COMMENTS ON SCHULZ

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Abstract. Schulz claims to have constructed an actively local stochastic theory which violates Bell’s inequality. This is unjustified.

In the conclusions of his paper [1], Schulz claims:

Specially designed stochastic processes can violate Bell’s inequality at two separated locations without any contact at all. We have mathematically analysed, what such a behaviour really does imply for a stochastic theory, and we have constructed a corresponding one.

This claim is unjustified. Schulz constructs a variant of Nelson’s stochastic interpretation proposed by Fritsche and Haugk [2], which he introduces with the words

In contrast to Nelson’s original contribution [3], the model of Fritsche and Haugk is an actively local theory with a non-markovian stochastic process.

The violation of Bell’s inequality would be a consequence of the recovery of full quantum theory (as in original stochastic mechanics) in his construction. But [1] contains the remarkable restriction (88)

\[
\vec{v}^{N,A(B)} = \left( v_{A(B)}^{1}(\vec{r}, t), \ldots, v_{N}^{A(B)}(\vec{r}, t) \right)
\]

\[
\vec{u}^{N,A(B)} = \left( u_{A(B)}^{1}(\vec{r}, t), \ldots, u_{N}^{A(B)}(\vec{r}, t) \right)
\]

that each \( \vec{v}^{i} \), \( \vec{u}^{i} \) depends only on \( \vec{r}^{i} \). Using his equation (66) and following

\[
\psi(\vec{r}, t) = \pm \sqrt{\rho(\vec{r}, t)} e^{i\phi(\vec{r}, t)}; \quad \vec{v} = \frac{\hbar}{m_0} \vec{\nabla} \phi(\vec{r}, t)
\]

we obtain

\[
\phi(\vec{r}) = \sum_{k} \phi_{k}(\vec{r}_{k}),
\]

and combining it with his equation (59)

\[
\vec{u}^{A(B)} = \pm \sqrt{\rho^{A(B)}} \ln \left( \rho^{A(B)}(\vec{r}, t)/\rho_{0} \right)
\]

one gets

\[
\ln \rho(\vec{r}) = \sum_{k} f_{k}(\vec{r}_{k})
\]

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so that as a consequence the wave function splits into a simple product state

\[ \psi(\vec{r}) = \prod_k e^{f_k(\vec{r}_k) + \phi_k(\vec{r}_k)} = \prod_k \psi_k(\vec{r}_k) \]

Given the otherwise quite unmotivated section “4.2 Quantum states in superpositions” (in pilot wave theory or Nelson’s stochastics there is no need to consider superpositions separately), one may guess that Schulz tries to overcome this restriction there. Unfortunately the whole construction, especially the considerations from (70) to (75), are nonsensical. And even if we would accept all this, we can use his equations

\[ \vec{v}_1 = \vec{u}_1 \equiv \vec{v}; \quad \vec{u}_1 = \vec{u}_2 \equiv \vec{u} \]

(in the text after equ. (83)) to obtain (modulo the integration constants in our (4), which become irrelevant constant factors) that

\[ \psi_1 = \psi_1 = \psi, \]

so that no nontrivial superposition has been obtained. Thus, full quantum theory is not recovered. The entangled quantum many-particle states which violate Bell’s inequality cannot be obtained in this construction. No direct proof that the construction nonetheless allows to violate Bell’s inequality is given.

Even more, one can even directly prove that the construction cannot violate Bell’s inequality. Schulz himself describes the way how to do this at p. 31:

The only constraint on the random forces was that they should have a Gaussian distribution. Now consider the following mechanism: A real random number generator sets a value \( \lambda_j \in \mathbb{R} \) for each \( j \)-th particle pair at preparation stage. This value \( \lambda_j \) is similar to the hidden parameter in Bells original work \([2]\), but we define it to be independent for each entangled particle pair generated by the source.

Now, Bell’s \( \lambda \) will be in general also different for each entangled pair, thus, using a different \( \lambda_j \) for each pair does not pose a problem if we want to fit into Bell’s framework.

In our model, \( \lambda_j \) serves as the starting value for two pseudo random number generators of the same type.

Pseudo-random number generators nicely fit into Bell’s original scheme, where all we need are well-defined deterministic functions \( A(a, \lambda), B(b, \lambda) \). So what could prevent us from applying Bell’s original proof? Schulz gives only the following argument:

At first sight, one may think that our procedure would set up a deterministic passively local theory. Indeed, for \( \vec{F}_{11}^{\text{Brown}}(t, \lambda_j) \) and \( \vec{F}_{22}^{\text{Brown}}(t, \lambda_j) \) there is a parameter \( \lambda_j \) at preparation stage which determines all later outcomes of these forces. However, one does not have access to the outcomes of the Gaussian distributed random forces in the probability space, but only to the two spin values \([1]\).

But nowhere in Bell’s theorem one can find a requirement that the \( \lambda \in \Lambda \) should be accessible in any way to anybody. Instead, they are usually named “hidden variables” because of their assumed inaccessibility. It is their pure existence which

\[ ^1 \text{[2] in our bibliography} \]
is already sufficient to prove Bell’s inequality. Once the existence is not questioned by Schulz, we can apply Bell’s theorem and obtain Bell’s inequalities for Schulz’s construction.

But maybe Bell’s theorem in its original form is simply wrong? Schulz implicitly suggests some lack of mathematical rigour in Bell’s proof:

In 1985, Nelson tried to analyse Bell’s theorem with full mathematical rigour. … Bells definition of locally causal [ ] could be divided into two separate conditions … one of them being “passive locality”. While this is far away from a claim that theorem itself is wrong, the claim

We will illustrate below that deterministic passive locality can be easily violated, even though the outcomes of the time dependent random force may be predetermined by a hidden parameter for each entangled particle pair.

suggests that Schulz may think that a theory which fits into Bell’s original assumptions may nonetheless violate Bell’s inequality. Thus, Bell’s original theorem would have to be plainly wrong, missing the important assumption of passive locality.

But the condition of passive locality (or output independence) makes sense only in the context of a stochastic description. If we consider, as in Bell’s original paper [2], deterministic functions $A(\lambda, a)$, $B(\lambda, b)$, passive locality becomes a triviality: Deterministic functions, considered as particular examples of stochastic functions, are always independent of every other stochastic function. On the other hand, if we consider the $A(\lambda, a)$, $B(\lambda, b)$ as stochastic functions $A(a)$, $B(b)$, with $\Lambda$ as the corresponding space of events, then they clearly violate passive locality. Thus, the violation of passive locality depends on the particular look at the same mathematical construction. But if for this same mathematical construction Bell’s inequality holds follows already from the existence of a proof for one of the ways to look at it. In our case, we can simply apply Bell’s original way.

Thus, the main result of the paper [1] appears invalid.

But, maybe, the very idea to save relativity and realism with stochastic theories which don’t violate active locality (parameter independence) but only passive causality (output independence) deserves future research? We don’t think so. Any theory of this type faces the good old EPR argument that it is simply incomplete – it doesn’t explain the observed correlations, because an explanation is the presentation of a cause for the correlation such that once the cause is taken into account, the correlation disappears.

In general, stochastic functions do not seem very promising. If stochastic functions $A(\lambda, a)$, $B(\lambda, b)$ have a parameter-independent probability measure $d\nu(\omega)$ on some sample space $\Omega$ a simple redefinition $\Lambda \rightarrow \Lambda' \cong \Lambda \times \Omega$, $d\rho(l) \rightarrow d\rho'(\lambda') = d\rho(\lambda) d\nu(\omega)$ makes Bell’s original theorem for deterministic functions $A(\lambda, a)$, $B(\lambda, b)$ applicable. Allowing parameter dependence $d\nu(\omega|a, b)$ leads effectively to solipsism – the observable results $d\rho(A, B|a, b)$ themself would already define a complete “realistic model”. Thus, Kolmogorovian stochastics does not help to avoid the alternative posed by Bell’s theorem – realism or relativity. And a non-Kolmogorovian generalization will automatically face the argument that it violates realism, does not give a realistic explanation.

On the other hand, there is no reason to give up realism. In the opinion of the author, the road to realism is to accept a hidden preferred frame, as in pilot wave
theory and Nelsonian stochastics. This is not only compatible with all existing evidence, but also allows new possibilities like a condensed matter interpretation for fundamental fields [5].

References

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