NON-SPECTATOR DIQUARK EFFECTS ON LIFETIMES OF \( \Lambda_b, \Omega_b^{(*)} \)

AND WEAK DECAY RATES OF \( \Sigma_b^{(*)}, \Xi_b^{(*)} \)

Wu-Sheng Dai\textsuperscript{1,2}, Xin-Heng Guo\textsuperscript{3,4}, Xue-Qian Li\textsuperscript{1,2} and Gang Zhao\textsuperscript{1,2}

1. CCAST (World Laboratory), P.O. Box 8730, Beijing 10080, P.R. China
2. Department of Physics, Nankai University, Tianjin 300071, P.R. China
3. Department of Physics and Mathematical Physics, and Special Research Center for the Subatomic Structure of Matter, University of Adelaide, SA 5005, Australia
4. Institute of High Energy Physics, Academia Sinica, Beijing 100039, P.R. China

Abstract
The difference of \( \tau_B \) and \( \tau_{\Lambda_b} \) indicates the role of the light flavors. We calculate the lifetimes of B-meson and \( \Lambda_b \) based on the weak effective Hamiltonian while assuming the heavy baryon is constructed by a heavy b-quark and a diquark containing two light quarks. In this scenario, we use the information of the measured ratio \( \tau_{\Lambda_b}/\tau_B \) as input to predict rates of the inclusive weak decays of \( \Sigma_b^{(*)} \) and \( \Xi_b^{(*)} \) into non-bottom final states. We find that these rates of \( \Sigma_b^{(*)} \) and \( \Xi_b^{(*)} \) are much larger than that of B-mesons and \( \Lambda_b \). We also give the predictions for the lifetimes of \( \Omega_b \) and \( \Omega_b^* \). Phenomenological implication of our result is discussed.

PACS numbers: 14.20.Mr, 13.30.-a, 12.39.-x, 13.20.He
I. Introduction

The present data of the lifetime ratio of $B$ and $\Lambda_b$ are \cite{1}: 

\[
\frac{\tau(B^-)}{\tau(B_d)} = 1.06 \pm 0.04, \\
\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.79 \pm 0.06. 
\] 

(1)

Deviation of the ratios from unity manifests some important issues missing in our present theoretical framework.

Since the new experiments on heavy flavor physics have accumulated more and more data, we have a chance to get better insight into the physics which governs the transition processes.

The heavy quark effective theory (HQET) has achieved great success in evaluating physical processes of heavy hadrons. One can expect that for the processes where heavy flavors (b and c) are involved, it is possible to establish a reasonable framework to make more accurate calculations and predictions \cite{2}. At least the leading contribution would be effectively obtained in the suggested scenario. It is generally believed that the decay processes at quark level occur via the heavy quark decays \cite{3}. Because the estimation of hadron lifetimes only refers to inclusive processes, where final states are only composed of free quarks and gluons, all calculations are more reliable except the binding effects of the initial hadron which may bring up some unfixed factors. In the heavy quark limit, the lifetime is mainly determined by the decay rate of the heavy flavors which can be accurately calculated in the framework of weak effective Hamiltonian. If it is true, the lifetime of $B^-$, $B_d$ and $\Lambda_b$ must be close up to some phase space kinematics, because the leading order of the expansion with respect to inverse powers of the b-quark mass \cite{4} is the same for all of them. The first order correction ($\Lambda_{\text{QCD}/m_b})^2$ can only result in very small change. However, a discrepancy between the measured data and theoretical prediction on $\tau(\Lambda_b)/\tau(B)$ is obvious.
As a matter of fact, the observation $\tau(D^{\pm}) \sim 2\tau(D^0)$ while $\tau(B^{\pm}) \sim \tau(B^0)$ attracts attentions of theorists for a long time. Bigi et al. attributed the difference to the Pauli interference $\Delta \Gamma_{PI}$ at quark level [5]. Voloshin and Shifman also discussed similar effects for charmed mesons and baryons [6]. They have noticed the significance of the non-spectator effects. In their work, $\tau(\Lambda_b)/\tau(B)$ was estimated as $\sim 0.9$ which still deviates from the present data [1].

Usually the light flavors are treated as spectators in the transitions, namely they do not participate in the quark level processes. Their existence only manifests at hadronization. Neubert and Sachrajda carefully studied these effects on the lifetime differences and other quantities, such as $N_c$ [7]. Obviously, the discrepancy about the ratios implies that the light flavors should get involved in the weak transition. In most cases, the non-spectator effects would be small, however, in some processes, they can play important roles.

As shown in most literatures, the non-spectator effects for B-meson exclusive decays are small and we have also calculated [8] these effects in B-decays based on the well-established weak effective Hamiltonian and found that they can only bring up at most 3 to 4 per cent of corrections.

In this work we turn to study if the non-spectator effects can more affect the lifetime of a b-baryon. Since a b-baryon contains a heavy b-quark and two light quarks, its binding structure and decay mechanism are not as certain as for B-meson, and we may have room to conceive some possibilities which involve the light flavors and increase the total decay width of b-baryons.

The diquark structure has been considered for a long time [9]. Especially because of the extra spin symmetry, the heavy baryons can be classified by the total spin of the light flavors, so that a diquark structure would be quite reasonable [10]. Parallel studies on the form factors of $\Lambda_b$ or $\Lambda_c$ decays have also been carried out in some literatures [11].

In this work, we assume that a b-baryon is constructed by a b-quark and a diquark
made of two light quarks. The diquark may have a total spin 0 or 1 and can be seen as scalar or axial-vector boson-like particles of color triplet. While evaluating weak transitions, an imposed postulation is needed that the diquark can undergo a flavor transition or spin-change, but does not dissolve during the process [12]. In this scenario, the diquark is somehow treated as an elementary particle. Actually, they are not point-like, but as some authors discussed, several form factors can be used to describe the inner structure of diquarks phenomenologically. The reactions involving light flavor diquarks were studied by Anselmino, Kroll and Pire [12]. We are going to employ the effective vertices and concerned parameters given in [12] to carry out the estimations of the non-spectator effects on b-baryon lifetimes. In the calculations, by comparing the derived result of $\tau(\Lambda_b)$ with data, we obtain a phenomenological parameter $\beta$, then with this value, we go on calculating the weak decay rates for $\Sigma^{(s)}_b$ and $\Xi^{(s)}_b$ and give predictions for lifetimes of $\Omega_b$ and $\Omega^*_b$. Since $\Lambda_b$ and $\Omega^{(s)}_b$ can only decay via weak interaction, their lifetimes are determined by weak decays. However, $\Sigma^{(s)}_b$ and $\Xi^{(s)}_b$ can decay strongly (for instance, $\Sigma_b \rightarrow \Lambda_b + \pi$), their lifetimes are mainly determined by the strong decay rates.

This paper is organized as following. After the introduction, we first briefly introduce the diquark structure of b-baryons and the effective interaction vertices of the diquarks given in [12], then we derive the formulae about the inclusive decay processes of B-meson and b-baryons. In Sec.III, we present our numerical results for $\tau(B)$, $\tau(\Lambda_b)$ and make predictions on weak decay rates of $\Sigma^{(s)}_b$ and $\Xi^{(s)}_b$ as well as $\tau(\Omega_b)$ and $\tau(\Omega^*_b)$. Then the last section is devoted to our conclusion and discussion.

II. Formulation

(i) For $B_d$ and $B^-$ decays we can easily evaluate the non-spectator effects in the well-established theoretical framework. The pure b-quark decay rate for $b \rightarrow c\bar{u}s + c\bar{u}d + c\bar{e}\nu_e +$
\(c\mu^-\bar{\nu}_\mu + c\tau^-\bar{\nu}_\tau\) has been carefully evaluated by Bagan et al \[13\] as

\[
\Gamma_b = \Gamma(b \to c\bar{s}s + c\bar{d}d) + \sum_{l=e,\mu,\tau} \Gamma(b \to c\bar{l}l),
\]

where

\[
\Gamma(b \to c\bar{s}s + c\bar{d}d) = (4.0 \pm 0.4)\Gamma(b \to c\bar{e}e),
\]

\[
\Gamma(b \to c\bar{\tau}\nu_\tau) = 0.25\Gamma(b \to c\bar{e}e),
\]

and

\[
\Gamma(b \to c\bar{e}e) = |V_{cb}|^2 \frac{G_F^2 m_b^5}{192\pi^3} [1 - 8\left(\frac{m_c}{m_b}\right)^2 - 12\left(\frac{m_c}{m_b}\right)^4 \ln \left(\frac{m_c}{m_b}\right) + 8\left(\frac{m_c}{m_b}\right)^6 - \left(\frac{m_c}{m_b}\right)^8].
\]

The quark level effective Hamiltonian for these processes has been evaluated to the next-to-leading order QCD corrections in the SM. It is given in \[14\] as

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} [V_{tb}V_{tq}^*(c_1O_1^{f_1f_2}(q) + c_2O_2^{f_1f_2}(q)) - V_{tb}V_{tq}^* \sum_{i=3}^{6} c_{i\text{eff}}O_i(q)],
\]

where

\[
O_1^{f_1f_2}(q) = \bar{f}_{1\alpha}\gamma_\mu(1 - \gamma_5)f_2\bar{q}\gamma_\beta\gamma^\mu(1 - \gamma_5)b_\alpha,
\]

\[
O_2^{f_1f_2}(q) = \bar{q}\gamma_\mu(1 - \gamma_5)f_2\bar{f}_1\gamma_\mu(1 - \gamma_5)b,
\]

\[
O_{3,5}(q) = \bar{q}\gamma_\mu(1 - \gamma_5)bq'\gamma^\mu(1 + \gamma_5)q',
\]

\[
O_{4,6}(q) = \bar{q}_{\alpha}\gamma_\mu(1 - \gamma_5)b_{\beta}\bar{q}_{\beta}\gamma^\mu(1 + \gamma_5)q'_{\alpha}.
\]

In eq.\(5\) we have neglected electroweak penguin contributions which are obviously small. \(q'\) is summed over \(u, d, s\) and \(c, q\) can be \(s\) or \(d\), and \(f_i\) \((i = 1, 2)\) can be \(u\) or \(c\), \(\alpha\) and \(\beta\) are color indices. In our later discussions we will use the Wilson coefficients evaluated in \[14\].

The lifetime of \(B_d\) is determined by the inclusive processes at quark level and the binding effects are taken into account when one correctly evaluates the transition hadronic matrix
elements of $B_d \rightarrow qq'$ where $q$ and $q'$ are free quark and antiquark. Thus the non-spectator processes occur via the W-boson exchange ($\bar{B}^0_d$) or annihilation ($B^-$) corresponding to $O_1$ and $O_2$ operators and the penguin-induced $O_3$ through $O_6$. We obtain

$$\Gamma(\bar{B}^0 \rightarrow c\bar{u}) = N_c \frac{G_F}{\sqrt{2}} V_{cb} V^*_{ud} (c_1 + 2 c_2) m_B^2 f_B^2 \frac{1}{4} m_B (1 - \frac{m^2}{m_B^2})^2 \frac{1}{\pi},$$

$$\Gamma(\bar{B}^0 \rightarrow s\bar{d}) = N_c \frac{G_F}{\sqrt{2}} V_{cb} V^*_{cs} \left( c_5^e + c_6^e \right) m_B^2 f_B^2 \frac{1}{4} m_B (1 - \frac{m^2}{m_B^2})^2 \frac{1}{\pi},$$

$$\Gamma(B^- \rightarrow s\bar{c}) = N_c \frac{G_F}{\sqrt{2}} V_{ub} V^*_{cs} (c_1 + c_2) m_B^2 f_B^2 \frac{1}{4} m_B (1 - \frac{m^2}{m_B^2})^2 \frac{1}{\pi},$$

$$\Gamma(B^- \rightarrow s\bar{u}) = N_c \frac{G_F}{\sqrt{2}} V_{ub} V^*_{cs} \left( c_5^e + c_6^e \right) m_B^2 f_B^2 \frac{1}{4} m_B (1 - \frac{m^2}{m_B^2})^2 \frac{1}{\pi}. \quad (7)$$

Then we can write the total decay width of $\bar{B}^0_d$ and $B^-$ as

$$\Gamma(\bar{B}^0_d) = \Gamma_b + \Gamma_{\bar{B}^0_d (\text{non})}; \quad \Gamma(B^-) = \Gamma_b + \Gamma_{B^- (\text{non})}, \quad (8)$$

where $\Gamma_{(\text{non})}$ denotes the non-spectator contribution to the total width as $\Gamma_{\bar{B}^0 (\text{non})} = \Gamma(\bar{B}^0 \rightarrow c\bar{u}) + \Gamma(\bar{B}^0 \rightarrow s\bar{d})$, $\Gamma_{B^- (\text{non})} = \Gamma(B^- \rightarrow s\bar{c}) + \Gamma(B^- \rightarrow s\bar{u})$. Our numerical evaluation indicate that such $\Gamma_{(\text{non})}$ can only change the total width of either $\bar{B}^0$ or $B^-$ by 3\~4\% at most.

(ii) The effective Hamiltonian of weak interaction for heavy quark-diquark interaction.

Since there lacks an available effective Hamiltonian for Q-D weak interaction where Q and D denote a heavy quark and a diquark composed of two light quarks, we have to construct it in some reasonable way. Here we do not intend to use the Renormalization Group Equation (RGE) to obtain a complete expression for the $QD \rightarrow Q'D'$ 4-body interactions, but only work at tree level.

(a) For the W-boson exchange between the heavy quark and the diquark

$$b + D \rightarrow c + D',$$

where D and D' are scalar or vector diquarks with two light quark constituents and they
reside in a color triplet. The effective vertex at the diquarks-W boson are

\[ V_S = -iG_S(q_1 + q_2)\mu W_\mu, \quad \text{for } SWS' \]  
\[ V_V = -i(G_1(q_1 + q_2)\mu g^{\alpha \nu} - G_2(q_2^\alpha g^{\mu \nu} + q_1^\mu g^{\alpha \nu}) + G_3(q_1 + q_2)^\mu q_1^\alpha q_2^\nu \epsilon_1^\alpha \epsilon_2^{\nu*} W_\mu, \quad \text{for } VWV', \]  

where \( S, S' \) and \( V, V' \) stand for scalar and vector diquarks, \( q_1, q_2 \) are momenta of \( D, D' \), \( \epsilon_1^\lambda, \epsilon_2^{\nu*} \) for polarizations of axial vectors \( D, D' \), respectively. \( G_S, G_1, G_2, G_3 \) are form factors which were determined by fitting data. For elementary vector particles \( G_1 = G_2, G_3 = 0 \). According to [12], in our case, the momentum transfer is small, so the \( G_3 \) term can be neglected and

\[ G_S = \frac{g_s}{2\sqrt{2}}F_S(Q^2), \quad G_1 = G_2 = \frac{g_s}{2\sqrt{2}}F_V(Q^2), \]  

where

\[ F_S(Q^2) = \frac{\tilde{\alpha}(Q^2)Q_0^2}{Q_0^2 + Q^2}, \]
\[ F_V(Q^2) = \frac{\tilde{\alpha}(Q^2)Q_1^2}{Q_1^2 + Q^2}, \quad \lambda_1 = \lambda_2 = 0, \]
\[ F_{V'}(Q^2) = \frac{Q_2^2}{Q_2^2 + Q^2}F_V(Q^2), \quad \text{otherwise.} \]  

These form factors are due to the QCD interactions which bind the quarks into a boson-like diquark. Therefore this \( \tilde{\alpha}(Q^2) \) corresponds to the effective non-perturbative QCD coupling which is not running and takes a reasonable value similar to that used in the potential model.

These transition amplitudes for \( b(p_1) + D(q_1) \rightarrow c(p_2) + D'(q_2) \) caused by the W-boson exchange read as

\[ T_{eff}^S = \frac{G_F}{\sqrt{2}}(V_{cb}V_{ud}^*)\bar{c}\gamma_\mu(1 - \gamma_5)b(q_1 + q_2)^\mu F_S(Q^2), \quad \text{for scalar diquarks} \]  
\[ T_{eff}^V = \frac{G_F}{\sqrt{2}}(V_{cb}V_{ud}^*)\bar{c}\gamma_\mu(1 - \gamma_5)[(q_1 + q_2)^\mu\epsilon_1^* + \epsilon_2^*\mu]\epsilon_2^\mu + q_2 \cdot \epsilon_1^*\mu] F_V(Q^2), \quad \text{for vector diquarks.} \]
(b) The penguin-induced effective vertices.

Now we turn to the transition \( b(p_1) + D(q_1) \to s(p_2) + D(q_2) \), where a virtual gluon bridges between the quark and diquark arms, all the formulation is similar to that for W-boson exchange, but only the coupling \( DWD' \) is replaced by \( DgD \).

The vertex for \( b \to s + g \) is given by Hou and Tseng \[15\] as

\[
V^a_{\mu} = \frac{G_F g_s}{\sqrt{2} 4\pi^2} V_t \bar{s} t^a[\Delta F_1(q^2 \gamma_\mu - q_\mu q)]L - F_2 i\sigma_{\mu\nu} q^\nu m_b R, \tag{15}
\]

with \( V_t = V_t^b V_{tb} \), \( \Delta F_1 = F_1^t - F_1^c \), \( F_1^t \approx 0.25 \), \( F_1^c = -\frac{2}{3} \ln(m_c^2/M_W^2) \approx 5.3 \), \( F_2 \approx 0.2 \).

Thus we can ignore the \( F_2 \) part, the transition amplitudes are

\[
T_{\text{eff}}^S = \frac{G_F \alpha_s}{\sqrt{2} \pi} V_t \bar{s} i \gamma_\mu b j t^a_{lm} \Delta F_1 F_S(Q^2)(q_1 + q_2)^\mu, \quad \text{for scalar diquarks} \tag{16}
\]

\[
T_{\text{eff}}^V = \frac{G_F \alpha_s}{\sqrt{2} \pi} V_t \bar{s} i \gamma_\mu b j t^a_{lm} \Delta F_1 F_V(Q^2)[(q_1 + q_2)^\mu \epsilon_1 \cdot \epsilon_2^* - (q_1 \cdot \epsilon_2^* \epsilon_1^\mu + q_2 \cdot \epsilon_1 \epsilon_2^\mu)], \quad \text{for vector diquarks.} \tag{17}
\]

Using

\[
t^a_{ij} t^a_{lm} = \frac{1}{2} [\delta_{im} \delta_{jl} - \frac{1}{N_c} \delta_{ij} \delta_{lm}],
\]

then eqs.(16) and (17) turn into the standard form in analog to that for mesons.

(iii) The contribution of the non-spectator effects to the decay width.

\( \Lambda_b \) can only decay via weak interaction to final states without bottom, so that the weak decay rate determines its lifetime.

Because there is no convenient way to evaluate the hadronic matrix elements like for mesons given in section (i), we can invoke a reasonable picture. \( \Lambda_b \) (or \( \Sigma_b^*, \Sigma_b^* \) etc.) can "decompose" into b-quark and a diquark D (scalar or axial vector, see next subsection), then \( b(p_1) \) and \( D(q_1) \) scatter into \( c(p_2) + D'(q_2) \) or \( s(p_2) + D(q_2) \) while \( p_1 + q_1 = p_2 + q_2 \) and \( (p_1 + q_1)^2 = M_{\Lambda_b}^2 \). In the scattering process both initial and final states only concern
free quarks and diquarks, and the confinement effects are reflected in a phenomenological parameter $\beta$ in analog to the wavefunction at origin $\psi(0)$ for the meson case. In the meson case, it is well known that the wavefunction at origin can be related to the decay constant $f_B$ as $\psi(0) = \sqrt{M_B/6f_B}$ [16], an analog of wavefunction of $Qq$ at origin $\psi^{Qq}_{\Lambda_b}$ for baryons has been discussed by many authors [17]. Even though such wavefunction at origin was employed for calculations, their values given in literatures are quite apart from each other. Here we use the scenario of quark-diquark, they cannot annihilate into a scalar or vector current but a fermionic one, and we keep $\beta$ as a free parameter to be fixed by fitting data.

The decay width of the non-spectator scattering process can be written as

$$\Gamma_{(\text{non})} = \frac{1}{2M_{\Lambda_b}} \int \frac{d^3p_2}{(2\pi)^3} \frac{1}{2E_2} \frac{d^3q_2}{(2\pi)^3} \frac{1}{2\omega_2} (2\pi)^4 \delta^4(p_1 + q_1 - p_2 - q_2)|\bar{T}|^2 \beta^2, \quad (18)$$

where $E_2$ and $\omega_2$ are the energies of the produced quark and diquark respectively, and $\beta$ corresponds to the unknown parameter.

(iv) The structure of the $b$-baryons.

The flavor configurations of the baryons containing a $b$-quark and a light color triplet subject which we identify as a diquark have been given in [10],

$$\Lambda_b = [(qq')_0b]_{1/2}, \quad \Xi'_b = [(qs)ob]_{1/2}, \quad \Sigma_b = [(qq')_1b]_{1/2}, \quad \Xi_b = [(qs)_1b]_{1/2},$$
$$\Omega_b = [(ss)_1b]_{1/2}, \quad \Sigma^*_b = [(qq')_1b]_{3/2}, \quad \Xi^*_b = [(qs)_1b]_{3/2}, \quad \Omega^*_b = [(ss)_1b]_{3/2}, \quad (19)$$

where the light quark pairs tend to constitute scalar or axial vector diquarks [3]. If one does not distinguish the mass difference between s-quark and $q(q') = u, d$, i.e., we can approximately assume the light flavor SU(3) symmetry, then the weak decay rates of $\Sigma_b^{(*)}, \Xi_b^{(*)}$ and $\Omega_b^{(*)}$ are the same.

By the heavy flavor $SU(2)$ symmetry, we can analyze the decay modes of $b$-baryons in analog to charm-baryons. Experimental data indicate that the mass difference of $\Omega_c$ and $\Xi_c$
is small and $M_{\Omega_c} - M_{\Xi_c} < M_K$ where $M_K$ is the kaon mass, so that the strong decay mode $\Omega_c \to \Xi_c + K$ is forbidden by the kinematic constraint. From $SU(2)_f$ symmetry, one can trust that $M_{\Omega_b} - M_{\Xi_b} < M_K$ also holds, therefore, such decay $\Omega_b \to \Xi_b + K$ cannot occur either. So, in principle, $\Omega_b^{(*)}$ does not have a strong decay mode. Moreover, since the mass splitting of $\Omega_b$ and $\Omega_b^*$ is so small (of the order $1/m_b$), the electromagnetic transition rate between them should be negligible. Therefore, the lifetimes of $\Omega_b^{(*)}$ are mainly determined by the weak interaction. On the contrary, $\Sigma_b^{(*)}(\Xi_b^{(*)})$ can decay into $\Lambda_b(\Xi'_b) + \pi$ via strong interaction, thus their lifetimes are determined by the strong interaction.

As we concern the transitions $b + D \to c + D'$ or $b + D \to s + D$, the diquark keeps its shape after the scattering, namely we only retain the two-body final states of quark and diquark. The case of breaking the diquark or that the produced quark may interchange with one of the light quarks inside the diquark to interfere with the amplitude without breaking the diquark is ignored. We will come to more discussions on this issue in the last section.

(v) The phenomenological assumption.

The previous derivations are all based on the commonly accepted physical picture with well-established theories and principles, later we will make an assumption based on our discussion given above. Namely, we attribute the lifetime difference between $B$-meson and $\Lambda_b$ to the light flavor involvement, in other words, the non-spectator scattering contributions are responsible for the difference. Including contributions from these non-spectator inclusive processes, we can write down the expression as

$$R^{th} = \frac{\Gamma_b^{(\Lambda_b)} + \Gamma_b^{(\Lambda_b)}}{\Gamma_b^{(B)} + \Gamma_b^{(B)}} = R^{exp} = \frac{1}{0.79}. \quad (20)$$

We can obtain the phenomenological parameter $\beta$ in eq. (18) from this equality.

$\Lambda_b$ is composed of a $b$-quark and a scalar diquark, whereas $\Sigma_b^{(*)}$, $\Xi_b^{(*)}$, and $\Omega_b^{(*)}$ are...
made of b and an axial-vector diquark, and their total spins are 1/2 and 3/2 respectively. The parameter $\beta$ corresponds to the effective wavefunction at origin in the meson case.

By SU(3) symmetry of light flavors, there can be two kinds of wavefunctions $\psi(0)$ for the b-baryons. They correspond to spin-0 scalar and spin-1 axial vector diquarks, respectively. Furthermore, $\psi(0) \propto f$ and the coupling constant $f$ is defined as

$$<0|\eta_F|F> = f_F u_F,$$
$$<0|\eta^\mu_F|F^*> = \frac{1}{\sqrt{3}} f_F^* u^\mu_{F^*},$$

where $F$, $F^*$ denote b-baryons with spin 1/2 and 3/2 respectively, $\eta_F$ and $\eta^\mu_{F^*}$ are proper baryonic currents, and $u^\mu_{F^*}$ is the Rarita-Schwinger spinor. In the leading order of HQET, $f_F = f_{F^*}$ \[18\]. With these structures of the currents, if the light flavors are treated as a mesonic object of color triplet, i.e., diquark, we can immediately have $\beta \propto \psi(0)$ which is proportional to $f_F$ or $f_{F^*}$. The numerical evaluations will be done in the next section.

### III. Numerical results

(i) For the $\Lambda_b$ lifetime.

Since $\Lambda_b$ is composed of b and a scalar diquark $(ud)_0$, the computation of $\Gamma_{(\Lambda_b)}^{(\text{non})}$ is relatively easier.

Taking $m_b = 4.8$ GeV, $m_c = 1.5$ GeV, $M_B = 5.3$ GeV, $M_{\Lambda_b} = 5.6$ GeV \[1\], $V_{ts} \sim V_{cb} \approx 0.036 \sim 0.042$, $V_{cs} \sim V_{tb} \sim 1$, and $m_D = 0.58$ GeV, $Q_0^2 = Q_1^2 = 3.22$ GeV$^2$, $Q_2^2 = 15$ GeV$^2$ \[12\] we can carry out all calculations\[12\].

The coupling constant at vertices $bgb$ and $DgD$ where b and D are the b-quark and

---

\[1\] In fact, there is another parameter set given in ref.\[12\] by fitting data, as $M_D = 0.7$ GeV, $Q_0^2 = Q_1^2 = 7.43$ GeV$^2$, $Q_2^2 = 20$ GeV$^2$, numerically this set produces results which only slightly deviate from that using the parameter set employed in text.
diquark of color-triplet, is running as

\[ \alpha_s(Q^2) = \frac{12\pi}{23 \ln(Q^2/\Lambda^2)}, \]

where \( \Lambda^2 \) is determined by \( \alpha_s(M_Z^2)(\sim 0.12) \). By contraries, the \( \alpha_s(Q^2) \) in the form factors of eq.(12) corresponds to the non-perturbative effects of QCD, so is an effective constant as that used in the potential model and does not need to be small. In fact, all the form factors in eq.(12) include this \( \alpha_s \), which can be incorporated into the parameter \( \beta \), namely \( \alpha_s \) does not show up explicitly in all expressions, as \( \beta \) is fixed by fitting data of the lifetime of \( \Lambda_b \). Thus with

\[ \frac{\tau(\Lambda_b)}{\tau(B)} \sim 0.79, \]

as input, we obtain

\[ \beta = 0.42 \text{ GeV}. \]

Since \( M_{\Lambda_b} < M_B + M_N \) where \( M_N \) is the mass of nucleon, it has no strong and electromagnetic decay channels. As discussed before, \( \Omega_b^{(*)} \) can only decay weakly. Thus their lifetimes are determined by weak interaction. For \( \Sigma_b^{(*)} \) and \( \Xi_b^{(*)} \), there are strong and electromagnetic channels available, so mainly their lifetimes are not owing to weak interactions.

According to the authors of [18, 19], \( f_{\Sigma_b^{*}} \approx f_{\Sigma_b} \) but

\[ \frac{\beta_1}{\beta} = \frac{f_{\Sigma_b}}{f_{\Lambda_b}} = 1.38 \sim 1.8, \]

where \( \beta_1 \) is the \( \beta \) parameter for \( \Sigma_b^{(*)} \).

(ii) Weak decay rates of \( \Sigma_b^{(*)} \) and \( \Xi_b^{(*)} \) and lifetime of \( \Omega_b^{(*)} \).

\( \Sigma_b \) and \( \Sigma_b^{*} \) are composed of a b-quark and an axial-vector diquark, so their wavefunctions are

\[ |\Sigma_b, 1/2, 1/2> = \sqrt{\frac{1}{3}} |b, 1/2, -1/2>_i |D, 1, 1>_i \]
where $b, D$ refer to the $b$-quark and diquark of spin-1, respectively, and "i" stands for the color index.

To calculate the cross sections of the scattering, we use the helicity-coupling amplitude method formulated by Chung [20], then the rest calculations are straightforward, even though a bit tedious. Thus we obtain

$$\frac{\Gamma^W(\Sigma_b)}{\Gamma(B)} \approx 4.15 \sim 5.02,$$

$$\frac{\Gamma^W(\Sigma^*_b)}{\Gamma(B)} \approx 2.87 \sim 3.37,$$

where $\Gamma^W$ denotes the weak decay width.

The reason of such large ratios is that the cross section for a scattering between a spinor and a spin-1 object is one order larger than that between a spinor and a scalar.

If this picture is right, we can expect that the inclusive weak decay rates of $\Sigma_b$ and $\Sigma^*_b$ are $4.2 \sim 5.0$ and $2.9 \sim 3.4$ times larger than the $B$-meson total width. It is a strong prediction. The same predictions hold for $\Xi_b^{(*)}$ and $\Omega_b^{(*)}$ if we assume the SU(3) light flavor symmetry. Since $\Omega_b$ and $\Omega^*_b$ decay only weakly, their lifetimes are determined by weak decay widths. Therefore, we predict that the lifetime ratio of $\tau(B)$ and $\tau(\Omega_b)$ is $4.2 \sim 5.0$ and that of $\tau(B)$ and $\tau(\Omega^*_b)$ is $2.9 \sim 3.4$. The prediction range is due to the uncertainty of $\beta_1$.

IV. Conclusion and discussion

(1) In this work we study the discrepancy between the measured lifetime ratio of $\tau(\Lambda_b)/\tau(\bar{B}^0(B^-))$ and the theoretical prediction based on the Standard Model and assume
that all the lifetime difference is due to the non-spectator effects where the light flavors get involved. The problem is how to correctly evaluate these non-spectator effects.

With the assumption of a quark-diquark structure of heavy baryons which contain a heavy quark and a diquark, the non-spectator effects for the baryons occur via interactions between the heavy quark and the diquark. In the case for $\bar{B}_d^0$ or $B^-$ mesons, the non-spectator effects are due to interactions between the heavy quark with a light antiquark, such processes can be evaluated in terms of the well-established weak effective Hamiltonian, all the calculation procedures are standard and have been tested, so that the results are relatively reliable. It is observed that a color matching factor $(C_1 + C_2/N_c)$ greatly suppresses contributions of $O_1$ and $O_2$ for $\bar{B}_d^0$, while the CKM entries also wash out their contributions to $B^-$. Thus the non-spectator effects caused by the W-boson exchange or annihilation can be neglected in either of $\bar{B}_d^0$ and $B^-$ lifetime calculations. Instead, we find that the penguin-induced operators $O_3 \sim O_6$ can make only $3\sim4\%$ contributions to the lifetime of both $\bar{B}_d^0$ and $B^-$. In total, such non-spectator effects cannot substantially change the lifetime of $\bar{B}_d^0$ and $B^-$. 

The baryon structure is less understood than for mesons, indeed the quark-diquark composition is a long-proposed picture. It is believed that the two light quarks constitute a scalar or axial-vector diquark of color-triplet, so it interacts with the heavy quark via W-boson exchange or a gluon-exchange where a penguin-effective-vertex stands for the transition of $b \rightarrow s + g$. The interaction vertices for the quark part are perfectly described by theories, on the contrary, the interaction forms for the diquarks are not well established yet. Since the diquark is not a real point-like elementary particles, even though the Lorentz structure and symmetries of the effective interaction are completely clear in analog to that for elementary particles, one needs to introduce some phenomenological form factors to describe the inner structure effects of the diquark.

We take the lifetime difference of $\Lambda_b$ and $\bar{B}_d^0$ as input and obtain a phenomenological
parameter $\beta$ which has a physical meaning in analog to the wavefunction at origin for meson case, namely describes the binding effects of the b-quark and the diquark.

Taking the $\beta$ value as input which is obtained by fitting $R = \tau(\Lambda_b)/\tau(B) \sim 0.79$, we further make predictions on $R' = \Gamma^W(\Sigma_b)/\Gamma(B)$ and $R'' = \Gamma^W(\Sigma'_b)/\Gamma(B)$, and find $R' \approx 4.15 \sim 5.02$, $R'' \approx 2.87 \sim 3.37$ respectively. Since $\Omega_b$ and $\Omega_b^*$ decay only weakly, these predictions show that the lifetimes of $\Omega_b$ and $\Omega_b^*$ are about $4.2 \sim 5.0$ and $2.9 \sim 3.4$ times shorter than that of $B^0(B^-)$.

In the derivations, we use the diquark structure of baryons which is accepted with some uncertainties. Its validity is to be confirmed or negated by the degree of closeness of theoretical predictions to the data. That, in fact, has been a subject of interest for quite a long time. Our work provides a way to test the diquark structure.

Indeed, because the diquark is not a real point-like particle, phenomenological form factors are needed. There exist several parameters in the form factors, which in principle, should be derived from some underlying theories, for example, the relativistic BS equation plus perturbative and non-perturbative QCD, but so far none of them has been well understood, thus we would rather keep them as parameters to be determined by fitting data. So we use the values of the parameters $Q^2_0, Q^2_1, Q^2_2$ given in [12] in our calculations.

Since the final states of the inclusive processes only involve free quark and diquark, the interaction forms and the transition amplitude evaluation are more reliable. However, we neglect possibilities that the diquark in the initial state might be broken at the scattering process, because in that the final state would be composed of three free quarks instead of a quark-diquark state, the three-body final state integration would much suppress its rate compared to two-body final state. We also neglect a possible situation that at the process of $b + D(ud) \rightarrow s + D(ud)$ may interfere with $b + D(ud) \rightarrow u(d) + D'(sd(u))$ where the produced s-quark replaces a quark in the diquark. Since it refers to an interference with breaking the diquark first, which needs to consume extra energies, we assume that such
effects are small.

All these treatments and approximations may bring up certain errors which would cause our theoretical evaluations deviate from real values of \( R' \) and \( R'' \). The ratios will be measured in the future. But, if the general picture is right, one has reason to believe that the qualitative conclusion of our results that the lifetimes of \( \Omega_b, \Omega_b^* \) should be much shorter than those of \( \bar{B}^0, B^- \) and \( \Lambda_b \). Our predictions will be tested in the future experiments.

(2) For observational sake, we can briefly discuss the phenomenological implication of the results.

Indeed, the branching fractions are actually an average over \( b \)-baryon weighted by their production rates in \( Z \) decays, branching ratios and detection coefficients. Therefore, the measured value is an admixture of \( \Lambda_b, \Xi_b, \Sigma_b, \Omega_b \)[1]. Considering the well-accepted mechanisms, we can assume the production rates of \( \Lambda_b, \Sigma_b \) should be larger than \( \Xi_b, \Omega_b \) and it is an admixture of \( \Lambda_b \) and \( \Sigma_b^{(*)} \). Therefore the theoretical estimation on the weak decay rates of \( \Lambda_b \) in fact is the corresponding value for the admixture.

Our numerical results show that the weak decay rates of \( \Sigma_b^{(*)} \) is much larger than that of \( \Lambda_b \), so an actual \( \tau(\Lambda_b)/\tau(B) \) may be not so serious as stated in the literatures. The problem for \( \Lambda_b \) is alleviated, but the non-spectator effects are still very important for the \( b \)-baryons with spin-1 diquarks.

This mechanism will be tested in the future precise measurements. However, because \( \Sigma_b^{(*)} \) has strong and electromagnetic decay modes, the complexity would make precise measurements very difficult.

First, the strong products and secondary products would contaminate the experimental environment and it is hard to identify if the products are directly from the \( b \)-baryons.

Moreover, for example, \( \Sigma_b \) can strongly decay into \( \Lambda_b + \pi \) and then \( \Lambda_b \) weakly decays into final states without bottom. It is also hard to distinguish between the products coming directly from \( \Sigma_b \) or as secondary products from \( \Lambda_b \). In principle, some clever cuts
may do the job, so for example, we just measure the two-body final states which cannot
be the secondary.

Anyhow, all these measurements are rather difficult, but since the diquark structure is
very important for understanding baryons, this study becomes interesting and necessary.
Also with rapid developments of detecting techniques and sophisticated experimental fa-
cilities, such measurements in the future will be possible and the prospect is optimistic.
We wish, that in near future, the B-factory can provide accurate data for the b-baryon
decays and then we can testify the diquark mechanism.

Acknowledgments:

This work was supported in part by the National Natural Science Foundation of China
and the Australian Research Council. One of us (Li) would like to thank Dr. X.G. He for
helpful discussions.

References

[1] The Particle Data Group, The Eur.Phys.J. C3 (198) 1.

[2] N. Isgur and M. Wise, Phys.Lett. B232 (1989) 113, B237 (1990) 527; H. Georgi,
   Nucl.Phys. B361 (1991) 349; M. Neubert, Phys.Rep. 245 (1994) 259.

[3] R. Rückl, Lectures on Weak decays of heavy flavors, (1983), Phys.lett. B120 (1983)
   448; D. Fakirov, B. Stech, Nucl.Phys. B133 (1978) 315; B. Stech, Phys.Lett. B130
   (1983) 189.
[4] A. Manohar and M. Wise, Phys.Rev. D49 (1984) 1310.

[5] I. Bigi, B. Blok, M. Shifman, N. Ural'tsev and A. Vainstein, B-Physics, edited by S. Stone, World Sci.Pub. Co. Singapore, (1994).

[6] M. Voloshin and M. Shifman, Sov.J.Nucl.Phys. 41 (1985) 120.

[7] M. Neubert and C. Sachrojda, Nucl.Phys. B483 (1997) 339.

[8] W.S. Dai, X.G. He, X.Q. Li and G. Zhao, in preparation.

[9] D. Ebert, T. Feldmann, C. Kettner and H. Reinhardt, Z.Phys. C71 (1996) 329.

[10] T. Mannel, W. Roberts and Z. Ryzak, Nucl.Phys. B355 (1991) 338.

[11] J. Körner and Kramer, Z.Phys. C2 (1979) 117; M. White and M. Savage, Phys.Lett. B271 (1991) 410; H. Georgi, B. Grinstein and M. Wise, Phys.Lett. B252 (1990) 456; N. Isgur, M. Wise and M. Youssfmir, Phys.Lett. B254 (1991) 215; X.-H. Guo and P. Kroll, Z. Phys. C59 (1993) 567; X.-H. Guo and T. Muta, Phys. Rev. D54 (1996) 4629.

[12] M. Anselmino, P. Kroll and B. Pire, Z.Phys. C36 (1987) 89; P. Kroll, M. Schürmann and W. Schweiger, Z.Phys. A338 (1991) 339.

[13] E. Bagan, P. Ball, V. Braun and P. Gasdzinsky, Nucl.Phys. B342 (1995) 362; erratum, ibid B374 (1996) 363.

[14] N. Deshpande and X.G. He, Phys.Lett. B336 (1994) 471.

[15] W-S. Hou and B. Tseng, Phys.Rev.Lett.80 (1998) 434.

[16] J. Donoghue, E. Golowich and B. Holstein, Dynamics of Standard Model, Cambridge University Press, (1992) London.
[17] N. Bilic, B. Gukerina and J. Trampedic, Nucl.Phys. B248 (1984) 261; M. Shifman and B. Voloshin, JETP 64 (1986) 698; P. Colangelo and F. DeFazio, Phys.Lett. B387 (1996) 371; J. Rosner, Phys.Lett. B379 (1996) 267.

[18] S. Groote and J. Körner, Phys.Rev. D55 (1997) 3016.

[19] S. Zhu and Y. Dai, hep-ph/9810243.

[20] S. Chung, Phys.Rev. D48 (1993) 1225.