Robust Fault Detection Based on $l_1$ Regularization

Young-Man Kim

Herein, $l_1$ regularization-based fault detection technique for stochastic discrete time systems in state space form is discussed. Compared with the deterministic nature of a fault which usually causes an abrupt change, the state of a system smoothly evolves in most cases and the disturbance in the process and the sensor measurement has a stochastic characteristic. The modeling uncertainty in the state space of a system can induce a bias in deterministic way and it can be combined to a fault. In the fault detection community, the modeling uncertainty is called a multiplicative fault and the abruptly-changing fault is called an additive fault. Inspired by the fact that $l_2$ norm is useful for estimating smoothly evolving states and reducing a bias and $l_1$ norm for detecting abrupt change with spare structure, this research develops the technique for detecting fault which changes abruptly under stochastic disturbance and modeling uncertainty. The $l_2$ norm for bias compensation (due to modeling uncertainty and/or additive fault) is combined with $l_1$ norm for fault detection (especially abruptly changing fault) and the two norms are set up as a regularization problem. The regularization problem is convex thus, global solution can be found in efficient way.

1. Introduction

Recently, intensive attention has been drawn to fault detection technique due to ever-increasing requirements on the reliability of the operation of complex dynamic systems. Fast and reliable fault detection in a system can prevent catastrophic disasters. Thus, vast amount of research efforts have been poured into that area and much of tackled.\textsuperscript{[1–10]}

However, technical developments outside fault detection community, such as machine learning and statistics, induced new insights about understanding the nature of faults and it has triggered this research. For example, the sparsity, which has been an important topic in estimating system state and controlling a system with minimal actuator operation,\textsuperscript{[11–14]} can represent the nature of abruptly changing fault. The theory of sparsity has been developed in the name of $l_1$-regularized least square\textsuperscript{[15]} or LASSO\textsuperscript{[11]} and it has been applied to machine learning, signal/image processing, seismic engineering, and so on. The $l_1$ regularization technique is used to force most of the decision variables to be equal to zero. It is useful for fault detection because fault signal is mostly zero under normal operation but it abruptly becomes nonzero when fault occurs. Compared with the deterministic nature of a fault which usually causes an abrupt change, the state of a system smoothly evolves in most cases and the disturbance in the process and the sensor measurement has a stochastic characteristic. The modeling uncertainty in the state space of a system can induce a bias in deterministic way and it can be combined to a fault. In the fault detection community, the modeling uncertainty is called a multiplicative fault and the abruptly changing fault is called an additive fault. Inspired by the fact that $l_2$ norm is useful for estimating smoothly evolving states and reducing a bias and $l_1$ norm for detecting abrupt change with spare structure, this research develops the technique for detecting fault which changes abruptly under stochastic disturbance and modeling uncertainty. In system identification literature, it is known as bias-variance tradeoff problem and in statistical learning, it is known as overfitting problem. The $l_2$ norm for bias compensation (due to modeling uncertainty and/or additive fault) is combined with $l_1$ norm for fault detection (especially abruptly changing fault) and the two norms are set up as a regularization problem through a regularization parameter.

In this research, disturbance is Gaussian. Under Gaussian noise, the state estimation using Kalman filter (KF) is well known as best linear estimator. The KF solves the optimization problem to minimize process and measurement noise which is in the form of $l_2$ norm. To detect fault under Gaussian noise and modeling uncertainty, the $l_2$ and $l_1$ norms are linearly combined through a regularization parameter. Thus, robust fault detection technique is formulated in convex optimization so that global solution is guaranteed.

In Section 2, a discrete time system in state space form under Gaussian process and measurement noise with additive fault (modeling uncertainty, abrupt change) is introduced. In Section 3, $l_1$ regularization is formulated to detect fault under noise. The formulation results to a convex optimization; hence, it can be solved effectively with open software. In Section 4, three simulations are used for demonstration using DC motor, wind turbines, and robot arm position control. The article is ended by a conclusion in Section 5.
The mathematical notation follows the standard way: a variable $v$ having normal distribution with zero mean and covariance matrix with $M$ is denoted as $v \sim \mathcal{N}(0, M)$. If needed, clarification is provided as we proceed.

2. Problem Setup

The following discrete time system is in state space form which is useful for designing various kinds of controller: Linear Quadratic Gaussian, Model Predictive Control, and so on.

$$
x(k + 1) = A_k x(k) + B_k u(k) + G_k f(k) + w(k)$$

$$
y(k) = C_k x(k) + e(k)$$

(1)

Each vector and matrix has a proper dimension. Here, $f$ is the fault signal, $w$ is the zero mean white Gaussian process noise, and $e$ is the zero mean white Gaussian measurement noise. $w$ and $e$ are independent each other. The covariance matrix for $w$ is defined as $Q_w(k)$. Likewise, $Q_e(k)$ for $e$. Thus, using standard notation, we can denote them as follows: $w \sim \mathcal{N}(0, Q_w(k))$, $e \sim \mathcal{N}(0, Q_e(k))$. If $B_k = G_k$, then $f$ can be considered as an actuator fault. In this research, modeling uncertainty is considered, which means $A_k \leftarrow (A + \Delta A)_k$, where $\Delta A$ represents modeling uncertainty and the same concept is applied to the following matrices: $B_k$, $G_k$, and $C_k$. It is easy that the modeling uncertainty term can be absorbed to fault term $f(k)$ by slightly modifying (1). Without loss of generality, $f$ represents a process fault which affects state change (abruptly or gradually). Thus, in this research, the fault $f(k)$ includes modeling uncertainty and abruptly changing fault. The modeling uncertainty induces bias and acts as an additive fault. It has a deterministic characteristic. The abrupt change can represent sudden change in control signal and/or actuator stuck. It can be considered as a sparse model which can be detected using $l_1$ regularization method.

3. Robust Fault Detection Based on $l_1$ Regularization

In this section, robust fault detection technique is developed by using $l_1$ regularization method for a given system of Equation (1). It is formulated in a convex optimization problem, which leads to an efficient global solution.

With the measurement (input $u$ and output $y$), the quality of state estimation is evaluated using the well-known criterion-of-fit

$$
\begin{align*}
\min_{x(1), w(k), f(k)} & \sum_{k=1}^{N} \left\| Q_w^{-1/2}(k)(y(k) - C_k x(k)) \right\|_2^2 \\
& + \sum_{k=1}^{N-1} \left\| Q_w^{-1/2}(k)w(k) \right\|_2^2
\end{align*}
$$

(2)

s.t. $x(k + 1) = A_k x(k) + B_k u(k) + G_k f(k) + w(k)$

where for a vector $v = [v_1, v_2, \ldots, v_n]^T$, $\|v\|_p \triangleq (\sum_{i=1}^{n}|v_i|^p)^{1/p}$ which is known as $l_p$. That is a minimization problem which computes the state estimation $\hat{x}(k)$ with a given $x(1)$. To avoid overfitting issue due to not only noise but also fault, this minimization problem is reformulated using regularization method

$$
\begin{align*}
\min_{x(1), w(k), f(k)} & \sum_{k=1}^{N} \left\| Q_w^{-1/2}(k)(y(k) - C_k x(k)) \right\|_2^2 \\
& + \sum_{k=1}^{N-1} \left\| Q_w^{-1/2}(k)w(k) \right\|_2^2 + \lambda \sum_{k=1}^{N-1} \left\| Q^{-1/2}(k)f(k) \right\|_2
\end{align*}
$$

(3)

s.t. $x(k + 1) = A_k x(k) + B_k u(k) + G_k f(k) + w(k)$

where $\lambda$ is a positive number which defines the amount of regularization whose term penalizes the state changes. In statistical learning, this regularization method is called bias-variance trade-off.$^{[16]}$ In this research, $l_1$ norm is applied for regularization because of its power to capture sparsity.$^{[15]}$ It is also useful for detecting fault because fault is often zero under normal operation but when fault occurs, the value suddenly changes to nonzero.

One key feature of Equation (3) is that it is a convex optimization form and it can be solved in $O(N)$ to find global solution.

The tuning parameter $\lambda$ should be properly determined to optimize the solution. In statistical learning$^{[13]}$ the estimated parameter sequence (in this research, $f(k)$) as a function of regularization parameter $\lambda$, is called the regularization path. If $\lambda \geq \hat{\lambda}_{\text{max}}$, then the estimated parameter sequence is identically to zero which means $f(k) = 0$. $\hat{\lambda}_{\text{max}}$ is called critical parameter value. The critical parameter value is useful for finding good starting value to determine the suitable value of $\lambda$. $\hat{\lambda}_{\text{max}}$ can be readily computed based on convex analysis as follows. First, we obtain $x(k), k = 1, \ldots, N$, with disregarding $w(k)$

$$
x(k) = A_{k-1} x(k-1) + B_{k-1} u(k-1) + G_{k-1} f(k-1)
$$

$$
= \sum_{t=1}^{k-1} \left( \prod_{s=t+1}^{k-1} A_s \right) \left( B_s u(r) + G_s f(r) \right) + \left( \prod_{s=1}^{k-1} A_s \right) x_1
$$

(4)

We define no-fault residual signal as given subsequently using Equation (4)

$$
r(k) \triangleq y(k) - C_k \left( \sum_{t=1}^{k-1} \left( \prod_{s=t+1}^{k-1} A_s \right) B_s u(r) + \left( \prod_{s=1}^{k-1} A_s \right) x_1 \right)
$$

(5)

The Equation (3) can be rewritten using Equation (4) and (5)

$$
\begin{align*}
\min_{x(1), w(k), f(k)} & \sum_{k=1}^{N} \left\| Q_w^{-1/2}(k)(y(k) - C_k x(k)) \right\|_2^2 \\
& + \sum_{k=1}^{N-1} \left\| Q_w^{-1/2}(k)w(k) \right\|_2^2 + \lambda \sum_{k=1}^{N-1} \left\| Q^{-1/2}(k)f(k) \right\|_2
\end{align*}
$$

(6)

s.t. $x(k + 1) = A_k x(k) + B_k u(k) + G_k f(k) + w(k)$

The gradient of (6) w.r.t. $Q^{-1/2}(k)f(k)$, $\nabla_{Q_{k}}^{-1/2}(k)f(k)$, is calculated as follows. For the first sum-of-squared $l_2$ norms term
\[
\mathbf{Q}_{k}^{1/2}(\mathbf{k}f(\mathbf{k})) = \mathbf{Q}_{k}^{1/2}(\mathbf{k})\mathbf{V}_{f}(\mathbf{k})
\]
\[
= -2\mathbf{Q}_{k}^{1/2}(\mathbf{k}) \sum_{k=1}^{N} \left( f(k) - C_{k} \prod_{r=1}^{k-1} A_{r} \right) G_{f}(r)
\]
\[
\times \mathbf{C}_{k}^{T} \left( \prod_{s=k+1}^{N} A_{s} G_{s} \right)^{T}
\]
\[
= -2\mathbf{Q}_{k}^{1/2}(\mathbf{k}) \frac{1}{C_{0}} \sum_{k=1}^{N} \left( f(k) - C_{k} \prod_{r=1}^{k-1} A_{r} \right) G_{f}(r)
\]
\[
\times \mathbf{C}_{k}^{T} \prod_{s=k+1}^{N} A_{s} G_{s}
\]
(7)

If Equation (7) is evaluated at \( f(k) = 0, k = 0, \ldots, N-1 \) to find the \( \lambda_{\text{max}} \), then the following Equation (8) is obtained as follows

\[
\mathbf{Q}_{k}^{1/2}(\mathbf{k}f(\mathbf{k})) = \mathbf{Q}_{k}^{1/2}(\mathbf{k})\mathbf{V}_{f}(\mathbf{k})
\]
\[
= -2\mathbf{Q}_{k}^{1/2}(\mathbf{k}) \sum_{k=1}^{N} r(k) \mathbf{Q}_{k}^{-1}(\mathbf{k}) \left( \mathbf{C}_{k} \prod_{s=k+1}^{N} A_{s} G_{s} \right)^{T}
\]
(when \( f(k) = 0, k = 0, \ldots, N-1 \))

The gradient of the second term of Equation (6) disappears. With Equation (8) and (9) at \( f(k) = 0, (k = 1, \ldots, N-1) \), the minimization condition of Equation (6) can be organized as

\[
\lambda\mathbf{Q}_{k}^{1/2}(\mathbf{k})\mathbf{V}_{f}(\mathbf{k}) \left( \sum_{k=1}^{N-1} \left\| \mathbf{Q}_{k}^{-1/2}(\mathbf{k}) f(k) \right\|_{1} \right)
\]
\[
= 2\mathbf{Q}_{k}^{1/2}(\mathbf{k}) \sum_{k=1}^{N} r(k) \mathbf{Q}_{k}^{-1}(\mathbf{k}) \left( \mathbf{C}_{k} \prod_{s=k+1}^{N} A_{s} G_{s} \right)^{T}
\]
\[
= \lambda\mathbf{Q}_{k}^{1/2}(\mathbf{k}) \mathbf{V}_{f}(\mathbf{k}) \left( \sum_{k=1}^{N} \left\| \mathbf{Q}_{k}^{-1/2}(\mathbf{k}) f(k) \right\|_{1} \right)
\]
\[
\lambda_{\text{max}} = \lambda\mathbf{Q}_{k}^{1/2}(\mathbf{k}) \mathbf{V}_{f}(\mathbf{k}) \left( \sum_{k=1}^{N} \left\| \mathbf{Q}_{k}^{-1/2}(\mathbf{k}) f(k) \right\|_{1} \right)
\]
(10)

It is well known that the dual norm of 1 norm is \( l_{\infty} \) norm and it should lie in the unit ball. If we apply this concept to the L.H.S. of Equation (10), then this is obtained

\[
\left\| \mathbf{Q}_{k}^{1/2}(\mathbf{k}) \mathbf{V}_{f}(\mathbf{k}) \left( \sum_{k=1}^{N} \left\| \mathbf{Q}_{k}^{-1/2}(\mathbf{k}) f(k) \right\|_{1} \right) \right\|_{\infty} \leq 1
\]
\[
(\mathbf{k} = 0, \ldots, N-1)
\]
(11)

Rbg In addition, the dual norm of 1 norm is \( l_2 \) norm. If we apply dual norm to both sides of Equation (10), \( \lambda_{\text{max}} \) can be found as follows

\[
\lambda_{\text{max}} = \max_{i=1, \ldots, N-1} \left\{ 2\mathbf{Q}_{k}^{1/2}(\mathbf{k}) \sum_{k=1}^{N} r(k) \mathbf{Q}_{k}^{-1}(\mathbf{k}) \right\}
\]
\[
\cdot \left( \mathbf{C}_{k} \prod_{s=k+1}^{N} A_{s} G_{s} \right)^{T}
\]
(12)

Equation (12) can be used as a tool to calculate the reasonable starting value, \( 0.01\lambda_{\text{max}} \) for tuning the optimization problem of Equation (6).

There is a technique called reweighting \( 1 \) minimization\textsuperscript{13} which estimates \( f(k) \) having more zeros with a slight increment in the criterion-of-fit term. With the idea, Equation (6) is modified by inserting the positive weighting term \( \alpha(k) \) as follows

\[
\min_{x(1), w(k), f(k)} \sum_{k=1}^{N} \left\| \mathbf{Q}_{k}^{-1/2}(\mathbf{k}) \right\|_{1} \left( r(k) - C_{k} \sum_{r=1}^{k-1} A_{r} G_{r} \right) + \lambda \sum_{k=1}^{N} \left\| w(k) \right\|_{2}^{2} + \lambda \sum_{k=1}^{N} \left\| \mathbf{Q}_{k}^{-1/2}(\mathbf{k}) f(k) \right\|_{1}
\]
(13)

For the first optimization stage \( (i = 1) \), \( \alpha_{i}(k) \) is initialized as \( 1 \) and the minimization of Equation (13) is done to find optimal \( f_{i}(k) \). Then, to find better \( f_{i+1}(k) \) which has more zeros than the previous one, we can reweight \( \alpha_{i}(k) \) as follows

\[
\alpha_{i+1}(k) = 1/(\varepsilon + \mathbf{Q}_{k}^{-1/2}(\mathbf{k}) f_{i}(k))
\]
(14)

where \( \varepsilon \) is a positive number. If convergence is noted, we can stop and find the optimal \( f(k) \). The Equation (13) is a convex optimization and several open softwares are available such as CVX\textsuperscript{17}, YALMIP\textsuperscript{18,19}, and L1.ks.

Main algorithm is summarized as follows:

**Algorithm 1**: Given \( A_{k}, B_{k}, C_{k}, G_{k}, Q_{k}(k), Q_{0}(k), \varepsilon, \delta, \) and \( \{ y(k), u(k) \}_{k=1}^{N} \).

Step 1 (Initialization): Find \( \lambda_{\text{max}} \) with Equation (12) and put \( \lambda = 0.01\lambda_{\text{max}} \) and \( \alpha(k) = 1 \). (i = 1).

Step 2: Find optimal \( f_{i}(k) \) using Equation (13).

Step 3: Reweight \( \alpha(k) \) as Equation (14).

Step 4: Increase \( i \) to \( i+1 \) and jump to step 2. If \( f_{i}(k) - f_{i-1}(k) \leq \delta \) as showing convergence, then stop. If not, jump to step 3.

### 4. Simulation

In this section, the previous algorithm is applied to DC motor system and wind turbines system to illustrate its effective fault detection. The first system is DC motor system in the study by Gustafson and Graebe\textsuperscript{20}.

With the sampling time of 0.1 s for the measurement of motor velocity and angle, the discrete time DC motor system is shown as follows

\[
x(k+1) = \begin{bmatrix} 0.7047 & 0 \\ 0.0843 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 11.81 \\ 0.625 \end{bmatrix} u(k) + f(k)
\]
\[
y(k) = [0 1] x(k) + e(k)
\]

where \( u(k) \sim N(0,1), e(k) \sim N(0,1), x(1) \sim N(0,1) \). An arbitrary fault signal \( f(k) = \left[ \begin{array}{c} 0 \\ 1 \end{array} \right] \) is injected only at \( k = 100 \). This kind of fault can represent an unexpected sudden movement of DC motor due to interruption from power line or surrounding electromagnetic noise. From Equation (12), \( \lambda_{\text{max}} \) is found as 31.4888. For the optimization problem of (13) with Algorithm 1, several parameters are set as follows: \( \lambda = 0.314888, N = 200, \varepsilon = 0.01, \delta = 1e-5 \).

Figure 1 shows the measured motor angle. At \( k = 100 \), fault is injected, but it is too hard to note any change in the output measurement.

© 2020 The Authors. Published by Wiley-VCH GmbH
change detection using the power of fault. It clearly shows the sparse nature of fault and its acute occurrence is correctly detected and otherwise it shows no occurrence of fault.

The benchmark model was used for the competition [21]. The transformation of Equation (18) to state space form is easily done with Matlab and it is shown as follows

\[
x(k+1) = \begin{bmatrix} 1.8640 & -0.8752 \\ 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \beta_r(k)
\]

With given sampling time, 0.01 s, the continuous time transfer function Equation (16) can be transformed to discrete time one with no fault parameter values in Equation (17)

\[
\frac{\beta_{m2}}{\beta_r} = \frac{0.0059z^{-1} + 0.005644z^{-2}}{1 - 1.864z^{-1} + 0.8752z^{-2}}
\]

The transformation of Equation (18) to state space form is easily done with Matlab and it is shown as follows

\[
x(k+1) = \begin{bmatrix} 1.947 & -0.9497 \\ 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \beta_r(k)
\]

\[
\beta_{m2}(k) = \begin{bmatrix} 0.001613 & 0.001586 \end{bmatrix} x(k)
\]

\[
\beta_{m2}(k) = 1.864\beta_{m2}(k-1) - 0.8752\beta_{m2}(k-2) + 0.0059\beta_r(k-1) + 0.005644\beta_r(k-2)
\]

Similarly, for the faulty case, each corresponding equations to (18)–(20) is shown as follows

\[
\frac{\beta_{m2}}{\beta_r} = \frac{0.001613z^{-1} + 0.001586z^{-2}}{1 - 1.947z^{-1} + 0.9497z^{-2}}
\]

\[
x(k+1) = \begin{bmatrix} 1.947 & -0.9497 \\ 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \beta_r(k)
\]

\[
\beta_{m2}(k) = 1.947\beta_{m2}(k-1) - 0.9497\beta_{m2}(k-2) + 0.001613\beta_r(k-1) + 0.001586\beta_r(k-2)
\]

From Equation (20) and (23), it is noted that the coefficient of AR form is changed due to fault. Thus, detection of the change in the coefficients is related with fault detection. For the unified labeling of the following figures, the coefficient of AR form is defined with \(b_1, b_2, a_1, a_2\)

\[
\beta_{m2}(k) = b_1\beta_{m2}(k-1) + b_2\beta_{m2}(k-2) + a_1\beta_r(k-1) + a_2\beta_r(k-2)
\]

The fault due to the air pressure drop is injected at \(k = 100\) by the transition from Equation (18) to (21). With \(\beta_r \sim \mathcal{N}(0, 1)\) and Equation (19), the output data, \(\beta_{m2}\), are collected and Algorithm 1 is applied. The following parameter values are used for this simulation: \(c = 0.01, \lambda_{\text{max}} = 66.6745, N = 200\).
the fault from the direct measurement of output itself shown in Figure 3 but, using the proposed technique, it is easy to identify the fault occurrence.

Figure 4–7 show the parameter change due to the fault. For example, Figure 4 shows the value of $b_1$ changes at $k = 100$.

Figure 8 shows the sample time when the fault occurred.

The proposed algorithm is applied to experimental data which is originated from the study by Torfs et al.\textsuperscript{[22]} It is the data file generated by the experiment of robot arm position control. Between the sample time of 100 and 103 with sampling period of 0.02 s, an abruptly changing fault occurred, as shown in Figure 9.

As shown in Figure 10, the fault is correctly detected using the proposed method in this article.

---

**Figure 3.** Pitch angle measurement with occurrence of fault at $k = 100$.

**Figure 4.** Coefficient ($b_1$) change at $k = 100$.

**Figure 5.** Coefficient ($b_2$) change at $k = 100$.

**Figure 6.** Coefficient ($a_1$) change at $k = 100$.

**Figure 7.** Coefficient ($a_2$) change at $k = 100$. 
5. Conclusion

In the proposal, fault detection problem is formulated based on $l_1$ regularization method for a system having modeling uncertainty and Gaussian disturbance. The modeling uncertainty is reformulated as additive fault and it is understood as inducing bias. Gaussian disturbance requires designed algorithm to perform as like the Kalman filter which is optimal in the sense of least squared regularization. In addition, it is required to detect fault which changes abruptly in the system.

The proposed algorithm shows that all the requirements described earlier are satisfied by setting the problem in mixed $l_2$ and $l_1$ norm. The illustrative examples show the detection of fault in abrupt change.

Conflict of Interest

The author declares no conflict of interest.

Keywords

convex optimization, fault detection, Gaussian noise, regularization, sparsity

Received: July 20, 2020  
Revised: October 17, 2020  
Published online: December 4, 2020

[1] J. Chen, R. J. Patton, Robust Model-Based Fault Diagnosis for Dynamic Systems, Kluwer Academic Publishers, Dordrecht, Netherlands 1999.
[2] S. X. Ding, Model-Based Fault Diagnosis Techniques—Design Schemes, Algorithms and Tools, 2nd ed., Springer-Verlag, London 2013.
[3] R. Isermann, Fault Diagnosis Systems, An Introduction from Fault Detection to Fault Tolerance, Springer, New York 2009.
[4] M. Basseville, I. Nikiforov, Digital Signal Processing: Detection of Abrupt Changes, Prentice Hall, Englewood Cliffs, NJ 1993.
[5] F. Gustafsson, Adaptive Filtering and Change Detection, Wiley, New York 2001.
[6] S. Boyd, L. Vandenberghe, Convex Optimization, Cambridge University Press 2004.
[7] D. Bertsekas, A. Nedic, A. Ozdaglar, Convex Analysis and Optimization, Athena Scientific, Nashua NH 2003.
[8] Y.-M. Kim, J. Franklin Inst. 2016, 353, 3104.
[9] Y.-M. Kim, Int. J. Syst. Control Inform. Process. 2014, 1, 298.
[10] L. Ljung, R. Singh, T. Chen, IFAC SYSDY, 2015, p. 745.
[11] R. Tibshirani, J. Roy. Stat. Soc. B 1996, 58, 267.
[12] D. L. Donoho, IEEE Trans. Inform. Theory 2006, 52, 1289.
[13] E. J. Candès, M. B. Wakin, S. P. Boyd, J. Fourier Anal. Appl. 2008, 14, 877.
[14] M. Gallieri, M. Maciejowski, in Proc. American Control Conf., 2012, pp. 1217–1222.
[15] S.-J. Kim, K. Koh, S. Boyd, D. Gorinevsky, SIAM Rev. 2009, 51, 339.
[16] H. Ohlsson, F. Gustafsson, L. Ljung, S. Boys, Automatica 2012, 48, 595.
[17] M. Grant, S. Boyd, Y. Ye, CVX: Matlab Software for Disciplined Convex Programming, http://cvxr.com/cvx/ (accessed: June 2009).
[18] J. Lofberg, in Proc. CACSD Conf., 2004.
[19] S.-J. Kim, K. Koh, M. Lustig, S. Boyd, D. Gorinevsky, IEEE J. Select. Topics Signal Process. 2007, 1, 606.
[20] F. Gustafson, S. F. Graebe, Automatica 1998, 34, 1311.
[21] P. F. Odgaard, J. Stoustrup, M. Kinnaert, in Proc. 7th IFAC Symp., 2009, pp. 155–160.
[22] D. E. Torfs, V. Rudi, S. Jan, J. Schoukens, IEEE Trans. Control Syst. Technol. 1998, 6, 2.