Directed transport in driven optical lattices by gauge generation

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We examine the dynamics of ultracold atoms held in optical lattice potentials. By controlling the switching of a periodic driving potential we show how a phase-induced renormalization of the intersite tunneling can be used to produce directed motion and control wavepacket spreading. We further show how this generation of a synthetic gauge potential can be used to split and recombine wavepackets, providing an attractive route to implementing quantum computing tasks.

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Introduction – In recent years enormous experimental progress has been made in creating and trapping ultracold atom gases [1]. When placed in an optical lattice potential these gases provide extremely clean and controllable implementations of interacting lattice systems, since parameters such as the interparticle interaction, the lattice depth and spacing are all readily tunable. Dissipation and decoherence effects are typically extremely weak, allowing the quantum coherent behaviour of these systems to be directly observed.

In contrast to electronic systems, however, trapped atoms are uncharged, and so electric or magnetic fields cannot easily be used to produce or regulate transport. Due to their excellent coherence properties, one means of controlling the dynamics of the atoms is via quantum interference effects. A notable example is termed “coherent destruction of tunneling” (CDT) in which a periodic driving of the lattice causes the amplitude of the intersite hopping to be renormalized [2]. This renormalization has been seen directly in the expansion of trapped Bose-Einstein condensates [3,4], and has been used very recently to produce the fascinating phenomenon of “super Bloch oscillations” [3,6], and to induce the quantum phase transition [7] between a superfluid and an insulator.

In this Letter we show that as well as controlling the amplitude of the hopping, a periodic driving field can also be used to produce a tunneling phase, equivalent to a $U(1)$ gauge potential. This gauge potential arises from the combined effect of the phase of the driving field, and the careful control of the switching condition. Although here we only consider one-dimensional lattices, the technique can also be similarly applied to create hopping phases in higher-dimensional systems. In this case the phases can be interpreted as Aharonov-Bohm phases picked up by a particle hopping from site to site, corresponding to a synthetic magnetic field threading the lattice [8]. Other schemes have been devised to produce such gauge potentials in cold atom systems, including lattice rotations, state-dependent optical potentials [9], or phase-imprinting [10]. Our procedure, however, has an appealing simplicity requiring only the periodic vibration of the lattice potential, which is easily produced in experiment. We show how the driving can be used to control both the spreading and position of an initial wavepacket, and in particular, how a directed current of non-dispersing wavepackets can be induced. We shall also demonstrate how wavepackets can be split, guided, and recombined in a controllable and robust manner, accessible to current experiment.

Model – A gas of weakly interacting ultracold bosonic atoms can be described well by the Gross-Pitaevskii equation (GPE). When a sufficiently deep optical lattice potential is applied, the wavefunction will localize mainly in the potential minima defining the lattice sites, making it convenient to use a discretised form of the GPE

$$i \frac{\partial \psi_j}{\partial t} = - \left( J \psi_{j+1} + J^\dagger \psi_{j-1} \right) + g \left| \psi_j \right|^2 \psi_j + jV(t)\psi_j.$$  (1)

Here $\psi_j$ denotes the system’s wavefunction on lattice site $j$, and $J$ describes the tunneling amplitude between nearest neighbor sites. Interactions between the bosons are given by a mean-field interaction, set by the nonlinearity parameter $g$. The time-dependent driving potential is assumed to rise linearly across the lattice [3,4,6], and has a time dependence given by $V(t) = \Delta + K \sin(\omega t + \phi)$, where $\Delta$ is a static tilt of the lattice potential, and $\omega$ and $K$ are the frequency and amplitude respectively of the oscillating component.

As an initial state we take a Gaussian wavepacket, $\psi_j = N \exp \left[ -j^2/2\sigma_0^2 + i\theta_j \right]$, where $\sigma_0$ is the initial width of the wavepacket measured in units of the lattice spacing, and $N$ normalizes the wavefunction to unity. This choice of initial state mimics the experimental situation [3,4,6], in which the condensate is prepared in a harmonic trap, and so typically has a Gaussian profile when transferred to the optical lattice. Note also that we explicitly include a site-dependent phase term $\theta_j$ in the wavefunction.

Analysis – We first consider the non-interacting case ($g = 0$). The Hamiltonian describing the system [11] is then $T$-periodic in time, where $T = 2\pi/\omega$, and the natural framework to describe its time evolution is Floquet theory. This reveals that in the high-frequency limit ($\omega > J$) the time-dependent driven system can be described by an effective static Hamiltonian, whose param-
eters can be systematically evaluated by using perturbation theory [1] on the Floquet states. While these states are explicitly time dependent, being $T$-periodic functions, in the high-frequency limit their time variation is rather weak. This is the origin, for example, of the well-known negligible time-dependence of CDT [2], as compared to the large oscillations observed at low driving frequencies when dynamical localization instead occurs [12].

To first-order the tunneling amplitudes are modified as

$$J \rightarrow J_{\text{eff}} = J \langle \exp \left[ -i \int_0^T V(t') dt' \right] \rangle,$$

where $\langle \cdots \rangle$ indicates a time-average over the driving period $T$. We restrict ourselves to considering the case of resonant driving, when $\Delta = n\omega$, which yields the result

$$J_{\text{eff}}/J = e^{-iK/\omega} \cos \phi e^{i n \pi/2} J_n (K/\omega),$$

where $J_n$ is the $n$th Bessel function of the first kind. For the case of $n = 0$, a similar expression was obtained in Ref.[13] for a driven Bose-Hubbard model. There a ramped driving potential was used to adiabatically transform the ground state of the Hamiltonian to a stroboscopically current-carrying Floquet state. Here, however, we use the tunneling phase in a very different way, to control the non-equilibrium dynamics of an expanding atomic wavepacket. This, in conjunction with the weak-time-dependence of the Floquet states in the high-frequency regime, means that the results we present are not stroboscopic, and have a negligible dependence on the moment within each driving period at which the system is measured.

From Eq. (2) we can immediately note the importance of the phase of the driving, $\phi$. For a sinusoidal driving ($\phi = \pm \pi/2$), the most frequently considered case in the literature, this result simplifies to yield $J_{\text{eff}}/J = (-1)^n J_n (K/\omega)$ – the well-known Bessel function renormalization of tunneling found in CDT. For sinusoidal driving ($\phi = 0$), the tunneling additionally acquires a phase $J_{\text{eff}}/J = \exp \left[ -i (K/\omega - n \pi/2) \right] J_n (K/\omega)$.

It is natural to ask whether this tunneling-phase has physical implications, since it would appear that $\phi$ can simply be eliminated by a shift of the time coordinate. It is important to note, however, that we consider the driving potential $V(t)$ to be switched on at a specific moment $t = 0$, in common with experimental implementations [2, 8]. This gives the time origin, and thus the driving-phase, an unambiguous definition, and consequently $\phi$ can indeed be of experimental relevance, as noted in Ref.[14]. This differs from many theoretical analyses [13], in which the steady-state properties of a driven system are considered, and the driving is implicitly assumed to have been turned on at $t \rightarrow -\infty$. In such cases the phase of the driving is indeed unimportant.

The expansion of an initially Gaussian condensate in a periodically-driven lattice was analyzed in Ref.[16] for real values of $J_{\text{eff}}$. Extending this analysis to complex $J_{\text{eff}}$ gives the result

$$\sigma(t) = \sigma_0 \sqrt{1 + (\Re[J_{\text{eff}}]/\sigma_0)^2},$$

We thus see that the spreading of the wavepacket is governed by the real component of $J_{\text{eff}}$. For $n = 0$, for example, $\Re[J_{\text{eff}}] = J_0 (K/\omega) \cos (K/\omega \cos \phi)$, and so as well as freezing at the “standard” CDT condition (when the Bessel function vanishes), expansion is also suppressed at an additional set of values where $\cos (K/\omega \cos \phi) = 0$.

As well as the expansion of the condensate, another useful experimental measurement is its center of mass motion. In the absence of driving our system has the standard spectrum of a non-interacting lattice model, $E_k = -2J \cos k$. When the system is driven, we can replace the energies $E_k$ with quasienergies, obtained as solutions of the Floquet equation, to obtain the new dispersion relation $\varepsilon_k = -2|J_{\text{eff}}| \cos(k - \theta_k)$, where $J_{\text{eff}} = |J_{\text{eff}}| \exp[i \theta_k]$. The effect of the tunneling phase is thus not to alter the quasienergy spectrum of the system, but to displace the wavepacket to another point in the first Brillouin zone. In analogy with the familiar semiclassical expression we can now define a mean group velocity, $\overline{v}_g = d\varepsilon_k / dk$, where the average is taken over one period of the driving, to obtain the final result

$$\overline{v}_g = -2K |J_{\text{eff}}|.$$

We thus arrive at the rather elegant result that the two quantities most accessible to experiment – the wavepacket expansion and its center of mass motion – are directly related to the real and imaginary parts respectively of $J_{\text{eff}}$.

**Directed transport** – To verify these results we numerically simulate the model [11] for a 200 site lattice with no static tilt ($\Delta = 0$), and take the onsite phases $\theta_j$ to be constant. In Fig.[4], we show the condensate’s expansion for a sinusoidal driving for several values of $K/\omega$. These curves consist of an initial quadratic dependence on $t$ followed by a linear ballistic expansion at long times [16], and clearly show how varying $K/\omega$ controls the condensate spreading. Eq.[3] can be used to extract the value of $|\Re[J_{\text{eff}}]|$ from these expansion curves, which we plot in Fig.[4]. The expected Bessel function dependence of $J_{\text{eff}}$ is clearly seen, with the condensate expansion being frozen at the zeros of $J_0$ (for $K/\omega = 2.40, 5.52 \ldots$). However, the corresponding expansion for sinusoidal driving shows an additional set of zeros at $K/\omega = \pi/2, 3\pi/2 \ldots$, in exact agreement with Eq.[2] for $n = 0$. At these values of driving the suppression of the expansion arises from a very different cause; the tunneling phase displaces the wavepacket to $\theta_0 = \pi/2$ in the first Brillouin zone where the quasienergy bands have an inflexion, causing the effective mass to diverge and so quenching the spreading of the wavepacket.
In Fig. 1 we show the motion of the center of mass of the condensate under sinusoidal driving. The wavepacket, initially at rest, begins to move at a constant rate, depending on $K/\omega$. In Fig. 1 we plot the velocity corresponding to this displacement, and find that it agrees excellently with the predicted mean group velocity $v_g$ (Eq. 3). Under sinusoidal driving, however, the velocity of the wavepacket is zero, also as predicted.

We note that to obtain Eqs. 3 and 4 we have assumed that $J_{\text{eff}}$ obtained. Experiments typically use driving frequencies of the order of kHz, which would thus demand ramp-times of ~ 10 µs, which are achievable.

**Directed motion** – We can thus see that $\phi$ can be used to cause an initially stationary wavepacket to move in a given direction with a precisely defined velocity, without requiring the spatial symmetry of the lattice to be broken. Two values of $K/\omega$ are of particular interest, and are marked in Fig. 1d. For $K/\omega = 2.404$ (the first zero of $J_0$) the expansion of the initial wavepacket is suppressed, and its induced velocity is zero for all values of the driving phase. This amounts to a complete suppression of the dynamics of the condensate. However, at $K/\omega = \pi/2$ a wavepacket that is sinusoidally driven will not expand, but will have a non-zero velocity – a directed current of non-dispersive wavepackets.

In Fig. 2 we show the motion of such a wavepacket. Initially we set $K/\omega = \pi/2$ to induce motion. The driving is then tuned to $K/\omega = 2.404$ to bring the wavepacket to a halt, and then to $K/\omega = -\pi/2$ to move the wavepacket in the opposite direction. It is clear that the spreading of the wavepacket is negligible, and that this technique indeed gives excellent control over the system. It is interesting to note that a similar form of control was reported in Ref. 17 for an amplitude modulated lattice, instead of the phase-modulated lattice we consider. An important difference between the two cases, however, is that phase modulation does not require the presence of a static lattice tilt, since the effects also occur for $n = 0$, whereas amplitude modulation is limited to the case of resonant driving ($n > 0$). In addition, amplitude modulation does not produce CDT, the intersite tunneling depending linearly on the driving amplitude instead of the Bessel function dependence given in Eq. 2.

**Wavepacket splitting** – We now consider the effect of the onsite phases $\theta_j$. It is well-known that imprinting a wavepacket with a uniform phase gradient $\theta_{j+1} - \theta_j = \theta$ has the effect of inducing motion of the center of mass, similar to the motion we have observed by manipulating the driving-phase $\phi$. By simulating the system with different values of $\phi$, we have confirmed that the two phases combine, so that the net motion of the wavepacket actually depends on the phase difference $\phi - \theta$. The driving field can thus be used to separate components of a wavepacket which possess different phase gradients. Let us consider the case of a superposition of a wavepacket with uniform phase ($\theta = 0$) and one with $\pi$-phase ($\theta = \pi$). If the components have equal weight, the
superposition will have the form $\psi_j = N \exp[-j^2/2\sigma_j^2]$ for $j$ odd, and $\psi_j = 0$ for $j$ even (with no loss of generality we can interchange the roles of the odd and even sites). Such a state can be prepared, for example, by patterned loading of a single uniform-phase condensate.

Under sinusoidal driving, for $K/\omega = \pi/2$, the component with uniform phase will move, without distortion, at a negative velocity, while the $\pi$-phase component will move identically but with a positive velocity. The initial wavepacket will thus split apart, as shown in Fig. 3. Tuning $K/\omega = 2.404$ will bring each component to a stop, and then setting $K/\omega = -\pi/2$ will bring the wavepackets together. For zero interaction ($g = 0$) the wavepackets will simply pass through each other. For small values of $g$ the splitting process occurs as before, but during the collision the interaction causes the wavepackets to distort and produces a slightly asymmetric final state as shown.

**Incoherent expansion** – An intriguing result seen in Ref. 6 is that unlike previous driven lattice experiments, the wavepacket deformed under resonant driving, developing pronounced edges during its expansion. As a possible explanation of this effect we now look at the expansion of a phase incoherent wavepacket, by averaging over many realizations of random onsite phases $\theta_j$. The result in Fig. 4 is strikingly similar to the experimental observation.

The phase effects we have discussed give a simple explanation of this behavior. A phase incoherent wavepacket can be expressed as a mixture of many wavepackets, each with a random, but constant, phase gradient $\theta$. Under the periodic driving each component will both develop a certain velocity and will spread, according to Eqs. 3, 4. As the components which spread least have the highest velocity, while those that move more slowly spread more rapidly (see Fig. 4b), the initial state will segregate with the rapidly moving components at the edges of the wavepacket remaining taller and narrower than the slower-moving components near the center. The edges of the wavepacket will move at the maximum speed, which for $n = 1$ is given by $v_{\text{max}} = |2J_1(K/\omega)|$. For the driving parameters used, our model predicts $v_{\text{max}} = 847 d_L/s$, where $d_L$ is the lattice spacing, which compares well with the experimentally measured value of $869 d_L/s$. We thus suggest that the unusual expansion seen in Ref. 6 is a consequence of the phase incoherence of the initial state, possibly arising from phase randomization produced by Wannier-Stark localization during the preparation of the system.

**Conclusions** – We have shown how the phase of a driving potential can be used to control the dynamics of an atomic wavepacket, both by regulating its rate of expansion, and by inducing a steady drift of its center of mass. Combining these effects allows the directed transport of non-dispersive wavepackets. Periodic driving also acts as a “prism” for the separation of different phase contributions within a wavepacket. This allows wavepackets to be divided and recombined, and also provides an appealing explanation for the unusual condensate expansion observed in Ref. 6. While these results have been obtained within mean field theory, probing the behavior of systems in the strongly-correlated regime remains an interesting subject for future research, holding out the enticing prospect of using these effects to generate and distribute entanglement in coherent lattice systems. We also note that since these directed currents require coherence across many lattice sites and driving cycles, their eventual decay may provide information on decoherence mechanisms.
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