The effect of inhomogeneities on dark energy constraints

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Abstract. Constraints on models of the late time acceleration of the universe assume the cosmological principle of homogeneity and isotropy on large scales. However, small scale inhomogeneities can alter observational and dynamical relations, affecting the inferred cosmological parameters. For precision constraints on the properties of dark energy, it is important to assess the potential systematic effects arising from these inhomogeneities. In this study, we use the Type Ia supernova magnitude-redshift relation to constrain the inhomogeneities as described by the Dyer-Roeder distance relation and the effect they have on the dark energy equation of state ($w$), together with priors derived from the most recent results of the measurements of the power spectrum of the Cosmic Microwave Background and Baryon Acoustic Oscillations. We find that the parameter describing the inhomogeneities ($\eta$) is weakly correlated with $w$. The best fit values $w = -0.933 \pm 0.065$ and $\eta = 0.61 \pm 0.37$ are consistent with homogeneity at $< 2\sigma$ level. Assuming homogeneity ($\eta = 1$), we find $w = -0.961 \pm 0.055$, indicating only a small change in $w$. For a time-dependent dark energy equation of state, $w_0 = -0.951 \pm 0.112$ and $w_a = 0.059 \pm 0.418$, to be compared with $w_0 = -0.983 \pm 0.127$ and $w_a = 0.07 \pm 0.432$ in the homogeneous case, which is also a very small change. We do not obtain constraints on the fraction of dark matter in compact objects, $f_p$, at the 95% C.L. with conservative corrections to the distance formalism. Future supernova surveys will improve the constraints on $\eta$, and hence, $f_p$, by a factor of $\sim 10$.

Keywords: dark energy experiments, supernova type Ia - standard candles

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1 Introduction

The discovery of the accelerated expansion of the universe [1, 2] indicates the existence of a cosmological fluid with negative pressure, dark energy. However, there are still significant problems in the standard model describing the current epoch of accelerated expansion (see [3] for a review). Current observations of Type Ia supernovae (SNe Ia; [4]) the cosmic microwave background (CMB; [5]) and baryon acoustic oscillation (BAO; [6]) have constrained the dark energy equation of state (EoS; the ratio of the pressure to density $w = P/\rho \sim -1$ with $\lesssim 5\%$ precision). Several models for cosmic acceleration e.g. scalar fields [7, 8] or a modification of general relativity could be a viable explanation of the observations (see [9] for a model comparison of the different scenarios).

The combination of SNe Ia luminosity distances with BAO angular scale and CMB geometric distance priors provides the most stringent constraints on the properties of dark energy [4, 6, 10, 11]. Apart from being crucial probes of the nature of dark energy, SNe Ia are also important tracers of structure in the universe. They have been used to independently measure $\sigma_8$, the rms linear fluctuation in the matter distribution [12], with future surveys expected to significantly improve the precision on growth of perturbations [13–15]. Several explanations of the late time acceleration (see [16–18] for a review), assume the cosmological principle, i.e. homogeneity and isotropy on large ($\sim 100$ Mpc) scales, however, as the universe is known to be inhomogeneous on smaller scales, the matter distribution in the line of sight to SNe Ia will effect the distance-redshift relation. An approximation was proposed to account for light traveling through emptier rather than denser regions of the universe in [19, 20] proposed a correction to the luminosity-redshift relation in homogeneous models and [21] derived a distance-redshift relation, known as the Dyer-Roeder (DR) relation. In this ansatz, the total matter density, $\rho_m$ contributes to the expansion of the universe, whereas a fraction of the density, $\eta \cdot \rho_m$, contributes to the focusing of the light in the line of sight to the source. The contribution of shear from inhomogeneities in the magnification is assumed to be negligible.
This approximate distance-redshift relation allows us to measure the inhomogeneity of the universe through, $1 - \eta$. The detection of gravitational waves (GW) from binary black hole mergers [22–24] has renewed interest in compact objects comprising a fraction of the dark matter (DM) content of the universe (see [25] for a review) and hence, observational limits from the magnitude-redshift relation of SNe Ia would be useful in testing this hypothesis. In the past, the study of the lensing distribution of SNe Ia has been proposed as a viable way to shed new light into this issue [26], and the recent discovery of the strongly lensed SN Ia iPTF16geu [27] further emphasizes the need to explore this possibility.

In this paper, we analyse the effect of inhomogeneities on the inferred properties of dark energy (density, EoS and its time dependence). We use constraints on inhomogeneities to evaluate the fraction of dark matter (DM) in compact objects from current and future SN Ia.

In section 2 we introduce the methodology and the data used in this work. In section 3 we present the parameter constraints for the different cosmological model scenarios in our study which are discussed in section 5.

2 Methodology and data

2.1 Type Ia supernovae

For our analysis we use the most recent SN Ia magnitude-redshift relation from the JLA compilation [28].

Theoretically, the distance modulus predicted by the homogeneous and isotropic, flat Friedman-Robertson-Walker (FRW) universe is given by

$$
\mu(z; \theta) = 5 \log_{10} \left( \frac{d_L}{10 \text{ Mpc}} \right) + 25
$$

where $z$ is the redshift, $\theta$ are the cosmological parameters and $d_L$ is given by

$$
D_L = \frac{c(1 + z)}{H_0 \sqrt{|\Omega_K|}} \sin\left( \sqrt{|\Omega_K|} \int_0^z \frac{dz'}{E(z')} \right)
$$

where $\Omega_K$ is the dimensionless curvature density and $\sin(x) = \{\sin(x), x, \sinh(x)\}$ for close, flat and open universes, respectively. $E(z) = H(z)/H_0$ is the normalised Hubble parameter.

$$
E(z)^2 = \Omega_M(1 + z)^3 + \Omega_{DE}(z) + \Omega_K(1 + z)^2,
$$

where

$$
\Omega_{DE}(z) = \Omega_{DE} \exp \left[ 3 \int_0^z \frac{1 + w(x)}{1 + x} dx \right],
$$

$w(z)$ is the dark energy EoS.

Observationally, the distance modulus is calculated from the SN Ia peak apparent magnitude ($m_B$), light curve width ($x_1$) and colour ($c$)

$$
\mu_{\text{obs}} = m_B - (M_B - \alpha x_1 + \beta c),
$$

where $M_B$ is the absolute magnitude of the SN Ia. Following [4], we apply a step correction ($\Delta M$) for the host galaxy stellar mass. We note that $\alpha$, $\beta$, $M_B$ and $\Delta M$ are nuisance parameters in the fit for the cosmology.

The $\chi^2$ is given by

$$
\chi^2_{\text{SN}} = \Delta^T C_{\text{SN}}^{-1} \Delta,
$$

where $\Delta = \mu - \mu_{\text{obs}}$ and $C$ is the complete covariance matrix described in [4].
Figure 1. Joint constraints on the present day matter density $\Omega_M$ and the equation of state of dark energy, $w$, under the assumption of homogeneity (left) and with $\eta$ as a free parameter (right). Although the constraints from the SN Ia are degraded, combining them with the CMB/BAO prior yields very similar results in both cases. The contours are at 1 and 2 $\sigma$ level. The plot was made using the python package corner [31].

2.2 Cosmic microwave background

Precise constraints on the expansion history can be derived by combining the SN Ia magnitude-redshift relation with complementary cosmological probes [29]. For the geometric constraints from the CMB we use the compressed likelihood from the Planck satellite, marginalised over the lensing amplitude ($A_L$) (see [5] for details). The CMB shift, $R$, position of the first acoustic peak in the power spectrum, $l_A$, and the baryon density at present day, $\Omega_b h^2$, comprise the data vector. The expression for the CMB shift and the position of the first acoustic peak are given by

$$R = \sqrt{\Omega_M H_0^2 d_A(z_*)/c},$$

and

$$l_A = \pi d_A(z_*)/r_s(z_*),$$

where $r_s(z)$ is the sound horizon at redshift, $z$, given by

$$r_s(z) = \frac{c}{\sqrt{3}} \int_0^{1+z} \frac{da}{a^2 H(a)\sqrt{(1 + a^3 \Omega_\gamma \Omega_b / 4)}},$$

where $\Omega_\gamma = 2.469 \cdot 10^{-5} h^{-2}$ and $h = H_0/100$ (see [30] for more details).

The value for $(R, l_A, \Omega_b h^2) = (1.7382, 301.63, 0.02262)$ with errors $(0.0088, 0.15, 0.00029)$ and covariance is

$$D_{\text{CMB}} = \begin{pmatrix} 1.0 & 0.64 & -0.75 \\ 0.64 & 1.0 & -0.55 \\ -0.75 & -0.55 & 1.0 \end{pmatrix},$$

such that the elements of the covariance matrix $C_{ij} = \sigma_i \sigma_j D_{ij}$. 

2.3 Baryon acoustic oscillation

The detection of the characteristic scale of the BAO in the correlation function of different matter distribution tracers provides a powerful standard ruler to probe the angular-diameter-distance versus redshift relation. BAO analyses usually perform a spherical average constraining a combination of the angular scale and redshift separation

\[ d_{z} = \frac{r_{s}(z_{\text{drag}})}{D_{V}(z)}, \quad (2.11) \]

with

\[ D_{V}(z) = \left( (1 + z)^{2}D_{A}(z)^{2} \frac{cz}{H(z)} \right)^{1/3}, \quad (2.12) \]

where \( D_{A} \) is the angular diameter distance. \( r_{s}(z_{\text{drag}}) \), is the sound horizon at the drag redshift given by equation (2.9).

For our analyses, we follow the method of [4] and use three measurements at \( z_{\text{eff}} = 0.106, 0.35 \) and 0.57 from [32–34] respectively. We consider a BAO prior of the form

\[ \chi^{2}_{\text{BAO}} = (d_{z} - d_{z}^{\text{BAO}})^{T}C_{\text{BAO}}^{-1}(d_{z} - d_{z}^{\text{BAO}}), \quad (2.13) \]

with \( d_{z}^{\text{BAO}} = [0.336, 0.1126, 0.07315] \) and \( C_{\text{BAO}}^{-1} = \text{diag}(4444., 215156., 721487.,) \).

We note that the WiggleZ team also presents three distance measurements (see [32, 35]). However, since the WiggleZ volume partially overlaps with that of the BOSS CMASS sample, and the correlations have not been quantified, we do not include the WiggleZ results in this study.

3 Parameter estimation

In this section, we quantify the parameter constraints on the dark energy EoS when using the DR distance redshift relation. We also compute the Hubble residuals for the SNe Ia and compare the skewness in each redshift bin to simulations with varying fractions of dark matter in compact objects.

3.1 Effect on dark energy constraints

The general differential equation for the distance between two light rays of the boundary of a small light cone propagating far away from all clumps of matter in an inhomogenous universe is developed in [19, 36–40]. This relation for angular diameter distance as a function of the inhomogeneities given in [21] is

\[ QD'' + \left( \frac{2Q}{1 + z} + \frac{Q'}{2} \right) D' + \frac{3}{2} \eta \Omega_{M} (1 + z) D = 0, \quad (3.1) \]

where \( \eta \) is the fraction of the matter density in opaque clumps and \( Q(z) = E(z)^{2} \), hence, \( Q(z) \) depends on the cosmological model. In previous studies (e.g. [41, 42]), the model has been fixed to the standard \( \Lambda \)CDM cosmology, assuming that the accelerated expansion is caused by a cosmological constant. In some cases, even the background cosmological parameters, e.g. \( \Omega_{M}, \Omega_{\Lambda} \) have been fixed to the values from concordance cosmology [42].
| Parameter | Prior |
|-----------|-------|
| $\Omega_M$ | U[0, 1] |
| $\eta^a$ | U[0, 5.] |
| $w$ | U[-2, 2] |
| $H_0$ | U[50, 100] |
| $\alpha$ | U[0, 1] |
| $\beta$ | U[0, 4] |
| $\Delta M$ | U[0, 0.2] |
| $M_B$ | U[-35, 15] |



\[Q(z) = \Omega_M(1 + z)^3 + \Omega_K(1 + z)^2 + \Omega_{DE}(z, w)\]  

with $w$, the EoS of dark energy being a free parameter. We test two simple cases, a constant $w$ and time-varying parametrisation $w(a) = w_0 + w_a(1 - a)$ [43, 44], however, this can be extended to any expression for $w(a)$. We assume flatness, i.e. $\Omega_M + \Omega_{DE} = 1$.

We fit the SN Ia Hubble diagram with the luminosity distance which is derived from the angular diameter distance using the distance duality relation [45–48].

\[D_L = D_A(1 + z)^2.\]  

The lower limit on the prior for $\eta$ is set by $\eta > 0$. If there are no small scale inhomogeneities, then $\eta = 1$. Finding $\eta > 1$ is possible if the average density in the beam is larger than the global density (such cases could be result of a selection effect).

The inhomogeneities investigated in this work are on scales much smaller than probed by the CMB and BAO observations [49, 50], hence, these data are fit with the assumption of homogeneity (i.e. $\eta = 1$). DM in compact objects affects the distances but not the expansion history (however, see backreaction models, e.g. [51]). Therefore, the only probe sensitive to $\eta$ is SN Ia. The masses considered here given by a comparison of Einstein radii and size of SNe [27], corresponding to $\gtrsim 10^{-2} M_{\odot}$ [52].

We fit equation (3.1) to the SN Ia Hubble diagram. The complementary probes help to constrain the background cosmological parameters. We impose a conservative, uniform prior of $\eta$ (see table 1). $\eta > 1$ corresponds to the average density in the beam being larger than the global density, but we do not strictly impose $\eta < 1$ at this stage of the inference. We constrain $\eta$ along with the parameters describing the dark energy density, equation of state, and its time-dependence.

### 3.1.1 Constant $w$

We infer $w = -0.933 \pm 0.065$ which is consistent with the cosmological constant and $\eta = 0.61 \pm 0.37$ consistent with homogeneity within the statistical accuracy of this study.
To investigate the effect of inhomogeneities on the inferred $w$, we fit all the datasets with $\eta = 1$ i.e. no inhomogeneities and find $w = -0.961 \pm 0.055$ slightly shifted from the case where $\eta$ is a free parameter. We also fit for $\eta$ fixing $w = -1$ i.e. concordance cosmology (red histogram, figure 2) and infer $\eta = 0.81 \pm 0.33$. Using the relation between $\eta$ and the fraction of DM in compact objects, $f_p$, from [53] we get $f_p < 0.73$ at 68% C.L., however, we cannot constrain $f_p$ at the 95% C.L. We note that without using the relation from the SNOC simulations, that conservatively corrects for Weyl focussing, we get a limit of $f_p < 0.81$ at the 95% C.L.

We note that the shift in $w$ in the case with inhomogeneities allowed is very small (0.03) and that the increase in the uncertainty is small. In both the $\Lambda$CDM and $w$CDM background cases, the value of $\eta$ is consistent with homogeneity.

### 3.1.2 Time-dependent equation of state

We note that in the constant $w$ model, there is only a small effect of $\eta$ being a free parameter. We test a general time-dependent equation of state parametrisation of $w(a) = w_0 + w_a(1-a)$ and present constraints in figure 3.

We find $w_0 = -0.954 \pm 0.112$ and $w_a = 0.059 \pm 0.418$ consistent with a cosmological constant, $\eta = 0.67 \pm 0.39$ consistent with homogeneity. Fixing $\eta = 1$ we find $w_0 = -0.982 \pm 0.127$ and $w_a = 0.070 \pm 0.432$. The change in the $w_0$ and $w_a$ is very small with a slight increase in the uncertainties.
3.2 Hubble residuals

A complementary method we employ to test for the presence of inhomogeneities involves a direct comparison of the observed SN Ia Hubble residuals and expected perturbations for a given fraction of DM in compact objects, $f_p$ [54]. For future surveys, forecasts for $f_p$ were evaluated in [26]. We simulate the perturbations using the ray-tracing software SNOC [55] for 8 different values of $f_p$, namely 0, 0.05, 0.10, 0.15, 0.20, 0.25, 0.50, 0.80, 1.

We calculate the Hubble residuals using an input ΛCDM universe with $\Omega_M = 0.3$ and take the nuisance parameters from [28], i.e. $\alpha = 0.141$, $\beta = 3.102$. We use only the Supernova Legacy Survey (SNLS) subsample [56, 57] of the JLA compilation since it covers a wide range of redshifts and is a uniformly observed set of SNe providing a “clean” dataset to constrain the value of $f_p$. For this subsample, we use an intrinsic scatter of 0.08 mag, as reported in [28].

We compute 10 000 Monte Carlo realisations from a distribution $\mathcal{N}(0, \sigma_i)$ where $\sigma_i$ is the measurement error for the $i^{th}$ SN which includes the error in $m_B$, $x_1$ and $c$ as well as the intrinsic scatter for the SNLS subsample. Each iteration of the realisations is added to the perturbations calculated from SNOC and a Kolmogorov-Smirnov (KS) test is used to determine whether both the realisations and the Hubble residuals are drawn from the same parent population, using the criterion that $p < 0.05$ suggests that they come from different parent populations. A value of $f_p$ can be excluded if more than 90% of the realisations are seen to be drawn from a different parent population from the Hubble residuals.

Applying the above test to the $f_p$ values from 0 to 1, we find that none of the simulations have a significant fraction of the realisations that appear to be from a different parent population than the residuals (for the most extreme case of $f_p = 1$ only $\sim 60\%$ of the realisations have a $p$-value $< 0.05$). Hence, all of the $f_p$ values are consistent with the data and

![Figure 3. Constraints on $w_0$ and $w_a$ (see section 3.1.2). The blue contours are the constraints with $\eta = 1$. The contours are at the 1 and 2 $\sigma$ level.](image)
Figure 4. A comparison of the current constraints on \( w \)-\( \eta \) (green) with forecasts (red) from a future SN Ia experiment \cite{60}. Using the best fit parameters from current fits we find that we can exclude \( \eta = 1 \) significantly (\( \sigma(\eta) = 0.03 \)). We also show the constraints for an underlying cosmology with \( w = -1 \) and \( \eta = 1 \), which will constrain the fraction of DM in compact objects, \( f_p < 12\% \) at 95\% C.L. The contours are at the 1 and 2 error level.

We cannot draw any conclusions on the fraction of DM in compact objects by just analysing the Hubble residuals of the SNe Ia.

We note that we do not obtain stringent constraints on \( f_p \) compared to recent studies, (e.g. \cite{58}) since the authors use the complete JLA catalog with the highest-\( z \) supernovae while we restrict our analyses to the largest uniform measured subsample since we conservatively don’t want to be affected by selection effects and photometric systematics. We note that \cite{59} find fraction being < 1.09 at 95\% C.L., which is similar to the constraints presented here. This is because their assumptions on the compact object mass distribution, SN physical size and background cosmology differ from the ones in \cite{58}.

4 Forecast for future SN survey

Although current constraints are consistent with homogeneity, the limits on the fraction of DM in compact objects are weak. Here, we investigate improvements in parameter estimates on \( \eta \) and \( w \) with future SN Ia surveys.

For our analyses we use the forecasts from the DESIRE survey \cite{60} which combines a ground-based low-\( z \) and intermediate-\( z \) with LSST with a space-based high-\( z \) lever extending to \( \sim 1.5 \). We combine the future SN Ia distances with current constraints from the CMB, BAO and \( H(z) \) data (see section 2). We test cosmological with input parameters \((\eta, w) = (0.61, -0.933)\), best fit values from current data and \((1, -1)\), a homogenous universe with a cosmological constant. The resulting contours are plotted in figure 4.
We obtain \( \eta = 0.61 \pm 0.030 \) and \( \eta = 1.00 \pm 0.036 \) (for the two input cases of \( \eta = 0.61 \) and \( \eta = 1 \)) which is a significant improvement on the current constraints (figure 4 has a comparison with current \( w \)CDM constraints). This would translate to a limit of < 12% of DM in compact objects. We also note that constraints on \( w \) improve to 4% compared to \( \sim 7\% \) with current data (note that the CMB and BAO data are the same as in the current analyses, hence, the improvement in \( w \) is only from the SNe and not the complementary probes). The forecast illustrates the power of an improved SN Ia Hubble diagram, beyond constraining dark energy models (e.g. [60, 61]), to test homogeneity and precisely estimate the fraction of DM in compact objects. We obtain similar limits on \( \eta \) (\( \sim 5\% \)) from the expected Hubble diagram of SN Ia observed with the WFIRST satellite [61].

We also constrain time-dependent dark energy EoS in the presence of inhomogeneities. We find that there is very little effect of \( \eta \) as a free parameter on the inferred values of \( w_0 \) and \( w_a \) which are \( -0.954 \pm 0.113 \) and \( 0.05 \pm 0.418 \) which is consistent with previous analyses (e.g. [6]). Hence, for various dark energy properties, namely, the density, EoS and time dependence, the presence of possible inhomogeneities does not significantly change the parameter constraints.

5 Discussion and conclusion

We studied the effect of non-uniformities on inferred properties of dark energy from the SNe Ia Hubble diagram. The impact of inhomogeneities on dark energy parameters was theoretically studied in [62–64] and of “Swiss-Cheese” cosmological models in [65–67]. The cumulative effect of the small scale inhomogeneities could (completely, or, in part) mimick a cosmological constant by altering the observational and dynamical relation on large scales [68]. Hence, it is critical to evaluate the effect of the clumpiness parameter, \( \eta \), on the inference of \( w \). We find that \( \eta \) is degenerate with \( w \), such that a higher value of \( \eta \) implies a more negative \( w \).

For a fixed \( \eta = 1 \), and with \( \eta \) as a free parameter \( w \) is consistent at < 2\( \sigma \) of the cosmological constant.

For a background concordance cosmology (i.e. \( w = -1 \)), we confirm previous analyses [42, 69] that find \( \eta = 0.81 \pm 0.33 \) at the 68% C.L. However, we stress that we do not a priori fix the cosmological parameter values as in [42], but rather we constrain it with data complementary to SN Ia distances. We also obtain more stringent constraints than [70] since we use complementary datasets that constrain \( \Omega_M \) very well. Using the relation between \( \eta \) and the fraction of DM in compact objects (\( f_p \)) [53] we find \( f_p < 0.73 \) at 68% C.L., but the data are not constraining at the 95% C.L. However, we note the relation between \( \eta \) and \( f_p \) accounts for the Weyl focussing from matter outside the beam. If we do not make this conservative correction we can constrain \( f_p < 0.81 \) at the 95% C.L. Some recent studies using less conservative constraints (e.g. [58]) find stronger constraints on \( f_p \), however [59], under another very different set of assumption, find the data to be consistent with all DM in compact objects.

Future SNe Ia, that aim to significantly increase the number of SNe at \( z > 1 \), would be crucial to precisely estimate the fraction of DM in compact objects and therefore, evaluate departures from homogeneity.

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