Non-Dimensional Scaling of Impact Fast Ignition Experiments

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Abstract. Recent experiments at the Osaka University Institute for Laser Engineering (ILE) showed that “Impact Fast Ignition” (IFI) could increase the neutron yield of inertial fusion targets by two orders of magnitude [1]. IFI utilizes the thermal and kinetic energy of a laser-accelerated disk to impact an imploded fusion target. ILE researchers estimate a disk velocity of $10^8$ cm/sec is needed to ignite the fusion target [2]. To be able to study the IFI concept using lasers different from that at ILE, appropriate non-dimensionalization of the flow should be done. Analysis of the rocket equation gives parameters needed for producing similar IFI results with different lasers. This analysis shows that a variety of laboratory-scale commercial lasers could produce results useful to full-scale ILE experiments.

1. Introduction
Recent “Impact Fast Ignition” (IFI) experiments have increased the fusion neutron yield by two orders of magnitude over Inertial Confinement Fusion (ICF) targets without fast ignition [1]. As shown in figure 1, with IFI, a laser heats and accelerates a spherical shell, which then impacts the collapsed ICF target. In contrast, using Fast Ignition (FI), whereby fast electrons are used to increase the fusion yield, the neutron production increased by three orders of magnitude. However, fast electron FI requires a petawatt laser system, the production of fast electrons or ions, and an understanding of fast electron interaction with the main ICF target. However, IFI is simpler, being primarily a high temperature hydrodynamics problem.

For potential IFI experiments at Osaka University, a high-power laser beam of the GEKKO XII facility can be utilized to drive their experiments. ILE researchers estimate an IFI shell velocity of $10^8$ cm/sec is needed to ignite the fusion target [2], but this velocity is specific to their IFI target design.
For fusion studies attempting to achieve this velocity, such a large and powerful laser is needed. However, with proper scaling, it might be possible to conduct similar experiments with a smaller laser.

By appropriately non-dimensionalizing important parameters for the IFI technique, data from different experiments could be consistently compared, and IFI targets could be designed for use with a much smaller laser system than the GEKKO XII. The purpose of this paper, therefore, is to determine whether scaling can be used to show that a typical university-scale laser could be utilized to provide data applicable to full-size IFI research efforts.

2. Analysis

The below methodology uses only non-dimensionalization to justify whether scaled models represent the IFI shell propagation of full-size experiments. True self-similarity is a more in-depth method [3], which simplifies partial differential equations (PDE’s) into solvable ordinary differential equations, and uses a similarity variable combining the two independent variables of space and time. Rigorous self-similarity would be needed if we wanted to simplify the PDE’s, or desired an understanding of shifting time and spatial scales, but this is not the case here. The present goal is only to determine the laser conditions needed for a scaled IFI model to represent the flow dynamics of the full-size system, which as will be shown below is satisfied by non-dimensionalization without resorting to a full self-similar analysis. Note that this is analogous to deriving the Reynolds number from the Navier-Stokes equation, which if the Reynolds number is maintained between the model and full-size system, then the flow field will be similar.

Following the analysis of Eliezer [4], the “rocket equation” for describing the velocity of an ablatively-driven payload can be utilized to non-dimensionalize the velocity and time scales of the IFI experiment. Newton’s Second Law, \( F = ma \), is used in the rocket equation for an ablatively-accelerated payload, in the form

\[
m(t) \frac{du}{dt} = P_a = \sqrt{m_0 I}
\]

where \( u \) is the IFI shell payload velocity. Note that \( m \) is in units of areal density (g/cm\(^2\)), and the areal mass ablation rate \( \dot{m}_0 \) has units of (g/cm\(^2\) sec). From Lindl [5], empirical relations are available for the mass ablation rate and ablation pressure as a function of the incident laser intensity and wavelength \( \lambda \), as given by

\[
\dot{m}_0 = 2.6 \times 10^5 \left( \frac{I_{15}}{\lambda} \right)^{\frac{1}{3}} \text{ (g/cm}^2\text{ sec)}
\]

\[
P_a = 4 \times 10^{13} \left( \frac{I_{15}}{\lambda} \right)^{\frac{2}{3}} \text{ (dyne/cm}^2\text{)}
\]

where \( I_{15} \) is the laser intensity divided by \( 10^{15} \) W/cm\(^2\), and the wavelength is expressed in \( \mu \)m.

Assuming the ablation rate to be constant, such that \( m(t) = m_0 - \dot{m}_0 t \), where \( m_0 \) is the initial areal mass of the target before ablation, equation (1) can be non-dimensionalized and integrated to give

\[
\tilde{u}_f = - \ln(1 - \tilde{\tau})
\]

Here, \( \tilde{u}_f \) is the non-dimensional final velocity of the payload using the characteristic velocity \( U = \sqrt{I/m_0} = I/P_a \), and \( \tilde{\tau} \) is non-dimensionalized by the characteristic time \( \tau = m_0/\dot{m}_0 = m_0 I/P_a \) (i.e. \( \tilde{u}_f = u_f/U ; \tilde{\tau} = t/\tau \)). For large aspect ratio spherical shell targets \( (R_0/\Delta R_0 >> 1) \), where \( R_0 \) is the initial shell radius, the initial areal mass is the IFI shell initial density \( \rho_0 \) multiplied by its thickness: \( m_0 = \rho_0 \Delta R_0 \). The characteristic time and velocity scales can thus be determined from the empirical relations of equations (2) or (3), and laser parameters.

Recognizing that \( 1 - \tilde{\tau} \) is equivalent to \( m(t)/m_0 \), equation (4) can immediately be rewritten into the form given by Lindl [5], as

\[
\tilde{u}_f = \ln\left[ m_0/m(\tilde{\tau}) \right]
\]
However, equation (5) is not so useful in this analysis since it is impossible to measure $m(t)$, whereas it is possible to measure the laser ablation time in equation (4). Note that for $\ddot{t} \sim 1$, one would expect from equation (4) to obtain an unrealistic very large target velocity (i.e. as $\ddot{t} \to 1$, $\dddot{u}_f \to \infty$).

3. Results

3.1. Non-dimensional time and velocity example

For the recent ILE experiments [6] using planar Br-doped targets, $\rho_0 = 1.35 \text{ g/cm}^3$, and $\Delta R_0$ was 14 $\mu$m. The laser intensity was a rectangular-shaped super-Gaussian pulse at $I = 4 \times 10^{14} \text{ W/cm}^2$, with a wavelength of $\lambda = 0.35 \mu$m. Using this data, a characteristic time scale of $\tau = 2.4 \text{ ns}$ and characteristic velocity of $U = 7.2 \times 10^7 \text{ cm/sec}$ are obtained. The maximum velocity realized in the experiment was estimated to be approximately $6 \times 10^7 \text{ cm/sec}$. By inspection of the streak camera data from this experiment, it is unclear precisely when the target achieved this speed. However, an estimate of $\ddot{t} \sim 1.8 \text{ ns}$ is made here from inspection of the x-ray image. Thus, these experiments achieved a non-dimensional speed of $\dddot{u}_f \sim 0.8$ in a non-dimensional time of $\dddot{t} \sim 0.7$.

From equation (4) with $\ddot{t} = 0.7$, one would expect a velocity of $\dddot{u}_f = 1.2$, but instead the planar experiments achieved only $\dddot{u}_f \sim 0.8$. However, it must be remembered that the above rocket equation analysis was idealized, assuming constant mass ablation and pressure, and no losses besides electron flux. Real experiments may also have other losses not considered here, such as turbulence and laser albedo. In any case, the above non-dimensionalization method is quite useful since experiments using different lasers and targets can thereby be consistently compared.

3.2. Laser Scaling

As a guide, assume that the maximum hydrodynamic efficiency is always desired. The hydrodynamic efficiency $\eta_h$ for ablatively accelerating a target is given by

$$\eta_h = \frac{\frac{1}{2}m(t)\dddot{u}(t)^2}{It} = \frac{1-\dddot{t}}{2\dddot{t}} \left[\ln(1-\dddot{t})\right]^2$$

(6)

From equation (6), the maximum hydrodynamic efficiency occurs at approximately $\dddot{t} = 0.77$, which from equation (4) corresponds to a non-dimensional velocity of $\dddot{u}_f = 1.5$. Thus, to achieve the highest kinetic energy of an IFI target for a given available laser energy, the laser should accelerate the target up to $\dddot{t} = 0.77$. Thus,

$$\dddot{t} = \frac{t}{m_0/m_0} = 2.6 \times 10^3 \left(\frac{I_{15}}{\lambda^3} \right)^{1/3} t / \rho_0 \Delta R_0 = 0.77$$

(7)

Rearranging equation (7) and defining $t_{ns}$ as the laser pulse time in nanoseconds, the following scaling equation is obtained

$$\rho_0 \Delta R_0 = \frac{I_{15}^{1/3} t_{ns}}{3000 \lambda^{1/3}} \text{ (g/cm}^2)$$

(8)

Note that the right side of equation (8) is completely dependent only on the laser parameters, and the left side describes the IFI shell target parameters, specifically the initial areal density of the target. The $I_{15}^{1/3} t_{ns}$ parameter in equation (8) is related to the maximum available energy of a laser system. Thus, each laser system can be categorized by its available $I_{15}^{1/3} t_{ns}$ and wavelength, which then defines the target parameters necessary to produce similar non-dimensional velocities between experiments.

The PINOCO code developed by ILE was used to design an IFI target specifically for the GEKKO XII laser system [6], showing that it may be possible to achieve the desired $10^8 \text{ cm/sec}$ IFI shell velocity necessary for ICF ignition. Assuming the same target parameters as ILE PINOCO code
results \((\rho_0 = 1 \text{ g/cm}^3, \Delta R_0 = 35 \mu\text{m})\), the required pulse lengths at different laser wavelengths and intensities can be calculated using equation (8). These results are shown in Table 1, which illustrates that an excimer laser (~0.20 \mu m wavelength) reduces the required laser pulse length by a factor of 2 compared with using a laser with wavelength of 0.35 \mu m. Similarly, the same pulse length can be used with an excimer laser operating at an order of magnitude lower intensity than a 0.35 \mu m laser. Note that commercial, university laboratory-scale lasers are available at the lower intensities and longer pulse times shown in Table 1.

Table 1. Laser pulse lengths required for maximum hydrodynamic efficiency using 0.35 \mu m and 0.20 \mu m wavelength lasers at varying intensities using the same shell target as ILE PINOCO code results \((\rho_0 = 1 \text{ g/cm}^3, \Delta R_0 = 35 \mu\text{m})\). GEKKO XII-scale lasers would be in the first column for \(I = 10^{15}\).

| \(\lambda\) (\mu m) | \(I = 10^{15}\) (W/cm\(^2\)) | \(I = 10^{14}\) (W/cm\(^2\)) | \(I = 10^{13}\) (W/cm\(^2\)) | \(I = 10^{12}\) (W/cm\(^2\)) | \(I = 10^{11}\) (W/cm\(^2\)) |
|-------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 0.35              | 2.6 ns                        | 5.6 ns                        | 12 ns                         | 26 ns                         | 56 ns                         |
| 0.20              | 1.2 ns                        | 2.6 ns                        | 5.7 ns                        | 12 ns                         | 26 ns                         |

4. Conclusions

Using a characteristic velocity scale of \(U = \sqrt{I/m_0} = I/\sqrt{P_a}\) and time scale of \(\tau = m_0/\dot{m}_0 = \rho_0 \Delta R_0/\dot{m}_0\), IFI shell velocities as a function of time can be appropriately non-dimensionalized. This non-dimensionalization then allows results from different researchers to be consistently compared. The rocket equation, in conjunction with assuming optimal hydrodynamic efficiency, defines the IFI shell properties required as a function of the available laser energy using equation (8). In this way, similar IFI experiments and numerical simulations can be designed utilizing laser drivers of different energies. Specifically, it is shown that laser systems much smaller than the scale of the GEKKO XII laser system at Osaka University could be used to produce experimental results relevant to furthering the IFI research effort. The present analysis is meant to encourage other researchers to pursue the IFI concept for achieving ICF ignition using their existing laser systems, which likely are much smaller than GEKKO XII of ILE, or other large-scale facilities.

5. References

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