Controllable Coupling between Flux Qubits

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We propose an experimentally realizable method to control the coupling between two flux qubits. In our proposal, the bias fluxes are always fixed for these two inductively-coupled qubits. The detuning of these two qubits can be initially chosen to be sufficiently large, so that their initial interbit coupling is almost negligible. When a time-dependent magnetic flux (TDMF) is applied to one of the qubits, a well-chosen frequency of the TDMF can be used to compensate the initial detuning and to couple two qubits. This proposed method avoids fast changes of either qubit frequencies or the amplitudes of the bias magnetic fluxes through the qubit loops, and also offers a remarkable way to implement any logic gate as well as tomographically measure flux qubit states.

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Introduction.— Superconducting Josephson junction circuits currently provide one of the best qubit candidates, and experiments have been performed for charge, flux, phase, and charge-flux 1 qubits. Quantum coherent oscillations and conditional gate operations have been demonstrated using two-coupled superconducting charge qubits 2. Further, entangled macroscopic quantum states have been experimentally verified in systems of coupled flux 3, and phase 4, 5 qubits.

Quantum computing requires that the interaction between different qubits can be selectively switched on and off. This is an extremely difficult and important issue. Several schemes have been proposed to realize controllable couplings and local qubit operations. One is a controllable coupling by dynamically tuning the qubit frequencies, e.g., in Refs. 6, 7, 8, 9, 10. This tunable proposal requires that different qubits have the same frequencies (i.e., resonant interaction) when they are coupled. When they are decoupled, one of their frequencies should be suddenly changed by an external bias variable such that two coupled subsystems have a larger detuning (i.e., non-resonant interaction). The second approach uses switchable couplings in charge qubit circuits by changing the bias magnetic flux, e.g., in Refs. 11, 12, 13. In practice, the switching time of the magnetic flux should be less than the inverse single-qubit Josephson energy (less than a nanosecond), which is a challenge for present experiments. The third proposal requires additional subcircuits, e.g., in Refs. 14, 15, 16. These additional elements increase the complexity of the circuits and add additional uncontrollable noise.

To easily switch on and off the coupling among qubits is one of the most important open problems in quantum information hardware. Here, we propose a way to overcome this severe problem plaguing experiments. Specifically, we present a proposal on how to achieve a controllable interaction between flux qubits by virtue of time-dependent magnetic fluxes (TDMFs). Here, we make the same assumption as in the decoupling experiments 15, 16, 17, which require the two qubits to be in the large detuning regime. However, here, the two-qubit coupling and decoupling are controlled by the frequency (not the dc component) of the applied TDMF. So we completely avoid having to quickly change the bias magnetic flux—a severe problem faced by many previous proposals for superconducting qubits. Moreover, our proposal does not require additional subcircuits. These merits also make our proposal potentially useful for a variety of other types of qubit experiments, and could solve a central problem in this field.

Controllable Hamiltonian.— Two flux-qubits interact with each other through a mutual inductance $M$, as shown in Fig. 1. Each qubit loop contains three junctions, one of them has an area $\alpha$ times smaller than that of the two identical junctions. The larger junction in the $l$th qubit loop has Josephson energy $E_{J,l}$ ($l = 1, 2$). The gauge-invariant phases (of the two identical junctions and the smaller one) in the $l$th qubit are $\varphi_{l}^{(1)}$, $\varphi_{l}^{(2)}$, and $\varphi_{l}^{(3)}$. We assume that a static (dc) magnetic flux $\Phi_{l}^{(1)}$ and a time-dependent magnetic flux (TDMF) $\Phi_{l}^{(1)}(t) = A_{l} \cos(\omega_{l}^{(1)} t)$ are applied through the $l$th qubit. Using the phase constraint condition through the $l$th qubit loop $\sum_{i=1}^{3} \varphi_{i}^{(l)} + 2(2\pi \Phi_{l}^{(1)}/\Phi_{0}) + 2(2\pi \Phi_{l}^{(1)}(t)/\Phi_{0}) = 0$, the total Hamiltonian of the two qubits can be written as [12]

$$H = \sum_{l=1}^{2} (H_{I} + H_{D}^{(l)}) + \sum_{l\neq m=1}^{2} H_{Ilm} + H_{C} + H_{A}. \quad (1)$$

Here, the single qubit Hamiltonian is $H_{I} = P_{P,l}^{2}/2M_{P,l} + P_{Q,l}^{2}/2M_{Q,l} + 2E_{J,l}(1 - \cos \varphi_{l}^{(1)} \cos \varphi_{l}^{(1)}) + \alpha E_{J,l}(1 - \cos(2\varphi_{l}^{(1)} + 2\pi f_{l}))$ with the redefined phases $\varphi_{l}^{(1)} = (\varphi_{l}^{(1)} + \varphi_{l}^{(2)})/2$, $\varphi_{l}^{(3)} = (\varphi_{l}^{(1)} - \varphi_{l}^{(2)})/2$, and reduced bias magnetic flux $f_{l} = \Phi_{l}^{(1)}/\Phi_{0}$. The effective masses are $M_{Q,l} = 2(\Phi_{0}/2\pi)^{2}C_{1,l}$ and $M_{P,l} = (1 + 2\alpha)M_{Q,l}$, which correspond to the effective momenta $P_{Q,l} = -i\partial/\partial \varphi_{l}^{(3)}$ and $P_{P,l} = -i\partial/\partial \varphi_{l}^{(1)}$. The capacitances in the $l$th qubit loop satisfy the condition $C_{1}^{(l)} = C_{2}^{(l)} = C_{3,l}$ and $C_{3}^{(l)} = \alpha C_{1,l}$.
The Hamiltonian $H_D^{(l)} = -(A_l/2) \left( I^{(l)}_3 + i \beta P_{Pl} \right) e^{-i \omega(l)t} + H.c.$ represents the interaction between the $l$th qubit and its TDMF. Here, the parameter $\beta = 2 \pi \alpha \omega^{(l)} / \left| \Phi_0 (1 + 2 \alpha) \right|$, and $I^{(l)}_3 = -(2 \pi \alpha E_{3,1} / \Phi_0) \sin(2 \varphi^{(l)} + 2 \pi f_1)$ is the supercurrent through the smaller junction of the $l$th qubit without applying the TDMF. So a TDMF-controlled single-qubit rotation can be realized by the Hamiltonian $H_D^{(l)}$. The qubit-qubit interaction $H_m$, controlled by one of the TDMFs $(\Phi^{(1)}_c(t) or \Phi^{(2)}_c(t))$, can be described by $H_m = -\beta \bar{m} e^{-i \omega l t} \cos(2 \varphi^{(m)} + 2 \pi f_1) + H.c.$, where $\beta = M (2 \pi / \Phi_0)^2 (A_l C_{l,1} E_{1,3,1} / 2 C_{1,1,1})$. and $I^{(m)} = C_m \sum \iota_i (I^{(m)}_3 / C^{(m)}_i) \sin \varphi^{(m)}_i$ is the qubit loop-current of the $m$th qubit without an applied TDMF, and $C^{-1}_m = \sum \iota_i (1 - C^{(m)}_i)$. However, the qubit-qubit interaction $H_C$, controlled by simultaneously applying two TDMFs $(\Phi^{(1)}_c(t) and \Phi^{(2)}_c(t))$ through the two qubits, respectively, is $H_C = B \prod_{l=1}^2 \Phi^{(l)}_c(t) \cos(2 \varphi^{(l)} + 2 \pi f_1) with B = M (2 \pi / \Phi_0)^4 (C_{l,1} C_{l,2} E_{1,3,1} E_{1,2,3})$. The Hamiltonian $H_A = M I^{(1)} I^{(2)}$ denotes an always-on interaction between the two fluxes, without applying the TDMF.

In the two-qubit basis $\{ |e_i \rangle, |g_i \rangle \} \otimes \{ |e_2 \rangle, |g_2 \rangle \}$, where $|g_i \rangle$ and $|e_i \rangle$ are the two lowest eigenstates (ground and first excited states) of $H_l$, Eq. (1) can become

$$
H = \sum_{l=1}^2 \frac{\hbar}{2} \omega_l \sigma_z^{(l)} - \sum_{l=1}^2 \left( \kappa_{l+} \sigma_+^{(l)} e^{-i \omega^{(l)} t} + H.c. \right) - \sum_{l \neq m=1}^2 \left( \Omega_{lm}^{(1)} \sigma_+^{(l)} \sigma_-^{(m)} + H.c. \right) \left( e^{i \omega^{(l)} t} + e^{-i \omega^{(l)} t} \right) - \sum_{l \neq m=1}^2 \left( \Omega_{lm}^{(2)} \sigma_+^{(l)} \sigma_+^{(m)} e^{-i \omega^{(l)} t} + H.c. \right) + \left( \lambda_1 \sigma_+^{(1)} \sigma_-^{(2)} + \lambda_2 \sigma_+^{(2)} \sigma_+^{(2)} + H.c. \right). \tag{2}
$$

Here, the terms $\kappa^*, \sigma^{(l)} \sigma^{(l)} e^{-i \omega^{(l)} t}$ and $\Omega_{lm}^{(2)} \sigma_+^{(l)} \sigma_+^{(m)} e^{-i \omega^{(l)} t}$, as well as their complex conjugates, have been neglected by considering energy conservation. The operators of the $l$th qubit are defined as $\sigma_{l}^{(z)} = |e_i \rangle \langle e_i| - |g_i \rangle \langle g_i|$, $\sigma_{l}^{+} = |e_i \rangle \langle g_i|$, and $\sigma_{l}^{-} = |g_i \rangle \langle e_i|$. The qubit frequency $\omega_l$ in Eq. (2) can be expressed as $\omega_l = \sqrt{\Omega^{(1)}_l (I^{(1)}_3 - \Phi_0 / 2)^2 + t_l^2}$ with the loop-current $I^{(1)}_3$ and the bias flux $\Phi^{(l)}_c(t)$. Here, the parameter $t_l$ denotes the tunnel coupling between two wells in the $l$th qubit. The controllable coupling constants are $\kappa_l = A_l (\lambda_l (I^{(l)}_3 + i \beta P_{Pl}) / g_i) / 2$, $\Omega_{lm}^{(1)} = A_l \beta (\lambda_l (g_i, g_m) I^{(m)} \cos(2 \varphi^{(m)} + 2 \pi f_1) g_i, e_m) / 2$, and $\Omega_{lm}^{(2)} = A_l \beta (\lambda_l (e_l, e_m) I^{(m)} \cos(2 \varphi^{(m)} + 2 \pi f_1) g_i, g_m) / 2$. The hard-to-control parameters are $\lambda_1 = M (e_1, g_2) (I^{(1)}_3 (g_1, e_2)$, and $\lambda_2 = M (e_1, e_2) (I^{(1)}_3 (g_1, g_2)$. It is not difficult to derive that $\lambda_l = \Omega_{lm}^{(1)} = \Omega_{lm}^{(2)} = 0$ when no TDMFs. Then, since both bias magnetic fluxes $f_1$ are near 1/2 (the optimal point is at $f_1 = 1/2$), the Hamiltonian $H_C$ can revert to the case in Refs. [3, 4], where the Pauli operators are defined by the states of the two potential wells.

**Decoupling mechanism and logic gates.**—We assume that the two qubits work at the fixed frequencies $\omega_1$ and $\omega_2$, which mean that their reduced bias magnetic fluxes $f_1$ and $f_2$ remain fixed. If the detuning $\Delta$ is initially chosen to be sufficiently large (such that it satisfies the condition: $| \Delta | \gg | \lambda_1 | / \hbar = | \lambda_2 | / \hbar = | \lambda | / \hbar$), then the two qubits can be approximately treated as two decoupled subsystems when the TDMFs are not applied.

By applying the TDMF with the frequency-matching condition $\omega^{(l)}_c = \omega_l$, we can easily derive from Eq. (2) that any single-qubit operation of the $l$th qubit can be performed via the dynamical evolution $U^{(l)} (\theta_l, \phi_l) = \exp [i \theta_l (e^{-i \phi_l} \sigma^{(l)} + H.c.)]$. Here, $\theta_l = | \lambda_l | / \tau$ and $\phi_l$ depends on the Rabi frequency $| \lambda_l | / \hbar$ and duration $\tau$; $\phi_l$ is related to the TDMF phase applied to the $l$th qubit. For example, $\pi/2$ rotations of the $l$th qubit around the $x$ or $y$ axes can be performed by using $U^{(l)} (\theta_l, \phi_l)$, with $\tau = h \pi / | \lambda_l |$ and $\phi_l = \pi \alpha / \pi$. Unless specified otherwise, hereafter, we work in the interaction picture, and all non-resonant terms have been neglected because their contributions to the transitions between different states are negligibly small.

To couple two qubits with the assistance of the TDMF: i) a TDMF needs to be applied through one of the qubits, and its frequency should be equal to the detuning (or summation) of the two qubit frequencies; ii) the reduced bias flux $| \lambda |$ on the qubit, which is addressed by the TDMF, should be near but not equal to 1/2. Without loss of generality, below, the TDMF is assumed to be always applied through the first qubit, so the bias for the first qubit is $f_1 = 1/2 \pm \epsilon$, with small $\epsilon$; however, the bias for the second qubit is taken as $f_2 = 1/2$.

Considering two new frequency-matching conditions in Eq. (2), produces different kinds of Hamiltonians for implementing two-qubit operations with the assistance of TDMF: One is $H_1 = \Omega_{12}^{(1)} \sigma_+^{(1)} \sigma_-^{(2)} + H.c.$, with the condition: $\omega_1 - \omega_2 \pm \omega_c^{(1)} = 0$. Here, the sign is possible (negative) when $\Delta < 0 (\Delta > 0)$. Another one is $H_2 = \Omega_{12}^{(2)} \sigma_+^{(1)} \sigma_-^{(2)} + H.c.$, when the frequencies satisfy the condition: $\omega_1 + \omega_2 - \omega_c^{(1)} = 0$. Using the Hamiltonian $H_1$ and $H_2$, two qubit gates can be implemented. For example, a TDMF is applied through...
for the first qubit is given in Eq. (3) by setting

\[ \psi_+ = \frac{1}{\sqrt{2}} (|g_1, e_1 \rangle \pm |g_2, e_2 \rangle) \]

where \( \psi_+ \) is the ground state of a flux qubit in an ISW AP gate \[ 19 \], with the phase \( \theta \) about several hundred MHz, e.g. \( |\theta|/\hbar \sim 0.4 \) GHz in Refs. 3, 4. the loop-current operator \( \hat{I} \) is set to \( \pi \) by an applied TDMF. Then an ISW AP gate \[ 19 \], denoted by \( U_{IS} \), can be realized by \( \hat{H}_{XY} \) with an evolution time \( t = \pi \hbar/(2|\Omega_{12}|) \). The CNOT gate can be constructed by combining the ISW gate with a few single-qubit operations.

Experimentally, it is found that the always-on coupling strength \( |\theta|/\hbar \Delta \) is about several hundred MHz, e.g. \( |\theta|/\hbar \sim 10 \) GHz. This means that the ratio \( |\theta|/\hbar \Delta \) cannot be infinitesimally small, and the always-on interaction needs to be considered when all TDMFs are switched off. Up to the first order in \( |\theta|/\hbar \Delta \), the effect of the always-on interaction, without the TDMF, can be described by the effective Hamiltonian \( \hat{H}_{IS} = (|\theta|^2/\hbar \Delta) \hat{e}_1 \hat{e}_1 \otimes |g_2, e_2 \rangle \langle g_1, e_2 | + \text{h.c.} \) with the phase \( \theta \). It is possible to write the Bell states \( |\psi_\pm \rangle = ( |g_1, e_2 \rangle \pm |e_1, g_2 \rangle)/\sqrt{2} \) can be generated with \( t_2 = \hbar \pi/(4|\Omega_{12}|) \) by setting the phase \( \theta' \) of \( \Omega_{12} \) as \( \pi/2 \) or \( 3\pi/2 \). Similarly, if both qubits are in the ground states \( |g_1 \rangle \) and \( |g_2 \rangle \), then another two Bell states \( |\psi_\pm \rangle = ( |g_1, g_2 \rangle \pm |e_1, e_2 \rangle)/\sqrt{2} \) can also be obtained with \( t_2 = \hbar \pi/(4|\Omega_{12}|) \) by setting the phase \( \theta' \) of \( \Omega_{12} \) as \( \pi/2 \) or \( 3\pi/2 \) through the Hamiltonian \( \hat{H}_{IS} \).

State tomography allows us to experimentally determine a quantum state \[ 20 \]. Qubit state tomography can be implemented by measuring the supercurrent through the qubit loop, which is inductively coupled to, e.g., a dc SQUID magnetometer or high-quality tank circuit. For the qubit in the qubit basis \( \{ |g_i \rangle, |e_i \rangle \} \), its loop-current operator can be written \[ 21 \] as

\[ \hat{I}^{(i)} \equiv \hat{I}_x^{(i)} = a_i \hat{\sigma}_x^{(i)} + b_i |e_i \rangle \langle g_i + b_i^* |g_i \rangle \langle e_i | \]

where \( a_i = \langle e_i |\hat{I}^{(i)}|e_i \rangle \) and \( b_i = \langle e_i |\hat{I}^{(i)}|g_i \rangle \), then the bias \( f_1 \) is near (but not equal to) 1/2. However, at the optimal point \( f_1 = 1/2 \), the supercurrent operator in Eq. \[ 21 \] can be reduced to \( \hat{I}^{(i)} \equiv \hat{I}_x^{(i)} = b_i \hat{\sigma}_x^{(i)} \), with a real number \( b_i \) and the Pauli operator \( \hat{\sigma}_x^{(i)} = |e_i \rangle \langle g_i + |g_i \rangle \langle e_i | \).

If the simultaneous joint measurement of two qubits can be performed in flux qubit circuits as in phase circuits, then single qubit operations are enough to realize the fifteen different measurements \[ 22 \] on the two-qubit states \( \rho = (1/4) \sum_{i,j} r_{ij} \hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_j^{(2)} \), with the Pauli operators \( \hat{\sigma}_i^{(l)} \) (\( i, j = x, y, z \) and \( l = 1, 2 \)) and the identity operator \( \hat{\sigma}_0^{(l)} \), where \( r_{90} \) is set to 1 by normalization. The loop-current operator for the first qubit is given in Eq. \[ 21 \] by setting \( l = 1 \) due to the assumption \( f_1 \neq 1/2 \), but it is reduced to \( \hat{I}_x^{(2)} = b_2 \hat{\sigma}_x^{(2)} \) for the second qubit with \( f_2 = 1/2 \). So the fifteen measurements on state \( \rho \) are given as \( \hat{I}_x^{(1)} \) and \( b_2 \hat{\sigma}_x^{(2)} \) (denoted as single-qubit measurements), as well as \( b_2 I_x^{(1)} \otimes \hat{\sigma}_x^{(2)} \) (called two-qubit or joint measurements), with \( i, j = x, y, z \), \( I_y^{(1)} = Y_1^{\dagger} I_1^{(1)} Y_1 \), and \( I_z^{(1)} = Z_1^{\dagger} I_1^{(1)} Z_1 \). It is clear that three measurements \( (I_x^{(1)}, b_2 \sigma_x^{(2)}, \text{and} b_2 I_x^{(1)} \otimes \sigma_x^{(2)}) \) on the input two-qubit state \( \rho \) can be directly performed. Other twelve measurements can be equivalently obtained by measuring \( (I_x^{(2)}, b_2 \sigma_x^{(2)}, \text{or both of them at the same time}) \) on the rotated state \( \rho \). For example, \( \pi/2 \) single-qubit rotations \( Y_1 \) around the y axis for the first qubit and \( Z_2 \) around the z axis for the second qubit are simultaneously performed on the state \( \rho \), then the measurement \( b_2 I_x^{(1)} \otimes \sigma_x^{(2)} \) on the rotated state \( Y_1 Z_2 \rho Z_2^\dagger Y_1^\dagger \) is equivalent to the measurement \( b_2 I_x^{(1)} \otimes \sigma_y^{(2)} \) on the original state \( \rho \). Similarly, other joint measurements can also be obtained. Finally, for the fifteen measured results, we solve a set of equations for the parameters \( r_{ij} \), and a two-qubit state is reconstructed.

If only a single-qubit measurement can be made at a time, besides single-qubit measurements mentioned above, a suitable nonlocal two-qubit operation \[ 22 \] is required to obtain the coefficients (e.g., \( r_{xy} \)) of the nine joint measurements on the state \( \rho \). Here, this is an ISW gate \[ 19 \], which can be implemented as described above. For example, if an operation \( U_{IS} \) is made on the input state \( \rho \), then the loop-current of the second qubit should be \( (I_x^{(2)} = \text{Tr}(U_{IS} \rho U_{IS}^\dagger I_x^{(2)}) = -b_2 \text{Tr}(\rho \sigma_y^{(1)} \otimes \sigma_y^{(2)}) = -b_2 r_{yx} \), and the coefficient \( r_{xy} \) is determined. Combining the ISW gate and single-qubit operations for two qubits, the coefficients of other eight joint measurements can also be determined by only measuring the loop-current \( I_x^{(2)} \).

Tomographically measured states are different for completely decoupled (CD) and large detuning (LD) two-qubit systems after two-qubit states are created, if we consider a duration \( t \) before measuring the generated two-qubit states. As an example, a schematic representation of a Bell state \( |\psi_\pm \rangle \) is given in Fig. 2 for the above two cases. There is only the real part for the reconstructed state \( \rho = \langle \psi_+ | \psi_+ \rangle \) in the CD system, shown in Fig. 2(a). However, due to the effect of \( \hat{H}_E \) for the LD system, the reconstructed state \( \chi = e^{-i \hat{H}_E t/\hbar} |\psi_+ \rangle \langle e^{i \hat{H}_E t/\hbar} \) includes both real and imaginary parts, shown in Fig. 2(b) and 2(c), respectively. In Fig. 2 we consider a longer duration \( t \sim 10^{-9} \) s; the detuning and the coupling constant are, e.g., \( \Delta \sim 5 \) GHz, and \( |\theta|/\hbar \sim 0.4 \) GHz. So if we consider the always-on interaction effect, the relative error with these parameters is \( \sim 0.08 \) for the non-diagonal parts of the reconstructed CD state \( |\psi_+ \rangle \). Here, the qubit free-evolution is neglected. In practice, considering unavoidable environmental effects and statistical errors, the experimentally measured data should be further optimized by other methods \[ 22 \].

Conclusions.— The controllable coupling of two inductively-coupled flux qubits can be realized, when the large detuning condition is satisfied, by the frequency of
the TDMF matching/mismatching to the detuning (or summation) of the two qubit frequencies; not by changing qubit biases (e.g., as in Ref. [7]). Our proposal is also different from the coupling/decoupling method by using dressed states [23]. We emphasize that the deviation $\epsilon$ from the optimal point 1/2 for the reduced bias $f_1$ through the first qubit will make the decoherence time $T_2$ short. However, our proposal can work for a small deviation, e.g., $\epsilon \sim 10^{-4}$, in which $T_2 \gtrsim 20$ ns in Refs. [24] or $T_2 \gtrsim 100$ ns with spin-echo signals [24]. At this point, the qubit coupling constant $\Omega^{(i)}_1$ ($i = 1, 2$) can reach [13] about several hundred MHz. Based on numerical estimates [24], the longest time for the single-qubit operation $Z_l$ is less than 5 ns, so the tomographic measurements can be performed within $T_2$. Our proposal can be scalable to a chain of many inductively coupled flux qubits, if all of qubits satisfy the large detuning condition. We need to note: i) we can use one LC circuit as a common information bus to couple many qubits, with the qubit-bus coupling controlled by externally variable frequencies [25]; ii) this circuit can be modified to work at the optimal point; iii) this method using frequency-controlled couplings can be applied to control one-junction flux qubits. It can also be modified to control phase, charge-flux, and charge qubits.

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