Effects of Weak Decay and Hadronic Scattering on the Proton Number Fluctuations in Au + Au Collisions at $\sqrt{s_{NN}} = 5$ GeV from JAM Model

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Proton number fluctuation is sensitive observable to search for the QCD critical point in heavy-ion collisions. To estimate the non-critical contributions, we studied the effects of weak decay and hadronic scattering on proton number fluctuation and its correlation functions in Au+Au collisions at $\sqrt{s_{NN}} = 5$ GeV from a microscopic hadronic transport model (JAM). The JAM model calculation is also performed with different equation of states (EoS), which are cascade, mean field and attractive scattering orbit mode. The attractive scattering orbit mode is to simulate the softening of EoS in a first-order phase transition. In our study, we found that the effects of weak decay and hadronic scattering on the observables are small. This work can serve as a study for non-critical baseline for further QCD critical point search.

I. INTRODUCTION

Exploring the QCD phase structure is one of the main goals of heavy ion collision experiments. The conjecturing QCD phase structure can be plotted in a $T - \mu_B$ phase diagram. In this diagram, the transition from Quark-Gluon Plasma (QGP) phase to hadronic phase at the zero baryon chemical potential ($\mu_B = 0$) has been confirmed to be crossover by Lattice QCD calculation [1]. However, the attempts to apply Lattice QCD at non-zero $\mu_B$ region meets the sign problem. Several models predict a first-order transition at larger $\mu_B$ by using various methods to avoid the sign problem in lattice QCD simulation [2]. The QCD critical point is the end point of the first-order phase boundary. The experimental and/or theoretical confirmation of the QCD critical point would be a milestone in studying the QCD phase structure.

To study the QCD phase structure, the fluctuation of conserved charged such net-baryon number, net-charge number and net-strangeness number are considered to be sensitive probes to the QCD critical point and the phase transition [3–7]. They have been extensively studied experimentally [8–10] and theoretically [11–32]. In Relativistic Heavy-Ion Collisions (RHIC) experiments, the fluctuation of conserved quantity such as net-baryon number and its experimental proxy net-proton number has been served as observables in the search of critical point [33].

In the year of 2010-2014, RHIC has finished the first phase of beam energy scan (BES) and took the data of Au+Au collisions at $\sqrt{s_{NN}} = 7.7, 11.5, 14.5, 19.6, 27, 39, 62.4, 200$ GeV. With those experimental data, STAR experiment has measured the higher order fluctuations of net-proton, net-charge and net-kaon multiplicity distributions [9, 10, 34]. One of the striking observation is the behavior of the fourth-order cumulants or kurtosis of the net-proton fluctuation $\kappa^2$ in most central (0-5%) Au+Au collisions as a function of beam energy. It was observed that the fourth order net-proton fluctuation is close to unity above 39 GeV but deviates significantly below unity at 19.6 and 27 GeV, then becomes above unity at lower energies. This non-monotonic structure is consistent with the results from QCD based models assuming the existing of QCD critical point. This may suggest that the created system skims close by the CP, and received positive and/or negative contributions from critical fluctuations. On the other hand, the enhancement of the $\kappa\sigma^2$ cannot be explained by transport model UrQMD, which does not contain the physics of critical point. In the UrQMD, $\kappa\sigma^2$ of proton and net-proton show monotonic decrease when decreasing the collision energy. However, non-critical contributions from different physics, such as resonance weak decay and hadronic scattering, are not yet presented in the calculation.

In this work, we studied the effects of resonance weak decay and hadronic scattering on the proton number fluctuations and correlation functions in Au + Au collision at $\sqrt{s_{NN}} = 5$ GeV. This is the top energy covered by the future fixed target experiment CBM at FAIR. By utilizing a transport model JAM, we first test how fluctuations response to different equation of state (EoS) of QCD matter. In the test, we use a soft EoS by applying attractive scattering orbit, which simulates the softening of EoS in first-order transition. And a stiffer EoS by applying mean-field potential. Our calculation can serve as a study for non-critical baseline for further experiments.

This paper is organized as follow, we will first introduce the fluctuation observables: cumulants and correlation functions. Then we will introduce the JAM model and the effects applied in the simulation in section III. In section IV, we will present the result of cumulants and correlation functions calculation. Finally, we will give a summary.

II. CUMULANTS AND CORRELATIONS

In the study of QCD phase structure, fluctuations of conserved charge are ideal observables, as they are sensitive to the correlation length of the medium. To characterize the multiplicity fluctuations, one can measure cumulants of the event-by-event particle multiplicity distributions. In
eliminate the volume term in equations, where

\[ C_n = \langle N^n \rangle \]

\[ C_2 = \langle N^2 \rangle - \langle N \rangle^2 \]

\[ C_3 = 2\langle N^3 \rangle - 3\langle N \rangle\langle N^2 \rangle + \langle N^3 \rangle \]

\[ C_4 = -6\langle N \rangle^4 + 12\langle N^2 \rangle^2 - 3\langle N^2 \rangle^2 - 4\langle N \rangle^2 \langle N^3 \rangle + \langle N^4 \rangle \]

where the \( \langle N^n \rangle \) is the \( n \)-th order moments of net-proton or proton number. The higher order cumulants \( C_n \) are more sensitive to correlation \( \chi_n \) of medium as [35]

\[ C_n = VT^3 \chi_n \]  \hspace{1cm} (2)

The ratio of different order of cumulants are calculated to eliminate the volume term in equations, where

\[ S = \frac{C_3}{C_2^2}, \quad \kappa = \frac{C_4}{C_2^2} \]

\[ S\sigma = \frac{C_3}{C_2}, \quad \kappa\sigma^2 = \frac{C_4}{C_2} \]  \hspace{1cm} (3)

The error estimation of cumulants and cumulants ratio is introduce in reference [36, 37]. Sorts of techniques [38, 39] have also been applied to suppress volume fluctuations and auto-correlation in analysis.

We can extract the strength of multi-particle correlation from different order of single particle cumulants (i.e. the proton cumulants, but not the net-proton cumulants) [40–42]

\[ c_2 = -\langle N \rangle + C_2 \]

\[ c_3 = 2\langle N \rangle - 3C_2 + C_3 \]

\[ c_4 = -6\langle N \rangle + 11C_2 - 6C_3 + C_4 \]  \hspace{1cm} (4)

and we can also express cumulants by various order of multi-particle correlations \( c_n \)

\[ C_2 = \langle N \rangle + c_2 \]

\[ C_3 = \langle N \rangle + 3c_2 + c_3 \]

\[ C_4 = \langle N \rangle + 7c_2 + 6c_3 + c_4 \]  \hspace{1cm} (5)

here we denote the \( c \) in lower case as correlation functions (or factorial cumulants), the \( \kappa \)-th order correlation function measures the strength of \( k \)-particle correlation. It can be demonstrated that \( c_n \) \((n > 2)\) of Gaussian or Poisson statistics is always zero. Thus, we can measure Non-Gaussian fluctuations from correlation functions. The correlation functions can be computed by factorial moments \( F_n \)

\[ F_n = \langle N^n \rangle \equiv \langle N(N-1) \cdots (N-n+1) \rangle \]  \hspace{1cm} (6)

The relations between factorial moments and correlation functions are equivalent to the relation between moments and cumulants. Compare to Equation 1, we have

\[ c_2 = F_2 - F_1^2 \]

\[ c_3 = 2F_3 - 3FF_2 + F_3 \]  \hspace{1cm} (7)

It has been reported that the enhancement of \( \kappa\sigma^2 \) at low energies observed by the STAR experiments is mainly dominated by the four-particle correlation function [40]. And the critical fluctuations can be encoded in the acceptance dependence of cumulants and/or correlation functions [41, 43]. In our previous calculation with UrQMD model [44], we observed large deviations from experimental results in second and fourth order correlation functions. Thus, it is important to include the correlation functions in our calculation to study the contributions from non-critical effects.
III. NON-CRITICAL EFFECTS IN JAM MODEL

A. Equation of state

Methods to study the effect of various type of equation of state has been included in the transport JAM (Jet AA Microscopic Transportation Model) [45–48]. Generally, the EoS of medium can be expressed in the relation between pressure and the energy density of system: $p = p(\epsilon)$. There are two approaches to change the EoS in this study. The first is to vary the scattering style by using an attractive scattering orbit. And the second EoS is generated by employing mean-field potential in JAM model. As a comparison, we firstly perform the calculation in cascade mode which do not include the changes to EoS.

For a cascade mode, the scattering angle of the two-body collision is chosen randomly. The pressure of system can be given by virial theorem [49]

$$P = P_f + \Delta P$$  \hspace{1cm} (8)

where $P_f$ is free stream part and the $\Delta P$ is determined by the momentum transfer in two-body collision. The $\Delta P$ is reduced by introducing the attractive scattering orbit. In this mode, the momentum transfer is chosen to be attractive. The attractive scattering orbit simulates the effect of softening of EoS for the first-order phase transition.

Meanwhile, we use a Skyrme type density dependent and Lorentzian-type momentum dependent mean-field potential [50] in our simulation. In this mean-field potential, nucleons feel a repulsive interaction with other particles. Therefore, the $\Delta P$ in Equation 8 is enhanced and we get a stiffer EoS. We illustrate the results of applying two EoS in model by plotting the $dN/dy$ distribution in Fig. 1a.

Comparing to cascade model, with the softening of EoS, more protons are stopped in mid-rapidity due to the attractive scattering. In contrast, due to the repulsive scattering, lower $dN/dy$ distribution observed in the mean-field mode.

| Identify          | Weak decay | Hadronic scattering |
|-------------------|------------|---------------------|
| Mean-Field        | NO         | YES                 |
| Mean-Field (Weak decay) | YES       | YES                 |
| Mean-Field (No scattering) | NO        |                     |

TABLE I. Three types of data used to study the effects of weak decay and hadronic scattering. The data “without” hadronic scattering means we disable only the meson-baryon and meson-meson scattering, but the baryon-baryon interaction is remained.

B. Weak decay and hadronic scattering

To study the effects of weak decay and hadronic scattering, we compare three types of data in JAM model listed in Table I. The details of these data have been introduced in the following.

First, we are interest in how the weak decay of resonance changes the final state proton number fluctuation. To study this effect, we base on the mean-field data and switch on the weak decay in the model. The two data sets in this work are identified as “Mean-field” and “Mean-field (Weak decay)”. With the weak decay switched on, we found there is an increase of proton number as shown in Fig. 1b. In order to study the hadronic scattering, we compare the mean-field (weak decay) data with mean-field (no scattering) data, in which we disable the meson-baryon and meson-meson interaction. In heavy ion collisions, the time for nuclei passing through each other get longer for lower collision energies [46]. Therefore, there is also interaction of produced particles and initial nucleons (Spectator effect). In Fig. 1b, we can find that the $dN/dy$ distribution of no hadronic scattering data become flatter. We deduce that this is the consequence of a weaker effect of baryon stopping, as the proton number at mid-rapidity region become smaller.
IV. RAPIDITY DEPENDENCE OF PROTON CUMULANTS AND CORRELATION FUNCTIONS

We show the event-by-event proton number distributions in 0-5% most central Au+Au collisions at $\sqrt{s_{NN}} = 5$ GeV with various non-critical effects in Fig. 2. In Fig. 2a, we can find that a softer EoS (attractive scattering) tends to have more protons stopped in mid-rapidity region and the distribution has been shifted rightward comparing with the results from cascade mode, while a stiffer EoS (mean-field, or repulsive scattering) leads to a leftward shift [51]. In Fig. 2b, it shows that the weak decay could enhance the proton multiplicity in mid-rapidity region as well as the hadronic scattering. However, various EoS and hadronic scattering seem not to vary the width of distribution significantly.

In Fig. 3a we show the rapidity dependence of various order proton cumulants and correlation functions with different EoS. We plot the results in different rapidity cut. The $\Delta y$ is the proton rapidity acceptance (for cut $y < |y|$, $\Delta y = 2|y|$) in calculation, and the $y_{beam}$ is the beam rapidity. (for $\sqrt{s_{NN}} = 5$ GeV, $y_{beam} = 1.63$). In the first row of Fig. 3a, we plot the proton cumulants up to fourth order. For the rapidity dependence of the mean value $C_1$, we can find that the attractive scattering orbit and mean-field potential have slightly changes comparing to cascade mode. At small $\Delta y$ acceptance, mean value of proton number is larger with attractive scattering and smaller with mean-field potential due to stronger/weaker baryon stopping. The variance $(C_2)$ have similar behavior as $C_1$. However, there is significant change in $C_3$, mean-field potential reduces the $C_3$ magnitude in all rapidity acceptance. For the fourth order cumu-
lant ($C_4$), the results are consistent within errors for all of the three modes. In addition, the third and fourth order cumulants show strong suppression at large $\Delta y$ due to the baryon number conservation [52]. The effect of baryon number conservation (BNC) will be stronger when the fraction of proton number in the analysis get larger. In the second row of Fig. 3a, we show the proton correlation functions up to forth order. The $c_1$ is equal to $C_1$ in the first row. And the $c_2$ measures the strength of two-particle correlation. The rapidity dependence of $c_2$ is negative with various EoS and the results from different EoS is close to each other. It indicates the BNC leads to anti-correlation of proton in different rapidity range. The $c_2$ become more negative with the increase of $\Delta y$, which is dominated by the BNC effect.

Figure 3b shows the effects of weak decay and hadronic scattering on proton cumulants and correlation function. In the first row of Fig. 3b, we found that the $C_1$ values from the weak decay and/or hadronic scattering are larger than the results without decay and scattering. Regarding with the $C_2$, the situation is similar to $C_1$. Interestingly, we found that the hadronic scattering can substantially suppress the $C_3$ values, while the weak decay only reduces $C_3$ slightly at large $\Delta y$. Due to the BNC, the $C_3$ and $C_4$ show strong suppression at large rapidity. In the second row of Fig. 3b, we compare the correlation functions within different effects. The weak decay and hadronic scattering can lead to stronger suppression effect for $c_2$. But these effects are small for third ($c_3$) and four particle ($c_4$) correlation function. The rapidity dependence of $c_3$ and $c_4$ is almost flat and close to zero at small rapidity window, which indicates that the higher order correlation functions are less sensitive to the effect of BNC.

In Fig. 4, we show the rapidity dependence of various cumulant ratios with different EoS and show the effects of weak decay and hadronic scattering. In general, we found the proton cumulant ratios decrease when increasing the rapidity window. This can be explained by the effects of BNC. In addition, the results from different EoS are consistent with each other within uncertainties, which means the effects of EoS studied here is small on the proton cumulant ratios. In Fig. 4b, we found that the hadronic scattering will suppress the second and third order cumulant ratios ($C_2/C_1$, $S\sigma$ and $C_3/C_1$). Meanwhile, proton cumulant ratios are hardly affected by the resonance weak decay.

V. SUMMARY

We studied non-critical effects (EoS, weak decay, hadronic scattering) to proton cumulant and correlation functions in Au+Au collisions at $\sqrt{s_{NN}} = 5$ GeV within JAM model. In our analysis, we found the baryon number conservation is the dominant background effect to the rapidity dependence of proton number fluctuations. It leads to the suppression of cumulants and cumulant ratios, as well as the negative-correlation of protons. The higher order correlation functions seem to be receive less affects from BNC. On the other hand, the effect of EoS, weak decay and hadronic scattering can change the magnitude of higher order cumulants and correlation functions. Attractive scattering mode in JAM model, which simulate the soften EoS can cause more proton stopped in the mid-rapidity region, and enlarge the $C_1$ values. The mean-field repulsive potential can result in stronger suppression for $C_3$. The effect from weak decay is not as pronounced as the effect from hadronic scattering, which can significantly suppress the third order cumulants and the two-particle correlation as well as the corresponding cumulants ratios. Our work provides a non-critical baseline for the future QCD critical point search in heavy-ion collisions.
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