Spin transport in a graphene normal-superconductor junction in the quantum Hall regime

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The quantum Hall regime of graphene has many unusual properties. In particular, the presence of a Zeeman field opens up a region of energy within the zeroth Landau level, where the spin-up and spin-down states localized at a single edge propagate in opposite directions. We show that when these edge states are coupled to an s-wave superconductor, the transport of charge carriers is spin-filtered. This spin-filtering effect can be traced back to the interplay of specular Andreev reflections and Andreev retro-reflections in the presence of a Zeeman field.

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I. INTRODUCTION

Monolayer graphene has remarkable electronic transport properties. One of them is a peculiar quantum Hall effect, which can be observed even at room temperature. Inducing superconductivity via the proximity effect further enriches these transport properties. Recently, a number of experiments have performed conductance measurements in the quantum Hall regime in monolayer graphene, using superconducting electrodes. Moreover, coupling the helical edge states within the zeroth Landau level in graphene to an s-wave superconductor can also give rise to Majorana bound states.

Low-energy excitations in graphene reside in two disconnected regions in the first Brillouin zone, known as valleys. In the quantum Hall regime, the energy spectrum has an unconventional Landau level structure, where the Landau level energies are proportional to $\pm \sqrt{n}$ with integer $n$. This discrete set of flat Landau levels develops into dispersive edge states towards the edge of the sample. In the low-energy approximation, the bulk Landau level (LL) energies in graphene are given by

$$E_n^\pm = \hbar \omega_c \sqrt{n},$$

where the valley index $\xi = \pm$ denotes the $K^\pm$ valley, and $\lambda = \pm 1$ labels the conduction and valence band, respectively. The cyclotron frequency is given by $\hbar \omega_c = \sqrt{2} \hbar v_F / \ell_B$, where $v_F$ is the Fermi velocity, $\ell_B = \sqrt{\hbar / (eB)}$ is the magnetic length and $B = |B|$ is the absolute value of the applied magnetic field; $n$ is a non-negative integer. These bulk LLs are four-fold degenerate: two-fold for the spin and two-fold for the valley degree of freedom. In the presence of a Zeeman field that splits the energies for spin-up (red) and spin-down (blue) electrons, the spin degeneracy is lifted as the valley index is changed.

If the spin degeneracy is lifted by, e.g., a Zeeman field, each of the LLs splits into two with energy difference $2\Delta_z$, where $\Delta_z = \frac{1}{2} g^* \mu_B B$. Here, $g^*$ is the effective $g$-factor of an electron in graphene and $\mu_B$ is the Bohr magneton. The energy difference between the spin-up and spin-down bulk LLs is $2\Delta_z \approx 2.3 \text{ meV}$ at $B \approx 10 \text{ T}$ for the interaction-enhanced $g$-factor, $g^* = 4$, see Refs. The authors propose a four-terminal device where the spin-filtering effect can be achieved by inducing backscattering between the counter-propagating edge states locally (using gates) in just one part of the system. The spin-filtering effect takes place due to the presence of an in-plane magnetic field and spin-orbit coupling.

Here, we suggest a different mechanism for the spin-filtering effect. We couple the edge states to an s-wave superconductor and consider only subgap transport. The Andreev-reflected hole can have the same or the opposite direction of propagation as the electron impinging on the interface with the superconductor at energy $E$. Which

![FIG. 1: Electron band structures of the few lowest Landau levels](image-url)
We discuss and summarize our results in Sec. IV and this structure are introduced and determined in Sec. III.

Hence, if an incoming spin-down electron is specularly reflected while the spin-up electron is retro-reflected, spin-reflection takes place. We demonstrate this effect in the three-terminal device shown in Fig. 2. This is done by employing a tight-binding model on a honeycomb lattice within the Bogoliubov-De Gennes (BdG) framework and taking into account the orbital and spin effect of the magnetic field.

The rest of this article is organized as follows. In Sec. II we describe the setup of a three-terminal device and introduce its Hamiltonian. The transport coefficients of this structure are introduced and determined in Sec. III. We discuss and summarize our results in Sec. IV and conclude in Sec. V.

II. MODEL

We investigate spin transport in the three-terminal device shown in Fig. 2. The underlying honeycomb lattice with lattice constant $a$ is exposed to a quantizing out-of-plane magnetic field. The upper edge of the system is coupled to an s-wave superconductor (S) with a sizeable critical field, such that the quantum Hall effect and superconductivity coexist\cite{3,12}. There are two normal leads $L_0$ and $L_1$ of widths $W_0$ and $W_1$, respectively, which serve to probe the spin-resolved transmission through the scattering region. In the rest of the paper, $W_0 = W_1 = W$.

The superconducting lead $L_2$ effectively creates a normal-superconducting interface of length $W_2$ that converts electrons to holes. The geometry of the system is motivated by a recent experiment\cite{4}.

The tight-binding Hamiltonian of the system can be written as

$$H = H_0 + H_\Delta + H_Z,$$ \hspace{1cm} (2)

where

$$H_0 = \sum_{\langle ij \rangle} \psi_i^{\dagger} \left[ -t e^{i\varphi_{ij}} \frac{1}{2} \left( \eta_0 + \eta_z \right) ight. 
+ t e^{-i\varphi_{ij}} \frac{1}{2} \left( \eta_0 - \eta_z \right) \left. \right] \otimes s_0 \psi_j + E_F \sum_i \psi_i^{\dagger} (\eta_z \otimes s_0) \psi_i,$$

$$H_\Delta = \sum_i \Delta_i \psi_i^{\dagger} (\eta_x \otimes s_0) \psi_i,$$

$$H_Z = \sum_i \Delta_i \psi_i^{\dagger} (\eta_0 \otimes s_z) \psi_i.$$

The four-spinor field $\psi_i$ is in the standard Nambu basis $\psi_i = (c_{i\uparrow}, c_{i\downarrow}, c_{i\downarrow}^\dagger, -c_{i\uparrow}^\dagger)^T$, where $\psi_i^\dagger$ creates a particle localized at site $i$ with a four-component wavefunction $(\chi_{x\uparrow}(r-r_i), \chi_{x\downarrow}(r-r_i), \chi_{h\uparrow}(r-r_i), -\chi_{h\downarrow}(r-r_i))^T$. Here, the index $es$ ($hs$) denotes an electron (hole) with spin $s \in \{ \uparrow, \downarrow \}$.

The two sets of Pauli matrices, $\eta_r$ and $\eta_s$ with $r \in \left\{ 0, x, y, z \right\}$, describe the electron-hole and spin degree of freedom, respectively. Finally, $\sum_i$ and $\sum_{\langle ij \rangle}$ denote sums over all sites and over nearest neighbors.

The first (second) term in $H_0$ describes the nearest-neighbor hopping of electrons (holes) in an out-of-plane magnetic field with a hopping amplitude $-te^{i\varphi_{ij}}$ ($te^{-i\varphi_{ij}}$). The Peierls phase is given by

$$\varphi_{ij} = -\frac{2}{\phi_0} \int_{x_i}^{x_j} \frac{y_i - y_j}{2} (x_j - x_i), \hspace{1cm} (4)$$

where $\phi_0 = \hbar/e$ is the magnetic flux quantum and $(x_i, y_i)$ are the real-space coordinates of site $i$. The vector potential in the Landau gauge is chosen to be constant along the $x$-axis, $A = (-By, 0, 0)$. The third term in $H_0$ describes the Fermi energy $E_F$ of the system. In undoped graphene, $E_F = 0$.

The s-wave superconducting pairing is represented by $H_\Delta$ and couples an electron with spin $s$ to a hole with spin $s$ on the same lattice site. The Zeeman field described by $H_Z$ splits each energy level into two with energy difference $2\Delta Z$.

For simplicity, we assume the spatial dependence of the pair potential $\Delta_i$ and of the magnetic field $B_i$ to be a step function. That is, $\Delta_i = \Delta(y) (B_i = B(y))$ is assumed to be a non-zero constant (zero) in the graphene sheet below the superconducting electrode and zero (a non-zero constant) otherwise. The magnitude of the Zeeman term has the same spatial dependence as the magnetic field.

In the following, we will calculate the scattering matrix for the system shown in Fig. 2. All the numerical results for the conductances and spin polarizations presented below were obtained using Kwant\cite{10}.

III. TRANSPORT COEFFICIENTS

In Figs. 3(a)–(c) we plot the relevant transport coefficients in the case when $E_F < \Delta Z < \Delta$ and the gap be-
The parameters are $\Delta = 10$ meV, $L_1 = 10^{-2}$, $E_F = 0.3\Delta$, $Z_2 = 0.5\Delta$, $B = 10T$, $W = 600a$, $W_2 = 510a$, and $y_0 = 300a$.

The horizontal dashed lines mark the energies where the edge states change the direction of propagation, while the thick ones correspond to $|E| = \Delta$. Here, the edge terminations of $L_0$ and $L_1$ are zigzag while the edge termination of $L_2$ is armchair.

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It is interesting to look at the spin polarization of the carriers in the subgap regime, where $|E| < \Delta$. Since $H$ conserves the spin projection along the $z$-axis, we define the spin polarization as

$$P = \frac{T_{ee} + T_{he} - T_{\bar{e}e} - T_{\bar{h}e}}{T_{ee} + T_{he} + T_{\bar{e}e} + T_{\bar{h}e}},$$

(5)

where $T_{\alpha\alpha',ss'}$ is the transmission coefficient for a particle $\alpha$ with spin $s$ in lead $L_0$ to a particle $\alpha'$ with spin $s'$ in lead $L_1$. To avoid numerical artifacts, we set $P = 0$ if the denominator in Eq. (5) is smaller than $10^{-3}$, i.e., if almost no particle is transmitted from $L_0$ to $L_1$. The numerically calculated spin polarization is non-zero in the energy region $E_F - \Delta < E < E_F + \Delta$ and zero otherwise, see Fig. 3(e). This can be understood by looking at the bandstructure and the propagation direction of the particles along the edges of the sample as illustrated in Figs. 4(a) and (b), respectively. In the energy region II, a spin-up electron $e \uparrow$ travels undisturbed along the lower edge into $L_1$, however a spin-down electron $e \downarrow$ propagating along the upper edge is backscattered to $L_0$ as a spin-down hole $h \downarrow$ because a superconductor is coupled to the upper edge. This results in the accumulation of spin-up particles in $L_1$. The situation in the energy region I is the same for spin-down electrons $e \downarrow$. However, here a spin-up electron $e \uparrow$ also travels along the upper edge and encounters the superconductor. Since an Andreev-reflected spin-up hole $h \uparrow$ has the same propagation direction as a spin-up electron $e \uparrow$, the particle propagates along the GS interface via Andreev edge states, and, depending on the geometry, ends up with a certain probability as a spin-up electron $e \uparrow$ or spin-up hole $h \uparrow$ in $L_1$. Thus, injecting spin-unpolarized particles in $L_0$ results in spin-polarized particles in $L_1$ in the energy region $E_F - \Delta < E < E_F + \Delta$.

We would now like to discuss the charge conductance. In the presence of hole excitations, the (differential) charge conductance from $L_0$ to $L_1$ is defined as

$$G_{10} = \frac{e^2}{h} \sum_{s \uparrow,\downarrow} (T_{ee,ss} - T_{he,ss}),$$

(6)

which is shown for our system in Fig. 3(f). In the energy region (I) the carrier ending in $L_1$ is a hole and $G_{10} = -e^2/h$, while in the region II it is an electron and $G_{10} = e^2/h$. In the energy region III there is a spin-up electron $e \uparrow$ along the lower edge and a spin-up hole $h \uparrow$ along the upper edge propagating into $L_1$, which results in zero.
FIG. 5: (a) Spin polarization and (b) charge conductance for three different interface lengths that correspond to three different valley polarizations. The charge conductance depends on the angle between the valley isospins, while the spin polarization is (almost) independent of the interface length. We see that the spin polarization is (almost) independent of the interface length $W_2$, while the charge conductance has a threefold character, depending upon the total number of hexagons across the width of the armchair ribbon being a multiple of three, or a multiple of three plus/minus one. Besides that, a set of dips (peaks) in the spin polarization (conductance) for energies close to $E_F - \Delta_Z$ can be observed. This feature is due to a spin-down electron $e \downarrow$ leaking from $L_0$ to $L_1$ through the interface (without being Andreev-reflected). This can be understood as follows. Without the superconductor, there are edge states propagating in opposite directions for opposite spins. When we couple the superconductor to the upper edge, the electron impinging on the interface will be reflected as a hole (in the case of non-zero Andreev reflection probability). However, this hole propagates in the direction opposite to the electron edge state (for both spin projections) in this energy region. Hence, the transport along the interface should be blocked. But if the Andreev reflection probability is less than one, the electron has a finite chance to leak along the interface onto the other side. In other words, edge states along the upper edge contacted to a superconductor develop an effective gap $\Delta^*$ that is smaller than the naively expected gap $2(\Delta_Z - E_F)$ (for $(\Delta_Z - E_F) < \Delta$). The bigger the pairing $\Delta$, the higher the Andreev reflection probability. Thus, on increasing $\Delta$, $\Delta^*$ approaches $2(\Delta_Z - E_F)$ as shown in Fig. 6.

The spin-filtering effect for $|E| < \Delta$ is lost once the gate voltage shifts the Fermi energy such that it exceeds $\Delta + \Delta_Z$, i.e., the propagation direction of the electron and hole states is the same within the subgap region.

We obtain similar results if leads $L_0$ and $L_1$ have armchair orientation and lead $L_2$ has zigzag orientation, see Fig. 7. The spin polarization in Fig. 7(e) is again (nearly) perfect for $E_F - \Delta_Z < E < E_F + \Delta_Z$. This is expected since, unlike the valley structure, the spin structure of the ZLL in graphene is independent of the type of the edge termination. The conductance profile in Fig. 7(f) matches the one in Fig. 5(b) for $W_2/a = 1$ mod 3, which is the result of the same valley structure for the states at the edges of the NS interface for the two cases. The dip in the spin polarization is present for the same reason as in Fig. 5.
V. CONCLUSION

We have shown that spin filtering can be achieved by coupling the edge states of the spin-split zeroth Landau level in graphene to a superconductor. The spin-filtering effect can be switched on and off by applying a (global) gate voltage that shifts the Fermi energy. Unlike the charge conductance, the spin polarization is independent of the edge termination. The device can be put in different regimes by tuning the Zeeman energy independently of the gap between the zeroth Landau level and the other Landau levels. This can be achieved by applying an in-plane magnetic field\textsuperscript{17–19}. The spin filtering effect discussed here does not require the presence of spin-orbit coupling and its experimental verification is within the current technological capabilities.

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