Do Ceasefires Work? A Bayesian autoregressive hidden Markov model to explore how ceasefires shape the dynamics of violence in civil war

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Abstract

Despite a growing body of literature focusing on ceasefires, it is unclear if most ceasefires achieve their primary objective of stopping violence. Motivated by this question and the new availability of the ETH-PRIO Civil Conflict Ceasefires Dataset, we develop a Bayesian hidden Markov modeling (HMM) framework for studying the dynamics of violence in civil wars. This ceasefires data set is the first systematic and globally comprehensive data on ceasefires, and our work is the first to analyze this new data and to explore the effect of ceasefires on conflict dynamics in a comprehensive and cross-country manner. We find that ceasefires do indeed seem to produce a significant decline in subsequent violence. However, the pre-ceasefire period (the period typically after a ceasefire agreement has been negotiated but before it is in effect) is shown to be prone to periods of intensified violence that are most likely a cause and effect of the subsequent ceasefire. This finding has significant implications for the research and practice community. Moreover, manifesting from the ubiquity of modern data repositories combined with a deficiency in meaningful labels, HMM-based semi-supervised data labeling strategies could pave the way for the next decade of conflict research progress.

Keywords: state space model; multi-state model; discrete-valued time series; count-valued time series
1 Introduction

Ceasefires are arrangements through which conflict parties commit to stop fighting. They are a common part of intra-state conflict, each year occurring in about a third of all conflicts.\(^1\) Between 1989 and 2020 there were at least 2202 ceasefires across 66 countries, in 109 civil conflicts (Clayton et al., 2021). Surprisingly, despite their frequency, it remains unclear to what extent ceasefires really work, i.e. we do not know to what extent they shift a conflict from a more violent to a less violent state. To illustrate this point, in early 2003 a ceasefire between the Government of Sudan and the SPLA/M marked a transition from a long period of sustained violence into a relatively non-violent state that remained in place until the parties reached a comprehensive peace agreement in 2005. Yet in Syria, for example, where there have been more than 130 ceasefires in the ongoing civil conflict, many ceasefires seem to have produced an escalation rather than deescalation in violence, or had no effect at all (Karakuş and Svensson, 2020). From existing research it is not possible to determine if the Syrian or Sudanese case are indicative of the general effect of ceasefires on conflict violence.\(^2\)

The lacuna in understanding the role ceasefires play in conflict is a result of conflict researchers lacking the necessary statistical tools to properly model violent dynamics and to study and understand the covariates that influence these dynamics. Existing models employed in the conflict research literature are not particularly adept at distinguishing when an event, such as a ceasefire, fundamentally shifts the violent dynamics of a conflict.

Motivated by the new availability of the ETH-PRIO Civil Conflict Ceasefires Dataset (Clayton et al., 2021), we address this critical need. This ceasefires data set is the first

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\(^1\)For simplicity we refer to ‘armed conflict’ as conflict or armed conflict. We follow the UCDP/PRIO Armed Conflict Database and define an armed conflict as: ‘a contested incompatibility that concerns government and/or territory where the use of armed force between two parties, of which at least one is the government of a state, results in at least 25 battle-related deaths in one calendar year’ (Gleditsch et al., 2002). In this article we only focus on internal armed conflicts.

\(^2\)In contrast, a burgeoning body of literature explores the drivers of ceasefire onset, and the impact that ceasefires have on other outcomes such as peace processes, crime, and state-building; see, Akebo (2016); Waterman (2020); Bara, Clayton and Rustad (2021); Clayton et al. (2020a)
systematic and globally comprehensive data on ceasefires, and our work is the first to analyze this new data and to explore the effect of ceasefires on conflict dynamics in a comprehensive and cross-country manner. As an interdisciplinary team of conflict researchers and statisticians, the authors of this paper are uniquely qualified to both develop the necessary statistical methods and tools and provide domain-expert analysis of the resulting statistical inferences. Furthermore, part of our aim is that we invite the attention and interest of other statisticians to the unique statistical methodological challenges that exist in conflict research areas; very few statisticians currently work on these applications.

Our contributions are the following. We study the new ceasefires dataset, combined with the violence data from the Uppsala Conflict Data Program’s geo-referenced event dataset (UCDP GED) (Sundberg and Melander [2013]). We develop a discrete-time Bayesian HMM using weekly battle death counts as the emitted response variable, combined with conflict-domain-theory motivated, country-specific covariates, to make inference on how violence dynamics evolve in time over a theorized (hidden) discrete conflict-related state space. The weekly battle death count data are modeled, conditional on the underlying state, using a negative-binomial distribution with an autoregressive mean structure. This is a natural choice because conflict-related death count data are time series that are commonly characterized by both overdispersion and zero-inflation. See Weiß (2018) for a focused account of discrete-valued time series models, and the recent review paper Davis et al. (2021). A custom Metropolis-within-Gibbs Markov chain Monte Carlo (MCMC) algorithm is constructed to fit the HMM, and the repeated sampling coverage of all HMM parameter estimates is provided via the construction of posterior credible sets. We also construct a second MCMC algorithm to estimate the conditional posterior distribution of the latent conflict-state sequence for any country, given the posterior HMM parameters (estimated with our first algorithm). This second algorithm is used to predict future conflict dynamics and counts of weekly battle deaths.
Next, building on existing conflict research literature, we discuss how our analyses and inferences are motivated from and translate to the existing theory for how ceasefires shape conflict violence. Most notably, a major finding of ours is surprising evidence for an escalation in the state of violence in the pre-ceasefire period (i.e., the two weeks prior to a ceasefire) of a conflict. Prior to a ceasefire parties have incentives to fight harder to gain the strongest relative position prior to the ceasefire suspending the violence. Escalated violence also increases the incentives for a ceasefire. Once a ceasefire enters into effect, we find that conflict tends to transition from a violent to a non-violent state, which might be explained as the benefits accrued from a ceasefire, whether peaceful or military/strategic, require some immediate shift in violent dynamics. Finally, we find evidence for three underlying states of conflict, which we describe as ‘non-violent’, ‘stable violence’, and ‘intensified violence’. We illustrate the utility of the constructed HMM for both inferential purposes and as a tool for predicting (the intensity of) conflict violence. Documented R code for the use of our methods by other researchers and policy analysts on their data, along with the workflow for reproducing our results is available at https://jonathanpw.github.io/research.html.

Within the conflict research community the HMM has been studied from a variety of different perspectives, and with varying degrees of sophistication. Early applications of HMMs in the conflict research literature are investigated as case studies for various countries. They are largely motivated by a perceived need for the conflict research community to explore its data beyond what can be provided by linear models, arguing that the dynamics exhibited by these data are complex, non-linear political systems (Petroff, Bond and Bond 2013; Schrodt 1997b; a 2006).

Schrodt (1997b) investigates the adequacy of an HMM to distinguish between crises involving war versus those not involving war in the Middle East (circa 1979-1997), as one of the first conflict research papers to apply an HMM. Two, six-state HMMs are fit (one using “war crises” examples and one using “non-war crises” examples), and predictions are...
assessed based on which of the two models fit a future sequence of events better. The subsequent work, Schrodt (1997a) extends the HMM to allow for transitions to adjacent states, termed a “left-right-left model”, and considers “tit-for-tat” escalation in South Lebanon (Circa 1979-1997). This progression of research evolved into the work of Schrodt (2006). This final chapter constructed an HMM to forecast “high-conflict-weeks” and “low-conflict-weeks” in the former Yugoslavia by estimating two, six-state HMMs and determining which of the two models fit better to a future sequences of events. See also Shearer (2007) for a similar analysis applied to Israeli-Palestinian conflict data. Significant drawbacks, noted in Schrodt (2006), of the HMM was an indeterminacy of parameter estimates obtained by the likelihood-maximizing Baum-Welch algorithm, and obtaining meaningful estimates of the standard errors of the maximum likelihood parameter estimates. We mitigate these issues by implementing Bayesian computational strategies.

A few years after the preliminary investigations of HMM in the conflict research literature, the book chapter Petroff, Bond and Bond (2013) summarizes the best practices with particular emphasis on forecasts/predictions of violence. Overall, the ideas about, and implementations of, HMM strategies for explaining/predicting conflict data have not advanced beyond the ideas and strategies prescribed in the classical tutorial paper, (Rabiner, 1989). That is, there is a set of theoretically motivated (unobserved) conflict states (in the range of 3-6) linked by way of an HMM to pre-process and organize sequences of event-coded symbols from a large repository of international news summaries (provided by an agency such as Reuters). Locally maximum likelihood estimates are obtained from a Baum-Welch algorithm, and the most plausible (unobserved) states-sequences are inferred via the Viterbi algorithm.

This segues into a second point of concern, that it is not clear whether data has been properly discretized in the existing studies. For example, Petroff, Bond and Bond (2013) describes the ability to tailor the length/number of time intervals to the precision of the
data available, and discourages aggregation of data (i.e., hours, days, weeks, etc.). While it is true that a discrete-time Markov process can be defined on any grid, the a priori chosen grid must apply to all data sequences (training and forecast). The use of time stamps of observed data sequences, as they were actually recorded in time, requires adherence to a continuous-time Markov process. Such a process can be modeled with a continuous-time HMM, and has been studied extensively in the disease progression literature (e.g., Satten and Longini Jr (1996); Williams et al. (2020)).

More recent conflict research developments of HMM frameworks includes Qiao et al. (2017) and Anders (2020). Still in a largely Rabiner (1989)-based approach, Qiao et al. (2017) embeds a Bayes classification rule to predict, similar to Schrodt (2006), the best fitting of two constructed HMM, one consistent with sequences prone to social unrest and the other for sequences not prone to social unrest. Anders (2020) undertakes an interesting exploitation of an HMM strategy to estimate territorial control in civil wars as a latent state space with the five states, ranging from “rebel control” and “government control”.

The remainder of the paper is organized as follows. In the next section, we provide an overview of the political science theory for conflict and ceasefires. Section 2.3 then reflects on existing attempts to model violent dynamics in conflict research. The data are described in Section 3 and Section 4 provides the technical details of our proposed HMM, along with a simulation study in Section 4.2. Model estimates and accompanying results and analysis are discussed in Section 5. Then the paper concludes with final remarks and avenues for future research directions in Section 6. Our computer code and additional plots are available in the supplementary material.

2 Political science theory for conflict and ceasefires

All ceasefires share the same immediate objective: to stop violence (Clayton, Nathan and Wiehler, 2021). Stopping violence can however serve various purposes. Firstly, ceasefire
can help to support conflict management efforts: creating breaks in the fighting to facilitate humanitarian assistance (Aary, 1995); helping to contain conflict when resolution is not yet possible (Clayton et al., 2020a); and terminating violence in such a way that does not require the resolution of the incompatibility (Hanson, 2020; Kreutz, 2010). Second, ceasefires can also help with conflict resolution efforts: helping to build trust (Åkebo, 2016); signalling control and cohesion (Höglund, 2011); and creating an environment more conducive to negotiations (Smith, 1995; Mahieu, 2007a; Chounet-Cambas, 2011; Clayton and Sticher, 2021). Third, not all ceasefires are conceived for peaceful purposes. Ceasefires can also be used to gain some strategic advantage, including: buying time to rearm and regroup (Clayton et al., 2020b; Smith, 1995); support statebuilding efforts (Woods, 2011; Sosnowski, 2020), or undertaking illicit activity (Kolás, 2011; Dukalskis, 2015).

Nevertheless, in almost all cases, it is logical to assume that in order to achieve their purpose, ceasefires must first achieve the immediate objective, i.e. shift conflict from a violent to a non-violent state. Surprisingly, we know relatively little about how effective ceasefires tend to be at achieving this objective. To date, ceasefire research is largely limited to case studies (Palik, 2021; Pinaud, 2020; Åkebo, 2016), or analysis tailored for the policy and practice community (e.g. Brickhill, 2018; Buchanan, Clayton and Ramsbotham, 2021). The research in this area details a number of cases in which ceasefires have ultimately proved successful (e.g., De Soto, 1999), but also shows that in many cases violence does not end with the onset of a ceasefire (Kolás, 2011; Jarman, 2004; Höglund, 2005; Åkebo, 2016), and that some ceasefires can even make the violence worse (Luttwak, 1999; Kolás, 2011; Mahieu, 2007b).

In-depth qualitative analysis has many advantages (see, George and Bennet, 2005), but is ill-suited to systematically identifying broad trends, such as whether ceasefires in general

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3 A ceasefire might prove to be successful according to one purpose (e.g., humanitarian aid), but unsuccessful in another (e.g., promoting peace talks), or successful in the eyes of one conflict party, while representing an abject failure in the eyes of another. Ceasefires might also achieve their purpose, but produce other unintended effects (e.g., promoting the splintering of a non-state group (Plank, 2017)).
produce a significant shift in violent dynamics. This instead requires comparative quantitative analysis which has, to date, been limited for questions surrounding ceasefire onset (Clayton et al., 2019) and design (Clayton and Sticher, 2021). There is some evidence that ceasefires stop violence (Fortna, 2003, 2004), but this is limited to inter-state conflict, and as we describe below, the analysis suffers a number of serious methodological limitations.

Accordingly, perhaps the most fundamental question on ceasefires remains largely unanswered: Do ceasefires stop violence?

2.1 Ceasefire period

A ceasefire, by definition, includes a statement of intent whereby the conflict parties commit to stopping violence. That conflict parties declare a commitment to stop violence does of course not mean they actually intend to do so. In some cases external pressure can push parties to accept ceasefires that they have no incentives or willingness to follow. In the midst of armed conflict actors are likely to exploit any opportunity to secure an advantage, and that might include deceiving their opponent about their intention to honor a ceasefire.

In other cases, problems of asymmetric information and a lack of communication can lead to the unintentional failure of a ceasefire, or internal divisions lead to spoilers that act to undermine a deal.

The breakdown of a ceasefire does however carry costs. If a conflict party fails to deliver the promised suspension in violence they are likely to suffer significant audience costs (Fortna, 2003b, p.343). This might include condemnation and sanction from the international community and domestic groups supporting the ceasefire. It is also likely to further sour relations between the conflict parties, undermining what little trust may have been built in the process that led to the ceasefire and complicating future peace efforts. This is particularly the case if an agreement breaks down immediately.

At the same time, the benefits of a ceasefire, both peaceful and strategic, take time and some minimal compliance to accrue. For example, if conflict parties intend for a ceasefire to
develop trust and build commitment, this can only be achieved if the parties cease violence in the manner stipulated in the agreement. For it is not the declaration of a ceasefire, but the subsequent period of honoring the commitment that develops trust between the parties. Similarly, at the other end of the spectrum, gaining a military advantage from a ceasefire also likely requires a sufficient pause in the fighting. For example, if an actor enters a ceasefire with the intention of using the break in violence to rearm and regroup, this requires a sufficient period of mutual compliance to be achieved.

Given that in most cases the benefits of ceasefires take time to accrue, and that the costs associated with (immediately) reneging on an agreement are high, we expect that all else being equal, in most cases the initiation of a ceasefire is likely to produce some minimal stop in the violence.

2.2 Pre-Ceasefire period

Periods of intense violence raise the financial, material, and human costs of conflict. This increases the incentives for a ceasefire, which can temporarily reduce many of these conflict costs (Clayton et al., 2020). Further, the prospect of a ceasefire dramatically changes the immediate time horizon. Whereas a normal conflict dyad must balance resources on both short and long term basis, a ceasefire means that the long term considerations become dramatically less important for both parties at the same time. Motivated by our empirical findings, a question of central focus from our research is: Are conflicts more likely to enter into state of intensified violence in the period directly preceding a ceasefire?

Civil conflict involves strategic actors (Cetinyan, 2002; Walter, 2006) that are likely to consider the probable response of their opponent when making decisions (Lake and Powell, 1999; Walter, 2006). Prior to entering into a ceasefire, conflict parties are therefore likely to consider how any such agreement might influence their position relative to their opponent. In this way, ceasefires are not self-contained political negotiations, but constitute a tool to be used as part of the larger conflict bargaining process. As ceasefires, by definition, commit
parties to the cessation of violence in a subsequent period, from a bargaining perspective both parties should seek to secure the strongest possible position on the battlefield to strengthen their hand in negotiations once the ceasefire is underway. That being so, both parties actually have an incentive to escalate violence prior to the ceasefire in order to have the strongest relative position once violence stops.

Conversely, there are also good theoretical reasons to believe that the anticipation of a ceasefire might lead to reduced violence. Conflict parties (and individual fighting units) might begin to deescalate hostilities in anticipation of the ceasefire, keen to avoid unnecessary military engagements (i.e., no one wants to be the last person to die in a conflict). Alternatively, the cumulative fatigue and/or hurting stalemate that led to the ceasefire might not allow for any further escalation, and a ceasefire might reflect, rather than create, a new less violent reality on the ground. Yet whilst these factors no doubt play an important role in specific context, it may be the case that, in general, the incentives that a ceasefire creates for violent escalation (and the incentives that escalated violence create for a ceasefire) mean that, on average, the pre-ceasefire period is more likely to experience intensified violence.

2.3 The challenges of modelling conflict dynamics

In order to assess the impact of ceasefires on conflict violence, we need to model violent conflict dynamics. Studies of intra-state conflict dynamics, however, tend to either focus on single countries or on primarily conceptual issues such as how patterns should be understood (c.f. Gutiérrez-Sanín and Wood, 2017). Yet, these patterns, and the behaviors and factors that influence them, are neither empirically nor theoretically well understood. This in no way implies that particular aspects of the violent dynamics have not been studied. Lacina and Gleditsch (2005) and Lacina, Russett and Gleditsch (2006) made crucial first contributions to our understanding of the intensity or severity of war. Since then a number of factors shaping conflict dynamics have been identified, including inter alia: the struc-
ture and composition of armed groups (Christia, 2012; Cunningham, 2006; Cunningham, Bakke and Seymour, 2012; Green, 2018), territorial control (Kalyvas, 2006), technologies of rebellion (Balcells and Kalyvas, 2014; Kalyvas and Balcells, 2010), contagion and diffusion (Salehyan and Gleditsch, 2006), different forms of violence (Eck and Hultman, 2007), and conflict management tools, most notably peacekeeping (Hegre, Hultman and Nygård, 2019; Hultman, Kathman and Shannon, 2014; Fortna and Howard, 2008; Walter, Howard and Fortna, 2020). While these studies all contribute to our understanding of conflict dynamics, in contrast to the literature on the onset of armed conflict, this literature is less coherent.

The literature on the onset of armed conflict has benefited tremendously from having a, more or less, standard statistical model (a logistic cross-section time series model) and a standard set of covariates (in particular related to socio-economic development, political institutions, and demography). This standard model has allowed the literature to accumulate knowledge as more and more pieces of the puzzle have been added. Unfortunately, this has not been the case for the conflict dynamics literature, which has developed in a much less coordinated fashion.

Fundamentally, modeling conflict dynamics requires formulating a statistical model to capture the latent, or underlying, ‘state’ of the conflict. To be a useful representation of the underlying process, this model needs to be able to capture and re-create the intensity of conflict, with its spikes and lulls, as well as more enduring patterns of violence. We argue that an HMM state space framework (introduced in Section 4), trained using recorded battle deaths in a conflict over time, is a reasonable statistical model formulation for this purpose. In the sections that follow, we illustrate how the HMM can first be used to describe conflict dynamics, and then to study how ceasefires effect these dynamics – in a manner similar to how the literature on the onset of armed conflict has evolved.

Figure 1 illustrates the statistical challenge at hand. It shows daily aggregated number of battle deaths in internal armed conflicts over the 1989 to 2018 period for the Democratic
Republic of Congo “Congo” and Colombia. Our goal is to build a model that can reproduce both the localized (in time) spikes, seen in both panels, and the more enduring ‘mountains’ seen in the bottom panel. To properly understand conflict dynamics, we need to account for both these phenomena. The typical statistical models employed by conflict researchers, overwhelmingly logistic or ordinary least squares regression, count models such as Poisson and negative-binomial, and time-to-event models such as Cox proportional hazard models, are not particularly well suited for such purposes (a recent review is Davenport et al., 2019).

3 Data

The violence data we consider come from the UCDP GED (Sundberg and Melander, 2013), which reports all events with at least one battle-related casualty. Each event record shows where and when an event took place, which actors where involved, and how many battle-related deaths that ensued. We aggregate the frequency of events weekly per country.

For ceasefires we rely on the ETH-PRIO Civil Conflict Ceasefires Dataset (Clayton et al., 2021), which represents the first systematic and globally comprehensive data on ceasefires. Our work is the first to use this new data to explore the effect of ceasefires on conflict dynamics in a comprehensive and cross-country manner. The ceasefires data defines a ceasefire as ‘an arrangement that includes a statement by at least one conflict party to stop violence temporarily or permanently from a specific point in time’. This broad conceptualization of a ceasefire captures the full range of security arrangements through which belligerents might agree to temporarily suspend and/or terminate hostilities. We include unilateral and bi/multi-lateral ceasefires. A unilateral ceasefire occurs if one group alone declares the cessation of hostilities. For example, in December 2018 the Tatmadaw (army) in Myanmar declared a unilateral ceasefire towards a number of armed ethnic orga-

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4We study the number of people killed due to intrastate conflict and/or internationalized intrastate conflicts per week in each country. This means that the data for countries with several parallel on-going conflicts are collapsed into one country time series.

5This definition covers arrangements that might be labelled truces, cessation of hostilities, armistices, and preliminary and definitive ceasefire agreements.
nizations that was not reciprocated. A bi/multi-lateral ceasefire occurs when two or more parties jointly declare a ceasefire towards one another. For example, in November 2018, Israel and Hamas jointly agreed to simultaneously halt hostilities. As our aim is to explore the impact of ceasefires on violence dynamics, rather than peace agreements, we consider only non-definitive ceasefires (i.e., ceasefires that attempt to suspend rather than permanently terminate a conflict). We also exclude ceasefires that only cover a partial part of the conflict area (i.e., so-called local ceasefires), as these agreements seek to only reshape violence in a limited area, and so it does not makes sense to assess their impact on the conflict as a whole. Finally, we exclude ceasefires that extend or renew prior agreements, based on the assumption that any shift in violent dynamics is likely to have occurred in response to the original agreement.

The ceasefires data includes the date on which the arrangement enters into effect. We define the week containing the start date of the ceasefire, together with the following four

![Congo](image1)

![Colombia](image2)

Figure 1: Weekly number of battle deaths in the Democratic Republic of Congo “Congo” and Colombia, 1989–2018. Note that the Democratic Republic of Congo suffered 3,000 deaths the week of December 14, 1998, a value beyond the plot range chosen for illustration.
weeks (i.e., five weeks in total) as a ceasefire period. We do this to focus the analysis and the attention of the model on the dynamics that we are most confident relate to the ceasefire. This helps to mitigate the record-keeping uncertainty surrounding the effective duration of a ceasefire. Further, we define the two weeks prior to the start of a ceasefire (i.e. the two weeks before the week that contains the start date) as the pre-ceasefire period. Ceasefires tend to be negotiated fairly quickly, and once agreed to often take a few days to implement. Thus, we believe two-weeks represents a sufficiently long period so as to capture the direct period in which the ceasefire is under consideration, but short enough to avoid picking up other conflict-related factors.\textsuperscript{6}

There are several countries in the dataset that do not contain any battle deaths or ceasefires. Out of the 170 countries, there are 74 without any battle deaths, and 124 countries with less than a 1000 battle deaths throughout the entire time period 1989–2018. These countries are important for estimating the baseline state of ‘non-violent’, also referred to as ‘state 1’. We partially label the data using the following definition: a week is labelled as state 1, without error, if it is at least 60 days (in both directions in time) removed from an observation of at least one battle death, and if it is also part of a consecutive period of at least 2 years without any battle-related deaths. This is the only state labelling used for model estimation.

3.1 Control covariates

There are a collection of standard covariate types that are known in the conflict research community to affect the likelihood of conflict, such as political regime, economic development, and population. For these covariate types we use the following. Political regime is measured with the polyarchy index from the V-Dem project (Coppedge et al. 2019), used to control for alternative conflict management opportunities within Dahl’s (1971)\textsuperscript{6}

\textsuperscript{6}Since we are aggregating the data at a country level, it means that it is possible for a country to simultaneously be in a pre-ceasefire and a ceasefire period. This is relatively rare (only 244 out of a total 3872 weeks have multiple events) thus we leave modelling this challenge to future work (see Section for additional discussion).
conceptualization of democracy. It is understood that countries somewhere in the middle of the polyarchy spectrum are most vulnerable to violent conflict, and so we include polyarchy in nominal value, as well its squared and cubed values, as covariates in our analysis. V-Dem’s indices consist of a mix of variables measured either at the end of a year, at the maximum value throughout a year, or as the average over a year. That being true, we use lagged polyarchy values to prevent mixing cause and effect.

Economic development is measured as gross domestic product (GDP) per capita. GDP relates directly to the feasibility of armed conflict because potential rebels in wealthy countries have more to lose from a rebellion, and wealthy governments have a larger capacity to co-opt a population through public goods and coerce a potential rebel through a strong security apparatus. Poor countries are more likely to have poor citizens, often more willing to engage in risky warfare, less ability to co-opt, and weaker militaries.

Population affects both the risk of conflict in the first place and the likelihood of a ceasefire in a specific conflict. Larger countries are more likely to have conflicts simply because of the larger number of people able to start a rebellion. Consequently, larger countries are more likely to have several parallel conflicts, which creates a complex situation where a rebel group might seek a ceasefire with the government to actively harm competing rebel groups. The economic development and population variables are obtained from the World Development Indicators\footnote{http://data.worldbank.org/indicator} and are included in log-lag-values in our analyses.

Lastly, we include as covariates, indicators for ceasefire and pre-ceasefire periods (as defined in Section \ref{section:dataset}) for each country/week.

4 Statistical methodology

In this section we develop our discrete-time Bayesian HMM. As described previously, we regard the data resolution on a weekly grid, but the details would be the same for any
discretization over time. Note that if the data cannot be meaningfully organized on a grid of time points (no matter how precise), then a continuous-time HMM must be developed (e.g., see Williams et al., 2020) otherwise the transition matrix cannot be properly computed.

For a data set consisting of \( N \) countries, denote each county by an index \( i \in \{1, \ldots, N\} \), and let \( y_{i,k} \) be the number of observed battle deaths for week \( t_{i,k} \), for \( k \in \{1, \ldots, n_i\} \), where \( n_i \) is the number of weeks included in the data set for country \( i \). The value of \( y_{i,k} \) results from a multitude of circumstances, and we summarize these circumstances by a latent state \( s_{i,k} \in \{1, 2, 3\} \), corresponding to the ‘state’ of conflict dynamics at week \( t_{i,k} \). These three underlying conflict-related states are defined as ‘non-violent’, ‘stable violence’, and ‘intensified violence’, respectively.8

The underlying state sequence, \( s_{i,1}, \ldots, s_{i,n_i} \), defines a stochastic process, and we assume for computational feasibility, as required for an HMM, that it is a Markov chain in that the state of the process at week \( t_{i,k} \) only depends on the state of the process (and possibly covariates) at week \( t_{i,k-1} \). That being so, the conditional distribution of the random variable \( S_{i,k} \mid S_{i,k-1} \) is determined by a \( 3 \times 3 \) probability transition matrix \( P \), which we express as,

\[
P := \begin{pmatrix}
(1 + e^{q_1} + e^{q_2})^{-1} & 0 & 0 \\
0 & (1 + e^{q_3} + e^{q_4})^{-1} & 0 \\
0 & 0 & (1 + e^{q_5} + e^{q_6})^{-1}
\end{pmatrix}
\begin{pmatrix}
1 & e^{q_1} & e^{q_2} \\
e^{q_3} & 1 & e^{q_4} \\
e^{q_5} & e^{q_6} & 1
\end{pmatrix},
\]

where \( q_1, q_2, q_3, q_4, q_5, q_6 \) are real-valued parameters which determine the rates of respective state transitions. Note that the matrix on the left simply re-scales (as row-wise multivariate logistic function transformations) the matrix on the right to have unit row sums, making it a proper probability transition matrix. The column and row indices correspond to the state space \( \{1, 2, 3\} \). For example, the (1,2) component of \( P \) expresses the value of the probability of transition from state 1 to state 2, for any two successive weeks. Since the matrix \( P \) is not indexed by \( t_{i,k} \) it is implied that the Markov chain is time-homogeneous.

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8We limit our focus to three states, as Petroff, Bond and Bond (2013) argue that beyond three states (in an HMM of conflict dynamics) it becomes exceedingly difficult to interpret the empirical results, and finds distinctions accounted for by including additional states to be vague at best.
Accordingly, for weeks $t_i, 1, \ldots, t_i, n_i$ the probability mass function of the latent state sequence $s_i, 1, \ldots, s_i, n_i$ has the form,

$$\ell(\{s_i, 1, \ldots, s_i, n_i\} \mid P) = \pi_{s_i, 1} \cdot \prod_{k=2}^{n_i} P_{s_{i, k-1}, s_{i, k}},$$

(1)

where $A_{k,l}$ denotes the $k^{th}$ row and $l^{th}$ column of a matrix $A$, and $\pi$ is a 3-dimensional column vector of initial state probabilities. Furthermore, we express the transition probability parameters as,

$$
\begin{pmatrix}
q_1 & q_2 & q_3 & q_4 & q_5 & q_6
\end{pmatrix} := \begin{pmatrix}
(x^{(i)}_k)
\end{pmatrix}' \zeta,
$$

where $x^{(i)}_k$ is a column vector of the geopolitical, region specific control covariates from Section 3.1 as well as the indicators for ceasefire and pre-ceasefire periods, for country $i$ at week $t_i, k$, and $\zeta$ is a coefficient matrix. The feature-rich coefficient matrix $\zeta$ is a crucial component of the HMM for the purpose of studying dynamics of conflict. In theory, the transition rate parameters can be made arbitrarily conflict specific by including as many features (i.e., covariates and coefficient parameters) as necessary. These features determine the rate at which the underlying state of conflict evolves or ceases to progress at all.

This Markov process defined for the latent conflict state sequences is embedded as a structural component of the following data-generating model for the observed battle death data. These data, $y_{i,k}$, for $k \in \{1, \ldots, n_i\}$ and $i \in \{1, \ldots, n\}$, are reasonably described as counts of rare events. Empirically, it is clear that several of the country-specific battle death time series are subjected to both overdispersion and zero-inflation. Since we also observe varying degrees of positive correlation – a type of escalating behavior in which violence leads to more violence – models arising in the discrete-valued time series literature are natural candidates (see e.g., [Weiß, 2018] for a general introduction). In particular, after much empirical study and analysis of the data we define the data-generating model as the so-called NB-DINARCH(4), i.e. negative-binomial dispersion integer autoregressive conditional heteroskedasticity of order four, with a special constraint on the parameters for lagged observations. We have found empirically that averaging over the previous four
weeks values, \( y_{i,(k-4):(k-1)} \) (i.e., \( y_{i,k-4}, \ldots, y_{i,k-1} \)), gives a value that provides a meaningful association between current and lagged battle death values. This amounts to modelling weekly battle death counts as having a mean structure as a function of lagged monthly battle death counts. The data-generating model is expressed as,

\[
Y_{i,k} \mid Y_{i,(k-4):(k-1)}, s_{i,k} \sim \text{negative-binomial}(r_{i,k}, p),
\]

where \( p := c(1 + c)^{-1}, \)

\[
r_{i,k} := a_{i,k} + \rho \cdot \frac{1}{4} \sum_{l=1}^{4} Y_{i,k-l},
\]

\[
a_{i,k} := a_1 1\{s_{i,k} = 1\} + a_2 1\{s_{i,k} = 2\} + a_3 1\{s_{i,k} = 3\},
\]

and

\[
\rho := 1\{s_{i,k} \neq 1\} \cdot e^{(\beta_1 1\{s_{i,k}=2\} + \beta_2 1\{s_{i,k}=3\})^T x_{i,k}},
\]

where \( a_1, a_2, a_3, \) and \( c \) are positive parameters, \( \beta_1 \) and \( \beta_2 \) are coefficient column vectors, and \( 1\{\cdot\} \) is the indicator function. Denote \( a := (a_1, a_2, a_3) \) and \( \beta := (\beta_1, \beta_2) \). Note that for this model to be weakly stationary, it is sufficient that \( 0 \leq \rho < 1 \) (see Theorem 4.1 Xu et al., 2012).

An implication of this data-generating model specification is that if \( s_{i,k} = 1 \), then \( Y_{i,k} \sim \text{negative-binomial}(a_{i,k}, p) \). Accordingly, the parameters \( a_1, a_2, a_3, \) and \( c \) describe the model for rare conflict related deaths that may occur during weeks when a country is not experiencing a substantial conflict (e.g., isolated terrorist attacks). Moreover, this forces the rate parameter \( \rho \) to be identified with an increased mortality rate relating specifically to a defined period of conflict (i.e., when the system is in state 2 or state 3). With the negative-binomial rate parameter defined as in (3),

\[
E(Y_{i,k} \mid Y_{i,(k-4):(k-1)}, S_{i,k}) = \frac{a_{i,k}}{c} \cdot \frac{\rho}{c} \cdot \frac{1}{4} \sum_{l=1}^{4} Y_{i,k-l}.
\]

From the definition of \( \rho \) in (4), this implies that the number of conflict deaths during state 1 has a mean of \( a_{i,k}/c \), whereas in the conflict states the mean structure is autoregressive.
To distinguish between states 2 and 3, the constraint that $\beta_{11} \leq \beta_{12}$ is imposed (i.e., baseline $\rho$ for state 2 does not exceed that for state 3). This constraint helps to facilitate the identification of states 2 and 3, respectively, as associated with ‘stable’ versus ‘intensified’ conflict violence. Furthermore, we also impose the constraints that $a_1 \leq a_2 \leq a_3$ for the purpose of state space identification.

Finally, combining components [1] and [2] gives a full likelihood function for the HMM for each country $i \in \{1, \ldots, N\}$. For efficient estimation of the parameters, the likelihood can be expressed as a marginal mass function, resulting from integrating over all possible state space sequences. That is,

$$p(y_{i,5}, \ldots, y_{i,n_i}) = \sum_{s_{i,1}=1}^3 \cdots \sum_{s_{i,n_i}=1}^3 \sum_{y_{i,1} \geq 0}^{} \cdots \sum_{y_{i,4} \geq 0}^{} p(y_{i,1}, \ldots, y_{i, n_i}, s_{i,1}, \ldots, s_{i,n_i})$$

$$= \sum_{s_{i,1}=1}^3 P(s_{i,1}) \cdots \sum_{s_{i,5}=1}^3 p(s_{i,5} \mid s_{i,4}) \cdot p(y_{i,5} \mid y_{i,1:4}, s_{i,5}) \cdots$$

$$\times \sum_{s_{i,n_i}=1}^3 p(s_{i,n_i} \mid s_{i,n_i-1}) \cdot p(y_{i,n_i} \mid y_{i,(n_i-4):(n_i-1)}, s_{i,n_i})$$

$$= \pi' \cdot P(i,2) \cdots P(i,4) \cdot P(i,5) D(i,5) \cdots P(i,n_i) D(i,n_i) 1,$$

where $\pi$ is the common initial state probability column vector, $P(i,k)$ is the probability transition matrix, $P$, evaluated for the covariates values for country $i$ at week $k$,

$$D(i,k) := \begin{pmatrix} p(y_{i,k} \mid y_{i,(k-4):(k-1)}, s_{i,k} = 1) \\ p(y_{i,k} \mid y_{i,(k-4):(k-1)}, s_{i,k} = 2) \\ p(y_{i,k} \mid y_{i,(k-4):(k-1)}, s_{i,k} = 3) \end{pmatrix},$$

and using the negative-binomial mass function,

$$p(y_{i,k} \mid y_{i,(k-4):(k-1)}, s_{i,k}) = \binom{r_{i,k} + y_{i,k}}{y_{i,k}} p^{r_{i,k}} (1 - p)^{y_{i,k}}.$$

Note that if further state space information is available, such as partial labels, then the number of state sequences that are integrated over is reduced. For example, if it is known that $s_{i,k} \in \{2, 3\}$ for some $k \in \{1, \ldots, n_i\}$, then $\sum_{s_{i,k}=1}^3 \cdot = \sum_{s_{i,k}=2}^3 \cdot$ within expression (6). Equivalently, the $(1, 1)$ component of $D(i,k)$ is set to zero.

Finally, the joint posterior density including the data from all countries $i \in \{1, \ldots, N\}$
then has the form,
\[
\pi(\zeta, \beta, a, c \mid \{y_{i,k}\}) \propto \left[ \prod_{i=1}^{N} p(y_{i,5}, \ldots, y_{i,n_i}) \right] \cdot \pi(\zeta, \beta, a, c) \cdot 1\{\beta_{11} \leq \beta_{12}, a_1 \leq a_2 \leq a_3\},
\] (7)
where \( \pi(\zeta, \beta, a, c) \) is a prior density. Our Bayesian computations based on this density are discussed next.

### 4.1 Remarks on computation

To draw posterior samples from (7) we have written and implemented a custom Metropolis-within-Gibbs MCMC algorithm. In a Gibbs sampler fashion, user-defined groups of the parameters (denoted by \( \zeta, \beta, a, c \); 70 parameters in total) are updated via a Metropolis-Hasting step. We have found in our experiments on our real data that organizing the parameters into three groups yields an efficient trade-off between adequate mixing of the MCMC samples versus the ability of the chain to sufficiently explore the high posterior density regions. Further, during the initial “burnin” phase of the MCMC algorithm, an adaptive proposal strategy is implemented that tunes the variance of the proposal distributions so as to achieve an optimal acceptance proportion approximately in the range of 0.3 – 0.5. These computational strategies are based on the strategies proposed in Williams et al. (2020), and further details can be found in the supplement to that paper.

With the posterior samples of the parameters in (7) obtained from our MCMC algorithm, we then develop and implement a custom computational strategy to estimate the conditional posterior distribution of the state space for a given country in our data set. That is, in a series of Gibbs sampler steps, two consecutive weeks at a time, the algorithm proposes a pair of associated latent states, \( s_{i,k}, s_{i,k+1} \), to iteratively update the entire latent sequence of states for a given country using the target density defined by the right side of (6). We refer to this algorithm as the state space sampler algorithm.

In the state sampler algorithm, one full update of the entire latent state sequence \( s_{i,1}, \ldots, s_{i,n_i} \) represents one iteration of the MCMC algorithm. The posterior means of the
samples of HMM parameters, $\zeta, \beta, a, c$, from the previous algorithm are used to evaluate the target mass function (1) in the state space sampler algorithm. The proportion of each state visited in the state space sampler algorithm, for each country/week, is an estimate of the posterior probability of each state for a given country/week. The state space sampler algorithm is a tool for evaluating or predicting the most likely state (i.e., non-violent, stable violence, or intensified violence) at any given country/week, based on the HMM fitted to the real data. In Section 5 we present a visual representation of the posterior distribution of the latent state sequence for countries both in our training data and in our held-out test data.

4.2 Simulation study of synthetic battle death data

The purpose of this section is to verify that synthetic data generated by our HMM resembles important features of the real battle death count data, and that our Bayesian estimation procedure produces credible sets for the HMM parameters that achieve the corresponding frequentist coverage (e.g., for each parameter, approximately 95 percent of the estimated 95 percent credible sets contain the ‘true’ parameter value). The ‘true’ HMM parameter values used to generate the synthetic data are set as the posterior means estimated from the real data set; these values are presented in Table 2.

We generate synthetic data for each of $N = 167$ countries in our training data set (leaving data from 3 countries as test data), based on the following procedure. For each country/week with recorded covariates a ‘true’ latent state $s_{i,k}$ is sampled from either the initial state probability vector $\pi$ if $k = 1$ or $P_{s_{i,k-1}}$, if $k > 1$, and a count of battle deaths $y_{i,k}$ is sampled from (2). Note that the first four values, $y_{i,1:4}$, are sampled from (2) with $\rho = 0$. Furthermore, the maximum number of battle deaths in any one week is restricted to not exceed the highest week-death-count to country-population proportion of any country in the data set. The highest such proportion is approximately 0.0006 for Congo. If a simulated death count $y_{i,k}$ exceeds 0.0006 times the country population, then the generated data for
that country is discarded and re-simulated until the restriction is satisfied. This was not a problem for any of the 167 countries in the data set, with the notable exception of India. For India, it took an excessive amount of time to generate battle death data that satisfy the maximum weekly death restriction, and so India was omitted from our simulation study.

India is an outlier country in our data set in the sense that it has an uncharacteristically large population size which, based on the fitted HMM parameters (see Table 2) is associated with markedly less time spent in state 1, and battle death sequences represented with an explosive or non-stationary series (i.e., \( \rho > 1 \)). Additionally, India is often in a state of ceasefire which the fitted HMM has associated with increased instability (i.e., states 2 and 3). One possible explanation for why conflict dynamics are not explained so well by our fitted HMM for countries with populations as

![Figure 2: The top panels for each country display 1000 synthetic realizations of the response variable sequence simulated from the HMM fitted with the posterior means of all parameters presented in Table 2, the maximum a posteriori latent state sequence, and all covariate values observed in the real data set. The red points represent the upper 0.025 percentile while the black points represent the lower 0.025 percentile. For reference, the bottom panel displays the real numbers of observed battle deaths.](image)
large as India is the greater possibility for numerous unrelated conflicts ongoing at any point in time. For the average country, conflict dynamics are more likely limited to a single conflict at a time.

As with our real data set, for our synthetic data sets we apply a single rule-based partial label. That is, any week that is at least two months after or two months before a week with one or more battle deaths, and is within a two year sequence of weeks with no battle deaths is labeled as state 1, without error.

| Parameter | baseline | declared | ceasefire | v2x | v2x^2 | v2x^3 | GDP | pop |
|-----------|----------|----------|-----------|-----|-------|-------|-----|-----|
| ζ_1^t (1 → 2) | .66 | .83 | .91 | .92 | .93 | .94 | .90 | .91 |
| ζ_2 (1 → 3) | .94 | .97 | .93 | .94 | .92 | .94 | .95 | .94 |
| ζ_3 (2 → 1) | .93 | .96 | .94 | .97 | .99 | .94 | .95 | .92 |
| ζ_4 (2 → 3) | .95 | .96 | .97 | .93 | .91 | .97 | .95 | .93 |
| ζ_5 (3 → 1) | .96 | .96 | .93 | .93 | .96 | .95 | .92 | .95 |
| ζ_6 (3 → 2) | .95 | .93 | .97 | .94 | .91 | .94 | .97 | .98 |
| β_1 (state 2) | .90 | .97 | .95 | .96 | .95 | .96 | .92 | .94 |
| β_2 (state 3) | .96 | .97 | .97 | .96 | .95 | .93 | .97 | .93 |
| a_1 | .91 | | | | | | | |
| a_2 | .66 | | | | | | | |
| a_3 | .95 | | | | | | | |
| c | .96 | | | | | | | |
| π_2 | .90 | | | | | | | |
| π_3 | .95 | | | | | | | |

Table 1: Proportion of 100, .95 posterior credible sets that contains the true parameter value for each of the HMM parameters (constructed by excluding the upper and lower .025 tails of each marginal posterior distribution). Synthetic data for N = 166 countries are generated for each of the 100 data sets in this simulation study. Note that parameter values reflect covariate values for lag v2x polyarchy (linear, quadratic, and cubic), lag and log GDP per capita, and lag and log population. These variables have all been centered and scaled to have unit standard deviation. See the supplementary material for box plots over the 100 posterior medians for each parameter.

A total of 100 synthetic data sets are generated based on the described procedure. We implement a simple independent components Gaussian prior density with mean zero and excessively diffuse standard deviation 20 for all parameters. To enforce the constraint that a_1, a_2, a_3, and c are positive-valued, we place the Gaussian prior on log(a_1), log(a_2), log(a_3), and log(c). Similarly, the Gaussian prior is placed on the logit transforms of the components of the initial state probability vector π. The MCMC algorithm described in Section 4.1 is used to estimate the posterior distributions for all 70 HMM parameters, for each of the 100 synthetic data sets. The coverage at the .95 level of significance for each
parameter is stated in Table 1. Box plots of the 100 posterior medians for each parameter are presented in our supplementary material.

Figure 2 gives a visual representation of the synthetic data we generated for the held-out test set countries, Sudan and Afghanistan. Note that the HMM parameter values were fit using only the training data consisting of 166 countries.

5 Analyses and Results

For our analysis of the real data, we focus on three inferential aspects of the fitted model. First, the posterior mean estimates of all 70 HMM parameters described in Section 4 are summarized in Table 2. The MCMC trace plots and histograms of the posterior samples are provided in the supplementary material. The posterior means presented in Table 2 quantify the effect of the various parameters/features in the model, but they are not dynamic in the sense that they represent the model at a weekly resolution whereas the HMM is fitted to the data as a system that evolves over many years of accumulated weeks. For this purpose, second, we present probability evolution curves for a small selection of countries in Figures 3, 5, and 7. For these figures, the transition probabilities are computed and plotted over the same time periods observed in the training data, using the covariate values associated with each country/week. Furthermore, these probability evolution plots ignore the HMM response data (i.e., the counts of conflict deaths each week) to provide inference purely on the effects of the state transition probability covariates. In particular, they demonstrate the role that ceasefires have played in the de-escalation of violence for the countries in our data set. Third, we use the posterior mean estimates from Table 2 to sample state sequences via the state space sampler discussed in Section 4.1. These are displayed, for the same small selection of countries, in Figures 4, 6, and 8. The probability evolution and state space sampler plots for all 170 countries included in our data set are available with our supplementary material.
The parameter estimates in Table 2 suggest a variety of interesting findings. First, we note that the state 2 (stable violence) to state 1 (non-violent) transition probability increases by about a factor of 3.5 when a ceasefire is in effect, taking all other covariates at mean value. This is strong evidence that ceasefires do seem to be associated with the cessation of violent conflict within the five-week period in which the ceasefire indicator is defined. Over time, the factor of 3.5 effect of ceasefires is visually displayed in the top panels of Figures 3, 5, and 7, within the vertical bars that indicate ceasefires are in effect. Note that it is also observed in the figures that this effect carries momentum for diminishing state 2 or 3 transition probabilities even after the ceasefire period.

| parameter | baseline | pre-ceasefire | ceasefire | v2x | v2x² | v2x³ | GDP | pop |
|-----------|----------|---------------|-----------|-----|------|------|-----|-----|
| ζ′₃ (2 → 1) | -4.714* | -0.322 | 1.243* | -0.938* | 1.748* | -0.899* | -0.554* | -0.588* |
| ζ′₄ (2 → 3) | -5.965* | 1.693* | 0.524* | -0.375 | -0.572* | 0.421 | -0.486* | -0.516* |
| ζ′₅ (3 → 1) | -0.986* | -1.550* | 0.367 | -1.617* | 0.153 | 0.317 | -0.554* | -0.005 |
| ζ′₆ (3 → 2) | 0.993* | -0.289 | 0.161 | -0.734* | 0.134 | 0.245 | 0.143 | 0.305 |
| β¹₁ (state 2) | -4.228* | 0.078 | -0.099* | 1.784* | -3.945* | 2.389* | 0.238* | 0.337* |
| β¹₂ (state 3) | -3.849* | 0.660* | 0.061 | -0.986* | -0.851* | 0.392 | 0.724* | 0.948* |
| a₁ | 0.0004* |
| a₂ | 0.0911* |
| a₃ | 5.8714* |
| c | 0.0246* |
| π₂ | 0.0279* |
| π₃ | 0.0140* |

Table 2: Posterior means of the HMM parameters. Note that parameter values reflect covariate values for lag v2x polyarchy (linear, quadratic, and cubic), lag and log GDP per capita, and lag and log population. These variables have all been centered and scaled to have unit standard deviation. Boldface* indicates that the .95 credible region, formed by excluding posterior samples in the upper and lower .025 tails, excludes the value 0. Note that we omit the state 1 → 2 transitions from this table because the inferential focus of our application to conflict research is restricted to the other transitions. See the supplementary material for the MCMC trace plots and histograms of the posterior samples.

Second, observe that statistically significant pre-ceasefire indicator variable coefficients appear for transitions 2 → 3 and 3 → 1, showing a sign that indicates heightened or sustained levels of violence. Thus, there seems evidence to suggest that there are escalatory conflict dynamics distinct to weeks associated with the pre-ceasefire period. This is a major finding of our analysis. It highlights the lag time between when a ceasefire is negotiated and when it actually begins, and that negotiating and preparing for a ceasefire is associated with an immediate short-term escalation in violence (which in turn is likely to also increase

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the likelihood of a ceasefire).

Furthermore, the estimated mean autoregressive coefficient, $\rho/c$ (recall equation (5)), increases from 0.8659 in state 3 to 1.6753, taking all other covariates at mean value, for all pre-ceasefire weeks. However, in either case, this coefficient will be explosive (i.e., the autoregressive process is not (weakly) stationary) for countries with larger GDP per capita and/or larger population, as demonstrated by the significant coefficient estimates 0.724 and 0.948, respectively. The explosive value of this coefficient is consistent with our interpretation of state 3 as ‘intensified’ violence. Conversely, the ‘stable’ violence interpretation for state 2 comes from the fact that it has an autoregressive coefficient, taking all other covariates at mean value, estimated to be less than 1, which describes a weakly stationary process. Such processes revert to a stationary mean, and it is in this sense that the ‘stable’ violence state describes both non-escalating and de-escalating violence.

Our final major finding is a statistically significant effect of a third degree polynomial in the $v_2x$ polyarchy variable on the state 2 (stable violence) to state 1 (non-violent) transition probability, with a leading negative coefficient. To the best of our knowledge, this is the first time that the effect of a third degree polynomial in the polyarchy variable has been studied on violence dynamics, so such a finding is significant to the conflict research community.

The fact that the data exhibit patterns of both stable and unstable models is a strong indication that conflicts endure distinct phases, and that intensified phases marked by explosive violence are only sustained for short periods of time. Namely, observe that, taking all other covariates at mean value and in the absence of ceasefires or negotiations, the baseline transition probabilities from $3 \rightarrow 1$, $3 \rightarrow 2$, and $3 \rightarrow 3$ are 0.0916, 0.6628, and 0.2456, respectively; once in state 3 (intensified violence) it is about 0.75 probability of transitioning to another state in the next week.
Figure 3: The top panel displays the evolution of the probabilities of transition on a week-by-week resolution. Counts of weekly conflict related deaths are omitted from the computation of these probabilities. Instead, the probabilities exclusively reflect the effects on the transition rates of the covariates observed for the labelled country, over time.

Figure 4: The top panel displays the observed data for South Sudan, and the bottom panel displays the estimated posterior probability of each state for each week.
Figure 5: The top panel displays the evolution of the probabilities of transition on a week-by-week resolution. Counts of weekly conflict related deaths are omitted from the computation of these probabilities. Instead, the probabilities exclusively reflect the effects on the transition rates of the covariates observed for the labelled country, over time.

Figure 6: The top panel displays the observed data for Israel, and the bottom panel displays the estimated posterior probability of each state for each week.
Figure 7: The top panel displays the evolution of the probabilities of transition on a week-by-week resolution. Counts of weekly conflict related deaths are omitted from the computation of these probabilities. Instead, the probabilities exclusively reflect the effects on the transition rates of the covariates observed for the labelled country, over time.

Figure 8: The top panel displays the observed data for the Afghanistan, and the bottom panel displays the estimated posterior probability of each state for each week.
Our last remark on the estimates in Table 2 is that the fitted expected number of battle deaths per week, in the absence of battle deaths in previous weeks, during state 1, state 2, and state 3 are 0.0163, 3.7033, and 238.6748, respectively. Recall from the expected value of the negative-binomial distribution in (5) that these are computed as $a_j/c$, for $j \in \{1, 2, 3\}$. This statistic serves as a simple, rudimentary description of how the HMM has fitted the mean behavior of each of the three states.

Finally, we note that (although not displayed in Table 2) the baseline probabilities for transition from state 1 → 2 and 1 → 3, in one week, are estimated to be 0.0006276 and $6 \times 10^{-7}$, respectively. This is consistent with the fact that periods of violent conflict are rare events, and are not part of the natural progression of the state of affairs for the average country, (average in the sense of the covariates we consider). However, we find that the transition probability from state 1 → 2 increases by a factor of about 52 for a country with a declared ceasefire, and by a factor of about 18 for a country with a ceasefire in effect, taking all other covariates at mean value. Mostly, this reflects the fact that ceasefires occur in the midst of violent conflict, and so the likelihood of future violence is higher than if a country was in state 1 not having recently transitioned from a state of violent conflict.

6 Concluding Remarks

The results are striking. Ceasefires do indeed seem to produce a significant decline in subsequent violence, shifting conflict from a violent to a non-violent state. However, the pre-ceasefire period is shown to be prone to periods of intensified violence that are most likely a cause and effect of the subsequent ceasefire. There are many research directions we hope to investigate in furthering this work. Namely, we hope to distinguish between ceasefires of different types and scope, separating those ceasefires that have markedly different characteristics (e.g., cessation of hostility arrangements vis-a-vis preliminary ceasefires). A complimentary research direction is to include a covariate to account for the duration of a
given conflict (i.e., time in state 2 and/or state 3). This would allow us to address questions relating to whether conflicts tend to have limited persistence and perhaps dissipate in time.

A challenge in our present analysis has been constructing an HMM over a theorized state space which provides context and meaning to the states that we have suggestively named as ‘non-violent’, ‘intensified violence’, and ‘stable violence’, respectively. This is particularly difficult due to the fact that conflicts can arise from such a heterogeneous collection of countries. For example, we fit the HMM to training data including those from Colombia, Congo, and India, as well as data from the United Kingdom and Spain. We will need to incorporate additional structure into our HMM to better define and identify states 1, 2, and 3, but deciding how to do so is an open topic of future research.

Next, there are a variety of potential utilities for policy making using the fitted HMM. With sufficient data and intervention relevant predictors it is possible to conduct analysis using the state space sampler to quantify the effect of changes in policy (e.g., how the risk or duration of a conflict changes if interventions are implemented).

A key insight we demonstrate is that HMM frameworks can be developed to assign labels for latent variables in data sets, such as various underlying states of conflict. While this is not new to the statistical research community, it is not well understood in the conflict research community that an HMM can be used as a tool to construct semi-supervised data sets for use in more conventional statistical models. This insight is also demonstrated in the recent article [Anders (2020)](https://doi.org/10.1007/s11211-020-00748-4) to identify territorial control during a civil war, that are intrinsically difficult (if not impossible or infeasible) to manually label. Arguably, manifesting from the ubiquity of modern data repositories combined with a deficiency in meaningful labels, HMM-based semi-supervised data labeling strategies could pave the way for the next decade of conflict research progress.
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