ASTROPHYSICAL IMPLICATIONS OF
NEUTRINO MASS AND MIXINGS *

A.B. Balantekin
Physics Department, University of Wisconsin
Madison, WI 53706

Abstract

Astrophysical implications of neutrino mass and mixings are discussed. The status of solar and atmospheric neutrino problems, and recent developments concerning nuclear physics input to solar models and solar opacities are reviewed. Implications of neutrino mass and mixings in supernova dynamics are explored. The effects of supernova density fluctuations in neutrino propagation is described.

INTRODUCTION

Neutrinos, Pauli’s little neutral ones, have already reached adolescence, and they may eventually be regarded as the most elucidative indicators of new astrophysical phenomena. In this review, some of the astrophysical implications of neutrino mass and mixings are discussed.

Only certain values of neutrino mass and mixings give rise to interesting astrophysical effects. One should raise the question if these values are realistic. Unfortunately we still do not have much experimental information about neutrino properties. The only measured neutrino property is the number of light neutrino flavors. It is determined from the invisible width of Z to be

\[ N_\nu = 2.99 \pm 0.04. \]  

(1)

This number is consistent with the primordial nucleosynthesis limit from the observed He abundance \[ 2 \]. We have upper bounds for all the other neutrino

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properties: masses, flavor mixing angles, electromagnetic moments, charges, and charge radii. In particular, the mass of the heaviest neutrino, $\nu_\tau$, is very poorly known: upper limit is 35 MeV. A tau neutrino with a mass in the range of 1 to 10 eV could possibly be a significant component of dark matter and, as we shall see later, could also have very interesting implications for supernova dynamics.

In the next two sections first the status of solar neutrino problem is briefly reviewed, and then some recent developments concerning physics input to solar models are discussed. After reviewing the status of the atmospheric neutrino anomaly, implications of neutrino mass and mixings in supernova dynamics are explored. Finally neutrino propagation in inhomogeneous media is described and its implications for the sun and supernovae are considered.

SOLAR NEUTRINOS

Some of the energy released in the thermonuclear reactions in the solar interior is emitted in the form of neutrinos and this neutrino flux can be calculated with relatively high precision $[3, 4]$. This flux was measured $[5]$ and its directionality (i.e. coming from the sun) was established $[6]$. However, the observed solar neutrino flux is deficient relative to what is predicted by the standard solar model. A summary of the current status of the solar neutrino experiments is given in Table 1. Since the same thermonuclear reactions produce photons as well as neutrinos, assuming the dominance of the $pp$ reaction, one can roughly estimate the flux of the neutrinos coming from this reaction from the solar photon luminosity to be

$$\phi_\nu \sim \frac{2L_\odot}{26.73 \text{MeV} - 2E_\nu} \cdot \frac{1}{4\pi r^2} \sim 6.5 \times 10^{10} \text{cm}^{-2} \text{s}^{-1}, \quad (2)$$

which indicates a Ga capture rate greater than 78 SNU’s. Inclusion of $^7\text{Be}$ and $^8\text{B}$ neutrinos would increase this number.

The data from these experiments appear to be consistent with a stronger suppression of the intermediate-energy neutrinos ($^7\text{Be}$) than the lower energy ($pp$) or higher energy ($^8\text{B}$) neutrinos. It was recognized that, if neutrinos observed in physical processes are mixtures of mass eigenstates, coherent forward scattering of neutrinos in electronic matter gives rise to a density-dependent effective mass resulting in an almost complete conversion of electron neutrinos into neutrinos of a different flavor. This scenario,
dubbed the Mikheyev-Smirnov-Wolfenstein (MSW) effect, does not require fine-tuning of mass differences and mixing angles and could possibly provide an elegant solution to the solar neutrino problem within current theoretical prejudices \cite{13, 14}. The non-adiabatic MSW solution with $\delta m^2 \sim 10^{-5} \text{ eV}^2$ and $\sin^2 \theta \sim 0.1$ is consistent with the data from those four experiments. This MSW solution is still plausible after incorporating the uncertainties in the solar model \cite{13}. An astrophysical solution for the solar neutrino problem is unlikely, but still not ruled out \cite{15, 16}.

Solar neutrinos are not the only experimental probes of the sun. Information from helioseismological p-wave observations complement information obtained by solar neutrino experiments. The p-waves cause the solar surface to vibrate with a characteristic period of about five minutes. By observing red- and blue-shifts of patches of the solar surface, projecting them on spherical harmonics, and finally Fourier transforming with respect to the observation time one can obtain eigenfrequencies of the solar p-modes. (One should exercise a little bit of caution with regard to using spherical harmonics, since we only observe half the sun). For very high overtones (for a spherically-symmetric three-dimensional object such as the sun these are characterized by two large integers), the equations describing p-modes simplify \cite{17} and one can reliably obtain a sound velocity profile for the outer half of the sun. The sound density profile obtained this way agrees with the predictions of the standard solar model. By studying discontinuities in the sound velocity profile, it is also possible to reliably extract the location of the bottom of the convective zone \cite{18}.

| Experiment | Data | SSM1 \cite{11} | SSM2 \cite{12} |
|------------|------|----------------|----------------|
| Homestake \cite{5} | $2.3 \pm 0.3 \text{ SNU}$ | $8.0 \pm 1.0 \text{ SNU}$ | $6.4 \pm 1.4 \text{ SNU}$ |
| Kamioka \cite{6} | $0.50 \pm 0.04 \pm 0.06$ | 1 | 0.78 |
| SAGE \cite{7, 8} | $74^{+11}_{-12} \text{ (stat.)}^{+7}_{-7} \text{ (sys.) SNU}$ | $131.5^{+7}_{-7} \text{ SNU}$ | $122.5 \pm 7 \text{ SNU}$ |
| Gallex \cite{9, 10} | $79 \pm 10 \text{ (stat.)} \pm 6 \text{ (syst.) SNU}$ | $131.5^{+7}_{-6} \text{ SNU}$ | $122.5 \pm 7 \text{ SNU}$ |

Table 1: The present status of solar neutrino experiments. The standard solar models SSM1 and SSM2 are taken from Refs. \cite{11} and \cite{12} respectively. The Gallex and SAGE quotes are combined results of initial and more recent runs.
Physics input into solar models is extensively discussed in the literature, see for example Ref. [4]. Here I concentrate on nuclear physics input and the opacity.

There are two places where input from nuclear physics is needed for solar neutrino studies: i) to solar models, such as the rates of the reactions leading to neutrinos, ii) to the detector cross sections, especially cross sections for chemical detectors, such as chlorine, gallium, and iodine. The latter kind of input is not trivial to obtain, for example the utility of (p,n) reactions to extract Gamow-Teller strengths was criticized [19]. Here I will discuss the former kind of input, since some recent measurements raised questions about previously-used values of $S_{17}$.

A measurement at RIKEN of the Coulomb breakup cross section of $^8$B by $^{208}$Pb into p and $^7$Be was used to extract the cross section for the reaction $^8$B $+\gamma \rightarrow ^7$Be $+ p$, from which one can obtain the cross section for the inverse reaction [20]. The resulting astrophysical S-factor $S_{17}$ is less than those obtained from previous measurements which would reduce the predicted flux of $^8$B neutrinos. The reaction measured at RIKEN is an electromagnetic (Coulomb breakup) process. In this experiment, to eliminate contributions from the strong nuclear force data are taken at very forward angles (i.e., at large impact parameters). Hence only the $E1$ contribution is measured. Caution should be exercised when extracting $S_{17}$ at low energies since other multipoles, not measured at RIKEN, are expected to contribute to it as well. There are sources of uncertainties in the method of virtual quanta used here (such as determining the value of the lowest impact parameter) [21]. It is probably too early to tell if this experiment warrants a lower $S_{17}$ before more data are taken.

Opacity is another parameter in solar models which had to be recently modified [22]. In the radiative zone of the sun, heat transfer by radiation is controlled by a single opacity parameter, which is a measure of photoabsorption. To calculate opacity one needs to include photoabsorption cross sections for many types of atomic configurations; hence in principle, one needs to know all atomic levels of all the isotopes. Heavier elements (those with a higher Z) contribute more to the opacity. For example iron alone, which is basically a trace element in the sun, contributes about 20%. Changing the opacity alters the rate of heat transfer and consequently the core temperature. By lowering the opacity one can homologously lower [23] the temperature profile. Since the rates of the neutrino-producing subbarrier
fusion reactions in the sun are governed by quantum-mechanical tunneling, lowering the temperature lowers the rate almost exponentially. The flux of higher-energy neutrinos coming from reactions with higher Coulomb barriers rapidly decreases as the core temperature falls. (This exponential dependence is usually expressed as a power law for a limited range of temperatures near the standard solar model temperature. This practice can sometimes be misleading.)

Until recently, most researchers used the Los Alamos Opacity Library. However a number of problems with pulsating stars indicate a need for increasing these opacities in the convective zone. It was also recently shown that for agreement between the results of the standard solar model and the helioseismologically observed p-mode frequencies, the solar opacity needs to be increased by about 30% from the Los Alamos Opacity Library results. Indeed, recent calculations at Livermore increase the opacity by about 15% in the convective zone and about 5% in the core. Increasing the opacity, however, raises the core temperature, and consequently increases the neutrino flux and could eliminate the effect of a lowered $S_{17}$.

ATMOSPHERIC NEUTRINOS

Atmospheric neutrinos arise from the decay of secondary pions, kaons, and muons produced by the collisions of primary cosmic rays with the O and N nuclei in the upper atmosphere. For energies less than 1 GeV all the secondaries decay:

\[
\pi^\pm(K^\pm) \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu), \\
\mu^\pm \rightarrow e^\pm + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu).
\]  

(3)

Consequently one expects the ratio

\[
r = (\nu_e + \bar{\nu}_e)/(\nu_\mu + \bar{\nu}_\mu)
\]

(4)

to be approximately 0.5 in this energy range. Detailed Monte Carlo calculations, including the effects of muon polarization, give $r \sim 0.45$. Since one is evaluating a ratio of similarly calculated processes, systematic errors are significantly reduced. Different groups estimating this ratio, even though they start with neutrino fluxes which can differ in magnitude by up to 25%, all agree within a few percent. The ratio (observed to predicted) of ratios
was determined in several experiments as summarized in Table 2. There seems to be a persistent discrepancy between theory and experiment. Oscillations of $\nu_\mu$ into $\nu_\tau$ are generally invoked to explain this discrepancy [34].

Experimentally the ratio of ratios, $R$, appears to be independent of zenith angle. The observed zenith angle distribution of low energy atmospheric neutrinos is consistent with no oscillations or with a large number of oscillations for all source-detector distances. Explanations of the low energy atmospheric neutrino anomaly based on the oscillation of 2 neutrino flavors require that the oscillating term $\cos(\Delta m^2 L/2E)$, average to zero for even the shortest source-detector distances ($L < 50$ km for neutrinos from directly overhead.) For neutrinos in the energy range 0.1 to 1 GeV this condition is satisfied for $\Delta m^2 > 10^{-3} eV^2$. If the atmospheric neutrino anomaly is resolved by the oscillations of muon into tau neutrinos, this value of $\Delta m^2$ is consistent with a tau neutrino mass relevant to hot dark matter and supernova dynamics. It is also possible to make a search in the three-neutrino-flavor parameter space and identify regions in this parameter space compatible with the existing atmospheric and solar neutrino data within the vacuum oscillation scheme [36].

**NEUTRINO FLAVOR MIXING IN SUPERNOVAE**

Understanding neutrino transport in a supernova is an essential part of understanding supernova dynamics. In this review, I will only concentrate on the effects of neutrino flavor mixing on supernova dynamics. In a core-

$$R = \frac{(\nu_\mu/\nu_e)_{\text{data}}}{(\nu_\mu/\nu_e)_{\text{MonteCarlo}}} \quad (5)$$

| Experiment  | $R$       |
|-------------|-----------|
| Kamioka [29]| $0.60^{+0.07}_{-0.06} \pm 0.05$ |
| IMB [30]    | $0.54 \pm 0.05 \pm 0.12$ |
| Soudan [31] | $0.55 \pm 0.27$ |
| Nusex [32]  | $0.99^{+0.35}_{-0.25}$ |
| Frejus [33] | $0.87 \pm 0.16 \pm 0.08$ |

Table 2: Ratio of Ratios, $R$ of Eq. (5), as observed in different experiments.
collapse driven supernova, the inner core collapses subsonically, but the outer part of the core supersonically. At some point during the collapse, when the nuclear equation of state stiffens, the inner part of the core bounces, but the outer core continues falling in. The shock wave generated at the boundary loses its energy as it expands by dissociating material falling through it into free nucleons and alpha particles. For a large initial core mass, the shock wave gets stalled at $\sim 200$ to $500$ km away from the center of the proto-neutron star \[37\]. Meanwhile, the proto-neutron star, shrinking under its own gravity, loses energy by emitting neutrinos, which only interact weakly and can leak out on a relatively long diffusion time scale. The question to be investigated then is the possibility of regenerating the shock by neutrino heating.

The situation at the onset of neutrino heating is depicted in Figure 1. The density at the neutrinosphere is $\sim 10^{12}$ g cm$^{-3}$ and the density at the position of the stalled shock is $\sim 2 \times 10^7$ g cm$^{-3}$ \[37\]. Writing the MSW resonance density in appropriate units:

Figure 1: Neutrinosphere and the stalled shock in a core-collapse driven supernova.
\[ \rho_{\text{res}} = 1.31 \times 10^7 \left( \frac{\delta m^2}{\text{eV}^2} \right) \left( \frac{\text{MeV}}{E_\nu} \right) \left( \frac{0.5}{Y_e} \right) \text{ g cm}^{-3}, \]  

(6)

one sees that, for small mixing angles, \( E_\nu \sim 10 \text{ MeV} \), and cosmologically interesting \( \delta m^2 \sim 1 - 10^4 \text{ eV}^2 \), there is an MSW resonance point between the neutrinosphere and the stalled shock.

Neutrinos emitted from the core are produced by a neutral current process, and so the luminosities are approximately the same for all flavors. The energy spectra are approximately Fermi-Dirac with a zero chemical potential characterized by a neutrinosphere temperature. The \( \nu_\tau, \overline{\nu}_\tau, \nu_\mu, \overline{\nu}_\mu \) interact with matter only via neutral current interactions. These decouple at relatively small radius and end up with somewhat high temperatures, about 8 MeV. The \( \overline{\nu}_e \)’s decouple at a larger radius because of the additional charged current interactions with the protons, and consequently have a somewhat lower temperature, about 5 MeV. Finally, since they undergo charged current interactions with more abundant neutrons, \( \nu_e \)’s decouple at the largest radius and end up with the lowest temperature, about 3.5 to 4 MeV. An MSW resonance between the neutrinosphere and the stalled shock can then transform \( \nu_\tau \leftrightarrow \nu_e \), cooling \( \nu_\tau \)’s, but heating \( \nu_e \)’s. Since the interaction cross section of electron neutrinos with the matter in the stalled shock increases with increasing energy, it may be possible to regenerate the shock. Fuller et al. found that for small mixing angles between \( \nu_\tau \) and \( \nu_e \) one can get a 60% increase in the explosion energy [37].

There is another implication of the \( \nu_\tau \) and \( \nu_e \) mixing in the supernovae. Supernovae are possible r-process sites [38], which requires a neutron-rich environment, i.e., the ratio of electrons to baryons, \( Y_e \), should be less than one half. \( Y_e \) in the nucleosynthesis region is given approximately by [39]

\[ Y_e \simeq \frac{1}{1 + \lambda_{\nu_e p}/\lambda_{\nu_e n}} \simeq \frac{1}{1 + T_{\overline{\nu}_e}/T_{\nu_e}}, \]  

(7)

where \( \lambda_{\nu_e n} \), etc. are the capture rates. Hence if \( T_{\overline{\nu}_e} > T_{\nu_e} \), then the medium is neutron-rich. As we discussed above, without matter-enhanced neutrino oscillations, the neutrino temperatures satisfy the inequality \( T_{\nu_\tau} > T_{\nu_e} \). But the MSW effect, by heating \( \nu_e \) and cooling \( \nu_\tau \) can reverse the direction of inequality, making the medium proton-rich instead. Hence the existence of neutrino mass and mixings puts severe constraints on heavy-element nucleosynthesis in supernova. These constraints are investigated in Ref. [39].
EFFECTS OF DENSITY FLUCTUATIONS

If a completely polarized particle beam travels through a random magnetic field for a long enough time, it will be completely depolarized. In a similar way, if different neutrino flavors mix, a completely “polarized” neutrino beam (say all $\nu_e$) may become completely “depolarized” (half $\nu_e$, half $\nu_x$) after passing through a medium with fluctuating matter density [40]. Note that this can happen without a neutrino magnetic moment, it is simply a new aspect of the MSW mechanism. There is an extensive discussion of neutrino oscillations in inhomogeneous media in the literature [40, 41].

![Figure 2: MSW effect in the sun with (dashed lines) and without (solid lines) density fluctuations.](image)

We take the total electron density to be $\rho_e(r) + \delta \rho_e(r)$ where $\delta \rho_e(r)$ is the fluctuating part. One can assume that the average fluctuation vanishes
\[ <\delta \rho_e(r)> = 0, \]  

but the two-body correlations are non-zero:

\[ <\delta \rho_e(r)\delta \rho_e(r')> = \rho_0^2 f(|r - r'|), \]  

where the correlation function \( f(|r - r'|) \) has a finite correlation length. The effect is significant if the correlation length, \( r_c \), is small [40].

In the white noise limit,

\[ f(|r - r'|) \rightarrow 2r_c \delta(r - r'), \]  

a number of simplifications make numerical calculations particularly easy. The effects of possible solar density fluctuations are presented in Figure 2. In this figure the electron neutrino survival probability is plotted for two values of the mixing angle, chosen to provide non-adiabatic solutions for 2% (left column) and 4% density fluctuations (right column). The density fluctuations are taken to be proportional to the local density predicted by the standard solar model [11]. One observes that the effect of density fluctuations in the sun are small, but can conceivably give rise to annual variations in the solar neutrino flux.

The MSW mechanism may also be relevant in collapsing pre-supernova stellar cores, where adiabatic conversion of electron neutrinos into massive (e.g. \( \nu_\tau \)) neutrinos could result in readjustment of lepton numbers and small entropy generation [43]. A consequent drop in the electron fraction, \( Y_e \), could be significant for the mechanism of supernova explosion.

The effect of density fluctuations in a collapsing star is plotted in Figure 3. In this figure the upper panel exhibits the neutrino survival probability as a function of the mass difference squared between the electron and heavy neutrino with (solid line) and without (dashed line) density fluctuations. The fractional density fluctuations are taken to be \( 5 \times 10^{-3} \). One observes that there is a complete depolarization (50% conversion), especially for heavy neutrino mass (assuming a very small electron neutrino mass) greater than \( \sim 30 \text{ MeV} \). In the lower panel the \( \nu_e \rightarrow \nu_\tau \) transition probability contours are plotted. Here the lowest solid line indicates 10%, dashed lines 70%, and the dotted lines 80% conversion, with intermediate lines being in steps of 10%. For a detailed study of density fluctuations in supernova, both during the infall and at the hot-bubble after the bounce, the reader is referred to Ref. [42].
Figure 3: The effect of density fluctuations at the infall [42]. a) Neutrino survival probability with (solid line) and without (dashed line) fluctuations. b) $\nu_e \rightarrow \nu_{\tau}$ transition probability contours (see text).

CONCLUSIONS

Neutrino astrophysics is a field with considerable prospects. In this review I only discuss low energy neutrino sources. There are many physics questions, such as the nature of the central engines of active galactic nuclei, which can be explored by doing high energy astrophysics [44]. Most of the time the information obtained from low- and high-energy neutrino astrophysics is complementary. For example, it is possible to look for high-energy neutrino signatures of cold dark matter. Supernova neutrino observations and long-baseline neutrino experiments at somewhat lower energies probe hot dark matter. These experiments together may help us assess the role of the dark matter in the structure of the universe. I believe that in the years
to come, neutrino astrophysics will answer many fundamental questions and will pose many new ones.

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