Capture of cosmic objects by central gravitational field of a galaxy cluster

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Abstract

The effect of capture of a cosmic object by the central gravitational field of a galaxy cluster is described in the expanding Universe. The cosmic evolution can be the origin of the capture explaining formation of galaxies and their clusters from the homogenous distribution of matter in the Universe. The Newton–like equation of the capture is derived for arbitrary equations of state in terms of the red shift parameter, and the influence of the Hubble velocities on the rotational curves is studied. The obtained rotational curves show us that the deficit of the visible matter for superclusters $M \sim 10^{15} M_\odot$ can be increased in the version of cosmology where observational quantities are identified with the conformal ones.

Key words: Cosmology, Constant rotational curves

Introduction

Mechanism of formation of galaxies and their clusters from homogenous distribution of matter in the Universe is not yet understood. The main problem is to clear up how the energy of two interaction particles increases to be negative so that the formation of a bound state is possible. The evolution of the Universe as a mechanism of braking of a cosmic object in the central gravitational field was probably firstly considered by Einstein and Strauss in 1945 [1]. This idea was developed in terms of conformal variables and coordinates in [2, 3] where it was shown that the cosmic evolution can give rise to increasing of...
energy, and it can be the origin of formation of galaxies and their clusters due to a capture of cosmic objects by the central gravitational field. In this paper, we consider the phenomenon of capture of objects by gravitational fields in the expanding Universe for any equations of state and the central fields to compare different cosmological models. The article starts with the determination of the gravitational potential as a component of the metrics by the cosmological perturbation theory (Section 1). In Section 2, cosmic evolution is described. Section 3 describes the dynamics of a free particle in the expanding Universe with the Friedman–Lemaitre–Robertson–Walker (FLRW) metrics. Sections 4 and 5, are devoted to dynamics and capture of a particle in the Newton–like potential. The conclusion completes this paper.

1 Metrics

We consider the motion of a test particle in the space with an interval

$$ds^2 = a(\eta)^2 \left[ (1 - \Phi) d\eta^2 - (1 + \Phi) \left( dx^i + N^i d\eta \right)^2 \right],$$

where the metric components can be determined by the Einstein equations in the cosmological perturbation theory [4–6]:

$$a^2 \Delta \Phi = -8\pi G T^0_0,$$

$$\left[ \frac{a^2}{2} \left( 2\partial_i N^i - 3\Phi' \right) \right]' = 8\pi G T^k_k,$$

where $\Phi' = d\Phi/d\eta$ and $T^0_0$ and $T^k_k$ are the components of an energy–momentum tensor of the gravitational center. Equations (2), (3) show us that there is a reference frame with a zero pressure $T^k_k = 0$ and nonzero shift vector $2\partial_i N^i = 3\Phi'$ in which the solution of equations (2), (3) take the Newton-like form

$$\Phi(x) = 2G \int d^3 y \frac{T_{00}(y)}{|y - x|}, \quad N^i(x) = \partial^i \int d^3 y \frac{3 \Phi'(y)}{2|y - x|}$$

in the case of an arbitrary equation of state. For a point source $T_{00} = M\delta^3(x)$ it takes the form $\Phi_{\text{point}} = r_g/r$, where $r_g = 2GM$. The cylindric symmetry of a source $T_{00} = \frac{M}{2l} \delta(x_1) \delta(x_2) \theta(l - |x_3|)$ leads to another potential with the derivative

$$\frac{d\Phi_{\text{cyl}}}{dr} = -\frac{r_g}{r\sqrt{l^2 + r^2}}$$
instead of \(d\Phi_{\text{point}}/dr = -r_g/r^2\).

## 2 Cosmic evolution

The dependence of the scale factor \((a)\) on the conformal time \((\eta)\) is given by the Einstein—Friedmann equation [7]:

\[
H_0^{-1} \left(\frac{da}{d\eta}\right)^2 = \Omega(a) \equiv \Omega_{\text{Stiff}} a^{-2} + \Omega_{\text{Radiation}} + \Omega_{\text{Matter}} a + \Omega_{\Lambda} a^4
\]

where \(\Omega_i\) is the sum of the partial densities: stiff, radiation, matter, and \(\Lambda\)-term—state, respectively, normalized, by the unit \((\Omega_{\mid a=1} = 1)\); \(H_0\) is the present—day value of the Hubble parameter. The best fit to the Supernova data [8] requires a cosmological constant \(\Omega_{\text{Stiff}} = 0, \Omega_{\Lambda} = 0.7\) and \(\Omega_{\text{Matter}} = 0.3\) in the case of Standard cosmology (SC), where the measurable distance is identified with the world space interval

\[
R^C_{\text{measurable}} = R = a(t)r; \quad r = (x^1)^2 + (x^2)^2 + (x^3)^2.
\]

In the conformal cosmology [9,10], measurable time and distance are identified with the conformal quantities \((r, \eta)\)

\[
R^C_{\text{measurable}} = r; \quad d\eta = dt/a(t).
\]

In this case the Supernova data [8] are consistent with the dominance of the stiff state [10,11], \(\Omega_{\text{Stiff}} \simeq 0.85 \pm 0.15, \Omega_{\text{Matter}} = 0.15 \pm 0.15\). In the case \(\Omega_{\text{Stiff}} = 1\), we have the square root dependence of the scale factor on conformal time

\[
a(\eta) = \sqrt{1 + 2H_0(\eta - \eta_0)}
\]

that does not contradict the standard description of nucleosynthesis [11,12] where the Friedmann time is replaced by the conformal one.
3 “Free” motion of a particle in FRWL metrics

A free motion of a particle in the conformal–flat metrics

\[(ds^2) = (dt)^2 - a^2(t)(dx^i)^2 = a^2(\eta) \left[ d\eta^2 - (dx^i)^2 \right] \]

follows from the definition of the corresponding one–particle energy in the field theory [13]

\[ E = p_0 = \sqrt{p_i^2 + m_0^2 a^2(\eta)} = m_0 a(\eta) + \frac{p^2}{2 m_0 a(\eta)}. \]

The nonrelativistic action

\[ S_0 = \int_{\eta_0}^{\eta_1} d\eta \left[ p_i x_i' - p_0 + m_0 a(\eta) \right] \approx \int_{\eta_0}^{\eta_1} d\eta \left[ p_i x_i' - \frac{p_i^2}{2 m_0 a(\eta)} \right] \]

coincides, in terms of the “world time” \( t \), with the action considered in [14]

\[ S_0 = \int_{t_1}^{t_0} dt \left[ p_i \dot{x}_i - \frac{p_i^2}{2 m_0 a^2(t)} \right] \quad \left( \dot{x} = \frac{dx_i}{dt} \right). \]

All solutions of the equations derived from (12) can be obtained by the conformal transformation of the solutions

\[ x_i(\eta) = x_i^{(0)} + \frac{p_i^{(0)}}{m_0} \int_{\eta_0}^{\eta} d\eta \frac{a(\eta)}{a(\eta)} = x_i^{(I)} + \frac{p_i^{(I)}}{m_0} \int_{\eta_1}^{\eta} d\eta \frac{a_I(\eta)}{a_I(\eta)} \]

of the equations derived from the action (11), where \( a_I = a(\eta = \eta_I) \), \( p_i^{(I)} \) and \( x_i^{(I)} \) can be arbitrary initial data at the moment \( \eta = \eta_I \).

Physical results do not depend on the choice of variables in the action (11), if observable quantities are defined unambiguously. But physical results depend on the choice of observables (as we have seen above in Section 1).

4 Newtonian motion of particles in the expanding Universe

The energy of a particle (moving on the geodesic line in space with set metrics) can be found by solving the mass–shell equation. Consider metrics (1) that
denote $g_{\mu\nu}$. Calculating the integral of motion of a test particle in this metric:

$$p^2 = g^{\mu\nu} p_\mu p_\nu = m^2,$$

we can find an expression for energy $p_0$

$$p_0 \approx -N^r p_r \pm \left(1 - \frac{r_g}{2r}\right) \left[\left(1 - \frac{r_g}{2r}\right) m + \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2}\right],$$

where $N^r = -\frac{3}{4} r_g H$ is the radial component of the shift vector. For a positive sign we get the action

$$S = \int d\eta \left[p_r r' + p_\theta \theta' - E\right],$$

where $E = p_0 - m$. In the nonrelativistic limit of small velocities the energy takes the form

$$E \approx E_{\text{classic}} = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} - \frac{r_g m}{2r},$$

here $m = m_0 a(\eta)$ is the mass a test particle that depends on time. The product $r_g m$ is a conformal invariant and does not depend on time.

The action (16) shows us a possibility of capturing in a particle with the running mass $m(\eta) = m_0 a(\eta)$ at the moment when the kinetic term $\sim 1/a$ takes a value equal to the potential $(\alpha/r)$. To study this effect of capture, we pass to the scale factor $a(\eta)$ as the time-like parameter, using the cosmological equations (6), dimensionless variables $y = r/r_0$, $p_y = p_r/m_0$, and three velocities: orbital ($v_0$), Newtonian ($w_0$), and cosmic ($c_0$) defined as $v_0 = P_\theta/(r_0m_0)$, $w_0 = \sqrt{\alpha/(r_0m_0)}$, and $c_0 = H_0 r_0$.

In this case, the equation derived from (16) takes the form

$$\frac{v_0^2}{y^2} = \frac{w_0^2 a}{y^2} + c_0^2 (\Omega^{1/2}(a)a) \frac{d\left[\Omega^{1/2}(a)a\right]}{da}.$$  

(18)

The general solution of their equation is given in [3] for the stiff state $\Omega^{1/2}(a)a = 1$. This solution allows us to check the results of the numerical calculation of eq. (18) for arbitrary parameters $\Omega_{\text{Stiff}}$, $\Omega_{\text{Radiation}}$, $\Omega_{\text{Matter}}$, and $\Omega_{\Lambda}$.

For the case $c_0 = 0$, we get the class of circular trajectories
with the constraint of the initial velocities $v_0^2 = w_0^2$ well known as the "virial theorem" [15]: $v_0 = r_g/(2r_0)$. In the case of the cylindric symmetry of a sources (5) we get $v_0 = \sqrt{r_g/\left(2\sqrt{l^2 + r_0^2}\right)}$. The last formula can explain the constant rotational curves in the region $r_0 \lesssim l$.

To estimate the role of the last term in eq. (18) in the class of circular trajectories, we substitute (19) in eq. (18) for $c_0^2 \neq 0$ and $\Omega = \Omega_{\text{Stiff}}a^{-2} + \Omega_{\text{Matter}}a + \Omega_{\Lambda}a^4$. In this way, we get the following relation between the velocities $v_0$, $w_0$, and $c_0$:

$$v_0^2 = w_0^2 + c_0^2 \left[2 - \left(\frac{3}{2}\Omega_{\text{Matter}} + 3\Omega_{\Lambda}\right)\right].$$

(20)

In the case of the Conformal Cosmology (CC), $\Omega_{\text{Matter}} = \Omega_{\Lambda} = 0$, these equations become

$$v_0^2 = w_0^2 + 2c_0^2,$$

(21)

and the last term $2c_0^2$ plays the role of the Dark Matter, whereas in the case of Standard Cosmology (SC): $\Omega_{\text{Matter}} = 0.3$, $\Omega_{\Lambda} = 0.7$, the last term is negative so that $v_0^2 = w_0^2 - 0.5c_0^2$. Thus the standard cosmology requires one more Dark Matter [2] in contrast to the conformal cosmology [10].

5 Capture of cosmic objects by the central field

Let us consider the action (16)–(17) in terms of the Friedmann coordinates $R = ra(\eta)$, $P_R = p/a(\eta)$, and $dt = a(\eta)d\eta$:

$$S_A = \int_{t_l}^{t_0} dt \left[ P_R(\dot{R} - HR) + P_\theta \dot{\theta} - \frac{P_R^2}{2m_0} - \frac{P_\theta^2}{2m_0R^2} + \frac{\alpha}{R}\right].$$

(22)

The “energy” of a particle in (22)

$$E = HRP_R + \frac{P_R^2}{2m_0} + \frac{P_\theta^2}{2m_0R^2} - \frac{\alpha}{R}$$

(23)
is not conserved in contrast to the energy of a particle with a constant mass in the Newtonian mechanics. This energy can change sign at \( t = t_1 \). It is known that the change of a sign of the energy means the change of an unrestricted motion of a particle by a finite motion in the central field. Therefore, the time \( t_0 \) can be treated as the time of the capture of a particle (cosmic object) by the central gravitational field [2].

![Graph](image.png)

Fig. 1. Effect of capture of a cosmic object by the central field is described by the dynamics of a radial coordinate \( R = ar \) and its energy \( E \) (23).

For the local Group \( M_{LC} = 10^{13} \div 10^{14}, R_{LC} = 8 \) Mpc, \( c_0 \sim 500 \) km/s, and \( v_0 \sim 100 \) km/s \( \div 200 \) km/s. The class of ellipsoidal trajectories of capture explains the anisotropic local velocity field of nearby galaxies by their Newtonian motion [3, 16].

### 6 Conclusion

It is shown that the cosmic evolution can be the mechanism of a capture of cosmic objects that can form halos of galaxies and their clusters. This capture can replay one of the fundamental questions of modern theory of formation of galaxies: How is a system of unbounded “particles” with positive energy converted to the system of bounded “particles” with negative energy? The range of validity of the Newtonian mechanics for description of galaxies is restricted by the critical radius \( R_{cr} \approx 10^{20} \) cm \( (M/M_\odot)^{1/3} \) that is closed to the radius of surface of the zero–velocity, separating a cluster from the cosmological evolution. We managed to express the Newton-like equation in terms of the red shift parameter for an arbitrary equations of state and to obtain modification of the rotational curves. It shows us that the cosmological evolution can plays the role ascribed to the Dark Matter. The deficit of the visible matter can be increased in the version of cosmology where observational quantities are identified with the conformal ones; whereas the standard cosmological model requires one more Dark Matter.
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