Multiscale Pressure-Balanced Structures in Three-dimensional Magnetohydrodynamic Turbulence

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Abstract

Observations of solar wind turbulence indicate the existence of multiscale pressure-balanced structures (PBSs) in the solar wind. In this work, we conduct a numerical simulation to investigate multiscale PBSs and in particular their formation in compressive magnetohydrodynamic turbulence. By the use of the higher-order Godunov code Athena, a driven compressible turbulence with an imposed uniform guide field is simulated. The simulation results show that both the magnetic pressure and the thermal pressure exhibit a turbulent spectrum with a Kolmogorov-like power law, and that in many regions of the simulation domain they are anticorrelated. The computed wavelet cross-coherence spectra of the magnetic pressure and the thermal pressure, as well as their time series, indicate the existence of multiscale PBSs, with the small PBSs being embedded in the large ones. These multiscale PBSs are likely to be related to the highly oblique-propagating slow-mode waves, as the traced multiscale PBS is found to be traveling in a certain direction at a speed consistent with that predicted theoretically for a slow-mode wave propagating in the same direction.

Key words: solar wind – turbulence – waves

1. Introduction

In the solar wind, pressure-balanced structures (PBSs), which show an inherent anticorrelation between the fluctuations of the magnetic pressure ($P_{mag}$) and the thermal pressure ($P_{th}$), have often been observed in past decades and are considered major ingredients of solar wind compressible turbulence (see the review by Tu & Marsch 1995 and references therein). For observations near 1 au, Burlaga & Ogilvie (1970) examined the correlation between magnetic and thermal pressures and found that they are negatively correlated, indicating the existence of PBSs. In the Voyager data, Burlaga et al. (1990) identified that the PBSs in the outer heliosphere increase with the heliocentric distance. Using the Helios data, Marsch & Tu (1993) and Tu & Marsch (1994) extensively studied the correlations between such quantities as density, temperature, and magnetic field magnitude. They found an anticorrelation between the magnetic pressure and the thermal pressure for fluctuations at scales less than 1 hr. On the basis of Ulysses observations of the high-latitude solar wind, Reisenfeld et al. (1999) showed that at timescales of less than 1 day, PBSs dominate the compressive component of the plasma turbulence of the high-latitude solar wind (see also McComas et al. 1996), and variations of the plasma $\beta$ within PBSs appear to be positively correlated with the variations of the helium abundance. In high-resolution Cluster data, Kellogg & Horbury (2005) found that PBSs are possible on the scale of seconds. Yao et al. (2011) selected a segment of data obtained by the Cluster C1 spacecraft in quiet solar wind and found a dominant anticorrelation between the time series of the electron density and magnetic field strength, indicating the existence of multiscale PBSs, whose durations can last from 1 hr down to 10 s.

Though PBSs were frequently identified in the multiscale fluctuations in the solar wind, their nature and origin have not yet been understood. In the inner heliosphere, the PBSs progressively fade away with increasing heliocentric distance, which suggests that they may “spaghetti-like” magnetic flux tubes stemming from the Sun (Mariani et al. 1973; Thieme et al. 1989, 1990; Borovsky 2008), while in the outer heliosphere the PBSs build up increasingly, which means that they may also be generated in the solar wind, such as by slow-mode magnetosonic waves or mirror-mode instability (Tu & Marsch 1994; Kellogg & Horbury 2005; Yao et al. 2013a, 2013b), or by pick-up ions (Burlaga et al. 1994). Thieme et al. (1989) used the data measured by the two Helios spacecraft to investigate properties of high-speed streams, and they presented clear cases that the variability of the fluid parameters, which characterizes the fine-stream tubes, is due to spatial structures that may be viewed as remnants of the underlying coronal structures. Thieme et al. (1990) further pointed out that these fine-stream structures are likely to be PBSs. However, this solar-origin explanation is challenged by the recent observation of PBSs with small scales of seconds, as their source region on the Sun when being mapped out to the heliosphere would be too small to be observable. By analyzing the plasma and magnetic field data obtained by Wind, Yao et al. (2013b) investigated the dependence of the properties of small-scale PBSs on the background magnetic field direction and showed that the small-scale PBS may be related to highly oblique-propagating, slow-mode waves in the solar wind. Based on the proton temperatures $T_{perp}$ and $T_{par}$, Yao et al. (2013a) also gave another example where the small-scale PBS in the quiet solar wind may have been formed as a result of mirror-mode instability. Complete evidence of oblique/quasi-perpendicular propagating slow magnetosonic
waves is provided by He et al. (2015), in which not only is the anticorrelation between $|B|$ and density presented but also the phase relation between density and parallel velocity $(V_y)$ is investigated, helping us to diagnose the propagation direction with respect to the magnetic field vector.

Apart from these observational evidences, numerical simulations have been conducted to investigate the nature of the PBSs and discontinuities (Greco et al. 2008, 2009; Servidio et al. 2011; Zhdkanin et al. 2012a, 2012b; Verscharen et al. 2012; Yang et al. 2015; Zhang et al. 2015a). By comparing solar wind data from the Advanced Composition Explorer with simulations of magnetohydrodynamic (MHD) turbulence, Greco et al. (2009) examined the link between discontinuity identification and intermittency analysis, which supports that many or most of the intermittent solar wind structures are discontinuities that are expected to be produced in MHD turbulence. Verscharen et al. (2012) presented a 2D model of solar wind turbulence in the dissipation range and illustrated some signatures of small-scale PBSs and could attribute their formation to oblique-propagating, slow-mode waves. In a simulation of the 3D decaying compressive MHD turbulence, Yang et al. (2015) examined how the rotational discontinuities can evolve from the steepening of the Alfvén wave with moderate amplitude, and this steepening is caused by the inhomogeneity of the Alfvén speed in the ambient turbulence. However, these works have not reported multiscale PBSs, especially in the sense that the small-scale PBSs are embedded in the larger ones. Their possible formation in MHD turbulence has not been demonstrated.

Yet in the present work, based on a simulation of the driven compressible MHD turbulence, we identified frequent occurrences of multiscale PBSs and could attribute their formation to oblique-propagating, slow-mode waves. In Section 2, a general description of the numerical MHD model is given. Section 3 describes the results of the numerical simulation and their analysis. Section 4 is reserved for the summary and discussion.

## 2. Numerical MHD Model

The plasma is described by a compressible 3D MHD model, which involves fluctuating variables and a uniform large-scale field $B_0$ in the $\hat{z}$-direction. The equations are written in the following nondimensional form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0, \quad (1)$$

$$\frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot (\rho \vec{u} \otimes \vec{u} + (p + \frac{1}{2} \vec{B}^2) \mathbf{I} - \vec{B} \vec{B}) = \vec{f}_v, \quad (2)$$

$$\frac{\partial \rho e}{\partial t} + \nabla \cdot \left[ \rho \left(e + p + \frac{1}{2} \vec{B}^2\right) \mathbf{I} - (\rho \vec{B}) \mathbf{I} \right] = \nabla \cdot (\vec{B} \times \eta \vec{f}), \quad (3)$$

$$\frac{\partial \vec{B}}{\partial t} + \nabla \cdot \left( \vec{u} \vec{B} - \vec{B} \vec{u} \right) = \eta \nabla^2 \vec{B} + \vec{f}_b, \quad (4)$$

where

$$e = \frac{1}{2} \rho \vec{u}^2 + \frac{p}{\gamma - 1} + \frac{1}{2} \vec{B}^2, \quad \vec{j} = \nabla \times \vec{B},$$

which correspond to the total energy density and current density, respectively. Here, $\rho$ is the mass density; $\vec{u} = (u_x, u_y, u_z)$ is the flow velocity composed of $x, y,$ and $z$ components; $p$ is the thermal pressure; $\vec{B}$ denotes the magnetic field; $t$ is time; $\gamma = 5/3$ is the adiabatic index; $\eta$ is the magnetic resistivity; and $\vec{f}_v$ and $\vec{f}_b$ are the random large-scale drivers. The distributions of $\vec{f}_v$ in the $y = 3.14$ plane at $t = 0$ are shown in Figure 2. An example of the temporal behavior of $\vec{f}_v$ at $x = 0.63, y = 0.63,$ and $z = 0.63$ is presented in Figure 3. The amplitude of $\vec{f}_v, \vec{f}_y,$ and $\vec{f}_z$ at high frequency (corresponding to the time period of about 0.5) is about 0.05, which is far smaller than the magnetic (thermal) pressure gradient at the grid size (i.e., $\nabla p_{mag(0)}$), as shown in Figure 4. Therefore, the behaviors of $\vec{f}_b$ and $\vec{f}_v$ do not affect the generation of PBSs directly without nonlinear cascading processes.

Specifically, the components of $\vec{f}_v$ and $\vec{f}_b$ are defined in Fourier space as $Pm(k)$ and $\phi$, where the profile of $Pm(k)$ is shown in Figure 1, and $\phi$ is the uniform-distributed random phase angle between $[0, 2\pi]$. Both $\vec{f}_v$ and $\vec{f}_b$ are noncompressive (although the driven turbulence is compressive), satisfying $\nabla \cdot \vec{f}_v = 0$ and $\nabla \cdot \vec{f}_b = 0$. To introduce the cross-helicity (indicator of imbalance) and an Alfvén ratio less than 1, the amplitude of $\vec{f}_b$ is designed to be about 3 times larger than that of $\vec{f}_v$. The driven turbulence is not pure Alfvénic, as the direction of forces is demanded to be only perpendicular to the wavevector $\vec{k}$, not the large-scale field $B_0$.

The distributions of $\vec{f}_v$ in the $y = 3.14$ plane at $t = 0$ are shown in Figure 1. An example of the temporal behavior of $\vec{f}_v$ at $x = 0.63, y = 0.63,$ and $z = 0.63$ is presented in Figure 2. The amplitude of $\vec{f}_v, \vec{f}_y,$ and $\vec{f}_z$ at high frequency are shown in Figure 4. Therefore, the behaviors of $\vec{f}_b$ and $\vec{f}_v$ do not affect the generation of PBSs directly without nonlinear cascading processes.

Under typical solar wind conditions, the Alfvén ratio is less than 1, and the perturbed magnetic field strength is less than the mean large-scale magnetic field strength. To be comparable to these observations, the amplitudes of the forcing components are tuned to maintain that the rms values for the velocity and magnetic field in a statistically stationary state are approximately 0.67 and 0.75, respectively. The large-scale field $B_0$ is set to be 1.00 $\hat{z}$.

We consider periodic boundary conditions in a cube with a side length of $2\pi$ and a resolution defined by $512^3$ grid points and run a simulation from an initial state with only the guide field $B_0$. The initial density and thermal pressure are set to be uniform. Hence, the simulation domain is initially without...
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3. Numerical Results

Figure 4 presents the calculated distributions of the magnetic pressure $P_{\text{mag}}$, the thermal pressure $P_{\text{th}}$, and the total pressure $P_{\text{tot}}$ in the $y = 3.14$ and $z = 3.14$ planes at $t = 22$. This figure shows that as a result of the well-known anisotropic behavior of fluctuations in MHD with a uniform guide field, the structures preferentially align along the $z$-direction and become much more varied in the perpendicular cross section. Both the magnetic pressure $P_{\text{mag}}$ and the thermal pressure $P_{\text{th}}$ are fluctuating, while the total pressure $P_{\text{tot}}$ changes not so chaotically. Taking the region marked by the white rectangle in this figure as an example, both $P_{\text{mag}}$ and $P_{\text{th}}$ alter from 0.2 to 2.2 and show fine structures, while $P_{\text{tot}}$ is between 2.25 and 2.75, without such fine structures. In many regions we can see such great changes of both $P_{\text{mag}}$ and $P_{\text{th}}$, yet $P_{\text{tot}}$ only changes slightly.

Figure 5 shows the spectra of the magnetic pressure $P_{\text{mag}}$ and the thermal pressure $P_{\text{th}}$. To calculate the spectra of $P_{\text{mag}}$ and $P_{\text{th}}$, we first transform them into 3D Fourier space and then add together the squared Fourier amplitudes over $\Delta k$ (here set to be 1) within a spherical shell $(k - \Delta k/2, k + \Delta k/2)$, that is, $\int_{\Delta k/2}^{\Delta k/2} \int_{0}^{2\pi} \int_{0}^{\pi} |\text{FFT}(P_{\text{mag/\text{th}}}(k)))|^2 \sin \theta \, \mathrm{d} \theta \, \mathrm{d} \phi \, \mathrm{d}k / \Delta k$. Although the simulation models a compressible magnetofluid, both the magnetic pressure and the thermal pressure exhibit a turbulent spectrum with a Kolmogorov-like power law that is characteristic of an incompressible hydrodynamic fluid. This result is comparable to in situ solar wind observations (Tu & Marsch 1994; Yao et al. 2011) and previous simulations (Shaikh & Zank 2010). Also, in the inertial range, the spectra of $P_{\text{mag}}$ and $P_{\text{th}}$ are close, which indicates that their amplitudes change similarly over the various spatial scales.

To study the possible correlation between $P_{\text{mag}}$ and $P_{\text{th}}$ at different scales, we first sample $P_{\text{mag}}$ and $P_{\text{th}}$ along a straight line and then calculate the wavelet cross-coherence spectrum of $P_{\text{mag}}$ and $P_{\text{th}}$ (Yao et al. 2011). In that research, the data sampled by the Cluster C1 spacecraft can be interpreted, in considerations of Taylor’s hypothesis, as a sampled result along a line with an angle of about $45^\circ$ to the interplanetary mean magnetic field direction. For numerical simulation data reported here, similarly we sample simulated data in a direction with an angle of $45^\circ$ to the direction of the large-scale $B_0 (\hat{z})$. We have tried other orientations and still detected the multiscale PBSs.

To study the correlation between the magnetic pressure $P_{\text{mag}}$ and the thermal pressure $P_{\text{th}}$ at different scales, we first calculate the wavelet coefficients $\omega_{\text{mag}}(l, \tau)$ and $\omega_{\text{th}}(l, \tau)$ as a function of the point $l$ and the scale $\tau$ for $P_{\text{mag}}$ and $P_{\text{th}}$ and then get the coherence coefficient as $\frac{|\omega_{\text{mag}}(l, \tau) \times \omega_{\text{th}}(l, \tau)|}{\sqrt{\text{imaginary}(\omega_{\text{mag}}(l, \tau) \times \omega_{\text{th}}(l, \tau))}}$, as well as the absolute phase as $\tan^{-1}\left(\frac{\text{imaginary}(\omega_{\text{mag}}(l, \tau) \times \omega_{\text{th}}(l, \tau))}{\text{real}(\omega_{\text{mag}}(l, \tau) \times \omega_{\text{th}}(l, \tau))}\right)$ (Torrence & Compo 1998).

Figure 6 shows the product of the coherence coefficient spectrum and the absolute phase spectrum, as well as the space fluctuations, and the turbulence is generated by the drivers alone.

For the calculation presented herein, we employ a higher-order Godunov code, Athena (Gardiner & Stone 2005, 2008; Stone et al. 2008). Specifically, we apply a third-order piecewise parabolic method to the reconstruction and the approximate Riemann solver of Harten–Lax–van Leer discontinuities to the calculation of the numerical fluxes. The constrained transport algorithm is applied for ensuring the divergence-free state of the magnetic field.
series of $P_{\text{mag}}$, $P_{\text{th}}$, and $P_{\text{tot}}/2$. This figure highlights the regions with significant anticorrelation, where the coherence coefficient is close to 1 and the absolute phase angle is close to $\pi$. Like the observations in the solar wind (Yao et al. 2011), this anticorrelation extends over the whole series of points located inside the region framed by the black line. As the scale decreases, the regions with sharp changes of $P_{\text{mag}}$ and $P_{\text{th}}$ become smaller, and more PBSs appear. The black line shown here is defined as the $e$-folding time ($\sqrt{2}\tau$ for Morlet; see Table 1 in Torrence & Compo 1998) for the autocorrelation of wavelet power at each scale and confines a bell-shaped area, beyond which the effect of the finiteness of data may influence the wavelet spectra (Torrence & Compo 1998).

Along the path we sample 8000 points and form four segments, with smaller ones embedded in larger ones, respectively, to display the variation of $P_{\text{mag}}$, $P_{\text{th}}$, and $P_{\text{tot}}/2$ on different scales. Like for the solar wind observations (Yao et al. 2011), simulation results reveal that a multiscale anticorrelation between $P_{\text{mag}}$ and $P_{\text{th}}$ exists, and that multiscale PBSs are generated, whereby the small PBSs are found to be embedded in the large ones. On the scale of 8000 points, a strong anticorrelation between $P_{\text{mag}}$ and $P_{\text{th}}$ appears, with a correlation coefficient of $-0.96$. When zooming in to the last 2000 points, the variations of $P_{\text{mag}}$ and $P_{\text{th}}$ are seen to be also negative, with a correlation coefficient of $-0.95$. For the subsequent scale of only 500 points, the anticorrelation with the coefficient of $-0.97$ continues. Finally, we zoom in to an even smaller scale of 125 points, where we find an anticorrelation with a coefficient of $-0.99$.

To investigate the nature of waves occurring possibly in the turbulence for each $k$, the perturbation speeds, corresponding to the fast ($\delta u_{\text{FM}}$) and the slow ($\delta u_{\text{SM}}$) magnetosonic waves and the Alfvén waves ($\delta u_{A}$), are computed via projections, based on the fact that the three MHD wave mode velocities $\delta u_{\text{FM}}(=v_{p,F}^2k - v_A^2 \cos \theta_0)$, $\delta u_{\text{SM}}(=v_{p,S}^2j_{\kappa} - v_A^2 \cos \theta_0)$, and $\delta u_{A}(=k \times b_0)$ form an orthogonal coordinate system for a given $k$, where $v_{p,F}$, $v_{p,S}$, and $v_A$ are the phase speed of the fast and slow
magnetosonic waves and Alfvén waves, respectively, and θ is the angle between unit vector $\mathbf{k}$ and $\mathbf{b}_0$ (Marsch 1986; Zhang et al. 2015b). Specifically, $\delta u_{\text{FMS}} \sim u(\mathbf{k}) \cdot \delta \psi_{\text{FMS}} / |\delta \psi_{\text{FMS}}|$, $\delta u_{\text{SMS}} \sim u(\mathbf{k}) \cdot \delta \psi_{\text{SMS}} / |\delta \psi_{\text{SMS}}|$, and $\delta u_\perp \sim u(\mathbf{k}) \cdot \delta \psi / |\delta \psi|$. Please note that $u(\mathbf{k})$ is assumed to be the superposition of velocity fluctuations of three kinds of MHD waves. In fact, a certain fraction of $u(\mathbf{k})$ may not be associated with any of the three types of MHD waves. Moreover, from the calculated $\delta u_{\text{SMS}}$, we define the rms perpendicular and parallel wavenumber by $K_{\text{SMS,perp}} = \sqrt{\sum \delta u_{\text{SMS}}^2 / \sum \delta u_{\text{SMS}}}$ and $K_{\text{SMS,para}} = \sqrt{\sum \delta u_{\text{SMS}}^2 / \sum \delta u_{\text{SMS}}}$.

In Figure 7, we plot the evolution of the rms perturbation speeds of fast ($\delta U_{\text{FMS}}$) and slow ($\delta U_{\text{SMS}}$) magnetosonic waves relative to that of Alfvén waves ($\delta U_\perp$) (left panel) and the rms perpendicular wavenumber $K_{\text{SMS,perp}}$ and parallel wavenumber $K_{\text{SMS,para}}$ of the slow magnetosonic waves (right panel).

Figure 6. Anticorrelation between the magnetic pressure $P_{\text{mag}}$ and the thermal pressure $P_\text{th}$ at different scales. The first panel is the wavelet cross-coherence spectrum of $P_{\text{mag}}$ and $P_\text{th}$, showing the product of the coefficient spectrum and the absolute phase spectrum. The other panels plot the space series of $P_{\text{mag}}$ (black), $P_\text{th}$ (red), and $P_{\text{tot}}/2$ (blue), annotated with correlation coefficients.

Figure 7. Time evolution of the rms perturbation speeds of fast ($\delta U_{\text{FMS}}$) and slow ($\delta U_{\text{SMS}}$) magnetosonic waves relative to that of Alfvén waves ($\delta U_\perp$) (left panel) and the rms perpendicular wavenumber $K_{\text{SMS,perp}}$ and parallel wavenumber $K_{\text{SMS,para}}$ of the slow magnetosonic waves (right panel).
and $\delta U_A$, and after the simulated turbulence reaches a statistically stationary state, $\delta U_{\text{SMS}}$ approaches $\delta U_A$. Therefore, the computation domain is full of slow magnetosonic waves and Alfvén waves. The compressive fluctuations behave as slow-mode waves, as the negative correlation between the magnetic pressure and the thermal pressure indicates.

The evolution of the $K_{\text{SMS,perp}}$ and $K_{\text{SMS,para}}$ shows that before $t = 10$, most of the slow magnetosonic waves propagate more and more obliquely, and after that most of them keep propagating highly obliquely.

To see whether the multiscale PBSs detected here could be related to the obliquely propagating slow-mode waves, Figure 7 displays the calculated distributions of the density $\rho$, the magnetic pressure $P_{\text{mag}}$, and the thermal pressure $P_{\text{th}}$ at $t = 21.5$ (first row) and $t = 21.7$ (second row), respectively. The purple arrows show the velocity fields, and the pink lines show the magnetic field lines. The black cycle marks the movement of the chunk of the high density at the right, and the black dashed line is the slice for the time–distance diagrams of Figure 9 in the $z$-direction.

**Figure 8.** Distributions of the density $\rho$, the magnetic pressure $P_{\text{mag}}$, and the thermal pressure $P_{\text{th}}$ at $t = 21.5$ (first row) and $t = 21.7$ (second row), respectively. Purple arrows show the velocity fields, and pink lines show the magnetic field lines. The black cycle marks the movement of the chunk of the high density at the right, and the black dashed line is the slice for the time–distance diagrams of Figure 9 in the $z$-direction.

**Figure 9.** Time–distance diagrams of the density $\rho$, the magnetic pressure $P_{\text{mag}}$, the thermal pressure $P_{\text{th}}$, and the total pressure $P_{\text{tot}}$. The spatial direction is along the propagation direction defined by the black dashed line in Figure 8, and the slopes of the black dashed lines indicate the propagation speeds.
To estimate the propagation speed of the chunk of the high density, the values of $\rho$, $P_{\text{mag}}$, $P_{\text{th}}$, and $P_{\text{tot}}$ along the black dashed line in Figure 8 are extracted and are stacked in time sequence. This gives the time–distance diagrams as displayed in Figure 9, showing the recurrent stripes, whose slopes correspond to the propagating speeds and are measured to be about 0.52. By averaging the magnetic fields in the region associated with the chunk of the high density, it could be calculated that the angle between the mean magnetic field and the $z$-direction is $\theta \sim 50^\circ$. We trace the position of the high-density chunk through visual inspection in the 3D subvolume and estimate its propagation direction to be mainly along the $z$-direction. With the use of the average values of thermal pressure and density, we could compute the local slow-mode wave speed along the propagation direction (i.e., basically the $z$-direction) as
$$\frac{1}{2} \left[ C_s^2 + V_A^2 \right] - \sqrt{\left( C_s^2 + V_A^2 \right)^2 - 4 C_s^2 V_A^2 \cos^2(\theta)} \quad (C_s \text{ is sonic speed, and } V_A \text{ is Alfvén speed}),$$
which is $\sim 0.55$.

Also, at both $t = 21.5$ and $t = 21.7$, in this region, the velocity is small (less than 0.5), and its directions are diverse, giving a global velocity (averaged over this chunk of the high density at either time point) of $\sim 0$. Considering that the magnetic pressure keeps being negatively correlated with the density and with the thermal pressure, it can be said that this chunk of the high density is probably associated with an obliquely propagating, slow-mode wave. At the same time, the magnetic pressure is well in balance with the thermal pressure for this wave, and the multiscale PBS forms here.

### 4. Summary and Discussion

In this work we used a higher-order Godunov code to simulate the generation of compressible turbulence in a plasma with an imposed uniform guide field and to explore the formation of the multiscale PBSs in compressive MHD turbulence.

The numerical simulation shows that in many regions the magnetic pressure is negatively correlated with the thermal pressure, and both $P_{\text{mag}}$ and $P_{\text{th}}$ alter greatly, yet the total pressure $P_{\text{tot}}$ only changes slightly. $P_{\text{mag}}$ and $P_{\text{th}}$ exhibit a turbulent spectrum with a Kolmogorov-like power law, although this spectrum applies here to compressible turbulence. In the inertial range, the spectra of the magnetic pressure and the thermal pressure are close, and the amplitudes of $P_{\text{mag}}$ and $P_{\text{th}}$ at various $k$ are nearly equal.

From the computed wavelet cross-coherence spectrum of the magnetic pressure and the thermal pressure, it is found that the anticorrelation between them extends over the whole range of covered scales. As the scale decreases, the regions with sharp changes of $P_{\text{mag}}$ and $P_{\text{th}}$ become small, and more PBSs are generated. The simulated spatial series of the magnetic pressure and the thermal pressure on different scales also show that here exists a multiscale anticorrelation of them, and thus that multiscale PBSs are generated, whereby the small PBSs are embedded in the large ones.

By analysis of the nature of waves in the turbulence, it is found that the rms perturbation of the velocity corresponding to the fast magnetosonic waves is marginal compared with that of the slow magnetosonic waves and Alfvén waves, and most of the compressive fluctuations behave as slow-mode waves. Also, most of these slow magnetosonic waves reveal a highly oblique propagation. On the basis of the negative correlation between the magnetic pressure and the density, the negative correlation between the magnetic pressure and the thermal pressure, and the approximate equality between the inferred propagation speed and the predicted phase speed of the slow-mode wave, we present a single example that the multiscale PBS is likely to be related to the highly oblique-propagating slow-mode waves.

In this study, we presented simulation data indicating that the multiscale PBSs could frequently appear in the compressible turbulence, and that their formation is likely associated with the oblique-propagating, slow-mode waves. In the simulated turbulence, the slow magnetosonic wave amplitudes are seen to be much stronger than the fast magnetosonic wave amplitudes. This is comparable to the solar wind observation (Tu & Marsch 1994) and earlier simulations (Cho & Lazarian 2002). However, the slow magnetosonic wave amplitudes seem to be comparable to the Alfvén wave amplitudes, which is unlike the typical solar wind conditions, in which the amplitude in the compressive fluctuations is smaller than the amplitude in the noncompressive Alfvénic fluctuations. This may be related to the nature of the nonpure Alfvénic drivers $f_1$ and $f_2$. In the future, we may investigate the factors that control the generation and evolution of waves, as well as their influence on the formation of the multiscale PBSs.

Another issue concerns the question why the slow-mode waves are highly oblique propagating. As mentioned by Chen et al. (2012), is it a result of the strong damping of the parallel-propagating slow-mode wave, or is it determined by the turbulent circumstances? The potential nonlinear interaction between Alfvén waves and slow magnetosonic waves, which has been inferred to exist in the compressible solar wind turbulence and may be related to the observed signature of Landau resonance heating (He et al. 2015), is another interesting topic worthy of further research. We may answer these questions in our subsequent works. In interplanetary space, there exist non-pressure-balanced structures of turbulence in boundary layers of magnetic clouds (e.g., Wei et al. 2003a, 2003b, 2006). Comparative studies of pressure and non-pressure balanced structures are worthy of further investigation in the future.

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