Research Article

Initial Value Determination of Chua System with Hidden Attractors and Its DSP Implementation

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In this paper, a method for determining the initial value of the hidden attractors in the Chua system is studied. The initial value of the hidden attractors can be calculated quickly and accurately by the proposed method, and the hidden attractors can be found by numerical simulation. Then, the initial values of the hidden attractors are set accurately by digital signal processor (DSP), so as to the circuit realization of the chaotic system with hidden attractors is performed. The results show that the numerical simulation results of Matlab are consistent with the experimental results of DSP.

1. Introduction

In the last three decades, chaos has been widely used in neural networks [1–5], electronic circuits [6], image processing [7–10], random number generators [11], system synchronization [12–14], and secure communication [15–18] because of its characteristics of aperiodic, continuous broadband, noise-like, and unpredictable for a long time. Since Lorenz puts forward the first chaotic system in the study of atmospheric motion [19], the research and exploration of the chaotic system composed of ordinary differential equations have attracted researchers’ great attention, and many new chaotic systems with complex dynamic attractors, such as multiscroll attractors [20] and coexistence attractors [21–23] have been constantly produced.

Because the domain of attraction of the hidden attractor does not intersect with any small neighbourhood of the equilibrium point, there is no general method to predict the existence of the hidden attractor, so it is of great theoretical and practical significance to study the hidden attractor in the field of machinery and so on [24, 25]. In 2011, Leonov et al. proposed a locating algorithm for hidden attractors [26] and used the algorithm to find hidden attractors of the Chua system. Since then, the research on hidden attractors has attracted extensive interest of scholars. In 2012, Leonov et al. used the algorithm to find the hidden attractor in the Chua system with hyperbolic tangent function as nonlinear function [27]. In 2014, Zhao et al. used the algorithm to find the hidden attractor in a generalized autonomous Pol–Duffing system [28]. In the same year, Li et al. found the twin hidden attractors in the Chua system [29]. In 2017, Zhao et al. found the hidden attractor in a modified Chua system [30]. In the same year, Kuznetsov et al. also found coexistence limit cycle and symmetric hidden attractors in the Chua system [31]. Stankevich et al. analysed the scenario of the birth of the hidden attractor from its attractor basin in the Chua system [32]. These attractors are different from the classical Lorenz attractors, Chua attractors, and Chen attractors. They are not near the equilibrium point and cannot be calculated by traditional methods. Leonov et al. proposed an algorithm to determine the initial value of the hidden attractors and found the hidden attractors.

In 2016, Bao et al. designed the chaotic circuit of the chaotic system and found the hidden attractor of the system by PSIM simulation [33]. In [34], a three-dimensional autonomous chaotic circuit is designed, and the hidden attractor of the system is found by PSpice simulation. In
[35], a 5-D memristor chaotic circuit is designed. The hidden multiscroll attractors and hidden multiwing attractors are found by PSpice simulation. In [36], a 5D extreme multi-stable chaotic circuit is designed, and the hidden attractor of the system is found by PSpice simulation. In [37], a four-dimensional chaotic circuit is designed, and the coexistence hidden attractor of the system is found by PSpice simulation. In [38], a new chaotic circuit is designed by introducing the cosh function into the system in [37], and the coexistence hidden attractor of the system is found by PSpice simulation. However, the hidden attractor is studied by circuit simulation software, but the initial state of the experimental circuit is random, so the initial value of the hidden attractor cannot be set accurately.

In this paper, we study the method to determine the initial value of the hidden attractors in the Chua system. Its initial value of the hidden attractors can be set accurately by DSP, and the circuit realization of the chaotic system with hidden attractors is performed. The results show that the numerical simulation results of Matlab are consistent with the experimental results of DSP.

The rest of this work is organized as follows. Section 2 describes initial value determining algorithm for the Chua system with hidden attractors. Section 3 calculates initial values of hidden attractors and finds its hidden attractors. The Chua system with hidden attractors is implemented by DSP in Section 4. Finally, we conclude in Section 5.

2. Initial Value Determining Algorithm for Hidden Attractors

According to the initial value determining algorithm for the chaotic system with hidden attractors in [26], the Chua system with hidden attractors is

$$\frac{dx}{dt} = a(y - x) - a f(x),$$
$$\frac{dy}{dt} = x - y + z,$$
$$\frac{dz}{dt} = -by - cz,$$

(1)

where \( f(x) = mx + 0.5(n - m) (|x + 1| - |x - 1|) \), \( x, y, \) and \( z \) are system variables, and \( a, b, c, m, \) and \( n \) are system constants.

Now, system (1) is rewritten as lure system:

$$\frac{dx}{dt} = Px + q \psi (r^* x), \quad x \in R^3,$$

(2)

where \( P = \begin{pmatrix} -a (1 + m + k) & a & 0 \\ 1 & -1 & 1 \\ 0 & -b & -c \end{pmatrix} \), \( q = \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} \), \( r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \), and \( \psi (\sigma) = 0.5 (n - m) (|\sigma + 1| - |\sigma - 1|) \).

Let \( k \) be the coefficient of harmonic linearization, and \( \epsilon \) be an infinitesimal number, and equation (2) can be rewritten as

$$\frac{dx}{dt} = P_0 x + q e \psi (r^* x),$$

(3)

where \( P_0 = \begin{pmatrix} -a (1 + m + k) & a & 0 \\ 1 & -1 & 1 \\ 0 & -b & -c \end{pmatrix} P_1, \quad \lambda_{1,2} = \pm i \omega_0, \quad \lambda_3 P_1 = -d < 0, \) and \( \delta (\sigma) = \psi (\sigma) - k \sigma. \)

Using nonsingular linear transformation \( x = Sy, \) equation (3) can be transformed as

$$\frac{dy}{dt} = H x + e c \psi (u^* y),$$

(4)

where \( H = \begin{pmatrix} 0 & -\omega_0 & 0 \\ \omega_0 & 0 & 0 \\ 0 & 0 & -d \end{pmatrix}, \quad e = \begin{pmatrix} e_1 \\ e_2 \\ 1 \end{pmatrix}, \) and \( u = \begin{pmatrix} 1 \\ 0 \\ -h \end{pmatrix}. \)

The transfer function of equation (4) can be expressed as

$$W_H (p) = \frac{-e_1 p + e_2 \omega_0 + h}{p^2 + \omega_0^2 + \frac{h p}{d}}.$$

(5)

The transfer functions of system (3) can be expressed as

$$W_{P_0} (p) = r^* (P_0 - pl)^{-1} q,$$

(6)

where \( p \) is complex variables, \( \omega_0 \) is the initial frequency, which can be calculated by \( \text{Im} W_H (\omega_0) H \), and \( k \) is the harmonic linearization coefficient, which can be calculated by \( k = -\text{Re} W_H (\omega_0) H \). From the equivalence of the transfer functions of systems (3) and (4), it can be concluded:

$$k = \frac{-a (m + mc + c + \omega_0^2 - b - c)}{a (1 + c)},$$
$$d = \frac{\omega_0^2 - b + 1 + a + c + c^2}{(1 + c)},$$
$$h = \frac{a (c + b - (1 + c) d + d^2)}{\omega_0^2 + d^2},$$
$$e_1 = \frac{a (c + b - (1 + c) d - \omega_0^2)}{\omega_0^2 + d^2},$$
$$e_2 = \frac{a ((1 + c - d) \omega_0^2 + d (c + b))}{\omega_0^2 + d^2}.$$

(7)

System (3) is transformed by nonsingular linear transformation, and it can be concluded:

$$H = S^{-1} P_0 S,$$
$$e = S^{-1} q,$$
$$u^* = r^* S.$$

(8)

Let \( S = \begin{pmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{pmatrix} \). We can obtain \( s_{11} = 1, s_{12} = 0, s_{13} = -h, s_{21} = m + k + 1, s_{22} = -\omega_0^2 / a, s_{23} = -(h (a (m + k + 1) - d) / a), s_{31} = (a (m + k) - \omega_0^2) / a, s_{32} = -(a (c + b) (m + k) + ab - c \omega_0^2) / a \omega_0, \) and \( s_{33} = h ((a (m + k) (d - 1) + d (1 + a - d)) / a). \)
For the infinitesimal number $\varepsilon$, the initial value of (4) is
given by
\[
y(0) = \begin{pmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \end{pmatrix} = \begin{pmatrix} g_0 \\ 0 \\ 0 \end{pmatrix}. \tag{9}
\]

From equation (9), the relationship between the initial values of equations (3) and (4) can be obtained:
\[
x(0) = sy(0) = S \begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} g_0s_{11} \\ 0 \\ 0 \end{pmatrix}. \tag{10}
\]

In this way, the initial value of system (1) is
\[
x(0) = g_0, \\
y(0) = -g_0(m + k + 1), \\
z(0) = g_0\frac{a(m + k) - \omega_0^2}{a},
\]
where the description function of $a_0$ can be calculated as
\[
\Phi(g) = \int_{\omega_0}^{2\pi/\omega_0} \left[ \delta_1((\cos \omega_0 t)g, (\sin \omega_0 t)g, 0)\cos \omega_0 t \\
+ \delta_2((\cos \omega_0 t)g, (\sin \omega_0 t)g, 0)\sin \omega_0 t \right]dt,
\]
and the description function satisfies $\Phi(g_0) = 0$ and $b_1(d\Phi(g)/dg)|_{g=g_0} \neq 0$.

3. Numerical Simulation

According to the algorithm in [26], the initial values are $x_0 = [5.9, 0.3720, -8.4291]$ and $y_0 = [-5.9, -0.3720, 8.4291]$. Based on the calculated initial value, the phase diagram is shown in Figure 1, and the attractor basin is shown in Figure 2.

From Figure 1, it can be seen that the hidden attractor can be found according to the initial value calculated by the algorithm in [26]. From Figure 2, the blue centre region is stable equilibrium, the red region is period-1 limit cycle, and the cyan region is divergent.

4. DSP Implementation of Chaotic System with Hidden Attractors

The realization of the chaotic system by hardware circuit is the most common method to verify the design of new chaotic system, including analog circuit and digital circuit. Analog circuit mainly adopts discrete components [13] and integrates circuit (IC) [40–42] design method, while digital circuit mainly adopts FPGA [43] and DSP [44–46]. It is difficult to design and debug chaotic circuit with discrete components, and the circuit is bulky. When using IC to design the chaotic oscillator, the chip area is greatly reduced, but the design of IC requires high chip technology, and the number of wings or scrolls of the attractor is difficult to control. Because the analog circuit cannot accurately set the initial state of the system and cannot reach the calculated initial value of the hidden attractors. FPGA and DSP have high-speed data processing capability and can realize various processing algorithms through software programming [47, 48], which can conveniently realize the nonlinear characteristics of the chaotic system with hidden attractors.

4.1. Implementation of the Chaotic System

In this part, the chaotic system with hidden attractors is implemented on DSP platform. The block diagram of the working principle is shown in Figure 3. In the experiments, the Texas Instrument DSP device TMS320F28335 is employed. It is a 32 bit DSP running at 150 MHz with floating point operations. Such a high-speed clock rate is considered to be sufficient. In order to observe the phase diagrams of the attractor on the oscilloscope, the digital chaotic sequences generated on DSP are converted into analog signals. DAC8552, a 16 bit digital-to-analog converter with dual channels, is adopted. It connects DSP through SPI (serial peripheral interface).

The flowchart of the programming is shown in Figure 4. In the program, in order to reduce the effect of finite computing precision in digital circuits, all data types are defined as long double. After initializing DSP, we set the initial conditions, including initial values of state variables, and the system parameters. Iterative computation is started according to the initial values $[x_0, y_0, z_0]$. To keep the iteration not being affected by data processing, it is necessary to push the results of each iteration into the stack. Data processing includes two steps. Firstly, an appropriate positive number is added to all data to make sure all data is greater than zero. Secondly, all data is rescaled and truncated to make the output adapting the 16 bit digital-to-analog converter.

4.2. Runge–Kutta4 (RK4) Algorithm

According to the required iterative equation in Figure 4, we use the RK4 algorithm to realize the iterative equation of the Chua system with hidden attractors. RK4 is a derivative of Runge–Kutta basic model, which is used to solve ordinary differential equations with high accuracy, and mostly has proved itself superior to other solutions. RK4 algorithm is expressed as

\[
\begin{align*}
y_{i+1} &= y_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4), \\
k_1 &= f(x_i, y_i), \\
k_2 &= f(x_i + \frac{h}{2}y_i + \frac{h}{2}k_1), \\
k_3 &= f(x_i + \frac{h}{2}y_i + \frac{h}{2}k_2), \\
k_4 &= f(x_i + h, y_i + hk_3).
\end{align*}
\tag{13}
\]

According to the values of $h$, $x_i$, and $y_i$ and calculating the values of $k_1$, $k_2$, $k_3$, and $k_4$, we can attain the value of $y_{i+1}$. Three equations of system (1) are substituted into equation (13), and the three state variables $(x, y, z)$ of system (1) are solved, respectively:
Figure 1: Phase diagrams of system (1); initial values: (5.9 0.3720 −8.4291) (magenta) and (−5.9 −0.3720 8.4291) (green).

Figure 2: Continued.
Figure 2: Attractor basin of system (1): (a) initial value x-y plane; (b) initial value x-z plane; (c) initial value y-z plane.

Figure 3: Working principle for implementing the Chua system with hidden attractors on DSP.

Figure 4: Flowchart for DSP implementation of the Chua system with hidden attractors.

Figure 5: Hardware part of DSP implementation.
Figure 6: Hidden attractors by DSP implementation for $a = 8.4562; \ b = 12.0732; \ c = 0.0052; \ m = -1.1468; \ n = -0.1768$: (a) $x$-$y$ plane; (b) $x$-$z$ plane; (c) $y$-$z$ plane.

Figure 7: Continued.
\[
K_{x1} = a - x_i - mx_i - 0.5(n - m)(|x_i + 1| - |x_i - 1|) + y_i, \\
K_{y1} = x_i - y_i + z_i, \\
K_{z1} = -by_i - cz_i, \\
K_{x2} = a\left(-x_i + mx_i + 0.5(n - m)(|x_i + 1| - |x_i - 1|) + 0.5hK_{x1}\right) + (y_i + 0.5hK_{y1}), \\
K_{y2} = (x_i + 0.5hK_{x1}) - (y_i + 0.5hK_{y1}) + (z_i + 0.5hK_{z1}), \\
K_{z2} = -b(y_i + 0.5hK_{y1}) - c(z_i + 0.5hK_{z1}), \\
K_{x3} = a\left(-x_i + mx_i + 0.5(n - m)(|x_i + 1| - |x_i - 1|) + 0.5hK_{x2}\right) + (y_i + 0.5hK_{y2}), \\
K_{y3} = (x_i + 0.5hK_{x2}) - (y_i + 0.5hK_{y2}) + (z_i + 0.5hK_{z2}), \\
K_{z3} = -b(y_i + 0.5hK_{y2}) - c(z_i + 0.5hK_{z2}), \\
K_{x4} = a\left(-x_i + mx_i + 0.5(n - m)(|x_i + 1| - |x_i - 1|) + 0.5hK_{x3}\right) + (y_i + 0.5hK_{y3}), \\
K_{y4} = (x_i + 0.5hK_{x3}) - (y_i + 0.5hK_{y3}) + (z_i + 0.5hK_{z3}), \\
K_{z4} = -b(y_i + 0.5hK_{y3}) - c(z_i + 0.5hK_{z3}), \\
\]

\[
x_{i+1} = x_i + h(k_{11} + 2k_{12} + 2k_{13} + k_{14})/6, \\
y_{i+1} = y_i + h(k_{21} + 2k_{22} + 2k_{23} + k_{24})/6, \\
z_{i+1} = z_i + h(k_{31} + 2k_{32} + 2k_{33} + k_{34})/6. 
\]

Figure 7: Hidden attractors by DSP implementation; initial values: (5.9 0.3720–8.4291) (magenta) and (–5.9 –0.37208.4291) (green); (a) x-y plane; (b) x-z plane; (c) y-z plane.
4.3. Circuit Implementation Using DSP. We set \( h = 0.001 \), initial values \( x_0 = [5.9 \ 0.3720 \ -8.4291] \) and \( x_{00} = [5.9 \ 0.3720 \ 8.4291] \), when \( a = 8.4562, \ b = 12.0732, \ c = 0.0052, \ m = -1.1468, \) and \( n = -0.1768 \). The system is realized by the DSP platform. Figure 5 shows the hardware part of DSP. Phase diagrams of the system are captured randomly by the oscilloscope, as shown in Figure 6. When \( a = 8.4, \ b = 12.1, \ c = 0.005, \ m = -1.1, \) and \( n = 0.1 \), its phase diagrams is shown in Figure 7. It indicates that the Chua system with hidden attractors is realized successfully on the DSP platform.

From Figures 5–7, it is observed that the DSP circuit can generate two hidden attractors.

5. Conclusions

In this paper, we calculate the initial values of the Chua system with hidden attractors, find its hidden attractors, and obtain its phase diagram and attractor basin. Since the analog circuit cannot accurately set its initial state and cannot achieve the calculated initial value of the hidden attractor, this paper uses DSP to realize the chaos system with hidden attractors. The results show that the numerical simulation is consistent with the experimental results of DSP, which provides a practical method for the circuit implementation of the chaotic system with hidden attractors. In the next work, we will study the hidden attractor applied to secure communication.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

The work presented in this paper was a collaboration of all authors. Xianming WU contributed the idea and wrote the paper. Weijie Tan and Huhai Wang did the simulation analysis and reviewed the paper.

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