Constraining the break of spatial diffeomorphism invariance with Planck data

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Abstract. The current most accepted paradigm for the early universe cosmology, the inflationary scenario, shows a good agreement with the recent Cosmic Microwave Background (CMB) and polarization data. However, when the inflation consistency relation is relaxed, these observational data exclude a larger range of red tensor tilt values, prevailing the blue ones which are not predicted by the minimal inflationary models. Recently, it has been shown that the assumption of spatial diffeomorphism invariance breaking (SDB) in the context of an effective field theory of inflation leads to interesting observational consequences. Among them, the possibility of generating a blue tensor spectrum, which can recover the specific consistency relation of the String Gas Cosmology, for a certain choice of parameters. We use the most recent CMB data to constrain the SDB model and test its observational viability through a Bayesian analysis assuming as reference an extended ΛCDM+tensor perturbation model, which considers a power-law tensor spectrum parametrized in terms of the tensor-to-scalar ratio, \( r \), and the tensor spectral index, \( n_t \). If the inflation consistency relation is imposed, \( r = -8n_t \), we obtain a strong evidence in favor of the reference model whereas if such relation is relaxed, a weak evidence in favor of the model with diffeomorphism breaking is found. We also use the same CMB data set to make an observational comparison between the SDB model, standard inflation and String Gas Cosmology.

Keywords: cosmological parameters from CMBR, physics of the early universe

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1 Introduction

The large amount of observational data recently released [1–8] have allowed to test the viability of a wide range of models of the early universe. Much more precise observations are expected to become available in the next years, and much of the future efforts will be focused on the tensor data [9–15]. The current constraints on the cosmological parameters are compatible with both the simplest slow-roll scenarios of inflation [16] as well as with some alternative scenarios [17–19]. The prospects for improvement of these constraints with the next generation of experiments suggest us to keep investigating different possibilities. In particular, a possible detection of a blue tensor tilt in the future would rule out a large class of models of inflation, since the so-called consistency relation would not be fulfilled in the standard scenario without violating the Null Energy Condition (NEC). It is worth mentioning that the recent polarization data have excluded a much larger range of red tensor tilt values than of blue ones [20, 21].

In the inflationary context, the violation of the NEC without incurring in instabilities was achieved in some class of models [22, 23]. At the same time, models with production of particle during inflation [24, 25], models with higher-curvature corrections to the gravitational effective action [26, 27], axion inflation [28, 29] and some other non-standard models of inflation [30, 31] are able to change the consistency relation, allowing a blue tensor tilt. It was recently suggested that by breaking the assumption of spatial diffeomorphism invariance in the context of the Effective Field Theory (EFT) of inflation, a blue tensor spectrum can be achieved without violating the NEC [32–36]. In this Spatial Diffeomorphism Breaking (SDB) model, the temporal diffeomorphism invariance is broken due to the time-dependence of the background. However, in the high energy regime of the early universe it is also possible that the spatial diffeomorphism invariance is broken (we refer the reader to [33] for a detailed discussion).

The EFT approach corresponds to the description of a system through the lowest dimension operators compatible with the underlying symmetries. This approach allows to characterise all the possible high energy corrections to simple inflationary scenario, whose sizes are constrained by experiment. It also has the advantage of being able to describe in a common language all single field models of inflation using only symmetry principles. This theory has been applied in recent works to describe the theory of fluctuations around an inflating cosmological background [37], and in this context was built the model considered here.
In ref. [32] it was shown that the scalar power spectrum of the EFT model with SDB exhibits characteristic signatures which allow it to be easily confronted with the data. In ref. [34] it was also examined the statistics of scalar and tensor modes in this class of models (EFT with SDB). The authors obtained the three point functions and point out the possibility that this symmetry breaking pattern can lead to an enhanced amplitude for the squeezed bispectra, and to a distinctive angular dependence between their three wavevector. Nevertheless the amplitude of the bispectrum is highly dependent on a non-trivial combination of the free parameters. Therefore, at the observational level, more work is needed in order to fully characterize the properties of higher point functions in these scenarios. In what concerns the tensor power spectrum, in ref. [33] it was shown that the extra terms that arise from the SDB invariance can produce a blue tilt, \( n_T > 0 \), for a certain range of values of the parameter.

Another viable model that predicts a blue tensor spectrum is the String Gas Cosmology (SGC) [38–43], an alternative scenario for the early universe that arises from a combination of basic principles of superstring theory and cosmology. In the SGC model of refs. [44, 45], a consistency relation between the scalar spectral index \( n_s \) (which corresponds to a red spectrum) and that of the tensor modes was derived, i.e.,

\[
 n_T \approx -(n_s - 1) .
\]  

(1.1)

On the other hand, current observations indicate that \( n_s = 0.9655 \pm 0.0062 \ (1\sigma) \) [20]. Therefore, if the ratio \( r \) of the tensor spectrum to the scalar spectrum is not too small, then future CMB polarisation experiments may have the potential to differentiate between the simple inflationary consistency relation and the one from SGC.

The consistency relation above can be reproduced by an EFT of inflation with breaking of spatial diffeomorphism invariance for a certain combination of the parameters [32]. In order to distinguish these models with future observations it is important to analyse if such a combination of parameters is allowed by the current data. This is one of the goals of the present work. By performing a Bayesian analysis using the most recent Planck and BICEP data, we test the observational viability of SDB model in the context of the EFT of inflation. We also discuss a possible observational distinction between the SDB model and the String Gas Cosmology. This paper is organized as follows. Section 2 reviews the basic assumptions of the SDB model. In section 3 we evaluate the power spectrum predicted by this model. The method, observational data sets and priors used in the Bayesian analysis are discussed in section 4. In section 5 we discuss our main results. We summarize our conclusions in section 6.

## 2 Spatial diffeomorphism breaking in the EFT

In what follows, we will consider the effective field theory for cosmological perturbations around a quasi-de Sitter background, with both temporal and spatial diffeomorphism invariance broken (but with isotropy preserved). We consider the following perturbed metric

\[
d s^2 = g_{\mu\nu}(x, \eta) dx^\mu dx^\nu .
\]  

(2.1)

The metric fluctuations are defined by

\[
h_{\mu\nu}(x, \eta) = g_{\mu\nu}(x, \eta) - \bar{g}_{\mu\nu}(\eta) ,
\]  

(2.2)

where \( \eta \) is the conformal time and \( \bar{g}_{\mu\nu}(\eta) \) is the background metric. We will focus on operators at most quadratic in the fluctuations for simplicity. The approach usually adopted in the
effective theory considers the unitary gauge, in which there are no perturbations of the inflaton field and all the fluctuations degrees are in the metric.

Following refs. [32–34] the SDB invariance will be described through effective mass terms in the action for cosmological perturbations. These terms simply correspond to the most general way to express quadratic non-derivative operators in the fluctuations that break the spatial diffeomorphism symmetry. To the usual Einstein-Hilbert action expanded to second order, we add generic operators with no derivatives that are quadratic in the metric fluctuations $h_{\mu\nu}$. We consider the following action

$$S = \int d^4x \sqrt{-g} M_{pl}^2 \left[ R - 2\Lambda - 2cg^{00} \right]$$

$$+ \frac{1}{4} M_{pl}^2 \int d^4x \sqrt{-g} [m_0^2 h_{00}^2 + 2m_1^2 h_{0i}^2 - m_2^2 h_{ij}^2 + m_3^2 h_{ii}^2 - 2m_4^2 h_{00} h_{ii}] .$$

The terms in the first line are the ones which contribute to the homogeneous and isotropic background. The parameters $c$ and $\Lambda$ can be expressed as functions of the Hubble parameter ($H$) and its time derivative. The term proportional to $m_0^2$ breaks time diffeomorphism invariance and the other mass terms break spatial diffeomorphism invariance. The invariance is restored for the limit $m_i \to 0$ with $i \neq 0$.

One may consider the mass terms in the above equation as arising from couplings between the metric and fields with a time-dependent profile during inflation. Following [32, 33], we assume as an approximation that their coefficients are effectively constant during inflation, and go to zero after inflation ends. However a small time-dependence proportional to the slow-roll parameter ($\epsilon$) should be expected for these coefficients, which can be neglected for our purposes here.

As in [32], we can define a particular combination of masses parameters,

$$M^2 = \frac{-2m_2^2 H^2 (m_2^2 - 3m_3^2 + (3 + \epsilon)m_4^2)}{(m_0^2 + 2cH^2)(m_2^2 - m_3^2) + m_4^2} .$$

As it will become clear latter, there is an important quantity which appears in the predictions of the model, which we define as

$$\alpha \equiv \sqrt{2cM^2 H^2 + 3\epsilon + \frac{M^2}{H^2} + 9/4} .$$

As usual, we can decompose the fluctuations in terms of scalar, vector and tensor perturbations, i.e.,

$$h_{00} = \psi,$$

$$h_{0i} = u_i + \partial_i \psi,$$

$$h_{ij} = \chi_{ij} + \partial_i \sigma + \partial_j \sigma + \delta_{ij} \tau,$$

with $\partial_i u_i = 0$, $\partial_i \chi_{ij} = 0$.

From the tensor part of the action it was obtained in [33] the following tensor power spectrum

$$P_T = \frac{2H^2}{\pi^2 M_{pl}^2 c_T} \left( \frac{k}{k_*} \right)^{n_T} ,$$

$$n_T = -2\epsilon + \frac{2m_2^2}{3H^2} \left( 1 + \frac{4}{3} \epsilon \right) .$$
to first order in the slow-roll parameter. In the equation above \( c_T = 1 \) [32, 33] and \( k_* \) denotes the \( k \) pivot. We can see that the parameters \( m_0, m_1, m_3 \) and \( m_4 \) do not appear in the action for tensor perturbations. We can also see that if \( m_2^2/H^2 \) is positive and sufficiently larger than the slow-roll parameter, then we have a positive tensor spectral index.

On the other hand, from the scalar part of the action the following power spectrum was obtained in ref. [32]:

\[
P_R = \frac{k^3 |v|^2}{a^2 N^2} = \frac{k^{3-2\alpha}}{8\pi^2} \left( \frac{c_s^{-2\alpha}}{2\alpha^2 N^2} \right) \left( \frac{|\eta|}{\sqrt{\alpha^2 - \frac{1}{4}}} \right)^{1-2\alpha},
\]  

(2.9)

where

\[
c_s^2 = 2\epsilon H^2 \left( \frac{m_2^2 - m_3^2}{m_0^2 + 2\epsilon H^2 \left( m_2^2 - m_3^2 \right) + m_4^2} \right),
\]

(2.10)

and

\[
N^2 \equiv \left( \frac{M_{pl}^2}{H^2} \right) \frac{(m_0^2 + 2\epsilon H^2 \left( m_2^2 - m_3^2 \right) + m_4^2)}{2(m_2^2 - m_3^2)}.
\]

(2.11)

Note that in the limit when the extra terms in the action go to zero, \( c_s^2 \to 1 \) and \( N^2 \to \epsilon \). Note also that \( N^2 \) can be written as a function of \( c_s^2 \), as \( N^2 = M_{pl}^2 \epsilon / c_s^2 \). Since in this model the curvature perturbation is not constant after Hubble radius crossing [33], an explicit time-dependence appears in eq. (2.9). We need to evaluate the result at the end of inflation (\( \eta = \text{const.}, \ a = \text{const.} \)).

We can write a simplified expression for the spectral index by expanding the square root in the expression 2.5 for \( \alpha \) as follows,

\[
n_s - 1 = 3 - 2\alpha = 3 - 3\sqrt{\frac{8\epsilon M^2}{9 H^2} + \frac{4}{3} \epsilon + \frac{4 M^2}{9 H^2} + 1}
\]

\[
\approx -2\epsilon - \frac{2 M^2}{3 H^2}.
\]

(2.12)

Although the scalar spectrum must be calculated at the end of inflation, it is possible to show that the slow-roll parameter that appears in the above equations corresponds to the one at the moment of Hubble radius crossing [32]. In general, quantities calculated at horizon crossing are marked with “\( \ast \)” but, for simplicity, we will omit such a symbol. For instance, \( \epsilon_* \) will be denoted just by \( \epsilon \). On the other hand, quantities calculated at the end of inflation will be indicated as such.

As shown in [32], the approximated SGC consistency relation (1.1) can be recovered in this model if the parameters satisfy the following relation,

\[
-2\epsilon + \frac{2 m_2^2}{3 H^2} (1 + 2\epsilon) = +2\epsilon + \frac{2 M^2}{3 H^2} (1 + 2\epsilon),
\]

(2.13)

which can be written as a function of the parameter \( \alpha \) by the approximated expression

\[
\frac{m_2^2}{H^2} \approx \frac{9}{2} + 3\epsilon + 3\alpha.
\]

(2.14)

In section IV we shall use current observational data to obtain the allowed range of values for \( M^2 \) and \( m_2^2 \) (or equivalently for \( \alpha \) and \( m_2^2 \)).
3 Theoretical analysis

Let us analyse the expression of the power spectrum (2.9). Since we suppose the SDB to be small (and \( c_s^2 \rightarrow 1 \) in the absence of this breaking), then in many cases of interest we can express \( c_s \) as \( c_s = 1 - \delta \), where \( \delta \) is much smaller than 1. By expressing the quantity \( N^2 = M^2 c_s^2 / c_s^2 \) and considering \( c_s^2 \approx 1 \), we can obtain a much simpler expression for the power spectrum,

\[
P_R = \frac{k^3 |v|^2}{a^2 N^2} = \frac{k^3}{8\pi^2} \left( \frac{1}{2a_{\text{end}}^2 c_s^2} \right) \left( \frac{|\eta_{\text{end}}|}{\sqrt{a^2 - \frac{1}{4}}} \right)^{1 - 2\alpha},
\]

which is a function of a single free parameter \( \alpha \) encoding the effects of the SDB. Although this approximation for the amplitude only contemplates a class of the SDB models, it almost does not affect the constraints we will derive on the spectral indexes, so that the results for these quantities remain quite general. We write the conformal time at the end of inflation \( \eta_{\text{end}} \) and the scale factor at the end of inflation \( a_{\text{end}} \) as a function of the respective quantities in the moment of Hubble radius exit and also as a function of the number of e-folds \( N \), i.e.,

\[
a_{\text{end}} \approx a_s e^N, \quad \eta_{\text{end}} \approx e^{-N} \eta_s \approx -e^{-N}/H_s a_s.
\]

By substituting the above expressions in eq. (3.1), and omitting the Hubble radius crossing index *, we obtain

\[
P_R \approx \frac{1}{8\pi^2} \left( \frac{k}{k_s} \right)^{3 - 2\alpha} \left( \frac{H^2}{2\epsilon^2} \right) \left( \frac{e^{-N}}{\sqrt{a^2 - \frac{1}{4}}} \right)^{1 - 2\alpha},
\]

In the case in which spatial diffeomorphism is preserved, the curvature perturbation is conserved after Hubble radius crossing, and the power spectrum is then computed at this crossing moment. Therefore in order to recover the usual case we substitute \( a_{\text{end}} \rightarrow a_s \) and \( \eta_{\text{end}} \rightarrow \eta_s \), which can be done by making \( N = 0 \) in the above equation. Also, in this limit, we can see from eq. (2.12) that by making \( M = 0 \) (i.e. neglecting the contribution from SDB), we obtain \( \alpha \approx 3/2 + \epsilon \). We can see that by considering these values in the above equation we recover the usual expression for the power spectrum of inflation, as expected.

For the purpose of making a theoretical analysis here let us write \( \alpha \) as \( \alpha \approx 3/2 + \delta \alpha \), where \( \delta \alpha \) encompasses the contribution from \( \epsilon \) plus the contribution coming from the SDB (\( \delta \alpha \approx \epsilon + M^2/(3H^2) \)) from eq. (2.12)).\(^1\) Using this notation in the above equation we obtain

\[
P_R \approx \frac{1}{8\pi^2} \left( \frac{k}{k_s} \right)^{-2\delta \alpha} \left( \frac{H^2}{\epsilon} \right) e^{2N\delta \alpha},
\]

where the number of e-folds between the Hubble radius crossing and the end of inflation is usually considered to be \( N \approx 60 \). Note that, if \( \alpha = 3/2 \), and consequently \( \delta \alpha = 0 \), a Harrison-Zeldovich spectrum is obtained. For \( \delta \alpha = \epsilon \) we have an almost scale invariant spectrum with the small red tilt, as predicted by usual inflationary models. In this case, the amplitude of the power spectrum is given by \( A_s \approx H^2/(8\pi^2\epsilon) \). We can see from the above equation that, since \( \delta \alpha \) is in the exponent and it is multiplied by \( N \), the effect of the SDB is exponentially

\(^1M^2 \) can be either positive or negative, just like \( m_i^2 \).
amplified in the amplitude of the scalar power spectrum. Therefore, one must expect that the combination of mass parameters described by \( M^2 \) will be quite constrained around zero. The same, however, is not expected for the parameter \( m_2^2 \), which enters in the expression of the tensor tilt.

4 Analysis method

In order to compute the CMB anisotropies spectrum for a given range of values of the \( \alpha, H^2 \) and \( n_t \) parameters, we use a modified version of the CAMB code [46] where the primordial power spectra are given by eqs. (2.7) and (3.3). We performed a Monte Carlo Markov chain analysis using the available package CosmoMC [47] and implementing the nested sampling algorithm of MULTINEST code [48–50] to obtain our results and calculate the Bayesian evidence factor. In our Bayesian analysis we used the most accurate Importance Nested Sampling (INS) [50, 51] instead of the vanilla Nested Sampling (NS), requiring a INS Global Log-Evidence error of \( \leq 0.02 \).

In addition to the above primordial parameters, we vary the usual set of cosmological variables, namely the baryon density, the cold dark matter density, the ratio between the sound horizon and the angular diameter distance at decoupling, and the optical depth: \( \{ \Omega_b h^2, \Omega_c h^2, \theta, \tau \} \). We consider purely adiabatic initial conditions, fix the sum of neutrino masses to 0.06 eV, and also vary the nuisance foregrounds parameters [2].

It is important to emphasize that in SDB model the scalar primordial potential is not parameterized with the usual primordial scalar amplitude, \( A_s \), and primordial spectral index \( n_s \), as is done for the minimal ΛCDM model. We consider as reference for our comparison two models, namely: the ΛCDM+\( r \), where \( r \) is the tensor-to-scalar ratio parameter, which assumes inflation relation consistency (\( r = -8n_t \)) and the ΛCDM+\( r + n_t \), where the inflationary relation consistency is relaxed.

In our analysis we work with the following priors on the SDB primordial parameters:

\[
-2 < \ln(10^{10} H^2) < 3.5, \\
1.48 < \alpha < 1.54, \\
-1.5 < n_t < 5.5.
\]

We also assume that the \( \epsilon \) and \( N \) values do not change significantly with respect to the canonical inflationary chaotic model, and fix them to \( \epsilon = 1/121 \) and \( N = 60 \). The parameter \( \alpha \) mimics the ΛCDM spectral index (\( n_s \)). Thus, from eq. (3.3) one may expect values around \( \alpha \simeq 1.51 \). At the same time, joint effects of \( H^2 \) and \( \alpha \) parameters replace the canonical primordial scalar amplitude. In such a degeneration of parameters, the amplitude increases considerably if a small increase in the value of \( \alpha \) is made. Therefore, the \( H^2 \) value must compensate it by varying in a larger interval.

We use the CMB data set from the latest Planck Collaboration release [20], considering the high multipoles Planck temperature data (\( TT \)) from the 100-, 143-, and 217-GHz half-mission T maps, and the low multipoles data (\( low-P \)) by the joint TT, EE, BB and TE likelihood, where EE and BB are the E- and B-mode CMB polarization power spectrum and TE is the cross-correlation temperature-polarization. We also use B-mode polarization data from the BICEP2 Collaboration [3, 4] to constraint the parameters associated to the tensor spectrum, using the combined BICEP2/Keck-Planck likelihood, hereafter BKP. Finally, we perform our analysis for a scalar and tensor pivot scale \( k_s = 0.01 \, \text{Mpc}^{-1} \), since at this value
the BKP release data are most sensitive and it is close to the decorrelation scale between the
tensor amplitude and slope for Planck and BKP joint constraints [20].

In order to make an appropriate comparison between the SDB model and the ΛCDM
scenario we use the Bayesian model comparison (see refs. [52, 53] for a more detailed discus-
sion). The Bayesian evidence \( \mathcal{E} \) is defined as the marginal likelihood for the model \( M_i \):

\[
\mathcal{E}_{M_i} = \int d\theta p(x|\theta, M_i) \pi(\theta|M_i).
\]

(4.1)

where \( x \) stands for the data, \( \theta \) is the parameters vector and \( \pi(\theta|M_i) \) the prior probability
distribution function. The ratio of the Bayesian evidence of the two models (the so-called
Bayes Factor) can be defined as \( B_{ij} = \mathcal{E}_{M_i}/\mathcal{E}_{M_j} \), where \( M_j \) is the reference model. We
assume uniform (hence separable) priors in each parameter, such that its possible to write
\( \pi(\theta|M) = (\Delta \theta_1 \ldots \Delta \theta_n)^{-1} \) and

\[
B_{ij} = \frac{\int d\theta p(x|\theta, M_i) (\Delta \theta_1 \ldots \Delta \theta_n)}{\int d\theta p(x|\theta', M_j) (\Delta \theta'_1 \ldots \Delta \theta'_n)}.
\]

(4.2)

In order to classify the models we use the Jeffreys scale [54] which gives empirically
calibrated levels of significance for the strength of evidence. Here we will use a revisited
version of the Jeffreys convention suggested in [55] where \( \ln B_{ij} = 0 \) − 1, \( \ln B_{ij} = 1 \) − 2,
\( \ln B_{ij} = 2.5 \) − 5, and \( \ln B_{ij} > 5 \) indicate an inconclusive, weak, moderate and strong preference
of the model \( M_i \) with respect to the model \( M_j \).²

5 Results

Figure 1 shows the good agreement between the cosmological parameters constrained in
the context of the SDB model (cyan) and the reference models one, namely the ΛCDM+r
(red) and the ΛCDM+r+nt (black dashed). In the left panel of figure 2, the posterior
probability comparison between the scalar spectral index of the ΛCDM+tensor models and
the \( \alpha \) parameter of the SDB model shows a comparable goodness of constraint, while the
shift of the mean value (as discussed in the previous section) depends on our parametrization
choice. In the right panel of figure 2, we show the posterior curves for the logarithmic values
of the primordial scalar amplitude of the reference models and the \( H^2 \) parameter of the SDB.
As expected, the degeneracy between the parameters \( H^2 \) and \( \alpha \) (see figure 3) discussed in
the previous section causes a worsening in the \( H^2 \) constraint. Our analysis show that the
difference between the \( H^2 \) constraints and the primordial amplitude of is significant.

In figure 4, we show the posterior density probability for the tensor spectral index
and the tensor-to-scalar ratio parameters. The latter is treated as derived parameter in the
SDB model and is calculated as the ratio between the tensor and the scalar power spectrum
amplitude at the pivot scale \( k_{0,01} \). Firstly, we note that the data allow a larger range of blue
tensor tilt, excluding negative values at 68% C.L. Such result is fully consistent with previous
analysis from the Planck Collaboration [20] using both temperature and polarization CMB
data, and with the recent analysis of ref. [21], where joint CMB and BKP data are used. In
the right panel of figure 4 we also note the very sharp constraints on \( r_{0,01} \) for the SDB model.
Such a narrow bound (\( r_{0,01} = 0.017 \pm 0.007 \) at 1σ) is due to our parametrization choice, since
the tensor amplitude is directly linked to the scalar one (both depending on \( H^2 \)). We stress

²Note that \( \ln B_{ij} < 0 \) means support in favor of the reference model \( M_j \).
that our parametrization worsens the constraints on the scalar amplitude but improves the tensor amplitude bound.

In table 1 we summarize the results of our analysis for the $H^2$, $\alpha$ and $n_t$ parameters of the SDB model. The parameter $\alpha$ is constrained to be $1.517 \pm 0.003$ at 68% confidence level,
Table 1. 68% confidence limits for the cosmological parameters using TT+lowP+BKP data. The $\Delta \chi^2_{\text{best}}$ and the $\ln B_{ij}$ refers to the difference with respect to the $\Lambda$CDM+$r$ with fixed consistency relation. The subscript “($n_t$)” refers to the difference with respect to the $\Lambda$CDM+$r+n_t$ (with relaxed consistency relation).

| Parameter                  | $SDB$ Model |
|----------------------------|--------------|
| $\ln(10^{10}H^2)$         | $0.605 \pm 0.384$ |
| $\alpha$                  | $1.517 \pm 0.003$ |
| $n_t$                     | $1.914 \pm 1.150$ |
| $\Delta \chi^2_{\text{best}}$ | $0$          |
| $\ln B_{ij}$              | $-15.8$      |
| $\Delta \chi^2_{\text{best}(+n_t)}$ | $+3.9$      |
| $\ln B_{ij(+n_t)}$        | $+1.6$       |

which entails a corresponding $n_s$ value very similar to the standard inflationary prediction. This implies that the parameter $M^2/H^2$, which quantifies the effect of the diffeomorphism breaking in the scalar spectrum, is constrained to be very small. However, from eq. (2.4) we can see that it is possible to have $M \approx 0$ along with not so small values of $m_2$ ($m_2^2/H^2$ of order $\epsilon$, for example).

We emphasize that positive values of $n_t$ correspond to values of $m_2^2$ such that $m_2^2/H^2 > 3\epsilon$ (see eq. (2.8)). A combined analysis involving the scalar and tensor spectral indexes show that a red tensor tilt ($m_2^2/H^2 < 3\epsilon$) is only marginally allowed by the data at 1$\sigma$, while a larger range of positive values for $m_2^2$ is supported.\(^3\)

In the last lines of the table 1, we show the $\Delta \chi^2$ and the Bayes factor of such SDB model with respect to the $\Lambda$CDM+tensor scenarios, when the inflationary consistency relation is considered (first lines) or not (latest lines). In the first case, while the two best fit models describe the data with the same goodness ($\Delta \chi^2 = 0$), the Bayesian analysis shows a strong preference of the data for the $\Lambda$CDM+$r$ model. This result can be well explained and understood by considering two factors. Firstly, the degeneration between $\alpha$ and $H^2$ parameters and the consequent deterioration of the $H^2$ constraint with respect to the standard model primordial amplitude parameter. Secondly, and more important, the fact that we are comparing our model (with tensor index free to vary) with a $\Lambda$CDM+$r$ one (which assumes the inflation consistency relation). If we relax such relation in the $\Lambda$CDM+$r+n_t$ model, we obtain a $n_t$ value of $1.6 \pm 0.9$ and a Bayes factor of $\ln B_{ij} = +1.6$. This implies a weak evidence in favor of the SDB model with respect to the $\Lambda$CDM. We also note that, if the tensor index is left free to vary in the standard cosmological model, a significant deterioration in its Bayesian evidence is produced. Moreover, as said above, the improved evidence of the SDB model with respect to the latter is due to our parametrization choice on the tensor amplitude (e.g. in terms of $H$), which leads to the sharp constraints on the parameter $r$.

Finally, in order to compare the predictions of the SGC and SDB models, we derive the scalar spectral index parameter $n_s$ for the SDB scenario, from the $\alpha$ constraints, using eq. (2.12). In figure 5, we show the observational bounds on $n_s$ and $n_t$ plane. The red and the

\(^3\)We must point out that although the data prefer higher values of $m_2^2/H^2$ this theoretical model is justified only for values of $m_2^2/H^2 \ll 1$. However, for illustrative purposes, we show in the figures all values for the tensor tilt allowed by the data. Nevertheless, the two lines shown in figure 5 are associated with values of $m_2^2$ lying within the range of strict validity of the theory.
blue lines show, respectively, the approximated consistency relations of inflation \((n_t \approx n_s - 1)\) and SGC \((n_t \approx -(n_s - 1))\), where both can be recovered by specific values of the SDB model parameters. As can be seen, both models are compatible with the data at 1\(\sigma\), although the SGC (and SDB) combination of parameters lies closer to the mean value. In other words, this amounts to saying that the current CMB data cannot distinguish between standard inflation and String Gas Cosmology (or the specific SDB model associated to it).

6 Conclusions

The assumption of the breaking of spatial diffeomorphism invariance changes the prediction of simple slow-roll inflationary models, allowing to obtain values of \(n_t > 0\). Through an observational analysis we have obtained constraints on the parameters of the model and also tested the specific combinations of values which recover the consistency relation of the SGC. While the scalar predictions of the SDB model are very similar to the ones expected from usual inflation, the tensorial predictions can be quite different. The parameter space of tensor parameters allowed within 1\(\sigma\) are mostly dominated by values of the SDB parameter \(m_2 > 0\). Also, we have shown that the parameters of the SDB model, which recover the SGC and inflationary consistency relation, are compatible with the data at 1\(\sigma\).
Figure 4. Posterior density probability distribution for the SDB (cyan line) and ΛCDM+r primordial tensor parameters with (red line) and without (black dashed line) inflation consistency relation arising from TT+lowP+BKP data.

Figure 5. The 68% and 95% confidence regions for the parameters $n_s$ and $n_t$. The blue and the red lines correspond to the approximated consistency relations for SGC and standard inflation, respectively. The figure on the right corresponds to a zoom of the figure on the left.

We have obtained an upper limit on the tensor-to-scalar ratio for the SDB model that is smaller than the one predicted by the usual inflation. Moreover, through a Bayesian analysis we have shown that, by comparing the SDB model and ΛCDM+r scenario, which assumes the inflation consistency relation, a strong evidence in favor of the latter is found. On the other hand, when the consistency relation is relaxed we have found a weak evidence in favor of the model with diffeomorphism breaking. Finally, it is worth emphasizing that there are great prospects that the next generation of experiments will improve the results obtained here, which encourages further analyzes of the SDB and SGC models. Furthermore, since the SGC consistency relation lies within the range of allowed values of the SDB parameters (within 1σ), we stress the importance of analyzing other predictions of the model, like the non-Gaussianities for instance [34, 35].
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References

[1] WMAP collaboration, G. Hinshaw et al., Nine-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: cosmological parameter results, Astrophys. J. Suppl. 208 (2013) 19 [arXiv:1212.5226] [SPIRE].

[2] PLANCK collaboration, N. Aghanim et al., Planck 2015 results XI. CMB power spectra, likelihoods and robustness of parameters, Astron. Astrophys. 594 (2016) A11 [arXiv:1507.02704] [SPIRE].

[3] BICEP2 and PLANCK collaborations, P.A.R. Ade et al., Joint analysis of BICEP2/Keck Array and Planck data, Phys. Rev. Lett. 114 (2015) 101301 [arXiv:1502.00612] [SPIRE].

[4] BICEP2 and Keck Array collaborations, P.A.R. Ade et al., Improved constraints on cosmology and foregrounds from BICEP2 and Keck Array cosmic microwave background data with inclusion of 95 GHz band, Phys. Rev. Lett. 116 (2016) 031302 [arXiv:1510.09217] [SPIRE].

[5] LIGO Scientific collaboration, B.P. Abbott et al., LIGO: the Laser Interferometer Gravitational-wave Observatory, Rept. Prog. Phys. 72 (2009) 076901 [arXiv:0711.3041] [SPIRE].

[6] LIGO Scientific collaboration, J. Abadie et al., A gravitational wave observatory operating beyond the quantum shot-noise limit: squeezed light in application, Nature Phys. 7 (2011) 962 [arXiv:1109.2295] [SPIRE].

[7] VIRGO collaboration, F. Acernese et al., Status of VIRGO, Class. Quant. Grav. 22 (2005) S869 [gr-qc/0406123] [SPIRE].

[8] VIRGO collaboration, F. Acernese et al., Advanced VIRGO: a second-generation interferometric gravitational wave detector, Class. Quant. Grav. 32 (2015) 024001 [arXiv:1408.3978] [SPIRE].

[9] R.D. Ferdman et al., The European Pulsar Timing Array: current efforts and a LEAP toward the future, Class. Quant. Grav. 27 (2010) 084014 [arXiv:1003.3405] [SPIRE].

[10] T. Matsumura et al., Mission design of LiteBIRD, J. Low Temp. Phys. 176 (2014) 733 [arXiv:1311.2847] [SPIRE].

[11] A. Kogut et al., The Primordial Inflation Explorer (PIXIE): a nulling polarimeter for cosmic microwave background observations, JCAP 07 (2011) 025 [arXiv:1105.2044] [SPIRE].

[12] CORE collaboration, F.R. Bouchet et al., COrE (Cosmic Origins Explorer) a white paper, arXiv:1102.2181 [SPIRE].

[13] LIGO Scientific collaboration, J. Aasi et al., Advanced LIGO, Class. Quant. Grav. 32 (2015) 074001 [arXiv:1411.4547] [SPIRE].

[14] E. Calabrese et al., Precision epoch of reionization studies with next-generation CMB experiments, JCAP 08 (2014) 010 [arXiv:1406.4794] [SPIRE].

[15] M.D. Niemack et al., ACTPol: a polarization-sensitive receiver for the Atacama Cosmology Telescope, Proc. SPIE Int. Soc. Opt. Eng. 7741 (2010) 77411S [arXiv:1006.5049] [SPIRE].
[16] J. Martin, C. Ringeval and V. Vennin, Encyclopædia inflationaris, Phys. Dark Univ. 5-6 (2014) 75 [arXiv:1303.3787] [inSPIRE].

[17] M. Benetti and J.S. Alcaniz, Bayesian analysis of inflationary features in Planck and SDSS data, Phys. Rev. D 94 (2016) 023526 [arXiv:1604.08156] [inSPIRE].

[18] L.L. Graef, W.S. Hipolito-Ricaldi, E.G.M. Ferreira and R. Brandenberger, Dynamics of cosmological perturbations and reheating in the anamorphic universe, JCAP 04 (2017) 004 [arXiv:1701.02654] [inSPIRE].

[19] W.S. Hipolito-Ricaldi, R. Brandenberger, E.G.M. Ferreira and L.L. Graef, Particle production in ekpyrotic scenarios, JCAP 11 (2016) 024 [arXiv:1605.04670] [inSPIRE].

[20] Planck collaboration, P.A.R. Ade et al., Planck 2015 results XX. Constraints on inflation, Astron. Astrophys. 594 (2016) A20 [arXiv:1502.02114] [inSPIRE].

[21] G. Cabass, L. Pagano, L. Salvati, M. Gerbino, E. Giusarma and A. Melchiorri, Updated constraints and forecasts on primordial tensor modes, Phys. Rev. D 93 (2016) 063508 [arXiv:1511.05146] [inSPIRE].

[22] T. Kobayashi, M. Yamaguchi and J. Yokoyama, G-inflation: inflation driven by the Galileon field, Phys. Rev. Lett. 105 (2010) 231302 [arXiv:1008.0603] [inSPIRE].

[23] P. Creminelli, M.A. Luty, A. Nicolis and L. Senatore, Starting the universe: stable violation of the null energy condition and non-standard cosmologies, JHEP 12 (2006) 080 [hep-th/0606090] [inSPIRE].

[24] A. Berera, Warm inflation, Phys. Rev. Lett. 75 (1995) 3218 [astro-ph/9509049] [inSPIRE].

[25] M. Benetti and R.O. Ramos, Warm inflation dissipative effects: predictions and constraints from the Planck data, Phys. Rev. D 95 (2017) 023517 [arXiv:1610.08758] [inSPIRE].

[26] N. Kaloper, M. Kleban, A.E. Lawrence and S. Shenker, Signatures of short distance physics in the cosmic microwave background, Phys. Rev. D 66 (2002) 123510 [hep-th/0201158] [inSPIRE].

[27] D. Baumann, H. Lee and G.L. Pimentel, High-scale inflation and the tensor tilt, JHEP 01 (2016) 101 [arXiv:1507.07250] [inSPIRE].

[28] N. Barnaby, E. Pajer and M. Peloso, Gauge field production in axion inflation: consequences for monodromy, non-Gaussianity in the CMB and gravitational waves at interferometers, Phys. Rev. D 85 (2012) 023525 [arXiv:1110.3327] [inSPIRE].

[29] S. Mukohyama, R. Namba, M. Peloso and G. Shiu, Blue tensor spectrum from particle production during inflation, JCAP 08 (2014) 036 [arXiv:1405.0346] [inSPIRE].

[30] E. Dimastrogiovanni, M. Fasiello and T. Fujita, Primordial gravitational waves from axion-gauge fields dynamics, JCAP 01 (2017) 019 [arXiv:1608.04216] [inSPIRE].

[31] M. Biagetti, M. Fasiello and A. Riotto, Enhancing inflationary tensor modes through spectator fields, Phys. Rev. D 88 (2013) 103518 [arXiv:1305.7241] [inSPIRE].

[32] L. Graef and R. Brandenberger, Breaking of spatial diffeomorphism invariance, inflation and the spectrum of cosmological perturbations, JCAP 10 (2015) 009 [arXiv:1506.00896] [inSPIRE].

[33] D. Cannone, G. Tasinato and D. Wands, Generalised tensor fluctuations and inflation, JCAP 01 (2015) 029 [arXiv:1409.6568] [inSPIRE].

[34] N. Bartolo, D. Cannone, A. Ricciardone and G. Tasinato, Distinctive signatures of space-time diffeomorphism breaking in EFT of inflation, JCAP 03 (2016) 044 [arXiv:1511.07414] [inSPIRE].

[35] G. Domènech, T. Hiramatsu, C. Lin, M. Sasaki, M. Shiraishi and Y. Wang, CMB scale dependent non-Gaussianity from massive gravity during inflation, JCAP 05 (2017) 034 [arXiv:1701.05554] [inSPIRE].
[36] C. Lin and L.Z. Labun, Effective field theory of broken spatial diffeomorphisms, JHEP 03 (2016) 128 [arXiv:1501.07160] [SPIRE].

[37] C. Cheung, P. Creminelli, A.L. Fitzpatrick, J. Kaplan and L. Senatore, The effective field theory of inflation, JHEP 03 (2008) 014 [arXiv:0709.0293] [SPIRE].

[38] R.H. Brandenberger and C. Vafa, Superstrings in the early universe, Nucl. Phys. B 316 (1989) 391 [SPIRE].

[39] C. Cheung, P. Creminelli, A.L. Fitzpatrick, J. Kaplan and L. Senatore, The effective field theory of inflation, JHEP 03 (2008) 014 [arXiv:0709.0293] [SPIRE].

[40] R.H. Brandenberger and C. Vafa, Superstrings in the early universe, Nucl. Phys. B 316 (1989) 391 [SPIRE].

[41] R.H. Brandenberger, A. Nayeri, S.P. Patil and C. Vafa, Tensor modes from a primordial Hagedorn phase of string cosmology, Phys. Rev. Lett. 98 (2007) 231302 [hep-th/0604126] [SPIRE].

[42] R.H. Brandenberger, A. Nayeri, S.P. Patil and C. Vafa, Tensor modes from a primordial Hagedorn phase of string cosmology, Phys. Rev. Lett. 98 (2007) 231302 [hep-th/0604126] [SPIRE].

[43] R.H. Brandenberger, A. Nayeri, S.P. Patil and C. Vafa, Tensor modes from a primordial Hagedorn phase of string cosmology, Phys. Rev. Lett. 98 (2007) 231302 [hep-th/0604126] [SPIRE].