Black hole, string ball, and p-brane production at hadronic supercolliders

Kingman Cheung
National Center for Theoretical Sciences, National Tsing Hua University, Hsinchu, Taiwan, R.O.C.

In models of large extra dimensions, the string and Planck scales become accessible at future colliders. When the energy scale is above the string scale or Planck scale a number of interesting phenomena occur, namely, production of stringy states, p-branes, string balls, black hole, etc. In this Proceedings, we summarize a recent work [1] on the production of black holes, string balls, and p-branes at hadronic supercolliders, and discuss their signatures.

I. INTRODUCTION

In a model of large extra dimensions (ADD model) [2], the fundamental Planck scale can be as low as a few TeV, which is made possible by localizing the SM particles on a brane while gravity is free to propagate in all dimensions. The observed Planck scale ($\sim 10^{19}$ GeV) is then a derived quantity. Signatures for the ADD model can be divided into two categories: sub-Planckian and trans-Planckian. The former has been studied extensively, while the latter just recently received more attention, especially black hole production in hadronic collisions.

The fact that the fundamental Planck scale is as low as TeV opens up an interesting possibility of producing a large number of black holes at collider experiments (e.g. LHC) [3, 4, 5]. Reference [6] showed that a BH localized on a brane will radiate mainly in the brane, instead of radiating into the Kaluza-Klein states of gravitons of the bulk. In this case, the BH so produced will decay mainly into the SM particles, which can then be detected in the detector. This opportunity has enabled investigation of the properties of BH at terrestrial collider experiments.

An important quantity of a BH is its entropy $S_{\text{BH}}$. To fulfill the thermodynamical description, a BH requires a large entropy of order of 25 [3]. Such an entropy requirement implies that the BH mass must be at least five times the fundamental Planck scale [7, 8]. This mass requirement makes the BH production not as large as previously calculated in a number of works [3-5], first pointed out in Ref. [8]. In addition, the signature of large multiplicity decay of a BH can only happen when the entropy is large. Even taking into account this mass requirement, the event rate is still large enough for detection. A typical signature of the BH decay is a high multiplicity, isothermal event, very much like a spherical “fireball.”

Other interesting trans-Planckian phenomena include string balls [9] and p-branes [10]. Dimopoulos and Emparan [9] pointed out that when a BH reaches a minimum mass, it transits into a state of highly excited and jagged strings – a string ball (SB). The transition point is at

$$M_{\text{min}}^\text{SB} = \frac{M_s}{g_s^2},$$

where $M_s$ is the string scale and $g_s$ is the string coupling. Naively, SB’s are stringy progenitors of BH’s. The BH correspondence principle states that properties of a BH with a mass $M_{\text{BH}} = M_s/g_s^2$ match those of a string ball with $M_{\text{SB}} = M_s/g_s^2$. We can then equate the production cross sections of SB and BH at the transition point. The existence of string balls could be argued from the string point of view. When the energy of the scattering reaches the string scale, the scattering of particles is no longer described by point-particle scattering but replaced by string-string scattering. As the energy goes further up, the strings become highly excited, jagged and entangled string states, and become like a string ball. When

* Talk presented in the SUSY02, DESY, Hamburg, Germany, June 2002
the energy reaches the transition point, it turns into a BH. In the above we mentioned a large entropy requirement on the BH in order for the object to be a BH. Such a large mass requirement makes the production cross section smaller than previously thought. Here in the case of SB’s, the mass requirement is substantially lower, thus the production rate is significantly higher. Hence, an SB is more interesting in the experimental point of view if it decays with a distinct signature. Dimopoulos and Emparan argued that the decay of a SB is similar to that of a BH, i.e., a high multiplicity decay into the SM particles, though in some intermediate stages the SB decays more likely into bulk particles.

A \( p \)-brane is a solution to the Einstein equation in higher dimensions. A BH can be considered a 0-brane. In fact, the properties of \( p \)-branes reduce to those of a BH in the limit \( p \to 0 \). In extra dimension models, in which there are large extra dimensions and small extra dimensions of the size of the Planck length, let a \( p \)-brane wrap on \( r \) small and \( p-r \) large dimensions. It was found that the production of \( p \)-branes is comparable to BH’s only when \( r = p \), i.e., the \( p \)-brane wraps entirely on the small dimensions only. If \( r < p \), the production of \( p \)-branes would be suppressed by powers of \( (M_* / M_{Pl}) \), where \( M_* \) is the fundamental scale of the \( 4+n \) dimensions. Therefore, here we only consider the case in which \( r = p \). The decay of \( p \) branes is not well understood. One interesting possibility is cascade into branes of lower dimensions until they reach the dimension of zero. Whether the zero brane is stable depends on the model. Another possibility is the decay into brane and bulk particles, thus experimentally the decay can be observed. Or it can be a combination of cascade into lower-dimensional branes and direct decays.

\( p \)-brane production in the unconfined scenario (e.g. in universal extra dimensions or fat-brane scenario) was studied in Ref. [11].

BH production has also been studied in cosmic ray experiments and neutrino telescopes [12], which are complementary to or even better than the LHC for BH discovery.

II. PRODUCTION

A. Black holes

A black hole is characterized by its Schwarzschild radius \( R_{BH} \) and its entropy, which depend on the mass \( M_{BH} \) of the BH. A simplified picture for BH production is as follows. When the colliding partons have a center-of-mass (c.o.m.) energy above some thresholds of order of the Planck mass and the impact parameter less than the Schwarzschild radius \( R_{BH} \), a BH is formed and almost at rest in the c.o.m. frame. The BH so produced will decay thermally (regardless of the incoming particles) and thus isotropically in that frame.

The Schwarzschild radius \( R_{BH} \) of a BH of mass \( M_{BH} \) in \( 4+n \) dimensions is given by [13]

\[
R_{BH} = \frac{1}{M_D} \left( \frac{M_{BH}}{M_D} \right)^{\frac{n+1}{n+3}} \left( \frac{2^n \pi^{\frac{n-3}{2}} \Gamma(\frac{n+3}{2})}{n+2} \right)^{\frac{1}{n+3}} = \frac{1}{M_D} \left( \frac{M_{BH}}{M_D} \right)^{\frac{n+1}{n+3}} f(n) ,
\]

where \( f(n) \) is introduced for convenience and \( M_D \) is the fundamental Planck scale. The radius \( R_{BH} \) is much smaller than the size of the extra dimensions. When the colliding partons with a center-of-mass energy \( \sqrt{s} \gtrsim M_{BH} \) pass within a distance less than \( R_{BH} \), a black hole of mass \( M_{BH} \) is formed and the rest of the energy, if there is any, is radiated as ordinary SM particles [13]. This semiclassical argument calls for a geometric approximation for the cross section for producing a BH of mass \( M_{BH} \) as

\[
\sigma(M_{BH}^2) \approx \pi R_{BH}^2 .
\]

In the \( 2 \to 1 \) subprocess, the c.o.m. energy of the colliding partons is just the same as the mass of the
BH, i.e., $\sqrt{s} = M_{BH}$, which implies a subprocess cross section

$$\hat{\sigma}(\hat{s}) = \int d\left(\frac{M_{BH}^2}{\hat{s}}\right) \pi R_{BH}^2 \delta\left(1 - \frac{M_{BH}^2}{\hat{s}}\right) = \pi R_{BH}^2 .$$

On the other hand, for the $2 \to k (k \geq 2)$ subprocesses the subprocess cross section is

$$\hat{\sigma}(\hat{s}) = \int_{(M_{BH})_{\min}/\hat{s}}^1 d\left(\frac{M_{BH}^2}{\hat{s}}\right) \pi R_{BH}^2 .$$

The entropy of a BH is given by

$$S_{BH} = \frac{4\pi}{n + 2} \left(\frac{M_{BH}}{M_D}\right)^{n+2} \left(\frac{2^{n+3}}{\pi n} \Gamma\left(\frac{n+3}{2}\right)\right)^{\frac{1}{n+2}} .$$

To ensure the validity of the above classical description of a BH, the entropy must be sufficiently large, of order 25 or so. We verified that when $M_{BH}/M_D \geq 5$, the entropy $S_{BH} \gtrsim 25$. Therefore, to avoid getting into the nonperturbative regime of the BH and to ensure the validity of the semiclassical formula, we restrict the mass of the BH to be $M_{BH} \geq 5 M_D$.

### B. String balls

According to the BH correspondence principle, the production cross section of a string ball or a BH should be smoothly joined at $M_{BH} = M_s/g_s^2$, i.e.,

$$\sigma(SB)|_{M_{BH}=M_s/g_s^2} = \sigma(BH)|_{M_{BH}=M_s/g_s^2} .$$

The production cross section for string balls with mass between the string scale $M_s$ and $M_s/g_s$ grows with $s$ until $M_s/g_s$, beyond which, due to unitarity, it should stay constant. Therefore, we can use the BH cross section and match to the string ball cross section at the transition point $M_s/g_s^2$. This string ball cross section then stays constant between $M_s/g_s$ and $M_s/g_s^2$. Then below $M_s/g_s$ the string ball cross section grows like $M_{SB}^2/M_D^4$.

The cross sections for the SB or BH are given by

$$\hat{\sigma}(SB/BH) = \begin{cases} \frac{\pi}{M_D^2} \left(\frac{M_{BH}}{M_D}\right)^{\frac{n+2}{2}} |f(n)|^2 & M_{s/g_s^2} \leq M_{BH} \\ \frac{\pi}{M_D^2} \left(\frac{M_s/g_s}{M_D}\right)^{\frac{n+2}{2}} |f(n)|^2 = \frac{\pi}{M_D^2} |f(n)|^2 & M_{s/g_s} \leq M_{SB} \leq M_{s/g_s^2} \\ \frac{\pi}{M_D^2} \left(\frac{M_s/g_s^2}{M_D}\right)^{\frac{n+2}{2}} [f(n)]^2 & M_s \ll M_{SB} \leq M_{s/g_s^2} , \end{cases}$$

in which we have set $M_D^{n+2} = \frac{M_{s/g_s^2}^{n+2}}{g_s^2}$.

### C. $p$-Branes

Consider an uncharged and static $p$-brane with a mass $M_{PB}$ in $(4 + n)$-dimensional space-time ($m$ small Planckian size and $n - m$ large size extra dimensions such that $n \geq p$). Suppose the $p$-brane wraps on $r(\leq m)$ small extra dimensions and on $p - r(\leq n - m)$ large extra dimensions. Then the “radius” of the $p$-brane is

$$R_{PB} = \frac{1}{\sqrt{\pi M_s}} \gamma(n, p) V_{pB}^{\frac{1}{n+1-p}} \left(\frac{M_{PB}}{M_s}\right)^{\frac{n+1}{n+1-p}} ,$$

(8)
where $V_{pB}$ is the volume wrapped by the $p$-brane in units of the Planckian length. The $M_s$ is related to $M_D$ by

$$M_{D}^{n+2} = \frac{(2\pi)^n}{8\pi} M_{s}^{n+2}.$$ 

Recall $M_{D}^{n} = M_{s}^{n} l^{m}/l^{m}$, where $l_{n-m} \equiv L_{n-m} M_s$ and $l_{m} \equiv L_{m} M_s$ are the lengths of the size of the large and small extra dimensions in units of Planckian length ($\sim 1/M_s$). Then $V_{pB}$ is given by

$$V_{pB} = \frac{l_{n-m} l_{m}^{p-r}}{l_{m}^{r}},$$

where we have taken $l_{m} \equiv L_{m} M_s \sim 1$. The function $\gamma(n, p)$ is given by

$$\gamma(n, p) = \left[ \frac{8 \Gamma(3 + n - p)}{2} \right] \sqrt{\frac{1 + p}{(n + 2)(2 + n - p)}}^{\frac{1}{1 + n - p}}.$$ 

The $R_{pB}$ reduces to the $R_{BH}$ in the limit $p = 0$.

The production cross section of a $p$-brane is similar to that of BH’s, based on a naive geometric argument [10], i.e.,

$$\hat{\sigma}(M_{pB}) = \pi R_{pB}^2.$$ 

Therefore, the production cross section for a $p$-brane is the same as BH’s in the limit $p = 0$ (i.e., a BH can be considered a 0-brane). In $2 \to 1$ and $2 \to k$ ($k \geq 2$) processes, the parton-level cross sections are given by similar expressions in Eqs. (11) and (12), respectively. Assuming that their masses are the same and the production threshold $M_{min}$ is the same, the ratio of cross sections is

$$R \equiv \frac{\hat{\sigma}(M_{pB} = M)}{\hat{\sigma}(M_{BH} = M)} = \left( \frac{M_s}{M_D} \right)^{\frac{n-4(p-r)}{n+2(2+n-p)}} \left( \frac{M}{M_s} \right)^{\frac{2p}{1+n(2+n-p)}} \left( \frac{\gamma(n, p)}{\gamma(n, 0)} \right)^2.$$ 

In the above equation, the most severe suppression factor is in the first set of parentheses on the right-hand side. Since we are considering physics of TeV $M_s$, the factor $(M_s/M_D) \sim 10^{-16} - 10^{-15}$. Thus, the only meaningful production of a $p$-brane occurs for $r = p$, and then their production is comparable.

### III. PRODUCTION AT THE LHC AND VLHC

The production of BH’s and SB’s depends on $M_s, n, M_D, g_s$. Since we also require the transition point $(M_s/g_s^2)$ at $5 M_D$, we can therefore solve for $M_s$ and $g_s$ for a given pair of $M_D$ and $n$. We present the results in terms of $M_D$ and $n$. The minimum mass requirement for the SB is set at $2 M_s$. The production of a $p$-brane also depends on $m$ and $r$. For an interesting level of event rates, $r$ has to be equal to $p$, i.e., the $p$-brane wraps entirely on small (of Planck length) extra dimensions.

In Fig. [a], we show the total production cross sections for BH’s, SB’s, and $p$-branes, including the $2 \to 1$ and $2 \to 2$ subprocesses (when computing the $2 \to 2$ subprocess we require a $p_T$ cut of 500 GeV to prevent double counting). Typically, the $2 \to 2$ subprocess contributes at a level of less than 10%. For the BH, SB, and $p$-brane, we show the results for $n = 3$ and $n = 6$. The results for $n = 4, 5$ lie in between. Since we require $M_{min}^{BH}, M_{min}^{SB} = 5 M_D$, their production is only sizable when $\sqrt{s}$ reaches about 10 TeV, unlike the SB, which only requires $M_{min}^{SB} = 2 M_s$. The $p$-brane cross section is about a few times larger than the BH, as we have chosen $r = p = m = n - 2$. String ball production is, on average, two orders of magnitude larger than that of a BH in the energy range between 20 and 60 TeV. Below 20 TeV (e.g., at
the LHC), the SB cross section is at least three orders of magnitude larger than the BH. Integrated cross sections for the LHC in Table I. Sensitivity information can be drawn from the table. The event rates for BH and p-brane production are negligible for $M_D = 2.5$ TeV and only moderate at $M_D = 2$ TeV. At $M_D = 2$ TeV, the number of BH events that can be produced in one year running (100 fb$^{-1}$) is about 120 – 340 for $n = 3 – 7$ while the number for p-brane events is 210 – 1300. Therefore, the sensitivity for a detectable signal rate for a BH and a p-brane is only around 2 TeV, if not much larger than 2 TeV. The SB event rate is much higher. Even at $M_D = 3$ TeV, the cross section is of order of 30 pb. In Table I, we also show the $\sigma$(SB) for $M_D = 4$ – 6 TeV. Roughly, the sensitivity is around 6 TeV.

The VLHC (very large hadron collider) is another pp accelerator under discussions [14] in the Snowmass 2001 [15]. The preliminary plan is to have an initial stage of about 40–60 TeV center-of-mass energy, and later an increase up to 200 TeV. The targeted luminosity is $(1 – 2) \times 10^{34}$ cm$^{-2}$s$^{-1}$. In Fig. 1(b), we show the total production cross sections for BH’s, SB’s, and p-branes for $\sqrt{s} = 60 – 200$ TeV and for $n = 3$ and 6. We found that the sensitivity reaches for BH and p-brane production are roughly between 6 and 7 TeV for $\sqrt{s} = 50$ TeV, 10 and 13 TeV for $\sqrt{s} = 100$ TeV, 14 and 18 TeV for $\sqrt{s} = 150$ TeV, and 20 and 25 TeV for $\sqrt{s} = 200$ TeV. These estimates are rather crude based on the requirement that the number of raw events is $\gtrsim 50 – 100$.

IV. DECAY SIGNATURES

The main phase of the decay of a BH is via the Hawking evaporation. An important observation is that the wavelength $\lambda$ of the thermal spectrum corresponding to the Hawking temperature is larger than the size of the BH. This implies that the BH evaporates like a point source in s-waves, therefore it decays equally into brane and bulk modes, and will not see the higher angular momentum states available in the extra dimensions. Since on the brane there are many more particles than in the bulk, the BH decays dominantly into brane modes, i.e., the SM particles in the setup. Furthermore, the BH evaporates “blindly” into all degrees of freedom. The ratio of the degrees of freedom for gauge bosons, quarks, and leptons is 29 : 72 : 18 (the Higgs boson is not included). Since the W and Z decay with a branching ratio of about 70% into quarks, and the gluon also gives rise to hadronic activities, the final ratio of hadronic
TABLE I: Total cross sections in pb for the production of BH, SB, and p-brane, for various values of \( n \) and \( M_D \) at the LHC. The minimum mass on the BH and p-brane is \( M_{\text{BH}}^{\text{min}}, M_{pB}^{\text{min}} = 5M_D \), while that on SB is \( M_{SB}^{\text{min}} = 2M_s \).

| \( M_D \) (TeV) | \( n = 3 \) | \( n = 5 \) | \( n = 7 \) |
|-----------------|-------------|-------------|-------------|
| \( BH \)        |             |             |             |
| 1.5             | 0.70        | 1.3         | 1.9         |
| 2.0             | \( 1.2 \times 10^{-3} \) | \( 2.2 \times 10^{-3} \) | \( 3.4 \times 10^{-3} \) |
| 2.5             | \( 1.3 \times 10^{-8} \) | \( 2.4 \times 10^{-8} \) | \( 3.6 \times 10^{-8} \) |
| \( SB \)        |             |             |             |
| 1.5             | 3300        | 4100        | 4900        |
| 2.0             | 590         | 670         | 760         |
| 2.5             | 130         | 130         | 140         |
| 3.0             | 33          | 29          | 28          |
| 4.0             | 2.4         | 1.5         | 1.1         |
| 5.0             | 0.16        | 0.060       | 0.033       |
| 6.0             | 0.0091      | 0.0015      | 0.00044     |
| \( p\text{-brane} \)|             |             |             |
| 1.5             | 1.2         | 4.0         | 7.6         |
| 2.0             | \( 2.1 \times 10^{-3} \) | \( 6.9 \times 10^{-3} \) | 0.013       |
| 2.5             | \( 2.3 \times 10^{-8} \) | \( 7.3 \times 10^{-8} \) | \( 1.4 \times 10^{-7} \) |

to leptonic activities in the BH decay is about 5 : 1 \[4\].

Another important property of the BH decay is the large number of particles, in accord with the large entropy in Eq. (6), in the process of evaporation. It was shown \[4, 5\] that the average multiplicity \( \langle N \rangle \) in the decay of a BH is order of 10 \(-30\) for \( M_{\text{BH}} \) being a few times \( M_D \) for \( n = 2 - 6 \). Since we are considering the BH that has an entropy of order 25 or more, it guarantees a high multiplicity BH decay. The BH decays more or less isotropically and each decay particle has an average energy of a few hundred GeV. Therefore, if the BH is at rest, the event is very much like a spherical event with many particles of hundreds of GeV pointing back to the interaction point (very much like a fireball). On the other hand, if the BH is produced in association with other SM particles (as in a \( 2 \rightarrow k \) subprocess), the BH decay will be a boosted spherical event on one side (a boosted fireball), the transverse momentum of which is balanced by a few particles on the other side \[8\]. Such spectacular events should have a negligible background.

Highly excited long strings emit massless quanta with a thermal spectrum at the Hagedorn temperature. At \( M_{SB} \lesssim M_s/g_s^2 \), the wavelength \( \lambda \) corresponding to the thermal spectrum at the Hagedorn temperature is larger than \( R_{SB} \). This argument is very similar to that of the BH, and so the string ball radiates like a point source and emits in s-waves equally into brane and bulk modes. With many more particles (SM particles) on the brane than in the bulk, the SB radiates mainly into the SM particles. When \( M_{SB} \) goes below \( M_s/g_s^2 \), the SB has the tendency to puff up to a random-walk size as large as the \( \lambda \) of the emissions \[9\]. Therefore, it will see more of the higher angular momentum states available in the extra dimensions. Thus, it decays more into the bulk modes, but it is only temporary. When the SB decays further, it shrinks back to the string size and emits as a point source again \[9\]. Most of the time the SB decays into SM particles. On average, a SB decays into invisible quanta somewhat more often than a BH does.
The decay of $p$-branes is not well understood, to some extent we do not even know whether it decays or is stable. Nevertheless, if it decays one possibility is the decay into lower-dimensional branes, thus leading to a cascade of branes. Therefore, they eventually decay to a number of 0-branes, i.e., BH-like objects. This is complicated by the fact that when the $p$-branes decay, their masses might not be high enough to become BH’s. Therefore, the final 0-branes might be some excited string states or string balls. Whether the zero brane is stable or not depends on models. Another possibility is decay into brane and bulk particles, thus experimentally the decay can be observed. Or it can be a combination of cascade into lower-dimensional branes and direct decays. Since the size $R_p$ is much smaller than the size of the large extra dimensions, we expect $p$-branes to decay mainly into brane particles. However, the above is quite speculative.

This research was supported in part by the National Center for Theoretical Science under a grant from the National Science Council of Taiwan R.O.C.

[1] K. Cheung, Phys. Rev. D66, 036007 (2002); and references therein.
[2] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett B429, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B436, 257 (1998); N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D59, 086004 (1999).
[3] P. Argyres, S. Dimopoulos, and J March-Russell, Phys. Lett. B441, 96 (1998).
[4] S. Giddings and S. Thomas, Phys. Rev. D65, 056010 (2002).
[5] S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. 87, 161602 (2001).
[6] R. Emparan, G. Horowitz, and R. Myers, Phys. Rev. Lett. 85, 499 (2000).
[7] S.B. Giddings, hep-ph/0110127.
[8] K. Cheung, Phys. Rev. Lett. 88, 221602 (2002).
[9] S. Dimopoulos and R. Emparan, Phys. Lett. B526, 393 (2002).
[10] E.-J. Ahn, M. Cavaglia, and A. Olinto, hep-th/0201042.
[11] K. Cheung and C.-H. Chou, Phys. Rev. D66, 036008 (2002).
[12] See, e.g., L. Anchordoqui, J. Feng, and H. Goldberg, and A. Shapere, hep-ph/0207138; P. Jain, D. McKay, S. Panda, and J. Ralston, Phys. Lett. B484, 267 (2000); A. Ringwald and H. Tu, Phys. Lett. B525, 135 (2002); and references therein.
[13] R. Myers and M. Perry, Ann. Phys. 172, 304 (1986).
[14] “M4 Working Group/Hadron Colliders”, plenary talk given by M. Syphers at the Snowmass 2001, available online at http://www.vlhc.org/M4finalPlenary.pdf.
[15] Snowmass 2001 “The future of particle physics” meeting, Snowmass, Summer 2001.