Baryon Instability in SUSY Models

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Abstract

A review is given of nucleon instability in SUSY models. The minimal SU(5) model is discussed in detail.

1. Introduction

We begin by discussing proton instability in non-supersymmetric grand unification. The simplest unified model that accommodates the electro-weak and the strong interactions is the SU(5) model\cite{1} and the instability of the proton arises here from the lepto-quark exchange with mass $M_V$. The dominant decay is the $e^+ \pi^0$ mode and its lifetime can be written in the form\cite{2}

$$\tau(p \to e^+ \pi^0) \approx \left(\frac{M_V}{3.5 \times 10^{14} \text{GeV}}\right)^4 10^{31\pm1} \text{yr}$$ (1)

The current experimental limit of this decay mode is\cite{3}

$$\tau(p \to e^+ \pi^0) > 9 \times 10^{32} \text{yr}, \quad (90\% CL)$$ (2)

In non-SUSY SU(5) the $e^+ \pi^0$ mode has a partial lifetime of $\tau(p \to e^+ \pi^0) \leq 4 \times 10^{29\pm2} \text{yr}$. Thus the non-SUSY SU(5) is ruled out because of the p-decay experimental limits. It is expected that the Super Kamiokande will increase the sensitivity of this mode to $1 \times 10^{34}$\cite{4}. That would imply that theoretically the $e^+ \pi^0$ mode would be observable if $M_V \leq 5 \times 10^{15} \text{ GeV}$. In supersymmetric grand unification current analyses based on unification of couplings constants already put a constraint on $M_G$ of about $10^{16}$\cite{5}. Thus it seems not likely that the $e^+ \pi^0$ mode would be observable in supersymmetry even in the next generation of proton decay experiments. Infact, reasonable estimates indicate that $\tau(p \to e^+ \pi^0) > 1 \times 10^{37\pm2} \text{yr}$.

In supersymmetric unification the dominant instability of the proton arises via baryon number violating dimension five operators\cite{6,7,8,9}. In SUSY
SU(5) operators of this type arise from the exchange of Higgs triplet fields and they have chiral structures LLLL and RRRR in the superpotential after the superheavy Higgs triplet field is eliminated. The main decay mode of the proton in these models is $\tau(p \to \bar{\nu}K)$. The current experimental limit for this decay mode is \( \tau(p \to \bar{\nu}K) > 1.0 \times 10^{32} \text{yr} \) \(^{(3)}\).

It is expected that Super Kamiokande will reach a sensitivity of $2 \times 10^{33} \text{yr} \) \(^{(4)}\) while ICARUS will reach a sensitivity of $5 \times 10^{33} \text{yr} \) \(^{(10)}\). Thus it is an interesting question to explore to what extent the new generation of proton decay experiments will be able to test SUSY unified models. Actually we shall show that, unlike the prediction for the $e^+\pi^0$ mode, it is not possible to make any concrete predictions for the $\bar{\nu}K$ mode in SUSY models without inclusion of the low energy SUSY mass spectra which depends on the nature of supersymmetry breaking. Such an explicit supersymmetry breaking mechanism is provided in supergravity grand unification \(^{(11, 12)}\), but not in MSSM. Thus it is only in supergravity grand unification \(^{(11)}\) that one can make detailed meaningful predictions of proton decay lifetimes.

2. GUT Varieties

Even within supersymmetric framework there are many possibilities that may occur. The simplest of these is the minimal SU(5) model. However, one can have extended gauge groups such as SU(3)$^3$, SO(10),...etc. and also string inspired models such as SU(5)$\times$U(1) \(^{(13)}\). There has been several works in the literature where there is a suppression of dimension five proton decay operators. There are a variety of ways in which a suppression of p-decay can occur \(^{(13, 14, 15)}\). One possibility is that matter is embedded in some unusual fashion in the basic particle multiplets. Such a situation arises in the flipped SU(5)$\times$U(1) model where one has an interchange $u \leftrightarrow d$ and $e \leftrightarrow \nu$ relative to the usual embeddings. The other possibility is the presence of some discrete symmetry which might forbid the baryon number violating dimension five operators. In the following we discuss the condition that would forbid such operators in the general case. Let us assume that one has several Higgs triplets 1,2,..N that couple with the matter fields. We make a field redefinition so that the linear combination that couples with matter is labelled $H_1, \bar{H}_1$ while the remaining Higgs triplet field have no couplings. We may write their interactions in the form

$$\bar{H}_1 J^x + \bar{K}_x H_1^x + \bar{H}_{iz} M_{ij}^{xy} H_{jy}$$

where $J$ and $K$ are given by

$$J^x = \lambda^2 \bar{M}_y M^{xy}, \quad K_x = \lambda^1 \epsilon_{xyzw} M^{yz} M^{uw}$$

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Now the condition that the dimension five operators be suppressed is given by

\[(M^{-1})_{11} = 0\]  \hspace{1cm} (6)

Of course satisfaction of the above condition would require either a finetuning or a discrete symmetry. It is generally found that the imposition of discrete symmetries can lead to unwanted light higgs doublets or light Higgs triplets which would spoil the consistency of the unification of couplings with the LEP data. It is possible that string theory may generate the desired discrete symmetries which suppress proton decay without producing undesirable features alluded to above. However, more generally one can expect proton decay to occur in both supergravity and string models. The detailed nature of proton decay modes, their signatures and partial lifetimes would depend on the specifics of the model.

Another problem that surfaces in supersymmetric unified models is that of the doublet-triplet splitting. That is one needs a mechanism that makes the Higgs triplets which mediate proton decay heavy and the Higgs doublet which generate electro-weak breaking light. Normally one simply finetunes the parameters to generate this splitting. Other possibilities consist of the so-called missing partner mechanism, where one uses 75, 50 and 50 representations in SU(5) instead of the usual 24 plet to break SU(5). Here 50 contains a (3, 1) and the 50 contains a (3,1) part in SU(3) x SU(2) decomposition but no (1,2) pieces. Thus the Higgs triplets from the 50 and 50 will match up with the Higgs triplet states from the 5 and 5 when the 75 plet develops a superheavy VEV, leaving the Higgs doublets light. More recently a mechanism has been discussed in the literature which makes use of higher global symmetries such as SU(6) in the GUT sector which lead to light Higgs as pseudo-Goldstone doublets when the local SU(5) symmetry breaks. However, there is as yet no complete model which also gives acceptable pattern of masses to fermions for this mechanism. Several other mechanisms have also been discussed mostly in SO(10) frameworks and make use of vacuum alignment, discrete symmetries etc to achieve the doublet-triplet splitting. In the following we shall assume that the doublet-triplet splitting is resolved and discuss the case of the minimal SU(5) model in detail.

3. Nucleon Instability in Supergravity SU(5)

We discuss now the details of proton decay in the minimal SU(5) model to get an idea of the sizes of the lifetimes of the various decay modes. The
invariant potential of this model is given by

\[ W_Y = -\frac{1}{8} f_{1ij} \epsilon_{uvwxy} H_1^u M_{i}^{uv} M_{j}^{xy} + f_{2ij} \tilde{H}_2 u \tilde{M}_{i}^{uv} M_{j}^{uv} \] (7)

Here the \( M_{xi}, M_{i}^{xy} \) stand for the three generations \( (i=1,2,3) \) of \( \bar{5}, 10 \) plet of quarks and leptons and \( H_1, H_2 \) are the \( \bar{5}, 5 \) plet of Higgs which give masses to the down and up quarks. After spontaneous breaking of the GUT group \( SU(5) \rightarrow SU(3) \times SU(2) \times U(1) \) and integration over the heavy fields one has the effective dimension five operators with baryon number violation given by \( L_5 = L_5^L + L_5^R \) where\[7, 8\]

\[ L_5^L = \frac{1}{M} \epsilon_{abc} (P f_{1}^u V)_{ij} (f_{2}^d)_{kl} (\bar{u}_{Lba} \bar{d}_{Lcj} (e_{Lk}^c (V u_{L})_{al} - \nu_k^c d_{Lal}) + ...) + H.c. \]

\[ L_5^R = -\frac{1}{M} \epsilon_{abc} (V^\dagger f_{u}^d)_{ij} (PV f_{d}^d)_{kl} (\bar{e}_{cRi}^c u_{Raj} \bar{u}_{Rck} \bar{d}_{Rbm} + ...) + H.c. \] (8)

Here the Yukawa couplings \( f_{u}, f_{d} \) are related to the quark masses \( m_{u} \) and \( m_{d} \) as

\[ m_{u}^i = f_{u}^i (\sin 2 \theta_W / e) M_Z \sin \beta \]

\[ m_{d}^i = f_{d}^i (\sin 2 \theta_W / e) M_Z \sin \beta \] (9)

where \( \theta_W \) is the Weak angle and \( \beta \) is defined by \( \tan \beta = \frac{v_2}{v_1} \) where \( v_2 = < H_2^5 > \) and \( v_1 = < H_1^5 > \). Further, \( V \) is the Kobayashi-Maskawa (KM) matrix and \( P \) is a diagonal phase matrix with elements

\[ P_i = (e^{i \gamma_i}), \sum_i \gamma_i = 0; \quad i = 1, 2, 3 \] (10)

The dimension five operators must be dressed by the exchange of gluinos, charginos and neutralinos to produce dimension six operators which produce proton decay. Of all these exchanges the chargino exchange is the most dominant and is governed by the interaction

\[ L_{ui}^\tilde{W} = \frac{i g_2}{\sqrt{2}} (\cos \gamma_- \tilde{W}_1 + \sin \gamma_- \tilde{W}_2) (V \gamma^0 d_L)_{i} - i g_2 (2 \cos \beta M_W)^{-1} (E \cos \gamma_+ \tilde{W}_1 - \sin \gamma_+ \tilde{W}_2)(V m^d \gamma^0 d_R)_{i} \] (11)

where \( \gamma_{\pm} \) are defined in the text preceding eq(30). The dressing loop diagrams which convert dimension five into dimension six operators also include squark and slepton exchanges. However, the sfermion states that are exchanged are
not pure L or R chiral states. As will be discussed later, in Supergravity one has soft susy breaking terms which mix the L and the R terms so that one has a \((\text{mass})^2\) matrix of the form

\[
\begin{pmatrix}
m^2_{\tilde{\nu}Lui} & m_u^2(A_{ui}m_0 - \mu \text{ctn}\beta) \\
m_u^2(A_{ui}m_0 - \mu \text{ctn}\beta) & m^2_{\tilde{\nu}Rui}
\end{pmatrix}
\]  

(12)

where \(A, \mu\) are parameters which will be discussed in sec5. The mass diagonal states are denoted by the scalar squark fields \(\tilde{u}_{i(1,2)}\). These are related to the L and R chiral states as

\[
\tilde{u}_{Ri} = \cos\delta_{ui}\tilde{u}_{i1} + \sin\delta_{ui}\tilde{u}_{i2}, \quad \tilde{u}_{Li} = -\sin\delta_{ui}\tilde{u}_{i1} + \cos\delta_{ui}\tilde{u}_{i2}
\]  

(13)

where \(\delta_{ui}\) is defined by

\[
sin2\delta_{ui} = -2m_{ui}(A_{ui}m_0 - \mu \text{ctn}\beta)/(\tilde{m}^2_{ui1} - \tilde{m}^2_{ui2})
\]  

(14)

The chiral structure of the dimension six operators involves operators of the type LLLL, LLRR, RRLL and RRRR. Of these it is the operators of the first type, i.e., LLLL which are the most dominant. In general one finds many SUSY decay modes for the proton, i.e.,

\[
\bar{\nu}_iK^+, \bar{\nu}_i\pi^+; i = e, \mu, \tau
\]

\[
e^+K^0, \mu^+K^0, e^+\pi^0, \mu^+\pi^0, e^+\eta, \mu^+\eta
\]  

(15)

The dependences of the branching ratios on quark mass factors and on KM matrix elements is shown in Table 1. Also exhibited are the enhancement factors, denoted by \(y_{ik}\) etc, from the third generation squark and slepton exchange contributions in the dressing loop diagrams.

| SUSY Mode | quark factors | CKM factors | 3rd generation enhancement |
|-----------|---------------|-------------|----------------------------|
| \(\bar{\nu}_eK\) | \(m_d m_e\) | \(V_{11}^V V_{21}^V V_{22}^V\) | \((1 + y_{11}^{(K)})\) |
| \(\bar{\nu}_\mu K\) | \(m_s m_\mu\) | \(V_{21}^V V_{21}^V V_{22}^V\) | \((1 + y_{21}^{(K)})\) |
| \(\bar{\nu}_\tau K\) | \(m_b m_\tau\) | \(V_{31}^V V_{21}^V V_{22}^V\) | \((1 + y_{31}^{(K)})\) |
| \(\bar{\nu}_e\pi, \bar{\nu}_e\eta\) | \(m_d m_e\) | \(V_{11}^V V_{21}^{V^2}\) | \((1 + y_{11}^{(\pi)})\) |
| \(\bar{\nu}_\mu\pi, \bar{\nu}_\mu\eta\) | \(m_s m_\mu\) | \(V_{21}^{V^2} V_{21}^{V^2}\) | \((1 + y_{22}^{(\pi)})\) |
| \(\bar{\nu}_\tau\pi, \bar{\nu}_\tau\eta\) | \(m_b m_\tau\) | \(V_{31}^{V^2} V_{21}^{V^2}\) | \((1 + y_{32}^{(\pi)})\) |
| \(eK\) | \(m_dm_e\) | \(V_{11}^V V_{12}^V\) | \((1 + y_{11}^{(K)})\) |
| \(\mu K\) | \(m_s m_\mu\) | \(1 - V_{12}^V V_{21}^V - y_{11}^{(K)}\) |
| \(e\pi, e\eta\) | \(m_dm_e\) | \(1 - V_{11}^V V_{12}^V - y_{11}^{(\pi)}\) |
| \(\mu\pi, \mu\eta\) | \(m_s m_\mu\) | \(V_{11}^V V_{21}^{V^2}\) | \((1 + y_{11}^{(\pi)})\) |
The dependence of $y$ factors, which contain the third generation contributions, on quark masses and KM matrix elements is shown in Table 2. The factors $R_e, R_\mu$, etc that enter in the evaluation of $y$ in Table 2 are the dressing loop integrals and their explicit form is given in ref. [8].

Table 2: Third generation factors.

| $y$ factor | Evaluation of $y$ |
|------------|-------------------|
| $y^l_1$    | $\frac{P_1 m_e V_{31} V_{12}^*}{P_1 m_e V_{32} V_{12}^*} R_e$ |
| $y^l_2$    | $\frac{P_2 m_e V_{31} V_{12}^*}{P_2 m_e V_{32} V_{12}^*} R_\mu$ |
| $y^{lK}_1$ | $\frac{P_1 m_e V_{31} V_{32}^*}{P_1 m_e V_{32} V_{31}^*} R_\tau$ |
| $y^{lK}_2$ | $\frac{P_2 m_e V_{31} V_{32}^*}{P_2 m_e V_{32} V_{31}^*} R_\tau$ |
| $y^{lK}_3$ | $\frac{P_3 m_e V_{31} V_{32}^*}{P_3 m_e V_{32} V_{31}^*} R_\tau$ |
| $y^{lK}_4$ | $\frac{P_4 m_e V_{31} V_{32}^*}{P_4 m_e V_{32} V_{31}^*} R_\tau$ |
| $y^{lK}_5$ | $\frac{P_5 m_e V_{31} V_{32}^*}{P_5 m_e V_{32} V_{31}^*} R_\tau$ |
| $y^{lK}_6$ | $\frac{P_6 m_e V_{31} V_{32}^*}{P_6 m_e V_{32} V_{31}^*} R_\tau$ |

From tables 1 and 2 one finds that there is a hierarchy in the partial decay branching ratios of these modes which can be read off from the quark mass factors and the KM matrix elements. In making order of magnitude estimates for lifetimes it is useful to keep in mind that

$$m_2 V_{11} : m_e V_{21} : m_\tau V_{31} \approx 1 : 50 : 500$$ (16)

One can then roughly order the partial decay branching ratios for the various modes listed in Table 1 as follows

$$BR(\mu K) >> BR(\mu K), BR(\mu \pi) >> BR(\mu \pi)$$

$$BR(\bar{\nu}K) > BR(\bar{\nu} \pi) > BR(l \bar{k}) > BR(l \pi)$$

$$BR(\bar{\nu}_\mu K) > BR(\bar{\nu}_\tau K) > BR(\bar{\nu}_e K), BR(\bar{\nu}_\mu \pi) > BR(\bar{\nu}_\tau \pi) > BR(\bar{\nu}_e \pi)$$ (17)

One finds from the above that the most dominant decay modes of the proton are the $\bar{\nu}K$ modes. The dimension six operators which govern these are given by $[\bar{\nu}]$ [3]

$$L_6(N \rightarrow \bar{\nu}i K) = [(\alpha_2)^2 (2 M M_{11}^2 \sin 2\beta)^{-1} P_2 m_e m_2 V_{11} V_{21} V_{22}][F(c, \tilde{d}_i, \tilde{W}) + F(\tilde{c}, \tilde{d}_i, \tilde{W})] + [(1 + y_1^{IK} + (y_{\bar{\nu}} + y_{\bar{\nu}d\bar{e}Z})\delta_{12} + \Delta^K_i \alpha_i^L + [1 + y^{IK}_1 - (y_{\bar{\nu}} - y_{\bar{\nu}})]\delta_{12} + \Delta^K_i \beta_i^L + (y_1(R)\alpha_3^R + y_2(R)\beta_3^R)\delta_{13}]$$ (18)
In the above $\alpha_i^{L,R}, \beta_i^{L,R}$ are defined by

$$\alpha_i^L = \epsilon_{abc}(d_{aL}\gamma^0u_{bL})(s_{cL}\gamma^0\nu_{iL}) \tag{19}$$

and $\alpha_i^R = \alpha_i^L(d_L, u_L \rightarrow d_R, u_R)$, and $\beta_i^{L,R} = \alpha_i^{L,R}(d \leftrightarrow s)$. Further $y_{iK}^I$ gives the dominant contribution from the third generation and is defined by\[4,5\]

$$y_{iK}^I = \frac{P_2(m_sV_{31}V_{22})}{P_3(m_cV_{21}V_{32})}(F(\bar{t}, \bar{d}, \bar{W}) + F(\bar{t}, \bar{c}, \bar{W})) \tag{20}$$

where the functions $F$ are dressing loop integrals and would be defined explicitly below. The remaining contributions represented by $\Delta_i^K$, $y_{\beta}\alpha$ (from gluino exchange) and $yZ$ (from neutralino exchange) are all relatively small.

The decay branching ratios of the $p$ into the $\tilde{\nu}K$ modes are given by the relation

$$\Gamma(p \rightarrow \tilde{\nu}K^+) = (\frac{\beta_p}{M_{H_3}})^2|A|^2|B_i|C \tag{21}$$

where $\beta_p$ is the three quark - vacuum matrix element of the proton and is defined by

$$\beta_p U_L^\gamma = \epsilon_{abc}\epsilon_{\alpha\beta} < 0|d_{aL}\gamma^0 u_{\beta L}u_{cL}|p > \tag{22}$$

The most recent evaluation of $\beta_p$ is from lattice gauge calculations\[23\] and is

$$\beta_p = (5.6 \pm 0.5) \times 10^{-3} GeV^3 \tag{23}$$

The factors $A$ and $B_i$ of eq(21) are defined by

$$A = \frac{\alpha_2^2}{2M_W^2}m_sm_cV_{21}^\dagger V_{21}A_LA_S \tag{24}$$

$$B_i = \frac{1}{sin2\beta}m_tV_{11}^\dagger[P_2B_{2i}+m_tV_{31}V_{32}P_3B_{3i}] \tag{25}$$

$$B_{ji} = F(\bar{u}_i, \bar{d}_j, \bar{W}) + (\bar{d}_j \rightarrow \bar{c}_j) \tag{26}$$

where

$$F(\bar{u}_i, \bar{d}_j, \bar{W}) = [Ecos\gamma_+sin\gamma_+\tilde{f}(\bar{u}_i, \bar{d}_j, \bar{W}) + cos\gamma_+sin\gamma_+\tilde{f}(\bar{u}_i, \bar{d}_j, \bar{W})]$$

$$-\frac{1}{2}\frac{\delta_{ui}m_{u_i}sin2\delta_{ui}}{\sqrt{2}M_Wsin\beta}[Esin\gamma_+sin\gamma_+\tilde{f}(\bar{u}_{i1}, \bar{d}_j, \bar{W}) - cos\gamma_+cos\gamma_+\tilde{f}(\bar{u}_{i1}, \bar{d}_j, \bar{W})] - (\bar{u}_{i1} \rightarrow \bar{u}_{(2)}) \tag{27}$$

In the above $\tilde{f}$ is given by

$$\tilde{f}(\bar{u}_i, \bar{d}_j, \bar{W}) = sin^2\delta_{ui}\tilde{f}(\bar{u}_{i1}, \bar{d}_j, \bar{W}) + cos^2\delta_{ui}\tilde{f}(\bar{u}_{i2}, \bar{d}_j, \bar{W}) \tag{28}$$
where
\[ f(a, b, c) = \frac{m_c}{m_a - m_c^2} \left[ \frac{m_b^2}{m_a^2 - m_b^2} \ln \left( \frac{m_a}{m_b} \right) - (m_a \rightarrow m_c) \right] \] (29)

and \( \gamma_\pm = \beta_+ \pm \beta_- \) where
\[ \sin 2\beta_\pm = \frac{(\mu \pm \bar{m}_2)}{[4\nu_\pm + (\mu \pm \bar{m}_2)^2]^{1/2}} \] (30)

and
\[ \sqrt{2}\nu_\pm = M_W (\sin \beta \pm \cos \beta) \] (31)
\[ \sin 2\delta_{u3} = \frac{-2(A_t + \mu \cot \beta)m_t}{m_{t_1}^2 - m_{t_2}^2} \] (32)
\[ E = 1, \sin 2\beta > \mu \bar{m}_2/M_W^2 \]
\[ = -1, \sin 2\beta < \mu \bar{m}_2/M_W^2 \] (33)

Finally C that enters eq(25) is a current algebra factor and is given by
\[ C = \frac{m_N}{32\pi f_\pi^2} \left[ (1 + \frac{m_N(D + F)}{m_B}) \right] (1 - \frac{m_K^2}{m_N^2})^2 \] (34)

where the chiral Lagrangian factors \( f_\pi, D, F, \ldots \) etc that enter the above equation have the numerical values: \( f_\pi = 139 \text{ MeV}, D=0.76, F=0.48, m_N=938 \text{ MeV}, m_K=495 \text{ MeV}, \) and \( m_B=1154. \)

4. Vector Meson Decay Modes of the Proton

The same baryon number violating dimension six quark operators that lead to the decay of the proton into lepton and pseudoscalar modes also lead to decay modes with lepton and vector mesons[24]. Although the vector mesons are considerably heavier than their corresponding pseudoscalar counterparts, decay modes involving \( \rho, K^*, \omega \) are still allowed. We list these below
\[ \bar{\nu}_i K^*, \bar{\nu}_i \rho, \bar{\nu}_i \omega; i = e, \mu, \tau \]
\[ eK^*, \mu K^*, e\rho, \mu \rho, e\omega, \mu \omega \] (35)

The quark, KM and third generation enhancement factors for the allowed vector meson decay modes is exhibited in table3. The branching ratios for the vector meson decay modes are typically smaller than the corresponding pseudo-scalar decay modes.
Table 3: lepton + vector meson decay modes of the proton

| SUSY Mode | quark factors | CKM factors | 3rd generation enhancement |
|-----------|---------------|-------------|---------------------------|
| $\bar{\nu}_eK^*$ | $m_dm_c$ | $V^\dagger_{11}V_{21}V_{22}$ | $(1 + y^K_{11})$ |
| $\bar{\nu}_\mu K^*$ | $m_sm_c$ | $V^\dagger_{21}V_{21}V_{22}$ | $(1 + y^K_{22})$ |
| $\bar{\nu}_\tau K^*$ | $m_bm_c$ | $V^\dagger_{31}V_{21}V_{22}$ | $(1 + y^K_{33})$ |
| $\bar{\nu}_e\rho, \bar{\nu}_e\omega$ | $m_dm_c$ | $V^\dagger_{11}V^2_{21}$ | $(1 + y^R_{11})$ |
| $\bar{\nu}_\mu\rho, \bar{\nu}_\mu\omega$ | $m_sm_c$ | $V^\dagger_{21}V^2_{21}$ | $(1 + y^R_{22})$ |
| $\bar{\nu}_\tau\rho, \bar{\nu}_\tau\omega$ | $m_bm_c$ | $V^\dagger_{31}V^2_{21}$ | $(1 + y^R_{33})$ |
| $\rho K^*$ | $m_dm_u$ | $V^\dagger_{11}V_{12}$ | $(1 + y^R_{11})$ |
| $\mu K^*$ | $m_dm_u$ | | | |
| $e\rho, e\omega$ | $m_dm_u$ | | | |
| $\mu\rho, \mu\omega$ | $m_dm_u$ | $V^\dagger_{11}V^2_{21}$ | $(1 + y^R_{11})$ |

5. Details of Analysis in Supergravity Unification

Next we discuss the details of the proton decay analysis in supergravity unification. As already indicated the low energy SUSY spectrum plays a crucial role in proton decay lifetime. In fact the spectrum that enters consists of 12 squark states, 9 slepton states, 4 neutralino states, 2 chargino states, and the gluino. There are thus 28 different mass parameters alone. In globally supersymmetric grand unification one has no way to meaningfully control these parameters and thus detailed predictions of p decay lifetimes in globally supersymmetric theories cannot be made. In supergravity unified models one has a well defined procedure of breaking supersymmetry via the hidden sector and the minimal supergravity unification contains only 4 SUSY parameters in terms of which all the SUSY masses can be predicted. Thus supergravity unification is very predictive. We give below a brief review of the basic elements of supergravity grand unification. These are: (1) supersymmetry breaks in the hidden sector by a superhiggs phenomenon and the breaking of supersymmetry is communicated gravitationally to the physical sector; (2) the superhiggs coupling are assumed not to depend on the generation index, and (3) one assumes the spectrum to be the MSSM spectrum below the GUT scale. After the breaking of supersymmetry and of the gauge group one can integrate over the superhiggs fields and the heavy fields and the following supersymmetry breaking potential in the low energy domain results [11, 12]:

$$V_{SB} = m_0^2 z_a z_a^\dagger + (A_0 W(3) + B_0 W(2) + h.c.)$$

(36)

where $W^{(2)}, W^{(3)}$ are the bilinear and trilinear parts of the superpotential. There is also a gaugino mass term $L^\lambda_{mass} = -m_{1/2} \tilde{\lambda}^\alpha \lambda^\alpha$. At this stage the theory has five SUSY parameters $m_0, m_{1/2}, A_0, B_0,$ and $\mu_0$. Here $\mu_0$ is the
Higgs mixing term which along with the other low energy quark-lepton-Higgs interactions is given by

\[
W = \mu_0 H_1 H_2 + [\lambda^{(u)}_{ij} q_i H^0 C_j + \lambda^{(d)}_{ij} d_i H^0 C_j + \lambda^{(e)}_{ij} \ell_i H^0 C_j]
\]

(37)

In the above \(H_1\) is the light Higgs doublet which gives mass to down quark and leptons and \(H_2\) give mass to the up quark. The number of SUSY parameters can be reduced after radiative breaking of the electro-weak symmetry. The radiative electro-weak symmetry breaking is governed by the potential

\[
V_H = m_1^2(t)|H_1|^2 + m_2^2(t)|H_2|^2 - m_3^2(t)(H_1 H_2 + h.c.)
\]

\[+ \frac{1}{8}(g^2 + g^2_\text{Y})(|H_1|^2 - |H_2|^2)^2 + \Delta V_1
\]

(38)

where \(\Delta V_1\) is the correction from one loop, and \(m_1^2(t)\) etc are the running parameters and satisfy the boundary conditions \(m_i^2(0) = m_0^2 + \mu^2_0; i = 1, 2, m_3^2(0) = -B_0 \mu_0, \alpha_2(0) = \alpha_G = (5/3)\alpha_Y(0)\). The breaking of the electroweak symmetry is accomplished by the relations \(\frac{1}{2} M_2^2 = (\mu^2 - \mu_0^2 tan^2 \beta)/(tan^2 \beta - 1)\) and \(sin2\beta = (2m_3^2)/(\mu^2 + \mu_0^2)\), where \(\mu^2 = m_i^2 + \Sigma_i\) and \(\Sigma_i\) is one loop correction from \(\Delta V_1\). Using the above relations one can reduce the low energy SUSY parameters to the following:

\[m_0, m_{1/2}, A_0, tan\beta\]

(39)

Another result that emerges from radiative breaking of the electro-weak symmetry is that of scaling. One finds that over most of the parameter space of the theory \(\mu^2 >> M_Z^2\) which gives \([23, 26]\)

\[m_{\tilde{\chi}_1} \sim \frac{1}{3} m_{\tilde{\chi}_1} (\mu < 0); m_{\tilde{\chi}_1} \sim \frac{1}{4} m_{\tilde{\chi}_1} (\mu > 0)
\]

\[2m_{\tilde{\tau}_1} \sim m_{\tilde{\tau}_1} \sim m_{\tilde{\nu}_2}; m_{\tilde{\tau}_3} \sim m_{\tilde{\nu}_2} >> m_{\tilde{\tau}_1}
\]

\[m_{\tilde{\chi}_1}^0 \sim m_A \sim m_{H^\pm} >> m_h
\]

(40)

Corrections to the above are typically small \(O(1/\mu)\) over most of the parameter space.

We discuss now the effects of the top quark which play an important role in limiting the parameter space of the model. Constrains from the top quark arise because there is a Landau pole in the top quark Yukawa coupling, i.e.,

\[Y_0 = Y_t/(E(t)D_0)\] where \(D_0 = 1 - 6Y_t F(t)/E(t)\), \(Y_t = \lambda_t^2/4\pi\), \(\lambda_t(Q)\) is the top-quark Yukawa coupling and is defined by \(m_t = <H_2> \lambda_t(m_t)\), and the functions \(E(t)\) and \(F(t)\) are as defined in ref [27]. We see from the above that the top Yukawa has a Landau pole which appears at

\[m_t = (8\pi/\alpha_2(t))^{1/2}(Y_t(t))^{1/2} M_Z \cos\theta_W \sin\beta
\]

(41)
where $\theta_W$ is the weak mixing angle. For some typical values of $\alpha_G$ and $M_G$ one has $m_f^T \sim 200 \sin \beta$. Now it is found that the same Landau singularity also surfaces in the other SUSY parameters because of the coupled nature of the renormalization group equations. Thus, for example, the trilinear soft SUSY parameter develops a Landau singularity: $A_0 = A_R/D_0 + A_0(\text{nonpole})$, and $A_R = A_t - 0.6m_{\tilde{g}}$, where $A_0$ is the value of $A_t$ at the GUT scale. A similar analysis shows that $\mu^2$ and thus the stop masses become singular. Specifically one finds that $m^2_{\tilde{t}_1} = -2x/D_0 + m^2_{\tilde{t}_1} (NP)$ and $m^2_{\tilde{t}_2} = -x/D_0 + m^2_{\tilde{t}_2} (NP)$ where $x = Y_t A_R^0 F/E$. We note that the Landau pole contribution is negative definite and thus drives the stops towards their tachyonic limit. Especially the Landau pole contributions to $\tilde{t}_1$ are rather large and so its transition to the tachyonic limit is very rapid. Thus the condition that there be no tachyons puts a strong limit on the parameter space. One finds that the allowed values of $A_t$ lie in the range $-0.5 < A_t < 5.5$.

6. Discussion of Results

Figure 1a gives the maximum lifetime of the $p \to \nu K^+$ mode for $\mu < 0$ as function of $m_0$ when all other parameters are varied over the allowed parameters space consistent with radiative breaking of the electro-weak symmetry and with the inclusion of the LEP1.4 constraints. The solid curve gives the maximum without the imposition of the cosmological relic density constraint while the dashed curve includes the relic density constraint. The solid horizontal line is the current experimental lower limit for this mode from IMB and Kamiokande. We see that the analysis shows that there exists a considerable part of the parameter space not yet explored by the current experiment, which will be accessible to SuperKamiokande and ICARUS. Figure 1b gives the same analysis when $\mu > 0$. Comparison of figs 1a and 1b shows that the current experiment excludes a somewhat larger region of the parameter space in $m_0$ for $\mu > 0$ than for $\mu < 0$. Thus for $\mu > 0$ one eliminates the region $\mu < 400 GeV$ while for $\mu < 0$ only the values $m_0 < 300 GeV$ are eliminated. Figure 1c gives the plot of the maximum lifetime for the $p \to \tilde{\nu} K$ mode as a function of gluino mass for the case $\mu < 0$ corresponding to Fig 1a. We see that regions of the parameter space with lifetimes above the current limits lie below approximately 400 GeV when the dark matter constraint is imposed. Figure 1d is similar to Fig 1c except that $\mu > 0$.

7. Conclusion

In the above we have given a brief review of nucleon instability in supersymmetric unified theories. We have pointed out that no concrete predictions
on proton decay lifetimes are possible unless the nature of the low energy SUSY mass spectrum which enters in the dressing loop diagrams is assumed. Thus no concrete predictions on proton lifetime can be made in globally supersymmetric grand unified theories. In contrast one can make predictions in supergravity unification since the SUSY breaking spectrum of the theory is characterised by four parameters. Further since there are 32 supersymmetric particles one has 28 predictions in the model, and thus supergravity grandunification is very predictive. An updated analysis of p-decay in the minimal SU(5) model was given including the constraints of LEP1.4. It is found that there exits a significant part of the parameter space which is not yet explored by the current proton lifetime limits on $p \rightarrow \bar{\nu}K$ from IMB and Kamiokande but which would become accessible to SuperKamiokande and ICARUS experiments. Finally we note that the minimal model can correctly accomodate the $b/\tau$ mass ratio[29]. However, it does not predict other quark lepton mass ratios correctly and non-minimal extensions are needed for this purpose. These non-minimal extensions also affect the proton lifetime predictions.

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