Deep sub-Doppler cooling of Mg in MOT formed by light waves with elliptical polarization

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Abstract. We study magneto-optical trap (MOT) of Mg atoms operating on the closed triplet \(^3\)P\(_2\)→\(^3\)D\(_3\) (\(\lambda = 383.3\) nm) transition formed by the light waves with elliptical polarization (\(\varepsilon - \theta - \bar{E}\) configuration). Compare to well-known trap formed by light waves with circular polarization (\(\sigma - \sigma\) configuration) the suggested \(\varepsilon - \theta - \bar{E}\) configuration offer the lower sub-Doppler temperature for trapped Mg atoms, that can’t be reached in conventional \(\sigma - \sigma\) MOT.

Mg atoms one of perspective candidates for realization of new-generation of atomic clock, based on optical lattices. These atoms have the narrow spectroscopic lines due to forbidden optical transition from the ground state \(^1\)S\(_0\) to the lowest excited states \(^1\)P\(_{0,2}\) (see figure 1). The closed singlet optical transition \(^3\)S\(_0\)→\(^3\)P\(_1\) can be used for Doppler cooling of Mg atoms down to the temperature \(T = 1.2\) mK. The ground state of this optical transition has angular momentum \(J = 0\) that forbids the possibility of sub-Doppler cooling here. In order to get the lower cooling temperature, the cooling on closed triplet \(^3\)P\(_2\)→\(^3\)D\(_3\) (\(\lambda = 383.3\) nm) optical transition was suggested in [1, 2]. In principle, the ground state here \(^3\)P\(_2\) is degenerated over angular momentum (\(J = 2\)) that should allow sub-Doppler cooling [3]. However, the experimental realization of laser cooling operating on \(^3\)P\(_2\)→\(^3\)D\(_3\) in \(\sigma - \sigma\) MOT did not bring significant progress to reaching lower temperature. In particular, Mg atoms were cooled by laser radiation only down to the temperature \(1\) mK that much higher than the Doppler limit for this optical transition \(T_D = 425\) \(\mu\)K. The recent analysis [2] has revealed the limitation of temperature of laser cooling for \(^25\)Mg atoms in \(\sigma - \sigma\) light field configuration and explained the results of previous experiments [1].

The evolution of atoms in the resonant light waves in presence of a static magnetic field is described by equation

\[
\frac{\partial}{\partial t} \hat{\rho} = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}] - \hat{\Gamma} \{\hat{\rho}\}
\]

for atomic density matrix \(\hat{\rho}\). Here \(\hat{H}\) is the Hamiltonian and term \(\hat{\Gamma} \{\hat{\rho}\}\) describes the relaxation of atomic levels in the process of spontaneous decay (e.g., see for details [4, 5]). Nowadays a lots of methods have been developed for solving this problem, and they can be separated to semiclassical [4, 6–9] and quantum [10–17] methods. The semiclassical methods require some conditions to be fulfilled [4, 6–9] that allow reducing the quantum equation for atomic density matrix to the Fokker-Plank type equation for atomic density function \(W(r, p) = Tr[\hat{\rho}(r, p)]\):
\[
\left( \frac{\partial}{\partial t} + \sum_i \frac{p_i}{m} \nabla_i \right) W = -\sum_i \frac{\partial}{\partial p_i} F_i(\vec{r}, \vec{p}) W + \sum_j \frac{\partial^2}{\partial p_i \partial p_j} D_{ij}(\vec{r}, \vec{p}) W
\]  

(2)

where \( F(\vec{r}, p) \) and \( D(\vec{r}, p) \) are light force on moving atom and diffusion coefficient in light field.

\( 24 \text{Mg} \)

\[
\begin{align*}
\lambda &= 383.9 \text{ nm} \quad \gamma/2\pi = 26.7 \text{ MHz} \\
\lambda &= 285.3 \text{ nm} \quad \gamma/2\pi = 78 \text{ MHz} \\
3s^2 \quad ^1S_0 \\
3s3p \quad ^1P_1 \\
3s4s \quad ^3S_0 \\
3s3d \quad ^3D_2 \\
3s3d \quad ^1D_2 \\
3s4p \quad ^1P_0 \\
3s3p \quad ^1S_1 \\
3s3d \quad ^3P_0,1,2 \\
3s3d \quad ^1P_0,1,2 \\
3s3d \quad ^3P_0,1,2 \\
3s3d \quad ^1D_{1,2,3} \\
\end{align*}
\]

\text{Figure 1. Diagram of low energy levels of } 24\text{Mg. Solid lines denote cooling transitions, and dashed lines denote possible “clock” transitions.}

The quantum methods as well utilize different approaches and simplifications. In particular, the method described in [16, 17], is so called band theory, use secular approximation that assumes enough large detuning \( \delta = \omega - \omega_0 \) of laser light with frequency \( \omega \) from atomic resonance frequency \( \omega_0 \). \( U_0/E_R \ll 36 \delta^2/\gamma^2 \) with \( U_0 \) is depth of optical potential and \( E_R = \hbar^2 k^2/2M \) is recoil energy (see for details [16]). Additionally developed approach is based on simplified equation for atomic density matrix of ground state only \( \hat{\rho}^{gg} \) that also limits applications of the results [18].

Recently we suggest a new method that allows to find solution of equation (1) for 1D problem with taken into account quantum recoil effects without limitations to field parameters and configuration [18, 19]. Our method allows one to compare different field configurations to find a minimum temperature of laser cooling. Strictly speaking, the distribution of cold atoms in electromagnetic field is significantly non-Maxwellian one. However, we define the temperature here as the average kinetic energy that for 1D case results \( k_B T = \langle p^2 \rangle / M \) that is measured in \( \hbar \gamma \) units (\( k_B T/\hbar \gamma = 1 \) corresponds \( T \approx 1.28 \text{ mK} \) for \(^3P_2 \rightarrow ^3D_3 \) optical transition linewidth \( \gamma \)).
Figure 2. Laser cooling temperature of $^{24}$Mg atoms in $\sigma_-\sigma_+$ field configuration (a) and fraction of cold atoms (b) as function of intensity of light waves. $k_{B}T/\hbar\gamma=1$ corresponds to $T \approx 1.28$ mK.

Figure 3. Laser cooling temperature of $^{24}$Mg atoms in lin lin $\perp$ field configuration (a) and fraction of cold atoms (b) as function of intensity of light waves. $k_{B}T/\hbar\gamma=1$ corresponds to $T \approx 1.28$ mK.

Additionally, we introduce the fraction of cold atoms $N_{|p|<3\hbar k}$, i.e. the atoms with momentum $|p|<3\hbar k$, that for $^{24}$Mg atoms cooled by the light field resonance to closed triplet transition $^{3}P_{2} \rightarrow ^{3}D_{3}$ ($\lambda = 383.3$ nm) corresponds to an effective temperature $(3\hbar k)^{2}/M \approx 48.4$ μK.

First we find the temperature of laser cooling $^{24}$Mg atoms on $^{3}P_{2} \rightarrow ^{3}D_{3}$ optical transitions in conventional $\sigma_-\sigma_+$ field configuration commonly used in MOT do not reach deep sub-Doppler values (see figure 2a). The lin $\perp$ lin configuration of light (formed by counterpropagating waves with orthogonal linear polarization) offers much dipper laser cooling temperatures (figure 3a) and much large fraction of cold atoms $N_{|p|<3\hbar k}$, but cannot be used for MOT, because of zero magneto-optical trapping force in lin $\perp$ lin field.

Here to get the deep sub-Doppler laser cooling temperature for $^{24}$Mg atoms in MOT we suggest $\varepsilon - \theta - \varepsilon$ light field configuration to use in MOT for Mg atoms (figure 4). The first study of MOT in $\varepsilon - \theta - \varepsilon$ light field was done by us in [5], where we discovered some peculiarities of laser cooling.
dealing with an elliptical polarization of the light waves, differ from linear \( \varepsilon_0 = 0 \) and circular polarization \( \varepsilon_0 = \pm \pi/4 \) [20].

Figure 5 shows the temperature of laser cooling in \( \varepsilon - \theta - \vec{E} \) field configuration as function of light intensity for particular case of \( \varepsilon_0 = 0 \) (linear polarization of counterpropagating waves). Figure 6 shows the temperature of laser cooling and the fraction of cold atoms (i.e. the atoms with momentum \(|p| < 3\hbar k\)) in \( \varepsilon - \theta - \vec{E} \) field configuration as function of light ellipticity parameter \( \varepsilon_0 \). As one can see, the temperature dependence is not symmetrical function of light wave ellipticity parameter \( \varepsilon_0 \).

Figure 4. \( \varepsilon - \theta - \vec{E} \) field configuration formed by counterpropagating light waves with opposite elliptical polarization with parameters \( \varepsilon_0 \) and \( -\varepsilon_0 \) \((-\pi/4 < \varepsilon_0 < \pi/4 \) and \( \varepsilon_0 = 0 \) corresponds to linear polarization and \( \varepsilon_0 = \pm \pi/4 \) corresponds to right and left circular polarizations). The \( \theta \) is the mutual orientation angle between main polarization axis of light waves.

Figure 5. Laser cooling temperature of \( ^{24}\text{Mg} \) atoms in \( \text{lin} - \theta - \text{lin} \) (\( \theta = -\pi/4 \)) field configuration (a) and fraction of cold atoms (b) as function of intensity of light waves. \( k_B T/\hbar \gamma = 1 \) corresponds to \( T \approx 1.28 \) mK.
Figure 6. Laser cooling temperature of $^{24}$Mg atoms in $\varepsilon - \theta - \varepsilon$ ($\theta = -\pi/4$) field configuration (a) and fraction of cold atoms (b) as function of intensity of parameter ellipticity of light waves $\varepsilon_0$.

The anomaly terms in the friction force results to minimum temperature is reached for an elliptical polarization. However, the anomaly force terms dominant for small detuning $|\delta| \ll \gamma$ only [20], thus for the considered case of $\delta = -\gamma$ the anomaly force effect is small and results to temperature of laser cooling get the minimum values for ellipticities are close to linear polarization: for $I = 100$ mW/cm$^2$ minimum of temperature is reached at $\varepsilon_0 \approx -0.28^8$, for $I = 200$ mW/cm$^2$ at $\varepsilon_0 \approx -0.86^8$, and for $I = 300$ mW/cm$^2$ at $\varepsilon_0 \approx -1.15^8$. The maximum fraction of cold atoms $N_{|p|<3\hbar k}$ for $I = 100$ mW/cm$^2$ is reached at $\varepsilon_0 \approx -2^8$, for $I = 200$ mW/cm$^2$ at $\varepsilon_0 \approx -5.16^8$, and for $I = 300$ mW/cm$^2$ at $\varepsilon_0 \approx -5.26^8$.

Nonlinear dependence of magneto-optical trapping force as function of Zeeman shift $\Omega_H$ of the ground energy sublevel $|J_g = 2, \mu_g = 1\rangle$ is shown on figure 7a for conventional $\sigma_+ - \sigma_-$ field configuration and figure 8a for $\varepsilon - \theta - \varepsilon$ field configuration ($\theta = -\pi/4$) at different waves ellipticity parameters $\varepsilon_0$. The Zeeman shift $\Omega_H = \gamma$ corresponds to magnetic field $H = 12.7$ G.

Assuming the linear dependence of magnetic field on position near the center of the trap $H(z) \approx \partial_z H z$ for $|z| < R_w$, the depth of magneto-optical trap can be estimated as:

$$U^{(H)} = -\frac{R_w}{\Omega_H(R_w)} \int_0^{\Omega_H(R_w)} \left\langle F^{(H)}(v = 0, \Omega_H) \right\rangle d\Omega_H,$$

(3)

where $R_w$ is the radius of the light beams forming the trap and $\Omega_H(R_w)$ is the Zeeman shift on the trap boarder (entrance into the trap). The depth of magneto-optical trap is determined by the gradient of magnetic field $\partial_z H$ and can be expressed as function of magnetic field at the trap boarder (figure 7b and figure 8b) for the area of linear grow of magnetic field (3).
The depth of magneto-optical trap, formed by $\varepsilon - \theta - \varepsilon$ field configuration, is significantly less the depth of magneto-optical trap formed by conventional $\sigma_+ - \sigma_-$ field configuration, but still much deep to capture cold atoms. As an example, the depth of the trap, formed by field with $\varepsilon_0 < 0$ at considered $\theta=-\pi/4$ for $R_W = 0.5$ cm and gradient of magnetic field of $\partial_z H = 12.7$ G/cm (that corresponds to $\Omega_H/\gamma \approx 0.5$ at the trap boarder, figure 8b) reaches $U^{(H)} = 0.094 \hbar \gamma R_W/\lambda \approx 1.56$ K that much exceed the temperature of cold atoms ($T = 124$ $\mu$K for considered parameters).

The elliptical polarization of the light waves that could results to smaller temperature due to anomaly parts in the friction force (i.e. for $\varepsilon_0 < 0$ at considered $\theta=-\pi/4$) unfortunately leads to decrement of the depth of magneto-optical potential (figure 8b). As an example for $\varepsilon_0 = -3^\circ$ the magneto-optical force reverse when magnetic field exceeds 9 G ($\Omega_H/\gamma \approx 0.71$).

For estimation of number of trapped atoms in MOT one should analysis of nonlinear dependence of magneto-optical force for slow atoms in magnetic field. The number $N_c$ of trapped atoms $N_c \propto v_c^4$ is determined by the critical velocity $v_c$, i.e. the maximum velocity of slow atom can have and be captured in MOT [21]. By using the methods we developed in [5], we study magneto-optical force on slow $^{24}$Mg atoms as function of magnetic field. For atoms in $\sigma_+ - \sigma_-$ MOT the critical velocity $v_c \approx 3.5 \gamma/k$ [21] for magneto-optical trap formed by waves with $R_W = 0.5$ cm results to $N_c \approx 7 \times 10^7$ atoms. For atoms in $\varepsilon - \theta - \varepsilon$ MOT at some large values of magnetic field the magneto-optical force is reversed (see figure 9) for some range of velocity of slow atoms. Thus the atoms in this range of parameters $(v, \Omega_H)$ cannot be captured by the trap, that define $v_c$ as a function of magnetic field. Here for lin $- \theta -$ lin field configuration $(\theta=-\pi/4)$ at large magnetic field ($\Omega_H/\gamma > 1.5$) the critical velocity decreased to $v_c \approx 0.014 \gamma/k$ (0.14 m/s) that almost nullify the number of trapped atoms.

Thus for stable work of $\varepsilon - \theta - \varepsilon$ MOT the low gradients of magnetic field is required, i.e the magnetic field within the trap area $r < R_W$ should never reach the critical values $H_c$ when the force on the atoms moving with some velocity range is reversed (figure 9). As an example, for stable work of $\varepsilon - \theta - \varepsilon$ MOT for $\varepsilon_0 = -1^\circ$ magnetic field should not exceed $H_c = 9.3$ G ($\Omega_H/\gamma \approx 0.73$).
Figure 8. Magneto-optical trapping force (in $\hbar k\gamma$ units) on $^{24}$Mg atoms (a) and depth of magneto-optical potential in $\hbar \gamma R_w/\lambda$ units (b) as function of Zeeman shift $\Omega_H$ in $\varepsilon - \theta - \bar{\varepsilon}$ field configuration for different parameters ellipticity of light waves $\varepsilon_0$ ($\theta = -\pi/4$, $I = 100$ mW/cm$^2$ and $\delta = -\gamma$).

Figure 9. The zones of parameters ($v, \Omega_H$) where magneto-optical force on $^{24}$Mg atoms is repulsive force in $\varepsilon - \theta - \bar{\varepsilon}$ field configuration ($\theta = -\pi/4$) for different parameters ellipticity of light waves $\varepsilon_0$ ($I = 100$ mW/cm$^2$ and $\delta = -\gamma$).
Figure 10 shows the critical values of magnetic field as function of intensity of light waves forming the trap for different detunings and ellipticity of light waves for $^{24}\text{Mg}$ atoms in $\varepsilon - \theta - \xi$ MOT ($\theta = \pi/4$).

**Conclusion**

In this paper we have considered a magneto-optical trap for $^{24}\text{Mg}$ atoms operating on the closed triplet $^3\text{P}_2 \rightarrow ^3\text{D}_3$ transition, formed by the light waves with elliptical polarizations ($\varepsilon - \theta - \xi$ configuration, see figure 2). In the limit of a 1D model we have studied the magneto-optical potential, temperature and fraction of atoms extremely cooled below the momentum $p=3\hbar k$ (12.9 cm/s) as the functions of intensity, frequency detuning and polarizations of light waves that form the MOT. For our simulations we have used recently suggested method [18, 19] that allows taking into account the quantum recoil effects of interaction of atoms with a light field and correctly taking into consideration the slow atoms localized in the optical potential wells as well as the atoms moving above the potential wells.

We have found that the $\varepsilon - \theta - \xi$ light field configuration formed by the waves with elliptical polarizations and orientation angle $\theta = -\pi/4$ with parameters ellipticity are close to linear polarization can offer the lowest cooling temperatures on the level $T \approx 100 \, \mu K$ together with enough depth of the magneto-optical potential. In comparison with the conventional MOT, formed by waves with $\sigma_+ - \sigma_-$ polarizations, the suggested $\varepsilon - \theta - \xi$ MOT should operate with lower gradient of a static magnetic field. Indeed, in comparison with the $\sigma_+ - \sigma_-$ MOT, the proposed $\varepsilon - \theta - \xi$ MOT is more exacting to the field parameters, because of strong magnetic field may reverse magneto-optical trap force for slow
atoms in $\epsilon - \theta - \pi$ configuration that limit the number of atoms captured in the trap. Parameters of the critical magnetic fields have been also figured out.

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