The Loudest Gravitational Wave Events

Hsin-Yu Chen\textsuperscript{1} and Daniel E. Holz\textsuperscript{2}
\textsuperscript{1}Department of Astronomy and Astrophysics, University of Chicago, Chicago, IL 60637
\textsuperscript{2}Enrico Fermi Institute, Department of Physics, and Kavli Institute for Cosmological Physics University of Chicago, Chicago, IL 60637

As first emphasized by Bernard Schutz, there exists a universal distribution of signal-to-noise ratios for gravitational wave detection. Because gravitational waves (GWs) are almost impossible to obscure via dust absorption or other astrophysical processes, the strength of the detected signal is dictated solely by the emission strength and the distance to the source. Assuming that the space density of an arbitrary population of GW sources does not evolve, we show explicitly that the distribution of detected signal-to-noise (SNR) values depends solely on the detection threshold; it is independent of the detector network (interferometer or pulsar timing array), the individual detector noise curves (initial or Advanced LIGO), the nature of the GW sources (compact binary coalescence, supernova, or some other discrete source), and the distributions of source variables such as the binary masses and spins (only non-spinning neutron stars of mass exactly 1.4 $M_\odot$ or a complicated distribution of masses and spins). We derive the SNR distribution for each individual detector within a network as a function of the relative detector orientations and sensitivities. While most detections will have SNR near the detection threshold, there will be a tail of events to higher SNR. We derive the SNR distribution of the loudest (highest SNR) events in any given sample of detections. We find that in 50% of cases the loudest event out of the first four should have an SNR louder than 22 (for a threshold of 12, appropriate for the Advanced LIGO/Virgo network), increasing to a loudest SNR of 47 for 40 detections. We expect these loudest events to provide particularly powerful constraints on their source parameters, and they will play an important role in extracting astrophysics from gravitational wave sources. These distributions also offer an important internal calibration of the response of the GW detector networks.

INTRODUCTION

Gravitational waves (GWs) couple very weakly to matter. The downside of this is that they are difficult to detect, and almost a century after they were first predicted by Einstein\textsuperscript{1} they remain to be directly detected on Earth. The upside is that GWs propagate with little interference, being almost impossible to absorb or scatter, and thus cleanly carry information from the source to the observer. As a result both the amplitude and the measured signal-to-noise (SNR) ratio of GWs scale inversely with luminosity distance, leading to a universal SNR distribution of GW events as a function of the SNR detection threshold\textsuperscript{2}. This follows directly from the simple relationship between distance and volume, and applies so long as the source population does not evolve with distance.

In the first several years of Advanced LIGO/Virgo operations we expect to have tens of detections per year of gravitational waves from compact binary coalescence\textsuperscript{3–5}. These detections must follow the universal SNR distribution, and this offers an important internal self-calibration of the GW detector network. Additionally, the distribution offers a simple internal test of whether the first events are statistically consistent with expectations. Although most of these detections will be found with SNRs close to the detection threshold, there will exist a tail to higher SNR. Fisher matrix calculations show that the timing, chirp mass, and amplitude measurement all improve as $\sim 1/\text{SNR}$\textsuperscript{6,7}. The highest SNR events will likely offer the best constraints on both intrinsic and extrinsic parameters of their sources, and thereby enable important physics and astrophysics\textsuperscript{7}. For example, accurate determination of binary masses helps distinguish between neutron stars and stellar mass black holes, and elucidates the “mass gap” problem\textsuperscript{8,9}. Higher SNR measurement of waveforms may help probe the neutron star equation-of-state\textsuperscript{10–12}. These loudest events are likely to have improved sky localization, increasing the probability of observing electromagnetic (EM) counterparts to the GW events and leading to the birth of GW/EM multi-messenger astronomy\textsuperscript{13}. In particular, joint detections would confirm binary systems as the progenitors of short-hard gamma-ray bursts\textsuperscript{14–15}, probe the Hubble constant, and potentially measure the dark energy equation of state\textsuperscript{16,17}. We argue that these loudest events must exist, and will play an important role in the coming age of gravitational-wave astrophysics.

THE UNIVERSAL SNR DISTRIBUTION

A given GW network will detect some number of GW events, with each event characterized by a measured signal-to-noise ratio (SNR), $\rho$. We are interested in the distribution of $\rho$. We assume that the space density and intrinsic properties of the source population do
not evolve. This is justified given that the Advanced LIGO/Virgo network is only able to probe the nearby universe, \( z \lesssim 0.2 \) [4] (although see [20] for an example where this is not the case). For the sake of definiteness and to enable Monte Carlo comparisons, in what follows we will assume that the GW sources are merging compact binaries, although our results are independent of this assumption and are valid for any discrete distribution of sources. Following [6, 21, 22], we compute the SNR of a binary inspiral and merger assuming a restricted post-Newtonian waveform observed by a network of ground-based GW detectors:

\[
\rho^2 = \frac{4A^2}{D_l^2} \left[ F_+^2 (\theta, \phi, \psi)(1 + \cos^2 \iota)^2 + 4F_\times (\theta, \phi, \psi) \cos^2 \iota \right] I_7,
\]

where \( A = \sqrt{5/96}\pi^{-2/3}(GM_z/c^5/6c), \ M_z = (1 + z)(m_1m_2)^{3/5}/(m_1 + m_2)^{1/5} \) is the redshifted chirp mass, \( D_L \) is the luminosity distance, \( \psi \) is the orientation of the binary within the plane of sky, and \( F_+ \) and \( F_\times \) are the detector antenna power patterns, which are themselves functions of the source sky location, \((\theta, \phi)\). The inclination angle between the binary’s rotation axis and the line of sight is given by \( \iota \). The noise curve of the detector is encapsulated in \( I_7 \), which is an integral over the detector’s power spectral density \( S_h(f) \):

\[
I_7 = \int_{f_{\text{low}}}^{f_{\text{high}}} \frac{f^{-7/3}}{S_h(f)} df.
\]

The probability of a merger lying within an infinitesimal interval \( dD \) near the comoving distance \( D \) is given by \( f_D dD \propto D^2 dD \), where \( f_D = f(D) \) is the distribution of the source comoving distances [27]. In the nearby universe we can approximate the luminosity distance by the comoving distance. We see from Eq. [1] that the SNR scales as \( \rho \propto 1/D_L \); we note this is true for all GW sources, not just for binary coalescences (e.g., [21]). The other extrinsic parameters (sky location, binary orientation, and inclination) are randomly distributed and do not impact the final distributions, as shown explicitly below. If we assume that the chirp mass distribution and the space density do not evolve with distance, then the resulting distribution of SNR only depends upon the distance. We find

\[
f_\rho = f_{DL} \left| \frac{dD_L}{d\rho} \right| = f_D \left| \frac{dD}{d\rho} \right| \propto \frac{1}{\rho^2} \frac{1}{\rho^2} = \frac{1}{\rho^4}.
\]

where the second equality is only true at low redshift. Normalizing this for a given network SNR threshold, \( \rho_{th} \), we find that the distribution of SNRs for sources in the local universe is exactly described by

\[
f_\rho = \frac{3\rho_{th}^3}{\rho^4}. \tag{2}
\]

This is identical to Eq. 24 of [2].

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**FIG. 1: Universal distribution of SNR (Eq. [2]), for \( \rho_{th} = 12 \), plotted as the solid black curve. The colored histograms show the results from our Monte Carlo simulations of 5,000 detections. “All Random” is from the basic simulation described in the text, and verifies our analytic prediction. The results are independent of the network properties (“Different Network” has a different number of detectors, with different relative orientations and noise curves), the chirp mass distribution (“Fixed Chirp Mass”), and the sky location and inclination distribution (“Optimal Angles” has all binaries “overhead” and face-on). This shows explicitly that the SNR distribution is universal.**

In Fig. 1 we plot this distribution assuming \( \rho_{th} = 12 \). The distribution peaks at the threshold value, and has a tail to higher SNR events. For explicit comparison we have also performed Monte Carlo simulations of the detection of a binary population, sampling over the full parameter space \( (D_L, \theta, \phi, \psi, \iota) \) with random sky locations and binary orientations, and with the total mass of the binaries, \( M_{tot} = m_1 + m_2 \), drawn uniformly between \( 2M_\odot \) and \( 20M_\odot \) and \( m_1 \) drawn uniformly between \( 1M_\odot \) and \( 1M_\odot + 1M_\odot \). For each randomly drawn binary we use Eq. [1] to calculate the SNR of the simulated events for a given GW network. As shown in Fig. 1 the histograms of SNR for our various simulated populations follow our predictions. The distribution of SNR presented in Eq. [2] is universal, and is what will be found in all GW detectors for all non-evolving, low-redshift GW source populations.

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**THE UNIVERSAL LOUDEST EVENT DISTRIBUTION**

We now turn our attention to the high-SNR tail of events, and make predictions for the highest SNR event out of any detected sample of GW events. We assume that within any arbitrary GW network we have \( N \) compact binary detections with SNR values given by \( \{\rho_1, \rho_2, ..., \rho_N\} \). We define the loudest event as \( \rho_{\text{max}} = \max\{\rho_1, \rho_2, ..., \rho_N\} \). The probability of \( \rho_{\text{max}} \) being less
than a given value $\rho$ is

$$P(\rho_{\text{max}} < \rho) = P(\rho_1 < \rho, \rho_2 < \rho, \ldots, \rho_N < \rho)$$

$$= P(\rho_1 < \rho)P(\rho_2 < \rho)\ldots P(\rho_N < \rho)$$

$$= (F_\rho)^N,$$

where the second line follows from the assumption that each event is independent. $F_\rho$ is the cumulative distribution function of $\rho$ and can be computed by integrating Eq. 2 from $\rho_{\text{th}}$ to any desired value of $\rho$. $P(\rho_{\text{max}} < \rho)$ is equivalent to the cumulative distribution function, $F_{\rho_{\text{max}}}$, of the loudest event, $\rho_{\text{max}}$. The probability distribution function of the loudest event, $f_{\rho_{\text{max}}}$, is obtained by taking a derivative:

$$f_{\rho_{\text{max}}} = \frac{dF_{\rho_{\text{max}}}}{d\rho_{\text{max}}} = \frac{d(F_\rho)^N}{d\rho} \bigg|_{\rho=\rho_{\text{max}}},$$

$$= \frac{3N}{\rho_{\text{max}}} \left( \frac{\rho_{\text{th}}}{\rho_{\text{max}}} \right)^3 \left[ 1 - \left( \frac{\rho_{\text{th}}}{\rho_{\text{max}}} \right)^3 \right]^{N-1}. \quad (3)$$

We have verified this distribution explicitly using Monte Carlo techniques. See [23] for an alternative approach to deriving this distribution.

We are now able to forecast the distribution of the loudest events, as shown in Fig. 2. The probability that the loudest event is louder than $\rho$, out of $N$ detections above a detection threshold $\rho_{\text{th}}$, is given by $P(\rho_{\text{max}} > \rho)$, and can be calculated by integrating Eq. 2 from $\rho$ to infinity. For example, if we set the network detection threshold to $\rho_{\text{th}} = 12$ (appropriate for the case of Advanced LIGO/Virgo) we find that 90% of the time the loudest event out of the first 4 detections will have $\rho_{\text{max}} > 15.8$. The loudest event out of the first 40 detections, corresponding roughly to one year of observation with Advanced LIGO/Virgo [3][5], will have $\rho_{\text{max}} > 31$. Half the time we will find the loudest event to have $\rho > 22$ for 4 events and $\rho > 47$ for 40 events. We emphasize that these statements are independent of the specific noise curves or configurations of the detector network or even the nature of the source population.

The distribution of the loudest events given in Eq. 3 depends upon only two parameters: the number of detections, $N$, and the detection threshold, $\rho_{\text{th}}$. Since the shape of the distribution is similar for all $N$ and $\rho_{\text{th}}$, we are able to find a scaling to produce a universal distribution. We define a new variable, $y \equiv \rho_{\text{max}}/a$, where $a$ is an arbitrary scaling. The distribution of $y$ values becomes

$$f_y = \frac{d\rho_{\text{max}}}{dy} f_{\rho_{\text{max}}} = \frac{3N}{a} \rho_{\text{th}}^3 \left[ 1 - \frac{\rho_{\text{th}}^3}{a^3 y^3} \right]^{N-1}. \quad (4)$$

If we set $a = \rho_{\text{th}} N^{1/3}$ this distribution becomes

$$f_y = \frac{3}{y^4} \left[ 1 - \frac{1}{N y^3} \right].$$

We note that this distribution is independent of $\rho_{\text{th}}$ and $N$ when $N$ is large. In Fig. 3 we explicitly show that this scaling produces a universal form for the distribution of the loudest events. Furthermore, from this distribution we are able to produce generic, simple, and powerful statistical predictions. For example, we conclude that in 90% of cases, $\rho_{\text{max}} > 0.76 \rho_{\text{th}}^{1/3} N^{1/3}$, while in 50% of cases, $\rho_{\text{max}} > 1.13 \rho_{\text{th}} N^{1/3}$. Comparing to the exact analytic form, these expressions are good to 8% for 4 detections and 0.6% for 40 detections.

**THE SNR DISTRIBUTION IN INDIVIDUAL DETECTORS**

In Eq. 2 we show the distribution of SNR for an arbitrary detector network, where the network SNR is the root of the sum of the squares of the SNRs in each individual detector comprising the network. In addition to the overall source amplitude, the signal strength in each detector depends on the individual detector’s sensitivity and the relative orientation between the source and each detector. For a detector network with a given network threshold, the distribution of SNR in each individual de-
The legend for reference. In all cases the distributions of $\rho$ and standard deviation of $\rho$ pattern, nary orientation depends upon the antenna power pattern, $P_i$, where $\rho_i$ > 100$\rho_{\text{th}}$. We find that Virgo tends to detect lower SNR values compared to the LIGO detectors when operated within the HLV network, even if the sensitivities of all three instruments are comparable. This is because the H and L detector arms are more closely aligned, and therefore more sources will be detected in the optimal directions for H and L (“overhead” for those detectors), leading to weaker SNR in Virgo.

We note that in practice GW searches often use a complicated detection threshold to better handle the presence of non-Gaussian noise (i.e., glitches). To get a sense of the importance of this, we have implemented a combined coherent/coincident threshold approach, where we demand $\rho_{\text{net}}$ > 12 and also implement an individual threshold of $\rho_i$ > 5 in at least two detectors. We find that this additional restriction eliminates less than 1% of events in the HLV network, and therefore does not substantially impact any of our predicted SNR distributions.

**DISCUSSION**

Bernard Schutz has emphasized that there exists a universal distribution of signal-to-noise (SNR) that will be measured for gravitational wave sources \([2]\). This distribution is presented in Eq. \([2]\) and assumes only that the spatial density of the sources does not evolve; it makes no assumptions about the nature of the sources (e.g., binary coalescence or supernovae or something else entirely), the properties of the sources (e.g., mass distribution of binaries, inclination distribution, sky locations), or the properties of the GW network (e.g., pulsar timing arrays or interferometers of any number, sensitivity, or location [including ground or space]). We have derived the universal distribution for the loudest (highest SNR) events, for any given number of detected events. When there are $N$ detections above network threshold $\rho_{\text{th}}$, 90% of the time the loudest event will have SNR larger than $0.76\rho_{\text{th}}N^{1/3}$. This loudest event may play an important role in binary parameter estimation, and is expected to be particularly well localized on the sky, since localization scales roughly as $1/\text{SNR}^2$. If we consider the first four detections by the Advanced LIGO network (or any sources within any network with a network threshold of $\rho_{\text{net}}$ = 12), we find that half the time the loudest event will be louder than $\rho = 22$, and the localization area will shrink by a factor of $\sim 3$ compared to threshold events.

Our results are similar to the $V/V_{\text{max}}$ test, which is a geometric test used for electromagnetic astronomical sources \([23]\). For any population one calculates the volume enclosed to each individual source, $V$, and the maximum volume to which that source could have been observed, $V_{\text{max}}$. If there is no evolution in the source pop-

**FIG. 3:** Rescaled histogram of $\rho_{\text{max}}/(\rho_{\text{th}}N^{1/3})$ from $N = 100$ and $N = 1,000$ detections, repeating each random sample ten thousand times, and with $\rho_{\text{th}}$ taken to be 8 or 12. The average and standard deviation of $\rho_{\text{max}}$ before rescaling are shown in the legend for reference. In all cases the distributions of $\rho_{\text{max}}$ follow a similar shape after rescaling. For 90% of the cases (right dotted line), $\rho_{\text{max}} > 0.76\rho_{\text{th}}N^{1/3}$, while for 50% of the cases (right dotted line), $\rho_{\text{max}} > 1.13\rho_{\text{th}}N^{1/3}$.

The detector can be calculated:

$$f_{\rho_i} = \int_{\rho_i \geq \rho_{\text{det}}} f(\rho_i, \theta, \phi, \psi, \iota) d\phi d\psi d\iota$$

where $f(\rho_i, \theta, \phi, \psi, \iota) = 3\rho_i^3/\rho_i^4$ and

$$\rho_{\text{eff}} = \rho_{\text{th}} \left( \sum_{j=1}^{N} R_{ji} \right)^{1/2}$$

with $R_{ji} \equiv \rho_j^2/\rho_i^2$. The prior on sky location and binary orientation depends upon the antenna power pattern, $P_j$ \([2]\):

$$f_{\text{det}}(\theta, \phi, \psi, \iota) = \frac{1}{n} \left( \sum_{j=1}^{N} P_j \right)^{3/2} \sin \theta \sin \iota$$

where the normalization factor $n$ is integrated over $d\theta d\phi d\psi d\iota$. We simplify Eq. \([4]\) by rewriting the individual SNR, $\rho_i$, as $y_i \equiv \rho_i/\rho_{\text{th}}$. We assume each detector has identical (arbitrary) sensitivity, finding:

$$f_{y_i} = \frac{3}{n} \int_{\sum_{j=1}^{N} R_{ji} \geq \frac{1}{4}} \frac{1}{y_i^{3/2}} P_i^{3/2} \sin \theta \sin \iota \rho_{\text{eff}} d\phi d\psi d\iota$$

This expression gives the distribution of SNR detected by each individual detector as part of a given detector network. In Fig. \([4]\) we show the SNR distribution for each detector within a network composed of two (LIGO-Hanford [H] and LIGO-Livingston [L]) and three (H, L, and Virgo [V]) detectors. For any two detector network, the SNR distributions for the individual detectors will be identical if the detectors operate at the same sensitivities.
ulation, simple geometric arguments imply that the ob-
served values of $V/V_{\text{max}}$ must be uniformly distributed
between 0 and 1. The same test can be applied to non-
evolving GW sources at low redshift: since SNR scales
inversely with $D$, and since volume scales as $D^3$, we find
$V/V_{\text{max}} \sim (D/D_{\text{max}})^{3/2} \sim (\rho_{\text{th}}/\rho)^{3/2}$ distributes uniformly
between 0 and 1. We have focused on the SNR distribution
instead, since this quantity is directly measured by GW detectors. However, the $V/V_{\text{max}}$ distribution re-
mains true to arbitrary redshift (modulo gravitational
lensing, which adds noise and may also introduce magni-
fication bias to all high-$z$ distributions). This is not
true for the SNR distributions discussed above, since at
high redshift two additional effects come in: luminosity
distance (which sets the SNR) and comoving distance
(which is relevant for the comoving volume) start to de-
viate from each other, and the source redshift affects
where the source is found relative to the frequency re-
sponse of the GW detectors. Both of these effects break
the universality of the SNR distributions. The latter ef-
teffect depends upon properties of the source population
and detector noise curves; for binary systems the effect
is encapsulated in the redshift dependence of $I_7$. For
example, using the Einstein Telescope noise curve [25] we
find a $\sim 10\%$ suppression from the form in Eq. 2 for
binary neutron stars detected at $\rho_{\text{th}} = 12$ (correspond-
ing to a horizon of $z \sim 1.2$). This effect grows to 25% and
60% as the binary masses increase to $3M_\odot-3M_\odot$ and
$10M_\odot-10M_\odot$, respectively (corresponding to horizons of
$z \sim 2.8$ and $z \sim 4.8$). In principle, precise measurements
of the distribution of SNR could be used to infer the
intrinsic mass distribution of binary systems, as well as
probe the cosmological parameters by measuring directly
the evolution of the cosmological volume. In practice the
evolution in the rate density of the source populations
dominates over the cosmological effects, and we are more
likely to be able to measure the former than the latter.

These universal distributions are a robust prediction
for all GW sources and for all GW networks, and there-
fore serve as an important internal consistency check for
the detectors. For example, in LIGO’s 6th and Virgo’s 3rd science run there was a “blind” hardware injection
event intended to test the data analysis procedures. This
event is presented in Fig. 3 of [26]: the “false” coincidence
events are found at SNR below 9.5, while the single injec-
tion event stands out at SNR of $\sim 12.5$. Given our univer-
sal distribution, we can calculate the probability of hav-
ing a single event at $\rho = 12.5$ with no other events down
to a threshold of $\rho = 9.5$. Instead of the loudest event,
we are now interested in the “quietest” event; following
the approach in Eq. 3 we find that the distribution of
the lowest SNR for $N$ events is: $P_{\text{min}} = 3N \rho_{\text{th}}^{N} / \rho_{\text{min}}^{3N+1}$. For the blind injection we have $N = 1$, and the proba-
bility that the first event will have $\rho \gtrsim 12.5$ when the
threshold is $\rho_{\text{th}}$ is $9.5$ is $(9.5/12.5)^3 = 44\%$. We con-
clude that the injection event was not unlikely, even in
the absence of any other events down to $\rho = 9.5$ [28].

The first LIGO/Virgo events must follow statistical ex-
pectations from our universal distributions, and this will
be an important sanity check.

In conclusion, all non-evolving low redshift populations
found in all GW detectors must follow the SNR distribu-
tion presented in Eq. 2. This distribution serves as an
important internal consistency check, and offers the
opportunity to test instrumental calibration and sample
completeness, as well as testing for source and cosmo-
logical evolution. In addition, we robustly predict the
distribution of the loudest events. These events must be
found, and will play an important role in gravitational
wave astrophysics.

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