Cost-Driven Ontology-Based Data Access
(Extended Version)

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Abstract

In ontology-based data access (OBDA), users are provided with a conceptual view of a (relational) data source that abstracts away details about data storage. This conceptual view is realized through an ontology that is connected to the data source through declarative mappings, and query answering is carried out by translating the user queries over the conceptual view into SQL queries over the data source. Standard translation techniques in OBDA try to transform the user query into a union of conjunctive queries (UCQ), following the heuristic argument that UCQs can be efficiently evaluated by modern relational database engines. In this work, we show that translating to UCQs is not always the best choice, and that, under certain conditions on the interplay between the ontology, the mappings, and the statistics of the data, alternative translations can be evaluated much more efficiently. To find the best translation, we devise a cost model together with a novel cardinality estimation that takes into account all such OBDA components. Our experiments confirm that (i) alternatives to the UCQ translation might produce queries that are orders of magnitude more efficient, and (ii) the cost model we propose is faithful to the actual query evaluation cost, and hence is well suited to select the best translation.

1 Introduction

An important research problem in Big Data is how to provide end-users with effective access to the data, while relieving them from being aware of details about how the data is organized and stored. The paradigm of Ontology-based Data Access (OBDA) [21] addresses this problem by presenting to the end-users a convenient view of the data stored in a relational database. This view is in the form of a virtual RDF graph [18] that can be queried through SPARQL [15]. Such virtual RDF graph is realized by means of the TBox of an OWL 2 QL ontology [20] that is connected to the data source through declarative mappings. Such mappings, associate to each predicate in the TBox (i.e., a concept, role, or attribute) an SQL query over the data source, which intuitively specifies how to populate the predicate from the data extracted by the query.

Query answering in this setting is not carried out by actually materialising the data according to the mappings, but rather by first rewriting the user query with respect to the TBox, and then translating the rewritten query into an SQL query over the data. In state-of-the-art OBDA systems [8], such SQL translation is the result of structural optimizations, which aim at obtaining a union of conjunctive queries (UCQ). Such an approach is claimed to be effective because (i) joins are over database values, rather than over URIs constructed by applying mapping definitions; (ii) joins in UCQs are performed by directly accessing (usually, indexed)
database tables, rather than materialized and non-indexed intermediate views. However, the requirement of generating UCQs comes at the cost of an exponential blow-up in the size of the user query.

A more subtle, sometimes critical issue, is that the UCQ structure accentuates the problem of redundant data, which is particularly severe in OBDA where the focus is on retrieving all the answers implied by the data and the TBox: each CQ in the UCQ can be seen as a different attempt of enriching the set of retrieved answers, without any guarantee on whether the attempt will be successful in retrieving new results. In fact, it was already observed in [2] that generating UCQs is sometimes counter-beneficial (although that work was focusing on a substantially different topic).

As for the rewriting step, Bursztyn et al. [3, 5] have investigated a space of alternatives to UCQ perfect rewritings, by considering joins of UCQs (JUCQs), and devised a cost-based algorithm to select the best alternative. However, the scope of their work is limited to the simplified setting in which there are no mappings and the extension of the predicates in the ontology is directly stored in the database. Moreover, they use their algorithm in combination with traditional cost models from the database literature of query evaluation costs, which, according to their experiments, provide estimations close to the native ones of the PostgreSQL database engine.

In this work we study the problem of alternative translations in the general setting of OBDA, where the presence of mappings needs to be taken into account. To do so, we first study the problem of translating JUCQ perfect rewritings such as those from [3], into SQL queries that preserve the JUCQ structure while maintaining property (i) above, i.e., the ability of performing joins over database values, rather than over constructed URIs. We also devise a cost model based on a novel cardinality estimation, for estimating the cost of evaluating a translation for a UCQ or JUCQ over the database. The novelty in our cardinality estimation is that it exploits the interplay between the components of an OBDA instance, namely ontology, mappings, and statistics of the data, so as to better estimate the number of non-duplicate answers.

We carry out extensive and in-depth experiments based on a synthetic scenario built on top of the Wisconsin Benchmark [11], a widely adopted benchmark for databases, so as to understand the trade-off between a translation for UCQs and JUCQs. In these experiments we observe that: (i) factors such as the number of mapping assertions, also affected by the number of axioms in the ontology, and the number of redundant answers are the main factors for deciding which translation to choose; (ii) the cost model we propose is faithful to the actual query evaluation cost, and hence is well suited to select the best alternative translation of the user query; (iii) the cost model implemented by PostgreSQL performs surprisingly poorly in the task of estimating the best translation, and is significantly outperformed by our cost model. The main reason for this is that PostgreSQL fails at recognizing when different translations are actually equivalent, and may provide for them cardinality estimations that differ by several orders of magnitude.

In addition, we carry out an evaluation on a real-world scenario based on the NPD benchmark for OBDA [19]. Also in these experiments we confirm that alternative translations to the UCQ one may be more efficient, and that the same factors already identified in the Wisconsin experiments determine which choice is best.

The rest of the paper is structured as follows. Section 2 fixes our notation conventions. Sections 3 introduces the relevant technical notions underlying OBDA. Section 4 we place our
paper w.r.t. the field literature. Section 5 provides our characterization for SQL translations of JUCQs. Section 6 presents our novel model for cardinality estimation, and Section 7 the associated cost model. Section 8 provides the evaluation of the cost model on the Wisconsin and NPD Benchmarks. Section 9 concludes the paper.

2 Notation

From now on, we will denote tuples in bold font, e.g. \( \mathbf{x} \) is a tuple (when convenient we might treat tuples as sets). Given a predicate symbol \( P \), a tuple of function symbols \( \mathbf{f} \), and a tuple of variables \( \mathbf{x} \), we denote respectively by \( P(\mathbf{f}(\mathbf{x})) \) an atom where each argument is of the form \( g(y) \), \( y \subseteq \mathbf{x} \), and by \( P(\mathbf{f}, \mathbf{x}) \) a generic atom over function symbols \( \mathbf{f} \) and variables \( \mathbf{x} \). Given an atom \( a = P(\mathbf{f}, \mathbf{x}) \), we denote by \( \text{pred}(a) \) its predicate symbol \( P \).

3 Preliminaries

In this section we introduce basic notions and notation necessary for the technical development of this work.

We rely on the OBDA framework of [21], which we formalize here through the notion of OBDA specification, which is a triple \( S = (T, M, \Sigma) \) where \( T \) is an ontology TBox, \( M \) is a set of mappings, and \( \Sigma \) is the schema of a relational database. In the remainder of this section, we formally introduce such elements of an OBDA specification.

3.1 Syntax and Semantics of DL-Lite\(_R\)

An ontology is a structured description of a domain of interest. We here consider ontologies formulated in DL-Lite\(_R\) [9], which is the DL providing the formal foundations for OWL 2 QL, the W3C standard ontology language for OBDA [20]. The building block of an ontology is vocabulary \( V = (N_C, N_R) \), where \( N_C, N_R \) are respectively countably disjoint sets of concept names and property names. A predicate is an element from \( N_C \cup N_R \). Roles \( R \), basic concepts \( B \), and concepts \( C \) in DL-Lite\(_R\) are constructed according to the following grammar rules:

\[
\begin{align*}
B &\rightarrow \bot \mid A \mid \exists R.T \\
C &\rightarrow B \mid \exists R.B \\
R &\rightarrow P \mid P^- 
\end{align*}
\]

where \( A \in N_C, P \in N_R \). A DL-Lite\(_R\) TBox is a finite set of inclusion assertions of the form \( B \equiv C, R_1 \equiv R_2, B_1 \sqcap B_2 \equiv \bot, R_1 \sqcap R_2 \equiv \bot \). Let \( N_I \) be an infinite set of individuals, disjoint from \( N_R \) and \( N_C \). An ABox is a finite set of assertions of the form \( A(i), P(i_1, i_2) \), where \( i, i_1, i_2 \in N_I \). A DL-Lite\(_R\) ontology is a pair \( (T, A) \) where \( T \) is an DL-Lite\(_R\) TBox and \( A \) is an ABox.

Semantics. Semantics for DL-Lite ontologies is given through Tarski-style interpretations. Intuitively, an interpretation assigns to each concept a set of individuals from a given interpre-
Table 1: Semantics for DL-Lite concepts.

| Syntax Element | Semantics |
|----------------|-----------|
| ⊤              | ∆²       |
| ⊥              | ∅        |
| ⊓              | C Ì D ²  |
| ∃r.C           | \{x ∈ ∆² | ∃y ∈ ∆² : (x,y) ∈ r² \} |

Interpretation domain. Formally, an interpretation is a pair \( I = (∆², ·²) \) where \( ∆² \) is a non-empty set, called domain, and \( ·² \) is a function (called interpretation function) that assigns:

- To every concept name \( A \in N_C \), a set \( A² \subseteq ∆² \)
- To every role name \( R \in N_R \), a set of pairs \( r² \subseteq ∆² \times ∆² \)
- To every individual \( a \in N_I \), an element \( a² \in ∆² \)

The function is extended to concepts as described in Table 1.

Let \( I \) be an interpretation. \( I \) satisfies an inclusion axiom \( C ⊑ D \) (denoted \( I \models C ⊑ D \)) if \( C² \subseteq D² \).

\( I \) satisfies a TBox \( T \) (denoted \( I \models T \)) iff it satisfies all inclusion axioms in \( T \).

\( I \) satisfies an assertion \( C(a) \) (resp., \( R(a,b) \)) if \( a² \in C² \) (resp., \( (a²,b²) ∈ R² \)). \( I \) satisfies an ABox \( A \) (denoted \( I \models A \)) if it satisfies all assertions in \( A \).

Finally, \( I \) satisfies an ontology \( O = (T,A) \) if \( I \models T \) and \( I \models A \).

3.2 Ontology-based Data Access (OBDA)

We consider here first-order (FO) queries [1], and we use \( q^D \) to denote the evaluation of a query \( q \) over a database \( D \). We use the notation \( q^A \) also for the evaluation of \( q \) over the ABox \( A \), viewed as a database. For an ontology \( O \), we use \( \text{cert}(q,O) \) to denote the certain answers of \( q \) over \( O \), which are defined as the set of tuples \( a \) of individuals such that \( O \models q(a) \) (where \( \models \) denotes the DL-Lite entailment relation). We consider also various fragments of FO queries, notably conjunctive queries (CQs), unions of CQs (UCQs), and joins of UCQs (JUCQs) [1].

Mappings specify how to populate the concepts and roles of the ontology from the data in the underlying relational database.

**Definition 3.1.** A mapping \( m \) is an expression of the form \( L(f(x)) ⇝ q_m(x) \): the target part \( L(f(x)) \) of \( m \) is an atom over function symbol \( f \) and variables \( x \) whose predicate name \( L \) is a concept or role name; the source part \( q_m(x) \) of \( m \) is a FO query with output variables \( x \).

1For conciseness, we use here abstract function symbols in the mapping target. We remind that in concrete mapping languages, such as R2RML [10], such function symbols correspond to IRI templates used to generate object IRIs from database values.

2In general, the output variables of the source query might be a superset of the variables in the target, but for our purposes we can assume that they coincide.
Given a mapping \( m \), we sometimes denote its target part as \( \text{target}(m) \), and its source part as \( \text{source}(m) \). We say that the signature \( \text{sign}(m) \) of \( m \) is the pair \((L, f)\), and that \( m \) defines \( L \). We also define \( \text{sign}(\mathcal{M}) = \{\text{sign}(m) | m \in \mathcal{M}\} \). Given a signature \( s = (X, f) \), we denote by \( \text{pred}(\mathcal{M}) \) the predicate symbol \( X \). We denote by \( \text{pred}(\text{sign}(m)) \) the set \( \{\text{pred}(\text{sign}(m)) | m \in \mathcal{M}\} \).

Following [2], we split each mapping \( m = L(f(x)) \leadsto q_m(x) \) in \( \mathcal{M} \) into two parts by introducing an intermediate view name \( V_m \) for the FO query \( q_m(x) \). We obtain a low-level mapping of the form \( V_m(x) \leadsto q_m(x) \), and a high-level mapping of the form \( L(x) \leadsto V_m(x) \).

In the following, we abstract away the low-level mapping parts, and we consider \( \mathcal{M} \) as consisting directly of the high-level mappings. In other words, we directly consider the intermediate view atoms \( V_m \) as the source part, with the semantics \( V_m^D = q_m^D \), for each database instance \( D \). We denote by \( \Sigma_\mathcal{M} \) the virtual schema consisting of the relation schemas whose names are the intermediate view symbols \( V_m \), with attributes given by the answer variables of the corresponding source queries.

From now on we fix an OBDA specification \( \mathcal{S} = (\mathcal{T}, \mathcal{M}, \Sigma) \). Given a database instance \( D \) for \( \Sigma \), we call the pair \((\mathcal{S}, D)\) an OBDA instance.

**Definition 3.2.** Let \( D \) be a database instance for \( \Sigma \). The virtual ABox exposed by \( D \) through \( \mathcal{M} \) is the set of assertions

\[
\mathcal{A}(\mathcal{M}, D) = \{ L(f(a)) | L(f(x)) \leadsto V(x) \in \mathcal{M} \text{ and } a \in V(x)^D \}
\]

Intuitively, such an ABox is obtained by evaluating, for each (high level) mapping \( m \), its source view \( V(x) \) over the database \( D \), and by using the returned tuples to instantiate the concept or role \( L \) in the target part of \( m \).

### 3.2.1 Query Answering in OBDA

The certain answers \( \text{cert}(q, (\mathcal{S}, D)) \) to a query \( q \) over an OBDA instance \((\mathcal{S}, D)\) are defined as \( \text{cert}(q, (\mathcal{T}, \mathcal{A}(\mathcal{M}, D))) \).

In the virtual approach to OBDA, such answers are computed without actually materializing \( \mathcal{A}(\mathcal{M}, D) \), by transforming the query \( q \) into a FO query \( q_{fo} \) formulated over the database schema \( \Sigma \) such that \( q_{fo}^D = \text{cert}(q_s, (\mathcal{S}, D')) \), for every OBDA instance \((\mathcal{S}, D')\). To define the query \( q_{fo} \), we introduce the following notions:

- A query \( q_r \) is a perfect rewriting of a query \( q' \) with respect to a TBox \( \mathcal{T} \), if \( \text{cert}(q'_r, (\mathcal{T}, \mathcal{A})) = q'_r^A \) for every ABox \( \mathcal{A} \) [9].
- A query \( q_l \) is an \( \mathcal{M} \)-translation of a query \( q' \), if \( q_l^D = q'^{\mathcal{A}(\mathcal{M}, D)} \), for every database \( D \) for \( \Sigma \) [21].

Notice that, by definition, all perfect rewritings (resp., translations) of \( q' \) with respect to \( \mathcal{T} \) (resp., \( \mathcal{M} \)) are equivalent. Consider now a perfect rewriting \( q_T \) of \( q \) with respect to \( \mathcal{T} \), and then a translation \( q_T^M \) of \( q_T \) with respect to \( \mathcal{M} \). It is possible to show that such a \( q_{T,M} \) satisfies the condition stated above for \( q_{fo} \).

Many different algorithms have been proposed for computing perfect rewritings of UCQs with respect to \( DL\text{-Lite}_R \) TBoxes, see, e.g., [9] [16]. As for the translation, [21] proposes an algorithm that is based on non-recursive Datalog [11], extended with function symbols in the
head of rules, with the additional restriction that such rules never produce nested terms. We consider Datalog queries of the form \((G, \Pi)\), where \(G\) is the answer atom, and \(\Pi\) is a set of Datalog rules following the restriction above. We abbreviate a Datalog query of the form \((q(x), \{q(x) \leftarrow B_1, \ldots, B_n\})\), corresponding to a CQ (possibly with function symbols), as \(q(x) \leftarrow B_1, \ldots, B_n,\) and we also call it \(q\).

**Definition 3.3 (Unfolding of a CQ)**. Let \(q(x) \leftarrow L_1(v_1), \ldots, L_n(v_n)\) be a CQ. Then, the *unfolding* \(\text{unf}(q, \mathcal{M})\) of \(q\) w.r.t. \(\mathcal{M}\) is the Datalog query \((q_{\text{unf}}(x), \Pi)\), where \(\Pi\) is a (up to variable renaming) minimal set of rules having the following property:

If \(((m_1, \ldots, m_n), \sigma)\) is a pair such that \(\{m_1, \ldots, m_n\} \subseteq \mathcal{M}\), and

- \(m_i = L_i(f_i(x_i)) \leadsto V_i(z_i),\) for each \(1 \leq i \leq n,\)
- \(\sigma\) is a most general unifier for the set of pairs \(\{(L_i(v_i), L_i(f_i(x_i))) \mid 1 \leq i \leq n\},\)

then the query \(q_{\text{unf}}(\sigma(x)) \leftarrow V_1(\sigma(z_1)), \ldots, V_n(\sigma(z_n))\) belongs to \(\Pi\).

**Definition 3.4 (Unfolding of a UCQ \(q\))**. Given an UCQ \(q(x) = \bigvee_i q_i(x_i)\) and a set of mappings \(\mathcal{M}\), the *unfolding* of \(\text{unf}(q, \mathcal{M})\) w.r.t \(\mathcal{M}\) is the datalog query

\[
\text{unf}(q, \mathcal{M}) = (q_{\text{unf}}(x), \bigcup_i \Pi_i)
\]

where \(\Pi_i\) is the datalog program of \(\text{unf}(q_i(x_i), \mathcal{M}).\)

**Theorem 3.1 (An Unfolding is a Translation)**. Let \(q\) be a UCQ. Then, \(\text{unf}(q, \mathcal{M})^D = q^{A_{\mathcal{M}, D}}\).

We now present some definitions and results laying down important technical foundations for this work.

**Definition 3.5 (Equivalent Sets of Mappings)**. Two sets of mapping assertions \(\mathcal{M}\) and \(\mathcal{M}'\) are said to be equivalent, denoted as \(\mathcal{M} \equiv \mathcal{M}'\), if

\[
\forall D: A_{\mathcal{M}, D} = A_{\mathcal{M}', D}
\]

A direct consequence of Theorem 3.1 is that unfoldings over equivalent mappings must be equivalent.

**Lemma 3.1.** Let \(\mathcal{M}'\) be a set of mappings such that \(\mathcal{M}' \equiv \mathcal{M}\). Then, for every UCQ \(q_i\), it holds

\[
\text{unf}(q, \mathcal{M}) \equiv \text{unf}(q, \mathcal{M}')
\]

**Proof.** We prove the contrapositive of the statement.
Without loss of generality, let \(a\) is such that \(a \in \text{unf}(q, \mathcal{M})^D\), and \(a \notin \text{unf}(q, \mathcal{M}')^D\), for some data instance \(D\) of \(\Sigma\). For Theorem 3.1 the above implies that

\[
a \in q^{A_{\mathcal{M}, D}}, a \notin q^{A_{\mathcal{M}', D}}.
\]

Hence, \(A_{\mathcal{M}, D} \neq A_{\mathcal{M}', D}\), and therefore \(\mathcal{M} \neq \mathcal{M}'\).
4 Background

Bursztyn et. al. [4, 7, 6] studied a space of alternatives to UCQ rewritings, in a setting without mappings and in which the ABox of the ontology is directly stored in the database. Their works provided empirical evidence supporting the hypothesis that UCQ rewritings can be evaluated less efficiently than alternatives forms of rewritings, under certain assumptions on how the ABox is stored in the relational database. In particular, they explore a space of alternatives to the traditional UCQ rewriting in shape of join of unions of conjunctive queries (JUCQs), and provide an algorithm that, given a cost model, selects the best choice.

Example 4.1. Consider the TBox $\mathcal{T} = \{ C \sqsubseteq D \}$, and the query $q(x, y) \leftarrow D(x), P(x, y)$. An UCQ perfect rewriting for $q$ w.r.t. $\mathcal{T}$ is

$$q_{\text{rew}} = \{(x, y) \mid (C(x) \land P(x, y)) \lor (D(x) \land P(x, y))\}$$

An alternative rewriting can be obtained by applying well-known distributive rules for $\lor$ and $\land$ connectives over the calculus formula in $q_{\text{rew}}$. By doing so, we obtain the alternative rewriting

$$q'_{\text{rew}} = \{(x, y) \mid (C(x) \lor D(x)) \land P(x, y)\}$$

Observe that $q'_{\text{rew}}$ is a JUCQ. In particular, it corresponds to the query

$$q_{\text{cover}}(x, y) \leftarrow q_{\text{rew}, D}(x), q_{\text{rew}, P}(x, y)$$

where

- $q_{\text{rew}, D}(x)$ is an UCQ perfect rewriting of $q_D(x) \leftarrow D(x)$ with respect to $\mathcal{T}$, and
- $q_{\text{rew}, P}(x, y)$ is an UCQ perfect rewriting of $q_P(x, y) \leftarrow P(x, y)$ with respect to $\mathcal{T}$.

In this work we study the problem of alternative translations in a full-fledged OBDA scenario with mappings. A classical pitfall when considering alternatives to UCQ translations in a context with mappings, is that it is very easy to lose ability of performing joins over database values, rather than over URIs constructed by applying mapping definitions. The next Example shows this issue.

Example 4.2. Consider the same ontology and queries from Example 4.1. Consider a set of mappings

$$\mathcal{M} := \begin{cases} C(f(a)) & \leadsto V_1(a) \\ D(f(b)) & \leadsto V_2(b) \\ D(g(n)) & \leadsto V_3(c) \\ P(f(d), h(e)) & \leadsto V_4(d, e) \end{cases}$$

An $\mathcal{M}$-translation $q_{\text{trans}}$ of $q'_{\text{rew}}$ can be obtained by applying the unfolding procedure to queries $q_{\text{rew}, D}$ and $q_{\text{rew}, P}$, and joining the resulting queries. That is,

$$q_{\text{trans}}(x, y) \leftarrow q_{\text{unf}, D}(x), q_{\text{unf}, P}(x, y)$$
where \( q_{unf_\mathcal{D}}(x) \) is a datalog query of program

\[
\Pi_{unf_\mathcal{D}} := \begin{cases} 
q_{unf_\mathcal{D}}(f(a)) & \leftarrow V_1(a) \\
q_{unf_\mathcal{D}}(f(b)) & \leftarrow V_2(b) \\
q_{unf_\mathcal{D}}(g(c)) & \leftarrow V_3(c)
\end{cases}
\]

and \( q_{unf_\mathcal{P}}(x, y) \) is a datalog query of program

\[
\Pi_{unf_\mathcal{P}} = \{ q_{unf_\mathcal{P}}(f(d), h(e)) \leftarrow V_4(d, e) \}
\]

The SQL corresponding to \( q_{\text{trans}} \), in algebra notation, would be a query of the form

\[
\pi_{x,y}(\pi_{x/f}(V_1) \cup \pi_{x/f}(V_2) \cup \pi_{x/f}(V_3)) \bowtie \pi_{x/f}(d, y/g)(V_4)
\]

where each expression of the form \( x/f(y) \) appearing in the projections denotes the application of the function symbol \( f \) over database values instantiating the attributes in \( y \), and that such applications construct individuals under the answer variable \( x \).

Observe that \( q_{\text{trans}} \) is a JUCQ. Moreover, it contains a join over the result of the application of function symbols to database values, that in Ontop correspond to an inefficient join over columns whose values are constructed from the concatenation of URI templates and database values.

We now introduce some terminology from [4], that formalize to the intuitions given in the Example above, and that we use in our technical development.

**Definition 4.1** (Cover [4]). Let \( q \) be a query consisting of atoms \( F = \{L_1, \ldots, L_n\} \). A cover for \( q \) is a collection \( C = \{f_1, \ldots, f_m\} \) of non-empty subsets of \( F \), called fragments, such that (i) \( \bigcup_{f_i \in C} f_i = F \) and (ii) no fragment is included into another one.

**Definition 4.2** (Fragment Query [4]). Let \( C \) be a cover and \( q \) a query. The fragment query \( q|_f(x_f) \), for \( f \in C \), is the query whose body consists of the atoms in \( f \) and whose answer variables \( x_f \) are given by the answer variables \( x \) of \( q \) that appear in the atoms of \( f \), union the existential variables in \( f \) that are shared with another fragment \( f' \in C \), with \( f' \neq f \).

**Definition 4.3** (Cover-based Perfect Rewriting [4]). Consider the query \( q_{C}(x) \leftarrow \bigwedge_{f \in C} q_f^{\text{acq}}(x_f) \), where \( q_f^{\text{acq}}(x_f) \), for each \( f \in C \), is a CQ-to-UCQ perfect rewriting of the query \( q_f \) w.r.t. \( \mathcal{T} \). Then \( q_{C} \) is a cover-based JUCQ perfect rewriting of \( q \) w.r.t. \( \mathcal{T} \) and \( C \), if it is a perfect rewriting of \( q \) w.r.t. \( \mathcal{T} \).

In DL-Lite\(_R\), not every cover leads to a cover-based JUCQ perfect rewriting. Authors in [4] gave a sufficient condition for cover rewritable covers, called “safety”.

**Theorem 4.1** (Cover-based query answering [4]). Applying Definition 4.3 on a rewritable cover \( C \) of \( q \) w.r.t. \( \mathcal{T} \), and any rewriting technique, yields a cover-based JUCQ perfect rewriting \( q_r \) of \( q \) w.r.t. \( \mathcal{T} \).
5 Cover-based Translation in OBDA

**Definition 5.1** (Unfolding for a JUCQ 1). For each \( f \in C \), let \( \text{Aux}_f \) be an auxiliary predicate for \( q^{\text{ucq}}_f(x_f) \), and let \( U_f \) be a view symbol for the unfolding \( \text{unf}(q^{\text{ucq}}_f(x_f), M) \), for each \( f \in C \). Consider the set of mappings \( M^{\text{aux}} = \{ \text{Aux}_f(x_f) \rightharpoonup U_f(x_f) \mid f \in C \} \) associating the auxiliary predicates to the auxiliary view names. Then, we define the unfolding \( \text{unf}(q_C, M) \) of \( q_C \) with respect to \( M \) as \( \text{unf}(q_C, M) \) = \( \text{unf}(q^{\text{ucq}}_C(x) \rightharpoonup \bigwedge_{f \in C} \text{Aux}_f(x_f), M^{\text{aux}}) \).

**Theorem 5.1** (Translation 1). Let \( q_C(x) \rightharpoonup \bigwedge_{i=1}^n q^{\text{ucq}}_{f_i} \) be a JUCQ cover-based perfect rewriting. Then, the query \( \text{unf}(q_C, M) \) is an \( M \)-translation for \( q_C \).

**Proof.** We need to prove that, for each OBDA instance \( D \) of \( S \), it holds

\[
\text{unf}(q_C, M)^D = q^{A,M,D}_C.
\]

By Definition 5.1 \( \text{unf}(q_C, M) := \text{unf}(q^{\text{ucq}}_C(x) \rightharpoonup \bigwedge_{f \in C} \text{Aux}_f(x_f), M^{\text{aux}}) \), with \( M^{\text{aux}}, \text{Aux}_f \) as in Definition 5.1. For each database instance \( D \) of \( \Sigma \), it is not hard to verify that \( q_C(x)^{A,M,D} = q^{\text{ucq}}_C(x)^{A,M^{\text{aux}}, D} \), by applying Definition 3.4 and using the fact the distributive property of the join operation over the union operation. By using Theorem 3.1 it holds that \( q^{\text{ucq}}_C(x)^{A,M^{\text{aux}}, D} = \text{unf}(q^{\text{ucq}}_C, M^{\text{aux}})^D \), for every data instance \( D \) of \( \Sigma \). By applying the transitive property of \( \rightharpoonup \), we obtain the thesis \( q_C(x)^{A,M,D} = \text{unf}(q^{\text{ucq}}_C, M^{\text{aux}})^D \), for every data instance \( D \) of \( \Sigma \).

The above unfolding characterization for JUCQs corresponds to a translation containing SQL joins over URIs resulting from the application of function symbols to database values, in a similar fashion as in Example 4.2, rather than over (indexed) database values directly. In general, such joins cannot be evaluated efficiently by RDBMSs [24].

In the remainder of this section we propose a solution to this issue by defining an alternative characterization for the unfolding of a JUCQ, that ensures that joins are always performed on database values. We first introduce a number of auxiliary notions and lemmas, that will be useful in the sequel of our discussion.

**Definition 5.2** (Restriction of a Mapping to a Signature). Let \( (L, f) \in \text{sign}(M) \) be a signature in \( M \). Then, the restriction \( M|_{(L,f)} \) of \( M \) w.r.t. the signature \( (L, f) \) is the set of mappings \( M|_{(L,f)} = \{ m \in M \mid m = L(f(v)) \rightharpoonup V(v) \} \).

**Definition 5.3** (Wrap of a Mapping). Let \( M|_{(L,f)} = \{ L(f(v_i)) \rightharpoonup V_i(v_i) \mid 1 \leq i \leq n \} \) be the restriction of \( M \) w.r.t. the signature \( (L, f) \), and \( f(v) \) be a tuple of terms over fresh variables \( v \). Then, the wrap of \( M|_{(L,f)} \) is the (singleton) set of mappings \( \text{wrap}(M|_{(L,f)}) = \{ L(f(v)) \rightharpoonup W(v) \} \) where \( W \) is a fresh view name for the Datalog query \( W(v), \{ W(v_i) \rightharpoonup V_i(v_i) \mid 1 \leq i \leq n \} \).

The wrap of \( M \) is the set \( \text{wrap}(M) = \bigcup_{(L,f) \in \text{sign}(M)} \text{wrap}(M|_{(L,f)}) \) of mappings.

**Example 5.1.** Consider the following set of mapping assertions

\[
\mathcal{M}|_{(A,f)} = \begin{cases} 
A(f(x, y)) \rightharpoonup V_1(x, y) \\
A(f(a, b)) \rightharpoonup V_2(a, b)
\end{cases}
\]
Let \( v,v' \) be fresh variables. Then,

\[
\text{wrap}(\mathcal{M}|(A,f)) = \{ A(f(v,v')) \sim W(v,v') \}
\]

where

\[
W(v,v') = (W(v,v'), \Pi)
\]

where

\[
\Pi = \begin{cases} 
W(x,y) \leftarrow V_1(x,y) \\
W(a,b) \leftarrow V_2(a,b)
\end{cases}
\]

**Lemma 5.1.**

\( \forall \mathcal{M}, (L,f) \in \text{sign}(\mathcal{M}) : \mathcal{M}|(L,f) \equiv \text{wrap}(\mathcal{M}|(L,f)) \)

**Proof.** Let

\[
\mathcal{M}|(L,f) := \{ L(f(v_i)) \sim V_i(v_i) \mid i = 1, \ldots, n \}
\]

Then, for every data instance \( D \),

\[
\mathcal{A}_{\mathcal{M}|(L,f), D} := \{ L(f(o)) \mid \exists 1 \leq i \leq n : o \in V_i(v_i)^D \} = \{ L(f(o)) \mid o \in (W(v), \{ W(v_i) \leftarrow V_i(v_i) \mid i = 1, \ldots, n \}^D \}
\]

for some fresh view symbol \( W \). The thesis follows from the observation that such set corresponds to the Definition of \( \mathcal{A}_{\text{wrap}(\mathcal{M}|(L,f), D)}. \)

**Lemma 5.2.** \( \forall \mathcal{M} : \mathcal{M} \equiv \text{wrap}(\mathcal{M}) \).

**Proof.** It follows directly from Lemma 5.1 and from the fact that fresh view symbols are introduced for each signature.

The wrap operation produces a mapping in which there do not exist two mapping assertions sharing the same signature. The next lemma formalizes this property.

**Lemma 5.3.** Consider the wrap mapping \( \text{wrap}(\mathcal{M}) \) of \( \mathcal{M} \). Then, for every mapping \( m,m' \in \text{wrap}(\mathcal{M}) \), it holds

If \( \text{sign}(\text{target}(m)) = \text{sign}(\text{target}(m')) \), then \( m = m' \)

**Proof.** It follows directly from the definition of the wrap operation.

The wrap operation groups the mappings for a signature into a single mapping. We now introduce an operation that \( \text{splits} \) a mapping according to the function symbols adopted on its source part.

**Definition 5.4 (Split).** Let \( m = L(x) \sim U(x) \) be a mapping where \( U \) is the name for the query \( (U(x), \{ U(f_i(x)) \leftarrow V_i(x_i) \mid 1 \leq i \leq n \}) \). Then, the \( \text{split of} \) \( m \) is the set \( \text{split}(m) = \{ L(f_i(x_i)) \sim V_i(x_i) \mid 1 \leq i \leq n \} \) of mappings. We denote by \( \text{split}(\mathcal{M}) \) the \( \text{split of} \) the set \( \mathcal{M} \) of mappings.
Example 5.2. Consider a mapping \( m = A(x) \leftrightarrow U(x) \), where \( U(x) = (U(x), \Pi) \) and

\[
\Pi := \begin{cases} 
U(f(a, b)) & \leftrightarrow V_1(a, b) \\
U(f(c, d)) & \leftrightarrow V_2(c, d) \\
U(g(e, f, g)) & \leftrightarrow V_3(e, f, g). 
\end{cases}
\]

Then,

\[
\text{split}(M) := \begin{cases} 
A(f(a, b)) & \leftrightarrow V_1(a, b) \\
A(f(c, d)) & \leftrightarrow V_2(c, d) \\
A(g(e, f, g)) & \leftrightarrow V_3(e, f, g). 
\end{cases}
\]

Lemma 5.4. \( \forall M : M = \text{split}(M) \).

Proof. Let \( M \) be an arbitrary mapping. We need to prove that

\[ M = \text{split}(M) \]

which amounts to prove that

\[ \forall D : A_{\text{(M, D)}} = A_{\text{(split(M), D)}} \]

W.l.o.g., let \( D \) be a database instance. Then

\[ A_{\text{(M, D)}} := \{ L(f_i(a_i)) : f_i(a_i) \in U(x)^D, \text{ for rules } L(x) \leftrightarrow U(x) \in M \} \]

W.l.o.g., let \( U(x) = (U(x), \Pi) \). Then, the set above is equal to

\[ \{ L(f_i(a_i)) : \exists q(f_i(x_i)) \leftarrow V_i(x_i) \in \Pi \text{ s.t. } a_i \in V_i(x_i)^D \} \]

\[ = \{ L(f_i(a_i)) : \exists q(f_i(x_i)) \leftarrow V_i(x_i) \in \text{split}(M) \text{ s.t. } a_i \in V_i(x_i)^D \} \]

\[ \square \]

Definition 5.5 (Unfolding of a JUCQ 2). Let \( q^{\text{aux}} \) be a query and \( M^{\text{aux}} \) a set of mappings as in Definition 5.1. Then, the \textbf{optimized unfolding} \( \text{unf}_{\text{opt}}(q^{\text{C}}(x), M) \) of \( q^{\text{C}} \) w.r.t. \( M \) is defined as \( \text{unf}(q^{\text{aux}}(x), \text{wrap}(\text{split}(M^{\text{aux}}))) \).

Observe that the optimized unfolding of a JUCQ is a \textit{union of JUCQs} (UJUCQ). Moreover, where each JUCQ produces answers built from a single tuple of function symbols, if all the attributes are kept in the answer. The next example, aimed at clarifying the notions introduced so far, illustrates this.

Example 5.3. Let \( q(x, y, z) \leftarrow P_1(x, y), C(x), P_2(x, z) \), and consider a cover \( \{ f_1, f_2 \} \) generating fragment queries \( q|_{f_1} = q(x, y) \leftarrow P_1(x, y), C(x) \) and \( q|_{f_2} = q(x, z) \leftarrow P_2(x, z) \). Consider the set of mappings

\[
\mathcal{M} = \left\{ 
\begin{array}{ll}
P_1(f(a), g(b)) & \leftarrow V_1(a, b) \\
P_1(h(a), i(b)) & \leftarrow V_2(a, b) \\
P_2(f(a), k(b)) & \leftarrow V_3(a, b) \\
P_1(f(a), g(b)) & \leftarrow V_2(a, b) \\
C(f(a)) & \leftarrow V_4(a) \\
P_2(f(a), h(b)) & \leftarrow V_6(a, b) 
\end{array}
\right\}
\]

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Translation I. According to Definition 5.1 the JUCQ \( q(x, y, z) \leftarrow q_1(x, y), q_2(x, z) \) can be rewritten as the auxiliary query \( q^{aux}(x, y, z) = AUX_1(x, y), AUX_2(x, z) \) over mappings

\[
M^{aux} = \{ \begin{array}{c}
AUX_1(x, y) \leadsto U_1(x, y) \\
AUX_2(x, z) \leadsto U_2(x, z)
\end{array} \}
\]

where \( U_1 \) is a view name for \( unf(q_1(x, y), M) = (U_1(x, y), \Pi_1) \), and \( U_2 \) is a view name for \( unf(q_2(x, z), M) = (U_2(x, z), \Pi_2) \), such that

\[
\Pi_1 = \{ \begin{array}{c}
U_1(f(a), g(b)) \leftarrow V_1(a, b), V_4(a) \\
U_1(f(a), g(b)) \leftarrow V_2(a, b), V_4(a)
\end{array} \}
\Pi_2 = \{ \begin{array}{c}
U_2(f(a), k(b)) \leftarrow V_5(a, b) \\
U_2(f(a), b(h)) \leftarrow V_6(a, b)
\end{array} \}
\]

Translation II. By Definition 5.4 we compute the split of \( M^{aux} \):

\[
\text{split}(M^{aux}) = \{ \begin{array}{c}
AUX_1(f(a), g(b)) \leftarrow W_3(a, b) \\
AUX_1(f(a), g(b)) \leftarrow W_4(a, b) \\
AUX_2(f(a), k(b)) \leftarrow W_5(a, b) \\
AUX_2(f(a), h(b)) \leftarrow W_6(a, b)
\end{array} \}
\]

By Definition 5.3 we compute the wrap of \( \text{split}(M^{aux}) \):

\[
\text{wrap}(\text{split}(M^{aux})) = \{ \begin{array}{c}
AUX_1(f(a), g(b)) \leftarrow W_3(a, b) \\
AUX_2(f(a), k(b)) \leftarrow W_4(a, b) \\
AUX_2(f(a), h(b)) \leftarrow W_5(a, b)
\end{array} \}
\]

where \( W_3(a, b), W_4(a, b), W_5(a, b) \) are Datalog queries whose programs are respectively

\[
\Pi_3 = \{ \begin{array}{c}
W_3(a, b) \leftarrow V_1(a, b), V_4(a) \\
W_3(a, b) \leftarrow V_2(a, b), V_4(a)
\end{array} \}
\Pi_4 = \{ \begin{array}{c}
W_4(a, b) \leftarrow V_5(a, b)
\end{array} \}
\Pi_5 = \{ \begin{array}{c}
W_5(a, b) \leftarrow V_6(a, b)
\end{array} \}
\]

Finally, by Definition 5.2 we compute the optimized unfolding of \( q_C \) w.r.t. \( M \):

\[
\text{unf}_{opt}(q_C(x, y, z), M) = \text{unf}(q^{aux}(x, y, z), \text{wrap}(\text{split}(M^{aux}))) = (q^{aux}_{unf}(x, y, z), \Pi_{unf})
\]

where

\[
\Pi_{unf} = \{ \begin{array}{c}
q^{aux}_{unf}(f(a), g(k(b')) \leftarrow W_3(a, b), W_4(a, b') \\
q^{aux}_{unf}(f(a), h(b')) \leftarrow W_3(a, b), W_5(a, b')
\end{array} \}
\]

Observe that \( \text{unf}_{opt}(q_C(x, y, z), M) \) is a UJUCQ. Moreover, each of the two JUCQs in \( q^{aux}_{unf} \) contributes with answers built out of a specific tuple of function symbols.

Theorem 5.2 (Translation 2). For any cover \( C = \{ f_1, \ldots, f_n \} \), the cover query \( q_C(x) \leftarrow \bigwedge_{i=1}^n q_{f_i} \), for some cover \( C \) of fragments \( \{ f_1, \ldots, f_n \} \). The query \( \text{unf}_{opt}(q_C, M) \) is an \( M \)-translation for \( q_C \).

Proof. By Definition 5.3

\[
\text{unf}_{opt}(q(x), M) := \text{unf}(q^{aux}(x), \text{wrap}(\text{split}(M^{aux}))).
\]

By Lemmas 5.2 and 5.4 \( \text{wrap}(\text{split}(M^{aux})) \equiv M^{aux} \). Hence, by Lemma 3.1

\[
\text{unf}(q^{aux}(x), \text{wrap}(\text{split}(M^{aux}))) \equiv \text{unf}(q^{aux}(x), M^{aux}).
\]

The thesis is proved by observing that, according to Theorem 5.1 \( \text{unf}(q^{aux}(x), M^{aux}) \) is a translation.
6 Unfolding Cardinality Estimation

For convenience, in this section, we use relational algebra notation for CQs. To deal with multiple occurrences of the same predicate in a CQ, the corresponding algebra expression would contain renaming operators. However, in our cardinality estimations we need to understand when two attributes actually refer to the same relation, and this information is lost in the presence of renaming. Instead of introducing renaming, we first explicitly replace multiple occurrences of the same predicate name in the CQ by aliases (under the assumption that aliases for the same predicate name are interpreted as the same relation). Specifically, we use alias $V[i]$ to represent the $i$-th occurrence of predicate name $V$ in the CQ. Then, when translating the aliased CQ to algebra, we use fully qualified attribute names (i.e., each attribute name is prefixed with the (aliased) predicate name). So, to reconstruct the relation name $V$ to which an attribute $V[i].x$ refers, it suffices to remove the occurrence information $i$ from the prefix $V[i]$. When the actual occurrence of $V$ is not relevant, we use $V[j]$ to denote the alias.

Moreover, in the following, we consider only the restricted form of CQs, which we call basic CQs, whose algebra expression is of the form

$$E = V^0_1 \Join_{\theta_i} V^1_1 \Join_{\theta_{i+1}} \cdots \Join_{\theta_n} V^n_1,$$

where, the $V$'s denote predicate names, and for each $i \in \{1, \ldots, n\}$, the join condition $\theta_i$ is of the form $V^i_1.x = V^i_1.y$, for some $j < i$. Arbitrary CQs, allowing for projections and arbitrary joins, will be considered in the remainder of this section.

Given a basic CQ $E$ as above, we denote by $E^{(m)}$, for $1 \leq m \leq n$, the sub-expression of $E$ up to the $m$-th join operator, namely $E^{(m)} = V^0_1 \Join_{\theta_1} V^1_1 \Join_{\theta_2} \cdots \Join_{\theta_m} V^m_1$.

In the following, in addition to an OBDA specification, we also fix a database instance $\mathcal{D}$ for $\Sigma$. We use $V$ and $W$ to denote relation names (with an associated relation schema) in the virtual schema $\mathcal{M}_\Sigma$, whose associated relations consist of (multi)sets of labeled tuples (see the named perspective in [11]). Given a relation $S$, we denote by $|S|$ the number of (distinct) tuples in $S$, by $\pi_L(S)$ the projection of $S$ over attributes $L$ (under set-semantics), by $S|_L$ the restriction of $S$ over attributes $L$ (under bag-semantics), and by $\pi_L(S_1) \cap \pi_L(S_2)$ intersection of relations disregarding attribute names, i.e., $\pi_L(S_1) \cap \rho_L \rightarrow_L \pi_L(S_2)$. We also use the classical notation $P(\alpha)$ to denote the probability that an event $\alpha$ happens.

6.1 Background on Cardinality Estimation

In this subsection we list a number of assumptions that are commonly made by models of cardinality estimations proposed in the database literature (e.g., see [26]). Some of these assumptions will be maintained also in our cardinality estimator, while others will be relaxed or dropped due to the additional information given by the structure of the mappings and the ontology, which is not available in a traditional database setting.

Uniform distribution in the interval. Values are usually assumed to be uniformly distributed across one interval, formally

$$P(C < v) = (v - \min(C)) / (\max(C) - \min(C))$$

(a1)

for each value $v$ in a column $C$. 

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Independent Distributions. When the distributions between different attributes are independent, as in Assumptions (a1) and (a4), as in Formula (1):

\[
P(T|_x = v) = P(\pi_x(T) = v) = 1/|T|_x, \forall v \in \pi_x(T|_x)
\]

Hence, the number of repetitions per element can be estimated as:

\[
|T| \cdot P(T|_x = v) = \frac{|T|}{|\pi_x(T)|}
\]

Independent Distributions. When the distributions between different attributes are independent, formally

\[
P(C_1 = v_1 | C_2 = v_2) = P(C_1 = v_1)
\]

Facing Values. Attributes join "as much as possible", that is, given a join \( \Join_{x=y} S \), it is assumed that

\[
|\pi_x(T) \cap \pi_y(S)| = \min(|\pi_x(T)|, |\pi_y(S)|).
\]

Under these assumptions, the cardinality of a join \( V \Join_{x=y} W \) can be estimated \([27]\) to be:

\[
k_D(V \Join_{x=y} W) = \frac{|V^D|}{\text{dist}(V, x)} \cdot \frac{|W^D|}{\text{dist}(W, y)}
\]

where \( k_D \) is an estimation of the number of distinct values satisfying the join condition (i.e., \( k_D \) estimates \( |\pi_x(V^D) \cap \pi_y(W^D)| \)), and \( \text{dist}(V, x) \) (resp., \( \text{dist}(W, y) \)) corresponds to the estimation of \( |\pi_x(V^D)| \) (resp., \( |\pi_y(W^D)| \)), both calculated according to the aforementioned assumptions. Note that the fractions such as \( \frac{|V^D|}{\text{dist}(V, x)} \) estimate the number of tuples associated to each value that satisfies the join condition, and derive directly from Assumption (a2).

6.2 Cardinality Estimation of CQs

Cardinality Estimator. Given a basic CQ \( E' \), \( f_D(E') \) estimates the number \( |E'^D| \) of distinct results in the evaluation of \( E' \) over \( D \). We define it as

\[
f_D(E \Join_{V[p]} x=W[p] \Join_{V[q]} y W[q]) = \begin{cases} 
    k_D(V[p] \Join_{V[p]} x=W[p] \Join_{V[q]} y W[q]) \cdot |V^D| \cdot |W^D|, & \text{if } E = V \\
    k_D(E \Join_{V[p]} x=W[p] \Join_{V[q]} y W[q]) \cdot f_D(E) \cdot |W^D|, & \text{otherwise.}
\end{cases}
\]

Our cardinality estimator exploits assumptions (a2) and (a3) above, and relies on our definitions of the facing values estimator \( k_D \) and of the distinct values estimator \( \text{dist}_D \), which are based on additional statistics collected with the help of the mappings, instead of being based on assumptions (a1) and (a4), as in Formula (1).
Facing Values Estimator. Given a basic CQ \( E' = E \bowtie_{V[p], x = W[q], y} \ W[q] \), the estimation \( k_D(E') \) of the cardinality \( |\pi_{V,x}(E') \cap \pi_{W,y}(W^D)| \) is defined as

\[
k_D(E \bowtie_{V[p], x = W[q], y} \ W[q]) = \begin{cases} |\pi_x(V^D) \cap \pi_y(W^D)|, & \text{if } E = V \\ \left| \frac{|\pi_x(V^D) \cap \pi_y(W^D)|}{\text{dist}(E, V[p], x)} \right|, & \text{otherwise} \end{cases}
\]

where \( |\pi_x(V^D) \cap \pi_y(W^D)| \) is assumed to be a statistic available after having analyzed the mappings together with the data instance. The fraction \( \frac{\text{dist}(E, V[p], x)}{\text{dist}(V, V[p], \pi)} \) is a scaling factor relying on assumption [a2].

Distinct Values Estimator.

Definition 6.1 (Equivalent Attributes). Let \( Q \) be a set of qualified attributes, and \( E \) be basic CQ. We define the set \( \text{ea}(E, Q) \) of equivalent attributes of \( Q \) in \( E \) as \( \bigcup_{i \geq 0} C_i \), where

- \( C_1 := \{Q\} \)
- \( C_{n+1} := C_n \cup \{Q' \mid \exists Q'' \in C_n \text{ s.t. } Q' = Q'' \text{ or } Q'' = Q' \text{ is a join condition in } E\}, n \geq 1 \)

Definition 6.2 (Join Sub-Expression). Given a basic CQ \( E \) and a set \( V[p], x \) of qualified attributes, the expression \( \text{se}(E, V[p], x) \) denotes the longest sub-expression \( E^{(n)} \) in \( E \), for some \( n > 0 \), such that \( E^{(n)} = E^{(n-1)} \bowtie_{W[q], y = U[r], z} U[r] \), for some relation name \( W \), tuples of attributes \( y \) and \( z \) such that \( U[r] \in \text{ea}(E, V[p], x) \), if \( E^{(n)} \) exists, and \( \perp \) otherwise.

For \( E \) and \( V[p], x \), the estimation \( \text{dist}(E, V[p], x) \) of the cardinality \( |\pi_{V[p], x}(E^D)| \) is defined as

\[
\text{dist}(E, V[p], x) = \begin{cases} |\pi_x(V^D)|, & \text{if } E = V \\ \min \left\{ \left[ k_D(E') \cdot \frac{f_D(E)}{f_D(E')} \right], k_D(E') \right\}, & \text{if } \text{se}(E, V[p], x) = E' \neq \perp \\ \min \left\{ \left| \frac{|\pi_x(V^D)|}{f_D(E)} \right|, |\pi_x(V^D)| \right\}, & \text{otherwise} \end{cases}
\]

where \( |\pi_x(V^D)| \) is assumed to be a statistic available after having analyzed the mappings together with the data instance. Observe that the fractions \( \frac{f_D(E)}{f_D(E')} \) and \( \frac{f_D(E)}{f_D(E)} \) are again scaling factors relying on assumption [a2]. Also, \( \text{dist}(E, V, x) \) must not increase when the number of joins in \( E \) increases, which explains the use of \( \text{min} \) for the case where the number of distinct results in \( E \) increases with the number of joins.

Example 6.1. Consider the data instance \( D \) from Figure[7] Relevant statistics are:

- \( |T_1^P| = 5, \ |T_2^P| = |T_3^P| = 10 \)
- \( |\pi_a(T_1^P)| = |\pi_a(T_2^P)| = 5, \ |\pi_c(T_1^P)| = |\pi_c(T_2^P)| = 10, \ |\pi_e(T_1^P)| = |\pi_e(T_2^P)| = |\pi_e(T_3^P)| = 10 \)
- \( |\pi_a(T_1^P) \cap \pi_c(T_2^P)| = 3, \ |\pi_a(T_2^P) \cap \pi_e(T_3^P)| = 5, \ |\pi_a(T_1^P) \cap \pi_e(T_3^P)| = 1. \)
We calculate $f_D(E)$ for the basic CQ $E = T_1 \bowtie_{T_1.a = T_2.c} T_2 \bowtie_{T_1.d = T_3.e} T_3 \bowtie_{T_1.a = T_3.f} T_3'$, where $T_3'$ is an alias (written in this way for notational convenience) for the table $T_3$. To do so, we first need to calculate the estimations $f_D(E^{(1)})$ and $f_D(E^{(2)})$.

$$
f_D(E^{(1)}) = f_D(T_1 \bowtie_{T_1.a = T_2.c} T_2) = \frac{k_D(T_1 \bowtie_{T_1.a = T_2.c} T_2) \cdot |T_2'| \cdot |T_2|}{\text{dist}(T_1, a) \cdot \text{dist}(T_2, c)}
$$

$$
= \frac{|\pi_a(T_1) \cap \pi_c(T_2)| \cdot |T_2'| \cdot |T_2|}{|\pi_a(T_1)| \cdot |\pi_c(T_2)|} = \frac{(3 \cdot 5 \cdot 10)}{(5 \cdot 10)} = 3
$$

$$
f_D(E^{(2)}) = f_D(E^{(1)} \bowtie_{T_2.d = T_3.e} T_3) = \frac{k_D(E^{(1)} \bowtie_{T_2.d = T_3.e} T_3) \cdot f_D(E^{(1)}) \cdot |T_3'|}{\text{dist}(E^{(1)}, T_2, d) \cdot \text{dist}(T_3, e)}
$$

(5)

By Formula (4), $\text{dist}(E^{(1)}, T_2, d)$ in Formula (5) can be calculated as

$$
\text{dist}(E^{(1)}, T_2, d) = \min \left\{ \left\lfloor \frac{|\pi_d(T_2)|}{|\pi(T_2)|} \cdot f_D(E^{(1)}) \right\rfloor, |\pi_d(T_2)| \right\}
$$

$$
= \min \left\{ \left\lfloor \frac{5}{10} \cdot 3 \cdot 5 \right\rfloor, \left\lfloor \frac{3}{2} \right\rfloor \right\} = 2
$$

By Formula (3), $k_D(E^{(1)} \bowtie_{T_2.d = T_3.e} T_3)$ in Formula (5) can be calculated as

$$
k_D(E^{(1)} \bowtie_{T_2.d = T_3.e} T_3) = \frac{k_D(T_2 \bowtie_{T_2.d = T_3.e} T_3)}{\text{dist}(T_2, d)} \cdot \text{dist}(E^{(1)}, T_2, d)
$$

$$
= \frac{|\pi_d(T_2) \cap \pi_e(T_3)|}{|\pi_d(T_2)|} \cdot \text{dist}(E^{(1)}, T_2, d) = \frac{5 \cdot 2}{5} = 2
$$

By plugging the values for $k_D$ and $\text{dist}_D$ in Formula (5), we obtain

$$
f_D(E^{(2)}) = \frac{(2 \cdot 3 \cdot 10)}{(2 \cdot 10)} = 3
$$

We are now ready to calculate the cardinality of $E$, which is given by the formula

$$
f_D(E) = f_D(E^{(2)} \bowtie_{T_1.a = T_3.f} T_3') = \frac{k_D(E^{(2)} \bowtie_{T_1.a = T_3.f} T_3') \cdot f_D(E^{(2)}) \cdot |T_3'|}{\text{dist}(E^{(2)}, T_1, a) \cdot \text{dist}(T_3, \ell)}
$$

(6)
By Formula (4), \( \text{dist}(E^{(2)}, T_{1,a}) \) in Formula (6) can be computed as
\[
\text{dist}(E^{(2)}, T_{1,a}) = \min \left\{ \left[ \frac{k_D(E^{(1)})}{f_D(E^{(2)})} \right] \cdot f_D(E^{(1)}) \right\} = \min \left\{ \left[ \frac{3}{3} \cdot 3 \right], 3 \right\} = 3
\]

Then, by Formula (3), \( k_D(E^{(2)} \bowtie_{T_1,a=T_{1',a}} T_3') \) in Formula (6) can be computed as
\[
k_D(E^{(2)} \bowtie_{T_1,a=T_{1',a}} T_3') = \left[ \frac{k_D(T_1 \bowtie_{T_1,a=T_{1',a}} T_3')}{\text{dist}(T_1, a)} \right] \cdot \text{dist}(E^{(2)}, T_{1,a}) = \frac{3}{5} = 1
\]

By plugging the values for \( k_D \) and \( \text{dist}_D \) in (6), we finally obtain
\[
f_D(E) = \lfloor (1 \cdot 3 \cdot 10)/(3 \cdot 10) \rfloor = 1
\]

Observe that, in this example, our estimation is exact, that is, \( f_D(E) = |E_D| \).

6.3 Extending to Non-basic CQs

6.3.1 Extending to Arbitrary Join Conditions.

We now extend the cardinality estimation to the case of a non-basic CQ. For this purpose we can exploit the following property of theta-joins:

\[
\sigma_{\theta_2}(T_1 \bowtie_{\theta_1} T_2) = T_1 \bowtie_{\theta_1 \land \theta_2} T_2
\]

We first assume that the CQ does not contain existential variables. The cardinality of a CQ query without existential variables, and with \( n \) atoms and \( m \) joins \((m > n)\) can be estimated in two steps. In the first step, we estimate through the equations in the previous section the cardinality of a basic CQ of \( n \) atoms over \( n-1 \) joins. In the second step, we multiply the number obtained in the first step by the probability that the additional \( \theta_{n+1}, \ldots, \theta_m \) conditions are all satisfied. Such probabilities can be easily calculated through the uniformity assumptions.

Example 6.2.

\[
f((T_1 \bowtie_{T_1,a=T_{2,d}} T_2) \bowtie_{\{t_1,h=T_3,i \land T_2,c=T_3,e\}} T_3) =
\]

\[
f((T_1 \bowtie_{T_1,a=T_{2,d}} T_2) \bowtie_{\{t_1,h=T_3,i \land T_2,c=T_3,e\}} T_3) \cdot \frac{k(T_2 \bowtie_{T_2,c=T_3,e} T_3)}{\text{dist}(T_2, T_2,c)}
\]

where \( \frac{k(T_2 \bowtie_{T_2,c=T_3,e} T_3)}{\text{dist}(T_2, T_2,c)} \) is an estimation for \( P(T_2,c = T_3,e) \) under our assumptions.

6.3.2 Extending Estimation to CQs with Existential Variables (Projection)

Let \( \pi_{V_0^{(1)}, \ldots, V_m^{(1)}, x_n}(E) \) be a CQ, where \( E \) is a basic CQ. Then, by relying on the uniformity assumptions, we can estimate the cardinality \( |\pi_{V_0^{(1)}, \ldots, V_m^{(1)}, x_n}(E)| \) through the formula

\[
f(E) - \left[ f(E) \cdot \left( 1 - \frac{\text{dist}(V_0^{(1)}, x_1, E)}{f(E)} \right) \right] \cdots \left( 1 - \frac{\text{dist}(V_m^{(1)}, x_n, E)}{f(E)} \right)
\]

(7)
6.4 Collecting the Necessary Base Statistics

The estimators introduced above assume a number of statistics to be available. We now show how to compute such statistics on a data instance by analyzing the mappings. Consider a set of mappings \( \mathcal{M} = \{ L_i(f_i(v_i)) \leftarrow V_i(v_i) \mid 1 \leq i \leq n \} \) and a data instance \( D \). We store the statistics:

- \( S_1 \mid |V_i^D|, \) for each \( i \in \{1, \ldots, n\} \);  
- \( S_2 \mid \pi_x(V_i^D), \) if \( f(x) \) is a term in \( f_i(v_i) \), for some function symbol \( f \) and \( i \in \{1, \ldots, n\} \);  
- \( S_3 \mid \pi_x(V_i^D) \cap \pi_y(V_j^D), \) if \( f(x) \) is a term in \( f_i(v_i) \), and \( f(y) \) is a term in \( f_j(v_j) \), for some function symbol \( f \) and \( i, j \in \{1, \ldots, n\}, i \neq j \).

Statistics \( S_1 \) and \( S_2 \) are required by all three estimators that we have introduced, and can be measured directly by evaluating source queries on \( D \). Statistics \( S_3 \) can be collected by first iterating over the function symbols in the mappings, and then calculating the cardinalities for joins over pairs of source queries whose corresponding mapping targets have a function symbol in common. It is easy to check that Statistics \( S_1 \)–\( S_2 \) suffice for our estimation, since all joins in a CQ are between source queries, and moreover, every translation calculated according to Definition 3.3 contains only joins between pairs of source queries considered by Statistics \( S_3 \). The same intuitions apply for the possible projections applied to a basic CQ in an unfolding.

6.5 Unfolding Cardinality Estimator

We now show how to estimate the cardinality of an unfolding by using the formulae (2), (3), and (4) introduced for cardinality estimation. The next theorem shows that such estimation can be calculated by summing-up the estimated cardinalities for each CQ in the unfolding of the input query, provided that (i) the unfolding is being calculated over wrap mappings, and (ii) the query to unfold is a CQ.

We start our discussion with an auxiliary general lemma about rules in a Datalog program with functions.

**Lemma 6.1.** Consider a datalog query \((q(v), \Pi)\) and two rules \( r_1, r_2 \in \Pi \). Then, if there does not exist a substitution \( \sigma \) such that \( \sigma(\text{head}(r_1)) = \sigma(\text{head}(r_2)) \), it holds \( \forall D : (q(v), \{r_1\})^D \cap (q(v), \{r_2\})^D = \emptyset \).

**Proof.** Let \( A_1 := (q(v), \{r_1\})^D \), and \( A_2 := (q(v), \{r_2\})^D \). W.l.o.g., let \( \text{head}(r_1) = q(f(x)) \), and \( \text{head}(r_2) = q(g(y)) \) such that \( f(x) \) and \( g(y) \) are not unifiable. By definition of answer over an instance \( D \), \( A_1 \subseteq \{f(a) \mid a \in \text{adom}(D)\} \), and \( A_2 \subseteq \{g(a) \mid a \in \text{adom}(D)\} \). The theorem follows by observing that, for every database instance \( D \), \( \{f(a) \mid a \in \text{adom}(D)\} \cap \{g(a) \mid a \in \text{adom}(D)\} = \emptyset \), since in databases it holds the standard name assumption for which \( f^D = f \neq g = g^D \). \( \square \)

Regardless of whether it is of a UCQ or a JUCQ, as they must be equivalent.
Theorem 6.1. Consider a CQ \( q(x) \leftarrow X_1(v_1), \ldots, X_n(v_n) \) such that \( x \supseteq \bigcup_{i=1}^n v_i \). Let \( M \) be a mapping. Then,
\[
|\text{unf}(q(x), M)^D| = \sum_{q_u \in \text{unf}(q, \text{wrap}(M))} |q_u(x)^D|
\]

Proof. Since \( M \equiv \text{wrap}(M) \), it suffices to prove that for each pair of rules \( q_u, q_u' \) in \( \text{unf}(q(x), \text{wrap}(M)) \), it holds that \( q_u^D \cap q_u'^D = \emptyset \). Consider two arbitrary rules \( q_u, q_u' \) in \( \text{unf}(q(x), \text{wrap}(M)) \). By Definition 3.3, there must exist two distinct lists of mappings \( (m_1, \ldots, m_n) \) and \( (m'_1, \ldots, m'_n) \) in \( \text{wrap}(M) \) and two substitutions \( \sigma, \sigma' \) such that \( \text{head}(q_u) = q_u(\sigma(x)) \), and \( \text{head}(q_u') = q_u'(\sigma'(x)) \), and \( \text{pred}(\text{target}(m_i)) = X_i \) (resp., \( \text{pred}(\text{target}(m'_i)) = X_i \)) for each \( 1 \leq i \leq n \). W.l.o.g., let \( \sigma(x) = f(y) \), and \( \sigma'(x) = f'(y') \).

W.l.o.g., let \( m_i \neq m'_i \). By Lemma 5.3, it is directly follows that \( f_i \neq f'_i \). Hence, \( \sigma(v_i) = f_i(v_i) \) and \( \sigma'(v'_i) = f'_i(v'_i) \) do not unify. Since, by assumption, \( x \supseteq v_i, v'_i \), we conclude that \( \sigma(x) \) does not unify with \( \sigma'(x') \). Then, by Lemma 6.1, it follows that \( q_u(\sigma(x))^D \cap q_u'(\sigma'(x'))^D = \emptyset \).

Since the choice of \( q_u, q_u' \) was arbitrary, we conclude that
\[
\forall 1 \leq i \neq i' \leq m : \forall D : q_u(\sigma_i(x))^D \cap q_{i'}(\sigma_{i'}(x))^D = \emptyset
\]
from which the theorem directly follows.

The previous theorem states that the cardinality of the unfolding of a query over a wrap mapping corresponds to the sum of the cardinalities of each CQ in the unfolding, under the assumption that all the attributes are kept in the answer. Intuitively, the proof relies on the fact that, when wrap mappings are used, each CQ in the unfolding returns answer variables built using a specific combination of function names. Hence, to calculate the cardinality of a CQ \( q \), it suffices to collect statistics as described in the previous paragraph, but over \( \text{wrap}(M) \) rather than \( M \), and sum up the estimations for each CQ in \( \text{unf}(q, \text{wrap}(M)) \).

The method above might overestimate the actual cardinality if the input CQ contains non-answer variables. In Section 6.7 we show how to address this limitation by storing, for each property in the mappings, the probability of having duplicate answers if the projection operation is applied to one of the (two) arguments of that property.

Also, the method above assumes a CQ as input to the unfolding, whereas a rewriting is in general a UCQ. This is usually not a critical aspect, especially in practical applications of OBDA. By using saturated (or T-)mappings \[22\] \( M_T \) in place of \( M \), in fact, the rewriting of an input CQ \( q \) is almost always \[17\] coincides with \( q \) itself \[4\]. Hence, in most cases we can directly use in Theorem 6.1 the input query \( q \), if we use \( \text{wrap}(M_T) \) instead of \( \text{wrap}(M) \). In the following subsection we provide a fully detailed example on how this is done, as well as more details on T-mappings and related techniques.

\[4\] Always, if the CQ is interpreted as a SPARQL query and evaluated according to the OWL 2 QL entailment regime, or if the CQ does not contain existentially quantified variables.
6.6 A Note on Rewriting

Modern OBDA systems such as Ontop or Ultrawrap use advanced rewriting techniques for the rewriting phase. In particular, Ontop uses a combination of T-mappings and tree-witness rewriting. We now give basic notions about these two techniques that will be useful in the rest of this paper.

Definition 6.3. The saturation of \( A \) w.r.t. \( T \) is the ABox

\[
A_T := \{ L(a) \mid (T, A) \models L(a), \ a \text{ is a tuple of individuals} \}
\]

In OBDA, saturated virtual ABoxes can be obtained by compiling the ontology in the mappings so as to obtain T-mappings. Intuitively, T-mappings expose a saturated ABox.

Definition 6.4. Given an OBDA specification \( S = (T, M, \Sigma) \), the mappings \( M_T \) are T-mappings for \( S \) if, for every OBDA instance \( D \) of \( S \), \( A_{(M,D)_T} = A_{(M_T,D)} \).

The saturation of an ABox is also known as a forward chaining technique, as opposed to backward chaining techniques such as rewriting algorithms. It is easy to see that, for each saturated ABox \( A_T \) and single-atom query \( q(x) \leftarrow X(x) \), it holds that \( q(x)^{A_T} = \text{cert}(q(x), (T, A)) \). However, such equality does not hold in case \( q \) is a general CQ.

Authors from [14] gave a characterization of queries for which query answering w.r.t. an ontology corresponds to query evaluation over saturated ABoxes. The tree-witness rewriting technique [13] exploits this characterization so as to produce rewritings that are minimal w.r.t. a saturated ABox. In short, queries for which query answering w.r.t. an ontology does not correspond to query evaluation over saturated ABoxes are the ones whose answers are influenced by existentials in the right-hand-side of DL-Lite\(_R\) TBox axioms. Such queries are called queries with tree-witnesses.

Example 6.3. Consider the data instance \( D \) from Figure 2. Consider the query

\[
q(x, y, z) = C(x), R_1(x, y), R_2(x, z)
\]

and the set of mappings

\[
M = \{ A(l(a)) \leftarrow T_1(a), P_2(l(g), m(h)) \leftarrow T_5(g, h), P_1(l(c), m(d)) \leftarrow T_3(c, d) \}
\]

Let \( O = \{ A \subseteq C, P_1 \subseteq R_1, P_2 \subseteq R_2 \} \). Since all the variables in \( q \) are answer variables, to compute \( \text{cert}(q, (T, A_{(M,D)})) \) it suffices to compute the evaluation \( q^{A_{(M_T,D)}} \), where \( M_T \) are the T-mappings of \( M \) w.r.t. \( T \). Such T-mappings \( M_T \) are:

\[
M_T = M \cup \{ C(l(a)) \leftarrow T_1(a), R_1(l(e), m(f)) \leftarrow T_4(e, f), R_2(l(i), m(j)) \leftarrow T_6(i, j), R_2(l(g), n(h)) \leftarrow T_5(g, h) \}
\]

The wrap of \( M_T \) is:

\[
\text{wrap}(M_T) = M \cup \{ C(l(a)) \leftarrow V_1(a), R_1(l(c), m(d)) \leftarrow V_2(c, d), R_2(l(g), n(h)) \leftarrow V_5(g, h) \}
\]

where \( V_1, \ldots, V_5 \) are view names for Datalog queries of programs respectively.
\[ \Pi_1 = \{ V_1(a) \leftarrow T_1(a), V_1(b) \leftarrow T_2(b) \} \]
\[ \Pi_2 = \{ V_2(c, d) \leftarrow T_3(c, d), V_2(e, f) \leftarrow T_4(e, f) \} \]
\[ \Pi_3 = \{ V_3(g, h) \leftarrow T_5(g, h), V_3(i, j) \leftarrow T_6(i, j) \} \]

Now, necessary statistics to estimate the cardinality of \( q(A(M_T : D)) \) by using our cardinality estimator are:

- \(|\pi_a(V_1)| = 5\)
- \(|\pi_{c, d}(V_2)| = 11\)
- \(|\pi_{g, h}(V_3)| = 11\)
- \(|\pi_{g, h}(T_5)| = 10\)
- \(|\pi_a(V_1)| = 5\)
- \(|\pi_c(V_2)| = |\pi_d(V_2)| = |\pi_g(V_3)| = |\pi_h(V_3)| = 11\)
The unfolding \( q_{\text{unf}} \) of \( q \) w.r.t. \( \text{wrap}(M_T) \) is:

\[
\Pi_{\text{unf}} = \left\{ q(l(a), m(d), m(h)) \leftarrow V_1(a), V_2(a, d), V_3(a, h) \right\}
\]

In relational algebra:

\[
E_1 := \pi_{x/l(a), y/m(d), z/m(h)}(V_1 \bowtie_{V_1.a = V_2.a} V_2 \bowtie_{V_1.a = V_3.a} V_3)
\]

\[
E_2 := \pi_{x/l(a), y/m(d), z/n(h)}(V_1 \bowtie_{V_1.a = V_2.a} V_2 \bowtie_{V_1.a = T_5.a} T_5)
\]

According to Theorem 6.7, the cardinality of \( q_{\text{unf}} \) in \( D \) can be calculated as \(|E_1| + |E_2|\). Such cardinalities can be computed in the same fashion as in Example 7.

6.7 Dealing With Projection

In presence of projection, our estimation for the cardinality of an unfolding is an upper-bound of the actual cardinality, if the cardinalities for each single CQ are estimated correctly. In this section we discuss a lower-bound of the actual cardinality of an unfolding.

We first recall some basic properties of sets.

**Theorem 6.2 (Cardinality of Set Union).** Given sets \( A, B \), \( |A \cup B| = |A| + |B| - |A \cap B| \).

In the general case, given \( n \) sets \( A_1, \ldots, A_n \),

\[
|\bigcup_{i=1}^{n} A_i| = \sum_{i=1}^{n} \left( \sum_{|J|=i} \left( -1 \right)^{i-1} |\bigcap_{j \in J} A_j| \right).
\]

Recall that our estimator stores the cardinalities for the intersection of pairs of joinable columns. An idea could be to estimate the cardinality of a union between \( n \) sets by exploiting such cardinalities. Observe that, according to Theorem 6.2, the information about intersection between pairs is sufficient for the union of two sets, but it is already not enough in the case of three sets. To estimate a lower bound for such union, we assume the following:

\[
\forall A, B, C : |A \cap B \cap C| = \min \{ |A \cap B|, |A \cap C|, |B \cap C| \}
\]

(8)

where \( A, B, C \) are sets. Basically we are assuming that values are shared as much as possible between the three sets \( A, B, C \), given the known information about how elements are shared between pairs of sets.

In our next Example we show how to calculate the cardinality for a union of \( n \) sets by using a set of linear equations.
**Example 6.4.** Consider the sets \( A = \{1, 2, 3, 4\} \), \( B = \{1, 2, 5, 6\} \), \( C = \{1, 2, 3, 4\} \), and \( D = \{5, 6, 7, 8\} \). We consider the following cardinalities to be available:

- \(|A| = |B| = |C| = |D| = 4\)
- \(|A \cap B| = 2\), \(|A \cap C| = 4\), \(|A \cap D| = 0\), \(|B \cap C| = 2\), \(|B \cap D| = 2\), \(|C \cap D| = 0\).

For each pair \( |S_1 \cap S_2| \) above, we introduce a variable \( x_{s_1,s_2} \).

In order to calculate the cardinality for the union of the four sets, by applying Theorem 6.2 we first need to calculate the cardinalities for \(|A \cap B \cap C|\), \(|A \cap B \cap D|\), \(|B \cap C \cap D|\), \(|A \cap C \cap D|\), and \(|A \cap B \cap C \cap D|\). Boundaries for such values can be trivially found by using a linear system of inequalities:

\[
\begin{cases}
  x_{ABC} \leq \min\{x_{AB}, x_{AC}, x_{BC}\} \\
  x_{ABD} \leq \min\{x_{AB}, x_{AD}, x_{BD}\} \\
  x_{BCD} \leq \min\{x_{BC}, x_{CD}, x_{BD}\} \\
  x_{ACD} \leq \min\{x_{AC}, x_{CD}, x_{AD}\} \\
  x_{ABCD} \leq \min\{x_{ABC}, x_{ABD}, x_{BCD}, x_{ACD}\} \\
  x_{AB} = 2, x_{AC} = 4, x_{AD} = 0, x_{BC} = 2, x_{BD} = 2, x_{CD} = 0
\end{cases}
\]

By using our Assumption \( 8 \), we obtain:

\[
\begin{cases}
  x_{ABC} = \min\{x_{AB}, x_{AC}, x_{BC}\} = \min\{2, 4, 2\} = 2 \\
  x_{ABD} = \min\{x_{AB}, x_{AD}, x_{BD}\} = \min\{2, 0, 2\} = 0 \\
  x_{BCD} = \min\{x_{BC}, x_{CD}, x_{BD}\} = \min\{2, 0, 2\} = 0 \\
  x_{ACD} = \min\{x_{AC}, x_{CD}, x_{AD}\} = \min\{4, 0, 0\} = 0 \\
  x_{ABCD} = \min\{x_{ABC}, x_{ABD}, x_{BCD}, x_{ACD}\} = \min\{2, 0, 0, 0\} = 0 \\
  x_{AB} = 2, x_{AC} = 4, x_{AD} = 0, x_{BC} = 2, x_{BD} = 2, x_{CD} = 0
\end{cases}
\]

By applying on the results of the system to the Formula in Theorem 6.2 we obtain:

\[
4 \times 4 - (2 + 4 + 0 + 2 + 2 + 0) - 2 = 16 - 10 - 2 = 8 \leq |A \cup B \cup C \cup D| \leq |A| + |B| + |C| + |D|
\]

Observe that, in this Example, the found lower-bound value \( 8 \) corresponds to the actual cardinality of \(|A \cup B \cup C \cup D|\). □

However, observe that the previous approach comes at the cost of calculating an expression of length exponential in the size of sets in the union. Thus, this approach for calculating the cardinality of the union of sets is feasible only if the number of sets for which such cardinality should be calculated is small.

We now show how the approach above, or any other approach able to estimate the cardinality for the union of sets, can be used to calculate the cardinality of an unfolding in the presence of projection. To do so, we first need to introduce the auxiliary notion of *answer template matrix*.

**Definition 6.5.** Let \( q(x) \leftarrow L_1(v_1), \ldots, L_n(v_n) \) be a CQ, and consider the unfolding \( \text{unf}(q, \mathcal{M}) = (\text{unf}(x), \Pi) \) of \( q \) w.r.t. \( \mathcal{M} \). W.l.o.g., we assume \( \Pi \) to be of the form

\[
\Pi = \left\{ \text{unf}(f_j, y_j) \leftarrow \wedge_i V_i^j \mid 1 \leq j \leq m, 1 \leq i \leq n \right\}.
\]
Then, the answer template matrix $\text{templ}(q_{\text{unf}})$ of $q_{\text{unf}}$ is the matrix

$$
\begin{bmatrix}
  f_1 \\
  \vdots \\
  f_n
\end{bmatrix}
$$

of tuples of function templates $f_j$, $1 \leq j \leq n$.

By Theorem 6.1, it is immediate to see that the answer template matrix of an unfolding of a CQ $q$ w.r.t. $\text{wrap}(M)$ does not contain repeated rows if $q$ does not contain existentially quantified variables. By exploiting the notion of answer template matrix, we can simply extend Theorem 6.1 to a more general case.

**Theorem 6.3.** Consider a CQ $q(x) \leftarrow L_1(v_1), \ldots, L_n(v_n)$. Let $M$ be a mapping. Then, if $\text{templ}(\text{unf}(q(x), \text{wrap}(M)))$ does not contain repeated rows, it holds:

$$
|\text{unf}(q(x), M)|^D = \sum_{q_u \in \text{unf}(q, \text{wrap}(M))} |q_u(x)|^D
$$

**Proof.** It follows the same argument as in the proof for Theorem 6.1 and observing that the condition $x \supseteq v_i, v_i'$ in the proof necessarily holds given our precondition for the answer template matrix of $\text{unf}(q(x), \text{wrap}(M))$. \qed

Theorem 6.3 gives us a less restrictive condition for calculating the cardinality of an unfolding, as it essentially says that projecting out variables does not change the number of results as long as duplicate rows do not appear in the answer template matrix of the unfolding.

**Estimation with Duplicate Rows in the Answer Template Matrix** In this paragraph we study the problem of estimating the number of results of an unfolding having duplicate rows in its answer template matrix. First, observe that any unfolding can be partitioned into several different UCQs where either the answer template matrix does not contain duplicate rows, or there is a single distinct row $r$ and all other rows are a duplicate of that row $r$. Summing up the cardinalities for such partitions would give the cardinality for the original unfolding. Hence, the problem of calculating the cardinality of a general unfolding can be solved by providing methods for such two cases. In the previous sections we have already seen how to deal with the case when there are no duplicate rows in the answer template matrix. In this section we study the case when all the rows in such matrix are duplicate.

We exploit another useful property of the unfolding over wrap mappings.

**Theorem 6.4.** Let $q(x) \leftarrow L_1(v_1), \ldots, L_n(v_n)$ be a CQ, and for which the unfolding $\text{unf}(q, \text{wrap}(M)) = (q_{\text{unf}}(x), \Pi)$ of $q$ w.r.t. $\text{wrap}(M)$ is such that $|\{r \mid r \text{ is a row in the answer matrix of } \text{unf}(q, \text{wrap}(M))\}| = 1$. Let $L_i(v_i)$ be an atom in $q$ such that $v_i \subseteq x$. Then, for each pair of rules

\[
\begin{align*}
  r_1 & : = q_{\text{unf}}(f_j(x_j)) \leftarrow V_j^1(y_1^j), \ldots, V_j^j(y_j^j), \ldots, V_j^n(y_n^j), \\
  r_2 & : = q_{\text{unf}}(f_k(x_k)) \leftarrow V_k^1(y_1^k), \ldots, V_k^k(y_k^k), \ldots, V_k^n(y_n^k)
\end{align*}
\]

in $\Pi$, it holds that $V_j^j = V_k^k$. 24
Proof. Without loss of generality, assume \( i = 1 \). Since the set cardinality of the answer template matrix is 1, there must exist a tuple of function symbols \( f \) such that \( f_j = f = f_k \). Since \( r_1, r_2 \in \Pi \), then by Definition 3.3 of unfolding for a CQ there must exist two pairs \( (\sigma_a, (m^a_1, \ldots, m^a_n)) \) and \( (\sigma_b, (m^b_1, \ldots, m^b_n)) \) such that \( (f_j(x_j)) = (\sigma_a(v_1) \ldots \sigma_a(v_n), f_k(x_k)) = (\sigma_b(v_1) \ldots \sigma_b(v_n)) \), and such that \( \sigma_a(\text{source}(m^a_1)) = V^j_1(y^j_1) \), \( \sigma_b(\text{source}(m^b_1)) = V^k_1(y^k_1) \). Let \( \sigma_a(v_1) = f_a(x_a) \) and \( \sigma_b(v_1) = f_b(x_b) \), for tuples of function symbols \( f_a, f_b \) and variables \( x_a, x_b \). Since \( f_j = f = f_k \), then it must be \( f_a = f_b \). Hence, \( \text{sign}(m^a_1) = \text{sign}(m^b_1) \). Then, by Lemma 5.3 it must be \( m_a = m_b \). Hence, \( \text{pred}(\sigma_a(\text{source}(m^a_1))) = \text{pred}(\sigma_b(\text{source}(m^b_1))) \). Since \( \sigma_a(\text{source}(m^a_1)) = V^j_1(y^j_1) \) and \( \sigma_b(\text{source}(m^b_1)) = V^k_1(y^k_1) \), it then must be \( \text{pred}(V^j_1(y^j_1)) = \text{pred}(V^k_1(y^k_1)) \), our thesis.

Example 6.5. Consider the CQ \( g(x, y) \leftarrow P_1(x, y), P_2(y, z), P_3(z, w) \) for which the answer template matrix of \( \text{unf}(q, \text{wrap}(M)) = (p_{unf}(x, y), \Pi) \) contains a single distinct row, repeated \( n \geq 1 \) times. Then, by Theorem 6.4 above, \( \Pi \) is of the following shape:

\[
eq \begin{align*}
\text{cq}_1 & : \quad q_{\text{unf}}(f(x_1), g(y_1)) \leftarrow V_{P_1}(x_1, y_1), V_{P_2}^1(y_1, z_1), V_{P_3}^1(z_1, w_1) \\
\vdots \\
\text{cq}_n & : \quad q_{\text{unf}}(f(x_n), g(y_n)) \leftarrow V_{P_1}(x_n, y_n), V_{P_2}^n(y_n, z_n), V_{P_3}^n(z_n, w_n)
\end{align*}
\]

That is, the relation \( V_{P_i} \) appears in each CQ. Due to this fact, coupled with the fact that atoms \( V_{P_1}^1(z_1, w_1), \ldots, V_{P_3}^n(z_1, w_1) \) do not contain answer variables, we have the interesting property that any duplicate across different CQs in the blue occurrences would produce a duplicate result in \( q \). Under the uniformity assumption (a2), we then get

\[
|q(x, y)| = \frac{1}{n} \sum_{j=1}^{n} |cq_j| \cdot \frac{|\pi_{y_1}(cq^*_1) \cup \cdots \cup \pi_{y_n}(cq^*_n)|}{\sum_{i=1}^{n} \text{dist}(cq^*_i, V_{P_2}^i, y_n)}.
\]

where \( cq^*_i \) represents the query \( cq_i \), where all the variables are answer variables\(^3\), and the fraction

\[
\frac{|\pi_{y_1}(cq^*_1) \cup \cdots \cup \pi_{y_n}(cq^*_n)|}{\sum_{i=1}^{n} \text{dist}(cq^*_i, V_{P_2}^i, y_n)}
\]

denotes the ratio of distinct values across the different CQs over the answer variable \( y \), calculated by assuming that each value is repeated the same amount of times (Assumption (a2)).

For estimating the cardinality of the unfolding from the previous Example, we need to be able estimate the quantity \( |\pi_{y_1}(cq^*_1) \cup \cdots \cup \pi_{y_n}(cq^*_n)| \). We here try to provide a lower-bound estimation for it. To perform this calculation, we make use of the following assumption:

If \( V_{P_2}^j = V_{P_2}^k \), then \( |\pi_{y_1}(cq^*_1) \cup \pi_{y_2}(cq^*_2)| = \max\{\text{dist}(cq^*_1, V_{P_2}^j, y_n), \text{dist}(cq^*_2, V_{P_2}^k, y_n)\} \).

Observe that the above is just an alternative way of writing the facing values assumption (a4), and it captures the scenario in which values are redundant\(^6\). To calculate the cardinality for the union, we can then use the formula from Theorem 6.4 as in Example 6.4 by using the available statistics on intersections between pairs of attributes sets arguments for some function symbol.

\(^3\)We abuse the notation and ignore the function symbols in the head of each CQ

\(^6\)We remind the reader that we are interested in finding a lower-bound for the cardinality of the union.
Example 6.6. Recall the scenario from Example 6.5. Assume that $n = 6$, and that

- $V_{P_1}^1 = V_{P_2}^2 = W_1$
- $V_{P_1}^3 = V_{P_2}^4 = W_2$
- $V_{P_1}^5 = V_{P_2}^6 = W_3$

Then,

- $|\pi_{y_1}(cq_1^1) \cup \pi_{y_4}(cq_2^1)| = \max\{\text{dist}(cq_1^1, V_{P_2}^1.y_1), \text{dist}(cq_2^1, V_{P_2}^2.y_2)\} := \text{max}_1$
- $|\pi_{y_5}(cq_3^4) \cup \pi_{y_4}(cq_4^5)| = \max\{\text{dist}(cq_3^4, V_{P_2}^1.y_3), \text{dist}(cq_4^5, V_{P_2}^2.y_4)\} := \text{max}_2$
- $|\pi_{y_5}(cq_5^6) \cup \pi_{y_4}(cq_6^6)| = \max\{\text{dist}(cq_5^6, V_{P_2}^3.y_5), \text{dist}(cq_6^6, V_{P_2}^4.y_6)\} := \text{max}_3$

Now, by using Theorem 6.2, we get the formula:

$$|\pi_{y}(W_1) \cup \pi_{y}(W_2) \cup \pi_{y}(W_3)| = |\pi_{y}(W_1)| + |\pi_{y}(W_2)| + |\pi_{y}(W_3)| - |\pi_{y}(W_1) \cap \pi_{y}(W_2)| - \cdots$$

$$+ |\pi_{y}(W_1) \cap \pi_{y}(W_2) - \pi_{y}(W_3)|$$

We use such formula to compute the ratio

$$r = \frac{|\pi_{y}(W_1) \cup \pi_{y}(W_2) \cup \pi_{y}(W_3)|}{|\pi_{y}(W_1)| + |\pi_{y}(W_2)| + |\pi_{y}(W_3)|}.$$

giving the ratio of distinct values. Hence, we estimate $|\pi_{y_1}(cq_1^1) \cup \cdots \cup \pi_{y_6}(cq_6^6)|$ to be

$$r \cdot (\text{max}_1 + \text{max}_2 + \text{max}_3).$$

The previous Example makes use of Theorem 6.2, which is practical only if the union is on a small number of sets. An alternative way is to store additional statistics over the projection on one of the two arguments of each property in the wrap T-mappings, and to use this value and the uniformity assumption to estimate $|W_1 \cup W_2 \cup W_3|$.

7 Unfolding Cost Model

We are now ready to estimate the actual costs of evaluating UJUCQ and UCQ unfoldings, by exploiting the cardinality estimations from the previous section. Our cost model is based on traditional textbook-formulae for query cost estimation [26], and it assumes source parts in the mappings to be CQs.
7.1 Cost for the Unfolding of a UCQ.

Recall from Section 5 that the unfolding of a UCQ produces a UCQ translation \( q^{ucq} = \bigvee_i q_i^{ucq} \).

We estimate the cost of evaluating \( q^{ucq} \) as

\[
c_{eval}(q^{ucq}) = \sum_i c_{eval}(q_i^{ucq}) + c_u(q^{ucq})
\]

where

- \( c_{eval}(q_i^{ucq}) \) is the cost of evaluating each \( q_i^{ucq} \) in \( q^{ucq} \);
- \( c_u(q^{ucq}) \) is the cost of removing duplicate results.

In the remainder of this paragraph we explain how we calculate each of these addends.

Value of \( c_{eval}(q_i^{ucq}) \).

Let \( q_i^{ucq} = V_1 \times \cdots \times V_{m_i} \), where \( V_i = T_{i1} \times \cdots \times T_{ini} \), where \( m_i \in \mathbb{N} \) and \( 1 \leq i \leq n \). Then, we define \( c_{eval}(q_i^{ucq}) \) as

\[
c_{eval}(q_i^{ucq}) = \sum_{1 \leq i \leq n} \sum_{1 \leq j \leq m_i} c_{scan}(T_{ij}) + c_{hjoin}(q_i^{ucq})
\]

where

- \( c_{scan}(T_{ij}) = |T_{ij}^D| \times c_t \)
  - where \( c_t \) is the fixed cost of retrieving one tuple from the database. Such constant, as well as other constants in this section, can be found through calibration techniques [13].
- \( c_{hjoin}(q_i^{ucq}) = (\sum_{i=1}^n m_i) \cdot f_D(q_i^{ucq}) \cdot c_j \)
  - where \( c_j \) is the fixed cost of joining one tuple.

Summing up, \( c_{eval}(q_i^{ucq}) \) is the cost of scanning each table in the conjunctive query and performing a hash join. We assume the statistics \( |T_{ij}^D| \), for \( 1 \leq i \leq n, 1 \leq j \leq m_i \), to be available after having analyzed the database instance according to the database schema.

Value of \( c_u(q^{ucq}) \). We assume the removal of duplicates at this level to be carried out by a sort-based strategy. Under this assumption, the cost of removal is

\[
c_u(q^{ucq}) = f_D(q^{ucq}) \cdot \log(f_D(q^{ucq})) \cdot c_u
\]

where \( f_D(q^{ucq}) \) is the cardinality estimation of the unfolding, calculated as described in Section 6 and \( c_u \) is a constant denoting the cost of eliminating a duplicate tuple.

Cost for the Unfolding of a JUCQ. Recall from Section 5 that the optimized unfolding of a JUCQ produces a UJUCQ. We estimate the cost of a single JUCQ \( q^{jucq} = \bigwedge_i q_i^{jucq} \) in the unfolding as

\[
c(q^{jucq}) = \sum_i c_{eval}(q_i^{jucq}) + \sum_{i \neq k} c_{mat}(q_i^{jucq}) + c_{mjoin}(q^{jucq})
\]

where

- \( c_{eval}(q_i^{jucq}) \) is the cost of evaluating each UCQ component \( q_i^{jucq} \);
- \( \sum_{i \neq k} c_{mat}(q_i^{jucq}) \) is the cost of materializing the intermediate results from \( q_i^{jucq} \), where the \( k \)-th UCQ is assumed to be pipelined [26] and not materialized;
• $c_{mjoin}(q^{jucq})$ is the cost of a merge join over the materialized intermediate results.

The cost for a UJUCQ $q^{u_{jucq}} = \bigvee_i q_i^{ucq}$, if all the attributes are kept in the answer, is simply the sum $\sum_i c(q_i^{ucq})$, since the results of all JUCQs are disjoint (c.f., Section 5). Otherwise, we need to consider the cost of eliminating duplicate results.

**Value of $c_{mat}(q_i^{ucq})$.** Let $q_i^{ucq} = q_1^{ucq} \triangledown \ldots \triangledown q_m^{ucq}$. Then

$$c_{mat}(q_i^{ucq}) = f_D(q_i^{ucq}) \cdot c_m$$

where $c_m$ is the fixed cost of materializing a tuple.

**Value of $c_{mjoin}(q^{jucq})$.** Let $q^{jucq} = q_1^{jucq} \triangledown \ldots \triangledown q_m^{jucq}$. Then

$$c_{mjoin}(q^{jucq}) = m \cdot f_D(q^{jucq}) \cdot c_j$$

where $f_D(q^{jucq})$ is the cardinality estimation of the unfolding, calculated as described in Section 6. Observe that we do not consider the cost of sorting each $q_i^{ucq}$, as this cost is already included in $c_{eval}(q_i^{ucq})$.

8 Experimental Results

In this section, we provide an empirical evaluation that compares unfoldings for UCQs and (optimized) unfoldings for JUCQs, as well as the estimated costs and the actual time needed to evaluate the unfoldings. We ran the experiments on an HP Proliant server with 2 Intel Xeon X5690 Processors (each with 12 logical cores at 3.47GHz), 106GB of RAM and five 1TB 15K RPM HDs. As RDBMS we have used PostgreSQL 9.6. the material to replicate our experiments is available online.

8.1 Wisconsin Experiment

We devised an OBDA benchmark based on the Wisconsin Benchmark [11]. In the following subsections we discuss each element of the benchmark, and their rationale.

Mappings. The mappings set consists of 1364 mapping assertions. Each mapping defines a property of the form $:J2OxxMyRzPropi$, where

• $J20$, read “join selectivity 20%”, denotes that each mapping for that property is on a query retrieving the 20% of the total tuples;

• $Oxx$, read “offset xx”, where $xx \in \{0, 5, 10, 15\}$, denotes the offset for the filter in the mappings based on column onePercent;

• $My$, $y \in \{1, \ldots, 6\}$ denotes the number of mapping assertions defining the property;

• $Rz$, $z \leq y$ denotes the number of redundant mapping assertions, that is, the number of mapping assertions whose source query does not produces new individuals for the property in the virtual RDF ABox;

[11] https://github.com/ontop/ontop-examples/tree/master/iswc-2017-cost
Prop, \(i \in \{1, \ldots, 3\}\). The tested BGP is made of three properties.

For instance, we the following listing provides the mapping definition for the property
\(\text{:S2000M2R0Prop1}\).

```sparql
target :number/{unique2} :S2000M2R0Prop1
:name/{evenOnePercent}/{stringu1}/{stringu2}.
source SELECT "unique2", evenOnePercent, stringu1, stringu2
FROM t1_1m
WHERE "onepercent" >= 0 AND "onepercent" < 20
```

**SparQL Queries.** Our test is on 84 queries, instantiations of the following template:

```sparql
SELECT DISTINCT * WHERE {?x :M\textsuperscript{m}R\textsuperscript{r}Prop1 \?y1; \:J\textsuperscript{j}M\textsuperscript{m}R\textsuperscript{r}Prop2 \?y2; \:J\textsuperscript{j}M\textsuperscript{m}R\textsuperscript{r}Prop3 \?y3}
```

where \(j \in \{5, 10, 15, 20\}\) denotes the selectivity of the join between the first property and each of the remaining two, expressed as a percentage of the number of retrieved rows for each mapping defining the property (each mapping retrieves 200k tuples); \(m \in \{1, \ldots, 6\}\) denotes the number of mappings defining the property (all such mappings have the same signature), and \(r \in \{0, \ldots, m - 1\}\) denotes the number of redundant mappings, that is, the number of mappings assertions retrieving the same results of another mapping defining the property, minus one. Queries are numbered as \(q_{m,r}\). Hence, query \(q_3\) is the query \(q_{m=2,r=1}\), with 2 mappings one of which is redundant.

For each query, we have tested a correspondent cover query of two fragments \(f_1, f_2\), where each fragment is an instantiation of the following templates:

```sparql
f_1: SELECT DISTINCT ?x ?y1 ?y2 WHERE { ?x :M\textsuperscript{m}R\textsuperscript{r}Prop1 \?y1; \:J\textsuperscript{j}M\textsuperscript{m}R\textsuperscript{r}Prop2 \?y2. }
f_2: SELECT DISTINCT ?x ?y3 WHERE { ?x a :M\textsuperscript{m}R\textsuperscript{r}Prop1; ?x :J\textsuperscript{j}M\textsuperscript{m}R\textsuperscript{r}Prop3 \?y3. }
```

Figure 3: Cost estimations vs evaluation running times

- Prop, \(i \in \{1, \ldots, 3\}\). The tested BGP is made of three properties.
Ontology. We have carried out our tests over an empty TBox. Observe that this is not a limitation, as the case with a TBox can be reduced to the case without a TBox by making use of T-mappings as in Example 6.3.

Data. We have created several copies of the wisconsin table, and populated each copy with ten million tuples.

Evaluation. In Figure 3, we present the cost estimation and the actual running time for each query. We have the following observations:

- In this experiment, for the considered cover, JUCQs are generally faster than UCQs. In fact, out of the 84 SPARQL queries, only one JUCQ was timed out, while 16 UCQs were timed out. The mean running time of successful UCQs and JUCQs are respectively 160 seconds and 350 seconds.
- In Figure 3a, where the fitted lines are obtained by applying linear regression over successful UCQ and JUCQ evaluations, we observe a strong linear correlation between our estimated costs and real running times. Moreover, the coefficients \( b_1 \) and \( b_0 \) for UCQs and JUCQs are rather close. This empirically shows that our cost model can estimate the real running time well.
- Figure 3b shows that the PostgreSQL cost model assigns the same estimation to many queries having different running times. Moreover, the linear regressions for UCQs and JUCQs are rather different, which suggests that PostgreSQL is not able to recognize when two translations are semantically equivalent. Hence, PostgreSQL is not able to estimate the cost of these queries properly.

In Figure 4, we visualize the performance gain of JUCQs compared with UCQs. The four subgraphs correspond to four different settings join selectivities. Each subgraph is a matrix in which each cell shows the value of the performance gain \( g = 1 - \text{jucq time/ucq time} \). When \( g > 0 \), we apply the red color; otherwise blue. These graphs clearly show that when there is
a large number of mappings and there is high redundancy, we have better performance gains. When the redundancy is low (0 or 1), and the number of mapping axioms is large, the join selectivity plays an important role in the performance gain, as discussed in [3]; in other cases, the impacts are non-significant.

Figures 5 and 6 report the cardinalities estimated by PostgreSQL divided by the actual sizes of the query answers for all UCQ and JUCQ queries. For UCQs, it shows that PostgreSQL normally underestimates the cardinalities, but it overestimates them when the redundancies are high. As for JUCQS, PostgreSQL always overestimates the cardinalities, ranging from 40 to 200K times. These numbers partially explains why PostgreSQL estimate the costs of both UCQs and JUCQs so badly in Figure 3b.

We obtained similar conclusions for a query with four atoms, and a cover of three fragments.
In this experiment we consider 84 queries, instances of the following template

\[
\text{SELECT DISTINCT } * \text{ WHERE } \{ ?x :MmRrProp1 ?y1; :JmMmRrProp2 ?y2; :JmMmRrProp3 ?y3; :JmMmRrProp3 ?y4 \}
\]

where \( j \in m, \) and \( r \) are defined as for the case with three atoms.

For each query, we have tested a correspondent cover query of three fragments \( f_1, f_2, f_3 \), where each fragment is an instantiation of the following templates:

\[
\begin{align*}
\text{f}_1: & \text{ SELECT DISTINCT } ?x ?y1 ?y2 \text{ WHERE } \{ \text{ ?x :MmRrProp1 ?y1; :JmMmRrProp2 ?y2. } \} \\
\text{f}_2: & \text{ SELECT DISTINCT } ?x ?y3 \text{ WHERE } \{ \text{ ?x a :MmRrProp1; ?x :JmMmRrProp3 ?y3. } \} \\
\text{f}_3: & \text{ SELECT DISTINCT } ?x ?y4 \text{ WHERE } \{ \text{ ?x a :MmRrProp1; ?x :JmMmRrProp3 ?y4. } \}
\end{align*}
\]

Evaluation. Figures 7, 8, and 9 are the counterparts for the Figures of the three atoms case. Observe that the results look extremely similar, thus confirming our comments from the previous Subsection.

8.2 NPD Experiment

The goal of this experiment is to verify that cost-based techniques can improve the performance of query answering over real-world queries and instances. This test is carried on the original real-world instance (as opposed to the scaled data instances) of the NPD benchmark [19] for OBDA systems. We pick the three most challenging UCQ queries (namely \( q_6, q_11, \) and \( q_{12} \)) from the query catalog, and create another even more difficult query (called \( q_{31} \)) by combining \( q_6 \) and \( q_9 \). Query \( q_{31} \), in the Listing below, retrieves information regarding wellbores (from \( q_6 \)) and their related facilities (from \( q_9 \)).
In Table 2 we show the evaluation results over the NPD benchmark for UCQs and JUCQs. The unfoldings for JUCQs are constructed using cover queries of 2 fragments, each guided by our cost model. We observe that the sizes of the unfoldings for JUCQs, measured in number of CQs, are sensibly smaller than the size of the unfoldings for UCQs. Finally, we observe that the unfoldings for the JUCQ version of the considered queries improve the running times up to a factor of 34.
| SPARQL Query | Unfolding for UCQs | Unfolding for JUCQs |
|--------------|-------------------|--------------------|
| name         | # triple patterns | time (s) | # CQs | time (s) | # Frags | # CQs |
| q₆           | 7                 | 2.18      | 48    | 1.20      | 2       | 14    |
| q₁₁          | 8                 | 3.39      | 24    | 0.40      | 2       | 12    |
| q₁₂          | 10                | 6.67      | 48    | 0.47      | 2       | 14    |
| q₁₄          | 10                | 54.27     | 3840  | 1.58      | 2       | 327   |

9 Conclusion and Future Work

In this paper, we have studied the problem of finding efficient alternative translations of a user query in OBDA. Specifically, we introduced a translation for JUCQ queries that preserves the JUCQ structure while maintaining the possibility of performing joins over database values, rather than URIs constructed by applying mappings definitions. We devised a cost model based on a novel cardinality estimation, for estimating the cost of evaluating a translation for a UCQ or JUCQ over the database. We compared different translations on both a synthetic and fully customizable scenario based on the Wisconsin Benchmark and on a real-world scenario from the NPD Benchmark. In these experiments we have observed that (i) our approach based on JUCQ queries can produce translations that are orders of magnitude more efficient than traditional translations into UCQs, and that (ii) the cost model we devised is faithful to the actual query evaluation cost, and hence is well suited to select the best translation.

As future work, we plan to implement our techniques in the state-of-the-art OBDA system Ontop and to integrate them with existing optimization strategies. This will allow us to test our approach in more and diversified settings. We also plan to explore alternatives beyond JUCQs. Finally, we plan to work on the problem of relaxing the uniformity assumption made in our cost estimator, by integrating our model with existing techniques based on histograms.

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