Charmless hadronic decays $B \to PP, PV, VV$ and new physics effects in the general two-Higgs doublet models

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Abstract

Based on the low-energy effective Hamiltonian with the generalized factorization, we calculate the new physics contributions to the branching ratios of the two-body charmless hadronic decays of $B_u$ and $B_d$ mesons induced by the new gluonic and electroweak charged-Higgs penguin diagrams in the general two-Higgs doublet models (models I, II and III). Within the considered parameter space, we find that: (a) the new physics effects from new gluonic penguin diagrams strongly dominate over those from the new $\gamma$- and $Z^0$- penguin diagrams; (b) in models I and II, new physics contributions to most studied B meson decay channels are rather small in size: from $-15\%$ to $20\%$; (c) in model III, however, the new physics enhancements to the penguin-dominated decay modes can be significant, $\sim (30 - 200)\%$, and therefore are measurable in forthcoming high precision B experiments; (d) the new physics enhancements to ratios $\mathcal{B}(B \to K\eta')$ are significant in model III, $\sim (35 - 70)\%$, and hence provide a simple and plausible new physics interpretation for the observed unexpectedly large $B \to K\eta'$ decay rates; (e) the theoretical predictions for $\mathcal{B}(B \to K^+\pi)$ and $\mathcal{B}(B \to K^0\pi^+)$ in model III are still consistent with the data within $2\sigma$ errors; (f) the significant new physics enhancements to the branching ratios of $B \to K^0\pi^0, K^+\eta, K^{*+}\pi^-, K^+\phi, K^{*0}\omega, K^{*+}\phi$ and $K^{*0}\phi$ decays are helpful to improve the agreement between the data and the theoretical predictions; (g) the theoretical predictions of $\mathcal{B}(B \to PP, PV, VV)$ in the 2HDM’s are generally consistent with experimental measurements and upper limits ($90\%C.L.$)

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I. INTRODUCTION

The main objective of B experiments is to explore in detail the physics of CP violation, to determine many of the flavor parameters of the standard model (SM) at high precision, and to probe for possible effects of new physics beyond the SM [1–3]. Precision measurements of B meson system can provide an insight into very high energy scales via the indirect loop effects of new physics. The B system therefore offers a complementary probe to the searches for new physics at Tevatron, LHC and NLC colliders [1].

In B experiments, new physics beyond the SM may manifest itself, for example, in the following two ways [1,3]: (a) decays which are expected to be rare in the SM are found to have large branching ratios; (b) CP-violating asymmetries which are expected to vanish or be very small in the SM are found to be significantly large or with a very different pattern with what predicted in the SM. These potential deviations may be induced by the virtual effects of new physics through loop diagrams.

It is well known that the two-body charmless hadronic decays $B \to h_1 h_2$ (where $h_1$ and $h_2$ are the light pseudo-scalar (P) and/or vector(V) mesons) play a very important role in studying CP violation and the heavy flavor physics [4,5]. Several groups [6–9] recently presented their systematic calculations for these B decay channels in the SM by using the low energy effective Hamiltonian [10–12] with the generalized factorization approach [7,13–15].

Theoretically, the effective Hamiltonian is our basic tool to calculate the branching ratios and CP-violating asymmetry $A_{CP}$ of B meson decays. The short and long distance quantum chromo-dynamical (QCD) effects in the hadronic decays are separated by means of the operator product expansion [16]. The short-distance QCD corrected Lagrangian at next-to-leading order (NLO) is available now, but we still do not know how to calculate hadronic matrix element from the first principles. One conventionally resort to the factorization ansatz [13]. However, we also know that the non-factorizable contribution really exists and can not be neglected numerically for most hadronic B decay channels. To remedy factorization hypothesis, some authors [7,11,13] introduced a phenomenological parameter $N_{eff}$ (i.e. the effective number of color) to model the non-factorizable contribution to hadronic matrix element, which is commonly called the generalized factorization. On the other hand, as pointed by Buras and Silvestrini [17], such generalization suffered from the problems of gauge and infrared dependence since the constant matrix $\hat{r}_V$ appeared in the expressions of effective Wilson coefficients $C_{i}^{eff}$ depends on both the gauge chosen and the external momenta. Very recently, Cheng et al. [18] studied and resolved above controversies on the gauge dependence and infrared singularity of $C_{i}^{eff}$ by using the perturbative QCD factorization theorem. In addition to the generalized factorization approach, a new approach, called as the QCD factorization [19], appeared recently [19,20], in which the decay amplitude is described by a kernel containing the ‘hard’ interaction given by a perturbatively evaluated effective Hamiltonian folded with form factors, decay constants and light-cone distributions of mesons into which the long distance effects are lumped. And some two-body hadronic B meson decays, such as $B \to \pi\pi$ and $K\pi$ modes, have been calculated in this approach [19,21].

On the experimental side, CLEO collaboration reported the observations of thirteen $B \to PP, PV$ decay channels and set new upper limits for many other decay modes [22–23].

The BaBar and Belle collaboration at SLAC and KEK also presented their first observation
for some $B \to PP, PV$ decays at the ICHEP 2000 conference\cite{20,21}. Except for the decay channels $B \to K\eta'$, the measured branching ratios for $B \to h_1h_2$ decays are generally in good agreement with the SM theoretical predictions based on the effective Hamiltonian with factorization. Unexpectedly large $B \to K\eta'$ rate was firstly reported by CLEO in 1997 \cite{28}, and confirmed very recently by CLEO and BaBar Collaborations \cite{23,24,26}. Although many possible mechanisms such as gluon and/or charm content in $\eta'$ and the hairpin diagram have been considered in order to increase the theoretical predictions of $B(B \to \eta')$, it is now still difficult to explain the observed large rate for $B \to K\eta'$ decays \cite{23,26,29}. This fact strongly suggests the requirement for additional contributions unique to the $\eta'$ meson in the framework of the SM, or large enhancements from new physics beyond the SM.

According to the studies in Refs. \cite{30–33}, we know that (a) an enhanced $b \to sg$ can lead to a large $\mathcal{B}(B \to \eta'X_s)$, and (b) the possible contributions to the ratio $b \to sg$ in both type I and II two-Higgs-doublet models (2HDM) are not large enough to meet the requirement \cite{30,31}. Very recently, we calculated \cite{34,35} the new physics enhancements to the branching ratios $\mathcal{B}(b \to sg)$ and $\mathcal{B}(b \to q'q\bar{q})$ with $q' \in \{d, s\}$ and $q \in \{u, d, s\}$ induced by charged-Higgs gluonic penguin diagrams in model III (the third type of 2HDM) with inclusion of NLO QCD corrections \cite{36}, and found that the rate of $b \to sg$ in model III can be enhanced significantly. The predicted charm multiplicity $n_c$ consequently become consistent with the measured $n_c$, while the agreement between the theoretical predictions and the data of $\mathcal{B}_{SL}$ is also improved by inclusion of the new physics effects.

In this paper we calculate the new physics contributions to the branching ratios of exclusive two-body charmless hadronic decays $B \to PP, PV, VV$ \cite{4} from new gluonic and electroweak charged-Higgs penguin diagrams in the general two-Higgs-doublet models (models I, II and III). We try to check the size and pattern of new physics effects on the exclusive two-body charmless B meson decays and to see if the new physics contributions in the model III can be large enough to provide the required enhancements for $B \to K\eta'$ decay modes. We will present our systematic calculation of branching ratios for seventy six $B \to h_1h_2$ decay modes by employing the effective Hamiltonian with the generalized factorization \cite{7,9}. We evaluate analytically all new strong and electroweak penguin diagrams induced by exchanges of charged Higgs bosons in the quark level processes $b \to qV^*$ with $q \in \{d, s\}$ and $V \in \{\text{gluon, } \gamma, Z\}$, and then combine the new physics contributions with their SM counterparts and finally calculate the branching ratios for all seventy six exclusive $B \to h_1h_2$ decay modes.

This paper is organized as follows. In Sec. \ref{2HDM}, we describe the basic structures of the 2HDM’s and examine the allowed parameter space of the general 2HDM’s from currently available data. In Sec. \ref{penguin}, we evaluate analytically the new penguin diagrams, combine the new physics contributions with their SM counterparts and find the effective Wilson coefficients $C_i^{eff}$. In Sec. \ref{formulae}, we present the formulae needed to calculate the branching ratios $\mathcal{B}(B \to h_1h_2)$. In the following three sections, we calculate and show numerical results of branching ratios for $B \to PP, PV$, and $VV$ decay modes, respectively. We concentrate on those decay modes with well-measured branching ratios and sizable yields. The conclusions

\footnote{1In the following, $B$ always means $B_u$ or $B_d$ mesons. We here do not consider the decays of $B_s$ meson.}
and discussions are included in the final section.

II. THE GENERAL 2HDM AND EXPERIMENTAL CONSTRAINTS

The simplest extension of the SM is the so-called two-Higgs-doublet models \[37\]. In such models, the tree level flavor changing neutral currents (FCNC’s) are absent if one introduces an ad hoc discrete symmetry to constrain the 2HDM scalar potential and Yukawa Lagrangian. Let’s consider a Yukawa Lagrangian of the form \[38\]

\[ L_Y = \eta_{ij}^U \bar{Q}_{i,L} \phi_1 U_{j,R} + \eta_{ij}^D \bar{Q}_{i,L} \phi_1 D_{j,R} + \xi_{ij}^U \bar{Q}_{i,L} \bar{\phi}_2 U_{j,R} + \xi_{ij}^D \bar{Q}_{i,L} \phi_2 D_{j,R} + h.c. , \]

(1)

where \( \phi_i \) \((i = 1, 2)\) are the two Higgs doublets of a two-Higgs-doublet model, \( \bar{\phi}_1, \phi_2 = i \tau_2 \phi_1^* \), \( Q_{i,L} (U_{j,R}) \) with \( i = (1, 2, 3) \) are the left-handed isodoublet quarks (right-handed up-type quarks), \( D_{j,R} \) are the right-handed isosinglet down-type quarks, while \( \eta_{ij}^U, D \) and \( \xi_{ij}^U, D \) (\( i, j = 1, 2, 3 \) are family index) are generally the non-diagonal matrices of the Yukawa coupling. By imposing the discrete symmetry

\[ \phi_1 \rightarrow -\phi_1, \phi_2 \rightarrow \phi_2, D_i \rightarrow -D_i, U_i \rightarrow \mp U_i \]

(2)

one obtains the so called models I and II. In model I the third and fourth term in Eq.(1) will be dropped by the discrete symmetry, therefore, both the up- and down-type quarks get mass from Yukawa couplings to the same Higgs doublet \( \phi_1 \), while the \( \phi_2 \) has no Yukawa couplings to the quarks. For model II, on the other hand, the first and fourth term in Eq.(1) will be dropped by imposing the discrete symmetry. Model II has, consequently the up- and down-type quarks getting mass from Yukawa couplings to two different scalar doublets \( \phi_1 \) and \( \phi_2 \).

During past years, models I and II have been studied extensively in literature and tested experimentally, and model II has been very popular since it is the building block of the minimal supersymmetric standard model. In this paper, we focus on the third type of the two-Higgs-doublet model \[38\], usually known as model III \[38,39\]. In model III, no discrete symmetry is imposed and both up- and down-type quarks may have diagonal and/or flavor changing couplings with \( \phi_1 \) and \( \phi_2 \). As described in Ref. \[38\], one can choose a suitable basis \( (H^0, H^1, H^2, H^\pm) \) to express two Higgs doublets \[38\]

\[ \phi_1 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \sqrt{2} \chi^+ \\
 v + H^0 + i\chi^0 \end{array} \right), \phi_2 = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \sqrt{2} H^+ \\
 H^1 + iH^2 \end{array} \right), \]

(3)

and take their vacuum expectation values as the form

\[ \langle \phi_1 \rangle = \left( \begin{array}{c} 0 \\
 v/\sqrt{2} \end{array} \right), \langle \phi_2 \rangle = 0, \]

(4)

where \( v = (\sqrt{2} G_F)^{-1/2} = 246 \text{ GeV} \). The transformation relation between \( (H^0, H^1, H^2) \) and the mass eigenstates \( (H^+, h^0, A^0) \) can be found in Ref. \[38\]. The \( H^\pm \) are the physical charged Higgs boson, \( H^0 \) and \( h^0 \) are the physical CP-even neutral Higgs boson and the \( A^0 \) is the physical CP-odd neutral Higgs boson. After the rotation of quark fields, the Yukawa Lagrangian of quarks are of the form \[38\]
where $\eta_{ij}^{U,D}$ correspond to the diagonal mass matrices of up- and down-type quarks, while the neutral and charged flavor changing couplings will be \cite{38}:

$$\xi_{ij}^{U,D} = \frac{\sqrt{m_i m_j}}{v} \lambda_{ij}, \quad \xi_{\text{neutral}}^{U,D} = \xi_{\text{charged}}^{U,D} = \xi^{U,D}_{\text{CKM}}, \quad \xi_{\text{charged}}^{D} = V_{\text{CKM}} \xi^{D},$$

where $V_{\text{CKM}}$ is the Cabibbo-Kobayashi-Maskawa mixing matrix \cite{40}, $i,j = (1,2,3)$ are the generation index. The coupling constants $\lambda_{ij}$ are free parameters to be determined by experiments, and they may also be complex.

In model II and setting $\tan \beta = v_2/v_1 \geq 1$ ( $v_1$ and $v_2$ are the vacuum expectation values of the Higgs doublet $\phi_1$ and $\phi_2$), the constraint on the mass $M_{H^+}$ due to CLEO data of $b \to s\gamma$ \cite{11} is $M_{H^+} \gtrsim 200$ GeV for the charged Higgs boson in the 2HDM at the NLO level \cite{12}. For model I, however, the limit can be much weaker due to the possible destructive interference with the SM amplitude.

For model III, the situation is not as clear as model II because there are more free parameters here. As pointed in Ref. \cite{38}, the data of $K^0 - \bar{K}^0$ and $B^0_d - \bar{B}^0_d$ mixing processes put severe constraints on the FC couplings involving the first generation of quarks. Imposing the limit $\lambda_{ij} = 0$ for $j = (1,2,3)$ and assuming all other $\lambda_{ij}$ parameters are of order 1, Atwood et al. \cite{43} found a very strong constraint of $M_{H^+} > 600$ GeV by using the CLEO data of $b \to s\gamma$ decay available in 1995. But this constraint can be lowered to $M_{H^+} \gtrsim 400$ GeV by using the new CLEO data of $b \to s\gamma$ decay \cite{35}. In Ref. \cite{14}, Aliev et al. studied the $b \to s\gamma$ decay in model III by extending the NLO results of model II \cite{12} to the case of model III, and found the constraint on the FC couplings.

In a recent paper \cite{15}, Chao et al. studied the decay $b \to s\gamma$ by assuming that only the couplings $\lambda_{tt} = |\lambda_{tt}|e^{i\theta_t}$ and $\lambda_{bb} = |\lambda_{bb}|e^{i\theta_b}$ are nonzero. They found that the constraint on $M_{H^+}$ imposed by the CLEO data of $b \to s\gamma$ can be greatly relaxed by considering the phase effects of $\lambda_{tt}$ and $\lambda_{bb}$. From the studies of Refs. \cite{35,43}, we know that for model III the parameter space

$$\lambda_{ij} = 0, \quad \text{for } i,j \neq tt, \text{ or } bb, \quad |\lambda_{tt}| = 0.3, \quad |\lambda_{bb}| = 35, \quad \theta = (0^0 - 30^0), \quad M_{H^+} = (200 \pm 100) \text{GeV},$$

are allowed by the available data, where $\theta = \theta_b - \theta_t$.

From the LEP and Tevatron searches for charged Higgs bosons \cite{10,47}, the new combined constraint in the $(M_{H^+}, \tan \beta)$ plane has been given, for example, in Ref. \cite{15}: the direct lower limit is $M_{H^+} > 77 \text{GeV}$, while $0.5 \leq \tan \beta \leq 60$ for a relatively light charged Higgs boson with $M_{H^+} \sim 100$ GeV. Combining the direct and indirect limits together, we here conservatively consider the range of $100 \text{GeV} \leq M_{H^+} \leq 300$ GeV, while take $M_{H^+} = 200 \text{ GeV}$ as the typical value for models I, II and III. For models I and II we consider the range of $1 \leq \tan \beta \leq 50$, while take $\tan \beta = 2$ as the typical value. In the following sections,
we calculate the new physics contributions to the exclusive two-body charmless decays of B meson in the Chao-Cheung-Keung (CCK) scenario of model III \cite{15}. Model III in the CCK scenario has the following advantages:

1. Since we keep only the couplings $\lambda_{tt}$ and $\lambda_{bb}$ nonzero, the neutral Higgs bosons do not contribute at tree level or one-loop level. The new contributions therefore come only from the charged Higgs penguin diagrams with the heavy internal top quark.

2. The new operators $O_{9,10}$ and all flipped chirality partners of operators $O_{1,\ldots,10}$ as defined in Ref. \cite{14} do not contribute to the decay $b \to s\gamma$ and the exclusive two-body charmless hadronic B decays under study in this paper.

3. The free parameters are greatly reduced to $\lambda_{tt}$, $\lambda_{bb}$ and $M_{H^+}$ in model III, and $\tan \beta$ and $M_{H^+}$ in models I and II.

III. EFFECTIVE WILSON COEFFICIENTS IN THE SM AND 2HDM’S

In this section we evaluate the new gluonic and electroweak penguin diagrams and present the well-known effective Hamiltonian for the two-body charmless decays $B \to h_1 h_2$ with the inclusion of new physics contributions. For more details about the effective Hamiltonian with generalized factorization for B decays one can see, for example, Refs. \cite{7,9}.

A. Operators and Wilson coefficients

The standard theoretical frame to calculate the inclusive three-body decays $b \to s\bar{q}q$ \footnote{For $b \to d\bar{q}q$ decays, one simply makes the replacement $s \to d$.} is based on the effective Hamiltonian \cite{12,13}.

$$
\mathcal{H}_{\text{eff}}(\Delta B = 1) = \frac{G_F}{\sqrt{2}} \left\{ \sum_{j=1}^{2} C_j \left[ V_{ub} V_{us}^* Q_j^u + V_{cb} V_{cs}^* Q_j^c \right] - V_{tb} V_{ts}^* \left[ \sum_{j=3}^{10} C_j Q_j + C_g Q_g \right] \right\}
$$

(8)

where $C_j$ and $C_g$ are Wilson coefficients, and the operator basis reads:

$$
Q_1 = (\bar{s}q)_{V-A}(\bar{q}b)_{V-A}, \quad Q_2 = (\bar{s}_\alpha q_\beta)_{V-A}(\bar{q}_\beta b_\alpha)_{V-A},
$$

(9)

with $q = u$ and $q = c$, and

$$
Q_3 = (\bar{s}b)_{V-A} \sum_{q'} (\bar{q}'q')_{V-A}, \quad Q_4 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A},
$$

(10)

$$
Q_5 = (\bar{s}b)_{V-A} \sum_{q'} (\bar{q}'q')_{V+A}, \quad Q_6 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A},
$$

(11)

$$
Q_7 = \frac{3}{2} (\bar{s}b)_{V-A} \sum_{q'} e_{q'} (\bar{q}'q')_{V+A}, \quad Q_8 = \frac{3}{2} (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A},
$$

(12)
\[ Q_9 = \frac{3}{2} (sb)_{V-A} \sum_{q'} e_{q'} (q'q')_{V-A}, \quad Q_{10} = \frac{3}{2} (\bar{s}_{a}b_{\beta})_{V-A} \sum_{q'} e_{q'} (q'q'_{\beta})_{V-A}, \quad (13) \]

\[ Q_g = \frac{g_s}{8\pi^2} m_b \bar{s}_a \sigma^{\mu\nu} (1 + \gamma_5) T_{\alpha\beta} b_{\beta} G_{\mu\nu} \quad (14) \]

where \(\alpha\) and \(\beta\) are the \(SU(3)\) color indices, \(T_{a\beta}(a = 1, \ldots, 8)\) are the Gell-Mann matrices. The sum over \(q'\) runs over the quark fields that are active at the scale \(\mu = O(m_b)\), i.e., \(q' \in \{u, d, s, c, b\}\). \(Q_1\) and \(Q_2\) are current-current operators, \(Q_{3,4,5,6,7,8,9,10}\) are QCD and electroweak penguin operators, and \(Q_g\) is the chromo-magnetic dipole (CMD) operator. Following Ref. [7], we also neglect the effects of the electro-magnetic penguin operator and do not consider the effects of the weak annihilation and exchange diagrams.

In the SM, the Wilson coefficients \(C_1(M_W), \ldots, C_{10}(M_W)\) at NLO level and \(C_g(M_W)\) at leading order (LO) have been defined, for example, in Refs. [11,12]. The explicit expressions of the coefficients in the naive dimensional regularization (NDR) scheme can also be found easily in Refs. [11,12].

**B. Contributions of the charged-Higgs penguin diagrams**

For the charmless hadronic decays of B meson under consideration, the new physics will manifest itself by modifying the corresponding Inami-Lim functions [13] \(C_0(x), D_0(x), E_0(x)\) and \(E'_0(x)\) which determine the coefficients \(C_1(M_W), \ldots, C_{10}(M_W)\) and \(C_g(M_W)\) in the SM. These modifications, in turn, will change for example the standard model predictions for the branching ratios of decays \(B \to h_1h_2\). The new strong and electroweak penguin diagrams can be obtained from the corresponding penguin diagrams in the SM by replacing the internal \(W^\pm\) lines with the charged-Higgs \(H^\pm\) lines, as shown in Fig. 1. In the analytical calculations of those penguin diagrams, we use the dimensional regularization to regulate all the ultraviolet divergence in the virtual loop corrections and adopt the \(\overline{MS}\) renormalization scheme. It is easy to show that all the ultraviolet divergence is canceled after summing up all Feynman diagrams.

By evaluating analytically the new \(Z^0\), \(\gamma\) and gluonic penguin diagrams induced by the exchanges of charged-Higgs boson \(H^\pm\) in the model III, we find the new \(C_0, D_0, E_0\) and \(E'_0\) functions

\[ C_0^{III} = -\frac{x_t}{16} \left[ \frac{y_t}{1-y_t} + \frac{y_t}{(1-y_t)^2} \ln[y_t] \right] \cdot |\lambda_t|^2, \quad (15) \]

\[ D_0^{III} = -\frac{1}{3} H(y_t)|\lambda_t|^2, \quad (16) \]

\[ E_0^{III} = -\frac{1}{2} I(y_t)|\lambda_t|^2, \quad (17) \]

\[ E_0'^{III} = \frac{1}{6} J(y_t)|\lambda_t|^2 - K(y_t)|\lambda_t\lambda_{bb}|e^{i\theta}, \quad (18) \]

with

\[ H(y) = \frac{38 y - 79 y^2 + 47 y^3}{72(1-y)^3} + \frac{4y - 6y^2 + 3y^4}{12(1-y)^4} \ln[y], \quad (19) \]
\[ I(y) = \frac{16y - 29y^2 + 7y^3}{36(1-y)^3} + \frac{2y - 3y^2}{6(1-y)^4} \log[y], \] (20)

\[ J(y) = \frac{2y + 5y^2 - y^3}{4(1-y)^3} + \frac{3y^2}{2(1-y)^4} \log[y], \] (21)

\[ K(y) = \frac{-3y + y^2}{4(1-y)^2} - \frac{y}{2(1-y)} \log[y], \] (22)

where \( x_t = m_t^2/M_W^2 \), \( y_t = m_t^2/M_{H^+}^2 \), and the small terms proportional to \( m^2_t/m^2_t \) have been neglected.

In models I and II, one can find the corresponding functions \( C_i, D_i, E_0 \) and \( E_0' \) by evaluating the new strong and electroweak penguin diagrams in the same way as in model III

\[ C_i^I = C_i^{II} = \frac{-x_t}{8 \tan^2 \beta} \left[ \frac{y_t}{1 - y_t} + \frac{y_t}{(1 - y_t)^2} \ln[y_t] \right], \] (23)

\[ D_i^I = D_i^{II} = -\frac{2}{3 \tan^2 \beta} H(y_t), \] (24)

\[ E_0^I = E_0^{II} - \frac{1}{\tan^2 \beta} I(y_t), \] (25)

\[ E_i^I = \frac{1}{3 \tan^2 \beta} [J(y_t) - 6K(y_t)], \] (26)

\[ E_i^{II} = \frac{1}{3 \tan^2 \beta} [J(y_t) + 2K(y_t)], \] (27)

where \( y_t = m_t^2/M_{H^+}^2 \).

We combine the SM part and the new physics part of the corresponding functions to define the functions at the scale \( \mu = M_W \) as follows

\[ F_0(M_W) = F_0^{SM} + F_0^{NP}, \] (28)

where \( F_0 \in \{ C_0, D_0, E_0, E_0' \} \). The explicit expressions of the functions \( C_0, D_0, E_0 \) and \( E_0' \) in the SM can be found, for example, in Ref. [12].

Since the heavy new particles appeared in the 2HDM’s have been integrated out at the scale \( M_W \), the QCD running of the Wilson coefficients \( C_i(M_W) \) down to the scale \( \mu = O(m_h) \) after including the new physics contributions will be the same as in the SM. By using QCD renormalization group equations [12], it is straightforward to run Wilson coefficients \( C_i(M_W) \) from the scale \( \mu = 0(M_W) \) down to the lower scale \( \mu = O(m_h) \). Working consistently to the NLO precision, the Wilson coefficients \( C_i \) for \( i = 1, \ldots, 10 \) are needed in NLO precision, while it is sufficient to use the leading logarithmic value for \( C_9 \);

\[ C(\mu) = U(\mu, M_W)C(M_W), \] (29)

\[ C_9(\mu) = \eta^{14/23} C_9(M_W) + \sum_{i=1}^{8} \tilde{h}_i \eta^{a_i}, \] (30)

where \( C(M_W) = (C_1(M_W), \ldots, C_{10}(M_W))^T \), \( U(\mu, M_W) \) is the five-flavor 10 × 10 evolution matrix at NLO level as defined in Ref. [11], \( \eta = \alpha_s(M_W)/\alpha_s(\mu) \), and the constants \( \tilde{h}_i \) and \( a_i \) can also be found in Ref. [11].
At the NLO level, the Wilson coefficients are usually renormalization scheme (RS) dependent. In the NDR scheme, by using the input parameters as given in Appendix and Eq. (7), and setting $M_{H^+} = 200$ GeV, $\theta = 0^0$, $\tan \beta = 2$ and $\mu = 2.5$ GeV, we find the Wilson coefficients $C_{g}^{eff}(\mu) = C_g + C_5$ and $C_i(\mu)$ with $i = 1, \ldots, 10$ in the SM and models I, II and III, and list them in Table I. From the numerical results as listed in Table I, one can easily see that:

- The values of $C_i(\mu)$ ($i = 1, \ldots, 10$) in models I, II and III are almost identical with those in the SM. Only the coefficient $C_{g}^{eff}$ in models II and III are clearly different from that in the SM.

- It is the coefficient $C_{g}^{eff}$ partially induced by the new gluonic penguin diagrams which dominate the total new physics corrections to the decay processes under study.

C. The effective Wilson coefficients

We know that the unphysical RS dependence of Wilson coefficients will be cancelled by the corresponding dependence in the matrix elements of the operators in $\mathcal{H}_{eff}$, as shown explicitly in Refs. [12,50]. Very recently, Cheng et al. [18] studied and resolved the so-called gauge and infrared problems [17] of generalized factorization approach [1]. They found that the gauge invariance is maintained under radiative corrections by working in the physical on-mass-shell scheme, while the infrared divergence in radiative corrections should be isolated using the dimensional regularization and the resultant infrared poles are absorbed into the universal meson wave functions [18].

The one-loop matrix elements can be rewritten in terms of the tree-level matrix elements of the effective operators [7]

$$\langle sq'\bar{q}'|\mathcal{H}_{eff}|b\rangle = \sum_{i,j} C_{i}^{eff}(\mu)\langle sq'\bar{q}'|O_j|b\rangle_{tree},$$

(31)

where $C_{i}^{eff}(\mu)$ ($i = 1, \ldots, 10$) are the effective Wilson coefficients. In the NDR scheme and for $SU(3)_C$, the effective Wilson coefficients $C_{i}^{eff}$ can be written as [4,8],

$$C_i^{eff} = \left[1 + \frac{\alpha_s}{4\pi} \left(r_V^T + \gamma_V^T \log \frac{m_b}{\mu}\right)\right]C_j + \frac{\alpha_s}{24\pi} A'_i \left(C_t + C_p + C_g\right) + \frac{\alpha_{ew}}{8\pi} B'_i C_e,$$

(32)

where $A'_i = (0, 0, 1, 3, -1, 3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0)^T$, $B'_i = (0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 1, 0)^T$, the matrices $r_V$ and $\gamma_V$ contain the process-independent contributions from the vertex diagrams. Like Ref. [2], we here include vertex corrections to $C_7 - C_{10}$ [4]. The anomalous dimension matrix $\gamma_V$ has been given explicitly, for example, in Eq.(2.17) of Ref. [9]. Note that the correct value of the element $(\hat{r}_{NDR})_{66}$ and $(\hat{r}_{NDR})_{88}$ should be 17 instead of 1 as pointed out in Ref. [51]. $\hat{r}_V$ in the NDR scheme takes the form

\footnote{The reliability of the generalized factorization approach is improved by this progress.}

\footnote{Numerically, such corrections are negligibly small.}
have been given, for example, in Refs. [7,9] to include $C_0$ and infrared safe. The explicit expressions of functions Eq.(32) are now scheme and scale independent in NLO precision, and also gauge invariant. $Q^2$ is the momentum transferred by the virtual gluon, photon or $Z$ to the $q\bar{q}$ quark pair in the inclusive three-body decays $b \to q\bar{q}'$, and $m$ is the mass of internal up-type quark in the penguin diagrams. For $k^2 > 4m^2$, an imaginary part of $g(z)$ will appear because of the generation of a strong phase at the $\bar{u}u$ and $\bar{c}c$ threshold [52, 54].

For the two-body exclusive B meson decays any information on $k^2$ is lost in the factorization assumption, and it is not clear what “relevant” $k^2$ should be taken in numerical calculation. One usually uses the ”physical” range for $k^2$: $m_b^2/4 \lesssim k^2 \lesssim m^2_b/2$. Following Refs. [6,7], we also use $k^2 = m_b^2/2$ in the numerical calculation and will consider the $k^2$-dependence of branching ratios of charmless B meson decays for several typical decay channels.

$$
\hat{t}^{NDR}_V = \left( \begin{array}{cccccccccc}
3 & -9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-9 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3 & -9 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -9 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -3 & 17 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & -9 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -9 \\
\end{array} \right). \quad (33)
$$

The function $C_t$, $C_p$, and $C_g$ describe the penguin-type corrections to the operators $Q_{1,2}$, $Q_{3...6}$, and the tree-level diagram of the operator $Q_9$ respectively. We here follow the procedure of Ref. [15] to include $C_g$ in (32). The effective Wilson coefficients $C^{\text{eff}}_i$ in Eq.(32) are now scheme and scale independent in NLO precision, and also gauge invariant and infrared safe. The explicit expressions of functions $C_t$, $C_p$, and $C_g$ in the NDR scheme have been given, for example, in Refs. [4,4]

$$
C_t = \left[ \frac{2}{3} + \frac{\lambda_u}{\lambda_t} G(m_u) + \frac{\lambda_c}{\lambda_t} G(m_c) \right] C_1, \quad (34)
$$

$$
C_p = \left[ \frac{4}{3} - G(m_q) - G(m_b) \right] C_3 + \sum_{i=u,d,s,c,b} \left[ \frac{2}{3} - G(m_i) \right] (C_4 + C_6), \quad (35)
$$

$$
C_g = \frac{2m_b}{\sqrt{<k^2>}} C^{\text{eff}}_g, \quad (37)
$$

with $\lambda_q \equiv V_{q'b}V^{*}_{q'q}$. The function $G(m)$ is of the form [52]

$$
G(m) = \frac{10}{9} - \frac{2}{3} \ln \frac{m^2}{\mu^2} + \frac{2\mu^2}{3m^2} - \frac{2(1 + 2z)}{3z} g(z) \quad (38)
$$

where $z = k^2/(4m^2)$, and

$$
g(z) = \begin{cases} \\
\sqrt{\frac{1-z}{2z}} \arctan \left( \frac{1}{\sqrt{1-z}} \right), & z < 1, \\
\sqrt{\frac{1-z}{4z}} \left[ \ln \left( \frac{\sqrt{z} + \sqrt{1-z}}{\sqrt{z} - \sqrt{1-z}} \right) - i\pi \right], & z > 1.
\end{cases} \quad (39)
$$

where $k$ is the momentum transferred by the virtual gluon, photon or $Z$ to the $q\bar{q}'$ quark pair in the inclusive three-body decays $b \to q\bar{q}'$, and $m$ is the mass of internal up-type quark in the penguin diagrams. For $k^2 > 4m^2$, an imaginary part of $g(z)$ will appear because of the generation of a strong phase at the $\bar{u}u$ and $\bar{c}c$ threshold [52, 54].
In numerical calculations, two sets of form factors at the zero momentum transfer from the Bauer, Stech and Wirbel (BSW) model [13], as well as Lattice QCD and Light-cone QCD sum rules (LQQSR) [55] will be used respectively. Explicit values of these form factors can be found in Ref. [7] and have also been given in Appendix. Following Ref. [7], the seventy six decay channels of $B_u$ and $B_d$ mesons are classified into five classes according to their $N^{\text{eff}}$-dependence:

- **Class-I**: including four decay modes, $B^0 \to \pi^-\pi^+$, $\rho^\pm\pi^\mp$ and $B^0 \to \rho^-K^+$, the large and $N^{\text{eff}}$ stable coefficient $a_1$ plays the major role.

- **Class-II**: including ten decay modes, for example $B^0 \to \pi^0\pi^0$, and the relevant coefficient for these decays is $a_2$ which shows a strong $N^{\text{eff}}$-dependence.

- **Class-III**: including nine decay modes involving the interference of class-I and class-II decays, such as the decays $B^+ \to \pi^+\eta'$.

- **Class-IV**: including twenty two $B \to h_1h_2$ decay modes such as $B \to K\eta(')$ decays. The amplitudes of these decays involve one (or more) of the dominant penguin coefficients $a_{4,6,9}$ with constructive interference among them. The Class-IV decays are $N^{\text{eff}}$ stable.

- **Class-V**: including twelve $B \to h_1h_2$ decay modes, such as $B \to \pi^0\eta(')$ and $B \to \phi K$ decays. Since the amplitudes of these decays involve large and delicate cancellations due to interference between strong $N^{\text{eff}}$-dependent coefficients $a_{3,5,7,10}$ and the dominant penguin coefficients $a_{4,6,9}$, these decays are generally not stable against $N^{\text{eff}}$.

With the factorization ansatz [13,56,57], the three-hadron matrix elements or the decay amplitudes $<XY|H_{\text{eff}}|B>$ can be factorized into a sum of products of two current matrix elements $<X|J_\mu|0>$ and $<Y|J_\mu|B>$ (or $<Y|J_\mu'|0>$ and $<X|J_\mu|B>$). The explicit expressions of the matrix elements in terms of decay constants ($f_X, g_X$) and the Lorentz-scalar form factors $A_{0,1,2}(k^2)$ and $F_{0,1}(k^2)$ can be found, for example, in Refs. [13,58,7].

In the B rest frame, the branching ratios of two-body B meson decays can be written as

$$B(B \to XY) = \tau_B \frac{|p|}{8\pi M_B^2} |M(B \to XY)|^2$$  \hspace{1cm} (40)$$

for $B \to PP$ decays, and

$$B(B \to XY) = \tau_B \frac{|p|^3}{8\pi M_V^2} |M(B \to XY)/(\epsilon \cdot p_B)|^2$$  \hspace{1cm} (41)$$

for $B \to PV$ decays. Here $\tau(B_u^-) = 1.65 \text{ ps}$ and $\tau(B_d^0) = 1.56 \text{ ps}$ [59], $p_B$ is the four-momentum of the B meson, $M_V$ and $\epsilon$ is the mass and polarization vector of the produced light vector meson respectively, and $|p|$ is the magnitude of momentum of particle X and Y in the B rest frame

$$|p| = \frac{1}{2M_B} \sqrt{[M_B^2 - (M_X + M_Y)^2][M_B^2 - (M_X - M_Y)^2]}.$$

\hspace{1cm} (42)
For $B \to VV$ decays, one needs to evaluate the helicity matrix elements $H_\lambda = \langle V_1(\lambda)V_2(\lambda)|H_{eff}|B \rangle$ with $\lambda = 0, \pm 1$. The branching ratio of the decay $B \to V_1V_2$ is given in terms of $H_\lambda$ by

$$
\mathcal{B}(B \to V_1V_2) = \tau_B \frac{|p|}{8\pi M_B^2} \left[ |H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2 \right].
$$

(43)

The three independent helicity amplitudes $H_0$, $H_{+1}$ and $H_{-1}$ can be expressed by three invariant amplitudes $a, b, c$ defined by the decomposition

$$
H_\lambda = i\epsilon^\mu(\lambda)\eta^\nu(\lambda) \left[ a g_{\mu\nu} + \frac{b}{M_1M_2} p_\mu p_\nu + \frac{i c}{M_1M_2} \epsilon_{\mu\nu\alpha\beta} p_\alpha p^\beta \right].
$$

(44)

where $p_{1,2}$ and $M_{1,2}$ are the four momentum and masses of $V_{1,2}$, respectively. $p = p_1 + p_2$ is the four-momentum of B meson, and

$$
H_{\pm 1} = a \pm c\sqrt{x^2 - 1}, \quad H_0 = -ax - b \left( x^2 - 1 \right)
$$

(45)

$$
x = \frac{M_B^2 - M_1^2 - M_2^2}{2M_1M_2}
$$

(46)

In the generalized factorization ansatz, the effective Wilson coefficients $C_i^{eff}$ will appear in the decay amplitudes in the combinations:

$$
a_{2i-1} \equiv C_{2i-1}^{eff} + \frac{C_{2i}^{eff}}{N^{eff}}, \quad a_{2i} \equiv C_{2i}^{eff} + \frac{C_{2i-1}^{eff}}{N^{eff}}, \quad (i = 1, \ldots, 5)
$$

(47)

where the effective number of colors $N^{eff}$ is treated as a free parameter varying in the range of $2 \leq N^{eff} \leq \infty$, in order to model the non-factorizable contribution to the hadronic matrix elements. It is evident that the reliability of generalized factorization approach has been improved since the effective Wilson coefficients $C_i^{eff}$ appeared in Eq. (47) are now gauge invariant and infrared safe. Although $N^{eff}$ can in principle vary from channel to channel, but in the energetic two-body hadronic B meson decays, it is expected to be process insensitive as supported by the data [1]. As argued in Ref. [14], $N^{eff}(LL)$ induced by the $(V-A)(V-A)$ operators can be rather different from $N^{eff}(LR)$ generated by $(V-A)(V+A)$ operators. In this paper, however, we will simply assume that $N^{eff}(LL) \equiv N^{eff}(LR) = N^{eff}$ and consider the variation of $N^{eff}$ in the range of $2 \leq N^{eff} \leq \infty$ since we here focus on the calculation of new physics effects on the studied B meson decays induced by the new penguin diagrams in the two-Higgs-doublet models. For more details about the cases of $N^{eff}(LL) \neq N^{eff}(LR)$, one can see for example Ref. [4]. We here will also not consider the possible effects of final state interaction (FSI) and the contributions from annihilation channels although they may play a significant rule for some B meson decays.

Using the input parameters as given in Appendix, and assuming $k^2 = m_b^2/2$, $M_{H^+} = 200 \text{ GeV}$, $\theta = 0^0$ and $\tan \beta = 2$, the theoretical predictions of effective coefficients $a_i$ are calculated and displayed in Table I and Table II for the transitions $b \to d \left( \bar{b} \to \bar{d} \right)$ and $b \to s \left( \bar{b} \to \bar{s} \right)$, respectively. For coefficients $a_3, \ldots, a_{10}$, the first, second and third entries in tables (I,II) refer to the values of $a_i$ in the SM and models II and III, respectively. $a_i$ in model I are very similar with those in the SM and hence was not given explicitly.

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All branching ratios in the following three sections are the averages of the branching ratios of $B$ and anti-$B$ decays. The ratio $\delta B$ describes the magnitude of new physics corrections on the SM predictions of the decay ratios and is defined as

$$\delta B(B \to XY) = \frac{B(B \to XY)^{2HDM} - B(B \to XY)^{SM}}{B(B \to XY)^{SM}}$$

(V. $B \to PP$ DECAYS)

Using formulae as given in last section, it is straightforward to find the decay amplitudes of $B \to PP$ decays. As an example, we present here the decay amplitude $M(B^- \to \pi^-\pi^0) = |\langle \pi^-\pi^0 | H_{eff} | B^- \rangle|$, with

$$M(B^- \to \pi^-\pi^0) = \frac{G_F}{2} \left\{ V_{ub}V_{ud}^* \left( a_1 M_{udud}^{\pi^-\pi^0} + a_2 M_{duu}^{\pi^-\pi^0} \right) ight. 
- V_{tb}V_{td}^* \left[ (a_4 + a_10 + (a_6 + a_8)R_1) \right. 
\left. \left( a_4 + \frac{3}{2} (a_7 - a_9) - \frac{a_10}{2} + (a_6 - \frac{a_8}{2})R_2 \right) \right] M_{uud}^{\pi^-\pi^0} \left] \right\}$$

(49)

with

$$R_1 = \frac{2m_{\pi^-}^2}{(m_b - m_u)(m_u + m_d)},$$

$$R_2 = \frac{m_{\pi^0}^2}{m_d(m_b - m_d)},$$

$$M_{uud}^{\pi^-\pi^0} = -i(m_b^2 - m_{\pi^-}^2) f_{\pi} F_0^{B \to \pi}(m_{\pi^0}^2),$$

$$M_{duu}^{\pi^-\pi^0} = -i(m_b^2 - m_{\pi^0}^2) f_{\pi} F_0^{B \to \pi}(m_{\pi^-}^2)$$

(52) (53)

where $f_{\pi}$ is the decay constant of $\pi$ meson. The form factor $F_0^{B \to \pi}(m^2)$ can be found in Appendix. Under the approximations of setting $m_u = m_d$ and $m_{\pi^0} = m_{\pi^-}$, the decay amplitude $M(B^- \to \pi^-\pi^0)$ in Eq.(49) will be reduced to the form as given in Eq.(80) of Ref. [7]. In the following numerical calculations, we use the decay amplitudes as given in Appendix A of Ref. [7] directly without further discussions about details of individual amplitude.

In tables [IV]-[VI], we present the numerical results of the branching ratios for the twenty $B \to PP$ decays in the framework of the SM and models I, II and III by using the BSW and LQQSR form factors, respectively. Theoretical predictions are made by using the central values of input parameters as given in Eq.(7) and Appendix, and assuming $M_{H^+} = 200$ GeV, $\theta = 0$, tan $\beta = 2$ and $N_{eff} = 2, 3, \infty$ in the generalized factorization approach. The $k^2$-dependence of the branching ratios is small in the range of $k^2 = m_b^2 / 2 \pm 2$ GeV$^2$ and hence the numerical results are given by fixing $k^2 = m_b^2 / 2$.

The currently available CLEO data [22, 24] are listed in the last column of table [IV]. From the numerical results, we see that:
• For $B \to K\eta'$ decays, the observed branching ratios are clearly much larger than the SM predictions [25,29]. All other estimated branching ratios in Table IV are, however, consistent with the new CLEO, BaBar and Belle measurements or upper limits.

• In model III, the new physics corrections to most class-II, IV and V decay channels can be rather large and insensitive to the variations of the mass $M_{H^+}$ and the color number $N^{eff}$: from 20% to 90% w.r.t the SM predictions for both cases of $\theta = 0^0, 30^0$. For tree-dominated decay modes $B \to \pi^+\pi^-,\pi^+\pi^0,\pi^+\eta(1')$, the new physics corrections are small in size.

• In models I and II, however, the new physics corrections to all $B \to PP$ decay modes are small in size within the considered parameter space: less than 3% in model I, and $\approx (-20 - 0)^0$% in model II, as shown in Table VI. So small corrections will be masked by other large theoretical uncertainties.

• In model III, the new gluonic penguins will contribute effectively through the mixing of chromo-magnetic operator $Q_5$ with QCD penguin operators $Q_3 - Q_6$. The $C^{eff}_g$ will strongly dominate the new physics contributions to all $B \to h_1 h_2$ decay modes.

• The central values of the branching ratios obtained by using the LQQSR form factors will be increased by about 15% when compared with the results using the BSW form factors, as can be seen from Table IV. We therefore use the BSW form factors only to calculate the new physics effects on the ratios $\mathcal{B}(B \to h_1 h_2)$ and treat the difference induced by using different set of form factors as one kind of theoretical uncertainties.

A. $B \to \pi\pi, K\pi$ decays

There are so far seven measured branching ratios of $B \to PP$ decays: one $B \to \pi^+\pi^-$ decay, four $B \to K\pi$ and two $B \to K\eta'$ decays [25,26,29,31]:

$$\mathcal{B}(B \to \pi^+\pi^-) = \begin{cases} (4.3^{+1.6}_{-1.2} \pm 0.5) \times 10^{-6} & [\text{CLEO}], \\ (9.3^{+2.8}_{-2.1} \pm 1.4) \times 10^{-6} & [\text{BaBar}], \end{cases}$$

(54)

$$\mathcal{B}(B \to K^+\pi^0) = \begin{cases} (11.6^{+3.0}_{-2.7} \pm 1.4) \times 10^{-6} & [\text{CLEO}], \\ (18.8^{+5.5}_{-4.9} \pm 2.3) \times 10^{-6} & [\text{Belle}], \end{cases}$$

(55)

$$\mathcal{B}(B \to K^+\pi^-) = \begin{cases} (17.2^{+2.5}_{-2.4} \pm 1.2) \times 10^{-6} & [\text{CLEO}], \\ (12.5^{+3.0}_{-2.6} \pm 1.3 \pm 2.3) \times 10^{-6} & [\text{BaBar}], \\ (17.4^{+5.1}_{-4.6} \pm 3.4) \times 10^{-6} & [\text{Belle}], \end{cases}$$

(56)

$$\mathcal{B}(B \to K^0\pi^+) = (18.2^{+4.6}_{-4.0} \pm 1.6) \times 10^{-6} & [\text{CLEO}],$$

(57)

$$\mathcal{B}(B \to K^0\pi^0) = \begin{cases} (14.6^{+5.9}_{-5.1} \pm 3.3) \times 10^{-6} & [\text{CLEO}], \\ (21^{+9.3}_{-7.8} \pm 2.3) \times 10^{-6} & [\text{Belle}], \end{cases}$$

(58)

$$\mathcal{B}(B \to K^+\eta') = \begin{cases} (80^{+10}_{-9} \pm 7) \times 10^{-6} & [\text{CLEO}], \\ (62 \pm 18 \pm 8) \times 10^{-6} & [\text{BaBar}], \end{cases}$$

(59)

$$\mathcal{B}(B \to K^0\eta') = (89^{+18}_{-16} \pm 9) \times 10^{-6} & [\text{CLEO}].$$

(60)
The measurements of CLEO, BaBar and Belle Collaborations are in good agreement with each other within errors. These decays are sensitive to the relevant form factors $F_{0}^{B \rightarrow \pi}, F_{0}^{B \rightarrow \eta}, F_{0}^{B \rightarrow \eta'}$, etc., and to the value of $N_{\text{eff}}$.

As a Class-I decay channel, the $B^0 \rightarrow \pi^+ \pi^-$ decay is dominated by the $b \rightarrow u$ tree diagram. The band between two dots lines in Fig.2 shows the CLEO measurement. Since the new physics corrections are very small in size, less than 3% within the considered parameter space, the four curves for the SM and 2HDM’s are close together and can not be separated clearly. The theoretical predictions look higher than the CLEO measurement, but they are still consistent with BaBar measurement because of very large error of BaBar data. In fact the theoretical predictions for $\mathcal{B}(B \rightarrow \pi^+ \pi^-)$ in the SM and 2HDM’s are still consistent with the CLEO data at the $2\sigma$ level if we consider currently still large theoretical and experimental uncertainties. On the other hand, if we take the average of CLEO and BaBar measurements, $\mathcal{B}(B^0 \rightarrow \pi^+ \pi^-) = (5.5 \pm 1.5) \times 10^{-6}$, as the experimental result, then the constraint on $F_{0}^{B \rightarrow \pi}(0)$ from the data will be $F_{0}^{B \rightarrow \pi}(0) = 0.25 \pm 0.03$ by setting $A = 0.2205$, $\lambda = 0.81; \rho = 0.12; \eta = 0.34$, $N_{\text{eff}} = 3$, and by neglecting FSI also.

In the SM, the four Class-IV decays $B \rightarrow K \pi$ are dominated by the $b \rightarrow sg$ gluonic penguin diagrams, with additional contributions from $b \rightarrow u$ tree and electroweak penguin diagrams. Measurements of $B \rightarrow K \pi$ decays are particularly important to measure the angle $\gamma$. In model III, the new physics enhancements to the branching ratios $\mathcal{B}(B \rightarrow K\pi)$ are significant, $\sim (50 - 60)\%$, and show a moderate dependence on the variations of other parameters, as illustrated in figures (3-6). In models I and II, however, the new physics corrections are always very small in size.

For the decays $B \rightarrow K^+ \pi^-, K^0 \pi^+$, the theoretical predictions in model III are higher than the CLEO data as shown in Fig.4 and Fig.5, but they are still consistent with the CLEO data at the $2\sigma$ level if we consider currently still large theoretical uncertainties. As a simple illustration of effects of the theoretical uncertainties, we recalculate the branching ratios of $B \rightarrow K^+ \pi^-$ and $K^0 \pi^+$ decays by using $F_{0}^{B \rightarrow \pi}(0) = 0.25$ instead of the ordinary BSW value $F_{0}^{B \rightarrow \pi}(0) = 0.33$ while keeping all other input parameters unchanged, and find numerically that

$$\mathcal{B}(B \rightarrow K^+ \pi^-) = \begin{cases} (11.3 +_{-2.5}^{+2.9}) \times 10^{-6} & \text{in SM}, \\ (17.2 +_{-3.9}^{+4.3}) \times 10^{-6} & \text{in Model III}, \end{cases} \quad (61)$$

$$\mathcal{B}(B \rightarrow K^0 \pi^+) = \begin{cases} (13.3 +_{-3.0}^{+3.4}) \times 10^{-6} & \text{in SM}, \\ (19.9 +_{-4.5}^{+5.9}) \times 10^{-6} & \text{in Model III}, \end{cases} \quad (62)$$

for $N_{\text{eff}} = 3$ and $M_{H^+} = 200$ GeV. Here the first and second error correspond to $F_{0}^{B \rightarrow \pi}(0) = 0.25 \pm 0.03$ and $2 \leq N_{\text{eff}} \leq \infty$, respectively. It is evident that the theoretical predictions of the two ratios $\mathcal{B}(B \rightarrow K^+ \pi^-)$ and $\mathcal{B}(B \rightarrow K^0 \pi^+)$ in the SM and model III can lie within the CLEO data.

Figures (3-6) show the mass and $N_{\text{eff}}$-dependence of the branching ratios for four $B \rightarrow K\pi$ decay modes in the SM and models I, II and III, using the input parameters as given in Eq.(7) and Appendix, and assuming $\theta = 0^0, \tan \beta = 2$, and $k^2 = m_t^2/2$. For Figs.(3a-b) we set $N_{\text{eff}} = 3$ and assume that $M_{H^+} = 100 - 300$ GeV. For Figs.(3b-6b), we set $M_{H^+} = 200$ GeV, and assume that $1/N_{\text{eff}} = 0 - 0.5$. In all four figures, the band between two dots lines shows the corresponding CLEO measurements with 2$\sigma$ errors. For $B \rightarrow K^0 \pi^0$ decay, the inclusion of new physics contribution will improve the agreement between the
data and theoretical prediction, as illustrated in Fig.6. For other three \( B \to K \pi \) decays, the theoretical predictions in the model III are still consistent with the data if the theoretical uncertainties are taken into account.

**B. \( B \to K\eta' \) decays and the new physics effects**

For \( B^+ \to K^+\eta \) and \( B^0 \to K^0\eta \) decay modes, the new physics corrections are large (small) in model III (models I and II). The theoretical predictions in the SM and 2HDM’s are consistent with the new CLEO upper limits.

For \( B \to K\eta' \) decay modes, the situation is very interesting now. In 1997, CLEO firstly reported the unexpectedly large \( B \to K\eta' \) rates \([28]\), which is confirmed very recently by CLEO with the full CLEO II/II.V data sample of 19 million produced B mesons \([23,29]\). The \( K\eta' \) signal is large, stable and has small error (\( \sim 14\% \)). Those measured ratios are clearly much larger than the SM predictions as given in table (IV). In \([51]\), Cheng and Yang considered various possible enhancements to \( K\eta' \) decay modes in the framework of the SM, but found that the net enhancement is not very large: \( \mathcal{B}(B^\pm \to K^\pm\eta') = (40 - 50) \times 10^{-6} \), which is smaller than the CLEO data \([51]\). At present, it is indeed difficult to explain the observed large rate for \( B \to K\eta' \) \([23,29]\). This fact strongly suggests the requirement for additional contributions unique to the \( \eta' \) meson in the framework of the SM, or from new physics beyond the SM.

In models I and II, the new physics contributions are too small (or negative) to provide the required enhancement. This feature remains unchanged within the considered range of \( \tan \beta = 1 - 50 \).

In model III, however, the new physics enhancements are significant, \( \sim 60\% \), and have a moderate dependence on \( M_{H^+} \) and \( N_{eff} \), as illustrated by the solid curves in Figs.(7-10) where only the central values of theoretical predictions in model III are shown. If we take into account other theoretical uncertainties, the theoretical predictions for ratios \( \mathcal{B}(B \to K\eta') \) in model III will become consistent with the CLEO data:

\[
\mathcal{B}(B^+ \to K^+\eta') = \begin{cases} (69 - 92) \times 10^{-6} & \text{[CLEO]}, \\ (20 - 52) \times 10^{-6} & \text{[SM]}, \\ (34 - 74) \times 10^{-6} & \text{[Model III]}, \end{cases} \quad (63)
\]

\[
\mathcal{B}(B^0 \to K^0\eta') = \begin{cases} (71 - 109) \times 10^{-6} & \text{[CLEO]}, \\ (19 - 53) \times 10^{-6} & \text{[SM]}, \\ (33 - 73) \times 10^{-6} & \text{[Model III]}. \end{cases} \quad (64)
\]

---

6 As discussed in \([3]\), \( B \to K\eta' \) decay may get enhanced due to (i) a small \( m_s \) at the scale \( m_b \), (ii) the sizable \( SU(3) \) breaking, (iii) large \( F_{B \to \eta'} \), (iv) the \( \eta' \) charm content, and (v) constructive interference in tree amplitudes. But these possible enhancements are partially washed out by the anomaly effect in the matrix element of pseudoscalar densities \([3,32]\).

7 Although this prediction is consistent with BaBar measurement, one should note that the error of BaBar measurement is still much larger than that of CLEO data. More statistics is clearly required for BaBar to make a definite conclusion.
Here the major theoretical uncertainties induced by using different set of form factors and varying $k^2, \eta$ and $N_{\text{eff}}$ in the ranges of $\delta k^2 = \pm 2 \text{GeV}^2$, $\delta \eta = \pm 0.08$ and $N_{\text{eff}} = 2 - \infty$ have been taken into account.

Figures (7,9) show the mass and $N_{\text{eff}}$ dependence of $\mathcal{B}(B \to K\eta')$ in the SM and 2HDM’s. The upper dots band shows the CLEO measurements with $2\sigma$ errors. The short-dashed, dot-dashed, long-dashed and solid curve refers to the theoretical predictions in the SM, models I, II and III, respectively. As shown explicitly in Figs. (8,10), in which the short-dashed, long-dashed and solid curve corresponds to the model III predictions for $N_{\text{eff}} = 2, 3, \infty$ respectively, the theoretical predictions become now consistent with the CLEO measurement due to the inclusion of new physics enhancement in model III.

VI. $B \to PV$ DECAYS

In tables (VII-IX) we present the branching ratios for the thirty seven $B \to PV$ decay modes involving $b \to d$ and $b \to s$ transitions in the SM and models I, II and III by using the BSW form factors and by employing generalized factorization approach. Theoretical predictions are made by using the same input parameters as those for the $B \to PP$ decays in last section.

For studied thirty seven $B \to PV$ decays, two general features are as follows:

- The theoretical predictions for those seven measured decay rates are consistent with the CLEO data within $2\sigma$ errors. All other estimated branching ratios in the SM and 2HDM’s as given in tables (VII-IX) are all consistent with the new CLEO upper limits.
- For most decay modes, the differences induced by using whether BSW or LQQSR form factors are small, $\sim 15\%$. We therefore use the BSW form factors only in the calculation of new physics effects.

There are so far seven measured branching ratios of $B \to PV$ decays. For the first three decay modes, $B \to \rho^\pm \rho^\mp, \rho^0 \pi^+, \omega \pi^+$, the new physics corrections are small in size, $< 5\%$, and have a weak dependence on $M_{H^+}$ and $N_{\text{eff}}$, as shown in tables (VII-IX). Consequently, the theoretical predictions in the SM and models I, II and III agree well with CLEO measurements.

Because of the appearance of very large $\mathcal{B}(B \to K\eta')$, the decay modes $B \to K^*\eta'$ also draw more attentions now. Very recently, CLEO and Belle reported their first observation \cite{22,23,27} of $B \to K^*\eta, K^{*+}\pi^-$ and $B \to K^+\phi$ decays:

\begin{align}
\mathcal{B}(B^+ \to K^{*+}\eta) &= (26.4^{+9.6}_{-8.2} \pm 3.3) \times 10^{-6},
\mathcal{B}(B^0 \to K^{*0}\eta) &= (13.8^{+5.5}_{-4.6} \pm 1.6) \times 10^{-6},
\mathcal{B}(B^0 \to K^{*+}\pi^-) &= (22^{+8}_{-6}+4_{-5}) \times 10^{-6},
\mathcal{B}(B^+ \to K^{+}\phi) &= (17.2^{+6.7}_{-5.4} \pm 1.8) \times 10^{-6},
\end{align}

while the theoretical predictions in the SM and model III are

\[\text{17}\]
\[
\mathcal{B}(B^+ \to K^{*+} \eta) = \begin{cases} (2 - 4) \times 10^{-6} & \text{[SM]}, \\ (2 - 5) \times 10^{-6} & \text{[Model III]}, \end{cases}
\]

(69)

\[
\mathcal{B}(B^+ \to K^{*0} \eta) = \begin{cases} (2 - 5) \times 10^{-6} & \text{[SM]}, \\ (3 - 6) \times 10^{-6} & \text{[Model III]}, \end{cases}
\]

(70)

\[
\mathcal{B}(B^+ \to K^{*+} \pi^-) = \begin{cases} (7 - 16) \times 10^{-6} & \text{[SM]}, \\ (10 - 22) \times 10^{-6} & \text{[Model III]}, \end{cases}
\]

(71)

\[
\mathcal{B}(B^+ \to K^+ \phi) = \begin{cases} (0.5 - 28) \times 10^{-6} & \text{[SM]}, \\ (1 - 39) \times 10^{-6} & \text{[Model III]}, \end{cases}
\]

(72)

where the uncertainties induced by using the BSW or LQSE form factors, and setting \( k^2 = m_b^2/2 \pm 2\text{GeV}^2 \), \( \eta = 0.34 \pm 0.08 \) and \( N^{\text{eff}} = 2 - \infty \), have been taken into account.

Although the central values of the theoretical predictions in the SM are much smaller than the corresponding central values of the CLEO measurements, the theoretical predictions are still consistent with the data within 2\( \sigma \) errors because current experimental error is still large. Further improvement of experimental measurements about the decay modes \( B \to K^{*} \eta \), \( K^{*+} \pi^- \) will tell us whether there is any discrepancy between the theory and experiments for these three decay modes. At present, any positive contributions to the above three branching ratios from new mechanisms in the SM or from new physics beyond the SM are clearly preferred by the CLEO data.

In models I and II, the new physics contributions are small in size: from \(-15\%\) to \(20\%\) for most \( B \to PV \) decay modes, and have weak dependence on \( M_{H^+}, \tan \beta \) and \( N^{\text{eff}} \), as shown in table (IX), and illustrated in Figs.(11-14) where the long-dashed line shows the theoretical predictions in the model II \(^8\). This feature remains unchanged within the considered range of \( \tan \beta = 1 - 50 \). When \( \tan \beta \) becomes larger, the size of new physics corrections will become even smaller.

In model III, however, the new physics contributions are significant, from \(30\%\) to \(110\%\), and have also weak dependence on \( M_{H^+}, \theta \) and \( N^{\text{eff}} \). These new physics enhancements are very helpful to improve the agreement between the theoretical predictions and the data, as shown in Eqs.(69-72) and illustrated in Figs.(11-14).

Figs.(11-14) show the mass and \( N^{\text{eff}} \) dependence of the branching ratios for \( B \to K^{*+} \eta, K^{*0} \eta, K^{*+} \pi^- \) and \( B \to K^+ \phi \) decays. The dot-dashed line is the SM prediction, while the long-dashed and solid curve corresponds to the predictions in models II and III, respectively. The theoretical uncertainties are not shown in these figures. The dots band in Figs.(11-13) (Fig.14) corresponds to the CLEO data with 1\( \sigma \) (2 - \( \sigma \)) error.

From Fig.(11), it can be seen that the CLEO measurement of the ratio \( \mathcal{B}(B^+ \to K^{*+} \eta) \) is much larger than theoretical predictions in the SM and 2HDM’s. More positive contributions to this decay mode are needed to improve the agreement between the data and theoretical prediction. For \( B \to K^{*0} \eta \) decay, the inclusion of new physics contribution in the model III leads to a better agreement between data and theory if we take into account still large theoretical uncertainties. For \( B \to K^{*+} \pi^- \) and \( K^+ \phi \) decays, the theoretical prediction

\(^8\)Because the lines for the SM and model I are too close to be separated clearly, we do not draw the line for model I in all four figures for \( B \to PV \) decays.
becomes now consistent with the CLEO and Belle measurements within 1σ error due to the large new physics enhancement in the model III.

For $B \to K^{*+}\eta'$ and $K^{*0}\eta'$ decays, the new physics contributions in model III are large in size, from $-77\%$ to $200\%$, as shown in Table VIII. But the theoretical predictions for these two decay modes are $N_{	ext{eff}}$-dependent and still far below the current CLEO upper limits.

**VII. $B \to V V$ DECAYS**

Using the formulae as given in Sec. [IV], it is straightforward to calculate the branching ratios of nineteen $B_{u,d} \to V V$ decays. As an example, we show here the calculation of the branching ratio for the Class-V decay $B^- \to \rho^-\omega$ ($b \to d$ transition). We firstly find the explicit expressions of the helicity amplitude $H_\lambda = \langle \rho^-\omega | H_{\text{eff}} | B^- \rangle$, and then compare this amplitude with the standard form as defined in Eq. (44) to extract out the process dependent coefficients $a, b$ and $c$

\[
a = -\frac{1}{\sqrt{2}} \left[ f_1 f_\omega M_\omega (M_B + M_\rho) A_1^{B\to \rho}(M_\omega^2) + f_2 f_\rho M_\rho (M_B + M_\omega) A_1^{B\to \omega}(M_\rho^2) \right],
\]

\[
b = f_1 \cdot \frac{\sqrt{2} f_\omega M_\omega^2 M_\rho}{M_B + M_\rho} A_2^{B\to \rho}(M_\omega^2) + f_2 \cdot \frac{\sqrt{2} f_\rho M_\rho^2 M_\omega}{M_B + M_\omega} A_2^{B\to \omega}(M_\rho^2),
\]

\[
c = f_1 \cdot \frac{\sqrt{2} f_\omega M_\omega^2 M_\rho}{M_B + M_\rho} V^{B\to \rho}(M_\omega^2) + f_2 \cdot \frac{\sqrt{2} f_\rho M_\rho^2 M_\omega}{M_B + M_\omega} V^{B\to \omega}(M_\rho^2),
\]

with

\[
f_1 = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{ud}^* a_2 - V_{ub} V_{td}^* \left( 2a_3 + a_4 + 2a_5 + \frac{1}{2}a_7 + \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right) \right],
\]

\[
f_2 = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{ud}^* a_1 - V_{ub} V_{td}^* (a_4 + a_{10}) \right],
\]

where the coefficients $a_1, \ldots, a_{10}$ have been defined in Eq. (47), the form factors and other input parameters can be found in Appendix. With these coefficients $a, b$ and $c$, the branching ratio $B(B^- \to \rho^-\omega)$ can finally be written as

\[
B(B^- \to \rho^-\omega) = \frac{\tau_{B^-}}{8\pi M_B} \frac{|p|}{M_B^2} \left( |H_0|^2 + |H_+|^2 + |H_-|^2 \right),
\]

where $|p|$ and $H_{1,0,-1}$ have been given in Eqs. (42) and (43).

In tables (X-XII) we present the branching ratios for the nineteen $B \to V V$ decay modes involving $b \to d$ and $b \to s$ transitions in the SM and models I, II and III. Theoretical predictions are made by using the same input parameters as those for the $B \to P P, P V$ decays in last two sections.

For $B \to V V$ decay modes, the differences induced by using whether BSW or LQGSR form factors are around ten percent in the SM and models I, II and III. We therefore show the numerical results obtained by using the BSW form factors only for the cases of models I, II and III. For all nineteen $B \to V V$ decays under study, the theoretical predictions in
the SM and 2HDM’s are still under or far away from the current CLEO upper limits, as can be seen from tables (X-XII).

In models I and II, the new physics contributions to $B \rightarrow VV$ decays are small in size: from $-15\%$ to $\sim 10\%$ as shown in Table XI, and therefore will be masked by other large theoretical uncertainties. This feature remains unchanged within the considered range of $\tan \beta = 1-50$. When $\tan \beta$ becomes larger, the size of new physics corrections will become smaller.

In model III, however, the new physics contributions to different channels are varying greatly: from $-11\%$ to $\sim 110\%$, assuming $M_{H^+} = 200\text{GeV}$, $N^{eff} = 2-\infty$ and $\theta = 0^\circ-30^\circ$. For decay modes $B \rightarrow K^{*0}\omega$, $K^{*+}\phi$, $K^{*0}\phi$, for example, the new physics enhancements are significant: $\sim (60-110)\%$. And hence the theoretical predictions in model III are close to or slightly surpass the current CLEO upper limits, as illustrated in Figs.(15-17) where the upper dots line shows the corresponding CLEO upper limits at 90$\%$C.L.. These decay modes will be observed soon.

Figures (15-17) show the mass and $N^{eff}$ dependence of the ratios $B(B \rightarrow K^{*0}\omega)$ and $B(B \rightarrow K^{*+}\phi)$. The dot-dashed line is the SM prediction, while the long-dashed and solid curve corresponds to the predictions in models II and III, respectively. As the Class-V decays, these three decays show strong $N^{eff}$ dependence as illustrated in Figs.(15-17).

VIII. SUMMARY AND DISCUSSIONS

In this paper, we calculated the branching ratios of two-body charmless hadronic $B$ meson decays $B_{u,d} \rightarrow PP, PV, VV$ in the SM and the general two-Higgs-doublet models by employing the NLO effective Hamiltonian with the generalized factorization.

In Sec. I with the help of previous works [38,43–45,35], we gave a brief review about the 2HDM’s and studied corresponding experimental constraints on models I, II an III. In Sec. II, we evaluated analytically all new gluonic and electroweak charged-Higgs penguin diagrams and found the effective Wilson coefficients $C^{eff}_i$ in the SM and models I, II and III. In Sec. III, we presented the formulae needed to calculate the branching ratios $B(B \rightarrow PP, PV, VV)$.

In sections IV-VII, we calculated the branching ratios for seventy six $B \rightarrow PP, PV, VV$ decays in the SM and models I, II and III, presented the numerical results in tables (IV-XII) and displayed the $M_{H^+}$ and $N^{eff}$-dependence for several phenomenologically interesting decay modes in Figs.(2-17).

From the numerical results, we find following general features about the new physics effects on the exclusive charmless hadronic $B \rightarrow PP, PV, VV$ decays studied in this paper:

1 The SM predictions for the $B$ meson decay rates presented in this paper agree well with those appeared in Refs. [7,9].

2 The new physics effects due to new gluonic penguin diagrams strongly dominate over those from the $\gamma-$ and $Z^0-$ penguin diagrams induced by exchanges of charged-Higgs bosons appeared in models I, II and III.
3 For models I and II, the new physics contributions to the decay rates $\mathcal{B}(B \to h_1 h_2)$ are always small in size: from $-15\%$ to $20\%$ for most decay modes. So small contributions will be masked by other still large theoretical uncertainties.

4 For model III, however, the new physics enhancements to penguin-dominated decay modes can be significant, $\sim (30 - 200)\%$, and therefore can be measured in high precision B experiments. In general, the new physics contributions in model III are large (small) for penguin-dominated (tree-dominated) B meson decay channels.

5 The uncertainties of the theoretical predictions for the branching ratios of $B \to h_1 h_2$ decays induced by varying $k^2$, $\eta$, $\theta$, $\tan \beta$, and $M_{H^+}$ are varying from $\sim 10\%$ to $\sim 50\%$ within the range of $k^2 = m_b^2/2 \pm 2 \text{GeV}^2$, $\eta = 0.34 \pm 0.08$, $\theta = 0^\circ - 30^\circ$, $\tan \beta = 1 - 50$, and $M_{H^+} = 200 \pm 100 \text{GeV}$. The dependence of decay rates on whether using the BSW or LQSSR form factors are weak, $\sim 10\%$. The $N^{\text{eff}}$-dependence of branching ratios, however, are varying greatly for different decay modes.

6 For phenomenologically interesting $B \to K\eta'$ decay modes, the new physics enhancements are significant in model III: $\sim (35 - 70)\%$, and have a moderate dependence on $M_{H^+}$ and $N^{\text{eff}}$. The theoretical predictions for $\mathcal{B}(B \to K\eta')$ therefore turn to be consistent with the CLEO data in model III, as illustrated in Figs. (8,10). For other $B \to PP$ decays, the theoretical predictions are still consistent with the measurements if one takes into account still large theoretical and experimental uncertainties.

7 For penguin-dominated $B \to PV$ decays, the new physics contributions in model III are significant, from $30\%$ to $60\%$, and have a weak or moderate dependence on $M_{H^+}$, $\theta$ and $N^{\text{eff}}$, as illustrated in the tables (VII-VIII) and figures (11-14). The CLEO measurements of $\mathcal{B}(B \to K^{*+}\eta, K^{*0}\eta)$ are much larger than theoretical predictions in the SM and hence large new physics enhancements in model III are indeed helpful to lead to or improve the agreement between the data and theoretical predictions.

8 In model III, the new physics contributions to different $B \to VV$ decay modes are varying greatly: from $-11\%$ to $\sim 110\%$. For decay modes $B \to K^{*0}\omega, K^{*+}\phi, K^{*0}\phi$, for example, the new physics enhancements are significant: $\sim (60 - 110)\%$, and hence the theoretical predictions in model III are close to or slightly surpass the current CLEO upper limits. These decay modes will be observed soon.

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APPENDIX A: INPUT PARAMETERS AND FORM FACTORS

In this appendix we present relevant input parameters. We use the same set of input parameters for the quark masses, decay constants, Wolfenstein parameters and form factors as Ref. \[7\].

- Input parameters of electroweak and strong coupling constant, gauge boson masses, B meson masses, light meson masses, \ldots, are as follows (all masses in unit of GeV ) \[7,59\]

\[
\begin{align*}
\alpha_{em} &= \frac{1}{128}, \quad \alpha_s(M_Z) = 0.118, \quad \sin^2\theta_W = 0.23, \quad G_F = 1.16639 \times 10^{-5}(GeV)^{-2}, \\
M_Z &= 91.187, \quad M_W = 80.41, \quad m_{B_d^0} = m_{B_s^+} = 5.279, \quad m_{\pi^\pm} = 0.140, \\
m_{\pi^0} &= 0.135, \quad m_{\eta} = 0.547, \quad m_{\eta'} = 0.958, \quad m_{\rho} = 0.770, \quad m_{\omega} = 0.782, \\
m_{\phi} &= 1.019, \quad m_{K^\pm} = 0.494, \quad m_{K^0} = 0.498, \quad m_{K^{*\pm}} = 0.892, \quad m_{K^{*0}} = 0.896, \\
\tau(B^\pm_u) &= 1.64ps, \quad \tau(B^0_d) = 1.56ps, \\
\end{align*}
\]

(A1)

- For the elements of CKM matrix, we use Wolfenstein parametrization, and fix the parameters $A, \lambda, \rho$ to their central values, $A = 0.81, \lambda = 0.2205, \rho = 0.12$ and varying $\eta$ in the range of $\eta = 0.34 \pm 0.08$.

- We first treat the internal quark masses in the loops in connection with the function $G(m)$ as constituent masses,

\[
m_b = 4.88GeV, \quad m_c = 1.5GeV, \quad m_s = 0.5GeV, \quad m_u = m_d = 0.2GeV.
\]

(A2)

Secondly, we will use the current quark masses for $m_i$ ($i = u, d, s, c, b$) which appear through the equation of motion when working out the hadronic matrix elements. For $\mu = 2.5GeV$, one finds \[7\]

\[
m_b = 4.88GeV, \quad m_c = 1.5GeV, m_s = 0.122GeV, \quad m_d = 7.6MeV, \quad m_u = 4.2MeV.
\]

(A3)

For the mass of heavy top quark we also use $m_t = m_{\overline{t}}(m_t) = 168GeV$.

- For the decay constants of light mesons, the following values will be used in the numerical calculations (in the units of MeV):

\[
\begin{align*}
f_\pi &= 133, \quad f_K = 158, \quad f_{K^*} = 214, \quad f_\rho = 210, \quad f_\omega = 195, \quad f_\phi = 233, \\
f^u_\eta &= f^d_\eta = 78, \quad f^u_{\eta'} = f^d_{\eta'} = 68, \quad f^c_\eta = -0.9, \quad f^c_{\eta'} = -0.23, \\
f^u_\eta &= -113, \quad f^s_\eta = 141.
\end{align*}
\]

(A4)

where $f^u_{\eta',\eta}$ and $f^s_{\eta',\eta}$ have been defined in the two-angle-mixing formalism with $\theta_0 = -9.1^\circ$ and $\theta_8 = -22.2^\circ$ \[15\]. For more details about the mixings between $\eta$ and $\eta'$, one can see \[60,15\].
The form factors at the zero momentum transfer in the BSW model [13] have been collected in Table 2 of Ref. [7]. For the convenience of the reader we list them here:

\[ F_{B \to \pi^0}(0) = 0.33, \quad F_{B \to K^0}(0) = 0.38, \quad F_{B \to \eta^0}(0) = 0.145, \quad F_{B \to \eta'}(0) = 0.135, \]

\[ A_{B \to \rho^0}(0) = A_{B \to \omega}(0) = 0.28, \quad A_{B \to K^+}(0) = 0.32, \quad A_{B \to K^0}(0) = 0.33, \]

\[ V_{B \to \rho^0}(0) = V_{B \to \omega}(0) = 0.33, \quad V_{B \to K^0}(0) = 0.37, \]

In the LQKSR approach, the form factors at zero momentum transfer being used in our numerical calculations are,

\[ F_{B \to \pi^0}(0) = 0.36, \quad F_{B \to K^0}(0) = 0.41, \quad F_{B \to \eta^0}(0) = 0.16, \quad F_{B \to \eta'}(0) = 0.145, \]

\{A_0, A_1, A_2, V\}(B \to \rho) = \{0.30, 0.27, 0.26, 0.35\},

\{A_0, A_1, A_2, V\}(B \to K^*) = \{0.39, 0.35, 0.34, 0.48\},

\{A_0, A_1, A_2, V\}(B \to \omega) = \{0.30, 0.27, 0.26, 0.35\}. \quad (A5)

The form factors \(F_{0,1}(k^2), A_{0,1,2}(k^2)\) and \(V(k^2)\) were defined in Ref. [13] as

\[ F_0(k^2) = \frac{F_0(0)}{1 - k^2/m^2(0^+)}, \quad F_1(k^2) = \frac{F_1(0)}{1 - k^2/m^2(1^-)}, \]

\[ A_0(k^2) = \frac{A_0(0)}{1 - k^2/m^2(0^-)}, \quad A_1(k^2) = \frac{A_1(0)}{1 - k^2/m^2(1^+)}, \]

\[ A_2(k^2) = \frac{A_2(0)}{1 - k^2/m^2(1^-)}, \quad V(k^2) = \frac{V(0)}{1 - k^2/m^2(1^-)}. \quad (A7)\]

The pole masses being used to evaluate the \(k^2\)-dependence of form factors are

\{m(0^-), m(1^-), m(1^+), m(0^+)\} = \{5.2789, 5.3248, 5.37, 5.73\}, \quad (A8)

for \(\bar{u}b\) and \(\bar{d}b\) currents, and

\{m(0^-), m(1^-), m(1^+), m(0^+)\} = \{5.3693, 5.41, 5.82, 5.89\}, \quad (A9)

for \(\bar{s}b\) currents.
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TABLE I. Wilson coefficients $C_i(\mu)$ and $C_{g}^{eff}(\mu)$ in the SM and models I, II and III at the scale $\mu = 2.5$ GeV, with $M_{H^+} = 200$ GeV, $\tan \beta = 2$ and $\theta = 0^0, 30^0$.  

|     | SM   | Model I | Model II | Model III: $\theta = 0^0$ | Model III: $\theta = 30^0$ |
|-----|------|---------|----------|--------------------------|--------------------------|
| $C_1$ | 1.1245 | 1.1245 | 1.1245 | 1.1245 | 1.1245 |
| $C_2$ | $-0.2662$ | $-0.2662$ | $-0.2662$ | $-0.2662$ | $-0.2662$ |
| $C_3$ | 0.0186 | 0.0187 | 0.0187 | 0.0186 | 0.0186 |
| $C_4$ | $-0.0458$ | $-0.0458$ | $-0.0458$ | $-0.0458$ | $-0.0458$ |
| $C_5$ | 0.0113 | 0.0113 | 0.0113 | 0.0113 | 0.0113 |
| $C_6$ | $-0.0587$ | $-0.0585$ | $-0.0585$ | $-0.0587$ | $-0.0587$ |
| $C_7$ | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 |
| $C_8$ | 0.0007 | 0.0007 | 0.0007 | 0.0007 | 0.0007 |
| $C_9$ | $-0.0095$ | $-0.0099$ | $-0.0099$ | $-0.0096$ | $-0.0096$ |
| $C_{10}$ | 0.0026 | 0.0027 | 0.0027 | 0.0026 | 0.0026 |
| $C_{g}^{eff}$ | $-0.1527$ | $-0.1321$ | $-0.2487$ | 0.3364 | $0.2708 + 0.2448i$ |
TABLE II. Numerical values of $a_i$ for the transitions $b \rightarrow d$ [\( \bar{b} \rightarrow \bar{d} \)]. The first, second and third entries for $a_3, \ldots, a_{10}$ refer to the values of $a_i$ in the SM and models II and III, respectively. All entries for $a_3, \ldots, a_{10}$ should be multiplied with $10^{-4}$.

| $N^{\ell f} = 2$ | $N^{\ell f} = 3$ | $N^{\ell f} = \infty$ |
|------------------|------------------|------------------|
| $a_1$            | 0.995 [0.995]    | 1.061 [1.061]    | 1.192 [1.192] |
| $a_2$            | 0.201 [0.201]    | 0.003 [0.003]    | -0.395 [-0.395] |
| $a_3$            | $-16 - 7i$ [-25 - 23i] | 77 [77]          | 261 + 13i [280 + 47i] |
| $a_4$            | $-423 - 33i$ [-470 - 117i] | $-467 - 35i$ [-517 - 125i] | $-554 - 39i$ [-610 - 141i] |
| $a_5$            | $-193 - 7i$ [-202 - 23i] | $-71$ [-71]      | 171 + 13i [190 + 47i] |
| $a_6$            | $-642 - 33i$ [-689 - 117i] | $-671 - 35i$ [-721 - 125i] | $-728 - 39i$ [-784 - 141i] |
| $a_7$            | 8.1 - 0.9i [7.7 - 1.7i] | 6.8 - 0.9i [6.4 - 1.7i] | 4.3 - 0.9i [3.9 - 1.7i] |
| $a_8$            | 9.3 - 0.9i [8.9 - 1.7i] | 8.0 - 0.9i [7.5 - 1.7i] | 5.3 - 0.9i [4.9 - 1.7i] |
| $a_9$            | 8.3 - 0.9i [7.9 - 1.7i] | 7.0 - 0.9i [6.6 - 1.7i] | 4.5 - 0.9i [4.1 - 1.7i] |
| $a_{10}$         | 9.7 - 0.5i [9.5 - 0.8i] | 9.0 - 0.3i [8.8 - 0.6i] | 7.5 [7.5] |
|                  | 11 - 0.5i [11 - 0.8i] | 9.9 - 0.3i [9.7 - 0.6i] | 8.1 [8.1] |
|                  | 9.9 - 0.5i [9.7 - 0.8i] | 9.1 - 0.3i [9.0 - 0.6i] | 7.6 [7.6] |
|                  | $-84 - 0.9i$ [-84 - 1.7i] | $-90 - 0.9i$ [-90 - 1.7i] | $-102 - 0.9i$ [-102 - 1.7i] |
|                  | $-87 - 0.9i$ [-87 - 1.7i] | $-93 - 0.9i$ [-94 - 1.7i] | $-106 - 0.9i$ [-106 - 1.7i] |
|                  | $-84 - 0.9i$ [-85 - 1.7i] | $-90 - 0.9i$ [-91 - 1.7i] | $-103 - 0.9i$ [-103 - 1.7i] |
|                  | $-14 - 0.5i$ [-15 - 0.8i] | 2.6 - 0.3i [2.5 - 0.6i] | 37 [37] |
|                  | $-15 - 0.5i$ [-15 - 0.8i] | 2.8 - 0.3i [2.7 - 0.6i] | 38 [38] |
|                  | $-15 - 0.5i$ [-15 - 0.8i] | 2.7 - 0.3i [2.5 - 0.6i] | 37 [37] |
| \( a_j \) | \( N_{\text{eff}} = 2 \) | \( N_{\text{eff}} = 3 \) | \( N_{\text{eff}} = \infty \) |
|---|---|---|---|
| \( a_1 \) | 0.995 [0.995] | 1.061 [1.061] | 1.192 [1.192] |
| \( a_2 \) | 0.201 [0.201] | 0.026 [0.026] | -0.395 [-0.395] |
| \( a_3 \) | -21 - 14i [-19 - 14i] | 77 [77] | 272 + 29i [269 + 29i] |
| \( a_4 \) | -449 - 72i [-442 - 72i] | -494 - 77i [-487 - 77i] | -585 - 86i [-576 - 86i] |
| \( a_5 \) | -198 - 14i [-196 - 14i] | -71 [-71] | 181 + 29i [179 + 29i] |
| \( a_6 \) | -667 - 72i [-660 - 72i] | -698 - 77i [-691 - 77i] | -758 - 86i [-750 - 86i] |
| \( a_7 \) | 7.9 - 1.3i [7.9 - 1.3i] | 6.6 - 1.3i [6.7 - 1.3i] | 4.1 - 1.3i [4.2 - 1.3i] |
| \( a_8 \) | 9.6 - 0.6i [9.6 - 0.6i] | 8.9 - 0.4i [8.9 - 0.4i] | 7.5 [7.5] |
| \( a_9 \) | -84 - 1.3i [-84 - 1.3i] | -90 - 1.3i [-90 - 1.3i] | -102 - 1.3i [-102 - 1.3i] |
| \( a_{10} \) | -15 - 0.6i [-14 - 0.6i] | 2.6 - 0.4i [2.6 - 0.4i] | 37 [37] |
TABLE IV. $B(B \to PP)$ (in units of $10^{-6}$) in the SM using the BSW [LQSSR] form factors, with $k^2 = m_b^2/2$ and $N^{eff} = 2, 3, \infty$. The last column shows the CLEO measurements and upper limits at 90\%C.L. [22-25].

| Channel | Class | $N^{eff} = 2$ | $N^{eff} = 3$ | $N^{eff} = \infty$ | Data |
|---------|-------|--------------|--------------|-------------------|------|
| $B^0 \to \pi^+\pi^-$ | I     | 9.10 [10.8]  | 10.3 [12.3]  | 13.0 [15.5]       | 4.3^{+1.6}_{-1.5} \pm 0.5 |
| $B^0 \to \pi^0\pi^0$ | II    | 0.28 [0.33]  | 0.15 [0.18]  | 0.92 [1.09]       | < 9.3 |
| $B^+ \to \pi^+\pi^0$ | III   | 6.41 [7.62]  | 5.06 [6.02]  | 2.85 [3.39]       | < 12.7 |
| $B^0 \to \eta\eta$ | II    | 0.14 [0.17]  | 0.10 [0.13]  | 0.29 [0.36]       | < 18  |
| $B^0 \to \eta\eta'$ | II    | 0.14 [0.17]  | 0.08 [0.09]  | 0.38 [0.45]       | < 27  |
| $B^0 \to \eta\eta'$ | II    | 0.04 [0.05]  | 0.01 [0.01]  | 0.13 [0.15]       | < 47  |
| $B^+ \to \pi^+\eta$ | III   | 3.51 [4.25]  | 2.78 [3.37]  | 1.75 [2.13]       | < 5.7 |
| $B^+ \to \pi^+\eta'$ | III   | 2.49 [2.90]  | 1.88 [2.17]  | 1.02 [1.17]       | < 12  |
| $B^0 \to \pi^0\eta$ | V     | 0.26 [0.31]  | 0.29 [0.35]  | 0.39 [0.47]       | < 2.9 |
| $B^0 \to \pi^0\eta'$ | V     | 0.06 [0.07]  | 0.08 [0.09]  | 0.14 [0.17]       | < 5.7 |
| $B^+ \to K^+\pi^0$ | IV    | 12.0 [14.3]  | 13.5 [16.0]  | 16.7 [19.8]       | 11.6^{+6.0}_{-2.7} \pm 1.3 |
| $B^0 \to K^+\pi^-$ | IV    | 17.8 [21.2]  | 19.8 [23.5]  | 24.0 [28.5]       | 17.2^{+2.5}_{-2.4} \pm 1.2 |
| $B^+ \to K^0\pi^+$ | IV    | 19.9 [23.7]  | 23.2 [27.7]  | 30.6 [36.4]       | 18.2^{+4.6}_{-4.0} \pm 1.6 |
| $B^0 \to K^0\pi^0$ | IV    | 7.27 [8.68]  | 8.31 [9.92]  | 10.7 [12.7]       | 14.6^{+5.9}_{-5.1} \pm 3.3 |
| $B^+ \to K^+\eta$ | IV    | 3.91 [4.37]  | 4.56 [5.10]  | 6.07 [6.80]       | < 6.9 |
| $B^+ \to K^+\eta'$ | IV    | 22.6 [26.2]  | 28.5 [33.1]  | 42.4 [49.2]       | 80^{+10}_{-9} \pm 7 |
| $B^0 \to K^0\eta$ | IV    | 3.22 [3.57]  | 3.63 [4.02]  | 4.58 [5.07]       | < 9.3 |
| $B^0 \to K^0\eta'$ | IV    | 21.9 [25.5]  | 28.2 [32.7]  | 43.0 [49.9]       | 89^{+18}_{-16} \pm 9 |
| $B^+ \to K^+K^0$ | IV    | 1.16 [1.35]  | 1.35 [1.58]  | 1.78 [2.07]       | < 5.1 |
| $B^0 \to K^0\bar{K}^0$ | IV    | 1.10 [1.28]  | 1.28 [1.49]  | 1.68 [1.96]       | < 17  |
TABLE V. $B(B \rightarrow PP)$ (in units of $10^{-6}$) in model III using the BSW form factors, with $k^2 = m_b^2/2$, $N_{eff} = 2, 3$, $\infty$, $M_{H^+} = 200\text{GeV}$ and $\theta = 0^\circ, 30^\circ$, respectively.

| Channel | $\theta = 0^\circ$ | $\delta B$ [%] | $\theta = 30^\circ$ | $\delta B$ [%] |
|---------|-----------------|----------------|-----------------|----------------|
|         | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $B^0 \rightarrow \pi^+\pi^-$ | 9.33 | 10.6 | 13.3 | 2.5 | 2.5 | 2.4 | 8.83 | 10.0 | 12.6 | -3.0 | -3.1 | -3.1 |
| $B^0 \rightarrow \pi^0\pi^0$ | 0.36 | 0.25 | 1.03 | 30 | 61 | 13 | 0.39 | 0.23 | 0.92 | 40 | 52 | -0.5 |
| $B^+ \rightarrow \pi^+\pi^0$ | 6.41 | 5.06 | 2.85 | 0.0 | 0.0 | 0.0 | 6.41 | 5.06 | 2.85 | 0.0 | 0.0 | 0.0 |
| $B^0 \rightarrow \eta\eta$ | 0.18 | 0.15 | 0.36 | 29 | 47 | 21 | 0.16 | 0.14 | 0.38 | 15 | 39 | 29 |
| $B^0 \rightarrow \eta\eta'$ | 0.19 | 0.13 | 0.46 | 29 | 68 | 20 | 0.16 | 0.12 | 0.506 | 9.6 | 57 | 30 |
| $B^0 \rightarrow \eta'\eta'$ | 0.05 | 0.02 | 0.15 | 20 | 127 | 17 | 0.04 | 0.02 | 0.16 | -5.4 | 103 | 30 |
| $B^+ \rightarrow \pi^+\eta$ | 3.82 | 3.13 | 2.20 | 8.7 | 13 | 26 | 3.48 | 2.81 | 1.95 | -1.1 | 1.2 | 12 |
| $B^+ \rightarrow \pi^+\eta'$ | 2.63 | 2.05 | 1.27 | 5.4 | 9.0 | 24 | 2.37 | 1.81 | 1.09 | -4.8 | -3.5 | 6.5 |
| $B^0 \rightarrow \pi^0\eta$ | 0.39 | 0.44 | 0.59 | 50 | 51 | 49 | 0.36 | 0.42 | 0.57 | 39 | 43 | 46 |
| $B^0 \rightarrow \pi^0\eta'$ | 0.11 | 0.14 | 0.25 | 92 | 91 | 72 | 0.10 | 0.13 | 0.25 | 65 | 76 | 70 |
| $B^+ \rightarrow K^+\pi^0$ | 17.4 | 19.6 | 24.4 | 45 | 45 | 46 | 17.2 | 19.3 | 23.8 | 43 | 43 | 43 |
| $B^0 \rightarrow K^+\pi^-$ | 26.8 | 29.9 | 36.5 | 51 | 51 | 53 | 26.5 | 29.5 | 36.1 | 49 | 49 | 51 |
| $B^+ \rightarrow K^0\pi^+$ | 29.8 | 34.6 | 45.3 | 50 | 49 | 48 | 28.6 | 33.3 | 43.5 | 44 | 43 | 42 |
| $B^0 \rightarrow K^0\pi^0$ | 11.4 | 13.0 | 16.7 | 57 | 57 | 56 | 10.8 | 12.5 | 16.1 | 49 | 50 | 51 |
| $B^+ \rightarrow K^+\eta$ | 5.69 | 6.63 | 8.78 | 45 | 45 | 45 | 5.30 | 6.22 | 8.34 | 36 | 36 | 37 |
| $B^+ \rightarrow K^+\eta'$ | 38.0 | 46.9 | 67.5 | 68 | 65 | 59 | 36.8 | 45.2 | 64.9 | 63 | 59 | 53 |
| $B^0 \rightarrow K^0\eta$ | 4.86 | 5.50 | 6.93 | 51 | 51 | 51 | 4.63 | 5.27 | 6.73 | 44 | 45 | 47 |
| $B^0 \rightarrow K^0\eta'$ | 36.7 | 45.9 | 67.3 | 67 | 63 | 57 | 35.1 | 43.8 | 64.2 | 60 | 56 | 49 |
| $B^+ \rightarrow K^+K^0$ | 1.73 | 2.01 | 2.62 | 49 | 48 | 47 | 1.64 | 1.91 | 2.49 | 41 | 41 | 40 |
| $B^0 \rightarrow K^0K^0$ | 1.64 | 1.90 | 2.48 | 49 | 48 | 47 | 1.55 | 1.80 | 2.36 | 41 | 41 | 40 |
TABLE VI. $B(B \to PP)$ (in units of $10^{-6}$) in models I and II using the BSW form factors, with $k^2 = m_b^2/2$, $N_{eff} = 2, 3, \infty$, tan $\beta = 2$ and $M_{H^+} = 200$GeV.

| Channel        | Model I | $\delta B$ [%] | Model II | $\delta B$ [%] |
|----------------|---------|----------------|----------|----------------|
| $B^0 \to \pi^+\pi^-$ | 9.11    | 10.3 13.0 | 0.1 0.1 0.1 | 9.1 10.3 13.0 | -0.5 -0.4 -0.4 |
| $B^0 \to \pi^0\pi^0$ | 0.28    | 0.15 0.92 | -0.1 -0.1 0.1 | 0.3 0.1 0.9 | -6.2 -12.6 -2.5 |
| $B^+ \to \pi^+\pi^0$ | 6.41    | 5.06 2.85 | 0.0 0.0 0.0 | 6.4 5.1 2.9 | 0.0 0.0 0.0 |
| $B^0 \to \eta\eta$ | 0.14    | 0.11 0.30 | 1.5 2.4 1.1 | 0.1 0.1 0.3 | -4.7 -7.5 -3.4 |
| $B^0 \to \eta\eta'$ | 0.14    | 0.08 0.38 | 1.0 2.3 0.7 | 0.1 0.1 0.4 | -4.8 -11.4 -3.5 |
| $B^0 \to \eta'\eta'$ | 0.04    | 0.01 0.13 | 0.3 2.0 0.2 | 0.04 0.04 0.1 | -2.7 -20.2 -3.1 |
| $B^+ \to \pi^+\eta$ | 3.53    | 2.79 1.77 | 0.4 0.5 1.0 | 3.5 2.7 1.7 | -1.5 -2.1 -4.4 |
| $B^0 \to \pi^+\eta'$ | 2.50    | 1.88 1.03 | 0.1 0.2 0.4 | 2.5 1.9 1.0 | -0.9 -1.6 -4.5 |
| $B^0 \to \pi^0\eta$ | 0.26    | 0.30 0.40 | 1.5 1.6 1.7 | 0.2 0.3 0.4 | -9.0 -9.1 -8.6 |
| $B^0 \to \pi^0\eta'$ | 0.06    | 0.08 0.15 | 0.9 0.9 0.7 | 0.05 0.1 0.1 | -15.9 -16.4 -13.5 |
| $B^+ \to K^+\pi^0$ | 12.3    | 13.8 17.1 | 2.4 2.3 2.2 | 11.2 12.5 15.4 | -7.1 -7.3 -7.5 |
| $B^0 \to K^+\pi^-$ | 18.0    | 20.0 24.2 | 1.3 1.3 1.2 | 16.2 17.9 21.6 | -9.3 -9.4 -9.7 |
| $B^+ \to K^0\pi^+$ | 20.2    | 23.6 31.1 | 1.4 1.5 1.6 | 18.1 21.2 28.0 | -9.0 -8.8 -8.5 |
| $B^0 \to K^0\pi^0$ | 7.28    | 8.33 10.7 | 0.1 0.2 0.5 | 6.4 7.4 9.5 | -11.6 -11.5 -11.1 |
| $B^+ \to K^+\eta$ | 3.91    | 4.58 6.12 | 0.0 0.3 0.8 | 3.5 4.1 5.6 | -9.6 -9.3 -8.7 |
| $B^+ \to K^+\eta'$ | 23.0    | 28.9 43.0 | 1.6 1.5 1.4 | 19.8 25.2 37.8 | -12.1 -11.6 -10.8 |
| $B^0 \to K^0\eta$ | 3.24    | 3.66 4.63 | 0.6 0.8 1.1 | 2.9 3.3 4.1 | -9.9 -9.8 -9.5 |
| $B^0 \to K^0\eta'$ | 22.3    | 28.5 43.5 | 1.4 1.4 1.3 | 19.3 24.9 38.5 | -12.2 -11.5 -10.5 |
| $B^+ \to K^+\bar{K}^0$ | 1.18    | 1.37 1.81 | 1.4 1.5 1.6 | 1.1 1.2 1.6 | -8.9 -8.7 -8.4 |
| $B^0 \to K^0\bar{K}^0$ | 1.11    | 1.30 1.71 | 1.4 1.5 1.6 | 1.0 1.2 1.5 | -8.9 -8.7 -8.4 |
TABLE VII. $B \to PV$ branching ratios (in units of $10^{-6}$) using the BSW [ LQSSR ] form factors in the SM, with with $k^2 = m_b^2/2$, $N_{eff}^{ij} = 2, 3, \infty$. The last column shows the CLEO measurements and upper limits (90% C.L.) [22-25,27].

| Channel | Class | $N_{eff}^{ij} = 2$ | $N_{eff}^{ij} = 3$ | $N_{eff}^{ij} = \infty$ | Data |
|---------|-------|--------------------|--------------------|--------------------|------|
| $B^0 \to \rho^0\pi^-$ | I | 21.1 [25.1] | 24.0 [28.5] | 30.3 [36.0] | 27.6_{-7.4}^{+8.4} \pm 4.2 |
| $B^0 \to \rho^-\pi^+$ | I | 5.7 [6.5] | 6.5 [7.4] | 8.2 [9.4] | < 5.1 |
| $B^0 \to \rho^0\pi^0$ | II | 0.49 [0.58] | 0.06 [0.07] | 2.05 [2.41] | < 10 |
| $B^+ \to \rho^0\pi^+$ | III | 5.72 [6.63] | 3.46 [3.97] | 0.71 [0.78] | 10.4_{-3.4}^{+3.3} \pm 2.1 |
| $B^+ \to \rho^+\pi^0$ | III | 13.5 [16.0] | 12.6 [15.0] | 10.9 [13.1] | < 33 |
| $B^0 \to \rho^0\eta$ | II | 0.01 [0.02] | 0.02 [0.02] | 0.06 [0.08] | < 12 |
| $B^0 \to \rho^0\eta'$ | II | 0.01 [0.01] | 0.002 [0.003] | 0.03 [0.03] | < 9 |
| $B^+ \to \rho^0\eta$ | III | 5.44 [6.57] | 4.75 [5.79] | 3.54 [4.38] | < 15 |
| $B^+ \to \rho^0\eta'$ | III | 4.35 [5.02] | 3.81 [4.40] | 2.85 [3.29] | < 12 |
| $B^0 \to \omega\pi^0$ | II | 0.29 [0.35] | 0.08 [0.09] | 0.15 [0.19] | < 5.4 |
| $B^+ \to \omega\pi^+$ | III | 6.32 [7.35] | 3.75 [4.31] | 0.78 [0.85] | 11.3_{-3.9}^{+3.3} \pm 1.4 |
| $B^0 \to \omega\eta$ | II | 0.32 [0.38] | 0.03 [0.04] | 0.82 [0.98] | < 12 |
| $B^0 \to \omega\eta'$ | II | 0.20 [0.23] | 0.001 [0.002] | 0.68 [0.79] | < 9 |
| $B^0 \to \phi\pi^0$ | V | 0.03 [0.04] | 0.002 [0.002] | 0.23 [0.27] | < 10 |
| $B^+ \to \phi\pi^+$ | V | 0.06 [0.08] | 0.004 [0.005] | 0.49 [0.58] | < 10 |
| $B^0 \to \phi\eta$ | V | 0.01 [0.01] | 0.001 [0.001] | 0.09 [0.10] | < 9 |
| $B^0 \to \phi\eta'$ | V | 0.01 [0.01] | 0.001 [0.001] | 0.07 [0.08] | < 33 |
| $B^+ \to K^{*0}K^+$ | IV | 0.42 [0.49] | 0.53 [0.61] | 0.78 [0.90] | < 5.3 |
| $B^0 \to K^{*0}K^0$ | IV | 0.40 [0.46] | 0.50 [0.58] | 0.73 [0.89] | < 17 |
| $B^+ \to K^{*+}K^0$ | V | 0.005 [0.007] | 0.002 [0.003] | 0.001 [0.001] | < 1.6 |
| $B^0 \to K^{*0}K^0$ | IV | 0.004 [0.006] | 0.002 [0.003] | 0.001 [0.001] | < 38 |
| $B^0 \to \rho^0K^+$ | IV | 0.52 [0.60] | 0.53 [0.62] | 0.71 [0.83] | < 27 |
| $B^+ \to \rho^0K^+$ | IV | 0.39 [0.46] | 0.31 [0.36] | 0.31 [0.36] | < 17 |
| $B^0 \to \rho^-K^+$ | I | 0.54 [0.62] | 0.59 [0.68] | 0.70 [0.81] | < 25 |
| $B^+ \to \rho^+K^+$ | IV | 0.11 [0.12] | 0.05 [0.05] | 0.01 [0.01] | < 12 |
| $B^+ \to K^{*+}\eta$ | IV | 2.43 [3.12] | 2.39 [3.04] | 2.32 [2.89] | < 27 |
| $B^+ \to K^{*+}\eta'$ | III | 0.66 [1.14] | 0.36 [0.61] | 0.24 [0.23] | < 5.4 |
| $B^0 \to K^{*0}\eta$ | IV | 2.32 [2.98] | 2.54 [3.23] | 3.06 [3.82] | 13.8_{-1.6}^{+5.5} \pm 1.6 |
| $B^0 \to K^{*0}\eta'$ | V | 0.33 [0.69] | 0.09 [0.23] | 0.31 [0.26] | < 20 |
| $B^0 \to K^{*+}\pi^-$ | IV | 8.59 [10.2] | 9.67 [11.5] | 12.0 [14.3] | 22_{-6.5}^{+8.4} |
| $B^0 \to K^{*0}\pi^0$ | IV | 2.44 [2.77] | 3.02 [3.43] | 4.42 [5.01] | < 3.6 |
| $B^+ \to K^{*+}\pi^0$ | IV | 4.95 [6.09] | 5.55 [6.84] | 6.91 [8.52] | < 31 |
| $B^+ \to K^{*0}\pi^+$ | IV | 7.35 [8.75] | 9.23 [11.0] | 13.6 [16.2] | < 16 |
| $B^+ \to \phi K^+$ | V | 22.1 [25.7] | 11.5 [13.4] | 0.60 [0.70] | 17.3_{-5.4}^{+6.7} \pm 1.8 |
| $B^0 \to \phi K^0$ | V | 20.9 [24.3] | 10.9 [12.6] | 0.57 [0.66] | < 28 |
| $B^0 \to \omega K^0$ | V | 3.31 [3.86] | 0.002 [0.003] | 13.3 [15.4] | < 21 |
| $B^+ \to \omega K^+$ | V | 3.53 [4.11] | 0.25 [0.28] | 16.5 [19.2] | < 7.9 |
TABLE VIII. \( B \to PV \) branching ratios (in units of \( 10^{-6} \)) using the BSW form factors in model III, assuming \( M_{H^+} = 200 \text{GeV} \), \( \theta = 0^\circ \) and \( N_{\text{eff}} = 2, 3, \infty \).

| Channel \( \rightarrow \) | Class | SM | \( \infty \) | Model III | \( \infty \) | \( \delta B \) [%] |
|------------------------|------|-----|-----|----------|-----|-------------|
| \( B^0 \to \rho^+ \pi^- \) | I | 21.1 | 24.0 | 30.3 | 21.2 | 24.1 | 30.5 | 0.7 | 0.7 | 0.7 |
| \( B^0 \to \rho^- \pi^+ \) | I | 5.70 | 6.48 | 8.19 | 5.70 | 6.48 | 8.19 | 0.0 | 0.0 | 0.0 |
| \( B^0 \to \rho^0 \pi^0 \) | II | 0.49 | 0.06 | 2.05 | 0.54 | 0.11 | 2.12 | 9.8 | 99.6 | 3.5 |
| \( B^+ \to \rho^0 \pi^+ \) | II | 5.72 | 3.46 | 0.71 | 5.79 | 3.54 | 0.81 | 1.3 | 2.3 | 14.0 |
| \( B^+ \to \rho^+ \pi^0 \) | III | 13.5 | 12.6 | 10.9 | 13.6 | 12.7 | 11.0 | 0.4 | 0.5 | 0.7 |
| \( B^0 \to \rho^0 \eta \) | II | 0.01 | 0.02 | 0.06 | 0.03 | 0.03 | 0.08 | 86.0 | 100 | 40.1 |
| \( B^0 \to \rho^0 \eta' \) | II | 0.01 | 0.003 | 0.03 | 0.004 | 0.001 | 0.03 | 47.3 | 54.4 | 18.1 |
| \( B^+ \to \rho^+ \eta \) | III | 5.44 | 4.75 | 3.54 | 5.46 | 4.79 | 3.59 | 0.5 | 0.7 | 1.4 |
| \( B^+ \to \rho^+ \eta' \) | III | 4.35 | 3.81 | 2.85 | 4.34 | 3.81 | 2.86 | 0.2 | 0.08 | 0.4 |
| \( B^0 \to \omega \pi^0 \) | II | 0.29 | 0.08 | 0.15 | 0.45 | 0.14 | 0.15 | 54.4 | 77.0 | 0.8 |
| \( B^+ \to \omega \pi^+ \) | III | 6.32 | 3.75 | 0.78 | 6.63 | 3.86 | 0.79 | 5.0 | 3.1 | 1.1 |
| \( B^0 \to \omega \eta \) | II | 0.32 | 0.03 | 0.82 | 0.37 | 0.05 | 0.83 | 16.3 | 67.1 | 0.2 |
| \( B^0 \to \omega \eta' \) | II | 0.20 | 0.001 | 0.68 | 0.22 | 0.004 | 0.69 | 9.5 | 155 | 0.5 |
| \( B^0 \to \phi \pi^0 \) | V | 0.03 | 0.002 | 0.23 | 0.05 | 0.002 | 0.33 | 59.1 | 1.9 | 42.3 |
| \( B^+ \to \phi \pi^+ \) | V | 0.06 | 0.004 | 0.49 | 0.10 | 0.004 | 0.69 | 59.1 | 1.9 | 42.3 |
| \( B^0 \to \phi \eta \) | V | 0.01 | 0.001 | 0.09 | 0.02 | 0.001 | 0.12 | 59.1 | 1.9 | 42.3 |
| \( B^0 \to \phi \eta' \) | V | 0.01 | 0.001 | 0.07 | 0.01 | 0.001 | 0.10 | 59.1 | 1.9 | 42.3 |
| \( B^+ \to K^{*0}K^+ \) | IV | 0.42 | 0.53 | 0.78 | 0.68 | 0.83 | 1.19 | 61.0 | 57.9 | 53.3 |
| \( B^0 \to K^{*0}K^0 \) | IV | 0.40 | 0.50 | 0.73 | 0.64 | 0.79 | 1.12 | 61.0 | 57.9 | 53.3 |
| \( B^+ \to K^{*+}K^0 \) | V | 0.005 | 0.002 | 0.001 | 0.002 | 0.001 | 0.003 | -58.4 | -72.5 | 256 |
| \( B^0 \to K^{*0}K^0 \) | IV | 0.004 | 0.002 | 0.001 | 0.002 | 0.001 | 0.003 | -58.4 | -72.5 | 256 |
| \( B^0 \to \rho^0 K^0 \) | IV | 0.52 | 0.53 | 0.71 | 0.43 | 0.44 | 0.60 | -17.1 | -18.3 | -16.5 |
| \( B^+ \to \rho^0 K^+ \) | IV | 0.39 | 0.31 | 0.31 | 0.43 | 0.36 | 0.40 | 8.0 | 16.4 | 30.6 |
| \( B^0 \to \rho^- K^+ \) | I | 0.54 | 0.59 | 0.70 | 0.47 | 0.52 | 0.62 | -13.1 | -12.7 | -11.8 |
| \( B^+ \to \rho^+ K^0 \) | IV | 0.11 | 0.05 | 0.01 | 0.05 | 0.01 | 0.02 | -50.8 | -70.1 | 363 |
| \( B^+ \to K^{*+} \eta \) | IV | 2.43 | 2.39 | 2.32 | 3.27 | 3.29 | 3.34 | 34.4 | 37.4 | 43.5 |
| \( B^+ \to K^{*+} \eta' \) | III | 0.66 | 0.36 | 0.24 | 0.31 | 0.24 | 0.65 | -52.2 | -34.2 | 170 |
| \( B^0 \to K^{*0} \eta \) | IV | 2.32 | 2.54 | 3.06 | 3.15 | 3.47 | 4.20 | 35.8 | 36.5 | 37.4 |
| \( B^0 \to K^{*0} \eta' \) | V | 0.33 | 0.09 | 0.31 | 0.08 | 0.10 | 0.96 | -77.3 | 6.9 | 204 |
| \( B^0 \to K^{*+} \pi^- \) | IV | 8.59 | 9.67 | 12.0 | 13.6 | 15.4 | 19.1 | 58.6 | 58.8 | 59.3 |
| \( B^0 \to K^{*0} \pi^0 \) | IV | 2.44 | 3.02 | 4.42 | 4.26 | 5.18 | 7.34 | 74.9 | 71.6 | 66.0 |
| \( B^+ \to K^{*+} \pi^0 \) | IV | 4.95 | 5.55 | 6.91 | 7.42 | 8.38 | 10.5 | 49.9 | 50.9 | 52.2 |
| \( B^+ \to K^{*0} \pi^+ \) | IV | 7.35 | 9.23 | 13.6 | 11.9 | 14.7 | 21.0 | 62.0 | 58.8 | 54.1 |
| \( B^+ \to \phi K^+ \) | V | 22.1 | 11.5 | 0.60 | 35.7 | 19.0 | 1.29 | 61.5 | 65.3 | 113 |
| \( B^0 \to \phi K^0 \) | V | 20.9 | 10.9 | 0.57 | 33.7 | 18.0 | 1.21 | 61.5 | 65.3 | 113 |
| \( B^0 \to \omega K^0 \) | V | 3.31 | 0.002 | 13.3 | 5.33 | 0.01 | 19.4 | 60.9 | 175 | 46.5 |
| \( B^+ \to \omega K^+ \) | V | 3.53 | 0.25 | 16.5 | 5.57 | 0.23 | 23.6 | 57.8 | -7.7 | 42.9 |
TABLE IX. $B \rightarrow PV$ branching ratios (in units of $10^{-6}$) using the BSW form factors in models I and II, assuming $M_{H^+} = 200 \text{ GeV}$, $\tan\beta = 2$ and $N^{\text{eff}} = 2, 3, \infty$.

| Channel       | Model I   | $\delta B$ [%] | Model II  | $\delta B$ [%] |
|---------------|-----------|----------------|-----------|----------------|
| B$^0 \rightarrow \rho^+ \pi^-$ | 21.1      | 24.0           | 0.0       | 0.0            |
| B$^0 \rightarrow \rho^- \pi^+$ | 5.70      | 6.48           | 0.0       | 0.0            |
| B$^0 \rightarrow \rho^0 \pi^0$  | 0.49      | 0.06           | 0.0       | 0.0            |
| B$^+ \rightarrow \rho^0 \pi^0$  | 5.72      | 3.46           | 0.1       | 0.0            |
| B$^+ \rightarrow \rho^+ \pi^+$  | 13.5      | 12.6           | 0.0       | 0.0            |
| B$^0 \rightarrow \rho^+ \eta$   | 0.02      | 0.02           | 0.0       | 0.0            |
| B$^0 \rightarrow \rho^0 \eta'$  | 0.01      | 0.02           | 0.0       | 0.0            |
| B$^+ \rightarrow \rho^+ \eta$   | 5.44      | 4.76           | 0.0       | 0.0            |
| B$^+ \rightarrow \rho^+ \eta'$  | 4.35      | 3.81           | 0.0       | 0.0            |
| B$^0 \rightarrow \omega \pi^0$   | 0.29      | 0.08           | 0.0       | 0.0            |
| B$^+ \rightarrow \omega \pi^0$   | 0.32      | 0.03           | 0.0       | 0.0            |
| B$^0 \rightarrow \omega \eta$    | 0.209     | 0.01           | 0.0       | 0.0            |
| B$^0 \rightarrow \phi \pi^0$     | 0.03      | 0.002          | 0.0       | 0.0            |
| B$^+ \rightarrow \phi \pi^0$     | 0.06      | 0.004          | 0.0       | 0.0            |
| B$^0 \rightarrow \phi \eta$      | 0.01      | 0.001          | 0.0       | 0.0            |
| B$^+ \rightarrow \phi \eta'$     | 0.01      | 0.001          | 0.0       | 0.0            |
| B$^+ \rightarrow K^{*0}K^+$     | 0.43      | 0.54           | 1.8       | 1.9            |
| B$^0 \rightarrow K^{*0}K^0$     | 0.40      | 0.51           | 1.8       | 1.9            |
| B$^+ \rightarrow K^{*+}K^{0}$   | 0.005     | 0.002          | 3.5       | 7.1            |
| B$^0 \rightarrow K^{*+}K^0$     | 0.004     | 0.002          | 3.5       | 7.1            |
| B$^0 \rightarrow \rho^0 K^0$     | 0.53      | 0.55           | 3.1       | 3.3            |
| B$^+ \rightarrow \rho^0 K^+$     | 0.40      | 0.32           | 2.4       | 4.9            |
| B$^0 \rightarrow \rho^- K^+$     | 0.53      | 0.58           | 0.7       | 1.8            |
| B$^+ \rightarrow \rho^+ K^+$     | 0.10      | 0.04           | 0.0       | 0.0            |
| B$^+ \rightarrow K^{*+}\eta$     | 2.49      | 2.45           | 2.5       | 2.3            |
| B$^+ \rightarrow K^{*+}\eta'$   | 0.64      | 0.35           | 0.2       | 0.2            |
| B$^0 \rightarrow K^{*0} \eta$    | 2.37      | 2.60           | 3.1       | 3.3            |
| B$^0 \rightarrow K^{*0} \eta'$  | 0.32      | 0.09           | 0.3       | 0.3            |
| B$^0 \rightarrow K^{*+} \pi^-$   | 8.76      | 9.84           | 12.2      | 2.0            |
| B$^0 \rightarrow K^{*0} \pi^0$  | 2.44      | 3.03           | 4.45      | 0.1            |
| B$^+ \rightarrow K^{*+} \pi^0$  | 5.11      | 5.72           | 7.10      | 3.2            |
| B$^+ \rightarrow K^{*0} \pi^0$  | 7.48      | 9.41           | 13.9      | 1.8            |
| B$^+ \rightarrow \phi K^+$       | 22.3      | 11.6           | 0.61      | 1.1            |
| B$^0 \rightarrow \phi K^0$      | 21.1      | 11.0           | 0.57      | 1.1            |
| B$^0 \rightarrow \omega K^0$    | 3.35      | 0.002          | 13.4      | 1.2            |
| B$^+ \rightarrow \omega K^+$    | 3.59      | 0.25           | 16.7      | 1.6            |
TABLE X. \( B \rightarrow VV \) branching ratios (in units of \( 10^{-6} \)) using the BSW [ LQQSR ] form factors in the SM, with \( k^2 = m_b^2/2 \), \( N_{\text{eff}} = 2, 3, \infty \). The last column shows the CLEO upper limits (90\% C.L.) [22-25].

| Channel | Class | \( N_{\text{eff}} = 2 \) | \( N_{\text{eff}} = 3 \) | \( N_{\text{eff}} = \infty \) | Data |
|---------|-------|----------------|----------------|----------------|-------|
| \( B^0 \rightarrow \rho^+ \rho^- \) | I | 17.8 [19.8] | 20.2 [22.5] | 25.5 [28.4] | < 2200 |
| \( B^0 \rightarrow \rho^0 \rho^0 \) | II | 0.39 [0.44] | 0.09 [0.10] | 1.56 [1.73] | < 4.8 |
| \( B^0 \rightarrow \omega\omega \) | II | 0.81 [0.90] | 0.15 [0.17] | 1.22 [1.35] | < 19 |
| \( B^+ \rightarrow \rho^+ \rho^0 \) | III | 12.8 [14.3] | 10.1 [11.3] | 5.69 [6.33] | < 120 |
| \( B^+ \rightarrow \rho^+ \omega \) | III | 15.7 [17.4] | 12.2 [13.5] | 6.69 [7.45] | < 47 |
| \( B^0 \rightarrow K^{*+} \rho^- \) | IV | 6.17 [6.82] | 6.95 [7.68] | 8.64 [9.55] | – |
| \( B^0 \rightarrow K^{*0} \rho^0 \) | IV | 1.73 [1.82] | 2.01 [2.11] | 2.79 [2.91] | < 16.1 |
| \( B^+ \rightarrow K^{*+} \rho^0 \) | IV | 5.22 [5.97] | 5.97 [6.82] | 7.76 [8.91] | < 52 |
| \( B^+ \rightarrow K^{*0} \rho^+ \) | IV | 6.65 [7.35] | 8.36 [9.24] | 12.4 [13.7] | – |
| \( B^+ \rightarrow K^{*+} K^{*0} \) | IV | 0.38 [0.49] | 0.48 [0.61] | 0.70 [0.90] | < 62 |
| \( B^0 \rightarrow K^{*0} \bar{K}^{*0} \) | IV | 0.36 [0.47] | 0.46 [0.58] | 0.67 [0.86] | < 7.4 |
| \( B^0 \rightarrow \rho^0 \omega \) | V | 0.45 [0.50] | 0.24 [0.27] | 0.02 [0.02] | < 11 |
| \( B^0 \rightarrow K^{*0} \omega \) | V | 13.5 [16.1] | 4.52 [5.03] | 1.04 [1.79] | < 19 |
| \( B^+ \rightarrow K^{*+} \omega \) | V | 13.4 [16.1] | 3.94 [4.39] | 2.74 [4.01] | < 52 |
| \( B^+ \rightarrow K^{*+} \phi \) | V | 21.8 [27.8] | 11.3 [14.5] | 0.60 [0.76] | < 41 |
| \( B^0 \rightarrow K^{*0} \phi \) | V | 20.6 [26.2] | 10.7 [13.6] | 0.56 [0.72] | < 21 |
| \( B^+ \rightarrow \rho^+ \phi \) | V | 0.06 [0.07] | 0.004 [0.005] | 0.47 [0.52] | < 16 |
| \( B^0 \rightarrow \rho^0 \phi \) | V | 0.03 [0.03] | 0.001 [0.002] | 0.22 [0.25] | < 13 |
| \( B^0 \rightarrow \omega \phi \) | V | 0.03 [0.03] | 0.001 [0.002] | 0.22 [0.24] | < 21 |
TABLE XI. $B \to VV$ branching ratios (in units of $10^{-6}$) using the BSW form factors in model III, assuming $M_{R^+} = 200$ GeV, $N_{eff} = 2, 3, \infty$ and $\theta = 0^0, 30^0$.

| Channel                      | $\theta = 0^0$ | $\delta B [\%]$ | $\theta = 30^0$ | $\delta B [\%]$ |
|------------------------------|----------------|------------------|-----------------|-----------------|
| $B^0 \to \rho^+\rho^-$       | 17.9           | 20.3             | 25.7            | 0.7             |
| $B^0 \to \rho^0\rho^0$       | 0.46           | 0.16             | 1.65            | 16              |
| $B^0 \to \omega\omega$       | 1.05           | 0.24             | 1.23            | 29              |
| $B^+ \to \rho^+\rho^0$       | 12.8           | 10.1             | 5.69            | 0.0             |
| $B^+ \to \rho^+\omega$       | 13.8           | 10.9             | 6.12            | 12              |
| $B^0 \to K^{*+}\rho^-$       | 9.78           | 11.0             | 13.8            | 59              |
| $B^0 \to K^{*0}\rho^0$       | 3.18           | 3.70             | 4.98            | 84              |
| $B^+ \to K^{*+}\rho^0$       | 7.61           | 8.75             | 11.4            | 46              |
| $B^+ \to K^{*0}\rho^+$       | 10.8           | 13.3             | 19.0            | 62              |
| $B^+ \to K^{*+}\rho^+$       | 0.61           | 0.75             | 1.08            | 61              |
| $B^0 \to K^{*0}\bar{K}^{*0}$ | 0.59           | 0.72             | 1.03            | 61              |
| $B^0 \to \rho^0\omega$       | 0.72           | 0.39             | 0.04            | 61              |
| $B^0 \to K^{*0}\omega$       | 20.8           | 6.97             | 1.51            | 53              |
| $B^+ \to K^{*+}\omega$       | 20.8           | 6.23             | 3.47            | 55              |
| $B^+ \to K^{*+}\phi$         | 35.2           | 18.8             | 1.27            | 62              |
| $B^0 \to K^{*0}\phi$         | 33.2           | 17.7             | 1.20            | 62              |
| $B^+ \to \rho^+\phi$         | 0.10           | 0.004            | 0.67            | 59              |
| $B^0 \to \rho^0\phi$         | 0.05           | 0.002            | 0.32            | 59              |
| $B^0 \to \omega\phi$         | 0.05           | 0.002            | 0.32            | 59              |
TABLE XII. $B \to VV$ branching ratios (in units of $10^{-6}$) using the BSW form factors in models I and II, assuming $M_{H^+} = 200$ GeV, $N^\text{eff} = 2, 3, \infty$ and $\tan \beta = 2$.

| Channel   | 2     | 3     | $\infty$ | $\delta B$ [%] | 2     | 3     | $\infty$ | $\delta B$ [%] |
|-----------|-------|-------|----------|----------------|-------|-------|----------|----------------|
| $B^0 \to \rho^+ \rho^-$ | 17.7  | 19.8  | 24.4     | 0.0            | 17.7  | 20.2  | 24.3     | -0.1          |
| $B^0 \to \rho^0 \rho^0$  | 0.53  | 0.10  | 1.21     | 0.1            | 0.52  | 0.08  | 1.20     | -1.8          |
| $B^0 \to \omega \omega$  | 0.90  | 0.16  | 0.93     | 0.6            | 0.86  | 0.14  | 0.93     | -4.2          |
| $B^+ \to \rho^+ \rho^0$  | 13.5  | 10.7  | 6.04     | 0.0            | 13.6  | 10.1  | 6.04     | 2.1           |
| $B^+ \to \rho^+ \omega$  | 14.6  | 11.6  | 6.50     | -10.9          | 14.6  | 10.9  | 6.50     | -11           |
| $B^0 \to K^{*+} \rho^-$ | 6.42  | 7.13  | 8.66     | 2.1            | 5.70  | 6.25  | 7.71     | -9.3          |
| $B^0 \to K^{*0} \rho^0$  | 1.70  | 1.97  | 2.72     | 0.3            | 1.47  | 1.70  | 2.36     | -13           |
| $B^+ \to K^{*+} \rho^0$  | 5.44  | 6.08  | 7.63     | 2.8            | 4.90  | 5.55  | 6.83     | -7.5          |
| $B^+ \to K^{*0} \rho^+$  | 6.55  | 8.13  | 11.8     | 1.8            | 5.78  | 7.51  | 10.5     | -10           |
| $B^0 \to K^{*0} \bar{K}^{*0}$ | 0.37  | 0.46  | 0.67     | 1.8            | 0.33  | 0.43  | 0.60     | -10           |
| $B^0 \to K^{*0} \rho^-$  | 0.41  | 0.23  | 0.02     | 1.1            | 0.37  | 0.21  | 0.02     | -10           |
| $B^0 \to K^{*0} \omega$  | 12.3  | 4.15  | 0.92     | 1.3            | 11.1  | 4.07  | 0.83     | -9.3          |
| $B^+ \to K^{*+} \omega$  | 12.6  | 3.85  | 2.39     | 1.6            | 11.3  | 3.52  | 2.24     | -9.4          |
| $B^+ \to K^{*+} \phi$    | 20.4  | 10.8  | 0.65     | 1.4            | 18.0  | 10.0  | 0.54     | -11           |
| $B^0 \to K^{*0} \phi$    | 19.2  | 10.1  | 0.61     | 1.1            | 17.0  | 9.41  | 0.51     | -11           |
| $B^0 \to \rho^0 \phi$    | 0.1   | 0.01  | 0.44     | -0.7           | 0.05  | 0.005 | 0.40     | -11           |
| $B^0 \to \omega \phi$    | 0.02  | 0.002 | 0.21     | -0.7           | 0.02  | 0.002 | 0.19     | -11           |


FIG. 1. Typical one-loop Feynman diagrams for the quark level decays $b \rightarrow (s, d)V^*$ ($V = \gamma, Z^0, g$), with $W^\pm$ (internal wave lines) and charged-Higgs exchanges (internal dashed lines) in the SM and two-Higgs-doublet models. The internal quarks are the upper type quark $u, c$ and $t$. 
FIG. 2. Branching ratios $\mathcal{B}(B \to \pi^+\pi^-)$ versus $M_{H^+}$ and $1/N^{\text{eff}}$ in the SM and 2HDM’s. For (a) and (b), we set $N^{\text{eff}} = 3$ and $M_{H^+} = 200\text{GeV}$, respectively. The four adjacent curves are the theoretical predictions in the SM and models I, II and III respectively. The band between two dots lines shows the CLEO data with $1\sigma$ error: $\mathcal{B}(B \to \pi^+\pi^-) = (4.3^{+1.7}_{-1.6}) \times 10^{-6}$.
FIG. 3. Branching ratios $\mathcal{B}(B \rightarrow K^+\pi^0)$ versus $M_{H^+}$ and $1/N_{\text{eff}}$ in the SM and 2HDM’s. For (a) and (b), we set $N_{\text{eff}} = 3$ and $M_{H^+} = 200$GeV, respectively. The dot-dashed, short-dashed, long-dashed and solid curve correspond to the theoretical predictions in the SM and models I, II and III, respectively. The theoretical uncertainties are not shown here. The band between two dots lines shows the CLEO data with $2\sigma$ errors: $\mathcal{B}(B \rightarrow K^+\pi^0) = (11.6^{+6.6}_{-6.0}) \times 10^{-6}$. 
FIG. 4. Same as Fig.3, but for the decay $B \rightarrow K^+\pi^-$. The dots band corresponds to the CLEO data with $2\sigma$ errors: $B(B \rightarrow K^+\pi^-) = (17.2^{+5.6}_{-5.4}) \times 10^{-6}$. 
FIG. 5. Same as Fig.3, but for the decay $B \rightarrow K^0\pi^+$. The dots band corresponds to the CLEO data with $2\sigma$ errors: $B(B \rightarrow K^0\pi^+) = (18.2^{+9.8}_{-8.6}) \times 10^{-6}$. 
FIG. 6. Same as Fig. 3, but for the decay $B \to K^0\pi^0$. The dots band corresponds to the CLEO data with 1σ error: $\mathcal{B}(B \to K^0\pi^0) = (14.6^{+6.4}_{-6.1}) \times 10^{-6}$. 
FIG. 7. Plots of $\mathcal{B}(B^+ \to K^+\eta')$ versus $M_{H^+}$ and $1/N_{\text{eff}}$ in the SM and 2HDM’s. For (a) and (b), we set $N_{\text{eff}} = 3$ and $M_{H^+} = 200\text{GeV}$, respectively. The dot-dashed curve and the closely adjacent short-dashed curve refer to the theoretical predictions in the SM and model I; while the long-dashed and solid curve correspond to the theoretical predictions in models II and III, respectively. The theoretical uncertainties are not shown here. The dots band corresponds to the CLEO data with $2\sigma$ errors: $\mathcal{B}(B^+ \to K^+\eta') = (80^{+24}_{-22}) \times 10^{-6}$.
FIG. 8. Plots of $\mathcal{B}(B^+ \to K^+\eta')$ versus $M_{H^+}$ in the SM and model III. The dot-dashed line shows the SM prediction with $N_{\text{eff}} = 3$. The short-dashed, long-dashed and solid curve correspond to model III predictions for $N_{\text{eff}} = 2, 3, \infty$, respectively. Other theoretical uncertainties are not shown here. The dots band corresponds to the CLEO data with $2\sigma$ errors: $\mathcal{B}(B^+ \to K^+\eta') = (80^{+24}_{-22}) \times 10^{-6}$.
FIG. 9. Same as Fig. 7, but for the decay $B \to K^0\eta'$. The dots band corresponds to the CLEO data with 2σ errors: $B(B^0 \to K^0\eta') = (89^{+40}_{-36}) \times 10^{-6}$. 
FIG. 10. Same as Fig. 8, but for the decay $B \rightarrow K^0\eta'$. The dots band corresponds to the CLEO data with $2\sigma$ errors: $B(B^0 \rightarrow K^0\eta') = (89^{+40}_{-36}) \times 10^{-6}$. 
FIG. 11. $B(B \to K^{*+}\eta)$ versus $M_{H^+}$ and $1/N^{eff}$ in the SM and 2HDM’s. For (a) and (b), we set $N^{eff} = 3$ and $M_{H^+} = 200\text{GeV}$, respectively. The dot-dashed, long-dashed and solid curve show the theoretical predictions in the SM and models II and III, respectively. The theoretical uncertainties are not shown here. The dots band corresponds to the CLEO data with 1$\sigma$ error: $B(B^+ \to K^{*+}\eta) = (26.4^{+10.2}_{-8.8}) \times 10^{-6}$. 
FIG. 12. Same as Fig.11, but for the decay $B \to K^{*0}\eta$. The dots band corresponds to the CLEO data with $1\sigma$ error: $\mathcal{B}(B^0 \to K^{*0}\eta) = (13.8^{+5.7}_{-4.9}) \times 10^{-6}$.
FIG. 13. Same as Fig. 11, but for the decay $B \to K^{*+}\pi^-$. The dots band corresponds to the CLEO data with $1\sigma$ error: $B(B^0 \to K^{*+}\pi^-) = (22^{+8.9}_{-7.8}) \times 10^{-6}$. 
FIG. 14. Same as Fig.11, but for the decay $B \rightarrow K^+\phi$. The dots band corresponds to the Belle data with 2σ errors: $B(B^+ \rightarrow K^+\phi) = (17.2^{+13.8}_{-11.4}) \times 10^{-6}$. 
FIG. 15. $B(B \to K^{*0} \omega)$ versus $M_{H^+}$ and $N_{\text{eff}}$ in the SM and 2HDM’s. For (a) and (b), we set $N_{\text{eff}} = 3$ and $M_{H^+} = 200\text{GeV}$, respectively. The upper dots line shows the CLEO upper limit: $B(B \to K^{*0} \omega) \leq 19 \times 10^{-6}$. The dot-dashed, long-dashed and solid curve correspond to the theoretical prediction in the SM and models II and III, respectively. The theoretical uncertainties are not shown here.
FIG. 16. Same as Fig. 15, but for the decay $B \to K^{*+}\phi$. The upper dots line shows the CLEO upper limit: $B(B \to K^{*+}\phi) \leq 41 \times 10^{-6}$. 
FIG. 17. Same as Fig. 15, but for the decay $B \to K^{*0}\phi$. The upper dots line shows the CLEO upper limit: $B(B \to K^{*0}\phi) \leq 21 \times 10^{-6}$.