Measurement surface plasmon polariton assisted optical force using a carbon nanowhisker mechanical resonator

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Abstract. Optical tweezers are widely used for manipulating nano- and micro sized particles. The presence of a special plasmonic substrate can significantly affect the optical forces acting on an object. This paper discusses the possibility of experimental measuring optical plasmon forces using a mechanical resonator based on a carbon nanowhisker. A mathematical model describing the effect of optical forces on nanowhisker oscillations with an additional nanoparticle at the end is presented with taking into account the effect of surface plasmon waves on the substrate.

1. Introduction

Focused laser beams have long been very successfully used as optical tweezers [1]. Nowadays there lots of different modifications and auxiliary structures are used to provide flexible control over the dynamics of micro- and nanoparticles [2-6]. Much effort has been made to study the spatial distribution of the trapping force and its magnitude. Most often, these works are based on a combination of viscosity and a statistical analysis of the particles Brownian motion in a medium. However, these methods are difficult and may not always provide the necessary sensitivity, especially when manipulating and trapping nanoparticles.

To increase the sensitivity when measuring an optical force, it is possible to use nanoscale mechanical resonators. As such a resonator, various nanoscale whiskers can be used, in particular, carbon nanotubes [7, 8]. Carbon nanotube resonators are useful for highly sensitive mass determination [9,10]. In [11], this type of mechanical resonator was used to measure the optical force of trapping a microparticle by a laser beam in empty space.

Earlier, it was shown in [12-14] that the optical forces acting on nanoobjects can be enhanced and inverted when a plasmonic or metamaterial substrate is introduced into the system. In our work, the possibility of experimentally measuring the optical plasmon trapping and anti-trapping forces is discussed, and a mathematical model of the system under consideration is given.
2. Implementation of an experiment to measure the optical plasmonic force

Previously, the possibility of creating an elastic carbon nanowhisker with a particle at the end was experimentally shown [15]. The idea of measuring the plasmon force is to shake a nanowhisker with a particle at the resonant frequency, bring it to the plasmon substrate and illuminate the system with a focused laser beam (Figure 1). In this case, by moving the focus of the beam relative to the substrate, it is possible to switch between the trapping and anti-trapping modes of the nanoparticles with respect to the beam [12]. It is expected that the plasmon force acting on the nanoparticle will lead to a shift in the resonant frequency.

![Figure 1. Scheme of the proposed experiment to measure surface plasmon polariton assisted optical force. The inset shows a micrograph of nanowhisker oscillations with a nanoparticle at the end [15].](image1)

![Figure 2. Typical profile of the x-component optical force acting on a nanoparticle in the field of a focused laser beam. Blue shows the area where the force is linear.](image2)

3. Formalism

The mathematical model is based on the non-stationary Euler-Bernoulli equation with Voigt damping for calculating the vibrations of an elastic cantilever beam:

\[
\frac{\partial^2}{\partial y^2} \left[ E J \frac{\partial^2 u(y,t)}{\partial y^2} \right] + \rho S \frac{\partial^2 u(y,t)}{\partial t^2} + \alpha E J \frac{\partial^2 u(y,t)}{\partial y^2 \partial t} = 0,
\]

where \(E\) - elastic modulus, \(J\) - moment of inertia of the beam cross section, \(u\) - deflection of beam, \(\alpha\) - damping factor, \(y_0\) - oscillation amplitude of the whisker fixed end and \(\rho\) - material density.

Boundary conditions:

- fixed end \(u(0,t) = y_0 \cdot e^{\alpha t}; \frac{\partial u(0,t)}{\partial y} = 0\)
- free end \(-EJ \frac{\partial^2 u(L,t)}{\partial y^2} = 0; -EJ \frac{\partial^2 u(L,t)}{\partial y^3} = -m \frac{\partial^2 u(L,t)}{\partial t^2} + F_p(u(L,t))\).

In the last expression, the first term is the force of inertia, which is due to the presence of additional mass \(m\) at the free end. The second term is the optical force acting on the nanoparticle at the end of the whisker when it is illuminated by a laser near the plasmonic surface. An analytical expression for the optical plasmon forces based on self-consistent expressions for the total field at the location of the particle obtained using the dyadic Green function is given in [12].

To simplify the calculations, the whisker oscillations are considered in the range of displacements, where the optical force has an almost linear dependence (Figure 2). Then the general solution of the problem will be:
where

\[
\begin{align*}
\frac{d}{dt} (x, y, t) &= \mathbf{M} \cdot \mathbf{v} \\
\mathbf{v} &= \frac{1}{m} \mathbf{F}_{\text{opt}}(x, y, t)
\end{align*}
\]

To increase the value of optical forces you need to increase the size of the nanoparticles. On the other hand, it is necessary to remain within the framework of the dipole approximation. This can be achieved using the ITO substrate in which surface plasmons are excited by light with a wavelength of 1580 nm. Below are the calculations of the optical force (Figure 3) and the amplitude-frequency characteristic of the first resonance (Figure 4) for a gold nanoparticle with a radius of 100 nm, oscillating above the plasmon substrate of ITO in the field of a laser beam focused above (f>0) and under the substrate (f<0).

**Figure 3.** The distribution along the x axis of the x-component optical force acting on a gold nanoparticle in the field of a laser beam focused above and below the ITO substrate.

**Figure 4.** Amplitude-frequency characteristic of a nanowhisker with a gold nanoparticle fixed at the end in the laser beam field focused above and below the ITO substrate. The results were obtained with nanowhisker parameters:

\[
E = 8.5 \times 10^{10} \text{ N/m}^2, \quad L = 5 \mu m, \quad \rho_S = 2200 \text{ kg/m}^3, \quad R_w = 50 \text{ nm}, \quad \alpha = 2 \times 10^{-10} \text{ s}.
\]

This force on the system "nanowhisker+nanoparticle" leads to a shift of the resonance frequency: for the particle trapping mode \( \Delta \nu_1 = -240 \text{ Hz} \), for the particle anti-trapping mode \( \Delta \nu_2 = 64 \text{ Hz} \).
5. Conclusions
The paper shows the fundamental possibility of experimentally measuring plasmon optical forces acting on a nanoparticle near a plasmonic substrate using a mechanical resonator. Also, a mathematical model of nanowhisker oscillations with a nanoparticle at the end in the field of a laser beam is given. It is shown that a force effect on the “nanowhisker + nanoparticle” system from the side of optical radiation leads to a shift in the resonance frequency. In this case, depending on the position of the focus of the laser beam with respect to the substrate, the shift of the resonance frequency can be either greater or less than zero.

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