Simulation of Snow Mass Movement on the Roof of Cylindrical Shells

A Siyanov

Technical Institute, North-Eastern Federal University, St. Kravchenko 16, Neryungri, 678960, Republic of Sakha (Yakutia), Russia

E-mail: vntusiyanov@gmail.com

Abstract. A simulation was performed and a method was developed for determining the kinematic and dynamic characteristics of the elements of a mobile snow mass at the stage of their movement on the surface and at the moment of separation from the roof of cylindrical mesh coverings. The developed method allows us to determine the pressure of the snow mass on the shell, calculate the acceleration of the snow elements of the array at the time of sliding off the roof and calculate the time of separation of snow elements from the surface of the shell. It is possible to calculate the speed and value of the angle at which an element of the snow mass is separated from the roof and falls along a ballistic curve.

1. Introduction
Modern numerical calculations use different models of snow mass movement [1-7]. However, in the complex, none of them takes into account its condition and the geometry of the roof. Cases of smooth and homogeneous surfaces are usually the basis for constructing widely used mathematical models [8]. It is possible that the snow layers are displaced relative to each other and there are minor surface deformations [4, 9]. Some researchers take into account certain parameters that characterize the properties of the snow mass and the surface of the coating and change over time depending on climatic factors [1, 5, 10-13]. The snow mass movement model is characterized by disorder and properties of a loose heterogeneous medium, liquid and solid bodies [14]. Taking into account additional circumstances and these factors, it is necessary to preserve the shape of the structure and mitigate the avalanche danger [15, 16]. Therefore, determining the parameters of the movement of snow mass from the roof is of significant practical importance.

2. Background for modeling
To perceive the snow load, take a cylindrical shell with a smooth uniform roof without taking into account defects. The layer of snow mass in comparison with the size of the shell is characterized by a small thickness and without interference orderly moves on the surface. Under this condition, it is obvious that the movement of the elements of the snow mass has little effect. Based on this, we use the model of an idealized loose medium.

3. Ratio of parameters
Consider the problem of moving a netted snow mass on a smooth cylindrical roof under the action of gravity. The width of the calculated layer is 1 m. The snow mass is characterized by a loose state with
unbound elements, the movement of which is independent and does not violate the integrity of the layer. The distributed mass of the stationary snow cover at the initial moment of time \( t = 0 \)

\[
m_{s,i}(\alpha_{s,i}) = \gamma \cdot m_{s,i}^{\text{max}} \cdot \mu_2 (\alpha_{s,i}),
\]

where \( \alpha_{s,i} \) – angular coordinate of the element of the snow array in the initial (stationary) state; \( m_{s,i}(\alpha_{s,i}) \) – function of mass distribution on the roof of the shell in a stationary state; \( m_{s,i}^{\text{max}} \) – the maximum value of the snow mass distribution at the lower point of the array; \( \gamma \) – dimensionless coefficient; \( \mu_2 \) – dimensionless function of the law of distribution of snow mass in the initial state.

In Fig. 1 shows a fragment of the shell surface with geometric parameters.

![Figure 1. Geometric scheme of the snow mass in the initial state.](image1)

Given the value of a dimensionless function, one can write

\[
m_{s}(\alpha) = \gamma \cdot m_{s}^{\text{max}} \cdot 2.4 \sin(1.4 \cdot \alpha) ; \quad m_{i}(\alpha) = 0.
\]

Elements of the snow mass move at different speeds, which leads to an increase in the angular size \( \Delta \alpha_i \) and weight loss \( m_i(\alpha_i, t) \) (show in Figure 2).

![Figure 2. Geometric scheme of the snow mass at the moment \( t_d \) of separation of the element.](image2)
When the element reaches the speed limit \( v_{d,i} \), that is the centrifugal force of inertia, what acts on it becomes an equal component of gravity, which is normal to the shell surface

\[ \Delta m_i v_i^2 / R = \Delta m_i g \cos \alpha_i n_i. \] (3)

Then the element is out of contact with the surface of the roof, which provides it with free fall until it collides with the ground surface, a specially provided obstacle or surrounding object.

4. Materials and methods of research

Modeling methods are used to establish the kinematic and dynamic characteristics of all elements of the snow mass at the stages of their movement on the surface of the shell and free fall.

It is necessary to calculate the normal and tangential components of the forces of their impact on the roof and the horizontal and vertical components of their impact on possible surrounding objects or special obstacles.

All components of the forces acting on the shell are shown in Figure 3.

![Figure 3. Model of the dynamic state of a snow array element.](image)

After a series of transformations found

\[ \Delta m_i = m_i(\alpha_i, t) \cdot \Delta m_{ij}(t) = m_i(\alpha_i, t) \cdot R \Delta \alpha_i(t). \] (4)

The study of the motion of a mass element \( \Delta m_i \) is carried out within the framework of the theorem on the change of its kinetic energy in the differential form

\[ \frac{\partial T}{\partial t} = F^i + F^e, \] (5)

where \( T \) – kinetic energy of the mass element \( \Delta m_i \); \( F^i \) – power of internal forces; \( F^e \) – power of external forces.

5. Results and discussion

Since the element is under the influence of gravity and normal pressure...
\[ F_{gr}^e = \Delta m \cdot g \cdot v_i (\alpha, t) \; ; \; F_{fr}^e = \Delta f_{fr} \cdot v_i (\alpha, t), \]  
\[ F_i^e = F_{gr}^e + F_{fr}^e. \]

where \( F_{gr}^e, F_{fr}^e \) – the power of gravity and friction, respectively; \( g \) – the vector of gravity acceleration; \( v_i (\alpha, t) \) – the velocity vector of the element; \( \Delta f_{fr} \) – the vector of friction forces.

Taking into account the coefficient of friction \( \mu \) between the snow and the surface of the shell

\[ F_i^e = \Delta m_i \cdot g \cdot \sin \alpha_i \cdot v_i - \mu \cdot \Delta m_i \left( g \cdot \cos \alpha_i = \frac{v_i^2}{R} \right) v_i. \]

Then the kinetic energy of the snow mass element will have the form

\[ T = \Delta m_i \cdot \frac{v_i^2}{2}. \]

Using a series of substitutions in equation (2), one can find

\[ \frac{dv_i}{dt} = g \cdot \sin \alpha_i - \mu (g \cdot \cos \alpha_i = \frac{v_i^2}{R}). \]

The expression for determining the speed is known and has the form

\[ v_i = R \cdot \dot{\alpha}_i. \]

Then the equation of motion of the mass element

\[ \dot{\alpha}_i = \frac{g}{R} \sin \alpha_i - \frac{\mu}{R} \left[ g \cdot \cos \alpha_i - R \cdot (\dot{\alpha}_i)^2 \right], \]

with the specified initial conditions

\[ \alpha_i (0) = \alpha_{i,0}, \; \dot{\alpha}_i (0) = 0. \]

The Runge-Kutta numerical method is suitable for integrating.

The solution of the equation (12) with the conditions (13) is carried out until the moment \( t_{dl} \), when, under the action of the force \( F_{cf} \), the element comes out of contact with the surface of the shell and makes a fall along the ballistic curve.

The condition for the onset of such a state has the form

\[ \Delta m_i g \cdot \cos \alpha_i = \Delta m_i \cdot \frac{v_i^2}{R}, \]

where the action of the centrifugal force of inertia is described by the expression

\[ F_{cf} = \Delta m_i \cdot \frac{v_i^2}{R}. \]
The obtained ratios of parameters reflect the behavior of the snow mass when it moves on the surface of cylindrical shells. The first stage of the movement is considered, the importance of which is of practical importance in the process of designing large-span structures. In the future, it is necessary to obtain the ratios of parameters for the second and third stages, which respectively characterize the free fall and the impact of the snow mass on the surrounding objects [17-21].

6. Conclusions
Theoretical modeling was carried out and a method was developed for determining the kinematic and dynamic characteristics of the elements of the mobile snow mass at the stage of their movement on the surface of the cylindrical shell. The developed method allows us to determine the pressure of the snow mass on the shell, calculate the acceleration of the snow elements of the array at the time of sliding off the roof and calculate the time of separation of snow elements from the surface of the shell.

7. References
[1] Fischer J-T, Kofler A, Fellin W, Granig M and Kleemayr K 2015 Multivariate parameter optimization for computational snow avalanche simulation Journal of Glaciology 61/229 pp 875–888
[2] Boutanios Z and Jasak H 2017 Two-way coupled Eulerian-Eulerian simulations of drifting snow with viscous treatment of the snow phase Journal of Wind Engineering and Industrial Aerodynamics 169 pp 67–76
[3] Okaze T, Niiya H and Nishimura K 2018 Development of a large-eddy simulation coupled with Lagrangian snow transport model Journal of Wind Engineering and Industrial Aerodynamics 183 pp 35–43
[4] Wever N, Fierz C, Mitterer C, Hirashima H and Lehning M 2014 Solving Richards Equation for snow improves snowpack meltwater runoff estimations in detailed multi-layer snowpack model The Cryosphere 8 pp 257–274
[5] Wever N, Valero C V and Fierz C 2016 Assessing wet snow avalanche activity using detailed physics based snowpack simulations Geophysical Research Letters 43(11) pp 5732–40
[6] Bartelt P and Buser O 2018 Avalanche dynamics by Newton. Reply to comments on avalanche flow models based on the concept of random kinetic energy Journal of Glaciology 64(243) pp 1–6
[7] Zhou X, Zhang Y, Kang L and Gu M 2019 CFD simulation of snow redistribution on gable roofs: Impact of roof slope Journal of Wind Engineering and Industrial Aerodynamics 185 pp 16–32
[8] Wang J, Liu H, Chen Z and Ma K 2019 Probability-based modeling and wind tunnel test of snow distribution on a stepped flat roof Cold Regions Science and Technology 163 pp 98–107
[9] Cao Z, Liu M and Wu P 2019 Experiment Investigation and Numerical Simulation of Snowdrift on a Typical Large-Span Retractable Roof Hindawi Complexity pp. 1–14
[10] Buser O and Bartelt P 2015 An energy-based method to calculate streamwise density variations in snow avalanches Journal of Glaciology 61(227) pp 563–575
[11] Wang J, Liu H, Xu D, Chen Z and Ma K 2019 Modeling snowdrift on roofs using Immersed Boundary Method and wind tunnel test Building and Environment 160 pp 16–32
[12] Cherepanov G P 2019 Snow Avalanches. In: Invariant Integrals in Physics pp 197–214
[13] Sun X, He R and Wu Y 2018 Numerical simulation of snowdrift on a membrane roof and the mechanical performance under snow loads Cold Regions Science and Technology 150 pp 15–24
[14] Wang W, Liao H, Li M and Huang H 2013 Similarity Study on Snowdrift Wind Tunnel Test Open Journal of Civil Engineering 3 pp 13–17
[15] Naaim M, Durand Y, Eckert N and Chambon G 2013 Dense avalanche friction coefficients: influence of physical properties of snow Journal of Glaciology 59 pp 771–782
[16] De Biagi V, Chiaia B and Frigo B 2015 Impact of snow avalanche on buildings: Forces estimation from structural back-analyses Engineering Structures 92 pp 15–28
[17] Liu M, Zhang Q, Fan F and Shen S 2018 Experiments on natural snow distribution around simplified building models based on open air snow-wind combined experimental facility Journal of Wind Engineering and Industrial Aerodynamics 173 pp 1–13

[18] Flaga A, Bosak G, Pistol A and Flaga L 2019 Wind tunnel model tests of snow precipitation and redistribution on rooftops, terraces and in the vicinity of high-rise buildings Archives of Civil and Mechanical Engineering 19/4 pp 1295–03

[19] Tominaga Y 2018 Computational fluid dynamics simulation of snowdrift around buildings: Past achievements and future perspectives Cold Regions Science and Technology 150 pp 2–14

[20] Yan K, Cheng T and Zhang Y 2018 A new method in measuring the velocity profile surrounding a fence structure considering snow effects Measurement 116 pp 373–381

[21] Hashim H M, Sokolova E, Derevianko O, Solovev D B 2018 Cooling Load Calculations IOP Conference Series: Materials Science and Engineering 463 Paper № 032030. [Online]. Available: https://doi.org/10.1088/1757-899X/463/3/032030

Acknowledgments
This research paper is of applied significance and was carried on the program to improve the competitiveness of North-Eastern Federal University among the world's leading research and education centres in the 2020-2024.