Rare interactions of neutrinos with matter as contraction of the Electroweak Model

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Abstract.
The modification of the electroweak model with contracted gauge group $SU(2; \epsilon) \times U(1)$ is suggested. The field space of the model is fibered under contraction in such a way that neutrino fields are in the fiber. Properties of the fibered field space are understood in context of semi-Riemannian geometry. Contraction of gauge group is connected with the limit properties of the cross-section for interactions of neutrinos with matter, when neutrinos energy tends to zero. For small contraction parameter this model explain already at the level of classical gauge fields why neutrinos so rarely interact with anything and why their cross-section with matter increase with energy. Small contraction parameter is connected with the universal Fermi constant of weak interactions and neutrino energy as $\epsilon^2(s) = \sqrt{G_F s}$. The modified model need be considered at low energies much less then gauge bosons masses.

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1. Introduction
The standard Electroweak Model based on gauge group $SU(2) \times U(1)$ gives a good description of electroweak processes. Due to this model the W- and Z-bosons was predicted and experimentally observed at the end of the last century. Higgs boson is now searched at the modern LHC. The gauge group of the model is the product of two simple groups. In physics it is well known the operation of group contraction [1], which is connected with introduction of special zero tending contraction parameter. This operation transforms, for example, a simple or semisimple group to a nonsemisimple one. Usually for better understanding of a physical system it is useful to investigate its limits for limiting values of its physical parameters. In this paper we discuss the modified Electroweak Model with the contracted gauge group $SU(2; \epsilon) \times U(1)$. We explain at the level of classical fields the vanishingly small interactions neutrinos with matter especially for low energies and the decrease of the neutrinos-matter cross-section when energy tends to zero with the help of contraction of gauge group. We connect dimensionless contraction parameter $\epsilon \to 0$ with neutrinos energy.

2. Standard Electroweak Model
We shall follow the books [2]–[4] in description of standard Electroweak Model. The Lagrangian of this model is given by

$$L = L_B + L_L + L_Q,$$

(1)
where boson sector \( L_B = L_A + L_\phi \) involve two parts:

\[
L_A = \frac{1}{8g^2} \text{Tr}(F_{\mu \nu})^2 - \frac{1}{4}(B_{\mu \nu})^2 = -\frac{1}{4}((F_{\mu \nu})^1)^2 + (F_{\mu \nu}^2)^2 + (F_{\mu \nu}^3)^2 - \frac{1}{4}(B_{\mu \nu})^2
\]

(2)
is the gauge field Lagrangian and

\[
L_\phi = \frac{1}{2} (D_\mu \phi) \bar{D}_\mu \phi - \frac{\lambda}{4} (\phi^\dagger \phi - v^2)^2,
\]

(3)

where \( \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \) \( \in \mathbb{C}_2 \) are the matter fields, represents the matter field Lagrangian.

The fermion sector is the sum of the lepton \( L_L \) and quark \( L_Q \) Lagrangians. The lepton Lagrangian is taken in the form

\[
L_L = L_1^L \bar{\tau}_\mu D_\mu \tau_l + e^L_1 \bar{\tau}_\mu D_\mu e_r - h_e [e^L_1(\phi^\dagger L_l) + (L_1^L \phi)e_r],
\]

(4)

where \( L_1 = \begin{pmatrix} e_l \\ e_r \end{pmatrix} \) is the SU(2)-doublet, \( e_r \) is the SU(2)-singlet, \( h_e \) is constant, \( \tau_0 = \bar{\tau}_0 = 1 \), \( \bar{\tau}_k = -\tau_k \), \( \tau_\mu \) are the Pauli matrices, \( \phi \in \mathbb{C}_2 \) are the matter fields and \( e_r, e_l, v_l \) are the two component Lorentzian spinors.

The quark Lagrangian is given by

\[
L_Q = Q_1^L \bar{\tau}_\mu D_\mu Q_l + u^L_1 \bar{\tau}_\mu D_\mu u_r + d^L_1 \bar{\tau}_\mu D_\mu d_r - h_d[d^L_1(\phi^\dagger Q_l) + (Q_1^L \phi)u_r]
\]

(5)

where left quark fields form the SU(2)-doublet \( Q_l = \begin{pmatrix} u_l \\ d_l \end{pmatrix} \), right quark fields \( u_r, d_r \) are the SU(2)-singlets, \( \tilde{e} = 2 \), \( e_{\tilde{q}} = e_{\bar{q}} \), \( e_{00} = 1 \), \( e_{ii} = -1 \) is the conjugate representation of SU(2) group and \( h_e, h_d \) are constants. All fields \( u_l, d_l, u_r, d_r \) are two component Lorentzian spinors.

The covariant derivatives are given by the formulas:

\[
D_\mu e_r = \partial_\mu e_r - ig'Qe_r \cos \theta_w + ig'QZ e_r \sin \theta_w,
\]

\[
D_\mu L_l = \partial_\mu L_l - ig \bar{\tau}_k T_k \left( W^+ T_+ + W^- T_- \right) L_l - ig' \bar{\tau}_k Z_\mu \left( T_3 - Q \sin^2 \theta_w \right) L_l - i e A_\mu Q L_l,
\]

(6)

where \( T_k = \frac{1}{2} \tau_k \), \( k = 1, 2, 3 \) are the generators of SU(2), \( T_\pm = T_1 \pm i T_2 \), \( Q = Y + T_3 \) is the electrical charge, \( Y \) is the hypercharge, \( e = g g' (g^2 + g'^2)^{-\frac{1}{2}} \) is the electron charge and \( \sin \theta_w = eg^{-1} \). The gauge fields

\[
W^\pm_\mu = \frac{1}{\sqrt{2}} \left(A^1_\mu \mp i A^2_\mu\right), \quad Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g A^3_\mu - g' B_\mu\right), \quad A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g' A^3_\mu + g B_\mu\right)
\]

(7)

are expressed through the fields

\[
A_\mu (x) = -ig \sum_{k=1}^3 T_k A^k_\mu (x), \quad B_\mu (x) = -ig' B_\mu (x),
\]

(8)

which take their values in the Lie algebras \( su(2) \) and \( u(1) \), respectively.

From the viewpoint of electroweak interactions all known leptons and quarks are divided on three generations. Next two lepton and quark generations are introduced in a similar way to (4) and (5). Full lepton and quark Lagrangians are obtained by the summation over all generations. In what follows we shall regarded only first generations of leptons and quarks.
3. Modified Model

We consider a model where the contracted gauge group $SU(2;\epsilon) \times U(1)$ acts in the boson, lepton and quark sectors. The contracted group $SU(2;\epsilon)$ is obtained \cite{5} by the consistent rescaling of the fundamental representation of $SU(2)$ and the space $C_2$

$$ z'(\epsilon) = \begin{pmatrix} \epsilon z'_1 \\ \epsilon z'_2 \end{pmatrix} = \begin{pmatrix} \alpha & \epsilon \beta \\ -\epsilon \beta & \alpha \end{pmatrix} \begin{pmatrix} \epsilon z_1 \\ z_2 \end{pmatrix} = u(\epsilon)z(\epsilon), $$

$$ \det u(\epsilon) = |\alpha|^2 + \epsilon^2|\beta|^2 = 1, \quad u(\epsilon)u^\dagger(\epsilon) = 1 $$

in such a way that the hermitian form

$$ z^\dagger z(\epsilon) = \epsilon^2|z_1|^2 + |z_2|^2 $$

remains invariant, when contraction parameter tends to zero $\epsilon \to 0$ or is equal to the nilpotent unit $\epsilon = \iota, \epsilon^2 = 0$. The actions of the unitary group $U(1)$ and the electromagnetic subgroup $U(1)_{em}$ in the fibered space $C_2(\iota)$ with the base $\{z_2\}$ and the fiber $\{z_1\}$ are given by the same matrices as on the space $C_2$.

The space $C_2(\iota)$ of the fundamental representation of $SU(2;\epsilon)$ group can be obtained from $C_2$ by substituting $z_1$ by $\epsilon z_1$. Substitution $z_1 \to \epsilon z_1$ induces another ones for Lie algebra generators $T_1 \to \epsilon T_1, T_2 \to \epsilon T_2, T_3 \to \epsilon T_3$. As far as the gauge fields take their values in Lie algebra, we can substitute the gauge fields instead of transforming the generators, namely:

$$ A^1_\mu \to \epsilon A^1_\mu, \quad A^2_\mu \to \epsilon A^2_\mu, \quad A^3_\mu \to \epsilon A^3_\mu, \quad B_\mu \to B_\mu. $$

Indeed, due to commutativity and associativity of multiplication by $\epsilon$

$$ SU(2;\epsilon) \ni g(\epsilon) = \exp \left\{ (\epsilon A^1_\mu)T_1 + (\epsilon A^2_\mu)T_2 + (\epsilon A^3_\mu)T_3 \right\} = \exp \left\{ (A^1_\mu)T_1 + (A^2_\mu)T_2 + (A^3_\mu)T_3 \right\}. $$

For the gauge fields (7) these substitutions are as follows:

$$ W^\pm_\mu \to \epsilon W^\pm_\mu, \quad Z_\mu \to Z_\mu, \quad A_\mu \to A_\mu. $$

The fields $L_l = \begin{pmatrix} v_l \\ e_l \end{pmatrix}, \quad Q_l = \begin{pmatrix} u_l \\ d_l \end{pmatrix}$ are $SU(2)$-doublets, so their components are transformed in the similar way as components of the vector $z$, namely:

$$ v_l \to \epsilon v_l, \quad e_l \to \epsilon e_l, \quad u_l \to \epsilon u_l, \quad d_l \to \epsilon d_l. $$

The right lepton and quark fields are $SU(2)$-singlets and therefore are not transformed.

After transformations (13), (14) and spontaneous symmetry breaking with $\phi^{vac} = \begin{pmatrix} 0 \\ \nu \end{pmatrix}$ the boson Lagrangian (2), (3) can be represented in the form

$$ L_B(\epsilon) = L_B^{(2)}(\epsilon) + L_B^{int}(\epsilon) = $$

$$ = \frac{1}{2} (\partial_{\mu} \chi)^2 - \frac{1}{2} m_\chi^2 \chi^2 - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} m_Z^2 Z_\mu Z_\mu - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + $$

$$ + \epsilon^2 \left\{ -\frac{1}{2} W^\pm_\mu W^\pm_\mu + m_W^2 W^+_\mu W^{-}_\mu \right\} + L_B^{int}(\epsilon), $$

(15)
where as usual second order terms describe the boson particles content of the model and higher order terms $L_{B}^{m}$ are regarded as their interactions. So Lagrangian (15) include charded $W$-bosons with identical mass $m_{W} = \frac{1}{2}gv$, massless photon $A_{\mu}$, neutral $Z$-boson with the mass $m_{Z} = \frac{v}{2}\sqrt{g^{2} + g'^{2}}$ and Higgs boson $\chi$, $m_{\chi} = \sqrt{2}\lambda v$. The lepton Lagrangian (4) in terms of electron and neutrino fields takes the form

$$L_{L}(\epsilon) = e_{\epsilon}^{\dagger}i\tilde{\tau}_{\mu}\partial_{\mu}\epsilon_{\epsilon} + e_{\epsilon}^{\dagger}i\tilde{\tau}_{\mu}\epsilon_{\mu} - m_{\epsilon}(e_{\epsilon}^{\dagger}\epsilon_{\epsilon} + e_{\epsilon}^{\dagger}e_{\epsilon}) + \frac{g}{2}\cos\theta_{\nu}\epsilon_{\epsilon}^{\dagger}\tilde{\tau}_{\mu}Z_{\mu}\epsilon_{\epsilon} - g'\cos\theta_{\nu}\epsilon_{\epsilon}^{\dagger}\tau_{\mu}A_{\mu}\epsilon_{\epsilon} + g'\sin\theta_{\nu}\epsilon_{\epsilon}^{\dagger}\tau_{\mu}Z_{\mu}\epsilon_{\epsilon} + \epsilon_{\epsilon}^{\dagger}\left[\nu_{\epsilon}^{\dagger}\tilde{\tau}_{\mu}W_{\mu}^{\pm}\epsilon_{\epsilon} + e_{\epsilon}^{\dagger}\tilde{\tau}_{\mu}W_{\mu}^{-}\nu_{\epsilon}\right]\epsilon_{\epsilon}.$$  

(16)

The quark Lagrangian (5) in terms of u- and d-quarks fields can be written as

$$L_{Q}(\epsilon) = d_{L}^{\dagger}i\tilde{\tau}_{\mu}\partial_{\mu}d_{L} + d_{L}^{\dagger}i\tilde{\tau}_{\mu}\partial_{\mu}d_{r} - m_{d}(d_{L}^{\dagger}d_{L} + d_{L}^{\dagger}d_{r}) - \frac{e}{3}d_{L}^{\dagger}\tilde{\tau}_{\mu}A_{\mu}d_{L} - \frac{2}{3}\sin^{2}\theta_{\nu}d_{r}^{\dagger}\tilde{\tau}_{\mu}Z_{\mu}d_{r} - \frac{1}{3}g'\cos\theta_{\nu}d_{r}^{\dagger}\tau_{\mu}A_{\mu}d_{r} + \frac{1}{3}g'\sin\theta_{\nu}d_{r}^{\dagger}\tau_{\mu}Z_{\mu}d_{r} - \frac{2}{3}\sin^{2}\theta_{\nu}d_{r}^{\dagger}\tilde{\tau}_{\mu}Z_{\mu}d_{r} - \frac{2}{3}g'\cos\theta_{\nu}d_{r}^{\dagger}\tau_{\mu}A_{\mu}u_{r} - \frac{2}{3}g'\sin\theta_{\nu}d_{r}^{\dagger}\tau_{\mu}Z_{\mu}u_{r} = L_{Q,b} + e^{2}L_{Q,f}. \quad (17)$$

where $m_{e} = h_{e}v/\sqrt{2}$ and $m_{u} = h_{u}v/\sqrt{2}$, $m_{d} = h_{d}v/\sqrt{2}$ represents electron and quark masses.

The full Lagrangian of the modified model is the sum

$$L(\epsilon) = L_{B}(\epsilon) + L_{Q}(\epsilon) + L_{L}(\epsilon) = L_{b} + e^{2}L_{f}. \quad (18)$$

The boson Lagrangian $L_{B}(\epsilon)$ was discussed in [6], where it was shown that masses of all particles of the Electroweak Model remain the same under contraction $\epsilon^{2} \rightarrow 0$. In this limit the contribution $e^{2}L_{f}$ of neutrino, $W$-boson and $u$-quark fields as well as their interactions with other fields to the Lagrangian (18) will be vanishingly small in comparison with contribution $L_{b}$ of electron, d-quark and remaining boson fields. So Lagrangian (18) describes very rare interaction neutrino fields with the matter for low energies. On the other hand, contribution of the neutrino part $e^{2}L_{f}$ to the full Lagrangian is risen when the parameter $\epsilon^{2}$ is increased, that again corresponds to the experimental facts. The dependence of $\epsilon$ on neutrino energy can be obtained from the experimental dates.

In the mathematical language the fields space of the standard electroweak model is fibered after the contraction in such a way that neutrino, $W$-boson and $u$-quark fields are in the fiber, whereas all other fields are in the base. In order to avoid terminological misunderstanding let us stress that we regard locally trivial fiberization, which is defined by the projection in the field space. This fiberization is understood in the context of semi-Riemannian geometry [7, 8] and has nothing to do with the principal fiber bundle. The simple and best known example of such fiber space is the nonrelativistic space-time with one a dimensional base, which is interpreted as time, and a three dimensional fiber, which is interpreted as proper space. It is well known, that in nonrelativistic physics the time is absolute and does not depend on the space coordinates, while the space properties can be changed in time. The space-time of the special relativity is transformed to the nonrelativistic space-time when dimensionfull contraction parameter — velocity of light $c$ — tends to the infinity and dimensionless parameter $\frac{v}{c} \rightarrow 0$. 


4. Rarely neutrino-matter interactions
To establish the physical meaning of the contraction parameter we consider neutrino elastic scattering on electron and quarks. The corresponding diagrams for the neutral and charged currents interactions are represented in Fig. 1 and Fig. 2.

**Figure 1.** Neutrino elastic scattering on electron.

**Figure 2.** Neutrino elastic scattering on quarks.

Under substitutions (13),(14) both vertex of diagram in Fig. 1, a) are multiplied by $\epsilon^2$, as it follows from lepton Lagrangian (16). The propagator of virtual fields $W$ according to boson Lagrangian (15) is multiplied by $\epsilon^{-2}$. Indeed, propagator is inverse operator to operator of free field, but the later for $W$-fields is multiplied by $\epsilon^2$.

So in total the probability amplitude for charged weak current interactions is transformed as $\mathcal{M}_W \to \epsilon^2 \mathcal{M}_W$. For diagram in Fig. 1, b) only one vertex is multiplied by $\epsilon^2$, whereas second vertex and propagator of $Z$ virtual field do not changed, so the corresponding amplitude for neutral weak current interactions is transformed in a similar way $\mathcal{M}_Z \to \epsilon^2 \mathcal{M}_Z$. A cross-section is proportionate to an squared amplitude, so neutrino-electron scattering cross-section is proportionate to $\epsilon^4$. For low energies $s \ll m_W^2$ this cross-section is as follows [3]

$$\sigma_{\nu e} = G_F^2 s f(\xi) = \frac{g^4}{m_W^2} \tilde{f}(\xi),$$

where $G_F = 10^{-5} \frac{1}{m_p^2}$ = 1,17 $\cdot$ 10$^{-5}$ GeV$^{-2}$ is Fermi constant, $s$ is squared energy in c.m. system, $\xi = \sin \theta_w$, $\tilde{f}(\xi) = f(\xi)/32$ is function of Weinberg angle. On the other hand, taking into account that contraction parameter is dimensionless, we can write down

$$\sigma_{\nu e} = \epsilon^4 \sigma_0 = (G_F s)(G_F f(\xi))$$

and obtain

$$\epsilon^2(s) = \sqrt{G_F s} \approx \frac{g\sqrt{s}}{m_W}.$$
Neutrino elastic scattering on quarks due to neutral and charged currents are pictured in Fig. 2. Cross-sections for neutrino-quarks scattering are obtained in a similar way as for the lepton case and are as follows \[3\]

\[
\sigma^W_\nu = G_F^2 s \hat{f}(\xi), \quad \sigma^Z_\nu = G_F^2 s h(\xi).
\] (22)

Nucleons are some composite construction of quarks, therefore some form-factors are appeared in the expressions for neutrino-nucleons scattering cross-sections. The final expression

\[
\sigma_{\nu n} = G_F^2 s \hat{F}(\xi)
\] (23)

coincide with (19), i.e. this cross-section is transformed as (20) with the contraction parameter (21). At low energies scattering interactions make the leading contribution to the total neutrino-matter cross-section, therefore it has the same properties (20),(21) with respect to contraction of the gauge group.

5. Conclusion

We have suggested the modification of the standard Electroweak Model by the contraction of its gauge group. At the level of classical (non-quantum) gauge fields the very weak neutrino-matter interactions especially at low energies can be explained by this model. The zero tending contraction parameter depend on neutrino energy in accordance with the energy dependence of the neutrino-matter interaction cross-section.

The limit transition \(c \rightarrow \infty\) in special relativity was resulted in the notion of group contraction \[1\]. In our model on the contrary the notion of group contraction is used to explain the fundamental limit process of nature.

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