Effects of doping on thermally excited quasiparticles in the high-$T_c$ superconducting state

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The physical properties of low energy superconducting quasiparticles in high-$T_c$ superconductors are examined using magnetic penetration depth and specific heat experimental data. We find that the low energy density of states of quasiparticles of La$_{2-x}$Sr$_x$CuO$_4$ scales with $(x-x_c)/T_c$ to the leading order approximation, where $x_c$ is the critical doping concentration below which $T_c = 0$. The linear temperature term of the superfluid density is renormalized by quasiparticle interactions and the renormalization factor times the Fermi velocity is found to be doping independent.

In high-$T_c$ superconductors (HTS), the low energy density of states (DOS) is linear due to the $d_{x^2−y^2}$-wave pairing symmetry. This linear DOS leads to a linear in-plane superfluid density (which is proportional to the inverse square of the magnetic penetration depth $\lambda_{ab}$) and a quadratic electronic specific heat at low temperatures. The experimental observations of these power-law temperature dependences provided some of the early evidence for the unconventional pairing symmetry and lent support to the Fermi liquid description of the high-$T_c$ superconducting state. Exploring the physical properties of the low energy quasiparticle excitations is of fundamental importance for understanding the high-$T_c$ mechanism. A central issue which is currently under debate is the nature of quasiparticle interactions and their effect on the physics of the superconducting state. Recently Mesot et al. calculated the slope of the superfluid stiffness using parameters obtained from angular resolved photoemission (ARPES) measurements for Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (BSCCO). They found that the renormalization factor to the superfluid stiffness is doping dependent and about a factor of 2 to 3 smaller than that for non-interacting quasiparticle systems.

In this paper we provide further evidence for the existence of strong quasiparticle interaction in the superconducting state and present a detailed analysis for the low energy DOS and other fundamental parameters of HTS. We calculate the doping dependence of these parameters using high quality penetration depth and specific heat data and discuss an interesting correlation between an energy scale derived from the low temperature superfluid response and the normal state pseudogap.

Let us first consider the low energy DOS of high-$T_c$ quasiparticles $N(\omega)$. If we denote the linear coefficient of $N(\omega)$ by $\eta$, then at low energies $N(\omega)$ is given by

$$N(\omega) \approx \eta \omega.$$  (1)

Since the low temperature penetration depth and specific heat are governed by thermally excited quasiparticles, it can be shown that the slopes of $\lambda^2$ and $\gamma$ at zero temperature are given by

$$\frac{d\lambda_{ab}^{-2}}{dT}|_{T\to 0} = (4\pi \ln 2)\eta k_B \left(\frac{e\beta v_F}{c}\right)^2,$$  (2)

$$\frac{d\gamma}{dT}|_{T\to 0} = 5.4\eta k_B^3,$$  (3)

where $\gamma = C_v/T$ is the specific heat coefficient and $v_F$ is the Fermi velocity. $\beta$ is a renormalization factor to the paramagnetic term in the superfluid density due to quasiparticle interactions or vertex corrections. For non-interacting quasiparticles $\beta = 1$, and Eq. (2) is the standard BCS mean-field result. The above equations show that from experimental data of $\lambda_{ab}$ and $C_v$, we can determine the values of two important parameters: the linear coefficient of DOS $\eta$, and the product of the renormalization constant and Fermi velocity $\beta v_F$.

Figure 1 shows the experimental result of $T_c d\gamma/dT$ at $T = 0K$, as a function of doping for La$_{2-x}$Sr$_x$CuO$_4$ (LSCO). Note here we plot the product $T_c d\gamma/dT$ rather than $d\gamma/dT$ since in an ideal BCS superconductor with $d$-wave pairing, $T_c d\gamma/dT|_{(0K)}$ is proportional to the charge concentration. We find that $T_c d\gamma/dT|_{(0K)}$ increases monotonically with doping and the slope is smaller in the underdoped regime than in the overdoped one. The value of $\eta$ can be obtained from the data shown in Fig. 1. In the whole doping regime, we find that $\eta$ can be approximately fitted by

$$\eta \approx \eta_1 (x - x_c) + \eta_2 (x - x_c)^2,$$  (4)

where $x_c \approx 0.058$ is approximately equal to the critical doping concentration below which $T_c = 0$, $\eta_1 = 13$ $mJ/\left(K^2 k_B^4 mol\right)$ and $\eta_2 = 87$ $mJ/\left(K^2 k_B^6 mol\right)$. For other high-$T_c$ materials, the low temperature data for $\gamma$ generally contain a significant contribution from magnetic impurities and it is difficult, especially in the underdoped...
regime, to determine accurately the value of $d\gamma/dT(0K)$. However, from the data available we find that, within experimental errors, the doping dependence of $\eta$ is similar to that shown in Fig. 1.

Figure 2 shows the extrapolated experimental results of $T_c d\gamma_{ab}/dT$ at 0K as a function of doping, $x$, for LSCO and YBa$_2$Cu$_{3-x}$O$_6$ (YBCO). The magnetic penetration depth data for LSCO and the specific heat data of Loram et al shown in Fig. 1 were obtained from samples of the same batch. The similar doping dependence in $T_c d\lambda_{ab}^2/dT$ and $T_c d\gamma/dT$ indicates that it is indeed the thermally excited quasiparticles which are responsible for the low temperature thermodynamic response of HTS. Two sets of penetration depth data are shown for YBCO. One set comes from the $\chi$-susceptibility measurements of grain-aligned YBCO and the other from microwave measurements of detwinned YBCO single crystals. The $\chi$-susceptibility technique measures the effective in-plane penetration depth, $\lambda_{ab}$, whereas the microwave experiment measures the penetration depth along two principal axes, $\lambda_a$ and $\lambda_b$. In order to compare these two sets of data, we have converted the single crystal data to the effective in-plane penetration depth by assuming $\lambda_{ab}^2 = \lambda_a \lambda_b$. The agreement between the two YBCO data sets is striking considering the difference in the type of samples and measurement techniques used by the two groups. For both LSCO and YBCO we find that within experimental errors $T_c d\lambda_{ab}^2/dT$ increases almost linearly with doping.

Using the penetration depth and specific heat data for LSCO we have estimated $\beta v_F$ from the ratio of $d\lambda_{ab}^{-2}/dT$ and $d\gamma/dT$. As shown in Fig. 3, $\beta v_F$ is almost doping independent and of the order $5 \times 10^6 \text{cm/sec}$. If we assume that the Fermi velocity of LSCO is equal to the Fermi velocity of BSCCO, i.e. $v_F = 2.5 \times 10^5 \text{cm/sec}$, Fig. 3 suggests that $\beta$ is approximately equal to 0.2, a value much smaller than that for non-interacting quasiparticles where $\beta = 1$. Obviously this estimate for $\beta$ is crude since the Fermi velocity for LSCO may not be the same as that for BSCCO. Nevertheless, it indicates that the quasiparticle interaction is quite strong, in qualitative agreement with the analysis of Mesot et al for BSCCO. Furthermore, if $v_F$ in LSCO is doping independent as in BSCCO the result in Fig. 3 suggests that $\beta$ is also doping independent, which disagrees with the conclusions of Mesot et al. for BSCCO. We see no apparent physical reason why the doping dependence of $\beta v_F$ in these two high-$T_c$ systems should be different. It is likely that the difference is just due to the experimental uncertainty. To clarify this problem, more measurements of the doping dependence of the Fermi velocity and low energy DOS of quasiparticles for
both systems are required.

Since $\beta_{VF}$ for LSCO is approximately doping independent, the above results indicate that in the underdoped regime $\lambda_{ab}^{-2}(T) - \lambda_{ab}^{-2}(0) \sim (x - x_c)T/T_c$. Experimentally, it was found that $\lambda_{ab}^{-2}(0)$ is approximately proportional to $T_c[2]$, this therefore gives

$$\frac{\lambda_{ab}^{-2}(T)}{\lambda_{ab}^{-2}(0)} \approx 1 - \alpha \frac{x - x_c}{T_c^2} T,$$  \hspace{1cm} (5)$$

where $\alpha$ is a doping independent constant. This equation holds only at low temperatures and low doping. However, if we extrapolate $\lambda_{ab}^{-2}(T)$ to high temperatures using this equation, we find that $\lambda_{ab}^{-2}(T)$ becomes zero at a temperature $T_0 \sim T_c^2/(x - x_c)$. It is interesting to note that $T_0$ has approximately the same doping dependence and order of magnitude as the temperature scale $T^*$ of the normal state gap obtained from tunnelling, ARPES, and other measurements[2]. If we interpret $T_0$ as the energy scale at which Cooper pairs begin to form and the difference between $T_0$ and $T_c$ is due to the pair phase fluctuations this result seems to be consistent with the widely discussed phase fluctuation picture[2]. However, as both the XY order parameter fluctuations and the gauge fluctuations are important in the critical phase transition regime, this interpretation is not unique. Nevertheless, it suggests that from low temperature measurements of the superconducting state, one can also obtain information on phase fluctuations in the normal state.

In conclusion, using low temperature experimental data of the magnetic penetration depth and electronic specific heat of HTS, we have estimated the effect of doping on the low energy DOS of quasiparticles and $\beta_{VF}$. Both $T_c d\gamma/dT$ and $T_c d\lambda_{ab}^{-2}/dT$ are found to increase with increasing doping whereas $\beta_{VF}$ is doping independent. The low temperature superfluid density $\lambda_{ab}^{-2}(T)$ extrapolates to zero at a temperature $T_0$ which has approximately the same doping dependence and order of magnitude as the onset temperature of the normal state gap.

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\begin{figure}[h]
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\includegraphics[width=0.5\textwidth]{figure3.png}
\caption{$\beta_{VF}$ estimated from penetration depth and electronic specific heat measurements for La$_{2-x}$Sr$_x$CuO$_4$.}
\end{figure}

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