Flavor and CP violation in the SUSY SU(5) GUT
with the right-handed neutrinos

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ABSTRACT

The neutrino flavor mixing may induce rich flavor and CP violating phenomena in the SUSY GUTs. In this paper we discuss the hadronic EDMs and the correlation among low-energy processes induced by the mixings between second and third generations in the SUSY SU(5) GUT with the right-handed neutrinos.

1. Introduction

The supersymmetric grand unified models (SUSY GUTs) predict rich flavor and CP violation in the SUSY breaking terms for squarks and sleptons, when origin of the SUSY breaking is dynamics above the GUT scale, such as the gravity mediation [1]. This may give a chance to probe the interactions at the GUT scale by the low-energy flavor and CP violating processes, such as $K^0 - \bar{K}^0$ mixing, $B$ physics, and lepton flavor violation, and EDMs. The recent results for CP asymmetries in $b\to s$ penguin processes, including $B \to \phi K_s$, by the Belle and Babar experiments [2,3] are 2.4 and 2.7 $\sigma$ deviated from the Standard Model (SM) prediction, respectively. The deviation might come from the flavor violation induced by the GUT-scale interactions.

While the flavor and CP violating phenomena at low energy are indirect probes to the SUSY GUTs, it is important to study the characteristic signatures. One of them is the correlation between hadronic and leptonic processes. In the SUSY GUTs quarks and leptons are embedded into the common multiplets, and the SUSY breaking terms for the squarks and sleptons is related to each others. For example, the $b\to s$ transition in the right-handed current is correlated with the $Br(\tau \to \mu \gamma)$ in the models.

Another one is the leptonic and hadronic electric dipole moments (EDMs) [6,7,8]. While the processes are flavor diagonal, they depend on the flavor violation in the internal sfermion lines in the SUSY models. Especially, when both the left-handed and right-handed sfermions have flavor violation, the EDMs are enhanced by the heavier fermion masses. In the minimal SUSY standard model (MSSM), the sizable flavor violating SUSY breaking terms for the left-handed squarks are induced by the large top quark Yukawa coupling, and those for the left-handed sleptons may be also generated by the neutrino Yukawa interaction in the SUSY seesaw mechanism [4]. In the SUSY GUTs the flavor violating SUSY breaking terms for the SU(5) partners of the left-handed squarks or sleptons, right-handed squarks and sleptons, are generated by the flavor violating interactions for the colored Higgs multiplets, that are also SU(5) partners of the doublet Higgs multiplets. Thus, non-vanishing EDMs are also a good signature of the SUSY GUTs.
In this article we discuss some flavor and CP violating phenomena in the SUSY SU(5) GUT with the right-handed neutrinos. The neutrino mixing generates new flavor and CP violation in squark and slepton sectors. First, we discuss the hadronic EDMs. The neutrino Yukawa coupling generates the flavor violating SUSY breaking terms for the right-handed down-type squarks. This implies that we can investigate the neutrino sector by the hadronic EDMs. Next, we show the correlations among flavor and CP violating processes in the model, assuming that the mixing between the second and third generations is induced by the neutrino Yukawa coupling. The hadronic EDMs and $Br(\tau \to \mu \gamma)$ are correlated with the CP asymmetry in $b \to s$ penguin processes, and they give constraints on the deviation from the SM prediction. If one of them are discovered, others are good tests for the model.

This article is organized as follows. We review the hadronic EDMs in next section, and discuss the prediction for the hadronic EDMs in the SUSY SU(5) GUT with the right-handed neutrinos in Section 3. In Section 4 we show the correlation among the hadronic EDMs, the CP asymmetries in the $b \to s$ penguin processes, and $Br(\tau \to \mu \gamma)$. Section 5 is devoted to summary.

2. Hadronic EDMs

Now the most stringent bounds on the hadronic EDMs are those of neutron [9] and $^{199}$Hg atom [10], and they give constraints on the SUSY models. In addition to them, the improvement of the deuteron EDM is proposed recently [11], and the sensitivity may reach to $d_D \sim (1 - 3) \times 10^{-27} e \, cm$. If it is realized, one or two orders of magnitude improvement may be achieved relative to the current bounds on the CP violating parameters. In this section we review the hadronic EDMs from the theoretical points of view.

The CP violation in the strong interaction of the light quarks is dictated by the following effective operators,

$$\mathcal{L}_{GP} = \frac{\bar{\theta}}{8\pi} G \tilde{G} + \sum_{q=u,d,s} i \frac{d_q}{2} q g_s (G\sigma)\gamma_5 q,$$

up to the dimension five ones. Here, $G_{\mu\nu}$ is the SU(3) gauge field strength, $\tilde{G}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}$ and $G\sigma = G_{\mu\nu}^a \sigma^{\mu\nu} T^a$. The first term in Eq. (1) is the effective QCD theta term, and the second term is for the quark CEDMs. The $\bar{\theta}$ parameter can be $O(1)$ generically, however, it is strongly constrained by the neutron EDM experiments. One of the most elegant solution is to introduce the Peccei-Quinn (PQ) symmetry [12], since the axion makes $\bar{\theta}$ vanish dynamically. On the other hand, when the quark CEDMs are non-vanishing, the theta term is induced again even if the PQ symmetry is introduced [13]. Thus, in the evaluation of the effect of the quark CEDMs, we need to include the QCD theta term systematically. In the following we show the results in a case that the PQ symmetry is imposed.

We need to translate the quark-level interactions into the hadronic interactions in order to calculate the hadronic EDMs. This is a rather difficult task, and the sizable theoretical uncertainties are still expected for evaluation of hadronic matrix elements. The PCAC
relation and the SU(3) chiral Lagrangian technique are useful for the purpose. However, we still need some matrix elements. The matrix elements for the scalar operators $\bar{q}q$ are determined by the baryon mass splittings and the nucleon sigma term \[14\] as
\[
\langle p|\bar{u}u|p\rangle = 3.5, \quad \langle p|\bar{d}d|p\rangle = 2.8, \quad \langle p|\bar{s}s|p\rangle = 1.4.
\]
(2)
The non-negligible strange quark component implies that that strange CEDM gives sizable contributes to the hadronic EDMs.

For the evaluation of those of the dipole operators $\bar{q}g_s(G\sigma)q$, we need theoretical consideration. In Ref. \[14,15\] it is argued that saturation with the lightest $0^{++}$ state leads to the following relation in the QCD sum rule,
\[
\langle B_a|\bar{q}g_s(G\sigma)q|B_b\rangle \simeq \frac{5}{3}m_0^2\langle B_a|\bar{q}q|B_b\rangle,
\]
(3)
for each quark. This relation is, of course, approximate one. The latest QCD sum analysis of the CP violating pion-nucleon coupling \[16\] shows that the result by evaluation of the leading-order OPE term is almost consistent with that of the saturation with the lightest $0^{++}$ state. When the next and next-to-next leading order term are included, the couplings are suppressed by a factor and a possibility of the accidental cancellation in the isoscalar pion coupling still remains. In the following we assume the saturation with the lightest $0^{++}$ state while keeping the uncertainties in mind.

The neutron EDM is induced by the one-loop diagrams of charged mesons (Fig. 1(b)) while the EDM of $^{199}$Hg atom comes from the isovector channel in the meson exchange between nucleons (Fig. 1(a)). $^{199}$Hg atom is a diamagnetic atom, in which electrons make a close shell. In such atoms, the atomic EDMs are primary sensitive to the CP violation in nucleons and represented by the Schiff moments. The recent evaluation of the Schiff momentum of $^{199}$Hg atom shows that the isoscalar and isotensor channel contributions in the $\pi$ and $\eta^0$ exchanges are suppressed \[17\]. Thus, the contribution of $d_s^C$ to the EDM of $^{199}$Hg atom is generated by the $\pi^0-\eta^0$.

\[\text{Figure 1: a) Isovector contribution to the Schiff momentum of }^{199}\text{Hg atom. Here, } N, N' = (p, n). \text{ b) One-loop diagrams contributing to the neutron EDM. Large bobs represent the CP violating nucleon-meson coupling.}\]

When the Peccei-Quinn symmetry is imposed, the EDMs for neutron and $^{199}$Hg atom are given as \[18\]
\[
d_n = -1.6 \times e(d_u^C + 0.81 \times d_d^C + 0.16 \times d_s^C),
\]
\[ d_{\text{Hg}} = -8.7 \times 10^{-3} \times e(d_u^C - d_d^C + 0.005 \times d_s^C), \]  
respectively. Here, we do not include the contribution of the local counter terms to the neutron EDM since they cannot be not fixed by symmetry argument. The contribution of \( d_s^C \) to the \(^{199}\text{Hg} \) atom EDM, which is suppressed by the \( \pi^0-\eta^0 \) mixing \(^{13}\), is much smaller than that those of other light quarks. The current experimental bounds on the EDMs for neutron \(^9\) and \(^{199}\text{Hg} \) atom \(^{10}\) are

\[ |d_n| < 6.3 \times 10^{-26} e \text{ cm}, \]
\[ |d_{\text{Hg}}| < 1.9 \times 10^{-28} e \text{ cm}, \]  
respectively \((90\%\text{C.L.})\). Thus, the upperbounds on the quark CEDMs are

\[ e|d_u^C| < 3.9(2.2) \times 10^{-26} e \text{ cm}, \]
\[ e|d_d^C| < 4.8(2.2) \times 10^{-26} e \text{ cm}, \]
\[ e|d_s^C| < 2.4(44) \times 10^{-25} e \text{ cm}, \]  
from the EDM of neutron \(^{199}\text{Hg} \) atom). Here, we assume that the accidental cancellation among the CEDMs does not suppress the EDMs. Notice that we do not include the theoretical uncertainties which comes from the matrix elements here.

The deuteron EDM is represented as

\[ d_D = (d_n + d_p) + d_{\text{NN}}^N, \]  
where \( d_p \) is the proton EDM and the second term comes from the CP violating nuclear force. The nuclear dynamics in deuteron is rather transparent, and the theoretical uncertainty is expected to be small. When the PQ symmetry is imposed, each components in Eq. \((7)\) are given as \(^{13}\)

\[ (d_p + d_n) = (-5.1 \times \tilde{d}_u + 2.1 \times \tilde{d}_d + 0.32 \times \tilde{d}_s) e \text{ cm}, \]
\[ d_{\text{NN}}^N = (-11 \times \tilde{d}_u + 11 \times \tilde{d}_d - 0.063 \times \tilde{d}_s) e \text{ cm}, \]  
respectively. From these equations, the measurement of the deuteron EDM will be a stringent test for the SM. If the sensitivity of \( d_D \sim (1-3) \times 10^{-27} e cm \) is established, \( e\tilde{d}_u \sim e\tilde{d}_d \sim 10^{-28} e cm \) can be probed from the nuclear force. Also, for the strange quark CEDM, \( e\tilde{d}_s \sim 10^{-26} e cm \) may be possible from the nucleon EDMs.

3. Hadronic EDMs in the SUSY SU(5) GUT with the right-handed neutrinos

Let us discuss the hadronic EDMs in the SUSY SU(5) GUT with the right-handed neutrinos. First, we review the flavor structure in the squark and slepton mass matrices in the SUSY SU(5) GUT with the right-handed neutrinos. The Yukawa interactions for quarks and leptons and the Majorana mass terms for the right-handed neutrinos in this model are given by the following superpotential,

\[ W = \frac{1}{4} f_{ij}^u \Psi_i \Psi_j H + \sqrt{2} f_{ij}^d \Psi_i \Phi_j H + f_{ij}^e \Phi_i \overline{\Psi}_j H + M_{ij} \overline{\Psi}_i \overline{\Psi}_j, \]
where $\Psi$ and $\Phi$ are for $10$- and $5$-dimensional multiplets, respectively, and $\mathcal{N}$ is for the right-handed neutrinos. $H$ ($\overline{H}$) is $5$- ($\overline{5}$-) dimensional Higgs multiplets. After removing the unphysical degrees of freedom, the Yukawa coupling constants in Eq. (9) are given as follows,

$$
\begin{align*}
    f^u_{ij} &= V_{ki} f_{uk} e^{i\varphi_{uk}} V_{kj}, \\
    f^d_{ij} &= f_d \delta_{ij}, \\
    f^e_{ij} &= e^{i\varphi_{ij}} f^* \nu_j,
\end{align*}
$$

Here, $\varphi_u$ and $\varphi_d$ are CP violating phases inherent in the SUSY SU(5) GUT. They satisfy $\sum_i \varphi_{fi} = 0$ ($f = u$ and $d$). The unitary matrix $V$ is the CKM matrix in the extension of the SM to the SUSY SU(5) GUT, and each unitary matrices $U$ and $V$ have only a phase. When the Majorana mass matrix for the right-handed neutrinos is diagonal in the basis of Eq. (10), $U$ is the MNS matrix observed in the neutrino oscillation. In this paper we assume the diagonal Majorana mass matrix in order to avoid the complexity due to the structure. In this case the light neutrino mass eigenvalues are given as $m_{\nu_i} = f^2_{\nu_i} \langle H_f \rangle^2 / M_N$, where $H_f$ is a doublet Higgs in $H$.

The colored Higgs multiplets $H_c$ and $\overline{H_c}$ are introduced in $H$ and $\overline{H}$ as SU(5) partners of the Higgs doublets in the MSSM, respectively. They have new flavor violating interactions in Eq. (9). If the SUSY-breaking terms in the MSSM are generated by dynamics above the colored Higgs masses, such as in the gravity mediation, the sfermion mass terms may get sizable corrections by the colored Higgs interactions. The interactions are also baryon-number violating, and then proton decay induced by the colored Higgs exchange is a serious problem, especially in the minimal SUSY SU(5) GUT [19]. However, it depends on the detailed structure in the Higgs sector [20,21]. Thus, we ignore the proton decay while we adopt the minimal Yukawa structure in Eq. (9).

In the minimal supergravity scenario the SUSY breaking terms are supposed to be given at the reduced Planck mass scale ($M_G$). In this case, the flavor violating SUSY breaking mass terms at low energy are induced by the radiative correction, and they are qualitatively given in a flavor basis as

$$
\begin{align*}
    (m^2_{u_{ij}})_{ij} &\approx -V_{i3} V^*_{j3} f^2_{lh}(4\pi)^2 (3m_0^2 + A_0^2) (2 \log \frac{M_G^2}{M_{H_c^2}} + \log \frac{M_{H_s^2}}{M_{SUSY^2}}), \\
    (m^2_{d_{ij}})_{ij} &\approx -e^{-i\varphi_u} V_{i3} V^*_{j3} \frac{2 f^2_{lh}(4\pi)^2}{(3m_0^2 + A_0^2)} (3m_0^2 + A_0^2) \log \frac{M^2_G}{M_{H_c^2}}, \\
    (m^2_{d_{ij}})_{ij} &\approx -V_{i3} V^*_{j3} \frac{f^2_{lh}(4\pi)^2}{(3m_0^2 + A_0^2)} (3 \log \frac{M_G^2}{M_{H_c^2}} + \log \frac{M_{H_s^2}}{M_{SUSY^2}}), \\
    (m^2_{d_{ij}})_{ij} &\approx -e^{-i\varphi_d} U_{ik} U_{jk} \frac{f^2_{lh}(4\pi)^2}{(3m_0^2 + A_0^2)} \log \frac{M_G^2}{M_{H_c^2}}, \\
    (m^2_{\ell_{ij}})_{ij} &\approx -U_{ik} U^*_{jk} \frac{f^2_{lh}(4\pi)^2}{(3m_0^2 + A_0^2)} \log \frac{M_G^2}{M_{H_c^2}}, \\
    (m^2_{\ell_{ij}})_{ij} &\approx -e^{-i\varphi_d} V_{i3} V^*_{j3} \frac{3 f^2_{lh}(4\pi)^2}{(3m_0^2 + A_0^2)} \log \frac{M_G^2}{M_{H_c^2}}.
\end{align*}
$$

(11)
with \( i \neq j \), where \( \varphi_{uij} \equiv \varphi_{ui} - \varphi_{uj} \) and \( \varphi_{dij} \equiv \varphi_{di} - \varphi_{dj} \), and \( M_{hc} \) is the colored Higgs mass. Here, \( M_{SUSY} \), \( m_0 \) and \( A_0 \) are the SUSY-breaking scale in the MSSM and the universal scalar mass and trilinear coupling, respectively. \( f_t \) is the top quark Yukawa coupling constant while \( f_b \) is for the bottom quark. As mentioned above, the off-diagonal components in the right-handed squarks and slepton mass matrices are induced by the colored Higgs interactions, and they depend on the CP violating phases in the SUSY SU(5) GUT with the right-handed neutrinos \[22\].

When both the left-handed and right-handed squarks have the off-diagonal components in the mass matrices, the EDMs and CEDMs for the light quarks are enhanced significantly by the heavier quark masses. The CEDMs of the down-type quarks are generated by the diagrams in Fig. 2. In the SUSY SU(5) GUT with the right-handed neutrinos, the neutrino Yukawa coupling induces the flavor violating mass terms for the right-handed down-type squarks. Since the flavor violating mass terms for the left-handed down-type squarks are expected to be dominated by the radiative correction induced by the top quark Yukawa coupling as in Eq. (11), we can investigate or constrain the structure in the neutrino sector.

![Figure 2: Dominant diagram contributing to the CEDMs of down and strange quarks when both the left-handed and right-handed squarks have flavor mixings.](image)

The CEDMs of the down and strange quarks derived by the flavor violation in the both the left-handed and right-handed quark mass matrices are given by the following dominant contribution, which is enhanced by the heavier quark masses,

\[
d_{di}^C = \frac{c \alpha_s m_{\tilde{g}}}{4\pi m_d^2} f \left( \frac{m_{\tilde{d}}^2}{m_d^2} \right) \text{Im} \left[ \left( \delta_{d}^d \right)_{L/R} \left( \delta_{d}^d \right)_{L/R} \right],
\]

(12)

where \( m_{\tilde{g}} \) and \( m_{\tilde{d}} \) are the gluino and averaged squark masses and \( c \) is the QCD correction, \( c \sim 0.9 \). The mass insertion parameters are defined as

\[
(\delta_{ij}^d)_{L/R} \equiv \frac{(m_{\tilde{d}}^2)_{ij}}{(m_d^2)}, \quad (\delta_{i}^d)_{LR} \equiv \frac{m_{d_1} (A_{d_1}^{(d)} - \mu \tan \beta)}{m_d^2},
\]

(13)

The function \( f(x) \) is given as

\[
f(x) = \frac{177 + 118x - 288x^2 - 6x^3 - x^4 + (54 + 300x + 126x^2) \log x}{18(1 - x)^6},
\]

(14)
and \( f(1) = -11/180 \).

In Fig. 3 we show the CEDMs for the down and strange quarks in the SUSY SU(5) GUT with the right-handed neutrinos. The figures come from Ref. \[8\]. In Fig. 3(a) the strange quark CEDM is shown as a function of the right-handed tau neutrino mass. We take \( M_{H_c} = 2 \times 10^{16}\text{GeV}, \ m_{\nu_\tau} = 0.05\text{eV} \) and \( U_{\mu 3} = 1/\sqrt{2} \). For the SUSY breaking parameters to be fixed, the minimal supergravity scenario is assumed, and \( m_0 = 500\text{GeV}, \ A_0 = 0, \ m_{\tilde{g}} = 500\text{GeV} \) and \( \tan\beta = 10 \), which lead to \( m_{\tilde{q}} \approx 640\text{GeV} \). While the electron and muon neutrino Yukawa interactions contribute to the flavor violation in the right-handed down-type squark mass matrix, they are ignored here. They are bounded by the constraints from the \( K^0 - \bar{K}^0 \) mixing and \( Br(\mu \to e\gamma) \) when \( |U_{e2}| \sim 1/\sqrt{2} \). From this figure, the right-handed tau neutrino mass should be smaller than \( \sim 3 \times 10^{14}\text{GeV} \).

In Fig. 3(b) the down quark CEDM is shown as a function of the right-handed tau neutrino mass. This comes from non-vanishing \( U_{e3} \) in our assumption that the right-handed neutrino mass matrix is diagonal. The current bound is not significant even when \( U_{e3} = 0.2 \).

The new technique for the measurement of the deuteron EDM has a great impact on the quark CEDMs as mentioned in the previous section. The sensitivity of \( d_D \sim 10^{-27}\text{cm} \) may imply that we can probe the structure in the neutrino sector even if \( M_{N_3} \sim 10^{13}\text{GeV} \) or \( U_{e3} \sim 0.02 \) \[8\].

![Figure 3: CEDMs for the strange quark in (a) and for the down quark in (b) as functions of the right-handed tau neutrino mass, \( M_{N_3} \). Here, \( M_{H_c} = 2 \times 10^{16}\text{GeV}, \ m_{\nu_\tau} = 0.05\text{eV}, \ U_{\mu 3} = 1/\sqrt{2}, \) and \( U_{e3} = 0.2 \) and 0.02. For the MSSM parameters, we take \( m_0 = 500\text{GeV}, \ A_0 = 0, \ m_{\tilde{g}} = 500\text{GeV} \) and \( \tan\beta = 10 \).](image)

We have not discussed the up quark CEDM in the model. The right-handed up-type squarks also have the flavor violating mass terms, which depend on the GUT CP violating phases, and the magnitudes are controlled by the CKM matrix at the GUT scale. (See Eq. \[11\].) They also contribute to the hadronic EDMs in the SUSY SU(5) GUT \[7\]. However, the off-diagonal terms in both the left-handed and right-handed up-type squark mass matrices are induced by the bottom quark Yukawa coupling, and the up quark CEDM and EDM induced by them are proportional to \( \tan^4\beta \). It is found that the CEDM for the up quark can reach to \( 10^{-28}\text{cm} \) when \( \tan\beta \simeq 35 \) and \( m_{\text{SUSY}} \simeq 500\text{GeV} \). Thus, if we observe the non-vanishing deuteron EDM larger than \( 10^{-27}\text{cm} \), it might be interpreted as the contribution of the down or strange quark CEDM in the SUSY GUT.
with the right-handed neutrinos.

4. Correlation among processes induced by mixings between the second and third generations

The flavor and CP violating processes predicted in the SUSY GUTs are indirect probes. Thus, it is important to take correlations among various processes. The recent results for CP asymmetries in $b$–$s$ penguin processes, such as $B \to \phi K_s$, by the Belle and Babar experiments may be explained by introduction of the flavor violation between the second and third generations in the right-handed down-type squark mass matrix, which is suggested by the atmospheric neutrino oscillation. The $b$–$s$ penguin processes are sensitive to the models beyond the SM [24], and the various studies are done for the processes in the SUSY SU(5) GUT with the right-handed neutrinos. Other processes induced by the mixing between the second and third generations are given in Ref. [23].

First, we show the correlation between $\tilde{d}_s$ and $S_{\phi K_s}$. When $(\delta^{(d)}_{32})_R$ is not vanishing, $S_{\phi K_s}$ may have a sizable deviation from the SM prediction. The dominant contribution to $S_{\phi K_s}$ is supplied by a penguin diagram with the double mass insertion of $(\delta^{(d)}_{32})_R$ and $(\delta^{(d)}_3)_{LR}$ in Fig. 4(b). The contribution is represented as $H = -C^R_8 g_s/(8\pi^2) m_b s_R (G\sigma) b_L$, where

$$C^R_8 = \frac{\pi \alpha_s m_\tilde{q}}{m_\tilde{q}} \frac{m_\tilde{q}}{m_b} (\delta^{(d)}_{32})_{LR} (\delta^{(d)}_3)_{LR} g \left( \frac{m_\tilde{q}^2}{m_\tilde{q}} \right),$$

up to the QCD correction. Here,

$$g(x) = -\frac{53 - 9x - 45x^2 + x^3 + (18 + 66x + 12x^2) \log x}{6(1-x)^5}$$

In a limit of $x \to 1$, $g(x)$ is 7/60. On the other hand, since the left-handed down-type squarks have the flavor violating mass terms induced by the top quark Yukawa coupling, the strange quark CEDM is also predicted by the diagram in Fig. 4(a). From Eqs. (12) and (15), we find a strong correlation between $\tilde{d}_s$ and $C^R_8$ as

$$\tilde{d}_s = -\frac{m_b}{4\pi^2 21} \text{Im} \left[ (\delta^{(d)}_{23})_L C^R_8 \right]$$

for $x = 1$, up to the QCD correction. The coefficient 11/21 in Eq. (17) changes from 1 to 1/3 for $0 < x < \infty$.

In Fig. 5, the correlation between $\tilde{d}_s$ and $S_{\phi K_s}$ is shown, assuming Eq. (17) up to the QCD correction. This figure comes from Ref. [18]. In this figure, we take $(\delta^{(d)}_{23})_L = -0.04$, $\arg[C^R_8] = \pi/2$ and $|C^R_8|$ corresponding to $10^{-5} < |(\delta^{(d)}_{32})_R| < 0.5$. The matrix element of chromomagnetic moment in $B \to \phi K_s$ is parameterized by $\kappa$. Since $\kappa$ may suffer from the large hadron uncertainty, we show the results for $\kappa = -1$ and $-2$. From this figure, the deviation of $S_{\phi K_s}$ from the SM prediction due to the gluon penguin contribution should be
suppressed when the constraints on $\tilde{d}_s$ from the hadronic EDMs, especially, the neutron EDM, are applied. Therefore, the hadronic EDMs give an important implication to $S_{\phi K_s}$.

Here, we stress the theoretical uncertainties in the evaluation for the hadronic EDMs, again. In this paper we assume the saturation with the lightest $0^{++}$ state for the evaluation of the matrix element $\langle B_a | g \bar{g}_a(G\sigma)q | B_b \rangle$ as in Eq. (3), and our result relies on this assumption strongly. Also, the neutron EDM evaluation does not include the local counter term contribution. These uncertainties might still allow the sizable deviation of $S_{\phi K_s}$ from the SM prediction. The further experimental and theoretical improvement of the bounds on the hadronic EDMs is very important.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{(a) The dominant diagram contributing to the strange quark CEDM when both the left-handed and right-handed squarks have flavor mixings. (b) The dominant SUSY diagram contributing to the CP asymmetry in $B \rightarrow \phi K_s$ when the right-handed squarks have a mixing.}
\end{figure}

Next, we show the correlation between $S_{\phi K_s}$ and $Br(\tau \rightarrow \mu \gamma)$. The LFV processes are originally predicted in the SUSY seesaw model $^{[4,27]}$. As shown in Eq. (11), the $(2,3)$ components in the left-handed slepton and right-handed down-type squark mass matrix are related as $^{[23]}

$$
(m_{d_R}^2)_{23}^* \simeq e^{-i\phi_{23}} \frac{\log M_G}{\log M_{N_3}} \frac{M_{\mu}}{M_{\tau}} (m_{l_L}^2)_{23}.
$$

Thus, the observables are also correlated.

In Fig. 6 we show $Br(\tau \rightarrow \mu \gamma)$ as a function of $S_{\phi K_s}$ for fixed gluino masses $m_{\tilde{g}}$ in the SUSY SU(5) GUT with the right-handed neutrinos. This figure comes from Ref. $^{[23]}$. Here, tan $\beta$ is 30, $200 \text{ GeV} < m_0 < 1 \text{ TeV}$, $A_0 = 0$, $m_{\nu_e} = 5 \times 10^{-3} \text{ eV}$, $M_{N_3} = 5 \times 10^{14} \text{ GeV}$, and $U_{32} = 1/\sqrt{2}$. The deviation of $S_{\phi K_s}$ is maximized when $m_{\tilde{g}}$ is lighter and $m_0$ is comparable to $m_{\tilde{g}}$. While larger tan $\beta$ enhances the deviation of $S_{\phi K_s}$ from the SM prediction ($\sim 0.7$), it is also bounded by the constraint from $Br(\tau \rightarrow \mu \gamma)$. If the results of the Belle and Babar experiments are confirmed in the future, the search for $\tau \rightarrow \mu \gamma$ is also a good check for the SUSY SU(5) GUT with the right-handed neutrinos.

5. Summary

While the low-energy flavor and CP violating observables are predicted in the SUSY
Figure 5: The correlation between $\tilde{d}_s$ and $S_{\phi K_s}$ assuming $\tilde{d}_s = -m_b/(4\pi^2)(11/21)\text{Im}[(\delta_{23}^{(d)})_L C_R^s]$. Here, $(\delta_{23}^{(d)})_L = -0.04$ and $\arg[C_R^s] = \pi/2$. $\kappa$ comes from the matrix element of chromomagnetic moment in $B \to \phi K_s$. The dashed (dotted) line is the upperbound on $\tilde{d}_s$ from the EDM of $^{199}$Hg atom (neutron).

GUTs, they are the indirect probes. Thus, it is important to study the characteristic signatures. One way is to check the correlation among various processes. Especially, hadronic and leptonic processes are related to each others due to the GUT relation. Second is the EDMs. They are induced when both mixings of the left-handed and right-handed sfermions are non-vanishing. In this paper we discuss the hadronic EDMs and the correlation among low-energy processes induced by mixing between the second and third generations in the SUSY SU(5) GUT with the right-handed neutrinos.

6. References

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Figure 6: $Br(\tau \to \mu\gamma)$ as a function of $S_{\phi K_S}$ for fixed gluino masses $m_{\tilde{g}} = 400, 600, 800,$ and $1000\text{GeV}$. $\tan \beta$ is $30$. Also, $200\text{GeV} < m_0 < 1\text{TeV}$, $A_0 = 0$, $m_{\nu_\tau} = 5 \times 10^{-2}\text{eV}$, $M_N = 5 \times 10^{14}\text{GeV}$, and $U_{32} = 1/\sqrt{2}$. $(\varphi_{d_3} - \varphi_{d_2})$ is taken for the deviation of $S_{\phi K_S}$ from the SM prediction to be maximum. The constraints from $b \to s\gamma$ and the light-Higgs mass are imposed.

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