Implication of $\theta_{13}$ on Majorana Neutrino mass matrices with One Texture Zero and One Vanishing Eigenvalue

Madan Singh*, Radha Raman Gautamb, Manmohan Guptaa

a Department of physics, Centre of Advanced Study, P.U., Chandigarh, India.
b Department of physics, Himachal Pradesh University, Shimla 171005, India.

*email id: singhmadan179@gmail.com

Abstract. The latest neutrino mixing data has established the relatively large value of reactor mixing angle, $\theta_{13} \sim 9^0$. In view of that and assuming Majorana nature of neutrinos, we have re-examined the texture one–zero neutrino mass matrices with one vanishing neutrino mass in the flavor basis. We observe that for $m_1 = 0$ (Normal mass ordering), all the six one zero classes are now phenomenologically ruled out and for $m_3 = 0$ (Inverted mass ordering), only four classes out of total six are viable with the latest neutrino oscillation data at 3σ confidence level.

1. Introduction

In recent times, experiments like Double Chooz, Daya Bay and RENO Collaborations [1-3] have established a non-zero and relatively large value of the reactor mixing angle $\theta_{13}$. The result has motivated the experimentalists to pin down the long-standing problem of CP violation and mass ordering in neutrino sector. During past one year, several interesting experimental features have been observed, which though at low CL ($<2\sigma$CL), trigger optimism in lepton sector. For instance, according to recent global fit results of neutrino oscillation [4, 5], best fit value of Dirac CP-violating phase ($\delta$) lies near $270^0$.

In the absence of any convincing theory, several phenomenological schemes [6, 7, 8] have been adopted in the literature to reduce the number of free parameters in neutrino mass matrix, however texture zeros [6, 7, 8] have been the most successful among them. In flavor basis, the texture three zero was found to be incompatible with experimental data [6], whereas the analysis of two-zero textures as propounded by Frampton, Glashow, Marfatia (FGM) [6], restrict the number of experimentally compatible classes to seven. In case of texture one zero, all the six possible classes are experimentally viable [9]. The condition of one texture zero in the neutrino mass matrix is less restrictive (and hence less predictive) than the condition of two texture zeros. However, one can further reduce the number of free parameters of one texture zero Majorana neutrino mass matrices by assuming one of the neutrino masses to be zero, which is still an experimentally viable situation.
The Majorana neutrino mass matrix contains nine parameters which include three neutrino masses \( m_1, m_2, m_3 \), three mixing angles \( \theta_{12}, \theta_{23}, \theta_{13} \) and three CP violating phases \( \delta, \rho, \sigma \). The condition of one vanishing neutrino mass allows reducing the number of free parameters of neutrino mass matrix from nine to seven. In Ref. [9, 10], a particular attention has been paid on texture one zero with one vanishing neutrino mass. In the light of the precise measurement of reactor mixing angle \( \theta_{13} \) [1-3], we have re-investigated the texture one zero with one vanishing neutrino mass.

Assuming Majorana nature of neutrinos, we consider neutrino mass matrix having one zero element with one vanishing lightest neutrino mass \( (m_1 = 0 \text{ or } m_3 = 0) \). In the flavor basis (i.e. charged lepton mass matrix is diagonal), the Majorana mass matrix, being complex symmetric, contains six independent entries. If one of the elements is assumed to be zero, then we obtain six possible one-zero classes:

\[
P_1 = \begin{pmatrix} 0 & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad P_2 = \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix}, \quad P_3 = \begin{pmatrix} \times & \times & \times \\ \times & \times & 0 \\ \times & \times & \times \end{pmatrix}, \quad P_4 = \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}, \quad P_5 = \begin{pmatrix} \times & \times & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad P_6 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}
\]

where ‘\( \times \)’ denotes non-zero element.

The rest of the paper is structured as follows: In section 2, we discuss the methodology used to reconstruct the Majorana mass matrix and hence to obtain the constraints on mixing angles \( (\theta_{12}, \theta_{23}, \theta_{13}) \) and CP violating phases \( (\delta, \rho, \sigma) \), by imposing the condition of texture one zero and vanishing mass eigen value. In section 3, we present the numerical analysis. In section 4, we summarize our work.

2. Methodology

In the flavor basis, the Majorana neutrino mass matrix \( M_\nu \), depending on three neutrino masses \( (m_1, m_2, m_3) \) and the flavor mixing matrix \( U \) can be expressed as [6,7],

\[
M_\nu = U \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} U^T,
\]

where \( \lambda_i = m_i e^{2i\rho} \). \( \lambda_1 = m_1 \), \( \lambda_2 = m_2 \), \( \lambda_3 = m_3 \) and \( \rho, \sigma \) are two Majorana phases. The flavor mixing matrix \( U \) can be parameterized [6,7] like:

\[
U = \begin{pmatrix} c_{12} c_{13} & s_{13} & c_{13} s_{12} e^{-i\rho} \\ c_{12} s_{13} & c_{13} & s_{13} c_{12} e^{-i\rho} \\ -s_{12} c_{13} & s_{12} s_{13} & c_{12} s_{13} c_{13} + c_{12} c_{23} e^{-i\delta} \\ -c_{12} s_{13} & s_{12} s_{13} & -s_{13} c_{12} c_{13} + s_{13} c_{23} e^{-i\delta} \\ -s_{12} c_{13} & -c_{12} s_{13} & s_{13} c_{12} c_{13} + s_{13} c_{23} e^{-i\delta} \\ s_{12} c_{13} & -c_{12} s_{13} & -s_{13} c_{12} c_{13} + s_{13} c_{23} e^{-i\delta} \end{pmatrix}
\]

where \( c_{ij} = \cos \theta_{ij} \), \( s_{ij} = \sin \theta_{ij} \) (i, j = 1, 2, 3).

If one of the element of \( M_\nu \) is considered zero, i.e. \( M_{im} = 0 \), we obtain the following constraint equation

\[
\sum_{i=1}^{3} U_{im} U_{in} \lambda_i = 0.
\]

where \( l, m \) run over e, \( \mu \) and \( \tau \). The one vanishing neutrino mass condition give rise to two possibilities either \( m_1 = 0 \) or \( m_3 = 0 \), corresponding to normal or inverted mass ordering.
Case I: $m_1 = 0$ (normal mass ordering)

Using Eq. (4), one can deduce the relation for neutrino mass ratio $\frac{m_2}{m_3}$ and Majorana phase ($\sigma$) as

$$m_2 \quad m_3 = \left| \frac{U_{13}U_{m3}}{U_{12}U_{m2}} \right|, \quad \sigma = \frac{1}{2} \arg \left[ -\frac{m_3}{m_2} \times \frac{U_{13}U_{m3}}{U_{12}U_{m2}} \right]. \quad (5)$$

Since $m_1$ is zero, therefore Majorana phase $\rho$ become irrelevant in this case and can be dropped out.

The neutrino masses ($m_1, m_2, m_3$) can be expressed in terms of experimentally known mass squared differences ($\delta m^2 = |m_2^2 - m_1^2|, \Delta m^2 = |m_3^2 - m_2^2|$) as

$$m_1 = 0, \quad m_2 = \sqrt{\delta m^2}, \quad m_3 = \sqrt{\Delta m^2 + \delta m^2}. \quad (6)$$

Using Eqs. (5) and (6), we can constrain the ratio of solar and atmospheric neutrinos mass squared differences, $R_{\nu} (=\delta m^2/\Delta m^2)$ in terms of mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$) and Dirac CP violating phase ($\delta$)

$$R_{\nu} = \left[ 1 - \left( \frac{|U_{12}U_{m2}|^2}{|U_{13}U_{m3}|^2} \right) \right]^{-1}. \quad (7)$$

Case II: $m_3 = 0$ (Inverted mass ordering)

Using Eq. (4), one can derive the relations for neutrino mass ratio $\frac{m_2}{m_1}$ and Majorana phase difference ($\rho - \sigma$) as

$$m_2 \quad m_1 = \left| \frac{U_{13}U_{m1}}{U_{12}U_{m2}} \right|, \quad \rho - \sigma = \frac{1}{2} \arg \left[ -\frac{m_2}{m_1} \times \frac{U_{13}U_{m2}}{U_{12}U_{m1}} \right]. \quad (8)$$

Here, $\rho - \sigma$ is only Majorana physical phase. From Eq. (8), it is clear that Majorana phases ($\rho, \sigma$) are linearly co-related. The neutrino mass spectrum is given as

$$m_i = \sqrt{\Delta m^2 - \delta m^2}, \quad m_2 = \sqrt{\Delta m^2}, \quad m_3 = 0. \quad (9)$$

From Eqs. (8) and (9), the relation for $R_{\nu}$ can be obtained as

$$R_{\nu} = 1 - \left| \frac{U_{12}U_{m2}}{U_{13}U_{m1}} \right|^2. \quad (10)$$

The expression for Jarlskog rephasing parameter $J_{CP}$, which is a measure of CP violation

$$J_{CP} = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin \delta. \quad (11)$$

The effective neutrino mass term for neutrinoless double beta decay ($0\nu\beta\beta$) is given as

$$\langle m \rangle_{ee} = m_1 c_{12}^2 c_{13}^2 e^{2i\rho} + m_2 s_{12}^2 c_{13}^2 e^{2i\sigma} + m_3 s_{13}^2. \quad (12)$$
The observation of neutrinoless double beta decay would imply lepton number violation and the Majorana nature of neutrinos.

3. Numerical analysis

In this section, we discuss the numerical procedure for our calculations based on the approach given in the section 2. For singular patterns, we consider either \( m_1 = 0 \) or \( m_3 = 0 \) corresponding to normal and inverted mass ordering, respectively. To begin with, we span the parameter space of neutrino oscillation parameters \( (\delta m^2, \Delta m^2, \theta_{12}, \theta_{23}, \theta_{13}, \delta) \) lying in \( 3\sigma \) range by randomly generated points of the order of \( 10^5 \). Since the Dirac CP-violating phase \( (\delta) \) is unconstrained at \( 3\sigma \) level, the full range of \( \delta \) \( [0^0, 360^0] \) is considered. Taking into account the five oscillation parameters, namely \( \theta_{12}, \theta_{23}, \theta_{13}, \delta m^2, \Delta m^2 \) as inputs along with the constraint of one texture zero and a vanishing neutrino mass and the ranges of other experimentally unknown parameters can be constrained. Eqs. (7) and (10) incorporate the important constraints of a vanishing mass eigenvalue and one texture zero.

In the analysis, we consider a conservative limit on sum of neutrino masses \( (\Sigma) \) (i.e. \( \Sigma < 1 \text{eV} \)) at \( 3\sigma \) C.L. In addition, for effective neutrino mass term for 0\( \nu \beta \beta \) decay, we have chosen relatively large value of upper bound at \( 3\sigma \) C.L. i.e. \( \langle m \rangle_{ee} < 0.5 \text{eV} \).

In our numerical analysis, the main result is that for texture one- zero model with \( m_1 = 0 \) (normal mass ordering), all the six classes are now found to be incompatible with latest neutrino oscillation data at \( 3\sigma \) level. This is due to the relatively large value of precisely measured reactor mixing angle \( \theta_{13} \). On the other hand, only classes \( P_2, P_3, P_5, P_6 \) with \( m_3 = 0 \) are found to be compatible with latest neutrino mixing data at \( 3\sigma \) C.L. [4]. The numerical results for all these classes \( (P_2, P_3, P_5, P_6) \) are found to be approximately similar to the analysis given by Lashin et al. [9]. However, we have provided a significant discussion in order to distinguish these classes from each other. The correlation plots of these classes are given in Figures 1 and 2. For Class \( P_1 \) with \( m_3 = 0 \), with the help of Eq. (10), we find the exact expression of mass-squared difference ratio \( (R_{\nu}) \) as \( R_{\nu} = 1 - \tan^2 \theta_{12} \), showing dependence on mixing angle \( \theta_{12} \). Using \( 3\sigma \) range of \( \theta_{12} \) [4], we obtain \( 0.40 \leq R_{\nu} \leq 0.61 \), which is in conflict with the experimental range of \( R_{\nu} \). Therefore, class \( P_1 \) is ruled out with latest mixing data. Similarly, one can check that class \( P_4 \) also remains ruled out with current experimental data as also predicted in Ref. [9]. The two independently viable classes \( (P_2, P_5) \) have been discussed below:

**Class \( P_2 \) with \( m_3 = 0 \):** Using best fit value of current experimental data [5], we obtain, \( |\rho - \sigma| \approx 0.1^0 \), \( \delta \approx 87.61^0 \). Therefore, \( \rho \approx \sigma \), as also shown in Figure 1 (a). The best fit value of effective mass term, \( \langle m \rangle_{ee} \approx 0.0487 \text{eV} \), is found to be \( O \left( 10^{-5} \right) \), which lies well within the sensitivity limit of future neutrinoless double beta decay experiment. The phenomenological implications of class \( P_1 \) is similar to class \( P_2 \) due to permutation symmetry [9].

**Class \( P_5 \) with \( m_3 = 0 \):** The correlation plot [Figure 2 (b)] predicts \( \delta \approx 90^0, 270^0 \). Hence, this class allows approximate maximal CP violation. The preliminary hint given by latest global fits data of neutrino oscillation [5, 6], on the best fit value of \( \delta \) coincidently supports our numerical result. From Figure 2(d), it is clear that the parameter space of Jarlskog rephasing parameter \( (J_{CP}) \) excludes the possibility of \( J_{CP} = 0 \). On the other hand, Majorana phases \( (\rho, \sigma) \) remain unrestricted with latest experimental data, allowing the full range \( [0^0, 180^0] \) [Figure 2(a)]. The phenomenological implications of class \( P_5 \) is similar to class \( P_3 \) due to permutation symmetry. All these features discussed above are quite helpful in distinguishing the two independent experimentally viable classes \( (P_2, P_5) \).

Interestingly, out of four viable classes \( (P_2, P_3, P_5, P_6) \) at \( 3\sigma \) C.L, classes \( (P_2, P_5) \) are found to be ruled out at \( 1\sigma \) C.L, while remaining viable classes \( (P_3, P_6) \) predict \( \delta \approx 270^0 \) at same confidence level.
Figure 1. Correlation plots for class $P_2$ with $m_3=0$ (inverted mass ordering) at $3\sigma$ confidence level.

Figure 2. Correlation plots for class $P_5$ with $m_3=0$ (inverted mass ordering) at $3\sigma$ confidence level.
4. Summary and conclusions
We have done a systematic study of texture one-zero classes with one vanishing neutrino mass using latest global fits for neutrino oscillation parameters. We observe that the all the six classes with normal mass ordering (NO) are now ruled out at 3σ confidence level, whereas in case of inverted mass ordering, only four classes (P2, P3, P5, P6) are found to be viable with latest experimental data [4]. For these classes, the CP violating Majorana phases (ρ, σ) remain unconstrained; however, they exhibit strong linear correlation. In particular, for classes P5 and P6, the Dirac CP-violating phase (δ) is observed to be very close to maximal CP-violation i.e. δ ≈ 90°, 270° (δ ≈ 270°) at 3σ (1σ) C.L. This prediction is significant considering the fact that latest hints on best fit value of CP-violating phase (δ) lies in proximity of 270°. Since no experiment has been able to fix absolute neutrino mass scale, therefore, the possibility of vanishing lowest neutrino mass cannot be ruled out at present. The future experiments will test the survivability of singular texture one zero classes along with their predictions on neutrino masses and mixings.

References

[1] Y. Abe et al., [Double Chooz collaboration], Phys. Rev. Lett. 108, 131801 (2012), arXiv: 1112.6353 [hep-ex].
[2] F. P. An et al., [Daya Bay collaboration], Phys. Rev. Lett. 108, 171803 (2012), arXiv: 1203.1669 [hep-ex].
[3] Soo-Bong Kim, for RENO collaboration, Phys. Rev. Lett. 108, 191802 (2012), arXiv:1204.0626[hep-ex].
[4] D. V. Forero, M. Trlota, J. W. F. Valle, Phys. Rev. D 90, 093006 (2014), arXiv:1405.7540 [hep-ph].
[5] M. C. Gonzalez-Garcia, M. Maltoni, J. Salvado and T. Schwetz, JHEP 1411, 052 (2014) , arXiv:1409.5439 [hep-ph].
[6] Paul H. Frampton, Sheldon L. Glashow and Danny Marfatia, Phys. Lett. B 536, 79 (2002), hep-ph/0201008.
[7] Zhi-zhong Xing, Phys. Lett. B 530, 159 (2002), hep-ph/0201151.
[8] P. O. Ludl, S. Morisi, E. Peinado, Nucl. Phys. B 857, 411 (2012); D. Meloni, G. Blankenburg, Nucl. Phys. B, 867, 749 (2013); W. Grimus, P. O. Ludl, J. Phys. G, 40, 055003 (2013); X.Liu, S. Zhou, Int. J. Mod. Phys. A, 28 (2013) 1350040; J.Liao, D. Marfatia and K. Whisnant, JHEP 09, 013 (2014); L. M. Cebola, D. E. Costa, R. G. Felipe, arXiv: 1504.06594 [hep-ph], H. Fritzsch, Zhi-zhong Xing, S. Zhou, JHEP 1109, 083 (2011), arXiv: 1108.4534 [hep-ph].
[9] E. I. Lashin and N. Chamoun, Phys. Rev. D85, 113011 (2012), arXiv:1108.4010 [hep-ph].
[10] Z.Z. Xing; Phys. Rev. D 69, 013006 (2004).