Breaking the centrifugal barrier to giant planet contraction by magnetic disc braking

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ABSTRACT

During the runaway phase of their formation, gas giants fill their gravitational spheres of influence out to Bondi or Hill radii. When runaway ends, planets shrink several orders of magnitude in radius until they are comparable in size to present-day Jupiter; in 1D models, the contraction occurs on the Kelvin–Helmholtz time-scale \(t_{\text{KH}}\), which is initially a few thousand years. However, if angular momentum is conserved, contraction cannot complete, as planets are inevitably spun up to their breakup periods \(P_{\text{break}}\). We consider how a circumplanetary disc (CPD) can de-spin a primordially magnetized gas giant and remove the centrifugal barrier, provided the disc is hot enough to couple to the magnetic field, a condition that is easier to satisfy at later times. By inferring the planet’s magnetic field from its convective cooling luminosity, we show that magnetic spin-down times are shorter than contraction times throughout post-runaway contraction:

\[
\frac{t_{\text{mag}}}{t_{\text{KH}}} \sim \left(\frac{P_{\text{break}}}{t_{\text{KH}}}\right)^{1/21} \lesssim 1.
\]

Planets can spin down until they corotate with the CPD’s magnetospheric truncation radius, at a period \(P_{\text{max}}/P_{\text{break}} \sim \left(\frac{t_{\text{KH}}}{P_{\text{break}}}\right)^{1/7}\). By the time the disc disperses, \(P_{\text{max}}/P_{\text{break}} \sim 20–30\); further contraction at fixed angular momentum can spin planets back up to \(\sim 10P_{\text{break}}\), potentially explaining observed rotation periods of giant planets and brown dwarfs.

Key words: planets and satellites: formation – planets and satellites: gaseous planets – planets and satellites: magnetic fields – planet–disc interactions

1 INTRODUCTION

Gas giants are thought to form when gas from an ambient circumstellar disc cools atop a rocky or icy core (e.g. Bodenheimer & Pollack 1986; Pollack et al. 1996; Ikoma et al. 2000). The planet’s proto-atmosphere extends to the smaller of the Bondi radius and the Hill radius, both of which are several orders of magnitude larger than present-day Jupiter. Atmospheric gas cools and contracts on a Kelvin–Helmholtz (KH) time-scale, allowing fresh nebular gas to flow in and take its place. Once the atmosphere outweighs the core, the KH time-scale begins to decrease with increasing mass, and the planet cools and grows at an ever faster, ‘runaway’ rate.

Accretion is also limited by the nebula’s ability to supply gas at a sufficient rate to the Bondi/Hill radius to keep up with the planet’s increasingly shorter cooling and contraction time. The rate of supply is capped by the radial transport rate through the disc, and further limited by the opening of annular gaps (depleted cavities) around the planet’s orbit. Both of these effects can put an end to runaway growth (Lin & Papaloizou 1993; Kley 1999; Tanigawa & Ikoma 2007; Lissauer et al. 2009; Tanigawa & Tanaka 2016; Ginzburg & Chiang 2019b; Lee 2019). Post-runaway planetary accretion is not zero; it persists up to the eventual dispersal of the disc in a few million years’ time (Mamajek 2009; Pfalzner et al. 2014), and can be responsible for doubling the planet’s mass or more (Mordasini et al. 2012; Ginzburg & Chiang 2019a).

In the post-runaway phase, because of insufficient gas supply, the planet no longer fills the Bondi/Hill radius but detaches from the nebula to contract on the KH timescale (Bodenheimer et al. 2000; Mordasini et al. 2012, 2017; Ginzburg & Chiang 2019a). Previous one-dimensional models assumed the planet is free to shrink by orders of magnitude until its radius is comparable to that of Jupiter. However, planets spin up as they contract as a consequence of angular momentum conservation. Eventually they hit up against a centrifugal barrier: when the planet spins at breakup velocity, it cannot contract further without shedding spin angular momentum. To aggravate the problem, the spin of the planet at the end of runaway growth will
already be close to breakup if it accretes mass with the Keplerian shear velocity across the Hill sphere.

Possible mechanisms to dispose of excess angular momentum include expulsion of material, once breakup is reached, into a circumplanetary disc (CPD; Ward & Canup 2010) and magnetic interaction between the planet and such a disc (Takata & Stevenson 1996). Batygin (2018) demonstrated that magnetic planet–CPD interaction can explain the observed sub-breakup spins of young extra-solar gas giants (Bryan et al. 2018). The same spin regulation mechanism can be invoked for isolated brown dwarfs and T-Tauri stars (objects without a stellar companion); in these cases the CPD and primary disc are one and the same (see Koenigl 1991; Armitage & Clarke 1996).1 While Batygin (2018) focused on the terminal rotation of planets and evolved them from an initial condition of twice the radius of Jupiter, here we expand the scope of the theory to cover earlier times, and ask whether planets can contract starting from as large a radius as the Bondi/Hill radius. We utilize the Christensen et al. (2009) scaling to relate a planet’s magnetic field to its convective luminosity and thence to its KH contraction time, thereby self-consistently evolving the planet’s radius and magnetic field over the entire duration of post-runaway accretion.

The rest of this letter is organized as follows. We describe the planetary magnetic field and its coupling to the CPD in Section 2, and calculate the planet’s spin evolution in Section 3. We compare our theory with observations of rotation periods in Section 4, and summarize in Section 5.

2 MAGNETIC COUPLING

We present a model for the magnetic interaction between a nascent planet and its CPD. The treatment is similar to that of Batygin (2018). We omit order-unity coefficients to focus on scaling relations.

2.1 Magnetic field

Christensen et al. (2009) proposed that the magnetic field strength $B$ of a planet of mass $M$ and radius $R$ is determined by equipartition of energy, with the magnetic energy density comparable to the kinetic energy density of the convective flow that transports the planet’s internal luminosity $L$:

$$B^2 \sim \rho v_{\text{conv}}^2 \sim \rho^{1/3} \left(\frac{L}{R^2}\right)^{2/3},$$

where $\rho \sim M/R^3$ is the planet’s mean density, $v_{\text{conv}}$ is the convective velocity, and $L/R^2 \sim \rho v_{\text{conv}}^3$ is the convective flux. Christensen et al. (2009) demonstrated that this scaling fits both solar system planets and rapidly rotating stars. Recent observations seem to also validate this scaling for hot Jupiters; these emit a higher internal luminosity compared to Jupiter, and are therefore expected to maintain a stronger field (Yadav & Thorngren 2017; Cauley et al. 2019).

Throughout most of the evolution discussed here, planets remain larger than $2R_J$, where $R_J$ is the radius of Jupiter (Ginzburg & Chiang 2019a). Under such conditions, electron degeneracy and electrostatic interactions are negligible, and the planets can be modelled with an ideal gas equation of state. The effects of degeneracy, which might play a role during the latest stages of contraction, are discussed in the appendix. For non-degenerate objects, the KH time-scale is given by

$$t_{\text{KH}} \sim \frac{GM^2}{RL},$$

where $G$ is the gravitational constant. We rewrite equation (1) as

$$B \sim \left(\frac{GM^2}{R^4}\right)^{1/2} \left(\frac{P_{\text{break}}}{t_{\text{KH}}}\right)^{1/3},$$

with $P_{\text{break}} \sim (GM/R^3)^{-1/2}$ denoting the planet’s breakup rotation period.

2.2 Truncation radius

If the magnetic field is strong enough, the CPD does not extend all the way to the planet’s surface. Close enough to the planet, the magnetic energy density exceeds the kinetic energy density of the accretion flow, truncating the disc at an inner radius

$$R_t \sim \left(\frac{\mu^4}{GMM^2}\right)^{1/7},$$

where $\mu = BR^3$ is the planet’s magnetic dipole moment and $M$ is the mass accretion rate (Elsner & Lamb 1977; Ghosh & Lamb 1979; Koenigl 1991; Ostriker & Shu 1995; Mohanty & Shu 2008). Equation (4) is appropriate for spherically symmetric accretion, or for accretion through an equatorial disc in which the flow transitions from Keplerian to magnetospheric over a radial length scale comparable to the disc radius (e.g. Shapiro & Teukolsky 1983, section 15.2). The geometry of our problem is different; three-dimensional simulations of planets embedded in circumstellar discs find that a planet accretes mainly through its poles, and forms a decretion disc at the equator (Tanigawa et al. 2012; Morbidelli et al. 2014; Fung et al. 2015; Fung & Chiang 2016; Szulágyi et al. 2016). We ignore these complications and assume that equation (4) gives the truncation radius of the equatorial decetion disc. The same assumption was made by Batygin (2018). If nothing else, the dipole geometry of the magnetic field, whose energy density dominates at $r < R_t$, should help to enforce a roughly spherical magnetosphere.

In Ginzburg & Chiang (2019a) we explained that during post-runaway accretion the planet adjusts its contraction to satisfy $t_{\text{KH}} = M/\dot{M}$. This condition was derived neglecting spin angular momentum; however, we will find below that magnetic spin-down is efficient and that contraction can proceed on the KH time-scale, with the planet radiating away the accretion luminosity $L = GMM/R$. Using this relation, and substituting $B$ from equation (3), we rewrite equation

1 Magnetic braking also occurs when spin angular momentum is carried away by a magnetized wind emanating from the object. Wind braking seems too weak to explain the slow rotation of low-mass stars and brown dwarfs that are a few Myr old (Kawaler 1988; Bouvier et al. 2014; Moore et al. 2019).
magneto-rotational turbulence; e.g. Fleming et al. 2000), and a larger one which replaces the disc scale height with the disc radius (this \( R_m \) was used by Turner & Sano 2008 to gauge whether toroidal fields could be generated by disc Keplerian shear from radial fields).

\[ \frac{R}{R} \sim \left( \frac{t}{t_{KH}} \right)^{2/21} > 1. \]

The magnetic field always truncates the disc \( (R_t > R) \) in our case because the thermal time-scale \( t_{KH} \) is orders of magnitude longer than \( P_{break} \) (see Section 3).

### 2.3 Magnetic torque

Magnetic field lines that originate from the planet rotate at its spin frequency \( \omega \) and puncture the CPD. At any given radius \( r \) in the disc, if the Keplerian orbital angular velocity \( (GM/R^3)^{1/2} \) differs from \( \omega \), the field lines are twisted by the disc, generating a counter-torque. The total torque that the disc exerts on the planet is given by

\[ T \sim \int R_k \mu^2 dR \sim B^2 R^3 \left( \frac{R}{R_k} \right)^3. \]

(6)

(for details, see Livio & Pringle 1992; Armitage & Clarke 1996; Spalding & Batygin 2015; Batygin 2018). The magnitude and sign of the torque are dictated by the inner edge of the disc at \( R_k \). If the inner edge rotates slower (faster) than the planet, the torque will spin the planet down (up). Following Batygin (2018), we assume that the CPD is connected to the larger circumsellar disc such that whatever angular momentum (of whatever sign) is transferred from the planet to the CPD is subsequently transferred to the nebula at large. In equilibrium, the planet nearly corotates with the disc’s edge: \( \omega^2 \sim GM/R_k^3 \).

### 2.4 Electrical conductivity

We have assumed in the above that the disc is sufficiently electrically conductive that it couples to the planetary magnetosphere. A measure of the coupling is the magnetic Reynolds number \( R_m \) which needs to be \( \gg 1 \) (otherwise, field lines diffuse rather than advect and twist under the Keplerian disc flow). Batygin (2018) evaluated \( R_m \) using the Keplerian velocity and the disc scale height (their equation 4). \(^2\) We will adopt this same definition, and use their result that \( R_m > 1 \) when the temperature \( T > 750 \) K, which follows from thermal ionization of trace alkali metals (their fig. 1). This prescription overestimates the electrical conductivity insofar as charge-adsorbing dust grains are ignored, but also underestimates the conductivity because it neglects non-thermal sources of ionization, e.g. ultraviolet radiation. For simplicity we take \( T \) to be that of the planet’s photosphere. Our value of \( T \) may be an underestimate because it ignores other sources of heating such as shocks or dissipative

\[^2\) This choice yields a value intermediate between other choices for \( R_m \): a smaller one which replaces the Keplerian velocity with the disc sound speed (this \( R_m \) is typically used to gauge whether the disc can sustain magneto-rotational turbulence; e.g. Fleming et al. 2000), and a larger one which replaces the disc scale height with the disc radius (this \( R_m \) was used by Turner & Sano 2008 to gauge whether toroidal fields could be generated by disc Keplerian shear from radial fields).
of possible evolutions, Fig. 1 demonstrates that our conclusions for spin appear robust. Despite a wide range of possible magnetic field strengths, the magnetic spin-down time $t_{\text{mag}}$ is nominally always shorter than the KH cooling time $t_{\text{KH}}$. As the planet contracts, its breakup period $P_{\text{break}}$ decreases, whereas $t_{\text{KH}}$ increases. Thus $P_{\text{break}}/t_{\text{KH}}$ decreases with time, and by extension so does $t_{\text{mag}}/t_{\text{KH}}$ by equation (8).

How much slower does the planet spin relative to breakup? As discussed in Section 2.3, planets seek an equilibrium where they corotate with the disc’s magnetic truncation radius $R$. Using equation (5), corotation sets a limiting rotation period given by

$$P_{\text{max}}/P_{\text{break}} \sim \left( \frac{R_t}{R} \right)^{3/2} \sim \left( \frac{t_{\text{KH}}}{t_{\text{break}}} \right)^{1/7}$$

as plotted in the bottom panel of Fig. 1. The time-scale to establish corotation is comparable to $t_{\text{mag}}$ if the planet starts at breakup, and shorter than $t_{\text{mag}}$ if it starts slower than breakup. We argued from equation (8) that $t_{\text{mag}}/t_{\text{KH}} < 1$, which would imply that planets maintain corotation as they contract. However, the margin by which $t_{\text{mag}}$ is less than $t_{\text{KH}}$ is small and uncertain (owing to the weak exponent in equation 8 and the unknown coefficient), and so we allow for the possibility that corotation equilibrium might not quite be reached. Thus it is safer to regard $P_{\text{max}}$ as an upper limit on the true spin period (and $P_{\text{break}}$ as a lower limit). By the end of the disc’s lifetime, when $t_{\text{KH}} = t_{\text{disc}} = 3$ Myr (e.g. Ginzburg & Chiang 2019a), planets rotate up to 20–30 times slower than breakup.

### 4 OBSERVATIONS

Bryan et al. (2018) measured the rotation periods of several directly-imaged planets and low-mass brown dwarfs, compiling a sample that included theirs and previous observations. These objects have ages of 2–300 Myr, and their parent discs have all completely dissipated. We plot the rotation periods and masses of the Bryan et al. (2018) sample in Fig. 2, overlaying our theoretical estimates of the maximal rotation period $P_{\text{max}}$ relative to the breakup period $P_{\text{break}}$. Note that $P_{\text{break}} \propto R^{1/2}$, and the radius $R$ at the time of disc dispersal depends on atmospheric opacity; the theoretical curves in Fig. 2 employ two possible radii, 2 $R_\odot$ for dust-free atmospheres, and 7 $R_\odot$ for dusty ones (Ginzburg & Chiang 2019a).

Fig. 2 demonstrates that real-life gas giants and brown dwarfs respect the upper bound on spin periods set by magnetic coupling to CPDs. If we imagine that planets start at $P/P_{\text{break}} \sim P_{\text{max}}/P_{\text{break}} \approx 20–30$ at the time of disc dispersal, and subsequently preserve their spin angular momentum while contracting and spinning up as $P/P_{\text{break}} \propto R^{1/2}$, then the final observed $P/P_{\text{break}}$ will drop from its initial value by a factor of a few. This scenario may explain why many of the data points appear to cluster around $P/P_{\text{break}} \approx 10$.

Whether there is a trend in $P/P_{\text{break}}$ with $M$ is unclear. Bryan et al. (2018) did not find any of statistical significance. Fig. 2 gives the same impression, especially if we omit the one point with the lowest mass—and note that there is an observational bias against detecting slowly rotating, low-mass objects, as these will be among the faintest, with atmospheric lines hardest to spectrally resolve. The absence of a correlation appears consistent with theory; the 1/7 power
in equation (9) flattens all trends. More data are needed at low $M$ to see whether this agreement continues to hold.

5 SUMMARY

During the initial cooling-limited phase of its growth, a nascent gas giant fills its gravitational sphere of influence (out to the Bondi or Hill radius) as it accumulates mass at its periphery faster than it can contract. Accretion in this phase eventually runs away with the increasing self-gravity of the gaseous envelope. When runaway growth ends—because of gaps opened by the planet in its parent disc, and limitations in the rate at which the disc can transport mass—the planet’s radius is finally free to begin contracting on the Kelvin–Helmholtz cooling time-scale $t_{KH}$, which is initially as short as a few $10^3$ years. During this post-runaway phase, accretion continues even as the planet shrinks (Mordasini et al. 2012; Ginzburg & Chiang 2019a). The radius must decrease by several orders of magnitude before it attains the present-day observed value for Jupiter. The problem is that without a mechanism to remove the planet’s spin angular momentum, contraction stalls as the planet is inevitably spun up to breakup speed.

In this letter we considered magnetic coupling of the planet to a circumplanetary disc (CPD) as a means to shed angular momentum and spin the planet down (Takata & Stevenson 1996; Batygin 2018). New theoretical arguments and observations link the planet’s magnetic field to its luminosity (Christensen et al. 2009; Yadav & Thorngren 2017; Cauley et al. 2019) and thereby to $t_{KH}$. These connections enabled us to consistently compare the magnetic spin-down time $t_{mag}/t_{KH}$ throughout the planet’s contraction history, and to extend the calculation of Batygin (2018) to earlier times, when the planet is much larger than present-day Jupiter. The theory applies also for objects without a stellar host (e.g. isolated brown dwarfs); the only requirement is that the object be surrounded by a disc that can shuttle angular momentum away to large distance.

We found that at any given time during contraction, $t_{mag}/t_{KH} \propto (P_{\text{break}}/t_{KH})^{1/2}$, where $P_{\text{break}}$ is the breakup rotation period. Since $P_{\text{break}}/t_{KH} \ll 1$, $t_{mag}/t_{KH} \ll 1$, indicating that planets are marginally able to shed their angular momentum while contracting on the Kelvin–Helmholtz time-scale. That is, the angular momentum barrier to planetary contraction can be largely removed by CPDs interacting with primordially strong planetary magnetic fields. An underlying assumption is that the CPD is sufficiently ionized to couple to the planet’s magnetosphere; this assumption might fail at early times, especially for low-mass planets.

In addition to justifying the results of previous studies of giant planet formation that did not explicitly consider spin, we also calculated how the rotation periods of gas giants evolve during post-runaway accretion. We found that planets rotate slower than breakup, with a maximal rotation period given by $P_{\text{max}}/P_{\text{break}} \sim (t_{KH}/t_{KH})^{1/7}$. This ratio gradually increases with time as long as the CPD can transfer angular momentum from the planet to the nebula at large. When the parent disc finally expires at a time $t_{\text{disc}} \sim t_{KH} \sim 3$ Myr, $P_{\text{max}}/P_{\text{break}} \approx 20–30$ in a variety of post-runaway models (Fig. 1). Contraction at fixed angular momentum after the disc vanishes may spin planets back up to $P_{\text{max}}/P_{\text{break}} \sim 10$, potentially explaining observed rotation periods of young planets and low-mass brown dwarfs (Fig. 2).

While we have focused on the magnitude of planetary spin, future work can examine the direction of the spin vector, i.e. the relative orientations of CPDs and planetary magnetic/spin axes (Lai et al. 2011; Spalding & Batygin 2015). The CPD might not necessarily be aligned with the parent circumstellar disc, particularly in turbulent discs. Obliquities of sub-stellar objects are beginning to be measured (Bryan et al. 2019).

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Figure 2. Rotation periods relative to breakup of young substellar companions (blue squares) and isolated brown dwarfs (red triangles), taken from Bryan et al. (2018) with updated error bars (M. Bryan 2019, personal communication). The sloped dashed black lines show the longest rotation periods $P_{\text{max}}$, normalized to breakup periods $P_{\text{break}}$, to which objects may be spun down by magnetic coupling to circumplanetary (or circum-brown dwarf) discs. The ratio $P_{\text{max}}/P_{\text{break}}$, given by equation (9) with $t_{KH}$ equated to the disc lifetime $t_{\text{disc}} = 3$ Myr, is evaluated for two values of the object’s radius $R \approx 2R_J, 7R_J$; these give different breakup periods at the time of disc dispersal ($P_{\text{break}} \propto R^{1/2}$), with the smaller radius corresponding to contraction of a relatively low-opacity dust-free atmosphere. After the disc dissipates, continued contraction at fixed angular momentum lowers $P/P_{\text{break}} \propto R^{1/2}$ by a factor of a few below the sloped black lines, possibly explaining why the observations cluster there.

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APPENDIX A: DEGENERATE PLANETS

Electron degeneracy sets a floor on the radius $R_0 \approx R_1$ below which planets cannot contract (e.g., Zapolsky & Salpeter 1969). We therefore have to correct our scaling laws for the regime $R \lesssim 2R_0$, or equivalently $\Delta R/R_0 \lesssim 1$, where $\Delta R \equiv R - R_0$. This regime, in which degeneracy can no longer be neglected, is relevant for the late stages of contraction in the case of low opacities (see fig. 3 of Ginzburg & Chiang 2019a). The cooling time in this regime is given by

$$t_{KH} \sim \frac{G M^2}{R_0 L} \frac{\Delta R}{R_0}, \quad (A1)$$

which replaces equation (2). In addition, degenerate planets need lose only a fraction $\sim \Delta R/R_0$ of their angular momentum to contract. Taking these two differences into account, equation (8) is replaced by

$$\frac{P_{\text{mag}}}{t_{KH}} \sim \left(\frac{R_0}{R}\right)^{1/21} \frac{\Delta R}{R_0}^{19/21} < 1. \quad (A2)$$

We conclude that for a given cooling time $t_{KH}$ (in practice equal to the age of the system), degenerate planets spin down faster. However, using equations (A1), (1), and (4), their equilibrium rotation periods are closer to breakup:

$$\frac{P_{\text{max}}}{P_{\text{break}}} \sim \left(\frac{R_0}{R}\right)^{3/2} \left(\frac{t_{KH}}{P_{\text{break}}}\right)^{17/7} \left(\frac{\Delta R}{R_0}\right)^{2/7}, \quad (A3)$$

which replaces equation (9).

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