Some Analytical Considerations Regarding the Traveling Salesman Problem Solved with Wolfram Mathematica Applications

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Authors' contributions
This work was carried out in collaboration among all authors. Author BVC designed the study, performed the statistical analysis and wrote the first draft of the manuscript. Authors AL and MC managed the analyses of the study. All authors read and approved the final manuscript.

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ABSTRACT

The paper presents an introduction to the Ant Colony Optimisation (ACO) algorithm and methods for solving the Travelling Salesman Problem (TSP). Documenting, understanding and knowledge of concepts regarding the emergent behavior and intelligence swarms optimization, easily led on solving the Travelling Salesman Problem using a computational program, such as Mathematics Wolfram via Creative Demonstration Projects (*.cdf) module. The proposed application runs for a different number of ants, a different number of ants, a different number of leaders (elite ants), and a different pheromone evaporation index. As a result it can be stated that the execution time of the algorithm to solve the TSP is direct and strictly proportional to the number of ants, cities and elite ants considered, the increase of the execution time increasing significantly with the increase of the variables.

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1. INTRODUCTION

In recent years, interest in large groups of small and relatively inexpensive robots has grown significantly. In this context, the evolution of information processing equipment and the new trend of electronic devices miniaturization have imposed distributed systems as a viable alternative, as part of combinatorial optimization based on the emerging behavior of insect societies. Nature-inspired emergent behavior patterns, very suitable for embedded systems, have led to the development of a series of complex and efficient algorithms, such as the Ant Colony Optimization (ACO) algorithm [1], that solves various and multiple problems. As such, for additional information see Table 1.

The problems solved by the ACO algorithm are mainly management problems (investment portfolio, marketing, production, programming & planning, adjustment of stocks), or problems that occur in engineering disciplines (optimal road design, very large-scale integrated circuits design, resources planning, telecommunications), transport problems, and so on [1].

A relatively new algorithm, used in the field of discrete optimizations, is the one inspired by the life of ant colonies (ACO algorithm) [2,3] - able to solve, for example, the Traveling Salesman Problem (TSP), as suggested in Table 1 [4-8].

Table 1. Brief illustration for a list of problems to which the ACO has been applied

| Problem's name                      | Versions of the problem                                                                 | References                                      |
|-------------------------------------|----------------------------------------------------------------------------------------|-------------------------------------------------|
| Traveling Salesman Problem (TSP)    | -                                                                                      | Dorigo M., 1991-1997 Stützle T., 1997-2000      |
| Vehicle Routing Problem (VRP)       | Capacitated vehicle routing problem                                                    | Toth P. & Vigo D., 2002 Belenguer M., 2003      |
|                                     | Multi-depot vehicle routing problem                                                   | Salhi S. & Sari M., 1997                       |
|                                     | Period vehicle routing problem                                                        | Angelelli E. et al., 2002                       |
|                                     | Split delivery vehicle routing problem                                                | Ho S. et al., 2004                             |
|                                     | Vehicle routing problem with pick up and delivery                                      | Nanry W. et al., 2000                          |
|                                     | Vehicle routing problem with time windows                                             | Bent R. et al., 2003                           |
|                                     | Time dependent vehicle routing problem with time windows                               | Hong S. et al., 1999                           |
|                                     |                                                                                       | Rusell R. et al., 2006                          |
|                                     |                                                                                       | Donati A. et al., 2008                          |
| Sequential Ordering Problem (SOP)   | Generalized assignment problem                                                        | Gambardella et al., 1999                       |
| Quadratic Assignment Problem (QAP)  | Frequency assignment problem                                                          | Reimann et al., 2002                           |
|                                     | Redundancy allocation problem                                                         | Stützle T., 1997                               |
| Shop Scheduling Problem (SSP)       | Job(open)-shop scheduling problem                                                     | Lourenço R. et al., 2002                        |
|                                     | Permutation flow shop problem                                                         | Yagiura M., 2004                               |
|                                     | Group-shop scheduling problem                                                         |                                              |
|                                     | Resource-constrained project scheduling problem                                       |                                              |
| Single Machine Total Tardiness Problem (SMTTP) | Single machine total tardiness problem                                           | Salhi S. et al., 1997                          |
|                                     | Single machine total weighted tardiness problem                                       | Reimann et al., 1998                           |
|                                     | Single-machine total tardiness problem with sequence dependent setup times           | Salhi S. et al., 2002                           |
| Set Covering Problem (SCP)          | Partition problem                                                                     | Dorigo M. et al., 1995                          |
| Multiple knapsack problem (MKP)     | Weight constrained graph tree partition problem                                       | Gambardella L., 1997                           |
| Maximum Clique Problem (MCP)        | Maximum independent set problem                                                      | Dorigo M. et al., 2000                          |
All the solutions proposed to the problems mentioned above are, in one way or another, part of local and regional strategies based on various algorithms, most of them inspired from the social behavior of biological entities living in nature (swarm intelligence methods), such as ant nests, swarms of bees, swarms of termites, pack of wolves, schools of fish, as shown in Fig. 1 [9, 10].

Emerging structures are models that do not appear as a result of a single rule or a simple condition. No clearly defined control structure generates the appropriate signals to create emerging behavior. It is determined by the interactions that occur between the parts of the system, as well as between them and the environment. Thus it can be concluded that these systems that exhibit emerging behaviors are more than the sum of their components.

One of the most eloquent examples of emerging behavior in the biological field is ant colonies. The queen does not give direct orders to certain groups of ants to start looking for food or to defend the mound. Each ant reacts to chemical stimuli generated by larvae, other ants, intruders, etc. and in turn leaves a trail of pheromone chemistry that is a stimulus for the other ants.

Under these conditions, we can say that any ant reacts only to the local environment, and the rules of behavior for each type of ant are genetically coded. In the absence of centralized control, ants exhibit extremely complex behavior that allows an ant colony to solve geometric problems - usually, ants identify the longest distance from all mound entrances to store corpses. Several other emerging behaviors in which communication between individuals is in one way or another based on pheromones or takes place in another chemical way are represented by swarms of bees, termites, etc. In nature, a multitude of emerging behaviors can be identified: flocks of birds, herds of animals, packs of wolves, etc.

Strictly for the present study, we considered only the behavior of an ant community. The present approach begins with the definition of the emergent behavior and implicitly of the behavior of ant colonies and ends with the discussion of some results obtained as a result of running an ACO application made in W. Mathematica.

![Fig. 1. A non-exhaustive list of metaheuristics algorithms [9]](image-url)
2. MATERIALS AND METHODS

For the present paper, a series of articles were considered that expose in extenso the issue of emerging behavior, respectively the issue of behavior specific to ant colonies (as indexed via Table 1), with emphasis on the particular applications of ACO. An important place in the documentation and realization of this study had the articles that referred to various applications and implementations of ACO, especially those related to Wolfram Mathematica. At the same time, various mathematical formulations were used to substantiate the study and to determine the conditions for variation of the entire research approach.

For this paper, we chose to make a series of applications in the Wolfram Mathematica environment, to exemplify the ACO applied for TSP. The number of cities, the number of ants, the number of leaders, and the evaporation index of pheromones were considered as variables - all having an essential role in the factual solution of TSP.

3. RESULTS AND DISCUSSION

3.1 Emergent Behavior: Principles and Examples

Emergence is the process by which complex behaviors are formed starting from extremely simple rules. This process is dynamic, with entities presenting the emergency over a longer period [11]. At the same time, emergence can occur between entities of different sizes in many orders of magnitude. Emergent behavior can occur when several simple entities (agents) operate in an environment forming more complex behaviors than a whole. This complex behavior is not the sum of individual behaviors, nor can it be deduced or predicted based on these individual behaviors of low-level entities.

Emerging properties are unpredictable. In many cases, the manifestation of emerging behavior is a new phase in the evolution of the system of entities. One reason that may explain the emergence of emerging behaviors is that the number of interactions between individual entities increases exponentially with their number. This gives rise to the possibility of new subtle behaviors, which cannot be deduced or predicted based on a rigorous mathematical apparatus due to the extremely large number of possibilities.

However, a large number of interactions between entities do not individually explain the emergence. Situations were identified in which a large number of entities did not lead to the emergency, but, on the contrary, determined the complete elimination of the emerging behavior from the system [11]. At the same time, a large number of entities produce a proportional noise level that masks the useful signal until it is eliminated. Thus it is shown that not only a large number of entities in the system or the large number of interconnections made between them matter but also how these connections are organized. It seems, however, that emerging behavior occurs especially in the case of highly decentralized structures.

Systems that exhibit emerging behavior may appear to defy the principles of entropy or the second law of thermodynamics because they form and increase in size in the absence of centralized command and control structures [6]. This is possible because these open systems can retrieve information directly from their environment. The emerging behavior of autonomous systems together with the extreme scale of these systems leads to the impossibility of predicting all possible scenarios for configuration or errors. It is virtually impossible to create leadership or recovery strategies for all possible outcomes.

Emerging behavior, characterized as nondeterministic of natural systems, is called this because it is formed from the behaviors of many simple agents who follow a limited set of simple rules-based only on their local view of the system. Individual agents are not aware of the overall state of the system or the ultimate functional goals of the system. Communication is imperative in both types of distributed systems, both deterministic and emerging (natural).

In the natural field, the study of ant colonies revealed that the exchange of pheromones plays a very important role in communication inside the anthill. In natural systems, pheromone exchange is an important element that generates emerging patterns of behavior for the entire system. Individual ants form a localized image of the entire system based on the exchange of pheromones between them and the other ants they encounter. Different types of pheromones correspond to different information, ants also extract information from the intensity of pheromones. By extracting information from pheromones, ants determine the behavior of
neighboring ants and from here they can deduce their role, without establishing a dialogue or any other type of negotiation.

Pheromone exchange has a simple way of communicating through which information is transmitted using a series of chemical messages. The receiving entity can extract information from the chemical message itself. However, additional information (meta-data) can be extracted from the chemical message, such as the importance of the message (interpreted from the frequency of the pheromone transmission) or how recent the message is (deduced from the chemical power of the pheromone).

Natural pheromone communication is non-deterministic and yet extremely reliable, using a small number of simple messages, it is stable, robust, and scalable. It is thus tried to implement a type of communication for distributed systems in which the messages are implemented as pheromones; eliminates the need to establish a connection between communication partners. The disadvantages of using a non-deterministic system are the empirical determination of the optimal operating conditions: the use of the emerging pheromone-based communication protocol is not effective in the case of a small number of entities, as no anteater survives with a limited number of individuals. At the same time, an exaggerated number of individuals communicating through pheromones invariably leads to the bottleneck of interconnection networks.

The patterns of emergent behavior inspired by nature, presented in this subchapter, are very suitable for embedded systems. Usually, the patterns of emergent behavior together with the operating algorithms that can be derived from them are very simple, they require low computing power, suitable for their use in the case of intelligent sensors and sensor networks. The functionality of these patterns of emerging behavior requires looking at the system where they are applied as a whole that is greater than the sum of its parts. One of the main disadvantages of using emergent behavior is that it only works in the presence of a relatively large number of entities.

3.2 Ant Colony Optimization: Background and Main Features

Applications of emergent behavior patterns are found not only in practical, real cases of physical entities. Emerging behavior patterns inspired by the study of ant colonies are frequently used to solve complex problems whose solution is a heuristic one. Emerging phenomena result from simple interactions that occur between the component entities of a system. Consequently, there are several problems for which finding an optimal solution is difficult and sometimes impractical due to the lack of a mathematical apparatus. To solve these problems, it is preferable to identify heuristic techniques to generate appropriate solutions that are not always optimal.

There are normally many problems of combinatorial complexity to solve which finding an optimal solution is difficult and sometimes impractical due to too much solution space or due to too high computational requirements generated by existing constraints. To solve these problems, it is preferable to identify heuristic techniques to generate appropriate solutions that are not always optimal.

Metaheuristics are algorithms that are driven by a simple, basic heuristic to escape the trap of local optimal and a metaheuristic part. They either use a constructive heuristic that starts from an empty solution to which are then added elements necessary to reach an acceptable solution or a heuristic based on local searches that start from a complete solution to which it modifies the elements iteratively to reach a better solution [12].

The metaheuristic part allows the use together with the basic heuristic to obtain better results than in the absence of it, even by iterating the basic heuristic. The improvement mechanism is used either by introducing constraints, or by randomly choosing the solutions considered for the local search, or by combining elements extracted from different previous solutions.

The study of ant colonies showed that they can identify the shortest path between the anthill and the food source, without using visual evidence. These ants are also able to adapt to environmental changes, such as the invalidation of a known path between the moss and a food source through the appearance of an obstacle. In Fig. 2 the ants move in a straight line between the food source and the moss, as there is no obstacle in their way. However, communication between individuals involves the use (deposition) of a certain amount of pheromones on objects of interest, or even while walking, thus marking the path they follow.
Fig. 2. An uninterrupted path between the moss and the food

Ants prefer to follow the directions rich in pheromone traces, choosing in most cases the richest and most recent directions in pheromones. Pheromones excreted by ants can expire over time, which allows an unused direction to eventually be disregarded. This elementary behavior of ants is used to explain the method by which ants can identify the shortest path that connects two points of the pheromone pathway, interrupted by placing an obstacle, as seen in Fig. 3.

Once the obstacle appears, those ants that are just in front of it can no longer follow the path of pheromones, and therefore have to decide the direction in which they should move, respectively to the left or the right. Probably half of the ants will choose to bypass the obstacle on the right, and the rest will choose to bypass the obstacle on the left. A similar situation can be found on the other side of the obstacle, as shown in Fig. 4.

Those ants who have decided, by chance, the shorter way around the obstacle will reconstitute the path of pheromones interrupted by the obstacle faster, compared to the ants that have chosen the long way. Thus, on the shorter path, a greater amount of pheromones will be deposited, per unit time, than on the longer path, which will cause a greater number of ants to choose that path. In turn, that path will lead to an increase in the number of pheromones deposited. Due to this autocatalytic process (positive feedback), all ants will choose the shorter path very quickly, as shown in Fig. 5.

An interesting aspect of this autocatalytic process is that the identification of the shortest path that bypasses the obstacle is an emerging property of the interaction between the shape of the obstacle, the way the obstacle is placed, and the distributed behavior of the ants.
Although all ants move at about the same speed and deposit about the same amount of pheromones per unit time, the fact that a path is shorter leads to a faster accumulation of pheromones.

The ants of the artificial colony, considered for the present study, can generate successively shorter and shorter feasible rounds, using the information accumulated in the form of a pheromone strip, stored on the edges of the graph of a TSP problem. The colony of artificial ants can generate good solutions, both for symmetrical and asymmetrical instances of the TSP [5,6]. This combined with the ants' natural preference for following pheromone-rich directions, and the pheromone's expiration property shows that in a relatively short time the longer path is completely abandoned. ACO was first introduced using the Traveling Salesman Problem (TSP) - starting from the initial node, an ant moves iteratively from one node to another. When it reaches a node, an ant chooses to visit an unvisited node at time t, with a probability given by the relation below:

\[ p_{i,j}^k(t) = \frac{[\tau_{i,j}(t)]^\alpha [\eta_{i,j}(t)]^\beta}{\sum_{a \in N_i^k} [\tau_{i,a}(t)]^\alpha [\eta_{i,a}(t)]^\beta} \quad i, j \in N_i^k \]

where:

- \( N_i^k \) is the most feasible neighborhood for ant \( k \), but this neighborhood includes only the cities that have not been visited before;
- \( \tau_{i,j}(t) \) is the pheromone value of the \((i,j)\) arc, at the moment \( t \);
- \( \alpha \) is the weight of the pheromone;
- \( \eta_{i,j}(t) \) is a heuristic information about the \((i,j)\) arc, at the moment \( t \);
- \( \beta \) is the weight of the heuristic information considered.

The element \( \eta_{i,j}(t) \) is determined by the formulas:

\[ \tau_{i,j}(t) = p \cdot \tau_{i,j} + \sum_{k=1}^{n} \Delta \tau_{i,j}^k(t) \] and

\[ \Delta \tau_{i,j}^k(t) = \begin{cases} \frac{Q}{t_{a}(t)} & \text{if the contour}(i,j)\) is chosen by ant \( k \\ 0, \text{otherwise} \end{cases} \]

where \( p \) is the evaporation rate of the pheromone \((0<p<1)\) and \( Q \) is a constant for pheromone updating. The characteristics of the ACO algorithm variant proposed by Colomi and Dorigo in 1991 refer to versatility - the same algorithm can be applied to different variants of a problem, robustness - the same algorithm can be applied, with small modifications, to different problems, and positive feedback (from populations).

### 3.3 Using the ACO Algorithm in Solving the TSP

TSP involves finding a way to go through n cities, so that the traveling cost is minimal, and each city is visited only once. For the present study the Wolfram Mathematica was chosen as the implementation medium, which presents as a major advantage the existence of a useful graphic package for simulating, and exported in the Computable Document Format (working files with the extension *.cdf).

Fig. 6 presented some images of the application, made in W. Mathematica and exported as a .cdf file, running for different values of the parameters, of which we can mention the following: number of cities, number of ants, number of leaders (elite ants), and pheromone evaporation index. Based on the results obtained, a table with some of the used values was attached. Some comments and annotations were made, also based on the results obtained.

In Fig. 6 it can be notice that for a different number of cities different results were obtained, and the solving time increases substantial with the increase of the number of cities.

In Fig. 7 it can be noticed that for a different number of ants different results were obtained. Analogue, in Fig. 8, in the case of elite ants. In both cases, the solving time increases very low with the increase in the number of ants, or elite ants.

The case study (regarding the use of ACO to solve TSP) took into account different values of running parameters (no. of ants, pheromone evaporation index, no. of cities/nodes) for which it involved the analysis of the results obtained relative to various environmental changes. Also, the case study allowed the issuance of hypotheses on the possibilities of improving the presented algorithm (optimization of running...
Fig. 6. Solving the TSP for different cities values

Fig. 7. Solving the TSP for different ants values

Fig. 8. Solving the TSP for different elite ants (%) values
Table 2. Results on the TSP solved by ACO through Wolfram Mathematica apps

| ID | Cities [16-50] | Ants [10-200] | Elite ants [%] [0.5-100] | Pheromone evap. index [0-1] | Solving time (s) |
|----|----------------|----------------|--------------------------|--------------------------|-----------------|
| A  | 16             | 100            | 1                        | 0.8                      | 4.6             |
|    | 30             | 100            | 1                        | 0.8                      | 12.2            |
|    | 50             | 100            | 1                        | 0.8                      | 52.4            |
| B  | 16             | 10             | 1                        | 0.8                      | 1.5             |
|    | 16             | 50             | 1                        | 0.8                      | 5.0             |
|    | 16             | 200            | 1                        | 0.8                      | 6.6             |
| C  | 25             | 50             | 1                        | 0.8                      | 13.4            |
|    | 25             | 50             | 10                       | 0.8                      | 10.0            |
|    | 25             | 50             | 50                       | 0.8                      | 32.9            |
| D  | 20             | 75             | 1                        | 0.8                      | 5.1             |
|    | 20             | 75             | 1                        | 0.9                      | 14.0            |
|    | 20             | 75             | 1                        | 0.99                     | 41.5            |

parameters), so that it can move to a new stage of searching for a more efficient algorithm for economic problems occurring daily. Next, we present a series of discussions regarding the obtained results, through which we want to show that their correlation can be done with the execution time of the algorithm, as shown in Table 2. As such, we also referred to the working interval in which the considered parameters vary; so that the [16-50] interval was imposed for the number of cities, [10-200] range for the number of ants, [0.5-100] for the number of elite ants, and [0-1] for the pheromone evaporation index.

If the application runs for a different number of cities - as shown in subsection A of Table 2, it can be observed that for the same number of ants, the same number of leaders (elite ants) and the same pheromone evaporation index, the execution time of the algorithm to solve the TSP is direct and strictly proportional to the number of cities considered. In other words, the solving time increases with the increase in the number of cities, which is more than logical.

If the application runs for a different number of ants - as shown in subsection B of Table 2, it can be observed that for the same number of cities, the same number of leaders (elite ants) and the same pheromone evaporation index, the execution time of the algorithm to solve the TSP is direct and strictly proportional to the number of ants considered. In other words, the solving time increases with the increase of the ant's number, which is also more than logical.

If the application runs for a different number of leaders (elite ants) - as shown in subsection C of Table 2, as a percentage, it can be observed that for the same number of cities, the same number of ants and the same pheromone evaporation rate, the execution time of the algorithm to solve the TSP increases for values below 10% and values above 50%. In other words, if the number of elite ants is over 50% of the total ants then the time to solve the problem increases substantially. At the same time, the level of convergence varies over the range [0.85-0.92], in which case the application can no longer provide a viable solution.

If the application runs for a different value of the pheromone evaporation index - as shown in subsection D of Table 2, it can be observed that for the same number of cities, the same number of ants and the same number of leaders (elite ants), the execution time of the algorithm to solve the TSP is direct and strictly proportional to the value of the index. In other words, the solving time increases with the increase of the index, which makes that for an evaporation index equal to 1 the application to run more than 200s.

4. CONCLUSION

Currently, the interest in metaheuristic algorithms and the possibilities of using them to solve different daily problems is growing in more and more interest fields. The current economic context generates problems that are increasingly difficult to solve by classical methods, which increasingly requires a form of algorithm adaptation to environmental complexity. Starting from these hypotheses, the optimization algorithm based on the study of the emerging behavior of ant colonies was brought to the fore, by presenting the principles on which it is based and especially the variety of applications that ensure finding a reliable result for the most complex problems. The next stage of the
research will aim at improving the model of the optimization problem, by adding hypotheses that allow the observance of several conditions, such as the consideration of a multi-objective function, and the hypothesis of activities parallelization.

As it could be observed, as a result of the results obtained by running the program created in Wolfram Mathematica, the TSP is a particularly difficult problem to quantify and solve. The variation of the parameters within a certain imposed interval having a decisive word to say in choosing the best working variant, but especially in the optimization process of the problem considered.

COMPETING INTERESTS
Authors have declared that no competing interests exist.

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