Predictive power of strong coupling in theories with large distance modified gravity

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New Journal of Physics 8 (2006) 326
Received 10 July 2006
Published 19 December 2006
Online at http://www.njp.org/
doi:10.1088/1367-2630/8/12/326

Abstract. We consider theories that modify gravity at cosmological distances, and show that any such theory must exhibit a strong coupling phenomenon, or else it is either inconsistent or is already ruled out by solar system observations. We show that all the ghost-free theories that modify dynamics of spin-2 graviton on asymptotically flat backgrounds, automatically have this property. Due to the strong coupling effect, modification of the gravitational force is source-dependent, and for lighter sources sets in at shorter distances. This universal feature makes modified gravity theories predictive and potentially testable not only by cosmological observations, but also by precision gravitational measurements at scales much shorter than the current cosmological horizon.

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1. Introduction

The observed accelerated expansion of the Universe [1] could result [2] from a breakdown of the standard laws of gravity at very large distances, in particular, caused by extra dimensions becoming gravitationally accessible at distances of order the current cosmic horizon [3].

In this study, we shall formulate some very general properties of large distance modified gravity theories, that follow from the consistency requirements based on four-dimensional (4D) effective field theory considerations. These arguments allow us to give a simple parameterization and display some very general necessary properties of such theories, even without knowing their complete explicit form.

In this study, we shall be interested in theories that (i) are ghost free, (ii) admit the weak field linearized expansion on asymptotically flat backgrounds, and (iii) modify Newtonian dynamics at a certain crossover scale, \( r_c \). Motivated by cosmological considerations one is tempted (and usually has to) to take \( r_c \) of the order of the current cosmological horizon \( H_0^{-1} \sim 10^{28} \text{ cm} \), but in most of our discussions we shall keep \( r_c \) as a free parameter.

There exists a unique class of ghost-free linearized theories of a spin-2 field with the above properties. These theories have a tensorial structure of the Pauli–Fierz massive graviton, and are described by the following equation

\[
\varepsilon^{\mu\nu}_{\alpha\beta} h_{\alpha\beta} - m^2(\Box)(h_{\mu\nu} - \eta_{\mu\nu} h) = T_{\mu\nu}. \tag{1.1}
\]

Here,

\[
\varepsilon^{\mu\nu}_{\alpha\beta} h_{\alpha\beta} = \Box h_{\mu\nu} - \Box \eta_{\mu\nu} h - \partial^\alpha \partial_\mu h_{\alpha\nu} - \partial^\alpha \partial_\nu h_{\alpha\mu} + \eta_{\mu\nu} \partial^\beta \partial^\alpha h_{\beta\alpha} + \partial_\mu \partial_\nu h,
\]

is the linearized Einstein’s tensor, and as usual \( h \equiv \eta_{\alpha\beta} h^{\alpha\beta} \). \( T_{\mu\nu} \) is a conserved source. The gravitational coupling constant is set equal to one for simplicity. The difference of the above theory from the Pauli–Fierz massive graviton is that in our case \( m^2(\Box) \) is not a constant, but a more general operator, that depends on \( \Box \). In order to modify Newtonian dynamics at scale \( r_c \), \( m^2(\Box) \) must dominate over the first Einsteinian term at the scales \( r \gg r_c \), or equivalently for the momenta \( k \ll r_c^{-1} \). This implies that to the leading order

\[
m^2(\Box) \simeq r_c^2(\alpha-1) \Box^\alpha,
\]

with \( \alpha < 1 \).

The class of theories of interest thus can be parameterized by a single continuous parameter \( \alpha \). As we shall see in a moment, unitarity further restricts \( \alpha \) from below by demanding \( \alpha > 0 \). The limiting case \( \alpha = 0 \) corresponds to the massive Pauli–Fierz graviton.

Currently, the only theory that has a known ghost-free generally covariant non-linear completion is \( \alpha = 1/2 \), which is a 5D brane world model (Dvali–Gabadadze–Porrati (DGP)-model) [3]. The limiting case of massive graviton \( \alpha = 0 \), unfortunately, has no known consistent nonlinear completion, and it is very likely that such completions are impossible [4, 5], at least in theories with finite number of states.

The nonlinear completions for theories with other values of \( \alpha \) have not been studied. It is not the purpose of the present study to discover such completions. Instead, we shall display some necessary conditions that such completions (if they exist) must satisfy. As we shall see, the effective field theory considerations combined with consistency assumptions give us a powerful
tool for predicting some of the general properties of the theories, even without knowing all the details of the underlying dynamics.

The central role in our consideration is played by the strong coupling phenomenon, first discovered in [6], both for massive gravity ($\alpha = 0$) and for DGP model ($\alpha = 1/2$). We shall show that this phenomenon must be exhibited by theories with arbitrary values of $\alpha$ from the unitarity range $0 < \alpha < 1$, and that this fact predicts the following modification of the gravitational potential for an arbitrary localized source

$$\delta \sim \left( \frac{r}{r_c} \right)^{2(1-\alpha)} \frac{r}{r_g^\alpha},$$

(1.4)

where $r_g \equiv 2G_NM$ is the usual gravitational (Schwarzschild) radius of the source, and $G_N$ is Newton’s constant. Because of this predictive power, the theories of modified gravity are extremely restrictive and potentially testable by precision gravitational measurements. Our analysis complements and solidifies the earlier study of [7, 8], in which the corrections (1.4) were already suggested.

2. Unitarity constraints

We shall first derive the unitarity lower bound on $\alpha$. For this, consider a one-graviton exchange amplitude between the two conserved sources ($T_{\mu\nu}$ and $T'_{\mu\nu}$)

$$(\text{Amplitude})_h \propto \frac{T'_{\mu\nu}T^{\mu\nu} - \frac{1}{3} T'_{\mu} T'^{\mu}_{\nu}}{\Box - m^2(\Box)}. \quad (2.1)$$

The scalar part of the (Euclidean) graviton propagator then has the following form

$$\Delta(k^2) = \frac{1}{k^2 + m^2(k^2)}. \quad (2.2)$$

As said above, modification of gravity at large distances implies that the denominator is dominated by the second term, for $k \to 0$. To see how unitarity constrains such a behaviour, let the spectral representation of the propagator $\Delta(k^2)$ be

$$\Delta(k^2) = \int_0^\infty ds \frac{\rho(s)}{k^2 + s}, \quad (2.3)$$

where $\rho(s)$ is a bounded spectral function. The absence of the negative norm states demands that $\rho(s)$ be a semi-positive definite function. Evaluating (2.3) for the small $k$ and using parameterization (1.3), we get

$$\frac{1}{r_c^{2(\alpha-1)} k^{2\alpha}} = \int_0^\infty ds \frac{\rho(s)}{k^2 + s}. \quad (2.4)$$

Non-negativity of $\rho(s)$ implies that $\alpha$ cannot be negative. In the opposite case, $\Delta(k^2)$ would be zero for $k^2 = 0$, which is impossible for non-negative $\rho(s)$. Thus, from unitarity, it follows that $\alpha = 0$ is the lowest possible bound.
In general, a non-analytic behaviour of the propagator reflect the existence of the continuum of states, which could be interpreted as the signature of extra dimension. Thus, in a sense large distance modification of gravity leads to the existence of extra non-compact dimensions, as in DGP [3].

3. Extra polarizations

A graviton satisfying the equation (1.1), just as in the Palui-Fierz case, propagates five degrees of freedom. These include, two tensor, two vector and one scalar polarizations. The extra polarizations (especially the scalar) play the central role in the strong coupling phenomenon, and it is therefore useful to separate the ‘new’ states from the ‘old’ (massless spin-2) states.

We shall now integrate the extra three polarizations out and write down the effective equation for the two remaining tensorial ones. For this we shall first rewrite equation (1.1) in the manifestly gauge invariant form, using the Stückelberg method. We can rewrite $h_{\mu\nu}$ in the following form

$$h_{\mu\nu} = \hat{h}_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu,$$

(3.1)

where the Stückelberg field $A_\mu$ is the massive vector that carries two extra helicity-1 polarizations. The remaining extra scalar state resides partially in $A_\mu$ and partially in $\hat{h}_{\mu\nu}$. Written in terms of $\hat{h}_{\mu\nu}$ and $A_\mu$,

$$\varepsilon^{\alpha\beta}_{\mu\nu} \hat{h}_{\alpha\beta} - m^2(\square) (\hat{h}_{\mu\nu} - \eta_{\mu\nu} \hat{h}) + \partial_\mu A_\nu + \partial_\nu A_\mu - 2\eta_{\mu\nu} \partial^\rho A_\rho = T_{\mu\nu},$$

(3.2)

the equation (1.1) becomes manifestly invariant under the following gauge transformation

$$\hat{h}_{\mu\nu} \to \hat{h}_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \quad A_\mu \to A_\mu - \xi_\mu,$$

(3.3)

where $\xi_\mu$ is the gauge parameter. Note that the first, Einstein’s, term is unchanged under the replacement (3.1), due to its gauge invariance. We now wish to integrate out $A_\mu$ through its equation of motion,

$$\partial^\mu F_{\mu\nu} = -\partial^\mu (\hat{h}_{\mu\nu} - \eta_{\mu\nu} \hat{h}),$$

(3.4)

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. Before solving for $A_\mu$, note that by taking a divergence from the equation (3.4) we get the following constraint on $\hat{h}_{\mu\nu}$

$$\partial^\mu \partial^\nu \hat{h}_{\mu\nu} - \square \hat{h} = 0,$$

(3.5)

which means that $\hat{h}_{\mu\nu}$ is representable in the form

$$\hat{h}_{\mu\nu} = \tilde{h}_{\mu\nu} + \eta_{\mu\nu} \frac{1}{3} \Pi_{\alpha\beta} \tilde{h}^{\alpha\beta},$$

(3.6)

where $\Pi_{\alpha\beta} = \frac{\partial_\alpha \partial_\beta}{\square} - \eta_{\alpha\beta}$ is the transverse projector. $\tilde{h}_{\mu\nu}$ carries two degrees of freedom. Notice that, since the last term in (3.6) is gauge invariant, under the gauge transformations (3.3), $\hat{h}_{\mu\nu}$ shifts in the same way as $\tilde{h}_{\mu\nu}$ does.
Coming back to the equation (3.4) and solving it for $A_\mu$ we get

$$A_\nu = -\frac{1}{\Box} \partial^\mu (\hat{h}_{\mu\nu} - \eta_{\mu\nu} \hat{h}) - \partial_\nu \Theta,$$

(3.7)

where $\Theta$ is arbitrary. The gauge invariance demands that under the transformations (3.3), $\Theta$ shifts in the following way

$$\Theta \to \Theta + \frac{1}{\Box} \partial_\alpha \xi^\alpha.$$

(3.8)

Substituting (3.7) back to (3.2), expressing $\hat{h}_{\mu\nu}$ in terms of $\tilde{h}_{\mu\nu}$ through (3.6), and choosing the gauge appropriately, we can write the resulting effective equation for $\tilde{h}_{\mu\nu}$ in the following form

$$\left(1 - \frac{m^2}{\Box}\right) E^{\alpha\beta}_{\mu\nu} \tilde{h}_{\alpha\beta} = T_{\mu\nu},$$

(3.9)

which has the tensorial structure identical to the linearized Einstein’s equation. This latter fact indicates that $\tilde{h}_{\mu\nu}$ indeed propagates only two tensor polarizations, characteristic for the massless graviton.

Because $\tilde{h}_{\mu\nu}$ propagates only two degrees of freedom, the one-particle exchange amplitude between the two conserved sources $T_{\mu\nu}, T'_{\mu\nu}$ mediated by $\hat{h}$ is

$$\text{(Amplitude)}_{\tilde{h}} \propto T'_{\mu\nu} - \frac{1}{2} T^\mu_{\mu} T'^{\nu}_{\nu} \frac{\Box}{\Box - m^2(\Box)},$$

(3.10)

which continuously recovers the massless graviton result in the limit $m^2 \to 0$. This fact however does not avoid the well-known van Dam–Veltman–Zakharov (vDVZ) discontinuity [9], because (3.10) is only a part of a full one-particle exchange amplitude. This becomes immediately clear if we notice that the metric exitation that couples to the conserved $T_{\mu\nu}$ is $\hat{h}_{\mu\nu}$ (or equivalently $h_{\mu\nu}$), which depends on $\tilde{h}_{\mu\nu}$ through (3.6). The full physical amplitude is generated by the latter combination of $\tilde{h}_{\mu\nu}$ and is equal to (2.1), which clearly exhibits the vDVZ-type discontinuity in the $m^2(\Box) \to 0$ limit.

4. The strong coupling of the longitudinal gravitons

We have seen that in the considered class of theories the graviton should contain three extra longitudinal polarizations, which leads to vDVZ discontinuity in linearized theory. Hence, any theory of modified gravity that remains weakly coupled at the solar system distances is automatically ruled out by the existing gravitational data.

Thus, the only point that could save such theories is the breakdown of the linearized approximation at the solar system distances, that is, the strong coupling phenomenon [6]. In the other words, the only theories of modified gravity that can be compatible with observations are the strongly coupled ones. Any attempt to cure the strong coupling will simply rule out the theory.

Fortunately, the same extra polarization that creates a worrisome discontinuity in the gravitational amplitude also provides the strong coupling which invalidates the unwanted result at solar system scales. We shall now generalize the results of [6] to theories with arbitrary $\alpha$. 

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Consider a theory of gravitons that in the linearized approximation satisfies the equation (1.1). We shall assume that this theory has a generally-covariant nonlinear completion. The nonlinear completion of the first term is the usual Einstein’s tensor. The completion of the second term is unknown for general $\alpha$. In fact, extrapolation from the only known case of $\alpha = 1/2$ [3] tells us that if a completion exists, it will probably require going beyond 4D. Discovering such a completion is beyond our program. In fact, knowing the explicit form of this completion is unnecessary for our analysis, provided it meets some general requirements listed above. Then, we will be able to derive general properties of the strong coupling and the resulting predictions.

The propagator for the graviton satisfying equation (1.1) has the following form,

$$D_{\mu \nu; \alpha \beta} = \left( \frac{1}{2} \tilde{\eta}_{\mu \alpha} \tilde{\eta}_{\nu \beta} + \frac{1}{2} \tilde{\eta}_{\mu \beta} \tilde{\eta}_{\nu \alpha} - \frac{1}{3} \tilde{\eta}_{\mu \nu} \tilde{\eta}_{\alpha \beta} \right) \frac{1}{\Box + m^2(\Box)}; \quad (4.1)$$

where

$$\tilde{\eta}_{\mu \nu} \equiv \eta_{\mu \nu} + \frac{\partial_\mu \partial_\nu}{m^2(\Box)} = \eta_{\mu \nu} + r_c^{2(1-\alpha)} \frac{\partial_\mu \partial_\nu}{\Box^\alpha}, \quad (4.2)$$

The existence of the terms that are singular in $r_c^{-1}$ is the most important fact. This singularity is precisely the source of the strong coupling observed in [6] for $\alpha = 0$ and $1/2$ cases. The terms that are singular in $r_c^{-1}$ come from the additional, scalar polarization state of the resonance graviton. In the Stückelberg language given in (3.3), this scalar polarization resides partially in $A_\mu$ and partially in the trace of $\hat{h}_{\mu \nu}$. If we denote the canonically normalized longitudinal polarization by $\chi$, then, ignoring the spin-1 helicity vector, the full metric fluctuation can be represented as

$$h_{\mu \nu} = \tilde{h}_{\mu \nu} - \frac{1}{6} \eta_{\mu \nu} \chi + r_c^{2(1-\alpha)} \frac{\partial_\mu \partial_\nu}{3 \Box^\alpha} \chi. \quad (4.3)$$

Notice that $\tilde{h}_{\mu \nu}$ is the same as in (3.6).

The same state is responsible for the extra attraction, that provides factor of $1/3$ in the one graviton exchange amplitude, as opposed to $1/2$ in the standard gravity, leading to vDVZ discontinuity.

The strong coupling of the longitudinal gravitons, however, has a profound effect on discontinuity. The effect of the longitudinal gravitons becomes suppressed near the gravitating sources, where the linearized approximation breaks down. Due to the strong coupling effects, the gravitating sources of mass $M$, on top of the usual Schwarzschild gravitational radius $r_g \equiv 2G_N M$, acquire the second physical radius, which we shall call $r_s$. Breakdown of the linearized approximation near gravitating sources, was first noticed in the context of $\alpha = 0$ theory in [10], and the underlying strong coupling dynamics was uncovered in [6]. The key point of the present discussion is that both the strong coupling, and the resulting $r_s$-scale, is a property of theories with arbitrary $\alpha$.

For the longitudinal graviton $\chi$, the latter radius plays the role somewhat similar to the one played by $r_g$ for the transverse graviton $\tilde{h}_{\mu \nu}$. Namely, due to the strong coupling, at $r = r_s$ the nonlinear self interactions of $\chi$ become important, and the expansion in series of $G_N (r_g)$ breaks down.

The concept of $r_s$ plays the central role in any large distance modified theory of gravity, as we shall now discuss. Consider a localized static gravitating source $T_{\mu \nu} = \delta_{\mu}^0 \delta_{\nu}^0 M \delta(r)$ of gravitational
radius $r_g$. Then, sufficiently far from the source, the linearized approximation should be valid, and the metric created by the source can be found in one graviton exchange approximation to linear order in $G_N$,

$$h_{\mu\nu} = \frac{\delta^0_\mu \delta^0_\nu - \frac{1}{3} (\eta_{\mu\nu} + r_g^2 (1-\alpha) \frac{\partial^2}{\partial^2 r}) r_g}{\Box - r^2 (\alpha - 1) c} \frac{r_g}{2} \delta(r).$$  \hspace{1cm} (4.4)

The term in the numerator, that is singular in $1/r_c$, vanishes when convoluted with any conserved test source $T'_{\mu\nu}$, in accordance with (2.1). Hence, at the distances below $r_c$, the metric has an $r_g/r$ form, but with a wrong (scalar–tensor type) tensorial structure, manifesting vDVZ discontinuity. However, in nonlinear interactions, the singular in $1/r_c$ terms no longer vanish and in fact wash-out the linear effects, due to the strong coupling. The scale of the strong coupling can be figured out by generalizing the analysis of [6, 7]. The straightforward power counting then gives that the leading singularity in $r_c$ is of the order of $r_c^{4(1-\alpha)}$, and comes from the trilinear interaction of the longitudinal gravitons. This vertex has a momentum dependence of the form

$$(r_c k)^{4(1-\alpha)} k^2.$$  \hspace{1cm} (4.5)

Then, the scale $r_s$ corresponds to a distance from the source for which the contribution from the above trilinear vertex, becomes as important as the linear one given by (4.4). It is obvious that the corresponding distance is given by

$$r_s = \left( r_c^{4(1-\alpha)} r_g \right)^{-1/(1-\alpha)}$$  \hspace{1cm} (4.6)

For distances $r \ll r_s$ the correction to the Einsteinian metric coming from the longitudinal gravitons is suppressed by powers of $r_s$. The leading behaviour can be fixed from the two requirements. Firstly, $\chi(r)$ should become of order $r_g/r_s$ at $r = r_s$, in order to match the linear regime (4.4) outside the $r_s$-sphere. Secondly, the solution inside $r_s$ must be possible to approximate by the analytic series in $r^2 (\alpha - 1) c$. These two requirements fix the leading behaviour as

$$\chi(r \ll r_s) \sim \frac{r_g}{r_s} \left( \frac{r}{r_s} \right)^{3/2 - 2\alpha},$$  \hspace{1cm} (4.7)

which gives the relative correction to the gravitational potential given by (1.4).

As pointed out in [7], for some interesting values of $\alpha$ these corrections are strong enough to be tested in precision gravitational measurement experiments. For $\alpha = 1/2$ theory this was also pointed out in [12], with the correction from the cosmological self-accelerated background [2, 13] taken into the account.

In fact, existing Lunar Ranging constraints already rule out theories with $\alpha$ significantly above 1/2, and the new generation of the improved accuracy measurements [14] will probably test the $\alpha \simeq 1/2$ case.

Notice that for $\alpha = 1/2$, (1.4) matches the known explicit solutions derived both in $1/r_c$-expansion [15], as well as exactly [16].

\footnote{An identical result was obtained in [11] for $\alpha = 0$, agreeing with the result of [6].}
5. Conclusions

Possibility of large-distance modification of gravity is a fundamental question, motivated by the dark energy problem [2], [17]–[19]. In this paper, we tried to stress the important role played by the strong coupling phenomenon in this class of theories. In particular, it was already appreciated [6, 8] that the strong coupling is the only cure that saves any such theory from being immediately ruled out by solar system observations, due to the fact that any such ghost-free theory must contain an extra scalar polarization and is subject to vDVZ discontinuity at the linearized level. This fact makes impossible the existence of consistent weakly coupled theories of modified gravity (relevant for cosmic observations).

Interestingly, in the class of theories considered, the same scalar polarization that creates the problem at the linearized level, also invalidates it by exhibiting the strong coupling phenomenon.

Due to this strong coupling phenomenon, gravitating objects are ‘endowed’ with a physical scale \( r_* \), which for the longitudinal graviton plays the hole of the second ‘horizon’. At distances \( r_* \sim r \) the nonlinear interactions of \( \chi \) catch-up with the linear part, and expansion in terms of the Newtonian coupling constant can no longer be trusted. Hence, vDVZ-discontinuity is invalidated.

For scales \( r \ll r_* \), the metric can be found in series of \( (r/r_*)^{2(1-\alpha)} \)—expansion (or possibly exactly). The assumption that the true solution can be approximated in such analytic series, fixes the form of the leading correction to the Einsteinian metric in the form of (1.4).

This fact makes theories of modified gravity potentially testable by the precision gravitational experiments in the solar system, and in particular by Lunar Ranging experiments.

Acknowledgments

We thank G Gabadadze and M Redi for discussions. The study is supported in part by the David and Lucile Packard Foundation Fellowship for Science and Engineering, and by NSF grant PHY-0245068. We also thank the Galileo Galilei Institute for Theoretical Physics and Institut des Hautes Etudes Scientifiques for hospitality and INFN for partial support during the completion of this study.

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