Spectrum Sensing Method of Optimized D-S Evidence Theory

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Abstract. In order to reduce the reporting data and the bandwidth cost effectively, reduce the impact on the performance from high conflicting evidences, the D-S evidence theory of cooperative spectrum sensing algorithm is optimized. The uncertainty information of the local perception basic probability assignment is assigned into the certainty in proportion, and the weighted sum evidence is calculated using the evidence reliability of every cognitive user as the weight coefficient, which reduce the reporting data sent to the fusion center, effectively reduce the bandwidth cost, and reduce the influence of high conflict evidences on the fusion results. The simulation results show that the method proposed has obvious effect on reducing the reported bandwidth and the impact by high conflicting evidences, and the algorithm is simple.

1. Introduction
In Cognitive Radio (CR) spectrum sensing techniques, Cooperative Spectrum Sensing (CSS) [1][2][3] can effectively solve the problems that the single user spectrum sensing method is easily affected by the environmental factors[4]. At present, there are some hard decisions such as "AND", "OR" and "K optimal rank" (Majority) [3] for the data fusion methods of CSS, but these algorithms are only applicable to the non-actual conditions that have the same signal-to-noise ratio (SNR), and the same threshold [5][6]. The document [5] had an effective quantization for the transmission data according to the hypothesis distribution properties of Licensed User (LU) signal under different signal-to-noise ratios, reduced the transmission bandwidth, but the quantization method was more complex. Using D-S (Dempster-Shafer) evidence theory to fuse spectrum sensing data is a popular method at present. D-S evidence theory cooperative sensing algorithm can effectively deal with the uncertainty caused by the perceived environmental differences, which had aroused extensive research and attention. The document [7] was the first application of D-S evidence theory in spectrum sensing, and the results show that the performance is superior to the hard decision algorithms. In document [8] in order to solve the problem of evidence conflicts caused by the different perception and channel environment, the method of reliability source evaluation (RSE) was proposed to evaluate the reliability of the data source and reduce the misjudgment. The document [9] combined the real-time BPA and holistic credibility to defense against malicious user attacks. In document [10], the mean and variance of detection statistic were used as weights of evidence to reduce the influence of evidence conflict. The document [11] provided the similarity between evidences according the robustness. Fusion Center (FC) calculated the reliability of each evidence depending on the similarity matrix. If the reliability of some evidence is lower than the threshold value, it will be discarded. The document [12] assessed the reliability of the evidence combining the current and historical reliabilities, but this approach increases the complexity and cost of the algorithm. In order to reduce the computational cost and the sensing
process duration, the document [13] introduced a new BPA and cope with the small sample size, but the algorithm depended on the Tracy-Widom distribution. Since the performance of the preceding algorithms was affected greatly by high conflicting evidences, and more reporting data and wider transmission bandwidth were required, an optimized D-S evidence theory (ODS) of spectrum sensing algorithm is proposed. The BPA function of Cognitive User (CU) is adjusted through assigning the uncertainty information into the certainty in proportion, reducing the amount of reporting data. Then the reliability of each evidence, which will be used as weight coefficient to work out the weighted sum of BPA, is calculated. Last, the data fusion is carried out using D-S fusion rules, making a decision. The simulation results show that the algorithm can effectively reduce the impact by high conflicting evidences on the fusion results and obtain accurate results.

2. Network System Model And D-S Evidence Theory

2.1 Network System Model
CU determines the channel occupancy by FC. If the radio spectrum surrounding environment is idle, it will access and use the idle spectrum [4], to realize the effective sharing of the radio spectrum resources and improve the spectrum utilization. A typical Cognitive Radio Network (CRN) structure model [14] is shown in figure 1. The network contains one LU, M CU, and one FC. FC sends a request signal to spectrum sense as needed. After the signal is received, each CU will participate in the spectrum sensing process voluntarily. Each CU uses energy sensing to sense the local radio spectrum independently, and then sends the sensed information to FC through the reporting data. Finally, FC detects the sensing results.

![Figure 1: CR network model](image)

In the model, if the target frequency band is not being occupied by LU, only noise exists in the signal detection of CU; if the target frequency band is being occupied, the noise and the LU signal exist together. The detection signal can be described by the two element hypothesis testing problem [5]:

\[
H_0 : x_i(k) = n_i(k), \quad i = 1, 2, \ldots, M \\
H_1 : x_i(k) = h_i(k)x_i(k) + n_i(k), \quad i = 1, 2, \ldots, M
\]

(1)

\(H_0\) means the target frequency band is not being occupied, \(H_1\) means the target frequency band is being occupied. \(x_i(k)\) is the sampled signal by the \(i\)th CU at the time of \(k(k = 1, 2, \ldots, N)\); \(h_i(t)\) is the channel gain; \(n_i(t)\) is the additive white Gauss noise with zero mean and variance \(\sigma_n^2\). Normally, \(s(t)\) and \(n_i(t)\) is independent and \(n_i(t)\) is independent identically distributed.

2.2 D-S Evidence Theory
D-S evidence theory was proposed by Dempster in 1967 [15], and further supplemented by his student Shafer. It can deal with uncertain information caused by different conditions effectively.
D-S evidence theory describes an identification framework $\Phi$ [16], $\Phi$ is a finite hypothesis set which is mutual exclusive but complete. BPA function $m$ can be defined as a mapping of $2^{\Phi}$ to $[0, 1]$, and $A$ is a subset of $\Phi$:

$$
\begin{align*}
&m(\phi) = 0 \\
&\sum_{A \in \Phi} m(A) = 1
\end{align*}
$$

(2)

Where $\phi$ is an empty set, $m(A)$ means the precise trust on $A$, namely evidence. A combination of multiple evidences is more reliable than single evidence, and multiple evidences can be fused by BPA function combination rules:

$$
m(A) = m_1 \oplus m_2 \oplus \ldots \oplus m_u (A) = \begin{cases} 
\sum_{A_i \cap A_j \cap \ldots \cap A_u = \emptyset} \prod_{i=1}^{u} m_i (A_i), & A \subseteq \Phi \\
0, & A = \emptyset
\end{cases}
$$

(3)

Where $A_i \in \Phi$, $K$ is the conflict factor which decides how much the conflict is between evidences.

$$
K = \sum_{A_i \cap A_j \cap \ldots \cap A_u = \emptyset} \prod_{i=1}^{u} m_i (A_i)
$$

(4)

2.3 D-S Evidence Theory for Spectrum Sensing

In the CRN model, $\Phi = \{H_0, H_1, \Omega\}$, $\Omega$ represents the uncertainty of single user detected about $H_0$ or $H_1$. BPA function of the $i$th CU is defined as [8]

$$
m_i (H_0) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_u} \exp \left(-\frac{(x-\mu_0)^2}{\sigma_u^2}\right) \, dx
$$

(5)

which represents the exact trust of the $i$th CU to $H_0$;

$$
m_i (H_1) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_u} \exp \left(-\frac{(x-\mu_1)^2}{\sigma_u^2}\right) \, dx
$$

(6)

which represents the exact trust of the $i$th CU to $H_1$;

According to D-S evidence theory,

$$
m_i (\Omega) = 1 - m_i (H_0) - m_i (H_1)
$$

(7)

The $i$th CU will send $2$ of $\{m_i (H_0), m_i (H_1), m_i (\Omega)\}$ to FC. FC completes the data fusion according to formula (3), then $m(H_0)$ and $m(H_1)$ are obtained, here $m(H_0)$ and $m(H_1)$ are exact trust about $H_0$ or $H_1$ which FC received from all users. According to the relationship between the ratio of them and the threshold $\lambda$, FC can do the final verdict. The rule is:

$$
\begin{align*}
H_0 : & \frac{m(H_1)}{m(H_0)} \leq \lambda \\
H_1 : & \frac{m(H_1)}{m(H_0)} > \lambda
\end{align*}
$$

(8)

That is: when the trust to $m(H_i)$ is greater than that to $\lambda m(H_0)$, it is considered $H_1$ is true and LU is occupying the target frequency band at this time; on the contrary, $H_0$ is true and LU does not occupy the target band.
Therefore, in order to fuse the sensing data at some time, 2 of \( \{ m(H_0), m(H_1), m(\Omega) \} \) have to be sent from all involved CUs in the process of sensing to FC, which is a large amount of data. When there is high conflicting evidence, which means \( K \) is equal to 1, the data fusion cannot be carried out. It leads the fusion results will be reversing to the facts and the performance of sensing will be reduced. In the actual environment of CRN, the high conflicting evidence is often caused by bad communication environment, CU device abnormal work or CU malicious sending error information [17].

3. Optimized D-S Evidence Theory For Spectrum Sensing

In order to reduce the amount of sensing reporting data and bandwidth of the channel, as also as overcome the difficult of data fusion in high conflicting environment with D-S evidence theory, a spectrum sensing algorithm of optimized D-S evidence theory (ODS) is proposed. CUs detect BPA functions through the local sensing, and then the uncertain information is assigned to the BPA function of certainty, one part of which will be sent to FC to reduce the amount of reporting data. After FC receives all of the CUs data, it will calculate the reliabilities of the evidences, which will be the weighted coefficients of the evidence sum. It will reduce the high conflicting evidence effect on sensing performance. The last step is FC fuses the data and makes a decision. The process of spectrum sensing using the optimized D-S evidence theory is shown in figure 2.

![Figure 2: Spectrum sensing process of the optimized the D-S block diagram](image)

3.1 Allocation of Uncertain Information

According to the formula (7), \( m(\Omega) \) contains the uncertain information about the trust degree to \( H_0 \) or \( H_1 \). But actually it can be obtained from \( m(H_0) \) and \( m(H_1) \). So it also can be assigned to \( m(H_i) \) and \( m(H_o) \) in the proportional of \( m(H_i) / m(H_o) \), denoted by BPA', i.e.:

\[
\begin{align*}
\{ & m_{a_i} (\Omega) \} = \frac{m(H_i)}{m(H_o)} \\
\{ & m_{a_o} (\Omega) \} = \frac{m(H_o)}{m(H_i)} \\
\{ & m(\Omega) = m_{a_i} (\Omega) + m_{a_o} (\Omega) \}
\end{align*}
\]  

(9)

Where \( m_{a_i} (\Omega) \) is the uncertain information assigned to \( m(H_o) \), and \( m_{a_o} (\Omega) \) is the uncertain information assigned to \( m(H_i) \).

After the assignment of uncertain information, the BPA' function \( \{ m'(H_0), m'(H_1) \} \) is:

\[
m'(H_i) = m(H_i) + m_{a_i} (\Omega)
\]

(10)

\[
m'(H_o) = m(H_o) + m_{a_o} (\Omega)
\]

(11)

And meet the relationship:

\[
m'(H_0) + m'(H_1) = m(H_0) + m(H_1) + m(\Omega) = 1
\]

and

\[
\frac{m(H_i)}{m'(H_o)} = \frac{m'(H_i)}{m'(H_o)}
\]

(12)
Therefore, after the assignment, each CU only needs to send one value of \( m_i' (H_0) \) or \( m_i' (H_1) \) to FC. FC can be easily obtained other certain and uncertain information according to the formula (12). In the whole process, the amount of reporting data and the channel bandwidth are reduced effectively.

3.2 Reliability Calculation and Data Fusion
When the BPA’s are received, in FC, the similarity coefficients between the evidences are calculated according to the similarity relation, and so is the evidence support degree. Since the conflicting evidence and other evidences are different, the similarity coefficient \( s_{ij} \) [18] between the evidences \( m_i' \) and \( m_j' \) is defined as:

\[
s_{ij} = \frac{\sum A_j \cap A_i \neq \emptyset m_i' (A_j) m_j' (A_i)}{\sqrt{\left[ \sum m_i'^2 (A_j) \right] \left[ \sum m_j'^2 (A_i) \right]}}
\]

\( A_1 \), \( A_2 \), \( A_n \) \( \in \Phi \), and \( \Phi = \{ H_0, H_1 \} \).

FC calculates the similarity coefficients between BPA’s sent by \( M \) CU, and a similarity coefficient matrix \( S \) [9] can be obtained:

\[
S = \begin{bmatrix}
1 & s_{12} & \cdots & s_{1M} \\
&s_{21} & 1 & \cdots & s_{2M} \\
& & \vdots & \ddots & \vdots \\
&s_{M1} & s_{M2} & \cdots & 1 \\
\end{bmatrix}
\]

(14)

Each line in \( S \) represents the similarity between the \( i \)th CU BPA’s and the other users’, and the closer the value is to 1, the more similar the evidence is. The support degree for the evidence \( m_i \) can be defined as:

\[
Sup (m_i) = \sum_{j=1}^{M} s_{ij} \quad i, j = 1, 2, \ldots M
\]

(15)

The reliability of the evidence \( m_i \) is:

\[
Rel (m_i) = \frac{Sup (m_i)}{\max_{j=1,2,\ldots,M} \{ Sup (m_j) \}}
\]

(16)

The evidence with the greatest support \( \max_{j=1,2,\ldots,M} \{ Sup (m_j) \} \) is absolutely reliable. The greater the \( Rel (m_i) \) of an evidence, the smaller the conflict with the evidence.

FC defines a threshold value \( r \). When the reliability \( Rel (m_i) \) of an evidence is less than the threshold \( r \), the evidence is considered to be a high conflicting data which will be discarded in order to eliminate the impact on the fusion results. When the reliability \( Rel (m_i) \) is greater than the threshold, the evidence is not a conflicting data and will participate in the process of the fusion.

\( Rel (m_i) \) will be a weight factor for each evidence, and the weighted sums of BPA’s are:

\[
m_{\text{mean}} (H_0) = \sum_{i=1}^{M} Rel (m_i) m_i' (H_0)
\]

(17)

\[
m_{\text{mean}} (H_1) = \sum_{i=1}^{M} Rel (m_i) m_i' (H_1)
\]

(18)

Finally, FC fuses all the data according to the D-S fusion rule, and makes a decision according to the rule of decision (formula 12).

The selection of the threshold \( r \) should be determined according to the actual situation. If the threshold is large, the non conflicting evidence may be filtered out. If the threshold is small, it may not be able
to filter out the conflicting data. The ODS algorithm can reduce the impact of the conflicting evidences without considering the environmental factors such as the SNR of cognitive users.

4. Performance Simulation
The performance of CR spectrum sensing is mainly measured by detection probability and false alarm probability. The higher the detection probability is, the smaller the interference to LU; the lower the false alarm probability, the higher the accuracy of the spectrum sensing [10].

In order to verify the sensing performance of the ODS algorithm, the relationship of the detection probability and false alarm probability are analyzed respectively in 3 conditions: no conflicting evidence, one conflicting evidence, more than one conflicting evidences. For better analysis, the ODS algorithm is compared with other algorithms. 10000 Monte Carlo simulation were made in MATLAB R2013a (8.1.0.604), M=10 and the probability of LU signals occupying the channel is 0.5, the threshold value r=0.6, and LU signal is transmitted through the AWGN channel, the channel between CU and FC is ideal.

4.1 No Conflicting Evidence
When there is no high conflicting evidence, the detection probabilities of ODS and other algorithms at different false alarm probabilities are shown in Figure 3, where the received SNR for each CU is -22~ -10dB.

![Figure 3: The relationship between detection probability and false alarm probability with no conflicting evidence](image)

The simulation results show that, when there is no conflicting evidence, the sensing performance of ODS and RSE are better than that of D-S fusion algorithm. When the false alarm probability is 0.2, the detection probability of RSE is higher than that of D-S near 30%, while the detection probability of ODS is higher about 10%. However, in the ODS algorithm, when r=0.6, the data conflict is relatively small between -22dB and -10dB cognitive users, the improvement in performance is not obvious.

4.2 One Conflicting Evidence
When there are 1 conflicting evidences, the detection probabilities in different false alarm probabilities are shown in figure 4. The received SNR for each CU is -22~ -10dB, and the SNR for the high conflicting CU is -10 dB.

The simulation results show that: the sensing performances of several algorithms are all affected when there is 1 high conflicting evidence. When the false alarm probability is 0.2, the detection probability of D-S is only 0.47, compared to 0.81 when no conflicting evidence, reduced by 42%. It shows that D-S fusion algorithm can hardly handle conflicting data. The detection probability of RSE decreases.
from 0.92 to 0.66, reduced by 28%, while that of ODS is 0.78, only reduced by 9%. It is obvious that the ODS algorithm is the least affected by high conflicting evidence, and it does not need to know the SNR of the cognitive users, has certain robustness.

**Figure 4:** The relationship between detection probability and false alarm probability with 1 conflicting evidence

4.3 More Than one Conflicting Evidences
When there is no high conflicting evidence and there are more than one high conflicts, the relation curves of false alarm probabilities and detection probabilities of ODS are shown in figure 5. The SNRs of high conflicting CU are -10dB, and the others’ are -22~ -10dB.

**Figure 5:** The relationship between detection probability and false alarm probability with more than one conflicting evidences

The simulation results show that: when the number of high conflicting evidences is less than 3 (30%), the detection performance will decline with the increase of the number of conflicts, but the overall performance drops slightly. When there are 1 or 2 high conflicts and the false alarm probability is 0.2, compared with no conflicting evidence, the detection probability is decreased by 15.3%, 41.1%, but the sensing performance is still good. When the number of high conflicting evidences is more than 3 (30%), the sensing performance of the system will be greatly affected, and the detection probability is
reduced by 88.7%. When there are 4 conflicts, the detection performance has deteriorated to the point where it cannot be used. It shows that the system should take effective anti-conflicting measures.

### 4.4 Summary

Through the simulation we can see ODS has better sensing performance in the condition of less (less than 30%) high conflicting evidences compared with other algorithms, and is the least affected by the high conflicting evidence. Furthermore compared with other methods, it reduces the amount of transmission data by 33.3% and it needs least transmission bandwidth required for CU to transmit reporting data. In the CRN with a large number of CU, the advantages of the algorithm will be more obvious.

### 5. Conclusions

In summary, to reduce the impact of conflicting evidence on sensing performance and reduce the amount of data CU sends to FC and bandwidth, an optimized algorithm is proposed in this paper. First, each CU will assign the detected uncertain information to BPA, only a small amount of data sent to FC, reduced the reporting data and the transmission bandwidth; then through the similarity coefficients between evidences, FC calculates the evidence reliability, and calculates the weighted sum BPA using the reliability as the weight coefficient; FC finally makes a decision according to the D-S fusion rules. Simulation results show that the proposed algorithm can reduce the impact of high conflicting evidences on the fusion results, increase the robustness of the system, and effectively reduce the reported bandwidth.

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