Derivation of GPS TEC and receiver bias for Langkawi station in Malaysia

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Abstract. This paper presents the polynomial-type TEC model to derive total electron content (TEC) and receiver bias for Langkawi (LGKW) station in Malaysia at geographic latitude of 6.32° and longitude of 99.85°. The model uses a polynomial function of coordinates of the ionospheric piercing point to describe the TEC distribution in space. In the model, six polynomial coefficients and a receiver bias are unknown which can be solved by the least squares method. A reasonable agreement is achieved for the derivation of TEC and receiver bias for IENG station in Italy, as compared with that derived by the IGS analysis center, CODE. We process one year of LGKW data in 2010 and show the monthly receiver bias and the seasonal TEC variation. The monthly receiver bias varies between −48 and −24 TECu (10¹⁶ electrons/m²), with the mean value at −37 TECu. Large variations happen in the monthly receiver biases due to the low data coverage of high satellite elevation angle (60° < a ≤ 90°). Post-processing TEC approach is implemented which can resolve the wavy pattern of the monthly TEC baseline resulted from the large variation of the receiver bias. The seasonal TEC variation at LGKW exhibits a semi-annual variation, where the peak occurs during equinoctial months, and the trough during summer and winter months.

1. Introduction

The Earth’s ionosphere is a thin atmospheric layer consisting of charged particles (ions and electrons) and neutral particles. The charged particles are resulted from the photoionization process by solar EUV radiations. It is found that the electron density plays a crucial role in manipulating the ionospheric conditions. Since the Global Positioning System (GPS) satellites have been launched for positioning and velocity determination, the dual-frequency GPS signals have then been implemented for estimating the total electron content (TEC) of the ionosphere. Here TEC is denoted as a line integral of electron density along the signal path between GPS satellite and ground receiver. Over the past decades, the TEC parameter had been widely used for studying the ionospheric phenomena such as equatorial ionization anomaly, plasma bubbles, equatorial spread F, and ionospheric storms.

The TEC calculation is based on the group delay and phase advance of the dual-frequency GPS signals, which are caused by the dispersive characteristic of the ionosphere. The accuracy of TEC estimation is affected by systematic biases and random noises. The systematic biases can be attributed to satellite bias (e.g., orbital errors and clock bias) and receiver bias (e.g., clock bias, multipath, and antenna phase center variation). The Center for Orbit Determination in Europe (CODE), one of the International GPS Service (IGS) analysis centers, has provided the monthly differential code biases (DCBs) for all active GPS satellites and for all the IGS stations processed at CODE [1]. However, the GPS stations in Malaysia are not included in the list of the IGS stations.

For a single GPS station, there are three approaches in the literatures to estimate the TEC and the DCB for the receiver, namely, polynomial method [2], minimum variance method [3], and IONOLAB-bias method [4]. The first approach is using polynomial function of coordinates to model the TEC distribution, where the receiver bias and the polynomial coefficients are unknown which can be solved by the least squares method. The second approach is to minimize the standard deviation of...
vertical TEC for different satellites over a certain elevation angle by use of the trial receiver biases, for which the optimal receiver bias will yield the minimum deviation. The third approach is using the global ionosphere maps (GIMs) and the satellite DCBs from CODE to estimate the receiver bias. We note that the polynomial method is able to estimate the TEC along the non-measured lines of sight as compared to the second approach.

In this paper we apply the polynomial-type TEC model to derive the receiver bias and TEC for Langkawi (LGKW) station in Malaysia. We use one year of LGKW data for demonstration and show the variation of the monthly receiver bias and the seasonal TEC variation. The paper is organized as follows. In section 2, the polynomial-type TEC model is presented. In section 3, we apply the polynomial-type TEC model to the GPS data from IENG station in Italy and compare our results of TEC and receiver bias with that derived from the CODE. The results of receiver bias and TEC are presented for LGKW station in section 4, where the post-processing TEC approach are proposed for resolving the monthly TEC baseline issue. Finally, summary and discussion are given in section 5.

2. Polynomial-type TEC model
Two L-band frequencies ($f_1 = 1575.42$ MHz and $f_2 = 1227.60$ MHz) are used in the GPS transmitted signals. The GPS observables are code and phase pseudoranges which are deduced from measured time and phase differences between received signals and receiver generated signals, respectively. The code (phase) pseudoranges are denoted as $P_1$ and $P_2$ ($L_1$ and $L_2$) for the two different frequencies. The slant TEC ($S$) along the ray path between the satellite and the ground receiver can be calculated from $P_2-P_1$ or from $\lambda_1 L_1 - \lambda_2 L_2$, as shown in equations (1) and (2), where $\lambda_1$ and $\lambda_2$ are the wavelength at $f_1$ and $f_2$, respectively.

$$S_p = \frac{1}{40.3} \frac{(f_1 f_2)^2}{(f_1^2 - f_2^2)} (P_2 - P_1)$$

$$S_L = \frac{1}{40.3} \frac{(f_1 f_2)^2}{(f_1^2 - f_2^2)} (\lambda_1 L_1 - \lambda_2 L_2)$$

Due to the ambiguity of phase measurement, $S_L$ is a relative value in equation (2), but has higher precision than $S_p$. To retain the higher precision for slant TEC, a baseline value, $B_r = <S_p>-<S_L>$, is computed for $S_L$. Here $<$ > denotes the mean value. Therefore, a slant TEC is calculated as $S = S_L + B_r$. Note that the satellite biases from the CODE are used in the slant TEC calculation and the cycle slips in $L_1$ and $L_2$ phase measurements are processed. To convert slant TEC to vertical TEC, the ionosphere is assumed as a thin shell at a given height. By considering the receiver bias, the vertical TEC (VTEC) is then calculated as

$$VTEC = (S - b)m$$

$$m = \cos(\sin^{-1}(\frac{R_E \cos \alpha}{R_E + h}))$$

Here $b$ is the receiver bias, $m$ is mapping function, $\alpha$ is the elevation angle of the satellite, $R_E$ is the Earth’s radius, and $h = 400$ km is the height of the ionosphere.

In this paper, the VTEC can be expressed as a polynomial function of coordinates of the ionospheric pierce point (IPP), which is the intersection point of the ray path from satellite to receiver through the thin shell of the ionosphere. The polynomial function is defined as

$$E_{\phi}(\phi, \theta) = VTEC = c_1 \phi^2 + c_2 \theta^2 + c_3 \phi + c_4 \theta + c_5 \phi \theta + c_6$$

where $\phi$ and $\theta$ are the longitude and latitude of IPP, and $c_1$-$c_6$ are constant. One can then compute the variance as
\[ \sigma^2 = \frac{1}{N} \sum_{i=1}^{N} w_i^2 \left[ (S_i - b)m_i - E_i \right]^2 \]  

(6)

Here \( N \) is the number of measurements and \( w_i = \sin^2 \alpha_i \) is the weighting function. By minimizing \( \sigma^2 \) with respect to \( b \) and the coefficients \( c_1 \rightarrow c_6 \), one can then obtain equations which can be solved for the receiver bias and the polynomial coefficients. In the present study, we use all measurements from different satellites over one hour period to perform the minimization.

3. **Comparison results for IENG station**

Figure 1a shows the monthly receiver bias derived from the CODE and the polynomial model for IENG station in Italy for the year 2008. Figure 1b shows the difference of receiver bias (\( \Delta \text{DCB} \)) between the polynomial model and the CODE. The mean deviation is \( \sim 1.7 \) TECu (\( 10^{16} \) electrons/m\(^2\)), as shown by the dashed line in Figure 1b, while the maximum deviation is \( \sim 3.0 \) TECu. The overall result indicates that the receiver bias derived from the polynomial model agrees reasonably with the CODE. Figure 2 shows the daily TEC from the two different approaches for IENG station during February (low \( \Delta \text{DCB} \)) and October (high \( \Delta \text{DCB} \)). It is seen that these two approaches agree well in the TEC calculation, albeit with a bias shift in Figure 2b. This bias shift is attributed to the deviation of the receiver bias from the CODE.

![Figure 1a](image1a.png)  
![Figure 1b](image1b.png)  

**Figure 1.** (a) The monthly receiver bias derived from the CODE and the polynomial model for IENG station in Italy. (b) The difference of receiver bias between the CODE and the polynomial model. The dashed line denotes the mean value of the deviations.

![Figure 2a](image2a.png)  
![Figure 2b](image2b.png)  

**Figure 2.** Plots of the daily TEC computed from the CODE and the polynomial model for IENG station during (a) February and (b) October. Note that the \( \Delta \text{DCB} \) is low in February while it is high in October.

4. **Receiver bias and TEC for Langkawi station**

The GPS data from LGKW station is obtained from the Malaysia Real Kinematics GNSS Network under the Department of Survey and Mapping Malaysia (JUPEM). In this study, we use the data in the
year 2010 for demonstration. Note that the LGKW data is using C1 signal, which is less accurate than P1 signal in the TEC calculation from the code pseudoranges. Figure 3 shows the monthly receiver bias derived from the polynomial-type TEC model for LGKW station. The monthly receiver bias varies between −48 and −24 TECu, with a mean value at −37 TECu (as shown by dotted line in Figure 3). Large variations happen in the monthly receiver biases (e.g., August and September) which is likely due to the low data coverage of high satellite elevation angle ($60^\circ < \alpha \leq 90^\circ$).

Figure 4a shows the daily TEC data over an one year period for LGKW station. It is seen that the monthly TEC baseline is wavy, which is due to the large variation of the receiver bias. To tackle this issue, we implement five post-processing steps as follows:

1. Finding the daily peak (P) and trough (T) TEC over an one year period.
2. Calculating the mean value $\kappa$ of P and T, i.e., $\kappa = (P + T)/2$.
3. Calculating the median value of $\kappa$, $\kappa_0$.
4. Repeating step 1 to step 3 for monthly data to get $\kappa_M$.
5. Calculating the new TEC, $TEC_R = TEC_P - (\kappa_M - \kappa_0)$. Here $TEC_P$ is the TEC obtained from the polynomial model for M month.

![Figure 3. The variation of the monthly receiver bias derived from the polynomial-type TEC model for LGKW station.](image)

![Figure 4. (a) Daily TEC obtained from polynomial-type model for the whole year 2010. (b) Variations of the peak TEC, trough TEC, and $\kappa$. The green dashed line is the median value of $\kappa$. (c) Corrected TEC by the post-processing procedures.](image)
Figure 4b shows the variation of the peak TEC, the trough TEC, and $\kappa$ over the whole year period. As compared to $\kappa_0$, the $\kappa$ has large deviations at June, July, August, and September. The post-processing TEC is shown in Figure 4c. Evidently, the monthly TEC baseline has been improved significantly, indicating that the post-processing approach can resolve the monthly TEC baseline problem resulted from the large variation of the receiver bias.

Figure 5 shows the seasonal variation of the peak TEC at LGKW station. It is seen that the seasonal TEC variation exhibits a semi-annual variation, where the peak occurs during equinoctial months and the trough during summer and winter months. This pattern is consistent with the previous studies for the equatorial regions [5].

![Figure 5. Seasonal variation of the peak TEC at LGKW station for the year 2010.](image)

![Figure 6. Comparison of the original receiver bias with the corrected one.](image)

5. Summary and discussion

We have examined the polynomial-type TEC model to derive the receive bias and TEC for Langkawi station in Malaysia. In the model, six polynomial coefficients and a receiver bias are unknown, which can be solved by the least squares method. We have tested the performance of the polynomial model using GPS data from the IENG station in Italy. The comparison result indicates that the polynomial model is capable of deriving the receive bias and TEC, although there are some deviations between the model and the CODE. We have therefore applied the polynomial-type TEC model to the LGKW data in the year 2010. The monthly receiver bias of LGKW varies between $-48$ and $-24$ TECu, with the mean value at $-37$ TECu. Large variations in the monthly receiver biases are present due to the low data coverage of high satellite elevation angle ($60^\circ < \alpha \leq 90^\circ$). As a result, the TEC values obtained from the polynomial model are biased by the wavy pattern of the monthly TEC baseline. To tackle this issue, we implement the post-processing TEC procedures. Significant improvement is achieved for which the monthly TEC baseline is rectified over the whole year period. The seasonal variation of the peak TEC at LGKW exhibits a semi-annual variation, where the peak occurs during equinoctial months and the trough during summer and winter months.

It is found that the polynomial-type TEC model cannot efficiently extract the wavy pattern of the monthly TEC baseline from the TEC calculation. In doing the post-processing TEC, the $\kappa_M - \kappa_0$ is used for TEC correction. By considering that this corrected amount is treated as the receiver bias, one can recalculate the receiver bias by adding it into the original one. Figure 6 shows the comparison of the original receiver bias with the corrected one. Evidently, the corrected receiver biases become more stable as compared to the original one. The mean value of the corrected receiver biases is about $-38$ TECu, consistent with that obtained from the original one.

In Figure 5, one can find that the overall TEC is higher in the winter months than in the summer months. This result is closely related with the thermospheric neutral composition ratio O/N$_2$. During
solstice, the global thermospheric circulation is generated as a result of the asymmetric heating of the globe that induces a prevailing summer-to-winter wind with upwelling at low latitudes in the summer hemisphere and downwelling in the winter hemisphere [6]. The upwelling in the summer hemisphere enhances the concentration of molecular species and thus decreases the O/N\textsubscript{2} ratio, while the downwelling in the winter hemisphere increases the concentration of atomic oxygen and thus enhances the O/N\textsubscript{2} ratio. Rishbeth et al. [7] have shown that NmF\textsubscript{2} is closely related to the O/N\textsubscript{2} ratio, a large ratio of O/N\textsubscript{2} resulting in a large NmF\textsubscript{2} at low latitudes. As a result, the TEC is on average higher in winter than in summer.

In conclusion, we have successfully applied the polynomial-type TEC model to derive the receiver bias and TEC for LGKW station, with the post-processing TEC procedures. The seasonal TEC variation at LGKW is consistent with the previous studies for the equatorial regions.

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