Current redistribution effects on superconducting d.c. and microwave measurements

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Abstract. In the last two decades, non conventional behavior of the d.c. transport properties of superconductors, with the appearance of anomalous peaks at the transition, have been investigated and interpreted in different ways. In several cases it was recognized that the behavior can be due to current redistribution effects related to the non-homogeneous nature of the measured superconducting sample.

In this paper we will briefly review and discuss these effects and, referring to simple concentrated constant equivalent circuits, we will show that sample non-homogeneity can produce the observed features. Then, in the same framework, by performing specific simulations on planar resonators, we will show that anomalous peaks in temperature dependence of the resonant frequency and of the extracted surface reactance can occur at the transition temperature of minority, lower Tc, superconducting phases.

1. Introduction: current redistribution effects in d.c. transport measurements
In the past years, in various experiments dealing with transport (electrical or thermal) properties of superconductors, anomalous peaks, just above the critical temperature Tc, were observed and interpreted in different ways. It has been now largely recognized that anomalous peaks in resistance versus temperature measurements can be often due to a current redistribution effect related to the non-homogeneous nature of the measured superconducting sample.

In fact, in the transition region, the sample can be thought as consisting of a network of superconducting and normal zones. The bias current takes then complex minimum resistance paths, producing anomalous non local voltages in parts of the sample. Depending on contact arrangement this can produce voltage peaks at the transition, interpreted as resistance or resistivity peaks when normalized to the bias current. The effect can be easily reproduced using simple resistance network models, that in most cases efficiently describes the observed effects. The same explanation holds for “negative resistance” anomalies observed below the resistive transition.

In a recent review, Mrowka et al.5, following the basic approach of Ref.1, have analyzed many experiments showing excess voltage signals in resistivity measurements at the superconducting transition attributed, from time to time, to charge imbalance, presence of phase slip centers, vortex-antivortex interactions or other complex effects, and proved that in most cases the observed effects could be easily explained assuming inhomogeneous current distributions related to partially degraded superconducting areas in the measured sample and easily simulated by simple resistance arrays.

Anomalous peaks are observed in other transport measurements such as thermopower. Mosqueira et al.17 hypothesized that, also in this case, the observed anomalous behavior could be due to the presence of non-homogeneities.

Similarly, anomalous voltage peaks have been also observed, below the critical current, in transport measurements on superconducting YBCO coated conductor samples, for specific contact arrangements. In this case the effect have been easily explained in terms of current redistribution due to sample non uniformities and carefully reproduced using specific resistor networks.

Finally, microwave measurements performed on superconducting resonators have evidenced anomalous features in the temperature dependencies of the resonant frequencies and of the quality.
factor. In particular, anomalous peaks have been observed in the temperature dependence of the extracted penetration depth and surface resistance, below the critical temperature $T_C^{19,20}$. Scope of the present paper is to discuss the possibility that in microwave measurements non-homogeneities and current redistribution effects could explain in many cases the observed behaviour as already proved in d.c. transport measurements.

2. Current redistribution effects in microwave resonators

In our analysis, we focused our attention on a microstrip hairpin resonator. This type of resonator has been used as the base element to realize superconducting filters for mobile communications also for its capability to minimize the radiation losses $^{21}$. The full electromagnetic response of the resonator can be easily calculated using standard software packages giving perfect agreement with the measurements, so we used a computational approach instead of introducing concentrated current equivalent circuits.

To investigate the effect of non-homogeneities, we have introduced a localized macroscopic defect in the basic hairpin resonator (fig.1). In the simulation, the presence of the defect in the superconducting film has been modelled assuming that a small part of the resonator was constituted by a superconductor with a reduced $T_c$ value. In particular, the defect with a critical temperature $T_{cd} < T_c$ has been introduced in the centre of the hairpin resonator.

![Figure 1](image)

**Figure 1.** (a) Schematic sketch of the hairpin resonator with a small defect located at the centre (b) Current redistribution effect at the defect transition

Simulations of this structure have been performed by using IEE3D 7.0 from Zeland Software, a commercial EM simulator based on the Moment method. This simulator allows introducing specific parameters for superconducting materials as the penetration depth $\lambda$ and the real part of the conductivity $\sigma_1$. We have chosen, for the sake of simplicity, the standard “two fluid” expressions:

$$\lambda(T) = \frac{\lambda(0)}{\sqrt{1 - \left(\frac{T}{T_C}\right)^4}}$$

$$\sigma_1 = \sigma_0 \left(\frac{T}{T_C}\right)^4$$

In this way, it is quite easy to model the resonator behaviour as a function of temperature, by simply changing the values of the input settings of the numerical analysis. As a first step we have performed the simulation without introducing the defect and we have calculated the temperature dependence of the fundamental resonant frequency, $f_0(T)$ of the resonator.

From this, we have calculated the effective penetration depth, using the standard expression:

$$\lambda(T) - \lambda(0) = -\frac{2\Gamma}{\omega\mu_0} \left(\frac{f_0(T) - f_0(0)}{f_0(0)}\right)$$
Where $\Gamma$ is a geometrical factor of the microstrip that can be easily computed analytically. The extracted effective penetration depth, accounting for the finite film thickness, was the same of the input $\lambda(T)$ inside 2% error, fully validating the overall simulation procedure 22.

**Table 1. Parameters used for the simulation**

| Parameter                      | Value                  |
|--------------------------------|------------------------|
| Film Thickness                 | $t = 360\text{nm}$    |
| Normal state conductivity      | $\sigma_n = 1 \times 10^6 \text{ S/m}$ |
| Penetration depth              | $\lambda(0) = 150\text{nm}$ |
| Defect critical temperature    | $T_c/T_{cd} = 0.52 - 0.50$ (6 steps) |
| Central defect size            | 1%-5%-15% of the resonator length |

To simulate the presence of defect we have used the same procedure, but, in the central area of the resonator the transition to the normal state has been divided in equally spaced sequential phases (six in this case), going from the edge to the mid section of the microstrip. Thus, the critical temperature of the defect $T_{cd}$ is not constant but presents a decreasing trend from the centre to the edges. This means that just below $T_{cd}$ the current, that was flowing uniformly in the defect area, would redistribute, “preferring” to flow in the central region where the material has become superconducting (see fig.1 (b)). When all the strip is superconducting the current will again flow uniformly through the strip. In practice, the deviation of the current lines, which tend to concentrate in the superconducting zones, implies a strong inductive effect with a consequent increase of the total inductance of the strip around $T_{cd}$. This will in turn determine a dip in the resonant frequency and a peak in $\lambda(T)$ when determined using Eq. 2.

![Graph](image1)

**Figure 2.** Calculated resonant frequency dependence and extracted penetration depth for different central defect sizes

In fig. 2 the results for the fundamental resonant frequency of the hairpin resonator with central defect and the corresponding $\lambda(T)$ curves are reported, assuming the parameters reported in Table 1.

The well defined peak in $\lambda(T)$ at $T_{cd}$ certainly resembles the peaks observed in refs. 19, 20. In particular, in the case of TBCCO crystals, the peak was clearly associated to the transition of the 1212 minority phase.

The reported effects are due to the failure of the generally adopted model, based on the assumption that $\Gamma$ is constant. This circumstance is altered by the current redistribution effect. The anomalous
peak of $\lambda$ is generated by the attempt to attribute to it the change of the equivalent inductance, which instead is due to the current redistribution.

3. Conclusions
In conclusion, we have proved that, in analogy with d.c. transport measurements the simple assumption of current redistribution effects due to small sample non-homogeneities determines the presence of “anomalous” peaks in the temperature dependence of the microwave response of superconducting resonators. Size and shape of the peaks are directly related to the size and location of the nonuniform region introduced in the simulation. The model developed here might account for some anomalies observed in surface impedance measurements on thin superconducting films. Generally, it is our opinion that “current redistribution” effects, related to sample non-homogeneity, have to be seriously considered in any measurement where an anomalous behaviour is seen, above all close to the superconducting transition. Indeed, in most cases, lumped or distributed equivalent circuits, describing the sample non-homogeneous nature, easily account for the main features of the experimental observations.

4. References

[1] R. Vaglio, C. Attanasio, L. Maritato, A. Ruosi, *Phys. Rev. B* 47, (1993) 15302.
[2] J. Mosqueira, A. Pomar, A. Diaz, J.A. Viera, F. Vidal, *Physica C* 225, (1994) 34.
[3] C. Attanasio *et al*., *J. Phys. Cond. Mat.* 13, (2001) 3215.
[4] J. Jorritsma, J.A. Mydosh, *Phys. Rev. B* 62, (2000) 9703.
[5] F. Mrowka *et al*., *Physica C* 339, (2003) 22.
[6] T.L. Francavilla, R.A. Hein, *IEEE Trans. Magn.* 27, (1991) 1039.
[7] E. Spahn and K. Keck, *Solid State Comm.* 78, (1991) 69.
[8] P. Santhanam *et al*., *Phys. Rev. Lett.* 66, (1991) 2254.
[9] Y. K. Wong *et al*., *Phys. Rev. B* 44, (1991) 462.
[10] P. Lindqvist, A. Nordstrom, O. Rapp, *Phys. Rev. Lett.* 64, (1991) 2941.
[11] A. Nordstrom, O. Rapp, *Phys. Rev. B* 45, (1992) 12577.
[12] M. Park, M.S. Isaacson, J.M. Parpia, *Phys. Rev. Lett.* 75, (1995) 3740.
[13] C. Strunk *et al*., *Phys. Rev. B* 57, (1998) 10854.
[14] B. Burk *et al*., *J. Appl. Phys.* 83, (1998) 1549.
[15] Y. Kopelevich, F. Ciovacco, P. Esquinazi, M. Lorenz, *Phys. Rev. Lett.* 80, (1998) 4048.
[16] M. Putti, M.R. Cimberle, A. Canesi, C. Foglia, A.S. Siri, *Phys. Rev. B* 58, (1998) 12344.
[17] J. Mosqueira, J.A. Viera, J. Maza, O. Cabeza and F. Vidal, *Physica C* 253, (1995) 1.
[18] L. Gianni, A. Cassinese, R. Vaglio and S. Zannella, *SUST* 17, (2004) L38.
[19] M.R. Trunin, *J. of Superconductivity* 11 (1998) 381.
[20] V.M. Pan, *MSMW ’04 Symposium Proceedings*, Kharkov, Ukraine (2004) 85.
[21] J. Mateu, C. Collado, O. Menendez and J. M. O’Callaghan *SUST* 17 (2004) S359.
[22] M. Barra, A. Cassinese and R. Vaglio, *SUST* 18, (2005) 271.