A numerical approach for fish fillet modeling during freezing process _ Case study from Vietnamese catfish fillets

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Abstract. Freezing is one of the most complicated treatment in food processing due to the dependence of food properties on temperature. In addition, many kinds of food contain a high proportion of water which has some special thermal characteristics. In this study, the freezing process for a fish fillet from Vietnamese catfish fillets is considered using the Finite Element Method. The coupled heat and mass transfer problem is investigated in a discretized computational domain. Numerical results obtained from COMSOL solver then are compared with the experimental data to prove the efficiency of the proposed method.

1. Introduction
Freezing methodology plays a vital role in the fish industry due to the constantly increasing demand for freshness and quality of frozen food. It is proved that a rapid freezing scheme can create smaller and finer ice crystals reducing structural damage to the cells [1]. Protein denaturation is one of the most disadvantages of traditional slow freezing without temperature monitoring [2]. An appropriate freezing condition is essential for quality preservation supporting a longer distance of delivery in the fish raw market. Fish freezing exposes several physical phenomena in a complex process influenced by many factors [3]. Therefore, preserving food freshness and restricting denaturation are the most crucial strategies for technologists in the new era [4, 5]. Food experts have spent a lot of time in investigating freezing periods to design the refrigeration systems with essential power and optimize electrical consumption.

The phenomena in freezing can be described by initial value problems in mathematics with propagating properties. There is unsteady non-linear heat conduction in the physical model with changeful thermal coefficients and convective cooling [6]. Heat transfer is assumed as a primary principle for food multi-stage evolution. The validity of this assumption has been proved by numerical simulations in many cases [7]. However, the heat transfer is modified by a variation in water proportion before crystallization. Consequently, it is required to combine the mass transfer in the estimation models for the purpose of acquiring better results.
As the development of freezing processes and description models, there are four distinct methods for modeling freezing problem and estimating freezing times: analytical, empirical, approximate, and numerical. Analytical aspects consider the problem in a simple state (surrounding conditions, product shape, product features) such that the solution of governing equations can be obtained exactly. Due to the simplifications in the problem model, analytical methods are limited in the practical applications but it can be used as a reference for developing other methods. Empirical methods are hybrid schemes based on theory and data attained from numerical calculations or experiments. The method can return to a poor result if conditions do not belong to the input data set. Approximate methods use some equivalent models of reality to remove extreme simplifications in analytical approaches. Normally, the freezing process is separated into sensible cooling and phase change steps with avoiding any changes in physical properties. The predictions gained from these methods still acceptable to use in practice. In numerical methods, the computational space is discretized into elements to apply the differential equations depicting evolution of temperature field and other variables over the domain. As a result, this strategy can be expressed for any ranges of problems if the computational resources are sufficient.

2. The freezing process
The time-temperature relation of the overall process is described in Figure 1. In this diagram, food undergoes three main stages: precooling, phase change and subcooling. In particular, the temperature goes down remarkably in the first stage by releasing the product apparent heat. Next, the latent heat is removed to change the water state from liquid to solid. Afterward, the apparent heat is taken out in solid product making a noticeable drop of temperature in the final stage. At the end of precooling stage, in some cases, water in food can pass the freezing threshold to enter the supercooling domain for a short time without changing phase. Although supercooling is a special period in the freezing time, its effect typically neglected in most calculation approaches.

![Figure 1. Time-temperature curve during freezing stages.](image)

2.1. Thermal conduction problem
For the example of pure heat transfer in an arbitrary solid environment, the Fourier equation for heat conduction can be expressed as [8]:

$$\rho C_p \frac{\partial T}{\partial t} = \nabla \left( k \nabla T \right) + S$$  \hspace{1cm} (1)

where $\rho$ is the density, $C_p$ is the special heat, $T$ is the temperature, $k$ is the thermal conductivity, $S$ is the heat generation. In the case of single temperature for phase changing and latent heat releasing, it is
called as the Stefan problem. The analytical results have been found in simple cases for validation of numerical simulations.

2.2. Numerical approach for solving the conduction equation
As a mean to solve Eq. (1), the computational domain should be discretized to construct the ordinary differential equations (ODEs) expressing the temperature field through nodal values as,

$$ C \frac{dT}{dt} + KT = f $$

where $C$ is the global matrix for special heat value, $T$ is the temperature vector, $K$ is the global conductance matrix computed from the thermal conductivity, and the global forcing $f$ is derived from boundary conditions and heat generation. In Finite Element Method (FEM) [9, 10], the temperature at location $x$ is interpolated through approximation:

$$ T(x,t) = N^T(x)T(t) $$

where $N(x)$ is the basis functions for finite elements evaluated at $x$, $T(t)$ is the nodal vector of the temperatures. In the Galerkin version of FEM, the basic functions are employed as weighting functions:

$$ \int_{\Omega} N \left[ \rho C_p \frac{\partial T}{\partial t} - \nabla (k \nabla T) - S \right] d\Omega = 0 $$

Eq. (6) and Eq. (7) illustrate an influence between the nodes in each single element. The information at these are then assembled to construct the global matrices for the linear equation solving step.

2.3. Dealing with variable thermal properties
The ingredients in food (water, protein, fat, hydrocarbons, cellulose, etc.) are strongly depended on kind of food and temperature. The general formula for calculating the special heat of foods according to the componentional special heat is:

$$ C = \sum_i C_i x_i, $$

where $C_i$ is the componentional special heat, $x_i$ is the proportion of each ingredient in food. For the frozen food, the special heat calculation becomes more difficult due to the phase changing phenomenon. Therefore, the apparent and latent heats must be included in the estimation. In this study, the special heat is measured from experiment as depicted in Figure 2. It is obvious that there is a discontinuity at the crystalline point at is -2.2 °C where the equivalent special heat achieves a very high value because release of latent heat. This sudden increment makes the iterative solver become divergent at the end of precooling stage. In order to overcome this barrier, the relative error of the solver should be modified to a very small value (~10⁻⁸) to ensure the convergent rate.
Figure 2. Dependence of special heat to temperature for fish fillet.

The thermal conductivity exposes the relationship between heat transfer speed and the thermal gradient. For food substances, it depends on the components, the individual arrangement of physical structure, and the food temperature. It is quite hard to determine the thermal conductivity diagram for complex component food. For the case of fish fillet, this graph is drawn from measured data as shown in Figure 3(a). Besides, the thermal diffusion coefficient described in Fourier equation is an important factor in heat transfer process. It can also be defined as a function of temperature as depicted in Figure 3(b).

Figure 3. Variability of (a) thermal conductivity and (b) thermal diffusion coefficient during freezing process.

3. Numerical result

In this section, the special phenomena in freezing process are investigated at a standard slice of the fish fillet from Vietnamese catfish fillets. The freezing problem is set up in a rectangular domain which has 80x15 mm in dimensions as given in Figure 4(a). Due to the symmetric geometry and boundary conditions, a simplified domain is chosen to consider as shown in Figure 4(b). The heat
transfer coefficient takes the value 26.3 W/(m$^2$K) corresponding to the air velocity 5 m/s at -30 °C. At the beginning, the percentage of water in the fillet is 67.3% and the initial temperature is 12 °C. The solidification temperature of fish is -2.2 °C measured by experiment as mentioned in Section 2. In addition, the dependence of thermal properties of fish to temperature are indicated in Section 2.

![Diagram](image1)

**Figure 4.** Freezing problem, geometry domain and boundary description.

The diagram in Figure 5 demonstrates the changing of temperature at external surface (point A) and the centroid (point B) from precooling to subcooling stages. Overall, the interior region (green curve) need more time for solidification than the exterior one (blue curve). In the precooling step, there is a dramatic slump of temperature over the fillet when the temperature larger than -2.2 °C. From the beginning of freezing step to 1200 s, point B witnesses a moderate decline of temperature, whereas point A remains at -2.2 °C for a while due to the appearance of the latent heat in the ice crystallization. After that, the temperature at two points goes down noticeably until 2200 s before making a significant plummet to achieve the required temperature at 2900 s.

![Diagram](image2)

**Figure 5.** Freezing time (a) estimated at the surface (blue) and the centroid (green) of the fillet, and (b) obtained from experimental data at the centroid.

The series of temperature distribution results throughout this food treatment is shown in Figure 6. After each duration of 500 s, the simulation results are captured to analyse the transformation of physical properties over time. There is a trend that the decreasing rates of temperature are highest at four corners and lowest at the centroid. As a result, the first ice crystals appear at these corners corresponding to the higher ice proportion illustrated in Figure 7.
Figure 6. Temperature distribution during fish freezing process.
4. Conclusion
The food behaviours during the cooling periods have been considered and investigated based on mathematical models in numerical approach. For this purpose, the required freezing time predicted by simulation is quite conformable with the result from experiment. Although the simulation tools require the experimental data for material modeling, their predictions can be applied for many aspects in food processing. Therefore, the budget for instrumenting and the cost for testing could be moderated by the support of numerical predictions, while more perceptions from the mechanisms of freezing can also be obtained from numerical simulations. In addition, this research is a basis for other advanced studies related to adding electromagnetic fields during the freezing processes to improve not only food quality but also the energy performance for industrial scale production in the future.

5. References
[1] Petzold G and Aguilera J M 2009 Ice Morphology: Fundamentals and Technological Applications in Foods Food Biophysics 4 pp. 378–396, DOI 10.1007/s11483-009-9136-5
[2] Johnston W A, Nicholson F J, Roge A, and Stroud G D 1994 Freezing and Refrigerated Storage in Fisheries FAO Fisheries Technical Paper No. 340 Rome FAO 143p
[3] Pham Q T 2014 Food Freezing and Thawing Calculations in SpringerBriefs in Food Health and Nutrition Springer New York Heidelberg Dordrecht London 162p, DOI 10.1007/978-1-4939-0557-7
[4] Billiard F 1999 New Developments in the Food Cold Chain Worldwide. Proc. 20th Int. Congr. Refrig. IIR/IIF (Sydney, Australia)
[5] Nguyen T T T, Vo C C and Nguyen T V 2016 An Overview of Novel Food Freezing Technologies Applying Electromagnetic Fields Proc. Of the 2016 Int. Conf. on Advanced Technology and Sustainable Development (ICATSD2016) August 21-23 HoChiMinh City, Vietnam, pp. 313–322, ISBN: 978-604-920-040-3
[6] Cleland D J, Cleland A C Earle, R L and Byrne S J 1987 Prediction of freezing and thawing times for multi-dimensional shapes by numerical methods. Butterworth Co (Publishers) Ltd and IIR Int. J. of Refrig. Vol. 10 Jan pp. 32–39, 0140-7007/87/010032-08$03.00
[7] Pham Q T 1989 Effect of supercooling on freezing times due to dendritic growth of ice Crystals. Butterworth Co (Publishers) Ltd and IIR Int. J. of Refrig. Vol. 12 Sep pp. 295–300, 0140-7007/89/050295-06$03.00
[8] Carslaw H S and Jaeger J C 1959 Conduction of Heat in Solids 2nd Ed. Clarendon Press Oxford 517p
[9] Segerlind L J 1984 Applied Finite Element Analysis 2nd Ed. John Wiley and Sons, New York Chichester Brisbane Toronto Singapore, 222p
[10] Zienkiewicz O C and Taylor R L 2000 The Finite Element Method Vol. 3: Fluid Dynamics 5th Ed. Butterworth-Heinemann Oxford Auckland Boston Johannesburg Melbourne New Delhi 347p, ISBN: 0 7506 5050 8
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