Nearest-Neighbour Interaction from an Abelian Symmetry and Deviations from Hermiticity

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Abstract

We show that Nearest-Neighbour Interaction (NNI) textures for the quark mass matrices can be obtained through the introduction of an Abelian flavour symmetry. The minimal realisation requires a $\mathbb{Z}_4$ symmetry in the context of a two Higgs doublet model. It is further shown that the NNI textures can be in agreement with all present experimental data on quark masses and mixings, provided one allows for deviations of Hermiticity in the quark mass matrices at the 20% level.

Keywords: Quark masses and mixings, Flavour symmetries, Extension of Higgs sector

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1. Introduction

In the Standard Model (SM) the flavour structure of the Yukawa couplings is not constrained by gauge symmetry, which leads to an arbitrary flavour dependence for fermions mass matrices, after spontaneous symmetry breaking. Finding a framework which could explain the observed pattern of fermions masses and mixings, is one of the fundamental open questions in particle physics. In the last years, there has been great progress in the determination [1] of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2]. At present, essentially all data is in agreement [3] with the SM and there is clear evidence for a complex CKM matrix, even if one allows for the presence of New Physics [4]. One may be tempted to use this input from experiment, together with the knowledge of quark masses, to extract the flavour structure of the quark mass matrices. The hope is to obtain, through this bottom-up approach, a hint of a possible family symmetry constraining the flavour structure of the SM. This bottom-up approach to the flavour puzzle is rendered specially difficult, due to the freedom one has to make weak-basis (WB) transformations which change the flavour structure of the quark mass matrices $M_u, M_d$, while maintaining the gauge currents flavour diagonal. Needless to say, entirely analogous considerations apply to the leptonic sector [5]. Even if there is a symmetry principle controlling the flavour structure of the Yukawa couplings, in what WB will this family symmetry be transparent? One of the WB which has been proposed [6] in the
literature is the so-called Nearest-Neighbour-Interaction (NNI) basis, where the elements (1, 1), (1, 3), (2, 2) and (3, 1) vanish both in $M_u$ and $M_d$. It has been shown [6] that in the SM, starting from arbitrary quark mass matrices, one can always make WB transformations so that $M_u$, $M_d$ acquire the NNI form.

The NNI basis has been extensively studied in the literature [7] and is closely connected to the Fritzsch ansatz [8] which assumes the NNI structure, together with Hermiticity for both $M_u$ and $M_d$. Taken separately, these two assumptions do not have any physical consequences since they are just a choice of WB. But taken together, they do have physical implications and in fact the Fritzsch ansatz has been ruled out by the large value of the top quark mass and the experimental value of $V_{cb}$. In the present paper, we address the following two questions:

i) In a multi-Higgs extension of the SM, what is the minimal scenario to obtain the NNI structure for $M_u$ and $M_d$, as a result of an Abelian family symmetry?

ii) Assuming that $M_u$ and $M_d$ are in the NNI basis, what are the minimal deviations from Hermiticity in $M_u$, $M_d$ which are required in order to accommodate the presently available data on quark masses and the CKM matrix, including the experimental values of $B_d - \bar{B}_d$, $B_s - \bar{B}_s$ mixings and the measurement of the rephasing invariant phases $\beta$ and $\gamma$ of the unitarity triangle?

This paper is organised as follows. In section 2, we show that $Z_4$ is the minimal family symmetry which leads to the NNI structure. The implementation of this symmetry requires the introduction of a minimum of two Higgs doublets. In the section 3, we confront the non-zero entries of the NNI structure with experimental data. In particular, we show that, in the NNI framework, it is possible to obtain a quark mass spectrum and a CKM matrix consistent with all experimental data, if one allows for relatively small deviations from Hermiticity, at the 20% level. Finally, our conclusions are contained in the section 4.

2. Discrete Flavour Symmetries

In this section, we show that the NNI structure for the quark mass matrices,

\[
M_u = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix},
\]

(1)
can be achieved through the introduction of an Abelian family symmetry in the Lagrangian. The implementations of this symmetry requires the introduction of at least two Higgs doublets and the minimal symmetry is $Z_4$. Let $\phi_1$, $\phi_2$ denote the charges of two Higgs doublets $\Phi_1$, $\Phi_2$, under a $U(1)$ symmetry imposed on the Lagrangian:

\[
\phi_1 \equiv Q(\Phi_1), \quad \phi_2 \equiv Q(\Phi_2).
\]

(2)
In order to achieve the NNI structure under this symmetry, the quark fields must transform through the following charge assignments:

\[
\begin{align*}
(q_1, q_2) &= (q_3 + \phi_1 - \phi_2, q_3 - \phi_1 + \phi_2), \\
(u_1, u_2, u_3) &= (q_3 - \phi_1 + 2\phi_2, q_3 + \phi_1, q_3 + \phi_2), \\
(d_1, d_2, d_3) &= (q_3 - 2\phi_1 + \phi_2, q_3 - \phi_2, q_3 - \phi_1),
\end{align*}
\]

(3)

where \( q_i \equiv Q(Q_{L_i}) \), \( u_i \equiv Q(u_{R_i}) \) and \( d_i \equiv Q(d_{R_i}) \), with \( Q_{L_i} \) denoting the left-handed quark doublets.

The charge assignments given in Eq. (3) are not affected by an overall change, which can be absorbed in the definition of \( q_3 \). In order to preserve the zero entries of the NNI form, one must forbid some quark bilinears to couple to the Higgs doublets. The charges under \( U(1) \) of the quark bilinears \( Q_{L_i} u_{R_j}, Q_{L_i} d_{R_j} \) are given for the up sector by:

\[
\begin{pmatrix}
-2\phi_1 + 3\phi_2 & \phi_2 & -\phi_1 + 2\phi_2 \\
\phi_2 & 2\phi_1 - \phi_2 & \phi_1 \\
-\phi_1 + 2\phi_2 & \phi_1 & \phi_2
\end{pmatrix},
\]

(4)

and for the down sector by

\[
\begin{pmatrix}
-3\phi_1 + 2\phi_2 & -\phi_1 & -2\phi_1 + \phi_2 \\
-\phi_1 & \phi_1 - 2\phi_2 & -\phi_2 \\
-2\phi_1 + \phi_2 & -\phi_2 & -\phi_1
\end{pmatrix}.
\]

(5)

It is clear from Eqs. (4) and (5) that the non-vanishing entries in the NNI structure will be generated through the Yukawa couplings of the fermion bilinears with \( \Phi_j \equiv i\sigma_2\Phi_j^* \) and \( \Phi_j \), for the up and down quark sectors, respectively. One has to further guarantee that no couplings arise for the zero textures of the NNI structure. It can be readily verified that the minimal discrete symmetry leading to the NNI structure is a \( Z_4 \) symmetry, under which:

\[
\Phi_j \longrightarrow \Phi_j' = e^{i \frac{2\pi}{4}\phi_j} \Phi_j,
\]

(6)

with the following charges:

\[
(\phi_1, \phi_2) = (1, 2),
\]

(7)

leading to the following charge assignments for the quark fields by means of Eq. (3):

\[
\begin{align*}
(q_1, q_2, q_3) &= (2, 0, 3), \\
(u_1, u_2, u_3) &= (2, 0, 1), \\
(d_1, d_2, d_3) &= (3, 1, 2).
\end{align*}
\]

(8)

The choice for the charges \( \phi_1, \phi_2 \) in Eq. (7), among others possible charge assignments, is motivated by the possibility of embedding this \( Z_4 \) symmetry in the framework of a \( SU(5) \) Grand Unification [9]. It is worthwhile mentioning that in grand unified theories with extended fermionic content, the required minimal Abelian group may be larger than \( Z_4 \) [10]. It has also been considered in the literature an example of the NNI-type fermion masses in the context of supersymmetric flavour symmetry based on the non-Abelian group \( Q_6 \) [11].
The most general Yukawa couplings allowed by the $Z_4$ symmetry are then given by:

$$-L_Y = \Gamma_1^u \overline{Q}_L \Phi_1 u_R + \Gamma_2^u \overline{Q}_L \Phi_2 u_R + \Gamma_1^d \overline{Q}_L \Phi_1 d_R + \Gamma_2^d \overline{Q}_L \Phi_2 d_R + \text{H.c.},$$

(9)

where the Yukawa matrices $\Gamma_{u,d}^{1,2}$ have the following flavour structure:

$$\Gamma_u^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_u \\ 0 & b'_u & 0 \end{pmatrix}, \quad \Gamma_u^2 = \begin{pmatrix} 0 & a_u & 0 \\ a'_u & 0 & 0 \\ 0 & 0 & c_u \end{pmatrix},$$

(10a)

$$\Gamma_d^1 = \begin{pmatrix} 0 & a_d & 0 \\ 0 & 0 & c_d \end{pmatrix}, \quad \Gamma_d^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_d \\ 0 & b'_d & 0 \end{pmatrix}.$$  (10b)

After spontaneous gauge symmetry breaking, the NNI mass matrices are generated through the vacuum expectation values of the Higgs doublets $v_1 \equiv \langle \Phi_1 \rangle$ and $v_2 \equiv \langle \Phi_2 \rangle$ leading to:

$$M_u = \begin{pmatrix} 0 & v_2 a_u & 0 \\ v_2 a'_u & 0 & v_1 b_u \\ 0 & v_1 b'_u & v_2 c_u \end{pmatrix},$$

(11a)

$$M_d = \begin{pmatrix} 0 & v_1 a_d & 0 \\ v_1 a'_d & 0 & v_2 b_d \\ 0 & v_2 b'_d & v_1 c_d \end{pmatrix}.$$  (11b)

From Eqs. (4) and (5) it can be seen that some higher dimension operators allowed by the SM gauge group and the $Z_4$ symmetry can contribute to the vanishing elements on the quark mass matrices in Eqs. (11). One example is the following sixth dimensional operator,

$$\lambda \overline{Q}_{2L} \Phi_1 u_{2R} \Phi_1^\dagger \Phi_2,$$

(12)

which contributes to the (2,2) entry of $M_u$. Moreover, one can show by working out the matrices given in Eqs. (4) and (5) that neither $Z_2$ nor $Z_3$ can be invoked in order to have the NNI pattern as the result of a discrete symmetry imposed on the Lagrangian. This can been clearly seen by considering the (1,1)-element of the bilinear matrix given in Eq. (4) and noting that for the $Z_2$ symmetry one has

$$-2 \phi_1 + 3 \phi_2 = \phi_2 \pmod{2},$$

(13)

which allows the Higgs doublet $\Phi_2$ to couple to the bilinear $\overline{Q}_{L1} u_{R1}$. In the case of $Z_3$ symmetry one gets for the (1,1)-element from Eq. (4):

$$-2 \phi_1 + 3 \phi_2 = \phi_1 \pmod{3},$$

(14)

which then allows the doublet $\Phi_1$ to couple to the bilinear $\overline{Q}_{L1} u_{R1}$. This implies that $Z_2$ and $Z_3$ are excluded.
At this point, the following comment is in order. As we have emphasised, in the SM, the NNI structure for $M_u, M_d$, is just a choice of weak basis. On the other hand, we have shown that the NNI form for $M_u, M_d$, can arise as the result of a $Z_4$ symmetry, in the context of a two Higgs doublet extension of the SM. Note however, that in a general two Higgs doublet model, the form of the Yukawa couplings $\Gamma^{1,2}_{u,d}$, given in Eqs. (10), is not just a choice of WB, they do imply restrictions on the scalar couplings to quarks.

The requirement of renormalisability implies that $Z_4$ has to be imposed on the full Lagrangian, in particular on the Higgs potential. The most general renormalisable scalar potential consistent with $Z_4$ and gauge symmetry can be written:

$$V = \mu_1 |\Phi_1|^2 + \mu_2 |\Phi_2|^2 + \lambda_1 |\Phi_1|^4 + \lambda_2 |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 .$$

(15)

It is clear that the potential has acquired a new accidental global symmetry which, upon spontaneous symmetry breaking leads to a massless neutral scalar, at tree level. This can be avoided by soft-breaking of the $Z_4$ symmetry through the introduction of a term like

$$V' = \mu_{12} \Phi_1^\dagger \Phi_2 + \text{H.c.}$$

(16)

Alternatively, one may introduce a singlet Higgs field which transforms non-trivially under $Z_4$.

It can be readily verified that in this model with two Higgs doublets and a $Z_4$ symmetry, it is not possible to achieve spontaneous CP violation even if one allows for the $Z_4$ soft-breaking term of Eq. (16). This is essentially due to the absence of terms like $(\Phi_1^\dagger \Phi_2 \Phi_1^\dagger \Phi_2) + \text{H.c}$, which are forbidden by $Z_4$. Denoting the scalar vacuum by

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 e^{i\theta} \end{pmatrix},$$

(17)

one verifies easily that there are two minima of the potential, both CP conserving, corresponding to $\theta = 0$ or $\theta = \pi$, for $\mu_{12} < 0$ or $\mu_{12} > 0$, respectively. Therefore, in this model, CP violation arises from complex Yukawa couplings, leading to the Kobayashi-Maskawa mechanism, see KM in Ref. [2].

3. Minimal Deviation from Hermiticity

In this section, we investigate what are the minimal deviations of Hermiticity in $M_u, M_d$, written in the NNI form, in order to accommodate both our present knowledge of the CKM matrix from experiment and the value of quark masses. In the NNI basis, the quark mass matrices can be written as:

$$M_u = \begin{pmatrix} 0 & A_u & 0 \\ A'_u & 0 & B_u \\ 0 & B'_u & C_u \end{pmatrix}, \quad M_d = K \begin{pmatrix} 0 & A_d & 0 \\ A'_d & 0 & B_d \\ 0 & B'_d & C_d \end{pmatrix},$$

(18)

where $(A, A', B, B', C)_{u,d}$ are all real and the matrix $K$ can be parametrised as

$$K = \text{diag}(e^{i\xi_1}, e^{i\xi_2}, 1) ,$$

(19)
without loss of generality. In order to parametrize deviations from Hermiticity in the up and down sectors, we introduce the parameters:

\[
\epsilon_u^a \equiv \frac{A'_{u,d} - A_{u,d}}{A'_{u,d} + A_{u,d}}, \quad \epsilon_u^b \equiv \frac{B'_{u,d} - B_{u,d}}{B'_{u,d} + B_{u,d}}.
\] (20)

For a given set of quark mass matrices, a measure of how close to Hermiticity \(M_u, M_d\) are, is provided by the parameter \(\epsilon\), defined by

\[
\epsilon \equiv \frac{\sqrt{(\epsilon_u^a)^2 + (\epsilon_u^b)^2 + (\epsilon_d^a)^2 + (\epsilon_d^b)^2}}{2}.
\] (21)

Deviations from Hermiticity in the NNI framework have been previously considered [12], at the time where our experimental knowledge of the CKM matrix was very limited, in particular the rephasing invariant phases \(\beta, \gamma\) had not been measured.

In the evaluation of the quark masses and the CKM matrix it is useful to work with Hermitian matrices \(H_u, H_d\) defined by:

\[
H_{u,d} \equiv M_{u,d} M_{u,d}^\dagger,
\] (22)

which have the property of \((H_u)_{12} = (H_d)_{12} = 0\), a signature of the NNI basis. One can see from Eq. (18) that the matrix \(H_u\) is a real Hermitian matrix, while \(H_d\) can be written in terms of a real Hermitian matrix \(H_0^d\) and the phase matrix \(K\) as

\[
H_d = K H_0^d K^\dagger.
\] (23)

From Eqs. (19), (23), it follows that:

\[
\kappa_1 = \text{arg}(H_{d3}), \quad \kappa_2 = \text{arg}(H_{d23}).
\] (24)

Hence, the matrices \(H_u\) and \(H_0^d\) are diagonalized by two real orthogonal matrices, \(O_u\) and \(O_d\), in the following way:

\[
O_u^T H_u O_u = \text{diag}(m_u^2, m_c^2, m_t^2),
\] (25a)

\[
O_d^T H_0^d O_d = \text{diag}(m_u^2, m_s^2, m_b^2).
\] (25b)

Then the CKM matrix, \(V\), is simply given by

\[
V = O_u^T K O_d.
\] (26)

Observing the mass and mixing hierarchy of the quarks, the matrices \(O_u\) and \(O_d\) can be well
approximated by \([12, 13]\):

\[
(O)_{12} \approx -\sqrt{\frac{m_1}{m_2}} \left(1 - \epsilon_a - \frac{m_2}{m_3} \epsilon_b\right), \quad (27a)
\]

\[
(O)_{13} \approx \sqrt{\frac{m_1 m_2^2}{m_3}} (1 + \epsilon_b - \epsilon_a), \quad (27b)
\]

\[
(O)_{21} \approx \sqrt{\frac{m_1}{m_2}} \left(1 - \epsilon_a - \frac{m_1}{m_3} \epsilon_b\right), \quad (27c)
\]

\[
(O)_{23} \approx \sqrt{\frac{m_2}{m_3}} (1 - \epsilon_b), \quad (27d)
\]

\[
(O)_{31} \approx -\sqrt{\frac{m_1}{m_3}} (1 - \epsilon_a - \epsilon_b), \quad (27e)
\]

\[
(O)_{32} \approx -\sqrt{\frac{m_2}{m_3}} (1 - \epsilon_b + \frac{m_1}{m_2} \epsilon_a), \quad (27f)
\]

where \(\epsilon_a, \epsilon_d\) are assumed to be small and \(m_i\) denote the quark masses, for the up and down sectors. Notice that, we have dropped for convenience the up and down quark sector indices.

Our task is to find “small values” for the parameters \(|\epsilon_{u,d}^{a,b}|\) such that the resulting CKM matrix, \(V\), is in agreement with experiment while a correct value for the quark masses is obtained. The constraint of having a correct mass spectrum is easily achieved by using the invariants of \(H_u, H_d\), which can be readily expressed in terms of the parameters of \(M_u, M_d\):

\[
\text{Tr}(H) \equiv m_1^2 + m_2^2 + m_3^2 = 2\bar{A}^2(1 + \epsilon_a^2) + 2\bar{B}^2(1 + \epsilon_b^2) + C^2, \quad (28a)
\]

\[
\chi(H) \equiv m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_1^2 = \bar{A}^4(1 - \epsilon_a^2)^2 + \bar{B}^4(1 - \epsilon_b^2)^2 + 8\bar{A}^2\bar{B}^2\epsilon_a\epsilon_b
\]

\[
+ 2\bar{A}^2(1 + \epsilon_a^2)\left[\bar{C}^2 + \bar{B}^2(1 + \epsilon_b^2)\right], \quad (28b)
\]

\[
det(H) \equiv m_1^2 m_2^2 m_3^2 = \bar{A}^4 C^2(1 - \epsilon_a^2)^2, \quad (28c)
\]

where \(\bar{A}_{u,d}\) and \(\bar{B}_{u,d}\) are defined as

\[
\bar{A}_{u,d} = \frac{A_{u,d} + A_{u,d}'}{2}, \quad \bar{B}_{u,d} = \frac{B_{u,d} + B_{u,d}'}{2}. \quad (29)
\]

Once \(|\epsilon_{u,d}^{a,b}|\) and quark masses are given, the quantities \(\bar{A}_{u,d}, \bar{B}_{u,d}\) and \(C_{u,d}\) are just determined by means of Eqs. (28). Thus, the matrices \(M_u, M_d\) and consequently \(H_u, H_d\) are fully reconstructed with the knowledge of \(\kappa_1, \kappa_2\) in Eq. (19) and the CKM matrix can then be obtained from Eq. (26).
In our numerical search, we have performed a deep scan of small values of $|\epsilon_{u,a,b}|$ (allowing for positive and negative values), phases of $\kappa_1, \kappa_2$ and quark running masses within the allowed range at $M_Z$ scale taken from Refs. [14, 15]. Among the various solutions obtained, we have accepted only those corresponding to a correct CKM matrix. This include not only the allowed CKM moduli, but also the experimental limits on the strength of CP violation measured by $I \equiv |\text{Im}(V_{us}V_{cb}^*V_{ub}^*)|$ and on the angles $\beta \equiv \arg(-V_{cd}V_{cb}^*V_{td}^*)$ and $\gamma \equiv \arg(-V_{ud}V_{ub}^*V_{cd}^*)$ of the unitarity triangle.

We have checked that most of our numerical solutions require that $\epsilon_{u,a,b}$ be negative and $\epsilon_{d,a,b}$ be positive. Moreover, we have also verified that the NNI basis is experimentally compatible with having $\epsilon_a^u = 0$ or $\epsilon_b^u = 0$ (or even both), but this would imply a large deviation from Hermiticity in the down sector, $\epsilon_{a,b}^d \gtrsim 0.6$. We also emphasise that no numerical solution for $\epsilon_{a,b}^d = 0$ or $\epsilon_{b}^d = 0$ corresponding to $\epsilon_{a,b}^u \leq 0.3$ was found. Finally, the consistent set of $|\epsilon_{a,b}^{u,d}|$ obtained numerically are sketched in Figure 1, for $\epsilon \leq 0.2$.

![Figure 1](image-url)  
Figure 1: Plotting a set of solutions for $|\epsilon_{a,b}^{u,d}|$ corresponding to the constraint $\epsilon \leq 0.2$.

One numerical example of $|\epsilon_{a,b}^{u,d}|$ extracted from Figure 1 is given by,

$$
\begin{align*}
\epsilon_a^u &= -0.243, & \epsilon_b^u &= -0.144, \\
\epsilon_a^d &= 0.133, & \epsilon_b^d &= 0.236,
\end{align*}
$$

which corresponds to $\epsilon = 0.196$ by Eq. (21). Using as input the running quark masses taken at $M_Z$ scale within the allowed range:

$$
\begin{align*}
m_u &= 2.0 \text{ MeV}, & m_d &= 2.7 \text{ MeV}, \\
m_c &= 0.557 \text{ GeV}, & m_s &= 47 \text{ MeV}, \\
m_t &= 168.3 \text{ GeV}, & m_b &= 2.92 \text{ GeV},
\end{align*}
$$

(31)
together with the invariants in Eqs. \((28)\), one obtains for \(\bar{A}_{u,d}, \bar{B}_{u,d}, \bar{C}_{u,d}\):

\[
\bar{A}_u = 34.4 \text{ MeV}, \quad \bar{A}_d = 11.5 \text{ MeV}, \\
\bar{B}_u = 9.76 \text{ GeV}, \quad \bar{B}_d = 0.371 \text{ GeV}, \\
\bar{C}_u = 167.7 \text{ GeV}, \quad \bar{C}_d = 2.87 \text{ GeV}.
\]

The matrices \(H_u, H_d\) are then fully determined from Eqs. \((30), (32)\) by taking the input phases \(\kappa_1, \kappa_2, \kappa_1 = -121.7^\circ, \kappa_2 = -21.0^\circ\).

Thus, the CKM matrix, \(V\), is directly evaluated from Eq. \((26)\), obtaining:

\[
|V| = \begin{pmatrix}
0.9743 & 0.2253 & 0.0034 \\
0.2251 & 0.9734 & 0.0415 \\
0.0087 & 0.0407 & 0.9991
\end{pmatrix}.
\]

For the rephasing invariant angles and the strength of CP violation, one obtains:

\[
\alpha = 89.7^\circ, \\
\sin(2\beta) = 0.669, \\
\gamma = 69.3^\circ, \\
I = 2.92 \times 10^{-5}.
\]

The results of Eqs. \((31), (34), (35)\) for quark masses and their mixings are in agreement with experiment \(\text{[1]}\). They were obtained by exact numerical diagonalisation of the quark mass matrices, with no approximations involved. Yet, it is instructive to understand the reason why one can reproduce in the NNI framework a correct CKM matrix, \(V\), with relatively small deviations of Hermiticity. Note that if one assumes exact Hermiticity in the NNI basis, one is led to the Fritzsch Ansatz (FA) which has been ruled out by experiment. As previously mentioned, the main reason why FA has been excluded, has to do with the experimental value of \(|V_{cb}|\) and the fact that the top quark is very heavy. In the framework of the NNI with small deviations from Hermiticity, using Eqs. \((26)\) and \((27)\), one obtains:

\[
|V_{cb}| \approx |O_{22}^\mu O_{23}^{\mu d} e^{i\kappa_2} + O_{32}^{\mu u} O_{33}^{\mu}|.
\]

Taking into account that \(O_{23}^{\mu d} \approx \sqrt{m_u/m_b}(1 - \epsilon_b^d)\) and \(O_{32}^{\mu u} \approx -\sqrt{m_c/m_t}(1 - \epsilon_t^u)\), it is clear that in order to obtain \(|V_{cb}|\) consistent with experiment, one needs to obtain a “suppression” of the \(O_{23}^{\mu d}\) and an enhancement of \(O_{32}^{\mu u}\). This is achieved for the values of \(\epsilon_{a,b}^{u,d}\) given in Eq. \((30)\).

4. Conclusions

We have pointed out that the NNI form for the quark mass matrices can be obtained in the context of a two Higgs doublet extension of the SM, through the introduction of a \(Z_4\) symmetry. We have further shown that the NNI scheme, with small deviations from Hermiticity, can correctly reproduce the experimentally allowed values for quark masses and CKM mixings.
Most of the searches for allowed fermion mass textures have been conducted in the framework of Hermitian or symmetric quark mass matrices. In the SM, quark mass matrices need not be Hermitian or symmetric. But in the framework of a left-right symmetric theory [16] or SO(10) Grand Unification [17], Hermitian or symmetric quark matrices naturally arise. Non-Hermitian quark mass matrices have also been considered and their phenomenological implications analysed in the literature [18]. We find remarkable that a good fit of quark masses and mixing is obtained in the NNI framework, with small deviations of Hermiticity, specially taken into account the rather precise experimental information one has at present on the CKM matrix, including those resulting from the measurements of $B_d - \bar{B}_d$, $B_s - \bar{B}_s$ mixings and the rephasing invariant phases $\beta$ and $\gamma$.

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