An Improved $H_{\infty}$ Control Method for Mixed Time-Varying Delay Systems

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ABSTRACT

This paper handles the $H_{\infty}$ control problems of linear system with mixed time-varying delays. To deduce the conservatism of delay-dependent stability condition, a descriptor system transformation approach is first introduced to represent the original system into descriptor system form. Second, a novel Lyapunov-Krasovskii functional which consists of different kinds of integrals even the triple integral is proposed to obtain delay-dependent sufficient condition for closed-loop system. To further deduce the conservatism of the condition, a novel integrate inequality technique is introduced in forms of linear matrix inequality (LMI). Finally, simulations show the effectiveness of the approach.

INTRODUCTION

When it comes to practical systems, time-delay often appears in the system model [1]. Time-delay, in some circumstance, leads to instability and poor control performance, so the stability issue in control systems with delay (sometimes mixed time-varying delay) is theoretically and practically important [2].

Recently, increasing attention has been paid to the stability criteria for time-delay systems. They can be classified into delay-dependent and delay-independent criteria [3]. Since delay-dependent criteria makes use of information on the length of delays, they are less conservative than delay-independent ones. Delay-dependent criteria, however, leads to unequal transformation in varying degrees. So one of the goals in control work is reducing conservatism of the system. The conservatism of the delay-dependent conditions is twofold [4-6]: the transformed and the original system are not equivalent and the bounds placed upon certain terms are quite wasteful. For the above factors, the descriptor method shows its superiority [7].

After taking consideration of the superiority in the descriptor method, we, in the present paper, introduce a novel integrate inequality technique in forms of linear matrix inequality, and a descriptor system transformation approach is first intro-
duced to represent the original system into descriptor system form. Then a novel Lyapunov-Krasovskii functional which consists of different kinds of integrals even the triple integral is proposed to obtain delay-dependent sufficient condition for closed-loop system, and it can further deduce the conservatism of the system. The effectiveness in the above theory is demonstrated by a simulation.

A NEW STABILIZATION METHOD

Consider the following linear system with mixed time-delays:

\[ \dot{x}(t) = \sum_{i=0}^{2} A_i x(t - \tau_i(t)) + Bu(t) + \int_{-h}^{0} A_x(s)x(t + s)ds + B_0 \phi(t) \]

where \( x(t) \in \mathbb{R}^n \) is the system state, \( u(t) \in \mathbb{R}^n \) is the control input, \( \tau_i(t) \) with \( \tau_0 \equiv 0 \) is time delay, \( A_i, B, B_0 \) and \( C \) are the system matrices with appropriate dimension. Assume that \( h_i \) is an known upper-bound on the time-delays \( \tau_i \), that is \( 0 < \tau_i(t) \leq h_i \). \( \phi(t) \in L^2_{\omega}(0,\infty) \) is the exogenous disturbance signal and \( z(t) \in \mathbb{R}^m \) is the controlled output., \( h = \max_{i=1,2} \{ h_i \} \). \( A_x(t) \) is a continuous matrix on \([-d,0)\). Moreover, assume \( \tau_i(t) \) are differentiable function and satisfies for all \( t \geq 0 \), \( \dot{\tau}_i(t) \leq d, < 1, i=1,2 \).

Consider state feedback control in following form

\[ u(t) = Kx(t) \] (2)

Where \( K \) is the controller gain matrix to be designed. So the closed-loop system is

\[ \dot{x}(t) = (A_0 + BK)x(t) + \sum_{i=0}^{2} A_i x(t - \tau_i(t)) + \int_{-d}^{0} A_x(s)x(t + s)ds + B_0 \phi(t) \] (3)

The purpose of this paper is to design controller (2) such that the system (3) is asymptotically stable with prescribed \( H_{\infty} \) performance index \( \gamma \), which means that:

1) the system (3) with \( \phi(t) = 0 \) is asymptotically stable;
2) under the zero initial condition, the controlled output satisfies

\[ J = \int_{h_0}^{\infty} (z^T(\tau)z(\tau) - \gamma^2 \omega^T(\tau)\omega(\tau))d\tau \leq 0 \] (4)

for all nonzero \( \omega(t) = 0 \in L_2_{\omega}(h_0,\infty) \).

To obtain the main results, some useful lemmas are given.

Lemma 1 \[8\]. for any variables \( a \in \mathbb{R}^n \), \( b \in \mathbb{R}^m \), and matrices \( N \in \mathbb{R}^{2nxn} \), \( R \in \mathbb{R}^{nxn} \), \( Y \in \mathbb{R}^{nx2n} \), \( Z \in \mathbb{R}^{2nx2n} \), the following inequality holds:

\[ -2b^T Na \leq \begin{bmatrix} a^T & b \end{bmatrix} \begin{bmatrix} R & Y^T - N^T \\ Y - N & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \] (5)

Where, \( \begin{bmatrix} R & Y \\ * & Z \end{bmatrix} \geq 0 \).

Lemma 2 \[3\]. for any matrix \( R > 0 \) and a vector function \( x : [\alpha, \beta] \rightarrow \mathbb{R}^n \), if the integrals concerned are well defined, then the following inequality holds:

\[ \int_{\alpha}^{\beta} x^T(s)ds R \int_{\alpha}^{\beta} x(s)ds \leq (\beta - \alpha) \int_{\alpha}^{\beta} x^T(s)Rx(s)ds. \] (6)

Lemma 3 \[9\]. for any constant positive-definite matrix \( P = P^T > 0 \) and a scalar \( \gamma > 0 \) such that the following integrations are well-defined.
Lemma 4\cite{10}. Let $x$ be a differentiable function: $[\alpha, \beta] \rightarrow \mathbb{R}^n$. For symmetric matrices $R \in \mathbb{R}^{n\times n}$ and $M_i, M_i \in \mathbb{R}^{3\times 3}$, and any matrices $M_j \in \mathbb{R}^{3\times 3}$ and $N_i, N_j \in \mathbb{R}^{3\times 3}$ satisfying
\[
\overline{\Omega} = \begin{bmatrix} M_i & M_j & N_j \\
0 & M_i & N_j \\
* & * & R \end{bmatrix} \succeq 0,
\]
the following inequality holds:
\[
-\int_{\alpha}^{\beta} \dot{x}^T(s)R\dot{x}(s)ds \leq \sigma^T\Omega\sigma
\]
Where, $\Omega = \beta - \alpha)\{M_i + \frac{1}{3}M_j\} + N_i\overline{N}_i + N_j\overline{N}_j \geq 0$, $\sigma = \begin{bmatrix} x^T(\beta) & x^T(\alpha) \end{bmatrix}$, $\Omega = \begin{bmatrix} N_{i}^T_{11} & N_{i}^T_{12} & N_{i}^T_{13} \\
N_{i}^T_{21} & N_{i}^T_{22} & N_{i}^T_{23} \\
N_{i}^T_{31} & N_{i}^T_{32} & N_{i}^T_{33} \end{bmatrix}$, $\overline{N}_i = \begin{bmatrix} \overline{e}_1 - \overline{e}_2, \\
2\overline{e}_1 - \overline{e}_2, \\
\overline{e}_1 - \overline{e}_2 \end{bmatrix}$, $\overline{N}_j = \begin{bmatrix} \overline{e}_1 & \overline{e}_2 & \overline{e}_3 \end{bmatrix}$.

**Main Result**

In this section, we will design the state controller such that the closed-loop system is asymptotically stable and with $H_\infty$ performance. First, the equivalent descriptor form\cite{11} is as follows
\[
\dot{x}(t) = y(t)
\]

or: $E\ddot{x}(t) = \begin{bmatrix} 0 & I \\
\sum_{i=1}^{2} A_i & -I \end{bmatrix} \overline{x}(t) - \begin{bmatrix} 0 \\
\sum_{i=1}^{2} A_i \end{bmatrix} \int_{\tau_i(t)}^t y(s)ds + \begin{bmatrix} 0 \\
\sum_{i=1}^{2} A_i \end{bmatrix} \int_{\tau_i(t)}^t y(s)ds + \begin{bmatrix} 0 \\
B_i \end{bmatrix} \omega(t)
\]

with $\overline{x}(t) = col\{x(t), y(t)\}$, $E = \text{diag}\{I, 0\}$. The following Lyapunov-Krasovskii functional is employed:
\[
V(t) = \overline{x}^T(t)EP\overline{x}(t) + V_2 + V_3 + V_4 + V_5 + V_6
\]

Where, $P = \begin{bmatrix} P_1 & 0 \\
\frac{1}{3} P_2 & P_3 \end{bmatrix}$, $P_1 > 0$, $V_2 = \sum_{i=1}^{2} \int_{\tau_i(t)}^t x^T(\tau + \theta)A_i(\theta)x(\tau + \theta)d\tau \theta$, $V_3 = \sum_{i=1}^{2} \int_{\tau_i(t)}^t x^T(\tau)S_i x(\tau)d\tau$, $V_4 = \sum_{i=1}^{2} \int_{\tau_i(t)}^t \dot{x}^T(\tau + \theta)A_i(\theta)R_\theta A_i^T(\tau + \theta) x(\tau + \theta)d\tau \theta$, $V_5 = \sum_{i=1}^{2} \int_{\tau_i(t)}^t \dot{x}^T(\tau + \theta)A_i(\theta)R_\theta A_i^T(\tau + \theta) x(\tau + \theta)d\tau \theta$, $V_6 = \sum_{i=1}^{2} \int_{\tau_i(t)}^t \dot{x}^T(\tau + \theta)A_i(\theta)R_\theta A_i^T(\tau + \theta) x(\tau + \theta)d\tau \theta$, $EP = P^T E \succeq 0$.

**Theorem 1:** With $B = 0$, the system of(2.3) is asymptotically stable and for a described $\gamma > 0$, the function(4)satisfies $J(\omega) < 0$ for all nonzero $\omega \in L_{2}^{\infty}[0, \infty)$, if there exist $n \times n$ matrices $0 < P_i, P_2, P_3, S_i, Y_i, Z_i, R_j, W_i, i = 1, 2$, positive definite symmetric matrices $F_j \in \mathbb{R}^{n\times n}$, $G_j, j = 1, 2$, and $R_i, R_j \in \mathbb{R}^{n\times n}, i = 1, 2$, $M'_i, M''_i, M'_j, M''_j \in \mathbb{R}^{3\times 3}$, and any matrices $M'_i, M''_i \in \mathbb{R}^{3\times 3}$, $N'_i, N'_i, N'_i, N'_i \in \mathbb{R}^{3\times 3}$ satisfying:
\[
\overline{N} = \begin{bmatrix} M'_i & M'_j & N'_i \\
0 & M'_i & N'_i \\
* & * & R_i - R'_i \end{bmatrix} \succeq 0, \quad \overline{N} = \begin{bmatrix} M''_i & M''_j & N''_i \\
0 & M''_i & N''_i \\
* & * & R_i - R'_i \end{bmatrix} \succeq 0
\]
such that the following LMI holds.
where, \[ R_i^r Y_i Z_i \geq 0, \quad Y_i = [Y_{i1} \quad Y_{i2}], \quad Z_i = [Z_{i1} \quad Z_{i2}], \quad i = 1, 2. \]

\[ \Phi = P^T \begin{bmatrix} 0 & I \\ \sum_{i=0}^{2} A_i & -I \end{bmatrix} + \begin{bmatrix} \sum_{i=0}^{2} A_i & I \\ 0 & -I \end{bmatrix} P \begin{bmatrix} 0 \\ B_i \end{bmatrix} + \begin{bmatrix} \sum_{i=0}^{2} A_i & I \\ 0 & -I \end{bmatrix} P \begin{bmatrix} 0 \\ B_i \end{bmatrix} P^T \begin{bmatrix} 0 \\ B_i \end{bmatrix} \]

\[ \Xi = \begin{bmatrix} \sum_{i=1}^{\infty} h_k \Gamma \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ \sum_{i=0}^{2} A_i \\ -I \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ \sum_{i=0}^{2} A_i \\ -I \end{bmatrix} P \begin{bmatrix} 0 \\ B_i \end{bmatrix} + \begin{bmatrix} \sum_{i=0}^{2} A_i & I \\ 0 & -I \end{bmatrix} P \begin{bmatrix} 0 \\ B_i \end{bmatrix} + \begin{bmatrix} \sum_{i=0}^{2} A_i & I \\ 0 & -I \end{bmatrix} P \begin{bmatrix} 0 \\ B_i \end{bmatrix} P^T \begin{bmatrix} 0 \\ B_i \end{bmatrix} \]

\[ \Xi_0 = \tau_1 (t) (M'^{T}_{i1} + 2 \frac{1}{3} M'^{T}_{i2}) + \tau_3 (t) (M'^{T}_{i3}), \quad \Xi_0 = \tau_2 (t) (M'^{T}_{i4}) + \tau_3 (t) (M'^{T}_{i5}) \]

Proof:

Note that:
\[ \overline{X}'(t) E P \overline{X}(t) = x'(t) P \overline{x}(t) \]

Hence, by differentiating \( V(t) \) in (2.3), and from performance (4), we can get:
\[ \frac{dV}{dt} + \zeta(t) z(t) - Y \alpha'(t) a(t) = 2\overline{X}'(t) P \begin{bmatrix} 0 \\ B \end{bmatrix} u(t) + \eta_a + \eta_b + \eta_c + \eta_d + 2\overline{X}'(t) P \begin{bmatrix} 0 \\ I \end{bmatrix} \int_{-\tau}^{0} A_j(s)x(t+s)ds \]

\[ \zeta = \begin{bmatrix} \Phi & P^T \begin{bmatrix} 0 \\ B_i \end{bmatrix} \xi - \sum_{i=1}^{\infty} (1 - d_i)x'(t - \tau_i(t)) s, x(t - \tau_i(t)) + \tau_i(t) z(t) - \sum_{i=1}^{\infty} \int_{-\tau}^{0} y'(\tau) R \gamma(\tau)d\tau \end{bmatrix} \]

\[ + \sum_{i=1}^{\infty} x'(t) W \int_{-\tau}^{0} x(s)ds + \sum_{i=1}^{\infty} (1 - \tau_i(t)) x'(t - \tau_i(t)) W \int_{-\tau}^{0} x(s)ds \]

Where, \( \xi = \text{col}(\overline{x}(t), \alpha(t)) \) and \( \Phi = P^T \begin{bmatrix} 0 & I \\ \sum_{i=0}^{2} A_i & -I \end{bmatrix} + \begin{bmatrix} \sum_{i=0}^{2} A_i & I \\ 0 & -I \end{bmatrix} P \begin{bmatrix} 0 \\ B_i \end{bmatrix} + \begin{bmatrix} \sum_{i=0}^{2} A_i & I \\ 0 & -I \end{bmatrix} P \begin{bmatrix} 0 \\ B_i \end{bmatrix} P^T \begin{bmatrix} 0 \\ B_i \end{bmatrix} \]

\[ \eta_a = -2 \sum_{i=1}^{\infty} \overline{X}'(t) P A \int_{-\tau_i}^{0} y(s)ds, \quad \eta_b = -\int_{-\tau}^{0} \overline{x}'(t + \theta) A_j(t) R \gamma(t)d\theta, \]

\[ \eta_c = -\sum_{i=1}^{\infty} \int_{-\tau}^{0} \overline{x}'(t) F \gamma(t)d\theta, \quad \eta_d = -\sum_{i=1}^{\infty} \int_{-\tau_i}^{0} \overline{x}'(t) G \gamma(t)d\theta. \]

According to Lemma 1-4, make transformations on \( \eta_a, \eta_b, \eta_c, \eta_d \) and by integrating the resulting inequality in \( t \) from 0 to \( \infty \), we have:
\[ \int_{0}^{\infty} \zeta(t) z(t)dt = \int_{0}^{\infty} \overline{x}'(t) C^T C_x(t)dt \]

Then, from (2.4) to (2.6), it realizes that \( J < 0 \) if the following LMI (2.4) holds.

This completes the proof.
Next, we will design the controller gain matrix $K$. From theorem 1 and its proof, we know that when $B \neq 0$, $A_i\text{in}(2.4)$ turns into $A_i+ BK$. Correspondingly, the condition(2.4) in Theorem 1 turns into nonlinear.

To obtain linear matrix inequality form, the problem with(8) is that it is linear on its variables, only when the state-feedback gain $K$ is given. So define $P^{-1}=Q=[Q_1, Q_2, Q_3]$ and $\Delta=\text{diag}\{Q^T, I, Q_i, R_i^{-1}, 0\}$. From theorem 1 and its proof, 

$$P^{-1} = Q = \left[ \begin{array}{cc} Q_1 & 0 \\ Q_2 & Q_3 \end{array} \right]$$

Then $\Delta=\text{diag}\{Q^T, I, Q_i, R_i^{-1}, 0\}$. From theorem 1 and its proof, 

$$P^{-1} = Q = \left[ \begin{array}{cc} Q_1 & 0 \\ Q_2 & Q_3 \end{array} \right]$$

We multiply(8) by $\Delta$ and $\Delta^T$, on the left and on the right, respectively. Define 

$$\sum_{i=1}^{n} Q_i^T Q_i = \sum_{i=1}^{n} Q_i^T S_i Q_i = \sum_{i=1}^{n} \frac{1}{2} A_i^T (\theta) R_i A_i (\theta) d\theta Q_i = \tilde{A}_d, Q_i^T C Q_i = \tilde{C}, Q_i^T \tilde{Z}_i Q_i = \tilde{Z}_i, Q_i^T Q_i = \tilde{Z}_i,$$ 

$$Q_i^T \tilde{Z}_i Q_i = \tilde{Z}_i, Q_i^T Q_i = \tilde{Z}_i, Q_i W_i Q_i = \tilde{W}_i, Q_i^T \tilde{Z}_i Q_i = \tilde{Z}_i, Q_i G_i Q_i = \tilde{G}_i, Q_i \tilde{Z}_i Q_i = \tilde{Z}_i, Q_i W_i Q_i = \tilde{W}_i, Q_i^T \tilde{Z}_i Q_i = \tilde{Z}_i, Q_i G_i Q_i = \tilde{G}_i, Q_i \tilde{Z}_i Q_i = \tilde{Z}_i, Q_i W_i Q_i = \tilde{W}_i, Q_i^T \tilde{Z}_i Q_i = \tilde{Z}_i, Q_i G_i Q_i = \tilde{G}_i, Q_i \tilde{Z}_i Q_i = \tilde{Z}_i, Q_i W_i Q_i = \tilde{W}_i.$$ 

The state-feedback gain is then given by 

$$K=\tilde{Y}Q_i^{-1}$$  \hspace{1cm} (2.7) 

Proof:

Multiplying(2.4) by $\Delta$ and $\Delta^T$, on the left and on the right, respectively. Further, substituting $Q=[Q_1, Q_2, Q_3]$ and $Y_i=[Y_i, Y_i]=\epsilon_i A_i^T [P_2, P_1]$ into the above, and after
Schur complement, we transform “$-2\sum_{i=1}^{2}Q_{i}^{T}F_{i}Q_{i}$” into “$2\sum_{i=1}^{2}(F_{i}^{T} - Q_{i}^{T} - Q_{i}^{T})$”. And define

$$
\sum_{i=1}^{2}Q_{i}^{T}Z_{i}Q_{i} = \tilde{Z}_{i}, \quad \sum_{i=1}^{2}Q_{i}^{T}S_{i}Q_{i} = \tilde{S}_{i}, \quad Q_{i}^{T}A_{i}^{T}(\theta)R_{j}A_{i}(\theta)d\theta Q_{i} = \bar{A}_{i}, \quad Q_{i}C^{T}Q_{i} = \bar{C}_{i},
$$

where $Q_{i}^{T}Z_{i}Q_{i} = \tilde{Z}_{i}$, $Q_{i}^{T}S_{i}Q_{i} = \tilde{S}_{i}$, $Q_{i}^{T}A_{i}^{T}(\theta)R_{j}A_{i}(\theta)d\theta Q_{i} = \bar{A}_{i}$, $Q_{i}C^{T}Q_{i} = \bar{C}_{i}$, $Q_{i}^{T}Z_{i}Q_{i} = \tilde{Z}_{i}$, $Q_{i}^{T}S_{i}Q_{i} = \tilde{S}_{i}$, $Q_{i}^{T}A_{i}^{T}(\theta)R_{j}A_{i}(\theta)d\theta Q_{i} = \bar{A}_{i}$, $Q_{i}C^{T}Q_{i} = \bar{C}_{i}$.

Then the inequality (2.9) transforms to the following:

$$
\Phi^{*}(Q_{1}^{T}F_{1}Q_{1} + Q_{2}^{T}F_{2}Q_{2}) = \sum_{i=1}^{2}A_{i}(\theta)R_{j}A_{i}(\theta)d\theta Q_{i} + \sum_{i=1}^{2}Q_{i}C^{T}Q_{i} = \sum_{i=1}^{2}\bar{A}_{i} + \bar{C}_{i}.
$$

(2.10)

Where,

$$
\Phi^{*} = \sum_{i=1}^{2}A_{i} + BK - I
$$

For the accordance of the variables, we make the following transformations:

Multiplying $\tilde{X}$ by $\tilde{T}$ and $\tilde{X}$ ( $\tilde{X} = diag\{Q_{1},Q_{2},Q_{3},Q_{4},Q_{5},Q_{6}\}$ ) on the left and on the right, respectively. We can get:

$$
\tilde{X}_{i} = \begin{bmatrix}
Q_{i}^{T}M_{i}Q_{i} & Q_{i}^{T}N_{i}Q_{i} & \vdots & Q_{i}^{T}R_{i}Q_{i}
Q_{i}^{T}M_{i}^{T}Q_{i} & Q_{i}^{T}N_{i}^{T}Q_{i} & \vdots & Q_{i}^{T}R_{i}^{T}Q_{i}
\vdots & \vdots & \ddots & \vdots
Q_{i}^{T}M_{i}^{T}Q_{i} & Q_{i}^{T}N_{i}^{T}Q_{i} & \vdots & Q_{i}^{T}R_{i}^{T}Q_{i}
\end{bmatrix}
\geq 0 \quad \tilde{X}_{i}^{*} = \begin{bmatrix}
Q_{i}^{T}M_{i}Q_{i} & Q_{i}^{T}M_{i}^{T}Q_{i} & Q_{i}^{T}N_{i}Q_{i} & Q_{i}^{T}R_{i}Q_{i}
Q_{i}^{T}M_{i}^{T}Q_{i} & Q_{i}^{T}N_{i}^{T}Q_{i} & Q_{i}^{T}R_{i}^{T}Q_{i}
\vdots & \vdots & \ddots & \vdots
Q_{i}^{T}M_{i}^{T}Q_{i} & Q_{i}^{T}N_{i}^{T}Q_{i} & Q_{i}^{T}R_{i}^{T}Q_{i}
\end{bmatrix}
\geq 0.
$$

(2.11)

After multiplying $\begin{bmatrix}
R_{i} & Y
* & Z_{i}
\end{bmatrix}_{\geq 0}$ by $\begin{bmatrix}
Q_{i}^{T}Q_{i} & Q_{i}^{T}R_{i}Q_{i}
Q_{i}^{T}M_{i}Q_{i} & Q_{i}^{T}M_{i}^{T}Q_{i} & Q_{i}^{T}N_{i}Q_{i} & Q_{i}^{T}R_{i}Q_{i}
\end{bmatrix}$ and column($Q_{i}$, $Q_{i}$) , on the left and on the right, respectively. We can get:

$$
\begin{bmatrix}
Q_{i}^{T}R_{i}Q_{i} & Q_{i}^{T}Y_{i}Q_{i}
\end{bmatrix}_{\geq 0}
$$

Then the proof is completed.

EXAMLES

The system is described with:

$$
A_{i} = \begin{bmatrix}
1 & 0 & 1 & 0
0 & 0 & 0 & 0
0 & 2 & 0 & 0
0 & 0 & 1 & 0
\end{bmatrix}, \quad A_{i} = \begin{bmatrix}
1 & 0 & 1 & 0
0 & 0 & 0 & 0
0 & 2 & 0 & 0
0 & 0 & 1 & 0
\end{bmatrix}, \quad A_{i} = \begin{bmatrix}
1 & 0 & 0 & 0
0 & 0 & 0 & 0
0 & 0 & 0 & 0
0 & 0 & 0 & 0
\end{bmatrix}, \quad B_{i} = \begin{bmatrix}
1 & 0
0 & 0
\end{bmatrix}, \quad B_{i} = \begin{bmatrix}
0 & 0
0 & 0
\end{bmatrix}.
$$

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By applying theorem 2, the upper bound of system stability is obtained after 98 iterations, and the upper bound is $h \leq 1.203$. The above result is obtained in the situation of $\gamma_{\text{min}} = 21$, which comes from the LIMs. The value of the $\gamma$ also gives $K = [68.9632 \; -2.6737 \; -48.7291 \; -11.7802]$, which shows in robust control law $u(t) = K(x(t))$.

After applying the above state feedback control law in the system, the relationship between $a$ and $z$ can be plotted. The stability of the closed-loop system is guaranteed by the inequality in theorem 1, and the peak value in the latter relationship shows the practical limit that obtained by disturbance attenuation in the system. The results obtained by theorem 2 proposed in this paper is superior to the references, so the new conservatism method which combined with descriptor method is proved to be superior to other solutions.

CONCLUSION

The $H_{\infty}$ control problem of linear system with mixed time-varying delays is solved in this paper. The newly proposed Lyapunov-Krasovskii functional obtains delay-dependent sufficient condition for closed-loop system, and the descriptor system transformation approach deduces the conservatism of delay-dependent stability condition. A novel integrate inequality technique which is introduced in forms of linear matrix inequality (LMI) further deduces the conservatism of the condition. The superiority in conservatism of the proposed method is verified in the simulations.

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