On Pulsed Magnetic Field Measurements of Enhanced Sommerfeld Constant $\gamma$ and $A$-Coefficient around Metamagnetic Transition in UTe$_2$

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1. Introduction

The Sommerfeld constant $\gamma$ and the $A$-coefficient in the temperature dependence of the resistivity in the low temperature limit are crucial physical quantities in strongly correlated Fermi liquid metals. It is well recognized that both $\gamma$ and $A$ are highly enhanced in strongly correlated metals but the ratio $A/\gamma^2$ remains as the quasi-universal one from experimental results$^1$ and theoretical arguments.$^2$ It was a mystery in early stage of research in UTe$_2$ that both quantities exhibit sharp increase around the first-order metamagnetic transition under the magnetic field along $b$-axis. One of possible answer was proposed in Ref. 3 to explain such increase just at the metamagnetic field $H_b = H_M$ and the fact that the ratio $A/\gamma^2$ is given approximately by that of Kadowaki-Woods.$^1$ On the other hand, the $H_b$ dependence of around $H_b = H_M$ exhibits rather different aspects depending on details of experimental probes. The purpose of the present note is to clarify the origin of this fact by examining the character of those probes.

2. Summary of Experimental Facts

In this section, we summarize the characteristics of experimental facts of the metamagnetic transition observed in UTe$_2$. 

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2.1 Variations of the splitting width of the metamagnetic fields of up-sweep and down-sweep processes

In this subsection, we discuss the aspect of the splitting of the metamagnetic fields, $H_m$'s, for up-sweep and down-sweep processes. First, let us denote the splitting of the metamagnetic fields for up-sweep ($H_{m}^{up}$) and down-sweep ($H_{m}^{down}$) as $\Delta H_m \equiv H_{m}^{up} - H_{m}^{down} > 0$. The results for these splittings reported in Refs. 4 and 5 are

$$\Delta H_m^K \simeq 0.53 \text{ T}, \quad (1)$$

and

$$\Delta H_m^M \simeq 0.36 \text{ T}, \quad (2)$$

respectively. On the other hand, that reported in Ref. 6 is

$$\Delta H_m^I \simeq 0.10 \text{ T}. \quad (3)$$

2.2 Variations of the rate of change of the magnetic field at the metamagnetic transition in up-sweep process

Corresponding to the variations of ($\Delta H_m$)'s discussed in the previous subsection, the rate of change of the magnetic field, $(dH/dt)|_{H=H_{m}^{up}}$'s, at the metamagnetic field of the up-sweep process exhibits large variations as follows. The rates reported in Refs. 4, 7 and 5, 8 are

$$\left( \frac{dH}{dt} \right)_K^{H_{m}^{up}} \simeq \frac{68 - 34.9}{(50/2)} = 1.3 \text{ T/msec}, \quad (4)$$

and

$$\left( \frac{dH}{dt} \right)^M_{H_{m}^{up}} \simeq \frac{70 - 30}{(20/2)} = 4.0 \text{ T/msec}, \quad (5)$$

respectively. On the other hand, that in Ref. 6 is far smaller than those given by Eqs. (4) and (5) as

$$\left( \frac{dH}{dt} \right)^I_{H_{m}^{up}} \simeq \frac{40 - 36.1}{(14 \times 10/2)} = 5.6 \times 10^{-2} \text{ T/msec}. \quad (6)$$

2.3 Variations of the time interval between the metamagnetic transitions of up-sweep and down-sweep processes

Complementarily to the variations of ($\Delta H_m$)'s discussed in Subsec. 1.1, the time intervals ($\Delta t$)'s between two metamagnetic transitions, i.e., those corresponding to down-sweep and up-sweep processes, show large variations as follows. The intervals ($\Delta t$)'s reported in Refs. 4, 7 and 5, 8 are

$$\Delta t^K \simeq 50 \text{ msec}, \quad (7)$$

and

$$\Delta t^M \simeq 20 \text{ msec}, \quad (8)$$

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respectively. On the other hand, that reported in Ref. 6 is far longer than those given by Eqs. (7) and (8) as

\[(\Delta t)_1 \simeq 14 \times 10 \text{ msec.}\] (9)

Note, furthermore, that the measurements of the Sommerfeld constant \(\gamma\) in Ref. 9 were performed by the quasi-adiabatic method, in which each measurement at specific strength of magnetic field has been performed under the almost static magnetic field during about 400 msec which is longer moreover than \((\Delta t)_1 \simeq 140 \text{ msec}\) [Eq. (9)].

3. Interpretation of Experimental Facts

In this section, we discuss the meaning of experimental facts shown in the previous section. The facts shown in Subsects. 1.1 and 1.2 imply that the metamagnetic transition is controlled not only by the balance of the static free energy but it occurs through the metastable states whose life time must depend on the rates of magnetic field change. In general, an ideal first-order metamagnetic transition under the adiabatic process occurs when the free energies of the two stable states with \(M < M_m\) and the unstable state \(M > M_m\) coincide with each other as the magnetic field \(H\) increases adiabatically through \(H = H_m\). However, in reality, the state at \(H < H_m\) continues to stay as the metastable one even at \(H > H_m\) in a certain period of time, depending on the rate of increase of \(H\) unless the instability point is reached where the sponodal like process is forcibly started to occur, instantaneously giving rise to the discontinuous transition. Indeed, the correspondence between variations of \(\Delta H_m\)'s shown in Eqs. (1), (2), and (3), and those of \((dH/dt)|_{H=H_m}\)'s given approximately by Eqs. (4), (5), and (6), respectively, would support this point of view. Namely, this circumstantial evidence suggests the importance to take into account the time dependent process beyond the competition among static free energies, such as the extended Landau-type free energy used for understanding the enhancements of the Sommerfeld constant \(\gamma\) and the \(A\)-coefficient just at the metamagnetic transition discussed in Ref. 3.

3.1 Free energy shift due to thermodynamic magnetic fluctuations

In this subsection, we discuss a possible origin of excess free energy gain due to magnetic fluctuations beyond the static free energy of the extended Landau-type free energy, used in Ref. 3.

According to a general principle of statistical physics,\(^{10}\) the free energy is influenced also by the thermodynamic fluctuations, i.e., non-uniform magnetic fluctuations in the present case. Namely, the shift of the free energy \(\Delta F\) due to the fluctuations, under fixed temperature and volume, is given as follows:

\[\Delta F = -k_B \log \Delta Z_{\parallel},\] (10)
where $k_B$ is the Boltzmann constant and the partition function $\Delta Z_{fl}$ is defined as

$$\Delta Z_{fl} = \Pi_q \int dM'_q dM''_q \exp \left[-(\eta + Aq^2)|M_q|^2/k_B T N_F^*\right],$$

(11)

where $M_q = M'_q + iM''_q$. Substituting $\Delta Z_{fl}$ [Eq. (11)] into Eq. (10), $\Delta F$ is expressed as

$$\Delta F = -k_B \sum_q \log \left\{ \int dM'_q dM''_q \exp \left[-(\eta + Aq^2)|M_q|^2/k_B T N_F^*\right] \right\}$$

$$= -k_B \sum_q \log \left( \frac{\pi k_B T N_F^*}{\eta + Aq^2} \right),$$

(12)

which is negative in the thermodynamic limit, i.e., $N \to \infty$. Note that the form of Eq. (12) reflects the dynamical spin susceptibility of ferromagnetic fluctuations discussed in Ref. 11, i.e.,

$$\chi_s(q, i\omega_m) = \frac{q N_F^*}{\omega_s(q) + |\omega_m|}, \text{ for } q < q_c \sim p_F,$$

(13)

where $N_F^*$ is the density of states (DOS) of the quasiparticles per spin at the Fermi level and $\omega_s(q)$ is defined as

$$\omega_s(q) \equiv \frac{q}{C}(\eta + Aq^2),$$

(14)

where $\eta$ parameterizes the closeness to the ferromagnetic criticality. Note that $\mathrm{UTe}_2$ is considered to be located near the ferromagnetic criticality as discussed in Refs. 12–14. Therefore, the contribution of $\Delta F$ is considered to have a certain effect on the metamagnetic transition as discussed below.

### 3.2 Condition for metamagnetic transition in pulsed magnetic field experiments

In this subsection, we discuss the aspect of the metamagnetic transition in the up-sweep process in the pulsed magnetic field experiments.

In Ref. 3, the extended Landau-type free energy $F(M; H)$ under the magnetic field $H$ was adopted as

$$F(M; H) \simeq aM^2 - bM^4 + cM^6 - HM,$$

(15)

where coefficients $a$, $b$ and $c$ are assumed to be positive, and tuned to recover the gross characters of the metamagnetic transition and concomitant anomalies of the Sommerfeld constant $\gamma$ and the $A$-coefficient just at $H = H_m$. The condition for the metamagnetic transition discussed in Ref. 3 was given by the condition that the free energies at $M = M_-$ and $M = \bar{M}$ coincide, i.e., $F(M_-) = F(M)$, as schematically shown in Fig. 1. However, as noted above, the excess contribution $\Delta F$ [Eq. (12)] beyond $F(M; H)$ [Eq. (15)] is crucial to determine the ideol or adiabatic metamagnetic transition. Namely, its condition in the up-sweep process of the pulsed magnetic field experiment should have been replaced by

$$F(M_-; H_m^{up}) + \Delta F(H_m^{up}) = F(M; H_m^{up}),$$

(16)
where $\Delta F(H_{m}^{up})$ expresses the contribution from the non-uniform magnetic fluctuations around $M = M_{-}$, one of the degenerate local minima, while such a contribution from the fluctuations around $M = \bar{M}$ has been discarded because it stays as a virtual one before the adiabatic metamagnetic transition occurs, i.e., $H \leq H_{m}^{up}$.

Furthermore, the stable state at $H \leq H_{m}^{up}$ remains to be metastable before the instability point is reached and the spinodal like process is caused forcibly to reach the true stable higher field phase. However, according to the experimental facts shown in Subsects 1.1 and 1.2, the observed metamagnetic fields $H_{m}^{up}$’s are distributed depending on the rates of increasing the magnetic field, implying that the spinodal point is not reached in actual experiments. This is because the spinodal point must be determined, independently of the rate of increasing the magnetic field $H$, by the condition that the following two relations hold simultaneously for the first time as increasing $M$ together with $H$:

$$\frac{\partial}{\partial M} \left[ F(M; H_{sp}^{up}) + \Delta F(H_{sp}^{up}) \right] = 0,$$

(17)

and

$$\frac{\partial^2}{\partial^2 M} \left[ F(M; H_{sp}^{up}) + \Delta F(H_{sp}^{up}) \right] = 0.$$

(18)

Here, $H_{sp}^{up}$ is the magnetic field of starting the spinodal like transition in the up-sweep process. Note that $H_{m}^{up}$ must approach $H_{sp}^{up}$ from the lower side in the limit of $(dH/dt)_{H=H_{m}^{up}} = \infty$. Although the precise mechanism of the real transition from the metastable state to the stable one at $H \geq H_{m}^{up}$ has not been clarified yet, as a working hypothesis, we assume the existence of some mechanism to cause this transition on the basis of experimental observations above.

Needless to say, the manner of the inverse metamagnetic transition in the down-sweep process is similar to that in the up-sweep but in inverse way. Namely, the conditions [Eqs. (16), (17), and (18)] are replaced by

$$F(M; H_{m}^{down}) + \Delta F(H_{m}^{down}) = F(M_{-}; H_{m}^{down}),$$

(19)

$$\frac{\partial}{\partial M} \left[ F(M; H_{sp}^{down}) + \Delta F(H_{sp}^{down}) \right] = 0,$$

(20)

and

$$\frac{\partial^2}{\partial^2 M} \left[ F(M; H_{sp}^{down}) + \Delta F(H_{sp}^{down}) \right] = 0,$$

(21)

respectively.

### 3.3 Origin of difference of anomalies in $\gamma$ and $A$ coefficients around the metamagnetic transition in pulsed magnetic field measurements

Finally, in this subsection, we propose a possible scenario to understand why the difference arises in anomalies of $\gamma$ and $A$-coefficient around the first-order metamagnetic transition in pulsed magnetic field measurements. Namely, the discussions are focused on the reason why
Fig. 1. Schematic behavior of free energies, appearing in Eq. (15), as a function of the uniform magnetization. Solid curve and solid line represent the first three parts, free energy $F_0(M)$ [Eq. (15)] without the magnetic field $H$ and the Zeeman energy at the metamagnetic field $H_m$, respectively. The dotted curve represents the free energy $F_0(M) - H_m M$ which has two degenerate minima at $M = M_-$ and $\bar{M}$.

the $H$ dependence of the $A$-coefficient and the Sommerfeld constant $\gamma$ is almost symmetric around $H = H_m$, while $\gamma(H)$ at $H \gtrsim H_m$, reported in Ref. 6, is considerably suppressed compared to that at $H \lesssim H_m$.

A hint to solve this paradox must be the fact that the metamagnetic transition is determined not only by the extended Landau-type free energy [Eq. (15)] for the uniform ferromagnetic order parameter $M$, but is influenced by the excess free energy $\Delta F$ [Eq. (12)] arising from the non-uniform magnetic fluctuations. Indeed, this aspect was pointed out in the previous subsection in relation to the condition [Eq. (16)] for the ideal and adiabatic metamagnetic transition.

Another circumstantial aspect of the metamagnetic transition under the pulsed magnetic field measurements is that the actual metamagnetic field $H_{up}^{up}$'s in the up-sweep process depend on the rate of increasing magnetic fields as suggested by experimental facts shown in Subsects. 1.1 and 1.2. This aspect implies that the metamagnetic transition in question is quite a time dependent phenomenon.

The most crucial fact would be that the above-mentioned non-uniform magnetic fluctuations remain as non-vanishing in a certain period of time ($\Delta t$) even after the metamagnetic transition, i.e., in the region $H \gtrsim H_m^{up}$. Note that these magnetic fluctuations are described by the dynamical susceptibility [Eq. (13) with Eq. (14)] which are the same fluctuations giv-
ing rise to the enhancements of $\gamma$ and $A$-coefficient in the region $H \leq H_{m}^{up}$. Therefore, such enhancements of $\gamma$ and $A$-coefficient should remain even in the region $H \geq H_{m}^{up}$ in the period $(\Delta t)|H$ in general. Then, the question is how long $(\Delta t)|H$ is. Although the reliable theoretical estimation of $(\Delta t)|H$ has not been available, its possible range can be inferred so as to explain consistently experimental observations shown in Sect. 1.

The time intervals $(\Delta t)$’s between the metamagnetic transitions of up-sweep and down-sweep processes in Refs. 4 and 5 are given by Eqs. (4) and (5), respectively. The almost symmetric behaviors of $\gamma$ and $A$-coefficient imply that $(\Delta t)|H$ would be comparable to or longer than $(\Delta t)|K \simeq 50\text{msec} > [ (\Delta t)|M \simeq 20\text{msec}]$. Note that $\gamma$ reported in Ref. 5 was estimated by using the Maxwell relation between $M$ and the entropy $S$, i.e., $(\partial S/\partial H)_{T} = (\partial M/\partial T)_{H}$, so that the duration time of measurement is about $(\Delta t)|M \simeq 20\text{msec}$ [Eq. (5)], during of which the effect of the non-uniform magnetic fluctuations almost remains giving rise to a considerable enhancements of $\gamma$ and $A$-coefficient as really observed in Refs. 4 and 5. On the other hand, the specific heat measurements in Ref. 6 have been performed by the quasi-adiabatic method,\(^9\) whose duration time of each measurement at specific strength of magnetic field is about 400 msec which is far longer than $(\Delta t)|H$ estimated above. Therefore, the contribution from the remnant non-uniform magnetic fluctuations would be considerably suppressed in the high field region $H > H_{m}^{up}$ as observed in Ref. 6.

3.4 Conclusion of Sect. 3

As discussed in the previous subsection, the seemingly conflicting aspects of the enhancement of the Sommerfeld constant $\gamma$ and the $A$-coefficient around the metamagnetic transition, which is summarized in the first paragraph of this section (Sect. 3), can be understood naturally by taking carefully into account the deference in the time dependence of the pulsed magnetic field probes.

4. Conclusion and Perspective

A possible scenario has been proposed why the aspects of the enhancements of the Sommerfeld constant $\gamma$ anf the $A$-coefficient of the resistivity, near the first order metamagnetic transition of UT\(_2\) observed by a series of pulsed magnetic field measurements, are rather different. It was crucial that the metamagnetic transition under the pulsed magnetic field measurements is influenced by the rate of changing the magnetic field. Therefore, it became indispensable to consider the effect of the non uniform and time dependent magnetic fluctuations around the stationary point of the static Landau-type free energy for the uniform magnetization.

In order to verify the present scenario, a series of measurements of the $A$-coefficient are expected, with changing $(\Delta t)$, time interval between the metamagnetic transitions of up-sweep and down-sweep processes, to approach that used in Ref. 6.
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