An Integrated Inventory and Order Pickup Model in the MVSB System considering Capacities of Vendors

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Abstract. To reduce inventory and pick-up costs, a mutually beneficial collaboration between suppliers and manufacturers is the basis for developing an integrated inventory and order pick-up model, known as Inbound Inventory Routing Problems (IIRP). Unfortunately, the IIRP model developed so far has not been able to accommodate several manufacturing industry problems, including vendors' limited capacities. Concerning these conditions, this study aims to develop an IIRP model in the Multi-Vendor Single-Buyer (MVSB) systems considering suppliers' limited capacity. The total relevant cost consisting of set-up costs, ordering costs, holding costs, and the pick-up cost is kept to a minimum. The IIRP model developed in this study is classified as mixed-integer non-linear programming (MINLP), which can be solved using an analytic, heuristic, or metaheuristic approach. Model testing was carried out on the MVSB system by developing three numerical examples. Each of them involved three suppliers of similar parts, then solved using an analytic approach. The results obtained show that the three numerical examples' global optimum solutions were found. Based on these numerical models and criteria, it is recommended that parts pick-up from suppliers be carried out as often as possible in small lot sizes, consistent with the JIT procurement.

1. Introduction

To produce products according to consumer demand, manufacturing industries (such as automotive, heavy equipment, machine tools, and electronics) provide various types of parts in two ways, namely making in-house or buying (outsourcing). Global competition and the increasing demand often encouraged companies to outsource functions previously made [1]. In many industries, outsourcing ratios have reached 60 percent or more [2]. Companies tended to outsource several required parts because they chose to focus on core competencies [3].

The trend towards outsourcing encourages collaboration between suppliers, manufacturers, and other parties (including freight forwarders) in the supply chain. As a result, manufacturers prefer to leave traditional, self-serving, and short-term purchasing strategies. Instead, manufacturers employ a purchasing approach in a just in time (JIT) environment, known as JIT procurement, to procure parts as often as possible in small lot sizes [4, 5]. There was a trade-off between the cost of inventory and transportation costs. Small inventory costs can be achieved with small lot sizes, with the risk of incurring high transportation costs due to high pick-up frequency. Conversely, small transportation costs could be achieved with large pick-up lot sizes, potentially resulting in enormous inventory costs [6]. For JIT procurement to provide...
long-term benefits, suppliers’ determination of lot size and delivery policies should consider both parties’ interests simultaneously [6–8].

Determination of lot sizes that integrates all relevant costs between suppliers and buyers to minimize the total system costs is known as the Joint Economic Lot Size (JELS) [9,10]. Based on the structure of supplier and buyer relationships, JELS is divided into four types, namely single-vendor single-buyer (SVSB), single-vendor multi-buyer (SVMB), multi-vendor single-buyer (MVSB), and multi-vendor multi-buyer (MVMB) [10–12]. Of the four systems mentioned, this study focuses on the MVSB system because the manufacturing industry generally involves many suppliers to provide various parts. Although the supply of parts for manufacturers involves many suppliers as found in the manufacturing industry practice, from 155 papers related to vendor-buyer integration, it turns out that papers that discussed the JELS model in the MVSB system were still limited [12].

Several inventory models in the MVSB system proposed an integrated pick-up consolidation strategy to reduce pick-up costs [3,6,13,14]. Pick-up consolidation is carried out by supplying large capacity vehicles by manufacturers to pick up small lot size parts from each supplier and then collectively brought to the manufacturer. One form of pick-up consolidation strategy is the milk-run, which has been successfully implemented by Toyota and various other companies [15]. For the milk-run transportation mode, the vehicle visited several suppliers’ locations in a specific route to minimize the pick-up costs.

An integrated inventory and pick-up model using a milk-run transportation mode known as the inbound inventory routing problem (IIRP) has been proposed by several researchers [4,16–20]. However, the IIRP model in the various studies mentioned above does not consider suppliers’ limited capacity to be found in the manufacturing industry. Each supplier has a limited capacity, which results in un-meet demand of manufacturers. For example, in the automotive industry, one type of part should be purchased from 2 to 3 suppliers due to its limited capacity. Thus, this research focus on integrating inventory and pick-up that considered the limited capacity of suppliers.

The problem in this research is how to model the integration of inventory and pick-up orders using milk-run transportation mode considering the limited capacity of suppliers. The criteria used to minimize relevant costs consist of set-up costs, ordering costs, holding costs, and pick-up costs. This study aims to produce an integrated inventory and order pick-up model using milk-run transportation mode on the MVSB system that considers suppliers’ limited capacity to minimize the total relevant costs.

2. Formulation of the Problem
In this study, the MVSB system consists of several suppliers (vendors) who provide a single buyer. Each supplier acts as a manufacturer because it must carry out production activities to meet the parts demand for a single buyer. The purchaser is also the manufacturer that needs to assembly the parts purchased from the supplier. The proposed model used some decision variables, including the typical pick-up cycle time, pick-up frequency, pick-up batch size, production lot size for each supplier, number of vehicles, and vehicle routes. The model minimizes the total relevant costs of set-up costs, ordering costs, saving costs, and pick-up costs.

2.1. Assumptions and notations
The assumptions and notations that limit the problem need clearly defined.

(i) Each supplier offers only one kind of part to the assembler.
(ii) Production rates of parts at all suppliers and the demand rate of parts at assemblies are constant.
(iii) Each supplier’s production rate is lower than the buyer’s demand rate.
(iv) Shortage and backlog are not allowed.
(v) The system is assumed not to produce defective parts.
(vi) All suppliers use a JIT delivery policy with the same delivery cycle time.
(vii) The pick-up of parts from suppliers to manufacturers/assemblers involves third-party logistics (TPL) services, using a milk-run transportation mechanism.
(viii) One vehicle (truck) handles one transportation route.

| Parameters                          | Variables                                      |
|-------------------------------------|------------------------------------------------|
| Demand rate for part k (k = 1, ..., k), (unit/year) | Common pickup cycle time for n supplier (in year) |
| Production rate of supplier i for part k, (i = 1, ..., n; k = 1, ..., k), (unit/year) | Production lot size of supplier i to produce part k (unit/batch) = m_q[k] |
| Holding cost of parts k at manufacturer’s side ($.unit/year) | A binary variable set to 1 if supplier j on route v visited immediately after supplier and 0 otherwise, v = 1, 2, ..., V |
| Holding cost of parts k at the supplier i ($.unit/year) | The number of routes (i.e., the number of trucks used) |
| Fixed transportation cost for the delivery truck ($.truck) | A binary variable set to 1 if the route is used and 0 otherwise |
| Ordering cost of parts k from supplier i ($.order) | A binary variable set to 1 when supplier j is on route v and 0 otherwise |
| Set-up cost of part k at the supplier i, (i = 1, ..., n), ($.set-up) | The number of pick-ups of parts k from supplier i to assembler in each production cycle |
| Distance from supplier i to j, (i,j = 1, ..., n), (km) | Pick-up lot size part k from supplier i (unit) |
| Transportation capacity of each truck, (ton/truck) | Supplier indices, i, j = 1, ..., n |
| Variable transportation cost per unit distance | Part index, k = 1, ..., K |
| per unit weight ($/kg/km) | Vehicle index, v = 1, ..., V |
| Weight of unit part k (kg) | |

2.2. The objective function

This model’s objective function is to minimize the total relevant costs consisting of set-up costs, ordering costs, saving costs, and pick-up costs. Regarding [4], the objective function is formulated as follows:

\[
TC^C = \frac{1}{T} \left( \sum_{i=1}^{n} \frac{S_i}{m_i} + \sum_{i=1}^{n} A_i \right) + T \left\{ \sum_{i=1}^{n} D_i \left( H_i^M + H_i^S \left( \frac{2D_i}{\sum_{k=1}^{t} P_{ik}} - 1 \right) \right) \right\} + F_0 \sum_{v=1}^{V} \sum_{j=1}^{n} x_{ij}^v + F_y \sum_{v=1}^{V} \left\{ \sum_{i=0}^{n} \sum_{j=0}^{n} x_{ij}^v \left( \sum_{r \in \Omega_i} y_{ij}^r u_r D_r \right) \right\}
\]

(1)
2.3. The constraint functions

The constraint functions in this study refer to [4] by adding two new constraints. The addition of further restrictions, namely Constraint (2) and Constraint (3), is needed because the model [4] does not consider the limited capacity of each supplier. The formulation of all constraints is stated in Table 2.

| No | Constraint (2) | Constraint (3) | Constraint (4) | Constraint (5) | Constraint (6) | Constraint (7) | Constraint (8) | Constraint (9) | Constraint (10) | Constraint (11) | Constraint (12) |
|----|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 2  | $\sum_{i=1}^{n} q_{ik} = D_k T$; $k = 1, \ldots, n$ | $q_{ik} \leq P_k T$; $k = 1, \ldots, n$ | $\sum_{j=1}^{n} \sum_{k=1}^{n} y_{ij} = W$; $\sum_{v=1}^{V} x_{ij} = 1$, $j = 1, 2, \ldots, n$ | $\sum_{i=0}^{n} x_{ij} = y_{ij}$; $j = 1, 2, \ldots, n$ | $x_{ij} \epsilon \{0,1\}$, $y_{ij} \epsilon \{0,1\}$, $i, j = 0, 1, 2, \ldots, n$ | $T > 0$, $m_k > 0$, $i = 1, 2, \ldots, n$, $m_k \in Z^+$ |
| 3  | $q_{ik} \leq P_k T$; $k = 1, \ldots, n$ | $\sum_{v=1}^{V} x_{ij} = 1$; $j = 1, 2, \ldots, n$ | $\sum_{v=1}^{V} y_{ij} = \frac{1}{V}$; $j = 1, 2, \ldots, n$ | $\sum_{j=1}^{n} x_{ij} = y_{ij}$; $i = 0, 1, 2, \ldots, V$ | $\sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n} x_{ij}$; $i = 0, 1, 2, \ldots, V$ | $\sum_{j=1}^{n} \sum_{k=1}^{n} x_{ij} \leq [S] - 1$, $\sum_{i=1}^{i} x_{ij} = 1$, $v = 1, 2, \ldots, V$, $S \subseteq \{1, 2, \ldots, n\}$ |

Constraint (2) shows that the fulfillment of one type of part involves several suppliers because of the suppliers’ capacity limitation. Constraint (3) shows that each supplier’s production rate limits the order size to each supplier. Constraint (4) ensures that the load does not exceed the truck’s capacity for each truck (route) $v$. Constraint (5) provides that each supplier is assigned to one and only one truck. Constraint (6) ensures that each part produced by the supplier is transported by one truck, and all supplier parts are transported by as many as $V$ trucks. Constraints (7) and (8) ensure that each supplier can only be served by one truck. Constraint (9) provides that each route starts and ends at the buyer/assembler’s location. Constraint (10) means no sub-tours in any set $S$ from the supplier, excluding the starting point. Constraints (11) and Constraints (12) are a range of variables.

3. Result and Discussion

Concerning the results of the model formulation, it can be seen that the model developed is mixed-integer non-linear programming (MINLP). This paper uses an analytic approach with the help of LINGO 12.0 to solve the MINLP problem. To check the model’s feasibility and the model solution approach’s effectiveness, we tested the model using three numerical examples. Each example involves three limited-capacity suppliers providing similar parts ($k=1$), picked up by one vehicle ($V=1$) using milk-run transportation mode. The three numerical examples have several different parameter values. Table 3 shows the parameter data for the three numerical models.

Using LINGO 12.0, Figures 1 and 2 suggest the optimum solution for numerical example 1. It can be seen in Figure 1 that the optimum solution obtained is the global optimum solution, with the computation time are 16 minutes and 19 seconds. Figure 2 shows the pick-up vehicle’s optimum solution to the supplier’s location every 0.0022 years or every 19.2 hours. During each pick-up, the vehicle departs from the buyer’s location (0) to the supplier’s location 3 to pick up the parts’ 44,444 units. Then the vehicle headed for supplier 2 to pick up 133,333 units. The vehicle then should continue to the supplier 1 location to pick up 155,556 units. In the end, the vehicle carries all the parts to the buyer’s location (0). Thus the pick-up route is 0-3-2-1-0. In
Table 3: Parameter data for numerical example 1, 2 and 3

|   | D1  | H1|^M | A11 | S21 | D30 | C  |
|---|-----|----|-----|-----|-----|-----|----|
| 1 | 150000 | 25 | 1000 | 800 | 13  | 1000|   |
| 2 | 70000  | 10 | 1500 | 900 | 6   | 2000|   |
| 3 | 60000  | 12 | 2000 | 12  | 8   | 5   |   |
| 4 | 50000  | 11 | 700  | 16  | 7   | 3   |   |

|   | D1  | H1|^S | A21 | S31 | D12 | F_a  |
|---|-----|----|-----|-----|-----|------|
| 1 | 250000 | 12 | 1000 | 700 | 6   | 2000 |
| 2 | 70000  | 10 | 3000 | 500 | 6   | 2000 |
| 3 | 90000  | 11 | 900  | 16  | 7   | 3   |

|   | D1  | H2|^S | A31 | D10 | D31 | W_1 |
|---|-----|----|-----|-----|-----|-----|
| 1 | 800  | 11 | 700  | 12  | 8   | 3   |
| 2 | 700  | 10 | 500  | 16  | 7   | 3   |
| 3 | 700  | 11 | 600  | 16  | 7   | 3   |

one production cycle, the pick-up frequency from suppliers 1, 2, and 3 was 34 times, 33 times, and 37 times. Since the production lot size of supplier i, Q_{ik} = m_{ik} \times q_{ik}, the production lot sizes of supplier 1, 2, and 3 are 3289, 4400, and 1644 units, respectively. The vehicle's utility can be calculated by comparing the total cargo carried in one pick-up with the vehicle capacity. The total cargo carried in one pick-up is \((155.5556 + 133.3333 + 44.4444)\) units = \(333.3333\) units = \((333.3333 \text{ unit} \times 3 \text{ kg/unit}) = 999.9999 \text{ kg or near to 1000 kg. Thus the utility of all vehicles is 100 percent.}

The status of solutions, computation time, pick-up route, and vehicle utility for the three numerical examples are shown in Table 4. The type of solutions obtained is a global optimum. It means that an analytic approach using LINGO 12.0 is practical in finding optimal solutions.

![Lingo 12.0 Solver Status](image)

**Figure 1:** Solution status for numerical example 1

The results obtained indicate that in the MVSB system, the pick-up of parts from each supplier is carried out as often as possible with a small lot size to minimize the total relevant
Figure 2: Solutions for numerical example 1

| Status          | Computation times       | utility (%) | Route           |
|-----------------|-------------------------|-------------|-----------------|
| Numerical example 1 | global optimum          | 16 minutes, 19 seconds | 100 | 0-3-2-1-0 |
| Numerical example 2 | global optimum          | 17 minutes, 43 seconds | 100 | 0-3-2-1-0 |
| Numerical example 3 | global optimum          | 17 minutes, 08 seconds | 100 | 0-3-2-1-0 |

costs. These results are consistent with JIT procurement, namely receiving parts as often as possible in small lot sizes [4–6]. However, the optimal value of pick-up frequency (mi) and pick-up lot size (qi) depends on the parameter values used in the model.

Accommodating supplier capacity constraints are needed to develop an integrated inventory and order pick-up model in the MVSB system. Buyers must involve many suppliers to provide certain parts because of the suppliers’ capacity limitation. However, this study has limitations because it only can involve three suppliers of similar parts. As a result, it only needs one vehicle to pick-up. Manufacturing industrial companies generally transport various types of parts from many suppliers located in different locations, so it is necessary to involve several pick-up vehicles on other routes.

Consequently, increasing the number of suppliers will exponentially increase the number of decision variables and constraints (known as NP-hard). As a result, the computation time will be even greater. In this condition, this study’s analytic approach is no longer reliable, so it requires a different approach. For this reason, in future studies, it is suggested to develop more complex computational models and strategies to accommodate a larger number of suppliers and parts.

4. Conclusion
This study aims to produce an integrated inventory and order pick-up model using milk-run transportation mode on the MVSB system by considering suppliers’ limited capacity to minimize the total relevant costs. By modeling the problem as a mixed-integer non-linear programming (MINLP) with the help of LINGO 12.0, this research can accommodate the limited capacity of suppliers. Model outputs include typical pick-up cycle time, pick-up frequency, pick-up batch
size, the production lot size for each supplier, number of vehicles, and vehicle routes to produce a minimum total relevant cost. It is recommended to pick up from each supplier as often as possible with small lot sizes. In future studies, it is suggested to develop a model for a larger number of suppliers and parts.

References
[1] H. M. Wee, S. Y. Peng, and P. K. P. Wee, 2010 Modelling of outsourcing decisions in global supply chains. An empirical study on supplier management performance with different outsourcing strategies, International Journal of Production Research, vol. 48, no. 7, pp. 2081–2094.
[2] H. E. Müller, 2009 Supplier integration: An international comparison of supplier and automaker experiences, International Journal of Automotive Technology Management, vol. 9, no. 1, pp. 18–39.
[3] C. H. Glock and T. Kim, 2009 Shipment consolidation in a multiple-vendor-single-buyer integrated inventory model, Computer & Industrial Engineering, vol. 70, no. 1, pp. 31–42.
[4] Z. Chen and B. R. Sarker, 2014 An integrated optimal inventory lot-sizing and vehicle-routing model for a multisupplier single-assembler system with JIT delivery, International Journal of Production Research, vol. 52, no. 17, pp. 5086–5114.
[5] D. A. De Moura and R. C. Botter, 2016 Delivery and Pick-Up Problem Transportation - Milk Run and Conventional Systems, Independent Journal of Management & Production, vol. 7, no. 3, pp. 746–770.
[6] Z. X. Chen and B. R. Sarker, 2010 Multi-vendor integrated procurement-production system under shared transportation and just-in-time delivery system, Journal of the Operational Research Society, vol. 61, no. 11, pp. 1654–1666.
[7] S. K. Goyal and S. G. Deshmukh, 1992, Integrated procurement-production systems: A review, European Journal of Operational Research, vol. 62, no. 1, pp. 1–10.
[8] M. Ben-Daya, M. Darwish, and K. Ertogral, 2008 The joint economic lot sizing problem: Review and extensions, European Journal of Operational Research, vol. 185, no. 2, pp. 726–742.
[9] A. Banerjee, 1986 A Joint Economic Lot Size Model for Purchaser and Vendor, Decision Sciences, vol. 17, no. 3, pp. 292–311.
[10] C. H. Glock, 2011 A multiple-vendor single-buyer integrated inventory model with a variable number of vendors, Computer & Industrial Engineering, vol. 60, no. 1, pp. 173–182.
[11] F. G. Beck, C. H. Glock, and T. Kim, 2017 Coordination of a production network with a single buyer and multiple vendors with geometrically increasing batch shipments, International Journal of Production Economics, vol. 193, pp. 633–646.
[12] C. H. Glock, 2012 The joint economic lot size problem: A review, International Journal of Production Economics, vol. 135, no. 2, pp. 671–686.
[13] T. Kim and S. K. Goyal, 2009 A consolidated delivery policy of multiple suppliers for a single buyer, International Journal Procurement Management, vol. 2, no. 3, pp. 267–287.
[14] C. H. Glock, 2012 A comparison of alternative delivery structures in a dual sourcing environment, International Journal of Production Research, vol. 50, no. 11, pp. 3095–3114.
[15] T. Nemoto, K. Hayashi, and M. Hashimoto, 2010 Milk-Run logistics by Japanese automobile manufacturers in Thailand, Procedia - Social and Behavioural Science, vol. 2, no. 3, pp. 5980–5989.
[16] J. Stacey, M. Natarajarathinam, and C. Sox, 2007 The storage constrained, inbound inventory routing problem, International Journal of Physical Distribution & Logistic Management, vol. 37, no. 6, pp. 484–500.
[17] M. Natarajarathinam, J. Stacey, and C. Sox, 2012 Near-optimal heuristics and managerial insights for the storage constrained, inbound inventory routing problem, International Journal of Physical Distribution & Logistic Management, vol. 42, no. 2, pp. 152–173.
[18] I. K. Moon, B. C. Cha, and C. U. Lee, 2011 The joint replenishment and freight consolidation of a warehouse in a supply chain, International Journal of Production Economics, vol. 133, no. 1, pp. 344–350.
[19] H. Kuhn and T. Liške, 2011 Simultaneous supply and production planning, International Journal of Production Research, vol. 49, no. 13, pp. 3795–3813.
[20] L. Bertazzi, D. Laganà, J. W. Ohlmann, and R. Paradiso, 2020 An exact approach for cyclic inbound inventory routing in a level production system, European Journal of Operational Research, vol. 283, no. 3, pp. 915–928.