Frequency Extended-MUSIC Method for DOA Estimation in Indoor IR-UWB Environment

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Abstract: In recent years, many researchers have been developed in order to ensure an accurate localization. So many techniques have been used the direction of arrival to estimate and track of IR-UWB signals. In this study, we propose a new direction of arrival estimation method for impulse radio UWB signals. This technique based on Extended-MUSIC algorithm. The proposed method has high performances in the multipath channels. Simulation results show that it has more accuracy than others high resolution algorithms, moreover, It is compared the performance corresponding to the Cramer-Rae bound (CRB).

Keywords: Impulse Radio Ultra Wide Band (IR-UWB), Direction of Arrival Estimation (DOA), Extended-MUSIC, Cramer-Rae bound (CRB)

Introduction

Nowadays, the impulse radio ultra wideband technologies present a promising solution for the wireless data communication with the presence of many obstacles. In the literature many studies performed in this area and provide the high performances of this technology (Win and Scholtz, 1998; 2000; Lee et al., 2002).

The IR-UWB signals has many advantages: First, it’s low power consumption. Second, the nature of impulse it makes to penetrate obstacles and reduce interferences. Moreover, it characterized by the high accuracy. Furthermore, the ability of the UWB pulse to resolve individual multi-path components.

The IR-UWB signals has an angle resolution capabilities, it has an accurate location estimates.

The DOA is important parameters in IR-UWB signal, it has been addressed in the literature (Mani and Bose, 2010; 2009). Many studies have discussed the performances of some super resolution techniques for DOA estimation in frequency domain (Cao and Li, 2010; Zhang et al., 2006; Monica and Montse, 2011).

In this study, a new approach for DOA estimation is proposed. This method based on Extended Music algorithm in frequency domain for indoor localization application (Harabi et al., 2007; Meknessi et al., 2014). This method estimate the DOAs of the IR-UWB signals, we provide the performance of our proposed algorithm in term of RMSE on comparing with others super resolution techniques and the CRB.

This paper is structured as follows: Section 2 is dedicated to describe of the signal model. Section 3 gives the main features of our proposed Frequency Extended Music method for DOA estimation and developed Music, root Music, Esprit and Matrix Pencil. In section 4, the Cramer-Rae bound for DOA estimation is derived. Some simulation results are provided in section 5. Finally, section 6 concludes the paper.

These superscripts: (.)T, (.)*, (.)H and (.)-1 presents the matrix transpose, conjugate, hermitian and inverse operators, respectively.

Signal Model

In an IR-UWB system, we consider the sending signal model, it can be expressed as:

\[ s_k(t) = \sum_{k=0}^{N_f-1} p(t - (kN_f + j)T_f + c_kT_s + b_kT_s) \]  

where, transmission of an information symbol is typically implemented by the repetition of \( N_f \) pulses of very short duration (Win and Scholtz, 1998), \( b_k \in \{0, 1\} \) is the information symbols, \( T_f \) is the frame period, where a total of \( N_f \) pulses are sent, \( N_f \) is the number of frames.
per symbol, $T_c$ is chip duration, $T_d$ is the PPM modulation time shift, $\{\epsilon_j\}$ is the time hopping sequence, $q$ is the number of paths and $p(t)$ is the waveform of the transmitted pulse, which is the second order derivative of Gaussian function given by:

$$p(t) = (1 - 4\pi^2 / r_0^2) \exp(-2\pi^2 / r_0^2)$$

(2)

Therefore, the UWB multipath channel impulse model defined as:

$$h(t) = \sum_{j=1}^{q} \alpha_j \delta(t - \tau_j)$$

(3)

Where:

$\alpha_j = \text{The fading coefficient}$

$\tau_j = \text{The propagation delay of } j^{th} \text{ path}$

For UWB systems, the received wideband signal can be represented as the convolution of $s(t)$ and $h(t)$ as:

$$x_u(t) = s_j(t) * h(t) + w_u(t)$$

(4)

where, $w_u(t)$ is the additive Gaussian white noise, $M$ is the number of element antenna array and $m \in \{1, M-1\}$.

Then the received signal on the $m$ sensor antenna equidistant spaced d can be formulated as follow:

$$x_u(t) = \sum_{j=1}^{q} \left( t - \frac{(m-1)d \sin(\theta_j)}{c} \right) + w_u(t)$$

(5)

$d$: is the distance between antenna elements in the array, $c$ the speed of light and $\theta_j$ direction of arrival of the $q$th path.

After Fourier transformation to the received signal in (5), we obtain a Transforming signal in the frequency domain, it can be formulated as follow:

$$X_u(f_j) = \sum_{j=1}^{Q} S_j(f_j) e^{-j\frac{2\pi}{T_w} \sin(\theta_j)} + W_u(f_j)$$

(6)

where, $X_u(f_j)$, $S_j(f_j)$ and $W_u(f_j)$ are the Fourier transformation of $x_u(t)$, $s_j(t)$ and $w_u(t)$ respectively.

The received signals in can be given in matrix form as:

$$X(f_j) = A(\theta, f_j) S(f_j) + W(f_j)$$

(7)

Where:

$$X(f_j) = [X_1(f_j), X_2(f_j), ... X_M(f_j)]^T$$

(8)

$$S(f_j) = [S_1(f_j), S_2(f_j), ... S_M(f_j)]^T$$

(9)

$$A(\theta, f_j) = [a(\theta_1, f_j), a(\theta_2, f_j), ... a(\theta_M, f_j)]$$

(10)

$$W(f_j) = [W_1(f_j), W_2(f_j), ... W_M(f_j)]^T$$

(11)

$$a(\theta_j, f_j) = \begin{bmatrix} e^{-j\frac{2\pi}{T_w} (\sin(\theta_j))} & e^{-j\frac{2\pi}{T_w} (2\sin(\theta_j))} & ... & e^{-j\frac{2\pi}{T_w} (N\sin(\theta_j))} \end{bmatrix}$$

(12)

Finally the covariance matrix is expressed as:

$$R = E \left[ X(f_j) X(f_j)^T \right]$$

(13)

Let the number of element antenna array $M$ be larger than the number of paths $q$. Then, from the formulation of received signals and additive noise is obtained as:

$$R = AS \lambda + I \sigma^2$$

(14)

where, the $q \times q$ vector $S_k = E[S(f_k) S(f_k)^T]$ is the source covariance matrix, $I$ is the identity matrix and $\sigma^2$ is the noise variance.

**DOA Estimation Algorithms**

**Frequency Extended-MUSIC Method**

Frequency Extended-MUSIC is based on Extended MUSIC algorithm (Harabi et al., 2007; Meknessi et al., 2014). The following steps outline the procedure of estimation of the DOA using the proposed method:

- Perform Eigen values decomposition on $R \lambda, n = 1, ... , N$
- Calculate a positive scalar value as $m = \frac{\text{tr}(\hat{R})}{N}$
- Construct the new Frequency EXTENDED MUSIC function as:

$$G' = R + m a(\theta)a(\theta)^T$$

(15)

- Extract the Eigen values of the new extended correlation matrix $G'$ in decreasing order:

$$\mu_1 \geq \lambda_2 \geq \cdots \geq \lambda_L > \lambda_L + \cdots > \lambda_N$$

(16)

- Calculate the function:

$$D(\theta) = \frac{1}{N} \sum_{l=1}^{N} \left( \mu_l - \lambda_l \right)$$

(17)

**Music Method**

The following steps summarize the MUSIC algorithm (Stoica and Nehorai, 1989):

- Perform Eigenvector and Eigen value on $R$. Where $e_j$ is the eigenvector associated with Eigen value $\lambda_j$ which will organized as:

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_L = \lambda_{L+1} = \cdots = \lambda_N = \sigma^2$$

(18)
• Let \( U_s = [e_1 \ldots e_L] \) and \( U_n = [e_{L+1} \ldots e_N] \) be the signal subspace and noise subspace, respectively.
• Construction of the MUSIC pseudo spectrum function as:
\[
F(\theta) = \frac{1}{a(\theta)^H U_s U_n^H a(\theta)}
\quad \text{(19)}
\]
where, \( a(\theta) \) is the steering vector.
• Find the values of \( \theta \) that minimizes \( F(\theta) \), these values are the estimated DOA.

**ESPRIT Method**

ESPRIT stands for estimation of Signal Parameters via rotational Invariance Technique. It developed by ROY and KEILATH (Keshavaraz, 2015; Li and Jiang, 2009; Yilmazer et al., 2008). This subspace based method explores the rotational invariance property in the subspace signal created by two sub-arrays derived from original array with a translation invariance structure.

The ESPRIT algorithm is summarized as follows:
• Perform Eigen values decomposition on \( R \)
• Extraction of the subspace signal
• Decompose the subspace into 2 subarray \( E_1 \) and \( E_2 \)
• Estimate the matrix \( \Psi \) using the method of least squares:
\[
\Psi = (E_1^H E_1)^{-1} E_1^H E_2
\quad \text{(20)}
\]
• Calculate the Eigen values of \( \Psi \)
• The DOA is obtained from Eigenvectors of \( \Psi \)

**Matrix Pencil Method**

The Matrix PENCIL technique is based on the use of the received signal expression. It was developed in order to estimate the poles of a system (Bayat and Adve, 2003).

The Pencil Matrix algorithm is summarized as follows:
• Choose the Pencil parameter \( P \) as:
\[
\frac{N}{3} \leq P \leq \frac{2 \times N}{3}
\quad \text{(21)}
\]
• Construction of the received signals matrix
• Decompose the received signals matrix into 2 subarrays \( X_0 \) and \( X_1 \) as:
\[
X_0 = \begin{bmatrix}
    x(0) & x(1) & \ldots & x(P-1) \\
    x(1) & x(2) & \ldots & x(P) \\
    \vdots & \vdots & \ddots & \vdots \\
    x(L-P) & x(L-P) & \ldots & x(L-2)
\end{bmatrix}
\quad \text{(22)}
\]
\[
X_1 = \begin{bmatrix}
x(1) & x(2) & \ldots & x(P) \\
\vdots & \vdots & \ddots & \vdots \\
x(L-P) & x(L-P+1) & \ldots & x(L-1)
\end{bmatrix}
\quad \text{(23)}
\]
And:
\[
\Psi_{MP} = X_0^H X_0 X_0^H X_1
\quad \text{(23)}
\]
• Estimate the matrix \( \Psi_{MP} \) using the method of least square:
\[
\Psi_{MP} = (X_0^H X_0)^{-1} X_0^H X_1
\quad \text{(23)}
\]
• The DOA is obtained from Eigenvectors of \( \Psi_{MP} \)

**Root-Music**

ROOT-MUSIC is the polynomial copy of Multiple Signal Classification (MUSIC). This technique detected the DOA of the arriving signals on searching roots in the complex plane (Fang et al., 2014).

The following ROOT-MUSIC algorithm is:
• Perform Eigen vector and Eigen value on \( R \). Where \( e_i \) is the eigenvector associated with Eigen value \( \lambda_i \) and:
\[
\lambda_1 > \lambda_2 > \ldots > \lambda_k > \lambda_{k+1} = \lambda_{k+2} = \ldots = \lambda_N = \sigma^2
\quad \text{(25)}
\]
• Let \( U_s = [e_1 \ldots e_L] \) and \( U_n = [e_{L+1} \ldots e_N] \) be the signal subspace and noise subspace, respectively
• Construction the polynomial degree:
\[
f(z) = f_0 + f_1 z^{-1} + \ldots + f_L z^{-L}
\quad \text{(26)}
\]
Who’s the coefficients correspond to the trace of the noise subspace matrix:
• Finding roots of this polynomial and display them in the complex plane with the unit circle
• Find the value of \( \theta \) corresponding to the nearest root in the unit circle

**Cramer-Rao Bound**

The Cramér-Rao bound (CRB) is a limit mathematical on the performance of an unbiased estimator. Indeed, it’s a lower bound on the error variance of any methods. In order for the analytical formulations to be valid, the following assumptions are needed (Stoica and Nehorai, 1989), a general formula for the CRB on the covariance matrix of any unbiased estimate (Zhang et al., 2005; He and Blum, 2010):
where, $\alpha_i$ denotes the $i$th component of $\alpha$. $(\Gamma(\alpha))$ independent of $\alpha$ and in (Fang et al., 2014) $m(\alpha)$ Independent of $\alpha$. The general formula has been presented in (Zhang et al., 2005). The CRB corresponding to these parameters is (He and Blum, 2010):

$$\text{CRB} = \frac{\sigma^2}{2N} \left( R_{ij} [H \otimes P^T] \right)^{-1}$$

(28)

Where:

$$H = D^* [I - A(A^* A)^{-1}] D$$

(29)

$$D = [d_1, \ldots, d_N]$$

(30)

$$d_i = \frac{\partial a(\theta_i)}{\partial \theta}, \quad \theta = \theta_i$$

(31)

$$P = \frac{1}{K} \sum_{k=1}^{K} S_r(f) * S_r(f)$$

(32)

Simulation Results

We have investigated the performances of our proposed method against other algorithms in terms of accuracy and resolution. We consider an ULA antenna array with eight elements and the antenna separation is one half of the wavelength.

The simulation parameters are given in Table 1. Three IR-UWB pulses incident on the antenna element from 30, 50 and 70° respectively, with a Signal to Noise Ratio (SNR) of 10 dB and 1000 snapshots.

We will study the resolution of our proposed method with Root-Music, Esprit and Matrix Pencil.

The obtained spectrums of Frequency Extended MUSIC and Frequency MUSIC have been illustrated in Fig. 1. However, those of Esprit, Root-Music and Pencil Matrix are presented in Table 2.

As shown in Fig. 1 and Table 2, Frequency Extended MUSIC, Frequency MUSIC, Esprit and Root-Music are able to detect three paths, but Matrix Pencil separate only one. Our proposed method has performed an accurate detection of DOAs.
In order to investigate the performances of our approach in terms of resolution, we consider two rays of UWB arriving signals with directions of arrival are 40 and 43°, respectively, with respect of the same simulation condition. The spectrums of Frequency Extended MUSIC and Frequency MUSIC are plotted in Fig. 2, while the result simulation of Esprit and Root-Music methods are illustrated in Table 3.

Figure 2 and Table 3 show that our method, ESPRIT and the root-MUSIC algorithms are able to detect and separate these two paths. However we can note that the Frequency Music method shows only one peak. So, it can be pointed out that the obtained results prove the high resolution of Frequency Extended Music. Moreover, it can be separate these two neighboring paths scenario in this condition simulation.
To illustrate the performance of our proposed approach, we simulate the Mean Square Error (MSE) is defined as following:

$$MSE = \sqrt{\frac{1}{\text{nbr}} \sum_{i=1}^{\text{nbr}} (\hat{\theta} - \theta)^2}$$

(33)

where, $\hat{\theta}$ is the estimated direction of arrival for $i^{th}$ measurement.

In the next simulation we use four paths of UWB signal localized at 10, 20, 40 and 80° respectively, we have simulated the MSE of DOA estimation of our proposed approach with Frequency Music method, it has evaluated for different number of sensors levels with a Signal to Noise Ratio (SNR) of 5 dB and 1000 snapshots.

Figure 3 illustrates the MSE of FREQUENCY EXTENDED MUSIC AND FREQUENCY MUSIC methods for four paths located at 10, 20, 40 and 80° versus number of sensors.

As shown above, we can notice that the number of sensor has an influence on the estimation quality; as well, our method is the most efficient. It is clearly evident that performance of the FREQUENCY EXTENDED MUSIC is better than FREQUENCY MUSIC at low SNR with less number of sensors.

The high performance of our proposed method has been studied via a comparison with the CRB, so the last simulation, has been carried out with estimating two paths at, 20 and 50° with 1000 snapshots.

The MSE of the FREQUENCY EXTENDED MUSIC, FREQUENCY MUSIC and CRB is evaluated in different SNR as we show at Fig. 4.

Indeed, the capacity of our proposed approach is proved in the simulation results. In contrast, FREQUENCY EXTENDED MUSIC estimates seem to be less sensitive in the multipath channel, when it compared with FREQUENCY MUSIC and CRB.

Thus, we conclude that our proposed techniques have better performance than others methods. It is less biased and has better resolution on comparing to FREQUENCY MUSIC for ULA array processing in IR-UWB systems.

**Conclusion**

In this study, a frequency domain DOA estimation method for direction of arrival estimation in IR-UWB channels based on extended MUSIC algorithm is proposed. Simulation results shows that our new
approach has high performances in terms of accuracy and resolution. It characterized by an efficient and reliable estimation. Our method improves the DOA estimation of IR-UWB signals in dense environment. So, it has been compared with others like Music, Esprit and root Music. Our Frequency Extended ensures an accurate estimation at low SNR with less number of sensors in multipath environment.

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Author’s Contributions

Hajer Meknessi: Conception, design and writing of computer programs for Frequency Extended-MUSIC Method for DOA Estimation in Indoor IR-UWB Environment.

Ferid Harrabi: coordination of the realization of this work.

Ali Gharsallah: organization of the plan of this study.

Ethics

This work is new and presents unpublished material. All authors have examined and agreed the manuscript. We confirm that no ethical issues concerned.

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