Simulation of flow over high-lifted turbine cascade at low Reynolds numbers

A P Duben¹, T K Kozubskaya¹, O V Marakueva¹ and D V Voroshnin²

¹Keldysh Institute of Applied Mathematics of RAS, Moscow, Russia
²Numeca Russia, St. Petersburg, Russia

¹E-mail: aduben@keldysh.ru
²E-mail: contact@numeca.ru

Abstract. This paper is dedicated to the simulation of flow in the T106C high loaded turbine cascade. A presence of a laminar-turbulent transition at the suction side of the blade caused by the separation of the boundary layer leads to a significant increase in kinetic energy losses. Predictions using both RANS and scale-resolving methods are carried out. The RANS computations are performed by the SST and BSL-EARSM models with Abu-Ghannam and Shaw criterion of laminar-turbulent transition or the $\gamma - Re_{Bl}$ model with different closing correlation dependencies. The hybrid RANS-LES IDDES method is used for scale-resolving simulations. The validation is done by comparing the numerical results with the experimental data on the isentropic Mach number along the blade surface and the relative total pressure losses at different Reynolds numbers. The RANS and IDDES results are comparatively analyzed. A strong dependence of all the numerical results on the incoming turbulent flow conditions is observed.

1. Introduction

Nowadays in order to lower the engine weight, designers are forced to reduce the number of blades in the low-pressure turbine (LPT) while maintaining the load on the whole stage. The LPT normally operates in a wide range of rather low Reynolds numbers reaching $10^7$ and even less in the flight mode, so the boundary layer mostly remains laminar. Stronger pressure gradients arising from the higher loading on a blade may cause a separation of the laminar boundary layer and its turbulization downstream. The boundary layer may then reattach the surface forming a recirculation bubble. The process of laminar-turbulent (LT) transition on the blade surface is schematically presented in Figure 1. Here the effect of the bubble is local and the pressure distribution over the blade surface upstream and downstream the bubble remains the same. A significant increase in the angle of attack or a decrease in the Reynolds number results in the growth of the bubble. A further lowering of the Reynolds number may lead to the formation of an open bubble, i.e. a strong separation without reattachment to the blade surfaces [1-3]. In this case, the pressure distribution changes significantly, the peak value of the Mach number decreases, and one can observe a noticeable amplification of kinetic energy losses. The LT transition caused by a flow separation can appear both on the suction and pressure sides of the blade,
however up to 60% of losses in LPT happen on the suction side [4] and therefore the most studies focus on the transition on this side.

![Diagram of laminar-turbulent transition with bubble formation](image)

**Figure 1.** A scheme of laminar-turbulent transition with the formation of a short bubble [3].

The modelling of flow with LT transition in the framework of the RANS/URANS (Reynolds Averaged Navier-Stokes / Unsteady RANS) approach requires additional models for a correct mathematical description of the transition phenomenon. In turbomachinery, the most used models are the Abu-Ghanam and Shaw (AGS) criterion [5] and the $\gamma - \tilde{Re}_{\theta t}$ model [6-9]. They are based on the concept of "intermittency" ($\gamma$) which is introduced into a turbulence model as a weight factor of the kinetic energy generation. According to AGS, the criterion of transition which switches the intermittency from 0 to 1 is determined by the empirical correlations using the non-local flow parameters: turbulence intensity and pressure gradient at the edge of the boundary layer. To diffuse the criterion into the boundary layer, the $\gamma - \tilde{Re}_{\theta t}$ model is supplemented by the transport equation for the transition onset momentum thickness Reynolds number ($\tilde{Re}_{\theta t}$), to trigger the transition onset the intermittency transport equation is used. Thus, the estimation of the LT transition localization is performed based on the local flow parameters, while the intermittency changes smoothly. A lot of studies (see, for instance, [10-12]) demonstrate the efficiency of the RANS approach with the $\gamma - \tilde{Re}_{\theta t}$ model for the LT transition to simulate the flow including the separation and bubble formation in the turbine cascades. The authors of works [13-18] propose different correlation dependencies for estimating the critical momentum thickness Reynolds number and the transition length within the $\gamma - \tilde{Re}_{\theta t}$ model. As it is shown in [12], the choice of this or that correlation form can significantly influence the results of simulation.

To better capture the details of flow in the turbine cascade, the authors of works [10,20,21] use the scale-resolving simulation (SRS). The computational cost of the corresponding computations is significantly high due to the rigorous limitation for the mesh step not only in the region along the blade surface, but also in the wake downstream the trailing edge and the region upstream the blade to reproduce the incoming turbulent flow properly. For instance, paper [20] shows that at rather low Reynolds numbers (below $10^5$) the presence or absence of resolved turbulence at the inlet can significantly affect the resulting aerodynamic characteristics at the outlet, and therefore the energy losses are rather sensitive to the inflow data.

This paper considers the flow in the turbine high-loaded cascade T106C. At low Reynolds numbers the flow is characterized by the presence of LT transition caused by the separation of the laminar boundary layer with the formation of a short bubble. The flow simulations are carried out using both RANS and scale-resolving approaches. The RANS predictions are performed using the commercial software Numeca Fine/Turbo v. 14.1. We use both the AGS and $\gamma - \tilde{Re}_{\theta t}$ LT transition models within the turbulence models SST [22,23] and BSL-EARSM [24]. The correlations of Langtry [6], Malan [15], Sorensen [18], Kelterer [17], and the modification of Langtry [19] are considered to close the $\gamma - \tilde{Re}_{\theta t}$ model. The scale-resolving predictions are carried out using the in-house research code NOISEtte. We use the hybrid non-zonal RANS-LES method IDDES [25] to simulate the turbulent flow. The validation is done through the comparison of the numerical results with the available experimental
data [26]: the distribution of isentropic Mach number along the blade surface and relative total pressure losses for various Reynolds numbers. We comparatively analyse the results of RANS and SRS too.

2. Experimental setup

The main geometrical parameters of the T106C cascade are presented in Table 1 and Figure 2. In more detail the description of the experimental set-up is given in [26].

Table 1. Main parameters of the T106C cascade.

| Parameter                      | Value   |
|--------------------------------|---------|
| Design exit Mach number, \( Ma_{2th} \) | 0.65    |
| Chord length, \( l \)          | 100 mm  |
| Axial chord length, \( l_{ax} \) | 85.9 mm |
| Pitch to chord ratio, \( \tau \) | 0.95    |
| Inlet flow angle, \( \beta_1 \) | 127.7°  |
| Outlet flow angle, \( \beta_2 \) | 29.4°   |

![Figure 2. T106C profile.](image)

The flow parameters at the inlet and outlet are defined by the theoretical exit Mach number \( Ma_{2th} \) and Reynolds number \( Re_{2th} \) under the assumption of isentropic flow in the blade cascade:

\[
Ma_{2th} = \left\{ \frac{2}{\gamma - 1} \left[ \frac{P_{t1}}{P_k} \right]^{\gamma - 1} \right\}^{1/2} \tag{1}
\]

\[
Re_{2th} = \sqrt{\frac{\gamma l}{R C_s}} P_{t1} \cdot \frac{Ma_{2th} \cdot P_k \cdot \left( \frac{T_{t1}}{1 + (\gamma - 1) \cdot Ma_{2th}^2} + S \right)}{1 + (\gamma - 1) \cdot Ma_{2th}^2} \tag{2}
\]

Here \( P_{t1} \) is the inlet total pressure, \( P_k \) is the outlet static pressure, \( C_s \) and \( S \) are the constants of the Sutherland’s law for dynamic viscosity, \( R \) and \( \gamma \) are the universal gas constant and heat capacity ratio correspondingly. The total temperature at the inlet is equal to \( T_{t1} = 303.15 \) K.

We perform the numerical simulations for zero incidence angle at \( Re_{2th} = 90000, 200000 \) and \( 500000 \). The turbulence intensity \( Tu \) of the incoming flow in the experiment is in the range of 2.9-3.1%, the streamwise integral length scale is \( \Lambda = 20 \) mm. To reproduce the experimental flow conditions, we choose the parameters as follows: total pressure \( P_{t1} \) according to the regime (Table 2), and total temperature \( T_{t1} = 303.15 \) K at the inlet boundaries; flow direction \( (V_X = 0, V_Y = -0.61, V_Z = 0.79) \); turbulence intensity \( Tu = 3\% \) and turbulent length scale \( \Lambda = 20 \) mm. The static pressure values are fixed at the outlet according to the regime (see Table 2).

Table 2. Flow parameters upstream and downstream the cascade.

| \( Re_{2th}, \times 10^4 \) | \( P_{t1}, \text{Pa} \) | \( P_k, \text{Pa} \) |
|---------------------------|-------------------|----------------|
| 9                         | 7860.8            | 5900           |
| 20                        | 17467             | 13110          |
| 50                        | 43667.6           | 32775          |
The static pressure is measured in the middle section of the middle blade. The results are presented as a distribution of isentropic Mach number. To estimate the total pressure loss, the measurements are made in the section located at 40% of the axial chord downstream the trailing edge of the blade. The total pressure loss factor is calculated according to the formula:

$$\zeta = \frac{P_{t1} - P_{t2}}{P_{t1} - P_k} \cdot 100\%$$

(3)

3. Computational setup for RANS simulations

The T106C blade profile used in the simulations is obtained by scaling the T106A profile since, according to [26], they are identical. At the same time, the inlet blade angle of the final model exceeds the value indicated in the paper by 10°, however a significant difference between the computational and experimental data is observed only in the vicinity of the leading edge. Therefore, it is possible to assume that the profiles are similar with the exception of the inlet angle.

3.1. Computational mesh

The computational mesh we use is a block-structured hexagonal mesh built by the AutoGrid5 software [19]. The computational domain contains 1/10 part of the blade height with the symmetry boundary conditions at the lower and upper end and one blade passage with the periodicity conditions. The input and output boundaries are located at a distance longer than three axial chords far from the blade. The mesh and domain topology are shown in Figure 3 (every third mesh line is drawn). Since the mesh topology has no significant impact on the numerical results, topology 1 is chosen. Table 3 gives the basic information about the meshes in use. All the meshes have 9 cells in the spanwise direction, the maximum local value of \(y^+\) reaches 2 at for \(Re_{th} = 500000\). The preliminary computations show that the mesh independency is reached on the Mesh 2.

![Figure 3. Computational meshes used in RANS simulations: left – mesh Topology 1 (corresponds to Mesh 3 in Table 3), right – mesh Topology 2 with the mesh lines built along the flow streamlines downstream the blade.](image)

| Parameter                   | Mesh 1 | Mesh 2 | Mesh 3 |
|-----------------------------|--------|--------|--------|
| Number of nodes, \(\times 10^3\) | 206    | 554    | 1090   |
| Number of nodes in the O-block | 25     | 49     | 57     |
| Minimum skewness angle      | 45.1°  | 45.7°  | 46.6°  |
| Max. scale factor           | 1.9    | 1.9    | 1.6    |
| Near-wall region scale factor | 1.0-1.3 | 1.0-1.2 | 1.0-1.2 |
| Max. aspect ratio           | 571    | 571    | 571    |
3.2. Mathematical models and numerical methods

The RANS simulations are carried out using the commercial software *Numeca Fine/Turbo* v. 14.1.

Both the AGS and $\gamma-\bar{R}e_{th}$ LT transition models are used. The AGS criterion is considered within the SST turbulence model [22,23]. The binary distribution of intermittency which controls the generation of specific dissipation rate $\omega$, besides the generation of turbulence kinetic energy $k$, is considered. The $\gamma-\bar{R}e_{th}$ model is used with both the BSL-EARSM [24] and SST models. The coefficients of the $\gamma-\bar{R}e_{th}$ model correspond to those suggested in [6]. The BSL-EARSM turbulence model is used with the following coefficients: $c_{a1} = 1.5$, $c_{a2} = 0.08$. The correlations of Malan [15], Kelterer [17], Sorensen [18] and Langtry [6] are taken to compute the critical Reynolds number and the transition length. Additionally, the modification of Langtry correlations proposed by Numeca International [19] is tried within the $\gamma-\bar{R}e_{th}$ BSL-EARSM model.

The central 2nd order scheme is used for the spatial discretization, and the 4-stages Runge-Kutta method – for the time advancing.

In the most computations the convergence is observed when the global residual is reduced 5-6 times and a difference between the input and output flow rates becomes less than 0.02 %.

The computational time of one simulation with the Mesh 3 is about 45 minutes on 4 cores on a computer with Intel(R) Xeon (R) CPU E5-2690v3.

4. Computational setup for IDDES simulations

The scale-resolving simulations of the flow in the T106C turbine high-loaded cascade are carried out using the latest version of the hybrid non-zonal RANS-LES method IDDES [25] with the special shear-layer adapted subgrid lengthscale $\Delta_{SLA}$. The advantage of this method is the adaptive switching to different simulation regimes (wall-resolved LES, or wall-modelled LES / WMLES, or RANS) depending on the local flow characteristics and mesh resolution.

The preliminary simulations of the flow are conducted with no resolved turbulence at the inlet (the incoming turbulence intensity is $Tu=0$). The obtained results show a significant discrepancy with respect to the reference experimental data. That is why we introduce the synthetic turbulence generator [27] acting as a distributed volume source [28] (hereinafter referred to as VSTG) to provide the upstream unsteady turbulent flow corresponding to the given parameters (turbulence intensity $Tu=3\%$ and turbulence lengthscale scale $\Lambda=20$ mm), at a distance of $0.35l$ upstream the leading edge of the blade. The source form, size and other parameters are adjusted by the additional simulations of homogenous turbulent flow with the conditions like the experimental ones. As it is seen from Figure 4, the incoming synthetically-generated resolved turbulence cover both the suction and pressure regions near the blade surface. We assume that the turbulence far from the blade surface affect negligibly the kinetical energy losses studied in the case. Moreover, to impose the volume source over the entire height of the channel with the corresponding space resolution would require an unreasonably large mesh.

**Figure 4.** Computational domain and mesh in XY section for IDDES simulations. The mesh lines are colored by velocity magnitude. VSTG shows a zone of the volume source generating the turbulent pulsations. The dashed line corresponds to section $x/l_{ax}=1.4$ where the total pressure losses are measured.
Figure 4 demonstrates the computational domain together with the unstructured mesh in the XY cross section. It is built in such a way so that the key and sensitive areas of the flow field are covered with the mesh of high resolution. Among these areas are the region near the blade surface with the mesh wall-tangential step $\Delta t=1.5\cdot 10^{-3}l$, the wake region downstream the trailing edge with the isotropic mesh step $\Delta=10^{-3}l$, and the area upstream the leading edge of the blade starting from the zone of the synthetic turbulence generator. The size of the computational domain in the spanwise direction $Z$ is $L_z=0.15l$. The first wall-normal mesh step is chosen in such a way that the value $y^+$ does not exceed 1 for the maximal Reynolds number $Re_{268}=50\times 10^4$. Figure 5 shows the diagrams of dimensionless mesh step values in the wall-tangential ($\Delta x^+$) and spanwise ($\Delta z^+$) directions along the suction side of the blade for different Reynolds numbers. The mesh parameters are close to those that are recommended for confining LES simulations. A total size of the 3D computational mesh is about 83.2 million nodes with 0.53 million in the 2D XY cross section. The number of cells in the spanwise direction, except the buffer zones built for periodicity conditions, is 150.

![Figure 5](image)

The vertex-centered finite-volume numerical method implemented in the parallel research code NOISEtte [29,30] is used for the IDDES simulations. It is based on the original higher-accuracy EBR (Edge-Based Reconstruction) scheme [31]. Its hybrid modification [33] which blends the upwinding and central-differences approximations of fluxes minimizes the numerical dissipation. The time integration is performed using the implicit 2nd order scheme with the Newton iterations. For the cases considered, three iterations allowed for achieving the relative residual equal approximately to $10^{-2}$. The system of linear difference equations is solved by the method of bi-conjugated gradients. In the computations performed, the maximal Courant–Friedrichs–Lewy (CFL) number estimated globally in the whole domain was 10 and does not exceed 1 in the disturbed flow areas with the resolved turbulent structures.

The numerical results were obtained as follows. The 3D IDDES computation was started from the corresponding pre-computed 2D RANS solution with no LT transition model, then the simulation continued until a statistically established unsteady solution was obtained. After that the unsteady numerical data was accumulated to calculate the average flow characteristics. Note that besides time averaging space averaging along the homogeneous direction, i.e. along the $Z$ axis, was also completed. The interval of data accumulation of $5t/U_0$ time length was quite enough to obtain accurate average parameters of the simulated flow and aerodynamic characteristics of the blade.

At each considered regime, the computational cost of one simulation for $5t/U_0$ convective time units is about 13266 CPU hours if using the hybrid MPI+OpenMP parallel model on a supercomputer with Intel Xeon E5-2680v3 CPUs.
5. Numerical results and discussion

5.1. RANS results versus experimental data

Figure 6 presents a comparison of the experimental data with the numerical results obtained using the AGS SST and \( \gamma - \overline{Re}_{Bl} \) SST models. For convenience, the isentropic Mach number distributions are shown only for the most distinguishing and sensitive area located from middle of the blade to the trailing edge along the suction side of the profile.

![Graphs showing Mach number distributions](image)

**Figure 6.** RANS results obtained using AGS SST and \( \gamma - \overline{Re}_{Bl} \) SST models with correlations of Langtry – 1, Malan – 2, Sorensen – 3, and Kelterer – 4: isentropic Mach number \( Ma_{in} \) distribution along the blade suction side at Reynolds numbers \( Re_{2th}=9\times10^{4} \) (a), \( Re_{2th}=20\times10^{4} \) (b), \( Re_{2th}=50\times10^{4} \) (c), and total pressure loss coefficient (d).

As seen from Figure 6, the AGS SST criterion is decidedly consistent with the physical experiment on isentropic Mach number \( Ma_{in} \) at Reynolds number \( Re_{2th}=9\times10^{4} \) while for \( Re_{2th}=20\times10^{4} \) the LT transition occurs before the flow reaches the gradient sufficient for the boundary layer separation. The distributions obtained using the \( \gamma - \overline{Re}_{Bl} \) SST model with the Langtry and Malan correlations are identical for all the theoretical Reynolds numbers. At \( Re_{2th}=9\times10^{4} \) and \( Re_{2th}=20\times10^{4} \), these model combinations give an earlier transition and reattachment of the boundary layer with respect to the physical experiment and the other tried correlations, especially, with the Sorensen relations (Fig. 3). The usage of the Sorensen correlation causes a larger bubble with a later (than in the experiment) boundary layer reattachment at \( Re_{2th}=20\times10^{4} \), and at \( Re_{2th}=50\times10^{4} \) the Sorensen model is the only correlation predicting the flow separation.

The relative total pressure losses are shown in Figure 6 d. They are evaluated in terms of \( \zeta/\zeta_{ref} \) where \( \zeta_{ref} \) is the pressure loss coefficient at Reynolds number \( Re_{2th}=50\times10^{4} \). The experimental value of \( \zeta_{ref} \) is not available [26]. The best agreement with the experimental data is observed for the RANS results obtained using the Sorensen relations. With the other correlations the relative pressure losses are underestimated by 0.5-0.8 at \( Re_{2th}=9\times10^{4} \) and by 0.2-0.3 at \( Re_{2th}=20\times10^{4} \).
Similar results are obtained using the $\gamma - \tilde{Re}_{BT}$ BSL-EARSM model (see Figure 7). The Langtry, Malan and modified Langtry correlations (curves 1, 2 and 5, correspondingly) show identical distributions of isentropic Mach number for all the regimes. At Reynolds number $Re_{2th} = 9 \times 10^4$, the separation bubble zone is of larger size than that predicted using the SST model. The numerical results obtained using the Kelterer correlation (curve 4 in Figure 7) practically coincide with the curves 1, 2 and 5, except a slightly later transition observed at $Re_{2th} = 20 \times 10^4$. As for the $\gamma - \tilde{Re}_{BT}$ SST model, the results with the Sorensen relations (curves 3) are significantly different: a larger-size separation bubble is predicted both at $Re_{2th} = 9 \times 10^4$ and $Re_{2th} = 20 \times 10^4$. Moreover, at $Re_{2th} = 50 \times 10^4$ the results obtained using the BSL-EARSM with Sorensen correlation are characterized by the presence of separation bubble which is prominently larger than the one simulated by the SST model (see Figure 6 c, curve 3). The relative total pressure losses predicted with the Sorensen correlations are also close to the experimental data. The losses obtained using the other correlation dependencies are about the same and differ from the experimental data by 0.5 at $Re_{2th} = 9 \times 10^4$ and by 0.15 at $Re_{2th} = 20 \times 10^4$.

![Figure 7](image_url)

**Figure 7.** RANS results obtained using $\gamma - \tilde{Re}_{BT}$ BSL-EARSM model with correlations of Langtry – 1, Malan – 2, Sorensen – 3, Kelterer – 4, and modified Langtry – 5: isentropic Mach number $Ma_{is}$ distribution along the blade suction side at Reynolds numbers $Re_{2th} = 9 \times 10^4$ (a), $Re_{2th} = 20 \times 10^4$ (b), $Re_{2th} = 50 \times 10^4$ (c), and total pressure loss coefficient (d).

Integrally, the comparative analysis against the experimental data reveals a noticeable dependence of the numerical results on the turbulence model and, in the case of model $\gamma - \tilde{Re}_{BT}$, on the chosen correlations. The Sorensen correlation matches better the experimental data in terms of relative losses, but there is a significant discrepancy in the distribution of isentropic Mach number, larger recirculation zones are predicted and observed even at the maximal Reynolds number. Based on the results analyzed, the Kelterer correlation seems to be the most promising option for the problem at hand. Figure 8 shows the distribution of Mach number and turbulence kinetic energy $k$ obtained using this correlation dependency.
5.2. IDDES results versus experimental data
The results of scale-resolving simulations using the IDDES method compared with the experimental data are presented in Figure 9. It is clearly seen that the surface distributions of isentropic Mach number $Ma_{is}$ along the suction side of the blade fit well with the reference data for all the Reynolds numbers. The accuracy of the IDDES results is expectedly higher than those provided by the considered RANS models. We can conclude that if using a sufficiently refined mesh the IDDES method allows to predict the turbulent flow over the blade quite correctly. In the next subsection we consider the IDDES results as reference data to evaluate the results obtained by different RANS models. At the same time, it should be noted that while the isentropic Mach number $Ma_{is}$ is captured accurately, the relative total pressure loss coefficient obtained using the IDDES model slightly differs from the experimental value (see Figure 9 c). The reason of this observation is a rather strong sensitivity of this parameter to the incoming turbulent flow. However, the main studied phenomenon, i.e. the growth of total pressure losses with the Reynolds number lowering, is reproduced quite well both in quality and quantity (the difference between the computational and experimental data is about 0.15-0.25).

The instantaneous vorticity magnitude fields showing the main flow features are presented in Figure 10. The key process causing the total pressure losses is the LT transition on the suction side of the blade, followed by the boundary layer turbulization and the formation of the turbulent wake downstream the trailing edge. It is seen that the lowering of Reynolds number results in increasing a transverse size of the wake and its intensity. The coherent structures at $Re_{2th}=9\times10^4$ are indeed discernibly larger than those at $Re_{2th}=50\times10^4$. 

**Figure 8.** RANS results obtained using $\gamma = \tilde{Re}_{\theta t}$ BSL-EARSM model with Kelterer correlation at $Tu=3\%$: Mach number (top) and turbulent kinetic energy $k$ (bottom) fields at Reynolds numbers $Re_{2th}=9\times10^4$ (a, d), $Re_{2th}=20\times10^4$ (b, e), and $Re_{2th}=50\times10^4$ (c, f).
Figure 9. IDDES results: isentropic Mach number $Ma_{is}$ distribution along the blade suction side at Reynolds numbers $Re_{2th}=9\times10^4$ (a), $Re_{2th}=20\times10^4$ (b), $Re_{2th}=50\times10^4$ (c), and relative total pressure losses coefficient (d).

Figure 10. IDDES results: instantaneous vorticity magnitude fields at Reynolds numbers $Re_{2th}=9\times10^4$ (a), $Re_{2th}=20\times10^4$ (b), and $Re_{2th}=50\times10^4$ (c).

5.3. RANS results versus IDDES results
Figures 11-12 illustrate the comparison of the RANS and IDDES results. The absolute values of total pressure losses obtained numerically are collected in Table 4.

A comparative analysis of the numerical results obtained using the $\gamma-\tilde{Re}_t$ SST model (Figure 11) shows that the involvement of Langtry and Malan relations leads to an earlier LT transition while the point of separation remains the same. The results obtained using the Sorensen correlation demonstrate an early separation and delayed transition at Reynolds numbers $Re_{2th}=9\times10^4$ and $Re_{2th}=20\times10^4$. At $Re_{2th}=50\times10^4$, on the contrary, a later transition is observed. The results closest to the IDDES results are obtained using the Kelterer relations which confirms the conclusion made in subsection 5.1. The usage
of the AGS SST criterion leads to the prediction of earlier separation at $Re_{2\theta h}=9 \times 10^4$ in the same way as in the case of $\gamma - \tilde{Re}_{\theta t}$ SST model with the Sorensen correlation. As for the other model combinations considered, the LT transition occurs much earlier in respect to both the $\gamma - \tilde{Re}_{\theta t}$ and IDDES results.

**Figure 11.** Comparison of IDDES results (marked as “ID”) and RANS results obtained using AGS SST and $\gamma - \tilde{Re}_{\theta t}$ SST models with correlations of Langtry – 1, Malan – 2, Sorensen – 3 and Kelterer – 4: distributions of friction coefficient $C_f$ along the blade suction side (left) and total pressure at section $x/l_x=1.4$ where the losses are measured (right) at Reynolds numbers $Re_{2\theta h}=9 \times 10^4$ (a, d), $Re_{2\theta h}=20 \times 10^4$ (b, e), and $Re_{2\theta h}=50 \times 10^4$ (c, f).
Figure 12. Comparison of IDDES results (marked as “ID”) and RANS results obtained using $\gamma - \tilde{Re}_{\theta_t}$ BSL-EARSM model with correlations of Langtry – 1, Malan – 2, Sorensen – 3, Kelterer – 4, and modified Langtry – 5: distributions of friction coefficient $C_f$ along the blade suction side (left) and total pressure at section $x/l_x = 1.4$ where the losses are measured (right) at Reynolds numbers $Re_{2th} = 9 \times 10^4$ (a, d), $Re_{2th} = 20 \times 10^4$ (b, e), and $Re_{2th} = 50 \times 10^4$ (c, f).

The comparison of the pressure distribution downstream the blade in the section $x/l_x = 1.4$ (where the losses are measured) shows that the transverse size of the IDDES wake is discernibly wider at low Reynolds number $Re_{2th} = 9 \times 10^4$. At higher Reynolds numbers it is, on the contrary, narrower in comparison to the corresponding RANS results. Analysing the graphs in Figure 11 one can conclude that the RANS solution the closest to the IDDES results is achieved using the Kelterer correlation to model the LT transition.

The comparison of the RANS results obtained using the $\gamma - \tilde{Re}_{\theta_t}$ BSL-EARSM model (Figure 12) shows that if using the Langtry, or Malan, or Kelterer correlations the point of separation is predicted closely to the IDDES result, however the LT transition with the corresponding reattachment occurs
earlier at \(Re_{2th}=20 \times 10^4\) and especially at \(Re_{2th}=9 \times 10^4\). The \(\gamma - \Re_{\theta t}\) BSL-EARSM results obtained using the Sorensen correlation coincide with those obtained by the SST model.

In terms of the friction coefficient and total pressure, the RANS results produced by the \(\gamma - \Re_{\theta t}\) BSL-EARSM models are more consistent with the corresponding IDDES data than the \(\gamma - \Re_{\theta t}\) SST results. Both models reveal lower friction in the turbulent boundary layer downstream the LT transition, however in the BSL-EARSM the difference against the IDDES results is less. As mentioned in subsection 5.1, higher accuracy is provided by using the Kelterer relation in both turbulence models when the \(\gamma - \Re_{\theta t}\) LT transition model is involved. The SST model with the AGS criterion produces results significantly different from the IDDES data for all the considered cases. In terms of absolute losses (see Table 4), almost all the RANS models give an overestimation at the highest Reynolds number \(Re_{2th}=50 \times 10^4\) and an underestimation at the lower Reynolds numbers as compared with the IDDES method. The SST turbulence model predicts a lower loss level than the EARSM.

Almost all the considered RANS models, except the cases with \(\gamma - \Re_{\theta t}\) transition model and the Sorensen correlation, predict an early LT transition even if the location of separation point corresponds the IDDES results. This fact may be due to different reasons. For instance, note that the AGS criterion works in the range of dimensionless pressure gradients \(-0.1<\lambda_d<0.1\) while in the considered case the gradient at which the separation occurs reaches the value \(\lambda_d=0.25\). The same concerns the model \(\gamma - \Re_{\theta t}\) since it exploits a similar empirical ratio as the AGS criterion.

### Table 4. Absolute values of total pressure losses \(\zeta\) in \% depending on models

| Models                      | \(Re_{2th}=9 \times 10^4\) | \(Re_{2th}=20 \times 10^4\) | \(Re_{2th}=50 \times 10^4\) |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| IDDES                       | 4.37                        | 2.78                        | 2.18                        |
| \(\gamma - \Re_{\theta t}\) SST Langtry | 3.64                        | 2.67                        | 2.48                        |
| \(\gamma - \Re_{\theta t}\) SST Malan   | 3.68                        | 2.67                        | 2.41                        |
| \(\gamma - \Re_{\theta t}\) SST Sorensen | 4.61                        | 2.98                        | 2.04                        |
| \(\gamma - \Re_{\theta t}\) SST Kelterer | 3.77                        | 2.65                        | 2.20                        |
| \(\gamma - \Re_{\theta t}\) BSL-EARSM Langtry | 4.08                        | 2.98                        | 2.41                        |
| \(\gamma - \Re_{\theta t}\) BSL-EARSM Malan   | 4.12                        | 2.98                        | 2.40                        |
| \(\gamma - \Re_{\theta t}\) BSL-EARSM Sorensen | 5.34                        | 3.49                        | 2.39                        |
| \(\gamma - \Re_{\theta t}\) BSL-EARSM Kelterer | 4.12                        | 2.98                        | 2.39                        |
| \(\gamma - \Re_{\theta t}\) BSL-EARSM modified Langtry | 4.16                        | 2.98                        | 2.40                        |
| AGS SST                     | 4.03                        | 2.66                        | 2.43                        |

For comparison reasons, predictions with the lower intensity of incoming turbulence are carried out.

The corresponding results are presented in Figures 13-16. In the case of \(\gamma - \Re_{\theta t}\) BSL-EARSM model with the Kelterer correlations (see Figures 13 and 15) the introduction of lower turbulence leads to the better results in comparison to the experimental data on the isentropic Mach number. A decrease of the turbulence intensity level causes an increase of relative total pressure losses especially at Reynolds number \(Re_{2th}=9 \times 10^4\) (see Figure 13 d). As for the AGS criterion (see Figures 14 and 16), lowering the incoming turbulence results in a significantly better agreement of the RANS results with both the IDDES and experimental data. At \(Re_{2th}=9 \times 10^4\) and \(Re_{2th}=20 \times 10^4\), the LT transition is predicted much further than at the given turbulence intensity \(Tu=3\%\).

The absolute values of total pressure losses (Table 5) predicted by both considered RANS models with turbulence intensity \(Tu=2\%\) are noticeably closer to the corresponding values obtained by the IDDES method with respect to the case of \(Tu=3\%\).
Figure 13. RANS results obtained using $\gamma - \tilde{Re}_{\theta t}$ BSL-EARSM model with Kelterer correlations for different intensity of incoming turbulence: distribution of isentropic Mach number $Ma_{is}$ along the blade suction side at Reynolds numbers $Re_{2th}=9 \times 10^4$ (a), $Re_{2th}=20 \times 10^4$ (b), $Re_{2th}=50 \times 10^4$ (c), and relative total pressure losses (d).

Figure 14. RANS results obtained using AGS SST model for different intensity of incoming turbulence: distribution of isentropic Mach number $Ma_{is}$ along the blade suction side at Reynolds numbers $Re_{2th}=9 \times 10^4$ (a), $Re_{2th}=20 \times 10^4$ (b), $Re_{2th}=50 \times 10^4$ (c), and relative total pressure losses (d).
Figure 15. Comparison of IDDES results (marked as “ID\_3\%”) and RANS results obtained using γ − Re_bt BSL-EARS model with Kelterer correlation at different intensity of incoming turbulence: distribution of friction coefficient $C_f$ along the blade suction side (left) and total pressure at section $x/l_x = 1.4$ (right) at Reynolds numbers $Re_{2th} = 9 \times 10^4$ (a, d), $Re_{2th} = 20 \times 10^4$ (b, e), and $Re_{2th} = 50 \times 10^4$ (c, f).
Figure 16. Comparison of IDDES results (marked as “ID_3%”) and RANS results obtained using AGS SST model at different intensity of incoming turbulence: distributions of friction coefficient $C_f$ along the blade suction side (left) and total pressure at section $x/l_x=1.4$ (right) at Reynolds numbers $Re_{2th}=9\times10^4$ (a, d), $Re_{2th}=20\times10^4$ (b, e), and $Re_{2th}=50\times10^4$ (c, f).

Table 5. Absolute values of total pressure losses $\zeta$ in % depending on turbulence intensity

| Model               | $Re_{2th}=9\times10^4$ | $Re_{2th}=20\times10^4$ | $Re_{2th}=50\times10^4$ |
|---------------------|-------------------------|-------------------------|-------------------------|
| IDDES Tu_3%         | 4.37                    | 2.78                    | 2.18                    |
| $\gamma \sim \theta_{BT}$ BSL-EARSM Kelterer Tu_3% | 4.12                    | 2.98                    | 2.39                    |
| $\gamma \sim \theta_{BT}$ BSL-EARSM Kelterer Tu_2% | 4.26                    | 2.7                     | 2.13                    |
| $\gamma \sim \theta_{BT}$ BSL-EARSM Kelterer Tu_1% | 5.26                    | 2.57                    | 1.87                    |
| AGS SST Tu_3%       | 4.03                    | 2.66                    | 2.43                    |
| AGS SST Tu_2%       | 4.15                    | 2.6                     | 2.1                     |
6. Conclusion

Simulations of flow over the highly loaded turbine cascade T106C using both RANS and scale-resolving IDDES approaches are performed. The significant feature is the LT transition caused by the boundary layer separation at the suction side of the blade.

Overall, the RANS results obtained using different turbulence models with LT transition and correlation dependencies are of acceptable accuracy. The LT transition model $\gamma - \tilde{Re}_{BT}$ works more accurately than the Abu-Ghannam and Shaw criterion. The results closest to the experimental data are achieved using this transition model with the turbulence model BSL-EARSM. Among the different correlation dependencies, the Kelterer relations provide the best results. The Langtry and Malan correlations work about the same while the Sorensen relations predict a larger separation bubble and overestimate the losses. However, in all the considered cases, underprediction of losses is observed at low Reynolds numbers and overprediction — at higher numbers. The reduction of turbulence intensity at the input results in the better alignment of the computational and experimental data.

The scale-resolving IDDES method with synthetic turbulence at the inlet allows to capture the flow details quite well. The isentropic Mach number distributions along the blade surface practically coincide with the experimental data. A slight underestimation of total pressure losses may be due to high sensitivity of this value to the inlet flow conditions, more specifically, to the inlet turbulence. Small deviations with respect to the experimental data may be also caused by uncertainty in experimental flow conditions and not exhausted mesh resolution in IDDES simulations.

If comparing the IDDES and RANS methods as applied to simulating the flow over T106C cascade, we can say that the RANS results are underpredicted at low Reynolds numbers and overpredicted at higher Reynolds number in all the considered cases. This may be related to greater possibilities of scale resolving approaches to reproduce accurately the details of unsteady turbulent boundary layer which seems crucial for the considered problems. In particular, we observe a strong sensitivity of the numerical results to the inlet turbulence.

More detailed experimental data is needed to study the T106C case more accurately. Exhaustive information about the incoming turbulent flow is especially important. This condition may crucially affect the LT transition and therefore the evaluated data. Another piece of extremely important experimental data is the absolute values of total pressure loss coefficient. The numerical experiments show a different discrepancy of relative total pressure losses with respect to the experimental data dependently to the Reynolds numbers and the models in use. This complicates evaluating the results and assessing the methods used.

Acknowledgements

The scale-resolving computations have been carried out using the computing resources of the federal collective usage center Complex for Simulation and Data Processing for Mega-science Facilities at NRC “Kurchatov Institute”, http://ckp.nrcki.ru/.

References

[1] Mayle R E 1991 The role of laminar-turbulent transition in gas turbine engines ASME J. Turbomach. 113(4) pp 509–31
[2] Gaster M 1966 The Structure and Behaviour of Laminar Separation Bubbles Aerodynamics Division N.P.L. R&M 3595 (London: Her Majesty’s Stationery Office) pp 1-31
[3] Roberts W B 1975 The effect of Reynolds number and laminar separation on axial cascade performance Journal of Engineering for Power 97(2) pp 261-73
[4] Curtis E M, Hodson H P, Banieghbal M R, Denton J D, Howell R J and Harvey N W 1997 Development of blade profiles for low-pressure turbine applications ASME Journal of Turbomachinery 119(3) pp 531-38
[5] Abu-Ghannam B J and Shaw R 1980 Natural transition of boundary layers - the effects of turbulence, pressure gradient and flow history J. Mech. Eng. Sci. Vol 22 No. 5 pp 213-28
[6] Langtry R B and Menter F R 2009 Correlation-based transition modeling for unstructured
parallelized computational fluid dynamics codes AIAA Journal 47(12) pp 2894–906

[7] Langtry R B, Menter F R, Likki S R, Suzen Y B, Huang P G and Völker S 2006 A correlation-based transition model using local variables—part I: model formulation ASME J. Turbomach. 128(3) pp 413–422

[8] Langtry R B, Menter F R, Likki S R, Suzen Y B, Huang P G and Völker S 2006 A correlation-based transition model using local variables - part II: test cases and industrial applications ASME J. Turbomach. 128(3) pp 423–34

[9] Menter F R, Langtry R and Völker S 2006 Transition modelling for general purpose CFD codes Flow, Turbulence and Combustion 77 pp 277–303

[10] Bigoni F, Vagnoli S, Arts T and Verstraete T 2016 Detailed numerical characterization of the suction side laminar separation bubble for a high-lift low pressure turbine blade by means of RANS and LES Proc. of ASME Turbo Expo: Turbomachinery Technical Conference and Exposition June 13 – 17 2016 Seoul South Korea GT2016-56653

[11] Babajee J, and Arts T 2012 Investigation of the laminar separation-induced transition with the \( \gamma \)-Re\( \theta \) transition model on low-pressure turbine rotor blades at steady conditions Proc. of ASME Turbo Expo 2012 June 11-15 2012 Copenhagen Denmark GT2012-68687

[12] Pacciani R, Marconcini M, Arnone A and Bertini F 2013 Predicting high-lift LP turbine cascades flows using transition-sensitive turbulence closures Proc. of ASME Turbo Expo 2013 Turbine Technical Conference and Exposition June 3-7 2013 San Antonio Texas USA GT2013-95605

[13] Content C and Houdeville R 2010 Application of the \( \gamma \)-Re\( \theta \) laminar-turbulent transition model in Navier-Stokes computations Proc. 40th Fluid Dynamics Conference and Exhibit 28 June – 1 July 2010 Chicago Illinois AIAA 2010-4445

[14] Suluksa K, Dechaumphai P and Juntasaro E 2009 Correlations for modeling transitional boundary layers under influences of freestream turbulence and pressure gradient Int. J. Heat Fluid Flow 30 pp 66–75

[15] Malan P, Suluksa K and Juntasaro E 2009 Calibrating the \( \gamma \)-Re\( \theta \) transition model ERCOFTAC Bulletin 80 pp 53–7

[16] Elsner W, Piotrowski W and Warzecha P 2009 Transition modelling with intermittency transport equations. In ERCOFTAC Bulletin 80 pp 49–52

[17] Kelterer M E, Pecnik R and Sanz W 2010 Computation of laminar-turbulent transition in turbomachinery using the correlation-based \( \gamma \)-Re transition model Proc. of ASME Turbo Expo 2010: Power for Land, Sea and Air June 14-18 2010 Glasgow UK GT2010-22207

[18] Sørensen N N 2009 CFD modelling of laminar-turbulent transition for airfoils and rotors using the \( \gamma \) – Re\( \theta \) model Wind Energ. 12 pp 715–33

[19] FINE/Turbo Theoretical Manual NUMECA International Brussels Belgium

[20] Marty J. 2014 Numerical investigations of separation-induced transition on high-lift low-pressure turbine using RANS and LES methods. Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy 228 (8), 924-952

[21] Hu S., Zhou Ch., Xia Zh., Chen Sh. 2016 Bigoni F, Vagnoli S, Arts T and Verstraete T 2016 LES and CDNS Investigation of T106C Low Pressure Proc. of ASME Turbo Expo: Turbomachinery Technical Conference and Exposition June 13 – 17 2016 Seoul South Korea GT2016-56838

[22] Menter F R 1994 Two-equation eddy-viscosity turbulence models for engineering applications AIAA Journal 32(8) pp 1598-05

[23] Menter F R 1993 Zonal two equation k-\( \omega \), turbulence models for aerodynamic flows Proc. Of 24th Fluid Dynamics Conference July 6-9 1993 Orlando Florida AIAA 93-2906

[24] Menter F et al 2009 Explicit algebraic Reynolds stress models for anisotropic wall-bounded flows Proc. of EUCASS – 3rd European Conference for Aero-Space Sciences July 6-9 Versailles

[25] Guseva, E. K., Garbaruk, A. V., and Strelets, M. K. 2017 Assessment of Delayed DES and Improved Delayed DES Combined with a Shear-Layer-Adapted Subgrid Length-Scale in Separated Flows Flow, Turbulence and Combustion 98 pp 481–502.

[26] Stotz, S., Guendogdu, Y., and Niehuis, R. 2017 Experimental Investigation of Pressure Side Flow
Separation on the T106C Airfoil at High Suction Side Incidence Flow. *ASME. J. Turbomach.* 139(5): 051007

[27] Shur M.L., Spalart P.R., Strelets M.K., Travin A.K. 2014 Synthetic Turbulence Generators for RANS-LES Interfaces in Zonal Simulations of Aerodynamic and Aeroacoustic Problems *Flow, Turbulence and Combustion* 93 pp 63-92.

[28] Shur, M., Strelets, M., Travin, A., Probst, A., Probst, S., Schwamborn, D., Deck, S., Skillen, A., Holgate, J. and Revell, A. 2017 Improved Embedded Approaches. *Notes on Numerical Fluid Mechanics and Multidisciplinary Design* 134 pp 65-69

[29] Abalakin, I.V., Bakhvalov, P.A., Gorobets, A.V., Duben, A.P., and Kozubskaya, T.K. 2012 Parallel research code NOISEtte for large-scale computations of aerodynamics and aero-acoustics problems *Vychisl. Metody Programm.* 13(3), pp. 110–125 (in russian).

[30] Gorobets A.V. 2018 Parallel algorithm of the NOISEtte code for CFD and CAA simulations *Lobachevskii Journal of Mathematics* 39(4) pp 524–532.

[31] Bakhvalov P.A., Abalakin I.V., Kozubskaya T.K. 2016 Edge-based reconstruction schemes for unstructured tetrahedral meshes *Int. J. Numer. Methods Fluids* 81(6) pp 331–356.

[32] Duben A.P., Kozubskaya T.K. 2019 Evaluation of Quasi-One-Dimensional Unstructured Method for Jet Noise Prediction *AIAA J.* 57(12), pp 5142–5155