Dynamic Dark Energy Equation of State (EoS) and Hubble Constant analysis using type Ia supernovae from Union 2.1 dataset.

Syed Faisal ur Rahman

Institute of Space and Planetary Astrophysics (ISPA), University of Karachi (UoK), Karachi, Pakistan

ABSTRACT
This paper constraints dynamic dark energy equation of state (EoS) parameters using the type Ia supernovae from Union 2.1 dataset. The paper also discusses the dependency of dynamic dark energy EoS parameters on the chosen or assumed value of the Hubble Constant. To understand the correlation between the Hubble Constant values and measured dynamic dark energy EoS parameters, we used recent surveys being done through various techniques such as cosmic microwave background studies, gravitational waves, baryonic acoustic oscillations and standard candles to set values for different Hubble Constant values as fixed parameters with CPL and WCDM models. Then we applied trust region reflective (TRF) and dog leg (dogbox) algorithms to fit dark energy density parameter and dynamic dark energy EoS parameters. We found a significant negative correlation between the fixed Hubble Constant parameter and measured EoS parameter, \( w_0 \). Then we used two best fit Hubble Constant values (70 and 69.18474) km s\(^{-1}\) Mpc\(^{-1}\) based on Chi-square test to test more dark energy EoS parameters like: JBP, BA, PADE-I, PADE-II, and LH4 models and compared the results with \( \Lambda \)-CDM with constant \( w_{de}=-1 \), WCDM and CPL models. We conclude that flat \( \Lambda \)-CDM and WCDM models clearly provide best results while using the BIC criteria as it severely penalizes the use of extra parameters. However, the dependency of EoS parameters on Hubble Constant value and the increasing tension in the measurement of Hubble Constant values using different techniques warrants further investigation into looking for optimal dynamic dark energy EoS models to optimally model the relation between the expansion rate and evolution of dark energy in our universe.

Key words: Dark Energy EoS – Type Ia supernovae – Hubble Constant

1 INTRODUCTION

The discovery of the accelerated expansion of the universe (Riess et al. (1998) Perlmutter (1999) Perlmutter & Schmidt (2003)) revolutionized modern cosmology and answered many questions related to the evolution of our universe. However, we are still trying to understand the ingredient which is likely responsible for the accelerated expansion of the universe i.e. dark energy. Dark energy seems to be something which is not only overcoming the tendency of collapse of the matter in our universe but it is also providing a push for the accelerated expansion of our universe (Weinberg (2008)). After the discovery of the accelerated expansion of the universe in the late 1990s by HighZ Supernova and the Supernova Cosmology Project teams (Riess et al. (1998) Perlmutter (1999) Perlmutter & Schmidt (2003)) using the type Ia supernovae, several observations applying various signatures like cosmic microwave background radiation (CMB), baryonic acoustic oscillation (BAO), Cepheid Variables, large scale structures etc. (Bennett et al. (2013) Hinshaw et al. (2013) Planck (2018) Birrer et al. (2018) Macaulay et al. (2019) Riess et al. (2019)), confirmed the accelerated expansion of our universe. Although, these observations confirm that our universe is going through a phase of accelerated expansion but these different observations also presented some serious problems by getting variations in their measurements of the cosmological parameters based on the standard model of cosmology or the \( \Lambda \)-CDM model (Liddle (2003) Jackson (2015) Rahman (2018)) which is providing impetus towards the development
of greater interest in non Lambda-CDM model studies (Zhai et al. (2017) Khosravi et al. (2019) Solã­Aã­ et al. (2019)). New standard candles like active galactic nuclei (AGN) are also being explored to get better measurements of cosmological parameters at high redshifts (Watson et al. (2011)).

2 COSMOLOGY FROM TYPE IA SUPERNOVA

Type Ia Supernovae are useful tools to be used as standard candles because of their almost standard absolute magnitude values. Therefore observations of apparent magnitude (m) and redshift (z) for type Ia Supernovae can lead to measurements of key cosmological parameters: ΩΛ, ΩM, and Ωm, the dark energy, radiation and matter density parameters respectively within the Lambda-CDM cosmology framework. The difference between apparent magnitude (m) and absolute magnitude 'M' from the equations 2 and 3, as:

\[ m = M - M \]  

Given a set of assumed cosmological parameters (C), the redshift of an object, its apparent magnitude and luminosity distance DL are linked thus:

\[ m(C, z) = S\log(DL(C, z)) + M + 25 \]  

Thus luminosity distance and distance modulus are linked:

\[ \mu(C, z) = S\log(DL(C, z)) + 25 \]  

For a spatially flat universe, we can write luminosity distance as:

\[ DL(z) = (1 + z)\chi(z) \]  

Where,

\[ \chi(z) = c\theta(z) \]  

is the comoving distance and η(z) is conformal loop back time which can be calculated as:

\[ \eta(z) = \frac{c^2}{H(z)} \int_0^{z} \frac{dz'}{1 + wz'} \]  

Here, E(z) = \sqrt{\OmegaM(1+z)^3 + \Omega\Lambda(1+z)^2 + \Omega\delta(1+z)^3} for flat Lambda-CDM model. I(z) depends on the parametrization of the dark energy equation of state (EoS) and for standard Λ-CDM model with EoS as \( w_{de}(z) = -1 \) (constant), the multiplier I(z) becomes 1.

We can separate contribution of H0 and absolute magnitude 'M' from the equations 2 and 3, as:

\[ M = M + 25 + S\log(c/H0) \]  

Here 'c' is the speed of light in vacuum. This is often done to marginalize uncertainties arising from measurements of H0 and M. However, the dominant contributor in these uncertainties is H0. We are fixing different H0s from various surveys to test them for most suitable H0 for our dataset in relation with the equation of state (EoS) models in discussion which will minimize these uncertainties for the most suitable value of H0. Therefore instead of separating \( \bar{M} \), we can fit cosmologies using the equations 2 and 3. The contribution from absolute magnitude uncertainties is very minor if we apply proper fits for coefficients for stretch, color and probability of supernova in data are hosted by galaxies with less than certain threshold mass. We use Union 2.1’s compilation Suzuki et al. (2012) magnitude vs redshift table which used fitted values for coefficients of stretch, color and the probability that a particular supernova in dataset was hosted by a low-mass galaxy. The dataset also employs a constant M≈-19.31 with uncertainties in distance modulus arising from fitting values and systematic contributions mentioned separately as distance modulus error which we incorporated in our model fitting using TRF and dog leg (Voglis & Lagaris (2004)) and \( \chi^2 \) analysis, and so it is absorbed in the parameter error bounds provided H0 is set to an optimal value.

3 DATASET AND DATA ANALYSIS TECHNIQUES

For our study, we use Union 2.1 (Suzuki et al. (2012)) dataset publicly shared by Supernova Cosmology Project (SCP) (Perlmutter (1999) Perlmutter & Schmidt (2003) Amanullah et al. (2010)). The dataset is comprised of 580 type Ia supernovae which passed the usability cuts. The dataset is comprised of redshift range 0.015 ≤ z ≤ 1.414 with median redshift at z ≈ 0.294.

We use SciPy’s (Jones et al. (2001)) optimize package’s trust region reflective (TRF) and dog leg (dogbox) algorithms (Voglis & Lagaris (2004)), which are suitable for problems with constraints as in our case, to fit dark energy density parameter and dynamic dark energy EoS parameters for A-CDM, WCDM, CPL, JBP, BA, PADE-I, PADE-II and LH4 models (Barboza & Alcaniz (2008) Chevallier & Polarski (2001) Linder (2003) Jassal et al. (2005a) Jassal et al. (2005b) Linder & Huterer (2005) Wei et al. (2014)). We also apply grid method to obtain maximum likelihood (Davis & Parkinson (2016)) for WCDM and CPL to compare results obtained through TRF and dog leg methods (Voglis & Lagaris (2004)). We used TRF and dog box options simultaneously with our selected models and then used the best fit results based on the \( \chi^2 \) values.

4 DYNAMIC DARK ENERGY EQUATION OF STATE (EOS)

\[ I(z) = \exp\left(3\int_0^z \frac{1 + w_{de}(z')}{1 + z'} dz'\right) \]  

In order to extend the standard Lambda-CDM model to incorporate dynamic dark energy EoS, we can define I(z) as:

For the study we tested various dynamic dark energy EoS models.

We started with standard flat Lambda-CDM model with \( w_{de}=-1 \) and then tested WCDM model by treating \( w_{de} \) as free parameter. Then we moved towards more complex CPL, JBP, BA, PADE, I, PADE-II and LH4 models (Barboza & Alcaniz (2008) Chevallier & Polarski (2001) Linder (2003) Jassal et al. (2005a) Jassal et al. (2005b) Linder & Huterer (2005) Wei et al. (2014)) with model equations as:

\[ w_{de}(z) = w_0 + w_a \frac{z}{(1+z)} \]  

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\[ w_{de}(z) = \frac{w_0 + w_a}{1 + \frac{z}{(1 + z)^2}} \] (9)

BA (Barboza & Alcaniz (2008))

\[ w_{de}(z) = \frac{w_0 + w_a}{1 + \frac{z}{(1 + z)^2}} \] (10)

PADE-I (Wei et al. (2014))

\[ w_{de}(z) = \frac{w_0 + w_a}{1 + \frac{z}{(1 + z)^2}} \] (11)

For \( w_b = 0 \), PADE-I reduces to CPL model.

PADE-II (Wei et al. (2014))

\[ w_{de}(z) = \frac{w_0 + w_a ln \left( \frac{1}{1 + z} \right)}{1 + \frac{w_a}{1 + \frac{z}{(1 + z)^2}}} \] (12)

Linder-Huterer (LH4) (Linder & Huterer (2005))

\[ w_{de}(z) = w_0 + \frac{\left( w_a - w_0 \right)}{1 + \frac{z}{(1 + z)^2}} \] (13)

For parameter boundaries for TRF and dog leg analysis, we set \( \Omega A \) boundary between 0.65 and 0.75. For \( w_0 \), we set the upper boundary as \( w_0 < -1/3 \) which is a pre-condition for accelerated expansion of our universe but for lower limits we first set restrict it to \( w_0 \geq -1 \) to exclude phantom dark energy (Vikman (2005) Farnes (2018)) and keeping it we first set restrict it to for accelerated expansion of our universe but for lower limits we first set restrict it to \( w_0 \geq -1 \) to exclude phantom dark energy (Vikman (2005) Farnes (2018)) and keeping it we first set restrict it to

\[ DL(z) = \frac{cz}{H_0} \left[ 1 + \frac{(1 - q_0)z}{2} - \frac{(1 - q_0 - 3q_0^2 + j_0)z^2}{6} + O(z^3) \right] \] (14)

With \( q_0 = -0.55 \) and \( j_0 = 1 \).

We can see from tables 1 and 2 that our best measurements based on \( \chi^2 \) values for both CPL and WCDM are obtained through H0=70 km s\(^{-1}\) Mpc\(^{-1}\) which is measured by Abbott et al. 2017 by studying gravitational waves (GW170817) (LIGO (2017) Abbott et al. (2016) Abbott et al. (2017) from neutron stars collision and was also measured by Wilkinson Microwave Anisotropy Probe (WMAP) (Bennett et al. (2013)) with WMAP only dataset. Our second best measurements were obtained through the best fit H0=69.18473827 \( \pm \) 0.50179901 or approximately 69.185 km s\(^{-1}\) Mpc\(^{-1}\) value from Union 2.1 dataset using kinematic expression for luminosity distance which is closer to the value obtained by (Bennett et al. (2013)) using WMAP+eCMB+BAO+H0 data set (Hinshaw et al. (2013)). We applied TRF with bounds 65 \( \leq \) H0 \( \leq \) 75 to obtain the best fit H0 value. Both of these values are interestingly somewhat in the middle region of the H0 values obtained by early universe studies (Gorbunov & Rubakov (2011)) like Planck cosmic microwave background (CMB) (Planck (2014a) Planck (2014b) Planck (2015) Planck (2018) Wojtak & Adriano 2019 (2019) Vattis et al. (2019) Riess et al. (2019)). The problem has become even more interesting as the expansion rate is found to be same in all directions by Soltis et al. 2019 based on 1000 type Ia supernovae sample. (Soltis et al. (2019)) Therefore we considered it appropriate to measure CPL and WCDM model parameters by fixing H0 values from Planck 2018, Riess 2018, Abbott et al. 2017, Planck+SN+BAO-Planck 2018, Planck+BAO/RSD+WL-Planck 2018, H0LiCOW 2018 and DES 2018 (Abbott et al. (2017) Birrer et al. (2018)). We also fit our own value for Union 2.1 dataset (Suzuki et al. (2012)) using the kinematic expression from Riess et al. 2016 (Riess et al. (2016)) for luminosity distance with source redshift of \( z < 0.04 \). Figure 1 shows that luminosity distances from (14) is in good agreement with luminosity distances from (4) for \( z < 0.04 \) using various EoS models.

The kinematic expression from Riess et al. 2016 is written as:

5 HUBBLE CONSTANT VALUE

The value of Hubble Constant has recently been a topic of great interest in physics and astronomy community. It had been studied in the past like the first precise measurements by Sandage 1958 (Sandage (1958)) which gave H0=75 but recent interest has increased as the measurements of H0 from cosmic microwave background (CMB), baryon acoustic oscillations (BAO), standard candles and others do not seem to agree with each other (Freedman (2017) Jackson (2015) Planck (2018) Wojtak & Adriano 2019 (2019) Vattis et al. (2019) Riess et al. (2019)). The problem has become even more interesting as the expansion rate is found to be same in all directions by Soltis et al. 2019 based on 1000 type Ia supernovae sample. (Soltis et al. (2019)) Therefore we considered it appropriate to measure CPL and WCDM model parameters by fixing H0 values from Planck 2018, Riess 2018, Abbott et al. 2017, Planck+SN+BAO-Planck 2018, Planck+BAO/RSD+WL-Planck 2018, H0LiCOW 2018 and DES 2018 (Abbott et al. (2017) Birrer et al. (2018)). We also fit our own value for Union 2.1 dataset (Suzuki et al. (2012)) using the kinematic expression from Riess et al. 2016 (Riess et al. (2016)) for luminosity distance with source redshift of \( z < 0.04 \). Figure 1 shows that luminosity distances from (14) is in good agreement with luminosity distances from (4) for \( z < 0.04 \) using various EoS models.

The kinematic expression from Riess et al. 2016 is written as:

\[ DL(z) = \frac{cz}{H_0} \left[ 1 + \frac{(1 - q_0)z}{2} - \frac{(1 - q_0 - 3q_0^2 + j_0)z^2}{6} + O(z^3) \right] \] (14)

With \( q_0 = -0.55 \) and \( j_0 = 1 \).

We can see from tables 1 and 2 that our best measurements based on \( \chi^2 \) values for both CPL and WCDM are obtained through H0=70 km s\(^{-1}\) Mpc\(^{-1}\) which is measured by Abbott et al. 2017 by studying gravitational waves (GW170817) (LIGO (2017) Abbott et al. (2016) Abbott et al. (2017) from neutron stars collision and was also measured by Wilkinson Microwave Anisotropy Probe (WMAP) (Bennett et al. (2013)) with WMAP only dataset. Our second best measurements were obtained through the best fit H0=69.18473827 \( \pm \) 0.50179901 or approximately 69.185 km s\(^{-1}\) Mpc\(^{-1}\) value from Union 2.1 dataset using kinematic expression for luminosity distance which is closer to the value obtained by (Bennett et al. (2013)) using WMAP+eCMB+BAO+H0 data set (Hinshaw et al. (2013)). We applied TRF with bounds 65 \( \leq \) H0 \( \leq \) 75 to obtain the best fit H0 value. Both of these values are interestingly somewhat in the middle region of the H0 values obtained by early universe studies (Gorbunov & Rubakov (2011)) like Planck cosmic microwave background (CMB) (Planck (2014a) Planck (2014b) Planck (2015) Planck (2018) or baryon acoustic oscillations (BAO) (Grieb et al. (2017) Macaulay et al. (2019)) which give H0= 67 and standard candles studies like (Riess et al. (1998) Riess et al. (2007) Riess et al. (2016) Riess et al. 2018a (2018a) Riess et al. (2018b) Pietrzyński et al. (2019) Riess et al. (2019)) which give H0 > 73. Because of this discrepancy in the measurement of H0 higher redshift studies of type Ia supernovae and other standard candles are becoming important (Risaliti & Lusso 2019 (2019) Riess et al. 2018a (2018a) Daniel et al. (2019)). Like early universe studies, standard candles are also useful to study the nature of dark energy (Wood-Vasey et al. (2007)) which is still an open problem of cosmology (Davis et al. (2007) Davis & Parkinson (2016)).

In order to understand how H0 value affects the measurements of dynamic dark energy EoS model parameters, we simply cross-correlated the data in tables 1 and 2. Figures 2 and 3 show how the measurement or choice of the Hubble Constant can affect the measurements of dynamic dark energy EoS parameters in WCDM and CPL models. We can clearly observe significant negative cross-correlation between \( w_0 \) and H0 for both WCDM and CPL models.

These results are particularly interesting due to the Hubble Constant tension arising due to the differences in measurements of H0 through cosmic microwave background, standard candles and other techniques.
In figure 4, for WCDM model, the maximum likelihood fit values are \( \Omega \Lambda = 0.712^{+0.039}_{-0.021} \) and \( w_0 = -0.995^{+0.070}_{-0.092} \). Their corresponding mean likelihood fit values are \( \Omega \Lambda = 0.724 \pm 0.030 \) and \( w_0 = -1 \pm 0.065 \). Both maximum and mean likelihood values agree, within one sigma overlapping values, with the best fit values obtained by using TRF and dog leg methods for \( H_0=70 \). The best values from tables 1 and 8 are \( \Omega \Lambda = 0.720362 \pm 0.00626 \) and \( w_0 = -1.00449 \pm 0.1435 \). Values in table 1 are rounded off to fit in the columns.

In figure 5, for CPL model, the maximum likelihood fit values are \( \Omega \Lambda = 0.687^{+0.043}_{-0.014} \) and \( w_a = -0.35^{+0.97}_{-0.92} \). Their corresponding mean likelihood fit values are \( \Omega \Lambda = 0.731 \pm 0.080 \) and \( w_a = -1.02 \pm 0.015 \) and \( w_a = -0.0126 \pm 0.030 \). Both maximum and mean likelihood values agree, within one sigma overlapping values, with the best fit values obtained by using TRF and dog leg methods for \( H_0=70 \). The best values from tables 2 and 8 are \( \Omega \Lambda = 0.71933 \pm 0.07885 \) and \( w_a = -0.01126 \pm 3.033239 \). Values in table 1 are rounded off to fit in the columns. For wa, there is a relatively larger standard deviation in both likelihood estimates and in TRF and dog leg optimization approaches which is likely due to smaller redshift coverage from type Ia supernovae sample from Union 2.1. On very large redshifts, wa almost plays an equal role as \( w_0 \) in CPL model because on extremely large \( z' \) values, \( w_{de}(z) \) approximately becomes \( w_0 + wa \). However in case of a model like JBP, the model will be more or entirely dependent on \( w_0 \). This means higher redshift surveys especially highly sensitive all sky surveys like galaxy surveys to study the late time integrated Sachs-Wolfe effect (ISW) (Sachs & Wolfe 1967 [1967]Afshordi (2004)Rahman & Iqbal 2019 [2019]) or surveys studying the early universe signatures like cosmic microwave background radiation (CMB) or baryonic acoustic oscillations (BAO), can play an important part in estimating parameters like \( wa \) or other extended EoS

Table 1. Best fit values for WCDM model using union 2.1 dataset

| H0    | \( \Omega \Lambda \) | \( w_0 \) | \( \chi^2 \) | Bounds on \( \Omega \Lambda, w_0 \) |
|-------|-----------------|--------|---------|---------------------------|
| 67.400 | 0.75 | -0.7024 | 606.6761 | (0.65,0.75),(-1/3) |
| 73.520 | 0.75 | -1.7459 | 641.5908 | (0.65,0.75),(-1/3) |
| 70,000 | 0.72 | -1.0045 | 562.2257 | (0.65,0.75),(-1/3) |
| 72,500 | 0.65 | -1.5683 | 588.7033 | (0.65,0.75),(-1/3) |
| 67,770 | 0.75 | -0.7353 | 594.2718 | (0.65,0.75),(-1/3) |

Table 2. Best fit values for CPL model using union 2.1 dataset

| H0    | \( \Omega \Lambda \) | \( w_0 \) | \( wa \) | \( \chi^2 \) | Bounds on \( \Omega \Lambda, w_0, wa \) |
|-------|-----------------|--------|--------|---------|---------------------------|
| 66,300 | 0.65 | -0.333 | -3.601 | 620.9512 | (0.65,0.75),(-1/3),(-5.5) |
| 66,300 | 0.65 | -0.333 | -3.601 | 620.9512 | (0.65,0.75),(-1/3),(-5.5) |
| 66,300 | 0.75 | -0.567 | -0.300 | 650.2256 | (0.65,0.75),(-1/3),(-3.0) |
| 67,400 | 0.65 | -0.333 | -4.676 | 586.7344 | (0.65,0.75),(-1/3),(-5.5) |
| 67,400 | 0.75 | -0.333 | -4.676 | 586.7344 | (0.65,0.75),(-1/3),(-5.5) |
| 67,400 | 0.75 | -0.664 | -0.300 | 602.6112 | (0.65,0.75),(-1/3),(-3.0) |
| 67,770 | 0.65 | -0.419 | -4.326 | 579.2176 | (0.65,0.75),(-1/3),(-5.5) |
| 67,770 | 0.75 | -0.697 | -0.300 | 590.9692 | (0.65,0.75),(-1/3),(-3.0) |
| 67,770 | 0.75 | -0.697 | -0.300 | 590.9692 | (0.65,0.75),(-1/3),(-3.0) |
| 68,340 | 0.65 | -0.584 | -3.491 | 571.3333 | (0.65,0.75),(-1/3),(-5.5) |
| 68,340 | 0.65 | -0.584 | -3.491 | 571.3333 | (0.65,0.75),(-1/3),(-5.5) |
| 68,340 | 0.75 | -0.749 | -0.300 | 577.1210 | (0.65,0.75),(-1/3),(-3.0) |
| 69,185 | 0.65 | -0.830 | -2.278 | 564.2394 | (0.65,0.75),(-1/3),(-5.5) |
| 69,185 | 0.65 | -0.830 | -2.278 | 564.2394 | (0.65,0.75),(-1/3),(-5.5) |
| 69,185 | 0.75 | -0.827 | -0.300 | 565.2861 | (0.65,0.75),(-1/3),(-3.0) |
| 70,000 | 0.72 | -1.005 | -0.011 | 562.2257 | (0.65,0.75),(-1/3),(-3.0) |
| 70,000 | 0.72 | -1.005 | -0.011 | 562.2257 | (0.65,0.75),(-1/3),(-3.0) |
| 70,000 | 0.72 | -1.000 | 0.039 | 562.2260 | (0.65,0.75),(-1/3),(-3.0) |
| 70,000 | 0.72 | -1.000 | 0.039 | 562.2260 | (0.65,0.75),(-1/3),(-3.0) |
| 72,500 | 0.67 | -1.727 | 2.402 | 583.9811 | (0.65,0.75),(-1/3),(-5.5) |
| 72,500 | 0.65 | -1.598 | 0.300 | 587.6217 | (0.65,0.75),(-1/3),(-5.5) |
| 72,500 | 0.75 | -1.000 | -1.142 | 623.7649 | (0.65,0.75),(-1/3),(-5.5) |
| 72,500 | 0.75 | -1.000 | -1.142 | 623.7649 | (0.65,0.75),(-1/3),(-5.5) |
| 73,520 | 0.65 | -2.101 | 3.575 | 603.5328 | (0.65,0.75),(-1/3),(-5.5) |
| 73,520 | 0.65 | -1.773 | 0.300 | 613.0368 | (0.65,0.75),(-1/3),(-3.0) |
| 73,520 | 0.75 | -1.000 | -1.950 | 682.8480 | (0.65,0.75),(-1/3),(-5.5) |
| 73,520 | 0.75 | -1.000 | -1.950 | 682.8480 | (0.65,0.75),(-1/3),(-5.5) |
model parameters can make major contributions in higher redshifts in various dynamic dark energy equation of state (EoS) models which are in discussion in this study.

To see how dynamic dark energy EoS evolves in JBP, BA, PADE-I, PADE-II, and LH1 models especially in comparison the results from the flat \( \Lambda \)-CDM model with constant \( w_{de} = -1 \), WCDM and CPL models, we again applied TRF and dog leg methods (Voglis & Lagaris (2004)) simultaneously and selected the best fit values based on \( \chi^2 \) criteria.

We can see in figure 6 that for \( H_0 = 70 \) km \( s^{-1} Mpc^{-1} \), the results are closer to \( \Lambda \)-CDM model with constant \( w_{de} = -1 \) except for BA model which is in quintessence regime and PADE-II which is a bit farther than \( w_{de} = -1 \) in comparison with others. However, due to large standard deviations from mean for \( w_a \), \( a_t \) and \( T \) parameters in CPL, JBP,BA, PADE-I, PADE-II and LH4 models (Barboza & Alcaniz (2008)Chevallier & Polarski 2001 (2001) Linder (2003) Jassal et al. (2005a) Jassal et al. (2005b) Linder & Huterer (2005) Wei et al. (2014)) for relatively smaller redshift objects like in Union 2.1 dataset of type Ia supernovae, we still need to test these models using early universe signatures like CMB and BAO. For our type Ia supernova dataset with relatively smaller redshift coverage in comparison with they early universe studies, we can see that \( \Lambda \)-CDM model with \( w_{de} = -1 \) as fixed value is still the preferred model based on Bayesian information criterion (BIC) (Schwarz (1978) Arevalo et al. (2017) Liddle 2007 (2007)) especially if we consider \( \Delta BIC \) values which are basically the difference of BIC values from our models in discussion with the lowest BIC obtained from these models. \( \Delta BIC > 2 \) suggests positive evidence against a model with higher BIC and \( \Delta BIC > 6 \) suggests strong evidence against higher BIC value models (Kass & Raftery (1995)) as BIC heavily penalizes the inclusion of newer parameters (Liddle 2007 (2007)) despite having better \( \chi^2 \) scores for non-\( \Lambda \)-CDM models. This can change for higher redshift or early universe studies when extra parameters in dynamic dark energy EoS models are potentially going to play important role which will also be useful for \( H_0 \) studies (Gorbunov & Rubakov (2011) Planck (2018) Riess et al. (2019) Risaliti & Lusso 2019 (2019) Poulin et al. (2019) Liu et al. (2019)).

For \( H_0 = 69.185 \) km \( s^{-1} Mpc^{-1} \), we first look at figure 7 and observe that PADE-I is showing most deviation from \( w_{de} = -1 \) in comparison with the others especially at higher redshifts. This difference in scale of deviation towards \( w_{de} = -1 \) is due to the relatively higher contribution of \( w_a \) and \( w_b \) of PADE-I model with increasing redshift values. In figure 8, we remove PADE-I model to see the evolution of \( w_{de}(z) \) in other models. We can see that apart from JBP, which is moving towards quintessence regime, others are closer to phantom
regime (Vikman (2005)Farnes (2018)) with BA and PADE-II deviating away more from \(w_{de}=-1\) and towards phantom regime. Theoretically, all structures in our universe would be eventually ripped apart by the repulsive force associated with the phantom dark energy (Vikman (2005)Weinberg (2008)). It will be interesting to see if future high precision standard candles, early universe and other surveys can settle expansion rate debate and which \(w_{de}\) evolution or best fit value will be associated with it as we can observe from figures 2 and 3 that expansion rate and dark energy EoS parameters have significant cross-correlation with each other.

We can also see from figures 6, 7 and 8 that despite H0 values being <2% different from each other, their impact on \(w_{de}(z)\) evolution is significant for all the models. This difference is significant enough to impact our understanding of the scales and evolution of our universe which warrants the need to carefully model \(w_{de}(z)\) in observations of early universe signatures, galaxy surveys, standard candles, standard rulers and recently discovered gravitational waves which can be used as standard sirens (Schutz (1999)Jarvis et al. (2014)Rahman (2018)Chen et al. (2018)). Gravitational waves can also be used to study the gravitational wave strain signals from type Ia supernovae and we can use them to study cosmological parameters. For this purpose it will be useful to carefully study the progenitors of the type Ia supernovae (Keichi & Terada 2016 (2016)Rahman (2018)) as the mass profiles of the objects involved will be crucial in modeling the expected signal (Schutz (1999)Rahman (2018)).

7 CONCLUSION

We studied various dynamic dark energy EoS models and also discussed the key EoS parameter \(w_0\) in relation with the Hubble Constant. We also observed strong negative correlation between the Hubble Constant and EoS parameter \(w_0\). This relation is also studied in relation with different H0 values obtained from various surveys adopting different techniques to constraint the cosmological parameters especially H0. We found that the models we tested agreed mostly with standard cosmological model predictions. We also observed that the extended dynamic dark energy equation of state (EoS) models we tested agreed with the idea of a universe going through an accelerated expansion phase. We also observed that the value of \(w_0\), which provides value of \(w_{de}(z)\) at z=0 or the current epoch, is in quite close to the standard \(\Lambda\)-CDM constant value of \(w_{de}=-1\) with \(w_0=-1\) in the confidence interval of one sigma. For the Hubble Constant value of H0 ≈ 69.185, which we fit on Union 2.1 dataset using kinematic expression for luminosity distance, we found that best fit values for dynamic dark energy EoS models deviate from the constant \(w_{de}=-1\). However, \(\Lambda\)-CDM with constant value of \(w_{de}=-1\) still comes as the preferable model based on the BIC selection criteria. However this deviation, even in the EoS models with higher number of parameters, shows the importance of studying H0 in relation with \(w_{de}(z)\). Based on our results, we can also conclude that by carefully modeling and studying \(w_{de}(z)\), we can potentially resolve the Hubble Constant tension arising from the results obtained using different techniques.

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### Table 3. Best Fit Parameters for Dynamic Dark Energy EoS models using Union 2.1 SN type Ia Dataset

| EoS Models | p | q | r | s | t | x | y | T | Ch-Sign | Tau | H0C |
|------------|---|---|---|---|---|---|---|---|---|--------|-----|-----|
| AcEOM (n=0, a=1) | 0.732205 ± 0.0015 | -0.00069 ± 0.0004 | -1 | 5.2857 | 1 | 524.48 | 5 | 528.50 | 5 |
| WCQOM | 0.7166 ± 0.0325 | -0.00065 ± 0.0003 | -0.00069 ± 0.0004 | 5.2857 | 1 | 524.48 | 5 | 528.50 | 5 |
| M | 0.7166 ± 0.0325 | -0.00065 ± 0.0003 | -0.00069 ± 0.0004 | 5.2857 | 1 | 524.48 | 5 | 528.50 | 5 |
| R | 0.7166 ± 0.0325 | -0.00065 ± 0.0003 | -0.00069 ± 0.0004 | 5.2857 | 1 | 524.48 | 5 | 528.50 | 5 |
| PQR-0 | 0.7166 ± 0.0325 | -0.00065 ± 0.0003 | -0.00069 ± 0.0004 | 5.2857 | 1 | 524.48 | 5 | 528.50 | 5 |
| LEW | 0.7166 ± 0.0325 | -0.00065 ± 0.0003 | -0.00069 ± 0.0004 | 5.2857 | 1 | 524.48 | 5 | 528.50 | 5 |

### Note

The table above lists the best fit parameters for various Dynamic Dark Energy EoS models using the Union 2.1 SN type Ia dataset. Each model is represented by specific parameters (p, q, r, s, t, x, y, T) along with their uncertainties and other relevant values such as Ch-Sign, Tau, and H0C.
