CONSTRAINTS ON STIRRING AND DISSIPATION OF MHD TURBULENCE IN MOLECULAR CLOUDS

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ABSTRACT

We discuss constraints on the rates of stirring and dissipation of MHD turbulence in molecular clouds. Recent MHD simulations suggest that turbulence in clouds decays rapidly, thus providing a significant source of energy input, particularly if driven at small scales by, for example, bipolar outflows. We quantify the heating rates by combining the line-width–size relations, which describe global cloud properties, with numerically determined dissipation rates. We argue that if cloud turbulence is driven on small internal scales, the $^{12}$CO flux (enhanced by emission from weakly supersonic shocks) will be much larger than observed; this, in turn, would imply excitation temperatures significantly above observed values. We reach two conclusions: (1) small-scale driving by bipolar outflows cannot possibly account for cloud support and yield long-lived clouds unless the published MHD dissipation rates are seriously overestimated, and (2) driving on large scales (comparable to the cloud size) is much more viable from an energetic standpoint, and if the actual net dissipation rate is only slightly lower than what current MHD simulations estimate, then the observationally inferred lifetimes and apparent virial equilibrium of molecular clouds can be explained.

Subject headings: ISM: clouds — ISM: kinematics and dynamics — ISM: magnetic fields — MHD — turbulence — waves

1. INTRODUCTION

Millimeter-wave surveys of CO line emission have established the large-scale distribution and properties of molecular clouds in the Galaxy (Solomon, Sanders, & Scoville 1979; Leung, Kutner, & Mead 1982; Dame et al. 1986; Solomon et al. 1987). These surveys reveal two essential features of clouds: (1) supersonic line widths, with observed velocity dispersion $\sigma$ exceeding the isothermal sound speed ($c_s \approx 0.2$ km s$^{-1}$, corresponding to the measured temperature $T \approx 10$ K) by factors of $\sim 10$ or more, and (2) power-law relations between $\sigma$ and the cloud size $L$, of the approximate form $\sigma \propto L^{0.5}$, and between the mean density $n$ and $L$, of the approximate form $n \propto L^{-1}$ (hereafter collectively labeled the line-width–size–density relations). The magnitude of the observed line widths is essentially that required to put individual clouds in virial equilibrium, given the estimated sizes and masses (e.g., Larson 1981; Myers 1983; Solomon et al. 1987).

Turbulent motions are often invoked as an origin for the supersonic line widths since they can provide an effective pressure (Chandrasekhar 1951) that, unlike the much lower thermal pressure, is adequate to provide cloud support. However, supersonic hydrodynamic turbulence should decay rapidly through shocks (e.g., Mestel 1965; Goldreich & Kwan 1974). Since molecular clouds tend to have large-scale magnetic fields (see Crutcher 1999), an attractive possibility has been that the turbulence represents supersonic but sub-Alfvénic magnetohydrodynamic (MHD) waves, of which the noncompressive shear Alfvén mode may be a long-lived component (Arors & Max 1975; Mushovias 1975).

However, recent numerical MHD simulations suggest that a spectrum of MHD waves or MHD turbulence cannot persist in a cloud for many crossing times (e.g., Stone, Gammie, & Ostriker 1998; Mac Low et al. 1998). These simulations tend to show that compressive motions and shocks arise so readily that the turbulence decays in one (or even much less than one) turbulent crossing time of the cloud ($t_c = L/\sigma$). This result is particularly surprising in the simulations of Stone et al. (1998), who stir the system only through incompressible modes ($\mathbf{V} \cdot \mathbf{\nabla} = 0$). If confirmed, these results imply that molecular cloud turbulence must be constantly and strongly driven if clouds survive for up to several crossing times, as implied by the inferred virial equilibrium of the clouds and their estimated lifetimes (e.g., Blitz & Shu 1980; Williams & McKee 1997).

But are such high rates of dissipation really allowed, even if a steady state is maintained by equally strong stirring? In this paper, we consider this question by examining the energetics of clouds with turbulent dissipation. In § 2, we determine heating rates in clouds obeying the line-width–size–density relations based on dissipation rates determined from numerical simulations. In § 3, the corresponding luminosities are determined and compared with observed luminosities. Further constraints on stirring are given in § 4, and our conclusions are presented in § 5.

2. HEATING RATES

We define $\Gamma(r)$ as the heating rate per unit volume as a function of position $r$ in a molecular cloud. Various processes can contribute to $\Gamma(r)$. Thermal energy input from cosmic rays at the mean rate

$$\Gamma_{cr} = 6.4 \times 10^{-25} \left( n/10^3 \text{ cm}^{-3} \right) \text{ ergs cm}^{-3} \text{ s}^{-1}$$

has been thought to maintain clouds of mean number density $n$ at temperatures $T \sim 10$ K (e.g., Goldsmith & Langer 1978).

A second contribution could also come from the decay of the MHD turbulence that is thought to support molecular
clouds globally. The turbulent heating rate $\Gamma_{\text{turb}}$ can be written as the ratio of the energy density $U$ of the cloud to an arbitrary dissipation timescale $t_d$: $\Gamma_{\text{turb}} = U/t_d$. The results of recent numerical simulations (Stone et al. 1998; Mac Low 1999) suggest that the ratio of dissipation time to turbulent crossing time $\kappa \equiv t_d/t_c \leq 1$ under typical conditions in molecular clouds.

For a cloud with mean density $\rho$ and one-dimensional nonthermal velocity dispersion $\sigma$, the turbulent energy density is $U = 3/2 \rho \sigma^2$ so that the turbulent heating rate

$$
\Gamma_{\text{turb}} = \frac{3}{2} \frac{\rho \sigma^2}{t_d} = \frac{3}{2} \kappa^{-1} \frac{\rho \sigma^3}{L}.
$$

There are a variety of other processes that can contribute to the energy balance in molecular clouds. They include the Galactic ultraviolet background, which heats the edges of clouds, and the internal radiation field generated by the stars that form in localized regions of clouds. Here we neglect processes which might contribute only to localized regions of star formation and cloud boundaries.

2.1. Stirring Scale

An important property of the turbulent dissipation rate is its dependence on stirring scale. Dimensionally, the dissipation rate

$$
\Gamma_{\text{turb}} = \eta \frac{\rho \sigma^3}{\lambda},
$$

where $\lambda$ is the driving scale and $\eta$ is, in general, a dimensionless function of $\rho$, $\sigma$, and $\lambda$. Kolmogorov & Obukhov (e.g., Landau & Lifshitz 1987) originally derived the value of $\eta$ in a self-similar cascade which is appropriate to the decay of incompressible hydrodynamic turbulence.

By analogy, Stone et al. (1998) and Mac Low (1999) derive values of $\eta$ for their simulations of compressible MHD turbulence. In this case, $\rho$ is the mean density of the computational region. Their results indicate that $\eta \approx 1$ over a range of driving scales, implying that $t_d$ is of order the crossing time $t_c$ or less. In particular, comparing equations (2) and (3) shows that

$$
\kappa = \frac{3}{2} \eta^{-1} \frac{\lambda}{L}.
$$

Since driving may occur on scales $\lambda$ much smaller than typical cloud sizes $L$ few to 100 pc (e.g., through outflows), the results of numerical simulations suggest that $\kappa$ may be much less than unity. This can obviously result in very large cloud luminosities; below we determine the expected fluxes for these heating rates.

2.2. Dissipation in Clouds

The line-width–size relation is an empirically derived scaling between the velocity width of $^{12}$CO $(J = 1\rightarrow 0)$ lines observed in a particular molecular cloud and the size of that cloud. Clouds approximately obey the scaling determined by Solomon et al. (1987):

$$
\sigma = 0.72 \left( \frac{R}{ pc} \right)^{0.5} \text{km s}^{-1},
$$

where $R \approx \frac{1}{2} L$ is an effective radius of the cloud. The same study also established an approximate density-size relation

$$
\rho = 134 \left( \frac{R}{ pc} \right)^{-1} M_\odot \text{pc}^{-3}.
$$

These relations imply that the most massive molecular clouds are approximately gravitationally bound and in virial equilibrium. Although clouds have very complicated density structures, these relations give a reasonable estimate of the mean density, mean turbulent energy, and size of a cloud. Thus, they allow an estimate of the bolometric luminosity of a cloud, given a mean dissipation rate as determined, say, from numerical simulations.

By combining the line-width–size and density-size relations, we obtain the mean heating rate due to dissipation of energy in turbulence:

$$
\Gamma_{\text{turb}} = 7.1 \times 10^{-11} t_d^{-1} \text{ ergs cm}^{-3} \text{ s}^{-1} = 5.4 \times 10^{-25} \kappa^{-1} \times (n/10^3 \text{ cm}^{-3})^{1/2} \text{ ergs cm}^{-3} \text{ s}^{-1}.
$$

This relation should hold roughly for clouds with mean density in the range $10^2 \text{ cm}^{-3} \leq n \leq 10^4 \text{ cm}^{-3}$. Also note that the energy density (or pressure) is independent of cloud size.

3. CLOUD LUMINOSITIES

Assume only that the cloud exists in some definite volume and that it is in a steady state equilibrium. Under these circumstances, the total cooling rate within the volume must balance the total heating rate. Therefore, although the cloud may have a rather complex density structure and a variety of atomic and molecular species (each having a different optical depth) that contribute to the cooling, the bolometric luminosity $L_{\text{bol}}$ must balance the volume-integrated heating rate:

$$
\dot{U} = \int dr \Gamma(r) = L_{\text{bol}}.
$$

Note that we have neglected the possibility that mass loss can carry away excess kinetic energy from the cloud in the form of a global wind or outflow.

Of course, determining the bolometric luminosity of a cloud is no trivial task. Current observations usually consider only specific atomic and molecular line transitions, with $^{12}$CO $(J = 1\rightarrow 0)$ by far the most thoroughly investigated. In their study, Solomon et al. (1987) determine a $^{12}$CO $(J = 1\rightarrow 0)$ luminosity–line-width relation for the clouds in their sample (their eq. [9]). We use the line-width–size–density relations and convert units in their formula to express the empirical $^{12}$CO $(J = 1\rightarrow 0)$ luminosity as

$$
L'_{\text{CO}} = 3.1 \times 10^{30} (n/10^3 \text{ cm}^{-3})^{-5/2} \text{ ergs s}^{-1}.
$$

This luminosity represents only the flux through the portion of the cloud’s surface that is visible to the observer. Here we adopt a total CO luminosity $L_{\text{CO}} = 4L'_{\text{CO}}$ appropriate for a spherical cloud. Of course, real molecular clouds have irregular shapes and surfaces so that the multiplicative constant may be somewhat greater than 4, but it should be no more than an order unity correction.

For clouds whose average properties obey the line-width–size–density relations, the bolometric luminosity determined from equation (8) is

$$
L_{\text{bol}} = 8.5 \times 10^{22} \left( \frac{1.2 (n/10^3 \text{ cm}^{-3})^{-2}}{1} \right) \text{ ergs s}^{-1},
$$

where the first term in brackets gives the cosmic-ray heating from equation (1) and the second term gives the turbulent heating from equation (7).
The top panel of Figure 1 shows the bolometric luminosity (equivalent to the heating rate) compared with the expected $L_{\text{CO}}$ from Solomon et al. (1987). Dashed lines show $L_{\text{bol}}$ for two values of $\kappa$, and dotted lines show $L_{\text{bol}}$ for two values of a fixed driving scale $\lambda$ that are compatible with driving by bipolar outflows. The bottom panel shows the flux ratio $L_{\text{CO}}/L_{\text{bol}}$, thereby illustrating how much emission is required beyond the $^{12}\text{CO} (J = 1-0)$ transition to balance $L_{\text{CO}}$ and driving scale. Detailed models of molecular line cooling (Neufeld, Lepp, & Melnick 1995) suggest that $L_{\text{CO}}/L_{\text{bol}}$ typically falls in the range 1%–2%. Therefore, it may be difficult to account for an additional 1–2 orders of magnitude excess emission. Yet this is required if clouds are stirred from within, since bipolar outflows are usually invoked as the only viable stirring mechanism (Norman & Silk 1980; McKee 1989; Shu et al. 1999), and these act on relatively small scales $\lambda \approx 0.1–1$ pc.

Can the required excess emission be hidden in the high $J$ transitions of CO or in lines from other species? In the recent simulations of Stone et al. (1998), Smith, Mac Low, & Zuev (2000, hereafter SMZ), and Smith, Mac Low, & Heitsch (2000, hereafter SMH), some 50%–68% of the turbulent energy dissipates in shocks. In simulations of freely decaying MHD turbulence, SMZ find that most shocks are slow, with typical Mach number in the range 1–3, i.e., velocities in the range $v_t = 0.2–0.6$ km s$^{-1}$ for a kinetic temperature $T = 10$ K. Applying the Rankine-Hugoniot jump conditions for a nonradiative shock, this range of $v_t$ yields post-shock temperatures at best as high as 27 K. SMH find only slightly higher Mach number velocity jumps for driven MHD turbulence in systems with an energetically significant magnetic field. Even if shock velocities are as high as $v_t = 1$ km s$^{-1}$, the maximum (nonradiative) post-shock temperature is $T = 87$ K, which lies within the range in which emission from CO dominates the molecular emission for $n \leq 10^4$ cm$^{-3}$ (see Fig. 4 of Neufeld et al. 1995). Furthermore, the low-order transitions of CO are easily excited under these conditions, with the lowest transition of $^{12}\text{CO} (J = 1-0)$ accounting for $L_{\text{CO}} \sim 10^{-2} L_{\text{bol}}$ (Neufeld et al. 1995). Therefore, the high bolometric luminosities $L_{\text{bol}}$ shown in Figure 1 for small-scale driving (implying an observed $L_{\text{CO}}/L_{\text{bol}} \sim 10^{-4}$ to $10^{-3}$) are incompatible with molecular cooling models. It is likely that a detailed calculation based on the distribution of shock strengths in a typical simulation with small-scale driving will show a significant excess of $L_{\text{CO}}$ above the empirical values presented in Figure 1. This will also imply much greater values of the excitation temperature than actually observed.

Finally, we note the scalings of the luminosity and energy generation rate. Since cloud luminosities scale as $n^{2.5}$ (alternatively $L^{2.5}$, leading to the mass-luminosity scaling $M \propto L^{0.8}$ measured by Solomon et al. 1987), it may be difficult to attribute their radiative output to cosmic-ray heating, which, on average, scales as $n^{-2}$ (or $L^2$). Turbulent dissipation proportional to the crossing time provides the correct scaling for energy input, i.e., when the driving scale is proportional to the individual cloud size $L$ rather than a fixed internal scale $\lambda < L$. In fact, dissipation with $\kappa \gtrsim 1$, consistent with $\lambda \approx L$, yields $L_{\text{CO}}/L_{\text{bol}} \sim 10^{-3}$ (see Fig. 1), roughly compatible with the observed $L_{\text{CO}}$ and the application of molecular cooling models to post-shock conditions of weakly supersonic shocks.

4. OTHER CONSTRAINTS

In addition to the luminosity problem, another feature of rapid internal driving is that an amount of energy in excess of the gravitational binding energy of a cloud needs to be input and dissipated every crossing time (since $\lambda < L$). This is not likely to be a very stable situation, as any fractional imbalance between dissipation and cooling can lead to expansion and mass flow from the cloud.

Furthermore, the star formation rate required to support this energy input also appears to be too large. An estimate for the required star formation rate can be obtained in the manner first done by McKee (1989; see also Shu et al. 1999) but incorporating the published dissipation rates of Stone et al. (1998) and Mac Low (1999) and using the line-width–size–density relations. The local dissipation rate given by equation (3), with $\eta = 1$, yields a Galactic dissipation rate

$$L_{\text{turb}} = \frac{M \sigma^3}{\lambda},$$

where $M$ is the mass of molecular gas in the Galaxy. We compare this to the input rate from momentum-driven
bipolar outflows with launching speed $v_w$ and mass flux $M_{w}$, which equals a fraction $f$ of the star formation accretion rate $\dot{M}_{SF}$. If the swept-up shell effectively dissipates when the speed drops to $v_{\text{rms}} = \sqrt{3} \sigma$, the delivered energy is

$$L_{\text{out}} = \frac{1}{2} f \dot{M}_{SF} v_w v_{\text{rms}}.$$  

Equating equations (11) and (12), the required star formation rate is

$$\dot{M}_{SF} = \frac{2}{3^{1/2} f} \frac{\sigma^2}{\lambda v_w}.$$  

To obtain a numerical estimate, we use $f = \frac{1}{2}$ (e.g., Shu et al. 1999), $v_w = 200$ km s$^{-1}$, and a relatively large driving scale $\lambda = 1$ pc for the outflows. The total mass $M \approx 10^9 M_\odot$ of molecular gas in the Galaxy is concentrated in the giant molecular clouds (e.g., Solomon et al. 1987), for which we take $n = 10^2$ cm$^{-3}$ to be a representative density. Equations (5) and (6) then yield $\sigma = 3.48$ km s$^{-1}$. Finally, evaluating equation (13) yields $\dot{M}_{SF} = 143 M_\odot$ yr$^{-1}$, which is clearly too high compared to most estimates of the Galactic star formation rate $\approx 3$–5 $M_\odot$ yr$^{-1}$ (see McKee 1989). Altogether, recovering a low enough star formation rate given the fast dissipation rates can be done only with extreme fine tuning and therefore seems unlikely. Our conclusion is similar to that of McKee (1989), that bipolar driving is adequate if the dissipation occurs over many free-fall times $t_{ff}$ of the cloud but is inadequate if it occurs over $\sim t_{ff}$.

5. CONCLUSIONS

In molecular clouds, energy generation by turbulent dissipation can far exceed heating by cosmic rays (see eq. [10]), which have traditionally been viewed as the energy source responsible for the 10 K excitation temperatures measured in $^{12}$CO (Goldsmith & Langer 1978; Neufeld et al. 1995). In this paper, we have demonstrated that rapid internal turbulent driving on small scales, in clouds which obey the line-width–size–density relation, and in which MHD turbulence damps on essentially a crossing time over the driving scale, requires that the observed $^{12}$CO ($J = 1$–0) luminosity $L_{CO}$ undersample the bolometric cloud luminosity $L_{bol}$ by a far greater factor than predicted in standard molecular cooling models (Neufeld et al. 1995). We are left with two distinct possibilities: either current numerical simulations are seriously overestimating the dissipation rate of MHD turbulence, or the idea of long-lived molecular clouds supported for several global crossing times by small-scale internal driving is incorrect.

If stirring occurs on approximately the scale $L$ of each individual cloud, then the observed magnitude and scaling of $L_{CO}$ can be understood with MHD dissipation rates comparable to those calculated in recent simulations. This may mean that clouds are not long-lived and need not maintain a steady state through stirring equal to the dissipation rate, consistent with the conclusion (based on stellar ages) that star formation occurs on a single cloud crossing time (Elmegreen 2000). Alternatively, clouds need to survive for only $\approx 2$–3 crossing times to explain their estimated lifetimes and their apparent state of virial equilibrium. We note that the crossing time for clouds which obey the line-width–size–density relation is

$$t_c = L/\sigma = 1.3 \times 10^7 (n/10^2 \text{ cm}^{-3})^{-1/2} \text{ yr},$$  

which is $\approx 4$ times the free-fall time

$$t_{ff} = (3\pi/2G\rho)^{1/2} = 3.3 \times 10^6 (n/10^2 \text{ cm}^{-3})^{-1/2} \text{ yr}.$$  

The calculated $t_c$ is only a factor of $\approx 2$–3 lower than the estimated lifetime $\approx 3 \times 10^7$ yr of giant molecular clouds (Williams & McKee 1997) of mean density $n \approx 10^2$ cm$^{-3}$. Thus, we believe that future numerical simulations which yield slightly (a factor of $\approx 2$–3) lower net dissipation rates can reconcile turbulent dissipation (if driving occurs on the scale $L$ of each cloud) with the apparent evidence for long-lived (lifetime $\approx 2t_c$–$3t_c$) clouds that are in near virial equilibrium. These somewhat lower dissipation rates may be obtained by tapping previously unmodeled global sources of turbulence, such as gravitational contraction and participation in the differential rotation of the Galaxy, and/or dealing more carefully with heating and cooling of low-density molecular cloud envelopes. The latter requires dropping the isothermal assumption, which can overestimate radiative losses and enhance the development of shocks, particularly in low-density regions.

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