COLOR EXCHANGE IN NEAR-FORWARD HARD ELASTIC SCATTERING

Michael G. Sotiropoulos and George Sterman

Institute for Theoretical Physics, State University of New York, Stony Brook, N.Y. 11794-3840, U.S.A.

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Abstract

We study the large-\(t\) small angle behavior of quark-quark elastic scattering. We employ a factorization procedure previously developed for fixed angle scattering, which depends on the color structure of the factorized hard subprocess. We find an evolution in \(t\) that (in leading logarithmic approximation) becomes diagonal in a singlet-octet basis in the \(t\)-channel as \(s \to \infty\). Octet exchange in the hard scattering is associated with the familiar ‘reggeized’, \(s^{\alpha_g(t)}\) behavior, which arises from \(s\)-dependence in Sudakov suppression. In contrast, Sudakov suppression for \(t\)-channel singlet exchange in the hard scattering is \(s\)-independent. In general, these contributions are mixed by soft corrections, which, however, cancel in many experimental amplitudes and cross sections.

email: michael@max.physics.sunysb.edu
email: sterman@max.physics.sunysb.edu
1 Introduction

Elastic scattering has been extensively studied in perturbation theory at fixed, wide angles [1, 2, 3] and in the forward direction [4, 6]. Since quarks are confined, true on-shell quark-quark elastic scattering amplitudes are exactly zero. Yet, in proton-proton elastic scattering and in jet production, we do see strong evidence for subprocesses involving parton-parton scattering. Such a connection is possible because we factorize long-distance from short-distance behavior in the amplitude of the former, and the cross section of the latter.

In this paper, we shall employ the formalism of wide-angle hard scattering, and push it toward the forward direction, $s \gg -t$, but keeping $-t \gg \Lambda^2$, with $\Lambda$ the scale of the strong interactions. In this way, we shall try to separate truly long-distance effects from logarithms of $s/t$. Our main interest will be in the behavior of the short-distance part of our amplitude.

Our techniques are closely related to those discussed by Lipatov in the introductory sections of his study of quasi-elastic unitarity [5], but our aim here is complementary. Rather than study the role of soft gluons, we will focus on amplitudes and cross sections in which the effects of soft gluons cancel. In this respect, our results should be applicable to $t$-dependent corrections to jet cross sections of the sort investigated by Mueller and Tang [6] and del Duca, Peskin and Tang [8]. Another application is to the elastic scattering amplitude of protons at large $t$ and very large $s$ [2].

Unlike factorization in deeply inelastic scattering or inclusive jet production, our factorization procedure is carried out for fixed, rather than averaged, colors of the active quarks of the process. We will thus speak of the color exchange within the hard scattering, and, as we shall see below, we will find special, and quite different roles for hard ‘singlet’ and ‘octet’ exchange. We must emphasize that singlet and octet exchange in the hard part are not the same things as singlet and octet exchange in the full amplitude. Indeed, the color structure of the full amplitude is not infrared safe, since it is always possible to change octet to singlet, or vice-versa, by the exchange of
a single zero-momentum gluon in the distant past or in the distant future. Because our color structure will apply to short-distance functions, our results will be infrared safe.

We shall rederive a difference between the asymptotic behavior of hard color singlet and color octet exchange [5, 9] in the forward limit, with the ‘reggeized’ behavior of the latter arising from Sudakov suppression of elastic scattering at fixed angles. In addition, we shall see that the fixed-angle analysis is consistent with the summation of leading behavior in \( \ln(s/t) \) and the simultaneous summation of leading logarithms of \( \ln(t/\Lambda^2) \). Thus, we shall derive for singlet exchange in quark-quark scattering a result of the form \( \exp[\ln(t) \ln \ln(t)] H_s(s/t) \), with the exponential coming from Sudakov behavior in the momentum transfer, independent of \( s \) but dependent on the infrared cutoff, and with \( H_s \) a hard-scattering function that is free of all scales below \( t \). Because the infrared behavior has been factorized into overall Sudakov factors, it is possible systematically to incorporate our results on quark-quark scattering into hadron-hadron elastic scattering, as well as into inclusive cross sections involving singlet exchange [7, 8, 10].

Wide angle and forward scattering, although related, differ in several important respects. Fixed-angle scattering is a two-scale problem, with the angle simply a parameter of order unity. The scattering occurs through one [1] or a few [2] interactions that are well-localized in space-time, from which long-distance, nonperturbative behavior may be factorized into universal functions that describe the structure of the external hadrons in isolation [3, 11]. In contrast, near-forward scattering involves at least three scales: \( s, t, \) and hadronic masses. Similarly, leading high-energy behavior is associated with a ‘ladder’ structure in perturbation theory, in which there is a kinematic ordering, but no clearly identifiable hard scattering. And, in the limit of small \( t \), there seems no obvious way of separating long- and short-distance behavior. Formally, we shall assume that \( t \) remains large enough that \( \alpha_s(-t) \ln(s/t) \) remains small. Although we suspect that our arguments can be extended beyond this restriction, we shall not do so here.

The methods developed in [12] and [11] allow us to calculate the simpler process
of hadron-hadron fixed-angle scattering, including leading and, in principle, non-leading, powers of $s$. For forward scattering, most results are at the level of leading logarithm in $s$, although certain results are available for nonleading logarithms [6, 9, 13]. In this paper, we shall study quark-quark elastic scattering in the formalism of ref. [11], by taking the $\theta \rightarrow 0$ limit, with $\theta$ the center-of-mass (CM) scattering angle. We use dimensional regularization to handle the infrared and collinear divergences that are factorized, and which cancel, or are absorbed into wave functions when the external particles are hadrons [3, 11].

In Sect. 2, we begin our treatment by adapting the approach of ref. [11] to dimensionally regularized quark-quark elastic scattering, emphasizing the role of angle-dependent anomalous dimensions in the space of color flow. In Sect. 3, we examine the specific anomalous dimension matrix for quark-quark scattering calculated in ref. [11] in the forward-scattering limit. In this limit, we immediately rederive the leading trajectory for octet exchange, and show that at leading logarithm in $s$, hard singlet exchange remains uncorrected. We exhibit, however, for the singlet, factorized $t$ dependence, which becomes leading for large $\theta$, but which remains as a $t$-dependent Sudakov suppression for $t$ fixed, $s \rightarrow \infty$, independent of $s$. Sects. 4 and 5 discuss lowest-order singlet exchange, the first dealing with the one-loop diagrams in dimensional regularization, and the second discussing issues associated with the choices of finite parts in the factorization of short- from long-distance contributions to singlet exchange, treating $t$ as an ultraviolet scale, even though $|t| \ll s$. We suggest a somewhat unconventional way of treating singlet exchange, in which the lowest-order hard scattering may have a nonzero real contribution (we note that the remainder of our discussion is independent of this suggestion). Finally, we present our conclusions and hopes for future progress.

2 Quark-quark elastic scattering at high energy

Consider the amplitude $A(s,t)$ for quark-quark elastic scattering at high energy and fixed CM scattering angle $\theta(s/t)$. The quarks are taken massless. In refs. [12, 11],
it was shown that this amplitude may be factored into functions that summarize the effects of: (i) soft gluons exchanged between quarks, (ii) lines collinear to each external quark, and (iii) infrared safe, short-distance contributions. Let us first discuss some relevant details of this factorization applied to quark-quark scattering.

2.1 The factorized q-q amplitude

The general form of a quark-quark scattering amplitude is

$$A_{\{\alpha_i, a_i\}}(s, t, \epsilon; \lambda_i) = \prod_{i=1}^{4} u_{\alpha_i}(P_i, \lambda_i) z_i^{1/2} \left( \frac{P_i \cdot n}{\mu \sqrt{n^2}}, \epsilon \right) G_{\{\alpha_i, a_i\}} \left( \frac{P_i \cdot n}{\mu \sqrt{n^2}}, s, t, \epsilon \right),$$  \hspace{1cm} (1)

where $\alpha_i, a_i$ are Dirac and SU($N_c$) indices, $\lambda_i, u_{\alpha_i}$ are quark helicities and spinors, $z_i$ are the residues of the full quark propagators evaluated at $P_i^2 = 0$ and $G$ is the 1PI, 4-point, on-shell vertex function. As indicated, $A$ is regularized in $D = 4 - 2\epsilon$ dimensions. In an axial gauge the residues and vertex function depend on the gauge fixing vector $n^\mu$ in the manner shown.

The general leading momentum region producing IR singularities in $A$ (leading pinch singular surface) is shown in fig. 1 in the axial gauge. Here $J_i$ are subgraphs of on-shell lines nearly collinear to the external momenta $P_i$; $S$ is a subgraph attached to the $J_i$’s via soft gluons $q^\mu$, ($|q^\mu| \ll Q$), and $H$ is the hard subgraph whose internal lines are off-shell by $O(Q^2)$, $Q$ being the hard scale of the q-q scattering. To be specific, we shall take

$$Q^2 = -t.$$  \hspace{1cm} (2)

The applicability of pQCD requires $Q^2 \gg \Lambda^2$. Through the use of the soft approximation and Ward identities [14] the contribution of the leading region can be factorized in the following manner [12, 11].

First, soft gluon connections, $S$, between jets are factored into an eikonal function $U_{\{a_i, b_i\}}(\alpha_s(\mu), \mu, \theta, \epsilon)$ that carries no Dirac structure and can be expanded as a series in the coupling constant

$$U_{\{a_i, b_i\}} = \prod_{i=1}^{4} \delta_{a_i b_i} + O(\alpha_s).$$  \hspace{1cm} (3)
Next, the remaining connections of $S$ that are one-particle reducible are included in the jet functions $J_i((P_i \cdot n/\mu \sqrt{n^2}), \epsilon)$, where $\mu$ is the renormalization scale and $v_i$ are light-like vectors along the direction of motion of the external quarks,

$$P^\mu_i = v^\mu_i \sqrt{s}/2.$$  

(4)

In an arbitrary gauge, $n^\mu$ may be thought of as a space-like vector introduced to define the jet function as a specific matrix element in the manner described in ref. [15]. In the $n \cdot A = 0$ gauge, however, the jet functions $J$ are identical to the residues $z_i^{1/2}$ in eq. (1). To be specific, we shall work in axial gauge, but we emphasize that our results are gauge-independent.

With collinear and infrared contributions organized into the functions $z_i$ and $U$, the remaining factor, denoted $H$ below, is short-distance dominated. This separation is illustrated in fig. 2. In summary, the factorized form of the amplitude in axial gauge may be written as in eq. (1), with the vertex function $G$ given by

$$G_{\{a_i,a_i\}}(Q/\mu, \alpha_s(\mu), \theta, \epsilon) = U_{\{a_i,b_i\}}(\alpha_s(\mu), \theta, \epsilon)H_{\{a_i,b_i\}}(Q/\mu, \alpha_s(\mu), \theta).$$

(5)

Here, we have replaced the $P_i$- and $n$-dependence in $G$ by $Q$ and $\theta$. Again, in a more general gauge, the $z_i$ of eq. (1) would be replaced by a jet function $J_i$ which would equal the $z_i^{1/2}$ of the $n \cdot A = 0$ gauge. Note that $U$, although angular-dependent (through $v_i \cdot n$), depends on $\mu$ only through $\alpha_s$, and is otherwise independent of the mass scales of the problem.

As discussed in ref. [11], the functions $U$ and $H$ must be defined through a renormalization procedure [16]. To this end, it is now convenient to reexpress the color sums in eq. (5) in terms of a specific basis. As we see, this will also enable us to treat the $Q$-dependence of these functions by renormalization group methods.

### 2.2 Color flow and renormalization

Color flow in quark-quark scattering may always be decomposed according to a color basis $c_i$, defined by

$$(c_1)_{\{a_i\}} = \delta_{a_1a_4}\delta_{a_2a_3}, \quad (c_2)_{\{a_i\}} = \delta_{a_1a_3}\delta_{a_2a_4}.$$  

(6)
In terms of this basis, we may write

\begin{align}
A_{\{a_i\}} &= A_I (c_i)_{\{a_i\}}, \\
G_{\{a_i\}} &= G_I (c_i)_{\{a_i\}}, \\
H_{\{b_i\}} &= H_I (c_i)_{\{b_i\}},
\end{align}

and we define

\begin{equation}
U_{\{a_i, b_i\}} (c_j)_{\{b_i\}} \equiv U_{IJ} (c_i)_{\{a_i\}},
\end{equation}

Then the vertex function can be written as

\begin{equation}
G_I = U_{IJ} H_J.
\end{equation}

with perturbative expansion for \( U_{IJ} \) as

\begin{equation}
U_{IJ} = \delta_{IJ} + \mathcal{O}(\alpha_s).
\end{equation}

From eqs. (8) and (9) it is clear how soft gluon insertions lead to color mixing. The matrix \( U_{IJ} \) is generated by dressing the ‘hard’ color tensor \( c_j \) with soft gluons and projecting the result along the direction of \( c_i \).

We now discuss the renormalization of the eikonal and hard scattering functions in the above color basis. We denote by \( U_{IJ}^{(0)} \) and \( H_I^{(0)} \) the UV-divergent unrenormalized soft and hard functions. Their multiplicative renormalization [11] involves the same color mixing as in the amplitude,

\begin{align}
U_{IJ}^{(0)} (\alpha_s(\mu), \theta, \epsilon) &= U_{IK} (\alpha_s(\mu), \theta, \epsilon) (Z_U)_{KJ} (a_s(\mu), \theta, \epsilon), \\
H_I^{(0)} (Q/\mu, \alpha_s(\mu), \theta, \epsilon) &= (Z_{\psi}^{-1})^4 (Z_{U}^{-1})_{IJ} (a_s(\mu), \theta, \epsilon) H_J (Q/\mu, \alpha_s(\mu), \theta).
\end{align}

Hence, the renormalized \( H \) and \( U \) satisfy the complementary matrix renormalization group equations

\begin{equation}
\mathcal{D} U_{IJ} = -U_{IK} (\Gamma_U)_{KJ}, \quad \mathcal{D} H_I = [(\Gamma_U)_{IK} + 4\gamma_5 \delta_{IK}] H_K,
\end{equation}

with

\begin{equation}
\mathcal{D} = \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g},
\end{equation}

6
and
\[ \gamma_\psi(\alpha_s(\mu)) = -\frac{\alpha_s}{4\pi} N_c C_F + O(\alpha_s^2) \] (13)

the anomalous dimension of the quark wave-function renormalization in the axial gauge [17]. For minimal subtraction and \( D = 4 - 2\epsilon \) the anomalous dimension matrix \( \Gamma_U \) is given simply by
\[ \Gamma_U(\alpha_s(\mu), \theta) = -g \frac{\partial}{\partial g}\text{Res} Z_U(\alpha_s(\mu), \theta, \epsilon). \] (14)

We note that because, as mentioned above, \( U \) depends on no momentum scales, the unrenormalized \( U^{(0)} \) actually vanishes in dimensional regularization beyond zeroth order, eq. (10). As a result, the renormalized \( U \) is given by \( Z_U^{-1} \),
\[ U_{ij} = (Z_U^{-1})_{ij}. \] (15)

We are now ready to discuss the energy dependence of the amplitude in terms of its factorized form.

2.3 Energy dependence

At lowest order in \( \alpha_s \) the diagonalization of of the matrix \( \Gamma_U \) may be performed with a matrix \( R \) that is independent of \( \mu \) and \( \alpha_s(\mu) \).
\[ (R^{-1} \Gamma_U R)_{ij} = \lambda_i(\alpha_s(\mu), \theta) \delta_{ij}, \quad \hat{U}_{ij} = R^{-1}_{ik} U_{kl} R_{lj}, \quad \hat{H}_j = R^{-1}_{jk} H_k. \] (16)

In the new basis we find decoupled renormalization group equations (12) for the components of both \( H \) and \( U \) in the space of color flow. We emphasize that the eigenvalues \( \lambda_i \) are in general \( \theta \)-dependent.

To use the information contained in eqs. (12), we recall that the quark-quark scattering amplitude should be thought of as embedded in a physical process, involving hadron-hadron elastic or inelastic scattering. In any such process, the infrared divergences of the eikonal will cancel. For elastic scattering this will occur through the color singlet nature of the external hadrons; in inclusive cross sections, it will
occur through cancellation between real and virtual processes in perturbation theory. In either case, the divergences of the soft gluon function $U$ will be replaced by a dynamically-generated (Sudakov) transverse momentum scale $[11]$ or by an energy resolution characteristic of the inclusive cross section $[14]$. With this in mind, we shall use eq. (12) to evolve the scale of the coupling in $U$ from $\mu$ down to an ‘infrared’ scale that we shall denote by $1/b \gg \Lambda$, with $b$ a length, referring to the notation of ref. $[11]$. Similarly, for $H$, we evolve from the general scale $\mu$ up to the scale $Q$, eq. (2). Following this procedure, we may write the vertex function $G$ in eq. (5) in terms of the leading order $\lambda_I$ as

$$
\hat{G}_I(Q/\mu, \alpha_s(\mu), \theta, \epsilon) \hat{c}_I = \exp \left[ - \frac{2N_c C_F}{\beta_1} \ln \left( \frac{\ln(C_2 Q/\Lambda)}{\ln(\mu/\Lambda)} \right) \right] 
\times \sum_J \exp \left[ -\gamma_J(Q, b, \theta) \right] \hat{U}_{IJ}(\alpha_s(C_1/b), \theta, \epsilon) \hat{H}_{J}(1/C_2, \alpha_s(C_2 Q), \theta) \hat{c}_I, \tag{17}
$$

with $\hat{c}$ the basis in which $\Gamma_U$ is diagonal, and with exponent given by

$$
\gamma_J(Q, b, \theta) = \int_{C_1/b}^{C_2 Q} \frac{d\mu'}{\mu'} \lambda_J(\alpha_s(\mu'), \theta). \tag{18}
$$

The constants $C_1, C_2$ are order unity, chosen so as to optimize perturbative calculations $[16]$.

We denote the expansion of the eigenvalues as a series in the coupling constant as

$$
\lambda_I(\alpha_s(\mu), \theta) = \sum_n \left( \frac{\alpha_s}{\pi} \right)^n \lambda_I^{(n)}(\theta). \tag{19}
$$

Then from the 1-loop eigenvalues, with

$$
\alpha_s(\mu) = \frac{4\pi}{\beta_1 \ln(\mu^2/\Lambda^2)}, \quad \beta_1 = \frac{11}{3} N_c - \frac{2}{3} n_f, \tag{20}
$$

we obtain from eq. (17) the behavior

$$
\hat{G}_I(Q/\mu, \alpha_s(\mu), \theta, \epsilon) \hat{c}_I = \exp \left[ - \frac{2N_c C_F}{\beta_1} \ln \left( \frac{\ln(C_2 Q/\Lambda)}{\ln(\mu/\Lambda)} \right) \right] 
\times \sum_J \exp \left[ - \frac{2\lambda_J^{(1)}(\theta)}{\beta_1} \ln \left( \frac{\ln(C_2 Q/\Lambda)}{\ln(C_1/b \Lambda)} \right) \right] 
\times \hat{U}_{IJ}(\alpha_s(C_1/b), \theta, \epsilon) \hat{H}_{J}(1/C_2, \alpha_s(C_2 Q), \theta) \hat{c}_I. \tag{21}
$$
Here the $Q$-dependence is determined by the color flow in the *hard* scattering, $U_{ij}$, which summarizes the infrared behavior, may still mix colors and depend on the scattering angle. We shall assume that our q-q scattering amplitude will appear in an IR safe cross section in which this extra dependence cancels.

In order to complete the picture we must include the energy dependence of the four jet functions, $z_i$ in eq. (I). As in the case of the eikonal function $U$, we appeal to the eventual incorporation of the quark-quark amplitude into a physical process, and evolve the $z_i$ from scale $\mu$ to an infrared scale denoted, as above, by $1/b$. This may be done in precisely the manner outlined in ref. [11], with the result,

$$z_i^{1/2}(\sqrt{2s\nu_i/\mu^2}, b\mu, \alpha_s(\mu), \epsilon) = \exp \left[ -\frac{2C_F}{\beta_1} \ln(\sqrt{2s\nu_i/\Lambda^2}) \ln \left( \frac{\ln(\sqrt{2s\nu_i/\Lambda^2})}{\ln(C_1/b\Lambda)} \right) + \text{NL} \right] \times z_i^{1/2}(1, 1, \alpha_s(C_1/b), \epsilon),$$

(22)

where

$$\nu_i = \frac{(v_i \cdot n)^2}{|n|^2},$$

(23)

with $n$ the axial gauge vector chosen spacelike, $n^2 = -1$. ‘NL’ stands for non-leading logarithmic corrections coming from the RG evolution of $z_i$ [11] that we shall not need explicitly. The $\mu$-dependence of $z_i^{1/2}$ is now in the contribution to the exponent from the quark anomalous dimension, eq. (13), and is also contained in ‘NL’. Of course, $\prod_i z_i^{1/2} G_i$ is RG invariant.

Combining eqs. (I), (21) and (22), we may summarize the energy behavior of the quark-quark scattering amplitude as

$$\hat{A}_i = \hat{S}_{ij}(\alpha_s(1/b), \theta, \epsilon) \hat{H}_j(\alpha_s(Q), \theta, \epsilon; \lambda_i)$$

(24)

where

$$\hat{S}_{ij} = \prod_{i=1}^4 z_i^{1/2}(1, 1, \alpha_s(1/b), \epsilon) \hat{U}_{ij}(\alpha_s(1/b), \theta, \epsilon),$$

(25)

$$\hat{H}_j = \exp \left[ -\sum_{i=1}^4 \frac{2C_F}{\beta_1} \ln(\sqrt{2s\nu_i/\Lambda^2}) \ln \left( \frac{\ln(\sqrt{2s\nu_i/\Lambda^2})}{\ln(1/b\Lambda)} \right) + \text{NL} \right] \hat{H}_j(1, \alpha_s(Q), \theta; \lambda_i).$$
We set $C_1 = C_2 = 1$ for simplicity, and from now on we include the Dirac spinors in $H$. We distinguish between the IR regulator $\epsilon$ and the factorization scale $1/b$, which, as noted above, is a physical IR cutoff characteristic of the experiment. $\hat{S}_{IJ}$ contains all the IR structure of the q-q amplitude whereas $H_J$ is the Sudakov-evolved hard part that enters hadronic cross sections. For scattering at fixed angles, the leading logarithms are generated by the self energies, through the $\nu_i$. We shall see below, however, that for the hard octet component of forward scattering, the leading logarithms in $s/t$ are generated through the anomalous dimensions of the vertex function $G = UH$.

2.4 One-loop anomalous dimensions

We close this section with a summary of the one-loop anomalous dimension matrix $\Gamma_U$ in the basis defined by eq. (6).

The matrix $\Gamma_U$ has been calculated [1] to $O(\alpha_s)$ in ref. [11]. In terms of kinematic invariants it is

$$
\Gamma_{U11} = \frac{\alpha_s}{\pi} \left\{ C_F \left[ \ln \left( \frac{u^2}{4s^2} \right) - \frac{1}{2} \ln \prod_{i=1}^{4} \nu_i + 2 \right] - \frac{1}{2N_c} \left[ \ln \left( \frac{t^2}{s^2} \right) + 2\pi i \right] \right\},
$$

$$
\Gamma_{U12} = \frac{\alpha_s}{\pi} \frac{1}{2} \left[ \ln \left( \frac{u^2}{s^2} \right) + 2\pi i \right],
$$

$$
\Gamma_{U21} = \Gamma_{U12} |_{t \leftrightarrow u}, \quad \Gamma_{U22} = \Gamma_{U11} |_{t \leftrightarrow u}.
$$

(26)

Here the SU($N_c$) normalization is, as usual,

$$
\text{Tr}\{t_m t_n\} = \frac{1}{2} \delta_{mn} \quad C_F = \frac{N_c^2 - 1}{2N_c}.
$$

(27)

$\Gamma_U$ is gauge dependent via the combination $\ln \prod_{i=1}^{4} \nu_i$, where $\nu_i$ has been defined in eq. (23). In the following section, we shall use these results to study scattering in the limit of fixed $t, s \to \infty$.

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1These results correct a misprint in ref. [11], in which the (12) and (21) matrix elements of $\Gamma^{(3)}_U$ are too small by a factor of 2.
3 Color flow in the forward direction

In the kinematic region of forward elastic scattering with large momentum transfer, \( \ln(s/t) \gg 1 \), the eigenvalues of \( \Gamma_U \) are

\[
\begin{align*}
\lambda_1^{(1)} &= \frac{1}{N_c} \left[ \ln \left( \frac{s}{-t} \right) - 2\pi i \right] + 2C_F \left[ 1 - \ln 2 - \frac{1}{4} \ln \prod_{i=1}^4 \nu_i \right] + \mathcal{O}(\ln^{-1}(s/t)), \\
\lambda_2^{(1)} &= 2C_F \left[ -\ln \left( \frac{s}{-t} \right) + 1 - \ln 2 - \frac{1}{4} \ln \prod_{i=1}^4 \nu_i \right] + \mathcal{O}(\ln^{-1}(s/t)).
\end{align*}
\]

We observe that their difference is gauge independent:

\[
\Delta \lambda^{(1)} \equiv \lambda^{(1)} - \lambda^{(2)} = N_c \ln \left( \frac{s}{-t} \right) - \frac{2\pi i}{N_c} + \mathcal{O}(\ln^{-1}(s/t)) .
\]

Using this result in eq. (21) for \( \hat{G}_I \), the component of the vertex function \( G \) along the diagonalized color channel \( \hat{c}_1 \) is (Sudakov) suppressed relative to the component along \( \hat{c}_2 \) by a factor

\[
\exp \left[ -\frac{2\Delta \lambda^{(1)}}{\beta_1} \ln \left( \frac{\ln(Q/\Lambda)}{\ln(1/b \Lambda)} \right) \right] \approx \left( \frac{s}{-t} \right)^{-\frac{2N_c}{\beta_1} \ln \left( \frac{\ln(Q/\Lambda)}{\ln(1/b \Lambda)} \right)} .
\]

The eigenvectors are also gauge independent, and in the \( c_I \) basis of eq. (3) they are

\[
e_1 = \frac{1}{2} \begin{pmatrix}
1 \\
1/
\end{pmatrix} + \mathcal{O}(\ln^{-1}(s/t)), \quad e_2 = \begin{pmatrix}
0 \\
1
\end{pmatrix} + \mathcal{O}(\ln^{-1}(s/t)),
\]

where the factor 1/2 is a convenient normalization. In the \( s \to \infty \) limit, the matrix \( R = (e_1, e_2) \) generates the diagonalized basis \( \hat{c}_I \)

\[
\hat{c}_1 = (t_m)_{a_3a_1} (t_m)_{a_4a_2} \equiv c_{\text{adj}}, \quad \hat{c}_2 = \delta_{a_1a_3} \delta_{a_2a_4} \equiv c_s ,
\]

which corresponds to octet and singlet for SU(3) exchange in the \( t \)-channel and it is the octet that receives the additional suppression (30) relative to the singlet.

In the forward direction, the \( \lambda_I \), and hence \( G \), evidently include the leading logarithms in \( s \), in the form \( \ln(s/t) \). This is to be contrasted to the fixed angle
case, where the leading (double) logs appear as \( \ln(Q^2/\Lambda^2) \), with \( Q^2 \sim s, t \). Since \( s \)-dependence appears in a crucial manner in the color-dependence of \( G = UH \), while the \( z_i \)'s in eq. (24) are independent of color flow, it is natural to eliminate \( s \)-dependence from the external self-energies altogether. This we may accomplish by the choice of gauge

\[
n^\mu = (P_1 - P_3)^\mu / \sqrt{-t} = (v_1 - v_3)^\mu \sqrt{s/2t},
\]

which gives, (see eq. (23)),

\[
\frac{1}{4} \ln \prod_{i=1}^{4} \nu_i = - \ln \left( \frac{s}{-t} \right) - \ln 2.
\]

Then the eigenvalues become

\[
\lambda_{\text{adj}}^{(1)} = N_c \ln \left( \frac{s}{-t} \right) - \frac{1}{N_c} 2\pi i + \lambda_s^{(1)}, \quad \lambda_s^{(1)} = 2C_F + \mathcal{O}(\ln^{-1}(s/ - t)).
\]

In LLA with gauge choice (33), \( \lambda_{\text{adj}}^{(1)} \approx \Delta \lambda^{(1)} \) and we thus find for hard octet exchange in eq. (26) the combination

\[
\mathcal{H}_{\text{adj}} = \left( \frac{s}{-t} \right)^{-2N_c \pi i} \ln \left( \frac{\ln(\sqrt{-t}/\Lambda)}{\ln(1/\Lambda)} \right) H_{\text{adj}}(\alpha_s(-t), \theta; \lambda_i),
\]

where we again suppress non-leading terms in the exponent, and where, at lowest order,

\[
H_{\text{adj}}^{(1)} = A_{\text{Born}} = 4\pi \alpha_s(\bar{u}_3\gamma^\mu u_1)(\bar{u}_4\gamma_\mu u_2).
\]

Eq. (36) gives the reggeized form of the amplitude for hard octet exchange as derived from the fixed angle formalism of [11]. This is to be compared with the standard result [4, 5] for the amplitude from leading logarithms in \( s \) only,

\[
A_{\text{adj}} = \left( \frac{s}{-t} \right)^{\alpha_s(-t)} A_{\text{Born}},
\]

where, using notation \( q \) for \( q_\perp^\mu \),

\[
j_g(t) = 1 + \alpha_s(q^2) = 1 - \frac{N_c\alpha_s}{4\pi^2} q^2 \int \frac{d^2k}{k^2(q - k)^2}, \quad q^2 = -t,
\]
is the gluon Regge trajectory. The quantity $\alpha_g(-t)$ as defined in (39) is IR divergent at $k \to 0$ and $k \to q$. Dimensional regularization gives, for $\alpha_g(-t)$,

$$\alpha_g = \frac{N_c \alpha_s}{2\pi} \frac{1}{\epsilon} + O(\epsilon^0).$$  \hspace{1cm} (40)

The relationship of this leading $\ln(s)$ result with eq. (36) becomes clear when we substitute the octet eigenvalue in (35) into the exponent eq. (18), taking $b \to \infty$ and using the zeroth order ('fixed') running coupling in $D$ dimensions,

$$\alpha_s(\mu') = \alpha_s(\mu) \left( \frac{\mu}{\mu'} \right)^{2\epsilon}.$$  \hspace{1cm} (41)

The result is precisely eq. (40).

We should note that the correspondence of our eigenvectors to singlet and octet exchange is only exact for $s \to \infty$. According to eq. (31), the singlet-octet basis diagonalizes $\Gamma_U$ only to $O(\ln(s/t))$. Indeed

$$R^{-1} \Gamma_U R = \frac{\alpha_s}{\pi} \left( \frac{\lambda^{(1)}_{\text{adj}}}{N_c \pi i} \lambda^{(1)}_s \right) + O(\ln^{-1}(s/t)).$$  \hspace{1cm} (42)

The off-diagonal elements describe an $O(\ln^{-1}(s/t))$ mixing of the singlet and the octet in the basis that would diagonalize $\Gamma_U$ to $O(\ln^0(s/t))$. But in view of the octet reggeization, the contribution to the evolution from this mixing dies out at high energy and the amplitude is singlet dominated.

For the singlet vertex function we obtain in place of eq. (30) a suppression in $t$ only in LLA,

$$\exp \left[ -\frac{2\lambda_s^{(1)}}{\beta_1} \ln \left( \frac{\ln(\sqrt{-t}/\Lambda)}{\ln(1/b\Lambda)} \right) \right] = \left( \frac{\ln(\sqrt{-t}/\Lambda)}{\ln(1/b\Lambda)} \right)^{-\frac{4CF}{\beta_1}}.$$  \hspace{1cm} (43)

Evidently, for sufficiently large $s$ and fixed $t$, the singlet dominates the amplitude and we find from eq. (24) in LLA in $s$ and $t$ separately,

$$A_I = \exp \left[ -\frac{8CF}{\beta_1} \ln \left( \frac{\sqrt{-t}}{\Lambda} \right) \ln \left( \frac{\ln(\sqrt{-t}/\Lambda)}{\ln(1/b\Lambda)} \right) + NL \right] \times S_{I,s}(\alpha_s(1/b), \theta, \epsilon) H_s(\alpha_s(-t), \theta; \lambda_i),$$  \hspace{1cm} (44)
where $H_s$ is the ‘hard singlet amplitude’, IR finite and independent of all scales below the hard scale $-t$, and $S_{t,s} \equiv S_{t2}$ in the singlet-octet basis of eq. (32). Unlike the case of octet exchange, however, where the lowest order contribution is $A_{\text{Born}}$, for the singlet the lowest possible order contribution is $\mathcal{O}(\alpha_s^2)$. In order to use eq. (44) in its perturbative expansion we need to construct the lowest order IR finite amplitude $H_s^{(1)}$.

4 Lowest order singlet exchange

In this section we review singlet exchange at one loop. For massless q-q elastic scattering in the region

$$s \gg -t \gg \Lambda^2,$$

we have seen that it is natural to write the amplitude $A_{\{a_i\}}$ in the singlet-octet basis of eq. (32),

$$A_{\{a_i\}}(s, t; \lambda_i) = A_s(s, t; \lambda_i)(c_s)_{\{a_i\}} + A_{\text{adj}}(s, t; \lambda_i)(c_{\text{adj}})_{\{a_i\}}.$$

The lowest order contribution, $\mathcal{O}(\alpha_s^2)$, to $A_s$ comes only from graphs (a), (b) of fig. 3, whose color coefficients are

$$C^{(a)}_s = C^{(b)}_s = (N_c^2 - 1)/4N_c^2 \equiv C_s,$$

$$C^{(a)}_{\text{adj}} = -1/N_c, \quad C^{(b)}_{\text{adj}} = (N_c^2 - 2)/2N_c.$$

To this order $A_s^{(1)}$ is gauge invariant, since the one-loop vertex and gluon self-energy renormalization do not contribute to $A_s^{(1)}$. We perform the calculation in Feynman gauge. The graphs (a), (b) are UV finite. Their analytic structure is exhibited through the use of the parameter

$$z = \frac{-t}{s} - i\varepsilon.$$
The one-loop calculation gives for the singlet contribution of graphs (a), (b),

\[
A_s^{(a)} = C_s(\alpha_s \mu^\epsilon)^2 \left( \frac{4\pi \mu^2}{-t} \right) \epsilon \left( \frac{1}{\epsilon^2} - \frac{1}{\epsilon} - \frac{1}{2} \ln^2 (-z) - 4\zeta(2) + 1 \right) \cdot \nonumber
\]

\[
\times \frac{4}{t} (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_4 \gamma^\mu u_2) + R^{(a)},
\]

(50)

\[
A_s^{(b)} = -C_s(\alpha_s \mu^\epsilon)^2 \left( \frac{4\pi \mu^2}{-t} \right) \epsilon \left( \frac{1}{\epsilon^2} - \frac{1}{\epsilon} - \frac{1}{2} \ln^2 z - 4\zeta(2) + 1 \right) \cdot \nonumber
\]

\[
\times \frac{4}{t} (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_4 \gamma^\mu u_2) + R^{(b)}.
\]

(51)

The remainders \(R^{(a)}\) and \(R^{(b)}\) are IR finite and power suppressed by \(O(|z|)\). \(A_s^{(a)}, A_s^{(b)}\) manifestly satisfy s-channel two-particle unitarity. Before attempting further expansion in \(\epsilon\), let us identify the origin of the various pieces in eqs. (50), (51). The terms inside the braces come from integration over the Feynman parameters, and the \(\epsilon\)-poles are the soft and collinear divergences. The terms \(\Gamma(2 + \epsilon)(\pm z)^\epsilon\) come from integration over the loop momentum. Upon expanding in \(\epsilon\) and adding the contributions we obtain simply

\[
A_s^{(1)}(s, t, \epsilon; \lambda_i) = C_s(\alpha_s \mu^\epsilon)^2 \left( \frac{4\pi \mu^2}{-t e^{\gamma_E}} \right) \frac{i\pi}{\epsilon} \frac{4}{t} (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_4 \gamma^\mu u_2). \quad (52)
\]

For the renormalization scale, the \(\overline{\text{MS}}\) choice \(\mu^2 = Q^2\), with

\[
Q^2 = \exp(\gamma_E) / 4\pi \cdot (-t), \quad (53)
\]

sets the scale of the hard scattering and is compatible with the one-loop gluon self-energy and vertex renormalization that contribute to \(A_{\text{adj}}\) to \(O(\alpha_s^2)\) [18]. Then the amplitude becomes

\[
A_s^{(1)}(s, t, \epsilon; \lambda_i) = C_s(\alpha_s \mu^\epsilon)^2 \frac{i\pi}{\epsilon} \frac{4}{t} (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_4 \gamma^\mu u_2) = C_s \frac{i\alpha_s}{\epsilon} A_{\text{Born}}. \quad (54)
\]

The \(\overline{\text{MS}}\) choice for the hard scale can be easily included in eq. (44) by reinstating the \(C_2\) parameter as in eq. (21) and setting \((C_2)^2 = \exp(\gamma_E)/4\pi\). For any other scale
choice $\mu^2$ the amplitude becomes

$$A_s^{(1)}(s, t, \epsilon; \lambda_i) = C_s^i \alpha_s \left( \frac{1}{\epsilon} + \ln \frac{\mu^2}{Q^2} \right) A_{\text{Born}}.$$  \hspace{1cm} (55)

The leading IR singularities $1/\epsilon^2$ cancel in the singlet exchange. We observe that $A_s^{(1)}$ in region $\langle 13 \rangle$ is purely imaginary to leading power in $s/t$ and that there are no IR regular terms in $\overline{\text{MS}}$ scheme. All these features are well known. We will now discuss the IR subtraction procedure for the above amplitude.

5 IR subtractions

The IR singular part of $A_s^{(1)}$ comes from soft gluon exchange. The soft region in loop momentum space is defined in light cone coordinates as the region where gluon momenta scale as

$$q_1^\mu = (q_1^+, q_1^-; q_1\perp) \approx Q(\lambda, \lambda),$$ \hspace{1cm} (56)

with $\lambda \to 0$. Although collinear singularities are present in (a), (b) in the Feynman gauge, unlike in axial gauge, their net effect cancels in singlet exchange.

The contributions to the one-loop singlet amplitude, factorized as in eq. (3), are

$$A_s^{(1)} = U_{s, \text{adj}}^{(1)}(\alpha_s(\mu), \theta, \epsilon) A_{\text{Born}} + H_s^{(1)}(Q/\mu, \alpha_s(\mu), \theta; \lambda_i).$$  \hspace{1cm} (57)

As noted above $U_{s, \text{adj}}$ is the singlet-octet mixing term, whose unrenormalized value is zero in dimensional regularization (see eq. (15)). We may then write

$$U_{s, \text{adj}}^{(1)} = C_s(\omega^{(a)} + \omega^{(b)})_{\text{ren}},$$

$$H_s^{(1)} = A_s^{(1)} - C_s(\omega^{(a)} + \omega^{(b)})_{\text{ren}} A_{\text{Born}}.$$  \hspace{1cm} (58)

Here the $\omega^{(a,b)}$ are given by the eikonal graphs of fig. 4,

$$\omega^{(a)} = 4\pi\alpha_s\mu^{2\epsilon} \frac{(-i)v_1 \cdot v_2}{(2\pi)^D (q_1^2 + i\epsilon)(-v_1 \cdot q_1 + i\epsilon)(v_2 \cdot q_1 + i\epsilon)},$$

$$\omega^{(b)} = 4\pi\alpha_s\mu^{2\epsilon} \frac{(-i)v_1 \cdot v_4}{(2\pi)^D (q_1^2 + i\epsilon)(-v_1 \cdot q_1 + i\epsilon)(-v_4 \cdot q_1 + i\epsilon)}.  \hspace{1cm} (59)$$

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The factor of 2 is due to equal contributions from left-right mirror graphs. The subscript ‘ren’ in eq. (58) indicates that \( \omega^{(a,b)} \) vanish in \( D \) dimensions and are defined by a renormalization prescription that uncover their IR poles at \( D = 4 \). Thus we must evaluate the integrals formally to identify an IR pole term, which will serve as the one-loop UV counterterm, \( (Z_{\psi}^{-1})^{(1)} \), as in eq. (15). To facilitate this process, we multiply and divide the integrands in \( \omega^{(a,b)} \) by \( \mu^2/\kappa^2 \), with \( \kappa \) a dimensionless free parameter and use Feynman parametrization on the denominators \( \pm (\mu/\kappa)v_i \cdot q_1 + \iota \epsilon \).

This procedure gives the formal expressions for the unrenormalized integrals:

\[
\begin{align*}
\omega^{(a)} &= \frac{\alpha_s}{\pi} \left( \frac{8\pi\kappa^2}{-v_1 \cdot v_2} \right)^\epsilon \Gamma(1 + \epsilon) B(2\epsilon, -2\epsilon) B(-\epsilon, -\epsilon), \\
\omega^{(b)} &= -\frac{\alpha_s}{\pi} \left( \frac{8\pi\kappa^2}{v_1 \cdot v_4} \right)^\epsilon \Gamma(1 + \epsilon) B(2\epsilon, -2\epsilon) B(-\epsilon, -\epsilon). \tag{60}
\end{align*}
\]

Of course, since \( B(2\epsilon, -2\epsilon) = 0 \), these forms are not unique. The UV pole, however, may be isolated by applying the identity

\[
B(a, b) = B(1 + a, b) + B(a, 1 + b) \tag{61}
\]

to \( B(2\epsilon, -2\epsilon) \) and identifying poles from \( \Gamma(n\epsilon) \) as UV (convergent for \( \epsilon > 0 \)) and \( \Gamma(-n\epsilon) \) as IR (convergent for \( \epsilon < 0 \)). In this fashion, we identify purely IR pole terms for the \( \omega \)'s, which we denote by \( \omega_{\text{ren}}^{(a,b)} \):

\[
\begin{align*}
\omega_{\text{ren}}^{(a)} &= \frac{\alpha_s}{\pi} \left( \frac{8\pi\kappa^2}{-v_1 \cdot v_2} \right)^\epsilon \Gamma(1 + \epsilon) \left\{ \frac{1}{\epsilon^2} + 3\zeta(2) + \mathcal{O}(\epsilon) \right\}, \\
\omega_{\text{ren}}^{(b)} &= -\frac{\alpha_s}{\pi} \left( \frac{8\pi\kappa^2}{v_1 \cdot v_4} \right)^\epsilon \Gamma(1 + \epsilon) \left\{ \frac{1}{\epsilon^2} + 3\zeta(2) + \mathcal{O}(\epsilon) \right\}. \tag{62}
\end{align*}
\]

Here our choice of \( \kappa^2 \) will determine the precise renormalization of \( U_{s,\text{adj}} \). \( U_{s,\text{adj}} \) must be chosen to absorb the IR divergences of \( A_{s}^{(1)} \), eqs. (53), (54), but is otherwise arbitrary. Its general form at one loop is

\[
U_{s,\text{adj}}^{(1)} = C_s [\omega^{(a)} + \omega^{(b)}]_{\text{ren}} = C_s \frac{\alpha_s}{\pi} \frac{1}{\epsilon^2} \left[ (-v_1 \cdot v_2)^{-\epsilon} - (v_1 \cdot v_4)^{-\epsilon}\right][1 + \epsilon(\ln(8\pi\kappa^2) - \gamma_E)] + \mathcal{O}(\epsilon)
\]

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We may choose $\kappa^2$ to eliminate all or part of the finite terms that accompany the imaginary IR divergence in $A_s^{(1)}$. (Note that $v_1 \cdot v_2/v_1 \cdot v_4 = 1$ in the approximation $-t/s \ll 1$, in which we work.) In a strictly ‘minimal’ scheme, we might well choose $\kappa^2$ to cancel all the finite terms in $U_{s,\text{adj}}^{(1)}$, so that $U_{s,\text{adj}}^{(1)} A_{\text{Born}} = A_s^{(1)}$. In this case, we would have, by eq. (57), $H_{s}^{(1)} = 0$ for $\text{MS}$ scheme $\mu = \overline{\text{Q}}$. This choice is not unique, however. The optimal choice should be guided by the manner in which the factorized amplitude is embedded in an infrared-safe quantity. In such a quantity we may expect the IR divergences of $U$ to cancel. If other, finite, contributions to $A_s$ cancel as well, these may naturally be included in $U$, at least if they arise from regions of ‘soft’ loop momentum, as in eq. (56), with $0 < \lambda < 1$.

From the above considerations, we may identify an alternate definition of $\kappa^2$ in eq. (63),

$$\kappa^2 = \frac{1}{8\pi} e^{\gamma_E},$$

for which

$$U_{s,\text{adj}}^{(1)} \equiv C_s \frac{\alpha_s}{\pi} \left[ \frac{i\pi}{\varepsilon} + \ln\left(\frac{v_1 \cdot v_2}{v_1 \cdot v_4}\right) + \frac{i\pi}{\varepsilon} \ln(8\pi \kappa^2) - \frac{\pi^2}{2} \right] + O(\varepsilon).$$

That is, we keep $O(\varepsilon^0)$ terms that follow directly from the combination of the double IR pole with the analytic behavior of the graphs as functions of their external momenta. We conjecture that in the calculation of the proton-proton elastic scattering amplitude, the entire $U_{s,\text{adj}}$ contribution of this form cancels in the sum over soft corrections. When the IR function $U_{s,\text{adj}}^{(1)}$ is defined according to the latter procedure, we find the result

$$U_{s,\text{adj}}^{(1)} A_{\text{Born}} = C_s (\alpha_s \mu^2)^{\frac{4}{3}} \left( \frac{i\pi}{\varepsilon} - \frac{\pi^2}{2} \right) (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_4 \gamma^\mu u_2).$$
So, by eqs. (58) and (54), the lowest order IR finite amplitude for singlet exchange at scale $\mu^2$ is

$$H_s^{(1)}(Q/\mu, s/t; \lambda_i) = C_s(\alpha_s\mu^2)^2 \left( i\pi \ln \frac{\mu^2}{Q^2} + \frac{\pi^2}{2} \right) \frac{4}{t}(\bar{u}_3\gamma^\mu u_1)(\bar{u}_4\gamma^\mu u_2)$$

$$= C_s\alpha_s \left( i \ln \frac{\mu^2}{Q^2} + \frac{\pi}{2} \right) A_{\text{Born}}. \quad (67)$$

Notice again that had we not kept the $O(\epsilon^0)$ terms in eq. (58), the $\overline{\text{MS}}$ choice $\mu = Q$ would have given $H_s^{(1)} = 0$. The real part of $H_s^{(1)}$ is generated by the above oversubtraction procedure.

## 6 Summary

In this paper, we have found that in near-forward q-q elastic amplitude with large momentum transfer, the hard singlet component evolves at leading logarithm in $t$ according to eq. (44). The singlet hard-scattering function $H_s(s/t; \lambda_i)$ is by construction free from all scales below the hard scale $t$. We have observed that the value of the hard-scattering function depends on the infrared safe quantity in which the amplitude appears. One possibility for the $H_s(s/t; \lambda_i)$ at one loop is given in eq. (67). The singlet evolution shown in eq. (44) organizes all leading and non-leading Sudakov $\ln(t)$ behavior. The basis of this derivation is the factorization of the amplitude as shown in eqs. (1) and (5).

In future work we shall examine the $s$-dependence of $H_s$ beyond the lowest order in $\alpha_s$. This dependence cannot be captured by the Sudakov-oriented approach of [11]. We anticipate that the leading logarithmic $s/t$ dependence of $H_s$ is given by a ‘BFKL’ equation [3]. Of interest is the possible influence of infrared subtractions on the resulting evolution [7]. In addition, we hope to pursue applications to both proton-proton elastic scattering and to jet production. For proton-proton elastic scattering, our results indicate an important role for a perturbative singlet exchange in the independent scattering of quarks [3].
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Figure Captions

1. Leading radiative corrections to fixed angle quark-quark elastic scattering in the axial gauge.

2. Factorized form of the 4-point on-shell vertex function $G$. The internal lines of $H$ are off-shell by $\mathcal{O}(Q^2)$.

3. Lowest order graphs for singlet exchange in the $t$-channel.

4. Eikonal approximation applied to the graphs of fig. 3.
This figure "fig1-1.png" is available in "png" format from:

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