Swirling flow of fluid containing (SiO$_2$) and (MoS$_2$) nanoparticles analyze via Cattaneo-Christov theory

Juan Zhang$^{1,2}$, Awais Ahmed$^3$, Muhammad Naveed Khan$^4$, Fuzhang Wang$^5$, Shaimaa AM Abdelmohsen$^6$ and Hadia Tariq$^3$

Abstract
Investigation of heat transport mechanism in swirling flow of viscous fluid containing silicon dioxide (SiO$_2$) and molybdenum disulfide (MoS$_2$) nanoparticles is performed. The flow is engendered due to stretchable rotating cylinder which immersed in infinite fluid. The boundary layer assumption is applied to simplify the governing equations of the problem. The theory of Cattaneo-Christov for thermal energy transportation is employed in the present phenomenon under the heat and mass constraints. The flow is also influenced by Lorentz force. The results for flow field, temperature, and concentration field are produced by employing the bvp4c numerical technique to the similar differential equations. According to the observations, it is noted that in the presence of Lorentz force the reduction in velocity field of the nanofluid occurs. The thermal and solutal relaxation phenomena also declines the energy transport in nanofluid flow. The outcomes are validated through the comparison with previous published studies.

Keywords
Nanofluid (water-SiO$_2$/MoS$_2$), rotating cylinder, numerical solution, heat source or sink, Cattaneo-Christov theory

Introduction
Researchers have been interested in the fluid flow characteristics and heat transfer aspects of a stretching/shrinking surfaces for a few years. There is no doubt that the theory of fluid flow and heat transfer at a stagnation point may be applied to a wide range of industrial processes, such as plastic extrusion, wire coating, and so on. Sakiadis$^1$ proposed the concept of a boundary layer formed by a moving flow on a rigid sheet with constant velocity from a slit through a fluid at rest. The boundary layer assumption is applied to simplify the governing equations of the problem. The theory of Cattaneo-Christov for thermal energy transportation is employed in the present phenomenon under the heat and mass constraints. The flow is also influenced by Lorentz force. The results for flow field, temperature, and concentration field are produced by employing the bvp4c numerical technique to the similar differential equations. According to the observations, it is noted that in the presence of Lorentz force the reduction in velocity field of the nanofluid occurs. The thermal and solutal relaxation phenomena also declines the energy transport in nanofluid flow. The outcomes are validated through the comparison with previous published studies.

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There are several applications of swirling flow due to rotating surfaces in industry such as paper drying, plastic manufacturing and wire coating. Many scientists put their effort to investigate the mechanism of swirling flow of viscous fluid with heat transport in the recent decade. Swirl motion of Newtonian fluid was investigated by Fang and Yao. In this article authors established the suitable flow similarities for the conversion of governing three-dimensional flow equations. Naumov et al. investigated the thin layer in interface of two immiscible fluids using well-known finite element method. The studies in recent years put on the thermal characteristics of nanofluid. The focus of this article is to explore the thermal mechanism in the nanofluid. The source/sink and chemical reaction parameters are also utilized as heat and mass constraint. The MATLAB built-in solver bvp4c is used to achieve numerical results of the governing problem. The results of the velocity, temperature, and concentration field are illustrated graphically for various values of significant parameters.

**Mathematical formulation**

Consider an elastic cylinder of radius $R_i$ which is immersed in an infinite viscous fluid which contains $SiO_2$ and $MoS_2$ solid nanoparticles. The axisymmetric 3D flow produce due the constant rotation $E$ and stretching with rate $a$ of the cylinder. The flow field is assumed as $V=[u,v,w]$ in the $(z,\phi,r)$ directions. The magnetic field $B=(0,0,B_z)$ is applied normal to the cylinder axis (see Figure 1). Moreover, the heat source/sink and chemical reaction are also considered. Furthermore, it is supposed that the temperature at the cylinder surface $T_w$ and the ambient fluid temperature is $T_a$, are constant and $T_w > T_a$. In view of conservation laws and the Cattaneo-Christov theory for heat flux the governing equation for the present problem are:

$$\frac{\partial u}{\partial z} + \frac{w}{r} \frac{\partial u}{\partial r} + \frac{1}{\rho_n} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) = \frac{\sigma_n}{\rho_n} B_z^2 u,$$

$$\frac{\partial v}{\partial z} + \frac{w}{r} \frac{\partial v}{\partial r} + \frac{1}{\rho_n} \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} \right) = \frac{\sigma_n}{\rho_n} B_z^2 v,$$

$$\frac{\partial w}{\partial z} + \frac{w}{r} \frac{\partial w}{\partial r} + \lambda_s \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] = \frac{k_n}{\rho C_p} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \frac{Q_0}{(\rho C_p)_n} (T - T_a)$$

The above comprehensive literature review prove that there is space for the investigation of nanofluid swirling flow. The focus of this article is to explore the thermal transportation characteristics of flow of viscous fluid containing $SiO_2$/$MoS_2$ particles. The flow produced via a stretched and rotating cylinder with constant surface temperature. The realistic approach of Cattaneo-Christov theory is employed to predict the heat and mass transport.
the equation (1) is satisfied automatically and equations (2) – (5) yield
\[
\frac{A1}{A2}\left(f'''' + \frac{f'''}{\eta}\right) + \frac{Re}{\eta} f''' - \frac{Re}{A2} f' = 0, \quad (9)
\]
\[
\frac{A1}{A2}\left(2\eta^2 g'' + \eta g' - \frac{g}{2}\right) + 2Re\eta f'g' = 0, \quad (10)
\]
\[
\left(\eta\theta'' + \theta'\right) + \frac{Re}{A4} \left(\delta + Le f\theta'\right) = 0, \quad (11)
\]
\[
\left(\eta\phi'' + \phi'\right) + RePrLe\left(f\phi''' - \frac{\gamma}{2}\phi''\right) = 0, \quad (12)
\]
with transformed boundary conditions
\[
f(1) = 0, f'(1) = 1, g(1) = 1, \theta(1) = \phi(1) = 1, \quad (13)
\]
\[
f'(\infty) = 0, g(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0.
\]
Where \(Re\left(= \frac{aR_0^2}{2v}\right)\) is the Reynolds number, \(M\left(= \frac{\delta A^2}{\rho_f a}\right)\) the Magnetic field parameter, \(Pr\left(= \frac{v}{\alpha_f}\right)\) the Prandtl number, \(Le\left(= \frac{a}{D_B}\right)\) the Lewis number, \(\delta\left(= \frac{Q_0}{a(\rho c_p)_f}\right)\) the source/sink parameter, \(\gamma\left(= \frac{k_f}{a}\right)\) the chemical reaction parameter, \(\beta_1\left(= \lambda_1a\right)\) and \(\beta_3\left(= \lambda_3a\right)\) the thermal and mass relaxation parameters. Moreover the \(A1, A2, A3, A4, A5\) can be stated as follows
\[
A1 = \left(\frac{\mu_{nf}}{\mu_f}\right), A2 = \left(\frac{\rho_{nf}}{\rho_f}\right), A3 = \left(\frac{(\rho c_p)_{nf}}{(\rho c_p)_f}\right), \quad (14)
\]
\[
A4 = \left(\frac{k_{nf}}{k_f}\right), A5 = \left(\frac{\sigma_{nf}}{\sigma_f}\right).
\]
A further transformation \(\eta = e^\tau\) is employed to accelerate convergence of the solution. Thus equations (8) – (11) become
\[
\frac{A1}{A2}\left(f_{xx'} - 2f_{xx} + f_x\right) - Re\left(f_x'' + ff' - ff''\right) = 0, \quad (15)
\]
\[2 \frac{A_1}{A_2} g_{ss} + \frac{g}{2} + Re \left(2 f g_s + f g \right) \frac{A_5}{A_4} MRe \left(g e^x \right) = 0, \quad (16)\]

\[\theta_{ss} e^x + \frac{RePr}{A_4} \left(e^{2x} \left( \delta / 2 \right) + A3 f \theta_s e^x \right) \]

\[-2\beta, RePr \frac{A3}{A4} \left(f^2 \theta_{ss} - f^2 \theta_s + 2 f f \theta_s \right) = 0 \] \quad (17),

\[\phi_{ss} e^x + RePr Le \left(f \phi_s e^x - \frac{\rho_f}{2} \right) \]

\[-2\beta, RePr Le \left(f^2 \phi_{ss} - f^2 \phi_s + ff \phi_s \right) = 0, \quad (18)\]

with BCs

\[f(0) = 0, f_s(0) = 1, g(0) = 1, \theta(0) = 1, \phi(0) = 1 \] \quad (19)

\[\lim_{x \to x} e^{\varphi} f_s = 0, g(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0 \] \quad (19).

### Physical reasoning of the outcomes

In this portion of research, the graphical results which are computed numerically describe the impact of physical parameters such as Reynolds number \(Re\), magnetic field parameter \(M\), volume fraction coefficient \(\phi\), Prandtl number \(Pr\), Lewis number \(Le\), sourcing parameter \(\delta\), chemical reaction parameter \(\gamma\), and thermal and mass relaxation parameters \((\beta, \beta_c)\) for both nanofluids \((\text{SiO}_2 / \text{water} \text{and MoS}_2 / \text{water})\) with the provided mathematical relations for thermophysical properties of nanofluids.

### Table 1. Mathematical relations for thermophysical properties of nanofluids.

| Properties                | Nanofluid |
|---------------------------|-----------|
| Density \(\rho_{nf} = \rho_t \left(1 - \phi + \phi \frac{\rho_r}{\rho_t}\right)\) |
| Viscosity \(\mu_{nf} = \frac{\mu_t}{(1 - \phi)^{2.5}}\) |
| Heat Capacity \((\rho C_p)_t = \frac{1}{(1 - \phi + \phi \left(\frac{\rho C_p}{\rho_t}\right))}\) |
| Thermal Conductivity \(k_f = k_t + 2k_f - 2\phi (k_f - k_t)\) |
| Electric Conductivity \(\frac{\sigma_{nf}}{\sigma_t} = 1 + 3(\sigma - 1)\phi / (\sigma + 2)(\sigma - 1)\phi\) |

### Table 2. Thermophysical characteristics of nanoparticles

| Nanoparticles | Water | MoS\(_2\) | SiO\(_2\) |
|---------------|-------|------------|------------|
| \(\rho / \text{kgm}^{-3}\) | 997.1 | 5060       | 2650       |
| \(c_p / \text{kgK}^{-1}\) | 4179  | 397.746    | 730        |
| \(\sigma / \text{Sm}^{-1}\) | 0.613 | 34.5       | 1.5        |
| \(k / \text{WmK}^{-1}\) | 5.5x10^{-6} | 17.9x10^6 | 10^{-12} |

### Table 3. The comparison table for numerical values of \(f'(1)\) and \(g'(1)\).

| Re    | 0.1   | 0.2   | 0.5   | 1.0   | 2.0   | 5.0   | 10.0  |
|-------|-------|-------|-------|-------|-------|-------|-------|
| \(f'(1)\) | -0.4880 | -0.6164 | -0.7920 | -0.8348 | -0.8900 | -0.9560 | -0.9759 |
| \(g'(1)\) | -0.5102 | -0.5259 | -0.5859 | -0.6899 | -0.8726 | -0.9560 | -0.9759 |
| \(f'(1)\) Present | -0.4825 | -0.6161 | -0.884  | -1.1782 | -1.5978 | -2.4174 | -3.3446 |
| \(g'(1)\) Present | -0.5102 | -0.5259 | -0.5859 | -0.6879 | -0.8726 | -0.9560 | -0.9759 |
| \(f'(1)\) Fang and Yao12 | -0.4818 | -0.6175 | -0.8822 | -1.1775 | -1.5939 | -2.4173 | -3.3446 |
| \(g'(1)\) Fang and Yao12 | -0.5101 | -0.5260 | -0.5849 | -0.6877 | -0.8726 | -0.9560 | -0.9759 |
Table 4. Numerical values of $-\theta'(l)$ for various $Re, Pr, \delta$, and $\beta_t$.

| $Re$ | $Pr$ | $\delta$ | $\beta_t$ | $-\theta'(l)$ |
|------|------|----------|-----------|-------------|
| 0.5  | 6.2  | 2        | 0.1       | 2.242906    |
| 0.7  |      |          |           | 2.382541    |
| 0.9  |      |          |           | 2.552958    |
|      | 0.5  |          |           | 1.751631    |
|      | 0.7  |          |           | 1.747774    |
|      | 0.9  |          |           | 1.744803    |
|      |      | 0.5      |           | 2.842302    |
|      |      | 0.7      |           | 2.613992    |
|      |      | 0.9      |           | 2.242906    |
|      |      | 0.5      |           | 2.341122    |
|      |      | 0.7      |           | 1.740708    |
|      |      | 0.9      |           | 0.368323    |

thermophysical properties in Table 2 on flow field and temperature and concentration distributions. When we go through the numerical computation, we specify the values of relevant parameters as $Re = 1.0, M = 2.0, \phi = 0.025$, $Pr = 6.2, Le = 1, \gamma = \delta = 1.5, \beta_t = 0.1, \beta_c = 0.1$. Figure 2(a) to (c) represents the results for velocity field, temperature, and solutal distributions for various values of Reynolds number $Re$. It is noted that the axial velocity $f'(\eta)$, swirling velocity $g(\eta)$, and radial velocity $\frac{f}{\eta^{1/2}}$ are significantly decrease when $Re$ boosted. Physically, the opposing flow agent in the form of inertial force increase when the Reynolds number enhance with fixed viscosity of the fluid. Moreover, it is also observed that there is algebraic decay in flow field for higher $Re$ and the disturbance in the fluid occurs very far away to the thermophysical properties in Table 2 on flow field and temperature and concentration distributions. When we go through the numerical computation, we specify the values of relevant parameters as $Re = 1.0, M = 2.0, \phi = 0.025$, $Pr = 6.2, Le = 1, \gamma = \delta = 1.5, \beta_t = 0.1, \beta_c = 0.1$. Figure 2(a) to (c) represents the results for velocity field, temperature, and solutal distributions for various values of Reynolds number $Re$. It is noted that the axial velocity $f'(\eta)$, swirling velocity $g(\eta)$, and radial velocity $\frac{f}{\eta^{1/2}}$ are significantly decrease when $Re$ boosted. Physically, the opposing flow agent in the form of inertial force increase when the Reynolds number enhance with fixed viscosity of the fluid. Moreover, it is also observed that there is algebraic decay in flow field for higher $Re$ and the disturbance in the fluid occurs very far away to the

Figure 2. (a–c) Impact of Reynolds number $Re$ on $f'(\eta), g(\eta)$ and $f/\eta^{1/2}$. 


free stream from surface of cylinder when Re have small magnitude. As the transport mechanism of thermal and solutal energy in the present problem is exist due to the forced convection in the system and the fluid motion produced due cylinder rotation with stretching. The outcomes for flow field with higher Re prove that fluid motion limited to the surface of cylinder in this case. Thus, the forced convection also decreases for higher Re and diminishing temperature, concentration profiles are observed in Figure 3(a) and (b). The objective of Figure 4 is to demonstrate the effects of Prandtl number Pr on temperature. These results tell us that the thermal diffusivity of the fluid decreases as Pr values rise. Figure 5(a) to (c) demonstrate the effect of the magnetic field parameter $M$ on the velocity profile. The velocity profile is shown to declines as the magnetic field parameter is raised. Because the transverse magnetic field in the flow system of electrically conducting fluid generates a drag-like Lorentz force. This force tends to oppose the motion of nanofluid. Figure 6(a) to (c) depicts the decreasing behavior of axial velocity, swirling velocity, and radial velocity as nanoparticles volume fractions is enhanced. Physically, the higher concentration of nanoparticles in the viscous fluid produce the resistance to the fluid motion as fluid particles require more kinetic energy to flow. Thus, in results flow field decreases. The thermal and solutal conduction enhancement of base fluid is booted due to higher concentration of solid particles in it. These results are shown in Figure 7(a) and (b). The volume fraction parameter is an important one that plays a role in improving the thermal characteristics of fluids. As a conclusion, it may deduce that in many industrial processes, temperature can be adjusted by changing the nanoparticles volume fraction. Figure 8(a) and (b) displays the impact of Lewis number $Le$ and chemical reaction parameter $\gamma$ on concentration field. It is noted the mass diffusivity of the fluid decrease with more values of $Le$ and a chemical reaction the fluid declines the mass transport. The temperature and concentration patterns are displayed versus the thermal relaxation time parameter $\beta_r$ and the mass relaxation time parameter $\beta_c$, accordingly, in Figure 9(a) and (b). The temperature and concentration fields decline as the thermal relaxation time and thermal relaxation mass parameter values improve. In Cattaneo-Christov theory, higher values of the relaxation time parameter control the immediate propagation of heat waves in a given medium.
As a consequence, the fluid with increasing relaxation parameter values required more time to carry the thermal and solutal energy in the base fluid. Thus, the temperature and concentration fields drop as a result. The results of the present study validated through the comparison Table 3. The values of thermal gradient at the cylinder surface are given in Table 4 as well.

Concluding remarks

Thermal energy transport in the nanofluid flow caused by stretchable rotating cylinder investigated in this article. The water base fluid contains the two different nanoparticles (SiO$_2$ / MoS$_2$). The mechanism of thermal and solutal energy transport predicted via well-known Cattaneo-Christov theory. Thermal conduction controlled with heat source and Lorentz force in the system. Governing similar equations solved numerically. Some major points of the entire analysis are as follows:

- The flow confined to the surface of cylinder when Reynolds number $Re$ was increased.
- Forced convection declined in case of higher inertial force.
- Flow of nanofluid feels resistance when Lorentz force and solid particles concentration were boosted.
- The temperature of the fluid rises as $\phi$ increased.
- Thermal and solutal relaxation phenomenon limited the energy transportation in the nanofluid flow.

Future scope

The present work can be extend for the rheology of non-Newtonian fluids with heat transport. Moreover, entropy
Figure 6. (a–c) Impact of volume fraction coefficient $\phi$ on $f'(\eta)$, $g(\eta)$, and $f/\eta^2$.

Figure 7. (a and b) Impact of volume fraction coefficient $\phi$ on $\theta(\eta)$ and $\phi(\eta)$. 
generation analysis can also be performed in the present investigated phenomenon.

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