On $e^+e^-$ pair production by a focused laser pulse in vacuum

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The probability of electron-positron pair creation by a focused laser pulse is calculated. For description of the focused laser pulse we use a 3-dimensional model of the electromagnetic field which is based on an exact solution of Maxwell equations. There exists two types of focused waves: $c$- and $h$-polarized waves with only either electric, or magnetic vector being transverse respectively. It is shown that pair production is possible only in $c$-polarized electromagnetic wave. The dependence of the pair production probability on the intensity of the laser pulse is obtained. It is argued that there exists a natural physical limit for attainable focused laser pulse intensities. This limit is posed by the pulse energy loss due to the effect of pair creation.

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I. INTRODUCTION

The effect of electron-positron pair production in vacuum by a strong electric field was first considered by Sauter [1] as early as in 1931, see also [2]. Schwinger was the first who had derived the exact formula for the probability of pair creation in a static electric field [3] and since then this process was often called the Schwinger effect.

The probability of pair creation acquires its optimum value when the electric field strength is of the order of the "critical" for QED value $E_S = m^2c^3/\hbar \epsilon = 1.32 \times 10^{16}$ V/cm. Clearly, such field strength is unattainable for static fields experimentally in near future. Therefore attention of many researchers was focused on theoretical study of pair creation by time-varying electric fields [4, 5, 6, 7, 8, 9, 10, 11, 12]. Nowadays, a very strong time-varying electromagnetic field can be practically realized only with laser beams. First experiments dealing with nonlinear QED effects caused by high energy electrons and photons interacting with intense laser pulses have been carried out recently. First, the observation of nonlinear Compton scattering in the collision of 46.6 GeV electron with a laser pulse of $10^{18}$W/cm$^2$ intensity has been reported [12]. Then, the same group of researchers has observed electron-positron pair creation in collision of laser photons, backscattered to GeV energies by the 46.6 GeV electron beam, with an intense laser pulse [12]. The results of the latter experiment were the first laboratory evidence for inelastic light-by-light scattering involving only real photons. However, the intensities of existing now lasers are far lower than the value $I_S = \frac{\epsilon}{4\pi} E_S^2 \approx 4.6 \times 10^{20}$W/cm$^2$ corresponding to the critical QED field $E_S$. The estimations [6, 15, 16] show that pair creation by a single laser pulse in vacuum, or even in collision of two laser pulses, could be hardly observed with lasers of intensity $I \ll I_S$. Therefore the results of Refs. [1, 2, 3, 4, 5, 6, 7, 8, 9] were commonly believed to be of purely theoretical interest. Fortunately, the latest achievements of laser technology promise very rapid growth of peak laser intensities. Tajima and Mourou has suggested recently [17] a path to reach an extremely high-intensity level of $10^{26}$−$10^{28}$W/cm$^2$ already in the coming decade. Such field intensities are very close to the critical value, $E_S$. Hence the more detailed study of the Schwinger effect in time-varying electromagnetic fields, in particular in the field of a focused laser pulse, is becoming an urgent physical problem from experimental point of view also.

As it was shown in Ref. [3], a plane electromagnetic wave of arbitrary intensity and spectral composition does not create electron-positron pairs in vacuum because it has both field invariants $F = (E^2 - H^2)/2$ and $G = (E \cdot H)$ equal to zero. Therefore we consider the effect in the field of a focused circularly polarized laser pulse which is described by a realistic 3-dimensional model developed in Ref. [18]. Unlike the case of spatially homogeneous time-varying electric field used in Refs. [4, 5, 6, 7, 8, 9, 10, 11, 12], the utilized model is based on an exact solution of Maxwell equations. The model has been already successfully used in Ref. [19] for quantitative explanation of anisotropy of electrons accelerated by a high-intensity laser pulse which was observed in experiment of Malka et al. [20].

1 We assume that the laser beam is circularly polarized.

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The method we use in the present paper for calculation of the number of pairs created by the laser pulse is based on the fact that the characteristic length of the process is determined by the Compton length $l_c = \hbar/mc$ which is much less than the wavelength $\lambda$ of the laser field, $l_c \ll \lambda$. Therefore, at arbitrary point of the pulse we can calculate the number of created particles per unit volume and unit time according to the Schwinger formula for the static homogeneous field and then obtain the total number of created particles as the integral over the volume $V$ and duration $\tau$ of the pulse

$$N = \frac{e^2 E_S^2}{4\pi^2 \hbar^2 c} \int_V dV \int_0^\tau dt \, \epsilon \eta \coth \frac{\pi \eta}{\epsilon} \exp \left( \frac{\pi \epsilon}{\epsilon} \right).$$

(1)

Here $\epsilon = E/E_S$, $\eta = H/E_S$ are the reduced fields, and $E$ and $H$ are the field invariants which have the meaning of electric and magnetic fields in the reference frame where they are parallel. The invariants $E$ and $H$ can be expressed in terms of invariants $F$ and $G$, see, e.g., Ref. [3],

$$E = \sqrt{F^2 + G^2} \frac{1}{2} + F,$$

$$H = \pm \sqrt{F^2 + G^2} \frac{1}{2} - F.$$

(2)

The paper is organized as follows. In Sec. II we consider the model of the focused laser pulse field and calculate the field invariants. The qualitative discussion of the pair production process is carried out in Sec. III. We present the results of numerical calculations of the pair production probability in Sec. IV. The summary of the results and conclusions are presented in Sec. V.

II. MODEL OF THE FIELD.

It is well known that the electromagnetic field of a focused light beam is not transverse. Therefore, strictly speaking, one cannot ascribe some definite type of polarization to it. However, we can always represent the field of a focused beam as a superposition of fields with transverse either electric, or magnetic vector only, see, e.g., Ref. [21]. For each of these fields we can define the type of polarization with respect to that vector which is transverse. We will call such fields $e$- or $h$-polarized fields respectively.

It can be verified straightforwardly that there exists the following exact solution of Maxwell equations which describes a wave propagating along the $z$ axis [18]

$$E^e = i E_0 e^{-i\varphi} \left\{ F_1(e_x \pm ie_y) - F_2 e^{\pm i\phi} (e_x \mp ie_y) \right\},$$

$$H^e = \pm E_0 e^{-i\varphi} \left\{ \left( 1 - i\Delta^2 \frac{\partial}{\partial \chi} \right) \left[ F_1(e_x \pm ie_y) + F_2 e^{\pm i\phi} (e_x \mp ie_y) \right] + 2i \Delta e^{\pm i\phi} \frac{\partial F_1}{\partial \xi} e_z \right\}.$$ 

(3)

(4)

Here $\omega$ is the wave frequency, $x$, $y$, and $z$ are spatial coordinates, and

$$\varphi = \omega(t - z/c), \quad \xi = \rho/R, \quad \chi = z/L,$$

$$\rho = \sqrt{x^2 + y^2}, \quad \cos \phi = x/\rho, \quad \sin \phi = y/\rho,$$

$$\Delta \equiv c/\omega R = \lambda/2\pi R, \quad L \equiv R/\Delta.$$ 

(5)

The fields $E^e$ and $H^e$ are solutions of Maxwell equations if function $F_1(\xi, \chi; \Delta)$ obeys the equation [18]

$$2i \frac{\partial F_1}{\partial \chi} + \Delta^2 \frac{\partial^2 F_1}{\partial \chi^2} + \frac{1}{\xi} \frac{\partial}{\partial \xi} \left( \xi \frac{\partial F_1}{\partial \xi} \right) = 0,$$

(6)

and function $F_2(\xi, \chi; \Delta)$ is defined by the relation

$$F_2(\xi) = F_1(\xi) - \frac{2}{\xi^2} \int_0^\xi \xi' F_1(\xi')d\xi'.$$

(7)
The fields \( E^h \) and \( \mathbf{H}^e \) describe a focused beam if functions \( F_1, F_2 \) are chosen so that they tend to zero sufficiently fast when \( \xi, |\chi| \to \infty \) and satisfy the following conditions
\[
\lim_{\Delta \to 0} F_1(0,0;\Delta) = 1, \quad \lim_{\Delta \to 0} F_2(0,0;\Delta) = 0. \tag{8}
\]
In this case the parameter \( R \) can be interpreted as the radius of the focal spot, \( L \) as the diffraction length \cite{18}, and \( \Delta \) as the focusing parameter. If \( \Delta = 0 \) the solution \( \text{(3), (4)} \) describes a plane wave.

Even if the laser beam is focused so that \( R \sim \lambda \) (this corresponds to the diffraction limit), \( \Delta \sim 10^{-1} \). Therefore in our calculations we will always assume that \( \Delta \ll 1 \). Under this assumption the field \( \text{(3) in the spatial area} \)
\[
\xi \ll 1, \quad |\chi| \ll 1,
\]
in virtue of Eqs. \( \text{(3)} \), is very close to electric field of a circularly polarized plane wave. In this sense we will identify the field \( \text{(3), (4)} \) with circularly \( e \)-polarized focused light beam. The electric and magnetic fields in the circularly \( h \)-polarized beam are given by the following expressions \cite{18}
\[
E^h = \pm iH^e, \quad H^h = \mp iE^e. \tag{9}
\]

For discussion of other types of polarizations of focused light beams see Ref. \cite{18}.

The exploited model admits different field configurations, which are determined by two functions \( F_1, F_2 \). One of the possible variants for solutions of Eqs. \( \text{(6), (7)} \) satisfying constraints \( \text{(8)} \) can be expressed at \( \Delta \ll 1 \) as
\[
F_1 = (1 + 2i\chi)^{-2} \left( 1 - \frac{\xi^2}{1 + 2i\chi} \right) \exp \left( -\frac{\xi^2}{1 + 2i\chi} \right), \tag{10}
\]
\[
F_2 = -\xi^2(1 + 2i\chi)^{-3} \exp \left( -\frac{\xi^2}{1 + 2i\chi} \right),
\]
see Ref. \cite{18}. The waves of such type are commonly called Gaussian beams. We will work with expressions \( \text{(10)} \) for functions \( F_1, F_2 \) throughout the paper.

To describe a laser pulse with finite duration \( \tau \) one should introduce \( \text{Ref.} \) \cite{18} a temporal amplitude envelope \( g(\varphi/\omega\tau) \) making the following substitutions in Eqs. \( \text{(3), (4)} \)
\[
\exp(-i\varphi) \to if'(\varphi), \quad \Delta \exp(-i\varphi) \to \Delta f(\varphi), \tag{11}
\]
where
\[
f(\varphi) = g(\varphi/\omega\tau) \exp(-i\varphi).
\]
It is assumed that the function \( g(\varphi/\omega\tau) \) is equal to unity at the point \( \varphi = 0 \) and decreases exponentially at the periphery of the pulse for \( |\varphi| \gg \omega\tau \). In this case the electric and magnetic fields of the model constitute an approximate solution of Maxwell equations with the second-order accuracy with respect to small parameters \( \Delta \) and \( \Delta' = 1/\omega\tau \)
\[
\Delta' \ll \Delta \ll 1. \tag{12}
\]

Using Eqs. \( \text{(3), (4)} \) we can calculate the field invariants \( \mathcal{F}, \mathcal{G} \). For the sake of compactness we will give here explicit expressions for them only for the case of \( e \)-polarized pulses. In the lowest order with respect to parameter \( \Delta \) they read
\[
\mathcal{F}^e = \frac{1}{2} \left\{ (\text{Re}E^e)^2 - (\text{Re}H^e)^2 \right\} =
\]
\[
= 2E_0^2g^2(\varphi/\omega\tau)\Delta^2 \left\{ \text{Im} \left[ F_1 \frac{\partial F_1}{\partial \chi} + F_2 \frac{\partial F_2}{\partial \chi} \right] - \left| \frac{\partial F_1}{\partial \xi} \right|^2 + \text{Re} \left[ e^{-2i(\varphi+\phi)} \left( \frac{\partial F_1}{\partial \xi} \right)^2 + i \frac{\partial}{\partial \chi} (F_1F_2) \right] \right\}, \tag{13}
\]
\[
\mathcal{G}^e = \text{Re}E^e\text{Re}H^e = \pm 2E_0^2g^2(\varphi/\omega\tau)\Delta^2 \left\{ \text{Re} \left( F_2 \frac{\partial F_2}{\partial \chi} - F_1 \frac{\partial F_1}{\partial \chi} \right) - \text{Re} \left[ F_2 \frac{\partial F_2}{\partial \chi} - F_1 \frac{\partial F_1}{\partial \chi} \right) e^{-2i\varphi+2i\phi} \right\}.
\]

As it should be, at \( \Delta = 0 \), i.e. in the case of plane wave field, both invariants are equal to zero. The invariants \( \mathcal{E}, \mathcal{H} \) can be calculated by substitution of Eqs. \( \text{(13)} \) into Eqs. \( \text{(4)} \).
III. QUALITATIVE DISCUSSION.

Now, using the model described in the preceding section, we will give a qualitative estimate of the number of pairs created by a focused laser pulse in vacuum. Taking into account that pairs are created mainly near the focus, we assume that the spatial volume of the pulse is of the order of \( \pi R^2 c \tau \) and the number of created pairs is given by the following expression

\[
N \approx \frac{e^2 E_0^2}{4\pi^2 \hbar^2 c} \pi R^2 c^2 \tau \eta \coth \left( \frac{\eta}{\tau} \right) \exp \left( -\frac{\pi}{\tau} \right),
\]

instead of Eq. (1). Here \( \tau \) and \( \eta \) are the averaged over time values of dimensionless field invariants \( \epsilon \) and \( \eta \) in the focus.

Using Eqs. (13), (10) we easily find that in the focus \( \xi = 0, \chi = 0 \) invariants \( F \) and \( G \) in the lowest approximation with respect to \( \Delta \) are equal to

\[
F_e(0, 0) = 8\Delta^2 E_0^2 g^2 \left( \frac{t}{\tau} \right), \quad G_e(0, 0) = 0,
\]

\[
F_h(0, 0) = -8\Delta^2 E_0^2 g^2 \left( \frac{t}{\tau} \right), \quad G_h(0, 0) = 0.
\]

It immediately follows from Eqs. (2) that at the focus of the \( e \)-polarized pulse

\[
\epsilon = \left( \frac{2}{\sqrt{S_0^2}} \right)^{1/2}, \quad \eta = 0,
\]

while for the \( h \)-polarized pulse we have

\[
\epsilon = 0, \quad \eta = \left( \frac{2}{\sqrt{S_0^2}} \right)^{1/2}.
\]

So we conclude that pairs can be created only by a \( e \)-polarized but not \( h \)-polarized pulse. This statement is not exact however. We will see in the next section that \( h \)-polarized pulse also create pairs though their number is several orders of magnitude less then in the case of \( e \)-polarized pulse.

Now we can estimate the number \( N \) of pairs created by a \( e \)-polarized focused pulse. However, it is convenient to express \( N \) in terms of laser intensities instead of \( \tau \). To do this, we begin from the Pointing vector averaged over fast oscillations of electromagnetic field of the pulse

\[
< S > = \frac{c}{4\pi} E_0^2 g^2 \left( \frac{\varphi}{\omega \tau} \right) \left\{ |F_1|^2 + |F_2|^2 \right\} e_z.
\]

Using explicit expressions for functions \( F_1, F_2 \) we obtain for the energy flow through the focal plane

\[
\Phi = \int < S > \big|_{z=0} dx dy = \frac{c}{4\pi} E_0^2 g^2 \left( \frac{\varphi}{\omega \tau} \right) \frac{\pi R^2}{2}.
\]

The total energy carried by the pulse is

\[
W = \int_{-\infty}^{\infty} dt \Phi = G \frac{c}{4\pi} E_0^2 \pi R^2 \tau,
\]

where

\[
G = \frac{1}{2} \int_{-\infty}^{\infty} du g^2(u),
\]

and \( \tau \) is the pulse duration. Further on we will use the gaussian temporal amplitude envelope \( g \big|_{z=0} = e^{-4t^2/\tau^2} \), for which \( G = \frac{1}{4} \sqrt{\frac{\pi}{2}} \approx 0.31 \). By definition, the intensity \( I \) is

\[
I = \frac{W/\pi R^2 \tau = G \frac{c}{4\pi} E_0^2}. \tag{21}
\]
The factor $G$ in the right-hand side of the latter equality has arisen due to the fact that the mean value of the electric field in the pulse is less than its peak value $E_0$. For the averaged over time invariant $\tau$ we get using Eqs. (18) and (19)

$$\tau = \frac{1}{\tau} \int_{-\infty}^{\infty} e^t dt = 2\sqrt{\pi} \Delta \frac{E_0}{E_S}.$$

Finally we obtain from Eqs. (21), (22)

$$\frac{I}{I_S} = \frac{1}{\sqrt{2\pi}} \frac{\tau^2}{16\Delta^2}.$$

Let us estimate the number of pairs created by the $e$-polarized pulse with critical peak electric field $E_0 = E_S$ for $\Delta = 0.1$, $\lambda = 1 \mu m$ and $\tau = 10 \text{ fs}$. In this case $\tau \approx 0.35$, $I \approx 0.31 I_S \approx 1.5 \times 10^{20} \text{W/cm}^2$ and according to Eq. (14) $N_e \approx 5 \times 10^{20}$. If intensity of the same pulse is equal to critical value $I = I_S$ we have $\tau \approx 0.63$, and $N_e \sim 10^{25}$. We see that the number of created pairs grows very fast with intensity. Indeed, only three times increase of intensity yields three orders of magnitude in the number of created pairs.

Let us now compare the rest energy of created pairs with the total energy carried by the laser pulse which according to Eq. (20) for accepted values of parameters $\Delta$, $\lambda$ and $\tau$ is given by the following formula

$$W \approx 5 \cdot 10^{21} \frac{I}{I_S} mc^2.$$

We see that the energy of the pulse with intensity $I \approx 0.31 I_S$ is of the order $W \sim 10^{21} mc^2$ and hence is of the same order as the rest energy of created pairs $W_p \sim 10^{21} mc^2$. Thus, we conclude that the exploited method becomes inconsistent and one should take into account back reaction of the pair creation effect on the process of laser pulse focusing at such intensity. In other words, one cannot consider the electromagnetic field of the pulse at near critical intensities as a given external field and should take into account depletion of the pulse due to pair production. Moreover, it is clear that the critical intensity $I_S$ can hardly be achieved for at least the $e$-polarized focused laser pulse in optical range. We will discuss the situation with $h$-polarized pulses in Sec. V.

To conclude this section, we should emphasize that it follows from Eq. (14) that the number of created pairs $N_e \approx 30$ at $I = 5 \cdot 10^{27} \text{W/cm}^2$ for the same values of parameters $\Delta$, $\lambda$ and $\tau$ in $e$-polarized pulse. Hence we can say that the effect of pair creation becomes observable just at intensity of such order of magnitude. The peak value of electrical field for the pulse of such intensity is of the order of $E_0 \sim 0.18 E_S$, $\tau \sim 6 \cdot 10^{-2}$ and the rest energy of created pairs $W_p \sim 4mc^2 \ll W \approx 5 \cdot 10^{19} mc^2$. The last inequality approves the application of our method to the considered problem at $I = 5 \cdot 10^{27} \text{W/cm}^2$.

IV. NUMERICAL CALCULATIONS.

In this section we present the results of numerical calculations of the number of created pairs $11$ by the fields $3$, $4$.

Figs. 1, 2 demonstrate the dependencies of invariants $\mathcal{E}$ and $\mathcal{H}$ for a $e$-polarized wave on spatial coordinates $x$ and $y$ for the time moments $t = 0$, $t = \pi/2\omega$ respectively. It can be seen from these figures that the electric field $\mathcal{E}$ reaches its maximum in the focus ($x = 0$, $y = 0$), while the maximum of the field $\mathcal{H}$ moves with time along the circle of radius $\xi \approx 0.6$, and its value is less than the value of $\mathcal{E}$ maximum.

In Fig. 3 we present the dependencies of invariants $\mathcal{E}$ and $\mathcal{H}$ on spatial coordinate $\chi$. One can see that $\mathcal{E}$ has a maximum in the focus, while $\mathcal{H}$ is very close to zero there. The forms of $\mathcal{E}$ and $\mathcal{H}$ dependencies on spatial coordinates as well as on time (which can be deduced from comparing Fig. 1 and Fig. 2) justify the method of estimation used in the preceding section, which is based on the assumption that pairs are produced mainly near the focus where the values of $\mathcal{E}$ and $\mathcal{H}$ are independent of time apart the factor $q(\varphi/\omega\tau)$.

The coordinate dependencies of the invariants $\mathcal{E}$ and $\mathcal{H}$ in the $h$-polarized wave are also given by Figs. 1-2 if one makes the interchange $\mathcal{E} \rightleftharpoons \mathcal{H}$. These results explain why we have obtained the zero value for the number of created pairs by the $h$-polarized wave. Indeed, our method of estimation used in the preceding section was based on the value of the invariants in the focus and, as it is clear from the above, $\mathcal{E}$ is equal there to zero for the $h$-polarized wave. In fact, $\mathcal{E}$ is not equal to zero at the periphery of the focal plane. However its values are suppressed by the exponent in functions $F_1$, $F_2$, see Eq. (10). Hence the number of pairs produced by an $h$-polarized wave is several orders of magnitude less than by the $e$-polarized wave of the same intensity and parameter $\Delta$, see below.
FIG. 1: The dependencies of $E$ (a) and $H$ (b) on spatial coordinates $x$ and $y$ for the time moment $t = 0$. $E$ and $H$ are measured in units of $E_S$, and the other parameters are chosen $E_0 = 0.1$, $z = 0$, $\Delta = 0.1$.

FIG. 2: $E$ (a) and $H$ (b) as functions of spatial coordinates $x$ and $y$ for the time moment $t = \pi/2\omega$. The parameters are the same as in Fig. 1.

In Fig. 4a we present the dependence of the number $N_e$ of pairs created by a $e$-polarized laser pulse on intensity for different values of $\Delta$ and for $\lambda = 1 \mu m$, $\tau = 10^{-14}$ s. We see that the number of created pairs grows very rapidly when intensity increases from $10^{27}$ W/cm$^2$ to $10^{28}$ W/cm$^2$. In agreement with our estimation the number of created pairs reaches the value of the order 1 at intensity close to $5 \cdot 10^{27}$ W/cm$^2$. This means that the effect of pair creation can be experimentally observed for the laser pulse intensity two orders of magnitude less the critical value $I_S$. This is true for at least a $e$-polarized pulse and can be explained by the large value of the preexponential factor in (14), which can be represented as the ratio of the laser pulse 4-volume to the Compton 4-volume. This factor compensates small Schwinger exponent in the expression for the number of created pairs.

Fig. 4b illustrates the dependence of the number of created pairs on the parameter $\Delta$. One can see that for all values of intensity the number of pairs very rapidly approaches zero with decrease of $\Delta$. It is due to the fact that the less is $\Delta$, the better the focused laser pulse can be approximated by a monochromatic plane wave which cannot create pairs at all. This can be explicitly seen from the expressions (13) for the field invariants.

In Fig. 5 we show the dependence of number of created pairs on the laser pulse intensity for $e$- and $h$-polarized waves. Since in the case of the $h$-polarized wave pairs are created at the periphery of the pulse, where the field amplitude is exponentially suppressed, the number of pairs is several orders of magnitude less than in the case of the $e$-polarized wave. The dotted line in Fig 5 represents the number of pairs created by an equal mixture of $e$- and $h$-polarized pulses. We see that the number of created pairs by the mixture is several orders of magnitude less than even in the case of the $h$-polarized pulse. This happens due to nonlinear dependence of invariants $E$ and $H$ on the field.
For the calculation of the number of pairs created, we have used a 3-dimensional model of the electromagnetic field based on an exact solution of Maxwell equations [18]. It was shown that the number of created pairs strongly depends on the type of polarization and on the configuration of the electromagnetic field in laser focus and on the dimensionless focusing parameter ∆.

In the present paper we have studied the effect of pair creation by a circularly polarized focused laser pulse in vacuum. To describe the focused laser pulse we have used a 3-dimensional model of electromagnetic field based on an exact solution of Maxwell equations [18]. It was shown that the number of created pairs strongly depends on the configuration of the electromagnetic field in laser focus and on the dimensionless focusing parameter ∆.

Pairs are created most effectively by an e-polarized pulse. Indeed, for the e-polarized laser pulse with λ = 1μm, τ =

![Graph](image_url)

**FIG. 3:** The dependencies of $\mathcal{E}$ (a) and $\mathcal{H}$ (b) on spatial coordinate $\chi = z/L$ for the time moment $t = 0$. $\mathcal{E}$ and $\mathcal{H}$ are measured in units of $E_S$, and the other parameters are chosen $E_0 = 0.1$, $x = 0$, $y = 0$, $\Delta = 0.1$.}

| $I$, W/cm² | $E_0/E_S$ | $N_e$, $\Delta = 0.1$ estimate | $N_e$, $\Delta = 0.05$ estimate | $N_h$, $\Delta = 0.1$ estimate |
|------------|-----------|-------------------------------|-------------------------------|-------------------------------|
| 2 × 10²⁷  | 0.11      | $4.0 \times 10^{-21}$         | $3.4 \times 10^{-12}$         | $4.6 \times 10^{-42}$         |
| 4 × 10²⁷  | 0.16      | 0.09                          | 0.07                          | $6.8 \times 10^{-24}$         |
| 5 × 10²⁷  | 0.18      | 24                            | 30                            | $3.11 \times 10^{-19}$        |
| 6 × 10²⁷  | 0.20      | $1.5 \times 10^3$            | $2.7 \times 10^3$            | $8.8 \times 10^{-16}$         |
| 8 × 10²⁷  | 0.23      | $5.3 \times 10^5$            | $1.6 \times 10^6$            | $6.4 \times 10^{-11}$         |
| 1 × 10²⁸  | 0.25      | $3.0 \times 10^7$            | $1.2 \times 10^8$            | $1.4 \times 10^{-7}$          |
| 2 × 10²⁸  | 0.36      | $8.0 \times 10^{11}$         | $7.1 \times 10^{12}$         | $32$                          |
| 5 × 10²⁸  | 0.56      | $1.0 \times 10^{16}$         | $1.6 \times 10^{17}$         | $1.3 \times 10^{9}$           |

**TABLE I:** The number $N_{e,h}$ of produced pairs for different values of laser pulse intensity and parameter $\Delta$ for e- and h-polarized waves.

The numbers of pairs $N_{e,h}$ created by e- and h-polarized waves for different values of $I$ and $\Delta$, as well as the estimated values of $N_e$ for e-polarized wave, are given also in the TABLE I. We have limited the values of intensity in the TABLE I by $I = 5 \cdot 10^{28}$ W/cm² since at larger intensities the exploited method of calculations ceases to be valid.

We have calculated the number of pairs created by an arbitrary mixture of e- and h-polarized pulses and have found that e type of polarization is the optimal configuration of the focused laser pulse for observation of the pair creation effect.

V. CONCLUSIONS

In the present paper we have studied the effect of pair creation by a circularly polarized focused laser pulse in vacuum. To describe the focused laser pulse we have used a 3-dimensional model of electromagnetic field based on an exact solution of Maxwell equations [18]. It was shown that the number of created pairs strongly depends on configuration of the electromagnetic field in laser focus and on dimensionless focusing parameter $\Delta$.

Pairs are created most effectively by a e-polarized pulse. Indeed, for the e-polarized laser pulse with $\lambda = 1\mu m$, $\tau =$
FIG. 4: a) The dependence of the number of created pairs on laser pulse intensity for different values of $\Delta$ ($\Delta = 0.1$, 0.075, 0.05 from top to bottom) and for $\lambda = 1\mu m$, $\tau = 10^{-14}$ s. b) The dependence of the number of created pairs on $\Delta$ for different values of laser pulse intensity ($I = 0.5 \times 10^{28}$ W/cm$^2$, $1 \times 10^{28}$ W/cm$^2$, $2 \times 10^{28}$ W/cm$^2$ from bottom to top).

FIG. 5: Dependence of the number of created pairs versus laser pulse intensity for $e$-polarized (solid curve), $h$-polarized (dashed curve), and for equal mixture of $e$- and $h$-polarized pulses (dotted curve). $\lambda = 1\mu m$, $\tau = 10^{-14}$s, $\Delta = 0.1$.

10 fms, and $\Delta = 0.1$, the effect becomes observable at intensity $I \approx 5 \cdot 10^{27}$W/cm$^2$, while for the $h$-polarized pulse with the same set of parameters only at $I \approx 10^{28}$W/cm$^2$. In both cases the pair creation process begins at intensities essentially less than the characteristic value $I_S = 5 \cdot 10^{29}$W/cm$^2$. It is worth noting that the peak value of the electric field in the focus in both cases is less than the critical QED value $E_S$. As it was mentioned in the preceding section, this is explained by a very large value of the effective laser pulse 4-volume, where pairs are effectively created, in comparison with the characteristic Compton 4-volume, $l_4^C/c$. One can see from the TABLE that for the $e$-polarized pulse with the same $\lambda$ and $\tau$ but with $\Delta = 0.05$ the effect becomes observable only at intensity $I \approx 2 \cdot 10^{28}$W/cm$^2$.

A very important consequence of our investigation is existing of a natural physical limit for attainable focused laser pulse intensities. This limit is posed by the effect of pair creation and for the circularly $e$-polarized laser pulse with $\lambda = 1\mu m$, $\tau = 10$ fms, and $\Delta = 0.1$ is approximately $0.3I_S$, see Sec. III.

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