Extraction of Singlet States from Noninteracting High-Dimensional Spins

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We present a scheme for the extraction of singlet states of two remote particles of arbitrary quantum spin number. The goal is achieved through post-selection of the state of interaction mediators sent in succession. A small number of iterations is sufficient to make the scheme effective. We propose two suitable experimental setups where the protocol can be implemented.

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Achieving control at the quantum level is a pivotal requirement for the grounding of quantum technology and the development of reliable protocols for information processing. Frequently, state manipulation of a quantum device needs the connection of remote nodes of a network and the creation of their entangled state. Such a delocalized architecture has received strong experimental attention, especially at the quantum optics level. The heralded entanglement has received strong experimental attention, and the creation of their entangled state. Such a delocalized device needs the connection of remote nodes of a network. Frequently, state manipulation of a quantum requirement for the grounding of quantum technology way to distribute quantum channels [2].

A different approach exploits a mediated interaction between two remote nodes, 1 and 2, by means of their sequential coupling to the same ancillary system e: The ancilla can bring to system 2 the information that has been previously impressed on it by its interaction with system 1. Recently, this idea has been used in a solid-state context involving multiple electron scattering between magnetic impurities [3–5]. Interestingly, e can also be used so as to condition the state of 1 and 2. Once a three-body correlated state is established by means of bilocal 1 − e and 2 − e interactions, by measuring the state of e we could project the remote systems onto entangled states with a nonzero probability [3–6]. In these examples, 1 and 2 are embodied by two-level systems whose finite Hilbert space bounds the entanglement that can be shared [7]. Overcoming such a limitation is an important task deserving attention.

Here we present a scheme that allows the “extraction” of maximally entangled states via an effective nondemolition Bell measurement performed onto the state of two spin-s particles. This occurs through repeated injection and post-selection of simple mediators, each undergoing multiple scattering and spin-flipping between the two spins [8]. Besides achieving the maximum number of ebits allowed to two spin-s systems, the protocol provides a procedure for accumulating entanglement. Remarkably, our protocol does not require interaction-time tuning. In our scheme maximal entanglement is stable against the parameters of the conditioned dynamics, which is a clear advantage in experimental implementations. In order to fix the ideas, we first describe the protocol in terms of a system composed of a conduction electron and two magnetic impurities. This will allow us to clearly illustrate the relevant features of our scheme. Later, we show how a cavity-quantum electrodynamics (QED) system, consisting of two multilevel atoms interacting with a photon field, can also embody the desired dynamics and allows a prompt experimental implementation.

We consider a quasi one-dimensional (1D) wire, such as a semiconductor quantum wire [9] or a single-wall carbon nanotube [10], where two identical spin-s magnetic impurities 1 and 2 are embedded at positions \( x_1 = 0 \) and \( x_2 = x_0 \) [see Fig. 1(a)]. Left-incident single electrons undergo multiple scattering between the two impurities and simultaneous spin-flipping. Assuming that the electron’s coherence length exceeds \( x_0 \) and that each electron occupies only the lowest sub-band, the Hamiltonian reads (we set \( \hbar = 1 \))

\[
\hat{H} = \hat{p}^2/(2m^*) + J \vec{\sigma} \cdot (\vec{S}_1 \delta(x) + \vec{S}_2 \delta(x-x_0)).
\]

Here, \( \hat{p} = -i\nabla \), \( m^* \), and \( \vec{\sigma} \) are the electron momentum, effective mass, and Pauli spin operator, respectively. \( \vec{S}_i \) is the spin-s operator of the impurity \( i = 1, 2 \), and \( J \) is the Heisenberg exchange coupling constant whose dimensions are frequency times length. Because of the elastic nature of the interactions, the energy spectrum reads \( E = k^2/2m^* \).
(k is the electron wave vector). We label with $\hat{S} = \hat{\sigma} + \hat{S}_1 + \hat{S}_2$ the total spin of the system, while $m_i$ and $m_e$ = ±1/2 are the quantum numbers associated with $\hat{S}_1$ and $\hat{S}_2$, respectively. From now on, we denote $\{1/2, -1/2\}$ by $|\uparrow, \downarrow\rangle$ and, for convenience, we use the basis of product states $|m_e, \{m_i\}\rangle = |m_e, \{m_i\}, m_1, m_2\rangle$. We prepare the impurities in $|\{m_i\}\rangle_{12}$. An incoming electron of wave vector $k$ and spin state $|m_i\rangle_e$ is reflected (transmitted) in the state $|m_i\rangle_e$, while the impurities’ spin state changes into $|\{m_i\}\rangle_{12}$ with probability amplitude $r (t)$ (we omit the dependence of $r$ and $t$ on $m_{\epsilon(o)}$ and $m_{\epsilon(e)}$). As $\hat{S}_\perp$ is a constant of motion, the only nonzero amplitudes are those obeying the selection rule $m_{12} + m_{e} = m_{12} + m_{e} = m_{12} + m_{12} + m_{2}$. We solve this scattering problem by finding the steady states $|k, m'_{e}, \{m_i\}\rangle$ with input part $\langle x|k, m'_{e}, \{m_i\}\rangle_{in} = e^{ikx}\theta(-x)|m'_{e}, \{m_i\}\rangle$, where $\theta(x)$ is the Heaviside step function. Their output part reads $\langle x|k, m'_{e}, \{m_i\}\rangle_{out} = \sum_{x'}\langle x'|k, m'_{e}, \{m_i\}\rangle_{a} \delta (x' - x)$ with $\langle x'|k, m'_{e}, \{m_i\}\rangle_{a} = \sum_{m_{\epsilon(o)}}\alpha_{a}(x)|m_{\epsilon(o)}\rangle_{a} = \sum_{m_{\epsilon(o)}}\alpha_{a}(x)|m_{\epsilon(o)}\rangle_{a}$ and $\alpha_{a}(x) = e^{i\eta_{k}x}\theta(x - \eta_{k}) = e^{i\eta_{k}x}\theta(-x)$, $\eta_{k} = -\eta_{e} = -1$. The steady states are computed at all orders in $J$ solving the time-independent Schrödinger equation and imposing the matching of the wave function at $x_{i}$’s [4]. We now derive how an (in general mixed) initial state of the impurities $\rho_{12}$ is transformed after scattering of an electron incoming in an arbitrary statistical mixture $\rho_{e}$ of the spin states $|\uparrow\rangle_{e}$ and $|\downarrow\rangle_{e}$. To this aim, we consider the state having $|k\rangle |k\rangle |\rho_{e}\rangle |\rho_{12}\rangle$ as the input part, where $\langle x|k\rangle = e^{ikx}\theta(-x)$. The output part of such state is found by expanding it in the basis $|k, m'_{e}, \{m_i\}\rangle$ and replacing each component of this expansion with the corresponding output part. A further projection onto the electron’s position eigenstates far from the impurities $|x_{i}\rangle$ and $|x_{i}\rangle (x_{i} \ll 0, x_{i} \gg x_{0})$ yields $\sum_{\mu} \langle x_{a}|\mu\rangle_{a} |m_{\epsilon(o)}\rangle_{a} |k, m'_{e}, \{m_i\}\rangle_{a} |x_{a}\rangle |x_{a}\rangle$. After tracing over the electron’s degrees of freedom, the impurities’ state becomes

$$E_{\rho} (\rho_{12}) = \sum_{\mu} \rho_{e\mu} (\hat{R}_{\mu}^{\dagger} \rho_{12} \hat{R}_{\mu} + \hat{T}_{\mu}^{\dagger} \rho_{12} \hat{T}_{\mu}), \quad (1)$$

where $\sum_{\mu} (\hat{R}_{\mu}^{\dagger} \hat{R}_{\mu} + \hat{T}_{\mu}^{\dagger} \hat{T}_{\mu}) = 1_{12}$. Each Kraus operator $\hat{R}_{\mu}^{\dagger} (\hat{T}_{\mu}^{\dagger})$ depends only on $r$’s (t’s) and is physically interpreted as the effect on $\rho_{12}$ due to the detection in spin-state $|\mu\rangle_{e}$ of a reflected (transmitted) electron incoming in state $|\nu\rangle_{e}$. We want to show that, conditioning the map in Eq. (1) and iterating it for $n$ electrons (injected in succession in the same spin state), singlet-state extraction is efficiently performed. To achieve this, we first describe what is induced by post-selecting the state of $n = 1$ scattered electrons. Preparation and post-selection of a given electron-spin state, say $|\uparrow\rangle_{e}$, can be accomplished using spin-filtering contacts at the input or output ports of the wire [11], each selecting the same spin state. We obtain the final impurities’ state $\rho^{(n)} (\rho_{12}) = \rho^{(n)} (\rho_{12}) = \rho^{(n)} (\rho_{12})$, $\rho^{(n)} (\rho_{12}) = \rho^{(n)} (\rho_{12})$ with success probability $P (\rho_{12}) = P (\rho_{12}) = P (\rho_{12})$. The state $\rho^{(n)} (\rho_{12})$ corresponding to $n$ electrons being prepared and post-selected in $|\uparrow\rangle_{e}$ is obtained as $\rho^{(n)} (\rho_{12}) = \rho^{(n)} (\rho_{12})$ with conditional probability $P (\rho_{12}) = \prod_{n} P (\rho^{(n)} (\rho_{12}))$ and $\rho^{(n)} (\rho_{12}) = \rho^{(n)} (\rho_{12})$. Here, the rate of electron-injection is chosen so that, as an electron reaches the impurities, the previous one has been already scattered off.

Let $|\Psi_{+}\rangle$ be the singlet state of two spin-$s$ impurities. Using resonance conditions (i.e., $k_{x0}/\pi \in \mathbb{Z}$), in Figs. 2(a) and 2(b) we consider the case $s = 1/2$ and plot the fidelity $F^{(n)} (\rho_{12})$ with respect to the singlet $|\Psi_{+}\rangle$ together with $P^{(n)}$ as functions of $n$ and $J/\nu$ for the initial product state $|1/2, -1/2\rangle_{12}$ ($\nu = m^{*}$ is the electronic group velocity). Clearly, $F^{(n)} \rightarrow 1$ for a range of values around $J/\nu \approx 1.5$ that becomes a plateau when $n$ increases ($n < 7$ iterations are enough to get fidelity higher than 0.95). For a fixed value of $J/\nu$, such convergence is exponential in $n$. Remarkably, although our protocol is conditioned on the outcomes of $n$ projective measurements all with the same outcome, the probability of success converges exponentially to 0.5. Differently from [3–5], the scheme is still efficient for a nonoptimal $J/\nu$. Only a larger $n$ is required, for a fixed $s$. Moreover, the process is robust against discrepancies of $k$ with respect to resonance conditions and the use of a stream of mediators with mutually different wave vectors. In fact, by considering a Gaussian distribution of wave vectors centered at $k$ with variance $\sigma$, we have found that the fidelity (probability) is larger than 0.9 (0.35) for $k_{x0} \in [0.9, 1.03]\pi$ and $\sigma/k$ up to $\approx 5\%$.

We now address the dependence of our figures of merit on the dimensionality of the impurities’ spin. While the optimum ratio $J/\nu$ depends slightly on $s$, the efficiency of singlet extraction persists, as shown in Fig. 2(c) for

FIG. 2 (color online). (a) and (b) Fidelity and success probability vs. $J/\nu$ and $n$ for $s = 1/2$. (c) $F^{(n)}$ (filled symbols) and $P^{(n)}$ (empty symbols) vs $n$ for $s = 1/2$ and $J/\nu = 1.5$ ($\triangle$, $\bigtriangleup$), $s = 1$ and $J/\nu = 1.2$ ($\blacksquare$, $\square$), and $s = 3/2$ and $J/\nu = 1.1$ ($\bigcirc$, $\bullet$) at $k_{x0}/\pi \in \mathbb{Z}$ ($J/\nu$ is optimized for each $s$).

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\( \rho_{12} = |s, -s\rangle\langle s, -s| \) with \( s = 1/2, 1, 3/2 \). Evidently, \( \rho_{12}^{(n)} \) rapidly converges to the singlet state regardless of \( s \) (for instance, \( F(n>5) > 0.95 \) for \( s = 1 \)) while \( P_{11}^{(n)} \) approaches a finite value according to \( P_{11}^{(n\to\infty)}(\rho_{12}) = -\langle \Psi_0^- | s, -s \rangle^2 = (2s + 1)^{-1} \), exponentially in \( n \). Our scheme thus asymptotically performs an effective projective measurement onto the spin-\( s \) singlet state. As the singlet state has the maximum number of ebits allowed by the dimension of the Hilbert space of each impurity, the scheme provides a way to extract more than one ebit by considering sufficiently high-dimensional impurities' spins. Moreover, an entanglement accumulation mechanism is achieved [7]. For instance, for \( s = 2 \) and \( J/\nu = 1 \) the impurities' entanglement [measured by the logarithmic negativity, which is upper-bounded by \( \log_2(d) \) for a \( d^2 \)-dimensional Hilbert space] after \( n = 2, 4, \) and \( 5 \) is, respectively, 1.2, 1.8, and 2. These are larger than the bound given by \( \log_2(2s + 1) \) for \( s = 1/2, 1, \) and \( 3/2 \), making our system an iteratively exploitable quantum channel: The impurities' entanglement can be extracted to many pairs of qubits [7]. Similar results hold for any initial eigenstate of \( \hat{S}_{12} = \hat{S}_{1z} + \hat{S}_{2z} \) with null eigenvalue.

We now show how the efficiency of singlet-state extraction relies on resonance-induced selection rules. Let \( |s, s, s_1, m_{12}\rangle \) be the coupled basis of common eigenstates of \( \hat{S}_{1z}^2, \hat{S}_{2z}^2, \hat{S}_{12}^z \), and \( \hat{S}_{12}^z \) (the singlet state thus reads \( |\Psi_0^-\rangle = |s, s, s_1 = 0, m_{12} = 0\rangle \)). Let \( \xi_{12}(\rho_{12}) \) be the unconditioned map in Eq. (1) for \( \rho_{12} = |\xi_{12}\rangle\langle \xi_{12}| \). Clearly, with the additional output-filtering of \( |\xi_{12}\rangle, \xi_{12}(\rho_{12}) \) becomes \( \xi_{12}(\rho_{12}) \). Notice that in general the product state \( |s, s, s_1\rangle \) is the only fixed point of \( \xi_{12}(\rho_{12}) \). However, at resonance \((kx_0 = n \pi)\), \( \hat{S}_{12}^z \) is conserved due to the equal probabilities of the electron to be found at each of the \( x_i \)'s [4]. Thus, repeated applications of the unconditioned map cannot drive the system out of the eigenspace associated with a set value of \( s_{12} \). This and the conservation of \( \hat{S}_{12} \) imply that the singlet-state \( |\Psi_0^-\rangle \) becomes an additional fixed point of \( \xi_{12} \). Let \( p_{s_{12}} \) be the probability for an injected electron prepared in \( |\xi_{12}\rangle \) to be flipped down when the impurities are prepared in \( |s, s, s_1, 0\rangle \). The selection rules at resonance yield the evolved impurities' state \( \rho_{s_{12}} = |s, s, s_1, 1\rangle\langle s, s, s_1, 1| + (1 - p_{s_{12}})|s, s, s_1, 0\rangle\langle s, s, s_1, 0| \). If we post-select \( |\xi_{12}\rangle \) at the output ports, each state \( |s, s, s_1, 0\rangle \) with \( s_{12} \neq 0 \) is left uncharged with probability \( 1 - p_{s_{12}} \). Under application of \( \xi_{12}^{-1} \), it thus vanishes as \( (1 - p_{s_{12}})\xi_{12}^{-1}(0) \), which clarifies the exponential convergence exhibited by \( F^{(n)} \) and \( P_{11}^{(n)} \) (cf. Fig. 2). Differently, \( |s, s, s_1 = 0, 0\rangle = |\Psi_0^-\rangle \) survives to the application of \( \xi_{12}^{-1} \) since the selection rules ensure that \( p_{s_{12}} = 0 \) [4]. If we consider an element of the uncoupled basis \( |\xi\rangle \) such that \( \hat{S}_{12}, |\xi\rangle = 0 \) and expand it over \( |s, s, s_1, 0\rangle \)'s, we find that, under application of \( \xi_{12}^{-1} \), \( |\xi\rangle \langle \xi| \to |\Psi_0^-\rangle \) with a probability \( P_{11}^{(n\to\infty)} \) that asymptotically becomes \( |\Psi_0^-\rangle \). When \( |\xi\rangle = |s, -s\rangle \), as in Fig. 2, the asymptotic probability is \( (2s + 1)^{-1} \). Our clear interpretation of the physics behind our protocol is an important feature for the development of novel schemes.

Unlike previous proposals [3–5], a remarkable advantage of our protocol is that it can be applied to magnetic impurities of spin higher than 1/2. For instance, we could use a 1D semiconducting wire with embedded Mn impurities having \( s = 5/2 \). Although impressive progress has been made, a major obstacle in spintronics implementations is the current lack of high-efficiency electron-spin filters [11]. As a way to overcome such difficulties, we discuss an alternative system [see Fig. 1(b)] able to act as an accurate simulator of \( \hat{H} \) and holding the promises for not far-fetched experimental implementation. The basic idea is to replace the electron with a single photon propagating in a 1D photonic waveguide sustaining two frequency-degenerate orthogonally polarized modes. For consistency of notation, we denote circular polarizations by \( \uparrow \) and \( \downarrow \). Each impurity is now embodied by a multilevel atom [see Fig. 1(c)] having a \((2s + 1)\)-fold degenerate ground level spanned by \( \{|g_{s_{12}}\rangle, \ldots, |g_s\rangle\} \) and a \(2s\)-fold degenerate excited level spanned by \( \{|e_{s_{12}}\rangle, \ldots, |e_{s}\rangle\} \). The standard three-level \( \Lambda \) and five-level \( M \) configurations are recovered, for instance, by taking \( s = 1/2 \) and \( s = 1 \), respectively. Such a configuration may be found in the rich hyperfine spectrum of alkali atoms. We assume electric-dipole selection rules such that each \( |e_m\rangle \) \((m = -s, \ldots, s - 1)\) is connected to the pair of nearest-neighbor ground states \( \{|g_{s_{12}}\rangle, |g_{s_{1}}\rangle\} \) via coherent scattering of a photon between the two orthogonally polarized modes. To fix the ideas, we take the transition \( |e_{m}\rangle \leftrightarrow |g_{s_{1}}\rangle \leftrightarrow |g_{m+1}\rangle \) to be driven by the \( \uparrow \)-polarized (\( \downarrow \)-polarized) mode. Each atom can thus undergo a transition between two adjacent ground states \( \{|g_{s_{1}}\rangle \leftrightarrow |g_{m+1}\rangle \} \) via a two-photon Raman process with associated coherent scattering of a photon between states \( |\uparrow\rangle \leftrightarrow |\downarrow\rangle \). Assuming a linear dispersion law \( E = v_\phi k \) with \( v_\phi \) the group velocity of the photon and \( E \) its energy, the free Hamiltonian of the field in the waveguide is [13] \( \hat{H}_f = -i \sum_{\beta = \uparrow, \downarrow} \sum_{\gamma = 1}^{s_{12}} \int dx v_\phi \hat{c}_{\beta,y}^\dagger(x) \partial_x \hat{c}_{\beta,y}(x) \) with \( v_\phi = -v_L = v_\phi \), and \( \hat{c}_{\beta,y}^\dagger(x) \) the bosonic operator creating a right (left) propagating photon of polarization \( y \) at position \( x \). Considering dipole transitions with Rabi frequencies and natural excited-state linewidth smaller than the corresponding detuning from the excited state, each state \( |e_{m}\rangle \) is only virtually populated and the effective atom-photon coupling reads \( \hat{V} = \sum_{\beta = \uparrow, \downarrow} \int dx \hat{c}_{\beta,y}^\dagger(x) \hat{c}_{\beta,y}^\dagger(x + h_c) \delta(x - x_c) \) with \( c_{\gamma}(x) = \sum_{\beta = \uparrow, \downarrow} \hat{c}_{\beta,y}^\dagger(x) \hat{c}_{\beta,y}(x) = \sum_{n=0}^{s_{12}} \sum_{\lambda = s_{12}} |\lambda\rangle \langle \lambda| |g_{n\lambda}\rangle \langle g_{n\lambda}| s_{12} \rangle \). Here \( J_{s,m} \) is the effective transition rate of the Raman process leading the \( i \)th atom from \( |g_{n\lambda}\rangle \) to \( |g_{n_{\lambda+1}}\rangle \), assuming identical atoms. We map the photonic polarization into an effective pseudospin \( s \) as \( \hat{\sigma}(x) = \int dx \hat{\sigma}(x) \) with \( \hat{\sigma}(x) = \hat{c}_{\uparrow}(x) \hat{c}_{\downarrow}(x) = \frac{1}{2} \langle c_{\uparrow}(x) | c_{\downarrow}(x) \rangle - \langle c_{\downarrow}(x) | c_{\uparrow}(x) \rangle \rangle / 2 \). Provided that \( J_{s,m} = J_{X,s,m} \),
$X_{s,m} = [s(s + 1) - m(m + 1)]^{1/2}$, each $\hat{S}_{i,z}$ becomes the effective pseudospin $s$ operator $\hat{S}_{i,z} = J \hat{S}_{i,z}$, where $\hat{S}_{i,z}$ obeys the standard algebra of angular momentum. Under these conditions, this model can be regarded as the second quantization version of $\hat{H}$ with the exchange electron-impurity coupling replaced by an isotropic $XY$ interaction. It is easily checked that $[\hat{H}_{ph} + \hat{V}, \hat{\mathcal{S}}] = 0$ and, provided $k_x_0/m \in \mathbb{Z}$, $[\hat{H}_{ph} + \hat{V}, \hat{S}_{i,z}] = 0$. Through standard procedures [13], we have derived the stationary states $|k, m_{ph}^i, \{m_j^i\}|$ for a single photon with wave vector $k$ ($m_{ph}^i$ is the quantum number of $\hat{S}_{i,z}$). The input (output) part of $|k, m_{ph}^i, \{m_j^i\}|$ is formally analogous to $|k, m_{ph}^i, \{m_j^i\}|_w$ ($|k, m_{ph}^i, \{m_j^i\}|_w$). Here, $\mathcal{E}_{\rho(\rho_{12})}$ is obtained analogously to what is done for the previous model with photonic polarization detection used for the post-selection. Plots analogous to those in Figs. 2 are reproduced with only negligible quantitative differences. Practically, $\mathcal{E}_{\rho(\rho_{12})}$ is obtained using Geiger-like photodetectors at the input or output ports of the waveguide combined with polarizing beam splitters to realize $\mathcal{E}_{\rho(\rho_{12})}$. Each $X_{s,m}$ depends on the product of the Clebsch-Gordan coefficients associated with the far-detuned (one-photon) transitions involved in the process $|g_{m}\rangle \rightarrow |g_{m+1}\rangle$. The condition $J_{s,m} = JX_{s,m}$ is clearly fulfilled for $s = 1/2$, involving only $X_{1/2,-1/2} = 1$. For $s \geq 1$ the pattern of $J_{s,m}$’s might in general deviate from the ideal one dictated by the $X_{s,m}$’s. However, we have assessed $P^n(\alpha)$ and $P^n_{\parallel}$ finding that our scheme is strikingly robust against such deviations [14]. For instance, for $s = 3/2$, the ideal pattern yields $J_{3/2,1/2}/J_{3/2,-3/2} = 1$ and $J_{3/2/-1/2}/J_{3/2,-3/2} = 2/\sqrt{3}$. By taking $J_{3/2,-3/2}/\nu_{ph} = J_{3/2,1/2}/\nu_{ph} = \sqrt{3}$ and $J_{3/2,-1/2}/\nu_{ph} = 4\sqrt{3}$, which are far from ideal, we obtain $P(\alpha = 60) = 0.97$, and $P(\alpha = 90) = 0.26$. These values are basically identical to the values obtained with the ideal ratios. This alternative model turns out to be also robust against deviations of $k$ from the ideal resonance conditions [14]. Our protocol is thus resilient and flexible to the actual working conditions.

For a realization of the scheme in the case $s = 1/2$, the impurities can be embodied by $\Lambda$ configurations encompassed in the (single-electron charged) trionic picture of semiconductor quantum dots (QDs), which have been the center of extensive studies [15]. Positioning QDs within a waveguide or a cavity is now achievable with high accuracy ($\sim 30$ nm). A back-of-the-envelope calculation shows that for a photonic wavelength of 780 nm in a GaAs structure (400 nm in a GaN nanowire), $x_0 \sim 0.1 \mu m$ ($1 \mu m$) is required for the resonance condition, which is achievable. Strong coupling between a single QD and a cavity field has been demonstrated [15] and current experimental efforts make the achievement of $J/\nu \sim 1$ realistic in large refractive-index structures, without the need of a waveguide’s band gap. We consider GaN (InAs) QDs in GaN (GaAs) nanowires as potential candidates for our scheme. Their typical quality factor is $\sim 10^5$, implying a single-photon lifetime $\tau_p \sim 1$ ps at 400 nm wavelength. The refractive index of GaN is $\sim 2$, so that a photon travels $x_0 = 1 \mu m$ in $\tau_p/100$. Ongoing experimental progress makes the controlled growth and positioning of two QDs in $\mu m$-long waveguides quite realistic.

We have proposed a scheme for the conditional extraction of singlet states of two remote spin $s$’s based on projective measurements over interaction mediators. The protocol does not require the demanding recycling of the same mediator. It achieves $s + 1/2$ ebits with finite probability, a small number of steps, weak requirements on the parameters entering the dynamics, and no interaction-time tuning. We have proposed a realistic setup where the mediators are embodied by photons and the spins to be entangled by artificial atoms.

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