Calculation of distribution of potential near the surface of metal particle in the dust-electron thermal plasma

G Dautov¹, I Fayrushin², N Kashapov², and I Dautov²
¹General Physics Department, Kazan National Research Technical University by named A.N. Tupolev-KAI, Kazan, 420111, Russia
²Department of Thechnical Physics and Power Engineering, Kazan Federal University, Kazan, 420008, Russia

E-mail: fairushin_ilnaz@mail.ru

Abstract. We obtained the equation, which describes the distribution of the potential in an equilibrium dust-electron plasma taking into account parameters of the electron gas inside the dust particles. The inclusion of these parameters performed on the basis of the model of "solid-state plasma," considering the condensed particle system as the ion core and the free electron gas. The analytical expressions for the potential distributions in thermal equilibrium dust-electron plasma were obtained, using a number of simplifying assumptions. It is found that near the particle surface there is a large gradient of the electric potential and as a result the concentration of free electrons, i.e. electrons in the dust particle are located in a potential well. It is shown that the form of the potential barrier at the particle surface depends on the plasma temperature, particle size and concentration.

1. Introduction

One of the most important characteristics of dust-electron thermal plasma are potential and electron density distributions [1-7]. To determine these characteristics of the data which are used in this paper the model of "jelly", which previously has been widely used in the description of the electrical properties of atomic clusters [8-10]. According to this model, the dust particle is represented as two-components system. The first component is ion core, which create uniform positive background in the whole volume of the dust particle. The second component is the electronic gas. The density of this gas distribute in the area from the condition of equilibrium of forces internal pressure of the gas and electrostatic forces. Thus, in this model, a dust particle can be regarded as the area of solid state plasma [11-13]. From the above it follows that the distribution of the potential and the electron density within and around the dust particles are dependent on the temperature, type of the substance, size and the concentration of particles.

2. Theoretical background and results

Consider the system of equations describing the potential distribution and the electron density in solid-state plasma and in the space around it in a state of statistical equilibrium when the temperature is low and can be neglected of the ionization of the gas.

It is known from the statistical physics, the properties of the electron gas are described by the Fermi-Dirac distribution
\[ n_j = \frac{1}{1 + e^\frac{j\varepsilon - \mu}{\theta}} , \quad \text{(1)} \]

where \( \theta \) – statistical temperature equal \( kT \), \( k \) - Boltzmann constant, \( T \) - absolute temperature, \( \mu \) – Fermi energy, \( \varepsilon_j \) - electron energy in the \( j \)-th quantum state, \( n_j \) - average numbers of electrons in the same quantum state with energy of \( \varepsilon_j \).

In the semiclassical approximation, the number of electrons per unit volume in the energy range from \( \varepsilon \) to \( \varepsilon + d\varepsilon \) defined by the formula [14]

\[ n_e(\varepsilon) = \frac{4\pi \cdot (2m_e)^{3/2}}{h^3} \left( \frac{1}{1 + e^\frac{\varepsilon - \mu}{\theta}} \right) \cdot d\varepsilon . \quad \text{(2)} \]

Here \( h \) – Planck’s constant, \( m_e \) - mass of the electron, \( n_e \) - electron density.

When \( \theta = 0 \), the electron gas is completely degenerate and the equation (2) is obtained from the Fermi energy:

\[ \mu = \frac{h^2}{2m_e} \left( \frac{3n_e}{8\pi} \right)^{2/3} . \quad \text{(3)} \]

The distribution of the potential \( \phi \) of the electrostatic field is described by Poisson equation

\[ \Delta \varphi = \frac{n_e - n_i}{\varepsilon_0 \varepsilon_1} q , \quad \text{(5)} \]

where \( \varepsilon_1 \) - relative permittivity, \( \varepsilon_0 \) - electric constant, \( n_i \) - the concentration of positive charge of the ion core (the density of the positive background), \( q \) - the absolute value of the electron charge.

The condition of statistical equilibrium of the electron gas can be written as[14]

\[ \mu - q\varphi = \text{const} . \quad \text{(6)} \]

Equations (2), (5) and (6) give the boundary conditions completely determine the spatial distribution of the potential and the electron density.

The system of equations (2), (5) and (6) can be obtained by a single equation, which calculates the potential distribution and electron density. From (6) we find

\[ \Delta \varphi = \frac{\Delta \mu}{q} . \quad \text{(7)} \]

Substitution of expressions (2) and (7) into (5) leads to a nonlinear integro-differential equation.
\begin{equation}
\Delta \mu = \frac{q^2}{\varepsilon_0 \varepsilon_r} \left[ \int_0^\infty \frac{4\pi (2m_e)^{3/2}}{\hbar^3} \left( \frac{1}{1 + e^{\frac{e-\mu}{\theta}}} \right) d\varepsilon - n_i \right]. \tag{8}
\end{equation}

This equation is applicable to both degenerate and non-degenerate electron gas. This is particularly important in the case of dust particles from the metal. In this case, there are areas of the degenerate and non-degenerate electron gas.

In the case of degenerate gas \((2)\) we find

\[ n_e = \frac{16\sqrt{2\pi} n^{3/2}}{3\hbar^3} \mu^{3/2}. \tag{9} \]

Substituting this expression in \((8)\), we obtain

\[ \Delta \mu = \frac{q^2}{\varepsilon_0 \varepsilon_r} \left( \frac{16\sqrt{2\pi} n^{3/2}}{3\hbar^3} \mu^{3/2} - n_i \right). \tag{10} \]

If we introduce the dimensionless potential \(\psi = \frac{q\varphi}{\mu_0}\), for the case of spherical symmetry from the \((12)\) and \((13)\) we obtain the equation

\[ \frac{1}{x^2} \frac{d}{dx} \left( x^2 \cdot \frac{d\psi}{dx} \right) = a^2 \left[ (1 + \psi)^{3/2} - n_i \right]. \tag{11} \]

where \(x = \frac{r}{R}, \quad \frac{n_i}{n_{e0}} = n, \quad a^2 = \frac{q^2 R^2 n_{e0}}{\mu_0 \varepsilon_0 \varepsilon_r} \).

Consider the case, when \(|\psi|<1\). We expand the value \((1 + \psi)^{3/2}\) in the series and retain the first two terms: \((1 + \psi)^{3/2} = 1 + \frac{3}{2}\psi\).

Given of this expression of the equation\((15)\) obtained by

\[ \frac{1}{x^2} \frac{d}{dx} \left( x^2 \cdot \frac{d\psi}{dx} \right) = a^2 \left( 1 + \frac{3}{2}\psi - n_i \right). \tag{12} \]

The initial conditions for this equation is \(\psi(0) = 0, \quad \psi'(0) = 0\). Its solution is
If the concentration of dust particles is large; concentration of electrons in the region \( 1 < x \leq \lambda \) will be large and the electron gas is degenerate. In this case, the potential distribution in the area \( 1 < x \leq \lambda \) defined by the formula.

\[
\psi(x) = \frac{2}{3} \cdot \left( \frac{(a-1) \cdot e^a - \left( \frac{a\lambda-1}{a\lambda+1} \right) \cdot e^{a(a+1)} \cdot e^{(2a-1)} + \left( \frac{a\lambda-1}{a\lambda+1} \right) \cdot e^{(2a+1)}}{2a \cdot (a-1) \cdot e^a - (a+1) \cdot \left( \frac{a\lambda-1}{a\lambda+1} \right) \cdot e^{(2a+1)} - \frac{e^{a(a+1)} \cdot e^{(2a-1)}}{x} - 2a} \right) , \text{if } 0 \leq x \leq 1 \tag{13}
\]

If the concentration of dust particles is low; in the direction of the axis \( x \) degeneracy of the electron gas is gradually removed. In this case, the potential distribution is found by solving equation (8), which is valid for both case degenerate and non-degenerate electron gas.

In conclusion, we obtained the equation (8), which describes the distribution of the potential and the electron density in an equilibrium dust-electron plasma taking into account parameters of the electron gas inside the dust particles. The inclusion of these parameters performed on the basis of the model of "solid-state plasma", considering the condensed particle system as the ion core and the free electron gas. Solving equation (8) using the number of simplifying assumptions, we have been obtained the analytical expressions for the potential distributions and in thermal equilibrium dust-electron plasma. We have been found that near the particle surface there is a large gradient of the
electric potential and as a result, the concentration of free electrons, i.e. electrons in the dust particle are located in a potential well. The obtained expressions allow to calculate the influence of several parameters such as temperature, concentration and dust particle size, type of the substance of particles on properties of the equilibrium dust-electron plasma.

Acknowledgements
The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University.

References
[1] Complex and Dusty Plasmas, edited by V. E. Fortov and G. E. Morfill (CRC Press, Boca Raton, FL, 2010).
[2] V.I. Vishnyakov, Phy. Rev. E 85, 026402 (2012).
[3] V.I. Vishnyakov and G.S. Dragan, Phys. Rev. E 73, 026403 (2006).
[4] V.I. Vishnyakov, G.S. Dragan and V.M. Evtuhov, Phys. Rev. E 76, 036402 (2007).
[5] Dautov G, Kashapov N, Larionov V, Fayrushin I et al. 2013 J. Phys.: Conf. Series 479 011001
[6] Dautov G, Fayrushin I 2013 J. Phys.: Conf. Series 479 012013
[7] Dautov G, Dautov I, Fayrushin I and Kashapov N 2013 J. Phys.: Conf. Series 479 012014
[8] Dautov G, Dautov I, Fayrushin I and Kashapov N 2013 J. Phys.: Conf. Series 479 012001
[9] R.D. Smirnov, A.Y. Pigarov, M. Rosenberg, S.I. Krasheninnikov, and D.A. Mendis, Plasma Phys. Controlled Fusion 49, 347 (2007).
[10] J. Vaverka, I. Richterová, M. Vyšinka, J. Pavlů, J. Šafránková, and Z. Němeček, Plasma Phys. Controlled Fusion 56, 025001 (2014).
[11] V.I. Krauz, Yu.V. Martynenko, N.Yu.Svechnikov, V.P. Smirnov, V.G. Stankevich, L.N. Khimchenko, Phys. Usp. 53, 1015–1038 (2010).
[12] W. Ekardt, Phys. Rev. B. 29, 1558 (1984).
[13] V.K. Ivanov, A.N. Ipatov, V.A. Kharchenko, JETP, Vol. 82, No 3, p. 485 (1996)
[14] M.B. Smirnov, V.P. Krainov, JETP, Vol. 88, No 6, p. 1102 (1999)
[15] N.W. Ashcroft and N. D. Mermin, Solid State Physics (Saunders College, Philadelphia, 1976).
[16] L.A. Artsimovich, R.Z. Sagdeev, Plasma physics for physicists. Moscow, Atomizdat, 1979. 320 p.
[17] B.M. Smirnov, Phys. Usp. 45 1251–1286 (2002).
[18] L. D. Landau and E. M. Lifshitz, *Statisticheskaya Fizika* ( Nauka, Moscow, 1976), Vol. 1
*Statistical Physics* (Pergamon Press, Oxford, 1980)