Appropriately Incorporating Statistical Significance in PMI

Abstract

Two recent measures incorporate the notion of statistical significance in basic PMI formulation. In some tasks, we find that the new measures perform worse than the PMI. Our analysis shows that while the basic ideas in incorporating statistical significance in PMI are reasonable, they have been applied slightly inappropriately. By fixing this, we get new measures that improve performance over not just PMI but on other popular co-occurrence measures as well. In fact, the revised measures perform reasonably well compared with more resource intensive non co-occurrence based methods also.

1 Introduction

The notion of word association is used in many language processing and information retrieval applications and it is important to have low-cost, high-quality association measures. Lexical co-occurrence based word association measures are popular because they are computationally efficient and they can be applied to any language easily. One of the most popular co-occurrence measure is Pointwise Mutual Information (PMI) (Church and Hanks, 1989).

One of the limitations of PMI is that it only works with relative probabilities and ignores the absolute amount of evidence. To overcome this, recently two new measures have been proposed that incorporate the notion of statistical significance in basic PMI formulation. In (Washtell and Markert, 2009), statistical significance is introduced in PMI\textsubscript{sig} by multiplying PMI value with the square root of the evidence. In contrast, in (Damani, 2013), cPMId is introduced by bounding the probability of observing a given deviation between a given word pair’s co-occurrence count and its expected value under a null model where with each word a global unigram generation probability is associated. In Table 1, we give the definitions of PMI, PMI\textsubscript{sig}, and cPMId.

While these new measures perform better than PMI on some of the tasks, on many other tasks, we find that the new measures perform worse than the PMI. In Table 3, we show how these measures perform compared to PMI on four different tasks. We find that PMI\textsubscript{sig} degrades performance in three out of these four tasks while cPMId degrades performance in two out of these four tasks. The experimental details and discussion are given in Section 4.2.

Our analysis shows that while the basic ideas in incorporating statistical significance are reasonable, they have been applied slightly inappropriately. By fixing this, we get new measures that improve performance over not just PMI, but also on other popular co-occurrence measures on most of these tasks. In fact, the revised measures perform reasonably well compared with more resource intensive non co-occurrence based methods also.

2 Adapting PMI for Statistical Significance

In (Washtell and Markert, 2009), it is assumed that the statistical significance of a word pair association is proportional to the square root of the evidence. The question of what constitutes the evidence is answered by taking the lesser of the frequencies of the two words in the word pair, since at most that many pairings are possible. Hence the PMI value is multi-
Termialogy:

- \( W \) : Total number of words in the corpus
- \( f(x), f(y) \): Unigram frequencies of \( x, y \) respectively in the corpus
- \( f(x,y) \): Span-constrained \((x, y)\) word pair frequency in the corpus
- \( d(x), d(y) \): Total number of documents in the corpus containing at least one occurrence of \( x \) and \( y \) respectively
- \( d(x,y) \): Total number of documents in the corpus having at least one span-constrained occurrence of the word pair \((x, y)\)
- \( \delta \): A parameter varying between 0 and 1

| Method                  | Formula                                      | Revised Formula                      |
|-------------------------|----------------------------------------------|---------------------------------------|
| PMI (Church and Hanks, 1989) | \( \log \frac{f(x,y)}{f(x)f(y)/W} \)        | \( \log \frac{f(x,y)}{f(x)f(y)} \) |
| PMI_{sig} (Washtell and Markert, 2009) | \( \log \frac{f(x,y)}{f(x)f(y)/W} + \sqrt{\min(f(x), f(y))} \) | \( \log \frac{f(x,y)}{f(x)f(y)/W} + \sqrt{\delta} \) |
| cPMId (Damani, 2013)    | \( \log \frac{d(x,y)}{W + \sqrt{d(x)f(x)}} \) | \( \log \frac{d(x,y)}{W + \sqrt{\max(d(x),d(y))}} \) |
| sPMId                  | \( \log \frac{d(x,y)}{\max(d(x),d(y))\min(d(x),d(y)) + \sqrt{\delta}} \) | \( \log \frac{d(x,y)}{\max(d(x),d(y))\min(d(x),d(y)) + \sqrt{\delta}} \) |

Table 1: Definitions of PMI and its statistically significant adaptations. The sub-parts in bold represent the changes between the original formulas and the revised formulas. The product \( \max(d(x), d(y)) \) * \( \min(d(x), d(y)) \) in sPMId formula can be simplified to \( f(x) * f(y) \), however, we left it this way to emphasize the transformation from cPMId.

By setting \( t = \sqrt{\ln \delta/(-2 * f(x))} \), we get \( \delta \) as an upper bound on probability of observing more than \( f(x) * f(y)/W + f(x) * t \) bigram occurrences in the corpus, just by chance. Based on this Corpus Level Significant PMI (cPMI) is defined as:

\[
cPMI(x, y) = \log \frac{f(x,y)}{f(x)f(y)/W + f(x) * t}
= \log \frac{f(x,y)}{f(x)f(y)/W + \sqrt{f(x)} * \sqrt{\ln \delta/(-2)}}
\]

In (Damani, 2013), several variants of cPMI are introduced that incorporate different notions of statistical significance. Of these Corpus Level Significant PMI based on Document count (cPMId - defined in Table 1) is found to be the best performing, and hence we consider this variant only in this work.

2.1 Choice of Random Trial

While considering statistical significance, one has to decide what constitutes a random trial. When looking for a word-pair \((x, y)\)’s occurrences, \( y \) can potentially occur near each occurrence of \( x \), or \( x \) can potentially occur near each occurrence of \( y \). Which of these two set of occurrences should be considered the sites of random trial. We believe that the occurrences of the more frequent of \( x \) and \( y \) should be considered, since near each of these occurrences the other word could have occurred. Hence \( f(x) \) and \( f(y) \) in cPMI definition should be replaced with \( \max(f(x), f(y)) \) and \( \min(f(x), f(y)) \) respectively. Similarly, \( d(x) \) and \( d(y) \) in cPMId formula should be replaced with \( \max(d(x), d(y)) \) and \( \min(d(x), d(y)) \) respectively to give a new measure Significant PMI based on Document count (sPMId).

Using the same logic, \( \sqrt{\min(f(x), f(y))} \) in PMI_{sig} formula should be replaced with \( \sqrt{\max(f(x), f(y))} \) to give the formula for a new measure PMI-significant (PMIs). The definitions of sPMId and PMIs are also given in Table 1.

3 Related Work

There are three main types of word association measures: Knowledge based, Distributional Similarity...
### Table 2: Definition of other co-occurrence measures being compared in this work. The terminology used here is same as that in Table 1, except that \( E \) in front of a variable name means the expected value of that variable.

| Method               | Formula                                                                 |
|----------------------|-------------------------------------------------------------------------|
| ChiSquare (\( \chi^2 \)) | \( \sum_{i,j} \frac{(f(i,j) - Ef(i,j))^2}{Ef(i,j)} \)                  |
| Dice (Dice, 1945)    | \( f(x,y) \)                                                          |
| GoogleDistance (L.Cilibrasi and Vitany, 2007) | \( \max\{(\log d(x), \log d(y)) - \log \min(\log d(x), \log d(y))\} \)  |
| Jaccard (Jaccard, 1912) | \( f(x,y) \)                                                          |
| LLR (Dunning, 1993)  | \( \sum x' \in \{x, \neg x\} \sum y' \in \{y, \neg y\} f(x', y') \log \frac{f(x', y')}{f(x') f(y')} \) |
| nPMI (Bouma, 2009)   | \( \log \frac{f(x,y) f(y')} {f(x)' f(y)} \)                            |
| Ochiai (Janson and Vegelius, 1981) | \( \sqrt{ \frac{f(x,y)}{f(x) f(y)}} \)                                  |
| PMI\(^2\) (Daille, 1994) | \( \log \frac{f(x,y)}{f(x) f(y)} \)                                     |
| Simpson (Simpson, 1943) | \( \frac{f(x,y)}{f(x) \sqrt{f(y)}} \)                                    |
| T-test               | \( \frac{f(x,y) - Ef(x,y)}{\sqrt{Ef(x,y) \left(1 - \frac{f(x,y)^2}{W} \right)}} \) |

### Table 3: 5-fold cross-validation results for different co-occurrence measures. The results for the best, and second best co-occurrence measures for each data-set is shown in bold and underline respectively. Except GoogleDistance and LLR, all results for all co-occurrence measures are statistically significant at \( p = .05 \). For each task, the best known result for different non co-occurrence based methods is also shown.

| Task                     | Semantic Relatedness | Sentence Similarity | Synonym Selection |
|--------------------------|----------------------|---------------------|-------------------|
| Dataset                  | WordSim             | Li                  | ESL               | TOEFL              |
| Metric                   |                      |                     |                   |                    |
| PMI                      | 0.66                 | 0.69                | 0.62              | 0.59               |
| PMI\(_{sig}\)           | 0.67                 | 0.85                | 0.58              | 0.56               |
| cPMId                    | 0.72                 | 0.67                | 0.56              | 0.59               |
| PMIs                     | 0.66                 | 0.85                | 0.66              | 0.61               |
| sPMId                    | 0.72                 | 0.75                | 0.70              | 0.61               |
| ChiSquare (\( \chi^2 \)) | 0.62                 | 0.80                | 0.62              | 0.58               |
| Dice                     | 0.58                 | 0.76                | 0.56              | 0.57               |
| GoogleDistance           | 0.53                 | 0.75                | 0.59              | 0.19               |
| Jaccard                  | 0.58                 | 0.76                | 0.56              | 0.57               |
| LLR                      | 0.50                 | 0.18                | 0.18              | 0.27               |
| nPMI                     | 0.72                 | 0.35                | 0.54              | 0.54               |
| Ochiai/ PMI\(^2\)       | 0.62                 | 0.77                | 0.62              | 0.60               |
| SCI                      | 0.65                 | 0.85                | 0.62              | 0.60               |
| Simpson                  | 0.59                 | 0.78                | 0.58              | 0.57               |
| TTest                    | 0.44                 | 0.63                | 0.44              | 0.52               |
| Semantic Net (Li et al., 2006) |                      |                     |                   | 0.82               |
| ESA (Gabrilovich and Markovitch, 2007) | (reimplemented in (Yeh et al., 2009)) | | | |
| Distributional Similarity (on web corpus) (Agirre et al., 2009) | | 0.74 | 0.71 | 0.71 | 0.71 |
| Context Window based Distributional Similarity (Agirre et al., 2009) | | 0.65 | 0.65 | 0.65 | 0.65 |
| LateNet Semantic Analysis (on web corpus) (Finkelstein et al., 2002) | | 0.60 | 0.60 | 0.60 | 0.60 |
| WordNet::Similarity (Recchia and Jones, 2009) | | 0.56 | 0.56 | 0.56 | 0.56 |
| PMI-IR3 (using context) (Turney, 2001) | | 0.70 | 0.70 | 0.70 | 0.70 |

Table 3: 5-fold cross-validation results for different co-occurrence measures. The results for the best, and second best co-occurrence measures for each data-set is shown in bold and underline respectively. Except GoogleDistance and LLR, all results for all co-occurrence measures are statistically significant at \( p = .05 \). For each task, the best known result for different non co-occurrence based methods is also shown.
Based on Firth’s *You shall know a word by the company it keeps* (Firth, 1957), distributional similarity based measures characterize a word by the distribution of other words around it and compare two words for distributional similarity (Agirre et al., 2009; Wandmacher et al., 2008; Bollegala et al., 2007; Chen et al., 2006). They are also used for modeling the meaning of a phrase or a sentence (Grefenstette and Sadrzadeh, 2011; Wartena, 2013; Mitchell, 2011; G. Dinu and Baroni, 2013; Kartsaklis et al., 2013).

Knowledge-based measures use knowledge-sources like thesauri, semantic networks, or taxonomies (Milne and Witten, 2008; Hughes and Ramage, 2007; Gabrilovich and Markovitch, 2007; Yeh et al., 2009; Strube and Ponzetto, 2006; Finkelstein et al., 2002; Liberman and Markovitch, 2009).

Co-occurrence based measures (Pecina and Schlesinger, 2006) simply rely on unigram and bigram frequencies of the words in a pair. In this work, our focus is on the co-occurrence based measures, since they are resource-light and can easily be used for resource-scarce languages.

### 3.1 Co-occurrence Measures being Compared

Co-occurrence based measures of association between two entities are used in several domains like ecology, psychology, medicine, language processing, etc. To compare the performance of our newly introduced measures with other co-occurrence measures, we have selected a number of popular co-occurrence measures like ChiSquare ($\chi^2$), Dice (Dice, 1945), GoogleDistance (L.Cilibrasi and Vitany, 2007), Jaccard (Jaccard, 1912), LLR (Dunning, 1993), Simpson (Simpson, 1943), and T-test from these domains.

In addition to these popular measures, we also experiment with other known variations of PMI like nPMI (Bouma, 2009), PMI$^2$ (Daille, 1994), Ochiai (Janson and Vegelius, 1981), and SCI (Washtell and Markert, 2009). Since PMI$^2$ is a monotonic transformation of Ochiai, we present their results together. In Table 2, we present the definitions of these measures. While the motivation given for SCI in (Washtell and Markert, 2009) is slightly different, in light of the discussion in Section 2.1, we can assume that SCI is PMI adapted for statistical significance (multiplied by $\sqrt{|f(y)|}$), where the site of random trial is taken to be the occurrences of the second word $y$, instead of the less frequent word, as in the case of PMI$\text{sig}$.

When counting co-occurrences, we only consider the non-overlapping span-constrained occurrences. The span of a word-pair’s occurrence is the direction-independent distance between the occurrences of the members of the pair. We consider only those co-occurrences where span is less than a given threshold. Therefore, span threshold is a parameter for all the co-occurrence measures being considered.

### 4 Performance Evaluation

Having introduced the revised measures PMIs and sPMId, we need to evaluate the performance of these measures compared to PMI and the original measures introducing significance. In addition, we also wish to compare the performance of these measures with other co-occurrence measures. To compare the performance of these measures with more resource heavy non co-occurrence based measures, we have chosen those tasks and datasets on which published results exist for distributional similarity and knowledge based word association measures.

#### 4.1 Task Details

We evaluate these measures on three tasks: Sentence Similarity (65 sentence-pairs from (Li et al., 2006)), Synonym Selection (50 questions ESL (Turney, 2001) and 80 questions TOEFL (Landauer and Dutnais, 1997) datasets), and, Semantic Relatedness (353 words Wordsim (Finkelstein et al., 2002) dataset).

For each of these tasks, gold standard human judgment results exist. For sentence similarity, following (Li et al., 2006), we evaluate a measure by the Pearson’s correlation between the ranking produced by the measure and the human ranking. For synonym selection, we compute the percentage of correct answers, since there is a unique answer for each challenge word in the datasets. Semantic relatedness has been evaluated by Spearman’s rank correlation with human judgment instead of Pearson’s correlation in literature and we follow the same practice to make results comparable.
For sentence similarity detection, the algorithm used by us (Li et al., 2006) assumes that the association scores are between 0 and 1. Hence we normalize the value produced by each measure using max-min normalization:

$$v' = \frac{v - \text{min}}{\text{max} - \text{min}}$$

where max and min are computed over all association scores for the entire task for a given measure.

### 4.2 Experimental Results

We use a 1.24 Gigawords Wikipedia corpus for getting co-occurrence statistics. Since co-occurrence methods have span-threshold as a parameter, we follow the standard methodology of five-fold cross validation. Note that, in addition to span-threshold, cPMId and sPMId have an additional parameter $\delta$.

In Table 3, we present the performance of all the co-occurrence measures considered on all the tasks. Note that, except GoogleDistance and LLR, all results for all co-occurrence measures are statistically significant at $p = .05$. For completeness of comparison, we also include the best known results from literature for different non co-occurrence based word association measures on these tasks.

### 4.3 Performance Analysis and Conclusions

We find that on average, $\text{PMI}_{\text{sig}}$ and cPMId, the recently introduced measures that incorporate significance in PMI, do not perform better than PMI on the given datasets. Both of them perform worse than PMI on three out of four datasets. By appropriately incorporating significance, we get new measures PMIs and sPMId that perform better than PMI(also $\text{PMI}_{\text{sig}}$ and cPMId respectively) on most datasets. PMIs improves performance over PMI on three out of four datasets, while sPMId improves performance on all four datasets.

The performance improvement of PMIs over $\text{PMI}_{\text{sig}}$ and of sPMId over cPMId, is not random. For example, on the ESL dataset, while the percentage of correct answers increases from 58 to 66 from $\text{PMI}_{\text{sig}}$ to PMIs, it is not the case that on moving from $\text{PMI}_{\text{sig}}$ to PMIs, several correct answers become incorrect and an even larger number of incorrect answers become correct. As shown in Table 4, only one correct answers become incorrect while seven incorrect answers get corrected. The same trend holds for most parameters values, and for moving from cPMId to sPMId. This substantiates the claim that the improvement is not random, but due to the appropriate incorporation of significance, as discussed in Section 2.1.

PMIs and sPMId perform better than not just PMI, but they perform better than all popular co-occurrence measures on most of these tasks. When compared with any other co-occurrence measure, on three out of four datasets each, both PMIs and sPMId perform better than that measure. In fact, PMIs and sPMId perform reasonably well compared with more resource intensive non co-occurrence based methods as well. Note that different non co-occurrence based measures perform well on different tasks. We are comparing the performance of a single measure (say sPMId or PMIs) against the best measure for each task.

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References

Eneko Agirre, Enrique Alfonseca, Keith Hall, Jana Kravalova, Marius Pasca, and Aitor Soroa. 2009. A study on similarity and relatedness using distributional and wordnet-based approaches. In NAACL-HLT 2009, Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies.

Gerlof Bouma. 2009. Normalized (pointwise) mutual information in collocation extraction, from form to meaning: Processing texts automatically. In GSCL 2009, Proceedings of the Biennial International Conference of the German Society for Computational Linguistics and Language Technology.

Hsin-Hsi Chen, Ming-Shun Lin, and Yu-Chuan Wei. 2007. Measuring semantic similarity between words using web search engines. In WWW 2007, The World Wide Web Conference, pages 757–766.

J. R. Firth. 1957. A synopsis of linguistics theory. Studies in Linguistic Analysis, pages 1930–1955.

J. R. Firth. 1957. A synopsis of linguistics theory. Studies in Linguistic Analysis, pages 1930–1955.

O. S. O’Shea, and Keeley Crockett. 2006. Sentence similarity distance. Psychological review, 104(2):211–240.

P. Jaccard. 1912. The distribution of the flora of the alpine zone. New Phytologist, 11:37–50.

Svante Janson and Jan Vegelius. 1981. Measures of ecological association. Oecologia, 49:371–376.

Dimitrios Kartsaklis, Mehrnoosh Sadrzadeh, and Stephen Pulman. 2013. Separating disambiguation from composition in distributional semantics. In CoNLL 2013, Conference on Computational Natural Language Learning.

Thomas K Landauer and Susan T. Dutnais. 1997. A solution to platos problem: The latent semantic analysis theory of acquisition, induction, and representation of knowledge. Psychological review, 104(2):211–240.

Rudi L. Cilibrasi and Paul M.B. Vitany. 2007. The google similarity distance. Psychological review, 19(3).

Yuhua Li, David McLean, Zuhair A. Bandar, James D. O’Shea, and Keeley Crockett. 2006. Sentence similarity based on semantic nets and corpus statistics. IEEE Transactions on Knowledge and Data Engineering, 18(8):1138–1150, August.

Sonya Liberman and Shaul Markovitch. 2009. Compact hierarchical explicit semantic representation. In WikiAI 2009, Proceedings of the IJCAI Workshop on User-Contributed Knowledge and Artificial Intelligence: An Evolving Synergy, Pasadena, CA, July.

David Milne and Ian H. Witten. 2008. An effective, low-cost measure of semantic relatedness obtained from wikipedia links. In ACL 2008, Proceedings of the Annual Meeting of the Association for Computational Linguistics.

Edward Grefenstette and Mehrnoosh Sadrzadeh. 2011. Experimental support for a categorical compositional distributional model of meaning. In EMNLP 2011, Conference on Empirical Methods on Natural Language Processing, pages 1394–1404.

T Hughes and D Ramage. 2007. Lexical semantic relatedness with random graph walks. In EMNLP 2007, Conference on Empirical Methods on Natural Language Processing.

Evgeniy Gabrilovich and Shaul Markovitch. 2007. Computing semantic relatedness using wikipedia-based explicit semantic analysis. In IJCAI 2007, International Joint Conference on Artificial Intelligence.

Edward Grefenstette and Mehrnoosh Sadrzadeh. 2011. Experimental support for a categorical compositional distributional model of meaning. In EMNLP 2011, Conference on Empirical Methods on Natural Language Processing, pages 1394–1404.

Yuhua Li, David McLean, Zuhair A. Bandar, James D. O’Shea, and Keeley Crockett. 2006. Sentence similarity based on semantic nets and corpus statistics. IEEE Transactions on Knowledge and Data Engineering, 18(8):1138–1150, August.

Sonya Liberman and Shaul Markovitch. 2009. Compact hierarchical explicit semantic representation. In WikiAI 2009, Proceedings of the IJCAI Workshop on User-Contributed Knowledge and Artificial Intelligence: An Evolving Synergy, Pasadena, CA, July.

David Milne and Ian H. Witten. 2008. An effective, low-cost measure of semantic relatedness obtained from wikipedia links. In ACL 2008, Proceedings of the Annual Meeting of the Association for Computational Linguistics.

Jeffrey Mitchell. 2011. Composition in Distributional Models of Semantics. Ph.D. thesis, The University of Edinburgh.

Pavel Pecina and Pavel Schlesinger. 2006. Combining association measures for collocation extraction. In ACL 2006, Proceedings of the Annual Meeting of the Association for Computational Linguistics.
Gabriel Recchia and Michael N. Jones. 2009. More data trumps smarter algorithms: Comparing pointwise mutual information with latent semantic analysis. Behavior Research Methods, 3(41):647–656.

George G. Simpson. 1943. Mammals and the nature of continents. American Journal of Science, pages 1–31.

Michael Strube and Simone Paolo Ponzetto. 2006. Wikirelate! computing semantic relatedness using wikipedia. In AAAI 2006, Conference on Artificial Intelligence, pages 1419–1424.

P. Turney. 2001. Mining the web for synonyms: PMI-IR versus LSA on TOEFL. In ECML 2001, European Conference on Machine Learning.

T. Wandmacher, E. Ovchinnikova, and T. Alexandrov. 2008. Does latent semantic analysis reflect human associations? In ESSLLI 2008, European Summer School in Logic, Language and Information.

Christian Wartena. 2013. Hsh: Estimating semantic similarity of words and short phrases with frequency normalized distance measures. In SemEval 2013, International Workshop on Semantic Evaluation.

Justin Washtell and Katja Markert. 2009. A comparison of windowless and window-based computational association measures as predictors of syntagmatic human associations. In EMNLP 2009, Conference on Empirical Methods on Natural Language Processing, pages 628–637.

Eric Yeh, Daniel Ramage, Chris Manning, Eneko Agirre, and Aitor Soroa. 2009. Wikiwalk: Random walks on wikipedia for semantic relatedness. In TextGraphs 2009, Proceedings of the ACL workshop on Graph-based Methods for Natural Language Processing.