Path and Star Decomposition of Knodel and Fibonacci Digraphs

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Abstract. In broadcasting and gossiping, communication structures are modelled using graphs. In simplex model, a communication link sends messages in a particular direction. The simplex network is modelled by directed graphs. Certain Knodel graphs are optimal broadcast graphs. This article defines the two layer representation of Knodel digraphs and Fibonacci digraphs-both are symmetric regular bipartite digraphs. A Knodel digraph is a symmetric digraph produced by replacing each edge of an undirected Knodel graph by a symmetric pair of directed edges. A Fibonacci digraph is a symmetric digraph produced by replacing each edge of an undirected Fibonacci graph by a symmetric pair of directed edges. Decomposition of a given graph is the partitioning of edge set so that the given graph is split in to sub-structures. If all such sub-structures are isomorphic, the decomposition is isomorphic decomposition. The isomorphic decomposition of the Knodel digraphs and Fibonacci digraphs in to isomorphic paths and stars is presented. The star decomposition of both Knodel digraph and Fibonacci digraph leads to the decomposition of equi-bipartite complete digraphs in to two classes of stars.

1. Introduction
Broadcasting and Gossiping are two important problems in interconnection networks for disseminating information between nodes. A network can be modelled as a graph $G = (V,E)$, where $V$ is the set of nodes and $E$ is the set of communication lines. Walter Knodel [6] introduced these structures in 1975. Knodel graphs are regular bipartite graphs of even order $n$ with degree $1 \leq d \leq \lceil \log_2 n \rceil$ and are denoted by $W_{d,n}$. They are formally defined by Fraigniaud and Peters in 1993. A broadcast graph is a graph with broadcast time equal to that of complete graph. A broadcast graph with minimum number of edges is known to be a minimum broadcast graph. A gossip graph is a graph with gossip time equal to that of complete graph. A gossip graph with minimum number of edges is known to be a minimum gossip graph. For any $k$, the Knodel graph of order $2^k$ and degree $k$ is a minimum broadcast graph. The literature of $W_{k,2^k}$ is well developed and a detailed literature review is given by Fertin et al [5].

Communication models are classified based on their properties. Among them in the full duplex model communication flows in both directions and the graph model for such network is undirected, whereas in simplex model a communication link sends messages in a particular direction. The simplex network is modelled by directed graphs. This article defines the two layer representation of Knodel digraphs and Fibonacci digraphs-both are symmetric regular bipartite digraphs. A Knodel digraph is a symmetric digraph produced by replacing each edge of an
undirected Knodel graph [3] by a symmetric pair of directed edges. A Fibonacci digraph is a symmetric digraph produced by replacing each edge of an undirected Fibonacci graph by a symmetric pair of directed edges.

A graph \( D = (V, A) \) is a directed graph or digraph [1], if \( A \) is a collection of the ordered pairs of elements from \( V \). The ordered pair \( a = (v_i, v_j) \) is an arc. \( v_i \) is tail and \( v_j \) is head of \( a \). The head and tail are the end vertices of an arc. A pair of arcs with same head and same tail are parallel arcs. If \( G \) is a graph corresponding to a digraph \( D \), then \( G \) is the underlying graph of \( D \). Conversely, for a given graph \( G \) if we obtain a digraph \( D \) by specifying an order for each edge, such a digraph is an orientation of \( G \). The number of vertices is the order of \( D \) and the number of arcs is the size of \( D \). Order is denoted by \( n \) and size is represented by \( m \).

The in-degree \( d^−_D(v) \) of a vertex \( v \) in \( D \) is the number of arcs having head \( v \). The out-degree \( d^+_D(v) \) of \( v \) is the number of arcs having tail \( v \). The maximum and minimum in-degrees are denoted by \( \Delta^−(D) \) and \( \delta^−(D) \). Similarly the maximum and minimum out-degrees are denoted by \( \Delta^+(D) \) and \( \delta^+(D) \).

A digraph is strict if it has no loops or no two arcs with the same ends have the same direction. An orientation of a complete graph is a tournament. A directed Hamilton path is a directed path which passes through every vertex of \( D \). A directed Hamilton cycle of \( D \) is a directed cycle which passes through every vertex of \( D \). The associated digraph (symmetric digraph) \( D(G) \) of a graph \( G \) is a digraph obtained by replacing each edge \( e \) of \( G \) by two arcs with opposite directions. There is a one-one correspondence between paths in \( G \) and directed paths in \( D(G) \).

To analyze the efficiency of a communication network, many graph parameters such as Broadcast time, Diameter, degree centrality, vertex and edge connectivity are used [2].

Classical Topologies used in communication networks include path, cycle, complete graph, hypercube, cube connected cycles, shuffle exchange graph, De-Bruijn Graph, d-grid graph, d-torus graph, Recursive circulant graph, Star graph, Butterfly graph, Cayley Graph, Knodel Graph, Fibonacci Graph, r-modulo bipartite graph, complete bipartite graph etc. The analysis of these graphs and their parameters help to develop more efficient communication systems. The isomorphic path and star decomposition of Fibonacci graphs were given by Reji Kumar and Jasmine [7]. The decomposition of complete bipartite graphs and complete graphs in to some classes of stars have been studied. Reji Kumar and Jasmine defined Knodel and Fibonacci hypergraphs. The star decomposition of Knodel, Fibonacci and r-partite complete hypergraphs are studied by Reji and Jasmine. The main focus of this article is the path and star decomposition of Knodel and Fibonacci digraphs.

Decomposition of a given graph is the partitioning of edge set so that the given graph is split in to sub-structures. If all such sub-structures are isomorphic, the decomposition is isomorphic decomposition. The isomorphic decomposition of the Knodel digraphs and Fibonacci digraphs in to isomorphic paths and stars is presented. The star decomposition of both Knodel digraph and Fibonacci digraph leads to the decomposition of equi-bipartite complete digraphs in to two classes of stars.

2. Knodel Digraphs
Knodel graphs are optimal in the sense of minimum broadcast time and minimum gossip time. A graph is directed if each edge is an ordered pair of vertices. If the edges of an undirected graph are replaced by a symmetric pair of directed edges, the new graph is a symmetric digraph.
If \((u, v)\) is a directed edge or arc, then \(u\) is the source and \(v\) is the sink.

The decomposition possibilities of some major digraphs which are efficient communication structures in parallel computing are considered in this article. Among such graphs Knodel graphs have been studied widely for the last three decades. The decomposition problems of Knodel digraphs, Fibonacci Digraphs and complete bipartite digraphs are studied here.

Knodel digraph is a symmetric digraph produced by replacing each edge of an undirected Knodel graph by a symmetric pair of directed edges. Let \(|V^*_1| = |V^*_2| = \frac{n}{2}\). \(n\) is even; \(V^* = V^*_1 \cup V^*_2\) be a bipartition of \(V^*\). Then Knodel digraph can be defined as follows:

**Definition 2.1.** Each vertex is an ordered pair \((i, j)\); \(i = 0\) if \((i, j) \in V^*_1\) and \(i = 1\) if \((i, j) \in V^*_2\), \(j = 0, 1, 2, \ldots, \frac{n}{2} - 1\). The edge set consisting of all ordered pairs of vertices \\{\((0, j), (1, j)\)\}/\(j' \equiv j + 2^k - 1(\text{mod}\ \frac{n}{2})\} \cup \{((1, j'), (0, j''))\}/\(j'' \equiv j' - 2^k + 1(\text{mod}\ \frac{n}{2})\}, 0 \leq k \leq d, 1 \leq d \leq \lfloor \log_2 n \rfloor\).

\[
\begin{array}{cccc}
(0,0) & (0,1) & (0,2) & (0,3) \\
(1,0) & (1,1) & (1,2) & (1,3)
\end{array}
\]

**Figure 1.** Knodel digraph \(W^*_3,8\)

Knodel Digraphs are denoted by \(W^*_{d,n}\). It is a bipartite symmetric digraph in which indegree \((v)\) = outdegree \((v)\) = \(d\); where \(0 \leq d \leq \lfloor \log_2 n \rfloor\), for each vertex \((i, j)\); hence \(2d\)-regular.

**Definition 2.2.** An edge \((\text{or} (0, j), (1, j'))\) is of positive dimension \(k^+\) if \(j' \equiv j + 2^k - 1(\text{mod}\ \frac{n}{2})\) and an edge \((\text{or} (1, j'), (0, j''))\) is of negative dimension \(k^-\) if \(j'' \equiv j' - 2^k + 1(\text{mod}\ \frac{n}{2})\).

\(G_{k^+}\) denote the collection of all edges having dimension \(k^+\) and \(G_{k^-}\) denote the collection of all edges having dimension \(k^-\). The succeeding theorem decomposes \(W^*_{d,n}\) in to directed paths \(P^*_{d+1}\).

**Theorem 2.1.** \(W^*_{d,n}\) is \(P^*_{d+1}\)-decomposable.

**Proof.** Define a directed path \(P^*_{d+1}\) having \(d + 1\) vertices and \(d\) edges, each edge is of different dimension as follows:

choose any arbitrary vertex \((0, j_0)\) from \(V^*_1\). \(e_0\) be the positive edge of dimension \(0^+\) joining \((0, j_0)\) and \((1, j_1)\); where \(j_1 \equiv j_1 + 2^0 - 1(\text{mod}\ \frac{n}{2})\), \(e_1\) be the negative edge of dimension \(1^-\) joining \((1, j_1)\) and \((0, j_2)\); where \(j_2 \equiv j_1 - 2^1 + 1(\text{mod}\ \frac{n}{2})\), \(e_2\) be the positive edge of dimension \(2^+\) joining \((0, j_2)\) and \((1, j_3)\); where \(j_3 \equiv j_2 + 2^2 - 1(\text{mod}\ \frac{n}{2})\) and so on. \(e_k\) be the edge of dimension \(k\) joining \((i', j')\) and \((i'', j'')\). If \(k\) is odd, then \(e_k\) is an positive edge \(k^+\); that is \(i' = 0, i'' = 1, j'' = j' + 2^k - 1(\text{mod}\ \frac{n}{2})\). If \(k\) is even, then \(e_k\) is a negative edge \(k^-\); that is \(i' = 1, i'' = 0, j'' = j' - 2^k + 1(\text{mod}\ \frac{n}{2})\). The process is terminated when \(d\) edges are selected. As \(k\) takes
values from $0, 1, 2, \cdots, d - 1$, the edges $e_0, e_1, \cdots, e_{d-1}$, in sequence form $P^*_{d+1}$, since each $j_k$’s
are distinct. $j$ varies from 0 to $\frac{n}{2} - 1$, to produce $\frac{n}{2}$ isomorphic copies of $P^*_{d+1}$.

Again choose any arbitrary vertex $(1, j_1)$ from $V_2$. $e_1$ be the negative edge of dimension $0^-$ joining $(1, j_1)$ and $(0, j_1)$; where $j_1 \equiv j_1 - 2^0 + 1 (mod \frac{n}{2})$, $e_2$ be the positive edge of dimension $1^+$ joining $(0, j_1)$ and $(1, j_2)$; where $j_2 \equiv j_1 + 2^1 - 1 (mod \frac{n}{2})$, $e_3$ be the negative edge of dimension $2^-$ joining $(1, j_2)$ and $(0, j_3)$; where $j_3 \equiv j_2 - 2^2 + 1 (mod \frac{n}{2})$ and so on. $e_k$ be the edge of dimension $k$ joining $(i', j')$ and $(i'', j'')$. If $k$ is odd, then $e_k$ is an negative edge $k^-$; that is $i' = 1, i'' = 0, j'' = j' - 2^k + 1 (mod \frac{n}{2})$. If $k$ is even, then $e_k$ is an positive edge $k^+$; that is $i' = 0, i'' = 1, j'' = j' + 2^k - 1 (mod \frac{n}{2})$. The process is terminated when $d$ edges are selected. As $k$ takes values from $1, 2, \cdots, d$, the edges $e_1, e_2, \cdots, e_d$, in sequence form $P^*_{d+1}$, since each $j_k$’s are distinct. $j$ varies from 0 to $\frac{n}{2} - 1$, to give $\frac{n}{2}$ isomorphic copies of $P^*_{d+1}$. Hence $W^*_{d,n}$ is decomposed in to $n$, directed paths $P^*_{d+1}$.

\[ \begin{align*}
(0,0) & \quad (0,3) & \quad (0,1) & \quad (0,0) & \quad (0,2) & \quad (0,1) & \quad (0,3) & \quad (0,2) \\
(1,0) & \quad (1,2) & \quad (1,1) & \quad (1,3) & \quad (1,2) & \quad (1,0) & \quad (1,3) & \quad (1,1) \\
(1,0) & \quad (1,1) & \quad (1,1) & \quad (1,2) & \quad (1,2) & \quad (1,3) & \quad (1,3) & \quad (1,0) \\
(0,0) & \quad (0,2) & \quad (0,1) & \quad (0,3) & \quad (0,2) & \quad (0,0) & \quad (0,3) & \quad (0,1)
\end{align*} \]

Figure 2. $P^*_{4}$ decomposition of Knodel digraph $W^*_{3,8}$

Isomorphic star decomposition of the Knodel digraphs is follows:

**Theorem 2.2.** $W^*_{d,n}$ is $S^*_{d+1}$-decomposable.

**Proof.** Let $S^*_{d+1}$ be the $d+1$-star containing all edges begin from $(0, j)$. $j$ varies from 0 to $\frac{n}{2} - 1$, to produce $\frac{n}{2}$, isomorphic $S^*_{d+1}$-stars.
Let $S^*_{d+1}$ be the $d+1$-star containing all edges begin from $(1, j)$. $j$ varies from 0 to $\frac{n}{2} - 1$, to get $\frac{n}{2}$, isomorphic $S^*_{d+1}$-stars.

Hence $W_{d,n}^*$ is $S^*_{d+1}$-decomposable.

**Figure 3.** $S_4^*$ decomposition of Knodel digraph $W_{3,6}^*$

Define Bipartite complement of digraphs as given below.

**Definition 2.3.** Let $G^* = (V_1^*, V_2^*, E)$ be a bipartite digraph. $G^{*bc} = (V_1^*, V_2^*, E^{*bc})$ is the bipartite complement of $G^*$; where $E^{*bc} = \{(x, y) \in E(K^*_p,q) / x \in V_1^*, y \in V_2^*, and (x, y) \notin E^*, p = |V_1^*|, q = |V_2^*|\}$

The complete bipartite digraph $K^*_n,n$ is decomposed into two isomorphic families of stars in the upcoming theorem.

**Theorem 2.3.** $K^*_n,n$ is decomposable into $S^*_{d+1}$ and $S^*_{n-d+1}$, where $0 \leq d \leq n - 1$.

**Proof.** Let $K^*_n,n$ be the complete bipartite digraph with vertex bipartition $V_1^*$ and $V_2^*$, such that $|V_1^*| = n = |V_2^*|$. Label the vertices of the bipartite graph by the pairs $(i, j)$ with $i = 0, 1 (i = 0$ means that the vertex belongs to $V_1^*$ and $i = 1$ means that the vertex belongs to $V_2^*$) and $0 \leq j \leq n - 1$.

**Part I** Consider the spanning sub graph $G$ of $K^*_n,n$ with the following property. For every $j$, $0 \leq j \leq n - 1$, there is a pair of arcs between the vertex $(0, j)$ and every vertex...
(1, j + 2^k - 1(mod n)), for k = 0, 1, · · · d − 1; where 1 ≤ d ≤ ⌊log_2 n⌋. G = W_{d, 2n}^* is a Knodel digraph, which is a d-regular bipartite digraph. By theorem 2.2, W_{d, 2n}^* is S_d^*-decomposable.

Part II G^{*bc} is a (n − d)-regular bipartite digraph. Let S_{j+1}^* be the sub graph of G^{*bc} with (0, j) as the central vertex, containing all edges start from it, for all j = 0, 1, · · · , n−1. Each S_{j+1}^* is a S_{n−d+1}^* star. Let S_{j+1}^* be a subgraph of G^{*bc} with (1, j) as the central vertex containing all edges start from it. Each S_{j+1}^* is a S_{n−d+1}^* star. \{S_{j+1}^*\}_{j=0}^{n−1} and \{S_{j+1}^*\}_{j=0}^{n−1} gives an isomorphic decomposition of G^{*bc} into 2n, S_{n−d+1}^* stars.

Part I and Part II gives the required decomposition.

Analogous approach produce similar results to Fibonacci Digraphs. Fibonacci digraphs can be defined as follows.

3. Fibonacci Digraphs

Definition 3.1. Label the vertices of Fibonacci digraphs by ordered pairs (i, j); i = 0 if (i, j) ∈ V_1^* and i = 1 if (i, j) ∈ V_2^*, where V = V_1^* ∪ V_2^* and the arc set consisting of all ordered pairs of vertices \(A = \{(0, j), (1, j')\}/j' ≡ j + F(k) − 1(mod \frac{n}{2})\} ∪ \{(1, j'), (0, j'')\}/j'' ≡ j' − F(k) + 1(mod \frac{n}{2})\}.

\[
\begin{align*}
(0,0) & \quad (0,1) & \quad (0,2) & \quad (0,3) & \quad (0,4) \\
(1,0) & \quad (1,1) & \quad (1,2) & \quad (1,3) & \quad (1,4)
\end{align*}
\]

Figure 4. Fibonacci digraph F_{4,10}^*

Denote Fibonacci Digraphs by F_{d,n}^*. Fibonacci Digraph is bipartite symmetric digraph in which in-degree = out-degree = d; where 0 ≤ d ≤ F^−1(n), for each vertex (i, j)

Definition 3.2. An arc ((0, j), (1, j')) is of positive dimension k^+ if j' ≡ j + F(k) − 1(mod \frac{n}{2})

and an arc ((1, j'), (0, j'')) is of negative dimension k^− if j'' ≡ j' − F(k) + 1(mod \frac{n}{2}).

Isomorphic path decomposition of Fibonacci digraphs is given in the succeeding theorem.

Theorem 3.1. F_{d,n}^* is P_{d+1}^*-decomposable.

Proof. Define a path P_{d+1}^* having d + 1 vertices and d arcs, each arc is of different dimension as follows:

Choose an arbitrary vertex (0, j_0) from V_1^*. e_1 be the positive arc of dimension 1^+ joining (0, j_0) and (1, j_1); where j_1 ≡ j_0 + F(1) − 1(mod \frac{n}{2}), e_2 be the negative arc of dimension 2^− joining (1, j_1) and (0, j_2); where j_2 ≡ j_1 − F(2) + 1(mod \frac{n}{2}), e_3 be the positive arc of dimension 3^+ joining (0, j_2) and (1, j_3); where j_3 ≡ j_2 + F(3) − 1(mod \frac{n}{2}) and so on. e_k be the arc of dimension k joining (i', j') and (i'', j''). If k is odd, then e_k is a positive arc k^+; that is i' = 0,
$i^\prime = 1, j^\prime = j^\prime + F(k) - 1 (mod \frac{n}{2})$. If $k$ is even, then $e_k$ is a negative arc $k^-$; that is $i' = 1$, $i'' = 0$, $j'' = j' + F(k) + 1 (mod \frac{n}{2})$. If $k$ is odd, then $e_k$ is a positive arc $k^+$; that is $i' = 0$, $i'' = 1$, $j'' = j' + F(k) - 1 (mod \frac{n}{2})$. The process is terminated when $d$ arcs are selected. As $k$ takes values from 1, 2, · · ·, $d$, the arcs $e_1, e_2, · · ·, e_d$, in sequence form $P_{d+1}^*$, since each $j_k$’s are distinct. $j$ varies from 0 to $\frac{n}{2} - 1$, to give $\frac{n}{2}$ isomorphic copies of $P_{d+1}^*$.

Choose an arbitrary vertex $(1, j_1)$ from $V_{2}^*$. $e_1$ be the negative arc of dimension $1^-$ joining $(1, j_1)$ and $(0, j_1)$; where $j_1 \equiv j_1 - F(1) + 1 (mod \frac{n}{2})$, $e_2$ be the positive arc of dimension $2^+$ joining $(0, j_1)$ and $(1, j_2)$; where $j_2 \equiv j_1 + F(2) - 1 (mod \frac{n}{2})$, $e_3$ be the negative arc of dimension $3^-$ joining $(1, j_2)$ and $(0, j_3)$; where $j_3 \equiv j_2 - F(3) + 1 (mod \frac{n}{2})$ and so on. $e_k$ be the arc of dimension $k$ joining $(i^\prime, j^\prime)$ and $(i'', j'')$. If $k$ is odd, then $e_k$ is an negative arc $k^-';$ that is $i' = 1$, $i'' = 0$, $j'' = j' - F(k) + 1 (mod \frac{n}{2})$. If $k$ is even, then $e_k$ is an positive arc $k^+';$ that is $i' = 0$, $i'' = 1$, $j'' = j' + F(k) - 1 (mod \frac{n}{2})$. The process is terminated when $d$ arcs are selected. As $k$ takes values from 1, 2, · · ·, $d$, the arcs $e_1, e_2, · · ·, e_d$, in sequence form $P_{d+1}^*$, since each $j_k$’s are distinct. $j$ takes from 0 to $\frac{n}{2} - 1$, to give $\frac{n}{2}$ isomorphic copies of $P_{d+1}^*$.

Hence $F_{d,n}^*$ is $P_{d+1}^*$ decomposable.

**Figure 5.** $P_{5}^*$ decomposition of Fibonacci digraph $F_{4,10}^*$

Next theorem decomposes Fibonacci digraphs in to stars.

**Theorem 3.2.** $F_{d,n}^*$ is $S_{d+1}^*$-decomposable.

**Proof.** Let $S_{d+1}^*$ be the directed $d + 1$-star containing all edges start from $(0, j)$. $j$ varies from 0 to $\frac{n}{2} - 1$, to produce $\frac{n}{2}$, isomorphic $S_{d+1}^*$-stars.
Let $S^*_{d+1}$ be the directed $d + 1$-star containing all edges start from $(1, j)$. As $j$ varies from 0 to $\frac{n}{2} - 1$, we get $\frac{n}{2}$, isomorphic $S^*_{d+1}$-stars.

Hence $F^*_{d,n}$ is $S^*_{d+1}$-decomposable.

\[\begin{array}{c}
(0,0) & (0,1) & (0,2) & (0,3) \\
(1,0) & (1,1) & (1,2) & (1,3) & (1,4) \\
(0,0) & (0,1) & (0,2) & (0,3) & (0,4) \\
(0,0) & (1,0) & (1,1) & (1,2) & (1,3)
\end{array}\]

**Figure 6.** $S^*_5$ decomposition of Fibonacci digraph $F^*_{4,10}$

The succeeding theorem decomposes Complete bipartite digraphs $K^*_{n,n}$ in to two classes of isomorphic stars; $d$ is an integer $1 \leq d \leq F^{-1}(n)$.

**Theorem 3.3.** $K^*_{n,n}$ is decomposable in to two families of stars $S^*_{d+1}$ and $S^*_{n-d+1}$. 
Proof. Let $K^*_{n,n}$ be the complete bipartite digraph with vertex bipartition $V^*_1$ and $V^*_2$, such that $|V^*_1| = n = |V^*_2|$. Label the vertices of the bipartite graph by the pairs $(i,j)$ with $i = 0, 1$ ($i = 0$ represents that the vertex belongs to $V^*_1$ and $i = 1$ means that the vertex belongs to $V^*_2$) and $0 \leq j \leq n-1$. Consider the spanning sub graph $G$ of $K^*_{n,n}$ with the following property. For every $j$, $0 \leq j \leq n-1$, a pair of arcs between the vertex $(0,j)$ and every vertex $(1,j+F(k)-1(mod n))$, for $k = 1, 2, \cdots d$; where $1 \leq d \leq F^{-1}(n)$. Then $G = W^*_{d,2n}$ is a Fibonacci digraph, (which is a $2d$- regular bipartite digraph. Then, $G^{*bc}$ is a $2(n-d)$-regular bipartite digraph.

Let $S^*_j$ be the sub graph of $G^{*bc}$ with $(0, j)$ as the central vertex, containing all arcs start from it, for all $j = 0, 1, \cdots , n-1$. Then each $S^*_j$ is a $S^*_{n-d+1}$ star.

Let $S^{**}_j$ be the sub graph of $G^{*bc}$ with $(1, j)$ as the central vertex, containing all arcs start from it, for all $j = 0, 1, \cdots , n-1$. Then each $S^{**}_j$ is a $S^*_{n-d+1}$ star.

$\{S^*_j\}_{j=0}^{n-1}$ and $\{S^{**}_j\}_{j=0}^{n-1}$ gives an isomorphic decomposition of $G^{*bc}$ in to $2n$, $S^*_{n-d+1}$ stars. $G$, together with $\{S^*_j\}_{j=0}^{n-1}$, provides the required decomposition.

4. Conclusion

Knodel graphs have been generated from Knodel’s optimal algorithm construction among $n$ vertices in the study of broadcasting and gossiping. Research has been done on the communication properties of Knodel graphs. In the simplex model of communication, digraphs are used. Yet the literature for Knodel’s digraphs has not begun. This article provides two layer representation of Knodel digraphs and Fibonacci digraphs. The isomorphic path and star decomposition of them are given. The star decomposition of both Knodel digraphs and Fibonacci digraphs leads to the decomposition of $K^*_{n,n}$ in to two classes of stars.

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