A Comparative Study of Simulated Annealing and Genetic Algorithm Method in Bayesian Framework to the 2D-Gravity Data Inversion

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Abstract. The use of modern optimization method in geophysical inversion has effectively given a robust global solution in its application to solve a complex non-linearity problem. Here, we tested two artificial intelligent-based methods, the simulated annealing and the genetic algorithm to the gravity data. Using predicted anomalies geometries, these methods are addressed to invert a synthetical gravity data extracted from grav2d open source. Differences between these methods are observed in both single parameter inversion and simultaneous multi parameter inversion to evaluate the speed of computing and the use of Space in memory. The result give us an idea that the genetic algorithm are slower than the simulated annealing in solving a simple inversion problem (small data set and less parameter to be inverse) but efficient in a large data set. meanwhile, the simulated annealing faced some problem in locating a global minima of the misfit function for the large data, in the latter case, we simulated the methods in the Bayesian framework to see the distribution of posterior probability of the parameters.

Keywords: geophysical inversion problem, gravity method, simulated annealing, genetic algorithm, Bayesian distribution

1. Introduction

Many geophysical inverse problem still face some problem in escaping the local maxima. The conventional inverse processing formulate the cost function depending to physical properties which is to be determined by the optimization process. The usual technics for the inversion problem are Quasi-Newton method or conjugate Gradient method [1,3]. It performs a deterministic process to locate the local maxima by calculating a derivative formulation. However, it may allow some difficulties to determine a derivative equations in complex problems. To overcome this problem, geophysicists have been tried various technics for developing the optimization problem. Nowadays, they starts to leave
the conventional methods to modern methods which is called as a modern optimization method which adapted to natural intelligence based on artificial method, because of its ability to give a robustness and locate the global optima in non-linearity problem[2].

Two best modern optimization based-artificial methods which is very popular among geophysicists are the Simulated Annealing (SA) and The Genetic Algorithm (GA). Both can relocate the random search of solution into the guided search by setting the factors for each methods which leads to global solution. The main point which differs these methods to the conventional technics is that they are fully independent of calculating derivatives. They likely solve the problem in probabilistic mode[4]. Hence, for a complex systems, we do not need to play with many derivations anymore. As for substitution, we may 'play' with the operators for each methods.

Both of the methods seem to be very powerful to solve the geophysical problem regardless of how each methods work during the inverse process. However, they differ in the way they locate the global minima. The GA is working based on the survival of the 'fittest' adapted from biological evolution system [5,6]. It generate a large number of sampled model to create a 'population' of potential solution which is grouped in ‘Chromosomes’ containing ‘gen’ of parameters to be inverted. In geophysical problems, gen represents the physical/geological properties which are related to the exploration methods. (e.g., $p$ or $s$ wave velocity in seismic, density in gravity, or resistivity in geo-electricity methods). The fittest ‘individual’ (solution) depends on how good it fits the observed data. This steps continue in each ‘generation’ for generate a new fittest individual. This entire process is controlled by three operator parameters, selection probability, cross-over and mutation rate. number of models with better fitness increases as the generations advances[10]. On the other hand, the SA mimics the annealing process in thermodynamic. The slow cooling process related to the kinetic energy of the materials which allows the particle to form a crystal state. While, rapid cooling process results the formation of glass [6,7,8,9]. These are equivalent with the local and global maxima. The annealing process is controlled by the ‘temperature’ parameters. As the temperature decrease, the cooling schedule leads to the global solution.

Based on the above description, there is a very interesting thing to investigate, that is how both methods differ in their performance to solve the geophysical problem. Here, in this paper we present the results of inversion of gravity anomalies over 2D synthetic data using both genetic algorithm and simulated annealing and recorded the speed and the memory usage of the computer in order to evaluate the computational cost. The synthetic data has been granted by the free software of GRAV2DC.

2. Mathematical Model

2.1. Bayesian formulation

According to what we mention about the non-linearity of a geophysical model, we may face difficulties in estimating a robust solution. The real earth problems are complex. That is why most of the preferred model are not liniier. Hence, we cannot estimate a specific number of related parameters in any geophysical inversion. Nevertheless, we may forecast the range of allowed number of parameters. The best method to collaborate with the probability problem is the Bayesian solution which unites the potential solution into a normal distribution which draws a description how the probabilities of the solution behave in the model[11].

The basic concept of the Bayes’ formula is both simple and very powerful:

$$P(A)P(B \mid A) = P(B)P(A \mid B)$$ (1)

Let us call $A$ is the data and $B$ is the model, hence Bayes’ formula gives

$$P(B \mid A) = \frac{P(B)P(A \mid B)}{P(A)}$$ (2)
where \( P(B|A) \) is the posterior probability, \( P(B) \) is prior probability of the model, \( P(A|B) \) is data likelihood for model \( B \), and \( P(A) \) is normalization for data \( A \) which is equal to \( \int P(A,B)dB \).

Therefore,
\[
P(B|A) = \frac{P(B)P(A|B)}{\int P(A,B)dB}
\]

the Equation 3 shows the practical issues to obtain the Bayesian posterior probability where the data likelihood for model \( B - P(A|B) \) is obtained by computing the probability for the data to be actually observed if model \( B \) is the true model. This \( P(A|B) \) is the “forward problem” in probabilistic form which contains errors. The prior probability of the model \( P(B) \) is the ‘previously solved’ “inverse problem” by the Bayesian probability.

In the geophysical systems, the relation between the data \( d \) and the model \( G \) which contains parameters \( m \) is
\[
d = G(m)
\]
where the \( L^2 \) norm is \( L^2 = \|d_{\text{obs}} - d_{\text{cal}}\|^2 \), hence the misfit function of the model is
\[
\chi = (d - G(m))^T (d - G(m))
\]
according to Bayesian formula (eq. 3). The model could be formulated into
\[
P(m|d_{\text{obs}}) = \frac{P(m)P(d_{\text{obs}}|m)}{\int P(d_{\text{obs}},d_{\text{cal}})dm}
\]
or in mathematical form, could become
\[
P(m|d_{\text{obs}}) = \exp \left( -\frac{1}{2}\|d_{\text{obs}} - d_{\text{cal}}\|^2 / \sigma^2 \right)
\]
where \( \sigma^2 \) is the variance of the observed data. This describes a probability density function (pdf) defined over the whole model space (assuming exact data and theory). This pdf is also called the a posteriori probability. In the probabilistic sense the a posteriori pdf is the solution to the inverse problem.

Figure 1. A pdf distribution
2.2. Model Anomalies
In this paper, we model the anomalies as a square model buried in subsurface.

$$\Delta g = 2G\Delta \rho d \left( \log \frac{x^2 + h_2^2}{x^2 + h_1^2} \right)$$

(8)

$G$ is the gravity constant $(6.67 \times 10^{-3} \text{ N m}^2/\text{kg}^2)$, $\Delta \rho$ is the density contrast, $d$ is the width, and $h_1/h_2$ is the length between the lower and upper surface. $x$ is the length of the observed model (see figure 2.)

![Figure 2](image)

**Figure 2.** The scenario of the model

This model will be used for all the simulation in this research. However, the number of parameters and models may vary for each simulation. The model are assumed to be homogenous and isotropic.

2.3. Simulated Annealing Formulation
The SA method use random sampling for the initial condition. The SA method progressively deforms the pdf form, from prior pdf to posterior pdf by decreasing the controlled parameters, temperature. The temperature is set in high number (initially), all models are sampled based on the Monte-Carlo sampling. As the temperature decreases, fewer models are selected. Only those which give a lesser misfit does not get trapped in the local minima. The expression of pdf for the SA is

$$P(d^{\text{obs}} | m) = \exp \left( -\frac{1}{2} \frac{d^{\text{obs}} - d^2}{\sigma^2 T} \right)$$

(9)

where $T$ is temperature decreased from high to low.

The algorithm of the SA is to decrease the simulated kinetic energy of the annealing process by decreasing the temperature (cooling state). First we set the initial temperature while randomly set the initial place of the initial model. The annealing process works in a condition where the temperature is low. Hence the kinetic energy is low as well (See Figure 3.)
2.4. Genetic Algorithm Formulation

The minimization of the misfit function of the gravity model can be modeled as the maximization of the pdf model. Based on the equation 4 and 5 and the $L^2$ norm, $L^2 = \|d_{\text{obs}} - d_{\text{cal}}\|^2$ therefore

$$\mathcal{X} = \frac{2}{\sum \|g_{\text{obs}} - g_{\text{cal}}\| + \sum \|g_{\text{obs}} - g_{\text{cal}}\|}$$

where $g$ is the gravity anomalies which is used to define the fitness function $F$

$$F = 1 - \left( w_1 \mathcal{X}_1 + w_2 \mathcal{X}_2 + \cdots + w_n \mathcal{X}_n \right)$$

which is maximized. Here, $w$ is the weighting factor for each potential model. We will perform single model and two square model in the next chapter. Hence, the number of $w$ is two ($w_1, w_2$). The selection of the new individual follows this formula:

$$g_{\text{new}} = \frac{F_i}{\sum_{i=1}^n F_i}$$

where $g_{\text{new}}$ is the new individual after the last generation, and $F$ is the misfit function for each potential model.

The general structure of a genetic algorithm can be expressed as the following steps [10]:

a) Generating the initial population, this initial population is generated randomly so as to obtain an initial solution;

b) The population itself consists of a number of chromosomes that present the desired solution;

c) Forming a new generation, in forming a new generation used three operators, namely reproduction / selection, cross over and mutation;

d) Evaluation of the solution, this process will evaluate each population by calculating the fitness value of each chromosome, and evaluating it until the stop criteria are met. If the stopping criteria have not been met, it will be formed again new generation by repeating steps b).

The process of genetic algorithm will stop when the generation counter has reached the number of defined generations $c_g$ that is $c_g = 1000$, or the misfit function reach the limit of 0.0001.

The Chromosome Selection process is done using roulette wheel selection. We set the cross over and mutation rate at 0.25 and the probability of the reproduction at 0.5.
3. Result and discussion

The first simulation is to test both methods in inverting a single square model. The first step is to produce the synthetic data.

![Figure 4. Synthetic response](image)

We set the synthetic data as it is shown in figure 4. Then we perform the inversion using both methods to obtain the information of the computational performance and the pdf.

The computer specification which is used during the simulation is Intel Core 2 i5 MacBook Pro which has the clock speed of 2.55 GHz and 4 GB DDR3 ROM with 1600 MHz rotation speed. The computer performance for each methods can be seen in Table 1.

|                         | Simulated Annealing | Genetic Algorithm |
|-------------------------|---------------------|-------------------|
| Sampling size           | 100                 | 100               |
| Time consume            | 0.36 s              | 0.51 s            |
| Predicted density       | 0.66437 gr/cm³      | 0.7082 gr/cm³     |

Table 1. Performance for single parameter inversion

| Table of Prediction      |                          |                  |
|--------------------------|--------------------------|------------------|
| Temperature              | Tmax = 1000 Tmin = 0.0043|                  |
| Cross over Rate          | 0.25                     |                  |
| mutation Rate            | 0.25                     |                  |
| Selection function       | Controlled metropolis    | Roulette Wheels  |
From the figure 4, the density contrast is at 0.7 g/cm$^3$. By perform both parameter, we may see the pdf for each methods in figure 5

![Pdf Distribution of GA vs SA](image)

![Error Plot](image)

**Figure 5.** The pdf of a. The GA vs SA and b. The error plot

According to Table 1. And the figure 5, the SA performs faster than the GA method for the single parameter inversion. However, the GA’s pdf offers a tighter distribution than SA’s. it means that the GA’s draws a narrower distribution of the solution. It has a consequence that GA give fewer potential solution than SA. For this case, GA may give more robust solution than SA. However, it needs more computational cost.
The gravity response of the SA (left) and GA (right). The SA inversion result is 0.66437 g/cm$^3$, while GA gives 0.7082 g/cm$^3$.

The second simulation is similar to the first one, but adding new parameters to be inverted, width $d$. We set the synthetic model of the width, $d$ at 3000 m. The result of both methods can be seen in Figure 7.
Figure 7. The gravity response of the SA (top) and GA (bottom). The SA inversion result is 0.697 g/cm$^3$
For density contrast and 3423 for the width, while GA (based Monte Carlo) gives 0.677 g/cm$^3$ and 3212 m.

The performance of both method can be seen in the Table 2.

|                           | Simulated Annealing | Genetic Algorithm |
|---------------------------|---------------------|-------------------|
| Sampling size             | 100                 | 100               |
| Time consume              | 0.52 s              | 0.54 s            |
| Predicted width           | 3423 m              | 3212 m            |
| Predicted density         | 0.697 gr/cm$^3$     | 0.677 gr/cm$^3$   |

Table of Prediction

|                           | Tmax = 1000 Tmin = 0.12 |
|---------------------------|-------------------------|
| Cross over Rate           | 0.25                    |
| mutation Rate             | 0.25                    |
| Selection function        | Controlled metropolis   | Roulette Wheels    |

Here, the SA method performs faster than GA method. However, the GA give us a closer solution to the best model. Since the GA can perform parallelism in locating the initial solution, therefore, it may have larger sample of potential solution of the model. The population set in this simulation has six chromosomes which mean we may have 6 candidate which has best fitness for each parallel inversion process. It is different with the SA that can only have single initial ‘population’. Therefore, the solution is not various as GA’s. However, we may extend the size of the random sampling for SA. But, it cannot guarantee that it gives a better solution. Also, we spend more computational cost.
A further simulation has been done to test both methods using the observed data from free software GRAV2D as shown in Figure 8. From Figure 5, the response of gravity is high at two point. The first one is located in the left side while the second one has lower peak. Using the previous simulation to predict the model, we assume that the response appears as result of the two square anomalies buried in the subsurface which have different physical properties. The first step is to predict the physical properties of the model. The predictive value of the model can be seen in Table 3.

Table 3. Table of Prediction

| Anomaly | Square 1       | Square 2       |
|---------|----------------|----------------|
| h1      | 3000 m         | 10000 m        |
| h2      | 55000 m        | 90000 m        |
| d       | 7000 m         | 5000 m         |

We set the first square anomaly at 0.5-0.7 g/cm$^3$ and 5000-9000 m of width, while the second square anomaly at 0.8-1.0 g/cm$^3$ and 3000-7000m of width. The second step is to perform the inversion process which test both methods. The inversion result is shown in Figure 9.
Figure 9. The Inversion result for 2D-observed data: (a) Inversion Result (b) Predicted Model

Both of the methods can easily fit the observed model for the predicted ‘green’ square anomaly. But start to face difficulties while inverting the ‘orange’ anomaly square. However, both of them give a close solutions. However, here the GA’s show its privilege in solving complex phenomenon. GA give closer solution than SA. Also it performs faster than SA (Table 4.) with the same size of sampling, GA seems more efficient that SA. Again, we can presume that GA fit with a more complex problem than SA.

Table 4. Table of Prediction

|                          | Simulated Annealing | Genetic Algorithm |
|--------------------------|---------------------|-------------------|
| Sampling size            | 1000                | 1000              |
| Time consume             | 0.85 s              | .72 s             |
| Predicted width          | 8813/5155 m         | 7123/5101 m       |
| Predicted density        | 0.537/0.8825 gr/cm³| 0.665/0.8925 gr/cm³|

Table of Prediction

|                          | Tmax = 1000 Tmin = 0.21 |
|--------------------------|-------------------------|
| Temperature              |                         |
| Cross over Rate          | 0.25                    |
| mutation Rate            | 0.25                    |
| Selection function       | Controlled metropolis   | Roulette Wheels    |
4. Conclusions
We have tested two methods of Global optimization applied to the 2D-Gravity Inversion, The GA and SA method. Both of the method generate a random initial parameters to build the initial model, but differ in each technics in solving the inversion. From the simulation, we can conclude that GA has more advantages in solving a complex problem because of its ability in parallelism and treatment of the potential solution. While SA seems to fit with a simpler case. However, the result may differ for different simulation. Since these methods allows us to modified the controlled parameters (e.g., mutation, cross-over rate, roulette wheels in GA, and temperature in SA), thus the varieties of the objective function can be formulated in order to solve the inversion.

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