If the fundamental Planck scale is about a TeV and the cosmic neutrino flux is at the Waxman-Bahcall level, quantum black holes are created daily in the Antarctic ice-cap. We re-examine the prospects for observing such black holes with the IceCube neutrino-detection experiment. To this end, we first revise the black hole production rate by incorporating the effects of inelasticity, i.e., the energy radiated in gravitational waves by the multipole moments of the incoming shock waves. After that we study in detail the process of Hawking evaporation accounting for the black hole’s large momentum in the lab system. We derive the energy spectrum of the Planckian cloud which is swept forward with a large, $O(10^9)$, Lorentz factor. (It is noteworthy that the boosted thermal spectrum is also relevant for the study of near-extremal supersymmetric black holes, which could be copiously produced at the LHC.) In the semiclassical regime, we estimate the average energy of the boosted particles to be less than 20% the energy of the $r$-progenitor. Armed with such a constraint, we determine the discovery reach of IceCube by tagging on “soft” (relative to what one would expect from charged current standard model processes) muons escaping the electromagnetic shower bubble produced by the black hole’s light descendants. The statistically significant $5\sigma$ excess extends up to a quantum gravity scale $\sim 1.3$ TeV.

I. GENERAL IDEA

Over the past few years it has become evident that a promising approach towards reconciling the apparent mismatch of the fundamental scales of particle physics and gravity is to modify the short distance behavior of gravity at scales much larger than the Planck length, $l_{Pl} \sim 10^{-35}$ m. The key premise of such an approach entails that the weakness of gravity is indeed evidence of extra spatial compactified dimensions [1]. This is possible because standard model (SM) fields are confined to a 4-dimensional world (corresponding to our apparent universe) and only gravity spills into the higher dimensional spacetime bulk, without conflicting with experimental bounds [2]. Therefore, if this new approach is correct, gravity is not intrinsically weak, but of course appears weak at relatively large distances of common experience because its effects are diluted by propagation in the extra dimensions. The distance at which the gravitational and electromagnetic forces might have equal strength is unknown, but a particularly interesting possibility is that it is roughly at $10^{-19}$ m, the distance at which electromagnetic and weak forces are known to unify to form the electroweak force. This would imply a fundamental D-dimensional Planck mass, $M_D \sim M_W \sim 1$ TeV, considerably smaller than the macroscopic 4-dimensional value, $M_{Pl} \sim 10^{19}$ GeV.

If nature gracefully picked a sufficiently low-scale gravity, the first evidence for it would likely be the observation of microscopic black holes (BHs) produced in particle collisions [3]. According to Thorne’s hoop conjecture, a BH forms in a two-particle collision when and only when the impact parameter is smaller than the radius of a Schwarzschild BH of mass equal to the total center-of-mass energy [4]. Subsequent to formation a TeV-scale BH will promptly decay via thermal Hawking radiation [17] (for $M_D = 1$ TeV, the lifetime of a BH of mass 10 TeV is less than $10^{-25}$ s) into observable quanta [5]. Although the BH production cross section, $O(M_W^{-1})$, is about 5 orders of magnitude smaller than QCD cross sections, $O(\Lambda_{QCD}^{-1})$, in two well-known papers [6, 7] it was proposed that such BHs could be produced copiously at the LHC, and that these spectacular events could be easily filtered out of the QCD background. This is possible by triggering on BH events with prompt charged leptons and photons, each carrying hundreds of GeV of energy.

Cosmic ray collisions, with center-of-mass energies ranging up to $10^5$ GeV, certainly produce BHs if the LHC does. The question is, can they be detected? Most cosmic rays are protons, which generally collide with hadrons in the upper atmosphere, producing cascading showers which eventually reach the Earth’s surface. At energies of interest, however, the cosmic ray luminosity, $L \sim 10^{-24}$ cm$^{-2}$ s$^{-1}$, is about 50 orders of magnitude smaller than the LHC luminosity, thus making it futile to hunt for BHs in baryonic cosmic rays. On the other hand, neutrino interaction lengths are still far larger than the Earth’s atmospheric depth, although they would be greatly reduced by the cross section for BH production [8]. Cosmic neutrinos therefore would produce BHs with roughly equal probability at any point in the atmosphere. As a result, the light descendants of the BH may initiate low-altitude, quasi-horizontal showers at rates significantly higher than SM predictions [9]. Because of these considerations the atmosphere provides a buffer against contamination by mismeasured baryons, for which the electromagnetic channel is filtered out.

Neutrinos that traverse the atmosphere unscathed can produce BHs through interactions in the Antarctic ice-cap and be detected by the IceCube neutrino telescope [10]. This telescope, which is currently being deployed near the Amundsen-Scott station, comprises a cubic-kilometer of ultra-clear ice about a mile below the South Pole surface, instrumented with long strings of sen-
sitive photon detectors which record light produced when neutrinos interact in the ice [11]. The In-ice array is complemented by IceTop, a surface air shower detector consisting of a set of 160 frozen water tanks, which serves as a veto for atmospheric muon background. At the same time, the energy deposited by tagged muon bundles in air shower cores becomes an external source of energy calibration. Altogether, the expected energy resolution of the experiment is about ±0.1 on a log_{10} scale. Moreover, the energy reconstruction is optimized for neutrino energy $E_{\nu} > 10^{6}\text{ GeV}$, allowing sufficient precision (±0.2 on a log_{10} scale) to separately assign the energy fraction for emergent muons in neutrino interactions. Because of this, the inelasticity distribution of events becomes a unique tool for SM background rejection, providing powerful discrimination of resonant processes [12]. In this work we re-examine the prospects for discovering BH resonances at IceCube, by tagging on “soft” (relative to what one would expect from charged current SM processes) muons escaping the electromagnetic shower bubble triggered by the BH explosion.

The paper is organized as follows. In Sec. [11] we update the semi-classical BH production cross section considering a new estimate [13] of the energy radiated in gravitational waves by the multipole moments of the incoming shock waves. This is followed by a detailed discussion of Hawking evaporation taking into account that cosmic neutrinos produce BHs with large momenta in the lab system. Specifically, we derive the energy distribution of the Planckian cloud which is boosted in the forward direction with a large Lorentz factor. Armed with this distribution, we estimate the average energy of the BH light descendants to be less than 20% the energy of the $\nu$-progenitor. The Hawking radiated muons then provide a very clean signal with negligible SM background, as the production of “soft” leptons in charged current (CC) interactions occurs at a much smaller rate than BH production. Our results for event rates and discovery reach are presented in Sec. [11]. Conclusions are given in Sec. [LV]. In this last section we also entertain the possibility of producing TeV-scale near-extremal BHs in particle collisions.

II. BH PRODUCTION AND EVAPORATION

Analytic and numerical studies have revealed that gravitational collapse takes place at sufficiently high energies and small impact parameters, as conjectured years ago by Thorne [14]. In the case of 4-dimensional head-on collisions [14], as well as those with non-zero impact parameter [15], a horizon forms when and only when a mass is compacted into a hoop whose circumference in every direction is less than 2\pi times its Schwarzschild radius up to a factor of order 1. In the D-dimensional scenario the Schwarzschild radius still characterizes the maximum impact parameter for horizon formation [16]. In the course of collapse, a certain amount of energy is radiated in gravitational waves by the multipole moments of the incoming shock waves [14], leaving a fraction $y = M_{\text{BH}}/\sqrt{s}$ available for Hawking evaporation [17]. Here, $M_{\text{BH}}$ is a lower bound on the final mass of the BH and $\sqrt{s}$ is the center-of-mass energy of the colliding particles, taken as partons. This ratio depends on the impact parameter of the collision, as well as on the dimensionality of spacetime [18].

Of course, this work is purely in the framework of classical general relativity, which is valid only for sufficiently massive BHs, $M_{\text{BH}} \gg M_{D}$. For masses close to $M_{D}$, gravity becomes strong and the classical description can no longer be trusted. Hence, it is important to impose a lower cutoff on the mass of microscopic BHs for which the simple semiclassical arguments can reasonably be expected to hold. Following [19], we define $x_{\min} = M_{\text{BH}}^{\min}/M_{D} = 3$, where $M_{\text{BH}}^{\min}$ is the smallest BH mass for which we trust the semiclassical approximation.

String theory provides a promising route for understanding the regime of strong quantum gravity and in particular for computing cross sections at energies close to the Planck scale [20]. Therefore, the ensuing discussion will be framed in the context of string theory. To be specific we will consider an embedding of a 10-dimensional low-energy scale gravity scenario within the context of SO(32) Type I superstring theory, where gauge and charged SM fields can be identified with open strings localized on a 3-brane and the gravitational sector consists of closed strings that propagate freely in the internal dimensions of the universe [21]. After compactification on $T^{6}$ down to four dimensions, $M_{P_{1}}$ is related to the string scale, $M_{s}$, and the string coupling constant, $g_{s}$, by $M_{P_{1}}^{2} = (2\pi r_{\text{c}})^{6} M_{s}^{8}/g_{s}^{2}$, where $r_{\text{c}}$ is the compactification radius. Within this framework, the problem of avoiding fast baryon decay [22] or lepton flavor violation [23] is shifted to the examination of symmetries [24] in the underlying string theory which would suppress the appropriate non-renormalizable operators at low energies. Nevertheless, it is important to stress that for $x_{\min} \geq 3$ the typical decay involves a large number of particles. Therefore, though these symmetries constrain the decay of the BH, throughout this paper we ignore the constraints imposed by the few conservation laws and we assume that BHs decay with roughly equal probability into all SM particles. From now on we set $D = 10$.

The inclusive production of BHs proceeds through different final states for different classical impact parameters $b$ [18]. These final states are characterized by the fraction $y(z)$ of the initial parton center-of-mass energy, $\sqrt{s}$, which is trapped within the horizon. Here, $z = b/b_{\text{max}}$, and $b_{\text{max}} = \sqrt{F} r_{s}(\sqrt{s})$ is the maximum impact parameter for collapse, where

$$r_{s}(\sqrt{s}, M_{10}) = \frac{1}{M_{10}} \left[ \frac{\sqrt{s}}{M_{10}} 8 \pi^{3/2} \Gamma(9/2) \right]^{1/7}$$

(1)

is the radius of a Schwarzschild BH in 10-dimensions [25], and $F$ is a form factor.
A bound on the inelasticity and the form factor can be obtained by studying the formation of an apparent horizon, which (because of cosmic censorship [26]) guarantees the formation of a BH event horizon [27]. Such a study can be easily accomplished by modeling the incoming partons as two Aichelburg-Sexl [28] shock waves (i.e., by boosting the Schwarzschild solution to the speed of light at fixed energy). The scattering of partons is then simulated through the superposition of two shock waves coming from opposite directions, such that their union defines a closed trapped surface which provides a lower bound on $M_{\text{BH}}$ and $b_{\text{max}} [18]$. This lower bound, however, depends on the slice used to determine the apparent horizon, and becomes larger if the apparent horizon is taken on the future light cone of the collision plane [13]. This is because it is possible that for a given impact parameter an apparent horizon is not yet formed on the so-called “old slice”, but arises by the time a later “new slice” is reached. In our calculations we consider the estimates of $y$ and $F$ in both the “old” [18] and “new” [13] slices.

The $y$ dependence complicates the parton model calculation, since the production of a BH of mass $M_{\text{BH}}$ requires that $\hat{s}$ be $M_{\text{BH}}^2/y^2(z)$, thus requiring the lower cutoff on parton momentum fraction to be a function of impact parameter [29]. Because of the complexity of the final state, we assume that amplitude interference effects can be ignored and we take the $\nu N$ cross section as an impact parameter-weighted average over parton cross sections, with the lower parton fractional momentum cutoff determined by $x_{\text{min}}$. This gives a lower bound $X = (x_{\text{min}}M_{10})^2/[y^2(z)s]$ on the parton momentum fraction $x$, where $\sqrt{s}$ is the center-of-mass energy of the $\nu N$ collision. All in all, the $\nu N \to \text{BH}$ cross section reads [30]

$$\sigma = \int_0^1 2z dz \int_0^1 dx F \pi r_s^2(\sqrt{s}, M_{10}) \sum_i f_i(x, Q),$$

where $\hat{s} = xs = 2x m_N E_{\nu}$, $i$ labels parton species, and the $f_i(x, Q)$ are parton distribution functions (pdfs).

The choice of the momentum transfer $Q$ is governed by considering the time or distance scale probed by the interaction. Roughly speaking, the formation of a well-defined horizon occurs when the colliding particles are at a distance $\sim r_s$ apart. This has led to the advocacy of the choice $Q \simeq r_s^{-1}$ [31], which has the advantage of a sensible limit at very high energies. However, the dual resonance picture of string theory [32] would suggest a choice $Q \sim \sqrt{\hat{s}}$. Fortunately, as noted elsewhere [33], the BH production cross section is largely insensitive to the details of the choice of $Q$. In what follows we use the CTEQ6D pdfs [34] with $Q = \min\{r_s^{-1}, 10 \text{ TeV}\}$. In Fig. 1 we show the BH production cross section for $x_{\text{min}} = 3$ and $M_{10} = 1 \text{ TeV}$.

BHs produced in particle collisions have non-vanishing angular momenta determined by the impact parameter of the incoming partons. Since $\hat{b}$ is only non-zero along the brane directions, the angular momentum lies within the brane. Moreover, the initial horizon is likely to be very asymmetric as only gravity spills into the compactified dimensions. Therefore, the excited BH state carries additional hair corresponding to multipole moments for the distribution of gauge charges and energy momentum within the asymmetric configuration. The decay of an excited spinning BH state follows several stages. The initial configuration loses hair associated with multipole moments in a balding phase through emission of gravitational waves. In addition, gauge charges inherited from the initial state partons are discharged within this phase via Schwinger emission. The subsequent spinning BH evaporates in a two-step process: a short spin-down phase in which angular momentum is shed [35], followed by a long Schwarzschild phase of semi-classical Hawking radiation.

In the rest frame of the Schwarzschild BH, both the average number [17] and the probability distribution of the number [36] of outgoing particles in each mode obey a thermal spectrum. In 10-dimensions, the emission rate per degree of particle freedom $i$ of particles of spin $s$ with initial total energy between $(\omega, \omega + d\omega)$ is found to be [37]

$$\frac{dN_i}{d\omega} = \frac{\sigma_i(\omega)\Omega_{d-3}\omega^{d-2}}{(d-2)(2\pi)^{d-1}} \left[ e^{\omega/T} - (-1)^{2s} \right]^{-1},$$

where

$$T = \frac{7}{4 \pi r}$$

FIG. 1: BH production cross section for $x_{\text{min}} = 3$ and $M_{10} = 1 \text{ TeV}$. The solid line indicates the result obtained using the new estimates of $F$ and $y$ given in [13], whereas the dot-dashed line indicates the result obtained assuming the old values from Ref. [13].
is the instantaneous Hawking temperature,

\[ \Omega_{d-3} = \frac{2 \pi^{(d-2)/2}}{\Gamma[(d-2)/2]} \]

(5)
is the volume of a unit \((d - 3)\)-sphere,

\[ r = \frac{1}{M_{10}} \left[ \frac{M}{M_{10}} 8 \pi^{3/2} \Gamma(9/2) \right]^{1/7} \]

(6)
is the instantaneous Schwarzschild radius of mass \(M\), and \(\sigma_s(\omega)\) is the greybody absorption area due to the backscattering of part of the outgoing radiation of frequency \(\omega\) into the BH (a.k.a. the greybody factor) \[^{[38]}\]. Recall that SM fields live on a 3-brane \((d = 4)\), while gravitons inhabit the entire spacetime \((d = 10)\). The prevalent energies of the decay quanta are of \(O(T \sim 1/r)\), resulting in \(s\)-wave dominance of the final state. Indeed, as the total angular momentum number of the emitted field increases, \(\sigma_s(\omega)\) rapidly gets suppressed \[^{[39]}\]. In the low energy limit, \(\omega r \ll 1\), higher-order terms are suppressed by a factor of \(3(\omega r)^{-2}\) for fermions and by a factor of \(25(\omega r)^{-2}\) for gauge bosons. For an average particle energy \((\omega)\) of \(O(r^{-1})\), higher partial waves also get suppressed, although by a smaller factor. This strongly suggests that the BH is sensitive only to the radial coordinate and does not make use of the extra angular modes available in the internal space \[^{[38]}\]. Actually, a recent detailed analysis \[^{[40]}\] has explicitly shown that the relative emission rate of SM particles and the 10-dimensional bulk graviton is roughly 92:5. This implies that the power lost in the bulk is less than 15\% of \(M_{\text{BH}}\), largely favoring the dominance of visible decay. Therefore, in what follows we assume the Hawking evaporation process to be dominated by SM brane modes and we neglect graviton emission during the Schwarzschild phase. With this in mind, the average total emission rate for particle species \(i\) is,

\[ \frac{d(N)}{dt} = \frac{1}{2\pi} \left( \sum c_i g_i \Gamma_i \right) \zeta(3) \Gamma(3) r^2 T^3, \]

(7)

where \(c_i\) is the number of internal degrees of freedom of particle species \(i\), \(g_i = 1\ (3/4)\) for bosons (fermions),

\[ \Gamma_i = \frac{1}{4\pi r^2} \int \frac{\sigma_s(\omega) \omega^2 d\omega}{e^{\omega/T} \pm 1} \left[ \int \frac{\omega^2 d\omega}{e^{\omega/T} \pm 1} \right]^{-1}. \]

(8)
The rate of change of the BH mass in the evaporation process is

\[ \frac{dM}{dt} \bigg|_{\text{evap}} = -\frac{1}{2\pi} \left( \sum c_i f_i \Phi_i \right) \zeta(4) \Gamma(4) r^2 T^4, \]

(9)
where \(f_i = 1\ (7/8)\) for bosons (fermions) and

\[ \Phi_i = \frac{1}{4\pi T^4} \int \frac{\sigma_s(\omega) \omega^3 d\omega}{e^{\omega/T} \pm 1} \left[ \int \frac{\omega^3 d\omega}{e^{\omega/T} \pm 1} \right]^{-1}. \]

(10)
Dividing Eq. (17) by Eq. (9) and integrating, one obtains a compact expression for the average multiplicity

\[ \langle N \rangle = \frac{\pi}{2} \rho \left[ 8 \pi^{3/2} \Gamma(9/2) \right]^{1/7} \left[ \frac{M_{\text{BH}}}{M_D} \right]^{8/7} \approx \rho S_0, \]

(11)
where

\[ \rho = \sum c_i g_i \Gamma_i \zeta(3) \Gamma(3) \]

(12)
and

\[ S_0 = \frac{7}{8} \frac{M_{\text{BH}}}{T_{\text{BH}}^3} \]

(13)
is the initial value of the entropy in terms of the initial BH mass and Hawking temperature \(T_{\text{BH}}\) \[^{[41]}\].

Before proceeding, we comment briefly on the BH rate of absorption. The upper limit on the cross section for the particle absorption is \(^{[42]}\)

\[ \sigma_{\text{accr}} = \pi r_{\text{eff}}^2, \]

(14)
and so the accretion rate of the BH mass becomes

\[ \frac{dM}{dt} \bigg|_{\text{accr}} = \pi r_{\text{eff}}^2 \epsilon, \]

(15)
where

\[ r_{\text{eff}} = \sqrt{\frac{9}{7} \frac{1}{M_{10}} \left[ 36 \pi^{3/2} \Gamma(9/2) \frac{M}{M_{10}} \right]^{1/7}} \]

(16)
is the effective BH radius for capturing particles \[^{[43]}\] and \(\epsilon\) is the nearby parton energy density. The net change of the BH mass is therefore

\[ \frac{dM}{dt} = \frac{dM}{dt} \bigg|_{\text{accr}} + \frac{dM}{dt} \bigg|_{\text{evap}}. \]

(17)
Now, using the greybody parameters given in Table II it is easily seen that \(dM/dt > 0 \iff \epsilon > 10^{10} \text{ GeV}/\text{fm}^3\). The highest earthly value of energy density of partonic matter will be the one created at the LHC, \(\epsilon_{\text{LHC}} < 500 \text{ GeV}/\text{fm}^3\). This means that the BHs that could be produced at the South Pole would evaporate much too quickly to swallow the partons nearby. Contrary to collider experiments, these BHs are produced with large momentum in the lab system, and their decay products are swept forward with large Lorentz factors.

To perform a Lorentz transformation of the evaporating BH from its rest frame \(S\) to the rest frame \(S'\) of the observer (IceCube), we make use of the energy-momentum tensor \(T_{\mu\nu}\) of the outgoing Hawking radiation. In the rest frame of the BH, at a distance large with respect to the Schwarzschild radius of Eq. (6), the energy-momentum tensor of the outgoing particles with energies \(\omega\) in the range \(d\omega\) and with directions lying in a solid angle \(d\Omega\) is

\[ dT_{\mu\nu} = \frac{\rho \mu \nu}{\omega^2} \dot{N}_i \frac{d\Omega}{4\pi} d\omega, \]

(18)
where \(\dot{N}_i\) is defined by Eq. (6) as the emission rate per degree of freedom \(i\) of particles having energies \(\omega\) in the range \(d\omega\) and spin \(s\). As noted earlier, we are working
with $d = 4$. In spherical coordinates with the BH centered at the origin, the 4-momentum is

$$ p^\mu = \omega (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta, 1). \quad (19) $$

Integrating the $(0,0)$ component of Eq. (18) over all directions and energies, we see that $T^{00}$ is the rate at which the BH radiates energy per degree of freedom $i$ of a particle of spin $s$. This requirement dictated the expression that we wrote in Eq. (18) [44]. Explicitly, we have for $dT^{\mu\nu}$ in the BH’s rest frame (with $D = 10$ and $d = 4$):

$$ dT^{\mu\nu} = p^\mu p^\nu \frac{\sigma_s(\omega)}{32\pi^3} \omega \left[ \exp(\omega / T) - (-1)^{2s} \right]^{-1} \times \sin \theta \, d\theta \, d\phi \, d\omega. \quad (20) $$

Let the black hole move at speed $v_{BH}$ in the $z'$ direction relative to the rest frame $S'$ of the observer, and take the axes of $S$ and $S'$ to be parallel. Then one has $\phi' = \phi$,

$$ \omega' = \frac{\omega (1 + v_{BH} \cos \theta)}{(1 - v_{BH}^2)^{1/2}}. \quad (21) $$

and

$$ \cos \theta' = \frac{v_{BH} + \cos \theta}{1 + v_{BH} \cos \theta}. \quad (22) $$

from which it follows that

$$ \sin \theta' \, d\theta' \, d\phi' = (\omega / \omega')^2 \sin \theta \, d\theta \, d\phi. \quad (23) $$

Therefore, in the rest frame of the observer

$$ dT'^{\mu\nu} = p'^\mu p'^\nu \frac{\sigma_s(\omega')}{32\pi^3} \omega' \left[ \exp(\omega' / T') - (-1)^{2s} \right]^{-1} \times \sin \theta' \, d\theta' \, d\phi' \, d\omega', \quad (24) $$

where

$$ T' = \frac{(1 + v_{BH} \cos \theta)}{(1 - v_{BH}^2)^{1/2}} \, T. \quad (25) $$

This can also be written using the inverse Lorentz transformation as

$$ T' = \frac{(1 - v_{BH}^2)^{1/2}}{(1 - v_{BH} \cos \theta') \, T}, \quad (26) $$

where $\theta'$ is the angle between the direction of the black hole and of the emitted particle as measured in the observer’s rest frame. Similarly, we can write

$$ \omega = \frac{\omega' (1 - v_{BH} \cos \theta')}{(1 - v_{BH}^2)^{1/2}}. \quad (27) $$

Putting this into the grey body factor $\sigma_s(\omega)$ gives its dependence on the energy and direction of the emitted particle as measured in the observer’s rest frame. Figure 2 shows how particles tend to be emitted in the direction of motion of a moving black hole.

**FIG. 2**: Distribution of Particles emitted by a BH at the origin and moving in the $z'$ direction at speed 0.95c. In the BH’s rest frame, the particle directions are uniformly distributed and each particle has 1 unit of energy. The length and angle of each line represents the energy and angle of an emitted particle in the observer’s rest frame. Rotate the figure about the $z'$ axis for the 3-dimensional distribution.

**TABLE I**: Degrees of freedom of particle species and greybody parameters as defined in Eqs. (8) and (10).

| particle's spin $i$ | $c_i$ | $\Gamma_i$ | $\Phi_i$ |
|---------------------|------|-----------|---------|
| 0                   | 1    | 0.80      | 0.80    |
| $\frac{3}{4}$       | 90   | 0.66      | 0.62    |
| 1                   | 27   | 0.60      | 0.67    |
Recent HiRes data \[^{51}\]\. A similar enhancement in the neutrino flux is expected from "hidden" sources which are opaque to ultra-high energy cosmic rays \[^{52}\]\. IceCube is sensitive to both downward and upward coming cosmic neutrinos. However, to remain conservative with our statistical sample, here we select only downward going events. To a good approximation, the expected number of BH events at IceCube is given by

\[ N_{\text{BH}} = 2\pi n_T T \int dE_\nu \sigma(E_\nu) \phi_\nu(E_\nu), \] (31)

where \( n_T \) is the number of target nucleons in the effective volume and \( T \) is the running time of the experiment. In our analysis we are interested only in contained events, for which an accurate measurement of the inelasticity can be obtained. IceCube's effective volume for (background rejected) contained events is roughly 1 km\(^3\) \[^{53}\]\. The lower limit on this integral will be set so as to minimize the background from SM events and consequently depends on the infrared cutoff \( x_{\text{min}} \) (more on this below). To give an idea of the overall picture, the total number of BHs expected to be produced within the lifetime of the experiment, for different "beam luminosities" and considering \( E_{\text{min}} = 10^7 \) GeV, are summarized in table 11.

In the spirit of \[^{41}\] we consider the signal of BH events with total multiplicity \( N \geq 4 \) and at least one \( \mu^\pm \) in the final state. To implement the first cut we make use of the average multiplicities \( \langle N \rangle \) for the various particle species (incorporating evolution effects during Hawking radiation) summarized in the previous section. To implement the second cut we define the average multiplicity for any subset of states \( s \) as usual, \( \langle N_{s} \rangle = B_{s} \langle N \rangle \), where

\[ B_{s} = \frac{\sum_{i \in s} c_{i} g_{i} \Gamma_{i}}{\sum_{i} c_{i} g_{i} \Gamma_{i}}, \] (32)

is the so-called "branching fraction". Now, using the parameters given in Table 11, we find \( \langle N \rangle = 0.30 M/T \) and \( \langle N_{\mu^\pm} \rangle = 0.022 \langle N \rangle = 0.007 M/T \).

\( \langle N \rangle \) is the average value of a Poisson distribution. If all species are Poisson distributed, then the sum of particles.
in any subset is also Poisson distributed, and so $N, N_{\mu^\pm}$, and $N - N_{\mu^\pm}$ are all Poisson distributed, where $N_{\mu^\pm}$ is the total number of $\mu^\pm$ per event. The signal probability (i.e., that a given event has $N_{\mu^\pm} \geq 1$ and $N \geq 4$) is [19]

$$P_{\text{sig}} = e^{-\langle N_{\mu^\pm} \rangle} \left(1 - e^{-\langle N - N_{\mu^\pm} \rangle} \sum_{i=0}^{3} \frac{\langle N - N_{\mu^\pm} \rangle^i}{i!} \right) + \left(1 - e^{-\langle N \rangle} \sum_{i=0}^{3} \frac{\langle N \rangle^i}{i!} \right),$$

(33)

and so the number of signal events becomes

$$N_{\text{sig}} = 2\pi n_T T \int dE_{\nu} \, \sigma(E_{\nu}) \phi_{\nu}(E_{\nu}) \, P_{\text{sig}}.$$

(34)

The quarks and gluons emitted by the BH promptly fragment into hadrons (mostly $\pi^\pm$ and $\pi^0$). For $E_\pi > 1$ TeV, the interaction mean free path of $\pi^\pm$ in ice is orders of magnitude smaller than the pion decay length, and so nearly all the hadronic energy is channeled into electromagnetic modes through $\pi^0$ decay. The signal of such a hadronic/electromagnetic cascade is a bright, point like, source of Čerenkov light. The shower topology can be easily identified by the sphericity of the light pattern. The measurement of the radius of the lightpool mapped by the lattice of photomultiplier tubes determines the energy and turns IceCube into a total absorption calorimeter [55]. On the other hand, the muons emitted by the BH would produce a track moving outwards from the interaction vertex, providing very useful tags for the event.

As discussed in the previous section the SM background masking BH events are in the tail of the CC $y$ distribution: for $\langle N \rangle > 4$, the average energy of the emitted muon (after considering energy losses due to classical radiation) is less than 20% of the incoming neutrino energy. Therefore, to filter the background we evaluate Eq. (28) for $y > 0.8$, yielding

$$\sigma_{CC}^{y = 0.8} \simeq 1.2 \, (E_{\nu}/\text{GeV})^{0.458} \, \text{pb}.$$  

(35)

Now, substituting the CC SM cross section (with $y > 0.8$) into Eq. (31) leads to a straightforward calculation that shows that for $E_{\nu_{\text{min}}} = 10^8$ GeV the expected number of background events is negligible, less than 1 event in 15 years, independently of the selected (WB/AARGHW) beam luminosity [56]. The sensitivity of IceCube to probe the $x_{\text{min}}/M_{10}$ parameter space at the 3$\sigma$ level [57] is summarized in Table III.

The expected number of background events rises with decreasing the low-energy cutoff. For example, for $E_{\nu_{\text{min}}} = 10^7$ GeV and a beam luminosity at the AARGHW level, $N_{\text{bk}}/d = 10$. To remain conservative, we adopt this energy range and require a 5$\sigma$ excess for discovery. The resulting reach is shown in Fig. 3.

The BH entropy is a measure of the validity of the semiclassical approximation. For $x_{\text{min}} > 3, S_0 \gg 10$ yields small thermal fluctuations in the emission process [58]. Therefore, strong quantum gravity effects may be safely neglected in this “energy regime,” $M_{B0}/M_{10} > 3$. Moreover, gravitational effects due to brane back-reaction are expected to be insignificant for $M_{B0}$ well beyond the brane tension, which is presumably on the order of $M_{10}$ [19]. As noted in the previous section, string theory provides a more complete picture for $M_{B0}$ close to $M_{10}$. In string theory, the ultraviolet fate of the BH is determined by the string BH correspondence principle: when the Schwarzschild radius of the BH shrinks to the fundamental string length $l_s \approx 10^{-19}$ m an adiabatic transition occurs to a massive superstring mode [32]. Subsequent energy loss continues as thermal radiation at the unchanging Hagedorn temperature [59]. The continuity of the cross section at the correspondence point, parametrically in both the energy and the string coupling, provides independent support for this picture [20]. In the perturbative string regime, however, the parton-parton cross section

| $x_{\text{min}}$ | $M_{10}/\text{TeV}$ [WB] | $M_{10}/\text{TeV}$ [AARGHW] |
|-----------------|-------------------------|-------------------------|
| 3               | 1.5 (1.2)               | 1.5 (1.2)               |
| 5               | 1.3 (1.1)               | 1.3 (1.1)               |
| 7               | 1.2 (1.0)               | 1.2 (1.0)               |
| 9               | 1.1 (1.0)               | 1.1 (0.9)               |


FIG. 3: IceCube discovery reach of quantum BHs, assuming the new estimates of $F$ and $y$ given in [13]. For comparison the LHC discovery reach, assuming a cumulative integrated luminosity of 1 ab$^{-1}$ over the life of the collider, is also shown. The LHC discovery reach has been obtained by scaling up the results given in [19], to account for the new values of $y$ and $F$. 

TABLE III: Sensitivity of IceCube at the 3$\sigma$ level for the value of $M_{10}/\text{TeV}$ using fiducial beam luminosities. We have taken an integration time of 15 yr corresponding to the lifetime of the experiment and used the new (old) values of $F$ and $y$. 

| $x_{\text{min}}$ | $M_{10}/\text{TeV}$ [WB] | $M_{10}/\text{TeV}$ [AARGHW] |
|-----------------|-------------------------|-------------------------|
| 3               | 1.5 (1.2)               | 1.5 (1.2)               |
| 5               | 1.3 (1.1)               | 1.3 (1.1)               |
| 7               | 1.2 (1.0)               | 1.2 (1.0)               |
| 9               | 1.1 (1.0)               | 1.1 (0.9)               |
contains the Chan Paton factors which control the projection of the initial state onto the string spectrum. In general, this projection is not uniquely determined by the low-lying particle spectrum, yielding one or more arbitrary constants \([60]\). Interestingly, the parton-parton cross section derived in \([20]\) from the Virasoro-Shapiro amplitude leads to an enhancement of the predicted BH cross section for \(1 < x_{\text{min}} < 3\) \([61]\). This makes it plausible to adopt the BH cross section as a lower bound of the real Planckian cross section. However, it is important to stress that the proposed signal to search for BHs at the low-lying particle spectrum, yielding one or more arbitrary constants \([60]\). Interestingly, the parton-parton cross section derived in \([20]\) from the Virasoro-Shapiro amplitude leads to an enhancement of the predicted BH cross section for \(1 < x_{\text{min}} < 3\) \([61]\). This makes it plausible to adopt the BH cross section as a lower bound of the real Planckian cross section. However, it is important to stress that the proposed signal to search for BHs at IceCube strongly depends on the large density of quantum mechanical states of BHs with \(S_0 \gg 10\), and consequently it is only valid in the semi-classical regime. The LHC signal (hard photons and leptons in the collision rest frame), however, can be maintained under plausible hypotheses on the superstring decay modes. With this in mind, the LHC discovery reach for the quantum regime shown in Fig. 3 may be taken as a lower bound, derived on the assumption that the BH cross section provides a lower limit on the string cross section.

**IV. CONCLUSIONS**

In this work we have reviewed the possibility of searching for BHs using cosmic neutrino interactions in the Antarctic ice. We have shown that the ability of IceCube to accurately measure the inelasticity distribution of events provides a unique discriminator for SM background rejection \([62]\), allowing extremely sensitive probes of TeV-scale BH production. In the optimistic case that the neutrino flux is at the WB level, IceCube has a substantial discovery potential for BHs, well in the semi-classical regime where \(M_{\text{BH}}^{\text{min}} > 3M_{10}\). The statistically significant 5σ excess extends up to \(M_{10} \sim 1.3\) TeV.

A point worth noting at this juncture: In assessing the discovery potential of IceCube we have performed a Lorentz transformation of the evaporating BH from its rest frame to the lab frame, so as to obtain the boosted energy spectrum of the outgoing Hawking radiation. Such a spectrum is also relevant for the study of near-extremal supersymmetric BHs, which could be copiously produced at the LHC. These BHs can be associated with the massive superstring modes expected to populate the quantum regime \([63]\). Note that in order to maintain the configuration of the initial state with “zero supersymmetry,” the central charge conservation would force these BHs to be pair produced with non-zero transverse momentum \((p_T)\), travelling along the beam pipe. Therefore, the prompt decay of the BHs would produce a startlingly clean signal that should have very few backgrounds: their decay products (that of course may include sparticles \([64]\), would trigger high multiplicity fireworks with boosted spherical shape collimated into back-to-back pencil beams. A crude estimate of the event rates can be obtained from the analysis of “\(ij \to \text{BH} + \text{others}\)” subprocesses \([65]\). The results are encouraging: the production rate of BHs with large \(p_T > 500\) GeV (which is balanced by the momentum of an energetic parton) would still be large enough for detection at LHC (assuming an integrated luminosity 100 fb\(^{-1}\)). Of course the energy requirement for BH pair production would yield an additional suppression factor and its proper consideration requires a Monte Carlo simulation. Therefore, we strongly recommend to include in future versions of the BH event generators CHARYBDIS \([66]\) and CATFISH \([67]\) a detailed treatment of production and evaporation of near-extremal supersymmetric BHs.

In closing, it is important to stress that this analysis is meant to investigate the underlying principles and does not account for the details of the detector response. Though the devil is generally in the details, we believe our estimate of the IceCube discovery reach is conservative, as we have not included the \(\tau\)-channel which has the potential of nearly doubling our signal event background. Specifically, \(\tau\) leptons emitted by the BH may decay in flight inside the instrumented volume after escaping the electromagnetic shower bubble, thereby triggering a “second bang \([68]\)” : The ratio of the first to second bang fractional energy provides a clean direct signal of BH production, and like the muon channel is “independent” of the absolute neutrino flux. The inclusion of the \(\tau\) channel, however, requires a full blown Monte Carlo simulation to properly determine the acceptance for such events, in which the triggering probability for the second bang depends on \(T_{\text{BH}}\) and the corresponding \(\tau\) decay length.

In summary, over the next few years high-statistics high-energy precision data will be collected at the LHC. In addition, the IceCube neutrino telescope is coming on line with complementary information at ultra-high energies. This new arsenal of data will certainly provide an ideal testing ground for TeV-scale BH production, and, at the same time, a unique opportunity to view similar physics from two different points of view. Should the LHC find evidence of BHs a bit outside the range accessible for the baseline IceCube design, the ideas discussed in this paper could constitute another compelling reason for pursuing HyperCube \([69]\).

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For ice, the Cerenkov light generated by the shower particles spreads over a volume of radius 130 m at 10^4 GeV and 460 m at 10^9 GeV (i.e., the bubble radius grows by about 50 m per decade of energy). Therefore, a contained direct hit by a neutrino with energy \( \sim 10^9 \) GeV will not saturate the km^3 detector volume. For details the reader is referred to Ref. [54].

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