Fusion of light proton-rich exotic nuclei at near-barrier energies

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Abstract

We study theoretically fusion of the light proton-rich exotic nuclei \(^{17}\)F and \(^{8}\)B at near-barrier energies in order to investigate the possible role of breakup processes on their fusion cross sections. To this end, coupled channel calculations are performed considering the couplings to the breakup channels of these projectiles. In case of \(^{17}\)F, the coupling arising out of the inelastic excitation from the ground state to the bound excited state and its couplings to the continuum have also been taken into consideration. It is found that the inelastic excitation/breakup of \(^{17}\)F affect the fusion cross sections very nominally even for a heavy target like Pb. On the other hand, calculations for fusion of the one-proton halo nucleus \(^{8}\)B on a Pb target show a significant suppression of the complete fusion cross section above the Coulomb barrier. This is due to the larger breakup probability of \(^{8}\)B as compared to that of \(^{17}\)F. However, even for \(^{8}\)B, there is little change in the complete fusion cross sections as compared to the no-coupling case at sub-barrier energies.

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I. INTRODUCTION

The increasing availability of radioactive ion beams has made possible the study of the interactions and structure of exotic nuclei far from the line of stability. Nuclei located near the neutron or proton drip lines, for which the valence particles are very loosely bound, give rise to interesting new phenomena, e.g., formation of halo structures \cite{1}. The low binding energies of the valence nucleons in these nuclides result in large root mean square (rms) radii and thus, in increased probabilities for specific reaction channels such as breakup, nucleon transfer and fusion. Breakup reactions, in which the valence nucleons are removed from these exotic nuclei, have very large cross sections of a few barns. This is particularly true for the neutron-halo nuclei with well-developed halo structure, e.g., \(^{11}\)Be, \(^{11}\)Li. However, the breakup cross sections are quite significant also for a light one-proton halo nucleus like \(^{8}\)B \cite{2}.

Since a large flux goes into the breakup channel, many questions concerning the effects of breakup processes on sub-barrier fusion of drip-line nuclei have been raised recently, both from the experimental \cite{3–8} and theoretical \cite{9–12} points of view. The presence of halo structure in these exotic nuclei means a rms matter radius larger than the usual value deduced from the systematics. Consequently, the sub-barrier fusion cross section should be enhanced since the Coulomb barrier is lowered. On the other hand, one might think that increased breakup probabilities for these nuclei would remove a significant part of flux and thus cross sections for complete fusion would be hindered.

Very recently fusion at near-barrier energies has been investigated theoretically for the neutron halo nuclei \(^{11}\)Be and \(^{6}\)He on Pb. Coupled channel calculations have been performed by discretizing in energy the particle continuum states \cite{12}. The calculations show that the couplings to the breakup channels have appreciable effects. At energies above the Coulomb barrier, it is found that the cross sections for complete fusion are hindered compared to the no-coupling case. On the other hand, at sub-barrier energies the complete fusion cross sections are enhanced significantly. Recent measurements on fusion cross sections for the loosely bound nucleus \(^{9}\)Be \cite{7} and the two-neutron halo nucleus \(^{6}\)He \cite{8} on heavy targets at near-barrier energies are in general agreement with the theoretical findings in Ref. \cite{12}.

However, for the proton-rich systems, the effect of coupling to the breakup channels on the fusion process may not be as significant as the neutron-rich nuclei. This is because unlike the neutron-rich systems the valence proton in the loosely bound proton-rich exotic nuclei has to tunnel through the barrier resulting from the Coulomb repulsion due to the charged core, which hinders the formation of halo. In fusion reaction of such a nucleus with a target, this might not lead to a significant lowering of the Coulomb barrier.

Fusion reaction induced by a proton-rich nucleus has received attention only recently. First measurement of fusion cross sections has been done for \(^{17}\)F \cite{4}. Therefore, we present results of calculations on fusion cross sections of the proton drip-line exotic nucleus \(^{17}\)F and compare with the data. But it would also be quite interesting to study fusion induced by a proton-rich nucleus with large breakup probabilities, e.g., the one-proton halo nucleus \(^{8}\)B. In this paper, we also report first calculations on fusion reactions induced by the one-proton halo nucleus \(^{8}\)B. We follow the same theoretical formalism as in Ref. \cite{12}. Within this coupled channel method of calculation, we neglect continuum-continuum couplings. We assume the target to be inert in each case, i.e., possible target excitations are neglected. We
choose a heavy target because it would be ideal to study the effects of inclusion of coupling to the breakup channels, as breakup effects increase with target mass.

In section 2, we present the theoretical formalism. Results and discussions are given in section 3. Finally, we summarize and conclude in section 4.

II. FORMALISM

The coupled channel equations for the projectile($P$)-target($T$) system (with reduced mass $\mu$) in the iso-centrifugal approximation \[13\] are given by \[14\]

\[
\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V_N^{(0)}(r) + \frac{Z_P Ze^2}{r} + \epsilon_n - E\right]\psi_n(r) = -\sum_m V_{nm}(r)\psi_m(r). \tag{1}
\]

In writing the above equations, the angular momentum of the relative motion in each channel has been replaced by the total angular momentum $l$ \[13\]. In eqs.(1), $V_N^{(0)}$ is the nuclear potential in the entrance channel and $\epsilon_n$ is the excitation energy of the $n$–th channel. Here we assume that $n = 0$ labels the ground state of the projectile nucleus and all other $n$'s refer to bound excited states or particle continuum states of the projectile. $V_{nm}$ are the coupling form factors computed on the microscopic basis. They are obtained by folding the external nuclear and Coulomb fields with the proper single-particle transition densities, in which the last weakly bound nucleon is promoted from the ground state to excited states below the breakup threshold or in the continuum. More explicitly, the coupling form factor for the promotion of the single valence particle from the bound state $(n_1l_1j_1m_1)$ to the continuum state $(l_2j_2m_2)$ with continuum energy $E_c$ is given by

\[
V_{nm}(r) = \sum_{m_c} \langle j_1m_1j_cm_c | J_iM_i \rangle \langle j_2m_2j_cm_c | J_fM_f \rangle \\
\times f_{n_1l_1j_1m_1\rightarrow l_2j_2m_2}(r), \tag{2}
\]

where $J_i(f), M_i(f)$ are the total spin of the projectile and its projection quantum number in the initial (final) state. $j_c$ is the spin of the core and $m_c$ is its projection. We have assumed a core + single particle structure for the projectile. The single-particle form factor is given by

\[
f_{n_1l_1j_1m_1\rightarrow l_2j_2m_2}(r) \\
= \sqrt{\pi} \sum_\lambda (-1)^{m_2} \frac{\lambda}{2} \delta(l_1 + l_2 + \lambda, even) \langle j_1 \frac{1}{2}j_2 - \frac{1}{2} | \lambda 0\rangle \\
\times \frac{\sqrt{2j_1 + 1}\sqrt{2j_2 + 1}}{\sqrt{2\lambda + 1}} \langle j_1 - m_1j_2m_2 | \lambda(m_2 - m_1) \rangle \sqrt{\frac{2\lambda + 1}{4\pi}} \delta_{m_1,m_2} \\
\times \int_0^{\infty} r'^2 dr' \int_{-1}^{+1} du R_{E_i,l_2j_2}(r') R_{n_1l_1j_1}(r') V_T(\sqrt{r'^2 + r^2 - 2rr'}u) P_\lambda(u), \tag{3}
\]

where $P_\lambda$ is the Legendre polynomial. The functions $R_{E_i,l_2j_2}(r)$ and $R_{n_1l_1j_1}(r)$ are the radial parts of the single-particle wave functions for the continuum and bound states, respectively. In case of a transition from the ground state to a bound excited state, $R_{E_i,l_2j_2}(r)$ would be
replaced by \( R_{n^2l^2j^2}(r) \) in eq.(3). The potential \( V_T \), involving both a nuclear and a Coulomb component, is the mean field experienced by the single particle due to the presence of the target, responsible for the transition.

The coupled channel eqs.(1) are solved by imposing the incoming wave boundary condition (IWBC) [15], where there are only incoming waves at \( r_{\text{min}} \) which is taken to be the minimum position of the Coulomb pocket inside the barrier. This type of boundary condition is valid for heavy-ion reactions, for which there is a strong absorption inside the Coulomb barrier. The boundary conditions are expressed as

\[
\psi_n(r) \rightarrow T_n \exp(-i \int_{r_{\text{min}}}^r k_n(r')dr'), \quad r \leq r_{\text{min}}, \tag{4}
\]

\[
\rightarrow H_l^{(-)}(k_n r) \delta_{n,0} + R_n H_l^{(+)}(k_n r), \quad r > r_{\text{max}}, \tag{5}
\]

where

\[
k_n(r) = \sqrt{\frac{2\mu}{\hbar^2}(E - \epsilon_n - \frac{l(l+1)\hbar^2}{2\mu r^2} - V^{(0)}_N(r) - \frac{Z_P Z_T e^2}{r})} \tag{6}
\]

is the local wave number for the \( n \)–th channel and \( k_n \), in Eq.(5), is equal to \( \sqrt{2\mu (E - \epsilon_n) / \hbar^2} \). \( H_l^{(-)} \) and \( H_l^{(+)} \) are the incoming and outgoing Coulomb functions, respectively. \( r_{\text{max}} \) is taken to be a distance beyond which both the nuclear potential and the Coulomb coupling are sufficiently small.

The fusion probability is defined as the ratio between the flux inside the Coulomb barrier and the incident flux. For the boundary conditions given by eqs.(4) and (5), it becomes

\[
P_n = \frac{k_n(r_{\text{min}})}{k_0} | T_n |^2 \tag{7}
\]

for the \( n \)–th channel. Complete fusion is a process where all the nucleons of the projectile are captured by the target nucleus. We thus define cross section of complete fusion using the flux for the non-continuum channel (i.e. \( n = 0 \)) as [16]

\[
\sigma_{\text{CF}} = \frac{\pi}{k_0^2} \sum_l (2l + 1) P_0 = \frac{\pi}{k_0^2} \sum_l (2l + 1) \frac{k_0}{{k_0}} r_{\text{min}} | T_0 |^2. \tag{8}
\]

The flux for the particle continuum channel \((n \neq 0)\) is associated with incomplete fusion, whose cross section is given as

\[
\sigma_{\text{ICF}} = \frac{\pi}{k_0^2} \sum_l (2l + 1) \sum_{n \neq 0} P_n = \frac{\pi}{k_0^2} \sum_l (2l + 1) \sum_{n \neq 0} \frac{k_n(r_{\text{min}})}{k_0} | T_n |^2. \tag{9}
\]

Eqs.(8) and (9) are correct when there is only one bound state below the breakup threshold. If there are some bound excited states below the threshold other than the ground state, the summations in these equations are to be carried out accordingly.
III. RESULTS AND DISCUSSIONS

In recent measurements with the proton drip-line exotic nucleus $^{17}$F in fusion-fission reaction on Pb at energies around the Coulomb barrier no enhancement of the fusion cross section is observed \[4\]. In Fig. 1, we show our calculations on the fusion cross sections for $^{17}$F + Pb at near-barrier energies alongwith the data \[4\]. In our calculation, we have considered the effect of the transition from the ground state of $^{17}$F (600 keV below the breakup threshold) into the continuum. The bound excited state situated 495 keV above the ground state is also included in the calculation. This means that we have considered ground state ($0d_\frac{5}{2}$) to first excited state ($1s_\frac{1}{2}$) coupling (through quadrupole transition) and also couplings from the first excited state to the continuum. The continuum up to 2 MeV has been considered and it has been found that the results converge. The continuum has been discretized into 10 bins with a bin size of 200 keV. The continuum states have been taken to be situated at the middle of each bin. We consider transitions of multipolarity 1 and 2 for transitions into the continuum, with $s$, $p$, $d$- and $f$-waves in the continuum. The nuclear part of the valence proton-target interaction, causing the inelastic transition/breakup of $^{17}$F, has been taken to be the same as the neutron-target interaction in Ref. \[12\]. The nuclear part of the ion-ion interaction potential has been taken to be equal to that of the neighbouring nucleus $^{16}$O on Pb \[17\]. As far as the structure of $^{17}$F is concerned, we assume a single particle potential model in which the valence proton moves in a Coulomb and Woods-Saxon potential relative to $^{16}$O core. The depth of the Woods-saxon potential has been adjusted to reproduce the known one-proton separation energies for the bound states. The radius and diffuseness parameters of the Woods-Saxon potential have been taken to be 1.25 fm and 0.5 fm respectively. This gives a rms proton radius of 3.7 fm.

There is good overall agreement of our calculations with the data. But compared to the no-coupling case, there is no discernible change of the fusion cross sections when the channel coupling effects are considered (see Fig. 1).

However, observations are quite different when calculations on the fusion cross sections of $^{8}$B on the same target (Pb) have been done. We use the single-particle potential model used by Esbensen and Bertsch to describe the structure of $^{8}$B \[18,19\]. This yields a rms proton radius of 4.24 fm. We consider transitions into the continuum of multipolarities 1 and 2, and include $J = 1$, 2 and 3 final channels in the couplings. $s$, $p$, $d$- and $f$-waves in the continuum are considered for the appropriate transitions. The inclusion of the $J = 4$ final channel in the calculation has almost got no effect. The $J = 0$ final channel has been left out of calculation as the $0^+$ final state appears to be very weak in the low-lying excitation spectrum of $^{8}$B \[18\]. The continuum discretization scheme is the same as that for $^{17}$F. The $1^+$ and $3^+$ resonances at 0.637 MeV and 2.183 MeV have also been included in the calculations. The nuclear potential for the $^{8}$B + Pb system has been taken to be equal to that between the neighbouring nucleus $^{7}$Be and Pb as in Ref. \[7\]. As for $^{17}$F, the nuclear part of the valence proton-target interaction, causing the inelastic transition/breakup of $^{8}$B, has been taken to be the same as the neutron-target interaction in Ref. \[12\].

The results are displayed in Fig. 2. At sub-barrier energies, there is some 20% increase of the total (complete + incomplete) fusion cross sections (dashed line) compared to the zero coupling case. But results of channel coupling are nominal so far as complete fusion cross sections (thick solid line) in this region are concerned. However, at energies above
the Coulomb barrier, there is significant suppression of the complete fusion cross sections. The complete fusion cross section is \(~\sim 69\%\) of the total fusion cross section, for example, at 60 MeV centre-of-mass (c.m.) energy. This feature of fusion cross sections in \(^8\)B induced reaction on a heavy target at above-barrier energies is the same as that for the one-neutron halo nucleus \(^{11}\)Be. But compared to the appreciable enhancement of the complete fusion cross sections in case of \(^{11}\)Be \([13,20]\), the changes observed are small in the sub-barrier region.

If only dipole transitions are considered, the breakup effects will be smaller compared to the full (dipole + quadrupole) calculations and we expect that the suppression of the complete fusion cross sections will be less. For \(^{17}\)F + Pb reaction, practically no difference is observed as to this difference in calculation. However, for the \(^8\)B induced reaction on Pb, this effect is visible at above-barrier energies (see Fig. 3). But the increase in the complete fusion cross section is not much, e.g., \(~\sim 4\%\) at 60 MeV. This indicates that the quadrupole breakup contributions are small in case of \(^8\)B + Pb reaction.

The breakup probabilities of an exotic nucleus depend significantly on the separation energy of the valence nucleon and its orbital angular momentum configuration \([21]\). Any non-zero angular momentum with respect to the core will lead to a centrifugal barrier, which will restrict the extent of the wave function in the coordinate space. This, in turn, decreases the cross sections of breakup processes in which the valence particle is removed from the nucleus \([21]\). Increase of separation energy of the valence nucleon further decreases the breakup probabilities \([21]\).

The dominant ground state configuration of \(^8\)B is \(^7\)Be(\(\frac{3}{2}^-\)) \(\otimes\) \(\pi(0p_{\frac{3}{2}})\), whereas for \(^{17}\)F it is \(^{16}\)O(\(0^+\)) \(\otimes\) \(\pi(0d_{\frac{5}{2}})\). The valence proton in \(^{17}\)F feels more than two times the Coulomb barrier as the last proton in \(^8\)B. The \(\ell=2\) centrifugal barrier experienced by the valence proton in \(^{17}\)F is also almost 3.5 times larger compared to the \(\ell=1\) centrifugal barrier for the valence proton in \(^8\)B. The one-proton separation energy (0.6 MeV) in \(^{17}\)F is almost 4.5 times that (0.137 MeV) in \(^8\)B. Thus, breakup through one-proton removal is much more favoured in case of \(^8\)B compared to \(^{17}\)F. Dynamical calculations in Ref. \([4]\) also show that the chances of inelastic excitation/breakup of \(^{17}\)F are small. On the other hand, the breakup probabilities are quite significant for \(^8\)B on a heavy target \([3]\). In fact, within a semiclassical theory \([22]\), we compute the breakup cross section of \(^8\)B on Pb at 60 MeV in the laboratory system arising out of electric dipole transition to be as large as 5.1 b. This is close to 4.6 b, the \(E1\) dissociation cross section for the one-neutron halo nucleus \(^{11}\)Be on the same target at the same incident energy. On the other hand, for \(^{17}\)F on Pb target at near-barrier energy, the cross section for the same process comes out to be 46.46 mb only. Therefore, it is expected that compared to \(^{17}\)F, the breakup process would have larger effects on the fusion cross sections in \(^8\)B induced reactions.

IV. SUMMARY AND CONCLUSIONS

In summary, we have performed coupled channel calculations of fusion cross sections in reactions induced by the proton-rich exotic nuclei \(^{17}\)F and \(^8\)B on Pb target at near-barrier energies. Couplings to the inelastic and/or breakup channels have been taken into consideration. For \(^{17}\)F induced reaction, the channel coupling effects are minimal. However, for \(^8\)B induced reaction, there is appreciable suppression of the complete fusion cross sections
at energies above the Coulomb barrier. The difference in the features of the fusion cross sections in reactions involving these two nuclei is attributed to the difference in their ground state configurations. $^8\text{B}$ is a one-proton halo nucleus with larger breakup probabilities, most of its breakup cross sections originating due to dipole transitions. On the other hand, $^{17}\text{F}$, although proton-rich, is close to an ordinary nucleus so far as its size and breakup cross sections are concerned. We suggest measurement of the different components of the fusion cross sections at near-barrier energies in reaction involving $^8\text{B}$ and a heavy target.

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FIG. 1. Fusion cross sections in the $^{17}$F + Pb reaction at near-barrier energies. The thin solid line gives the cross sections with zero couplings. The thick solid line and the dashed line give the complete and the total (complete + incomplete) fusion cross sections respectively. The data have been taken from Ref. [4].
FIG. 2. Fusion cross sections in the $^8\text{B} + \text{Pb}$ reaction at near-barrier energies. The thin solid line gives the cross sections with zero couplings. The thick solid line and the dashed line give the complete and the total (complete + incomplete) fusion cross sections respectively.
FIG. 3. Complete fusion cross sections in the $^8$B + Pb reaction at near-barrier energies. The dot-dashed line gives the cross sections for dipole transitions only. The solid line gives the same for dipole and quadrupole transitions.