Research on the Method of Measuring Soil Water Content by Time Domain Reflectometry

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Abstract-Time domain reflectometry (TDR) can calculate the water content by testing the dielectric constant of the rock and soil medium, but the TDR measurement results under time domain conditions are susceptible to the influence of soil conductivity. This article introduces the basic theory of TDR measurement, then uses the transmission scattering matrix to describe the transmission equation, analyzes the dielectric constant in the frequency domain, simulates the TDR waveform with Matlab based on the Debye model, and fits the simulated waveform to the measured waveform, and then according to the simulated waveform the matching degree proves the feasibility of calibrating the dielectric constant of the soil. Taking clay as the measurement object, finally fitting a simulation waveform with a higher degree of matching with the measurement waveform, which proves the feasibility of the method and has certain reference value for the study of the TDR method of measuring soil dielectric constant and water content.

1. Introduction

With the advancement of engineering design and construction technology, on-site monitoring of the basic physical parameters of soil such as water content and density becomes more and more important [1-3]. Feller-Feldegg (1969) introduced a method that can measure the complex permittivity of liquids, called a time domain reflectometry (TDR) [4]. TDR testing technology is a reliable, fast and safe way to test the water content of soil. The measurement principle is that the signal source sends a step signal to propagate in the TDR system in the form of electromagnetic waves. If the impedance of the transmission line changes, electromagnetic waves will be emitted. The reflected wave returns to display on the oscilloscope. The difference between the reflected wave and the incident wave contains information that can reflect the dielectric properties of the soil [5-7]. Then the water content is obtained through the relationship equation between the dielectric constant and the water content.

At present, most TDR waveforms are based on time domain analysis, although it is simple and straightforward to analyze time domain reflectometry (TDR) waveforms in the time domain, in the case of electrical conductivity, time-domain analysis is difficult to accurately measure the dielectric constant of the soil, the reflected signal will disappear at the tip of the probe of the TDR system. In addition, the dielectric constant of the soil is related to the frequency domain. Even if the dielectric constant can be measured correctly in the time domain, for different soils, the real part of the dielectric constant obtained is for different frequencies. This uncertainty causes the dielectric constant to be ambiguous [8].

In order to solve the limitations of time domain analysis, this article uses frequency domain analysis of TDR waveforms. TDR waveforms under frequency domain analysis can better explain the results of time domain analysis [9-11]. The work of this paper is to obtain a simulation waveform with a higher degree of fit to the measured waveform, and prove the feasibility of calibrating the soil dielectric
2. Theory

2.1. Transmission line theory

Each part of the TDR system can be equivalent to a section of transmission line, the calculation of the transmission line is composed of voltage, current and length. In an ideal uniform transmission line, only transverse waves propagate. Ramo [12] et al. gave a general solution for a uniform transmission line:

\[ V(x) = V^+ e^{-\gamma x} + V^- e^{+\gamma x} \]  
\[ I(x) = \frac{1}{Z_0} (V^+ e^{-\gamma x} + V^- e^{+\gamma x}) \]  

Where \( x \) represents the position in the transmission line, \( V^+ \) and \( V^- \) are unknown constants, which can be determined by boundary conditions, \( \gamma \) is the propagation constant, which is a complex quantity, and \( Z_0 \) is the characteristic impedance, which is related to the geometric properties of the transmission line and the dielectric between the inner and outer conductors Related to the nature.

The transmission constant \( \gamma \) and characteristic impedance \( Z_0 \) are calculated by equations (3) and (4):

\[ \gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \]  
\[ Z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} \]  

Where \( j \) is the imaginary unit, \( L \) is the linear inductance in series with the resistance \( R \), which represents the skin effect of the transmission line; \( C \) is the transmission line capacitance per unit length and \( G \) is the reactance of the transmission line.

The TDR system consists of four parts, and each part can be regarded as a uniform transmission line, but they are not uniform and discontinuous with each other. The transmission of electromagnetic waves along each part of the coaxial system is based on the theory of transmission lines. Since the characteristic impedance of each part is different, the electromagnetic wave will be reflected multiple times in the TDR test system [13].

The reflection coefficient \( \rho \) is defined as the ratio of the reflected voltage \( V_r \) to the incident voltage \( V_0 \) at the impedance discontinuity surface, as shown in equation (5): The transmission coefficient \( \tau \) is defined as the ratio of the transmission voltage \( V_t \) to the incident voltage \( V_0 \) at the impedance discontinuity surface, as shown in equation (6)

\[ \rho = \frac{V_r}{V_0} = \frac{Z_1 - Z_0}{Z_1 + Z_0} \]  
\[ \tau = \frac{V_t}{V_0} = \frac{2Z_1}{Z_1 + Z_0} \]  

It can be seen from equation (5) that if the impedances of the transmission lines on both sides of the interface are equal, that is, \( Z_1=Z_0 \), then \( \rho=0 \) and no reflection occurs; if \( Z_1>Z_0 \), then \( \rho>0 \), resulting in regular reflection; if \( Z_1<Z_0 \), then \( \rho<0 \), negative reflection occurs; there are two limit states, when \( Z_1 \) tends to \( \infty \), that is, the end of the coaxial transmission line is open, then \( \rho=1 \); when \( Z_1 \) tends to 0, that is, the end of the coaxial transmission line is short-circuited, then \( \rho=-1 \).

2.2. Transmission Scattering Matrix

The transmission scattering matrix, also known as the T matrix, is one of the main observations used to describe the scattering process in physics. A network is described by a scattering matrix that first needs to have any number of ports, and the number of ports refers to the points where voltage enters or exits. The most commonly used is the scattering matrix of a two-port network, which is a basic component of some complex networks.

The relationship between the transmission scattering matrix and the input and output voltage is as
follows:

\[
\begin{bmatrix} V_1^+ \\ V_1^- \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} V_2^+ \\ V_2^- \end{bmatrix}
\] (7)

If the length of the transmission line is finite, the electromagnetic wave will be reflected infinitely, and the amplitude of the incident wave and the reflected wave will gradually decrease. Assuming that the length of the transmission line is \( L \), \( Y_1 \) is the characteristic admittance of the reflecting part, and \( Y_2 \) is the characteristic admittance of the incident part.

\[
\begin{bmatrix} e^{i\theta} & 0 \\ 0 & e^{i\theta} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_1^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} e^{-i\theta} & 0 \\ 0 & e^{-i\theta} \end{bmatrix} \begin{bmatrix} V_2^+ \\ V_2^- \end{bmatrix}
\] (8)

Where \( \theta = \gamma L \), \( \theta \) is the propagation factor, and the scattering matrix \( S \) is defined as:

\[
\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{2Y_1} \begin{bmatrix} Y_1 + Y_2 & Y_1 - Y_2 \\ Y_1 - Y_2 & Y_1 + Y_2 \end{bmatrix}
\] (9)

Multiply both sides of the equation by the inverse of the first matrix on the left side of the equation:

\[
\begin{bmatrix} V_1^+ \\ V_1^- \end{bmatrix} = \begin{bmatrix} e^{-i\theta} & 0 \\ 0 & e^{-i\theta} \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} e^{i\theta} & 0 \\ 0 & e^{i\theta} \end{bmatrix} \begin{bmatrix} V_2^+ \\ V_2^- \end{bmatrix}
\] (10)

Then we can get the transmission scattering \( T \) matrix:

\[
T = \begin{bmatrix} e^{-i\theta} & 0 \\ 0 & e^{-i\theta} \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} e^{i\theta} & 0 \\ 0 & e^{i\theta} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} e^{-2i\theta} & 0 \\ 0 & e^{-2i\theta} \end{bmatrix}
\] (11)

The TDR system is considered to be composed of four sections of transmission lines, which are: (1) test panel (2) coaxial cable (3) coaxial connector (4) probe. Due to the difference in the characteristic impedances of the four segments, the electromagnetic waves are more reflected when they propagate in the TDR system. The characteristic admittance of each segment is different, and the connections are not continuous, so the TDR system can be understood as a cascaded 2-port network. As shown in Figure 1.

Figure 1. Cascade port network of TDR

The T matrix of the entire system is

\[
\begin{bmatrix} V_1^+ \\ V_1^- \end{bmatrix} = [T_1][T_2][T_3][T_4] \begin{bmatrix} V_2^+ \\ V_2^- \end{bmatrix}
\] (12)

Since the end of the probe is open, \( \begin{bmatrix} V_1^+ \\ V_1^- \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \)

2.3. Debye model

Under the action of an external electric field, the polarization process of the dielectric always takes a certain time to reach a steady state, especially under the alternating electric field, the polarization process of the medium is relatively complicated [14].

Generally speaking, the polarization of a dielectric can be expressed as a generalized integral form of the dielectric constant \( \varepsilon \) and the angular frequency \( \omega \) of the applied alternating electric field as:

\[
\varepsilon(\omega) = \varepsilon_\infty + \int_0^\infty \alpha(t) e^{i\omega t} dt
\] (13)

\( \alpha(t) \) is the attenuation factor, which reflects the law of the attenuation of the dielectric polarization after the external electric field is suddenly removed, and the law that the dielectric polarization tends to the equilibrium state after the external electric field is rapidly applied. \( \varepsilon_\infty \) is called the instantaneous displacement polarization constant, also known as the high-frequency dielectric constant of the medium, and it represents the contribution of the displacement polarization. In general, \( \alpha(t) \) obeys exponential decay, that is

\[
\alpha(t) = \alpha_0 e^{-\frac{t}{t}}
\] (14)
Substitute equation (14) into equation (13), $\varepsilon(0) = \varepsilon_s$, the above formula is simplified to get:

$$\varepsilon(\omega) = \varepsilon' - \varepsilon'' = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 - j\omega\tau}$$

Equation (15) is called the Debye equation. Where $\varepsilon_\infty$ reflects the relaxation loss of the medium. It is not difficult to see from the equation that when $\omega\tau \gg 1$, the $\varepsilon' \to \varepsilon_\infty$ medium has only displacement polarization and no relaxation loss. When $\omega\tau \ll 1$, $\varepsilon' \to \varepsilon_s$ the various polarization processes of the medium have not been established yet, and the loss mainly depends on the conductivity of the dielectric material.

3. **Waveform simulation of TDR**

For the TDR measurement of clay, we can obtain the pulse waveform to provide to the simulation program. The pulse waveform is shown in Figure 2.

![Pulse waveform](image-url)

The pulse wave generated by the measuring instrument is transformed into a frequency domain wave by FFT; then the TDR system parameters and the initial value of the assumed soil dielectric parameter are used to calculate the output frequency domain voltage through various parameters and the scattering matrix is calculated. The frequency domain voltage is converted into the time domain voltage to obtain the simulation waveform, and then the measured waveform and the simulation waveform are optimized and matched to calibrate the real dielectric parameters of the soil. The optimization effect is measured by calculating the minimum mean square error (MSE) between the measured waveform and the simulated waveform, and the MSE expression is

$$\text{MSE} = \sum (S_i - M_i)^2$$

Run the simulation program, the simulation waveform obtained, and the measurement waveform fitting result is shown in Figure 3.
4. Conclusion
In this paper, the TDR system is analyzed in the frequency domain to avoid the influence of the conductivity on the TDR measurement dielectric constant under the time domain analysis, and a more accurate soil dielectric constant can be obtained. Finally, the obtained simulation waveform and the measured waveform have a high degree of matching, and the mean square error value is only 0.244, which proves the feasibility of using the simulation waveform to calibrate the soil dielectric constant. The method in this article provides a new reference for the measurement of soil dielectric constant and water content, and has certain practical significance. The calibration of the dielectric constant in this article is based on the Debye model, which has a good fitting effect for clay, and whether there is a good fitting result for other soils needs further investigation.

References
[1] Ramo, S., J. R. Whinnery, and T. vail Duzer. (2004) Fields and waves in communication electronics [D]. John Wiley&Sons, New York.
[2] M.G. Schaap, D. A. Robinsona, S. P. Friedman and A. Lazarb. (2003) Measurement and Modeling of the TDR Signal Propagation through Layered Dielectric Media [M]. Soil Science Society of America Journal 67: 1113-1121.
[3] P. Savi, I. A. Maio, I. S. Stievano. (2007) TDR response properties and their use in the estimation of soil permittivity [M]. Instrumentation and Measurement Technology Conference-IMTC 2007 Warsaw, Poland, May 1-3.
[4] Chart C. Y. and R. Knight. (2009) Laboratory measurements of electromagnetic wave velocity in layered sands [J]. Water Resour. Res. 37, 4: 1099-1105.
[5] Heimovaara, T. J., W. Bouten, and J. M. Vertraten. (2004) Frequency domain analysis of time domain reflectometry waveforms, 2, a four-component complex dielectric mixing model for soils [J]. Water Resour. Res. 30: 201-209.
[6] Jones, S. B., and D. Or. (2004) Frequency domain analysis for extending time domain reflectometry water content measurement in highly saline soils [J]. Soil. Sci. Soc. Am. J. 68: 1568. 1577.
[7] Heimovaara TJ, Bouten W, Verstraten JM. (1994) Frequency Domain Analysis of Time Domain Reflectometry Waveforms 2. A Four Component Complex Dielectric Mixing Model for Soils. Water Resources Research 30, 201-209.
[8] Whalley W. R. (2007) Considerations on the use of time domain reflectometry (TDR) for measuring soil water content [J]. J. Soil Sci. 44: 1-9.
[9] Nadler, A., S. Dasberg, and I. Lapid. (2010) Time domain reflectometry measurements of water content and electrical conductivity of layered soil columns [M]. Soil. Sci. Soc. Am. J. 55: 938-943.
[10] Chart, C. Y., and R. Knight. (2009) Determining water content and saturation from dielectric measurements in layered materials [J]. Water Resour. Res. 35:85-93.

[11] Ramo S, Whinnery JR, Van Duzer T. (1994) Fields and waves in communication electronics [D]. John Wiley, New York.

[12] Xiufu Shuai, Ole Wendroth, Caicheng Lu and Chittaranjan Ray. (2009) Reducing the Complexity of Inverse Analysis of Time Domain Reflectometry Waveforms. [M] Published in Soil Sci Soc Am J 73: 28. 36.

[13] Topp, G. C., J. L. Davis, and A. P. Annan. (1980) Electromagnetic determination of soil water content: Measurements in coaxial transmission lines [M]. Water Resour. Res. 16:574-582.