Distributed Stabilization by Probability Control for Deterministic-Stochastic Large Scale Systems: Dissipativity Approach
Koji Tsumura, Binh Minh Nguyen, Hisaya Wakayama, and Shinji Hara

Abstract—By using dissipativity approach, we establish the stability condition for the feedback connection of a deterministic dynamical system and a stochastic memoryless map. After that, we extend the result to the class of large-scale systems in which: the deterministic system consists of many sub-systems; and the stochastic map consists of many stochastic actuators. We will demonstrate the proposed approach by showing the design procedures to globally stabilize the manufacturing systems while locally balance the stock levels in any production process.

I. INTRODUCTION

In these decades, one of the most active research fields in the control community is the distributed/decentralized control of large-scale systems, and it is also promoted by several research projects such as “Industry 4.0”, “Smart factory,” or “IoT.” Our research group also have contributed to this field with a concept of “glocal control” to achieve both glocal and local objectives by using local measurements and local control actions [1]. To make the availability of the distributed control strategy to the actual systems higher, it is necessary to extend the class of objective plants or controllers close to realistic systems. When the number of subsystems becomes large, it is natural that the properties and the behavior emerged in the systems become heterogeneous and from above idea, we consider large scale hybrid systems composed of deterministic subsystems, which may include time delay, and stochastic subsystems in this paper. Especially, we deal with a case that the actions of control actuators are stochastic. We can find this class of systems in various applications such as electric power networks [2], manufacturing factories [3], chemical process systems [4], and biological systems [5], and so on.

A typical example is the case of manufacturing factories, which are composed of several manufacturing processes and transportation processes by a swarm of vehicles. One of strategies to operate the transportation is a centralized deterministic scheduling of all vehicles [6], however when the number of vehicles is large, it is not feasible. Another strategy is that each vehicle moves around autonomously and randomly according to guidance signals. This method is more realistic for large scale systems as discussed in this paper, however the behavior of the vehicles can be regarded as stochastic actions. As a result, the whole system is composed of deterministic manufacturing processes and stochastic transportation processes.

II. MOTIVATING EXAMPLE: MANUFACTURING SYSTEMS

A. Outline of the manufacturing systems

As shown in Fig. 2, we consider a network of N production processes (PPs). Each production process PPi includes a pair of an input buffer and an output buffer of materials where...
$x_i^{[i]}(t)$ and $x_o^{[i]}(t)$ represent their stock levels, respectively. Let $p_i^{[i]}(t)$ denote the production rate from the input-buffer to the output-buffer of the PP $i$, $u_k^{[i,j]}(t)$ the transportation rate from the output-buffer of PP $i$ to the input-buffer of PP $j$, $M^{[i]}$ the set of indices of PP from which materials are sent to input-buffer of PP $i$, and $\bar{M}^{[i]}$ the set of indices to which materials are sent from output-buffer of PP $i$. The control objectives are (i) to satisfy a given throughput of materials from the entrance to the exit of the manufacturing system, (ii) to keep all the stock levels of materials in the buffers equal in a normalized unit.

B. Deterministic dynamics of the stock level in PP

The stock level dynamics are given as follows

$$x_i^{[i]}(t+1) = x_i^{[i]}(t) - p_i^{[i]}(t) \sum_{k \in M^{[i]}} u_{k}^{[i,j]}(t)$$

$$x_o^{[i]}(t+1) = x_o^{[i]}(t) + p_i^{[i]}(t) \sum_{k \in \bar{M}^{[i]}} u_{k}^{[i,j]}(t)$$

(1)

(2)

To achieve the objective (ii) in the local PP $i$, the local production rate is given as

$$p_i^{[i]}(t) = -h_i^{[i]} \left( x_i^{[i]}(t) - x_i^{[i]}(t) \right)$$

(3)

where $h_i^{[i]}$ is a production control gain which is selected between 0 and 1 to stabilize the local agent PP $i$. Define

$$x_i^{[i]}(t) = \left[ x_i^{[i]}(t), x_o^{[i]}(t) \right]^T, u_i^{[i]}(t) = \sum_{k \in M^{[i]}} u_{k}^{[i,j]}(t) - \sum_{k \in \bar{M}^{[i]}} u_{k}^{[i,j]}(t)$$

(4)

the stock level dynamics of the PP $i$ is expressed as

$$\dot{x}_i^{[i]}(t+1) = A_i^{[i]} x_i^{[i]}(t) + u_i^{[i]}(t)$$

(5)

$$A_i^{[i]} = \begin{bmatrix}
1 - h_i^{[i]} & h_i^{[i]} \\
-h_i^{[i]} & 1 - h_i^{[i]}
\end{bmatrix}$$

(6)

We also assume that the stock levels can be measured by local sensor $q_i^{[i]}$, i.e., $\hat{x}_i^{[i]}(t) = q_i^{[i]}(x_i^{[i]}(t)) = x_i^{[i]}(t)$.

C. Stochastic transportation actuator

There exists $M$ local transportation actuators $\tau_j^{[i,j]}, j \in M^{[i]}, i \in [1, N]$. Each actuator can be treated as a memoryless operator that probabilistically outputs the quantized values $u_k^{[i,j]}(t) \in \{ 0, +1, -1 \}$. The input to each $\tau_j^{[i,j]}$ is a scalar signal $u_c^{[i,j]}(t)$ generated by the transportation controller $C_i^{[i,j]}$. We regard the value “-1” of negative transportation as a “virtual action” [3]. The stochastic operator is expressed as

$$\tau_j^{[i,j]}(u_c^{[i,j]}(t)) = \begin{cases}
+1 & \text{Prob.} = \pi_c(u_c^{[i,j]}(t)) \times u_c^{[i,j]}(t) \\
0 & \text{Prob.} = 1 - |u_c^{[i,j]}(t)| \\
-1 & \text{Prob.} = \pi_c(u_c^{[i,j]}(t)) \times u_c^{[i,j]}(t)
\end{cases}$$

(7)

where $u_c^{[i,j]}(t)$ is normalized between -1 and +1, and

$$\pi_c(u_c^{[i,j]}(t)) = \begin{cases}
+1 & \text{if } u_c^{[i,j]}(t) \geq 0 \\
0 & \text{if } u_c^{[i,j]}(t) < 0
\end{cases}$$

(8)

D. Model of the overall manufacturing system

Now we define

$$x(t) = \left[ \begin{array}{c} x_1^{[1]}(t) \\ x_2^{[1]}(t) \\ \cdots \\ x_N^{[1]}(t) \end{array} \right]^T$$

$$u_i(t) = \left[ u_i^{[1]}(t) u_i^{[2]}(t) \cdots u_i^{[N]}(t) \right]^T$$

(9)

$$u_c(t) = \left[ u_1^{[1]}(t) u_2^{[2]}(t) \cdots u_N^{[N]}(t) \right]^T$$

(10)

where $u_c^{[i]}(t) = \{ u_k^{[i]}(t) \}, k \in \bar{M}^{[i]}$, is a column vector of size $|\bar{M}^{[i]}|$. We have

$$\dot{x}(t+1) = Ax(t) + Bu_i(t) = Ax(t) + u(t)$$

(11)

where $A = \text{diag}\{ A_i^{[i]} \}$ and $B$ is an incidence matrix of the manufacturing network. The size of matrix $B$ is $2N \times M$ where $M = |\bar{M}^{[1]}| + \ldots + |\bar{M}^{[N]}|$. Due to space limitation, we neglect to discuss the general structure of matrix $B$ which was presented in our recent works [3].

In general case, $y_c(t) = -B^T x(t)$ is a column vector that consists of the components $\{ y_c^{[i,j]}(t) \}$. It measures the stock level unbalances between the PPs. In other words, it can be utilized to attain the aforementioned global objective (i). It should be treated as the input to the decentralized transportation controller $\{ D_j^{[i,j]} \}$. Each $D_j^{[i,j]}$ maps the input $y_c^{[i,j]}(t)$ to the scalar control signal $u_j^{[i,j]}(t)$ for controlling the local transportation actuator $\tau_j^{[i,j]}$. Based on (1)–(11), the manufacturing control system is established as in Fig. 3 which is equivalent to the feedback connection of a deterministic $\bar{P}$ and a stochastic $\hat{C}_\tau$.

III. DETERMINISTIC-STOCHASTIC DISSIPATIVITY APPROACH

A. Definitions and preliminary result

Firstly, we consider a class of feedback connection shown in Fig. 1 where $\Sigma$ is a linear discrete-time system
where the input vector \(u(t) \in \mathbb{R}^p\), the output vector \(y(t) \in \mathbb{R}^p\), and the state vector \(x(t) \in \mathbb{R}^n\); the matrices \(\{A, B, C, D\}\) are of appropriate sizes. \(\Psi\) is a stochastic memoryless map with the input vector \(y(t)\) and the output vector \(z(t)\) of the same size.

**Definition 1** [19]: \(\Sigma\) is \((Q_x, R_x, S_x)\) dissipative if there exists a positive semidefinite storage function \(V: \mathbb{R}^n \rightarrow \mathbb{R}_+\), such that \(V(x(t + 1)) - V(x(t)) \leq W_x(y(t), u(t))\) holds true \(\forall x(t) \in \mathbb{R}^n\) and \(u(t) \in \mathbb{R}^p\) where \(W_x(y(t), u(t)) = y(t)^T Q_x y(t) + u(t)^T R_x u(t) + 2y(t)^T S_x y(t)\) where \(Q_x = Q_x^T, R_x = R_x^T, S_x\) are the matrices of appropriate sizes.

To define the dissipativity of \(\Psi\) in a sense of probability, we firstly state the expectation definition.

**Definition 2**: A map \(g(\cdot)\) maps an input \(a \in \mathbb{R}^p\) to the limited set \([b_1^n, b_2^n, \ldots, b_k^n]\), \(b_k^n \in \mathbb{R}^p\). We let \(W\) be a function of \(a\) and \(g(a)\), \(W: \mathbb{R}^p \times \mathbb{R}^p \rightarrow \mathbb{R}\). \(W(a, g(a))\) might take one value in the limited set of \(\{W(a, b)\}\) with the probability \(\{P(a)\}\) where \(k \in [1, K]\). \(P(a)\) is obtained by the stochastic properties of \(g(\cdot)\) and it s.t. \(P(a) + \ldots + P(a) = 1, 0 \leq P(a) \leq 1\) for all \(k\). The expectation of \(W(a, g(a))\) is given as

\[
E[W(a, g(a))] = \sum_{k=1}^{K} W(a, b_k^n) P(a)
\]

**Definition 3**: \(\Psi\) is said to be \((Q_x, R_x, S_x)\) dissipative if the following inequality holds true

\[
\sum_{i=0}^{n} E[W_x(z(t), y(i))] \geq 0 \quad \forall y(i) \in \mathbb{R}^p, l \in \mathbb{Z}, l \geq 0
\]

where \(W_x(z(t), y(i)) = z(t)^T Q_x z(t) + y(i)^T R_x y(i) + 2z(t)^T S_x y(i)\) where \(Q_x = Q_x^T, R_x = R_x^T, S_x\) are the matrices of appropriate sizes. For instance, \(\Psi\) is said to be output strictly passive (OSP) if it is \((\Theta_p, \gamma I_p, 0.5 I_p)\) dissipative with a storage function \(V(t)\); and each map \(\Psi^{[k]}\) is an OSP system that satisfies \(E[z(t), y(t)] \geq \gamma(t)E[y(t)]\). If \(\gamma(t) \geq L^2(dg[I_p, I_p]L)\), then \(E[V(x(t + 1)) | x(t)] - V(x(t)) \leq 0\) holds true with a storage function \(V = V^{[1]} + V^{[2]} + \ldots + V^{[N]}\).

**Proposition 4**: If \(\Sigma = \mathbb{I}(\Theta, \gamma I_p, 0.5 I_p)\) dissipative with a storage function \(V\), if \(\gamma(t) \geq \gamma(t)\) then the following inequality holds true:

\[
\mathbb{E}[V(x(t + 1)) | x(t)] - V(x(t)) \leq 0
\]

B. Dissipativity for multi-agent systems

Next, we will extend **Proposition 4** to the class of multi-agent systems in Fig. 4 where \(\Sigma^{[i]} (i = 1 \text{ to } N)\) is a linear discrete time system with the input \(u^{[i]}(t) \in \mathbb{R}^p\), output \(y^{[i]}(t) \in \mathbb{R}^p\), and the state \(x^{[i]}(t) \in \mathbb{R}^p\). Each \(\Psi^{[i]} (k = 1 \text{ to } M)\) is a stochastic map with the scalar input \(u^{[i]}(t)\) and the scalar output \(z^{[i]}(t)\). Matrix \(L\) of size \(Np \times M\) represents the connections of the agents, and matrix \(L^T\) can be used for the purposes of consensus and system stabilisation. The system \(\Sigma = \mathbb{I}(\Theta, \gamma I_p, 0.5 I_p)\) has the input, output, and state vectors \(u^{[i]}(t) = [u^{[i]}(t) u^{[i]}(t) \ldots u^{[i]}(t)]^T \in \mathbb{R}^{np}\), \(y^{[i]}(t) = [y^{[i]}(t) y^{[i]}(t) \ldots y^{[i]}(t)]^T \in \mathbb{R}^{np}\), \(x^{[i]}(t) = [x^{[i]}(t) x^{[i]}(t) \ldots x^{[i]}(t)]^T \in \mathbb{R}^{np}\).

**Theorem**: The system \(\Sigma\) has the input \(u(t)\) and the output \(y(t)\). The map \(\Psi = \mathbb{I}(\Theta, \gamma I_p, 0.5 I_p)\) has the input vector \(\bar{y}(t)\) and the output vector \(\bar{z}(t)\) expressed as

\[
\bar{y}(t) = \begin{bmatrix} y^{[i]}(t) \\ y^{[i]}(t) \\ \vdots \\ y^{[i]}(t) \end{bmatrix}^T \in \mathbb{R}^M
\]

\[
\bar{z}(t) = \begin{bmatrix} z^{[i]}(t) \\ z^{[i]}(t) \\ \vdots \\ z^{[i]}(t) \end{bmatrix}^T \in \mathbb{R}^L
\]

**Proof**: Since each \(\Sigma^{[i]}\) is \((\Theta_p, \gamma I_p, 0.5 I_p)\) dissipative, the following inequality holds true for the system \(\Sigma\):

\[
V(x(t + 1)) - V(x(t)) \leq \mathbb{E}[\Psi^{[i]}(x(t)) | x(t)] - V(x(t)) \leq 0
\]

where the storage function \(V = V^{[1]} + V^{[2]} + \ldots + V^{[N]}\). Substitute \(u(t) = \tilde{L}u(t) = \tilde{L}y(t)\) into (16), we have

\[
V(x(t + 1)) - V(x(t)) \leq -z(t)^T \gamma(t)(z(t)) + z(t)^T L^T (dg[I_p, I_p]L) \mathbb{E}[\Psi^{[i]}(x(t)) | x(t)] - V(x(t)) \leq 0
\]

**IV. DEMONSTRATION OF THE DISSIPATIVITY APPROACH**

This Section is to demonstrate the dissipativity approach presented in Section III. We consider the cyclic manufacturing systems with homogeneous production control gains for simplicity. Due to the limitation of paper space, we only present the main theoretical results which can be derived without special difficulties from Definitions 1–3 and Propositions 4–5.

A. Manufacturing system without transportation delay

**Procedure 1 (without transportation delay)**

We consider a cyclic manufacturing system of \(N\) PPs with the homogeneous production control gain \(\gamma = 1/(2(1-h))\). The transportation controller \(\beta^{[i]}(x)\) is designed such that
(i) It is a SISO memoryless mapping with the input $y^{[i\to j]}(t)$ and the output $u^{[i\to j]}(t)$ which is a normalized number that varies between $-1$ and $+1$.

(ii) The cascade of $\tau^{[i\to j]}$ and $\varphi^{[i\to j]}$ is a memoryless map $\tilde{\mathcal{C}}^{[i\to j]}$ with the input $\gamma^{[i\to j]}(t)$ and the output $u^{[i\to j]}(t)$ that s.t.

$$E(u^{[i\to j]}(t) | \gamma^{[i\to j]}(t)) \geq \delta E(u^{[i\to j]}(t)^2)$$

where $\delta \geq 2\gamma_m$.

To discuss the stability and consensus issue, we firstly define the equilibrium state vector $\bar{x} = \frac{1}{2N} \sum_{i=1}^{2N} x_i(0)$, where $1_{2N}$ is the all-one column vector of size $2N$, and $x(0)$ is the initial state vector that includes all the stock levels at time step $t = 0$. The error state vector is defined as $\tilde{x}(t) = x(t) - \bar{x}$.

**Proposition 6:** If the cyclic manufacturing system is design to satisfy the Procedure 1, then the following inequality holds true for the total network $E(V(x(t+1)) | x(t)) - V(x(t)) \leq 0$ where $V(x(t)) = 0.5x^T(t) \cdot x(t)$. On the other hand, the error state dynamics s.t. $E(V(x(t+1)) | x(t)) - V(x(t)) \leq 0$, $\forall t$.

**Proof:** In Fig. 3, let $\tilde{\mathcal{P}}$ be the cascade connection of $\mathcal{P}$ and $\mathcal{J}$, with the input $u(t)$ and the output $y(t) = x_i(t)$. Applying the KYP Lemma [19], each $\tilde{\mathcal{P}}$ is shown $(\Theta_j, \gamma T_j, 0.5 I_j)$ dissipative with the storage function $V[x(t)] = 0.5x^T(t) \cdot x(t)$ and $\gamma \geq \gamma_m = 1/(2(1-h))$. The statement of Proposition 6 is obtained by applying Proposition 5 to Fig. 3 with the notice that $B^{\tilde{\mathcal{P}}} = 2I_2$ in case of cyclic manufacturing.

A class of transportation controller that satisfies the Procedure 1 is proposed in Fig. 5 where $L^{[i\to j]}$ is a positive gain and the function $f^{[i\to j]}(\cdot)$ is a normalizer with the following characteristics. (i) $f^{[i\to j]}(\theta) = \tanh(\theta)$ if $|\theta| \geq L$, (ii) $f^{[i\to j]}(\theta) = 0$ for $|\theta| < L$, (iii) $f^{[i\to j]}(\theta) \leq 1$ for all $\theta \in \mathbb{R}$.

Several normalizer candidates are

$$f^{[i\to j]}(\theta) = \begin{cases} \tanh(\theta) & \text{if } |\theta| \geq L, \\ 0 & \text{if } |\theta| < L \end{cases} \quad (19-a)$$

$$f^{[i\to j]}(\theta) = \begin{cases} 2 \tan^{-1}(\theta) & \text{if } |\theta| \geq L, \\ 0 & \text{if } |\theta| < L \end{cases} \quad (19-b)$$

**B. Cyclic manufacturing system with transportation delay**

Model of the system with transportation delay:

We notice that the stock level in the output-buffer of PP $i$ can be reduced instantly. We assume that the transportation route to the input-buffer of PP $i$ has a delay of $d[i]$ steps which is a positive integer known by the controller. Define the delay operator $\Gamma^{[i]} = \text{diag} \{\rho^{[i]}(\cdot), 1\}$ where $\rho^{[i]}(\cdot)$ is the SISO operator that delays the scalar input by $d[i]$ steps. The dynamics of the stock levels at PP $i$ can be written as

$$x^{[i]}(t+1) = A^{[i]} \cdot x^{[i]}(t) + \Gamma^{[i]} \cdot \gamma^{[i]}(t) + B^{[i]} u^{[i]}(t) + B^{[i]} d[i](t - d[i]) \quad (20)$$

**Fig. 6. Manufacturing factory control system with delay compensator.**

$$B_i = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_0 = I_2 - B_t$$

Due to the physical meaning of the transportation rate which is quantized to $\{0, 1, -1\}$, it is impossible to utilize several standard methods for handling the time delay, such as the scattering transformation [20] and passivation [21]. Instead, we design for each local PP $i$ a "delay compensator" $\Xi^{[i]}$. Each $\Xi^{[i]}$ utilizes the local stock levels measurement at time $t$ and the input signals at several previous steps to calculate a delay-handled signal $\tilde{y}^{[i]}(t)$. The overall manufacturing system with delay compensator is proposed as in Fig. 6. We can re-express the delay dynamics (20) as

$$\tilde{x}^{[i]}(t+1) = \tilde{x}^{[i]}(t) + \tilde{B}^{[i]} \tilde{u}^{[i]}(t) \quad (22)$$

$$\tilde{x}^{[i]}(t) = \begin{bmatrix} x_i^1(t) & \cdots & x_i^{d[i]}(t) \end{bmatrix} \quad (23)$$

$$\tilde{z}^{[i]}(t) = u(t - d[i]), \quad \tilde{z}^{[i]}(t) = u(t - d[i]) \quad (24)$$

$$\tilde{\Xi}^{[i]} = \begin{bmatrix} A^{[i]} & B_{(-)} & \cdots & B_{(-)} \\ \Theta_2 & \cdots & \cdots & \Theta_2 \\ \vdots & \ddots & \vdots & \vdots \\ \Theta_2 & \cdots & \cdots & \Theta_2 \\ \Theta_2 & \cdots & \cdots & \Theta_2 \\ \Theta_2 & \cdots & \cdots & \Theta_2 \\ \Theta_2 & \cdots & \cdots & \Theta_2 \\ I_2 & \vdots & \vdots & \vdots \end{bmatrix} \quad (25)$$

where $\Theta_2$ and $I_2$ are the $2 \times 2$ zero and unity matrices.

**Proposition 7:** For each PP $i$, the partitioned matrix $\Omega^{[i]} = [\Omega^{[i]}_{j,k}]$ is established by the following procedure

$$\Omega^{[i]} = \begin{bmatrix} I_x, & j=1,2, \ldots, d[i]+1 \end{bmatrix} \quad (26-a)$$

$$\Omega^{[i]} = A^{[i]} j, \quad \Omega^{[i]}_{j,k} = A^{[i]} j, [d[i] + 1] \quad (26-b)$$

$$\Omega^{[i]} = B_{(-)} j, \quad \Omega^{[i]}_{j,k} = B_{(-)} j, [d[i] + 1] \quad (26-c)$$

$$\Omega^{[i]} = \begin{bmatrix} \Omega^{[i]}_{j,k} \end{bmatrix}^{T} \quad (j = 3, \ldots, d[i] + 1) \quad (26-d)$$

Then, design for each PP $i$ a delay compensator as

$$\Xi^{[i]} : y^{[i]}(t) = C^{[i]} x_i(t) + \sum_{j=1}^{d[i]} \Gamma^{[i]} j, u^{[j]}(t - d[j] + 1 - j) \quad (27-a)$$

$$C^{[i]} = B_{(-)} A^{[i]} + \Omega^{[i]}_{d[i]+1} \quad (27-b)$$
\[ C_2^{[i]} = B_j \Omega_{1, i+1} B_i + \Omega_{1, i+1, i+1} B_i \]  
(27-c)

\[ C_2^{[i]} = B_j \Omega_{1, i+1} + \Omega_{1, i+1, i+1} B_i \]  
(27-d)

Define \( \bar{C}_2^{[i]} = [C_2^{[i]}, C_2^{[i]}, \ldots, C_2^{[i]}] \). Each system \( \bar{\theta}_2^{[i]} \) in Fig. 6 given as

\[
\bar{\theta}_2^{[i]}(t + 1) = A^T \bar{\theta}_2^{[i]}(t) + B^T_i \bar{u}_i^{[i]}(t)
\]

(28)

is \( (\Theta_2, \gamma_2^{[i]} I_2, 0.5 I_2) \) dissipative with the storage function

\[
\rho^2(\bar{x}_2^{[i]}(t)) = 0.5 \bar{\chi}^T \bar{\chi}(t) \Omega_2 \bar{\chi}_2^{[i]}(t)
\]

(29)

for all number \( \rho_0^{[i]} \) that satisfies

\[
\rho_0^{[i]} \geq \frac{3 + \sqrt{(1-2h^d)-1}}{4}
\]

(30)

**Proof:** Given a production control gain \( h \in (0, 1) \). Applying Cholesky decomposition, matrix \( \Omega_2^{[i]} \) is shown to be positive definite for all positive integer number of \( d^{(i)} \). This means we can select the class of storage function (29) for the augmented system \( \bar{\theta}_2^{[i]} \). From the KYP Lemma, \( \bar{\theta}_2^{[i]} \) is \( (\Theta_2, \gamma_2^{[i]} I_2, 0.5 I_2) \) dissipative if the following matrix is negative semi-definite

\[
\bar{\Theta}_2^{[i]} = \begin{bmatrix}
A^{T}[\Omega_2^{[i]} \tilde{A}^{[i]} - \tilde{A}^{[i]} \Omega_2^{[i]}] & -A^{T}[\tilde{\Omega}_2^{[i]} \tilde{B}_2^{[i]} - \tilde{C}_2^{[i]}] \\
B^{T}[\tilde{\Omega}_2^{[i]} \tilde{A}^{[i]} - \tilde{A}^{[i]} \tilde{\Omega}_2^{[i]}] & [\tilde{B}_2^{[i]} \tilde{B}_2^{[i]} - 2\gamma_2^{[i]} I_2]
\end{bmatrix}
\]

(31)

From (25), (26-a,b,c,d,e) and (27-a,b,c,d), the components \( \tilde{A}^{[i][[T]} \tilde{\Omega}_2^{[i][[T]} = \tilde{C}_2^{[i][[T]} \tilde{B}_2^{[i]} - \tilde{C}_2^{[i][[T]} \tilde{\Omega}_2^{[i][[T]} \tilde{C}_2^{[i]} \tilde{B}_2^{[i]} \tilde{C}_2^{[i]} \tilde{B}_2^{[i]} - 2\gamma_2^{[i]} I_2 \) are shown to be zero, and \( \tilde{C}_2^{[i][[T]} \tilde{\Omega}_2^{[i][[T]} \tilde{C}_2^{[i]} \tilde{B}_2^{[i]} \tilde{C}_2^{[i]} \tilde{B}_2^{[i]} - 2\gamma_2^{[i]} I_2 \) is a negative semi-definite matrix. We only need to take care of the negative semi-definiteness of \( \tilde{B}_2^{[i][[T]} \tilde{B}_2^{[i]} - 2\gamma_2^{[i]} I_2 \) which finally lets us to (30).

Based on Proposition 5 and 7, the procedure for designing the manufacturing system with transportation delay is proposed as follows. We neglect to show the **Procedure 2 (with constant transportation delay)**

Step 1: Design for each PP \( i \) a delay compensator \( \Xi_2^{[i]} \) by using (31-a,b,c,d,e) and (32-a,b,c,d).

Step 2: Design for each transportation route \( j \rightarrow i \) a transportation controller \( \theta_2^{[i]} \) such that:

(i) the output \( u_2^{[i]}(t) \) of \( \theta_2^{[i]} \) is a normalized number that varies between -1 and +1.

(ii) the cascade connection \( \theta_2^{[i]} \) of \( \theta_2^{[i]} \) and \( \tau^{(i)} \) with input \( y_i^{(i)}(t) \) and output \( u_2^{[i]}(t) \) is a memoryless map that satisfies

\[
E\left[u_2^{[i]}(t) y_i^{(i)}(t)\right] \geq \delta^{[i]} E\left[y_i^{(i)}(t)^2\right]
\]

(32)

where the set of \( \{\delta^{[i]}\} \) satisfies

\[
diag\{\delta^{[i]}\} \geq B^T\left(diag\left\{\rho_0^{[i]} I_2\right\}\right) B
\]

(33)

Stability and consensus of the overall system:

To discuss the consensus performance, we define \( \hat{x}^{[i]} = \left(\frac{1}{2N} \mathbf{1}_{N} \cdot x(0)\right) I_2 \) and \( \hat{x}^{[i]}(t) = \hat{x}^{[i]}(t) - \hat{x}^{[i]} \). Then, we construct for each local PP the augmented state

\[
\bar{x}^{[i]}(t) = \bar{x}(t)^T \begin{bmatrix} z_{d_1}^T(t) & z_{d_2}^T(t) & \ldots & z_{d_N}^T(t) \end{bmatrix}^T
\]

(34)

The following statement is straightforward. We neglect to show the proof which is very similar to that of Proposition 6.

**Proposition 8:** If the cyclic manufacturing system is designed to satisfy the **Procedure 2**, then the following inequality holds true \( E\left[V^{(i)}(\bar{x}(t + 1))\right] - V^{(i)}(\bar{x}(t)) \leq 0 \)

where \( V^{(i)} = V^{(1)} + V^{(2)} + \ldots + V^{(i)} \) with \( V^{(i)} \) given by (29) and \( \bar{x}(t) = \left[\bar{x}_{d-1}^{[i]}(t) \quad \bar{x}_{d}^{[i]}(t) \quad \ldots \quad \bar{x}_{d-1}^{[i]}(t)\right]^T \).

**V. SIMULATION VERIFICATION**

**A. Outline of the simulation**

We consider the cyclic manufacturing factory with 6 PPs. The simulation is performed from the initial time until \( t = 300 \). The initial values of the input-buffers and output-buffers are \( 15, 40, 30, 10, 5, 2 \) and \( 27, 25, 2, 15, 30, 17 \), respectively. We assume that at the input-buffer of PP 1, there is a rate of raw materials from the outside of the factory (\( \hat{C}_1 = 0.05 \)). We also assume there is a constant output rate (\( \zeta_0 = 0.05 \)) of the produced production at the output-buffer of the PP 6. To evaluate the performance of the overall system under disturbance, we assume that at time step \( t = 150 \), a large quantity (\( \zeta = 15 \)) of materials are taken from the output-buffer of the PP 4.

**B. Simulation results (Fig. 7)**

Three test cases are conducted. Their settings are summarized in TABLE 1, and their simulation results are summarized in Fig. 7.

Case 1 (Fig. 7(a)): We still use the consensus algorithm presented in our recent work [3]. The signal sent to the transportation controller is \( y_i(t) = -B^T x_i(t) \). Since \( \hat{C}_i \) is only passive and the delay compensator is not implemented in this case, the stock levels suffer fluctuation of large amplitudes.

Case 2 (Fig. 7(b)): We use the proposed **Procedure 1**. Thanks to the strict-passivity of \( \hat{C}_i \), the control performance of Case 2 is still better than that of Case 1. However, the fluctuations become noticeable. This shows that it is essential to handle the delay properly.

Case 3 (Fig. 7(c)): The system is designed using the **Procedure 2**. The delay compensator is implemented with the nominal delay \( d_a = \max\{d_i\} = 12 \). This means the control system suffers a certain uncertainty \( \Delta d = d - d_a \) for each transportation route. Even though, this simulation result shows that the consensus performance is successfully attained. This means the delay compensator might tolerate the delay uncertainty to a certain extent. Our hypothesis is as follows: The system is still stable if the nominal delay is selected as the upper-bound of the actual delays.

**VI. CONCLUSIONS**

Based on dissipativity, this paper establishes the stability condition for a class of large scale discrete-time systems
TABLE I. SUMMARY OF THE SIMULATION SETTING

| Delay uncertainty | Case | Delay compensator | Transportation system $g_f$ | $L_f$ | Normalizer $h_f$ | Design procedure |
|-------------------|------|-------------------|-----------------------------|------|----------------|-----------------|
| $(d_1 \in [8, 12])$ | 1    | No                | Passive                     | 0.75 | $(2/\pi)\tan^(-1)(\theta)$ | 0.1 | Algorithm [3]   |
|                   | 2    | No                | Strictly passive            | 0.75 | $(19-b)$       | 0.1 | Procedure 1     |
|                   | 3    | Yes               | Strictly passive            | 0.75 | $(19-b)$       | 0.1 | Procedure 2     |

Fig. 7. Cyclic manufacturing factory system with large transportation delay.

which consists of deterministic dynamical sub-systems and stochastic memoryless mapping. Our proposal is demonstrated by manufacturing systems as a case of study. The global and local objectives are attained in our dissipativity frameworks by only using the local measurements and local control actions. This paper, therefore, is an application of our local concept to the multi-agent systems. Moreover, the delay of material transportation between the agents can be handled properly by using the “delay compensator” designed based on dissipativity approach. In future study, we are interested in a theoretical explanation for the robustness of the proposed control system when it has to suffer delay uncertainty. We will also consider the case such that the time delay is randomly changed in time.

REFERENCES

[1] S. Hara, J. Imura, K. Tsumura, T. Ishizaki, and T. Sadamoto: “Glocal (Global/Local) Control Synthesis for Hierarchical Network System, Proceedings of 2015 IEEE Conference on Control Applications, pp.107-112, 2015.
[2] D. Angeli and P-A. Kountouriotis: “A Stochastic Approach to Dynamic-Demand Refrigerator Control,” IEEE Transactions on Control System Technology, Vol. 20, No. 3, pp. 581-592, 2012.
[3] K. Tsumura, B-M. Nguyen, H. Wakayama, Y. Maeno, and S. Hara: “Distributed Control of Stochastic Manufacturing Process Based on Consensus Algorithm,” Proceedings of the America Control Conference, 2019.
[4] U. M. Diwekar and E. S. Rubin: “Stochastic Modeling of Chemical Process,” Computers & Chemical Engineering, Vol. 15, Iss. 2, pp. 105-114, 1991.
[5] H. E. Samad, M. Khammash, L. Petzold and D. Gillespie: “Stochastic Modelling of Gene Regulatory Networks,” International Journal of Robust and Nonlinear Control, Vol. 15, pp. 691-711, 2005.
[6] E. B. Kosmatopoulos and M. A. Christodoulou: “A State Space Model for Modelling and Control of Manufacturing Process,” Proceedings of 1995 INRIA/IEEE Symposium on Emerging Technologies and Factory Automation, pp. 563-571, 1995.
[7] J. Liang, Z. Wang, and X. Liu: “On Passivity and Passification of Stochastic Fuzzy Systems with Delays: The Discrete-time Case,” IEEE Transactions on Systems, Man, and Cybernetics - Part B: Cybernetics, Vol. 40, No. 3, pp. 964-969, 2010.
[8] Y. Wang, V. Gupta, and P. J. Antsaklis: “Stochastic Passivity of Discrete-time Markovian Jump Nonlinear Systems,” Proceedings of 2013 American Control Conference, pp. 4879-4884, 2013.
[9] L. Wu, X. Yang, and H-K. Lam: “Dissipativity Analysis and Synthesis for Discrete-Time T-S Fuzzy Stochastic Systems with Time-Varying Delay,” IEEE Transactions on Fuzzy Systems, Vol. 22, No. 2, pp. 380-394, 2014.
[10] G. Nagamani and S. Ramasamy: “Stochastic Dissipativity and Passivity Analysis for Discrete-time Neural Networks with Probabilistic Time-varying Delays in the Leakage Term,” Applied Mathematics and Computation, Vol. 289, pp. 257-257, 2016.
[11] E. B. Kosmatopoulos and M. A. Christodoulou: “A State Space Model for Modelling and Control of Manufacturing Process,” Proceedings of 1995 INRIA/IEEE Symposium on Emerging Technologies and Factory Automation, pp. 563-571, 1995.
[12] M. Schwaninger and P. Vrhovec: “Supply System Dynamics: Distributed Control in Supply Chains and Networks,” Cybernetics and Systems: An International Journals, Vol. 37, pp. 375-415, 2006.
[13] W. B. Dunbar and S. Desa: “Distributed Model Predictive Control for Dynamic Supply Chain Managements,” International Workshop on Assessment and Future Directions of NMPC, Freundenstads-Lauterbad, Germany, August 26-30, 2005.
[14] L. Rodrigues and E-K. Boukas: “Piecewise-Linear $H_\infty$ Controller Synthesis with Applications to Inventory Control of Switched Production Systems,” Automatica, vol. 42, pp. 1245-1254, 2006.
[15] R. Olffi-Saber and R. M. Murray: “Consensus Problems in Networks of Agents with Switching Topology and Time-Delays,” IEEE Transactions on Automatic Control, Vol. 49, Iss. 9, pp. 1520-1533, 2004.
[16] R. Olffi-Saber, J. A. Fax, R. M. Murray: “Consensus and Cooperation in Networked Multi-Agent Systems,” Proceedings of the IEEE, Vol. 95, Iss. 1, pp. 215-233, 2007.
[17] M. Nabi-Abolyousefi and M. Mesahibi: “System Theory over Random Consensus Networks: Controllability and Optimality Properties,” 50th IEEE Conference on Decision and Control, pp. 2323-2328, 2011.
[18] S. R. Etesami and T. Basar: “Convergence Time for Unbiased Quantized Consensus Over Static and Dynamics Networks,” IEEE Transactions on Automatic Control, Vol. 61, Iss. 3, pp. 433-455, 2016.
[19] C. I. Byrnes and W. Lin: “Losslessness, Feedback Equivalence, and the Global Stabilization of Discrete-Time Nonlinear Systems,” IEEE Transactions on Automatic Control, Vol. 39, No. 1, pp. 83-98, 1994.
[20] N. Chopra and M. W. Spong: “Passivity-Based Control of Multi-Agent Systems,” in S. Kawamura and M. Svinin (Eds): “Advances in Robot Control,” Springer, 2006.
[21] M. Xia, P. J. Antsaklis, and V. Gupta: “Passivity Indices and Passivation of Systems with Application to Systems with Input/Output Delay,” Proceedings of 53rd IEEE Conference on Decision and Control, 2014.

1889