Non-Pertubative Quantum Corrections to a Born-Infeld Black Hole and its Information Geometry

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ABSTRACT: We study the non-perturbative quantum corrections to a Born-Infeld black hole in a spherical cavity. These quantum corrections produce a non-trivial short distances modification to the relation between the entropy and area of this black hole. This in turn modifies the thermodynamics of this system. We also investigate the effect of such non-perturbative corrections on the information geometry of this system. This is done using various different information metrics.

KEYWORDS: Quantum correction; Thermodynamics; Black hole
1 Introduction

It is known that black holes behave as hot objects, and have an entropy equal to the quarter of its area. In fact, this entropy of a black hole is the maximum entropy which can be associated with any object of the same volume \([1, 2]\). Thus, the maximum entropy that a region of space can contain scales with its area rather than its volume, and this observation has lead to the development of the holographic principle \([3, 4]\). However, it has also been argued that the holographic principle could get modified at Planck scale, due to the modification to the structure of spacetime by quantum fluctuations \([5, 6]\). Thus, we expect that the relation between the entropy and area would also get modified by quantum corrections to the structure of spacetime.

Such quantum corrections to the entropy of a black hole have been studied using various different approaches, and it has been demonstrated that the quantum corrections even modify other aspects of black holes at short distances. In fact, AdS/CFT has been used to analyze quantum corrected to the entropy of various AdS black holes \([7-11]\). Such quantum correction to the entropy have also been calculated using the extremal limit of a black hole \([12, 13]\). Non-perturbative quantum correction to the entropy of a black hole has also been obtained using the density of microstates associated with conformal blocks \([14]\). Such quantum corrections have also been obtained from the Cardy formula for black holes \([15]\). It is also possible to use Rademacher expansion to obtain corrections to the entropy of a black hole \([16]\). Thus, it is well established that the entropy of a black hole would get corrected at short distances due to the quantum fluctuations.

It has been argued in Jacobson formalism that the geometry of spacetime emerges from thermodynamics, as an emergent theory \([17]\). So, using the Jacobson formalism, it can be demonstrated that the thermal fluctuations in the thermodynamics of black holes produce quantum fluctuations in the geometry of spacetime \([18]\). The thermal fluctuations to the thermodynamics of various black holes has been thoroughly studied \([19-23]\). In fact, as in the Jacobson formalism, these thermal fluctuations are related to quantum fluctuations \([17, 18]\), quantum corrections to the entropy of various black holes have also been studied \([24-27]\).

As these corrections occur due to quantum fluctuations, they can be neglected for large black object. This is because large black holes have small temperature, and so the thermal fluctuations can be neglected for large black holes. As in the Jacobson formalism \([17, 18]\), these thermal fluctuations produce the quantum fluctuations in the geometry of spacetime, we can also neglect quantum fluctuations in the geometry of spacetime for large black holes. However, as these black holes
reduce in size due to Hawking radiation, we cannot neglect the effects of quantum fluctuations on them. Thus, we need to consider the leading order quantum corrections to the entropy of such black holes to analyze the short distance corrections to theory thermostodynamics [24–27]. However, as the black holes reduce further in size, we need to consider the full non-pertubative corrections (not just leading order) to the entropy of a black hole. It has been recently proposed that such corrections are represented by an exponential function of the original entropy, and they become important for black holes at short distances [28]. It has also been demonstrated that such non-pertubative corrections to the entropy of a black hole can be obtained using Kloosterman sums [29].

As these corrections are important corrections to the entropy of a black hole [30], it is important to analyze its effect on non-trivial generalization of a black hole solution, such as a Born-Infeld black hole. So, in this paper, we study the effects of such non-pertubative corrections on the thermodynamics of a Born-Infeld black hole in a cavity [31, 32]. This is because the Born-Infeld black hole is an interesting black object obtained from a non-linear source. As this is an exotic solution, it would be interesting to analyze the modifications to its thermostodynamics from such non-pertubative corrections. We will also analyze the non-pertubative corrections to the information theoretical geometry of this system. This will be done using different informational theoretical metrics for this system [33–39]. It is observed that these non-pertubative corrections produce a non-trivial modification to the information theoretical geometry of this system.

2 Born-Infeld Black Hole in a Cavity

Let us first briefly review a Born-Infeld black hole in a cavity [31, 32]. A Born-Infeld black hole inside a four-dimensional spacetime manifold \( \mathcal{M} \), with a time-like boundary \( \partial \mathcal{M} \) at \( r = r_B \) can be constructed by coupling gravity to a Born-Infeld electromagnetic field \( A_\mu \) with the field strength tensor \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). Thus, we can write the action for this system as [40]

\[
S = \int_\mathcal{M} d^4x \sqrt{-g} \left[ R + \frac{1}{2} \left( 1 - \sqrt{1 + \frac{a}{2} F_{\mu\nu} F^{\mu\nu}} \right) \right] \\
- 2 \int_{\partial \mathcal{M}} d^3x \sqrt{-\gamma} \left[ (K - K_0) + \frac{A_\mu n_\nu F^{\mu\nu}}{\sqrt{1 + \frac{a}{2} F_{\mu\nu} F^{\mu\nu}}} \right],
\]

(2.1)

in unit of \( 16\pi G \), with the last term being the surface term. This solution can be obtained from string theory, and in that case, the coupling parameter \( a \) is related to the string slope parameter \( \alpha' \) as \( a = (2\pi\alpha')^2 \). Here, \( K \) and \( K_0 \) represent the extrinsic curvature and a subtraction term, respectively. The subtraction term \( K_0 \) is placed in the surface term to make sure that the Gibbons-Hawking-York term vanish. Furthermore, \( \gamma \) is the metric on the boundary and \( n_\nu \) is the unit normal of the surface vector pointing in the outward direction.

The static spherically symmetric black hole (of mass \( M \) and charge \( Q \)) solution arising from the action (2.1) is given by [31, 32]

\[
ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2d\Omega_2^2,
\]

(2.2)

with

\[
f(r) = 1 - \frac{M}{8\pi r} - \frac{Q^2}{6\sqrt{r^4 + aQ^2 + 6r^2}} + \frac{Q^2}{3r^2} \left[ 2F_1\left( \frac{1}{4}, \frac{1}{2}, \frac{5}{4} \middle| -\frac{aQ^2}{r^4} \right) \right],
\]

(2.3)

where \( d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2 \), and \( 2F_1 \) is generalized hypergeometric function. In the limit \( Q \to 0 \), the above solution reproduces the Schwarzschild black hole, which is thermodynamically unstable. In Fig. 1, we demonstrate the horizon structure of the charged back hole (2.2). The black dotted line in this figure shows a naked singularity for a specific choice of \( M \) (\( M = 7 \)). A relatively larger
value of $M$ yields the extremal case (see, the solid blue line), where both the horizons coincide. By increasing $M$ further one can obtain both the inner ($r_-$) and outer ($r_+$) horizons, as shown by the red dashed line. For example, by choosing $Q = 0.4$, $a = 0.003$ and $M = 13$, we have $r_+ \approx 0.4$. Furthermore, larger values of $M$ (dash-dotted green line) yields only one horizon.

![Figure 1](image-url)

**Figure 1.** Horizon structure of Born-Infeld black hole in a cavity for $a = \frac{1}{300}$. The mass of the Born-Infeld black hole (2.2) for $f(r) = 0$ is given by

$$M = 8\pi r_+ \left[ 1 - \frac{Q^2}{6\sqrt{r_+^4 + aQ^2 + 6r_+^2}} + \frac{Q^2}{3r_+^2} 2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{aQ^2}{r_+^2}\right) \right].$$

The variation of the mass (2.4) with the horizon radius $r_+$ is plotted in Fig. 2 for different values of the charge $Q$. Eq. (2.4) indicates that the mass $M$ varies linearly with $r_+$ in the limit $Q \to 0$. In the case of a finite electric charge, the mass of the black hole decreases with the decrease of $r_+$ and reaches a minima at a critical value ($r_c$). For example, for $Q = 0.4$, $r_c \approx 0.2$ (see the minimum of red solid line of Fig. 2). By decreasing the horizon radius $r_+$ further, the mass of the black hole increases, which is in agreement with its behavior described in [31]. It is interesting to note that the black hole mass does not become zero when $r_+$ vanishes. There are some specified points with circle and square which will explain later (end of this section). It is possible for the horizon radius $r_+$ to vanish, when the mass is non-zero. In this paper, we shall analyze this case by studying the behavior of this black hole at small horizon radius. It has been argued that at such scales the black hole entropy should get corrected by an exponential term [28]. In the next section, we show the origin of such a correction and discuss about the modification of the thermodynamics due to the exponentially corrected entropy.

### 3 Quantum Correction

It is expected that the quantum corrections would correct the relation between area and entropy of a black hole [24–27]. In fact, such corrections have been studied mostly perturbatively [41–43]. However, it is important to analyze the full non-perturbative quantum corrections to this black hole. Now as the quantum corrections to geometry of spacetime can be obtained from thermal fluctuations in the Jacobson formalism [17, 18], we would like to analyze the such non-perturbative corrections to the area entropy relation of Born-Infeld black hole

$$S_0 = 16\pi^2 r_+^2.$$  

(3.1)
It may be noted that perturbative quantum correction in the form of logarithmic correction \([44, 45]\), and even higher order perturbative corrections \([46, 47]\) have been studied using thermal fluctuations. Here will analyze non-perturbative exponential corrections to the entropy of this system using thermal fluctuations. Now in statistical mechanics, the entropy of a given system is related to the number of measurable microstates

\[ S_0 = \ln \Omega, \tag{3.2} \]

where we have used the units of the Boltzmann constant \((k_B = 1)\). Now if the microstates of this black hole can be represented by a conformal field theory, then it is known that the conformal field theory corrects represents the entropy and its thermal fluctuations for such a black hole \([15, 16]\). Now, these perturbative thermal fluctuations seems to have a universal form, and this universal form would imply that the corrections to the entropy would also have a universal form \([19–23]\)

\[ S_{per} = \ln \Omega + f(\Omega) = S_0 + \frac{1}{2} \ln(T^2 S_0) + \ldots \tag{3.3} \]

It has also been argued that the non-perturbative corrections to a black hole will be expressed by an exponential function of the original entropy \([28, 29]\). Now these corrections should also be produced by suitable function of \(\Omega\), and so they can be written as

\[ S_m = \frac{1}{\Omega} = e^{-S_0}. \tag{3.4} \]

As the perturbative corrections are universal, we propose that this non-perturbative correction is also universal (as both are obtained as functions of \(\Omega\)). Hence, we can write the total non-perturbative quantum corrected entropy of Born-Infeld black hole as

\[ S = S_0 + S_m = 16\pi^2 r^2 + e^{-16\pi^2 r^2}. \tag{3.5} \]

The exponential term is negligible for large black hole, however, it becomes relevant when the area of the black hole is small. It is illustrated by Fig. 3. It may be noted that this correction can produce important quantum effect for black holes at short distances.
The Hawking temperature of black holes is obtained by using the relations (2.3) and (2.4) as follows:

\[ T_H = \frac{1}{4\pi} \left( \frac{df(r)}{dr} \right)_{r=r_+} = \frac{aQ^4}{30\pi r_+^4} \, _2F_1\left(\frac{5}{4}, \frac{3}{2}; \frac{9}{4}; \frac{aQ^2}{r_+^2}\right) - \frac{Q^2}{6\pi r_+^2} \, _2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \frac{aQ^2}{r_+^2}\right) \]

\[-\left(\sqrt{r_+^4 + aQ^2} - 2r_+^2 - 6a)Q^2 - 6r_+^4 (r_+^4 + aQ^2)\right)\]

\[24\pi r_+ (r_+^4 + aQ^2)\sqrt{r_+^4 + aQ^2} - Q^2 6\pi r_+^3 + 2\left(r_+^4 + aQ^2\right)\sqrt{r_+^4 + aQ^2}. \] (3.6)

In Fig. 4, we show the behavior of Hawking temperature with the horizon radius. Now when \( Q \neq 0 \), we can see there is a minimum value of the horizon radius \( (r_m) \), where the Hawking temperature vanishes. Previously, in Fig. 2, we showed the corresponding mass at this point, and they were depicted by a small square in the red solid line, and a circle in the green dashed line. Therefore, it is obvious that \( r_m > r_c \), which means, before the black hole reaches to the minimum mass, the Hawking temperature vanishes. The specific heat of black holes also is zero at this point, hence, the Hawking radiation will stop at this point, and the black hole will not evaporate any further. So, it may create a black remnant of finite mass in zero temperature and zero specific heat, which may solve the information loss paradox of black holes [48, 49]. However, a more important point which was discussed earlier is that the black hole entropy should get corrected by an exponential term at this range of horizon radius, modifying thermodynamics of this system.

4 Corrected Thermodynamics

Now we can analyze the effects of these corrections on the thermodynamics of this system. Using the expressions of entropy in (3.5) and temperature in (3.6), one can study the specific heat using the following relation

\[ C = T_H \frac{dS}{dT_H}. \] (4.1)

In Fig. 5, we show the behavior of specific heat of Born-Infeld black hole in a cavity for different black hole charges. The special case of \( Q = 0 \) is completely unstable like a Schwarzschild black hole. However, for a charged black hole, we notice a first order phase transition (unstable to stable). With the decrease of horizon radius, the black hole goes to unstable phase. Indeed, for \( r_+ = r_m \),
the specific heat become zero. So, a black remnant forms before the black hole enters the unstable phase.

It is interesting to calculate the Helmholtz free energy at the point $r_+ = r_m$. To the first order approximation ($e^{-16\pi^2 r_+^2} \approx 1 - 16\pi^2 r_+^2$), it becomes

$$F = -\int SdT_H \approx \frac{Q^2}{30\pi r_+^4} \left[ 5r_+^4 \sum \left( \frac{1}{1}, \frac{5}{2}, \frac{1}{4}, \frac{1}{r_+^4} \right) - aQ^2 \sum \left( \frac{3}{4}, \frac{9}{4}, \frac{1}{r_+^4} \right) \right]$$

$$+ \frac{3r_+^4 + aQ^2 - 3(r_+^2 + 2a) \sqrt{r_+^4 + aQ^2}}{24\pi r_+ \sqrt{r_+^4 + aQ^2}}. \quad (4.2)$$
In Fig. 6, we plot the behavior of Helmholtz free energy $F$ of Born-Infeld black hole in a cavity, with the variation of horizon radius $r_+$. We denote the uncorrected Helmholtz free energy by $F_0$, where the exponential term in (3.5) is neglected. In both of the panels of Fig. 6, we can see that the overall effect of the non-perturbative quantum correction is to reduce the Helmholtz free energy. The left panel ($Q = 0$) is more interesting, where we notice that the Helmholtz free energy is positive and grows linearly with the increasing horizon radius, if we neglect the quantum correction. However, in the presence of quantum corrections, it is completely negative approaching zero at larger $r_+$. Also, for the charged case (right panel), the Helmholtz free energy is positive which becomes zero at large $r_+$. This means that the black remnant has a positive energy.

![Figure 6](image)

Figure 6. Helmholtz free energy of Born-Infeld black hole in a cavity for $a = \frac{1}{300}$.

Helmholtz free energy is related to the partition function via (in the unit of Boltzmann constant),

$$ F = -T \ln Z, $$

where $T$ is the reciprocal temperature on the boundary of the cavity, and it is given by [40]

$$ T = \frac{T_H}{\sqrt{f(r_B)}}, $$

where

$$ f(r_B) \approx 1 - \frac{M}{8\pi r_B} + \frac{Q^2}{4r_B^2}. $$

Along with (4.2), (4.4) and (4.5), we obtain the corrected partition function from (4.3), which can be used to obtain all the thermodynamics quantities. Behavior of the corrected partition function (4.3) is illustrated in Fig. 7.

The corrected internal energy $U$ is obtained from the partition function as

$$ U = T^2 \frac{d \ln Z}{dT}. $$

Now we analyze the behavior of $U$ in Fig. 8. Just like the Helmholtz free energy, the internal energy $U$ is also zero at $r_+ = r_m$. Here, the asymptotic region shows the first order phase transition.

In the given scenario, the first law of thermodynamics is expressed as

$$ Y = dU - TdS + \Phi dQ - \lambda dA, $$
Figure 7. Partition function of Born-Infeld black hole in a cavity for $a = \frac{1}{300}$.

Figure 8. Internal energy of Born-Infeld black hole in a cavity for $a = \frac{1}{300}$ and $Q = 0.4$.

where $A = 4\pi r_B^2$ is the surface area of the cavity, and the corresponding chemical potential is given by

$$\Phi = \frac{4\pi A_t(r_B)}{\sqrt{f(r_B)}}$$

(4.8)

where the gauge field is given by

$$A_t(r) = \int \frac{Qdr}{\sqrt{r^2 + aQ^2}}$$

(4.9)

The first law of thermodynamics is satisfied if $Y = 0$. In Fig. 9, we can see that such a case comes out when $r_+ = r_m$.

5 Information geometry

Information geometry can be used to study stability of black holes. It can also be used to explore the points of type-one and type-two phase transitions. In this method, the phase transition points are identified through divergent points of the thermodynamic Ricci scalar. Thus, by starting from
a information metric one has to calculate the relevant Ricci scalar. The points at which the Ricci scalar diverges represent the thermodynamic phase transitions. Now we will analyze the effects of these non-perturbative quantum corrections on the information metrics for this system. The two proposed Quevedo metrics (\(QI\) and \(QII\)) are given by [33, 34]

\[
\begin{align*}
\text{for } QI: & \quad ds^2 = (SM_S + QM_Q) \left( -M_{S}dS^2 + M_{QQ}dQ^2 \right), \\
\text{for } QII: & \quad ds^2 = SM_S \left( -M_{S}SdS^2 + M_{QQ}dQ^2 \right),
\end{align*}
\]

The information metrics of Weinhold (W) and Ruppeiner (R) can be expressed as [35–38]

\[
\begin{align*}
\text{for } W: & \quad ds^2 = M g_{ab}^{(W)} dx^a dx^b, \quad \text{with } g_{ab}^{(W)} = \frac{\partial^2 M}{\partial X^a \partial X^b}, \\
\text{for } R: & \quad ds^2 = -\frac{M}{M_S} g_{ab}^{(W)} dx^a dx^b,
\end{align*}
\]

Recently, HPEM (Hendi, Panahiyan, Eslam Panah, and Momennia) metric has been proposed as a new metric [39]. The results of this metric are consistent with those of obtained using the canonical ensemble [50–52]. The information metric of HPEM can be written in the following form

\[
\begin{align*}
\text{for } HPEM: & \quad ds^2 = \frac{SM_S}{M_{QQ}} \left( -M_{S}SdS^2 + M_{QQ}dQ^2 \right),
\end{align*}
\]
corrected thermodynamics, we can see an asymptotic behavior.

On the other hand, for the Quevedo metric of the second kind we do not observe any important difference between corrected and uncorrected thermodynamics (see Fig. 11). It means that the exponential correction is negligible in this case. The first order phase transition also exist in this metric.

Ricci scalar of Weinhold metric represented by Fig. 12. Now for this metric we do not observe any phase transition. Similar behavior is observed for Ruppeiner metric. Now this is illustrated by Fig. 13.

Finally, we plot the Ricci scalar of HPEM metric in Fig. 14. The first and second order phase transition happen for both corrected and uncorrected thermodynamics. We observe separate curve near critical points for these cases.

6 Conclusions

We have analyzed non-perturbative quantum corrections to a Born-Infeld black hole in a spherical cavity. In the Jacobson formalism, the quantum corrections are related to the thermal fluctuations
Figure 12. Ricci scalar of Weinhold metric ($R_W$) versus $S$ for the corrected (red) and uncorrected (blue) entropy. $a = \frac{1}{300}$, $Q = 0.5$.

Figure 13. Ricci scalar of Ruppeiner metric ($R_R$) versus $S$ for the corrected (red) and uncorrected (blue) entropy. $a = \frac{1}{300}$, $Q = 0.5$.

Figure 14. Ricci scalar of HPEM metric ($R_{HPEM}$) versus $S$ for the corrected (red) and uncorrected (blue) entropy. $a = \frac{1}{300}$, $Q = 0.5$.

of a black hole, and so we have used thermal fluctuations to obtain the non-perturbative quantum corrections to this system. These corrections can be neglected at large distances, but they produce interesting modifications to the thermodynamics of this system at short distances. We have explicitly obtained the correction to the entropy of this system, and observed how such corrections to the entropy of this system can produce non-trivial modification to the thermodynamic behavior of this system. We have also analyzed the effects of these correction on various different information
metrics. Thus, these thermodynamic metrics have been used to obtain non-perturbative corrections produce to the information geometry of this system. We would like to point out that other interesting non-perturbative corrections to the thermodynamics of black holes have recently been studied. Thus, the corrections to the black hole thermodynamics from finiteness of string theory has been studied using the T-duality of center of mass Green’s function for bosonic strings [53, 54]. This has been generalized to higher dimensions, and it has been observed that such a modification the thermodynamics of black holes can explain the absence of mini black holes at the LHC [55]. In fact, similar arguments have also been made with the modification of black hole thermodynamics from gravity’s rainbow [56] and minimal length [57]. It would be interesting to analyze such effects using the modification to the black hole thermodynamics studied here. This is expected to produce interesting consequences for detection of mini black holes at the LHC.

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