Effect of Topology Structure on the Output Performance of an Automobile Exhaust Thermoelectric Generator

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Abstract. The majority of the thermal energy released in an automotive internal combustion cycle is exhausted as waste heat through the tail pipe. This paper describes an automobile exhaust thermoelectric generator (AETEG), designed to recycle automobile waste heat. A model of the output characteristics of each thermoelectric device was established by testing their open circuit voltage and internal resistance, and combining the output characteristics. To better describe the relationship, the physical model was transformed into a topological model. The connection matrix was used to describe the relationship between any two thermoelectric devices in the topological structure. Different topological structures produced different power outputs; their output power was maximised by using an iterative algorithm to optimize the series-parallel electrical topology structure. The experimental results have shown that the output power of the optimal topology structure increases by 18.18% and 29.35% versus that of a pure in-series or parallel topology, respectively, and by 10.08% versus a manually defined structure (based on user experience). The thermoelectric conversion device increased energy efficiency by 40% when compared with a traditional car.

1. Introduction

Approximately 30% of fuel energy can be converted into mechanical energy for a traditional internal combustion engine; the rest is rejected to the surrounding environment via cooling water or an exhaust. If waste heat can be handled by thermoelectric conversion technology, it can be used to generate electricity for use in a vehicle system [1]. The use of multiple thermoelectric devices in an automobile exhaust thermoelectric generator (AETEG) is a new means of recycling waste heat for power generation [2-4].

A high-power AETEG usually contains many thermoelectric devices because of the structural design constraints imposed by the internal flow field. When exhaust gas flows through the heat exchanger, the thermoelectric devices have difficulty in achieving a completely uniform surface temperature distribution [5], so the heat source temperature for each thermoelectric device is different under the same cold source condition. Thus, the hot side and cold side temperatures of each thermoelectric device are different. Because of processing and manufacturing limitations and
inconsistent clamping conditions in installation, the internal resistance of the thermoelectric devices can change significantly.

We took the automobile exhaust thermoelectric generator (AETEG) to be the focus of our research. Output characteristics for each thermoelectric device were found by testing each device with an open circuit voltage and internal resistance. A database was created that stored the output characteristics of each thermoelectric module, based on the highest heat source temperature reaching the thermoelectric device of maximum temperature resistance value. The 60 thermoelectric modules were combined in series and parallel, and represented as a matrix. The maximum output power of the whole AETEG was constructed as an objective function, and then optimised by using a recursive iterative algorithm to produce a series and parallel electrical topology. Finally, the model was verified by experiments.

2. **Design of an automobile exhaust thermoelectric conversion device**

2.1. Layout and number of thermoelectric device groups

The first and second thermoelectric device groups consisted of 32 single thermoelectric devices, which were distributed in four rows and eight columns on the top and bottom surface of the heat exchanger. Temperature resistant insulation and thermal insulation material were used to fill the gap between each thermoelectric device from the exhaust gas inlet to the outlet. In turn, the thermoelectric device group was defined as the first column to the eighth column. The layout and number of layers of the thermoelectric devices on the surface of the heat exchanger are shown in figures 1, 2.

![Figure 1. Number and distribution of thermoelectric modules in group 1.](image1)

![Figure 2. Number and distribution of thermoelectric modules in group 2.](image2)

2.2. The main performance parameters of the system

The test shows that after adopting the herringbone flow structure, the engine power output was 15 kW, the rotating speed was 3200 rpm, the surface temperature of the heat exchanger maximum was 350 °C, and the maximum engine back pressure was approximately 7.8 kPa. Therefore, the internal structure of the heat exchanger had no obvious effect on the engine performance. This paper focuses on the effects that the series-parallel topological structure of the devices has on the performance of the exhaust, and on the topology optimization.

3. **Topology optimization of the thermoelectric device group**

3.1. The output performance of the device

The uneven temperature distribution on the surface of the heat exchanger causes different heat source temperatures for each thermoelectric device. When the above-mentioned single-row type cold source structure of the external cooling system is adopted, the temperature difference of each thermoelectric device will vary (the open circuit voltage and the peak power corresponding to the maximum output current will be different); also there are nuances in the machining and installation method. The characteristics of each thermoelectric device (such as the internal resistance) are different. If all of the thermoelectric devices are in series, the total resistance of the system is a maximum. As a result, the
system’s maximum output current is small and is affected by the thermoelectric devices that have small temperature differences. If the different output characteristics of thermoelectric devices are randomly configured in parallel arrangements, circulation between the thermoelectric devices with different open circuit voltages can occur, because of Kirchhoff’s law. The thermoelectric devices will produce heat, which will increase the internal power dissipation and reduce the output performance of the system. We thus performed electrical topology optimization in series and in parallel, to ensure that the combination of output characteristics for each thermoelectric device resulted in an output voltage that is not zero or negative. This enabled the power generation potentials of the various thermoelectric devices to improve the overall efficiency of the device and its performance.

3.2. The equivalent circuit model of the thermoelectric device

By considering the structure of a single thermoelectric device and assuming that its internal series semiconductor galvanic arm (p-n junction) pair is n, the internal resistance can be represented as:

\[ r = n(l_p/\sigma_p A_p) + n(l_n/\sigma_N A_N) \]  

(1)

\[ l_p, \sigma_p, A_p, \text{ and } l_n, \sigma_N, A_N \text{ represent the p-type and n-type semiconductor galvanic arm length, conductivity, and cross-sectional areas, respectively.} \]

In figure 6, under different temperature conditions, the slope of the voltage and current characteristic curve for a single thermoelectric device remains the same, namely, the resistance can be thought of as being constant. The open circuit voltage \( U \), the actual output voltage \( U_0 \) and its internal resistance \( r \) for a single thermoelectric device can be represented as follows:

\[ U = n\alpha_{pN}(T_H - T_L) = n(\alpha_p - \alpha_N)(T_H - T_L) \]  

(2)

\[ U_0 = U - ir \]  

(3)

\[ r = U/I_{\text{max}} \]  

(4)

Among them, \( \alpha_{pN} \) is the relative Seebeck coefficient of the semiconductor (V/K), \( \alpha_p \) and \( \alpha_N \) are the Seebeck coefficient of the p-type and n-type semiconductors (V/K), respectively, \( T_H \) and \( T_L \) are the hot and cold source temperatures (K) of the thermoelectric devices’ internal semiconductor galvanic arm, \( I_{\text{max}} \) is the short-circuit current (A) of the thermoelectric devices, and \( i \) is the actual output current of the thermoelectric device (A).

The thermoelectric voltage unit [6][7] and the temperature detection unit were combined with an adjustable load under different working conditions to test the open circuit voltage and maximum peak power of thermoelectric devices.

Under working condition 1, the engine speed was 3100 rpm and the power output was 9 kw at 318 °C; under condition 2, the engine speed was 3200 rpm, and the power output was 12 kw. The maximum heat source temperature for all thermoelectric devices was 350 °C (the maximum heat resistance of the thermoelectric devices). Under the same cooling conditions, a larger temperature difference between the hot and cold sides[8] corresponded to a higher the open circuit voltage. We take condition 2 as an example, and gradually increase the output current of the thermoelectric devices. The current corresponding to the peak power of each thermoelectric device is shown in figure 3. The external load was connected to the thermoelectric module to form a loop. The output current of each thermoelectric device was gradually increased, and the corresponding output power was recorded. When the power was at a maximum, the corresponding current value was recorded and is shown in figure 3.

Our preliminary conclusion is that when the open circuit voltage and internal resistance is very close, and a low current corresponds to peak power, the devices should be connected in parallel. When the internal resistance is very low, and the current is low corresponding to peak power, the devices should be connected in series.
We used formula (4) to calculate the average resistance value of all of the thermoelectric devices, as shown in figure 4. Because of the differences between each thermoelectric device and installation under the pressure difference, the average resistance of each thermoelectric device was different and has no direct link to their distribution on the surface of the heat exchanger.

3.3. Optimization algorithm

By considering the output characteristic differences between the 60 thermoelectric devices described above, the appropriate topology in series and in parallel was adopted to obtain the maximum output power. $U_i$ is the open circuit voltage of each thermoelectric module, and $R_i$ is the internal resistance of each thermoelectric module. According to the principle of maximum output power, when the external load impedance was equal to the internal resistance of the thermoelectric module, the output power of the device was at a maximum.

The objective function was thus the maximum peak power ($P_{\text{max}}$) of the 60 thermoelectric modules in a series-parallel combination, as follows:

$$P_{\text{max}} = \frac{U_{\text{AETEG}}^2}{4R_{\text{AETEG}}^2}$$

$U_{\text{AETEG}}$ is the total equivalent voltage of the 60 series-parallel topology thermoelectric modules, and $R_{\text{AETEG}}$ is the total equivalent internal resistance. To obtain the optimal topological structure, the connection matrix is shown as follows:

| The module number | A1 | A2 | ... | A60 | B1 | B2 | ... | B60 |
|------------------|----|----|-----|-----|----|----|-----|-----|
| A1               | -1 | y_{1,2} | ... | y_{1,60} | -1 | x_{1,2} | ... | x_{1,60} |
| A2               | y_{2,1} | -1 | ... | y_{2,60} | x_{2,1} | -1 | ... | x_{2,60} |
| ...              | ... | ... | -1 | ... | ... | ... | ... | ... |
| A60              | y_{60,1} | y_{60,2} | ... | -1 | x_{60,1} | x_{60,2} | ... | -1 |
| B1               | -1 | x_{1,2} | ... | x_{1,60} | -1 | z_{1,2} | ... | z_{1,60} |
| B2               | x_{2,1} | -1 | ... | x_{2,60} | z_{2,1} | -1 | ... | z_{2,60} |
| ...              | ... | ... | ... | ... | ... | ... | ... | ... |
| B60              | x_{60,1} | x_{60,2} | ... | -1 | z_{60,1} | z_{60,2} | ... | -1 |
In the table 1, $A_i$ and $B_i$ are the rear node and front node of module $i$ respectively. $x_{i,j} = 1$ means that the module $i$ rear node and the module $j$ front node are connected in the opposing directions. $x_{i,j} = 0$ means that the module $i$ rear node and the module $j$ front node have no connection; $y_{i,j} = 1$ shows that the module $i$ rear node and the module $j$ rear node are connected, and $y_{i,j} = 0$ means no connection. $x_{i,j} + y_{i,j} = 1$ is used to avoid short circuits between modules. $z_{i,j} = 1$ indicates connections between the module $i$ front node and the module $j$ front node. $z_{i,j} = 0$ indicates no connections, and $z_{i,j} = -1$ means that there are no connection methods.

Here, $N$ modules are in a series-parallel combination, the algorithm for the total equivalent voltage and the total resistance of a series-parallel module is as follows:

1. All of the pure parallel modules (front nodes are linked together, and rear nodes are connected together) are equivalent to a module. Assuming that a parallel module has $K$ modules connected in parallel, the equivalent voltage and resistance are:

$$U_k = \frac{(U_1 R_2 \cdots R_N + \cdots + U_K R_N)}{(R_2 \cdots R_N \cdots + R_K \cdots R_N)}$$

(6)

$$R_k = \frac{1}{R_1 + R_2 + \cdots + R_K}$$

(7)

2. The series module in the parallel branch can be equivalent to a module (rear nodes connected with front nodes). Assuming that a parallel branch has $M$ modules in series, the equivalent voltage and equivalent resistance are:

$$U_j = U_{j1} + U_{j2} + U_{j3} + \cdots + U_{jM}$$

(8)

$$R_j = R_{j1} + R_{j2} + R_{j3} + \cdots + R_{jM}$$

(9)

3. Then, all of the parallel modules are equivalent to a module.

$$U_k = \frac{(U_1 R_2 \cdots R_N + U_2 R_3 \cdots R_N + \cdots + U_K R_N)}{(R_2 \cdots R_N \cdots + R_3 \cdots R_N \cdots + R_K \cdots R_N)}$$

(10)

$$R_k = \frac{1}{R_1 + R_2 + \cdots + R_K}$$

(11)

4. Now, all series modules are added to obtain the total equivalent voltage, $U$, and the equivalent resistance, $R$.

5. Calculating the peak power:

$$P = U^2/4R$$

(12)

6. Each 0-1 combination of $x_{i,j}$, $y_{i,j}$, and $z_{i,j}$ represents a series-parallel method, which has a maximum output power $P_{\text{max}}(i)$ corresponding to each module combination. The maximum $P_{\text{max}}(i)$ corresponding to the module combination represents the optimal module combination.

By considering that this is a NP-hard problem, we simplified it by taking a local step-by-step optimization method as follows:

1. First, all of the thermoelectric modules are in series according to the open circuit voltage of each thermoelectric module and the resistance to obtain $P1$;

2. We select two thermoelectric modules from $N$ thermoelectric modules as a parallel module, which is in series with other $N-2$ thermoelectric modules to achieve a maximum power $P1$.

3. In step 2, the two thermoelectric modules corresponding to the maximum power are equivalent and can be made into a new thermoelectric module; the rest of the $N$-2 thermoelectric modules make up a new $N'$ module. Then, we repeat process 2;

4. When $N' = 1$, the maximum power $P = MAX \{P1\}$; according to each parallel method, this determines the final topology.

The algorithm diagram process is shown in figure 5:
Algorithm process

(1). In the first round, we find the total power of the $N$ thermoelectric modules in series:

$$P = \sum_{i=1}^{i=N} \frac{U_i}{R_i}$$

(13)

(2). We find the two parallel thermoelectric modules that are in series with the rest of the modules for maximum power to obtain the greatest power:

$$P = \text{MAX} \left( \frac{\left( U_i R_j + U_j R_i \right)}{R_i + R_j} + \sum_{i=1}^{i=N} \frac{U_i - U_j}{\sum_{i=1}^{i=N} U_i - U_j} \right)$$

(14)

(3). We update the total number of thermoelectric modules: $N \leftarrow N-1$.

(4). While $N=1$, $P_{\text{MAX}} = \text{MAX} \{ P \}$.

3.4. Results

Currently, we have the open circuit voltage and the internal resistance of 60 thermoelectric modules through the experiment under certain temperature data (see annex). Here, we set the speed to 3000 rpm. According to the simplified algorithm model, the calculation results are as follows:

The maximum power output curve is as follows:
As shown in the figure 6, after the 18th iteration, the maximum output power for a speed of 3100 rpm was obtained. The best combinations of the 60 thermoelectric modules were: (3,33) modules in parallel, (5,38,43,8) modules in parallel, (13,7) modules in parallel, (14,42,31,32) modules in parallel, (6,34,37,39) modules in parallel, (1,2,29,30) modules in parallel, (9,10,52,54) modules in parallel, and (17,18) modules in parallel.

In table 2, 3, 4 and 5 the experimental results show that the output efficiency of the structure increased by 20% and 40% versus that of a pure in series or parallel topology, respectively. The topological structure of a pure in-series or parallel topology is chosen. In addition, we chose a special case where a structure was established by user experience; modules with a similar voltage and a similar resistance were in parallel, and the others were in series. The topological structure is shown in figure 7. The output power when the speed is 3100 rpm was 362.24 W.

![Figure 7. Structure established by the experience.](image)

To describe the output power of each structure in detail, we tested other speeds: 2800 rpm, 3100 rpm, and 3300 rpm. Under these different speeds, we tested the maximum output power for each structure. For the structure produced by the optimization algorithm, the output power and the tested output power are given at the same time as follows:

**Table 2. Output power of a pure in series topology under different speeds (structure 1).**

| Speed/rpm | output power (W) |
|-----------|------------------|
| 2800      | 312.23           |
| 3000      | 332.02           |
| 3100      | 347.60           |
| 3300      | 360.10           |

**Table 3. Output power of a pure in parallel topology under different speeds (structure 2).**

| Speed/rpm | output power (W) |
|-----------|------------------|
| 2800      | 270.00           |
| 3000      | 284.41           |
| 3100      | 306.12           |
| 3300      | 332.15           |

**Table 4. Output power of optimal structure under different speeds (structure 3)**

| Speed/rpm | Test output power (W) | Calculation output power (W) |
|-----------|-----------------------|------------------------------|
| 2800      | 367.12                | 362.63                       |
| 3000      | 402.59                | 398.14                       |
| 3100      | 415.23                | 411.06                       |
| 3300      | 440.10                | 432.89                       |
Table 5. Output power of the structure established by the rule of thumb under different speeds (structure 4)

| Speed/rpm | output power P/W |
|-----------|------------------|
| 2800      | 330.13           |
| 3000      | 362.24           |
| 3100      | 378.40           |
| 3300      | 401.19           |

Figure 8. Output curves for each structure under different speeds.

As shown in figure 8, the optimal structure can output the maximum power under different speeds.

4. Conclusions
This paper described a device to recycle waste automobile exhaust heat. By considering the different output characteristics of 60 thermoelectric modules, this paper tried to find an appropriate topology in series and in parallel to obtain the maximum output power. According to the law of circuit topology to obtain equivalent voltage and equivalent resistance, and based on Vein’s law, the topological model able to provide maximum power point was established. Two algorithms were used for the model in this paper: topology matrix and iterative optimization algorithm.

This paper adopted an iterative optimization algorithm: all of the thermoelectric modules were in series at first, and then each round selected the two optimal thermoelectric modules to be configured in parallel, and located in series with the rest of the module until the power value fell. The iterative optimization algorithm’s computational complexity is low, and it obtains the optimal topology.

In the experiments, we found three other solutions and compared them with the optimal topology. The experimental results show that the transformation output power of the structure increases by 18.18% and 29.35% versus that of a pure series or parallel topology, respectively, and by 10.08% versus the structure constructed by experience.

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