Analysis of Bose system in spin-orbit coupled Bose-Fermi mixture to induce a spin current of fermions

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Abstract. We found that a spin current of fermions could be induced in spin-orbit coupled Bose-Fermi mixture at zero temperature. Since spatial change of the spin structure of the bosons is necessary to induce the spin current of the fermions, we analyzed the ground state of the bosons in the mixture system, using a variational method. The obtained phase diagram indicated the presence of a bosonic phase that allowed the fermions to have a spin current.

1. Introduction
Spin current is the counterflow of particles with opposite spins and a number of methods to induce local and global spin currents have been suggested[1, 2, 3, 4, 5]. Techniques to control electrons using a spin current have a potential to develop a novel device, called spintronics device. On the other hand, a neutral cold atom system with a synthetic spin-orbit (SO) coupling was engineered between two atomic spin states with a pair of Raman lasers[6, 7]. Since SO coupling is known to induce various phenomena involving a spin current, a number of studies of SO coupled cold atom systems have been reported[8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. In particular, it is known that a spiral spin structure can be induced in the SO coupled Bose-Einstein condensate (BEC)[9, 10, 11, 12]. One can expect from these studies that a spin current of fermions could be induced in a mixture of SO coupled BEC and spin-1/2 fermions.

The purpose of this paper is to reveal a possibility of inducing the spin current of fermions in the SO coupled Bose-Fermi mixture. First, we explain that a certain-type spatial spin structure of the bosons is necessary to induce the spin current of the fermions. Next, we show the ground state phase diagram of the bosons in the SO coupled Bose-Fermi mixture by using a variational method and the presence of a bosonic phase that allowed the fermions to have a spin current. Finally, we discuss the density dependence on the phase diagram.
2. Model and spin current

Let us consider a mixture of SO coupled pseudospin-1/2 bosons and spin-1/2 fermions. The total Hamiltonian $H$ is given by

$$H = H^b + H^f + H^{bf}_0 + H^{bf}_S,$$

(1)

$$H^b = \sum_{j=1}^{N_b} h^{b}_{SO}(j) + \frac{1}{2} \sum_{\alpha, \beta} d\mathbf{r} \ g_{\alpha \beta} n^{(b)}_\alpha n^{(b)}_\beta,$$

(2)

$$H^f = \sum_\sigma \sum_{j=1}^{N_f} h^{f}_\sigma(j),$$

(3)

$$H^{bf}_0 = \frac{u_0}{2} \int d\mathbf{r} \ n^{(b)} n^{(f)},$$

(4)

$$H^{bf}_S = \frac{u_s}{2} \int d\mathbf{r} \ \mathbf{S}^{(b)} \cdot \mathbf{S}^{(f)}.$$  

(5)

In the Bose Hamiltonian $H^b$, $\alpha$ and $\beta$ (=$\uparrow$, $\downarrow$) are the spin indices, $g_{\alpha \beta}$ are the spin-dependent strengths of the Bose interaction, $n^{(b)}_\alpha(r)$ is the number density of the bosons with spin $\alpha$, and

$$h^{b}_{SO} = \frac{1}{2m_b} (p_x + \hbar k_0 \sigma_z)^2 + \frac{p_y^2 + p_z^2}{2m_b} + \frac{\Omega}{2} \sigma_x + \frac{\delta}{2} \sigma_z,$$

(6)

is the single-particle Hamiltonian of the bosons. Systems described by $h^{b}_{SO}$ have been already realized experimentally[6, 7]. In Eq. (6), $k_0$ is the strength of the SO coupling, $\Omega$ and $\delta$ are the strengths of the effective magnetic field, and $\sigma_i$, with $i=x, y, z$, is the $2 \times 2$ Pauli matrix. Equation (3) expresses the Fermi Hamiltonian where $h^{f}_\sigma = p^2/(2m_f)$ is the kinetic energy of the fermions with spin $\sigma$. $H^{bf}_0$ and $H^{bf}_S$ are the Bose-Fermi particle and spin interaction Hamiltonians, respectively, where $u_0$ and $u_s$ are the spin-independent strengths of the Bose-Fermi particle and spin interaction, respectively, $n^{(f)}$ is the number density of the fermions, and $\mathbf{S}^{(b)}$ and $\mathbf{S}^{(f)}$ are the spin densities of the bosons and fermions, respectively.

The single-particle energy of the fermions including the Bose-Fermi interaction is given by

$$\varepsilon^{(f+b)} = \int d\mathbf{r} \left[ \psi^{(f)} \right]^* \left[ -\frac{\hbar^2 \nabla^2}{2m_f} + \frac{u_0}{2} n^{(b)} + \frac{u_s}{2} \mathbf{S}^{(b)} \cdot \mathbf{\sigma} \right] \psi^{(f)},$$

(7)

where $\psi^{(f)}$ is the single-particle wave function of the fermions. This energy is just the one of the spin-1/2 Fermi system in the potential $u_0 n^{(b)}/2$ and the magnetic field $u_s \mathbf{S}^{(b)}/2$. The first point that we should discuss is the condition to induce the spin current in this Fermi system. To investigate the condition, we simply analyze the spin current by using a single-fermion spinor wave function[4]

$$\psi^{(f)}_1 = \sqrt{n^{(f)}_1} \left( \begin{array}{c} \cos \frac{\gamma}{2} e^{i\phi_1} \\ \sin \frac{\gamma}{2} e^{i\phi_1} \end{array} \right),$$

(8)

where $n^{(f)}_1$ is the Fermi density normalized to unity: $\int d\mathbf{r} n^{(f)}_1(r) = 1$. $\gamma$ is a parameter to determine the ratio between the two spinor components and $\phi_\uparrow$ and $\phi_\downarrow$ are the phases of the
components. Substituting Eq. (8) into Eq. (7), we obtain the free energy of the Fermi system

\[ F = \int d\mathbf{r} \left\{ \frac{n_1^{(f)} \hbar^2}{2m_f} \left[ \left( \nabla \phi \right)^2 - \frac{\nabla^2 \sqrt{n_1^{(f)}}}{\sqrt{n_1^{(f)}}} + \left( \nabla \phi_z \right)^2 - 2 \left( \nabla \phi \right) \left( \nabla \phi_z \right) \cos \gamma \right] \right. \]
\[ \left. + \frac{n_1^{(f)} u_0}{2} n_1^{(b)} + \frac{n_1^{(f)} u_2}{2} \mathbf{S}_1^{(f)} \cdot \mathbf{S}_1^{(f)} \right\}, \]

where \( \phi = \phi_\uparrow + \phi_\downarrow, \phi_z = \phi_\downarrow - \phi_\uparrow, \) and

\[ \mathbf{S}_1^{(f)} = \left[ \psi_1^{(f)} \right]^* \sigma \psi_1^{(f)} \frac{n_1^{(f)}}{n_1^{(f)}} = (\sin \gamma \cos \phi_z, \sin \gamma \sin \phi_z, \cos \gamma). \]

Note that the spin quantization axis is along the \( z \) axis. The spin current density is given by

\[ \mathbf{j}_s = -\frac{1}{\hbar} \frac{\partial F}{\partial (\nabla \phi_z)} = \frac{n_1^{(f)} \hbar}{m_f} \left\{ (\nabla \phi) \cos \gamma - \nabla \phi_z \right\}. \]

This equation shows that the spatial dependence of the phase \( \phi_\uparrow \) or \( \phi_\downarrow \) is necessary to induce the spin current. The spatial change of the phase of the fermions can be generated by a spatially-changing spin structure in the background. Therefore, for the realization of the spin current of the fermions, spatial dependence of the bosonic spins is at least necessary.

3. Phase diagram

Here, we analyze the ground state of the bosons in the SO coupled Bose-Fermi mixture by using a variational method and look for a state where the spin current could be induced. For simplicity, we assume that the bosons and fermions have the same mass, \( m_b = m_f = m \), and that \( \delta = 0 \) and \( g_{\uparrow \underline{\downarrow}} = g_{\downarrow \uparrow} > g_{\uparrow \uparrow} \). In addition, we use a variational wave function of the bosons in the form[11, 12]

\[ \psi^{(b)} = \left( \frac{\psi_1^{(b)}}{\psi_\downarrow^{(b)}} \right) = \sqrt{n_b} \left[ C_+ \left( \cos \theta \right) + i k_1 x \right] e^{i k_1 x} + C_- \left( \sin \theta \right) e^{-i k_1 x}, \]

where \( n_b \) is the mean number density of the bosons and \( C_\pm, k_1, \) and \( \theta \) are variational parameters satisfying \( |C_+|^2 + |C_-|^2 = 1 \) and \( 0 \leq \theta \leq \pi/4 \). For this variational function, we have the boson density

\[ n_1^{(b)} = \frac{n_1^{(b)}}{n_b} = [1 + 2 \beta \sin(2\theta) \cos(2k_1 x + \eta)], \]

and the spin density

\[ s^{(b)} = \left( -\sin(2\theta) - 2 \beta \cos(2k_1 x + \eta), 2 \beta \cos(2\theta) \sin(2k_1 x + \eta), \alpha \cos(2\theta) \right), \]

where \( \alpha = |C_+|^2 - |C_-|^2, \beta = |C_+||C_-|, \) and \( \eta \) is the phase difference between \( C_+ \) and \( C_- \). Substituting Eq. (12) into Eq. (2), we can obtain the energy of the bosons as a function of the variational parameters. Without the fermions, the ground state phase diagram of the SO coupled Bose system was already calculated as in Fig. 1[11, 12]. Figure 1 shows three different phases, Phase I to III. In Phase I \( (k_1 \neq 0, \alpha = 0, \beta = 1/2, 0 \leq \theta < \pi/4) \), the bosons
equally condensate in two finite-momentum states and have a spiral spin structure. In Phase II \((k_1 \neq 0, \alpha = \pm 1, \beta = 0, 0 \leq \theta < \pi/4)\), all the bosons condensate in a single finite-momentum state. In Phase III \((k_1 = 0, \alpha = \pm 1, \beta = 0, \theta = \pi/4)\), all the bosons condensate in a zero-momentum state. Since the spin structure does not change spatially in Phase II and III but do in Phase I, we can expect that, if fermions are introduced, the bosons in Phase I would induce a fermionic spin current. Therefore, for the Bose-Fermi mixture system to sustain a fermionic spin current, Phase I of the bosons has to remain even when the fermions are introduced to the system.

To find the ground state of the bosons in the SO coupled Bose-Fermi mixture, we calculate the energy of the Fermi system including the Bose-Fermi interaction by using the variational wave function of the bosons \((12)\). Substituting Eqs. \((13)\) and \((14)\) into Eq. \((7)\), Schrödinger equations of the fermions can be written as

\[
i \hbar \frac{\partial}{\partial t} \psi^{(f)} = \left( -\frac{\hbar^2 \nabla^2}{2m} + U_0 n_1^{(b)} + U_s s_z^{(b)} - \frac{\hbar^2 \nabla^2}{2m} + U_0 n_1^{(b)} - U_s s_z^{(b)} \right) \psi^{(f)},
\]

where \(U_0 = n_u u_0/2\) and \(U_s = n_u u_s/2\).

### 3.1. \( \Omega \sim 0 \) limit

To calculate the energy from Eq. \((15)\) analytically, we consider \( \Omega \sim 0 \) limit. In this limit, two spin components of the bosons are not coupled with each other and the variational wave function of the bosons can be written as

\[
\psi^{(b)} = \begin{pmatrix} \psi_+^{(b)} \\ \psi_-^{(b)} \end{pmatrix} = \sqrt{n_b} \begin{pmatrix} C_+ e^{i k_1 x} \\ -C_- e^{-i k_1 x} \end{pmatrix}.
\]

This function corresponds to Eq. \((12)\) with \( \theta = 0 \). Therefore, Eq. \((15)\) becomes

\[
\hbar \omega \psi^{(f)}_{\tilde{\sigma}, q}(\omega) = [\epsilon_q + U_0 + U_s \alpha] \psi^{(f)}_{\tilde{\sigma}, q}(\omega) - 2U_s \beta e^{i \eta} \psi^{(f)}_{\tilde{\sigma}, q-k_1}(\omega),
\]

\[
\hbar \omega \psi^{(f)}_{\tilde{\sigma}, q}(\omega) = -2U_s \beta e^{-i \eta} \psi^{(f)}_{\tilde{\sigma}, q+k_1}(\omega) + [\epsilon_q + U_0 - U_s \alpha] \psi^{(f)}_{\tilde{\sigma}, q}(\omega),
\]

where \( \psi^{(f)}_{\sigma}(r, t) = \frac{1}{2\pi} \int d\omega \sum_q \hat{\psi}^{(f)}_{\sigma, q}(\omega)e^{-i(\omega t - r \cdot q)}, \epsilon_q = \hbar^2 q^2/(2m), \) and \( k_1 = (k_1, 0, 0) \). Note that the fermions do not affect the phase transition of the bosons between Phases II and III because Eq. \((17)\) for Phases II and III have the same form. From Eq. \((17)\), we can obtain the
Figure 2. Phase diagram of the bosons in the SO coupled Bose-Fermi mixture for \( \Omega \sim 0 \) at zero temperature, where \( u_0 = u_s = g_{\uparrow \uparrow} \). The phase transition is the first-order transition. The parameters are the same as those in Fig. 1.

Figure 2 shows that Phase I where the spin current could be induced can persist even in the presence of the fermions and Phase II partially appears. Figure 1 shows that the bosons always prefer Phase I when \( \Omega \sim 0 \). On the other hand, the fermions always prefer Phase II since the bosonic spins apply the spiral magnetic field in Phase I and a uniform magnetic field in Phase II to the fermions. Therefore, Phase II appears when a small amount of the fermions are first introduced to the bosons. However, as the number of the fermions increases, the energy profit that the fermions have from Phase II under the uniform magnetic field generated by the bosonic spins would decrease. This is because two Zeeman-split fermionic bands in the uniform magnetic field begin to be both filled if the fermions are added more and more. So Phase II turns into Phase I with the fermions added more to the bosons in order to let the bosons have the lower energy.

4. Conclusion
We investigated a possibility to induce the spin current of fermions in an SO coupled Bose-Fermi mixture at zero temperature. We focused on a zero magnetic field limit (\( \Omega \sim 0 \)) and showed the ground state phase diagram of the bosons in the SO coupled Bose-Fermi mixture. As a result, it was found from the phase diagram that Phase I, where the bosons had a spiral spin structure and a fermionic spin current was expected, persisted even when the number of the spin-1/2 fermions increased. In addition, we discussed the density dependence on the ground state of the bosons. The present result showed that the spin current could be induced in the SO coupled Bose-Fermi mixture. The magnitude and controllability of the spin current will be studied elsewhere.

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