Application of Bar-Shalom and Fortmann’s Input Estimation for Underwater Target Tracking

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Abstract

Background/Objectives: The Extended Kalman Filter (EKF) using range and bearing measurements is analyzed for undersea target tracking. The Input estimation technique, developed by Bar-Shalom and Fortmann for radar applications is implemented for sonar applications. Methods/Statistical Analysis: Input estimation is used to estimate the target acceleration whenever the target makes a maneuver. Findings: The algorithm estimates target kinematics using zero mean chi-square distributed random sequence residual. Upon detection of target maneuver, this algorithm corrects the velocity and position components using acceleration components. Application/Improvements: Finally, the performance of this algorithm is evaluated in Monte-Carlo simulations and results conform the effectiveness of input estimation technique.

Keywords: Bearing Measurements, Estimation, Maneuvering, Range, Target Tracking

1. Introduction

In the underwater scenario, Target Motion Analysis (TMA) or target estimation analysis or contact motion analysis is used to estimate the dynamics of constant velocity target. A sonar positioned on a ship observes noisy bearing and range measurements of the target in active mode. The observer in straight line course and the target is mostly in straight line course with occasional maneuver as illustrated in Figure 1. The observer processes the measurements and estimates the target motion parameters. Rich literature is available to track a target using range and bearing measurements1–6. The input estimation technique is proposed to take care of target maneuver.

The difference between the measurements and the estimated measurements is termed as innovations. It is observed that the innovation sequence follows chi-square distribution. This process is done using a sliding window of the latest’s (s is a design factor) measurements. During this window period, the input is deemed to be constant. This procedure is called input estimation7. Input estimation developed by8,9 is used so far for radar applications, in which measurements are available continuously. Here effort is made to utilize the technique for underwater sonar applications, in which the measurements are available at discrete intervals.

Figure 1. Typical target observer encounter.

2. Mathematical Modelling

"The state equation is
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\[ X(k + 1) = \Phi(k + 1/k)X(k) + Fu(k) + \tau(k)\omega(k) \]  
\[ F = \begin{bmatrix} t^2 & t \\ t & 0 \end{bmatrix} \]  

The innovations corresponding to the correct filter are given by
\[ \nu(k + 1) = z(k + 1) - HX(k + 1/k) \]  
and the innovations for the non-maneuvering model (now mismatched model) are
\[ \nu^*(k + 1) = z(k + 1) - H\dot{X}^*(k + 1/k) \]  
\[ \psi(i + 1) = H\sum_{j=k}^{i} \prod_{m=j+1}^{i} \varphi(m)F \]  
\[ \varphi(i) = \Phi[1-G(i)H] \]  
\[ \dot{u} = (\psi^T S^{-1}\psi)^{-1}\psi^T S^{-1}y \]  

where \( S \) is a covariance matrix of \( Y(k) \) and it is given by \( S = \text{diag} \{ S(i) \} \)

The covariance matrix of \( \dot{u} \) is given by
\[ L = (\psi^T S^{-1}\psi)^{-1} \]

The significance test for the vector estimate \( \dot{u} \) for the purpose of maneuver declaration is
\[ d(\dot{u}) = \dot{u}^T L^{-1}\dot{u} \geq c \]  
\( c \) is a threshold.

As the estimate \( \dot{u} \) is a normal random variable with mean zero and covariance \( L \), then the statistic \( d \) is chi-square distributed with \( n \) degrees of freedom and \( c \) is chosen such that the probability of false alarm is
\[ P\{d(\dot{u}) \geq c\} = \alpha \quad \text{with} \quad \alpha = 10^{-2} \text{ or smaller.} \]

If a maneuver is detected, then the state has to be corrected as follows.
\[ \dot{x}^*(k+s+1/k+s) = \dot{x}^*(k+s+1/k+s) + Mu \]  

where
\[ M = \sum_{j=k}^{k+s} \prod_{m=j+1}^{k+s} \varphi(m)F \]  

The covariance associated with the estimate is
\[ P^u(k+s+1/k+s) = P(k+s+1/k+s) + MLM^T \]

A maneuver is considered finished when the input estimate based on measurements from the sliding window of length \( s \) becomes insignificant.

3. Simulation and Results

The initial estimate of the target state vector is chosen as follows.
\[ X(0/0) = [10 \ 15 \ R_m \sin(B_m) \ R_m \cos(B_m)]^T \]

Here the velocity components are assumed to be 10 m/s and 15 m/s. The initial covariance matrix \( P (0/0) \) can be comfortably taken as unit diagonal matrix. The observer is assumed to be moving in straight line, at a constant speed of 18 m/s at a course of 60°. The underwater target is assumed to be moving at a speed of 6 m/s and at an initial range of 20000 m with the initial bearing 60° relative to the observer. The noise in the bearing and range are assumed to be 0.33° r.m.s. and 7 m r.m.s. respectively. The plant noise is chosen as 0.01. The time interval between the measurements is initially around 27 seconds. It reduces as subsequently the range gets decreased. Here the transient matrix is not considered as a constant and it is updated along with the Kalman filter equations, whenever the measurements are obtained. The simulation is carried out for 30 minutes.

The results of these scenarios in Monte-Carlo Simulations are noted and for the purpose of analysis a scenario at a target course equal to 140° as shown in Figure 2 is considered. In the subsequent figures (Figure 3(a)-3(d)), the errors in the range, course, bearing and speed estimates are denoted by Range Error, Course Error, Bearing Error and Speed Error respectively. The range, course, bearing and speed are converged at 4th sample (105 seconds), 13th sample (310 seconds), 4th sample (105 seconds), and 6th sample (154 seconds) respectively. From the results, it is observed that the total solution with the required accuracy is obtained from the 13th sample (310 seconds) onwards.
The theoretical value of the chi-square variable with 5 degrees of freedom at 90% confidence level is 9.24 and the same value is considered for maneuver detection. The scenario is run 100 times in Monte-Carlo Simulation and it is observed that the solution is obtained around 14th sample (330 seconds). Let us say that by 330 seconds the process is stabilized. The statistic at this time is around 0.1. Thereafter it never increased to more than 0.4.

The geometry shown in Figure 4 and 5 is extended as follows. The target is assumed to do course maneuver at the time of 600 sec from 140° to 45° with a turning rate of 3°/sec as shown. The results after 100 Monte-Carlo runs are shown in Figure 3 (a)-(d).

The maneuver is given between 29th sample (591 seconds) and 30th sample (605 seconds). The maneuver is continued and completed between 31st (620 seconds) and 32nd sample (635 seconds). The change in statistic at various timings is shown in Table 1.

Table 1. The change in statistic at various timings

| Sample Number | Time | Statistic |
|---------------|------|-----------|
| 30            | 605  | 1.1       |
| 31            | 620  | 25.9      |
| 32            | 635  | 252       |
| 33            | 649  | 534       |
| 34            | 662  | 639       |
| 35            | 676  | 522       |
| 36            | 690  | 262       |
| 37            | 704  | 64        |

Figure 3. (a) Error in range estimates. (b) Error in course estimates. (c) Error in bearing estimates. (d) Error in speed estimates.
The maneuver is detected first at 31st sample and continued to show up to 37th sample (704 seconds). Actually the target maneuver is completed by 32nd sample but the statistic is reduced to below 9.4 after 37th sample (704 seconds). Thereafter, the statistic is reduced to around 0.4 and the process is stabilized. The range estimate is not disturbed due to maneuver. The course and bearing speed are converged at 34th sample (662 seconds) and speed at 32nd sample (635 seconds). More or less the practical requirement specifications are matching with that of theoretical threshold.

Figure 4. (a) TMA with single observation platform.

Figure 4. (b) Error in range estimates. (c) Error in course estimates. (d) Error in bearing estimates. (e) Error in speed estimates.

Figure 5. (a) TMA with single observation platform.
4. Conclusion

Extended Kalman Filter (EKF) using range and bearing measurements is analyzed for undersea target tracking. The Input estimation technique, developed by Bar-Shalom and Fortmann for radar applications is implemented for sonar applications. The algorithm estimates target kinematics using zero mean chi-square distributed random sequence residual. Upon detection of target maneuver, this algorithm corrects the velocity and position components using acceleration components.

5. References

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