Externalities in Socially-Based Resource Sharing Network

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\textbf{ARTICLE HISTORY}
Compiled November 27, 2018

\textbf{Abstract}
This paper investigates the impact of link formation between a pair of agents on resource availability of other agents in a social cloud network, which is a special case of socially-based resource sharing systems. Specifically, we study the correlation between externalities, network size, and network density.

We first conjecture and experimentally support that if an agent experiences positive externalities, then its closeness (harmonic centrality measure) should increase. Next, we show the following for ring networks: in less populated networks no agent experiences positive externalities; in more populated networks a set of agents experience positive externalities, and larger the distance between agents forming a link, more the number of beneficiaries; and the number of beneficiaries is always less than the number of non-beneficiaries. Finally, we show that network density is inversely proportional to positive externalities, and further, it plays a crucial role in determining the kind of externalities.

\textbf{KEYWORDS}
network externalities; sharing economy; network formation; data-backup service; social cloud

1. Introduction

Network externalities (or spillover) is a widely studied topic in various contexts, for example, in research and development (Miyazaki and Aman 2008), in LCD television market (Livingston et al. 2013), in strategic delegation (Hou, Hong, and Zhao 2017), and in urban and rural produced goods market (Pandey and Whalley 2009).

We study externalities in the context of endogenous network formation (where economic agents decide with whom they want to form social connections and with whom they do not). According to Jackson (2008), “externalities refer to situations where the utility or payoffs to one individual are affected by the actions of others, where those actions do not directly involve the individual in question”.

The endogenous network formation literature consists of several network formation models, which exhibit only positive externalities (Jackson and Wolinsky 1996; Goyal and Joshi 2006\textsuperscript{b}; Belleflamme and Bloch 2004), only negative externalities (Jackson and Wolinsky 1996; Goyal and Joshi 2006a,\textsuperscript{a}), both positive-negative externalities
(Möhlmeier, Rusinowska, and Tanimura 2016; Currarini 2007), or no externalities (Mane, Ahuja, and Krishnamurthy 2016).

In general, the role of externalities is usually studied in determining which network structure is likely to emerge, and the tension between network stability and its efficiency (Buechel and Hellmann 2012). Stable network is the one where no agent has an incentive to alter the network structure by either forming new or severing existing social connections. Efficient network is the one that maximises the overall benefit of everyone involved.

However, in this paper, we study externalities with a different motivation. In particular, we study how closeness (how much an agent is close to others in the network), network size (number of agents in the network), and network density (the proportion of actually present links to all potential links in the network) affect externalities.

To study the above aspects, we consider the socially based resource sharing network model proposed by Mane, Krishnamurthy, and Ahuja (2014), where the network is built endogenously. Social Cloud (Chard et al. 2012), BuddyBackup\(^1\) or CrashPlan\(^2\) are few examples of socially-based resource sharing systems. These systems follow the sharing economy model where agents share/trade storage resources owned by them with their social relatives for data backup purposes. In recent years, this area of research has emerged at the intersection of computer science (Chard et al. 2012; Blume et al. 2015) and economics\(^3\).

In Section 2, we revisit the above described model and refine it (without loss of generality) to make it relevant for our study. Section 3 states our motivation. In Section 4, we first provide our data and experimental setup. Next, we study how network size affects externalities through empirical analysis. Finally, we analyse how network density and externalities are related. Section 5 concludes our discussion.

2. Our Model

Social Cloud \(g = (N, L)\) is a resource (e.g., disk space) sharing (or trading) network consisting of two sets; \(N\) (a set of agents, who are engaged in resource sharing) and \(L\) (a set of links connecting these agents). A link \(\langle ij \rangle \in L\) represents that agents \(i, j \in N\) share computational resources with each other when needed. A link \(\langle ij \rangle\) is undirected, which implies resource trading is bidirectional. The number of agents is represented by \(n\) (i.e., \(|N| = n\)) and the number of links is represented by \(\ell\) (i.e., \(|L| = \ell\)). The shortest distance between two agents \(i\) and \(j\) in the network \(g\) is denoted by \(d_{ij}(g)\).

The diameter \(D_g\) of \(g\) is the maximum shortest path between any pair of agents. A network structure \(g\) updates to \(g + \langle ij \rangle\) when a pair of agents \(i, j \in g\) form a link \(\langle ij \rangle\) in \(g\).

In \(g\), agents perform computation tasks like data backup. If an agent does not have a resource to accomplish its computational task, then it is dependent on the network to get the required resource. Hence, resource availability is at the center for the discussion of externalities.

Agents could limit resource sharing with those friends who are close (important) to

\(^1\)http://www.buddybackup.com
\(^2\)https://support.crashplan.com
\(^3\)The recent report of kbv research (Report ID: 978-1-68038; On-line available at: https://kbvresearch.com/data-backup-and-recovery-market/) expects that the data backup and recovery market will grow at a Compound Annual Growth Rate (CAGR) of 10.2 during the forecast period and reach $12.9 billion by 2023.
them (Chard et al. 2012). In order to capture this notion of closeness, we make use of the harmonic centrality measure discussed in Boldi and Vigna (2014) and is given as follows:

$$\Phi_i(g) = \sum_{j \in g \setminus \{i\}} \frac{1}{d_{ij}(g)}.$$

where $\Phi_i(g)$ denotes how close an agent $i$ is to others in $g$ (or how close other agents are to $i$ in $g$).

In $g$, the chance that agent $i$ obtains a resource from $j$ is $\alpha_{ij}(g) = \frac{1}{d_{ij}(g)} \Phi_j(g)$ (Mane, Krishnamurthy, and Ahuja 2014). Further, let $\gamma_i(g)$ be the chance that the agent $i$ will get the resource from at least one agent in the network $g$. This is given as follows:

$$\gamma_i(g) = 1 - \prod_{j \in g} (1 - \alpha_{ij}(g)).$$

(1)

Next, we formally define externalities using the above terminology. This is motivated from a related definition given by Jackson and Wolinsky (1996).

**Definition 2.1.** Consider a network $g$, with agents $i, j, k \in g$ such that $i \neq j \neq k$ and $\langle jk \rangle \notin g$. Now, let agents $j$ and $k$ form a direct link. Then, all agents $i \in g \setminus \{j,k\}$ experience

1. no externalities if $\gamma_i(g + \langle jk \rangle) = \gamma_i(g)$,
2. negative externalities if $\gamma_i(g + \langle jk \rangle) < \gamma_i(g)$, and
3. positive externalities if $\gamma_i(g + \langle jk \rangle) > \gamma_i(g)$.

Informally, the above definition captures the effect of a newly formed link $\langle jk \rangle$ on the resource availability of all agents $i$ in $g$, except the pair of agents $j, k$ who are involved in this link formation.

### 3. Motivation

We expand our motivation with the following example: consider the network $g$ as in Figure 1(a). Let us assume that agents $j$ and $k$ decide to form a link. Then, network structure changes from $g$ to $g + \langle jk \rangle$ (see Figure 1(b)). The quantity $\gamma_i(g + \langle jk \rangle) - \gamma_i(g)$ (by using (1)) provides a basis for study of externalities. This computed quantity for

![Figure 1. Externalities Example](image-url)
all \( i \in g \) is shown in Table 1. This table shows that the link \( \langle jk \rangle \) is advantageous for agents \( f \) and \( g \) but it is disadvantageous for all the remaining agents\(^4\). In other words, agents \( f \) and \( g \) experience positive externalities (highlighted in blue) and the remaining agents experience negative externalities (highlighted in red). This example motivates us to investigate the aspects discussed in Section 1, that is, how closeness, network size, and network density affect externalities.

| agent \((x)\) | \(\Phi_x(g)\) | \(\gamma_x(g)\) | \(\Phi_x(g + \langle jk \rangle)\) | \(\gamma_x(g + \langle jk \rangle)\) | \(\gamma_x(g + \langle jk \rangle) - \gamma_x(g)\) |
|---|---|---|---|---|---|
| \(a\) | 6.25 | 0.654 | 6.25 | 0.646 | -0.008 |
| \(b\) | 6.25 | 0.654 | 6.33 | 0.650 | -0.004 |
| \(c\) | 6.25 | 0.654 | 6.33 | 0.650 | -0.004 |
| \(d\) | 6.25 | 0.654 | 6.25 | 0.646 | -0.008 |
| \(e\) | 6.25 | 0.654 | 6.25 | 0.644 | -0.011 |
| \(f\) | 6.25 | 0.654 | 6.50 | 0.657 | 0.003 |
| \(g\) | 6.25 | 0.654 | 6.50 | 0.657 | 0.003 |
| \(h\) | 6.25 | 0.654 | 6.25 | 0.644 | -0.011 |
| \(i\) | 6.25 | 0.654 | 6.25 | 0.637 | -0.017 |
| \(j\) | 6.25 | 0.654 | 7.00 | 0.687 | 0.033 |
| \(k\) | 6.25 | 0.654 | 7.00 | 0.687 | 0.033 |
| \(l\) | 6.25 | 0.654 | 6.25 | 0.637 | -0.017 |

Table 1. Externalities in a given network \(g\).

From Table 1 we further observe the following:

**Remark 1.** Due to the newly added link \( \langle jk \rangle \) in \(g\) leading \(g + \langle jk\rangle\), the closeness remains the same for a set agents and it increases for a set of agents (it does not decrease). For example, in Table 1, closeness of agents \(h\) and \(i\) remain the same, but the closeness of agents \(b\) and \(c\) increases.

**Remark 2.** We conjecture that for an agent to experience positive externalities, the increment in its closeness is a necessary but not a sufficient condition. For example, although the closeness of agents \(b\) and \(c\) increases, their chance of obtaining a resource does not improve.

4. Experimental Analysis

Here, we first discuss our setup. Next, we present our initial observations on the correlation between externalities, network size, and network density through experiments.

4.1. **Data and Experimental Design**

A recent review of the literature on experimental research on networks in economics by Choi, Gallo, and Kariv (2016) found that many studies focus on a small set of nodes and various classes of networks (e.g., complete, wheel, etc.). Frey, Corten, and Buskens (2012) carried out experiments to study a coordination game between 6-nodes (agents) with different network structures like complete, star, line, ring and bipartite. Gallo and Yan (2015) studied the game of strategic complements on ring and wheel networks of sizes 9, 15-nodes.

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\(^4\)By Definition 2.1, we do not study externalities for the agents directly forming the link, i.e. \(j\) and \(k\) here.
Thus, to begin with, we perform experiments on networks with a small \( n \) since it is limitless\(^5\). In our experiments, we focus on \( \eta \)-regular networks, where each agent has a \( \eta \) number of neighbours. Although regular networks can have various geometries, these do not have any considerable effect on the harmonic centralities distribution (or closeness, which we use). This is unlike the scale-free network. In fact, the ring network (the class of regular network where \( \eta = 2 \)) has uniform harmonic centralities distribution (i.e. all the agents have the same harmonic centrality). This property helps us to understand the correlation between externalities and network size. Hence, in our experiments, we predominantly focus on the ring network.

Due to unavailability of data of real world socially based resource sharing networks like BuddyBackup\(^1\) or CrashPlan\(^2\), we generate these networks by using SocNetV (Social Network Analysis and Visualization) Software\(^6\). This tool helps to generate different social networks and derive the shortest path between agents as well as the diameter of any given network.

Next, we propose an algorithm to analyse externalities. In Algorithm 1, we initially compute \( \gamma_i(g) \) from (1) for all agents in any input network \( g \) as well as given networks diameter \( D_g \). Then, we select an agent \( i \) in \( g \). Next, we form a link of agent \( i \) with another agent \( j \) whose distance is two hops from \( i \). We compute \( \gamma_i(g + \langle ij \rangle) - \gamma_i(g) \) to observe the effect of the newly formed link \( \langle ij \rangle \) on all other agents probability of getting resource in the new network \( g + \langle ij \rangle \). We do the above procedure for all agents who are located at distance two hops from agent \( i \) in \( g \). Then, we increment the distance by one and follow the same procedure to analyse further. We do this until we exhaust all the agents who are located at the maximum shortest path (i.e. the diameter \( D_g \) of the network) from agent \( i \). The agent \( i \) is chosen in one way for the network size experiments and in another way for the network density experiments, and is discussed in their respective sub-sections below.

**Algorithm 1. Algorithm for Analysing Externalities in a Network \( g \)**

\begin{algorithm}
\begin{align*}
\textbf{Input:} & \quad \text{Network } g \\
\textbf{Output:} & \quad \text{Number of Beneficiaries for an agent } i \ (\text{NOB}_i) \\
1: & \quad \text{for each agent } i \in g \ \text{do} \\
2: & \quad \quad \text{Compute } \gamma_i(g) \text{ by (1)} \\
3: & \quad \text{end for} \\
4: & \quad \text{Compute diameter } D_g \text{ of } g \\
5: & \quad \text{For an agent } i \in g \text{ set } \text{NOB}_i = 0 \\
\{& \text{NOB}_i \text{ is the number of beneficiaries due to agents } i \text{'s link formation with other agents}\} \\
6: & \quad \text{Set distance } = 2 \\
7: & \quad \text{while distance } \leq D_g \ \text{do} \\
8: & \quad \quad \text{for each agent } j \in g \text{ such that } d_{ij}(g) = \text{distance do} \\
9: & \quad \quad \quad \text{Add a link } \langle ij \rangle \text{ in } g \\
\{& \text{Network } g \text{ is updated and it becomes } g + \langle ij \rangle\} \\
10: & \quad \quad \text{Set } \text{NOB}_{ij} = 0 \\
\{& \text{NOB}_{ij} \text{ is the number of beneficiaries due to newly added link between } i \text{ and } j\} \\
11: & \quad \quad \text{for each } k \in g + \langle ij \rangle \setminus \{i, j\} \ \text{do} \\
12: & \quad \quad \quad \text{Compute } \gamma_k(g + \langle ij \rangle) \text{ by (1)} \\
13: & \quad \quad \quad \text{Compute } \gamma_k(g + \langle ij \rangle) - \gamma_k(g) \\
14: & \quad \quad \quad \text{if } \gamma_k(g + \langle ij \rangle) - \gamma_k(g) > 0 \ \text{then} \\
15: & \quad \quad \quad \quad \text{NOB}_{ij} = \text{NOB}_{ij} + 1 \\
16: & \quad \quad \quad \text{end if} \\
17: & \quad \quad \text{end if} \\
18: & \quad \quad \text{NOB}_i = \text{NOB}_i + \text{NOB}_{ij} \\
19: & \quad \text{Delete } \langle ij \rangle \text{ in } g + \langle ij \rangle \\
20: & \quad \text{end for} \\
\end{align*}
\end{algorithm}

\(^5\)For example, there are \( 2^{\frac{n(n-1)}{2}} \) number of labeled undirected networks possible with any given \( n \).

\(^6\)http://socnetv.org/
21: increase distance by 1
22: end while

end

4.2. Network Size

To study the correlation between externalities and network size, we execute Algorithm 1 on ring networks (i.e., \( \eta = 2 \)) with sizes varying from 4 to 30 agents. Note that, as earlier, here for a given input, all runs of (or calls to) Algorithm 1 give the same result since the ring network has uniform centralities distribution.

Figure 2 summarizes the results obtained. In Figure 2, the \( x - axis \) represents the shortest distance between two agents involved in a link formation. The \( y - axis \) represents the number of beneficiaries (NOB) (i.e. how many agents experience positive externalities due to a link addition in a network). The \( z - axis \) represents the network size (i.e. the number of agents in the network). Figure 2 helps answer questions like, in a network consisting of \( n \) agents, if a pair of agents \( i \) and \( j \) who are located at distance \( d \) from each other form a direct link, then this link introduces positive externalities for \( NOB_{ij} \) (Line 15 in Algorithm 1) agents.

**Finding 1.** In “less” populated ring networks, agents do not experience positive externalities.
In a network with size varying from 4 to 10 (see Figure 2(a)), agents do not experience positive externalities. This is because as conjectured in Remark 2, an increment in closeness is a necessary condition for experiencing positive externalities, which does not happen here. However, from network size 11 to 20 (see Figure 2(b)), we find that a substantial number of agents experience positive externalities. This is because the addition of a link between a pair of agents (who are located at the specific distance) increases the closeness of a certain number of agents. A similar pattern is observed for network sizes 21 to 30 (Figure 2(c)).

**Finding 2.** In ring networks of size greater than 10, as the distance between two agents (involved in a link formation) increases, the number of beneficiaries often increase.

For example, in the network of size 21 (see Figure 2(c)), if a pair of agents who have distance 2 form a link, we have 3 NOB. But, if a pair of agents who have distance 6 form a link we have 4 NOB. Most of the plots in Figure 2(b), 2(c) and 2(d) are of this type. However, sometimes with increment in the distance, we see a reduction in NOB as well although the percentage of these cases is very low.

**Finding 3.** In ring networks, the number of beneficiaries is always less than the number of non-beneficiaries.

In all our experiments, with network size varying from 4 to 30, the percentage of beneficiaries varies from 0% to 26% of the total number of agents in the network (the remaining percentage is of non-beneficiaries).

### 4.3. Network Density

Network density of a network $g$ is the ratio of the number of existing links in $g$ to the maximum number of possible links in $g$ (Wasserman and Faust 1994). That is, $\varpi(g) = \frac{\ell}{\frac{n(n-1)}{2}}$. Thus, $\varpi(g)$ goes from 0 (if each agent is isolated in $g$, i.e. $\ell = 0$) to 1 (if each agent has connections with all other $n-1$ agents in $g$, i.e. $\ell = \frac{n(n-1)}{2}$).

To study the correlation between externalities and network density, we focus on $\eta$-regular networks with varying $\eta$’s. As compared to the network size experiments above, where $\eta$ was 2 (or ring networks), this is done so that different agents have different closeness, which would help in our analysis (as earlier, for ring networks, closeness of all agents was the same). The range of number of agents $n$ is same as above, but for the sake of compactness we report data for the number of agents varying from 11 to 20, which form a representative set.

For a given $n$ and a given $\eta$ (which also gives the diameter of the network or $D_g$), we first compute its density ($\varpi(g)$). Computing NOB is more involved here because of the varying closeness of agents. Next, we compute closeness of all agents, and sort them into buckets corresponding to the same closeness\(^7\). Then, we randomly pick any agent from each bucket and run Algorithm 1. We report the maximum of NOB obtained from all buckets. Note that the combination of odd $n$ and odd $\eta$ is not valid (i.e., a network is not possible for this combination). The results of these experiments are given in Table 2.

**Finding 4.** The network density is inversely proportional to positive externalities.

\(^7\)The bucket logic is added for efficiency. Also, for ring networks, we had only one such bucket because all agents had the same closeness.
From Table 2, we observe that the network density and the NOB are inversely proportional. That is, as the network density increases, the number of agents who
experience positive externalities decreases. We analyse this finding further by using our earlier conjectures. From Remark 1, we know that by every link addition the closeness of all agents either remains same or increases. From Remark 2, we know that for an agent to experience positive externalities, its closeness must improve due to this new link (although this is not sufficient).

In a highly connected network (or a dense network, i.e., $\nabla(g) \to 1$), agents are already very close to each other, and hence, it is less likely that a newly added link improves their closeness. Thus, here the chance of agents experiencing positive externalities is also less.

On the contrary, in a less connected network (or a sparse network, i.e., $\nabla(g) \to 0$), when a pair of agents form a link, it is very likely that this newly added link improves the closeness of many agents. Thus, here the chance of agents experiencing positive externalities also increases.

**Finding 5.** The above relation between the network density and positive externalities, to a large extent, is independent of the number of agents in the network.

As in Table 2, for network sizes varying from 11 to 20, we see that for a linear increment in $\eta$, the number of neighbours (or a linear decrement in the maximum shortest path), the network density increases and the NOB decrease loosely following different arithmetic progressions, which is independent of the number of agents in the network. A similar behavior is observed for network sizes less than 11, but that data is not reported in this table for the sake of compactness.

5. Conclusion

In this paper, we study the correlation between externalities, network size, and network density. For this, we first use the harmonic centrality measure to capture the closeness of agents. With the focus on socially-based resource sharing systems, next we define our model. Further, we conjecture that for an agent to experience positive externalities, the increment in its closeness is a necessary but not a sufficient condition.

We perform experiments on different $\eta$-regular networks (specifically ring networks). The evidence from this experimental study suggests the following: the chance of experiencing positive externalities is more in larger networks; the number of beneficiaries is loosely proportional to the distance between agents forming a link; and the ratio of the number of beneficiaries to the number of non-beneficiaries is always less than one (when new links are formed). We also demonstrate that network density plays a crucial role in determining the kind of externalities, and it is inversely proportional to positive externalities.

We are aware that our findings are preliminary in nature and are limited by a small set of sample sizes. Nevertheless, we believe that this is the first attempt of its kind. Our work substantially adds to the understanding of externalities in socially-based resource sharing networks and serves as a springboard for further discussions. Our future investigations in this area, besides working with large social networks, will also include collaboration and peer-to-peer networks.

*https://snap.stanford.edu/data/#canets*
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