Otoacoustic emissions of the 4th kind: Nonlinear reflection

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Abstract: Otoacoustic emission (OAE) refers to acoustic waves that originate from the cochlea. Since its discovery, various ways have been developed to elicit OAEs; those elicited by short clicks are called transient-evoked (TE) OAEs, and the cubic distortion elicited by two tones are called distortion-product (DP) OAEs. In addition, spontaneous OAEs can be found from some ears without applying any external stimulus. Shera and Guinan proposed a taxonomy of OAEs that consists of three kinds: the linearly reflected emissions, the spontaneous emissions, and the distortion emissions. This article aims to introduce an additional 4th kind of OAEs to the taxonomy. We have shown theoretically that, when a high frequency, large-amplitude suppressor tone is present, it may set up a temporary and reversible impedance mismatch for the traveling waves that pass through its characteristic place. Because of this mismatch, the waves get partially reflected and going backward toward the stapes. The derivation of this “nonlinear reflection” mechanism is based on de Boer’s quasi-linear, equivalent system framework, and may help explain the controversial tone-burst evoked OAEs experiments obtained in recent years.

Keywords: Otoacoustic emissions, Cochlear mechanics, Nonlinearity

PACS number: 43.64.Kc, 43.64.Jb [doi:10.1250/ast.41.204]

1. INTRODUCTION

The nonlinearity in input/output (I/O) relation has been found in OAE since the day of its discovery; the relative intensity of click evoked (CE) OAEs with respect to the click stimuli was found to diminish as the stimulus level increased [1, Fig. 4]. Because the nonlinearity stems from normal functioning of the outer hair cells (OHCs), different OAE measurement techniques have been developed as non-invasive, objective tools for screening cochlea-related hearing impairment [2]. In practice, CEOAE can be used for fast screening — its presence indicates that the cochlea is not deaf, while distortion-product (DP) OAEs help pinpoint the frequency range and the severity of OHC impairment if there is any. Up to this date, OAEs are often explained as tiny waves generated by the OHCs and emitted to the ear canal. This explanation provides a vivid picture that is friendly for the general public to comprehend.

However, the picture has been found to be not entirely correct. DPOAEs, evoked by two tones at frequency $f_1 < f_2$ and typically detected at $2f_1 - f_2$, are certainly generated through cubic distortion by the OHC and re-emitted to the ear canal, but the stimulation of CEOAE (or its spectral equivalence [3], the stimulated-frequency (SF) OAE) does not involve such frequency mixing. Early on, CE/SFOAEs were regarded as re-emitted nonlinear distortion from different places along the cochlea, but such explanation failed to account for the rapid phase accumulation against the stimulus frequency. Thus, an alternative theoretical framework was developed, in which the cochlea was modeled with place-frequency scaling symmetry, and CE/SFOAE was predicted to be a result of random reflection due to perturbation of the mechanical properties along the cochlea [4]. In this framework, the rapid phase accumulation against frequency could be explained as a result of place-fixed wave reflection, in a linear manner that is akin to Bragg diffraction. The theory is referred to as coherent reflection.

To emphasize the different between coherent reflection and nonlinear re-emission, Shera and Guinan [5] proposed a taxonomy of OAEs according the generation mechanism; two major branches were shown in the taxonomy: the “linear reflection” branch, and the “nonlinear distortion” branch which includes DPOAEs. The linear reflection branch can be further classified into the evoked kind, which includes SF and TEOAEs, and the spontaneous emissions which are regarded as a result of standing waves due to multiple internal (linear) reflections.

The theory of coherent reflection has been subject to scrutiny over the years. Particularly being questioned is the
source location of SFOAE. The theory predicts that, for every spectral component of the stimulus, its reflection mainly comes from the characteristic-frequency (CF) place [4]. To estimate the breadth of source distribution of SFOAE inside the cochlea, an experimental paradigm using the suppressor tone was adopted [6]. The results astonishingly suggested that the distribution of sources was heavily extended basally. This series of experiments have been continually refined, most recently using tone-bursts (TB) as the stimuli, and similar conclusions were made [7]; briefly speaking, a suppressor tone at more than one octave above the TB frequency could still affect the OAE level.

Initially, Shera et al. [8] responded by pointing out that the suppressor tone in Siegel et al.’s experiment [6] might have created a component of OAE instead of eliminating an existing component. The generation mechanism of the additional OAE components was described as either due to “mechanical perturbation and/or sources of nonlinear distortion.” More recently, Liu and Liu [9] unified these two possibilities by showing that, in certain circumstances, the nonlinear distortion can be treated as reflection due to the mechanical perturbation produced by the suppressor tone in a level-dependent and highly localized manner. The theoretical formulation requires the mechanics of the cochlea to satisfy several constraints laid by de Boer [10] so equations can be linearized to construct equivalent models. In the remaining part of this article, we shall first review de Boer’s original equivalent nonlinearity (EQ-NL) theory, then present some of the effort to extend the theory by relaxing the constraints [9].

2. DE BOER’S EQ-NL THEOREM

At around the same time when the coherent reflection theory [4] was formulated, de Boer and colleagues developed a series of numerical techniques [11,12] that allow cochlear mechanics equations, particularly the relation between stimulus and responses, to be expressed in terms of linear transfer functions despite that the system is nonlinear. This thread of research culminated in what he called the EQ-NL theorem [10]; as it was stated originally, the theorem was about the existence of “linearized model of the same structure” (A1) with the same “I/O cross-correlation function” (A2) for “wide-band noise stimuli” (A3). For A1 to exist, de Boer set the following constraints to the class of nonlinear models of interest:

- certain elements (e.g., the OHCs) act to amplify the cochlear waves in a place- and frequency-specific way
- these elements are the sole sites of cochlear nonlinearity, and
- the nonlinear function associated with OHC transduction is memoryless.

Performing the actual linearization involves calculating a reduced efficiency factor \( \gamma(x) \leq 1 \) which varies along the direction of wave propagation, \( x \), and \( \gamma(x) \) would also depend on the input level. If \( \gamma(x) \) can be precisely estimated, then the entire cochlear-mechanics model can be replaced by an equivalent linearized model which uses \( \gamma(x) \) to scale down the amplification efficiency (of the OHC). Note that the linearized model will preserve the same mechanical structure as far as all the linear parts are concerned because, by assumption, nonlinearity resides solely in the OHCs.

What truly was novel of the 1997 paper [10] was to consider \( \gamma(x) \) when the stimulus is wideband random noise. The key idea was that, if the stimulus is wideband, every single wave component at frequency \( \omega \) is riding on top of all the others, so it could be treated as small perturbation. Consequently, the amplification efficiency \( \gamma(x) \) that it receives is proportional to the expected value of the slope of the memoryless OHC transduction function, which depends on the input rms value but not on \( \omega \),

\[
\gamma(x) = \int p(\xi; x) \left| \frac{d\eta(\xi)}{d\xi} \right| d\xi, \tag{1}
\]

where \( \xi \) denotes the input variable for the OHC transduction function \( \eta(\xi) \), and \( p(\xi; x) \) represents the probably density function of \( \xi \). Without loss of generality, we assume that \( \eta'(0) = 1 \) and \( \eta(\xi) \) describes a compressive type of nonlinearity, so \( 0 < \gamma(x) \leq 1 \) always. An example of the \( \eta(\cdot) \) function could be

\[
\eta(\xi) = I_0 \tanh(\xi/I_0), \tag{2}
\]

where \( I_0 \) denotes the saturation output level of this nonlinear function.

The 1997 paper seems to be de Boer’s final publication on this topic, and it was not explicit how to find \( \xi(x) \) along \( x \) given a random stimulus from outside the cochlea. It turned out that \( \xi(x) \) and \( \gamma(x) \) must be calculated successively by iteration [11], and this calculation was carried out [13] for Gaussian stationary input\(^2\) with the hyperbolic tangent \( \eta(\xi) \) specified in Eq. (2) [14]. The iteration converged in all cases that were tested, and by comparing with direct finite-different time-domain simulation, the equivalent linear model was able to predict the nonlinear output (the basilar-membrane and reticular-lamina motion) with high accuracy. This work showed that nearly-linear filtering can occur [15] across a wide range of stimulus levels even if the nonlinearities in cochlear mechanics are simply memoryless.

\(^1\)from the ARO poster pdf

\(^2\)for the first time as I am aware of
3. EXTENDING EQ-NL BY RELAXING THE ASSUMPTION A3

The controversy in explaining the experiments of high-frequency suppression of SF- and TB-OAEs [6,7] has sparked an interest to formulate a reflection theory based on the EQ-NL framework. Long after Shera et al.’s ARO poster [8], I started considering how a transient small-signal b(t)’s amplification efficiency β(x) may be reduced due to the presence of a dominating suppressor tone A(x)cos(ωt). The follow equation was presented [16] then later published [9],

\[
β(x) = \frac{1}{T} \int_0^T |η'(A(x)\cos(\omega t))|dT,
\]

where β(x) denotes the efficiency reduction factor and A(x) denotes the amplitude of the dominating Fourier component at the suppressor frequency ω = 2π/T. This equation preserves the idea in Eq. (1) of treating the signal of interest as perturbation so the gain equals the mean slope. The difference is that the mean-slope is averaged over one period in Eq. (3) but averaged across ensemble in Eq. (1).

Note that Eq. (3) assumes that the displacement magnitude A(x) due to the suppressor tone can be calculated beforehand and remains stable regardless of the presence of the small signal b(t). Indeed, once the cochlear-mechanics model is defined, A(x) can be calculated either by time-domain finite-difference simulation [14] or by Kanis and de Boer’s iteration techniques [11]. Liu and Liu [9] showed that the calculation of A(x) using both approaches agreed with high precision. A complete list of parameters of the cochlear-mechanics model can be found in [14] (their Table 1).

Equation (3) enables an equivalent linear model to be constructed by applying the efficiency reduction factor β(x); more specifically, this was done by multiplying the stereocilia mechanoelectrical transduction coefficient, which is a function of x, by β(x). Two details are worth noting here. First, β(x) generally varies along x. Secondly, all the intermodulation components between b(t) and the suppressor are neglected in favor of simplicity. Nevertheless, we thus extend the EQ-NL theory to accommodate non-stationary inputs, and are blessed with the advantage of preservation of model structure; by applying β(x), we obtain an equivalent linear system that is applicable for TBs, clicks, or any given small stimuli. Then, because the mechanical structure remains the same, one can further calculate wave scattering and analyze the results in terms of familiar quantities in the linear regime. Further, one can systematically study how these quantities vary as the suppressor intensity and frequency changes. For a transmission-line model in particular, such linear quantities include the wave impedance, the reflectance, and transfer functions.

Figure 1 shows an example of change of wave impedance when a dominating suppressor is present. A cochlear model that satisfies de Boer’s constraints was built [9], and in this example it receives a 7-kHz suppressor at 60 dB SPL measured in the ear canal. The figure shows how a small component at 4 kHz would ‘experience’ in the presence of a suppressor. In this example, Zc is the characteristic impedance for 4 kHz as it varies against x. Both the real (in blue) and the imaginary (in green) parts are shown. The solid lines show Zc without the influence from any suppressor, and the dashed lines show how the results change when a 7-kHz suppressor is present. Note that Re[Zc] dropped by more than 20% near the 7-kHz CF place, which is located near the basal end of the negative damping region [17] for 4 kHz.

![Fig. 1 Impedance change due to the presence of a suppressor.](image-url)
wave with or without being affected by the suppressor. Secondly, this reflection mechanism is fundamentally different from coherent reflection, since the model is smooth and does not produce significant reflection without the suppressor.

4. DISCUSSION

Here we suggest a revised taxonomy of OAEs as shown in Fig. 3. The category of nonlinear distortion emissions is divided into the distortion products and the nonlinear reflection emissions. Because of the newly added branch of nonlinear reflection as the 4th kind of OAEs, the apparent dichotomy between reflection emissions and distortion emissions have been resolved. Nevertheless, this should not be regarded as a revolution against the taxonomy promoted by Shera and Guinan [5]. Their effort was made mainly to emphasize that CEOAEs should not be regarded as a kind of distortion product generated by the OHCs and re-emitted to the ear canal. It was clearly stated that the taxonomy was not meant to prohibit further branching if different aspects of OAEs need to be compared and emphasized.

The newly added branch of OAE generation mechanism is a result of our attempt to explain specifically what happens when a high-frequency suppressor nonlinearly scatters other transient components at lower frequencies. This present formulation is concise because higher-ordered mixing components between the suppressor and the transient signal are ignored. If the suppressor level does not dominate, all these components need to be taken into consideration. The full picture becomes more complicated; even though an iterative technique could still be developed [12], the beauty of structure preservation might be lost. In this case, to refer to the nonlinear wave scattering as reflection would be an over-simplification and is not recommended.

Note that Eqs. (1) and (3) describe different ways of calculating the efficiency reduction factor due to the nature of the signals that participate in the scattering. Interestingly, yet another way of calculation needs to be used when one considers dynamic range compression of a single tone traveling along the basilar membrane. In their first paper on this topic, Kanis and de Boer [11] already described how exactly the efficiency factor \( \alpha(x) \) could be calculated, as expressed below:

\[
\alpha(x) = \frac{2}{T} \int_0^T \eta(A(x) \cos \omega t) \cdot \cos(\omega t) dt.
\]

Note that \( \alpha(x) \neq \beta(x) \); the latter describes how a tone suppresses other components, while the former describes how a tone “suppresses itself” [11, see the title]. An obscure result that has not received much attention was that, similarly, when such self-suppression causes sufficiently localized impedance mismatch, the wave is expected to reflect itself partially. This partial reflection was actually found in their simulation [11, Fig. 2]. It has been called the zeroth-order distortion product [5] and can be regarded as an early example of what would be called nonlinear reflection in this article.

Interestingly, the simulation predicted that “self-reflection” of a single tone happens only near the mid-level range. In this sense, it is partially correct to say SFOAEs generate from nonlinear distortion if mid-to-high stimulus levels are used to elicit the emissions. The situation for transient-evoked (TE) OAEs is more complicated. In my opinions, the self-scattering of transient waves is beyond the reach of EQ-NL theory, as components with a latency shorter than predicted by the coherent theory have been found in TEOAEs [18,19]. In this case, basal distribution of OAEs sources seem to be the simplest way to explain the data [20].
5. CONCLUSIONS AND FUTURE WORK

De Boer et al.’s iterative techniques were adopted and extended to model the strange phenomena of a high-frequency suppressor scattering low frequency waves in the cochlea. A new picture was created and here we suggest to refer to it as nonlinear-reflection emissions. By adding nonlinear reflection to Shera and Guinan’s taxonomy of OAE generation mechanisms, we somehow bring the picture a step closer back to Kemp’s original idea that click-evoked emission is a result of nonlinear distortion re-emitted to the ear canal.

Currently, the efficiency reduction factor techniques were only tested on a 1-dimensional transmission line model of cochlear mechanics. The field of cochlear mechanics has recently been gearing toward understanding 2D motion of the organ of Corti, particularly thanks to large amount of newly available data of optical coherent tomography vibrometry [21]. Different modes of wave propagation were observed [22], and studying how they couple and propagate [23] could be a daunting task. Nevertheless, the methods of EQ-NL may eventually provide a more concise way if mode coupling phenomena needs to be described in the nonlinear regime. This direction can be listed for future exploration.

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