Tackling Multiple Ordinal Regression Problems: Sparse and Deep Multi-Task Learning Approaches

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Abstract

Many real-world datasets are labeled with natural orders, i.e., ordinal labels. Ordinal regression is a method to predict ordinal labels that finds a wide range of applications in data-rich science domains, such as medical, social and economic sciences. Most existing approaches work well for a single ordinal regression task. However, they ignore the task relatedness when there are multiple related tasks. Multi-task learning (MTL) provides a framework to encode task relatedness, to bridge data from all tasks, and to simultaneously learn multiple related tasks to improve the generalization performance. Even though MTL methods have been extensively studied, there is barely existing work investigating MTL for data with ordinal labels. We tackle multiple ordinal regression problems via sparse and deep multi-task approaches, i.e., two regularized multi-task ordinal regression (RMTOR) models for small datasets and two deep neural networks based multi-task ordinal regression (DMTOR) models for large-scale datasets. The performance of the proposed multi-task ordinal regression models (MTOR) is demonstrated on three real-world medical datasets for multi-stage disease diagnosis. Our experimental results indicate that our proposed MTOR models markedly improve the prediction performance comparing with single-task learning (STL) ordinal regression models.

Keywords

Multiple ordinal regression problems, Multi-task learning, Deep learning, Multi-stage disease diagnosis.

I. INTRODUCTION

Ordinal regression is capable of exploiting ordinal labels to implement multi-ordered classification problems, which has been widely applied to diverse application domains [1], [2], e.g., medical diagnosis [3]–[6], social science [7]–[10], education [11], [12], computer vision [13] and marketing [14]–[16]. Specifically in medical diagnosis, many major diseases are multi-stage progressive diseases, for example, Alzheimer’s Disease (AD) progresses into three stages that are irreversible with orders, i.e., cognitively normal, mild cognitive impairment and AD [3]. Conventional methods either convert ordinal regression problems into multiple binary classification problems [17]–[19] (e.g., health and illness) or consider them as multi-class classification problems [20], [21]. However, these methods fail to capture the key information of ordinal labels (e.g., the progression of multi-stage diseases). Therefore, ordinal regression is vital as it incorporates the ordinal labels with multi-class classification [22]–[24].

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In the real-world scenario, there is an increasing need to build multiple related ordinal regression tasks. For instance, multi-stage disease diagnosis in multiple sub-groups of patients (e.g., various age groups, genders, races), student satisfaction questionnaire analysis in multiple subpopulations of students (e.g., various schools, majors), customer survey analysis in multiple communities (e.g., various incomes, living neighborhoods). However, in the field of ordinal regression, most of the prior works merely concentrate on learning a single ordinal regression task, i.e., either build a global ordinal regression model for all sub-groups, which ignores data heterogeneity among different sub-groups [25]–[28]; or build and learn an ordinal regression model for each sub-group independently, which ignores relatedness among these sub-groups [22]–[24].

To overcome the aforementioned limitations, multi-task learning (MTL) is introduced to learn multiple related tasks simultaneously [29]. MTL is a framework that has been extensively researched for classification, standard regression and clustering in the fields of data mining and machine learning. By building multiple models for multiple tasks and learning them collectively, the training of each task is augmented via the auxiliary information from other related sub-groups, which leads to an improved generalization performance of the prediction. Especially in the selected domain of biomedical informatics, where MTL has achieved significant successes recently, such as prediction of patients’ survival time for multiple cancer types [30] and HIV therapy screening [31]. However, MTL for data with ordinal labels, such as multi-stage disease diagnosis, remains a largely unexplored and neglected domain. Multi-stage progressive diseases are rarely cured completely and the progression is often irreversible, e.g., AD, hypertension, obesity, dementia and multiple sclerosis [3], [5], [6]. Hence, new approaches incorporating ordinal regression and MTL are urgently needed.

To train multiple correlated ordinal regression models jointly, [32] connects these models using Gaussian process (GP) prior within the hierarchical Bayesian framework. However, multi-task models within hierarchical Bayesian framework are not sparse or performed well in high dimensional data. In [33], forecasting the spatial event scale is targeted using the incomplete labeled datasets, which means not every task has a complete set of labels in the training dataset. The objective function in [33] is regularized logistic regression derived from logistic ordinal regression; therefore, their approach also suffer from the limitations of logistic regression, e.g., more sensitive to outliers comparing with our proposed methods based on maximum-margin classification [35], [36]. To overcome these limitations, we propose two regularized multi-task ordinal regression (RMTOR) models within the MTL framework. Moreover, we propose two concrete MTOR models based on deep neural networks (DNN) denoted as DMTOR. The proposed RMTOR under MTL framework belongs to the regularized MTL approach [37], where the assumption of task relatedness is encoded via regularization terms that have been widely studied in the past decade [38], [39]. In the proposed DMTOR, the task relatedness is encoded by shared representation layers. Note that in [18], the authors formulate a single ordinal regression problem as a multi-task binary classification problem. However, in our work we solve multiple ordinal regression problems simultaneously within the MTL framework.

In this paper, to solve the proposed models, we employ the alternating structure optimization to achieve an efficient learning process. In the experiment part of this paper, we demonstrate the prediction performance of proposed MTL ordinal regression models using three real-world datasets corresponding to three multi-stage progressive diseases, i.e., AD, obesity and hypertension from multiple age groups. The main contributions of this paper can be summarized as follows:
• We propose two regularized MTOR models (i.e., RMTOR using two different types of thresholds) for small datasets to encode the task relatedness of multiple ordinal regression tasks using structural regularization term;
• We propose two DNN based MTOR models (i.e., DMTOR using two different types of thresholds) for large-scale datasets to encode the task relatedness through shared hidden layers;
• We propose an alternating structure optimization framework to train RMTOR models, and within this framework the fast iterative shrinkage thresholding algorithm (FISTA) is employed to update the weights of RMTOR;
• Our comprehensive experimental studies demonstrate the advantage of MTOR models over single-task ordinal regression models.

The rest of this paper is organized as follows: Section II summarizes some relevant previous works regarding ordinal regression and MTL. In Section III, we review the preliminary knowledge on the ordinal regression. Section IV elaborates the details of RMTOR models using two types of thresholds. In Section V, we extend the MTOR models with deep learning using DNN. Section VI demonstrates the effectiveness of the MTL ordinal regression models using three real-world medical datasets for the multi-stage disease diagnosis. In Section VII we conclude our work with discussion and future work.

II. RELATED WORKS

In this section, we summarize the related works in the fields of ordinal regression and multi-task learning (MTL), and discuss the relationships and primary distinctions of the proposed methods compared to the existing methods that are available in the literatures.

A. Ordinal regression

Ordinal regression is an approach aiming at classifying the data with natural ordered labels and plays an important role in many data-rich science domains. According to the commonly used taxonomy of ordinal regression [40], the existing methods are categorized into: naive approaches, ordinal binary decomposition approaches and threshold models.

The naive approaches are the earliest approaches dealing with ordinal regression, which convert the ordinal labels into numeric and then implement standard regression or support vector regression [18], [41]. Since the distance between classes is unknown in this type of methods, the real values used for the labels may undermine regression performance. Moreover, these regression learners are sensitive to the label representation instead of their orders [40].

Ordinal binary decomposition approaches are proposed to decompose the ordinal labels into several binary ones that are then estimated by multiple models [17], [42]. For example, [17] transforms the data from \( U \)-classes ordinal problems to \( U - 1 \) ordered binary classification problems and then they are trained in conjunction with a decision tree learner to encode the ordering of the original ranks, i.e., train \( U - 1 \) binary classifiers using C4.5 algorithm.

Threshold models are proposed based on the idea of approximating the real value predictor followed with partitioning the real line of ordinal values into segments. During the last decade, the two most popular threshold models are support vector machines (SVM) models [25], [26], [28], [43] and generalized linear models for ordinal regression [44]–[47]; the former is to find the hyperplane that separates the segments by maximizing margin using the \textit{hinge} loss and the latter is to predict the ordinal labels by maximizing the likelihood given the training data.

In [43], support vector ordinal regression (SVOR) is achieved by finding multiple thresholds that partition the real line of ordinal values into several consecutive intervals for representing
ordered segments; however, it does not consider the ordinal inequalities on the thresholds. In [25], [26], the authors take into account of ordinal inequalities on the thresholds and propose two approaches using two types of thresholds for SVOR by introducing explicit constraints. To deal with incremental SVOR learning caused by the complicated formulations of SVOR, [28] proposes a modified SVOR formulation based on a sum-of-margins strategy to solve the computational scalability issue of SVOR.

Generalized linear models perform ordinal regression by fitting a coefficient vector and a set of thresholds, e.g., ordered logit [44], [45] and ordered probit [46], [47]. The margin functions are defined based on the cumulative probability of training instances’ ordinal labels. Different link functions are then chosen for different models, i.e., logistic cumulative distribution function (CDF) for ordered logit and standard normal CDF for ordered probit. Finally, maximum likelihood principal is used for training.

In this paper, we propose two novel ordinal regression threshold models under the MTL framework. Particularly, we implement two different types of thresholds in the loss functions under different assumptions and use alternating structure optimization for training RMTOR, which are different from existing threshold models using hinge loss or likelihood. Please refer to Section IV for details.

B. Multi-task learning

To leverage the relatedness among the tasks and improve the generalization performance of all machine learning models, MTL is introduced as an inductive transfer learning framework by simultaneously learning all the related tasks and transferring knowledge among the tasks. How task relatedness is assumed and encoded into the learning formulations is the central building block of MTL. In [37], the earliest MTL approach is to couple the learning process by using multi-task regularizations. Regularized MTL is able to leverage large-scale optimization algorithms such as proximal gradient techniques, so that the regularized MTL approach has a clear advantage over the other MTL approaches [39], [48]–[50]. As a result, the regularized MTL can efficiently handle complicated constraints and/or non-smooth terms in the objective function.

Recently, MTL has been combined with many deep learning approaches [51]. MTL can be implemented in the DNN based approaches in two ways, i.e., soft and hard parameter sharing of hidden layers. In the soft parameter sharing, all tasks do not share representation layers and the distance among their own representation layers are constrained to encourage the parameters to be similar [51], e.g., [52] and [53] use $l_2$-norm and the trace norm, respectively.

Hard parameter sharing is the most commonly used approach in DNN based MTL [51]. In the hard parameter sharing, all tasks share the representation layers to reduce the risk of overfitting [54] and keep some task-specific layers to preserve characteristics of each task [55]. In this paper, we use the hard parameters sharing for DMTOR.

In the all aforementioned methods and other related works, the learning tasks are either classification or standard regression. Here, in this paper, the learning tasks are multiple ordinal regression problems. We propose a set of novel MTOR models in Section IV and Section V to solve multiple multi-ordered classification problems simultaneously. Moreover, in the Section VI, the multi-stage disease diagnosis are handled for experiments using the proposed MTOR models, i.e., RMTOR and DMTOR models.
III. PRELIMINARY: LATENT VARIABLE MODEL IN ORDINAL REGRESSION

Given \( N \) training instances shown as \((X_i, Y_i)_{i \in \{1, \ldots, N\}}\), the latent variable model is used to predict the ordinal label \( Y^* \):

\[
Y^* = XW + b, \tag{1}
\]

\[
\hat{Y}_i = u \quad \text{if} \quad \vartheta_{\mu-1} < Y_i^* \leq \vartheta_\mu,
\]

where \( Y^* \) is the latent variable and \( \hat{Y}_i \) is the ordered predicted label (i.e., \( \hat{Y}_i = \mu \in \{1, \ldots, U\} \)) for the \( i^{th} \) training instance. \( \vartheta \) is a set of thresholds, where \( \vartheta_0 = -\infty \) and \( \vartheta_U = \infty \), so that we have \( U - 1 \) thresholds (i.e., \( \vartheta_1 < \vartheta_2 < \ldots < \vartheta_{U-1} \)) partitioning \( Y^* \) into \( U \) segments to obtain \( \hat{Y} \), which can be expressed as:

\[
\hat{Y} = \begin{cases} 
1 & \text{if} \quad \vartheta_0 < Y^* \leq \vartheta_1, \\
\vdots & \vdots \\
\mu & \text{if} \quad \vartheta_{\mu-1} < Y^* \leq \vartheta_\mu, \\
\vdots & \vdots \\
U & \text{if} \quad \vartheta_{U-1} < Y^* \leq \vartheta_U.
\end{cases} \tag{2}
\]

As we see in Eq. (1) and Eq. (2), \( U \) ordered predicted labels, i.e., \( \hat{Y} \), are corresponding to \( U \) ordered segments and each \( Y^* \) has the value within the range: \((\vartheta_{\mu-1}, \vartheta_\mu)\), the latter is immediate thresholds, for \( \mu \in \{1, \ldots, U\} \).

IV. REGULARIZED MULTI-TASK ORDINAL REGRESSION (RMTOR) MODELS

In this section, we formulate regularized multi-task ordinal regression (RMTOR) using two different types of thresholds: 1) Immediate thresholds: the thresholds between adjacent ordered segments including the first threshold \( \vartheta_0 \) and last threshold \( \vartheta_U \). In the real-world problems, \( \vartheta_0 = -\infty \) and \( \vartheta_U = \infty \), so that we can use the first and last thresholds to calculate the errors for training instances in the corresponding segments. 2) All thresholds: the thresholds between adjacent and nonadjacent ordered segments followed the traditional definition of the first and last thresholds, i.e., \( \vartheta_0 = -\infty \) and \( \vartheta_U = \infty \). Thus, the first and last thresholds can not be used for calculating the errors of training instances. Fig. [illustrates the differences between immediate thresholds and all thresholds.

A. Regularized multi-task learning framework

In the real-world scenario, multiple related tasks are more common comparing with many independent tasks. To achieve MTL, many studies propose to solve a regularized optimization problem. Assume there are \( T \) tasks and \( G \) input variables/features in each corresponding dataset, then we have the weight matrix as \( W \in \mathbb{R}^{G \times T} \) and regularized MTL object function as:

\[
J = \min_W \mathcal{L}(W) + \Omega(W), \tag{3}
\]

where \( \Omega(W) \) is the regularization/penalty term, which encodes the task relatedness.
Immediate (only adjacent) thresholds.

Y=1 Y=2 Y=3

Y*

Immediate thresholds (upper panel) only calculate the errors using the neighbor thresholds of each segment when first and last thresholds remain in finite range. All thresholds (lower panel) calculate the error using both neighbor and non-neighbor thresholds between segments when $\vartheta_0 = -\infty$ and $\vartheta_U = \infty$. Note that, in lower panel, thin dash lines represent the errors calculated using adjacent thresholds, while solid lines represent the errors calculated using nonadjacent thresholds.

All (adjacent and nonadjacent) thresholds.

Y=1 Y=2 Y=3

Fig. 1: Illustration figure of two different types of thresholds using three segments. Immediate thresholds (upper panel) only calculate the errors using the neighbor thresholds of each segment when first and last thresholds remain in finite range. All thresholds (lower panel) calculate the error using both neighbor and non-neighbor thresholds between segments when $\vartheta_0 = -\infty$ and $\vartheta_U = \infty$. Note that, in lower panel, thin dash lines represent the errors calculated using adjacent thresholds, while solid lines represent the errors calculated using nonadjacent thresholds.

B. RMTOR using immediate thresholds (RMTOR$_I$)

1) RMTOR$_I$ model: We define a function $M(D) := \log(1 + \exp(D))$, then the loss function of RMTOR with the immediate thresholds is formulated as:

$$\mathcal{L}_I = \sum_{t=1}^{T} \sum_{j=1}^{n_t} \left[ M(\vartheta_{(Y_{tj}-1)}) - X_{tj} W_t \right] + M(X_{tj} W_t - \vartheta_{Y_{tj}}) ,$$

where $t$ is the index of task, $n_t$ is the number of instances in the $t^{th}$ task, $j$ is the index of instance in the $t^{th}$ task, $Y_{tj}$ is the label of the $j^{th}$ instance in the $t^{th}$ task, $X_{tj} \in \mathbb{R}^{1 \times G}$, $W_t \in \mathbb{R}^{G \times 1}$ and $\vartheta \in \mathbb{R}^{T \times U}$. Note that, $\vartheta_{Y_{tj}}$ is a threshold in the $t^{th}$ task, which is a scalar and its index is $Y_{tj}$. [Diagram]

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**B. RMTOR using immediate thresholds (RMTOR$_I$)**

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where $t$ is the index of task, $n_t$ is the number of instances in the $t^{th}$ task, $j$ is the index of instance in the $t^{th}$ task, $Y_{tj}$ is the label of the $j^{th}$ instance in the $t^{th}$ task, $X_{tj} \in \mathbb{R}^{1 \times G}$, $W_t \in \mathbb{R}^{G \times 1}$ and $\vartheta \in \mathbb{R}^{T \times U}$. Note that, $\vartheta_{Y_{tj}}$ is a threshold in the $t^{th}$ task, which is a scalar and its index is $Y_{tj}$. [Diagram]
Thus, we have the objective function $RMTOR_l$ as:

$$RMTOR_l = \min_{W, \vartheta} \sum_{t=1}^{T} \sum_{j=1}^{n_t} \left[ M(\vartheta_{j_t, j_{t-1}}) - X_{t_j} W_t \right] + M(X_{t_j} W_t - \vartheta_{j_t}) \right] + \lambda \| W \|_{2,1},$$

(5)

where $\lambda$ is the tuning parameter to control the sparsity and $\| W \|_{2,1} = \sum_{g=1}^{G} \sqrt{\sum_{t=1}^{T} |w_{gt}|^2}$. Note that, $g$ is the index of feature and $w_{gt}$ is the weight for the $g^{th}$ feature in the $t^{th}$ task.

2) Optimization: Alternating structure optimization [56] is used to discover the shared predictive structure for all multiple tasks simultaneously, especially when the two sets of parameters $W$ and $\vartheta$ in Eq. (5) can not be learned at the same time.

   a) Optimization of $W$: With fixed $\vartheta$, the optimal $W$ can be learned by solving:

$$\min_{W} L_l(W) + \lambda \| W \|_{2,1},$$

(6)

where $L_l(W)$ is a smooth convex and differentiable loss function, and the first order derivative can be expressed as:

$$L_l'(W_t) = \sum_{j=1}^{n_t} X_{t_j} [G(X_{t_j} W_t - \vartheta_{j_t})] - G(\vartheta_{j_t, j_{t-1}} - X_{t_j} W_t)],$$

$$L_l'(W) = \left[ L_1'(W_1), \ldots, L_1'(W_t), \ldots, L_1'(W_T) \right],$$

(7)

where $G(D) := \frac{\partial M(D)}{\partial D} = \frac{1}{1+\exp(-D)}$.

To solve the optimization problem in Eq. (6), fast iterative shrinkage thresholding algorithm (FISTA) shown in Algorithm 1 is implemented with the general updating steps:

$$W^{(l+1)} = \pi_P(S^{(l)} - \frac{1}{\gamma^{(l)}} L_l'(S^{(l)})),$$

(8)

where $l$ is the iteration index, $\frac{1}{\gamma^{(l)}}$ is the possible largest step-size that is chosen by line search [57, Lemma 2.1, page 189] and $L_l'(S^{(l)})$ is the gradient of $L_l(\cdot)$ at search point $S^{(l)}$. $S^{(l)} = W^{(l)} + \alpha^{(l)}(W^{(l-1)} - W^{(l-1)})$ are the search points for each task, where $\alpha^{(l)}$ is the combination scalar. $\pi_P(\cdot)$ is $l_{2,1}$ regularized Euclidean project shown as:

$$\pi_P(H(S^{(l)})) = \min_{W} \frac{1}{2} \| W - H(S^{(l)}) \|_F^2 + \lambda \| W \|_{2,1},$$

(9)

where $\| \cdot \|_F$ is the Frobenius norm and $H(S^{(l)}) = S^{(l)} - \frac{1}{\gamma^{(l)}} L_l'(S^{(l)})$ is the gradient step of $S^{(l)}$. An efficient solution (Theorem 1) of Eq. (9) has been proposed in [39].

**Theorem 1:** Given $\lambda$, the primal optimal point $\hat{W}$ of Eq. (9) can be calculated as:

$$\hat{W}_g = \begin{cases} 
\left(1 - \frac{\lambda}{\| H(S^{(l)})_g \|_2} \right) H(S^{(l)})_g & \text{if} \quad \lambda > 0, \| H(S^{(l)})_g \|_2 > \lambda \\
0 & \text{if} \quad \lambda > 0, \| H(S^{(l)})_g \|_2 \leq \lambda \\
H(S^{(l)})_g & \text{if} \quad \lambda = 0,
\end{cases}$$

(10)

where $H(S^{(l)})_g$ is the $j^{th}$ row of $H(S^{(l)})$, and $\hat{W}_g$ is the $g^{th}$ row of $\hat{W}$.
Algorithm 1: Fast iterative shrinkage thresholding algorithm (FISTA) for training RMTOR.

Input: A set of feature matrices \( \{X_1, X_2, \cdots, X_T\} \), target value matrix \( Y \) for all \( T \) tasks, initial coefficient matrix \( W^{(0)} \) and \( \lambda \)

Output: \( \hat{W} \)

1. \textbf{Initialize:} \( W^{(1)} = W^{(0)} \), \( d_{-1} = 0 \), \( d_0 = 1 \), \( \gamma^{(0)} = 1 \), \( l = 1 \);
2. \textbf{repeat}
3. \hspace{0.5em} Set \( \alpha^{(l)} = \frac{d_{l-2} - 1}{d_{l-1}} \), \( S^{(l)} = W^{(l)} + \alpha^{(l)}(W^{(l)} - W^{(l-1)}) \);
4. \hspace{1em} \textbf{for} \( j = 1, 2, \cdots \) \textbf{do}
5. \hspace{1.5em} Set \( \gamma = 2^j \gamma^{(l-1)} \);
6. \hspace{1.5em} Calculate \( W^{(l+1)} = \pi_P(S^{(l)} - \frac{1}{\gamma^{(l)}} L_f'(S^{(l)})) \);
7. \hspace{1.5em} Calculate \( Q_{\gamma}(S^{(l)}, W^{(l+1)}) \);
8. \hspace{1.5em} \textbf{if} \( L_f(W^{(l+1)}) \leq Q_{\gamma}(S^{(l)}, W^{(l+1)}) \) \textbf{then}
9. \hspace{2em} \( \gamma^{(l)} = \gamma \), \textbf{break} ;
10. \hspace{1.5em} \textbf{end}
11. \hspace{0.5em} \textbf{end}
12. \hspace{0.5em} \( d_l = \frac{1 + \sqrt{1 + 4d_{l-1}^2}}{2} \);
13. \hspace{0.5em} \( l = l + 1 \);
14. \textbf{until} Convergence of \( W^{(l)} \);
15. \( \hat{W} = W^{(l)} \);

In lines 4-11 of Algorithm 1, the optimal \( \gamma^{(l)} \) is chosen by the backtracking rule based on \cite[Lemma 2.1, page 189]{57}, \( \gamma^{(l)} \) is greater than or equal to the Lipschitz constant of \( L_f(\cdot) \) at search point \( S^{(l)} \), which means \( \gamma^{(l)} \) is satisfied for \( S^{(l)} \) and \( \frac{1}{\gamma^{(l)}} \) is the possible largest step size.

In line 7 of Algorithm 1, \( Q_{\gamma}(S^{(l)}, W^{(l+1)}) \) is the tangent line of \( L_f(\cdot) \) at \( S^{(l)} \), which can be calculated as:

\[
Q_{\gamma}(S^{(l)}, W^{(l+1)}) = L_f(S^{(l)}) + \frac{\gamma}{2} \| W^{(l+1)} - S^{(l)} \|^2 + \langle W^{(l+1)} - S^{(l)}, L_f'(S^{(l)}) \rangle.
\]

\( b) \) Optimization of \( \vartheta \): With fixed \( W \), the optimal \( \vartheta \) can be learned by solving \( \min_{\vartheta} L_f(\vartheta) \), where \( L_f(\vartheta) \)'s first order derivative can be expressed as:

\[
L_f'(\vartheta) = \sum_{j=1}^{n_t} \sum_{y_{ij} - 1 = \mu} G(\vartheta_{t\mu} - X_{ij} W_t) - \sum_{j=1}^{n_t} \sum_{y_{ij} = \mu} G(X_{ij} W_t - \vartheta_{t\mu}),
\]

\[
L_f'(\vartheta) = \left[ \frac{L_f'(\vartheta_1)}{n_1}, \cdots, \frac{L_f'(\vartheta_i)}{n_i}, \cdots, \frac{L_f'(\vartheta_T)}{n_T} \right],
\]

where \( \vartheta_{t\mu} \) is the \( \mu^{th} \) threshold in task \( t \), so that \( \vartheta \) can be updated as:

\[
\vartheta^{(l)} = \vartheta^{(l-1)} - \varepsilon^{(l)} L_f'(\vartheta),
\]

where \( \varepsilon \) is the step-size of gradient descent.
C. RMTOR using all thresholds (RMTOR_A)

1) RMTOR_A model: RMTOR with the all thresholds, loss function is calculated as:

\[ \mathcal{L}_A = \sum_{t=1}^{T} \sum_{j=1}^{n_t} \left[ \sum_{\mu=1}^{Y_{tj}-1} M(\vartheta_{t\mu} - X_{tj}W_t) + \sum_{\mu=Y_{tj}}^{U-1} M(X_{tj}W_t - \vartheta_{t\mu}) \right], \tag{13} \]

where \( \sum_{\mu=1}^{Y_{tj}-1} M(X_{tj}W_t - \vartheta_{t\mu}) \) is the sum of errors when \( \mu < Y_{tj} \), which means the threshold’s index \( \mu \) is smaller than the \( j^{th} \) training instance label \( Y_{tj} \), while \( \sum_{\mu=Y_{tj}}^{U-1} M(\vartheta_{t\mu} - X_{tj}W_t) \) is the sum of errors when \( \mu \geq Y_{tj} \); thus, its objective function RMTOR_A is calculated as:

\[
RMTOR_A = \min_{W, \vartheta} \mathcal{L}_A + \lambda \|W\|_{2,1}. \tag{14}
\]

2) Optimization: We also implement alternating structure optimization method to obtain the optimal parameters \( W \) and \( \vartheta \), which is similar as we perform for RMTOR_I optimization.

a) Optimization of \( W \): With fixed \( \vartheta \), the optimal \( W \) can be learned by solving:

\[
\min_{W} \mathcal{L}_A(W) + \lambda \|W\|_{2,1}, \tag{15}
\]

where \( \mathcal{L}_A(W) \) is a smooth convex and differentiable loss function. First, we calculate its first order derivative w.r.t. \( W_t \):

\[
\mathcal{L}'_A(W_t) = \sum_{j=1}^{n_t} \left[ \sum_{\mu=Y_{tj}}^{U-1} X_{tj}G(X_{tj}W_t - \vartheta_{t\mu}) - \sum_{\mu=1}^{Y_{tj}-1} X_{tj}G(\vartheta_{t\mu} - X_{tj}W_t) \right]. \tag{16}
\]

We introduce an indicator variable \( z_{\mu} \):

\[
z_{\mu} = \begin{cases} +1, & \mu \geq Y_{tj} \\ -1, & \mu < Y_{tj} \end{cases} \tag{17}
\]

Then the updated formulation of Eq. (16) and the first order derivative w.r.t. \( W \) are calculated as:

\[
\mathcal{L}'_A(W_t) = \sum_{j=1}^{n_t} \sum_{\mu=1}^{U-1} X_{tj}^T [z_{\mu} \cdot G(z_{\mu} \cdot (X_{tj}W_t - \vartheta_{t\mu}))], \tag{18}
\]

\[
\mathcal{L}'_A(W) = \left[ \frac{\mathcal{L}'_A(W_1)}{n_1}, \ldots, \frac{\mathcal{L}'_A(W_t)}{n_t}, \ldots, \frac{\mathcal{L}'_A(W_T)}{n_T} \right].
\]

Similar as we did for RMTOR_I optimization of \( W \), we then use FISTA to optimize with the parameters in RMTOR_A updating steps:

\[
W^{(l+1)} = \pi_p(S^{(l)} - \frac{1}{\gamma^{(l)}} \mathcal{L}'_A(S^{(l)})), \tag{19}
\]

which is solved in Algorithm 1.
b) Optimization of $\vartheta$: With fixed $W$, the optimal $\vartheta$ can be learned by solving $\min_{\vartheta} L_A(\vartheta)$, where $L_A(\vartheta)$’s first order derivative can be expressed as:

$$L_A'(\vartheta_t) = -1^T [z_\mu \cdot G(z_\mu \cdot (X_{tj} W_t - \vartheta_{t\mu}))],$$

and hence $\vartheta$ can be updated as:

$$\vartheta^{(l)} = \vartheta^{(l-1)} - \epsilon^{(l)} L_A'(\vartheta).$$

V. DEEP MULTI-TASK ORDINAL REGRESSION (DMTOR) MODELS

In this section, we introduce two deep multi-task ordinal regression (DMTOR) models implemented using deep neural networks (DNN). Fig. 2 illustrates the basic structure of the DMTOR.

![Diagram of DMTOR structure](image)

Fig. 2: Illustration of the basic structure of the DMTOR. All tasks share the input and representation layers, while all tasks keep several task-specific layers. Note that, circles represent the nodes at each layer and squares represent layers.

A. DMTOR architecture

We denote input layer, shared representation layers and task-specific representation layers as $L_I$, $L_{(R)}$ and $L_{(S)}$, respectively. Thus, we have the shared representation layers as:

$$L_{R(1)} = ReLU(W_1 \cdot L_I),$$
$$L_{R(2)} = ReLU(W_2 \cdot L_{R(1)}),$$
$$\ldots,$$
$$L_{R(r)} = f(W_r, L_{R(r-1)}),$$
where \( \{ W_1, \ldots, W_r \} \) are the coefficient parameters at different hidden layers, \( \text{ReLU}(\cdot) \) stands for rectified linear unit that is the nonlinear activation function, \( r \) is the number of hidden layers and \( f(\cdot) \) is a linear transformation.

Task-specific representation layers are expressed as:
\[
L_{S(1)}^t = \text{ReLU}(B_{1t} \cdot L_{R(r)}),
\]
\[
\ldots,
\]
\[
L_{S(s)}^t = \text{ReLU}(B_{st} \cdot L_{S(s-1)}),
\]
where \( B^t \) is the coefficient parameter corresponding to the \( t^{th} \) task and \( s \) is the number of task-specific representation layers.

**B. Network training**

Forward propagation calculation for the output is expressed as:
\[
output^t = f(O^t, L_{S(s)}^t),
\]
where \( O^t \) is the coefficient parameter corresponding to the \( t^{th} \) task.

Then the loss function of \( \text{DMTOR}_I \) model can be calculated as:
\[
\mathcal{L}_I = \sum_{t=1}^{T} \sum_{j=1}^{n_t} [M(\vartheta_{Y_{ij} - 1} - output^t) + M(output^t - \vartheta_{Y_{ij}})].
\]

Similarly, the loss function of \( \text{DMTOR}_A \) model can be calculated as:
\[
\mathcal{L}_A = \sum_{t=1}^{T} \sum_{j=1}^{n_t} [\sum_{\mu=1}^{U-1} M(\vartheta_{\mu \cdot Y_{ij}} - output^t) + \sum_{\mu=Y_{ij}}^{U-1} M(output^t - \vartheta_{\mu})].
\]

We use mini-batch to train our models’ parameters for faster learning with partitioning the training dataset into small batches, and then calculate the model error and update the corresponding parameters.

Stochastic Gradient Descent (SGD) is used to iteratively minimize the loss and update all the model parameters (weights: \( W, B, O \) and thresholds: \( \vartheta \)):
\[
W^{(l)} = W^{(l-1)} - \varepsilon^{(l)} \nabla_W \mathcal{L},
\]
\[
\vdots
\]
\[
\vartheta^{(l)} = \vartheta^{(l-1)} - \varepsilon^{(l)} \nabla_{\vartheta} \mathcal{L}.
\]

**VI. Experiments and Results**

To evaluate the performance of our proposed multi-task ordinal regression (MTOR) models, we extensively compare them with a set of selected single-task learning (STL) models. We first elaborate some details of the experimental setup and then describe three real-world medical datasets used in the experiments. Finally, we discuss the experimental results using accuracy and mean absolute error (MAE) as the evaluation metrics.
A. Experimental setup

We demonstrate the performance of proposed RMTOR and DMTOR models on small and large-scale medical datasets, respectively: 1). We conduct experiment on a small medical dataset (i.e., Alzheimer’s Disease Neuroimaging Initiative) to compare $RMTOR_I$ and $RMTOR_A$ with their corresponding STL ordinal regression models denoted as $STOR_I$ and $STOR_A$. We also compare them with two SVM based ordinal regression (SVOR) models, i.e., support vector for ordinal regression with explicit constraints ($SVOREC$) [26] and support vector machines using binary ordinal decomposition ($SVMBOD$) [17]. Both SVOR models are implemented in Matlab within ORCA framework [58]. 2). Our experiments on two large-scale medical datasets (i.e., Behavioral Risk Factor Surveillance System and Henry Ford Hospital hypertension) compare $DMTOR_I$ and $DMTOR_A$ with their corresponding STL ordinal regression models denoted as $DSTOR_I$ and $DSTOR_A$. In addition, we compare them with a neural network approach for ordinal regression, i.e., $NNRank$ [59], which is downloaded from the Multicom toolbox. In our experiments, the models with DNN (i.e., $DMTOR_I$, $DMTOR_A$, $DSTOR_I$ and $DSTOR_A$) are implemented in Python using Pytorch and the other models without DNN ($RMTOR_I$, $RMTOR_A$, $STOR_I$ and $STOR_A$) are implemented in Matlab.

| Task/AG | No. of instances | MTOR | Global setting | Individual setting |
|---------|------------------|------|----------------|-------------------|
|         |                  | $RMTOR_I$ | $RMTOR_A$ | SVOREC | SVMBOD | $STOR_I$ | $STOR_A$ | SVOREC | SVMBOD | $STOR_I$ | $STOR_A$ |
| 50-59   | 72               | 0.791 (0.344) | 0.783 (0.307) | 0.572 (0.673) | 0.522 (0.691) | 0.493 (0.683) | 0.489 (0.629) | 0.554 (0.537) | 0.633 (0.501) | 0.473 (0.792) | 0.459 (0.690) |
| 60-69   | 104              | 0.739 (0.311) | 0.687 (0.362) | 0.583 (1.014) | 0.611 (0.892) | 0.429 (1.033) | 0.493 (1.098) | 0.638 (0.911) | 0.621 (0.837) | 0.633 (0.894) | 0.656 (1.063) |
| 70-79   | 142              | 0.764 (0.401) | 0.659 (0.561) | 0.533 (0.943) | 0.661 (0.798) | 0.572 (0.743) | 0.478 (0.832) | 0.602 (0.601) | 0.645 (0.592) | 0.674 (0.611) | 0.629 (0.975) |
| ≥ 80    | 83               | 0.747 (0.579) | 0.709 (0.619) | 0.623 (0.912) | 0.671 (0.939) | 0.523 (0.840) | 0.475 (0.983) | 0.693 (0.812) | 0.701 (0.727) | 0.677 (0.930) | 0.616 (1.091) |

1) MTL ordinal regression experimental setup: In three real-world datasets, tasks are all defined based on various age groups in terms of the predefined age groups in MTOR models for the consistency. Also, all tasks share the same feature space, which follows the assumption of MTL that the multiple tasks are related.

For $RMTOR_I$ and $RMTOR_A$, we use 10-fold cross validation to select the best tuning parameter $\lambda$ in the training dataset.

For $DMTOR_I$ and $DMTOR_A$, we use the same setting of DNN, i.e., three shared representations layers and three task-specific representation layers. For each dataset, we set the same hyper-parameters, e.g., number of batches and number of epochs; while these hyper-parameters

[http://sysbio.rnet.missouri.edu/multicom_toolbox/tools.html]
TABLE II: The accuracy and MAE of two proposed DNN based ordinal regression models, (i.e., $DMTOR_I$ and $DMTOR_A$), their corresponding STL ordinal regression models (i.e., $DSTOR_I$ and $DSTOR_A$) and a STL neural network approach for ordinal regression (i.e., $NNRank$) using a large-scale medical dataset, i.e., BRFSS. (The best performance results are in bold face.)

| Task/AG instances | MTOR | Global setting | Individual setting |
|------------------|------|----------------|--------------------|
|                  | DMTOR\textsubscript{I} | DMTOR\textsubscript{A} | NN\textit{Rank} | DSTOR\textsubscript{I} | DSTOR\textsubscript{A} | NN\textit{Rank} | DSTOR\textsubscript{I} | DSTOR\textsubscript{A} |
| 18-24 5,325      | 0.532 (0.479) | 0.431 (0.582) | 0.525 (0.793) | 0.405 (0.783) | 0.363 (1.020) | 0.507 (0.802) | 0.359 (0.745) | 0.328 (1.055) |
| 25-34 5,693      | 0.524 (0.521) | 0.452 (0.633) | 0.521 (0.757) | 0.432 (0.795) | 0.379 (0.839) | 0.513 (0.581) | 0.325 (0.935) | 0.389 (1.037) |
| 35-49 17,480     | 0.577 (0.755) | 0.513 (0.924) | 0.574 (0.915) | 0.455 (1.090) | 0.381 (0.927) | 0.563 (0.790) | 0.367 (0.954) | 0.328 (1.077) |
| 50-79 55,388     | 0.608 (0.536) | 0.529 (0.711) | 0.580 (0.875) | 0.421 (1.330) | 0.276 (1.033) | 0.585 (0.582) | 0.293 (1.503) | 0.284 (1.270) |
| $\geq$ 80 745    | 0.451 (0.630) | 0.443 (0.681) | 0.447 (0.833) | 0.410 (0.961) | 0.391 (0.902) | 0.425 (0.710) | 0.394 (1.027) | 0.374 (1.009) |

TABLE III: The accuracy and MAE of two proposed DNN based MTOR models, their corresponding STL models and $NNRank$ using a large-scale dataset, i.e., FORD. (The best performance results are in bold face.)

| Task/AG instances | MTOR | Global setting | Individual setting |
|------------------|------|----------------|--------------------|
|                  | DMTOR\textsubscript{I} | DMTOR\textsubscript{A} | NN\textit{Rank} | DSTOR\textsubscript{I} | DSTOR\textsubscript{A} | NN\textit{Rank} | DSTOR\textsubscript{I} | DSTOR\textsubscript{A} |
| 0-17 4,176       | 0.732 (0.277) | 0.709 (0.303) | 0.451 (0.654) | 0.532 (0.745) | 0.588 (0.894) | 0.455 (0.531) | 0.577 (0.845) | 0.591 (0.919) |
| 18-24 5,284      | 0.742 (0.298) | 0.697 (0.401) | 0.551 (0.537) | 0.530 (0.639) | 0.592 (0.792) | 0.479 (0.938) | 0.635 (0.862) | 0.671 (0.583) |
| 25-34 6,279      | 0.722 (0.435) | 0.720 (0.539) | 0.488 (0.680) | 0.497 (1.032) | 0.593 (0.794) | 0.452 (0.902) | 0.622 (0.883) | 0.530 (0.895) |
| 35-49 9,516      | 0.781 (0.301) | 0.737 (0.350) | 0.667 (0.548) | 0.649 (0.642) | 0.563 (1.055) | 0.619 (0.720) | 0.620 (0.860) | 0.565 (0.930) |
| 50-79 10,991     | 0.755 (0.379) | 0.734 (0.351) | 0.615 (0.537) | 0.534 (0.665) | 0.530 (0.995) | 0.598 (0.850) | 0.616 (0.990) | 0.613 (1.034) |
| $\geq$ 80 1,070  | 0.737 (0.383) | 0.733 (0.412) | 0.690 (0.731) | 0.570 (0.790) | 0.539 (1.077) | 0.658 (0.609) | 0.609 (1.073) | 0.579 (0.977) |

are not the same in different datasets. We use random initialization for parameters. Please refer to Section V-B to see the details of the network training procedures.

2) STL ordinal regression experimental setup: In our experiments, STL ordinal regression methods are applied under two settings: 1) Individual setting, i.e., a prediction model is trained for each task; 2) Global setting, i.e., a prediction model is trained for all tasks. In the individual setting the heterogeneity among tasks are fully considered but not the task relatedness; on the contrary, in the global setting all the heterogeneities have been neglected.

For $DSTOR_I$ and $DSTOR_A$, the setting of DNN uses three hidden representation layers, where each layer’s activation function is $ReLU(\cdot)$. During the training procedure, the loss functions use the same function $M(\cdot)$ with either immediate or all thresholds. Same as we did for DMTOR, we set the same hyper-parameters within each dataset and different ones among
In the training of \textit{NNRank}, we use the default setting, e.g., number of epochs is 500, random seed is 999 and learning rate is 0.01. In testing, we also use the default setting, e.g., decision threshold is 0.5.

B. Data description

In this paper, Alzheimer’s Disease Neuroimaging Initiative (ADNI) \cite{60} and Behavioral Risk Factor Surveillance System (BRFSS) are public medical benchmark datasets, while Henry Ford Hospital hypertension (FORD) is the private one. We divide these three datasets into training and testing using stratified sampling, more specifically, 80\% of instances are used for training and the rest of instances are used for testing.

Age is a crucial factor when considering phenotypic changes in disease \cite{61–64}. Thus, we define the tasks according to the disjoint age groups in ADNI, BRFSS and FORD datasets.

1) Alzheimer’s Disease Neuroimaging Initiative (ADNI): The mission of ADNI is to seek the development of biomarkers for the disease and advance in order to understand the pathophysiology of AD \cite{60}. This data also aims to improve diagnostic methods for early detection of AD and augment clinical trial design. Additional goal of ADNI is to test the rate of progress for both mild cognitive impairment and AD. As a result, ADNI are trying to build a large repository of clinical and imaging data for AD research.

We pick one measurement from the participants of diagnostic file in this project and delete two participants whose age information are missing, which leaves us 1,998 instances and 95 variables including 94 input variables that are corresponding to measurement of AD, e.g., FDG-PET is used to measure cerebral metabolic rates of glucose; plus one output variable that is phase used to represent three stages of AD (cognitively normal, mild cognitive impairment, and AD).

Since the age groups in ADNI dataset fall in mature adulthood and late adulthood, we divide mature adulthood into three sub-groups. Hence, the tasks are defined in ADNI based on different stages of people shown as the first column in Table I, i.e., mature adulthood 1 (50 years to 59 years), mature adulthood 2 (60 years to 69 years), mature adulthood 3 (70 years to 79 years) and late adulthood (equal or older than 80 years).

2) Behavioral Risk Factor Surveillance System (BRFSS): The BRFSS dataset is a collaborative project between all the states in the U.S. and the Centers for Disease Control and Prevention (CDC), and aims to collect uniform, state-specific data on preventable health practices and risk behaviors that affect the health of the adult population (i.e., adults aged 18 years and older). In the experiment, we use the BRFSS dataset that is collected in 2016\cite{https://www.cdc.gov/brfss/annual_data/annual_2016.html}.

The BRFSS dataset is collected via the phone-based surveys with adults residing in private residence or college housing. The original BRFSS dataset contains 486,303 instances and 275 variables, after deleting the entries with missing age information and the variables with all hidden values, the preprocessed dataset contains 459,156 with 85 variables including 84 input variables and one output variable, i.e., categories of body mass index (underweight, normal weight, overweight and obese).

The tasks are defined in BRFSS based on different stages of people shown in the first column in Table II, i.e., early young (18 years to 24 years), young (25 years to 34 years), middle-aged (35 years to 49 years), mature adulthood (50 years to 70 years) and late adulthood (equal or older than 80 years).

\cite{https://www.cdc.gov/brfss/annual_data/annual_2016.html}
3) Henry Ford Hospital hypertension (FORD): FORD dataset is collected by our collaborator from Emergency Room (ER) of Henry Ford Hospital. All participants in this dataset are all from metro Detroit. All variables except for the outcomes are collected from the emergency department at Henry Ford Hospital. Some diagnostic variables are collected from any hospital admissions that occurred after the ER visits. The index date in FORD dataset for each patient started in 2014 and went through the middle of 2015. They then collect outcomes for each patient for one year after that index date. So, the time duration from the date that a patient seen in ER to his/her diagnostic variable collection date may be longer than one year. For example, a patient may have been seen in the ER on July 2, 2015 and they would have had diagnosis variable collected date up to July 2, 2016.

Originally, this FORD dataset contains 221,966 instances and 63 variables including demographic, lab test and diagnosis related information. After deleting the entries with missing values, the preprocessed dataset contains 186,572 instances and 23 variables including 22 input variables and one output, i.e., four stages of hypertension based on systolic and diastolic pressure: normal (systolic pressure: 90-119 and diastolic pressure: 60-79), pre-hypertension (120-139 and 80-89), stage 1 hypertension (140-159 and 90-99) and stage 2 hypertension (≥ 160 and ≥ 160).

Since the number of instances in the age groups of infant, children and teenager are much less than other age groups, we combine these three age groups into one age group as minor. Hence, the tasks are defined in FORD based on different ages of people shown as the first column in Table III, i.e., minor (1 year to 17 years), early young (18 years to 24 years), young (25 years to 34 years), middle-aged (35 years to 49 years), mature adulthood (50 years to 70 years) and late adulthood (equal or older than 80 years).

C. Performance comparison

To access the overall performance of each ordinal regression method, we use both accuracy and MAE as our evaluation metrics. Accuracy reports the proportion of accurate predictions, so that larger value of accuracy means better performance. With considering orders, MAE is capable of measuring the distance between true and predicted labels, so that smaller value of MAE means better performance.

To formally define accuracy, we use $i$ and $j$ to represent the index of true labels and the index of predicted labels. A pair of labels for each instance, i.e., $(Y_i, \hat{Y}_j)$, is positive if they are equal, i.e., $Y_i = \hat{Y}_j$, otherwise the pair is negative. We further denote $N_T$ as the number of total pairs and $N_P$ as the number of positive pairs. Thus, accuracy $= \frac{N_P}{N_T}$. MAE is calculated as $MAE = \frac{1}{n_s} \sum_{i=1}^{n_s} |Y_i - \hat{Y}_i|$, where $n_s$ is the number of instances in each testing dataset.

We show the performance results of accuracy and MAE of different models using the aforementioned three medical datasets ADNI, BRFSS and FORD in Table I, Table II and Table III, respectively. Each task in our experiments is to predict the stage of disease for people in each age group. In the experiments of MTOR models, each task has its own prediction result. For each task, we build one STL ordinal regression model under the global and individual settings as comparison methods.

Overall, the experimental results show that the MTOR models perform better than other STL models in terms of both accuracy and MAE. MTOR models with immediate thresholds largely outperform the ones with all thresholds in both evaluation metrics, which confirms the assumption that first and last thresholds are always remaining in finite range in the real-world scenario.

Under the proposed MTOR framework, both deep and shallow models have descent performance for different types of datasets: RMTOR model with immediate thresholds performs better
for small dataset whereas DMTOR model with immediate thresholds is more suitable for large-scale dataset. More specifically, the $DMTOR_I$ model outperforms the competing models in the most tasks of BRFSS and FORD datasets. In ADNI dataset, $RMTOR_I$ outperforms other models in terms of accuracy and MAE. Note that, the accuracy and MAE do not always perform consistently for all tasks. For example in the experiment using ADNI dataset, for the first task with ages ranging in (50-59), $RMTOR_I$ shows the best (largest) accuracy whereas $RMTOR_A$ exhibits the best (lowest) MAE.

VII. CONCLUSION

In this paper, we tackle multiple ordinal regression problems by proposing two regularized and two DNN based MTOR models. The first two proposed models $RMTOR_I$ and $RMTOR_A$ belong to the regularized multi-task learning, where the ordinal regression is used to handle the ordinal labels and regularization terms are used to encode the assumption of task relatedness. The other two proposed models $DMTOR_I$ and $DMTOR_A$ are based on DNN with shared representation layers to encode the task relatedness. Therefore, the proposed MTOR models are comprehensively designed for both large-scale and small datasets, particularly, the DMTOR outperforms other models for the large-scale datasets and the RMTOR are appropriate for small datasets. In the future, we plan to implement our method to diverse applications, e.g., progressive disease risk factor analysis and customer rating behavior analysis.

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REFERENCES

[1] J. Domingo-Ferrer and V. Torra, “Ordinal, continuous and heterogeneous k-anonymity through microaggregation,” Data Mining and Knowledge Discovery, vol. 11, no. 2, pp. 195–212, 2005.
[2] R. Henriques, S. C. Madeira, and C. Antunes, “Multi-period classification: learning sequent classes from temporal domains,” Data Mining and Knowledge Discovery, vol. 29, no. 3, pp. 792–819, 2015.
[3] R. Brookmeyer, E. Johnson, K. Ziegler-Graham, and H. M. Arrighi, “Forecasting the global burden of alzheimer’s disease,” Alzheimer’s & dementia: the journal of the Alzheimer’s Association, vol. 3, no. 3, pp. 186–191, 2007.
[4] D. A. Davis, N. V. Chawla, N. A. Christakis, and A.-L. Barabási, “Time to care: a collaborative engine for practical disease prediction,” Data Mining and Knowledge Discovery, vol. 20, no. 3, pp. 388–415, 2010.
[5] D. S. Chan and T. Norat, “Obesity and breast cancer: not only a risk factor of the disease,” Current treatment options in oncology, vol. 16, no. 5, p. 22, 2015.
[6] T. M. Cruickshank, A. R. Reyes, and M. R. Ziman, “A systematic review and meta-analysis of strength training in individuals with multiple sclerosis or parkinson disease,” Medicine, vol. 94, no. 4, 2015.
[7] D. Kaplan, The Sage handbook of quantitative methodology for the social sciences. Sage, 2004.
[8] A. A. O’Connell, Logistic regression models for ordinal response variables. Sage, 2006, no. 146.
[9] H. Grosskreutz and S. Rüeping, “On subgroup discovery in numerical domains,” Data mining and knowledge discovery, vol. 19, no. 2, pp. 210–226, 2009.
[10] F. Lemmerich, M. Atzmueller, and F. Puppe, “Fast exhaustive subgroup discovery with numerical target concepts,” Data Mining and Knowledge Discovery, vol. 30, no. 3, pp. 711–762, 2016.
[11] C.-K. Chen and H. John Jr, “Using ordinal regression model to analyze student satisfaction questionnaires. ir applications, volume 1, may 26, 2004.” Association for Institutional Research (NJ1), 2004.
[12] D. Yar Hamidi, K. Wennberg, and H. Berglund, “Creativity in entrepreneurship education,” Journal of small business and enterprise development, vol. 15, no. 2, pp. 304–320, 2008.
[13] M. Kim, “Conditional ordinal random fields for structured ordinal-valued label prediction,” Data mining and knowledge discovery, vol. 28, no. 2, pp. 378–401, 2014.
[14] A. K. Menon and C. Elkan, “Predicting labels for dyadic data,” Data Mining and Knowledge Discovery, vol. 21, no. 2, pp. 327–343, 2010.
[45] G. Baetschmann, K. E. Staub, and R. Winkelmann, “Consistent estimation of the fixed effects ordered logit model,” *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, vol. 178, no. 3, pp. 685–703, 2015.

[46] K. M. Kockelman and Y.-J. Kweon, “Driver injury severity: an application of ordered probit models,” *Accident Analysis & Prevention*, vol. 34, no. 3, pp. 313–321, 2002.

[47] F. Ye and D. Lord, “Comparing three commonly used crash severity models on sample size requirements: multinomial logit, ordered probit and mixed logit models,” *Analytic methods in accident research*, vol. 1, pp. 72–85, 2014.

[48] Y. Nesterov, *Introductory lectures on convex optimization: A basic course*. Springer Science & Business Media, 2013, vol. 87.

[49] S. Ji and J. Ye, “An accelerated gradient method for trace norm minimization,” in *Proceedings of the 26th annual international conference on machine learning*. ACM, 2009, pp. 457–464.

[50] J. Zhou, J. Chen, and J. Ye, “Clustered multi-task learning via alternating structure optimization,” in *Advances in neural information processing systems*, 2011, pp. 702–710.

[51] S. Ruder, “An overview of multi-task learning in deep neural networks,” *arXiv preprint arXiv:1706.05098*, 2017.

[52] L. Duong, T. Cohn, S. Bird, and P. Cook, “Low resource dependency parsing: Cross-lingual parameter sharing in a neural network parser,” in *Proceedings of the 53rd Annual Meeting of the Association for Computational Linguistics and the 7th International Joint Conference on Natural Language Processing (Volume 2: Short Papers)*, vol. 2, 2015, pp. 845–850.

[53] Y. Yang and T. M. Hospedales, “Trace norm regularised deep multi-task learning,” *arXiv preprint arXiv:1606.04038*, 2016.

[54] J. Baxter, “A bayesian/information theoretic model of learning to learn via multiple task sampling,” *Machine learning*, vol. 28, no. 1, pp. 7–39, 1997.

[55] Y. Lu, A. Kumar, S. Zhai, Y. Cheng, T. Javidi, and R. Feris, “Fully-adaptive feature sharing in multi-task networks with applications in person attribute classification,” *arXiv preprint arXiv:1611.05377*, 2016.

[56] R. K. Ando and T. Zhang, “A framework for learning predictive structures from multiple tasks and unlabeled data,” *Journal of Machine Learning Research*, vol. 6, no. Nov, pp. 1817–1853, 2005.

[57] A. Beck and M. Teboulle, “A fast iterative shrinkage-thresholding algorithm for linear inverse problems,” *SIAM journal on imaging sciences*, vol. 2, no. 1, pp. 183–202, 2009.

[58] P. Gutiérrez, M. Pérez-Ortiz, J. Sánchez-Monedero, F. Fernandez-Navarro, and C. Hervás-Martínez, “Ordinal regression methods: survey and experimental study,” *IEEE Transactions on Knowledge and Data Engineering*, vol. 28, no. 1, 2016. [Online]. Available: [http://dx.doi.org/10.1109/TKDE.2015.2457911](http://dx.doi.org/10.1109/TKDE.2015.2457911)

[59] J. Cheng, Z. Wang, and G. Pollastri, “A neural network approach to ordinal regression,” in *Neural Networks, 2008. IJCNN 2008.(IEEE World Congress on Computational Intelligence). IEEE International Joint Conference on*. IEEE, 2008, pp. 1279–1284.

[60] S. G. Mueller, M. W. Weiner, L. J. Thal, R. C. Petersen, C. Jack, W. Jagust, J. Q. Trojanowski, A. W. Toga, and L. Beckett, “The alzheimer’s disease neuroimaging initiative,” *Neuroimaging Clinics*, vol. 15, no. 4, pp. 869–877, 2005.

[61] A. Buja, G. Damiani, R. Gini, M. Visca, B. Federico, D. Donato, P. Francesconi, A. Marinì, A. Donatini, S. Brugaletta et al., “Systematic age-related differences in chronic disease management in a population-based cohort study: a new paradigm of primary care is required.” *PLoS One*, vol. 9, no. 3, p. e91340, 2014.

[62] D. Duricova, J. Burisch, T. Jess, C. Gower-Rousseau, P. L. Lakatos, and ECCO-EpiCom, “Age-related differences in presentation and course of inflammatory bowel disease: an update on the population-based literature,” *Journal of Crohn’s and Colitis*, vol. 8, no. 11, pp. 1351–1361, 2014.

[63] M. T. Westbrook and L. L. Viney, “Age and sex differences in patients’ reactions to illness,” *Journal of health and social behavior*, pp. 313–324, 1983.

[64] N. Geifman, R. Cohen, and E. Rubin, “Redefining meaningful age groups in the context of disease,” *Age*, vol. 35, no. 6, pp. 2357–2366, 2013.