Quantum-Hall to insulator transition

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Abstract

The crossover from the quantum Hall regime to the Hall-insulator is investigated by varying the strength of the diagonal disorder in a 2d tight-binding model. The Hall and longitudinal conductivities and the behavior of the critical states are calculated numerically. We find that with increasing disorder the current carrying states close to the band center disappear first. Simultaneously, the quantized Hall conductivity drops monotonically to zero also from higher quantized values.

1 Introduction

The disappearance of the integer quantum Hall effect (QHE) at low magnetic fields or, equivalently, for strong disorder potentials, has been the subject of continuing interest since its discovery. Based on the continuum model where the energy spectrum is bounded only from below it was argued [1, 2] that the current carrying states continuously float up in energy when the magnetic field is decreased. The system becomes an insulator when the last critical state crosses the Fermi level. Therefore, a direct transition from the QHE-state to an insulating state is possible only from the lowest Landau band [3].

On the other hand, for a disordered tight-binding model (TBM) with lattice constant \( a \) this scenario seems not to be adequate. Here a symmetric energy band is formed which splits into \( q \) sub-bands when a perpendicular magnetic field \( B \) is applied which can be expressed by the number of flux quanta per lattice cell, \( \alpha = eBa^2/h = 1/q \). As in the Landau bands of the continuum model the localization length diverges near the center of the disorder broadened sub-bands [4]. These critical states exhibit a topological property, a nonzero Chern integer that leads to a quantized Hall conductivity [7].

Recently, it was found for the TBM [8] that with increasing disorder the critical states disappear one after another without changing their position in energy, starting with the states close to the band center. The reason for this behavior is that states with negative Chern number originally situated only near the band center move down in energy and annihilate those with Chern number +1 located in each sub-band. As a consequence of this scenario a direct QHE to insulator transition from higher Landau bands becomes possible [9, 10], which has already been observed in experiments [11, 12, 13, 14].

However, from a similar numerical investigation it was concluded [15] that for weak fields the crossover to the insulator is due to a floating up of critical levels into the band center, because a magnetic field independent critical disorder \( W_c \approx 6V \) was found where all current carrying states disappear.

In contrast to this view Sheng and Weng [16] inferred from their calculation of the thermodynamic localization length that the critical disorder \( W_c/V \sim \sqrt{\alpha} \) tends to zero as the magnetic field is lowered. This implies that for fixed magnetic field the longitudinal conductivity \( \sigma_{xx} \) increases in the tails of the sub-bands with increasing disorder strength. This increase of \( \sigma_{xx} \) should also go along with a decrease of the width of the quantum Hall plateaus.

In this paper, we investigate numerically this scenario by calculating the Hall and longitudinal conductivity using a recursive Greens function technique [16, 17] and extract the behavior of the current carrying states from the energy level statistics [18, 19].

2 Model and Method

The lattice model describing non-interacting electrons moving in a two-dimensional disorder potential and a perpendicular magnetic field is defined [20] by an Anderson Hamiltonian where the magnetic field is incorporated into the transfer terms \( V_{mn} \) via Peierls phase factors,

\[
H = \sum_{m} \varepsilon_m |m\rangle\langle m| + \sum_{<m\neq n>} V_{mn} |m\rangle\langle n|.
\]

The \(|m\rangle\) are lattice vectors associated with the sites \( m \) of a simple square lattice with spacing \( a \). The transfer is restricted to nearest neighbors only, \( V_{mn} = V \exp(i2\pi(e/h) \int_{r_m}^{r_n} A(\mathbf{r}) \, d\mathbf{r}) \), and for the vector

\[
1
\]
potential the Landau gauge, $A = (0, -Bx, 0)$, is chosen. The disorder potentials $\{\varepsilon_m\}$ are represented by a set of independent random numbers with uniform distribution, $|\varepsilon_m| \leq W/2$, where $W$ is the disorder strength.

The density of states, the Hall and longitudinal conductivities are calculated using a recursive Greens function technique developed previously [14, 17]. This method allows for a computation of large lattice sizes. In the present work we take $a = 1/8$ and consider sample widths $M$ up to $96a$ and lengths in the range $10^4a \leq L \leq 10^5a$. For $\rho(E)$ and $\sigma_{xx}$ periodic boundary conditions are applied across the sample while for $\sigma_{xy}$ Dirichlet boundary conditions are chosen.

The eigenvalues $\{E_i\}$ used for the energy spacing distribution $P(s)$ have been calculated by direct diagonalization of square systems using a Lanczos algorithm. Here, periodic boundary conditions are applied in both directions and the maximal size considered was $M \times M = (128a)^2$. A large number of realizations were computed resulting in a total of more than $5 \cdot 10^5$ eigenvalues per disorder strength. As usual the spacing of consecutive levels is divided by the mean level spacing, $|E_i - E_{i+1}|/\Delta \equiv s$, and the spectrum is properly unfolded. The second moment of the nearest neighbor level spacing distribution, $\langle s^2 \rangle = \int_0^\infty s^2P(s)\,ds$, serves as a measure of the electron localization length. We calculate the quantity $I_0 = 1/2\langle s^2 \rangle$ which in the thermodynamic limit is known to be $I_0^{\text{loc}} = 1$ for strongly localized states, and from random matrix theory $I_0^{\text{typ}} = 0.59$ for a metallic system with unitary symmetry. For the QHE system only localized and critical states are to be expected. The latter are easily distinguished because the level statistics is scale independent at the critical point $[21, 28, 19]$.

### 3 Results and Discussion

In Fig. 1 $I_0 = 1/2\langle s^2 \rangle$ is shown (upper part) as a function of energy for disorder strength $W/V = 1$ and 2. The corresponding density of states (DOS) is plotted below. Because of the symmetry with respect to $E/V = 0$ only half of the energy range is displayed. Increasing the disorder broadens the DOS and also the energy range where the localization length of the electronic states exceeds the system size. The 3rd and 4th minima of $I_0$ start to merge already at disorder $W/V = 2$ which eventually results in a disappearance of these critical states while the two lowest minima remain separated up to $W/V = 5$.

For the lowest Landau band, the disorder dependence of the position of the minimum of $I_0(E)$, $E_0^d$, is shown in Fig. 2 which coincides with the divergence of the localization length. A linear shift of $E_0^d$ down to smaller energies is seen which corresponds to the broadening of the total tight binding band.
An opposed shift within the lowest Landau band can be observed if one looks at the position of the critical filling factor $\nu^0_c$ instead, which also seems to be a more physical quantity than the critical energy when comparing with experiment. For $W/V > 2$, a linear increase of $\nu^0_c$ can clearly be seen in Fig. 3. This shift of the critical states to higher filling factors is understood as an effect of overlapping Landau bands $[8, 15, 9]$. The flattening for smaller disorders is due to a peculiar asymmetry of the Landau bands in the lattice model.

However, the opposite changes of $E^0_c$ and $\nu^0_c$ finally prove to be irrelevant for the fate of the current carrying states because the shifts are small compared to the downward movement of critical states from the band center that cause the disappearance of the current carrying states of the higher Landau bands. The imminent destruction of the last current carrying state is illustrated in Fig. 4 where $I_0(E)$ is plotted for disorder strengths $W/V = 0.5$ and 5. All minima of $I_0(E)$ exhibit scale invariant behavior for
Figure 4: The energy dependence of the second moment $I_0 = 1/2 \langle s^2 \rangle$ for disorder strengths $W/V = 0.5$ ($\Diamond$) and $W/V = 5$ ($\ast$).

$W/V = 0.5$ as expected for critical states. For $W/V = 5.0$, however, only the two lowest minima remain scale invariant while the states are scale dependent for energies larger than $\approx -2.8$. Increasing the system size shifts the value of $I_0$ to 1 as expected for localized states.

The small shift to lower energies of the first critical states has to be compared to the descent of the critical states originating from the band center that are now visible a little above $E/V = -3$. Further increase of the disorder eventually brings the two minima together, thereby annihilating the last conducting state so that the system becomes an insulator. This scenario is in accordance with the results of the calculation of the thermodynamic localization length by Sheng and Weng [9].

Figure 5: The Hall conductivity $\sigma_{xy}$ and the longitudinal conductivity $\sigma_{xx}$ versus energy $E$ for disorder strength $W/V = 1$ ($\Diamond$, $\ast$) and $W/V = 2$ ($\Diamond$, $\times$). The inset shows the transition from QHE to insulator for $E/V = -0.8$ with the crossing of $\sigma_{xx}$ ($\ast$) and $\sigma_{xy}$ ($\Diamond$) at the critical disorder $W_c/V \approx 2.8$. 
The disappearance of current carrying states, starting at the highest Landau bands, can also be seen in Fig. 5. Here the influence of increasing disorder on the Hall and longitudinal conductivities is shown. The decay of the longitudinal conductivity near the critical energies coincides with an increase of $\sigma_{xx}$ in the plateau regions which in turn destroys the quantized Hall conductivity that exhibits a monotonical decay with increasing disorder \[10\]. This direct QHE to insulator transition is shown in the inset for energy $E/V = -0.8$. The crossing point $\sigma_{xx}^c = \sigma_{xy}^c \approx 1.5$ lies at the critical disorder $W_c/V \approx 2.8$. For small $W$, both the conductivity $\sigma_{xx}$ and the resistivity $\rho_{xx}$ are zero while for $W > W_c$, $\sigma_{xx} \to 0$, but $\rho_{xx}$ tends to infinity.

In conclusion, the QHE to insulator transition has been investigated in a disordered tight binding model. From the energy level statistics the shift down in energy of states with negative Chern numbers becomes apparent. In contrast to the continuum model we find that direct transitions to the insulator are possible also from higher quantized Hall values. Our results support the scenario proposed recently by Sheng and Weng in Refs. \[8, 10\].

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