KIC 9451096: MAGNETIC ACTIVITY, FLARES AND DIFFERENTIAL ROTATION

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ABSTRACT

We present spectroscopic and photometric analysis of KIC 9451096, where the latter is based on very high precision long cadence photometry obtained by Kepler space craft. Combined spectroscopic and photometric modeling show that the system is a detached eclipsing binary in a circular orbit and composed of F5V + K2V components. Subtracting the best-fit light curve model from whole long cadence data reveals additional low (mmag) amplitude light variation in time and occasional flares, suggesting low, but still remarkable level of magnetic spot activity on the K2V component. Analyzing rotational modulation of light curve residuals enables us to estimate differential rotation coefficient of the K2V component as $k = 0.069 \pm 0.008$, which is 3 times weaker compared with the solar value of $k = 0.19$, assuming a solar type differential rotation. We find stellar flare activity frequency for K2V component as $0.000368411 \ h^{-1}$ indicating low magnetic activity level.

Key Words: stars: activity — (stars:) binaries: eclipsing — stars: flare — stars: fundamental parameters — stars: individual (KIC 9451096)

1. INTRODUCTION

Although the primary aim of Kepler mission is to detect transiting planets by obtaining very high precision photometric measurements, it provides further benefits, especially in terms of clear and reliable determination of very small amplitude light variation on eclipsing and intrinsic variable stars. About 150,000 targets have been observed in the mission, and apart from the exoplanets, which is the main purpose of the mission, numerous variable stars have been discovered. Unprecedented precision of Kepler photometry clearly revealed low amplitude (mmag) light variations, which is used in analysis of stellar flares, spot activity and differential rotation (Balona 2015; Balona et al. 2016; Reinhold & Reiners 2013; Reinhold et al. 2013a). Among these variable stars, 2876 eclipsing binary stars have been discovered (Prša et al. 2011; Slawson et al. 2011). Careful light curve modeling of the binaries with cool components ($T_{\text{eff}} < 6500$ K) revealed rotational modulation of light curves and flares in model residuals. KIC 09641031 (Yoldaş & Dal 2016), KIC 09761199 (Yoldaş & Dal 2017), and KIC 2557430 (Kamil & Dal 2017),

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GJ 1243, GJ 1245A and B (Hawley et al. 2014), KIC 2300039, KIC 4671547 (Balona 2015) are such stars.

The analyses of the patterns of magnetic activity exhibiting by these stars reveals some clues about their evolutionary stages. Although there are several indicators found in these analyses for the evolutionary stage, two of them are the energy spectra defined by Gershberg (1972) and flare frequencies described by Ishida et al. (1991). Both of them have been computed especially from the 1970’s to the 1980’s in order to figure out the magnetic activity levels for the stars, which the flares are detected from. In 1990’s, Leto et al. (1997) examined the flare frequency variation of EV Lac, a well-known UV Ceti type star. There are a few studies, in which the activity levels of three magnetic active stars discovered in Kepler Mission are discussed depending on their flare frequencies, recently published in the literature. Yolda¸s & Dal (2016) detected 240 flares from KIC 09641031, and Yolda¸s & Dal (2017) detected 94 flares from KIC 09761199. In addition, Kamil & Dal (2017) detected 69 flares from KIC 2557430. Yolda¸s & Dal (2016) derived the One Phase Exponential Association (hereafter OPEA) model, and the flare frequency \( N_1 \) was found to be 0.41632 \( h^{-1} \) for KIC 09641031. Yolda¸s & Dal (2017) computed \( N_1 \) as 0.01351 \( h^{-1} \) over 69 flares for KIC 09761199. However, an interesting situation occurs in case of KIC 2557430. Kamil & Dal (2017) found that some of the flares detected from KIC 2557430 come from a third body, which is unclear whether it is a component in the system or an undetected light source from background. Depending on the OPEA model derived from 69 flares, Kamil & Dal (2017) reveal that 40 flares (called Group 1) of them come from the secondary component, while 29 flares (called Group 2) come from a third body. They computed the flare frequency \( N_1 \) as 0.02726 \( h^{-1} \) for Group 1 and 0.01977 \( h^{-1} \) for Group 2. As it is discussed by Yolda¸s & Dal (2016) and Gershberg (2005), the flare frequency is one of the parameter which is an indicator about the nature of the flare mechanism that is in progress on the stellar atmosphere. Apart from the classical parameters described by Gershberg (2005), Dal & Evren (2010, 2011) have also described some new parameters derived from the OPEA models in order to determine the flare process running on the stellar surface.

Continuous photometry of variable single stars discovered in the scope of Kepler enabled to trace photometric period variation as a proxy of differential rotation via Fourier transform (see, e.g. Reinhold et al. 2013b, Reinhold & Reiners 2013). However, Fourier transform may not perfectly work in case of eclipsing binaries, where the amplitude of rotational modulation of star spots is usually embedded into the relatively large amplitude light variations caused by eclipses and break of spherical symmetry of the binary components. Furthermore, insufficient representation of light curve models, especially around mid-eclipse phases, may require discarding of data around those phases and causes regular gaps in light curve, which would lead to unwanted alias period and harmonics. In this case, alternative methods could be adopted to trace photometric period variation, such as \( O-C \) diagram based on minimum times
of rotationally modulated light curves (see, e.g., Özdarcan et al. 2010).

In case of eclipsing binary stars, additional intrinsic variations may not be determined at first look, due to the reasons explained above. KIC 9451096 is such an eclipsing binary in Kepler eclipsing binary catalog (Prša et al. 2011; Slawson et al. 2011) with a short period, and with a confirmed third body (Borkovits et al. 2016). Beyond the properties provided by the catalog, such as morphology and eclipse depths, Armstrong et al. (2014) provided physical information, estimated from spectral energy distribution based on photometric measurements. They estimated the effective temperature of the components of KIC 9451096 as 7166 K and 5729 K for the primary and the secondary component, respectively.

In this study, we carry out photometric and spectroscopic analysis of KIC 9451096, based on Kepler photometry and optical spectroscopic observations with intermediate resolution, described in Section 2. Section 3 comprises spectroscopic and photometric modeling of the system, and the analysis of out-of-eclipse variations. In the final section, we summarize and discuss our findings.

2. OBSERVATIONS AND DATA REDUCTIONS

2.1. Kepler photometry

Photometric data obtained by Kepler spacecraft cover a broad wavelength range between 4100 Å and 9100 Å, which has advantage of collecting much more photons in a single exposure and reaching sub-milimag precision, but also has disadvantage of having no "true" photometric filter, hence no photometric color information. There are two types of photometric data having different exposure times. These are short cadence data (having exposure time of 58.89 seconds) and long cadence data (having exposure time of 29.4 minutes). In this study we use long cadence data of KIC 9451096 obtained from Kepler eclipsing binary catalog. The catalog provides detrended and normalized intensities, which is obtained by application of procedures described by Slawson et al. (2011) and Prša et al. (2011). The whole data covers ~4 years of time span with 65,307 data points in total. MAST archive reports 0.9% contamination level in the measurements, practically indicating no additional light contribution to the measured fluxes of KIC 9451096.

2.2. Spectroscopy

We obtained optical spectra of KIC 9451096 by 1.5 m Russian - Turkish telescope equipped with Turkish Faint Object Spectrograph Camera (TFOSC) at Tubitak National Observatory. TFOSC enables one to obtain intermediate resolution optical spectra in échelle mode. In our case, the instrumental setup provides actual resolution of $R = \lambda / \Delta \lambda \sim 2500$ around 6500 Å, and observed

\[^2\]http://keplerebs.villanova.edu/
\[^3\]http://www.tug.tubitak.gov.tr/rtt150_tfosc.php
spectra covers usable wavelength range between 3900–9100 Å in 11 échelle orders. Back illuminated 2048 × 2048 pixels CCD camera, which has pixel size of 15 × 15 μm², was used to record spectra.

We obtained ten optical spectra of KIC 9451096 between 2014 and 2016 observing seasons. In order to obtain enough signal, we used 3600 s of exposure time for each observation. Estimated signal–to–noise ratio (SNR) of observed spectra is mostly between 80–100, except a few case, where the SNR is around 50. SNR estimation is based on photon statistic. Together with the target star, we also obtained high SNR optical spectrum of HD 225239 (G2V, \( v_r = 4.80 \) km s\(^{-1}\)) and \( \iota \) Psc (HD 222368, F7V, \( v_r = 5.656 \) km s\(^{-1}\)), and adopted them as radial velocity and spectroscopic comparison templates.

We reduce all observations by using standard IRAF packages and tasks. Typical reduction procedure starts with obtaining master bias frame from nightly taken several bias frames and subtracting master bias frame from all object, calibration lamp (Fe-Ar spectra in our case) and halogen lamp frames. Then bias corrected halogen frames are combined together to form average halogen frame and this average frame is normalized to the unity to produce normalized master flat frame. After that, all target and calibration lamp spectra are divided by the normalized flat field frame. Next, cosmic rays removal and scattered light corrections are applied to bias and flat corrected frames. At the end of these steps, reduced frames are obtained and these frames are used for extraction of spectra. In the final steps, Fe-Ar frames are used for wavelength calibration of extracted spectra and wavelength calibrated spectra are normalized to the unity by using cubic spline functions.

3. ANALYSIS

3.1. Radial velocities and spectroscopic orbit

The first step of our analysis is to determine radial velocities of the components and spectroscopic orbit of the system. We cross-correlate each observed spectrum of KIC 9451096 with spectra of template stars HD 225239 and \( \iota \) Psc, as described in Tonry & Davis (1979). In practice we use \textit{fxcor} task in IRAF environment. We achieve better cross-correlation signals (especially for the weak secondary component) when we use HD 225239 as template, thus we determine all radial velocities with respect to the HD 225239 spectrum. We obtain acceptable cross-correlation signals of both components in échelle orders 5 and 6, which cover wavelength range between 4900–5700 Å. Figure shows cross-correlation functions of two spectra obtained around orbital quadratures.

We list observational log and measured radial velocities of the components in Table 1. Note that we use ephemeris and period given by Borkovits et al. (2016) and listed in Table 2 to calculate orbital phases and for further analysis.

\footnote{The Image Reduction and Analysis Facility is hosted by the National Optical Astronomy Observatories in Tucson, Arizona at URL iraf.noao.edu.}
We achieve reasonable solution for spectroscopic orbit under non-eccentric orbit assumption, where the eccentricity is zero and the longitude of periastron is undefined. We check this assumption by inspecting Kepler light curve of the system, where we observe deeper and shallower eclipses at 0.0 and 0.5 orbital phases, respectively, indicating circular orbit (see Section 3.3, Figure 4). In order to reach the final spectroscopic orbital solution, we prepare a simple script written in python language, which applies Markov chain Monte Carlo simulations to the measured radial velocities, considering their measured errors. We list the final spectroscopic orbital elements in Table 2 and plot measured radial velocities, their observational errors, theoretical spectroscopic orbit and residuals from the solution in Figure 2.

3.2. Spectral type

We rely on our intermediate resolution TFOSC optical spectra to determine the spectral type of the components. Most of our spectra correspond to the phases around orbital quadratures, where we observe the signal of two components separated. However, there are two spectra obtained at phases close to the eclipses, where two components can not be resolved separately. One of these spectra corresponds to ~0.56 orbital phase (see Table 1), where we can not observe the radial velocity signal of the secondary component in cross-correlation. Even in the orbital quadratures, cross-correlation signal
TABLE 1
LOG OF SPECTROSCOPIC OBSERVATIONS TOGETHER WITH MEASURED RADIAL VELOCITIES AND THEIR CORRESPONDING STANDARD ERRORS ($\sigma$) IN km s$^{-1}$.

| HJD Orbital Exposure | Primary | Secondary |
|----------------------|---------|-----------|
| (24 00000+) Phase time (s) | $V_r$ | $\sigma$ | $V_r$ | $\sigma$ |
| 56842.5435 0.7794 3600 | 91.4 | 8.2 | -152.5 | 36.9 |
| 56844.4052 0.2682 3600 | -79.9 | 6.3 | 151.9 | 39.1 |
| 56844.4479 0.3024 3600 | -74.4 | 6.6 | 155.0 | 37.2 |
| 56889.4315 0.2781 3600 | -77.1 | 5.7 | 148.1 | 40.0 |
| 56890.2958 0.9693 3600 | 14.5 | 5.0 | — | — |
| 57591.4532 0.7199 3600 | 88.5 | 7.2 | -153.3 | 32.0 |
| 57601.4386 0.7058 3600 | 88.7 | 5.4 | -149.8 | 32.1 |
| 57616.4778 0.7333 3600 | 86.0 | 4.3 | -145.2 | 38.7 |
| 57617.5188 0.5659 3600 | 31.0 | 5.8 | — | — |
| 57672.3009 0.3779 3600 | -54.8 | 5.1 | 111.1 | 47.9 |

TABLE 2
SPECTROSCOPIC ORBITAL ELEMENTS OF KIC 9451096. $M_1$ AND $M_2$ DENOTE THE MASSES OF THE PRIMARY AND THE SECONDARY COMPONENT, RESPECTIVELY, WHILE $M$ SHOWS THE TOTAL MASS OF THE SYSTEM.

| Parameter | Value |
|-----------|-------|
| $P_{\text{orb}}$ (day) | 1.25039069 (fixed) |
| $T_0$ (HJD24 00000+) | 54954.72942 (fixed) |
| $\gamma$ (km s$^{-1}$) | 2.8±0.5 |
| $K_1$ (km s$^{-1}$) | 84.1±2.3 |
| $K_2$ (km s$^{-1}$) | 153.2±14.6 |
| $e$ | 0 (fixed) |
| $a \sin i$ ($R_\odot$) | 5.92±0.35 |
| $M \sin^3 i$ ($M_\odot$) | 1.79±0.25 |
| Mass ratio ($q = M_2/M_1$) | 0.55±0.05 |
| rms1 (km s$^{-1}$) | 3.7 |
| rms2 (km s$^{-1}$) | 4.9 |
of the secondary component is considerably weak compared to the primary component, indicating a very small light contribution from the secondary component to the total light of the system. Our preliminary light curve analysis shows that the contribution of the secondary component to the total light does not exceed $\sim$10%. In this case, the signal from the secondary component becomes almost negligible in the resolution of our observed spectrum at $\sim$0.56 orbital phase, therefore we assume that we only observe the spectrum of the primary component and adopt this spectrum as reference spectrum for the primary component. We confirm this assumption by calculating composite spectrum of the binary via final parameters of the components (see Section 3.3), where we observe that the contribution of the secondary component affects the theoretical composite spectrum less than 2% for wavelength range of 4900-5700 Å. We refrain from performing detailed analysis with spectral disentangling. Future studies could take advantage of this technique and derive atmospheric parameters of the secondary.

We first compare the reference spectrum with the template spectra of HD 225239 and $\iota$ Psc. We observe that $\iota$ Psc spectrum provides closer match to the reference spectrum but also indicates earlier spectral type and slightly lower metal abundances for the primary component. At that point, we switch to the spectrum synthesizing method. We use the latest version of python framework iSpec (Blanco-Cuaresma et al. 2014) which enables practical and quick calculation of a synthetic spectrum with a given set of atmospheric parameters via different radiative transfer codes. Among these codes we adopt
SPECTRUM code (Gray & Corbally 1994), together with ATLAS-9 (Castelli & Kurucz 2004) model atmospheres and actual line list from the third version of the Vienna atomic line database (VALD3) (Ryabchikova et al. 2015). Considering spectral type of ι Psc, we synthesize spectra for the effective temperatures between 6000 K and 7000 K in steps of 250 K, and metallicity values ([Fe/H]) between −1.0 and 0.0 in steps of 0.5. For all synthetic spectra we fix the gravity (log g) to 4.15, which we precisely calculate in light curve modeling (see Section 3.3). Since we do not have high resolution spectrum, we fix the microturbulence velocity to 2 km s$^{-1}$. We convolve all calculated spectra with a proper Gaussian line spread function in order to degrade their resolution to the resolution of TFOSC spectra. Instrumental broadening in TFOSC spectra is 2.2 Å, corresponding 119 km s$^{-1}$ for wavelengths around 5500 Å. Estimated projected rotational velocities of the components are 62 km s$^{-1}$ and 36 km s$^{-1}$ for the primary and the secondary component respectively (see Section 3.3). Since instrumental broadening is the most dominant broadening source in observed spectra, we do not consider rotational broadening and other line broadening mechanisms.

Among the calculated spectra we find that the model with 6500 K effective temperature and [Fe/H] value of −0.5 provides the closest match to the reference spectrum. The final effective temperature indicates F5 spectral type (Gray 2005). Considering the effective temperature and metallicity steps in model atmospheres, and resolution of TFOSC spectra, the final values and their estimated uncertainties are $T_{\text{eff}} = 6500 \pm 200$ K and [Fe/H] = −0.5±0.5 dex, respectively. Note that even we consider the neglected contribution of the secondary component in the reference spectrum, its effect would be fairly inside the estimated uncertainties above. The final $T_{\text{eff}}$ values is $\sim$670 K lower than the 7166 K value estimated in Armstrong et al. (2014). We show portions of reference spectrum and the model spectrum, calculated with the final parameters above, in Figure 3.

3.3. Light curve modeling and physical properties

Global visual inspection of KIC 9451096 Kepler photometry reflects properties of a typical close eclipsing binary. We start light curve modeling by phasing the whole long cadence data with respect to the ephemeris and period given by Borkovits et al. (2016), and re-binning the phased data with a phase step of 0.002 via freely available fortran code lcbin written by John Southworth. We plot the binned and phased light curves of the system in Figure 4 panel a and aa. The light curve indicates detached configuration for the system. Mid-eclipse phases are 0.0 and 0.5 phases, indicating circular orbit. There is no conspicuous asymmetry in the light curve.

We model the light curve with 2015 version of the Wilson-Devinney code (Wilson & Devinney 1971, Wilson & Van Hamme 2014). In the modeling, we
first fixed the most critical two parameters of the light curve modeling, i.e., mass ratio \( q \) and effective temperature of the primary component \( T_1 \). Since we have reliably derived these parameters in previous sections as \( q = 0.55 \) and \( T_1 = 6500 \) K, we adopt them as fixed parameters. Calculated atmospheric properties of the primary component reveal that both stars have convective envelopes, therefore we set albedo \( A_1, A_2 \) and gravity darkening \( g_1, g_2 \) coefficients of the components to 0.5 and 0.32, respectively, which are typical values for stars with convective outer envelopes. We also consider slight metal deficiency of the system, thus adopt internal stellar atmosphere formulation of the Wilson-Devinney code according to the determined [Fe/H] value of −0.5. We assume that the rotation of the components is synchronous to the orbital motion, thus fix the rotation parameter of each component \( F_1, F_2 \) to 1.0.
We adopt square root law \cite{KlingSmith1970} for limb darkening of each component, that is more appropriate for stars cooler than 9000 K. We take the limb darkening coefficients for *Kepler* passband \((x_1, x_2, y_1, y_2)\) and bolometric coefficients \((x_{bol1}, x_{bol2}, y_{bol1}, y_{bol2})\) from \cite{VanHamme1993}. In the modeling, we adjust inclination of the orbit \((i)\), temperature of the secondary component \((T_2)\), dimensionless omega potentials of the components \((\Omega_1, \Omega_2)\) and luminosity of the primary component \((L_1)\). We also include phase shift parameter as adjustable in the modeling since we expect a shift in ephemeris due to the light–time effect of the third body \cite{Borkovits2016}. The model quickly converged to a steady solution in a few iterations. We list the model output in Table 3 and we plot the best–model in Figure 4 panel \(a\), \(b\), and residuals from the model in panel \(c\).

In Figure 4 panel \(b\), one can easily see the model inconsistency around 0.25 orbital phases. The inconsistency indicates an additional light variation, which is known as \textit{O’Connell effect}, i.e. difference between light levels of
subsequent maxima in an orbital cycle. Possible sources of the difference may be Doppler beaming, hot spot or a cool spot on one of the component in the system. KIC 9451096 is a detached eclipsing binary, thus we can safely exclude possibility of mass transfer between components, i.e. hot spot possibility. Doppler beaming was detected observationally among some Kepler binaries (see, e.g. van Kerkwijk et al. 2010), which becomes important for systems with very low mass ratio, especially for systems with a compact component, such as white dwarf or hot sub-dwarf. In addition, if the effect is in progress, then it would change light levels of each maxima. However, we observe inconsistency only for 0.25 phase, while the model fairly represents light level at 0.75 phase, thus Doppler beaming should have negligible effect in case of KIC 9451096, if any. Remaining possibility is cool spots located preferably on the cooler component.

Here we do not prefer to model this inconsistency alone, which would only show cumulative effect of hundreds of light curves, but instead we subtract the best–fit model from whole long cadence data and inspect the residuals in order to investigate further light variations. We will focus on this in Section 3.4.

We complete light curve modeling section with calculation of absolute parameters of the system by combining spectroscopic orbital solution and light curve model results. In Table 4, we give physical properties of each component. Our analysis reveals that the system is formed by F5V primary and K2V secondary components.

3.4. The out-of-eclipse variations

In this section, we subtract the best–fit light curve model from the whole long cadence data and obtain residuals. Here, we first divide the whole long cadence data into subsets, where each subset covers only a single orbital cycle, resulting in 1026 individual light curve. Then we apply differential corrections routine of the Wilson-Devinney code and fix all parameters, except ephemeris reference time. In this way, we find precise ephemeris reference time for each individual subset, therefore eliminate any shift in the ephemeris time due to the third body reported by Borkovits et al. (2016) and obtain precise residuals. In Figure 5, we plot three different parts of the residuals. Note that we remove data points that correspond to the eclipse phases due to the insufficient representation of the model at those phases. This mainly arises from inadequacy of radiative physics used in light curve modeling for a very high photometric precision and can clearly be seen in Figure 4 panel c.

Inspecting residual brightness, we immediately see a variation pattern which changes its shape from time to time. Furthermore, we observe sudden increase and gradual decrease in residual brightness which occasionally occurs in four years of time span and has short time scale of a few hours. These patterns are traces of magnetic spot activity, which is very possibly from the K2V secondary component. Observational confirmation of this possibility can be done by inspecting magnetic activity sensitive spectral lines, such as Hα and Ca II H & K lines. We inspect these lines in our TFOSC
TABLE 3
LIGHT CURVE MODELING RESULTS OF KIC 9451096. $\langle R_1 \rangle$ AND $\langle R_2 \rangle$ DENOTE MEAN FRACTIONAL RADII OF THE PRIMARY AND THE SECONDARY COMPONENTS, RESPECTIVELY. INTERNAL ERRORS OF THE ADJUSTED PARAMETERS ARE GIVEN IN PARENTHESES FOR THE LAST DIGITS. ASTERISK SYMBOLS IN THE TABLE DENOTE FIXED VALUE FOR THE CORRESPONDING PARAMETER. NOTE THAT WE ADOPT THE UNCERTAINTY OF $T_1$ FOR $T_2$ AS WELL, SINCE THE INTERNAL ERROR OF $T_2$ IS UNREALISTICALLY SMALL (~1 K).

| Parameter | Value          |
|-----------|----------------|
| $q$       | 0.55*          |
| $T_1 (K)$ | 6500*          |
| $g_1$, $g_2$ | 0.32*, 0.32*   |
| $A_1$, $A_2$ | 0.5*, 0.5*     |
| $F_1 = F_2$ | 1.0*          |
| phase shift | 0.00108(2)     |
| $i (^\circ)$ | 79.07(4)       |
| $T_2 (K)$ | 5044(200)      |
| $\Omega_1$ | 4.4942(49)     |
| $\Omega_2$ | 4.8885(125)    |
| $L_1/(L_1 + L_2)$ | 0.897(1) |
| $x_1 \text{bol}, x_2 \text{bol}$ | 0.136*, 0.293* |
| $y_1 \text{bol}, y_2 \text{bol}$ | 0.583*, 0.401* |
| $x_1$, $x_2$ | 0.106*, 0.482* |
| $y_1$, $y_2$ | 0.670*, 0.313* |
| $\langle r_1 \rangle$, $\langle r_2 \rangle$ | 0.2557(3), 0.1506(5) |
| Model rms | $3.0 \times 10^{-4}$ |

spectra and do not notice any emission features, which could be considered as the sign of the activity. However, one should consider that the contribution of the secondary component to the total light does not exceed 10% at optical wavelengths and will steeply decrease towards the ultraviolet region of the spectrum. Furthermore, variation patterns observed in Figure 5 exhibit very small amplitude. Therefore the existence of magnetic spot activity can not be confirmed or excluded via spectral line inspection in case of KIC 9451096. Nevertheless, variation patterns and flares observed in the residuals indicate weak magnetic spot activity on the secondary component, which can still be detected with the very high precision of Kepler photometry.
TABLE 4

ABSOLUTE PHYSICAL PROPERTIES OF KIC 9451096. ERROR OF EACH PARAMETER IS GIVEN IN PARANTHESIS FOR THE LAST DIGITS.

| Parameter          | Primary | Secondary |
|--------------------|---------|-----------|
| Spectral Type      | F5V     | K2V       |
| [Fe/H]             | −0.5 ± 0.5 |
| Mass (M\(_\odot\))| 1.18(26) | 0.65(9)   |
| Radius (R\(_\odot\))| 1.53(10) | 0.90(6)   |
| \(\log L/L\odot\) | 0.574(76) | −0.327(88) |
| \(\log g\) (cgs) | 4.14(4)  | 4.34(1)   |
| \(M_{bol}\) (mag) | 3.31(19) | 5.57(22)  |

We analyze rotational modulation and flares of the secondary component via residuals by assuming that the source of all variation patterns is only the secondary component.

3.4.1. Photometric period and differential rotation

Conventional periodogram methods for determining rotational period do not perfectly work in our case because observed variation patterns exhibit quick changes in amplitude and mean brightness level in a short time scales of a few days, which is comparable to the orbital period. Moreover, since we remove data points at eclipse phases, this causes regular gaps in the data which repeats itself in every \(\sim 0.625\) day (i.e. half of the orbital period), thus causes alias period and its harmonics, and disturbs real periods. Furthermore, one can clearly see that the rotational modulation of residuals has asymmetric shape. Considering an individual light curve with an asymmetric shape, it is not possible to find a single period to represent whole light curve perfectly and additional periods (i.e. harmonics) are required to full representation. Therefore we apply an alternative method based on tracing the time of a minimum light observed in an orbital cycle, which was previously applied to RS CVn system HD 208472 (Özdarcan et al. 2010). For each orbital cycle, we find the time of the deepest minimum in the cycle by fitting a second or third order polynomial to the data points around the expected minimum time. The order of the polynomial depends on the light curve shape. After obtaining all minimum times, we construct an \(O-C\) diagram by adopting the first minimum time observed in the residuals as initial ephemeris reference time and orbital period as the initial period, and obtain \(O-CI\) values. Then we apply a linear fit to the \(O-CI\) values and calculate average ephemeris reference time and period given in Equation 1 together with statistical uncertainties given in parentheses for the last digits.
Fig. 5. a) Residuals from whole long cadence data. Remaining panels show different time ranges of residuals, where we observe different light curve shapes, and flares.

In the equation, $T_0(BJD)$ and $E$ denote ephemeris reference time and integer cycle number, respectively. We plot $O - CI$ values and linear fit in Figure 5 panel a. After obtaining average ephemeris and period, we subtract the linear fit from $O - CI$ data and obtain $O - CII$ data, which in principle shows real period variation for a given time range. Figure 5 panel b shows $O - CII$ data. We divide $O - CII$ data into 30 subsets by grouping data points that appear with a linear slope. Linear trend of a subset gives the difference between the best-fit photometric period of the subset and grand average photometric period given in Equation 1. Therefore we can calculate mean photometric period for each subset. We plot the calculated mean photo-

$$T_0(BJD) = 2,454,954.02(24) + 1^{d}24544(36) \times E. \quad (1)$$
metric periods versus time in Figure 6, panel c, together with their statistical uncertainties. We list photometric periods for 30 subsets in Table 5, and tabulate $O - C$ analysis results in Table 8.

Fig. 6. a) $O - CI$ diagram of observed minimum times (blue filled circles) and linear fit (red line). b) $O - CIH$ diagram obtained via residuals from the linear fit in panel a. Each color denotes a subset where data points appear on a linear trend. Linear fit to each subset is shown by black dashed line. c) Calculated mean photometric period for each subset (blue filled circles) and their statistical uncertainties. Note that the horizontal axis values are converted from E numbers to barycentric Julian date. Orbital period and grand average photometric period obtained from linear fit to the $O - CI$ data are shown with blue color in form of dashed line and dot–dashed line, respectively.

The average period given in Equation 1 represents average rotation period for magnetic activity features on the surface of the secondary component, which are typically cool and dark regions, i.e., star spots, and indicates a slightly (~0.5% day) shorter period compared to the orbital period. This is clearly observed in Figure 6 panel c, where the mean photometric periods of subsets are mostly shorter than the orbital period. Assuming solar type differential rotation, it means that the orbital period is slightly longer than the equatorial rotation period of the secondary component. Under the same assumption, differential rotation coefficient can be estimated from $(P_{max} -$
| Subset | BJD 24 00000+ | P (day) | σ(P) (day) |
|--------|---------------|--------|------------|
| 1      | 54994.8107    | 1.2456 | 0.0004     |
| 2      | 55048.8731    | 1.2326 | 0.0008     |
| 3      | 55094.1598    | 1.2441 | 0.0004     |
| 4      | 55139.0644    | 1.2260 | 0.0019     |
| 5      | 55169.9192    | 1.2459 | 0.0008     |
| 6      | 55208.0721    | 1.2489 | 0.0006     |
| 7      | 55250.0831    | 1.2584 | 0.0011     |
| 8      | 55314.8252    | 1.2484 | 0.0004     |
| 9      | 55366.4562    | 1.2355 | 0.0006     |
| 10     | 55425.0957    | 1.2470 | 0.0006     |
| 11     | 55478.0779    | 1.2517 | 0.0010     |
| 12     | 55507.4240    | 1.2437 | 0.0006     |
| 13     | 55539.3828    | 1.2216 | 0.0025     |
| 14     | 55629.1787    | 1.2430 | 0.0004     |
| 15     | 55702.5236    | 1.2447 | 0.0004     |
| 16     | 55740.2684    | 1.2522 | 0.0007     |
| 17     | 55793.0150    | 1.2485 | 0.0004     |
| 18     | 55840.9410    | 1.2223 | 0.0022     |
| 19     | 55868.2947    | 1.2534 | 0.0005     |
| 20     | 55894.6874    | 1.2712 | 0.0022     |
| 21     | 55924.7567    | 1.2494 | 0.0006     |
| 22     | 55960.4676    | 1.2391 | 0.0011     |
| 23     | 55996.8636    | 1.2507 | 0.0005     |
| 24     | 56026.2172    | 1.2474 | 0.0009     |
| 25     | 56073.0738    | 1.2528 | 0.0005     |
| 26     | 56136.3924    | 1.2449 | 0.0005     |
| 27     | 56258.6328    | 1.2509 | 0.0004     |
| 28     | 56333.3104    | 1.2323 | 0.0019     |
| 29     | 56359.5423    | 1.2565 | 0.0008     |
| 30     | 56400.8932    | 1.2504 | 0.0004     |
\[ \frac{P_{\text{min}}}{P_{\text{equ}}} = kf, \]

where \( P_{\text{max}} \), \( P_{\text{min}} \), \( k \) and \( f \) denote observed maximum and minimum period, differential rotation coefficient and a constant that depends on the range of spot forming latitudes, respectively [Hall & Busby 1990]. Considering small amplitude of rotational modulation of residuals, we assume that the secondary component is not largely spotted and total latitudinal range of spot distribution is 45 degrees, which puts the \( f \) constant takes values between 0.5 and 0.7 [Hall & Busby 1990]. Using maximum and minimum photometric periods from \( O-C \) analysis, and assuming the shortest period corresponds to the equatorial rotation period of the star, we find \( k = 0.081 \pm 0.011 \) and \( k = 0.058 \pm 0.006 \) for \( f = 0.5 \) and \( f = 0.7 \), respectively. Since these \( k \) values are calculated via boundary values of \( f \), the real differential rotation coefficient must lie in the range of \( k \) values calculated above. An average \( k \) is found as 0.069\( \pm \)0.008.

### 3.4.2. Flares

We detect 13 flares in the residuals from long cadence data. In flare analysis, it is critical to determine quiescent level, which denotes the brightness level in the absence of flare. In our case, we determine the quiescent level by applying Fourier analysis to single orbital cycle where the flare occurs. The Fourier analysis represents the rotational modulation of residuals in the cycle, and then we remove the Fourier representation from the data. The remaining residuals only shows quiescent level and flare itself. We show such a flare light curve in Figure [7].

The energy (\( E \)) is a very important parameter for a flare. However, the energy parameter has the luminosity \( L \) of the star as a factor in equation \( E = P \times L \) described by [Gershberg 1972]. Due to the disadvantages described in [Dal & Evren 2010], we use flare equivalent duration instead of flare energy, which is more proper. We compute the equivalent durations of flares via equation \( P = \int \left[ \frac{(I_{\text{flare}} - I_0)}{I_0} \right] dt \) [Gershberg 1972], where \( P \) is the flare equivalent duration in seconds, \( I_0 \) is the quiescent level intensity, and \( I_{\text{flare}} \) is the intensity observed at the moment of flare. Considering the quiescent level, the times of flare beginning, flare maximum and flare end are determined, together with flare rise duration, flare decay duration and flare amplitude. We list all computed values in Table [6] for each of 13 flare.

[Dal & Evren 2010, 2011] suggest that the best function to represent the relation between flare equivalent duration and flare total durations is the OPEA, where the flare equivalent duration is considered on a logarithmic scale. The OPEA function is defined as \( y = y_0 + (\text{Plateau} - y_0) \times (1 - e^{-kx}) \), where \( y \) is the flare equivalent duration on a logarithmic scale, \( x \) is the flare total duration, and \( y_0 \) is the flare equivalent duration in the logarithmic scale for the least total duration, according to the definition of [Dal & Evren 2010]. It should be noted that the \( y_0 \) does not depend on only flare mechanism, but also depends on the sensitivity of the optical system used in the mission. The most important parameter in the model is the \( \text{Plateau} \) value, which defines the upper limit for the flare equivalent duration on a logarithmic scale and
TABLE 6
THE PARAMETERS CALCULATED FOR EACH FLARE. NOTE THAT BJD COLUMN DENOTES THE MID–FLARE TIME. TR, TD AND AMP DENOTE FLARE RISE DURATION, FLARE DECAY DURATION AND FLARE AMPLITUDE, RESPECTIVELY.

| BJD     | P   | Tr  | Td  | Amp  |
|---------|-----|-----|-----|------|
| (24 00000+) | (s) | (s) | (s) | (mag) |
| 55021.2171 | 11.4 | 1763 | 15889 | -0.001516 |
| 55043.1016 | 5.6  | 1763 | 5296  | -0.002483 |
| 55310.6569 | 7.6  | 1763 | 8830  | -0.002047 |
| 55326.5140 | 2.7  | 1771 | 1763  | -0.001618 |
| 55412.0302 | 5.9  | 1763 | 7068  | -0.001648 |
| 55416.9343 | 12.1 | 1771 | 14118 | -0.002853 |
| 55824.2162 | 4.3  | 1763 | 5296  | -0.001578 |
| 55931.1213 | 4.5  | 3534 | 3534  | -0.001453 |
| 55971.7021 | 4.9  | 1763 | 5296  | -0.002152 |
| 56142.9809 | 6.0  | 3534 | 7059  | -0.001983 |
| 56284.8887 | 3.4  | 1771 | 3525  | -0.001806 |
| 56286.5642 | 4.4  | 1771 | 3525  | -0.001568 |
| 56375.4705 | 2.2  | 1763 | 1763  | -0.001429 |
Fig. 7. An example of a flare light curve. The filled black circles represent the observations, while the red line represents the quiescent level derived from the data out-of-flare.

defined as saturation level for a star (Dal & Evren 2011). Using the least squares method, the OPEA model leads to the results in Table 7. We plot the resulting model in Figure 8 with its 95% statistically sensitivity limit.

Fig. 8. The OPEA model obtained over 13 flares. The blue filled circles show each flare while the continuous red line shows the OPEA model and the dotted red lines show the sensitivity range of the model.

We tested the derived model by using method proposed by D'Agostino &
TABLE 7
PARAMETERS DERIVED FROM THE OPEA MODEL BY USING THE LEAST SQUARES METHOD.

| Parameter   | Value                          |
|-------------|--------------------------------|
| $Y_0$       | $-0.015961 \pm 0.13891$        |
| Plateau     | $1.2394 \pm 0.14441$          |
| $K$         | $0.00011438 \pm 0.000036715$  |
| Half-time   | 6060                           |
| $R^2$       | 0.94535                        |
| P value     | $\sim 0.10$                   |

[Stephens (1986)](10) to understand whether there are any other functions to model the distribution of flare equivalent durations on this plane. In this method, the probability value (P value), is found to be as $\sim 0.10$, which means that there is no other function to model the distributions ([Motulsky 2007](10) [Spanier & Oldham 1987](10)).

[Ishida et al. (1991)](10) described a frequency for the stellar flare activity as $N_1 = \frac{\Sigma n_f}{\Sigma T_k}$, where $\Sigma n_f$ is the total flare number detected in the observations, while $\Sigma T_k$ is the total observing duration from the beginning of the observing season to the end. In case of KIC 9451096 we find $N_1$ frequency as $0.000368411$ h$^{-1}$ adopting the total long cadence observing duration as 1470.2786 days from the times of the first and last long cadence data points.

4. SUMMARY AND DISCUSSION

Photometric and spectroscopic analysis of KIC 9451096 reveals that the system is composed of a F5V primary and a K2V secondary star on a circular orbit with a detached binary configuration. Medium resolution TFOSC spectra suggest that the system has one third of [Fe/H] of the Sun. Light curve modeling reasonably represents observations, however, we are able to catch the signals of additional light variation, which is very weak compared to the variations due to the binarity and eclipses, but still observable in the very high precision of Kepler photometry.

We observe occasional flares and rotational modulation of the light curve residuals from eclipsing binary model. Considering the physical and atmospheric properties of the components, we attribute these variations to the secondary component, which is a perfect candidate for magnetic star spot activity with its deep convective zone owing to its spectral type and very fast rotation caused by short orbital period. We inspect rotational modulations of the residuals to trace photometric period of the secondary component, and analyze its flare characteristics.
Photometric period analysis via $O-C$ diagrams shows us the average photometric period is shorter than the orbital period by $\sim 0.5\%$ day. Under any type of differential rotation (either solar like, or anti-solar like) assumption, it means that the orbital period does not correspond to the equatorial rotation period of the star. Following the method proposed by Hall & Busby (1990), we find an average differential rotation coefficient as $k = 0.069 \pm 0.008$, suggesting $\sim 3$ times weaker differential rotation compared to the solar value of 0.19. We note that the type of differential rotation can not be determined from photometry alone and we implicitly assume solar type differential rotation in case of KIC 9451096. However, $k = 0.069$ value, which is extracted from very high precision continuous photometry for a restricted time range (four years in our case), defines a lower limit for the strength of differential rotation on the star. Quick comparison of $k$ values for other stars can be done by looking at 17 stars listed in Hall & Busby (1990), where $k$ values are usually a few percent or less, except BY Dra with $k = 0.17$.

More reliable way of detecting differential rotation with its magnitude and type is Doppler imaging, which is based on high resolution time series spectroscopy. Considering other stars whose $k$ values were determined by Doppler imaging, we see mostly weak differential rotation with a $k$ value of a few percent, either among solar type differential rotators (HD 208472 $k = 0.015$ [Ozdarcan et al. 2016], XX Tri $k = 0.016$ [Künstler et al. 2015], ζ And $k = 0.055$ [Kovári et al. 2012], KU Peg $k = 0.04$ [Kovári et al. 2016]) or anti-solar type differential rotators (UZ Lib $k = -0.004$ [Vida et al. 2007], σ Gem $k = -0.04$ [Kovári et al. 2015], HU Vir $k = -0.029$ [Harutyunyan et al. 2016]). Due to the binary nature of KIC 9451096, considerable effect of tidal forces on redistribution of the angular momentum in the convective envelope of the components can be expected, which would alter the magnitude of differential rotation (Scharlemann 1982). Based on observational findings, Collier Cameron (2007) suggests suppression of differential rotation by tidal locking, which is possibly in progress for KIC 9451096.

We detect 13 flares in residuals from long cadence data, which are attributed to the secondary component with a corresponding $B-V$ value of $0^\mathrm{m}.92$ (Gray 2005). We apply OPEA model to analyze flare characteristic and find that the calculated flare parameters and resulting OPEA model parameters seem to be in agreement with parameters derived from stars analogous to the secondary component, except half-time value. Possible source of disagreement for half-time value is that there are not enough sample flares at the beginning of the OPEA model.

We find $N_1$ value of 0.000368411 $h^{-1}$ for KIC 9451096. $N_1$ was found to be 0.41632 $h^{-1}$ for KIC 09641031 (Yoldaş & Dal 2016), 0.01351 $h^{-1}$ for KIC 09761199 (Yoldaş & Dal 2017), and 0.02726 $h^{-1}$ for Group 1 and 0.01977 $h^{-1}$ for Group 2 of KIC 2557430 (Kamil & Dal 2017). Among these systems, KIC 9451096 has the lowest $N_1$ value, which indicates the magnetic activity level of the secondary component of KIC 9451096 is the lowest, according to Dal & Evren (2011).
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APPENDICES

A. \(O-C\) ANALYSIS RESULTS

We tabulate \(O-C\) analysis results in Table 8. \(N\) is the number of the minimum, beginning from the first observed minimum in the data set. \(E\) is the decimal cycle number and \(E \text{ rounded}\) is the rounded \(E\) number to the nearest integer or half integer. Note that as the time progress \(O-C\) differences approach to a cycle. When this is occurred, one needs to add an additional increment of 0.5 to the \(E \text{ rounded}\) value in order to see \(O-CI\) diagram on a trend without any discontinuity.
| N  | BJD   | K     | K  | O − C (day) | O − C (day) |
|----|-------|-------|---|------------|------------|
| 1  | 5495.5400 | 0.98  | 1.0 | 0.305035  | 0.377696   |
| 2  | 5495.5404 | 0.98  | 1.0 | 0.305038  | 0.377696   |
| 3  | 5495.5405 | 0.98  | 1.0 | 0.305039  | 0.377695   |
| 4  | 5495.5406 | 0.98  | 1.0 | 0.305040  | 0.377695   |
| 5  | 5495.5407 | 0.98  | 1.0 | 0.305041  | 0.377695   |
| 6  | 5495.5408 | 0.98  | 1.0 | 0.305042  | 0.377695   |
| 7  | 5495.5409 | 0.98  | 1.0 | 0.305043  | 0.377695   |
| 8  | 5495.5410 | 0.98  | 1.0 | 0.305044  | 0.377695   |

**TABLE 8**

*O − C ANALYSIS RESULTS.*
| N   | BJD   | E     | $O - CI$ (day) | $O - CI'$ (day) | N   | BJD   | E     | $O - CI$ (day) | $O - CI'$ (day) |
|-----|-------|-------|---------------|----------------|-----|-------|-------|---------------|----------------|
| 111 | 55146.8326 | 165.9 | -1.298693 | 0.035951 | 220 | 55304.6255 | 230.81 | 290.31 | 290.3 | -1.053145 | 0.315541 |
| 152 | 55146.8305 | 166.0 | -1.316199 | 0.024002 | 220 | 55107.0253 | 260.29 | 290.29 | 290.1 | -1.047761 | 0.327577 |
| 148 | 55146.8259 | 166.0 | -1.321659 | 0.031226 | 220 | 55109.2339 | 270.28 | 293.28 | 293.3 | -1.053560 | 0.326398 |
| 150 | 55146.8330 | 166.1 | -1.333439 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 157 | 55146.8230 | 166.0 | -1.338729 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 159 | 55146.8142 | 192.9 | -1.335297 | 0.064975 | 234 | 55304.8972 | 290.29 | 290.75 | 290.7 | -1.053560 | 0.315541 |
| 162 | 55146.8165 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 203 | 55146.8024 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 206 | 55146.8024 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 208 | 55146.8024 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 209 | 55146.8024 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 213 | 55146.8061 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 217 | 55146.8061 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 218 | 55146.8061 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 219 | 55146.8061 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 220 | 55146.8061 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 222 | 55146.8061 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 223 | 55146.8061 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 228 | 55146.8061 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 230 | 55146.8061 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 231 | 55146.8061 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 234 | 55146.8061 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 236 | 55146.8061 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 238 | 55146.8061 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 240 | 55146.8061 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |
| 242 | 55146.8061 | 192.9 | -1.347272 | 0.021311 | 220 | 55304.8985 | 290.05 | 293.05 | 293.5 | -1.048525 | 0.317976 |

*Continued.*
| N | BJD  | E   | O - CI (day) | O - CI (day) |
|---|------|----|-------------|-------------|
|   | (24 00000+) | |             |             |
| 1 | 55215.725 | 173.3 | 2.69045 | 2.01172 |
| 2 | 55231.100 | 173.6 | 2.69726 | 2.01910 |
| 3 | 55233.310 | 171.8 | 2.70220 | 2.02570 |
| 4 | 55234.360 | 172.8 | 2.67559 | 2.01049 |
| 5 | 55235.322 | 175.6 | 2.67455 | 2.01084 |
| 6 | 55236.400 | 176.7 | 2.67484 | 2.01127 |
| 7 | 55237.480 | 176.8 | 2.67455 | 2.01084 |
| 8 | 55238.560 | 176.9 | 2.67455 | 2.01084 |
| 9 | 55239.640 | 176.8 | 2.67455 | 2.01084 |
| 10 | 55240.720 | 176.5 | 2.67455 | 2.01084 |
| 11 | 55241.800 | 176.6 | 2.67455 | 2.01084 |
| 12 | 55242.880 | 176.8 | 2.67455 | 2.01084 |
| 13 | 55243.960 | 176.9 | 2.67455 | 2.01084 |
| 14 | 55245.040 | 176.8 | 2.67455 | 2.01084 |
| 15 | 55246.120 | 176.7 | 2.67455 | 2.01084 |
| 16 | 55247.200 | 176.6 | 2.67455 | 2.01084 |

TABLE 8 CONTINUED.
| N | BJD     | E    | K clipped (lag) | K clipped (lag) |
|---|---------|------|----------------|----------------|
| 161 | 55795.85 | 98.92 | -0.170446      | 0.036101       |
| 162 | 55795.89 | 98.82 | -0.156187      | 0.036101       |
| 163 | 55795.92 | 98.86 | -0.156187      | 0.036101       |
| 164 | 55795.99 | 99.92 | -0.170446      | 0.036101       |
| 165 | 55796.00 | 99.92 | -0.170446      | 0.036101       |
| 166 | 55796.02 | 99.92 | -0.170446      | 0.036101       |
| 167 | 55796.05 | 99.92 | -0.170446      | 0.036101       |
| 168 | 55796.10 | 99.92 | -0.170446      | 0.036101       |
| 169 | 55796.12 | 99.92 | -0.170446      | 0.036101       |
| 170 | 55796.14 | 99.92 | -0.170446      | 0.036101       |
| 171 | 55796.15 | 99.92 | -0.170446      | 0.036101       |

**TABLE 8.** (continued.)
| N   | BJD (24,000 +) | E   | E rounded (day) | O - C1 (day) | O - C1 (day) |
|-----|---------------|-----|----------------|-------------|-------------|
| 601 | 5498.00530   | 726.97 | 730.5 | -3.325095   | -3.319979   |
| 602 | 5498.54004   | 727.95 | 731.5 | -2.970265   | -2.956257   |
| 603 | 5498.93902   | 725.90 | 731.5 | -2.562240   | -2.549618   |
| 604 | 5499.24000   | 724.00 | 733.5 | -2.183255   | -2.170271   |
| 605 | 5499.52702   | 722.10 | 726.5 | -1.676467   | -1.666265   |
| 606 | 5499.77900   | 721.10 | 729.5 | -1.226502   | -1.216267   |
| 607 | 5499.96000   | 725.11 | 727.5 | -0.795837   | -0.785435   |
| 608 | 5500.12500   | 719.00 | 728.5 | -0.363838   | -0.353325   |
| 609 | 5500.26000   | 714.00 | 730.5 | 0.036136    | 0.038639    |
| 610 | 5500.36200   | 712.00 | 732.5 | 0.485837    | 0.495343    |
| 611 | 5500.43200   | 710.00 | 734.5 | 0.990838    | 0.995242    |
| 612 | 5500.56000   | 708.00 | 736.5 | 1.489837    | 1.485343    |
| 613 | 5500.66200   | 706.00 | 738.5 | 1.960838    | 1.975343    |
| 614 | 5500.73200   | 704.00 | 740.5 | 2.405837    | 2.415343    |
| 615 | 5500.86000   | 702.00 | 742.5 | 2.835838    | 2.845343    |
| 616 | 5500.94200   | 700.00 | 744.5 | 3.245837    | 3.255343    |
| 617 | 5501.03200   | 698.00 | 746.5 | 3.625838    | 3.635343    |
| 618 | 5501.10200   | 696.00 | 748.5 | 3.975837    | 3.985343    |
| 619 | 5501.17200   | 694.00 | 750.5 | 4.295838    | 4.295343    |
| 620 | 5501.25200   | 692.00 | 752.5 | 4.585837    | 4.575343    |
| 621 | 5501.32200   | 690.00 | 754.5 | 4.845838    | 4.855343    |
| 622 | 5501.39200   | 688.00 | 756.5 | 5.075837    | 5.085343    |
| 623 | 5501.46200   | 686.00 | 758.5 | 5.285838    | 5.295343    |
| 624 | 5501.52200   | 684.00 | 760.5 | 5.475837    | 5.465343    |
| 625 | 5501.58200   | 682.00 | 762.5 | 5.645838    | 5.635343    |
| 626 | 5501.64200   | 680.00 | 764.5 | 5.805837    | 5.795343    |
| 627 | 5501.70200   | 678.00 | 766.5 | 5.955838    | 5.945343    |
| 628 | 5501.76200   | 676.00 | 768.5 | 6.085837    | 6.075343    |
| 629 | 5501.82200   | 674.00 | 770.5 | 6.205838    | 6.195343    |
| N   | BID (24:00000+) | E       | E (calculated) | O - C (deg) | O - C (deg) |
|-----|-----------------|---------|----------------|-------------|-------------|
| T3  | 54119.5429      | 931.86  | 935.55         | -2.557145   | 0.357320    |
| T4  | 54120.9086      | 940.81  | 944.55         | -3.482061   | 0.322214    |
| T5  | 54120.7368      | 940.81  | 944.55         | -2.532111   | 0.197882    |
| T6  | 54120.1513      | 932.82  | 936.55         | -3.289088   | 0.185761    |
| T7  | 54120.5713      | 932.82  | 936.55         | -2.500188   | 0.161651    |
| T8  | 54120.7450      | 932.82  | 936.55         | -2.532111   | 0.197882    |
| T9  | 54120.7426      | 932.82  | 936.55         | -2.500188   | 0.161651    |
| T10 | 54120.7404      | 932.82  | 936.55         | -2.532111   | 0.197882    |
| T11 | 54120.7382      | 932.82  | 936.55         | -2.500188   | 0.161651    |
| T12 | 54120.7360      | 932.82  | 936.55         | -2.532111   | 0.197882    |
| T13 | 54120.7338      | 932.82  | 936.55         | -2.500188   | 0.161651    |
| T14 | 54120.7316      | 932.82  | 936.55         | -2.532111   | 0.197882    |
| T15 | 54120.7294      | 932.82  | 936.55         | -2.500188   | 0.161651    |
| T16 | 54120.7272      | 932.82  | 936.55         | -2.532111   | 0.197882    |
| T17 | 54120.7250      | 932.82  | 936.55         | -2.500188   | 0.161651    |
| T18 | 54120.7228      | 932.82  | 936.55         | -2.532111   | 0.197882    |
| T19 | 54120.7206      | 932.82  | 936.55         | -2.500188   | 0.161651    |
| T20 | 54120.7184      | 932.82  | 936.55         | -2.532111   | 0.197882    |
| T21 | 54120.7162      | 932.82  | 936.55         | -2.500188   | 0.161651    |
| T22 | 54120.7140      | 932.82  | 936.55         | -2.532111   | 0.197882    |
| T23 | 54120.7118      | 932.82  | 936.55         | -2.500188   | 0.161651    |
| T24 | 54120.7096      | 932.82  | 936.55         | -2.532111   | 0.197882    |
| T25 | 54120.7074      | 932.82  | 936.55         | -2.500188   | 0.161651    |
| T26 | 54120.7052      | 932.82  | 936.55         | -2.532111   | 0.197882    |
| T27 | 54120.7030      | 932.82  | 936.55         | -2.500188   | 0.161651    |
| T28 | 54120.7008      | 932.82  | 936.55         | -2.532111   | 0.197882    |
| T29 | 54120.6986      | 932.82  | 936.55         | -2.500188   | 0.161651    |
| T30 | 54120.6964      | 932.82  | 936.55         | -2.532111   | 0.197882    |
| T31 | 54120.6942      | 932.82  | 936.55         | -2.500188   | 0.161651    |
| T32 | 54120.6920      | 932.82  | 936.55         | -2.532111   | 0.197882    |
| T33 | 54120.6898      | 932.82  | 936.55         | -2.500188   | 0.161651    |
| T34 | 54120.6876      | 932.82  | 936.55         | -2.532111   | 0.197882    |
| T35 | 54120.6854      | 932.82  | 936.55         | -2.500188   | 0.161651    |
| T36 | 54120.6832      | 932.82  | 936.55         | -2.532111   | 0.197882    |
TABLE 8
CONTINUED.

| N  | BJD (24 00000+) | $E$  | $E$ rounded | $O - CI$ (day) | $O - CI$ (day) |
|----|-----------------|------|-------------|----------------|----------------|
| 901| 56408.3847      | 1162.86 | 1167.5     | -5.757700     | 0.365945       |
| 902| 56409.6487      | 1163.87 | 1168.5     | -5.744102     | 0.384496       |
| 903| 56410.8892      | 1164.86 | 1169.5     | -5.753967     | 0.379583       |
| 904| 56412.1522      | 1165.87 | 1170.5     | -5.741409     | 0.397095       |
| 905| 56413.4054      | 1166.87 | 1171.5     | -5.738612     | 0.404844       |
| 906| 56420.9048      | 1172.87 | 1177.5     | -5.741557     | 0.431616       |
| 907| 56422.1512      | 1173.87 | 1178.5     | -5.745583     | 0.432544       |
| 908| 56423.4023      | 1174.87 | 1179.5     | -5.744909     | 0.438170       |