Whistler Wave Turbulence in Solar Wind Plasma

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Abstract. Whistler waves are present in solar wind plasma. These waves possess characteristic turbulent fluctuations that are characterized typically by the frequency and length scales that are respectively bigger than ion gyro frequency and smaller than ion gyro radius. The electron inertial length is an intrinsic length scale in whistler wave turbulence that distinguishes the high frequency solar wind turbulent spectra into scales smaller and bigger than the electron inertial length. We present nonlinear three dimensional, time dependent, fluid simulations of whistler wave turbulence to investigate their role in solar wind plasma. Our simulations find that the dispersive whistler modes evolve entirely differently in the two regimes. While the dispersive whistler wave effects are stronger in the large scale regime, they do not influence the spectral cascades which are describable by a Kolmogorov-like $k^{-7/3}$ spectrum. By contrast, the small scale turbulent fluctuations exhibit a Navier-Stokes like evolution where characteristic turbulent eddies exhibit a typical $k^{-5/3}$ hydrodynamic turbulent spectrum. By virtue of equipartition between the wave velocity and magnetic fields, we quantify the role of whistler waves in the solar wind plasma fluctuations.

Keywords: MHD Plasma, Whistler waves, Space Plasmas, 3D Simulation

PACS: 96.50.Ci, 96.50.Tf, 96.50.Ya, 96.50.Zc

1. INTRODUCTION

The solar wind is an excellent in-situ laboratory for investigating nonlinear and turbulent processes in a magnetized plasma fluid since it comprises a multitude of spatial and temporal length-scales associated with an admixture of waves, fluctuations, structures and nonlinear turbulent interactions. The in-situ spacecraft measurements [1, 2] reveal that the solar wind fluctuations, extending over several orders of magnitude in frequency and wavenumber, describe the power spectral density (PSD) spectrum that can be divided into three distinct regions [2,3]. The frequencies, for instance, smaller than $10^5$ Hz lead to a PSD that has a spectral slope of -1. This follows the region that extends from $10^2$ Hz to or less than ion/proton gyrofrequency where the spectral slope exhibits an index of -3/2 or -5/3. Smaller than ion gyro radius ($k \rho_i \gg 1$) and temporal scales bigger than ion cyclotron frequency $\omega > \omega_c = eB_0/m_e c$, (where $k, \rho_i, \omega_c, e, B_0, m_e, c$ are respectively characteristic mode, ion gyroradius, ion cyclotron frequency, electronic charge, mean magnetic field, mass of electron and speed of light) exhibit a spectral break where the inertial range slope of the solar wind turbulent fluctuations varies between -2 and -5 [2,3]. The onset of the second or the kinetic Alfven inertial range is still elusive to our understanding of the solar wind turbulence and many other nonlinear interactions. Specifically, the mechanism leading to the spectral break has been thought to be either mediated by the kinetic Alfven waves (KAWs) [8], or by electromagnetic whistler fluctuations [5,4], or by a class of fluctuations that can be dealt within the framework of the HMHD plasma model [9,10]. Stawicki et al [7] argue that Alfven fluctuations are suppressed by proton cyclotron damping at intermediate wavenumbers so the observed power spectra are likely to consist of weakly damped magnetosonic and/or whistler waves which are dispersive unlike Alfven waves. Moreover, turbulent fluctuations corresponding to the high frequency and $k \rho_i \gg 1$ regime lead to a decoupling of electron motion from that of ion such that the latter becomes unmagnetized and can be treated as an immobile neutralizing background fluid. While whistler waves typically survive in the higher frequency (and the corresponding smaller length scales) part of the solar wind plasma spectrum, their role in influencing the inertial range turbulent spectral cascades is still debated [11,12,13,14,15].

In this paper, we focus on understanding the nonlinear turbulent cascades mediated by whistler waves in a fully three dimensional geometry. Our objective is to investigate the role of whistlers in establishing the turbulent equipartition amongst the modes that are responsible for the nonlinear mode coupling interactions which critically determine the inertial range power spectra.

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Whistler modes are excited in the solar wind plasma when the characteristic plasma fluctuations propagate along a mean or background magnetic field with frequency $\omega > \omega_{ci}$ and the length scales are $c/\omega_{pi} < l < c/\omega_{pe}$, where $\omega_{pi}, \omega_{pe}$ are the plasma ion and electron frequencies. The electron dynamics plays a critical role in determining the nonlinear interactions while the ions merely provide a stationary neutralizing background against fast moving electrons and behave as scattering centers. The whistler wave turbulence can be described by the electron magnetohydrodynamics (EMHD) model of plasma [16]. The three-dimensional equation of EMHD describing the evolution of the magnetic field fluctuations in whistler wave,

$$\frac{\partial}{\partial t}(B - d^2_k \nabla^2 B) + V_e \cdot \nabla (B - d^2_k \nabla^2 B) - (B - d^2_k \nabla^2 B) \cdot \nabla V_e = \mu d^2_k \nabla^2 B.$$  \hspace{1cm} (1)

The length scales in Eq. (1) are normalized by the electron skin depth $d_e = c/\omega_{pe}$ i.e. the electron inertial length scale, the magnetic field by a typical amplitude $B_0$, and time by the corresponding electron gyro-frequency. In Eq. (1), the diffusion operator on the right hand side is raised to 2. Here $n$ is an integer and can take $n = 1, 2, 3, \ldots$. The case $n = 1$ stands for normal diffusion, while $n = 2, 3, \ldots$ corresponds to hyper- and other higher order diffusion terms.

The linearization of Eq. (1) about a constant magnetic field $B_0$ yields the dispersion relation for the whistlers, the normal mode of oscillation in the EMHD frequency regime, and is given by

$$\omega_k = \omega_0 + \frac{d^2_k k^2}{1 + d^2_k k^2},$$

where $\omega_0 = eB_0/mc$ and $k^2 = k_x^2 + k_y^2$. From Eq. (1), it appears that there exists an intrinsic length scale $d_e$, the electron inertial skin depth, which divides the entire spectrum into two regions; namely short scale ($kd_e > 1$) and long scale ($kd_e < 1$) regimes. In the regime $kd_e < 1$, the linear frequency of whistlers is $\omega_k \sim k, k$ and the waves are dispersive. Conversely, dispersion is weak in the other regime $kd_e > 1$ since $\omega_k \sim k$, hence the whistler wave packets interact more like the eddies of hydrodynamical fluids.

3. SIMULATIONS

Turbulent interactions mediated by the coupling of whistler waves and inertial range fluctuations are studied in three dimensions (3D) based on a nonlinear 3D whistler wave turbulence code that we have developed at Center for Space Plasma and Aeronomic Research (CSPAR), the University of Alabama in Huntsville (UAH). Our code numerically integrates Eq. (1). The spatial discretization employs a pseudospectral algorithm [12, 14] based on a Fourier harmonic expansion of the bases for physical variables (i.e. the magnetic field, velocity), whereas the temporal integration uses a Runge Kutta (RK) 4th order method. The boundary conditions are periodic along the $x, y$ and $z$ directions in the local rectangular region of the solar wind plasma.

Electron whistler fluid fluctuations, in the presence of a constant background magnetic field, evolve by virtue of nonlinear interactions in which larger eddies transfer their energy to smaller ones through a forward cascade. According to [19], the cascades of spectral energy occur purely amongst the neighboring Fourier modes (i.e. local interaction) until the energy in the smallest turbulent eddies is finally dissipated gradually due to the finite dissipation. This leads to a damping of small scale motions. By contrast, the large-scales and the inertial range turbulent fluctuations remain unaffected by direct dissipation of the smaller scales. Since there is no mechanism that drives turbulence at the larger scales in our model, the large-scale energy simply migrates towards the smaller scales by virtue of nonlinear cascades in the inertial range and is dissipated at the smallest turbulent length-scales. The spectral transfer of turbulent energy in the neighboring Fourier modes in whistler wave turbulence follows a Kolmogorov phenomenology [19, 18, 20] that leads to Kolmogorov-like energy spectra. We find from our 3D simulations that whistler wave turbulence in the $kd_e < 1$ and $kd_e > 1$ regimes exhibits respectively $k^{-7/3}$ and $k^{-5/3}$ (see Fig 1) spectra. The inertial range turbulent spectra obtained from our 3D simulations are further consistent with 2D work [11, 21, 22]. Interestingly, the wave effects dominate in the large scale, i.e. $kd_e < 1$, regime where the inertial range turbulent spectrum depicts a Kolmogorov-like $k^{-7/3}$ spectrum. On the other hand, turbulent fluctuations in the smaller scale ($kd_e > 1$) regime behave like non magnetic eddies of hydrodynamic fluid and yield a $k^{-5/3}$ spectrum. The wave effect is weak, or negligibly small, in the latter. Hence the nonlinear cascades are determined essentially by the hydrodynamic like interactions. The observed whistler wave turbulence spectra in the $kd_e < 1$ and $kd_e > 1$ regimes (Figs 1) can be followed from the Kolmogorov-like arguments [13, 18, 20] that describe the inertial range spectral cascades. We elaborate on these arguments to explain our simulation results of Fig. (1) in the following section.


4. WHISTLER WAVE SPECTRA

The exact spectral indices corresponding to the whistler wave turbulent spectra, described by the ideal electron magnetohydrodynamic invariant, can be understood from the Kolmogorov’s dimensional arguments [18,20].

In the underlying whistler wave model of magnetized plasma turbulence, the inertial range eddy velocity is characterized typically by \( v_e \sim \nabla \times B \). Thus the typical velocity of the magnetic field eddy \( B_\ell \) with a scale size \( \ell \) can be represented by \( v_e \sim B_\ell / \ell \). The eddy turn-over time is then given by

\[
\tau \sim \frac{\ell}{v_e} \sim \frac{\ell^2}{B_\ell}.
\]

This is the time scale that predominantly leads to the nonlinear spectral transfer of energy in fully developed whistler wave turbulence.

In the regime where characteristic length scales are bigger than the electron skin depth \( (kd_e < 1) \), the inertial range whistler turbulent energy is dominated by the large scale fluctuations. The total energy corresponding to the turbulent fluctuations in this regime is then given as,

\[
E \sim |B|^2 \sim B_\ell^2 \sim v_e^2 \ell^2.
\]

The \( B_\ell \) represent magnetic field associated with the magnetic field eddy of length \( \ell \). The second similarity follows from the assumption of an equipartition of energy in the magnetic and velocity field components of whistler waves. The process of equipartition originates from the correlation between the velocity and magnetic field fluctuations \( v_e \sim k \times B \), where \( k = k_x \hat{x} + k_y \hat{y} + k_z \hat{z} \) is a three dimensional wave vector. The latter is further consistent with the electron flow speed, in combination with the wave perturbed magnetic field, that is used to derive the dynamical equation of whistler wave turbulence, i.e. Eq. (1). This velocity-magnetic field correlation essentially produces the velocity field fluctuations that are normal to the magnetic field in a whistler wave packet. Consequently, the energy associated with the velocity and magnetic field for each characteristic turbulent mode evolves toward a relationship that satisfies \( v_e^2 \sim k^2 B^2 \). To quantify our arguments, we follow the evolution of turbulent equipartition in our simulations by computing the following quantity,

\[
E_{\text{equi}}(t) \sim \sum_k (|v_e(k,t)|^2 - k^2 |B(k,t)|^2),
\]

Interestingly, our 3D simulations, describing the equipartition between the velocity and magnetic field fluctuations, are consistent with the 2D counterpart. It thus appears that the turbulent equipartition is a robust feature of whistler waves that is preserved in both 2D and 3D nonlinear mode coupling interactions. The spectral cascades of inertial range turbulent energy is nonetheless determined by the energy cascade per unit nonlinear time as follows,

\[
\epsilon \sim \frac{E}{\tau} \sim \frac{B_\ell^4}{\ell^2}.
\]

On assuming that the spectral energy cascade is local in the wavenumber space [18,19,20], the energy spectrum per unit mode yields \( E_k \sim \epsilon^\alpha k^\beta \). On substituting the energy and energy dissipation rates and equating the powers of \( B_\ell \) and \( \ell \), we obtain \( \alpha = 2/3 \) and \( \beta = -7/3 \). This, in the \( kd_e < 1 \) regime, leads to the following expression for the energy spectrum \( E_k \sim \epsilon^{2/3} k^{-7/3} \).
that are associated with the nonlinear term.

The convective time scales, which the wave magnetic and velocity fields are strongly correlated through the equipartition (\(\frac{v_e}{\omega_i} \sim k^2 B^2\)). The latter is employed in our simulations to quantify the role of whistler waves corresponding to the inertial range fluctuations that possess characteristic frequency bigger than the ion gyro frequency (\(\omega > \omega_i\)) and length scales smaller than the ion gyro radius (\(k\rho_i > 1\)). In this regime, the solar wind plasma fluctuations comprise of unmagnetized ions, hence the entire dynamics is governed by the electron fluid motions. The rotational magnetic field fluctuations in the presence of a background magnetic field lead to propagation of dispersive whistler waves in which the wave magnetic and velocity fields are strongly correlated through the equipartition (\(v_e^2 \sim k^2 B^2\)). The latter is employed in our simulations to quantify the role of whistler waves that are ubiquitously present in the inertial range in the high frequency (\(\omega > \omega_i\)) solar wind plasma. Interestingly we find that despite strong wave activity in the inertial range, whistler waves do not influence the inertial range turbulent spectra. Consequently, the turbulent fluctuations in the inertial range are described by Kolmogorov-like phenomenology [19].

It is to be noted that as long as the cascade of energy is concerned, kinetic [5,6] and fluid [12,13,14,15] like processes lead to a similar power law (i.e. \(E_k \sim k^{-5/3}\)) in the \(k\rho_e > 1\) regime. This is because the energy cascade is determined entirely by the convective time scales that are associated with the nonlinear term \(v_e, \nabla v^2, B\) in the electron fluid momentum equation. The breakdown of fluid-like behavior occurs for the characteristic scales \(k\rho_e \gg 1, k\rho_e \sim 1\), where \(\rho_e\) is electron gyro radius. The major difference in the fluid and kinetic models, however, arises from wave-particle resonances (or wave-particle interactions) which are a fully kinetic effect and it is beyond the capability of the fluid theory. Since the energy spectra are not critically dependent on the wave-particle resonances, our fluid model yields spectral laws similar to the kinetic model.

To conclude, consistent with the Kolmogorov-like dimensional argument [19], we find that turbulent spectra in the \(k\rho_e < 1\) and \(k\rho_e > 1\) regimes are described respectively by \(k^{-7/3}\) and \(k^{-5/3}\). Our results are important particularly in understanding turbulent cascade corresponding to the high frequency (\(\omega > \omega_i\)) solar wind plasma where characteristic fluctuations are comparable to the electron inertial skin depth.

The support of NASA(NNG-05GH38) and NSF (ATM-0317509) grants is acknowledged.

5. CONCLUSIONS

Three dimensional simulations of turbulent cascades in solar wind plasma are carried out to quantify the role of whistler waves corresponding to the inertial range fluctuations that possess characteristic frequency bigger than the ion gyro frequency (\(\omega > \omega_i\)) and length scales smaller than the ion gyro radius (\(k\rho_i > 1\)). In this regime, the solar wind plasma fluctuations comprise of unmagnetized ions, hence the entire dynamics is governed by the electron fluid motions. The rotational magnetic field fluctuations in the presence of a background magnetic field lead to propagation of dispersive whistler waves in which the wave magnetic and velocity fields are strongly correlated through the equipartition (\(v_e^2 \sim k^2 B^2\)). The latter is employed in our simulations to quantify the role of whistler waves that are ubiquitously present in the inertial range in the high frequency (\(\omega > \omega_i\)) solar wind plasma. Interestingly we find that despite strong wave activity in the inertial range, whistler waves do not influence the inertial range turbulent spectra. Consequently, the turbulent fluctuations in the inertial range are described by Kolmogorov-like phenomenology [19].

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