Abstract. In this paper, we introduce and study some new classes of $\alpha$ weakly generalized locally closed sets in the context of intuitionistic fuzzy topological spaces.

1. Introduction

Fuzzy set was proposed by Zadeh [8] in 1965 and fuzzy topology by Chang [4] in 1968, Atanassov [1] proposed intuitionistic fuzzy set (IFS) in 1986. In 1997, Coker [5] introduced the concept of intuitionistic fuzzy topological space. The first step of locally closedness topological space was done by Bourbaki [3]. Ganster and Reilly [6] used locally closed sets to define LC-continuity and LC-irresoluteness. Sundaram [2] introduced the concepts of generalized locally closed sets, GLC-continuous maps, GLC-irresolute maps and investigated some of their properties. The purpose of this paper is to introduce and study the concepts of new classes of intuitionistic fuzzy sets namely intuitionistic fuzzy $\alpha$ weakly generalized locally closed set, intuitionistic fuzzy $\alpha$ weakly generalized locally closed* set, intuitionistic fuzzy $\alpha$ weakly generalized locally closed** set and study some of their properties. For terms and notations used but left undefined we refer to [1, 5, 7].

2. Preliminaries

Definition 2.1: [7] An IFS $A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$ in an IFTS $(X, \tau)$ is said to be an intuitionistic fuzzy $\alpha$ weakly generalized closed set (IF$\alpha$WGCS in short) if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is an IF$\alpha$OS in $X$.

The family of all IF$\alpha$WGCSs of an IFTS $(X, \tau)$ is denoted by IF$\alpha$WGC$(X)$.

Definition 2.2: [7] Let $(X, \tau)$ be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in $X$. Then the intuitionistic fuzzy $\alpha$ weakly generalized interior and intuitionistic fuzzy $\alpha$ weakly generalized closure are defined by

$\alpha\text{wgint}(A) = \cup \{ G / G \text{ is an IF} \alpha \text{WGOS in } X \text{ and } G \subseteq A \}$,

$\alpha\text{wgcl}(A) = \cap \{ K / K \text{ is an IF} \alpha \text{WGCS in } X \text{ and } A \subseteq K \}$. 
Definition 2.3: [7] An IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy \(\alpha\ wgT_{1/2}\) space (IF\(\alpha\ wgT_{1/2}\) space in short) if every IF\(\alpha\ WGCS\) in \(X\) is an IFCS in \(X\).

Result 2.4: [7] If an IFTS \((X, \tau)\) is an IF\(\alpha\ wgT_{1/2}\) space, then for every subset \(A\) of \(X\), IF\(\alpha\ wgcl(A)\) is an IFCS in \(X\).

3. \(\alpha\) weakly generalized locally closed sets in intuitionistic fuzzy topological spaces

In this section, we introduce three new classes of intuitionistic fuzzy locally closed sets namely intuitionistic fuzzy \(\alpha\) weakly generalized locally closed sets, intuitionistic fuzzy \(\alpha\) weakly generalized locally closed* sets, intuitionistic fuzzy \(\alpha\) weakly generalized locally closed** sets and study some of their properties.

Definition 3.1: An IFS \(A = \langle x, \mu_A, v_A \rangle\) of an IFTS \((X, \tau)\) is said to be

(i) intuitionistic fuzzy \(\alpha\) weakly generalized locally closed set (IF\(\alpha\ WGlcs\) in short) if \(A = B \cap C\) where \(B = \langle x, \mu_B, v_B \rangle\) is an IF\(\alpha\ WGOS\) and \(C = \langle x, \mu_C, v_C \rangle\) is an IF\(\alpha\ WGCS\) in \(X\),

(ii) intuitionistic fuzzy \(\alpha\) weakly generalized locally closed* set (IF\(\alpha\ WGlc*s\) in short) if \(A = B \cap C\) where \(B = \langle x, \mu_B, v_B \rangle\) is an IF\(\alpha\ WGOS\) and \(C = \langle x, \mu_C, v_C \rangle\) is an IFCS in \(X\),

(iii) intuitionistic fuzzy \(\alpha\) weakly generalized locally closed** set (IF\(\alpha\ WGlc**s\) in short) if \(A = B \cap C\) where \(B = \langle x, \mu_B, v_B \rangle\) is an IFOS and \(C = \langle x, \mu_C, v_C \rangle\) is an IF\(\alpha\ WGCS\) in \(X\).

The family of all IF\(\alpha\ WGlcs\) (respectively IF\(\alpha\ WGlc*s, IF\(\alpha\ WGlc**s\)) of an IFTS \((X, \tau)\) is denoted by IF\(\alpha\ WGLC(X)\) (respectively IF\(\alpha\ WGLC*(X), IF\(\alpha\ WGLC**(X))\).

Theorem 3.2: For an IFTS \((X, \tau)\), the following inclusions hold.

(i) IFLC(X) \(\subseteq\) IF\(\alpha\ WGLC(X)\).

(ii) IFLC(X) \(\subseteq\) IF\(\alpha\ WGLC**(X) \(\subseteq\) IF\(\alpha\ WGLC(X)\).

(iii) IFLC(X) \(\subseteq\) IF\(\alpha\ WGLC**(X) \(\subseteq\) IF\(\alpha\ WGLC(X)\).

Proof: Obvious.

Example 3.3: Let \(X = \{a, b\}\) be a nonempty set. Let \(T_1 = \langle x, \left(\begin{array}{c} a \\ 0.1 \\ 0.2 \\ 0.4 \\ 0.5 \end{array}\right), \left(\begin{array}{c} a \\ 0.4 \\ 0.5 \end{array}\right)\rangle\) and \(T_2 = \langle x, \left(\begin{array}{c} a \\ 0.2 \\ 0.3 \\ 0.4 \end{array}\right), \left(\begin{array}{c} a \\ 0.3 \\ 0.4 \end{array}\right)\rangle\) be the IFSs of \(X\). Then the family \(\tau = \{0, T_1, T_2, 1\}\) is an IFT on \(X\). The IFSs \(\begin{array}{c} E = T_1 \cap T_2 \cap T_1 \\ F = T_2 \cap T_1 \end{array}\) are intuitionistic fuzzy \(\alpha\) weakly generalized locally closed set, intuitionistic fuzzy \(\alpha\) weakly generalized locally closed** set and intuitionistic fuzzy \(\alpha\) weakly generalized locally closed** set in \(X\).

Theorem 3.4: Let \((X, \tau)\) be an IF\(\alpha\ wgT_{1/2}\) space. Then the following statements hold.

(i) Every IF\(\alpha\ WGlcs\) is an IFICS in \(X\).

(ii) Every IF\(\alpha\ WGlc*s\) is an IFICS in \(X\).

(iii) Every IF\(\alpha\ WGlc**s\) is an IFICS in \(X\).

Proof: Obvious.
**Theorem 3.5:** Every intuitionistic fuzzy $\alpha$ weakly generalized locally closed* set and intuitionistic fuzzy $\alpha$ weakly generalized locally closed** set is an intuitionistic fuzzy $\alpha$ weakly generalized locally closed set but not conversely.

**Proof:** Obvious

The converse of the above Theorem need not be true in general as seen from the following examples.

**Example 3.6:** Let $X = \{a, b\}$ be a nonempty set. Let $T_1 = \left\langle x, \left(\begin{array}{c}
\frac{a}{0.3}, \frac{b}{0.4} \\
\frac{a}{0.6}, \frac{b}{0.5}
\end{array}\right)\right\rangle$ and $T_2 = \left\langle x, \left(\begin{array}{c}
\frac{a}{0.3}, \frac{b}{0.4} \\
\frac{a}{0.5}, \frac{b}{0.5}
\end{array}\right)\right\rangle$ be the IFSs of $X$. Then the family $\tau = \{0, T_1, T_2, 1\}$ is an IFT on $X$. Let the IFS $A = \left\langle x, \left(\begin{array}{c}
\frac{a}{0.3}, \frac{b}{0.3} \\
\frac{a}{0.6}, \frac{b}{0.6}
\end{array}\right)\right\rangle$ be an IFWGCS in $X$. The IFS $E = T_2 \cap A = \left\langle x, \left(\begin{array}{c}
\frac{a}{0.3}, \frac{b}{0.3} \\
\frac{a}{0.6}, \frac{b}{0.6}
\end{array}\right)\right\rangle$ is an intuitionistic fuzzy $\alpha$ weakly generalized locally closed set but not an intuitionistic fuzzy $\alpha$ weakly generalized locally closed* set in $X$.

**Example 3.7:** Let $X = \{a, b\}$ be a nonempty set. Let $T_1 = \left\langle x, \left(\begin{array}{c}
\frac{a}{0.7}, \frac{b}{0.4} \\
\frac{a}{0.5}, \frac{b}{0.5}
\end{array}\right)\right\rangle$ and $T_2 = \left\langle x, \left(\begin{array}{c}
\frac{a}{0.7}, \frac{b}{0.4} \\
\frac{a}{0.5}, \frac{b}{0.5}
\end{array}\right)\right\rangle$ be the IFSs of $X$. Then the family $\tau = \{0, T_1, T_2, 1\}$ is an IFT on $X$. Let the IFS $A = \left\langle x, \left(\begin{array}{c}
\frac{a}{0.8}, \frac{b}{0.7} \\
\frac{a}{0.5}, \frac{b}{0.5}
\end{array}\right)\right\rangle$ be an IF$\alpha$WGCS in $X$. The IFS $E = A^c \cap T_2^c = \left\langle x, \left(\begin{array}{c}
\frac{a}{0.4}, \frac{b}{0.7} \\
\frac{a}{0.5}, \frac{b}{0.5}
\end{array}\right)\right\rangle$ is an intuitionistic fuzzy $\alpha$ weakly generalized locally closed set but not an intuitionistic fuzzy $\alpha$ weakly generalized locally closed** set in $X$.

**Theorem 3.8:** Let $(X, \tau)$ be an IF$\alpha$WG$T_{1/2}$ space. Then the following statements hold.

(i) Every IF$\alpha$WGGlcs is an IF$\alpha$WGGlcs* in $X$.

(ii) Every IF$\alpha$WGGlcs is an IF$\alpha$WGGlcs** in $X$.

**Proof:** Obvious.

**Remark 3.9:** IF$\alpha$WGGlcs* and IF$\alpha$WGGlcs** are independent to each other in general as seen from the following example.

**Example 3.10:** Let $X = \{a, b\}$ be a nonempty set. Let $T_1 = \left\langle x, \left(\begin{array}{c}
\frac{a}{0.1}, \frac{b}{0.4} \\
\frac{a}{0.6}, \frac{b}{0.5}
\end{array}\right)\right\rangle$ and $T_2 = \left\langle x, \left(\begin{array}{c}
\frac{a}{0.1}, \frac{b}{0.4} \\
\frac{a}{0.5}, \frac{b}{0.6}
\end{array}\right)\right\rangle$ be the IFSs of $X$. Then the family $\tau = \{0, T_1, T_2, 1\}$ is an IFT on $X$. Let the IFS $A = \left\langle x, \left(\begin{array}{c}
\frac{a}{0.1}, \frac{b}{0.2} \\
\frac{a}{0.7}, \frac{b}{0.8}
\end{array}\right)\right\rangle$ be an IF$\alpha$WGCS in $X$. The
IFS $E = T_2 \cap A = \left\{ x \left( \begin{array}{l} \frac{a}{0.1} \cdot \frac{b}{0.2} \\ \frac{a}{0.7} \cdot \frac{b}{0.8} \end{array} \right) \right\}$ is an IF$\alpha$WGlc**s but not an IF$\alpha$WGlc*s in $X$. The
IFS $F = A^c \cap T_2^c = \left\{ x \left( \begin{array}{l} \frac{a}{0.5} \cdot \frac{b}{0.6} \\ \frac{a}{0.1} \cdot \frac{b}{0.4} \end{array} \right) \right\}$ is an IF$\alpha$WGlc*s but not an IF$\alpha$WGlc**s in $X$.

**Remark 3.11:** From the above discussions the following implications hold:

However none of the above implication is reversible.

**Remark 3.12:** The union of two IF$\alpha$WGlc*s need not be an IF$\alpha$WGlc*s in general as seen from the following example.

**Example 3.13:** Let $X = \{a, b\}$ be a nonempty set. Let $T_1$ be the IFS
of $X$. Then the family $\tau = \{0, T_1, 1\}$ is an IFT on $X$. Let the IFSs $A = \left\{ x \left( \begin{array}{l} \frac{a}{0.1} \cdot \frac{b}{0.2} \\ \frac{a}{0.8} \cdot \frac{b}{0.2} \end{array} \right) \right\}$ and $B = \left\{ x \left( \begin{array}{l} \frac{a}{0.6} \cdot \frac{b}{0.7} \\ \frac{a}{0.4} \cdot \frac{b}{0.3} \end{array} \right) \right\}$ be IF$\alpha$WGClcs in $X$. The IFS $E = T_1 \cap A = \left\{ x \left( \begin{array}{l} \frac{a}{0.6} \cdot \frac{b}{0.7} \\ \frac{a}{0.4} \cdot \frac{b}{0.3} \end{array} \right) \right\}$ and the IFS $F = T_1 \cap B = \left\{ x \left( \begin{array}{l} \frac{a}{0.6} \cdot \frac{b}{0.7} \\ \frac{a}{0.4} \cdot \frac{b}{0.3} \end{array} \right) \right\}$ are IF$\alpha$WGlc*s in $X$. But the IFS $E \cup F = \left\{ x \left( \begin{array}{l} \frac{a}{0.6} \cdot \frac{b}{0.8} \\ \frac{a}{0.4} \cdot \frac{b}{0.2} \end{array} \right) \right\}$ is not an IF$\alpha$WGlc*s in $X$.

**Theorem 3.14:** Let $A$ be an IF$\alpha$WGlc*s in $X$ and $B$ be an IF$\alpha$WGOS in $X$. Then $A \cap B$ is an IF$\alpha$WGlc**s in $X$ if $(X, \tau)$ is an IF$_{awg}$T$_{1/2}$ space.

**Proof:** Since $A$ is an IF$\alpha$WGlc*s in $X$, we have $A = P \cap Q$ where $P$ is an IF$\alpha$WGOS and $Q$ is an IF$\alpha$WGCS in $X$. Now

$A \cap B = (P \cap Q) \cap B = P \cap (Q \cap B) = P \cap (B \cap Q) = (P \cap B) \cap Q$.

Since $P$ and $Q$ are IF$\alpha$WGOS in $X$ and $(X, \tau)$ is an IF$_{awg}$T$_{1/2}$ space, $P \cap B$ is an IF$\alpha$WGOS in $X$. Hence $A \cap B$ is an IF$\alpha$WGlc*s in $X$.

**Theorem 3.15:** Let $A$ be an IF$\alpha$WGlc**s in $X$ and $B$ be an IF$\alpha$WGOS in $X$. Then $A \cap B$ is an IF$\alpha$WGlc**s in $X$ if $(X, \tau)$ is an IF$_{awg}$T$_{1/2}$ space.
Proof: Since A is an IF\alpha WGlc\ast{s} in X, we have A = P \cap Q where P is an IF\alpha WGOS and Q is an IFCS in X. Now

\[ A \cap B = (P \cap Q) \cap B = P \cap (Q \cap B) = P \cap (B \cap Q) = (P \cap B) \cap Q. \]

Since P and Q are IF\alpha WGOS in X and (X, \tau) is an IF\alpha wg T_{1/2} space, P \cap B is an IF\alpha WGOS in X. Hence A \cap B is an IF\alpha WGlc\ast{s} in X.

4. References
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