Perturbative Matching of the NRQCD Heavy-Light Axial Current

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A one-loop matching calculation between Lattice NRQCD and full QCD for the heavy-light axial current is described and the effects of renormalization on \(f_B\) are discussed.

1. Introduction

An important ingredient in lattice studies of \(B\) meson decays, is the matching between currents in full continuum QCD and current operators of the lattice theory being simulated. The GLOK collaboration, for instance, uses NRQCD to simulate \(b\)-quarks together with tadpole improved clover light quarks \(^1\). One goal is to calculate the pseudoscalar meson decay constant \(f_{PS}\),

\[
\langle 0 | A_\mu | PS \rangle = p_\mu f_{PS}.
\]  

(1)

In general, several operators of the effective theory are associated with a given full QCD current operator and one has, for instance,

\[
\langle A_0 \rangle_{QCD} = \sum_j C_j \langle J^{(j)} \rangle_{NRQCD}
\]  

(2)

Our task is to identify the relevant operators \(J^{(j)}\) and calculate the matching coefficients \(C_j\) to a given order in \(\alpha\) and \(1/M\). We present here results of a matching calculation through \(O(\alpha/M)\) \(^2\) and discuss their implications for \(f_B\).

2. The Lattice Action and Current Operators

The lattice NRQCD heavy quark action density used in the perturbative calculations is

\[
\mathcal{L}_{NRQCD} = \overline{\psi} \gamma^n \left(1 + \frac{cB}{2} \sigma \cdot \mathbf{B} \right) \gamma^n \frac{1}{2} \left(1 + \gamma_0\right) \gamma_0 \\Psi
\]  

\[
U_4^\dagger \left(1 - \frac{aH_0}{2n}\right)^n \left(1 - \frac{a\delta H}{2n}\right) \psi
\]  

(3)

with

\[
H_0 = \frac{D^{(2)}_{2M_0}}{2M_0}, \quad \delta H = -c_B \frac{\theta}{2M_0} \sigma \cdot \mathbf{B}
\]  

(4)

\(c_B = 1\) at tree level. The one-loop contribution to \(c_B\) is a higher order \(O(\alpha^2/M)\) effect in the matching calculation and hence can be ignored here. For the light quarks we use the clover action. A few comments will be made later, on perturbative calculations with Wilson light quarks.

Heavy-light currents in full QCD have the form \(\bar{q}h\). The four component Dirac spinor for the heavy quark, \(h\), is related to the two component NRQCD heavy quark (heavy anti-quark) fields, \(\psi (\bar{\psi})\), via an inverse Foldy-Wouthuysen transformation. Through \(O(1/M)\) one has,

\[
h = U_{FW}^{-1} \Psi_{FW} = \left[1 - \frac{1}{2M} (\gamma \cdot \mathbf{D})\right] \left(\begin{array}{c} \psi \\ \bar{\psi} \end{array}\right)
\]  

(5)

Hence at tree-level,

\[
J = J^{(0)} + J^{(1)} = \bar{q} \Gamma Q - \frac{1}{2M} \bar{q} \Gamma (\gamma \cdot \mathbf{D}) Q
\]  

(6)

\[
Q = \frac{1}{2} (1 + \gamma_0) \Psi_{FW}.
\]

At one-loop one needs to include, for \(\Gamma = \gamma_5 \gamma_0\) and massless light quarks, a third operator

\[
J^{(2)} = \frac{1}{2M} (\mathbf{D} \bar{q} \cdot \gamma) \Gamma Q
\]  

(7)
Furthermore, on the lattice there is a discretization correction to $J^{(0)}$,

$$J^{(0)} \rightarrow J^{(0)}_{\text{imp}} \equiv J^{(0)} + C_A J^{(\text{disc})}$$

(8)

with

$$J^{(\text{disc})} = a (D \bar{q} \cdot \gamma) \Gamma Q$$

(9)

$C_A$ is fixed by requiring that $\langle J^{(0)}_{\text{imp}} \rangle$ projects only onto matrix elements that exist in the continuum theory. In perturbation theory $C_A$ starts out $O(\alpha)$. $J^{(\text{disc})}$ is the analogue of the discretization correction $a \partial_\mu P$ ($P = \text{pseudoscalar density}$) to the axial current in light physics emphasized by the Alpha collaboration. Recent studies have shown that this term has a non-negligible effect on $f_\pi$ around $\beta = 6.0$. In heavy-light physics the improvement term $J^{(\text{disc})}$ to $J^{(0)}$ also leads to a significant correction to $f_B$.

3. Matching and One-Loop Correction Terms

In order to determine the matching coefficients $C_j$ we consider scattering of a heavy quark by the heavy-light current ($\bar{q} T h$ or the $J^{(i)}$'s) into a light quark. The calculation is carried out in both full QCD and in lattice NRQCD and one requires that the scattering amplitudes in the two theories agree through $O(\alpha/M)$. For the continuum QCD calculation we use NDR in the $\overline{\text{MS}}$ scheme. A gluon mass $\lambda$ is introduced at intermediate stages as an IR regulator. IR divergences cancel between the continuum and lattice contributions to the $C_j$'s. After taking mixing among the $J^{(i)}$'s into account one ends up with the following one-loop expression,

$$\langle A_0 \rangle_{\text{QCD}} = \sum_j C_j \langle J^{(j)} \rangle$$

$$= (1 + \alpha \rho_0) \left[ \langle J^{(0)} \rangle + C_A \langle J^{(\text{disc})} \rangle \right]$$

$$+ (1 + \alpha \rho_1) \langle J^{(1)} \rangle$$

$$+ \alpha \rho_2 \langle J^{(2)} \rangle$$

(10)

The matrix elements on the RHS are evaluated in the effective theory NRQCD. Writing $C_A = \alpha \rho_d$, one sees that there are four one-loop correction terms to $\langle A_0 \rangle$ and hence also to $f_{PS}$

$$\alpha \rho_0 \langle J^{(0)} \rangle , \quad \alpha \rho_1 \langle J^{(1)} \rangle , \quad \alpha \rho_2 \langle J^{(2)} \rangle$$

(11)

Figure 1. One-loop Corrections to $a^{3/2} f_{PS} \sqrt{M_{PS}}$ from $\rho_0 \langle J^{(0)} \rangle$ (Diamonds), $\rho_1 \langle J^{(1)} \rangle$ (Circles), $\rho_2 \langle J^{(2)} \rangle$ (Crosses), $\rho_d \langle J^{(\text{disc})} \rangle$ (Fancy Squares). Simple squares denote the total one-loop correction. Vertical lines mark the physical B.

$$\alpha \rho_d \langle J^{(\text{disc})} \rangle$$

(12)

In Fig.1 we show contributions from these terms to $a^{3/2} f_{PS} \sqrt{M_{PS}}$. Simulation results for the matrix elements were obtained on $\beta = 6.0$ quenched configurations and results are for $\kappa = \kappa_{\text{strange}}$ and $\alpha = \alpha_p(1/a)$. One should note the relative importance of the discretization correction term $\alpha \rho_d \langle J^{(\text{disc})} \rangle$. At the B-meson, due to cancellation among the other one-loop corrections, this is the dominant one-loop term which, as we will see in the next section, leads to a $\sim 12\%$ reduction of $f_B$ relative to the tree-level result. Both $\alpha \rho_d \langle J^{(\text{disc})} \rangle$ and $\alpha \rho_0 \langle J^{(0)} \rangle$ survive into the static limit. The other two terms are $O(\alpha/M)$ and van-
ish in that limit.

The results presented so far are for clover light quarks. For Wilson light quarks some modifications are necessary. In particular, if one tries to calculate $C_A = \alpha g$ with Wilson light quarks, one finds an uncancelled logarithmic IR divergence $\frac{-2\alpha}{\pi} a ln(a\lambda)$. This divergence can only be removed by including clover contributions to the calculation. Hence, it is not possible to include $\langle J^{\text{disc}} \rangle$ in a consistent way within perturbation theory if one is working with Wilson light quarks. (This is in addition to the fact that since Wilson light quarks have $O(\alpha)$ errors one normally would not include $O(\alpha \alpha)$ corrections). A figure similar to Fig.1 for Wilson quarks would only have contributions from $\rho_i \langle J^{\text{disc}} \rangle$, $i = 0,1,2$. Although the numbers change slightly upon going from clover to Wilson, the qualitative feature of cancellation among the three one-loop corrections around the B meson still holds. As a result the difference between tree-level and one-loop corrected $f_B$ is smaller for Wilson than for clover. On the other hand there will be large scaling violations due to the omission of the sizeable $J^{\text{disc}}$ improvement term.

4. Results for $f_B$

Fig. 2 shows preliminary GLOK collaboration results for,

$$a^{3/2} \Phi \equiv [ a^{3/2} f_{PS} \sqrt{M_{PS}} ]$$

with $ln(aM)$ in $C_0$ and $C_1$ set to $ln(aM_0) \approx ln(2.2)$ for all $M_0$. This ensures a smooth $aM \to \infty$ limit and contact with HQET scaling formulas while preserving the correct $f_B$ for the physical B meson. $q^*$ for the coupling $\rho(q^*)$ is not known yet for this calculation and we show results for $aq^* = \pi$ and $aq^* = 1$. Using the $\rho$-meson mass to fix $a^{-1}$ one finds,

$$f_B, = \begin{cases} 
0.207 (9) \text{ GeV} & \text{tree-level} \\
0.189 (8) \text{ GeV} & \text{aq}^* = \pi \\
0.178 (8) \text{ GeV} & \text{aq}^* = 1 
\end{cases}$$

Only statistical errors are shown. After extrapolating to $\kappa_{\text{light}}$,

$$f_B = \begin{cases} 
0.168 (13) \text{ GeV} & \text{tree-level} \\
0.153 (12) \text{ GeV} & \text{aq}^* = \pi \\
0.144 (11) \text{ GeV} & \text{aq}^* = 1 
\end{cases}$$

In both cases, one-loop corrections are a 9% (14%) effect for $aq^* = \pi (1)$. Taking an average, the preliminary estimate for quenched $f_B$ at $\beta = 6.0$ is,

$$f_B = 0.149 (12)(^{+22}_{-15}) (9) \text{ GeV}$$

The second error comes from scale uncertainties and the third from higher order relativistic and perturbative corrections.

REFERENCES

1. See contributions by Arifa Ali Khan and Joachim Hein to these proceedings.
2. C.Morningstar and J.Shigemitsu, in preparation. J.Shigemitsu, hep-lat/9705017.