We consider strongly interacting supersymmetric gauge theories which break dynamically the GUT symmetry and produce the light Higgs doublets naturally. Two models we proposed in the previous articles are reanalyzed as two phases of one theory and are shown to have desired features. Furthermore, employing nonabelian duality proposed recently by Seiberg, we study the dual theory of the above one and show that the low-energy physics of the original and dual models are the same as expected. We note that the Higgs multiplets in the original model are regarded as composite states of the elementary hyperquarks in its dual theory. Theories with other hypercolors and similar matter contents are also analyzed in the same way.
1 Introduction

The supersymmetric grand unified theory (SUSY-GUT) \cite{1} is one of the promising candidates for the physics beyond the standard model. In fact, the recent high-precision measurements on the standard-model parameters such as the Weinberg angle agree with some of its predictions \cite{2}. In spite of the remarkable success, there is a fatal fault in the SUSY-GUT: a fine-tuning problem. Since the GUT scale, typically $10^{16}$ GeV, is extremely high compared with the weak scale $\sim 10^2$ GeV, we have to adjust parameters in the GUT accurately in order to have a Higgs doublet in the standard model. Although a number of attempts have been made to solve this serious problem, there was no convincing model to explain the origin of the light Higgs doublet.

In recent papers \cite{3, 4, 5} we have proposed SUSY gauge theories whose interactions are strong at the GUT scale causing dynamical breaking of the GUT symmetry. These models also provide mechanisms which produce the light Higgs doublet naturally.

The main purpose of this paper is to examine the strongly interacting SUSY gauge theories more thoroughly. In addition to the method used to find quantum vacua in Ref.\cite{4, 5}, we employ nonabelian duality which has been proposed recently by Seiberg \cite{6} as a powerful tool to investigate nonabelian gauge theories. Since the nonabelian duality states that $SU(N_c)$ and $SU(N_f - N_c)$ gauge theories with the common $N_f$ flavors have the same low-energy behavior, especially on the vacuum structure, we may reduce the number of theories to study and also check consistency of the results using both theories.

In section 2 we review the results of our two models in Ref.\cite{4, 5} and show that these models are regarded as two phases of one theory. In section 3 we consider the dual theory of the model in section 2 and show that the low-energy physics of the dual model is the same as that of the original one. We, however, stress that short-distance structures of the original and dual models are different from each other and hence these models represent different physics above the GUT scale. We also note that the Higgs multiplets in the original model are composite states of the elementary hyperquarks in the dual theory. In section 4 we extend our analysis to theories with other hypercolors and similar matter contents. Section 5 is devoted to our conclusions. We also comment on some extensions and modifications of the models.
2 The original model

We review the models studied in Ref.[4, 5] in which light Higgs doublets are generated dynamically. We analyze these two models in a unified manner treating them as different phases of a single theory.

The model is based on a supersymmetric hypercolor SU(3)$_H$ gauge theory with six flavors of hyperquark chiral superfields $Q^I_\alpha$ and $\bar{Q}^I_\alpha$ ($\alpha = 1, \cdots, 3; A = 1, \cdots, 6$) in the fundamental representations $\mathbf{3}$ and $\mathbf{3}^*$ of SU(3)$_H$, respectively. The first five $Q^I_\alpha$ and $\bar{Q}^I_\alpha$ ($I = 1, \cdots, 5$) transform as $\mathbf{5}^*$ and $\mathbf{5}$ under the GUT gauge group SU(5)$_{GUT}$, respectively, while the last $Q^6_\alpha$ and $\bar{Q}^6_\alpha$ are singlets of SU(5)$_{GUT}$. We also introduce three kinds of SU(3)$_H$-singlet chiral superfields: a pair of $H_I$ and $\bar{H}_I$, $\Sigma^I J$ and $\Phi$ ($I, J = 1, \cdots, 5$) which are $\mathbf{5}$, $\mathbf{5}^*$, $\mathbf{24} + \mathbf{1}$ of SU(5)$_{GUT}$.

We impose a global U(1)$_A$ symmetry:

$$
\begin{align*}
Q^I_\alpha, \bar{Q}^I_\alpha &\rightarrow Q^I_\alpha, \bar{Q}^I_\alpha, \\
Q^6_\alpha, \bar{Q}^6_\alpha &\rightarrow e^{i\xi}Q^6_\alpha, e^{i\xi}\bar{Q}^6_\alpha, \\
H_I, \bar{H}^I &\rightarrow e^{-i\xi}H_I, e^{-i\xi}\bar{H}^I, \\
\Sigma^I J &\rightarrow \Sigma^I J, \\
\Phi &\rightarrow e^{-2i\xi}\Phi,
\end{align*}
$$

(1)

(1)

to forbid such terms as $\bar{H}^I H_I$ and $Q^6_\alpha \bar{Q}^6_\alpha$ in the superpotential. Then, the superpotential is given by

$$
W = \lambda \Sigma^I J \bar{Q}^I_\alpha Q^J_\alpha + hH_I \bar{Q}^I_\alpha Q^I_\alpha + h'\bar{H}^I \bar{Q}^I_\alpha Q^I_\alpha + f\Phi \bar{Q}^6_\alpha Q^6_\alpha + \frac{1}{2} m_\Sigma Tr(\Sigma^2) + \frac{1}{2} m'_\Sigma (Tr\Sigma)^2 - \mu \Sigma Tr\Sigma.
$$

(2)

Here, we have omitted trilinear self-coupling terms of $\Sigma$ for simplicity since they are irrelevant to the conclusion. The global U(1)$_A$ has a strong SU(3)$_H$ anomaly and hence it is broken by instanton effects at the quantum level. However, as shown in Ref.[4] the broken global U(1)$_A$ even plays a crucial role to protect a pair of massless Higgs doublets from having a mass.

---

1 We have chosen the normalization of the singlet field $Tr\Sigma$ so that the Yukawa term among $\Sigma^I J$, $\bar{Q}^I_\alpha$ and $Q^J_\alpha$ is written with a single coupling constant $\lambda$ as shown in Eq.1. The effect of the rescaling of the field $Tr\Sigma$ appears in the Kähler potential, but it is irrelevant to the present analysis.
Let us first consider a classical vacuum discussed in Ref.[4]:

\[
\langle Q^A \rangle = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
v & 0 & 0 \\
0 & v & 0 \\
0 & 0 & v
\end{pmatrix},
\langle \bar{Q}^\alpha \rangle = \begin{pmatrix}
0 & 0 & v & 0 & 0 \\
0 & 0 & v & 0 \\
0 & 0 & 0 & v & 0
\end{pmatrix},
\]

(3)

\[
\langle \Sigma^I J \rangle = \frac{\mu \Sigma}{m \Sigma + 2m \Sigma} \begin{pmatrix}
1 & 0 \\
1 & 0 \\
0 & 0
\end{pmatrix},
\langle H_I \rangle = \langle \bar{H}^I \rangle = 0,
\]

where

\[
v = \sqrt{\frac{m \Sigma \mu \Sigma}{\lambda (m \Sigma + 2m \Sigma)}}.
\]

(4)

Here, the vacuum-expectation value of \( \Phi \) is undetermined since its potential is flat for \( \langle Q^6 \rangle = \langle \bar{Q}^6 \rangle = 0 \). In this classical vacuum the gauge group is broken down as

\[SU(3)_H \times SU(5)_{GUT} \to SU(3)_C \times SU(2)_L.\]

(5)

There is no unbroken U(1)_Y, and we introduce an extra U(1)_H gauge symmetry in Ref.[3, 4] to have the standard-model gauge group unbroken below the GUT scale \( v \).

Remarkable is that the missing partner mechanism [7] does work very naturally in this classical vacuum [3]. Namely, the color triplets \( H_I \) and \( \bar{H}^I \) (\( I = 3, \cdots, 5 \)) acquire the GUT-scale masses together with \( Q^6_\alpha \) and \( \bar{Q}^6_\alpha \), respectively. On the other hand the SU(2)_L-doublet \( H_I \) and \( \bar{H}^I \) (\( I = 1, 2 \)) remain massless, since they have no partners to form massive chiral superfields with.

We now discuss quantum vacua where vacuum-expectation values of the Higgs fields \( \Sigma^I J \), \( H_I \) and \( \bar{H}^I \) take forms given in Eq.(3).

In these vacua two hyperquarks \( Q^I \alpha \) and \( \bar{Q}^\alpha_I \) (\( I = 1, 2 \)) become massive. The integration of the two massive hyperquarks leads to a low-energy effective theory having the other four massless hyperquarks \( Q^I \alpha \) and \( \bar{Q}^\alpha_I \) (\( I = 3, \cdots, 6 \)). We can then express the effective

\footnote{The GUT unification of three gauge coupling constants in the standard model is realized in the strong coupling limit of U(1)_H [3].}
superpotential with meson $M^i_j$, baryon $B_i$ and antibaryon $\bar{B}^i$ chiral superfields

$$
M^i_j \sim Q_i^\alpha \bar{Q}_j^\alpha,
B_i \sim \epsilon^{\alpha\beta\gamma} \epsilon_{ijkl} Q_j^\alpha Q_k^\beta Q_l^\gamma,
\bar{B}^i \sim \epsilon_{\alpha\beta\gamma} \epsilon^{ijkl} \bar{Q}_j^\alpha \bar{Q}_k^\beta \bar{Q}_l^\gamma,
$$

as follows \[5\]:

$$
W_{\text{eff}} = \Lambda^{-5}(B_i M^i_j \bar{B}^j - \det M^i_j) + \lambda \Sigma^a_b M^b_a
+ h H_a M^a_6 + h' \bar{H}^a M^6_a + f \Phi M^6_6
- \frac{h h'}{m}(\bar{H}^1 H_1 + \bar{H}^2 H_2) M^6_6
+ \frac{1}{2} m \Sigma^a_b \Sigma^b_a + \frac{1}{2} \frac{m \Sigma^a_b}{m \Sigma + 2 m' \Sigma} (\Sigma^a_a)^2 - \frac{m \Sigma \mu \Sigma}{m \Sigma + 2 m' \Sigma} \Sigma_a a,
$$

(7)

where $\Lambda$ denotes a dynamical scale of the low-energy SU(3)$_H$ interactions, $m = \frac{\lambda \Sigma}{m \Sigma + 2 m' \Sigma}$, $a, b = 3, \ldots, 5$ and $i, j, k, l = 3, \ldots, 6$. This superpotential implies a flat direction satisfying $\Lambda^{-5}(B_6 \bar{B}^6 - \det M^6_6) + f \Phi = 0$.

Let us consider, among the vacua of Eq.(7), the two vacua which satisfy $\langle \Phi \rangle = 0$ or $\langle B_6 \rangle = \langle \bar{B}^6 \rangle = 0$.

Vacuum (a): The vacuum with $\langle \Phi \rangle = 0$ is analyzed in Ref.[4]. We see that $B_6$ and $\bar{B}^6$ have non-vanishing vacuum-expectation values leading to breaking of the U(1)$_Y$ subgroup of SU(5)$_{GUT}$. Thus we need to introduce an extra U(1)$_H$ gauge symmetry so as to have the standard-model gauge group unbroken below the GUT scale.

Notice that this quantum vacuum is the same as the classical one, which is consistent with the fact that the classical moduli space is not altered by quantum corrections for the case of $N_f = N_c + 1$ \[8\] where $N_f$ and $N_c$ are the numbers of flavors and colors of the massless hyperquarks, respectively.

Vacuum (b): The vacuum with $\langle B_6 \rangle = \langle \bar{B}^6 \rangle = 0$ is analyzed in Ref.[5]. That is

$$
\langle B_i \rangle = \langle \bar{B}^i \rangle = 0,
\langle M^6_a \rangle = \langle M^a_6 \rangle = \langle M^6_6 \rangle = 0,
\langle M^a_b \rangle = \frac{m \Sigma \mu \Sigma}{\lambda (m \Sigma + 2 m' \Sigma)} \delta^a_b,
\langle \Phi \rangle = \frac{1}{f \Lambda^5} \left[ \frac{m \Sigma \mu \Sigma}{\lambda (m \Sigma + 2 m' \Sigma)} \right]^3,
$$

(8)

where the GUT gauge group is broken down to the standard-model one, namely SU(3)$_C \times$ SU(2)$_L \times$ U(1)$_Y$. Thus, there is no need to introduce an extra U(1)$_H$, differently from \[3\] although the model in Ref.[4] does not contain the singlets $Tr \Sigma$ and $\Phi$, the vacuum considered in Ref.[5] is equivalent to that with $\langle \Phi \rangle = 0$ in the present model.
the previous phase (a). An interesting point is that this quantum vacuum differs from the classical one which satisfies \( B_6 \bar{B}^6 - \det M^a_b = 0 \). This result agrees with the conclusion in Ref. [8] for the case of \( N_f = N_c \). Notice that the effective \( N_f \) is three (= \( N_c \)) in the present phase since the vacuum-expectation value of \( \Phi \) induces a mass for \( Q^a_6 \) and \( \bar{Q}^a_6 \).

As noted in Ref. [5], we have a pair of massless bound states \( B_6 \) and \( \bar{B}^6 \) in this vacuum. Since they have non-vanishing \( U(1)_Y \) charges, they contribute to the renormalization-group equations of three gauge coupling constants in the standard model. A change of running of couplings threatens to destroy the GUT unification of gauge coupling constants which is regarded as one of the motivations for considering the SUSY-GUT as a unified theory.

However, it seems quite reasonable to assume that there are nonrenormalizable operators in the superpotential suppressed by some scale \( M_0 \) higher than the GUT scale (originating from gravitational interactions, for example). Among such operators we consider the lowest-dimensional nonrenormalizable operator consistent with our gauge and global symmetries which is to contain baryon superfields. That is

\[
\delta W = \frac{f'}{M_0^3} \epsilon^{\alpha\beta\gamma} \epsilon_{\alpha'\beta'\gamma'} (Q^I_\alpha Q^J_\beta Q^K_\gamma) (\bar{Q}^I_{\alpha'} \bar{Q}^J_{\beta'} \bar{Q}^K_{\gamma'}). \tag{9}
\]

This interaction generates a mass term for \( B_6 \) and \( \bar{B}^6 \) in the effective superpotential as

\[
\delta W_{\text{eff}} = \frac{f'}{M_0^3} B_6 \bar{B}^6, \tag{10}
\]

which corresponds to the physical mass for \( B_6 \) and \( \bar{B}^6 \)

\[
m_{B_6} \simeq \frac{f' \Lambda^4}{M_0^3}. \tag{11}
\]

If one takes \( M_0 \) in Eq.(11) at the gravitational scale, i.e. \( M_0 \simeq 2 \times 10^{18} \) GeV, and \( \Lambda \simeq 3 \times 10^{16} \) GeV, for example, one has the mass for \( B_6 \) and \( \bar{B}^6 \sim 10^{11} \) GeV for \( f' \sim \mathcal{O}(1) \).

This mass is too small compared with the GUT scale and the presence of \( B_6 \) and \( \bar{B}^6 \) destroys the GUT unification of gauge coupling constants. However, since \( M_0 \) is given at the gravitational scale, it evolves as the change of scale by renormalization effects. Provided that the renormalized \( M_0^R \) becomes about \( 10^{17} \) GeV at the GUT scale, we

---

4 Above the GUT scale the number of effectively massless flavors is six and the present model lies in the conformal window [6]. Therefore, the model has a quasi-infrared fixed point. As pointed out in Ref. [5] the renormalization factor \( Z_Q \) for the wave functions of quarks \( Q \) and \( \bar{Q} \) goes to vanish in the long distance. This suggests the renormalized mass \( M_0^R = Z_Q^2 M_0 \) becomes smaller as the renormalization point is lowered. We also suspect that this kind of renormalization effects may be an origin of the GUT scale itself.
obtain $m_{B_6} \sim 10^{15}$ GeV for $\Lambda \simeq 3 \times 10^{16}$ GeV and $f' \sim \mathcal{O}(1)$. This result turns out to be consistent with the recent experimental data on the three gauge coupling constants $[3]$.

In both the phases (a) and (b), the colored Higgs $H_a$ and $\bar{H}^a$ acquire masses of the GUT scale with the composite states $M_6^a$ and $M^a_6$ from the interactions in Eq. (7), but the Higgs doublets $H_I$ and $\bar{H}^I$ ($I = 1, 2$) remain massless because of $\langle M_6^a \rangle = 0$.

It is remarkable that the operator in Eq. (9) changes the quantum moduli space. Actually, the vacuum (a) is no longer in the quantum moduli space, while the vacuum (b) still remains there. Since it is quite natural to consider that the operators suppressed by the gravitational scale exist, there is a doubt as to the presence of the vacuum (a).

3 The dual model

Nonabelian duality proposed in Ref. [4] enables us to study supersymmetric nonabelian gauge theories with different color groups. The duality means that an SU($N_c$) gauge theory with $N_f$ flavors of quarks and an SU($N_f - N_c$) gauge theory with the same number of quarks have the same behavior in the infrared limit, especially the same moduli space of vacua. Thus, one may investigate the dual theory instead of our original model, which may even clarify the structure of the original model. Furthermore, there is a possibility that we can throw a new light on the origin of the fields in the original model, such as $H_I$ and $\bar{H}^I$, as composite fields of the elementary hyperquarks in the dual theory.

Now let us consider the nonabelian dual of the model in section 4, which is expected to coincide with the original model as a low-energy effective theory.

The dual gauge group turns out to be SU(3), and thus we denote it as SU(3)$_\tilde{H}$ in distinction with the original gauge group SU(3)$_H$. The dual model is described by a supersymmetric SU(3)$_\tilde{H}$ gauge theory with singlet chiral superfields $\sigma^A B$ and six flavors of dual hyperquarks $\bar{q}^A$ and $q_A^A$ in the fundamental representations $3^*$ and $3$, respectively. In addition we have to introduce Yukawa couplings between hyperquarks and $\sigma^A B$ to obtain a correct global symmetry and correct vacua.

Then the dual superpotential to Eq. (2) is given by

$$\tilde{W} = \tilde{\lambda} \bar{q}_A^A \sigma^A B q^B + \lambda \rho \Sigma^I J \sigma^I J + h \rho H \sigma^I J + h' \rho \bar{H} \sigma^I J + f \rho \Phi \sigma^6_6$$

$$+ \frac{1}{2} m_{\Sigma} Tr (\Sigma^2) + \frac{1}{2} m_{\Sigma} Tr (T r \Sigma)^2 - \mu \Sigma Tr \Sigma,$$

$$I, J = 1, \cdots, 5),$$

(12)
where $\rho$ denotes the duality scale to match the operator dimensions in a correspondence

$$\rho \sigma^A B \sim Q^A_\alpha \bar{Q}^\alpha_B, \quad (A, B = 1, \cdots 6).$$  

(13)

To obtain Eq.(12) the hyperquarks $Q^A_\alpha$ and $\bar{Q}^\alpha_A$ in Eq.(2) are substituted with $\sigma^A B$ by means of the relation Eq.(13).

The fields in Eq.(12) transform under a global $U(1)_A$ symmetry as

$$
\begin{align*}
\bar{q}^\alpha_I, q^\alpha_I & \rightarrow \bar{q}^\alpha_I, q^\alpha_I, \\
\bar{q}^\alpha_6, q^\alpha_6 & \rightarrow e^{-i\eta} \bar{q}^\alpha_6, e^{-i\eta} q^\alpha_6, \\
\sigma^I J & \rightarrow \sigma^I J, \\
\sigma^I_6, \sigma^6 I & \rightarrow e^{i\eta} \sigma^I_6, e^{i\eta} \sigma^6 I \\
\sigma^6_6 & \rightarrow e^{2i\eta} \sigma^6_6,
\end{align*}
$$

(14)

with the transformation law in Eq.(1).

Since the dual superpotential in Eq.(12) looks complicated, we reduce it by integrating out the superfields $H_I, \bar{H}_I, \Sigma, \Phi, \sigma^I_6, \sigma^6 I$ and $\sigma^6_6$ to

$$
\tilde{W}' = \tilde{\lambda} \bar{q}^\alpha_I \sigma^I J q^\alpha_J + \frac{1}{2} m_\sigma Tr(\sigma^2) + \frac{1}{2} m'_\sigma (Tr \sigma)^2 - \mu_\sigma Tr \sigma,
$$

(15)

where

$$
m_\sigma = -\frac{(\lambda \rho)^2}{m_\Sigma}, \quad m'_\sigma = \frac{(\lambda \rho)^2 m'_\Sigma}{(m_\Sigma + 5m'_\Sigma) m_\Sigma}, \quad \mu_\sigma = -\lambda \rho \mu_\Sigma m_\Sigma + 5m'_\Sigma m_\Sigma.
$$

Notice that the sixth hyperquarks $\bar{q}_6^\alpha$ and $q_6^\alpha$ are massless. If we integrate out $\sigma^I J$, Eq.(13) becomes a superpotential including only $\bar{q}^\alpha_I$ and $q^\alpha_I$ with nonrenormalizable interactions $1/m_\sigma (\bar{q}^\alpha_I q^\alpha_I)^2$ and $1/m'_\sigma (\bar{q}^\alpha_I q^\alpha_I)^2$. From this viewpoint, the superfields $\sigma^I J$ are interpreted as composite states of $\bar{q}^\alpha_I$ and $q^\alpha_I$.

Let us study vacua corresponding to the original vacua given in the previous section which satisfy

$$
\langle \sigma^I J \rangle = \frac{\mu_\sigma}{m_\sigma + 3m'_\sigma} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.
$$

(17)

The integration of massive hyperquarks $\bar{q}^\alpha_I$ and $q^\alpha_I$ $(I = 1, 2, 3)$ leads to a low-energy effective theory. Since this effective theory is an $SU(3)$ gauge theory with three massless hyperquarks $\bar{q}^\alpha_A$ and $q^A_\alpha$ $(A = 4, 5, 6)$, its effective superpotential is obtained by means of

8
knowledge for the $N_f = N_c$ case in Ref. [8]. The effective superpotential is described by

\begin{align}
M_{ij} &\sim q^{i}_\alpha \bar{q}^{\alpha}_j, \\
B &\sim \epsilon^{\alpha\beta\gamma} q^{4}_\alpha q^{5}_\beta q^{6}_\gamma, \\
\bar{B} &\sim \epsilon^{\alpha\beta\gamma} \bar{q}^{\alpha}_4 \bar{q}^{\beta}_5 \bar{q}^{\gamma}_6,
\end{align}

as follows:

\begin{align}
\tilde{W}_{\text{eff}} &= X (B\bar{B} - \det M_{ij} - \tilde{\Lambda}^6) + \tilde{\lambda} \sigma_{ab} M^{ab} \\
&+ \frac{1}{2} \frac{m_{\sigma}}{m_{\sigma} + 3m'_{\sigma}} (\sigma_{a}^{a})^2 - \frac{m_{\mu\sigma}}{m_{\sigma} + 3m'_{\sigma}} \sigma_{a}^{a},
\end{align}

where $\tilde{\Lambda}$ denotes a dynamical scale of the low-energy SU(3)$_H$ interactions, $a, b = 4, 5$, and $i, j = 4, 5, 6$. This superpotential implies a flat direction satisfying $B\bar{B} - \det M_{ij} - \tilde{\Lambda}^6 = 0$.

We consider two vacua, $\langle M_{i6}^6 \rangle = 0$ and $\langle B \rangle = \langle \bar{B} \rangle = 0$, which have a pair of massless Higgs doublets. Here these Higgs doublets are all composite bound states of the dual hyperquarks $M_{6i} \sim q^{6}_\alpha \bar{q}^{\alpha}_i$ and $M_{i6} \sim q^{i}_\alpha \bar{q}^{\alpha}_6$ ($i = 4, 5$).

Vacuum (a'): The vacuum with $\langle M_{i6}^6 \rangle = 0$ corresponds to the vacuum (a) in the previous section, where the fields have the following vacuum-expectation values:

\begin{align}
\langle B \rangle &= \langle \bar{B} \rangle = \tilde{\Lambda}^3, \\
\langle M^a_{ab} \rangle &= - \frac{m_{\sigma \mu \sigma}}{\tilde{\Lambda} (m_{\sigma} + 3m'_{\sigma})} \delta^{a}_{b}, \\
\langle M_{i6}^{6} \rangle &= \langle M_{a6}^{a} \rangle = 0, \langle \sigma_{ab}^a \rangle = 0.
\end{align}

In this vacuum the SU(5)$_{\text{GUT}}$ breaks down to SU(3)$_C \times$ SU(2)$_L$ since the baryon $B$ and the antibaryon $\bar{B}$ have non-vanishing U(1)$_Y$ charges. Therefore, we need an extra U(1)$_H$ as in the vacuum (a). Interesting enough, although the point $\langle B \rangle = \langle \bar{B} \rangle = 0$ and $\det M_{ij} = 0$ with unbroken U(1)$_Y$ is in the classical moduli space, this point disappears from the moduli space by non-perturbative effects at the quantum level. This phenomenon contrasts with the vacuum (b) in section 2 in which the U(1)$_Y$ breaks down classically but is restored quantum mechanically.

Although there is no problem phenomenologically if the U(1)$_H$ is strong enough at the GUT scale, the U(1)$_H$ brings some theoretical problems. First of all, the U(1)$_H$ is not asymptotically free and its gauge coupling constant blows up at some higher scale. Secondly, the charge quantization is left unexplained.

\footnote{For $m_{\sigma} \sim 10^{18}$ GeV, the mass of the SU(2)$_L$ triplet in $M_{ab}^a$ is of the order of $\tilde{\Lambda}^2/m_{\sigma}$ which may be smaller than the GUT scale. In this case, the unification of the three gauge coupling constants is realized within the experimental errors, even if the gauge coupling $g_H$ of U(1)$_H$ is not so large. Thus, it is possible that the coupling $g_H$ does not diverge below the Planck scale.}
Vacuum (b’): The vacuum with $\langle B \rangle = \langle \bar{B} \rangle = 0$ corresponds to the vacuum (b) in the original model. We find the vacuum:

$$\langle M^6_6 \rangle = -\frac{\tilde{\lambda}^2 (m_\sigma + 3m'_\sigma)^2}{m_\sigma^2 \mu_\sigma^2},$$

and the other fields acquire the same expectation values as in the vacuum (a’) which breaks $SU(5)_{GUT}$ down to $SU(3)_C \times SU(2)_L \times U(1)_Y$. Therefore, we do not need to introduce an extra $U(1)_H$. As in (a’) this quantum vacuum is different from the classical one which satisfies $\det M^i_j = 0$.

As in the vacuum (b), there is a pair of massless baryon $B$ and antibaryon $\bar{B}$ with non-vanishing $U(1)_Y$ charges. Unlike in the original model, however, nonrenormalizable operators generating the mass for $B$ and $\bar{B}$ are forbidden by the global $U(1)_A$ symmetry in Eq.(14). The effective superpotential in Eq.(19) says that nonperturbative effects never generate the mass term of the baryon $B$ and the antibaryon $\bar{B}$ although the $U(1)_A$ is broken by instanton effects. Thus, there remains a pair of the baryons in the massless spectrum, which renders the vacuum (b’) unrealistic. It also suggests that the short-distance behaviors of the original and dual models are different. On the other hand, any nonrenormalizable operators with the $U(1)_A$ symmetry do not affect the stability of the vacuum (a’). Indeed, it is rather guaranteed by an introduction of the term $\tilde{\Phi} M^6_6$ which leads to $\langle M^6_6 \rangle = 0$.

We note an interesting relation between the original and the dual models. In the original model the Higgs multiplets are regarded as elementary fields. Whereas in the dual model the Higgs doublets (even including $\sigma^{I,J}$) are composite states of the dual hyperquarks $\bar{q}_A^\alpha$ and $q_A^\alpha$. Moreover, all the Higgs multiplets, $H_I$, $\bar{H}^I$, $\Sigma^{I,J}$ and $\Phi$, in the original model might be regarded as composite states of the elementary hyperquarks in the dual model.

To summarize, the dual model reproduces exactly the same low-energy physics as the original model does. This supports the correctness of the duality arguments. We proceed to use this nonabelian duality as a powerful tool to investigate supersymmetric gauge theories with other hypercolors in the next section.

---

If one adds a nonrenormalizable operator $\tilde{f}' \frac{M^6_6}{m_\sigma} q^4 q^5 q^6 q^4 q^5 q^6$, one reproduces the same low-energy physics as in the previous vacuum (b). However, short-distance physics are different from each other, since in the original model the global $U(1)_A$ is unbroken at the classical level whereas there is no such a symmetry in its dual model.
4 Other hypercolors

In this section we consider supersymmetric SU($N_c)_H$ hypercolor gauge theories other than the SU(3)$_H$. We continue to restrict ourselves to the minimal case of six flavors of hyperquarks. [4]

For the theories of SU($N_c)_H$ ($N_c \geq 5$) we find that there is no appropriate vacuum which breaks SU(5)$_{GUT}$ down to standard-model gauge group by a similar argument to that in Ref.[4, 5]. Since the case of $N_c = 3$ is already analyzed in the previous section, $N_c = 2$ and 4 remain as possible gauge groups.

(i) First we investigate an SU(4)$_H$ gauge theory.

Since this model is the same as the one in section 2 except that the index $\alpha$ runs from 1 to 4, the superpotential is written as Eq.(2). We consider a classical vacuum given by

$$\langle Q^4_{\alpha} \rangle = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
v & 0 & 0 & 0 \\
0 & v & 0 & 0 \\
0 & 0 & v & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}, \quad \langle \bar{Q}^4_{\alpha} \rangle = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & v & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & v & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & v & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & v & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & v & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & v & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & v & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & v \\
\end{pmatrix},$$

(22)

$$\langle \Sigma^I_J \rangle = \frac{\mu_{\Sigma}}{m_{\Sigma} + 2m_{\Sigma}'} \begin{pmatrix}
1 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\end{pmatrix},$$

$$\langle H^I \rangle = \langle \bar{H}^I \rangle = 0,$$

where $v = \sqrt{\frac{m_{\Sigma}\mu_{\Sigma}}{\lambda(m_{\Sigma} + 2m_{\Sigma}')}}$ and $\Phi$ remains undetermined. In this classical vacuum the gauge group is broken down as desired, but this vacuum does not survive quantum corrections as we will see below.

Since two hyperquarks $Q^I_{\alpha}$ and $\bar{Q}^I_{\alpha}$ ($I = 1, 2$) become massive, we can integrate them to obtain a low-energy effective theory with $N_f = 4$. The effective superpotential is described

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7We may consider more than six flavors. In fact, Ref.[8] deals with the case of seven flavors. Then the Peccei-Quinn symmetry is naturally accommodated, though the low-energy spectrum is rather involved with two pairs of light Higgs doublets.
by gauge invariant operators

\[ M_{ij} \sim Q_i^\alpha \bar{Q}_j^\alpha, \]
\[ B \sim \epsilon^{\alpha\beta\gamma\delta} Q_3^\alpha Q_4^\beta Q_5^\gamma Q_6^\delta, \]
\[ \bar{B} \sim \epsilon_{\alpha\beta\gamma\delta} \bar{Q}_3^\alpha \bar{Q}_4^\beta \bar{Q}_5^\gamma \bar{Q}_6^\delta, \]

as follows:

\[ W_{\text{eff}} = X (B \bar{B} - \det M_{ij} - \Lambda^6) + \lambda \Sigma^a \Sigma^b M_{ab}^6 + \frac{hh'}{m} (\bar{H}^1 H_1 + \bar{H}^2 H_2) M_6^0 \]
\[ + \frac{1}{2} m_{\Sigma} \Sigma^a \Sigma^b + \frac{1}{2} \frac{m_{\Sigma} m_{\Sigma'}}{m_{\Sigma} + 2m_{\Sigma'}} (\Sigma^a_{\alpha})^2 - \frac{m_{\Sigma} m_{\Sigma}}{m_{\Sigma} + 2m_{\Sigma'}} \Sigma^a_{\alpha}, \]

where \( \Lambda \) denotes a dynamical scale of the low-energy SU(4) \( _H \) interactions, \( m = \frac{\lambda m_{\Sigma}}{m_{\Sigma} + 2m_{\Sigma'}}, \)

\( a, b = 3, 4, 5 \) and \( i, j = 3, \ldots, 6. \)

From this effective superpotential in Eq.(24) we find a quantum vacuum given by

\[ \langle X \rangle = 0, \]
\[ \langle B \rangle = \langle \bar{B} \rangle = \Lambda^3, \]
\[ \langle M^a_{ab} \rangle = -\frac{m_{\Sigma} m_{\Sigma'}}{\lambda (m_{\Sigma} + 2m_{\Sigma'})} \delta^a_{\alpha}, \]
\[ \langle M^0_{ab} \rangle = \langle M^0_{ab} \rangle = \langle \Sigma^a_{\alpha} \rangle = 0, \]
\[ \langle H_a \rangle = \langle \bar{H}^a \rangle = \langle \Phi \rangle = 0. \]

Since \( \langle M^0_6 \rangle \) is vanishing, we obtain exactly massless Higgs doublets. On the other hand, the baryon \( B \) and the antibaryon \( \bar{B} \) get non-vanishing vacuum-expectation values and hence the U(1)\( _Y \) subgroup of SU(5)\( _{\text{GUT}} \) is broken down in this quantum vacuum in contrast to the classical vacuum.

If one removes the term \( f \Phi M^0_6 \) from Eq.(24), there is a vacuum with \( \langle M^0_6 \rangle \neq 0 \) and \( \langle B \rangle = \langle \bar{B} \rangle = 0 \), that is, U(1)\( _Y \) is unbroken. However, the Higgs doublets \( H_I \) and \( \bar{H}^I \) (\( I = 1, 2 \)) acquire masses from the interaction with \( M^0_6 \) in this vacuum, and hence there is no light Higgs doublet.

(ii) Next we consider an SU(2)\( _H \) gauge theory.

From the nonabelian duality, the SU(2)\( _H \) gauge theory with six flavors of hyperquarks is expected to be dual to the SU(4)\( _H \) gauge theory.

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\(^8\)If \( \Phi \) were non-vanishing, the sixth hyperquarks \( Q^6_\alpha \) and \( \bar{Q}^6_\alpha \) would become massive. Then the vacuum becomes unstable quantum mechanically because the number of effective massless hyperquarks is three which is less than \( N_c = 4 \) \(^1\). Therefore, \( \Phi \) is fixed at the origin (\( \langle \Phi \rangle = 0 \)) in the stable vacuum.
The superpotential in this theory is given by Eq. (15) with $\alpha = 1, 2$. The corresponding dual vacua satisfy

$$\langle \sigma^I J \rangle = \frac{\mu_\sigma}{m_\sigma + 3m'_\sigma} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$  

(26)

In these vacua three hyperquarks become massive and three remain massless. Integrating out the three massive hyperquarks $Q^I_\alpha$ and $\bar{Q}^\alpha_I$ ($I = 1, 2, 3$), we obtain the effective superpotential written in terms of meson $M^i_j$, baryon $B^i$ and antibaryon $\bar{B}^i$ as

$$\tilde{W}_{eff} = \tilde{\Lambda}^{-5}(B^i M^j_i \bar{B}^j - \text{det} M^i_j) + \tilde{\lambda}\sigma^a_b M^a_b$$

$$+ \frac{1}{2} m_\sigma \sigma^b_a \sigma^b_a + \frac{1}{2} m_\sigma m'_\sigma (\sigma^a_a)^2 - \frac{m_\sigma m'_\sigma}{m_\sigma + 3m'_\sigma} \sigma^a_a,$$  

(27)

where $\tilde{\Lambda}$ denotes a dynamical scale of the low-energy SU(2)$_H$ interactions, $a, b = 4, 5$, and $i, j = 4, 5, 6$. By the same analysis as in section 2, we find the following quantum vacuum:

$$\langle B^i \rangle = \langle \bar{B}^i \rangle = (0 0 v^2),$$

$$\langle M^i_j \rangle = v^2 \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix},$$

$$\langle \sigma^a_b \rangle = 0,$$  

(28)

where $v^2 = \frac{m_\sigma m'_\sigma}{\tilde{\Lambda}(m_\sigma + 3m'_\sigma)}$.

Thus, we have a pair of massless Higgs doublets but the U(1)$_Y$ subgroup of SU(5)$_{GUT}$ is broken by the non-zero expectation values of $B^6$ and $\bar{B}^6$. This low-energy behavior of the theory is precisely the same as that in the SU(4)$_H$ gauge theory as expected.

As for the vacuum stability, the perturbation such as $B^6 \bar{B}^6 \sim (q^4 q^5) (\bar{q}_4 \bar{q}_5)$ extinguishes the above vacuum. Accordingly the SU(2)$_H$ model seems unrealistic. On the other hand the corresponding vacuum in the SU(4)$_H$ model is stable due to the global U(1)$_A$ symmetry.

5 Conclusion

In this paper we have investigated supersymmetric SU($N_c)_H$ hypercolor gauge theories with six flavors of hyperquarks. There are two types of desirable vacua. One type requires no

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Their definitions are similar to those in Eq. (15) in the original SU(3)$_H$ model though there are no intrinsic difference between mesons and baryons in the SU(2)$_H$ model.
extra $U(1)_H$ to have the standard-model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ below the GUT scale. While the other needs an extra $U(1)_H$ gauge symmetry. We have found that the $SU(3)_H$ gauge theory is a unique model for the first type. As for the second type of the vacua, $SU(3)_H$, $SU(4)_H$ and $SU(2)_H$ are possible gauge groups. We have also shown that some pairs of models are dual to each other and thus they have the same low-energy physics. It is intriguing that the Higgs superfields in one model are regarded as the composite states of the elementary hyperquarks in another model.

Even though each type is phenomenologically consistent, it has its own shortcoming. The first one needs nonrenormalizable interactions to remove unwanted singlet baryons from the massless spectrum. To derive the precise form of these interactions, it is necessary to understand the physics at the Planck (gravitational) scale, yet still unknown. The second one requires an extra $U(1)_H$ gauge symmetry which seems to have theoretical difficulties explained in the text.

One way to avoid the problem arising from the introduction of $U(1)_H$ is to embed $U(1)_H$ and $SU(3)_H$ into a larger simple group such as $SU(4)_H$. Here we show briefly an $SU(4)_H$ extension of the present $SU(3)_H \times U(1)_H$ model.

The model contains five flavors of hyperquark chiral superfields $Q^I_{[\alpha\beta]}$ and $Q^{[\alpha\beta]}_I$ ($\alpha, \beta = 1, \cdots, 4; I = 1, \cdots, 5$) in the antisymmetric second rank tensor representation $6$ of $SU(4)_H$ which transform as $5^*$ and $5$ under the GUT gauge group $SU(5)_{GUT}$, respectively. We also introduce $SU(5)_{GUT}$ singlet chiral superfields $Q^6_{[\alpha\beta]}$ and $X^\alpha\beta$ which transform as $6$ and an adjoint $15$ of $SU(4)_H$, respectively. We assume that the field $X^\alpha\beta$ has a vacuum-expectation value

$$\langle X^\alpha\beta \rangle \sim \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix} V_X$$

(29)

at some scale $V_X$ above the GUT scale. Then the hypercolor $SU(4)_H$ is broken down to $SU(3)_H \times U(1)_H$ and the model includes the superfields in the original model in section 2. Thus, we may have the consistent $SU(3)_H \times U(1)_H$ model discussed in this paper. However, in addition to the GUT scale, we have to introduce a new scale $V_X$. An intriguing possibility is to identify the $V_X$ with the Planck (gravitational) scale. This could be done if the $SU(3)_H$ gauge coupling constant is closed to the infrared-stable fixed point. However, it is beyond the scope of this paper to examine if it is indeed the case.

We have always assumed the adjoint Higgs superfields $\Sigma^I_{\alpha\beta}$ of $SU(5)_{GUT}$ in this paper. One of the purposes to introduce such fields is to eliminate unwanted Nambu-Goldstone
multiplets \[3\]. We now comment that the Higgs fields \(X^{\alpha\beta}\) in the adjoint representation of the hypercolor gauge group \(SU(N_c)_{H}\) may play the same role as \(\Sigma^I_J\) \[12\]. A remarkable feature in this model is that if one imposes \(N = 2\) extended supersymmetry, one has an \(SU(6)\) global symmetry. The spontaneous breakdown of the global \(SU(6)\) naturally produces a pair of massless Higgs doublets as Nambu-Goldstone multiplets \[13\].

So far we have discussed only the \(SU(5)\) gauge group as the GUT group. We note, here, that our approach can also be applied to the \(SO(10)\) GUT. For example, we propose an \(SO(10)_{GUT} \times SO(6)_H\) gauge theory with eleven hyperquarks \(Q^I_{\alpha}\) (\(I = 1, \cdots, 11\)) in the vector \(6\) representation of the \(SO(6)_H\). The first ten \(Q^I_{\alpha}\) (\(I = 1, \cdots, 10\)) transform as \(10\) under the \(SO(10)_{GUT}\) and the last hyperquark \(Q^1_{\alpha}\) is a singlet of the \(SO(10)_{GUT}\). Instead of an adjoint Higgs \(\Sigma^I_J\) of \(SO(10)_{GUT}\), we introduce a Higgs field \(S^I_J\) in the symmetric second rank tensor \(54\) and \(S\) in the singlet \(1\) of the \(SO(10)_{GUT}\). It is very interesting that there is a vacuum in which the \(SO(10)_{GUT}\) breaks down to the Pati-Salam gauge group \(SU(4) \times SU(2)_L \times SU(2)_R\) without having any unwanted massless states except for the Higgs doublets transforming as \((2, 2)\) under the \(SU(2)_L \times SU(2)_R\). The details of this model will be given in a forthcoming paper \[14\].

Finally, we should note that future experiments of the proton decay might bring important informations to judge or distinguish the models discussed in this paper. The models without an extra \(U(1)_H\) have the dangerous dimension-five operators \[15\] for nucleon decays and hence we have unsuppressed proton decays \[3\]. On the other hand, the models with an extra \(U(1)_H\) have no such operators and the proton decays are suppressed as explained in Ref.\[3, 4\].
References

[1] E. Witten, Nucl. Phys. B188, 513 (1981);
S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D24, 1681 (1981);
S. Dimopoulos and H. Georgi, Nucl. Phys. B193, 150 (1981);
N. Sakai, Z. Phys. C11, 153 (1981).

[2] P. Langacker and M.-X. Luo, Phys. Rev. D44, 817 (1991);
U. Amaldi, W. de Boer and H. Fürstenau, Phys. Lett. B260, 447 (1991);
J. Ellis, S. Kelley and D.V. Nanopoulos, Phys. Lett. B260, 131 (1991);
W.J. Marciano, Brookhaven preprint BNL-45999 (April 1991).

[3] T. Yanagida, Phys. Lett. B344, 211 (1995).

[4] T. Hotta, Izawa K.-I. and T. Yanagida, hep-ph/9509201.

[5] T. Hotta, Izawa K.-I. and T. Yanagida, hep-ph/9511431.

[6] N. Seiberg, Nucl. Phys. B435, 129 (1995).

[7] A. Masiero, D.V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. B115, 380 (1982);
B. Grinstein, Nucl. Phys. B206, 387 (1982).

[8] N. Seiberg, Phys. Rev. D49, 6857 (1994).

[9] I.I. Kogan, M. Shifman and A. Vainshtein, hep-th/9507170.

[10] K. Intriligator and N. Seiberg, hep-th/9509066.

[11] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B241, 493 (1984).

[12] J. Hisano and T. Yanagida, hep-ph/9510277.

[13] W. Buchmüller, R. Peccei and T. Yanagida, Nucl. Phys. B227, 503 (1983);
K. Inoue, A. Kakuto and H. Takano, Prog. Theor. Phys. 75, 664 (1986);
R. Barbieri, G. Dvali and M. Moretti, Phys. Lett. B312, 137 (1993).

[14] T. Hotta, Izawa K.-I. and T. Yanagida, in preparation.

[15] N. Sakai and T. Yanagida, Nucl. Phys. B197, 533 (1982);
S. Weinberg, Phys. Rev. D26, 287 (1982).