Absence of slow transients, and the effect of imperfect vertical alignment, in turbulent Rayleigh-Bénard convection

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We report experimental results for the influence of a tilt angle $\beta$ relative to gravity on turbulent Rayleigh-Bénard convection of cylindrical samples. The measurements were made at Rayleigh numbers $R$ up to $10^{11}$ with two samples of height $L$ equal to the diameter $D$ (aspect ratio $\Gamma \equiv D/L \simeq 1$), one with $L \simeq 0.5$ m (the “large” sample) and the other with $L \simeq 0.25$ m (the “medium” sample). The fluid was water with a Prandtl number $\sigma = 4.38$.

In contrast to the experiences reported by Chillà et al. (2004) for a similar sample but with $\Gamma \simeq 0.5$ ($D = 0.5$ and $L = 1.0$ m), we found no long relaxation times.

For $R = 9.4 \times 10^{10}$ we measured the Nusselt number $\mathcal{N}$ as a function of $\beta$ and obtained a small $\beta$ dependence given by $\mathcal{N}(\beta) = \mathcal{N}_0[1 - (3.1 \pm 0.1) \times 10^{-2}|\beta|]$ when $\beta$ is in radian. This depression of $\mathcal{N}$ is about a factor of 50 smaller than the result found by Chillà et al. (2004) for their $\Gamma \simeq 0.5$ sample.

We measured side-wall temperatures at eight equally spaced azimuthal locations on the horizontal mid-plane of the sample and used them to obtain cross-correlation functions between opposite azimuthal locations. The correlation functions had Gaussian peaks
centered about $t^\circ_1 > 0$ that corresponded to half a turn-over time of the large-scale circulation (LSC) and yielded Reynolds numbers $R^e_\circ$ of the LSC. For the large sample and $R = 9.4 \times 10^{10}$ we found $R^e_\circ(\beta) = R^e_\circ(0) \times [1 + (1.85 \pm 0.21)|\beta| - (5.9 \pm 1.7)\beta^2]$. Similar results were obtained also from the auto-correlation functions of individual thermometers.

These results are consistent with measurements of the amplitude $\delta$ of the azimuthal sidewall temperature-variation at the mid-plane that gave $\delta(\beta) = \delta(0) \times [1 + (1.84 \pm 0.45)|\beta| - (3.1 \pm 3.9)\beta^2]$ for the same $R$. An important conclusion is that the increase of the speed (i.e. of $R_\circ$) with $\beta$ of the LSC does not significantly influence the heat transport. Thus the heat transport must be determined primarily by the instability mechanism operative in the boundary layers, rather than by the rate at which “plumes” are carried away by the LSC. This mechanism apparently is independent of $\beta$.

Over the range $10^9 < R < 10^{11}$ the enhancement of $R^e_\circ$ at constant $\beta$ due to the tilt could be described by a power law of $R$ with an exponent of $-1/6$, consistent with a simple model that balances the additional buoyancy due to the tilt angle by the shear stress across the boundary layers.

Even a small tilt angle dramatically suppressed the azimuthal meandering and the sudden reorientations characteristic of the LSC in a sample with $\beta = 0$.

For large $R$ the azimuthal mean of the temperature at the horizontal mid-plane differed significantly from the average of the top- and bottom-plate temperatures due to non-Boussinesq effects, but within our resolution was independent of $\beta$.

1. Introduction

Turbulent convection in a fluid heated from below, known as Rayleigh-Bénard convection (RBC), has been under intense study for some time [for reviews, see e.g. Siggia(1994)].
Absence of slow transients, and the effect of imperfect vertical alignment, in turbulent Rayleigh-Bénard convection [Kadanoff(2001), Ahlers, Grossmann & Lohse (2002)]. A central prediction of models for this system [Kraichnan(1962), Castaing et al. (1998), Shraiman and Siggia (1990), Grossmann & Lohse (2001)] is the heat transported by the fluid. It is usually described in terms of the Nusselt number

\[ N = \frac{QL}{A\lambda\Delta T} \]  

(1.1)

where \( Q \) is the heat current, \( L \) the cell height, \( A \) the cross-sectional area, \( \lambda \) the thermal conductivity, and \( \Delta T \) the applied temperature difference. The Nusselt number depends on the Rayleigh number

\[ R = \alpha g \Delta T L^3 / \kappa \nu \]  

(1.2)

and on the Prandtl number

\[ \sigma = \nu / \kappa . \]  

(1.3)

Here \( \alpha \) is the isobaric thermal expansion coefficient, \( g \) the acceleration of gravity, \( \kappa \) the thermal diffusivity, and \( \nu \) the kinematic viscosity.

An important feature of turbulent RBC is the existence of a large-scale circulation (LSC) of the fluid [Krishnamurty & Howard (1981)]. For cylindrical samples of aspect ratio \( \Gamma \equiv L/D \simeq 1 \) the LSC is known to consist of a single cell, with fluid rising along the wall at some azimuthal location \( \theta \) and descending along the wall at a location \( \theta + \pi \) [see, for instance, Qiu and Tong (2001a)]. As \( \Gamma \) decreases, the nature of the LSC is believed to change. For \( \Gamma \lesssim 0.5 \) it is expected [Verzicco & Camussi (2003), Stringano & Verzicco (2005), Sun et al. (2005)] that the LSC consists of two or more convection cells, situated vertically one above the other. Regardless of the LSC structure, the heat transport in turbulent RBC is mediated by the emission of hot (cold) volumes of fluid known as “plumes” from a more or less quiescent boundary layer above (below) the bottom (top) plate. These plumes are swept away laterally by the LSC and rise (fall) primarily near the side wall. Their buoyancy helps to sustain the LSC.
In a recent paper Chillà et al. (2004) reported measurements using a cylindrical sample of water with $\sigma \simeq 2.33$ and with $L = 1$ m and $D = 0.5$ m for $R \simeq 10^{12}$. Their sample thus had an aspect ratio $\Gamma \simeq 0.5$ at the borderline between a single-cell and a multi-cell LSC. They found exceptionally long relaxation times of $\mathcal{N}$ that they attributed to a switching of the LSC structure between two states. Multi-stability was observed also in Nusselt-number measurements by Roche et al. (2004) for a $\Gamma = 0.5$ sample (see also Nikolaenko et al. (2005) for a discussion of these data). Chillà et al. also found that $\mathcal{N}$ was reduced by tilting the sample through an angle $\beta$ relative to gravity by an amount given approximately by $\mathcal{N}(\beta)/\mathcal{N}(0) \simeq 1 - 2\beta$ when $\beta$ is measured in radian. A reduction by two to five percent of $\mathcal{N}$ (depending on $R$) due to a tilt by $\beta \approx 0.035$ of a $\Gamma = 0.5$ sample was reported as well recently by Sun et al. (2005), although in that paper the $\beta$-dependence of this effect was not reported. Chillà et al. developed a simple model that yielded a depression of $\mathcal{N}$ for the two-cell structure that was consistent in size with their measurements. Their model also assumes that no depression of $\mathcal{N}$ should be found for a sample of aspect ratio near unity where the LSC is believed to consist of a single convection cell; they found some evidence to support this in the work of Belmonte et al. (1995). Indeed, recent measurements by Nikolaenko et al. (2005) for $\Gamma = 1$ gave the same $\mathcal{N}$ within 0.1 percent for a level sample and a sample tilted by 0.035 rad.

In this paper we report on a long-term study of RBC in a cylindrical sample with $\Gamma \simeq 1$. As expected, we found no long relaxation times because the LSC is uniquely defined. The establishment of a statistically stationary state after a large change of $R$ occurred remarkably quickly, within a couple of hours, and thereafter there were no further long-term drifts over periods of many days.

We also studied the orientation $\theta_0$ of the circulation plane of the LSC by measur-
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ing the side-wall temperature at eight azimuthal locations [Brown et al. (2005b)]. With the sample carefully leveled (i.e. $\beta = 0$) we found $\theta_0$ to change erratically, with large fluctuations. There were occasional relatively rapid reorientations, as observed before by [Sreenivasan et al. (2002)]. The reorientations usually consisted of relatively rapid rotations, and rarely were reversals involving the cessation of the LSC followed by its re-establishment with a new orientation. This LSC dynamics yielded a broad probability distribution-function $P(\theta_0)$, although a preferred orientation prevailed. When the sample was tilted relative to gravity through an angle $\beta$, a well defined new orientation of the LSC circulation plane was established, $P(\theta_0)$ became much more narrow, and virtually all meandering and reorientation of the LSC was suppressed.

We found that $N$ was reduced very slightly by tilting the sample. We obtained $N(\beta) = N_0[1 - (3.1 \pm 0.1) \times 10^{-2}|\beta|]$. This effect is about a factor of 50 smaller than the one observed by Chillá et al. for their $\Gamma = 0.5$ sample.

From side-wall-temperature measurements at two opposite locations we determined time cross-correlation functions $C_{i,j}$. The $C_{i,j}$ had a peak that could be fitted well by a Gaussian function, centered about a characteristic time $t^e_{cc}$ that we interpreted as corresponding to the transit time needed by long-lived thermal disturbances to travel with the LSC from one side of the sample to the other, i.e. to half a turnover time of the LSC. We found that the $\beta$-dependence of the corresponding Reynolds number $R^e_{cc}$ is given by $R^e_{cc}(\beta) = R^e_{cc}(0) \times [1 + (1.85 \pm 0.21)|\beta| - (5.9 \pm 1.7)\beta^2]$. A similar result was obtained from the auto-correlation functions of individual thermometers. Thus there is an $O(1)$ effect of $\beta$ on $R_e$, and yet the effect of $\beta$ on $N$ was seen to be nearly two orders of magnitude smaller. We also determined the temperature amplitude $\delta$ of the azimuthal temperature variation at the mid-plane. We expect $\delta$ to be a monotonically increasing function of the speed of the LSC passing the mid-plane, i.e. of the Reynolds
number. We found $\delta(\beta) = \delta(0) \times [1 + (1.84 \pm 0.45)|\beta| - (3.1 \pm 3.9)\beta^2]$. Thus, for small $\beta$ its $\beta$-dependence is very similar to that of the Reynolds number.

From the large effect of $\beta$ on $R_e$ and the very small effect on $N$ we come to the important conclusion that the heat transport in this system is not influenced significantly by the strength of the LSC. This heat transport thus must be determined primarily by the efficiency of instability mechanisms in the boundary layers. It seems reasonable that these mechanisms should be nearly independent of $\beta$ when $\beta$ is small. This result is consistent with prior measurements by Ciliberto et al. (1997), who studied the LSC and the Nusselt number in a sample with a rectangular cross section. They inserted vertical grids above (below) the bottom (top) plate that suppressed the LSC, and found that within their resolution of a percent or so the heat transport was unaltered. Their shadowgraph visualizations beautifully illustrate that the plumes are swept along laterally by the LSC when there are no grids and rise or fall vertically due to their buoyancy in the presence of the grids. Ciliberto et al. (1997) also studied the effect of tilting their rectangular sample by an angle of 0.17 rad. Consistent with the very small effect of tilting on $N$ found by us, they found that within their resolution the heat transport remained unaltered.

We observed that the sudden reorientations of the LSC that are characteristic of the level sample are strongly suppressed by even a small tilt angle.

2. Apparatus and Data Analysis

For the present work we used the “large” and the “medium” sample and apparatus described in detail by Brown et al. (2005a). Copper top and bottom plates each contained five thermistors close to the copper-fluid interface. The bottom plate had imbedded in it a resistive heater capable of delivering up to 1.5 kW uniformly distributed over the plate. The top plate was cooled via temperature-controlled water circulating in a
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Figure 1. Time evolution of the top- and bottom-plate temperatures and of the heat current for the large sample. Initially the temperature difference was $\Delta T \simeq 0$. At $t = 0.6$ hours the top- and bottom-plate regulators were given new set points corresponding to $\Delta T = 20^\circ$C ($R = 9.43 \times 10^{19}$). (a): Top- (lower data set, $T_t$) and bottom- (upper data set, $T_b$) plate temperatures. (b): Heat current $Q$ delivered by the temperature regulator designed to hold $T_b$ at a specified value. (c): Heat current $Q$ on an expanded scale. The solid line represents a fit of an exponential function to the data for $t > 1.2$ h that gave a relaxation time $\tau_Q = 0.48$ h.

double-spiral channel. For the Nusselt-number measurements a temperature set-point for a digital feedback regulator was specified. The regulator read one of the bottom-plate thermometers at time intervals of a few seconds and provided appropriate power to the heater. The top-plate temperature was determined by the temperature-controlled cooling water from two Neslab RTE740 refrigerated circulators.

Each apparatus was mounted on a base plate that in turn was supported by three
legs consisting of long threaded rods passing vertically through the plate. The entire apparatus thus could be tilted by an angle $\beta$ relative to the gravitational acceleration by turning one of the rods. The maximum tilt angle attainable was 0.12 (0.21) rad for the large (medium) sample.

The Nusselt number was calculated using the temperatures recorded in each plate and the power dissipated in the bottom-plate heater. The side wall was plexiglas of thickness 0.64 cm (0.32 cm) for the large (medium) sample. It determined the length $L$ of the sample. Around a circumference the height was uniformly $50.62 \pm 0.01$ cm (24.76 ± 0.01 cm) for the large (medium) sample. The inside diameter was $D = 49.70 \pm 0.01$ cm ($D = 24.84 \pm 0.01$ cm) for the large (medium) sample. The end plates had anvils that protruded into the side wall, thus guaranteeing a circular cross section near the ends. For the large sample we made measurements of the outside diameter near the half-height after many months of measurements and found that this diameter varied around the circumference by less than 0.1%.

Imbedded in the side wall and within 0.06 cm of the fluid-plexiglas interface were eight thermistors, equally spaced azimuthally and positioned vertically at half height of the sample. They yielded a relatively high (low) temperature reading at the angular positions where there was up-flow (down-flow) of the LSC. A fit of

$$T_i = T_c + \delta \cos(i\pi/4 - \theta_0), \ i = 0, \ldots, 7$$  \hspace{1cm} (2.1)

yielded the mean center temperature $T_c$, the angular orientation $\theta_0$ of the LSC (relative to the location of thermistor 0), and a measure $\delta$ of the LSC strength.

We expect the size of $\delta$ to be determined by the heat transport across a viscous boundary layer separating the LSC from the side wall. Thus $\delta$ should be a monotonically increasing function of the LSC Reynolds number $Re$ because the boundary-layer thickness is expected to decrease with $Re$ as $1/Re^{1/2}$, and because the azimuthal temperature
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Variation carried by the LSC near the boundary layer increases with $R$ and thus with $R_e$. However, the precise relationship between $\delta$ and $R_e$ is not obvious. Experimentally we find, over the range $5 \times 10^9 < R < 10^{11}$ and for the large sample, that $\delta$ is related to $R$ by an effective power law $\delta \propto R^{0.81}$, whereas $R_e \propto R^{0.50}$ in this range, yielding $\delta \propto R_e^{1.62}$. We would then expect that $\delta$ and $R_e$ will have a similar dependence on $\beta$ (at least for small $\beta$), albeit possibly with somewhat different coefficients.

From time series of the $T_i(t)$ taken at intervals of a few seconds and covering at least one day we determined the cross-correlation functions $C^{i,j}(\tau)$ corresponding to signals at azimuthal positions displaced around the circle by $\pi$ (i.e. $j = i + 4$). These functions are given by

$$C^{i,j}(\tau) = \langle [T_i(t) - \langle T_i(t) \rangle_t] \times [T_j(t + \tau) - \langle T_j(t) \rangle_t] \rangle_t .$$  \hspace*{1cm} (2.2)

We also calculated the auto-correlation functions corresponding to $i = j$ in Eq. 2.2 for all eight thermometers.

Initially each sample was carefully leveled so that the tilt angle relative to gravity was less than $10^{-3}$ radian. Later it was tilted deliberately to study the influence of a non-zero $\beta$ on the heat transport.

The fluid was water at $40^\circ$C where $\alpha = 3.88 \times 10^{-4}$ K$^{-1}$, $\kappa = 1.53 \times 10^{-3}$ cm$^2$/s, and $\nu = 6.69 \times 10^{-3}$ cm$^2$/s, yielding $\sigma = 4.38$.

3. The Nusselt number of a vertical sample

3.1. Initial transients

In Fig. 1a we show the initial evolution of the top and bottom temperatures of the large sample in a typical experiment. Initially the heat current was near zero and $T_b$ and $T_t$ were close to $40^\circ$C. The sample had been equilibrated under these conditions for over one day. Near $t = 0.6$ h a new temperature set point of $50^\circ$C was specified for the bottom
Figure 2. The reduced Nusselt number $\mathcal{N}/R^{1/3}$ for $R = 9.43 \times 10^{10}$ for the large sample as a function of time during a single experimental run that lasted nine days. Each data point is based on temperature and heat-current averages over a time interval of two hours. The dotted line corresponds to the value estimated by using all the data. Note that the entire vertical axis covers only a change of 0.8%. The mean value is 0.06035, and the standard deviation from the mean is $5.1 \times 10^{-5}$ or 0.084%.

plate, and the circulator for the top plate was set to provide $T_t \simeq 30^\circ C$. From Fig. 1a one sees that there were transients that lasted until about 0.9 h (1.2 h) for $T_b$ ($T_t$). These transients are determined by the response time and power capability of the bottom-plate heater and the top-plate cooling water and are unrelated to hydrodynamic phenomena in the liquid. Figures 1b and c show the evolution of the heat current. After the initial rapid rise until $t \simeq 0.8$ h the current slowly evolved further to a statistically stationary value until $t \simeq 3$ h. A fit of the exponential function $Q(t) = Q_\infty - \Delta Q \exp(-t/\tau_Q)$ to the data for $t > 1.2$ h is shown by the solid line in Fig. 1c and yielded a relaxation time $\tau_Q = 0.48 \pm 0.04$ h. We attribute this transient to the evolution of the fluid flow. It is interesting to compare $\tau_Q$ with intrinsic time scales of the system. The vertical thermal diffusion time $\tau_v \equiv L^2/\kappa$ is 467 hours. Obviously it does not control the establishment of the stationary state. If we consider that it may be reduced by a factor of $1/\mathcal{N}$ with $\mathcal{N} = 263$, we still obtain a time scale of 1.78 hours that is longer than $\tau_Q$. We believe that the relatively rapid equilibration is associated with the establishment of the top and bottom boundary layers that involve much shorter lengths $l_t$ and $l_b$. It also is necessary
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Figure 3. The reduced Nusselt number $N/R^{1/3}$ as a function of the Rayleigh number $R$.

Stars: data from Chillà et al. (2004) for $\sigma = 2.3$ and $\Gamma = 0.5$. Solid circles: Data from Nikolaenko et al. (2005) for $\sigma = 4.38$ and $\Gamma = 0.43$. Open symbols: from Nikolaenko et al. (2005) for $\Gamma = 0.67$. Open squares: $\sigma = 5.42$. Open circles: $\sigma = 4.38$. Open diamonds: $\sigma = 3.62$.

for the large-scale circulation to establish itself; but, as we shall see, its precise Reynolds number is unimportant for the heat transport. In addition, the LSC can be created relatively fast since this is not a diffusive process.

3.2. Results under statistically stationary conditions

Figure 1 shows the behavior of the system only during the first six hours and does not exclude the slow transients reported by Chillà et al. that occurred over time periods of $O(10^2)$ hours. Thus we show in Fig. 2 results for $N/R^{1/3}$ from a run using the large sample that was continued under constant externally imposed conditions for nine days. Each point corresponds to a value of $N$ based on a time average over two hours of the plate temperatures and the heat current. Note that the vertical range of the entire graph is only 0.8%. Thus, within a small fraction of 1%, the results are time independent. Indeed, during nearly a year of data acquisition for a $\Gamma = 1$ sample at various Rayleigh numbers, involving individual runs lasting from one to many days, we have never experienced long-
term drifts or changes of $N$ after the first few hours. This differs dramatically from the observations of Chillà et al. who found changes by about 2% over about 4 days. We conclude that the slow transients observed by them for their $\Gamma = 0.5$ sample do not occur for $\Gamma \approx 1$.

To document further the stationary nature of the system, we compared results from the large sample for $N$ obtained from many runs, each of one to ten days’ duration, over a period of about five months [Nikolaenko et al. (2005), Funfschilling et al. (2005)]. The scatter of the data at a given $R$ is only about 0.1%. This excellent reproducibility would not be expected if there were slow transients due to transitions between different states of the LSC.

Although work in our laboratory with other aspect ratios has been less extensive, we also have not seen any evidence of drifts or transients for the larger $\Gamma = 1.5, 2, 3, and 6$ [Funfschilling et al. (2005), Brown et al. (2005a)] nor for the smaller $\Gamma = 0.67, 0.43,$ and 0.28 [Nikolaenko et al. (2005), Brown et al. (2005a)]. It may be that $\Gamma = 0.5$, being near the borderline between a single-cell LSC and more complicated LSC structures [Verzicco & Camussi (2003), Stringano & Verzicco (2005), Sun et al. (2005)], is unique in this respect.

In Fig. 3 we compare results for $N/R^{1/3}$ from our large sample [Nikolaenko et al. (2005)] with those reported by Chillà et al. (stars). Our results are larger by about 15%. To find a reason for this difference, we first look at the $\Gamma$ and $\sigma$ dependence. The open (solid) circles represent our data for $\sigma = 4.38$ and $\Gamma = 0.67$ (0.43) and show that the dependence of $N$ on $\Gamma$ is not very strong. The open squares (diamonds) are our results for $\Gamma = 0.67$ and $\sigma = 5.42$ (3.62) and indicate that $N$ actually increases slightly with $\sigma$. Thus the lower values of $N$ (compared to ours) obtained by Chillà et al. for $\sigma = 2.3$ and $\Gamma = 0.5$ can not be explained in terms of the $\Gamma$ and $\sigma$ dependence of $N$. Some of the difference can be
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Figure 4. The reduced Nusselt number $\mathcal{N}(\beta)/\mathcal{N}(0)$ as a function of time for the large sample and $R = 9.43 \times 10^{10}$. Open circles: tilt angle $\beta = 0$. Solid circles: $\beta = 0.087$ radians. Open squares: $\beta = 0.122$ radians. All data are normalized by the average $\mathcal{N}(0)$ of the data for $\beta = 0$. Each data point is based on temperature and heat-current measurements over a two-hour period. The vertical dotted lines indicate the times when $\beta$ was changed.

attributed to non-Boussinesq effects that tend to reduce $\mathcal{N}$ [Funfschilling et al. (2005)]. However, for the largest $\Delta T$ used by Chillá et al. (31°C) we expect this effect to be somewhat less than 1% [Funfschilling et al. (2005)]. Finally, the effect of the finite conductivity of the top and bottom plates comes to mind. This can reduce $\mathcal{N}$ by several % when $\Delta T$ is large [Chaumat et al. (2002), Verzicco (2004), Brown et al. (2005a)], but it is difficult to say precisely by how much. It seems unlikely that this effect can explain the entire difference, particularly at the smaller $R$ (and thus $\Delta T$) where it is relatively small.

4. Tilt-angle dependence of the Nusselt number

In Fig. 4 we show results for $\mathcal{N}$ from the large sample at $R = 9.43 \times 10^{10}$. Each data point was obtained from a two-hour average of measurements of the various temperatures
and of \( Q \). Three data sets, taken in temporal succession, for tilt angles \( \beta = 0.000, 0.087, \) and 0.122 are shown. All data were normalized by the mean of the results for \( \beta = 0 \). Typically, the standard deviation from the mean of the data at a given \( \beta \) was 0.13\%.

The vertical dotted lines and the change in the data symbols show where \( \beta \) was changed. One sees that tilting the cell caused a small but measurable reduction on \( N \). In Fig. 5 we show the mean value for each tilt angle, obtained from runs of at least a day’s duration at each \( \beta \), as a function of \( |\beta| \). One sees that \( N \) decreases linearly with \( \beta \). A fit of a straight line to the data yielded

\[
N(\beta) = N_0[1 - (3.1 \pm 0.1) \times 10^{-2} |\beta|]. \tag{4.1}
\]

with \( N_0 = 273.5 \). Similar results for the medium cell are compared with the large-cell results in Fig. 6. At the smaller Rayleigh number of the medium sample the effect of \( \beta \) on \( N \) is somewhat less. Because the effect of \( \beta \) on \( N \) is so small, we did not make a more detailed investigation of its Rayleigh-number dependence.

Chillà et al. proposed a model that predicts a significant tilt-angle effect on \( N \) for \( \Gamma = 0.5 \) where they assume the existence of two LSC cells, one above the other. They also assumed that there would be no effect for \( \Gamma = 1 \) where there is only one LSC cell. Although we found an effect for our \( \Gamma = 1 \) sample, we note that it is a factor of about 50 smaller than the effect observed by Chillà et al. for \( \Gamma = 0.5 \).

5. Tilt-angle dependence of the large-scale circulation

5.1. The orientation

In Fig. 7 we show the angular orientation \( \theta_0 \) (a) and the temperature amplitude \( \delta \) (b) of the LSC. For the first 8000 sec shown in the figure, the sample was level (\( \beta = 0.000 \pm 0.001 \)). One sees that \( \theta_0 \) varied irregularly with time. The probability-distribution
Figure 5. The Nusselt number $N(\beta)$ as a function of the tilt angle $\beta$ for the large sample with $R = 9.43 \times 10^{10}$. Each point is the average over an entire run of duration one day or longer. Solid circles: $\beta > 0$. Open circles: $\beta < 0$. The solid line is a least-squares fit of a straight line (Eq. 4.1) to the data for $\beta > 0$.

Function $P(\theta_0)$ is shown in Fig. 8 as solid dots. Essentially all angles are sampled by the flow, but there is a preferred direction close to $\theta_0/2\pi = 0.6$. At $t = 8000$ sec, the sample was tilted through an angle $\beta = 0.087$ radian. The direction of the tilt was chosen deliberately so as to oppose the previously prevailing preferred orientation. As a consequence one sees a sharp transition with a change of $\theta_0$ by approximately $\pi$. The temperature amplitude $\delta$ on average increased slightly, and certainly remained non-zero. From this we conclude that the transition took place via rotation of the LSC, and not by cessation that would have involved a reduction of $\delta$ to zero [see Brown et al. (2005b)]. We note that $\theta_0(t)$ fluctuated much less after the tilt. The results for $P(\theta_0)$ after the tilt are shown in Fig. 8 as open circles. They confirm that the maximum was shifted close to $\theta_0 = 0$, and that the distribution was much more narrow.

In Fig. 9 we show $P(\theta_0)$ for $\beta = 0.122$, (solid circles), 0.044 (open circles), and 0.026
Figure 6. The reduced Nusselt number $N(\beta)/N_0$ as a function of the tilt angle $\beta$ for the large sample with $R = 9.43 \times 10^{10}$ (solid circles) and the medium sample with $R = 1.13 \times 10^{10}$ (open circles).

(solid squares). One sees that a reduction of $\beta$ leads to a broadening of $P(\theta_0)$. The square root of the variance of data like those in Fig. 6 is shown in Fig. 10 on a logarithmic scale as a function of $\beta$ on a linear scale. Even a rather small tilt angle caused severe narrowing of $P(\theta_0)$.

5.2. The temperature amplitude

In Fig. 11 we show the temperature-amplitude $\delta(\beta)$ of the LSC as a function of $\beta$. As was the case for $N$, the data are averages over the duration of a run at a given $\beta$ (typically a day or two). The solid (open) circles are for positive (negative) $\beta$. The data can be represented well by either a linear or a quadratic equation. A least-squares fit yielded

$$\delta(\beta) = \delta(0) \times [1 + (1.84 \pm 0.45)|\beta| - (3.1 \pm 3.9)\beta^2] \quad (5.1)$$

with $\delta(0) = 0.164$ K.
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Figure 7. (a): The orientation $\theta_0$ of the plane of circulation and (b): the temperature amplitude $\delta$ of the large-scale circulation as a function of time for the large sample and $R = 9.43 \times 10^{10}$. At $t = 8000$ sec the sample was tilted by an angle $\beta = 0.087$ radians relative to gravity.

5.3. The Reynolds numbers

Using Eq. 2.2 we calculated the auto-correlation functions (AC) $C^{i,i}, i = 0, \ldots, 7$, as well as the cross-correlation functions (CC) $C^{i,j}, j = (i + 4) \% 8, i = 0, \ldots, 7$, of the temperatures measured on opposite sides of the sample. Typical examples are shown in Fig. 12. The CC has a characteristic peak that we associate with the passage of relatively hot or cold volumes of fluid at the thermometer locations. Such temperature cross-correlations have been shown, e.g. by Qiu and Tong (2001b) and Qiu and Tong (2002) to yield delay times equal to those of velocity-correlation measurements, indicating that warm or cold fluid volumes travel with the LSC. The function

$$C^{i,j}(\tau) = -b_0 \exp \left(-\frac{\tau - t_i^j}{\tau_0} \right) - b_1 \exp \left[ - \left( \frac{\tau - t_i^j}{\tau_1^j} \right)^2 \right],$$

(5.2)

consisting of an exponentially decaying background (that we associate with the random time evolution of $\theta_0$) and a Gaussian peak, was fitted to the data for the CC. The fitted
Figure 8. The probability distribution $P(\theta_0)$ of the orientation $\theta_0$ of the plane of circulation of the large-scale flow for the large sample and $R = 9.43 \times 10^{10}$ as a function of $\theta_0$. Solid circles: tilt angle $\beta = 0 \pm 0.001$. Open circles: $\beta = 0.087$ radians.

Figure 9. The probability distribution $P(\theta_0)$ of the orientation $\theta_0$ of the plane of circulation of the large-scale flow for the large sample and $R = 9.43 \times 10^{10}$ at three tilt angles $\beta$. Solid circles: $\beta = 0.122$. Open circles: $\beta = 0.044$. Solid squares: $\beta = 0.026$.

function is shown in Fig. 12 as a solid line over the range of $\tau$ used in the fit. It is an excellent representation of the data and yields the half turnover time $T/2 = t_{1,2}^{i,j}$ of the LSC. Similarly, we fitted the function

$$C^{i,j}(\tau) = b_0 \exp \left( -\frac{\tau}{\tau_0^{i,j}} \right) + b_1 \exp \left[ -\left( \frac{\tau}{\tau_1^{i,j}} \right)^2 \right] + b_2 \exp \left[ -\left( \frac{\tau - \tau_{i,j}^{1,2}}{\tau_{i,j}^{1,2}} \right)^2 \right]$$

(5.3)
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Figure 10. The square root of the variance $\sigma_{\theta}$ of the probability distribution $P(\theta_0)$ of the orientation $\theta_0$ of the plane of circulation of the large-scale flow for the large sample and $R = 9.43 \times 10^{10}$ as a function of the tilt angle $\beta$.

Figure 11. The time-averaged temperature amplitude $\delta(\beta)$ of the LSC for the large sample as a function of $|\beta|$. Solid circles: $\beta \geq 0$. Open circles: $\beta < 0$. For this example $R = 9.43 \times 10^{10}$.

to the AC data. It consists of two Gaussian peaks, one centered at $\tau = 0$ and the other at $\tau = t_2^{ij}$, and the exponential background. We interpret the location $t_2^{ij}$ of the second Gaussian peak to correspond to a complete turnover time $T$ of the LSC.
Figure 12. The cross-correlation function $C^{04}(\tau)$ between thermometers 0 and 4 and the auto-correlation function $C^{00}(\tau)$ of thermometer 0 for the large sample. The solid lines are fits of Eqs. 5.2 and 5.3 to the data. They also indicate the range of $\tau$ used for the fits. For this example the tilt angle was $\beta = -0.009$ and $R = 9.43 \times 10^{10}$.

In terms of the averages $\langle t_{i,j}^{1} \rangle$ and $\langle t_{i,i}^{1} \rangle$ over all 8 thermometers or thermometer-pair combinations we define [Qiu and Tong (2002), Grossmann & Lohse (2002)] the Reynolds numbers

$$R_{cc}^e = (L/\langle t_{i,j}^{1} \rangle)(L/\nu)$$

and

$$R_{ac}^e = (2L/\langle t_{i,i}^{1} \rangle)(L/\nu).$$

(5.4)

(5.5)

Here the length scale $2L$ was used to convert the turnover time $T$ into a LSC speed $2L/T$. For $\Gamma = 1$, the length $4L$ might have been used instead, as was done for instance by Lam et al. (2002). This would have led to a Reynolds number larger by a factor of two. In Fig. 13a and b we show $R_{cc}^e(|\beta|)$ and $R_{ac}^e(|\beta|)$ respectively. The solid circles are for positive and the open ones for negative $\beta$. One sees that $R_{cc}^e(|\beta|)$ and $R_{ac}^e(|\beta|)$ initially
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Figure 13. (a): The Reynolds number $R_{cc}^e(|\beta|)$ obtained from the temperature cross-correlation functions, and (b): the Reynolds number $R_{ac}^e(|\beta|)$ obtained from the temperature auto-correlation functions, of the LSC for the large sample and $R = 9.43 \times 10^{10}$ as a function of the absolute value $|\beta|$ of the tilt angle. Solid circles: $\beta \geq 0$. Open circles: $\beta < 0$. For this example $R = 9.43 \times 10^{10}$.

grow linearly with $\beta$, but the data also reveal some curvature as $|\beta|$ becomes larger. Thus we fitted quadratic equations to the data and obtained

\[
R_{cc}^e(\beta) = R_{cc}^c(0) \times [1 + (1.85 \pm 0.21)|\beta| - (5.9 \pm 1.7)\beta^2] \tag{5.6}
\]

and

\[
R_{ac}^e(\beta) = R_{ac}^c(0) \times [1 + (1.72 \pm 0.38)|\beta| - (4.1 \pm 3.2)\beta^2] \tag{5.7}
\]

with $R_{cc}^c(0) = 10467 \pm 43$ and $R_{ac}^c(0) = 10565 \pm 82$ (all parameter errors are 67% confi-
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Figure 14. The Reynolds-number ratio $R_e^{cc}(\beta)/R_e^{cc}(0)$ (solid circles) obtained from the temperature cross-correlation functions, and the amplitude ratio $\delta(\beta)/\delta(0)$ (open circles), of the LSC of the medium sample for $R = 1.13 \times 10^{10}$ as a function of $\beta$.

The results for $R_e^{cc}(0)$ and $R_e^{ac}(0)$ are about 10% higher than the prediction by [Grossmann & Lohse (2002)] for our $\sigma$ and $R$. The excellent agreement between $R_e^{cc}$ and $R_e^{ac}$ is consistent with the idea that the CC yields $T/2$ and that the AC gives $T$. As expected (see Sect. 2), the $\beta$-dependences of both Reynolds numbers are the same within their uncertainties. It is interesting to see that the coefficients of the linear term also agree with the corresponding coefficient for $\delta$ (Eq. 5.1). This suggests that there may be a closer relationship between $\delta$ and $R_e$ than we would have expected a priori. However, the coefficient of the linear term in Eq. 5.6 or 5.7 is larger by a factor of about 50 than the corresponding coefficient for the Nusselt number in Eq. 4.1.

In Fig. 14 we show measurements of $R_e^{cc}$ and of $\delta$, each normalized by its value at $\beta = 0$, as a function of $\beta$ for the medium sample and $R = 1.13 \times 10^{10}$. For this sample we were able to attain larger values of $\beta$ than for the large one. One sees that $\delta$ and $R_e^{cc}$ have about the same $\beta$ dependence for small $\beta$, but that $\delta$ then increases more rapidly.
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Figure 15. The Reynolds-number ratio $R_{e}^{cc}(\beta = 0.122)/R_{e}^{cc}(0)$ of the LSC as a function of $R$.

Open circles: medium sample. Solid circles: large sample. Solid line: powerlaw with an exponent of $1/6$.

than $R_{e}^{cc}$ as $\beta$ becomes large. Although we do not know the reason for this behavior, it suggests that the larger speed of the LSC enhances the thermal contact between the side wall and the fluid interior.

The Rayleigh-number dependence of $R_{e}^{cc}$ at constant $\beta$ is shown in Fig. 15. Here the open (solid) circles are from the medium (small) sample. There is consistency between the two samples, and the data can be described by a power law with a small negative exponent. The solid line is drawn to correspond to an exponent of $-1/6$.

5.4. A model for the enhancement of the Reynolds number

As seen in Fig. 8 the LSC assumes an orientation for which gravity enhances the velocity above (below) the bottom (top plate), i.e. the LSC flows “uphill” at the bottom where it is relatively warm and “downhill” at the top where it is relatively cold. This leads to an enhancement of the Reynolds number of the LSC. As suggested by Chillà et al. (2004), one can model this effect by considering the buoyancy force per unit area parallel to
the plates. This force can be estimated to be $\rho l g \beta \Delta T / 2$ where $l$ is the boundary-layer thickness. It is opposed by the increase of the viscous shear stress across the boundary layer that may be represented by $\rho \nu u'/l$ where $u'$ is the extra speed gained by the LSC due to the tilt. Equating the two, substituting

$$l = L/(2N) ,$$

(5.8)
solving for $u'$, using Eq. 1.2 for $R$, and defining $R'_e \equiv (L/\nu)u'$ one obtains

$$R'_e = \frac{R \beta}{8\sigma N^2}$$

(5.9)

for the enhancement of the Reynolds number of the LSC. From our measurements at large $R$ we found that $R_e$ [Ahlers et al. (2005)] and $N$ [Nikolaenko et al. (2005)] can be represented within experimental uncertainty by

$$R_e = 0.0345 R^{1/2} ,$$

(5.10)

$$N = 0.0602 R^{1/3} ,$$

(5.11)

giving

$$\frac{R'_e}{R_e} = 1.00 \times 10^3 R^{-1/6} \sigma^{-1} \beta .$$

(5.12)

For our $\sigma = 4.38$ and $R = 9.43 \times 10^{10}$ one finds $R'_e/R_e = 3.4 \beta$, compared to the experimental value $(1.9 \pm 0.2)\beta$ from $R'_e$ [Eq. 5.6] and $(1.7 \pm 0.4)\beta$ from $R''_{ac}$ [Eq. 5.7].

We note that the coefficient $1.00 \times 10^3$ in Eq. 5.12 depends on the definition of $R_e$ given in Eqs. 5.4 and 5.5 that was used in deriving the result Eq. 5.10. If the length scale $4L$ had been used instead of $2L$ to define the speed of the LSC, as was done for instance by Lam et al. (2002), this coefficient would have been smaller by a factor of two, yielding near-perfect agreement with the measurements. In Fig. 15 one sees that the predicted dependence on $R^{-1/6}$ also is in excellent agreement with the experimental results. However, such good agreement may be somewhat fortuitous, considering the
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Approximations that were made in the model. Particularly the use of Eq. \(5.8\) for the boundary-layer thickness is called into question at a quantitative level by measurements of \cite{Lui98} that revealed a significant variation of \(l\) with lateral position. In addition, it is not obvious that the thermal boundary-layer thickness \(l\) should be used, as suggested by \cite{Chilla04}, to estimate the shear stress; perhaps the thickness of the viscous BL would be more appropriate.

In discussing their \(\Gamma = 0.5\) sample, \cite{Chilla04} took the additional step of assuming that the relative change due to a finite \(\beta\) of \(N\) is equal to the relative change of \(R_e\). For our sample with \(\Gamma = 1\) this assumption does not hold. As we saw above, the relative change of \(N\) is a factor of about 50 less than the relative change of \(R_e\). The origin of the (small) depression of the Nusselt number is not so obvious. Naively one might replace \(g\) in the definition of the Rayleigh number by \(g \cos(\beta)\); but this would lead to a correction of order \(\beta^2\) whereas the experiment shows that the correction is of order \(\beta\), albeit with a coefficient that is smaller than of order one. The linear dependence suggests that the effect of \(\beta\) on \(N\) may be provoked by the change of \(R_e\) with \(\beta\), but not in a direct causal relationship.

6. Tilt-angle dependence of reorientations of the large-scale circulation

It is known from direct numerical simulation \cite{Hansen91} and from several experiments \cite{Cioni97,Niemela01,Sreenivasan02,Brown05b} that the LSC can undergo relatively sudden reorientations. Not unexpectedly, we find that the tilt angle strongly influences the frequency of such events. For a level sample \((\beta = 0)\) we demonstrated elsewhere \cite{Brown05b} that reorientations can involve changes of the orientation of the plane of circulation of the LSC through any
angular increment $\Delta \theta$, with the probability $P(\Delta \theta)$ increasing with decreasing $\Delta \theta$. Thus, in order to define a “reorientation”, we established certain criteria. We required that the magnitude of the net angular change $|\Delta \theta|$ had to be greater than $\Delta \theta_{\text{min}} = (2\pi)/8$. In addition we specified that the magnitude of the net average azimuthal rotation rate $|\dot{\theta}| \equiv |\Delta \theta/\Delta t|$ had to be greater than $\dot{\theta}_{\text{min}} = 0.1/T$ where $T$ is the LSC turnover time and $\Delta t$ is the duration of the reorientation (we refer to Brown et al. (2005b) for further details). Using these criteria, we found that the number of reorientation events $n(\beta)$ at constant $R = 9.43 \times 10^{10}$ decreased rapidly with increasing $|\beta|$. These results are shown in Fig. 16. It is worth noting that nearly all of these events are rotations of the LSC and very few involved a cessation of the circulation. A least-squares fit of the Gaussian function

$$n(\beta) = N_0 \exp\left[-(\beta - \beta_0)^2/w^2\right]$$

(6.1)
to the data yielded $N_0 = 1.23 \pm 0.06$ events per hour, $\beta_0 = 0.0093 \pm 0.0010$ rad, and $w = 0.0251 \pm 0.0015$ rad. It is shown by the solid line in the figure.

We note that the distribution function is not centered on $\beta = 0$. The displacement of
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7. Tilt-angle dependence of the center temperature

We saw from Fig. 11 that the increase of $R_e$ with $\beta$ led to an increase of the amplitude $\delta$ of the azimuthal temperature variation at the horizontal mid-plane. An additional question is whether the tilt-angle effect on this system has an asymmetry between the top and bottom that would lead to a change of the mean center temperature $T_c$ (see Eq. (2.1)). Chilla et al. (2004) report such an effect for their $\Gamma = 0.5$ sample. For a Boussinesq sample with $\beta = 0$ we expect that $T_c = T_m$ with $T_m = (T_t + T_b)/2$ ($T_t$ and $T_b$ are the top and bottom temperatures respectively), or equivalently that $\Delta_t = T_c - T_t$ is equal to $\Delta_b = T_b - T_c$. A difference between $\Delta_b$ and $\Delta_t$ will occur when the fluid properties have a significant temperature dependence \[Wu & Libchaber(1991)\] \[Zhang et al.(1997)\], i.e. when there are significant deviations from the Boussinesq approximation. For the sequence of measurements with the large apparatus and $R = 9.43 \times 10^{10}$ as a function of $\beta$ the mean value of $\Delta T = T_b - T_t$ was $19.808 \pm 0.018^\circC$ and $T_c - T_m$ was $0.97^\circC$, indicating a significant non-Boussinesq effect. In Fig. 17a we show $\Delta_t$ and $\Delta_b$ as a function of $\beta$. One sees that increasing $\beta$ does not have a significant effect for our $\Gamma = 1$ sample. This is shown with greater resolution in Fig. 17b where $T_c - T_m = (\Delta_t - \Delta_b)/2$ is shown. We believe that the small variation, over a range of about $5 \times 10^{-3}^\circC$, is within possible systematic experimental errors and consistent with the absence of a tilt-angle effect.
Figure 17. (a): The temperature difference in Kelvin between the bottom and the center ($\Delta_b = T_b - T_c$, solid circles) and the center and the top ($\Delta_t = T_c - T_t$, open circles) as a function of the tilt angle $\beta$. (b): The temperature difference $T_c - T_m = (\Delta_t - \Delta_b)/2$ between the center temperature $T_c$ and the mean temperature $T_m = (T_t + T_b)/2$. The center temperature is the average of the values given by the eight side-wall thermometers.

8. Summary

In this paper we reported on an experimental investigation of the influence on turbulent convection of a small tilt angle $\beta$ relative to gravity of the axes of two cylindrical Rayleigh-Bénard samples. The aspect ratios were $\Gamma \simeq 1$.

Where there was overlap, there were significant differences between our results and those obtained by Chillà et al. (2004) for a $\Gamma = 0.5$ sample. We found our system to establish a statistically stationary state quickly, within a couple of hours, after a Rayleigh-number change whereas Chillà et al. (2004) found long transients that they attributed to changes of the LSC structure. We found a very small depression of the Nusselt number.
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$N$ with increasing $\beta$, by about 4% per radian at small $\beta$. Chillà et al. (2004) found a decrease by 200% per radian for their sample.

In contrast to the very small effect of $\beta$ on $N$, we found an increase of the Reynolds number $R_e$ by about 180% per radian for small $\beta$. The small effect on $N$ in the presence of this large change of $R_e$ indicates that the heat transport does not depend strongly on the speed of the LSC sweeping over the boundary layers. Instead, $N$ must be determined by instability mechanisms of the boundary layers, and the associated efficiency of the ejection of hot (cold) volumes (so-called “plumes”) of fluid from the bottom (top) boundary layer.

It is interesting to note that the strong dependence of $R_e$ on $\beta$ in the presence of only a very weak dependence of $N$ on $\beta$ can be accommodated quite well within the model of Grossmann & Lohse (2002). The Reynolds number can be changed by introducing a $\beta$-dependence of the parameter $a(\beta)$ in their Eqs. (4) and (6). As pointed out by them, a change of $a$ has no influence on the predicted value for $N$.

We also measured the frequency of rapid LSC reorientations that are known to occur for $\beta = 0$. We found that such events are strongly suppressed by a finite $\beta$. Even a mild breaking or the rotational invariance, corresponding to $\beta \simeq 0.04$, suppresses re-orientations almost completely.

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