Tight focusing of a higher-order radially polarized beam transmitting through multi-zone binary phase pupil filters

Hanming Guo,1,2,* Xiaoyu Weng,1 Man Jiang,1 Yanhui Zhao,2 Guorong Sui,1 Qi Hu,1 Yang Wang,3 and Songlin Zhuang1

1Engineering Research Center of Optical Instrument and System, Ministry of Education, Shanghai Key Lab of Modern Optical System, School of Optical-Electrical and Computer Engineering, University of Shanghai for Science and Technology, 516 Jungong Rd, Shanghai 200093, China
2Department of Engineering Science and Mechanics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA
3Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China
hmguo@usst.edu.cn

Abstract: When the pupil filters are used to improve the performance of the imaging system, the conversion efficiency is a critical characteristic for real applications. Here, in order to take full advantage of the subwavelength focusing property of the radially polarized higher-order Laguerre-Gaussian (LG) beam, we introduce the multi-zone binary phase pupil filters into the imaging system to deal with the problem that the focal spot is split along the $z$ axis for the small size parameter of the incident LG beam. We provide an easy-to-perform procedure for the design of multi-zone binary phase pupil filters, where the zone numbers of $\pi$ phase are uncertain when the optimizing procedure starts. Based on this optimizing procedure, we successfully find the set of optimum structures of a seventeen-belt binary phase pupil filters and generate the excellent focal spot, where the depth of focus, the focal spot transverse size, the Strehl ratio, and the sidelobe intensity are 9.53\$\lambda$, 0.41\$\lambda$, 41.75\% and 16.35\% in vacuum, respectively. Most importantly, even allowing the power loss of the incident LG beam truncated by the pupil of the imaging system, the conversion efficiency is still as high as 37.3\%. Theoretical calculations show that we succeed to have sufficient conversion efficiency while utilizing the pupil filters to decrease the focal spot and extend the depth of focus.

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1. Introduction

In an imaging system with high numerical aperture (NA), because of the asymmetry of polarization with respect to the imaging system, the focal spot is not a circle but ellipsoidal elongating in the polarization direction for a linearly polarized beam [1,2]. If a circularly polarized beam is used, the focal spot is circularly symmetric, but the focal spot size is larger than that for a linearly polarized beam [2]. Compared with a linearly polarized beam, a radially polarized beam can produce a smaller focal spot with circularly symmetric intensity distribution for a high NA imaging system [3], which means that it cannot only improve the resolutions, but also make the resolutions in all transverse directions same of the imaging system [4]. From the view of vector plane wave spectrum, as there are only TM-wave (the polarization direction of magnetic field perpendicular to the plane formed by wave vector and longitudinal direction) [5], the surface plasmon mode is excited more efficiently by the radially polarized beam [6,7]. Another attractive characteristic of the radially polarized beam is that it can create the focal spot with a strong longitudinal electric field. This strong longitudinal electric field can be effectively used for particle manipulation [8], Raman spectroscopy [9], second-harmonic generation [10], material processing [11], and fluorescence imaging [12]. However, most of the studies on the radially polarized beam and its applications have concerned the so-called doughnut-shaped beam, which is represented as a fundamental radially polarized mode (01TM mode) [3,4,6–18]. Recently, Kozawa et al. report that the radially polarized higher-order Laguerre-Gaussian beam (LG) has higher performance than the low-order one and can generate the focal spot beyond the Abbe's diffraction limit ~0.52 λ [19–21]. Tian and Pu also utilize the double-ring-shaped azimuthally polarized LG beam to form a subwavelength focal hole near the focus [22]. This kind of
A higher-order LG beam has concentric multi ring-shaped intensity pattern with $\pi$ phase shift between the rings. Therefore, the higher-order LG beam is equivalent to the zero-order LG beam transmitting through the combination of a multi-zone binary phase and a successive amplitude pupil filters. However, for the higher-order LG generated directly by the laser, as the concentric multi ring-shaped intensity pattern is the inherent properties of the higher-order LG beam, the equivalent successive amplitude pupil filters have no loss of light energy when we cannot consider how to generate the higher-order LG. In other words, for the higher-order LG generated directly by the laser, the equivalent successive amplitude pupil filters not only can reduce the size of the focal spot, but also have no loss of light energy unlike the amplitude pupil filters placed at the pupil of the imaging system illuminated by the zero-order LG beam. Unfortunately, the equivalent combination of a multi-zone binary phase and a successive amplitude pupil filters and the relationship between the two cannot be adjusted arbitrarily for the higher-order LG beam. The natural question arises whether we can further reduce the focal spot size or extend the depth of focus by utilizing the multi-zone binary phase pupil filters to redistribute the phase shift between the rings of the radially polarized higher-order LG beam.

The binary phase pupil filters are widely used to reduce the focal spot size or extend the depth of focus, but, because of the complexity of optimizing multi-zone binary phase pupil filters, most of authors only focus on the design for the two- or three-zone binary phase pupil filters [23–25]. Wang et al. have an optimized design on the five zone binary phase filters, but the optimization method is not a strictly defined procedure [15]. To our knowledge, there is not an easy-to-perform procedure for the design of multi-zone binary pupil filters until now. However, as the focusing properties of the radially polarized higher-order LG beam are already excellent, i.e., beyond the Abbe's diffraction limit $-0.5\lambda$, and the intensity of each ring is successive and inhomogenous, we think that the multi-zone binary phase pupil filters are more likely to adjust finely the phase shift between the multi ring-shaped intensity patterns of the higher-order LG beam so that one might further improve its focusing properties.

In addition, when the pupil filters are used to improve the performance of the imaging system, the conversion efficiency is a critical characteristic for real applications. Wang et al. characterize the conversion efficiency as the ratio of the total energy within the focal volume after using the pupil filters to its original energy within the focal volume before using the pupil filters [15]. How to have sufficient conversion efficiency while utilizing the pupil filters to decrease the focal spot and extend the depth of focus is still a big challenge until now. For example, the narrow-width annular beams are used to decrease the focal spot and extend the depth of focus, but all have insufficient conversion efficiency (at the level of a few per cent) [16–18].

In this paper, we introduce the multi-zone binary phase pupil filter into the imaging system while taking full advantage of the subwavelength focusing property of the radially polarized higher-order LG beam. We provide an easy-to-perform procedure for the design of multi-zone binary phase pupil filters, where the zone numbers of $\pi$ phase are uncertain when the optimizing procedure starts. Based on this optimizing procedure, we successfully find the set of optimum structures of a seventeen-belt binary phase pupil filters and generate the excellent focal spot, where the depth of focus, the focal spot transverse size, the Strehl ratio, and the sidelobe intensity are $9.53\lambda$, $0.41\lambda$, $41.75\%$ and $16.35\%$ in vacuum, respectively. Most importantly, even allowing the power loss of the incident LG beam truncated by the pupil of the imaging system, the conversion efficiency is still as high as $37.3\%$. To the best of our knowledge, this is the first report of the focal spot with subwavelength transverse size ($0.41\lambda$), ultra long depth of focus ($9.53\lambda$), and high conversion efficiency (37.3%) until now.
2. Focusing model and the optimizing procedure

Shown in Fig. 1, we assume that the time dependence is \( \exp(-j\omega t) \) and the incident light propagating along the +z axis is focused by an aplanatic imaging system \( L \) obeying the sine condition. The ideal focus \( O \) is the origin of rectangular coordinates \((x, y, z)\) and its corresponding cylindrical coordinates \((r, \phi, z)\). On the basis of the classical vector diffraction theory of polarized beams [1], the electric fields in the focal region for the high-NA imaging system illuminated by a radially beam can be expressed as [14,19]

\[
E_z(r, \phi, z) = A \int_0^\alpha \cos^2 \theta \sin(2\theta) J_m(\theta) \Gamma_J(\theta) J_1(kr \sin \theta) \exp(jkz \cos \theta) d\theta, \quad (1a)
\]

\[
E_z(r, \phi, z) = j2A \int_0^\alpha \cos^2 \theta \sin^2 \theta J_m(\theta) \Gamma_J(\theta) J_1(kr \sin \theta) \exp(jkz \cos \theta) d\theta, \quad (1b)
\]

where the maximum aperture angle \( \alpha = a \sin(\text{NA}/n) \), \( n \) is the refractive index in image space, \( k \) is the wave number in image space, \( A \) is a constant, \( J_m \) is the first kind of Bessel functions of orders \( m \), and \( T(\theta) \) is transmission function of the pupil filters. \( I_m(\theta) \) is the relative amplitude of the electric field of the incident beam at the pupil plane. For the higher-order LG incident beam, its relative amplitude can be expressed as [19]

\[
I_m(\theta) = \frac{\beta_0^2 \sin \theta}{\sin^2 \alpha} \exp \left(-\frac{\beta_0^2 \sin^2 \theta}{\sin^2 \alpha} \right) \left[ \frac{2\beta_0^2 \sin^2 \theta}{\sin^2 \alpha} \right], \quad (2)
\]

where the size parameter \( \beta_0 \) of the incident beam is defined as the ratio of the pupil radius to the beam waist, \( L_p^1 \) is the generalized Laguerre polynomial, and \( p \) is the radial mode number.

In this paper, the multi-zone binary phase pupil filters is used (see Fig. 1). As the transmission function \( T(\theta) \) of the multi-zone binary phase pupil filters is irrelevant to the azimuthal angle \( \phi \) and only dependent on the angle \( \theta \) with \( 0 \leq \theta < \alpha \), \( T(\theta) \) is a one-dimensional function of the variable \( \theta \). Therefore, in order to clearly introduce the optimizing procedure for the multi-zone binary phase pupil filters, we consider the multi circular belts shown in Fig. 1 as the multi rectangular belts with variation of angle \( \theta \) (see Fig. 2), namely each rectangular belt denoting one circular belt. Shown in Fig. 2(a), the integral interval \( 0 \leq \theta < \alpha \) of angle \( \theta \) in Eqs. (1a) and (1b) is divided into \( N \) rectangular belts and each rectangular belt corresponds to the angle interval \( \theta_{i-1} \leq \theta < \theta_i \). The widths of each rectangular belt, i.e., the angle interval \( \theta_{i-1} \leq \theta < \theta_i \), may be same or different. We number all rectangular belts from left to right and utilize these numbers to denote the positions of the rectangular belts. Our optimizing procedure for the multi-zone binary phase pupil filters are...
divided into four steps. In the first step, we assume the binary phases 0 and \( \pi \) are used in this paper. We let \( E_{no} \) and \( E_{yes} \) denote the electric fields in the focal region for the imaging system without pupil filters and with circular belts of phase \( \pi \), respectively. Namely \( E_{no} \) is the results of the integrand of Eqs. (1a) and (1b) with \( T(\theta) = 1 \) in the whole integral interval \( 0 \leq \theta < \alpha \). \( E_{yes} \) is the results of the integrand of Eqs. (1a) and (1b) with \( T(\theta) = \exp(j\pi) = -1 \) in all integral intervals \( \theta_{i-1} \leq \theta < \theta_i \) corresponding to the circular belts with phase \( \pi \). As the integral interval \( 0 \leq \theta < \alpha \) of angle \( \theta \) in Eqs. (1a) and (1b) is divided into \( N \) circular belts and each circular belt corresponds to one angle interval \( \theta_{i-1} \leq \theta < \theta_i \), it is obvious that \( E_{no} = \sum_{i=1}^{N} E_i \), where \( E_i \) denotes the individual electric field in the focal region of the incident beam transmitting each circular belt with phase \( \pi \), i.e., the results of the integrand of Eqs. (1a) and (1b) with \( \theta_{i-1} \leq \theta < \theta_i \). When the numbers of the circular belt with phase \( \pi \) among the \( N \) circular belts is \( M \) \((1 \leq M \leq N)\), then \( E_{yes} = \sum_{i=1}^{M} E_i \). Therefore, the whole electric fields \( E_{whole} \) in the focal region of the incident beam transmitting the whole pupil filters are the coherent composition of the focusing electric fields of each circular belt, i.e., \( E_{whole} = E_{no} + 2E_{yes} \). The reason that we use the formula \( E_{whole} = E_{no} + 2E_{yes} \) is to reduce computation cost of our optimizing procedure because \( E_i \) can be calculated and recorded before the following recursive procedure starts.

\[
F(m+1, M, n_{\text{max}} + n_{\text{max}} + 1, \mathbf{R}) = F(m, M, n_{\text{min}} + n_{\text{max}} + 1, \mathbf{R}),
\]

(3)

where \( M \) still denotes the numbers of rectangular belts that will be added phase \( \pi \) indicated before. \( m \) denotes that we are adding the \( m \)th phase \( \pi \) into the pupil filters. \( n_{\text{min}} \) denotes the current position of the \( m \)th phase \( \pi \). \( n_{\text{min}} \) and \( n_{\text{max}} \) represent the minimum and the maximum positions at which the \( (m+1) \)th phase \( \pi \) might be placed, respectively. \( \mathbf{R} \) is a \( M \) row matrices used for recording the positions of \( M \) rectangular belts with phase \( \pi \). The recursive function (3) begins with \( L(1, M, 1, N - M + 1, \mathbf{R}) \) and ends with \( m = M \).

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Fig. 2. Diagram of multi-zone binary phase pupil filters viewed as the \( N \) rectangular belts (a) before and (b) after adding phase \( \pi \) for ease of analysis. The blue rectangular denotes the phase \( \pi \).

In the second step, shown in Fig. 2(a), we first assume all \( N \) rectangular belts have not phase \( \pi \) and the numbers of rectangular belts that will be added phase \( \pi \) are \( M \) \((1 \leq M \leq N)\). Then we start to add phase \( \pi \) into \( M \) rectangular belts of the total \( N \) rectangular belts. This adding phase \( \pi \) procedure are always carried out in sequence, from left to right, and can be described by the following recursive function

\[
F(m+1, M, n_{\text{max}} + 1, \mathbf{R}) = F(m, M, n_{\text{max}} + 1, \mathbf{R}),
\]

where \( M \) still denotes the numbers of rectangular belts that will be added phase \( \pi \) indicated before. \( m \) denotes that we are adding the \( m \)th phase \( \pi \) into the pupil filters. \( n_{\text{max}} \) denotes the current position of the \( m \)th phase \( \pi \). \( n_{\text{min}} \) and \( n_{\text{max}} \) represent the minimum and the maximum positions at which the \( (m+1) \)th phase \( \pi \) might be placed, respectively. \( \mathbf{R} \) is a \( M \) row matrices used for recording the positions of \( M \) rectangular belts with phase \( \pi \). The recursive function (3) begins with \( L(1, M, 1, N - M + 1, \mathbf{R}) \) and ends with \( m = M \).
In the third step, when the recursive function (3) stops, we start to calculate the electric fields \( E_{\text{act}} \) in terms of the actual positions of the \( M \) rectangular belts with phase \( \pi \) recorded by \( \mathcal{R} \). Then we examine whether \( E_{\text{whole}} \) meets the requirements determined by the merit parameters. The merit parameters we have selected are the Strehl ratio \( S \), the sidelobe intensity \( I_s \), and the superresolution gain factor \( G \), respectively. \( S \) is a relevant parameter for estimating conversion efficiency [15] and is defined as the ratio of the intensity at the focal point of the imaging system with to that corresponding to the one without pupil filters. \( I_s \) is defined as the second maximum intensity relative to the central peak intensity at the focal plane. \( G \) is defined as the ratio of the focal spot size of the imaging system with to that corresponding to the one without pupil filters, where the focal spot size is defined as the full width at half-maximum (FWHM) of the focal spot at the focal plane. As indicated by Wang et al., the conversion efficiency is a critical characteristic for real applications [15]. So, during optimizing procedure, \( S > 30\% \), \( I_s < 20\% \), and \( G < 1 \) must be met because \( S > 30\% \) can basically assure a conversion efficiency above 20\%. Through the above three steps, we can obtain the optimum structures of the multi-zone binary pupil filters when \( M \) rectangular belts are added phase \( \pi \).

In the final step, we further search the final optimum structures of the multi-zone binary phase pupil filters by changing \( M \). As the phases of the belts are either zero or \( \pi \), we only consider the case of \( M \) varying from one to the minimum integer bigger than \( N/2 \) because the optimizing results are same for the other half. After the optimizing procedure finished, the final optimum structures (i.e., the zone numbers and their radii) of the multi-zone binary phase pupil filters can be calculated by \( \mathcal{R} \). It is noted that \( M \) is not the zone numbers of the binary phase pupil filters because many rectangular belts with phase zero or \( \pi \) might be adjacent (see Fig. 2(b)).

The above optimizing procedure is not only a strictly defined procedure, but also an easy-to-perform. Moreover, the zone numbers of binary phase pupil filters are uncertain when the optimizing procedure starts. The final optimization structures of the multi-zone binary phase pupil filters are majorly determined by the merit parameters.

3. Simulations and discussions

As the higher the order of LG beam is, the smaller the focal spot size is in the case of the same NA [19], we will only focus on the focusing properties of the radially polarized fifth-order LG beam, namely \( p = 5 \) in the generalized Laguerre polynomial \( L_p^0 \) (hereafter called the LG_{5,1} beam). In addition, the NA = 0.95, the refractive index \( n = 1 \) in image space, and the constant \( A = 1 \) in Eqs. (1a) and (1b) are used in our calculations.

3.1 Effects of the size parameter \( \beta_0 \) of the incident beam on the distribution of the focal spot

In this Subsection, the multi-zone binary phase pupil filters are not used in all calculations, i.e., \( T(\theta) = 1 \) in Eqs. (1a) and (1b). Figure 3(a) and 3(b) show the intensity (\( |E|^2 \)) profiles along the transverse axis (\( r \)) at the focal plane (\( z = 0 \)) and the \( z \) axis for various size parameters \( \beta_0 \), respectively. The intensity profiles are normalized by the intensity of the focal point (\( x = y = z = 0 \)). In this paper, the focal spot size and the depth of focus are defined as the FWHMs of the focal spot at the focal plane and at the \( z \) axis, respectively. Shown in Figs. 3(a) and 3(b), the smaller the \( \beta_0 \) is, the smaller the focal spot size is, whereas the longer the depth of focus is, which means that the smaller \( \beta_0 \) is beneficial to generate the needles of longitudinally polarized light [15]. However, when the \( \beta_0 \) is smaller than 3.58 for the radially polarized LG_{5,1} beam, the focal spot has central peak intensity at the focal plane.
(see Fig. 3(a)), but start to be split along the $z$ axis (see Fig. 3(b)). Namely, the intensity of the focal point is no longer the peak intensity along the $z$ axis. Meanwhile, in terms of the definition of the focal plane, i.e., the plane with intensity being the peak intensity along the $z$ axis, the actual focal plane will shift the original focal plane ($z = 0$) when the $\beta_0$ is smaller than 3.58. In the actual focal plane, the focal spot size become bigger. Therefore, there is a discrepancy between the focal spot size reduced and the focal spot split along the $z$ axis when decreasing the $\beta_0$. When the imaging system illuminated by the radially polarized LG$_{5,1}$ beam, the $\beta_0$ cannot be smaller than 3.58.

As the incident beam is truncated by the pupil of the imaging system, we define the effective power of the incident beam as the ratio of the power of the incident beam after truncated by the pupil to that of the incident beam before truncated by the pupil. Shown in Fig. 4, the intensity of the focal point ($x = y = z = 0$) reaches the maximum value at $\beta_0 = 3.68$ and the effective power of the incident beam reaches 98.6% at the same time. However, the focal spot size is only about 0.46$\lambda$ for $\beta_0 = 3.68$. When $\beta_0 = 3.58$, the focal spot size is about 0.45$\lambda$ and slightly smaller than that for $\beta_0 = 3.68$. We note that the focal spot size is about 0.414$\lambda$ and the effective power of the incident beam also reaches 90% for $\beta_0 = 3.36$ (see Fig. 4). Therefore, when the radially polarized LG$_{5,1}$ beam is used to generate the small focal spot, one need not to focus on enhancing the effective power of the incident beam because the effective power of above 90% is enough for the real applications. Compared with the case of $\beta_0 = 3.68$, the focal spot size corresponding to $\beta_0 = 3.36$ is decreased by 10%, which is attractive for the high resolution imaging and lithography. Unfortunately, the $\beta_0 = 3.36$ cannot be used because the focal spot will be split along the $z$ axis.

Fig. 3. Intensity profiles along (a) the transverse axis ($r$) at the focal plane ($z = 0$) and (b) the $z$ axis for various size parameters $\beta_0$ of the radially polarized LG$_{5,1}$ beam. The intensity profiles are normalized by the intensity of the focal point ($x = y = z = 0$).
Based on the above discussions, we conclude that for the imaging system with $NA = 0.95$ illuminated by the radially polarized LG$_{5,1}$ beam, the smallest focal spot that can be achieved is about $0.45\lambda$ and the optimum size parameter $\beta_0$ of the incident beam should be 3.58. The smaller the $\beta_0$ is, the smaller the focal spot. However, when $\beta_0 < 3.58$, the focal spot will be split along the $z$ axis. The natural question arises whether we can deal with the problem that the focal spot is split for the smaller $\beta_0$ by utilizing the multi-zone binary phase pupil filters. In the following, we will deal with it.

3.2 Generate the excellent focal spot with multi-zone binary phase pupil filters

In our optimizing procedure of the multi-zone binary phase pupil filters given before, the zone numbers of binary pupil filters are uncertain when the optimizing procedure starts. Before the optimizing procedure runs, we need to set the numbers $N$ of rectangular belts and calculate the angles $\theta$ that are determined by the used method of dividing the integral interval $0 \leq \theta < \alpha$ of angle $\theta$. The simplest way is to divide the integral interval $0 \leq \theta < \alpha$ into $N$ equal parts. We do not use this way in this paper. In order to get some rectangular belts with narrower width, we first divide the integral interval $0 \leq \theta < \alpha$ into fifteen equal parts, and then divide the odd rectangular belts into four equal parts. So, the numbers of rectangular belts are $N = 39$. In addition, as indicated before, during optimizing procedure, the merit parameters $S > 30\%$, $I_s < 20\%$, and $G < 1$ must be met. Basing on the above optimizing procedure of the multi-zone binary phase pupil filters, for the $\beta_0 = 3.36$, we find the set of optimum structures of a seventeen-belt binary phase pupil filters:

$$
\begin{align*}
\theta_1 &= 14.36, \theta_2 &= 19.15, \theta_3 &= 22.74, \theta_4 &= 23.95, \theta_5 &= 28.72, \theta_6 &= 29.92, \\
\theta_7 &= 32.31, \theta_8 &= 33.51, \theta_9 &= 39.49, \theta_{10} &= 40.69, \theta_{11} &= 43.08, \theta_{12} &= 47.87, \\
\theta_{13} &= 52.66, \theta_{14} &= 57.44, \theta_{15} &= 58.64, \theta_{16} &= 59.84.
\end{align*}
$$

For each zone of the pupil filters, the corresponding radial positions are written as $r_i = \sin \theta_i / NA$ (normalized to the optical aperture). So, the corresponding positions $r_i$ are given by

$$
\begin{align*}
r_1 &= 0.2611, r_2 = 0.3453, r_3 = 0.4069, r_4 = 0.4271, r_5 = 0.5059, r_6 = 0.525, \\
r_7 &= 0.5627, r_8 = 0.5811, r_9 = 0.6695, r_{10} = 0.6863, r_{11} = 0.719, r_{12} = 0.7807, \\
r_{13} &= 0.8369, r_{14} = 0.8872, r_{15} = 0.8989, r_{16} = 0.9101.
\end{align*}
$$
The transmission function $T(\theta)$ of this binary phase pupil filters is shown in Fig. 5(e).

Figures 5(a)-5(c) describe the intensity distribution of the focal spot in the $rz$ plane for the total intensity, the transverse component, and the longitudinal component, respectively. Figure 5(d) describes the total intensity distribution of the focal spot along the $r$ (red dashed curve) and $z$ (blue solid curve) axes, respectively. The focal spot size (i.e., the FWHM of the total intensity of the focal spot at the focal plane) is 0.41$\lambda$ and the depth of focus is 9.53$\lambda$. The Strehl ratio $S$ and the sidelobe intensity $I_s$ are 41.75% and 16.35%, respectively. Compared with the case of no binary phase pupil filters (the focal spot size 0.414$\lambda$), the focal spot size is almost unchanged, but the binary phase pupil filters designed by us successfully deal with the problem that the focal spot is split for $\beta_0 = 3.36$ and make the depth of focus reach 9.53$\lambda$. Most importantly, even allowing the effective power of 90% for the radially polarized LG$_{s,1}$ beam with $\beta_0 = 3.36$, the conversion efficiency is still as high as 37.3%. In addition, the longitudinal component is dominant in this focal spot. To the best of our knowledge, this is the first report of the focal spot with subwavelength transverse size (0.41$\lambda$), ultra long depth of focus (9.53$\lambda$), and high conversion efficiency (37.3%) until now.

We also do some optimizing designs for various $\beta_0$ and find that for the radially polarized LG$_{s,1}$ beam, the multi-zone binary phase pupil filters can extend effectively the depth of focus, but have no obvious effects on the decrease of the focal spot size. The focal spot size is majorly determined by the radially polarized LG$_{s,1}$ beam itself (i.e., the $\beta_0$ when
the $p$ of $\text{LG}_{p,1}$ beam is given). Of course, the improper multi-zone binary phase pupil filters can still increase the focal spot size. For example, by utilizing the multi-zone binary phase pupil filters, the focal spot size is only decreased to about 0.42$\lambda$ from about 0.45$\lambda$ and the depth of focus usually can be extended to about $6\lambda$ for $\beta_0 = 3.58$. We find that the bigger the $\beta_0$ is, the more the optimization results of the multi-zone binary phase pupil filters are. Namely, we can find more sets of optimum structures of the multi-zone binary phase pupil filters that make the focal spot generated by the radially polarized $\text{LG}_{3,1}$ beam have nearly same focal spot size and depth of focus for a given $\beta_0$. When $\beta_0 < 3.36$, we almost cannot find the multi-zone binary phase pupil filters to deal with the problem that the focal spot is split along the $z$ axis. Meanwhile, the difference between the intensity of the focal point ($x = y = z = 0$) and the peak intensity at the $z$ axis is so big that the multi-zone binary phase pupil filter fail to remove the split of the focal spot along the $z$ axis.

4. Conclusion

In conclusion, we investigate the focusing properties of the high-NA imaging system illuminated by a radially polarized higher-order LG beam. We find that the focal spot size is majorly determined by the size parameter $\beta_0$ of the incident LG beam when the radial mode number $p$ is given ($p = 5$ is used in this paper). The smaller the $\beta_0$ is, the smaller the focal spot. However, when $\beta_0 < 3.58$, the focal spot will be split along the $z$ axis, which is unacceptable for many practical applications. We note that the focal spot sizes are about 0.414$\lambda$ and 0.45$\lambda$ for $\beta_0 = 3.36$ and $\beta_0 = 3.58$, respectively. Obviously, the smaller focal spot size (0.414$\lambda$) is more interesting. In order to take full advantage of the subwavelength focusing property of the radially polarized higher-order LG beam, we introduce the multi-zone binary phase filter into the imaging system. We provide an easy-to-perform procedure for the design of multi-zone binary phase pupil filters, where the zone numbers of $\pi$ phase are uncertain when the optimizing procedure starts. Based on this optimizing procedure, for the $\beta_0 = 3.36$, we successfully find the set of optimum structures of a seventeen-belt binary phase pupil filters. When this seventeen-belt binary phase pupil filters are used, we not only deal with the problem that the focal spot is split along the $z$ axis for $\beta_0 = 3.36$, but also make the depth of focus reach 9.53$\lambda$. Moreover, the focal spot size, the Strehl ratio $S$, and the sidelobe intensity $I_s$ are 0.41$\lambda$, 41.75% and 16.35%, respectively. Most importantly, even allowing the effective power of 90% for the radially polarized $\text{LG}_{3,1}$ beam with $\beta_0 = 3.36$, the conversion efficiency is still as high as 37.3%. To the best of our knowledge, this is the first report of the focal spot with subwavelength transverse size (0.41$\lambda$), ultra long depth of focus (9.53$\lambda$), and high conversion efficiency (37.3%) until now. Theoretical calculations show that we succeed to have sufficient conversion efficiency while utilizing the pupil filters to decrease the focal spot and extend the depth of focus. The work in this paper is important for nanoscale imaging, nanolithography, trapping and manipulating particles, etc.

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