Effect of inclusions' distribution on microwave absorbing properties of composites

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Abstract. Effect of inclusions' spatial distributions on the permeability and permittivity of composites is studied using the generalized Maxwell-Garnett equations. The result indicates that inclusions' orientation distribution can increase the longitudinal electromagnetic parameters. For inclusions' random and orientation distribution, single and three-layer absorbers are designed and optimized using genetic algorithm. The result shows that under a given absorbing requirement, absorber with inclusions' orientation distribution is lighter and thinner than absorber with inclusions' random distribution.

1. Introduction
With the fast improvement of electronic components, muti-phase composites are attracting more and more attentions in designing absorbers because of their lightweight broadband microwave absorbing properties [1, 2]. Many researchers have been focusing on using different types of inclusions to improve composites' permeability and permittivity [3-5]. Maxwell-Garnett and Bruggeman equations are two famous equations of effective medium theory [6]. However both equations are applied to randomly distributed inclusions and will become invalid if the inclusions are nonrandomly distributed. Duan Huiling and other scholars have proposed a generalized M-G equation based on the variational principles by introducing the concept of distribution factor [7, 8]. The shape factor and distribution factor are brought into represent inclusions’ shapes and distribution [9]. The generalized M-G equations have advantages in estimating the effective permeability and permittivity of composites, especially composites with nonrandomly distributed inclusions of fiber, flake and other nonspherical shapes.

An optimized function between inclusions' distribution and the reflection loss (RL) of composites can be built by combining the generalized M-G equation with transmission line approach. The main variables of the target function are concentration and thickness of composites. A certain algorithm is needed to find a suitable solution because the target function is not a linear equation. Genetic algorithm (GA) optimizers are robust and stochastic search methods modeled on the concepts of natural selection and evolution [10]. Single and three-layer absorbers considering inclusions’ random and orientational distribution are designed using GA. A new scheme of designing absorbers is proposed and discussed in this paper and it will be for reference to the following research in designing absorbers.

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2. Generalized M-G equations

Generalized M-G equations are given below [7]:

\[
\begin{align*}
\underline{\mu}_e &= \underline{\mu}_1 + \underline{\mu}_r \left( \sum_{r=2}^N \underline{h}_r - \underline{S}_v \right)^{-1} \\
\underline{h}_r &= \frac{1}{3} f_r \left[ 2 \left( \underline{\mu}_1 \cdot \left( \underline{\mu}_e - \underline{\mu}_1 \right)^{-1} + \underline{S}_r \right)^{-1} + \left( \underline{\mu}_1 \cdot \left( \underline{\mu}_r - \underline{\mu}_1 \right)^{-1} + 1 - 2 \underline{S}_r \right)^{-1} \right]
\end{align*}
\]  

(1)

(2)

\[\mu_1, \mu_r, \text{ and } \mu_e \text{ are the permeability tensors of the base, the } r \text{ th-phase inclusions and the composites, respectively. } S_r \text{ is the geometrical factor tensor of } r \text{ th-phase and } S_v \text{ is the inclusions’ distribution tensor. } f_r \text{ is the concentration of } r \text{ th-phase. As pointed out by Hashin and Strikman [11], the permittivity and permeability are mathematically analogous.} \]

![Figure 1](image)

**Figure 1.** (a) Randomly distributed inclusions. (b) Orientationally distributed inclusions.

For two-phase composites whose inclusions are randomly distributed as shown in figure 1a, its effective permeability tensor \( \underline{\mu}_e = \underline{\mu}_1^I + \underline{\mu}_e \) is given below:

\[
\begin{align*}
\underline{\mu}_e &= \underline{\mu}_1 + \underline{\mu}_r \left( \frac{1}{h_{2,\mu}} - \underline{S}_v \right)^{-1} \\
h_{2,\mu} &= \frac{1}{3} f_r \left[ 2 \left( \frac{\underline{\mu}_1}{\mu_2 - \mu_1} + \underline{S}_1 \right)^{-1} + \left( \frac{\underline{\mu}_1}{\mu_2 - \mu_1} + 1 - 2 \underline{S}_1 \right)^{-1} \right]
\end{align*}
\]

(3)

(4)

If the inclusions are fibers with an aspect ratio larger than 1, \( S_1 \) is given as:

\[
S_1 = \frac{1}{4(1-\gamma^2)^{3/2}} \left( 2\sqrt{1-\gamma^2} - \gamma^2 \ln \frac{1+\sqrt{1-\gamma^2}}{1-\sqrt{1-\gamma^2}} \right)
\]

(5)

\[\gamma = \frac{a_1}{a_3}, \quad a_1 \text{ and } a_2 \text{ denote the radial length and } a_3 \text{ denotes semi-axial length. For randomly distributed inclusions } S_v = \frac{1}{3}. \]
For two-phase composites with inclusions distributed orientationally, this paper focuses only on one situation for simplicity. The inclusions’ axes are parallel to the XY plane as shown in figure 1b. The transverse and longitudinal permeability is as follows:

\[
\mu_T = \mu_i + \mu_i \left( f \left( \frac{\mu_i}{\mu_2 - \mu_i} - S_v \right) \right)^{-1}
\]

\[
\mu_L = \mu_i + \mu_i \left( f \left( \frac{\mu_i}{\mu_2 - \mu_i} + 1 - 2S_v \right) \right)^{-1}
\]

(6) (7)

For orientationally distributed inclusions \(S_v = S_i\).

Numerical calculation is carried out to observe the effect of inclusions’ distribution on the EM parameters of composites. Let the base parameters be \(\varepsilon_i = 3, \mu_i = 1\). Inclusions are iron fiber with 4 mm length and 9 um diameter. Inclusions’ concentration is 10 % and its permeability is given by figure 2.

![Figure 2. Permittivity and permeability of inclusions.](image)

![Figure 3. Permeability of composites with orientationally and randomly distributed inclusions.](image)

As shown in figure 3, the longitudinal permeability of composites with orientational inclusions is larger than its transverse permeability. While the permeability of composites with randomly distributed inclusions is between the longitudinal and transverse one. If the plane wave incident along the Z axis, its electric and magnetic field will transmit in the XY plane, then the longitudinal...
parameters will mainly affect composites’ absorbing properties. So comparing with inclusions randomly distributed, orientational inclusions can apparently increase composites’ absorbing properties.

3. Reflection loss

For a plane wave normally incident on the air-composites-metal structure shown in figure 4, the reflection loss can be calculated using transmission-line approach [12]. The $i$ th input impedance of $N$-layer composites with metal backing is given by:

$$Z_{in}^i = \eta_i \frac{Z_{in}^{i+1} + \eta_i \tanh(jk_i d_i)}{\eta_i + Z_{in}^{i+1} \tanh(jk_i d_i)}$$

(8)

$d_i$, $k_i$ and $\eta_i$ is the thickness, propagation constant and characteristic impedance of the $i$ th layer, respectively:

$$k_i = \frac{2\pi f}{c} \sqrt{\mu_i \varepsilon_{\text{air}}} , \quad \eta_i = \frac{E_{in}}{H_{iy}} = \eta_0 \sqrt{\frac{\mu_i}{\varepsilon_{\text{air}}}}$$

(9)

$\eta_0$ is the characteristic impedance of the air. $\mu_i$, $\varepsilon_{\text{air}}$ are the relative permeability and permittivity of the $i$ th layer. The reflection coefficient of the $i$ th layer is calculated by equation (10).

$$R_i = \frac{Z_{in}^i - \eta_{i+1}}{Z_{in}^i + \eta_{i+1}}$$

(10)

For the $n+1$ th metal backing, $Z_{in}^{n+1} = 0$. For single layer with metal backing, $n = 1$

$$Z_{in}^1 = \eta_i \tanh(jk_i d_i) , \quad R_i = \frac{Z_{in}^1 - 1}{Z_{in}^1 + 1}$$

(11)

The reflection loss ($RL$)

$$RL = 20 \log |R_i|$$

(12)

**Figure 4.** Geometry of multi-layer absorber with perfectly conducting backing.

Composites with randomly distributed inclusions of different concentrations are simulated to emphasize the effect of concentrations on reflection loss. Let the base parameters be $\varepsilon_i = 3$, $\mu_i = 1$. Inclusions are iron fiber with 4mm in length and 9um in diameter.
The calculated RL of five different concentrations of 5%, 10%, 15%, 20%, 25% are shown in figure 5. It can be seen from figure 5 that composites’ absorbing properties are related to EM frequency and inclusions’ concentrations. There is an absorption peak for single layer absorber, and the peak frequency goes down as the concentration increases. The absorption peak is not a constant but first increases then decreases as concentration goes up. However, the absorber will gain a broader absorbing band and more reflection loss as concentration increases.

![Figure 5](image-url)  
**Figure 5.** Reflection loss of composites of different concentrations.

4. Design and optimization of absorber
For randomly and orientationally distributed inclusions, there is a relationship between its concentration and the reflection loss. In designing an absorber which meets certain requirement, its thickness and concentration are the main variables. This paper uses genetic algorithm for multi-variable optimization. Single and three-layer absorbers are designed using GA for random and orientational inclusions separately. The design and optimization flowchart is in figure 6:

![Figure 6](image-url)  
**Figure 6.** Design and optimization flowchart of absorber.

In the optimization process, the target absorber should be thinner than 4 mm and its RL should be less than -5 dB. For single layer absorbers, the optimization result using GA for random inclusions is 4 mm thickness and concentration of 27%, and the result for orientational inclusions is 4 mm thickness and 20% concentration. For three-layer absorbers, the results are showed in table 1 and table 2.
Table 1. Design parameters for three-layer absorber with randomly distributed inclusions.

| Layer  | d (mm) | f    |
|--------|--------|------|
| Layer 1| 1.01   | 0.14941|
| Layer 2| 1.81   | 0.23803|
| Layer 3| 1      | 0.14849|

Table 2. Design parameters for three-layer absorber with orientationally distributed inclusions.

| Layer  | d (mm) | f    |
|--------|--------|------|
| Layer 1| 1.1    | 0.127|
| Layer 2| 1.62   | 0.234|
| Layer 3| 1      | 0.13 |

The reflection loss of all four kinds of absorbers are shown in figure 7. For single layer absorber the optimized results of both kinds of inclusions have the same reflection loss above 5 GHz. However absorber with orientational inclusions is lighter and thinner than that with random inclusions, so absorber with orientational inclusions is better than the other one for designing single layer absorber.

For three-layer absorber the differences between two kinds of absorbers are more complicated. Absorber with orientational inclusions has a greater RL at the frequency range of 2-7 GHz and an absorbing peak at 5 GHz. At the range of 7-18 GHz absorber with random inclusions is better at absorbing than the other one. Though both kinds of absorbers can satisfy the design requirement the one with random inclusions is thicker and heavier than the other one. That is why the absorber with random inclusions is better than the other one at the range of 7-18 GHz.

Figure 7. Reflection loss of optimized absorbers.

5. Conclusions
Effect of inclusions' distribution on composites' absorbing properties is discussed by introducing the distribution tensor. The absorbing properties of composites are obviously affected by the distribution of inclusions. Composites with orientationally distributed inclusions have a greater absorbing ability than that with randomly distributed inclusions.
The relationship between inclusions' concentration and composites' reflection loss is brought in based on the transmission line approach. The absorber will have a lower peak frequency and broader absorbing frequency band as the inclusions’ concentration increases.

Single and three-layer absorbers of randomly and orientationally distributed inclusions are designed using genetic algorithm. The result shows that under a given absorbing requirement, absorber with inclusions’ orientation distribution is lighter and thinner than absorber with inclusions’ random distribution. For single layer absorber the two kinds of composites have almost the same absorbing ability. However for three-layer absorber composites with randomly distributed inclusions have a better absorbing ability at most of the frequencies.

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