The $\mu$ Problem and the Invisible Axion

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Abstract

The $\mu$ term in the supersymmetric standard model is known to be nonzero. Supersymmetry breaking at the intermediate scale may provide the needed $\mu$ term and the invisible axion. A possible solution of this $\mu$ problem in superstring models is also discussed.

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1. THE $\mu$ PROBLEM

Supersymmetry has been introduced to solve the scalar mass problem or the
gauge hierarchy problem. The minimal supersymmetric standard model (MSSM)
contains two Higgs doublets

$$H_1(Y = -\frac{1}{2}), \quad H_2(Y = \frac{1}{2}).$$

(1)

Due to the above hypercharge assignment, $H_1$ couples to $d^c_L$ and $H_2$ couples to
$u^c_L$. This type of couplings that $Q = -1/3$ quarks get masses from one type of
Higgs doublets and $Q = 2/3$ quarks get masses from the other Higgs field hints
a possibility of a Peccei-Quinn (PQ) symmetry. Introducing singlet superfields,
one can introduce Higgs quartic couplings. However, the quartic couplings are
absent in MSSM due to the absence of singlet superfields. The Higgs couplings in
supersymmetric theory can be conveniently given in terms of superpotential. The
superpotential $W$ in MSSM can contain a term

$$W_\mu = \mu H_1 H_2$$

(2)

where $\mu$ is a free parameter. If $\mu = 0$, there results a spontaneously broken global
PQ symmetry and an axion with $m_a \sim 0.1$ MeV, which is phenomenologically ruled
out. Also, the parameter space allowed by LEP experiments favors $\mu \sim 100$ GeV. In
supergravity models, the electroweak scale of 100 GeV arises as soft supersymmetry
breaking terms through the gravitino mass, $m_{3/2} \sim M_I^2/M_P$, where $M_P$ is the
Planck mass. The parameter for the electroweak symmetry breaking is pro-
vided if $M_I \sim 10^{11}$ GeV. The soft supersymmetry breaking terms are of order of
the gravitino mass

$$A m_{3/2} W_\mu + \text{h.c.} + B m_{3/2}^2 \sum_i \phi^* \phi + \cdots$$

(3)

where $A$ and $B$ are dimensionless numbers of $O(1)$. But the supersymmetric $\mu$
term in the superpotential is not of this kind of a soft term and must be put in by
hand, which is the so-called \( \mu \) problem. Eqs. (2) and (3) shows that PQ symmetry is broken by the presence of the \( \mu \) term. The \( \mu \) problem is a fine tuning problem of why \( \mu \) must be so small compared to a high energy scale. The best motivation for introducing supersymmetry has been to solve the gauge hierarchy problem. If we put in the \( \mu \) term by hand, we lose the original motivation of introducing the supersymmetry in particle physics. Since \( \mu \rightarrow 0 \) gives a PQ symmetry, any radiative generation scheme of \( \mu \) must start from a PQ symmetry in the beginning. Thus, studying the \( \mu \) problem from the PQ symmetry can be applied to most models generating an electroweak scale \( \mu \).

2. COMMON SCALE

The hidden sector scale in supergravity models and the invisible axion scale fall in the common region; it is desirable to have them tied together. Because the \( \mu \) term signals the breaking of the Peccei-Quinn symmetry, it is logical to understand it from the symmetry principle \cite{[1]}. Thus we introduce a PQ symmetry in supergravity models. Through the supergravity interaction,

\[
W \sim \frac{1}{M_P} S_1 S_2 H_1 H_2
\]

(4)
can be generated where \( S_1 \) and \( S_2 \) are \( SU(3) \times SU(2) \times U(1) \) singlet superfields carrying nonvanishing net Peccei-Quinn charge. The scale of the Peccei-Quinn symmetry breaking by vacuum expectation values of scalar components of \( S_1 \) and \( S_2 \) at \( 10^{10} \sim 10^{13} \text{ GeV} \) can lead to an electroweak scale \( \mu \) term,

\[
\mu \simeq \frac{\langle \tilde{s}_1 s_2 \rangle}{M_P} \simeq \frac{(10^{10-11} \text{ GeV})^2}{M_P} \sim 1000 \text{ GeV}.
\]

(5)

Supergravity interactions can generate the needed global symmetry preserving non-renormalizable superpotential Eq. (4).
3. PQ SYMMETRY WITH ANOMALOUS U(1)

It is well known that superstring models in ten dimensions do not allow any global symmetry. In addition, there exists the scale problem of the model-independent axion [2] in superstring models, $F_a \sim 10^{15-16}$ GeV [3]. This model-independent axion realizes a global symmetry nonlinearly. The scale problem of the model-independent axion was a serious problem in superstring models [3]. However, this was a story before the discovery of anomalous $U(1)$ gauge symmetry in some compactification schemes [4]. In other words, if a compactified theory does not have any gauge anomaly, the strong CP problem cannot be solved by an invisible axion because of no global symmetry available or too large axion decay constant. However, some compactification schemes allow anomalous $U(1)$ gauge symmetry, which is not inconsistent because of the model independent-axion [5]. The model-independent axion becomes the longitudinal degree of the anomalous $U(1)$ gauge boson. Then the theory becomes consistent below the $U(1)$ gauge boson mass scale, and there survives a global symmetry $U(1)_{PQ}$. This is due to the ’t Hooft mechanism [6] that a global symmetry survives if a gauge symmetry and a global symmetry is broken by the VEV of a single Higgs field rendering the gauge boson a mass. In the present case, the mechanism works as follows. Let the gauge boson of the anomalous $U(1)$ be $A_\mu$ and the model-independent axion $a_{MI}$. The corresponding currents are not divergenceless,

$$\partial^\mu J^A_\mu = \Gamma_A \frac{1}{32\pi^2} F \tilde{F}$$
$$\partial^\mu J^a_\mu = \Gamma_a \frac{1}{32\pi^2} F \tilde{F}$$

(6)

where $F \tilde{F}$ is an abbreviation for $2\text{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}$. Therefore, we can write an effective interaction in the form,

$$\mathcal{L}_{\text{int}} = A^\mu J^A_\mu - \frac{a_{MI}}{f} \frac{1}{32\pi^2} F \tilde{F}.$$  

(7)
The transformations of the anomalous $U(1)_A$ and global $U(1)_a$ are

$$
A_\mu \to A_\mu + \partial_\mu \Lambda(x) \\
a_{MI} \to a_{MI} + f \theta.
$$

(8)

Under these transformations, the shift of $\mathcal{L}_{int}$ is

$$
\delta \mathcal{L}_{int} = (-\Lambda \Gamma_A - \theta \Gamma_a) \frac{1}{32\pi^2} F \tilde{F}.
$$

(9)

Choosing

$$
\theta = -\frac{\Gamma_A}{\Gamma_a} \Lambda(x),
$$

(10)

$\mathcal{L}_{int}$ remains invariant. The the nonlinearly realized global transformation is identified as the gauge transformation, and $a_{MI}$ becomes the longitudinal degree of the original anomalous gauge boson. The mass of the gauge boson becomes

$$
M_A^2 = \left( \frac{\Gamma_A}{\Gamma_a} f \right)^2.
$$

(11)

Below the scale $M_A$, we can define a anomalous global current

$$
J_{gl}^\mu \equiv \frac{\Gamma_A}{\Gamma_a} J^A_\mu + J^a_\mu
$$

and the other orthogonal combination is anomaly free and corresponds to the broken gauge symmetry

$$
J_{ga}^\mu \equiv J^A_\mu - \frac{\Gamma_A}{\Gamma_a} J^a_\mu.
$$

(12.b)

Below the scale $M_A$, a global symmetry survives. Thus,

in superstring models with anomalous $U(1)$ there exists a possibility of solving the strong CP problem by an invisible axion with its decay constant at $F_a \sim 10^{12}$ GeV. Therefore, if there exists only one $\theta$ to remove below the anomalous gauge boson mass, then we solve the scale problem of the model independent axion. The $U(1)_{PQ}$ symmetry is broken when $\langle S_1S_2 \rangle$ get VEV around $10^{12}$ GeV.
4. SUPERSTRINGS AND THE \( \mu \) PARAMETER

For supersymmetry breaking by gaugino condensation, one needs an extra
confining gauge group at \( \sim 10^{13} \) GeV [7] and there exists one more \( \theta \). Thus
the global symmetry from the anomalous \( U(1) \) gauge symmetry is not enough to
remove two \( \theta \)'s. For example, an orbifold construction with the shift and Wilson
lines [8]

\[
v = (11112000)(20000000) = (00000002)(01100000) = (1121011)(11000000) = (0000200)(00011112)
\]

(13)
gives the gauge group

\[
SU(3) \otimes SU(2) \otimes U(1)^5 \otimes [SU(5) \otimes U(1)^4]'.
\]

(14)

Three generations of doublet quarks in addition to other light fields arise from the
untwisted sector

\[
3(3, 2, 1) + 3(\bar{3}, 1, 1) + 3(1, 2, 1) + 3(1, 1, 5').
\]

(15.a)

The light fermions from the twisted sector are

\[
9(3, 1, 1) + 12(\bar{3}, 1, 1) + 30(1, 2, 1) + 3(1, 1, 5') + \text{singlets}.
\]

(15.b)

There is no triangle anomalies except for one anomalous gauge \( U(1)_X \).

The explicit calculation of the sums of \( X \) charges for color triplets, \( SU(2) \)
doublets, and hidden color quintets are nonvanishing and equal,

\[
\partial^\mu J^X_\mu = c \left( F\tilde{F} + WW' + F'\tilde{F}' + \cdots \right)
\]

(16)

where \( \cdots \) denote the anomalies of \( U(1)'s \) whose coefficients are also 1. Thus the
current \( J^X_\mu \) has the same anomaly structure as that of the current corresponding
to the model-independent axion. Therefore, the anomalous $U(1)_X$ gauge boson obtains a mass, absorbing the model-independent axion as the longitudinal degree. Then, there results a global symmetry below the $U(1)_X$ gauge boson mass which is a PQ symmetry. To solve the $\mu$ problem, we identify $\Lambda SU(5)$ as $M_I$. $S_1$ and $S_2$, which are $SU(5)$ quintet and antiquintet, can condense to give $\langle S_1 S_2 \rangle \sim M_I^2$, which leads to the desired magnitude for the $\mu$ term. But the QCD $\theta$ problem is not solved automatically because there is only one available global symmetry. However, the possibility of the phase field of the hidden sector gaugino condensation providing another needed phase has been argued in Ref. [8].

5. CONCLUSION

The PQ symmetry has been introduced to solve the strong CP problem and the $\mu$ problem in supergravity models. At the supergravity level it works. Superstring extension of this idea also solves these two problems if there is no additional confining group except those of the standard model; but the mechanism for supersymmetry breaking is not clear. On the other hand, if there exists an additional confining group for supersymmetry breaking by gaugino condensation, the $\mu$ problem is solved; but the solution of the strong CP problem needs an extra phase field.

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