MOTION OF INERTIAL OBSERVERS THROUGH NEGATIVE ENERGY

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Abstract

Recent research has indicated that negative energy fluxes due to quantum coherence effects obey uncertainty principle-type inequalities of the form $|\Delta E| \Delta \tau \lesssim 1$. Here $|\Delta E|$ is the magnitude of the negative energy which is transmitted on a timescale $\Delta \tau$. Our main focus in this paper is on negative energy fluxes which are produced by the motion of observers through static negative energy regions. We find that although a quantum inequality appears to be satisfied for radially moving geodesic observers in two and four-dimensional black hole spacetimes, an observer orbiting close to a black hole will see a constant negative energy flux. In addition, we show that inertial observers moving slowly through the Casimir vacuum can achieve arbitrarily large violations of the inequality. It seems likely that, in general, these types of negative energy fluxes are not constrained by inequalities on the magnitude and duration of the flux. We construct a model of a non-gravitational stress-energy detector, which is rapidly switched on and off, and discuss the strengths and weaknesses of such a detector.
1 Introduction

It has been known for some time that quantum field theory allows violations of the weak energy condition (WEC) [1] in the form of locally negative energy densities and fluxes [2]. A classic example is the Casimir effect [3, 4]. However, if the laws of physics place no restrictions on such violations, then it would be possible to use negative energy to produce gross macroscopic effects. These effects might include violations of the second law of thermodynamics [5], causality [6], and cosmic censorship [7, 8].

Fortunately, quantum field theory does impose some restrictions on the extent of WEC breakdown. One such restriction takes the form of an average of the WEC taken over a null geodesic [9, 10, 11, 12, 13]. A version of this “averaged weak energy condition” (AWEC) holds for a quantized scalar field for physically reasonable quantum states in a wide variety of two and four-dimensional spacetimes [12, 13]. However, in general it does not appear to hold in an arbitrary curved four-dimensional spacetime [13]. Other constraints on WEC violation are uncertainty principle-type inequalities on the magnitude and duration of negative energy fluxes due to quantum coherence effects. Such “quantum inequalities” are satisfied by this type of negative energy flux, as seen by inertial observers in two and four-dimensional flat spacetime [3, 14]. A typical form of this kind of inequality (in two spacetime dimensions) is

$$|F| (\Delta \tau)^2 \lesssim 1,$$  

(1)
where $|F|$ is the magnitude of the negative energy flux and $\Delta \tau$ is its duration. Although they are not covariantly formulated, in at least some cases, constraints of the form Eq. (1) are stronger than those provided by AWEC [8]. A similar inequality was found to hold for a quantized massless, minimally-coupled scalar field propagating on two and four-dimensional extreme Reissner-Nordstrøm black hole backgrounds. For these cases, it was shown that the magnitude of the change in the mass of the black hole $|\Delta M|$, due to the absorption of an injected negative energy flux, and the effective lifetime, $\Delta T$, of the naked singularity thus produced were limited by a quantum inequality of the form:

$$|\Delta M| \Delta T \lesssim 1.$$  \hfill (2)

The constraint given by Eq. (2) prevents an unambiguous observation of a violation of cosmic censorship [7, 8].

Although quantum inequalities prevent the manipulation of negative energy to produce gross macroscopic effects, such as violations of the second law, they do not prevent the detection of negative energy [15]. However, recent calculations [16] indicate that negative energy densities in flat spacetime are subject to large fluctuations. This suggests that the semiclassical theory of gravity may not always yield an accurate description of the gravitational effects of negative energy.

The purpose of the present paper is to further explore the generality of these quantum inequality-type constraints on negative energy fluxes. An
apparent counterexample is the case of a static observer near the horizon of an evaporating black hole. The stress tensor of a quantized field in the frame of reference of such an observer corresponds to a constant negative energy flux directed into the black hole. (Here we are neglecting the backreaction of the Hawking radiation on the black hole.) One might attempt to interpret this ingoing negative flux as really being an outgoing positive flux. However, the energy density in this frame is negative, so it seems more natural to regard it as ingoing negative energy. A similar apparent counterexample was suggested by Davies [17]. A mirror in a two-dimensional spacetime which moves to the right with increasing acceleration will radiate a negative energy flux to the right [18]. The energy flux in the frame of reference of a (noninertial) observer who runs ahead of this mirror can be constant and negative. An inertial observer collides with the mirror in a finite time.

Ottewill and Takagi [19] have also shown that an observer who co-moves with a mirror which is slowly lowered in the vicinity of a black hole sees a net negative energy flux. They showed that the sign of this flux is directly due to the negativity of the static vacuum energy density swept through by the comoving observer. By lowering the mirror arbitrarily slowly, they found that one could obtain an arbitrarily large violation of the flux inequality.

However, these are all noninertial observers. It is well known that noninertial observers see Unruh radiation effects [20]. An accelerated detector in flat spacetime responds as if it were immersed in a thermal bath of particles,
even though the stress tensor is zero. Similarly, a detector suspended near the horizon of an evaporating black hole behaves as if it were surrounded by a positive energy thermal bath \[20, 21\]. Thus the Unruh effect dominates the effects of the negative energy density and flux. Therefore it appears unlikely that such observers would be able to use these negative energy fluxes to produce violations of the second law of thermodynamics. It would be quite surprising if it were otherwise, since the ingoing negative energy flux is required to maintain consistency with the generalized second law. Inertial observers do not encounter any such acceleration radiation which could otherwise mask the effects of the negative energy. Hence in this paper, we restrict our attention to geodesic observers.

In Sec. 2 we consider the effective negative energy flux seen by an inertial observer moving through the Casimir vacuum. We show that arbitrarily large violations of the flux inequality, Eq. (1), can be obtained for slowly moving observers and argue that such violations are likely to be generic for observers who move slowly through more general static negative energy regions. In Sec. 3, we discuss geodesic observers in black hole spacetimes. In a numerical analysis, we show that for observers moving along radial geodesics in two and four-dimensional black hole spacetimes, the quantum inequalities appear to be obeyed. However, geodesic observers in circular orbits near a black hole may experience a constant negative energy flux, and thus an arbitrarily large violation of the quantum inequalities.
In an attempt to better understand the physical implications of these violations, we present a model of a non-gravitational stress-energy detector in Sec. 4. We find that one can construct a local energy density detector, but that this detector appears to require rapid switching. Consequently, such detectors do not respond to the cumulative effects of a flux of negative energy. We discuss the difficulties inherent in the construction of a non-gravitational energy flux detector which would register such cumulative effects. A summary of our results is contained in Sec. 5. Unless otherwise noted, units with $G = \hbar = c = 1$ will be used.

2 Motion Through Casimir Energy

We wish to consider "effective" negative energy fluxes produced by the motion of observers through static negative energy backgrounds. Let $u^\mu$ be the four-velocity (or two-velocity in the case of two spacetime dimensions) of an inertial observer. The energy density and flux in this observer’s frame are given by

$$U = T_{\mu\nu}u^\mu u^\nu, \quad (3)$$

and

$$F = -T_{\mu\nu}u^\mu n^\nu, \quad (4)$$

respectively. Here $n^\mu$ is a spacelike unit normal vector, for which

$$n_\mu u^\mu = 0. \quad (5)$$
Consider the following simple scenario for the generation of an effective negative energy flux. If we take a quantized scalar field in a two-dimensional Minkowski spacetime and topologically identify the spatial dimension, then the ground state of the field will be the Casimir vacuum state. The vacuum expectation values of the stress-energy tensor in this state are given by \[22\]:

\[
\rho = \langle T_{tt} \rangle = -\frac{\pi}{6L^2},
\]

\[
p = \langle T_{xx} \rangle = -\frac{\pi}{6L^2},
\]

where \(L\) is the periodicity length. The closure of the spatial dimension introduces a preferred reference frame. Let us now consider an inertial observer who moves with velocity \(v\) relative to this frame so that \(u^\mu = \gamma(1, v)\) and \(n^\mu = -\gamma(v, 1)\), where \(\gamma = (1 - v^2)^{-1/2}\). This observer will see an effective negative energy flux given by

\[
F = v\gamma^2 (\rho + p) = -v\gamma^2\left(\frac{\pi}{3L^2}\right).
\]

Here we have chosen the sign of \(n^\mu\) so that \(F > 0\) when the energy density and pressure are both positive. Let

\[
\tau = \eta\left(\frac{L'}{v}\right) = \eta\left(\frac{L}{v\gamma}\right)
\]

be the proper time for the observer to traverse some fraction \(\eta\) of the closed space, where \(L' = L/\gamma\) is the periodicity length in the moving observer’s frame. Then from the above expressions it is easily shown that

\[
|F|\tau^2 = \eta^2\left(\frac{\pi}{3v}\right).
\]
where $|F|$ is the magnitude of the flux. Note that by making $v$ arbitrarily small we can make the right-hand side of Eq. (10) arbitrarily large compared to one, and hence produce an arbitrarily large violation of the flux inequality. This violation at small velocities was first noted by Ottewill and Takagi [19].

Klinkhammer has used this same spacetime and quantum state to construct a violation of the averaged weak energy condition [12]. An important feature of his example is the periodicity of the space, because causal curves can repeatedly traverse the same negative energy region. By contrast, our counterexample does not depend on this feature. Note that in our example, to achieve a violation of the flux inequality the observer is not required to make even one complete traversal of the space. A similar result to Eq. (10) is also true for plate-type boundary conditions in both two and four dimensions. By using the methods of Sec. 3, one can show that an analogous violation of the flux inequality occurs for an observer falling into a static “zero-tidal-force” Morris-Thorne [23] traversable wormhole [24].

Such violations should occur quite generally whenever an observer moves slowly through a region of static negative energy. For example, in a general two-dimensional case let the characteristic proper size of the region be $L$ and the magnitude of the negative energy be of order $L^{-1}$, as is the case in the Casimir effect. Then on dimensional grounds, the flux should $\sim v/L^2$, while the time taken to traverse the region is of order $L/v$. Therefore, $|F|\tau^2 \sim 1/v$.

Although the flux inequality may be violated, this does not necessarily
mean that there are no restrictions on the negative energy. Returning to our two-dimensional Casimir example, let the magnitude of the total vacuum energy over the region $L$ be $|E|$. Then from Eq. (11), we see that

$$|E| L = |\rho| L^2 = \left(\frac{\pi}{6}\right).$$

(11)

In the moving observer’s frame:

$$|E'| L' = |\rho'| L'^2 = \frac{\pi}{6}(1 + v^2).$$

(12)

Since $v < 1$, it follows that $|E'| L' < \pi/3$. Thus, at least for the case of the two-dimensional Casimir effect, there seems to be a restriction on the total negative energy and the size of the region occupied by it. This constraint is of the form originally suggested in Ref. [14]. However, in four-dimensions the issue is less clear due to the presence of additional length scales, e.g. the transverse dimensions of a pair of Casimir plates. It is also possible that even in two-dimensions the constraint might be circumvented due to the contributions by other additional fields to the vacuum energy.

3 Observers near Black Holes

In this subsection we consider inertial observers moving in two and four-dimensional black hole backgrounds. We will calculate the energy density and flux in the frame of such observers in order to check whether quantum inequalities of the form of Eq. (1) are satisfied.
3.1 Radially Moving Observers

In this section, we treat observers which are moving radially toward or away from a Schwarzschild black hole. First consider a two-dimensional Schwarzschild black hole, where closed form expressions for $T_{\mu\nu}$ are available. We understand $T_{\mu\nu}$ to denote a quantum expectation value in a specified vacuum state. The metric is

$$ds^2 = -Cdt^2 + C^{-1}dr^2,$$  \hspace{1cm} (13)

where $C = 1 - 2M/r$. The geodesic equations imply that

$$\frac{dr}{d\tau} = \pm \sqrt{k^2 - C}.$$  \hspace{1cm} (14)

A geodesic observer’s two-velocity is

$$u^\mu = (u^t, u^r) = \left(\frac{dt}{d\tau}, \frac{dr}{d\tau}\right) = \left(\frac{k}{C}, \pm \sqrt{k^2 - C}\right).$$  \hspace{1cm} (15)

Here the + and − refer to outgoing and ingoing observers, respectively. The constant $k$ is the energy per unit rest mass; $k \geq 1$ corresponds to unbound trajectories whereas $k < 1$ corresponds to bound trajectories. In the latter case, a particle is constrained to move in the region where $k^2 \geq C$. From Eqs. (13), (15) and the fact that $n^\mu$ is a unit vector, we have that

$$n^\mu = (n^t, n^r) = \left(C^{-1}\sqrt{k^2 - C}, k\right),$$  \hspace{1cm} (16)

for an outgoing observer, and

$$n^\mu = (n^t, n^r) = \left(-C^{-1}\sqrt{k^2 - C}, k\right)$$  \hspace{1cm} (17)
for an ingoing observer. In both cases, the sign of \( n^r \) is chosen so that the Hawking flux at infinity is positive.

In our two-dimensional discussion, we will consider observers moving in both the Unruh and Boulware vacua. The stress tensor components in the Unruh vacuum are [25]:

\[
T_{tt} = \frac{1}{24\pi} \left( \frac{7M^2}{r^4} - \frac{4M}{r^3} + \frac{1}{32M^2} \right),
\]

(18)

\[
T_{tr} = -\frac{1}{24\pi} \left( 1 - \frac{2M}{r} \right)^{-1} \frac{1}{32M^2},
\]

(19)

and

\[
T_{rr} = -\frac{1}{24\pi} \left( 1 - \frac{2M}{r} \right)^{-2} \left( \frac{M^2}{r^4} - \frac{1}{32M^2} \right).
\]

(20)

The corresponding components in the Boulware vacuum are given by:

\[
T_{tt} = \frac{1}{24\pi} \left( \frac{7M^2}{r^4} - \frac{4M}{r^3} \right),
\]

(21)

\[
T_{tr} = 0,
\]

(22)

and

\[
T_{rr} = -\frac{1}{24\pi} \frac{M^2}{r^4} \left( 1 - \frac{2M}{r} \right)^{-2}.
\]

(23)

The energy density and flux can be obtained by substituting the above expressions into Eqs. (3) and (4).

We want to be able to distinguish a “genuine” negative energy flux moving to the right from a positive flux moving to the left. Hence we will consider the flux to be negative when the energy density, \( U \), is simultaneously negative. Our goal is to determine whether quantum inequalities of the form Eq. (1)
hold for geodesic observers in these quantum states. The strategy is the
following. First specify the geodesic as ingoing or outgoing and choose a
value of $k$. Next, numerically determine the range in $r$ over which $U < 0$ for
that observer by using Eq. (3) and the appropriate components of the stress
tensor. We then numerically integrate the geodesic equation, Eq. (14), to
find $\tau(r)$. The flux as a function of $r$ is constructed using Eq. (4) and
the appropriate stress tensor components. Finally, $|F(r)|$ is graphed as a
function of $\tau(r)$ to yield $|F(\tau)|$.

In two dimensions, we can evaluate the relevant expressions for all values
of $r$ up to $r = 0$. A representative example of a graph of $|F|$ as a function
of $\tau$ is given in Fig. (1), for $k = 1$ in the Unruh vacuum. This value of $k$
corresponds to free fall from rest at infinity. Using Eq. (3), a numerical
computation shows that $U < 0$ and $F < 0$ for this observer in the range $0 \leq
r \lesssim 0.36M$. Note that although the flux diverges as the observer approaches
the singularity at $r = 0$, most of the negative energy flux is emitted in a
very small fraction of the total time interval shown in the graph. It therefore
seems reasonable to consider the time interval $\Delta \tau$ of Eq. (11) not to be
the entire time over which the flux is negative, but rather a timescale which
characterizes the change in the flux. For example, in Fig. (1) we see that
$|F|$ increases more rapidly as $\tau \approx 0.1$ is approached. We could take the
characteristic time to be that over which the flux changes by a factor of 2
or 3. This time will decrease as $|F|$ increases. However, near the maximum

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of $|F|$ in Fig. (1), we have $\Delta \tau \lesssim 0.01 M$. The maximum value of $|F|$ shown in the graph is less than about $150 M^{-2}$ and hence $|F|(\Delta \tau)^2 \lesssim 0.01$. From the graph it can be seen that as the singularity is approached, this timescale rapidly decreases as the flux increases, such that $|F|(\Delta \tau)^2 \lesssim 1$. We have performed the same analysis for the following values of $k$ for infalling observers in the Unruh vacuum: 0.001, 0.1, 0.5, 2, 10, and 1000. Note that $k > 1$ observers fall in from infinity with nonzero initial velocity, whereas the $k < 1$ observers fall from rest at a finite distance. Similar analyses have been performed for several values of $k$ for outgoing geodesic observers who originate very near the horizon. In all cases, we find that $|F|(\Delta \tau)^2 \lesssim 1$. A corresponding treatment for geodesic observers moving through the Boulware vacuum yields similar results.

We now consider four-dimensional black holes. Utilizing numerical data of Jensen, McLaughlin, and Ottewill [26] for the renormalized quantum stress-tensor of an electromagnetic field in the Unruh state of a four-dimensional evaporating Schwarzschild black hole, we have performed a similar analysis for several freely infalling radial observers in the more limited range from $r = 5M$ to $r = 2.1M$. The energy density and flux for the chosen observers are negative throughout this range. In four-dimensions, the analog of Eq. (1) is

$$|F| A (\Delta \tau)^2 \lesssim 1,$$

where $A$ is the collecting area for the negative energy. Here the time interval
\[ \Delta \tau \] will be taken to be the proper time to fall from \( r = 5M \) to \( r = 2.1M \) and \(|F|\) will be taken to be the maximum value of the magnitude of the negative flux, which occurs at \( r = 2.1M \). The area \( A \) must therefore be less than about \( 16\pi M^2 \). The following is a representative sample of our numerical results. For \( k = 10 \), \( \Delta \tau = 0.29M \) and \((|F|\Delta \tau^2) = 0.004/M^2\); for \( k = 1 \), \( \Delta \tau = 3.8M \) and \((|F|\Delta \tau^2) = 0.007/M^2\); for \( k = 0.775 \), \( \Delta \tau = 10M \), and \((|F|\Delta \tau^2) = 0.03/M^2\). The last value of \( k \) corresponds to free fall from rest at \( r = 5M \). Thus we see that Eq. (24) appears to be satisfied.

### 3.2 Orbiting Observers

We now examine geodesic observers in orbit around Schwarzschild and extreme \((Q = M)\) Reissner-Nordstrøm black holes. The line element is

\[ ds^2 = -C dt^2 + C^{-1} dr^2 + r^2(\theta^2 + \sin^2 \theta d\phi^2), \quad (25) \]

where \( C = (1 - 2M/r) \) for Schwarzschild and \( C = (1 - M/r)^2 \) for \( Q = M \) Reissner-Nordstrøm black holes. The four-velocity for such a geodesic observer is

\[ u^\mu = \left( 1/\sqrt{(C - rC'/2)}, 0, \sqrt{C''/(2rC - r^2C')}, 0 \right). \quad (26) \]

Since \( u' > 0 \) and we have chosen \( u^\theta > 0 \), we now wish to choose the observer’s normal vector so that \( n^\theta < 0 \). The relation \( u^\mu n_\mu = 0 \), and the fact that \( u^\mu \) and \( n^\mu \) are unit vectors then imply

\[ n^\mu = \left( -1/\sqrt{(2C^2/rC' - C)}, 0, -\sqrt{2C/[r^2(2C' - C')]} \right). \quad (27) \]
For an observer moving through “classical dust”, this choice would correspond to a positive energy flux (i.e., a positive energy flux flowing in the direction opposite to the direction of the observer’s motion). As before, we will regard the energy flux as negative when the energy density is also negative. Substitution of Eqs. (26) and (27) into Eqs. (3), (4) yields

\begin{equation}
U = \frac{2C T^t_t - C' r T^\theta_\theta}{(C' r - 2C)}, \tag{28}
\end{equation}

\begin{equation}
F = \frac{\sqrt{2} C C' (T^\theta_\theta - T^t_t)}{(2C - rC')} \tag{29}
\end{equation}

For a Schwarzschild black hole we have

\begin{equation}
U = (1 - 3M/r)^{-1} [T^\theta_\theta (M/r) - T^t_t (1 - 2M/r)], \tag{30}
\end{equation}

\begin{equation}
F = \frac{(M/r)^{1/2} (1 - 2M/r)^{1/2}}{(1 - 3M/r)} (T^\theta_\theta - T^t_t). \tag{31}
\end{equation}

Timelike circular orbits occur for \( r > 3M \). In Ref. [26], the numerical values of \( T^t_t \) and \( T^\theta_\theta \) are plotted for a quantized electromagnetic field in the Unruh vacuum state in the range from \( r = 5M \) to \( r = 2M \). From Figs. (1) and (3) of Ref. [26], one can see that \( T^\theta_\theta < 0 \) and \( T^t_t > 0 \) in the region \( 3M < r \lesssim 5M \). Thus from Eqs. (30) and (31), we see that an orbiting observer who manages to stay in a circular orbit (with small thrusts to prevent the growth of instabilities) will see \( U < 0 \) and \( F < 0 \). That is, the observer will see a constant negative energy flux. Therefore, it appears that the quantum inequality restrictions on the flux are violated.

In the case of the evaporating Schwarzschild black hole, it is not clear whether this violation is due to the existence of the Hawking radiation or
to the motion of the orbiting observer through the static negative energy background polarization surrounding the hole. It is therefore interesting to examine a case where the Hawking radiation is absent, such as a $Q = M$ Reissner-Nordstrøm black hole. A quantized field in its vacuum state on this background possesses more of the properties expected of a ground state than the same field on the Schwarzschild background. For this black hole

$$U = -\frac{[(r - M)T_{t}^{t} - MT_{\theta}^{\theta}]}{(r - 2M)}, \quad (32)$$

$$F = \frac{\sqrt{M(r - M)}}{(r - 2M)} (T_{\theta}^{\theta} - T_{t}^{t}). \quad (33)$$

In this case, timelike circular orbits occur for $r > 2M$. Anderson, Hiscock, and Samuel [27, 28] have recently given a numerical computation of the stress-energy tensor of quantized scalar fields in static black hole spacetimes. For $Q = 0.99M$, which we take to represent the extreme $Q = M$ case, they find [28] that $T_{\theta}^{\theta} < 0$ and $T_{t}^{t} > 0$ in the region $2M < r < 4M$. From Eqs. (32) and (33), we again find that an orbiting observer in this region will see $U < 0$ and $F < 0$. Thus in this case as well, the flux inequality restrictions are violated.

It is of interest to note that the counterexamples in this subsection depend upon the ability of the orbiting observer to make many orbits around the black hole. For example, for the observer orbiting a Schwarzschild black hole near $r = 3M$, one finds that

$$|F|(\Delta\tau)^{2} \approx \frac{0.04N^{2}}{M^{2}}, \quad (34)$$
where $\Delta \tau$ is the proper time required to complete $N$ orbits. The collecting area $A$ cannot exceed $M^2$ by a large factor, so for a single orbit one will have that

$$|F|A(\Delta \tau)^2 \lesssim 1.$$  \hfill (35)

A similar result holds in the case of the observers orbiting a Reissner-Nordstrøm black hole.

4 Switched Energy Detectors

Particle detector models are a useful probe of the operational significance of the constructs of quantum field theory. The simplest such model is the monopole detector \cite{20, 29} in which a quantum system is coupled linearly to the quantized field. A variation of the monopole detector was proposed by Grove \cite{30} in which the detector is switched on for only a finite time interval. Switched detectors were further discussed in Ref. \cite{31}. Even an inertial detector in the Minkowski vacuum state will undergo excitations as a result of the switching, and a flux of negative energy can be interpreted as tending to suppress these excitations. However, none of these model detectors is really a detector of the energy density or of other components of the energy momentum tensor. The response of the detector typically involves a double line integral of a two-point function over the detector’s world line, or a portion of the world line. In this section, we wish to propose a further modification of the monopole detector models which will enable one to measure operationally
the expectation value of the stress tensor of a quantized field.

Let us consider a quantized scalar field \( \phi \) which is coupled to a quantum system (the detector) via the interaction Lagrangian

\[
L_I = c \, m(\tau) \, g(\tau) \sum_{j=1}^{n} \ell_{j}^{\mu} \phi_{,\mu}.
\] (36)

Here \( \tau \) is the detector’s proper time, \( c \) is a coupling constant, \( m(\tau) \) is the monopole moment (an operator with nonvanishing matrix elements between the different quantum states of the detector), \( g(\tau) \) is a switching function which vanishes outside of some finite interval in \( \tau \), and \( \{ \ell_{j}^{\mu} \} \) is a set of vectors. The analysis of the response of this detector follows the same steps as in the case of the usual monopole detector \[32\]. The probability for the detector to be excited from energy eigenstate \( |E_0\rangle \) to energy eigenstate \( |E\rangle \) is found to be

\[
P_{E,E_0} = c^2 |\langle E|m(0)|E_0\rangle|^2 R(E - E_0),
\] (37)

where the detector’s response function, \( R \), is given by

\[
R(E) = \sum_{i,j=1}^{n} \ell_{j}^{\mu} \ell_{i}^{\nu} \int_{-\infty}^{\infty} d\tau d\tau' e^{-iE(\tau-\tau')} g(\tau) g(\tau') \langle \phi_{,\mu}(x) \phi_{,\nu}(x') \rangle.
\] (38)

Here \( x \) is the spacetime location of the detector at time \( \tau \), and \( \langle \rangle \) denotes the expectation value in \( |\psi\rangle \), the quantum state of the field.

Let us restrict our attention primarily to flat spacetime models, in which \( |\psi\rangle \) is either a nonvacuum state or a Casimir vacuum state. We cannot directly measure \( R \), but we can measure \( R - R_0 \), where \( R_0 \) is the response in
the Minkowski vacuum. This difference is given by

\[ R - R_0 = \sum_{i,j=1}^{n} \ell_j^\mu \ell_i^\nu \int_{-\infty}^{\infty} d\tau d\tau' e^{-iE(\tau-\tau')} g(\tau) g(\tau') \langle \phi, \mu(x) \phi, \nu(x') \rangle_R. \]  

(39)

Here \( \langle \rangle_R \) is the renormalized expectation value, the difference between that in \( |\psi\rangle \) and that in the Minkowski vacuum. The quantity \( R - R_0 \) is in general a nonlocal quantity which depends upon field fluctuations over an extended region of spacetime. However, we may make the switching function \( g(\tau) \) sharply peaked in order to probe a localized region. This function should satisfy

\[ \int_{-\infty}^{\infty} d\tau g(\tau) = 1. \]  

(40)

Thus in the limit that \( g(\tau) \) is sharply peaked at \( \tau = \tau_0 \),

\[ g(\tau) \rightarrow \delta(\tau - \tau_0). \]  

(41)

In this limit, we have

\[ R - R_0 = \sum_{i,j=1}^{n} \ell_j^\mu \ell_i^\nu \langle \phi, \mu(x) \phi, \nu(x) \rangle_R. \]  

(42)

The quantity \( \langle \phi, \mu(x) \phi, \nu(x) \rangle_R \) is finite and is evaluated at the location of the detector at time \( \tau_0 \). A negative value for \( \langle \phi, \mu(x) \phi, \nu(x) \rangle_R \) simply means that the detector’s response is less than the corresponding response in the Minkowski vacuum.

By combining the responses of a set of detectors we may measure the renormalized expectation value of the stress tensor, \( \langle T_{\mu\nu}(x) \rangle_R \). Consider, for
example, the massless minimally coupled scalar field for which

\[ T_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\rho}\phi_{,\rho}. \]  

(43)

To measure a diagonal component of \( \langle T_{\mu\nu}(x) \rangle_R \) in four dimensional spacetime, we need to combine the responses of four different detectors, each of which measures one diagonal component of \( \langle \phi_{,\mu}(x)\phi_{,\nu}(x) \rangle_R \). That is, to measure \( \langle [\phi_{,t}(x)]^2 \rangle_R \), we take \( n = 1 \) and \( \ell^\mu = t^\mu \), the unit vector in the t-direction. Similarly, we construct detectors for each of the three spatial directions. To measure an off-diagonal component, we need three detectors. For example, the flux in the x-direction is given by the expectation value of \( T_{tx} = \frac{1}{2}(\phi_{,t}\phi_{,x} + \phi_{,x}\phi_{,t}) \). We take the first detector to be specified by \( n = 2 \) and \( \ell_1^\mu = t^\mu \) and \( \ell_2^\mu = x^\mu \), where \( x^\mu \) is the unit vector in the x-direction. Equation (42) for this detector becomes

\[ R - R_0 = \langle [\phi_{,t}(x)]^2 + \phi_{,t}(x)\phi_{,x}(x) + \phi_{,x}(x)\phi_{,t}(x) + [\phi_{,x}(x)]^2 \rangle_R. \]  

(44)

By combining this with the response of detectors which measure \( \langle [\phi_{,t}]^2 \rangle_R \) and \( \langle [\phi_{,x}]^2 \rangle_R \), we obtain \( \langle T_{tx} \rangle_R \).

The various detectors which are required to measure a particular component of \( \langle T_{\mu\nu}(x) \rangle_R \) must all make measurements in a spacetime region which is small compared to the scales over which \( \langle T_{\mu\nu}(x) \rangle_R \) varies. One might be concerned that the different detectors could disturb one another. However, this effect may be made small by choosing the coupling constant \( c \) to be sufficiently small. From Eq. (37), we see that the effect of the quantized
field upon any individual detector is of order $c^2$. The different detectors are assumed to interact with one another only through their coupling to the field. Thus the disturbance in one detector due to another detector is necessarily of higher order in $c$ and can be minimized by making $c$ small.

The main result of this section is that we have given a prescription by which the stress tensor of a quantized scalar field may be measured by non-gravitational means. Note that this prescription answers a challenge proposed by Padmanaban and Singh [33], although the method we have used is rather different from that envisioned by these authors. We have restricted our attention to flat spacetimes, but this procedure could be generalized to curved spacetime. In order to do this, one would have to modify the definition of $\langle \phi_{,\mu}(x)\phi_{,\nu}(x) \rangle_R$ to include subtraction of the curvature-dependent divergences which arise in a nonflat spacetime.

Although the above model illustrates the possibility of constructing a non-gravitational stress-energy detector, we saw that measurement of $\langle T_{\mu\nu}(x) \rangle$ at a spacetime point necessarily entailed rapid switching of the detector. As a result, such a detector may tell us little about the (possibly large) cumulative effects of negative energy. An example of dramatic cumulative effects of negative energy is black hole evaporation. Conversely, a detector which is not rapidly switched involves an integral over the detector’s world line and hence measures a nonlocal quantity, rather than $\langle T_{\mu\nu}(x) \rangle$.
5 Summary and Discussion

In this paper, we have been concerned with effects due to the motion of an observer through a stationary distribution of negative energy, in contrast to dynamically generated negative energy fluxes of the sort which are created by quantum coherence effects in Minkowski spacetime. One of our goals has been to test the extent to which quantum inequalities like Eq. (1) are satisfied by the fluxes due to such motion. We have found that these inequalities appear to be satisfied for geodesic observers moving radially in two and four dimensional black hole spacetimes. However, they are not satisfied in all cases. We found that observers who move sufficiently slowly through the Casimir vacuum may violate the inequality arbitrarily. This is likely to be a general feature of slow motion through static negative energy regions. Similarly, observers in circular orbits around black holes may see a constant negative energy flux in their frame of reference. (Note that in the latter case, the observer’s velocity does not play a crucial role, as it is determined by the orbit and is not a free parameter.) In both cases, we have examples of inertial observers who see negative energy fluxes which are not constrained by a quantum inequality such as Eq. (1), which limits the magnitude and duration of the flux.

The physical implications of these counterexamples are still somewhat obscure. It is not clear whether the effective negative energy flux produced by motion relative to a stationary negative energy distribution can be absorbed
to create macroscopic effects. There may be fundamental differences between these quantum states and those involving negative energy fluxes produced by quantum coherence effects. In order to address this issue, one would want to construct model energy detectors. Most quantum detectors do not couple to the local energy density \[33\]. We have seen that non-gravitational stress-energy detectors can be devised, but they appear to require rapid switching. This prevents them from registering cumulative effects of negative energy. Thus it remains an open question as to whether negative energy fluxes can be manipulated to produce gross macroscopic effects.

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Figure Captions

- [1] Graph of the magnitude, $|F|$, of the negative energy flux in units of $M^{-2}$ as a function of proper time $\tau$ in units of $M$. Here the observer falls inward from rest at infinity, i.e. $k = 1$. The flux diverges as the observer approaches the singularity at $r = 0$. In this region, the magnitude of the flux doubles on a timescale of order $\Delta \tau \lesssim 0.01M$. 