Uncertainty Relations (UR) Have Nothing to do with Quantum Mechanics (QM)

Vladimir K. Ignatovich

Joint Institute for Nuclear Research Laboratory of Neutron Physics Dubna, Russia

Abstract

Uncertainty relations are shown to have nothing specific for quantum mechanics, being the general property valid for arbitrary function. A wave function of a particle having precisely defined position and momentum in QM simultaneously is demonstrated. Interference on two slits in a screen is shown to exist in classical mechanics. A nonlinear classical system of equations replacing QM Schrödinger equation is suggested. This approach is shown to have nothing in common with Bohmian mechanics.

1 Introduction

The title of the paper is not truth, because uncertainty relations are valid in QM, however we chose this title in a protest against the widely spread belief that UR are the cornerstone of QM. The real cornerstone of QM is the Schrödinger equation, which was a great guess, like Maxwellian ones. In the next section we remind to readers how UR are proven for an arbitrary function. It follows from this proof that UR have nothing specific for QM. In the third section we show that the such notions as position and momentum are a matter of definition for an extended object like a wave function, and demonstrate that nonsingular de Broglie wave packet describes a particle, which simultaneously has precisely defined momentum and position. In the forth section we show that interference is not an exclusive property of a wave mechanics. It takes place also in classical mechanics. In fifth section we discuss whether QM equation can be replaced with classical equations. We suppose that it is possible to define a system of equations for trajectory and field of the particle, propose for mathematicians to solve an electrodynamical problem for an electron moving through a slit in a conducting screen, and show that such a system of equations is not contained in so called “Bohmian mechanics”. And in conclusion we repeat our main points.

2 What are UR

UR is a mathematical theorem which relates ranges of a function and its Fourier image. This theorem is valid in all branches of physics and mathematics dealing with extended objects described with functions. Let us remind this well known theorem.

Take an arbitrary function $f(x)$ of finite range, and its Fourier image

$$F(p) = \int_{-\infty}^{+\infty} f(x) \exp(ipx) dx,$$  \hspace{1cm} (1)
and define
\[ \int_{-\infty}^{+\infty} |f(x)|^2 \, dx = \int_{-\infty}^{+\infty} |F(p)|^2 \, dp \equiv N < \infty, \quad (2) \]
\[ x_0 = \frac{1}{N} \int x |f(x)|^2 \, dx, \quad p_0 = \frac{1}{N} \int p |F(p)|^2 \, dp, \quad (3) \]

With this function we can write the nonnegative integral
\[ \frac{1}{N} \int |(\alpha(x - x_0) + d/dx - ip_0)f(x)|^2 = \alpha^2 A + \alpha B + C \quad (4) \]
for arbitrary \( \alpha \), where
\[ A = \frac{1}{N} \int (x - x_0)^2 |f(x)|^2 \, dx = \frac{1}{N} \int (x^2 - x_0^2) |f(x)|^2 \, dx \equiv < (\Delta x)^2 > \quad (5) \]
\[ B = \frac{1}{N} \int \frac{d}{dx} |f(x)|^2 \, dx = \frac{d}{dx} \int |f(x)|^2 - \int |f(x)|^2 - \frac{1}{N} \int |f(x)|^2 = -1 \quad (6) \]
\[ C = \frac{1}{N} \int (p - p_0)^2 |F(p)|^2 \, dp = \frac{1}{N} \int (p^2 - p_0^2) |F(p)|^2 \, dp \equiv < (\Delta p)^2 >. \quad (7) \]

Since eq. (4) is nonnegative for all \( \alpha \), we have
\[ \alpha^2 < (\Delta x)^2 > -\alpha + < (\Delta p)^2 > \geq 0 \]
which is possible only for
\[ < (\Delta p)^2 > < (\Delta x)^2 > \geq \frac{1}{4}, \quad (8) \]
which is just the uncertainty relation used in QM, however it is satisfied for arbitrary function \( f(x) \), and therefore is not related specifically to QM. Thus it cannot be a cornerstone of QM. The uncertainty relation takes place in all branches of physics. For example, in classical field theory, thermodynamics, hydrodynamics, plasma. It is valid even in classical mechanics, because for functions \( x(t) \) we have UR \((\Delta \omega)^2(\Delta t)^2 \geq 1/4\).

**UR contain nothing, specific to QM. QM is only a particular case, which is very alike to classical field theory.**

### 3 Position and momentum can be defined absolutely precisely simultaneously

Since the wave function in QM defines a particle and it is an extended object, the question arises: what is a position of the extended object?

The answer to this question is: position of the extended object is the matter of definition.

In classical electrodynamics position of the electron is the singularity of its field.

In classical mechanics position of, say, a ball is the matter of definition. You may choose its center or a point, where you touch it.
For a free particle of mass $m$ in QM we can use the nonsingular de Broglie’s wave-packet [1, 2, 3]:

$$\psi = j_0(s|r - vt|) \exp(ivr - i\omega t),$$

in which $j_0(x)$ is the spherical Bessel function, $s$ is a parameter determining the width of the function, and

$$\omega = (v^2 + s^2)/2. \tag{10}$$

Here we use unities $\hbar = m = 1$, so velocity $v$ of the particle is the same as its wave-vector $k$. The function (10) is a solution of the Schrödinger equation:

$$(i\partial_t + \Delta/2)\psi = 0.$$ 

We can define position of it as the position of maximum of $|\psi|^2$ and as a momentum it’s velocity $v$. They are defined absolutely precisely simultaneously in QM.

4 Interference in classical mechanics

Let’s consider an experiment on interference on two slits in a screen, shown in fig. 1. It is usually stated that particle goes through both slits in the screen, and transmitted parts of the particle wave function interfere on the screen of observation, which is manifested by the interference pattern. However the interference can be explained purely classically with particle going through only one exactly specified slit.

Let us consider the same experiment with a classical electron, moving through one specified slit in the target screen, as is shown in fig. 2. Because of interaction of the electron field with the screen, the electron trajectory changes after the screen. Interaction of the electron field with the screen $S_t$ depends on the screen structure. In particularly, it is different when there is one or two slits. It means that the direction of propagation of the electron after $S_t$ depends on whether the second slit is opened or closed. Thus the

![Figure 1: According to standard QM wave function of a particle transmitted through both slits in target screen $S_t$ interferes after $S_t$ and gives a diffraction pattern on observation screen $S_o$.](image)
second slit interfere with electron motion, even if the electron goes precisely through the chosen upper slit.

Our considerations permit to predict the change of direction of the electron after the screen $S_t$, if we perform an experiment shown in fig. 2, where the second slit can be closed with the shutter sh. With such simple considerations we cannot predict the diffraction pattern on the screen $S_o$, shown in fig. 1, because in classical physics there are no such a parameter as wave-length, however wavelength can enter, if we take into account relativistic retardation of the interaction of electron with its own field reflected from the screen $S_t$ or introduce a quantum of action. Indeed, we can suppose that the shift of the incident electron along distance $l$ can affect the total field of the electron in presence of the screen $S_t$, and consecutively electron motion only if $pl = h$. Just at this point the quantization can enter into the classical behavior, and give such a parameter as the wave-length.

## 5 Nonlinear classical system of equations instead of QM

All the usual equations in mathematical physics can be sorted into two groups:

1. **Field equations** of the type

   \[ \hat{L}\psi(r) = j(r), \quad (11) \]

   where $\hat{L}$ is an operator, which can be linear or nonlinear in field $\psi(r)$, and $j(r)$ is a source, which can depend on some particle trajectory $r(t)$, and this trajectory is supposed to be fixed. As an example we can mention Maxwell equations with given currents, and with determined boundary conditions.
2. **Trajectory equations** of the type

\[
\frac{d^2 r}{dt^2} = F(r(t), t),
\]

(12)

where the field of force \( F(r, t) \) is fixed.

However, above, we had another type of the problem. It differs from (11) and (12). In this problem one has a trajectory equation

\[
\frac{d^2 r_p}{dt^2} = F(\psi(r_p(t), t)),
\]

(13)

with the force \( F(\psi) \), which depends on unknown field \( \psi \). The field \( \psi \) is a solution of the field equation

\[
\hat{L} \psi(r, t) = j(r, r_p(t))
\]

(14)

with the source, which depends on yet unknown solution of equation (13).

Formally we can exclude \( \psi = \hat{L}^{-1}j(r, r_p(t)) \) from the equation (13), however then we obtain highly nonlinear equation for trajectory:

\[
\frac{d^2 r_p}{dt^2} = F(\hat{L}^{-1}j(r_p(t), r_p(t'))).
\]

(15)

Solution of (15) or of the system (13,14) is the **challenge for mathematicians**.

QM avoids solution of such a nonlinear system, however reduction of nonlinear system to a linear Schrödinger equation costs probabilities instead of determinism.

However it would be very interesting to try to solve such a nonlinear system, which can be easily formulated in classical electrodynamics.

### 5.1 The problem of classical electrodynamics

We have the Maxwell equation for 4-tensor \( F_{\mu\nu} \):

\[
\partial_\mu F_{\mu\nu}(r, t) = \frac{4\pi}{c}e u_\nu \delta(r - r(t)), \quad \mu, \nu = 0 \div 3,
\]

(16)

where \( u_\nu \) is speed with components \( u_0 = c, u_k = v_k(t) \) for \( k = 1 \div 3 \). The functions \( r(t) \) and \( v(t) \) are not known and are to be determined from the other equation — the trajectory one:

\[
m \frac{dv(t)}{dt} = eE(r, t) + \frac{e}{c}[v(t)H(r, t)],
\]

where

\[
v(t) = \frac{dr(t)}{dt},
\]

and electric and magnetic fields are the components of the 4-tensor \( F_{\mu\nu} \)

\[
E_k(r, t) = F_{0k}(r, t), \quad H_k = \epsilon_{ijk}F_{ij}(r, t),
\]

which are formed by field, reflected from the target screen, and the reflection is determined by boundary conditions for the field \( F_{\mu\nu} \). The screen can be accepted to be an infinitely
thin ideal conductor. Position of slits, their width and the distance between them can be arbitrary.

For the beginning it is sufficient to solve even non relativistic, pure Coulomb problem. In the case, when there are no slits, solution in nonrelativistic limit is trivial.

We want to remark that this nonlinear system has nothing to do with Bohmian mechanics. No quantum potential is introduced, and no Schrödinger equation is presupposed. In the next section we briefly review the Bohmian mechanics.

5.2 Bohmian mechanics and hydrodynamical interpretation

There are a lot of activity on interpretation of quantum mechanics in terms of classical trajectories and quantum potential (see, for example, [4] and references there in), which are known as Bohmian mechanics or hydrodynamical interpretation. However it is not a classical version, which replaces quantum mechanics, but only an alternative way of solving the Schrödinger equation. We can find the full wave function $\psi$ by solving the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t}\psi(r,t) = \left[ -\frac{\hbar^2}{2m}\Delta + V(r) \right]\psi,$$

or represent it as $\psi(r,t) = R(r,t)\exp(iS(r)/\hbar)$, where $R(r) = |\psi(r)|$, substitute into (17), and after separation of real and imaginary parts obtain two other equations [4] for them

$$\frac{\partial R^2}{\partial t} + \nabla \left( R^2 \frac{\nabla S}{m} \right) = 0,$$

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V - \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} = 0.$$

Solution of these two equations is equivalent to solution of the single (17) equation. When you find $R$ and $S$, you can find such things as

$$Q(r,t) = \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R},$$

which you call “quantum potential”, and

$$v(r,t) = \frac{\nabla S(r,t)}{m}$$

which you call speed. If you apply $\nabla$ to Eq. (19) and use definition (21), you obtain the equation

$$m\frac{\partial v(r,t)}{\partial t} + m(v \cdot \nabla)v = -\nabla (V(r) + Q(r,t)),$$

which is equivalent to

$$\frac{dv(r,t)}{dt} = -\nabla (V(r) + Q(r,t)).$$

However $v$ is not equal to $\dot{r}(t)$, because it is a field, which depends on both $r$ and $t$.

Now, if you have already solved Eq. (17), you can consider (23) as the Newton equation and find a family of trajectories. However, in this case you arrive at the problem of finding trajectories for given field (12). It has nothing in common with the proposed classical nonlinear system of equations.
6 Conclusion

We think that the wave function $\psi$ in QM represents some kind of a field, and the force of this field can be proportional to $|\psi|^2$. Then it will explain why in QM probability for a particle to be detected is proportional to $|\psi|^2$. If $\psi$ is a field, then the position and momentum of a particle which is the source of this field can be naturally defined simultaneously, and UR do not forbid it.

7 A history of referee reports

I think the science now is very alike to a religion, and because of this there is a strong theocratic like censorship. I am sure that publication of this paper will be almost impossible. I want to start here the history of all the referee reports and my replies to them. May be it will be useful for history of science.

7.0.1 From Phys.Lett. A

Dear Dr Ignatovich,

On 05-Mar-2004 you submitted a new paper number PLA-ignatovi.AT.jinr.ru-20040305/1 to Physics Letters A entitled:

“Uncertainty Relations (UR) Have Nothing to do with Quantum Mechanics (QM)”

We regret to inform you that your paper has not been accepted for publication. Comments from the editor are:

I enclose a report on your paper. In view of the referee’s remarks I regret that your article is not suitable for publication in Physics Letters A.

Thank you for submitting your work to our journal.

_ Report_ 

1) The fact that mathematical relations having the form of the uncertainty relations occur in many fields of physics does not imply that the uncertainty relations have nothing to do with quantum mechanics, as claimed in the title.

2) The remarks of sections 2, 3 and 5, where they are correct, are trivial and well known.

3) The claim to give a classical model in section 4 is just hand-waving. It is well known that one can retain the particle trajectory in the two-slit experiment and there is a huge literature on this.

4) A reference which touches on the points raised in this paper, and deals with them with considerably more rigour, detail and consistency, is P. Holland, The Quantum Theory of Motion (Cambridge UP, 1993).

The paper should be rejected.

_ Yours sincerely, _

J.P. Vigier

Editor Physics Letters A For queries about the E-submission website please contact: authorsupport@elsevier.com For queries specific to this submission please contact: c/o phys.letters@green.oxford.ac.uk
My comment Though the report is absolutely negative and does not suppose a discussion, we try to improve our paper.

1) In the first section we included a sentence, that UR are not a corner stone for QM. The real corner stone is the Schrödinger equation, which was genuinely guessed. We explained also, why did we choose such a title.

2) According to this remark my sections 2, 3 and 5 are trivial, because there are no incorrect remarks. So, the referee agrees that the position and momentum of a particle can be defined simultaneously. In that case he must accept that UR do not forbid it. Iam grateful to the referee for his support.

3) I agree that my model in section 4 is in some respect hand waving. However I can precisely formulate a mathematical problem for solution. And I included it in the section 5. As for trajectories in Bohmian mechanics, I included a paragraph also.

4) Thanks to referee for the reference. I included another one [4], more recent, which is related to the problem and to the point 3) of the referee report.

References

[1] L. de Broglie, Non-Linear Wave Mechanics: A Causal Interpretation. (Elsevier, Amsterdam, 1960).

[2] V.K.Ignatovich, The Physics of Ultracold Neutrons (UCN). Oxford Clarendon Press. 1990.

[3] Ignatovich V.K., Utsuro M. Tentative solution of UCN problem. PL A 255 (4-6) p. 195-202 1997.

[4] A.S.Sanz, F.Borondo, and S.Miret-Artes, Particle diffraction studied using quantum trajectories. J.Phys.: Condens. Matter, 14, p. 6109-6145, 2002.