Power spectrum of the rectified EMG: when and why is rectification beneficial for identifying neural connectivity?

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Abstract

Objective. The identification of common oscillatory inputs to motor neurons in the electromyographic (EMG) signal power spectrum is often preceded by EMG rectification for enhancing the low-frequency oscillatory components. However, rectification is a nonlinear operator and its influence on the EMG signal spectrum is not fully understood. In this study, we aim at determining when EMG rectification is beneficial in the study of oscillatory inputs to motor neurons. Approach. We provide a full mathematical description of the power spectrum of the rectified EMG signal and the influence of the average shape of the motor unit action potentials on it. We also provide a validation of these theoretical results with both simulated and experimental EMG signals. Main results. Simulations using an advanced computational model and experimental results demonstrated the accuracy of the theoretical derivations on the effect of rectification on the EMG spectrum. These derivations proved that rectification is beneficial when assessing the strength of low-frequency (delta and alpha bands) common synaptic inputs to the motor neurons, when the duration of the action potentials is short, and when the level of cancellation is relatively low. On the other hand, rectification may distort the estimation of common synaptic inputs when studying higher frequencies (beta and gamma), in a way dependent on the duration of the action potentials, and may introduce peaks in the coherence function that do not correspond to physiological shared inputs. Significance. This study clarifies the conditions when rectifying the surface EMG is appropriate for studying neural connectivity.

Keywords: motor unit, motor neuron, synaptic input, coherence, EMG

Introduction

During force contractions, electric signals due to cortical and muscular activity are correlated in specific frequency bands, as shown by EEG-electromyographic (EMG) or EMG–EMG coherence measures (Conway et al 1995, Baker et al 1999). This association is used as evidence for neural connectivity or shared synaptic inputs (Halliday et al 1995) on the basis of two main assumptions. First, the relation between synaptic input and output spike trains of the motor neuron pool should be linear because coherence can only detect linear associations (Cadzow and Solomon 1987). Second, the convolution of the motor neuron spike trains with the surface action potentials of individual motor units (a linear transformation) should not substantially influence the measure of correlation.
However, both assumptions are only partially satisfied, especially when the EMG is subjected to nonlinear pre-processing, such as rectification (Boonstra and Breakspear 2012).

Related to the first assumption, there is evidence that despite the nonlinear properties of the motor neurons, the synaptic input is approximately linearly transmitted to the motor neuron output (Lowery et al 2007, Stegeman et al 2010, Negro and Farina 2011a) and that this approximation is stronger as the number of active motor neurons increases (Negro and Farina 2011b). Because the surface EMG comprises the activity of a relatively large number of motor neurons, the first assumption is approximately satisfied (Farina et al 2013, Farina et al 2014a, Farina and Negro 2015). Violations of the second assumption are more problematic. Because motor unit action potentials have little power below 10–20 Hz, the action potentials convolving the spike trains of motor neurons act as a high-pass filter for coherence measures in these frequencies (Griep et al 1978, Farina et al 2004). Specifically, oscillatory inputs to motor neurons below these frequencies are filtered out in the raw EMG, but can be recovered to some extent by rectifying the EMG (Myers et al 2003, Farina et al 2013, Farina et al 2014b). However, rectification also distorts the spectral content of the signal, since it is a nonlinear operator (Neto and Christou 2010, Farina et al 2013). For these reasons, the appropriateness of rectifying the EMG for coherence analysis has generated a debate that is currently active (Boonstra 2010, Halliday and Farmer 2010, McClelland et al 2012, Farina et al 2013, McClelland et al 2013, Ward et al 2013, Dakin et al 2014, Farina and Negro et al 2014, Farmer and Halliday 2014, McClelland et al 2014). In this debate, some conclusions have been reached empirically by simulation studies (Myers et al 2003, Boonstra and Breakspear 2012) and a few have been provided on theoretical bases (Farina et al 2013).

An important step forward in understanding the effect of EMG rectification on the estimate of the proportion of shared inputs to motor neuron pools has been the observation that the rectified EMG can be represented as a noisy version of the EMG signal generated by the summation of the rectified motor unit action potentials, that we termed no-cancellation EMG (Keenan et al 2005, Farina et al 2008a, Farina et al 2013). The noise term corresponds to the amplitude cancellation signal, which is the difference between the no-cancellation EMG and the rectified EMG signal (Farina et al 2008a, Farina et al 2013) and results from the algebraical summation of positive and negative phases of the motor unit action potentials. This approach has led to several empirical observations on the effect of rectification to be explained on a theoretical basis. In the present study, we use the model of rectified EMG proposed in (Farina et al 2013) and we extend the derivations by analytically examining the filtering properties of the raw and rectified action potentials and the spectrum of the cancellation term. This additional analysis provides definitive answers to when and why rectification is beneficial for EMG coherence analysis.

Materials and methods

Theory

In this section, we explain analytically the influence of rectification on the information contained in the neural drive to the muscle that can be extracted from the EMG signal. We assume that the spike trains of N motor neurons have a given proportion of correlation that depends on the frequency band. In experimental recordings, significant correlation is usually found in the delta (common drive), alpha (physiological and pathological tremor), and beta/gamma (corticospinal pathways) frequency bands. If we indicate with $p_i(t)$ the motor unit action potential shape and $s_i(t)$ the spike train of the $i$th motor unit, we can express the noiseless EMG signal $y(t)$ as:

$$y(t) = \sum_{i=1}^{N} z_i(t) * p_i (t).$$  \hspace{1cm} (1)

where $*$ indicates the convolution operator.

As discussed previously (Farina et al 2013), under the assumption that the average duration of the motor unit action potentials is lower than the mean inter-spike interval (ISI) of the motor neurons, we can separate the rectified EMG into a non-cancellation term $z(t)$ and the cancellation signal $c(t)$:

$$|o(t)| = z(t) - c(t).$$  \hspace{1cm} (2)

The cancellation term $c(t)$ can be seen as (colored) additive noise, correlated to the term $z(t)$. In general, it has a relatively complex analytical form, but its spectral characteristics can be analyzed under approximation, as discussed later.

No-cancellation EMG

We will first focus on the no-cancellation term which can be written as:

$$z(t) = \sum_{i=1}^{N} z_i(t) * |p_i(t)|.$$  \hspace{1cm} (3)

This term corresponds to the summation of each spike train as filtered with an impulse response equal to the corresponding rectified motor unit action potential. The transfer function of the filter is the Fourier transform of the rectified action potential. If we exclude the influence of the noise term created by cancellation ($c(t)$ in equation (2)), the rectified EMG is the filtering of the neural information (motor neuron spike trains) with impulse responses equal to the rectified action potentials. Conversely, the raw EMG is the filtering of the same neural information with raw action potentials as impulse responses (equation (1)). In the frequencies of interest, the transfer functions are high-pass for raw action potentials and low-pass for rectified potentials. Therefore, the rectified action potentials are the preferable filters for oscillatory inputs at relatively low frequencies whereas raw action potentials are preferable for high frequency inputs. In the following, this general statement will be detailed by quantitatively estimating the cut-off frequencies of these filters. Since rectification is a nonlinear operation, it is not possible to
provide analytically the transfer function of the rectified action potential given any raw action potential. Rather, it is necessary to model the raw action potential with a general parameterized mathematical function that sufficiently approximates the shapes of raw action potentials by varying the parameter(s) and to analytically compute the Fourier transform of both this function and its rectification. This derivation is provided in appendix A.

As demonstrated in appendix A, the Fourier transform of a rectified action potential detected with bipolar electrodes represents a low-pass transfer function (Dakin et al 2014) with the first zero (dip) at a frequency that can be approximated as:

\[ f_0 \approx \frac{1}{\pi \lambda}. \]  
(4)

where lambda is a scale factor (parameter) that defines the duration of the action potential. Accordingly, the 3 dB low-pass cut-off frequency of the rectified action potential is approximated as (appendix A):

\[ f_{3dB}^{low} \approx \frac{1}{2 \pi \lambda}. \]  
(5)

From equations (4) and (5), the rectified action potentials act as low-pass filters on the neural drive to the muscle with cut-off frequencies inversely related to the action potential durations. The greater the duration, the smaller the cut-off frequency of the respective filter (more selective filter).

The 3 dB high-pass cut-off frequency of the transfer function associated to the raw action potential is approximated as (appendix A):

\[ f_{3dB}^{high} \approx \frac{1}{3 \pi \lambda}. \]  
(6)

Figure 1 shows these three parameters calculated for an experimental MUAP estimated from a recording with low selectivity (see below).

An increase in action potential duration decreases the low-pass cut-off frequency of the rectified EMG and the high-pass cut-off frequency of the raw EMG. A decrease in low-pass cut-off frequency negatively influences the transmission of the neural information whereas the contrary holds for the high-pass cut-off frequency. Therefore, the greater the average duration of the action potentials, the smaller (larger) is the frequency range of inputs for which the rectified (raw) EMG is suited. It is worth noting that an increase in action potential duration also increases EMG amplitude cancellation (Keenan et al 2005, Farina et al 2014a) which further negatively impacts on the effectiveness of EMG rectification in identifying common oscillatory inputs.

Cancellation in the EMG

We will now assess the properties of the cancellation term \( c(t) \). For this purpose, we will provide an analytical derivation, with approximations, for the signal square instead of the absolute value since this simplifies the analytical derivations and the notations. However, the absolute value of the signal is obtained as the square root of the squared signal and therefore all the properties that will be discussed in the extend directly to rectification. The squared EMG signal has the following expression:

\[
|y(t)|^2 = \sum_{i=1}^{N} x_i^2(t) + \sum_{i\neq j}^{N} x_i(t)x_j(t), \quad (7)
\]

where \( x_i(t) \) is the convolution between the \( i \)th spike train \( s_i(t) \) and the corresponding action potential shape \( p_i(t) \), and \( x_j(t) \) the convolution between the \( j \)th spike train and the corresponding action potential shape. The power spectrum of \( |y(t)|^2 \) is the Fourier transform of the autocorrelation function (Proukis et al 1992), given by:

\[
E \left\{ |y(t)|^2 |y(t - \tau)|^2 \right\} = E \left\{ \left[ \sum_{i=1}^{N} x_i^2(t) + \sum_{i\neq j}^{N} x_i(t)x_j(t) \right] \right. \\
\times \left. \sum_{i=1}^{N} x_i^2(t - \tau) + \sum_{i\neq j}^{N} x_i(t - \tau)x_j(t - \tau) \right\}. \quad (8)
\]

Appendix B describes the mathematical derivations in detail. Following those derivations, the power spectrum of the cancellation term, when only two spike trains are present, is obtained as:

\[
S_{c_{12,13,12}}(f) = F \left\{ R_{11}(\tau)R_{22}(\tau) \right\} = S_{s_1}(f)S_{s_2}(f) = \left[ S_{s_1}(f) |B(f)|^2 \right] \times \left[ S_{s_1}(f) |B(f)|^2 \right]. \quad (9)
\]

where \( S_{s}(f) \) are the power spectra of the original spike trains and \( P(f) \) the Fourier transform of the motor unit action potential.

Equation (9), which represents the oversimplified case of two spike trains under the assumptions made in appendix B, indicates that the power spectrum of the cancellation term is the convolution in the frequency domain of the spectra of the two trains of action potentials. Furthermore, since the spectrum of the spike trains \( S_{s}(f) \) is approximately constant for a large range of frequencies (Perkel et al 1967, Cox and Miller 1977), the spectrum of the cancellation term basically corresponds to the convolution between the modulus squared of the Fourier transforms of the two action potentials. In the range of frequencies below 10–20 Hz, there is an additional contribution due to the average discharge rate of the motor neuron (peak in the spectrum), but the conclusions are similar. This result can be extended, with approximation, to any number of action potentials since the shape of the bipolar surface action potentials is similar across motor units (Griep et al 1978). In conclusion, the spectral content of the cancellation term is approximately equal to the autocorrelation in the frequency domain of the squared modulus of the motor unit action potential transfer function:

\[
S_{pf}(f) \approx S_{s_1^2}(f) + S_{s_2^2}(f) + 4S_{s_1s_2}(f). \quad (10)
\]

Finally, it is important to note that the power spectrum of the rectified EMG is the sum of the (auto)spectra of the no-
cancellation EMG and of the cancellation signal, and of the cross-spectrum between these two components.

Simulations. The theoretical results were evaluated using a realistic motoneuron model and a model for the generation of the surface EMG.

The motoneuron model has been previously described (Negro and Farina 2011a, Negro and Farina 2012, Farina et al 2013) and it was a modification of that used by (Cisi and Kohn 2008). It consists of two compartments and six conductances (three voltage dependent, Na, Kf and Ks). The dynamic of the conductances were characterized by four state variables ($m$, $h$, $n$ and $q$). The motoneuron parameters were selected according to an exponential distribution of properties across the motor neuron pool (Fuglevand et al 1993) with the same ranges used by (Cisi and Kohn 2008) (their table 2). As in our previous publication (Farina et al 2013), the number of motoneurons was chosen equal to 300, which is similar to histological findings in the abductor digiti minimi muscle (ADM) (Santo Neto et al 1985). Two populations of motor neurons representing two muscles were simulated using similar parameters. This muscle was also used in the experimental protocol of the study (see below).

The synaptic input to the motor neuron pools was divided into two synaptic currents: one common component (CI) to all motor neurons resembling a shared synaptic input to the entire motoneuron pool (and between pools) and an independent component (IN) for each motor neuron. In all simulations, the CI was set to 40% of the total variance and the IN to 60%. Both inputs were modelled as a band-limited (0–100 Hz) white Gaussian noise (Maltenfort et al 1998). This broad band was selected in order to investigate the effect of rectification in the full frequency range. The means and standard deviations of the CI and IN inputs were the same for all motor neurons and therefore small motor neuron received the same synaptic current as the large ones. The mean excitation current was selected in order to have an average discharge rate of the motor neurons of $\sim 8$ pps. The total variance of the stochastic input (CI + IN) was selected as a percentage of the mean current in order to obtain a coefficient of variation for cancellation EMG and of the cancellation signal, and of the cross-spectrum between these two components.

Figure 1. Time and frequency representations of a raw and rectified MUAP estimated from the ADM muscle using a bipolar recording configuration with inter-electrode distance equal to 10 mm. (A) Raw MUAP. (B) Power spectrum of the raw MUAP with the location of the high-pass $-3$ dB frequency (equation (6)). (C) Rectified MUAP. (D) Power spectrum of the rectified MUAP with the locations of the low-pass $-3$ dB and dip frequencies (equations (4)–(5)).
the ISI of approximately 15% (Moritz et al. 2005). The total synaptic input was injected in the soma compartment of the model. Plateau potentials caused by persistent inward currents were not simulated. This simplification is acceptable in the current study since the discharge rates were selected in a range where the saturation has a limited effect.

The EMG model was based on a cylindrical volume conductor (Farina et al. 2004) and applied in several previous studies (e.g., Farina and Negro 2007, Farina et al. 2008a, Farina et al. 2008b, Farina et al. 2010, Farina et al. 2013). The parameters of the EMG model were the same as described in (Farina et al. 2008b). Single muscle fibers action potentials were independently simulated and the surface-recorded motor-unit potentials comprised the sum of the action potentials of the muscle fibers belonging to each motor unit. EMG signals were computed at 4096 samples s⁻¹. The signals were simulated as recorded with an array of nine electrodes of circular shape (radius 1 mm), arranged in bipolar derivation with 2.5 mm inter-electrode distance, thus similarly to the recording protocol used in the experimental session. Changes in the duration of the motor unit action potentials were simulated using a scaling factor in the time domain.

The neural-EMG model was implemented in Matlab. The system of differential equations for the motoneuron model was solved with the Adams–Bashforth–Moulton PECE solver (Shampine 1975), using optimized time steps within intervals of 1 ms. Each simulation was 100 s long.

Experimental recordings. Experimental recordings were used to support selected theoretical and simulation results. Specifically, we analyzed experimental data acquired in similar conditions as those simulated above but with EMG detection systems of different spatial selectivity. Spatial selectivity has a direct influence on the duration of the action potentials: more selective systems correspond to action potentials of shorter duration (Farina et al. 2003). The possibility of experimental tuning the duration of the action potentials allowed us to verify the theoretical prediction on the impact of the duration of the action potentials on the filtering of neural information by the raw and rectified EMG.

Experimental EMG signals were collected using two protocols that included EMG detection systems of different selectivity. Both experiments were performed on the ADM and the corresponding experimental data were used also in previous work (Negro et al. 2009, Negro and Farina 2011b). In the first experiment, surface EMG signals were recorded with an array of nine electrodes and 2.5 mm inter-electrode distance (Aalborg University, Denmark and Politecnico di Torino, Italy) in bipolar derivation. The surface EMG signals were amplified, band-pass filtered (3 dB bandwidth, 10–500 Hz), and sampled at 2048 Hz by 12-bit A/D converter (LISIN-OT Bioelettronica, Torino, Italy). In the second experiment, surface EMG signals were recorded using a pair of electrodes of dimensions 30×22 mm (Ambu Neuroline, Denmark) and 10 mm inter-electrode distance. The inter-electrode distance was calculated from the centers of the electrodes. The different inter-electrode distances and electrode sizes in the two experiments corresponded to different selectivity of the recordings. It was hypothesized that, through an effect on the resulting action potential durations, the two recording systems would influence the representation of oscillatory inputs in the raw and the rectified EMG.

In both experiments, single motor unit spike trains were extracted from intramuscular EMG signals recorded concurrently with the surface EMG using two pairs of Teflon-coated stainless steel wires (diameter 0.1 mm; A-M Systems, Carlsborg, WA) inserted into the muscle with 25-gauge hypodermic needles. The bipolar intramuscular EMG signal was amplified (Counterpoint EMG, Dantec Medical, Skovlunde, Denmark), band-pass filtered (500 Hz to 5 kHz), and sampled at 10 kHz.

In each experiment, five subjects (27±4 years in experiment 1 and 24±2 years in experiment 2) performed an isometric contraction at 10% MVC with duration of 60–100 s. Visual feedback of the exerted force level was provided to the subject during the experiment. In each contraction, individual motor units were identified from the intramuscular EMG signals by the use of a decomposition algorithm (McGill et al. 2005). The experiments were conducted in accordance with the Declaration of Helsinki and approved by the ethics committee of Nordjylland (approval number N-20090019). All participants signed a written informed consent form before inclusion. Additional details on the experimental methods are described in the previous publications (Negro et al. 2009, Negro and Farina 2011a).

Signal analysis. The surface EMG signals (simulated and experimentally recorded) were digitally band-pass filtered (10–500 Hz), rectified and divided in segments of 1 s duration and analyzed using the power spectrum and coherence functions between EMG signals (Hanning windowing). The window duration was chosen as a trade-off between the frequency resolution and the duration of the recorded or simulated EMG signals. The coherence function was calculated between the simulated EMG signals of two populations of motor neurons. The confidence limit for coherence values was computed as described by (Halliday et al. 1995) and it was defined as significant when above the 95% confidence limit. Motor unit action potential shapes were extracted using the spike triggered averaging (±50 ms) on the raw surface recordings using the discharge times estimated from the decomposition of the intramuscular EMG signals. For the experimental EMG signals, the two recording systems in the two experiments were compared. Moreover, from the first experiment it was possible to obtain from the linear array recordings at inter-electrode distances multiple of 2.5 mm.

Statistical analysis of experimental data. The 3 dB cut-off frequencies and the dip frequencies measured from the experimental action potentials were reported as mean and standard deviation. Unpaired t-tests were used to compare the parameters extracted from the two surface EMG derivations.
Results

Simulations

Figure 2 shows in simulations how the low-pass cut-off frequency (−3 dB) and the dip frequency for the rectified action potentials change as a function of the duration of the action potential shape. In the simulated condition, we changed the duration of the waveform using a scaling factor in the time domain. The reference duration was set to a scaling factor of one. Smaller factors refer to longer durations. As theoretically predicted (appendix A), the low pass cut-off frequency and the dip frequency for the rectified waveform and the high-pass frequency for the raw waveform depended proportionally on the inverse of the duration of the waveform (equations (4)–(6)).

According to equation (3), spectral dips in the transfer function of the rectified motor unit action potential are reflected in the power spectrum of the rectified EMG. Therefore, the coherence between two rectified EMG signals or between EEG and one rectified EMG will show similar dips. Figure 3 shows an example of the coherence between two rectified EMG signals that have been simulated as driven by two motor neuron pools that share a common synaptic noise in the frequency band 0–100 Hz. These simulations represent the experimental recording of surface EMGs from two different muscles. The results are presented for two cases with different average durations of the action potentials. Figure 3(A) shows the coherence between composite spike trains (CST) that is the reference measure of correlation as the result of the common synaptic input imposed to the two motor neuron pools. The average duration of the motor unit action potentials in this case was ~20 ms. The coherence between the rectified EMG signals shows a reduction in all the frequencies compared with that estimated from the spike trains.

This reduction is related to the effect of cancellation (47% in this case) that has been described in equation (2). Moreover, a low-pass effect is evident with a spectral dip at approximately 90–100 Hz. The frequency at which the dip occurred depended on the average duration of the motor unit action potential waveforms. Indeed, when increasing the duration of the action potential (figure 3(D)), the dip shifted to lower frequencies (~50 Hz). In this case, the coherence increased to ~54%. In comparison, the coherence calculated using the raw EMG (figures 3(B)–(E)) showed the typical high-pass characteristic with cut-off frequency decreasing with larger average motor unit action potential duration (figures 3(C) and (F), black dot lines). In this case, the average duration of the motor unit action potentials was simulated as ~40 ms. Interestingly, rectification resulted also in the amplification of coherence values at high frequencies (see figure 3(A)). This result confirms that the coherence profiles are highly dependent on the average transfer function of the rectified motor unit action potentials.

The previous example indicates that rectification of the surface EMG can introduce distortions in the coherence analysis for certain frequency bands. In particular, zeroes in the frequency spectrum are evident if the duration of the motor unit action potential waveform is relatively long. Since both the no-cancellation and cancellation terms are filtered by the transfer function of the motor unit action potential, their frequency domain amplitude is reflected in the coherence analysis. Additionally, for the rectified EMG, the cancellation term decreases the average coherence magnitude because the cross-spectra terms in equation (10) generate a distortion of the estimated common synaptic input.

Figure 4 shows the influence of cancellation in altering coherence magnitude at some frequencies. Figure 4(A) shows the power spectrum of a simulated EMG with only two motor units active. The spike trains have been selected randomly from the previous simulations (figures 3(A)–(B)). The two motor unit action potential trains that generated the EMG signal were weakly correlated (σ ≈ 3%) to fulfill the hypothesis of equation (10). Figure 4(A) shows a numerical example
of equation (10), as represented in the case of a weak correlation between spike trains and a low number of motor unit action potential trains. The motor unit action potential shapes were equal and had duration of 10 ms. In this case, the spectrum of the cancellation term (figure 4(B), black line) corresponds to the transfer function (figure 4(B)), blue line) derived by the product of the two motor unit action potentials (figure 4(B), grey lines). Since in this example the two motor unit action potentials are the same, the spectrum of the cancellation term has the same zero as the no-cancellation signal. In general, this may not be exact, although we can assume that the motor unit action potential shapes as detected by surface EMG are relatively similar for different motor units (Farina et al. 2008b). Figures 4(C)–(D) show the spectra of the rectified EMG and the cancellation term when the motor unit action potentials have different durations (10 and 25 ms). In this less realistic case, the spectrum of the cancellation term (figure 4(D)) shows attenuation below 100 Hz, but no dips.

From the theoretical and simulation results, the reduction in duration of the motor unit action potentials increases the effectiveness of rectification to accurately reflect the neural oscillatory drive to the muscle. A decrease in action potential duration can be achieved, for example, by spatially high-pass filtering the EMG signal before rectification, i.e. by applying highly selective recording systems (Farina et al. 2003). Conversely, this is not beneficial when using the raw EMG for the analysis. Figure 5 shows the power spectra of simulated rectified EMG signals detected with different inter-electrode distances, corresponding to different selectivities. With an inter-electrode distance of 12.5 mm, the EMG power spectra showed a cut-off frequency of approximately 33 Hz and a dip at approximately 80 Hz (figure 5, black line) whereas with smaller distances between the electrodes (2.5 mm; more selective), the cut-off and zeros frequencies were higher, ∼41 and ∼133 Hz respectively (figure 5, blue line).

Experiments

Figure 6 reports the results for the estimation of the cut-off and the dip frequencies for an experimentally recorded motor unit action potential. In addition to changing the selectivity of the recording, the duration of the action potentials can also be varied by filtering in the time domain the EMG signal (Farina et al. 2013). In this case, the duration of the motor unit action potential was changed using low-pass filtering. Similarly to the previous results, a longer duration (smaller low-pass cut-off frequency when filtering the EMG) of the motor unit action potentials corresponded to a lower cut-off frequencies in the filtering of the neural information for the rectified signal.

We further estimated the −3 dB cut-off frequency and the dip frequencies for the rectified motor unit action potential shapes obtained from the experimental recordings by the spike triggered averaging (see methods). Figure 7 shows these
parameters from experimental recordings of the ADM. For bipolar recordings with 10 mm inter-electrode distance, a total of 34 motor unit action potentials were extracted using spike triggered averaging from the discharge patterns estimated from the decomposition of intramuscular EMG signals. The average −3 dB low-pass cut-off frequency of the rectified action potentials was 19.7 ± 6.8 Hz, whereas the average dip frequency was 55.9 ± 22.6 Hz (figures 7(A)–(B)). This implies that frequencies above ∼20 Hz would be attenuated. For the raw EMG, in the same conditions, the −3 dB high-pass cut-off frequency was estimated on average as 13.2 ± 3.2 Hz, which implies that inputs at frequencies lower than ∼13 Hz would be attenuated (figure 7(C)). As a consequence, rectification would be beneficial in these conditions for input frequencies lower than 13 Hz. Using the most selective recording with inter-electrode distance of 2.5 mm (n = 43 motor units), the average −3 dB low-pass cut-off frequency and average dip frequency were significantly greater than for the less selective system (figures 7(A)–(B)) (26.8 ± 15.5 Hz and 103.2 ± 38.4 Hz, respectively). Conversely, when more selective EMG recordings were used, the raw motor unit action potentials resulted in a greater high-pass cut-off frequency (22.0 ± 7.1 Hz), limiting the use of the raw signal for analyzing alpha/beta bands.

The influence of the selectivity of the recording is exemplified by comparing the power spectrum of the rectified EMG with different inter-electrode distances (figure 8). The
results are in agreement with the simulations (figure 5), showing a shift in the −3 dB bandwidth and dip frequency toward higher frequencies when more selective recordings are used.

Discussion

Rectification of EMG signals is a common pre-processing procedure used for the study of neural connectivity. Several techniques make use of this pre-processing step, among which are corticomuscular and inter-muscular coherence (Grosse et al 2002). For coherence analysis, there has been an extensive discussion if this preprocessing step is needed or not. Recently, we demonstrated that amplitude cancellation is an important contribution to the distortion generated by the rectification process (Farina et al 2013). For this reason, the effectiveness of rectification in extracting an undistorted neural drive depends on the total number of action potentials. According to our previous results (Farina et al 2013), this phenomenon has been recently interpreted as an important justification for using (Holliday and Farmer 2010, Boonstra and Breakspear 2012, Ward et al 2013, Dakin et al 2014) or not using (Neto and Christou 2010, McClelland et al 2012, McClelland et al 2013) rectification as pre-processing before the calculation of the coherence function. Despite our previous results, there are open issues that make the debate on rectification still active (Farmer and Holliday 2014, McClelland et al 2014). Therefore, we have extended in this study the theoretical analysis of the problem by adding a detailed analytical analysis of the effect of action potential filtering and cancellation when rectifying the EMG signal.

Rectification changes the impulse response of the filter that acts on the neural drive to the muscle from the raw motor unit action potential to the rectified potential. These two filters, which have different properties (high-pass/low-pass) and cut-off frequencies, can be obtained experimentally by spike-triggered averaging (figures 2 and 6). Therefore, the influence of the motor unit action potential shapes on coherence analysis, that has been described qualitatively in previous studies (Stegeman et al 2010, Boonstra and Breakspear 2012), is now explained theoretically.

Influence of the duration of the motor unit action potential

The main factor that influences the cut-off frequency of the motor unit action potential filter is the duration of the motor unit action potentials (figure 2). The duration of the motor unit action potentials may change depending on the type of recordings or the muscle conditions, e.g. fatigue (Danna-Dos Santos et al 2010, Kattla and Lowery 2010, Yang et al 2010). For example, highly selective recordings provide shorter motor unit action potentials than less selective systems, which correspond to a greater bandwidth for the rectified EMG (figures 5 and 7). This is beneficial when using the rectified EMG (but not the raw EMG) for the study of neural connectivity. For example, the detection of an input frequency at 10–15 Hz may benefit from rectification, especially if the recording is selective (figure 6). However, it should be emphasized that this conclusion is valid only when neglecting the cancellation term that always influences the rectified EMG (but not the raw EMG). On the other hand, in the same conditions, if the input frequency to estimate is greater, e.g., 30 Hz, then the raw EMG is less influenced by filtering and allows a better identification of this oscillatory component. On the other hand, in case of intramuscular EMG recordings, where the high-pass frequency of the signals is typically >100 Hz, the rectification becomes a necessary pre-processing step to estimate low-frequency neural connectivity.

Dips and peaks in the transfer function of the rectified motor unit action potential

An interesting effect of rectification, not previously reported, is the generation of dips (and concurrent peaks) due to the rectification of the biphasic waveforms (for bipolar recordings) of the motor unit action potentials (figure 3). We demonstrated the effect analytically for a particular class of waveforms (equation (A3)) that are a good approximation of
bipolar motor unit action potentials (figure A1). Interestingly, this ‘dip’ effect in the coherence functions has been previously observed (e.g., see Stegeman et al 2010, figures 9(D) and 10(D)), but not explained with the theoretical explanation provided in the present study. The dips introduced by rectification are usually at frequencies greater (∼60 Hz, figure 5(B)) than those commonly investigated by coherence analysis (usually 0–35 Hz and occasionally up to 50 Hz; Conway et al 1995, Halliday et al 2003, Ward et al 2013). However, they still have a filtering influence in commonly used frequency bands. Moreover, some studies have also addressed the relevance of gamma-band (40–70 Hz) neural oscillations (Smith and Denny 1990, Shibata et al 1999, Schoffelen et al 2005) which are influenced by this effect.

Amplitude cancellation

The rectified EMG can be analytically separated in the summation of two correlated terms (equation (2)): the no-cancellation term that contains the information related to the spike trains low-pass filtered by the rectified motor unit action potential shapes and the cancellation term. The no-cancellation term is filtered by the transfer functions of the motor unit action potentials and therefore, as we described, its spectral information is shaped by those. For this reason, the motor unit action potentials have a multiplicative effect in the frequency domain on the neural information contained in the EMG signals. On the other hand, the cancellation term is an additive term. In this study, under approximations, we demonstrated that the cancellation term is a convolutive term in the frequency domain between the different spectra of the action potential trains. This supports the important finding that the cancellation signal depends on the motor unit action potential shapes, empirically observed in simulation (Farina et al 2008a, Keenan et al 2005). On the other hand, the convolution in the frequency domain can ‘spread’ the correlation between filtered spike trains across frequencies where correlation is not present (see figure 3(A)). This is the main nonlinear effect of rectification.

Conclusions

In conclusion, the rectification of EMG signals overcomes the high-pass nature of the raw EMG signal by filtering the neural information with a low-pass transfer function, corresponding to the rectified action potential impulse response. This allows for the analysis of low frequency oscillations that are not identified from the spectrum of the raw EMG. The main assumption behind the use of this nonlinear operator is that the spectral properties of the motor unit action potentials do not influence the frequency range usually analyzed with coherence techniques (0–50 Hz). However, this assumption is in general not met for this entire frequency range. The average duration of the motor unit action potentials indeed determines the low-pass cut-off frequency that defines the range of frequencies for which rectification is effective. The rectification of the EMG for coherence analysis is thus effective for studying low-frequency oscillatory inputs, especially when the duration of the action potentials is short, and when the...
level of cancellation is low. Conversely, rectification may distort the coherence function when studying higher frequencies, in a way dependent on the duration of the action potentials and may introduce peaks in the coherence function that do not correspond to physiological shared input. A final important result of the study is that EMG rectification is beneficial for studying oscillatory inputs when processing methods are used that decrease the duration of the action potentials (e.g., time-domain high-pass filters or high-selective recording systems).

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Appendix A

The motor unit action potential waveform of a single differential EMG can be well approximated by Hermite–Rodriguez (HR) functions (Conte and Merletti 1995). The HR functions are defined as:

\[ w_{2,n}(t) = \frac{1}{\sqrt{2^n n!}} \lambda^n \frac{1}{\sqrt{\pi \lambda}} e^{-t^2/\lambda^2} \]

\[ n \in [0, \infty), \quad (A1) \]

where \( n \) is the order of the function, \( \lambda \) is a width parameter and \( H_n(t) \) are the Hermite polynomials recursively defined as

\[ \begin{align*}
H_0(t) &= 1, \\
H_1(t) &= 2t, \\
H_n(t) &= 2tH_{n-1}(t) - 2(n-1)H_{n-2}(t).
\end{align*} \]

The HR function of order \( n \) is proportional to the \( n \)th derivative of a Gaussian function

\[ w_{2,n}(t) = \frac{(-1)^n \lambda^n}{\sqrt{2^n n!}} \frac{d^n}{dt^n} \left( \frac{1}{\sqrt{\pi \lambda}} e^{-t^2/\lambda^2} \right) \]

\[ \quad (A3) \]

In this study, we focus on HR functions of order \( n = 1 \), since they are a good approximation of bipolar action potentials (figure A1).

The rectified version of the HR function of order 1 is

\[ |w_{2,1}(t)| = \frac{2}{\sqrt{2\pi}} \frac{1}{\lambda^2} e^{-t^2/\lambda^2}. \]

\[ \quad (A4) \]

Since \( |w_{2,1}(t)| \) is an even function, its Fourier transform can be written as

\[ 2 \left( \int_{-\infty}^{\infty} |w_{2,1}(t)| \cos(2\pi f_0 t) dt \right). \]

\[ \quad (A5) \]

The integrated function is also even and the integral converges, thus we can write:

\[ 2 \left( \int_{-\infty}^{\infty} |w_{2,1}(t)| \cos(2\pi f_0 t) dt \right) = -\frac{2}{\sqrt{2\pi}} \int_0^{\infty} \frac{2t}{\lambda^2} e^{-t^2/\lambda^2} \cos(2\pi f_0 t) dt, \]

\[ = -\frac{2}{\sqrt{2\pi}} \int_0^{\infty} \frac{d}{dt} \left( e^{-t^2/\lambda^2} \right) \cos(2\pi f_0 t) dt. \]

\[ \quad (A6) \]
The integral can be resolved using the integration by parts:

\[ \int e^{-i\lambda z} \lambda e^{-\lambda^2} \cos(2\pi f_0 t) dt = -\frac{\lambda}{\sqrt{2\pi}} \frac{d}{dr} \left( e^{-i\lambda z} \right) \bigg|_0^{+\infty} - \frac{4\pi f_0}{\sqrt{2\pi}} \int_0^{+\infty} e^{-i\lambda z} \sin(2\pi f_0 t) dt, \]

\[ = \frac{2}{\sqrt{2\pi}} \left( \frac{\lambda}{\sqrt{2\pi}} - \frac{4\pi f_0}{\sqrt{2\pi}} \right) \int_0^{+\infty} e^{-i\lambda z} \sin(2\pi f_0 t) dt. \]

The last term has a closed form solution and equation (A7) can be rewritten as:

\[ W_{\lambda,1}^d(f) = \left( \frac{2}{\sqrt{2\pi}} \right) \int_0^{+\infty} e^{-\lambda^2 t^2} \cos(2\pi f_0 t) dt = \frac{2}{\sqrt{2\pi}} \frac{\lambda}{\sqrt{2\pi}} - \frac{4\pi f_0}{\sqrt{2\pi}} \sin(2\pi f_0 t) dt, \]

\[ = \frac{2}{\sqrt{2\pi}} \frac{\lambda}{\sqrt{2\pi}} + \frac{2\pi f_0}{\sqrt{2\pi}} \sin(2\pi f_0 t). \]

The frequency of the dip can be calculated imposing

\[ W_{\lambda,1}^d(f_0) = \frac{2}{\sqrt{2\pi}} \frac{a^2}{\sqrt{2\pi}} e^{-a^2} \left( 1 + \frac{a^2}{3} + \frac{a^4}{10} + \ldots \right) = 0 \]

with \( a = -\pi \lambda f_0 \).

(A10)

The frequency of the dip and the -3 dB cut-off frequency can be calculated solving the equations:

\[ \left[ W_{\lambda,1}^d(f_{\text{DBP}}) \right]^2 = 0 \Rightarrow f_{\text{DBP}} \approx \frac{49}{53} \frac{1}{\pi \lambda} \approx \frac{1}{\pi \lambda}. \]

(A11)

\[ \left[ W_{\lambda,1}^d(f_{\text{3dB}}) \right]^2 = \frac{1}{\pi} \Rightarrow f_{\text{3dB}} \approx \frac{1}{2} \frac{1}{\pi \lambda} \approx \frac{1}{2 \pi \lambda}. \]

(A12)

Using similar calculations, it is possible to obtain the -3 dB high-pass cut-off frequency for the raw HR waveform:

\[ \left[ W_{\lambda,1}^d(f_{\text{3dB}}) \right]^2 = \frac{1}{\pi} \Rightarrow f_{\text{3dB}} \approx \frac{17}{50} \frac{1}{\pi \lambda} \approx \frac{1}{3 \pi \lambda}. \]

(A13)

Figure A3 shows how the low-pass cut-off frequency (−3 dB) and the dip frequency for the rectified case change as a function of the duration of the motor unit action potential waveform. For the raw waveform, we report also the high-pass cut-off frequency. For the waveform represented by a HR function, an increase in \( \lambda \), which corresponds to the wave length, determines a decrease in the cut-off and the dip frequencies (figures A3(B)–(C)). Similarly, the high-pass cut-off frequency for the raw waveform also decreased with an increase in duration of the waveform (figure A3(D)). For example, for a duration of 30 ms, the high- and low-pass frequencies were 15.5 and 18.4 Hz and the dip frequency was
42 Hz whereas for a duration of 38.5 ms, the values were 12, 14.3 and 32.7 Hz, respectively. This is in agreement with the theoretical derivations reported above. In particular, the numerical solutions $A_{11}, A_{12}$ and $A_{13}$ are very close to the estimated value for the different variables (empty circles). The approximations reported in equations (4)–(6) deviate more compared with the estimated solutions, but the differences are still moderate, justifying the use of those approximations.

Appendix B

We simplify equation (8) by first considering only two spike trains and we will then discuss the extension to a greater number of spike trains. For two spike trains, equation (8) reduces to:

$$E \left\{ \left| y(t) \right|^2 \left| y(t - \tau) \right|^2 \right\} = E \left\{ \left[ x_1^2(t) + x_2^2(t) + 2x_1(t)x_2(t) \right] \left[ x_1^2(t + \tau) + x_2^2(t + \tau) + 2x_1(t + \tau)x_2(t + \tau) \right] \right\}$$

$$= E \left\{ x_1^2(t)x_1^2(t + \tau) + x_2^2(t)x_2^2(t + \tau) + 2E \left\{ x_1^2(t)x_2^2(t + \tau) + 2E \left\{ x_1(t)x_2(t) \right\} \left( x_1(t + \tau)x_2(t + \tau) \right) + 4E \left\{ x_1^2(t)x_1(t + \tau)x_2(t + \tau) \right\} \right\} + 4E \left\{ x_1(t)x_2(t)x_1(t + \tau)x_2(t + \tau) \right\} \right\} \right\}.$$  \hspace{1cm} (B1)

The power spectrum of the squared EMG is the Fourier transform of the autocorrelation function, provided by equation (B1):

$$S_{ff}(f) = S_{x_1^2,x_1^2}(f) + S_{x_2^2,x_2^2}(f) + 2S_{x_1^2,x_2^2}(f) + 4S_{x_1,x_2}(f) + 4S_{x_1,x_2}(f) + S_{x_2,x_2}(f) + 4S_{x_1,x_2}(f) + S_{x_2,x_2}(f).$$  \hspace{1cm} (B2)

The first term on the right hand side of equation (B2) is the power spectrum of the no-cancellation signal. The remaining terms represent the cancellation signal. If we assume that the stochastic processes $x_1(t)$ and $x_2(t)$ are only weakly correlated, the cross-spectra in equation (B2) are much smaller than the auto-spectra in the same expression. In this case, equation (B2) further simplifies as:

$$S_{ff}(f) = S_{x_1^2,x_1^2}(f) + S_{x_2^2,x_2^2}(f) + 4S_{x_1,x_2}(f).$$  \hspace{1cm} (B3)

The above assumptions (only two spike trains, weak correlation) seem very strong for a general result; however in the results section we demonstrate numerically that for a greater number of spike trains and for physiological levels of correlation, similar conclusions as those derived from equation (B3) can be drawn in the general case. According to all the assumptions made, the cancellation term can be written as:

$$S_{x_1,x_2}(f) = F \left\{ R_{x_1}(\tau)R_{x_2}(\tau) \right\} = S_{x_1}(f)^*S_{x_2}(f) \right\} \left\| P_{f}(f) \right\|^2.$$

$$= \left[ S_{x_1}(f) \right| P_{f}(f) \right|^2 \left[ S_{x_2}(f) \right| P_{f}(f) \right]^2.$$

(B4)

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