Beyond The Standard Model: This Time for Real

Frank Wilczek\textsuperscript{a} *

\textsuperscript{a}Institute for Advanced Study, School of Natural Sciences, Olden Lane, Princeton, New Jersey 08540

The value of the neutrino mass reported by the SuperK collaboration fits beautifully into the framework of gauge theory unification. Here I justify this claim, and review the other main reasons to believe in that framework. Supersymmetry and SO(10) symmetry are important ingredients; nucleon instability is a dramatic consequence.

It has been a great privilege to attend this conference, which I am sure the future will regard as historic. I want to thank the organizers for making it in every way a very enjoyable experience, as well.

Undoubtedly it will take us, collectively, many years to do full justice to the wonderful discovery announced here, that neutrinos have non-zero mass. Many important tasks remain at the level of pure phenomenology, most obviously perhaps that of integrating the firm atmospheric oscillation results with the long-standing but still confusing solar neutrino anomalies, and the possible hints from LSND of a third distinct effect. However I am going to indulge myself by leaping over these vital issues, to discourse and speculate on the larger implications of the discovery for fundamental physics. Some of us have been hoping for many years to see results of this kind. Now that they are coming in, we look forward with both eagerness and trepidation to the confrontation of our dreams with reality. Let me remind you what’s at stake.

1. A New Scale

It is important to realize that the degrees of freedom of the Standard Model permit neutrino masses. A minimal implementation of the construction requires an interaction of the type

$$\Delta \mathcal{L} = \kappa_{ij} L^{\alpha i} L^{\beta j} \epsilon_{\alpha \beta} \phi^{\dagger}_a \phi^b + \text{h.c.} ,$$

(1)

where $i$ and $j$ are family indices; $\kappa_{ij}$ is a symmetric matrix of coupling constants; the $L$ fields are the left-handed doublets of leptons, with Greek spinor indices, early Roman weak $SU(2)$ indices, and middle Roman flavor indices; and finally $\phi$ is the Higgs doublet, with its weak $SU(2)$ index. Two-component notation has been used for the spinors, to emphasize that this way of forming mass terms, although different from what we are used to for quarks and charged leptons, is in some sense more elementary mathematically. $\Delta \mathcal{L}$ becomes a neutrino mass term when the $\phi$ field is replaced by its vacuum expectation value $\langle \phi^a \rangle = v \delta^a_1$.

Although this Eq. (1) is a possible interaction for the degrees of freedom in the Standard Model, it is usually considered “beyond” the Standard Model, for a very good reason. The new term differs from the terms traditionally included in the Standard Model in that the product of fields has mass dimension 5, so that the coefficient $\kappa$ must have mass dimension -1. In the context of quantum field theory, it is a nonrenormalizable interaction. When one includes it in virtual particle loops, one will find amplitudes containing the dimensionless factors of the type $\kappa \Lambda$, where $\Lambda$ is an ultraviolet cutoff. In this framework, therefore, one cannot accept $\Delta \mathcal{L}$ as an elementary interaction. It can only be understood within a larger theoretical context.
Given a numerical value for the neutrino mass, we can infer a scale beyond which $\Delta \mathcal{L}$ cannot be accurate, and degrees of freedom beyond the Standard Model must open up. To get oriented, let us momentarily pretend that $\kappa$ is simply a number instead of a matrix, and that $m = 10^{-2}$ eV is the neutrino mass. Then, using $v = 250$ GeV for the vacuum expectation value, we calculate

$$1/M \equiv \kappa = m/v^2 = 1/(6 \times 10^{15} \text{ GeV}) .$$  \hfill (2)

When energy and momenta of order $M$ begin to circulate in loops the form of the interaction must be modified. Otherwise the dangerous factor $\kappa \Lambda$ will become larger than unity, inducing large and uncontrolled radiative corrections to all processes, and rendering the success of the Standard Model accidental.

Thus we trace the “absurdly small” value of the observed neutrino mass scale to an “absurdly large” fundamental mass scale. As I shall now discuss, this new scale provides, on the face of it, a wonderful confirmation of our best developed ideas for unification beyond the Standard Model.

Of course, experts will recognize that the foregoing argument is oversimplified; in due course, I shall revisit it in a more critical spirit.

### 2. Two Pillars of Unification

The standard model of particle physics is based upon the gauge groups $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$ of strong, electromagnetic and weak interactions acting on the quark and lepton multiplets as shown in Figure 1.

In this Figure I have depicted only one family (u,d,e,νe) of quarks and leptons; in reality there seem to be three families which are mere copies of one another as far as their interactions with the gauge bosons are concerned, but differ in mass. Actually in the Figure I have ignored masses altogether, and allowed myself the convenient fiction of pretending that the quarks and leptons have a definite chirality – right- or left-handed – as they would if they were massless. The more precise statement, of course, is that the gauge bosons couple to currents of definite chirality. The chirality is indicated by a subscript R or L. Finally the little number beside each multiplet is its assignment under the U(1) of hypercharge, which is the average of the electric charge of the multiplet.

$$\begin{align*}
\text{SU}(3) & \times \text{SU}(2) \times \text{U}(1) \\
8 \text{ gluons} & \quad W^\pm, Z, \quad \gamma \\
\text{SU}(3) & \quad (u^r_L, u^w_L, u^b_L) \quad 1/6 \\
\text{SU}(2) & \quad (u^r_R, u^w_R, u^b_R) \quad 2/3 \\
& \quad (d^r_R, d^w_R, d^b_R) \quad -1/3 \\
& \quad (\nu^L_L, e^L_L) \quad -1/2 \\
& \quad \nu^R_R, e^R_R \quad -1 \\
\end{align*}$$

Figure 1. The gauge groups of the standard model, and the fermion multiplets with their hypercharges.

While little doubt can remain that the Standard Model is essentially correct, a glance at Figure 1 is enough to reveal that it is not a complete or final theory. To remove its imperfections, while building upon its solid success, is a worthy challenge.

There are two improvements on the Standard Model that are so deeply suggested in its structure, that I think it is perverse to deny them. Let me briefly recall these two pillars of unification:

### 3. Gauge Group and Fermion Unification

Given that the strong interactions are governed by transformations among three colors, and the weak by transformations between two others, what could be more natural than to embed both theories into a larger theory of transformations among all five colors? This idea has the additional attraction that an extra U(1) symmetry commuting with the strong SU(3) and weak SU(2) symmetries automatically appears, which we can attempt to identify with the remaining gauge symmetry of the standard model, that is hypercharge. For while in the separate SU(3) and SU(2) theories we must throw
out the two gauge bosons which couple respectively to the color combinations R+W+B and G+P, in the SU(5) theory we only project out R+W+B+G+P, while the orthogonal combination (R+W+B)−3/2(G+P) remains.

Finally, the possibility of unified gauge symmetry breaking is plausible by analogy; after all, we know for sure that gauge symmetry breaking occurs in the electroweak sector.

Georgi and Glashow [2] showed how these ideas can be used to bring some order to the quark and lepton sector, and in particular to supply a satisfying explanation of the weird hypercharge assignments in the standard model. As shown in Figure 2, the five scattered SU(3)×SU(2)×U(1) multiplets get organized into just two representations of SU(5).

In making this unification it is necessary to allow transformations between (what were previously considered to be) particles and antiparticles of the same chirality, and also between quarks and leptons. It is convenient to work with left-handed fields only. Since the conjugate of a right-handed field is left-handed, we don’t lose anything by doing so – though we must shed traditional prejudices about a rigorous distinction between matter and antimatter, since these get mixed up. Specifically, it will not be possible to declare that matter is what carries positive baryon and lepton number, since the unified theory does not conserve these quantum numbers.

As shown in Figure 2, there is one group of ten left-handed fermions that have all possible combinations of one unit of each of two different colors, and another group of five left-handed fermions that each carry just one negative unit of some color. These are the ten-dimensional antisymmetric tensor and the complex conjugate of the five-dimensional vector representation, commonly referred to as the “five-bar”. In this way, the structure of the standard model, with the particle assignments gleaned from decades of experimental effort and theoretical interpretation, is perfectly reproduced by a simple abstract set of rules for manipulating symmetrical symbols. Thus for example the object RB in this Figure has just the strong, electromagnetic, and weak interactions we expect of the complex conjugate of the right-handed up-quark, without our having to instruct the theory further.

A most impressive, though simple, exercise is to work out the hypercharges of the objects in Figure 2 and checking against what you need in the Standard Model. These ugly ducklings of the Standard Model have matured into quite lovely swans.

4. Coupling Constant Unification

We have just seen that simple unification schemes are spectacularly successful at the level of classification. New questions arise when we consider dynamics.

Part of the power of gauge symmetry is that it fully dictates the interactions of the gauge bosons, once an overall coupling constant is specified. Thus if SU(5) or some higher symmetry were exact, then the fundamental strengths of the different color-changing interactions would have to be equal, as would the (properly normalized) hypercharge coupling strength. In reality the coupling strengths of the gauge bosons in SU(3)×SU(2)×U(1) are not observed to be equal, but rather follow the pattern \( g_3 \gg g_2 > g_1 \).
Fortunately, experience with QCD emphasizes that couplings “run” \[3\]. The physical mechanism of this effect is that in quantum field theory the vacuum must be regarded as a polarizable medium, since virtual particle-anti-particle pairs can screen charge. For charged gauge bosons, as arise in non-abelian theories, the paramagnetic (antiscreening) effect of their spin-spin interaction dominates, which leads to asymptotic freedom. As Georgi, Quinn, and Weinberg pointed out \[4\], if a gauge symmetry such as SU(5) is spontaneously broken at some very short distance then we should not expect that the effective couplings probed at much larger distances, such as are actually measured at practical accelerators, will be equal. Rather they will all have been affected to a greater or lesser extent by vacuum screening and anti-screening, starting from a common value at the unification scale but then diverging from one another. The pattern \(g_3 \gg g_2 > g_1\) is just what one should expect, since the antiscreening effect of gauge bosons is more pronounced for larger gauge groups.

The running of the couplings gives us a truly quantitative handle on the ideas of unification. To specify the relevant aspects of unification, one basically needs only to fix two parameters: the scale at which the couplings unite, (which is essentially the scale at which the unified symmetry breaks), and their common value when they unite. Given these, one calculates three outputs, the three \textit{a priori} independent couplings for the gauge groups in SU(3)×SU(2)×U(1). Thus the framework is eminently falsifiable. The astonishing thing is, how close it comes to working (Figure 3).

The GQW calculation is remarkably successful in explaining the observed hierarchy \(g_3 \gg g_2 > g_1\) of couplings and the approximate stability of the proton. In performing it, we assumed that the known and confidently expected particles of the standard model exhaust the spectrum up to the unification scale, and that the rules of quantum field theory could be extrapolated without alteration up to this mass scale – thirteen orders of magnitude beyond the domain they were designed to describe. It is a triumph for minimalism, both existential and conceptual.

On closer inspection, however, it is not quite good enough. Accurate modern measurements of the couplings show a small but definite discrepancy between the couplings, as appears in Figure 3. And heroic dedicated experiments to search for proton decay did not find it \[5\]: they currently exclude the minimal SU(5) prediction \(\tau_p \sim 10^{31}\) yrs. by about two orders of magnitude.

If we just add particles in some haphazard way things will only get worse: minimal SU(5) nearly works, so a generic perturbation will be deleterious. Even if some \textit{ad hoc} prescription could be made to work, that would be a disappointing outcome from what appeared to be one of our most precious, elegantly straightforward clues regarding physics well beyond the Standard Model.

Fortunately, there is a compelling escape from this impasse. That is the idea of supersymmetry \[6\]. Supersymmetry is certainly not a symmetry in nature: for example, there is certainly
no bosonic particle with the mass and charge of the electron. However there are several reasons for thinking that supersymmetry might be spontaneously, and only relatively mildly broken, so that the superpartners are no more massive than \( \approx 1 \text{ Tev} \). The most concrete arises in calculating radiative corrections to the \( (\text{mass})^2 \) of the Higgs particle from diagrams of the type shown in Figure 4. One finds that they make an infinite, and also large, contribution. By this I mean that the divergence is quadratic in the ultraviolet cutoff. No ordinary symmetry will make its coefficient vanish. If we imagine that the unification scale provides the cutoff, we will find, generically, that the radiative correction to the \( (\text{mass})^2 \) is much larger than the total value we need to match experiment. This is an ugly situation.

In a supersymmetric theory, if the supersymmetry is not too badly broken, it is possible to do better. For any set of virtual particles that might circulate in the loop there will be another graph with their supersymmetric partners circulating. If the partners were accurately degenerate, the contributions would cancel. Taking supersymmetry breaking into account, the threatened quadratic divergence will be cut off only at virtual momenta such that the difference in \( (\text{mass})^2 \) between the virtual particle and its supersymmetric partner is negligible. Notice that we will be assured adequate cancellation if and only if supersymmetric partners are not too far split in mass – in the present context, if the splitting times the square root of the fine structure constant is not much greater than the weak scale.

The effect of low-energy supersymmetry on the running of the couplings was first considered long ago [8], in advance of the precise measurements of low-energy couplings or of the modern limits on nucleon decay. One might have feared that such a huge expansion of the theory, which essentially doubles the spectrum, would utterly destroy the approximate success of the minimal SU(5) calculation. This is not true, however. To a first approximation since supersymmetry is a space-time rather than an internal symmetry it does not affect the group-theoretic structure of the calculation.

Thus to a first approximation the absolute rate at which the couplings run with momentum is affected, but not the relative rates. The main effect is that the supersymmetric partners of the color gluons, the gluinos, weaken the asymptotic freedom of the strong interaction. Thus they tend to make its effective coupling decrease and approach the others more slowly. Thus their merger requires a longer lever arm, and the scale at which...
the couplings meet increases by an order of magnitude or so, to about $10^{16}$ Gev.

I want to emphasize that this very large new mass scale has emerged unforced from the internal logic of the Standard Model itself.

Its value is important in several ways. First, it explains why the exchange of gauge bosons that are in SU(5) but not in SU(3)$\times$SU(2)$\times$U(1) does not lead to catastrophically quick nucleon decay. Second, it brings us close to the Planck scale $M_{\text{Planck}} \sim 10^{19}$ Gev at which exchange of gravitons competes quantitatively with the other interactions. Because $M_{\text{un}}$ is significantly smaller than the Planck mass, we need not be too nervous about the neglect of quantum gravity corrections to our calculation; but because it is not absurdly smaller, we can feel encouraged for the prospect of unification including both gravity and gauge forces.

Finally, as I shall be emphasizing, it can hardly be accidental that the unification scale found here is so close to the scale we previously gleaned from the neutrino mass.

There is another effect of low-energy supersymmetry on the running of the couplings, which although quantitatively small is of prime interest. There is an important exception to the general rule that adding supersymmetric partners does not immediately (at the one loop level) affect the relative rates at which the couplings run. That rule works for particles that come in complete SU(5) multiplets, such as the quarks and leptons, or for the supersymmetric partners of the gauge bosons, because they just renormalize the existing, dominant effect of the gauge bosons themselves. However there is one peculiar additional contribution, from the Higgs doublets. It affects only the weak SU(2) and hypercharge U(1) couplings. The net affect of doubling the number of Higgs fields (as, for slightly technical reasons, one must) and including their supersymmetric partners is a sixfold enhancement of the Higgs field contribution to the running of weak and hypercharge couplings. This causes a small, accurately calculable change in the unification of couplings calculation. From Figure 5 you see that it is a most welcome one. Indeed, in the minimal implementation of supersymmetric unification, it puts the running of couplings calculation right back on the money.

Since the running of the couplings with scale is logarithmic, the unification of couplings calculation is not terribly sensitive to the exact scale at which supersymmetry is broken, say between 100 Gev and 10 Tev. There have been attempts to push the calculation further, in order to address this question of the supersymmetry breaking scale, but there are many possibilities, and it is difficult to decide among them. An intriguing recent contribution is [10].

5. SO(10), and a Third Pillar

There is a beautiful extension of SU(5) to the slightly larger group SO(10). With this extension, one can unite all the observed fermions of a family, plus one more, into a single multiplet [11]. The relevant representation for the fermions is a 16-dimensional spinor representation. Some of its features are depicted in Figure 6.

In addition to the conventional quarks and leptons the SO(10) spinor contains an additional
SO(10): 5 bit register

\[
\begin{align*}
(\pm \pm \pm \pm) & : \text{even} \# \text{ of } - \\
(+-+-) & : 6 \ (u_L, d_L) \\
10 : (+-+-|+++) & : 3 \ u_R^i \\
(+-+-|--) & : 1 \ e_R^i \\
5 : (-+-|--) & : 3 \ d_L^i \\
(-+-|+|+) & : 2 \ (e_L, \nu_L) \\
1 : (+++|+++) & : 1 \ N_R
\end{align*}
\]

Figure 6. Unification of fermions in SO(10). The rule is that all possible combinations of 5 + and - signs occur, subject to the constraint that the total number of - signs is even. The SU(5) gauge bosons within SO(10) do not change the number of signs, and one sees the SU(5) multiplets emerging. However there are additional transformations in SO(10) but not in SU(5), which allow any fermion to be transformed into any other.

particle, an SU(3) × SU(2) × U(1) singlet. (It is even an SU(5) singlet.) Usually when a theory predicts unobserved new particles they are an embarrassment. But these N particles – there are three of them, one for each family – are a notable exception. Indeed, they are central to the emerging connection between neutrino masses and unification.

Because the \( N^i \) are singlets, mass terms of the type

\[
\Delta \mathcal{L}_N = \eta_{ij} N^i N^j \epsilon_{\alpha \beta}
\]  

(3)

with \( \eta_{ij} \) a symmetric coupling matrix, are consistent with \( SU(3) \times SU(2) \times U(1) \) symmetry. This term of course greatly resembles the effective interaction responsible for neutrino masses, Eq. (1), but the difference is conceptually crucial. Because the Ns are Standard Model singlets the Higgs doublets that occurred in Eq. (1) need not appear here. A consequence is that the operators appearing in Eq. (3) have mass dimension 3, so that the \( \eta_{ij} \) must have mass dimension +1. This interaction therefore does not bring in any ultraviolet divergence problems.

What sets the scale for \( \eta \)? Although Eq. (3) is consistent with Standard Model gauge symmetries, or even \( SU(5) \), it is not consistent with \( SO(10) \). Indeed for the product of spinor 16 we have the decomposition 16×16 = 10 + 120 + 126, where only the 126 contains an \( SU(5) \) singlet component. The most straightforward possibility for generating a term like Eq. (3) in the full theory is therefore to include a Higgs 126, and a Yukawa coupling of this to the 16s. If the appropriate components of the 126 acquire vacuum expectation values, Eq. (3) will emerge. The 126 is a five-index self-dual antisymmetric tensor under \( SO(10) \), which may not be to everyone’s taste. Alternatively, one can imagine that more complicated interactions, containing products of several simpler Higgs fields which condense, are responsible. These need not be fundamental interactions (they are, of course, non-renormalizable), but could arise through loop effects even in a renormalizable field theory.

At this level there are certainly many more options than constraints, so that without putting the discussion of N masses in a broader context, and making some guesses, one can’t very specific or quantitatively precise. Nevertheless, I think it is fair to say that these general considerations strongly suggest that \( \eta \) is associated with breaking of unified symmetries down to the Standard Model. Thus, if the general framework is correct, the expected scale for its entries is set by the one we met in the unification of couplings calculation, i.e. \( \eta \sim 10^{16} \) Gev.

The Ns communicate with the familiar fermions through the Yukawa interactions

\[
\Delta \mathcal{L}_{N-L} = g^a_j \bar{N}_i L^a j \phi^a_\alpha + \text{h.c.},
\]  

(4)

using the previous notations but now, in this ‘conventional’ term, suppressing the Dirac spinor indices. These interactions are of precisely the type that generate masses for the quarks and charged leptons in the Standard Model. If N were otherwise massless, the effect of Eq. (1) would be to generate neutrino masses, of the same order as ordinary quark and lepton masses. In \( SO(10) \), indeed, these masses would be related by simple Clebsch-Gordon and renormalization factors of order unity. Fortunately, as we have seen, N is far from massless.
Indeed, it is so massive that for purposes of low-energy physics we can and should integrate it out. This is easy to do. The effect of combining Eq. (3) and Eq. (4) and integrating out \( N \) is to generate

\[
\Delta L_{\text{eff}} = g_k^i g_j^l (\eta^{-1})_{kl} L^{aa_1} L^{bb_1} \epsilon_{\alpha \beta} \phi_a^\dagger \phi_b^\dagger + \text{h.c.} .
\]  

Thus we arrive back at Eq. (1), with

\[
\kappa_{ij} = g_k^i g_j^l (\eta^{-1})_{kl} .
\]  

This “seesaw” equation provides a much more precise version of the loose connection between unification scale and neutrino mass we discussed at the outset. There is much uncertainty in the details, since there is no reliable detailed theory for the \( g_k \) nor the \( \eta_s \). But if \( g \) has an eigenvalue of order unity pointing toward the third family (as suggested by symmetry and the value of the top quark mass), and if we set the scale for \( \eta \) using the logic above, then we get close to \( 10^{-2} \) eV for the \( \tau \) neutrino mass, as observed.

While at present it is less imposing than the others, this success promises to become the third pillar of unification.

The pattern of quark and charged lepton masses suggests that the other eigenvalues of \( g \) might be considerably smaller, thus generating a hierarchical pattern of neutrino masses. This is at least broadly consistent with proposed explanations of the solar neutrino anomalies, but will not readily accommodate the reported LSND results, nor neutrinos as cosmologically significant hot dark matter.

6. Summary and Prospect

A mass of approximately \( 10^{-2} \) eV for the heaviest neutrino fits beautifully into the framework of supersymmetric unification in SO(10). This sort of theory unifies the fermions in a particularly compelling way, with all the quarks and leptons in a generation fitting into a single multiplet, but requires the existence of new degrees of freedom, the \( N_s \) (one per family), which within the theory are predicted to be very heavy. The \( N_s \) themselves are not accessible, but they induce tiny masses for the observable neutrinos. Assuming supersymmetry is spontaneously and only mildly broken, this sort of theory also has impressive quantitative success in accounting for the disparate values of the gauge couplings of the Standard Model. Although I don’t have time to discuss it here, one also finds here an attractive mechanism for understanding why the standard model Higgs field, unlike the other ingredients of the Standard Model, forms an incomplete multiplet of the unified symmetry.

In this talk I have taken a minimalist approach, extrapolating straight weak-coupling quantum field theory and gauge symmetry up to near- (but sub-) Planckian mass scales, using only degrees of freedom that the facts more or less directly require. This approach has the advantage of allowing us to make some simple, definite predictions.

General consequences of the minimalist framework are that the neutrino masses are Majorana and that there are no light sterile neutrinos. Also, it is hard to avoid a hierarchical pattern of neutrino masses. This makes it difficult to accommodate a cosmologically significant contribution of neutrino dark matter. These are eminently falsifiable assertions. Indeed, at this conference some have argued, implicitly or explicitly, that they already have been falsified. We shall see. If the minimalist framework really does break down, we will have learned a profound lesson.

The large mixing angle indicated by the atmospheric oscillation results, though by no means problematic, does come as something of a surprise. To do justice to experimental information at this level of detail, we must consider it in conjunction with the whole complex of questions around how unified symmetry is broken and how the pattern of quark and lepton masses is set. Some general considerations that guide this sort of phenomenology were discussed here by Professor Pati, and in rather different ways by Professors Langacker, Mohapatra, Ramond and Yanagida. In working on this subject with Babu and Pati, I have been pleasantly surprised at how well so many diverse facts can be fit together. But as yet no insight comparable to the “pillars” has emerged from thinking about the pattern of masses and mixings, and here one longs for a deeper, more compelling theory.

In any case, the acid test for this whole line
of development is nucleon instability. Supersymmetric unification introduces new sources of nucleon instability that are precariously close to existing experimental limits. The large mixing indicated by the atmospheric neutrino oscillation results sharpens the problem from Higgsino exchange, because the dangerous Higgsino exchange is suppressed by the supposed smallness of its couplings to the light particles, and the straightforward relation of mass to coupling will be modified by mixing. Also, careful inclusion of the fields necessary to break the unified symmetry and generate neutrino masses brings to light additional potential sources of nucleon instability [14].

I hope and expect that at some future conference we will hear from SuperK – or their successors – reports of the other shoe dropping.

REFERENCES

1. J. Pati and A. Salam, Phys. Rev. Lett. 31, 661-664 (1973).
2. H. Georgi and S. Glashow, Phys. Rev. Lett. 32, 438 (1974).
3. D. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973).
4. H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).
5. LEP Electroweak Working Group, preprint CERN-PPE/96-183 (Dec. 1996).
6. See for example G. Blewitt, et al, Phys. Rev. Lett. 55, 2114 (1985), and the latest Particle Data Group compilations.
7. A very useful introduction and collection of basic papers on supersymmetry is S. Ferrara, Supersymmetry (2 vols.) (World Scientific, Singapore 1986). Another excellent standard reference is N.-P. Nilles, Phys. Reports 110, 1 (1984).
8. S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Rev. D 24, 1681 (1981).
9. J. Ellis, S. Kelley, and D. Nanopoulos, Phys. Lett. B260, 131 (1991); U. Amaldi, W. de Boer, and H. Furstenau, Phys. Lett. B260, 447 (1991); for more recent analysis see P. Langacker and N. Polonsky, Phys. Rev. D 49, 1454 (1994).
10. Barr, S.M., Preprint hep-ph/9806217.
11. H. Georgi, in Particles and Fields – 1974, ed. C. Carlson (AIP press, New York, 1975).
12. M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, ed. P. van Neuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979), p. 315; T. Yanagida, Proc. of the Workshop on Unified Theory and Baryon Number in the Universe, eds. O. Sawada and A. Sugamoto (KEK, 1979).
13. S. Dimopoulos and F. Wilczek, in The Unity of the Fundamental Interactions, Proceedings of the 19th Course of the International School of Subnuclear Physics, Erice, Italy, 1981, edited by A. Zichichi (Plenum, New York, 1983); K.S. Babu and S.M. Barr, Phys. Rev. D48 5354 (1993).
14. K.S. Babu, J. Pati and F. Wilczek, “Unification, Neutrino Masses, and Nucleon Instability,” to appear as IASSNS-HEP Preprint 98/80.