Nonlinear Coupling of Electromagnetic and Spin-Electron-Acoustic Waves in Spin-polarized Degenerate Relativistic Astrophysical Plasma

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Propagation of the finite amplitude electromagnetic wave through the partially spin-polarized degenerate plasmas leads to the instability. The instability happens at the interaction of the electromagnetic wave with the small frequency longitudinal spin-electron-acoustic waves. Strongest instability happens in the high density degenerate plasmas with the Fermi momentum close to \( m_e c \), where \( m_e \) is the mass of electron, and \( c \) is the speed of light. The increase of the increment of instability with the growth of the spin polarization of plasmas is found.

Keywords: relativistic plasmas, hydrodynamics, degenerate electrons, spin polarization, separate spin evolution.

I. INTRODUCTION

We consider the propagation of the large amplitude high frequency electromagnetic radiation through the high density plasmas surrounding the compact astrophysical objects, such as white dwarfs or neutron stars. Mostly these objects generate strong magnetic fields which modifies the trajectory of macroscopic flows or individual charged particles. It also strongly changes the spectrum of longitudinal and electromagnetic waves propagating in the highly magnetized plasmas, creating, for instance, the longitudinally-transverse waves. The magnetic fields create the spin-polarization of electrons as well, which can be macroscopically hidden by the diamagnetic effects of moving charges. Propagation of the electromagnetic waves in plasmas leads to the spin polarization of medium via the relativistic effects of the spin-torque \([1]\), the spin-orbit interaction \([2]\), or presence small amplitude low frequency electromagnetic radiation absorbed by electrons with further spin flipping.

The white dwarfs are the least compact of the compact astrophysical objects. But their density is enough to reach high density degenerate plasmas with the Fermi momentum close to \( m_e c \), where \( m_e \) is the mass of electron, and \( c \) is the speed of light. It is possible that the observed radiation which comes from the compact astrophysical objects can give information from nonlinear interactions, particularly about interaction of the high-frequency electromagnetic radiation with the matter waves existing in plasmas \([3]\), \([4]\), \([5]\). There are examples of plasmas composed of two species of electrons which demonstrate scattering of the large amplitude electromagnetic waves on the sound waves, particularly the decay instability and the modulation instability are found in Ref. \([5]\).

Plasmas can exist in extreme conditions, near the compact objects with magnetic field close to critical value, where the particle-antiparticle pairs can occur \([6]\). However, we do not consider these kinds of objects here.

The partially spin polarized degenerate plasmas show existence of the spin-electron-acoustic waves \([7]\), \([8]\), \([9]\), \([10]\), \([11]\), \([12]\). Particularly, the relativistic spin-electron-acoustic waves are studied in high-density degenerate spin-polarized plasmas using the separate-spin-evolution relativistic hydrodynamic model with the average reverse gamma factor evolution \([13]\). This hydrodynamic model is partially based on the quantum hydrodynamic models \([7]\), \([14]\), \([15]\), \([16]\). Variety of instabilities are studied in nonrelativistic degenerate plasmas, particularly effects appearing due to the separate spin evolution of electrons with different spin projections \([17]\), \([18]\), \([19]\), \([20]\), \([21]\).

In this paper, we combine the knowledge about the relativistic spin-electron-acoustic wave with the possibility nonlinear instabilities existing in degenerate plasmas surrounding the white dwarfs in order to find new regimes for the unstable plasma behavior.

This paper is organized as follows. In Sec. II the separate spin evolution relativistic hydrodynamic model with the average reverse gamma factor evolution is presented for the systems of degenerate partially spin polarized electrons. In Sec. III approximate wave equations for the coupled transverse and longitudinal waves are derived. In Sec. IV the analysis of stability of the spin-electron-acoustic waves under pumping of electromagnetic wave is analyzed. In Sec. IV a brief summary of obtained results is presented.

II. TWO FLUID RELATIVISTIC HYDRODYNAMIC MODEL WITH THE AVERAGE REVERSE GAMMA FACTOR EVOLUTION

In order to study waves in the high-density degenerate spin-polarized plasmas we apply the separate spin evolution relativistic hydrodynamic model with the average reverse gamma factor evolution, which is developed in Ref. \([13]\), as the generalization of Refs. \([22]\), \([23]\), \([24]\), \([25]\), \([26]\). This model is composed of four following equations.
for each species of charged particles which are obtained in the mean-field approximation. One is the continuity equation

$$\partial_t n_s + \nabla \cdot (n_s v_s) = 0. \quad (1)$$

Second equation is a form of relativistic Euler equation derived for the flux of particles and showing the evolution of the velocity field

$$n_s \partial_t v_s + n_s (v_s \cdot \nabla) v_s + \frac{1}{m_s} \nabla \tilde{p}_s = \frac{q_s}{m_s} \left( \Gamma_s - \frac{\tilde{\Gamma}}{c^2} \right) E + \frac{q_s}{m_s} \left( [\Gamma_s v_s + t_s] \times B \right)$$

$$- \frac{q_s}{m_s c^2} \left( \Gamma_s v_s (v_s \cdot E) + v_s (t_s \cdot E) + t_s (v_s \cdot E) \right), \quad (2)$$

where $m_s$ and $q_s$ are the mass and charge of particle of $s$ species, $c$ is the speed of light, tensor $\tilde{p}_s^{ab} = \tilde{p}_s \delta^{ab}$ is the flux of the velocities for electrons with fixed spin projection, tensor $t_s^{ab} = \tilde{t}_s \delta^{ab}$ is the flux of the average reverse gamma-factor for spin-$s$ electrons, $\varepsilon^{abc}$ is the three-dimensional Levi-Civita symbol, $\delta^{ab}$ is the three-dimensional Kronecker symbol. Moreover, we work in the Minkovskii space, hence the metric tensor has diagonal form with the following sings $g^{a\beta} = \{-1, +1, +1, +1\}$. We mostly use the three dimensional notations, therefore, we can change covariant and contravariant indexes for the three-vector indexes: $v^b_s = v_{b,s}$. The Latin indexes like $a, b, c$ etc describe the three-vectors, while the Greek indexes are deposited for the four-vector notations. We also have Latin index $s$ which refers to the species or subspecies of electrons with different spin projections. However, the indexes related to coordinates are chosen from the beginning of the alphabet, while other indexes are chosen in accordance with their physical meaning. The model under presentation and this Euler equation includes no effects related to change of the spin projections of electrons on the chosen direction.

The third equation is for the evolution for the average reverse relativistic gamma factor or the hydrodynamic Gamma function

$$\partial_t \Gamma_s + \nabla (\Gamma_s v_s + t_s)$$

$$= -\frac{q_s}{m_s c^2} n_s (v_s \cdot E) \left( 1 - \frac{1}{c^2} \left( \frac{v_s^2}{n_s} + \frac{5 \tilde{p}_s}{n_s} \right) \right). \quad (3)$$

The fourth and final material equation is the equation for the evolution of the flux of reverse gamma factor

$$(\partial_t + v_s \cdot \nabla) t_s^a + \nabla^a \tilde{t}_s + (t_s \cdot \nabla) v_s^a + t_s (\nabla \cdot v_s)$$

$$+ \Gamma_s (\partial_t + v_s \cdot \nabla) v_s^a = \frac{q_s}{m_s} n_s E^a \left[ 1 - \frac{v_s^2}{c^2} - \frac{3 \tilde{p}_s}{n_s c^2} \right]$$

$$+ \frac{q_s}{m_s c} \left[ n_s v_s \times B \right] \left[ 1 - \frac{v_s^2}{c^2} \right] - \frac{2 q_s}{m_s c^2} \left[ \frac{E_s \tilde{p}_s}{c^2} \right] \left[ 1 - \frac{v_s^2}{c^2} \right]$$

$$+ n_s v_s^a (v_s \cdot E) \left[ 1 - \frac{v_s^2}{c^2} - \frac{9 \tilde{p}_s}{n_s c^2} \right] - \frac{5 M_{s0}}{3c^2} E^a. \quad (4)$$

Some functions appearing in this set of equations are discussed below together with the necessary equations of state.

The Maxwell equations are used to couple interspecies and inspecies electromagnetic interaction

$$\nabla \cdot E = -\frac{1}{c} \partial_t B,$$  \quad (5)

$$\nabla \cdot E = 4 \pi (q_e n_{\text{e}^+} + q_{\text{e}^-} n_{\text{e}^-} + q_i n_i), \quad (6)$$

and

$$\nabla \times B = \frac{1}{c} \partial_t E + \frac{4 \pi q_e}{c} n_{\text{e}^+} v_{\text{e}^+} + \frac{4 \pi q_{\text{e}^-}}{c} n_{\text{e}^-} v_{\text{e}^-} + \frac{4 \pi q_i}{c} n_i v_i. \quad (7)$$

In this paper we consider the ions as the motionless background, hence below we have $v_i = 0$.

Presented hydrodynamic equations do not contain information about evolution of the spin density. However, there are nonrelativistic quantum hydrodynamic and kinetic models, where this effect included \cite{7, 15, 27, 28}. Moreover, some relativistic spin effects in plasmas are described by hydrodynamics in Ref. \cite{29}. The variety of other models describing the relativistic plasmas can be found in literature \cite{30, 31, 32, 33, 34, 35, 36, 37, 38, 39}.

### A. Equations of state for spin-up and spin-down electrons

We follow the results of Ref. \cite{13} to present the equations of state, which are necessary to get the closed set of the relativistic hydrodynamic equations. Let us repeat the method of derivation of equations of state. We consider the high density degenerate electron gas. The Fermi velocity is obtained for the relativistic regime $v_{F_s} = p_{F_s} / \sqrt{1 + p_{F_s}^2 / m_s^2 c^2}$, where $p_{F_s} = (6 \pi^2 n_s)^{1/3} h$.

Systems of degenerate fermions with the fixed spin projection are described within the Fermi-Dirac distribution, which simplifies to the Fermi step distribution for the zero-temperature limit

$$f_{s0} = \begin{cases} \frac{1}{(2 \pi h)^3} & \text{for } p \leq p_{F_s} \\ 0 & \text{for } p > p_{F_s} \end{cases} \quad (8)$$

The concentration of $s$-species can be expressed via the distribution function

$$n_s = \int f_{s0} d^3 p. \quad (9)$$
where $p = m_s v / \sqrt{1 - v^2 / c^2}$.

Here we ready to present the flux of the current of particles via the distribution function

$$p_s^{ab} = \int v^a v^b f_{s0} d^3 p. \quad (10)$$

We use distribution function \( \tilde{p} \) to calculate the equation of state \( p_s^{ab} = \tilde{p}_s \delta^{ab} \).

$$\tilde{p}_s = \frac{m_s^3 c^5}{6 \pi^2 h^3} \left[ \frac{1}{3} \xi_s^3 - \xi_s + \arctan \xi_s \right], \quad (11)$$

where \( \xi_s = p_{Fs} / mc \).

Next, we derive the flux of the current of the average reverse gamma factor via the distribution function

$$t_s^{ab} = \int \left( \frac{v^{a} v^{b}}{\gamma} \right) f_{s0} d^3 p, \quad (12)$$

with the following result \( t_s^{ab} = \tilde{t}_s \delta^{ab} \), and

$$\tilde{t}_s = \frac{m_s^3 c^7}{12 \pi^2 h^3} \left[ \xi_s \sqrt{\xi_s^2 + 1} + \frac{2 \xi_s}{\sqrt{\xi_s^2 + 1}^3} - 3 \arcsinh \xi_s \right]. \quad (13)$$

where \( \arcsinh \xi_s \equiv \text{ln} \left[ \xi_s + \sqrt{\xi_s^2 + 1} \right] \), and \( \text{sinh}(\arcsinh \xi_s) = \xi_s \).

The fourth rank tensor \( M_s^{abcd} \) is

$$M_s^{abcd} = \int v^a v^b v^c v^d f_{s0} d^3 p. \quad (14)$$

Expression \( \tilde{t}_s \) leads to \( M_s^{abcd} = (M_{s0}/3)(\delta^{ab} \delta^{cd} + \delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc}) \), where

$$M_{s0} = \frac{m_s^3 c^7}{60 \pi^2 h^3} \left[ 2 \xi_s (\xi_s^2 - 6) - 3 \xi_s \xi_s^2 + 1 + 15 \arctan \xi_s \right]. \quad (15)$$

The fourth rank tensor \( M_s^{abcd} \) enters equation \( \tilde{t}_s \) via its partial trace \( M_s^{abc} = M_s^{bac} \). We have the following nonzero elements of this tensor: \( M_s^{xxx} = M_s^{yyy} = M_s^{zzz} = M_{s0} \) and \( M_s^{xzy} = M_s^{yzz} = M_s^{xzx} = 0 \). The partial trace \( M_s^{abc} \) has the following presentation via elements of tensor \( M_{s0} \): \( M_s^{bac} = (5M_{s0}/3) \delta^{ab} \).

We also find the equilibrium expression for function \( \Gamma_{s0} \):

$$\Gamma_s = \int \frac{1}{\gamma} f_{s0} d^3 p, \quad (16)$$

with

$$\Gamma_{s0} = \frac{m_s^3 c^5}{4 \pi^2 h^3} \left[ \xi_s \sqrt{\xi_s^2 + 1} - \arcsinh \xi_s \right]. \quad (17)$$

### III. WAVE EQUATIONS FOR THE LONGITUDINAL SPIN-ELECTRON-ACOUSTIC WAVES IN PRESENCE OF THE FINITE AMPLITUDE ELECTROMAGNETIC WAVE

Presence of the strong magnetic field leads to change of the behavior of waves and plasmas. We neglect this effect considering white dwarfs with relatively small magnetic field. However, presented analysis can be considered as the rough estimation of possible instabilities which may occur in the plasmas surrounding the neutron stars.

We rewrite the electromagnetic field in terms of the scalar \( \varphi \) and vector \( A \) potentials \( E = -\nabla \varphi - \frac{1}{c} \partial_t A \), and \( B = \nabla \times A \). Moreover, we use the Coulomb Gauge \( \nabla \cdot A = 0 \). Therefore, the Maxwell equations have the following form

$$\partial_0^2 A - c^2 \Delta A + c \partial_0 \nabla \varphi - 4 \pi e c \left( n e u \nu_{eu} + n e d \nu_{ed} \right) = 0, \quad (18)$$

and

$$\Delta \varphi = -4 \pi (q_e n e u + q_n n e d + q_i n_i). \quad (19)$$

We assume that all hydrodynamic functions depend on coordinate \( z \) which is the direction of the electromagnetic wave propagation and time \( t \). For instance, the concentration is \( n = n(z,t) \). Moreover, the vector potential of the electromagnetic field has the following structure

$$A_{\perp} = \frac{1}{2} \left[ (e_x + i e_y) A(z,t) e^{i k_0 z - \omega_0 t} + c.c. \right], \quad (20)$$

where \( k_0 \) and \( \omega_0 \) are the wave vector and the frequency of the electromagnetic wave propagating through the plasmas, and c.c. is the complex conjugation.

As the result of suggested structure of functions we find \( A_z = e_z \cdot A = 0 \) from the Coulomb Gauge, \( E_z = -\partial_z \varphi \), \( E_{\perp} = -\frac{1}{c} \partial_0 A_{\perp}, B = -\partial_z A_{\perp}, e_x + \partial_z A_{\perp} e_y \).

For the suggested structure of functions we find the following equation for the transverse part of the vector potential

$$\partial_0^2 A_{\perp} - c^2 \Delta A_{\perp} - 4 \pi e c \left( n e u \nu_{eu} + n e d \nu_{ed} \right) = 0, \quad (21)$$

We find simplification of equation \( \tilde{t}_s \) for the longitudinal perturbations:

$$n_s \partial_t v_{sz} + n_s v_{sz} \partial_z v_{sz} + \partial_z p_s = -\frac{q_e}{m_e} \left( \Gamma_s - \frac{1}{c^2} \tilde{t}_s \right) \partial_z \varphi + \frac{q_e}{m_e} \left[ (\Gamma_s v_{sx} + t_{sx}) \partial_z A_x + (\Gamma_s v_{sy} + t_{sy}) \partial_z A_y \right]. \quad (22)$$

The transverse motion of the electrons is described by the following equation

$$n_s \partial_t v_{s\perp} + n_s v_{s\perp} \partial_z v_{s\perp} + \frac{q_e}{m_e} \left( \Gamma_s - \frac{1}{c^2} \tilde{t}_s \right) \partial_0 A_{\perp} + \frac{q_e}{m_e} \left( \Gamma_s v_{sz} + t_{sz} \right) \partial_z A_{\perp} = 0 \quad (23)$$

Equation \( 23 \) can be represented in the following form

$$\partial_t (n_s v_{s\perp}) + \partial_0 (n_s v_{s\perp} v_{s\perp}) + \frac{q_e}{m_e} \left( \Gamma_s - \frac{1}{c^2} \tilde{t}_s \right) \partial_0 A_{\perp}$$
\[
+ \frac{q_e}{m_e c} \left( \Gamma_s v_{sz} + t_{sz} \right) \partial_z A^\perp = 0 \tag{24}
\]

using the continuity equation

\[
\partial_t n_s + \partial_z (n_s v_{sz}) = 0. \tag{25}
\]

Next, we try to put coefficients in front of \(\partial_t A^\perp\) and \(\partial_z A^\perp\) in equation (24) under the derivatives. To do this step we need to consider the average reverse gamma factor evolution (3) in more details. As we stated above, we assume that we consider relatively small intensity electromagnetic waves and nonrelativistic flows. Hence, the right-hand side of equation (3) can be dropped and this equation reappears in the following form

\[
\partial_t \Gamma_s + \partial_z (\Gamma_s v_{sz} + t_{sz}) = 0. \tag{26}
\]

Equation (26) allows to give the following representation of equation (24)

\[
\partial_t (n_s v_{s\perp}) + \partial_z (n_s v_{sz} v_{s\perp}) + \frac{q_e}{m_e c} \partial_z \left( \Gamma_s - \frac{1}{c^2} t_s \right) A^\perp \]

\[
+ \frac{q_e}{m_e c} \partial_z \left( \Gamma_s v_{sz} + t_{sz} \right) A^\perp + \frac{1}{c^2} \frac{q_e}{m_e c} A^\perp \partial_t t_s = 0. \tag{27}
\]

So, we focus on the last term in the found equation:

\[
\frac{q_e}{m_e c} A^\perp \partial_t t_s = - \frac{q_e}{m_e c} \frac{1}{\delta n_s} \partial_t \Gamma_s n_s
\]

Here, we have \(v_{sx} A^\perp / c^2 \sim v^2 / c^2 \ll 1\). It shows that we can neglect the last term in equation (27) and find the following structure

\[
\partial_t \left[ n_s v_{s\perp} + \frac{q_e}{m_e c} \left( \Gamma_s - \frac{1}{c^2} t_s \right) A^\perp \right]
\]

\[
+ \partial_z \left[ n_s v_{sz} v_{s\perp} + \frac{q_e}{m_e c} \left( \Gamma_s v_{sz} + t_{sz} \right) A^\perp \right] = 0. \tag{28}
\]

Equation (28) shows the conservation of the vector function \(w_s \equiv n_s v_{s\perp} + \frac{q_e}{m_e c} \left( \Gamma_s - \frac{1}{c^2} t_s \right) A^\perp\), while vector function \(n_s v_{sz} v_{s\perp} + \frac{q_e}{m_e c} \left( \Gamma_s v_{sz} + t_{sz} \right) A^\perp\) gives the flux of function \(w_s\).

In our analysis we assume the zero value of the conserving constant \(w_s = 0\). It gives us the following expression for the transverse velocity \(v_{s\perp}\) in terms of the vector potential \(A^\perp\)

\[
v_{s\perp} = - \frac{q_e}{m_e c} \left( \Gamma_s - \frac{1}{c^2} t_s \right) A^\perp. \tag{29}
\]

Obtained expression for the transverse velocity allows to rewrite the equation for the transverse vector potential (21)

\[
\partial_t^2 A^\perp - \frac{\omega_{lu}^2}{n_{lu}} \left( \Gamma_{eu} + \Gamma_{ed} - \frac{1}{c^2} t_u - \frac{1}{c^2} t_d \right) A^\perp = 0, \tag{30}
\]

where \(\omega_{lu}^2 = \frac{4\pi e^2 n_{lu}}{m_e}\) is the partial Langmuir frequency for the spin-down electrons.

Expression (29) allows to make transformation of equation (22). However, equation (22) also contains the contribution of the transverse vector motion via the transverse part of the flux of the average reverse gamma factor \(t_{s\perp}\). Hence, we need to get relation between \(t_{s\perp} A^\perp\) and \(A^\perp\), which is found in Appendix A

\[
t_{s\perp} = \beta_s A^\perp \tag{31}
\]

using equation (1). Explicit form of parameter \(\beta_s\) is also given in Appendix A.

Let us to point out that considering the longitudinal motion we consider the small amplitude acoustic waves. Therefore, we can consider the linear response for the longitudinal perturbations.

This assumption allows to get further simplification of equation (30)

\[
\begin{align*}
+ \Gamma_{od} - \frac{1}{c^2} t_{od} + \delta \Gamma_{eu} + \delta \Gamma_{ed} - \frac{1}{c^2} \delta t_u - \frac{1}{c^2} \delta t_d \right) A^\perp = 0,
\end{align*}
\]

The equilibrium values of functions \(\Gamma_{0s}\) and \(t_{0s}\) can be found from the corresponding equations of state (17) and (18), correspondingly. Next, the perturbations of \(\delta t_s\) can be also found from equation (13). We can make expansion of expression (13) on the perturbations of the concentration and take the linear on \(\delta n_s\) part. Perturbations of relativistic gamma function \(\delta \Gamma_s\) can be found from equation (24). However, equation (24) requires the longitudinal part of the flux of the average reverse relativistic gamma factor \(t_{sz}\). Equation for \(t_{sz}\) can be found from general equation (4).

Before, we consider equation for \(t_{sz}\) let us discuss equation for \(v_{sz}\) and \(n_s\). Here, we ready to give representation of equation (22), which is also considered in the linear regime on perturbations of functions \(n_s, v_{sz}, \Gamma_s\) and \(\varphi\)

\[
\frac{n_{0s}}{n_s} \partial_t v_{sz} + \frac{\partial p_{0s}}{\partial n_{0s}} \partial_z n_s = - \frac{q_e}{m_e} \left( \Gamma_{0s} - \frac{1}{c^2} t_{0s} \right) \partial_z \varphi + \Upsilon_s \partial_z A^2_{\perp},
\]

where \(n_s = n_{0s} + \delta n_s, \Gamma_s = \Gamma_{0s} + \delta \Gamma_s, v_{sz} = \delta v_{sz}\) and \(\varphi = \delta \varphi\), and we also introduced parameter \(\Upsilon_s = \frac{1}{2} (\frac{m_e}{e})^2 \left( \frac{\Gamma_{0s}}{n_{0s} c^2} - \delta \Gamma_s \right)\). We take the derivative on \(z\) of equation (33) and use the continuity equation (25)

\[
\partial_t^2 \delta n_s + \frac{\partial p_{0s}}{\partial n_{0s}} \partial_z^2 \delta n_s
\]

\[
+ \frac{\omega_{lu}^2}{n_{lu}} \left( \Gamma_{0u} - \frac{t_{0u}}{n_{0us} c^2} \right) \left( \delta n_u + \delta n_d \right) - \Upsilon_s \partial_z^2 A^2_{\perp} = 0, \tag{34}
\]
where we use the simplified equation (35)
\[ \partial_t^2 \varphi = -4\pi q_e (\delta n_e + \delta n_e^c). \] (35)

Let us present equation for \( t_{sz} \) found from equation (3) under assumption of the small amplitude response of the longitudinal functions, and nonrelativistic fluxes \( v^2 \ll c^2 \):

\[ \partial_t \delta t_{sz} + \Gamma_{os} \partial_t \delta v_{sz} + \frac{\delta \Gamma_{os}}{\delta n_s} \partial_t \delta n_s \]
\[ = -\frac{q_e}{m_e} n_{os} \left( 1 - \frac{5p_{os}}{n_{os} c^2} + \frac{10M_{os}}{3n_{os} c^4} \right) \partial_z \varphi \]
\[ - \frac{1}{2} \left( \frac{q_e}{m_e c} \right)^2 n_{os} \left( 1 - \frac{5p_{os}}{n_{os} c^2} \right) \left( \frac{\Gamma_{os}}{n_{os}} - \frac{\tilde{t}_{os}}{n_{os} c^2} \right) \partial_z^2 A_\perp. \] (36)

To exclude \( \delta t_{sz} \) from final equations we take the derivative on \( z \) of equation (36) and use equation (20)
\[ \partial_t^2 \delta \Gamma_s - \frac{\delta \Gamma_{os}}{\delta n_s} \partial_z^2 \delta n_s + \frac{\omega^2_s}{\gamma_{F_s}} \left( 1 - \frac{5p_{os}}{n_{os} c^2} + \frac{10M_{os}}{3n_{os} c^4} \right) (\delta n_u + \delta n_d) \]
\[ - \frac{1}{2} \left( \frac{q_e}{m_e c} \right)^2 \left( 1 - \frac{5p_{os}}{n_{os} c^2} \right) \partial_z^2 A_\perp = 0. \] (37)

Equations presented above are shown in the way as they appear from the general hydrodynamic equations (1), (2), (3), and (4). However, the application of equations of state (11), (13), (15), and (17). However, some combinations of parameters can be represented in physically more meaningful notations: \( \left( \frac{\Gamma_{os}}{n_{os}} - \frac{\tilde{t}_{os}}{n_{os} c^2} \right) = \frac{1}{\gamma_{F_s}} \), where \( \gamma_{F_s} = 1/\sqrt{1 - v^2_{F_s}/c^2} = \sqrt{1 + p^2_{F_s}/m^2_{e}c^2} \), and \( \gamma_{F_s}^2 = \frac{q_e}{m_e c} \), \( \frac{p_{F_s}}{m_{e} c} \), with \( p_{F_s} = (3\pi^2 n_{0e})^{1/3} \hbar \), and We also find \( (1 - \frac{5p_{os}}{n_{os} c^2} + \frac{10M_{os}}{3n_{os} c^4}) = \frac{1}{\gamma_{F_s}} \).

IV. THE STABILITY ANALYSIS

Let us sum up the obtained wave equations in order to study the stability of the system. We repeat equations (38), (39), and with some small modifications:
\[ \partial_t^2 \delta n_s = \frac{\delta p_{us}}{\delta n_s} \partial_z^2 \delta n_s + \frac{\omega^2_s}{\gamma_{F_s}} (\delta n_u + \delta n_d) - \frac{\Xi_s}{\gamma_{F_s}} \partial_z^2 A_\perp = 0, \] (38)
\[ \partial_t^2 \delta \Gamma_s - \frac{\delta \Gamma_{os}}{\delta n_s} \partial_z^2 \delta n_s + \frac{\omega^2_s}{\gamma_{F_s}} (\delta n_u + \delta n_d) - \Xi_s \partial_z^2 A_\perp = 0, \] (39)
where \( \Xi_s = \frac{1}{2} \left( \frac{q_e}{m_e c} \right)^2 (1 - \frac{5p_{os}}{n_{os} c^2}) \), and
\[ \partial_t^2 A_\perp = c^2 \partial_z^2 A_\perp + \frac{\omega^2_{ld}}{\gamma_{F_d}} \left( \frac{\delta u}{\gamma_{F_d}} + \frac{\delta d}{\gamma_{F_d}} \right) \partial_z A_\perp = 0. \] (40)

Start analysis of obtained equation from the transformation of equation (41) explicitly using structure (20). We also include \( \partial_t A \ll \omega A \) since amplitude \( A \) is the slow amplitude. The linearized equation (41) gives approximate relation between \( \omega_0 \) and \( \delta_0 \): \( \omega_0^2 = k_0^2 c^2 + \frac{\omega^2_{ld}}{\gamma_{F_d}} \), \( \omega_0^2 = k_0^2 c^2 + \frac{\omega^2_{ld}}{\gamma_{F_d}} \). It is also used to cancel corresponding terms in equation (41):
\[ 2\omega_0 (\partial_t A + V_g \partial_z A) + U^2 \partial^2_z A \]
\[ - \frac{\omega^2_{ld}}{\gamma_{F_d}} \left( \delta \Gamma_{eu} + \delta \Gamma_{ed} - \frac{1}{c^2} \partial_t u - \frac{1}{c^2} \partial_t d \right) A_\perp = 0, \] (41)
where \( V_g = k_0 c^2/\omega_0 \), and \( U^2 = c^2 \).

Function \( A(z, t) \) in equation (41) is the slowly changing complex amplitude. We split it on the amplitude and the phase \( A(z, t) = a(z, t) e^{i\theta(z, t)} \), with further decomposition of the amplitude and the phase \( a(z, t) = \frac{1}{2} [a_0 + \delta a \cdot e^{i[kz - \delta t + c.c.]}] \), \( \delta A(z, t) = \frac{1}{2} [\delta a \cdot e^{i[kz - \delta t + c.c.]}] \), \( \delta \theta(z, t) = \frac{1}{2} [\theta_0 + \delta \theta \cdot e^{i[kz - \delta t + c.c.]}] \), where \( \delta a \ll a_0 \), and \( \delta \theta \ll \theta_0 \). Next, we make corresponding decomposition of the hydrodynamic functions \( \delta n_s = \frac{1}{2} [N_s \cdot e^{i[kz - \delta t + c.c.]}] \), and \( \delta \Gamma_s = \frac{1}{2} [\Gamma_s \cdot e^{i[kz - \delta t + c.c.]}] \).

We use presented decompositions in order to simplify equations (38), (39), and (41)
\[ -\Omega^2 \delta n_s + \frac{\delta p_{us}}{\gamma_{F_s}} k^2 \delta n_s + \frac{\omega^2_s}{\gamma_{F_s}} (\delta n_u + \delta n_d) + Y_s k^2 a_0 \delta A = 0, \] (42)
and
\[ -\Omega^2 \delta \Gamma_s + \frac{\delta \Gamma_{os}}{\gamma_{F_s}} k^2 \delta n_s + \frac{\omega^2_s}{\gamma_{F_s}} (\delta n_u + \delta n_d) + \Xi_s k^2 a_0 \delta A = 0, \] (43)
while equation (41) splits on two equations for the amplitude and the phase
\[ 2\omega_0 \partial_t a + 2\omega_0 V_g \partial_z a + U^2 a_0 \partial^2_z \theta = 0, \] (44)
and
\[ 2\omega_0 a_0 \partial_t \theta + 2\omega_0 V_g a_0 \partial_z \theta - U^2 \partial^2_z a + a_0 \delta H = 0, \] (45)
where
\[ \delta H = \frac{\omega^2_{ld}}{\gamma_{F_d}} \left( \delta \Gamma_{eu} + \delta \Gamma_{ed} - \frac{1}{c^2} \partial_t u - \frac{1}{c^2} \partial_t d \right). \] (46)

Equations (44) and (45) gives
\[ \left[ (\Omega - V_g k)^2 - \frac{1}{4} \frac{U^4 k^4}{\omega_0^4} \right] \delta a - a_0 \delta H = 0. \] (47)

We get expressions for the partial concentrations from equation (42)
\[ \delta n_u = \frac{a_0 k^2 \delta A}{\Delta_N} \left[ \gamma_u \left( \Omega^2 - u_{pd}^2 k^2 - \frac{\omega^2_{ld}}{\gamma_{F_d}} \right) + \gamma_d \frac{\omega^2_{ld}}{\gamma_{F_d}} \right]. \] (48)
and

\[ \delta n_d = \frac{a_0 k^2 \delta A}{\Delta_N} \left[ \frac{\omega_{Ld}^2}{\gamma_{Fd}} \right. \left. + \frac{\omega_{Ld}^2}{\gamma_{Fd}} \left( \Omega^2 - u_{pu}^2 k^2 - \frac{\omega_{Lu}^2}{\gamma_{Fu}} \right) \right], \quad (49) \]

where

\[ \Delta_N = (\Omega^2 - u_{pd}^2 k^2)(\Omega^2 - u_{pu}^2 k^2) \]

\[ - \frac{\omega_{Lu}^2}{\gamma_{Fu}} (\Omega^2 - u_{pu}^2 k^2) - \frac{\omega_{Ld}^2}{\gamma_{Fd}} (\Omega^2 - u_{pu}^2 k^2), \quad (50) \]

with \( u_{pu}^2 = \frac{\delta \rho_u}{\delta n_u} \).

We need to get the contribution of the partial hydrodynamic gamma functions \( \Gamma_s \). However, we do not need expressions for each of them. We need to find their sum \( \delta \Gamma_u + \delta \Gamma_d \) only, as we can see it from expression for \( \delta H \) (see equation (46)). We find required expression from equation (43)

\[ \delta \Gamma_u + \delta \Gamma_d = \frac{1}{\Omega^2} \left[ \frac{\delta \rho_u}{\delta n_u} \right. \left. k^2 \delta n_u + \frac{\delta \rho_d}{\delta n_d} k^2 \delta n_d \right. \]

\[ + (\Xi_u + \Xi_d) a_0 k^2 \delta A + \left( \frac{\omega_{Lu}^2}{\gamma_{Fu}} + \frac{\omega_{Ld}^2}{\gamma_{Fd}} \right) (\delta n_u + \delta n_d) \]. \quad (51) \]

We also need \( \delta n_u + \delta n_d \):

\[ \delta n_u + \delta n_d = \frac{a_0 k^2 \delta A}{\Delta_N} \left[ \gamma_{Fu} (\Omega^2 - u_{pu}^2 k^2) + \gamma_{Fd} (\Omega^2 - u_{pu}^2 k^2) \right]. \quad (52) \]

Obtained expressions for the partial concentrations \( \delta n_s \) and gamma functions \( \delta \Gamma_s \) allow us to represent function \( \delta H \) (46):

\[ \delta H = \frac{\omega_{Ld}^2}{n_{od} \Omega^2 c^2} \left( \frac{\delta \rho_u}{\delta n_u} - \frac{\delta \rho_d}{\delta n_d} \right) (k^2 c^2 - \Omega^2) \]

\[ + c^2(\Xi_u + \Xi_d) a_0 k^2 \delta A + c^2 \left( \frac{\omega_{Lu}^2}{\gamma_{Fu}} + \frac{\omega_{Ld}^2}{\gamma_{Fd}} \right) (\delta n_u + \delta n_d) \]. \quad (53) \]

Moreover, we get the following consequence of equation (47):

\[ \Delta_N \cdot [(\Omega - V_g k)^2 - \frac{1}{4} \frac{U_0^4 k^4}{\omega_0^2}] = -\frac{1}{2} \frac{q^2}{m_2 c^2} \frac{a_0^2 k^2}{\omega_{Ld}^2} \times \]

\[ \times \left\{ \frac{\delta \rho_u}{\delta n_u} \left[ (k^2 c^2 - \Omega^2) \left( \frac{n_{od} \omega_{Ld}^2}{\gamma_{Fd} \gamma_{Fu}} + \frac{n_{od} \omega_{Ld}^2}{\gamma_{Fd} \gamma_{Fu}} \left( \Omega^2 - U_0^2 k^2 - \frac{\omega_{Lu}^2}{\gamma_{Fu}} \right) \right) \right. \right. \]

\[ + \frac{\delta \rho_d}{\delta n_d} \left[ (k^2 c^2 - \Omega^2) \left[ \frac{n_{od} \omega_{Ld}^2}{\gamma_{Fd} \gamma_{Fu}} + \frac{n_{od} \omega_{Ld}^2}{\gamma_{Fd} \gamma_{Fu}} \left( \Omega^2 - U_0^2 k^2 - \frac{\omega_{Lu}^2}{\gamma_{Fu}} \right) + \frac{n_{od} \omega_{Ld}^2}{\gamma_{Fd} \gamma_{Fu}} \right) \right. \right. \]

\[ + c^2 \left( \frac{\omega_{Lu}^2}{\gamma_{Fu}} + \frac{\omega_{Ld}^2}{\gamma_{Fd}} \right) \left( \frac{n_{od} \omega_{Ld}^2}{\gamma_{Fd}^2} \right. \left. \Omega^2 - U_0^2 k^2 \right) + \frac{n_{od} \omega_{Ld}^2}{\gamma_{Fd}^2} \left( \Omega^2 - U_0^2 k^2 \right) \right\}, \quad (54) \]

where \((\Xi_u + \Xi_d) \cdot \Delta_N \approx 0\).

The high-frequency regime of the matter waves with the spectrum obtained from \( \Delta_N \approx 0 \) consists of two waves: the Langmuir wave and the spin-electron-acoustic wave. Depending on the equilibrium concentration of electrons \( n_{oe} = n_{ou} + n_{od} \) the frequency of these waves can be comparable or they can have considerable difference. For the large concentrations \( n_{oe}^{1/3} h/m_e c \sim 1 \) we have comparable frequencies of these waves in the high-frequency regime, which also corresponds to relatively large wave vectors.
Let us consider the low-frequency regime, where we have single spin-electron-acoustic wave. This regime of relatively small frequencies corresponds to frequencies high enough to neglect the motion of ions and the contribution of the ion-acoustic wave. The chosen small frequency regime corresponds to the small wave vectors, where \( \Omega_{SEAW} = c_A k \ll \frac{\omega_{pe}}{2 \gamma_p} \), and \( c_A \) has same order as \( U_{ps} \). It also gives restriction on the large spin polarizations.

We make simplification of \( \Delta_N \) and the right-hand side of equation (54) in the chosen range of parameters. First, we consider parameter \( \Delta_N \) (55). We present it as the superposition of two parts \( \Delta_N = \Delta_{N1} + \Delta_{N2} \), where \( \Delta_{N1} = \Omega^2 (\Omega^2 - (u_{pd}^2 k^2 + u_{pu}^2 k^2 + \omega_{Lu}/\gamma_F u^2 + \omega_{Ld}/\gamma_F d^2)) \cong -\Omega^2 (\omega_{Lu}^2/\gamma_F u^2 + \omega_{Ld}^2/\gamma_F d^2) \), and \( \Delta_{N2} = u_{pd}^2 u_{pu}^2 k^2 + u_{pd}^2 u_{pu}^2 k^2 + u_{pu}^2 k^2 \omega_{Lu}/\gamma_F u^2 + u_{pu}^2 \omega_{Ld}/\gamma_F d^2 \cong u_{pd}^2 \omega_{Lu}^2/\gamma_F u^2 + u_{pu}^2 \omega_{Ld}^2/\gamma_F d^2 \). Hence, we find

\[
\Delta_N \approx -\frac{\omega_{Lu}^2}{\gamma_F u} (\Omega^2 - u_{pd}^2 k^2) - \frac{\omega_{Ld}^2}{\gamma_F d} (\Omega^2 - u_{pu}^2 k^2).
\] (55)

Condition \( \Delta_N = 0 \) gives the following spectrum of acoustic waves \( \Omega = c_A k \), with

\[
e^2 = \frac{u_{pd}^2 \omega_{Lu}^2}{\gamma_F u} + \frac{u_{pu}^2 \omega_{Ld}^2}{\gamma_F d},
\] (56)

where \( u_{ps} = (c^2/3) p_{F_s}/(p_{F_s}^2 + m_e^2 c^2) \).

Using suggested range of parameters and spectrum of spin-electron-acoustic wave we find simplified form of equation (54)

\[
\left( \Omega^2 - c_A^2 k^2 \right) \cdot \left( \left( \Omega - V_g k \right) \frac{k^2}{U_g} - \frac{1}{4} \frac{U_g k^4}{\omega_0^2} \right) = \frac{1}{2} \frac{q^2 c^2}{m_e c^2} \frac{\omega_0^2}{\omega_0^2} \frac{c_A^2 k^2}{\Omega^2 c^2} \times
\]

\[
\times \frac{\omega_{Lu}^2 \omega_{Ld}^2}{\gamma_F u \gamma_F d} \left( \frac{\omega_{Lu}}{\gamma_F u} + \frac{\omega_{Ld}}{\gamma_F d} \right) \left( k^2 c^2 - \Omega^2 \right) \frac{\delta t_{ld}}{\delta n_{ld}} - \frac{\delta t_{lu}}{\delta n_{lu}}
\]

\[-k^2 c^2 (U_{pd}^2 - U_{pu}^2) \left( \frac{\omega_{Lu}}{\gamma_F u} + \frac{\omega_{Ld}}{\gamma_F d} \right) \right),
\] (57)

where we used \( \Delta_N = -\left( \Omega^2 - c_A^2 k^2 \right) (\omega_{Lu}^2/\gamma_F u + \omega_{Ld}^2/\gamma_F d) \).

In order to consider the instability of the system we assume \( \Omega = V_g k + \frac{1}{2} \frac{U_{Lu}}{\omega_0} + \frac{1}{2} \frac{U_{Ld}}{\omega_0} \) and \( \Omega = c_A k + \frac{1}{2} \frac{U_{Lu}}{\omega_0} + \frac{1}{2} \frac{U_{Ld}}{\omega_0} \). The low frequency branch of the electromagnetic wave can be chosen \( \Omega = V_g k + \frac{1}{2} \frac{U_{Lu}}{\omega_0} \), but it appears to be stable in the considered regime.

It leads to

\[
\Lambda^2 = -\frac{1}{2 c_A k} U_{Lu}^2 k^2 \times
\]

\[
\times \frac{\omega_{Lu}^2 \omega_{Ld}^2}{\gamma_F u \gamma_F d} \left( \frac{\omega_{Lu}}{\gamma_F u} + \frac{\omega_{Ld}}{\gamma_F d} \right) \left( k^2 c^2 - \Omega^2 \right) \frac{\delta t_{ld}}{\delta n_{ld}} - \frac{\delta t_{lu}}{\delta n_{lu}}
\]

(58)

Next, let us represent it in the dimensionless form

\[
\frac{\Lambda^2}{\Omega_{Le}^2} = \frac{\omega_0}{\omega_{Le} m_e^2 c^2} \frac{q^2 c^2}{\delta n_{lu}} \Theta,
\] (59)

where

\[
\Theta = \frac{1}{8} \frac{1}{c_A^2} \frac{1}{\gamma_{F u} \gamma_{F d}} \left( \frac{1 - u_{pu}^2 c^2}{\omega_{Lu}^2 \gamma_F u} + \frac{1 + u_{pu}^2 c^2}{\omega_{Ld}^2 \gamma_F d} \right) \left( \frac{k^2 c^2 - \Omega^2}{\omega_{Le}^2} \right) \left( \frac{u_{ld}^2 c^2}{\omega_{Lu}^2 \gamma_F u} + \frac{u_{lu}^2 c^2}{\omega_{Ld}^2 \gamma_F d} \right),
\] (60)

where \( \eta = |n_{ld} - n_{lu}|/(n_{ld} + n_{lu}) \), and \( u_{pu}^2 = \frac{\delta t_{lu}}{\delta n_{lu}} \), with \( u_{pu}^2 = (c^2/3) p_{F_s}^2/(p_{F_s}^2 + m_e^2 c^2)^{3/2} \).

Numerical analysis of the increment of instability \( \Lambda \) is made in Fig. 1, where it is demonstrated via study of behavior of \( \Theta \) (60). We see that the instability exists in the area of intermediate concentrations, which however correspond to the interval of relatively large concentrations \( n_{de} \sim 10^{30} \text{ cm}^{-3} \). Strong increase of function \( \Theta \) with the increase of the spin polarization is also can be seen in Fig. 1.

V. CONCLUSION

Propagation of strong electromagnetic waves through the high density degenerate plasmas has been considered. It has been considered in order to study the radiation of compact astrophysical objects, which propagates through the layer of plasmas, after the generation of radiation. Particularly, we have studied effect induced by the interaction of electromagnetic waves with plasmas, particularly, with the small frequency acoustic waves. Moreover, it has been assumed that the propagating radiation induced the spin polarization of plasmas. Hence, the conditions for existence of the spin-electron-acoustic waves has been created. We have considered the two-component electron-ion plasmas with the assumption of motionless ions. But, the electrons being spin polarized can be considered as two fluids with different spin projections. Hence, we have two active fluids interaction with the radiation. For relatively small spin polarization both subspecies of electrons are degenerate at the same conditions.

The strong nonlinear coupling between the electromagnetic waves and spin-electron-acoustic waves leads to decay instability. This instability exists in the interval of concentrations of electrons near \( n_F e = \sqrt{32\pi} m_e c \approx m_e c \) and depends on the spin polarization of electrons showing strong increase with the increase of the spin polarization.
Basically it exists in the nonrelativistic limit, but has small value. The instability also disappears at large concentrations corresponding to ultrarelativistic Fermi momentums.

VI. DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study, which is a purely theoretical one.

Appendix A: Calculation of transverse flux of the average relativistic gamma factor

Function $t_{s\perp}$ enters equation (22) in front of $\partial_s A_{\perp}$. Equation of (22) describes the longitudinal dynamics, which can be considered in the linear approximation. Therefore, we can find expression for $t_{s\perp}$ in the linear approximation as well. We can present the linearized equation for $t_{s\perp}$ from general equation (3):

$$\partial_t t_{s\perp} + \Gamma_0 s \partial_s v_{s\perp}$$

$$= -\frac{q_s}{m_s c} n_{0s} \partial_s A_{\perp} \left( 1 - \frac{5 p_{0s}}{n_{0s} c^2} + \frac{10 M_{0s}}{3 n_{0s} c^4} \right).$$

(A1)

It can be considered as the time derivative of one vector function. Hence, this function is the constant. Similarly to condition $w_s = 0$ we assume that this function is also equal to zero. We also include the expression for $v_{s\perp}$ via $A_{\perp}$ (29). Finally we find $t_{s\perp} = \beta_s A_{\perp}$, where

$$\beta_s = \frac{q_s}{m_s c} \left[ \Gamma_0 s \left( \frac{1}{n_{0s}} - \frac{\tilde{t}_{0s}}{n_{0s} c^2} \right) - \left( 1 - \frac{5 p_{0s}}{n_{0s} c^2} + \frac{10 M_{0s}}{3 n_{0s} c^4} \right) \right].$$

(A2)

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