Transverse momentum spectra of identified particles in high energy collisions with statistical hadronisation model

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A detailed analysis is performed of transverse momentum spectra of several identified hadrons in high energy collisions within the framework of the statistical model of hadronisation. The effect of the decay chain following hadron generation is accurately taken into account. The considered centre-of-mass energies range from \( \simeq 10 \) to \( 30 \) GeV in hadronic collisions (\( \pi p, pp \) and \( Kp \)) and from \( \simeq 15 \) to \( 45 \) GeV in \( e^+e^- \) collisions. A clear consistency is found between the temperature parameter extracted from the present analysis and that obtained from fits to average hadron multiplicities in the same collision systems. This finding indicates that in the hadronisation, the production of different particle species and their momentum spectra are two closely related phenomenons governed by one parameter.

1. Introduction

The idea of a statistical approach to hadron production in high energy collisions dates back to '50s [1] and '60s [2] and it has been recently revived by the observation that hadron multiplicities in \( e^+e^- \) collisions agree very well with a thermodynamical-like ansatz [3,4]. This finding has also been confirmed in hadronic collisions and it has been interpreted in terms of a pure statistical filling of multi-hadronic phase space of assumed pre-hadronic clusters (or fireballs) formed in high-energy collisions, at a critical value of energy density \( \varepsilon \). In this framework, temperature and other thermodynamical quantities have a statistical meaning which does not entail the existence of a thermalised hadronic system on an event-by-event basis. Stated otherwise, statistical equilibrium shows up only when comparing many different events, whilst in each of them the Gibbs law of equally likely multi-hadronic states applies. So far, this proposed statistical cluster hadronisation model has been mainly tested against measured abundances of different hadron species for a twofold reason. Firstly, unlike momentum spectra, they are quantities which are not affected by hard (perturbative) QCD dynamical effects but are only determined by the hadronisation process; indeed, in the framework of a multi-cluster model, they are Lorentz-invariant quantities which are independent of the cluster’s overall momentum. Secondly, they are quite easy to calculate and provide a very sensitive test of the model yielding an accurate determination of the temperature. However, in order to establish the validity of this approach, it is necessary to test further observables and to assess their consistency with the results obtained for multiplicities. Certainly, one of the best suited observables is the transverse momentum of identified hadrons, where transverse is meant to be with respect to beam line in high energy hadronic collision, and thrust or event axis in high energy \( e^+e^- \) collisions. Indeed, such projection of particle momentum is supposed to be the most sensitive to hadronisation or, conversely, the least sensitive to perturbative QCD dynamics.

Actually, it has been known for a long time that transverse momentum distributions are Boltzmann-like in hadronic collisions and this very observation was pointed out by Hagedorn as a major indication in favour of his statistical model of hadron production [5]. It must be emphasized that the prediction of a thermal-like shape in principle only applies to particles directly emitted from the hadronising source, whereas measured spectra also include particles produced by decays of heavier hadrons. However,
most analyses do not take into account the distortion of primordial hadronisation spectrum due to the hadronic decay chain and try to fit the data straight through it. This problem has been discussed in literature [6] and an analytical calculation has been developed to take into account the effect of two and three body decays [8,9], which has then been used both for pp [9] and heavy ion collision [8–10] including most abundant resonances. In this paper we introduce a method allowing to rigorously and exhaustively determine the contribution of all particle decays. Hence, by taking advantage of this technique, we have performed an analysis of many measured transverse momentum spectra of identified hadrons in a wide range of centre-of-mass energies for several kind of collisions.

2. Statistical hadronisation and transverse momentum spectra

The statistical hadronisation model [12] assumes that in high energy collisions, as a consequence of strong interaction dynamics, a set of colourless clusters (or fireballs) is formed having certain values of mass, volume, internal quantum numbers and momentum, the latter being inherited from the hard stage of the process. Those clusters are assumed to give rise to hadrons according to a pure statistical law in the multi-hadronic phase space defined by their mass, volume and quantum numbers. This approach differs from another popular cluster hadronisation model [13] mainly because it gives clusters a volume so that hadron production is ruled by the properly understood phase space rather than relativistic momentum space. In this framework, the use of statistical mechanics and thermodynamical quantities, such as temperature, which need spatial dimension besides momentum space in order to be meaningful, is allowed. We emphasize once more that the introduction of such quantities does not entail any thermalisation process of hadrons after their formation, nor the existence of a thermalised system event by event.

Although many clusters with different momenta, volumes, masses and quantum numbers are formed, it can be shown that the average values of many observables, e.g. particle multiplicities, are the same as those relevant to one equivalent cluster having suitable values of volume (namely the sum of all cluster volumes measured in the rest frame of the equivalent cluster itself), mass and quantum numbers (namely the sum of all cluster quantum numbers). The proof of this statement [14], a lengthy one, requires the assumption of special probabilities governing the fluctuations of cluster masses and quantum numbers for a given set of volumes. If the volume (or mass) of the equivalent cluster is large enough, it is then reasonable to take a canonical approach (i.e. introducing a temperature) in order to calculate mean quantities, instead of carrying out an involved microcanonical calculation. Therefore, even though actual clusters are small sized and microcanonical calculations would be needed to determine mean quantities within each of them, the choice of suitable mass fluctuation probabilities for the clusters allows one to calculate overall means dealing with only one large global structure and with much fewer parameters. Arguing the other way around, it is not difficult to be convinced that the fluctuation probabilities to be chosen in order to achieve such reduction of the problem are exactly the same as those of obtaining a given set of masses by splitting a large cluster into a given number of clusters with volumes \( (V_1, \ldots, V_N) \). This statement extends a reduction procedure to the microcanonical case which was proved in the canonical case [6], where, from the very beginning, clusters were given temperature and volume instead of mass and volume. Nevertheless, it is not obvious how large the equivalent cluster should be for the canonical approximation to apply. For the present, we have adopted a simple-minded \textit{a posteriori} method consisting in justifying the canonical framework by its capability of accordance with the data. A simple argument to support the canonical approximation even at moderate values of volumes is the very large number of states \( (O(10^2)) \), i.e. hadrons and resonances, which can be excited in a hadron gas. As far as single-particle transverse momentum spectra are concerned, a similar reduction theorem from many clusters to one equivalent cluster in the averaging procedure applies [14], though...
only approximately. In general, it can be shown that the primary spectrum of \( j \)th hadron species depends on transverse four-velocities \( u_{T_i} = \beta_i \gamma_{T_i} \) of the \( N \) clusters:

\[
\frac{dn_j}{dp_T} = \left[ \prod_{i=1}^{N} \int_0^\infty du_{T_i} \right] f(u_{T_1}, \ldots, u_{T_N}) \\
\times \sum_{i=1}^{N} \frac{dn_j}{du_{T_i}} (u_{T_i}).
\]

where \( f(u_{T_1}, \ldots, u_{T_N}) \) is the transverse four-velocities distribution function. If one expands all single-cluster spectra \( \frac{dn_j}{dp_T} |_{i}(u_{T_i}) \) in series of \( u_{T_i} \) starting from a common value \( \langle u_T \rangle \) for all clusters, it can be proved that, at the zeroth order, the spectrum in eq. (1) becomes the same as that obtained for the aforementioned equivalent cluster, endowed with a transverse four-velocity \( \langle u_T \rangle \). This reduction possibly allows taking the canonical approach in order to calculate the transverse momentum spectrum since the equivalent cluster size is much larger than single cluster’s:

\[
\frac{dn_j}{dp_T} \propto \frac{(2J_j + 1)}{\sqrt{1 + \langle u_T \rangle^2}} m_T p_T \\
\times K_1 \left( \frac{\sqrt{1 + \langle u_T \rangle^2} m_T}{T} \right) I_0 \left( \frac{\langle u_T \rangle p_T}{T} \right)
\]

where \( m_T = \sqrt{p_T^2 + m_j^2} \), \( T \) is the temperature, and \( K_1, I_0 \) are modified Bessel functions. Eq. (2) is the Boltzmann limit of quantum statistics and it is a very good approximation for all hadrons except pions \(^3\). For the zeroth order approximation in the \( u_{T_i} \) expansion to be sufficiently accurate, the involved transverse four-velocities should be small, i.e. \( f(u_{T_1}, \ldots, u_{T_N}) \rightarrow 0 \) already for \( u_{T_i} \ll 1 \). Since it is possible to choose the starting point \( \langle u_T \rangle \) of the series expansion to make the first order term vanishing \(^4\), the largest neglected term turns out to be \( O((u_{T_i} - \langle u_T \rangle)^2) \) which is very small provided \(^1\)In fact, we mean by transverse four-velocity the module of the spacial part of a velocity four-vector \( u = (\gamma, \beta_T) \) with vanishing longitudinal component.

\(^3\)In fact, the spectra of six identified particles in pp collisions at \( \sqrt{s} = 27.4 \text{ GeV} \) \(^2\). The measured \( \frac{dn}{dp_T^2} \) spectra (above) have been normalised so as to have the same value at \( p_T = 0 \). While slopes of \( \frac{dn}{dp_T^2} \) spectra are different, those of \( \frac{dn}{dm_T} \) (below) are approximately the same for all identified particles.
decays (e.g. is huge and also owing the presence of cascade ber of contributing resonances and decay modes work is a very complicated problem as the num-
side of eq. (3) within a statistical-thermal frame-
calculation of the second term in the right hand
determined the momentum spectrum \(d_n/dm_T\) directly or through a cascade process, we have
Carlo method by randomly generating 200,000
in the equivalent cluster’s rest frame via a Monte-
Whilst \(d_n/dp_T\) is given by eq. (2), the
primary spectrum of the equivalent cluster, to be
plugged into eq. (3), with an integral formula derived by the authors [14]:
\[
\frac{d\eta_j}{dp_T}((u_T)) = \frac{4p_T}{\sqrt{1+(u_T)^2 m_T}} \int_0^\infty \frac{dp}{dp'} \frac{d\eta_j}{dp'}|_{\beta=0}^{k-j} \times \frac{1}{2\pi p'}(z_{+}-z_{\text{min}})(z_{\text{max}}+1) \frac{F(\frac{\pi}{2}, r)}{}
\]
where \(F\) is the elliptic integral of the first kind and:
\[
r = \frac{\sqrt{(z_{+}-z_{\text{max}})(z_{\text{min}}+1)}}{\sqrt{(z_{+}-z_{\text{min}})(z_{\text{max}}+1)}}
\]
\[
z_{\text{max}} = \max(1, z_{-}) \quad z_{\text{min}} = \min(1, z_{-})
\]
\[
z_{\pm} = \frac{e' \pm (u_T)p_T}{\sqrt{1+(u_T)^2 m_T}} \quad e' = \sqrt{p'^2 + m_j^2}
\]

The main advantage of formula (6) is to allow a quick computation of transverse momentum spec-
tra associated with particle decays for any trans-
verse four-velocity once the corresponding mo-
momentum spectra at zero velocity are known. To
obtain it, we took advantage of the isotropy of \(d\eta_j/dp|_{\beta=0}\) distributions, which holds as long as
decay products distribution is isotropic, which is true for simple phase-space decays with no po-
larisation. The integration in the variable \(p'\) in
eq \text{(3)} has been performed numerically. The
Monte-Carlo generation of \(d\eta_j/dp|_{\beta=0}\) spectra,
depending on the temperature, has been per-
formed for all unstable hadrons and resonances
up to a mass of 1.8 GeV and it has been repeated for temperature values between 140 and 190 MeV
in steps of 1 MeV, so as to have a large enough
range of temperatures where to search for minima

3. Method of data analysis

A peculiar prediction of a statistical picture in high energy collisions which is relevant to trans-
verse momentum spectra is the so-called \(m_T\) scaling: there should be an apparent common slope for \(d\eta_j/dm_T\) spectra of identical hadrons at a
given centre-of-mass energy. By looking at eq. (3) one can easily realize that this holds in the limit
\(\langle u_T \rangle \rightarrow 0\). In principle, \(m_T\) scaling applies only
to primary hadrons, namely those directly emit-
ted from the hadronising source. On the other hand, as stated in Sect. 1, most observed or recon-
structed particles in experiments arise from de-
cays of heavier hadrons. Those secondary decays
may well distort the primary spectrum shape, thus spoiling \(m_T\) scaling. Nevertheless, it can be seen in fig. (3) that \(m_T\) scaling apparently also
holds for measured final hadrons, at least in the examined collision, implying seemingly little dis-
tortion from primary to final spectrum. Setting this and other related issues demands a thor-
ough analysis taking into account the effect of decays. Therefore, measured \(p_T\) spectrum of the
\(j\)th hadron species should be compared with the
sum of its primary spectrum and the contribution
arising from all heavier hadrons decaying into it:
\[
\frac{d\eta_j}{dp_T} = \frac{d\eta_j}{dp_T}\text{primary} + \sum k \frac{d\eta_j}{dp_T}|_{k-j}
\]
Table 1
Results of the fit to hadron average multiplicities for the chosen set of hadronic and $e^+e^-$ collisions. The $\chi^2$ value in pp collisions is very high due to lack of systematic errors [16] in the measurements of particle multiplicities [17].

| collision | $\sqrt{s}$ (GeV) | $T$ (MeV) | $VT^3$ | $\langle s\bar{s} \rangle$ | $\chi^2$/dof |
|-----------|------------------|-----------|--------|-----------------|-------------|
| K$^+$p    | 11.5             | 176.9±2.6 | 5.85±0.39 | 0.347±0.020 | 68.0/14     |
| $\pi^+$p  | 21.7             | 170.5±5.2 | 10.8±1.2  | 0.734±0.049  | 39.7/7      |
| K$^+$p    | 21.7             | 175.8±5.6 | 8.48±1.05 | 0.578±0.056  | 38.0/9      |
| pp        | 27.4             | 162.6±1.6 | 14.17±0.66| 0.644±0.018  | 313.9/29    |

In a fitting procedure. For any other temperature value, the spectra have been calculated by means of a linear interpolation. Overall, for hadronic collisions, we have generated about 1200 spectra associated to 144 unstable hadrons for each $T$ value.

The abundances of all hadrons, which are needed to assess the contribution of secondaries in the spectra of eq. (3), have been preliminary set to the model values calculated by fitting the parameters $T$, $V$ and $\langle s\bar{s} \rangle$ or $\gamma_S$ to the experimentally measured yields. This procedure amounts to exclude the overall normalisation dependence of $dn_j/dp_T|^{k→j}$ on $T$ in the fit so to keep only its shape dependence on $T$.

Whilst the $\gamma_S$ parameter has already extensively been used in multiplicity fits [3,4], a new parameter $\langle s\bar{s} \rangle$ is introduced here for the strangeness suppression. The particle yields are computed by constraining the number of newly created $s\bar{s}$ pairs to fixed integers which fluctuate according to a Poissonian distribution. The mean value of the Poissonian distribution, namely the mean value of newly created $s\bar{s}$ pairs, is the adjustable fit parameter $\langle s\bar{s} \rangle$. For the present, this new parametrisation has been used only for hadronic collisions (see Table 1) because $e^+e^-$ collisions present difficulties related to the production of heavy flavoured quarks which makes calculations extremely slow. The mathematical features of the $\langle s\bar{s} \rangle$ parametrisation are described in detail in ref. [14].

The fitted values, which update those in refs. [3,4] for hadronic and $e^+e^-$ collisions are quoted in Table 1. The fitted temperature in pp collision in the present analysis is somewhat lower compared with ref. [4] owing to an updated set of hadron parameters [19], an upgraded procedure of $\chi^2$ minimisation also taking into account correlations in the effective variance method [20], as well as the replacement of $\gamma_S$ with $\langle s\bar{s} \rangle$. The fitted temperature values show a very interesting trend consisting in a slight but significant increase as centre-of-mass energy diminishes; this is in agreement with the observation pointed out in ref. [21].

4. Data set and results for hadronic collisions

The collisions and relevant centre-of-mass energies to be analysed have been primarily chosen on the basis of a fairly large number of measured particle yields and spectra. Particle multiplicities are needed to establish with some accuracy the contribution of secondary decays while many identified hadron spectra allow performing a test of universality on the slopes governed by

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2The measurements used to perform this fit have been gathered through the Durham reaction data database [18]; the relevant references will be quoted in ref. [14].
Figure 2. χ² contour plot in the $T - \langle u_T \rangle$ plane for $\pi^+ p$ spectrum in pp collisions at $\sqrt{s} = 27.4$ GeV.

Figure 3. Contour plot in the $T - \langle u_T \rangle$ plane of $\sum_i \chi_i^2$ for $\pi^+ p$ collisions $\sqrt{s} = 21.7$ GeV. The identified particles involved in the sum are $\pi^-, \pi^0, K^0, \rho^0, K^*0$ and $\bar{K}^*0$. The two vertical lines define the 1-sigma band corresponding to the temperature fitted by using average particle multiplicities. The $\sum_i \chi_i^2$ quoted in the picture is its value in the local minimum pointed by the solid line.

The fit is performed by minimising the following "χ² function:

$$\chi^2 = \sum_{p_T \text{bins}} \left[ \frac{\int_{\text{bin}} d_{p_T} n_{d_{p_T}}}{\sigma_i} - (\text{Exp.value})_i \right]^2$$ (7)

The errors $\sigma_i$ in the denominator of eq. (7) have been taken as the sum in quadrature of experimental errors and errors arising from the uncertainty on masses, widths and branching ratios of hadrons decaying into the examined hadron, according to the effective variance method (see ref. [14] for a detailed description). Once the normalisation parameter $A$ is fitted, a study of $\chi^2$ contour lines is performed in the $T - \langle u_T \rangle$ plane in order to look for possible local minima. Indeed, the presence of several local minima along a band is a quite general feature of the $\chi^2$ (see fig. (3)) owing to the strong anticorrelation between $T$ and $\langle u_T \rangle$. Hence, different solutions are possible for the $(T, \langle u_T \rangle)$ pair. In order to enforce universality of such parameters among different hadron species we have chosen a solution for each particle according to the following procedure:

1. Fit $T$ and $\langle u_T \rangle$ (which criterion led to singling out mainly four collision systems for hadronic collisions: $K^+ p$ at $\sqrt{s} = 11.5$ [22] and 21.7 GeV [23], $\pi^+ p$ at $\sqrt{s} = 21.7$ GeV [24] and pp $\sqrt{s} = 27.4$ GeV [25]. For each identified particle transverse momentum spectrum, three free parameters have been determined by fitting eq. (3) to the data: $T$, $\langle u_T \rangle$ and an overall normalisation parameter $A$. The fits have been performed in the variables $p_T$ or $p_T^2$ according to the available experimental spectrum; henceforth, we will denote by $dn/dp_T$ both $dn/dp_T$ and $dn/dp_T^2$. As we have mentioned before, overall multiplicities of hadrons in eq. (3) have been fixed to model values by using fit parameters in Table 1.

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Figure 4. Contour plot in the $T - \langle u_T \rangle$ plane of $\sum \chi_i^2$ for $K^+ p$ collisions $\sqrt{s} = 21.7$ GeV. The identified particles involved in the sum are $\pi^-$, $\pi^0$, $K_0^0$, $\rho^0$, $K^*$ and $f_2(1270)$. The two vertical lines define the 1-sigma band corresponding to the temperature fitted by using average particle multiplicities. The $\sum \chi_i^2$ quoted in the picture is its value in the local minimum pointed by the solid line.

Figure 5. Contour plot in the $T - \langle u_T \rangle$ plane of $\sum \chi_i^2$ for $pp$ collisions $\sqrt{s} = 27.4$ GeV. The identified particles involved in the sum are $\pi^+$, $\pi^-$, $\pi^0$, $K^+$, $K^-$, $\eta$, $\rho^0$, and $f_2(1270)$. The two vertical lines define the 1-sigma band corresponding to the temperature fitted by using average particle multiplicities. The $\sum \chi_i^2$ quoted in the picture is its value in the local minimum pointed by the solid line.

1. firstly, we have determined the contour lines of $\sum_{i=1}^N \chi_i^2$ as a function of $T$ and $\langle u_T \rangle$ by fixing all normalisation parameters to their fitted values, where $N$ is the number of different hadron species spectra for a given collision. The minimisation of the sum of all $\chi_i^2$'s amounts to fit a common value of $T$ and $\langle u_T \rangle$ for all measured hadrons.

2. secondly, among the different local minima we have chosen the one lying in or closest to the band defined by $T$ fitted with average multiplicities along with its error (see figs. (3,4,5)).

3. finally, all single particle spectrum fits have been repeated seeking a local minimum as close as possible to the previously determined $\sum_{i=1}^N \chi_i^2$ minimum.

It must be noted that this procedure aims at achieving the best agreement between temperatures determined by using different observables, namely particle yields and transverse momentum spectra. The $\sum_{i=1}^N \chi_i^2$ contour plots are shown in figs. (3,4,5) for the examined collisions. There is a good agreement between the $T$ obtained from multiplicities and the location of minima in the $T - \langle u_T \rangle$ plane. For $K^+ p$ and $\pi^+ p$ at $\sqrt{s} = 21.7$ GeV the absolute minimum lies nearly about the centre of the band demonstrating an intriguing correlation between the slope of transverse momentum distributions and the slope of average multiplicities as a function of mass. Moreover,
the \langle u_T \rangle values turn out to be small, thus justifying the series expansion described in Sect. 2. Conversely, in K^+p collisions at \( \sqrt{s} = 11.5 \) GeV a clear disagreement emerges for K^+p at \( \sqrt{s} = 11.5 \) GeV between differently determined temperatures (see fig. (6)) which can be explained with the inadequacy of our used parametrisation at low energy where exact \( p_T \) conservation, neglected in the canonical framework, is expected to play a significant role. The summary plots of the single particle fits are shown in figs. (8) while an example of fitted spectrum in pp collisions is shown in fig. (6). In general, very good fits (low \( \chi^2 \)'s) are obtained with good agreement between temperatures and average transverse boost velocities fitted for different identified hadrons, especially in pp collisions. Some discrepancies show up in K^+p and \( \pi^+p \) collisions where K^*s seem to have slopes steeper than expected. It must be noted that error estimates for \( T \) and \( \langle u_T \rangle \) are still very rough.

5. Results for \( e^+e^- \) collisions

We have also performed a similar analysis for \( e^+e^- \) collisions at centre-of-mass energies between 14 and 44 GeV. Compared with hadronic collisions, the analysis of \( e^+e^- \) data presents several additional difficulties. First of all, the production of heavy flavoured hadrons can not be neglected and the inclusion of many more decay channels is implied. A major issue is the fact that \( p_T \) is defined with respect to the so-called thrust or event axis. This direction is not known \textit{a priori}, unlike in hadronic collisions where \( p_T \) is
defined with respect to the beam line, and must be determined on an event by event basis. Hence, whatever the algorithm being used, this very fact introduces a bias on transverse momentum spectra because momenta projection are used to determine the event or thrust axis itself; often, this is done by just minimising the sum of particle $p_T$’s. In spite of this problem, we have repeated the same analysis performed for hadronic collisions by using measured charged tracks transverse momentum spectra \cite{26} with respect to thrust axis. The $\chi^2$ contour plots are shown in fig. \cite{11} along with $T$ bands obtained from multiplicity fits and resulting $\chi^2$’s minima. It can be seen that no minimum is found within the band for $e^+e^-$ collisions at $\sqrt{s} = 14$ GeV, a situation looking very similar that found in $K^+p$ collision at $\sqrt{s} = 11.5$ GeV. It is also found that fitted $\langle u_T \rangle$ increases quite rapidly with centre-of-mass energy reflecting the rise of average $p_T$ of gluon radiation. Since our parametrisation of transverse momentum spectra with an average transverse four-velocity $\langle u_T \rangle$ requires $\langle u_T \rangle$ to be $\ll 1$ (see discussion in Sect. 2), it is quite natural that fit quality deteriorates as centre-of-mass energy increases. This is indeed found already at 44 GeV where $\langle u_T \rangle > 0.55$ cannot longer be considered small. As energy increases, particle $p_T$ spectra become sensitive to the shape of clusters $u_T$ distribution (related in turn to radiated gluon $p_T$ spectrum) and not only to its mean value. Further on, they become dominated by the shape of gluon radiation spectrum whilst hadronisation $p_T$ plays the role of a small superimposed noise.
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Figure 10. Summary of the fits to transverse momentum spectra of identified particles in pp collisions at $\sqrt{s} = 27.4$ GeV. From top to bottom: temperatures (the dashed line is the value obtained by fitting multiplicities), average transverse four-velocity $\langle u_T \rangle$ and minimum $\chi^2$/dof. The errors on $T$ and $\langle u_T \rangle$ are still very rough estimates.

6. Discussion and conclusions

The analysis of transverse momentum spectra of several identified hadrons, accurately taking into account the effect of hadron decays, indicates that they can be well reproduced within the statistical hadronisation model for several high energy hadronic collisions. Furthermore, a good agreement is found between the temperatures estimated by fits to average particle multiplicities and those extracted by fitting transverse momentum spectra. This can be seen especially by comparing figs. 3, 4, 5; in fact, the location of the best $T$ value for transverse momentum spectra, determined by $\chi^2$’s lowest contour line, is apparently correlated to the best $T$ value defined by particle multiplicities as spotted by the vertical band. Indeed, those temperatures move together from $\sim 162$ MeV in pp collisions at $\sqrt{s} = 27.4$ GeV to $\sim 175$ MeV in $\pi^+ p$ and $K^+ p$ collisions at $\sqrt{s} = 21.7$ GeV. In our view, this finding is a strong indication in favour of one of the key predictions of the statistical hadronisation model, namely the existence of a close relationship between the laws governing the production of particles as a function of their mass and, for each particle species, the production as a function of momentum (measured in the rest frame of the cluster they belong to) at the hadronisation.

Besides this main conclusion, there are many remarks to be made. First of all, our calculations assumed a canonical framework and so they are expected to have only a limited validity at low centre-of-mass energies where the effect of exact
transverse momentum conservation must show up. Indeed, in $e^+e^-$ collisions at a centre-of-mass energy of 14 GeV and $K^+p$ collisions at 11.5 GeV, a clear disagreement is found between the temperatures determined in the two fashions (see fig. 5). Secondly, whilst the results obtained in pp collisions for different particles are in very good agreement among them, there are some significant discrepancies between different particles in $\pi^+p$ and $K^+p$ collisions, especially for $K^*$’s, which are not understood at the present. Finally, it is evident in the analysis of moderately high energy $e^+e^-$ collisions, at $\sqrt{s} = 44$ GeV, that our parametrisation of transverse momentum spectra as a function of an average transverse four-velocity alone, is not accurate enough. As the average $p_T$ of radiated gluons and, as a consequence, of hadronising clusters, rises, particle spectra are influenced more by the shape of clusters transverse four-velocity spectrum than by primordial hadronisation $p_T$ spectrum so that more accurate calculations are needed involving perturbative QCD to perform such an analysis. Finally, in the very high energy regime, $p_T$ spectra become insensitive to hadronisation and, as a consequence, it is no longer interesting to study its properties. In summary, the presently used parametrisation of $p_T$ spectra is found to work well only in a limited centre-of-mass energy range (roughly between 20 and 30 GeV). At lower energies, the study of $p_T$ is still of great interest in probing the statistical features of hadronisation but complex microcanonical calculations are required. At higher energy, hadronisation is swamped by hard QCD dynamics in $p_T$ spectra and all it can do is to add a little smearing.

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REFERENCES

1. E. Fermi, Progr. Theor. Phys. 5 (1950) 570.
2. R. Hagedorn, Nuovo Cimento 15 (1960) 434.
3. F. Becattini, Z. Phys. C 69 (1996) 485.
4. F. Becattini, Proc. of XXXIII Eloisatron Workshop on "Universality Features in Multihadron Production" (1996) 74.
5. F. Becattini and U. Heinz, Z. Phys. C 76 (1997) 269.
6. R. Hagedorn, "Hot Hadronic Matter: Theory and Experiment" (1994) 13.
7. R. Hagedorn, Riv. Nuovo Cimento 6 (1984) 1983.
8. J. Sollfrank, P. Koch and U. Heinz, Phys. Lett. B 252 (1990) 256.
9. J. Sollfrank, P. Koch and U. Heinz, Z. Phys. C 52 (1991) 593.
10. U. A. Wiedemann and U. Heinz, Phys. Rev. C 56 (1997) 3265.
11. T. Peitzmann, Nucl. Phys. A 638 (1998) 415c.
12. for a description of the model in the canonical approximation see F. Becattini, Proc. of XI Chris Engelbrecht summer school "Hadrons in dense matter and hadrosynthesis" (1998) 71.
13. G. Marchesini and B. Webber, Nucl. Phys. B 238 (1984) 1.
14. F. Becattini, L. Bellucci and G. Passaleva, in preparation.
15. F. Becattini, L. Bellucci, G. Passaleva, Proc. of XXIX International Symposium on Multiparticle Dynamics (1999) 220.
16. P. Chliapnikov, private communication.
17. M. Aguilar-Benitez et al., Z. Phys. C 50 (1991) 405.
18. The Durham reaction data database, http://durpdg.dur.ac.uk/HEPDATA/REAC
19. Particle Data Book, C. Caso et al, Eur. Phys. J. C 3 (1998) 1.
20. F. Becattini, M. Gazdzicki and J. Sollfrank, Eur. Phys. J. C 5 (1998) 143.
21. M. I. Gorenstein, M. Gazdzicki and W. Greiner, Phys. Lett. B 483 (2000) 60.
22. M. Barth et al., Z. Phys. C 22 (1984) 23
23. M. Barth et al., Nucl. Phys. B 191 (1981) 39.
24. M. Adamus et al., Z. Phys. C 35 (1987) 7
25. N.M. Agababyan et al., Z. Phys. C 41 (1989)
539
M. Adamus et al., Z. Phys. C 39 (1988) 311
I.V. Azhinenko et al., Z. Phys. C 46 (1990) 525.

24. M. Adamus et al., Z. Phys. C 35 (1987) 7
N.M. Agababyan et al., Z. Phys. C 46 (1990) 387
M. Adamus et al., Z. Phys. C 39 (1988) 311
I.V. Azhinenko et al., Z. Phys. C 46 (1990) 525.

25. M. Aguilar-Benitez et al., Z. Phys. C 50 (1991) 405; numerical values of $p_T$ spectra can be found in Durham reaction data database
A. Suzuki, Lett. Nuovo Cimento 24 (1979) 31.

26. M. Althoff et al., TASSO Coll., Z. Phys. C 22 (1984) 307
D. Bender et al., HRS Coll., Phys. Rev. D 31 (1985) 1
W. Braunschweig et al., TASSO Coll., Z. Phys. C 47 (1990) 187.