$\eta'$ production in $B$-decays: Standard Model vs. New Physics

Alexander L. Kagan

Department of Physics, University of Cincinnati
Cincinnati, OH 45221

Alexey A. Petrov

Department of Physics and Astronomy
University of Massachusetts
Amherst MA 01003

Abstract

Standard model factorization can account for the large rate for $B^\pm \rightarrow \eta' K^\pm$. The large rate for $B \rightarrow \eta' X_s$ is more problematic and requires a “cocktail” solution. Individual contributions from factorization, $b \rightarrow \eta' sg$, and “intrinsic charm” fall significantly short of the observed rate. In contrast, the observed $\eta'$ yields are easily explained in models with $\mathcal{B}(b \rightarrow sg) \sim 15\%$. Factorization can account for the entire semi-inclusive $\eta'$ yield without violating rare decay constraints. Implications for $CP$ violation are discussed.
I. INTRODUCTION

The CLEO collaboration [1] has reported sizable production of fast $\eta'$ mesons,

$$\mathcal{B}(B \to \eta' X_s) = (7.5 \pm 1.5 \pm 1.1) \times 10^{-4} \ (2.0 < p_{\eta'} < 2.7 \text{ GeV})$$  \hspace{1cm} (1)

and a correspondingly large exclusive rate,

$$\mathcal{B}(B^\pm \to \eta' K^\pm) = (7.8^{+2.7}_{-2.2} \pm 1.0) \times 10^{-5}.$$  \hspace{1cm} (2)

There is no evidence for $\eta' K^*$ modes in the inclusive analysis, with most events lying at large recoil mass, $m_{X_s}$. The experimental cut on $p_{\eta'}$ corresponds to $m_{X_s} < 2.35 \text{ GeV}$. There is also an exclusive branching fraction upper limit for $\eta$ production,

$$\mathcal{B}(B \to \eta K^\pm) < .8 \times 10^{-5} \ (90\% \text{ c.l.}).$$  \hspace{1cm} (3)

In the standard model (SM) factorization estimates for semi-inclusive [2,3,4,5] and exclusive [6,7] rare $B$ decay branching ratios ($Br$'s) are typically of order $10^{-4}$ and $10^{-5}$, respectively, so it is tempting to speculate that the large $\eta'$ yields are due to the intervention of new flavor physics. A natural candidate to consider is enhanced $b \to sg$ chromomagnetic dipole operators which can explain several apparent discrepancies between SM expectations and experimental results in $B$ decays: the low semileptonic $Br$ and charm deficit [8,9,10,4,11], the kaon deficit [10], and the low $\tau(\Lambda_b)/\tau(\Lambda_d)$ lifetime ratio [12]. All can be accounted for if $\mathcal{B}(b \to sg) \sim 10\% - 15\%$, which is an interesting range because it is naturally associated with $TeV$ scale dynamics for quark mass generation [4].

In what follows we critically examine possible mechanisms for explaining the large $\eta'$ yields in the standard model and in models with enhanced $b \to sg$. We find that standard model factorization can, in principle, account for the exclusive $\eta'$ yield. For a liberal range of factorization model parameters $\mathcal{B}(B^{\pm} \to \eta' K^{\pm})$ lies in the range $(1.1 - 5.8) \times 10^{-5}$ for $m_s = .2 \text{ GeV}$ and $(2.3 - 12.1) \times 10^{-5}$ for $m_s = .1 \text{ GeV}$. It is worth noting in this regard that the recent values of $m_s$ obtained in lattice and QCD sum rule studies are quite low, e.g., $\overline{m}_s(2 \text{ GeV}) \approx 128 \pm 18 \text{ MeV}$ [13] and $\overline{m}_s(2 \text{ GeV}) \approx 100 \pm 21 \pm 10 \text{ MeV}$ [14].
inclusive yield is more problematic. For the same range of parameters SM factorization gives \( \mathcal{B}(B^\pm \to \eta'X_s) \sim (0.5 - 2.5) \times 10^{-4} \) including the experimental cut, with the largest yields corresponding to a fairly limited region of parameter space. Of course there could be additional non-factorizable corrections, e.g., at the 10% – 30% level.

Atwood and Soni (AS) [15] have suggested that the large inclusive rate is connected to the standard model QCD penguins via the gluon anomaly, leading to the subprocess \( b \to sg^* \to \eta'sg \). Taking a constant \( gg\eta' \) vertex form factor \( H(0, 0, m_{\eta'}^2) \), extracted from \( J/\Psi \to \eta'\gamma \), they obtain agreement with Eq. (1). More recently, Hou and Tseng (HT) [16] have argued that the factor \( \alpha_s \) implicit in \( H \) should be running, which would lower AS’s result by roughly a factor of 3. However, both AS and HT have overlooked the leading \( m_{\eta'}^2/(q^2 - m_{\eta'}^2) \) dependence of the form factor, where \( q \) is the virtual gluon’s momentum. Including this dependence nominally reduces AS’s result to \( \mathcal{B}(B \to \eta'X_s) \sim 1.6 \times 10^{-5} \) including the experimental cut, more than an order of magnitude below what is observed.

AS have estimated the contribution from decay of intermediate charmonia and from \( \eta' - \eta_c \) mixing to be \( \mathcal{B}(B \to \eta'X_s) \sim 1.1 \times 10^{-4} \), including the cut. Halperin and Zhitnitsky [17] have suggested that the elevated glue content of the \( \eta' \) gives it a large intrinsic charm component which simultaneously accounts for the exclusive and inclusive \( \eta' \) yields via \( b \to c\bar{c}s \to \eta'X \). However, as already noted in [16], their prediction of \( \mathcal{B}(B \to \eta'K) \sim 2\mathcal{B}(B \to \eta'K^*) \) is inconsistent with the absence of \( \eta'K^* \) modes in the inclusive analysis. This alone implies that the intrinsic charm \( \eta' \) yields can not account for more than about a tenth of Eqs. (1) and (2).

Finally, as a by-product of their SM analysis of \( b \to sgg \) Wyler and Simma

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1 Throughout we obtain results for inclusive \( \eta' \) production for charged \( B \) decays. We will update our results in the near future to include the average over charged and neutral \( B \) decays. The branching ratio results are not significantly modified and our conclusions remain the same.

2 It was recently suggested [15] that the anomalously large \( Br \) in Eq. (1) can be explained by color-octet \( c\bar{c} \) contributions to \( \eta' \) production, in analogy with the color-octet mechanism in the
have obtained an $O(10^{-6})$ $Br$ estimate for $B \to K + \text{glueball}$ $^{19}$, indicating that the $Br$ for $b \to sgg \to s\eta'$ should not exceed $O(10^{-5})$. In our view, although each of the above mechanisms falls short by itself, a standard model ‘cocktail solution’ for the inclusive $\eta'$ yield is still possible given the large hadronic uncertainties involved, with factorization providing the largest contribution.

In contrast, we find that models with enhanced $b \to sg$ readily account for the observed inclusive yield in the factorization model alone, obtaining $\mathcal{B}(B \to \eta'X_s) \approx (1.0 - 9.0) \times 10^{-4}$. The actual value largely depends on the choice of weak phases in the dipole operator coefficients, and $B \to \eta'$ form factors. Moreover, we have checked that the large $\eta'$ yields are compatible with the potentially restrictive CLEO upper limits on $B \to \phi X_s$ $^{20}$ and $B \to \phi K$ $^{21}$ for a significant range of parameters in the factorization approximation. The recoil spectrum increases with $m_{X_s}$, in qualitative agreement with observation. In the case of the anomaly mediated process $b \to \eta' sg$, HT argued that the entire inclusive yield can be reproduced if $b \to sg$ is enhanced. Taking into account the previously overlooked $m_{\eta'}^2/(q^2 - m_{\eta'}^2)$ dependence of the $gg\eta'$ form factor, but ignoring the subleading $q^2$ dependence of $\alpha_s$ in HT, we find $\mathcal{B}(B \to \eta'X_s) \sim (0.3 - 1.6) \times 10^{-4}$ for $\mathcal{B}(b \to sg) \approx 15\%$. Although significantly smaller than the rates obtained in the factorization approach it is clear that the sum of the two can easily reproduce Eq. (1) if $b \to sg$ is enhanced.

An important feature of quark model $B$ decays pointed out by Lipkin $^{22}$ is the interference between spectator ($b \to s\bar{u}u$) and non-spectator ($b \to s\bar{s}s$) production of $\eta'$ and $\eta$ mesons. It is constructive for exclusive $\eta'K$ modes and destructive for exclusive $\eta K$ modes, leading to order of magnitude smaller rates for the latter, in accord with the recent measurements. We have checked this explicitly in the factorization approximation for the standard model and for enhanced $b \to sg$. The constructive interference can also account for the larger NRQCD description of charmonium production. However, the $c\bar{c}$ pair must be far off shell inside of the $\eta'$, invalidating the use of the NRQCD formalism.
rates for $\eta'K$ versus $K\pi, \phi K$. Lipkin has noted that the same interference pattern persists generally for orbital parity even final states, e.g., constructive for $\eta'K^{**}$ and destructive for $\eta K^{**}$, while the reverse is true for orbital parity odd final states, e.g., destructive for $\eta'K^*$ and constructive for $\eta K^*$. This most likely can explain the absence of $\eta'K^{*\pm}$ events in the inclusive analysis. We leave a factorization model analysis of $B \to \eta'K^*$ for future work.

The weak phases associated with new physics contributions to the dipole operator coefficients generally lead to large $CP$ violating asymmetries in charmless $B$ decays in the factorization approximation \[2\] when strong phases in the penguin amplitudes from $c\bar{c}$ rescattering \[24,25\] are taken into account. For SM factorization we obtain CP asymmetries in the range $4.5\% - 8\%$ for $B^{\pm} \to \eta'K^{\pm}$ and $2\% - 5\%$ for $B \to \eta'X_s$ with, as usual, the larger asymmetries corresponding to smaller $Br$’s. Including enhanced $b \to sg$ can give asymmetries of order $10\%$ (exclusive) and $5\%$ (inclusive) for $Br$’s in the observed range. These are not dramatically larger than the SM asymmetries, unlike the ‘pure penguin’ processes $B^{\pm} \to \phi K^{\pm}$ and $B \to K^{0}\pi^{\pm}$ for which large asymmetries are possible compared to $\leq 1\%$ in the standard model. We note that enhanced $b \to sg$ can also lead to large isospin violating asymmetries in radiative $B$ decays \[26\].

In the next section we discuss $\eta'$ production in the standard factorization approach. In Section 3 we focus on the gluon anomaly subprocess and comment on other ‘exotic’ mechanisms. We conclude with discussion of our results in Section 4.

II. $\eta'$ AND $\eta$ PRODUCTION IN THE FACTORIZATION APPROACH.

The effective Hamiltonian for non-leptonic charmless $b \to s$ transitions is given below$^3$

\[ O_{11}, O'_{11} \] correspond to use of $-ig_sT^a$ for the quark - gluon vertex Feynman rule \[27\], leading to destructive interference in the standard model between penguin and chromomagnetic dipole operator contributions to charmless $b \to sq\bar{q}$ decays.

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$^3$The sign of the operators $O_{11}, O'_{11}$ corresponds to use of $-ig_sT^a$ for the quark - gluon vertex Feynman rule \[27\], leading to destructive interference in the standard model between penguin and chromomagnetic dipole operator contributions to charmless $b \to sq\bar{q}$ decays.
\[ H_{eff} = \frac{4G_F}{\sqrt{2}} \left[ V_{ub}V_{us}^* \sum_{i=1}^{2} c_i O_i^u - V_{tb}V_{ts}^* \sum_{i=3}^{11} c_i O_i + c_1' O_{11}' \right], \]

\[ O_1^u = \bar{s}\gamma_\mu L u_\beta \bar{u}_\beta \gamma^\mu L b_\alpha, \quad O_2^u = \bar{s}\gamma_\mu L u_\beta \gamma^\mu L b, \]

\[ O_{3(5)} = \frac{3}{2} s_\alpha \gamma_\mu L b \sum_q \bar{q} \gamma^\mu L(R)q, \quad O_{4(6)} = s_\alpha \gamma_\mu L b \sum_q \bar{q} \beta \gamma^\mu L(R)q_\alpha, \]

\[ O_{7(9)} = \frac{3}{2} \bar{s} \gamma_\mu L b \sum_q \bar{q} \gamma_\beta \gamma^\mu L(R)q, \quad O_{8(10)} = \frac{3}{2} \bar{s} \gamma_\mu L b \sum_q \bar{e} q \beta \gamma^\mu L(R)q_\alpha, \]

\[ O_{11} = \frac{g_s}{32\pi^2} m_b(\mu) \bar{s} \sigma_{\mu\nu} R t^a b G_{\alpha}^{\mu\nu}, \quad O_{11}' = \frac{g_s}{32\pi^2} m_b(\mu) \bar{s} \sigma_{\mu\nu} R t^a b G_{\alpha}^{\mu\nu}, \]

(4)

where the standard notations are used and \( q \) is summed over \( u, d, s \).

In our numerical analysis we take \( \alpha_s(M_Z) = .117 \), \( m_t = 174 \text{ GeV} \), \( m_b = 5.0 \text{ GeV} \), \( m_c = 1.63 \text{ GeV} \) (HQET parameter \( \lambda_1 = -.2 \)), and \( m_s = .1 \) to .2 GeV. For KM matrix entries we take \( V_{ts}V_{td}^* = .037 \), and Wolfenstein parameters \( \rho = -.15 \), \( \eta = .33 \) which are in the range favored by the SM analysis of Ali and London \cite{AliLondon}. We use the Next-to-Leading Order (NLO) scheme-independent effective Wilson coefficients \cite{AliLondon,Maruhn} at \( \mu = m_b \), obtaining

\[ c_1 = -0.306, \quad c_2 = 1.15, \quad c_3 = 0.017 - 0.37 P_s, \quad c_4 = -0.037 + 1.11 P_s, \]

\[ c_5 = 0.01 - 0.37 P_s, \quad c_6 = -0.045 + 1.11 P_s, \quad c_7 = 1.03 \times 10^{-5} + 0.33 P_s, \]

\[ c_8 = 3.82 \times 10^{-4}, \quad c_9 = -0.01 + 0.33 P_s, \quad c_{10} = 1.97 \times 10^{-3}. \]

(5)

The \( P_{s,e} \) are given by \( P_s = \frac{8\pi}{9} \left( \frac{10}{9} - G(m_c, \mu, q^2) \right) \), \( P_e = \frac{8\pi}{9} \left( \frac{10}{9} - G(m_c, \mu, q^2) \right) \), where

\[ G(m, \mu, q^2) = -4 \int_0^1 x(1-x) \ln \frac{m^2 - x(1-x)q^2}{\mu^2} dx. \]

(6)

\( q \) is the momentum of the virtual gluon in the penguin diagram. \( G \) becomes imaginary when \( q^2 > 4m_c^2 \) and a strong phase is generated \cite{AliLondon,Maruhn}.

The Leading Order (LO) dipole coefficient is \( c_{11}^{m}(m_b) \approx -.295 \), which corresponds to \( B(b \to sg) \approx .2\% \). For enhanced dipoles we’ll consider \( B(b \to sg) \approx 15\% \), corresponding to \( 4( |c_{11}|^2 + |c_{11}'|^2)^{1/2} \approx 2.26 \). In obtaining \( Br's \) we consider two total \( B \) decay widths: the

\footnote{This value is determined using the total NLO standard model, inclusive \( b \) decay width, see next footnote, and the LO expression for \( B(b \to sg) \).}
NLO SM value\(^{\text{\footnote{This includes the NLO corrected } b \rightarrow c \text{ and } b \rightarrow u \text{ transitions }^{\footref{footnote:30}}, \text{ taking into account } O(\alpha_s c_s^2) \text{ corrections to the LO penguin contributions to } b \rightarrow c\bar{c}s, \text{ and } O(1/m_b^3) \text{ HQET corrections }^{\footref{footnote:31}}. \text{ The charmless } b \rightarrow s \text{ transitions and } O_{11} \text{ have not been included but their total correction is } \sim 1\%.}}\) for the above inputs, \( \Gamma_{SM} \approx 3.3 \times 10^{-13} \text{ GeV}, \) and \( 3.88 \times 10^{-13} \text{ GeV} \) for \( \mathcal{B}(b \rightarrow s g) \approx 15\%. \) The dipole coefficients are parametrized as

\[
c_{11} = -|c_{11}| e^{i\theta_{11}}, \quad c'_{11} = -|c'_{11}| e^{i\theta_{11}}. \tag{7}\]

Generally, large direct CP violation effects result from interference between new amplitudes containing the weak phases \( \theta_{11}, \theta'_{11} \) and standard penguin amplitudes containing strong phases from the \( G \) function.

In what follows the dipole operators are included by allowing the off-shell gluon to turn into a quark-antiquark pair. For \( O_{11} \) this leads to the effective Hamiltonian

\[
\mathcal{H}_{11} = - \sum_{q=u,d,s} i \frac{G_F}{\sqrt{2} \pi q^2} V_{tb} V_{ts}^* C_{11}(m_b(\mu) \bar{s}(p_s) \sigma_{\mu\nu} R t^a b(p_b) \bar{q}(p_2) \gamma^\mu t_a q(p_1) q^\nu, \tag{8}\]

where \( q^2 = (p_b - p_s)^2 \). In the standard model \( \mathcal{H}_{11} \) interferes destructively with the penguin amplitudes. In practice, we take \( \alpha_s = g_s(m_b) g_s(q)/4\pi \), identifying one factor of \( g_s \) with the virtuality of the gluon. This procedure models the fact that in approaching the ‘on-shell’ gluon limit \( \mathcal{H}_{11} \) becomes increasingly non-local so that gluon fragmentation must take place at non-perturbative size scales. Performing Fiertz rearrangement yields\(^{\text{\footnote{This result differs from the result in }^{\footref{footnote:3}}.}}\)

\[
\mathcal{H}'_{11} = + \sum_{q=u,d,s} \frac{G_F}{\sqrt{2}} \alpha_s(q^2) V_{tb} V_{ts}^* C_{11}(m_b(\mu)) \frac{N_c^2 - 1}{N_c^2} \left[ \delta_{\alpha\beta} \delta_{\alpha'\beta'} - \frac{2N_c}{N_c^2 - 1} t^a \tau^a a \tau^a a \right] \left\{ 2m_b \bar{s}_\alpha \gamma_\mu L q_\beta \bar{q}_\alpha \gamma^\mu L b_{\beta'} - 4m_b \bar{s}_\alpha R q_\beta \bar{q}_\alpha \gamma_\mu L b_{\beta'} + 2m_s \bar{s}_\alpha \gamma_\mu R q_\beta \bar{q}_\alpha \gamma^\mu R b_{\beta'} - 4m_s \bar{s}_\alpha L q_\beta \bar{q}_\alpha \gamma^\mu R b_{\beta'} + (p_b + p_s)_{\mu} \left[ \bar{s}_\alpha \gamma_\mu L q_\beta \bar{q}_\alpha \gamma_{\mu} R b_{\beta'} + \bar{s}_\alpha R q_\beta \bar{q}_\alpha \gamma^\mu R b_{\beta'} + i\bar{s}_\alpha \epsilon_{\mu\nu} R q_\beta \bar{q}_\alpha \gamma_{\nu} R b_{\beta'} - i\bar{s}_\alpha \gamma_{\nu} L q_\beta \bar{q}_\alpha \epsilon_{\mu\nu} R b_{\beta'} \right] \right\} \tag{9}\]

The effective Hamiltonian \( \mathcal{H}'_{11} \) corresponding to \( O'_{11} \) is obtained by substituting \( C_{11} \rightarrow C'_{11} \) and \( L \rightarrow R \) in the above. In all of our factorization model results we take \( N_c = 3. \)
A. Semi-inclusive \( \eta' \), \( \eta \) production.

Factorization model contributions to \( B^- \to \eta^{(i)} X_s \) involve two classes of amplitudes distinguished by their hadronization pattern, which we loosely refer to as “two-body” and “three-body” decays. The matrix element is of the form

\[
M = M^{2b} + M^{3b}
\]

so that

\[
|M|^2 = |M^{2b}|^2 + |M^{3b}|^2 + M^{2b\dagger} M^{3b} + M^{3b\dagger} M^{2b}.
\]

In the two-body decays an \( \eta' \) or \( \eta \) is formed from an \( s\bar{s} \) pair via the subprocess \( b \to s\bar{s} s \), which in the parton model looks like \( b \to \eta^{(i)} s \). Applying the Gordon identity and Dirac equation the matrix element for \( \eta^{(i)} \) production can be written in the simplified form

\[
M^{2b} = \zeta_1^{\eta^{(i)}} \langle X_s | \bar{s} L b | B \rangle + \zeta_2^{\eta^{(i)}} \langle X_s | \bar{s} R b | B \rangle.
\]

For \( \eta' \) production the coefficients depend on the matrix elements

\[
\langle \eta' | \bar{s} \gamma^{\mu} \gamma_5 s | 0 \rangle = -i p_{\eta'}^{\mu} \sqrt{6} \left( F_0 \cos \theta - \sqrt{2} F_8 \sin \theta \right)
\]

\[
\langle \eta' | \bar{s} \gamma_5 s | 0 \rangle = i \sqrt{3} \frac{m_{\eta'}}{2 m_s} F_8 \sin \theta
\]

\[
\langle \eta' | \bar{u} \gamma^{\mu} \gamma_5 u | 0 \rangle = -i p_{\eta'}^{\mu} \sqrt{2} \left( F_0 \cos \theta + \frac{1}{\sqrt{2}} F_8 \sin \theta \right),
\]

where \( F_0 \) and \( F_8 \) are the decay constants for the \( SU(3) \) singlet and octet axial vector currents, and the anomaly has been taken into account in \( \langle \eta' | \bar{s} \gamma_5 s | 0 \rangle \) following [32]. For the \( \eta \) the corresponding matrix elements are

\[
\langle \eta | \bar{s} \gamma^{\mu} \gamma_5 s | 0 \rangle = i p_{\eta}^{\mu} \sqrt{6} \left( F_0 \sin \theta + \sqrt{2} F_8 \cos \theta \right)
\]

\[7\] A similar treatment of \( B \to K X \) is presented in [3].
\[ \langle \eta | \bar{s} \gamma_5 s | 0 \rangle = i \frac{\sqrt{3} m_\eta^2}{2 m_s} F_8 \cos \theta \]  

(14b)

\[ \langle \eta | \bar{u} \gamma^\mu \gamma_5 u | 0 \rangle = i \bar{p}_\eta \sqrt{\frac{2}{3}} (F_0 \sin \theta - \frac{1}{\sqrt{2}} F_8 \cos \theta). \]  

(14c)

Phenomenological fits in [32] give \( F_0 \approx F_8 \approx F_\pi \), where \( F_\pi = \frac{132}{\sqrt{2}} \) MeV, and an \( \eta' - \eta \) mixing angle \( \theta_{\eta \eta'} \approx -17^\circ \). Chiral perturbation theory favors \( F_8 \approx 1.25 F_\pi \) and \( \theta_{11} \sim -21^\circ \) [33][32]. \( Br' \)’s are given for both choices. We make the usual quark model approximation, taking equal momenta for the two \( s \) quarks in the \( \eta'(t) \), which gives

\[ q^2 = m_b^2/2 - m_{\eta'(t)/4}^2/4 + m_s^2/2. \]  

(15)

For simplicity we take the \( b \) quark and spectator at rest so that the \( \eta'(t) \) energy is fixed. Taking their momenta into account as in Ref. [34] would not have a significant impact on the overall \( Br \) but would smear the “two-body” \( \eta'(t) \) spectrum by a few hundred MeV.

SM \( b \to \eta's \) \( Br' \)’s can be found in Table I. Associated CP asymmetries are \( \sim +1\% \). The large \( m_s \) dependence is traceable to the \( 1/m_s \) factor in \( \langle \eta' | \bar{s} \gamma_5 s | 0 \rangle \) which, as we’ll see shortly, can lead to large exclusive \( Br' \)’s in the SM. The corresponding \( b \to \eta s \) \( Br' \)’s range from \( (3.4 - 9.0) \times 10^{-5} \), with CP asymmetries of \( \sim -1\% \). Enhanced \( b \to sg \) can increase these contributions by a factor of 2 to 3, but the dominant contribution will come from the “three-body” decays.

The “three-body” decays involve hadronization of the spectator into the \( \eta(t) \) via the subprocess \( b \to s\bar{u}u \). In the parton model this is interpreted as \( B^- \to \eta(t) \bar{s}u \), with \( p_X = p_s + p_\bar{u} \). Using the equations of motion and again, for simplicity, taking the \( b \) quark at rest[8], the matrix element can be simplified to the form

\[ M^{3b} = \xi_1^{\eta(t)\mu} \langle X_{\eta s} | \bar{s} \gamma_\mu Lu | \overline{B} \rangle + \xi_2^{\eta(t)} \langle X_{\eta s} | \bar{s} Ru | \overline{B} \rangle 

+ \xi_3^{\eta(t)\mu} \langle X_{\eta \bar{u}} | \bar{s} \gamma_\mu Ru | \overline{B} \rangle + \xi_4^{\eta(t)} \langle X_{\eta \bar{u}} | \bar{s} Lu | \overline{B} \rangle \]  

(16)

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[8]There are in general two additional terms which vanish in this limit, being proportional to \( p_B^{\mu} p_B^{\nu} \bar{s} \sigma_{\mu\nu} Ru \) and \( p_B^{\mu} p_B^{\nu} \bar{s} \sigma_{\mu\nu} Lu \) following application of the equations of motion (EOM).
The matrix elements entering the $\xi_i$ are parameterized as \[35\]:

\[
\begin{align*}
\langle \eta^{(\nu)} | \bar{u} \gamma^\mu b | B^- \rangle &= (p_B + p_{\eta^{(\nu)}}) \mu F_1^{\eta^{(\nu)}} (m_{X_s}^2) + (p_B - p_{\eta^{(\nu)}}) \mu f_{\eta^{(\nu)}} (m_{X_s}^2) \\
\langle \eta^{(\nu)} | \bar{u} b | B^- \rangle &= \frac{1}{m_b - m_u} (m_{B^-}^2 - m_{\eta^{(\nu)}}^2) F_1^{\eta^{(\nu)}} (m_{X_s}^2) + m_{X_s}^2 f_{\eta^{(\nu)}} (m_{X_s}^2) \\
\langle \eta | \bar{u} \sigma^{\mu\nu} b | B^- \rangle &= -i \frac{(F_1^{\eta^{(\nu)}} (m_{X_s}^2) - f_{\eta^{(\nu)}} (m_{X_s}^2))}{2m_b} (p_B^\mu p_{\eta^{(\nu)}}^\nu - p_B^\nu p_{\eta^{(\nu)}}^\mu),
\end{align*}
\]

where,

\[
f_{\eta^{(\nu)}} (p^2) = \frac{m_{\eta^{(\nu)}}^2 - m_{B^-}^2}{p^2} [F_1^{\eta^{(\nu)}} (p^2) - F_0^{\eta^{(\nu)}} (p^2)].
\]

With a simple monopole parameterization assumed for the form factors they are determined by fixing $F_{1^-}$ and $F_{0^+}$ at zero momentum transfer, where they must be equal. It appears, based on various models in the literature \[35,7,36,37\], that a reasonably broad range to consider for all $B$ to light pseudoscalar form factors is $F_{1^-} (0) = .25 - .5$.

To gain insight into the factorization approach it is useful to study the quark level NLO Dalitz plots for $b \to s \bar{q} q$ shown in Fig. (1), where $q^2 \equiv (p_b - p_s)^2$ and $m^2 \equiv (p_s + p_q)^2$. For simplicity we only consider $b \to s \bar{d} d$, which does not contain tree-level contributions. The same conclusions can be drawn from the Dalitz plots for $b \to s \bar{u} u$ and $b \to s \bar{s} s$. We have blown up the regions relevant to factorization for two-body or semi-inclusive “quasi” two-body decays, i.e., $q^2 \sim m_b^2 / 2$ and $m \leq 2 \text{GeV}$. It is clear why models with enhanced $b \to s g$ readily evade constraints from rare $B$ decays in the factorization model. Although the charmless $Br$ is an order of magnitude larger than in the SM it is peaked at low $q^2$, i.e., the “on-shell” gluon limit.\[9\] In the region relevant to rare decays, e.g., $B \to K \pi, K \phi, \phi X_s$, etc.,

\[9\] The singular low $q^2$ dependence of $b \to s \bar{q} q$ originates from the pure dipole contributions, e.g., $O(c_{11}^2)$. Although these contributions come in at $O(\alpha_s^2)$ they are clearly important in the case of enhanced dipole coefficients and should be included, particularly since we will be interested in comparison of inclusive quark level and factorization model Dalitz plots. This dependence will be canceled in the total charmless $Br$ by $O(\alpha_s)$ corrections to $b \to sg$. 

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the pure dipole amplitude is only of same order as in the standard model. This means that in general large interference is possible between the penguin and dipole amplitudes. Generally this will lead to large CP violation\[23\] because at \(q^2 \sim m_b^2/2\) the penguin amplitude should develop large strong phases\[24,25\], and the dipole amplitude can have arbitrarily large weak phases.\[10\]

Three-body \(B^- \to \eta's\bar{u}\) Dalitz plots are shown in Fig. (4). Note that the ‘quark level’ definition for \(q\) is used, i.e., \(p_b - p_s\) rather than \(p_B - p_s\). The plots do loosely resemble the corresponding inclusive plots in Fig. (1). The regions surviving the cut \(m_{X_s} < 2.35\ GeV\) are blown up. The corresponding recoil spectra, shown in Fig. (3), show a characteristic rise with \(m_{X_s}\), like the measured spectrum [1]. Including the two-body spectra and interference between two-body and three-body \(\eta'\) production (2b-3b interference), particularly the destructive interference expected for exclusive \(\eta'K^*\) production [22], should improve agreement with the shape of the observed spectrum at low \(m_{X_s}\).

To address the question of whether 2b-3b interference can have a significant impact on the semi-inclusive \(\eta^{(l)}\) yields we have blown up the regions of Figs. (1a) and (1c) which are relevant for three-body \(\eta'\) production, with experimental cut, in Fig. (4). The three-body amplitude can interfere with two-body \(\eta^{(l)}\) production when \(q \sim m_{\eta^{(l)}}\) and \(m_{X_s} \sim 1\ GeV\). A glance at Fig. (4a) will convince the reader that in the SM the relative contribution of this region to the overall three-body \(Br\) is not important, so that 2b-3b interference should have little impact on the relative \(\eta'\) vs. \(\eta\) yields. However, hierarchies of 2 to 4 between the \(\eta'\) and \(\eta\) yields may result from reasonable differences between the \(B \to \eta\) and \(B \to \eta'\)

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10It is also clear that it is not possible to obtain an \(O(10\%)\) charmless \(Br\) via new physics contributions to the penguin operators without violating rare \(B\) decay constraints, since this would necessarily increase the differential \(Br's\) in the factorization region by an order of magnitude or more relative to the SM. For example, \(\mathcal{B}(B \to \phi X_s)\) would exceed the CLEO upper limit by a factor of 20 - 40 in the factorization approach.
form factors. In the case of enhanced $b \to sg$, Fig. (4b), the relative contribution of this region to the three-body yield appears to be significant, particularly for $\eta$ production, due to the singular behaviour at low $q^2$, so that the interference could turn out to be relevant. For now only the pure two-body and three-body factorization estimates will be considered, with a low $q^2$ cutoff imposed on the latter. We make no pretense that factorization is valid at low $q^2$ for dipole operator contributions, recognizing that this further contributes to the uncertainty in $\eta'$ production in models with large $b \to sg$.

In Table II we study the dependence of $\mathcal{B}(B^- \to \eta's\bar{u})$ with experimental cut on choice of low $q^2$ cutoff. The singular behavior is apparent below 1 GeV$^2$ for enhanced $b \to sg$, while there is relatively little change in the SM $Br$’s below 1 GeV$^2$. The three-body $B \to \eta$ $Br$’s are very close to the $\eta'$ yields in the SM and $\sim 5\% - 15\%$ larger with enhanced $b \to sg$. In the ensuing discussion we fix the $q^2$ cutoff at 1 GeV$^2$ for both $\eta'$ and $\eta$ production. We should note that the three-body $\eta'(0)$ yields are not very sensitive to $m_s$, unlike the two-body yields.

In Table III the two-body and three-body $CP$ averaged SM $Br$’s are summed for the range of parameters previously discussed. We find that $\mathcal{B}(B \to \eta'X_s)$ lies in the range $(0.5 - 2.5) \times 10^{-4}$, which is significantly smaller than the observed rate even for the largest $Br$’s. The $CP$ asymmetries range from 2% to 5% with the larger asymmetries corresponding to the smaller $Br$’s. In the case of $\eta$ production the SM $Br$ sums are slightly larger, lying in the range $\mathcal{B}(B \to \eta X_s) \approx (0.7 - 2.6) \times 10^{-4}$, with $CP$ asymmetries of 1.5% to 4.5%. As already noted, we do not expect significant changes in the ratio of $\eta$ to $\eta'$ yields when SM 2b-3b interference is taken into account. Finally, with no momentum cut we obtain $\mathcal{B}(B \to \eta'X_s) \approx (0.25 - 1.0) \times 10^{-3}$ in the SM.

Taking $\mathcal{B}(b \to sg) \approx 15\%$, the dependence of $\eta'$ yields on the relative magnitudes of $c_{11}$ and $c_{11}'$, corresponding phases, and factorization model parameters is explored in Fig. 5. Sums of pure two-body and pure three-body contributions have been plotted for three illustrative cases: $c_{11}' = 0$, $c_{11} = 0$, and $|c_{11}| = |c_{11}'|$. It is worth noting that in models of quark mass generation a large hierarchy between $c_{11}$ and $c_{11}'$ is not expected in the absence of
some special flavor symmetry. According to Fig. (3) factorization alone could well account for the observed inclusive yield in Eq. (1) if dipoles were enhanced.\textsuperscript{11} This conclusion persists when constraints from other rare decays are taken into account, as we’ll see shortly. The large range of $\eta'$ yields possible is in marked contrast to the low SM range, shown in grey. The $\eta'$ $Br$ with no cut is $\sim (2.5 - 10) \times 10^{-3}$ for maximal constructive interference between dipole and penguin contributions. The $\eta$ yields are about 10\% larger for $F_{1-}^{\eta}(0) = F_{1-}^{\eta'}(0)$ and $2b$-$3b$ interference not taken into account.

The $CP$ asymmetries for inclusive $\eta'$ production in Fig. (3) are not much larger than the SM range from Table III, despite the presence of significant new amplitudes with arbitrary weak phases. This is because the dominant three-body dipole contributions are weighted towards low $q^2$, where little or no strong phase is generated for the interfering three-body penguin amplitudes.

\textbf{B. Exclusive $\eta^{(l)}$ production.}

It is straightforward to obtain factorization amplitudes for $B^\pm \to \eta^{(l)} K^\pm$ from $H_{eff}$ and $H_{11}$ in terms of the $\eta^{(l)}$ hadronic matrix elements listed in Eqs. (13, 14, 17), the kaon matrix elements

\begin{align}
\langle K^-|\bar{s}\gamma^\mu\gamma_5 u|0 \rangle &= -i\sqrt{2}F_Kp_K^\mu \tag{19} \\
\langle K^-|\bar{s}\gamma_5 u|0 \rangle &= -i\sqrt{2}F_K\frac{m_K^2}{m_s + m_u} \tag{20}
\end{align}

and the analogs of the $B \to \eta'$ matrix elements in Eq. (17) for $B^- \to K^-$ transitions \textsuperscript{35}. Again we consider the range $F_{1-}^{\eta'}(0) = .25 - .5$ for the $B \to \eta^{(l)}$ and $B \to K$ form factors at zero momentum transfer. Although we are looking at exclusive decays, individual quark momenta enter the amplitudes because of $H_{11}$. For simplicity we will identify $p_6$ with

\textsuperscript{11}For $F_{1-}^{\eta'}(0) = .5$ but $m_s = .15$ $GeV$, $f_8 = f_\pi$, $\theta_{\eta\eta'} \approx -17^\circ$ the maximum yields would be about 15\% lower than in Fig. (3).
the $B$ meson momentum and make the usual quark model kinematical assumptions, taking $p_s = p = p_{\eta^{(i)}}/2$ for non-spectator $\eta^{(i)}$ production and $p_s = p_{\bar{u}} = p_{K^-}/2$ for spectator $\eta^{(i)}$ production. With these assumptions one obtains $q^2 = m_{B^-}^2/2 - m_{\eta^{(i)}}^2/4 + m_{K^-}^2/2$ for the former and $q^2 = m_{B^-}^2/2 - m_{K^-}^2/4 + m_{\eta^{(i)}}^2/2$ for the latter.

SM $Br$’s and CP asymmetries for $B^\pm \to \eta'K^\pm$ are summarized in Table IV. The $Br$’s range from $1.1 - 12.1 \times 10^{-6}$, with significant enhancement occurring at low values of $m_s$ due to the $1/m_s$ factors in $\langle \eta'|\bar{s}\gamma_5 s|0 \rangle$ and $\langle K^-|\bar{s}\gamma_5 u|0 \rangle$. It is clear that SM factorization can reproduce the measured $Br$ in Eq. (4) given a reasonably broad range of input parameters. The CP asymmetries can also be significant, varying from 4.5% - 8%. For $\eta$ production we find $B(B^\pm \to \eta K^\pm) \approx (1.5 - 4.8) \times 10^{-6}$ for $F_{1-}^{\eta'}(0) = F_{1-}^{\eta}(0)$, in accord with the order of magnitude suppression expected relative to the $\eta'K^\pm$ yield from Lipkin’s interference mechanism [22]. The corresponding $CP$ asymmetries are large, at the 15% - 20% level.

Enhanced dipole operators can lead to significant increases in the exclusive $\eta'$ yield, as is to be expected given the increases we saw are possible for the semi-inclusive yield. Eq. (2) can therefore be reproduced for a larger region of factorization model parameter space. This is illustrated in Fig. (3), where exclusive $\eta'$ production with enhanced dipoles included is compared to SM yields for intermediate choices of form factors and $m_s$. According to Fig. (3) significantly larger $CP$ asymmetries than in the SM are also possible, although differences in magnitude decrease in regions of $\theta_{11}$ and/or $\theta_{11}'$ corresponding to large $\eta'$ production. Finally, it is important to note that Lipkin’s interference mechanism for exclusive decays persists in the case of enhanced $b \to sg$. For example, the $\eta$ yields corresponding to Fig. (3) lie in the range $B(B \to \eta K^\pm) \sim (1.0 - 8.0) \times 10^{-6}$ if $F_{1-}^{\eta'}(0) = F_{1-}^{\eta}(0)$.

C. Comparison with other rare $B$ decays.

It is important to check that large factorization model $\eta'$ yields obtained with enhanced $b \to sg$ are consistent with constraints from other rare decays. We have considered three processes: $B \to \phi X_s$, which is the most restrictive semi-inclusive decay, $B^\pm \to \phi K^\pm$, which
is probably the most restrictive exclusive decay, and $B^\pm \to K^0\pi^\pm$ which is not particularly restrictive but, like the other two, is least cumbersome to calculate because there are no tree-level contributions (pure-penguin decays). Recently the CLEO collaboration has presented the following bounds: $\mathcal{B}(B \to \phi X_s) < 2.2 \times 10^{-4}$ (90\% c.l.) \cite{20}, $\mathcal{B}(B^\pm \to \phi K^\pm) < 1.2 \times 10^{-5}$ (90\% c.l.) \cite{21}, and $\mathcal{B}(B^\pm \to K^0\pi^\pm) = 2.5_{-1.0}^{+1.4} \times 10^{-5}$ \cite{38}. In the SM we obtain $\mathcal{B}(B \to \phi X_s) \approx 1.1 \times 10^{-4}$ (also see \cite{2,3,4}), and $\mathcal{B}(B^\pm \to \phi K^\pm) \approx (0.32 - 1.3) \times 10^{-5}$ as $F_{1-}^K(0)$ is varied from .25 to .5. Neither $Br$ is sensitive to $m_s$. For $F_{1-}^\pi(0) = .333$ \cite{35} we obtain $\mathcal{B}(B^\pm \to K^0\pi^\pm) = (1.0 - 2.1) \times 10^{-5}$ as $m_s$ is varied from .2 to .1 GeV. It is interesting that the largest SM $\eta'$ yields in Tables \[1\] and \[4\] are consistent with the above constraints. For the exclusive decays this is due to Lipkin’s mechanism, whereas in the semi-inclusive case this is because there is no spectator contribution to $\phi$ production, whereas it constitutes the bulk of the $\eta'$ yield. Another semi-inclusive process for which there is a spectator contribution, $B \to K_s X$, is very weakly bounded, $\mathcal{B}(B \to K_s X) < 7.5 \times 10^{-4}$ for $p_K > 2$ GeV \cite{39}.

In Fig. (6) we show the $\phi X_s$ and $\phi K^\pm$ yields with enhanced $b \to sg$ corresponding to the examples in Fig. (3). Comparison of Figs. (3 - 7) shows that within the factorization model there is significant overlap of regions in $(c_{11}, c'_{11})$ space with exclusive and semi-inclusive $\eta'$ yields in the measured ranges and acceptable $\phi$ production. No additional constraints arise from $B^\pm \to K^0\pi^\pm$.

III. EXOTIC MECHANISMS

It is well known that the $\eta'$ is not a Goldstone boson – its mass does not vanish in the chiral limit $m_{u,d,s} \to 0$. This is believed to be due to a large glue component in the $\eta'$. In the present context the enhanced glue content leads to a significant coupling of the $\eta'$ to two gluons, which participates in $b \to s$ transitions.

It is important to notice that because of large cancellations \cite{19,40}, the contribution of the $b \to sg g g$ mode followed by a $g g \to \eta'$ transition is extremely small. On the other hand, the $\mathcal{O}(1\%)$ $Br$ of the $b \to sg^* \to \eta'$ transition in the SM could, in principle, be responsible for the
anomalously large $b \to \eta' X_s$ decay rate. This fact was exploited by AS [13], who considered the contribution from the $b \to sg^* \to \eta'sg$ subprocess. They parameterized the effective $gg\eta'$ coupling as

$$V_{\mu\nu} e_1^\mu e_2^\nu = H(q^2, k^2, q_{\eta'}^2) \epsilon_{\alpha\beta\mu\nu} q^\alpha k^\beta \epsilon_1^\mu \epsilon_2^\nu$$  \hspace{1cm} (21)

where $q^2 \equiv (p_b - p_s)^2$. A constant form factor was assumed, i.e., $H(q^2, k^2, q_{\eta'}^2) \approx H(0, 0, m_{\eta'}^2) \equiv H_0$, and $H_0$ was extracted directly from the decay rate for $J/\psi \to \eta'\gamma$. For $\alpha_s(m_{J/\psi}) = 0.25$ and $\theta_{\eta'\eta} = -17^o$, the value $H_0 = 1.8 \text{ GeV}^{-1}$ is obtained.

With the central assumption of weak $q^2$ dependence in $H$, AS found that the observed $\eta'$ yield could be fully accounted for. However, it is clear that in order to obtain the total decay rate, the differential distribution for this subprocess must be integrated over a wide range of $q^2$, spanning approximately 1 GeV$^2$ to $m_b^2$. It is therefore of paramount importance to investigate the off-shell dependence of the form-factor describing the $g^* \to \eta'g$ transition.

Determination of the exact $q^2$-dependence of $H(q^2, 0, m_{\eta'}^2)$ is a difficult task. We therefore employ a semi-phenomenological description of the form-factor, picking up the leading $q^2$-dependence and parametrizing the rest by an appropriate constant. To set the stage we recall the leading $q^2$-dependence of the $\gamma^* \to \eta'\gamma$ form-factor in the limit of large momentum transfer [11]. In this limit, a perturbative QCD description is possible and it is not difficult to convince oneself that the simplest two-particle irreducible kernel yields a form-factor that scales like $1/Q^2$ as $Q^2 \to \infty$ ($Q^2 = -q^2$) [11]:

$$\Gamma_{\mu\nu} e_1^\mu e_2^\nu = ie^2 F_{\eta'}(Q^2) \epsilon_{\alpha\beta\mu\nu} q^\alpha k^\beta \epsilon_1^\mu \epsilon_2^\nu, \quad F_{\eta'}(Q^2) = 4\sqrt{2} F_{\pi} \sqrt{3} Q^2 \xi.$$  \hspace{1cm} (22)

with $\xi \simeq 0.5$. While the corresponding calculation for $g^* \to \eta'g$ is more involved, the lowest order $Q^2$ dependence in the $Q^2 \to \infty$ limit should be the same, so it is clear that the constant form factor approximation $H(q^2, 0, m_{\eta'}^2) \approx H_0$ is not acceptable.

In order to obtain a more realistic parameterization we consider a model of the $gg\eta'$ vertex in which we couple a pseudoscalar current to two gluons through quark loops. This approximation is believed to be a good one since the “direct” $gg\eta'$ coupling is suppressed –
the quantum numbers of $\eta'$ are $0^-$, while the lowest possible bound state of two gluons is $0^+$. The calculation yields \[12\]

$$V_{\mu\nu} = \sum_{f=u,d,s} \frac{a_f g_s^2 m_f}{2\pi^2} \varepsilon_{\alpha\beta\mu\nu} q^\alpha k^\beta \int_0^1 dx \int_0^{1-x} \frac{dy}{m_f^2 - (1-x-y)(xq^2 + yk^2) - xym_{\eta'}^2 - i\epsilon}$$

$$= \sum_{f=u,d,s} \frac{4\pi\alpha_s m_f}{2\pi^2} q^2 - m_{\eta'}^2 F_\Delta^{(f)}(q^2) \varepsilon_{\alpha\beta\mu\nu} q^\alpha k^\beta, \quad F_\Delta^{(f)}(q^2) = I\left(\frac{m_{\eta'}^2}{m_f^2}\right) - I\left(\frac{q^2}{m_f^2}\right),$$

(23)

where the $I(x^2)$-functions are defined as

$$I(x^2) = \begin{cases} -2\arcsin\left(\frac{|x|}{2}\right), & 0 \leq x^2 \leq 4, \\ 2\left[\ln\left(\frac{|x|}{2} + \sqrt{\frac{x^2}{4} - 1}\right) - \frac{i\pi}{2}\right]^2, & x^2 > 4. \end{cases}$$

(24)

$a_f$ is the coupling of the $\eta'$ to the quarks in the loop, i.e., $a_f\eta' \bar{f} \gamma_5 f$. One has to keep in mind that this vertex is in general non-local, so that $a_f$ will depend on the momentum transferred through the vertex. Since this dependence is at most logarithmic, i.e. subleading, it will be ignored in what follows.

Motivated by the above result it is now reasonable to parametrize the effective $gg\eta'$ coupling as

$$V_{\mu\nu} = -\frac{H_0 m_{\eta'}^2}{q^2 - m_{\eta'}^2} \varepsilon_{\alpha\beta\mu\nu} q^\alpha k^\beta, \quad \text{and} \quad H(q^2, 0, m_{\eta'}^2) = -\frac{H_0 m_{\eta'}^2}{q^2 - m_{\eta'}^2},$$

(25)

Note that $H_0$ does depend on $q^2$. We drop this dependence based on the following observations. First of all, $q^2 > 4m_f^2$ for any quark flavor over the whole range of $q^2$ contributing to the total decay rate. This implies that $F_\Delta^{(f)}(q^2)$ depends on $q^2$ only logarithmically. Also, as noted by HT \[16\], the factor $\alpha_s(\mu^2)$ implicit in $H_0$ must in general be running. They argued that the scale at which $\alpha_s$ is evaluated should be associated with the momentum transferred through the $gg\eta'$ vertex, introducing another logarithmic dependence on $q^2$ in $H_0$. It is clear that these dependences are subleading with respect to the strong power dependence in Eq. \(25\) and to first approximation can be modeled by a constant $H_0$, which we identify with the value extracted from $J/\psi \to \eta' \gamma$.

The strong $q^2$ dependence of Eq. \(25\) implies additional suppression of the $b \to \eta' sg$ anomaly subprocess at large $q^2$. It is easy to estimate the resulting contribution to $b \to \eta' X_s$.  

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The calculation is a straightforward generalization of the one carried out by AS [15] and HT [16] so we do not include any details here. We only point out that in the SM the interference between the charge radius and dipole form factor contributions is destructive.

Our SM result for the total branching ratio is $\mathcal{B}(b \to \eta'/sg) \approx 5.3 \times 10^{-5}$, corresponding to $\mathcal{B}(b \to \eta'/sg) \approx 1.6 \times 10^{-5}$ with experimental cut. The latter is more than an order of magnitude smaller than the observed $\eta'$ yield in Eq. (4). The recoil spectrum is shown in Fig. 8. As already pointed out in Refs. [15,16], the anomaly subprocess exhibits a rise with $m_{X_s}$ in the low $m_{X_s}$ region which is characteristic of three-body decays. The $q^2$ dependence of the $gg\eta'$ formfactor significantly distorts the $q^2$ distribution so that the maximum is shifted towards lower values of $q^2$ compared to constant $gg\eta'$ formfactor.

In the case of enhanced $b \to sg$ we again consider $\mathcal{B}(b \to sg) \approx 15\%$, and for simplicity take $c'_{11} = 0$. The resulting $Br$’s with experimental cut are presented in Fig. 9 as a function of $\theta'_{11}$. These are more than an order of magnitude smaller than obtained by HT under the assumption of mild $q^2$-dependence of the form factor. The corresponding CP violating asymmetries are also systematically smaller than those obtained by HT [15]. This is because of the shift in the decay distribution towards lower values of $q^2$, leading to smaller strong phases in the charge radius form factor.

We also remark that the short-distance $b \to \eta'/sg$ subprocess most probably does not affect the exclusive $B \to \eta'/K$ branching ratios. If the gluon attaches itself to the $s$ quark then the exclusive decay can be interpreted as being due to the short distance subprocess $b \to sg$ which we already know is suppressed [19]. The other two possibilities, that the gluon attaches itself to the spectator quark, or that it contributes to the process via higher Fock states of the participating mesons, must be additionally suppressed by the dynamics of the process.

Another interesting explanation of the anomalously large $\eta'$ production rate is to assume a large ‘intrinsic charm’ content for the $\eta'$ [17]. Phenomenologically, this can be described as a mixing of the $\eta_c$ and $\eta'$. Since the value of the mixing angle is obtained by a fit to the experimental branching ratios for radiative charmonia decays to $\eta'$ this phenomenological
estimate should already contain a contribution from the higher Fock states containing charm quarks \[18\]. It is worth mentioning that intrinsic heavy quarks of the light mesons must be far from their mass shells in order to satisfy energy-momentum conservation. Therefore, NRQCD methods developed for weakly bound quarks in charmonia can not be used to describe the dynamics of intrinsic heavy quark states.

The heavy-light quark mixing for the \(\eta - \eta' - \eta_c\) system was considered in \[43\] and found to be small, \(\sin \alpha_P \simeq 2.4 \times 10^{-2}\). This leads to the estimate \(Br(B \to \eta' X_s) \sim \sin^2 \alpha_P Br(B \to \eta_c X_s) < 10^{-6}\) so that the ‘intrinsic charm’ content of the \(\eta'\) is not likely to be responsible for the observed large branching ratio. In addition to phenomenological estimates limiting the size of the \(\bar{c}c\) contribution, a large ‘intrinsic charm’ explanation for the \(\eta'\) yields would lead to \(B(B \to \eta' K^*) \sim 2B(B \to \eta' K)\) and a quasi two-body momentum spectrum for \(B \to \eta' X_s\) \[17\], neither of which is observed.

### IV. CONCLUSIONS

We have considered production of \(\eta'\) in \(B\) meson decays in the Standard Model and in models with enhanced \(b \to sg\). It is clear that SM factorization can, in principle, account for the exclusive \(\eta'\) yield. For a liberal range of factorization model parameters the SM \(B^\pm \to \eta' K^\pm\) branching ratio lies in the range \((1.1 - 5.8) \times 10^{-5}\) for \(m_s = .2\) GeV and \((2.3 - 12.1) \times 10^{-5}\) for \(m_s = .1\) GeV. Sizable \(CP\) violating asymmetries at the 5% - 8% level are expected.

The inclusive yield requires a more complicated solution. For the same range of parameters SM factorization gives \(B(B^\pm \to \eta' X_s) \sim (0.5 - 2.5) \times 10^{-4}\) including the experimental cut, with the largest yields corresponding to a fairly limited region of parameter space. In addition, the branching ratio of the QCD anomaly mediated subprocess \(b \to \eta' sg\) considered in \[15\] is reduced to \(\sim 1.6 \times 10^{-5}\) when taking proper account of the leading \(q^2\)-dependence of the \(\eta' gg\) coupling form factor, \(H(q^2) \sim m^2_{\eta'}/(q^2 - m^2_{\eta'})\). The large rate reported in \[15\] for constant form factor would require a factor of 50 enhancement due to non-perturbative ef-
fects, e.g., resonances, of the rate obtained with leading $q^2$-dependence included - an unlikely possibility. However, a SM ‘cocktail’ solution for large $B(B^\pm \to \eta' X_s)$ involving contributions from several mechanisms, e.g., factorization pushed to the limit (the largest single contribution), large non-factorizable contributions, charmonia decays [15], ‘intrinsic charm’ [17], and $b \to \eta' sg$, is still a possibility.

On the other hand, the intervention of Non-SM physics in the form of enhanced chromo-magnetic dipole operators provides a simple and elegant solution to the puzzle of large $\eta'$ production in $B$ decays. This explanation can be realized in a broad region of the available parameter space. In fact, factorization model contributions alone can account for both the semi-inclusive and exclusive $\eta'$ yields without violating contraints from other rare decays. The $b \to \eta' sg$ rate, although significantly smaller, can be an order of magnitude larger than in the SM. $CP$ violating asymmetries in exclusive and semi-inclusive $B \to \eta'$ decays due to new weak phases in the dipole operator coefficients can be larger than in the SM and of either sign. However, large asymmetries in the ‘pure-penguin’ decays $B^\pm \to \phi K^\pm, K^0\pi^\pm$ would provide a more definitive signal for new physics because of the small asymmetries expected for these decays in the SM.

**Acknowledgements.** We would like to thank David Atwood, Tom Browder, Gerhard Buchalla, John Donoghue, Gene Golowich, Barry Holstein, George Hou, Uli Nierste, Joao Soares and Amarjit Soni for useful discussions.
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TABLE I. Two-body contributions to $\mathcal{B}(B^{\pm} \to \eta' X_s)$ (average of CP conjugate decays) and corresponding CP asymmetries for (i) standard model without $O_{11}$, (ii) standard model with $O_{11}$. $F_0 = F_\pi$.

| Inputs | no $O_{11}$ | with $O_{11}$ |
|--------|-------------|---------------|
| $m_s = .1, F_8 = F_\pi, \theta_{\eta\eta'} = -17^\circ$ | $3.7 \times 10^{-5}, .51\%$ | $3.2 \times 10^{-5}, .59\%$ |
| $m_s = .2, F_8 = F_\pi, \theta_{\eta\eta'} = -17^\circ$ | $1.3 \times 10^{-5}, 1.5\%$ | $1.04 \times 10^{-5}, 1.8\%$ |
| $m_s = .1, F_8 = 1.25 F_\pi, \theta_{\eta\eta'} = -21^\circ$ | $8.4 \times 10^{-5}, .21\%$ | $7.4 \times 10^{-5}, .24\%$ |
| $m_s = .2, F_8 = 1.25 F_\pi, \theta_{\eta\eta'} = -21^\circ$ | $2.8 \times 10^{-5}, .63\%$ | $2.34 \times 10^{-5}, .75\%$ |

TABLE II. Three-body contributions to $\mathcal{B}(B^- \to \eta' X_s)$, with experimental cut, versus $q_{\text{min}}^2$ [GeV$^2$] for (i) standard model with $O_{11}$, (ii) $\mathcal{B}(b \to sg) \approx 15\%$ with $|c_{11}| = |c'_{11}|$, $\theta_{11} = 180^\circ$, $\theta'_{11} = 0$. $F_1^{\eta'}(0) = .5$, $m_s = .1$ GeV.

| $q_{\text{min}}^2$ | SM | $\mathcal{B}(b \to sg) \approx 15\%$ |
|----------------|----|-----------------|
| .4            | $1.71 \times 10^{-4}$ | $12.1 \times 10^{-4}$ |
| .6            | $1.68 \times 10^{-4}$ | $9.61 \times 10^{-4}$ |
| .8            | $1.66 \times 10^{-4}$ | $8.38 \times 10^{-4}$ |
| 1.0           | $1.65 \times 10^{-4}$ | $7.61 \times 10^{-4}$ |
| 1.4           | $1.64 \times 10^{-4}$ | $6.64 \times 10^{-4}$ |
| 2.0           | $1.62 \times 10^{-4}$ | $5.77 \times 10^{-4}$ |
TABLE III. Sum of two-body and three-body contributions to $\mathcal{B}(B^\pm \to \eta^'X_s)$ (average of CP conjugate decays) in SM for (i) $F_{1^-}(0) = .25$, (ii) $F_{1^-}(0) = .5$. $F_0 = F_\pi$.

| Inputs                        | $F_{1^-}(0) = .25$ | $F_{1^-}(0) = .50$ |
|-------------------------------|---------------------|---------------------|
| $m_s = .1, F_8 = F_\pi, \theta_{\eta\eta'} = -17^o$ | $7.6 \times 10^{-5}$ | $2.1 \times 10^{-4}$ |
| $m_s = .2, F_8 = F_\pi, \theta_{\eta\eta'} = -17^o$ | $5.4 \times 10^{-5}$ | $1.8 \times 10^{-4}$ |
| $m_s = .1, F_8 = 1.25F_\pi, \theta_{\eta\eta'} = -21^o$ | $1.2 \times 10^{-4}$ | $2.5 \times 10^{-4}$ |
| $m_s = .2, F_8 = 1.25F_\pi, \theta_{\eta\eta'} = -21^o$ | $6.7 \times 10^{-5}$ | $2.0 \times 10^{-4}$ |

TABLE IV. $\mathcal{B}(B^\pm \to \eta^'K^\pm)$ (average of CP conjugate decays) and corresponding $CP$ asymmetries [%] in SM. $F_0 = F_\pi$.

| Inputs                        | $m_s = .1$ GeV | $m_s = .2$ GeV |
|-------------------------------|---------------|---------------|
| $F_8 = F_\pi, \theta_{\eta\eta'} = -17^o, F_{1^-}^{\eta}(0) = F_{1^-}^{K}(0) = .25$ | $2.3 \times 10^{-5}$, 5.4% | $1.1 \times 10^{-5}$, 8.0% |
| $F_8 = F_\pi, \theta_{\eta\eta'} = -17^o, F_{1^-}^{\eta}(0) = F_{1^-}^{K}(0) = .5$ | $9.1 \times 10^{-5}$, 5.4% | $4.6 \times 10^{-5}$, 8.0% |
| $F_8 = 1.25F_\pi, \theta_{\eta\eta'} = -21^o, F_{1^-}^{\eta}(0) = F_{1^-}^{K}(0) = .25$ | $3.0 \times 10^{-5}$, 4.5% | $1.5 \times 10^{-5}$, 6.9% |
| $F_8 = 1.25F_\pi, \theta_{\eta\eta'} = -21^o, F_{1^-}^{\eta}(0) = F_{1^-}^{K}(0) = .5$ | $1.2 \times 10^{-4}$, 4.5% | $5.8 \times 10^{-5}$, 6.9% |
FIG. 1. $10^3 \frac{dR}{dmdq}(b \rightarrow s\bar{d}d)$ for: (a) SM, (b) Enhanced $O_{11}$ by itself, (c) SM with enhanced $O_{11}$ for $\theta_{11} = 180^\circ$ (constructive interference). The regions relevant to two-body or quasi two-body decays in the factorization approximation are blown up.
FIG. 2. $10^3 \frac{d^3 \sigma}{dm_X dq}(B^- \rightarrow \eta's\bar{u})$ for: (a) SM, (b) SM with enhanced $O_{11}$ for $\theta_{11} = 180^\circ$ (constructive interference). The regions within the experimental cut are blown up.
FIG. 3. $10^3 \frac{dR}{dm_{X_s}}(B^- \to \eta's\bar{u})$ for SM (lower curves), and SM with enhanced $O_{11}$ and $\theta_{11} = 180^\circ$ (upper curves). Solid curves are for $m_s = .15$ GeV, $m_u = .005$ GeV, dashed curves are for $m_s = .5$ GeV and $m_u = .1$ GeV.

FIG. 4. $10^3 \frac{dR}{dm_{d\bar{q}}}(b \to sdd)$ for: (a) SM, (b) SM with enhanced $O_{11}$ and $\theta_{11} = 180^\circ$, in the regions relevant to $\eta'$ production with experimental cut.
FIG. 5. $B(B^- \to \eta's\bar{u})$ vs. $\theta$ and CP asymmetries vs. $\theta$ for $B(b \to sg) \approx 15\%$. Shaded regions are the corresponding SM ranges. Solid curves are for $c'_{11} = 0$ ($\theta = \theta_{11}$), long dashed curves are for $|c_{11}| = |c'_{11}|$ and $\theta'_{11} = 180^\circ$ ($\theta = \theta_{11}$), short dashed curves are for $c_{11} = 0$ ($\theta = \theta'_{11}$). The larger $Br$'s (lower asymmetries) are for $F_{1^-}^{\eta'}(0) = .5, f_8 = 1.25f_\pi, \theta_{\eta'\eta} = -21^\circ, m_s = .1 \text{ GeV}$, the lower $Br$'s (larger asymmetries) are for $F_{1^-}^{\eta'}(0) = .25, f_8 = f_\pi, \theta_{\eta'\eta} = -17^\circ, m_s = .2 \text{ GeV}$. 

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FIG. 6. (a) $10^5 \mathcal{B}(B^\pm \rightarrow \eta' K^\mp)$ vs. $\theta$ and SM value (horizontal line) for $F_{1-}^{\eta'}(0) = .35$, $F_{1-}^{K}(0) = .38$, $m_s = .15 \text{ GeV}$, $F_8 = F_\pi$, $\theta_{\eta' K} = -17^\circ$. Solid, long dashed, and short dashed curves are for same $c_{11}$ and $c'_{11}$ as in Fig. 5. (b) CP asymmetries vs. $\theta$ corresponding to (a). Horizontal line is SM value. (c) $\mathcal{B}(B^\pm \rightarrow \eta' K^\mp)$ in $(\theta_{11}, \theta'_{11})$ plane for $|c_{11}| = |c'_{11}|$. 
FIG. 7. (a) $10^4 B(B \to \phi X_s)$ vs. $\theta$ for $B(b \to sg) \approx 15\%$. Solid curve is for $c'_1 = 0$ ($\theta = \theta_1$), dashed curve is for $c_1 = 0$ ($\theta = \theta'_1$), horizontal solid curve is SM value, and dot-dashed curve is the CLEO upper limit. (b) $B(B \to \phi X_s)$ in the $(\theta_1, \theta'_1)$ plane for $|c_1| = |c'_1|$. (c) and (d) are analogous to (a) and (b) for $10^5 B(B^\pm \to \phi K^\pm)$. In (c) solid and dashed curves coincide.
FIG. 8. $\frac{d\mathcal{B}}{dm_{X_s}}(b \rightarrow \eta'sg)$ vs. $m_{X_s}$ in the SM. Curves from top to bottom are for $m_b = 5.2$, 5.0, and 4.8 GeV. $m_g = 0.5$ GeV and $m_s = 0.2$ GeV.
FIG. 9. $\mathcal{B}(b \to \eta' sg)$ vs. $\theta_{11}$ for $\mathcal{B}(b \to sg) \approx 15\%$ and $c'_{11} = 0$. 