Calculated estimation of influence of convection rate of molten working and heating medium during hardening of restored parts

V N Buylov¹, F Ya Rudik¹, I V Lyulakov¹, A V Danilin¹ and A V Pavlov¹

Saratov State Agricultural University n. a. N.I. Vavilov, 1, Teatralnaya Square, Saratov, 410012, Russia

E-mail: saratov-79@list.ru

Abstract. The paper presents a computational and mathematical assessment of the convection rate of the molten working and heating medium during the strengthening of the restored parts. The calculated estimate is based on the theory of multidimensional heat and mass transfer processes occurring in the electrode furnace during the strengthening of the restored parts. Computational modeling is carried out based on the choice of initial data and characteristics. The calculation results made it possible to establish the effect of the convection rate of the molten working and heating medium on the hardening of the restored parts. The theoretical description of the conditions for the onset of convective motion of a molten continuous medium is based on the Rayleigh–Bénard, Navier–Stokes, and Lorenz equations for the flow of a viscous continuous medium using the Oberbeck-Boussinesq approximation. The description of the internal space of the tank furnace is carried out by a system of differential equations of the balances of elementary volumes into which it is divided. Transformations into a system of differential equations are carried out on the basis of the Galerkin method. The use of this transition method leads to the fact that the sought functions are represented in a variable-separated form that exactly satisfies the boundary conditions. The introduction of assumptions and boundary conditions made it possible to obtain the equations of motion of mass forces and a viscous medium in a polar coordinate system. A formula describing the configuration of the temperature field is obtained.

1. Introduction

Restoration of parts of agricultural machines and units using advanced technologies includes surfacing operations, thermal and chemical-thermal hardening of the restored surfaces. These operations are carried out in thermal installations. For this, the design of an electrode tank furnace is proposed [3, 5, 8].

The aim of the study is to develop a computational model of the motion of the molten working and heating medium of the tank furnace to determine its speed.

2. The object and method of research

Since the molten working and heating medium is affected by gravity and the temperature difference characteristics of a viscous continuous medium, with a sufficiently large coefficient of thermal expansion, it generates a density difference [4, 6, 7]. Thus, thermal convection takes place.
There are two possible thermal convection schemes in the tank furnace working environment. According to the first theoretical scheme, the working medium is enclosed between horizontal surfaces with different temperatures. The lower surface has a temperature higher than that of the upper surface. With a small difference in surface temperatures, convection does not occur due to the presence of viscous friction in the molten medium. With an increase in the temperature gradient between the surfaces, thermal convection of the continuous medium occurs.

According to the second theoretical scheme, the working environment is enclosed between vertical surfaces with different temperatures. In this case, the molten medium rises along the warmer surface and descends along the colder one. Convection in this case will occur at any temperature gradient.

In a tank furnace, with electrode heating of the molten working medium, vertical and horizontal temperature gradients are formed.

The mathematical description of the conditions for the occurrence of convective motion of a molten continuous medium is based on the Rayleigh–Bénard, Navier–Stokes, and Lorenz equations for the flow of a viscous continuous medium using the Oberbeck-Boussinesq approximation [8, 9, 10].

The tank furnace workspace is described by a system of ordinary differential equations for the balances of elementary volumes into which it is divided.

To solve this problem, we divide it into two subproblems, which take into account horizontal and vertical temperature gradients.

Let us assume that the sources of heat release in the furnace melt bath create vertical components of temperature gradients, which allows us to start solving this flat problem.

We suppose that the system has translation invariance along the y-axis, therefore the variables in equations (1) and (2) depend on two spatial coordinates [11, 12, 13]: the height z and the horizontal coordinate y, perpendicular to the axis of convective rolls (Figure 1):

\[
\overrightarrow{V} = \overrightarrow{V}(u(y,z,t), w(y,z,t)),
\]

\[
T(y,z,t) = T_o + \Delta T - \frac{\Delta T}{h} z + \theta(y,z,t),
\]

where \( \overrightarrow{V} \) - flow velocity field, m/s; \( T \) - temperature field, °C; \( \theta(y,z,t) \) - deviation of the temperature field profile along the z axis, °C.

We introduce a stream function \( \psi(y,z,t) \), such that the relations will be satisfied (3):

\[
u = -\frac{\partial \psi}{\partial z} = -\psi_z, \quad w = \frac{\partial \psi}{\partial y} = \psi_y.
\]

We obtain the continuity equation (4):
\frac{\partial u}{\partial y} + \frac{\partial w}{\partial z} = 0. \quad (4)

Taking into account the rotor operator and the representation for the temperature field, we obtain the equations written in terms of the stream function and the deviation from the temperature field profile linear along the z-axis:

\frac{\partial}{\partial t} (\nabla^2 \psi) = -\frac{\partial \psi}{\partial y} \frac{\partial}{\partial z} (\nabla^2 \psi) + \frac{\partial \psi}{\partial y} (\nabla^2 \psi) + \nu \nabla^2 \psi + g \beta \frac{\partial \theta}{\partial y}, \quad (5)

\frac{\partial \theta}{\partial t} = -\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial z} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial z} + \frac{\Delta T}{h} \frac{\partial \psi}{\partial z} + k^p \nabla^2 \theta, \quad (6)

where \( g \) – free fall acceleration vector, \( m/s^2 \); \( \beta \) – the thermal expansion coefficient of the medium, \( 1/\circ C \); \( k^p \) – the thermal diffusivity of molten medium, \( m^2/s \); \( \nu \) – the kinematic viscosity of the molten medium, \( m^2/s \).

We transform (5) and (6) into a system of differential equations based on the Galerkin method [13, 14]. The use of this transition method leads to the fact that the sought functions are represented in a variable-separated form that exactly satisfies the boundary conditions.

We introduce the Lorenz representation of the functions \( \psi(y, z, t) \) and \( \theta(y, z, t) \):

\[ \psi(y, z, t) = \psi_1(t) \sin \frac{\pi y}{\ell} \sin \frac{\pi z}{h}, \quad (7) \]

\[ \theta(y, z, t) = \theta_1(t) \cos \frac{\pi y}{\ell} \sin \frac{\pi z}{h} - \theta_2(t) \sin \frac{2\pi z}{h}, \quad (8) \]

\[ u = -\psi_y = -\frac{\pi}{h} \psi_1(t) \sin \frac{\pi y}{\ell} \cos \frac{\pi z}{h}, \quad (9) \]

\[ w = \psi_z = \frac{\pi}{\ell} \psi_1(t) \cos \frac{\pi y}{\ell} \sin \frac{\pi z}{h}. \]

In accordance with the Galerkin method, functions (7) - (9) satisfy boundary conditions (10), taking into account the continuity of the walls and the absence of shear stresses at \( z = h \):

\[ w \big|_{z=0,h} = 0, \quad \frac{\partial u}{\partial z} \big|_{z=0,h} = 0, \quad \theta \big|_{z=0,h} = 0, \quad \psi \big|_{z=0,h} = 0. \quad (10) \]

Neglecting higher-order harmonics and after some transformations, we obtain the differential equations (11), (12) and (13):

\[ \psi_1 = -\frac{\pi^2}{\ell^2 h^2} (\ell^2 + h^2) \psi_1 + \frac{g \beta \ell h^2}{\pi (\ell^2 + h^2)} \theta_1, \quad (11) \]

\[ \dot{\theta}_1 = -\frac{\pi^2}{\ell h} \psi_1 \theta_1 + \Delta T \frac{\pi}{\ell h} \psi_1 - \frac{k^p \pi^2}{\ell^2 h^2} \theta_1, \quad (12) \]

\[ \dot{\theta}_2 = \frac{\pi^2}{2 \ell h} \psi_1 \theta_1 - k^p \frac{4 \pi^2}{h^2} \theta_2. \quad (13) \]

3. Results

The resulting equations represent a mathematical model of the first approximation for solving the problem. This model is consistent and takes into account the main features of the original equations.

To correctly determine the velocities of movement and distribution, the temperature fields of a continuous medium, it is necessary to take into account the horizontal temperature gradients.
The determination of the flow rates of a continuous medium caused by horizontal temperature gradients is based on the theory of the flow of a viscous medium in an annular channel [6, 7]. In this case, the configuration of the temperature field with a horizontal component is as follows (Figure 2).

This problem is modeled by systems of equations that are a particular case of the system of Rayleigh–Bénard, Navier–Stokes, and Lorenz equations for the flow of a viscous continuous medium using the Oberbeck-Boussinesq approximation [6, 7] without taking into account nonlinear terms.

The introduction of the necessary assumptions makes it possible to obtain the following equations: the equation of motion of a viscous medium in a polar coordinate system (14), the equation of mass forces (15), the formula for the configuration of the temperature field, and boundary conditions (16) and (17):

\[
\frac{\partial^2 V_\varphi(z^\circ, \varphi)}{\partial z^2} - \frac{1}{\mu^p R^o} \frac{\partial P}{\partial \varphi} - \frac{F_\varphi(\varphi)}{V^p}, \quad (14)
\]

\[
F_\varphi = -g \left[ 1 - \beta \Delta T \right] \cos \varphi, \quad (15)
\]

\[
\Delta T = \Delta T^o \cos \varphi, \quad (16)
\]

\[
V_\varphi(0, \varphi) = V_\varphi(h^o, \varphi) = 0, \quad (17)
\]

where \(\mu^p\) - dynamic viscosity of the molten continuous medium, Pa·s.

\[\begin{align*}
\frac{\partial^2 V_\varphi(z^\circ, \varphi)}{\partial z^2} &= \frac{1}{\mu^p R^o} \frac{\partial P}{\partial \varphi} - \frac{F_\varphi(\varphi)}{V^p}, \\
F_\varphi &= -g \left[ 1 - \beta \Delta T \right] \cos \varphi,
\end{align*}\]

Integrating (14) over \(z^o\) with regard to (15), (16) and boundary conditions (17), we obtain the following expression for the velocity component:

\[V_\varphi(z^o, \varphi) = \left( \frac{1}{\mu^p R^o} \frac{dP}{d\varphi} - \frac{F_\varphi}{V^p} \right) z^o (z^o - h^o) / 2. \quad (18)\]
Averaging (18) over $\varphi$ from 0 to $2\pi$ and over $z^o$ from 0 to $h_k$ and taking into account that $P(2\pi) - P(0) = 0$, we obtain formula (19):

$$<V_\varphi> = \frac{g \beta \Delta \Delta^0}{\nu^p} \frac{h_k^2}{12}.$$  \hspace{1cm} (19)

This component of the flow rate of the working fluid is due to the action of horizontal temperature gradients.

Thus, to estimate the maximum velocity components of the convective flow of a molten continuous medium caused by horizontal temperature gradients in the gravity field, taking into account the continuity of the flow, the following relations can be used:

$$<V_\varphi> = \frac{g \beta \Delta \Delta^0}{\nu^p} p^\varphi,$$

$$<W_\varphi> = \frac{g \beta \Delta \Delta^0 h}{\nu^p \ell} p^\varphi.$$  \hspace{1cm} (20)

The values of the velocities determined by formulas (20) are included as additional terms in the representations of the velocity field (9). The obtained formulas (21) are basic for refining the constructed mathematical model for the qualitative and quantitative analysis of dynamic thermal processes in the electrode tank furnace:

$$u = \left(-\frac{\pi}{h} \psi_1(t) \mp <V_\varphi>\right) \sin\frac{\pi y}{\ell} \sin\frac{\pi z}{h},$$

$$w = \left(\frac{\pi}{\ell} \psi_1(t) \pm <W_\varphi>\right) \cos\frac{\pi y}{\ell} \sin\frac{\pi z}{h}.$$  \hspace{1cm} (21)

4. Conclusion

Built on the Rayleigh–Bénard, Navier–Stokes, and Lorenz equations for the flow of a viscous continuous medium using the Oberbeck-Boussinesq approximation, the mathematical model of heat and mass transfer processes in an electrode tank furnace makes it possible to determine the velocities of convective motion of a molten continuous medium, as well as to refine the parameters of the furnace design, thermophysical characteristics of materials and environments, temperature and other necessary characteristics and parameters. By changing the input data for the program, it is possible to simulate the processes occurring in the working space of the tank furnace for various positions of the electrodes and their switching on, which made it possible to calculate the tank furnace, both with the bottom location of the electrodes and with the electrodes located both on the bottom and on the side walls. The convection rate of a continuous medium increases by a factor of 4 in comparison with a tank furnace with a bottom arrangement of electrodes.

References

[1] Bartenev I M and Pozdnyakov E V 2013 The deteriorating ability of soils and its effect on the durability of the working bodies of tillage machines Forestry magazine 3 (11) 114
[2] Builov V N 2019 Calculation assessment of the use of electrolyte melts in strengthening the restored working bodies of tillage and sowing aggregates Agricultural scientific journal 5 77
[3] Buylov V N, Lyulyakov I V, Eremenko V S and Pronin S A 2017 To the question of working conditions and wear of lancet paws of cultivators Actual problems of scientific and technological progress in the agro-industrial complex: Sat. scientific articles under the general ed A T Lebedev (Stavropol: AGRUS) pp 3–5.
[4] Builov V N and Lyulyakov I V 2018 Investigation into the Process of Electrolysis Borating of Steel Parts Surface Engineering and Applied Electrochemistry 54(4) 338
[5] Lebedev K A and Lebedev A L 2015 Increasing the resource of cultivator paws Scientific Review 3 50
[6] Lyalyakin V P, Soloviev S A and A V Aulov 2014 The state and prospects of hardening and restoration of parts of tillage machines by welding and surfacing methods Welding production 7
32

[7] Novikov V S and Petrovsky D I 2017 Increasing the durability of lancet paws of cultivators *Bulletin of FGOU VPO MGAU* n.a. V P Goryachkina. 4 55

[8] Rusinov A V and Slyusarenko V V 2015 Change of physical and mechanical properties of reclamation soils as a result of mechanical action *Innovations in environmental management and protection in emergency situations: Materials of the II international scientific and practical conference* ed A V Rusinov (Saratov: KUBiK) pp 30–33

[9] Tenenbaum M M and Shamshetov S N 1986 *Wear resistance and durability of agricultural machines* (Nukus: Karakalpakstan)

[10] Tkachev V N, Kazintsev N V and Zagrebin A V 1991 Increasing the durability of tillage working bodies *Tractors and agricultural machines* 12 46

[11] Kragelsky I V, Dobychin M N and Kombalov V S 1977 *Fundamentals of calculation for friction and wear* (Mockow: Mechanical Engineering)

[12] Akhmetshin T F 2014 Prediction of the durability of the blades of lancet paws of cultivators *Bulletin of the Orenburg State Agrarian University* 5 (49) 74

[13] Mikhailchenkov A M, Feskov S A and Yakushenko N A 2014 Restoration of lancet paws *Rural machine operator* 3 25

[14] Titov N V, Kolomeychenko A V and Logachev V N 2014 Study of hardness and wear resistance of the working bodies of machines hardened by vibration arc surfacing using cermet materials *Welding production* 9 36