Extracting $p\Lambda$ scattering lengths
from heavy ion collisions

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Abstract
The $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ and $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$ correlation functions for 10% most central Au+Au collisions at top RHIC energy $\sqrt{s_{NN}} = 200$ GeV are modeled with Lednicky and Lyuboshitz analytical formula using the source radii extracted from the hydrokinetic model (HKM) simulations. For the baryon-antibaryon case the corresponding spin-averaged strong interaction scattering length is obtained by fitting the STAR correlation function. In contrast to the experimental results, where extracted $p\Lambda$ source radius value was found $\sim 2$ times smaller than the corresponding $p\Lambda$ one, the calculations in HKM show both $p\Lambda$ and $p\bar{\Lambda}$ effective source radii to be quite close, as expected from theoretical considerations. To obtain the satisfactory fit to the measured baryon-antibaryon correlation function at this large source radius value, the modified analytical approximation to the correlation function, effectively accounting for the residual correlations, is utilized.

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I. INTRODUCTION

The heavy ion collision experiments provide a good possibility for a study of the baryon-
non-baryon strong interactions using the Final State Interaction (FSI) correlation technique \[1\textsuperscript{3}\]. The latter is based on the analysis of the momentum correlations caused by the final state
interaction between corresponding baryons produced in the collision. This activity is espe-
cially interesting in view of the ongoing nuclear collision experiments at the LHC, which
produce great amounts of various particles, including exotic multi-strange, charmed and
beauty ones. It allows one to study the fundamental interactions between specific hadron
species, which can hardly be achieved by other means. The extraction of this information
makes it possible to check the correctness of hadron-hadron strong interaction models, con-
strain corresponding interaction potentials, and also improve existing cascade models (like
UrQMD) by including into them the information about still unknown baryon-antibaryon
annihilation cross-sections.

In paper \[4\] the experimental $p\Lambda$ and $p\bar{\Lambda}$ correlation functions were fitted with Lednický
and Lyuboshitz analytical model \[1\] allowing, in principle, to extract scattering lengths char-
acterizing the two-particle strong interaction. However, apart from the interaction charac-
teristics, the correlation function depends also on the source spatial structure, described in
terms of emission source function, being the time-integrated relative distance distribution in
the pair rest frame. This fact complicates a study of the particle interaction, as it increases
the number of free parameters which enter the fit formula.

To simplify this study, one could calculate the corresponding source functions in real-
istic models of the collision process, which are known to describe well the experimental
observables. The hydrokinetic model \[5\textsuperscript{7}\] provides successful simultaneous description of a
wide class of bulk observables in the heavy ion collision experiments at RHIC and LHC \[8\].
Moreover, it reproduces well \[9\] the pion and kaon source functions for semi-central Au+Au
collisions at the top RHIC energy \[10\], including the specific non-Gaussian tails observed in
the pair momentum and beam direction projections of the experimental source function. In
this article we present the results of fitting the experimental data from \[4\] within the analyti-
cal model [1] where the Gaussian parametrization for the emission source function is utilized, and the corresponding Gaussian radii are extracted from the HKM model simulations.

II. MODELS DESCRIPTION

The STAR collaboration studied [4] baryon-baryon $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ and baryon-antibaryon $p - \bar{\Lambda} \oplus \bar{p} - \Lambda$ correlation functions in 10% most central RHIC Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Protons & antiprotons in transverse momentum range $0.4 < p_T < 1.1$ GeV/c with the rapidity $|y| < 0.5$, and lambdas & antilambdas with $0.3 < p_T < 2.0$ GeV/c and $|y| < 1.5$ were selected for the analysis.

The experimental correlation function is constructed as the ratio of the distribution of particle momentum in the pair rest frame, $k^*$, in the same events to the analogous distribution in mixed events. Then the measured correlation function $C_{meas}$ is corrected for the pair purity, defined as the fraction of correctly identified primary particle pairs among all the selected ones, to give the corrected function $C_{corr}$

$$C_{corr}(k^*) = \frac{C_{meas}(k^*) - 1}{\lambda(k^*)} + 1,$$

where $\lambda(k^*)$ is the pair purity. The estimated mean pair purity in the experiment is $\lambda = 17.5 \pm 2.5\%$.

To fit the experimental correlation function the Lednický and Lyuboshitz analytical model [1] is used, which connects the two-particle correlation function $C(k^*)$ with the particle emission source size $r_0$ and the s-wave strong interaction scattering amplitudes $f^S(k^*)$ at a given total pair spin $S$. In the equal-time approximation, valid on condition $|t^*_1 - t^*_2| \ll m_{2,1}r^{*2}$ for sign$(t^*_1 - t^*_2) = \pm 1$ respectively, the correlation function can be calculated as a square of the wave function $\Psi_{-k^*}^S$, representing the stationary solution of the scattering problem with the opposite sign of the vector $k^*$, averaged over the total spin $S$ and the distribution of the relative distances $S(r^*)$:

$$C(k^*) = \left\langle |\Psi_{-k^*}^S(r^*)|^2 \right\rangle.$$

In typical nuclear collisions the source radius can be considered much larger than the range of the strong interaction potential, so $\Psi_{-k^*}^S$ at small $k^*$ can be approximated by the s-wave solution in the outer region:

$$\Psi_{-k^*}^S(r^*) = e^{-ik^* \cdot r^*} + \frac{f^S(k^*)}{p^*} e^{ik^* \cdot r^*}.$$
The effective range approximation for the s-wave scattering amplitude is utilized

\[ f^S(k^*) = \left( \frac{1}{f_0^S} + \frac{1}{2} d_0^S k^*^2 - i k^* \right)^{-1}, \tag{4} \]

where \( f_0^S \) is the scattering length and \( d_0^S \) is the effective radius for a given total spin \( S = 1 \) or \( S = 0 \).

The particles are assumed to be emitted unpolarized (i.e. with the polarization \( P = 0 \)), so that the fraction of pairs in the singlet state \( \rho_0 = 1/4(1 - P^2) = 1/4 \), and in the triplet state \( \rho_1 = 1/4(3 + P^2) = 3/4 \). The pair separation distribution (source function) \( S(r^*) = d^3N/d^3r^* \) is assumed to be Gaussian one

\[ d^3N/d^3r^* \propto e^{-\frac{r^*^2}{4r_0^2}}, \tag{5} \]

where \( r_0 \) is considered as the effective radius of the source.

Under such assumptions the correlation function can be calculated analytically \([1]\): \n
\[ C(k^*) = 1 + \sum_S \rho_S \left[ \frac{1}{2} \left| \frac{f^S(k^*)}{r_0} \right|^2 \left( 1 - \frac{d_0^S}{2 \sqrt{\pi} r_0} \right) + \frac{2 \Re f^S(k^*)}{\sqrt{\pi} r_0} F_1(2k^* r_0) - \frac{3}{r_0} F_2(2k^* r_0) \right], \tag{6} \]

where \( F_1(z) = \int_0^z dx e^{x^2 - z^2}/z \) and \( F_2(z) = (1 - e^{-z^2})/z \). The term \(-\frac{d_0^S}{2 \sqrt{\pi} r_0}\) in this expression corresponds to the correction accounting for a deviation of \( \Psi^S_{-k^*} \) from the true wave function inside the range of the strong interaction potential. So, the model has quite a large number of parameters, being the scattering lengths \( f_0^S \), which may be complex in general case, the effective radii \( d_0^S \) and the source radius \( r_0 \). Although in principle all of them can be determined from the measured data, in each concrete situation the number of free parameters can be reduced by making certain reasonable assumptions about the values of some of them.

In our study the source radius \( r_0 \) is extracted from the Gaussian fit to the source functions calculated in hybrid HKM model. The simulation of the full process of evolution of the system formed in nuclear or particle collision in hHKM consists of two stages. The first one is hydrodynamical expansion of thermally and chemically equilibrated matter described within ideal hydrodynamics approximation with the lattice-QCD inspired equation of state \([11]\) (corrected for small but nonzero chemical potentials), which is matched with the hadron-resonance gas in chemical equilibrium via cross-over type transition. The second stage consists in gradual system decoupling after loosing chemical and thermal equilibrium. It
can be described either within hydrokinetic approach with switching to UrQMD cascade at some space-like hypersurface situated behind the hadronization one, or with sudden switch to UrQMD cascade at the hadronization hypersurface. In current study we choose the second variant of switching to cascade, basing on [8], where the comparison of one- and two-particle spectra, calculated at both types of matching hydro and cascade stages for RHIC and LHC energies, showed a fairly small difference between them.

The model provides particle distribution functions $d^α N/dxd^4p$ at the chosen switching hypersurface. Using the Monte-Carlo procedure, one generates particle momenta and coordinates according to these distributions, which serve as the input for the UrQMD hadronic cascade.

To perform a specific calculation one should specify the initial conditions for the hydrodynamics stage attributed to the starting proper time $τ_0$. These conditions are the initial energy density (or entropy) profile $ε(χ)$ and the initial rapidity profile (initial flow) $y(χ)$. Here we suppose longitudinal boost-invariance and use $ε(χ_T)$ corresponding to the MC-Glauber model calculated with GLISSANDO code [12]. The maximal energy density $ε_0$ is chosen to reproduce the experimental mean charged particle multiplicity, and the initial flow is supposed to be $y_T = α r_T/ε(χ_T)$, with $α = 0.45$ fm for top RHIC energy. Thus, model has only two free parameters $ε_0$ and $α$. We start hydrodynamics at $τ_0 = 0.1$ fm/c and work in mid-rapidity region. Sudden switch from hydrodynamics to UrQMD is performed at the isotherm $T = 165$ MeV. The hadron distribution functions (for each hadron sort $i$) at the switching hypersurface $σ_{sw}$ are calculated according to the Cooper-Frye formula

$$p_0 \frac{d^3N_i}{p_T dp_T dφ_μ dy} = \int_{σ_{sw}} p_μ dσ_μ f^{eq}_i(p·u(x), T(x), μ_i(x)).$$  \hspace{1cm} (7)

The source functions $S(χ^*)$ are calculated as

$$S(χ^*) = \frac{\sum_{i\neq j} δ_Δ(χ^* - χ_i^* + χ_j^*)}{\sum_{i\neq j} 1}. \hspace{1cm} (8)$$

Here $χ_i^*$ and $χ_j^*$ are the particles space positions, and $χ^*$ is the particle separation in the pair rest frame, $δ_Δ(x) = 1$ if $|x| < Δr/2$ and 0 otherwise, $Δr$ is the size of the histogram bin.

### III. RESULTS AND DISCUSSION

The $pA$ source function projections calculated in HKM together with the corresponding Gaussian fits are presented in Fig. 1. Here the out-side-long coordinate system is used, where
the out axis is directed along the pair total momentum in longitudinally co-moving system, the long direction coincides with the beam axis, and the side axis is perpendicular to the latter two ones. One can notice that non-Gaussian tails, observed in pion source function \[10\], are also present in pΛ case. They are partially related with the averaging over a wide \( p_T \) interval.

In Figs. 2-3 we present experimental \( p - \Lambda \oplus \bar{p} - \bar{\Lambda} \) and \( \bar{p} - \Lambda \oplus p - \bar{\Lambda} \) correlation functions, measured by STAR collaboration in 10% most central Au+Au collisions at top RHIC energy \( \sqrt{s_{NN}} = 200 \text{ GeV} \) together with the fits performed within Lednický and Lyuboshitz analytical model.

For baryon-baryon case (Fig. 2) the scattering lengths \( f^S_0 \) and effective radii \( d^S_0 \) values are taken from \[13\] (\( f^S_0 = 2.88 \text{ fm}, f^t_0 = 1.66 \text{ fm}, d^S_0 = 2.92 \text{ fm}, d^t_0 = 3.78 \text{ fm} \)), leaving only one parameter \( r_0 \) free in the STAR fit \[4\] (red curve) and making all the parameters fixed in our own fit (blue curve), where \( r_0 \) value is determined from a Gaussian fit to calculated in HKM separation \( r^* \) distribution in the pair rest frame for \( r^* < 70 \text{ fm} \). One can see that our fitting curve describes the data well. The experimental and HKM source radius values are \( r^\text{exp}_0 = 3.09 \pm 0.30^{+0.17}_{-0.25} \pm 0.2 \text{ fm} \) and \( r^\text{HKM}_0 = 3.637 \pm 0.001 \text{ fm} \) respectively.

In the baryon-antibaryon case (Fig. 3) to reduce the number of free parameters both singlet and triplet scattering amplitudes are assumed to be equal, \( f^s = f^t = f \) (approximately corresponding to spin-averaged scattering length \( f_0 \)), and both effective radii are set to zero \( d^s_0 = d^t_0 = 0 \). The scattering length should have a positive imaginary part \( \Im f_0 > 0 \) describing the contribution of annihilation channels and leading to a wide dip in the correlation function at intermediate \( k^* \)-values. Thus, the model has three free parameters \( \Re f_0, \Im f_0 \) and \( r_0 \) in \[4\] and two free parameters \( \Re f_0, \Im f_0 \) in our fit. The STAR has obtained pΛ source radius value \( r_0 = 1.50 \pm 0.05^{+0.10}_{-0.12} \pm 0.3 \text{ fm} \) (red curve), which is \( \sim 2 \) times smaller than the pΛ one, although there is no apparent physical reason for such a difference. Both radii can be expected to have similar values, and the HKM source radius for the baryon-antibaryon case \( r^\text{HKM}_0 = 3.621 \pm 0.001 \text{ fm} \) is expectedly close to the corresponding baryon-baryon one. But at this source radius value the fitting curve (blue) is too narrow to describe the data points. Still, we think that the real baryon-antibaryon source size is close to the baryon-baryon one, and the apparent difference between them is caused by some additional effect.

The possible reason for the small fitted baryon-antibaryon source radius could be the influence of residual correlations and imperfection of purity correction \[4, 14\]. Constructing
the experimental correlation function one usually supposes that only the pairs composed of two primary particles are correlated, and the rest of the pairs, which include secondary or misidentified particles, are supposed to be uncorrelated. However, the correlation can exist between two parents of secondary particles or between the parent of secondary particle and the primary one. In case, when the secondary particle carries most of the momentum of its parent, such a particle can be “residually” correlated with another particle (or its daughter), which was correlated with its parent. The interactions in most of such pairs are unknown, so at the moment there is no possibility to reliably refine the constructed experimental correlation function from the effect of residual correlations. However, one can try to account for the residual correlations at least phenomenologically in analytical model used for fitting of the correlation function.

In case when the measured correlation function is not corrected for purity, the fitted uncorrected correlation function is expressed through the true one in (6) similar to (1):

\[ C_{\text{uncorr}}(k^*) = \lambda(k^*)C(k^*) + (1 - \lambda(k^*)), \]

(9)

The pair purity \(\lambda(k^*)\) in our calculations is taken in the form \(\lambda(k^*) = a\lambda_{\text{exp}}(k^*),\) where \(\lambda_{\text{exp}}(k^*)\) is extracted from the plots provided in [13] as the ratio \(\lambda_{\text{exp}}(k^*) = (C_{\text{uncorr}}(k^*) - 1)/(C(k^*) - 1).\) To account for possible imperfection of purity determination in the experiment, we also introduce here a normalizing constant \(a\), which is in fact a fitting parameter in our analysis.

The first term in formula (9) corresponds to the pairs of correlated (primary only) particles, and the second one represents the contribution of the uncorrelated pairs, where one or both particles are secondary (or misidentified) ones. Assuming that among the latter there can be residually correlated pairs and taking into account that the effect of baryon-antibaryon residual correlations is dominated by the effect of annihilation dip in parent correlations and that this dip is essentially widen in residual correlations, it can be effectively described by a factor \((1 - \beta e^{-4k^* R^2})\) multiplying the second term in (9) for baryon-antibaryonic system:

\[ C(k^*) = \lambda(k^*)C(k^*) + (1 - \lambda(k^*))(1 - \beta e^{-4k^* R^2}), \]

(10)

thus introducing additional two parameters \(\beta > 0\) and \(R \ll r_0.\)

In Fig. 4 one can see the result of fitting the experimental \(\bar{p} - \Lambda \oplus p - \bar{\Lambda}\) correlation function not corrected for purity (the data are taken from [14]) using the analytical expressions
FIG. 1. The pA source function projections calculated in HKM (blue markers) and the Gaussian fits to them (red lines). The simulations correspond to Au+Au collision STAR experiment at RHIC top energy [4].

The source radius was fixed at the value obtained from HKM simulations, \( r_0^{HKM} = 3.621 \pm 0.001 \) fm, and five parameters, \( a, \Re f_0, \Im f_0, \beta, \) and \( R \) were left to vary freely. The extracted parameter values are the following: \( a = 1.28 \pm 0.84, \Re f_0 = -0.05 \pm 0.68 \) fm, \( \Im f_0 = 1.41 \pm 1.07 \) fm, \( \beta = 0.029 \pm 0.005 \), and \( R = 0.45 \pm 0.06 \) fm. The obtained fitting curve describes the data quite well. The scattering length with \( \Re f_0 = 0.45 \pm 1.23 \) fm, \( \Im f_0 = 2.02 \pm 3.91 \) fm, extracted from our fit neglecting purity \( k^* \)-dependence (with \( \lambda = const \) as a free parameter) is consistent with the result, obtained in recent paper [14], where the purity \( k^* \)-dependence is also neglected. The radius \( r_0 \) in [14] is treated as a free parameter, and the account for residual correlations is performed by summarizing the contributions from different parent pairs to the full correlation function, making, however, a series of simplifying assumptions about the parameters \( r_{0i}, f_{0i} \) and \( d_{0i} \), which describe each residual correlation.
FIG. 2. The $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ correlation function measured by STAR (black markers), the corresponding fit from the STAR paper [4] within the Lednický and Lyuboshitz analytical model (red line) and our fit within the same model with the source radius $r_0$ extracted from the HKM calculations (blue line).

FIG. 3. The same as in Fig. 2 for the $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$ correlation function.

IV. CONCLUSIONS

Study of baryon and antibaryon correlations provides a powerful tool for measuring space-time evolution of heavy ion collisions and for extracting the parameters of strong interaction between emitted particles.

We reproduced the $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ and $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$ correlation functions, measured in 10% most central Au+Au collisions by STAR at $\sqrt{s_{NN}} = 200$ GeV, using Lednický and Lyuboshitz analytical formalism with the source radii extracted from the hydrokinetic model (HKM). To take into account the residual correlations influencing baryon-antibaryon femtoscopic effects, a modified analytical approximation has been applied. The values of the $p\Lambda$ and $p\bar{\Lambda}$ source radii calculated in HKM are similar, in agreement with theoretical expectations, and consistent with experimental result for $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$. The significantly smaller system size measured by STAR for $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$ pairs can be attributed to the residual correlations. The real and imaginary parts of the spin averaged scattering lengths have been extracted for baryon-antibaryon pairs. The hydrokinetic model including a detailed description of particle correlations allows for a precise study of heavy ion collisions. New
The purity uncorrected $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$ correlation function measured by STAR [15] (black markers) and our fit according to (10) and (6) (blue line), with the account for the residual correlations. The source radius $r_0$ was fixed at a value extracted from the HKM calculations.

High statistics data from RHIC and LHC will provide measurements of various particle pairs, including baryon-antibaryon ones, allowing to investigate the particle interactions in these pairs. A consistent approach for a wide class of observables will help to understand complex and unknown features of the evolution of heavy ion collisions.

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