SOURCE VACUUM FLUCTUATIONS OF BLACK HOLE RADIANCE

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Abstract

The emergence of Hawking radiation from vacuum fluctuations is analyzed in conventional field theories and their energy content is defined through the Aharonov weak value concept. These fluctuations travel in flat space-time and carry transplanckian energies sharply localized on cisplanckian distances. We argue that these

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features cannot accommodate gravitational nonlinearities. We suggest that the very emission of Hawking photons from tamed vacuum fluctuations requires the existence of an exploding set of massive fields. These considerations corroborate some conjectures of Susskind and may prove relevant for the back-reaction problem and for the unitarity issue.
1. Introduction

The remarkable discovery by Hawking\cite{1} of the thermal radiation of an incipient black-hole has raised many questions about quantum gravity but has as yet not delivered conclusive answers. The very existence of the radiation confirmed the Bekenstein conjecture relating the area of the event horizon to entropy\cite{2} but did not yield the identification of the quantized matter-gravity states building up this entropy. The back-reaction of the radiation on the metric should lead to the evaporation of the hole but its precise mechanism is far from being understood and the end point of the evaporation itself poses in an acute way the consistency of quantum physics with general relativity. Is unitarity violated within our universe as initially suggested by Hawking\cite{3} or does the planckian black hole turn into a (infinitely?) long lived remnant\cite{4} correlated to the distant radiation? This question is related to the value of a possible additive constant to the area entropy which would count the remnant degeneracy; attempts to understand the nature of the constant\cite{5} have been made but no definite conclusion has been reached. Other more revolutionary attempts to save unitarity through a breakdown of large scale physics have been proposed\cite{6,7} but remain conjectural.

The heart of the difficulty lies in the quantum back-reaction. The semi-classical treatment, whereby the expectation value of the energy-momentum tensor of the radiation is taken as the source to the classical Einstein equations\cite{8}, is questionable in view of the importance of the fluctuations. In particular the fact that the original derivation of the emission process\cite{1} requires vacuum fluctuations of frequencies much higher than the Planck scale may change completely the nature of the back-reaction. The occurrence of these transplanckian* frequencies, although consistent in the free field description of the vacuum, poses a moot problem when gravitational nonlinearities are introduced\cite{9}. To clarify the issue, it would be helpful to have a complete description of the pair creation process generating Hawking quanta out of the free field vacuum. However despite the earlier work of Unruh\cite{10}

\* A picturesque adjective we heard from 't Hooft at a meeting.
and Wald[11], and the more recent clarifications of Parentani and Brout[12], the explicit history of the correlated pairs for free massless fields have not been brought to light. We shall display the history in a simplified collapse situation which we think contains the relevant physical properties. Our result is that nearly all the quanta are generated from dipolar vacuum fluctuations in flat Minkowski space-time with transplanckian frequencies sharply localized at cisplanckian distances. For s-waves these dipoles are spherical; they travel from past light-like infinity towards the centre of the star in an essentially flat background and then separate. Their energy content is displayed following reference [13] where a systematic analysis of the energy momentum tensor in terms of weak value[14] is given. The positive energy pole flows towards an eventual horizon while the negative pole, composed of a positive energy core followed by an oscillatory tail with overcompensating negative energy, crosses the star surface, loses its tail and gets converted into a Hawking quantum by reducing its core energy to confront the gravitational background outside the star. The history of higher angular momenta modes is essentially the same, except that after crossing the star’s surface most of the outgoing modes get reflected. The result is a thermal distribution close to equilibrium in the vicinity of the horizon.

This analysis will lead us to the conclusion that while independent arguments suggest quite convincingly that Hawking radiation does occur, the mechanism that produces it, taking gravitational effects into account, cannot be realized out of the conventional cisplanckian physics, except for the emission of the very first quanta.

We shall suggest that the emission process requires, in addition to the massless fields generally considered, an exploding set of massive fields such as those encoded in weakly interacting closed string theories. Such structure, in four dimensional space-time, exhibit a high temperature phase transition when the energy stored in the massless modes exceeds a critical energy density, the excess energy condensing into large massive strings with huge entropy. This provides a dynamical mechanism to reduce transplanckian frequencies to planckian and cisplanckian ones. Further investigations along these lines may prove valuable for uncovering some features of quantum gravity and maybe of the string theory approach itself. They could shed
light on the back-reaction process and on the unitarity issue.

The presentation is as follows. In section 2, the conventional derivation of the Hawking radiation is reviewed for a simple idealized collapsing star in the restricted space-time available to external observer. In this way, we avoid, in the subsequent discussions, unnecessary references to an eventual horizon. This formalism is used in section 3 to uncover the history of s-wave vacuum fluctuations giving birth to Hawking quanta in absence of back-reaction. Their energy content is obtained. The shortcomings of this history, generalized to include higher angular momentum waves, are discussed in section 4. A possible remedy based on elements contained in closed string theories is presented.

2. Pair creation in the external observers space-time.

The Hawking radiation due to a collapsing star is often analysed in the framework of the global space-time background generated by the collapse, extending through the future event horizon up to the classical singularity. We shall find it convenient to phrase it in the restricted space-time available to the outside observer which is limited by the horizon. Our analysis will therefore not rely on the existence of the event horizon itself but rather on the geodesic motion of the star at late Schwartzschild times. In this way no a priori assumption about the formation of a horizon is needed and the analysis is easily extended to the case where the star, after having followed for some time an “asymptotic” geodesic motion, would slow down to rest or would bounce back\cite{6}. Hawking radiation would still be emitted during the geodesic motion\cite{12} but its subsequent alterations would automatically contain the required correlations to render the radiation process unitary. As will be later discussed, the inclusion of gravitational effects raises a fundamental difficulty which is essentially the same for a stopped collapse and for a real one.

Following the pioneering work of Unruh\cite{10} and the more recent analysis of reference \cite{12}, we consider a spherically symmetric star of mass $M$ idealized by a shell of the same mass collapsing along the geodesic trajectories of the points located at
the surface of the star, that is geodesic trajectories of a Schwarzschild geometry with mass $M$. Tensions in the shell must be adjusted accordingly. This idealization simplifies the mathematics without affecting qualitatively the conclusions.

For the external observer, in absence of back-reaction, space-time is limited to the shaded region of the Penrose diagram of Fig.1 depicting the classical collapse of the star. This region can be described by tortoise coordinates outside the shell and by Minkowskian ones inside. Thus, outside,

$$
ds^2 = \left(1 - \frac{2M}{r}\right) du \, dv - r^2 d\Omega^2
$$

$$
u = t - r^* \quad v = t + r^*
$$

$$
dr = (1 - \frac{2M}{r})dr^*,
$$

where $r$, understood as a function of $v - u$, is the “radius” which measures the invariant surface $4\pi r^2$ of a sphere. Inside the shell one may write

$$
ds^2 = dU \, dV - r^2 d\Omega^2
$$

$$
U = \tau - r \quad V = \tau + r,
$$

where the general spherically symmetric solution of Einstein’s equations imposes that $\tau$ be a function of $t$ only.

One can choose a single $(u, v)$ coordinate system covering the whole space-time available to the external observer which coincides with the metric defined by Eq.(1) outside the shell. Keeping the “conformal gauge” of Eq.(1) in the two dimensional $r, t$ subspace, this is entirely fixed by continuity of the metric across the shell and by the continuity of $r$. Asymptotically close to the Schwarzschild radius $2M$ one has

$$
(1 - \frac{2M}{r}) \simeq \exp \left(\frac{v - u}{4M}\right)
$$

and at the surface of the star $v$ tends to a constant $v = v_\infty$. 

6
In this asymptotic region, the trajectory of the shell is described, up to exponentially small corrections, by

\[ v_s = v_\infty - 4Mf \exp \left( \frac{v_\infty - u_s}{4M} \right) \]  

(4)

where \( f \) is a positive constant whose precise value depends on the initial conditions but which remains of order one if the shell collapses from rest at distances large compared to the Schwartzschild radius. In fact a straightforward computation of the geodesic motion yields \( f = 1/4 \) for a shell at rest at \( r = +\infty \). The value of \( f \) then decreases if the shell has an initial velocity at \( \infty \) and reaches zero in the limit of a light-like shell. The continuity of \( ds^2 \) yields, in the vicinity of the shell surface,

\[ dU = \lambda \exp \left( \frac{v_\infty - u}{4M} \right) du \quad dV = \lambda^{-1} dv \]  

(5)

where the constant \( \lambda \) is fixed by the continuity of \( r \). From Eqs.(1),(2) and (3), we write on the trajectory Eq.(4)

\[ 2dr = dV - dU = \exp \left( \frac{v_\infty - u_s}{4M} \right) (dv - du). \]  

(6)

Using Eq.(5) and differentiating Eq.(4), Eq.(6) reduces to

\[ \lambda^{-1} f = \lambda - 1. \]  

(7)

We see that \( \lambda \) remains of order one in the above range of initial conditions.

We have used here, inside the shell, a time \( \tau \) different from the Schwartzschild time \( t \). But one may synchronize the time inside and outside the shell by parametrizing it everywhere by \( t \); then inside, one writes

\[ d\tau = \sqrt{g_{00}^{in}(t)} dt. \]  

(8)

As \( dV + dU = 2d\tau \), we now get from Eqs.(4) and (5),

\[ \sqrt{g_{00}^{in}(t_s)} = (2\lambda - 1) \exp \left( \frac{v_\infty - u_s}{4M} \right) = (2\lambda - 1)g_{00}^{out}(t_s) \]  

(9)

where \( t_s \) is the synchronized time on the shell. Eq.(9) measures the redshift expen-
rienced by a photon emitted from inside the star to infinity and crossing the shell at time $t_s$. This redshift suffers a discontinuity across the shell as a consequence of the geodesic motion imposed on the shell to mimic the surface of a realistic collapsing star: if the shell were at rest, no such discontinuity would appear.

We shall need the $u,v$ parametrization in the neighbourhood of the event horizon, not only in the vicinity of the shell surface where it is close to the Schwartzschild radius, but also deep inside the shell, where $r$ goes to zero. There, Eq.(5) still correctly defines $u$ in terms of $U$ but not $v$ in terms of $V$. The latter relation is indeed fixed by the surface of the star at its intersection with the line $V =$constant and this point gets too far from the Schwartzschild radius $2M$ to use the asymptotic form Eq.(3). Rather, $(1 - 2M/r)$ gets closer to its limiting value at $r \to \infty$, namely one, and in this Minkowskian limit $V = v$. Thus, the second equation in Eq.(5) should be replaced by $dV = \lambda^{-1}(v)dv$ where $\lambda^{-1}(v)$ is a slowly varying function of $v$. This slow variation introduces unessential complications due to the fact that an incoming photon does not travel in an exactly flat space-time before entering inside the shell. To avoid these, we shall therefore take $\lambda = 1$ and independent of $v$. This amounts to put $f = 0$ in Eq.(7), and thus to consider the limit of a light-like shell. Solving then Eq.(5) with $U = 0$ at the event horizon ($u = \infty$) and $V = v = 0$ at $r = U = 0$, we get

$$U = -4M \exp \left( \frac{v_\infty - u}{4M} \right) \quad V = v$$

and the trajectory $(u_0,v_0)$ of the centre of the star, $2r = V - U = 0$, can be written as

$$v_0 = -4M \exp \left( \frac{v_\infty - u_0}{4M} \right) = -A \exp \left( \frac{-u_0}{4M} \right)$$

and, from Eqs.(2) and (10),

$$v_\infty = 4M.$$
taken. The reader may verify that all relevant equations below would similarly only be affected by such factors. The general classical collapse, as seen by an external observer, is depicted in Fig.2.

We now analyse the radiation emitted from the vacuum fluctuations of a massless scalar field by the time-dependent metric. In sections 2 and 3, we consider only s-waves and neglect the residual potential barrier. The Heisenberg scalar field operator rescaled by $r$ obeys then $\partial_u \partial_v \Phi = 0$. It is expanded into a complete set of solutions $\phi_k$, that is

$$\phi_k = f_k(u) + g_k(v)$$  \hspace{1cm} (13)

such that

$$(\phi_j | \phi_i) \equiv i \int_\Sigma [\phi_j^* \partial_v \phi_i - \phi_j \partial_u \phi_i] dv = \delta_{ij} \text{ or } \delta(i - j)$$  \hspace{1cm} (14)

where $\Sigma$ is an arbitrary Cauchy surface which does not cross the horizon (i.e. $0 \leq r < \infty, U < 0$), and the $\phi_k$ vanish at $r = 0$. From Eq.(11) such a complete set up to hermitian conjugation is

$$| - \omega_{\text{out}} \rangle = \frac{1}{\sqrt{4\pi \omega}} \left[ \exp(-i\omega u) - \Theta(-v) \exp(i4M\omega \ln \frac{-v}{A}) \right]$$  \hspace{1cm} (15)

$$| + \omega_{\text{out}} \rangle = \frac{1}{\sqrt{4\pi \omega}} \Theta(v) \exp(-i4M\omega \ln \frac{v}{A})$$  \hspace{1cm} (16)

where the frequencies $\omega$ span the positive real axis. The out-modes $| - \omega_{\text{out}} \rangle$ have positive frequencies with respect to the Killing vector on $\mathcal{I}^+$ so that we write

$$\Phi(u, v) = \int_0^\infty d\omega \left[ | - \omega_{\text{out}} \rangle a_{-\omega} + \text{h.c.} \right]$$

$$+ \int_0^\infty d\omega \left[ | + \omega_{\text{out}} \rangle a_{+\omega} + \text{h.c.} \right].$$  \hspace{1cm} (17)

The creation operators $a_{-\omega}^\dagger$ then create quanta of energy $\omega$ on $\mathcal{I}^+$ in a Hilbert space $H_1$: these we call Hawking “photons”. However the creation operators $a_{+\omega}^\dagger$
cannot be associated with well defined negative frequencies; they creates states in a Hilbert space $H_2$ orthogonal to $H_1$ which describes vacuum fluctuations propagating towards the horizon.

A convenient complete set of positive frequency in-modes on $I^-$ is

$$| \pm \omega_{in} \rangle = \frac{1}{\sqrt{8\pi \omega \sinh(\omega 4\pi M)}} \left[ \Theta(v) \exp( \mp i 4M \omega \ln \frac{v}{A} ) \exp(\pm \omega 2\pi M) + \Theta(-v) \exp( \mp i 4M \omega \ln \frac{-v}{A} ) \exp(\mp \omega 2\pi M) \right],$$

(18)

so that on $I^-$, $\Phi(v)$ can be written as

$$\Phi(v) = \int_0^\infty d\omega \left[ | - \omega_{in} \rangle a_{-\omega}^{in} + | + \omega_{in} \rangle a_{+\omega}^{in} + h.c. \right].$$

(19)

The Heisenberg vacuum $|0\rangle$ is annihilated by the operators $a_{1\omega}^{in}$. Identifying Eq.(19) with Eq.(17) on $I^-$, we get the Bogoliubov transformation relating in- and out- operators

$$a_{+\omega}^{in} = \alpha_\omega a_{+\omega}^{out} - \beta_\omega a_{-\omega}^{out} \dagger,$$

$$a_{-\omega}^{in} = \alpha_\omega a_{-\omega}^{out} - \beta_\omega a_{+\omega}^{out} \dagger,$$

(20)

where

$$\alpha_\omega = \frac{\exp(\omega 2\pi M)}{\sqrt{2 \sinh(\omega 4\pi M)}} \quad \beta_\omega = \frac{\exp(-\omega 2\pi M)}{\sqrt{2 \sinh(\omega 4\pi M)}}.$$

(21)

Equivalently, if $|\Omega\rangle$ is the vacuum annihilated by the operators $a_{1\omega}^{out}$, and $|\Omega\rangle$ is the tensor product $|\Omega_1\rangle |\Omega_2\rangle$ of vaccua for $H_1$ and $H_2$,

$$U a_{1\omega}^{out} U^{-1} = a_{1\omega}^{in}$$

$$|0\rangle = U |\Omega\rangle = U |\Omega_1\rangle |\Omega_2\rangle$$

(22)

(23)
with
\[ U = \exp \int_{0}^{\infty} d\omega \gamma_\omega \left[ a_{-\omega}^{\dagger} a_{+\omega}^{\dagger} - a_{+\omega}^{\dagger} a_{-\omega}^{\dagger} \right] \]
\[ \tanh \gamma_\omega = \exp(-\omega 4\pi M) = \frac{\beta_\omega}{\alpha_\omega}. \] (24)

Normal ordering the \( U \) operator in Eq.(24) yields
\[ |0\rangle = \langle \Omega | 0 \rangle \exp \int_{0}^{\infty} d\omega \frac{\beta_\omega}{\alpha_\omega} a_{-\omega}^{\dagger} a_{+\omega}^{\dagger} |\Omega\rangle \] (25)
\[ \langle \Omega | 0 \rangle = \exp - \int_{0}^{\infty} d\omega \ln \alpha_\omega \delta(0) \]
\[ = \exp - \frac{T}{2\pi} \int_{0}^{\infty} d\omega \ln \alpha_\omega \] (26)

where \( \delta(0) = T/2\pi \) is the (infinite) time during which the black hole radiates quanta in a fixed background.

One sees from Eq.(25) that the state describing a Hawking photon of frequency \( \omega \ a_{-\omega}^{\dagger} |\Omega_1\rangle \) has an Einstein-Rosen-Podolsky (EPR) correlation with the state \( a_{-\omega}^{\dagger} |\Omega_2\rangle \). Upon tracing the pure state density matrix \( |0\rangle \langle 0| \) over \( H_2 \), one recovers the outgoing s-wave thermal flux at the Hawking temperature
\[ T = \frac{1}{8\pi M}. \] (27)

Clearly, as stated before, if the collapse is brought to a halt, the states \( a_{+\omega}^{\dagger} |\Omega_2\rangle \) get converted through late “reflexion” on a bended \( r = 0 \) curve, into a linear superposition of real quanta on \( I^+ \). The correlation of these late non thermal photons with earlier Hawking photons is of course maintained and reflects the purity of the quantum state \( |0\rangle \). For genuine collapse, back-reaction is expected to reduce the total initial mass \( M \) to a Planck mass in a retarded time of order \( M^3 \); of course,
the above computation is then at best valid for a retarded time \( u = O(M^3) \). In both cases Hawking photons emerge from the vacuum through production of correlated pairs which should determine the back-reaction. We now further analyse the nature of these pairs.

### 3. Hawking Radiation from Transplanckian Dipoles.

The Hawking process, in absence of back reaction is entirely described by the Heisenberg state \( |0\rangle \) reexpressed as \( U |\Omega\rangle \) by Eq.(25). To understand the source of the back-reaction, it is interesting to uncover from this equation the detailed history of the vacuum fluctuations leading to the emission of real quanta. We shall do this in two steps. First we shall use a very simple method to get a qualitative picture of this history which will then be made quantitatively precise at the expense of a more sophisticated formalism.

Let us isolate the single pair contributions \( |p\rangle \) from \( |0\rangle \) by expanding \( U |\Omega\rangle \) to first order in \( \beta \omega / \gamma \omega \). Up to a normalization factor we get

\[
|p\rangle = \int_\epsilon^\infty d\omega \exp(-\omega^4 \pi M) a_{-\omega}^{\text{out}} a_{\omega}^{\text{out}} |\Omega\rangle
\] (28)

where \( \epsilon \) is an infrared cut-off. The first quantized wave-function \( \Psi(u_1, v_1; u_2, v_2) \equiv \langle \Omega| \Phi(u_1, v_1)\Phi(u_2, v_2) |p\rangle \) describing \( |p\rangle \) becomes at late times

\[
\lim_{u_2 \to \infty, v_1 \to \infty} \Psi(u_1, v_1; u_2, v_2) = \Psi(u_1, v_2) = \int_\epsilon^\infty d\omega \frac{\exp(-\omega^4 \pi M)}{4\pi \omega} \exp \left[-i\omega(u_1 + 4M \ln \frac{v_2}{A})\right]
\] (29)

where \( (u_1, v_1) \) refers to the Hawking quanta and \( (u_2, v_2) \) to its partner. One can form 2-point conserved currents out of \( \Psi(u_1, v_1; u_2, v_2) \) and its complex conjugate but their time components do not define positive definite probabilities; nevertheless one expects that the wave packets described by Eq.(29) correspond to the
localisation of the vacuum fluctuations at late times and therefore at any time via the conservation law. This expectation will be proven correct in our quantitative method.

The wave-function Eq.(29) of the correlated pair at late times is a sum over frequencies in a range $\Delta \omega = O(M^{-1})$; it is peaked at $u_1 = \bar{u}$ and $v_2 = \bar{v}$ such that

$$\bar{u} + 4M \ln \frac{\bar{v}}{A} = 0$$

(30)

and spread over a range

$$\Delta u + 4M \Delta \ln \frac{\bar{v}}{A} \simeq (\Delta \omega)^{-1} = O(M).$$

(31)

Comparing the curve Eq.(30) with the $r=0$ trajectory Eq.(11) we see that the former is the symmetric of the latter with respect to $v = 0$ in Fig.2. It is thus a space-like curve which is readily identified to a $t =$constant curve, namely $U(\bar{u}) + V(\bar{v}) = 2\tau = 0$.

We may express the correlated pair at late times as a superposition of pairs of Hawking photons localized within their wavelength

$$\Delta u_1 = O(M)$$

(32)

correlated to a partner which from Eq.(32) is localized in a region

$$\Delta v_2 = O(\bar{v}).$$

(33)

Extrapolating back in time, we see that all these correlated wave packets meet in the flat Minkowski space-time inside the shell, simultaneously in the Schwartzschild time $t$ along the curve Eq.(30), up to spreads Eqs.(32) and (33). The local frequency
\( \tilde{\omega} \) of the partner, in the Lorentz frame fixed by the spherical shell, is obtained from Eq. (16) by expanding \( v \) around \( \bar{v} \). Thus

\[
\tilde{\omega} = \omega \frac{4M}{|\bar{v}|}.
\] (34)

Similarly, the local frequency of the “Hawking fluctuation” itself, that is the vacuum fluctuation generating the Hawking photon of frequency \( \omega \), is from Eq. (10) \( \omega 4M/U(\bar{u}) \), and thus equals the partner frequency \( \tilde{\omega}^\star \). Extrapolating further back in time, we can trace the correlated wave-packets back to \( \mathcal{I}^- \). Thus, starting from there, the Hawking fluctuation and its partner form concentric spheres traveling with the velocity of light and separated at a given time \( t \) by a radial distance \( v \) of the order of their local wavelength. After crossing \( r = 0 \), the Hawking fluctuation merges on the curve Eq. (30) with its partner which was chasing it; the fluctuations then separate: one member propagates towards \( \mathcal{I}^+ \) and converts to a real Hawking photon while its partner propagates towards the horizon.

The above description of the Hawking radiation is expected to apply for retarded times \( O(1) < u_1 < O(M^3) \) and thus for \( O(M) > \bar{v} > O(M \exp -M^2) \) which means that the local frequencies of a pair goes up from \( \tilde{\omega} = O(M^{-1}) \) to transplanckian values \( \tilde{\omega} = O(M^{-1} \exp +M^2) \). Correspondingly the localization is focused for late retarded time \( u_1 \) down to a radial spread \( \Delta r = O(M \exp -M^2) \). Planckian frequencies and distances are reached a very short time after the onset of Hawking radiation, namely after a time \( u_p \) such that

\[
u_p = O(M \ln M).
\] (35)

More generally one would get a quantum superposition of such sharply defined correlated pairs.

* Similar conclusions were reached in reference [12] by examining local Bogoliubov coefficients.
The common local frequency of the Hawking fluctuation and of its partner has a deep significance. Comparing Eq.(34) to Eq.(10), we see that (the equality is exact for $\lambda = 1$)

$$\frac{\omega}{\tilde{\omega}} = \sqrt{g_{00}(t)}$$  \hspace{1cm} (36)

where $t$ is the time at which the Hawking fluctuation emanating from a point of the curve Eq.(30) crosses the shell. Eq.(36) expresses then the redshift experienced by this fluctuation when it moves with the velocity of light from inside the shell to $I^+$ and seem to indicate that the Hawking fluctuation required to make a Hawking photon must, on the average, carry the energy necessary to overcome the gravitational potential energy to reach infinity. However, the above qualitative considerations do not explain the relation between local frequencies and local energies in the vacuum. In particular the fact that the equality between the frequencies of the Hawking fluctuation and of its partner is consistent with the fact that the total Minkowskian energy is zero on $I^-$ remains mysterious. To clarify these issues and get a complete picture, we now turn to a quantitative description of the structure and the history of the pair.$^*$

Let us consider a normalized state describing a Hawking photon on $I^+$ localized on the size of its wavelength; its state vector is

$$|P_1\rangle = \int_0^\infty d\omega f(\omega) a_{-\omega}^{\text{out}} |\Omega_1\rangle$$  \hspace{1cm} (37)

where $f(\omega)$ is a complex function whose modulus is centred around a frequency $\omega_0$ of order $M^{-1}$ and spreads over a comparable range. This state is EPR correlated to $\langle P_1 | 0 \rangle$ and the pair can be represented, up to a normalization factor by $|P_1\rangle \langle P_1 | 0\rangle$.

$^*$ The forthcoming discussion follows the analysis of reference [13] where the energy content of vacuum fluctuations is expressed in terms of weak values$^{[14]}$. Weak values were previously used to describe the creation of a pair of charged particles in an external electric field$^{[15]}$ and a clear picture of the emergence of the pair out of vacuum fluctuations was obtained in this way.
Using Eq.(25), we have

\[ |P_1 \rangle \langle P_1| 0 \rangle = \langle \Omega | 0 \rangle \int d\omega d\omega' \frac{\beta_{\omega}}{\alpha_{\omega}} f^*(\omega) f(\omega') a^\dagger_{-\omega} a^\dagger_{+\omega} \Omega_1 \rangle |\Omega_2 \rangle . \quad (38) \]

The matrix element of the energy momentum tensor operator \( \hat{T}_{\mu\nu}(x) \) between the vacuum state \(|0\rangle\) and the correlated pair, suitably normalized, is called the weak value of the operator for the post selected state \(|P_1\rangle\), namely

\[ T^\text{weak}_{\mu\nu}(x) \equiv \frac{\langle 0 | P_1 \rangle \langle P_1 | \hat{T}_{\mu\nu}(x) | 0 \rangle}{\langle 0 | P_1 \rangle \langle P_1 | 0 \rangle}. \quad (39) \]

Aharonov and al have shown that, if a future measurement were to yield the post selected state, the real part of the weak value of a hermitian operator can be recorded by a “weak” non demolition measurement and its imaginary part induces a shift of the conjugate variable of the measuring device\(^{[14]}\).

By post selecting a rare event one can gain information about quantum fluctuations which are averaged out in expectation values. In this way Eq.(39) selects out of the full wave function the contribution of the pair considered and provides the quantitative elements lacking in our previous description. It yields indeed the values of the energy-momentum tensor hidden in the vacuum fluctuations needed to generate the final state \(|P_1\rangle\) on \(\mathcal{I}^+\) out of the initial state \(|0\rangle\).

\( \hat{T}_{\mu\nu}(x) \) operating on \(|0\rangle\) is a sum of a multiple of the unit operator and a bilinear form in \(a^{in \dagger}\). It follows from Eq.(20) that

\[ a^\dagger_{-\omega} a^\dagger_{+\omega} a_{-\omega} a_{+\omega} \langle 0 | = \frac{1}{\alpha_{\omega}\alpha_{\omega'}} a^\dagger_{-\omega} a^\dagger_{+\omega} \langle 0 | - \frac{\beta_{\omega}}{\alpha_{\omega}} \delta(\omega - \omega') \langle 0 | \quad (40) \]

so that it can also be expressed as a sum of a multiple \(-\beta_{\omega}/\alpha_{\omega}\) of the unit operator and a bilinear form in \(a^{out \dagger}\). From the definition Eq.(39) of \(T^\text{weak}_{\mu\nu}(x)\), we see that the unit operator yields a contribution independent of the post selected state \(|P_1\rangle\).
We therefore define
\[
\tilde{T}_{\mu\nu}^{\text{weak}}(x) = \frac{\langle 0 | P_1 \rangle \langle P_1 | \hat{T}_{\mu\nu}(x) | 0 \rangle}{\langle 0 | P_1 \rangle \langle P_1 | | 0 \rangle} - \frac{\langle \Omega | \hat{T}_{\mu\nu}(x) | 0 \rangle}{\langle \Omega | 0 \rangle}
\] (41)
which does not depend on the second term of Eq.(40) and truly characterizes the pair. Note that \(\tilde{T}_{\mu\nu}^{\text{weak}}(x)\) is independent of the subtraction needed to renormalize the vacuum expectation value of \(\hat{T}_{\mu\nu}(x)\).\(^{[15]}\)

We first compute on \(I^+ \tilde{T}_{uu}^{\text{weak}}\) which measures the energy density carried by the Hawking photon. From Eq.(38), the weak value Eq.(39) is given by
\[
\tilde{T}_{\mu\nu}^{\text{weak}}(x) = \int d\omega d\omega' \frac{\beta_\omega}{\alpha_\omega} f(\omega) f^*(\omega') \langle \Omega | a_{\omega+\omega'} a_{\omega-\omega'}^{\text{out}} \hat{T}_{\mu\nu}(x) | 0 \rangle \frac{\omega}{\int d\omega \left[ \frac{\beta_\omega}{\alpha_\omega} \right]^2 |f(\omega)|^2} \langle \Omega | 0 \rangle.
\] (42)

Using Eqs.(40) and (41) and the energy momentum tensor \(\hat{T}_{\mu\nu}(x)\) of the rescaled field \(\Phi\) (corresponding to an energy density \(\hat{T}_{\mu\nu}(x)/4\pi r^2\)), we get
\[
\lim_{v \to +\infty} \tilde{T}_{uu}^{\text{weak}} = \frac{1}{2\pi} \int d\omega d\omega' \left[ \frac{\beta_\omega}{\alpha_\omega} \right]^2 f(\omega) f^*(\omega') \sqrt{\omega\omega'} \exp[-i(\omega - \omega')u] \int d\omega \left[ \frac{\beta_\omega}{\alpha_\omega} \right]^2 |f(\omega)|^2 \langle \Omega | 0 \rangle.
\] (43)

and
\[
\int du \lim_{v \to +\infty} \tilde{T}_{uu}^{\text{weak}} = < \omega > = \frac{\int d\omega \left[ \frac{\beta_\omega}{\alpha_\omega} \right]^2 |f(\omega)|^2 \omega}{\int d\omega \left[ \frac{\beta_\omega}{\alpha_\omega} \right]^2 |f(\omega)|^2}.
\] (44)

Eq.(44) expresses that the total average energy of the Hawking photon is \(< \omega >\). Note that the average is taken not only with respect to the quantum weight \(f(\omega)\) but also over a thermal distribution at the Hawking temperature \((1/8\pi M)\). The distribution is here maxwellian because we have post selected a single pair; this is an interesting result which deserves further analysis\(^{[13]}\). Eq.(43) give the local (complex) energy content of the quantum and we learn that to localize it around
a retarded time $u_0$ on its wavelength scale we must choose the phase of $f(\omega)$ accordingly, for instance

$$f(\omega) = |f(\omega)| \exp(i\omega u_0). \quad (45)$$

We now evaluate the incident distribution of vacuum fluctuation energies building the Hawking quantum and its partner. To this effect we compute $\tilde{T}_{vv}^{weak}$ on $\mathcal{I}^-$, taking into account Eq.(45). We get

$$\lim_{u \to -\infty} \tilde{T}_{vv}^{weak} = \frac{1}{2\pi} \int d\omega d\omega' \frac{\beta_\omega}{\alpha_\omega} \frac{\beta_{\omega'}}{\alpha_{\omega'}} |f(\omega)f^*(\omega')|^{4M^2 \omega_{\omega'}^2} \exp\{i(\omega - \omega')|u_0 + \ln \frac{\omega_{\omega'} + i\epsilon}{A}|\} \int d\omega \frac{\beta_\omega}{\alpha_\omega}^2 |f(\omega)|^2 \quad (46)$$

We see that $\lim_{u \to -\infty} \int dv \tilde{T}_{vv}^{weak}$ is zero. From (41), we verify that this must indeed be the case because $|0\rangle$ is an eigenstate of total energy with eigenvalue zero. We also see that for $v > 0$, $\tilde{T}_{vv}^{weak}$ is real and positive and therefore that the locally complex contribution for $v < 0$, which is the Hawking fluctuation, must integrate to a negative real value. We thus have two overlapping wave packets located around $v_\pm = \pm A \exp(-u_0/4M)$, that is at the values of $v$ corresponding to the intersection of the ray travelling along $u = u_0$ and of the curves $r = 0$ and $r = 0$, Eqs (11) and (30). Their spread is of order $|v_\pm| = |\bar{v}|$ and they carry an energy of the order of $\bar{\omega} = 4M\omega_0/|\bar{v}|$ as in Eq.(34). It is easy to verify, comparing $\tilde{T}_{vv}^{weak}$ and $\tilde{T}_{UU}^{weak}$ that these wave packets travel in empty space with the velocity of light, keeping their shape. The qualitative behaviour previously described is entirely recovered but we now have a precise picture of the energy densities carried by the fluctuations. It remains to understand how the negative total energy carried by the Hawking fluctuation on $\mathcal{I}^-$ gets converted into a positive energy photon on $\mathcal{I}^+$. We see from Eq.(46) that the post selection of a monochromatic photon by a $\delta$-function peaking $|f(\omega)|$ at $\omega_0$, would render $\tilde{T}_{vv}^{weak}$ real and positive both for positive and negative $v$, leaving a negative energy singularity at $v = 0$ to restore
the vanishing of the total energy of the pair. This singularity is avoided in the normalized wave packet formulation and the resulting behaviour is illustrated in Fig.3. The negative (and the imaginary) contributions to the Hawking fluctuation are mostly concentrated in the oscillatory tail of the wave packet near $v = 0$, or after reflection, at large $u$. The “core” of the wave packet, centered at $-\bar{v}$ with local frequency $\tilde{\omega} = 4M\omega_0/|\bar{v}|$ remains real and positive. The negative tail energy overcompensates the positive energy of the core. The conversion of the vacuum Hawking fluctuation depicted in Fig.3b to the Hawking photon depicted in Fig.3a is achieved by the differential redshift $du/dU = -4M/U$. This redshift converts the frequency $\tilde{\omega}$ of the core to $\omega$ and damps exponentially the oscillatory tail near $v = 0$ (while enhancing the oscillations on the other side of the core).

The differential redshift encodes the loss of core energy in the gravitational background outside the star. In addition it changes the sign of the total energy of the Hawking fluctuation and may be viewed as the detailed mechanism underlying pair production through frequency sign shift in the Bogoliubov transformation. This can made even more explicit by using local Bogoliubov transformation[17] to interpolate smoothly between the $U$-description inside the star to the $u$-description on $I^+$. We now have a complete description for the emission of a (s-wave) Hawking quantum. At $t = -\infty$, on $I^-$, we select among the vacuum fluctuations of empty Minkowski space, a dipolar spherically symmetric distribution with total energy zero, moving with light velocity, carrying positive energy on the outer pole and such that the inner pole carries a total negative energy but with a positive energy core. The outer pole never reaches $r = 0$ for the external observer. After passing the curve $r = 0$ the inner pole meets the outer one and then separates, propagates towards $I^+$ and converts to a real quantum of positive energy by loosing its negative tail and redshifting its positive core. The real quantum is still correlated in an EPR

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* Such a singular behaviour for the expectation value of $T_{\mu\nu}$ is encountered in the Fulling-Rindler vacuum.[16]
fashion with the positive energy fluctuation moving towards an eventual horizon. The local frequency $\tilde{\omega}$ of the dipole pair is determined by the redshift required to realize the conversion of the core of the Hawking fluctuation into a real quantum, or equivalently by energy conservation. The distance between the two poles before their separation is of order $\tilde{\omega}^{-1}$ and hence comparable to their spread. For quanta emitted a very short time after the start of the Hawking emission, namely after a time $t_p$ given by Eq.(30), the dipole ancestors acquire very large transplanckian frequencies and are localized over very short cisplanckian distances. This remains true if after a time long compared to $t_p$ the geodesic collapse is brought to a halt. The only difference is that now the outer pole does reach the bended $r = 0$ curve and then propagates towards $I^+$ giving birth to a superposition of quanta whose correlations ensure the pure state character of the final radiation state.

Although transplanckian dipoles are totally consistent in the framework of the free field theory used here, their use in a theory where gravitational back-reaction should be included is puzzling and raises many questions. We now discuss these points.

4. Gauge invariance, back-reaction and the unitarity issue.

The gravitational interaction experienced by transplanckian dipoles in the empty Minkowski background space cannot be deduced from Einstein equations because of the non renormalizability of the quantized theory: we do not know the effective coupling at such small distances and at the high energy densities encountered in the dipoles histories, nor do we understand how to depart from a non fluctuating classical background metric. Therefore the pair production mechanism in presence of these gravitational nonlinearities seems to escape our ken, and so does therefore the back-reaction they induce on the metric. The fact that the dipoles are part of vacuum fluctuations does not help: it simply means that although they would not contribute to expectation values of the energy, they will enter general matrix elements, as for instance in the evaluation of the weak value $T_{\mu\nu}^{\text{weak}}(x)$ or in the vacuum expectation values of correlators $\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\sigma\tau}(y)|0\rangle$. 
It is important to realize that it is impossible to reduce transplanckian dipoles to planckian ones by some gauge transformation as long as a background exists. One would have to scale \( \tilde{\omega} \) down to the Planck scale everywhere on the dipole spheres and between them so that their separation \( \tilde{\omega}^{-1} \) would be stretched accordingly to a Planck length. This cannot be achieved through local Lorentz transformations without reintroducing transplanckian Unruh frequencies. Global Lorentz transformations are not available either. Indeed, in the extended coordinates \((u, v)\) which can be used by the distant observer, the frequency \( \tilde{\omega} \) depends only on the Lorentz boost parameter \( \lambda \) in Eq.(5) which is fixed from the continuity equation for \( r \) at the star boundary (it is equal to one in the limit of a light-like collapse). Thus the invariant meaning of \( \tilde{\omega} \) is explicitly related to the existence of the collapsing star. This should be contrasted with the dependence of Minkowskian frequencies on Lorentz boosts in the description of the Hawking radiation from the Unruh vacuum where the effect of the star collapse is mimicked by suitable boundary conditions on a fictitious past horizon. The latter description introduces a spurious symmetry which is just such a Lorentz boost and which is equivalent to a global Killing symmetry \( t \rightarrow t + \text{constant} \). This global symmetry is clearly broken inside the collapsing star.

We now examine whether the transplanckian frequencies can be reduced dynamically, in absence of gravitational back-reaction, by using more realistic field theoretical models. We shall argue that this is not possible because the s-wave transplanckian frequencies are for free fields part of a thermal bath containing higher angular momenta at the same local temperature \( T_{loc} \) and that this thermal bath at transplanckian temperatures survive conventional renormalizable interacting field theories.

The free scalar field radial partial wave equation

\[
\frac{\partial^2 \Phi^{(l)}}{\partial t^2} - \frac{\partial^2 \Phi^{(l)}}{\partial r^2} - \left(1 - \frac{2M}{r}\right) \left(\frac{2M}{r^3} + \frac{l(l+1)}{r^2}\right) \Phi^{(l)} = 0
\]  

(47)
illustrates that, outside the star, at coordinate distances
\[ \eta \equiv (r - 2M) \ll 1 \] (48)
of the horizon, the centrifugal barrier for the radial partial wave of angular momentum \( l \), centred at \( r = 3M \), goes down as \( \eta l(l+1)/8M^3 \). This is an exponential drop in \( r^* \), and thus for a mode of frequency \( \omega \), \( \Phi^{(l)} \) will, outside the star, propagate roughly as a free wave for coordinate distances smaller than \( \eta(l) \) given by
\[ \omega^2 = \frac{\eta l(l+1)}{8M^3}. \] (49)

Outside the star, in the immediate vicinity of the shell, the local frequency is \( \omega \sqrt{2M/\eta} \) and, from Eq. (9), \( \Phi^{(l)} \) propagates inside the star at a still higher frequency \( \omega 2M/\eta \). It is therefore quite insensitive to the centrifugal barrier \( l(l+1)/r^2 \) there until it reaches very small distances from the centre of the star ( \( r \leq \sqrt{2\eta M} \)) whereupon it is reflected. Thus the analysis of sections 2 and 3 is essentially valid for all angular momenta except that most of the high angular momentum modes are reflected outside the star at a radius \( \eta(l) \) towards the horizon. Transplanckian dipoles exist for all angular momenta. Outside the shell the distribution of Hawking photons ancestors of frequency \( \omega \sqrt{2M/\eta} \) is the result of a balance of outgoing and reflected “hot photons”. They are nearly in thermal equilibrium close to the horizon outside the star\(^{18}\) with a local blueshifted temperature
\[ T_{loc} = \frac{1}{8\pi M} \left( 1 - \frac{2M}{r} \right)^{-1/2} = \Theta \left( \frac{1}{\sqrt{\eta M}} \right). \] (50)

Let us explain qualitatively how this picture arises. The number of hot photons \( N \) in the thermal distribution Eq. (50), crossing a sphere of radius \( \eta \) per unit Schwarzschild time is roughly
\[ \frac{dN}{dt} = O \left( M^2 T_{loc}^3(\eta) \sqrt{\frac{\eta}{M}} \right) \] (51)
where the last factor measures the dilation from local time to time at infinity. The ratio \( \rho \) of such modes reaching \( I^+ \) to the number of reflected modes with
\( \omega = O(1/M) \) is of order \( l^{-2} \) where \( l \) is given by Eq.(49). Thus

\[
\rho = O \left[ \frac{\eta}{M} \right]. \tag{52}
\]

Each mode passing the barrier carries to infinity an energy of order \( 1/M \), so that the total amount of radiated energy per unit time is

\[
\frac{dM}{dt} = O \left[ M T_{loc}^3(\eta) \sqrt{\frac{\eta}{M}} \rho \right] = O \left[ \frac{1}{M^2} \right]. \tag{53}
\]

This result describes correctly the radiated flux but the important point is that the dimensional parameter \( \eta \) has canceled out in Eq.(53), expressing the fact that the total flux at infinity can be estimated from any sphere sufficiently close to the horizon by replacing the sphere by heat source of hot photons at the local temperature Eq.(50). Note that the thermal cloud of hot photons is consistent with the finite limit of the expectation value of the energy on the horizon because of the compensating divergence of the negative Schwartzschild vacuum energy.

The inclusion of higher angular momentum for free fields does not change the transplanckian character of the production process. Rather, it imbeds, outside the star, the s-wave transplanckian frequencies in a transplanckian thermal bath, which at this stage is essentially kinematical as it does not rely on interactions. Interactions due do asymptotically free interacting renormalizable field theory, mixing different angular momenta of single hot photons will be weak at the Planck energy and could only contribute to stabilize the temperature. Hence, as stated above, they cannot reduce the frequencies \( \tilde{\omega} \) to planckian or cisplanckian values.

Thus, to describe correctly the vacuum fluctuations responsible for the Hawking process, gravitational interactions must be taken into account at energies and distance scales where the classical theory does not seem to make any sense. In other words, understanding correctly the production of Hawking photons requires at least some genuine properties of quantum gravity.
At this point, one might question whether the Hawking radiation should at all exist: a Planck scale cut-off of vacuum fluctuations would clearly wipe out production of Hawking quanta except for the very few emitted before the time $t_p$. We believe that this would be an unreasonable conclusion. In presence of an horizon, the Hawking radiation appears indeed to have thermodynamical significance. If one admits the Bekenstein conjecture that the area of the event horizon is a measure of entropy, then the black hole entropy must be, for dimensional reasons, inversely proportional to the Planck constant. This in turn requires that an eternal black hole should have a global temperature proportional to $\hbar$. Consider indeed the classical Killing identity\cite{19} (it can be viewed as the integrated constraint equation over a static coordinate patch) which can be written as

$$-\frac{\kappa}{2\pi} \frac{\delta A}{4} = \delta H - \delta M_\infty$$

(54)

where $\kappa = 1/4M$ is the surface gravity of the hole of mass $M$, $M_\infty$ the total mass at infinity and $\delta H$ is the variation of all non gravitational parameters in the matter hamiltonian outside the horizon. The Bekenstein conjecture implies that there exists a global temperature proportional to the surface gravity. But (54) being a classical equation, this temperature should be proportional to $\hbar$ to cancel the $\hbar^{-1}$ in the entropy. This gives credence to the estimate of this temperature via euclidean continuation of the metric, either for Green’s functions, for partition functions\cite{20} or for tunneling amplitudes\cite{5}, because euclidean continuation always leads to the required dependence on $\hbar$ and because all these methods yield the same result, namely the Hawking temperature Eq.(27). The fact that the thermodynamical argument refers more directly to hypothetical eternal black holes than to incipient one does not weaken its significance. The pair production mechanism in an incipient black hole results as shown above in a local thermalization close to the horizon which is consistent with the global Hawking temperature and this is presumably a general feature. In other words, close to its horizon, incipient black holes tend to behave as eternal ones. The thermodynamic significance of the Hawking result Eq.(27) suggest that its derivation through the dynamics of free field,
thus in absence of gravitational interactions, is a particular realisation of a more
general phenomenon and that the Hawking radiation is a necessary concomitant
of a geodesic collapse.

The taming of the transplanckian dipoles and the related transplanckian local
temperatures poses then a fundamental problem whose solution apparently does
not lie in conventional physics. Despite our ignorance, a quest on how to achieve
this without a trivial cut-off at the Planck scale may provide useful clues for en-
tering the unsafe land of quantum gravity.

We shall argue that such a taming mechanism can be found in a general feature
of the closed string theory approach to gravity, independent of the particular model
used and in fact independent to a large extend to the detailed structure of the
string theory itself in the limit of weak coupling. This is the reason why we
shall restrict our considerations to this limit although interactions may lead to
interesting consequences\(^{[21]}\) but rely on more detailed aspects of string theories
whose theoretical foundations are at best incomplete. The assumption of weak
coupling means that a gas of strings can be approximatively described by a set of
free massless and massive fields with an exponential asymptotic density of states \(\rho(m)\) of mass \(m\):

\[
\rho(m) = A m^{-D} \exp(\beta_0 m).
\]

In string theory, the inverse Hagedorn temperature \(\beta_0\) depends, for a given string
tension, on the left and right central charges and \(A\) depends in addition on the
dimensionality of space-time \(D\) \(^{[22]},[24]\). The string tension must be chosen such
that \(\beta_0\) is larger than unity.

Such weakly interacting closed strings in four dimensional flat space-time ex-
hibit, above a critical energy density, a phase transition whereby any additional
increase of density in the massless modes would condense at fixed temperature
into infinite strings of classical Hausdorff dimension two\(^{[22]},[23]\). As explained be-
low, these results rely on the value of the prefactor \(m^{-D}\), as always in the case
of exponential energy spectrum\textsuperscript{[25]}. It is therefore important to realize that the exponent $-D$ is independent of the particular closed string theory used (provided the theory does not allow open strings) and even of the particular compactification scheme. In fact it requires only one property of closed string theory: namely that the classical length of massive free closed strings has the shape of a random walk; one can indeed verify that the form of Eq.(55), including the value $(-D)$ of the exponent in the prefactor, follows from the central limit theorem as applied to closed random walk\textsuperscript{[26]}.

We briefly review how the phase transition arises\textsuperscript{[22]}. Consider the following toy model: a box of arbitrary large volume $V$ in $D - 1$ spatial dimensions contains a total energy $E$ shared between two constituents in thermal equilibrium; a gas of massless particles and a macroscopic “string” of finite energy density characterized only by a density of states growing with its energy $E_s$ as $\exp(\beta_0 E_s)$. Denoting by $S_g, S_s, S_{g+s}$ respectively the entropies of the gas, the string, and the string-gas system, we have

\begin{align*}
S_g &= \frac{D}{D-1} V'^{1/D} E^{(D-1)/D} \\
S_s &= \beta_0 E \\
S_{g+s} &= \frac{D}{D-1} V'^{1/D} (E - E_s)^{(D-1)/D} + \beta_0 E_s.
\end{align*}

Here $V' = \xi V$ where $\xi$ is a number and the temperature of the gas is $\beta^{-1} = [(E - E_s)/V']^{1/D}$. Equilibrium of the two phase system implies $\beta = \beta_0$ and it is a stable one. Thus one can rewrite Eq.(58) for $E > \beta_0^{-D} V'$ as

$$S_{g+s} = \beta_0 E + \frac{1}{D-1} \beta_0^{1-D} V'. $$

Following Eqs.(56), (57) and (59), we have plotted in Fig.4, as a function of the total energy density $\sigma = E/V'$, the corresponding entropy densities $s_g, s_s$, and $s_{g+s}$. Clearly, for $\sigma$ exceeding $\sigma_c \equiv \beta_0^{-D}$, $s_{g+s}$ becomes greater than $s_g$ and any increase of density in the gas above $\sigma_c$ would condense into the (infinite) string at the Hagedorn temperature $T = \beta_0^{-1}$. 
Consider now, in the weak coupling limit, genuine strings with total energy $E$ enclosed in the volume $V$. The main feature ignored in the toy model is the existence of a spectrum of massive modes extrapolating between the zero mass states and macroscopic strings encoded in Eq. (55). If $D$ is large enough ($D \geq 4$), small massive closed strings are strongly favoured with respect to larger ones and the total contribution of finite energy strings (in an infinite volume) gives only a finite contribution to the energy and to the entropy densities when $T$ reaches $\beta_0^{-1}$. The qualitative results of the toy model remains then valid: if additional energy is poured into the system it will condense into infinite strings at the temperature $\beta_0^{-1}$. Large and infinite strings are, classically, random walked shaped. Infinite strings play the role of an entropy reservoir in thermal equilibrium which massless modes and with a universal distribution of closed strings.

We have seen previously that vacuum fluctuations of massless fields outside the star are well described by thermal distributions with local transplanckian temperatures when it approaches the horizon. Let us now assume that the spectrum contains a large, presumably exponential, degeneracy of massive states. The toy model and the theory of weakly coupled closed strings suggest that entropy considerations would favour a kind of condensation of these hot photons into an extended object similar to a macroscopic string enveloping the horizon. Residual photons would be distributed at the cisplanckian temperature $\beta_0^{-1}$. In this way a hot thermal distribution at a fixed temperature at a fixed distance of the horizon, from which the Hawking photon can be generated as in conventional field theory, could still exist. But the feeding of this thermal distribution by a still higher temperature at still smaller radius could be avoided and transplanckian frequencies might disappear from the spectrum of vacuum fluctuations.

Such a reconditioning of the vacuum close to the horizon could cut off transplanckian frequencies while precipitating extended structures whose energy density is hopefully not transplanckian. Further analysis is of course required to inquire

* The appearance of an infinite string occurs also for $D = 3$ but below $\beta_0^{-1}$ because of large energy fluctuations.
into the consistency and the stability of such a condensation in a finite volume with a non trivial metric, but progress along these lines is possible. Note that this scenario could be consistent with weak coupling at the Planck scale as is the case for weakly coupled closed string theories which do include gravitons.

This scheme is related to recent conjectures of Susskind\cite{27} and may be viewed as an attempt towards formulating in dynamical terms the brick wall model\cite{28} or the streched horizon model\cite{7}, or alternatively an achronon\cite{5}. At this stage, it is however far from obvious that, as proposed in references \cite{6} and \cite{7}, the classical collapse would be observer dependent. Hopefully, in view of the fact that our dynamical approach may be consistent with a weak coupling to gravity, one could investigate whether or not this strong form of complementarity\cite{7},\cite{27} is needed or if and how a more conventional approach to the unitarity issue, with or without remnants, is still available.

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Figure Captions

Figure 1. Penrose Diagram of a Collapsing Shell.

The shaded region is the space-time available to the external observer. The dashed lines represent the motion of the centres of two correlated vacuum fluctuation wave packets; these are the ancestor of a Hawking photon localized on the scale of its wavelength and its partner.

Figure 2. Collapsing Shell in the \((u, v)\) coordinate system.

The figure depicts the shaded region of Fig. 1. The dashed line reflected on the curve \(r = 0\) is the Hawking photon ancestor and the straight dashed line is its partner. They meet on the curve \(\tau = 0\).

Figure 3. The Vacuum Fluctuation Generating a Hawking Photon.

Fig. 3a represents the real part of \(\tilde{T}_{uu}\) on \(\mathcal{I}^+\) that corresponds to a post-selected Hawking photon emitted in a gaussian wave packet centered on \(u = u_0\) with frequency \(\omega \simeq (2M)^{-1}\). The two crosses on the vertical axis correspond to \(\tilde{T}_{uu} = \pm \frac{1}{4\pi r^2(4M)^2}\). The positive energy core is centered around \(u = u_0\). Fig. 3b represents the real part of \(\tilde{T}_{vv}\) corresponding to the same Hawking photon. The Hawking photon ancestor is located at negative \(v\) (it carries negative total energy) whereas its partner is located at positive \(v\) (it carries poistive energy). Their are oscillations of \(\tilde{T}_{vv}\) near \(v = 0\) for \(v < 0\) which have not been represented. The region of positive \(\tilde{T}_{vv}\) on the left of the drawing centered on \(v \simeq -4Me^{-u_0/4M}\) becomes the positive energy core of Fig. 3a after reflection at \(r = 0\). The remaining negative and oscillatory parts of the ancestor correspond to the oscil 3a. The crosses on the vertical axis correspond to \(\tilde{T}_{vv} = \pm \frac{1}{4\pi r^2(4M)^2}e^{u_0/2M}\). The energy density becomes transplanckian after a time \(u_0 = t_p = O(M\log M)\) and is localized on a cisplanckian distance \(v = \pm 4Me^{-u_0/4M}\).

Figure 4. Phase diagram of closed string theory.
The equilibrium curve is the dark solid line. It coincides with $s_g$ for $\sigma < \beta_0^{-4}$ and with $s_{g+s}$ for $\sigma > \beta_0^{-4}$. 
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