Fiber Organization has Little Effect on Electrical Activation Patterns during Focal Arrhythmias in the Left Atrium

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Abstract—Over the past two decades there has been a steady trend towards the development of realistic models of cardiac conduction with increasing levels of detail. However, making models more realistic complicates their personalization and use in clinical practice due to limited availability of tissue and cellular scale data. One such limitation is obtaining information about myocardial fiber organization in the clinical setting. In this study, we investigated a chimeric model of the left atrium utilizing clinically derived patient-specific atrial geometry and a realistic, yet foreign for a given patient fiber organization. We discovered that even significant variability of fiber organization had a relatively small effect on the spatio-temporal activation pattern during regular pacing. For a given pacing site, the activation maps were very similar across all fiber organizations tested.

Index Terms—Cardiac electrical activation propagation, Focal arrhythmia activation pattern, Fiber organization, Heart modeling, Left atrium, Patient-specific

I. INTRODUCTION

Over the past two decades, there has been steady progress towards the development of realistic electrophysiological models [1]–[3] of cardiac propagation with detailed description of gross cardiac anatomy [4]–[6], myocardial fiber organization [7], [8], and ionic currents involved in the generation of cardiac action potentials [9]–[11]. At the same time, the focus of modeling studies has been gradually shifting from the investigation of fundamental mechanisms of cardiac arrhythmias [12]–[16] towards clinical applications [17]–[19]. The range of applications that are actively been explored includes screening anti-arrhythmic [20], [21] and pro-arrhythmic [22], [23] drugs, optimizing anti-tachycardia [24] and anti-fibrillation [25], [26] pacing protocols, and aiding arrhythmia ablation procedures [27], [28], among others.

One of the key obstacles towards the development of constructing patient-specific models is not only speed in simulations [11] but adequate quantification, verification and information for such models [29], [30]. Detailed models have a very high-dimensional parameter space; moreover, most of the parameters cannot be measured on an individual basis and need to be postulated based on some generic values or recomputed libraries [31], [32]. In this study, we investigate the problem of incorporating realistic myocardial fiber organization into an atrial conduction model, and their effect in the electrical propagation.

The velocity of action potential propagation along fibers is typically 2-3 times faster than across fibers [33], which makes fiber organization one of the key factors determining activation patterns in cardiac tissue [34], [35]. Unlike atrial geometry, which can be readily obtained in a clinical setting via electroanatomical mapping [36] or from clinical imaging, fiber organization cannot be obtained with a sufficient level of detail in live tissue [37]. To date, the best available resource for real patient fiber data is an ex-vivo Diffusion Tensor Magnetic Resonance Imaging (DT-MRI) fiber database [38], [39], which required about 50 hours to scan each atrium [38].

The focus of our study is on modeling the left atrium, which is known to harbor the most common sources of atrial fibrillation and thus is particularly important with regard to potential clinical applications. Here we explore 3D chimeric models of the left atrium informed by a patient-specific DT-MRI-derived geometry and various fiber direction patterns adopted from the DT-MRI fiber database. Specifically, we investigate models with identical atrial geometry but with different fiber organization. By assuming one of these models as the ground-truth, we assess the capability of the others for predicting activation patterns produced by focal excitation sources located in different areas of the left atrium.

The rest of this paper is organized as follows: We first describe the procedure for constructing in-silico heart models utilizing clinically derived patient-specific left atrial geometry and realistic fiber organization. We then show our results of heart models with different fiber organization in terms of Local Activation Time (LAT) and spatial activation patterns. In the discussion, we validate our models and explain the results in terms of a cancellation effect across the atrium. Finally, we describe the potential application to fiber-independent patient-
specific heart models, list the limitations of our work, and summarize the main contributions.

II. METHOD

A. Ground-Truth and Chimeric Models

To generate ground-truth and chimeric models of action potential propagation in the left atrium, we used a DT-MRI fiber database [39]. It contains data sets from seven left atria, which are denoted as LA1, LA2, . . . , LA7. The different fiber organizations are denoted as F1, F2, . . . , F7, each can be subdivided into endocardium and epicardium fiber organizations, and are denoted as Fendo1, Fendo2, . . . , Fendo7 and Fepi1, Fepi2, . . . , Fepi7, respectively. Endocardial and epicardial triangular meshes are denoted as Mendo1, Mendo2, . . . , Mendo7 and Mepi1, Mepi2, . . . , Mepi7, respectively.

We used Mendo1 and F1 to generate ground-truth models of action potential propagation and surrogate clinical activation maps. Mendo1 was chosen because LA1’s fiber organization was found to be the optimal one, giving the least LAT errors among the seven left atria for bi-layer model atrial fibrillation simulations [39]. We used Mendo1 and F2, F3, . . . , F7 to generate six chimeric models. We called them chimeric models because foreign fibers were registered onto the left atrium.

Fig. 1 illustrates the generation of all models. The process involves generation of 3D Cartesian nodes around Mendo1, mapping of the respective fiber organizations onto the nodes, simulation of action potential propagation from stimuli in different locations, and generation of surrogate clinical activation maps. The latter were then used for evaluation of chimeric model performance in predicting activation patterns across different fiber patterns.

B. Atrium mesh and fiber processing

Triangular mesh Mendo1 has 63,112 vertices, 125,501 faces, and its average triangle edge length is 0.67 mm. The seven fiber organizations are registered onto Mendo1, resulting in seven different models (one ground-truth model and six chimeric models). This procedure requires morphing other meshes and fibers onto Mendo1. Details in Appendix A.

The next step is to create 3D Cartesian nodes wrapped around Mendo1, as shown in Fig. 2. First, a cubic space that contains Mendo1 is filled with equally spaced nodes (1 mm spacing), then nodes that are further than a distance threshold from the Mendo1 surface are removed. Here, the left atrium tissue thickness is set at 2 mm, which is in the range of clinically observed values [40], [41], and the total number of Cartesian nodes is 60,691. The endocardium and epicardium layers split the tissue thickness in half, each having a thickness of 1 mm. The spatial resolution is chosen to be adequate (refer to Appendix C) for accurate simulation without being computationally demanding.

Each node of the grid was attributed to either the epicardial or endocardial layer, based on its location with respect to Mendo1. The appropriate layer was determined by the angle between the vector pointing to the respective node from the center of the closest triangle and the normal vector of this
Fig. 2. Labeling of endocardial and epicardial nodes. (a) $V_1$ and $V_2$ are vectors pointing from the nearest triangle center to the respective node. $V_N$ is the surface normal vector. Because the angle between $V_1$ and $V_N$ is less than 90°, the magenta node was labeled as an epicardial node; if the angle is larger than 90°, the blue node was labeled as an endocardial node. If the angle is equal to 90°, then randomly assign it to endocardial or epicardial node. (b) All left atrial nodes labeled as epicardial (magenta) or endocardial (blue) nodes.

C. Heart model equations

To simulate action potentials, we used the Mitchell-Schaeffer model [42], as shown in (1). This model was chosen because its simplicity makes it efficient in 3D numerical simulations, and the parameters provide direct insight into changes in electrophysiological behavior. It models the inward current caused by sodium and calcium ion channels, outward changes in electrophysiological behavior. It models the inward current caused by sodium and calcium ion channels, outward changes in electrophysiological behavior. It models the inward current caused by sodium and calcium ion channels, outward changes in electrophysiological behavior.

\[
\begin{align*}
\frac{du}{dt} &= \frac{hu^2(1-u)}{\tau_{in}} - \frac{u}{\tau_{out}} + J_{stimulus} + \nabla \cdot (D \nabla u) \\
\frac{dh}{dt} &= \begin{cases} 
\frac{1-h}{\tau_{open}} & \text{if } u < u_{gate} \\
\frac{-h}{\tau_{close}} & \text{if } u \geq u_{gate}
\end{cases}
\end{align*}
\]

For each node, fiber anisotropy is introduced via a $3 \times 3$ diffusion tensor $D$ according to (2).

\[
D = d \begin{pmatrix} I & (1 - r) f f^T \\
(1 - r) f f^T & I \end{pmatrix}
\]

\[
r = \frac{r_{f} - r_{L}}{r_{L}} = \left( \frac{CV_T}{CV_L} \right)^2
\]

where $d$ is the diffusion coefficient, $r$ is the anisotropy ratio, a ratio of fiber’s transverse to longitudinal diffusion coefficients, or the ratio of transverse to longitudinal conduction velocities squared. $I$ is the identity matrix, and $f$ is a $3 \times 1$ unit vector pointing along the fiber direction.

The differential equations (1) were solved using the explicit Euler method on the Cartesian nodes. We followed [44] that assumed no-flux boundary conditions and used a 19-node stencil. The time step is 0.01 ms and the spatial resolution is 1 mm. Such resolution is adequate, refer to Appendix B for details. The parameters are given nominal values as shown in Table I [45], [46]. With this setting, the Conduction Velocity (CV) will be around 0.69 m/s, which is a typical value for the atrium. The model is implemented in MATLAB (MathWorks, Natick, Massachusetts, United States) and accelerated with GPU computing using CUDA kernels (Nvidia, Santa Clara, California, United States).

| Parameter | Value |
|-----------|-------|
| $\tau_{in}$ | 0.3 ms |
| $\tau_{out}$ | 6 ms |
| $\tau_{open}$ | 120 ms |
| $\tau_{close}$ | 150 ms |
| $u_{gate}$ | 0.13 mV |
| $r_{f}$ | 0.2 |
| $r_{L}$ | 1 |

III. Results

A. Fibers vary significantly across different atria

To compare fiber organization in different models of the left atrium, all fiber patterns were registered onto the same mesh ($M_{endo}$). To compare the different fibers, we made a reference frame transformation, details refer to Appendix B. The comparison was done separately for the endocardial and epicardial fibers. The correlations of fiber orientations are in the range between -0.12 and 0.18 (with the exception of auto-correlation). Low correlation indicates significant variation in fiber organization across different atria, which is consistent with earlier observations [7], [38].

B. Fibers vary within the left atrium

Importantly, we found that in all atria the fiber pattern of the endocardium is very different from that of the epicardium. We define $\Delta \theta$ as the fiber angle difference between endocardial and epicardial fibers in a given location. Fig. 3 shows the $\Delta \theta$ maps in all seven left atria analyzed in this study; red represents small and blue represents large $\Delta \theta$. No large regions have the same $\Delta \theta$ value, and there is no regularity in the spatial distribution of the small and large $\Delta \theta$ regions in all atria tested.

Table II shows the relative area occupied by smaller $\Delta \theta$ regions ($\Delta \theta \leq 45^\circ$) in different atria. Areas with smaller $\Delta \theta$ manifest higher effective anisotropy, which contributes to increased sensitivity of activation patterns to fiber organisation.
C. Fibers do not significantly affect activation pattern

1) Local activation time: To test the effects of fiber organization on activation patterns, we generated 42 simulations: six pacing sites each with seven different fiber patterns. The six pacing sites, shown in Fig. 1(e), are P1: Right Inferior Pulmonary Vein (RIPV), P2: Left Inferior Pulmonary Vein (LIPV), P3: Right Superior Pulmonary Vein (RSPV), P4: Left Superior Pulmonary Vein (LSPV), P5: Mitral Valve (MV), and P6: Bachmann’s Bundle (BB).

The resulting LAT maps are shown in Fig. 4. All maps are on $M_{endo}$. The first column shows the results obtained in the ground-truth model built upon the intrinsic left atrium geometry and the fibers $F_{endo}$ and $F_{epi}$. Columns 2-7 are LAT maps obtained in models in which the intrinsic fiber was substituted with other fibers. The LAT maps of each row are similar, indicating that differences in fiber organization do not significantly affect the activation pattern.

To compare quantitatively the activation maps generated using the chimeric models with the ground-truth we calculated LAT errors (see Table III). The average LAT error is 7.8 ms, which is relatively small compared to the time it takes for the activation to travel through the entire left atrium (4.3% of 180 ms). The LAT correlation is also quite high ranging between 0.94 and 0.99 for different pacing sites and across all models. Also, we noticed that average LAT error (Table III bottom row) correlates with the relative size of the region with small $\Delta \theta$ (Table II).

2) Latest activation location: Fig. 5 shows the latest activation locations in the left atrium. Blue regions indicate where LAT values are within 5 ms of the maximum LAT; The first column shows the results obtained in the ground-truth model with the intrinsic fiber pattern. Other columns are results obtained with different fiber organizations. Notably, in most of cases, the regions of late activation have similar locations, indicating that different fiber organizations do not alter the start-to-end activation wave patterns significantly. For example, rows P3 and P6 have consistent latest activation locations regardless of fiber organization; rows P2 and P4 show some instances with dilated regions, and row P1 has some different locations, but the clusters are still on the opposite side of the pacing site, which means the general activation pattern remains the same.

The quantitative analysis is summarized in Table IV. For each simulation illustrated in Fig. 5 columns F2-F7, we compute the distance of the center of the blue area to the truth. For most scenarios, the distances are small (median: 9.43 mm; average: 13.47 mm) compared to the size of the left atrium mesh $M_{endo}$ (84 mm $\times$ 97 mm $\times$ 88 mm). Notably, the average across different pacing sites does not change significantly from model to model (see the last row in Table IV). Yet, for some pacing sites, they are consistently better than for other pacing sites, for example, compare rows P5 and P6.

D. Fibers have local effects on activation propagation

Although differences in fiber organization have little effect on the large scale, they can be observed locally. Fig. 6 shows
The regions of the latest activation derived from the activation maps shown in Fig. 4. The view angles are adjusted for each of pacing sites for better visualization. The first column (boxed) shows the results in the ground-truth model. Blue regions show the sites with the LAT values within 5 ms from the maximum. The average maximum LAT of the 42 scenarios is 180 ms. The last column shows the superposition of the latest activation regions for all fiber organizations. The magenta dots show the pacing sites on the back side of the atrium. For a given pacing site, the position of the blue regions are reasonably well preserved across different fiber organizations.

**TABLE IV**

|     | F2  | F3  | F4  | F5  | F6  | F7  | Avg  |
|-----|-----|-----|-----|-----|-----|-----|------|
| P1  | 3.37| 34.63| 38.75| 16.80| 38.30| 24.17| 26.00 |
| P2  | 6.95| 1.86 | 1.73 | 8.02 | 1.38 | 3.17 | 3.85  |
| P3  | 17.28| 14.10| 15.83| 10.72| 10.60| 12.62| 13.52 |
| P4  | 2.95| 3.16 | 4.24 | 6.13 | 6.32 | 17.12| 6.65  |
| P5  | 26.09| 18.61| 26.81| 26.09| 23.61| 27.40| 24.77 |
| P6  | 9.72| 4.68 | 5.23 | 2.49 | 8.33 | 5.78 | 6.04  |
| Avg | 11.06| 12.84| 15.43| 11.71| 14.76| 15.04| 13.47 |

Latest activation location difference is the distance between the center of the blue area for F2-F7 and F1 (the truth). The mean is 13.47 mm and the median is 9.43 mm, which are relatively small compared to the left atrium size, indicating that the latest activation locations for different fiber organizations are similar. Note that the values in rows P5 and P6 are very different, which demonstrates that the latest activation location differences depend on pacing location.

Activation maps generated using models F2 and F3 paced at P2. The activation propagates in the direction of the red-dashed arrow. In the F2 row, the activation propagation slows down in the orange circled area (it has a smaller red region) because the fiber direction is perpendicular to the activation direction. In the F3 row, because fiber direction is along the activation direction, it accelerates propagation and results in a larger red area. Such acceleration and slow-down effects occur throughout the left atrium. The cyan-circled area is another example: propagation is slower in the F2 row than F3 row, resulted in more blue area.

**E. Fiber effects with different anisotropy ratios**

**TABLE V**

| r    | 0.1 | 0.2 | 0.5 |
|------|-----|-----|-----|
| LAT Error (ms) | 10.6 or 5.5% | 7.8 or 4.3% | 3.15 or 2.0% |

The percentage is with respect to the time it takes the activation to travel through the left atrium. For \( r = 0.1, 0.2, 0.5 \), that time is 192, 180, 155 ms respectively.

The majority of simulations of atrial propagation use values of \( r \) between 0.1 and 0.2. Some publications use \( r = 0.11 \) \cite{47, 48}, while some others use \( r = 0.2 \) \cite{49}. To investigate
Fig. 6. Local effect of fiber organization. Upper and lower panels show LAT maps (left) and fiber organization (center and right) in models F2 and F3, respectively. Red arrows: propagation direction. Black lines: prevailing fiber direction in the areas of interest (orange and cyan ovals). The LAT gradient inside the orange oval is conspicuously greater in F2 than in F3 (red-green vs red-yellow). This is because the endocardial and epicardial fibers in F2 are perpendicular to the activation direction, resulting in slower propagation. This is not the case for F3 where propagation is parallel to fibers. Such local effects exist throughout the entire atrium, but they do not alter the overall activation pattern much.

The robustness of our conclusions, we performed the same experiment as described in Section III-C with $r = 0.1$ and $r = 0.5$, and the results are summarized in Table V. The small errors suggest that the anisotropy ratio is not the main factor responsible for the low sensitivity of the large scale activation patterns to specific fiber organization.

F. Uncouple the two fiber layers

The reason for fiber organization having little effect on the activation patterns at the large scale could be partly due to significant differences in fiber orientation on the epicardial and endocardial layers (see Fig. 5) which cause reduction in apparent anisotropy. But the left atrial wall was not always two layers in all parts [5], [50]. To evaluate the contribution of this effect on the large scale activation we generated seven models in which epicardial layers were assigned the endocardial fiber orientations, thus producing identical fiber orientation in both layers, which is equivalent to completely uncouple the endocardial layer from the epicardium.

We hypothesized that this modification should amplify the apparent anisotropy, and consequently the differences between activation patterns in different models. Using these modified models, we performed the same experiment as described in Section III-C and compared the results. It might be expected, the errors in LAT in the models with identical epicardial and endocardial fiber orientations was greater than in the original models, however, the difference was not very large (9.1 ms vs 7.8 ms).

This suggests that the difference in the endocardium and epicardium fiber orientations is not the main factor responsible for low sensitivity of the large scale activation patterns to specific fiber organization.

IV. DISCUSSION

A. Contemporary heart modeling with fibers

Electrophysiological heart modeling is becoming increasingly realistic with an aim to guide atrial fibrillation ablation procedures. One important element of a heart model is the fiber organization. However, accurate and high-resolution fiber data is not available for clinical use. The major difficulty of obtaining such in-vivo fiber organization is the time it takes to scan the fibers. The current DT-MRI technology can scan high-resolution fibers in about 50 hours [38], which is clinically impractical.

One way to quickly obtain fiber organization is to mathematically calculate synthetic fibers based on atrial geometry [8], [51]–[54]. However, these fibers are artificial and do not represent the truth. Another option is to register foreign fibers. It has been found that one certain patient’s atrial fiber organization can be generalized to many different patients [39].
Fig. 7. The cancellation effect explained via the fibers: there is no macro level consistent fiber organization, therefore local fiber effects cancel each other. (a, b) show fiber’s local effect. Blue lines represent endocardium fiber, magenta lines represent epicardium fiber. Aniso region has a small $\Delta \theta$ (about 0°), and Iso region has a large $\Delta \theta$ (about 90°). Red arrows are activation wave traveling directions. For activation scenario (a), The Aniso region has a slow down effect on the activation; However, for activation scenario (b), The Aniso region has a speed up effect on the activation. And for both (a) and (b), the propagation speed in the Iso region are similar. (c, d, e) show that the local effects cancel each other at the macro level. (c) is endocardium fiber, (d) is epicardium fiber, and (e) is $\Delta \theta$ map. Along the activation path marked by the red dashed arrow, the propagation speed increases in the top Aniso region, then it goes trough an Iso region that does not change the speed much, finally it enters the bottom Aniso region and the speed decreases. In summary, activation speed first increases then decreases, therefore the effects cancel each other.

In this paper, we tried an alternative way, we ask the question: how big an effect does fiber organization have on activation patterns? If the effects are small, then we could create a clinically practical heart model that does not involve fiber data.

\subsection*{B. Cancellation effect}

Our data suggest that the effects of fiber organizations are cancelled because fiber organizations vary across the left atrium. This cancellation effect can be explained at two levels.

At the micro level, depending on the location of the activation origin, wave propagation can be shaped by the local fiber organization. As shown in Fig. 7(a, b), they represent the same tissue region but has different activations. Aniso region has small $\Delta \theta$ (about 0°), it is highly anisotropic, and have the strongest local effect on activation propagation; Iso region has large $\Delta \theta$ (about 90°), it is highly isotropic, and have the least effect on activation propagation. For Iso region, activation waves will travel through them in a similar manner regardless of the direction of travel. For Aniso region, depends on the direction of the activation wave, its travelling speed can be decreased (as shown in (a)) or increased (as shown in (b)).

At the macro level, the local effects from the micro level will be cancelled by each other. As shown in Fig. 7(c, d, e), along a global path, the activation wave can travel through areas that increase and areas that decrease propagation speed, resulting in an overall zero effect.

Quantitatively, the propagation speed along a path is shown in Fig. 8. The path is the geodesic line (minimum distance path) from the activation origin (point A) to the latest activation location (point B) as shown in (a), and it can be found via Dijkstra’s algorithm. The path has a length of about 120 mm. The propagation speed of a point along the path is calculated as distance divided by time, where distance is the geodesic length from that point to point A, and the time is the LAT values difference between that point and point A. We can see that the propagation speeds vary around its average value along the path as shown in (b), and the average propagation speeds are similar regardless of fiber organizations as shown in (c). Such phenomenon happened for all 42 scenarios. Because of this phenomenon, the fiber organization’s local effect does not accumulate along the path, resulting in a small macro effect.

\subsection*{C. Potential applications: fiber-independent model}

In our previous research [56], we showed that we can construct a model of the left atrium to reproduce clinical electroanatomical mapping data [57] without incorporating fiber organization. We validated our fiber-independent model
with data from 15 patients and the performance was good: the average absolute LAT error was 5.47 ms for sinus rhythm and 10.97 ms for flutter and tachycardia, and the average correlation was 0.95 for sinus rhythm and 0.81 for flutter and tachycardia. There are also other studies support the use of heart model without fibers [58]–[60].

In this work, because we found that fibers did not significantly affect activation patterns, we expect that the fiber-independent model can be useful for predicting fibrillation source locations. Because the fiber-independent model only requires heart geometry and electrograms, it could be integrated into contemporary electroanatomical mapping systems to provide real-time atrial fibrillation ablation guidance.

V. LIMITATIONS

We found that fiber organization does not significantly affect activation patterns. However, there are important limitations to the scenarios we considered that informed this finding.

1) We focused only on sustained sources, such as focal atrial tachycardia, which may occur following atrial fibrillation ablation. It is possible that non-stationary sources such as drifting rotors could be affected by fibers.

2) We did not consider scars, which could play an important role in atrial fibrillation dynamics [61].

3) The Mitchell-Schaeffer model we used is not a detailed ionic model, and it may not be a good choice to model complex rhythms such as atrial fibrillation. Still such a two-component model is good for modeling periodic propagation.

We utilized the computationally more efficient mono-domain model. It is well established that more detailed bi-domain models are required for accurate simulation of electrical activity in the immediate vicinity of the stimulating electrodes and for modelling electrical defibrillation [62]. With regard to accuracy, the bi-domain models have practically no advantages over mono-domain models for simulations of propagation patterns of external stimulus [63].

4) We simplify fiber organization into only two layers. The real left atrium has many more layers, and the number of layers also varies in different regions, as does the atrial thickness. If more layers were incorporated, we would need to study if the effects of fibers would become stronger as well as whether the fiber direction changes abruptly or gradually through the thickness.

5) We examined fiber organizations of the left atrium, because the most common atrial fibrillation sources are in the left atrium; therefore, it has more available clinical electroanatomical mapping data than the right atrium. We have not examined whether our findings would hold in the right atrium.

VI. CONCLUSION

In this paper, we found that 1) Fiber organization varies significantly across different left atria and within a left atrium. 2) Fibers have local effects on activation propagation but such local effects cancel each other at the macro level. And 3) fibers do not significantly affect activation pattern.

In summary, for left atrial focal arrhythmia, we found that the global activation patterns do not seem to be significantly affected by fiber organization. Therefore, fiber organization may not be essential for accurate heart modeling of arrhythmias in the left atrium, and thus more practical heart models for some clinical applications could be fiber-independent models. It has been shown that there is an asynchrony and dissociation between activation on the epicardium and endocardium during atrial fibrillations [64]–[66]; these and our results further suggest that it may be more important to consider the role of the complex trabecular network [10], [66], [67] during arrhythmias.

APPENDIX A

Fiber registration

The seven endocardium and seven epicardium fibers are registered onto the left atrium 1’s endocardium mesh. Name left atrium 1’s endocardium the target mesh, other endocardium or epicardium mesh a moving mesh. First, rotate the moving mesh to roughly align with the target mesh. Next, apply a rigid ICP algorithm to optimize the alignment. Lastly, apply a non-rigid ICP algorithm to morph the moving mesh into the shape of the target mesh [68]. Fig. 9 shows an example of the mesh morphing. The blue mesh is the target mesh, the red mesh (can be endocardium or epicardium) is the moving mesh. (a) shows the original meshes. (b) shows the red mesh is morphed into the shape of the blue mesh. We can see that the morphed mesh matches the target well, on average, the distance between the nearest vertex pairs between the morphed mesh and the target mesh is: 1.4 +/- 0.5 mm. Then we need to morph the fiber data.
Fig. 9. Fiber registration. (a) and (b) are the before and after mesh morphing. (c) shows how to transform fiber from the original mesh (red mesh in (a)) to the morphed mesh (red mesh in (b)). Given xyz coordinates of $A$, $B$, $C$, $A_1$, $B_1$, $C_1$, and the fiber (3D vector $f$), calculate $f_1$. We have $f = aA_1B_1 + bA_1C_1$, by solving for $a$ and $b$, we can obtain $f_1 = aA_1B_1 + bA_1C_1$, then make $f_1$ unit length, it will be the morphed fiber. (Note that $f = aA_1B_1 + bA_1C_1$ will give 3 equations, since $AB$, $AC$ and $f$ are co-plane, any 2 of the 3 equations will give the same $a$ and $b$.)

(c) shows the process: it is a reference frame transformation. The final step is to register the morphed fiber to the target mesh, this can be done by copying the fibers that are nearest to the target mesh’s triangles.

Appendix B

Compare Different Fiber Organizations

Fig. 10. Transform fiber data. (a) Fiber $f$ resides in a triangle of the mesh. The reference vector $e$ created from a fixed edge, all the seven different fibers will reference this vector $e$. (b) The 3D fiber vector is transformed into a 1D value. Examples of fibers ($f_1$, $f_2$, and $f_3$) and their transformed values ($\theta_1$, $\theta_2$, and $\theta_3$).

There are seven different fiber organizations registered onto the left atrium 1’s endocardium mesh. Fibers are 3D vectors as shown in Fig. 10(a), it is not convenient to directly compute correlations among 3D vectors. Therefore, we make a reference frame transformation so that the 3D fibers ($f_1$, $f_2$, and $f_3$) are represented as 1D values ($\theta_1$, $\theta_2$, and $\theta_3$) as shown in (b). The reference frame is a vector created by a fixed edge of each triangle (vector $e$ in (a)), and the transformation is to compute the angle between the fiber and that fixed edge ($\theta$ in (b)) follow the right hand rule of the triangle face normal vector. (Note that it is a property of a triangular mesh, that the sequence of the 3 vertices of a triangle are in such a way that we can utilize the right-hand-rule to find out the triangular face normal vector that points outwards of the mesh.) After this transformation, we can easily compute the correlations between different fiber organizations.

Appendix C

Spatial and Temporal Resolutions

Fig. 11. Top: Conduction velocity (CV) values (the dots) of implementing different resolutions. The dashed blue line is a quadratic fit of the dots. Bottom: CV percentage differences compare to the asymptotic value, which is the value of the dashed blue line at 0 mm spatial resolution. For spatial resolution of 1 mm and temporal resolution of 0.01 ms (the big red dot), the CV percentage difference is 3.6%, much smaller than the adequate 10%.

We ran simulations on a slab of 100 mm $\times$ 4 mm $\times$ 4 mm. (The long length of the slab is to help increase CV computational accuracy.) The heart model parameters are chosen such that the CV values are close to the physical values \cite{9}. Assume isotropic conduction, or no fiber organization. Spatial resolutions are set to 0.2, 0.3, ..., 2.0 mm. Temporal resolution $dt$ is set to 0.01 ms. Results are summarized in Fig. 11. In this paper, spatial resolution is 1 mm and temporal resolution is 0.01 ms. Using such resolutions, the accuracy of CV is adequate with a deviation from the asymptotic value of 3.6%, which is much smaller than the usual required 10%.

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J. D. Bayer et al. [32] and M. Valderrásabo [39] examined the influence of anisotropic conduction properties in the propagation of the cardiac action potential. A. Wachter [31] and G. R. Krueger [31] explored the Mesh structure-independent modeling of patient-specific atrial fiber orientation.

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J. Whitaker et al. [38] described the role of myocardial wall thickness in atrial arrhythmogenesis. S. Ho et al. [7] and J. Whitaker et al. [38] also contributed to the understanding of the heart's functional architecture.

R. McFarlane, C. M. Sanchez-Quijano, and E. Kayvanpour [10] explored the creation and application of virtual patient cohorts of heart models. H. J. Arevalo et al. [17] and S. Ho et al. [7] contributed to the synchronization of the heart's electrical activity.

I. Elaff [44] modeled the role of myocardial wall thickness in atrial arrhythmogenesis. C. M. Sanchez-Quijano, E. Kayvanpour, and H. J. Arevalo [17] contributed to the development of personalized virtual-heart technology for guiding the form for studying clinical atrial fibrillation.

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