New insight into the Hall effect

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In this paper, we develop a unified theory for describing Hall effect in various electronic systems based on a pure electron picture (without the hole concept). We argue that the Hall effect is the magnetic field induced symmetry breaking of the charge carrier’s spatial distribution. Due to the interaction of the charge carriers and the ion lattice, there are two possible symmetry breaking mechanisms which cause different signs of Hall coefficient in a Hall material. The scenario provides an explicit explanation of the sign different of the Hall coefficient in the N-type and P-type semiconductors, the sign reversal induced by both temperature and magnetic field in different materials, and the integer and fractional quantum Hall effect (QHE) in two-dimensional electron gas (2DEG) of GaAs/AlGaN heterostructures.

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I. INTRODUCTION

Since the discovery of the Hall effect over a century ago [1], much effort have been made to elucidate this phenomenon both experimentally and theoretically. In the experiment, a number of Hall effect-related phenomena have been uncovered. Of these, the most remarkable achievements are the observation of the integer quantum Hall effect (IQHE) [2] and the fractional quantum Hall effect (FQHE) [3] in the two-dimensional electron systems. In the theory, different approaches have been developed and applied to the study of these peculiar experimental results, however, the physical interpretation of the Hall effects is made difficult even in relatively simple metals. It is well known that in many cases the simple semiclassical expression of the Hall coefficient \( R_H = 1/qn \) does not work, where \( n \) is the density of mobile charges and \( q \) is the charge of the charge carriers. In strongly correlated systems the Hall effect is even more difficult to interpret because more factors can have a large influence on the Hall resistivity. As a result, some unexpected reversals of the sign (the so-called anomalous Hall effect) of the Hall coefficient (or voltage) have been reported in these systems.

Despite many works have been put into this field, the mechanisms which lead to the Hall effects are not completely understood. In fact, the theoretical investigations are almost hampered by the complexity of the system. In this paper, we would like to address that the theoretical difficulties encountered in the Hall effect do not arise from the complexity of the studied systems, but from a fatal misunderstanding of the most fundamental mechanism that rules the Hall effects. It will be shown clearly that a pure electron picture (without the hole concept) can provide a unified explanation of the Hall effects for different electronic systems, for example, the sign different of the Hall coefficient in the N-type and P-type semiconductors, the sign reversal in different materials, and the appearance of the plateaux in the Hall conductivity in two-dimensional electron systems at low temperatures and in strong magnetic fields.

FIG. 1: The schematic plot of Hall effect with different kinds of charge carriers. (a) Negative charge carriers: electrons, and (b) positive charge carriers: holes.

II. AN OLD LOOK FOR HALL EFFECT

According to the traditional viewpoint of the Hall effect, the basic physical reason underlying the Hall effect is the Lorentz force. As shown in Fig. 1 when an electric current flows through a conductor in a direction perpendicular to an applied magnetic field which exerts a transverse force on the moving charge carriers (the electric current), this force tends to push the moving charges to one side of the conductor and a Hall voltage \( V_H = V_A - V_B \) builds up. Note that the Fig. 1(a) is for N-type materials with the negative carriers (electrons) while Fig. 1(b) for P-type materials with the positive charge carriers (holes). Normally, the N-type materials have a negative Hall voltage, while the P-type materials have a positive Hall voltage.

Figure 2 illustrates the Hall effect for a simple metal where there is only one type of charge carrier (electrons). Without an external magnetic field, there is no Hall effect \( (V_H = 0) \) in the sample, as shown in Fig. 2(a). When
a magnetic field is applied in Z-direction, a buildup of charge at the sides of the conductors (characterized by an static electric field $E$ in $Y$-direction) will balance this magnetic influence, producing a measurable Hall voltage $V_H$ which is given by

$$V_H = \frac{1}{n_e} \frac{IB}{d} = R_H \frac{IB}{d}, \tag{1}$$

where $n_e$ is the charge carrier density of the carrier electrons, $B$ is the magnetic flux density, $d$ is the depth of the sample, $e$ is the electron charge, and the electric current can be expressed in terms of the drift velocity $I = -n_e e v_{bd}$. It is easy to find that Eq. (1) leads to the following expression

$$n_e e R_H = -1. \tag{2}$$

Eq. (2) implies that the Hall coefficient $R_H$ of the simple metals should always be negative. Table I shows some experimental values of $1/n_e e R_H$ for some simple metals. It is not difficult to find that all the experimental data are inconsistent with the theoretical prediction. However, probably the most surprising result is that the sign of $n_e e R_H$ (or $R_H$) is positive for Li and Na. We refer this situation of the Hall effect as “sign catastrophe”. To the best of our knowledge, this “catastrophe” is a rather common experimental fact that can be found easily in any Hall materials. Many researchers believe that this problem can be easily overcome by introducing the concept of the positive “hole”. However, our point of view is quite different in this paper. We consider that one should be cautious when drawing conclusions based on the man-made concept of “hole”. In fact, using the language of ‘hole’ rather than ‘electron’ can obscure the essential physics of the Hall effect. There is no reason for us to overlook one basic fact that it is the real electrons, not the artificial holes, carrying the electric current in the materials. In other words, we need a new look for Hall effect.

### III. NEW LOOK FOR HALL EFFECT

Figure 3 shows our new idea of Hall effect. For the case without an external magnetic field, there will be no Hall effect ($V_H = V_A - V_B = 0$) as shown in Fig.

![Diagram](image-url)
When a magnetic field is present in $Z$-direction, the mobile electrons have an unitary displacement ($\xi$) along $+Y$-direction because of the Lorentz force on the electrons, leading to a negative Hall effect ($V_H < 0$) as shown in Fig. 3(b). Here we assume that, under some special circumstance, the mobile electrons can move in an opposite direction ($-Y$) under the interaction of the magnetic field and shows a positive Hall effect ($V_H > 0$) as shown in Fig. 3(c). A more detailed explanation for such an inexplicable assumption will be given in the following.

Evidently, as compared to the old picture of Hall effect (see the previous section), our new idea described in Fig. 3 contains four main innovations:

1. The new picture does not involve any quasiparticle (for example, the well-known ‘hole’), electrons are the only charge carriers in any Hall materials.

2. In Fig. 3, the mobile electrons are no longer assigned to be the “free electrons”, the interaction between the mobile electrons and the fixed positive lattice ions should be taken into account.

3. We do not think that the magnetic field would cause the accumulation of the charge carriers at the sides of the Hall materials, in our opinion, the application of the magnetic field can only result in an overall shift of the mobile electrons.

4. The new mechanism is based on the viewpoint of the symmetry breaking, the magnetic field can induce two different symmetry breaking effects which contribute to two different Hall effects: the negative Hall effect of Fig. 3(b) and the positive Hall effect of Fig. 3(c).

Of course, the key point of the new concept is: As illustrated in Fig. 3(c), how can the electrons move in a “wrong direction” under the magnetic field?

We believe that the sign of Hall coefficient $R_H$ is dominated by the strength of restriction the mobile electrons in a Hall material. When the mobile electrons are strongly confined inside a material, the corresponding sample tends to have a negative Hall coefficient, while for a weak-restricted system, the Hall coefficient most probably has a positive sign. To illustrate this idea more clearly, I would like to present in Fig. 4(a) why a material can exhibit a positive Hall coefficient. Figure 4(a) shows a Hall material of a large number of valence electrons, such as $Mn^{3+}$, $Al^{3+}$, $In^{3+}$, and $Be^{2+}$, which has a negative sign of the Hall effect ($V_H < 0$), or the $N$-type Hall effect. As can be seen from Fig. 4(a), when applied magnetic field, the mobile electrons will move from the old equilibrium positions (the dash green lines) to new ones ($F_D = F_U + F_B$) of the solid green lines which are very close to the positive quasi-one-dimensional ion chains, owing to the strong electromagnetic interactions between the mobile electrons and the positive ions. For a small number of valence electrons’ Hall material (for example, $Li^{1+}$, $Na^{1+}$), the influence of an external magnetic field may be different. As shown in Fig. 4(b), the mobile electrons nearest to the electrode (A) are most likely to enter into the electrode directly due to a relatively weak electromagnetic interaction between these electrons and the ion chain. In this situation, the effect (indicated by the black solid curved arrows) of the external magnetic field (+$Z$-direction) on the mobile electrons is equivalent to that (indicated by the blue dash curved arrows) of a $-Z$-direction magnetic field. Obviously, Fig. 4(b) favors the positive sign of the Hall effect ($V_H > 0$), or the $P$-type Hall effect.

Based on the new picture of Fig. 4, we now continue to discuss the dependence of the sign of Hall effect on the properties of the Hall materials and external factors (for example, the magnetic field and the temperature).
It’s now become very clear that to obtain a positive Hall effect ($P$-type), the mobile electrons inside the Hall material should be less confined by the positive lattice ions. By decreasing the number of valence electrons is an effective way to achieve the positive Hall effect as discussed above. In the semiconductors, it is well known that the intrinsic silicon may exhibit a $N$-type semiconductor and $P$-type behavior through the doping of pentavalent impurities (such as antimony, arsenic or phosphorous) and trivalent impurities (such as boron, aluminum or gallium), respectively. From the viewpoint of Fig. 4, the adding of the pentavalent impurities greatly enhance the confinement ability of the positive ion lattice on the mobile electrons, hence the material should have a negative Hall coefficient ($N$-type semiconductor), while the situation for the later case is different, the trivalent impurities will decrease the confinement ability on the mobile electrons and lead to a $P$-type Hall behavior.

Besides, there are two important factors (magnetic field and temperature) which could lead to the $N$-type to $P$-type transition in a Hall material. We suppose that a material appears to be a $N$-type Hall material under an external magnetic field $B$. With the increasing strength of the magnetic field, some mobile electrons may be free from the restriction of the positive lattice ions and cause the $N$-type to $P$-type transition in the material. The same discussion can be used to explain the temperature induced $N$-type to $P$-type transition, it is quite clear that a higher temperature would decrease the stability of the ion lattice which in turn make the mobile electrons less confinement, this fact implies that high temperature may favor a $P$-type Hall phase. Let us mention that these discussions can be applied to interpret the observed sign reversal induced by both temperature and magnetic field in different materials.

**IV. ANALYTICAL RESULTS**

Our scenario of the Hall effect offers a totally new picture of Hall effect and shows that electric currents in Hall materials are carried only by moving electrons, not by holes. In other words, it is inappropriate to think that there are two kinds of charge carriers (electrons and holes) coexist in materials. In this section, we try to establish the relationships between Hall voltage $V_H$, Hall resistance $R_{xx}$, charge carrier density $n_e$ and magnetic field $B$ using the new mechanism.

For simplicity, it is assumed that two electrons (denoted by $i$ and $-i$) are moving along the $X$-direction at velocity $v$ in a positive-charge background, as shown in Fig. 5. Without the magnetic field, these two electrons locate in the $Y$-direction at $y_i$ and $-y_i$, respectively. If a magnetic field is present in $Z$-direction, the electrons will be driven $y_i + \xi$ and $-y_i + \xi$ by the Lorentz force (see Fig. 5). As a result, the voltage of the electrode $A$ and $B$ can be directly given by

\[ V_A^i = -\frac{e^2}{4\pi\varepsilon_0(b/2 - y_i - \xi)} - \frac{e^2}{4\pi\varepsilon_0(b/2 + y_i - \xi)} \]  
\[ V_B^i = -\frac{e^2}{4\pi\varepsilon_0(b/2 - y_i + \xi)} - \frac{e^2}{4\pi\varepsilon_0(b/2 + y_i + \xi)} \]  

Because $\xi \ll b/2$, then Eqs. (3) and (4) lead to the following expression for the Hall voltage:

\[ V_H^i = V_A^i - V_B^i \approx -\frac{4e^2}{\pi\varepsilon_0} \frac{\xi}{(b^2 - 4y_i^2)^2}. \]  

For a real Hall system with a electron density $n_e$, the total Hall voltages is approximately given by

\[ V_H = \sum_i V_H^i \propto n_e^\alpha \frac{\xi}{b^2}, \]  

where $0 < \alpha < 1$ is a material related constant. The shift of the mobile electrons $\xi$ is described by the Lorentz force

\[ \xi \propto vB = \frac{n_e e v B d}{n_e e B d} = -\frac{IB}{n_e e B d}. \]
voltage
\[ V_H = -\kappa \frac{1}{n_e^{\alpha} e b^3} \frac{IB}{d}, \quad (8) \]
and the Hall resistance
\[ R_{xx} = \frac{V_H}{I} = -\kappa \frac{B}{n_e^{\alpha} e b^3 d}. \quad (9) \]
where \( \kappa \) is a constant. Although our theoretical analysis above requires some mathematical approximation, it is easily seen that our result of Eq. (8) contains more reasonable terms than the traditional result of Eq. (1). Normally, the Hall resistance \( R_{xx} \) increases linearly with magnetic field.

V. INTEGER AND FRACTIONAL QUANTUM HALL EFFECTS

Undoubtedly, the discoveries of the integer and fractional quantum Hall effects (QHE) are remarkable achievement in condensed matter physics. However, the theoretical explanation of these experiments is far beyond expectations. In fact, a general theoretical understanding of the QHE is still lacking, many existing theories of QHE have endowed these findings with some mystery and some apparent contradictory behavior. We are confident that the new picture of this paper can provide the most simple straightforward description of the QHE.

The quantum Hall phenomena can only be observed in the two-dimensional electron systems (as illustrated in Fig. 6) subjected to extreme low temperatures and very strong magnetic fields. Figure 7 shows the experimental results of the integer quantum Hall effect (IQHE) and the fractional quantum Hall effect (FQHE) in a GaAs/GaAlAs heterojunction with a surface mobile electron density \( n_s \) about \( 1.0 \times 10^{11} \text{cm}^{-2} \). From the figure it is easy to see that there are two classes of Hall resistance plateaux, one class is the so-called the integer quantum Hall effect and the corresponding plateaux can be well described by
\[ R_{xx} = \frac{h}{e^2N}, \quad N = 1, 2, 3, 4, 5... \quad (10) \]
The other class is referred to as the fractional quantum Hall effect. Here the Hall resistance plateaux are seen at the so-called fractional filling factors, especially \( 1/3, 2/3, 4/3, 5/3, 2/5 \) and \( 3/5 \).

The majority researchers hold that IQHE and FQHE are two absolutely different types of physical effects which are caused by different physical reasons, therefore, different theories are needed to explain them. We think that such a viewpoint is physically inappropriate, or even wrong. We insist that these two effects share exactly the same physical mechanism. Furthermore, the Landau theory of the quantization of the cyclotron orbits of charged particles in magnetic fields cannot be used to explain the quantum Hall effects, since the interactions between the mobile electrons and the ion lattice had been ignored totally in the theory. As we have discussed above, these interactions play an essential role in the Hall effects.
Here, we will try to provide a unified explanation of the IQHE and FQHE based on our hypothesis. In a GaAs/GaAlAs heterojunction as shown in Fig. 6, the mobile electrons (or electron gas) is effectively restricted within the heterojunction of XY-plane but their movement in XY-plane is assumed to be free (the interactions between the mobile electrons and ion lattice are neglected) in all the existing theories of Hall effect. One most important feature of our theory is that without the interactions between the mobile electrons and ion lattice, there is no Hall effect in the system. In other words, to study the Hall effect we must take into account the interactions between the mobile electrons and ion lattice.

Figure 8 shows two kinds of interactions between the mobile electrons and ion lattice in a GaAs/GaAlAs heterojunction. According to the values of \( n_s \approx 1.0 \times 10^{11}/cm^2 \) and the lattice constant of GaAs \( a = 5.56 \text{Å} \), there is only one electron per unit supercell \( (A \times A \approx 18a \times 18a) \) of the superlattice, as shown in Fig. 8(a). Therefore, the mobile electrons will interact with the fixed positive charges in AlGaAs layer. At the same time, these electrons are confined by the electropositive Ga atoms in GaAs layer, see the enlarged figure of Fig. 8(b). For a given external magnetic field, some real space ballistic trajectories (or real space Landau levels) can be formed along X-direction (the green and cyan solid lines), where the Lorentz force \( F_B \) can be well balanced by the electromagnetic interactions \( (F_D = F_U + F_E) \), as indicated in the figure. The combined effects of the magnetic field and the positive ion lattice on the mobile electrons lead to the “localization” of the electrons in Y-direction with a real space real-space separation of approximately \( a/2 \) while extended in X-direction. But for now the most interesting question is: How can such a picture explain the formation of Hall resistance plateaux of Fig. 7? In our theoretical framework, the Hall resistance satisfies

\[
R_{xx} = \frac{V_H(B)}{I} \propto \xi(B).
\]

As the electric current \( I \) remains unchanged in the experiment, the emergence of the Hall resistance plateaux means that the corresponding Hall voltage \( V_H \) does not change with magnetic field \( B \). From Eq. (11), this imply that the applied magnetic field cannot induce a significant displacement of the mobile electrons in some special positions along Y-direction. This conclusion is in agreement with the discussion above, when the mobile electrons are moving along the restricted real space Landau channels, they are very difficult to be shifted by the external magnetic field \( (\Delta \xi(B) \rightarrow 0) \) and the Hall resistance plateaux emerge. Furthermore, a wide plateau reveals a more intense confinement on the electrons, if all the mobile electrons are traveling along the most confined channels [indicated by cyan solid lines in Fig. 8(b)], a wide plateau of the Hall resistance will naturally appear.

The final secret of the Hall effect: Why the Hall resistance plateaux take on the quantized values related to the Planck’s constant \( h = 6.626196 \times 10^{-34} J \cdot s \)? For the quasi-two-dimensional Hall system, Eq. (9) of the Hall resistance can be reexpressed as

\[
R_{xx} = \frac{V_H}{I} = -\kappa \frac{B}{n_s} = -\frac{\kappa eB}{n_s^2} \frac{1}{e^2},
\]

where \( n_s \) is the surface mobile electron density.

Because the Planck’s constant \( h \) is an infinitely small amount, from Eq. (12), with the increasing of the magnetic field \( B_1 \) makes the following formula true

\[
\frac{\kappa eB_1}{n_s^2} = h.
\]

Inserting Eq. (13) into Eq. (12), one gets the Hall resistance plateau \( R_{xx}(B_1) = h/e^2 \). When the magnetic field \( B > B_1 \), one can obtain a set of Hall resistance plateaux which are given by

\[
R_{xx}(B) = \frac{h}{\eta e^2},
\]
where $\eta < 1$ is a fractional number. When the applied magnetic field $B < B_1$, another set of Hall resistance plateaux can be described as

$$R_{xx}(B) = \frac{h}{\zeta e^2},$$

where $\zeta > 1$ is an integer or a fractional number. It should be pointed out that these Hall resistance plateaux are closely related to the Landau channels of Fig. 8(b). The integer $\zeta$ corresponds to the integer quantum Hall effect, while the fractional $\eta$ and $\zeta$ contribute to the fractional quantum Hall effect.

VI. CONCLUSION

In this paper, we have developed a new theory for describing Hall effect in various electronic systems based on a pure electron picture. It has been shown solidly that the interaction of the charge carriers and the ion lattice play an essential role in the Hall effects. Our theory provides an explicit explanation of the sign different of the Hall coefficient in the $N$-type and $P$-type semiconductors. We have attempted to uncover the physical nature of the sign reversal Hall phenomena induced by both temperature and magnetic field in different materials. Furthermore, we have considered that the integer and fractional quantum Hall effects should share exactly the same physical mechanism and a unified and simple picture of these two effects has been provided. We are confident that the research may shed light on the fundamental of the Hall effect.

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