Secondary convective flows in the rectangular tank with non-uniform heating

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Abstract. Horizontal rolls, generated in convective flow above a partially heated bottom in a rectangular box are studied experimentally for a wide range of the Prandtl number ($7 \leq Pr \leq 1020$), the Rayleigh number ($300 \leq Ra \leq 2.8 \times 10^7$) and the aspect ratio ($0.08 \leq a \leq 0.7$). Experimental studies are supported by direct numerical simulations, which made possible the examination of the regimes inaccessible in the experiment, and also to investigate in detail the heat transfer in the convective flow. A variety of regimes with longitudinal helical rolls, with transverse rolls and with mixed structures has been observed. The structure of secondary flows is defined by the level of convective supercriticality in the boundary layer (Rayleigh number) and the intensity of the throughflow, defined by the Reynolds number, which depends itself on the heating and size, i.e. on the Rayleigh number. Most of the studied regimes were characterized by the appearance of longitudinal rolls. The transverse rolls appear in the flow only under the conditions of the large vertical drop in the temperature and weak large-scale flow (that is possible only at large values of the Prandtl number). Both longitudinal and transverse rolls lead to remarkable heat transfer enhancement. The formation and characteristics of horizontal rolls are described in details.

1. Introduction

Secondary flows in the form of horizontal rolls are a common feature of a large variety of flows of different nature and scale. Roll vortices are often observed in the atmospheric boundary layer (Morrison et al., 2005). Depending on their size and strength, rolls play a significant role in transporting momentum, heat and moisture through the atmospheric boundary layer. In practical applications horizontal rolls appear in mixed (forced plus natural) convection in channels and enhance heat transfer in heat exchangers, chemical vapour depositions (CVD), coolers of electronic equipments and nuclear reactors (Benderradji et al., 2008).

One of the interesting problems is the generation of convective rolls in the boundary layer when the cold fluid flows over the region with warm surface. The aim of this study is to investigate experimentally and numerically the structure of the secondary flows in a basic flow above a partially heated bottom in a closed domain for a wide parameters range. The experiments were carried out in a rectangular cell. The two heat exchangers were placed on the bottom of the cell, and the temperature has high gradient drop near the boundary between the exchangers. The basic flow is driven by a horizontal temperature gradient.
2. Experimental Setup and Direct Numerical Simulations

The scheme of the model and the structure of the flow is presented in Fig. 1. The cell bottom consisted of two identical copper heat exchangers inside which there were channels for circulating water at a controlled temperature. Pure distilled water, transformer oil and glycerin-water solution characterised by different values of Prandtl number (at $T = 25^\circ$ C, $Pr = 7$ for distilled water, $Pr = 263$ for transformer oil, $Pr = 1020$ for 89% glycerin-water solution) were used as the working fluid. The velocity field measurements were made with a 2D particle image velocimetry (PIV) system. The temperature in the layer was measured by array of 7 copper-constantan thermocouples.

The numerical code solves the Boussinesq equations for free convection of incompressible fluid which are written in dimensionless form

$$\frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{Pr} [(\mathbf{v} \cdot \nabla) \mathbf{v} + \nabla P] + \Delta \mathbf{v} + RaT e_z,$$

$$Pr \frac{\partial T}{\partial t} = -(\mathbf{v} \cdot \nabla) T + \Delta T,$$

$$\nabla \cdot \mathbf{v} = 0,$$

where the units of scale, time $t$, velocity $\mathbf{v}$, pressure $P$ and temperature $T$ are $h$, $h^2/\nu$, $\chi/h$, $\rho_0 \nu \chi/h^2$, and $\theta h/D$. Here $\rho_0$ is the reference density of the fluid, $\nu$ is the kinematic viscosity; $\chi$ is the thermal diffusivity; $h$ is the layer depth; and $\theta$ is the applied temperature difference. The reference temperature, $T_0$, is the room temperature and the reference pressure is the hydrostatic pressure, $P_0$. The Rayleigh number is defined as

$$Ra = \frac{g \beta \theta h^4}{\nu \chi D}.$$

Impermeable and no-slip velocity conditions are applied at all boundaries ($\mathbf{v} = 0$). The temperature at the bottom is $T = -\theta/2$ for $x < 0$ and $T = \theta/2$ for $x > 0$. Lateral walls are heat–insulated ($\partial T/\partial n = 0$), and at the upper border, the heat exchange condition is specified as $\lambda_{fluid} (\partial T/\partial n)_{internal} = \lambda_{air} (\partial T/\partial n)_{external}$ ($\lambda$ is the heat conductivity). Simulations start at equilibrium state $\mathbf{v}(x,y,z,0) = 0$, $T(x,y,z,0) = 0$, $P(x,y,z,0) = 0$. 

**Figure 1.** Coordinate system and scheme of the experimental setup: 1 – rectangular cell ($D = 100$ mm, $L = 205$ mm, and $H = 100$ mm), 2 – hot heat exchanger, 3 – cold heat exchanger, 4 – basic advective flow, 5 – longitudinal convective rolls.
Eqs. (1)–(3) were solved numerically by a high performance multiprocessor system using the finite difference method on a staggered grid in a primary variable formulation. The integration domain was a rectangular box \((-D < x < D, -D/2 < y < D/2, 0 < z < h)\). Two grids were used for simulations: \(200 \times 100 \times 60\) and \(200 \times 100 \times 100\) nodes, which correspond to laboratory experiments in a cell with \(D = 100\) mm and two layer depths \(h = 30\) mm and \(h = 50\) mm.

3. Results

At the first stage, the formation of basic advective flow was studied. The mean velocity fields in the \(XZ\) plane were obtained by averaging over a large number of measurements, and then the stream function was reconstructed. Fig. 2 shows how the flow structure changes with an increase in the layer depth, \(h\), at a fixed temperature difference, \(\theta = 1.7\)K. For small values of \(h\), an asymmetric vortex is formed in the vicinity of the boundary between the cold and hot plates (Fig. 2(a)). The vortex is characterized by an intense upward flow over the hot plate (on the right) and a less intense, but more extended, downward flow over the cold plate (on the left). With an increase in the layer depth both the velocity within the vortex and the vortex size increase (Fig. 2(b)). At \(h = 20\) mm, the advective flow occupies the entire domain (Fig. 2(c)). The flow evolution with an increase in \(Ra\) due to variation of the temperature difference, \(\theta\), occurs in the same way as when the layer depth grows.

The advective flow velocity profiles for different values of \(\theta\) and \(Pr\) were reconstructed from laboratory and numerical results. Laboratory and numerical velocity profiles are in a good agreement (Fig. 3(a)). Note that conditions on the upper boundary in the experiments are not stress-free (see upper part of the velocity profile in Fig. 3(a)) despite the fact that upper boundary was open. The possible explanation of evident stress on the upper boundary is the influence of surface tension on the liquid-air border.
The intensity of the large-scale advective flow increases with the growth of Rayleigh number and decreases with the growth of Prandtl number (velocity profiles are shown in Figs. 3(b)).

Figure 3. (a) – Velocity profiles over hot plate at $x = 50$ mm, $y = 0$ mm for $Pr = 263$, $h = 30$ mm and $\theta = 33.7K$: open squares – experiment, circles – numerical simulation. (b) – Velocity profiles over hot plate at $x = 50$ mm, $y = 0$ mm for $h = 30$ mm, $\theta = 10K$ and different values of Prandtl number. $Pr = 510$ (1); $Pr = 263$ (2); $Pr = 116$ (3); $Pr = 7$ (4). DNS.

The large-scale advective flow leads to the formation of boundary layers with potentially unstable temperature stratification near the upper and lower surfaces and makes possible the generation of secondary convective flows. Thus, the formation of secondary flows is governed by the structure of the temperature field. In Fig. 4(a),(b) we show the averaged temperature field together with the velocity field in the central plane $XZ$, calculated in numerical simulations. The cold stream provides in the temperature field a dark tongue in the lower part of the layer. This flow forms a thermal boundary layer with a considerable negative (unstable) temperature gradient near the lower surface. The negative temperature gradient leads to the appearance of thermal plumes and as a consequence to the formation of horizontal rolls.

The ascending currents of hot liquid are well visible in cross-sections, transverse to the main flow. In Fig. 4(c), we show the instant temperature field in a cross-section over the hot plate at $x = 65$ mm, where thermal plumes are clearly seen. The distribution of vorticity in the same cross-section is shown in Fig. 4(d) which displays explicitly the set of strong longitudinal rolls near the bottom of the layer and much weaker rolls near the upper surface.

The centre (axis) of rotation is tied to the position of the temperature minimum which hardly changes along the flow. To illustrate this, in Fig. 4(a),(b) we have marked the positions of the points with maximum values of the vorticity (roll centre) by black points, connected by a curve. This curve is practically parallel to the lower boundary of the domain. In the same figure, the points corresponding to the upper boundary of the rolls are marked by white points. The total roll height is related with the thickness of the entire boundary layer and increases along the flow until it becomes limited by the border between the main flows in the lower and upper layers. Thus, the rolls are nonsymmetric in the vertical direction. The increase in the temperature difference on the heat exchangers leads to an increase in the flow velocity and squeezes the boundary layer, thus reducing the roll height and increasing the roll rotation rate.

One of the important characteristics of longitudinal rolls is the wavelength, $\lambda$, (the distance between centre of the two nearest co-rotating rolls). In a horizontal converging channel heated from below (Chiu et al., 2000), the wavelength of the rolls in the longitudinal roll regime is insensitive to the flow rate or heating rate. On the contrary, $\lambda$ in a horizontal parallel plate
Figure 4. Average temperature (a) and absolute velocity value (b) fields in the $XZ$ plane over the hot plate; center and upper boundary of rolls are shown by black circles and white squares. Instant temperature (c) and vorticity $\omega_x$ (d) fields in $YZ$ plane for $x = 65$ mm ($h = 30$ mm, $Pr = 263$ and temperature difference $\theta = 10K$). Temperature is given in K, velocity – in mm/s, vorticity – in s$^{-1}$. Numerical simulation.

channel with uniformly heated bottom (Maughan & Incropera, 1987) strongly depends on the heating (Grashof number). Although we can not make direct comparison, qualitatively our results are in agreement with the study of (Maughan & Incropera, 1987). The dependence of non-dimensional roll wavelength $\lambda/D$ for a wide range of $Ra$ decreases as $\lambda \sim Ra^{-0.40}$.

The transverse rolls appear in the flow only under the conditions of the combination of the large vertical drop in temperature and weak large-scale flow. In the advective flow under discussion, this combination is possible only in the liquids with a large Prandtl number.

It was shown that both longitudinal and transverse rolls lead to remarkable heat transfer enhancement and the average Nusselt number grows with increasing of Rayleigh number.

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