Coupled-channel evaluations of cross sections for scattering involving particle-unstable resonances

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(Dated: July 18, 2008)

How does the scattering cross section change when the colliding bound-state fragments are allowed particle-emitting resonances? This question is explored in the framework of a multi-channel algebraic scattering method of determining nucleon-nucleus cross sections at low energies. Two cases are examined, the first being a gedanken investigation in which $n^{16}\text{C}$ scattering is studied with the target states assigned artificial widths. The second is a study of neutron scattering from $^8\text{Be}$: a nucleus that is particle unstable. Resonance character of the target states markedly varies evaluated cross sections from those obtained assuming stability in the target spectrum.

PACS numbers: 24.10.Eq,24.30.-v,25.40.-h,25.60.-t

The ready availability of radioactive ion beams allows experimental information to be obtained on many exotic nuclei, allowing for study of novel structures, such as skins and halos. Of particular interest are the data obtained from scattering exotic nuclei from hydrogen targets, which equates to proton scattering from those nuclei in the inverse kinematics. Such data have been analysed, the first being a gedanken investigation in which $n^{16}\text{C}$ scattering is studied with the target states assigned artificial widths. The second is a study of neutron scattering from $^8\text{Be}$: a nucleus that is particle unstable. Resonance character of the target states markedly varies evaluated cross sections from those obtained assuming stability in the target spectrum.

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MCAS is used to find solutions of the coupled-channel, partial-wave expanded Lippmann-Schwinger equations for each total system spin-parity ($J^\pi$),

$$T^{J^\pi}_{cc}(p,q;E) = V^{J^\pi}_{cc}(p,q) + \mu \left[ \sum_{c'=1}^{\text{open}} \int_0^{\infty} V^{J^\pi}_{cc'}(p,x) \frac{x^2}{k_{c'}^2 - x^2 + i\epsilon} T^{J^\pi}_{c'c'}(x,q;E) dx \right] - \sum_{c'=1}^{\text{closed}} \int_0^{\infty} V^{J^\pi}_{cc'}(p,x) \frac{2}{h_{c'}^2 + x^2} T^{J^\pi}_{c'c'}(x,q;E) dx \right],$$

where a finite set of scattering channels, denoted $c$, are considered, and where $\mu = \frac{2\pi\hbar^2}{4\pi}$, $\hbar$ being the reduced mass. There are two summations as the open and closed channel components are separated, with wave numbers

$$k_c = \sqrt{\mu(E - \epsilon_c)} \quad \text{and} \quad h_c = \sqrt{\mu(\epsilon_c - E)},$$

for $E > \epsilon_c$ and $E < \epsilon_c$ respectively, $\epsilon_c$ is the energy threshold at which channel $c$ opens (the excitation energies of the target nucleus). Henceforth the $J^\pi$ superscript is to be understood. Expansion of $V_{cc'}$ in terms of a finite number ($N$) of sturmians leads to a separable represen-
tation of the scattering matrix \[ \mathbf{S}_{cc'} = \delta_{cc'} - i^{(l_c' - l_c + 1)} \pi \mu \]
\[ \times \sum_{n,n'\leq 1} \sqrt{k_c \hat{\chi}_{cn}(k_c)} \left[ (\mathbf{\eta} - \mathbf{G}_0)_{nn'}^{-1} \right] \hat{\chi}_{cn'}(k_c' \sqrt{k_c'}), \quad (3) \]
where \( c \) and \( c' \) refer now only to open channels, \( l_c \) is the partial wave with channel \( c \) and the Green’s function matrix is
\[
[\mathbf{G}_0]_{nn'} = \mu \left[ \sum_{c=1}^{\text{open}} \int_0^\infty \sqrt{k_c \hat{\chi}_{cn}(k_c)} k_c^2 - x^2 \hat{\chi}_{cn'}(x) dx \right.
- \sum_{c=1}^{\text{closed}} \int_0^\infty \sqrt{k_c \hat{\chi}_{cn}(k_c)} k_c^2 + x^2 \hat{\chi}_{cn'}(x) dx \right], \quad (4)
\]
\( \mathbf{\eta} \) is a column vector of sturmian eigenvalues and \( \hat{\chi} \) are form factors determined from the chosen sturmian functions. Details are given in Ref [5].

Traditionally, all target states are taken to have eigenvalues of zero width and the (complex) Green’s functions are evaluated using the method of principal parts. This assumes time evolution of target states is given by
\[
|x, t\rangle = e^{-iH_0t/\hbar} |x, t_0\rangle = e^{-iE_0t/\hbar} |x, t_0\rangle. \quad (5)
\]
However, if states decay, they evolve as [3]
\[
|x, t\rangle = e^{-\frac{1}{2}t} e^{-iE_0t/\hbar} |x, t_0\rangle. \quad (6)
\]
Thus, in the Green’s function, channel energies become complex, as do the squared channel wave numbers,
\[
k_c^2 = \mu \left( E - \epsilon_c + \frac{i\Gamma_c}{2} \right); \quad \hat{\hbar}_c^2 = \mu \left( \epsilon_c - E - \frac{i\Gamma_c}{2} \right), \quad (7)
\]
where \( \frac{i\Gamma_c}{2} \) is half the width of the target state associated with channel \( c \). Thus, the Green’s function matrix elements are
\[
[\mathbf{G}_0]_{nn'} = \mu \left[ \sum_{c=1}^{\text{open}} \int_0^\infty \sqrt{k_c \hat{\chi}_{cn}(k_c)} k_c^2 - x^2 \hat{\chi}_{cn'}(x) dx \right.
- \sum_{c=1}^{\text{closed}} \int_0^\infty \sqrt{k_c \hat{\chi}_{cn}(k_c)} k_c^2 + x^2 \hat{\chi}_{cn'}(x) dx \right], \quad (8)
\]
where \( k_c \) and \( \hat{\hbar}_c \) are as in Eq. (2). Thus, poles are moved significantly off the real axis, and integration of a complex integrand along the real momentum axis is feasible. This has been done; however, for any infinitesimal-width target state, or resonance so narrow that it can be treated as such, the method of principal parts has been retained.

As previously [3, 7], the \( ^{13}\text{C} \) \((n + ^{12}\text{C})\) system is studied using the MCAS approach with a rotational model prescription of the matrix of interaction potentials connecting three states of \( ^{12}\text{C} \) (the \( 0^+_{\text{g.s.}} \), \( 2^+ (4.43 \text{ MeV}) \) and \( 0^+_2 (7.64 \text{ MeV}) \)), using the same interaction Hamiltonian and allowing for Pauli blocking via the OPP scheme. In the first instance, all three states are considered zero-width, giving the elastic scattering cross section of neutrons to 6 MeV as previously published. Additionally, evaluations are made for the same interaction allowing the \( 2^+_1 \) and \( 0^+_2 \) states of \( ^{12}\text{C} \) to have particle emission widths of varying size; the ground state kept with zero width. Results are displayed in Fig. 1.

![FIG. 1: (Color online) Calculated cross sections for hypothetically n-(unstable) \( ^{12}\text{C} \) scattering as functions of neutron energy. Labels as per Table I](image)

Cross sections follow a marked trend as hypothetical state widths increase. Widths used are listed in Table I. Ascribing the first widths to the excited states (set (b)),

| Curve | \( 0^+_1 \) width | \( 2^+_1 \) width | \( 0^+_2 \) width |
|-------|-----------------|-----------------|-----------------|
| a     | —               | —               | —               |
| b     | 0.00            | 0.20            | 0.60            |
| c     | 0.00            | 0.40            | 1.20            |
| d     | 0.00            | 0.60            | 1.80            |
| e     | 0.00            | 0.80            | 2.40            |

very narrow resonances in the original cross section disappear. From the earlier studies [3, 7], it is noted that those (narrow) compound resonance states are dominated by the coupling of an \( sd \)-shell nucleon to the \( 2^+_1 \) state in \( ^{12}\text{C} \). The broader resonances remain evident in the cross section as the state widths are artificially increased. However, with these increases the remaining resonances smear out. In the case of the broadest target states (set (e)) the
cross section has very little remnant of the compound system resonances. Clearly only the cross section from evaluation with three zero-width target states replicates measurement.

Table II displays the widths of states in the compound nucleus, $^{13}$C, found using MCAS when attributing the diverse widths to the excited states of $^{12}$C listed in Table I. The first column after $J^\pi$ lists the bound state and resonance centroid energies obtained from the calculation made with the physically reasonable, zero-width excitation energies of the $2^+_1$ and $0^+_2$ states of $^{12}$C. Allowing those states to be resonances with the widths selected alters the state energy centroids by at most a few tens of keV, so these are not listed. Thus, it is observed that allowing these target states to be resonances mostly affects widths of the resulting compound nucleus resonances. Those variations are consistent with changes noted in the cross section, with sharp resonances found for the zero-width state case rapidly disappearing and the others broadening to an extent that only a few are left distinguishable from a background. It is important to note, though, that all states in the compound system defined by the coupled-channel evaluations remain present, with, in this case, centroid energies little affected but widths increased.

The low excitation $^8$Be spectrum has a $0^+$ ground state that has a small width for its decay into two $\alpha$-particles ($6 \times 10^{-6}$ MeV), a broad $2^+$ resonance state with centroid at 3.03 MeV and width of 1.5 MeV, followed by a broader $4^+$ resonance state with centroid at 11.35 MeV and width $\sim 3.5$ MeV $^{11}$. Two evaluations of the $n+^8$Be cross section are obtained with MCAS; in both calculations, the same nuclear interaction is considered. It is taken from a rotor model with parameter values chosen in the finite-width states calculation to reproduce some aspects of the experimentally determined structure of $^9$Be $^{10}$, shown graphically in Fig. 2.

![FIG. 2: Experimental $^9$Be spectrum and that calculated from neutron scattering with stable and unstable $^8$Be.](image)

![FIG. 3: (Color online) Calculated cross sections for neutron scattering from (f) stable and (g) unstable $^8$Be as functions of neutron energy.](image)
The results for the scattering cross sections are shown in Fig. 3. Upon introducing target state widths, as found in the \( n+{^8}_{12}\text{C} \) investigation, the resonances are suppressed but still present; their widths increasing and magnitudes decreasing so as all but the \( \frac{5}{2}^+ \) cannot be discerned from the background. Compound system resonances of both calculations are at essentially the same energies. These effects are further illustrated by the centroid energies and widths of the resonances listed in Table III. Column 1

| \( J^\pi \) | \( E(f) \) | \( \Gamma(f) \) | \( E(g) \) | \( \Gamma(g) \) | \( \Gamma(\text{exp.}) \) |
| --- | --- | --- | --- | --- | --- |
| \( 1^- \) | -1.67 | — | -1.66 | — | — |
| \( 1^+ \) | -0.31 | — | -0.31 | — | 0.217\pm0.001 |
| \( 2^- \) | 0.118 | 1.39 | 0.244 | 0.282\pm0.011 |
| \( 2^+ \) | 3.7\times10^{-9} | 1.70 | 0.694 | 7.8\times10^{-4} |
| \( 3^+ \) | 0.222 | 1.74 | 0.682 | 1.330\pm0.360 |
| \( 4^- \) | 0.003 | 2.96 | 1.144 | N/A |
| \( 4^\pm \) | 0.009 | 3.16 | 1.856 | 0.743\pm0.055 |
| \( 5^\pm \) | 0.009 | 3.32 | 0.786 | 1.210\pm0.230 |
| \( 6^- \) | 0.072 | 4.23 | 0.873 | N/A |
| \( 6^\pm \) | 0.189 | 5.08 | 1.261 | 1.330\pm0.090 |

Table III: \(^9\text{Be} \) centroid energies and widths (\( E \& \Gamma \) in MeV) from calculation with \(^8\text{Be} \) states taken as zero-width and then with known resonance widths, and experimental widths.

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Acknowledgements

This research was supported in part by Melbourne University Scholarship Office PORES program. P.F. acknowledges the gracious hospitality of the Department of Physics and Astronomy, University of Manitoba; the INFN, Sezione di Padova, Padova and the Dipartimento di Fisica dell’ Universita di Padova; and the Department of Physics and Electronics, Rhodes University. S.K. acknowledges support from the National Research Foundation (South Africa). J.P.S. acknowledges support from the Natural Sciences and Engineering Research Council (NSERC), Canada.

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