Calculation of call option using trinomial tree method and black-scholes method case study of Microsoft Corporation

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Abstract. Options are one of the most commonly known financial instrument used in financial market. Options are expected to minimize risk or even increase profits in stock trading. Therefore, it is important for options traders to take into account the contract price of options. This study discusses the determination of call option prices using the Trinomial Tree method and the Black-Scholes method with the data of Microsoft Corporation's stock price and strike price. From the result of the calculations, a conclusion will be drawn from the comparison of the use of the two methods. In conclusion, the call option price using the trinomial tree method is close to the call option price using the Black-Scholes method. Therefore, the Trinomial tree method and the Black-Scholes method is worth using for the base calculation of option pricing.

1. Introduction

At this time, many people invest to gain profit. One of the most widely used financial market instrument of investment is stock. Stock may be defined as a sign of capital inclusion by a person or business entity in a company. In the process of capital inclusion, the most commonly known financial instrument used in financial markets is option. Option is a contract between the writer and the holder. The holder only has the right, without obligation, to sell or buy the underlying asset at the strike price, on the expiration date or before expiration date [1].

There are several types of options based on the price and exercise of option, but European option and American option are the most often used in option trading. The European option is an option that restricts the holder from selling or buying shares only on the expiration date. Meanwhile, the American option is an option that gives the holder the freedom to sell or buy shares right on the expiration date or before the expiration date [2].

In investing in an option, there is certainly a high risk, so it is necessary to take into account on whether the option contract (that is going to be purchased) is appropriate to obtain profit in the exercise of the option by calculating the option price. Option pricing can be done using the Lattice method and the Black-Scholes method. The Lattice method consists of the Binomial Tree method, the Trinomial Tree method, and the Multinomial Tree method. The Trinomial Tree method is a developed version of the Binomial Tree method with three possible stock price movements. The three possible movements is that either the stock is increasing, stable, or falling. The Black-Scholes method is a well-known method and is still widely used by the general investor in setting option prices. Black and Scholes built the model assuming option prices, implicitly, determined by developments in the
underlying asset prices. Black and Scholes consider ideal market conditions, such as a known and constant short-term interest rate, the option type is European option, no arbitrage, no dividend stock, no transaction costs or taxes, and so on [3].

Several researchers have previously conducted research related to option prices using the Lattice method or Black-Scholes method with different research objects. Puspita, et al observed the convergence of the Trinomial model by Boyle and the Binomial model built on the theory of Cox, et al. Based on these results, the convergence of European option pricing using the Trinomial model is still not monotonous but more stable than the Binomial model, and option pricing with the Trinomial model converges faster to the Black-Scholes call option pricing than the binomial model [1]. Krznaric observed the comparison of option prices obtained from the Black-Scholes method with market prices. The conclusion from this research is that the Black-Scholes method is not very accurate in determining the price of call option in real life, but the Black-Scholes method can identify the overall distribution of option over time and is a viable starting point for determining the price of stock option [4]. In this paper, the authors conduct further research to determine the price of the European type call option using the Trinomial Tree method and the Black-Scholes method.

2. Method

The data used in this study are the stock price for the January 1, 2019 – December 31, 2019 period, and the strike price of Microsoft Corporation (MSFT) taken from www.yahoofinance.com, and risk-free interest rates as of January 1, 2020, taken from www.treasury.gov.

Before calculating the price of a call option using the Trinomial Tree method and the Black-Scholes method, it is necessary to find the value of stock volatility to determine the level of fluctuation in stock prices. Stock volatility can be obtained as follows [5]:

1. Calculate the return ($R_t$) using the following formula:
   \[ R_t = \ln \left( \frac{S_t}{S_{t-1}} \right) \]  
   where $S_t$ is the stock price at time $t$.

2. Calculate the expected return ($\bar{R}_t$) using the following formula:
   \[ \bar{R}_t = \frac{1}{n} \sum_{i=0}^{n} R_t \]  
   where $n$ is the amount of observed data.

3. Calculate the volatility ($\sigma$) using the following formula:
   \[ \sigma = \left( \frac{1}{n-1} \sum_{i=1}^{n} (R_t - \bar{R}_t)^2 \right)^{1/2} \]  
   where $k$ is the number of stock trading periods in a year (daily, $k = 252$).

2.1. Trinomial Tree Method

The Trinomial Tree method was introduced by Boyle in 1986. In the Trinomial Tree method, there are three possible stock price movements, namely rising stock prices ($u$), falling stock prices ($d$), and stable stock prices ($m = 1$) [6]. Calculating the call option price using the Trinomial Tree method can be done as follows:

1. Determine the value of parameter $u$ and $d$ using the following formula [6]:
   \[ u = e^{\sigma(\Delta t)^{1/2}} \]  
   \[ d = e^{-\sigma(\Delta t)^{1/2}} \]  
   where $\Delta t$ is the time interval at maturity $t = T$ of a call option is divided into $N$ sub intervals of equal magnitude ($\Delta t = \frac{T}{N}$) and $\sigma$ is the volatility of the stock.
2. Determine the probability value of \( u (p_u) \) and the probability of \( d (p_d) \) using the following formula [6]:

\[
p_u = \left( \frac{\Delta t}{2\sigma^2} \right)^{1/2} \left( r - \frac{\sigma^2}{2} \right) + \frac{1}{6} \\
p_d = -\left( \frac{\Delta t}{2\sigma^2} \right)^{1/2} \left( r - \frac{\sigma^2}{2} \right) + \frac{1}{6}
\]

where \( r \) is the risk-free interest rate as of January 1, 2020 obtained from the daily treasury yield curve rates (\( r = 1.56\% \)).

3. Determine the stock price for each period \((S_{ij})\) using the following formula [7]:

\[
S_{ij} = \begin{cases} 
  u^i S_0, & \text{with } i \geq 1 \\
  S_0, & \text{with } i = 0 \\
  d^j S_0, & \text{with } i \leq -1
\end{cases}
\]

where \( i \) is the rate of change in shares and \( j \) is the option trading period.

4. Determine the payoff value \((C_{iN})\) using the following formula [7]:

\[
C_{iN} = \text{Max} \left[ \left( S_{ij} - K \right), 0 \right]
\]

where \( K \) is the strike price (\( K = $50 \)).

5. Determine the price of call option \((C_i)\) using the following formula [7]:

\[
C_i = e^{r\Delta t} \left( C_{i+1,j+1} \cdot p_u + C_{i,j+1} \cdot p_m + C_{i-1,j+1} \cdot p_d \right)
\]

where \( p_m \) is the probability of \( m (p_m = \frac{2}{3}) \).

2.2. **Black-Scholes Method**

The Black-Scholes method was introduced by Black and Scholes in 1973. Apart from the very limiting assumptions in the calculation of option prices, this method is still widely used by the general investor.

The Black-Scholes formula is considered to be a pricing method that provides a roughly correct estimation of the theoretical option price [3].

There are several assumptions that need to be considered in pricing option using the Black-Scholes method, such as log-normally distributed shares, underlying volatility and interest rates are known and constant throughout the option contract period, no dividend payment during the option contract period, method Black-Scholes is only used for pricing European type call option, and there is no commission in trading these option [8].

2.2.1. **Normality Test**

Before determining the option price using the Black-Scholes method, it is necessary to test for normality to ensure that the data used is normally distributed. The normality test used in this study is the Kolmogorov-Smirnov test. The statistical hypothesis on the Kolmogorov-Smirnov test was used:

\( H_0 : \) Data are normally distributed  
\( H_1 : \) Data isn’t normally distributed

The Kolmogorov-Smirnov test can be done in the following steps [9]:

1. Sort data from smallest to largest and create a cumulative frequency \((F_{cum})\) of the data.
2. Calculate the value of the observed cumulative frequency distribution \((S_{\bar{N}}(X_i))\) using the following formula:

\[
S_{\bar{N}}(X_i) = \frac{F_{cum}}{N}
\]

where \( X_i \) is the \( i \)th data and \( N \) is the amount of data observed.

3. Calculate the value of the defined cumulative frequency distribution function \((F_0(X_i))\) using the following formula:

\[
F_0(X_i) = F \left( \frac{X_i - \bar{X}}{\sigma} \right)
\]
where: $\bar{X}$ is the average of all data, 
$\sigma$ is standard deviation, 
$F\left(\frac{X_i - \bar{X}}{\sigma}\right)$ is the cumulative distribution function of $\left(\frac{X_i - \bar{X}}{\sigma}\right)$.

4. Calculate the deviation value ($D$) using the following formula:

$$D = \max \left| F_{0\%}(X_i) - S_{N}(X_i) \right|$$

(13)

5. Calculate the value of $D_{(0.05)(N)}$ using the following formula [10]:

$$D_{(0.05)(N)} = \frac{1.36}{\sqrt{N}}$$

(14)

6. Perform a normality test with the following criteria:
   If $D > D_{(0.05)(N)}$, then $H_0$ is rejected
   If $D \leq D_{(0.05)(N)}$, then $H_0$ is accepted

2.2.2. Call Option Pricing Using Black-Scholes Method

After ensuring that the data is normally distributed, it can be continued to calculate the price of the European type call option using the Black-Scholes method with the following formula [5]:

$$C = S_0 N(d_1) - Ke^{-rT}N(d_2)$$

(15)

with

$$d_1 = \frac{\ln \left(\frac{S_0}{K}\right) + \left(\frac{r + \sigma^2}{2}\right) T}{\sigma \sqrt{T}}$$

(16)

$$d_2 = \frac{\ln \left(\frac{S_0}{K}\right) + \left(\frac{r - \sigma^2}{2}\right) T}{\sigma \sqrt{T}} = d_1 - \sigma \sqrt{T}$$

(17)

where: $T$ is the time to expiration ($T = \frac{DTE}{k}$).

$DTE$ is the many periods of stock trading before expiration date.

$N(d_1)$ is the cumulative distribution function of $d_1$.

$N(d_2)$ is the cumulative distribution function of $d_2$.

3. Result and Discussion

From the stock closing price data that has been obtained, the stock return value will be calculated using equation (1). So that the results are obtained as in Table 1.

| $t$  | Price      | $R_t$    |
|------|------------|----------|
| 0    | 101.120003 | -        |
| 1    | 97.400002  | -0.037482|
| 2    | 101.93     | 0.045460 |
| 3    | 102.059998 | 0.001275 |
| 251  | 157.699997 | 0.000698 |

Source: https://finance.yahoo.com/quote/MSFT/history?period1=1546300800&period2=1577836800&interval=1d&filter=history&frequency=1d&includeAdjustedClose=true
Then, calculate the expected return using equation (2), so that obtained the expected return value of 0.001770. Then calculate the value of stock volatility using equation (3), so that obtained the volatility of 0.199168.

### 3.1. Calculation of Call Option Using the Trinomial Tree Method
To determine the call option price using the Trinomial Tree method, the first step is to calculate the $u$ and $d$ values using equations (8) and (9), so that the following results are obtained:

$$u = e^{0.199168 \left( \frac{3 \times 1}{12} \right)^{1/2}} = 1.104711,$$

$$d = e^{-0.199168 \left( \frac{3 \times 1}{12} \right)^{1/2}} = 0.905214.$$

Next, calculate the probability of $u$ and $d$ using equations (10) and (11), so that the following results are obtained:

$$p_u = \left( \frac{1}{12 \times (0.199168)^2} \right)^{1/2} \left( 0.0156 - \frac{0.199168^2}{2} \right) + \frac{1}{6} = 0.164895,$$

$$p_d = -\left( \frac{1}{12 \times (0.199168)^2} \right)^{1/2} \left( 0.0156 - \frac{0.199168^2}{2} \right) + \frac{1}{6} = 0.168438.$$

The next step is to calculate the stock price for each period ($S_{ij}$) using equation (12). Then the payoff value will be calculated using equation (13) with $K = $ 50. After getting the payoff value, the call option price can be calculated at $t = 0$ using equation (14). So that the results of $S_{ij}$, $C_{IN}$, and $C_{ij}$ are obtained as in Table 2.

### Table 2. Calculation of Call Option Using the Trinomial Tree Method

| $(i, j)$ | $S_{ij}$      | $C_{IN}$      | $C_{ij}$      |
|---------|---------------|---------------|---------------|
| (0,0)   | 157.699997    | 107.699997    | 108.537769    |
| (1,1)   | 174.212942    | 124.212942    | 124.986759    |
| (0,1)   | 157.699997    | 107.699997    | 108.473826    |
|         | ...           | ...           | ...           |
| (-12,12)| 47.736098     | 0.000000      | 0.450336      |

The value that will be used as the final call option price, with the contract made on January 1, 2020 is the value of $C_{0,0}$, which is the call option price in the 0th period which is the final result of recursively calculating the call option price. So that the price of the European type call option at Microsoft Corporation using the Trinomial Tree method with $K = $ 50 is $ 108.537769.

### 3.2. Calculation of Call Option Using the Black-Scholes Method
#### 3.2.1. Normality Test
The data used in the normality test is the return value. To perform the Normality Test using the Kolmogorov-Smirnov method, the first step that must be taken is to sort the data from smallest to largest, then create a cumulative frequency value. Furthermore, we will look for the value of the observed cumulative frequency distribution using equation (4) and the value of the cumulative distribution function which is determined using equation (5). To obtain the value of the specified cumulative distribution function, it will be determined using the NORMSDIST function of $\frac{X - \bar{X}}{\sigma}$ for
every $i$ in Microsoft Excel. After obtaining the values of $S_N(X_i)$ and $F_0(X_i)$, the deviation value for each $i$ will be determined by finding the difference between $S_N(X_i)$ and $F_0(X_i)$, so that the results are as in Table 3.

| $R_i$ | $F_{cum}$ | $S_N(X_i)$ | $\frac{X_i - \bar{X}}{\sigma}$ | $F_0(X_i)$ | $D_i$ |
|-------|-----------|-------------|---------------------------------|-------------|-------|
| -0.037482 | 1 | 0.003984 | -3.128565 | 0.000878 | 0.003106 |
| -0.034859 | 2 | 0.007968 | -2.919528 | 0.001753 | 0.006215 |
| -0.033902 | 3 | 0.011952 | -2.843215 | 0.002233 | 0.009719 |
|           |   |            |        |           |       |
| 0.045460  | 251 | 1 | 3.482245 | 0.999751 | 0.00249 |

Then, the overall deviation value will be determined using equation (6), so that the deviation value is 0.066936. Then the value of $D(0.05)(N)$ will be calculated using equation (7), so that the value of $D(0.05)(N)$ is 0.085842.

From the results of $D$ and $D(0.05)(N)$ that have been obtained, normality will be tested based on the criteria previously described. Because $D = 0.066936 < D(0.05)(N) = 0.085842$, it can be concluded that the data is normally distributed.

3.2.2. Calculation of Call Option Price
To determine the price of a call option using the Black-Scholes method, the first step that must be taken is to calculate the $d_1$ and $d_2$ values using equations (16) and (17).

$$d_1 = \frac{\ln \left( \frac{157.7}{50} \right) + \left( \frac{0.0156 + 0.199168^2}{2} \right) 0.297619}{0.199168 \sqrt{0.297619}} = 7.743327$$

$$d_2 = \frac{\ln \left( \frac{157.7}{50} \right) + \left( \frac{0.0156 - 0.199168^2}{2} \right) 0.297619}{0.199168 \sqrt{0.297619}} = 7.592923$$

Next, the values for $N(d_1)$ and $N(d_2)$ will be calculated with the help of the NORM.DIST function in Microsoft Excel. After obtaining values of $N(d_1)$ and $N(d_2)$, the call option price can be calculated using equation (15).

$$C = 157.7 \times 1 - 50e^{0.0156 \times 0.297619} \times 1 = 107.931602.$$ The price for the European type call option at Microsoft Corporation using the Black-Scholes method with $K = $ 50 is $107,931,602.

4. Conclusion
After conducting the calculations, it is obtained that the European type call option price of Microsoft Corporation using the Trinomial Tree method is $108,537,769 and using the Black-Scholes method is $107,931,602 with a strike price of $50. The call option price using the Trinomial Tree method approaches the call option price using the Black-Scholes method. The calculation of option prices using the Trinomial Tree method has a longer and more detailed stage, compared to the Black-Scholes method, so the use of the Black-Scholes method is considered more efficient in calculating option prices, while the use of the Trinomial Tree method is considered more flexible. However, these two methods are suitable for use as a basic calculation in determining the option price.
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