Self-Organized Criticality and earthquakes

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Abstract. We discuss recent results on a new analysis regarding models showing Self-Organized Criticality (SOC), and in particular on the OFC one. We show that Probability Density Functions (PDFs) for the avalanche size differences at different times have fat tails with a q-Gaussian shape. This behavior does not depend on the time interval adopted and it is also found when considering energy differences between real earthquakes.

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INTRODUCTION

In the study of earthquake dynamics, the Self-Organized Criticality (SOC) paradigm proposed by Bak and coworkers [1] has been lengthly debated during the last decade in order to clarify the controversial earthquakes predictability [2]. In this short paper, we discuss recent results [3, 4] and we show that it is possible to reproduce statistical features of earthquake catalogs within a SOC scenario, if one takes into account long-range interactions. Here we consider the dissipative Olami-Feder-Christensen model [5] on a small world topology and we show that the Probability Density Functions (PDFs) for the avalanche size differences at different times have fat tails with a q-Gaussian shape [6] only if finite-size scaling (FSS) is present. This behavior does not depend on the time interval adopted and it is found also after reshuffling the data. Similar results have been obtained if energy differences between real earthquakes are considered.

MODEL AND RESULTS

The Olami-Feder-Christensen (OFC) model [5] is one of the most interesting models displaying Self-Organized Criticality. Despite of its simplicity, it exhibits a rich phenomenology resembling real seismicity. In its original version the OFC model consists of a two-dimensional square lattice of \( N = L^2 \) sites, each one connected to its 4 nearest neighbours and carrying a seismogenic force represented by a real variable \( F_i \), which initially takes a random value in the interval \((0, F_{th})\). In order to mimic a uniform tectonic loading all the forces are increased simultaneously and uniformly, until one of them reaches the threshold value \( F_{th} \) and becomes unstable \((F_i \geq F_{th})\). The driving is then stopped and an "earthquake" (or avalanche) starts, i.e. each unstable site \( i \) releases
a part of its force, proportional to \( F_i \) to its four neighbours. The number of topplings during an avalanche defines its size \( S \), while the dissipation level of the dynamics is controlled by the parameter \( \alpha \in [0, 0.25] \). The model is conservative if \( \alpha = 0.25 \), while it is dissipative for \( \alpha < 0.25 \). In the following we consider the dissipative version of the OFC model with \( \alpha = 0.21 \) on a regular lattice with \( L = 64 \) and open boundary conditions (i.e. we impose \( F = 0 \) on the boundary sites). In order to improve the model in a more realistic way, we introduced a small fraction of long-range links in the lattice so to obtain a small world topology. Just a few long-range edges create short-cuts that connect sites which otherwise would be much further apart. Long-range connections allow the system to synchronize and to show both FSS and universal exponents as was shown in Ref.[3]. Furthermore, a small world topology is expected to model more accurately earthquakes spatial correlations, taking into account long-range as well as short-range seismic effects. In our version of the OFC model the links of the lattice are rewired at random with a probability \( p \). The transition to obtain small world features and criticality is observed at \( p = 0.02 \) [3]. In Ref. [4] we calculated the distribution of the avalanche size time-series \( S(t) \) for the OFC model on a small world topology and on a regular lattice. In our case the time \( t \) is a progressive discrete index labelling successive events and is analogous to the "natural time" successfully used in Ref. [7]. In order to have a good statistics, we considered up to \( 10^9 \) avalanches in our numerical experiments. In both cases, as shown in Fig.1(a) of Ref. [4], the PDFs follow a power-law decay \( y \sim S^{-\tau} \) with a slope \( \tau = 1.8 \pm 0.1 \) even if real criticality is present only for the small world topology [3]. In the last years SOC models have been intensively studied considering time intervals between avalanches in the critical regime. Here we discuss a different approach which reveals interesting information on the eventual criticality of the model under examination. We focus our attention on the "returns" \( x(t) = S(t+\Delta) - S(t) \), i.e.
on the differences between avalanche sizes calculated at time $t + \Delta$ and at time $t$, $\Delta$ being a discrete time interval. The resulting signal is extremely intermittent at criticality, since successive events can have very different sizes. On the other hand, if the system is not in a critical state, this intermittency character is very reduced.

In Fig. 1 (a) we plot as open circles the Probability Density Function (PDF) of the returns $x(t)$ (with $\Delta = 1$) obtained for the critical OFC model on small world topology. The returns are normalized in order to have zero mean and unitary variance. The curves reported have also unitary area. A behavior very different from a Gaussian shape (plotted as dashed curve) is observed: the PDF is very peaked and exhibits fat tails. On the other hand, for the model on regular lattice, even if power laws for the avalanche size are found, the model is not critical since no FSS is observed [3]. In this latter case no fat tails emerge, although a sensible departure from Gaussian behavior is evident (see full circles). These findings thus indicate a new powerful way for characterizing the presence of criticality. Another remarkable feature which we have found is that such a behavior does not depend on the interval $\Delta$ considered for the avalanche size difference. Even after reshuffling the data, i.e. changing in a random way the time order of the avalanches, no change in the PDFs was observed. Moreover, the data reported in Fig. 1 (a) for the critical OFC model on a small world can be well fitted by a $q$-Gaussian curve $f(x) = A[1 - (1 - q)x^2/B]^{1/(1-q)}$ which is typical of Tsallis $q$-statistics [6]. This function generalizes the standard Gaussian curve, depending on the parameters $A, B$ and on the exponent $q$. For $q = 1$ the normal distribution is obtained again, so $q \neq 1$ indicates a departure from Gaussian statistics. The q-Gaussian curve, reported as full line, reproduces very well the model behavior in the critical regime, yielding in our case a value of $q = 2.0 \pm 0.1$.

In order to compare these theoretical results with real earthquakes data, we repeated the previous analysis for several catalogs [4]. Here we discuss the Northern California catalog for the period 1966-2006 [8]. The latter is a very extensive and complete seismic data set on one of the most active and studied faults on the Earth, i.e. the San Andreas Fault. The total number of earthquakes considered is almost 400000. Actually the energy, and not the magnitude, is the quantity which should be considered equivalent to the avalanche size in the OFC model. Therefore we studied the quantity $S = \exp(M)$, where $M$ is the magnitude of a real earthquake. This quantity is simply related to the energy dissipated in an earthquake, the latter being an increasing exponential function of the magnitude.

In Fig. 1 (b) we consider the released energy $S$ for the Northern California catalog and we plot the PDFs of the corresponding returns $x(t) = S(t + \Delta) - S(t)$ (with $\Delta = 1$). Also for real data $t$ is a progressive discrete index labelling successive events. As for the critical OFC model previously discussed, fat tails and non-Gaussian probability density functions are observed. In both cases the experimental points can be fitted by a q-Gaussian curve, obtaining an exponent $q = 1.75 \pm 0.15$, a value which is compatible, within the errors, to that one found for the OFC model. In Ref. [4] also the world catalog was considered with similar results. As for the OFC model also for the real earthquakes data, by varying the interval $\Delta$ of the energy returns $x$, or by reshuffling the time-series $S(t)$, no change in the PDFs was observed. The above results give further support to the argument that a SOC mechanism underlies earthquake dynamics and that, actually,
although the system is in a strongly correlated regime in space, there is no correlation in time between the magnitude of successive events, as also explained by means of a simple analytical derivation in Ref. [4].

CONCLUSIONS

We have discussed a new type of analysis which is able to discriminate in a quantitative way real SOC dynamics. The method, here applied to the OFC model and to real earthquakes data, gives further support to the argument that seismicity can be explained within a dissipative self-organized criticality scenario when long-range interactions are taken into consideration.

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