About agreement of PYTHIA and the experimental results in $e^+e^-$ annihilation to hadrons

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Abstract

The experimental charged particles multiplicity distributions in $e^+e^-$ annihilation to hadrons are compared with the distributions obtained by PYTHIA. The ratio $\chi^2$/degrees of freedom is calculated for 6 energies at $\sqrt{s}$ 14 – 206.2 GeV. The necessity of more subtle tuning of PYTHIA at the energy of $Z^0$ peak is discussed.

1 Introduction

In the last decades both experimentalists and theorists have applied significant forces to understand character of hadron interactions at high energies. As result many aspects of hadron processes at high energies became clear not only qualitatively but also quantitatively. The last corresponds to quantum chromodynamics which was able to describe quantitatively many the simplest processes occurring at short distances. However, it is impossible to count on QCD calculations of the multiple hadron production, these processes have enormous number of degrees of freedom. Thus one has to use different model approaches. A quantitative description of high energy processes by Monte Carlo generators can be considered as one of these approaches. Monte Carlo generators are perspective method of theoretic study at all energies of the existing accelerators including LHC, since any other approaches, theoretical as the BFKL theory or phenomenological as the Regge theory, are only valid to the values of $1/\ln \sqrt{s}$ order of magnitude, where $s$ is the square of total center-of-mass energy. Since even for LHC $\ln \sqrt{s} \simeq 19$, then it is necessarily to take into account the next to $1/\ln \sqrt{s}$ corrections what is an inexecutable task at the most cases. Therefore it is essentially to construct such MC generator which will describe well processes both in hard and soft regions.

Amongst generators describing well hard processes there is known PYTHIA [1]. Still soft processes, i.e. processes with small transverse momenta, are included in PYTHIA only on basis of the simplest phenomenological models, the values of cross sections are not generated. The multiparticle processes are described with the simplest pomeron model in which only one string is stretched between quark and diquark. Pomeron cuts are not taken into account. The weakness of PYTHIA is also in absence of the

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AGK sum rules [2]. An oscillation structure in the multiplicity distribution function $P_n = \sigma_n / \sigma_{tot}$, where $\sigma_n$ is the topological cross section of $n$ hadrons production, $\sigma_{tot}$ is the total cross section of hadron-hadron scattering, that can be observed at the LHC energies, follows from the AGK rules. Since PYTHIA is the most advanced program then the future MC generator describing both hard and soft regions, in our opinion, should be based on PYTHIA.

In the present work we will study how multiplicity distributions obtained with PYTHIA agree with experimental data.

2 The charged particles multiplicity distribution in $e^+e^-$ annihilation to hadrons and PYTHIA predictions

We consider the process of $e^+e^-$ annihilation at 6 values of total center-of-mass energy from 14 to 206.2 GeV. Wide coverage of the experimental data and sufficient statistical provision of them were criteria for the choice.

Electron-positron annihilation to hadrons was simulated by PYTHIA at $\sqrt{s}$ 14, 29, 34.8, 91.2, 188.6, 206.2 GeV. The decay process of $\gamma^* (Z^0)$ was considered without the initial state radiation, values of the other parameters were set by default. There were generated one million events for every energy, and the charged particles multiplicity distributions were analyzed.
2.1 $\sqrt{s} = 14$ GeV

Experimentally there were obtained 2704 events (TASSO). The experimental values of $P_n$ and the corresponding probabilities obtained by PYTHIA are shown in Fig.1. Both statistical and systematical errors are included. The $\chi^2$ is presented in Table 1.

![Figure 1](image_url)

**Table 1. $\sqrt{s} = 14$ GeV**

| $n$   | number of events, experiment | number of events, PYTHIA | $\chi^2$ in bin |
|-------|------------------------------|--------------------------|-----------------|
| 0-2   | 15.64                        | 12.20                    | 0.33            |
| 4     | 145.98                       | 116.08                   | 3.43            |
| 6     | 445.47                       | 414.55                   | 1.20            |
| 8     | 720.66                       | 720.78                   | 0.00            |
| 10    | 677.63                       | 707.34                   | 0.71            |
| 12    | 405.05                       | 443.66                   | 2.06            |
| 14    | 181.75                       | 194.74                   | 0.55            |
| 16    | 70.79                        | 66.74                    | 0.15            |
| 18    | 23.53                        | 20.51                    | 0.28            |
| 20-30 | 17.51                        | 7.40                     | 2.64            |

**total $\chi^2$/degrees of freedom = 11/9**
2.2 $\sqrt{s} = 29$ GeV

Experimentally there were obtained 29649 events (HRS) \(^4\). The experimental values of $P_n$ and the corresponding probabilities obtained by PYTHIA are shown in Fig.2. Both statistical and systematical errors are included. The $\chi^2$ is presented in Table 2.

![Fig.2.](image)

| $n$ | number of events, experiment | number of events, PYTHIA | $\chi^2$ in bin |
|-----|-----------------------------|--------------------------|-----------------|
| 0 – 2 | 14.82 | 12.87 | 0.05 |
| 4 | 186.79 | 169.59 | 0.08 |
| 6 | 1055.50 | 952.57 | 1.98 |
| 8 | 2795.90 | 2811.24 | 0.01 |
| 10 | 5200.43 | 5122.86 | 0.10 |
| 12 | 6854.85 | 6313.39 | 2.91 |
| 14 | 5760.80 | 5724.38 | 0.02 |
| 16 | 3949.25 | 4057.80 | 0.32 |
| 18 | 2188.10 | 2397.37 | 2.73 |
| 20 | 1064.40 | 1212.53 | 2.82 |
| 22 | 367.65 | 545.37 | 11.70 |
| 24 | 148.25 | 213.92 | 4.70 |
| 26 | 50.40 | 79.13 | 2.76 |
| 28 – 36 | 11.86 | 35.99 | 7.29 |

Total $\chi^2$/degrees of freedom = 37/13
2.3 $\sqrt{s} = 34.8$ GeV

Experimentally there were obtained 52832 events (TASSO) [3]. The experimental values of $P_n$ and the corresponding probabilities obtained by PYTHIA are shown in Fig.3. Both statistical and systematical errors are included. The $\chi^2$ is presented in Table 3.

![Graph showing data points for $P_n$ vs $n$ for $\sqrt{s} = 34.8$ GeV, with lines for both experiment and PYTHIA.]

**Fig.3.**

| $n$ | number of events, experiment | number of events, PYTHIA | $\chi^2$ in bin |
|-----|-----------------------------|-------------------------|-----------------|
| 0 - 2 | 23.62 | 22.93 | 0.00 |
| 4 | 302.89 | 302.20 | 0.00 |
| 6 | 1673.45 | 1697.39 | 0.08 |
| 8 | 4427.16 | 5009.39 | 19.14 |
| 10 | 8090.16 | 9128.50 | 33.12 |
| 12 | 10456.88 | 11249.92 | 14.78 |
| 14 | 10167.52 | 10200.36 | 0.03 |
| 16 | 7625.51 | 7230.66 | 5.23 |
| 18 | 4850.98 | 4271.90 | 18.46 |
| 20 | 2674.51 | 2160.62 | 27.65 |
| 22 | 1434.97 | 971.79 | 43.62 |
| 24 | 652.37 | 381.18 | 33.72 |
| 26 | 273.30 | 141.01 | 19.60 |
| 28 | 104.45 | 47.23 | 9.52 |
| 30 - 36 | 74.28 | 16.91 | 4.48 |

**Table 3.** $\sqrt{s} = 34.8$ GeV

Total $\chi^2$/degrees of freedom = 229/14

5
2.4 $\sqrt{s} = 91.2$ GeV

Experimentally there were obtained 248100 events (L3) [5]. The experimental values of $P_n$ and the corresponding probabilities obtained by PYTHIA are shown in Fig.4a. Both statistical and systematical errors are included. The $\chi^2$ is presented in Table 4a. Unlike the previous ones, the ratio of $\chi^2$/degrees of freedom = 1155/25 is very large for the results of L3 col., although visually the experimental and generated points are in good agreement (especially if one takes the curve fitting the generated points but not the points themselves, as it was done in some articles, for example [5], [6]). This is bound to very large statistics in the L3 col. and its small errors. The multiplicity distributions for the same energy obtained in DELPHI [7] (25364 events) and OPAL [8] (82941 events) experiments are given below (Fig.4b, Table 4b and Fig.4c, Table 4c accordingly). In these experiments statistics is lower and the values of $\chi^2$/degrees of freedom are correspondingly smaller. The result of multiplicity distributions from all collaborations combined together is given in Fig.4d.

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![Fig.4a.](image-url)
Table 4a. $\sqrt{s} = 91.2$ GeV, L3 col.

| $n$  | number of events, experiment | number of events, PYTHIA | $\chi^2$ in bin |
|------|-------------------------------|--------------------------|----------------|
| 2 – 4 | 70.96                         | 39.94                    | 21.15          |
| 6    | 509.60                        | 343.37                   | 54.22          |
| 8    | 2314.28                       | 1602.97                  | 218.37         |
| 10   | 6852.77                       | 5177.60                  | 203.97         |
| 12   | 14495.49                      | 11806.33                 | 151.89         |
| 14   | 23280.71                      | 20305.99                 | 90.10          |
| 16   | 30198.48                      | 27721.70                 | 48.02          |
| 18   | 33190.57                      | 32102.16                 | 8.85           |
| 20   | 32138.63                      | 32437.34                 | 0.96           |
| 22   | 28183.66                      | 29575.26                 | 24.48          |
| 24   | 22970.59                      | 24935.54                 | 104.10         |
| 26   | 17573.42                      | 19586.50                 | 88.30          |
| 28   | 12708.92                      | 14481.85                 | 56.91          |
| 30   | 8805.07                       | 10245.29                 | 28.70          |
| 32   | 5840.52                       | 6955.73                  | 21.36          |
| 34   | 3703.64                       | 4480.69                  | 13.01          |
| 36   | 2280.04                       | 2734.06                  | 7.44           |
| 38   | 1338.50                       | 1619.84                  | 5.33           |
| 40   | 773.82                        | 920.45                   | 3.18           |
| 42   | 430.45                        | 514.31                   | 1.97           |
| 44   | 224.03                        | 258.77                   | 0.76           |
| 46   | 112.14                        | 134.97                   | 0.81           |
| 48   | 56.57                         | 68.23                    | 0.51           |
| 50   | 25.31                         | 31.51                    | 0.35           |
| 52   | 12.65                         | 12.90                    | 0.00           |
| 54 – 62 | 8.93                        | 6.70                     | 0.07           |

Total $\chi^2$/degrees of freedom = 1155/25
Fig. 4b.
Table 4b. $\sqrt{s} = 91.2$ GeV, DELPHI col.

| $n$  | number of events, experiment | number of events, PYTHIA | $\chi^2$ in bin |
|------|------------------------------|--------------------------|----------------|
| 2 – 6| 45.91                        | 39.19                    | 0.23           |
| 8    | 170.95                       | 163.88                   | 0.14           |
| 10   | 578.30                       | 529.32                   | 1.10           |
| 12   | 1230.15                      | 1207.00                  | 0.09           |
| 14   | 2084.92                      | 2075.94                  | 0.01           |
| 16   | 2815.40                      | 2834.07                  | 0.01           |
| 18   | 3271.96                      | 3281.90                  | 0.00           |
| 20   | 3322.68                      | 3316.17                  | 0.00           |
| 22   | 2967.59                      | 3023.57                  | 0.12           |
| 24   | 2483.14                      | 2549.23                  | 0.23           |
| 26   | 1909.91                      | 2002.39                  | 0.70           |
| 28   | 1460.97                      | 1480.52                  | 0.05           |
| 30   | 1050.07                      | 1047.41                  | 0.00           |
| 32   | 743.17                       | 711.11                   | 0.40           |
| 34   | 476.84                       | 458.07                   | 0.28           |
| 36   | 309.44                       | 279.51                   | 1.30           |
| 38   | 191.50                       | 165.60                   | 1.83           |
| 40   | 121.24                       | 94.10                    | 1.00           |
| 42   | 63.66                        | 52.58                    | 0.43           |
| 44   | 36.27                        | 26.45                    | 0.92           |
| 46   | 20.80                        | 13.80                    | 1.16           |
| 48   | 5.07                         | 6.98                     | 0.39           |
| 50 – 62 | 4.31                      | 5.22                     | 0.02           |

Total $\chi^2$/degrees of freedom = 10/22
Fig. 4c.

91.2 GeV, OPAL, $\chi^2$/dof=20/24

- Experiment
- Pythia
Table 4c. $\sqrt{s} = 91.2$ GeV, OPAL col.

| $n$ | number of events, experiment | number of events, PYTHIA | $\chi^2$ in bin |
|-----|------------------------------|--------------------------|----------------|
| 2 – 4 | 14.10                        | 13.35                     | 0.00          |
| 6    | 132.71                       | 114.79                    | 0.04          |
| 8    | 564.00                       | 535.88                    | 0.03          |
| 10   | 1725.17                      | 1730.90                   | 0.00          |
| 12   | 3889.93                      | 3946.91                   | 0.06          |
| 14   | 6635.28                      | 6788.39                   | 0.48          |
| 16   | 8949.33                      | 9267.50                   | 0.71          |
| 18   | 10458.86                     | 10731.90                  | 0.99          |
| 20   | 10657.92                     | 10843.96                  | 0.29          |
| 22   | 9811.92                      | 9887.15                   | 0.09          |
| 24   | 8285.81                      | 8336.07                   | 0.02          |
| 26   | 6510.87                      | 6547.86                   | 0.04          |
| 28   | 4934.99                      | 4841.35                   | 0.24          |
| 30   | 3607.93                      | 3425.05                   | 1.06          |
| 32   | 2463.35                      | 2325.33                   | 1.41          |
| 34   | 1675.41                      | 1497.91                   | 2.75          |
| 36   | 1069.94                      | 914.01                    | 2.08          |
| 38   | 671.82                       | 541.52                    | 3.58          |
| 40   | 389.82                       | 307.71                    | 2.16          |
| 42   | 215.65                       | 171.94                    | 1.01          |
| 44   | 141.00                       | 86.51                     | 1.43          |
| 46   | 73.82                        | 45.12                     | 0.68          |
| 48   | 34.84                        | 22.81                     | 0.38          |
| 50   | 20.74                        | 10.53                     | 0.47          |
| 52 – 62 | 12.44                       | 6.55                      | 0.20          |

Total $\chi^2$/degrees of freedom = 20/24
Fig. 4d. The comparison between the results of different experiments at the energy of $Z^0$ peak.
2.5 $\sqrt{s} = 188.6$ GeV

Experimentally there were obtained 4479 events (L3) [5]. The experimental values of $P_n$ and the corresponding probabilities obtained by PYTHIA are shown in Fig.5. Both statistical and systematical errors are included. The $\chi^2$ is presented in Table 5.

![Image of Fig.5](image-url)

Fig.5.
Table 5. $\sqrt{s} = 188.6$ GeV

| $n$ | number of events, experiment | number of events, PYTHIA | $\chi^2$ in bin |
|-----|-----------------------------|--------------------------|-----------------|
| 2 – 10 | 17.92 | 18.16 | 0.00 |
| 12 | 58.23 | 44.23 | 2.32 |
| 14 | 125.41 | 101.23 | 2.23 |
| 16 | 214.99 | 186.12 | 1.87 |
| 18 | 322.49 | 279.53 | 3.41 |
| 20 | 389.67 | 364.89 | 1.09 |
| 22 | 434.46 | 425.13 | 0.11 |
| 24 | 452.38 | 447.01 | 0.04 |
| 26 | 434.46 | 441.80 | 0.05 |
| 28 | 394.15 | 413.00 | 0.47 |
| 30 | 353.84 | 366.51 | 0.21 |
| 32 | 309.05 | 317.03 | 0.09 |
| 34 | 255.30 | 260.58 | 0.03 |
| 36 | 201.56 | 210.87 | 0.11 |
| 38 | 152.29 | 165.96 | 0.25 |
| 40 | 116.45 | 125.96 | 0.13 |
| 42 | 80.62 | 93.32 | 0.37 |
| 44 | 58.23 | 68.70 | 0.27 |
| 46 | 40.31 | 48.67 | 0.18 |
| 48 | 22.40 | 33.54 | 2.32 |
| 50 | 17.92 | 23.68 | 0.09 |
| 52 – 82 | 8.96 | 43.07 | 8.12 |

total $\chi^2$/degrees of freedom = 24/21
2.6 $\sqrt{s} = 206.2$ GeV

Experimentally there were obtained 4146 events (L3) [5]. The experimental values of $P_n$ and the corresponding probabilities obtained by PYTHIA are shown in Fig.6. Both statistical and systematical errors are included. The $\chi^2$ is presented in Table 6.

![Fig.6.](chart)
Table 6. $\sqrt{s} = 206.2$ GeV

| $n$ | number of events, experiment | number of events, PYTHIA | $\chi^2$ in bin |
|-----|-------------------------------|--------------------------|-----------------|
| 2–10| 12.44                         | 13.43                    | 0.02            |
| 12  | 41.46                         | 32.33                    | 1.25            |
| 14  | 91.21                         | 77.09                    | 1.22            |
| 16  | 165.84                        | 146.40                   | 1.02            |
| 18  | 257.05                        | 229.01                   | 1.74            |
| 20  | 344.12                        | 306.95                   | 2.89            |
| 22  | 385.58                        | 366.97                   | 0.53            |
| 24  | 402.16                        | 395.36                   | 0.07            |
| 26  | 389.72                        | 400.01                   | 0.14            |
| 28  | 364.85                        | 381.23                   | 0.31            |
| 30  | 319.24                        | 347.40                   | 1.00            |
| 32  | 281.93                        | 303.93                   | 0.60            |
| 34  | 240.47                        | 259.44                   | 0.51            |
| 36  | 194.86                        | 214.66                   | 0.55            |
| 38  | 157.55                        | 172.92                   | 0.35            |
| 40  | 128.53                        | 135.30                   | 0.07            |
| 42  | 99.50                         | 103.13                   | 0.02            |
| 44  | 74.63                         | 78.00                    | 0.03            |
| 46  | 58.04                         | 56.78                    | 0.00            |
| 48  | 49.75                         | 40.26                    | 0.42            |
| 50  | 33.17                         | 28.50                    | 0.05            |
| 52–88| 24.88                         | 56.91                    | 4.48            |

Total $\chi^2$/degrees of freedom = 24/21
3 Conclusion

As it was shown, PYTHIA describes well the experimental data on the multiplicity distribution in $e^+e^-$ annihilation to hadrons. So it can be used as a reliable method of theoretical studies.

As it was noticed in PYTHIA manual [1], the default values of all its parameters were tuned to describe the events at the energy of $Z^0$ peak. The ratio of $\chi^2$/degrees of freedom is equal to 10/22 for DELPHI col. with 25364 events, 20/24 for OPAL col. with 82941 events and it is surprisingly large, 1155/25, for L3 col. with 248100 events. The ratio is approximately equal to 1 for other discussed energies, although for 34.8 it is 229/14 (52832 events, TASSO col.). This behavior of $\chi^2$ is quite obvious. Thus, more subtle tuning is necessary for the values of PYTHIA parameters, so that PYTHIA will be able to predict quantity results.

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