Tracking resistivity changes using suboptimal fading extended Kalman filter in electrical resistance tomography

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Abstract. This paper presents a suboptimal fading extended Kalman filter to track fast resistivity changes inside the heart as well as the lungs during the cardiac cycle. The performance of the proposed algorithm is evaluated through the numerical simulation of a cardiac cycle. The results demonstrate that the proposed method is more robust as compared to the extended Kalman filter in tracking fast changes in the resistivity distribution.

1. Introduction

Electrical resistance tomography (ERT) is a fast and non-invasive imaging modality in which a cross sectional image of resistivity distribution inside the human body can be obtained based on the applied currents and the resulting voltages through the electrodes attached to the skin of the body [1]. In order to enhance temporal resolution, dynamic techniques have been proposed for situations where the resistivity changes rapidly. In dynamic imaging, the inverse problem is formulated as a nonlinear state estimation problem in which the time-varying state is estimated with the aid of the linearized Kalman filter [2] or extended Kalman filter [3-4].

Usually the exact knowledge of the state evolution model in ERT is not available, so the so-called random-walk model is used instead. The modeling uncertainty of the random-walk model is usually not so small that there may be significant negative effects on the quality of the reconstructed image. Therefore, when the process noise covariance matrix is chosen improperly, these dynamic algorithms give poor estimation performance.

In this paper, an suboptimal fading extended Kalman filtering (SFEKF) algorithm [5] is proposed to track fast time-varying resistivity inside the heart as well as the lungs during the cardiac cycle. The performance can be improved with the use of the SFEKF in which a fading factor is introduced to reduce the modeling uncertainty and leads to a better estimation of the state-error covariance matrix. In particular, pre-grouping technique is employed to stabilize the inverse solver. The performance of the proposed algorithm is evaluated through the numerical simulation of the cardiac cycle. The results demonstrate that the proposed method is more robust as compared to the EKF in tracking fast changes in the resistivity distribution.
2. Method

The physical relationship between the resistivity and the voltage is governed by a partial differential equation with appropriate boundary conditions based on the complete electrode model [6]. For more details of the formulation of the forward problem, see Vauhkonen [1].

2.1. Integration of the state

The internal structure of the human body can be obtained a priori from anatomical imaging [2,4]. The major regions considered are the heart, the left and right lungs, backbone and background (other tissues), called regions of interest (ROIs). The elements in each ROI can be grouped together before inverse procedure. The pre-grouping can be written in the form \( \rho \equiv G\rho \) where \( \rho \in \mathbb{R}^{N_f \times 1} \) and \( \rho \in \mathbb{R}^{N_g \times 1} \) are the resistivities in the forward and inverse meshes, respectively, and \( G \in \mathbb{R}^{N_g \times N_f} \) is a grouping matrix.

2.2. Formulation of the dynamic model in the inverse problem

In the state estimation problem, the dynamic model is necessary, which consists of the state and observation equations. Here, the state equation is assumed to be of the following linear form

\[
x_{k+1} = F_k x_k + w_k
\]

where \( x_k \) is the state vector at time \( k \), \( x_k = \left[ \rho_1^k, \rho_2^k, \ldots, \rho_{N_f}^k \right]^T \), \( F_k = I_{N_g} \) (the identity matrix) is the state transition matrix for the random-walk model and \( N_g \) is the number of states. It is assumed that \( w_k \) is white Gaussian noise with known covariance \( Q_k = E \left[ w_k w_k^T \right] \).

Next, let \( V_k = \left[ V^1_k, V^2_k, \ldots, V^L_k \right]^T \in \mathbb{R}^{L \times 1} \) be the actual boundary voltage measured by the \( k \)th current pattern. Then the observation equation can be described as the nonlinear mapping \( V_k = U_k (\rho_k) + v_k \). Linearizing it about the predicted state \( x_{k|k-1} \), we have

\[
V_k = U_k + J_k \left( x_k - x_{k|k-1} \right) + \bar{v}_k
\]

where \( U_k \) is the calculated voltage and \( J_k \in \mathbb{R}^{L \times N_g} \) is the Jacobian matrix [3]. Therefore, we obtain the following linearized observation equation as

\[
y_k = J_k x_k + \bar{v}_k, \quad y_k = V_k - U_k + J_k x_{k|k-1}
\]

where \( \bar{v}_k \) is assumed to be white Gaussian noise with known covariance \( R_k = E \left[ \bar{v}_k \bar{v}_k^T \right] \).

2.3. Suboptimal fading extended Kalman filter

The key of the suboptimal fading extended Kalman filter (SFEKF) is to introduce a fading factor into the EKF, to adjust the state-error covariance matrix and the Kalman gain matrix. The derivation of the SFEKF is presented in Zhou et al [5] in detail. With the Gaussian assumptions the recursive SFEKF algorithm in one single sampling cycle can be expressed as follows:

\[
\hat{x}_{k|k-1} = F_{k-1} \hat{x}_{k-1|k-1}, \quad r_k = y_k - J_k \hat{x}_{k|k-1}
\]

\[
\lambda_k = \begin{cases} 
\lambda_0, & \lambda_0 \geq 1 \\
1, & \lambda_0 < 1 \end{cases}, \quad \lambda_0 = \frac{\text{tr} [N_k]}{\text{tr} [M_k]}, \quad W_k = \begin{cases} 
\frac{r_1 r_1^T}{1+\eta}, & k = 1 \\
\frac{\eta W_{k-1} + r_k r_k^T}{1+\eta}, & k \geq 2
\end{cases}
\]

\[
M_k = J_k F_{k-1} P_{k-1|k-1} F_{k-1|k-1}^T + J_k^T, \quad N_k = W_k - J_k Q_{k-1} J_k^T - R_k
\]

\[
P_{k|k-1} = \lambda_k F_{k-1} P_{k-1|k-1} F_{k-1|k-1}^T + Q_{k-1}
\]
\[
K_k = P_{k|k-1} J_k^T \left[ J_k P_{k|k-1} J_k^T + R_k \right]^{-1}
\]

\[
\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k r_k,
\]

\[
P_{k|k} = (I_{N_g} - K_k J_k) P_{k|k-1}
\]

where \(\lambda_k\) is the suboptimal fading factor, \(0 \leq \eta \leq 1\) is a forgetting factor, \(P_{k|k-1}\) is the state-error covariance matrix and \(K_k\) is the gain matrix. Here, the striking feature is that this algorithm makes the state-error covariance to be minimum and then adaptively adjusts the gain matrix forcing the residual \(r_k\) to be orthogonal. This makes to alleviate the modeling uncertainty.

3. Numerical simulation

In order to illustrate the estimation performance of the proposed algorithm, we simulated the cardiac cycle of the human heart. It is assumed that the volumes of the heart and the lungs are fixed during the simulation, which corresponds to breath holding. However, the blood volume causes the average resistivities of the heart and the lungs to change slightly. For the generation of the cardiac cycle, it is assumed that the resistivities of each ROI in figure 1(a) are increasing or decreasing in time due to the changes in the blood volume during the cardiac cycle as shown in figure 1(b). The average resistivity values are 1.78 \(\Omega\) for the heart, 10.35 \(\Omega\) for the left lung, 10.44 \(\Omega\) for the right lung, 10 \(\Omega\) for the backbone and 4 \(\Omega\) for the background [2],[4].

![Figure 1](image)

(a) The meshes used for forward solver with 4204 elements and inverse solver with 5 ROIs which correspond to the heart, lungs, backbone and background. (b) Simulated resistivity changes in the heart and in the lungs. The resistivity of the background is set to change vary slightly. The resistivity of the backbone is set to constant. The range of the resistivity values is given in \(\Omega\).
resistivities by EKF1, EKF2, EKF3 and SFEKF3, respectively. From figure 2, it is noted that the tracking accuracy of the EKF heavily depends on the magnitude of the process noise covariance in the random-walk model due to its modeling uncertainty. Also, it should be pointed out that the estimation performances of three SFEKFs are very similar to each other and are not shown, and the result of the EKF1 is close to that of the SFEKF. As can be seen clearly, the performance of the SFEKF is enhanced than that of the EKF. The results for the other ROIs are similar and are not shown.

![Figure 2](image-url)

**Figure 2.** The results from (a) the right lung and (b) the heart. True resistivity (—), EKF1 (–×–), EKF2 (–◦–), EKF3 (–△–) and SFEKF3 (–+–).

4. Conclusions
In this paper, we formulated the dynamic inverse problem based on ERT. The inverse problem was treated as a nonlinear state estimation problem and the unknown time-varying resistivity was estimated on-line with the aid of the SFEKF which reduced the modeling uncertainty. In particular, pre-grouping technique was also employed to stabilize the inverse solver. The numerical simulation was provided to illustrate the reconstruction performance in terms of the spatial and temporal resolutions. The results showed that the proposed algorithm was more robust as compared to the EKF in tracking fast changes in the resistivity distribution.

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