Localization of matter fields that interact with gravity in a non-minimal derivative coupling in the Randall-Sundrum model

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Abstract. In a brane-world model matter fields are assumed to localize on a (3+1)-dimensional brane imbedded in a higher dimensional space-time. However, this assumption is not always fulfilled. In fact, vector fields that couple minimally with gravity as well as matter fields that interact with gauge fields, in addition to gravity in a minimal coupling way are not localized on a brane in the five-dimensional Randall-Sundrum (RS) model \cite{1,2,3}. In this paper we will report our analysis on localization properties in the RS brane-world model for the case of matter fields that interact with gravity in a non-minimal derivative coupling. By expressing matter fields as a multiplication of fields in a four-dimensional space-time and a function of the extra dimension coordinate we derive localization conditions and field equations in the function of the extra coordinate and solve the equation. The solution is then applied to analyze field localizations. We obtain that the massless scalar and spinor fields are localizable respectively on the RS brane for decreasing and increasing warp factor respectively.

1. Introduction

The quantum field and general relativity theories explain natural phenomena in different domain areas which have been well tested with phenomenal accuracy. The quantum theory describes very tiny objects with dynamical interactions include weak, strong and electromagnetic interactions while general relativity describes macroscopic objects such as solar system, galaxies and other large-scale structures. The most outstanding problem in physics is to unify these two theories: the quantum field and general relativity theories which can explain all phenomena in the universe with a single theory called the theory of everything. Started in 1920, Theodore Kaluza and Oscar Klein \cite{4,5} tried to unify gravity and electromagnetism with a consequence of the existence of a space-like extra dimension. The size of the extra dimension was compactified in the Planck length. The new paradigm of extra
dimension came from the revolution of string theory i.e. M theory [6,7] that the extra dimension could be large. The M theory is the most prominent theory to unify quantum and gravity which is established within 11 dimensions. The eleventh dimension is compact and periodic: \(x_{11} \in [-\pi R, \pi R]\) and at the fixed points of the orbifold \(x_{11} = 0\) and \(x_{11} = \pi R\) are placed the ten-dimensional boundary called “brane-orbifold” which on it gauge fields corresponding to open strings are stuck and gravity corresponding to a closed string is spread out over large extra dimensions. The other six-dimensions in these brane orbifolds are in the Planck length compactified in a Calabi-Yau manifold and finally, the effective space-time is 5 dimensions. This was the birth of a braneworld scenario.

One motivation of the braneworld scenario is to solve hierarchy problem i.e. the huge ratio between Planck and electroweak scales. Lisa Randall and Raman Sundrum introduced the braneworld model known as the Randall-Sundrum (RS) model which was set to address this problem [8,9]. One important aspect of braneworld model is the localizability of fields on the brane. References [1-3, 10-14] reveal the localization properties of fields on the brane. In this paper we will analyze the localization properties for the case of scalar and spinor fields that interact with gravity in a non-minimal derivative coupling in the RS model in which the model is specified by the metric [9]

\[
ds^2 = a^2(x^5)\eta_{\mu\nu}dx^\mu dx^\nu - dx^5 dx^5 ,
\]

where \(a(x^5)\) is a warp factor \(a(x^5) = e^{-k|x^5|}\), \(x^5 = y\) is the fifth coordinate, \(k\) is a warp parameter and \(\eta_{\mu\nu}\) is the Minkowski metric with signature \((+,-,-,-,\ldots)\).

### 2. Localization of Scalar Field

The five-dimensional action of scalar field derivative coupled non-minimally to gravity reads

\[
S = \int d^5x \sqrt{|g|} g^{MN} \partial_M \Phi^* \partial_N \Phi + \xi g^{MN} R \partial_M \Phi^* \partial_N \Phi ,
\]

where \(g\) is the determinant of the RS metric, \(R_{[y]} = -20k^2 + 16k \delta(y)\) is the Ricci scalar, and \(\xi\) is a coupling constant. Decomposing \(\Phi = \varphi(x^\mu) \chi(x^5)\), the action becomes

\[
S = \int_0^\infty dy a^2(y) \eta^{\mu\nu} \chi^* \chi [1 + \xi R] \int d^4x \partial_\mu \varphi^* \partial_\nu \varphi - \int_0^\infty dy a^4(y) \partial_\nu \chi^* \partial_\nu \chi [1 + \xi R] \int d^4x \varphi^* \varphi .
\]

The scalar field is localizable on the brane if the action integrals over the fifth coordinate are finite giving the following localization conditions

\[
\int_0^\infty dy a^2(y) \eta^{\mu\nu} \chi^* \chi [1 + \xi R] = N \eta^{\mu\nu} ,
\]

\[
\int_0^\infty dy a^4(y) \partial_\nu \chi^* \partial_\nu \chi [1 + \xi R] = m_\varphi^2 ,
\]

where \(N\) is a finite constant and \(m_\varphi\) is the mass of the scalar field \(\varphi\).

The equation of motion for scalar field corresponding to the action (2) reads

\[
\partial_M (\sqrt{|g|} g^{MN} \partial_N \Phi) + \xi \partial_M (\sqrt{|g|} g^{MN} R \partial_N \Phi) = 0 .
\]

The general solution of the above equation for massless mode reads

\[
\chi(y) = A + Be^{\frac{80k^3 \xi + 4k}{20k^2 (-1)}} y + D U(y) \left(-1 + e^{\frac{80k^3 \xi + 4k}{20k^2 (-1)}} y\right) ,
\]
where \( A = -4k\chi(0)[1 - 20k^2\xi]/\left[\frac{80k^3\xi + 4k}{20k^2\xi - 1}\right], B = 8k\chi(0)/\left[\frac{80k^3\xi + 4k}{20k^2\xi - 1}\right] \) and
\[ D = 16k\xi\chi''(0)/\left[\frac{80k^3\xi + 4k}{20k^2\xi - 1}\right] \]
and \( \mathcal{U}(y) \) is a Heaviside function i.e.
\[
\begin{align*}
\mathcal{U}(x^5) &= 1, \quad x^5 \geq 0 \\
\mathcal{U}(x^5) &= 0, \quad x^5 < 0
\end{align*}
\]
Inserting the solution (5) into the localization condition (3a) gives
\[
\int_0^\infty dy \left[ A^2 e^{-2ky} + B^2 e^{2\left(\frac{80k^3\xi + 4k}{20k^2\xi - 1}\right)y} + D^2\mathcal{U}^2(y) \left(e^{-2ky} - 2e^{\left(\frac{80k^3\xi + 4k}{20k^2\xi - 1}\right)y} + e^{2\left(\frac{80k^3\xi + 4k}{20k^2\xi - 1}\right)y}\right)\right]
+ 2ABe^{\left(\frac{80k^3\xi + 4k}{20k^2\xi - 1}\right)y} + 2AD\mathcal{U}(y) \left(-e^{-2ky} + e^{\left(\frac{80k^3\xi + 4k}{20k^2\xi - 1}\right)y}\right) + 2B^2 e^{\left(\frac{80k^3\xi + 4k}{20k^2\xi - 1}\right)y} D\mathcal{U}(y) \left(-e^{-2ky} + e^{\left(\frac{80k^3\xi + 4k}{20k^2\xi - 1}\right)y}\right) \left(1 - 20k^2\xi + 16k\delta(y)\xi\right)
\]

The finiteness of above integrals is satisfied for \( k > 0 \) if \( B = D = 0 \) and for \( k < 0 \) if \( A = 0 \) and \( \frac{80|k|^3\xi + 2|k|}{20|k|^2\xi - 1} > 0 \). The former gives \( \chi(y) = A \) which satisfies the condition (3b) for massless scalar field. The latter, on the other hand, does not satisfy the condition (3b) for massless scalar field. Thus, a massless scalar can be localized on the brane for increasing warp factors and cannot be localized for decreasing warp factors.

3. Localization of Spinor Field

The five-dimensional action corresponding to the spinor field derivative coupled non-minimally to gravity reads
\[
S = \int d^5x \sqrt{|g|} \left[ \psi\Gamma^M \sigma_{\psi} + \beta R \psi\Gamma^M D_{\sigma} \psi \right],
\]
where \( R \) is Ricci scalar, \( \Gamma^M = e^M_\mu \tilde{\Gamma}^\mu \) are gamma matrices in a curved spacetime, \( \gamma^\mu \) are the gamma matrices in Minkowski spacetime, with \( e^M_\mu \) and \( e^\mu_M \) are funfbeins and their inverses respectively, \( \gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3 \) and \( \bar{\psi} = \psi^+ \Gamma^0 = (\psi^\dagger)^T e^0_0 \). \( D_M = \partial_M + \frac{1}{4} \omega^M_N \sigma_{\tilde{M}N} \) are covariant derivatives, where \( \sigma_{\tilde{M}N} \) are Dirac tensors \( \sigma_{\tilde{M}N} = \frac{1}{2} [\gamma_M, \gamma_N] \) and \( \omega^M_N \) are spin connections, see refs. [3,10].

For the metric considered, the covariant derivatives have the forms
\[
D_\mu = \partial_\mu + \frac{ik}{2} a\gamma_\mu \gamma_5, \quad D_5 = \partial_y.
\]

Decomposing the five-dimensional spinor as follows
\[
\psi(x^\mu, y) = \begin{pmatrix} \psi^{(1)}_R(x_\mu) P^{(1)}_R(y) \\ \psi^{(2)}_R(x_\mu) P^{(2)}_R(y) \\ \psi^{(1)}_L(x_\mu) P^{(1)}_L(y) \\ \psi^{(2)}_L(x_\mu) P^{(2)}_L(y) \end{pmatrix},
\]
the action (7) becomes
\[ S = \int d^5x \sqrt{g} e_0^0 e_0^0 \left[ \psi_R^{(1)*} P_R^{(1)} (i\partial_0) \psi_R^{(1)*} P_R^{(1)} + \psi_R^{(2)*} P_R^{(2)} (i\partial_0) \psi_R^{(2)*} P_R^{(2)} + \psi_L^{(1)*} P_L^{(1)} (i\partial_0) \psi_L^{(1)*} P_L^{(1)} + \psi_L^{(2)*} P_L^{(2)} (i\partial_0) \psi_L^{(2)*} P_L^{(2)} \right] \] 
\[ + \int d^5x \sqrt{g} e_0^0 e_0^0 \left[ \psi_R^{(2)*} P_R^{(2)} (i\partial_3) \psi_R^{(2)*} P_R^{(2)} + \psi_R^{(2)*} P_R^{(2)} (i\partial_3) \psi_R^{(2)*} P_R^{(2)} - \psi_L^{(1)*} P_L^{(1)} (i\partial_3) \psi_L^{(1)*} P_L^{(1)} + \psi_L^{(2)*} P_L^{(2)} (i\partial_3) \psi_L^{(2)*} P_L^{(2)} \right] \] 
\[ + \int d^5x \sqrt{g} e_0^0 e_0^0 \left[ \psi_R^{(2)*} P_R^{(2)} (i\partial_3) \psi_R^{(2)*} P_R^{(2)} - \psi_L^{(1)*} P_L^{(1)} (i\partial_3) \psi_L^{(1)*} P_L^{(1)} + \psi_L^{(2)*} P_L^{(2)} (i\partial_3) \psi_L^{(2)*} P_L^{(2)} \right] \] 
\[ - 2k \int d^5x \sqrt{g} e_0^0 e_0^0 \left[ \psi_R^{(1)*} P_R^{(1)} (i\partial_0) \psi_R^{(1)*} P_R^{(1)} + \psi_L^{(1)*} P_L^{(1)} (i\partial_0) \psi_L^{(1)*} P_L^{(1)} - \psi_R^{(2)*} P_R^{(2)} (i\partial_0) \psi_R^{(2)*} P_R^{(2)} + \psi_L^{(2)*} P_L^{(2)} (i\partial_0) \psi_L^{(2)*} P_L^{(2)} \right] \] 
\[ + \int d^5x \sqrt{g} e_0^0 e_0^0 \left[ \psi_R^{(1)*} P_R^{(1)} (i\partial_0) \psi_R^{(1)*} P_R^{(1)} + \psi_L^{(2)*} P_L^{(2)} (i\partial_0) \psi_L^{(2)*} P_L^{(2)} \right] \] 

The localization are
\[ \int_0^\infty dx^5 \sqrt{g} e_0^0 e_0^0 \left[ P_R^{(1)} P_R^{(1)} (1 + \beta R) = N_{R0}^{(1)} , R \leftrightarrow L ; \right] \] 
\[ \int_0^\infty dx^5 \sqrt{g} e_0^0 e_0^0 \left[ P_R^{(2)} P_R^{(2)} (1 + \beta R) = N_{R1}^{(2)} , R \leftrightarrow L ; i \neq j ; \right] \] 
\[ \int_0^\infty dx^5 \sqrt{g} e_0^0 e_0^0 \left[ P_L^{(1)} P_L^{(1)} (1 + \beta R) = N_{L0}^{(1)} , R \leftrightarrow L ; \right] \] 
\[ \int_0^\infty dx^5 \sqrt{g} e_0^0 e_0^0 \left[ P_L^{(2)} P_L^{(2)} (1 + \beta R) = N_{L1}^{(2)} , R \leftrightarrow L ; \right] \] 
\[ 2k \int_0^\infty dx^5 \sqrt{g} e_0^0 e_0^0 \left[ P_R^{(1)} P_L^{(1)} (1 + \beta R) = M_{RL}^{(1)} , R \leftrightarrow L ; \right] \] 
\[ - \int_0^\infty dx^5 \sqrt{g} e_0^0 e_0^0 \left[ P_R^{(1)} P_L^{(1)} (1 + \beta R) = N_{RL}^{(1)} , R \leftrightarrow L . \right] \]

Recalling that for the RS model \( \sqrt{g} = a^4(y) \), \( g^{\mu\nu} = a^{-2}(y)\eta^{\mu\nu} \), \( g^{55} = -1 \), \( e_\mu = \frac{1}{a} e_5^\mu = 1 \) and that the sum of the last two conditions corresponds to a mass term, the localization conditions reduce into
\[ \int_0^\infty dy a^2(y) P_R^{(1)} P_R^{(1)} (1 + \beta R) = N_{R0}^{(1)} = N_{R3}^{(1)} , R \leftrightarrow L ; \] 
\[ \int_0^\infty dy a^2(y) P_R^{(2)} P_R^{(2)} (1 + \beta R) = N_{R1}^{(2)} = N_{R3}^{(2)} , R \leftrightarrow L ; i \neq j ; \] 
\[ \int_0^\infty dy a^3(y) 2k P_R^{(1)} P_L^{(1)} (1 + \beta R) - \int_0^\infty dy a^3(y) P_R^{(1)} P_L^{(1)} (1 + \beta R) = -m , R \leftrightarrow L . \]

where all constants on the RHS are finite.

The equation of motion for spinor field corresponding to the action (7) reads
\[ i\Gamma^R D_R \psi(1 + \beta R) = 0. \] 

Inserting the covariant derivatives, gamma matrices, the decomposition (9) into (13) and using the 4D equation for spinor field \( iv^\mu \partial_\mu \psi(x) = m\psi \), the equation of motion (13) can be written as follows \( (i=1,2) \)
\[ \left[ mP_R^{(1)} (y) - 2akP_R^{(1)} (y) + a\partial_y P_R^{(1)} (y) \right] (1 + \beta R) = 0, \]
(14a)
\[ [mP_L^{(i)}(y) + 2akP_L^{(i)}(y) - a\partial_y P_L^{(i)}(y)](1 + \beta R) = 0. \] \hspace{1cm} (14b)

The solutions of the above equation for right-handed spinor and left-handed spinor respectively are

\[ P_R^{(1)}(y) = P_R^{(2)}(y) = b_{1/2}e^{(\frac{m}{k}e^{ky} + 2ky)}; P_L^{(1)}(y) = P_L^{(1)}(y) = c_{1/2}e^{(\frac{m}{k}e^{ky} + 2ky)}. \] \hspace{1cm} (15)

Feeding the above solutions into the localization conditions, the integral in (12a) and (12b) for right-handed and left-handed spinors respectively become

\[ b_{1/2}^2 \int_0^\infty dy e^{(-2m/k)e^{ky} + 2ky})(1 + \beta R) \] and \[ c_{1/2}^2 \int_0^\infty dy e^{(2m/k)e^{ky} + 2ky)}(1 + \beta R). \] \hspace{1cm} (16)

Since \( R[y] = -20k^2 + 16k\delta(y) \) the integrals consist of the Dirac delta function term and the non-Dirac delta function term. The former has a finite value while the latter has the form:

\[ \int_0^\infty dy e^{(\pm2m/k)e^{ky} + 2ky)} = \frac{1}{k^2} \left( \frac{k}{2m} \right)^2 \int_{u=2m/k}^\infty du e^{u} \] \hspace{1cm} (17)

The integral with (−) sign, corresponding to right-handed spinor fields, has a finite value of \( \frac{1}{k^2} \left( \frac{k}{2m} \right)^2 (2m + k)e^{-2m/k} \), for either a positive or negative \( k \), while the integral with (+) sign, corresponding to left-handed spinor fields, has an infinite value. Thus, the right-handed spinor fields are localizable while the left-handed spinor fields are not. The whole spinor fields are localizable only if \( P_L = 0 \). From condition (12c), the choice \( P_L = 0 \) is equivalent to \( m = 0 \). Accordingly, massive spinor fields are not localizable on the RS brane.

### 4. Conclusions

We have analysed the localization properties of the scalar and spinor fields derivative-coupled non-minimally to gravity. The fields are localizable if all integrals over the fifth coordinate are finite. For the case of scalar fields, only the constant solution of massless field (5), i.e. \( B = D = 0 \), fulfils the condition (3a) for positive warp. Thus, massless scalar fields are localizable for a constant solution and a positive warp parameter. For the case of spinor fields, the finiteness of conditions (12a) and (12b) are only satisfied by for right-handed massless spinors. This means that, in general, massive spinor fields are not localizable on the RS brane while massless spinor fields are localizable on the RS brane for negative warp parameters.

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