A Brief Overview of Planet Formation

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Abstract The initial conditions, physics, and outcome of planet formation are now constrained by detailed observations of protoplanetary disks, laboratory experiments, and the discovery of thousands of extrasolar planetary systems. These developments have broadened the range of processes that are considered important in planet formation, to include disk turbulence, radial drift, planet migration, and pervasive post-formation dynamical evolution. The N-body collisional growth of planetesimals and protoplanets, and the physics of planetary envelopes—key ingredients of the classical model—remain central. I provide an overview of the current status of planet formation theory, and discuss how it connects to observations.

Introduction

Solar System and astronomical evidence of the origin of planets is most naturally interpreted in terms of a bottom-up theory (Safronov 1972), in which planetary systems form within largely gaseous protoplanetary disks from initially microscopic solid material. Different physical processes dominate as growth proceeds. The earliest phases (corresponding to particle sizes of $s \sim \mu$m-m) involve primarily aero-dynamic and material physics. Gravitational forces become increasingly important later on, first between growing planetesimals ($s \sim$ km) and later between protoplanets and gas in the disk (for masses $M \gtrsim 0.1 M_\oplus$, where $M_\oplus$ is the mass of the Earth). Giant planet growth from $\sim 3 - 20 + M_\oplus$ cores is limited initially by the ability of their gaseous envelopes to cool, and subsequently by how fast the surrounding disk can supply mass. Finally, the planetary systems that we observe—often after an interval of several Gyr—can be profoundly modified from their initial state by dynamical instabilities, secular evolution, and tides.
This review is an introduction to the processes that matter during planet formation, how those processes may combine to yield planetary systems, and where the theory can be tested against observations. To the extent that there is a theme, it is mobility—of gas in the disk, of dust and pebbles under aerodynamic forces, and of planets due to gravitational torques against the gas and interactions with other bodies. Mobility, particularly in the guise of the radial drift of particles or the migration of low-mass planets through gaseous disks, was once seen as a “problem” to be ideally solved or otherwise ignored. The current view is more positive. Mobility is due to clearly defined physical processes, opens up new routes for rapid growth, and is key to the architecture of many observed extrasolar planetary systems.

**Protoplanetary disks**

The kinematics of protoplanetary disks, their thermal and chemical structures, and their evolutionary histories are all key to planet formation. Most attention focuses on Class II Young Stellar Objects (YSOs) (Lada 1987), when the star has attained close to its final mass and the disk is low mass ($M_{\text{disk}} \ll M_*$) and relatively long-lived (several Myr). (It remains possible, however, that significant particle growth occurs during prior embedded phases.) Observational inferences of the stellar mass accretion rate $\dot{M}$ in Class II sources are moderately robust, and typically yield $\dot{M} \sim 10^{-8.5} M_\odot \text{ yr}^{-1}$ for $M_* \approx M_\odot$, with a super-linear scaling with stellar mass (Alcalá et al 2017). Disk mass estimates are problematic, because H$_2$ is not detected directly. For a very small number of disks (including TW Hya; Bergin et al 2013) HD emission in the far-infrared has been observed, and mass estimates based on this tracer (Trapman et al 2017) provide calibration for more accessible estimators. Estimates based on scaling the mm continuum emission from dust yield a median ratio $M_{\text{disk}}/M_* \sim 10^{-2.5}$ for disks in Taurus (Andrews et al 2013). Modeling of CO isotopologue line emission gives on average lower values (Williams and Best 2014).

Figure 1 shows a cartoon version of disk structure. In the “vertical” direction (perpendicular to the disk plane) the profile of the gas density $\rho$ is determined by a hydrostatic balance between the gradient of pressure $P$ and the vertical component of stellar gravity $g_z$,

$$\frac{dP}{dz} = -\rho g_z. \tag{1}$$

Protoplanetary disks are observed to be thin, in that their vertical thickness is a modest fraction of the distance to the star, and hence we can approximate $g_z \approx \Omega^2 z$, where $\Omega = \sqrt{GM_* / r^3}$ is the Keplerian angular velocity. For an isothermal gas the pressure is given in terms of the sound speed $c_s$ via $P = \rho c_s^2$, and the above equation is easily solved. An isothermal thin disk has a gaussian density profile, $\rho(z) \propto \exp(-z^2/2h^2)$, with a scale height $h = c_s/\Omega$. In the radial direction force balance,

$$\frac{\dot{v}_\phi^2}{r} = \frac{GM_*}{r^2} + \frac{1}{\rho} \frac{dP}{dr}, \tag{2}$$
implies an orbital velocity $v_\phi = v_K [1 - \theta(h/r)^2]$ that is close to Keplerian, though pressure support leads to a slight deviation, typically by tens of meters per second in the sense of sub-Keplerian rotation. This sub-Keplerian rotation has important consequences for particle dynamics.

Disks are heated by stellar irradiation and by dissipation of potential energy as gas accretes. Irradiation leads to a temperature profile roughly given by $T(r) \propto r^{-1/2}$ ([Kenyon and Hartmann] [1987]), and a disk that flares. The vertical structure in irradiation-dominated regions has an isothermal interior in which $T_{\text{dust}} = T_{\text{gas}}$, a warm layer of surface dust directly exposed to starlight ([Chiang and Goldreich] [1997]), and a hot gas atmosphere with photon-dominated chemistry. At small radii (typically at AU scales) accretion heating becomes more important, producing higher temperatures and replacing the isothermal interior with one in which $T(z)$ decreases with height. In the simple limit of radiative transfer of energy and heating in a narrow mid-plane slice the ratio of central to effective temperatures depends on the optical depth via $T_c/T_{\text{eff}} \approx \tau^{1/4}$ (e.g. [Armitage] [2010]), and the mid-plane is substantially hotter than a non-accreting disk. Accretion heating is needed to reproduce the location of the water snow line (at $T \approx 150$ K) in the Solar System, which is inferred from meteoritic evidence to have fallen at $r \approx 2.7$ AU. The radius of the snow line changes over time as the importance of accretion heating wanes (moving inside 1 AU at low accretion rates; [Garaud and Lin] [2007]), so its observed location in the Solar System suggests that the bodies in the asteroid belt formed relatively early. Critically, the positioning of the snow line in the asteroid belt implies that Earth did not acquire its water in situ ([Morbidelli et al] [2000]).

The radial distribution and evolution of the gas defy simple predictions. Dust continuum observations in Ophiuchus (at $r \geq 20$ AU scales; [Andrews et al] [2009]) and $^{13}$C$^{18}$O line emission from TW Hya (at 5-20 AU; [Zhang et al] [2017]) suggest
a surface density profile $\Sigma \propto r^{-0.9}$, but this cannot be predicted from first principles. Disk initial conditions are set by the angular momentum distribution of the collapsing cloud, while evolution can occur due to turbulent torques (either fluid or magnetohydrodynamic), large-scale laminar torques, and either thermal or magnetohydrodynamic (MHD) winds. In the turbulent case the disk evolves as if it has a large kinematic viscosity $\nu$, and it is conventional to express the efficiency of the transport by a dimensionless Shakura-Sunyaev $\alpha$ parameter, defined via,

$$\nu = \alpha c_s h.$$  \hspace{1cm} (3)

Order of magnitude estimates suggest that values of $\alpha = 10^{-3} - 10^{-2}$ would suffice to drive significant disk evolution on Myr time-scales.

Self-gravity may be the dominant angular momentum transport agent at early times, when the disk is massive and the Toomre $Q$ parameter $Q = c_s \Omega / \pi G \Sigma$ that describes the linear stability of a disk (Toomre 1964) is low ($Q \approx 1$). Self-gravitating disks can fragment—either when cooling of an isolated disk is too rapid (Gammie 2001; Rice et al 2005) or when an embedded disk is over-fed with mass (Kratter et al 2010)—but this process is now considered unlikely to form a significant population of planets; the unstable radii and resultant masses are both predicted to be too large (Kratter and Lodato 2016).

At later times, as the disk mass drops, MHD transport due to the magnetorotational instability (Balbus and Hawley 1998), the Hall shear instability (Kunz 2008), and MHD disk winds (Blandford and Payne 1982; Pudritz and Norman 1986), is likely to dominate. Except in the innermost disk, thermally ionized at $T > \sim 10^3$ K, the available sources of non-thermal ionization (X-rays, UV photons, and possibly cosmic rays if they are not screened) are weak enough that non-ideal MHD processes are important. There are three non-ideal effects (Wardle and Ng 1999):

- **Ohmic diffusion**, in the regime where frequent collisions couple the charged species (ions, electrons, and possibly charged grains) and the magnetic field to the neutrals, but there is finite conductivity.
- **Ambipolar diffusion**, where the charged species are tied to the magnetic field, but less frequent collisions allow the neutrals to drift relative to the field.
- **The Hall effect**, when electrons are well-coupled to the field but ions are decoupled due to collisions with neutrals.

The relative importance of these effects depends on location within the disk (for a review, see Armitage 2011). Ambipolar diffusion provides strong damping under the low density conditions of the outer disk ($r \gtrsim 30$ AU), where a weak net vertical magnetic field is needed to stimulate any significant transport (Simon et al 2013). At the higher densities on AU-scales the Hall and Ohmic terms are controlling. The action of the Hall term depends upon the sign of the net field with respect to the disk’s rotation, and depending upon the polarity either a quiescent solution resembling the Gammie (1996) dead zone, or an accreting solution driven by laminar MHD torques, is possible (Lesur et al 2014; Bai 2014; Simon et al 2015; Béthune et al 2017; Bai 2017). The same net fields that play a major role in setting the level of ambipolar and Hall-dominated transport also support MHD winds, carrying away
both mass and angular momentum (Bai and Stone 2013; Gressel et al. 2015). Photoevaporative winds allow surface gas, heated by X-ray or UV photons, to escape at radii where $c_s > v_K$. Photoevaporation alone can disperse disks on reasonable time scales (Alexander et al. 2014), though if net magnetic flux remains at late times hybrid winds driven by thermal and magnetic forces are expected (Bai et al. 2016).

Elements of this rather complex picture find observational support, though not yet highly constraining tests. Tobin et al. (2016) observe the spiral structure characteristic of gravitational instability, and fragmentation (into stars), in the L1448 IRS3B system. Flaherty et al. (2015, 2017), analyzing molecular line profiles from the HD 163296 disk, show that turbulence is weak on scales where ambipolar damping would be a strong effect. Finally, a variety of studies find evidence for disk winds (Simon et al. 2016b), though discrimination between thermal and MHD wind solutions is difficult. Open theoretical questions include the role of hydrodynamic instabilities—the most important of which may be the Vertical Shear Instability (Nelson et al. 2013)—which would provide a baseline level of turbulence in magnetically dead regions. The strength and evolution of net disk magnetic fields arising from star formation is another difficult open problem.

**Aerodynamically controlled collisional growth**

The growth of particles from $\mu$m sizes up to scales of at least mm occurs almost everywhere within the disk via adhesive 2-body collisions. (A possible exception is near ice lines, where vapor condensation can be competitive; Ros and Johansen 2013). The rates and outcomes of growth in this regime are set by aerodynamic and material physics considerations that are reasonably well understood.

Key to understanding the aerodynamic evolution of solid particles in disks is the realization that, almost always, the particles are smaller than the mean free path of gas molecules. This means that drag occurs in the Epstein regime, with a drag force that is linear in the relative velocity $\Delta v$ between particle and gas,

$$\mathbf{F}_{\text{drag}} = -\frac{4\pi}{3}\rho s^3 v_{th} \Delta \mathbf{v}. \quad (4)$$

Here $\rho$ is the gas density, $v_{th}$ is the thermal speed of molecules, and we have assumed that particles are spheres of radius $s$, mass $m$, and material density $\rho_m$ (more realistically, they would be irregular aggregates of small monomers). Because of the linearity, we can define a stopping time $t_s \equiv m\Delta v/|\mathbf{F}_{\text{drag}}|$ that expresses the strength of the aerodynamic coupling and which depends only on basic particle and gas properties, $t_s = (\rho_m/\rho)(s/v_{th})$. Often, the physical quantity that matters most is a dimensionless version of the stopping time,

$$\tau_s \equiv t_s \Omega, \quad (5)$$
obtained by multiplying through by an angular frequency (which might be the Keplerian frequency, or the turnover frequency of a fluid eddy). \( \tau_s \) is also known as the Stokes number.

Aerodynamic forces have both local and global effects on particle evolution. Locally, the aerodynamic coupling of particles to turbulence (on small scales where we expect a universal Kolmogorov description to be valid) largely determines collision velocities, which peak for \( \tau_s \sim 1 \) at \( \sim \sqrt{\alpha c_s} \) \( \text{Ormel and Cuzzi} \ 2007, \text{Johansen et al} \ 2014 \). Globally, aerodynamic effects lead to vertical settling and radial drift. Vertical settling is opposed by any intrinsic turbulence in the gas, leading to an equilibrium thickness of the particle disk given approximately by \( h_d/h \sim \sqrt{\alpha/\tau_s} \) \( \text{Dubrulle et al} \ 1995 \). In the absence of turbulence, particles in principle settle until either vertical shear ignites the Kelvin-Helmholtz instability \( \text{Cuzzi et al} \ 1993 \), or until conditions become favorable for the streaming instability \( \text{Youdin and Goodman} \ 2005 \).

Simultaneously, particles drift radially because of the slightly non-Keplerian gas rotation profile (equation 2). For \( \tau_s \ll 1 \) one can think of this drift as being due to the unbalanced radial force felt by tightly coupled particles forced to orbit at a non-Keplerian velocity, whereas for \( \tau_s \gg 1 \) one thinks instead of a boulder orbiting at Keplerian speed and experiencing a headwind or tailwind from the non-Keplerian gas. In the general case, if the gas has orbital velocity \( v_\phi = (1 - \eta)^{1/2} v_K \) and radial velocity \( v_r, \text{gas} \), the particle drift speed is (Takeuchi and Lin 2002),

\[
v_r = \frac{\tau_s^{-1} v_{r, \text{gas}} - \eta v_K}{\tau_s^{-1} + \tau_s^{-1}}.
\]

(6)

For typical disk parameters drift can be rapid, peaking at \( \tau_s = 1 \) where the drift time scale \( r/|v_r| \) is only \( \sim 10^3 \) orbits. The direction of drift is inward if \( dP/dr < 0 \), because in this (usual) case the radial gas pressure gradient partially supports the gas against gravity leading to sub-Keplerian rotation. Inverting this argument, however, one finds that \( dP/dr > 0 \) would lead to outward drift, and hence it is possible to slow or avert inward loss of solids in disks that have local pressure maxima. Absent such effects particles with \( \tau_s \gtrsim 10^{-2} \) (roughly of mm-size and larger) are expected to drift inward and develop a time-dependent surface density profile that differs from that of the gas \( \text{Youdin and Chiang} \ 2004 \). Andrews & Birmstiel’s chapter in this volume discusses these effects in detail.

The material properties of aggregates mean that some combinations of particle masses \((m_1, m_2)\) and collision velocities \( \Delta v \) lead to bouncing or fragmentation rather than growth. If—given some physically plausible distribution of particle masses and collision speeds—no net growth occurs beyond some mass we speak of a barrier to coagulation. The existence of barriers is material-dependent because, at a microscopic level, the forces required to separate or rearrange aggregates differ for, e.g. ices and silicates \( \text{Dominik and Tielens} \ 1997 \). For aggregates of \( \mu \text{m}-\text{sized} \) silicates experiments suggest that a fragmentation barrier sets in for \( \Delta v \gtrsim 1 \) m s\(^{-1} \), while bouncing may set in at lower velocities \( \text{Güttler et al} \ 2010 \). Water ice aggregates may be able to grow in substantially more energetic collisions, up to at least \( \Delta v \sim 10 \) m s\(^{-1} \) \( \text{Gundlach and Blum} \ 2015, \text{Wada et al} \ 2009 \).
The known barriers do not preclude growth up to at least mm-sizes, and models predict the rapid establishment of a coagulation-fragmentation equilibrium for \( \mu m \lesssim s \lesssim mm \) in which most of the mass is in large particles (Birnstiel et al 2011). At \( s \sim \) mm radial drift is already important, especially in the outer regions of the disk, and hence the gas-to-dust ratio will change as a function of radius and time. Beyond the snow line, particles plausibly grow until their growth time scale matches the local radial drift time (“drift-limited growth”; Birnstiel et al 2012). This is due both to the intrinsic propensity of icy particles to grow to larger sizes, and to the fact that radial drift becomes significant at smaller physical sizes in the low density gas further out. In the inner disk the greater fragility of silicates means that growth may instead be frustrated by bouncing or fragmentation at mm-cm scales.

Multi-wavelength observations of resolved disks support part of the above picture (Tazzari et al 2016), suggesting a radius-dependent maximum particle size in the cm (close to the star) to mm range (further out). There is more tension between observations and models of radial drift, with models of drift in smooth disks predicting faster depletion of mm-sized grains than is observed (Pinilla et al 2012). Indeed, although the radial extent of resolved dust disks often appears markedly smaller than that of gas disks (e.g. in TW Hya; Andrews et al 2012), detailed modeling of dust evolution and disk thermochemistry is needed to reliably infer the true radial variation of the dust to gas ratio (Facchini et al 2017). The observed outer radius of a gas disk in \(^{12}\)CO, for example, varies substantially with \( \alpha \) (which is not normally known), and there is a strong coupling between turbulence levels, particle sizes, and gas temperature. From analysis of meteorites, the fact that chondritic meteorites are largely made up of chondrules—0.1-1 mm-sized spheres of rock that were once molten—is pertinent and could be taken to imply a preferred size-scale for Solar Nebula solids in the asteroid belt. This interpretation is, however, model-dependent, and chondrule formation may involve processes (e.g. planetesimal collisions) unrelated to primary particle growth (for a review see, e.g. Connolly and Jones 2016).

The possibility of feedback loops that couple disk chemistry to disk dynamics requires further investigation. It is easy to sketch out a number of possible feedback mechanisms. The ionization state, for example, can depend sensitively upon the abundance of small dust grains (which soak up free charges), and may in turn determine the level of turbulence driven by MHD processes. A disk rich in small dust grains could then promote a low level of turbulence, rapid settling, and efficient coagulation. The depletion of dust might then trigger stronger levels of turbulence, and enhanced fragmentation, potentially leading to a limit cycle. Ideas in this class are physically possible, but it remains to be seen whether the details of disk chemistry and physics work in such a way as to realize them in disks.

**Planetesimal formation**

Bridging from the aerodynamically dominated regime of mm-sized particles to gravitationally dominated km-scale planetesimals poses dual challenges. Growth
must be fast because at intermediate scales where $\tau_s \sim 1$ radial drift is rapid, and must occur via a mechanism that avoids material barriers. No observations directly constrain the population of m-km sized bodies within primordial gas disks, and the observed populations of planetesimal-scale bodies (in the asteroid and Kuiper belts, and in debris disks where larger bodies must be present to produce the observed dust) are often heavily modified by collisions. Assessment of planetesimal formation models thus relies on theoretical considerations and circumstantial evidence.

The leading hypothesis for how planetesimals form is anchored by one of the most surprising and consequential theoretical discoveries of recent years, the streaming instability (Youdin and Goodman 2005). The streaming instability is a linear instability of aerodynamically coupled mixtures of particles and gas that leads to small-scale clustering of the solids (generally on scales $\ll h$). Although the physical interpretation is maddeningly subtle, the instability is robust across a broad range of stopping times and dust-to-gas ratios, with growth time scales that are substantially longer than dynamical but still faster than radial drift.

The streaming instability could play a role in the collisional growth of planetesimals (if there are no insurmountable material barriers), but the most direct channel relies on clustering that is strong enough to locally exceed the Roche density, $\rho \sim M_* / r^3$. Simulations suggest that this strength of clustering is possible but not necessarily trivial to attain, requiring a minimum dust-to-gas ratio that is a function of $\tau_s$ (Carrera et al. 2015; Yang et al. 2017) but always greater than the fiducial disk value of 0.01 (Johansen et al. 2009b). Exceeding the Roche density allows clumps of relatively small (mm-cm) particles to gravitationally collapse, bypassing entirely the problematic scales where radial drift is rapid and material barriers lurk. The resulting initial mass function of planetesimals can be fit by a truncated power-law, $dN/dM \propto M^{-1.6}$ (Johansen et al. 2012; Simon et al. 2016a; Schäfer et al. 2017), whose slope appears to be independent of the size of the particles participating in the instability (Figure 2; Simon et al. 2017). This is a top-heavy mass function with most of the mass in the largest bodies. Their size in the inner disk, for reasonable estimates of disk properties, could be comparable to large asteroids.
Solar System constraints on streaming-initiated planetesimal formation are inconclusive. No observed small body population has the shallow slope that results from a single burst of planetesimal formation via streaming, though Morbidelli et al. (2009) argue that the size distribution of the asteroid belt is consistent with large primordial planetesimals and Nesvorný et al. (2010) suggest that gravitational collapse could explain the high binary fraction among classical Kuiper Belt Objects. A potentially important consequence of large planetesimals arises because they suffer less aerodynamic damping than small ones, leading to less efficient gravitational focusing and slower growth of giant planet cores in the classical (planetesimal dominated) formation scenario (Pollack et al. 1996).

The role of large scale disk structure in growth through to planetesimals is not clear. Several flavors of structure are observed in disks, including axisymmetric rings (ALMA Partnership et al. 2015; Andrews et al. 2016; Isella et al. 2016), spiral arms (Pérez et al. 2016), and horseshoe-shaped dust structures (van der Marel et al. 2013). These structures could be related to zonal flows, self-gravitating spiral arms, and vortices, which may develop spontaneously in gas disks and which trap particles (Johansen et al. 2009a; Béthune et al. 2017; Rice et al. 2006; Barge and Sommeria 1995). If this interpretation is right, large scale structure could be a critical pre-requisite to attaining conditions conducive to planetesimal formation. Alternatively, however, the same observed structures might be caused by planets. The planet hypothesis is most compelling in the case of horseshoe-shaped structures (e.g. Zhu and Stone 2014), but both possibilities are likely realized in nature.

**Terrestrial and giant planet formation**

Once planetesimals have formed, the outcome of collisions depends upon the energy or momentum of impacts relative to their strength—set by material properties for \(s \lesssim \text{km}\) and by gravity thereafter. Scaling laws derived from simulations (Leinhardt and Stewart 2012) can be used as input for N-body simulations. Collisions lead to accretion if the planetesimals are dynamically cold, while high velocity impacts in dynamically excited populations (the current asteroid belt, debris disks, etc) lead to disruption and a collisional cascade that grinds bodies down to small particles (in the simplest case, with a size distribution \(n(s) \propto s^{-7/2}\); Dohnanyi 1969).

The classical model for forming protoplanets and giant planet cores assumes that growth occurs within an initially cold disk of planetesimals. Consider a body of mass \(M\), radius \(R\), and escape speed \(v_{\text{esc}}\), embedded within a disk of planetesimals that has surface density \(\Sigma_p\) and velocity dispersion \(\sigma\). The eccentricity and inclination of the planetesimals are given by \(e \sim i \sim \sigma/v_K\), so the disk thickness is \(\sim \sigma/\Omega\). In the dispersion dominated regime (i.e. ignoring 3-body tidal effects) elementary collision rate arguments yield a growth rate (Lissauer 1993; Armitage 2010),

\[
\frac{dM}{dt} = \frac{\sqrt{3}}{2} \Sigma_p \Omega \pi R^2 \left(1 + \frac{v_{\text{esc}}^2}{\sigma^2}\right).
\]
The term in parenthesis describes the effect of gravitational focusing. It can vary by orders of magnitude, and hence growth in the classical picture is essentially controlled by the evolution of $\sigma$. Two regimes can be identified,

- A small growing body does not affect the velocity dispersion (typically, $\sigma$ is set by a balance between excitation by planetesimal-planetesimal scattering encounters and aerodynamic damping). Let us assume that $\sigma = \text{const} \ll v_{\text{esc}}$. Then for two bodies in the same region of the disk, with masses $M_1 > M_2$, equation (7) gives $d(M_1/M_2)/dt \propto (M_1/M_2)(M_1^{1/3} - M_2^{1/3}) > 0$. Any initially small mass differences are amplified. This is the runaway growth phase (Greenberg et al 1978).

- Eventually the fastest growing bodies start to excite the velocity dispersion of planetesimals in their immediate vicinity, slowing their own growth and allowing their radial neighbors to catch up. A number of planetary embryos then grow at comparable rates in a phase of oligarchic growth (Kokubo and Ida 1998).

If there is no migration the outcome of these phases is a system of protoplanets on near-circular orbits. The dynamical stability of the system against planet-planet perturbations that drive orbit crossing and collisions is determined, approximately, by the planetary separation measured in units of the Hill radius. The Hill radius is defined for a planet of mass $M$, orbiting at distance $a$, as,

$$r_H = \left(\frac{M}{3M_\ast}\right)^{1/3} a. \tag{8}$$

Physically, it specifies the distance out to which the planet’s gravity dominates over the tidal gravitational field of the star. In the context of Solar System terrestrial planet formation we expect the initial growth phases to lead to a system of protoplanets separated by 5-10 Hill radii, with a similar amount of mass surviving in planetesimals. Simulations based on these initial conditions go on to form plausible analogs of the Solar System’s terrestrial planets on $\sim 100$ Myr time scales (Chambers and Wetherill 1998; Raymond et al 2009).

Extension of this model to giant planet formation is conceptually straightforward (Pollack et al 1996). Beyond the snow line growth is faster and planetary cores can readily reach masses in excess of $M_\oplus$. If we continue to ignore migration, one possible limit to growth is the finite supply of nearby planetesimals. A growing core can perturb planetesimals onto orbit-crossing trajectories within an annulus $\Delta a$ whose width scales with the Hill radius, $\Delta a = C r_H$, with $C$ a constant. The mass in planetesimals within this feeding zone is $2\pi a \times 2\Delta a \times \Sigma_p$, where $\Sigma_p$ is the surface density in planetesimals. This reservoir of planetesimals increases with the planet mass, but only weakly due to the $M_1^{1/3}$ dependence of $\Delta a$ on mass. Growth ceases when the planet reaches the isolation mass, when the mass of the protoplanet equals the mass of planetesimals in the feeding zone. A simple calculation shows that $M_{\text{iso}} \propto \Sigma_p^{3/2} a^3$, so this consideration favors growth to larger masses at greater orbital radii. At radii beyond about 10 AU, however, scattering dominates over accretion, and it becomes increasingly hard to build large cores through planetesimal accretion.
Once the mass of a planetary core reaches a few $M_⊕$, it can bind a hydrostatic gas envelope, forming a planet that resembles an ice giant. Above some critical core mass—probably in the $M_{\text{core}} \approx 5 - 20 M_⊕$ range—hydrostatic envelope solutions cease to exist (Mizuno 1980). Disk gas can thereafter be accreted rapidly, forming a gas giant planet. How the later stages of core accretion work depends in detail on how the envelope cools (by convection and radiative diffusion; Rafikov 2006; Piso et al. 2015), and is uncertain because the appropriate opacity is poorly known. A floor value to the opacity is provided by the value calculated for dust-free gas, and adopting this value minimizes the time scale for forming a gas giant. A much larger opacity is possible if the envelope contains grains with the size distribution inferred for the interstellar medium, though coagulation (either in the disk, or in the envelope itself) can reduce the opacity of even dusty gas by a large factor (Podolak 2003).

Classical (i.e. planetesimal dominated) models show that giant planet formation is possible on Myr time scales at 3-10 AU (Movshovitz et al. 2010). Classical models involve two key assumptions whose validity has been challenged by recent work. The first is that that planetesimal formation consumes most or all of the disk’s solid inventory. This is false; observations show that significant masses of small solids, observable at mm wavelengths, are present whenever there is evidence for a gas disk. Due to radial drift these approach growing planets with some velocity $\Delta v$, and can be captured. The resulting growth rate can be large, with an optimal limit in which a substantial fraction of particles entering the Bondi radius $r_B \equiv GM/\Delta v^2$ are accreted (Ormel and Klahr 2010; Lambrechts and Johansen 2012). Pebble accretion can be substantially faster than planetesimal-driven growth, depending upon the mass and size of surviving pebbles and on the dynamics of planets and planetesimals (Levison et al. 2015). The second assumption is that the core binds a static envelope that extends out to either the gaseous Bondi radius $r_{B,\text{gas}} \equiv GM/c_s^2$ or to the Hill sphere. Simulations, however, show that three dimensional flows continually cycle gas into and out of the region that is assumed to be bound in one dimensional models (D’Angelo and Bodenheimer 2013; Ormel et al. 2015; Lambrechts and Lega 2017). These flows affect the thermodynamics of the outer envelope, and will alter the growth tracks of ice giants and mini-Neptunes embedded within gaseous disks.

Migration is the wildcard in most planet formation models. Gas disk migration occurs because of gravitational torques between planets and the disk (Goldreich and Tremaine 1979), exerted at corotation and Lindblad resonances. In the Type I regime, relevant to planet masses $M \lesssim 10 M_⊕$, the disk surface density is only weakly perturbed and the torque scales as $T \propto M^2$ (hence, the migration time scale $\propto M^{-1}$). The Lindblad torque is proportional to surface density, weakly dependent on gradients of density or temperature, and in isolation would invariably lead to inward planet migration (Ward 1997). The corotation torque, by contrast, is a complex function of the disk’s structure and thermodynamics (Paardekooper et al. 2011). It can more than offset the Lindblad torque, leading to outward migration. Crucially, the two components of the torque have different dependencies on the disk structure, and while the sum may coincidentally cancel at one or a few locations within the disk (Hasegawa and Pudritz 2011; Bitsch et al. 2014) it is not generally zero. Type
I migration will therefore be important whenever planets grow to masses of at least $0.1 - 1 \, M_\oplus$ while the gas is still present. The Solar System’s terrestrial planets grow slowly enough to avoid migration (this would not be true for similar mass planets around lower mass stars), but giant planet cores and Kepler systems of super-Earths and mini-Neptunes will inevitably be affected. This line of reasoning favors models in which planet cores form at migration null points (Hasegawa and Pudritz 2011; Hellary and Nelson 2012; Cossou et al 2014), and those in which a substantial fraction of Kepler multi-planet systems were once in resonant configurations that later break (Goldreich and Schlichting 2014).

Planets with masses of $M \gtrsim 3\,(h/r)^3 M_\oplus$ and above can start to open a gap in the disk. Once a gap forms migration occurs in the Type II regime, at a rate that depends upon the disk’s evolution and on how rapidly the planet accretes (Durmann and Kley 2017). The existence of resonant pairs of massive extrasolar planets provides strong evidence for the importance of this flavor of migration (Lee and Peale 2002). The argument is simple: a pair of massive planets is unlikely to either form or be scattered into a resonant configuration (Raymond et al 2008), because such configurations are a small subset of stable orbital elements. Convergent Type II migration, however, can readily form resonant systems because initially well-separated planets have a non-zero probability of becoming locked when they encounter mean motion resonances (Goldreich 1965).

**Long term evolution of planetary systems**

The physical processes outlined above are not fully understood, but even if they were there would still be considerable freedom in chaining them together to make a complete planet formation model. (Therein lies the promise and peril of population synthesis models.) It is clear, however, that some generic expectations—for example that massive planets should have near-circular orbits—are grossly in error. This mismatch points to the importance of dynamical processes that reshape planetary systems between the disk dispersal epoch and the time at which they are observed.

Two planet systems are unconditionally stable against close encounters if the orbital separation exceeds a critical number of Hill radii (Gladman 1993), but richer planetary systems have a “soft” stability boundary and are typically unstable on a time scale that is a steep function of the separations (Chambers et al 1996; Obertas et al 2017). Unstable systems of massive planets stabilize by physical collisions (at small radii) and ejections (further out), leaving survivors with eccentric orbits. If we assume that a large fraction of giant planet systems form in ultimately unstable configurations, simple scattering experiments show good agreement with the observed eccentricity distribution of massive extrasolar planets (Chatterjee et al 2008). The low eccentricities of the Solar System’s giant planets would then imply either that scattering never occurred, or that it was followed in the Solar System by a phase in which dynamically excited orbits were damped back down.
The existence of debris disks (Wyatt 2008) indicates that disks of planetesimal-scale bodies are common in outer planetary systems (see Wyatt’s chapter in this volume). Scattering of this material by gas or ice giants leads to smooth orbital migration that tends to circularize orbits. In the Solar System, scattering of a massive primordial Kuiper Belt would have led to the outward migration of Neptune (Fernandez and Ip 1984), and the capture (and excitation of eccentricity) of Pluto and other KBOs into resonance with Neptune (Malhotra 1995). The Nice Model, discussed further in Morbidelli’s chapter, embeds this evolution into a broader framework for outer Solar System evolution, in which the giant planets formed in a compact resonant configuration that was subsequently disrupted, leading to planet-planet scattering and planetesimal-driven migration (Levison et al 2011). Similar dynamics in extrasolar planetary systems would reduce the eccentricities of giant planets at larger orbital radii to Solar System-like values (Raymond et al 2010).

The typical star is part of a binary system. Binary companions—if misaligned to the planetary orbital plane—can excite large-amplitude oscillations in $e$ and $i$ via the Kozai-Lidov effect (Naoz 2016). When coupled to tidal evolution, Kozai-Lidov migration provides an obvious (though not unique) channel for the formation of misaligned hot Jupiters (Wu and Murray 2003).

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