Can the Equivalence Principle Survive Quantization?*

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Abstract

It is well known that Einstein gravity is non-renormalizable; however a generalized approach is proposed that leads to Einstein gravity after renormalization. This then implies that at least one candidate for quantum gravity treats all matter on an equal footing with regard to the gravitational behaviour.

“It stands to the everlasting credit of science that by acting on the human mind it has overcome man’s insecurity before himself and before nature.” – Albert Einstein

1 Introduction

1.1 Is Einstein gravity incomplete?

It is intrinsic from the start that in Einstein gravity all matter (anti-matter included) responds equally to the gravitational field. However, Einstein gravity cannot be quantized [reviewed in Isham, 1981], and as a result one might wonder if it was the equivalence principle that was hindering quantization. Nevertheless, the fact that it seems impossible to quantize, does not exclude the existence of a quantum form of Einstein gravity; in the same way as prehistoric man had a source of fire (lightning strikes on trees), though not necessarily a means to produce it [Shiekh, 94; 96]. It is all too easy to forget that the world is quantum, and that the classical picture is but a special case, so we are actually working backwards when we try to derive the more general case from a restrictive form, and there is no reason to believe that this is always possible. However, we are compelled to follow this route out of a lack of choice.

Even if there were to exist a quantum form of Einstein gravity, this would prove very little except to present an example of a complete theory of gravity that made a solid prediction, namely that matter and anti-matter respond equally to the gravitational field. It would reduce the compulsion, though not the need, to ask nature her opinion.

There is also the line of reasoning that since the equivalence principle is a founding part of Einstein gravity, it is biased in this matter. But it is just such inflexibility that makes a theory predictive, and without this behaviour a theory is only descriptive. We should not forget that we are happy to build in other factors such as the conservation of momentum and energy into most of our theories. Physics is not so much explaining things per se, but rather explaining things.

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in terms of the least number of reasonable unknowns (reductionism). One might still ask why momentum and energy are conserved, and can partly answer this through Noether’s theorem, which tells us it follows from space and time invariance (the physics of here and now is the same as that of there and yesterday, or that of tomorrow). But at some point the basic question remains open; for example, nobody really knows the mechanism behind the instantaneous collapse of the wave function, or the reality behind the infinities we turn back upon the construct in the case of quantum field theory. At some point we fall away from hard science and are left with a few dangling philosophical questions. However, there is progress in this reductionism, and we should have a certain reluctance to adopting a less predictive theory, unless compelled to do so by experiment. Einstein gravity is based upon the aesthetically pleasing picture of free flight in a curved space-time, where the forces of Newton’s picture are gone. Why or how it is curved is not known, only by how much and by what. Perhaps one day the back tracking will end, and all will be explained in the maybe unsatisfying response that the world is the way it is on the grounds that there is only one self-consistent way to formulate it.

If we found ourselves unable to build a complete theory without letting go of the underlying principle, we would have very good reason to consider abandoning it; and so, the existence of a candidate is thought to have some significance, in having something to take to the ‘lab’. One might even take the stance that a proposal amplifies the need to test the validity of the underlying principles.

It is this approach I wish to take, with neither two open a mind, nor one too closed. It would be wrong to so quickly abandon all the efforts of the past, just as it would be dangerous to adhere to them with excessive rigidity. And so this will not be a revolutionary, scorched earth, proposal; but a more evolutionary move. But we should not rush ahead, for the proposal might be made that gravity need not be quantized at all, that the classical theory is the complete theory.

1.2 Need gravity be quantize at all?

Is it not possible that although the other forces of nature are seen to be quantized, that perhaps gravity, which is presently not seen as a force at all, need not be quantized [Feynman, 1963; Kibble, 1981]? The concept of force is a classical one that does not even make an appearance in quantum theory, where distribution is governed by the overlap of wave functions. Newtonian gravity certainly carries the notion of force, while the Einstein view is of matter in free fall in a curved space-time. This picture of gravity might lead one to propose leaving the curved space time unquantized, and to have the quantum fields play out on this arena. It is an appealing scenario, and would present a resolution to the problem of quantization, claiming it is not necessary at all.

However the gravitational field, if left classical, could be used to make measurements on a quantum field, and being classical would not be subject to the Heisenberg uncertainty principle.
Thus if one were to look at a quantum field using gravity as a probe, one would be able to extract information about the quantum field that defied the Heisenberg principle. Such arguments are far from water-tight, but do make a strong case for the quantization of gravity.

Despite the very geometric picture that is usually assigned to Einstein gravity, it can be made to look like the other perturbative field theories, with the graviton a spin two particle. It is this very traditional, particle physics perspective that we will be following. Unfortunately, the whole structure now takes on a very mathematical structure, and at times it is hard to keep contact with the physics.

Actually, we will be adding a second theme to the quantization that is not strictly necessary, namely renormalization by analytic continuation. In these methods there is no subtraction of infinities, but rather a reinterpretation of the formulae. This is a formulation devoid of infinities, so completely by-passing the problem of interpreting the divergences. At the risk of mixing concepts, the quantization is illustrated using this novel method, which can be shown to be equivalent, in effect, to the older techniques.

1.3 Tweaking gravity

Einstein gravity is sometimes rather interesting in its intolerance to tweaking. Normally one might suppose that one is always free to add a small modification to a theory, so long as it does not imply an intrinsic inconsistency. One usually has the impression that experiment can never do more than put limits on a small deviation. For example, the photon in electrodynamics could be assigned a small rest-mass, so small that the effect goes unnoticed. It is worth noting that this freedom is not always present. One might suggest a very small rest mass for the graviton; but in general relativity, the mass goes to zero limit gives a differing predictions from the massless theory [van Nieuwenhuizen, 1973], and for this reason alone, we will not be employing a rest mass for the graviton to tame the infra-red divergences that are encountered.

1.4 Can anti-matter fall upwards?

We are used to thinking of matter and anti-matter as complementary. That is to say the charges of a particle and it’s anti-partner are normally either the same or opposite. This opposition could not be the case for gravity, for I recall an accelerator engineer saying that gravity must be taken into account when designing today’s particle accelerators. This being so, a direct anti-gravity response of anti-matter would have been observed long ago. So any effect, if any, would be expected not to display the anti-symmetry we are biased to think of, and would need to be at a smaller level, perhaps even only as a quantum correction. Also, this effect must some how not be present in the particles we think of as being their own anti-particles (the photon, for instance), unless one is willing to modify not just gravity, but the entire spectrum of forces (particles).
1.5 Should we look?

No matter how beautifully the gears of a theory turn, it is little more than a piece of art till it has gone out on the field and proved its worthiness. After all, the null results of Michelson and Morley supported special relativity, while the null result of Eötvös and Dicke lent support to general relativity. So a null result of the violation on the equivalence principle might be viewed as significant support for quantum Einstein gravity.

Any non-zero deviation would make a turmoil of the usual matter/anti-matter symmetry. Of course, one could propose there are two types of mass, one (like the photon) that is its own anti-particle, and so does not change sign; and the other which does. The small deviations would then be explained as a dominance of the first type. But this approach lacks predictive power, with a tuning parameter to fit experimental results.

1.6 The problem with traditional quantization (a lightning review)

Those that know the traditional methods and problems with quantizing are not in need of an introductory review, while those that have not tinkered with the guts cannot really be properly shown the approach in an hour. It is with this contradiction in mind that we set out on a lightning tour of the problem.

The normal approach is to start with a classical theory and try to quantize it. In reality the world is quantum, and the classical view is just a special limit. In this sense we are starting from the top and trying to work down to the more fundamental. It is an approach fraught with dangers, but it is the best we can do for now. There is no guarantee that the attempt to derive the more general from the special case will bear fruit, and the fact that Einstein gravity can’t be quantized should not be taken to imply that there is no quantum Einstein gravity.

We have been quantizing non-relativistic systems with success for some time now, but relativistic theories seem to demand the use of field theories (to allow for particle creation and destruction). However, the infinite number of degrees of freedom tends to be accompanied by infinite quantities in the theory, and this, very crudely, is the source of the problem when quantizing field theories. However, some field theories are quantizable, despite the presence of these infinities. It turns out that in some very special cases it is possible to re-absorb the infinities into the coupling constants of the original, starting (classical) theory. It is a mathematical technique that is difficult to interpret physically; but despite this difficulty it leads to very good physical predictions for most of the forces of nature (the electric, weak and strong forces). The fact that this so called process of renormalization is only successful for as small class of theories is what makes it predictive. In fact, it is so successful that it has permitted the unification of the electric and weak forces and even has a lesser constrained proposal for uniting also the strong nuclear force. However, the one remaining force, namely gravitation, does not succumb to quantization so easily.
So long as this is the case, speculation about the quantum regime has a free rein and can run unhindered, and unguided. A convincing theory should make hard predictions and act as a guide to what should be tested by experiment. The more a theory gambles in taking a hard stance, and risking being shown wrong, the more it stands to gain should it be found to stand in harmony with nature. The first World War saved Einstein some embarrassment in this respect, for he was able to locate a factor of two error in his calculation for the bending of light, before Eddington was able to confirm his prediction. A correction after the fact would have been a lot less convincing.

The traditional methods of quantization that have worked so very well in taming the theories of the other forces of nature, have not fared so well when taking on the task of quantizing gravity. Here the infinities of quantum field theory don’t match the original coupling constants. This suggests the need for something novel, but should not be taken as reason to totally abandon the past thinking as a complete failure, and so devoid of usefulness. It is a habit most every generation makes politically, and this tends to hinder progress.

Despite this need for change, there seems to be a proposal for quantizing gravity that is unexpectedly conservative in its lack novelty. In fact, we will be so traditional as to investigate the perturbative quantization of gravity. This means that much of the presentation can be versed in the now rather old language of field theory, and we can embark on a more detailed investigation of the problems obstructing the traditional quantization of Einstein gravity.

2 The Perturbative Quantization of Einstein Gravity

The usual scheme of field quantization is plagued by divergences, but in some special cases those infinities can be consistently ploughed back into the theory to yield a finite end result with a small number of arbitrary constants remaining; these then being obtained from experiment [Ramond, 1990; Collins, 1984]. This is the renown scheme of renormalization, disapproved of by some, but reasonably well defined and yielding results in excellent agreement with nature. For those disturbed by the appearance of infinities, there now exists a finite perturbative version employing analytic continuation (a generalization of the Zeta function, one loop, technique), that goes under the deceptive name of ‘operator regularization’. The fact that after renormalization some factors, such as mass and charge, are left undetermined should perhaps not be viewed as a predictive shortcoming, since the fundamental units of nature are relative. That is to say, the choice of reference unit (be it mass, length, time, or charge) is always arbitrary, and then everything else can be stated in terms of these units. In this sense the final theory of everything should not, and cannot, predict all.

The fact that only some theories are renormalizable has the beneficial effect of being selective, and so predictive. This follows the line of reasoning that is more than descriptive, but predictive by virtue of being limited by the requirement of self consistency.
Unfortunately, in the usual sense, general relativity is not renormalizable [Veltman, 1976], and we will run quickly over the failure of Einstein gravity to quantize perturbatively, by considering the example of a massive scalar field with gravity. The starting theory in Euclidean space would be characterized by:

\[ L = -\sqrt{g} \left( R + \frac{1}{2}g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2}m^2 \phi^2 \right) \]  

(1)  

(using units where $16\pi G = 1, c = 1$)

One discovers, upon perturbatively quantizing both the matter and gravitational fields, that the counter terms do not fall back within the original Lagrangian, so the infinities cannot be reabsorbed.

One natural thought might be to generalize Einstein gravity by extending the starting Lagrangian to accommodate the anticipated counter terms. Here symmetry can be employed, and by using the most general starting Lagrangian consistent with the original symmetries one arranges that the counter terms (which also retain the symmetry in the absence of an anomaly) fall back within the Lagrangian. One would not anticipate an anomaly, as these arise from a quantum conflict between two or more symmetries, when one must choose between one or the other. Thus one is lead to the infinitely large Lagrangian:

\[ L_0 = -\sqrt{g_0} \left( -2\Lambda_0 + R_0 + \frac{1}{2}p_0^2 + \frac{1}{2}m_0^2 \phi_0^2 + \frac{1}{4!} \phi_0^4 \lambda_0(\phi_0^2) + p_0^2 \phi_0^2 \kappa_0(\phi_0^2) + R_0 \phi_0^2 \gamma_0(\phi_0^2) 
+ p_0^4 a_0(p_0^2, \phi_0^2) + R_0 p_0^2 b_0(p_0^2, \phi_0^2) + R_0^2 c_0(p_0^2, \phi_0^2) + R_0 \phi_0^2 R_0^{\mu\nu} d_0(p_0^2, \phi_0^2) + ... \right) \]  

(2)

where $p_0^2$ is shorthand for $g_0^{\mu\nu} \partial_\mu \phi_0 \partial_\nu \phi_0$ and not the independent variable of Hamiltonian mechanics. $\lambda_0, \kappa_0, \gamma_0, a_0, b_0, c_0, d_0$ ... are arbitrary analytic functions, and the second line carries all the higher derivative terms. Strictly this is formal in having neglected gauge fixing and the resulting presence of ghost particles.

The price for having achieved ‘formal’ renormalization, is that the theory (with its infinite number of arbitrary renormalized parameters) is devoid of predictive content. The failure to quantize has been rephrased from a problem of non-renormalizability to one of non-predictability.

Despite this, after renormalization we are lead to:

\[ L = -\sqrt{g} \left( -2\Lambda + R + \frac{1}{2}p^2 + \frac{1}{2}m^2 \phi^2 + \frac{1}{4!} \phi^4 \lambda(\phi^2) + p^2 \phi^2 \kappa(\phi^2) + R \phi^2 \gamma(\phi^2) 
+ p^4 a(p^2, \phi^2) + R p^2 b(p^2, \phi^2) + R^2 c(p^2, \phi^2) + R \phi^2 R^{\mu\nu} d(p^2, \phi^2) + ... \right) \]  

(3)

However, there remain physical criterion to pin down some of these arbitrary factors. Since in general the higher derivative terms lead to acausal classical behavior, their renormalized coefficient can be put down to zero on physical grounds. This still leaves the three arbitrary functions: $\lambda(\phi^2), \kappa(\phi^2)$ and $\gamma(\phi^2)$, associated with the terms $\phi^4, p^2 \phi^2,$ and $Ro^2$ respectively. The last may be abandoned on the grounds of defying the equivalence principle. To see this,
begin by considering the first term of the Taylor expansion, namely $R\phi^2$; this has the form of a mass term and so one would be able to make local measurements of mass to determine the curvature, and so contradict the equivalence principle (charged particles, with their non-local fields have this term present with a fixed coefficient). The same line of reasoning applies to the remaining terms, $R\phi^4$, $R\phi^6$, ... etc.

This leaves us the two remaining infinite families of ambiguities with the terms $\phi^4\lambda(\phi^2)$ and $p^2\phi^2\kappa(\phi^2)$. In the limit of flat space in 3+1 dimensions this will reduce to a renormalized theory in the traditional sense if $\lambda(\phi^2) = \text{constant}$, and $\kappa(\phi^2) = 0$. So one is lead to proposing that the physical parameters should be:

$$\Lambda = \kappa(\phi^2) = \gamma(\phi^2) = 0$$

$$a(p^2, \phi^2) = b(p^2, \phi^2) = c(p^2, \phi^2) = d(p^2, \phi^2) = ... = 0$$

$$\lambda(\phi^2) = \lambda = \text{scalar particle self coupling constant}$$

$$m = \text{mass of the scalar particle}$$

and so the renormalized theory of quantum gravity for a scalar field should have the form:

$$L = -\sqrt{-g} \left(-2\Lambda + R + \frac{1}{2}p^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4\right)$$

One might now worry about the renormalization group pulling the coupling constants around. This is an open point to which I feel one of several things might happen:

- **The couplings, set to zero at a low energy scale, might reappear around the Plank scale.** Whether the resulting theory then makes sense is a matter for dispute.

- **Certain coupling constants (beyond those already set to zero) should be related, in order that the beta functions of the zeroed couplings be zero (a fixed point), so ensuring that all their couplings remain at zero.** This consistency condition could be the basis of a unification scheme, although its implementation might not be possible within the perturbative formulation.

This is a highly non-trivial matter that needs looking at more closely.

### 2.1 Regularization Method

On a diverse, but related track, one might wonder which renormalization scheme to choose for implementing the scheme proposed above. In this context analytic continuation [Bollini et al., 1964; Speer, 1968; Salam and Strathdee, 1975; Dowker and Critchley, 1976; Hawking, 1977] is very appealing in being finite, and in this context there is an ‘unsung hero’ in the guise of operator regularization, which I think deserves a mention [McKeon and Sherry, 1987; McKeon
et al., 1987; McKeon et al., 1988; Mann, 1988; Mann et al., 1989; Culumovic et al., 1990; Shiekh, 1990).

In operator regularization one avoids the divergences by using the analytic continuation:

\[ \Omega^{-m} = \lim_{\epsilon \to 0} \frac{d^n}{d\epsilon^n} \left( \frac{\epsilon^n}{n!} \Omega^{-\epsilon - m} \right) \]  

(6)

where \( n \) is chosen large enough to eliminate the infinities (the loop order is sufficient). Actually, operator regularization is a bit of a misnomer, since it need not be applied to an operator and does not just regulate, but also renormalizes all in one. However, under this form of the method all theories are finite and predictive (gravity included). A little playing shows the above is simply an automated system for minimal subtraction, and this realized, the general form is easily located, and is given by:

\[ \Omega^{-m} = \lim_{\epsilon \to 0} \frac{d^n}{d\epsilon^n} \left( 1 + \alpha_1 \epsilon + \ldots + \alpha_n \epsilon^n \right) \frac{\epsilon^n}{n!} \Omega^{-\epsilon - m} \]  

(7)

\( \text{(the alphas being ambiguous)} \)

This form is not too powerful, and gravity must again be dealt with as before, setting most of the final renormalized parameters to zero on physical grounds.

That the earlier special form actually just locates and zeros the divergences is perhaps most clearly seen by how it treats some terms of the Maclaurin expansion. The taming of 1 yields:

\[ \lim_{\epsilon \to 0} \frac{d}{d\epsilon} (\epsilon.1) = 1 \]

while, on the other hand, the taming of 1/\( \epsilon \) yields:

\[ \lim_{\epsilon \to 0} \frac{d}{d\epsilon} \left( \frac{1}{\epsilon} \right) = 0 \]

The method of operator regularization has the strength of explicitly maintaining invariances, further even than dimensional regularization, for dimension dependent invariances are not disturbed. It is further blessed with the feature of being finite throughout, as the Zeta function technique [Salam and Strathdee, 1975; Dowker and Critchley, 1976; Hawking, 1975]. But unlike the Zeta function method, it is not limited in applicability to one loop, being valid to all orders.

To quickly see this method and its simplicity in action we might look at a typical divergent one loop integral intermediate result:

\[ I(p, m) = \int_{-\infty}^{\infty} \frac{d^4 x}{(2\pi)^4} \int_0^1 dx \frac{(p^2 l^2 + 2m^2 p.l - 2m^4)}{[l^2 + m^2 x + p^2 (1 - x) + 2p.l(1 - x)]^2} \]  

(8)

which we tame with:

\[ \Omega^{-2} = \lim_{\epsilon \to 0} \frac{d}{d\epsilon} \left( (1 + \alpha \epsilon)\epsilon \Omega^{-\epsilon - 2} \right) \]  

(9)

to yield the finite object.
$$I(p, m) = \int_0^1 dx \lim_{\varepsilon \to 0} \frac{d}{d \varepsilon} \int_{-\infty}^\infty \frac{d^4 x}{(2\pi)^2} \left( \varepsilon(1 + \alpha \varepsilon) \frac{p^2 l^2 + 2m^2p.l - 2m^4}{[l^2 + m^2 x + p^2(1 - x) + 2p.l(1 - x)]^{\varepsilon + 2}} \right)$$ (10)

from which we proceed; to yield:

$$\frac{m^4}{(4\pi)^2} \left( 3 + 2 \frac{p^2}{m^2} + \frac{m^2}{p^2} \right) \ln(1 + \frac{p^2}{m^2}) - 1 - \frac{5}{2} \frac{p^2}{m^2} - \frac{1}{6} \frac{p^4}{m^4} + 2 \left( 1 + \frac{p^2}{m^2} \right) \left( \ln(m^2/\mu^2) - \alpha \right)$$ (11)

which actually has no divergence at $p = 0$. The factor $\mu$ which leads to the renormalization group appears on dimensional grounds. As is typical at one loop (but not beyond), the arbitrary factor $\alpha$ can be reabsorbed into the parameter $\mu$.

### 2.2 Discussion

We are now left with a finite theory that has few arbitrary constants, and so is predictive. Despite the present lack of experimental data to test it against, and regardless of the patch work line of reasoning invoked to arrive at this hypothesis, one might alter perspective and simply be interested in investigating the consequences of such a scheme for its own sake, where many of the arbitrary factors have been set to zero, for whatever reason. At this stage any well behaved, finite theory, is worth investigating; and it is unfortunate that we don’t have the guiding hand of mother nature to assist us in this guessing game.

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