An Ethnomathematics Case Study of Candrasengkala: A Reversed Order Chronogram in Bali-Java Tradition

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Abstract. Chronogram is a method to represent year by using a phrase. It is known in some cultures in the world, including Indonesia. Not only to record years, chronogram also be used as an encryption code in ancient cryptography. The present study aims at explaining the chronogram practiced in old Bali and Java traditions called Candrasengkala. Specifically, this is an ethnomathematics case study to understand how indigenous people of Bali and Java, two closely related cultures in Indonesia, practicing their ancient chronogram. To gain the data, we employed literature review and expert interview method to deeply learn the relation and arrangement among the numbers in Candrasengkala. The collected data were analysed qualitatively using descriptive method. From the analysis, it was found that Bali-Java chronogram performed a unique place value system. It means the represented year in Candrasengkala is written in ascending (unit-tens-hundreds-thousands) rather than descending (thousands-hundreds-tens-unit) order which is commonly used nowadays.

1. Introduction

Chronogram is the art and knowledge to assign figurative word to a number. It is known in many different ancient cultures and civilizations across the world, such as Rome \cite{1}, Urdu \cite{2}, Czech \cite{3}, China \cite{4} and India \cite{5}. It gained popularity because the use of alphabets or words combination to produce a beautiful phrase or sentence which, in other hand, hold a hidden message about certain number. Due to its unique characteristics, some argue that chronogram was the ancient cryptography, provides encryption code when people want to send a secret message \cite{6}.

The knowledge about chronogram is also practiced in Bali-Java tradition, two closely related regions in Indonesia in term of distance and culture. Not just any word numerals, chronogram is employed to represent a year. It is very likely that Bali and Java got the influence from India, who comes to Indonesia in 456 CE \cite{7}. As the assimilation occurred, some knowledge also developed. It is including the system used in the ancient Bali-Java calendar.

The ancestor of Bali and Java are known to have a complex calendar which combines the arithmetic and lunar-solar astronomy based to enumerate the days. The arithmetic calendar is called \textit{pawukon}, which has 210 days in a year. The limitation of this calendar is that it cannot be used to record the year. Every cycle ends, the next cycle starts. Later, the influence of Indian astronomy brings a new method to count the year continuously. The calendar is called Caka Calendar, named after the name of King Aji Caka.
Besides to reckon the years continuously, Indian also influence the tradition in Bali and Java to write the year number in chronogram. The chronogram used in Bali and Java is called Candrasengkala. Some previous studies only mentioned Candrasengkala as a part of the historical background of certain places or phenomena [8] & [9]. In other words, the discussion about the mathematical procedures in the numbers’ arrangement of Candrasengkala was not given adequate proportion. These available resources will not be enough to provide the complete picture of Candrasengkala as it falls in the category of the intersection of mathematics, anthropology and history; or what it is usually called as Ethnomathematics [10]. It is the mathematics that emerges and be practiced by a non-mathematical group of people, might be certain profession, indigenous society or others [11] & [12].

Reflect the aforementioned background, this paper attempts to provide elaboration of Candrasengkala from ethnomathematics point of view. The result of the present study will be beneficial to enrich the ethnomathematics discussion globally. Also, further study about Candrasengkala may lead to a design and development study in preparing a culture-based mathematics teaching and learning. It will produce a new perspective of ethnomathematics learning activities which implement integrative mathematics approach in the classroom [13], [14] & [15].

2. Method

The present study is a case study. The purpose of the study is to understand and explain the mathematics behind Candrasengkala, the ancient chronogram practiced in Bali and Java. Bali and Java are two neighbourhood islands in Indonesia. Not only close by distance, Bali and Java also share some traditions. It can be seen in the language, written alphabet, folklore, building and calendar system.

The data were gathered from literature which discusses about Candrasengkala and its relation with chronogram system worldwide, especially India. Besides literature review, this study also supported by interviewing a calendar maker specialist in Bali. The interviewee is a 72 years old man who lives in Buleleng, Bali-Indonesia, who started studying traditional astronomy based on Bali tradition since 1979. The collected data were analyzed qualitatively using descriptive method to answer the research question.

3. Results and Discussion

The first method in calculating days in Bali and Java tradition is based on arithmetic calculation. It has a year with 210 days called pawukon that consist of 30 wuku. Each wuku will be combined with seven type of wewaran, ‘week’ with different lengths from one to ten. The arithmetic calendar was not used to record year, only to reckon the days. According to the belief in Bali and Java tradition, there will be a perfect day to start doing certain activities (e.g. farming, cutting hair, building house, starting community) or religious-based ceremonies [16] & [17].

By the influence of India who came to Indonesia in 456 CE, the people, especially those in Bali and Java, started to learn astronomy and added the lesson to their calendar system without replacing the arithmetic one. Indian calendar system is based on Sûrya-Siddāntha, a 3rd BCE astronomical-manuscript [18]. From this knowledge, the calendar system in Bali and Java developed to have year recording system. The calendar with continuous year is called Caka Calendar, it differs 78 years with Gregorian Calendar (e.g. year 2020 CE in Gregorian Calendar equals to year 1942 in Caka Calendar).

Another important influence from Indian is the ancient chronogram system called Bhutasamkhya [5]. In Bali-Java tradition, the chronogram is known as Candrasengkala. The difference with chronogram that developed in Rome, or Czech, Candrasengakala used word to represent a digit. There are several words that can be used to represent digit from 0 to 9 as can be seen in Table 1 (summarized from [19]).
| Digit | Possible Words | Meaning          |
|-------|---------------|------------------|
| 0     | Byoma, Langit, Awang-awang, Gegana, Luhur, Kos, Akasa, Tawang Wiyat, Widik-widik | Sky, Honorable |
|       | Musna, Nis, Sirna, Ilang, Mesat, Ngles, Moksa, Pejah, Nir, Surud | Lost, Die, Gone |
|       | Mletik, Kombul, Muluk, Oncat, Suwung, Sonya Swarga | Upward, Empty |
|       | Rusak, Brasta, Swuh, Wuk, Sempal Tanpa, Tan, Walang Windu | Broken, Ugly, Without, Insect, Very |
|       | Tumenga, Nenga, Doh, Tebih Barakan | Look up, Far, Friend |
|       | Sat | Dry |
| 1     | Tunggal, Gusti, Buddha, Eka, Hyang Sujana | One, God, King Scholar |
|       | Rupa | Various |
|       | Maha, Luwih | Very |
|       | Wong, Badan | Human |
|       | Buweng | Circle |
|       | Rat, Jagad | Universe |
|       | Surya | Sun |
|       | Candra, Lek | Moon |
|       | Kartika, Sasa, Dhara | Star |
|       | Bumi | Earth |
|       | Wiji | Seeds |
|       | Urip | Live |
|       | Ron | Leaves |
|       | Pamase, Prabu, Raja, Nata | King |
|       | Kenya, Dhara | Women |
|       | Nekung | Meditation |
|       | Niyata | Real |
|       | Iku | That, Tail |
|       | Putra | Child |
|       | Peksi, Dara | Bird |
|       | Tyas | Feeling |
|       | Wungkul | Complete |
|       | Sudira, Wani | Courage |
| 2     | Kalih, Ro, Dwi Asta | Two, Hand |
|       | Nembah, Ngabekti | Worship |
|       | Netra, Nayana, Caksuh, Mata, Paninggal, Locana | Eyes |
|       | Kembar, Apasang | Twin, Pair |
|       | Myat, Mandeng | See |
|       | Swiwi, Lar Sikara | Feather, Intervention |
|       | Gandheng, Khanti | Connect |
|       | Paksa, Ama | Obligation |
|       | Sungu | Thorn |
|       | Athi-athi, Carana | Temples’ Hair |
|       | Talingan, Karnan | Ear |
|       | Dresth | Eyebrow |
|       | Bau | Shoulder |
|       | Suk | Leg |
|       | Ngrengga | Decorate |
| Digit | Possible Words                                                                 | Meaning                  |
|-------|-------------------------------------------------------------------------------|--------------------------|
| 3     | Wahni, Ujwala, Murub, Dahana, Nala, Pawaka, Api, Apyu, Brahma, Utawaka, Kobar, Agni | Fire, Light, Rays        |
|       | Tiga, Tri                                                                      | Three                    |
|       | Kaaksi, Katon, Katingalan                                                       | Vision                   |
|       | Payudan, Rana, Rananggana                                                       | War                      |
|       | Kukus                                                                         | Steam                    |
|       | Weda, Kawruh                                                                   | Knowledge                |
|       | Kaya, Lir                                                                      | Similar                  |
|       | Benter                                                                        | Heat                     |
|       | Uninga, Wrin                                                                   | Know                     |
|       | Naut, Nauti                                                                    | Answer                   |
|       | Teken, Siking                                                                  | Stick                    |
|       | Rana                                                                           | Woman                    |
|       | Uta                                                                            | Leech                    |
|       | Ujel                                                                           | Eel                      |
|       | Wignya                                                                         | Smart                    |
|       | Gunu                                                                           | Able                     |
|       | Jatha                                                                          | Hair, Place              |
| 4     | Catur, Papat                                                                    | Four                     |
|       | Kerta                                                                          | Swallow                  |
|       | Wahana                                                                         | Vehicle                  |
|       | Warih, Toya, Udaka, We, Sindu, Her, Tirta                                       | Water                    |
|       | Jladri, Waudadi, Nadi, Tasik, Udaya, Samodra, Jalaniddhi, Segara               | Sea                      |
|       | Tlaga                                                                          | Lake                     |
|       | Yoga                                                                           | Child, Era               |
|       | Woh                                                                            | Fruit                    |
|       | Suci, Wening, Marta                                                            | Pure                     |
|       | Dadya                                                                          | Being                    |
|       | Gawe, Karya, Karti                                                             | Create                   |
|       | Sumber                                                                         | Origin                   |
|       | Karta                                                                          | Wealth                   |
|       | Sruti                                                                          | Knowledge                |
|       | Tawa                                                                           | Offer, Bargain           |
| 5     | Pandhawa                                                                       | Sons of Pandu            |
|       | Lima, Panca                                                                    | Five                     |
|       | Wisik                                                                          | Whisper                  |
|       | Wrayang, Warasta, Astra, Lungid, Sara, Bana, Jemparing, Cakra, Hru, Margana    | Arrow, Sharp             |
|       | Gati, Tata, Nata                                                               | Rules                    |
|       | Indri, Bayu, Bajra, Samirana, Pawaka, Maruta, Angin                            | Wind                     |
|       | Indriya                                                                        | Feeling, senses          |
|       | Sare, Guling                                                                   | Sleep                    |
|       | Raseksa, Diyu, Buta, Wil, Yaksa, Yaksi                                         | Giant                    |
|       | Saya, Wisaya,                                                                  | Trick                    |
|       | Galak                                                                          | Savage                   |
|       | Bayu                                                                           | Strong, Muscle           |
|       | Marga                                                                          | Road                     |
| 6     | Nenem, Nem, Sad                                                                | Six                      |
|       | Rasa, Rinaras, Karaseng                                                        | Feeling                  |
|       | Artati, Madura, Sarkara, Manis                                                 | Sweet                    |
|       | Lona                                                                           | Spicy                    |
|       | Tikta, Retu                                                                    | Bitter                   |
|       | Amla, Kayasa                                                                   | Sour                     |
|       | Oyag, Obah, Lindhu                                                             | Shake                    |
The possible words in Table 1 is based on the characteristic of digit which is found in daily live on the society in ancient Bali and Java. For instance, the word *sirna* which means gone is employed to represent the digit 0 and *apasang* which means a pair is dedicated for digit 2. Another example is *indriya* which means basic senses (usually 5) and therefore it represents the digit 5. Hence, we can infer that the figurative words is having certain characteristics that makes it is suitable to stand with certain digit.
Mathematically, from Table 1 we can observe that in writing Candrasengkala people aware of the notion of place value. It can be seen from the possible words that appear only for digit, not number. Instead of having words for every number, Candrasengkala provide digit 1 to 9 and let people combine it to create the intended number. Therefore, to represent a number, one word or more will be arranged together as the concept in place value. Place value is considered as one of the important concepts in mathematics [20] & [21]. The mastery of place value will allow people to do computation effectively.

Usually, the phrase will be created by considering the year that wishes to be represented, and the important event occurred on the year. Therefore, besides being true mathematically, the phrase also has literal meaning and historical nuance. The ancient kingdom in Java and Bali employed the Candrasengkala method to record certain circumstance, can be celebration or crisis.

The oldest inscription in Indonesia that used Candrasengkala is Canggal inscription which was found in Gunung Wukir Temple, Magelang, Central Java [22]. In Canggal inscription, there is a phrase sruti indriya rasa. Looking at the words given in Table 1, we can notice that sruti stands for 4, indriya stands for 5 and rasa stands for 6. In fact, the year represented in Canggal inscription is 654 Caka (732 CE). There is also a Candrasengkala of Warmadewa’s clan namely sara (5) wahn (3) murti (8) which represent the 835 Caka (913 CE). The phrase itself means sharp (powerful) and bright as a God (of Siwa). Another famous example of Candrasengkala is sirna (0) ilang (0) kertaning (4) bumi (1). The literal meaning of the phrase is “the wealth of earth disappeared and diminished” which refers to the year 1400 Caka (1478 CE) where Majapahit Kingdom, one of the most powerful empire in Indonesia, declined.

From the aforementioned examples, one remarkable note is that the arrangement of the digits in Candrasengkala were not from left to right as is commonly used to represent year in Gregorian Calendar. The digit is written from right to left, by which the units of the year will be in the very beginning, followed by tens, hundreds, thousands and so on.

Mathematically, we can express the digit of Candrasengkala in Canggal inscription as \(4 \times 10^0 + (5 \times 10^1) + (6 \times 10^2)\), while the Majapahit one as \((0 \times 10^0) + (0 \times 10^1) + (4 \times 10^2) + (1 \times 10^3)\). It makes the order unique since in the present mathematics textbook, the format of digits should be started from the highest place value (for example, in this case, is hundreds) to the unit. For instance, 654 usually be represented as \((6 \times 10^2) + (5 \times 10^1) + (4 \times 10^0)\). Although the order was reversed, Candrasengkala holds the core concept of place value in mathematics. Indeed, it is showing the commutative properties in addition to numbers. Mathematically it can be used to illustrate how the indigenous people understanding \(a + b = b + a\).

4. Conclusion

From the analysis it can be seen that the word numerals of Candrasengkala is using the combination of phrase that associate with certain number. It was arranged to represent a number and describe the situation happened in the year. The thorough analysis of Candrasengkala showed that this ancient chronogram practiced in Bali and Java were having an adequate understanding of place value. It can be seen that each number used in Candrasengkala can taking role as units, tens, hundreds or thousands depend on the location. However, it is not exactly same as the present common place value representation, since it was using reverse order when stating year. It means the numbers were arranged from units to the larger one.

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