Spin-orbit-coupled cold atomic gases are proposed as a model system for observing the signatures of quantum chaos.

Such energy-level statistics are primarily related to the static properties of a quantum system. Chaotic behavior, however, emerges in the first place in the dynamics of a system, i.e., in how it changes with time. From an experimental perspective, the observation of quantum dynamics is challenging, requiring exquisite control of system parameters and good isolation from the environment, in order to protect quantum mechanical wave functions from decoherence. As is often the case, the physics of cold atomic gases comes to the rescue, offering a highly controllable platform for the realization of theoretically predicted effects.

Over recent decades, researchers have learned some important things about the relationship between quantum mechanics and classical chaos. One of the most solid conclusions concerns the statistical properties of the energy levels of large quantum systems [3], such as heavy atomic nuclei. If a classically nonchaotic system is quantized, the resulting discrete energy values (eigenvalues) tend to cluster, while eigenvalues in quantum systems whose classical counterparts are chaotic typically repel each other and tend to be distributed more uniformly within the available energy range. These signatures of chaos have been experimentally observed in the energy levels obtained from atomic and molecular spectra.

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Larson et al. analyze the dynamical behavior of a dilute spin-orbit coupled gas of atoms. To investigate the quantum-classical correspondence and to base their analysis on the firmer ground of classical chaos, the authors first address the classical dynamics of the system. Since the spin degree of freedom is inherently quantum mechanical, the system does not have a trivially identifiable classical counterpart. However, Larson et al. show that a classical counterpart in the usual sense can be found by describing the system within a Born-Oppenheimer approximation (in which the spin and orbital motions are separated). Furthermore, they show that the behavior of this classical counterpart can be switched from nonchaotic, if the spin orbit coupling is isotropic, to chaotic, if the coupling is directionally dependent (anisotropic).

The authors go on to analyze the quantum dynamics, initiated by what is called a “quantum quench.” After preparing the system in its ground state in the optical trap, they imagine slightly moving the trap, thus creating a nonequilibrium condition that evolves with time. They then simulate the long-term dynamics of the perturbed system for the two spin-orbit coupling symmetries that correspond to a chaotic and nonchaotic classical counterpart, searching for the fingerprints of chaos.

The key chaos-related question addressed by the author is whether the quantum system thermalizes, e.g., relaxes to a state in which expectation values no longer change and which is distributed more or less evenly over the allowed energy manifold. In isolated classical systems, thermalization is associated to chaos: In the presence of chaos, the particles’ motions can explore the entirety of the phase space, which allows reaching a fully thermal redistribution. The details of quantum thermalization are still poorly understood, but by the correspondence principle, one would expect that the quantum counterparts of classically chaotic systems also show thermalization [7]. This is indeed what the authors find in their calculations. Thermalization of the quantum system is only observed when the classical counterpart is chaotic. The authors further calculate the expected timescales of such thermalization processes, and their findings support previous speculations that these are directly related to the rate by which initially close trajectories separate in the corresponding classical system. Thermalization thus seems to be a key signature of quantum chaos.

A compelling aspect of the work of Larson et al. is the predicted observation of so-called “quantum scars” [8,9]. In a classically chaotic system, dynamic trajectories—due to their instability to perturbations—eventually explore the whole space. One would thus expect that quantum trajectories, as smeared-out versions of their classical counterparts, would spread out in time all over the available space. However, in a classical system, there are specific initial conditions for which the system moves back and forth along a closed, periodic trajectory, hidden in the chaotic sea. Such closed trajectories leave high-density “scars” in some steady-state eigenfunctions of the corresponding quantum system (see Fig. 1). Larson et al. have argued that similar scars can be observed in the thermalized states of spin-orbit-coupled cold atoms after a long time evolution. In their theorized cold-atom experiment, the scarred distribution could be made directly visible, providing a spectacular demonstration of the quantum signatures of chaos.

The system proposed by Larson et al. would allow a comprehensive investigation of the physics of quantum chaos and the recent demonstrations of spin-orbit-coupled gases of bosons suggest that a realization of

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their ideas may soon be within reach. Their approach might then help address a key unresolved question: What drives—at the quantum mechanical level—the process of thermalization? If, e.g., an ensemble of molecules thermalizes, it must be because quantum mechanics tells them to. But how? Quantum chaos may provide one of the key missing pieces for solving this important puzzle.

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