Portfolio return using Black-litterman single view model with ARMA-GARCH and Treynor Black model

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Abstract. Establishing an optimal portfolio is a method that can help investors to minimize risk and optimize profits. Some models for optimal portfolio include Black-litterman Model and Treynor Black Model. The Black-litterman Model combines the elements of historical data and views of investors to form a new prediction of the portfolio as the basis for the preparation of weighted asset models. Predicted views in this study using time series ARMA-GARCH. The stock return data mostly have high volatility causing heteroscedasticity problems, so the GARCH model is chosen to overcome the problem. Treynor Black Model is active and passive portfolio. An active portfolio is that investors allocate their investment funds to individual securities in the capital market, while passive portfolios are investors allocating their investment funds to the stock market indices. In this research we will use Treynor-Black Model with active portfolio, so that investors can choose stock according to their allocation of funds. The purpose of this research is to form the weight of Black-litterman model portfolio with a single view investor using ARMA-GARCH and compare the profit result obtained from the formation of portfolio Black-litterman Model and Treynor-Black Model. The selected shares are PT. Bank BCA (BBCA), PT. Gudang Garam (GGRM), and PT. Waskita Karya (WSKT).

1. Introduction

Stock according to Bodie (2009) is a securities that indicate the existence of a company and is. Based on Hiriyappa (2008) establishing an optimal portfolio is a method that can help investors to attract risk and optimize profits.

In optimal portfolio required a model that is Black-litterman Model and Treynor Black Model. Black-litterman model is a model used for input prediction portfolio optimization. This model repeats two types of historical data on equilibrium conditions with the views of investors so as to meet the results (Black & Litterman, 1990). In Black-litterman Model, estimates investors will use GARCH (General Autoregressive Conditional Heteroskedasticity) method. Stock return data mostly have high volatility so GARCH method selected to overcome the problem.

The next problem is how to determine the prior market parameters of the Bayesian example. The Bayesian framework on the Black-litterman model is the equilibrium return on the Capital Asset Pricing Model (CAPM) as an option prior to the investor's view so as to produce a new view of the return expectation (posterior distribution). To get the optimal benefit from the Black-litterman
portfolio model, investors need to determine which combination to use from an investor's perspective. The combination of investor views can be one type of view (single-view) or some kind of combination of views.

Fortunately to find the optimal portfolio, then there is another model called the Treynor-Black Model. If the model portfolio of Black-litterman Model is viewed from view investor, then the Treynor-Black Model portfolio is divided into two, namely the active and passive portfolio. An active portfolio is the investor who allocates his investment funds to individual securities in the capital market, while the passive portfolio is the investor allocating his investment funds on the capital market index.

2. Black-litterman Model
Black-litterman Model is a model used to estimate inputs for portfolio optimization. This model combines two types of estimates, namely historical data on equilibrium conditions with the views of investors so as to update the estimation results (Black & Litterman, 1990).

The advantage of the Black-litterman Model is that investors can combine views in both absolute and relative terms with prior prior estimates, to generate new posterior estimates that include all view. The equation below is the Black-litterman equation and represents the expected return vector formed by the bayes rule of the equilibrium return vector \( \pi \) and the vector from the investor view \( V \).

\[
E(R) = [(\tau \Sigma)^{-1} + P' \Omega^{-1} P]^{-1} [(\tau \Sigma)^{-1} \Pi + P' \Omega^{-1} Q] = \mu_{bl} 
\]

\( \mu_{bl} \): the average vector estimation of the Black-litterman model is sized \( p \times 1 \)
\( \tau \): the level of investor confidence in his views (constants)
\( \Sigma \): variance-covariance matrix of return-sized \( p \times p \)
\( P \): vector / matrix from the investor's view of size \( q \times p \)
\( \Omega \): the diagonal matrix of covariance from the investor's view with certain sized \( q \times q \)
\( \Pi \): equilibrium return vector of CAPM sized \( p \times 1 \)
\( Q \): vector return value expectations according to investor's view \( q \times 1 \)

2.1 CAPM (Capital Asset Pricing Model)
There is a model that can be used to estimate the return of a securities stock that is Capital Asset Pricing Model (CAPM) pioneered by Sharpe, Lintner, and Mossin in 1964-1966. According to Bodie et al (2014), the CAPM model is an important part of the financial field used to predict the expected return balance and the risk of an asset under equilibrium conditions. CAPM is an important part of finance that is used to predict the relationship between return and asset expectations of an asset. the equilibrium formation formula of each share according to CAPM is as follows:

\[
E(R_{i, CAPM}) = R_f + \beta_i [E(R_m) - R_f] ; i = 1,2,..., p 
\]

\( E(R_{i, CAPM}) \): return equilibrium stock
\( R_f \): risk-free return value
\( \beta_i \): estimation of stock beta coefficient value
\( E(R_m) \): value of expectation of return from the market
\( p \): the number of stocks studied

2.2 The views of investor
\( P \) is a vector / matrix of investor's view. Each matrix row represents an investor's view either certain or relative to a stock. While \( Q \) is a vector of expectation return value according to investor's view. According to He & Litterman (1999), the diagonal matrix is the covariance of the view with a certain degree of tau.
\[ \Omega = P^T \sum P \]  

\( \tau \) : the level of investor confidence in his views (constants)
\( \sum \) : variance-covariance matrix of return-sized p x p
\( P \) : vector / matrix from the investor's view of size q x p
\( \Omega \) : diagonal matrix of covariance of investor's view with a certain sized q x q
\( p \) : the number of stocks
\( q \) : the number of investors' views

2.2 Investor's view with time series GARCH method

Bollerslev (1986) refined the ARCH model to GARCH to facilitate the assessment of parameters at higher order. Bollerslev states that residual variance depends not only on the last period's residual but also the residual variance of the past period. Generally GARCH model can be written as follows:

\[ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i e_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}^2 \]  

where 
\( \sigma_t^2 \) : variance of the residual t-period
\( p \) : ARCH order
\( q \) : GARCH order
\( \alpha_0 \) : constants
\( \alpha_i \) : parameters from ARCH
\( \beta_j \) : parameters from GARCH
\( e_{t-1}^2 \) : variance of residual period to t-1
\( h_{t-j}^2 \) : squared from residual period to t-j

2.3 Portfolio of the Black-litterman Model

After forming an equilibrium return value with CAPM and estimating the views of investors, the next step is to establish the weight of the Black-litterman model portfolio. The stages of establishing a portfolio of Black-litterman models are as follows:

1. Calculating the return value vector return of the Black-litterman model

\[ \mu_{bl} = \left( (\tau \sum)^{-1} + P^T \Omega^{-1} P \right)^{-1} \left( (\tau \sum)^{-1} + P^T \Omega^{-1} Q \right) \]  

2. Determine the weight of assets / stocks on the Black-litterman model

\[ W_{bl} = (\delta \sum)^{-1} \mu_{bl} \]  

where
\( \mu_{bl} \) : the average vector estimate on the Black-litterman model is p x 1.
\( \tau \) : the level of investor confidence in his views (constants)
\( \sum \) : variance-covariance matrix of return p x p size
\( P \) : vector or matrix of investor sized view q x p
\( \Omega \) : diagonal matrix of covariance of investor's view with a certain sized q x q
\( \Pi \) : vector return equilibrium of CAPM sized p x 1
\( Q \) : vector return expectation value according to investor sized view q x 1
\( W_{bl} \) : vector weight portfolio model Black-litterman p x 1 size
\( \delta \): value of stock coefficient
\( p \): the number of stocks
\( q \): the number of investors’ views

2.4 Treynor Black Model
Treynor and Black deal with a scenario in which the mean-variance criterion (the Sharpe ratio) is used by investors; a specified market index is taken as the default efficient (passive) strategy, and the security analysts of a portfolio management firm cover a limited number of securities. Under these conditions, securities that are not analyzed are assumed to be efficiently priced, and a portfolio of only the covered securities cannot be efficient. The optimal portfolio must be a mix of the covered securities and the index portfolio. Treynor Black Model identify the portfolio of only the covered securities (the efficient Active Portfolio, A) that can be mixed with the index (Passive Portfolio, M) to obtain the optimal risky portfolio. Here are the Treynor-Black model optimization step.

1. Calculate the initial position of each of the securities in the active portfolio

\[
W_i^0 = \frac{\alpha_i}{\sigma^2(e_i)}; \quad W_i = \frac{W_i^0}{\sum W_i^0}
\]

\( W_i^0 \): the initial position of securities i on the active portfolio
\( \alpha_i \): alpha securities i
\( \sigma^2(e_i) \): variance of i-th security
\( W_i \): the weight scale of the securities i from the initial position

2. Calculates the active portfolio alpha and beta

\[
\alpha_A = \sum W_i \alpha_i; \quad \beta_A = \sum W_i \beta_i
\]

\( \alpha_A \): alpha active portfolio
\( W_i \): the weight scale of the securities i from the initial position
\( \alpha_i \): alpha securities i
\( \beta_A \): the sensitivity of the active portfolio to the market
\( \beta_i \): the sensitivity of securities to the market

3. Calculates the starting position of active portfolio and optimal weight of active portfolio

\[
W_A^0 = \left[ \frac{\alpha_A}{\sigma^2(e_A)} \right] \frac{E(R_m)}{\sigma_M^2} ; \quad W_A^* = \frac{W_A^0}{1 + (1 - \beta_A)W_A^0}
\]

3. Portfolio Analysis of Black-litterman Model
3.1 Establishment of Return Equilibrium with CAPM
The shares used in this research are PT. Bank BCA, PT. Gudang Garam and PT. Waskita Karya shares with daily period from January 2014 to May 2017. The result of CAPM equilibrium return from each stock are PT. Bank BCA 0.000842 percent, PT. Gudang Garam 0.001016 percent, and PT. Waskita Karya 0.002987 percent. The result will then be used to form the equilibrium return vector of CAPM as follows
3.2 Investor’s view with Time Series Approach
Before stepping on the establishment of the Black-litterman model, we must identify investor views with the time series GARCH. The assumption that must be stationary test, normality test, autocorrelation test and heteroscedasticity test. The three shares have met the assumption test so that the ARMA-GARCH order used is BBCA ARMA (1.2)-GARCH (1.1), GGRM ARMA (2.1)-GARCH (1.1) and WSKT ARMA (2.2)-GARCH (1.2) with result as follows

Table 1. ARMA-GARCH models.

| Parameter | BBCA | GGRM | WSKT |
|-----------|------|------|------|
| ar1       | p-value = 0.000150 | p-value = < 2 x 10^{-16} | p-value = < 2 x 10^{-16} |
|           | estimates = 0.05455 | estimates = 0.08160 | estimates = 0.05942 |
| ar2       | - | p-value = 0.005547 | p-value = < 2 x 10^{-16} |
|           | estimates = -0.0102 | estimates = -0.09359 | estimates = 0.06111 |
| ma1       | p-value = 0.000197 | p-value = < 2 x 10^{-16} | p-value = < 2 x 10^{-16} |
|           | estimates = -0.05506 | estimates = -0.07944 | estimates = 0.05942 |
| ma2       | p-value = 0.053192 | - | p-value = < 2 x 10^{-16} |
|           | estimates = -0.008245 | | estimates = 0.09080 |
| omega     | p-value = 0.001818 | p-value = 0.086378 | p-value = 0.02341 |
|           | estimates = 0.000004405 | estimates = 1.20 x 10^{-5} | estimates = 7.7 x 10^{-5} |
| alpha1    | p-value = 0.00000373 | p-value = 0.000613 | p-value = 0.00180 |
|           | estimates = 0.0149 | estimates = 0.00632 | estimates = 0.01558 |
| beta1     | p-value = 7.89 x 10^{-10} | p-value = < 2 x 10^{-16} | p-value = 0.00841 |
|           | estimates = 0.06116 | estimates = 0.09078 | estimates = 0.03596 |
| beta2     | - | - | p-value = 0.01971 |
|           | | | estimates = 0.03215 |
| LM Arch   | Statistic = 3.334952 | Statistic = 9.356863 | Statistic = 10.29474 |
|           | p-Value = 0.9926816 | p-Value = 0.6721851 | p-Value = 0.5901208 |
| AIC       | -5.834237 | -5.041847 | -4.878264 |

Furthermore, investors’ views on the three stock returns will be estimated using GARCH method with the following results:

Table 2. Return Estimation

| Stock | Return Estimation |
|-------|------------------|
| BBCA  | 0.000130         |
| GGRM  | 0.000220         |
| WSKT  | 0.000210         |

next is formed vector / matrix from investor’s view (P) and vector return expectation value according to investor’s view (Q). Whereas (Ω) is diagonal matrix the covariance of the view investor with the (τ = 1). The variance-covariance matrix of return of each share are:
\[
\sum = \begin{bmatrix}
4.018831e-04 & 9.071965e-05 & 1.067316e-04 \\
9.071965e-05 & 1.880227e-04 & 8.243658e-05 \\
1.067316e-04 & 8.243658e-05 & 4.713472e-04 \\
\end{bmatrix}
\]

The matrix (P), vector (Q) and matrix (Ω) values for each view investor are as follows

1. **Single View 1**
   \[
P = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}; \quad Q = [0.00013]; \quad \Omega = [4.018831e-04]
   \]

2. **Single View 2**
   \[
P = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}; \quad Q = [0.00022]; \quad \Omega = [1.880227e-04]
   \]

3. **Single View 3**
   \[
P = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}; \quad Q = [0.00021]; \quad \Omega = [4.713472e-04]
   \]

3.3 **Portfolio Strategy with Single View Investor**

The comparison between Treynor Black model portfolio weight as with the Black-litterman model generated from each investor’s view is shown in Table 3. Black-litterman model with a Single View Investor and Treynor Black Model consistently allocates the biggest weight to the stock of PT. Waskita Karya

| Stock              | Treynor Black Model | View 1 | View 2 | View 3 |
|--------------------|---------------------|--------|--------|--------|
| PT. Bank BCA       | 0.2933              | 0.0676 | 0.0640 | 0.0677 |
| PT. Gudang Garam   | 0.1716              | 0.0520 | 0.0522 | 0.0516 |
| PT. Waskita Karya  | 0.5350              | 0.8804 | 0.8838 | 0.8807 |

4. **Conclusion**

ARMA-GARCH model can be used to determine investor’s view on Black-litterman model with the assumption that time series assumption can be fulfilled. Black Litterman and Treynor Black Model models consistently allocate the greatest weight to the shares of PT. Waskita Karya, PT. Bank BCA and PT. Gudang Garam with different weight characteristics. However, weight generated by both models can be used as a reference for choosing an optimal portfolio.

5. **References**

[1] Alex Kane, T.-H. K. 2003. Active Portfolio Management: The Power of the Treynor-Black Model.
[2] Black, F., & Robert, L. 1992. Global Portfolio Optimization. *Financial Analysis Journal*.
[3] Bodie, K. M. 2014. *Investments*. McGraw-Hill Education.
[4] Bollerslev, T. 1986. Generalized Autoregressive Conditional Heteroskedasticity.
[5] Hiriyappa, B. 2008. *Investment Management*. New Age International.
[6] Idzroek, T. M. 2002. A Step by Step Guide To The Black Litterman Model.