CAUSAL EXTRACTION OF BLACK HOLE ROTATIONAL ENERGY BY VARIOUS KINDS OF ELECTROMAGNETIC FIELDS

SHINJI KOIDE AND TAMON BABA
Department of Physics, Kumamoto University, 2-39-1, Kurokami, Kumamoto 860-8555, Japan
Received 2013 November 22; accepted 2014 June 25; published 2014 August 20

ABSTRACT
Recent general relativistic magnetohydrodynamics (MHD) simulations have suggested that relativistic jets from active galactic nuclei (AGNs) have been powered by the rotational energy of central black holes. Some mechanisms for extraction of black hole rotational energy have been proposed, like the Penrose process, Blandford–Znajek mechanism, MHD Penrose process, and superradiance. The Blandford–Znajek mechanism is the most promising mechanism for the engines of the relativistic jets from AGNs. However, an intuitive interpretation of this mechanism with causality is not yet clarified, while the Penrose process has a clear interpretation for causal energy extraction from a black hole with negative energy. In this paper, we present a formula to build physical intuition so that in the Blandford–Znajek mechanism, as well as in other electromagnetic processes, negative electromagnetic energy plays an important role in causal extraction of the rotational energy of black holes.

Key words: black hole physics – galaxies: nuclei – gamma-ray burst: general – magnetic fields – magnetohydrodynamics (MHD) – plasmas – stars: black holes

1. INTRODUCTION
A number of observations suggest that phenomena in the most active regions in the universe are related to black holes. Some of the most active objects in the universe—for example, active galactic nuclei (AGNs), microquasars (black hole binaries), and gamma-ray bursts (GRBs)—emit relativistic jets (Biretta et al. 1999; Pearson & Zensus 1987; Mirabel & Rodriguez 1994; Tingay et al. 1995; Kulkarni 1999). It is believed that these relativistic jets are caused by the drastic phenomena around the black holes at the centers of these objects. The possible energy sources of the drastic phenomena are the gravitational energy of the matter falling toward the black hole and the rotational energy of the black hole itself. Recently, numerical simulations of general relativistic magnetohydrodynamics (GRMHD) have suggested that the relativistic jet is launched from the vicinity of the black hole, i.e., inside of the ergosphere (Koide 2004; Koide et al. 2002; Koide 2003). However, because of the short time duration of the simulation, the numerical solution is far from a stationary state. Komissarov (2005) performed a long-term GRMHD simulation with a similar initial situation to that of Koide (2003) and confirmed the MHD Penrose process in the early stage. Furthermore, he found that the MHD Penrose process is a transient phenomenon, and alternately, the outward electromagnetic energy flux through the horizon continues to appear almost everywhere with the exception of a very thin equatorial belt. He remarked that the pure electromagnetic mechanism with ideal MHD condition continues to operate to extract the rotational energy of the black hole. Strictly speaking, this electromagnetic mechanism should be distinguished from the original Blandford–Znajek mechanism because the original mechanism is derived with the force-free condition, while the electromagnetic energy extraction mechanism was shown with the ideal MHD simulations. In this paper, we call the mechanism shown by the simulations the “MHD Blandford–Znajek mechanism,” while the original mechanism is called the “force-free Blandford–Znajek mechanism.” Considering the numerical results, Komissarov (2009) discussed the electromagnetic extraction mechanism of the black hole energy, including the force-free Blandford–Znajek mechanism, MHD Penrose mechanism, and superradiance in the wide view. However, unfortunately, a convincing explanation with respect to the causality of these mechanisms, which should also yield the conditions of the mechanisms, is not given, except for the MHD Penrose process (Komissarov 2009). Koide (2003) pointed out that the force-free Blandford–Znajek mechanism uses the negative electromagnetic energy at infinity to extract the spin energy of the black hole. This point of view was discussed extensively by
Table 1

| Mechanism                                      | Form of Negative Energy | Torque for Redistribution of Angular Momentum | Output Energy                        | References                                      |
|------------------------------------------------|-------------------------|-----------------------------------------------|--------------------------------------|------------------------------------------------|
| Penrose process                                | Mechanical energy of particle | Force of particle fission                     | Mechanical energy of particle       | Penrose (1969)                                  |
| Magnetic Penrose process                       | Mechanical energy of electrically charged particles | Electromagnetic force                         | Mechanical energy of electrically charged particles | Wagh (1989)                                    |
| Force-free Blandford–Znajek mechanism          | Electromagnetic energy | Electromagnetic tension force (force-free)    | Electromagnetic energy              | Blandford & Znajek (1977)                       |
| MHD Blandford–Znajek mechanism                 | Electromagnetic energy | Electromagnetic tension force (MHD)           | Electromagnetic energy and kinetic energy (Alven wave) | Takahashi et al. (1990); Koide (2003); Komissarov (2005) |
| MHD Penrose process                            | Mechanical energy of plasma | Lorentz force (magnetic tension, MHD)         | Electromagnetic energy and kinetic energy (Alven wave) | Takahashi et al. (1990); Hirotsuyu et al. (1992); Koide et al. (2002); Koide (2003) |
| Energy extraction with magnetic reconnection   | Mechanical energy of plasmoid | Magnetic tension due to magnetic reconnection | Mechanical energy of plasmoid       | Koide (2009)                                    |
| Superradiance                                  | Electromagnetic energy of electromagnetic wave | “Half-mirror” effect due to quantum tunneling | Electromagnetic energy of electromagnetic wave | Press & Teukolsky (1972); Teukolsky & Press (1974); Lightman et al. (1975) |

Krolik et al. (2005) and Lasota et al. (2014) for ideal MHD and force-free Blandford–Znajek mechanisms, respectively. However, it is often difficult to build physical intuition regarding the MHD/force-free Blandford–Znajek mechanisms with causality. Here we present an intuitive formula for the electromagnetic mechanism of the energy extraction from the rotating black hole to aid in building physical intuition regarding the mechanisms. The formula is also applicable to other electromagnetic mechanisms such as the MHD Penrose process (Takahashi et al. 1990; Hirotsuyu et al. 1992; Koide et al. 2002; Koide 2003) and superradiance (Press & Teukolsky 1972; Teukolsky & Press 1974; Lightman et al. 1975).

In Section 2, we review the energy and angular momentum transport of the electromagnetic field around the black holes briefly but sufficiently. In Section 3, we explain the electromagnetic mechanisms of black hole energy extraction that is, the force-free Blandford–Znajek mechanism, MHD Blandford–Znajek mechanism, and superradiance within causality. We summarize our explanation about the energy extraction mechanisms from the black hole, including both Blandford–Znajek mechanisms, in Section 4.

2. ELECTROMAGNETIC ENERGY AND ANGULAR MOMENTUM TRANSPORT NEAR A ROTATING BLACK HOLE

We review the electromagnetic energy and angular momentum transport in the spacetime $(x^0, x^1, x^2, x^3)$ around a spinning black hole based on the so-called “3+1 formalism.” The scale of a small element in the spacetime around the rotating black hole is given by

$$ds^2_{\text{E}} = g_{\mu\nu}dx^\mu dx^\nu = -h_0^2dt^2 + \sum_{i=1}^{3}\left[h_i^2(dx^i)^2 - 2h_i^2\dot{\omega}_i dt dx^i\right].$$

Here we have $g_{ij} = 0 (i \neq j)$, $g_{00} = -h_0^2$, $g_{ii} = h_i^2$, and $g_{0i} = g_{i0} = -h_i^2\dot{\omega}_i$, where Greek indices $(\mu, \nu)$ run from 0 to 3 and Roman indices $(i, j)$ run from 1 to 3. Throughout this paper, we use the natural unit system, where the light speed, electric permittivity, and magnetic permeability in a vacuum are unity: $c = 1$, $\varepsilon_0 = 1$, and $\mu_0 = 1$. When we define the lapse function $\alpha$ and shift vector $\beta^i$ by

$$\alpha = \sqrt{\frac{1}{h_0^2} + \sum_{i=1}^{3}\left(h_i\dot{\omega}_i\right)^2}, \quad \beta^i = \frac{h_i\dot{\omega}_i}{\alpha},$$

the line element $ds$ is written as

$$ds^2_{\text{E}} = -\alpha^2dt^2 + \sum_{i=1}^{3}\left(h_i dx^i - \alpha \beta^i dt\right)^2.$$}

The determinant of the matrix with elements $g_{\mu\nu}$ is given by

$$\sqrt{-g} = \alpha h_1 h_2 h_3,$$

and the contravariant metric is written explicitly as $g^{00} = -(1/\alpha^2)$, $g^{0i} = g^{i0} = -(\beta^i/\alpha h_3)$, and $g^{ij} = (1/\alpha h_1 h_2)\delta^{ij} - \beta^i \beta^j$, where $\delta^{ij}$ is the Kronecker $\delta$ symbol.

The relativistic Maxwell’s equations are

$$\nabla^* F_{\mu\nu} = 0,$$

$$\nabla_\mu F^{\mu\nu} = -J^\nu,$$

where $\nabla_\nu$ is the covariant derivative, $F_{\mu\nu}$ is the electromagnetic field-strength tensor, and $F^{\mu\nu}$ is the dual tensor of $F_{\mu\nu}$.

Table 1

| Mechanism                                      | Form of Negative Energy | Torque for Redistribution of Angular Momentum | Output Energy                        | References                                      |
|------------------------------------------------|-------------------------|-----------------------------------------------|--------------------------------------|------------------------------------------------|
| Penrose process                                | Mechanical energy of particle | Force of particle fission                     | Mechanical energy of particle       | Penrose (1969)                                  |
| Magnetic Penrose process                       | Mechanical energy of electrically charged particles | Electromagnetic force                         | Mechanical energy of electrically charged particles | Wagh (1989)                                    |
| Force-free Blandford–Znajek mechanism          | Electromagnetic energy | Electromagnetic tension force (force-free)    | Electromagnetic energy              | Blandford & Znajek (1977)                       |
| MHD Blandford–Znajek mechanism                 | Electromagnetic energy | Electromagnetic tension force (MHD)           | Electromagnetic energy and kinetic energy (Alven wave) | Takahashi et al. (1990); Koide (2003); Komissarov (2005) |
| MHD Penrose process                            | Mechanical energy of plasma | Lorentz force (magnetic tension, MHD)         | Electromagnetic energy and kinetic energy (Alven wave) | Takahashi et al. (1990); Hirotsuyu et al. (1992); Koide et al. (2002); Koide (2003) |
| Energy extraction with magnetic reconnection   | Mechanical energy of plasmoid | Magnetic tension due to magnetic reconnection | Mechanical energy of plasmoid       | Koide (2009)                                    |
| Superradiance                                  | Electromagnetic energy of electromagnetic wave | “Half-mirror” effect due to quantum tunneling | Electromagnetic energy of electromagnetic wave | Press & Teukolsky (1972); Teukolsky & Press (1974); Lightman et al. (1975) |
The electromagnetic energy–momentum tensor $T_{\mu\nu}^{EM}$ is given by
\[ T_{\mu\nu}^{EM} = F_{\mu\rho} F_{\nu}^{\rho} - \frac{1}{4} g_{\mu\nu} F^{\rho\sigma} F_{\rho\sigma}. \] (6)
The total energy–momentum tensor $T^{\mu\nu}$ is
\[ T^{\mu\nu} = \rho g^{\mu\nu} + h U^{\mu} U^{\nu} + T_{\mu\nu}^{EM}. \] (7)
where $\rho$, $h$, and $U^{\mu}$ are the proper pressure, the proper enthalpy density, and the four-velocity of the plasma, respectively. The energy–momentum conservation law is given by
\[ \nabla_{\mu} T^{\mu\nu} = 0. \] (8)
The force-free condition is
\[ J^{\mu} F_{\mu\nu} = 0, \] (9)
and the general relativistic Ohm’s law is
\[ F_{\mu\nu} U^{\nu} = \eta [J_{\mu} + (U^{\nu} J_{\nu}) U_{\mu}], \] (10)
where $\eta$ is the resistivity of the plasma. The ideal MHD condition is given by setting $\eta = 0$, $F_{\mu\nu} U^{\nu} = 0$.

Here we introduce a local coordinate frame, the so-called “fiducial observer (FIDO) frame,” ($\hat{x}^0, \hat{x}^1, \hat{x}^2, \hat{x}^3$). Using the local coordinates of the frame $\hat{x}^{\mu}$, the line element becomes
\[ ds_{\text{EM}}^2 = \eta_{\mu\nu} d\hat{x}^{\mu} d\hat{x}^{\nu} = -dt^2 + \sum_{i=1}^{3} (d\hat{x}^i)^2, \]
where $\eta_{\mu\nu}$ is the metric of Minkowski spacetime. Comparing this metric with Equation (3), we get
\[ ds_{\text{EM}}^2 = dt^2 - \sum_{i=1}^{3} (dx^i)^2, \]
and we have partial derivative relations,
\[ \frac{\partial}{\partial \hat{t}} = \frac{\partial}{\partial t}, \quad \frac{\partial}{\partial \hat{x}^i} = h_i \frac{\partial}{\partial x^i}. \]
(12)
Then a contravariant vector $\hat{a}^{\mu}$ in the FIDO frame of an arbitrary contravariant vector $a^{\mu}$ in the global coordinates $x^{\mu}$ is written as
\[ \hat{a}^{0} = \alpha a^{0}, \quad \hat{a}^{i} = h_i a^i - \alpha \beta^i a^0 \] (13)
and the covariant vector $\hat{a}_{\mu}$ is
\[ \hat{a}_{0} = \frac{1}{\alpha} a_{0} + \sum_{i} \frac{\beta^i}{h_i} a_i, \quad \hat{a}_{i} = \frac{1}{h_i} a_i. \] (14)
We use the quantities observed by the FIDO frame because they can be treated intuitively and yield formulae more easily. This is because the relations between the variables in the FIDO frame are the same as those in the special theory of relativity and similar to the Newtonian relation.

Using the quantities of the electromagnetic field in the FIDO frame, Maxwell’s equations are written using the following 3+1 formalism,
\[ \frac{\partial \hat{B}^i}{\partial t} = - \sum_{j,k} \frac{h_j}{h_1 h_2 h_3} \epsilon^{ijk} \frac{\partial}{\partial x^j} \left[ \alpha h_k \left( \hat{E}_k - \sum_{l,m} \epsilon^{lmm} \beta^l \hat{E}^m \right) \right], \]
(15)
\[ \alpha (\hat{J}^j + \hat{\rho} \beta^j) + \frac{\partial \hat{E}_i}{\partial t} = \sum_{j,k} \frac{h_j}{h_1 h_2 h_3} \epsilon^{ijk} \frac{\partial}{\partial x^j} \left[ \alpha h_k \left( \hat{B}^k + \sum_{l,m} \epsilon^{lmm} \beta^l \hat{E}^m \right) \right], \] (16)
\[ \sum_{i} \frac{1}{h_1 h_2 h_3} \frac{\partial}{\partial x^i} \left( \frac{h_i h_2 h_3}{h_i} \hat{E}_i \right) = 0, \] (17)
\[ \hat{\rho}_e = \sum_{i} \frac{1}{h_1 h_2 h_3} \frac{\partial}{\partial x^i} \left( \frac{h_i h_2 h_3}{h_i} \hat{E}_i \right), \] (18)
where $\epsilon^{ijk} = \epsilon^{0ijk}$.

For convenience, we introduce the derivatives of an arbitrary three-vector field $\hat{a}$ and an arbitrary scalar field $\phi$ measured by the FIDO frame as
\[ \nabla \cdot \hat{a} = \sum_{i} \frac{1}{h_1 h_2 h_3} \frac{\partial}{\partial x^i} \left( \frac{h_i h_2 h_3}{h_i} \hat{a}^i \right), \] (19)
\[ (\nabla \phi)_{\mu} = \frac{1}{h_i} \frac{\partial \phi}{\partial x^i}, \] (20)
\[ (\nabla \times \hat{a})_{i} = \sum_{j,k} \frac{h_i}{h_1 h_2 h_3} \epsilon^{ijk} \frac{\partial}{\partial x^j} (h_k \hat{a}^k). \] (21)
We express Maxwell’s equations in vector form as
\[ \frac{\partial \hat{B}}{\partial t} = -\nabla \times [\alpha (\hat{E} - \hat{B} \times \hat{B})], \] (22)
\[ \alpha (\hat{J} + \hat{\rho} \beta) + \frac{\partial \hat{E}}{\partial t} = \nabla \times [\alpha (\hat{B} + \beta \times \hat{E})], \] (23)
\[ \nabla \cdot \hat{B} = 0, \] (24)
\[ \hat{\rho}_e = \nabla \cdot \hat{E}, \] (25)
where $\beta = (\beta_1, \beta_2, \beta_3)$, $\hat{E} = (\hat{E}_1, \hat{E}_2, \hat{E}_3)$, $\hat{B} = (\hat{B}_1, \hat{B}_2, \hat{B}_3)$, and $\hat{J} = (\hat{J}_1, \hat{J}_2, \hat{J}_3)$.

The 3+1 form of the force-free condition is
\[ \hat{J} \cdot \hat{E} = 0, \quad \hat{\rho}_e \hat{E} + \hat{J} \times \hat{B} = 0, \] (26)
and Ohm’s law is written as
\[ \hat{E} + \hat{\nu} \times \hat{B} = \frac{1}{\gamma} \eta [\hat{J} - \rho_e \hat{\nu} \hat{\nu}], \] (27)
where $\gamma = \hat{U}^0$ is the Lorentz factor, $\hat{\nu} = (\hat{U}/\gamma, \hat{U}^2/\gamma, \hat{U}^3/\gamma)$ is the three-velocity, and $\rho_e' = -\hat{J} \cdot \hat{U}_e$ is the electric charge density observed by the plasma rest frame (the proper electric charge density). The conservation equation of the electric charge is derived using Equations (23) and (25) as
\[ \frac{\partial \hat{\rho}_e}{\partial t} + \nabla \cdot [\alpha (\hat{J} + \hat{\rho}_e \beta)] = 0. \] (28)
We present the equations of energy and angular momentum conservation around a spinning black hole. When \( \delta \mu \) is a Killing vector, we have a conservation law associated with Equation (8)

\[
\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} (\sqrt{-g} T^{\mu \nu} \xi_\nu) = 0.
\]

Because \( g = -(a h_1 h_2 h_3)^2 \), this equation yields

\[
\frac{\partial}{\partial t} (a T^{0 \nu} \xi_\nu) = \frac{1}{h_1 h_2 h_3} \sum_i \frac{\partial}{\partial x^i} (a h_1 h_2 h_3 T^{i \nu} \xi_\nu).
\]

Using the Killing vector \( \chi^\nu = (-1, 0, 0, 0) \), we have the law of energy conservation

\[
\frac{\partial e^\infty}{\partial t} = -\hat{\nabla} \cdot \mathbf{S},
\]

where \( e^\infty = a(\epsilon + \rho \hat{\gamma}) + \sum_i a \beta^i \hat{Q}^i \),

\[
S^i = S^i_{\text{hyd}} + S^i_{\text{EM}},
\]

where

\[
e^\infty_{\text{hyd}} = a(\gamma^2 - p) + \sum_i a \beta^i \gamma^2 \hat{v}^i,
\]

\[
e^\infty_{\text{EM}} = a \left( \frac{\hat{B}^2}{2} + \frac{\hat{E}^2}{2} \right) \sum_i a \beta^i (\hat{E} \times \hat{B})_i,
\]

\[
S^i_{\text{hyd}} = a^2 \gamma^2 \left( 1 + \sum_j \beta^j \hat{v}^j \right) (\hat{v}^i + \beta^i),
\]

\[
S^i_{\text{EM}} = a^2 [ (\hat{E} - \beta \times \hat{B}) \times (\hat{B} + \beta \times \hat{E}) ]^i,
\]

where the subscripts “hyd” and “EM” indicate hydrodynamic and electromagnetic components, respectively, and \((\hat{E})^2 = (\hat{E}_1)^2 + (\hat{E}_2)^2 + (\hat{E}_3)^2\), \((\hat{B})^2 = (\hat{B}_1)^2 + (\hat{B}_2)^2 + (\hat{B}_3)^2\). Here, \(S_{\text{EM}} = (S^1_{\text{EM}}, S^2_{\text{EM}}, S^3_{\text{EM}})\) can be regarded as the Poynting vector.

The general relativistic Maxwell’s Equations (22)–(25) read

\[
\frac{\partial e^\infty_{\text{EM}}}{\partial t} = -\hat{\nabla} \cdot \mathbf{S}_{\text{EM}} - a(\hat{\nabla} \cdot \hat{v} + \beta \cdot \mathbf{f}_L),
\]

where \(\mathbf{f}_L = \hat{\rho}_L \hat{E} + \hat{j} \times \hat{B}\) is the Lorentz force density.

If \( \eta^\mu = (0, 0, 0, 1) \) is the Killing vector for the azimuthal direction, we have the equation of angular momentum conservation

\[
\frac{\partial l}{\partial t} = -\hat{\nabla} \cdot \mathbf{M},
\]

where \( l \equiv a T^{0 \nu} \eta_\nu \) and \( M^i \equiv a h_i T^{0 \nu} \eta_\nu \) are the total angular momentum density and the angular momentum flux density, respectively. Using the quantities measured in the FIDO frame, we have

\[
l = h_3 \hat{Q}^3,
\]

\[
M^i = a h_3 (\hat{T}^{i3} + \beta^i \hat{Q}^3).
\]

These variables also can be divided into the hydrodynamic and electromagnetic components, denoted by the subscripts “hyd” and “EM,” as follows:

\[
l_{\text{hyd}} = h_3 \gamma^2 \hat{v}^3,
\]

\[
l_{\text{EM}} = h_3 (\hat{E} \times \hat{B})_3.
\]

\[
M^i_{\text{hyd}} = a h_3 \left( p \hat{\delta}^3 + h \gamma^2 \hat{v}^3 \right) + c \beta^i \gamma^2 \hat{v}^3,
\]

\[
M^i_{\text{EM}} = a h_3 \left( \frac{(\hat{B})^2}{2} + \frac{(\hat{E})^2}{2} \right) \hat{\delta}^3 - \hat{B}^i \hat{B}^3
\]

In this case, from Equations (32) and (42), we have a relation of the energy and the angular momentum,

\[
e^\infty = a(\epsilon + \rho \hat{\gamma}) + \omega^3 l = a \left[ \epsilon + \rho \hat{\gamma} + \frac{\beta^3}{h_3} \right],
\]

when \( \omega_1 = \omega_2 = 0 \). Furthermore, Equations (36), (37), (46), and (47) yield

\[
e^\infty_{\text{hyd}} = a(\gamma^2 - p) + \omega^3 l_{\text{hyd}},
\]

\[
e^\infty_{\text{EM}} = a \left( \frac{\hat{B}^2}{2} + \frac{\hat{E}^2}{2} \right) + \omega^3 l_{\text{EM}}.
\]

The general relativistic Maxwell’s Equations (22)–(25) read

\[
\frac{\partial l_{\text{EM}}}{\partial t} = -\hat{\nabla} \cdot \mathbf{M}_{\text{EM}} = h_3 f^3_L.
\]

Hereafter, we consider the electromagnetic energy transport when we have the relation between the electric field and magnetic field as

\[
\hat{E} = -\hat{\nu}_E \times \hat{B}.
\]
the electric field \( E, \hat{v}_E = (\hat{E} \times \hat{B})/\hat{B}^2 \), can be used as one of the vectors of \( \hat{v}_F \). Intuitively, \( \hat{v}_F \) is regarded as the velocity of the magnetic field lines, while this intuition is not rigorous because we cannot identify the magnetic field lines at the different times. However, we do not have such a serious contradiction with the interpretation, and we often recognize \( \hat{v}_F \) as the velocity of the field line implicitly. Using Equation (54), we have

\[
e_{\text{EM}}^\infty = \alpha \left( \frac{(\hat{B}^2)^2}{2} + \frac{(\hat{E}^2)^2}{2} \right) + \alpha \hat{B} \cdot (\hat{E} \times \hat{B})
\]

\[
= \alpha \left[ \frac{1}{2} \left( 1 + \hat{v}_{F\perp}^2 \right) + \beta \cdot \hat{v}_{F\perp} \right] (\hat{B})^2, \tag{55}
\]

where \( \hat{v}_{F\perp} \) is the component of \( \hat{v}_F \) perpendicular to the magnetic field \( \hat{B} \). \( \hat{v}_F = \hat{v}_{F\perp} + \hat{v}_{F\parallel} \parallel \hat{B} \). Here we used the relations \( (\hat{E})^2 = (\hat{B})^2 \hat{v}_{F\perp}^2 \), \( \hat{E} \times \hat{B} = (\hat{B})^2 \hat{v}_{F\perp} \). With respect to the energy transport flux density, we have

\[
S_{\text{EM}} = \alpha^2 \left[ \frac{1}{2} \left( 1 + \hat{v}_{F\perp}^2 \right) + \beta \cdot \hat{v}_{F\perp} \right] (\hat{B})^2 \hat{v}_{F\perp} + \beta + (1 - \hat{v}_{F\perp}^2) \times \left[ \frac{1}{2} (\hat{B}^2 \hat{v}_{F\perp} + \beta) - (\beta \cdot \hat{B}) \hat{B} \right]. \tag{56}
\]

Using Equations (55) and (56), we obtain

\[
S_{\text{EM}} = \alpha^2 e_{\text{EM}}^\infty \hat{v}_{F\perp} + \beta + \alpha^2 \left( 1 - \hat{v}_{F\perp}^2 \right) \times \left[ \frac{1}{2} \left( \hat{B}^2 \right) \hat{v}_{F\perp} + \beta \right] - (\beta \cdot \hat{B}) \hat{B}. \tag{57}
\]

With respect to the angular momentum of the electromagnetic field, assuming Equation (54), we have

\[
l_{\text{EM}} = h_3 \hat{B}^2 \hat{v}_{F\perp}^3, \tag{58}
\]

\[
M_{\text{EM}}^i = \alpha h_3 \left[ \frac{1}{2} \left( 1 + \hat{v}_{F\perp}^2 \right) \hat{B}^2 \hat{v}_{F\perp}^3 + \beta \hat{B}^2 \hat{v}_{F\perp}^3 + \hat{B}^2 \hat{v}_{F\perp}^3 - \hat{B}^2 \hat{v}_{F\perp}^3 - \hat{E}^i \hat{E}^3 \right]. \tag{59}
\]

In this case, we also have

\[
e_{\text{EM}}^\infty = \alpha \left( \frac{B^2}{2} \right) (1 + \hat{v}_{F\perp}^2) + \alpha^3 l_{\text{EM}}. \tag{60}
\]

3. CAUSAL ENERGY EXTRACTION FROM BLACK HOLES WITH SEVERAL KINDS OF ELECTROMAGNETIC FIELDS

3.1. Force-free Electromagnetic Field Case: Blandford–Znajek Mechanism

In this subsection, we consider the energy transport near the horizon in the force-free limit case, which is assumed in the original work on the Blandford–Znajek mechanism (Blandford & Znajek 1977). Here we use the Kerr metric for spacetime \((x^0, x^1, x^2, x^3) = (t, r, \theta, \phi)\) with \(\omega^\phi \geq 0\) in this section. The force-free condition, \( J^\mu F_{\mu \nu} = 0 \), reads

\[
\hat{J} \cdot \hat{E} = 0, \tag{61}
\]

\[
\hat{J} = \hat{E} \times \hat{B}, \hat{E} \cdot \hat{B} = 0.
\]

This means there are no energy and momentum transforms between the electromagnetic field and plasma. In such a case, we can write the electromagnetic field by Equation (54). This is because when \( \hat{v}_F \neq 0 \), we have \( \hat{E} = -(\hat{J} / \hat{\rho}_E) \times \hat{B} \), and confirm Equation (54) with \( \hat{v}_F = (\hat{J} / \hat{\rho}_E) \). When \( \hat{v}_F = 0 \), we have \( \hat{J} \times \hat{B} = 0 \), i.e., \( \hat{J} \parallel \hat{B} \). Furthermore, because of Equation (61), we have \( \hat{J} \perp \hat{E} \), and then \( \hat{E} \perp \hat{B} \). We confirm Equation (54) with \( \hat{v}_F = (1 / \hat{B}^2) \hat{E} \times \hat{B} \).

In the steady-state and axisymmetric case, Equations (15), (17), (61), and (62) yield

\[
\hat{v}_F = \frac{h_\phi (\Omega_F - \omega_\phi) e_\phi}{\alpha (\Omega_F - \omega_\phi) e_\phi}, \tag{63}
\]

where \(\Omega_F\) is a constant along the magnetic flux surface. \( R \equiv h_\phi = h_3 \) corresponds to the distance from the \( z \) axis, and \( e_\phi \) is the unit vector for azimuthal direction (Blandford & Znajek 1977). Because the triangle of \( \hat{v}_{F\perp} \) and \( \hat{v}_F \) and the triangle of \( \hat{B}_F \) and \( \hat{B} \) are similar (Figure 1), we found the following relation,

\[
\frac{\hat{v}_{F\perp}}{\hat{B}} = \frac{\hat{B}_P}{\hat{B}}, \tag{64}
\]

Here we define \( \hat{B}_P \) and \( \hat{B}_\phi \) as the poloidal and azimuthal components of the magnetic field \( \hat{B} \), respectively. We then have

\[
\hat{v}_{F\perp} = \frac{\hat{v}_F}{\sqrt{1 + \hat{v}_F^2/\hat{E}^2}}, \tag{65}
\]

The Znajek boundary condition at the horizon (Znajek 1977) is expressed as

\[
\hat{B}_P = \hat{B}_\phi = \hat{v}_F. \tag{66}
\]

Then, very near the horizon, we also have

\[
\hat{v}_{F\perp} \approx \frac{\hat{v}_F}{\sqrt{1 + \hat{v}_F^2}}, \tag{67}
\]

where "\( \approx \)" means asymptotic equivalence. In the limit toward the horizon \( r \to r_H \), \( r_H \) is the radius of the black hole), we have \( \hat{v}_F \to \infty \) when \( \Omega_F \neq \Omega_H \); we then find \( \hat{v}_{F\perp} \to 1 \). Here we write the value of \( \omega^\phi \) at the horizon by \( \Omega_H \). Eventually, using Equation (57) we obtain, very near the horizon,

\[
S_{\text{EM}} = \alpha e_{\text{EM}}^\infty (\hat{v}_{F\perp} \hat{B} + \hat{B}). \tag{68}
\]

Figure 1. Geometric relation between the vectors \( \hat{B}, \hat{B}_P, \hat{B}_\phi, \hat{v}_F, \hat{v}_{F\perp}, \hat{v}_{F\parallel} \), and \( \hat{v}_{F\perp} \).

\[
\hat{\rho}_E \hat{E} + \hat{J} \times \hat{B} = 0. \tag{62}
\]
The directions of \( \hat{v}_F \) of the cases of \( \Omega_F < \Omega_H \) and \( \Omega_F > \Omega_H \) are opposite because of Equation (63), and the slopes of the magnetic field lines in the two cases are also opposite (Figure 2). Then, the direction of \( \hat{v}_{F\perp} \) is always directed toward the inner region of the black hole when \( \Omega_F \neq \Omega_H \). Then, when \( e_{EM}^\infty < 0 \), the electromagnetic energy flux is directed outward, and the energy of the black hole is extracted through the horizon.

Because the triangle of \( \hat{v}_F \) and \( \hat{F} \) of the cases of \( \Omega_F < \Omega_H \) and \( \Omega_F > \Omega_H \) are similar (Figure 1), we found \((\hat{v}^\Phi_{F\perp}/\hat{v}_{F\perp}) = (\hat{B}_F/\hat{B})\), and we then obtain

\[
\hat{v}_{F\perp} = \left(\frac{\hat{B}_F}{\hat{B}}\right)^2 \hat{v}_F = \frac{\hat{v}_F}{1 + \hat{v}^\Phi_{F\perp}}. \tag{69}
\]

Finally, using the second equation of Equation (2) and Equation (63), we get

\[
e_{EM}^\infty = \left[\frac{1}{2} \left(1 + \frac{\hat{v}^2_{F\perp}}{1 + \hat{v}^\Phi_{F\perp}}\right) + \frac{\hat{B}^\Phi_{F\perp}}{1 + \hat{v}^\Phi_{F\perp}}\right] \alpha \hat{B}^2 \\
= \frac{1}{2} \alpha^2 + 2R^2\Omega_F(\Omega_F - \hat{\omega}_p) - \frac{\alpha^2}{2} R^2(\Omega_F - \hat{\omega}_p)^2 \alpha \hat{B}^2. \tag{70}
\]

At the horizon \((\alpha \to 0, \omega_p \to \Omega_H)\), Equations (63) and (66) yield \( \hat{B} = \sqrt{\hat{B}^\Phi_{F\perp}^2 + (\hat{B}_F)^2} \approx |\hat{B}^\Phi_{F\perp}| = |(R/\alpha)(\Omega_F - \Omega_H)|\hat{B}_{PH} \), because \( |\hat{B}^\Phi_{F\perp}| \gg \hat{B}_F \), where \( \hat{B}_{PH} \) is the value of \( \hat{B}_F \) at the horizon. Eventually, at the horizon we found

\[
e_{EM}^\infty = \frac{K^2_H}{\alpha}(\Omega_F - \Omega_H)(\hat{B}_{PH})^2, \tag{71}
\]

\[
S_{EM} = R^2_H(\Omega_F - \Omega_H)(\hat{B}_{PH})^2(\hat{v}_{F\perp} + \beta). \tag{72}
\]

where \( R_H \) is the value of \( R = h_p \) at the horizon. It is noted that the radial component of the electromagnetic energy flux is identical to the simple equation given by McKinney & Gammie (2004, Equation (34) in the paper), if we set the force-free condition at the horizon, \( \hat{v}_F = \epsilon_r \). Then, when \( 0 < \Omega_F < \Omega_H \), the negative energy of the electromagnetic field is realized \((e_{EM}^\infty < 0)\), and the rotational energy of the black hole is extracted. This is exactly the same condition of the Blandford–Znajek mechanism. This suggests even in the Blandford–Znajek mechanism, to extract the black hole rotational energy, the negative energy of the electromagnetic field is utilized as a mediator. In conclusion, putting the negative electromagnetic energy into the black hole, the black hole rotational energy is extracted causally in the Blandford–Znajek mechanism.

Sometimes the energy extraction of the rotating black hole is intuitively explained by the torque of the magnetic field at the horizon. This intuitive explanation is not appropriate with respect to causality because at the horizon, no torque affects the matter and field at the horizon outward. Equations (71) and (72) suggest that the falling of the negative (electromagnetic) energy into the black hole could decrease the black hole energy to extract the black hole energy.

3.2. Ideal MHD Case: MHD Blandford–Znajek Mechanism/MHD Penrose Process

We consider the ideal MHD case in the spacetime around the spinning black hole. We assume the situation is stationary and axisymmetric, the same as the force-free case in the previous section. In such a case, the magnetic flux surfaces are stationary and axisymmetric and are expressed as a constant azimuthal component of the vector potential, \( A_\Phi \). We introduce the new coordinate system \((t, s, \Psi, \phi)\), where \( t \) is the time of Kerr spacetime, \( \phi \) is the azimuthal coordinate, \( \Psi = A_\phi \), and the coordinate \( s \) is set outward along the intersection line of a magnetic surface and the meridian plane \((\phi = \text{const.})\) (Figure 3). Here we set the coordinates \( s \) so that it is perpendicular to the coordinate \( \Psi \). The \( s \) coordinate at the...
horizon is $s_{11}$. Essentially, this coordinate system corresponds to the Boyer–Lindquist coordinate $(t, r, \theta, \phi)$ where $t = t$, $s = s(r, \theta)$, $\Psi = \Psi(r, \theta)$, and $\phi = \phi$. Then, the length of a line element in the spacetime of the rotating black hole is given by

$$ds^2 = -h_t^2dt^2 + h_r^2dr^2 + h_\theta^2d\Psi^2 + h_\phi^2d\phi^2 - 2h_\theta\omega_\theta dtd\phi.$$ 

We assume the ideal MHD condition, $U^\mu F_{\mu\nu} = 0$, which yields

$$\dot{E} + \dot{\mathbf{v}} \times \dot{\mathbf{B}} = 0.$$  

Using the coordinates $(s, \Psi, \phi)$, Equations (15)–(18), (31), (41), and (73) yield the following conservation variables along the magnetic surface:

$$M(\Psi) = h_\phi h_\Psi \rho a U^s = \frac{\alpha \rho \dot{U}^s}{B^s},$$  

$$B^s(\Psi) = h_\Psi h_\phi \dot{B}^s = 1,$$  

$$\Omega_F(\Psi) = \frac{\alpha}{h_\phi} \left[ \dot{\psi}^\phi + \dot{\psi}^\theta - \frac{\dot{\mathbf{B}}^\phi}{B^s} \dot{\mathbf{v}} \right],$$  

$$L(\Psi) = h_\Psi \left[ \frac{\hat{h}}{\rho} \dot{\Psi}^\phi - \frac{\alpha}{M} \dot{\mathbf{B}}^\phi \right],$$  

$$H(\Psi) = \frac{\hat{h}}{\rho} [\alpha \dot{\mathbf{v}} - h_\phi (\Omega_F - \omega_\phi)] = \frac{\hat{h}}{\rho} (\dot{\mathbf{v}} - \dot{\psi}^\phi \dot{\mathbf{U}}^\phi).$$

It is noted that quantities with hats are variables observed by the FIDO frame. It is also noted that the distribution of $\Psi$ is determined by the transverse equation called the “Grad–Shafranov equation” (Beskin & Kuznetsova 2000). Recently, numerical simulations of GRMHD have provided a more complete feature of the mechanism such as the distribution of Poynting flux over the event horizon, the relative importance of negative energy at infinity fluid and electromagnetic field, the energy flux from the black hole to the disk through the magnetic field lines, etc. (McKinney et al. 2012; Hawley & Krolik 2006). It is noted that the numerical, time-dependent simulations showed that magnetorotational instability (MRI) always causes fluctuations, and no steady state of plasma and the magnetic field is found.

At the black hole horizon, the lapse function $\alpha$ becomes 0, $h_i$ becomes infinite, while $\omega^\phi = (\alpha \beta^\phi / h_\phi) \to \Omega_{H1}$, $h_\Psi \to h_{\Psi H1}$, and $h_\phi \to R_{H1}$ are finite except on the $z$ axis. Hereafter, we discuss the quantities along a certain fixed magnetic flux surface $\Psi = \Psi_1$.

Because the magnetic field is not a real singular surface, and the density $\rho$ and pressure $p$ are measured by the plasma rest frame, $\rho$ and $p$ should be finite at the horizon. Then, from Equations (74) and (75), $\alpha \dot{U}^s$ and $\dot{B}^s$ must be finite at the horizon, where we write $\dot{B}^s$ at the horizon by $\dot{B}^s_H$. At the horizon, the plasma falls vertically to the horizon at the light velocity, $\dot{\psi}^\phi = \dot{\psi}^\Psi = 0$, and the second equation of Equations (2) and (76) yield

$$\alpha \dot{B}^\phi \dot{B}^s = \frac{R_{H}}{\alpha} (\Omega_F - \Omega_{H1}).$$  

Using Equations (73) and (76), we have

$$\dot{E} = -\dot{\mathbf{v}} \times \dot{\mathbf{B}} = \dot{\mathbf{B}}^s \left( -\dot{\psi}^\phi + \dot{\psi}^\theta \frac{\dot{\mathbf{B}}^\phi}{B^s} \right) e^\phi \times e_\Psi = -\dot{v}_F \times \dot{B},$$

where we put $\dot{v}_F = (R/\alpha)(\Omega_F - \omega_\phi)e^\phi$ and $e^\phi, e_\Psi$ are the unit base vectors along the $\phi$ and $s$ coordinates, respectively. Very near the horizon, we have

$$\dot{v}_F \approx \frac{R_{H}}{\alpha} (\Omega_F - \Omega_{H1})e^\phi.$$  

Equations (79) and (81) present the geometrical disposition of vectors $\dot{B}$ and $\dot{v}_F$, as shown in Figure 2. When $\Omega_F \neq \Omega_{H1}$, we found that the vector of $v_F$ is always directed toward the black hole inner region.

Intuitively, at the horizon of the rotating black hole, the plasma falls into the black hole radially with the speed of light ($\dot{\psi}^\phi = -1$, $\dot{\psi}^\theta = 0$ at $s = s_{11}$). When the azimuthal component of the magnetic field is finite outside of the horizon and stationary, the magnetic field lines are twisted extremely strongly in appearance near the horizon because of Equation (76) where $\alpha \dot{B}_\phi$ is uniform along the magnetic surface and $\alpha$ vanishes at the horizon. This is due to the difference in the lapse of time near the black hole and is an apparent feature in the Kerr metric. In such a case, the perpendicular component of the velocity to the magnetic field is identical to the plasma velocity; we then have

$$\dot{v}_F = \dot{v}^\phi = 1$$

at the horizon.² Then, the electromagnetic energy flux density at the horizon is given by

$$S_{EM} = \alpha e_{EM}^\infty (\dot{v}_{F\perp} + \dot{B}),$$

from Equation (57). When $e_{EM}^\infty$ becomes negative at the horizon, the electromagnetic energy is transported outward through the horizon when $\Omega_F \neq \Omega_{H1}$, because $\dot{v}_{F\perp}$ is always directed inward toward the black hole’s inner region (see Figure 2). Here, because $\dot{v}_{F\perp}$ vanishes if $\Omega_F = \Omega_{H1}$, no electromagnetic output is expected; we then consider only the case of $\Omega_F \neq \Omega_{H1}$.

As shown in Equation (55), the density of electromagnetic energy at infinity is given by

$$e_{EM}^\infty = \alpha \left[ \frac{1}{2} (1 + \dot{\psi}_{F\perp}^2) + \beta^\phi \dot{\psi}_{F\perp}^\phi \right] (\dot{B})^2.$$  

---

1 This is also derived as follows. Extremely near the horizon, $\alpha \dot{B}^\phi$ is finite, because $\dot{\psi}^\phi$ is finite. Then, from Equation (77), $\dot{U}^s$ is finite. At the horizon, because $\alpha \dot{U}^s$ is finite and $\alpha \to 0$, $\dot{U}^s$ and $\dot{\psi}^\phi = \sqrt{1 + (\dot{U}^s)^2 + (\dot{\psi}^\phi)^2}$ are infinite. Then, $\dot{\psi} = (\dot{U}^s)/\dot{\psi}^\phi$ becomes zero at the horizon, and $\dot{\psi}^\phi = [\dot{U}^s]/\dot{\psi}^\phi$. Finally, at the horizon, $\dot{\psi}^\phi = (\dot{U}^s)/\dot{\psi}^\phi = -1$.

2 This equation is also derived as follows. Using the similarity of the triangle of $\dot{v}_{F\perp}$ and $\dot{v}_F$ and the triangle of $\dot{B}^s = \dot{\mathbf{B}}^s$ and $\dot{B}$ (Figure 1), we found

$$\frac{(\dot{v}_{F\perp}^2)}{(\dot{v}_F^2)} = \frac{(\dot{B}^s/\dot{\mathbf{B}}^s)}{(\dot{B}/\dot{\mathbf{B}})}.$$  

Very near the horizon, because $\dot{B}^s$ is much larger than $\dot{\mathbf{B}}$, we have

$$\dot{B}^s \approx (\dot{B}^s/\dot{\mathbf{B}}^s),$$

and then $(\dot{v}_{F\perp}^2)/(\dot{v}_F^2) \approx (\dot{B}^s/\dot{\mathbf{B}}^s)$. Using Equations (79) and (81), at the horizon we confirm $\dot{v}_{F\perp} = (\dot{B}^s/\dot{\mathbf{B}}^s)|_{\dot{B}^s} = 1$. 

KOIDE & BABA

The Astrophysical Journal, 792:88 (10pp), 2014 September 10
With Figure 1, we found \((\hat{v}_F^\parallel /\hat{v}_F^\perp) = (\hat{v}_F^\parallel /\hat{v}_F^\perp)\); and we then have \(\hat{v}_F^\parallel \approx (\hat{v}_F^\parallel + \hat{v}_F^\perp)^2 / \hat{v}_F^\parallel\). At the horizon, using Equations (81) and (82), we have

\[
\hat{v}_F^\parallel \approx \frac{\alpha}{R_H(\Omega_F - \Omega_H)}.
\]  

(85)

Using Equation (79), we also have \(\hat{B} \approx |\hat{B}^\parallel | \approx |(R_H/\Omega)(\Omega_F - \Omega_H)(\hat{B}_H)^2|\) at the horizon because \(|\hat{B}^\parallel | \approx |\hat{B}|^2\). Eventually, we obtain

\[
e_{EM}^\infty \approx \frac{R_H^2}{\alpha} \Omega_F(\Omega_F - \Omega_H)(\hat{B}_H)^2.
\]  

(86)

\[
S_{EM} = \frac{R_H^2}{\alpha} \Omega_F(\Omega_F - \Omega_H)(\hat{B}_H)^2(\hat{v}_F^\parallel + \hat{\beta}).
\]  

(87)

This clearly shows that when \(0 < \Omega_F < \Omega_H\), \(e_{EM}^\infty \) becomes negative and the electromagnetic energy flux is directed outward through the horizon. It is surprising that not only the condition of the electromagnetic energy extraction from the black hole but also the expression of energy density and the energy flux density at the horizon are the same as those of the Blandford–Znajek mechanism (force-free case).

In the above two cases of electromagnetic extraction of the black hole rotational energy, the negative electromagnetic energy at infinity is required as a mediator to extract the black hole rotational energy through the horizon causally. As shown in Equation (60), we have \(\hat{e}_{EM}^\infty = \alpha \hat{e}_{EM} + \omega \hat{l}_{EM}\), where \(\hat{u}_{EM} = (\hat{E}^2 / 2) + (\hat{B}^2 / 2)\) is the electromagnetic energy density in the FIDO frame. To realize the negative electromagnetic energy, the angular momentum of the electromagnetic field \(\hat{l}_{EM}\) should become less than \(-\omega u_{EM} / \omega^2\). Locally, the angular momentum should be conserved because of Equation (41), and then redistribution of the angular momentum is required. In the Penrose process, fission of a particle is utilized for redistribution of the angular momentum and production of a particle with negative energy at infinity. Equation (53) indicates that dynamically only the magnetic force (the magnetic tension in the axisymmetric case) and the Lorentz force can redistribute the electromagnetic angular momentum. In the ideal MHD case, magnetic tension plays an important role to redistribute the electromagnetic angular momentum and realize the negative electromagnetic energy. This mechanism of energy extraction with negative electromagnetic energy is often confused with the (original) Blandford–Znajek mechanism, where the force-free condition is used, as we did in Section 1. However, strictly speaking, they should be distinguished. Hereafter, in this paper, we call the ideal MHD process with negative electromagnetic energy the “MHD Blandford–Znajek mechanism.”

In the MHD Blandford–Znajek mechanism, we have to take the hydrodynamic energy flux of the plasma flow into account to discuss the net energy flux from/in to the black hole.

In fact, in the ideal MHD case, the black hole rotational energy can be also extracted with the negative hydrodynamic energy of the plasma. The hydrodynamic energy flux density is

\[
S_{hyd} = \alpha (e_{hyd}^\infty + \omega p)(\hat{\omega} + \hat{\beta}).
\]

Then, near the horizon if the plasma with \(\alpha e_{hyd}^\infty < 0\) falls into the black hole, the energy is transported outward through the horizon because \(\alpha \to 0\) at the horizon. If \(S_{EM}\) is directed outward, \(\alpha e_{hyd}^\infty\) must be smaller than zero to extract the black hole rotational energy. This extraction mechanism of black hole rotational energy is called the “MHD Penrose process” (Takahashi et al. 1990; Hirota et al. 1992; Koide et al. 2002; Koide 2003). The hydrodynamic energy is given by \(e_{hyd}^\infty = \alpha(\omega y^2 - \rho) + \omega l_{hyd}\) where \(\omega = (\omega_1, \omega_2, \omega_3)\) and \(l_{hyd} = h^2 y^2 b^3\) is the hydrodynamic angular momentum density. To realize the negative hydrodynamic energy, \(l_{hyd}^2 < -\omega(\omega y^2 - \rho) / \omega^2\). Angular momentum is conserved, and redistribution of the hydrodynamic angular momentum is also required. Redistribution of the hydrodynamic angular momentum is caused by the Lorentz force shown in Equation (53).

To distinguish between the MHD Blandford–Znajek mechanism and the MHD Penrose process, we should observe the density of the electromagnetic and hydrodynamic energy at infinity \((e_{EM}^\infty\) and \(e_{hyd}^\infty\)). If the electromagnetic energy plays a main role in extracting the black hole energy, we recognize the process as the MHD Blandford–Znajek mechanism. On the other hand, hydrodynamic or plasma energy has an important role in the extraction; this is recognized as the MHD Penrose process. In actual cases, both are possible, while some long-term simulations indicate that the MHD Penrose process is transient, and the MHD Blandford–Znajek mechanism is dominant in the late phase of the simulations (Komissarov 2005; McKinney 2006).

The electromagnetic extraction mechanisms of black hole rotational energy outlined in this paper are restricted to those in the steady-state, axisymmetric cases. Recently, the long-term GRMHD simulations showed 3D dynamics of plasma interacting with the magnetic field around the rotating black hole (McKinney et al. 2012). Strictly speaking, the results of this paper are not applicable to the time-dependent, axisymmetric numerical results. Generalization of the results of this paper for such time-dependent, axisymmetric numerical results is required.

### 3.3. Electromagnetic Wave Case: Superradiance

We briefly mention the electromagnetic wave energy transport through the horizon. We use the Kerr metric for the spacetime \((x^0, x^1, x^2, x^3) = (t, r, \theta, \phi)\) around the spinning black hole, where we set \(\omega^\parallel \geq 0\). We consider the stationary solution of the electromagnetic wave in a vacuum, where each component of the electromagnetic field is proportional to \(f(r, \theta) e^{-i\omega t + i\alpha \phi}\) (\(f\) is a function of \(r\) and \(\theta\)). We use the short-wavelength limit of the electromagnetic wave, \(|k| / ((1/h)|g_{\mu\nu} / \delta x|^i (i = 1, 2, 3, \mu, \nu = 0, 1, 2, 3)\), where \(k\) is the wave number of the electromagnetic wave in a local region, which is fixed at the global coordinates. In a vacuum \((\hat{J} = 0\), \(\hat{\rho}_p = 0\), Equations (22)–(25) in the FIDO frame yield

\[
\hat{E} = -\frac{\hat{k}}{\omega} \times \hat{B}, \quad \hat{B} = \frac{\hat{k}}{\omega} \times \hat{E},
\]  

(88)

where \(\hat{k}\) and \(\hat{\omega}\) are the wave number and angular frequency of the electromagnetic wave in the FIDO frame, respectively. These equations give us the dispersion relation, \(\hat{\omega} = \pm \hat{k}\) and the relation \(\hat{k} \perp \hat{B}\). In this case, we identify

\[
\hat{v}_F^\perp = -\frac{\hat{k}}{\omega}.
\]  

(89)

Because \(\hat{v}_F^\perp = |\hat{k}/\hat{\omega}| = 1\), using Equation (57), we have

\[
S_{EM} = \alpha e_{EM}^\infty(\hat{u} + \hat{\beta}).
\]  

(90)
When an electromagnetic wave passes through the horizon and enters into the black hole, if $\alpha e_{\text{EM}}^\infty$ is negative, the rotational energy of the black hole decreases. In this case, Equations (58), (60), and (89) read

$$e_{\text{EM}}^\infty = \alpha (\hat{B})^2 + \omega^3 l_{\text{EM}} = \alpha \left( 1 + \omega^3 \frac{\hat{k}_\phi}{\omega} \right) (\hat{B})^2. \quad (91)$$

Very near the horizon, we have $e_{\text{EM}}^\infty \approx \omega^3 R (\hat{k}_\phi/\omega) (\hat{B})^2$. Because the four-wave-number $k_\alpha = (-\omega, k_1, k_2, k_3)$ is the covariant vector, using Equation (14), we have

$$-\hat{\omega} = \frac{1}{\alpha} (-\omega) + \frac{\beta^3}{h_3} k_3 = -\frac{1}{\alpha} (\omega - \omega_3 k_3), \quad \hat{k}_3 = \frac{1}{h_3} k_3 = \frac{m}{h_3}. \quad (92)$$

Then, the energy density of the electromagnetic wave very near the horizon is approximately given by

$$e_{\text{EM}}^\infty \approx \Omega_H \alpha \frac{m}{\omega - m\Omega_H} (\hat{B})^2. \quad (93)$$

When $\omega < m\Omega_H$, negative energy appears at the horizon, and the rotational energy of the black hole is extracted. This extraction mechanism corresponds to the “superradiance.” To produce the negative energy of the electromagnetic wave, redistribution of the angular momentum is required. To understand the redistribution process, we have to consider the structure of the solution of the electromagnetic wave in the ergosphere.

4. DISCUSSION

In this paper, we showed simple formulae (Equations (57) and (60)) to aid in building physical intuition for the causal extraction mechanism of the energy from black holes by electromagnetic fields with negative electromagnetic energy produced in the ergosphere. In three cases of force-free, ideal MHD conditions and electromagnetic wave in a vacuum, at the horizon we found that $\hat{v}_{\text{F+L}} = 1$; we then have $s_{\text{EM}} = \alpha e_{\text{EM}}^\infty (\hat{v}_{\text{F+L}} + \hat{B})$. To extract the black hole rotational energy causally, we have to put the negative electromagnetic energy down into the black hole through the horizon. To produce the negative electromagnetic energy, because of the angular momentum conservation (60), we should redistribute the angular momentum of the electromagnetic field, where we require the negative electromagnetic angular momentum density,

$$l_{\text{EM}} < -\frac{\alpha (\hat{B})^2}{\beta_\phi} = - \frac{R(\hat{B})^2}{\beta_\phi} < 0, \quad (94)$$

at the horizon (see Equation (60)). To realize the negative angular momentum azimuthal component, the angular momentum should be redistributed because the total angular momentum is conserved. Redistribution of the angular momentum of the electromagnetic field is caused by the electromagnetic torque (Koide 2003; Gammie et al. 2004; Hawley & Krolik 2006; Krolik et al. 2005).

This point of view originates with the Penrose process (Penrose 1969), which uses the negative mechanical energy of a particle. In fact, equations of the energies of matter and the electromagnetic field have similar forms as shown in Equations (51) and (52). With this viewpoint, in general, we classify the known mechanisms of energy extraction from the black hole as shown in Table 1. The Penrose process is well known and is briefly mentioned in Section 1. The Blandford–Znajek mechanism, MHD Blandford–Znajek mechanism, and MHD Penrose process were explained in the previous sections. We showed that in all electromagnetic mechanisms of energy extraction from the spinning black hole, the negative electromagnetic energy is utilized as a mediator for the causal energy extraction through the horizon. We confirmed that the condition of energy extraction is given by the realization condition of the negative energy at the horizon. The magnetic Penrose process was not discussed in this paper. In the magnetic Penrose process, a particle interacts with the electromagnetic field and falls to the negative energy orbit. The negative energy of the particle is used to extract the black hole rotational energy. This is just the Penrose process with electromagnetic interaction instead of fission. Superradiance was mentioned in Section 3.3. We found that the electromagnetic wave with negative energy is used to extract the black hole rotational energy. We also add the energy extraction mechanism by magnetic reconnection in the ergosphere as shown in Table 1 (Koide 2009).

We discuss the coincidence of the formulae of the energy density and the energy flux density of the electromagnetic field at the horizon for the force-free and MHD Blandford–Znajek mechanisms as shown by Equations (71), (72), (86), and (87) in Sections 3.1 and 3.2, although the conditions of the two mechanisms are different. Reasoning a posteriori, we have coincident expressions of the electric field $\hat{E} = -\hat{v}_{\text{F}} \times \hat{B}$, $\hat{v}_{\text{F}} = (R_H/\alpha)(\Omega_H - \Omega_0)\hat{e}_\phi$ in the assumption of stationary, axisymmetric conditions for both cases. Furthermore, we have the coincident boundary condition at the horizon $\hat{v}_{\text{F+L}} \rightarrow 1$ and $\hat{B} = B_{\phi}(\hat{v}_{\text{F+L}})$ for both cases. These leading equations for both cases are the same; we then have the coincident formulae for both mechanisms.

Here we remark on the overlap of the ideal MHD and force-free conditions. The ideal MHD (Equation (73)) and force-free (Equation (62)) conditions can both be satisfied if $\vec{J} = \rho_i \vec{v} + \vec{J}_H$ and $\rho_v \neq 0$, where $\vec{J}_H$ is a vector parallel to the magnetic field $\vec{B}$. The vector $\vec{J}_H$ corresponds to the net current density along the magnetic field lines at the plasma rest frame. Alternatively, in ideal MHD simulations, the “force-free” condition is often defined by $B^2/(2\rho_H) \gg 1$ even if $\vec{J} - \rho_i \vec{v}$ is not parallel to $\vec{B}$.

In an astrophysical situation such as in AGNs, which mechanism is most expected to extract the black hole rotational energy and activate the region near the black hole? We think the MHD Blandford–Znajek mechanism, rather than the original Blandford–Znajek mechanism, is the most promising process. Because the plasma near the black hole is expected to be relativistically hot, the plasma beta $\beta_p = 2p/B^2$ never vanishes. Of course, the original Blandford–Znajek mechanism is applicable as an approximation with respect to the very strong magnetic field case. Such very low plasma beta is expected at the higher latitude of the black hole magnetosphere and the fast component of a relativistic jet.

We are grateful to Mika Koide for her helpful comments on this paper.

REFERENCES

Beskin, V. S., & Kuznetsova, I. V. 2000, NCimB, 115, 795
Biretta, J. A., Sparks, W. B., & Macchetto, F. 1999, ApJ, 520, 621
Blandford, R. D., & Znajek, R. 1977, MNRAS, 179, 433
Gammie, C. F., Shapiro, S. L., & McKinney, J. C. 2004, ApJ, 602, 312
Hawley, J. F., & Krolik, J. H. 2006, ApJ, 641, 103
Hirotani, K., Takahashi, M., Nitta, S.-Y., & Tomimatsu, A. 1992, ApJ, 386, 455
Jackson, J. D. 1979, Classical Electrodynamics (New York: Wiley)
Koide, S. 2003, PhRvD, 67, 104010
Koide, S. 2004, ApJL, 606, L45
Koide, S. 2009, ApJ, 696, 2220
Koide, S., Kudoh, T., & Shibata, K. 2006, PhRvD, 74, 044005
Koide, S., Shibata, K., Kudoh, T., & Meier, D. L. 2002, Sci, 295, 1688
Komissarov, S. S. 2005, MNRAS, 359, 801
Komissarov, S. S. 2009, JKPS, 54, 2503
Krolik, J. H., Hawley, J. F., & Hirose, S. 2005, ApJ, 622, 1008
Kulkarni, S. R. 1999, Natur, 398, 389
Lasota, J.-P., Gourgoulhon, E., Abramovicz, M., Tchekhovskoy, A., & Narayan, R. 2014, PhRvD, 89, 024041
Lightman, A. P., Press, W. H., Price, R. H., & Teukolsky, S. A. 1975, Problem Book in Relativity and Gravitation (Princeton: Princeton Univ. Press)
McKinney, J. C. 2006, MNRAS, 368, 1561
McKinney, J. C., & Gammie, C. F. 2004, ApJ, 611, 977
McKinney, J. C., Tchekhovskoy, A., & Blandford, R. D. 2012, MNRAS, 423, 3083
Mirabel, I. F., & Rodriguez, L. F. 1994, Natur, 371, 46
Pearson, T. J., & Zensus, J. A. 1987, in Superluminal Radio Sources, ed. J. A. Zensus & T. J. Pearson (Cambridge: Cambridge Univ. Press), 1
Penrose, R. 1969, Nuovo Cimento Riv. Ser., 1, 252
Press, W. H., & Teukolsky, S. A. 1972, Natur, 238, 211
Punsly, B., & Coroniti, F. V. 1989, PhRvD, 40, 3834
Punsly, B., & Coroniti, F. V. 1990a, ApJ, 350, 518
Punsly, B., & Coroniti, F. V. 1990b, ApJ, 354, 583
Takahashi, M., Nitta, S., Tatematsu, Y., & Tomimatsu, A. 1990, ApJ, 363, 206
Teukolsky, S. A., & Press, W. H. 1974, ApJ, 193, 443
Tingay, S. J., Jauncey, D. L., Preston, R. A., et al. 1995, Natur, 374, 141
Wagh, S. M., & Dadhich, N. 1989, PhR, 183, 137
Znajek, R. 1977, MNRAS, 179, 457