DMT analysis and optimal scheduling for FSO relaying communications

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Abstract
In this paper, two-hop parallel N-relay networks are considered and the diversity-multiplexing tradeoff (DMT) is derived over Gamma-Gamma free-space optical (FSO) channels with identical average received signal to noise rations in all links. In the derivations, both local and global channel state information (CSI) are investigated. In the local CSI case, the node only knows its incoming link conditions, while in the global CSI case, the nodes are aware of all the CSIs in the network. The listening and transmitting times at the relays as variables in the DMT derivation are further considered which are later used to optimize the performance of the network. It is demonstrated that the optimal DMT is obtained with the static quantize map and forward (SQMF) and dynamic quantize map and forward (DQMF) strategies for different ranges of the multiplexing gain. In addition, the optimal schedule of relays in the DQMF strategy is determined as a function of the local channel conditions in the relays.

1 | INTRODUCTION
Free-space optical (FSO) communication provides high channel bandwidth at low cost and inherent security using near-infrared (NIR) carrier frequencies in the optical band [1]. These benefits make FSO systems appealing for various applications in terrestrial and satellite communication links. However, despite their important advantages, the performance of terrestrial FSO systems are affected by atmospheric fading. Parallel relay networks are one of the ways to eliminate this defect in FSO systems [2]. One of the fundamental metrics to characterize the performance of an FSO system is the optimality of the diversity-multiplexing tradeoff (DMT). In [3], the authors introduce the DMT metric in an FSO system for Log-normal, Gamma-Gamma, and negative exponential channel models.

Parallel relaying connects the source and the destination through several parallel sets of relays that are assumed to use orthogonal channels for relaying the information simultaneously [4, 5]. Also, a direct link might exist based on the propagation conditions. Since the destination node receives the same data via multiple independent different paths, parallel relaying systems can enhance the reliability and/or capacity in FSO systems. This model of cooperative communications was presented first in radio-frequency (RF) by Schein and Gallager [6], where upper and lower bounds of the capacity were derived over the additive white Gaussian noise (AWGN) channel. Later, various researches have investigated the potential benefits of parallel relaying over different types of fading channels [7–9]. In [7], the authors derived the DMT for a class of parallel relay RF networks in which the direct link between the source and destination is not available, and signals over different paths can overlap with each other. They demonstrated that the proposed scheme with half-duplex (HD) amplify-and-forward relays can approach the full-duplex (FD) DMT when the number of paths is bigger than 3. In [8], the authors studied the impact of inter-relay cooperation on the performance of decode-and-forward (DF) cooperative FSO communication systems with any number of parallel relays. The DMT of a cooperative parallel relay network in FSO communications over Gamma-Gamma fading channels was calculated in [9], where it was shown that using a parallel relay network can increase the DMT in FSO systems. Tight bounds for the capacity of a FSO system is required for calculating the DMT metric. In [10], the authors discuss the capacity of the single-input single-output channel in optical systems. In [11], the authors have considered the multiple-input multiple-output model of the FSO systems, and review capacity.
bounds for this channel model. They also study multi-user channels which are modeled as a broadcast channel (downlink) or a multiple-access channel (uplink) and then they provide capacity bounds for the channels, but the exact capacity of the parallel relay channel is still unknown. In this paper, we use the max-flow min-cut bound [12] for the FSO system similar to [2, 9].

In [13], the authors considered a dual-hop parallel relay network with the same average signal-to-noise ratio (SNR) in links. They demonstrated that the optimal DMT for different multiplexing gains is achieved by the quantize-map-and-forward (QMF) strategy. In this scheme, the relay listens to the message from the source for a shorter time than the decoding time, required typically for decode-and-forward (DF) relaying. Then, it quantizes the received message at the noise level and assigns a codeword to the quantized signal, and finally forwards it to the destination [14]. In [15], it was shown that the QMF scheme can provide the rates within a fixed gap of the capacity for an arbitrary network, and thus it can obtain the optimal DMT in the network.

In this paper, we study the proposed FSO system from the information theory aspect by the DMT metric and do not analyze from the higher layers of the network’s point of view, where most of the relevant literature in this category evaluate FSO systems through utilizing a secure performance metric. We consider a parallel FSO relaying scheme for an identical average received SNR in all links under the assumptions of local and global CSI. In the local CSI case, the node only knows its incoming link conditions, while for the global CSI case, the nodes are aware of all the CSIs in the network [16, 17]. We further consider the listening and transmitting times at the relays as variables in our DMT derivation which are later used to optimize the performance of the parallel FSO relay system. We prove that the optimal DMT is achieved by the DQMF and SQMF strategies for different multiplexing gain ranges and using a proper schedule depending on the local CSIs in relays. Since the SQMF communication strategy only uses the average link SNRs and not the instantaneous channel gains to optimize the listening and transmitting times, we propose an upper limit on the ratio of DMTs for the DQMF and SQMF schemes and show that this ratio is upper bounded by $2$. In fact, we can select a simpler protocol with a fixed schedule to achieve at least half the optimal DMT in a two-hop network with $N$ half-duplex relays in the FSO system with Gamma-Gamma fading channels.

The rest of the paper is organized as follows. Some basic concepts and definitions along with the system model are introduced in Section 2. In Section 3, we derive the DMT of the network for global CSI. In addition, we present the DMT for a two-hop parallel half-duplex FSO relay network with local CSI in Section 4. In Section 5, the DMT of the FSO network with the local received CSI is derived. Simulation results are presented in Section 6 to evaluate the performance of the proposed strategy and schedules for relays of the network. Finally, in Section 7, an overview of the results is provided.

**Notation:** Throughout this paper, we define $x^+ = \max\{0, x\}$ for a real number $x$, $\eta_{ij} = \frac{1}{2} \min\{m_{ij}, n_{ij}\}$ and $\eta_i = \min\{\eta_{i1}, \eta_{i2}\}$. For simplicity, we assume that $\eta_1 \geq \eta_2 \geq \cdots \geq \eta_N$, and we arrange parallel paths based on this assumption.

### 1.1 Diversity multiplexing tradeoff

In radio technology, multiplexing gain is achieved when a system transmits different streams of data from the same radio resource in separate spatial dimensions. Diversity gain, on the other hand, is defined as the increase in the signal-to-interference plus noise ratio due to some diversity schemes, or how much the transmission power can be reduced when a diversity scheme is introduced without a performance loss.

Achieving a full multiplexing gain requires fully independent usage of the antennas or each data pipe; however, this reduces the diversity gain, and increases the outage probability of the network. Additional coding may incur some overhead, but the resulting diversity gain can enhance the BER performance, even though the data rate is kept at the same level. Therefore, it is of much importance to consider the tradeoff between the multiplexing gain and the diversity gain. More precisely, DMT correlates the tension between the rate and the reliability over fading channels.

For the transmission rate of $R$ and the outage probability of $P_{\text{out}}$ in the network, the multiplexing and diversity gains, denoted by $r$ and $d$, respectively, are given by [13]

$$ r = \lim_{\rho \to \infty} \frac{R}{\log \rho}, \quad d = -\lim_{\rho \to \infty} \frac{\log P_{\text{out}}}{\log \rho}. $$

For a specified multiplexing gain $r$, the supremum over the diversity gain of all codes is shown by $d(r)$ which represents the DMT in the network.

### 1.2 Relaying protocols

- The QMF strategy was extended to the static QMF (SQMF) and dynamic QMF (DQMF) communication protocols [13]. In the DQMF strategy, relays listen to the coded message from the source for a fraction of time, which is a function of their received CSI. Then, relays quantize the message at the noise scale and assign a codeword to the quantized signal and finally forward it to the destination [18]. The destination node chooses the most likely message in the source codebook given its observation. Clearly, the above protocol is a generalized version of the DDF scheme, since, if the received time is sufficient to decode, the difference between decoding and quantizing of the received signal is the removal or forwarding of the additive noise correspondingly, which does not matter at high SNR regimes.

- The SQMF strategy is similar to DQMF with optimal fixed schedules that depend only on the statistics of channels in the network and not on the instantaneous channel states [19].
2 | SYSTEM MODEL AND ASSUMPTIONS

In this work, we consider a parallel FSO relaying scheme where a laser/LED source (S) transmits data via N relays \(R_i, i = 1, \ldots, N\) to a photo-detector in the destination (D). Relay \(R_i\), \(i = 1, \ldots, N\), listens to the transmitted message of the source for the duration of \(t_i\). In this time, relays turn off their receiver antennas and they transmit data for the duration of \(1 - t_i\).

All channels are assumed to be independent of each other and are modeled as frequency non-selective Gamma-Gamma fading channels. The channel gains of \(S - R_i\) and \(R_i - D\) FSO links are of the form \(g_{SR_i} = |h_{SR_i}|^2\) and \(g_{R_iD} = |h_{R_iD}|^2\), respectively, where \(h_{SR_i}\) and \(h_{R_iD}\) denote the atmospheric fading power coefficients of \(S - R_i\) and \(R_i - D\) links and \(\rho\) is the average received optical power in the absence of the fading. We assume that these coefficients follow \(\Gamma\) distribution [9]. Dropping the subscripts for the sake of simplicity, the probability density function (pdf) of \(b\) is given by

\[
f_{b_{i,j}}(x) = \frac{2(m_{i,j}n_{i,j}/\mu)(m_{i,j}+n_{i,j}/\mu)^{m_{i,j}+n_{i,j}}}{\Gamma(m_{i,j})\Gamma(n_{i,j})} \times K_{i,j}(2\sqrt{m_{i,j}n_{i,j}/\mu}),
\]

for \(i = 1, \ldots, N, j = 1, 2\), where \(K_{i,j}(\cdot)\) is the modified Bessel function of the second kind of order \(\epsilon\) and \(\mu\) is the mean of the random variable \(b_{i,j}\) for \(i = 1, \ldots, N,\) and \(j = 1, 2\). In addition, \(m_{i,j}\) and \(n_{i,j}\) denote the distribution shaping parameters. The exponential orders of the instantaneous SNRs for the links in the network are defined as

\[
\alpha_i = \frac{\log(|h_{SR_i}|^2)}{\log(\rho)}, \quad i = 1, \ldots, N,
\]

\[
\beta_i = \frac{\log(|h_{R_iD}|^2)}{\log(\rho)}, \quad i = 1, \ldots, N.
\]

3 | DMT FOR GLOBAL CSI

In this section, we assume that global channel states are available at all nodes, and relay nodes can optimize their listening and transmitting times accordingly for such a global CSI. The DMT of the DQMF with the global CSI is an upper bound on the DMT of HD \(N\)-relay for FSO systems as shown in Proposition 1 and therefore the DQMF strategy with the global CSI can be used as a benchmark for the performance of other relaying schemes.

Proposition 1. Let \(d_{DQMF-G}(r)\) and \(d(r)\) denote the DMTs of the DQMF communication scheme in an FSO system with HD \(N\)-relay for the multiplexing gain of \(r\) in global CSI. Then, we have

\[
d(r) \leq d_{DQMF-G}(r).
\]

Proof. Similar to the proof in [13], for a sequence of codes with the transmission rate of \(R\) and the outage probability \(P_{out}\) in the network, the multiplexing and the diversity gains, denoted by \(r\) and \(d\), are given by

\[
r = \lim_{\rho \to \infty} \frac{R}{\log(\rho)},
\]

\[
d = -\lim_{\rho \to \infty} \frac{\log(P_{out})}{\log(\rho)},
\]

respectively [21]. For a specified multiplexing gain \(r\), the supremum over the diversity gain of all codes is shown by \(d(r)\) that represents the DMT of the network. Similar to [15], we assume that in a parallel HD \(N\)-relay in FSO systems, \(C\) represents the cut-set upper bound with \(i.i.d.\) sources. Denoting \(t_i\), \(i = 1, \ldots, N\), as the listening time of the relay \(R_i\), \(C\) for a two-hop parallel \(N\)-relay in FSO system is characterized as [13]:

\[
C = \frac{1}{2} \min \left\{ t_1 c(|h_{SR_1}|^2\rho), (1-t_1) c(|h_{R_1D}|^2\rho) \right\}
\]

where \(\Re\) is the responsivity of the photo-detector [20], \(x_{i,1}\) and \(n_{i,2}\) denote the transmitted electrical signals of relay \(R_i\) and the noise at the destination, respectively, with \(E[|x_{i,1}|^2] = 1\) and \(E[|n_{i,2}|^2] = 1\). Each noise term is the superposition of the thermal noise and the background light-induced shot noise and is modeled as the zero-mean signal-independent Gaussian noise. In addition, if the relay \(R_i\) is in the receiving phase, then the electrical signals received at the relay \(R_i\) and the destination, denoted by \(y_{i,1}\) and \(y_{i,2}\), are given by

\[
y_{i,1} = \Re h_{SR_i} \rho^{1/2} x_{i,1} + n_{i,1},
\]

\[
y_{i,2} = 0,
\]

\[
y_{i,2} = \Re h_{R_iD} \rho^{1/2} x_{i,2} + n_{i,2},
\]

with \(x_{i,1}\) and \(n_{i,1}\) denote the intensity-modulated source signal and the noise at the relay \(R_i\), respectively, with \(E[|x_{i,1}|^2] = 1\) and \(E[|n_{i,1}|^2] = 1\)
Then for the limited SNR regime, we have

\[ K = \frac{1}{2} \min \{ t_2 c(b_{\text{SN}}^2 \rho), (1 - t_2) c(b_{\text{SN}} D^2 \rho) \} + \cdots + \frac{1}{2} \min \{ t_N c(b_{\text{SN}}^2 \rho), (1 - t_N) c(b_{\text{SN}} D^2 \rho) \}. \]  

(8)

where as it is shown in [22], the capacity of an intensity modulation/direct-detection optical link at high SNR asymptotically satisfies \( c(\gamma) \approx \frac{1}{2} \log(\gamma) \). Using the results in [15], an upper bound on the capacity of the two-hop parallel \( N \)-relay in FSO systems with the global CSI at the relays can be derived as

\[ \max_{\{t_1, \ldots, t_N\}} C + K_1, \]  

(9)

where \( K_1 \) is a constant value independent of SNRs in the network. All parameters of this expression are independent of the type of transmitted signal and the transmitter modulation and receiver detection, and this equation only depends on the channel gain, signal power, and the average SNR in links. Then, a lower bound on the outage probability in the network for a specified transmission rate \( r \) can be derived from the event that the channel is in outage which is equivalent to the case when the transmission rate is greater than the capacity of the channel, that is,

\[ P_{\text{out}} \geq \epsilon \Pr( \max_{\{t_1, \ldots, t_N\}} C + K_1 \leq r \log \rho ) \triangleq P_{\text{out}-L}, \]  

where \( \epsilon \) is a constant value greater than zero. Also, it is shown that the DQMF scheme utilizing the global CSI can achieve all rates less than

\[ \max_{\{t_1, \ldots, t_N\}} C - K_2, \]  

(10)

where \( K_2 \) is another constant [15]. Then, an upper bound on the outage probability for the DQMF strategy is given by

\[ P_{\text{out}} \leq \Pr( \max_{\{t_1, \ldots, t_N\}} C - K_2 \leq r \log \rho ) + \epsilon \triangleq P'_{\text{out}-U}. \]  

(11)

We denote the upper bound on the DMT of HD \( N \)-relay in FSO systems and the DMT achieved by the DQMF protocol with the global CSI as \( d_{1\text{DGF}}(r) \) and \( d_{\text{DQMF-G}}(r) \), respectively. Then for the limited SNR regime, we have

\[ - \frac{\log P_{\text{out}-L}}{\log \rho} \geq d_{1\text{DGF}}(r) \geq d_{\text{DQMF-G}}(r) \geq - \frac{\log P'_{\text{out}-U}}{\log \rho}. \]  

(12)

Since \( \epsilon, K_1, \) and \( K_2 \) are constant values and independent of SNR, then \( d_{1\text{DGF}} \) is equal to \( d_{\text{DQMF-G}} \) for high SNRs. Thus, we get an upper bound on the DMT using the DQMF protocol with the global CSI and this completes the proof of the proposition.

We present the DMT of the DQMF protocol with the global CSI as an optimization problem given in Proposition 2.

**Proposition 2.** The DMT of the DQMF strategy under the global CSI for a two-hop parallel HD \( N \)-relay in FSO systems is characterized by the following optimization problem:

\[ d_{\text{DQMF-G}}(r) = \min_{(\alpha_1, \ldots, \alpha_N, \beta_1, \ldots, \beta_N) \in O(r)} \sum_{i=1}^{N} \eta_i (1 - \alpha_i)^+ + \eta_i (1 - \beta_i)^+, \]  

(13)

where the outage region is given by

\[ O(r) = \{ (\alpha_1, \ldots, \alpha_N, \beta_1, \ldots, \beta_N) : 0 \leq \alpha_1 \leq 1, \ldots, 0 \leq \alpha_N \leq 1, \]  

\[ 0 \leq \beta_1 \leq 1, \ldots, 0 \leq \beta_N \leq 1, \]  

\[ \frac{\alpha_i \beta_1}{(\alpha_1 + \beta_1)} + \cdots + \frac{\alpha_i \beta_N}{(\alpha_N + \beta_N)} \leq 2r \}, \]  

(14)

where \( r \) denotes the multiplexing gain.

**Proof.** We obtain an upper bound on the DMT of the DQMF strategy similar to Lemma 7 in [13] as

\[ d_{\text{DQMF-G}}(r) = \min_{(\alpha_1, \ldots, \alpha_N, \beta_1, \ldots, \beta_N) \in O(r)} \sum_{i=1}^{N} \eta_i (1 - \alpha_i)^+ + \eta_i (1 - \beta_i)^+, \]  

(15)

where

\[ O(r) = \{ (\alpha_1, \ldots, \alpha_N, \beta_1, \ldots, \beta_N) : 0 \leq \alpha_1 \leq 1, \ldots, \]  

\[ 0 \leq \alpha_N \leq 1, 0 \leq \beta_1 \leq 1, \ldots, 0 \leq \beta_N \leq 1, \]  

\[ \max_{\{t_1, \ldots, t_N\}} \left( \frac{1}{2} \min \{ t_1 \alpha_1, (1 - t_1) \beta_1 \} + \cdots + \frac{1}{2} \min \{ t_N \alpha_N, (1 - t_N) \beta_N \} \right) \leq r \}. \]  

(16)

The optimal choices for \( t_1, \ldots, t_N \) are identified by setting \( t_1 \alpha_1 = (1 - t_1) \beta_1, \ldots, t_N \alpha_N = (1 - t_N) \beta_N \) which results in (14).

## 4 DMT FOR LOCAL CSI

In this section, the nodes can track the instantaneous realization of their incoming links and communicate them to the destination, but they cannot track their outgoing links. In fact, we suppose that the local CSIs are known at the receivers but not at the transmitters. When the nodes of the network only know their incoming channel conditions, the upper bound on the DMT of the HD \( N \)-relay in FSO systems can be achieved by the
DMT of the DQMF scheme with the local CSI as shown in Proposition 3.

**Proposition 3.** Let \(d_{DQMF-L}(r)\) and \(d(r)\) denote the DMT of the DQMF communication scheme in an FSO system for the multiplexing gain of \(r\) in local CSI, then, we have

\[
d(r) \leq d_{DQMF-L}(r).
\]

**Proof.** Notice that the relays of the network only are aware of instantaneous conditions of their incoming channels. Thus, the relays \(R_1, \ldots, R_N\) select the listening times based on the channel coefficients \(b_{SR_1}, \ldots, b_{SR_N}\), denoted by \(t_i(b_{SR_1}), \ldots, t_i(b_{SR_N})\), respectively. It is proved in [15] that \(\max_{t_i(b_{SR_1}), \ldots, t_i(b_{SR_N})} C + K_1\) is an upper bound on the capacity of the network for the local CSI case, and also it is demonstrated that the DQMF strategy with local CSI can achieve the rate \(\max_{t_i(b_{SR_1}), \ldots, t_i(b_{SR_N})} C + K_2\) in the network, where both \(K_1\) and \(K_2\) are constant and independent of SNR. Similar to the proof of Proposition 1, we can show that the upper bound on the DMT of the network, based on an upper bound on the capacity, is equal to the DMT for the achievable rate of the DQMF strategy in the high SNR regime. Thus, it is proved that the upper bound on the DMT of an HD \(N\)-relay network with local CSI can be provided by the DQMF protocol with the local CSI. \(\square\)

In Proposition 4, we describe the DMT of the network with the local CSI as an optimization problem.

**Proposition 4.** An upper bound on the DMT of a two-hop HD parallel \(N\)-relay network under the local CSI over the Gamma-Gamma fading channel is given by

\[
d(r) = \min_{0 \leq \alpha_1 \leq 1, \ldots, 0 \leq \alpha_N \leq 1, 0 \leq \beta_1 \leq 1, \ldots, 0 \leq \beta_N \leq 1} \min_{0 \leq \beta_{1N} \leq 1, 0 \leq \beta_{2N} \leq 1, (\beta_1, \ldots, \beta_N, t_1, \ldots, t_N) \in \mathcal{O}(\alpha_1, \ldots, \alpha_N, t_1, \ldots, t_N)} 
\sum_{i=1}^{N} \eta_i (1 - \alpha_i)^+ + \eta_i (1 - \beta_i)^+,
\]

where \(\alpha_1, \ldots, \alpha_N\) and \(\beta_1, \ldots, \beta_N\) denote the exponential orders of the instantaneous received powers for the links \(S - R_i\) and \(R_i - D\), \(i = 1, \ldots, N\), respectively. In addition, the outage region in the network is characterized by

\[
O(\alpha_1, \ldots, \alpha_N, t_1, \ldots, t_N) \times \left\{ (\beta_1, \ldots, \beta_N) : 0 \leq \beta_1 \leq 1, \ldots, 0 \leq \beta_N \leq 1, \frac{1}{2} \right\}
\]

\[
\times \left\{ \sum_{i=1}^{N} \min\{t, \alpha_i, (1 - t)\beta_i\} \leq r \right\},
\]

where \(t_i, i = 1, \ldots, N\), denotes the listening time of the relay \(R_i\) and \(r\) is the multiplexing gain.

**Theorem 1.** The DMT of the two-hop half-duplex parallel \(N\)-relay network on the Gamma-Gamma fading channel in an FSO system is given by

\[
d(r) = \begin{cases} 
\sum_{i=1}^{N} \eta_i (1 - \frac{2r}{1 - 2r}), & 0 \leq r \leq \frac{1}{4}, \\
\sum_{i=1}^{N} \eta_i (1 - 4r), & \frac{1}{4} \leq r \leq \frac{1}{2}, \\
\sum_{i=1}^{N} \eta_i (1 - 4r), & \frac{1}{2} \leq r \leq 1.
\end{cases}
\]

**Proof.** See Appendix A for the proof. \(\square\)

In Theorem 1, we present the DMT of a two-hop parallel \(N\)-relay in FSO systems with the local CSI. We also propose the optimal strategies and schedules for this DMT.

**Theorem 2.** Let \(d_{SQMF}(r)\) and \(d_{DQMF}(r)\) denote the DMTs of the SQMF and DQMF communication schemes in a two-hop parallel \(N\)-relay FSO system over Gamma-Gamma fading channel for the multiplexing gain \(0 \leq r \leq 1/4\). Then, we have

\[
d_{DQMF}(r) \leq 2.
\]

**Proof.** See Appendix C for the proof. \(\square\)
5 | DMT PERFORMANCE OF OPTIMAL STRATEGIES AND SCHEDULING

In this section, we compute the DMT of a two-hop parallel $N$-relay in FSO systems under the local CSI by solving the optimization problem (18) and hence prove the results of Theorem 1 in Section 4. In addition, we show that the optimal DMT is achieved by the proposed scheme in Theorem 1. We assume that only receivers are aware of their incoming channel conditions. Relays $R_i$ shall optimize $t_i$ based on the observed $S-R_i$ channel gain, $i = 1, \ldots, N$. In other words, the transmit time $t_i(\alpha_i)$ chosen by relay $R_i$ must be proper for all instantaneous channel conditions represented by $\beta_i$, respectively, for $i = 1, \ldots, N$.

We analyze the DMT for two ranges of the multiplexing gains $0 \leq r \leq 1/4$ and $1/4 \leq r \leq N/4$.

5.1 | Case $0 \leq r \leq 1/4$

Substituting the optimum listening time of the first case in (21), that is, the range $0 \leq r \leq 1/4$, in problem (18), the DMT of the network with the local CSI when relays employ the DQMF strategy is obtained as

$$d_{\text{DQMF-L}}(r) = \min_{(\alpha_1, \ldots, \alpha_N, \beta_1, \ldots, \beta_N) \in O(r)} \times \sum_{i=1}^{N} \eta_{i,1}(1 - \alpha_i)^+ + \eta_{i,2}(1 - \beta_i)^+,$$

where the outage region is given by

$$O(r) = \left\{ (\alpha_1, \ldots, \alpha_N, \beta_1, \ldots, \beta_N) : 0 \leq \alpha_1 \leq 1, \ldots, 0 \leq \alpha_N \leq 1, 0 \leq \beta_1 \leq 1, \ldots, 0 \leq \beta_N \leq 1, \right.$$

$$\times \lim_{t \to -(1-(1-r)\alpha_i)^+} \min\{t \alpha_i, (1 - t) \beta_i\} \leq 2r \right\}.$$

Firstly, we solve the problem (23) by considering $N = 2$, and finally based on the results, we obtain the DMT of the network for any $N \geq 2$.

The instance $N = 2$: In this instance, the problem (23) changes as follows:

$$d_{\text{DQMF-L}}(r) = \frac{2}{k_1} \eta_{1,1}(1 - \alpha_1)^+ + \eta_{1,2}(1 - \beta_1)^+,$$

where

$$O(r) = \left\{ (\alpha_1, \alpha_2, \beta_1, \beta_2) : 0 \leq \alpha_1 \leq 1, 0 \leq \beta_1 \leq 1, 0 \leq \alpha_2 \leq 1, 0 \leq \beta_2 \leq 1, \right.$$

$$\times \left( \lim_{t_1 \to -(1-(1-r)\alpha_1)^+} \min\{t_1 \alpha_1, (1 - t_1) \beta_1\} \right.$$  

$$+ \left. \lim_{t_2 \to -(1-(1-r)\beta_2)^+} \min\{t_2 \alpha_2, (1 - t_2) \beta_2\} \right) \leq 2r \right\},$$

and $k_1 = k_2 = 1 - 2r$. We consider three different cases for this instance.

Case 1: If $\alpha_1 + \beta_1 \leq \frac{1}{k_1}$ and $\alpha_2 + \beta_2 \leq \frac{1}{k_2}$, the outage region becomes

$$O(r) = \left\{ (\alpha_1, \alpha_2, \beta_1, \beta_2) : 0 \leq \alpha_1 \leq 1, 0 \leq \beta_1 \leq 1, 0 \leq \alpha_2 \leq 1, 0 \leq \beta_2 \leq 1, \right.$$  

$$\alpha_1 + \beta_1 \leq \frac{1}{k_1}, \alpha_2 + \beta_2 \leq \frac{1}{k_2}, k_1 \alpha_1 \beta_1 + k_2 \alpha_2 \beta_2 \leq 2r \right\}.$$

Defining $C = \frac{2r - k_2 \alpha_2 \beta_2}{k_1}$ and under condition $k_1 \alpha_1 \beta_1 + k_2 \alpha_2 \beta_2 \leq 2r$, for any values of $(\alpha_2, \beta_2)$ such that $\alpha_2 \beta_2 \leq \frac{2r}{k_2}$, it can be proven that the function $(\eta_{1,1} \alpha_1 + \eta_{1,2} \beta_1)$ is maximized over the boundary points $A$, $A'$, $B$, $B'$ with coordinates $A: (\alpha_1 = 1, \beta_1 = \frac{1}{k_1} - 1), B: (\alpha_1 = 1, \beta_1 = \frac{2r - k_2 \alpha_2 \beta_2}{k_1})$, $A': (\alpha_1 = \frac{1}{k_1} - 1, \beta_1 = 1)$ and $B': (\alpha_1 = \frac{2r - k_2 \alpha_2 \beta_2}{k_1}, \beta_1 = 1)$, respectively. For simplicity, we assume that $\eta_{i,1} \geq \eta_{i,2}$ for $i = 1, 2$. It can be shown that $\beta_1$ in point $B$ and $\alpha_1$ in point $B'$ are smaller than points $A$ and $A'$, respectively, because

$$1 \frac{k_1 - 1}{k_1} = \frac{2r}{1 - 2r} \geq \frac{2r - k_2 \alpha_2 \beta_2}{k_1} = \frac{(2r - k_2 \alpha_2 \beta_2)}{(1 - 2r)},$$

or

$$0 \leq -k_2 \alpha_2 \beta_2.$$

The inequality (28) is always true. Only two points $B$ and $B'$ meet all conditions of the outage region in (26). Also, it can be shown that the function $(\eta_{1,1} \alpha_1 + \eta_{1,2} \beta_1)$ at point $B$ is always smaller than at point $B'$. As a result, the problem (24) is optimized over $\alpha_1 = 1$ and similarly, $\alpha_2 = 1$, where, in either case the conditions $\alpha_1 + \beta_1 \leq \frac{1}{k_1}$ and $\alpha_2 + \beta_2 \leq \frac{1}{k_2}$ are true. Thus, the optimization problem in (24) for $k_1 = k_2 = 1 - 2r$ reduces to

$$d_{\text{DQMF-L}}(r) = \min_{(\beta_1, \beta_2) \in O(r)} \eta_{1,2}(1 - \beta_1)^+ + \eta_{2,2}(1 - \beta_2)^+,$$

where the outage region is

$$O(r) = \left\{ (\beta_1, \beta_2) : \beta_1 + \beta_2 \leq \frac{2r}{1 - 2r}, 0 \leq \beta_1 \leq \frac{2r}{1 - 2r}, 0 \leq \beta_2 \leq \frac{2r}{1 - 2r} \right\}.$$
In can be shown that $d_{\text{DQMF-L}}(r) = \eta_{1,2} + \eta_{2,2} - \max\{\eta_{1,2}, \eta_{2,2}\} \frac{2r}{1 - 2r}$ for $0 \leq r \leq 1/4$. For other assumptions about $\eta_{1,1}$ and $\eta_{1,2}$, we can prove that $d_{\text{DQMF-L}}(r) = \eta_{1} + \eta_{2} - \max\{\eta_{1}, \eta_{2}\} \frac{2r}{1 - 2r}$, where $\eta_{i} = \min\{\eta_{i,1}, \eta_{i,2}\}$ for $i = 1, 2$.

**Case 2:** For $\alpha_{1} + \beta_{1} \leq \frac{1}{k_{1}}$ and $\alpha_{2} + \beta_{2} > \frac{1}{k_{2}}$, the outage region is

$$O(r) = \left\{ (\alpha_{1}, \alpha_{2}, \beta_{1}, \beta_{2}) : 0 \leq \alpha_{1} \leq 1, 0 \leq \beta_{1} \leq 1, 0 \leq \alpha_{2} \leq 1, 0 \leq \beta_{2} \leq 1, \alpha_{1} + \beta_{1} \leq \frac{1}{k_{1}}, \alpha_{2} + \beta_{2} > \frac{1}{k_{2}}, \lim_{t \to \infty} \frac{t}{1 - \alpha_{1} k_{1}} + \lim_{t \to \infty} \frac{t}{1 - \alpha_{2} k_{2}} \leq 2r \right\}.$$  \hspace{1cm} (31)

Considering the above outage region, we can see in optimization (24) that we have $\beta_{2} = 1$ and as a result $\alpha_{2} > \frac{1}{k_{2}} - 1$. Therefore, the outage condition is changed to

$$\beta_{1} \lim_{h \to \infty} \frac{h}{1 - \alpha_{1} k_{1}} \leq \frac{r}{1 - \alpha_{2} k_{2}} \leq 2r(1 - \alpha_{2} k_{2}),$$  \hspace{1cm} (32)

and consequently the first condition in the outage region is changed to $\beta_{1} \lim_{h \to \infty} \frac{h}{1 - \alpha_{1} k_{1}} \leq k_{2} \alpha_{2}^{2} - \alpha_{2} + 2r$. We have $k_{2} = 1 - 2r$ and the right hand side of the above inequality must be non-negative, thus, $\alpha_{2} \leq \frac{2r}{1 - 2r}$ or $\alpha_{2} = 0$. For $\frac{1}{k_{2}} - 1 < \alpha_{2} \leq 1$, the right hand side of (32) is always non-negative, therefore $\alpha_{2} = 1$ and so, $k_{2} \alpha_{2}^{2} - \alpha_{2} + 2r = 0$. Since $(1 - h_{1}) \geq 0$, there is no feasible value for $\beta_{1}$. Hence, there is no solution for the optimization problem (24) in this case.

**Case 3:** For $\alpha_{1} + \beta_{1} > \frac{1}{k_{1}}$ and $\alpha_{2} + \beta_{2} > \frac{1}{k_{2}}$, the outage region is

$$O(r) = \left\{ (\alpha_{1}, \alpha_{2}, \beta_{1}, \beta_{2}) : 0 \leq \alpha_{1} \leq 1, 0 \leq \beta_{1} \leq 1, 0 \leq \alpha_{2} \leq 1, 0 \leq \beta_{2} \leq 1, \alpha_{1} + \beta_{1} > \frac{1}{k_{1}}, \alpha_{2} + \beta_{2} > \frac{1}{k_{2}}, \lim_{t \to \infty} \frac{t}{1 - \alpha_{1} k_{1}} + \lim_{t \to \infty} \frac{t}{1 - \alpha_{2} k_{2}} \leq 2r \right\}.$$  \hspace{1cm} (33)

Similar to the arguments in Case 2, we must set $\beta_{1} = 1$ and $\beta_{2} = 1$ in the optimization problem in (24), where the outage condition is changed to

$$\lim_{t \to \infty} \frac{t}{1 - \alpha_{1} k_{1}} \leq 2r - t_{2} \alpha_{2} < 2r - \alpha_{2}(1 - \alpha_{2} k_{2})$$  \hspace{1cm} (34)

and similar to the previous case, there is no feasible value for $\alpha_{1}$ meaning that the optimization problem is infeasible. Note that because of the symmetry of the problem, the cases $\alpha_{1} + \beta_{1} > \frac{1}{k_{1}}$ and $\alpha_{2} + \beta_{2} > \frac{1}{k_{2}}$ are similar to Case 2.

**Lemma 1.** The DMT of the DQMF strategy in the HD parallel relay network with the local CSI in the range $0 \leq r \leq 1/4$ reaches the upper bound on the DMT with the global CSI given in Proposition 1, that is, $d_{\text{DQMF-L}}(r) = d_{\text{DQMF-G}}(r)$.

**Proof.** We need to show that the outage point $(\alpha_{1}, \beta_{1}, \alpha_{2}, \beta_{2}) = (1, \frac{2r}{1 - 2r}, 1, 0)$ for $\eta_{1,1} \geq \eta_{1,2}, \eta_{2,1} \geq \eta_{2,2}$ and $\eta_{1,2} \geq \eta_{2,2}$ of the DQMF scheme with the local CSI is a feasible point for the DMT of the network with the global CSI in (14), that is,

$$\frac{\alpha_{1} \beta_{1}}{\alpha_{1} + \beta_{1}} + \frac{\alpha_{2} \beta_{2}}{\alpha_{2} + \beta_{2}} = \frac{1}{(1 + \frac{2r}{1 - 2r})} + \frac{1}{(1 + 0)} \leq 2r.$$  \hspace{1cm} (35)

Then, we have $(\alpha_{1}, \beta_{1}, \alpha_{2}, \beta_{2}) \in O(r)$ in (14). Thus,

$$d_{\text{DQMF-G}}(r) \leq \eta_{1} + \eta_{2} - \max\{\eta_{1,2}, \eta_{2,2}\} \frac{2r}{1 - 2r} = d_{\text{DQMF-L}}(r).$$  \hspace{1cm} (36)

On the other hand, we proved that $d_{\text{DQMF-L}}(r) \leq d_{\text{DQMF-G}}$ in Proposition 1, thus, $d_{\text{DQMF-G}}(r) = d_{\text{DQMF-L}}(r)$, and this proves that the DQMF scheme with the local CSI reaches the optimal DMT with the global CSI in State 1.

The instance $N \geq 2$: Similar to the instance $N = 2$, to solve the problem (23), the outage region is divided into $2^{N}$ different cases. Among all these cases only one case provides the feasible outage region expressed as follows:

$$O(r) = \left\{ (\alpha_{1}, \ldots, \alpha_{N}, 0, 0) : 0 \leq \alpha_{1} \leq 1, \ldots, 0 \leq \alpha_{N} \leq 1, 0 \leq \beta_{1} \leq 1, \ldots, 0 \leq \beta_{N} \leq 1, \sum_{i=1}^{N} \alpha_{i} \beta_{i} \leq 2r \right\}.$$  \hspace{1cm} (37)

According to the above outage region and similar to the proposed solution in Case 1 of the instance $N = 2$, the DMT of the parallel $N$-relay network in (23) is computed as $d(r) = \sum_{i=1}^{N} \eta_{i} - \max\{\eta_{1}, \ldots, \eta_{N}\} \frac{2r}{1 - 2r}$, for the range $0 \leq r \leq 1/4$. In addition, we can show that this DMT is optimal using Lemma 1.

### 5.2 Case $1/4 \leq r \leq N/4$

In this range of the multiplexing gain, the relays $R_{i}$ use the SQMF scheme. We consider the next lemma for this case.
Lemma 2. The optimal DMT of the two-hop parallel HD $N$-relay in FSO systems with the local CSI for 
\[ \frac{1}{4} \leq r \leq \frac{j}{4} \] is obtained as 
\[ d_{\text{HD}}(r) = \sum_{i=j}^{N} \eta_i + \eta_j(j - 1 - 4r) \] for $j = 2, \ldots, N$.

Proof. We set $t_i$ to $f(r)$ where $f(\cdot)$ is an arbitrary function. For a fixed $r$, the value of $f(r)$ belongs to $[0,1]$. We consider the DMT of the network in the following cases for different ranges of $f(r)$ with 
\[ \frac{1}{4} - 2r \leq r \leq \frac{j}{4} \] for $j = 2, \ldots, N$, as follows:

- In the case $0 \leq f(r) \leq 1/2$, we select $\alpha_i$ and $\beta_i$, $i = 1, \ldots, N$, as follows:

\[
\alpha_i = \begin{cases} 
1, & i < j \\
\frac{2(1-r)j}{r(1-f(r))}, & i = j, j \eta_{i,1} \geq \eta_{i,2} \\
1, & i = j, j \eta_{i,1} < \eta_{i,2} \\
0, & i > j, j \eta_{i,1} \geq \eta_{i,2} \\
0, & i > j, j \eta_{i,1} < \eta_{i,2}
\end{cases}
\]

and

\[
\beta_i = \begin{cases} 
1, & i < j \\
\frac{2(1-r)j}{r(1-f(r))}, & i = j, j \eta_{i,1} \geq \eta_{i,2} \\
1, & i = j, j \eta_{i,1} < \eta_{i,2} \\
1, & i > j, j \eta_{i,1} \geq \eta_{i,2} \\
0, & i > j, j \eta_{i,1} < \eta_{i,2}
\end{cases}
\]

where $(\alpha_1, \ldots, \alpha_N, \beta_1, \ldots, \beta_N) \in O(r)$. The functions \[\frac{2r - (j - 1)(1-f(r))}{f(r)}\] and \[\frac{2r - (j - 1)(1-f(r))}{1-f(r)}\] in $f(r) = 1/2$ are minimal for \(0 \leq f(r) \leq 1/2\) and thus \(d_{\text{HD}}(r) \leq \sum_{i=j}^{N} \eta_i + \eta_j(j - 1 - 4r)\) for $j = 2, \ldots, N$. Whereas $0 \leq \alpha_i \leq 1$ and $0 \leq \beta_i \leq 1$, then, \[\frac{2r - (j - 1)(1-f(r))}{f(r)}\] and \[\frac{2r - (j - 1)(1-f(r))}{1-f(r)}\] in $f(r) = 1/2$ are smaller than 1 for \(\frac{j}{4} - 2r \leq r \leq \frac{j}{4}\).

- In the case $1/2 \leq f(r) \leq 1$, we select $\alpha_i$ and $\beta_i$ for $i = 1, \ldots, N$, as follows:

\[
\alpha_i = \begin{cases} 
1, & i < j \\
\frac{2(1-r)j}{r(1-f(r))}, & i = j, j \eta_{i,1} \geq \eta_{i,2} \\
1, & i = j, j \eta_{i,1} < \eta_{i,2} \\
0, & i > j, j \eta_{i,1} \geq \eta_{i,2} \\
0, & i > j, j \eta_{i,1} < \eta_{i,2}
\end{cases}
\]

and

\[
\beta_i = \begin{cases} 
1, & i < j \\
\frac{2(1-r)j}{r(1-f(r))}, & i = j, j \eta_{i,1} \geq \eta_{i,2} \\
1, & i = j, j \eta_{i,1} < \eta_{i,2} \\
1, & i > j, j \eta_{i,1} \geq \eta_{i,2} \\
0, & i > j, j \eta_{i,1} < \eta_{i,2}
\end{cases}
\]

where \((\alpha_1, \ldots, \alpha_N, \beta_1, \ldots, \beta_N) \in O(r)\). The functions \[\frac{2r - (j - 1)(1-f(r))}{f(r)}\] and \[\frac{2r - (j - 1)(1-f(r))}{1-f(r)}\] in $f(r) = 1/2$ are minimal for \(0 \leq f(r) \leq 1/2\) and thus \(d_{\text{HD}}(r) \leq \sum_{i=j}^{N} \eta_i + \eta_j(j - 1 - 4r)\) for $j = 2, \ldots, N$. Whereas $0 \leq \alpha_i \leq 1$ and $0 \leq \beta_i \leq 1$, then, \[\frac{2r - (j - 1)(1-f(r))}{f(r)}\] and \[\frac{2r - (j - 1)(1-f(r))}{1-f(r)}\] in $f(r) = 1/2$ are smaller than 1 for \(\frac{j}{4} - 2r \leq r \leq \frac{j}{4}\).

We also prove that the optimal DMT can be obtained by the SQMF strategy in Lemma 3.

Lemma 3. The optimal DMT of two-hop parallel N-relay in FSO systems in Lemma 2 for \(\frac{j}{4} \leq r \leq \frac{j}{4}\), $j = 2, \ldots, N$, is achieved by the SQMF scheme.

Proof. See Appendix B.

6 | SIMULATION RESULTS

In this section, we numerically evaluate the DMT of the proposed strategies and schedules in a two-hop parallel N-relay in FSO systems for different cases of $\eta_{1,1}, \eta_{1,2}, \eta_{2,1}$, and $\eta_{2,2}$. We assume that there is no direct link between $S$ and $D$. In this model, the source transmits data signals accompanied by pilot signals, and the relays send them to the destination based on their own relaying protocol. The channels are modeled as independent Gamma-Gamma fading channels. There is a local awareness of channel conditions at the nodes. We investigate our proposed optimal scheme in accordance with the following two scenarios. We assume an FSO system with the wavelength of $\lambda = 1.5 \mu m$, a refractive-index structure constant of $1 \times 10^{-14} m^{-2/3}$, and attenuation of 0.44 dB/km, where the total receive aperture diameter at each node is 25 cm. It is assumed that six Zernike modes are compensated at each receive aperture.

- **Scenario 1:** In this scenario, we evaluate the DMT of the parallel N-relay network versus multiplexing gain $r$ in FSO
systems for different values of minimum distribution shaping parameter values over different paths. In Figure 1, we see that if shaping parameters over links are the same, then, the performance of the FSO system will be better for higher multiplexing gains.

- **Scenario 2:** In this scenario, we compare the DMT of the proposed DQMF scheme with the SQMF for the multiplexing gains \(0 ≤ r ≤ 1/4\). Figure 2 illustrates the DMT of the network versus multiplexing gain \(r\) for these strategies. This figure shows that the DQMF protocol outperforms the SQMF scheme for different values of minimum distribution shaping parameter values over different paths. In addition, we see that the two strategies have the same DMT at \(r = 1/4\). We have repeated our analysis for various \(\eta_1, \eta_2, \eta_3,\) and \(\eta_4\), or equivalently, for different values of the minimum distribution shaping parameter over different paths, and we obtained similar results.

- **Scenario 3:** In this scenario, we compare the DMT of the proposed optimal scheme with the DDF for a two-hop parallel relay in FSO systems versus multiplexing gain \(r\) for \(N = 5\) and (a) \(\eta_1 = 4, \eta_2 = 2.5, \eta_3 = 1.5, \eta_4 = 1.5, \eta_5 = 1\), and (b) \(\eta_1 = 1, \eta_2 = 1, \eta_3 = 1, \eta_4 = 1, \eta_5 = 1\)
in the performance between the two methods become more apparent.

- **Scenario 4**: In this scenario, we examine the DMT of HD and FD modes of the relay network versus different values of minimum distribution shaping parameter values over different paths in an FSO system. We consider the DMT of an FD relay network without self-interference as an upper bound on the DMT of the HD relay system. The DMT of the parallel N-relay in the FSO system is calculated in Appendix E. In Figure 4, we see that the DMT of the HD relay network is closer to the FD relay network for the multiplexing gains in region \( r \in [0, 0.25] \) when compared to any other multiplexing gains. This comes from the fact that the proposed scheme with the local CSI has DMT that is equal to the DMT with the global CSI for multiplexing gains in this region of the multiplexing gain.

7 | CONCLUSIONS

In this paper, we derived the DMT in a two-hop parallel HD N-relay in FSO systems for an identical average received SNR in all links. We derived an upper limit on the achievable DMT under the global CSI. However, the global awareness of the channel conditions is usually unavailable in many networks. We considered the parallel FSO relay model and analytically derived the DMT of the network with the local CSI. We showed that the optimal DMT is achieved by the QMF strategy for a range of multiplexing gains. In addition, we proved the optimal performance by the SQMF strategy in terms of DMT for other ranges of the multiplexing gain. We derived the optimal schedules of the relays in different strategies as functions of the channel realizations.

PERMISSION TO REPRODUCE MATERIALS FROM OTHER SOURCES

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APPENDIX A: DMT FOR LOCAL CSI

We calculate the DMT of a two-hop half-duplex parallel relay in FSO systems under the local CSI over the Gamma-Gamma fading channel. For simplicity, we select $\gamma_{ij} = \frac{\rho |h_{ij}|^2}{N_0}$, $i = 1, \ldots, N$, $j = 1, 2$. Using (2), the pdf of the channel power gain $\gamma_{ij}$, $i = 1, \ldots, N$, $j = 1, 2$, is obtained as

$$f_{\gamma_{ij}}(x) = f_{\gamma}(x) |d\gamma/dx|$$

(A.1)

Let express the modified Bessel function

$$K_{(m_{i,j}-m_{i,j})}(2 \sqrt{m_{i,j}n_{i,j}} \sqrt{\frac{x}{\rho}}),$$

according to the Meijer-G function as [24]:

$$K_{(m_{i,j}-m_{i,j})}(2 \sqrt{m_{i,j}n_{i,j}} \sqrt{\frac{x}{\rho}}) = \frac{1}{2} G_{0,2}^{2,0}(m_{i,j}n_{i,j} \sqrt{\frac{x}{\rho}}; m_{i,j} - n_{i,j}, 0; 0, 1).$$

(A.2)

Using the hyper-geometric function in Meijer-G function [25] and simplifying it, we have

$$f_{\gamma_{ij}}(x) = \frac{\sum_{k=0}^{\infty} c_{i,j,k}(n_{i,j}, m_{i,j})}{\rho} \left(\frac{x}{\rho}\right)^{\frac{k+m_{i,j}-2}{2}} + \frac{\sum_{k=0}^{\infty} c_{i,j,k}(n_{i,j}, n_{i,j})}{\rho} \times \left(\frac{x}{\rho}\right)^{\frac{k+n_{i,j}-2}{2}}, \quad i = 1, 2, \ldots, N, \ j = 1, 2.$$  

(A.3)

where

$$c_{i,j,k}(n_{i,j}, m_{i,j}) = \frac{\pi (m_{i,j}n_{i,j})^{n_{i,j}+k}}{2 \sin \left( (m_{i,j} - n_{i,j}) \pi \right) k! \Gamma(m_{i,j}) \Gamma(n_{i,j}) \Gamma(n_{i,j} - m_{i,j} + k + 1)}$$

(A.4)

$$i = 1, \ldots, N, j = 1, 2.$$

Having $\gamma_{i,1} = \rho^{\alpha_i}$, $\gamma_{i,2} = \rho^{\beta_i}$, $i = 1, \ldots, N$, and for the high SNR values, the pdf of Gamma-Gamma distribution over channel $S = R_i - D$ can be obtained as

$$f_{\gamma_{i,1}}(\alpha_i, \beta_i) \propto \frac{\Gamma(\alpha_i-1)}{\Gamma(\alpha_i-1)\Gamma(\beta_i-1)} \rho^{\alpha_i-1} \rho^{\beta_i-1}$$

(A.5)

Using (11), the outage probability for the high SNR region is obtained as

$$\lim_{\rho \to \infty} P_{out}(\rho^{\alpha_1}, \rho^{\beta_1}, \ldots, \rho^{\alpha_N}, \rho^{\beta_N})$$

where

$$O(r, \tau) = \{(\alpha_1, \beta_1, \ldots, \alpha_N, \beta_N) \mid 0 \leq \alpha_1 \leq 1, 0 \leq \beta_1 \leq 1, \ldots, 0 \leq \alpha_N \leq 1, 0 \leq \beta_N \leq 1,$$

and $c'(Y) \simeq \frac{1}{2} \log(Y)$ at high SNR [10]. By substituting the pdf of Gamma-Gamma distribution in (A.6) for the high SNR region, we have

$$\lim_{\rho \to \infty} P_{out}(\rho^{\alpha_1}, \rho^{\beta_1}, \ldots, \rho^{\alpha_N}, \rho^{\beta_N})$$

where

$$O(r, \tau) = \{(\alpha_1, \beta_1, \ldots, \alpha_N, \beta_N) \mid 0 \leq \alpha_1 \leq 1, 0 \leq \beta_1 \leq 1, \ldots, 0 \leq \alpha_N \leq 1, 0 \leq \beta_N \leq 1,$$

and $c'(Y) \simeq \frac{1}{2} \log(Y)$ at high SNR [10]. By substituting the pdf of Gamma-Gamma distribution in (A.6) for the high SNR region, we have

$$\lim_{\rho \to \infty} P_{out}(\rho^{\alpha_1}, \rho^{\beta_1}, \ldots, \rho^{\alpha_N}, \rho^{\beta_N})$$

and

$$\lim_{\rho \to \infty} P_{out}(\rho^{\alpha_1}, \rho^{\beta_1}, \ldots, \rho^{\alpha_N}, \rho^{\beta_N})$$

The above integral is conquered by the term with the largest SNR exponent when the SNR tends to infinity. Therefore, using
Laplace’s principle in [14], the DMT for the two-hop HD parallel N-relay in FSO systems can be obtained as

\[ d_{\text{HD}}(r) = \min_{(\alpha_1, \beta_1, \ldots, \alpha_N, \beta_N) \in O(r)} \sum_{j=1}^{N} \eta_{j1}(1 - \alpha_j)^+ + \eta_{j2}(1 - \beta_j)^+, \quad (A.8) \]

and this completes the proof of the DMT of two-hop HD parallel N-relay in FSO systems in Proposition 4.

**APPENDIX B: PROOF OF LEMMA 3**

First, we present the DMT of the SQMF strategy in an optimization problem given in Proposition B.1.

**Proposition B.1.** The DMT of the SQMF strategy for a two-hop parallel N-relay in FSO systems is characterized by the following optimization problem

\[
d_{\text{SQMF}}(r) = \max_{O(r)} \min_{(\alpha_1, \ldots, \alpha_N, \beta_1, \ldots, \beta_N) \in O(r)} \left( \sum_{j=1}^{N} \eta_{j1}(1 - \alpha_j)^+ + \eta_{j2}(1 - \beta_j)^+ \right), \quad (B.1)\]

where the outage region is given by

\[
O(r) = \{ (\alpha_1, \ldots, \alpha_N, \beta_1, \ldots, \beta_N) : 0 \leq \alpha_1 \leq 1, \ldots, 0 \leq \alpha_N \leq 1, 0 \leq \beta_1 \leq 1, \ldots, 0 \leq \beta_N \leq 1, \right. \\
\left. \quad \times \sum_{i=1}^{N} N \min t_i \alpha_i (1 - t_i) \beta_i \leq 2r \}, \quad (B.2)\]

where \( r \) is the desired multiplexing gain.

**Proof.** Similar to Appendix D in [13]. \( \square \)

It can be easily shown that the DMT \( d_{\text{HD}}(r) = \sum_{j=1}^{N} \eta_j + \eta_j (j - 1 - 4r) \), for \( j = 2, \ldots, N \), is obtained by the SQMF scheme by substituting the listening and transmitting times \( t_i = 1/2 \) in optimization problem (B.1) for \( -\frac{1}{4} \leq r \leq \frac{1}{4} \), \( j = 2, \ldots, N \).

**APPENDIX C: PROOF OF COROLLARY 1**

To compute the ratio \( \zeta(r) = \frac{d_{\text{SQMF}}(r)}{d_{\text{DQMF}}(r)} \) in a two-hop N-relay in FSO systems over Gamma-Gamma fading channel for \( 0 \leq r \leq 1/4 \), we calculate the DMT of the DQMF and SQMF strategies in Section 5 and then, we prove that \( \zeta(r) \leq 2 \).

The DMTs of the DQMF and SQMF schemes for \( 0 \leq r \leq 1/4 \) are achieved in (20) as

\[
d_{\text{DQMF}}(r) = \sum_{j=2}^{N} \eta_j + \eta_j \left( 1 - \frac{2r}{1 - 2r} \right), \\
d_{\text{SQMF}}(r) = \sum_{j=2}^{N} \eta_j + \eta_j (1 - 4r). \quad (C.1)\]

Inequality \( \frac{d_{\text{DQMF}}(r)}{d_{\text{SQMF}}(r)} \leq 2 \) is represented as

\[
\zeta(r) = \frac{\sum_{j=1}^{N} \eta_j + \eta_1 \left( 1 - \frac{2r}{1 - 2r} \right)}{\sum_{j=2}^{N} \eta_j + \eta_1 (1 - 4r)} \leq 2. \quad (C.2)\]

Then, we can obtain

\[
\sum_{j=1}^{N} \eta_j \geq \eta_1 \left( 8r - \frac{2r}{1 - 2r} \right). \quad (C.3)\]

Since \( 8r - \frac{2r}{1 - 2r} \) is always smaller than one, then, the above inequality is valid for \( 0 < r < 1/4 \) and this completes the proof of Corollary 1 in Section 4.

**APPENDIX D: THE DMT OF DDF STRATEGY**

We present the DMT of the HD N-relay in FSO systems for the DDF relaying protocol. For the DDF scheme, the listening time of each relay is set such that the relay can decode the incoming message. The listening and transmitting times of relays \( R_i, i = 1, \ldots, N \), are \( t_i = \frac{2r}{\alpha_i} \). The outage event in the DDF strategy for each path \( i \) happens if either of the following two events occurs:

- Event 1: Relay \( R_i \) cannot decode the received signal over its listening time. In other words, the decoding time for the relay \( R_i \) is more than one, that is, \( \frac{2r}{\alpha_i} > 1 \).
- Event 2: Relay \( R_i \) decodes its data, that is, \( \frac{2r}{\alpha_i} < 1 \), but link is too weak to send data to destination \( D \).

The DMT of the DDF scheme in the parallel N-relay network based on [26] is given by

\[
d_{\text{DDF}}(r) = \min_{(\alpha_1, \ldots, \alpha_N, \beta_1, \ldots, \beta_N) \in O(r)} \sum_{i=1}^{N} \eta_{i1}(1 - \alpha_i)^+ \\
+ \eta_{i2}(1 - \beta_i)^+, \quad (D.1)\]

where the outage event \( O(r) \) occurs for

\[
O(r) = \{ (\alpha_1, \ldots, \alpha_N, \beta_1, \ldots, \beta_N) : 0 \leq \alpha_1 \leq 1, \ldots, \\
0 \leq \alpha_N \leq 1, 0 \leq \beta_1 \leq 1, \ldots, 0 \leq \beta_N \leq 1, \\
\times \left\{ \frac{2r}{\alpha_1} > 1 \text{ or } \left( \frac{2r}{\alpha_1} < 1, \left( 1 - \frac{2r}{\alpha_1} \right) \beta_1 < 2r \right) \right\}, \ldots, \\
\times \left\{ \frac{2r}{\alpha_N} > 1 \text{ or } \left( \frac{2r}{\alpha_N} < 1, (1 - \frac{2r}{\alpha_N}) \beta_N < 2r \right) \right\}. \quad (D.2)\]
We can split the optimization (D.1) into $2^N$ separate optimizations and solve them in similar with [26], then, we select the minimum DMT, and thus we have

$$d_{DDF}(r) = \sum_{i=1}^{N} \eta_i \left(1 - \frac{2r}{1 - 2r}\right)^+. \quad (D.3)$$

**APPENDIX E: DMT OF THE HD N-RELAY IN FSO SYSTEMS**

For an FD parallel relay network, the DMT is calculated according to Proposition E.1.

**Proposition E.1.** The DMT of the HD N-relay in FSO systems is characterized by the following optimization problem:

$$d_{FD}(r) = \min_{(\alpha_1, \ldots, \alpha_N, \beta_1, \ldots, \beta_N) \in O(r)} \sum_{i=1}^{N} \eta_i (1 - \alpha_i)^+ + \eta_i (1 - \beta_i)^+, \quad (E.1)$$

where the outage region is given by

$$O(r) = \left\{ (\alpha_1, \ldots, \alpha_N, \beta_1, \ldots, \beta_N) : 0 \leq \alpha_i \leq 1, \ldots, \right. \left. 0 \leq \alpha_N \leq 1, 0 \leq \beta_1 \leq 1, \ldots, 0 \leq \beta_N \leq 1, \right\}$$

$$\times \left( \frac{1}{2} \min\{\alpha_1, \beta_1\} + \cdots + \frac{1}{2} \min\{\alpha_N, \beta_N\} \right) \leq r \right\} \quad (E.2)$$

**Proof.** Similar to [13].

By solving the problem (E.1), it can be simply demonstrated that

$$d_{FD}(r) = \begin{cases} \sum_{i=2}^{N} \eta_i + \eta_1 (1 - 2r), & 0 \leq r \leq 0.5, \\ \sum_{i=j}^{N} \eta_i + \eta_j (j - 2r), & \frac{j-1}{2} \leq r \leq \frac{j}{2}, \end{cases} \quad (E.3)$$

$$j = 2, \ldots, N.$$