Simulation of the propagation of the vortex eigenfunctions of the two-lens system in the parabolic fiber

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Abstract. In this paper, the limited optical two-lens system, which describes the transmission of light beams in the parabolic profile fiber is considered. Vortex eigenfunctions, calculated for the given system form the set of the orthogonal functions, through which another optical field can be represented. Numerical simulation of the passage of the eigenfunctions through the system of the multiple lenses is carried out. It is shown, that eigenfunctions, whose eigenvalues are close to unity by absolute value, can pass through a large number of consecutive two-lens systems with practically no distortion.

1. Introduction
The current level of use of the optical fiber in terms of the time and frequency domain characteristics tends to the bandwidth limit [1]. However, the requirements for increasing the volume of global traffic are constantly growing. To ensure the correspondence of communication networks to ever-growing bandwidth requirements, additional approaches are considered for the capacity increasing of the optical fiber channels. One of such approaches is the mode division multiplexing (MDM) [2,3]. A special advantage for increasing the capacity of the information channel is achieved with the help of optical beams with orbital angular momentum and having an infinite number of possible quantum states [4, 5]. The significant success of this method has already been demonstrated in optical fibers [6] and in free space [7,8]. For generation and analysis of the vortex beams, the diffractive optical elements are used [9-11], and the lens systems are used to introduce them into the optical fiber [12-15].

The propagation of the vortex laser beam of the m-th order through the spherical lens can be described using the Hankel transform of the m-th order. In real lens systems, there is a spatial limitation, and finite (spatially-restricted) propagation operators are used to describe the propagation of the optical signal [16, 17]. Due to the spatial limitation, both in the object and spectral regions, it is impossible to obtain an ideal image in the two-lens system. In order to understand how the optical signal is distorted, it is necessary to expand them according to the eigenmodes of the lens system. In connection with this, the concept of communication modes is widely used [18, 19]. The communication modes for square apertures and Fresnel transformations are an elongated angular spheroidal functions [20,21], which are widely studied and applied in optics [22-25]. The communication modes for round apertures and finite Hankel transform are the circular [26] and generalized [27] spheroidal functions.

In this paper, the set of vortex eigenfunctions of the bounded circular two-lens system is calculated. The set allows analyzing the distortion of the optical signal during transmission on the basis of approximation by eigenfunctions.
2. Theoretical foundations

The propagation of the optical signal in the parabolic profile fiber is analogous to its propagation through the successive system of lenses. The passage of the vortex beam through the bounded by radius two-lens system can be described as follows [28]:

\[
H_{R, P} \left[ f(r) \exp(im\varphi) \right](r', \varphi') = \frac{kP}{2\pi f} \int_0^P L(r, r'; m, P) f(r) r \, dr \cdot \exp(im\varphi'),
\]

\[
L(r, r'; m, P) = \frac{2J_1 \left( \frac{kP}{f} \sqrt{r^2 + r'^2 + 2rr' \cos \theta} \right)}{\sqrt{r^2 + r'^2 + 2rr' \cos \theta}} \exp(im\theta) d\theta,
\]

where \( J_1 \) is the Bessel functions, \( k \) is the wavenumber, \( f \) is the focal length of both lenses, \( R \) is the radius of the first aperture, \( P \) is the radius of the second aperture, \( m \) is the order of the optical vortex. The eigenfunctions of such system have modal properties and are orthogonal.

As the radius of the aperture increases, an increase in the number of significant eigenvalues (eigenvalues close to unity by absolute value) is observed, which means an increase in the number of orthogonal states supported by the system in the spatial degree of freedom. In other words, when the aperture radius is increasing, the more number of eigenfunctions can be transmitted without losses. So, more complex optical signals propagate through the system without distortions and with less energy losses.

3. Numerical simulation

Simulation was carried out with the given parameters: \( R = 1 \) \( \mu \)m, \( P = 5 \) \( \mu \)m, \( m = 2 \), and \( k / 2\pi f = 1 \). It should be noted, that with increasing parameters \( R, P \) and \( m \), the calculations can also be performed, but they become more complex and may require the use of high-performance supercomputers. The simulation of the propagation of the vortex eigenfunctions of the operator (1) in the bounded parabolic fiber is carried out with the aid of the finite Hankel transform:

\[
H_{R} \left[ f(r) \exp(im\varphi) \right](r', \varphi') = \frac{k}{f} \int_0^P f(r) J_n \left( \frac{k}{f} rr' \right) r \, dr \cdot \exp(im\varphi'),
\]

In this case, each odd transformation \( H_{R} \) occurs in the region, bounded by the radius \( R \), and is equivalent to passing the beam through the first half of the two-lens system. Every even transformation \( H_{P} \) takes place in the region, bounded by the radius \( P \), and corresponds to the passage of the optical beam through the second half of the two-lens system.

Figure 1 shows the graph of the moduli of eigenvalues of the operator (1) with the parameters described above. The trajectory of the graph has step behavior: starting from a certain moment all eigenvalues are close to zero, whereas before that their moduli are approximately equal to 1. The graph shows only the first 15 values, since the remaining ones are approximately equal to zero.

![Figure 1](image)

Figure 1. Modules of eigenvalues of the operator (1).

Figure 2 shows one-dimensional graphs of the original eigenfunction \( \psi_{nm}(r, \varphi) = \psi_n(r) \exp(im\varphi) \) with the index \( n = 2 \), as well as its passage through the system of 10 lenses (the left graph). It can be
seen from the Figure 2, that the shape of the function remains practically unchanged. This is because the absolute value of its eigenvalue is close in value to 1. On the right, the spectrum of this eigenfunction, i.e. its passage through only 1 lens is shown. It can be seen, that the spectrum differs greatly from the original function.

![Figure 2. Amplitude of the eigenfunction $\psi_2(r)$ (blue colour, left) and its passage through 10 lenses (black colour, left): the graphs coincide; the amplitude of the spectrum of the original function (right).](image)

As the index of eigenfunctions increases, the moduli of their eigenvalues decrease, therefore, they will be less resistant to the diffraction effects, associated with domain limitation. Figure 3 shows analogous to the previous graphs, but for the function with the index $n = 7$. Unlike the previous case, the graphs do not coincide already, but differ slightly from each other.

![Figure 3. Amplitude of the eigenfunction $\psi_7(r)$ (blue colour, left) and its passage through 10 lenses (black colour, left): the graphs slightly diverge; the amplitude of the spectrum of the original function (right).](image)

As can be seen from Figure 1, the modulus of the eigenvalue of the function with index $n = 8$ is less than one, so the corresponding eigenfunction will be distorted even more, than the previous one, as shown in Figure 4. Part of the energy is lost and goes beyond the system when passing each lens. In this case, the spectrum pattern becomes similar to the original function.
Figure 4. Amplitude of the eigenfunction $\psi_n(r)$ (blue colour, left) and its passage through 10 lenses (black colour, left): the graphs vary greatly; the amplitude of the spectrum of the original function (right).

For clarity, Table 1 lists the 2D-images of the regarded eigenfunctions.

| $n$ | Eigenfunction | Spectrum |
|-----|---------------|----------|
|     | Amplitude     | Phase    | Amplitude | Phase |
| 2   | ![Image](image1.png) | ![Image](image2.png) | ![Image](image3.png) | ![Image](image4.png) |
| 7   | ![Image](image5.png) | ![Image](image6.png) | ![Image](image7.png) | ![Image](image8.png) |
| 8   | ![Image](image9.png) | ![Image](image10.png) | ![Image](image11.png) | ![Image](image12.png) |

Thus, proposed propagation operator (1) allows us to simulate the propagation of vortex beams through the bounded by radius two-lens system or the optical fiber with parabolic refractive index. Vortex eigenfunctions of the propagation operator (1) with eigenvalues close to 1 propagate through these systems without deformation, energy loss or information distortion. So, the proposed calculation algorithm makes it possible to estimate the number of orthogonal states supported by the system in the spatial degree of freedom or the number of optical vortices that can be transmitted by the systems under consideration. The number of the system states defines the information capacity (each vortex in this case is a bit of an information word). Such estimations can be useful for communication systems based on vortex MDM.
4. Conclusion
The calculation of the set of vortex eigenfunctions of the bounded circular two-lens system is performed. A simulation of the propagation of the functions, obtained in the bounded optical fiber with parabolic refractive index, was conducted. The results showed, that the beams, corresponding to the eigenfunctions with small indices, remain in the fiber while passing and retain the properties of orthogonality. This means, that any signals, that can be represented as a superposition of such eigenfunctions, can be transmitted without losses.

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