Entropy Analysis of Magnetized Carbon Nanofluid over Axially Rotating Stretching Disk

Hossam A. Nabwey 1,2,*, Uzma Sultana 3, Muhammad Mushtaq 3, Muhammad Ashraf 4, Ahmed M. Rashad 5, Sumayyah I. Alshber 1 and Miad Abu Hawsah 1

1 Department of Mathematics, College of Science and Humanities in Al-Kharj, Prince Sattam bin Abdulaziz University, Al-Kharj 11942, Saudi Arabia
2 Department of Basic Engineering Science, Faculty of Engineering, Menoufia University, Shebin El-Kom 32511, Egypt
3 Department of Mathematics, COMSATS University Islamabad, Islamabad 44000, Pakistan
4 Department of Mathematics, University of Sargodha, Sargodha 40100, Pakistan
5 Department of Mathematics, Faculty of Science, Aswan University, Aswan 81528, Egypt
* Correspondence: eng_hossam21@yahoo.com or h.mohamed@psau.edu.sa

Abstract: Nanofluids receive recognition from researchers and scientists because of their high thermal transfer rates. They have impactful industrial and technological modules in daily activities. In recent times, the heat transfer rate has been strengthened even more by a certain type of nanofluid known as "carbon nanotubes". The water-based magnetohydrodynamic flow with the nanoparticles MWCNT and SWCNT over an axially rotating stretching disk is highlighted in this article. In addition, the perspectives of viscous dissipation and MHD were taken into consideration. In order to formulate the physical problem, Xue’s model is considered with the thermophysical properties and characteristics of carbon nanofluid. The current modeled system of partial differential equations is transformed into an ordinary differential equation system by the suggesting of the best similarity technique. Later, the transformed system of ordinary differential equations is solved numerically by using the Keller box method and the shooting method. Figures and charts are used to study and elaborate the physical behavior of the key subjective flow field parameters. The saturation in the base fluid is considered in both kinds of carbon nanotubes, the single-wall (SWCNTs) and the multiwall (MWCNTs). It is noted that the heat transfer mechanism shows some delaying behavior due to the increase in the Eckert number and the volume fraction elevation values. For the larger volume fraction values and the magnetic parameter, the skin friction increases. In addition, while the temperature profile increases with the Biot numbers, it falls for the increasing values of the Prandtl number. Furthermore, it is noted that the irreversibility of the thermal energy is influenced by the Biot number, temperature difference, Brinkmann number, and magnetic field, which all have dynamic effects on the entropy and the Bejan number.

Keywords: carbon nanofluid; magnetohydrodynamics; Joule heating; entropy; rotating stretching disk; carbon nanotubes

1. Introduction

The nanofluid study is currently a leading scientific area because of its broad range of activities in oils, water, solar power, and mechatronics. Chemotherapy with nanoparticles is used to kill infected censorial cells. Nano-liquids offer enhanced thermophysical properties, such as thermal diffusivity and conductivity, and are crucial in many industrial applications, such as shipbuilding, nuclear power stations, thermosyphons, pulsing heat pipes, and biotechnology. Nanomaterials are innovative because they are potentially helpful in various mass transportation systems, heat transportation applications such as cooling machines, pharmaceuticals, nuclear reactors, electronics, solar collectors, fuel, and residential cooling processes. Although metal has a higher thermal conductivity rate in solids than in water...
and oil, its thermal conductivity can be improved by dispersing metallic nanoparticles in base fluids. This notion was also postulated by Choi [1] as a mix of basic fluids and colloidal nanoparticles and is called nanofluids. The physical properties of nano-liquids change dramatically as the temperature and viscosity are increased. Nanofluids have been developed to produce thermal fluids. Ayodeji et al. [2] studied the flow of nanofluid MHD across a stretching surface with dissipation effects and slip. Khan et al. [3] investigated the magnetic properties of Newtonian fluids as a result of the transition of paraboloid and chemically bonded bioconvection species. Khan took into consideration the phenomena of fluid flow with cylinder-shaped CNTs on a flat plate with heat transfer and the Navier slip [4]. In order to examine the flow of nanotubes in the rotating medium, Seth [5] chose porous media. The results were derived by the flow phenomena for heat dissipation and Joule heating. Such flows had a magnificent effect on industrial and mechanical processes. Hayat et al. [6] evaluated the fluid flow of the CNTs to a stagnation point on a nonlinear stretching sheet with uniform thickness. This caused both homogeneous and heterogeneous reactions in the melting of the heat and chemical reactions of an autocatalyst. It was ultimately concluded that multiwall nanotubes of carbon were accelerating the flow at higher speeds than the single-wall nanotubes. In an investigation of a spinning stretching channel exhibiting radiative heat flux, the flow of CNTs was also investigated by Ghadikolaei et al. [7]. Many researchers [8–13] have demonstrated numerically the magnifying of the heat conductivity in different aspects after Eastman [14] and Choi [15].

The fluid flow as a result of a rotating disk has attracted a lot of attention; the reason for this is that it covers a wide range of geophysical applications, including Earth rotation, fluid movement in the Earth’s mantle near the crust, surface rotation, rotors, and reactors. Von Karman [16] was the first to explain the impact of fluid flow on a rotating disk. Following Karman’s influential work, eminent researchers have addressed these transformations in a variety of physical situations. The flow is required in the expulsion of polymers, metals, paper, and fiber glass, among other things, due to the stretched surface. Heat transfer and the stretching rate have a huge impact on the output product quality. Heat transfer causes heated metals to cool to a specific temperature. It is ensured that the metals do not approach the melting point in a heat process. These processes are carried out in order to make metals highly resistant. The three-dimensional flow caused by a rotating, stretching surface was explored [17], and Wang introduced a parameter \( \lambda \) that denoted the comparison of rotation and stretching. Fang [18] mainly examined the steady flow on a rotating stretching disk. On a rotating disk that stretches radially at the same time, three-dimensional steady flow was also investigated. Weidman [19] envisioned a flow of axisymmetric stagnation points affecting a rotating disk while radially stretching. Weidman investigated two types of stagnation flows: the Homann stagnation flow and the rotating Agarwal stagnation flow. Entropy is the amount of inaccessible energy in a closed thermodynamic system. Quality and energy are very important parameters for the design and development of engineering products. The second law of thermodynamics gives us a way to determine the quality and extent of energy degradation. Irreversibility or entropy is the important approach to determining energy quality. In accordance with the second thermodynamics law, transforming energy into useful work is an energy loss that reduces the efficiency of devices for energy conversion. The energy degradation is equivalent to the production of entropy. Therefore, entropy production in a system causes the amount of energy available due to the disordered behavior of the fluid. Through reduced entropy generation, the productivity of the thermal system can be enhanced technically. In order to reduce entropy production, it is therefore very important to know how the entropy production is distributed during the thermodynamic process. Entropy production minimization is necessary for the maximal utility of the flow problems and thermal systems [20–28]. The flow of spinning nanofluid with entropy generation incorporated in the thermal slip was studied by Rehman et al. [29]. Recent research has looked at the effects of various constraints on liquid flow and heat transmission in a spinning fluid across a stretched disk [30–33]. Biswas et al. [34,35] studied
the several cases of nanofluid heat transfer in a W-shaped geometry. Ashraf et al. [36–39] investigated the transient mixed convection flow along different surfaces numerically.

In order to eliminate the energy wasted, scientists took drastic actions. They revised the energy conversion equipment and created products and methods to use the established resources better. The current study aims to obtain a numerical solution for a steady, MHD, and incompressible rotating flow of carbon nanotubes over a rotating stretching disk with a viscous dissipation effect using the shooting method and the Keller box scheme. The single-wall and double-wall carbon nanotubes in the water base are taken into consideration. The formulated problem is resolved, and the results are thoroughly investigated. Finally, it presents and analyzes the behaviors of the physical parameters on the temperature and velocity profiles. Furthermore, Xue’s model is used to formulate the problem. In accordance with this model, thermal conductivity is given by:

\[
\frac{k_{nf}}{k_f} = \frac{1 - \phi + 2\phi \left( \frac{k_{CNT}}{k_{CNT} - k_f} \right) \ln \left( \frac{k_{CNT} + k_f}{2k_f} \right)}{1 - \phi + 2\phi \left( \frac{k_f}{k_{CNT} - k_f} \right) \ln \left( \frac{k_{CNT} + k_f}{2k_f} \right)}
\]

2. Problem Statement

The steady three-dimensional flow of an incompressible carbon nanofluid over a radially stretching and rotating disk is investigated. The disk is considered to be in a plane with \(z \geq 0\). Figure 1 illustrates the flow geometry as well as the coordinate system.

Figure 1. Geometry and coordinates of model.

The flow is assumed to have a radial velocity \(u = ar\) and an azimuthal velocity \(v = \Omega r\), where \(a\) is the strain rate and \(\Omega\) is the anticlockwise angular velocity of the disk. In the transverse direction of flow, the uniform magnetic field \(B_0\) is produced. Because of the small Reynold number, the electric field is absent. It is also assumed that a heated fluid just below the disk is used to change the temperature of the disk via convective heat transfer, which yields the heat transfer coefficient \(h_f\). Joule heating is considered. We have the following equations for the above flow by Weidman [19]:

The mass conservation equation:

\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0,
\]
The radial momentum equation:

\[ u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} = v_{nf} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{\sigma_{nf} B_0^2}{\rho_{nf}} u, \]  

(2)

The azimuthal momentum equation:

\[ u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + \frac{u v}{r} = v_{nf} \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2} \right) - \frac{\sigma_{nf} B_0^2}{\rho_{nf}} v, \]  

(3)

The axial momentum equation:

\[ u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = v_{nf} \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right), \]  

(4)

The energy equation:

\[ u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \frac{K_{nf}}{(\rho c_p)_{nf}} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\sigma_{nf} B_0^2}{(\rho c_p)_{nf}} (u^2 + v^2) \]  

(5)

The boundary equations:

\[ u = a r, v = \Omega r, w = 0, -k_{nf} \frac{\partial T}{\partial z} = h_f (T_f - T), \text{ at } z = 0. \]  

(6)

\[ u \rightarrow 0, v \rightarrow 0, T \rightarrow T_\infty \text{ as } z \rightarrow \infty. \]  

(7)

where the radial, azimuthal, and axial components of velocity are \((u, v, w)\), and \(v_{nf}, \sigma_{nf},\) and \((\rho c_p)_{nf}\) are the efficient kinematic viscosity, thermal diffusion, and heat capacity of the nanofluid, respectively. \(T_f\) is the temperature of the heated fluid, and \(T_\infty\) is the ambient fluid temperature. Xue’s model is utilized for the thermal conductivity of the carbon nanofluid, and is expressed in Equation (11).

The efficient properties of the nanofluid are expressed as:

\[ \rho_{nf} = (1 - \phi) \rho_f + \phi \rho_{CNT}, \]  

(8)

\[ \mu_{nf} = \frac{\mu_f}{(1 - \phi)^2}, \quad \nu_{nf} = \frac{\nu_f}{\rho_{nf}}, \quad \kappa_{nf} = \frac{k_f}{(\rho c_p)_{nf}} \]  

(9)

\[ (\rho c_p)_{nf} = (1 - \phi) (\rho c_p)_f + \phi (\rho c_p)_{CNT} \]  

(10)

\[ \frac{k_{nf}}{k_f} = \frac{1 - \phi + \phi \left( \frac{k_{CNT}}{k_{CNT-f}} \right) \ln \left( \frac{k_{CNT+f}}{2k_f} \right)}{1 - \phi + \phi \left( \frac{k_f}{k_{CNT-f}} \right) \ln \left( \frac{k_{CNT+f}}{2k_f} \right)} \]  

(11)

Here, \(\rho_{nf}\) is the density, \(\mu_{nf}\) is the dynamic viscosity, \(\kappa_{nf}\) is the thermal conductivity, and \(\phi\) is the solid volume fraction of the nanofluid, \(\rho_{CNT}\) is the density, and \((\rho c_p)_{CNT}\) is the heat capacity of the carbon nanotubes. \(\rho_f\) is the density, \(\mu_f\) is the dynamic viscosity, and \((\rho c_p)_f\) is the heat capacity of the base fluid. The following similarity transitions are used to obtain the non-linear ordinary differential equations.

\[
\begin{align*}
  u(r,z) &= u_0 f'(\eta) = ar f'(\eta), v(r,\eta) = u_0 g(\eta) = arg(\eta), \\
  w(r,z) &= -2\sqrt{\pi r^2 f'(\eta)} \text{, } \theta(\eta) = \frac{1 - T_{inf}}{T_f - T_{inf}}, \eta = \sqrt{\frac{z}{r^2}}
\end{align*}
\]  

(12)
Equation (1) is satisfied, and Equations (2)–(7) will take the form after applying Equation (12) as:

\[ f'' - M^2(1 - \phi)^2 f'' + (1 - \phi + \phi \left( \rho \right)_{\text{CNT}} / \rho_f) (2f f'' - f'') = 0 \]  
\[ s'' - M^2(1 - \phi)^2 s'' + (1 - \phi + \phi \left( \rho \right)_{\text{CNT}} / \rho_f) (2f s'' - f'' s) = 0, \]  
\[ \frac{k_{nf}}{k_f} \theta'' + 2Pr(1 - \phi + \phi \left( \rho \right)_{\text{CNT}} / \left( \rho c_p \right)_f) f \theta' + PrEc M^2 \left( u^2 + v^2 \right) = 0, \]
\[ f(0) = 0, g(0) = S, f'(0) = 1, \theta'(0) = \frac{k_f}{k_{nf}} \sigma [1 - \theta(0)] \]  
\[ f'(\infty) \to 0, g(\infty) \to 0, \theta(\infty) \to 0. \]  

The skin friction coefficient and Nusselt number for the fluid flow due to the rotating stretching disk are:

\[ C_f = \frac{1}{2 \rho f u_w^2 \left[ \tau_0^2 + \tau_0^2 \right]^{1/2}}, \quad Nu = \frac{r}{k_f \left( T_f - T_\infty \right)} \left[ -k_{nf} \frac{\partial T}{\partial z} \right]_{z=0} \]  

Applying the similarity transformations, Equations (17) and (18) will take the form

\[ \frac{1}{2} Re^{1/2} C_f = \frac{1}{(1 - \phi)^{1/2}} \left[ f''(0) + s''(0) \right]^{1/2}, \]  
\[ \frac{1}{2} Re^{-1/2} Nu = -\frac{k_{nf}}{k_f} \theta'(0) \]  

The non-dimensional parameters are the Reynold number, the magnetic field parameter, the Eckert number, the rotational parameter, the Biot number, and the Prandtl number, as follows:

\[ Re = \frac{a u_w}{\nu_f}, \quad M^2 = \frac{\sigma_f}{a(\rho)_f}, \quad Ec = \frac{u_w^2}{(\epsilon_p)_f (T_f - T_\infty)}, \quad S = \frac{\Omega}{a}, \quad \sigma = \frac{h_f}{k_f} \frac{\sqrt{T_f}}{a}, \quad Pr = \frac{a_f}{\nu_f} \]  

3. Entropy Generation

To understand the irreversibility of a system’s thermal energy, it is necessary to investigate entropy generation. The rate at which entropy is produced per unit volume for the three-dimensional carbon nanotube flow on a stretched and rotating disk is provided in accordance with approximated boundary layers.

\[ E'_{Gc} = \frac{k_{nf}}{T_\infty^2} (\Delta T)^2 + \frac{H_{nf}}{T_\infty^2} f \frac{\partial T}{\partial z}^2 + \frac{\sigma_{nf}}{T_\infty^2} B_0^2 \left( u^2 + v^2 \right) \]  

The entropy related to heat transfer is represented by the term (1) on the right-hand side of the expression (22), while the entropy due to viscous dissipation is indicated by the second term. The entropy production characteristics are defined by the boundary conditions (14) and (15).

\[ \left( E'_{Gc} \right)_o = \frac{k_f}{T_\infty^2} \left( T_f - T_\infty \right) \]  

The induced similarity variables define the dimensionless entropy production:

\[ N_G = \frac{E'_{Gc}}{\left( E'_{Gc} \right)_o} = \frac{k_{nf}}{k_f} Re \theta' + \frac{H_{nf}^2}{A} B r \left( f'' + s'' \right) \]
where

$$B_r = Pr \cdot Ec = \frac{\mu_f u_w^2}{k_f(T_f - T_\infty)} \text{ (Brickman number)}, \quad (25)$$

$$\Lambda = \frac{(T_f - T_\infty)}{T_\infty} \text{ (Temperature difference parameter)}$$

The distribution of the entropy generation in the flow domain is determined by the entropy generation number $N_G$. To solve the problem, the involvement of thermal conductivity in the entropy production compared to the total entropy production must be calculated. The Bejan number indicates the relevance of the thermal irreversibility in comparison to the overall irreversibility and is defined as

$$B_e = \frac{k_n f}{k_f} Re \theta^2 \frac{U^2}{\Lambda B_r (f'^2 + g'^2)} \quad (26)$$

### 4. Numerical Scheme

Two numerical schemes are used to solve the transformed problem described by Equations (13)–(17): the Keller box method (an implicit finite difference scheme) and the shooting method. The differential equations are modified to a system of the first order to enact both numerical methodologies. We explain each step further in detail.

#### 4.1. Step 1

Firstly, all the differential equations are contracted to first-order equations.

$$f' = U, \quad f'' = V, \quad g' = P, \quad \theta' = Q, \quad (27)$$

$$V' - A_1 U + A_2 f V - A_2 U^2 + A_2 g^2 = 0, \quad (28)$$

$$P' - A_1 g + A_2 P f - A_2 g U = 0 \quad (29)$$

$$A_3 Q' + A_4 f Q + A_5 U^2 + A_5 V^2 = 0 \quad (30)$$

$$A_1 = M^2 (1 - \phi)^{-25}, \quad A_2 = \frac{2Pr(1 - \phi + \phi \rho_{CNT} \rho_f)}{\rho_c p_{CNT}}, \quad A_3 = \frac{k_n f}{k_f} Ec M^2, \quad A_4 = Ec Pr / [(1 - \phi + \phi \rho_{CNT} \rho_f)] \quad \{31\}$$

The boundary conditions are modified as:

$$f(0) = 0, \quad U(0) = 1, \quad Q(0) = -\frac{k_f}{k_n f} \sigma [1 - \theta(0)]$$

$$g(0) = S, \quad U(\infty) = 0, \quad g(\infty) = 0, \quad \theta(\infty) = 0. \quad (32)$$

#### 4.2. Step 2

The following important points are explored to discretize Equations (27)–(30).

$$\eta_0 = 0, \quad \eta_j = \eta_{j-1} + h_{j-1}, \text{ } j = 1, 2, 3, \ldots, J, \quad \eta_J = \eta_{\infty} \quad (33)$$

We differentiated Equations (27)–(30) at $\eta_{j-\frac{1}{2}}$ in the following way.

$$h_{j-1}^{-1} (f_j - f_{j-1}) = U_{j-\frac{1}{2}} \quad (34)$$

$$h_{j-1}^{-1} (U_j - U_{j-1}) = V_{j-\frac{1}{2}} \quad (35)$$

$$h_{j-1}^{-1} (V_j - V_{j-1}) - A_1 U_{j-\frac{1}{2}} + A_2 (fV)_{j-\frac{1}{2}} - A_2 (U^2)_{j-\frac{1}{2}} + A_2 (g^2)_{j-\frac{1}{2}} = 0. \quad (36)$$
$$f_{j} = f_j + \delta f_j, U_j = U_j + \delta U_j, V_j = V_j + \delta V_j, \quad g_j = g_j + \delta g_j, P_j = P_j + \delta P_j, \theta_j = \theta_j + \delta \theta_j,$$

By swapping expressions (42) in Equations (34)–(40) and ignoring the second and higher order of \( \delta \), we obtain the result shown in step 4.

4.4. Step 4

As a result, in order to solve the linear equations, the entire system is expressed in a matrix form using the block tridiagonal approach.

$$\delta f_j - \delta f_{j-1} - \frac{1}{2} h_{j-1} (\delta U_j + \delta U_{j-1}) = (Y_1)_{j-\frac{1}{2}},$$

$$\delta U_j - \delta U_{j-1} - \frac{1}{2} h_{j-1} (\delta V_j + \delta V_{j-1}) = (Y_2)_{j-\frac{1}{2}},$$

$$\xi_1 \delta V_j + (\xi_2) \delta V_{j-1} + (\xi_3) \delta f_j + (\xi_4) \delta f_{j-1} + (\xi_5) \delta U_j + (\xi_6) \delta U_{j-1} + (\xi_7) \delta g_j + (\xi_8) \delta g_{j-1} = (Y_3)_{j-\frac{1}{2}},$$

$$\xi_9 \delta P_j + (\xi_{10}) \delta P_{j-1} + (\xi_{11}) \delta f_j + (\xi_{12}) \delta f_{j-1} + (\xi_{13}) \delta g_j + (\xi_{14}) \delta g_{j-1} + (\xi_{15}) \delta U_j + (\xi_{16}) \delta U_{j-1} = (Y_4)_{j-\frac{1}{2}},$$

$$\delta \theta_j - \delta \theta_{j-1} - \frac{1}{2} h_{j-1} (\delta Q_j + \delta Q_{j-1}) = (Y_6)_{j-\frac{1}{2}},$$

$$\xi_{17} \delta Q_j + (\xi_{18}) \delta Q_{j-1} + (\xi_{19}) \delta f_j + (\xi_{20}) \delta f_{j-1} + (\xi_{21}) \delta V_j + (\xi_{22}) \delta V_{j-1} + (\xi_{23}) \delta Q_j + (\xi_{24}) \delta Q_{j-1} = (Y_7)_{j-\frac{1}{2}},$$

The boundary conditions are

$$\delta f_0 = 0, \delta g_0 = 0, \delta U_0 = 0, \delta Q_0 = 0, \delta U_1 = 0, \delta g_1 = 0, \delta \theta_1 = 0,$$

also

$$(Y_1)_{-\frac{1}{2}} = f_j - f_j + h_{j-1} U_{j-\frac{1}{2}},$$

$$(Y_2)_{-\frac{1}{2}} = U_{j-1} - U_j + h_{j-1} V_{j-\frac{1}{2}},$$

$$(Y_3)_{-\frac{1}{2}} = -(h_{j-1} V_j - V_{j-1}) - A_1 U_{j-\frac{1}{2}} + A_2 (fV)_{j-\frac{1}{2}} - A_2 (U^2)_{j-\frac{1}{2}} + A_2 (g^2)_{-\frac{1}{2}},$$

$$(Y_4)_{-\frac{1}{2}} = g_j - g_j + h_{j-1} P_{j-\frac{1}{2}},$$

$$A_1 h_{j-1} (Q_j - Q_{j-1}) + A_4 (fQ)_{j-\frac{1}{2}} + A_5 (V^2)_{j-\frac{1}{2}} + A_5 (U^2)_{j-\frac{1}{2}} = 0,$$
\[ (Y_5)_{j-\frac{1}{2}} = -[h_{j-1}(P_j - P_{j-1}) - A_1g_{j-\frac{1}{2}} + A_2(fP)_{j-\frac{1}{2}} - A_2(Ug)_{j-\frac{1}{2}}], \]  
\[ (Y_6)_{j-\frac{1}{2}} = \theta_{j-1} - \theta_j + h_{j-1}Q_{j-\frac{1}{2}}, \]  
\[ (Y_7)_{j-\frac{1}{2}} = -\left[A_3h^{-1}\left(Q_j - Q_{j-1}\right) + A_4(fQ)_{j-\frac{1}{2}} + A_5(V^2)_{j-\frac{1}{2}} + A_5(U^2)_{j-\frac{1}{2}}\right], \]

The coefficients are
\[ (\zeta_1)_j = h^{-1}_{j-1} + A_2f_j, \]  
\[ (\zeta_2)_j = -h^{-1}_{j-1} + A_2f_{j-1}, \]  
\[ (\zeta_3)_j = 0.5A_2V_j, \]  
\[ (\zeta_4)_j = 0.5A_2V_{j-1}, \]  
\[ (\zeta_5)_j = -0.5A_1 + A_2U_j, \]  
\[ (\zeta_6)_j = -0.5A_1 - A_2U_{j-1}, \]  
\[ (\zeta_7)_j = -A_2g_j, \]  
\[ (\zeta_8)_j = -A_2g_{j-1}, \]  
\[ (\zeta_9)_j = h^{-1}_{j-1} + 0.5A_2f_j, \]  
\[ (\zeta_{10})_j = -h^{-1}_{j-1} + 0.5A_2f_{j-1}, \]  
\[ (\zeta_{11})_j = 0.5A_2P_j, \]  
\[ (\zeta_{12})_j = 0.5A_2P_{j-1}, \]

Equations (43)–(49) are a linear system of algebraic equations. To improve the solution, the system of equations is sought iteratively. The iterative procedure is stopped when the specified tolerance is obtained.

5. Numerical Results and Discussion

In the presence of viscous and magnetic effects, the second laws and the thermal transfer analysis of the CNT water-based nanofluid are managed to perform the theoretical behavior of above-mentioned mechanism. The Keller box scheme is used to solve the dimensionless set of nonlinear differential equations numerically. The nonlinear differential equations are also numerically solved by a shooting method for the validation of our numerical code. The appropriate estimations that satisfy all the boundary conditions are chosen to solve Equations (12)–(16) and to obtain a more accurate approximation of the solution. Table 1 shows the thermal properties of the nanofluids and water.

| Table 1. Characteristics (thermophysical) of water and nanofluid. |
|---------------------------------|----------------|----------------|----------------|
| Properties                      | Water (Base)   | SWCNT          | MWCNT          |
| \(\rho\) (kg/m\(^3\))          | 997.1          | 2600           | 1600           |
| \(c_p\) (J/kgK)                 | 4179           | 425            | 796            |
| \(k\) (W/mK)                    | 0.613          | 6600           | 3000           |

Table 2 includes the impact of the magnetic and rotation parameters on \(f'(0)\), \(-g'(0)\), and \(-\theta'(0)\) at two chosen magnetic parameters when viscous dissipation and Joule heating are not present. According to the previously shown figures, the larger the rotation, the larger all the physical quantities, together with the heat transfer. When these values are compared to those of Mustafa [36], it is revealed that the calculated results have excellent promise.
Table 2. Comparison of radial and azimuthal velocities and heat transfer rate at Ec = φ = 0 and Pr = 1.

| M  | S  | \( f'(0) \) Mustafa [36] | Present | \( g'(0) \) Mustafa [36] | Present | \( \theta'(0) \) Mustafa [36] | Present |
|----|----|-------------------------|---------|-------------------------|---------|-------------------------|---------|
| 0  | 0  | -1.1737207             | -1.1737207 | 0.00000000              | 0.00000000 | 0.8519914              | 0.8519914 |
| 1  | -0.9483137 | -0.94831384 | 1.4869526 | 1.4869526 | 0.8756621 | 0.8756619 |
| 2  | -0.3262439 | -0.3262440 | 3.1278281 | 3.1278281 | 0.9304111 | 0.9304107 |
| 5  | 3.1937329 | 3.1937329 | 9.2535411 | 9.2535411 | 1.1291404 | 1.1291401 |
| 10 | 12.7208997 | 12.7208997 | 22.9134072 | 22.9134071 | 1.4259266 | 1.4259262 |
| 20 | 40.9056723 | 40.9056723 | 60.0129305 | 60.0129300 | 1.8944305 | 1.8944300 |

5.1. Velocity Profiles

The current study investigated the effect of the parameter \( M^2 \) on the velocities \( f'(\eta) \) and \( g(\eta) \) profiles and are described in Figures 2 and 3. The magnetic field parameter has a negative impact on the \( f'(\eta) \) and \( g(\eta) \) profiles according to the theory of the Lorentz force, i.e., the increments in the magnetic field parameter which reduce the \( f'(\eta) \) and \( g(\eta) \) profiles can be seen in the above-mentioned figures. The behavior of the velocities under volume fraction \( \phi \) and the rotation \( S \) influences is indicated in Figures 4–7. As shown in Figures 4 and 5, an increasing solid volume value increases both velocities \( f'(\eta) \) and \( g(\eta) \). Figures 6 and 7 show the effect of rotation parameter ‘S’ on the velocity profiles. Figure 7 depicts the increasing behavior as the rotation parameter is increased, whereas Figure 6 depicts the reverse effect.

Figure 2. Magnetic parameter verses velocity \( f'(\eta) \).
1. Velocity Profiles

The current study investigated the effect of the parameter ($M$) on the velocities $f'(\eta)$ and $g(\eta)$ profiles and are described in Figures 2 and 3. The magnetic field parameter has a negative impact on the $f'(\eta)$ and $g(\eta)$ profiles according to the theory of the Lorentz force, i.e., the increments in the magnetic field parameter which reduce the $f'(\eta)$ and $g(\eta)$ profiles can be seen in the above-mentioned figures. The behavior of the velocities under volume fraction $\phi$ and the rotation $S$ influences is indicated in Figures 4–7. As shown in Figures 4 and 5, an increasing solid volume value increases both velocities $f'(\eta)$ and $g(\eta)$. Figures 6 and 7 show the effect of rotation parameter 'S' on the velocity profiles. Figure 7 depicts the increasing behavior as the rotation parameter is increased, whereas Figure 6 depicts the reverse effect.

Figure 2. Magnetic parameter verses velocity $f'(\eta)$.

Figure 3. Magnetic parameter verses velocity $g(\eta)$.

Figure 4. Volume fraction verses velocity $f'(\eta)$.

Figure 5. Volume fraction verses velocity $g(\eta)$.

Figure 6. Rotation $S$ verses velocity $f'(\eta)$.

Figure 5. Volume fraction verses velocity $g(\eta)$. 
5.2. Temperature Profiles

The influence of \( \text{Pr} \) on the temperature profile \( \theta(\eta) \) is portrayed in Figure 8. Physically, nanofluids have a high thermal diffusivity while having a low \( \text{Pr} \), and vice versa. As a result, the liquid temperature drops. From here, we can see that the rising \( \text{Pr} \) has a decrease in \( \theta(\eta) \). Figure 9 describes the temperature distribution impact of the volume fraction of the nanoparticles for both the SWCNTs and the MWCNTs. Higher \( \phi \) results can be observed in both CNTs and in an improved thermal boundary layer thickness at a stronger temperature field. The influence of the Biot number on both CNTs is seen in Figure 10; an increase in the temperature is noticed when the amount of the Biot number is increased. The Biot number is well defined as the thermal resistance ratio of a solid to the thermal resistance of the boundary layer. Greater Biot numbers imply better convection and thicker thermal layers, resulting in more even temperature distribution in both types of CNTs. The effects of the magnetic parameter and the Eckert number are seen in Figures 11–13. Figure 11 shows that temperature increases with the increasing magnetic values. The reason for this is that with the increasing magnetic values the ohmic heating grows, and thus, the fluid temperature is increased. The Eckert number is a measurement of the friction force in between fluid layers. Thus, the frictional heating increases by increasing the Eckert numbers, and this results in an increase in the fluid temperatures, as shown in Figures 12 and 13. The temperature increases with the increasing rotating parameter when there is viscous dissipation and a magnetic parameter, as shown in Figure 14.
5.2. Temperature Profiles

The influence of $\text{Pr}$ on the temperature profile $\theta(\eta)$ is portrayed in Figure 8. Physically, nanofluids have a high thermal diffusivity while having a low $\text{Pr}$, and vice versa. As a result, the liquid temperature drops. From here, we can see that the rising $\text{Pr}$ has a decrease in $\theta(\eta)$. Figure 9 describes the temperature distribution impact of the volume fraction of the nanoparticles for both the SWCNTs and the MWCNTs. Higher $\phi$ results can be observed in both CNTs and in an improved thermal boundary layer thickness at a stronger temperature field. The influence of the Biot number on both CNTs is seen in Figure 10; an increase in the temperature is noticed when the amount of the Biot number is increased. The Biot number is well defined as the thermal resistance ratio of a solid to the thermal resistance of the boundary layer. Greater Biot numbers imply better convection and thicker thermal layers, resulting in more even temperature distribution in both types of CNTs. The effects of the magnetic parameter and the Eckert number are seen in Figures 11–13. Figure 11 shows that temperature increases with the increasing magnetic values. The reason for this is that with the increasing magnetic values the ohmic heating grows, and thus, the fluid temperature is increased. The Eckert number is a measurement of the friction force in between fluid layers. Thus, the frictional heating increases by increasing the Eckert numbers, and this results in an increase in the fluid temperatures, as shown in Figures 12 and 13. The temperature increases with the increasing rotating parameter when there is viscous dissipation and a magnetic parameter, as shown in Figure 14.
5.3. Variations of Skin Friction and Nusselt Number

Figures 15–17 show the variation of the skin friction as a function of the rotating parameter $S$ and the magnetic parameter $M_s$. The skin friction has been observed to increase for both $S$ and $M_s$ in both SWCNTs and MWCNTs. Figures 18–21 show the Nusselt number behavior for the number of relevant parameters for both CNT types. According to Figure 18, the Nusselt number appears to increase for $(Pr)$ in both CNTs. When the value of $(Pr)$ is increased, the coefficient of heat transfer increases rapidly. These observations...
5.3. Variations of Skin Friction and Nusselt Number

Figures 15–17 show the variation of the skin friction as a function of the rotating parameter S and the magnetic parameter $M^2$. The skin friction has been observed to increase for both S and $M^2$ in both SWCNTs and MWCNTs. Figures 18–21 show the Nusselt number behavior for the number of relevant parameters for both CNT types. According to Figure 18, the Nusselt number appears to increase for (Pr) in both CNTs. When the value of (Pr) is increased, the coefficient of heat transfer increases rapidly. These observations clearly show that the Prandtl number and the Nusselt number are directly proportional to each other. When looking at Figures 19–21, it has been determined that the Nusselt number is a decreasing function of the magnetic parameters $M^2$ and Ec, whereas the increasing behavior is shown against rotation S, implying that as these parameters increase the rate of heat transfer declines.

![Figure 14. Rotation verses $\theta(\eta)$.](image)

![Figure 15. $M^2$ verses Prandtl and $Re^{1/2}C_f$.](image)
clearly show that the Prandtl number and the Nusselt number are directly proportional to each other. When looking at Figures 19–21, it has been determined that the Nusselt number is a decreasing function of the magnetic parameters $M^2$ and $Ec$, whereas the increasing behavior is shown against rotation $S$, implying that as these parameters increase the rate of heat transfer declines.

Figure 15. $M^2$ verses Prandtl and $Re^{1/2}C_f$.

Figure 16. $S$ verses $Re^{1/2}C_f$.

Figure 17. $M^2$ verses $Re^{1/2}C_f$.

Figure 18. Prandtl verses $Re^{-1/2}Nu$. 
5.4. Effects of Parameters on Entropy and Bejan Number

As shown in Figure 22a, the temperature difference has a great impact on the entropy generation in the closed and as well in the open system. When the temperature difference parameter is increased, the entropy production is reduced. This indicates that lowering the operating temperature can reduce the entropy production. As can be observed in Figure 22b, the Bejan number decreases as the value of $\Lambda$ is increased. The heat transfer effects are also observed to be completely dominant at the surface of the rotating stretching disk. As $\Lambda$ increases, the irreversibility of the fluid friction is increased. This is based on the fact that the viscous dissipation parameter enhances with the lowering operating temperature.
temperature. Figure 23a,b depict the behavior of $N_G$ and the Bejan number when the Br is varied. According to Figure 23a,b, the rapid expansion of the Brinkman number significantly increases the irreversibility of the thermal energy in relation to the entropy generation and the Bejan number. The effect of the Biot number on $N_G$ and the Bejan number is depicted in Figure 24a,b. On the rotating stretching disk surface, it was observed that higher values of $\sigma$ resulted in higher thermal energy irreversibility. The enhanced $N_G$ and Bejan number cause an increase in the dominating effects of the fluid friction and heat transfer. In Figure 25a,b, the influence of the rotation parameter $S$ can be seen. The rapid expansion of the Hartmann number, as shown in Figure 26a,b, significantly increases the irreversibility of the thermal energy in relation to the entropy generation and the Bejan number. It was revealed that the increase in $S$ enhances both $N_G$ and Bejan number. As a result, all of these characteristics give a good understanding of how to calculate the thermal energy irreversibility in the boundary layer thickness.

Figure 22. (a) $\Lambda$ verses entropy $N_G$. (b) $\Lambda$ verses Bejan (Be).

Figure 23. (a) Brickman verses entropy $N_G$. (b) Brickman verses Bejan (Be).
6. Conclusions

In this study, two numerical approaches, Keller box and shooting, are used to simulate the three-dimensional carbon nanotube flow on a rotating stretching disk through a convective boundary condition. The following are the key findings:

- As the values of the nanoparticle volume fraction, magnetic number, Eckert number, and Biot number increase, the temperature \( \theta \) decreases for both forms of CNTs by the increasing Prandtl number.
- The increase in the rotation parameter causes an increase in velocity \( f'(\eta) \) and velocity \( g(\eta) \).
- With increasing magnitudes of rotation \( S \), the Brinkman number \( Br \), the Biot number, and Hartmann number increase significantly in the immediate vicinity of the rotating and stretching disk.
- The skin friction coefficient is improved for both forms of CNTs by the increasing Hartmann number, the entropy, and Bejan number.
- The current work can be extended for entropy analysis over a magnetized axially rotating stretching disk. The authors declare no conflict of interest.

**Nomenclature**

- \( Ec \): Eckert number
- \( Re \): Reynolds number
- \( Ha \): Hartmann number
- \( Br \): Brinkman number
- \( S \): Rotation parameter
- \( \phi \): Nanoparticle volume fraction
- \( Pr \): Prandtl number
- \( f'(\eta) \): Velocity
- \( g(\eta) \): Velocity

**Conflicts of Interest:** The authors declare no conflict of interest.
• With an increase in the values of the regulating flow variables, such as the magnetic parameter, the velocities \( f'(\eta) \) and \( g(\eta) \) for both the SWCNTs and the MWCNTs decrease.

• The increase in the rotation parameter causes an increase in velocity \( f'(\eta) \) and velocity \( g(\eta) \) in both types of carbon nanotubes.

• As the values of the nanoparticle volume fraction, magnetic number, Eckert number, and Biot number increase, the temperature (\( \theta \)) of the fluid increases for both the SWCNTs and the MWCNTs, although inverse behavior is observed against the Prandtl number.

• The skin friction coefficient is improved for both forms of CNTs by the increasing values of the parameters S and \( M^2 \).

• As the Prandtl number and the rotation are increased, the heat transfer rate of the fluid increases for the SWCNTs and the MWCNTs, whereas the contrary trend is shown for the magnetic parameter and the Eckert number.

• With increasing magnitudes of rotation S, the Brinkman number \( Br \), the Biot number, the Hartmann number, the entropy, and Bejan number increase significantly in the immediate vicinity of the rotating and stretching disk.

• When the temperature difference is increased, the entropy generation is decreased.

• The current work can be extended for entropy analysis over a magnetized axially rotating stretching disk to save the surface from excessive heating.

Author Contributions: Methodology, H.A.N., U.S., M.M. and M.A.; Software, M.A.H.; Investigation, A.M.R.; Resources, H.A.N. and S.I.A.; Data curation, S.I.A.; Writing—original draft, H.A.N., U.S. and A.M.R.; Writing—review & editing, U.S., M.M. and M.A.; Visualization, M.A.H.; Supervision, M.A. and A.M.R.; Funding acquisition, H.A.N. All authors have worked equally. All authors have read and agreed to the published version of the manuscript.

Funding: The authors extend their appreciation to the Deputyship for Research & Innovation, Ministry of Education in Saudi Arabia for funding this research work through the project number (IF2/PSAU/2022/01/22970).

Data Availability Statement: Data are available upon request.

Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

| Symbol | Definition |
|--------|------------|
| \( a \) | Strain rate |
| \( k_1 \) | Porosity |
| \( k_{nf} \) | Nanofluid thermal conductivity |
| \( \Omega \) | Angular velocity |
| \( Re \) | Reynolds number |
| \( C_f \) | Skin friction coefficient |
| \( T_{\infty} \) | Ambient fluid temperature |
| \( \nu_{nf} \) | Nanofluid kinematic viscosity |
| \( \mu_{nf} \) | Nanofluid dynamic viscosity |
| \( \beta^* \) | Non-dimensional thermal relaxation time |
| \( (\rho c_p)_{nf} \) | Heat capacity of nanofluid |
| \( (\rho c_p)_{CNT} \) | Heat capacity of carbon nanotubes |
| \( \kappa_{nf} \) | Thermal diffusion of nanofluid |
| \( S \) | Rotation parameter |
| \( Pr \) | Prandtl number |
| \( (u, v, w) \) | Velocity components in radial, azimuthal, and axial directions |
| \( (r, \phi, z) \) | Cylindrical coordinates in radial, azimuthal, and axial directions |
| \( k_f \) | Fluid thermal conductivity |
| \( v_f \) | Fluid viscosity |
| \( \mu_f \) | Fluid dynamic viscosity |
| \( \lambda^* \) | Dimensional thermal relaxation time |
| \( (\rho c_p)_f \) | Heat capacity of fluid |
| \( \sigma \) | Biot number |
| \( \rho_{CNT} \) | Density of carbon nanotubes |

References

1. Choi, S.; Eastman, J.A. Enhancing thermal conductivity of fluids with nanoparticles. In Proceedings of the 1995 ASME International Mechanical Engineering Congress and Exposition, Argonna National Laboratory, Lemont, IL, USA, 12–17 November 1995; pp. 99–105.

2. Ayodeji, F.; Tope, A.; Pele, O. Magneto-hydrodynamics (MHD) Bioconvection Nanofluid Slip Flow over a Stretching Sheet with Thermophoresis, Viscous Dissipation and Brownian Motion. Mach. Learn. Res. 2020, 4, 51. [CrossRef]
3. Khan, M.; Salahuddin, T.; Malik, M.Y.; Alqarni, M.S.; Alqahtani, A.M. Numerical modeling and analysis of bioconvection on MHD flow due to an upper paraboloidal surface of revolution. *Phys. A* 2020, 27, 124231. [CrossRef]

4. Khan, W.A.; Khan, Z.H.; Rahi, M. Fluid flow and heat transfer of carbon nanotubes along a flat plate with Navier slip boundary. *Appl. Nanosci.* 2014, 4, 633–641. [CrossRef]

5. Seth, G.S.; Kumar, R.; Bhattachay, A. Entropy generation of dissipative flow of carbon nanotubes in rotating frame with Darcy-Forchheimer porous medium: A numerical study. *J. Mol. Liq.* 2018, 268, 637–646. [CrossRef]

6. Hayat, T.; Hussain, Z.; Alsaeedi, A.; Ashghar, S. Carbon nanotubes effects in the stagnation point flow towards a nonlinear stretching sheet with variable thickness. *Adv. Powder Technol.* 2016, 27, 1677–1688. [CrossRef]

7. Ghadikolaei, S.S.; Hosseinzadeh, K.; Hatami, M.; Ganji, D.D.; Armin, M. Investigation for squeezing flow of ethylene glycol (C$_2$H$_5$O$_2$) carbon nanotubes (CNTs) in rotating stretching channel with nonlinear thermal radiation. *J. Mol. Liq.* 2018, 263, 10–21. [CrossRef]

8. Nabwery, H.A.; Boumazgour, M.; Rashad, A.M. Group method analysis of mixed convection stagnation-point flow of non-Newtonian nanofluid over a vertical stretching surface. *Indian J. Phys.* 2017, 91, 731–742. [CrossRef]

9. Alwawi, F.A.; Alkasasbeh, H.T.; Rashad, A.M.; Idris, R. MHD natural convection of Sodium Alginate Casson nanofluid over a solid sphere. *Results Phys.* 2020, 10, 102818. [CrossRef]

10. Eid, M.R. Chemical reaction effect on MHD boundary layer flow of two phase nanofluid model over an exponentially stretching sheet with a heat generation. *J. Mol. Liq.* 2016, 220, 718–725. [CrossRef]

11. Mustafa, M.; Mushtaq, A.; Hayat, T.; Alsaedi, A. Rotating flow of magnetite-water nanofluid over a stretching surface inspired by non-linear thermal radiation. *PloS ONE* 2016, 11, e0149304. [CrossRef]

12. Beg, O.A.; Rao, A.S.; Nagendra, N.; Amanulla, C.H.; Reddy, M.S.N. Computational analysis of non-Newtonian boundary layer flow of nanofluid past a vertical plate with partial slip. *Model. Meas. Control B* 2017, 86, 271–295. [CrossRef]

13. Hassan, M.; Tabar, M.M.; Nemati, H. An analytical solution for boundary layer flow of a nanofluid past a shrinking sheet with a prescribed heat flux. *Int. J. Therm. Sci.* 2011, 50, 2256–2263. [CrossRef]

14. Eastman, J.A.; Choi, S.U.S.; Li, S.; Yu, W.; Thompson, L.J. Anomalously increased effective thermal conductivity of ethylene glycol-based nanofluids containing copper nanoparticles. *Appl. Phys. Lett.* 2001, 78, 718–720. [CrossRef]

15. Choi, S.U.S.; Zhang, Z.G.; Yu, W.; Lockwood, F.E.; Gruulke, E.A. Anomalous thermal conductivity enhancement in nanotube suspension. *Appl. Phys. Lett.* 2001, 79, 2225–2228. [CrossRef]

16. Karman, T.V. Über laminate und turbulente Reibung. *J. Appl. Math. Mech.* 1921, 1, 233–252. [CrossRef]

17. Wang, C.Y. Stretching a surface in a rotating fluid. *Z. Angew. Math. Phys.* 1988, 39, 177–185. [CrossRef]

18. Fang, T.G. Flow over a stretching disk. *Phys. Fluids* 2007, 19, 128105. [CrossRef]

19. Weidman, P. Axisymmetric stagnation point flow on a spiraling disk. *Phys. Fluids* 2014, 26, 073603. [CrossRef]

20. Kumam, P.; Shah, Z.; Dawar, A.; Rasheed, H.U.; Islam, S. Entropy Generation in MHD Radiative Flow of CNTs Casson Nanofluid in Rotating Channels with Heat Source/Sink. *Math. Probl. Eng.* 2019, 2019, 9158093. [CrossRef]

21. Adesanya, S.O.; Makinde, O.D. Entropy generation in couple stress fluid flow through porous channel with fluid slippage. *Int. J. Exergy* 2014, 15, 344–362. [CrossRef]

22. Khan, W.A.; Culham, R.; Aziz, A. Second Law Analysis of Heat and Mass Transfer of Nanofluids along a Plate with Prescribed Surface Heat Flux. *ASME J. Heat Transf.* 2015, 137, 081701. [CrossRef]

23. Makinde, O.D. Thermodynamic second law analysis for a gravity-driven variable viscosity liquid film along an inclined heated plate with convective cooling. *J. Mech. Sci. Technol.* 2010, 24, 899–908. [CrossRef]

24. Sheremet, M.A.; Oztop, H.F.; Pop, I.; Hamdeh, N.A. Analysis of entropy generation in natural convection of nanofluid inside a square cavity having hot solid block: Tiwari and das‘ model. *Entropy* 2016, 18, 9. [CrossRef]

25. Dawar, A.; Shah, Z.; Khan, W.; Idrees, M.; Islam, S. Unsteady squeezing flow of MHD CNTs nanofluid in rotating channels with Entropy generation and viscous Dissipation. *Adv. Mech. Eng.* 2019, 10, 1–18. [CrossRef]

26. Das, S.; Jana, R.N.; Chamkha, A.J. Entropy generation due to unsteady hydromagnetic Couette flow and heat transfer with asymmetric convective cooling in a rotating system. *J. Math. Model.* 2015, 3, 107–128.

27. Chamkha, A.J.; Ismael, M.; Kasaei, A.; Armaghani, T. Entropy generation and natural convection of CuO-Water nanofluid in C-shaped cavity under magnetic field. *Entropy* 2016, 18, 50. [CrossRef]

28. Rashidi, M.M.; Freidoonimehr, N. Analysis of entropy generation in MHD stagnation-point flow in porous media with heat source/sink. *Int. J. Comput. Methods Eng. Sci. Mech.* 2014, 15, 345–355. [CrossRef]

29. Rehman, A.U.; Mehmood, R.; Nadeem, S. Entropy analysis of radioactive rotating Nanofluid with thermal Slip. *Appl. Therm. Eng.* 2017, 112, 832–840. [CrossRef]

30. Sahoo, B.; Shevchuk, I.V. Heat transfer due to revolving flow of Reiner-Rivlin fluid over a stretchable surface. *Therm. Sci. Eng. Prog.* 2019, 10, 327–336. [CrossRef]

31. Shevchuk, I.V. Unsteady conjugate laminar heat transfer of a rotating non-uniformly heated disk. *Int. J. Heat Mass Transf.* 2006, 49, 3530–3537. [CrossRef]

32. Turkylmazoglu, M. MHD fluid flow and heat transfer due to a stretching rotating disk. *Int. J. Therm. Sci.* 2012, 51, 195–201. [CrossRef]

33. Banik, S.; Mirja, A.S.; Biswas, N.; Ganguly, R. Entropy analysis during heat dissipation via thermomagnetic convection in a ferrfluid filled enclosure. *Int. Commun. Heat Mass Transf.* 2022, 138, 106323. [CrossRef]

34. Biswas, N.; Mandal, D.K.; Manna, N.K.; Benim, A.C. Magneto-hydrothermal triple-convection in a W-shaped porous cavity containing oxytactic bacteria. *Sci. Rep.* 2022, 12, 18053. [CrossRef] [PubMed]
35. Mandal, D.K.; Biswas, N.; Manna, N.K.; Gorla, R.S.R.; Chamkha, A.J. Hybrid nanofluid magnetohydrodynamic mixed convection in a novel W-shaped porous system. *Int. J. Numer. Methods Heat Fluid Flow* 2022, ahead-of-print. [CrossRef]

36. Ashraf, M.; Ilyas, A.; Ullah, Z.; Abbas, A. Periodic magnetohydrodynamic mixed convection flow along a cone embedded in a porous medium with variable surface temperature. *Ann. Nucl. Energy* 2022, 175, 109218. [CrossRef]

37. Ullah, Z.; Ashraf, M.; Sarris, I.E.; Karakasidis, T.E. The impact of reduced gravity on oscillatory mixed convective heat transfer around a non-conducting heated circular cylinder. *Appl. Sci.* 2022, 12, 5081. [CrossRef]

38. Ashraf, M.; Ahmad, U.; Zia, S.; Gorla, R.S.R.; Al-Johani, A.S.; Khan, I.; Andualem, M. Magneto-exothermic catalytic chemical reaction of convective heat transfer along a curved surface. *Math. Probl. Eng.* 2022, 2022, 8439659. [CrossRef]

39. Ullah, Z.; Ashraf, M.; Ahmad, S. The Analysis of Amplitude and Phase Angle of Periodic Mixed Convective Fluid Flow across a Non-Conducting Horizontal Circular Cylinder. *Partial Differ. Equ. Appl. Math.* 2022, 5, 100258. [CrossRef]