CONSTRaining THE properties OF DARK matter WITH observations OF THE COSMIC MICROWAVE BACKGROUND

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Received 2016 February 8; revised 2016 July 29; accepted 2016 August 2; published 2016 October 19

ABSTRACT

We examine how the properties of dark matter, parameterized by an equation-of-state parameter \( w \) and two perturbative generalized dark matter (GDM) parameters, \( c^2_s \) (the sound speed) and \( c^2_{\text{vis}} \) (the viscosity), are constrained by existing cosmological data, particularly the Planck 2015 data release. We find that the GDM parameters are consistent with zero, and are strongly constrained, showing no evidence for extending the model of dark matter beyond the cold dark matter (CDM) paradigm. The equation of state of dark matter is constrained to be within \(-0.000896 < w < 0.00238 \) at the 99.7% confidence level (CL), which is several times stronger than constraints found previously using WMAP data. The parameters \( c^2_s \) and \( c^2_{\text{vis}} \) are constrained to be less than \( 3.21 \times 10^{-6} \) and \( 6.06 \times 10^{-6} \) respectively at the 99.7% CL. The inclusion of the GDM parameters does significantly affect the error bars on several \( \Lambda \)CDM parameters, notably the dimensionless dark matter density \( \omega_g \), and the derived parameters \( \sigma_8 \) and \( H_0 \). This can be partially alleviated with the inclusion of data constraining the expansion history of the universe.

Key words: cosmic background radiation – cosmological parameters – cosmology: observations – dark matter

1. INTRODUCTION

The \( \Lambda \) cold dark matter (\( \Lambda \)CDM) cosmological model provides a good phenomenological fit to current cosmological data. It matches the current expansion history of the universe including supernovae measurements (Riess et al. 1998; Perlmutter et al. 1999) and baryon acoustic oscillations (BAO) (Anderson et al. 2014; Ross et al. 2015), local measurements (Riess et al. 2011), the matter power spectrum (Gil-Marín et al. 2015), and the cosmic microwave background (CMB) data from the Planck satellite (Planck Collaboration et al. 2016a).

In this standard picture the CDM is a crucial component and is described as a non-interacting, initially pressureless perfect fluid. This perfect fluid has an equation-of-state parameter \( w \) that is identically zero as well as zero sound speed and zero viscosity. These CDM characteristics make it an extremely simple system to model physically, either perturbatively or through cosmological \( N \)-body simulations.

Astrophysical systems also provide important evidence for dark matter. Galaxy rotation curves (Persic et al. 1996) were one of the earliest suggestions of dark matter’s existence (Zwicky 1933; Rubin & Ford 1970). Furthermore, gravitational-lensing measurements have been used to infer “mass maps” (see for example Lefor et al. 2013), which show the distribution of the CDM under the assumption that it traces the gravitational-lensing potential. These maps are consistent with what is expected from theory (Massey et al. 2007). One of the most studied gravitational-lensing systems is the “bullet cluster” (Clowe et al. 2006), a system consisting of two colliding galaxy clusters. The “bullet cluster” system shows a clear displacement between the map of the gravitational-lensing potential and the map of the luminous matter, in a way that is naturally explained within the CDM paradigm. Since the discovery of the original system, further examples have been observed (Harvey et al. 2015), providing more evidence in support of the CDM component of the standard cosmological model.

Despite the promising concordance of these results, no dark matter candidate has been found experimentally. Moreover, many physical models of dark matter are not as idealized as the pure CDM model. Thus, it seems timely, with the precise cosmological data currently available, to examine constraints on departures from this idealized model.

Work has been carried out to constrain the properties of dark matter using astrophysical systems, particularly galaxy rotation curves; see, e.g., Faber & Visser (2006), Serra & Domínguez Romero (2011), Barranco et al. (2015), and Boehm et al. (2014). Interestingly, it has also been pointed out that halo properties deviate from expectations of \( \Lambda \)CDM (Moore 1994; Boylan-Kolchin et al. 2011; Jee et al. 2014; Papastergis et al. 2015). As we are only interested in the properties of dark matter on linear scales in this paper, we will not discuss halos further.

Cosmological constraints on the properties of dark matter beyond CDM have been investigated within the context of specific models. In Yang (2015), Planck data were used to constrain the properties of dark matter, focusing on its mass and decay rate and its thermally averaged cross section for annihilation. CMB data from Planck as well as large-scale structure observations have been used to place constraints on interactions of dark matter with other particles of the Standard Model (SM), for instance, possible dark matter interactions with photons (Wilkinson et al. 2014b) or with neutrinos (Wilkinson et al. 2014a). The elastic scattering of dark matter with other SM particles also leaves an imprint on spectral distortions on the CMB, as investigated lately in Ali-Hamoud et al. (2015), where constraints were put on the interaction cross section. Dark matter may also interact with a dark radiation component (Buen-Abad et al. 2015; Lesgourgues et al. 2015), which may further have its own dark recombination and dark atomic structure (Cyr-Racine & Sigurdson 2013). Other well motivated models are axions (Hlozek et al. 2015), collisionless warm dark matter (Armendariz-Picon &
Neelakanta 2014; Piattella et al. 2016), and collisionless massive neutrinos (Shoji & Komatsu 2010).

In this paper we are interested in constraining the properties of dark matter by modeling the dominant component of structure formation beyond a simple pressureless perfect fluid. This could either be due to a more complicated dark matter model, or just a more precise modeling of CDM, as in the case of the effective field theory of large-scale structure (EFTofLSS: Baumann et al. 2012). For convenience throughout this paper, we consider “CDM” to be the modeling of the dark matter component as a pressureless perfect fluid. Thus, both more complicated models and a more precise modeling of the evolution of CDM are considered to be “beyond CDM,” in the sense that they are a change to how the dark matter is usually modeled.

We parameterize the departures from CDM according to the generalized dark matter (GDM) model (Hu 1998). As we discuss in more detail in a companion paper (Kopp et al. 2016), the GDM parameterization naturally arises in more realistic models, for instance, the EFTofLSS (Baumann et al. 2012), non-equilibrium thermodynamics (Landau & Lifshitz 1987), the effective theory of fluids (Ballesteros 2015), tightly coupled fluids, and scalar fields. In Kopp et al. (2016) we also study more closely the physical effects and the interpretation of the GDM parameters.

Several authors have used the GDM model to put cosmological constraints on the dark matter properties (Müller 2005; Calabrese et al. 2009; Kumar & Xu 2014; Wei et al. 2013; Xu & Chang 2013; Li & Xu 2014; Xu 2014), as we also do in this paper. We perform a more detailed comparison with these works in Section 4.3. Apart from describing dark matter, the GDM parameterization, or a subset of it, has been used in some form in the literature for several different purposes including neutrinos (Trotta & Melchiorri 2005), dark energy (Weller & Lewis 2003; Bean & Doré 2004), and unified models of dark matter/dark energy (Kumar & Xu 2014; Yang & Xu 2013).

This paper is organized as follows. Section 2 contains a brief introduction to the GDM model. Section 3 details the method and data that were used in our analysis, and our results are presented in Section 4. We conclude in Section 5.

2. A SHORT OVERVIEW OF THE GDM MODEL

In this section we present the basic ingredients of the GDM model (Hu 1998). We work in the synchronous gauge as this is the gauge most commonly used in numerical Boltzmann codes. We are interested only in scalar perturbations, so the metric takes the form

\[ ds^2 = a^2 \left\{ -d\eta^2 + \left[ \left( 1 + \frac{1}{3} h \right) \gamma_{ij} + D_{ij} \nu \right] dx^i dx^j \right\} \]  

where \( a \) is the scale factor in conformal time \( \eta \), \( \gamma_{ij} \) is a flat spatial metric with covariant derivative \( \nabla_i \), \( h \) and \( \nu \) are the two scalar metric perturbations in this gauge, and \( D_{ij} = \nabla_i \nabla_j - \frac{1}{2} \nabla^2 \gamma_{ij} \) is a traceless spatial operator. The GDM has background density \( \bar{\rho}_g \) and isotropic pressure \( \bar{P}_g \), related by an equation of state \( w \), such that

\[ \bar{P}_g = -3H(1+w)\bar{\rho}_g \]  

Unlike CDM, the GDM is allowed to have a pressure perturbation \( \Pi_g \) and shear perturbation \( \Sigma_g \) (Hu 1998), in addition to the usual perturbations of density (\( \delta_g \)) and velocity (\( \theta_g \)). The perturbations obey the Euler and continuity equations, as well as two postulated closure equations for the pressure perturbation and the shear (Hu 1998); see also Kopp et al. (2016) for an extended discussion and an explanation of our notation. The equations that the GDM obeys in a spatially flat universe are as follows:

\[ \delta_g = 3H(w\delta_g - \Pi_g) - (1 + w) \left( \frac{1}{2} \dot{h} - \nabla^2 \delta_g \right) \]  

\[ \dot{\theta}_g = (3c_a^2 - 1)H\dot{\theta}_g + \frac{\Pi_g}{1 + w} + \frac{2}{3} \nabla^2 \Sigma_g \]  

\[ \Sigma_g = -3H\Sigma_g + \frac{4}{(1 + w)c_{\text{vis}}^2} \left( \theta_g - \frac{1}{2} \dot{\nu} \right) \]  

\[ \Pi_g = c_a^2 \delta_g + 3H(1 + w)(c_a^2 - c_{\text{vis}}^2)\theta_g, \]  

where we have introduced two sound speeds and a viscosity (the equivalent equations in a spatially curved universe and in a general gauge can be found in Kopp et al. 2016). The sound speed \( c_a^2 \) and viscosity \( c_{\text{vis}}^2 \) are parameters of the GDM model, and the adiabatic sound speed \( c_a^2 \) is defined as

\[ c_a^2 = \frac{\bar{P}_g}{\bar{\rho}_g} = w - \frac{w}{3H(1+w)}. \]  

In the present work we consider only a constant equation of state \( w \) so that \( c_a^2 = w \).

In general, the sound speed \( c_a^2 \) causes oscillations in the density perturbation \( \delta_g \) below the Jeans length, although if \( c_a^2 \) and \( c_{\text{vis}}^2 \) become comparable in size then there are no sound waves. For convenience, we refer to \( c_a^2 \) as the sound speed, although this is only true for \( c_a^2 \gg c_{\text{vis}}^2 \) (Kopp et al. 2016). The viscosity \( c_{\text{vis}}^2 \) damps the density perturbations. For details of the model and investigations into which physical models map to GDM, see Hu (1998) and Kopp et al. (2016).

To summarize, replacing CDM with GDM amounts to introducing three parametric functions to the model: the GDM equation of state \( w(\eta) \), the sound speed \( c_a^2(\eta, \mathbf{x}) \), and the viscosity \( c_{\text{vis}}^2(\eta, \mathbf{x}) \). In this work we shall assume that all three take constant values, i.e., with no time or space dependence. While this is not the case for many of the physical models that GDM relates to, these constant parameters can be considered as a null test of whether there is any evidence for departures from CDM.

3. METHOD AND DATA

In order to perform our analysis, we modified the Cosmic Linear Anisotropy Solving System code (CLASS: Lesgourgues 2011). CLASS numerically solves the Boltzmann equation for each relevant component coupled to the Einstein equations and calculates the CMB and matter power spectra given a set of model parameters. The CLASS code already includes an additional dark energy fluid component with an equation of state and sound speed (Lesgourgues & Tram 2011), which we further modified to include the viscosity \( c_{\text{vis}}^2 \) and to allow this
fluid to work as a replacement for dark matter rather than for dark energy. We also independently modified a different Boltzmann code (DASH: Kaplinghat et al. 2002) to include the full GDM parameterization. We performed a full comparison between the codes, including the background evolution, perturbation evolution, the C_\ell s, matter power spectrum, and lensing potential. The numerical difference in the two codes in the case of the GDM model is similar to the corresponding difference in the case of \Lambda CDM, within \sim 0.1%. This level of agreement holds for all quantities in both the synchronous gauge and the conformal Newtonian gauges.1

We investigated the constraints on the GDM parameters using a Markov chain Monte Carlo (MCMC) approach, carried out using the publicly available MontePython code (Audren et al. 2013), which implements the Metropolis–Hastings algorithm. The MontePython code calls the CLASS code through a Python wrapper. Our covariance matrix for the final runs was generated using the standard methodology: first some initial runs were carried out with a simple estimated diagonal covariance matrix, and these were used to generate a covariance matrix that was used for the next run. After several iterations of this we had a covariance matrix that gave an appropriate acceptance rate for the steps in the chains and was suitable for the final runs.

We considered three types of model: the standard \Lambda CDM model (used for comparison), the \Lambda-wDM model where CDM is replaced with GDM but the speed of sound and viscosity are set to zero, and the \Lambda-GDM model where all three GDM parameters are included. Note that in all three models, the dark energy component is always modeled as a cosmological constant \Lambda and only the dark matter component is modified.

In all models we varied the standard \Lambda CDM parameters with the MCMC algorithm: the dark matter dimensionless density \omega_d,2 the baryon dimensionless density \omega_b, 100 \times \theta_i where \theta_i is the ratio of the sound horizon to the angular diameter distance at decoupling, the optical depth \tau, \ln(10^{10}A_s) where A_s is the amplitude of scalar perturbations, and the spectral index of scalar perturbations n_s (six parameters in total). For the \Lambda-wDM model we varied the GDM equation of state w in addition to the \Lambda-CDM parameters (seven parameters in total), and for the \Lambda-GDM model we further varied the sound speed c_s^2 and the viscosity c_{\text{vis}}^2 (nine parameters in total). In addition to the main MCMC parameters we considered three more derived parameters: the Hubble constant H_0 (in units of km s^{-1} Mpc^{-1}), the relative density of the cosmological constant \Lambda, and the rms matter density fluctuation \sigma_8. We used flat priors on all of the parameters and restricted c_s^2 and c_{\text{vis}}^2 to be non-negative as dictated by theoretical models (Kopp et al. 2016) and \tau to be greater than 0.01. The remaining fiducial cosmology was set as follows. The spatial curvature was set to zero with a cosmological constant \Omega_\Lambda making up the remainder of the matter content, and the primordial helium fraction was set to Y_{\text{He}} = 0.2477.3 We used two massless neutrinos and one massive neutrino with mass 0.06 eV, keeping the effective number of neutrinos to N_{\text{eff}} = 3.046.2

The main data set that we used was the Planck 2015 data release (Planck Collaboration et al. 2016b) of the CMB anisotropies. We used the low-l likelihood and the full TT, EE, and TE high-l likelihood with the complete “not-lite” set of nuisance parameters. The low-l likelihood consists of the TT, EE, TE, and BB spectra up to l = 29, whereas the high-l spectra are from l = 30 upwards. See the Planck papers and wiki\textsuperscript{5} for full details of these likelihoods. As we always used the high-l and low-l likelihoods together, this combination will be referred to simply as the Planck power spectrum (PPS). We included Gaussian priors on the nuisance parameters (also varied as MCMC parameters) as recommended by the Planck Collaboration and implemented in the MontePython code.

We ran further chains that included the Planck lensing potential likelihood (hereby referred to as “Lens”) in addition to the low-l and high-l likelihoods, and found that this made a significant difference to the constraints on c_s^2 and c_{\text{vis}}^2. Finally, we also investigated the effect on the constraints of other cosmological data sets that constrain the expansion history of the universe. The two data sets used for this were the Hubble Space Telescope (HST) key project (Riess et al. 2011) and BAO data (Beutler et al. 2011; Anderson et al. 2014). The HST likelihood was implemented by a Gaussian prior on H_0 around H_0 = 73.8 \pm 2.4, whereas the BAO data constrain the distance combination

\[ D_V(z) = [cz(1+z)^2D_A^2(z)H^{-1}(z)]^2, \]

where D_A(z) is the angular diameter distance to redshift z, H(z) is the Hubble parameter, and c is the speed of light.

For the final results we generated a set of eight chains for each combination of data sets within each of the three models considered. Convergence of the chains was tested using the Gelman–Rubin 1 – R test (Gelman & Rubin 1992), which compares the variance within each chain to the variance between the chains in order to assess whether the individual chains have converged to the same posterior distribution. Convergence was determined by requiring 1 – R to be smaller than 0.01 for all parameters. Note that GDM is defined for linear perturbations only, thus we have made no use of halofit to model nonlinearities. In \Lambda CDM, halofit makes a small difference to the lensing potential and thus also to the lensed C_\ell s.\textsuperscript{6} For parameter values around the 99.7% confidence level (CL) of our constraints, the GDM effects are approximately four times larger than the effect of halofit for \Lambda CDM, but in the opposite direction. Thus, we expect that the constraints would not undergo a significant change if a full nonlinear analysis were performed.

4. RESULTS

4.1. Constraints on the GDM Parameters

Our main results are displayed in Table 1, where we tabulate the 95.5% and 99.7% credible regions for w, c_s^2, and c_{\text{vis}}^2 for the

1 The actual difference between the codes in the case of \Lambda CDM ranges from around 10^{-3} on small scales to around 10^{-4} on large scales. However, as DASH is an older code and not as optimized as CLASS, we believe CLASS to be more accurate.

2 For \Lambda CDM, the parameter \omega_d is equal to \omega_d the CDM dimensionless density, as in that case w = c_s^2 = c_{\text{vis}}^2 = 0.

3 The value for Y_{\text{He}} is the (rounded) value quoted by Planck 2013 (Planck Collaboration et al. 2014). It was verified by a preliminary Fisher-matrix analysis that including Y_{\text{He}} as a parameter would lead to minimal changes to the constraints on the GDM parameters.

4 In CLASS, this required us to set the parameter for the effective number of massless neutrinos to N_{\text{eff}} = 2.0328 and the neutrino temperature parameter to T_{\text{vmin}} = 0.71611. Note that this is slightly larger than the ratio of the instantaneous decoupling value to the photon temperature of (4/11)^{1/3}; see the CLASS explanatory parameter file or Mangano et al. (2005) for details.

5 http://wiki.cosmos.esa.int/planckpla2015/index.php/

6 We thank Steffen Hogstatz for bringing this to our attention.
Table 1

Constraints on the GDM Parameters

| Models | \(\Lambda\)-wDM | \(\Lambda\)-GDM |
|--------|----------------|---------------|
|        | 1D posteriors  | 1D posteriors  |
|        | 95.5% CL   | 99.7% CL  | 95.5% CL   | 99.7% CL  | 95.5% CL   | 99.7% CL  |
| PPS    |            |            |            |            |            |            |
|        | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) |
|        | 3.31        | 6.31         | 5.70        | 11.3       | 5.70        | 11.3       |
| PPS+Lens |            |            |            |            |            |            |
|        | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) |
|        | 3.31        | 6.31         | 5.70        | 11.3       | 5.70        | 11.3       |
| PPS+Lens+HST |            |            |            |            |            |            |
|        | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) |
|        | 3.31        | 6.31         | 5.70        | 11.3       | 5.70        | 11.3       |
| PPS+Lens+BAO |            |            |            |            |            |            |
|        | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) | \(-0.040\times10^{-7}\) |
|        | 3.31        | 6.31         | 5.70        | 11.3       | 5.70        | 11.3       |

\(\Lambda\)-wDM and \(\Lambda\)-GDM models for each of the different choices of data sets: PPS, PPS+Lens, PPS+Lens+HST, and PPS+Lens+BAO. The constraints on the common parameters to \(\Lambda\)CDM are displayed in Table 2.

The first result to note from Table 1 is that all three GDM parameters are consistent with zero in all cases. The constraints on the GDM parameters are strong regardless of the data set combination used. The strongest constraints come from the PPS+Lens+BAO combination where \(-0.00109 < w < 0.00225\) at the 99.7% CL in the case of \(\Lambda\)-wDM and \(-0.000896 < w < 0.000238\) at the 99.7% CL in the case of \(\Lambda\)-GDM. For the latter model the parameters \(c_2\) and \(c_3\) are constrained to be less than \(3.21 \times 10^{-6}\) and \(6.06 \times 10^{-6}\) respectively at the 99.7% CL. The 1D posteriors for the \(w\) parameter in the \(\Lambda\)-GDM model are plotted in Figure 1 for all four combinations of data sets. We now examine more closely how these constraints vary between the different choices of experiment.

Consider first the constraints from the Planck power spectra. As discussed in Hu (1998) and Kopp et al. (2016), the main effect of increasing (decreasing) \(w\) is to shift the radiation–matter equality to earlier (later) times, which in turn decreases (increases) the acoustic driving effect caused by the time variation of the potential wells. The result is a decrease (increase) in the anisotropies around the first and second peaks. As further explained in Kopp et al. (2016), potential decay after recombination is related only to a time-varying total equation of state (that is, the combined equation of state from all species), hence changing \(w\) has only a mild effect on the early integrated Sachs–Wolfe (ISW) term and the lensing

Table 2

Constraints on Common Parameters to \(\Lambda\)CDM

| Models       | PPS+Lens | PPS+Lens+HST | PPS+Lens+BAO |
|--------------|----------|--------------|--------------|
| Likelihoods  | Parameters | Likelihoods  | Parameters | Likelihoods  | Parameters |
| \(\Lambda\)-wDM |            | \(\Lambda\)-GDM |            | \(\Lambda\)-wDM |            | \(\Lambda\)-GDM |            |
| \(\Lambda\)-wDM |            | \(\Lambda\)-GDM |            | \(\Lambda\)-wDM |            | \(\Lambda\)-GDM |            |
| \(\Lambda\)-wDM |            | \(\Lambda\)-GDM |            | \(\Lambda\)-wDM |            | \(\Lambda\)-GDM |            |
| \(\Lambda\)-wDM |            | \(\Lambda\)-GDM |            | \(\Lambda\)-wDM |            | \(\Lambda\)-GDM |            |
| \(\Lambda\)-wDM |            | \(\Lambda\)-GDM |            | \(\Lambda\)-wDM |            | \(\Lambda\)-GDM |            |

As further explained in Kopp et al. (2016), potential decay after recombination is related only to a time-varying total equation of state (that is, the combined equation of state from all species), hence changing \(w\) has only a mild effect on the early integrated Sachs–Wolfe (ISW) term and the lensing
potential. This means that the ISW effects on the CMB TT and TE power spectra and the lensing effects on all the CMB power spectra are of similar strength to ΛCDM and do not drive the constraints on \( w \). Finally, changing \( w \) results in a change to \( \theta_s \) that induces a lateral shift in the location of the CMB peaks, although this is overshadowed by the changes in their heights.

In Figure 2, we plot the temperature \( C_\ell \) for the best-fit ΛCDM parameters, plus two curves with \( w \) values around our 99.7% CL limits. In the bottom panel, the standard six ΛCDM parameters are held fixed when \( w \) is varied. However, in the upper panel all of the ΛCDM parameters were varied in addition to \( w \), guided by the results from our chains. Comparing the top and bottom plots shows the importance of the degeneracies of \( w \) with the standard cosmological parameters; these degeneracies act to mask the effects of \( w \). Thus, if the standard cosmological parameters were known, then the constraints on \( w \) would be significantly tighter. Note that in these plots the residuals of the data points and the \( u \) curves with respect to the ΛCDM best-fit curve have been multiplied by 10 in order to make the differences clearer, by using the transformation \( C_\ell \to C_\ell + 10(C_{\ell}^{\Lambda \text{CDM}} - C_\ell) \).

The addition of CMB lensing to the temperature and polarization \( C_\ell \) has little effect on the posterior distribution for \( w \), other than a slight shift of the mean to positive values without changing the width of the distribution. This is not surprising because the lensing is generally less constraining than the other \( C_\ell \) from Planck due to the larger noise. Furthermore, although \( w \) does have some effects on the lensing potential, such as causing the gravitational potentials to freeze out at different values (see Kopp et al. 2016), these effects can be partially compensated, as for the temperature \( C_\ell \) in Figure 2.

The addition of the data related to the expansion history does have a strong affect on \( w \), as expected. The 95.5% and 99.7% CL constraints on \( w \) are tighter by up to a factor of two when the HST prior is included. In addition, the mean value is also increased to a value just outside the 68.3% credible region, although this is not strongly significant. Note that in this case, \( w = 0 \) is excluded at the 95.5% CL. There is a known tension between the \( H_0 \) value from Planck and the HST prior, so that one cannot yet safely conclude that this is a signature of new physics. We have repeated this run using the alternative value \( H_0 = 70.6 \pm 3.3 \) from the analysis in Efstathiou (2014), with the result that the posterior shifts to the left and widens slightly compared to the standard HST value. In this case \( w \) is consistent with zero.

The change to the mean value does not appear when BAO data are used instead of the HST prior. Furthermore, with the addition of the BAO data to the Planck combination of PPS + Lens, the constraints on \( w \) are improved by a factor of four. These are the tightest constraints on \( w \) presented in this paper. The 1D posteriors for \( w \) for the Λ-GDM model can be seen in Figure 1, where the green (dotted–dashed) curve shows the posterior for PPS only, the black (solid) curve is for the PPS + Lens combination, and the red (dotted) and blue (dashed) curves show the posteriors for the addition of the HST and BAO data respectively. The changes to the constraints, and the shift of the peak when the HST data are included, are all clearly visible.

We now turn to the Λ-GDM model, and in particular to the constraints on the GDM perturbative parameters \( c_2^2 \) and \( c_{\text{vis}}^2 \). First, the constraints on \( w \) are not significantly affected by the inclusion of the other GDM parameters, \( c_2^2 \) and \( c_{\text{vis}}^2 \), as can be seen by the similarity of the constraints on \( w \) between the Λ-wDM and Λ-GDM models for each data set combination. This is to be expected because our discussion above shows that \( w \) affects the CMB differently than the two perturbative GDM parameters. In the right panels of Figure 3 we show the 2D contours in the \( w-c_2^2 \) (upper right) and \( w-c_{\text{vis}}^2 \) (lower right) planes for the PPS + Lens data set combination. The other data set combinations give similar-looking contours. These plots show that \( w \) is not strongly correlated with either of the other GDM parameters. This lack of correlation indicates a clear split
between the perturbative GDM parameters and background GDM parameters.

Moving on to the perturbative GDM parameters themselves, the main effect of non-zero $c_s^2$ and $c_{\text{vis}}^2$, as found in Kopp et al. (2016), is to cause the gravitational potential to decay. This decay results in two main effects on the CMB power spectrum, which drive the constraints on $c_s^2$ and $c_{\text{vis}}^2$. The first is a continuous ISW effect after recombination until the present time, which becomes stronger with increasing values of the perturbative GDM parameters. The second effect is on the lensing potential, also examined in Kopp et al. (2016). Since the lensing potential is directly sourced by the gravitational potential, potential decay leads to a smaller CMB lensing signal on the CMB power spectrum. As the CMB lensing decreases the peak heights and increases the peak troughs (without changing their location), the reduced lensing potential in Λ-GDM results in higher peaks and deeper troughs compared to either Λ-wDM or ΛCDM models.

The scale $k_d^{-1}(\eta)$ at conformal time $\eta$ at which potential decay in a pure GDM universe is approximately

$$k_d^{-1}(\eta) = \eta \sqrt{c_s^2 + \frac{8}{15} c_{\text{vis}}^2}.$$  \hspace{1cm} (10)

For length scales larger than $k_d^{-1}$, the effects of these two parameters on the gravitational potential are indistinguishable. For the scales relevant to the CMB, this induces a degeneracy between the two parameters. Naively, the expression for $k_d$ suggests a negative correlation between the parameters such that the errors on $c_s^2$ should be about half the size of those on $c_{\text{vis}}^2$. This is approximately what is found for the errors in Table 1. In addition, see Figure 4 where we show the 2D contours in the $c_s^2$-$c_{\text{vis}}^2$ plane. In this figure, we have plotted lines that correspond to constant $k_d^{-1}$. The direction of these lines is a good fit to the direction of the contours, providing further evidence that this is the cause of the degeneracy between these parameters.

Since $c_s^2$ and $c_{\text{vis}}^2$ do not affect the expansion history, the inclusion of either HST or BAO data has little effect on their constraints, and this is precisely what is observed in Table 1. However, the inclusion of the CMB lensing potential data (Lens) does have a significant effect on the $c_s^2$ and $c_{\text{vis}}^2$ constraints, as opposed to what was found for the constraints on $w$. In particular, since potential decay in Λ-GDM after recombination leads to shallower lensing potentials, the result is a smaller lensing potential autocorrelation power spectrum plus a larger temperature lensing potential cross-correlation due to the larger ISW contribution to the temperature spectrum. As the lensing potential probes low redshifts, the result of the lensing potential is stark. This is why the constraints on these parameters greatly improve when the Planck lensing data are included.

In Figures 5 and 6, we show the TT and lensing potential $C_8$ for the best-fit ΛCDM values, and also for values of $c_s^2$ and $c_{\text{vis}}^2$ that are close to our 99.7% CL. In the TT plot, the residuals (differences between the GDM models and the best-fit ΛCDM models) are shown for the following combinations: $\omega_b-c_s^2$ (upper left); $\omega_s-c_{\text{vis}}^2$ (lower left); $w-c_s^2$ (upper right); $w-c_{\text{vis}}^2$ (lower right). The lack of correlation shown in these plots indicates a clear split between the perturbative GDM parameters and background GDM parameters.
model) have been multiplied by 50 in order to make them visible as in Figure 2. The lensing potential plot shows how it changes significantly as a result of GDM parameters. In the TT plot, the differences from ΛCDM also arise from lensing: the smaller lensing potential results in reduced smoothing of the peaks. These plots confirm that, for constant values of these parameters, it is predominantly the lensing that is generating the constraints. This might be different if the parameters were not constant. For example, the effect of the lensing would be smaller if the parameters scaled as $a^{-3}$, as is the case for warm dark matter (Lesgourgues & Tram 2011). The values of $c^2_{\text{vis}}$ and $c^2_{s\text{vis}}$ here were chosen to reflect the $k_s$ degeneracy, and indeed there is little difference between the two GDM curves.

We can translate the upper bound on $c^2_{\text{vis}}$ and $c^2_{s\text{vis}}$ into an upper bound on the ratio of $k_s^{-1}$ to the Hubble scale. At the 99.7% CL, this is approximately $2.13 \times 10^{-3}$ (using $\eta \approx H^{-1}$), so the largest currently allowed scale on which GDM can modify cosmology is significantly below the Hubble Scale. In this sense we consider our constraints on the GDM parameters to be strong.

4.2. Constraints on the Standard ΛCDM Parameters

As expected, the inclusion of the GDM parameters worsens the constraints on some of the ΛCDM parameters, notably $\omega_g$, $\tau$, and the derived parameters $H_0$ and $\sigma_8$, as seen in Table 2.

The increased error bars for $\omega_g$ and $H_0$ are due to the strong degeneracies with $w$, because all three parameters primarily affect the background expansion. These degeneracies can be seen in Figure 7, which shows the 2D contours in the $w$-$\omega_g$ and $w$-$H_0$ planes. As explained in the previous subsection, $\omega_g$ and $w$ shift the radiation–matter equality, which in turn affects the heights of the first and second peaks in the CMB temperature spectrum (see Kopp et al. 2016). This results in a negative correlation between the two parameters, hence the degeneracy seen in the left panel of Figure 7.

Changing $w$ also has an effect on the expansion history, so we expect to get a degeneracy with $H_0$. In particular, as is well known $H_0$ and $\omega_g$ are negatively correlated even in ΛCDM (Hinshaw et al. 2013), and since $w$ and $\omega_g$ are also negatively correlated, we expect $w$ and $H_0$ to be positively correlated. Indeed this is verified in the right panel of Figure 7.

The full 1D posteriors for the PPS+Lens combination of data sets for $\omega_g$ and $H_0$ can be found in Figure 8. In this plot, the black (solid) curves show the posteriors for the ΛCDM model (upper panel) and Λ-GDM model (lower panel). In addition to the broadening of the constraints, there is also a slight shift away from the mean value found in the ΛCDM model; however, this shift remains within the 68.3% credible region for both parameters. The posterior for Λ-wDM is similar to the Λ-GDM case, as is expected from the lack of correlation of the perturbative GDM parameters with either $w$ or $\omega_g$, as shown in Figure 3, and we choose not to plot it.

As the parameters $\omega_g$ and $H_0$ primarily affect the background expansion history, their constraints are influenced by the inclusion of either the HST or BAO data. In Figure 8 we also plot the 1D posteriors for $\omega_g$ and $H_0$, for the combination of PPS+Lens with the inclusion of the HST prior, depicted by the red (dotted) curve. Once again, there is little difference in these parameters between the Λ-wDM and Λ-GDM models, and we do not plot the former. We see that the addition of the HST prior significantly improves the constraints on these parameters. There is still an offset of the mean value of the posterior

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Figure 6. Lensing potential power spectrum plotted for the ΛCDM best-fit values and for values of $c^2_{\text{vis}}$ and $c^2_{s\text{vis}}$ that are close to our 99.7% CL. The red points show the Planck data with error bars.

Figure 7. 68.3% and 95.5% credible regions in the $w$-$\omega_g$ plane (left panel) and in the $w$-$H_0$ plane (right panel). The blue (outer) contours are for the Λ-GDM run for the PPS+Lens data set combination, and the yellow (inner) contours are for the PPS+Lens+BAO data set combination. The left panel shows a strong negative correlation between these parameters due to their effects on the radiation–matter equality, whereas the right panel shows a strong positive correlation between these parameters due to their effect on the background expansion history.

Figure 8. 1D posteriors for $\omega_g$ (left panel) and $H_0$ (right panel) in the case of ΛCDM. The black (solid) curve is with the combination of PPS+Lens, the blue (dashed) curve is with PPS+Lens+BAO, and the red (dotted) curve is for PPS+Lens with the addition of the HST prior.
Figure 9. 68.3% and 95.5% credible regions of \( \sigma_8 \) with the perturbative GDM parameters \( c_2^2 \) (left) and \( c_{\text{vis}}^2 \) (right) for the \( \Lambda \)-GDM model using the PPS+Lens data set combination. The strong negative correlation in these two pairs of parameters is due to the strong reduction in structure growth induced by the GDM parameters.

compared to \( \Lambda \)CDM, which is now more significant due to the reduced width of the posteriors.

Figure 8 also shows the posteriors for the inclusion of the BAO data rather than the \textit{HST} prior, depicted by the blue (dashed) curve. For this combination, the constraints on the parameters are even tighter, with the constraints from the \( \Lambda \)-GDM model (and similarly for the \( \Lambda \)-wDM model, which is not plotted) now being almost as strong as those from the \( \Lambda \)CDM runs, despite the extra parameters. Moreover, for this data set combination, there is no significant offset of the means compared to \( \Lambda \)CDM. The shrinking of the \( w - \omega_k \) and \( w - H_0 \) contours with the inclusion of BAO is also clearly visible in Figure 7.

In the \( \Lambda \)-wDM model, \( w \) has a degeneracy with \( \sigma_8 \). However, in the \( \Lambda \)-GDM model, this is subdominant to the stronger degeneracy between \( \sigma_8 \) and the perturbative parameters \( c_2^2 \) and \( c_{\text{vis}}^2 \). This is due to the strong effect that these two parameters have on the growth of structure, as found in Kopp et al. (2016). These two parameters both greatly reduce the growth of the matter perturbations on length scales below \( \lambda_\text{d}^{-1} \). Since \( \sigma_8 \) is defined to be the amplitude of this spectrum, this results in a strong degeneracy between these two GDM parameters and \( \sigma_8 \), which is sufficiently strong to replace the usual degeneracy between \( \sigma_8 \) and \( \Lambda_8 \). In Figure 9, we show the 2D contours in the \( \sigma_8 - c_2^2 \) and \( \sigma_8 - c_{\text{vis}}^2 \) planes. Fixing either of \( \sigma_8 \) and \( c_2^2 \) (or \( c_{\text{vis}}^2 \)), and increasing the other, results in \( \Lambda_8 \) increasing. Thus we see a negative correlation between \( \sigma_8 \) and these GDM parameters.

It is interesting to look more closely at \( \sigma_8 \) for the combination of \textit{Planck}+\textit{Planck} lensing data. The one-dimensional posteriors for \( \sigma_8 \) are shown in Figure 10 for \( \Lambda \)CDM (blue, dashed), \( \Lambda \)-wDM (red, dotted), and \( \Lambda \)-GDM (black, solid). From this plot it is easily seen that, although the errors increase significantly for the \( \Lambda \)-wDM run, the mean value stays close to the \( \Lambda \)CDM value. In contrast, in the \( \Lambda \)-GDM run, in addition to the increase in the errors, the mean shifts compared to \( \Lambda \)CDM. We saw before that increasing the GDM parameters reduces the value of \( \sigma_8 \) for fixed cosmological parameters (including fixed \( \Lambda_8 \)). Since these two GDM parameters only take positive values, \( \sigma_8 \) can only be reduced, i.e., it becomes biased toward smaller numbers, relative to the \( \Lambda \)CDM value. Thus, the inclusion of these two parameters results in the posteriors being shifted, even though the GDM parameters themselves are found to be consistent with zero.

The mean value for the parameter \( \tau \) is affected by the inclusion of the GDM parameters for the combination of PPS+Lens data. The one-dimensional posteriors for \( \tau \) from the different models are also plotted in Figure 10. We can see that for the \( \Lambda \)-wDM model, the mean and width of the posterior have changed little from the \( \Lambda \)CDM values. For the \( \Lambda \)-GDM case, the width of the posterior is very similar to the \( \Lambda \)CDM value; however, the mean value has increased, albeit by less than the 68.3% CL constraint. Since \( \tau \) and \( A_8 \) are positively correlated, while \( A_8 \) and \( \sigma_8 \) are anticorrelated, we expect \( \tau \) and \( \sigma_8 \) to be anticorrelated. Therefore \( \tau \) is biased for the same reason as \( \sigma_8 \) but in the opposite direction.

Before closing this discussion let us remark that we also ran chains with the low-\( l \) \textit{Planck} likelihood and the TT-only high-\( l \) likelihood, again with the complete “not-lite” set of nuisance parameters. For these runs the constraints on \( c_2^2 \) and \( c_{\text{vis}}^2 \) become about 50% worse than for the PPS run, as expected from the reduction in information. The constraints on \( w \) worsen significantly, as do those on the parameters that it has its key degeneracies with (\( \omega_k \) and \( H_0 \), as discussed above). This is because there is simply not enough information in the temperature spectrum to constrain the expansion history once the extra degeneracies introduced by \( w \) are included. The extra information provided by the polarization is sufficient to constrain the \( \Lambda \)CDM parameters to values that are approximately in line with \textit{Planck}, with the exceptions noted above.

4.3. Comparison with Previous Work

Although a full comparison of the GDM parameters as a replacement for CDM has not been performed previously, several works have looked at including an equation of state for dark matter.

One of the first works to constrain the dark matter equation of state with cosmological data was Müller (2005). In that work, two cases for \( w \) were constrained using a combination of background data and data on the matter power spectrum. Their first case corresponds to setting \( w = c_2^2 \) in our notation (note that this means a negative value of \( c_2^2 \) was allowed in their analysis, which is unphysical). For that case, they found strong constraints on \( w \), \( \sim 10^{-6} \) at the 99.7% CL, due to the strong effect of the sound speed on matter clustering and the inclusion of the matter power spectrum data to constrain \( \sigma_8 \). These constraints are comparable to those obtained in this work on \( c_2^2 \), as would be expected. Their second case is similar to our \( \Lambda \)-wDM case, although not identical because of the difference in the definition of the non-adiabatic pressure. In that case the
constraints are much more non-Gaussian than ours, with $-8.78 \times 10^{-3} < w < 1.86 \times 10^{-3}$ at the 99.7% CL. Their constraints are at a similar level to those obtained in this paper with PPS+Lens+HST data, and less constraining than the tightest constraints in this paper obtained with the PPS+Lens+BAO data set combination.

The equation of dark matter’s state (the $\Lambda$-wDM model) was constrained in Calabrese et al. (2009) using data from WMAP (Hinshaw et al. 2009). They found $w$ to be consistent with zero, with the constraints from WMAP alone being $(-0.35^{+0.17}_{-0.98}) \times 10^{-2}$ at the 95.5% CL, which is more than a factor of two worse than those we obtain using PPS or PPS+Lens. The inclusion of additional data sets including extra CMB data sets, Supernova Legacy Survey data (SNLS: Astier et al. 2006), and matter power spectrum data from the Sloan Digital Sky Survey (Tegmark et al. 2006) improved their constraints to $(0.07^{+0.42}_{-0.33}) \times 10^{-2}$ at the 95.5% CL, which is very similar to the constraints we obtain from PPS+Lens. The strongest constraints in this paper, obtained using PPS+Lens+BAO data, are approximately three times tighter than the tightest constraints in Calabrese et al. (2009).

One of the more well-known models that fits into the GDM framework is the Chaplygin gas, and this model has been compared with Planck data previously (Li & Xu 2014). However, due to the difference between the forms of the GDM parameters in the Chaplygin gas model and those adopted in this work, our constraints are not directly comparable.

Two works (Kumar & Xu 2014; Wei et al. 2013) examine the equation of state of dark matter using a Taylor expansion around $a = 1$ as $w_\text{eff} = w_{\text{ref}} + w_{\text{m}}(1 - a)$. In Wei et al. (2013), the authors constrain the equation of state of dark matter using only background quantities, obtaining error bars that are typically larger than those obtained here. In Kumar & Xu (2014), further data sets including Planck and measurements of the matter power spectrum are also included, and the resulting error bars are typically slightly smaller than those obtained here, albeit highly non-Gaussian. However, as the equation of state was parameterized differently to our work, the results are not easy to compare directly with ours.

In a series of papers (Xu & Chang 2013; Xu 2014), the authors constrain the equation of state of dark matter (the $\Lambda$-wDM model) using different data sets in addition to Planck, including BAO data and data from SNLS3 (Guy et al. 2010; Sullivan et al. 2011) to constrain the background expansion; they also examine the effect of including WIGGLEz data on the matter power spectrum (Parkinson et al. 2012). At the 99.7% CL, the constraints are found to be $\sim 2 \times 10^{-3}$, which is very similar to the constraints on $w$ that we find here for the combination PPS+Lens+BAO. The inclusion of the WIGGLEz data has only a small effect on the constraints on $w$, again similarly to what was found in this paper with the effect on $w$ of including the Planck lensing (Lens) likelihood.

5. CONCLUSION AND DISCUSSION

The main results of this paper comprise Tables 1 and 2. These show the results of an MCMC analysis of the Planck satellite data, where the standard $\Lambda$CDM model has been extended to include the three (constant) GDM parameters, namely the equation of state $w$, the sound speed $c_s^2$, and the viscosity $c_w^2$ of the dark matter fluid. We found that these additional parameters are all consistent with zero, and are strongly constrained from the combination of high-$l$ and low-$l$ Planck likelihoods. The inclusion of Planck lensing data, and either BAO data or the HST prior, further tightens the constraints. The equation of state $w$ is constrained to be $-0.000896 < w < 0.00238$ at the 99.7% CL and the parameters $c_s^2$ and $c_w^2$ are constrained to be less than $3.21 \times 10^{-6}$ and $6.06 \times 10^{-6}$ respectively at the 99.7% CL. We found that the CMB lensing likelihood is important for constraining the parameters $c_s^2$ and $c_w^2$, whereas the inclusion of an additional data set that constrains the background expansion of the universe, either BAO or HST, is important for constraining $w$.

We uncovered a degeneracy between the two perturbative GDM parameters $c_s^2$ and $c_w^2$ that remains for all types of data sets used in this work. This is due to the way these two parameters affect the decay of the gravitational potential on length scales smaller than $k_{\text{d}}^{-1}$ as in (10). However, this degeneracy is broken well below the scale $k_d^{-1}$. In this regime, for a fixed $k_d$, $c_s^2$ causes a faster decay of the potential as well as resulting in oscillations of the matter power spectrum that are not present in a universe with $c_w^2 = 0$ and $c_w^2 = 0$. This suggests that including additional data on the matter power spectrum at late times, typically at even lower redshift than that probed by the CMB lensing potential, could improve the constraints on these parameters further. In addition, data on the matter power spectrum may probe the region where $c_s^2$ and $c_w^2$ have different effects, thus breaking degeneracies. We intend to investigate this in future work.

We have also examined the effects of including the GDM parameters on the constraints of the standard $\Lambda$CDM parameters. For the full Planck data set including lensing potential reconstruction, the main effect is to significantly increase the error bars on $\omega_d$ as well as on the derived parameters $\sigma_8$ and $H_0$. The parameters $c_s^2$ and $c_w^2$ inhibit the growth of structure below the length scale $k_{\text{d}}^{-1}$, reducing the matter power spectrum on those scales. Thus, the inclusion of $c_s^2$ and $c_w^2$ loosens the constraint on $\sigma_8$, as well as shifting the posterior to smaller values. This shift is due to the requirement that these parameters be non-negative; therefore they can only act to decrease $\sigma_8$ (and not to increase it) relative to $\Lambda$CDM. The parameter $\tau$ is affected as well. When only $w$ is added to the standard $\Lambda$CDM parameters, the posterior remains similar to that for the $\Lambda$CDM analysis. However, when all three GDM parameters are included, the mean of the posterior is shifted to higher values. We note that the constraints on $\tau$ are nonetheless similar for all three cases.

We examined the effect of using HST and BAO data in addition to the Planck data set. These improved the constraints on $w$, with the BAO data having a larger effect than the HST prior. For both the $\Lambda$-wDM and $\Lambda$-GDM models these additional data sets significantly improved the constraints on $\omega_d$ and $H_0$. Again, the inclusion of the BAO data has a larger effect, such that in this case $H_0$ and $\omega_d$ are nearly as well constrained as in a $\Lambda$CDM cosmology without the additional GDM parameters.

We have considered only the case of constant values of the GDM parameters in this work. This should act as a null test for whether there are any significant effects on the CMB from dark matter properties. Nonetheless, it is possible that a more sophisticated parameterization, perhaps following a specific
model, may result in non-zero values of the GDM parameters being preferred. In addition, we note that in our analysis we have not included many of the additional parameters that are unnecessary in a $\Lambda$CDM analysis, such as the curvature $\Omega_k$ and isocurvature perturbations. It may be that the inclusion of these parameters would allow for a non-zero value of the GDM parameters, which, combined with the effects on $r_{\delta}$, may allow the tension between Planck and observations of the late universe to be resolved. We leave this to future work.

We thank C. Brehm for useful discussions and T. Tram and B. Audren for help regarding the CLASS and MontePython codes. We also thank all of the authors of these codes for making them publicly available. The research leading to these results has received funding from the European Research Council under the European Union’s Seventh Framework Programme (FP7/2007-2013)/ERC Grant Agreement no. 617656 “Theories and Models of the Dark Sector: Dark Matter, Dark Energy and Gravity.”

REFERENCES

Ali-Hamoud, Y., Chluba, J., & Kamionkowski, M. 2015, PhRvL, 115, 071304
Anderson, L., Aubourg, É., Bailey, S., et al. 2014, MNRAS, 444, 24
Armendariz-Picon, C., & Neelakanta, J. T. 2014, JCAP, 3, 049
Astier, P., Guy, J., Regnault, N., et al. 2006, A&A, 447, 31
Audren, B., Lesgourgues, J., Benabed, K., & Prunet, S. 2013, JCAP, 2, 001
Ballesteros, G. 2015, JCAP, 3, 001
Bartrum, J., Bernal, A., & Nunez, D. 2015, MNRAS, 449, 403
Baumann, D., Nicolis, A., Senatore, L., & Zaldarriaga, M. 2012, JCAP, 7, 051
Bean, R., & Doré, O. 2004, PhRvD, 69, 083503
Beutler, F., Blake, C., Colless, M., et al. 2011, MNRAS, 416, 3017
Boehm, C., Schewitschenko, J. A., Wilkinson, R. J., Baugh, C. M., & Pascoli, S. 2014, MNRAS, 445, L31
Boylan-Kolchin, M., Bullock, J. S., & Kaplinghat, M. 2011, MNRAS, 415, L40
Buen-Abad, M. A., Marques-Tavares, G., & Schmaltz, M. 2015, PhRvD, 92, 023531
Calabrese, E., Migliaccio, M., Pagano, L., et al. 2009, PhRvD, 80, 063539
Clowe, D., Bradac, M., Gonzalez, A. H., et al. 2006, ApJL, 648, L109
Cyr-Racine, F.-Y., & Sigurdson, K. 2013, PhRvD, 87, 103515
Efstathiou, G. 2014, MNRAS, 440, 1138
Fabre, T., & Visser, M. 2006, MNRAS, 372, 136
Gelman, A., & Rubin, D. B. 1992, StaSc, 7, 457
Gil-Marín, H., Noreña, J., Verde, L., et al. 2015, MNRAS, 451, 539
Guy, J., Sullivan, M., Conley, A., et al. 2010, A&A, 523, A7
Harvey, D., Massey, R., Kitching, T., Taylor, A., & Tittley, E. 2015, Sci, 347, 1462
Hinshaw, G., Larson, D., Komatsu, E., et al. 2013, ApJS, 208, 19
Hinshaw, G., Weiland, J. L., Hill, R. S., et al. 2009, ApJS, 180, 225
Hlozek, R., Grin, D., Marsh, D. J. E., & Ferreira, P. G. 2015, PhRvD, 91, 103512
Hu, W. 1998, ApJ, 506, 485
Jee, M. J., Hoekstra, H., Mahdavi, A., & Babul, A. 2014, ApJ, 783, 78
Kaplinghat, M., Knox, L., & Skordis, C. 2002, ApJ, 578, 665
Kopp, M., Skordis, C., & Thomas, D. B. 2016, PhRvD, 94, 043512
Kumar, S., & Xu, L. 2014, PhLB, 737, 244
Landau, L. D., & Lifshitz, E. M. 1987, in Fluid Mechanics, Vol. 6 ed. L. D. Landau & E. M. Lifshitz (2nd ed.; New York: Elsevier), 512
Lefor, A. T., Futamase, T., & Akhlaghi, M. 2013, NewAR, 57, 1
Lesgourgues, J. 2014, PhLB, 737, 244
Lesgourgues, J., Marques-Tavares, G., & Schmaltz, M. 2016, JCAP, 2, 037
Lesgourgues, J., & Tram, T. 2011, JCAP, 9, 032
Li, W., & Xu, L. 2014, EPJC, 74, 2765
Mangano, G., Miele, G., Pastor, S., et al. 2005, NuPhB, 729, 221
Massey, R., Rhodes, J., Ellis, R., et al. 2007, Natur, 445, 286
Moore, B. 1994, Natur, 370, 629
Müller, C. M. 2005, PhRvD, 71, 047302
Papastergis, E., Giovanelli, R., Haynes, M. P., & Shankar, F. 2015, A&A, 574, A113
Parkinson, D., Riemer-Sorensen, S., Blake, C., et al. 2012, PhRvD, 86, 103518
Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, ApJ, 517, 565
Percia, M., Saluci, P., & Stel, F. 1996, MNRAS, 281, 27
Piattella, O. F., Casarini, L., Fabris, J. C., & de Freitas Pacheco, J. A. 2016, JCAP, 2, 024
Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2014, A&A, 571, A16
Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2016a, A&A, 594, A13
Planck Collaboration, Aghanim, N., Arnaud, M., et al. 2016b, A&A, 594, A11
Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, AJ, 116, 1009
Ross, A. J., Samushia, L., Howlett, C., et al. 2015, MNRAS, 449, 835
Rubin, V. C., & Ford, W. K., Jr. 1970, ApJ, 159, 379
Riess, A. J., Scolnic, D., Macri, L. M., et al. 2016, ApJ, 826, 10
Serra, A. L., & Domínguez Romero, M. J. L. 2011, MNRAS, 415, L74
Shoji, M., & Komatsu, E. 2010, PhRvD, 81, 123516
Sullivan, M., Guy, J., Conley, A., et al. 2011, ApJ, 737, 102
Tegmark, M., Eisenstein, D. J., Strauss, M. A., et al. 2006, PhRvD, 74, 123507
Trotta, R., & Melchiorri, A. 2005, PhRvL, 95, 011305
Wei, H., Chen, Z.-C., & Liu, J. 2013, PhLB, 720, 271
Weller, J., & Lewis, A. M. 2003, MNRAS, 346, 987
Wilkinson, R. J., Boehm, C., & Lesgourgues, J. 2014a, JCAP, 1405, 011
Wilkinson, R. J., Lesgourgues, J., & Boehm, C. 2014b, JCAP, 1404, 026
Xu, L. 2014, MPLA, 29, 1440004
Xu, L., & Chang, Y. 2013, PhRvD, 88, 123507
Yang, W., & Xu, L. 2013, PhRvD, 88, 023505
Yang, Y. 2015, PhRvD, 91, 083517
Zwicky, F. 1933, AChPh, 6, 110