010 for the case of 2D MHD turbulence). On the other hand, the magnetic complexity is anti-correlated with magnetic energy (Jafari & Vishniac 2019; Jafari et al. 2020b), therefore these ubiquitous local reversals imply magnetic energy conversion. The range of scales \([l, L]\) is arbitrary in the above arguments. Hence, at larger scales in the inertial range, these reversals will in general enhance turbulent diffusion whereas at the smaller scales they will enhance the
heating process in the dissipative range. All in all, this picture suggests that the magnetic field in a turbulent fluid will be spatially complex, in the sense that there will exist intense local magnetic shears which either annihilate the magnetic energy and/or cause local reversals. In fact, the diffusion of these small-scale effects by means of super-linear Richardson diffusion in turbulence (Jafari et al. 2019) can lead to a global reconnection event—stochastic reconnection (Lazarian & Vishniac 1999; Eyink et al. 2013). In the next subsection, we use scaling laws in MHD turbulence to show that these reversals are indeed efficient in both maintaining the turbulence on an arbitrary inertial scale \( l \) and also enhancing the magnetic-to-thermal energy conversion at smaller scales.

### 3.2. Scaling Laws and Local Reversals

In the preceding subsection, based on analytical and numerical arguments, we reasoned that local reversals occur frequently in MHD turbulence. In this subsection, we use conventional scaling laws in MHD turbulence to support our previous arguments, and also to show the efficiency of local reversals in enhancing the turbulence and heating the medium. For simplicity, in this section, we will use the Goldreich–Sridhar model (Goldreich & Sridhar 1995, 1997) of MHD turbulence, in particular the critical balance condition given by Equation (7) below.

Let us consider a local reconnection event to see how it interacts with the local turbulent cascade. Suppose that turbulence is generated by energy injection at some scale \( l \), which creates an rms turbulent velocity \( V_T \) at the largest scales of the ensuing cascade. This energy can be injected by, e.g., a global reconnection, or the source may be external. In any case, the kinetic energy of large-scale motions, \( V_T^2 \), will be much larger than that associated with any smaller-scale \( k^{-1} \), which we denote by \( \nu_k^2 \). Thus, in order to enhance, or sustain for that matter, the turbulence at a scale \( \lambda < l \), a local reconnection event would only have to inject a small amount of energy \( \lesssim \nu_k^2 \), much smaller than that contained at larger scales, \( \nu_k^2 < V_T^2 \). The available energy for this local event comes from the local magnetic energy, \( b^2 \). Since the local mean magnetic energy, unlike the kinetic energy, depends only slightly on scale, a local reconnection has enough magnetic energy to provide the local turbulence with a kinetic energy comparable to the turbulent energy at that scale. Stochastic reconnection feeds turbulence at all scales.

In order to quantify the above argument, we start by noting that MHD turbulence is anisotropic in general. Suppose energy is injected at a scale \( l_i \), parallel to the mean magnetic field, with corresponding perpendicular scale \( l_p = l_i(V_T/\nu_k) \), which creates the rms turbulent velocity \( V_T \) at this scale.\(^{10}\) The timescale corresponding to the nonlinear energy transfer rate in the turbulence cascade, in Goldreich–Sridhar MHD turbulence (Goldreich & Sridhar 1995), is given by\(^ {11} \) \( \tau \approx k_i V_A/(k_i^2 V_T^2) \) with the rms eddy velocity \( v_k \), which is clearly scale-dependent as its subscript indicates (Lazarian & Vishniac 1999). The critical balance condition,

\[
k_l V_A \approx k_l v_k
\]

then leads to the energy transfer rate

\[
e \approx \frac{V_T^2}{(l/l_A)^2} \tau \approx \frac{v_k^2}{\tau} \approx k_l v_k^3.
\]

The assumption of constant energy transfer rate (Kolmogorov 1941), \( e \approx k_l v_k^3 \approx \text{const.} \), leads to \( v_k \propto k_l^{-1/3} \), corresponding to the famous, Kolmogorov-type, energy power spectrum \( E_{kG}(k_l) \propto k_l^{-5/3} \) in the Goldreich–Sridhar MHD turbulence model. Putting this together, we find

\[
v_k \approx V_T \left( k_l l_A V_T V_A \right)^{-1/3},
\]

and

\[
k_l \approx l_A^{-1} \left( k_l l_A V_T V_A \right)^{2/3}.
\]

Let us first focus on a local reconnection at a parallel scale \( k_l^{-1} < l_i \), which is still much larger than the dissipation scale. Mass conservation leads to a local reconnection speed of order \( v_R \approx V_A(k_i/k_l) \). This is of order the local turbulent velocity \( v_k \), if we use the critical balance condition in the Goldreich–Sridhar model (Goldreich & Sridhar 1995); \( v_R \propto v_k \). The ejection velocity \( v_e \) will be in general larger than the local reconnection velocity, \( v_e > v_R \), since \( v_e/\nu_k \approx k_i/k_l > 1 \). This is how a local stochastic reconnection enhances the local turbulent cascade and enhances particle diffusion in the inertial range. Some part of the magnetic energy may also cascade down to the dissipative range where it is converted into thermal energy (Jafari et al. 2020a). Note that turbulent diffusion (Richardson diffusion) is super-linear and thus much more efficient than normal (linear) diffusion (Jafari et al. 2019). In the absence of turbulence, the rms particle separation would be

\(^{10}\) An implicit assumption here is that the magnetic diffusivity is of the same order as the viscosity, i.e., a magnetic Prandtl number of order unity. This is to assure that the viscous damping scale, below which hydrodynamic motions but not necessarily magnetic structures are dissipated, is of order the resistive dissipation scale, below which magnetic field is dissipated.

\(^{11}\) Numerically, the cascade timescale and the correlation timescale will be different in general. However, in strong turbulence, assumed throughout this paper, these quantities are expected to scale together. Although there has been some controversy over this point in magnetized turbulence, recent numerical simulations seem to support the idea that magnetized turbulence can be strongly nonlinear; see Beresnyak (2019) and references therein. In any case, the main results of this paper, extracted from a statistical analysis based on spatial complexities, remain independent of any MHD turbulence model used to support them in terms of scaling laws like those presented in this subsection.
governed by much slower normal diffusion. On the other hand, at smaller scales, where resistivity drives reconnection, the local reconnection speed scales as $v_R \sim \eta k_{\perp}$. Using $v_R \simeq V_A(k_{\parallel}/k_{\perp})$, the local reconnection speed is of order

$$v_R \sim V_A^2 \left( \frac{\eta V_A}{k_{\parallel}} \right)^{1/4}. \quad (11)$$

This is basically the local rms turbulent eddy velocity $v_R$ given by Equation (9) with $k_{\perp} \simeq \eta/v_R$ as the outflow width, which is set by the resistivity. The largest perpendicular wavenumber in the turbulent cascade is given by

$$k_{\perp} \sim \frac{v_R}{\eta} \propto \eta^{-3/4}. \quad (12)$$

Note that $k_{\perp}$ is the largest wavenumber expected in the existing turbulent cascade before the reconnection proceeds to generate its own local turbulence, which may affect the initial turbulent cascade (Lazarian & Vishniac 1999). At these small scales, local reconnections interact with the turbulence; they may generate a local cascade or enhance the turbulence if already present.

Next, let us compare the global magnetic heating with the turbulent heating in stochastic reconnection. Consider a fully turbulent reconnection zone, of spatial size $\Delta$ and with an rms turbulent velocity $V_T$ at large scales, embedded in large-scale field $B$. The energy dissipation rate at the reconnection zone is $J.E$, with current $J = \left| \nabla \times B \right| \simeq B/\Delta$ and electric field $E \simeq v_R B$ where $v_R$ is the reconnection speed. Consequently, the magnetic energy dissipation rate is roughly of order

$$\epsilon_b \simeq \frac{B^2 V_T^2}{\Delta} \simeq \frac{B^2}{(l/h)^{1/2}}, \quad (13)$$

where we have used Equation (2) for the reconnection speed. This is basically $\epsilon_b \simeq B^2/\tau_R$ with reconnection rate $\tau_R \simeq \Delta/V_R$. On the other hand, the kinetic energy dissipation rate in sub-Alfvénic turbulence (Goldreich & Sridhar 1995; Lazarian & Vishniac 1999; Jafari & Vishniac 2018a) scales as

$$\epsilon_v \simeq \frac{V_T^2}{(l/h V_A)}. \quad (14)$$

Because we expect $V_T/V_A < (l/h)^{1/2}$ in the presence of a large-scale magnetic field reversal, i.e., a global reconnection event, the magnetic heating rate is larger or at least of order the turbulent heating rate $\epsilon_b \gtrsim \epsilon_v$. Thus, a global reconnection in a turbulent medium might enhance the continuous heat generation by the turbulence, and Alfvén wave dissipation by the enhanced turbulence can increase this to even higher rates.

What is the heating rate associated with a local stochastic reconnection event above the dissipation scale? The magnetic heating rate is estimated as

$$\epsilon_{b,\text{loc}} \sim b^2 (k_{\parallel} v_R) \sim b^2 (v_R k_{\perp}). \quad (15)$$

Also, from Equation (8), the turbulent energy dissipation rate is given by

$$\epsilon_{v,\text{loc}} = \epsilon_v \sim v_R^2 (v_R k_{\perp}). \quad (16)$$

The magnetic heating rate associated with a local event, above the dissipation scale, is at least as efficient as the turbulent heating. Here, we have assumed a magnetic Prandtl number of order unity, i.e., $Pr_m \simeq 1$, as larger values are expected to lead to complications.

In conventional reconnection models, the kinetic energy injected into the medium at large scales, during a global reconnection event, has a long way to reach down the dissipation scale, which simply means long timescales. Above, we argued that this energy injection may enhance diffusion at large scales without directly affecting small-scale turbulent motions. Closely related is the notion that, because of the long timescales involved, the final conversion of this kinetic energy into thermal energy may occur long after a global reconnection ceases. In contrast, with energy injection at all scales in a stochastic model, a considerable fraction of kinetic energy should be converted into heat at shorter timescales, i.e., during a global reconnection. The democratic participation of all scales in stochastic reconnection, therefore, translates into a faster furnace down the cascade. Super-linear (Richardson) diffusion broadens the outflow zone during a reconnection event, and increases the flux of the ejected matter. Since the ejection velocity and reconnection zone’s length are almost fixed observables in all models, and since stochastic reconnection is fast, this implies more efficient kinetic energy injection into the turbulent cascade.

### 3.3. Energy Conversion and Local Reversals

In the preceding subsections, we used analytical and numerical results from previous work as well as conventional scaling laws of MHD turbulence to illustrate the efficiency of local reversals in enhancing turbulence and heat generation. In this subsection, we test these results numerically by looking at the correlation between the magnetic energy dissipation rate $\epsilon_m(t) = \eta|\nabla B|^2$ and the rate of change of the magnetic complexity, $\partial_t S_m(t)$. We use the homogeneous, incompressible MHD numerical simulation archived in the online, web-accessible database of Johns Hopkins University12 (Perlman et al. 2007; Li et al. 2008). This is a direct numerical simulation (DNS), using 1024$^3$ nodes, which solves incompressible MHD equations using a pseudospectral method. The simulation time is 2.56, equivalent to about two magnetic and one velocity large-scale eddy turnover times, and 1024 time steps are available (the frames are stored at every 10 time steps of the DNS). Energy is injected using a Taylor–Green flow-stirring force. We divide up the simulation box into subvolumes with randomly selected coordinates and sizes in order to obtain a larger number of statistical samples. Figures 6 and 7, for example, are produced in such randomly selected regions of the box. As for the scales $l$ and $L$, we take typical values $3 \lesssim l < L \lesssim 21$ in (grid units).

Figure 6 plots both the rate of change of magnetic complexity $\partial_t S_m$ and magnetic energy dissipation rate $\eta|\nabla B|^2$ in one randomly selected subvolume of the simulation box far away from the region considered in Figure 6. A strong correlation is observed between these quantities on average (a cross-correlation about 0.6). In this case, it seems that we are dealing with a region in the simulation box where, instead of magnetic reversals, the small-scale magnetic field gradients annihilate the field converting magnetic energy mostly to thermal energy, as in the nanoflare theory of Parker. In regions where magnetic energy dissipation is not strongly correlated with the

---

12 Forced MHD Turbulence Dataset, Johns Hopkins Turbulence Databases, doi:10.7281/T1930RBS (2008).
rate at which the magnetic complexity changes, the latter is usually correlated with the rate at which kinetic energy changes, i.e., $\partial_t (u \times u) / 2$. This quantity is important in the considerations related to magnetic reconnection, but in any case its trend over time closely resembles that of $(d/dt)(u^2/2)$ (the same argument applies to magnetic field; see Figure 3). Therefore, our qualitative discussion here is not sensitive to this choice; see also Jafari et al. (2020b). Figure 7 plots the rate of change of magnetic complexity $\partial_t S_m$ and $\partial_t (u \times u) / 2$. In this region, unlike that corresponding to Figure 6, the change in magnetic complexity shows a strong correlation with the change in kinetic energy, suggesting magnetic to kinetic, rather than magnetic to thermal, energy conversion.

We should emphasize that the correlations between different quantities discussed in this section, such as the rate of change in magnetic complexity and magnetic dissipation rate, should be understood as statistical cross-correlations between time series constructed from randomly selected samples in a simulation box, therefore they are meaningful only in a statistical sense in terms of the trends of these time series. In fact, more detailed numerical analyses based on a larger number of samples, i.e., subvolumes of the simulation box or even independent runs, are required to carefully test the analytical arguments advanced here. Our short treatment in this section should be regarded only as a self-consistency check rather than such a detailed numerical study.

4. Summary and Conclusions

In this paper, we have invoked analytical and numerical results from previous work (Jafari & Vishniac 2019; Jafari et al. 2019, 2020b) to argue that ubiquitous local magnetic reversals in MHD turbulence, which by the way play a major role in the stochastic model of magnetic reconnection (Lazarian & Vishniac 1999), efficiently enhance the turbulence in the inertial range and the heat generation in the dissipative range (Jafari et al. 2020a). Reconnection events seem to be ubiquitous in turbulent environments including the solar corona, therefore local reversals associated with nanoflares in Parker’s theory (Parker 1988), and their collective outcome as stochastic reconnection events, may at least partly explain the coronal heating problem. The main difficulty with Parker’s model lies basically in the detection of individual nanoflares observationally. Our approach here does not of course address this problem directly; however, relating local reversals to the recently formulated notion of magnetic complexity (Jafari & Vishniac 2019) and stochastic reconnection (Lazarian & Vishniac 1999) may in fact provide an indirect way to better understand coronal heating.

Previous work (Jafari & Vishniac 2019) has established a statistical formalism to study the spatial complexity/stochasticity level of a given vector field such as turbulent magnetic and velocity fields, which can be used to study reconnection and small-scale magnetic reversals in MHD turbulence. In this picture, magnetic reversals are studied in terms of the time evolution of magnetic and kinetic complexities and energies at arbitrary inertial scales. The correlation between the Lorentz forces responsible for local reconenctions and magnetic spatial complexity has also been analytically and numerically studied recently (Jafari et al. 2020a). Based on these recent developments, in this paper we have argued that small-scale magnetic reversals in the inertial range of turbulence result from tangling of the magnetic field by the turbulent motions. This can be understood in terms of stochastic flux freezing (Eyink 2011), which is a generalization of the conventional flux-freezing theorem (Alfvén 1942) in turbulent fluids. The more spatially complex the magnetic field becomes by statistically following the turbulent flow, the larger number of small-scale current sheets will be present. In these regions, magnetic energy will be converted into heat or they will undergo small-scale magnetic reversals thereby injecting energy to the flow. As a consequence, we expect that the rate at which magnetic complexity changes, $\partial_t S_m(t)$, will be positively correlated with the magnetic dissipation rate, $\eta |\nabla B|^2$, in the former case and with the rate at which the kinetic energy changes, $\partial_t u^2/2$, in the latter. Numerical simulations of incompressible, homogeneous MHD turbulence seem to be in agreement with this picture, although more detailed numerical studies are required to establish firm evidence. We have also backed up our analytical arguments by...
conventional scaling laws of MHD turbulence to show that small-scale reversals are indeed efficient in enhancing the turbulence and heat generation.

All in all, the arguments advanced in this paper suggest that small-scale, local magnetic reversals, i.e., reconnection events on any inertial scale \( l \), are ubiquitous in MHD turbulence and may play an important role in heating magnetized fluids such as the solar corona. The other implication is that stochastic magnetic reconnection, which results from many simultaneous local scale reversals are indeed efficient in enhancing the turbulence and heating the fluid than conventional reconnection models. The statistical picture presented in this paper, based on the spatial complexities of velocity and magnetic fields, can be regarded as a modern reformulation of the nanoflare theory (Parker 1972, 1987, 1988) and its connection with stochastic reconnection (Lazarian & Vishniac 1999).

### References

Afşin, E. H. 1942, Ark. Mat. Astron. Fys., 1, 9

Beresnyak, A. 2019, LRCA, 5, 2

Bowness, R., Hood, A. W., & Parnell, C. E. 2013, A&A, 560, A89

Bowers, R., Hood, A. W., & Parnell, C. E. 2013, A&A, 560, A89

Cranmer, S. R., van Ballegooijen, A. A., & Edgar, R. J. 2007, ApJS, 171, 520

Duchon, J., & Robert, R. 1999, CRASM, 329, 243

Einaudi, G., & Velli, M. 1999, PhRvE, 102, 035001

Edlén, B. 1946, Natur, 157, 297

Eyink, G. L. 2011, PhRvL, 107, 045001

Eyink, G. L., Vishniac, E. T., & Vaikundaram, V. 2018b, ApJ, 854, 2

Goldreich, P., & Sridhar, S. 1995, ApJ, 438, 763

Gosling, J. T., & Farrugia, C. J. 1988, GRL, 15, 1151

Hood, A. W., Browning, P. K., & van der Linden, R. A. M. 2009, A&A, 506, 913

Jafari, A., & Vishniac, E. T. 2018b, ApJ, 854, 2

Jafari, A., Vishniac, E. T., Kowal, G., & Lazarian, A. 2018, ApJ, 860, 52

Jokipii, J. R. 1966, ApJ, 146, 480

Kowal, G., Falceta-Gonçalves, D. A., Lazarian, A., & Vishniac, E. T. 2017, ApJ, 838, 91

Kowal, G., Lazarian, A., Vishniac, E. T., & Otmiannowska-Mazur, K. 2009, ApJ, 700, 63

Kowal, G., Lazarian, A., Vishniac, E. T., & Otmiannowska-Mazur, K. 2012b, NPGeo, 19, 297

Kowal, G., Vishniac, E. T., & Vaikundaram, V. 2020a, PhRvL, 108, 021102

Kowal, G., Falceta-Goncalves, D. A., Lazarian, A., & Vishniac, E. T. 2017, ApJ, 838, 91

Kowal, G., & Vishniac, E. 2019, NJPh, 19, 025008

Levine, R. H. 1974, ApJ, 190, 457

Li, Y., Perlman, E., Wan, M., et al. 2008, JTurh, 9, N31

Liu, B. F., Mineshige, S., & Shibata, K. 2002, ApJL, 572, L173

Longcope, D. W., & Tarr, L. A. 2015, RSPTA, 373, 20140263

Low, B. C. 2003, in ASP Conf. Ser. 286, Magnetic Reconnection and the Solar Corona, ed. A. A. Pevtsov & H. Uitenbroek (San Francisco, CA: ASP), 335

Lyu, Y., Guo, F., Kiliaris, et al. 2020, arXiv:2004.02277

Maclean, R. C., Haynes, A. L. 2010, ApJL, 725, L214

McIntosh, S. W., de Pontieu, B., Carlsson, M., et al. 2011, NatCor, 475, 477

Menou, A. S., & Agnon, A. M. 1971, Statistical Fluid Mechanics; Mechanics of Turbulence (Cambridge, MA: MIT Press)

Moriyasu, S., Kudoh, T., Yokoyama, T., & Shibata, K. 2004, ApJL, 601, L107

Murphy, G. C., Zanni, C., & Ferreira, J. 2009, ASSP, 13, 117

Onsager, L. 1949, NCim, 6, 279

Roald, C. B., Sturrock, P. A., & Wolfson, R. 2000, ApJ, 538, 960

Servidio, S., Matthaeus, W. H., & Dmitruk, P. 2018, PhPl, 25, 012304

Shay, M. A., Cassak, P. A., & Dmitruk, P. 2018, PhPl, 25, 012304

Snodin, A. P., Ruffolo, D., & Matthaeus, W. H. 2016, ApJ, 827, 115

Vlahos, L., & Isliker, H. 2019, PPCF, 61, 014020

Yang, K. E., Longcope, D. W., Ding, M. D., & Guo, Y. 2018, NatCo, 9, 692

### ORCID iDs

Amir Jafari @ https://orcid.org/0000-0003-3370-105X

Ethan T. Vishniac @ https://orcid.org/0000-0002-2307-3857

Siyao Xu @ https://orcid.org/0000-0002-0458-7828

### Acknowledgments

This work is supported by the National Key Research and Development Program of China (2016YFA0400604), the National Natural Science Foundation of China (11933007, 11873023, 11733009), and the Strategic Priority Research Program of Chinese Academy of Sciences (XDB23040401).