Supersymmetry Tests from a Combined Analysis of Chargino and Charged Higgs Boson Pair Production at a 1 TeV Linear Collider

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We consider the production of chargino and charged Higgs boson pairs at future linear colliders for c.m. energies in the one TeV range. Working in the MSSM under the assumption of a "moderately" light SUSY scenario, we compute the leading (double) and next-to leading (linear) supersymmetric logarithmic terms of the so-called "Sudakov expansion" at one loop. We show that a combined analysis of the slopes of the chargino and of the charged Higgs production cross sections would offer a simple possibility for determining $\tan \beta$ for large ($\gtrsim 10$) values and an allowed strip in the $(M_2, \mu)$ plane. This could provide a strong consistency test of the considered supersymmetric model.

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\section{I. INTRODUCTION}

One of the goals of a future electron-positron Linear Collider \cite{1} will be the performance of precision tests, hopefully of models of new physics beyond the Standard Model, under the assumption that a preliminary direct discovery of, at least, some of the new particles requested by the candidate model has been achieved at TEVATRON and/or LHC. These precision tests would then be particularly relevant for models that involve a large number of parameters, some of which might be different from the produced particle masses and might turn out to be determined with a relatively low accuracy. In alternative, a number of competitor models might exist, showing a certain number of properties (e.g. an identical set of particles), but differing in a number of fine details that might remain invisible, or partially hidden, in the process of direct production. In these cases, the need of a precision test of the candidate model would become mandatory, and should have the same relevance that the memorable precision tests performed at LEP and SLC \cite{2} had for a spectacular confirmation of several predictions of the electroweak component of the Standard Model.

If Supersymmetry were discovered at hadron colliders, the previous discussion would immediately apply to the simplest existing supersymmetric model, the MSSM. Being of perturbative nature, this model would offer a large number of possibilities of performing precision tests via the observation (and confirmation) of the virtual effects that it predicts at the simplest higher perturbative order, the so-called one loop level. Due to the rather large number of parameters that the model involves these tests would require the preparation and the use of dedicated theoretical calculations and numerical programs, that are actually already available in the literature. As a general remark, it can be noticed that a general feature of these programs, and of related precision tests as well, is that they are not simple, and require dedicated efforts and attention.

The aim of this short paper is to show that, for a reasonable light SUSY scenario and for c.m. energy values in the one TeV range, a rather simple test of the MSSM might be performed. This would imply the combined analyses of the production cross sections of chargino and charged Higgs boson pairs at a number of different energies in the one TeV region, in particular a determination of the slopes of the cross sections. The output of the effort would be, at a realistic optimal experimental accuracy, the determination (a) of $\tan \beta$ with an accuracy that would increase with increasing values of this parameter, and (b) of an allowed strip in the plane of the two quantities $(M_2, \mu)$, independently of the values of the remaining SUSY parameters of the chosen scenario. The details of this possibility will briefly be discussed in what follows.
II. DETERMINATION OF THE LOGARITHMIC SUSY SUDAKOV TERMS

Our analysis will be devoted to the two processes of production of charged Higgs and of chargino pairs in the MSSM with real input parameters. For the first process, a complete calculation has been given at one-loop in two papers [3,4], that is supposed to be valid for arbitrary values of energy and parameters. To perform the calculations presented in the second paper [4] the authors have also completed a numerical program (SESAMO). The latter has been used to test the validity of a logarithmic Sudakov expansion in a scenario of reasonably light SUSY, i.e. one in which all the relevant SUSY masses are sufficiently smaller than the chosen c.m. energy. This has been fixed at about one TeV with a corresponding limit on masses of about 400 GeV, since from previous studies [5] one knows that in this configuration a one-loop logarithmic expansion does not require higher order corrections. From a comparison of the exact SESAMO results with the Sudakov logarithmic expansion it was concluded that the latter provides a satisfactory description of the process in the chosen scenario, with the only request of an additional constant (i.e. energy independent) term in the expansion, that will depend in general on the several SUSY parameters of the model in a way that it was tried to analyze in a qualitative stage. On the contrary, the SUSY loop contribution consists only of linear logarithms stemming from Yukawa interactions. This term depends only on the SUSY parameter \( \tan \beta \). Therefore, a determination of the slope of the cross section, where the unknown SUSY constant term disappears (the other double and linear Sudakov logarithms are of pure SM origin and considered as known, together with the linear logarithms of RG origin for which the SUSY dependence is given by existing formulae), could provide a competitive [6,7] determination of \( \tan \beta \). Since a detailed description of this analysis has been given [4] we shall only write in this paper the relevant one-loop expressions for the cross section of the process. Following the notations of [4] we shall thus write for the charged Higgs case (\( \sqrt{s} \) is the c.m. energy): \[
\sigma^{\text{Born+1loop}} = \sigma^{\text{Born}} (1 + \Delta(q^2))
\] where \( \sigma^{\text{Born+1loop}} \) we are retaining only the genuine Sudakov one loop terms. The logarithmic expansion of \( \Delta \) is given by the expression

\[
\Delta(q^2) = -\frac{\alpha}{2\pi s_W} \left[ 1 + \frac{2 s_W^4}{1 + 4 s_W^4} \log^2 \frac{q^2}{M_W^2} \right] - \frac{\alpha}{4\pi s_W^2 c_W} \left[ 1 + \frac{2 s_W^4 + 8 s_W^6}{1 + 4 s_W^4} \log^2 \frac{q^2}{M_Z^2} \right] - \frac{3\alpha}{4\pi s_W^2 c_W^2} \left[ (m_t^2 \cot^2 \beta + m_t^2 \tan^2 \beta) \log \frac{q^2}{m_t^2} \right] + \frac{\alpha}{3\pi s_W^2 c_W^2} \left[ 11 - 16 s_W^4 + 32 s_W^4 + 72 s_W^6 \right] \log \frac{q^2}{M_Z^2}
\] where the third line contains the \( \tan \beta \) dependent logarithms of Yukawa origin and we have neglected any remainder terms not growing with energy.

For the process of chargino pair production, a complete one-loop calculation is also available [8]. In this preliminary paper, we shall be interested in the study of the same scenario that was considered for the Higgs pair production study previously discussed. In this spirit, we shall thus compute the two leading terms of the logarithmic SUSY expansion and assume, supported by the mentioned study, that the only extra term to be added to the expansion is, again, a (SUSY dependent) constant. We cannot, at this stage, prove this statement as we did for charged Higgs production; it appears rather reasonable to us, and we shall devote a more complete following paper to a detailed investigation of its validity. For the moment, we shall consider it as a working Ansatz, and show which relevant consequences might be derived under our assumption.

An essential difference between the processes of chargino and of charged Higgs pair production appears already at the Born level. In the process \( e^-e^+ \rightarrow \chi^+ \chi^- \), the analytic expression of the scattering amplitude contains already at lowest order the SUSY parameters \( M_2, \mu \) and \( \tan \beta \) appearing in the chargino mixing matrices \( Z_{ij}^c \) (our notation is described in [5])

\[
A_{ij}^{\text{Born}} = - e^2 / (4 s_W^2 c_W^2) \{ (\gamma^\mu P_L)^{\chi_i \chi_i} (\gamma^\mu P_L)^{\chi_j \chi_j} \left[ \delta_{ij} + (1 - 2 s_W^2) Z_{i j}^{++} Z_{i j}^{-+} \right] + (\gamma^\mu P_R)^{\chi_i \chi_i} (\gamma^\mu P_R)^{\chi_j \chi_j} \left[ 2 s_W^2 \delta_{ij} - Z_{i j}^{++} Z_{i j}^{-+} \right] + (\gamma^\mu P_R)^{\chi_i \chi_i} (\gamma^\mu P_R)^{\chi_j \chi_j} \left[ 2 s_W^2 \delta_{ij} - Z_{i j}^{++} Z_{i j}^{-+} \right] - e^2 / (2 s_W^2) [ Z_{i j}^{++} Z_{i j}^{-+} / u ] (\gamma^\mu P_L)^{\chi_i \chi_i} (\gamma^\mu P_L)^{\chi_j \chi_j}
\]

where \( s \equiv q^2 \) and \( u \) are Mandelstam variables and we used the following short-hands for the external spinors \( (\gamma^\mu P_L)^{R \chi_i \chi_j} \equiv \bar{\psi}(e^-) \gamma^\mu P_L u(e^-), (\gamma^\mu P_R)^{R \chi_i \chi_j} \chi_i \chi_j \equiv \bar{\psi}(e^-) \gamma^\mu P_R u(\chi^+ \chi^-) \). The Sudakov corrections at leading order are written in full details in [5] and can be expressed in a compact form by splitting the amplitude as \( A_{ij}^{ab,H} = A_{ij}^{ab,H,Born} + A_{ij}^{ab,W} \) where \( a,b \) are the initial and final chiralities and \( H, W \) stand for the Higgsino and Wino components proportional to \( Z_{2i}^\pm Z_{2j}^\pm \) and \( Z_{1i}^\pm Z_{1j}^\pm \) respectively. Omitting for simplicity the known renormalization group correction (of course, we included it in the analysis) the leading order Sudakov expansion of the amplitude reads

\[
A_{ij}^{ab,H} = A_{ij}^{ab,H,Born} (1 + \Delta A_{ij}^{ab,H}(q^2, \vartheta) + c_b \log \frac{q^2}{M_W^2})
\]

\[
A_{ij}^{ab,W} = A_{ij}^{ab,W,Born} (1 + \Delta A_{ij}^{ab,W}(q^2, \vartheta))
\]

In the above expressions \( \Delta A_{ij}^{ab}(q^2, \vartheta) \) is a logarithmic correction fully described in [5] depending on the c.m. energy and scattering angle \( \vartheta \). It contains the following
parts: (i) a universal angular independent term proportional to the combination $2 \log(q^2/M_W^2) - \log^2(q^2/M_W^2)$, (ii) an additional term $\sim \log(q^2/M_W^2)$ in the Wino component only, (iii) an angular term proportional to $\log(q^2/M_W^2)$ times a function of $\theta$. The Yukawa contribution appearing in the Higgsino component depends on $\tan \beta$ through the coefficient

$$Y_{\alpha k} = -\frac{3\alpha}{8\pi s_W^2 M_W^2} \left( \frac{m^2_\ell \sin^2 \beta}{\cos^2 \beta} \delta_{b,L} + \frac{m^2_b \cos^2 \beta}{\sin^2 \beta} \delta_{b,R} \right)$$

Before we proceed, it seems opportune at this point to make our working strategy completely clear. As one sees from Eqs. (3-4) the logarithmic expansion contains as starting input the Born amplitudes of the process. The latter ones depend on the bare parameters $Z^{\pm}_{\ell \ell}$, $Z^{\pm}_{\mu \mu}$ that must be expressed in terms of physical quantities defined in a chosen renormalization scheme. This would be necessary in a conventional treatment at one loop, and we shall pursue the rigorous approach in a more complete forthcoming paper. In this preliminary analysis, performed at next-to-leading (UV finite) one-loop logarithmic order, the problem disappears (it would affect the constant term of the expansion), and we can simply replace the bare input with an identical physical one, whose meaningful definition will be given in the complete one-loop treatment.

The tree-level cross section depends on $\tan \beta$ but in the cases we studied with $\tan \beta > 10$ and $M_2, \mu > M_Z$ this dependence vanishes (actually, it will be generated in a sensible way in our description from the Yukawa vertex effects). Note that, still as a welcome consequence of our one-loop treatment, we can factorize out all the QED IR divergent terms and consider them as a known quantity.

We performed a standard $\chi^2$ analysis of the Sudakov expansion of the cross sections for the above processes in order to determine bounds for the three parameters $M_2$, $\mu$ and $\tan \beta$. With this aim, we assumed that a certain number $N$ of cross sections measurements $\sigma(q^2_{11}), \ldots, \sigma(q^2_{1N})$ are available and computed the quantities $q^2_{1j} \sigma(q^2_{1j}) - q^2_{1j} \sigma(q^2_{1j})$ where the unknown subleading constant in the Sudakov expansion cancels. In Fig. (1), we show the results of the analysis of our benchmark point at the TESLA benchmark point RR2 [9] where $M_2 = 150$ GeV, $\mu = 263$ GeV and $\tan \beta = 30$. We took 10 points at equal distances for the four cross sections (pair production of $H^+H^-$ and three chargino channels $\chi^+_i\chi^-_j$ with $i,j = 11, 22$ and $12 + 21$) at energies between 700 and 1200 GeV. The assumed experimental errors of this (optimistic) example are 1% for charginos and 2% for charged Higgs boson cross section. The chargino masses are consistent with the chosen energy range i.e. sufficiently smaller than the lowest considered energy of 700 GeV. This is the only requirement in our approach while, in a more conventional analysis, chargino masses would be input observables depending on a set of MSSM parameters larger than simply $M_2, \mu$ and $\tan \beta$. The allowed region is a strip in the plane $(M_2, \mu)$ and comes from the parameter dependence of the mixing matrices in the chargino cross sections. The bounds on $\tan \beta$ are conversely the combination of those coming from Higgses and charginos. They are mainly due to the Yukawa terms whose relative weight in the Higgs case is bigger (roughly, a factor of two) than in the chargino cases (this motivates our choice of the experimental uncertainties). The relative errors on the three parameters in the planes $(M_2, \tan \beta)$ and $(\mu, \tan \beta)$ are shown in Fig. (2) where we also increased $\tan \beta$ to evidence the improvement due to the larger Yukawa contribution.

The same analysis can be repeated at the high $\tan \beta$ Snowmass benchmark point SPS4 [10] characterized by $M_2 = 233$ GeV, $\mu = 377$ GeV and $\tan \beta = 50$. Now the heaviest chargino mass is around 400 GeV and we performed the $\chi^2$ optimization starting at 850 GeV. The results are shown in Fig. (3).
III. CONCLUDING REMARKS

In the assumed logarithmic expansion, the slopes of the Higgs and of the chargino cross sections depend only on three SUSY parameters i.e. \( \tan \beta, M_2, \mu \). The parameter \( \tan \beta \) appears in both cross sections, with a relative weight that is more significant in the Higgs process. To improve the determination that would be provided by the Higgs data alone, chargino data of an experimental accuracy better than that achievable for Higgs data are requested, as we simulated in our example. If this better accuracy were not achieved, the relevant information on \( \tan \beta \) in our proposed method would come almost entirely from the Higgs pair production. For large values of the parameter, this would reach in any case interestingly small error percentages, depending on the experimental accuracy and on the number of measured points in a way that it is easy to calculate [4] and that seems to provide a competitive determination of this fundamental parameter [7]. On the contrary, the two remaining parameters \( (M_2, \mu) \) enter in the chargino cross sections alone. Those measurements will not provide a separate determination of the two parameters, but will force them to lie within an allowed strip in the \((M_2, \mu)\) plane. Thus, knowing the value of one of the parameters will fix the other one, with a precision that we have shown in the (optimistic) accuracy assumptions that were adopted (a more conservative strip can easily be drawn for different experimental assumptions). As we said in the Introduction, our constraints on \( \tan \beta, M_2, \mu \) could be obtained from a relatively simple analysis, in particular from minimization programs that only contain these three quantities as unknown terms to be fitted. In this sense, we believe to have shown in this (we repeat, preliminary) paper that from a combined investigation of charged Higgs boson and chargino pair production at a future LC a strong consistency test of the considered supersymmetric model would be, possibly and hopefully, derivable.

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