Stability Analysis of a Magneto Micropolar Fluid Layer by Variational Method

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Abstract This Paper deals with the Stability of a micropolar fluid layer heated from below in the presence of uniform magnetic field. The basic hydrodynamic equations of magneto-micropolar fluid layer heated from below are modified by using Boussinesq approximation and then the linearized perturbation equations are converted into a characteristic value problem with the help of Normal mode analysis. The expressions for Rayleigh number are obtained by using the variational principle. The effect of magnetic field and micropolar parameter on the Rayleigh numbers are discussed and upper bounds of critical Rayleigh number for all type of boundaries are obtained by using variational method.

Keywords: Rayleigh number, principle of exchange of stabilities, Variational principle, micro rotation, Boussinesq approximation, magnetic induction

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1. Introduction

Thermal convection occurs in nature in so many forms and over such a wide range of scales that it could be claimed with some justification that convection represents the most common fluid flow in the universe.

The experiments by Bénard [1] particular have attracted great attention and are today considered as classical in fluid mechanics. The Bénard stability problem was first formulated and mathematically solved by Lord Rayleigh [2] for the case of free boundaries with a linear temperature gradient. A comprehensive account of the linearized stability theory of Rayleigh-Bénard convection problem in the presence of uniform rotation has been given in Chandrasekhar [3].

Some years ago, Eringen [4] developed the theory of micropolar fluids in which local effects arising from micro-structure and intrinsic motions of the fluid elements are taken into account. Ahmadi [5] studies the stability of a layer of micropolar fluid heated from below using linear theory as well as energy method the derived lower bounds for critical Rayleigh number. The profiles on Benard convection in micropolar fluids is also discussed by Narasimha, MY [6]. Dattav and Sastry [7] also discussed the theory of micropolar fluid layer heated from below and obtained exact solution of the eigen value problem. Joginder Singh Dhiman, Praveenkumar Sharma and Gurdeep Singh.

[8] studied the stability of micropolar fluid layer heated from below by variational principle.

The stability of magneto-micropolar fluid motion was studied by G. Ahmadi and M. Shahinpoor [9]. The stability method employed by G. Ahmadi and M. Shahinpoor was an energy technique due to James Serrin [10].

Our aim in this paper is to verify the stability of magneto-micropolar fluid motion by using the variational method and we have established this by finding the upper bounds for critical Rayleigh numbers for all combinations of different type of boundaries.

2. Mathematical Analysis

2.1. Physical Problem

A viscous finitely heat and electrically conducting, micropolar fluid is statically confined between two horizontal boundaries Z = 0 and Z = d of infinite extension and finite vertical depth which are maintained at uniform temperatures T₀ and T₁ (T₀ > T₁) respectively in the presence of uniform magnetic field acting antiparallel to the force field of gravity.

2.2. Hydrodynamic Equations and Boundary Conditions

The basic thermodynamic equations of the problem of thermal stability of magnetic micropolar fluid layer heated from below are modified by using the usual steps of Boussinesq of approximation and Normal mode. Analysis the non dimensional linearized perturbation equations with boundary conditions are as follows:
\[
\left(D^2 - a^2\right) \left[D^2 - a^2 - \frac{p}{\sigma(1 + K)}\right] w \\
= -\frac{K}{1 + K} \left(D^2 - a^2\right)G + \frac{Ra^2 \theta}{1 + K} \frac{Q}{1 + K} \left(D^2 - a^2\right) h_z \\
\left(D^2 - a^2 - p\right) \theta = -w
\]  
\tag{1}

\left(D^2 - a^2 - \frac{n_p + 2k \sigma}{\sigma C_0}\right) G = \frac{K}{C_0} \left(D^2 - a^2\right)W \\
\left(D^2 - a^2 - \frac{p \sigma}{\sigma}\right) h_z = -Dw 
\tag{2}

The boundary conditions are
\[w = 0 = q = G = DW \text{ at } z = 0, z = 1.\]  
\tag{5}

(When both the boundaries are rigid).
\[w = 0 = q = G = D^2 w \text{ at } z = 0, z = 1\]  
\tag{6}

(When both boundaries are free) and
\[h_z = 0 \text{ at } z = 0, z = 1\]  
\tag{7}

(When both boundaries are either rigid or free, but boundaries are conducting).

In the foregoing equations;
\[K = k \mu, n_i = \frac{J}{d^2}, C_0 = \frac{\gamma}{\mu d^2}\]

\[D \left(\frac{d}{dz}\right)\] is the differentiation with respect to \(z\), \(a^2\) is the square of wave number, \(\sigma\) is the thermal Prandtl number, \(R = \frac{gab\alpha^4}{k_0 \mu}\),
\[K_0 = \frac{k_1}{k_0}\] is the heat diffusivity is the Rayleigh number, \(g\) is the gravitational Acceleration, \(\alpha\) is the coefficient of thermal gradient, \(d\) is the depth of layer, \(k^1\) is the thermometric conductivity, \(\nu\) is the kinematic \(\mu\) is viscosity, \(w, \theta, G\) and \(a\) are the perturbations in vertical velocity, temperature and microrotation respectively and \(Q = \frac{\mu h_z^2 d^2}{4 \pi p \eta}\).

The system of equations (1) — (4) together with boundary condition (5) — (7) constitutes an eigen value problem for \(R\) for given values of other parameters \(\sigma, C_0, n_1, a^2K\) and \(Q\).

2.3. Principle of Exchange of Stabilities (PES)

Firstly in the present problem, we shall see that whether the PES is valid or not for magneto micropolar fluid heated from below.

So multiplying equation (1) by \(w^*\) (the complex conjugate of \(w\)) and integrating over the range 0 ≤\(z\)≤ 1, a suitable number of times by using relevant boundary conditions (5)-(7) we get
\[\int_0^1 \left[D^2 w^2 + a^2 |w|^2 + 2a^2 |Dw|^2\right] dz\]
\[+ \frac{p}{\sigma(1 + k)} \int_0^1 \left[Dw^2 + a^2 |w|^2\right] dz\]
\[= \frac{K}{k + 1} \int_0^1 \left(D^2 - a^2\right) \theta dz + \frac{Q}{1 + K} \int_0^1 \left(D^2 - a^2\right) h_z dz.\]  
\tag{8}

Taking the complex conjugate of equation (3) and multiplying the resulting equation by \(G\) on both sides and integrating it over the range of \(z\), a suitable number of times by using boundary conditions (5) – (7), we get
\[\int_0^1 \left[DG^2 + a^2 |G|^2 + \frac{n_p + 2k \sigma}{\sigma C_0} |G|^2\right] dz\]
\[= -\frac{K}{C_0} \int_0^1 \left(D^2 - a^2\right) \theta dz\]  
\tag{9}

By using equation (9) is equation (8) and integrating by parts the resulting equation, we get
\[\int_0^1 \left[DG^2 + a^2 |G|^2 + \frac{n_p + 2k \sigma}{\sigma C_0} |G|^2\right] dz\]
\[+ \frac{p}{\sigma(1 + k)} \int_0^1 \left[Dw^2 + a^2 |w|^2\right] dz\]
\[= \frac{C_0}{1 + k} \int_0^1 \left[DG^2 + a^2 |G|^2 + \frac{n_p + 2k \sigma}{\sigma C_0} |G|^2\right] dz\]
\[+ \frac{Ra^2}{1 + k} \int_0^1 \left[\theta^2 + a^2 |\theta|^2 + |\theta|^2\right] dz\]
\[+ \frac{Q}{1 + k} \left[2a^2 \Gamma + \int_0^1 \left[Dh_z^2\right]^2 + a^2 |Dh_z^2 + a^4 |h_z|^2\right] dz\]
\[+ \frac{Qp \sigma}{\sigma(1 + k)} \left[\Gamma + \int_0^1 \left[Dh_z^2\right]^2 + a^2 |h_z|^2\right] dz\]
\[\Gamma = a \left(\left[Dh_z^2\right]_0^1 + \left|h_z^2\right|_0^1\right) \geq 0.\]

Where \(\Gamma = a \left(\left[Dh_z^2\right]_0^1 + \left|h_z^2\right|_0^1\right) \geq 0.\)

Now equating real and imaginary parts of both sides of equation (10) and canceling \(p, (\neq 0)\) (supposition) throughout from imaginary part, we get;
\[\frac{1}{\sigma(1 + k)} \left[\left[Dw^2\right]^2 + 2a^2 |w|^2\right] dz\]
\[+ \frac{C_0}{1 + K} \int_0^1 \left|\theta\right|^2 dz\]
\[+ \frac{Ra^2}{1 + K} \left|\theta\right|^2 dz = \frac{Q\sigma}{\sigma(1 + k)} \left[\Gamma + \int_0^1 \left[Dh_z^2\right]^2 + a^2 |h_z|^2\right] dz.\]
Now, multiplying the equation (4) by \( h_z^* \) (the complex conjugate of \( h_z \)) and integrating the resulting equation over the vertical range of \( z \), a suitable number of times and making use of either boundary conditions (5) – (7), we have:

\[
\Gamma + \int_0^1 \left( |Dh_z|^2 + \sigma^2 |h_z|^2 \right) dz + \frac{p\sigma}{\sigma} \int_0^1 |h_z|^2 dz
\]

\[
= -\int_0^1 w \, Dh_z^* dz
\]

Equating real parts from both sides of above equation, we have:

\[
\Gamma + \int_0^1 \left( |Dh_z|^2 + \sigma^2 |h_z|^2 \right) dz + \frac{p\sigma}{\sigma} \int_0^1 |h_z|^2 dz
\]

\[
= \text{Real part of} \left( \int_0^1 w \, Dh_z^* dz \right)
\]

\[
\leq \int_0^1 W |Dh_z| \, dz \leq \int_0^1 |W| |Dh_z| \, dz
\]

\[
\leq \frac{1}{2} \left[ \int_0^1 |w|^2 \, dz + \int_0^1 |Dh_z|^2 \, dz \right] .
\]

Since \( p \geq 0 \) from above inequality, we have

\[
\Gamma + \int_0^1 \left( |Dh_z|^2 + \sigma^2 |h_z|^2 \right) dz \leq \int_0^1 |w|^2 \, dz - \sigma^2 \int_0^1 |h_z|^2 \, dz
\]

Further, since \( w(0) = 0 = w(1) \), we have the Reyleigh – Ritz inequality namely;

\[
\int_0^1 |w|^2 \, dz \leq \frac{1}{\Pi^2} \int_0^1 |Dw|^2 \, dz
\]

Using inequalities (14) and (15) is equation (11), we get

\[
\left[ \frac{\Pi^2}{\sigma(1+K)} - \frac{Q\sigma}{\sigma(1+K)} \right] \int_0^1 |w|^2 \, dz + \frac{Q\sigma^2}{\sigma(1+K)} \int_0^1 |h_z|^2 \, dz
\]

\[
+ \frac{Ra}{1+K} \int_0^1 |y|^2 \, dz + \frac{n_1}{\sigma(1+K)} \int_0^1 |G|^2 \, dz
\]

\[
< 0.
\]

It clearly follows from above inequality that

\[
\frac{\Pi^2}{\sigma(1+K)} \frac{Q\sigma}{\sigma(1+K)} < 0
\]

\[
Q > \frac{\Pi^2}{\sigma_1},
\]

Which is a contradiction to our supposition that \( p \neq 0 \). Hence, we must have \( p = 0 \). In particular \( p = 0 \Rightarrow p = 0 \), hence PES is Valid.

Therefore, we have the result that PES is not valid or have over stability when \( Q\sigma_1 > \Pi^2 \), which can also be seen in classical magneto hydrodynamic Bénard convection.

### 2.4. Variational Principle

When instability sets as stationary convection, the marginal state will be characterized by \( p = 0 \) and basic equations (1) – (4) reduce to

\[
\left( D^2 - \alpha^2 \right) w + \frac{K}{1+K} \left( D^2 - \alpha^2 \right) G + \frac{1}{1+K} \alpha^2 R \theta
\]

\[
= -\frac{Q}{1+K} D \left( D^2 - \alpha^2 \right) h_z (17)
\]

\[
\left( D^2 - \alpha^2 \right) \theta = -w (18)
\]

\[
\left( D^2 - \alpha^2 \right) h_z = -Dw. (19)
\]

From equation (17), letting

\[
F = \left( D^2 - \alpha^2 \right)^2 w + \frac{K}{1+K} \left( D^2 - \alpha^2 \right) G
\]

\[
+ \frac{Q}{1+K} D \left( D^2 - \alpha^2 \right) h_z
\]

implies,

\[
F = \frac{1}{1+K} a^2 R \theta. (22)
\]

Operating the above equation lay \( (D^2 - \alpha^2) \) and using equation (18), we get:

\[
(D^2 - \alpha^2) F = -\frac{a^2}{1+K} Rw. (23)
\]

Multiplying the above equation by \( F \) and integrating the resulting equation over the range of \( z \), we have

\[
\int_0^1 F \left( D^2 - \alpha^2 \right) F dz = -\frac{a^2}{1+K} \int_0^1 w \, F dz. (24)
\]

Since \( F(0) = 0 = F(1) \) because of equation (22), therefore, we have

\[
\int_0^1 F \left( D^2 - \alpha^2 \right) F dz = -\frac{1}{2} \left( (DF)^2 + a^2 F^2 \right) dz. (25)
\]

Now integrating the equation (24) by using \( F \) from equation (22) and integrating a suitable number of times by using relevant boundary conditions, we get:

\[
\frac{K}{1+K} \int_0^1 w \left( D^2 - \alpha^2 \right) G dz = \frac{K}{1+K} \left[ \int_0^1 G \left( D^2 - \alpha^2 \right) dz \right]. (26)
\]

By using equation (19) in above equation and integrating the resulting equation by parts a suitable number of times and using boundary conditions on \( G \), we get

\[
\frac{k}{1+k} \int_0^1 w \left( D^2 - \alpha^2 \right) G dz
\]

\[
= -\frac{C_0}{1+k} \left( (DG)^2 + \left( a^2 + \frac{2k}{C_0} \right) G^2 \right) dz. (27)
\]
Now, by using all these equations (25), (27), in the
equation (24), we have:
\[
\int_0^1 \left( (DF)^2 + a^2 F^2 \right) dz \\
= a^2 R \left[ \int_0^1 \left( (D^2 - a^2) \right)^2 dz \right] \\
- \frac{C_0}{1+K} \left[ \int_0^1 (DG)^2 + \left( a^2 + \frac{2K}{C_0} \right) G^2 \right] dz \\
+ \frac{Q}{1+K} \left[ \int_0^1 (DW)^2 dz \right]
\]
Or
\[
R = \frac{(K+1) \left[ \int_0^1 \left( (DF)^2 + a^2 F^2 \right) dz \right]}{a^2 \left[ \int_0^1 \left( (D^2 - a^2) \right)^2 dz \right] \\
- \frac{C_0}{1+K} \left[ \int_0^1 (DG)^2 + \left( a^2 + \frac{2K}{C_0} \right) G^2 \right] dz \\
+ \frac{Q}{1+K} \left[ \int_0^1 (DW)^2 dz \right]}
\]
which may be written as
\[
R = \frac{(K+1) I_1}{a^2 I_2}
\]
Where
\[
I_1 = \int_0^1 \left[ \int_0^1 DF^2 + a^2 F^2 \right] dz
\]
And
\[
I_2 = \int_0^1 \left[ \int_0^1 \left( D^2 - a^2 \right) w^2 dz - \frac{C_0}{1+K} \int_0^1 (DG)^2 \right] dz \\
+ \left( a^2 + \frac{2K}{C_0} \right) G^2 \right] dz + \frac{Q}{1+K} \left[ \int_0^1 (DW)^2 dz \right]
\]

2.5. Stationary Property

The stationary property is R given by equation (28) is
checked just by giving a small variation \( \delta W \) to w, \( \delta G \) to G
and \( \delta h \) to h, which are again compatible with boundary
conditions i.e.
\[
dG = 0, dw = 0, dhz = 0 and dF = 0 for z = 0 and z = 1.
\]
This can be easily checked that \( \delta R = 0 \) if \( (D^2 - a^2) \) F = 
\(-Ra^2 w \) and conversely if \( \delta R = 0 \) for any arbitrary
Variation, then \( (D^2 - a^2) \) F = \(-Ra^2 w \).

2.6. Minimum Property

Now, we shall show that the lowest characteristic value
of R is indeed, a true minimum.
Let R be a characteristic value and let corresponding
characteristic function be distinguished by a subscript i.
Then from equation (23), we have
\[
\left( D^2 - a^2 \right) F_i = \frac{R}{K+1} a^2 w_i
\]
and
\[
W_i = F_i = 0 at z = 0 and z = 1
\]
and either
\[
D^2 w_i = 0 or D^2 w_j = 0 at z = 0 and z = 1.
\]
Multiplying equation (29) by \( F \) and integrating the
resulting equation over vertical range of z, we have
\[
\int_0^1 F_j \left( D^2 - a^2 \right) F_i dz = -\frac{R a^2}{K+1} \int_0^1 F_j W_i dz.
\]
By using equation (21), we have
\[
\int_0^1 F_j \left( D^2 - a^2 \right) F_i dz
\]
\[
= -\frac{R a^2}{K+1} \int_0^1 \left[ \left( D^2 - a^2 \right) W_j + \frac{K}{1+K} \left( D^2 - a^2 \right) G_j \right] dz
\]
\[
+ \frac{Q}{1+K} \left( D^2 - a^2 \right) Dhz
\]
Integrating the integrants on right hand side of above
equation by parts, a suitable number of times with the help
of boundary conditions(30) – (31), We have
\[
\int_0^1 W_j F_i dz = \int_0^1 W_j F_i dz
\]
Now, \[
\int_0^1 F_j \left( D^2 - a^2 \right) F_i dz = \int_0^1 \left( DF_i DF_i + a^2 F_i F_i \right) dz
\]
using this equation in (29) and interchanging i and j then
subtracting the resulting equations, we have
\[
(R_i - R_j) \frac{a^2}{K+1} W_i F_j dz = 0.
\]
Since, \( R_i \neq R_j \) we have;
\[
\int_0^1 W_i F_j dz = 0 if i \neq j
\]
if i = j.
The functions $W_i$ form an orthogonal set. Let

$$ W = \sum_{i=1}^{\infty} A_i W_i, \text{ Where } A_i = \lim_{0}^{\infty} \int_{0}^{\infty} W_i dz $$ (33)

Similarly using the expanded forms $W$ terms of basic set functions for $G, F,$ and $h$, the values of integrals $I_1$ and $I_2$ and equation (28) having the following values,

$$ I_1 = \frac{a^2}{1 + k} \sum_{i=1}^{\infty} A_i^2 R_i \quad (\text{for } i \neq j) $$

$$ I_2 = \sum_{i=1}^{\infty} A_i^2 \quad (\text{for } i = j) $$

Using these values of $I_1$ and $I_2$, is equations (28), we have

$$ R = \frac{(1 + K) a^2}{1 + k} \sum_{i=1}^{\infty} A_i^2 $$

$$ R = \frac{\sum_{i=1}^{\infty} A_i^2 R_i}{\sum_{i=1}^{\infty} A_i^2} = \sum_{i=1}^{\infty} R_i B_i^2 $$ (34)

When $\sum_{i=1}^{\infty} B_i^2 = 1$.

In follows from equation (34) that

$$(R \cdot R) = \sum_{i=1}^{\infty} (R_i - R_c) R_i^2 \geq 0.$$ (35)

Since $R_i \geq R_c \forall i$.

Thus, we have

$$ R_c \leq R = \frac{(1 + K) I_1}{a^2 I_2} $$

This shows that the quantity on R.H.S has a true minimum.

### 2.7 Upper Bounds for Critical Raylight Number for All Combination of Boundary Conditions

Let

$$ F = \cos \Pi z $$ (36)

Which obviously satisfies the boundary conditions

$$ F = 0 \text{ at } z = -\frac{1}{2} \text{ and } z = +\frac{1}{2}, $$

where the origin has been shifted to midway for convenience in computations.

Using the value of $F$ from equation (36) in equation (21), we obtain

$$ (D^2 - a^2) F + \frac{K}{1 + K} (D^2 - a^2) G $$

$$ + \frac{Q}{1 + K} (D^2 - a^2) H = \cos \Pi z $$

Operating the above equation by $(D^2 - a^2 - \frac{2K}{C_o})$ and using the equations (19) – (20), We get

$$ \left( \Pi^2 + \left( a^2 + \frac{2K}{C_o} \right) \right) \cos \pi Z $$

$$ = \left( D^2 - a^2 - \frac{2K}{C_o} \right) \left( D^2 - a^2 - \frac{2K}{C_o} \right)^2 w $$

$$ + \frac{Q}{1 + K} \left( D^2 - a^2 + \frac{2K}{C_o} \right) w $$

The general solution of above differential equation is given by

$$ W = B_1 \cosh x_1 z + B_2 \cosh x_2 z $$

$$ + B_3 \cosh x_3 z + A \cos \Pi z $$ (39)

Where $x_1^2, x_2^2, x_3^2$ are roots of auxiliary equation of (38) and

$$ A = \left\{ \begin{array}{l}
\Pi^2 + a^2 + \frac{2K}{C_0} \\
\Pi^2 + a^2 + \frac{K^2 + 2K}{C_0(1 + K)} \\
+ \frac{Q}{1 + K} \left( \Pi^2 + a^2 + \frac{2K}{C_0} \right) 
\end{array} \right\} $$

**Case I: when both the boundaries are dynamically free.**

By using equation (39), we have

$$ Dw = B_1 x_1, \sinh x_1 z + B_2 x_2 \sinh x_2 z $$

$$ + B_3 x_3 \sinh x_3 z - \Pi A \sin \Pi z $$

$$ D^2 w = B_1 x_1^2 \cosh x_1 z + B_2 x_2^2 \cosh x_2 z $$

$$ + \cosh x_3 z - \pi A \cos \Pi z $$

And

$$ \theta = -\left\{ \begin{array}{l}
\frac{B_1}{x_1^2 - a^2} \cos h x_1 z + \frac{B_2}{x_2^2 - a^2} \cos h x_2 z \\
+ \frac{B_3}{x_3^2 - a^2} \cos h x_3 z - \frac{A \cos \Pi z}{\Pi^2 + a^2} 
\end{array} \right\} $$ (42)

Since, both the boundaries are dynamically free, so boundary conditions are $W = 0, D^2 w$ at $z = \pm \frac{1}{2}$ and $\theta = 0$ at $z = \pm \frac{1}{2}$. 
By using these boundary conditions is above equations, we have
\[ B_1 = B_2 = B_3 = 0. \]

From equation (39), we get \( W = A \cos \Pi z \), when \( F = \cos \Pi z \).

Now, evaluating \( I_1 \) and \( I_2 \) in equation (28), we have
\[ B_1 = B_2 = B_3 = 0. \]

From equation (39), we get
\[ W A \cos z = \prod \]

Now, evaluating \( I_1 \) and \( I_2 \) is equation (28), we have
\[ \left( \prod ^2 + a^2 \right) \]

Using these values of \( I_1 \) and \( I_2 \) in equations (35), we get
\[ \left( \prod ^2 + a^2 \right) \]

Now, using the value of \( A \), we have
\[ \left( \prod ^2 + a^2 \right) \]

Letting \( k \rightarrow \infty \) in the above inequality, we have:
\[ \lim_{k \rightarrow \infty} \frac{R_c}{K} \leq \frac{\left( \prod ^2 + a^2 \right)^3}{2a^2} \]

Case II One Rigid – One Free Boundary:

Let us take lower boundary as rigid and upper dynamically free, so using these boundary conditions in the equations (39) – (41) we get:
\[ B_1 \cos h \left( \frac{x_1}{2} \right) + B_2 \cos h \left( \frac{x_2}{2} \right) + B_3 \cos h \left( \frac{x_3}{2} \right) = 0 \]
\[ x_1 B_1 \sin h \left( \frac{x_1}{2} \right) + x_2 B_2 \sin h \left( \frac{x_2}{2} \right) + B_3 x_3 \sin h \left( \frac{x_3}{2} \right) = 0 \]
\[ = \frac{\prod A}{K} \]
\[ x_1^2 B_1 \cos h \left( \frac{x_1}{2} \right) + x_2 B_2 \cos h \left( \frac{x_2}{2} \right) \]
\[ + B_3 x_3 \cos h \left( \frac{x_3}{2} \right) = 0 \]

Now, by using the values of \( B_1, B_2, B_3, \ A, \ A \) and letting \( K \rightarrow \infty \) in above inequality, we have
\[ \lim_{k \rightarrow \infty} \frac{R_c}{K} \leq \frac{\left( \prod ^2 + a^2 \right)^3}{2a^2} \]
\[
\frac{Li}{K} \rightarrow \infty \quad \frac{R_c}{K} \leq \frac{\left(\pi^2 + a^2\right)^3}{2a^2} \quad (49)
\]

Which is upper bound for \( R_c \) for both rigid-rigid boundaries.

3. Conclusion

In this paper it is verified that PES is valid for magneto-micropolar fluid motion for all type of boundary conditions when heated from below. There is over stability only when \( Q \sigma_1 > \Pi^2 \), which can also be seen in classical magneto hydrodynamic Bénard convection. The upper bounds for Critical Rayleigh number, that is

\[
\frac{Li}{K} \rightarrow \infty \quad \frac{R_c}{K} \leq \frac{\left(\Pi^2 + a^2\right)^3}{2a^2}
\]

are obtained by using variational principle having same value for all type of boundary conditions namely, both free-free, one rigid-one free and both rigid-rigid. Further, these upper bounds depend upon the value of micro rotation coefficient \( K \) in magneto-micropolar fluid heated from below. The inequality for upper bounds clearly shows that the increasing value of \( K \) increases the Rayleigh number \( R \). Thus \( K \) plays a stabilizing role in the system.

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