Finslerian Post-Lorentzian Kinematic Transformations in Anisotropic-Space Case

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Abstract

The Finslerian post-Lorentzian kinematic transformations can explicitly be obtained under uni-directional breakdown of spatial isotropy, provided that the requirement that the relativistic unit hypersurface (indicatrix or mass shell) be a space of constant negative curvature is still fulfilled. The method consists in evaluating respective Finslerian tetrads and then treating them as the bases of inertial reference frames. The Transport Synchronization has rigorously been proven, which opens up the ways proper to favour the concept of one-way light velocity. Transition to the Hamiltonian treatment is straightforward, so that the Finslerian transformation laws for momenta and frequencies, as well as due Finslerian corrections to Doppler effect, become clear. An important common feature of the ordinary pseudo-Euclidean theory of special relativity and of the Finslerian relativistic approach under study is that they both endeavour to establish a universal prescription for applying the theory to systems in differing states of motion.

Key words: Finsler metric, special relativity, non-Lorentzian transformations, relativistic effects.

Abbreviations: SR, FG, RS, and RF will be used for special relativity, Finsler geometry, reference system, and (inertial) reference frame, respectively.
1. Introduction

Kinematically, the FG-generalization of the pseudo-Riemannian relativistic theory of space-time may be seen to consist in that the metric tensor is admitted to depend on motion velocity of local observer. Involved SR-ideas will be discussed and contrasted with their possible FG-extensions in Section 2.

When going beyond square-root metric, it occurs constructive to keep the fundamental principle:

\[\mathcal{P}_1: \text{Indicatrix (mass shell) is a space of constant negative curvature.}\]

The associated indicatrix extends ordinary relativistic hyperboloid on a tiny deformation estimated by a small characteristic parameter \(g\).

The pseudo-Euclidean metric function has long been a convenient theoretical cornerstone of relativistic physics and our intuition is well developed for such a metric. Nevertheless, the empirical constraints on the explicated Lorentz invariance, as well as on the very metric, are difficult to express in an exact way:

“...in practice, however, it is often impossible to disentangle Lorentz invariance from other theoretical issues and experimental complications” (M.P. Haugan and C.M. Will [1], p. 69).

That is to say, the degree of agreement can be estimated but outwardly, on the basis of a particular successful extension enabling one to think of the agreement in terms of upper bounds of the characteristic parameters of the extension.

In this vein, we start with stipulation of a local preferred RF, to be denoted as \(\Sigma\), in which Lorentz invariance is expected to breakdown because of a physically-distinguished spatial direction. To get a systematic Finslerian spatially-anisotropic SR-framework, we are to substitute a relevant FMF with the ordinary pseudo-Euclidean function at the starting point of relativity theory. To this end we act in several proper steps.

First, we set forth the concept

**Finslerian Geometrically-Distinguished Direction**, to be denoted as \(\mathcal{FD}\)-direction.

Namely, let a small space-time local region \(U\) be chosen and endowed with ordinary square-root relativistic metric \(S(T, X, Y, Z) = \sqrt{T^2 - X^2 - Y^2 - Z^2}\). We interpret the coordinate set \(\{T, X, Y, Z\}\) as being related to a “rest frame” \(\Sigma\) meaningful over \(U\). Let \(\vec{D}\) be a vector in \(U\). We say that \(\vec{D}\) assigns a \(\mathcal{FD}\)-direction in \(U\) if the account of such direction deforms the metric \(S\):

\[S(T, X, Y, Z) \xrightarrow{\mathcal{FD}} \Phi(\vec{D}; T, X, Y, Z) \quad (0.1)\]

to get a Finslerian metric function \(\Phi\).

Second, we fix the case when \(\vec{D}\) is of spatial nature and imply the correspondence principle

\[\Phi(\vec{D} = 0; T, X, Y, Z) = S(T, X, Y, Z) \quad (0.2)\]

to hold safely.

Afterward, the rigorous isotropy is assumed to take place around the \(\mathcal{FD}\)-direction. That is, choosing the \(\vec{R}^1\)-axis to point in the the \(\mathcal{FD}\)-direction, and re-labelling the coordinates whenever convenient: \(\{T, X, Y, Z\} \rightarrow \{R^0, R^1, R^2, R^3\}\), we ascribe to the dependence the representation

\[\Phi(\vec{D}; R^0, R^1, R^2, R^3) = F \left( g; R^1, \sqrt{(R^0)^2 - (R^2)^2 - (R^3)^2} \right) \quad (0.3)\]
where $g$ is a small parameter for due experimental estimations.

Finally, we follow the Principle $P_1$ formulated above.

This way proves to fix the FMF $F$ and the associated Finslerian Hamiltonian function $H$ as follows.

\[ F(g; R) = |R^1 - g - q|^{G+/2} |R^1 - g + q|^{-G-/2} \]  
(0.4)

and

\[ H(g; P) = |P_1 - g + \hat{q}|^{-G-/2} |P_1 - g - \hat{q}|^{G+/2}, \]  
(0.5)

where

\[ q = q(R) = \sqrt{|(R^0)^2 - (R^2)^2 - (R^3)^2|} \]  
(0.6)

and

\[ \hat{q} = \hat{q}(P) = \sqrt{|(P_0)^2 - (P_2)^2 - (P_3)^2|}; \]  
(0.7)

we use the notation $G = g/h$, $h = (1 + \frac{1}{4}g^2)^{1/2}$, $g_+ = -\frac{1}{2}g + h$, $g_- = -\frac{1}{2}g - h$, $G_+ = g_+/h \equiv -\frac{1}{2}G + 1$, $G_− = g_-/h \equiv -\frac{1}{2}G - 1$; more detailed description can be found in [2].

In the timelike sector,

\[ g - q < R^1 < g + q, \]  
(0.8)

it is convenient to introduce the notation

\[ v = R^1/R^0 \equiv v^1, \quad u = \sqrt{(R^2)^2 + (R^3)^2}/R^0 \equiv v^\perp \]  
(0.9)

together with

\[ M = \sqrt{1 - u^2} \]  
(0.10)

and use the functions

\[ Q(g; v, u) = 1 - v^2 - u^2 - g v M \equiv M^2 - g v M - v^2 > 0 \]  
(0.11)

and

\[ V(g; v, u) = (v - g_+ M)^{G+/2} (g_+ M - v)^{-G-/2}, \]  
(0.12)

so that

\[ F = R^0 V. \]  
(0.13)

We find

\[ \frac{\partial V}{\partial v} = -(g M + v) \frac{V}{Q}, \quad \frac{\partial V}{\partial u} = -u \frac{V}{Q}. \]  
(0.14)

In the FG-approach under development, we may (and shall) truly follow the ordinary fundamental special-relativistic view, which matches also the general-relativistic view, that the light front is defined by the isotropic surface of the basic metric function of the physical space-time. Namely, if in the traditional SR we take

\[
\text{the pseudoEuclidean light front equation} : \quad S = 0, \tag{0.15}
\]

then under the Finslerian extensions we should merely insert here $F = F_{\{Finslerian\}}$ in place of $S = F_{\{Riemannian\}}$ to get

\[
\text{the Finslerian light front equation} : \quad F = 0. \tag{0.16}
\]

Faced with feasibility of the FG-way of consistent treatment, it is also worth adhering straightforwardly to the view that the kinematic relativistic transformations are
of but a pure-passive meaning. Namely they merely specify the variation rules for the vector components in going from one RF to another RF, – the four-dimensional vectors themselves remain unchanged, keeping their directions in the four-dimensional space-time. Therefore, to develop Finslerian kinematics a researcher is invited to follow

**The Tetrad-Kinematic Method:**

\[ F \rightarrow \{ g_{pq} \} \rightarrow \{ H_q^{(P)} \} \rightarrow \text{the kinematic transformations} \quad (0.17) \]

which works well in purely-inductive way. That is, given a FMF \( F \), one should calculate the associated metric tensor, then find for the tensor the orthonormal tetrads, and after that treat them as RSs for inertial observers to deduce the precise formulae for kinematic transformations (replicating in fact the method applied ordinarily in tetradic topics of general relativity; see [3-5]). For the relevant FMF (1.4), the method leads directly to the extended kinematic transformations presented explicitly in Section 3 (intermediate calculations will not be shown, for they are closely similar to those presented earlier in [6-9]).

On so doing, we apply direct calculations to arrive in Section 4 at the important conclusion that, for the Finslerian kinematic transformations obtained, the following Proposition is valid.

**PROPOSITION 1.1.** The Transport Synchronization holds fine.

Therefore, no conventionalistic “vicious circularities” would enter the Finslerian approach under development, for it is quite obvious that one can state

**PROPOSITION 1.2.** Given the light-front equation (1.16) with a particular metric function \( F \). If the Transport Synchronization holds as basic then the Light Signal Synchronization Method, founded on Eq. (1.16), can be verified against it and thus gets empirical, and vice versa.

The truth of these two Propositions, when taken in concert, leads obviously to justifiable utilization of the one-way light velocity.

The value of \( g \) should thus characterize the degree to which Lorentz invariance is broken in \( U \). The speed of light, of either one-way type or round-trip type, is no more a universal constant, being expected to be anisotropic even in the input rest frame \( \Sigma \).

Indeed, by comparing (1.4) and (1.16) we get

**PROPOSITION 1.3.** The firesurface in the input rest frame \( \Sigma \) is given by the equations

\[
\left( \sqrt{1 + \frac{1}{4}g^2 + \frac{1}{2}g} \right)^2 (R^1)^2 = (R^0)^2 - (R^2)^2 - (R^3)^2, \quad \text{if} \quad R^1 \geq 0, \quad (0.18)
\]

and

\[
\left( \sqrt{1 + \frac{1}{4}g^2 - \frac{1}{2}g} \right)^2 (R^1)^2 = (R^0)^2 - (R^2)^2 - (R^3)^2, \quad \text{if} \quad R^1 \leq 0. \quad (0.19)
\]

In particular, whenever \( R^1 = 0 \), propagation of light is going in accord with the ordinary isotropic law:

\( (R^0)^2 - (R^2)^2 - (R^3)^2 \) for the FD-perpendicular directions, \( (0.20) \)
so \( c^+ = 1 \) holds true.

Alternatively, if propagation takes place along, resp. opposite to, the \( \mathcal{F} \mathcal{D} \)-direction, so that \( R^2 = R^3 = 0 \) and \( R^1 > 0 \), resp. \( R^1 < 0 \), then for the light velocity value \( c^+ \), resp. \( c^- \), we have

\[
c^+ = \sqrt{1 + \frac{1}{4} g^2 - \frac{1}{2} g}, \quad c^- = \sqrt{1 + \frac{1}{4} g^2 + \frac{1}{2} g},
\]

which entails

\[
\frac{c^+ - c^-}{2} = -\frac{1}{2} g
\]

and

\[
\frac{c^+ + c^-}{2} = \sqrt{1 + \frac{1}{4} g^2}.
\]

After which in Section 5 we propose the Finslerian relativistic extensions for the dispersion relations and for the four-momentum. Basic conclusions will be summarized in the last Section 6.

2. ON INVOLVED PRINCIPLES AND IDEAS

**Ghost of Synchrony** has plagued the SR over a century. The so-called “clock synchronization problem” was given a strong impetus by the critical essays by Reichenbach [10], Grünbaum [11-12], and Jammer [13], published to overcome “insufficient attention to the conventions hidden in procedures of length measurements”. These authors motivated the view that no observational difference would arise if the one-way velocity of light were assumed to be anisotropic.

Indeed, when one simply raises the claim: “Measuring light velocity implies synchronized clocks” VS the opposed claim: “In order to synchronize clocks one needs the one-way velocity of light”, the logical situation looks circular. An urgent analysis and discussion of the differing views of synchronism can be found in the recent survey [14] which has embraced numerous books and papers that bear directly to the question.

However, the fact is that, though the synchrony-treatments certainly refined many important aspects of relativity, circling various possible re-synchronization methods is impotent to lead to a new fundamental relativistic theory. For they merely redone clock setups without affecting the primary metric function, the latter was fixed to be of the traditional square-root type. No new relativistic physics is stemming from re-synchronization procedures.

More than that, in the Lorentzian SR there exists

**THE CANONICAL SYNCHRONIZATION**

as presented by the Einstein synchronization on the basis of the isotropic and symmetric radar-way method [15]. Apart that the method is highly practical, that synchronization in the Lorentzian SR is equivalent to The Transport Synchronization provided that one follows the Tetrad Kinematic Method (1.17) (which entails the ordinary Lorentz transformations in case of the pseudo-Euclidean basic metric).

If one takes the position that “geometry is a convention”, than slow clock transport method and any context of light signal prescriptions are logically independent of each other. Under such a free position, one remains uncertain of a physical way to avoid
conventionalistic “vicious circularities” and is doomed to make the one-way speed of light indeterminate.

Conversely, we can undermine that position to argue against the logistics of “the geometrical conventionalism” in simple terms: “A physicist needs a particular metric function to be a physicist”, - and then to raise the simple question: ”Would The Canonical Synchronization remains being meaningful under a Finslerian metric extension?”.

The true answer is “Yes” in view of Propositions 1.1 and 1.2 formulated above in Introduction. The due method can be properly-motivated by geometrical reasons.

Realism of one-way light velocity concept is seriously supported by intuition of pragmatic-party physicists. Indeed, though

QUESTION: Whether or not a one-way velocity of light is physically meaningful quantity?

is deeply perplexing with anarchy of possible re-synchronizations of distant clocks, the photon is a particle like other quantized particles.

“After all light goes from a point to another in a well-defined way, and it would be very strange if the true velocity were forever inaccessible to us.” [F.Selleri [16], p. 44]

In this connection, many keen and important questions can be raised for. First of all, we should ask: If one admits that an electron, when going from one point to another, is given a certain physical velocity in an ordinarily accepted sense, why a photon is not? If a neutrino has a zero mass, it should be uncertain with the velocity like photons! But if a neutrino has acquired any small rest mass, it should have been described with a precise physically measurable values of an operator of velocity?

Moreover, one is fraught with

COINING: A conventionality of the one-way light velocity opposite to a reality of the photon momentum.

Indeed, on the one hand, a conventionality is a strong philosophical and logistic argument. However, if the velocity of photon is indeed “a conventional entity”, then the momentum of photon should also be of some discriminating nature. On the other hand, whenever physicists evaluate relativistic-particle experiments, they apply faithfully the conservation law of momentum, particularly for the Compton Effect:

\[ p_\gamma + p_e = p'_\gamma + p'_e, \]  

(2.1)

where the photon momenta \( \{p_\gamma, p'_\gamma\} \) are certain and measurable physical quantities as well as the electron ones \( \{p_e, p'_e\} \). If the one-way light velocity is conventional, why the momentum of photon is not? Eclecticism of such a situation is apparent from various physical standpoints, and any claim to favour synchronism is coining: the speed of light in one direction was never measured in not a single experiment - but the photon momenta were directly detected in everyday established practice in numerous laboratories?

Alternatively, the right-sense realism was assigned to the one-way light velocity by C.M.Will [17] in terms of measurable quantities: whenever the latters are properly considered to be “true physical arguments of the light velocity”, the experimental data are independent of the clock synchronization procedures. If one endeavours to analyze experiments on the basis of such a clear physical principle, as was done in [17], the one-way light velocity is fully vindicated and stops being an “intelligible convention”.

In any case, the Light Conventionality is deeply connected with the question of Nature of Geometry. The fact is that, traditionally, the SR-works presupposed the square-root metric. If, however, the question is investigated in the light of the FG-approach, we
may follow the realistic geometry-motivated way of reasoning actually in accord with the famous as well as pragmatic claims:

“Laws of geometry and nature are complementary.” (E.A.Milne [18], p. 17)

and

“... one cannot maintain that distant simultaneity is conventional without also maintaining that such basic quantities as the proper time metric are conventional as well.” (M.Friedman [19], p. 77).

Indeed it is simply obvious that, because The General Theoretical Relativistic Framework indispensably involves the Light Behaviour and the Space-Time Metric as being important parts, then “testing one part means a test for the second part”. That is to say, the two Parts should go hand in hand over the SR-physics!

**Interferometer-type experiments of post-Lorentzian appeal** can conventionally be devided in two following distinguished groups.

$I_1$: The turntable experiments have played a prominent role in the testing of light behaviour. They include: first, the historically-valuable Michelson-Morley type experiments (of which the most precise one was carried out by Joos in 1930 [20]) used optical interferometers and, second, the modern highly-monochromatic maser experiments”.

Proposals of respective maser-used techniques can be traced back to 1961:

“It can be hoped because of the striking monochromaticity of the radiation produced by infrared and optical masers that they will contribute to a number of fundamental experiments which involve very precise measurements or comparisons of two lengths or of two frequencies. One such example would be a highly improved experiment of the Michelson-Morley type.” [C.H.Townes, [21], p. 9].

This general program was specified as follows:

“If one maser is rotated with respect to the other and changes in the relative frequency of the two measured, any change in the effective optical path for the two can be detected. Thus one has the equivalent of a Michelson-Morley experiment.” [op. cit, p. 11],

and was first designed in the ammonia-beam maser *Cedarholm and Townes* experiment [22-23], - with two masers had oppositely directed molecular beams and mounted on a “rack which could be rotated about a vertical axis”. The measurements of changes in frequency upon rotation of equipment through 180° were repeated at intervals throughout a year during 1958-1959. The experiment was motivated by the nineteenth century ether-drift concept (and was based on the analysis of Møller [24]), so that in the fraction $v/c$ the numerator was associated to the earth’s orbital velocity around the Sun, and hence the estimation $(v/c)^2 \approx 10^{-8}$ was used.

The improved *Brillet and Hall* experiment [25], which used two He-Ne highly monochromatic masers mounted with axes perpendicular on a turning “granite slab” to examine frequency changes upon the slab rotation through 90°, was reported in 1979. Analytically, the Robertson’s framework [26] was applied. The instruments measured the relative change of the optical path $ct = l$ between the two end points of the interferometer and the wave interference effect was exploited. The experiment is often highlighted as “a more sensitive, modern laser version of the Michelson-Morley experiment”. The sensitivity was reported as $\Delta l/l = 2.5 \cdot 10^{-15}$.

In all the above experiments the zero changes in frequencies have been fixed. The *Brillet and Hall* experiment seems to have closed the epoch of turntable interferometer experiments.
The monitoring-type experiments involve the old ether-minded Kennedy and Thorndike experiment [27], which used an unequal-arm Michelson interferometer and a mercury lamp, and the modern-type Hill and Hall experiment [28] which utilized instead two He-Ne lasers together with computer store methods and started theoretically with the so-called Mansouri-Sexl test theory [29], so that the numerator in the fraction $v/c$ was taken to be the ($\approx 400 \text{ km/s}$)-velocity of the earth with respect to the Cosmic Microwave Background.

The Kennedy and Thorndike experiment reported in 1932 was differed from the Michelson-Morley experiment in the significant respect that the apparatus was fixed in an earth-grounded laboratory, so the idea was to observe the interference fringes due to motion of earth over a period of months.

Similarly, in the Hill and Hall experiment the earth itself was regarded as a “carrying platform”, if not as a “rotating optical bench”. Siderial signals were searching for during periods of 1986-1989 years. In this “laser-based Kennedy and Thorndike experiment” the high sensitivity of $2 \cdot 10^{-13}$ for a siderial term was gained.

The expectation in these experiments was to look for the variation in the length as the laboratory apparatus is rotated in a cosmic space. Positive effects were absent: no fringe shifts due to either the diurnal or the seasonal changes in the motion of the earth laboratory were observed.

Anisotropies of the earth’s local scale are, obviously, unobservable in the $I_2$-type experiments which are entirely of cosmos-born interest to trace possible diurnal or sidereal effects of the earth’s motion or rotation. Oppositely, as instruments are actively changing local orientation in the earth-bound laboratory, the outputs of the relevant $I_1$-type observations, in which special rotational platforms are used to mount equipments on, may be sensitive to search for such anisotropies.

Aimed to study the latter possibility on the basis of a convenient FG-metric, it is worth applying the concept of $\mathcal{FD}$-direction (introduced in Section 1), which trace is a tiny deformation of the square-root metric. In general, there may be examined various situations which don’t comply with the spatial isotropy, including the following nearest possibilities.

$A_1$: The earth-rotational anisotropic effects might be expected to meet in a local RF $\Sigma$ firmly anchored to the earth;

$A_2$: The radial-space anisotropies may be caused by the dominating spherical-mode distribution of matter inside the planet, so the radial direction is a real candidate for the role of the $\mathcal{FD}$-direction and the terrestrial-lab interpretation of the preferred RF $\Sigma$ is possible;

$A_3$: The cosmos-conditioned anisotropies may come to play, as caused by the non-uniformity of distribution of matter and energy in the embient space; as well as the customary-sense case:

$A_4$: The Cosmic Microwave Background retains its relevance, provided that the experimental evidence would make us to conclude that isotropy of the Background is not rigorously valid.

Of course, there is no a priori reason to judge which of them is of $\mathcal{FD}$-nature proper. In each particular case one of the central experimental questions is what is “a degree of anisotropy”, due answers thereto would involve estimating the characteristic Finslerian parameter $g$ and thereby provide an experimental testing of the Lorentz transformations.
3. EXPLICATED FINSLERIAN KINEMATIC TRANSFORMATIONS

Using the convenient notation (1.9)-(1.10) and assuming, without any loss of generality, that motion is going in the plane $R^1 \times R^2$, straightforward calculations (which intermediate steps were presented in [2]) lead to the following generalized kinematic transformations:

\[ R^0 = \frac{1}{V(g; v, u)} \left[ t + \left( \frac{v}{M} + g \right) x + \frac{u}{M} \sqrt{Q(g; v, u)} y \right], \]  \hspace{1cm} (3.1)

\[ R^1 = \frac{1}{V(g; v, u)} (vt + Mx), \]  \hspace{1cm} (3.2)

\[ R^2 = \frac{1}{V(g; v, u)} \left[ ut + \left( \frac{v}{M} + g \right) ux + \frac{1}{M} \sqrt{Q(g; v, u)} y \right], \]  \hspace{1cm} (3.3)

\[ R^3 = \frac{\sqrt{Q(g; v, u)}}{V(g; v, u)} z, \]  \hspace{1cm} (3.4)

which inverse reads

\[ t = \frac{V(g; v, u)}{Q(g; v, u)} \left[ R^0 - (v + gM)R^1 - uR^2 \right], \]  \hspace{1cm} (3.5)

\[ x = \frac{V(g; v, u)}{Q(g; v, u)} \left[ -vM R^0 + MR^1 + \frac{vu}{M} R^2 \right], \]  \hspace{1cm} (3.6)

\[ y = \frac{V(g; v, u)}{\sqrt{Q(g; v, u)}} \frac{1}{M} (-uR^0 + R^2), \]  \hspace{1cm} (3.7)

\[ z = \frac{V(g; v, u)}{\sqrt{Q(g; v, u)}} R^3. \]  \hspace{1cm} (3.8)

Here, $Q$ and $V$ are the functions (1.11) and (1.12);

\[ \{ R^0, R^1, R^2, R^3 \} \in \Sigma, \quad \{ t, x, y, z \} \in S_{\{v, u\}}, \]  \hspace{1cm} (3.9)

where $\Sigma$ is the input preferred rest frame and $S_{\{v, u\}}$ is an inertial RF moving with the velocity $\{ v^1 = v, v^2 = u, v^3 = 0 \}$ relative to $\Sigma$; the instantaneously common origin of the frames being implied.

Similarly for the momenta,
\[ P_0 = \frac{V(g; v, u)}{Q(g; v, u)} [p_0 - \frac{v}{M} p_1 - \frac{u}{M} \sqrt{Q(g; v, u)} p_2], \quad (3.10) \]

\[ P_1 = \frac{V(g; v, u)}{Q(g; v, u)} [-(v + gM)p_0 + Mp_1], \quad (3.11) \]

\[ P_2 = \frac{V(g; v, u)}{Q(g; v, u)} [-up_0 + \frac{vu}{M} p_1 + \frac{1}{M} \sqrt{Q(g; v, u)} p_2], \quad (3.12) \]

\[ P_3 = \frac{V(g; v, u)}{\sqrt{Q(g; v, u)}} p_3, \quad (3.13) \]

and its inverse

\[ p_0 = \frac{1}{V(g; v, u)} (p_0 + vP_1 + uP_2), \quad (3.14) \]

\[ p_1 = \frac{1}{V(g; v, u)} [\left( \frac{v}{M} + g \right) p_0 + MP_1 + \left( \frac{v}{M} + g \right) uP_2], \quad (3.15) \]

\[ p_2 = \frac{\sqrt{Q(g; v, u)}}{V(g; v, u)} \frac{1}{M} (uP_0 + P_2) \quad (3.16) \]

\[ p_3 = \frac{\sqrt{Q(g; v, u)}}{V(g; v, u)} p_3, \quad (3.17) \]

where

\[ \{P_0, P_1, P_2, P_3\} \in \Sigma, \quad \{p_0, p_1, p_2, p_3\} \in S_{(v,u)}. \quad (3.18) \]

The invariance of the contraction:

\[ R^0 P_0 + R^1 P_1 + R^2 P_2 + R^3 P_3 = tp_0 + xp_1 + yp_2 + zp_3 \quad (3.19) \]

can be verified directly.

Inversely, one may postulate (3.19) to explicate (3.10)-(3.17) from (3.1)-(3.8).

\( g \) is the characteristic Finslerian parameter, so that the above transformations reduce exactly to the ordinary special-relativistic Lorentz transformations whenever \( g = 0 \).

4. TRANSPORT SYNCHRONIZATION

If a clock moves in the RF \( S_{(v,u)} \) with a velocity \( \{\alpha, \beta, 0\} \), so that

\[ \Delta x = \alpha \Delta t, \quad \Delta y = \beta \Delta t, \quad \Delta z = 0, \quad (4.1) \]
we may consider the velocity \( \{ w^1 = \Delta R^1/\Delta R^0, w^2 = \Delta R^2/\Delta R^0, 0 \} \) of the clock as viewed from the RF \( \Sigma \) and find from (3.1)-(3.3) the values

\[
w^1 = \frac{v + M\alpha}{1 + (\frac{v}{M} + g)\alpha + \frac{u}{M}\sqrt{Q(g; v, u)} \beta} \quad (4.2)
\]

and

\[
w^2 = \frac{u + (\frac{v}{M} + g)u\alpha + \frac{1}{M}\sqrt{Q(g; v, u)} \beta}{1 + (\frac{v}{M} + g)\alpha + \frac{u}{M}\sqrt{Q(g; v, u)} \beta}. \quad (4.3)
\]

This entails for the time which the moving clock shows in its rest frame:

\[
\Delta t' = \Delta R^0 V(g; w^1, w^2) \frac{1 - (w^1)^2 - (w^2)^2}{Q(g; w^1, w^2)} - gw^1 \sqrt{1 - (w^2)^2} \quad (4.4)
\]

The fraction here is unity (see (1.11)), so

\[
\frac{\Delta t'}{\Delta R^0} = V(g; w^1, w^2). \quad (4.5)
\]

Applying here (3.1) yields

\[
\frac{\Delta t'}{\Delta t} = V(g; w^1, w^2) \left[ 1 + (\frac{v}{M} + g)\alpha + \frac{u}{M}\sqrt{Q(g; v, u)} \beta \right]. \quad (4.6)
\]

Let us follow the ideology of transport synchronization and consider the clock motion to be infinitesimally slow with respect to the RF \( S_{(v,u)} \). Then we approximate (4.2) and (4.3) to first order in \( \alpha \) and \( \beta \),

\[
w^1 \approx v + M\alpha - v \left( \frac{v}{M} + g \right)\alpha + \frac{u}{M}\sqrt{Q(g; v, u)} \beta \]

\[
= v + \frac{1}{M}Q(g; v, u)\alpha - \frac{uv}{M}\sqrt{Q(g; v, u)} \beta \quad (4.7)
\]

and

\[
w^2 \approx u + M\sqrt{Q(g; v, u)} \beta. \quad (4.8)
\]

Since

\[
\frac{\partial V(g; v, u)}{\partial v} = -(gM + v)\frac{V(g; v, u)}{Q(g; v, u)}, \quad \frac{\partial V(g; v, u)}{\partial u} = -u\frac{V(g; v, u)}{Q(g; v, u)} \quad (4.9)
\]

(see (1.14)), we arrive at the vanishing

\[
\left. \frac{\partial (\Delta t'/\Delta t)}{\partial \alpha} \right|_{\alpha=\beta=0} = 0, \quad \left. \frac{\partial (\Delta t'/\Delta t)}{\partial \beta} \right|_{\alpha=\beta=0} = 0, \quad (4.10)
\]

which just proves

**PROPOSITION 4.1.** The Finslerian post-Lorentzian kinematic transformations written out in Section 3 fulfill the Transport Synchrony.

5. **FINSLER-GENERALIZED HAMILTONIAN AND MOMENTUM**
Given a particle of rest mass $m$. For the Finslerian four-momentum
\[ P = \{P_0, P_1, P_2, P_3\} \] (5.1)
of the particle, $P_0$ denotes the energy, $P_1$ is the projection of the four-momentum on the preferred $FD$-direction, and $\{P_2, P_3\}$ is the respective perpendicular component. It is convenient to put
\[ P_\perp = \sqrt{(P_2)^2 + (P_3)^2}. \] (5.2)
In the time-like and isotropic cases of the Finslerian approach under study, from (1.5) we infer the Finslerian relativistic Hamiltonian
\[ H(g; P_0, P_1, P_2, P_3) = \left(-P_1 + g_+ \sqrt{(P_0)^2 - (P_\perp)^2}\right)^{-G_-/2} \left(P_1 - g_- \sqrt{(P_0)^2 - (P_\perp)^2}\right)^{G_+/2} \] (5.3)
which is meaningful over the range of variation
\[ g_- \sqrt{(P_0)^2 - (P_\perp)^2} \leq P_1 \leq g_+ \sqrt{(P_0)^2 - (P_\perp)^2}, \] (5.4)
where the equality relates to the isotropic case, for which $m = 0$, and otherwise the members $P$ of the inequality (5.4) are timelike.

So we get the Finslerian dispersion relation
\[ H(g; P_0, P_1, P_2, P_3) = m \] (5.5)
to use it in place of the ordinary pseudo-Euclidean SR-prescription $\sqrt{(P_0)^2 - (P_1)^2 - (P_2)^2 - (P_3)^2} = m$.

The equation (5.5) defines in an implicit way the energy-momentum function
\[ P_0 = P_0(g; m; P_1, P_\perp) \] (5.6)
which can be characterized by the derivatives
\[ \frac{\partial P_0}{\partial P_1} = \frac{P_1 + g\sqrt{(P_0)^2 - (P_\perp)^2}}{P_0}, \quad \frac{\partial P_0}{\partial P_\perp} = \frac{P_\perp}{P_0} \] (5.7)
(obtainable on direct differentiating (5.5) for the case (5.3)). In the slow-relativistic case, from (5.6)-(5.7) we get the approximation
\[ P_0 \approx m + gP_1 + \frac{1}{2m}((P_1)^2 + (P_\perp)^2) + ... \] (5.8)
in which the first-order Finslerian term, $gP_1$, has appeared.

In the Lorenzian SR-limit, the definition (5.3) reduces to the ordinary pseudo-Euclidean one:
\[ H|_{g=0} = \left(-P_1 + \sqrt{(P_0)^2 - (P_\perp)^2}\right)^{1/2} \left(P_1 + \sqrt{(P_0)^2 - (P_\perp)^2}\right)^{1/2} = \sqrt{(P_0)^2 - (P_1)^2 - (P_\perp)^2}, \]
that is,
\[ H|_{g=0} = \sqrt{(P_0)^2 - (P_1)^2 - (P_2)^2 - (P_3)^2}. \] (5.9)
We have

**PROPOSITION 5.1.** In the Finslerian framework under study, the occurrence of the first-order term in the slow-relativistic expansion (5.8) of the energy-momentum dependence is characteristic.
Indeed, discarding such a term would entail \( g = 0 \) and, hence, return us back to the ordinary pseudo-Euclidean relativistic dynamic relations.

6. CONCLUSIONS

Strictly speaking, the Finslerian relativistic framework proposed is not an implication of a free intelligible assumption about possibility of generalization of the pseudo-Euclidean SR-theory, but rather is an accurate continuation of the latter theory in the Finslerian domain with respect to the single parameter, \( g \). Actually, the principal setups listed in Introduction allow the relativistic FMF and the Finslerian kinematic transformations to be introduced in a straightforward manner. The key parameter \( g \) appears on keeping the condition that the relativistic indicatrix, as well as the mass shell which is given by (5.5), be a space of constant negative curvature, which implies for the curvature \( R_{\text{indicatrix}} = R_{\text{mass-shell}} = -(1 + \frac{g^2}{4}) \) (see [2]). By comparing the latter fact with (1.23), we can conclude the remarkable relation

\[
\frac{c^+ + c^-}{2} = \sqrt{-R_{\text{mass-shell}}}.
\]

A careful analysis has shown that, as the nearest possibilities, there are two alternative ways for such a continuation.

The first way, which was developed in previous publications [6-9], starts with the assumption that the spatial isotropy is kept valid in a preferred rest frame. The latter assumption is typically accepted in the modern post-Lorentzian experimental as well as theoretical works used the Cosmic Microwave Background to be such a frame. In this vein, a failure of Lorentz invariance would imply existence, in any given terrestrial ground-based laboratory, of a particular distinguished cosmos-born vector which would point the velocity of the earth with respect to the Cosmic Microwave Background. Experimental searches have been made for such a preferred RF in various modern high-precision post-Lorentzian experiments, – which, however, have so far shown no evidence for such a vector.

In the second, alternative, way formulated in the present paper, the spatial anisotropy, rotational around a distinguished spatial direction, is admitted in a preferred rest frame, \( \Sigma \), so that the parameter \( g \) measures a degree (intensity) of anisotropy. The Principle of Correspondence is implied: light signals are assumed to travel relative to \( \Sigma \) with the common, isotropic and universal, speed \( c_0 \), unless FG-corrections are switched on (put \( g = 0 \) in Eqs. (1.18) and (1.19)). At low velocities of relative motions, the Finslerian relativistic relations differ by but minor corrections from respective relations of ordinary SR-theory.

The Finslerian kinematic transformations obtained may serve as providing various attractive direct theoretical bases for post-Lorentzian estimations in situations capacious of revealing spatial isotropies, and particularly for study the local-space anisotropies of “the earth’s origin”. Although \( a \text{ priori} \) we may have little idea of the magnitude of the respective characteristic Finslerian parameter \( g \), the developed Finslerian framework can well be adapted to the needs of relevant post-Lorentzian SR-experiments when one is requested to create self-consistent relativistic programs for testing local-scale, as well as cosmos-scale, anisotropies.

Generally, the slow clock transport method can be viewed as allowing a determination of The Canonical Synchronization without any vicious circularity. Having the transport synchrony proven for the Finslerian kinematic transformations under study (Proposition 4.1), we can undermine the pure-conventionalistic, or neatly-speculative, claim that “the
speed of light is illusion”. The respective anisotropy in the one-way light velocity may be subjected to experimental search (in particular, via measurements of due Finslerian corrections to Doppler-effect phenomena), provided the FMF is fixed accordingly. Simultaneously, experimental verification of the Finslerian kinematic transformations which we have undertaken to propose might show whether the primary geometry of space-time is Finslerian or not.

In the post-Lorentzian context proper, importance of “the deep connection between kinematics and dynamics” has been emphasized in the highly readable program by M.P.Haugan and C.M.Will [1]. Dynamically, the FG-approach is certain in that all the basic dynamic relativistic ingredients are explicable in a clear Finsler-geometrical way. Indeed, for the FMF introduced by the definition (1.4), the Finslerian Hamiltonian (for a free relativistic particle) is obtainable in an explicit form, presented by (5.3), and then the respective Finslerian four-momentum is derivable directly to be the momentum \( P \) given by (5.1) subject to the Finslerian dispersion relation (5.5). Therefore, a direct way is opening to stretch all the body of the Relativistic Dynamics in due Finslerian way by using but a single characteristic parameter, \( g \). The principle of momentum and energy conservation is maintained, but the equation relating energy to momentum does change in due Finslerian way (5.5). In particular, for the Compton effect the respective Finslerian extension of the four-momentum conservation law, as shown by (2.1), can be written down in terms of the Finslerian \( P \)-momenta. We postpone generating required Finslerian analysis of particular physical phenomena for further papers.

Obviously, the theoretical structure and experimental limitations of the special relativity cannot be tested in all its aspects unless a self-consistent alternative theory is applied. We expect that it is the Finsler-type geometry that may born such theories, advancing simultaneously new philosophico-physical sights on the relationship between Relativity and Geometry.

An essential limitation of the \( \mathcal{FD} \)–anisotropy formulated in the present paper is that the anisotropy has been taken of “uni-directional type”, – such that a preferred vector of spatial type singles out one distinguished direction. Generally, anisotropies may be of complicated structure, not predetermined by any single vector. Of course, applying the uni-directional approach is the easest way to think of Finslerian correction to, or violation of, the pseudo-Euclidean theory of relativity.

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