Two-Field Quintessential Higgs Inflation

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We study a two-field quintessential Higgs inflation model in which a quintessence field with an exponential potential $e^{-\beta \phi/M_P}$ is coupled to the Higgs field from the beginning of inflation. The Higgs field itself is also non-minimally coupled to gravity. The inflationary predictions of this model for $n_s$ and $r$ are in good agreement with Planck 2018 data. We calculate the observables $n_s$ and $r$ against the free parameter $\beta$. Comparing these parameters with the observed $n_s$ and $r$ in Planck 2018 paper, we find $\beta \lesssim 8 \times 10^{-3}$ that strongly disfavors the Swampland conjecture.

I. INTRODUCTION

Cosmologists consider two accelerating eras of cosmic expansion history. The first is an inflationary era where a scalar field, called inflaton, with negative pressure and a slow-roll evolution, is responsible for accelerating expansion in the early universe \cite{1}. Based on the latest Planck papers \cite{2, 3}, single field potentials with plateau-like shapes are the most favored inflationary models by Planck temperature, polarization, and lensing data combined with the BICEP2/Keck Array BK15 data. In between the single field inflationary models, the Higgs model \cite{4, 5} has the best consistency with the Planck 2018 data. It belongs to a general class of inflationary models where a scalar field has a non-minimal coupling to gravity \cite{6}.

The second accelerating era is the late time acceleration, discovered in 1998 \cite{7}. The...
most accepted premise for explaining this discovery is the dark energy (DE) theory. In this theory, dark energy is assumed as an unknown form of energy responsible for the presently observed acceleration of the galaxies. The dark energy would need to be very homogeneous and very low-density. It also has negative pressure. There are two general proposals for dark energy. The first is the cosmological constant, $\Lambda$, added to the Einstein equation. This constant represents a constant energy density filling space homogeneously. Indeed, this proposal leads to Lambda cold dark matter ($\Lambda$CDM) cosmological model, which has excellent agreement with the cosmological data \cite{2}. Following the exotic nature of dark energy, a second DE scenario has been proposed as a dynamical scalar field with infinitesimal energy density, called quintessence, which its slow-roll evolution at the late time can accelerate the universe \cite{8}. This scalar field avoids the extreme fine-tuning of the cosmological constant \cite{9}. Although, $\Lambda$CDM model with dark energy equation of state $\omega_\Lambda = -1$ satisfies the cosmological data very well without invoking any other explanation for dark energy, if future cosmological experiments show a deviation from $\omega_\Lambda = -1$, then quintessence may help to explain this deviation \cite{10}.

In recent years, some string theorists have attempted to study dark energy in the context of string Swampland \cite{11}. They have studied constraints imposed by two proposed Swampland conjectures on cosmology. The first conjecture says that de Sitter vacua belong to the Swampland. Consequently, the dark energy cannot result from a positive cosmological constant or being at the minimum of potential with positive energy density. The second conjecture is that dark energy is a rolling quintessence field $\phi$ which it’s potential $V_Q(\phi)$ should satisfy the universal Swampland conjecture \cite{12}

$$M_P \frac{\left| \nabla_\phi V_Q \right|}{V_Q} > c \sim O(1) , \quad (1)$$

where $M_P$ is the reduced Planck mass and $\left| \nabla_\phi V_Q \right|$ is the norm of the gradient of $V_Q$. However, the results of some quintessence and inflation models are in tension with these conjectures, especially with the second one. For example, a famous quintessence potential for describing dark energy is $V_Q(\phi) = V_0 e^{-\beta\phi/M_P}$, where $\beta$ is a dimensionless positive constant and $V_0$ is potential energy constant. Interestingly, substituting this potential in the conjecture (1) gives $\left| \nabla_\phi V_Q \right|/V_Q = c$ (with $c = \beta$). However, based on the current observational constraints on dark energy, it has been shown in \cite{13} that the marginalized upper bounds give $c < 0.6$ with 68% confidence level (CL) and $c < 1.35$ with 99.7% CL. In another study, Akrami et
al. showed that the quintessence exponential models with $c \gtrsim 1.02$ are ruled out at the 99.7% CL \[14\]. It means that for these models, the data puts an upper bound on $c$ less than unity which contradicts conjecture \(\text{(1)}\). On the other hand, it has been demonstrated in \[15\] that increasing $c$ only exacerbates the so-called Hubble tension problem where there is a disagreement between the late-time and the early-universe measurements of the Hubble constant \[16\].

The situation is worse for the inflationary models. For most of them, the condition $|\nabla \varphi V(\varphi)|/V(\varphi) \to 0$ is met, where $\varphi$ is the inflaton field and $V(\varphi)$ is the inflation potential. Indeed, the current CMB constraint on the inflationary observables gives $|\nabla \varphi V(\varphi)|/V(\varphi) < 0.09$ \[3\]. Therefore, if one can not find a new inflationary model to solve this challenge, either the Swampland conjectures are wrong, or the inflation is not a suitable mechanism for solving the horizon problem, flatness, and density perturbation spectrum of the observable universe \[12\].

Although the quintessence avoids the extreme fine-tuning of $\Lambda$, it has another tuning problem, namely its initial condition. Therefore, explaining inflation and dark energy in unified scenarios with a single dynamical scalar field may overcome the quintessence difficulties \[10,17,20\]. In these scenarios, named single field quintessential inflation, the scalar field needs two plateau shoulders with an extremely large difference in their heights. The scalar field slowly rolls down from its inflationary plateau to its quintessential ones without oscillating around the potential minimum. It means that the reheating of the universe must have happened at the end of inflation via a mechanism called gravitational reheating \[10,17,18,21\], instead of inflaton field decay. Indeed, through a mechanism called kination, almost all of the potential energy of the inflaton becomes converted to its kinetic energy to prevent the field decays completely. After inflation, rolling the field towards its large negative values freezes at some $\phi_F$. In the late time, where the density of the radiation and the matter drops significantly, and the dark energy eventually dominates the energy density of the universe, the scalar field starts rolling down again and acts as the quintessence field \[10,17,18\].

Although the exponential potentials are interesting for describing dark energy, they cannot describe both the inflation and the quintessence in a single field quintessential inflationary model. Because, they support the inflation for $c < 0.09$ that not only strongly disfavors the conjecture \(\text{(1)}\) with $c > 1$, but also inflation never ends \[10,12,14\]. Therefore, one
should consider another adequate field that acts as the inflaton. On the other hand, if later studies confirm the Swampland conjecture \cite{1}, the single-field slow-roll models of inflation in the landscape will be ruled out, and multi-filed ones with steep potentials will be replaced \cite{22}.

The above discussions may motivate moving to the multi-field inflation and dark energy models. For this purpose, a new scenario, called two-field quintessential inflation, assumes that from the early times, there has been a two-dimensional potential in which one field is responsible for the dark energy. In contrast, inflation is driven by another field \cite{10}. During inflation, the inflaton field is central to the universe’s evolution, while the quintessence field is sub-dominant. When the inflation ends, the inflaton field falls to the quintessence valley. In this model, the reheating occurs due to oscillations of the inflaton field near the potential minimum. Because of the small potential slope along the quintessence field axis, the quintessence field is sub-dominant until the density of reheating products (radiation and matter) decreases significantly. Eventually, the quintessence field undergoes the slow-roll evolution, which leads to the late time accelerated expansion. One of the benefits of considering the quintessence field from the beginning of inflation is to remove its initial condition problem.

This paper aims to study a new model of two-field quintessential inflation. This new model is an extension of a proposed model of quintessence $\phi$, which is coupled to the Higgs boson \cite{23}. We suppose that the quintessence field is coupled to the Higgs field from the beginning of inflation. The Higgs field also has a non-minimal coupling to gravity during inflation. The paper is organized as follows: In Sec. II, we introduce the model and study its potential behavior in the Einstein frame during inflation. Next, we deal with the slow-roll evolution of the model and its inflationary observables in Sec. III. In Sec. IV, we calculate the number of e-folds to the end of inflation, which depends on some quantities like the energy density of the model, the reheating temperature, and the equation of the state of reheating. We review a proposal for solving the Higgs instability problem in Sec. V that puts a constraint on the $\beta$ parameter in the quintessence model. Sec. VI is devoted to studying the CMB bounds on the inflationary observables of the model. Finally, this paper concludes with a summary in Sec. VII.
II. THE MODEL

Our two-field quintessential inflation model includes the Higgs and quintessence fields coupled with each other. Before constructing our model, we introduce these two fields separately.

The Higgs potential is as the following form

$$V_{\mathcal{H}} = \lambda (|\mathcal{H}|^2 - v^2)^2,$$

where $\mathcal{H} = \begin{pmatrix} 0 \\ h/\sqrt{2} + v \end{pmatrix}$ is the Higgs field, $\lambda$ is the self-coupling constant, and $v$ is the Higgs vacuum. The Higgs potential has a global minimum at $|\mathcal{H}| = v$ and a local maximum at $|\mathcal{H}| = 0$. The Higgs field is accompanied by a fundamental particle known as the Higgs boson, which was discovered in 2012 at the Large Hadron Collider (LHC) lab [24]. Although the Higgs model plays an essential role in the standard particle physics (SM) model, it does not work well in describing inflation. That is because the large self-coupling $\lambda$ gives matter fluctuations larger than the observation. Fortunately, one can solve this problem by coupling the Higgs field non-minimally to gravity [5]. This solution provides an inflation model that, on the one hand, has a root in the SM theory and, on the other hand, has excellent consistency with the CMB data [3].

One of the well-known quintessence models for describing the late time dark energy is the model with the following potential

$$V_Q(\phi) = V_0 e^{-\beta \phi/M_P},$$

where $\beta$ is a dimensionless positive constant and $V_0$ is potential energy constant. It is shown that for this model, the scale factor of the expanding universe grows as $a \sim t^{2/\beta^2}$. To provide an accelerating expansion, one should set $\beta < \sqrt{2}$ [28, 29].

One also can extend the potential [3] and assume a quintessence potential with two exponential terms as

$$V_Q(\phi) = V_1 e^{-\beta_1 \phi/M_P} + V_2 e^{-\beta_2 \phi/M_P},$$

where $\beta_1$ and $\beta_2$ are positive dimensionless constants and $V_1$ and $V_2$ are potential energy constants [30, 31]. During radiation- and matter-dominated eras in which the quintessence field is sub-dominant, the potential [41] is approximated as $V_Q(\phi) \simeq V_1 e^{-\beta_1 \phi/M_P}$ giving rise to
the quintessence density parameter \( \Omega_i = 3(1 + \omega_i)/\beta_i^2 \ll 1 \), where \( \omega_i \) is the equation of state of the background fluid in matter and radiation eras. For the late time, the second term \( V_2 e^{-\beta_2 \phi/M_P} \) with the quintessence equation of state \( \omega_\phi = -1 + \beta_2^2/3 \) dominates. Considering the \( 2\sigma \) observational contour based on the data of Supernovae type Ia (SN Ia), Baryon Acoustic Oscillations (BAO) and CMB, the potential (4) with \( \beta_1 \gg 1 \) and \( \beta_2 \ll 0.1 \) is favored [31].

In the neighborhood of the local maximum of Higgs potential \( V_H(h) \), we obtains \( |\nabla_h V_H(h)| \sim 0 \) and hence the conjecture [11] is violated. Supposing the Higgs and the quintessence as the only scalar fields at the Electroweak (EW) scale [23, 32], a simple combination of (2) and (3) as

\[
V(h, \phi) = V_H(h) + V_Q(\phi)
\]

(5)
gives a non-vanishing \( |\nabla V| \). For this potential, one finds

\[
M_P |\nabla V| \sim 10^{-55}
\]

(6)
which is in significant tension with [11] [23, 32]. A special combination of (2) and (3), which may remove this tension, is

\[
V(h, \phi) = e^{-\beta(\phi - \phi_0)/M_P} (V_H(h) + \Lambda)
\]

(7)
where \( \phi_0 \) is the value of \( \phi \) at the present time [23]. In this model, one assumes a trilinear coupling \( v^2 M_P \phi h^2 \) between the Higgs and quintessence fields in the early universe.

Now, we use (7) to construct a two-field quintessential-Higgs inflation model. To this end, we use a generalized action in the Jordan frame for two scalar fields non-minimally coupled to gravity as follows [33]

\[
S_J = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_P^2 \left[ 1 + f(h, \phi) \right] R - \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(h, \phi) \right]
\]

(8)
where \( R \) is the Ricci scalar and \( f(h, \phi) \) is a non-minimal coupling term. The metric signature of this action is \((-,-,+,+)\).

To achieve a canonical form for the action (8), we use the following conformal transformation

\[
\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}
\]

(9)
with the conformal factor, \( \Omega^2 \), defined as

\[
\Omega^2 = 1 + f(h, \phi)
\]

(10)
where the non-minimal coupling \( f(h, \phi) \) is a polynomial function. Here, we assume that the quintessence field has minimal coupling to gravity. Therefore, the function \( f(h, \phi) \) is only a function of the Higgs field, \( f(h, \phi) = f(h) \). We choose a quadratic form for \( f(h, \phi) \) as

\[
\Omega^2 = 1 + \xi \frac{h^2}{M_P^2},
\]

(11)

where \( \xi \) is the coupling constant between the Higgs field and gravity, to have a non-minimal model which is compatible with the particle physics and inflation scenario simultaneously, we need \( 1 \ll \sqrt{\xi} \ll 10^{17} \) \[5\].

The conformal factor (11) leads to a non-minimal kinetic term for the Higgs field in\[8\] \[35\]. To find a conformal form for this kinetic term, we assume that

\[
\frac{d\chi}{dh} = \sqrt{\frac{1}{\Omega^2} + 6 \left( \frac{\Omega'}{\Omega} \right)^2},
\]

(12)

where \( \chi \) is the canonically normalized form of \( h \) and the prime denotes derivative with respect to \( h \) \[5\] \[25\]. Finally, using (11) and (12), one obtains the following well-known form for the action \( \mathcal{S}_E \) in the Einstein frame

\[
\mathcal{S}_E = \int d^4x \sqrt{-\hat{g}} \left[ \frac{1}{2} M_P^2 \hat{R} - \frac{1}{2} \partial_\mu \chi \partial^\mu \chi \right. \\
\left. - \frac{1}{2} e^{2b} \partial_\mu \phi \partial^\mu \phi - \hat{V}(\chi, \phi) \right],
\]

(13)

with

\[
\hat{V}(\chi, \phi) = \frac{V(h, \phi)}{\Omega^4} = \frac{e^{-\beta(h-\phi_0)/M_P} \left( \frac{\chi}{\sqrt{6}} \right)^2 + \Lambda}{\left( 1 + \frac{\xi}{M_P^2} h^2 \right)^2},
\]

(14)

and

\[
b = -\frac{1}{2} \ln(\Omega^2).
\]

(15)

For large field values of \( h \gg M_P/\sqrt{\xi} \) or equivalently for \( \chi \gg \sqrt{6} M_P \), the solution of (12) can be approximated as

\[
h = \frac{M_P}{\sqrt{\xi}} \exp \left( \frac{\chi}{\sqrt{6} M_P} \right).
\]

(16)

Therefore, by substituting (16) into (14), we find a product-separable potential in the limit \( h \gg v \gg \Lambda \)

\[
\hat{V}(\phi, \chi) = \hat{V}(\phi)\hat{V}(\chi),
\]

(17)
where

\[
\dot{V}(\phi) = e^{-\beta(\phi-\phi_0)/M_P},
\]

and

\[
\dot{V}(\chi) = \frac{\lambda M_P^4}{4 \xi^2} \left(1 + e^{-\sqrt{2}\chi/M_P}\right)^{-2}.
\]

Inflation is driven when the inflaton field with nearly flat potential rolls down very slowly compared to the expansion of the Universe [36]. At large field values \(\chi \gg \sqrt{6} M_P\) and for \(\beta \ll 1\), the potential (17) as shown in Fig. 1 is exponentially flat. Therefore, inflation occurs at the pink plateau along the \(\chi\)-axis via slow-roll evolution of the \(\chi\) field. In the next sections, we will discuss the inflationary observables of (17) and compare their results with the CMB data.

Approximating (11) and (12) for small field values \(h \ll M_P/\sqrt{\xi}\) or equivalently for \(\chi \ll \sqrt{6} M_P\), one finds \(\Omega^2 \simeq 1\) and \(\chi \simeq h\). In this limit, the two-field potential (14) approaches the potential (7), which is applicable for the eras after the end of inflation.

**FIG. 1**: The shape of the potential (17) for \(\beta = 0.005\) and \(\lambda/\xi^2 = 1 \times 10^{-10}\).

**III. SLOW-ROLL INFATION AND INFLATIONARY OBSERVABLES**

First, we discuss the background evolution of the two-field quintessential-Higgs inflation. The dynamics of the two scalar fields \(\chi\) and \(\phi\) during inflation is described by the Klein-
Gordon and Friedmann equations as follows

\[ \ddot{\chi} + 3H \dot{\chi} + \dot{V}_\chi(\phi, \chi) = b_\chi e^{2b} \dot{\phi}^2 , \]  
\[ (20) \]

\[ \ddot{\phi} + (3H + 2b_\chi \dot{\chi}) \dot{\phi} + e^{-2b} \dot{V}_\phi(\phi, \chi) = 0 , \]  
\[ (21) \]

\[ H^2 = \frac{1}{3M_p^2} \left( \frac{1}{2} \dot{\chi}^2 + \frac{1}{2} e^{2b} \dot{\phi}^2 + \dot{V}(\phi, \chi) \right) , \]  
\[ (22) \]

\[ \dot{H} = -\frac{1}{2M_p^2} (\dot{\chi}^2 + e^{2b} \dot{\phi}^2) . \]  
\[ (23) \]

where \( H \) is the Hubble parameter, and dot and subscript comma denote partial derivative concerning time and scalar fields, respectively.

During the inflation era, \( H \) is nearly constant, which corresponds to the condition

\[ \epsilon^H \equiv -\frac{\dot{H}}{H^2} \ll 1 , \]  
\[ (24) \]

where \( \epsilon^H \) is known as the Hubble slow-roll parameter. In this era, the potential energy of the model dominates over the kinetic energy as

\[ \dot{\chi}^2 \ll \dot{V}(\phi, \chi) , \quad e^{2b} \dot{\phi}^2 \ll \dot{V}(\phi, \chi) . \]  
\[ (25) \]

Moreover, the scalar fields vary slowly during the inflation phase if

\[ |\ddot{\chi}| \ll |3H \dot{\chi}| , \quad |b_\chi e^{2b} \dot{\phi}^2| \ll |3H \dot{\chi}| , \]  
\[ (26) \]

and

\[ |\ddot{\phi}| \ll |3H \dot{\phi}| , \quad |b_\chi \dot{\chi} \dot{\phi}| \ll |3H \dot{\phi}| . \]  
\[ (27) \]

Equivalently, the slow-roll conditions for a single field \( \phi \) with a nearly flat inflationary potential \( V(\varphi) \) are as

\[ \epsilon^\phi \equiv \frac{M_p^2}{2} \left( \frac{V_{,\varphi}(\varphi)}{V(\varphi)} \right)^2 \ll 1 , \quad |\eta^\phi| \equiv |M_p^2 V_{,\varphi\varphi}(\varphi) / V(\varphi)| \ll 1 . \]  
\[ (28) \]

For the two-field model, the slow-roll conditions become as the following

\[ \epsilon^\chi \equiv \frac{M_p^2}{2} \left( \frac{\dot{V}_\chi(\phi, \chi)}{V(\phi, \chi)} \right)^2 \ll 1 , \quad \eta^\chi \equiv M_p^2 \frac{\dot{V}_\chi(\phi, \chi)}{V(\phi, \chi)} \ll 1 , \]  
\[ (29) \]

\[ \epsilon^\phi \equiv \frac{M_p^2}{2} \left( \frac{\dot{V}_\phi(\phi, \chi)}{V(\phi, \chi)} \right)^2 e^{-2b} \ll 1 , \quad \eta^\phi \equiv M_p^2 \frac{\dot{V}_\phi(\phi, \chi)}{V(\phi, \chi)} e^{-2b} \ll 1 . \]  
\[ (30) \]
Substituting (17) in (29) and (30), the slow roll conditions take the forms as
\[ e^\chi = \frac{4}{3} \left( 1 - e^{\frac{\sqrt{2E}}{6M_p}} \right)^{-2} \ll 1, \quad \eta^\chi = \frac{4}{3} e^{-\frac{\sqrt{2E}}{6M_p}} \frac{-1 + 2e^{-\sqrt{\frac{\chi}{M_p}}}}{\left( 1 - e^{-\sqrt{\frac{\chi}{M_p}}} \right)^2} \ll 1, \] \hspace{1cm} (31)
and
\[ e^\phi = \frac{1}{2} \beta^2 \left( 1 + e^{\sqrt{\frac{2E}{3M_p}}} \right) \ll 1, \quad \eta^\phi = \beta^2 \left( 1 + e^{\sqrt{\frac{2E}{3M_p}}} \right) \ll 1. \] \hspace{1cm} (32)

Inflation ends when the slow-roll conditions (31) are broken as
\[ e^\chi = \frac{4}{3} \left( 1 - e^{\frac{\sqrt{2E}}{6M_p}} \right)^{-2} \simeq 1, \] \hspace{1cm} (33)
where \( \chi_e \) is the value of the inflaton field at the end of inflation. Using (33), one can calculate \( \chi_e \) numerically as
\[ \chi_e \simeq 0.94 M_P. \] \hspace{1cm} (34)

Imposing potential slow-roll conditions (25)-(27), the equations (20)-(23) are simplified as the following forms
\[ 3H \dot{\chi} + \ddot{V}_\chi(\phi, \chi) \simeq 0, \] \hspace{1cm} (35)
\[ 3H \dot{\phi} + e^{-2b} \ddot{V}_\phi(\phi, \chi) \simeq 0, \] \hspace{1cm} (36)
\[ H^2 \simeq \frac{1}{3M_P^2} \ddot{V}(\phi, \chi), \] \hspace{1cm} (37)
\[ \dot{H} = -\frac{1}{2M_P^2} \left( \dot{\chi}^2 + e^{2b} \dot{\phi}^2 \right). \] \hspace{1cm} (38)

Substituting (37) and (38) into (24), one obtains
\[ e^H = \frac{3}{2} \frac{\dot{\chi}^2 + e^{2b} \dot{\phi}^2}{\dot{V}}. \] \hspace{1cm} (39)

By deriving \( \dot{\chi} \) and \( \dot{\phi} \) from (35) and (36) and substituting them into (39), we obtain the relation between the Hubble and potential slow-roll parameters of the model as
\[ e^H = e^\chi + e^\phi. \] \hspace{1cm} (40)

Substituting \( e^\chi \) and \( e^\phi \) from (31) and (32) into (40), we find
\[ e^H = \frac{4}{3} \left( 1 - e^{\frac{\sqrt{2E}}{6M_p}} \right)^{-2} + \frac{1}{2} \beta^2 \left( 1 + e^{\sqrt{\frac{2E}{3M_p}}} \right). \] \hspace{1cm} (41)
Later, we use this formula to calculate the inflationary observables of our model.

In the inflationary era, the quantum fluctuations of the scalar and gravitational fields produce scalar and tensor perturbations, respectively. The scalar spectral index defines the scale-dependence of the scalar power spectrum $\Delta^2_\zeta$

$$n_\zeta - 1 = \frac{d \ln \Delta^2_\zeta}{d \ln k},$$

and the normalized tensor-to-scalar ratio of the power spectrum is

$$r = \frac{\Delta^2_T}{\Delta^2_\zeta},$$

where $\Delta^2_T$ determines the amplitude of gravitational waves. In the case of a two-field model with product-separable potential, the inflationary observables $\Delta^2_\zeta$, $n_\zeta$ and $r$ have been calculated exactly in \[37, 38\]. Therefore, we rewrite these observables for the potential (17) as

$$\Delta^2_\zeta = \frac{\dot{V}}{24 \pi^2 M_P^4} e^{4(b_0 - b_*)} \left( \frac{u^2 \alpha^2}{e^*_\chi} + \frac{v^2}{e^*_\phi} \right),$$

$$n_\zeta - 1 = -2 \epsilon_H - 4 \frac{e^{-4(b_0 - b_*)}}{u^2 \alpha^2 / e^*_\chi + v^2 / e^*_\phi} - \frac{1}{12} \frac{\left( \sqrt{e^*_\phi / e^*_\chi} u \alpha - \sqrt{e^*_\chi / e^*_\phi} v \right)^2}{u^2 \alpha^2 / e^*_\chi + v^2 / e^*_\phi} \left( \eta^b + 2 \epsilon^b \right) + \frac{2 u^2 \alpha^2 \eta^\chi / e^*_\chi + v^2 \eta^\phi / e^*_\phi + 4 uv \alpha}{u^2 \alpha^2 / e^*_\chi + v^2 / e^*_\phi} + \frac{\text{sign}(b_\chi) \text{sign}(\dot{V}_\chi)}{u^2 \alpha^2 / e^*_\chi + v^2 / e^*_\phi} \left( \frac{v}{e^*_\phi} - 2 \frac{u}{e^*_\chi} \right),$$

$$r = \frac{8 H^2_0}{(2 \pi)^2 M_P^2 \Delta^2_\zeta} = \frac{2 \dot{V}}{3 \pi^2 M_P^4 \Delta^2_\zeta} = 16 \frac{e^{-4(b_0 - b_*)}}{u^2 \alpha^2 / e^*_\chi + v^2 / e^*_\phi},$$

where

$$u \equiv \frac{e^*_\chi}{\dot{e}_H}, \quad v \equiv \frac{e^*_\phi}{\dot{e}_H},$$

$$e^b \equiv 8 (b_\chi)^2, \quad \eta^b \equiv 16 b_{\chi \chi},$$

$$\alpha \equiv e^{-2(b_0 - b_*)} \left[ 1 + \frac{e^\phi}{\epsilon^\chi} (1 - e^{2(b_0 - b_*)}) \right].$$

Planck data determine the observed scalar spectral index to be

$$n_\zeta = 0.9649 \pm 0.0042,$$

at 68% CL \[3\]. Moreover, the Planck data combined with the BICEP2/Keck Array BK15 data put an upper limit on the observed tensor-to-scalar ratio as

$$r_{0.002} < 0.056.$$
at 95% CL.

To calculate $n_\zeta$ and $r$, we need the values of $\chi$ and $\phi$ at the time of horizon crossing. In the next sections, we will calculate them. Then, we will compare $n_\zeta(\phi, \chi)$ and $r(\phi, \chi)$ with (50) and (51) to constraint the free parameters of our model.

IV. NUMBER OF E-FOLDS TO THE END OF INFLATION

The amount of inflation expansion during the time of horizon crossing, $t_*$, and the end of inflation, $t_e$, that is called the number of e-folds, $N_*$, is given in terms of the Hubble parameter $H$ as $N_*= \int_{t_*}^{t_e} H dt$, where the subscripts “*” and “e” indicate the value of quantities at the horizon exit and at the end of inflation, respectively. Assuming slow-roll conditions, one finds $N_*$ as a function of $\chi_*$ (or $\phi_*$) as

$$N_* = \frac{1}{M_P^2} \int_{\chi_*}^{\chi_e} \frac{\dot{V}(\chi)}{V(\chi)} d\chi.$$  \hspace{1cm} (52)

Substituting (19) into (52), one finds

$$N(\chi_*) = \frac{\sqrt{6}}{4M_P} (\chi_* - \chi_e) + \frac{3}{4} \left( e^{\sqrt{\frac{2}{3}} M_P} - e^{\sqrt{\frac{2}{3}} M_P} \right).$$  \hspace{1cm} (53)

By using the lower Lambert function, this equation can be inverted to

$$\chi_* (N_*) \simeq \sqrt{\frac{3}{2}} M_P \ln \left[ \frac{2N_* - \sqrt{2} \chi_e M_P}{3M_P e^{\sqrt{\frac{2}{3}} M_P}} \right].$$  \hspace{1cm} (54)

To determine $\chi_*$, one should calculate $N_*$. For this purpose, one may consider the connection between the time of horizon crossing of the observable cosmological scales and the time of their re-entering to the Hubble horizon as

$$\frac{k}{a_0 H_0} = \frac{a_* H_*}{a_* H_*} = e^{-N_*} \frac{a_e}{a_{re}} \frac{a_{eq}}{a_{eq} H_{eq}} H_{eq} a_0 H_0,$$  \hspace{1cm} (55)

where the comoving wavenumber $k$ equals the Hubble scale $a_* H_*$, and the subscripts refer to different eras, including the reheating (re), radiation-matter equality (eq), and the present time (0). By assuming the entropy conservation from the end of inflation to today and using the slow-roll condition in which $H_*^2 \simeq V_*/3M_P^2$, one obtains

$$N_* = 67 - \ln \left( \frac{k}{a_0 H_0} \right) + \frac{1}{4} \ln \left( \frac{V_2}{M_P^4 \rho_c} \right) + \frac{1 - 3 \omega_{int}}{12(1 + \omega_{int})} \ln \left( \frac{\rho_{re}}{\rho_c} \right) - \frac{1}{12} \ln g_{re}.$$  \hspace{1cm} (56)
where $V_*$ is the potential energy of an inflationary model when $k$ leaves the Hubble horizon during inflation, $\rho_e$ and $\rho_{re}$ are the energy densities at the end of inflation and reheating, respectively, $\omega_{int}$ is the e-fold average of the equation of state between the end of inflation and the end of reheating and $g_{re}$ is the number of effective bosonic degrees of freedom at the end of reheating.

We now begin evaluating the quantities on the right-hand side of (56) for the case of our two-field inflationary model. In the third term of (56), $V_* = \hat{V}_*(\phi, \chi)$ can be calculated through the normalization of the power spectrum [42]

$$\Delta^2_\zeta = \frac{k^3}{2\pi^2} P_\zeta(k) = \frac{1}{24\pi^2 M_P^4} \frac{\hat{V}_*(\phi, \chi)}{e^H}.$$  

(57)

Therefore, one obtains

$$\hat{V}_*(\phi, \chi) = 24\pi^2 M_P^4 \Delta^2_\zeta e^H.$$  

(58)

Assuming zero acceleration at the end of inflation, $\ddot{a}_e = 0$, one finds the condition $\chi^2 = \hat{V}_e(\phi, \chi)$ by solving the Friedmann equations. Using this condition, the energy density of the $\chi$ field at the end of inflation is given by

$$\rho_e = \frac{1}{2} \chi^2 + \hat{V}_e(\phi, \chi) = \frac{3}{2} \hat{V}_e(\phi, \chi).$$  

(59)

In the fourth term of (56), $\rho_{re}$ is related to the reheating temperature $T_{re}$ through

$$\rho_{re} = \frac{\pi^2}{30} g_{re} T_{re}^4.$$  

(60)

Moreover, it is well known that $\omega_{int}$ gives of the reheating phase for large field models citeTurner

$$\omega_{int} = \frac{p - 2}{p + 2},$$  

(61)

where $p$ is the power of the inflaton field in the corresponding potential during the oscillating phase around its minimum. In our model, the potential (17) can be approximate near its minimum using Taylor series expansion as

$$\hat{V}(\chi, \phi) \sim \chi^2,$$  

(62)

where

$$\chi \simeq \sqrt{\frac{3}{2} \frac{\xi}{M_P} h^2}.$$  

(63)
As a result, the behaviour of \( \hat{V}(\chi, \phi) \) around its minimum as a large field potential \( h^4 \) (p=4) leads to

\[
\omega_{\text{int}} = 1/3 .
\]  

(64)

Therefore, the fourth term on right hand side of (64) vanishes for \( \omega_{\text{int}} = 1/3 \). As a consequence, deriving \( \chi^* \) and \( N^* \) for the present model is independent of reheating temperature.

Finally, by replacing (53), (58), (59) and (64) into (56) we get

\[
\ln \left( \frac{k}{a_0 H_0} \right) = 67 - \frac{\sqrt{6}}{4M_P} (\chi^* - \chi_e) + \frac{3}{4} \left( e^{\frac{\sqrt{2} \chi^*}{M_P}} - e^{\frac{\sqrt{2} \chi_e}{M_P}} \right) + \frac{1}{4} \ln \left( \frac{16\pi^2 \Delta^2}{\hat{V}_e(\phi, \chi)} \right) - \frac{1}{12} \ln g_{\text{re}} .
\]  

(65)

This equation is the basis for our subsequent analysis of the observables \( n_\zeta \) and \( r \), which are dependent on \( \chi^* \). To obtain \( \chi^* \) from (65), we need to know the values of the \( \phi \) field and \( \beta \) parameter during the inflation. In the next section, we study the bounds of these two values.

V. DOES QUINTESSENCE SAVE HIGGS INSTABILITY?

The Higgs inflation model suffers from Higgs instability since the quantum fluctuations of the Higgs field may have exceeded the instability EW scale during inflation [5]. One proposal to overcome this problem is to couple a quintessence field \( \phi \) to the Higgs field as the potential (7) and define a new Higgs self coupling as \( \tilde{\lambda} = \lambda e^{-\beta (\phi - \phi_0)/M_P} \). Calculating the running of \( \tilde{\lambda} \), one finds that the EW vacuum will be stable when

\[
e^{-\beta (\phi - \phi_0)/M_P} \gtrsim 1.08 .
\]  

(66)

To find the upper limit for \( e^{-\beta (\phi - \phi_0)/M_P} \) and the allowed range of \( \beta \) that leads to Higgs stability, the authors of [44] studied the evolution of \( \phi \) field from the era of inflation until today. Supposing ultra-slow-roll evolution of the \( \phi \) field in the early universe and using the Klein-Gordon and Friedmann equations, they derived the initial condition of the \( \phi \) field during inflation as a function of \( \beta \). Comparing this initial condition with (66), they found a lower bound

\[
\beta > 0.35 \pm 0.05 ,
\]  

(67)
that leads to Higgs stability.

We will use (66) in our subsequent calculations as one of the initial conditions to obtain \( \chi_\ast \). However, we will check in the following whether the bound (67) is consistent with the CMB constraint to \( \beta \) or not.

VI. CMB CONSTRAINTS TO QUINTESSENTIAL HIGGS INFLATION OBSERVABLES

Using the previous results and setting \( g_{re} = 1000, \Delta_\xi^2 = 2 \times 10^{-9} \) \([2]\), \( \chi_e \simeq 0.94 \, M_P \) and 
\begin{equation}
\begin{aligned}
\chi \equiv e^{-\beta(\phi-\phi_0)/M_P} = 1 \text{, } \end{aligned}
\end{equation}

is simplified as
\begin{equation}
\begin{aligned}
\ln \left( \frac{k}{a_0 H_0} \right) &= 65.2 - \frac{\sqrt{6}}{4 M_P} \chi_\ast + \frac{3}{4} e^{\sqrt{3} \chi_M^{\ast}} + \frac{1}{4} \ln \left( \frac{4}{3} \left(1 - e^{\sqrt{3} \chi_M^{\ast}} \right)^{-2} + \frac{\beta^2}{2} \left(1 + e^{\sqrt{3} \chi_M^{\ast}} \right) \right) \\
&- \frac{1}{2} \ln \left(1 - e^{-\sqrt{3} \chi_M^{\ast}} \right). 
\end{aligned}
\end{equation}

This is a relation between the wave number \( k \) and \( \chi_\ast \). Hence, by setting \( \beta \) in (68) as a free parameter and obtaining \( \chi_\ast \) as a function of \( \beta \) for pivot scale \( k = 0.002 \, \text{Mpc}^{-1} \), it is easy to calculate the observables \((45)\) and \((46)\) numerically.

Figure 2: The spectral index, \( n_\zeta \) (left panel) and the tensor-to-scalar ratio, \( r \) (right panel) against \( \beta \). The blue band in the left panel represents the 68\% CL region of the Planck data combined with the BICEP2/Keck Array BK15 data \([3]\). The orange line in the right panel is due to the upper bound on \( r_{0.002} < 0.056 \) at 68\% CL \([3]\).

Figure 2 shows the dependence of \( n_\zeta \) and \( r \) to \( \beta \) for our model. The upper and lower bound on \( \beta \), which keeps \( n_\zeta \) within 68\% CL region, is
\begin{equation}
5 \times 10^{-5} \lesssim \beta \lesssim 8 \times 10^{-3}. \tag{69}
\end{equation}
Replacing $\chi_e$ and $\chi_*$ of our model into (53) and (46), we find numerically the permitted values of $N_*$ and $r$ as

$$58.70 \lesssim N_* \lesssim 59.35, \quad \text{for} \quad 5 \times 10^{-5} \lesssim \beta \lesssim 8 \times 10^{-3},$$

$$0.0014 \lesssim r \lesssim 0.0015, \quad \text{for} \quad 5 \times 10^{-5} \lesssim \beta \lesssim 8 \times 10^{-3},$$

which are in a very good agreement with CMB data. As a result, the CMB constraint (69) rules out the bound (67). It means that not only the quintessence does not remove Higgs instability, but also the Swampland Conjecture (1) is not satisfied, which is the opposite of the claim of the authors in [44].

### VII. CONCLUSION

In this work, we studied a new model of quintessential inflation where two different fields describe quintessence and inflaton. In this model, the quintessence field is coupled with the Higgs field from the beginning of the inflationary era, and the Higgs field has a non-minimal coupling with gravity. Considering the coupling between the quintessence and inflaton from the early times helps improve the quintessence models’ initial condition problem.

Moving to the Einstein frame through conformal transformation, we found the plateau-like two-field potential (17). Calculating the inflationary observables of this potential, $n_\zeta$ and $r$ against the free parameter of the model, $\beta$ and comparing them with the observed values of these observables in the Planck+BICEP2/Keck Array BK15 data, we found $\beta \lesssim 8 \times 10^{-3}$ that strongly disfavors the Swampland conjecture (1). Besides, this bound rules out the bound (67), which means the quintessence does not save the Higgs instability problem, and one may search for other proposals to solve it.

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[1] A. Guth, “Inflationary universe: A possible solution to the horizon and flatness problems,” Phys. Rev. D 23, 347 (1981); A. Linde, “A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems,” Phys. Lett. B 108, 389 (1982); A. Linde, “Chaotic Inflation,” Phys. Lett. B 129, 177 (1983).

[2] N. Aghanim et al. [Planck], “Planck 2018 results. VI. Cosmological parameters,” [arXiv:1807.06209 [astro-ph.CO]].

[3] Y. Akrami et al. [Planck], “Planck 2018 results. X. Constraints on inflation,” [arXiv:1807.06211 [astro-ph.CO]].

[4] D. S. Salopek, J. R. Bond and J. M. Bardeen, “Designing density fluctuation spectra in inflation,” Phys. Rev. D 40, 1753 (1989) doi.org/10.1103/PhysRevD.40.1753.

[5] F. L. Bezrukov and M. Shaposhnikov, “The Standard Model Higgs boson as the inflaton,” Phys. Lett. B 659, 703-706 (2008) doi:10.1016/j.physletb.2007.11.072 [arXiv:0710.3755 [hep-th]].

[6] T. Futamase and K. -i. Maeda, “Chaotic Inflationary Scenario in Models Having Nonminimal Coupling With Curvature,” Phys. Rev. D 39, 399 (1989); R. Fakir and W. G. Unruh, “Improvement on cosmological chaotic inflation through nonminimal coupling,” Phys. Rev. D 41, 1783 (1990); D. I. Kaiser, “Primordial spectral indices from generalized Einstein theories,” Phys. Rev. D 52, 4295 (1995) [astro-ph/9408044]; N. Makino and M. Sasaki, “The Density perturbation in the chaotic inflation with nonminimal coupling,” Prog. Theor. Phys. 86, 103 (1991); R. Kallosh, A. Linde and D. Roest, “Universal Attractor for Inflation at Strong Coupling,” Phys. Rev. Lett. 112, no. 1, 011303 (2014) [arXiv:1310.3950 [hep-th]]; M. Galante, R. Kallosh, A. Linde and D. Roest, “The Unity of Cosmological Attractors,” Phys. Rev. Lett. 114, no. 14, 141302 (2015) [arXiv:1412.3797 [hep-th]]; M. Eshaghi, M. Zarei, N. Riazi and A. Kiasatpour, “A Non-minimally Coupled Potential for Inflation and Dark Energy after Planck 2015: A Comprehensive Study,” JCAP 11, 037 (2015) doi:10.1088/1475-7516/2015/11/037 [arXiv:1505.03556 [hep-th]].

[7] A. G. Riess et al. [Supernova Search Team], “Observational evidence from supernovae for
an accelerating universe and a cosmological constant,” Astron. J. 116, 1009-1038 (1998) doi:10.1086/300499 [arXiv:astro-ph/9805201 [astro-ph]]; S. Perlmutter et al. [Supernova Cosmology Project], “Measurements of Ω and Λ from 42 high redshift supernovae,” Astrophys. J. 517, 565-586 (1999) doi:10.1086/307221 [arXiv:astro-ph/9812133 [astro-ph]].

[8] E. J. Copeland, A. R. Liddle and D. Wands, “Exponential potentials and cosmological scaling solutions,” Phys. Rev. D 57, 4686-4690 (1998) doi:10.1103/PhysRevD.57.4686 [arXiv:gr-qc/9711068 [gr-qc]]; I. Zlatev, L. M. Wang and P. J. Steinhardt, “Quintessence, cosmic coincidence, and the cosmological constant,” Phys. Rev. Lett. 82, 896-899 (1999) doi:10.1103/PhysRevLett.82.896 [arXiv:astro-ph/9807002 [astro-ph]]; B. Ratra and P. J. E. Peebles, “Cosmological Consequences of a Rolling Homogeneous Scalar Field,” Phys. Rev. D 37, 3406 (1988) doi:10.1103/PhysRevD.37.3406.

[9] S. Tsujikawa, “Quintessence: A Review,” Class. Quant. Grav. 30, 214003 (2013) doi:10.1088/0264-9381/30/21/214003 [arXiv:1304.1961 [gr-qc]].

[10] Y. Akrami, R. Kallosh, A. Linde and V. Vardanyan, “Dark energy, α-attractors, and large-scale structure surveys,” JCAP 1806 (2018) 041 doi:10.1088/1475-7516/2018/06/041 [arXiv:1712.09693 [hep-th]].

[11] C. Vafa, “The String landscape and the swampland,” [arXiv:hep-th/0509212 [hep-th]]; T. D. Brennan, F. Carta and C. Vafa, “The String Landscape, the Swampland, and the Missing Corner,” PoS TASI2017, 015 (2017) doi:10.22323/1.305.0015 [arXiv:1711.00864 [hep-th]].

[12] P. Agrawal, G. Obied, P. J. Steinhardt and C. Vafa, “On the Cosmological Implications of the String Swampland,” Phys. Lett. B 784, 271-276 (2018) doi:10.1016/j.physletb.2018.07.040 [arXiv:1806.09718 [hep-th]].

[13] L. Heisenberg, M. Bartelmann, R. Brandenberger and A. Refregier, “Dark Energy in the Swampland,” Phys. Rev. D 98, no.12, 123502 (2018) doi:10.1103/PhysRevD.98.123502 [arXiv:1808.02877 [astro-ph.CO]].

[14] Y. Akrami, R. Kallosh, A. Linde and V. Vardanyan, “The Landscape, the Swampland and the Era of Precision Cosmology,” Fortsch. Phys. 67, no.1-2, 1800075 (2019) doi:10.1002/prop.201800075 [arXiv:1808.09440 [hep-th]].

[15] E. Ó. Colgáin and H. Yavartanoo, “Testing the Swampland: H0 tension,” Phys. Lett. B 797, 134907 (2019) doi:10.1016/j.physletb.2019.134907 [arXiv:1905.02555 [astro-ph.CO]]; A. Banerjee, H. Cai, L. Heisenberg, E. Ó. Colgáin, M. M. Sheikh-Jabbari and T. Yang, “Hubble Sinks
In The Low-Redshift Swampland,” [arXiv:2006.00244 [astro-ph.CO]].

[16] A. G. Riess, L. M. Macri, S. L. Hoffmann, D. Scolnic, S. Casertano, A. V. Filippenko, B. E. Tucker, M. J. Reid, D. O. Jones and J. M. Silverman, et al. “A 2.4% Determination of the Local Value of the Hubble Constant,” Astrophys. J. 826, no.1, 56 (2016) doi:10.3847/0004-637X/826/1/56 [arXiv:1604.01424 [astro-ph.CO]]; A. G. Riess, S. Casertano, W. Yuan, L. M. Macri and D. Scolnic, “Large Magellanic Cloud Cepheid Standards Provide a 1% Foundation for the Determination of the Hubble Constant and Stronger Evidence for Physics beyond ΛCDM,” Astrophys. J. 876, no.1, 85 (2019) doi:10.3847/1538-4357/ab1422 [arXiv:1903.07603 [astro-ph.CO]]; N. Aghanim et al. [Planck], “Planck 2018 results. VI. Cosmological parameters,” Astron. Astrophys. 641, A6 (2020) doi:10.1051/0004-6361/201833910 [arXiv:1807.06209 [astro-ph.CO]].

[17] P. Peebles and A. Vilenkin, “Quintessential inflation,” Phys. Rev. D 59, 063505 (1999) doi:10.1103/PhysRevD.59.063505 [arXiv:astroph-ph/9810509 [astro-ph]].

[18] K. Dimopoulos and C. Owen, “Quintessential Inflation with α-attractors,” JCAP 06, 027 (2017) doi:10.1088/1475-7516/2017/06/027 [arXiv:1703.00305 [gr-qc]].

[19] L. Aresté Saló, D. Benisty, E. I. Guendelman and J. d. Haro, “Quintessential inflation and cosmological seesaw mechanism: reheating and observational constraints,” JCAP 07, 007 (2021) doi:10.1088/1475-7516/2021/07/007 [arXiv:2102.09514 [astro-ph.CO]].

[20] K. Dimopoulos, A. Karam, S. Sánchez López and E. Tomberg, “Palatini $R^2$ Quintessential Inflation,” [arXiv:2206.14117 [gr-qc]].

[21] L. H. Ford, “Gravitational particle creation and inflation,” Phys. Rev. D 35, 2955 (1987) doi.org/10.1103/PhysRevD.35.2955.

[22] A. Achúcarro and G. A. Palma, “The string swampland constraints require multi-field inflation,” JCAP 02, 041 (2019) doi:10.1088/1475-7516/2019/02/041 [arXiv:1807.04390 [hep-th]]; Y. Akrami, M. Sasaki, A. R. Solomon and V. Vardanyan, “Multi-field dark energy: cosmic acceleration on a steep potential,” [arXiv:2008.13660 [astro-ph.CO]]; J. R. Eskilt, Y. Akrami, A. R. Solomon and V. Vardanyan, “Cosmological dynamics of multifield dark energy,” Phys. Rev. D 106, no.2, 023512 (2022) doi:10.1103/PhysRevD.106.023512 [arXiv:2201.08841 [astro-ph.CO]].

[23] F. Denef, A. Hebecker and T. Wrase, “de Sitter swampland conjecture and the Higgs potential,” Phys. Rev. D 98, no.8, 086004 (2018) doi:10.1103/PhysRevD.98.086004
[24] S. Chatrchyan et al. [CMS], “Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC,” Phys. Lett. B 716, 30-61 (2012) doi:10.1016/j.physletb.2012.08.021 [arXiv:1207.7235 [hep-ex]]; G. Aad et al. [ATLAS], “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC,” Phys. Lett. B 716, 1-29 (2012) doi:10.1016/j.physletb.2012.08.020 [arXiv:1207.7214 [hep-ex]].

[25] D. I. Kaiser, “Primordial spectral indices from generalized Einstein theories,” Phys. Rev. D 52, 4295-4306 (1995) doi:10.1103/PhysRevD.52.4295 [arXiv:astro-ph/9408044 [astro-ph]].

[26] E. Komatsu and T. Futamase, “Complete constraints on a nonminimally coupled chaotic inflationary scenario from the cosmic microwave background,” Phys. Rev. D 59, 064029 (1999) doi:10.1103/PhysRevD.59.064029 [arXiv:astro-ph/9901127 [astro-ph]].

[27] F. Bezrukov, D. Gorbunov and M. Shaposhnikov, “On initial conditions for the Hot Big Bang,” JCAP 06, 029 (2009) doi:10.1088/1475-7516/2009/06/029 [arXiv:0812.3622 [hep-ph]].

[28] A. R. Liddle, “Power Law Inflation With Exponential Potentials,” Phys. Lett. B 220, 502-508 (1989) doi:10.1016/0370-2693(89)90776-4.

[29] R. Kallosh and A. D. Linde, “Dark energy and the fate of the universe,” JCAP 02, 002 (2003) doi:10.1088/1475-7516/2003/02/002 [arXiv:astro-ph/0301087 [astro-ph]].

[30] T. Barreiro, E. J. Copeland and N. J. Nunes, “Quintessence arising from exponential potentials,” Phys. Rev. D 61, 127301 (2000) doi:10.1103/PhysRevD.61.127301 [arXiv:astro-ph/9910214 [astro-ph]].

[31] T. Chiba, A. De Felice and S. Tsujikawa, “Observational constraints on quintessence: thawing, tracker, and scaling models,” Phys. Rev. D 87, no.8, 083505 (2013) doi:10.1103/PhysRevD.87.083505 [arXiv:1210.3859 [astro-ph.CO]].

[32] H. Murayama, M. Yamazaki and T. T. Yanagida, “Do We Live in the Swampland?,“ JHEP 12, 032 (2018) doi:10.1007/JHEP12(2018)032 [arXiv:1809.00478 [hep-th]].

[33] A. A. Starobinsky, S. Tsujikawa and J. Yokoyama, “Cosmological perturbations from multifield inflation in generalized Einstein theories,” Nucl. Phys. B 610, 383-410 (2001) doi:10.1016/S0550-3213(01)00322-4 [arXiv:astro-ph/0107555 [astro-ph]]; F. Vernizzi and D. Wands, “Non-gaussianities in two-field inflation,” JCAP 05, 019 (2006) doi:10.1088/1475-7516/2006/05/019 [arXiv:astro-ph/0603799 [astro-ph]]; D. I. Kaiser, “Conformal Transformations with Multiple Scalar Fields,” Phys. Rev. D 81, 084044 (2010)
doi:10.1103/PhysRevD.81.084044 [arXiv:1003.1159 [gr-qc]]; J. Kim, Y. Kim and S. C. Park, “Two-field inflation with non-minimal coupling,” Class. Quant. Grav. 31, 135004 (2014) doi:10.1088/0264-9381/31/13/135004 [arXiv:1301.5472 [hep-ph]].

[34] A. D. Linde, “Chaotic Inflation,” Phys. Lett. B 129 177-181 (1983) doi:10.1016/0370-2693(83)90837-7.

[35] R. Kallosh, A. Linde and D. Roest, “Universal Attractor for Inflation at Strong Coupling,” Phys. Rev. Lett. 112, no.1, 011303 (2014) doi:10.1103/PhysRevLett.112.011303 [arXiv:1310.3950 [hep-th]].

[36] A. Linde, “Inflationary Cosmology after Planck 2013,” doi:10.1093/acprof:oso/9780198728856.003.0006 [arXiv:1402.0526 [hep-th]].

[37] F. Di Marco and F. Finelli, “Slow-roll inflation for generalized two-field Lagrangians,” Phys. Rev. D 71, 123502 (2005) doi:10.1103/PhysRevD.71.123502 [arXiv:astro-ph/0505198 [astro-ph]].

[38] K. Y. Choi, L. M. Hall and C. van de Bruck, “Spectral Running and Non-Gaussianity from Slow-Roll Inflation in Generalised Two-Field Models,” JCAP 02, 029 (2007) doi:10.1088/1475-7516/2007/02/029 [arXiv:astro-ph/0701247 [astro-ph]].

[39] J. Ellis, M. A. G. Garcia, D. V. Nanopoulos and K. A. Olive, “Calculations of Inflaton Decays and Reheating: with Applications to No-Scale Inflation Models,” JCAP 07, 050 (2015) doi:10.1088/1475-7516/2015/07/050 [arXiv:1505.06986 [hep-ph]].

[40] A. R. Liddle and S. M. Leach, “How long before the end of inflation were observable perturbations produced?,” Phys. Rev. D 68, 103503 (2003) [astro-ph/0305263].

[41] J. Martin and C. Ringeval, “First CMB Constraints on the Inflationary Reheating Temperature,” Phys. Rev. D 82, 023511 (2010) [arXiv:1004.5525 [astro-ph]].

[42] D. Baumann, “Inflation,” [arXiv:0907.5424 [hep-th]].

[43] M. S. Turner, “Coherent scalar field oscillations in an expanding universe,” Phys. Rev. D 28, 1243 (1983); J. Martin, “Inflation and precision cosmology,” Braz. J. Phys. 34, 1307 (2004) [astro-ph/0312492]; L. Kofman, A. D. Linde and A. A. Starobinsky, “Towards the theory of reheating after inflation,” Phys. Rev. D 56, 3258 (1997) [hep-ph/9704452]; M. Eshaghi, M. Zarei, N. Riazi and A. Kiasatpour, “CMB and reheating constraints to $\alpha$-attractor inflationary models,” Phys. Rev. D 93, no.12, 123517 (2016) doi:10.1103/PhysRevD.93.123517 [arXiv:1602.07914 [astro-ph.CO]].
[44] C. Han, S. Pi and M. Sasaki, “Quintessence Saves Higgs Instability,” Phys. Lett. B 791, 314-318 (2019) doi:10.1016/j.physletb.2019.02.037 [arXiv:1809.05507 [hep-ph]].