A Study on Developing Percussion Instrument with Tobe Pottery

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Abstract—The present study is aimed to develop percussion instruments such as wind bell, hand bell, etc. with Tobe pottery which is a traditional product of Ehime prefecture in Japan. Not only the fundamental natural frequency corresponding to the desired music scale is designed, but also the higher order natural frequencies are considered in order to obtain a pleasant sound color. The material properties and vibration characteristics of the Tobe pottery are investigated by impact test and spectral analysis at first. And then, using the finite element method software, the shape of each musical scale model is estimated by changing the sizes of the Tobe pottery numerically. Since the second natural vibration is a major factor influencing the tone color, two approaches are used to change the tone color: (1) optimizing the shape of the pottery so that the second natural frequency changes to an objective frequency under a specified fundamental natural frequency, and (2) optimizing the shape of the pottery so that the contribution ratio of the second vibration changes under a specified fundamental natural frequency. Both approaches are developed with the basis vector method which is a technique of shape optimization. Finally, because of the shrinkage of material after firing, the initial shape of the pottery before the firing is also estimated. Numerical examples are given to show the effectiveness of the proposed approach.

Index Terms—Percussion Instrument, Shape optimization, Tobe Pottery, Natural Vibration, Sound.

I. INTRODUCTION

Tobe pottery is a type of pottery produced in Tobe, Ehime Prefecture of Japan. With hundreds of years of history, it is extremely popular in Japan for its simple and beautiful design, rustic touches. In recent years, more and more people begin to enjoy not only the design but also the sound of it. Since it sounds a clear and rustic sound, it is often designed to wind bells or hand bells. However, the design almost depends on the experience of the producers, and designing the sound with correct pitch (fundamental natural frequency) and desirable timbre (sound color) is an assignment to be solved.

The purpose of this study is to develop a design procedure for making the percussion instruments such as wind bell, hand bell, etc. with Tobe pottery. For this purpose, the followings are studied. Firstly, to design the sizes of the pottery so that it has correct pitch. Secondly, to change the sound color under a specific pitch so that it can have a desirable timbre. Finally, to estimate the initial shape of the pottery before firing because of the shrinkage of material during firing process.

To design the pitch of the sound of a pottery means to design the fundamental natural frequency of it. It can be realized simply by changing the sizes such as the thickness, the height and the diameter. In the case of considering the sound color, since higher orders natural frequencies should be taken into account, the sizes design is not enough, and the shape optimization approaches about the natural frequencies are necessary.

The researches of optimal design in natural vibration problem were stated several decades before. The studies of the early years focused the optimization of limited shape such as bar, beam and plate [1][2][3]. As the progress of computer technology and optimization theory, general designs of solid continua have been reported [4][5]. One of the authors also proposed an approach to solve the optimization problems of natural vibration with Traction Method [6], an analytical sensitivity based method for shape optimization of two- and three-dimensional continua. However, since the pottery has to be designed to a rotational symmetry shape for convenience of manufacturing, the analytical sensitivity function which is derived for free boundary design cannot be applied. To this problem, we use the basis vector method [7], an approximate approach for shape optimization.

II. IMPACT TEST FOR MATERIAL PROPERTIES AND VIBRATION CHARACTERISTICS IDENTIFICATION

In order to design sound pitch and timbre with computer analysis, the material properties and vibration characteristics of the Tobe pottery are investigated at first.

A. Material properties identification

As shown in Fig.1, 4 different sized specimens made of the same material with Tobe pottery are used to impact tests. Hung by a thread, each specimen is hit with a small hammer at the center. The sound of it is picked up by a microphone and inputted to a computer. The frequency spectrum of the sound is analyzed by Wave Spectra, a sound analysis software, and
the natural frequencies are identified. Table 1 shows sizes, densities, natural frequencies of the specimens.

**TABLE I. SIZES, DENSITIES AND NATURAL FREQUENCIES OF SPECIMENS**

| Specimen | Ave. length (mm) | Ave. width (mm) | Ave. height (mm) | density (g/cm³) | 1st natural frequency (Hz) | 2nd natural frequency (Hz) |
|----------|-----------------|-----------------|------------------|-----------------|--------------------------|--------------------------|
| 1        | 91.1            | 37.4            | 3.925            | 2.391           | 2953.1                   | 4453.1                   |
| 2        | 90.85           | 37.67           | 4.075            | 2.392           | 2812.5                   | 4265.6                   |
| 3        | 89.125          | 37.22           | 8.72             | 2.384           | 6780.8                   | 9659.3                   |
| 4        | 86.65           | 36.52           | 8.89             | 2.377           | 6334.7                   | 9046.9                   |

B. Vibration characteristics identification of a sample

As shown in Fig. 2, a cup-shaped Tobe pottery is made as a sample of percussion instrument. The sample is hung by a thread, and is hit with a hammer near the upper edge. Its sound is picked up and analyzed by the combination the microphone, the computer and Wave Spectra. Figure 3 shows the frequency spectrum of the sound after hitting immediately. It is found...
that the 1st, the 2nd and the 3rd natural vibration are excited after impact, and their frequencies can be identified. However, as shown in Fig. 4, each natural vibration shows different damping behavior: (1) the 3rd natural vibration attenuated and vanished after 0.1 second, (2) the 2nd natural vibration attenuated and vanished after 0.3 second, and (3) the 1st natural vibration continues for about 1 second.

From this impact test, it is obvious that: the fundamental natural vibration contributes most of the sound and determines the pitch of the sound; the second and the third natural vibration have influence on tone color; and the second one is a major factor for tone color.

III. SIZE DESIGN OF POTTERY FOR EACH MUSIC SCALE

Before considering the tone color, at first we design the sound pitch, i.e. fundamental natural frequency of the pottery for each music scale, from "do" (or C) to "si" (or B) in one octave. It can be realized by changing the sizes such as the thickness, the height and the diameter.

Based on the sizes of the sample, various potteries are created by changing the thickness, height and diameter from original design of the sample. This process is performed by Solid Works, a kind of CAD software. The geometries and material properties obtained from experiment are inputted to MSC Nastran to create FEM model and analyze the vibration characteristics. Based on the numerical results, an approximate equation is constituted for predicting the fundamental natural frequency from sizes of the pottery. As an example, the relation of the fundamental natural frequency and the amount of uniform thickness change from sample model can be expressed as

\[ y = -0.1699x^3 + 18.697x^2 + 556.31x + 2330.796 \] (1)

Where \( y \) is the fundamental natural frequency, and \( x \) is the amount of thickness change from original model, respectively. As shown in Fig. 5, the estimated frequencies correspond reasonably well with the fundamental frequencies of music scales in one octave. The maximum error is less than 0.3%, which is a limit of human to distinguish.

IV. GOALS AND METHODOLOGY FOR DESIGN OF SOUND COLOR

As we have understood from the impact test, the second and the third natural vibration have influence on tone color, and the second one is a major influential factor. Therefore, we focus on the second natural vibration, and try to change its vibration characteristics for a desirable tone color.

A. Goals for design of sound color

The vibration characteristics which have influence on tone color are considered to be frequency, contribution ratio in sound and attenuation. Attenuation property is not discussed here since it mainly depends on the material of the pottery which we cannot change. So we focus our efforts to change the frequency of the second natural vibration and its contribution ratio.

It is well known that it may be a good sound when the higher frequencies of a sound are the harmonic of the fundamental frequency. So, the first goal of sound color design is to change the second natural frequency to a harmonic of the fundamental frequency.

To the contribution ratio of the second natural vibration, a questionnaire was conducted. The sound of an original pottery is recorded and the sound pressure of the second natural vibration is modified from low level to high level relative to that of the fundamental vibration. The questionnaire results show that reducing the level of sound pressure of the second natural vibration can create a better sound. So, the second goal of sound color design is decided to reduce the contribution ratio of the second natural vibration.

B. Basis vector method

Since both the design goals concern with multi-natural frequencies or multi-natural modes, to realize the design goals sizes design only is not enough, and the shape optimization approaches about the natural frequencies are necessary.

In general, for a shape optimization problem, it is necessary to derive the design sensitivity that expresses the relation between the variation of an objective function and the variation of the design variables. However, as mentioned in the Introduction Chapter, analytical sensitivity function cannot support the design of rotation symmetry object. In this study, we use the basis vector method for shape optimization.

Shape changing in the basis vector method is calculated as a linear combination of perturbation vectors, each weighted with its respective design variable \( \alpha_i \). The perturbation vector is the difference between a basis vector \( C_i \) and the original locations of grids \( C_0 \). That is

\[ C = C_0 + \alpha_1(C_1 - C_0) + \alpha_2(C_2 - C_0) + \cdots + \alpha_N(C_N - C_0) \] (2)

Where \( N \) is the number of basis vectors, which is usually smaller than degrees of freedom of design in order to reduce the computation time. Shape optimization with the basis vector method, means that select a suitable set of basis vectors and find the optimal solution of weight coefficient \( \alpha_i \) to refine the shape so that the objective function gets to a maximum (or minimum).
V. SHAPE OPTIMIZATION FOR CHANGING THE 2ND NATURAL FREQUENCY

As the first step of design of tone color, the shape optimization of the pottery is considered so that the second natural frequency changes to a harmonic of the fundamental natural frequency under a specified fundamental natural frequency.

A. Formulation of optimization problem

Updating the shape with the basis vector method, and assuming that the shape variation is small enough in one cycle of optimization, the change of the fundamental natural frequency $f_1$ and the second natural frequency $f_2$ can be expressed as approximate linear equations as

$$
\Delta f_1 = \sum_{i=1}^{N} \alpha_i \Delta f_{1i}
$$

(3)

$$
\Delta f_2 = \sum_{i=1}^{N} \alpha_i \Delta f_{2i}
$$

(4)

Where $\Delta f_{1i}$ denotes the change of $f_1$, and $\Delta f_{2i}$ denotes the change of $f_2$, while the initial shape is changed with the $i$th basis vector.

The optimization problem of changing $f_2$ to a target value $f_{2t}$, a harmonic frequency of $f_1$, under the constraint of fixing $f_1$ to the initial value $f_1^0$ can be formulated as

Find $\alpha_i, \ i = 1,2,\cdots,N$

(5)

Minimize $\left| f_2 - f_{2t} \right| = \sum_{i=1}^{N} \alpha_i \Delta f_{2i} + f_2^0 - f_{2t}$

(6)

Subject to $\Delta f_1 = f_1 - f_1^0 = \sum_{i=1}^{N} \alpha_i \Delta f_{1i} = 0$

(7)

Where $f_2^0$ is the initial value $f_2$, and $\alpha_0 > 0$ is used to control the length of one step, and to guarantee a unique solution. This is a linear programming problem, which can be solved by standard LP algorithm. One of the authors has also proposed another approach to solve the solution of $\alpha_i$ using Lagrange multiplier method. [8]

Using the weight coefficient $\alpha_i$, the shape is modified as Equation 2 for one cycle. The shape is modified until the object function is convergent.

B. Numerical example 1

The initial shape of the pottery is determined by Equation 1. The fundamental frequency is designed to the sound of “do” (C7, 2093Hz), and the second natural frequency is 5008.9Hz. The objective of this numerical analysis is to change the second natural frequency to 4186Hz, one octave upper C, while keeping the fundamental frequency constant.
9 basis vectors which describe the change of thickness distribution are created by Solid Works. Figure 6 shows the cross section of the initial design, and figure 7 shows the cross sections of two examples of basis vectors. All the basis vectors are created to a rotational symmetry shape so that they can constitute a rotational symmetry design.

Figure 8 shows the optimized shape after 32 iterations. It is found that, the fundamental frequency is changed to 2095.0 Hz, and the second natural frequency is changed to 4187.5 Hz.

Maximizing

\[ \Delta f_i = f_i - f_i^0 = \sum_{i=1}^{N} \alpha_i \Delta u_i = 0 \]  \hfill (12)

and

\[ \alpha_1^2 + \alpha_2^2 + \cdots + \alpha_N^2 = \alpha_0 \]  \hfill (13)

Where \( \Delta u_i \) denotes the change of \( u_1 \), and \( \Delta u_2 \) denotes the change of \( u_2 \), while the initial shape is changed with the \( i \)th basis vector.

B. Numerical example 2

The initial shape is the same with numerical example 1. 18 basis vectors which describe change of thickness distribution are created by Solid Works. All the basis vectors are created to a rotational symmetry shape so that they can constitute a rotational symmetry design. Hitting point is selected near the upper edge. The ratio \( R \) of the initial design is 0.866.

Figure 10 shows the optimized shape after 35 iterations. It is found that the ratio \( R \) increased to 1.0 while the fundamental frequency is almost constant. From the optimized shape, it is also found that increasing the thickness at upper edge area and reducing the thickness at middle area can reduce the contribution of the second natural vibration.

VII. CORRECTION OF SHRINKAGE AND OTHER ERRORS

To manufacture the pottery products precisely is difficult. The main reason is shrinkage and deformation during drying and firing process. To produce a pottery with a desired natural frequency, it is necessary to estimate sizes before shrinkage. There are also some other uncertainty factors such as the variation of material, temperature and sizes. The correction of these factors are also necessary to be considered.

A. Correction of shrinkage

As shown in Fig. 11, to measure the shrinkage ratio of the material of Tobe pottery, a marking line with its length of 100mm is carved on a rectangular plate before drying and firing. After firing, the line changes its length to 88.8mm. So the shrinkage ratio is calculated as 11.2%. To correct the shrinkage, it is necessary to expand the design by 12.6%.
Figure 12 shows an after firing design and its estimated shape before firing. It is expanded by Solid Works.

![After firing and Before firing](image)

**Fig. 12. Design of a pottery and its estimated shape before firing**

**Fig. 11. A specimen carved with a marking line**

**Fig. 13. An example of sensitivity distribution of 1st natural frequency**

### B. Correction of frequencies errors

Since the influences of uncertainty factors are unavoidable during pottery manufacturing, an approach of correcting frequency by sandblasting is considered. Sandblasting is process of smoothing and cleaning a hard surface, such as pottery, cast metal and glass by forcing solid particles across that surface at high speed using compressed air. It is reasonable to use it for adjusting the frequency by changing the local thickness of the pottery. The area for sandblasting can be indicated by counter line of sensitivity distribution. The sensitivity function of rth natural frequency is given as [6]

\[
G^{(r)} = C_{ij} \nabla_k u_i^{(r)} u_j^{(r)} - \lambda^{(r)} \rho u_i^{(r)} u_j^{(r)} + A
\]

Where \(u_i^{(r)}\) is the rth natural mode vector, and \(\lambda^{(r)}\) is the eigenvalue. Figure 13 shows an example of sensitivity distribution of the fundamental natural frequency. The red color area denotes the high sensitivity area. Sandblasting this area can reduce the frequency efficiently.

### VIII. CONCLUSIONS

In this study, a design procedure for making the percussion instruments with Tobe pottery is developed. The material properties and vibration characteristics of the Tobe pottery are indentified, and the fundamental natural frequency corresponding to the desired music scale is determined by sizes design. In order to obtain a pleasant tone color, the shape optimizations are performed for: changing the second natural frequency of the pottery to a harmonic frequency; and changing its contribution ratio, under a specified fundamental natural frequency. The methods for shrinkage and other errors correction are also proposed. Numerical examples are given to show the effectiveness of the proposed approach.

### REFERENCES

[1] F. Niordson, "On the optimal design of a vibrating beam," Quart. Appl. Math., Vol. 23, pp. 47-54, 1965.

[2] N. Olhoff, "Optimal design of vibrating rectangular plates," Int. J. Solids Structures, Vol. 10, No. 1, pp. 93-109, 1974.

[3] N. Olhoff, "Optimal design of vibrating rectangular plates," Int. J. Solids Structures, Vol. 10, No. 1, pp. 93-109, 1974.

[4] J. Haug, K. Choi and V. Komkov, "Design sensitivity analysis of structural system," Academic Press, 1986.

[5] A. Diaz, N. Kikuchi, "Solutions to shape and topology eigenvalue optimization problems using a homogenization method," Int. J. num. meth. eng., Vol. 35, No. 7, pp. 1487-1502, 1992.

[6] Z. Wu, H. Azegami, “Domain optimization analysis in natural vibration problems. (Approach using Traction Method),” Tran. Japan Soc. Mech. Eng., Series C, Vol. 61, No. 583, pp. 930-937, 1995. (in Japanese)

[7] L. Harzheim, G. Graf, “A method to create basis vectors for shape optimization using solution 200," MSC European Users' Conf., Paper No. 12, 1996.

[8] Z. Wu, K. Nakai, Y. Sogabe and Y. Arimitsu, “Optimization of thickness distribution of a golf club face,” Tran. Japan Soc. Mech. Eng., Series C, Vol. 70, No. 698, pp. 2870-2876, 2004. (in Japanese)