Accelerating medium effect as a general wave phenomenon

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Abstract. Already at the end of the last century theory predicted that the wave number and frequency of any wave will change when passing an accelerating refractive medium. The effect was calculated both for electromagnetic and neutron waves. As a refractive index may be introduced for waves of any nature one can speak about a very general Accelerating Medium Effect. As far as we know this effect has not yet been observed for light. Here we report on a neutron-optics experiments performed with ultra-cold neutrons where this effect has been demonstrated for the first time ever. The maximum energy transform in the experiment was \( \pm (2\div6) \times 10^{-10} \) eV which agrees with theory within less than 10%. Possibilities for future investigations of the Accelerating Medium effect will be discussed.

1. Introduction

Similarities in the description of light optics and quantum mechanics have given rise to particle optics as an independent field of research in the middle of the last century. In the forties E. Fermi introduced the index of refraction for neutron waves [1]. Later the work of Foldy [2] and Lax [3,4] allowed to understand that an index of refraction can be introduced for any material and waves, if the material contains scattering centers for the considered wave. Of course that is also true for matter-waves. In this case the index of refraction is given as

\[
n^2 = 1 + 4\pi\rho k_0^2 f(0 \rightarrow 0),
\]

where \( k_0 \) – is the wave number of the incoming wave, \( \rho \) - the spatial density of the scattering centers and \( f(0 \rightarrow 0) \) - the forward scattering amplitude.

In neutron optics the majority of phenomena studied so far were such previously investigated and known from light optics. This was also the case, when during the seventies of the 20th century Horn and Zeilinger [5] started to investigate the interaction of neutron waves with moving matter. Due to
their analogies with light optics the experiments were called neutron Fizeau experiments. However, regardless the large number of similarities, there are substantial differences in the passage of electromagnetic and neutron waves through moving matter [6].

The phase shift of electromagnetic waves in Fizeau experiments can be explained by relativistic transformations of the velocity terms into a moving coordinate system. Neutron optics considers non-relativistic particles and the transformation into a coordinate system moving with the particle is a Galilean transformation. Further, in the most frequent cases, the forward scattering amplitude for neutrons on atomic nuclei can be considered as constant \( f(0 \rightarrow 0) = -b \) and the interaction of the neutron with matter is described by means of an effective potential

\[
U = 2\pi \hbar^2 \rho b, 
\]

where \( b \) – is the volume average of the coherent scattering length. Any movement of the matter itself which does not change its effective potential has no effect on the neutron and a phase shift only occurs when the sample boundaries move.

However, if a layer of material moves together with its boundaries, it will cause the wave frequency in the matter, measured in the laboratory system, to be different from the one in vacuum. This is valid –in light [7] as well as in neutron optics [8]. In both cases the frequency shift is caused by the Doppler effect

\[
\Delta \omega = (n-1) k_0 v , 
\]

where \( n \) – is the index of refraction in the moving reference system, in which the matter is at rest, \( k_0 \) – is the wave number of the incoming wave, \( v \) – is the speed of the moving matter.

For a constant motion of the matter the Doppler shifts during the passage of the two boundaries have the same amplitudes but different signs. The overall effect is zero and the frequency of the wave transferred through uniformly moving matter does not change. For a long time the condition that the matter moves with constant velocity was underestimated, although it is decisive. In the case of non-constant motion, however, the frequency shifts at the sample boundary do not cancel and hence, the frequency of the transmitted wave differs from the incoming one. This was firstly shown by Tanaka [9], who solved the theoretical problem of electromagnetic wave transmission through linearly accelerated dielectric matter on the basis of covariant generalization of the Maxwell equations. Corresponding experiments in light optics have never been carried out although different possibilities have been discussed [10].

The transmission of neutrons through a layer of matter moving with linear acceleration was considered by Kowalski [11] in the context of a new type of experiment for the verification of the equivalence principle. The author concludes that the energy of the neutrons must change after transmission through such a linearly accelerated layer. This was confirmed later by Nosov and Frank [12] who calculated the velocity of neutrons transmitted through the accelerated boundaries of a sample.

The first experimental evidence of a neutron energy change after transmission through accelerated matter can be found in [13], while a more detailed investigation of the accelerated matter effect (AME) is given in [14]. Recently we observed the acceleration and deceleration of neutrons after transmission through an oscillating sample for the first time. The results of these experiments are given in section 3.

The AME is a general optical effect, which has only been demonstrated so far in neutron-optical experiments. Some new and more general aspects of this effect will be discussed in section.
2. The accelerating medium effect and the equivalence principle

As shown in ref. [14] the energy change due to the AME can be derived in good approximation from the equivalence principle without too detailed calculations. We will follow this derivation here, as it demonstrates the universality of the effect. According to Kowalski [11] the following Gedanken experiment is considered (fig. 1):

Assume the wave moves from the source towards the detector in the field of gravity such that its frequency increases during flight (fig. 1a). As there are no dissipative processes involved, the introduction of a refracting sample between source and detector cannot change the wave frequency when its detected. Any frequency change can be excluded as one imagines the wave to be elastically back reflected (without passage through the sample). In case of a frequency change the energy conservation law would be violated. Now one assumes (see figure 1b) the source, the detector and the refracting sample move all together with an acceleration $w=g$. Here $g$ is the free fall acceleration in the system of inertia and the wave, measured in the laboratory system, has a constant frequency. According to the equivalence principle the frequencies measured in both systems have to be the same.

![Figure 1. Neutron propagation through a refractive sample (a) in an inertial reference frame in the presence of gravity and (b) in a noninertial reference frame](image)

However, the refracting sample in the flight path of the wave acts on its propagation time as the phase velocity inside the sample differs from the velocity in vacuum. For simplicity we assume that the inserted sample increases the propagation time by $\Delta t$. During this delay time the detector continues to accelerate and the additional (relative to the situation without sample) change of its velocity is $\Delta v = w \Delta t$. Therefore, if the refracting sample would only cause a time delay, the speed of the detector at the time when the wave is detected would be different in the laboratory system with and without refracting sample. Therefore, due to the Doppler-effect, the registered wave frequency would also depend on the delay time and therefore on the sample properties. As this contradicts the equivalence principle we have to conclude that during the transmission through the refracting sample also the frequency of the wave changes. This frequency change should compensate the Doppler-shift.

For neutrons and other non-relativistic particles the delay time is

$$\Delta t_{(n)} = \frac{d}{\sqrt{n} - 1}, \quad (2)$$


where \( t \) – is the time of flight, \( v \) – is the particle velocity and \( d \) – is the sample thickness. Accordingly, the energy, necessary for to compensate the Doppler-shift is \( \Delta E = mv \cdot \omega \Delta t \). Inserting equation (2) into this expression, we obtain

\[
\Delta E \equiv mvd \left( \frac{1}{n} - 1 \right),
\]

which is in complete agreement with the results in [11,12]. For photons and ultra-relativistic particles the delay time is

\[
\Delta t_{(\gamma)} = \frac{d}{c} (n - 1),
\]

and the Doppler-shift of the frequency (in first order of \( \frac{v}{c} \)) is \( \Delta \omega \approx \omega \Delta = \omega \Delta t/c \). Inserting equation (4) into this expression we get the same expressions as Tanaka [9] for light

\[
\Delta \omega = \frac{\omega}{c^2} (n - 1)vd.
\]

It is worth mentioning that for the derivation above we were only exploring the fact that the wave number inside the accelerated matter is different to the one in vacuum. Therefore the effect should occur not only for accelerating matter containing scattering centers but much more generally for any volume containing force fields.

3. Observation of the accelerating matter effect in experiments with ultracold neutrons.

3.1. Spectroscopy with UCN

The change of energy by transmission through an accelerating sample was firstly measured several years ago in experiments with ultracold neutrons. Neutrons passed through a silicon sample, which oscillates. Accordingly the energy change of the quasi-monochromatic neutrons is given as

\[
\Delta E \equiv -mA \Omega^2 d \left( \frac{1-n}{n} \right) \sin \Omega t, \quad \Omega << \frac{v_i}{d},
\]

where \( A \) and \( \Omega \) - denote the oscillation amplitude and frequency of the sample and \( v_i \) – their velocity inside the sample material. The layout of the experimental setup, a modified spectrometer of [15,16], is shown in Figure 2. Ultra-cold neutrons arrive at the main part of the device, passing through an annular corridor. At the exit of the corridor, there is a monochromator \( I \), realized via a neutron interference filter – a Fabry-Perrot interferometer (FPI). It transmits UCNs with a narrow spectrum of vertical velocities. Sample 2, a silicon plate of 0.6 or 1.85 mm thickness, can execute harmonic motion by means of the special driver \( J \) and is placed just below the monochromator. Sample oscillation frequencies of 40 or 60 Hz were used. The variable acceleration of the sample reached 90m/s².

Passing the monochromator and sample, neutrons arrive at a vertical mirror neutron guide, where the second FPI, analyzer \( 4 \), is located and whose position can be varied in height. A scintillation detector for UCNs \( 5 \) is placed below the analyzer. The transmission line maxima of the monochromator and analyzer correspond to 107 and 127neV, respectively. The dependence of the count rate on the distance between the filters is qualitatively presented in Figure 3. The sign-alternating change in the neutron energy caused by passage through a sample moving with variable
acceleration leads to the periodic variation in the count rate, as shown by the dashed straight lines in Figure 3.

![Figure 2. Layout of the experimental setup](image1)

![Figure 3. Count rate vs. the distance between the filters (scanning curve) and the detection principle of periodical variation of energy](image2)

The maximum change in the energy of UCNs, which is determined by Eq. (6), reaches approximately 0.2 neV in a first experiment \[13\] and 0.6 neV in the second experiment \[14\].

Unfortunately, the effect of accelerating matter is not the only origin of count rate modulation. Obviously the velocity \( V(t) = \Delta \Omega \cos(\Omega t) \), as well as the acceleration of the plate \( \alpha(t) = -\Delta \Omega^2 \sin(\Omega t) \), change simultaneously when the plate was moved. The periodic variation in the sample transparency modulates the intensity with a relative amplitude of about 8%. Therefore the relative change in the intensity is given by the expression

\[
P(t) = \frac{I(t)}{I_0} = 1 - B \cdot \cos(\Omega t + \alpha), \quad B = \sqrt{a^2 + b^2}, \quad \alpha = \arcsin \left( \frac{a}{B} \right),
\]

Here, the amplitude \( a \) determined by the acceleration of the sample is proportional to the derivative of the scanning curve and depends on the analyzer position. At the maximum of this curve, \( a = 0 \) and the count rate oscillation is determined only by the velocity effect in the transmission with amplitude \( b \). Formula (7) determines the experimental procedure. Measuring the modulation phase for various analyzer positions and using the known scanning curve, one can determine the amplitude of the periodic change in the energy of the neutron passing through the accelerating sample.

The phase and amplitude of the count rate modulation were measured in the experiment for various positions of the analyzer filter. The time dependence of the count rate was determined in a time interval equal to the oscillation period. The origin of the scale was specified by a generator controlling the sample motion. The data were normalized to the averaged count rate and fitted to the function \( f(t) = 1 + B \sin(\Omega t - \varphi) \). The amplitude \( B \) and phase \( \varphi \) of the count rate oscillation were
determined by such a way in each value of the position of the analyzer. Some results from the phase measurements are shown in Figure 4.

![Figure 4. Measured phase of the counting-rate oscillation versus the analyzer position. Solid (red) curve – Monte-Carlo simulation. Dashed (blue) line – theoretical prediction for the phase in the absence of the Accelerating medium effect](image)

Monte Carlo simulation of the oscillation phases were repeated using the relation

$$\Delta E = K nA_\Omega^2 L \left[ \frac{(1 - n)}{n} \sin \Omega t \right]$$

which differs from equation (7) by the correction factor $K$. For the latter one a value of $K = 0.94 \pm 0.06$ was found. Thus, the results of the measurements of the counting-rate oscillation phase are in reasonable agreement with the existing theory of the accelerating medium effect.

3.2. The Accelerating Medium Effect and time focusing of neutrons.

3.2.1. Experimental Principle

Recently a new type of experiment for the observation of the AME was carried out [18]. Here a plate, vibrating in space, was used as periodic modulator to change the velocity of ultra-cold neutrons. The main idea of this experiment is rather similar to experiments on time focusing [19] and its main principle is illustrated in Figure 5. Monochromatic neutrons are transmitted through a modulator – acting as a time lens. The neutron velocity was changed periodically such that in the ideal case all neutrons arrive simultaneously in the detector L. The lens is working in a cyclic regime and time focuses those neutrons passing through it in one cycle. In reference [18] an aperiodic diffraction grating moving across the neutron beam was used as a lens. In the current experiment we used a plate of silicon oscillating along the propagation direction of the ultra-cold neutron beam. Moving with periodic acceleration it was periodically accelerating or slowing down the neutrons due to the AME. Here the momentum transfer to the neutrons was insufficient for an efficient focusing and the focal plane was significantly away (to the right side) from the plane L. However, in the detection plane there occurs a substantial concentration in time of flight of the incoming neutrons. This leads to a time modulation of the count rate at the detector. It can be shown that in this case the time dependence of the detector count rate is determined by the derivative of the modulation function $\Delta v(t)$. The change of velocity by passing the oscillating plate is according to equation (6) given by
\[ \Delta N_a = -A \Omega^2 \frac{d}{v_0} \left(1 - \frac{n}{n_0}\right) \sin \Omega t \]

and therefore the weak focusing leads to the following time dependence of the count rate

\[ N_a(t) = N_0 + C_a A \Omega \left(1 - \frac{n}{n_0}\right) \cos(\Omega(t + \tau)) \]  

(8)

where \( N_0 \) - is the mean count rate, \( \tau \) - is the average time of flight and \( C_a \) – is a constant coefficient.

\[ t \]

\[ L \]

\[ T \]

\[ a \]

\[ \text{Figure 5. A coordinate versus time diagram illustrating the idea of time focusing.} \]

Obviously the harmonic movement of the sample leads to a periodic variation of the neutron velocity with respect to the sample. As the dependence of all absorbing processes in the case of ultra-cold neutrons is proportional to \( 1/v \) the transmission of the sample is also changing accordingly. Therefore, additionally to the AME given by equation (8) there is another systematic effect described by the following equation

\[ N_f(t) = N_0 + C_f A \Omega \cos(t + \tau) \]

As both effects are synchronous, the harmonic modulation of the count rate is obtained by summing the amplitudes of both

\[ \Delta N(\Omega) = A \Omega^2 \left( C_a \Omega + \frac{C_f}{\Omega} \right) \]  

(9)

Expression (9) is written such, that it underlines the main principle of the given experiment: the measurement is carried out such that the quantity \( A \Omega^2 \) stays constant and the AME is growing proportionally while the systematic velocity effect if inversely proportional to the frequency of modulation.

The systematic velocity effect was encountered in former experiments [13,14]. In these measurements it was already found that if the monochromatisation happens after transmission through the sample the AME is excluded. All experimentally observed modulations are due to the systematic velocity effect. This finding has been explored for the current measurement.

The experimental strategy consisted in the measurement of the modulation amplitude of the ultra-cold neutron count rate transmitted through an oscillating sample for a large set of frequencies \( \Omega \) and...
fixed values of $A \Omega^2$. The measurement was carried out in two geometries. In the first case the monochromatic neutrons were transmitted through the sample and the amplitude modulation was given by equation (9). In the second case the neutrons were transmitted through the oscillating sample and only after through the monochromator. In the second case the modulation amplitude is defined by the systematic velocity effect only. The frequency dependence of the difference of both effects is defined exclusively by the AME and should depend linearly on the frequency of modulation.

3.2.2. Experimental realisation and results.

For the experiment the same spectrometer as in reference [14] was used. A schematic illustration is given in Figure 2. The only difference consists in the absence of the analyzer 4. As in earlier works we used a Fabri-Perot interferometer as monochromator. It transmits a single wave length at about $107 neV$ with a relative width of $\Delta E/E \approx 0.04$. As a sample we used a wafer with $1.85 mm$ thickness, which was put into oscillation by the same driving stage as in [14]. The time depending acceleration was permanently measured by a piezoelectric sensor, the sinusoidal signal of which was also used for the stabilization of the amplitude of the driving stage. This allowed to stabilize the quantity $w_{max} = A \Omega^2$ on the level of 2%. The measurement was carried out for two values $w_{max} = 57 m/s^2$ and $w_{max} = 72 m/s^2$ and for a frequency range of $f = 20 \pm 100Hz$. The obtained results are shown in Figure 6.

![Figure 6](image)

Figure 6. The count rate oscillation amplitude as a function of frequency for a fixed value of $w_{max} = A \Omega^2$. On the left hand side $w_{max} = 57 m/s^2$, while on the right hand side $w_{max} = 72 m/s^2$. The measurements were done in two geometries, which symbolically are indicated on the very right. Here M – is the monochromator and S – is the sample.

The results show without any doubt the presence of weak time focusing, i.e. the acceleration/slowing down of neutrons when passing the oscillating sample. The modulation amplitude in the geometry excluding the AME effect (lower blue points) is substantially lower than the amplitude of modulation sensitive to both effects (upper red points). Further, it can be seen that the difference between both effects is growing with an increase of the modulation frequency.

Unfortunately it is difficult to directly compare these data with calculations. The reason is that the modulation amplitude is also depending on the dispersion of the time of flight, i.e. on the width of the spectrum transmitted by the monochromator. Additionally it turned out that the background of the spectrometer is slightly changing, when the monochromator is changed from one position into another. Therefore a number of calibration measurements were added, in which the modulation of the ultra-cold neutron flux was realized via a mechanical chopper. In these measurements the non moving silicon sample was also present and the monochromator was changed between different positions.
Such calibrations allowed to normalize the data of Figure 6. The difference of the two curves should be a straight line, corresponding to the first term of equation (9). These results are shown in Figure 7 and it is possible to compare them directly to theoretical calculations. The main parameter here is the inclination angle of the straight line. The calculations were done by assuming several origins of the background. The dispersion of the calculated background values was included into a systematic error.

Figure 7. The difference between normalized amplitudes of the count rate oscillation versus frequency. The value \( w_{\text{max}} = \Delta \Omega^2 \) was fixed. On the left \( w_{\text{max}} = 57 \text{m/s}^2 \), on the right \( w_{\text{max}} = 72 \text{m/s}^2 \).

The error of the linear fit of the data in Figure 7 was considered to be the statistical error. From the obtained results it was possible to estimate the agreement of the calculated velocity change when neutrons are transmitted through an oscillating refractive sample.

For the factor \( K = \Delta \nu_{\text{exp}}/\Delta \nu_{\text{exp}} \) and values of \( w_{\text{max}} = \Delta \Omega^2 \) as mentioned above the agreement of experiment and calculation was: \( K_1 = 0.95 \pm 0.10_{\text{stat}} \pm 0.05_{\text{syst}} \) and \( K_2 = 0.95 \pm 0.05_{\text{stat}} \pm 0.05_{\text{syst}} \)

4. The Accelerating Matter Effect in the case of double refraction

In the present works concerning the AME the polarization of waves was not considered so far. In order to partially fill this gap we will consider briefly the case of matter with double refraction, which is characterized by two indices of refraction \( n_{\pm} \), according to two polarizations of the incoming wave.

In light optics double refraction (birefringence) is a commonly known phenomenon. Obviously, when light is moving through double refracting matter, the phase shift given by the Tanaka equation (5) becomes depending on the direction of the E-vector with respect to the sample. It is possible that a comparably low frequency of rotation of the polarization can be easier detected than the small shift of frequency.

In neutron optics the quantities \( n_{\pm} \) correspond to two different projection of the neutron spin on a physical axis. Accordingly we will rewrite equation (3) as

\[
\Delta E_{\pm} \approx m \omega d \left( 1 - n_{\pm} \right) / n_{\pm}, \quad \Delta \omega_{\pm} = \Delta E_{\pm} / \hbar
\]

After transmission through an accelerated birefringent sample the two spin components of the neutron wave function differ by a frequency and form a non-stationary superposition. In the case of an arbitrary polarization of the original wave function \( \Psi_0(x,t) \) the final state will have the form

\[
\Psi(x,t) = \left\{ \exp \left[ -i \left( \Delta k_{\pm} x + \Delta \omega_{\pm} t + \chi_{\pm} \right) \right] \uparrow + \exp \left[ -i \left( \Delta k_{\pm} x + \Delta \omega_{\pm} t + \chi_{\pm} \right) \right] \downarrow \right\} \Psi_0(x,t)
\]

(10)
where

$$\Delta k_z = \frac{m \Delta E_z}{\hbar^2 k_0}.$$ 

The constant phase angles, $\chi\pm$, that are irrelevant to what follows, determine spin directions after transmission through a moving sample [19]. The wave function in equation (10) describes the state with spin precession. The precession angle is obtained from the difference between the phase angles of two spin components:

$$\varphi(x,t) = \left(\Delta k_+ - \Delta k_-\right)x - \left(\Delta \omega_+ - \Delta \omega_-\right)t + \chi_+ - \chi_-.$$ 

Assuming here the simplifying condition $\Delta E_z/E << 1$ it is possible to write that $(\Delta k_+ - \Delta k_-) x = (\Delta \omega_+ - \Delta \omega_-) x / v$, from which follows that in the reference system moving with the velocity of the neutron the direction of the spin vector is unchanged. However, in a fixed point of observation, $x=L$, the spin direction changes periodically in time with the frequency $\Omega = \Delta \omega_+ - \Delta \omega_-$. This periodic change of the polarization direction can be measured. The beat frequency $\Omega$ and respective energy transfers $h\Omega$ can be quite small.

In neutron optics there can be several physical reasons for double refraction. First of all one should mention the rather simple case of interaction of the magnetic momentum of the neutron $\mu$ with a magnetic field $B$. Obviously any space in which the magnetic field has a nonzero value is acting with two refraction indexes $n_z = \left(1 \mp \mu B / E\right)^{1/2}$. If this space moves together with its field boundaries it will generate according to what was mentioned above a non-stationary state (10). Sample material, put into a magnetic field can also act as double refracting material for neutron waves [20,21]. This will be caused by different neutron wave numbers due to the presence of the magnetic field and the dispersion of the material itself. An accelerated motion of the sample in a constant magnetic field will also lead to states with different frequencies for the different spin components of the wave function. Double refraction might also occur in the absence of a magnetic field. First of all one might focus here on nuclear pseudomagnetism, which takes place when a neutron wave is propagating in matter with polarized nuclei. [22,23]. Due to the spin dependence of the nuclear interaction the coherent scattering length $b_\pm$ is different for the two values of the total spin. As a result, and in accordance with (1) the medium has two refractive indices.

Finally, double refraction might be caused by parity violation in neutron-nucleon interaction. The forward scatter length and consequently the refractive index will be depending on the neutron spin orientation [24,25].

5. Conclusion

It was shown, that the Accelerating Medium Effect is closely related to the equivalence principle (EP). Consequently, the equations which describe the frequency change after their passage through accelerated refractive samples, may be derived not only from first principles but from the EP too. This can be interpreted as additional evidence for the general nature of this effect which exists for waves and particles of different nature. Two experiments detecting the AME in neutron optics were described. The measured energy variations were equal or less than $5 \times 10^{-10}$ eV while the velocities changed by about 1 cm/s. The measurements agree with theoretical predictions better than 10%. New possibilities for the detection and for applications of the AME may be possible by the use of birefringent material.

This work is partly supported by the Russian Foundation for Basic research (project 11-02-00271).
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