Abstract. I briefly review the symmetries and the associated low energy effective Lagrangian for two light flavor Color Superconductivity (2SC).

2SC SYMMETRIES AND EFFECTIVE LAGRANGIAN

Quark matter at very high density is expected to behave as a color superconductor [1]. Possible phenomenological applications include the description of quark stars, neutron star interiors, the physics near the core of collapsing stars and supernova explosions [1, 2, 3]. The color superconductive phase is characterized by its gap energy ($\Delta$) associated to quark-quark pairing which leads to the spontaneous breaking of the color symmetry.

To describe low energy physical processes, where perturbation theory is not applicable, effective Lagrangians based on the global symmetries of the underlying theory are known to play a relevant role. In the case of Color superconductivity effective Lagrangians describe the interactions among the excitations near the fermi surface. The three flavor case (CFL) has been developed in [4]. The low-energy effective Lagrangian for the in medium fermions and the broken sector of the $SU_c(3)$ color group for 2SC has been constructed in Ref. [5]. The effective theories encoding also the electroweak interactions for the low-energy excitations in the 2SC and CFL case can be found in [6]. The light glueball Lagrangian of the unbroken $SU_c(2)$ Yang-Mills sector of the 2SC phase has been constructed in [7].

Here I summarize the effective low energy Lagrangian for two flavors which contains all of the relevant degrees of freedom. First I review the low-energy effective Lagrangian for the 2SC phase of QCD [5, 6]. The latter describes the, in medium, fermions and the broken $SU_c(3)$ gluon sector. I then show how to build the effective Lagrangian describing the light glueballs associated with the unbroken $SU_c(2)$ color subgroup by using the information inherent to the trace anomaly and the medium effects related to a non-vanishing dielectric constant first presented in [8] and confirmed within a different formalism in [9]. Finally the, in medium, glueball to two photon decay process is estimated. The present talk is based on the papers [5, 6, 7, 10, 11].

Quantum Chromo Dynamics with two flavors has gauge symmetry $SU_c(3)$ and global symmetry

\[ SU_L(2) \times SU_R(2) \times U_V(1) . \]  

(1)
At high matter density a color superconductive phase sets in and the associated diquark condensate leaves invariant the following symmetry group:

\[ [SU_c(2)] \times SU_L(2) \times SU_R(2) \times \tilde{U}_V(1), \]

where \([SU_c(2)]\) is the unbroken part of the gauge group. The \(\tilde{U}_V(1)\) generator \(\tilde{B}\) is the following linear combination of the previous \(U_V(1)\) generator \(B = \frac{1}{3}\text{diag}(1, 1, 1)\) and the broken diagonal generator of the \(SU_c(3)\) gauge group \(T^8 = \frac{1}{2\sqrt{3}}\text{diag}(1, 1, -2)\):

\[ \tilde{B} = B - \frac{2\sqrt{3}}{3}T^8. \]

The quarks with color 1 and 2 are neutral under \(\tilde{B}\) and consequently the condensate too (\(\tilde{B}\) is \(\sqrt{2}S\) of Ref. [5]). The superconductive phase for \(N_f = 2\) possesses the same global symmetry group of the confined Wigner-Weyl phase [10]. In Reference [10], it was shown that the low-energy spectrum, at finite density, displays the correct quantum numbers to saturate the ’t Hooft global anomalies [12]. It was also observed in Reference [11] it was then seen, by using a variety of field theoretical tools, that global anomaly matching conditions hold for any cold but dense gauge theory.

The lowest lying excitations are protected from acquiring a mass by the aforementioned constrains and dominate the low-energy physical processes. The low-energy theorems governing their interactions can be usefully encoded in effective Lagrangians. The dynamics of the Goldstone bosons is efficiently encoded in a non-linear realization framework. Here, see [5], the relevant coset space is \(G/H\) with \(G = SU_c(3) \times U_V(1)\) and \(H = SU_c(2) \times \tilde{U}_V(1)\) is parameterized by

\[ \mathcal{V} = \exp(i\overline{\xi}^i X^i), \]

where \(\{X^i\} i = 1, \cdots , 5\) belong to the coset space \(G/H\) and are taken to be \(X^i = T^{i+3}\) for \(i = 1, \cdots , 4\) while \(X^5 = B + \sqrt{3}T^8 = \text{diag}(\frac{1}{2}, \frac{1}{2}, 0)\). \(T^a\) are the standard generators of \(SU(3)\). The coordinates

\[ \xi^i = \frac{\Pi^i}{f} \quad i = 1, 2, 3, 4, \quad \xi^5 = \frac{\Pi^5}{f}, \]

\(\Pi\) describe the Goldstone bosons.

\(\mathcal{V}\) transforms non linearly

\[ \mathcal{V}(\overline{\xi}) \rightarrow u_V g \mathcal{V}(\overline{\xi}) h^\dagger(\overline{\xi}, g, u) h^\dagger_V(\overline{\xi}, g, u), \]

with \(u_V \in U_V(1)\), \(g \in SU_c(3)\), \(h(\overline{\xi}, g, u) \in SU_c(2)\) and \(h_V(\overline{\xi}, g, u) \in \tilde{U}_V(1)\). It is, also, convenient to define:

\[ \omega_\mu = i\mathcal{V}^\dagger D_\mu \mathcal{V} \quad \text{with} \quad D_\mu \mathcal{V} = (\partial_\mu - ig_3 G_\mu) \mathcal{V}, \]

with gluon fields \(G_\mu = G^{m}_\mu T^m\). Following [5] we decompose \(\omega_\mu\) into:

\[ \omega^\parallel_\mu = 2S^a \text{Tr} [S^a \omega_\mu] \quad \text{and} \quad \omega^\perp_\mu = 2X^i \text{Tr} [X^i \omega_\mu], \]
where $S^a$ are the unbroken generators of $H$ with $S^{1,2,3} = T^{1,2,3}$, $S^4 = \tilde{B}/\sqrt{2}$. Summation over repeated indices is assumed.

To be able to include the in medium fermions in the picture we define:

$$\bar{\psi} = \mathcal{V}^\dagger \psi,$$

transforming as $\bar{\psi} \rightarrow h \mathcal{V}(\xi, g, u) h(\xi, g, u) \bar{\psi}$ and $\psi$ possesses an ordinary quark transformations (as Dirac spinor).

The simplest non-linearly realized effective Lagrangian describing in medium fermions, the five gluons and their self interactions, up to two derivatives and quadratic in the fermion fields is:

$$
L = f^2 a_1 \text{Tr} \left[ \omega^0_\perp \omega^0_\perp - \alpha_1 \bar{\omega}^\perp \bar{\omega}^\perp \right] + f^2 a_2 \left[ \text{Tr} \left[ \omega^0_\perp \right] \text{Tr} \left[ \omega^0_\perp \right] - \alpha_2 \text{Tr} \left[ \bar{\omega}^\perp \right] \text{Tr} \left[ \bar{\omega}^\perp \right] \right] + b_1 \bar{\psi} \gamma^0 (\partial_0 - i \omega^\parallel_0) \psi + b_2 \bar{\psi} \gamma^0 \omega^\perp_0 \psi + m_M \bar{\psi} C \gamma^5 \left( iT^2 \right) \psi + \text{h.c.},
$$

where $\bar{\psi}^C = i \gamma^2 \bar{\psi}^*$, $i, j = 1, 2$ are flavor indices and

$$
T^2 = S^2 = \frac{1}{2} \begin{pmatrix} \sigma^2 & 0 \\ 0 & 0 \end{pmatrix},
$$

$a_1$, $a_2$, $b_1$ and $b_2$ are real coefficients while $m_M$ is complex. The breaking of Lorentz invariance to the $O(3)$ subgroup, following [4], has been taken into account by providing different coefficients to the temporal and spatial indices of the Lagrangian, and it is encoded in the coefficients $\alpha$s and $\beta$s. For simplicity, the flavor indices are omitted. From the last two terms, representing a Majorana mass term for the quarks, we deduce that the massless degrees of freedom are the $\psi_{a=3,i}$ which possess the correct quantum numbers to match the ’t Hooft anomaly conditions [10]. The generalization to the electroweak processes relevant for the cooling history of compact stars has been investigated in [6].

**THE SU$_C$(2) GLUEBALL EFFECTIVE LAGRANGIAN**

The SU$_c$(2) gauge symmetry does not break spontaneously and it is expected to confine. If the new confining scale is lighter than the superconductive quark-quark gap the associated confined degrees of freedom (light glueballs) [7] can play, together with the true massless quarks a relevant role for the physics of Quark Stars featuring a 2SC superconductive surface layer [3].

Indeed, according to the findings in [8], the medium does lead to partial SU$_c$(2) screening. In other words the medium is polarizable, i.e., acquires a dielectric constant $\varepsilon$ different from unity (in fact $\varepsilon \gg 1$ in the 2SC case [8]) leading to an effectively reduced gauge coupling constant. By assuming locality the SU$_c$(2) effective action takes the form [8]:

$$
S_{eff} = \int d^4x \left[ \frac{\varepsilon}{2} \tilde{E}^a \cdot \tilde{E}^a - \frac{1}{2\lambda} \tilde{B}^a \cdot \tilde{B}^a \right]
$$

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with \( a = 1,2,3 \) and \( E_i^a \equiv F_{0i}^a \) and \( B_i^a \equiv \frac{1}{2} \epsilon_{ijk} F_{jk}^a \). Here one assumes an expansion in powers of the fields and derivatives. The gluon speed in this regime is \( v = 1/\sqrt{\epsilon \lambda} \). In Reference [8] the \( \epsilon \) and \( \lambda \) were obtained:

\[
\epsilon = 1 + \frac{g_0^2 \mu^2}{18 \pi^2 \Delta^2} , \quad \lambda = 1 . \tag{12}
\]

Equation (12) than suggests that a 2SC color superconductor can have a large positive dielectric constant. This implies that the Coulomb potential between \( SU_c(2) \) color charges is reduced in the 2SC medium. \( SU_c(2) \) glueballs like particles are expected to emerge. These particles are light with respect to \( \Delta \). So, the low-energy \( SU_c(2) \) theory should be well represented by the effective Lagrangian describing its hadronic low lying states. This Lagrangian has to be added to the one of Eq. (9) [5] and it has been constructed in [7].

We first rescale the coordinates and the \( SU_c(2) \) fields as follows:

\[
\hat{x}^0 = \frac{x^0}{\sqrt{\epsilon \lambda}} , \quad \hat{g} = g_s \left( \frac{\lambda}{\epsilon} \right)^{1/2} \quad \hat{A}_0^a = \lambda^{-\frac{1}{2}} \epsilon^{-\frac{1}{4}} A_0^a , \quad \hat{A}_i^a = \lambda^{-\frac{1}{2}} \epsilon^{-\frac{1}{4}} A_i^a . \tag{13}
\]

The \( SU_c(2) \) action now becomes:

\[
S_{SU(2)} = -\frac{1}{2} \int d^4x \text{Tr} \left[ \hat{F}_{\mu \nu} \hat{F}^{\mu \nu} \right] \tag{14}
\]

and \( \hat{F}_{\mu \nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + i \hat{g} \left[ \hat{A}_\mu , \hat{A}_\nu \right] \) with \( \hat{A}_\mu = \hat{A}_\mu^a T^a \) and \( a = 1, 2, 3 \). The low-energy effective 3 gluon dynamics in the color superconductor medium (with non-vanishing dielectric constant and magnetic permeability) is similar to the in vacuum theory. The expansion parameter is: \( \hat{\alpha} = \frac{g_s^2}{4\pi} = \frac{g_s^2}{4\pi} \sqrt{\frac{\lambda}{\epsilon}} \). Notice that \( g_s \) is the \( SU_c(3) \) coupling constant evaluated at the scale \( \mu \) while we now, following Ref. [8], interpret \( \hat{g} \) as the \( SU_c(2) \) coupling at \( \Delta \). The matching of the scales is encoded in \( \sqrt{\lambda/\epsilon} \).

The, in medium, anomaly-induced effective Lagrangian is based on the trace anomaly arising from the rescaled \( SU_c(2) \) [13]:

\[
\hat{\theta}_\mu^a = -\frac{\hat{\beta}(\hat{g})}{2\hat{g}} \hat{F}_{\mu \nu}^a \hat{F}^{\mu \nu : a} \equiv \frac{2b}{v} H , \tag{15}
\]

with \( a = 1, 2, 3 \) and we have defined \( \hat{\beta}(\hat{g}) = -b \hat{g}^3 / 16\pi^2 \). At one loop \( b = \frac{11}{3} N_c \) with \( N_c = 2 \) the color number. \( H \) is the composite field describing, upon quantization, the scalar glueball [14] in medium and possesses mass-scale dimensions 4. The specific velocity dependence is introduced to properly account for the velocity factors.

The complete simplest light glueball action in the unrescaled coordinates for the, in medium, Yang-Mill theory is:

\[
S_{G-ball} = \int d^4x \left\{ \frac{c}{2} \sqrt{b} H^{-\frac{3}{2}} \left[ \partial^0 H \partial^0 H - v^2 \partial^0 H \partial^0 H \right] - \frac{b}{2} H \log \left[ \frac{H}{\Lambda^2} \right] \right\} . \tag{16}
\]

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The glueballs move with the same velocity \( v \) as the underlying gluons in the 2SC color superconductor. \( \Lambda \) is the intrinsic scale associated with the theory and can be less than or of the order of few MeVs \([8, 7]\) while \( c \) is a constant of order unity. The glueballs are light (with respect to the gap) and might barely interact with the un-gapped fermions. They are stable with respect to the strong interactions unlike ordinary glueballs. We define the mass-dimension one glueball field \( h \) via

\[
H = \langle H \rangle e^{\frac{h}{\Lambda}}. \tag{17}
\]

By requiring a canonically normalized kinetic term for \( h \) one finds \( F_{h}^{2} = \frac{c}{\sqrt{2}} \sqrt{2b\langle H \rangle} \), while the glueball mass term is \( M_{h}^{2} = \frac{c}{\sqrt{2}} \sqrt{\langle H \rangle} = \frac{c}{\sqrt{2}} \sqrt{e\Lambda^{2}} \), which is clearly of the order of \( \Lambda \) since \( c \) is a positive constant of order unity.

Once created, the light \( SU_{c}(2) \) glueballs are stable against strong interactions but not with respect to electromagnetic processes. Indeed, the glueballs couple to two photons via virtual quark loops.

The relevant Lagrangian term, at non zero baryon density, obtained by saturating the electromagnetic trace anomaly is \([7]\):

\[
L_{h\gamma\gamma} = \frac{\bar{e}^{2}}{4\pi^{2}} \frac{M_{h}}{\sqrt{2b\langle H \rangle}} \sum_{\text{quarks}} \bar{Q}_{\text{quarks}}^{2} \bar{h} \bar{F}_{\mu\nu} \bar{F}^{\mu\nu}, \tag{18}
\]

with \( \bar{F}_{\mu\nu} = \partial_{\mu} \bar{A}_{\nu} - \partial_{\nu} \bar{A}_{\mu} \). Here \( \bar{A}_{\mu} \) is the in medium photon field corresponding to the following massless linear combination of the old photon and the eighth gluon \([15, 6]\):

\[
\bar{A}_{\mu} = \cos \theta_{Q} A_{\mu} - \sin \theta_{Q} G_{8\mu}, \tag{19}
\]

with \( \tan \theta_{Q} = e/(\sqrt{3}g_{s}) \). The new electric charge constant is related to the in vacuum one via \( \bar{e} = e \cos \theta_{Q} \). \( \bar{Q} \) is the new electric charge operator associated with the field \( \bar{A}_{\mu} \) with \( \bar{Q} = \tau^{3} \times 1 + \frac{\bar{B} - L}{\sqrt{3}} = Q \times 1 - \frac{1}{\sqrt{3}} 1 \times T^{8} \), where \( L = 0 \) is the lepton number, \( \tau^{3} \) the standard Pauli’s matrix, \( Q \) the quark matrix, while the new baryon number is \( \bar{B} \), and following the notation of Ref. \([6]\) we have flavor\(_{2 \times 2} \times \) color\(_{3 \times 3} \). This leads to the following decay width of the glueballs into two photons in medium:

\[
\Gamma[h \rightarrow \gamma\gamma] \approx 1.2 \times 10^{-2} \cos \theta_{Q}^{4} \left[ \frac{M_{h}}{1 \text{ MeV}} \right]^{5} \text{ eV}, \tag{20}
\]

where \( \alpha = e^{2}/4\pi \approx 1/137 \). For illustration purposes we consider a glueball mass of the order of 1 MeV which leads to a decay time \( \tau \approx 5.5 \times 10^{-14} \text{ s} \). We used \( \cos \theta_{Q} \approx 1 \) since \( \theta_{Q} \approx 2.5^{o} \) \([7]\). While we are aware of the possible contribution from other hadrons to the saturation of the electromagnetic trace anomaly \([16, 17]\), here we assume it to be dominated by the \( SU_{c}(2) \) glueballs. In any case, it is hard to imagine the photon decay process to be completely switched-off. This shows that a consistent portion of the glue (\(3/8 \) or \(37.5\%\)) filling the 2SC medium is very rapidly and efficiently converted into electromagnetic radiation.
CONCLUSION

I reviewed the symmetries and the low energy effective Lagrangian for two flavor Color Superconductivity. The effective Lagrangian describes the, in medium, fermions and the broken $SU_c(3)$ gluon sector. The theory has then been extended to incorporate the relevant confining and light (with respect to the gap), $SU_c(2)$ degrees of freedom, i.e. glueballs. It is shown that the light glueballs are unstable to photon decay and estimated the, in medium, two photon decay rate. The present analysis is limited to the zero temperature and high matter density case. However it might be relevant to investigate the role played by a non zero temperature [18].

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