Quasi-particle spectrum around half-quantum vortices (HQVs) in triplet superconductors

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Abstract. It is shown recently triplet superconductors like Sr₂RuO₄, UPt₃ and PrOs₄Sb₁₂ have Majorana fermion, d-soliton and half-quantum vortices (HQVs). The Bogoliubov-de Gennes (BdG) equation necessary for the present purpose is derived following Ivanov. For $H_P$ and d-vector in the ab plane the one of HQVs has a series of bound states including the zero-energy-mode or Majorana fermion while the other has no bound state for a single spin component. We illustrate this for p-wave SC.

1. Introduction

Triplet superconductivity is well established in many materials. They are UPt₃, Sr₂RuO₄, PrOs₄Sb₁₂. For these superconductors, superconducting symmetry is $^3$He-A phase like chiral p- or chiral f-wave state [1,2]. The order parameter for the chiral p-wave is written as,

$$\Delta_{\alpha\beta}(k) = d(k) \cdot (\sigma_i \sigma_j)_{\alpha\beta},$$

(1)

with

$$d(k) = \Delta \hat{d}(k, \pm i k),$$

(2)

where $\sigma_{\mu}$ are Pauli spin matrices and $\alpha$ and $\beta$ are spin indices.

H.-Y. Kee et al. [3,4] showed in the chiral $p(k, \pm i k)$-wave superconductors, there appears $\hat{d}$-soliton with two half-quantum vortices at both end of the soliton. This half-quantum vortex was predicted by Salomaa and Volovic [5] for $^3$He-A phase. The stability of the half-quantum vortex was studied by Kee and Maki [6].

Also, in the chiral triplet superconductors, zero-energy bound states around a single vortex appears [4,7]. It is also expected around a half-quantum vortex.

In this study, using the Bogoliubov-de Gennes equation, we analyze the quasi-particle structures around a half-quantum vortex.
2. Model

In this study, we only consider $p \left( k_x + i k_y \right)$-wave case, because for another $p \left( k_x - i k_y \right)$-wave case, the difference appears in the dependence on the direction of external magnetic field. And we consider a large disk with a radius $R$ and put only a half quantum vortex, not the pair of half-quantum vortices which is connected by $\hat{d}$-soliton, at the center of the disk. For simplicity, we assume the d-vectors as

$$\hat{d} = \cos \theta \hat{x} + \sin \theta \hat{y},$$

where $\theta$ is the azimuthal angle and $\hat{x}$ and $\hat{y}$ are unit vectors in the plane of the disk, and the order parameter as,

$$\Delta_{\sigma \beta} = \begin{pmatrix} -e^{i \theta} & 0 \\ 0 & e^{-i \theta} \end{pmatrix} \Delta_\sigma (r),$$

where $\alpha$ is the winding number of vortex, which shows the direction of magnetic field. In this case, the Bogoliubov-de Gennes equation becomes as,

$$E_n u_{\sigma n}(r) = \frac{1}{2m} \left( p - \frac{e}{c} A \right)^2 - \mu \left[ u_{\sigma n}(r) + \frac{1}{k_F} \begin{pmatrix} (i \partial_x + \partial_y) \Delta_\sigma \left( -e^{-i (\alpha \sigma \sigma)} \right) \end{pmatrix} v_{\sigma n}(r) + 2 \Delta_\sigma \begin{pmatrix} -e^{-i (\alpha \sigma \sigma)} (i \partial_x + \partial_y) \end{pmatrix} u_{\sigma n}(r) \right],$$

and

$$E_n v_{\sigma n}(r) = 2 \Delta_\sigma \begin{pmatrix} -e^{-i (\alpha \sigma \sigma)} (i \partial_x + \partial_y) \end{pmatrix} v_{\sigma n}(r) + \frac{1}{k_F} \begin{pmatrix} (i \partial_x + \partial_y) \Delta_\sigma \left( -e^{i (\alpha \sigma \sigma)} \right) \end{pmatrix} u_{\sigma n}(r) + 2 \Delta_\sigma \begin{pmatrix} -e^{i (\alpha \sigma \sigma)} (i \partial_x + \partial_y) \end{pmatrix} u_{\sigma n}(r),$$

where $\sigma$ is the spin index and $u_{\sigma n}(r)$ and $v_{\sigma n}(r)$ are electron and hole parts of quasi-particle wave function, respectively. Note that if winding number and spin are opposite, i.e. $\alpha = -\sigma$, then the order parameter does not depend on $\theta$ and shows no phase singularity. Therefore, the order parameter for such spin does not becomes zero at the center of vortex. This was pointed out by Ivanov [8]. So, we assume the order parameter is uniform for non-singular phase and $\Delta_0 \tanh \left( \gamma/\xi \right)$ for singular phase, respectively.

Using the Fourier-Bessel expansion as Gygi and Schlüter did for conventional s-wave superconductors [9], we can solve the eigenvalue equation numerically, and obtain the local density of states as,

$$N(r, E) = \sum_n \begin{pmatrix} u_\sigma (r) \end{pmatrix} f \left( E - E_n \right) + \begin{pmatrix} v_\sigma (r) \end{pmatrix} f \left( E + E_n \right).$$

3. Results

Here, we show the numerical results for $p \left( k_x + i k_y \right)$-wave triplet superconductors with the conditions $R/\xi_0 = 10$ and $k_F \xi_0 = 5.0$, where $k_F$ is the Fermi wave number and $\xi_0$ is the coherence length.
In figure 1, we show the local density of states (LDOS) around a half-quantum vortex with winding number $\alpha = 1$. This LDOS is similar to that around a single vortex. The quasi-particle bound state peak locates at the $E = 0$. This is because the Majorana fermion appears at exactly zero energy. But its peak height becomes rather low compared to the single quantum vortex case. In figure 2, we show the LDOS around a half-quantum vortex with winding number $\alpha = -1$. As the single vortex case, central peak moves toward positive energy. The difference of these two winding numbers vortices comes from the chirality of $p(\vec{k}_x + i\vec{k}_y)$-wave state.

Figure 3 Local density of states around a half-quantum vortex with winding number $\alpha = 1$ for up spin (a) and down spin (b).

Figure 4 Local density of states around a half-quantum vortex with winding number $\alpha = 1$ for up spin (a) and down spin (b).
In order to clarify the difference between half-quantum vortex state and single vortex states, we show the spin-dependent LDOS in figure 3 for winding number $\alpha = 1$ and in figure 4 for $\alpha = -1$. In figure 3, for up spin quasi-particles LDOS there appear bound states, but no such state appears for down spin LDOS, vice versa in figure 4.

4. Summary
We have obtained the quasi-particle structures around a half-quantum vortex in the chiral $p(k_x \pm ik_y)$-wave triplet superconductors. Similarly, chiral $f(k_x \pm ik_y)\cos ck_z$-wave case is also calculated. These results will be published elsewhere. In this study, we assume the $\hat{d}$ vector changes with circular symmetry around the vortex and also assumes order parameter variation with $r$ is assumed. Inclusion of $\hat{d}$-soliton structure and self-consistent calculation are future problems.

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