Human gait modeling method

S V Sivolobov¹, A V Khoperskov¹ and V V Bumagin¹
¹Volgograd State University, 100 Prospect Universitetskty, Volgograd, 400062, Russia
E-mail: sivolobov@volsu.ru

Abstract. The article deals with problem of human gait modelling. Human gait analysis is used for various human identification tasks and for various purposes in medicine. Lagrange equations of the second kind in many research are used to model gait. We use a model of an anthropomorphic mechanism of five links to describe human movements. The mathematical model is implemented in the form of software that uses input data on the kinematics of a real gait. An important result of the study is the creation of an algorithm for calculating model parameters that best reproduce the gait of a particular person. Our approach is based on the application of a genetic algorithm to minimize a new special objective function, which depends on discrete generalized coordinates that depend on time. The dynamic picture significantly improves the quality of the model, which is able to reproduce the subtle features of the human gait.

1. Introduction
Gait is an important biometric indicator of a person along with a face photograph, voice or handwriting [1]. It can be used to identify a person [2, 3], for diagnostic purposes in medicine [3]. The practice of clinical studies of gait shows promising prospects for a sample of children with developmental disorders with cerebral palsy and myelomeningocele. DeLuca and others [4] examined 91 patients who had a recommendation for surgery from experienced doctors. Gait analysis was the reason for reviewing surgical recommendations for 52 percent of patients, which reduced the cost of surgery and patients avoided unnecessary procedures [5]. Gait modeling is of considerable interest for identifying people, as well as for studying gait pathologies [6, 7]. Gait modeling is used in the construction of stably moving anthropomorphic systems with two legs, when creating prostheses and orthoses of the lower extremities [8], medical exoskeletons for the rehabilitation of disabled people, patients with paraplegia [9].

The one-limb support and two-limb support phases of movement alternate in the person walking process. During a one-limb support movement, the support leg is on the surface and the second leg is in the process of being transferred. During the two-limb support phase, both legs rest on the surface [10]. The first phase is more than 76 percent of the step duration and it is being actively studied in both experimental and theoretical approaches [5, 11, 12].

The work [12] describes the modeling of a person’s gait by analytically calculating the trajectories of the lower extremities in a single support phase using the Lagrange equations of the second kind, which are fitted with real trajectories using the MPC controller in MATLAB / Simulink. We offer an alternative way to calculate the trajectories of a particular gait by varying the parameters of the model using dynamic data.
2. Mathematical model

The Lagrange equations for a system of a planar anthropomorphic mechanism of five pivotally connected massive elements are the basis of our model [10]. All parts of the mechanism can only move in the sagittal plane (Figure 1).

The OC element denotes the body, and the identical (identical) two-link (OBA and ODE) are legs.

The distance between the hip joints of the legs in the frontal plane is small, and therefore we will consider a flat spatial structure. The links OB and OD are called the hips, BA and DE are the lower legs, and O is the hip joint. All joints are assumed to be ideal, since friction in the joints of a person is small [11]. We study only the single-limb support phase of movement, therefore, the OBA support leg is pivotally connected to the surface at point A, and the double-link system designates a moving leg that is not connected to the surface. Feet and hands are not considered in this model. The research work [10] describes a seven-link mechanism, which additionally contains two feet, which are considered massless.

The five generalized coordinates are angles \( \varphi, \alpha_1, \alpha_2, \beta_1, \beta_2 \). They denote the angles that form the body, thighs and lower legs with a vertical line, respectively. Non-conservative forces acting on the mechanism are shown in Figure 1b. We distinguish moments of forces in the knee joints \( \vec{u}_1 \) and \( \vec{u}_2 \), between the body and hips \( \vec{q}_1 \) and \( \vec{q}_2 \). The vector \( \vec{R}_2 \) denotes the external force applied to the end of the leg \( E \) (\( R_{2x} \) and \( R_{2y} \) are its horizontal and vertical components), and \( \Pi_1 \) and \( \Pi_2 \) are the moments of external forces applied to the legs.

The equations of motion in matrix form for such a five-link are as follows [10]:

\[
B(z)\ddot{z} + gA|\sin z_1| + D(z)|\dot{z}_1|^2 = C(z)w,
\]

where \( g \) is the gravity,

\[
z = |z_1| = \begin{bmatrix} \varphi \\ \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta_2 \\ \sin \varphi \\ \sin \alpha_1 \\ \sin \alpha_2 \\ \sin \beta_1 \\ \sin \beta_2 \end{bmatrix}, \quad |\sin z_1| = \begin{bmatrix} \sin \varphi \\ \sin \alpha_1 \\ \sin \alpha_2 \\ \sin \beta_1 \\ \sin \beta_2 \end{bmatrix}, \quad |\dot{z}_1|^2 = \begin{bmatrix} \varphi^2 \\ \alpha_1^2 \\ \alpha_2^2 \\ \beta_1^2 \\ \beta_2^2 \end{bmatrix}, \quad w = \begin{bmatrix} \vec{u}_1 \\ \vec{u}_2 \\ \vec{q}_1 \\ \vec{q}_2 \\ \Pi_1 \\ \Pi_2 \\ R_{2x} \\ R_{2y} \end{bmatrix}.
\]

The eight-dimensional column \( w \) is the vector of forces and moments of forces. The matrices \( B(z) \), \( A \), \( D(z) \), \( C(z) \) have the form:
The parameters $D$ formulas:

$$D(z) = \begin{bmatrix}
L_0 K_r \cos(\varphi - \alpha_1) & J_a - 2L_0 K_a + L_0^2 M & -L_0 K_r \cos(\alpha_1 - \alpha_2) & L_0 K_r \cos(\varphi - \beta_1) & 0 & 0 \\
-\frac{L_0 K_r \sin(\varphi - \alpha_1)}{L_0} & 0 & -L_0 K_r \sin(\alpha_1 - \alpha_2) & (J_{ab} - L_0 K_b - L_0 K_a) + \frac{L_0 L_0 M \cos(\alpha_1 - \beta_1)}{L_0} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

where $B(z)$ is the symmetric and positive definite kinetic energy matrix, $A$ is the diagonal matrix of potential energy, $D(z)$ is the matrix of Christoffel symbols of the first kind for the matrix $B(z)$, $L_a$ and $L_b$ are the lengths hips and shanks, respectively, $J$ is the moment of inertia of the body, relative to the point $O$, $K_r = m_1 r$, ($m_1$ is the body mass), $r$ is the distance from the hip joint to the center of mass of the body, $J_b$ is the moment of inertia of the shank relative to the knee joint ($B$ for the lower leg $BA$), $K_b = m_2 b$, $b$ is the distance from the knee joint to the center of mass of the shank, $J_b^0$ is the moment of inertia of the hip relative to point $O$. The mass of the entire five link $M$ is calculated by summing the mass of the body and the double mass of the same legs: $M = m_k + 2m_a + 2m_b$, where $m_a$ and $m_b$ are the masses of the thigh and shank, respectively. The parameters $K_a$, $J_a$, and $J_{ab}$ are calculated by the formulas:

$$K_a = m_a a + m_b L_a, \quad J_a = J_0^a + m_b L_a^2, \quad J_{ab} = K_b L_a,$$

where $a$ is the distance from the hip joint to the center of mass of the hip.

The control has an impulsive pattern at the beginning ($t = 0$) and the end of the step ($t = T$), and the mechanism moves along the ballistic trajectory in the time interval $0 < t < T$ in the absence of any control actions ($w(t) = 0$). The simulated mechanical system passes from the state $z(t)$ to the solution $z(T)$, which the moving leg is on the surface. Model of impulse control of an anthropomorphic mechanism was proposed in [10] and experimentally confirmed in [8]. Thus, the solution $z(t)$ satisfies a homogeneous system in the time interval $0 < t < T$:

$$B(z)\ddot{z} + gA|\sin z_1| + D(z) \left| \dot{z}^2 \right| = 0.$$  \hspace{1cm} (8)

Equation (8) resolved with respect to the second derivatives has the form:

$$\ddot{z} = -gB^{-1}(z)A|\sin z_1| - B^{-1}(z)D(z) \left| \dot{z}^2 \right|.$$  \hspace{1cm} (9)

To solve the boundary value problem, which involves finding the initial angular velocity vector $\dot{z}(0)$, in which the solution (8) starting from the initial state $z(0)$, $\dot{z}(0)$ passes through $z(T)$, it was proposed in [10] to minimize the residual function:

$$I = \sum_{i=1}^{T} k_i \left( \Delta z_i(T) \right)^2,$$  \hspace{1cm} (10)
where $\Delta z_i(T)$ is the difference between the given and calculated values of the generalized coordinate, and $k_i$ are some weight coefficients. The resulting trajectories of motion with this solution of equations (8) are very different from the real trajectories obtained from data on the movement of a person. To approximate the calculated trajectories to the real ones, a scheme in MATLAB / Simulink using the MPC controller was proposed in [12]. The authors estimated the error of the calculated trajectories within 5 percent. This article further proposes a method for constructing solutions of equations (8), in which the calculated trajectories are closer to real ones.

3. The method of calculating trajectories

We used the Biped file for the 3Ds Max program from open sources as a source of real motion paths of the anthropomorphic mechanism using motion capture technology. The control values of the generalized coordinates $\hat{z}(t)$ are calculated on the basis of 17 frames of human movement in the single-limb support phase. In contrast to the previously described method, it is proposed to use the following function as a residual function that must be minimized to find a solution to equations (8):

$$I = \sum_{i=1}^{n} \sum_{t=0}^{T} (z_i(t) - \hat{z}_i(t))^2$$

Minimization of function (10) ensures that only the initial and final positions of the mechanism match, while minimization (11) also ensures passage through points $\hat{z}(t)$. In the process of minimization, it is necessary to repeatedly solve the Cauchy problem. To solve the Cauchy problem, the fourth-order Runge-Kutta method with a constant integration step was used, as proposed in [10].

In the beginning, it was supposed to calculate the trajectory closest to the real one by finding only the suitable $\dot{z}(0)$. Since the biometric parameters of the person in the frames used were unknown, the values from [10, p. 86]

$$M = 75 \, \text{kg}, \quad m_a = 8.6 \, \text{kg}, \quad m_b = 4.6 \, \text{kg}, \quad J = 11.3 \, \text{kg} \cdot \text{m}^2, \quad J_a = 0.535 \, \text{kg} \cdot \text{m}^2,$$

$$J_b = 1.02 \, \text{kg} \cdot \text{m}^2, \quad r = 0.386 \, \text{m}, \quad a = 0.18 \, \text{m}, \quad b = 0.324 \, \text{m}, \quad L_a = 0.41 \, \text{m}, \quad L_b = 0.497 \, \text{m}.$$  

The initial value of $z(0)$ is equal to $\dot{z}(0)$. To minimize, we used the $\text{fminunc}$ function from the MATLAB package, for which it is necessary to specify an optimized function and a vector of initial values. The initial values of $\dot{z}(0)$ for optimization were calculated as follows:

$$\dot{z}_{10}(0) = \frac{\dot{z}_{10}(dt) - \dot{z}_{10}(0)}{dt}$$

where $dt$ is the time step. The result of optimization is the value $\dot{z}(0)$, minimizing the value of function (11):

$$\dot{z}(0) = [0.5136 \quad -4.4374 \quad 2.3095 \quad -0.0857 \quad -0.7415]^T$$

However, the residual $I$ with such a value of $\dot{z}(0)$ is 2.4298, which is a sufficiently large value and the trajectories of the mechanism are very different from real, which is clearly visible in the kinogram of the five-link system in Figure 2.

![Figure 2. Comparisons of the real (gray dashed lines) and modelled (solid black lines) five-link system motion kinograms.](image-url)
the values of $J, J_a^0, J_b, M, m_a, m_b, r$ were selected for optimization. Optimization results with a different number of parameters to be optimized are shown in Table 1. In addition to the fminunc function, the fminsearch function from the MATLAB package was also used for optimization. A dash in the table indicates an error while optimizing this function.

Table 1. Optimization results

| No | Optimized parameters | fminunc   | fminsearch |
|----|----------------------|-----------|-----------|
| 1  | $\dot{z}(0)$         | 2.4298    | 3.3181    |
| 2  | $\dot{z}(0), J$      | 2.4296    | 2.4413    |
| 3  | $\dot{z}(0), J, J_a^0$ | 1.8036    | 2.0484    |
| 4  | $\dot{z}(0), J, J_a^0, J_b$ | 0.5576    | 1.0271    |
| 5  | $\dot{z}(0), J, J_a^0, J_b, M$ | 0.3218    | 0.9379    |
| 6  | $\dot{z}(0), J, J_a^0, J_b, M, m_a$ | 0.2826    | 0.3565    |
| 7  | $\dot{z}(0), J, J_a^0, J_b, M, m_a, m_b$ | 0.2826    | 1.0098    |
| 8  | $\dot{z}(0), J, J_a^0, M, m_a$ | 0.6807    | 1.4156    |
| 9  | $\dot{z}(0), J, M, m_a$ | —         | 0.5912    |
| 10 | $\dot{z}(0), J, J_a^0, J_b, M, m_a, r$ | —         | 8.4122    |
| 11 | $\dot{z}(0), J, J_a^0, J_b, M, m_a, m_b, r$ | —         | 14.5106   |

Options 7 - 11 show that an increase in the number of parameters often leads to an increase in the discrepancy due to the fact that the objective function has several local minima in conditions of a large number of variable parameters. To solve this problem, global optimization methods are more suitable, and we used a genetic algorithm to optimize for 14 parameters: $\dot{z}(0) = |\varphi, \alpha_1, \alpha_2, \beta_1, \beta_2|^T, J, J_a^0, J_b, M, m_a, m_b, r, a, b$. Optimization was carried out in the MATLAB R2010b package with standard settings for the genetic algorithm (the size of the population is 50 individuals). To increase the accuracy and reduce the working time of the genetic algorithm, the hybrid method was used: in the beginning, a genetic algorithm was used to approach the global minimum, and after reaching a certain small residual value (after the genetic algorithm was completed), a minimum was searched using the fmincon function. As a result of the genetic algorithm, the following values of the model parameters are obtained:

$$
\begin{align*}
\dot{z}(0) &= (-0.04755, -1.72029, 2.13511, -3.14159, -3.14159)^T, M = 28.251 \text{ kg}, \\
m_a &= 0.100 \text{ kg}, m_b = 9.996 \text{ kg}, J = 250 \text{ kg} \cdot \text{m}^2, J_a^0 = 25.765 \text{ kg} \cdot \text{m}^2, \\
J_b &= 2.062 \text{ kg} \cdot \text{m}^2, r = 0.100 \text{ m}, a = 0.100 \text{ m}, b = 0.497 \text{ m}.
\end{align*}
$$

(15)

Values unchanged during optimization of the parameters $L_a$ and $L_b$ were taken as in (12). The discrepancy calculated by formula (11) is $I = 0.3917$ under the above calculation conditions. The result of optimization using the fmincon function following:

$$
\begin{align*}
\dot{z}(0) &= (-0.0484, -1.4885, 2.0153, -3.6294, -3.9709)^T, M = 8.734 \text{ kg},
\\m_a &= 1.122 \text{ kg}, m_b &= 2.910 \text{ kg}, J &= 250 \text{ kg} \cdot \text{m}^2, J_a^0 = 50 \text{ kg} \cdot \text{m}^2,
\\J_b &= 0.513 \text{ kr} \cdot \text{m}^2, r &= 0.717 \text{ m}, a = 0.010 \text{ m}, b = 0.492 \text{ m}.
\end{align*}
$$

(16)

The discrepancy decreased and amounted to $I = 0.2844$ for the above parameters. As can be seen in (15) and (16), some parameters have unrealistic values, however, precisely at such values, it was possible to obtain trajectories close to real ones. In addition, it may be worthwhile to set restrictions on the parameter values during optimization, however, at the same time, it may not be possible to achieve such small residual values. Figures 3a – 3d show a comparison of the graphs of the dynamics of the angles $\varphi, \alpha_1, \alpha_2, \beta_1, \beta_2$ obtained for a real person and by modeling.

Figure 4 shows the time dependences of the coordinates of the knee of the supporting leg (Figs. 4a, 4b) and the knee of the moving leg (Figs. 4c, 4d) to compare our results with the data [12]. The solid lines are the simulation results, the dashed lines are the experimental coordinates of the knee joints.
The coordinates of the joints were calculated according to the formulas published in [10, p. 88]. Figure 5 shows a visual comparison of the kinogram of a modelled and real human gait [13].

Figure 3. Comparison of the dynamics in angles for a real person (dash lines) and results of modeling (solid lines)
Figure 4. Time dependences of the coordinates of the knee of the supporting (top) and moving (bottom) legs; solid lines are the result of modeling, dashed lines are the coordinates of the movement of a real person.

Figure 5. Kinograms of modelled (solid lines) and real gait (dashed lines)
4. Conclusion
This article is aimed at improving methods for modeling the trajectories of the five-link mechanism close to the gait of a real person by finding the appropriate model parameters using the genetic algorithm to solve the optimization problem. The simulation results show that the mathematical model of the five-link described in [14], [15] can quite accurately describe the gait of a real person. Modeling errors are caused by several factors: the use of a flat model instead of a spatial model, feet and hands are not taken into account in the motion model. In addition, finding the optimal solution requires the use of complex algorithms due to a large number of arguments of the objective function.

Acknowledgment
This work was supported by the Ministry of Science and Higher Education of the Russian Federation when creating software for the numerical simulation (government task No. 2.852.2017/4.6).

References
[1] Bulgakov V G and Bumagin V V 2011 Forensic examination 3
[2] Sokolova A I and Konushin A S 2019 Proceedings of the Institute for System Programming of the RAS 31(1)
[3] Elif Surer and Alper Kose 2011 Computer Analysis of Human Behavior p 412
[4] DeLuca P A, Davis R B and Ounpuu S 1997 Journal of Pediatric Orthopaedics 17(5)
[5] Chambers H G and Sutherland David H 2002 The Journal of the American Academy of Orthopaedic Surgeons 10(3)
[6] Hicks J L, Schwartz M H and Delp S L 2009 The Identification and Treatment of Gait Problems in Cerebral Palsy p 660
[7] Martinez F, Cifuentes C and Romero E 2013 Journal of NeuroEngineering and Rehabilitation 10(73)
[8] Chigarev A V and Borisov A V 2010 Russian Journal of Biomechanics 15(1)
[9] Ivanov A V and Formalskiy A M 2015 Journal of Computer and Systems Sciences International 2
[10] Formalskiy A M 1982 Movement of anthropomorphic mechanisms (Moscow : Science) p 368
[11] Kolesnikova G P and Formalskiy A M 2014 About one way to model a human’s gait Engineering Journal: Science and Innovation 1
[12] Ovchinnikov I A, Kovalenko P P and Minh Vu 2016 Human gait modeling in MatLab/Simulink. Izvestiâ vysših učebnyh zavedenij 58(8)
[13] Ergunova O T, Lizunkov V G, Malushko E Yu, Marchuk V I, Ignatenko A Yu 2017 OP Conference Series: Materials Science and Engineering 177(1) 012046. DOI: 10.1088/1757-899X/177/1/012046
[14] Shamne A N 2019 IOP Conf. Ser.: Mater. Sci. Eng. 483 012099 DOI: https://doi.org/10.1088/1757-899X/483/1/012099
[15] Lizunkov V, Politsinskaya E, Malushko E, Kindaev A, and Minin M 2018 International journal of energy economics and policy 8 (3) 250-257