Robust Kalman Filter with Application to State Estimation of a Nuclear Reactor

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1. Introduction

State estimation algorithm deals with recovering some desired state variables of a dynamic system from available noisy measurements, and estimation of the state variables is one of the fundamental and significant problems in control and signal processing areas, and many significant progresses have been made in this area. In 1940s, Wiener, the founder of the modern statistical estimation theory, established the Wiener filtering theory which solves the minimum variance estimation problem for stationary stochastic processes. It was not until late 1950s and early 1960s that Kalman filtering theory, a novel recursive filtering algorithm, was developed by Kalman and Bucy which did not require the stationarity assumption (Kalman, 1960; Kalman & Bucy, 1961). Since Kalman filter theory is only applicable for linear systems and almost all practical dynamic systems are nonlinear, Bucy and some other researchers were engaged in extending Kalman filtering theory to nonlinear systems in the following 10 years (Bucy & Senne, 1971). The most celebrated and widely used nonlinear filtering algorithm is the extended Kalman filter (EKF), which is essentially a suboptimal nonlinear filter. The key idea of the EKF is using the linearized dynamic model to calculate the covariance and gain matrices of the filter. The Kalman filter (KF) and the EKF are all widely used in many engineering areas, such as aerospace, chemical and mechanical engineering.

However, it is well known that both the KF and EKF are not robust against modelling uncertainties and disturbances. This has motivated many studies on extending the Kalman filtering theory to its robust version, which may yield a suboptimal state estimator with respect to the nominal system model, but will guarantee an upper bound to the estimation error variance in spite of large disturbances and modelling uncertainties. In recent years, several results have been made on the design of such robust state estimators. Based on the fuzzy dynamic programming technique, a bounding estimator (Jian, 1975) for uncertain nonlinear systems was developed, which gave an upper bound to the error for any allowed system parameter variation. Petersen & McFarlane (1991, 1994) converted a robust Kalman filter design problem to a $H_{\infty}$ controller design problem of a certain linear continuous time-invariant system based on the concept of quadratic stability of linear continuous time-invariant systems. Xie & Soh (1994) converted the existence of robust Kalman filters to the existence of two differential Riccati equations, and then established a design approach of robust estimators for linear continuous time systems with uncertainties in the state and the
output matrices. Yang & Wang (2001) presented a robust Kalman filter design approach for uncertain linear continuous time-invariant systems with norm-bounded uncertainties not only in the state and the output matrices but also in the estimator’s gain matrix, and this design approach were given in terms of solutions to algebraic Riccati equations. The continuous-time case was fully discussed in Shaked & de Souza (1995) where both the finite and infinite-horizon filtering problems were addressed, and necessary and sufficient conditions for the existence of robust Kalman filters with an optimized upper bound of the error variance were established.

In most engineering applications, both the KF and the EKF are implemented as program codes in computers. Therefore, the design of robust estimators for uncertain discrete-time systems is more important. Recently, there have been some promising results in the design of discrete-time robust Kalman filters. Xie, Petersen and Shaked extended their results on the design of continuous-time Kalman filters to that of discrete-time Kalman filters (Xie et al., 1994; Petersen & McFarlane, 1996; Theodor & Shaked, 1996). Fu, de Souza and Luo (2000) presented a design approach for finite-horizon robust Kalman filters, and the scalar parameters of the robust Kalman filter can be obtained by solving a semidefinite program. Shaked, Xie and Soh (2001) established a new robust Kalman filter design approach through which all the parameters of the designed robust Kalman filter are given by solving a semidefinite program subjected to a set of linear matrix inequalities (LMIs). Zhu, Soh and Xie (2002) gave necessary and sufficient conditions to the design properties of the robust Kalman filters over finite and infinite horizon in terms of a pair of parameterized difference Riccati equation. Garcia, Tabouriech and Peres (2003) showed that if the original systems satisfied some particular structural conditions, such as minimum-phase, and if the uncertainty had a specific structure, then the formula of the robust Kalman filter only involved the original system matrices. The discrete-time systems discussed in the aforementioned papers are all linear discrete-time systems with uncertainty only in the state and the output matrices. Very recently, results on some new types of linear uncertain discrete-time systems have also been given. Yang, Wang and Hung (2002) presented a design approach of a robust Kalman filter for linear discrete-time varying systems with multiplicative noises. Since the covariance matrices of the noises cannot be known precisely, Dong and You (2006) derived a finite-horizon robust Kalman filter for linear time-varying systems with norm-bounded uncertainties in the state matrix, the output matrix and the covariance matrices of noises. Based on the techniques presented in Zhu, Soh and Xie (2002), Lu, Xie and et al. (2007) gave a robust Kalman filter design approach for the linear discrete-time systems with measurement delay and norm-bounded uncertainty in the state matrix. Hounkpevi and Yaz (2007) proposed a robust Kalman filter for linear discrete-time systems with sensor failures and norm-bounded uncertainty in the state matrix. Though there have been many types of promising robust Kalman filters, new engineering demand can stimulate the birth of novel robust Kalman filters.

From the viewpoint of the robust state estimator design based on the dynamical model for a nuclear reactor, there is not only uncertainty in the state and the output matrices but also nonlinear perturbation and uncertainty in the input and the direct output matrices. The robust Kalman filters presented in the aforementioned papers cannot deal with this case. In this chapter, a finite-horizon robust Kalman filter for discrete-time systems with nonlinear perturbation and norm-bounded uncertainties in the state, the output, the input and the direct output matrices is presented. Moreover, the robust Kalman filter is applied to solve...
the state-estimation problem of a low temperature pressurized water reactor (LTPWR), and a numerical experiment with a contrast to the EKF is done. Simulation results show that the state estimation performance provided by the robust Kalman filter is higher than that provided by the EKF.

This chapter is organized as follows: In Section 2, the model of the discrete-time system with nonlinear perturbation and norm bounded uncertainties in all the system matrices is presented, and the corresponding robust Kalman filtering problem is formulated. This problem is solved, for the finite-horizon case, in Section 3. In Section 4, the dynamic model of the aforementioned LTPWR is introduced, and then numerical simulation results are given to demonstrate the feasibility of applying the new robust Kalman filter to the state estimation for the LTPWR and the high performance of this estimator.

2. Problem formulation

Consider the following class of uncertain discrete time-varying systems defined on \( k = 0, 1, \ldots, N \):

\[
\begin{align*}
x_{k+1} &= (A_k + \Delta A_k) x_k + (B_k + \Delta B_k) w_k + f_k(x_k) \\
y_k &= (C_k + \Delta C_k) x_k + (D_k + \Delta D_k) v_k
\end{align*}
\]

(1)

where \( x_k \in \mathbb{R}^n \) is the state vector, \( y_k \in \mathbb{R}^p \) is the system output, \( w_k \in \mathbb{R}^m \) is the process noise, \( v_k \in \mathbb{R}^q \) is the measurement noise, \( A_k, B_k, C_k \) and \( D_k \) are given real time-varying matrices with proper dimensions, the matrices \( \Delta A_k, \Delta B_k, \Delta C_k \) and \( \Delta D_k \) correspond to the uncertainties in the matrices \( A_k, B_k, C_k \) and \( D_k \) respectively, and \( f_k(x_k) \) denotes the nonlinear perturbation.

Before formulating the robust Kalman filter design problem, we firstly make the following three assumptions to system (1).

**Assumption 1** Suppose the uncertainty matrices \( \Delta A_k, \Delta B_k, \Delta C_k \) and \( \Delta D_k \) satisfying the following condition

\[
\begin{bmatrix}
\Delta A_k & \Delta B_k \\
\Delta C_k & \Delta D_k
\end{bmatrix} = \begin{bmatrix}
H_{1,k} & F_k \end{bmatrix} \begin{bmatrix} E_{1,k} & E_{2,k} \end{bmatrix}
\]

(2)

where \( H_{1,k} \in \mathbb{R}^{nxr} \), \( H_{2,k} \in \mathbb{R}^{pxr} \), \( E_{1,k} \in \mathbb{R}^{sxr} \) and \( E_{2,k} \in \mathbb{R}^{sxm} \) are given matrices, and \( F_k \in \mathbb{R}^{r,s} \) denotes the norm-bounded time-varying uncertainty satisfying

\[
F_k^T F_k \leq I.
\]

**Assumption 2** Suppose the nonlinear perturbation \( f_k(x_k) \) satisfying

\[
f_k(O) = 0.
\]

(4)

Therefore, from equation (4), the stochastic vector \( f_k(x_k) \) can be expressed as

\[
f_k(x_k) = P_k(x_k) x_k.
\]

(5)

Moreover, we suppose that the stochastic matrix \( P_k(x_k) \) satisfies
where the matrix $P$ is a given positive definite symmetric matrix.

**Assumption 3** Suppose the process noise $w_k$, the measurement noise $v_k$ and the initial state vector $x_0$ have the following statistical properties:

$$
E\left[\begin{bmatrix} w_k \\ v_k \\ x_0 \end{bmatrix}\right] = \begin{bmatrix} O \\ O \\ \bar{x}_0 \end{bmatrix},
$$

$$
E\begin{bmatrix} w_k \\ v_i \\ w_j \\ v_j \\ x_o \\ x_o \end{bmatrix} = \begin{bmatrix} W_i\delta_{ij} & O & O \\ O & V_i\delta_{ij} & O \\ O & O & X_0 \end{bmatrix},
$$

where $E\{\cdot\}$ stands for the mathematical expectation operator, $W_i$, $V_i$ and $X_0$ represents the covariance matrices of the process noise, the measurement noise and the initial state vector respectively, $\delta_{ij}$ is the Kronecker function which is equal to unity for $k=j$ and zero for the other case.

Moreover, let the state estimator of system (1) take the form

$$
\hat{x}_{k+1} = A_{\delta_k}\hat{x}_k + K_{\delta_k}(y_k - C_{\delta_k}\hat{x}_k)
$$

where $\hat{x}_k \in \mathbb{R}^n$ is the state estimation with $\hat{x}_0 = O$, $A_{\delta_k}$ and $K_{\delta_k}$ are respectively the state and the gain matrix. In the following part, we shall design a finite-horizon Kalman filter for structure (8), such that for all allowed nonlinear perturbation and model uncertainty, there exists a sequence of symmetric positive-definite matrices $\Sigma_k$ satisfying

$$
E\left[ (x_k - \hat{x}_k)(x_k - \hat{x}_k)^T \right] \leq \Sigma_k.
$$

Since

$$
E\left[ (x_k - \hat{x}_k)^T (x_k - \hat{x}_k) \right] \leq \text{tr}(\Sigma_k),
$$

the robust Kalman filter design problem to be solved in the next section is

$$
A_{\delta_k}, K_{\delta_k} = \arg \min_{A_{\delta_k}, K_{\delta_k}} \text{tr}(\Sigma_k)
$$

s.t. (1), (2), (5), (6), (7), (8) and (9).

3. Robust Kalman filter design

In this section the robust Kalman filter for the uncertain system (1) is presented. Firstly, the state space model of the augmented system consisting of system (1) and estimator (8) will be established.

Define the state vector for the augmented system as
\[
\hat{x}_k = \left[\begin{array}{c}
  x_k^T \\
  \hat{x}_k^T
\end{array}\right]^T
\]

then the state-space model of the augmented system can be represented as

\[
\dot{\hat{x}}_{k+1} = \left(\tilde{A}_k + \tilde{H}_{1,k} F_k \tilde{E}_{1,k}\right) \hat{x}_k + \left(\tilde{B}_k + \tilde{H}_{2,k} \tilde{F}_k \tilde{E}_{2,k}\right) \tilde{w}_k + \tilde{f}_k(\hat{x}_k)
\]

(12)

where

\[
\tilde{A}_k = \begin{bmatrix}
  A_k & 0 \\
  K_{ob} C_k & A_{ob} - K_{ob} C_k
\end{bmatrix},
\quad
\tilde{B}_k = \begin{bmatrix}
  B_k & 0 \\
  O & K_{ob} D_k
\end{bmatrix},
\quad
\tilde{H}_{1,k} = \begin{bmatrix}
  H_{1,k} & 0 \\
  K_{ob} H_{2,k}
\end{bmatrix},
\quad
\tilde{F}_k = \begin{bmatrix}
  F_k & 0 \\
  O & F_k
\end{bmatrix},
\quad
\tilde{w}_k = \begin{bmatrix}
  w_k \\
  v_k
\end{bmatrix},
\quad
\tilde{f}_k(\hat{x}_k) = \begin{bmatrix}
  f_k(x_k) \\
  O
\end{bmatrix}.
\]

Let the covariance matrix of the augmented system (12) to be

\[
\tilde{\Sigma}_{k+1} = \left(\tilde{A}_k + \tilde{H}_{1,k} F_k \tilde{E}_{1,k}\right) \tilde{\Sigma}_k \left(\tilde{A}_k + \tilde{H}_{1,k} F_k \tilde{E}_{1,k}\right)^T + \left(\tilde{B}_k + \tilde{H}_{2,k} \tilde{F}_k \tilde{E}_{2,k}\right) \tilde{W}_k \left(\tilde{B}_k + \tilde{H}_{2,k} \tilde{F}_k \tilde{E}_{2,k}\right)^T + \left(\tilde{\Sigma}_k \tilde{G}_k \tilde{P} \tilde{G}_k^T \tilde{\Sigma}_k\right)^T
\]

(13)

where

\[
\tilde{W}_k = \begin{bmatrix}
  W_k & O \\
  O & V_k
\end{bmatrix}.
\]

From Assumption 2,

\[
\tilde{\Sigma}_{k+1} = \left(\tilde{A}_k + \tilde{H}_{1,k} F_k \tilde{E}_{1,k}\right) \tilde{\Sigma}_k \left(\tilde{A}_k + \tilde{H}_{1,k} F_k \tilde{E}_{1,k}\right)^T + \left(\tilde{B}_k + \tilde{H}_{2,k} \tilde{F}_k \tilde{E}_{2,k}\right) \tilde{W}_k \left(\tilde{B}_k + \tilde{H}_{2,k} \tilde{F}_k \tilde{E}_{2,k}\right)^T + \left(\tilde{\Sigma}_k \tilde{G}_k \tilde{P} \tilde{G}_k^T \tilde{\Sigma}_k\right)^T
\]

(14)

where

\[
\tilde{P} = \begin{bmatrix}
  P & O \\
  O & O
\end{bmatrix},
\quad
\tilde{G}_k = \begin{bmatrix}
  G_k & O \\
  O & O
\end{bmatrix},
\quad
\tilde{\Sigma}_k = \begin{bmatrix}
  \tilde{\Sigma}_k & 0 \\
  0 & \tilde{\Sigma}_k
\end{bmatrix}.
\]

and the matrix \(G_k\) satisfies \(G_k^T G_k \leq I\). Moreover, if we define

\[
\tilde{H}_{1,k} = \begin{bmatrix}
  H_{1,k} & P \\
  K_{ob} H_{2,k} & 0
\end{bmatrix},
\quad
\tilde{F}_k = \begin{bmatrix}
  F_k & O \\
  O & G_k
\end{bmatrix}.
\]
and

\[ \bar{E}_{1,k} = \begin{bmatrix} E_{1,k} & O \\ I & O \end{bmatrix}, \]

then \( \bar{\Sigma}_{k+1} \) can be rewritten as

\[ \bar{\Sigma}_{k+1} = \begin{bmatrix} \hat{A}_k + \hat{H}_{1,k} \hat{F}_1 \bar{E}_{1,k} \\ \hat{A}_k + \hat{H}_{1,k} \hat{F}_1 \bar{E}_{1,k} \end{bmatrix}^T \bar{\Sigma}_k \begin{bmatrix} \hat{A}_k + \hat{H}_{1,k} \hat{F}_1 \bar{E}_{1,k} \\ \hat{A}_k + \hat{H}_{1,k} \hat{F}_1 \bar{E}_{1,k} \end{bmatrix} + \begin{bmatrix} \hat{B}_k + \hat{H}_{2,k} \hat{F}_2 \bar{E}_{2,k} \\ \hat{B}_k + \hat{H}_{2,k} \hat{F}_2 \bar{E}_{2,k} \end{bmatrix} \bar{W}_k \begin{bmatrix} \hat{B}_k + \hat{H}_{2,k} \hat{F}_2 \bar{E}_{2,k} \\ \hat{B}_k + \hat{H}_{2,k} \hat{F}_2 \bar{E}_{2,k} \end{bmatrix}^T \] (15)

For the convenience of the following discussion, we now recall some useful lemmas.

**Lemma 1** (Xie & de Souza, 1993) For given matrices \( A, H, E \) and \( F \) with compatible dimensions such that \( F^T F \leq I \), let \( X \) be a symmetric positive-definite matrix and \( \alpha > 0 \) be an arbitrary positive constant such that \( \alpha^{-1} I - EXE^T > O \), then the following matrix inequality holds:

\[ (A + HFE)X(A + HFE)^T \leq AXA^T + AXE^T \left( \alpha^{-1} I - EXE^T \right)^{-1} EXA^T + \alpha^{-1} HH^T. \]

**Lemma 2** (Theodor & Shaked, 1996) Let \( f_k(\cdot) : \mathbb{R}^{nn} \rightarrow \mathbb{R}^{nn}, k = 0, 1, \ldots, N, \) be a sequence of matrix functions so that

\[ f_k(A) = f_k^T(A), \forall A = A^T > 0 \]

and

\[ f_k(B) \geq f_k(A), \forall B = B^T > A = A^T \]

and let \( g_k(\cdot) : \mathbb{R}^{nn} \rightarrow \mathbb{R}^{nn}, k = 0, 1, \ldots, N, \) be a sequence of matrix functions so that

\[ g_k(A) = g_k^T(A) \geq f_k(A), \forall A = A^T > 0. \]

Then, the solutions \( \{A_k\}_{k=0,1,\ldots,N} \) and \( \{B_k\}_{k=0,1,\ldots,N} \) to the difference equations

\[ A_{k+1} = f_k(A_k), B_{k+1} = g_k(B_k), A_0 = B_0 \geq 0 \]

satisfy \( A_k \leq B_k \) for \( k = 0, 1, \ldots, N. \)

**Lemma 3 (Matrix Inversion Lemma)** (Anderson & Moore, 1979) For given matrices \( A, B, C \) and \( D \) with compatible dimensions where \( A \in \mathbb{R}^{nn} \) and \( C \in \mathbb{R}^{nn} \) are all nonsingular submatrices, then

\[ (A \pm BCD)^{-1} = A^{-1} \mp A^{-1} B \left( C^{-1} \pm D^{-1} A B \right)^{-1} D^{-1}. \]

**Lemma 4 (Matrix Inversion Lemma for Matrices in Block Form)** (Chui & Chen, 1999) Let

\[ A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \]

where \( A_{11} \in \mathbb{R}^{nn} \) and \( A_{22} \in \mathbb{R}^{nn} \) are all nonsingular submatrices, such that \( A_{11} - A_{12} A_{22}^{-1} A_{21} \)

and \( A_{22} - A_{21} A_{11}^{-1} A_{12} \) are also nonsingular. Then \( A \) is a nonsingular with
\[ A^{-1} = \begin{bmatrix}
A_{11}^{-1} + A_{11}^{-1}A_{12}\left(A_{22} - A_{21}A_{11}^{-1}A_{12}\right)^{-1}A_{21}A_{11}^{-1} - A_{11}^{-1}A_{12}\left(A_{22} - A_{21}A_{11}^{-1}A_{12}\right)^{-1}A_{21}^{-1} \\
\left(A_{22} - A_{21}A_{11}^{-1}A_{12}\right)^{-1}A_{21}^{-1} + A_{22}^{-1}A_{21}\left(A_{22} - A_{21}A_{11}^{-1}A_{12}\right)^{-1}A_{22}^{-1} \\
\end{bmatrix} \]

**Lemma 5 (Schur Complement)** (Boyd, Ghaoui & al., 1994) For a given symmetric matrix 

\[ S = \begin{bmatrix} S_{11} & S_{12} \\
S_{12}^T & S_{22} \end{bmatrix}, \]

where \( S \in \mathbb{R}^{(n+m)\times(n+m)} \), \( S_{11} \in \mathbb{R}^{n\times n} \) and \( S_{22} \in \mathbb{R}^{m\times m} \), the following statements are equivalent:

i. \( S < 0 \)

ii. \( S_{11} < 0 \) and \( S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0 \)

iii. \( S_{11} - S_{12}^T S_{12}^{-1} S_{11} < 0 \)

Based on the state-space model of the augmented system, Lemma 1 and Lemma 2, we can get the following Theorem 1.

**Theorem 1** If there exist a symmetric positive-definite matrix sequence \( \Sigma_k \), and two positive scalar sequences \( \alpha_k \) and \( \beta_k \) such that

\[
\alpha_k^{-1}I - \tilde{E}_{1,k}^T \tilde{\Sigma}_k \tilde{E}_{1,k}^T > 0, (16)
\]

\[
\beta_k^{-1}I - \tilde{E}_{2,k}^T \tilde{W}_k \tilde{E}_{2,k}^T > 0, (17)
\]

\[
\Sigma_{k+1} = \tilde{A}_k \Sigma_k \tilde{A}_k^T + \tilde{A}_k \Sigma_k \tilde{E}_{1,k}^T \left( \alpha_k^{-1}I - \tilde{E}_{1,k} \tilde{\Sigma}_k \tilde{E}_{1,k}^T \right)^{-1} \tilde{E}_{1,k} \Sigma_k \tilde{A}_k^T + \alpha_k^{-1} \tilde{H}_{1,k} \tilde{H}_{1,k}^T + \beta_k^{-1} \tilde{H}_{2,k} \tilde{H}_{2,k}^T + \tilde{B}_k \tilde{W}_k \tilde{B}_k^T + \tilde{B}_k \tilde{W}_k \tilde{E}_{2,k}^T \left( \beta_k^{-1}I - \tilde{E}_{2,k} \tilde{W}_k \tilde{E}_{2,k}^T \right)^{-1} \tilde{E}_{2,k} \tilde{W}_k \tilde{B}_k^T, (18)
\]

and

\[
\Sigma_0 = \tilde{\Sigma}_0 = \begin{bmatrix} X_0 & O \\
O & O \end{bmatrix}, (19)
\]

then

\[
\tilde{\Sigma}_k \leq \Sigma_{\pi,k}, k = 0,1,\ldots,N, (20)
\]

where \( \tilde{\Sigma}_k \) satisfies equation (15).

**Proof:** If matrix inequalities (16) and (17) hold, then from Lemma 1

\[
\tilde{\Sigma}_{k+1} \leq \left( \tilde{A}_k + \tilde{H}_{1,k} \tilde{E}_{1,k} \right) \tilde{\Sigma}_k \left( \tilde{A}_k + \tilde{H}_{1,k} \tilde{E}_{1,k} \right)^T + \tilde{B}_k \left( \tilde{W}_k + \tilde{B}_k \tilde{E}_{2,k} \right) \left( \beta_k^{-1}I - \tilde{E}_{2,k} \tilde{W}_k \tilde{E}_{2,k}^T \right)^{-1} \tilde{E}_{2,k} \tilde{W}_k \tilde{B}_k^T + \alpha_k^{-1} \tilde{H}_{1,k} \tilde{H}_{1,k}^T + \beta_k^{-1} \tilde{H}_{2,k} \tilde{H}_{2,k}^T.
\]
Moreover, from the initial condition (19) and Lemma 2, \( \Sigma_k \leq \Sigma_k, k = 0, 1, \ldots, N \), where \( \Sigma_k, k = 0, 1, \ldots, N \), satisfies equation (18). Thus the theorem has been proved.

Suppose
\[
\Sigma_k = [I - I] \Sigma_k [I - I]^T, k = 0, 1, \ldots, N.
\]

It is clear that the matrix sequence \( \Sigma_k (k = 0, 1, \ldots, N) \) satisfies matrix inequalities (9) and (10).

In the following, we shall show how to determine the parameters \( A_{ok}, K_{ok}, \alpha_k \) and \( \beta_k \) in order to minimize \( \text{tr}(\Sigma_k) \). The following Theorem 2 solves the problem of how to select the state matrix \( A_{ok} \) and the gain matrix \( K_{ok} \) of the robust Kalman filter taking the form (8) when the positive scalar sequences \( \alpha_k \) and \( \beta_k \) are given.

**Theorem 2** For given positive scalar sequences \( \alpha_k \) and \( \beta_k \) which satisfy inequalities (16) and (17) respectively, if
\[
A_{ok} = A_k + (A_k - K_{ok} C_k) \Sigma_{1,k} E_{1,k}^T \left( \alpha_k^{-1} I - E_{1,k} \Sigma_{1,k} E_{1,k}^T \right)^{-1} E_{1,k}
\]
and
\[
K_{ok} = \left( A_k S_k C_k^T + \alpha_k^{-1} \bar{H}_{1,k} \bar{H}_{2,k}^T \right)^{-1} \bar{C}_k S_k C_k^T + D_k T_k D_k^T + \left( \alpha_k^{-1} + \beta_k^{-1} \right) H_{2,k} H_{2,k}^T,
\]
then
\[
\Sigma_n = \begin{bmatrix} \Sigma_{1,n} & \Sigma_{2,n} \\ \Sigma_{2,n} & \Sigma_{2,n} \end{bmatrix}, n = 0, 1, \ldots, N
\]
and \( \text{tr}(\Sigma_k) \) is minimal, where
\[
\bar{H}_{1,k} = \begin{bmatrix} H_{1,k} & P \end{bmatrix},
\]
\[
\bar{H}_{2,k} = \begin{bmatrix} H_{2,k} & O \end{bmatrix},
\]
\[
\bar{E}_{1,k} = \begin{bmatrix} E_{1,k} \\ I \end{bmatrix},
\]
\[
\bar{S}_k = \Sigma_k + \Sigma_k E_{1,k}^T \left( \alpha_k^{-1} I - E_{1,k} \Sigma_k E_{1,k}^T \right)^{-1} E_{1,k} \Sigma_k,
\]
and
\[
\bar{T}_k = V_k + V_k E_{2,k}^T \left( \beta_k^{-1} I - E_{2,k} V_k E_{2,k}^T \right)^{-1} E_{2,k} V_k.
\]
Moreover, the covariance matrices of the state and the estimation error can be represented respectively as

\[
\Sigma_{1,k+1} = A_k \Sigma_{1,k} A_k^T + A_k \Sigma_{1,k} \bar{E}_{1,k}^T (\alpha_k^{-1} I - \bar{E}_{1,k} \Sigma_{1,k} \bar{E}_{1,k}^T)^{-1} \bar{E}_{1,k} \Sigma_{1,k} A_k^T + \alpha_k^{-1} \bar{H}_{1,k} \bar{H}_{1,k}^T + B_k U_k B_k^T + \beta_k^{-1} H_{1,k} H_{1,k}^T,
\]

and

\[
\bar{\Sigma}_{k+1} = A_k \bar{\Sigma}_k A_k^T - (A_k \bar{\Sigma}_k C_k^T + \alpha_k^{-1} \bar{H}_{1,k} H_{2,k}^T) \left( \bar{C}_k \bar{\Sigma}_k \bar{C}_k^T + \bar{R}_k \right)^{-1} \left( A_k \bar{\Sigma}_k C_k^T + \alpha_k^{-1} \bar{H}_{1,k} H_{2,k}^T \right)^T + \alpha_k^{-1} \bar{H}_{1,k} \bar{H}_{1,k}^T + B_k U_k B_k^T + \beta_k^{-1} H_{1,k} H_{1,k}^T,
\]

where

\[
U_{k+1} = W_k + W_{E_{2,k}}^T \left( \beta_k^{-1} I - E_{2,k} W_{E_{2,k}}^T \right)^{-1} E_{2,k} W_k,
\]

\[
\bar{C}_k = \begin{bmatrix} \bar{E}_{1,k} \\ C_k \end{bmatrix},
\]

\[
\bar{H}_{2,k} = \begin{bmatrix} O \\ \bar{R}_{2,k} \end{bmatrix},
\]

and

\[
\bar{R}_k = \begin{bmatrix} -\alpha_k^{-1} I & O \\ O & D_k T_k D_k^T + (\alpha_k^{-1} + \beta_k^{-1}) H_{2,k} H_{2,k}^T \end{bmatrix}.
\]

**Proof:** In the following, the theorem will be proved based on the mathematical induction. From equation (19), it is clear that equation (23) holds when \( n = 0 \). Suppose (23) holds when \( n = k \). In the following, we shall prove that equation (23) also holds when \( n = k + 1 \). From equation (18),

\[
\Sigma_{k+1} = \begin{bmatrix} \Sigma_{1,k+1} & \Sigma_{12,k+1} \\ \Sigma_{12,k+1}^T & \Sigma_{2,k+1} \end{bmatrix},
\]

where \( \Sigma_{1,k+1} \) satisfies equation (24),

\[
\Sigma_{12,k+1} = A_k \begin{bmatrix} \Sigma_{1,k} + \Sigma_{1,k} \bar{E}_{1,k}^T (\alpha_k^{-1} I - \bar{E}_{1,k} \Sigma_{1,k} \bar{E}_{1,k}^T)^{-1} \bar{E}_{1,k} \Sigma_{1,k} \\ I + \Sigma_{1,k} \bar{E}_{1,k}^T (\alpha_k^{-1} I - \bar{E}_{1,k} \Sigma_{1,k} \bar{E}_{1,k}^T)^{-1} \bar{E}_{1,k} \Sigma_{1,k} \end{bmatrix} C_k K_{ok}^T + A_k \begin{bmatrix} \Sigma_{1,k} \bar{E}_{1,k}^T (\alpha_k^{-1} I - \bar{E}_{1,k} \Sigma_{1,k} \bar{E}_{1,k}^T)^{-1} \bar{E}_{1,k} \Sigma_{1,k} \\ I + \Sigma_{1,k} \bar{E}_{1,k}^T (\alpha_k^{-1} I - \bar{E}_{1,k} \Sigma_{1,k} \bar{E}_{1,k}^T)^{-1} \bar{E}_{1,k} \Sigma_{1,k} \end{bmatrix} \begin{bmatrix} A_{ok} - K_{ok} C_k \end{bmatrix}^T + \alpha_k^{-1} \bar{H}_{1,k} \bar{H}_{1,k}^T K_{ok}^T,
\]

and

\[
\Sigma_{2,k+1} = A_{ok} \Sigma_{2,k} A_{ok}^T + K_{ok} C_k \Sigma_{1,k} \bar{C}_k K_{ok}^T + (\alpha_k^{-1} + \beta_k^{-1}) K_{ok} H_{2,k} H_{2,k}^T K_{ok} + K_{ok} D_k T_k D_k^T K_{ok} + \left( A_{ok} \Sigma_{2,k} + K_{ok} C_k \Sigma_{1,k} \right) \bar{E}_{1,k}^T (\alpha_k^{-1} I - \bar{E}_{1,k} \Sigma_{1,k} \bar{E}_{1,k}^T)^{-1} \bar{E}_{1,k} \left( A_{ok} \Sigma_{2,k} + K_{ok} C_k \Sigma_{1,k} \right)^T.
\]
Then, from equations (26) and (27), we can derive that

\[
\bar{\Sigma}_{t+1} = [I - I] \Sigma_{t+1} [I - I]^T
\]

\[
= A_k \left[ \Sigma_{t, k} + \Sigma_{1, k} E_{1, k}^T (\alpha_k^{-1} I - E_{1, k} \Sigma_{1, k, t+1} E_{1, k}^T) + \bar{E}_{1, k} \bar{E}_{1, k}^T \right] A_k^T + \alpha_k^{-1} \bar{H}_{1, k} \tilde{H}_{1, k}^T + \beta_k^{-1} H_{1, k}^T H_{1, k}^T +
\]

\[
B_k U_k B_k^T + A_k \Sigma_{t, k} A_k^T + K_{ok} C_k \Sigma_{t, k} C_k^T K_{ok}^T + (\alpha_k^{-1} + \beta_k^{-1}) K_{ok} H_{2, k}^T K_{ok}^T + K_{ok} D_k D_k^T K_{ok}^T +
\]

\[
(A_{ok} \Sigma_{t, k} + K_{ok} C_k \Sigma_{t, k}) E_{1, k}^T (\alpha_k^{-1} I - E_{1, k} \Sigma_{1, k, t+1} E_{1, k}^T) - [I + \Sigma_{t, k} E_{1, k}^T (\alpha_k^{-1} I - E_{1, k} \Sigma_{1, k, t+1} E_{1, k}^T) + \bar{E}_{1, k} \bar{E}_{1, k}^T \Sigma_{1, k, t+1}] A_k^T -
\]

\[
\alpha_k^{-1} \left( \bar{H}_{1, k} \bar{H}_{1, k}^T + K_{ok} \Sigma_{t, k} \right) - (A_{ok} - K_{ok} C_k) \Sigma_{t, k} [I + E_{1, k}^T (\alpha_k^{-1} I - E_{1, k} \Sigma_{1, k, t+1} E_{1, k}^T) + \bar{E}_{1, k} \bar{E}_{1, k}^T \Sigma_{1, k, t+1}] A_k^T -
\]

\[
\alpha_k^{-1} (\bar{H}_{1, k} \bar{H}_{1, k}^T + K_{ok} A_k \Sigma_{t, k}) + (A_{ok} - A_{ok}) \Sigma_{t, k} (A_k - A_{ok})^T +
\]

\[
(A_{ok} \Sigma_{t, k} + K_{ok} C_k \Sigma_{t, k} - A_k \Sigma_{t, k}) E_{1, k}^T (\alpha_k^{-1} I - E_{1, k} \Sigma_{1, k, t+1} E_{1, k}^T) - \bar{E}_{1, k} \bar{E}_{1, k}^T (A_{ok} \Sigma_{t, k} + K_{ok} C_k \Sigma_{t, k} - A_k \Sigma_{t, k})^T +
\]

\[
\alpha_k^{-1} (\bar{H}_{1, k} \bar{H}_{1, k}^T + K_{ok} \Sigma_{t, k}) \Sigma_{2, k} [I + \Sigma_{2, k} E_{1, k}^T (\alpha_k^{-1} I - E_{1, k} \Sigma_{1, k, t+1} E_{1, k}^T) + \bar{E}_{1, k} \bar{E}_{1, k}^T \Sigma_{1, k, t+1}] A_k^T +
\]

\[
(\bar{H}_{1, k} + K_{ok} \bar{H}_{1, k}) (\bar{H}_{1, k} + K_{ok} \bar{H}_{1, k})^T + K_{ok} C_k \Sigma_{t, k} - A_k \Sigma_{t, k}) + (A_k - K_{ok} C_k) \Sigma_{t, k} [I + E_{1, k}^T (\alpha_k^{-1} I - E_{1, k} \Sigma_{1, k, t+1} E_{1, k}^T) + \bar{E}_{1, k} \bar{E}_{1, k}^T \Sigma_{1, k, t+1}] A_k^T +
\]

\[
B_k U_k B_k^T + \beta_k^{-1} H_{1, k}^T H_{1, k}^T.
\]

(28)

Since

\[
\frac{1}{2} \frac{\partial^2 \text{tr} \left( \bar{\Sigma}_k \right)}{\partial A_{ok}^2} = \Sigma_{2, k} + \Sigma_{2, k} E_{1, k}^T (\alpha_k^{-1} I - E_{1, k} \Sigma_{1, k, t+1} E_{1, k}^T) + \bar{E}_{1, k} \Sigma_{2, k} > O,
\]

and

\[
\frac{1}{2} \frac{\partial^2 \text{tr} \left( \bar{\Sigma}_k \right)}{\partial K_{ok}^2} = C_k \left[ \Sigma_k + \Sigma_k E_{1, k}^T (\alpha_k^{-1} I - E_{1, k} \Sigma_{1, k, t+1} E_{1, k}^T) + \bar{E}_{1, k} \bar{E}_{1, k}^T \right] C_k^T + \left( \alpha_k^{-1} + \beta_k^{-1} \right) H_{2, k}^T H_{2, k}^T + D_k T_k D_k^T > O,
\]

\[
\text{tr} (\bar{\Sigma}_k) \text{ is minimal for the given } \alpha_k \text{ and } \beta_k \text{ if and only if } \frac{\partial \text{tr} \left( \bar{\Sigma}_k \right)}{\partial A_{ok}} = 0 \text{ and } \frac{\partial \text{tr} \left( \bar{\Sigma}_k \right)}{\partial K_{ok}} = 0.
\]

Moreover, we can derive that

\[
\frac{1}{2} \frac{\partial \text{tr} \left( \bar{\Sigma}_k \right)}{\partial A_{ok}} = (A_k - A_{ok}) \left[ I + \Sigma_{2, k} E_{1, k}^T (\alpha_k^{-1} I - E_{1, k} \Sigma_{1, k, t+1} E_{1, k}^T) + \bar{E}_{1, k} \bar{E}_{1, k}^T \right] \Sigma_{2, k} +
\]

\[
(A_k - K_{ok} C_k) \Sigma_{t, k} E_{1, k}^T (\alpha_k^{-1} I - E_{1, k} \Sigma_{1, k, t+1} E_{1, k}^T) + \bar{E}_{1, k} \bar{E}_{1, k}^T \Sigma_{1, k, t+1}
\]

and

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\[
\frac{1}{2} \frac{\partial \text{tr}(\Sigma_k)}{\partial K_{ok}} = K_{ok} \left\{ C_k \Sigma_k C_k^T + C_k \Sigma_k \left[ I - E_{1,k}^T \left( \alpha_k^{-1} I - E_{1,k} \Sigma_{1,k} E_{1,k}^T \right)^{-1} \right] E_{1,k}^T \left( \alpha_k^{-1} I - E_{1,k} \Sigma_{1,k} E_{1,k}^T \right)^{-1} \right\} \left[ I - \Sigma_{1,k} \Sigma_{1,k}^T \left( \alpha_k^{-1} I - E_{1,k} \Sigma_{1,k} E_{1,k}^T \right)^{-1} \right] E_{1,k}^T \left( \alpha_k^{-1} I - E_{1,k} \Sigma_{1,k} E_{1,k}^T \right)^{-1}.
\]

Therefore, from
\[
\frac{\partial \text{tr}(\Sigma_k)}{\partial A_{ok}} = O,
\]

\[A_{ok} = A_k + (A_k - K_{ok}) \Sigma_k E_{1,k}^T \left( \alpha_k^{-1} I - E_{1,k} \Sigma_{1,k} E_{1,k}^T \right)^{-1} E_{1,k}^T \left[ I + \Sigma_{2,k} \Sigma_{1,k} \Sigma_{1,k}^T \left( \alpha_k^{-1} I - E_{1,k} \Sigma_{1,k} E_{1,k}^T \right)^{-1} \right] \Sigma_k \]
Thus \( \Sigma_{12,k+1} = \Sigma_{2,k+1} \), and equation (23) holds when \( n = k + 1 \).

In the following, we shall derive the iterative expression for the covariance matrix of the state estimation error. Based on Lemma 4 and equations (24) and (30), we can derive that

\[
\begin{align*}
\Sigma_{k+1} &= A_k \Sigma_k A_k^\top - (A_k \Sigma_k C_k^\top + \alpha_k^\top \tilde{H}_{1,k}^\top \tilde{H}_{2,k}^\top) \left[ C_k \Sigma_k C_k^\top + D_k T_k D_k^\top + (\alpha_k^\top + \beta_k^\top) H_{2,k} H_{2,k}^\top \right]^{-1} \cdot \\
&\quad \left( A_k \Sigma_k C_k^\top + \alpha_k^\top \tilde{H}_{1,k}^\top \tilde{H}_{2,k}^\top \right)^\top + B_k U_k B_k^\top + \alpha_k^\top \tilde{H}_{1,k} \tilde{H}_{1,k}^\top + \beta_k^\top H_{1,k} H_{1,k}^\top \\
&= A_k \Sigma_k A_k^\top - \left[ A_k \Sigma_k E_{k,1}^\top \ A_k \Sigma_k C_k^\top + \alpha_k^\top \tilde{H}_{1,k} \tilde{H}_{1,k}^\top \right] \left[ \left( \alpha_k^\top I - E_{k,1} \Sigma_{1,1} E_{k,1}^\top \right) \left( \alpha_k^\top I - E_{k,1} \Sigma_{1,1} E_{k,1}^\top \right)^\top \right]^{-1} \left( \alpha_k^\top I - E_{k,1} \Sigma_{1,1} E_{k,1}^\top \right) \left( \alpha_k^\top I - E_{k,1} \Sigma_{1,1} E_{k,1}^\top \right)^\top .
\end{align*}
\]

(31)

This completes the proof of Theorem 2. \( \Box \)

From equations (21) and (22), both \( A_{ok} \) and \( K_{ok} \) have very close relationships of the positive scalar sequences \( \alpha_k \) and \( \beta_k \). The following Theorem 3 gives the approach of determining \( \alpha_k \) and \( \beta_k \) through solving a semidefinite program subject to LMIs.

**Theorem 3** Suppose that the covariance matrices \( \Sigma_k \) and \( \Sigma_{1,1} \) are given, and the direct output matrix \( D_k \) is nonsingular. Then, the positive scalar sequences \( \alpha_k \) and \( \beta_k \) can be determined by solving the following semidefinite program:

\[
\begin{align*}
\min \ \text{tr} \ (X) \\
\text{s.t.}
\end{align*}
\]

\[
\begin{align*}
\left[ \begin{array}{cc}
-\alpha_k^\top I + E_{k,1} \Sigma_{1,1} E_{k,1}^\top & E_{k,1} \Sigma_{1,1} C_k^\top \\
C_k \Sigma_k C_k^\top + D_k T_k D_k^\top & (\alpha_k^\top + \beta_k^\top) H_{2,k} H_{2,k}^\top \\
A_k \Sigma_k E_{k,1}^\top & A_k \Sigma_k C_k^\top + \alpha_k^\top \tilde{H}_{1,k} \tilde{H}_{1,k}^\top \\
A_k \Sigma_k E_{k,1}^\top & A_k \Sigma_k C_k^\top + \alpha_k^\top \tilde{H}_{1,k} \tilde{H}_{1,k}^\top + B_k U_k B_k^\top + \alpha_k^\top \tilde{H}_{1,k} \tilde{H}_{1,k}^\top + \beta_k^\top H_{1,k} H_{1,k}^\top
\end{array} \right] &\preceq 0
\end{align*}
\]

(32)
\[
\begin{bmatrix}
-X & A_k & H_{1,k} & O & A_kE_{1,k} & A_kC_k^T + H_{1,k}R_{2,k} & B_k & H_{1,k} \\
A_k^T & -\Sigma_{k-1} & O & O & O & O & O & O \\
H_{1,k} & 0 & -\alpha_kI & O & O & O & O & O \\
O & O & O & -\beta_kI & O & O & O & O \\
E_{1,k}A_k^T & O & O & O & -\alpha_kI & O & O & O
\end{bmatrix} < O, \quad (33)
\]

\[
C_kA_k^T + \bar{H}_{2,k} \bar{H}_{1,k} = 0 \quad (34)
\]

and

\[
\begin{bmatrix}
-I & \alpha_kE_{1,k}^T \\
\alpha_kE_{1,k} & \alpha_k^{-1} \Sigma_{k-1}
\end{bmatrix} < O, \quad (35)
\]

where \( X = X^T > O \).

**Proof:** It is obviously that \( \alpha_k \) and \( \beta_k \) can be determined by the semidefinite program given as follows:

\[
\min_{A', \bar{A}} \text{tr}(X)
\]

s.t.

\[
\bar{\Sigma}_{k+1} < X, \quad (37)
\]

\[
\alpha_k^{-1}I - \bar{E}_{1,k} \Sigma_{k+1} \bar{E}_{1,k}^T > O, \quad (38)
\]

and

\[
\beta_k^{-1}I - E_{2,k}W_k^i E_{2,k}^T > O, \quad (39)
\]

From equation (25), matrix inequality (37) is equivalent to

\[
\begin{bmatrix}
\bar{A}_k & \bar{A}_kC_k^T & \bar{B}_k \\
0 & R_k^i & 0 & \bar{C}_kA_k^T & \bar{B}_k^i
\end{bmatrix} X < O, \quad (41)
\]

where
Kalman Filter

\[ \dot{A}_k = [A_k \ H_{1,k} \ O], \]

\[ \dot{C}_k = [C_k \ H_{2,k} \ \hat{H}_{2,k}], \]

\[ \dot{B}_k = [B_k \ H_{1,k}], \]

\[ \tilde{Q}_k = \begin{bmatrix} \Sigma \alpha_k^{-1} I & O \\ O & \beta_k^{-1} I \end{bmatrix}, \]

\[ \tilde{U}_k = \begin{bmatrix} u_k \\ O \\ O & \beta_k^{-1} I \end{bmatrix}, \]

and

\[ \bar{R}_k = \begin{bmatrix} \alpha_k^{-1} I & O \\ O & D_kT_kD_k^T \end{bmatrix}. \]

From Lemma 5, matrix inequality (41) is equivalent to the following LMI

\[ \begin{bmatrix} -X & \bar{A}_k \bar{C}_k^T & \bar{B}_k \\ \bar{A}_k^T & -\bar{Q}_k^{-1} & O \\ \bar{C}_k \bar{A}_k^T & O & -\bar{R}_k^{-1} \\ \bar{B}_k^T & O & O & -\bar{U}_k^{-1} \end{bmatrix} < O, \]

which is just (33). Also from Lemma 5, matrix inequalities (38), (39) and (40) are equivalent to LMIs (34), (35) and (36) respectively. This completes the proof of this theorem. □

Now, the new robust Kalman filtering algorithm is summarized as follows:

\[ \hat{x}_{k+1} = A_{ok} \hat{x}_k + K_{ok} (y_k - C_{ok} \hat{x}_k), \]

\[ A_{ok} = A_k + \left( A_k - K_{ok} C_k \right) \hat{E}_{1,k} \Sigma_k \hat{E}_{1,k}^T \left( \alpha_k^{-1} I - \hat{E}_{1,k} \Sigma_k \hat{E}_{1,k}^T \right)^{-1} \hat{E}_{1,k}, \]

\[ K_{ok} = \left( A_k S_k C_k^T + \alpha_k^{-1} \bar{H}_{1,k} \bar{H}_{2,k} \right) \left[ C_k S_k C_k^T + D_k T_k D_k^T + \left( \alpha_k^{-1} + \beta_k^{-1} \right) H_{2,k} H_{2,k}^T \right]^{-1}, \]

\[ \Sigma_{x+1} = A_k \Sigma_{x} A_k^T - \left( A_k \Sigma_{x} A_k^T + \alpha_k^{-1} \bar{H}_{1,k} \bar{H}_{2,k} \right) \left( \bar{C}_k \Sigma \bar{C}_k^T + \bar{R}_k \right)^{-1} \left( A_k \Sigma_{x} A_k^T + \alpha_k^{-1} \bar{H}_{1,k} \bar{H}_{2,k} \right)^T + \alpha_k^{-1} \Sigma_{k+1} \Sigma_{k+1}^T + \Sigma_{u} \Sigma_{u}^T + \beta_k^{-1} \Sigma_{u} \Sigma_{u}^T, \]

\[ \Sigma_0 = X_0, \]

where \( \alpha_k \) and \( \beta_k \) are determined by the semidefinite program (32).

3. Application to state estimation of a low temperature pressurized water nuclear reactor

Nowadays, nuclear fission reaction provides more and more energy required for generating electrical power in the world. Safety demand and economic feasibility of a nuclear power
plant requires smooth and uninterrupted plant operation in the face of varying electrical power level. However, some state variables associated with the dynamics of a nuclear reactor are not available for measurement. If it is necessary to implement a state-feedback control or a fault detection scheme for the reactor, then these state variables could be required and consequently some observation structure should be utilized to reconstruct them. Moreover, the model nonlinearity, the variation of dynamical model parameters and the noisy character of measurement signals requires a state estimator which is robust to model uncertainty and nonlinearity, and, at the same time, must be sensitive to changes in the state variables. Therefore, the new robust Kalman filter presented in Section 2 can be applied to solve the state estimation problem for nuclear reactors. In this section, the dynamical model for the robust state estimator design of the LTPWR, which is a small research reactor designed by INET, is established, and then the robust Kalman filter derived in the last section is applied to estimate the state variables of the LTPWR.

3.1 Dynamical model

The LTPWR is a small research nuclear reactor, and Fig. 1 is the schematic drawing of the LTPWR.

![Schematic drawing of the LTPWR](image_url)
The dynamical model of the nuclear reactor for designing the robust Kalman filter in this chapter is the point kinetics with one delayed neutron group and temperature feedback from the fuel and the coolant, which is summarized as follows:

\[
\begin{align*}
\frac{dn_r}{dt} &= \frac{\delta \rho_r}{A} - \beta n_r + \frac{\beta}{A} c_r + \frac{\alpha_r n_r}{A} (T_f - T_{f0}) + \frac{\alpha_c n_c}{A} (T_{cav} - T_{cav0}) \\
\frac{dc_r}{dt} &= \Lambda n_r - \lambda c_r \\
\frac{dT_f}{dt} &= -\frac{\Omega}{\mu_f} T_f + \frac{\Omega}{\mu_f} T_{cav} + \frac{P_0}{\mu_f} n_r \\
\frac{dT_{cav}}{dt} &= -\frac{2M + \Omega}{\mu_c} T_{cav} + \frac{\Omega}{\mu_c} T_f + \frac{2M}{\mu_c} T_{cin} \\
\frac{d\delta \rho_t}{dt} &= G_t z_t \quad \text{(43)}
\end{align*}
\]

where \( n_r \) is the neutron density relative to density at rated condition, \( c_r \) is the precursor density relative to density at rated condition, \( \beta \) is the fraction of delayed fission neutrons, \( \Lambda \) is the effective prompt neutron lifetime, \( \lambda \) is the effective precursor radioactive decay constant, \( \alpha_r \) and \( \alpha_c \) are respectively the average fuel and coolant temperature reactivity coefficient, \( T_f \) is the average reactor fuel temperature, \( T_{f0} \) is the equilibrium average reactor fuel temperature, \( T_{cav} \) and \( T_{cin} \) are respectively the average temperature of the coolant inside the reactor core and temperature of the coolant entering the reactor, \( T_{cav0} \) is the equilibrium average temperature of the coolant, \( \Omega \) is the heat transfer coefficient between fuel and coolant, \( M \) is the mass flow rate times heat capacity of the coolant, \( P_0 \) is the rated power level, \( \delta \rho_t \) is the reactivity due to the control rods, \( G_t \) is the total reactivity worth of the rod, and \( z_t \) is the control input, i.e. control rod speed. The parameters of the reactor at the initial of the fuel cycle in 100% power are given in Table 1.

| Symbol | Quantity | Symbol | Quantity |
|--------|----------|--------|----------|
| \( \beta \) | 0.0069 | \( \mu_c \) | 25151(kW/°C) |
| \( \Lambda \) | 1.0367e-4(s) | \( \alpha_f \) | -3.85e-4(1/°C) |
| \( n_{f0} \) | 1 | \( \alpha_c \) | -2.3e-5(1/°C) |
| \( \lambda \) | 0.08(1/s) | \( G_t \) | 0.0048 |
| \( P_0 \) | 10(MW) | \( M \) | 304.89(kW/°C) |
| \( \mu_f \) | 588.544(kW/°C) | \( \Omega \) | 437.15(kW/°C) |

Table 1. Parameters of the LTPWR at the Initial of the Fuel Cycle in 100% Power

Let

\[
\begin{align*}
\delta n_r &= n_r - n_{r0}, \\
\delta c_r &= c_r - c_{r0}, \\
\delta T_f &= T_f - T_{f0}, \\
\delta T_{cav} &= T_{cav} - T_{cav0}, \\
\delta T_{cin} &= T_{cin} - T_{cin0} \\
\end{align*}
\]

(44)
where the symbol $\delta$ indicates the deviation of a variable from an equilibrium value, and $n_{r0}$, $c_{r0}$, $T_{r0}$, $T_{cav0}$ and $T_{cin0}$ correspond to the values of $n_r$, $c_r$, $T_r$, $T_{cav}$ and $T_{cin}$ at the given equilibrium condition respectively. Choose the state vector as

$$x = [\delta n_r \quad \delta c_r \quad \delta T_r \quad \delta T_{cav} \quad \delta T_{cin}]^T,$$  \hfill (45) 

and then the state-space model, which is utilized to design the robust Kalman filter, is given as follows:

$$\begin{cases}
\frac{dx(t)}{dt} = A(t)x(t) + B[u(t) + w] + f(x), \\
y = Cx(t) + Du, 
\end{cases} \hfill (46)$$

where

$$A(t) = \begin{bmatrix}
\frac{\beta}{A} & 0 & 0 & 0 \\
0 & \frac{\beta}{A} & 0 & 0 \\
\lambda & -\lambda & 0 & 0 \\
\frac{P_0}{\mu_r} & 0 & \frac{\Omega}{\mu_c} & -\frac{\Omega}{\mu_c} \\
0 & \frac{\Omega}{\mu_c} & 0 & 2M + \frac{\Omega}{\mu_c} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ C_r \end{bmatrix}^T,$n

$$C = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$D = 1,$$n

$$f(x) = \begin{bmatrix}
\frac{\alpha_i}{A} \delta n_r \delta T_r + \frac{\alpha_c}{A} \delta n_r \delta T_{cav} + \frac{1}{A} \delta n_r \delta T_{cin} \\
0 \\
0 \\
\frac{2M}{\mu_c} \delta T_{cin} \\
0
\end{bmatrix},$$

and $w$ and $v$ are the process noise and the measurement noise respectively, which are all zero-mean Gaussian white noises.

As we have stated as above, the parameters of the reactor vary with the power level. Since the state-space model (46) is established near a specific working point, the sensors and the actuators are also not precise indeed, there must be uncertainties in $A(t)$, $B$, $C$ and $D$.

Let $\Delta A(t)$, $\Delta B(t)$, $\Delta C(t)$ and $\Delta D(t)$ denote the norm-bounded uncertainties of $A(t)$, $B$, $C$ and $D$ respectively, and suppose

$$\begin{bmatrix}
\Delta A(t) \\ \Delta B(t) \\ \Delta C(t) \\ \Delta D(t)
\end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} F(t) \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \hfill (47)$$

where
\[ H_1 = \begin{bmatrix} 0 & 0 & \frac{\alpha_t}{\Lambda} & \frac{\alpha_c}{\Lambda} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\Omega}{\mu_t} & \frac{\Omega}{\mu_t} & 0 \\ 0 & 0 & \frac{\Omega}{\mu_c} & \frac{2M + \Omega}{\mu_c} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \]

\[ E_i = \text{diag} (0.1,0.1,0.1,0.1,0), \]

\[ H_2 = [1 \ 0 \ 0 \ 1 \ 0]^T, \]

\[ E_2 = [0.01 \ 0 \ 0 \ 0 \ 0.01], \]

\[ F(t) = \text{diag} (\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4,\epsilon_5), \]

and the real scalars \( \epsilon_i, i = 1,2,\cdots, 5 \), satisfy \( |\epsilon_i| \leq 1, i = 1,2,\cdots, 5 \) respectively. Therefore, the state-space model containing model uncertainties and nonlinear perturbation can be written as follows:

\[
\begin{aligned}
\frac{dx(t)}{dt} &= [A(t) + \Delta A(t)]x(t) + [B + \Delta B(t)]u(t) + w + f(x), \\
y &= [C + \Delta C(t)]x(t) + [D + \Delta D(t)]v.
\end{aligned}
\]

(48)

Furthermore, it is very easy to obtain the discrete-time state-space model corresponding to system (48), which takes the form as (1).

### 3.2 Numerical simulation

Both the robust Kalman filter (8) and the EKF are applied to the situation that the power level of the LTPWR rises from 50% to 100% rated power. The feedback control law for nuclear power and coolant temperature control is a static output feedback control taking the following form

\[
u(t) = -(F_n \delta m_i + F_T \delta T_{iw})
\]

(49)

where \( F_n \) and \( F_T \) are both positive scalars. The parameter values for the numerical simulation are given in the following Table 2, where \( T_s \) is the sampling period to obtain the discrete-time state-space model (1) from the continuous-time state-space model (48).

| Symbol | Quantity |
|--------|----------|
| \( F_n \) | 0.05 |
| \( F_T \) | 0.005 |
| \( W \) | 1e-6 |
| \( V \) | 1e-6 |
| \( X_0 \) | 1e-8I_5 |
| \( P \) | 1e-2 I_5 |
| \( T_s \) (s) | 0.1 |

Table 2. Parameter values for the numerical simulation
Moreover, the simulation model of the LTPWR utilized in this numerical simulation, which was developed by INET based on MATLAB /SIMULINK®, contains point kinetics model with six delayed neutron groups (7th order), 2nd order heat exchange dynamics of the reactor core, 2nd order dynamics of primary heat exchanger, 5th order dynamics of a U-tube steam generator (UTSG) with a water-level controller and bypass system, 2nd order dynamics of the pump of UTSG, and 6th order dynamics of other heat transmission pipe or volume cells. The mathematical model of the LTPWR is introduced briefly in Section 5 (Appendix).
Simulation results are illustrated as the follows. The estimation values of the state variables, i.e. $\delta n_r$, $\delta c_r$, $\delta T_r$, $\delta T_{cw}$ and $\delta \rho_r$, provided by the robust Kalman filter (RKF) and the EKF, and the simulated values of these state variables provided by the simulation model are all given in Fig. 1. The values of the estimation error variance of the EKF and the upper-bound of the estimation error variance of the robust Kalman filter are given in Fig. 2. The mean squared errors (MSE) of the EKF and the RKF are illustrated in Fig. 3, and the MSE is defined as:

$$
\begin{align*}
&MSE = MSE_1 + MSE_2 \\
&MSE_1 = (\delta n_r - \delta \hat{n}_r)^2 + (\delta T_{cw} - \delta \hat{T}_{cw})^2 + (\delta \rho_r - \delta \hat{\rho}_r)^2 \\
&MSE_2 = (\delta c_r - \delta \hat{c}_r)^2 + (\delta T_r - \delta \hat{T}_r)^2
\end{align*}
$$

(50)

The values of the positive scalar sequences $\alpha_k$ and $\beta_k$ are given in Fig. 4.

![Fig. 2. Values of the estimation error variances](image1)

![Fig. 3. MSEs of the RKF and the EKF](image2)
3.3 Discussion
As we can see from Fig. 2, the upper-bound of the state estimation error variance provided by the RKF is larger than the state estimation error variance provided by the EKF. However, this does not show that the estimation performance of the EKF is higher than that of the RKF because the state estimation error variance of the EKF is obtained from the nominal linearized dynamic model of the LTPWR. Moreover from Fig. 1, the estimation values of the state variables with the RKF track the variation of the corresponding simulated values faster than that with the EKF do, and this also shows the high performance of the RKF derived in this chapter. Fig. 3 shows that the MSE of the EKF is larger than that of the RKF, though they are nearly equal to each other when the power level has been lifted to 100%. The reason of the phenomenon illustrated in Figs. 1, 2 and 3 is: During the procedure of power lift from 50% to 100%, there must be model uncertainties and nonlinear perturbations due to variations of the parameter values of the LTPWR. Therefore, the MSE of the RKF can be smaller than that of the EKF to the system model with nonlinear perturbation and uncertain parameter values.

4. Conclusions
Motivated by the robust state estimation of nuclear reactors, a new finite-horizon robust Kalman filter for discrete-time systems with nonlinear perturbation and norm-bounded uncertainties in the state, the output, the input and the direct output matrices was presented in this chapter. After deriving the mathematical expressions of the robust Kalman filter, the newly presented filter was then applied to the state estimation for a low temperature pressurized water reactor designed by INET. Simulation results show that the performance of the robust Kalman filter is higher than that of the celebrated EKF. The future research work lies in two aspects. The first one is to extend the robust Kalman filter in this chapter to time-delayed systems since there are many time-delays in a nuclear reactor system, and the second one is to estimate the density of poisons, such as xenon 135 and samarium 149, for a nuclear reactor.
5. Appendix: brief Introduction of the mathematical model for the LTPWR

The mathematical model of the aforementioned LTPWR will be introduced briefly as follows. The model is constructed upon the fundamental conservation of mass, energy and momentum. Here, only the dynamical equations are given, and the derivation procedures are all omitted.

5.1 Neutron kinetics

Point kinetics model with six delayed neutron groups is considered for the LTPWR, and the dynamical equations are given as follows.

\[
\begin{align*}
\frac{dn_i}{dt} &= \frac{\delta \rho_i}{A} - \frac{\beta_i}{A} n_i + \sum_{j=1}^{6} \frac{\beta_j}{A} c_{n,j} + \frac{\alpha_i}{A} n_i (T_t - T_{in}) + \\
&\quad + \frac{\alpha_i}{A} (T_{cav} - T_{cav,i}) \\
\frac{dc_{n,i}}{dt} &= \lambda_i (n_i - c_{n,i}) (i = 1, 2, \ldots, 6) \\
\frac{d\delta \rho_i}{dt} &= G_i z_i
\end{align*}
\]

(51)

where \( c_{n,i} \) is the precursor density relative to density of the \( i \)th group delayed fission neutrons at rated condition, \( \beta_i \) is the fraction of the \( i \)th group delayed fission neutrons, and \( \lambda_i \) is the effective precursor radioactive decay constant of the \( i \)th group delayed fission neutrons.

5.2 Reactor and primary loop

The primary loop is divided into seven volume units and two transport units. As illustrated in Fig. 1, the seven volumes are the reactor core, the reactor outlet, the top tank, the Primary Heat-Exchanger (PHE), the PHE outlet, the bottom tank, and the core inlet, and the two transport units are the chimney and the down tube.

(1) Reactor Core

Some assumptions are given to provide a simple model: (i) the fuel element is homogeneous, so one-dimensional thermo-dynamic equations can be achieved; (ii) the thermal power is produced by the fission products and radiation process such as \( \beta \)-radiation and \( \gamma \)-radiation, but here the heat produced by the radiation process is neglected; (iii) the temperatures of the fuel element and the coolant inside the reactor core are assumed even. Thus a lumped parameter approach can be given as follows:

\[
\begin{align*}
\frac{dT_t}{dt} &= -\frac{\Omega}{\mu_i} T_t + \frac{\Omega}{\mu_i} T_{cav} + \frac{P_i}{\mu_i} n_i, \\
\frac{dT_{cav}}{dt} &= \frac{2M + \Omega}{\mu_c} T_{cav} + \frac{\Omega}{\mu_c} T_t + \frac{2M}{\mu_c} T_{cav}. \quad (52)
\end{align*}
\]

(2) Volume Units

Assume that the coolant in these units is well mixed. Energy balance equations for each unit are given as follows.

Core outlet:
Robust Kalman Filter with Application to State Estimation of a Nuclear Reactor

\[ \rho_{21} V_{21} \frac{dT_{21}}{dt} = W_1 (T_{\text{cout}} - T_{21}), \]  

Top tank:

\[ \rho_{23} V_{23} \frac{dT_{\text{elpin}}}{dt} = W_1 (T_{22} - T_{\text{elpin}}), \]  

PHE outlet:

\[ \rho_{41} V_{41} \frac{dT_{41}}{dt} = W_1 (T_{\text{elpout}} - T_{41}), \]  

Bottom tank:

\[ \rho_{43} V_{43} \frac{dT_{43}}{dt} = W_1 (T_{42} - T_{43}), \]  

Core inlet:

\[ \rho_{43} V_{44} \frac{dT_{\text{cin}}}{dt} = W_1 (T_{43} - T_{\text{cin}}). \]

Here the subscripts 21, 22, 23, 41, 42, 43 and 44 denote the core outlet, chimney, top tank, PHE outlet, down tube, bottom tank, and core inlet respectively. \( T_{21}, T_{22}, T_{23}, T_{41}, T_{42} \) and \( T_{43} \) are the outlet temperatures of these units. \( \rho_{21}, \rho_{22}, \rho_{23}, \rho_{41}, \rho_{42}, \rho_{43} \) denote the densities of the coolant in these units. \( T_{\text{elpin}} \) and \( T_{\text{elpout}} \) denote the inlet and outlet temperatures of the PHE. \( W_1 \) is the natural circulation flow-rate of the coolant.

(3) Transport Units

It is assumed that there is no mixing within the chimney and the down tube. Therefore, the coolant flowing through these two units will lead to delay in the temperature variations. The equations are:

Chimney:

\[ T_{22} (t) = T_{21} \left( t - \frac{\rho_{22} V_{22}}{W_1} \right), \]  

Down tube:

\[ T_{42} (t) = T_{41} \left( t - \frac{\rho_{42} V_{42}}{W_1} \right). \]

(4) Natural Circulation Flux

The primary coolant flow of the LTPWR is natural circulation which is driven by the density difference between the hot and cold coolant. During the modeling procedure, the gravity pressure drop in the loop, the frictional pressure drop in the core and PHE, and the entrance pressure drop at the core inlet are considered, while all the resistance pressure drops in the other units as well as the acceleration pressure drop in the loop are neglected. Then the natural circulation flow-rate of the coolant \( W_1 \) is the determined by the following algebraic equation:
\[ \rho_2 g h_3 + \rho_{22} g h_{22} - \rho_3 g h_1 = \frac{1}{2} k_{e1p} \rho_{e1} \left( \frac{W_1}{\rho_{e1} A_{e1p}} \right)^2 + \frac{1}{2} (k_{cin} + k_c) \rho_{e1} \left( \frac{W_1}{\rho_{e1} A_c} \right)^2 \]  

(60)

where \( \rho_1 \) and \( \rho_3 \) are the coolant density in the reactor core and the PHE, \( h_1, h_{22}, h_3 \) and \( h_{42} \) are the lengths of the reactor core, the chimney, the PHE and the down tube respectively, \( k_{e1p}, k_{cin} \) and \( k_c \) are the resistance pressure coefficient in the primary side of the PHE, the core inlet and the core, \( A_c \) and \( A_{e1p} \) are the flow cross section of the core and the primary side of the PHE, and \( g \) is the gravitational acceleration.

(5) Primary Heat Exchanger

In order to obtain an accurate yet simple model, two approximations are introduced. The first approximation is to lump the tube metal thermal inertia with the primary side coolant, and the second one is the PHE outlet temperatures of both sides are calculated assuming a single node and perfect mixing. The dynamic equations are given as follows.

Primary side:

\[
\begin{align*}
\frac{dT_{e1p}}{dt} &= \frac{W_1 C_3 (T_{e1pin} - T_{e1pout}) - K_{el} A_{el} \Delta T_{el}}{M_{e1m} C_{e1m} + \rho_3 V_3 C_3} \\
T_{e1p} &= \frac{1}{2} \left( T_{e1pin} + T_{e1pout} \right) \\
\Delta T_{el} &= \ln \left( \frac{T_{e1pout} - T_{e1sin}}{T_{e1pin} - T_{e1sin}} \right) \\
\end{align*}
\]  

(61)

Secondary side:

\[
\begin{align*}
\frac{dT_{e2s}}{dt} &= \frac{K_{el} A_{el} \Delta T_{el} + W_2 C_{e2s} (T_{e2sin} - T_{e2soute})}{M_{e2s} C_{e2s}} \\
T_{e2s} &= \frac{1}{2} \left( T_{e2sin} + T_{e2soute} \right) \\
\end{align*}
\]  

(62)

where \( M_{e1m} \) is the mass of the heat exchange tube, \( K_{el} \) denotes the overall heat transfer coefficient, \( A_{el} \) is the heat exchange area, \( \rho_3 \) and \( V_3 \) are respectively the density and volume of the coolant in the PHE, \( C_{e1m} \) and \( C_3 \) are the specific heat of the coolant in the primary side and the metal of the heat exchange tube.

5.3 Steam generator

The steam generator of the LTPWR is a U-tube steam generator (UTSG). The dynamical equations are established based on mass balance, energy balance and momentum balance. The structure of the UTSG discussed in this paper is given in Fig. 8. Two assumptions are made to derive the mathematical model. The first one is a linear dependence on the steam pressure \( p \) for the saturate liquid and steam, and the second is that the flow quality of the homogenous two-phase flow is a linear function of the length from the top of the U-tubes to the outlet of the riser.

(1) Mass Balances
Steam Dome:
\begin{equation}
V_{st} \frac{d\rho_s}{dp} \frac{dp}{dt} = x_M W_{sep} - W_{st},
\end{equation}

Downcomer:

\begin{equation}
\rho_d A_d \frac{dL_d}{dt} = W_{fw} + (1 - x_M) W_{sep} - W_d,
\end{equation}

Riser:

\begin{equation}
\frac{k_s V_s}{v_s^2} \left( v_{lg} \frac{dx_M}{dt} + x_M \frac{dv_{lg}}{dp} \frac{dp}{dt} \right) = W_{sep} - W_d.
\end{equation}

Fig. 8. Structure of UTSG

Here $V_{st}$ is the volume of the steam dome, $\rho_s$ is the density of the saturated steam inside the steam dome, $p$ is the steam pressure, $x_M$ is the steam quality of the homogeneous two phase flow that reaches the steam separator, $W_{sep}$ is the total mass flow leaving the riser region and entering the steam separator region, $W_{st}$ is the steam mass flow rate leaving the steam dome, $\rho_d$ is the downcomer water density, $A_d$ is the downcomer flow area, $L_d$ is the downcomer water level, $W_{fw}$ is the flow rate of feed water, $W_d$ is the downcomer flow rate, $V_s$ is the volume of the riser, $v_s$ is the average specific volume of the two-phase flow inside the riser, $k_1 = 1 - \frac{L_1}{2L_2}$, $L_1$ is the height of the U-tube bundle, $L_2$ is the length of the riser, $v_{lg} = v_g - v_l$, $v_l$ and $v_g$ are the specific volume of the liquid and steam inside the riser respectively.
(2) Energy Balances
Downcomer:

\[ \rho_d A_d \left( h_d \frac{dL_d}{dt} + L_d \frac{dh_d}{dt} \right) = h_w W_{lw} + (1 - x_M) h_t W_{sep} - h_d W_d , \]  

(66)

Riser:

\[ \frac{V_k h_t}{\nu_w} \left[ \left( \frac{h_{lg}}{\nu_w} - \frac{v_{lg}}{\nu_s} \right) \frac{dx_M}{dt} - \frac{x_M}{\nu_s} \frac{d\nu_g}{dp} \frac{dp}{dt} \right] = -h_{sep} W_{sep} + h_d W_d + Q , \]  

(67)

Here \( h_t \) is the enthalpy of the liquid inside the riser, \( h_{lg} \) is the latent heat of vaporization, \( h_s \) is the average enthalpy of the two-phase flow inside the riser, \( \nu_s \) is the average specific volume of the two-phase flow inside the riser, \( h_{sep} \) is the enthalpy of two-phase flow leaving the riser region and entering the steam separator region, \( h_d \) is the enthalpy of the liquid inside the downcomer, and \( Q \) is the total heat input to the UTSG.

(3) Momentum Balance for the Downcomer Flow
Assume that the mass flow rate \( W_d \) from the downcomer to the riser is simply due to the static head between water level \( L_w \) inside the riser and the water level \( L_d \) inside the downcomer, i.e.

\[ \begin{cases} W_d = k_d \frac{L_d - L_w}{\nu_d}, \\ L_w = (1 - k_f x_M) \frac{\nu_t A_1}{\nu_w A_w} L_2. \end{cases} \]  

(68)

where \( k_d \) is a given constant, \( \nu_d \) is the specific volume of the water inside the downcomer, \( \nu_t \) is the specific volume of the liquid inside the riser, \( A_1 \) is the cross section area of the steam separator, and \( A_w \) is the cross section area of the riser.

6. References
Anderson, B. D. O. & Moore, J. B. (1979). Optimal filtering, Prentice-Hall Inc., 0-13-638122-7, Englewood Cliffs, N. J., USA
Boyd, S.; Ghaoui, L. E.; Feron, E. & Balakrishnan, V. (1994). Linear matrix inequalities in system and control theory, Society for Industrial and Applied Mathematics, 0-89-871485-0, Philadelphia, USA
Bucy, R. S. & Senne, K. D. (1971). Digital synthesis of nonlinear filters, Automatica, Vol. 7, No. 3, pp. 287-289, 0005-1098
Chui, C. K. & Chen, G. (1999). Kalman filtering with real-time applications (third edition), Springer, 3-540-64611-6, Germany
Dong, Z. & You, Z. (2006). Finite-horizon robust Kalman filtering for discrete time-varying systems with uncertain-covariance white noises, IEEE Signal Processing Letters, Vol. 13, No. 8, pp. 493-496, 1070-9908
Fu, M.; de Souza, C. E. & Luo, Z.-Q. (2001). Finite-horizon robust Kalman filter design. *IEEE Transactions on Automatic Control*, Vol. 49, No. 9, pp. 2103-2112, 0018-9286

Garcia, G.; Tarbouriech S. & Peres, P. L. D. (2003). Robust Kalman filtering for uncertain discrete-time linear systems, *International Journal of Robust and Nonlinear Control*, Vol. 13, pp. 1225-1238, 1099-1239

Hounkpevi, F. O. & Yaz, E. E. (2007). Robust minimum variance linear state estimators for multiple sensors with different failure rates, *Automatica*, Vol. 43, pp. 1274-1280, 0005-1098

Jian, B. N. (1975). Guaranteed estimation in uncertain systems, *IEEE Transactions on Automatic Control*, Vol. 23, pp. 230-232, 0018-9286

Kalman, R. E. (1960). A new approach to linear filtering and prediction problems, *Journal of Basic Engineering Transactions of the ASME, Series D*, Vol. 82, No. 1, pp. 35-45, 0021-9223

Kalman, R. E. & Bucy R. S. (1961). New results in linear filtering and prediction problems, *Journal of Basic Engineering Transactions of the ASME, Series D*, Vol. 83, No. 3, pp. 95-108, 0021-9223

Lu, X.; Xie, L; Zhang, H. & Wang, W. (2007). Robust Kalman filtering for discrete-time systems with measurement delay, *IEEE Transactions on Circuits and Systems---II: Express Briefs*, Vol. 54, No. 6, pp. 522-526, 1549-7747

Petersen, I. R. & McFarlane, D. C. (1991). Robust state estimation for uncertain systems, Proceedings of the 30th Conference on Decision and Control, pp. 1630-1631, Brighton, England, December 1991, IEEE

Petersen, I. R. & McFarlane, D. C. (1994). Optimal guaranteed cost control and filtering for uncertain linear systems, *IEEE Transactions on Automatic Control*, Vol. 39, No. 9, pp. 1971-1977, 0018-9286

Petersen, I. R. & McFarlane, D. C. (1996). Optimal guaranteed cost filtering for uncertain discrete-time systems, *International Journal of Robust and Nonlinear Control*, Vol. 6, pp. 267-280, 1099-1239

Shaked, U. & de Souza, C. E. (1995). Robust Minimum Variance Filtering, *IEEE Transactions on Signal Processing*, Vol. 43, No. 11, pp. 2474-2483, 1053-587X

Shaked, U.; Xie, L. & Soh, Y. C. (2001). New approaches to robust minimum variance filter design. *IEEE Transactions on Automatic Control*, Vol. 49, No. 11, pp. 2620-2629, 0018-9286

Theodor, Y. & Shaked, U. (1996). Robust discrete-time minimum-variance filtering. *IEEE Transactions on Signal Processing*, Vol. 44, No. 2, pp. 181-189, 1053-587X

Xie, L. & de Souza, C. E. (1993). H∞ state estimation for linear periodic systems. *IEEE Transactions on Automatic Control*, Vol. 38, pp. 1704-1707, 0018-9286

Xie, L. & Soh, Y. C. (1994). Robust Kalman filtering for uncertain systems. *Systems and Control Letters*, Vol. 22, pp. 123-129, 0167-6911

Xie, L.; Soh, Y. C. & de Souza, C. E. (1994). Robust Kalman filtering for uncertain discrete-time systems. *IEEE Transactions on Automatic Control*, Vol. 39, No. 6, pp. 1310-1314, 0018-9286
Yang, G.-H. & Wang, J. L. (2001). Robust nonfragile Kalman filtering for uncertain linear systems with estimator gain uncertainty. *IEEE Transactions on Automatic Control*, Vol. 46, No. 2, pp. 343-348, 0018-9286

Yang, F.; Wang, Z. & Hung, Y. S. (2002). Robust Kalman filtering for discrete time-varying uncertain systems with multiplicative noises. *IEEE Transactions on Automatic Control*, Vol. 47, No. 7, pp.1179-1183, 0018-9286

Zhu, X.; Soh, Y. C. & Xie, L. Design and analysis of discrete-time robust Kalman filters. *Automatica*, Vol. 38, pp. 1069-1077, 0005-1098
The Kalman filter has been successfully employed in diverse areas of study over the last 50 years and the chapters in this book review its recent applications. The editors hope the selected works will be useful to readers, contributing to future developments and improvements of this filtering technique. The aim of this book is to provide an overview of recent developments in Kalman filter theory and their applications in engineering and science. The book is divided into 20 chapters corresponding to recent advances in the field.

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