Discovery of doubly-charmed $\Xi_{cc}$ baryon implies a stable $bb\bar{u}\bar{d}$ tetraquark

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ABSTRACT

Recently LHCb discovered the first doubly-charmed baryon $\Xi_{cc}^{++} = ccu$ at 3621.40 $\pm$ 0.78 MeV, very close to our theoretical prediction. We use the same methods to predict a doubly-bottom tetraquark $T(bb\bar{u}\bar{d})$ with $J^P=1^+$ at 10,389 $\pm$ 12 MeV, 215 MeV below the $B^-\bar{B}^{*0}$ threshold and 170 MeV below threshold for decay to $B^-\bar{B}^{0}\gamma$. The $T(bb\bar{u}\bar{d})$ is therefore stable under strong and electromagnetic (EM) interactions and can only decay weakly, the first exotic hadron with such a property. On the other hand, the mass of $T(cc\bar{u}\bar{d})$ with $J^P=1^+$ is predicted to be 3882 $\pm$ 12 MeV, 7 MeV above the $D^0D^{*+}$ threshold and 148 MeV above $D^0D^{*+}\gamma$ threshold. $T(bc\bar{u}\bar{d})$ with $J^P=0^+$ is predicted at 7134 $\pm$ 13 MeV, 11 MeV below the $\bar{B}^0D^0$ threshold. Our precision is not sufficient to determine whether $bc\bar{u}\bar{d}$ is actually above or below the threshold. It could manifest itself as a narrow resonance just at threshold.

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I INTRODUCTION

The question whether $QQ\bar{q}\bar{q}$ tetraquarks with two heavy quarks $Q$ and two light antiquarks $\bar{q}$ are stable or unstable against decay into two $Q\bar{q}$ mesons has a long history. It has been largely undecided, mainly due to lack of experimental information about the strength of the interaction between two heavy quarks.

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The very recent discovery of the doubly charmed baryon $\Xi_{cc}$ by the LHCb Collaboration at CERN has now provided the crucial experimental input which allows this issue to be finally resolved.

LHCb has observed the doubly-charmed baryon $\Xi_{cc}^{++} = ccu$ with a mass of $3621.40 \pm 0.78$ MeV [1]. This value is consistent with several predictions, including our value of $3627 \pm 12$ MeV [2,3].

Here we use similar methods to those in Ref. [2] and earlier works [4] to predict the mass of the ground-state $bb \bar{u} \bar{d}$ tetraquark with spin-parity $J^P = 1^+$, $M(T(bb \bar{u} \bar{d})) = 10,389 \pm 12$ MeV.

Angular momentum and parity conservation in strong and EM interactions forbid a state with $J^P = 1^+$ from decaying strongly or electromagnetically into two pseudoscalars in any partial wave. Therefore $bb \bar{u} \bar{d}$ with $J^P = 1^+$ cannot decay into $BB$. The lowest-mass hadronic channel allowed by angular momentum and parity is $BB^*$, most favorably in $S$-wave. This channel is however kinematically closed, because the $T(bb \bar{u} \bar{d})$ mass is 215 MeV below $BB^*$ threshold at 10,604 MeV. $M(T(bb \bar{u} \bar{d}))$ is also 170 MeV below $2m_B$, the relevant threshold for EM decay to $B^- \bar{B}^0 \gamma$.

The $B$ mesons are the lightest states that carry open bottom, so the $bb \bar{u} \bar{d}$ tetraquark cannot decay through strong or EM interactions which conserve heavy flavor. It can only decay weakly, when one of the $b$ quarks decays into and $c$ quark and a virtual $W^+$. A typical decay is therefore $(bb \bar{u} \bar{d}) \rightarrow \bar{B}D\pi^+(\rho^+)$, etc.

The main challenge in the prediction of the $T(bb \bar{u} \bar{d})$ mass is the estimate of binding energy between the two $b$ quarks [5–7]. Table IX of Ref. [7] provides a compilation of earlier mass estimates of various $QQ \bar{q} \bar{q}$ tetraquarks. In Ref. [2] we estimated the binding energy between two heavy quarks $Q$ by assuming that $QQ$ binding is one-half of the $\bar{Q}Q$ binding which can be obtained from quarkonia. When applied to the $ccu$ baryon $\Xi_{cc}$ this led to the prediction $M(\Xi_{cc}) = 3627 \pm 12$ MeV, very close to the experimentally measured $ccu$ mass of $3621.40 \pm 0.78$ MeV.

The above relation between quark-quark and quark-antiquark binding is exact in the one-gluon-exchange weak-coupling limit. Its successful extension beyond weak coupling implies that the heavy quark potential factorizes into a color-dependent and a space-dependent part, with the space-dependent part being the same for $QQ$ and $\bar{Q}Q$. The relative factor $1/2$ is then automatic, just as in the weak coupling limit, resulting from the color algebra.

II CALCULATION OF THE $bb \bar{u} \bar{d}$ MASS

In the present work we build on the accuracy of the $\Xi_{cc}$ mass prediction and assume the same relation is true for $bb$ binding energy in a $bb \bar{u} \bar{d}$ tetraquark. In order to obtain a state with the lowest possible mass, we further assume that all four quarks are in a relative $S$-wave and that the $\bar{u}$ and $\bar{d}$ light antiquarks bind into a color-triplet “good” antidiquark with spin and isospin zero. The $bb$ diquark must then be a color antitriplet and Fermi statistics dictates it has spin 1. The total spin and parity are then $J^P = 1^+$. The upshot is that we are considering a configuration very similar to a heavy-light meson $Qq$, where instead of the heavy antiquark we have a doubly-heavy color antitriplet diquark and instead of the quark we have a light color triplet antidiquark. The rest of the calculation is straightforward and proceeds in a way entirely analogous to Ref. [2].

The contributions to the mass of the lightest tetraquark $T(bb \bar{u} \bar{d})$ with two bottom
Table I: Contributions to the mass of the lightest tetraquark $T(bb\bar{u}\bar{d})$ with two bottom quarks and $J^P = 1^+$. 

| Contribution Value (MeV) |
|--------------------------|
| $2m_b^2$            | 10087.0 |
| $2m_{\rho}^b$         | 726.0   |
| $a_{bb}/(m_b^b)^2$    | 7.8     |
| $-3a/(m_b^b)^2$       | -150.0  |
| $bb$ binding          | -281.4  |
| Total                  | 10389.4 ± 12 |

quarks and $J^P = 1^+$ are listed in Table I. The notation and the numerical values of all the parameters are the same as in Table VI and Table IX of Ref. [2]. In particular, the subscripts on masses $m$ denote flavor, while the superscripts $b$ indicate that these are effective masses in baryons.

The central value of the resulting mass 10,389 MeV ± 12 is 215 MeV below $B\bar{B}^*$ threshold at 10,604 MeV, and 170 MeV below $B^-\bar{B}^0\gamma$ threshold at 10,559 MeV.

### III $cc\bar{u}\bar{d}$ and $bc\bar{u}\bar{d}$ MASSES

The calculation of the masses of the lightest $cc\bar{u}\bar{d}$ and tetraquark masses proceeds analogously to $bb\bar{u}\bar{d}$. In Tables [II] and [III] we provide the corresponding contributions to the $cc\bar{u}\bar{d}$ and $bc\bar{u}\bar{d}$ masses.

The mass of $cc\bar{u}\bar{d}$ turns out to be 3882 ± 12 MeV, with the central value only 7 MeV above the $D^0D^{*+}$ threshold at 3875 MeV and 148 MeV above $D^{0}D^+\gamma$ threshold. Moreover, as the central value of our prediction of $M(\Xi_{cc}^+)$ is 6 MeV above the observed central value, if we were to increase the $cc$ binding energy by 6 MeV to force agreement between prediction and observation, the mass $cc\bar{u}\bar{d}$ would be lowered to 3876 MeV, only 1 MeV above $D^0D^{*+}$ threshold. As $M(D^0) + M(D^{*+}) = 3875.09\pm0.07$ MeV while $M(D^+) + M(D^{*0})$ is $1.35\pm0.12$ MeV higher at 3876.44 ± 0.10 MeV [8], there may be some interesting violations of isospin in the hadronic decays of such a state, in analogy with isospin violations in decays of $X(3872)$ [9].

Unlike $bb\bar{u}\bar{d}$ and $cc\bar{u}\bar{d}$, the lowest mass $bc\bar{u}\bar{d}$ tetraquark has $J^P = 0^+$, because the minimal energy $bc$ diquark has spin zero. The $bc\bar{u}\bar{d}$ mass is 7133 ± 13 MeV, with the central value about 11 MeV below the $\bar{B}^0D^0$ threshold at 7144.5 MeV.

The precision of our calculation is not sufficient to determine whether the $bc\bar{u}\bar{d}$ tetraquark is actually above or below the corresponding two-meson threshold. It could manifest itself as a narrow resonance just at threshold.
Table II: Contributions to the mass of the lightest tetraquark $T(cc\bar{u}\bar{d})$ with two charmed quarks and $J^P = 1^+$.  

| Contribution | Value (MeV) |
|--------------|-------------|
| $2m^b_c$     | 3421.0      |
| $2m^b_q$     | 726.0       |
| $a_{cc}/(m^b_c)^2$ | 14.2       |
| $-3a/(m^b_q)^2$ | -150.0     |
| $cc$ binding | -129.0      |
| **Total**    | 3882.2 ± 12 |

Table III: Contributions to the mass of the lightest tetraquark $T(bc\bar{u}\bar{d})$ with one bottom and one charmed quark and $J^P = 0^+$.  

| Contribution | Value (MeV) |
|--------------|-------------|
| $m^b_b + m^b_c$ | 6754.0     |
| $2m^b_q$     | 726.0       |
| $-3a_{bc}/(m^b_b m^b_c)$ | -25.5     |
| $-3a/(m^b_q)^2$ | -150.0     |
| $bc$ binding | -170.8      |
| **Total**    | 7133.7 ± 13 |

Fig. 1 shows the distance in MeV between the masses of the $cc\bar{u}\bar{d}$, $bc\bar{u}\bar{d}$ and $bb\bar{d}$ tetraquarks and the corresponding thresholds, $D^0D^+\gamma$, $\bar{B}^0D^0$, and $\bar{B}^0B^-\gamma$, respectively, plotted against the reduced mass of the doubly-heavy diquark.

The main reason $bb\bar{u}\bar{d}$ is deeply bound, while $cc\bar{u}\bar{d}$ is above threshold and $bc\bar{u}\bar{d}$ is borderline below threshold, is the big jump in the QQ binding energy as the heavy quarks' mass increases: 129 MeV for $cc$ vs. 281 MeV for $bb$. This increase in the binding energy can be understood qualitatively by noting that the two heavy quarks are non-relativistic and their interaction can be described by a Coulomb + linear potential, or by a logarithmic potential, both of which are singular at the origin. The mean distance between the two heavy quarks scales like $1/\alpha_s m_Q$ and is significantly smaller than the typical hadronic scale $\sim 1/\Lambda_{QCD}$. At such small distances as $m_Q$ increases, the $QQ$ binding energy grows rapidly with shrinking distance, due to the singularity of the potential.

While the above provides a qualitative understanding of the phenomenon, we stress again that the actual numerical value of the $QQ$ binding energy employed here is not computed from any particular potential, but rather taken directly from experiment, using the previously-discussed correspondence between binding in $\bar{Q}Q$ quarkonia and in $QQ$ diquarks which led to the accurate prediction of the $\Xi_{cc}$ mass.
Figure 1: Distance in MeV of the $cc\bar{d}$, $bc\bar{d}$ and $bb\bar{d}$ tetraquark masses from the corresponding thresholds $D^0D^+\gamma$, $B^0D^0$, and $\bar{B}^0B^-\gamma$, plotted against the reduced masses of the doubly-heavy diquarks $\mu_{\text{RED}}(QQ')$, $Q,Q' = c,b$.

IV $bb\bar{u}\bar{d}$ DECAY MODES AND LIFETIME

We focus on the decay of the $bb\bar{u}\bar{d}$ tetraquark which is deeply bound, unlike $cc\bar{u}\bar{d}$ and $bc\bar{u}\bar{d}$ which are, respectively, above and close to their relevant thresholds.

A crude estimate of the lifetime can be obtained similarly to Ref. [2]. We assume an initial state with mass 10,389.4 MeV, a final state with $M(\bar{B}) + M(D) = 7,144.5$ MeV, a charged weak current giving rise to $e\bar{\nu}_e$, $\mu\bar{\nu}_\mu$, $\tau\bar{\nu}_\tau$ and three colors of $\bar{u}d$ and $\bar{c}s$, a kinematic suppression factor

$$F(x) = 1 - 8x + 8x^3 - x^4 + 12x^2\ln(1/x) , \quad x \equiv \left\{[M(\bar{B}) + M(D)]/M(bb\bar{u}\bar{d})\right\}^2,$$

a value of $|V_{cb}| = 0.04$ as in Ref. [2], and a factor of 2 to count each decaying $b$ quark. The resulting decay rate is

$$\Gamma(bb\bar{u}\bar{d}) = \frac{18G_F^2M(bb\bar{u}\bar{d})^5}{192\pi^3}F(x)|V_{cb}|^2 = 17.9 \times 10^{-13} \text{ GeV},$$

leading to a predicted lifetime $\tau(bb\bar{u}\bar{d}) = 367$ fs.

The $bb\bar{u}\bar{d}$ decay can occur through one of two channels:
(a) The “standard process” $bb\bar{u}\bar{d} \to cb\bar{u} + W^+$. Typical reactions include

$$T(bb\bar{u}) \to D^0 B^0 \pi^-, \ D^+ B^- \pi^- \text{ and } T(bb\bar{u}) \to J/\psi K^- B^0, \ J/\psi K^0 B^-.$$  

In addition, there is a rare process where both $b$ quarks decay into $c\bar{c}s$,

$$T(bb\bar{u}) \to J/\psi J/\psi K^- \bar{K}^0.$$  

The signature for events with two $J/\psi$'s coming from the same secondary vertex might be sufficiently striking to make it worthwhile to look for such events against a large background.

(b) The $W$-exchange process $b\bar{d} \to c\bar{u}$, again involving either one of the two $b$ quarks, which ought to shorten the lifetime further. The latter process can involve a two-body final state, e.g., $T(bb\bar{u}) \to D^0 B^-$, which may partially compensate for suppression due to the small wave function of the $b\bar{d}$ pair at zero separation. However, the comparable process in $B^0$ meson decay does not seem to shorten its lifetime much with respect to $\tau(B^+)$.  

\section*{V PRODUCTION}  

Production will be difficult because in addition to two $b$ quarks one will need two $\bar{b}$ antiquarks. The probability for producing two heavy quark pairs can be estimated as the square of the probability for producing one pair. In the case of the doubly-charmed baryon $\Xi_{cc}^{++}$ observed by LHCb \cite{lipkin}, this difficulty appears to have been overcome.

The signature for decay of a $bb\bar{u}\bar{d}$ state will be a final state involving $b$ and $c$, whereas a $bb$ state will give rise to a $b$ and $\bar{c}$. The mixing transition $D^0 \leftrightarrow \bar{D}^0$ in the latter process will induce a small background contribution to the former process in final states containing a neutral $D$. Similarly, final states containing a $B^0$ will not be easily distinguishable from those containing a $\bar{B}^0$ because of the mixing transition $B^0 \leftrightarrow \bar{B}^0$.

Lipkin \cite{lipkin} has made the interesting point that in the limit of very heavy $Q$, the $QQ\bar{u}\bar{d}$ color structure is that of an antibaryon. On a related note, one can compare $QQ\bar{u}\bar{d}$ production with $QQq$ production achieved by LHCb in the discovery of $\Xi_{cc}^{++}$ \cite{lipkin}. We assume that the relative fragmentation of the heavy $QQ$ diquark into a light quark and into a light $(\bar{u}\bar{d})_{I=0,S=0}$ diquark is analogous to the relative fragmentation of a heavy antiquark (say, $\bar{b}$) into $u$ and into $(\bar{u}\bar{d})_{I=0,S=0}$. These latter fragmentation fractions have been measured for $b$ quarks produced at $\sqrt{s} = 1.96$ TeV at the Fermilab Tevatron \cite{lipkin}, with the results shown in Table \ref{tab:fracs}. These imply central values $f_u = 0.356$, $f_d = 0.338$, $f_s = 0.111$, and $f_{\Lambda_c} = 0.195$, assuming $f_u + f_d + f_s + f_{\Lambda_c} = 1$. In other words, the fragmentation of a heavy $Q$ into $\bar{u}\bar{d}$ in a state with $I = J = 0$ occurs about half as frequently as fragmentation into $u$. One can expect the same ratio for $bb\bar{u}\bar{d}$ relative to $bbu$.

\section*{VI SUMMARY}  

The calculation in Ref. \cite{lipkin} makes use of a scheme in which quarks in baryons are endowed with effective masses about 55 MeV heavier than those in mesons. An alternative approach with universal quark masses compensates for this difference by adding a term $S = 165$ MeV associated with a “string junction,” of which baryons possess one while mesons possess none \cite{lipkin}. Unless the $QQ$ system can be thought of as a pointlike object equivalent to a heavy antiquark, a $QQ\bar{u}\bar{d}$ tetraquark will have two string junctions, with a corresponding
Table IV: Fragmentation fractions of $b$ quarks produced at the Fermilab Tevatron with $\sqrt{s} = 1.96$ TeV [11]. Errors are statistical, systematic, and associated with branching fractions.

| Ratio          | Value                        |
|----------------|------------------------------|
| $f_u/f_d$      | $1.054 \pm 0.018^{+0.025}_{-0.045} \pm 0.058$ |
| $f_s/(f_u+f_d)$| $0.160 \pm 0.005^{+0.011+0.057}_{-0.010-0.034}$ |
| $f_{\Lambda_b}/(f_u+f_d)$ | $0.281 \pm 0.012^{+0.058+0.128}_{-0.056-0.087}$ |

increase in its mass. An explicit calculation shows that this increase is not enough to push the $bb\bar{u}\bar{d}$ ground state mass above $B^+B^0\gamma$ threshold.

We have used the recent discovery by LHCb of a doubly-charmed baryon $\Xi^{++}_{cc}$ [1] to confirm an assumption about the interaction energy of two heavy quarks $Q$ in a tetraquark $T = QQ\bar{u}\bar{d}$. This has enabled us to estimate the mass of the $J^P = 1^+$ ground state to be $10,389 \pm 12$ MeV, or $215$ MeV below the threshold to decay strongly to a $BB^*$ pair and $170$ MeV below $B^-B^0\gamma$ threshold. Such a state will then decay only weakly, initially via the subprocess $b \rightarrow cW^{*-}$. The other $b$ eventually also will decay via this subprocess, leaving a final state with two charmed quarks, unless flavor-changing mixing of neutral $B$ or $D$ mesons has taken place. The challenge will be to distinguish such a final state from one with $cc$.

Our approach is the first to use the discovery of $\Xi^{++}_{cc}$ to “calibrate” the binding energy in a $QQ$ diquark. However, many other estimates exist of masses of systems containing more than one heavy quark, by methods such as QCD sum rules, potential models, heavy quark effective theory, and lattice gauge theory, in addition to Ref. [7] and the many references cited therein.

An early paper to tackle such problems is Ref. [13] which states in the Abstract “[...] there is no [QQQQ] state below the threshold corresponding to the spontaneous dissociation into two mesons.” Various potential ways out are discussed, including unequal masses. According to ref. [10], “Four-quark states containing two identical heavy quarks are shown to have a good probability of being stable against strong decays.” Recent treatments using QCD sum rules [14,15] have an error of $200$ MeV on a central value $M(bb\bar{u}\bar{d}) = 10300$ MeV and therefore do not deliver a crisp answer regarding stability of the $bb\bar{u}\bar{d}$ tetraquark against strong decays. An early estimate [16] concluded that the $bb\bar{u}\bar{d}$ ground state tetraquark is stable against strong and EM interactions, with a mass of $130 \pm 15$ MeV below $BB^*$ threshold and $85 \pm 15$ MeV below $B^0B^+\gamma$ threshold. Ref. [17] finds that the ground state $cc\bar{u}\bar{d}$ tetraquark “may be bound”; there is no discussion of the $bb\bar{u}\bar{d}$ tetraquark. This gives an idea of the spread of results.

Lattice QCD calculations have matured to the point that their calculated masses of heavy-quark systems are stable to better than 10 or 20 MeV, and agree with experiment to at least that accuracy. An encouraging result [18] finds the lowest $bb\bar{u}\bar{d}$ state $144 \pm 10$ MeV below $B^-B^0\gamma$ threshold, not far from our value of $170 \pm 12$ MeV. The lowest $bb\bar{s}\bar{q}$ state ($q = u$ or $d$) is found to be $52 \pm 8$ MeV below $B_sB\gamma$ threshold.

Experimental search for the $bb\bar{u}\bar{d}$ tetraquark is a challenge well worth pursuing, because it is the first manifestly exotic hadron stable under strong and EM interactions.
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