Anisotropic Star on Pseudo-Spheroidal Spacetime

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ABSTRACT

A new class of exact solutions of Einstein’s field equations representing anisotropic distribution of matter on pseudo-spheroidal spacetime is obtained. The parameters appearing in the model are restricted through physical requirements of the model. It is found that the models given in the present work is compatible with observational data of a wide variety of compact objects like 4U 1820-30, PSR J1903+327, 4U 1608-52, Vela X-1, PSR J1614-2230, SMC X-4, Cen X-3. A particular model of pulsar PSR J1614-2230 is studied in detail and found that it satisfies all physical requirements needed for physically acceptable model.

Subject headings: General relativity; Exact solutions; Anisotropy; Relativistic compact stars
1. Introduction

The study of interior solutions of Einstein’s field equations play a significant role in predicting the nature of the star in the last stages of evolution. There is an emerging interest among researchers to develop mathematical models of superdense stars that are compatible with observational data. A number of articles appeared recently in literature that are in good agreement with observational data Pandya et al. (2014); Gangopadhyav et al. (2013); Hansraj and Maharaj (2006); Tikekar and Jotania (2009); Banerjee et al. (2013). The spacetime metric representing these models may not have a known 3-space geometry. Many researchers used spacetimes having known 3-space geometry to develop superdense star models. The spheroidal spacetime used by Vaidya and Tikekar (1982) Tikekar (1990) and the paraboloidal spacetime used by Finch and Skea (1989), Tikekar and Jotania (2007), Sharma and Ratanpal (2013) Pandya et al. (2014) are examples of spacetimes with definite 3-space geometry. The spacetime metric used by Tikekar and Thomas (1998), Thomas et al. (2005), Thomas and Ratanpal (2007), Tikekar and Jotania (2005) Paul and Chattopadhyav (2010) spheroidal geometry for its physical three space. It is found that all the above spacetimes are useful to describe compact objects like neutron stars and quark stars.

The matter content of astrophysical objects in ultra high densities may not be in the form of perfect fluid. Theoretical study of Ruderman (1972) and Canuto (1974) about more realistic stars suggest that in a very high density regime, matter may not be isotropic. Anisotropy arises due to the existence of solid stellar core or by presence of type-3A superfluid Kippenhahn and Weigert (1990), Sokolov (1980), phase transitions Sawyer (1972) or pion condensation in a star. Study of anisotropic distributions of matter got wide attention after the pioneering work of Bowers and Liang (1974). A number of articles appeared in literature related to compact anisotropic stars. The
anisotropic model developed by Maharaj and Marteens (1989) have uniform density for its matter content. Gokhroo and Mehra (1994) gave a more realistic model incorporating non-uniform density. Tikekar and Thomas (1998, 2003), Thomas et al. (2005) developed superdense anisotropic distributions on pseudo-spheroidal spacetime. The spacetime used to study the non-adiabatic gravitational collapse of anisotropic distribution has a pseudo spheroidal geometry. Dev and Gleiser (2002, 2003, 2004) have study the impact of anisotropy on the stability of stars. Anisotropic distributions with linear equation of state have been studied by Sharma and Maharaj (2007), Thirukkanesh and Maharaj (2008). Komathiraj and Maharaj (2007) studied charged distributions using linear equation of state. Sunzu et al. (2014) used linear equation of state for describing charged quark star models. Feroze and Siddiqui (2011) and Maharaj and Takisa (2012) developed models of anisotropic distributions using quadratic equation of state. In the MIT bag model of quark stars, Paul et al. (2011) have shown that introduction of anisotropy can affect bag constant. Thirukkanesh and Ragel (2012), Maharaj and Takisa (2013b) used polytropic equation of state for developing anisotropic models. The anisotropic charged star models given by Malaver (2013a, b, 2014) and Thirukkanesh and Ragel (2014) have Van der Waals equation of state. Pandya et al. (2014) have given anisotropic compact stars compatible with observational data by generalizing Finch and Skea (1989) model. The anisotropic model given by Sharma and Ratanpal (2013) is a particular case of this model. It is found that the models given by Pandya et al. (2014) agree with the recent observational data given by Gangopadhyay et al. (2013).

In this paper we have studied anisotropic stellar models on pseudo-spheroidal spacetime. The geometric parameter $R$ plays the role of radius of the star. The geometric parameter $K$ appearing in the model is bounded between two limits to comply with the physical requirements. It is found that the anisotropic model developed can accommodate wide range of pulsars like 4U 1820-30, PSR J1903+327, 4U 1608-52, Vela X-1, PSR J1614-2230,
SMC X-4, Cen X-3.

In section 2 we have given the spacetime metric and obtained solution of Einstein’s field equations. The bounds of model parameter $K$ is obtained in section 3 by imposing the physical requirements a physically viable model is expected to satisfy. In section 4 we have shown that the anisotropic compact star models are in agreement with the observational data of Gangopadhyay et al. (2013) and discuss the main results obtained at the end.

2. Space-time Metric

We shall take the pseudo-spheroidal spacetime metric for describing the anisotropic matter distribution in the form

$$ds^2 = e^{\nu(r)}dt^2 - \left(1 + \frac{K r^2}{R^2}\right)dr^2 - r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2\right),$$

(1)

where $K$, $R$ are geometric parameters and $K > 1$. The energy-momentum tensor for the anisotropic matter distribution (Maharaj and Marteens 1989) is taken as

$$T_{ij} = (\rho + p) u_i u_j - pg_{ij} + \pi_{ij},$$

(2)

where $\rho$, the proper density, $p$ denotes fluid pressure and $u_i$ denotes the unit four-velocity of the fluid. The anisotropic stress tensor $\pi_{ij}$ is given by

$$\pi_{ij} = \sqrt{3}S \left[c_i c_j - \frac{1}{3} (u_i u_j - g_{ij})\right].$$

(3)

Here $S = S(r)$ is the magnitude of the anisotropic stress and $c^i = (0, -e^{-\lambda/2}, 0, 0)$ denotes a radial vector.

We have now the following expressions for radial and transverse pressures

$$p_r = -T^1_1 = p + \frac{2S}{\sqrt{3}}.$$
and
\[ p_\perp = -T_2^2 = p - \frac{S}{\sqrt{3}} \]  \hfill (5)

The difference in radial and transverse pressures is taken as the measure of anisotropy and is given by
\[ S = \frac{p_r - p_\perp}{\sqrt{3}}. \]  \hfill (6)

The Einstein’s field equations
\[ R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij}, \]  \hfill (7)

are equivalent to following set of three equations
\[ 8\pi \rho = \frac{1 - e^{-\lambda}}{r^2} + \frac{e^{-\lambda} \lambda'}{r}, \]  \hfill (8)
\[ 8\pi p_r = \frac{e^{-\lambda} - 1}{r^2} - \frac{e^{-\lambda} \nu'}{r}, \]  \hfill (9)
\[ 8\pi p_\perp = e^{-\lambda} \left( \frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu' \lambda'}{4} + \frac{\nu' - \lambda'}{2r} \right). \]  \hfill (10)

Equation (8)-(10) can be further modified to the following form,
\[ e^{-\lambda} = 1 - \frac{2m}{r}, \]  \hfill (11)
\[ \left(1 - \frac{2m}{r}\right) \nu' = 8\pi p_r r + \frac{2m}{r^2}, \]  \hfill (12)
\[ -\frac{4}{r} \left(8\pi \sqrt{3} S\right) = (8\pi \rho + 8\pi p_r) \nu' + 2 \left(8\pi p'_r\right), \]  \hfill (13)

where,
\[ m(r) = 4\pi \int_0^r u^2 \rho(u)du. \]  \hfill (14)

Using
\[ e^\lambda = \frac{1 + K \frac{r^2}{R^2}}{1 + \frac{r^2}{R^2}}, \]  \hfill (15)
in (8), we obtain the expression for \( \rho \) in the form
\[ 8\pi \rho = \frac{K - 1}{R^2} \frac{3 + K \frac{r^2}{R^2}}{(1 + K \frac{r^2}{R^2})^2}. \]  \hfill (16)
The metric potential $\nu$ can be obtained from equation (12) once we know the expression for $p_r$. We shall take the expression for radial pressure as

$$8\pi p_r = \frac{K - 1}{R^2} \frac{1 - \frac{r^2}{R^2}}{(1 + K \frac{r^2}{R^2})^2}. \quad (17)$$

It can be noticed from equation (17) that the central pressure is $p_r(0) = \frac{K-1}{R^2}$, which is directly related to the geometric parameters $K$ and $R$. This choice of $p_r$ facilitates the integration of equation (12) and obtain $e^\nu$ in the form

$$e^\nu = CR^{\frac{K^2 - 2K + 1}{K}} \left(1 + K \frac{r^2}{R^2}\right)^{\frac{K+1}{2K}} \left(1 + \frac{r^2}{R^2}\right)^{\frac{K-3}{2}}, \quad (18)$$

where $C$ is a constant of integration.

Differentiating equation (17) with respect to $r$, we get

$$8\pi \frac{dp_r}{dr} = -\frac{2r(K - 1)}{R^4} \frac{1 + K \left(2 - \frac{r^2}{R^2}\right)}{(1 + K \frac{r^2}{R^2})^3}. \quad (19)$$

It can be noticed from equation (19) that

$$8\pi \frac{dp_r}{dr} (r = 0) = 0, \quad (20)$$

$$8\pi \frac{dp_r}{dr} (r = R) = -\frac{2(K - 1)}{R^2(1 + K)^2} < 0. \quad (21)$$

Further $8\pi \frac{dp_r}{dr} < 0$ for all values of $r$ in the range $0 \leq r \leq R$. Hence the radial pressure is decreasing radially outward and becomes zero at $r = R$, which is taken as the radius of the anisotropic fluid distribution.

From equation (16), we get

$$8\pi \frac{d\rho}{dr} = -\frac{2rK(K - 1)}{R^4} \frac{5 + K \frac{r^2}{R^2}}{(1 + K \frac{r^2}{R^2})^3} < 0, \quad (22)$$

indicating that the density is a decreasing function of $r$. 
The spacetime metric (1) now takes the explicit form

$$ds^2 = CR^{K^2-2K+1} \left( 1 + K \frac{r^2}{R^2} \right)^{\frac{K+1}{2}} \left( 1 + r^2 \frac{R^2}{R^2} \right)^{\frac{k-1}{2}} dt^2 - \left( 1 + K \frac{r^2}{R^2} \right) \frac{1}{1 + \frac{r^2}{R^2}} dr^2 - r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right). \quad (23)$$

For a physically acceptable relativistic distribution of matter, the interior spacetime metric (23) should continuously match with Schwarzschild exterior metric

$$ds^2 = \left( 1 - \frac{2M}{r} \right) dt^2 - \left( 1 - \frac{2M}{r} \right)^{-1} dr^2 - r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right), \quad (24)$$

across the boundary $r = R$. This gives the constants $M$ and $C$ as

$$M = \frac{R (K - 1)}{2 (K + 1)}, \quad (25)$$

and

$$C = R^{-\left( \frac{K^2-2K+1}{2} \right)} (1 + K)^{-\left( \frac{2K+1}{2} \right)} 2^{-\left( \frac{5-K}{2} \right)}. \quad (26)$$

The expression for anisotropy $S$ can be obtained using (16), (17), (18) and (19) in (13). We have

$$8\pi \sqrt{3}S = \frac{(K - 1)r^2 \left[ 12 + (-6K^2 + 16K - 2) \frac{r^2}{R^2} + (-K^3 + 3K^2 - 7K + 1) \frac{r^4}{R^4} \right]}{4R^4 \left( 1 + K \frac{r^2}{R^2} \right)^3 \left( 1 + \frac{r^2}{R^2} \right)}, \quad (27)$$

which takes the value zero at $r = 0$. The expression for $8\pi p_\perp = 8\pi p_r - 8\pi \sqrt{3}S$, now takes the form

$$8\pi p_\perp = \frac{(K - 1) \left[ 4 + (4K - 12) \frac{r^2}{R^2} + (6K^2 - 16K - 2) \frac{r^4}{R^4} + (K^3 - 3K^2 + 3K - 1) \frac{r^6}{R^6} \right]}{4R^2 \left( 1 + K \frac{r^2}{R^2} \right)^3 \left( 1 + \frac{r^2}{R^2} \right)}, \quad (28)$$

3. Physical Analysis

The anisotropic matter distribution described on the background of pseudo-spheroidal spacetime contains two geometric parameters $R$ and $K$. Since $p_r(r = R) = 0$, the parameter
$R$ represents the radius of the distribution. The bounds on the other parameter $K$ is to be determined using the physical plausibility conditions stipulated below:

(i) $\rho(r), p_r(r), p_\perp(r) \geq 0$ for $0 \leq r \leq R$;

(ii) $\rho - p_r - 2p_\perp \geq 0$ for $0 \leq r \leq R$;

(iii) $\frac{dp_r}{dr}, \frac{dp_r}{dp}, \frac{dp_\perp}{dr} < 0$ for $0 \leq r \leq R$;

(iv) $0 \leq \frac{dp_r}{dp} \leq 1; 0 \leq \frac{dp_\perp}{dp} \leq 1$, for $0 \leq r \leq R$;

(v) The adiabatic index $\Gamma(r) > \frac{4}{3}$, for $0 \leq r \leq R$.

The conditions $\rho(r) \geq 0$, $p_r \geq 0$, $\frac{dp_r}{dr} < 0$ and $\frac{dp_\perp}{dr} < 0$ are evidently satisfied in the light of equations (16), (17), (22) and (19), respectively.

The condition $p_\perp > 0$ imposes a restriction on the value of $K$, namely,

$$K \geq 2.4641.$$  \hspace{1cm} (29)

In order to examine the strong energy condition, we evaluate the expression $\rho - p_r - 2p_\perp$ at $r = 0$ and at $r = R$. It is easy to see that

$$(\rho - p_r - 2p_\perp) (r = 0) = 0,$$  \hspace{1cm} (30)

and $(\rho - p_r - 2p_\perp) (r = R) \geq 0$, imposes an upper bound for $K$, namely,

$$K \leq 4.1231.$$  \hspace{1cm} (31)

It has been suggested by Canuto (1974) that the velocity of sound should be monotonically decreasing for matter distribution with ultra-high densities. This demands that $\frac{d}{dr} \left( \frac{dp_r}{dp} \right) < 0$.

The expressions for $\frac{dp_r}{dp}$ and $\frac{dp_\perp}{dp}$ are given by

$$\frac{dp_r}{dp} = \frac{1 + 2K - K\frac{r^2}{R^2}}{K \left(5 + 2K \frac{r^2}{R^2}\right)},$$  \hspace{1cm} (32)
It can be noticed from (32) that \( \frac{dp}{d\rho} \leq 1 \) throughout the distribution.

\[
\frac{dp}{d\rho} = \frac{\left(1 + K \frac{r^2}{R^2}\right)^3 \left[X_1 + X_2 \frac{r^2}{R^2} + X_3 \frac{r^4}{R^4} + X_4 \frac{r^6}{R^6} + X_5 \frac{r^8}{R^8}\right]}{2K(K-1) \left(5 + K \frac{r^2}{R^2}\right) \left[2 + Y_1 \frac{r^2}{R^2} + Y_2 \frac{r^4}{R^4} + Y_3 \frac{r^6}{R^6} + Y_4 \frac{r^8}{R^8} + Y_5 \frac{r^{10}}{R^{10}} + 2K^4 \frac{r^{12}}{R^{12}}\right]},
\]

(33)

where, \( X_1 = 8K^2 + 8K - 16, \ X_2 = -4K^3 + 28K^2 - 20K - 4, \ X_3 = 3K^4 - 4K^3 - 30K^2 + 36K - 5, \ X_4 = 10K^4 - 36K^3 + 16K^2 + 12K - 2, \ X_5 = K^5 - 4K^4 + 6K^3 - 4K^2 + 4 \) and \( Y_1 = 8K + 4, \ Y_2 = 12K^2 + 16K + 2, \ Y_3 = 8K^3 + 24K^2 + 8K, \ Y_4 = 2K^4 + 16K^3 + 12K^2, \ Y_5 = 4K^4 + 8K^3. \)

The condition \( \frac{dp}{d\rho} \leq 1 \) at \( r = 0 \) and \( r = R \) gives the following bounds on \( K \), viz.,

\[ K > 1.3333, \]

(34)

and

\[ 1 \leq K \leq 14.7882. \]

(35)

The expression for adiabatic index \( \Gamma \) is given by

\[
\Gamma = \frac{\left(4 - \frac{r^2}{R^2} + K \frac{r^2}{R^2}\right)\left(1 + 2K - K \frac{r^2}{R^2}\right)}{K \left(1 - \frac{r^2}{R^2}\right) \left(5 + K \frac{r^2}{R^2}\right)},
\]

(36)

The necessary condition for the model to represent a relativistic star is that \( \Gamma > \frac{4}{3} \) throughout the star. \( \Gamma > \frac{4}{3} \) at \( r = 0 \) imposes a condition on \( K \), viz.,

\[ K > -3. \]

(37)

Considering all the relevant inequalities, we find that the admissible bound for \( K \) is given by

\[ 2.4641 \leq K \leq 4.1231. \]

(38)
4. Application to Compact Stars and Discussion

In order to examine the suitability of our model to fit into the observational data, we have considered the masses and radii of some well known pulsars given by Gangopadhyay et al. (2013). We have considered PSR J1614-2230 whose estimated mass and radius are $1.97 M_\odot$ and 9.69 km. If we set these values in equation (25) we get $K = 3.997$ which is well inside the valid range of $K$. We have further verified that our model is in good agreement with the estimated mass and radii of a number of compact stars like Vela X-1, 4U1608-52, PSRJ1903+327, 4U1820-30 SMC X-4 and Cen X-3. The value of $K$, mass, radius and other relevant quantities like $\rho_c$, $\rho_R$, $u = \frac{M}{R}$ and $\frac{dp_c}{d\rho}(r=0)$ are shown in Table-1.

| STAR              | $K$   | $M$  | $R$  | $\rho_c$ (MeV fm$^{-3}$) | $\rho_R$ (MeV fm$^{-3}$) | $u(=\frac{M}{R})$ | $\left(\frac{dp_c}{d\rho}\right)_{r=0}$ |
|-------------------|-------|------|------|--------------------------|--------------------------|-------------------|---------------------------------|
| 4U 1820-30        | 3.100 | 1.58 | 9.1  | 2290.97                  | 277.12                   | 0.256             | 0.465                           |
| PSR J1903+327     | 3.176 | 1.667| 9.438| 2206.89                  | 260.52                   | 0.261             | 0.463                           |
| 4U 1608-52        | 3.458 | 1.74 | 9.31 | 2561.92                  | 277.50                   | 0.276             | 0.458                           |
| Vela X-1          | 3.407 | 1.77 | 9.56 | 2379.27                  | 261.63                   | 0.273             | 0.459                           |
| PSR J1614-2230    | 3.997 | 1.97 | 9.69 | 2883.52                  | 269.33                   | 0.300             | 0.450                           |
| SMC X-4           | 2.514 | 1.29 | 8.831| 1753.84                  | 261.06                   | 0.215             | 0.480                           |
| Cen X-3           | 2.838 | 1.49 | 9.178| 1971.21                  | 260.42                   | 0.239             | 0.470                           |

In order to have detailed analysis of various physical conditions throughout the star we have considered a particular star PSR J1614-2230 having mass $M = 1.97 M_\odot$ and radius $R = 9.69$ km along with the geometric parameter $K = 3.997$. The variation of density and pressure from centre to the boundary of the star is shown graphically in Figure 1.
and Figure 2 respectively. It can be seen that density and pressures are monotonically decreasing functions of the radial variable $r$. In Figure 3, we have shown the variation of anisotropy $S$ throughout the star. The anisotropy increases initially and after reaching a maximum at $r = 2.54$, then it starts decreasing till the boundary of the star. The variation of square of sound speed and strong energy condition are displayed in Figure 4 and Figure 5 respectively. It can be noticed that the square of sound speed is less than 1 and the strong energy condition is satisfied throughout the star.

In Figure 6, we have shown the equation of state for matter distribution in graphical form. For a stable relativistic star, the adiabatic index $\Gamma$ should be greater than $\frac{4}{3}$ throughout the configuration. We have plotted the graph of $\Gamma$ against $r$ in Figure 7. The graph clearly indicates that $\Gamma > \frac{4}{3}$ throughout the star. For a physically acceptable model, the gravitational redshift, $z = \sqrt{e^{-\nu(r)}} - 1$, should be a decreasing function of $r$. Further the central redshift $z_c$ and boundary redshift $z_R$ should be positive and finite. From Figure 8 it can be seen that these conditions are satisfied throughout the star.

We have studied spherically symmetric anisotropic distributions of matter on pseudo-spheroidal spacetime. The model we have developed is in good agreement with the observational data of pulsars recently studied by Gangopadhyay et al. (2013). The model parameters are carefully selected so that the models satisfy all the physical requirements throughout the distribution. We have studied a particular model of PSR J1614-2230 and have shown that various physical requirements stipulated earlier are satisfied throughout the star.
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Fig. 1.— Variation of density against radial variable \( r \).

Fig. 2.— Variation of pressures against radial variable \( r \).
Fig. 3.— Variation of anisotropy against radial variable $r$.

Fig. 4.— Variation of $\frac{1}{c^2} \frac{dp_r}{d\rho}$, $\frac{1}{c^2} \frac{dp_\perp}{d\rho}$ against radial variable $r$. 
Fig. 5.— Strong energy condition Vs radial variable $r$.

Fig. 6.— Variation of $p_r$ and $p_\perp$ against $\rho$.
Fig. 7.— Variation of $\Gamma$ against radial variable $r$.

Fig. 8.— Variation of gravitational redshift against radial variable $r$. 
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