Analysis of Stress Intensity Factor in a Surface Cracked Plate under Convective Thermal Loading

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Abstract. A pre-existing surface cracked stainless steel specimen is brought to light to the cyclic temperature of convective medium. The history of cracked body’s stress intensity factor (SIF) is obtained by using appropriate weight function method is presented in this paper. Numerical simulation is performed with FEM based software ABAQUS. Cyclic temperature induces the compressive and tensile stresses in the specimen which in turn becomes the main cause of cyclic fatigue in the specimen. The other intensity function of body induced stress is stress intensity. The result shows that the behavior of Fatigue does not based only on the temperature range but also on frequency, Biot number and Initial temperature of the specimen.

1. Introduction
In aerospace, structural, nuclear, civil engineering, the components are subjected to wide range of thermal as well as mechanical loading. The fatigue has become the major cause of many accidents happening around us. The study of fatigue is needed because it is necessary to calculate the exact life period of the components under working conditions. If it exceeds the limit of fatigue life period then structure fails and causes many dangerous and hazardous effects to humans.

Crack propagation is the main cause of fatigue failure and crack propagation occurs when the threshold stress intensity factor limit exceeds through the local stress intensity feature [15, 16]. The factor of threshold stress intensity is fixed quantity for a particular material, it is a material property called ‘fracture toughness’ [11, 14, 17]. So the prognosis of fatigue life of the component requires the exact calculation of stress intensity factor to check whether it is crossing the limit or not.

In this paper the numerical simulation of pre-existing surface cracked body subjected to convective medium of cyclic temperature variation was done on finite element based ABAQUS software [4, 5, 6]. The calculation was done to calculate the thermal stress intensity factor to check whether the loading was appropriate to the specimen or higher than that. The results showed that the cyclic characteristic of thermal loading play very important role on crack propagation. The type of loading considered here was sinusoidal. Study was done on various parameters affecting the stress intensity factor.
2. Literature Review

It is very important to calculate the transient temperature distribution, stress induced and stress intensity factor induced for the thermal fatigue calculation [9, 12, 13]. Some authors use Duhamel’s and Green function using superposition theorem method for study of stress intensity factor [3, 8]. In practical application, finite element methods were used to study the stress intensity factor history for cyclic temperature loading but complicated numerical calculations were repeated for different loading for the same cracked body [2, 18]. The order of computation is step by step method required to create the whole model of crack growth and stress intensity factor variations at different loading. So scientists developed the finite element based softwares e.g. ABAQUS, ANSYS, AUTO-DYN, FE-SAFE etc in which the numerical simulation of any type of engineering application became very easy.

3. Numerical Analysis

3.1. Stress intensity and stress factor variations

A cracked infinite plate of crack length $a$, thickness $2L$, having material properties as thermal conductivity $\lambda$, density $\rho$, specific heat $C_p$, is initially at temperature of $T_i$, and the plate is unexpectedly exposed to the convective medium at temperature $T_f$. The coefficient of heat transfer was assumed to be constant. Stress state induces within the solid plate due to the heat flow. The $\sigma_{yy}$ stresses determination which are responsible of mode I opening for crack in xz plane. Fig.1 shows the cracked plate under different boundary conditions:

![Cracked plate](image)

**Fig.1.** Cracked plate induced to thermal cyclic loading

| Table 1. Thermal and Mechanical properties of AISI 304L stainless steel [1, 10] |
|-------------------|------------------|-----------------|------------------|------------------|------------------|------------------|
| Density $\rho$ (Kg/m$^3$) | Young’s modulus E (GPa) | Poisson’s ratio $\nu$ | Threshold SIF $K^\text{th}$ (MPa(m)$^{1/2}$) | Specific heat $C_p$ (J/Kg K) | Thermal Conductivity $\lambda$ (W/m K) | Coefficient of thermal expansion $\alpha$ |
| 7800 | 196 | 0.3 | 5.2 | 500 | 19 | $18\times10^{-6}$ |

3.1.1. Mechanical Boundary Condition:
- No displacement along the axis of symmetry (x axis).
- To avoid the overall translation the left point of this axis is implanted (i.e. $U_x=0$ and $U_y=0$)

3.1.2. Thermal Boundary Condition:
• The condition of adiabatic thermal is used with specimen of top surface and symmetry line including crack surface.
• The specimen of left and right side (x=±L) are explored to convective medium (Biot Number =1.57) of temperature T_f(t) with a heat transfer of coefficient H.

3.1.3. Temperature dependent boundary conditions:

\[ T_f(t) = B + A \cdot \cos(\omega t) \]  \[ \text{[7]} \]  \[ \ldots (1) \]

Where, the mean temperature is B as \( (T_{\text{max}} + T_{\text{min}})/2 \), A is the amplitude = \( (T_{\text{max}} - T_{\text{min}})/2 \) and \( \omega \) is the angular frequency. Material is considered as isotropic, stainless steel AISI 304L, wall thickness is \( 2L=10 \) mm, crack length \( a=0.2 \) mm. Heat transfer coefficient is taken as 6000 W/m²K⁻¹, for cyclic temperature variation, the temperature \( T_{\text{max}}=350 \) °C and \( T_{\text{min}}=100 \) °C, angular frequency is 0.2512 rad/s, ABAQUS is used to measure the stresses induced and stress intensity factor induced. The element used in the analysis is CPE8T, coupled temperature displacement plain strain 8 noded quadratic. Dimension in y direction was taken such that it does not impact the proximity of the crack. The mesh was précised around the crack tip to get the accurate variation of stresses induced and stress intensity factor induced. Analysis was done on half of the plate in ABAQUS.

![Adiabatic zone](image)

Convective medium

H, \( T_f(t) \)

Adiabatic zone

Convective medium

H, \( T_f(t) \)

Crack

U_x

U_y

Fig. 2. Meshed plate under thermal and mechanical boundary conditions

4. Results and Discussion

The cracked plate’s stress intensity factor under thermal loading can be calculated by using method of weight function. The weight function characteristic is autonomous of the loading and SIF is given by:

\[ K_1(t) = \int_{L=\alpha}^{L} f(x, \alpha) \cdot \sigma_{yy}(x, t)dx \]

where \( f(x, \alpha) \) is the weight function \[ \text{[3]} \]  \[ \ldots (2) \]
A primary small transient evolution is organized inadequate to first cycle, followed by steady one. For upper value of frequency, no. of cycles needed to reach the higher steady state. The cyclic gradient in temperature is subject to the stress induced in the plate. Fig. 4 shows stress and stress intensity factor variation when the circular frequency of the applied cyclic temperature loading changes from 0.2512 rad/s to 0.6280 rad/s and we see that both the stress and SIF values becomes higher. SIF value increased higher than the threshold stress intensity factor thereby crack growth will occur.

The initial temperature of the body was having a significant effect on the variations of stresses and SIF. Fig. 5 shows the effect when the initial temperature was changed from 350 °C to 500 °C on the stresses and SIF.
Fig. 5. Numerical variations of stress induced and stress intensity factor by cyclic temperature distribution when $T_i$ changes from 350 °C to 500 °C.

The initial value of stress intensity factor increases with $T_i$. The study evolutions depend only on the characteristics of convective medium.

5. Conclusions
The SIF value is directly proportional to the temperature range i.e. $(T_{max} - T_{min})$ of the applied convective medium. An increase in temperature range there is increase in SIF value. Increase in frequency implies the increase in SIF value and if it exceeds the threshold SIF then the specimen will fail because crack propagation occurs. It requires more time to reach steady state means the transient zone becomes large which is dangerous to the specimen. Increase in initial temperature has significant effect on the transient zone of the stress and SIF variations. With the increase in $T_i$, SIF value will increase and approaches the threshold value if initial temperature is kept high. Also effect of Biot number is counted, an increase in Biot number the SIF value increases.

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