Integrated optimization of multiproduct multiperiod transportation and inventory under a carbon cap constraint for online retailers

Yaorong Cheng* and Yijun Li

School of Traffic & Transportation Engineering, Central South University, Changsha 410075, China

*Corresponding author. E-mail: yaorong@csu.edu.cn

Abstract

This study aims to solve the problem of multiproduct multiperiod integrated transportation and inventory optimization for online retailers. A carbon cap constraint and multitype of capacitated trucks are simultaneously incorporated into the proposed mixed-integer program. A simulated annealing (SA) algorithm is designed. CPLEX 12.9.0 is used to solve the submodel obtained from the neighbourhood search and is also used to get the optimal solutions for instances. Experimental results show that the simulated annealing algorithm can find satisfactory solutions within a reasonable time. When the problem size increases, the growth of the computational time of the SA algorithm is significantly smaller than that of the CPLEX. A sensitivity analysis for the carbon cap is also conducted. The results indicate that if the carbon cap is gradually tightened, the total cost increases first with a gentle slope, and then with a remarkable slope, same as for the total number of trucks used; the total carbon emissions first decrease with a gentle slope, and then decline with a significant slope. When the carbon cap is strict, only a few different types of trucks will be considered. The percentage changes of the total cost increase and the total carbon emission reduction are also compared. When the allowed carbon emissions are gradually reduced, situation of a higher amount of carbon emission reduction and a lower amount of cost increase can be achieved. Additionally, using multiple types of trucks in the integrated optimization of transportation and inventory decisions can achieve greater cost savings with lower increments of carbon emission.

Keywords: integrated optimization; multiperiod; multiproduct; carbon emission; simulated annealing

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1. Introduction

In 2020, the online retail sales in China reached 11.76 trillion yuan, taking up nearly a quarter of total retail sales [1]. The online retail market brings opportunities to enterprises that are also accompanied by challenges. Transportation and inventory decisions account for a large proportion of the operating costs for online retailers. Factoring transportation decisions into the inventory replenishment model can further achieve greater cost savings than separately optimizing the two parts [2, 3]. Thus, integrated optimization of transportation and inventory for online retailers is crucial. As the concept of sustainable development has received increasing emphasis and attention from the public, China committed in the 2016 Paris Agreement that the national carbon dioxide emissions per unit of GDP will be reduced by 60%–65% compared to 2005 by 2030 [4]. Thus, carbon emissions limits should be regarded as an ethical boundary or a threshold over which companies might be punished. The carbon emissions generated during transportation and inventory are mainly determined by transportation frequency, energy used and inventory control decisions [5]. A reasonable transportation and inventory decision is a significant measure to reduce carbon emissions and achieve sustainable development [6–8].

Until now, 46 cities have been chosen for the green freight distribution pilot project in China [9, 10]. In this project, the transportation and distribution links will be reconstructed from four dimensions, namely equipment update, management optimization, carbon emission measurement and restrictions, to achieve the goal of green and low-carbon development. The ‘Development Plan for the New Energy Vehicle Industry (2021–2035)’ issued by the State Council of the People’s Republic of China stipulates that starting from 2021, public areas in the National Ecological Civilization Pilot Area and the key areas for air pollution prevention and control will add or update no less than 80% new-energy vehicles for public transportation, taxis and logistics vehicles [11]. In this process, more and more new-energy vehicles will be put into use in the retail industry. In China, the online retail industry, with huge development potential, has three main characteristics: (i) the customer base is very large; (ii) their products can be sold in 24 hours; and (iii) both the trading frequency and the delivery frequency are very high. Thus, the transportation and inventory replenishment behaviour of online retail must be placed within the scope of government carbon restriction supervision. Based on the online retail industry, an optimization model and a heuristic for online retailers are proposed, incorporating carbon emissions and multitype truck selection. Further, the analysis of the sensitivity of carbon caps also helps the government to set reasonable carbon caps.

The dynamic lot-sizing model is one of the classic models in the field of inventory control. This model was first proposed by Wagner and Whitin [12], and mainly solves the lot-sizing problem under time-varying demand. It is also the core module of the MRP (material requirement planning) system, and has practical applications in the field of MRP confronting dynamic demands [13]. Many online retailers use advanced resource management systems in their order fulfillment process, which contains some management modules, similar to MRP, related to decisions, such as replenishment and transportation [14]. Because the transaction time and location of online retail are determined by customers, the occurrence of online retail business is not restricted by time and space. Thus, studying the dynamic lot-sizing transportation and replenishment problem for online retailers is significant from both theoretical and practical perspectives. We will conclude the related studies about integrated optimization of transportation and inventory based on the dynamic lot-sizing model, and also the related researches considering carbon emissions based on the dynamic lot-sizing model.

Transportation decisions have been incorporated into inventory models in several studies (eg. [15–18]). Anily and Tzur [16] adopted dynamic programming to solve the multiproduct dynamic lot-sizing problem with capacitated vehicles. Lu et al. [18] considered multiproduct replenishment strategies through self-operated and outsourced transportation modes, and designed a heuristic algorithm based on Lagrangian relaxation. Kim and Lee [17] analysed the multiproduct dynamic lot-sizing problem with carrying capacity, and considered the differences in the volume of different products. They designed three metaheuristics, namely the simulated annealing algorithm, the genetic algorithm and the self-evolution algorithm to solve the model. Palak et al. [19], and Akbalik and Rapine [15] investigated integrated optimization of transportation and inventory through multiple types of trucks. Akbalik and Rapine [15] explored the dynamic lot-sizing problem for one type of product, but with multiple
replenishment modes, and designed a two-approximation algorithm to solve this problem.

Existing researches regarding carbon emissions mainly focus on three aspects: one is to add the constraints of carbon emissions according to specific policy requirements; the second is to include carbon tax into the objective function of the integrated optimization model of transportation and inventory; the third is to treat carbon emission as a separate objective function, together with the cost objective function, solving a multiobjective optimization problem [5, 20–26]. To our knowledge, there are only three works that incorporate carbon emissions into the dynamic lot-sizing model. Absi et al. [20] proposed four forms of carbon emission constraints based on the single product dynamic lot-sizing model, and proved that the single product dynamic lot-sizing problem with total carbon cap is NP-hard. This work is further extended by Absi et al. [21]. They investigated the dynamic lot-sizing problem of a single product with a carbon emission constraint in each period, and used dynamic programming to solve it. Helmrich et al. [23] also analysed the single product dynamic lot-sizing problem under the carbon cap constraint. They first used the Lagrangian relaxation heuristic to obtain a feasible solution and the lower bound of the original problem, and then proposed two approximate algorithms.

Engebrethen and Dauzère-Pérès [27] concluded that there were few studies based on the multiproduct dynamic lot-sizing model. The existing researches on inventory management of online retailers are mainly focused on macro policies, such as inventory coordination and distribution, while there are few studies on micro-level activities directly related to inventory management, such as replenishment decisions [14]. We also find that based on the dynamic lot-sizing model, the study related to integrated optimization of transportation and inventory in the background of online retail is even rarer. Among the inventory control studies, most of the transportation costs are only expressed as the linear function of the transportation volume, and the restriction of transportation capacity is ignored. In terms of the lot-sizing model with environmental factors, there are only three papers, which indicate that this area is really unexplored. Thereby, this study aims to help online retailers to make scientific decisions on multiproduct multiperiod replenishment plans when various types of trucks can be selected to complete a replenishment task under a carbon cap constraint.

The contributions of this research include: (i) the model proposed can be improved and embedded in the MRP system of online retailers to help them make scientific decisions on multiproduct multiperiod multitask replenishment plans under a carbon cap constraint. It has good extendibility, and can provide a basis for studying other transportation and inventory integrated optimization problems; (ii) the designed simulated annealing algorithm has three neighbourhoo strategies and can find a feasible solution, if it exists, for each lot-sizing plan with a little computational resource. The proposed algorithm provides a solution method for this NP-hard problem in solving large-scale instances; (iii) the sensitivity analysis of carbon cap can provide theoretical support for the government to establish a reasonable and economical carbon cap; (iv) the sensitivity analysis also indicates that if the carbon cap is very strict, only a few types of trucks will be considered. So, it motivates online retailers to adopt more environmentally friendly trucks and incorporate carbon emissions into their decision-making.

2. Problem description and mathematical model

Online retailers are currently facing increasingly personalized and diversified customer needs. This study considers an online retailer who needs to determine the replenishment quantity of each product in each period, and determine which type of truck selected to complete a replenishment task in order to minimize the total cost and respect the constraint of the carbon cap. This retailer needs to decide three key issues: (i) the replenishment period and quantity for each product; (ii) the type of trucks to complete a replenishment task and the corresponding number of trucks that need to be used; and (iii) how to incorporate carbon emissions into the integrated optimization model. In practical applications, the integrated optimization of retailers’ transportation and inventory is confronted with the selection of transportation trucks with different capacities, different freight rates and different energy consumption. When the government or relevant departments set a cap on the transportation and inventory carbon emissions, the retailers need to incorporate carbon cap constraints into their decision-making, and strive to achieve the lowest total cost of transportation and inventory under the carbon constraint.
2.1 Assumptions

Assume that a planning period $T$ contains multiple discrete periods $t$. $d_{mt}$ denotes the deterministic demand of product $m$ in period $t$. The unit inventory holding cost of product $m$ is $h_m$. $I_m$ denotes the inventory level of product $m$ at the end of period $t$. If the initial or ending inventory is not 0, for example when the initial inventory for product $m$ is $e_{m0}$, it can be easily transformed into a situation where the initial inventory is 0 by changing the demand of product $m$ in the first period by: $d_{m1} = d_{m1} - e_{m0}$. Similarly, when the ending inventory of the product $m$ needs to be maintained at $h_m$, only the demand for the last period of product $m$ needs to be changed as: $d_{mT} = d_{mT} + h_m$. So, it can be transformed into a situation where the ending inventory is 0. Therefore, without loss of generality, we assume that the initial and ending inventory of all products are all 0, which is $l_{m1} = l_{mT} = 0$ for all product $m$.

The set load capacity of truck $n$ is $v_n$. One truck can load different types of products replenished during one period. $x_{mtn}$ is the quantity of product $m$ transported by truck $n$ in period $t$. $z_n = \sum_{m \in M} x_{mtn}/v_n$. The fixed cost for truck $n$ to complete one replenishment task is $S_n$. The unit cost for using one truck is $A_n$; this cost is linearly related to the number of trucks used. Therefore, the transportation cost that the retailer needs to pay can be expressed as $S_n + A_n z_n$. As each truck type has different weights and fuel-use efficiency, the carbon emission of each truck type is different. Therefore, both Konur [24] and Absi et al. [21] consider representing the transportation carbon emissions by the emissions generated by different truck types and the load they carry. The carbon emissions of completing one replenishment task are constituted by the fixed carbon emission $e_{0n}$ and the carbon emission generated by transporting one unit product $e_{1n}$ ($e_{0n}$ can denote the fixed carbon emission when truck $n$ backhauls without any products; $e_{1n}$ is the carbon emission that is linearly related to the quantity transported). Thus, the total carbon emission during period $t$ can be expressed as: $\sum_{n=1}^{N} (2e_{0n} z_n + e_{1n} \sum_{m=1}^{M} x_{mtn})$. $f_{mtr}$ and $y_n$ are binary variables, respectively. $f_{mtr}$ represents whether the demand for the product $m$ in period $r$ is satisfied by the replenishment lot-sizing in period $t$. Because demand cannot be split, $\sum_{t=1}^{T} f_{mtr} = 1$. $y_n$ indicates whether truck $n$ is selected to complete the task during the period $t$.

2.2 Notations

Sets and parameters

- $T$: set of discrete time periods, indexed by $t = 1, 2, ..., T$
- $M$: set of products, indexed by $m = 1, 2, ..., M$
- $N$: set of different types of trucks, indexed by $n = 1, 2, ..., N$
- $d_{mt}$: demand of product $m$ in period $t$
- $h_m$: unit inventory holding cost of product $m$
- $S_n$: fixed cost of truck $n$ to complete a replenishment task
- $A_n$: unit cost for using one truck of type $n$
- $v_n$: available capacity of truck $n$
- $e_{0n}$: fixed carbon emission of truck $n$
- $e_{1n}$: fixed carbon emission of truck $n$ generated by transporting one unit product $n$
- $E$: carbon cap during the whole planning period
- $\eta$: a sufficiently large number

Decision variables

- $y_n$: binary variable, if truck $n$ is selected to complete the task in period $t$, $y_n = 1$; otherwise, $y_n = 0$
- $f_{mtr}$: binary variable, if the demand of product $m$ in period $r$ is satisfied by the replenishment lot-sizing in period $t$ ($1 \leq t \leq r \leq T$), $f_{mtr} = 1$; otherwise, $f_{mtr} = 0$
- $x_{mtn}$: the replenishment quantity of product $m$ in period $t$
- $l_{mt}$: inventory level of product $m$ in the end of period $t$
- $z_n$: number of trucks used for type $n$ in period $t$

2.3 Mathematical model

The problem described above can be expressed as a mixed-integer program (MIP):

$$\min \sum_{t \in T} \sum_{m \in M} (S_n y_n + A_n z_n) + \sum_{t \in T} \sum_{m \in M} h_m l_{mt}$$

(1)

$$\sum_{t \in T} \sum_{m \in M} (2e_{0n} z_n + e_{1n} \sum_{m \in M} x_{mtn}) \leq E$$

(2)

$$l_{m, t-1} + \sum_{n \in N} x_{mtn} - d_{mt} = l_{mt} \quad \forall m \in M, t \in T$$

(3)

$$\sum_{m \in M} x_{mtn} \leq z_{n} v_n \quad \forall n \in N, t \in T$$

(4)

$$z_n \leq \eta y_n \quad \forall n \in N, t \in T$$

(5)
The objective function (1) represents the total cost in the planning period, comprised of two parts: the transportation cost and the inventory holding cost. Constraint (2) represents the restriction on the carbon emissions generated by the transportation progress during the whole planning period. Equations (3) are the inventory balance equations. The ending inventory of this period is equal to the ending inventory of the last period plus the replenishment quantity in this period minus the demand in this period. Constraints (4) represent the load capacity for truck \( n \). Constraints (5), force \( z_{tn} \) to be positive when \( y_{tn} \) is one. Constraints (6) indicate that only one type of truck can be chosen to complete the replenishment task in one period. Constraints (7) ensure that the demand cannot be split. Constraints (8) depict that the maximum replenishment quantity for each product in each period does not exceed the sum of all demands in this period and the future periods. Constraints (9) indicate that when there is a replenishment task, the retailer will choose one type of truck to complete the task. Constraints (10) and (11) state binary and nonnegativity restrictions.

\[
\begin{align*}
\sum_{n \in N} y_{tn} & \leq T \quad \forall t \in T \\
\sum_{t=1}^{T} f_{mtr} & = 1 \quad \forall m \in M, 1 \leq t \leq r \leq T \\
\sum_{n \in N} x_{ntr} & \leq \sum_{r=t}^{T} d_{mtr} f_{mtr} \quad \forall m \in M, 1 \leq t \leq r \leq T \\
f_{mtr} & \leq \sum_{n \in N} y_{tn} \quad \forall m \in M, 1 \leq t \leq r \leq T \\
y_{tn}, f_{mtr} & \in \{0, 1\} \quad \forall n \in N, t \in T \\
x_{ntr}, y_{tn}, z_{tn} & \geq 0 \quad \forall m \in M, n \in N, t \in T
\end{align*}
\]

The simulated annealing algorithm is adopted and designed according to the researched problem. The simulated annealing algorithm (SA) was first proposed by Metropolis et al. [28], and has been widely used to solve combinatorial optimization problems [29–31]. This algorithm starts from an initial high temperature, a sufficient neighbourhood search will be implemented in a temperature layer, and then the temperature is continuously reduced by the cooling coefficient. The algorithm terminates when the temperature reaches the specified minimum value. The SA designed in this paper is described as follows.

1. Initial solution: the initial solution is generated lot-for-lot (\( x_{mtr} = 0 \)). We set the initial solution as the current solution and the current best solution.

2. Neighbourhood search: random selection of neighbourhood search strategy helps to achieve quick access to multiple neighbourhoods in a few steps. In order to enhance the richness of neighbourhood solutions, avoid premature convergence of the algorithm. The discrete probability allocation method is adopted to select one of the three neighbourhood search strategies. The three neighbourhood search strategies are as follows: (i) forward strategy: the replenishment quantity of multiple products moves in batches in the sequential direction of the time axis, which means adding a new replenishment period: randomly select one period \( t = 1, 2, \ldots, T-1 \), select the products that will be replenished in this period to form a set \( M_t \). If there is only one element \( i \) in the set \( M_t \), then the replenishment quantity of product \( i \) is partly moved to the next period. First perform transformation for demand of product \( i \) in period \( t+1 \): (a) \( x_{it} = x_{it+1} + x_{it} \). If there are multiple elements in set \( M_t \), implement transformation (a) and (b) for any \( \epsilon - 1 \) products (\( \epsilon \) denotes the number of elements in set \( M_t \)). (ii) Backward strategy: the replenishment quantity of multiple products moves in batches in the reverse direction of the time axis, which means reducing a replenishment period. Randomly select one period \( t = 1, 2, \ldots, T-1 \), select the products
that will be replenished in this period to form
a set \( M_2 \). For all \( i \in M_2 \), move their replenish-
ment quantity to the previous period \( j \) \((1 \leq j < t)\)
where there is a replenishment schedule. First
perform transformation for demand of product \( i \) in period \( j \): (c) \( x_{ij} = x_{ij} + x_{ij} \); then perform transformation for demand of product \( i \) in
period \( t \): (d) \( x_{it} = 0 \). (iii) Fine-tuning strategy: the fine-tuning strategy of a single product includes the forward and backward strategies for the replenishment quantity: randomly choose one product \( i \in M \) and randomly choose one period \( t \in T \). If the period selected is the first period \((t=1)\), then adopt the forward strategy and perform the transformation (a) and (b); if the period selected is the last period \((t=T)\), then adopt the backward strategy and perform the transformation (c) and (d). If the period selected is between the first and the last period \((2, \ldots, T−1)\), then the forward or backward strategy is randomly selected with a probability of 0.5. The selection probabilities of the three neighbourhood search strategies are, respectively, \( p_1 \), \( p_2 \), \( p_3 \). Here, we set \( p_1 = 0.2 \), \( p_2 = 0.3 \), \( p_3 = 0.5 \). After each neighbour-
hood search, for the new solution \( x' \), we get the replenishment arrangement for each period \( q \). \( q \) is a variable introduced here that represents the replenishment arrangement for period \( t \). If there exists one product \( m \) that satisfies \( x_{mt} > 0 \), then \( q_t = 1 \); otherwise, \( q_t = 0 \). Thus, once the replenishment matrix \( x_{mt} \) is determined, the variable \( q_t \) for each period is a fixed value. Subsequently, calculate the number of trucks required for each type of truck to complete the replenishment task in each period. Compute the corresponding transportation cost \( g_{tn} \) and carbon emissions \( E_{tn} \) for each type of truck in order to complete the replenishment task. We use CPLEX Optimization Studio 12.9 to solve the model P1

\[
P1 \quad \min \sum_{t \in T} \sum_{n \in N} g_{tn} y_{tn} \quad (12)
\]

\[
\sum_{n \in N} y_{tn} = q_t \quad \forall t \in T \quad (13)
\]

\[
\sum_{t \in T} \sum_{n \in N} E_{tn} y_{tn} \leq E \quad (14)
\]

\[
y_{tn} \in \{0, 1\} \quad \forall n \in N, t \in T \quad (15)
\]

The objective function (12) is to minimize the transportation cost. Constraints (13) repre-
sent that only one type of truck can be selected
in one period to complete a replenishment task. Constraints (14) ensure that the carbon emissions generated will not exceed the specified carbon cap. Constraints (15) represent the 0–1 integer restrictions.

If there is any feasible solution, for model
P1, return the binary variable matrix of \( y_{tn} \), and then obtain the total cost and total car-
bon emissions corresponding to the new solution. If there is no feasible solution for model
P1, save the new solution and obtain its cor-
responding total cost and total carbon emis-
sions, prepared for the next acceptance step. The effectiveness of this method is that, based
on the current replenishment arrangement, if there are multiple types of transportation trucks that can complete the replenishment task under the carbon cap constraint, the truck type with the lowest transportation cost can be quickly found.

Acceptance criteria: if the new solution is feasible and the corresponding total cost is lower than that of the current solution \((C(x') < C(x))\), replace the current solution with the new solution and check whether the current solution is better than the current best solution. If so, replace the current best solution with the current solution. If \( C(x') \geq C(x) \), then accept the new solution with the probability of \( e^{−(C(x')−C(x))/tem} > r \) (\( r \) is a random num-
ber generated between 0 and 1). For infeasible solutions (the solutions where the total carbon emissions exceed the carbon cap), we adopt the strategy of temporarily accepting the infeasible solution. If the corresponding total cost of the infeasible solution \( x'' \) is lower than the current solution \((C(x'') < C(x))\), then replace the current solution with the infeasible solution; otherwise, replace the current solution with the infeasible solution with the probability of \( e^{−(C(x')−C(x))/tem} > r \). The purpose of accepting infeasible solutions in this paper is to strive to transition to a better feasible solution through the infeasible solution [32].

(3) Stopping criterion: when the temperature \( tem < tem_{\text{min}} \), this algorithm terminates. Here we set \( tem_{\text{min}} = 0.001 \).

The flow chart of the simulated annealing algo-
rithm designed in this paper is shown in Fig. 1. In the following expressions, the algorithm is repre-
sented by SA.
4. Numerical experiment

4.1 Data sets

There are no standard classical data sets for the researched problem. Therefore, the parameters of the data sets in this paper are mainly set based on the research results of three well-known scholars [19, 25, 33] who are engaged in the research on the dynamic lot-sizing integrated optimization of transportation and inventory.

For the test data sets used in this paper, the product parameters and the cost parameters of various types of transportation trucks refer to Palak et al. [19]. They set the demands of various products generated from two ranges: high-U[500, 1 000]; low-U[100, 300], and set the inventory holding costs for products generated from two specific values, namely 1 and 5. So, we adopt the two-generation ranges for products’ demands, but we set the inventory holding costs generated from two ranges, namely high-U[1, 5] and low-U[1, 1]. It means that parameter \( h_m \) has the standard uniform distribution with a minimum of 1 and a maximum of 5 for the first range. Besides, we also refer to Palak et al. [19] to generate the truck cost parameters. Because the cost parameters might vary in different areas, so they might be adjusted as the values in China in the future. Based on the assumptions of truck capacity made by Mansini et al. [33], we set the capacity for three types of trucks to 1 000, 2 000 and 1 500, respectively. When considering six types of transport trucks, two truck types are generated according to the parameter for one truck type as presented in Tables 1 and 2.

The carbon emission parameters of different types of trucks \((e_{0n}, e_{1n})\) refer to Konur and Schaefer [25]. The carbon emission generated by transporting one unit of product, \(e_{1n}\), is rounded to two decimal places, the other parameters are all integers (see Table 1). The test data sets in this

**Table 1. Parameters for products used for instance generation**

| Product | Value |
|---------|-------|
| \(d_{sc}\) | High: U[500, 1 000]\<br>Low: U[100, 300] |
| \(h_m\) | High: U[1, 5]\<br>Low: U[1, 1] |

**Table 2 Parameters for trucks used for instance generation**

| Parameter | Truck type 1 | Truck type 2 | Truck type 3 |
|-----------|--------------|--------------|--------------|
| \(s_n\)   | U[150, 250]  | U[250, 350]  | U[200, 300]  |
| \(A_n\)   | U[600, 700]  | U[900, 1 100]| U[750, 850]  |
| \(v_{sn}\) | 1 000        | 2 000        | 1 500        |
| \(e_{0n}\) | U[25, 50]    | U[25, 50]    | U[25, 50]    |
| \(e_{1n}\) | U[0.5, 1]    | U[0.5, 1]    | U[0.5, 1]    |
Table 3. Problem size and type

| Problem type | Size | M | T | N | d<sub>nc</sub> | h<sub>nc</sub> |
|--------------|------|---|---|---|-------------|-------------|
| 1            | 1    | 1 | 10| 5 | Low         | Low         |
| 2            | 1    | 1 | 10| 5 | Low         | High        |
| 3            | 1    | 1 | 10| 5 | High        | Low         |
| 4            | 1    | 1 | 10| 5 | High        | High        |
| 5            | 2    | 1 | 10| 5 | Low         | Low         |
| 6            | 2    | 1 | 10| 5 | Low         | High        |
| 7            | 2    | 1 | 10| 5 | High        | Low         |
| 8            | 2    | 1 | 10| 5 | High        | High        |
| 9            | 3    | 1 | 10| 12| Low         | Low         |
| 10           | 3    | 1 | 10| 12| Low         | High        |
| 11           | 3    | 1 | 10| 12| High        | High        |
| 12           | 3    | 1 | 10| 12| High        | High        |
| 13           | 4    | 1 | 10| 12| Low         | Low         |
| 14           | 4    | 1 | 10| 12| Low         | High        |
| 15           | 4    | 1 | 10| 12| High        | Low         |
| 16           | 4    | 1 | 10| 12| High        | High        |

The parameters in each instance are randomly generated within the specified range. The transportation trucks with high fixed carbon emissions and high carbon emissions per unit load do not mean that their transportation costs are low. There is no direct connection between carbon emissions and transportation costs. There is also no direct connection between the carbon emissions e<sub>0n</sub> and e<sub>1n</sub>. This setting refers to Konur and Schaefer [25]. All mathematical models are solved by CPLEX Optimization Studio 12.9.0 (2-hour time limit). For small-size problems (T = 5), all instances can get optimal solutions; however, for large-size problems (T = 12, N = 6), the solving time for CPLEX increases significantly, and it is difficult to find optimal solutions within 2 hours. We use formula (16) to evaluate the algorithm performance.

\[
\text{Gap(heuristic)} = \left( \frac{C_{\text{heuristic}} - C_{\text{cplex}}}{C_{\text{cplex}}} \right) \times 100\% \tag{16}
\]

For formula (16), C<sub>cplex</sub> is the objective function value obtained by CPLEX, and C<sub>heuristic</sub> is the objective function value obtained by the SA algorithm. Two sets of experiments are implemented to explore: (1) the effectiveness of the SA algorithm proposed in this study; (2) the effects of different problem sizes (number of products, number of periods, number of truck types) on the proposed SA algorithm; (3) the impact of the changes in the carbon cap on the total cost, total carbon emissions, total number of trucks used and different types of trucks used; and (4) the comparison of the total cost increase percentage and the total carbon emission decrease percentage when the carbon cap changes.

4.2 The effectiveness of the SA algorithm

All algorithms in this study are coded in Python and implemented in Spyder 3.3.6. All the experiments are done on a laptop personal computer running Windows 7 with an Intel Core i5-4200M processor with 2.5 GHz and 4 GB of main memory. Based on preliminary tests, for the designed SA algorithm, when the initial temperature is 100, the termination temperature is 0.001, the cooling coefficient is 0.93, and the iteration number of each temperature layer (the number of inner loops) is 6. Its optimization performance is better. Table 4 summarizes the average optimality gap, total carbon emissions, total number of trucks used and the number of different types of...
Table 4. The results from the SA algorithm and CPLEX

| Problem type | SA Gap (%) | CPLEX Gap (%) | Carbon emission | SA Number of trucks used | CPLEX Number of trucks used | Time (s) | SA Number of different trucks used | CPLEX Number of different trucks used |
|--------------|------------|---------------|-----------------|--------------------------|-----------------------------|---------|--------------------------------------|---------------------------------------|
| 1            | 3.38%      | 0.00%         | 7 772.84        | 7 683.22                 | 7.00                        | 6.32    | 1.84                                 | 1.92                                  |
| 2            | 3.78%      | 0.00%         | 6 851.36        | 6 801.46                 | 6.52                        | 6.12    | 1.80                                 | 1.80                                  |
| 3            | 0.08%      | 0.00%         | 30 828.32       | 30 822.69                | 23.92                       | 23.92   | 1.52                                 | 1.52                                  |
| 4            | 0.00%      | 0.00%         | 27 294.70       | 27 296.56                | 27.44                       | 27.44   | 1.80                                 | 1.80                                  |
| 5            | 1.85%      | 0.00%         | 7 365.37        | 7 468.20                 | 6.96                        | 6.68    | 1.96                                 | 2.00                                  |
| 6            | 3.07%      | 0.00%         | 7 517.01        | 7 420.57                 | 7.60                        | 7.00    | 2.12                                 | 2.04                                  |
| 7            | 0.18%      | 0.00%         | 26 433.51       | 26 430.72                | 23.04                       | 22.96   | 1.28                                 | 1.28                                  |
| 8            | 0.00%      | 0.00%         | 27 247.70       | 27 247.85                | 23.96                       | 23.96   | 1.60                                 | 1.60                                  |
| 9            | 2.87%      | 0.01%         | 17 939.15       | 17 909.09                | 18.08                       | 16.36   | 2.12                                 | 1.92                                  |
| 10           | 3.43%      | 0.01%         | 18 605.08       | 18 559.87                | 17.72                       | 16.88   | 1.88                                 | 1.88                                  |
| 11           | 0.34%      | 0.01%         | 66 370.01       | 66 374.81                | 58.80                       | 58.44   | 1.80                                 | 1.96                                  |
| 12           | 0.03%      | 0.01%         | 19 198.81       | 19 127.58                | 16.60                       | 14.44   | 2.36                                 | 2.48                                  |
| 13           | 5.08%      | 0.26%         | 16 390.45       | 16 419.61                | 15.92                       | 15.04   | 1.92                                 | 2.00                                  |
| 14           | 2.64%      | 0.01%         | 69 514.91       | 69 513.16                | 56.08                       | 55.76   | 1.96                                 | 2.04                                  |
| 15           | 0.13%      | 0.01%         | 71 100.42       | 71 104.29                | 61.60                       | 61.56   | 1.92                                 | 1.96                                  |
| 16           | 0.02%      | 0.01%         | 30 170.51       | 30 153.94                | 26.71                       | 26.18   | 1.86                                 | 1.89                                  |
| Average      | 1.68%      | 0.02%         | 30 170.51       | 30 153.94                | 26.71                       | 26.18   | 1.86                                 | 1.89                                  |

Conclusion 1: (a) the proposed SA algorithm has great optimization performance. The average relative difference between the SA algorithm and the solutions found by CPLEX in terms of the total cost is 1.68; (b) when the problem size increases, the computational time for SA algorithm increases smoothly, while the CPU time for CPLEX increases significantly; (c) as for total carbon emissions, for the 4th, 5th, 14th and 16th problem types, the SA algorithm we proposed can find solutions with slightly lower carbon emissions than those of CPLEX.

Conclusion 2: (a) as the number of available trucks increases (for example: from N = 3 to N = 6), more different types of transport trucks can be used to achieve greater cost savings; (b) the designed SA algorithm performs well on different types of problems, and the algorithm is robust.

Conclusion 3: the proposed integrated optimization model of multiproduct multiperiod multitype of transport trucks under a carbon cap constraint is effective. The proposed simulated annealing algorithm can find satisfactory solutions to this problem within a reasonable time. The mixed-integer programming model and the algorithm provide scientific analysis methods and tools for integrated optimization of multiproduct, and multiperiod transportation, and inventory for online retailers.

4.3 Sensitivity analysis for the carbon cap

For each problem size, its 100 instances are divided into five categories according to five different carbon caps (E_1 < E_5). We take the average of the 20 instances in each category, in order to analyse the impact of the changes of carbon caps on the solutions. First, set the carbon cap E = E_5; the carbon cap constraint has the least impact on the solution at this moment, so the total cost obtained is also the minimum. Then the carbon cap constraint is gradually tightened, and we discuss the corresponding changes in total costs, total carbon emissions, the number of total trucks used, and the number of different types of trucks used. Further, we compare the percentage of total cost increase and the percentage of total carbon emissions decrease when the allowed carbon emissions decline.

4.3.1 Analysis of the changes on solutions. Here, a line chart is used to show the trend of changes in total cost, total carbon emissions, total number of trucks used, and different types of trucks used when the value of carbon cap changes.

From Figs. 2 and 3, it can be seen that when the allowed carbon emissions are gradually reduced (from E_5 to E_1), no matter for the optimal solutions found by CPLEX or the satisfactory solutions found by the SA algorithm, both will lead to higher...
Figure 2 reflects the changes in the number of different types of transport trucks used during the entire planning period when the allowed carbon emissions gradually decrease. From Fig. 5, the following conclusions can be drawn: (1) the number of different types of trucks first experiences an upward trend, and then is followed by a slight decrease as the carbon cap constraint changes; (2) when the value of carbon cap $E$ equals $E_2$, the number of different types of trucks used increases to the peak. If the carbon cap is continuously tightened ($E = E_1$), for problem types 1 and 2 (the problem setting when considering three different types of trucks), only one type of truck will tend to be selected during the entire planning period; for problem types 3 and 4, different types of trucks are rarely used throughout the planning period to complete replenishment tasks (from Fig. 5b, the value is around 1.2).
4.3.2 Analysis of the percentage changes. Fig. 6 shows the percentage changes of the total cost increase and the total carbon emissions reduction when the carbon cap constraint is gradually tightened. We first divide the 100 instances corresponding to the four problem sizes into five categories according to the five carbon caps ($E_1 < E_5$), and then take the average of the 20 instances within each category. Then change the carbon cap from $E_5$ to $E_4$, $E_4$ to $E_3$, $E_3$ to $E_2$, $E_2$ to $E_1$, respectively, and calculate the percentage changes of the total cost increase and the total carbon emissions reduction. Figs. 6a and 6b represent the total cost and carbon emissions percentage changes corresponding to the CPLEX solutions for four problem sizes. Figs. 6c and 6d show the total cost and carbon emissions percentage changes corresponding to the SA solutions for four problem sizes. The horizontal axis labels 4, 3, 2, 1 correspond to the carbon caps from $E_5$ to $E_4$, $E_4$ to $E_3$, $E_3$ to $E_2$, $E_2$ to $E_1$, which means starting from the maximum carbon cap $E_5$ and then gradually reducing the carbon caps. When the allowed carbon emissions decrease, from $E_5$ to $E_4$ or from $E_4$ to $E_3$ (corresponding to the horizontal axis labels 4, 3), it can be seen that no matter the solutions obtained by CPLEX or the solutions obtained by the SA algorithm, the percentage increase for total costs is less than the percentage reduction for total carbon emissions. This observation can be further illustrated when incorporating carbon emission factors into the integrated optimization of transportation and inventory: the percentage change of the total cost increase is less than the percentage change of the carbon emission reduction. However, when the allowed carbon caps are very small, such as from $E_2$ to $E_1$ (corresponding to the horizontal axis label 1), the percentage increase in the total cost and the percentage reduction in total carbon emissions are the same, and the former can even exceed the latter. Therefore, too strict carbon cap restrictions will have a great effect on the economic benefits of companies. This conclusion can provide
theoretical support for the government to establish a reasonable and economical carbon cap. It also illustrates that it is necessary and meaningful to take carbon emissions into the integrated optimization model.

5. Conclusions

Based on the dynamic lot-sizing model, this paper constructs a mixed-integer program for the researched problem, and designs a simulated annealing (SA) algorithm to solve it. The solutions obtained from the SA algorithm are compared with the solutions obtained from CPLEX 12.9.0. Experimental results show that the SA algorithm designed can find satisfactory feasible solutions within a reasonable time. Through a sensitivity analysis for the carbon cap, we found that when the carbon cap constraint is gradually tightened, total costs initially rise with a gentle slope, and then are followed by a sharp increase; total carbon emissions first decline marginally, and then decrease significantly. If the carbon cap is very strict, for example, $E$ is equal to $E_1$, there are very few truck types that can be selected, which will cause a rapid increase in total costs. Further, compare the percentage changes of the total cost increase and the carbon emissions reduction with the carbon cap constraint being tightened. Numerical results show that a reasonable carbon cap can reduce more carbon emissions while maintaining less cost growth. As the government’s policies on carbon emission control become more stringent, if online retailers do not take any corresponding measures, their economic interests will be affected to a certain extent. Therefore, this conclusion can help the government to set a reasonable and economical carbon cap. This study provides a scientific analysis tool and method for online retailers to incorporate carbon emissions into the integrated optimization of transportation and inventory. For small-size problems, the exact solutions can be obtained directly by solving the presented model from some commercial solvers (such as CPLEX). For large-size problems, our SA algorithm can find satisfactory solutions within a reasonable time. Promising areas for future research include considering the
situation where the demands in one period can be satisfied by multiple replenishment lot sizes; explore more efficient algorithms to find better solutions with fewer computational resources; investigating other carbon emission restriction strategies, such as carbon cap and trade or carbon tax.

Conflict of interest statement

None declared.

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