From Single to Multiple-Photon Decoherence in an Atom Interferometer

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We measure the decoherence of a spatially separated atomic superposition due to spontaneous photon scattering. We observe a qualitative change in decoherence versus separation as the number of scattered photons increases, and verify quantitatively the decoherence rate constant in the many-photon limit. Our results illustrate an evolution of decoherence consistent with general models developed for a broad class of decoherence phenomenon.

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Decoherence is the result of entanglement between a quantum system and an unobserved environment, and manifests as the reduction of coherent superpositions into incoherent mixtures. This reduction occurs more quickly as the number of particles comprising a quantum system increases, establishing decoherence as a fundamental limit to large-scale quantum computation. In the high-temperature or many scatterer limit, an isotropic distribution of scatterers have been studied. Progress in these fields therefore relies upon understanding and correcting for decoherence effects. On a macroscopic scale, decoherence is unavoidable and explains the emergence of classical behavior in a world governed by quantum mechanical laws.

Theoretical treatments of decoherence provide a description for the evolution of a system’s density matrix under the influence of a specific environment. For spatial decoherence, various environments including a thermal bath of harmonic oscillators, a scalar field, and an isotropic distribution of scatterers have been studied. In the high-temperature or many scatterer limit, these models all yield a diffusion-like master equation for the system’s spatial density matrix, $\rho(x, x')$:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar} [H, \rho] - D^2 |x - x'|^2 \rho,$$

where $H$ is the Hamiltonian for the isolated system and $D$, the diffusion constant, depends on the details of the system-environment coupling. Assuming negligible internal dynamics, this equation predicts an exponential reduction in coherence with time and with separation squared:

$$\rho(x, x', t) = e^{-D^2 |x - x'|^2 t} \rho(x, x', 0).$$

Similar decoherence behavior arises and has been studied in the context of an atom interacting with a high-Q cavity, and trapped ions interacting with a fluctuating electric field.

To investigate the distinct case of decoherence due to scattering processes, we have studied the loss of spatial coherence of atoms within an atom interferometer due to spontaneous scattering of photons. In the many photon limit, this represents a simple case of the general models above; we observe coherence loss consistent with Eq. (2) and are able to derive the decay constant from first principles. The few photon limit is of a qualitatively different character, and we have followed the smooth transition between these two regimes.

The atom interferometer is realized by passing a collimated, supersonic beam of Na atoms (velocity $\approx 3000$ m/s using a He carrier gas) through three diffraction gratings arranged in the Mach-Zehnder geometry (Fig. 1). Prior to the first grating, the atoms are collimated and optically pumped into the $^3S_1/2 (F = 2, m_F = +2)$ ground state. Two paths through the interferometer, separated by up to 20 µm, overlap at the position of the third grating, forming a spatial interference pattern. This pattern is masked by the third grating and the total transmitted flux is detected using a 50 µm hot wire. The interference pattern is measured as an oscillating atomic flux versus grating position. Because the contrast of the interference pattern is proportional to the coherence between the two paths, reduction in contrast is direct evidence of coherence loss.

FIG. 1. A schematic of our apparatus: a Mach-Zehnder interferometer comprised of three, evenly spaced, transmission gratings. Within the interferometer, sodium atoms continuously absorb and spontaneously emit photons from a variable intensity laser beam. Decoherence due to spontaneous emission results in reduced contrast interference fringes.

The effective dechering environment consists of photons from a laser beam directed along the $\hat{x}$ axis which intersects both interfering paths. The circularly polarized laser light is tuned to the $^3S_1/2 (2, +2) \rightarrow ^3P_2 (3, +3)$ transition with wavelength $\lambda = 2\pi / k_0 = 590$ nm. Because the atoms are dipole forbidden from decaying to
any state other than $3S_1(2, +2)$, they can continuously scatter photons without falling out of resonance (the natural linewidth is $\sim 200$ photon recoils wide).

At the intersection of the atomic beam and scattering laser, each atom’s transverse wavefunction is peaked at two positions which we label $x$ and $x + d$. If a photon, initially in momentum state $|k_0\rangle$, scatters from this atom, the two become entangled:

$$\psi_i = (|x\rangle + |x + d\rangle) |k_0\rangle \xrightarrow{\text{scat}} |x\rangle \otimes |\phi_x\rangle + |x + d\rangle \otimes e^{ik_0d}|\phi_{x+d}\rangle,$$

where $|\phi_x\rangle$ is the wavefunction of a photon spontaneously emitted from position $x$ and the factor $e^{ik_0d}$ accounts for the difference in spatial phase of the initial photon at the two positions. Generalizing the entangled wavefunction in Eq. (2) to a density matrix and tracing over a basis of scattered photon states, the net effect of scattering on the atom’s spatial density matrix is:

$$\rho(x, x + d) \xrightarrow{\text{scat}} \rho(x, x + d) \beta(d),$$

where $\beta(d)$ is known as the decoherence function and has the properties $|\beta(d)| \leq 1$ and $\beta(0) = 1$. The decoherence function thus defined is equal to the inner product of the two final photon states, which are identical apart from an overall translation:

$$\beta(d) = e^{ik_0d} \langle \phi_x \mid \phi_{x+d} \rangle = e^{ik_0d} \langle \phi_x \mid e^{-i\hat{k}_z d} \phi_x \rangle = \int d\Delta k P(\Delta k) e^{-i\Delta kd},$$

where the operator $\hat{k}_z$ is the generator of photon translations along the $z$ axis. The resulting decoherence function is the Fourier transform of a probability distribution $P(\Delta k)$, with $\Delta k = k_z - k_0$ being the change in momentum of the photon along the $z$ axis.

Previous experiments [11,12] have measured the decoherence function for an atom which spontaneously scatters a single photon. The theoretical prediction which these experiments confirm is displayed as the solid line in Fig. 2. Beneath an overall decay in coherence with distance, periodic coherence revivals are observed. This shape follows directly from the Fourier transform of the dipole radiation pattern for spontaneous emission. It has also been explained in terms of the ability of a single photon to provide which-path information [12]; the contrast drops to zero when the path separation is approximately equal to the resolving power of an ideal Heisenberg microscope $d \approx \lambda/2$, with revivals resulting from path ambiguity due to diffraction structure in the image.

If several photons are scattered, and if successive scattering events are independent, the total decoherence function includes one factor of $\beta$ for each scattered photon:

$$\beta_{\text{total}}(d) = \sum_{n=0}^{\infty} P(n) \beta^n(d).$$

In our experiment, the total number of photons scattered by an individual atom is intrinsically uncertain, but is described by the distribution $P(n)$ which can be measured or calculated. The sum in Eq. (3) is a trace over this additional degree of freedom of the environment.

**FIG. 2.** The total decoherence function, $|\beta_{\text{total}}|$, measured as the normalized contrast after spontaneous photon scattering. The solid line is the single photon decoherence function. Also displayed are the best fits from which we determine $\bar{n} = 0.9 \ (\Delta), 1.4 \ (\circ), 1.8 \ (\diamond), 2.6 \ (\triangledown)$, and $8.2 \ (+)$.

Figure 2 shows measurements of the decoherence function for laser intensities corresponding to an average number of scattered photons, $\bar{n}$, ranging from $\sim 1$ to $\sim 8$. At each intensity, a reference contrast and phase was measured, with the scattering laser positioned such that the interfering paths were completely overlapped ($d = 0$). We then adjusted the longitudinal position of the scattering laser, $z$, to select specific path separations in the range $0 < d < 1.4\lambda$ at which to measure the decoherence function (see Fig. 4). For each path separation, the ratio of the measured atom interference contrast to the reference contrast yields the magnitude of the decoherence function, $|\beta_{\text{total}}(d)|$. The difference between the measured atom interference phase and the reference phase yields the phase of the decoherence function.

We fit the data using Eq. (2) and taking $P(n) \sim \exp(-n/2)/\sigma_n^2$. This form was chosen as a good approximation to Monte-Carlo Wavefunction calculations of $P(n)$ for our laser parameters. From the best fit curves displayed in Fig. 4, values were extracted for $\bar{n}$ and $\sigma_n$ which were consistent with, and more accurate than, independent measurements of $P(n)$ based on the deflection and broadening of the atomic beam with the scattering laser blocked versus unblocked.

In the regime $d \gg \lambda$, a single scattered photon suffices to completely destroy the coherence between paths. Thus, the non-zero asymptotic value (for $n = 0.9$ in Fig. 2) of the decoherence function at large path separation is equal to the fraction of atoms which scatter zero photons (i.e. decoherence is proportional to the atom-photon interaction cross-section). This phenomenon is a simple example of saturation of decoherence (4,4): the loss
of coherence becomes independent of path separation at a characteristic length scale of the environment. A recent experiment by Cheng and Raymer [14], involving loss of optical coherence due to a disordered collection of polystyrene microspheres, has features similar to our own: contrast loss was observed to saturate when the path separation reached roughly the diameter of the microspheres, and the asymptotic contrast was proportional to the microsphere-light scattering cross section.

As the average number of scattered photons increases, the overall amount of decoherence increases, and the contrast revivals disappear. This behavior can be formalized as the Fourier transform of the total momentum distribution of all scattered photons:

$$\beta^n(d) = \int d\Delta K \, P(\Delta K) \, e^{i\Delta K d},$$  \hspace{1cm} (7)

where $\Delta K = \sum_{i=1}^{n} \Delta K_i$. As $n \to \infty$, the central limit theorem predicts that $P(\Delta K)$ will tend towards a Gaussian with mean $nk_0$ and variance $n\sigma_e^2$ (where $\sigma_e = \frac{\Delta k}{2\kappa}$ is the rms transverse momentum of an emitted photon). In the case of spontaneous emission, $P(\Delta K)$ is approximately Gaussian for $n > 3$ and the decoherence function reduces to:

$$\beta^n(d) = \int d\Delta K \left[ e^{-\frac{1}{2}(\Delta K - nk_0)^2/n\sigma_e^2} \right] e^{i\Delta K d} \hspace{1cm} (8)$$

Inserting this expression into Eq. (2) and taking $d/\lambda \ll 1$, we find:

$$\lim_{\bar{n} \to \infty} \beta_{total}(d) = e^{-\frac{1}{2}\kappa^2 d^2} e^{-i\bar{n}k_0d}, \hspace{1cm} (9)$$

where

$$\kappa^2 = \bar{n}\sigma_e^2 + \sigma_n^2 k_0^2 \hspace{1cm} (10)$$

is the variance of the total momentum transferred to the atom from the scattered photons. The first term in Eq. (10) comes from the trace over modes available to the spontaneously emitted photon, while the second is related to the uncertainty in number of absorbed photons combined with the fixed phase $k_0d$ imparted by each.

If $\sigma_n = \sqrt{\bar{n}}$ (i.e. Poissonian statistics), Eq. (9) predicts an exponential decay in contrast with number of scattered photons ($\kappa^2 \propto \bar{n}$). If in addition the scattering rate, $\Gamma$, is constant, then $\bar{n} = \Gamma t$ and the decoherence has exactly the exponential form derived from a master equation like Eq. (1).

We have measured this exponential reduction of spatial coherence by varying the average number of scattered photons, leaving the path separation fixed (Fig. 3). Theory curves (solid lines) are based on Eq. (9) with $\sigma_n$ determined from the broadening of the atomic beam due to the momentum of the scattered photons. The product of the two remaining free parameters, $\bar{n}d$, was obtained from the measured phase of the decoherence function.

The data follow a nearly exponential decay with $\bar{n}$. The upward trend at large $\bar{n}$ is a result of the finite size of our hot-wire: the trace over final photon states (Eq. 8) must be restricted to those states which allow the atom to reach the detector. As a result $\kappa$ in Eq. (9) is replaced with $\kappa'$ where $1/\kappa'^2 = 1/\kappa^2 + 1/\kappa_d^2$ and $\kappa_d = 3.3(1)k_0$ is our detector’s effective momentum acceptance.

In the previous single-photon experiment of Chapman et al. [13] lost coherence was similarly “recovered” by positioning a hot-wire detector to count only atoms which had scattered photons into a small range of momentum states. This scheme required [13] that the atomic beam width, $\sigma_x$, be greater than the path separation, $d$, so that the two interfering paths partially overlapped at the point of scattering, and a scattered photon could not have provided complete which-path information, even if $d \gg \lambda$. The condition $\sigma_x < d$ need not be satisfied to demonstrate the features of decoherence in the current experiment, however. Even when it is in principle possible to recover some coherence by measuring the environment, if no such attempt is made then the predicted loss of contrast is independent of $\sigma_x$.

In the many photon limit, the decoherence function we have derived agrees with the solution to the master equation presented in the introduction. Comparing Eqs. (2) and (9), taking into account the time varying intensity profile, $I(t)$, of the scattering light as experienced by atoms in the beam, we identify: $\kappa^2 = D^2\tau$ where $\tau$ is the amount of time needed to scatter $\bar{n}$ photons ($\bar{n} = \int_0^\tau \Gamma(I(t)) \, dt$). Because the atom-photon scattering interaction is well defined, and our decohering environment well controlled, we can accurately calculate the constant $\kappa$ (equivalently $D$) for any laser parameters.

Displayed in Figure 3 are data which demonstrate Gaussian reduction in contrast as a function of path separation for two different laser intensities. As before, we independently determined $\bar{n}$ and $\sigma_n$ for each.
components shifted by an amount $\Delta$ superposition state with the phase between the two components. In this expression for the entangled atom-photon wavefunction, a photon state $\ket{\psi}$ yields $\kappa' = 2.39(5)k_0$ and $\kappa' = 1.71(5)k_0$, within error of the calculated values.

Our system exhibits what have been referred to as the “naive” generalizations of decoherence phenomenon: exponential loss of contrast with path separation squared and with number of scattered particles. The similarity of Eq. (1) to a diffusion equation [16], invites identification of this type of decoherence with phase diffusion or a random phase walk. To make the identification explicit, we use the identity $|x+d\rangle = e^{-ikd} |x\rangle$ to rewrite Eq. (8) as:

$$\psi_{scat.} = \left( |x\rangle + |x+d\rangle \otimes e^{ikd} e^{-ikd} |\phi_x\rangle \right) \otimes |\overline{k}\rangle |\overline{k}\rangle |\phi_x\rangle$$

In this expression for the entangled atom-photon wavefunction, a photon state $|\overline{k}\rangle$ corresponds to an atomic superposition state with the phase between the two components shifted by an amount $\Delta\phi = (k_x - k_0)d$. Correlating interference data with measurements of each scattered photon momentum (effectively a randomly sampled element of the distribution $P(k)$) would allow complete recovery of lost contrast. In the absence of such post-processing, however, the phase of each atom’s interference fringes will vary randomly, and their sum, the measured interference pattern, will have reduced contrast. The phase diffusion and (previously discussed) which-path pictures are equally valid when the experimenter does not measure the scattered photons [17].

In conclusion, we have studied the decoherence of a spatial superposition due to photon scattering. Our data confirm theoretical predictions, and in the many-photon limit exhibit features of decoherence which are quite general. We have observed the exponential coherence loss with time and path separation squared characteristic of this general behavior, and we have for the first time predicted and experimentally verified the decoherence rate constant $\kappa$. The particular model we have explored is not only the most relevant for macroscopic systems but also applies generally to situations in which decoherence arises slowly though a series of independent, mildly decohering interactions, a situation of interest for decoherence avoidance or correction protocols.

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