NONCOMMUTATIVE SUPER YANG-MILLS THEORIES
WITH 8 SUPERCHARGES AND BRANE CONFIGURATIONS

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Abstract

In this paper we consider $D = 4$ NCSYM theories with 8 supercharges. We study these theories through a proper type IIA (and M-theory) brane configuration. We find the one loop beta function of these theories and show that there is an elliptic curve describing the moduli space of the theory, which is in principle the same as the curve for the commutative counter-part of our theory. We study some other details of the dynamics by means of this brane configuration.
1 Introduction

Recently it has been shown that the noncommutative gauge theories can arise as the low energy effective open string theory in the presence of D-branes with non-zero NSNS two form, B-field, on them [?, ?, ?, ?, ?].

The action for the noncommutative gauge theories is obtained from the usual (commutative) Yang-Mills theories by replacing the ordinary product of the fields by the *-product (for a review see [?]). From now on we call them NCYM theories. It has been shown that the noncommutative $U(1)$ theory can be understood as the commutative gauge theory with a deformed gauge group, so that the dynamics of this new gauge theory is governed by a series of dipoles in addition to the usual photons [?]. Similar statements can be made for NCYM with any gauge group. It has been discussed that NCYM theories are renormalizable [?, ?, ?]. Besides the pure gauge theories, the noncommutative scalar field theories have also been considered and it has been argued that these noncommutative gauge theories are renormalizable if and only if their commutative limit (noncommutative parameter, $\theta$, equal to zero) is renormalizable [?, ?].

From the string theory side, the branes with constant B-field on them preserve 16 supercharges, half of the supercharges of the type II theories, similar to the usual D-branes, and hence when we study the low energy effective theory of such branes we actually deal with a NC (Super)YM (NCSYM) theory with 16 supercharges. Although the perturbative fermionic fields in a noncommutative background have not been well elaborated, since the noncommutative field theories can be understood as a commutative field theory with definite interaction terms, it is believed that these theories should have the usual feature of the gauge theories with 16 supercharges. As an example, it has been shown that the large $N$ limit of the NCSYM can be studied through the gauge theory/gravity on definite background correspondence [?, ?, ?]. The conceptual new point of these NCSYM theories is that, despite being maximally supersymmetric, unlike the usual SYM (with 16 supercharges), they are not conformal invariant and this is because the deformation parameter, $\theta$, is a dimensionful parameter with dimensions of (length)$^2$.

For the $\mathcal{N} = 4, D = 4$ SYM theories large amount of supersymmetries kill all the interesting dynamics. In this paper we try to study more interesting theories with less supersymmetries, namely $\mathcal{N} = 2, D = 4$ NCSYM. After the work of [?] we learnt how we can study the four dimensional $\mathcal{N} = 2$ SYM theories, their BPS spectrum, their moduli space of vacua and its singularities, and how we can extract all of these physics from a proper elliptic curve. In another paper, [?], it was shown how the curve itself, and hence all of the above mentioned physics of $\mathcal{N} = 2, D = 4$ SYM theories, can be obtained from the definite type IIA string and M theories brane configuration $^\dagger$.

$^\dagger$In that paper only the $\prod_{\alpha=1}^{\mathcal{N}} SU(k_\alpha)$ gauge group was considered, later the same procedure generalized to SP and SO gauge theories in extensive papers, which we are not going to list them here.
In this paper, we show how the brane configuration method should be extended and modified for studying the $\mathcal{N} = 2, D = 4$ NCSYM with $SU(N)$ gauge group. In this way we study some of the physical aspects of the deformed SYM theories, and discuss that most of the physics expected in the commutative $\mathcal{N} = 2, D = 4$ SYM hold in the noncommutative case too. Intuitively this should be related to the fact that the number of degrees of freedom of noncommutative gauge theories, at least for the planar diagrams, are the same as the commutative gauge theories [? , ?].

The paper is organized as follows. In the next section we fix our conventions and notations and review the preliminaries we need. In section 3, we build the proper brane configuration from type IIA NS5-brane and (D4-D2)-brane bound states so that the brane system preserves 8 supercharges. In section 4 we relate the (pure) $D = 4$ NCSYM to the brane configuration of the section 3 and identify the parameters of gauge theory in terms of the brane model parameters. In this way we find the one loop beta function of the theory. By lifting the brane configuration to M-theory we study the Coulomb branch of the theory. Moreover we discuss different phases of our noncommutative theory in different energies. The last section is devoted to concluding remarks and open questions.

2 Conventions and Preliminaries

a) String Theory

It has been shown that turning on B-fields polarized parallel to a $D_p$-brane, is equivalent to building various bound states of $p$, $p - 2$, $p - 4$, ... branes, depending on the rank of the B-field [?]. For the B-field of rank one, these bound states are composed of a $p$-brane and a uniform distribution of $(p-2)$-branes on it. So that if the worldvolume of the $p$-brane is located along $012...p$ directions and $B_{p-1,p}$ is the non-zero component, then the $(p-2)$-branes span $012...(p-2)$ directions. This argument can be simply generalized to any higher rank B-fields. In our notations $B$ is a dimensionless parameter and $l_s^{-2} B$ is the $(p-2)$-branes density. The mass density of the bound state in this notation is $(l_s^{p+1} g_s)^{-1} \sqrt{1 + B^2}$ [?].

The perturbative dynamics of these bound states, similar to the individual D-branes, is governed by the open strings ending on them, but this time because of the B-field, they should satisfy mixed boundary conditions. Studying such open strings we learn that the brane worldvolume is in fact a noncommutative plane:

$$[x^\mu, x^\nu] = i l_s^2 \left( \frac{B}{1 - B^2} \right)^{\mu\nu} \equiv i \theta^{\mu\nu}.$$  \hspace{1cm} (2.1)

As we see $\theta$ is a parameter with dimensions of $(\text{length})^2$.

b) Noncommutative Field Theories

Field theories on a noncommutative plane are obtained from their commutative version by replacing the product of the fields by the $*$-product defined as follows[?]
Besides the field products, to couple the noncommutative theories to a NCYM, we should replace the ordinary derivative with a "covariant derivative":

\[ \partial_{\mu} \to \partial_{\mu} + \{ A_\mu, * \}_{M.B.}, \]

where the "Moyal Bracket" in the above relation is \( \{ f, g \}_{M.B.} = f \ast g - g \ast f \). As we see for the slowly varying fields \( (|\partial|)^2 << |\theta|^{-1} \), these noncommutative field theories effectively behave like the commutative theories.

It is worth noting that for the case of noncommutative SU(N) gauge theory, when \( A_\mu \) take values in SU(N) algebra, the field strength \( F_{\mu\nu} \) defined by (2.3) is not algebra valued unlike the usual gauge theories (it is sitting in the SU(N) group).

3 The Model with IIA branes

In order to reduce the number of supersymmetries, as it has been shown and discussed in [?, ?], we consider the brane intersections.

Since we are interested in the four dimensional theories, like [?], we consider the IIA theory and its NS5- and (D4-D2)- branes. Let us consider the following brane configuration:

(N+1) NS5-branes labelled by \( \alpha = 0, 1, \ldots, N \), spanning 012345 directions. These fivebranes are located at different values of \( x^6 \) direction, and a number of D4-branes with their worldvolume along 01236 so that there are \( k_\alpha \) number of them between the \( (\alpha - 1) \)th and \( \alpha \)th NS5-branes. As we see the intersection of these branes is a 3+1 dimensional space, 0123, on which we have our \( \mathcal{N} = 2 \) gauge theory.

To find a NCSYM, we turn on B-field along the D4-branes. We should recall that turning on a B-field will not change the number of conserved supercharges (see the Appendix for some explicit calculations). From the superalgebra point of view, turning on the B-field corresponds to choosing another central extension of the algebra (different from that of the individual brane). In other words, when we turn on the B-field, we again find a BPS solution which preserves \( \frac{1}{2} \) of supersymmetries.

There are various choices for the B-field, it can be of rank one or two, and also it can have different polarizations. In this paper we present calculations for the rank one case, with non-zero \( B_{23} = B \), which is the most interesting case. We will discuss the \( B_{36} \) and rank two cases briefly in the discussion section. With a non-zero \( B_{23} \), our brane configuration consists of NS5-branes along 012345 and D4-branes along 01236 and a distribution of D2-branes having their worldvolume along 016. These D2-branes are open D2-branes as discussed in [?].

In order to find a system with finite energy we should compactify the \( x^6 \) direction, or equivalently we should identify the \( (N + \alpha)^{th} \) fivebrane with \( \alpha^{th} \).
Along the lines of [?], the $x^6$ coordinate as a function of $v \equiv x^4 + ix^5$, is obtained by minimizing the total fivebrane worldvolume. For the large $v$ equation of $x^6$ reduces to a source free Laplace equation,

$$\nabla^2 x^6 = 0. \tag{3.1}$$

Since $x^6$ is only a function of the directions normal to the brane bound state,

$$x^6 = \mathcal{K} ln|v| + \text{constant.} \tag{3.2}$$

The parameter $\mathcal{K}$ is actually the ratio of the tensions (or mass densities) of two intersecting branes:

$$\mathcal{K} \sim \frac{(D4-D2) \text{ mass density}}{\text{NS5-brane mass density}} \sim \frac{l_s^{-5} g_s^{-1} \sqrt{1 + B^2}}{l_s^{-6} g_s^{-2}} = l_s g_s \sqrt{1 + B^2}. \tag{3.3}$$

For the case with $k$ number of these bound states on top of each other, obviously our $\mathcal{K}$ factor should be multiplied by a factor of $k$. It is worth noting that (??) is the $B = 0$ result with $g_s$ replaced with the open string coupling [?] and, it reduces to the results of [?] for the $B = 0$. Since the $(D4-D2)$-branes ending on the fivebrane on its left pulls it in the opposite direction compared to those ending on its right, the $\mathcal{K}$ factor for them should have different relative sign.

If we have $q_L$ (and $q_R$) $(D4-D2)$-branes ending on the fivebrane on its left (and right), which are located at $a_i, i = 1, ..., q_L$ (and $b_j, j = 1, ..., q_R$), then the asymptotic form of $x^6$ is

$$x^6 = \mathcal{K} \left( \sum_{i=1}^{q_L} ln|v - a_i| - \sum_{j=1}^{q_R} ln|v - b_j| \right) + \text{constant.} \tag{3.4}$$

A well-defined $x^6$ for $v \rightarrow \infty$ is obtained if and only if $q_L = q_R$.

To study the gauge theory dynamics we also need to consider the moving $(D4-D2)$-branes. This motion is realized by letting $a_i$ and $b_j$ become functions of the space-time coordinates $(0123)$. This motion as explained in [?], corresponds to the question of IR divergences of the NCSYM[1]. This motion contributes to the fivebrane kinetic energy as $\int d^4x d^2v \partial_\mu x^6 \partial^\mu x^6$, $\mu = 0, 1, 2, 3$. We should note that the metric on our space-time is the open string metric. With $x^6$ given by (??), it becomes

$$\int d^4x d^2v |\text{Re} \left( \sum_i \partial_\mu a_i \left( \frac{1}{v - a_i} \right) - \sum_j \partial_\mu b_j \left( \frac{1}{v - b_j} \right) \right) |^2. \tag{3.5}$$

The integral converges if and only if

$$\sum_i a_i - \sum_j b_j = q_\alpha, \tag{3.6}$$

where $q_\alpha$ is a constant, and is a characteristic of the $\alpha^{th}$ fivebrane. The $q_\alpha$'s are determined by the separation between $(D4-D2)$ branes, and hence we expect them to be related to the "bare mass" of the gauge theory hypermultiplets.

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1 As we will discuss in the next chapter, the IR behaviour of the NCSYM is the same as the commutative SYM.
The quantum mechanical treatment of our noncommutative gauge theory corresponds to the M-theory limit of our brane model. In that limit type IIA on $R^{10}$ is actually the M-theory on $R^{10} \times S^1$, with radius $R = l_s g_s$. But, this is only the compactification radius for closed strings, i.e. $R$ is the radius which relates the 10 and 11 dimensional supergravities. As we discuss in the following, the open string compactification radius is different$^\text{5}$. 

In the M-theory limit, our (D4-D2)-bound state is lifted to a (M5-M2)-bound state. Such bound states as discussed in [7, 8] are formed by turning on a non-zero C-field background of 11 dimensional theory on the M5-brane. This corresponds to turning on a self-dual three form of the six-dimensional M5-branes worldvolume theory. In other words, in the M-theory limit $B_{23}$ will be replaced by $C_{23(10)}$ and $C_{016}$, and these fields are equal because of the self-duality condition. From the 10 dimensional point of view, this self-duality is related to the fact that, D2-branes are the sources for the $RR$ three form of type IIA theory, and in our case their density is given by $B_{23}$. Having this in mind, we learn that the actual compactification radius viewed from the fivebrane worldvolume theory, and hence from the open M2-branes point of view, is not $R$, but $R\sqrt{1 + C^2}$. Since the eleven dimensional C-field and the ten dimensional B-field are supergravity fields they should be related by closed string parameters, which in our conventions are 

$$l_p^{-3} RC = l_s^{-2} B.$$  \hspace{1cm} (3.7) 

So, the effective compactification radius for the open strings, is 

$$R_o = R\sqrt{1 + B^2}.$$  \hspace{1cm} (3.8) 

If we denote the eleventh dimension by $x^{10}$, which is periodic with periods $2\pi R$, we have 

$$x^6 + ix^{10} = R\sqrt{1 + B^2} \{ \sum_{i=1}^{q_L} \ln|v - a_i| - \sum_{j=1}^{q_R} \ln|v - b_j| \} + i\phi,$$  \hspace{1cm} (3.9) 

where $0 \leq \phi = \frac{x^{10}}{R_o} < 2\pi$, and hence 

$$s \equiv \frac{x^6 + ix^{10}}{R} = \sqrt{1 + B^2} \{ \sum_{i=1}^{q_L} \ln(v - a_i) - \sum_{j=1}^{q_R} \ln(v - b_j) \} + \text{constant}. \hspace{1cm} (3.10)$$ 

The fact that $s$ is holomorphic in $v$, is expected from the supersymmetry. Let us concentrate on the imaginary part of (3.9). It states that circling around one of $a_i$ and $b_j$’s in the $v$ complex plane, $x^{10}$ jumps by $\pm 2\pi R_o$. From the fivebrane worldvolume effective theory, the end points of (D4-D2)-brane bound states are viewed as "dyonic vortices". In the T-dual version, where our brane configuration is composed of IIB NS5-branes (spanning 012345) and (D3-D1) bound state (with D3-branes along 0126 and D-strings along 06), the end point of the (D3-D1)-brane looks like "dyons" of fivebrane effective theory. With our conventions in definition of $K$, (3.9), these

$^\text{5} \text{The fact that open string compactification radius, } R_o, \text{ is effectively different, in the work of Connes, Douglas and Schwarz [7], which was the light-like compactification of M-theory with C-field background, is reflected in their } \text{dim } \mathcal{H} = \text{Tr } 1 \text{(the dimension of Schwartz space), and in our case is related to } \frac{l_p}{R} \text{.}$
dyons carry one unit of magnetic charge and \( l_s^{-2} B V_2 \) units of electric charge \((V_2\) is the volume of 12 plane).

## 4 Four Dimensional Noncommutative Gauge Theory

Now let us return to our main question: what can we learn about the \( D = 4 \) NCSYM from the above brane configuration.

Since we have considered the \( B_{23} \neq 0 \), according to (??) we deal with a NCSYM with non-zero \( \theta_{23} \),

\[
\theta \equiv \theta_{23} = l_s^2 \frac{B}{1 + B^2}. \tag{4.1}
\]

If we consider the brane configuration we built in the previous section, because of (??), we deal with \( \prod_{\alpha=1}^{N} SU(k_{\alpha}) \) NCSYM theory.

Now we should identify our gauge theory coupling constant through the string theory parameters. The naive answer to this question can be understood in light of the commutative/noncommutative gauge theory correspondence proposed in [?], by expanding the Born-Infeld action for the D4-branes with non-zero B-field, up to the first order in \( \alpha' \). Then the gauge coupling \( g_{\alpha} \) of the \( SU(k_{\alpha}) \) should be given by

\[
\frac{1}{g_{\alpha}^2(v)} = \frac{x_{\alpha}^0(v) - x_{\alpha-1}^0(v)}{l_s g_s}. \tag{4.2}
\]

Replacing \( x_{\alpha}^0 \) from (??) we find that the above relation is exactly the same as its commutative counter-part, with \( g_s \) which is the closed string coupling, replaced with open string coupling.

According to (??) we propose that \( g_{\alpha}^{-2} \) for the NCSYM to be logarithmic divergent for large \( v \). This is the familiar asymptotic behaviour of the commutative theory at high energies. This proposal is expected since we believe that the noncommutative gauge theories can be mapped into a commutative gauge theory through the Fourier expansion [? , ?] and also, we know that the planar degrees of freedom of the NCSYM is the same as its commutative counter-part [?]. Moreover we propose that this logarithmic divergence corresponds to the one loop beta function of our four dimensional NCSYM. This is in exact agreement with perturbative results of [? , ? , ?].

The theta angle of the \( SU(k_{\alpha}) \) theory, \( \theta_{\alpha} \), is then naturally related to the imaginary part of (??) as

\[
\theta_{\alpha} = \frac{x_{\alpha}^{10} - x_{\alpha-1}^{10}}{R}. \tag{4.3}
\]

Hence the

\[
\tau_{\alpha}(v) = \frac{\theta_{\alpha}}{2\pi} + \frac{4\pi i}{g_{\alpha}^2} = i(s_{\alpha}(v) - s_{\alpha-1}(v)). \tag{4.4}
\]

In the large \( v \) limit:

\*\( \theta_{\alpha} \) should not be confused with \( \theta \), the noncommutativity parameter.
\[ \tau_{\alpha}(v) = i \sqrt{1 + B^2}(2k_{\alpha} - k_{\alpha-1} + k_{\alpha+1})lnv. \] (4.5)

As we see the only difference of (??) with the commutative version is the coefficient in front.

Comparing the standard one loop asymptotic freedom formula, \( \tau = ib_0 lnv \), we find the important result that the one loop beta function for NCSYM theories with the above mentioned gauge group is proportional to the one loop beta function of the commutative counter-part.

Analogous to the commutative case, the open strings having their end points on the open (D4-D2)-bound states on the opposite sides of fivebranes form the hypermultiplets of our NC-SYM theory, obviously in the \((k_1, \bar{k}_2) \oplus (k_2, \bar{k}_3) \oplus \ldots (k_{N-1}, \bar{k}_N)\) representation. The only point which one should note here is that the mass of these hypermultiplets should be calculated by squaring the momentum with the open string metric \([?]\). These hypermultiplets become massless classically when (D4-D2)-bound states on the right of a fivebrane end on the same point as the (D4-D2)-branes on the left. Like the commutative case, when one these hypermultiplets becomes massless, namely \((k_i, k_{i+1})\), for the \(k_i = k_{i+1}\) we expect the related factor of the gauge group to become a \(SU(k_i)\) instead of \(SU(k_i) \times SU(k_{i+1})\).

Up to here we have identified and related the parameters of NCSYM with those of our brane configuration model and briefly discussed the similarities of noncommutative and commutative theories at one loop. Now we want to study different phases of the noncommutative gauge theories.

As we mentioned the noncommutativity parameter, \(\theta\), is a dimensionful parameter and hence there are two parameters of energy dimensions, \(\theta^{-1/2}\) and \(R_0^{-1}\) (the "open string" eleven dimensional radius), with which we should compare our energy scales. We distinguish two different phases:

a) \(\theta/R_0^2 \lesssim 1\)

In this case our noncommutative theory effectively behaves as a commutative SYM because, before the energies that noncommutativity effects show themselves, \(\theta E^2 \sim 1\), we should uplift our theory to M-theory, where we only have (M5-M2)-bound state effective theory which is not a noncommutative one.

If we replace the \(\theta\) and \(R_0\) by their string theoretic values we have

\[ \frac{\theta}{R_0^2} \sim \frac{l_s^2}{g_s^2} \frac{B}{(1 + B^2)^2}. \] (4.6)

Since \(\frac{B}{1 + B^2} \leq 1/2\), for \(\theta/R_0^2 \lesssim 1\) cases we have

\[ g_{\text{open string}} \gtrsim 1, \] (4.7)

and hence this case for generic B, corresponds to the strong coupling of our noncommutative gauge theory (which as we argued above) is not a noncommutative theory. For very large B, (??) reads as \(g_{YM} B \gtrsim 1\) which corresponds to the commutative gauge theory weak coupling.
b) $\theta/R^2_{\alpha} \geq 1$

This is the more interesting case. From (??) we see that for generic B, we are actually dealing with weakly coupled gauge theory, and the theory shows different behaviours at different energies:

b-1) IR limit ($\theta E^2 << 1$):

In this regime the noncommutative effects can totally be neglected and we effectively see the commutative SYM theory. So, all the arguments of IR limit of commutative SYM holds for the NCSYM too.

b-2) Intermediate energies ($\frac{1}{\sqrt{\theta}} \lesssim E << \frac{1}{R}$):

This is the phase which we actually deal with a NCSYM theory.

b-3) UV limit ($E \gtrsim \frac{1}{R_0}$):

In this case we should uplift our brane configuration to the M-theory, where there is no noncommutative description.

M-Theory Interpretation and Seiberg-Witten Curves

In order to study the quantum structure of NCSYM, we need to lift the brane configuration we built in IIA theory (which describes the classical regime) to the M-theory. In the M-theory, as we stated earlier the (D4-D2)-bound state coincides with the (M5-M2)-bound state in the M-theory and NS5-branes are viewed as M5-branes. The (M5-M2)-brane, is actually a fivebrane with a non-zero self-dual three form on it. Hence, in the M-theory limit our IIA brane configuration is reinterpreted as a single fivebrane with a definite C-field on it, whose worldvolume sweeps $R^4 \times \Sigma$ where $\Sigma$ is a hyper-surface given by a holomorphic function in the $v$ and $s$ complex planes. Since $s$ is not single valued, it is more convenient to introduce the $t = \exp(-s/\sqrt{1 + B^2})$ instead. Analogue of the Seiberg-Witten elliptic curve, is exactly the curve defining $\Sigma$ in the $t$ and $v$ plane, namely, $F(t,v) = 0$. Compared to the commutative case the only difference is in the definition of $t$ with respect to $s$. However, the powers in $v$ will remain the same as the commutative case.

One can study the low energy four dimensional noncommutative theory from the six dimensional effective theory of M5-branes, if we denote the field strength of the two-form field living on the M5-brane by $T$, we know that $T$ is self-dual and its equation of motion is $dT = 0$. One can decompose $T$ as follows:

$$T = F \wedge \Lambda + F^* \Lambda + C,$$

where $C$ is the self-dual constant background and in our case has non-zero components in 016 (and 23(10)). $\Lambda$ is a harmonic one form defined on the $\Sigma$, satisfying, $d\Lambda = d^*\Lambda = 0$. As discussed in [?], to see the reasonable low energy theory we need to define $\Lambda$ on a point-wise
compactified version of $\Sigma$, $\tilde{\Sigma}$, which is obtained by adding $(N+1)$ points to the $\Sigma$. $\tilde{\Sigma}$ is a surface of genus $g = \sum_{\alpha=1}^{N}(k_\alpha - 1)$. $F$ is a two form and noncommutative field defined on $R^4$, obeying $DF = D^*F = 0$, where $D$’s are the noncommutative "covariant derivative". The low energy theory for the $g$ dimensional $\Lambda$, the Coulomb branch of our theory, is $(\text{NC } U(1))^g$. Apart from the changes mentioned above, all the other results and discussions of [?] are valid in our case too.

5 Conclusion Remarks and Discussions

In this paper we studied the $\mathcal{N} = 2, D = 4$ NCSYM theories with gauge group $\prod_{\alpha=1}^{N} SU(k_\alpha)$ through the brane configuration method. In order to find the NCSYM gauge theory, we considered the D4-branes with a B-field background. We should emphasize that actually the noncommutative gauge theories we are discussing are formulated on a space with open string metric [?]. By means of our brane system, we found the one loop beta function of these noncommutative theories to be logarithmically divergent. Moreover, we argued that one can still find a holomorphic curve which describes the moduli space of our theory. For the case at hand, the pure gauge theory, we briefly discussed the behaviour of the theory in different energy regimes and its Coulomb branch.

In this paper we discussed the rank one B-field along 23 directions. For the $B_{16}$ case, since the noncommutative coordinates are 3 and 6 directions, our four dimensional space-time (0123 directions) is not noncommutative and hence we have our usual commutative field theory. In addition, since instead of NS5-branes we have (NS5-D2)-brane bound states, we expect the equation of motion for $x^6$ not to be altered compared to the commutative case. For the same reasons for the case of rank two B-field, i.e. $B_{23}, B_{16} \neq 0$, we expect to see the physics similar to $B_{23} \neq 0$. However, the $B_{23} = B_{16}$ seems to be an interesting special case to be discussed.

Another interesting extension of this work is adding matter fields and the Higgs branch of NCSYM. We believe that like the commutative version this can be done by adding D6-brane to the brane configuration. We hope to come back to this question in future works.

There are many other interesting problems which can be addressed, e.g. the generalization of Hanany-Witten work[?], and (2+1) dimensional NCSYM, the noncommutative two dimensional $\mathcal{N} = (4, 4)$ theories, theories with lower supersymmetries (four supercharges) which are obtained by rotating one of the NS5-branes.

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The explicit form of the supercharges for a (D4-D2) bound state, resulting from a non-zero $B_{23} = B$ component, can be written as a linear combination of the type IIA supercharges, $Q_R$ and $Q_L$ ($\Gamma^{01...9} Q_R = Q_R$ and $\Gamma^{01...9} Q_L = -Q_L$) : 

$$Q = \epsilon^R Q_R + \epsilon^L Q_L,$$

where $\epsilon^R$ and $\epsilon^L$ are 16 component spinors satisfying the following conditions:

$$(I) \left\{ \begin{array}{c} \frac{1}{\sqrt{1+B^2}} + \frac{B}{\sqrt{1+B^2}} \Gamma^{23} \epsilon^R = \epsilon^L \\ \Gamma^{45789} \epsilon^R = \epsilon^L \end{array} \right.$$ 

The above conditions can be obtained from the T-dual version of the rotation matrices in 23 plane. The $\Gamma^2$ is rotated by $R(\phi)\Gamma^2$, with $R(\phi) = exp(\phi \Gamma^{23}) = \cos \phi + \Gamma^{23} \sin \phi$. By T-duality $\tan \phi$ is replaced by $B$. It can easily be shown that relations (I) allow 16 supersymmetries. Moreover one can check that for the $B = 0$ and $B \to \infty$ these conditions coincide with the supersymmetries preserved by a D4- and D2- brane respectively. When we also consider the NS5-branes as well as (I), we should consider

$$(II) \left\{ \begin{array}{c} \Gamma^{012345} \epsilon^R = \epsilon^R \\ \Gamma^{012345} \epsilon^L = \epsilon^L \end{array} \right.$$ 

The simultaneous solutions of (I) and (II) give the allowed 8 supercharges.

References

[1] M. R. Douglas, C. Hull, "D-branes and Noncommutative Torus", JHEP 9802 (1998) 008, hep-th/9711165.

[2] Y.-K. E. Cheung, M. Krogh, "Noncommutative Geometry From 0-Branes in a Background $B$ Field", hep-th/9803031.

[3] C. Hofman and E. Verlinde, "U-duality of Born-Infeld on the Noncommutative Two Torus", JHEP 9812 (1998) 010, hep-th/9810116.

C. Hofman and E. Verlinde, "Gauge Bundles And Born-Infeld On the noncommutative Torus", hep-th/9810219.

[4] M.M. Sheikh-Jabbari, "Super Yang-Mills Theory on Noncommutative Torus From Open Strings Interactions", Phys. Lett. B450 (1999) 119, hep-th/9810179.

[5] N. Seiberg, E. Witten, "String Theory and Noncommutative Geometry", JHEP 09 (1999) 032, hep-th/9908142, and references therein.

[6] M.M. Sheikh-Jabbari, "One Loop Renormalizability of Noncommutative Yang-Mills on Two Torus", JHEP 06 (1999) 015, hep-th/9903166.
[7] C.P. Martin, D. Sanchez-Ruiz, *Phys. Rev. Lett.* **83** (1999) 476.

[8] T. Krajewski, R. Wulkenhaar, "Perturbative Quantum Gauge Fields on the Noncommutative Torus", hep-th/9903187.

[9] I. Chepelev, R. Roiban, "Renormalization of Quantum Field Theories on Noncommutative $R^d$", hep-th/9911098.

[10] S. Minwalla, M.V. Raamsdonk and N. Seiberg, "Noncommutative Perturbative Dynamics", hep-th/9912072.

[11] D. Bigatti, L. Susskind, "Magnetic Fields, Branes and Noncommutative Geometry", hep-th/9908056.

[12] J. Maldacena, J.G. Russo, "Large N Limit of Noncommutative Gauge Theories", *JHEP* **09** (1999) 025, hep-th/9908134.

[13] A. Hashimoto, N. Itzhaki, "Noncommutative Yang-Mills and AdS/CFT Correspondence", hep-th/9907166.

[14] M. Alishahiha, Y. Oz, M.M. Sheikh-Jabbari, "Supergravity and Large-N Noncommutative Field Theories", *JHEP* **11** (1999) 007, hep-th/9909215.

[15] N. Seiberg, E. Witten, "Electric-Magnetic Duality, Monopole Condensation, And Confinement In $\mathcal{N} = 2$ SYM theories”, *Nucl. Phys.* **B426** (1994) 19.

N. Seiberg, E. Witten, "Monopoles, Duality, And Chiral Symmetry Breaking, In $\mathcal{N} = 2$ QCD”, *Nucl. Phys.* **B341** (1994) 484.

[16] E. Witten, "Solutions of Four Dimensional Field Theories Via M-Theory”, *Nucl. Phys.* **B500** (1997) 3.

[17] H. Arfaei, M.M. Sheikh-Jabbari, "Mixed Boundary conditions and Brane-String Bound States”, *Nucl. Phys.* **B526** (1998) 278, hep-th/9709054.

M.M. Sheikh-Jabbari, "More on Mixed Boundary Conditions and D-branes Bound States’, *Phys. Lett.* **B425** (1998) 48, hep-th/9712199.

[18] A. Hanany, E. Witten, "Type IIB Superstrings, BPS Monopoles, And Three Dimensional Gauge Dynamics”, *Nucl. Phys.* **B492** (1997) 152, hep-th/9611230.

[19] A. Strominger, "Open p-Branes”,*Phys. Lett.* **B383** (1996) 44, hep-th/9512059.

[20] D. Sorokin, P.K. Townsend, "M-theory Superalgebra From the M5-Branes”, *Phys. Lett.* **B412** (1997) 265.

P.K. Townsend, "Four Lectures on M-theory”, hep-th/9612121.
[21] A. Connes, M.R. Douglas, A. Schwarz, ”Noncommutative Geometry and Matrix Theory: Compactification on Tori”, *JHEP 9802* (1998) 003, hep-th/9711162.

[22] P.K. Townsend, ”M-branes at Angles”, hep-th/9708074.

M. Berkooz, ”Branes Intersecting at angles”, *Nucl. Phys. B480* (1996) 265.

M.M. Sheikh-Jabbari, ”Classification of Branes at Angles”, *Phys. Lett. B420* (1998) 279, hep-th/9710121.