The natural properties analysis of a power turret gear system based on the varying mesh stiffness

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Abstract. As the time-varying contact stiffness is one of the most important nonlinear factors, which not only has great influence on dynamic properties, but also on the natural properties. In this paper, the natural properties of a power turret gear system were studied based on varying stiffness. The influence was concluded by natural frequencies.

1. Introduction
The dynamic characters of a power turret gear system has great influence on a turret. So, the study of a power turret gear system can reveal the sensitive design parameters and increase the tool's working speed effectively.

The natural properties are the basic dynamic characters of a gear system which can illustrated the vibration modes and location while motived by the external excitation. At present, many scholars have researched the natural properties of gear systems at home and abroad. Kahraman A.[1] researched the natural properties of a planetary gear system by the torsional vibration model. Wang Z.P et al.[2] presented a coupling model of a two-stage planetary gear rotor system, demonstrated the interaction between multistage planetary gears and external rotors, and proposed the lateral-torsional coupling properties. Liu W. et al.[3] analyzed the modal properties by a new elastic-discrete model of a planetary gear system. Sensitivity analysis of the natural properties were conducted in many papers[4,5]. But, the majority of papers used mean mesh stiffness when studying the natural properties of gear systems. In this paper, the varying mesh stiffness values are considered when computing the natural properties and the effects were shown out.

2. Dynamic model of the power turret gear train
The gear train is an idler drive system composed of four standard straight involute cylindrical gears as shown in figure 1 and the equivalent lumped-parameter dynamic model is established as shown in figure 2.

The gears are considered as rigid bodies with the mass \( m_i \), inertia \( I_i (i=p,s,b,q) \). Each gear is supported by the bearing with stiffness \( k_{ix}, k_{iy} \) and damping \( c_{ix}, c_{iy} \) (x, y are the two directions of the gear coordinate system). The connections between gears are illustrated by time varying mesh stiffness \( k_n(t) \) and linear damping \( c_n \) \((n=1,2,3)\). Each gear has two translational degrees \( x_i, y_i \) and one rotational degree \( \theta_i \). The backlash and error functions are \( f_{bn}(x_n), e_n(t) \).

Hence, the nonlinear equation of motions are
The motion differential equation of gear system is expressed as a matrix as follows
\[
M\ddot{q} + C\dot{q} + K(t)\dot{f}_h(q) = F(t)
\]  
(1)

Where \( M \) is mass matrix, \( C \) is damping matrix, \( K(t) \) is varying mesh stiffness, \( F(t) \) is load vector, \( q = \{x_p, y_p, \theta_p, x_s, y_s, \theta_s, x_b, y_b, \theta_b, x_q, y_q, \theta_q\} \). All the values used in the functions are listed in Table 1.

| parameter | gear p | gear s | gear b | gear q |
|-----------|--------|--------|--------|--------|
| gear number | 25 | 35 | 160 | 35 |
module (mm) 2
pressure angle (°) 20
mass (kg) 0.280 0.403 1.600 0.254
rotational inertia (kg.m²) 9.528E-5 1.104E-4 1.854E-2 6.236E-5
bearing stiffness (N/m) 6.88E7 4.20E7 9.82E7 6.12E7
bearing damping (N.s/m) 438.907 402.500 682.583 394.269
backlash (2bn) (m) 103E-6 145E-6 142E-6
mean value of stiffness (N/m) 1.639E8 1.783E8 1.821E8
meshphase (rad) 0 -1.961 -2.255
error (μm) 11.041 14.956 14.956
contact ratio 1.649 1.791 1.754
position angle φ (°) 91.899 265.967 32.487

tooth with (m) 0.01
input load (N.m) 40
output load (N.m) 40

3. Natural properties analysis considering varying stiffness

The three meshing rigidity of the gear system is divided into five combinations, as shown in table 2. In the table, "double" means double tooth mesh, and "single" means single tooth mesh. The first word in the combination represents the mesh pair 1, the second word represents the mesh pair 2, and the third word represents the mesh pair 3. Taking combination 1 as an example, "double-double-single" means that pair 1 is double-tooth mesh, pair 2 is double-tooth mesh, and pair 3 is single-tooth mesh. The mesh stiffness of single tooth g is the minimum mesh stiffness \( k_{\text{min}} \), The double teeth mesh stiffness is the maximum mesh stiffness \( k_{\text{max}} \).

Table 2. Combination mode of time-varying mesh stiffness

| a               | b               | c               | d               | e               |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| double-double-single | double- double - double | single - double - double | double - single - double | double - single - single |

In general, the effects of external load, damping, backlash, error and other nonlinear factors are not considered when the natural characteristics of gear system are studied. So, The undamping free vibration equation of the system is

\[
M\ddot{q} + K(t)q = 0
\]  

Thus, solving the natural properties of the system is transformed into solving the eigenvalues and eigenvectors, that is

\[
(K(t)\bar{q} - \omega^2 M)\bar{q} = 0
\]  

The mesh stiffness combination is substituted into the stiffness matrix \( K(t) \) of the system, and the natural characteristics are calculated to obtain the natural frequency and mode under different combination of mesh stiffness. First, the influence of combination stiffness on natural frequency is analyzed, as shown in table 3 and figure 3.

Table 3. Natural frequencies of the system considering the varying stiffness.

| number | Mean stiffness (Hz) | a (Hz) | b (Hz) | c (Hz) | d (Hz) | e (Hz) |
|--------|---------------------|--------|--------|--------|--------|--------|
| 1      | 946.1               | 947.7  | 947.9  | 944.2  | 943.0  | 942.8  |
| 2      | 1615.6              | 1613.3 | 1616.1 | 1616.0 | 1616.0 | 1613.3 |
| 3      | 1630.5              | 1630.5 | 1630.5 | 1630.5 | 1630.5 | 1630.5 |
| 4      | 1688.0              | 1688.0 | 1688.1 | 1688.0 | 1688.0 | 1688.0 |
Figure 3. The natural frequencies of the system under the combined mode of mesh stiffness.

It can be seen that mesh stiffness combinations have little impact on the first eight natural frequencies of the system, while the three natural frequencies of 9, 10 and 11 have great change, and the natural frequency fluctuates in a large range. The 9th natural frequencies of the system are analyzed here.

When the stiffness combination mode is a and b, the natural frequency is higher than that of the mean mesh stiffness. As can be seen from Table 3, in the stiffness combination mode a, the stiffness of meshing pair 1 and meshing pair 2 are taken as double tooth meshing. In combination b, the three meshing pairs are all double-tooth. The double tooth mesh stiffness of pair 1 and 2 is larger than the mean stiffness. It can be seen that the 9th natural frequency increases due to the increase of the mesh stiffness of pair 1 and 2. In these two combination modes, the meshing pair 3 is single tooth and double tooth respectively. It can be seen that the mesh stiffness of the pair 3 has little influence on the 9th natural frequency. Therefore, the mesh stiffness of pair 1 and pair 2 has a great influence on the 9th natural frequency.

When the combined stiffness modes are d and e, the 9th natural frequency is greatly reduced compared with that of the mean mesh stiffness. In the stiffness combination d, the meshing pair 2 is taken as single tooth mesh. In the combination e, meshing pair 2 is single tooth meshing. The 9th natural frequency decreases as the stiffness of pair 2 decreases. The mesh stiffness of pair 3 in these two combination modes is single tooth and double tooth respectively, and it can also be judged that the mesh stiffness of pair 3 has little influence on the 9th natural frequency. The meshing stiffness of pair 2 has a great influence on the natural frequency.

Based on the above two cases, it can be concluded that the mesh stiffness of pair 1 and pair 2 has great influence on the 9th natural frequency. At the same time, the meshing pair 1 in combination D and E is double-tooth meshing, but it does not cause the increase of natural frequency. It can be seen that the influence of mesh stiffness pair 2 on the natural frequency is greater than that of pair 1.

The 10th and 11th natural frequencies are taken the same analyzed method. The results show that the mesh stiffness of pair 3 has a great influence on the 10th natural frequency and the he mesh stiffness of pair 1 and pair 2 has great influence on the natural frequency of the 11th order.
4. Conclusion

The lumped parameter dynamic model of power turret gear train is established and computed the natural properties based on the varying mesh stiffness. The results show that the values of mesh stiffness have little influence on the first eight frequencies while have great effect on the last three frequencies which are helpful for the subsequent dynamic optimization design.

Acknowledgments

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References

[1] Kahraman A. 2001 Free torsional vibration characteristics of compound planetary gear sets Mechanism and Machine Theory vol. 36, pp 953–971
[2] Wang Z.P, Yuan Y.B, Wang Z.W et al. 2018 Lateral-Torsional Coupling Characteristics of a Two-Stage Planetary Gear Rotor System Shock and Vibration pp 1-15
[3] Liu W., Shuai Z.J, Guo Y.B and Wang D.H 2019 Modal properties of a two-stage planetary gear system with sliding friction and elastic continuum ring gear Mechanism and Machine Theory vol 135 pp 251-270
[4] Lin J, Parker RG 1999 Sensitivity of planetary gear natural frequencies and vibration modes to model parameters Journal of Sound and Vibration vol 228 pp 109-128
[5] Ayoub M, Ahmed H et al. 2019 Effect of load and meshing stiffness variation on modal properties of planetary gear Applied Acoustics vol 147 pp 32-43