Experimental realization of a quantum autoencoder via a universal two-qubit unitary gate

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ABSTRACT

As a ubiquitous aspect of modern information technology, data compression has a wide range of applications. Therefore, quantum autoencoder which can compress quantum information into a reduced space is fundamentally important to achieve atomatical data compression in the field of quantum information. Such a quantum autoencoder can be implemented through training the parameters of a quantum device using machine learning. In this paper, we experimentally realize a universal two-qubit unitary gate and achieve a quantum autoencoder by applying machine learning. Also, this quantum autoencoder can be used to discriminate two groups of nonorthogonal states.

INTRODUCTION

The compression of information, one of the fundamental tasks in classical information theory, has been studied for many years [1–6]. With the development of Internet, massive data are generated and transferred within very short time. Thus, compressing data into the smallest possible space is of crucial importance in present-day digital technology. Various compression methods have found a wide range of applications such as text coding [3, 4], image compression [5, 6]. Correspondingly, in the quantum domain, the compression of quantum information has aroused widespread attentions [7–8]. Many methods of compressing quantum information based on different specific assumptions on the structure of quantum data have been proposed [9–14]. Apart from specific assumptions on quantum data, devices called quantum autoencoders, which are capable of learning the data structure, have been proposed and studied recently [15–17].

A traditional autoencoder can compress classical data into a lower-dimensional space. As shown in Fig.1(a), the input information represented by yellow dots can be compressed into fewer dots after the encoder ε and a decoder D can reconstruct the input data at the output. Autoencoders form one of the core issues in machine learning and have many applications [15–20]. In recent years, quantum machine learning, which combines both quantum physics and machine learning, shows powerful capability in more and more applications [14, 21–27] and has become a booming research area attracting worldwide attention. Autoencoders for quantum data, which belong to the field of quantum machine learning, have aroused great interest in the field of quantum information recently [15, 17]. For a quantum device to realize an autoencoder, as illustrated in Fig.1(b), a parameterized unitary $U^j(p_1, p_2, \cdots, p_n)$ is trained as a quantum autoencoder where measurement results are considered and an optimization algorithm is employed.

In this paper, we experimentally realize a universal two-qubit unitary gate and achieve a quantum autoencoder based on the theoretical model in Ref. [16]. Our quantum autoencoder can encode two 2-qubit pure states $|\varphi_1\rangle, |\varphi_2\rangle$ into two qubit states without any other restriction. Besides encoding qubits, our device can also be used to discriminate two groups of nonorthogonal states.

RESULTS

Quantum-classical hybrid scheme

Here we first introduce the quantum-classical hybrid scheme for quantum autoencoder proposed in Ref. [16]. As shown in Fig.1(b), the state preparation, operation and measurement are performed by quantum means while the optimization of parameters is realized via a classical algorithm. Fig.1(b) also indicates the scheme for our experiment: the core issue is to use the same 2-qubit unitary operator $U$ to encode two 2-qubit states $|\varphi_1\rangle, |\varphi_2\rangle$ into two qubit states. In the classical scheme, we use stochastic gradient descent to optimize the parameterized unitary gate. In a single iteration of our algorithm, we perform the following steps:

1. Choose a random parameter $p_k$ to optimize $U^j$. Set new parameters $p_1, \cdots, p_k + a, \cdots, p_n$ for $U^j$. Here $a$ is a preset parameter indicating the extent of change at each step.

2. Prepare the input states $|\varphi_i\rangle$, and let it evolve under the encoding unitary $U^j$.

3. Measure and record the overlap between the trash state and the reference state. In our experiment,
Universal two-qubit unitary operator

Now, our task turns to realize a multi-qubit parameterized unitary operator. It is well known that any binary quantum alternative of a photon can serve as a qubit. Thus, by choosing polarization and path degrees of freedom as two qubits, we can achieve the 2-qubit universal parameterized unitary gate combining path unitary gate with polarization gate [28].

The setup for a universal two-qubit unitary gate is shown in Fig.1(c). In a few words, for any given 2-qubit unitary operator:

\[ U = \begin{bmatrix} U_{RR} & U_{RL} \\ U_{LR} & U_{LL} \end{bmatrix} \]

where \( U_{RR}, U_{RL}, U_{LR}, U_{LL} \) is a 2 \( \times \) 2 matrix referring to the path R/L alternative, they can be written as:

- \( U_{RR} = \frac{1}{2}V_2(V_R + V_L)V_1 \)
- \( U_{LL} = \frac{1}{2}(V_R + V_L) \)
- \( U_{RL} = -\frac{1}{2}V_2(V_R - V_L) \)
- \( U_{LR} = \frac{1}{2}(V_R - V_L)V_1 \)

Here, \( V_1, V_2, V_R, \) and \( V_L \) are unitary polarization operators which can be easily realized by a set of quarter-wave plates, half-wave plates and phase shifters. Thus, universal 2-qubit unitary operator \( U \) can be achieved. See more details in supplementary materials.

Experimental setup

In the part of state preparation, since the Mach–Zehnder interferometer in Fig.1(c) is difficult to realize and keep phase stable, we use two phase stable Sagnac interferometers to separately implement state preparation and M–Z interferometer. At the beginning (Fig.2(a)), photon pairs with wave length \( \lambda = 808 \) nm are created by type-I spontaneous parametric down-conversion (SPDC) in a nonlinear crystal (BBO) which is pumped by a 40-mW beam at 404 nm. The two photons pass through two interference filters whose full width at half maximum is 3 nm. One photon is detected by a single-photon counting module (SPCM) as a trigger, another photon is prepared in the state of very pure horizontal polarization noted as |H⟩ through a polarizer beam splitter (PBS). Then a half-wave plate (HWP) along with a PBS can control the path-bit of the photon. In each path, a HWP and a quarter-wave plate (QWP) are used to control the polarization of the photon, as shown in Fig.2(b). Thus, we can produce any expected phase stable two-qubit state thanks to the first Sagnac interferometer.

The parameterized unitary \( U \) is realized with the help of the second Sagnac interferometer in Fig.2(c). It is worth noting that there is a special beam-splitter cube which is half PBS-coated and half coated by non-polarizer beam splitter (NBS) in the junction of two Sagnac interferometers. The second...
Once we realize the universal two-qubit unitary gate, it is natural to ask how well our unitary gate performs. For characterization of our unitary gate, we estimate the process matrix using the maximum-likelihood method for many different but significant gates such as identity gate, controlled-not gate, controlled-Z gate, controlled-Hadamard gate, SWAP gate, √iSWAP gate and so on. Since the maximum-likelihood method needs joint measurement basis of path and polarization, we use a trick to reduce the difficulty of realizing joint measurements. We absorb the basis in the parameterized unitary gate. For example, if we want to set path measurement basis as $U^\dagger$, we actually set our $U$ as $U^\dagger \times U$. Some results of the process tomography are shown in Fig.3. Here we show the process matrix $\tilde{\chi}$ of controlled-not gate (polarization control path) and SWAP gate with the fidelity being 0.957 and 0.948, respectively. The real elements and the imaginary elements are plotted respectively, with ideal theoretical values overlaid. For clarity, we use red to represent positive and blue to represent negative. Full statistics are available in supplementary materials. Our fidelity is computed by $\text{tr} \sqrt{\chi_{\text{exp}} \chi_{\text{exp}}^\dagger}$. Here $\chi_{\text{exp}}$ is the experimental process matrix and $\chi$ is the theoretical process matrix. The average fidelity of our gates is 0.953.

**FIG. 3:** Characterization of experimentally realized gates. A two-qubit gate can be described by its process matrix $\tilde{\chi}$. Specifically, each input state $\rho$ is mapped to an output $\sum_m \tilde{\chi}_{mn} E_m \rho E_n^\dagger$, where the summation is over all possible two-qubit Pauli operators $E_k$. Here we plot the real elements in Fig.3(a) (Fig.3(c)) and the imaginary elements in Fig.3(b) (Fig.3(d)) of CNOT (SWAP), with ideal theoretical values overlaid. For clarity, we use red to represent positive and blue to represent negative. The fidelity of CNOT/SWAP is 0.957/0.948.

**Experimental results:** encoding quantum infor-
mation into lower dimension

Now we turn to the core issue of encoding the quantum information into lower dimension. Our goal is to achieve a 2-qubit unitary operator $U$ which can encode two 2-qubit states $|\varphi_1\rangle, |\varphi_2\rangle$ into two qubit states $|\varphi'_1\rangle, |\varphi'_2\rangle$. For example, we encode two 2-qubit states $|RH\rangle, |LV\rangle$ into states $|\varphi'_1\rangle|R\rangle, |\varphi'_2\rangle|L\rangle$. Here $|R\rangle/|L\rangle$ stands for path qubit and $|H\rangle/|V\rangle$ stands for polarization qubit. Thus, we can trash the path qubit and obtain the compressed states $|\varphi'_1\rangle, |\varphi'_2\rangle$ which maintain the original quantum information totally in polarization qubit. Similarly, encoding the information into path qubit is also feasible. Using the algorithm mentioned before, we can train our parameterized unitary operator $U$ efficiently to accomplish the goal. Fig.4(a) (Fig.4(b)) shows the result of encoding $\{|RH\rangle, |LV\rangle\}$ into path (polarization) qubit. Here infidelity is the cost function in our algorithm and iterations indicate the train process. Results of encoding another set of states $\{\frac{1}{2}|RD\rangle + \frac{\sqrt{3}}{2}|L\rangle, |LV\rangle\}$ into path (polarization) qubit is shown in Fig.4(c) (Fig.4(d)). The performance of our quantum autoencoder is related to the experimental conditions such as imperfect NBS-coated surface, unbalanced coupling efficiency, and uneven wave plates. Though under these imperfect conditions, the cost function can still approach 0 after a few iterations. See more data in supplementary materials.

Apart from encoding quantum information into lower dimension, we find our quantum autoencoder can also realize the discrimination between two different groups of nonorthogonal states. Discrimination between nonorthogonal states has been an important task in quantum information [30, 32]. Many methods have been proposed to solve this problem under different conditions. The most well-known ones are min-error discrimination [31, 33] and unambiguous discrimination [34–37]. It is natural to ask whether we may discriminate groups of nonorthogonal states. Some excellent works have been done [38, 39]. In our experiment, we find that just need to encode different groups into orthogonal path/polarization qubits. For example, encode two groups of nonorthogonal states $\{|\phi_i\rangle\}, \{|\varphi_j\rangle\}$ into states $\{|\varphi'_i\rangle|R\rangle, \{|\varphi'_j\rangle|L\rangle\}$. Thus we can realize the bound of min-error discrimination between two different groups of nonorthogonal states after some iterations.

Experimental results: discrimination between two different groups of nonorthogonal states

We follow the core principle in Ref. [31] to derive the error bound and the optimal strategies to realize the min-error discrimination between two groups. Similar to the case of compressing information, we can train our parameterized unitary operator $U$ efficiently to accomplish the goal and reach the bound of min-

FIG. 4: The results of encoding two 2-qubit states into two qubit states. Here we show the results of encoding different initial states into different qubits (path/polarization). (a) encode $\{|RH\rangle, |LV\rangle\}$ into path qubit. (b) encode $\{|RH\rangle, |LV\rangle\}$ into polarization qubit. (c) encode $\{|\frac{1}{2}|RD\rangle + \frac{\sqrt{3}}{2}|L\rangle, |LV\rangle\}$ into path qubit. (d) encode $\{|\frac{1}{2}|RD\rangle + \frac{\sqrt{3}}{2}|L\rangle, |LV\rangle\}$ into polarization qubit. Here infidelity is the cost function in our algorithm and iterations indicate the train process.

FIG. 5: The results of discriminating two different groups of nonorthogonal states. The bound for (a)-(b) is plotted in blue dashed line. (a) encode $\{\cos \theta_{1/2}|RH\rangle + \sin \theta_{1/2}|RV\rangle, \theta_{1/2} = \pm 45^\circ\}$ & $\{\cos \theta_{3/4}|RH\rangle + \sin \theta_{3/4}|RV\rangle, \theta_{3/4} = 60^\circ \pm 45^\circ\}$ into different polarization qubits. (b) encode $\{\cos \theta_{1/2}|RH\rangle + \sin \theta_{1/2}|RV\rangle, \theta_{1/2} = \pm 2^\circ\}$ & $\{\cos \theta_{3/4}|RH\rangle + \sin \theta_{3/4}|RV\rangle, \theta_{3/4} = 30^\circ \pm 2^\circ\}$ into different polarization qubits. (c) encode $\{\cos \theta_{1}|RH\rangle + \sin \theta_{1}|LH\rangle, \theta_{1} \in [-50^\circ, 50^\circ]\}$ & $\{\cos \theta_{2}|RV\rangle + \sin \theta_{2}|LV\rangle, \theta_{2} \in [-50^\circ, 50^\circ]\}$ into different path qubits. (d) encode $\{\cos \theta_{1}|RH\rangle + \sin \theta_{1}|RV\rangle, \theta_{1} \in [-20^\circ, 20^\circ]\}$ & $\{\cos \theta_{2}|LH\rangle + \sin \theta_{2}|LV\rangle, \theta_{2} \in [-20^\circ, 20^\circ]\}$ into different polarization qubits. Here infidelity is the cost function in our algorithm and iterations indicate the train process.
error discrimination between two different groups of nonorthogonal states. Some of the results are shown in Fig. 5. The blue dashed line is the bound of min-error discrimination between two different groups of nonorthogonal states. We also show our agent’s learning ability by encoding groups of path/polarization orthogonal states into orthogonal polarization/path states in Fig. 5(c)-(d). Full data are available in supplementary materials.

DISCUSSION
In summary, we have experimentally demonstrated a simple but important scheme for a quantum autoencoder. Our device is able to compress two 2-qubits into two qubits without knowing the initial states. We can also use the autoencoder to discriminate different groups of quantum states. We believe that the valuable ability of adjusting unknown data structures can process quantum information very well.

Furthermore, with larger size of the unitary operator being realized, one can extend this scheme to compress higher-dimensional quantum information into a reduced space.

During the writing of this article, we became aware of a similar experimental realization of a quantum autoencoder, reported in Ref. [40]. We use the similar cost function defined in terms of the probability of trash qubits in Ref. [40]. Different from Ref. [40], our quantum autoencoder can compress two 2-qubits into two qubits. Our device can also realize the discrimination between two groups of nonorthogonal 2-qubit states and reach the theoretical bound.

SUPPLEMENTARY MATERIALS
Section S1. Universal two-qubit unitary gate
Section S2. Bound of min-error discrimination
Section S3. All experimental data
Fig. S1. The setup for universal two-qubit unitary gate
Fig. S2. - Fig. S8. Process matrices of different two-qubit unitary gates
Fig. S9. Results of encoding different initial states into path qubit
Fig. S10. Results of encoding different initial states into polarization qubit
Fig. S11. - Fig. S12. Results of discriminating different initial groups of states
Reference [28,31]

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Supplementary Materials for
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two-qubit unitary gate

S1. UNIVERSAL TWO-QUBIT UNITARY GATE

The setup for a universal two-qubit unitary gate [28] which combines path unitary gate with
polarization gate is shown in Fig. S1.

FIG. S1: (a) Universal two-qubit unitary gate [28] which is composed of two beam splitters, two mirrors and
four same single-qubit parts (V1, V2, VR, VL). (b) Each part is composed of two quarter-wave plates (QWP),
a half-wave plate (HWP), and a phase shifter (PS).

The unitary operation of a symmetric beam splitter is given by:

\[ U_{BS} = \frac{1}{\sqrt{2}} (|R\rangle\langle R| + |L\rangle\langle L| + i|L\rangle\langle R| + i|R\rangle\langle L|). \]

Here \(|R\rangle\langle R|\) is path qubit which means the alternative of traveling to the right or to the left. Likewise,
the operation of the mirrors inside the M-Z setup is:

\[ U_{mirror} = -i(|L\rangle\langle R| + |R\rangle\langle L|), \]

where the phase factor \(-i\) is necessary to maintain \(U_{BS}U_{mirror}U_{BS} = I\).

On one hand, the unitary gate \(U\) in Fig. S1(a) can be expressed as follows:

\[
U = \begin{bmatrix} V_2 & 0 \\ 0 & I \end{bmatrix} U_{BS} \begin{bmatrix} V_R & 0 \\ 0 & V_L \end{bmatrix} U_{mirror} U_{BS} \begin{bmatrix} V_1 & 0 \\ 0 & I \end{bmatrix} = \frac{1}{2} \begin{bmatrix} V_2 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} I & i \\ i & I \end{bmatrix} \begin{bmatrix} V_R & 0 \\ 0 & V_L \end{bmatrix} \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} \begin{bmatrix} I & i \\ i & I \end{bmatrix} \begin{bmatrix} V_1 & 0 \\ 0 & I \end{bmatrix}. \]

Here, \(I\) means \(2 \times 2\) identity matrix for polarization qubit and \(0/i\) means \(0/i \neq I\). On the other hand,
the unitary gate \(U\) can also be written as:

\[
U = \begin{bmatrix} U_{RR} & U_{RL} \\ U_{LR} & U_{LL} \end{bmatrix}
\]

where \(U_{RR} (U_{RL}, U_{LR}, U_{LL})\) is a \(2 \times 2\) matrix referring to the path R/L alternative. Since the two
expressions above are actually the same form of the unitary gate \(U\), the entries of this matrix can be
written as:

\[
U_{RR} = \frac{1}{2} V_2 (V_R + V_L) V_1, \quad U_{LL} = \frac{1}{2} (V_R + V_L), \quad U_{RL} = -i \frac{1}{2} V_2 (V_R - V_L), \quad U_{LR} = i \frac{1}{2} (V_R - V_L) V_1.
\]

Thus, for any given 2-qubit unitary operator \(U\), one can find four unitary polarization operators
\(V_1, V_2, V_R, V_L\), which can be easily realized by a set of QWPs, HWP s and phase shifters, such that
the universal 2-qubit unitary operator \(U\) can be achieved.
S2. BOUND OF MIN-ERROR DISCRIMINATION

We follow the core principle in Ref. [31] to derive the error bound and the optimal strategies to realize the min-error discrimination between two groups. For simplicity, we consider two groups of the quantum states. Group a contains \{\{|\Psi_{a1}\rangle, |\Psi_{a2}\rangle\}\} and group b contains \{\{|\Psi_{b1}\rangle, |\Psi_{b2}\rangle\}\}. Our goal is to figure out the strategy to minimize the probability of making an error in identifying the group with the probabilities \{P_{a1}, P_{a2}, P_{b1}, P_{b2}\} for \{\{|\Psi_{a1}\rangle, |\Psi_{a2}\rangle, |\Psi_{b1}\rangle, |\Psi_{b2}\rangle\}\}. Here \(P_{a1} + P_{a2} + P_{b1} + P_{b2} = 1\) and \{\{|\Psi_{a1}\rangle, |\Psi_{a2}\rangle, |\Psi_{b1}\rangle, |\Psi_{b2}\rangle\}\} belong to a Hilbert space of \(d=2\). We take the measurements as \{\Pi_a, \Pi_b\} and outcome a/b (associated with the probability operator \(\Pi_a/\Pi_b\)) is taken to indicate that the state belongs to group a/b. The probability of making an error in classifying the state is given by:

\[
P_{\text{error}} = P_{a1}P(b|\Psi_{a1}) + P_{a2}P(b|\Psi_{a2}) + P_{b1}P(a|\Psi_{b1}) + P_{b2}P(a|\Psi_{b2})
\]

\[
= P_{a1}\langle \Psi_{a1}|\Pi_b|\Psi_{a1}\rangle + P_{a2}\langle \Psi_{a2}|\Pi_b|\Psi_{a2}\rangle + P_{b1}\langle \Psi_{b1}|\Pi_a|\Psi_{b1}\rangle + P_{b2}\langle \Psi_{b2}|\Pi_a|\Psi_{b2}\rangle
\]

\[
= P_{a1} + P_{a2} - P_{a1}\langle \Psi_{a1}|\Pi_a|\Psi_{a1}\rangle - P_{a2}\langle \Psi_{a2}|\Pi_a|\Psi_{a2}\rangle + P_{b1}\langle \Psi_{b1}|\Pi_a|\Psi_{b1}\rangle + P_{b2}\langle \Psi_{b2}|\Pi_a|\Psi_{b2}\rangle
\]

\[
= P_{a1} + P_{a2} - \text{tr}\{(P_{a1}|\Psi_{a1}\rangle\langle \Psi_{a1}| + P_{a2}|\Psi_{a2}\rangle\langle \Psi_{a2}| - P_{b1}|\Psi_{b1}\rangle\langle \Psi_{b1}| - P_{b2}|\Psi_{b2}\rangle\langle \Psi_{b2}|)\Pi_a\}.
\]

This expression has its minimum value when the term \(\text{tr}\{\cdots\}\) reaches a maximum, which in turn is achieved if \(\Pi_a\) is a projector onto the positive eigenket of the operator \(P_{a1}|\Psi_{a1}\rangle\langle \Psi_{a1}| + P_{a2}|\Psi_{a2}\rangle\langle \Psi_{a2}| - P_{b1}|\Psi_{b1}\rangle\langle \Psi_{b1}| - P_{b2}|\Psi_{b2}\rangle\langle \Psi_{b2}|\). We can obtain the solution using numerical calculation. For a specific solution, we limit the form of the states \{\{|\Psi_{a1}\rangle, |\Psi_{a2}\rangle, |\Psi_{b1}\rangle, |\Psi_{b2}\rangle\}\} as follows: Without loss of generality we can choose the basis as \{\{|0\rangle, |1\rangle\}\} such that the components of each state in this basis are real. Thus we can express the states \{\{|\Psi_{a1}\rangle, |\Psi_{a2}\rangle, |\Psi_{b1}\rangle, |\Psi_{b2}\rangle\}\} as follows:

\[
\begin{cases}
|\Psi_{a1}\rangle = \cos \theta_1 |0\rangle + \sin \theta_1 |1\rangle \\
|\Psi_{a2}\rangle = \cos \theta_2 |0\rangle + \sin \theta_2 |1\rangle \\
|\Psi_{b1}\rangle = \cos \theta_1 |0\rangle - \sin \theta_1 |1\rangle \\
|\Psi_{b2}\rangle = \cos \theta_2 |0\rangle - \sin \theta_2 |1\rangle.
\end{cases}
\]

Here we assume \(\theta_1 > \theta_2\) and \{\{|0\rangle, |1\rangle\}\} are orthogonal bases of the Hilbert space. Hence, we can obtain the matrix expression of \(P_{a1}|\Psi_{a1}\rangle\langle \Psi_{a1}| + P_{a2}|\Psi_{a2}\rangle\langle \Psi_{a2}| - P_{b1}|\Psi_{b1}\rangle\langle \Psi_{b1}| - P_{b2}|\Psi_{b2}\rangle\langle \Psi_{b2}|:

\[
\begin{bmatrix}
A\cos^2 \theta_1 + B\cos \theta_2 & C\sin \theta_1 \cos \theta_1 + D\sin \theta_1 \cos \theta_2 \\
C\sin \theta_1 \cos \theta_1 + D\sin \theta_1 \cos \theta_2 & A\sin^2 \theta_1 + B\sin \theta_2
\end{bmatrix}
\]

Here \(A = P_{a1} - P_{b1}, B = P_{a2} - P_{b2}, C = P_{a1} + P_{b1}, D = P_{a2} + P_{b2}\). The expression can be translated to:

\[
\frac{1}{2}\begin{bmatrix}
A\cos 2\theta_1 + B\cos 2\theta_2 + A + B & C\sin 2\theta_1 + D\sin 2\theta_2 \\
C\sin 2\theta_1 + D\sin 2\theta_2 & -A\cos 2\theta_1 - B\cos 2\theta_2 + A + B
\end{bmatrix}.
\]

The eigenvalues can be calculated as:

\[
\lambda_{\pm} = \frac{1}{2} \ast (A + B \pm \sqrt{(A \cos 2\theta_1 + B \cos 2\theta_2)^2 + (C \sin 2\theta_1 + D \sin 2\theta_2)^2})
\]

\[
= \frac{1}{2} \ast (A + B \pm \sqrt{A^2 \cos^2 2\theta_1 + B^2 \cos^2 2\theta_2 + 2ABE + C^2 \sin^2 2\theta_1 + D^2 \sin^2 2\theta_2 + 2CDF})
\]

\[
= \frac{1}{2} \ast (A + B \pm \sqrt{(A^2 - C^2) \cos^2 2\theta_1 + (B^2 - D^2) \cos^2 2\theta_2 + 2ABE + (C + D)^2 + 2CDF - 2CD})
\]

where we denote \(\cos 2\theta_1 \cos 2\theta_2 = E, \sin 2\theta_1 \sin 2\theta_2 = F\). We take \(A = P_{a1} - P_{b1}, B = P_{a2} - P_{b2}, C = P_{a1} + P_{b1}, D = P_{a2} + P_{b2}\) and \(P_{a1} + P_{a2} + P_{b1} + P_{b2} = 1\) back to the expression above. Using the relation: \(2AB \cos 2\theta_1 \cos 2\theta_2 + 2CDF \sin 2\theta_1 \sin 2\theta_2 = AB(\cos(2\theta_1 + 2\theta_2) + \)
\[
\cos(2\theta_1 - 2\theta_2) + CD(\cos(2\theta_1 - 2\theta_1) - \cos(2\theta_1 + 2\theta_2)) = 2((P_{a1}P_{b2} + P_{b1}P_{b2}) \cos(2\theta_1 - 2\theta_2) - (P_{a2}P_{b1} + P_{a1}P_{b2}) \cos(2\theta_1 + 2\theta_2)),
\]

we have:
\[
\lambda_\text{err} = \frac{1}{2}((A + B) \pm \{1 - 4P_{a1}P_{b1}\cos^2\theta_1 - 4P_{a2}P_{b2}\cos^2\theta_2 + 2((P_{a1}P_{b2} + P_{b1}P_{b2})\cos(2\theta_1 - 2\theta_2) - (P_{a2}P_{b1} + P_{a1}P_{b2})\cos(2\theta_1 + 2\theta_2)) - 2CD\}^{1/2}).
\]

It is clear that the equation in the radical expression must be larger than 0. We have \(P_{\text{error}} = \frac{1}{2}(1 - (1 - 4P_{a1}P_{b1}|\langle \psi_1 | \psi_{b1} \rangle|^2 - 4P_{a2}P_{b2}|\langle \psi_2 | \psi_{b2} \rangle|^2 + 2(P_{a1}P_{b2} + P_{b1}P_{b2})(2|\langle \psi_1 | \psi_2 \rangle|^2 - 1) - 2(P_{a2}P_{b1} + P_{a1}P_{b2})(2|\langle \psi_1 | \psi_{b1} \rangle|^2 - 1 - 2CD)^{1/2}).\)

For a simple example, let \(P_{a1} = P_{a2} = P_{b1} = P_{b2} = \frac{1}{4},\) and we can obtain \(P_{\text{error}}\) as:
\[
P_{\text{error}} = \frac{1}{2}(1 - \frac{1}{2}\sqrt{2 - |\langle \psi_1 | \psi_{b1} \rangle|^2 - |\langle \psi_2 | \psi_{b2} \rangle|^2 - 2|\langle \psi_1 | \psi_2 \rangle|^2 - 2|\langle \psi_1 | \psi_{b1} \rangle|^2}).
\]

The simplified \(P_{\text{error}}\) is the limited bound for our experiments. The optimal measurement is a projective measurement onto the states \(|\Phi_0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |\Phi_6\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\).

### S.3. Complete Experimental Data

**Process tomography.** Here we plot all the other process matrices in Fig.S2-S8, with ideal theoretical values overlaid. Data in the main text are not shown here. For clarity, we use red to represent positive and blue to represent negative. Our fidelity is computed by \(\text{tr}\sqrt{\chi_{\text{exp}}\chi\sqrt{\chi_{\text{exp}}}}\). Here \(\chi_{\text{exp}}\) is the experimental process matrix and \(\chi\) is the theoretical process matrix. The average fidelity of our gates is 0.9532.

**Compression.** The results of encoding different initial states into path or polarization qubit are shown in Fig.S9-S10. Data in the main text are not shown here. The blue dashed line is the bound of min-error discrimination between two different groups of nonorthogonal states.

**Discrimination.** The results of discriminating different initial groups of states are shown in Fig.S11-S12. The blue dashed line is the bound of min-error discrimination between two different groups of nonorthogonal states. Data in the main text are not shown here.

**FIG. S2:** Identity gate. (a) the real elements. (b) the imaginary elements. The fidelity is 0.9637.
FIG. S3: Controlled-Z gate. (a) the real elements. (b) the imaginary elements. The fidelity is 0.9612.

FIG. S4: Controlled NOT gate (path control polarization). (a) the real elements. (b) the imaginary elements. The fidelity is 0.9463.

FIG. S5: Controlled Hadamard gate (polarization control path). (a) the real elements. (b) the imaginary elements. The fidelity is 0.9587.
FIG. S6: Controlled Hadamard gate (path control polarization). (a) the real elements. (b) the imaginary elements. The fidelity is 0.9467.

FIG. S7: iSWAP gate. (a) the real elements. (b) the imaginary elements. The fidelity is 0.9538.

FIG. S8: $\sqrt{SWAP}$ gate. (a) the real elements. (b) the imaginary elements. The fidelity is 0.9430.
FIG. S9: The results of encoding different initial states into path qubit. (a) encode \( \frac{1}{\sqrt{2}} |RH\rangle + \frac{1}{\sqrt{2}} |RV\rangle + \frac{1}{\sqrt{2}} |LH\rangle \) into path qubit. (b) encode \( \frac{1}{\sqrt{2}} |RH\rangle + \frac{1}{\sqrt{2}} |RV\rangle, \frac{1}{\sqrt{2}} |LH\rangle + \frac{1}{\sqrt{2}} |LV\rangle \) into path qubit. (c) encode \( \frac{1}{4} |RH\rangle - \frac{i}{4} |RV\rangle - \frac{1}{4} |LH\rangle + \frac{i}{4} |LV\rangle, \frac{1}{4} |RH\rangle + \frac{i}{4} |RV\rangle + \frac{1}{4} |LH\rangle + \frac{i}{4} |LV\rangle \) into path qubit.

FIG. S10: The results of encoding different initial states into polarization qubit. (a) encode \( \frac{1}{\sqrt{2}} |RH\rangle + \frac{1}{\sqrt{2}} |RV\rangle, \frac{1}{\sqrt{2}} |LH\rangle + \frac{1}{\sqrt{2}} |LV\rangle \) into polarization qubit. (b) encode \( \frac{1}{\sqrt{2}} |RH\rangle + \frac{1}{\sqrt{2}} |RV\rangle, \frac{1}{\sqrt{2}} |LH\rangle + \frac{1}{\sqrt{2}} |LV\rangle \) into polarization qubit. (c) encode \( \frac{1}{\sqrt{2}} |RH\rangle + \frac{1}{\sqrt{2}} |RV\rangle \) into polarization qubit. (d) encode \( \frac{1}{4} |RH\rangle - \frac{i}{4} |RV\rangle - \frac{1}{4} |LH\rangle + \frac{i}{4} |LV\rangle, \frac{1}{4} |RH\rangle + \frac{i}{4} |RV\rangle + \frac{1}{4} |LH\rangle + \frac{i}{4} |LV\rangle \) into polarization qubit. (e) encode \( \frac{1}{4} |RH\rangle - \frac{i}{4} |RV\rangle + \frac{1}{4} |LH\rangle + \frac{i}{4} |LV\rangle, \frac{1}{4} |RH\rangle + \frac{i}{4} |RV\rangle + \frac{1}{4} |LH\rangle + \frac{i}{4} |LV\rangle \) into polarization qubit.
FIG. S11: The results of discriminating different initial groups of states. Here we encode different group into different path qubits. (a) encode {\( \cos\theta_1/2|RH\rangle + \sin\theta_1/2|LV\rangle, \theta_1/2 = \pm 4^\circ \} \& {\cos\theta_{3/4}|RH\rangle + \sin\theta_{3/4}|LV\rangle, \theta_{3/4} = 60^\circ \pm 4^\circ \} \) into different path qubits. (b) encode {\( \cos\theta_1|RH\rangle + \sin\theta_1|LV\rangle, \theta_1 \in [-4^\circ, 4^\circ] \) \& {\cos\theta_2|RH\rangle + \sin\theta_2|LV\rangle, \theta_2 \in [56^\circ, 64^\circ] \) \} into different path qubits. (c) encode {\( \cos\theta_1|RH\rangle + \sin\theta_1|RV\rangle, \theta_1 \in [-2^\circ, 2^\circ] \) \& {\cos\theta_2|RH\rangle + \sin\theta_2|RV\rangle, \theta_2 \in [58^\circ, 62^\circ] \) \} into different path qubits.

FIG. S12: The results of discriminating different initial groups of states. Here we encode different group into different polarization qubits. (a) encode {\( \cos\theta_1|RH\rangle + \sin\theta_1|RV\rangle, \theta_1 \in [-2^\circ, 2^\circ] \) \& {\cos\theta_2|RH\rangle + \sin\theta_2|RV\rangle, \theta_2 \in [58^\circ, 62^\circ] \) \} into different polarization qubits. (b) encode {\( \cos\theta_1|RH\rangle + \sin\theta_1|RV\rangle, \theta_1 \in [-2^\circ, 2^\circ] \) \& {\cos\theta_2|RH\rangle + \sin\theta_2|RV\rangle, \theta_2 \in [28^\circ, 32^\circ] \) \} into different polarization qubits.