Interleaving classical and reversible

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Abstract. Given a simple recursive function, we show how to extract two interacting processes from it. The two processes can be described by means of iterative programs, one of which is intrinsically reversible, in a language that, up to minor details, belongs to the core of widely used imperative programming languages. We implement the two processes as interleaving synchronous JAVA threads whose interaction is equivalent to the recursive function they are extracted from.

1 Introduction

Typically, scientific works on reversible computations promote reversibility as an interesting topic, for many situations exist in which a computational activity has to be able to retrace its steps; [10, Part I] is a standard reference to a first list of those situations. As a reinforcement, we think that knowing how to program in a reversible way, over time, could become a natural mental scheme, analogous to recursive and iterative programming schemes, when the computational problem to be solved contemplates its necessity. This work contributes to such an idea.

We deal with reversible computing in the lines of those techniques that [10, Chapter 9] identifies as “Adding Reversibility to Irreversible Programs”.

Mainly, in this work, we add reversibility to irreversible programs according to a basic scheme that we can intuitively describe in some steps. Given a classical recursive definition, we extract from it iterative reversible and classical parts that we can make to collaborate according to a synchronous Producer/Consumer pattern in order to implement the initially given recursion. Using Perumalla’s terminology [10, Chapter 9], it turns out that we can identify reversible aspects in a classical computation by means of a method that sits in between “Source-to-Source Translator” and “Library-Based” approaches.

In order to drive the intuition, let recF[p,b,h] (see Listing 1.1) be a recursive function defined in a reasonable programming formalism on top of a predecessor function p, a step function h, and a base function b. We decompose recF[p,b,h] into two processes itFCl[b,h] and itFRev[p,pInv] such that:

\[
\text{recF}[p,b,h] \simeq \text{itFCl}[b,h] \parallel \text{itFRev}[p,p\text{Inv}] ,
\]
where (i) “≃” stands for “equivalent to”, and “∥” for “interaction/parallel-composition” between its two arguments; (ii) itFCls\([b, h]\) sequentially composes \(b\) and a for-loop that iteratively applies \(h\); (iii) itFRev\([p, pInc]\) sequentially composes two for-loops one iterating \(b\), the other its inverse \(bInv\).

On one side, (1) says that we can translate the classical language construct \(\text{recF}[p, b, h]\) as composition of two interacting parts, one of which naturally exposes a reversible nature; so we see (1) as instance of “Source-to-Source Translator” approach. On the other, concerning the “Library-Based” approach, (1) allows to look at the reversible part \(\text{itFRev}[p, pInv]\) as the component of a library, with only reversible code in it, that produces the values that the classical part relies on to accomplish the overall task to implement \(\text{recF}[p, b, h]\) together with \(\text{itFCls}[b, h]\).

In order to realize the Producer/Consumer pattern that (1) summarizes, we use a programming syntax that, up to minor syntactic details, is a nucleus common to C-style programming languages (nowadays ubiquitous, e.g. C++, JAVA, C#, . . .) which can be implemented by the control structures of the two reversible languages SRL \[6, 7]\ and RPP \[8, 9, 7\]. We recall that SRL and RPP are intrinsically reversible programming notations with finite iterations only, as expressive as the class of primitive recursive functions. This essential correspondence allows us to implement the Producer/Consumer pattern in terms of JAVA synchronous threads that the interested reader can experiment with \[1\].

The reminder of this introduction deepens, but at an intuitive level, the observations that, starting from the idealized formalization in (1) eventually leads to the JAVA prototypical implementation in \[1\].

```java
1 Fix recF(x) {
2 if (x==0) { b(x); }
3 else { h(x, recF(x-1)); } }
```

Listing 1.1: The recursive function \(\text{recF}\).
/* Assumption: the initial value of x is 3 */

x = p(x) // == 2
x = p(x) // == 1
x = p(x) // == 0
y = b(x) // == b(p(p(p(3))))
y = h(x, y) // == h(p(p(p(3))), b(p(p(p(3)))))
x = pInv(x) // == pInv(p(p(p(3)))) == p(p(3))
y = h(x, y) // == h(p(3), h(p(3)))

Listing 1.2: Iterative unfolding recF(3); the bottom-up part.

of the final value of recF(3) by applying the base case of recF, i.e. b(x). By
definition, let pInv denote the inverse of p, i.e. pInv(p(z)) = p(pInv(z)) = z, for
any z. Clearly, in our running example, the function pInv(x) is x+1. Lines 6–13
alternate h(x, y), whose result y, step by step, gets better and better the final
value recF(3), and pInv(x), which produces a new value for x.

/* Assumptions. s == 0, e == 0, g == 0, w == 0 */

w = w + x;
for (i = 0; i <= w; i++) {
    if (x > 0) { g++; }
    else if (x == 0) { e++; }
    else { s++; }
    x = p(x);
}

Listing 1.3: Iterative itF equivalent to recF.

Let us call itF the code in Listing 1.3. It implements recF by means of finite
iterations only. Continuing with our running example, if we run itF here above
starting with x == 3, then x == 0 holds at line 8, just after the first for-loop;
after the second for-loop y == recF(3) holds at line 14.
/**/ Assumptions. s = 0, e = 0, g = 0, w = 0 */

w = w + x;
for (i=0; i<=w; i++) {
  if (x > 0) { g++; } /* number of times x is ‘g’reater than 0 */
  else if (x==0) { e++; } /* number of times x is ‘e’qual to 0 */
  else { s++; } /* number of times x is ‘s’maller than 0 */
  x = p(x);
}

for (i=0; i<=w; i++) {
  x = pInv(x);
  if (x > 0) { g--; /* Value of x for h available here */ }
  else if (x==0) { e--; /* Value of x for b available here */ }
  else { s--; }
  w = w - x;
}

Listing 1.4: Reversible side of itF.

Let us call reversible side of itF the code in Listing 1.4 i.e. it is Listing 1.3 without h(x,y) and b(x) at lines 11, 12. It is reversible because, when it stops, the value that every variable contains, but i, is identical to the one it contained at line 2, independently from the assumptions at line 1. The variable i does not contradict that Listing 1.4 is reversible, for a finite iteration can hide it [8,9].

The structure of the reversible side itF has two parts. Through lines 2–7, the variable g counts how many times x remains positive, the variable e how many it stays equal to 0, and the variable s how many it becomes negative. In our running example, it will never be the case that x assumes a negative value because the whole iteration at lines 3–7 is driven by the initial value of x which, initially, is assumed to be non negative, and p(x) decreases its argument of a single unity. We will discuss about the relevance of variable s later. Lines 9–13 simply undo what lines 2–7 do; the reason is that the execution of p(x), g++, e++, s++ is annihilated by executing their inverses pInv(x), g--, e--, s--, respectively, in reversed order. This is why the correct values of x are available at lines 12, 11 and we can use them as arguments of b(x) and h(x,y) in order to update y as in Listing 1.3 according to the results we obtain by the recursive calls to recF(x) in Listing 1.1.

The reason not to call b(x) and h(x,y) at lines 12, 11 of Listing 1.4 as compared to Listing 1.3 is that we want Listing 1.4 to be the reversible side of itF; calling b(x) and h(x,y) in it would generate the result y, so preventing the possibility to reset the value of every involved variable to their initial value. This is why we also need a classical side of itF that generates y in collaboration with the reversible side in order to implement recF(x) correctly.

We claim that Listing 1.5 and 1.6 are the classical side itFCls and the reversible side itFCRev of itF which, suitably interacting, make the scheme (1) almost fully concrete. We see itFCRev as the producer of values that the consumer itFCls consumes as soon as it uses them as arguments of the base b(x) and
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the step function $h(x,y)$. The following points illustrate how $itFCls$ and $itFRev$ synchronously interact.

1. The starting point of the synchronous interaction between $itFCls$ and $itFRev$ is line 2 of $itFCls$. The comment:

   ```
   /* Inject the current x at line 2 of itFRev to let it start */
   ```

   describes what, in a fully implemented version of $itFCls$, we expect in that line of code. The comment says that $itFCls$ injects (sends, puts) its input value $x$ to line 2 of the reversible side $itFRev$ (cf. Listing 1.6). Once $itFRev$ obtains that value at its line 2, as outlined by the comment:

   ```
   /* Inject here the value of x from line 2 of itFCls */
   ```

   its for-loop at lines 4–8 is executed.

2. After executing its line 2, $itFCls$ stops at line 3. It waits for $itFRev$ to produce the number of times that $itFCls$ has to iterate line 7. The comment:

   ```
   /* Probe line 9 of itFRev to get the number of iterations to execute */
   ```

   says that we see the consumer as probing the producer $itFRev$ until it delivers that value; $itFRev$ let that value be available in its variable $g$ at line 9, as outlined by comment:

   ```
   /* itFCls probes here g which has the number of iterations */.
   ```

3. Once gotten the value in iterations, $itFCls$ proceeds to line 5 and stops again. It waits for $itFRev$ to produce the argument of $b$ which is eventually available for probing at line 14 of $itFRev$.

4. Once the argument becomes eventually available, then $b$ is applied, and $itFCls$ enters its for-loop, stopping at line 7 at every iteration. The reason is that $itFCls$ waits for line 12 in $itFRev$ to produce the value of the first argument of $h(x,y)$. This interleaved dialog between line 7 of $itFCls$ and line 12 of $itFRev$ lasts iterations times.

Identifying the reversible side $itFRev$ and the classical side $itFCls$ of the classical recursive $recG$ in accordance with [1] recalls Girard’s decomposition $A → B ≃ !A → B$ [3] which looks at a classical computation $A → B$ as an interaction between a linear (ideally reversible) part with type $C → B$, for some $C$, and a non-linear !$A$ that replicates computational resources; conclusions will discuss this briefly again.

Contributions and road map.

– Let a predecessor function $p(x)$ be given. By definition, let $\Delta_p$ be defined as $p(x)-x$ which, forcefully, is negative and which we assume constant. Let also $b(x)$ and $h(x,y)$ be base and step functions, respectively. Section 2 identifies the structure of the reversible side that we can extract from any recursive classical function $recG[p,b,h]$, built on $b(x)$, $h(x,y)$, $p(x)$, and with a condition $c(x)$ with form $x ≤ 0$. This means that $recG$ weakens the structure of $recF$ in Listing 1.1 generalizing its use for programming.
Listing 1.5: Classical side: the consumer \textit{itFCl}\textsubscript{s} to implement \textit{itF}.

s = 0, e = 0, g = 0, w = 0;
x = /* Inject here the value of x at line 2 of \textit{itFCl}\textsubscript{s} */
w = w + x;
for (i = 0; i<=w; i++) {
    if (x> 0) { g++; }  
    else if (x==0) { e++; }  
    else { s++; }  
    x = p(x);
}  /* \textit{itFCl}\textsubscript{s} probes here g which has the number of iterations */
for (i = 0; i<=w; i++) {
    x = pInv(x);
    if (x> 0) { g--; /* \textit{itFCl}\textsubscript{s} probes here the first argument value of h */}  
    else if (x==0) { e--; /* \textit{itFCl}\textsubscript{s} probes here the argument value of b */}  
    else { s--; }  
    w = w - x;
Listing 1.6: Reversible side \textit{itFRev}: the producer to implement \textit{itF}.

Fix \textit{recG}(x)  
{
    if (x<=0) { b(x);  }  
    else { h(x,\textit{recG}(p(x))); }  
}
Listing 1.7: The generic structure of \textit{recG}.  

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Section 3 illustrates and implements the above Producer/Consumer pattern that corresponds to (1) by means of JAVA classes. The class that implements the producer has the structure of the reversible side that we will see in Section 2, the class that implements the producer synchronously interacts with the consumer, whose structure itFCls with minor structural updates. Sources of the JAVA classes and packages are available at [1].

Section 4 will address natural developments, open questions, and related works.

2 From recursion to iteration

We show how, and justify why a recursive function recG slightly more general than recF in Listing 3 requires to generalize the structure of the reversible side we already commented in Listing 6.

The structure of recG that we consider is in Listing 7 where b(x) and h(x,y) are a base and a step function, respectively. Let us recall from the introduction that, given a predecessor p(x), we let Δp be the negative difference defined as Δp = p(x)-x. The value of Δp that we consider is an arbitrary and constant k <= -1, not only k == -1, which requires to consider the slightly more general condition x <= 0. For example, let p(x) be x-2. The computation of recG(3) is h(3,h(p(3),h(p(p(3)),b(p(p(p(3))))))), which looks for the least n of iterated applications of p(x) such that p(...p(3)...) <= 0 holds: in our case 2 == n < 3.

We call itG in Listing 8 the generalization of itF in Listing 3. The scheme itG iteratively implements any recursive function whose structure can be brought back to the one of recG. Please, remark that line 1 initializes ancillae s, e, g, and w, like for the namesake variables of itF, a standard initialization in reversible computing [8,10], and line 2 adds new ancillae z, predDivX, and predNotDivX.

We also assume an initial non negative value for x. The reason is twofold. Firstly, it keeps our discussion as simple as possible, with no need to use the absolute value of x to set the upper limit of every index i in the for-loops that occur in the code. Second, negative values of x would widen our discussion about what a classical recursive function on negative values is and about what its reversible equivalent iteration has to be; we see this as a very interesting subject connected to [2].

We start observing that line 3 of itG sets w to the initial value of x; the reason is that every for-loop, but the one at lines 10–13, has to last x+1 iterations, and x changes in the course of the computation; so, w stores the initial value of x and stays constant from line 4 through line 21. In fact it can change at lines 23–33. We will see why, but eventually w is reset to its initial value 0 at line 34.

With the previous premises, given a non negative x, in analogy to itF, the for-loop at lines 4–8 of itG iterates the application of p(x) as many times as w+1, i.e. the initial value of x plus 1. So, the value of x at line 9 is equal to w+(w+1)*Δp, which cannot be positive. In particular, all the values that x assumes in the for
s = 0, e = 0, g = 0, w = 0;
z = 0, predDivX = 0, predNotDivX = 1;
w = w + x; /* x is assumed to be the input */
for (i = 0; i <= w; i++) {
    if (x > 0) { g++; }
    else if (x == 0) { e++; }
    else { s++; }
    x = p(x);
}
for (i = 0; i < e; i++) {
    predDivX = predDivX + predNotDivX;
predNotDivX = predDivX - predNotDivX;
}
for (j = 0; j < predDivX; j++) {
    for (i = 0; i <= w; i++) {
        x = pInv(x);
        if (x > 0) { g--; y = h(x,y); }
        else if (x == 0) { e--; y = b(x); }
        else { s--; }
    }
}
for (j = 0; j < predNotDivX; j++) {
    w++;
    for (i = 0; i <= w; i++) {
        x = pInv(x);
        if (x > 0) { g--; }
        else if (x < 0) { }
        else if (x == 0) { y = b(x); z++; }
        else { y = h(x,y); }
        x = pInv(x);
        if (z < 0) { }
        else if (z == 0) { y = b(x); z++; }
        else { y = h(x,y); }
        x = pInv(x);
        else if (x == 0) { e--; }
        else { s--; }
    }
    w--;
}
w = w - x;
/* y carries the output */

Listing 1.8: The iterative reversible function itG equivalent to recG.
-loop at lines 4–8 belong to the following interval:

\[ I(w) \triangleq \left[ w^+ (w+1)^+ \Delta_p, w+\Delta_p, \ldots, w+\Delta_p, w \right] \]  

(2)

from the least to the greatest; the counters \( g, e, s \) say how many elements of \( I(x) \) are greater, equal or smaller than 0, respectively. Depending on 0 to belong to \( I(x) \) determines the behavior of the reminder part of \( \text{itC} \), i.e. lines 10–34.

We need to distinguish two cases in order to illustrate them.

As a first case, let \( w\%\Delta_p = 0 \), i.e. \( \Delta_p \) divides the initial value of \( x \) which \( w \) stores in it. So, \( 0 \in I(x) \), which implies the following relations hold at line 9:

\[ e = 1 \quad g = \frac{w}{\Delta_p} \quad s = (u+1)-g-e \]  

(3)

Lines 10–12 execute exactly once, swapping \( \text{predDivX} \) and \( \text{predNotDivX} \). We observe that we could have well used the if-selection in Listing 1.9 here below, which is a construct of RPP, in place of the for-loop at lines 10–12; we opt for a more compact code with no empty branches.

```plaintext
if (e < 0) { }
else if (e == 0) { \text{predDivX} = \text{predDivX}+\text{predNotDivX}; \text{predNotDivX} = \text{predDivX} - \text{predNotDivX}; }
else { }
```

Listing 1.9: A possible replacement of lines 10–12 in Listing 1.8.

Swapping \( \text{predDivX} \) and \( \text{predNotDivX} \) let \( \text{predDivX} = 1 \) and \( \text{predNotDivX} = 0 \), which corresponds to computationally exploit that \( \Delta_p \) divides \( w \): the for-loop body at lines 15–19 becomes accessible, while lines 22–33, with for-loops among them, do not. Lines 15–19 are identical to lines 10–16 of \( \text{itF} \) in Listing 1.4 which we already know to correctly apply \( b(x) \) and \( h(x, y) \) in order to simulate the recursive function we start from.

As a second case, let \( w\%\Delta_p \neq 0 \), i.e. \( \Delta_p \) does not divide the initial value of \( x \) which \( w \) stores in it. So, \( 0 \notin I(x) \), implying that:

\[ e = 0 \quad g = -\left\lfloor \frac{w}{\Delta_p} \right\rfloor \quad s = (u+1)-g-e \]  

(4)

hold at line 9. Lines 11–12 cannot execute, leaving \( \text{predDivX} \) and \( \text{predNotDivX} \) as they are: lines 22–33 become accessible and the for-loop at lines 13–19 does not. Line 22 increments \( w \) to balance the information loss that the rounding of \( g \) in [4] introduces; line 33 recovers the value of \( w \) when the outer for-loop starts. The if-selection at lines 25–32 allows to identify when to apply \( b(x) \), which must be followed by the required applications of \( h(x, y) \). We know that \( 0 \notin I(x) \), so \( x == 0 \) can never hold. Clearly, \( s-- \) is executed until \( x > 0 \). But the first time \( x > 0 \) holds true we must compute \( b(p(x)) \), because the base function \( b(x) \) must be used the last time \( x \) assumes a negative value, not the first time it gets positive;
lines 26–30 implement our needs. Whenever $x > 0$ is true, the value of $x$ is one step ahead the one we need: we get one step back with line 26 and, if it is the first time we step back, i.e. $z == 0$ holds, then we must execute line 28. If not, i.e. $z != 0$, we must apply the step function at line 29. Line 30, restores the right value of $x$.

Finally, we observe that $itG$ does not reset $z$ to its initial value 0. The lines:

```java
for (i = 0; i < predNotDivX; i++) {
    z--; }
```

Listing 1.10: Code to set $z$ back to 0.

which we could place before current line 34, are missing. The reason not to introduce them is that we want to proceed according to the initial scheme (1). We identify a purely reversible part $itGRev$ and a classical part $itGCls$ in $itG$ which interact, with no need of $z$ anymore. The reason is that we delegate the control over which between $b(x)$ and $h(x, y)$ to apply to the classical part $itGCls$. This results from framing $itGRev$ and $itGCls$ as two JAVA threads that synchronously collaborate, under a Producer/Consumer template that Section 3 introduces.

## 3 Reversible vs. Classical as Producer/Consumer

This section comments parts of the JAVA code in [1], that compile and run with JAVA SE 14 - ORACLE. The section is meant to be self-contained; an intuitive idea about what a (JAVA) thread should be more than sufficient to catch the essential points. Endless literature on how using JAVA threads in details exists and [1] is based on one of those possible “infinite” instances.

Our goal is to describe how JAVA classes in [1] implement independent threads which, synchronously collaborating in accordance with a Producer/Consumer template by means of a couple of communication channels, fully implement both $itGCls$ in Listing 1.5 and $itGRev$ in Listing 1.6 in order implement any recursive function traceable to $recG$ in Listing 1.7. The classes are $ItGCls$ (Listing 1.11), and $ItGRev$ (Listing 1.12); they define methods $ItGCls.itGCls$ and $ItGRev.itGRev$, respectively. In particular, $ItGRev.itGRev$ fully implements the purely reversible part of $itG$ in Listing 1.8 which, we recall, extends the pattern that we find in Listing 1.6 meant for introductory purposes, and corresponding to $recF$ in Listing 1.1 simpler than $recG$.

As a global assumption, $B$ and $H$ are two JAVA classes with (static) methods $b(x)$ and $h(x, y)$, respectively, one implementing a base function, the other a step function. For example, downloading them from [1], one finds a $B.b(x)$ which is the identity, and a $H.h(x, y)$ that returns $x+y$.

As already outlined, $ItGCls$ in Listing 1.11 details out the classical side of $itG$, coherently with Listing 1.5. Lines 15–17 of $ItGCls$ are a classical iteration which, given a number of iterations, and the right sequence of values in the
public class ItGCls {
    private final Inject inject;
    private final Probe probe;
    private int out = 0;
    private int in = 0;
    public ItGCls(Inject inject, Probe probe, int x) {
        this.inject = inject;
        this.probe = probe;
        this.in = x;
    }
    public int getOut() { // Let out be available outside ItGCls
        return this.out;
    }
    public void itFCls() throws InterruptedException {
        inject.put(in);
        int iterations = probe.get();
        out = B.b(probe.get());
        for (int i = 0; i < iterations; i++) {
            out = H.h(probe.get(), out);
        }
    }
}

Listing 1.11: The consumer class ItGCls.

argument variable in, executes the sequence of assignments:

\[
\begin{align*}
\text{out} &= B.b(\text{probe.get()}) \\
\text{out} &= H.h(\text{probe.get()}, \text{out}) \\
\text{out} &= H.h(\text{probe.get()}, \text{out})
\end{align*}
\]

... until out contains the result, i.e. what we call y in Listing 1.5. At lines 7–9, ItGCls sets the instances inject and probe of the two communication channels Probe and Inject. Lines 15, 17 obtain the required argument values of the base and of the step functions by calling probe.get(). Also line 14 calls probe.get(). It is to obtain the number of iterations. Instead, line 13 sends the initial value of in to the reversible side, i.e. (an instance of) the producer ItGRev; ItGRev needs such a value to start producing the sequence of arguments values that ItGCls requires to execute the sequence (5).

As told, Listing [1.12] is a possible implementation of Listing [1.8] up to the parts that make Listing [1.8] irreversible. The table:

| itG | ItGRev |
|-----|-------|
| 4–8 | 12–16 |
| 10–12 | 17–19 |
| 14–19 | 20–30 |
| 21–33 | 31–43 |

list code lines in itG of Listing [1.8] that correspond in ItGRev of Listing [1.12]
public class ItGRev {
    private final Inject inject;
    private final Probe probe;

    ItGRev(Inject inject, Probe probe) {
        this.inject = inject;
        this.probe = probe;
    }

    public void itGRev() throws InterruptedException {
        int s = 0, e = 0, g = 0, w = 0, x = 0;
        int predDivX = 0, predNotDivX = 1;
        x = inject.swapIn(x); // read x from itGCls
        w = w + x;
        for (int i = 0; i <= w; i++) {
            if (x > 0) { g++; }
            else if (x == 0) { e++; }
            else { s++; }
            x = Pred.pred(x);
        }
        for (int j = 0; j < predDivX; j++) {
            probe.put(g); // send g to itGCls for iterations
            for (int i = 0; i <= w; i++) {
                x = Pred.predInv(x);
                if (x > 0) { g--; }
                else if (x == 0) { e--; }
                else { s--; }
            }
            w++; }
        for (int i = 0; i <= w; i++) {
            x = Pred.predInv(x);
            if (x > 0) { g--; }
            else if (x == 0) { e--; }
            else { s--; }
        }
        w = w - x;
        x = inject.swapOut(x); // restore initial x }
    }
}
We now trace the behavior of $\text{ItGRev}$ in analogy to the one of $\text{itG}$. Let $w_k \Delta_p = 0$, so lines 21–30 are accessible. Line 21 sets the value of `iterations` in the consumer $\text{ItGCls}$, so it can move to its line 15 waiting for the argument value of $B.b(x)$. Lines 26, and 29 call `probe.put()`; in both cases the call sets the value that the consumer $\text{ItGCls}$ waits in order to feed $H.h(x,\text{out})$ or $B.b(x)$.

Let, instead, $w_k \Delta_p \neq 0$, so lines 32–43 are accessible. Line 32 sets the value of `iterations` in the consumer $\text{ItGCls}$ exactly like line 21. Lines 36–42 simplify lines 25–32 of $\text{itG}$ in Listing 1.8 for they do not need neither an `if`-selection nor $z$ to identify which between $B.b(x)$ and $H.h(x,\text{out})$ to use, operation delegated to $\text{ItGCls}$ which receives $x$ from `probe.put(x)` at line 39.

Finally, the producer $\text{ItGRev}$ is triggered to start at line 10: `inject.swapIn()` swaps the content of $x$, ancilla local to $\text{ItGRev}$, with the input value, (possibly) available in `Inject.inject`, shared between $\text{ItFRev}$ and $\text{ItGCls}$. Line 45 restores the initial value of $x$ by swapping it with the one it gets at line 10.

```java
public class Probe {
    private int x = 0;
    private boolean xAvailable = false;
    public synchronized int get() throws InterruptedException {
        while (!xAvailable) { wait(); } // producer has not produced
        int out = x;
        xAvailable = !xAvailable;
        notify();
        return out;
    }

    public synchronized void put(int in) throws InterruptedException {
        while (xAvailable) { wait(); } // consumer has not consumed
        x = in;
        xAvailable = !xAvailable;
        notify();
    }
}
```

Listing 1.13: Channel $\text{Probe}$.

Instances of classes $\text{Probe}$ and $\text{Inject}$ model channels for synchronous dialogues between (instances of) $\text{ItGCls}$ and $\text{ItGRev}$.JAVA’s `synchronized` directive let the bodies of every method in $\text{Probe}$ and $\text{Inject}$ be critical regions, each one accessed by a single thread at a time, one running $\text{ItGCls}$, the other $\text{ItGRev}$. Let us comment on method `get` in $\text{Probe}$, all others, those ones of $\text{Inject}$ included, behaving analogously.\(^3\) Let `probe.get()` be a call in $\text{ItGCls}$ of an instance `probe` of $\text{Probe}$. The consumer enters `probe.get()` and waits() until `xAvailable` is set to `true`, meaning that the producer $\text{ItGRev}$ made a value available in `probe.x` by

\(^3\) We address to the official documentation on JAVA classes `Runnable` and `Threads` on possible implementations of critical regions.
means of a call to `probe.put(x)`. As soon as `ItGRev` negates `xAvailable`, setting it to `true`, and calls `notify()` at line 16 in `probe.put(x)`, `ItGCls` leaves line 6 of `probe.get()` in order to: (i) set `out` with the value in `x`; (ii) negate `xAvailable` again; (iii) notify that `ItGRev` can produce a new value for `probe.x`; (iv) return the value of `probe.out`. Method `prob.put(x)` sets a symmetric behavior on `ItGRev`.

```java
public class Inject {
    private int xInitial = 0;
    private boolean notSet = true;

    public synchronized int get() throws InterruptedException {
        while (notSet) { wait(); }
        int out = xInitial;
        notify();
        return out;
    }

    public synchronized void put(int in) throws InterruptedException {
        while (!notSet) { wait(); }
        xInitial = in;
        notSet = !notSet;
        notify();
    }

    public synchronized int swapIn(int in) throws InterruptedException {
        while (notSet) { wait(); }
        int out = xInitial;
        xInitial = in;
        notify();
        return out;
    }

    public synchronized int swapOut(int in) throws InterruptedException {
        int out = xInitial;
        xInitial = in;
        notSet = !notSet;
        notify();
        return out;
    }
}
```

Listing 1.14: Channel `Inject`.

### 4 Conclusions, related and future work

In this work we show that given a (unary) base function `b(x)`, a (binary) step function `h(x,y)`, a (unary) predecessor `p(x)` that decreases every of its input `x` by a constant value `Δp`, so the inverse `pInv(x)` forcefully exists, we can decompose every recursive (classical) function `recG[p,b,h]`, built on `p(x), b(x), and`
h(x, y) as in Listing 1.7 into two iterative sides: the reversible \( \text{itGRev}[p, p\text{Inv}] \), and the classical \( \text{itGCl}s[b, h] \), following the notation in [1]. Both \( \text{itGRev}[p, p\text{Inv}] \) and \( \text{itGCl}s[b, h] \) synchronously cooperate to implement \( \text{recG}[p, b, h] \) as Producer/Consumer. Despite the simplicity of \( \text{recG}[p, b, h] \), its decomposition requires some work in order to soundly model the recursive unfolding process by means of finite iterations available in intrinsically reversible programming formalism like SRL, and RPP which \( \text{itGRev}[p, p\text{Inv}] \) can be traced to. The decomposition into two parts finds implementation as JAVA threads which fit both Perumalla’s “Source-to-Source Translator” and “Library-based” approaches to reversibility.

The transformation we propose shows the role that reversibility can have in everyday programming. Let \( \text{recG} \) be matching Listing 1.7 for a possible example scenario which follows. Being \( \text{recG} \) recursive, typically it is simpler to prove correct, as compared to an equivalent iterative one. A compiler should generate \( \text{ItGCl}s \) and \( \text{ItGRev} \) equivalent to \( \text{recG} \), such that (instances of) \( \text{ItGRev} \) could run on a true reversible hardware, possibly contributing to leverage the promised greener foot-print of reversible computing, as compared to the classical one.

A first natural step beyond this work is to identify a class \( R \) of recursive schemes more general than the one in Listing 1.7. Of course, \( R \) must contain recursive functions with arbitrary arity. Second, the recursive functions of \( R \) should be able to be based on predecessors \( p \) such that, at least:

1. \( \Delta_p \) is not necessarily a constant, as in Section 2. For example, \( \Delta_p = -3 \) on even arguments, and \( -2 \) on odd ones can be useful;
2. \( p(x) \) is an integer division \( x/k \), for some given \( k > 0 \), like in a dichotomic search, that has \( k = 2 \).

More generally, we aim at an \( R \) with recursive functions only, of which every \( \text{recG} \) is defined on at least one predecessors \( p \) such that, for every argument \( x \), a finite iterated application \( p(\ldots p(x)\ldots) \) exists which let the condition \( c(p(\ldots p(x)\ldots)) \) of \( \text{recG} \) be true. So, for every \( \text{recG}[p, b, h], p, b \) and \( h \) in \( R \) the introductory scheme [1] would become a compilation scheme [1] from \( R \) to, just as an example, JAVA threads “defined” as:

\[
[p] = \text{required (reversible) code} \\
[p\text{Inv}] = !\!\!\![p] \\
[\text{recG}[p, b, h]] = \text{ltGCl}s[[b], [h]] \parallel \text{ltGRev}[[p], [p\text{Inv}]] ,
\]

where \( !\!\!\![.\!] \) inverts the code that its argument \([.\!\!]\) produces, and \( \parallel \) models some kind communication.

Also, we see the above compilation scheme a good starting point to abstract away from the concreteness oriented perspective taken so far in this work. The new goal would be to investigate if the decomposition we have seen has some analogies with Girard’s decomposition \( A \rightarrow B \simeq !A \rightarrow B \). To us, decomposing \( \text{recG}[p, b, h] \) in terms of \( \text{ltGCl}s[b, h] \) and \( \text{ltGRev}[p, p\text{Inv}] \) suggests that the relation between reversible and classical computations can be formalized by a linear isomorphism \( A^n \rightarrow B^n \) between tensor products \( A^n \), and \( B^n \) of \( A \) and \( B \), in analogy to [5]; then we can get classical computations by applying a functor, say
\( \gamma \), whose purpose is, at least, to forget, or to inject replicas of, parts of \( A^n \), and \( B^n \), in a way that \( (\gamma A^n \rightarrow \gamma A^n) \uplus (\gamma A^n \leftarrow \gamma A^n) \) can be a proposal for their type; the type says that we pass from a reversible computation to a classical one by choosing which is input and which is output, and by introducing freedom in the use of the computational resources.

In the same foundational vein, we conclude by observing that the decomposition of a given \( \text{revG} \) that this work introduces, can possibly lead to identify a hierarchy inside the class \( R \), once \( R \) is precisely identified. The hierarchy would characterize a function \( \text{revG} \) depending on the computational space that the reversible component \( \text{itGRev} \) we would extract from \( \text{revG} \) requires to work; intuitively, the space would depend on how complex the inverse \( \text{plinv} \) of the predecessor \( p \) is, and that \( \text{revG} \) is defined on. We see this as related to \( [12,13] \).

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