Misconception in Theory of Quantum Key Distribution  
-Reply to Renner-

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It has been pointed out by Yuen that the security theory of quantum key distribution (QKD) guided by Shor-Preskill theory has serious defects, in particular their key rate theory is not correct. Theory groups of QKD tried to improve several defects. Especially, Renner employed trace distance and quantum leftover Hash Lemma. However, the present theory encountered a problem of a quantitative evaluation of security. To cope with it, he uses a wrong interpretation on the trace distance and its level $\varepsilon_{\text{sec}}$, and justifies the unconditional security of own system when $\varepsilon_{\text{sec}}$ is $10^{-6} \sim 10^{-20}$.

In this paper, we discuss the following problems. What is the origin of the misconception of the present theory? How does the present theory lead to the misconception? To show their process toward the misconception, Koashi-Preskill’s theory which has a typical misconception is examined. A main point of our comment is that QKD theory ignores “the security requirement against attacker” which is necessary to compare whole encryption schemes from classical to quantum. To clarify it, we emphasize that the trace distance itself cannot have any operational meaning such as failure probability, and it is only mathematical tool as a measure of closeness. As a result, it is given that the security with above values derived from their formulation means nothing in the general cryptological sense. In addition, I point out that a comment by Bennett and Riedel on unconditional security of QKD is not correct. Also, I point out that the experimental systems of groups of Los Alamos, Toshiba-UK, NICT, and others cannot have security guarantee even in future.

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I. INTRODUCTION

The purpose of this paper is to stimulate the real discussion among researchers who are interested in quantum key distribution (QKD).

So far, taking the recent progress of theory of QKD [1,2] into account, I have tried to explain “What is the main point of misconception on the security theory of QKD” in meetings of communities consisting of Information theory, Cryptography, Mathematics, and Physics [3]. Recently, Renner has responded [4] to my paper, Yuen has responded to it and gave the detailed explanation on Renner’s fault [5]. Here, however, it is useful for the general researchers again to explain “What is a main point of the argument?”, because Renner still believes that the system has sufficient security when the level of the trace distance $\varepsilon_{\text{sec}}$ as the security criterion has a order of $10^{-20}$ [4]. His reasoning is that this is very small as a probability in physical world. A main subject in this paper is to explain that his $\varepsilon_{\text{sec}}$ is not a probability for event.

Our claim is as follows:

Claim

(a) The present theory does not deal with the security evaluation for QKD correctly. At beginning, the correct concept in evaluation of secrecy for key distribution had not been employed. That is, they did not care “what are they evaluating?”. Just it was “information of Eve” which is not an operationally meaningful concept in cryptology. The information theoretic security in key distribution has to be measured by success probability of estimation by Eve according to Shannon [6] and by her bit error rate (BER) [5] when she makes a wrong estimate of the key.

(b) The present security theory is originated by Shor-Preskill [7] suggesting post processing. Renner introduced a trace distance [8] as the security criterion to replace the mutual information criterion which is not good quantum mechanically. To justify the formulation, he was forced to employ the failure probability interpretation of the trace distance (or its level). However, it has been already pointed out that such an interpretation does not work in view of probability theory. That is, the trace distance (or its level) itself cannot have interpretation as probability.

(c) Why does Renner insist such interpretations in the security evaluation? The reason is that the present QKD protocol by post processing can bound the trace distance or other criteria only by very large value in the sense of security, depending on phase error estimation and others in BB-84 protocol. So, in order to justify the quantitative value in their theory, they insist on the wrong interpretation of the trace distance such as “failure probability”. By the correct theory of the trace distance, the present theory can be re-evaluated. So the present theory of QKD does not sufficiently ensure the
security and uniformity (against Eve) of generated key sequence by QKD.
(d) Following our observations on the present theory on QKD, the protocol based on post processing cannot provide sufficient “security and uniformity” which is required in the common cryptology [3].

The research groups of information theory and cryptography in Japan already understood our claim by my explanations in several meetings, and they also understood that they should take into account the serious defect of the security concept in QKD. However, still the research group of physics ignores the main claim.

To justify our claim, in the following, I will verify concretely the misconceptions of the formalism for “the security” of QKD groups in physics by using Koashi’s theory as an example of the theory. To proceed it, I analyze the following technical points.

(1) Theory groups of QKD in physics do not consider the problem “What is a measure of security in Shannon’s information theoretics cryptography?”
(2) The origin of the misconception comes from the fact that they do not deal with a concept of security requirement against attacker Eve, despite that “near uniformity of generated key sequence against the attacker Eve” has to be ensured. This is related with (1).
(3) The present theory deals only with privacy amplification to claim the uniformity. But, the idea of privacy amplification was introduced to create a scale change or scale shift of mutual information between the key of Alice and Eve’s measurement results in the original papers. Although it seems that Shannon’s mutual information can be reduced by such an operation, it does not mean that the real information of Eve is deleted. Also they ignore the fact that the privacy amplification does not improve the success probability of estimation on whole key (or uniformity of key sequence against Eve).

II. MAIN LOGIC OF SECURITY THEORY OF QKD

A. Basic concept of the present theory

Here several important sentences claimed by the QKD groups are listed as follows [9]:
(1) The task of the quantum key distribution theory in the case of finite key is to prove that the key is secure against any wiretapping of Eve, up to a small failure probability.
(2) Eve’s knowledge can be bounded by the probability that she correctly guesses Alice’s measurement outcomes. This is expressed by the conditional smooth mini-entropy of the data from which the key is generated given Eve’s quantum system.
(3) The finite case is different from the case of asymptotic rate in which perfect security in the limit for n to infinite is considered.

B. Structure of security theory of QKD

According to Tomamichel et al [9], the final structure of the security theory to do the above sentences is as follows:
In order to define the secrecy of a key, we consider

\[ d = \min \frac{1}{2} ||\rho_{SE} - \omega_s \otimes \sigma_E||_1 \leq \Delta \]  

(1)

A key is called \( \Delta \)-secret from E if it is \( \Delta \)-close to the ideal uniform key.

**Definition**[9]: A QKD protocol is called \( \varepsilon_{\text{sec}} \)-secret if it is \( \varepsilon_{\text{sec}} \)-indistinguishable from a secret protocol where

\[ (1 - p_{\text{abort}})\Delta \leq \varepsilon_{\text{sec}} \]  

(2)

The main misconception is this definition. The concept of the secrecy of the protocol is not appropriate to evaluate the security of the real cryptological function, according to the concept of Shannon’s information theoretical security. This is a kind of play in mathematical school. However, in order to justify this logic, they provided the following structure.[10]

(1) The secrecy of BB-84 follows from the observation that, Alice has a choice of encoding a \( n \) uniform bits in either the X or Z basis, then only one of the following two things can be true: either Bob is able to estimate Alice’s bits accurately if she prepared in the Z basis or Eve is able to guess Alice’s bits accurately if she prepared in the X basis.

(2) Our analysis employs the Quantum Leftover Hash Lemma, which gives a direct operational meaning to the smooth mini-entropy. It asserts that, using a random Hash function, it is possible to extract a \( \Delta \)-secret key of length from X, where

\[ \Delta = \min \frac{1}{2} \sqrt{2^{l - H_E'(X|E')} + \epsilon'} \]  

(3)

\( \epsilon' \) is a smoothing parameter.

(3) Following theorem gives a sufficient condition for which a protocol \( \Phi \) is \( \varepsilon_{\text{sec}} \)-secret. The minimum value \( \varepsilon_{\text{sec}} \) for which it is \( \varepsilon_{\text{sec}} \)-secret is called the secrecy of the protocol.

**Theorem**(R. 1)[10]: The protocol \( \Phi \) is \( \varepsilon_{\text{sec}} \)-secret for some \( \varepsilon_{\text{sec}} > 0 \) if \( l \) satisfies

\[ l \leq \max [n(q-h(Q_{tol} + \mu(\epsilon))) - 2 \log \frac{1}{2\epsilon} - \text{leack}_{EC} - \log \frac{2}{\varepsilon_{\text{cor}}} ] \]  

(4)

where \( q, Q, \mu, \) and others are given in [10].
III. FUNDAMENTAL DEFECT OF QKD

A. Security

I point out repeatedly here that they have two misconceptions as follows:

(a) The information theoretic security can be evaluated by the concept of the secrecy of protocol.

(b) There is a reasoning for an interpretation of failure probability of the trace distance in their formulation which justifies the concept of the secrecy of protocol.

The main purpose of this section is to explain why the above two claims are misconception? To do so, I will make clear first that theory groups of QKD misuse the probability theory.

QKD groups assign an interpretation of binary decision to all probability measures in their logical process. A reason of why is that they start from the probability for the abort of the protocol in the stage of threshold decision based on quantum bit error and they are forced to reach own definition of the security: “the secrecy of protocol”. Thus, they stick to binary events in all logical process. This is an origin of the misconception as explained below.

Unfortunately, Eq(1) means neither the binary decision nor the security of protocol. Just it means the closeness. However, they had started the formulation, keeping the above binary concept. After formulation, they realized that they have to include all concepts such as “uniformity”, “Eve’s guessing probability”, and “indistinguishability” which are not binary event. To proceed it, first, they employ the smooth min entropy which characterizes the average probability that Eve guess X correctly. Then this is used to give so called Quantum Leftover Hash Lemma which discuss a relation between privacy amplification and obtainable key length. Such a theory is also a measure of uniformity of key sequence.

**Theorem(Y − 1)[5]FLet \(|K_G|\) be the length of key. The success probability of estimation for the whole key can be bounded as follows:**

\[ P_{\text{suc}}(K_G) \leq 2^{-|K_G|} + d \] (5)

\[ d = \min \frac{1}{2} ||\sigma_{SE} - \omega_s \otimes \sigma_E||_1 \] (6)

where \(d\) is the trace distance.

Thus, one can re-evaluate the QKD system (designed by the present theory) based on the above relation between the trace distance and the correct measure of ITS.

Here one can consider two cases such as

\[ A: \quad d \sim 2^{-|K_G|}, \] (7)

\[ B: \quad d \gg 2^{-|K_G|} \] (8)

If the system has the case A, it can be regarded as sufficiently secure. But in real situation, we have the case B. Then we have

**Theorem(Y − 2)[5]FLet \(\varepsilon_{sec}\) be the average level of \(d\) in the Renner’s theory.**

\[ d = \min \frac{1}{2} ||\sigma_{SE} - \omega_s \otimes \sigma_E||_1 \leq \varepsilon_{sec} \] (9)

Then one has

\[ P_{\text{suc}}(K_G) \leq \varepsilon_F \] (10)

where \(\varepsilon_F = \varepsilon_{sec}^{(1/3)}\) from Markov inequality.

Thus, in the case \(\varepsilon_{sec} = 10^{-20}\) for \(|K_G| = 10^4\),

\[ P_{\text{suc}}(K_G) \leq 10^{-20/3} \sim 10^{-7} \] (11)

If one wants to apply it to one time pad to guarantee \(BER = \frac{1}{2}\) for any segment of the key, the uniformity is required as follows:

\[ P_{\text{suc}}(K_G) = 2^{-10000} \sim 10^{-3000} \] (12)

Even if one can provide \(\varepsilon = 10^{-20}\) as individual value in Renner’s theory, one has \(P_{\text{suc}}(K_G) \sim 10^{-3000}\) vs \(P_{\text{suc}}(K_G) \sim 10^{-20}\).

Of course, one can relax the uniformity in practice, but the difference is so large. Thus one cannot say it is secure. On the second misconception, I will explain in the next section IV, showing the concrete example.
B. Key rate

The rate theory is formulated as follows: They believe that the level of the trace distance assumed as $\varepsilon_{\text{sec}} = 10^{-6} \sim 10^{-10}$ is good enough already. So they say that the problem is how many key bits can be extracted under this level of the security. The solution is Eq(4) in their formalism, and the privacy amplification plays the main role. Thus one can see that the result of key rate in their discussions is effective under the value of $d$(trace distance). However, the uniformity of key against Eve is given by her $M$-ary quantum detection procedure before the privacy amplification. The $P_{\text{suc}}(K_G)$ cannot be improved by any privacy amplification. So the uniformity against Eve is fixed before the privacy amplification. Thus, the relation between the key rate and privacy amplification has to be reconsidered. Here one has the following result.

**Theorem (Y - 3)** [5] Let $\varepsilon_{\text{sec}}$ be the level of $d$ in the Renner’s theory and $n$ be the key length. Then one has

$$P_{\text{suc}}(K_G) \leq \varepsilon_F$$

The key rate $R$ under the uniformity guarantee is given by

$$R \sim \lambda$$

where

$$P_{\text{suc}}(K_G) \sim \varepsilon_F \equiv 2^{-\lambda n}$$

This is completely different from the key rate theory based on the present theory. Thus, Eq(4) is not able to provide the sufficient condition for extracted key by the theorem(R-1).

C. Asymptotic vs non-asymptotic key rate

The present theory has a strange problem such as the title of this section. This is because the key rate is discussed by the ECC and PAC. However, as one can see in the above section, the uniformity is given without ECC and PAC, so the problem such as the title of this section has no meaning. If $\varepsilon_{\text{sec}} = 10^{-20}$ for the key length $10^4$, the key length under the uniformity is about 30 bits and the rate becomes zero when bits for the leak from ECC and for the authentication are subtracted.

In sum, we emphasize that the present theory is not treating the true key rate of QKD.

IV. CONCRETE EXAMPLE OF PROCESS OF MISCONCEPTION

**Koashi-Preskill theory**

Here we describe the structure of Koashi-Preskill theory on QKD [11]. First Koashi explains the concept of security criterion as follows: “The target of the security theory of QKD is to analyze an information such as a function of parameters estimated from quantum channel” to design privacy amplification.”

In order to proceed such an idea, he provides the following mathematical basis.

1. Let $\rho_1, \rho_2$ be two density operators. In the decision of two density operators, the maximum success probability is given by

$$\frac{1}{2} + \frac{1}{4||\rho_1 - \rho_2||}$$

From this, one can understand that the trace distance has an operational meaning such as “how much the maximum value of the success probability can exceed $\frac{1}{2}$.”

2. There is a following relation between fidelity and trace distance.

$$1 - F(\rho - \sigma)^{1/2} \leq \frac{1}{2}||\rho - \sigma|| \leq (1 - F(\rho - \sigma))^{1/2}$$

3. Let us assume that there is a little different state from the maximum entanglement state, and assume that the fidelity is given by

$$F(\rho_{AB}, \rho_{\text{max}}) \geq 1 - \epsilon$$

The fidelity between state measured by Alice-Bob and ideal state $\tau_{ABE}$ is given by

$$F(\sigma_{ABE}, \tau_{ABE}) \geq 1 - \epsilon$$

Koashi says that the fidelity has a strong meaning as follows:

Let us use the generated key with the relation of Eq(19) in any system. But, the probability “that different situation from the situation that the ideal key is used outbreaks” is $\epsilon$.

Based on such a concept, he proceeds the formulation of the security analysis. That is, Keys of $n$ bits are described by $M = 2^n$ dimensional Hilbert space. The density operator for the whole system is given by

$$\sigma_{ABE} = \sum_i \sum_j p_{i,j} |i, j \rangle \langle i, j |_{AB} \otimes \rho_E^{(i,j)}$$

Here two protocols : X-protocol and Z-protocol are defined as follows:

1. Z-protocol: Alice and Bob determine values of keys by classical communication. The failure probability of the protocol is evaluated by

$$\eta_Z = 1 - \frac{1}{M} \sum_i \sum_i p_{i,i}$$

(This is average error probability among $M$ events, and it does not mean the binary event.)
(2) X-protocol: The state of Alice is purified by classical communication. The ideal state is $|0\rangle$, and the final state of Alice is $\sigma_A$. Then the the failure probability is given by

$$\eta_X = 1 - <0|\sigma_A|0>$$ \hspace{1cm} (22)

Here he employs the trace distance as the measure of difference between the real one and the ideal as follows:

$$\eta_{key} = ||\rho_{ABE} - \rho_{ABE}^{(key)}||$$ \hspace{1cm} (23)

By using these concepts, he shows the following theorem.

**Theorem(K - 1)**[10]: Let $\eta_X$ and $\eta_Z$ be the failure probabilities of both protocols. The key in the Z-protocol satisfies

$$\eta_{key} \leq 2\eta_Z + 2\sqrt{\eta_X}$$ \hspace{1cm} (24)

Thus, he showed a relation between the trace distance and some $\eta$ and interpreted it as the failure probability of the protocol. In fact, Tamaki who is a group member of Koashi claims in his paper that the protocol will fail just one time at the million trials when the level of the trace distance is $10^{-6}$ in that scheme.

Again, one cannot see the reason that each probability measure in the process of the security proof means binary such as failure or success.

Let us mention the problem of the key rate. In this stage, one needs the error correction code and privacy amplification. Here values for $\eta_Z$ and $\eta_X$ are assumed, and the security measure is fixed by Eq(24). The problem is how many bits can be extracted from the shifted key under the Eq(24). Koashi says that the privacy amplification reduces the correlation between key sequence and Eve’s measurement results. However, this correlation provides only an apparent uniformity of key against Eve which can be given by a success probability of estimation to the key. This has clearly an operational meaning. Consequently, “They ignored the Shannon’s security concept”.

As I emphasized in this paper, the relation between such a true criterion and trace distance has been given (see Eq(5)). Thus, one can re-evaluate the quantitative performance of security in the present theory, because they discuss the level of the trace distance. As a result, one can see that the security by the present theory is meaningless from Eqs(11),(12).

In addition, the final key rate $R_F$ after subtractions of the leak from ECC and message authentication is as follows:

$$R_F \sim 0$$ \hspace{1cm} (26)

(2) iid assumption and active attack

In the present theory and its application, so many researchers keep in mind a serious assumption so called “iid” in own model, where “iid” means independent and identically distributed. Especially, they believe the effectiveness of privacy amplification under such an assumption. However, in the QKD model with imperfection, one has to allows Eve to employ an attack so called active attack such as bit change or correlation creation. So one cannot expect “iid” in shared key sequence. Some researchers take it into account, but still their concern is limited to effect for privacy amplification only [12]. One should go back to the discussion of the trace distance under the active attack, and start under the understanding of the fact that the trace distance describes a degree of uniformity of the real qubit sequence, not failure probability.
VI. COMMENT ON EXPERIMENTS

I here point out that the experimental groups of R.Hughes [13], Z.Yuan and A.J.Shields [14], M.Sasaki, A.Zeilinger and others do not provide the quantitative security evaluation for own systems, and also do not compare own system with the present theory of the security. That is, the experiments are independent of the theory, and the progress of the theory is ignored. Especially A.J.Shields claims [15] that “High bit rate QKD with 100dB security”. Why 1 Mbit/sec is high bit rate nowadays. Also DB means the relative, but the security requires absolute value. The technical requirement in the real world is at least 1 Gbit/sec ~ 10 Gbit/sec if they want to apply the QKD to one time pad encryption. Even if the present scheme of QKD are implemented, these insecure and insufficient systems can be easily replaced by other technology.

On the other hand, recently Bennett and Riedel gave a comment in March 2013 [16] such that “QKD boasts unconditional security even in the presence of realistic noise”, referring Mayers [17] et al, and “that the techniques have matured enough that small commercial implementations have been explored.” As I have clarified in this paper, there is no proof even for own definition of unconditional security. Thus, these comments are not justified. I hope that we have an occasion for discussion on this issue each other.

VII. CONCLUSIONS

I have described the origin of the misconception in the security theory of quantum key distribution. Especially, I pointed out that there are many misconceptions in reasonsings in the process of the security proof and that the security designed by the present theory may be weaker than that of the classical cipher, when the system is evaluated correctly.

In order to develop the quantum key distribution and related subjects, one should consider more carefully theoretical formulation based on quantum detection theory, following “the original meaning of information theoretical security of Shannon”. I hope that the fruitful discussion begins by this article.

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Appendix

A. Discussions in QIT meeting

[Q-1]: I do not agree with your talk. There are only two results for security concept in QKD. That is, it is success or failure of protocol. It is enough. (QKD theorist)
[A-1]: You should prove it mathematically, because I am showing that the trace distance cannot have the meaning of the probability.

[Q-2]: Although my field is not QKD, I agree with your comment for the mathematical issue on the trace distance. In general, the trace distance is just value as closeness, it is not probability indeed. (Theoretical physicist-1)

[Q-3]: I think that “the concept of the security” is vague in any field. How to define the concept of security in physics. (Experimental physicist)

[A-2]: QKD is cryptology and not physics. So the researcher of QKD have to formulate the security of system as the cryptology, and compare it with whole schemes in cryptology. In fact, the definition of unconditional security by QKD community has no meaning in cryptology.

[Q-4]: What is the difference between unconditional security and information theoretic security. (Theoretical physicist-2)

[A-3]: The term of “unconditional security” was invented by QKD community such that “when mutual information of A-E channel or trace distance is bounded (sufficiently small) by certain physical parameter, the system is called unconditionally secure.” The community thought that this is equivalent to the information theoretic security, because it seems to be independent of the computational power. But this is valid only in the community of QKD. That is, it has no operational meaning as the security in the cryptology.

[Q-5]: Do you mean that the mutual information (MI) and also trace distance are not appropriate criteria. (Theoretical physicist-2)

[A-4]: The meaning of the information theoretic security cannot be changed in mathematical scheme and physical scheme.

The success probability of estimation for whole shared key by QKD is a natural measure of information theoretic security from Shannon’s concept. MI and also trace distance can be used as mathematical tool to bound the correct criteria, but these cannot have the operational meaning. The present theory of QKD is trying to directly give the operational meaning to them. That is, they insist that the trace distance is the failure probability.

[Q-6]: What is the success probability of estimation? (Theoretical physicist-3)

[A-5]: In “one time pad” without key distribution, the channel model between plaintext sequence X and ciphertext sequence C is described by conditional probability (success probability of estimation). If the channel matrix elements of X-C channel are completely uniform for all kinds of ciphertext sequences, it means the perfect secrecy. That is, \( P(X|C) = P(X), \forall C \). If the X-C channel has partial uniformity, it has partial information theoretic security. So it provides the quantitative evaluation. Here, a main origin of the security comes from the uniformity of pre-shared key sequence.

In the case of QKD, the measurements on key sequence of Eve \( Y_E \) on \( K_G \) suffer the error by quantum effects. This is also described by the channel model based on success probability of estimation \( P(K_G|Y_E) \). The origin of the security comes from quantum effects of \( K_G-Y_E \) channel. Thus, by following Shannon theory, QKD community should employ the criterion such as \( P(K_G|Y_E) \) based on the detection to each sequence in whole possibility \( 2^n = M \) at beginning.

[Q-7]: You mean when the present QKD is re-evaluated by the correct criterion, it was extremely bad or weaker than the classical one, even if QKD groups claim the unconditional security (by own definition) or unbreakable. (Theoretical physicist-3)

[A-6]: Yes, it is true. They want to enjoy the terms of intrusion detection by uncertainty principle, quantum loophole, and so on, but these are parameters only to quantify the security criterion. They do not care the real evaluation in cryptology.

[Q-8]: It seems that QKD groups do not follow Shannon’s basic concept, and that they want to lock the security issue only in physics. Can you analyze why the community of QKD had taken the present way? (Information theorist-1)

[A-7]: I am not sure. They thought that QKD is physics. As you know, QKD is a function in the cryptology. That is, it provides only key distribution for symmetric key cipher, though the origin of the basic security comes from just physical(quantum) phenomena. As a result, they ignored the regular course of the cryptology, as you say.