A Computation Offloading Model over Collaborative Cloud-Edge Networks with Optimal Transport Theory

Zhuo Li\textsuperscript{1,2}, Xu Zhou\textsuperscript{1}, Yang Liu\textsuperscript{3}, Congshan Fan\textsuperscript{1}, Wei Wang\textsuperscript{4}

\textsuperscript{1}Computer Network Information Center, Chinese Academy of Sciences, Beijing 100190, China
\textsuperscript{2}University of Chinese Academy of Sciences, Beijing 100049, China
\textsuperscript{3}School of Mathematical Sciences, Beijing Normal University, Beijing 100875, China
\textsuperscript{4}Knet Technologies Co. Ltd., Beijing 100190, China
{lizhuo, zhouxu, fcs, wangwei}@cnic.cn, liu.yang@mail.bnu.edu.cn

Abstract—As novel applications spring up in future network scenarios, the requirements on network service capabilities for differentiated services or burst services are diverse. Aiming at the research of collaborative computing and resource allocation in edge scenarios, migrating computing tasks to the edge and cloud for computing requires a comprehensive consideration of energy consumption, bandwidth, and delay. Our paper proposes a collaboration mechanism based on computation offloading, which is flexible and customizable to meet the diversified requirements of differentiated networks. This mechanism handles the terminal's differentiated computing tasks by establishing a collaborative computation offloading model between the cloud server and edge server. Experiments show that our method has more significant improvements over regular optimization algorithms, including reducing the execution time of computing tasks, improving the utilization of server resources, and decreasing the terminal's energy consumption.

Index Terms—computation offloading, computational optimal transport, cloud computing, edge computing

I. INTRODUCTION

Technological innovation represented by the 5th generation of communication (5G) and Artificial Intelligence (AI) has brought about the booming of emerging industries, including the Industrial Internet of Things (IIoT), Internet of Vehicles (IoV), and AR/VR. Starting from the traditional PC era dominated by stand-alone applications, the Internet has entered the era of mobile web, and it is about to face the era of the Internet of Everything (IoE) \cite{1}.

With the widespread use of the terminal, mobile data that needs to be processed has exploded, while the computing power, memory scale, and battery capacity of the terminal are minimal. Cloud computing brought about by the development of wireless communication technologies such as 3G, 4G, and Wi-Fi effectively solves the above challenges by plentiful computing resources and more substantial computing capacity. Computation offloading through offloading computationally intensive tasks to the cloud data center to improve the utilization of cloud server resources and reduce the energy consumption of the terminal \cite{2}. However, the emergence of new edge applications such as telemedicine, autonomous driving, unmanned aerial vehicle (UAV), and other delay-sensitive tasks put forward higher demands on the response delay in the service \cite{3}. With the trend of network services migrating to the edge, edge computing emerges as a critical technology of the 5G network architecture. It satisfies the three characteristics of high-speed, large-capacity, and low-latency of the 5G network, and distributes resources to provide services of computing, communication, control, and storage on the user side or nearby edge devices and systems \cite{4}.

Because of the mode of cloud computation offloading (CCO) is extended to the edge network, edge computation offloading (ECO) reduces the significant transmission delay and energy consumption caused by offloading tasks to remote cloud data centers \cite{2, 5}. Existing offloading strategies focus on using cloud computing or edge computing to offload tasks without considering collaboration between them, whose optimization goal is restricted by terminal equipment, energy consumption, or response delay. With the rapid development of battery storage technology, the impact of energy consumption is gradually decreasing, whereas computing power is playing an increasingly important role in processing tasks.

Therefore, the research goal of this paper is to introduce the collaboration between cloud computing and edge computing, establish a computation offloading model, and perform different offloading tasks in combination with offloading requirements and the use of server resources. The primary contributions of this paper are:

- We proposed a computation model offloading over a collaborative cloud-edge network, which takes full account of offloading requirements and server resource utilization.
- We creatively introduced the classic transmission optimization problem into the design of the offloading strategy. The convex optimization problem can get a unique and optimal solution.
- We analyzed the system’s total energy consumption, the success rate of offloading, and the server resource usage when using the collaborative offloading strategy.

We organize the rest of the paper as follows. In the next
section, we review background and related works on cloud computing and edge computing and computation offloading. Section III and IV present the system model of computation offloading and the major algorithms used in the collaborative system. It conducts evaluation and analysis of algorithms in Section V, and we conclude the paper in Section VI.

II. RELATED WORKS

This section provides details of the current work in the cloud and edge computing and computation offloading.

A. Cloud Computing and Edge Computing

Because of the terminal limitations in computation and storage resources, many applications have reduced the quality of users’ experience, while cloud computing in a mobile environment can provide enhanced computation capabilities and reduce computing latency [6]. Cloud computing adopts the migration of computing-intensive applications such as video and games to a cloud server to solve the problem of insufficient device resources and provides many advantages for mobile devices by migrating the computation and storage requirements of tasks from restricted terminal to the free cloud server. Although many instances under cloud computing can improve execution speed and reduce the energy consumption of devices, cloud computing also brings immense latency problems that latency-sensitive applications cannot execute. The authors in [7] proposed edge computing and defined it as a model for performing calculations on the network, where edge refers to any computing and network resource from the terminal to the cloud data center. Therefore, edge computing can significantly reduce latency and jitter when compared to cloud computing. Table I summarizes the characteristics of cloud and edge computing in different indicators. Resources near the terminal can no longer be limited by resources when computing tasks while using edge computing alone cannot adequately meet users’ massive offloading requests. Edge computing should not wholly replace cloud computing, and the two should complement each other to meet the offloading requests of computing tasks better.

B. Computation Offloading

Computation offloading is a technology of assigning a sizeable amount of computation to a computation node with sufficient resources for processing and then retrieving the computing results from the computation node. As one of the critical technologies of edge computing, computation offloading is mainly divided into offloading decision and resource allocation. Researchers focus on whether to offload tasks, how many tasks will offload, and where to offload tasks from the perspective of the offloading decision. From the perspective of resource allocation, communication and computing resources will be allocated after the offloading decision is completed, which needs to consider the transmission delay and terminals’ energy consumption caused by task offloading.

Aiming at the problem of multi-user computation offloading, the authors in [8] formulated the dynamic optimization problem into an infinite-horizon average-reward continuous-time Markov decision process (CTMDP) model. They proposed a joint computation offloading and multi-user scheduling algorithm that minimizes the long-term average weighted sum of delay and power consumption under stochastic traffic arrival. Kuang et al. [9] studied the problem of multi-user computation offloading and divided mobile users into active and inactive categories. The dynamic offloading decision process of mobile users was regarded as a random game, and an effective multi-agent random learning algorithm was proposed based on this. However, the above two algorithms only consider the edge server’s computing resources and do not pay attention to the more substantial cloud data center.

In terms of centralized cloud computation offloading, Guo et al. [10] proposed a location-aware offloading scheme in a two-layer cloud environment. The cloud environment includes edge servers and a centralized cloud server. It is necessary to choose an offloading strategy for mobile devices that can guarantee the quality of service and save the energy consumption of the equipment, and define it as a constrained optimization problem, and minimize the total energy consumption of all mobile devices as the optimization goal, as giving a centralized approximate algorithm and distributed collaborative algorithm.

Most of the documents, as mentioned above, aim at minimizing the energy consumption of mobile devices. It only gives the processing time of computing tasks a threshold. Few documents individually optimize it, but the processing time of computing tasks is a vital indicator of assessing the Quality of Service (QoS), especially for differentiated services and unexpected services. This article considers the importance of task processing time and minimizes the total processing time of all computing tasks as the optimization goal.

III. SYSTEM MODEL AND PROBLEM FORMULATION

We consider an offloading system over collaborative cloud-edge networks, whose model is shown in Fig. 1. The remote cloud server is connected to the base station that provides wireless access points in cellular networks through a wired connection. At the same time, numerous edge servers are deployed near the base stations. When the terminal generates a continuous stream of computation tasks and can compute some simple or necessary tasks that cannot be offloading according to its own capabilities, the remote cloud servers and edge servers allocate their own resources to process computing tasks

| Indicators            | Cloud computing | Edge computing |
|-----------------------|-----------------|----------------|
| Deployment            | Centralized     | Distributed    |
| Server location       | Cloud center    | Edge network   |
| Computing capacity    | Unlimited       | Restricted     |
| Storage capacity      | Unlimited       | Restricted     |
| Network access        | Wired connections| Wireless connections|
| Processing delay      | Higher          | Lower          |
| Distance              | Remote          | Nearby         |
| Scenario              | Compute-intensive| Latency-intensive|
computing tasks on edge servers, which can avoid the long transmission delay of cloud computing.

B. Computation Model

For the computing tasks generated by the terminal equipment, the network requirements are the amount of calculation $M_i$, the required CPU clock cycle $C_i$ and the maximum allowable delay $D_i$, and the maximum allowable energy consumption $E_i$.

$$\text{Task}_i = \{M_i, C_i, D_i, E_i\}.$$  \hfill (2)

The computing model can divide into local computing, edge computing, and cloud computing, the computation capabilities of mobile devices, edge servers, and cloud services are $CA_i^{local}$, $CA_i^{mec}$, and $CA_i^{mcc}$ respectively \[^{10}\], and energy consumption and transmission power of mobile device is $CP_i$ and $TP_i$.

a) Local Computing: In local computing, the processing time and energy consumption calculated locally are $T_i^{local}$ and $E_i^{local}$.

$$T_i^{local} = \frac{C_i}{CA_i^{local}},$$  \hfill (3)

$$E_i^{local} = CP_i \times T_i^{local}. \hfill (4)$$

b) Edge Computing: In edge computing, the terminal’s computing task is offloaded to the edge server near the base station, increasing the wireless channel transmission time of the task offloading from the terminal to the edge server. The processing time and energy consumption are $T_i^{mec}$ and $E_i^{mec}$.

$$T_i^{mec} = \frac{C_i}{CA_i^{mec}} + \frac{M_i}{R}, \hfill (5)$$

$$E_i^{mec} = TP_i \times T_i^{mec}. \hfill (6)$$

c) Cloud Computing: In cloud computing, the computing tasks of the terminal are offloaded to a remote cloud data center, and usually have sufficient computing resources to handle the terminal’s computing tasks. The upload time of data from the base station to the cloud data center is the uplink transmission delay $t_i^{\text{uplink}}$, and cloud computing time and energy consumption are $T_i^{mcc}$ and $E_i^{mcc}$.

$$T_i^{mcc} = \frac{C_i}{CA_i^{mcc}} + \frac{M_i}{R} \times N + t_i^{\text{uplink}}, \hfill (7)$$

$$E_i^{mcc} = TP_i \times T_i^{mcc}. \hfill (8)$$

The computing time and energy consumption generated by the edge computing model and cloud computing model as the basis for offloading are combined with the computing amount of each task. The maximum tolerable delay, the required number of clock cycles and the maximum energy consumption are tolerated for each task. The calculation model is classified, and the calculation method of the task classification’s comparison weight $\Delta$ is as follows:

$$\Delta = \gamma_1 \times T_i^{mec,mcc} + (1 - \gamma_1) \times E_i^{mec,mcc}, \hfill (9)$$

A. Communication Model

The edge base station for wireless access can be a wireless network access point or a base station in a cellular network, which can be used to manage the mobile device’s upload or download communication link. Let $S_n = \{0, 1\}$ denote the offloading decision of mobile user, $S_n = 0$ indicates that mobile user chooses to offload tasks to the edge server through a wireless channel, and $S_n = 1$ indicates that mobile user $n$ chooses cloud server to offload tasks. Given that the decision set $a = \{a_1, a_2, a_3, \cdots, a_N\}$ of all users, $N$ is the number of users. According to Shannon’s law, the transmission data rate $R$ of mobile user can be calculated as:

$$R = W\log_2(1 + \frac{P_n H_n}{\omega_n + \sum_{m=1}^{N} P_m H_m}), \hfill (1)$$

among them, $W$ is the channel bandwidth, $P_i (i = m, n)$ is the transmission powers, $H_m$ and $Y_m$ are the channel gains between the mobile device users $n$ and $m$ and the base station respectively, and $\omega$ is the background interference power consumption, including noise power consumption $\omega_n = \omega_n^0 + \omega_n^1$ and the interference power consumption of wireless transmission from other mobile devices $\omega_n^0$. It can be seen from the above formula (1) that if multiple mobile devices simultaneously perform calculation and offloading through the wireless access channel, serious interference and a decrease in data transmission rate will result.

According to the communication model, the offloading decisions among mobile users are interrelated. If too many mobile users simultaneously offload tasks to the cloud through wireless channel computing, it will inevitably lead to low data transmission rates. When mobile users’ data rate is lower, the backhaul link will bring higher energy consumption and longer transmission time when offloading tasks. It is more helpful in
where $\gamma_1$ is the weight value of delay energy consumption, representing the weight value of the several variables in the unloading decision index, and its initial value is 0.5. In calculating the offload, the size can be adjusted according to the computing task’s actual situation, with the computing amount and the maximum tolerable delay as indicators. Taking computationally intensive tasks as an example, when the calculation of a task is astronomical, the weight of delay energy consumption can be increased. When the computing task is a delay-sensitive task, the weight of delay energy consumption can be reduced.

IV. OPTIMAL TRANSPORT COMPUTATION OFFLOADING

A. Computation Optimal Transport

After getting the offloading request of the task, it is necessary to determine the computation node determined by each computation task. The computation optimal transport is introduced, whose purpose is to find the transmission scheme with the least overall cost [12]. For the set of tasks $\varphi$ that should be offloaded, setting a discrete probability measure of $\varphi$ so that the cost of a discrete measure corresponding to the computing task $\varphi$ that should be performed is minimized, where the computation task that should be allocated is equal to the computing task that should be performed:

$$\varphi = \sum_{i=1}^{n} \alpha_i \delta_{x_i} = \sum_{i=1}^{n} \beta_i \delta_{y_i} = \phi. \quad (10)$$

With reference to the transport optimization problem, the above task allocation model can be described as a task allocation problem:

$$\min \int_{M \times M} C(x, F(x))d\psi(x, y), \quad (11)$$

where $C(x, F(x))$ is the transmission delay here, and $\psi(x, y)$ is the computation power of the terminal. The Kantorovich optimal transport problem is a special linear programming problem, so advanced linear programming algorithm can be used to solve it. When faced with large-scale offloading, time complexity is a crucial factor, and the linear programming algorithm based on the interior point method has great limitations in execution time.

B. L1 Regularization and SinkHorn Algorithm

In the original definition of the Monge-Kantorovich Problem [12], its constraint requires that for each element in it, it corresponds to an element of equal quality in it. Since this constraint is not linear, the problem is challenging to solve. The introduction of Kantorovich relaxation relaxes the fundamental requirements and allows the mass of each element to be distributed to multiple elements in the target distribution instead of one-to-one transportation in the Monge-Kantorovich Problem [13], [14]. The simplified constraint condition becomes linear, which greatly reduces the difficulty of solving. Construct the optimal transmission cost $L_c(a, b)$ as follows:

$$L_c(a, b) = \min_{P \in U(a, b)} \langle C, P \rangle = \sum_{i,j} C_{i,j} P_{i,j}. \quad (12)$$

Here, $P_{i,j}$ is the decision matrix, $a$ is the computing task to be uninstalled, $b$ is the computing task that is offloaded to the computing node, and $C_{i,j}$ is the delay matrix spent processing the task. Sparsity regularization includes L0 regularization, L1 regularization and L2 regularization [15]. The optimization problem of L0 regularization is an NP hard problem, and there is a theoretically proven that L1 norm is the optimal of L0 norm convex approximation, so L1 norm is usually used instead of L0 norm.

$$L^L_C(a, b) = \min_{P \in U(a, b)} \langle P, C \rangle - \varepsilon H(P), \quad (13)$$

$$H(P) = \sum_{i,j} |P_{i,j}|. \quad (14)$$

After a round of regularization, the solution of Monge-Kantorovich Problem can be written in the following form:

$$P_{i,j} = \mu_i K_{i,j} \nu_j, \forall (i, j) \in [n] \times [m] \quad (15)$$

The vector $\mu$ and $\nu$ are the variables required by the SinkHorn algorithm [16]. When $\mu$ and $\nu$ obtained, the dual problem $f$ and $g$ of Kantorovich’s solution is obtained, and the solution of the optimal transport is completed. At each step, update $\mu$ to satisfy the equation on the left, and then update $\nu$ to satisfy the equation on the right.

C. Optimal Solution for Computation Offloading

In most cases, it is not necessary to find the standard Kantorovich solution [17]. An approximate algorithm based on entropy can be used to find the approximate solution. Then the computation cost of optimal transport will be significantly reduced. Therefore, when solving the optimal transmission of a task, the computation node formula for extension and unloading is:

$$C(x, F(x)) = ||x - F(x)||_2 \quad (16)$$

Here, $C(x, F(x))$ is the optimal transmission delay that can be achieved by performing ECO or CCO in the current state. It can be proved that the offloading optimization function $L_c(a, b)$ in (9) is strictly convex, its optimal solution must be unique, that is, the solution space contains only one element, so the offloading decision is the optimal solution.

V. SIMULATION AND ANALYSIS

In this section, we will set up a variable simulation experiment and select three typical algorithms to compare and verify the proposed computational offloading model.

A. Simulation Parameter Settings

The algorithms are all implemented in the Matlab language [18], and the running platform is Matlab r2019a. In the experiment, the wireless channel bandwidth is set to 50MHz, the number of channels is 50, and the number of mobile devices is 100, and they are randomly distributed within the coverage of multiple base stations. The fibers’ upload rate is set to 1Gbps, the computing capabilities of mobile devices, edge servers, and cloud servers are set to 1GHz, 10GHz, and 100GHz, respectively, the computation power of the mobile devices is 100, and they are randomly distributed within the coverage of multiple base stations.
device is set to 0.5W, the background noise power is -100dBm. The data size of computing tasks follows a uniform distribution of [1, 500]. The number of CPU cycles required by edge servers and cloud servers is 200 cycles/bit and 50 cycles/bit. The following Table II shows the main experimental parameter settings [10], [19].

| Devices          | Parameter          | Value                  |
|------------------|--------------------|------------------------|
| Mobile device    | Computation capability | 1(GHZ)                |
|                  | Clock cycles       | 1000 cycles/bit        |
|                  | Computation power  | 0.5(W)                 |
|                  | Data upload power  | 0.1(W)                 |
|                  | Data download power| 0.15(W)                |
|                  | Number of devices  | 100                    |
| Edge Server      | Computation capability | 10(GHZ)               |
|                  | Clock cycles       | 200 cycles/bit         |
|                  | Data upload power  | 0.2(W)                 |
|                  | Data download power| 0.3(W)                 |
| Cloud Server     | Computation capability | 100(GHZ)              |
|                  | Clock cycles       | 50 cycles/bit          |
| Network          | Wireless channels  | 50                     |
|                  | Wireless channel bandwidth | 50MHz            |
|                  | Upload rate        | 1Gbps                  |

**B. Simulation Results and Analysis**

Take the three latest task offloading strategy work as the analysis object. The algorithm based on the Markov model to optimize the task delay is referred to as "Markov" [8]. The algorithm based on game theory to solve how to select partial tasks for offloading in multi-user scenarios is called "Game" [9]. By constructing a Lyapunov-optimized offloading framework, the algorithm that comprehensively considers the cost of transmission, execution time, and task failure is referred to as "Cross-Edge" [20]. The strategy mentioned in our work is referred to as "Cloud-Edge".

The experiment first got the effect of the data arrival rate on the average task delay, as shown in Fig.2. In most cases, the method proposed in this paper performs best, ensuring that the system selects priority resources to minimize the system task delay and ensure that the system is non-blocking. When the task arrival rate increases, it reduces the system blockage as much as possible by reasonably cooperating with cloud server and edge server resources to minimize the system task delay under blockage conditions.

Fig.3 shows the effect of different data reaching rates on the system processing speed. In the case of low data arrival rate, the advantage of task processing speed compared to the other two algorithms is not obvious due to idle resources, while in the case of high data arrival rate, the other two algorithms enter the blocked state due to the resource bottleneck of a single node, and the task processing reaches the upper limit. Even with the blessing of cloud services, Cross-Edge has gradually reached the processing limit due to the cost of mission failure.

Fig.4 shows the impact of the data arrival rate on the system blocking queue, which is obtained by adding the waiting data packets of all nodes. In the Matlab simulation experiment, it represents the block queue’s size when the system is blocking the worst case after performing specific processing tasks. Compared with the other three algorithms, the method proposed in this paper reduces a load of bottleneck nodes in the system by balancing the communication resources and
computing resource consumption of cloud computing and edge computing. Because it can improve the computing throughput and reduces the accumulation of the blocking data packet in the system, the blocking situation of this data packet is the lightest.

VI. CONCLUSION

In this paper, we have proposed a computation offloading strategy based on optimal transport theory in a collaborative cloud-edge environment. By setting the Monge–Kantorovich transportation problem, the paper transforms the offloading over the collaborative cloud-edge networks into an optimization problem that only involves resource of the current time slot on the server. To solve the convex optimization problem, we propose an algorithm based on semidefinite programming, which is designed to decide to offload the task to an edge server near the base station, an adjacent edge server, or a cloud data center. Experimental results show that computing tasks are offloaded to the edge and central cloud through edge computing, which provides a practical way for resource allocation and application scheduling.

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