OPTIMAL PRICING AND ADVERTISING DECISIONS WITH SUPPLIERS’ OLIGOPOLY COMPETITION: STAKELBERG-NASH GAME STRUCTURES

ALI NAIMI-SADIGH

Electronic Business Research Group, Information Technology Research Department
Iranian Research Institute for Information Science and Technology (IRANDOC), Tehran, Iran

S. KAMAL CHAHARSOOGHI

Faculty of Industrial and Systems Engineering
Tarbiat Modares University, Tehran, Iran

MARZIEH MOZAFARI

Department of Industrial Engineering, Electronic Branch
Islamic Azad University, Tehran, Iran

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Abstract. This paper addresses the coordination of pricing, advertising, and production-inventory decisions in a multi-product three-echelon supply chain composed of multiple suppliers, single manufacturer, and multiple retailers. The demand of each product is considered to be non-linearly influenced by the retail price and advertising expenditure. Taking into account the dominant power of the manufacturer and the suppliers’ oligopoly competition, this paper aims at obtaining the equilibrium prices at each level of the supply chain and comparing two different scenarios of competitions and cooperation: The former focuses on the situation where the single manufacturer has the dominant power in the supply chain and acts as the leader followed by the retailers and the suppliers simultaneously. The latter implies the situation in which the dominant manufacturer enters cooperation with each independent retailer to boost sales while the suppliers play the role of the followers simultaneously. We develop the Stackelberg-Nash game (SNG), and the Stackelberg-Nash game with cooperation (SNGC) formulations to model the two market structures. The equilibrium decisions are achieved through the optimization methods and the existence and uniqueness properties are explored. Finally, analytical and computational analyses are carried out through a numerical example, and a comprehensive sensitivity analysis is conducted to discuss some managerial insights such as increasing competition among suppliers leads to reducing retail prices.

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* Corresponding author: Ali Naimi-Sadigh.
1. Introduction. In a world of increasingly global competition, supply chains have become more complex with several activities spread over multiple functions or organizations. Accordingly, designing a coordination system which aims at improving supply chain performance by aligning the different activities and decisions of individual organizations is a critical issue that has been addressed in the study of supply chain management. Malone and Crowston [32] defined supply chain coordination as "the act of managing dependencies between entities and the joint effort of entities working together towards mutually defined goals". A supply chain generally consists of numerous functions including production, procurement, inventory control, logistics, product design, and etc.

Companies usually apply pricing and promotion mechanisms to enhance the demand and boost the benefit. When demand is sensitive to the retailers’ decisions at the downstream, an important problem is aligning the production-inventory and procurement policies at the upstream with the achieved customer demand. In fact, coordination of production, inventory, pricing and advertising activities may lead to a remarkable saving in global costs, balancing production rate and demand rate, and reducing total inventory and stockout. Price-coordination mechanisms such as quantity discount [27], and two-part tariffs [33, 26] have been [27] primarily presented for two-echelon supply chains. Little [31] is one of the early researches discussed the effects of advertising, pricing and retail distribution on customer’s demand in a marketing mix model. A price and advertisement sensitive demand investigated by Dube et al. [17] considering a Markov process. There has been increasing literature on pricing-inventory coordination such as Whitin [58], Yano and Gilbert [62], Elmaghraby and Keskinocak [18], Karakul [28], and Webster and Weng [57]. Furthermore, the extant literature on joint advertising-inventory or advertising-production problem is substantial. As some instance we can refer to Sogomonian and Tang [48], Sethi and Zhang [47], Cheng and Sethi [12], Sajedinejad and Chaharsooghi [44], Tsao and Sheen [52], and Tsao and Sheen [54].

Traditionally, the literature considers the coordination of different functions in isolation or at the central level of the supply chain. However, the extant literature has scarcely addressed the coordination of business relationships among complex network of supply chain members [2]. This paper contributes the literature by providing effective models to simultaneously coordinate pricing, advertising, and production-inventory activities among multiple suppliers, a manufacturer, and multiple retailers in a three-echelon supply chain and indicating how such coordination affects the members’ optimal decisions considering different power structures. The under studied supply chain includes multiple products with nonlinear price and promotion sensitive demand. Applying game theory approach, two power structures are investigated and the results are compared. (1) Stackelberg-Nash game (SNG) in which the manufacturer has the dominant power and acts as the leader followed by the retailers and the suppliers who compete at the bottom-level simultaneously in a price oligopolistic market. (2) Stackelberg-Nash game with cooperation (SNGC) where the manufacturer and the retailers are cooperating at the upper level as the leader and the suppliers compete with each other at the follower’s level. These models can be applied for automobile industry, food industry, and handicraft industry in which the manufacturer investigates her sales representative (retailers) are independent or under cooperation. Analytical and computational methods are applied to obtain the equilibrium decisions of the players while the existence and uniqueness properties are explored under each scenario. The models which are presented
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in this paper can be applicable for supply chains where manufacturers possess the dominant power such as automobile industry and electronic products industry.

The reminder of this paper is organized as follows. The related literature is briefly reviewed in Section 2. The problem definition, notations, and assumptions are presented in Section 3, and then the formulations of the SNG, and SNGC scenarios are given to characterize the interactions among the supply chain members in Sections 4, and 5, respectively. Moreover, in Sections 4, and 5 the equilibrium decisions are calculated while the existence and uniqueness properties are explored. Analytical results and sensitivity analysis are reported in Section 6 in order to discuss rewarding managerial sights. Finally, concluding remarks and some directions for future research are included in Section 7.

2. Literature review. Our paper related to the extensive stream of researches on supply chain coordination. There is a vast literature analyzing coordination of different activities of the supply chain such as logistics, inventory, production, pricing, advertising, etc. Analyzing coordination mechanisms is becoming a critical issue in the presence of vertical and horizontal competition between supply chain members. Various interface such as supplier-manufacturer, manufacturer-retailer, etc. can be effectively managed in decentralized supply chains using coordination. We refer the reader to Arshinder et al. [2] for a review on supply chain coordination problem. In this section, we study production, pricing, and advertising decisions as coordination mechanisms in both the two-echelon, and three-echelon supply chain. Also, we survey game theory approach as a tool to model the competition of supply chain members.

The literature on channel coordination has traditionally focused on joint production-inventory decisions assuming that price is given. Arreola-Risa [1] considered an integrated multi-item production-inventory system under uncertain demand and capacitated production. Yang and Wee [61] proposed a production-inventory policy in a single-supplier-multi-buyer supply chain with deteriorating item while the supplier offers quantity discounts to minimize the holding and ordering costs. Hwarng et al. [25] applied a simulation approach to deal with complexities in synchronizing production cycles and risk pooling effects in a multi-echelon supply chain which results in a decline in holding costs. Kim et al. [29] coordinated production and ordering policies in a single-manufacturer-single-retailer supply chain considering common production cycle length, delivery frequency and quantity. Sana [45] presented an integrated production-inventory model in a three-level supply chain consisting of one supplier, one manufacturer, and one retailer considering perfect and imperfect quality items. The impact of business strategies such as optimal order size of raw materials, production rate and unit production cost, and idle times in different sectors on cooperating marketing system has been examined. A production-inventory system investigated by Ghiami and Williams [20] for a two-echelon supply chain with multiple buyers and deteriorating items. Dong et al. [15] proposed a mathematical model to determine optimal price of the existing product and the inventory level of the new product.

Besides, supply chain members can coordinate by sharing price information. Researches on pricing coordination can be traced back to Jeuland and Shugan [27] considering a channel with one supplier and one retailer facing a deterministic and price sensitive demand rate. Chen et al. [9] studied coordination mechanisms in a supply chain with one supplier and multiple independent retailers. They assumed
that the demand for a retailer only depends on his own price. Their proposed model has been then extended by Bernstein and Federgruen [5] to the case that the retailers compete in retail price. Increasing literature has studied the coordination of pricing decisions, separately or jointly with other supply chain functions. Coordination of wholesale pricing and lot sizing decisions for one wholesaler and one or more geographically dispersed retailers has been investigated by Boyaci and Gallego [6]. Zhao and Wang [66] considered the coordination of dynamic joint pricing-production/ordering decisions in a supply chain where the manufacturer outsources retailing function to an independent retailer. Mukhopadhyay et al. [34] developed a joint pricing and ordering model for items with finite lifetime taking into consideration the nonlinear price dependent demand rates. Sajadieh and Akbari-Jokar [43] proposed an integrated production-inventory-pricing model for two-echelon supply chains. Chen and Chang [10] dealt with the problem of jointly determining the optimal retail price, the replenishment cycle, and the number of shipments for exponentially deteriorating items under conditions of channel coordination, joint replenishment program, and pricing policy. A joint pricing and inventory control problem studied by Chen et al. [11] for a perishable product with a fixed lifetime over a finite horizon where the demand depends on the price of the current period plus an additive random term. Applying bargaining theory, Saha and Goyal [42] discussed three kind of contracts namely joint rebate, wholesale price discount, and cost sharing contracts for a two-echelon supply chain. They proposed a stock and price induced demand and found that the stock elasticity plays an important role in contract selection problem. Rezapour et al. [41] proposed an integrated mathematical model to coordinate manufacturers and merchandisers of the pre- and aftersales operations considering price and warranty sensitive stochastic demand. A manufacturer-retailer supply chain considered by Lin [30] analyzing price promotion taking into account the reference price effects of consumers.

Advertising cost sharing is also a mechanism to reach supply chain coordination. Tsao and Sheen [52] studied the problem of dynamic pricing, promotion, and replenishment for a deteriorating item subject to the supplier’s trade credit and retailer’s promotional effort while the demand depends on price and time. Zhang et al. [65] provided an analytical model for jointly pricing, promotion, and inventory control decisions. Assuming a price and promotion sensitive demand function, they characterized the optimal policy for coordinating advertising, pricing and inventory replenishment. Tsao and Sheen [54] considered promotion cost sharing as a mechanism to achieve coordination in a two-echelon multiple-retailer distribution channel and obtained the retailers’ promotion and replenishment decisions under retailer competition and promotional effort with the sales learning curve. Cardenas-Barron and Sana [46] investigated the production-inventory coordination problem in a one-manufacturer-one-retailer centralized channel with promotion-based demand. An analytical method has been employed to achieve optimal production rate, production lot size, backlogging and the initiatives of sales teams. Tsao and Lu [53] addressed the manufacturer-retailer supply chain in which the manufacturer provides trade promotions to the retailer. Considering a linear stochastic price sensitive demand, they compared four different trade promotions. The results indicated that the unsold discount policy is mutually beneficial while only manufacturer can benefit from the target rebate policy. Navarro et al. [38] proposed an
inventory model for a three-echelon supply chain with multiple products and multiple members considering the demand as an increasing function of the marketing effort.

Recently, the tendency of the researches in the area of supply chain coordination is to apply game theory approach to deal with competition of the members under different power structures or different decision sequences. Szmerekovsky and Zhang [50] developed pricing options and advertising actions between one manufacturer and one retailer where demand is dependent on the retail price and advertising by both players. The optimal decisions obtained by solving a manufacturer Stackelberg game. Xie and Wei [60] and Xie and Neyret [59] investigated the optimal cooperative advertising strategies and equilibrium pricing in a two-echelon supply chain. Yu et al. [64] investigated optimal pricing, advertising, and inventory decisions in a single manufacturer Stackelberg supply chain with multiple independent retailers. A supply chain game with a buyer and a seller considered by Cai et al. [7] to coordinate pricing and ordering decisions with partial lost sales. Wang et al. [56] modeled the coordination of pricing and lot sizing strategies in a manufacturer-retailer supply chain considering price sensitive demand and finite production rate. Cooperative and manufacturer Stackelberg games developed to find the equilibrium decisions. The paper presented by Yin et al. [63] aims at providing an optimal discount policy derived from Stackelberg equilibrium to coordinate production, price, and inventory in one-manufacturer-multi-suppliers supply chain under uncertainty. Qi et al. [40] analyzed the interactions among members in a single-manufacturer-two-retailer supply chain considering the customer market search behavior using game theoretic approach. Hong et al. [21] developed Stackelberg game models to investigate the optimal decisions of local advertising, used-product collection and pricing in centralized and decentralized closed-loop supply chains composed of a manufacturer and an independent retailer. A pricing competition and cooperation problem discussed in Huang et al. [22] for a two-echelon supply chain with one manufacturer and duopoly retailers. They built six decentralized game models to examine pricing strategies and power structures impacts on the supply chain members’ performance. Soleimani et al. [49] applied a game theoretical method to drive optimal wholesale and retail price in a dual channel under disruptions. Parsaeifar et al. [39] presented a Stackelberg game theory approach to coordinate pricing, advertising, and inventory decisions for green products.

The literature review by Arshinder et al. [2] showed that most of the studies on coordination are done for two-echelon supply chains. Besides, Aust and Buscher [3] presented a comprehensive review on advertising coordination models and showed that there is relative paucity of researches considering multi-echelon supply chains with more than one member operating at each level. Chiefly, in case where game theory is applied to analyze the coordination of several activities in supply chain interface with competition and different power structures, researches are considerably few. Chung et al. [13] studied the price markdown scheme for a three-level supply chain consists of one supplier, one manufacturer, and one retailer. They identified the optimal discount pricing strategies, capacity reservation, and the stocking policies for the supplier and the retailer, and the optimal inventory decision for the manufacture, under both demand and delivery uncertainties. The coordination of pricing, component selection and inventory decisions is investigated by Huang et al. [24] using a Nash game approach. Sana et al. [46] addressed the coordination
of inventory and production decisions in a three-layer supply chain including multiple suppliers, manufacturers, and retailers comparing between the collaborating system and Stackelberg game structure while the demand is uncertain. Taleizadeh and Noori-daryan [51] developed a decentralized pricing-manufacturing-inventory model in a three-layer supply chain including a supplier, a producer and several independent retailers under a Stackelberg structure. A three-echelon supply chain consisting of multiple suppliers, a single manufacturer, and multiple retailers has been modeled by Naimi Sadigh et al. [35, 36] under a Nash game structure to coordinate pricing, advertising and inventory decisions considering discrete and continuous replenishment settings.

To the best knowledge of the authors, there has been no work that explicitly addressed the pricing and advertising coordination in multi-echelon supply chains specially when there is competition at each level between multiple members with different power structures. To fill this gap, the current paper studies a three-echelon supply chain composed of multiple suppliers, one manufacturer, and multiple retailers which sells multiple products to the end consumers. The demand is considered as a nonlinear function of retail prices and advertising costs. This nonlinearity of the demand function represents the reality more closely, however the explicit analytical results are more difficult to obtain [23]. We investigate the equilibrium pricing, advertising, production, and supplier selection decisions while the suppliers operating in an oligopolistic market with price competition. Two market power scenarios are focused to compare the players’ market shares and payoffs in a manufacturer Stackelberg and manufacturer-retailers-cooperation Stackelberg structures.

3. Problem definition and formulation. In this paper we focus on the coordination of pricing and advertising decisions in a multi-product multi-echelon supply chain consisting of multiple suppliers, one manufacturer, and multiple retailers with asymmetric power structures which choose their strategies over an infinite planning horizon. The manufacturer has the dominant power and makes his decisions ahead. These decisions are then followed by the retailers and the suppliers through their best reactions. On the other words, we face with a bi-level problem at the first level of which a leader (i.e. the manufacturer) predicts the best responses of the followers and selects his optimal actions considering these responses. At the second level the followers observe the leader’s actions and then decide on their optimal strategies simultaneously. It is presumed that the retailers are the sales representatives of the manufacturer performing in different countries and thus no competitive relationships is considered among the multiple retailers. However, the suppliers, on the other end of the channel, are competing in an oligopolistic market to gain more share of providing raw materials required by the manufacturer. Besides, each retailer faces with a price sensitive market demand which can also be influenced by the amount of money the retailer spends on the advertisement. The demand of each market is modeled as a nonlinear function of price and advertising expenditure. The notations used for problem formulation are listed in the following.

Notations:
Decision variables:
$T$ shared production cycle time
$\psi_i$ wholesale price for product $i = \{1, 2, \cdots, n\}$
$Q_j$ required quantity of raw material $j = \{1, 2, \cdots, J\}$
$p_{ir}$ selling price for product $i$ by retailer $r = \{1, 2, \cdots, R\}$
ad_{ir} advertising expenditure for product i by retailer r
D(p_{ir}, a_{ir}) demand for product i selling by retailer r
F_{js} price of material j purchasing from supplier s = \{1, 2, \ldots, S\}
v_{js} production quantity of material j by supplier s
Parameters:
P_i production capacity for product i
Cm_i unit production cost for product i
As_i setup cost for product i
hm_i unit holding cost for product i
B_m available budget in each production cycle
w_{ji} required quantity of material j in a unit of product i
f_{ir} market scale for product i selling by retailer r (f_{ir} > 0)
h_{ir} unit holding cost for product i selling by retailer r
Ar_{ir} unit ordering cost for product i selling by retailer r
\alpha_{ir} price elasticity for product i selling by retailer r (\alpha_{ir} > 1)
\beta_{ir} advertising elasticity for product i selling by retailer r (\beta_{ir} > 0, \alpha_{ir} > 1 + \beta_{ir})
\omega_j subset of suppliers who produce the material j
Cs_{js} unit production cost of material j for supplier s
Ca_{js} production capacity of supplier s for producing material j
\eta_{js} direct price elasticity for material j selling by supplier s
\theta_{js} rival price elasticity for material j selling by supplier s

3.1. Formulation of the manufacturer’s sub-model. The manufacturer is producing multiple products with a shared cycle time. In fact, the time interval between two production run is assumed to be the same for all the products (i.e. \( T = T_i, \forall i \)). Hence, the batch size for each product can be calculated as \( Q_{ir} = D_{ir} \times T \) and the manufacturer aims at optimizing variable \( T \). It is assumed that the retailers have the less ordering cost and the more holding cost compared to the manufacturer’s setup and holding costs.

The manufacturer’s sub-model is a multi-product mathematical model aims at maximizing his individual benefit as well as determining the optimal values for the ordering quantity of raw materials, the whole sale price and the shared production cycle time variables. In addition, the manufacturer has a limited budget which affects the number of production cycles or the number of deliveries to the retailers. Thereby, the mathematical formulation of the manufacturer’s sub-model can be defined as follows:

\[
\max_{\psi, T, Q_{j}} \Pi_M (\psi, T, Q_{j}) = \sum_{i=1}^{n} \left[ \psi_i \sum_{r=1}^{R} D_{ir}^* \right] - \sum_{i=1}^{n} \left[ Cm_i \sum_{r=1}^{R} D_{ir}^* \right] - \sum_{j=1}^{J} \sum_{s=1}^{S} F_{js}^* v_{js}^* - \frac{\sum_{i=1}^{n} As_i}{T} - \frac{1}{2} \sum_{i=1}^{n} \left[ hm_i \sum_{r=1}^{R} D_{ir}^* \left( \frac{\sum_{r=1}^{R} D_{ir}^*}{P_i} \right) \right] \\
\text{s.t.} \\
T \sum_{i=1}^{n} \left[ Cm_i \sum_{r=1}^{R} D_{ir} \right] \leq B_m \\
Q_{j} = u_{ji} \left( \sum_{r=1}^{R} D_{ir}^* \right) \quad \text{for} \quad j = 1, 2, \ldots, J
\]

The manufacturer’s benefit is the total revenue of selling products to the different retailers minus the total material, production, setup, and holding costs. \( D_{ir}^* \) denotes the demand of product i from retailer r, which will be known when the retailers
choose their pricing and advertising strategies. \( F_{js}^* \) and \( v_{js}^* \) are the suppliers’ decisions. Constraint (2) balances the production budget with the production cycles. Constraint (3) specifies the required quantity of material \( j \). It is assumed that the production capacity is large enough (i.e. \( \sum_{i=1}^{n} \frac{\sum_{r=1}^{R} D_{ir}}{P_i} \leq 1 \)) and the setup time is equal to zero. Hereupon, \( \sum_{i=1}^{n} \frac{T \sum_{r=1}^{R} D_{ir}}{P_i} \leq T \) is always true.

### 3.2. Formulation of the retailers’ sub-models

Each retailer faces with a variable demand for each product \( i \) which is defined by a nonlinear function of the selling price and the advertising expenditure as follows:

\[
D_{ir}(p_{ir}, ad_{ir}) = f_{ir} p_{ir} - \alpha_{ir} p_{ir} ad_{ir} \beta_{ir} ad_{ir}, \quad p_{ir} \geq 0
\]  

(4)

The demand function implies that each retailer’s market can be extended by reducing the prices with the elasticity \( \alpha_{ir} \) and/or enhancing the advertising expenditure with the elasticity \( \beta_{ir} \).

Retailers have the objective of maximizing their individual benefits while determining the selling prices and advertising expenditures for the different products. The price and advertising expenditure strategies chosen by the retailers then influence on the production batch sizes of the manufacturer and the raw materials ordering quantities from the suppliers. The mathematical formulation of each retailer’s sub-model can be defined as follows:

\[
\max \Pi_{Rr}(p_{ir}, ad_{ir}) = \sum_{i=1}^{n} p_{ir} D_{ir} - \sum_{i=1}^{n} \psi_{ir} D_{ir} - \sum_{i=1}^{n} ad_{ir} D_{ir} - T \sum_{i=1}^{n} D_{ir} h_{ir} - \sum_{i=1}^{n} A_{ir} T^*)
\]

(5)

S.t.

\[
D_{ir} = f_{ir} p_{ir}^{\alpha_{ir}} ad_{ir}^{\beta_{ir}}
\]

\[
ad_{ir}, \quad p_{ir} \geq 0
\]

(6)

The benefit function (5) is the total revenue of selling products minus purchasing, advertising, holding, and ordering costs. It is worth mentioning that despite the demand is dependent on price and advertising expenditure, but because the demand rate is constant over an infinite planning horizon, we can still use the inventory models with constant demand rate. \( \psi_i \) and \( T \) are the wholesale price and the shared production cycle time which determined by the manufacturer and influence the retailer’s benefit.

### 3.3. Formulation of the suppliers’ sub-models

The demand of each supplier \( s \) can be described as a linear function of the supplier’s own prices as well as the prices of the rival suppliers which produces the same materials in the market and can be formulated as follows:

\[
v_{js}(F_{js}, F_{(jS)}) = Q_j - \eta_{js} F_{js} + \sum_{(s=1/s)}^{S} \theta_{(js)} F_{(jS)},
\]

(7)

where \( F_{(jS)} \) denotes the price of material \( j \) set by the rival suppliers.

The suppliers aim at maximizing their own benefits while determining the material production batch sizes and the material selling prices. Each supplier’s sub-model is a capacitated multi-product model which can be formulated as follows:

\[
\max \Pi_{Ss}(F_{js}, v_{js}) = \sum_{j=1}^{J} F_{js} v_{js} - \sum_{j=1}^{J} C_{js} v_{js}
\]

(8)
S.t.
\[ v_{js} = Q_j - \eta_{js} F_{js} + \sum_{s=1}^{S} \theta_{js} F_{js} \quad g = \{1, 2, \ldots, s-1, s+1, \ldots, S\} \]  \tag{9}
\[ \sum_{s=1}^{S} v_{js} = Q_j \text{ for } j = 1, 2, \ldots, J \]  \tag{10}
\[ v_{js} \leq C_{aj} \text{ for } j = 1, 2, \ldots, J \]  \tag{11}

The benefit function (8) for each supplier is the revenue minus the production costs. Eq. (9) states the supplier's demand function. Constraint (10) ensures that the quantity required by the manufacturer is completely supplied by the suppliers. Constraint (11) expresses the production capacity limit.

4. The Stackelberg-Nash game. In this section, we study the interactions between the supply chain members as a Stackelberg-Nash game where the manufacturer, as the leader, chooses his strategies and impose them to the retailers and suppliers, as the followers, which simultaneously decide on their best responses at the second priority. Therefore, there exists a Nash game between the followers. On the other words, the manufacturer first picks out the best batch sizes and wholesale prices for the product as well as the best shared production cycle time.

![Figure 1. The schematic view of SNG setting](image)

Then, each retailer in each separate market tries to determine the best retail prices and advertising expenditures in response while the suppliers are competing to each other to gain more share of the required raw materials by choosing their prices in a Nash game. In SNG setting, the leader aims at maximizing his benefit while considering the followers’ best responses. Figure 1 illustrates the schematic view of the players’ interactions in SNG.
Hence, the mathematical model of SNG can be formulated as a bi-level programming model as follows:

Upper Level:

\[
\begin{align*}
\text{Max} & \quad \Pi_M(\psi_i, T, Q_j) = \sum_{i=1}^{n} \left[ \psi_i \sum_{r=1}^{R} D_{ir} \right] - \sum_{i=1}^{n} Cm_i \sum_{r=1}^{R} D_{ir} - \sum_{j=1}^{J} \sum_{s=1}^{S} F_{js} v_{js} \\
& \quad - \sum_{i=1}^{n} A_{si} T - \frac{T}{2} \sum_{i=1}^{n} h_{mi} \sum_{r=1}^{R} D_{ir} \left( \frac{\sum_{r=1}^{R} D_{ir}}{P_t} \right) \end{align*}
\]

(12)

Lower Level:

\[
(p_{ir}, a_{ir}) \in \arg \text{Max} \quad \Pi_R(p_{ir}, a_{ir}) = \sum_{i=1}^{n} p_{ir} D_{ir} - \sum_{i=1}^{n} \psi_i D_{ir} - \sum_{i=1}^{n} a_{ir} D_{ir} \]

\[
- \frac{T}{2} \sum_{i=1}^{n} D_{ir} h_{ir} - \sum_{i=1}^{n} A r_{ir} T \]

\[
D_{ir} = f_{ir} p_{ir}^{-\alpha_{ir}} a_{ir}^{-\beta_{ir}} \]

\[
a_{ir}, p_{ir} \geq 0 \]

\[
(F_{js}, v_{js}) \in \arg \text{Max} \quad \Pi_S(F_{js}, v_{js}) = \sum_{j=1}^{J} F_{js} v_{js} - \sum_{j=1}^{J} C s_{js} v_{js} \]

\[
v_{js} = Q_j - \eta_{js} F_{js} + \sum_{s=1}^{S} \theta_{js} F_{js} \]

(13)

\[
\sum_{s=1}^{S} v_{js} = Q_j \]

\[
v_{js} \leq C a_{js} \]

In order to solve the problem, we first need to investigate the equilibrium conditions for the lower level sub-problems. If the Nash equilibrium in the lower level exists uniquely, then we can transform the bi-level programming model to an equivalent single level nonlinear programming problem which can be solved by using the commercial solvers. Note that in case of non-unique lower level solutions, the leader cannot be allowed to force the followers to take the one or the other of their optimal solutions. In this sense, when multiple equilibriums exist in the lower level, the result of the bi-level problem can be distorted. Hence, the leader cannot predict the true value of his objective function until the followers have communicated their choices. To overcome this ambiguity two approaches including optimistic position and pessimistic position have been suggested by Dempe [14]. In the following, we study existence and uniqueness of the lower level Nash equilibrium.

4.1. The optimality conditions for the retailers. To determine the optimal values for the retailers’ actions, it is assumed that the other players’ strategies are known. Since the retailers’ sub-models have no mutual interactions, we can obtain the best strategies for each individual retailer using the first order optimality conditions of each retailer’s sub-model.

**Proposition 1. (Convexity Conditions)** For every retailer \((r \in R)\) the objective function is a strongly pseudo-concave function. Function \(f\) is strongly pseudo-convex if for each distinct point \(x_1, x_2\) with \(f(x_2) \leq f(x_1)\), the inequality \(\nabla f(x_1) . (x_2 - x_1) < 0\) holds. Also, \(f\) is strongly pseudo-concave if \(-f\) is strongly pseudo-convex [4].

**Proof.** See Naimi Sadigh et al. [37].
According to proposition 1, since the objective function is pseudo concave, the optimal values of the decision variables are obtained when the partial derivatives of the objective function vanish. In this way, we have:

\[ ad_{ir}^* = \frac{\beta_{ir} (p_{ir} - \psi_i - Th_{ir}/2)}{\beta_{ir} + 1} \] (14)

\[ p_{ir}^* = \frac{\alpha_{ir} (ad_{ir}^* + \psi_i + Th_{ir}/2)}{\alpha_{ir} - 1} \] (15)

By replacing \( ad_{ir}^* \) in Eq. (15), \( p_{ir}^* \) and \( ad_{ir}^* \) will be calculated independently as follows:

\[ p_{ir}^* = \frac{\alpha_{ir} (\psi_i + Th_{ir}/2)}{\alpha_{ir} - \beta_{ir} - 1} \] (16)

\[ ad_{ir}^* = \frac{\beta_{ir} (\psi_i + Th_{ir}/2)}{\alpha_{ir} - \beta_{ir} - 1} \] (17)

4.2. The Nash equilibrium conditions for the suppliers. The game among the competing suppliers is a generalized Nash equilibrium problem (GNEP) with joint constraint, where both the objective function and the constraint set of each player depend on the actions taken by the rival players [19].

In order to obtain the Nash equilibrium decisions for the suppliers, we first ignore the capacity constraint (11), solve the Nash game, and check the capacity constraint satisfaction at the end. In this way, we face with optimizing \( s \) objective functions with regard to an equality joint constraint, for which the Nash equilibrium values can be obtained through concatenating the Karush-Kuhn-Tucker (KKT) optimality conditions of the suppliers.

**Proposition 2.** (Convexity Conditions) For every supplier \((s \in S)\), the sub-model is a convex programming model.

**Proof.** See Naimi Sadigh et al. [35].

Due to the proposition 2, we can write the Lagrangian of the sub-model for supplier \( s \) as follows:

\[ L_{S_s} (F_{js}) = \sum_{j=1}^{J} F_{js} \left( Q_j - \eta_{js} F_{js} + \sum_{s=1}^{S} \theta_{js} F_{js} \right) - \sum_{j=1}^{J} C_{s_j} \left( Q_j - \eta_{js} F_{js} + \sum_{s=1}^{S} \theta_{js} F_{js} \right) - \mu_j \left( \sum_{s=1}^{S} v_{js} - Q_j \right) \] (18)

where, \( \mu_j \) is a free variable.

Then, the K.K.T identity of supplier \( s \) can be calculated as:

\[
\frac{\partial L_{S_s} (F_{js})}{\partial F_{js}} = 0 \rightarrow \left( Q_j - \eta_{js} F_{js} + \sum_{s=1}^{S} \theta_{js} F_{js} \right) - \eta_{js} (F_{js} - C_{s_{js}}) - \mu_j (-\eta_{js} + (s - 1) \theta_{js}) = 0
\] (19)

Concatenating the K.K.T identity of the suppliers together with the joint constraint forms a linear system of equations. Solving this system of equations will result in the Nash equilibrium solutions. Note that all the equations involved in this system are linearly independent and therefore the equilibrium point exists uniquely. From this point, we need to check the capacity constraint satisfaction. If the resulted Nash equilibrium values meet the capacity constraint, they are optimal decisions, otherwise, the production quantities for raw materials should be set equal to the maximum capacities and the system of equations should be resolved to calculate the optimal raw material prices.
4.3. The SNG equilibrium conditions. In order to find the SNG equilibrium of the multi-echelon supply chain, we can transform the bi-level programming model, Eqs. (12) to (13), into an equivalent single level model by adding the optimality conditions of the retailers and the Nash equilibrium conditions of the suppliers to the manufacturer’s set of constraints. As discussed in section 4.1, the optimality conditions for the retailers can be expressed implicitly as a unique function of the upper level variables. Also as mentioned in section 4.2, the Nash equilibrium solutions for the suppliers exists uniquely. Thus, we can convert the bi-level model of SNG into the following single level optimization model:

\[\max \Pi_M(\psi, T) = \sum_{i=1}^{n} (\psi_i - Cm_i) \sum_{r=1}^{R} f_r \left( \frac{\alpha_{ir}}{\alpha_{ir} - \beta_{ir} - 1} \right)^{-\alpha_{ir}} \left( \frac{\beta_{ir}}{\alpha_{ir} - \beta_{ir} - 1} \right)^{\beta_{ir}} \left( \psi_{ir} + \frac{T h_{ir}}{2} \right)^{-\alpha_{ir} + \beta_{ir}} - \eta_{j} \left( F_{js} - Cs_{js} \right) - \mu_j \left( -\eta_{js} + (s - 1) \theta_{js} \right) = 0 \]

for \( j = 1, 2, \ldots, J \)

\[\sum_{i=1}^{n} Cm_i \sum_{r=1}^{R} f_r \left( \frac{\alpha_{ir}}{\alpha_{ir} - \beta_{ir} - 1} \right)^{-\alpha_{ir}} \left( \frac{\beta_{ir}}{\alpha_{ir} - \beta_{ir} - 1} \right)^{\beta_{ir}} \left( \psi_{ir} + \frac{T h_{ir}}{2} \right)^{-\alpha_{ir} + \beta_{ir}} - B_m = 0 \]

\[u_{js} \left( \sum_{r=1}^{R} f_r \left( \frac{\alpha_{ir}}{\alpha_{ir} - \beta_{ir} - 1} \right)^{-\alpha_{ir}} \left( \frac{\beta_{ir}}{\alpha_{ir} - \beta_{ir} - 1} \right)^{\beta_{ir}} \left( \psi_{ir} + \frac{T h_{ir}}{2} \right)^{-\alpha_{ir} + \beta_{ir}} \right) - Q_j = 0 \]

Without loss of generality, it is assumed that the production costs, the holding costs and the whole sale prices are the same for each retailer. Also, to meet convexity assumption, we replace the first two equality constraints (20) with the following inequalities:

\[\sum_{i=1}^{n} Cm_i \sum_{r=1}^{R} f_r \left( \frac{\alpha_{ir}}{\alpha_{ir} - \beta_{ir} - 1} \right)^{-\alpha_{ir}} \left( \frac{\beta_{ir}}{\alpha_{ir} - \beta_{ir} - 1} \right)^{\beta_{ir}} \left( \psi_{ir} + \frac{T h_{ir}}{2} \right)^{-\alpha_{ir} + \beta_{ir}} - B_m \leq 0 \]

\[u_{js} \left( \sum_{r=1}^{R} f_r \left( \frac{\alpha_{ir}}{\alpha_{ir} - \beta_{ir} - 1} \right)^{-\alpha_{ir}} \left( \frac{\beta_{ir}}{\alpha_{ir} - \beta_{ir} - 1} \right)^{\beta_{ir}} \left( \psi_{ir} + \frac{T h_{ir}}{2} \right)^{-\alpha_{ir} + \beta_{ir}} \right) - Q_j \leq 0 \] for \( j = 1, 2, \ldots, J \)

Since the manufacturer’s objective is a maximization objective, the above inequalities always hold as equalities at the optimal points.

The shared production cycle implies that all the products should be produced in \( T \) and no shortage is allowed by the manufacturer. Therefore, by introducing \( w_{ir}^{-1} = \frac{\sum_{i=1}^{n} B_{ir}}{P} \), we need the inequality (22) to be hold.

\[\sum_{i=1}^{s} \sum_{r=1}^{R} w_{ir}^{-1} \leq 1 \]

Finally, the equivalent single level model is a nonlinear optimization problem that can be solved exactly through the nonlinear optimization solvers.
4.4. The existence and uniqueness of SNG equilibrium. Since SNG model is transformed to a single level optimization problem, to explore the existence and uniqueness properties of the equilibrium, we need to show that the leader’s objective function is strongly pseudo concave with respect to the wholesale price, the shared production cycle, and the required raw material variables. Furthermore, the convexity of the solution space should be explored.

**Lemma 1.** The manufacturer’s objective function in the upper level is strongly pseudo concave with respect to the wholesale price variable.

*Proof. See appendix A.*

**Lemma 2.** The manufacturer’s objective function in the upper level is strongly pseudo concave with respect to the shared cycle variable.

*Proof. See appendix B.*

**Lemma 3.** The solution space of SNG is a convex set.

*Proof. See appendix C.*

Bringing together all we discussed in Lemmas 1, 2, and 3, it is inferred that SNG equilibrium of the proposed game uniquely exists.

5. The Stackelberg-Nash game with cooperation. In this section, we investigate the cooperative relationships between the manufacturer and each retailer at the upper level. This kind of relationship is common when retailers are the sales representatives of the manufacturer.

It is presumed that multiple retailers each enters a cooperative contract with the dominant manufacturer and they set their strategies cooperatively. These strategies include shared production cycle ($T$), required quantity of raw materials ($Q$), retail prices ($p$) and advertising expenditure ($ad$). The wholesale prices ($\psi$) are not important anymore in the cooperative setting and can be easily omitted from the manufacturer’s sub-model. In this scenario there are no competition among manufacturer and the multiple retailers. In this cooperative scenario, the manufacturer and the retailers as the leaders negotiate on their best strategies at the first priority. Then, the suppliers decide on their best responses to the manufacturer-retailer coalition while they are competing in a Nash game. Therefore, we face with the Stackelberg-Nash game with cooperation (SNGC) with the manufacturer-retailer coalition as the leader and multiple competing suppliers as the followers. Figure 2 illustrates the schematic view of the players’ interactions in SNGC scenario.

Maximizing the joint benefit function of the retailers and the manufacturer forms a single objective for the upper level. The constraint set consists of the manufacturer’s and the retailers’ constraints along with the Nash equilibrium conditions of the suppliers. In this way, the bi-level model is reduced to an equivalent single level model formulate as follows:

$$
\begin{align*}
\max_{p_{ir}, \, ad_{ir}, \, T, \, Q_j} \Pi_{MR} &= \sum_{r=1}^{R} \sum_{i=1}^{n} p_{ir} D_{ir} - \sum_{r=1}^{R} \sum_{i=1}^{n} ad_{ir} D_{ir} \\
&- \frac{T}{2} \sum_{i=1}^{n} D_{ir} h_{ir} - \frac{A r_{ir}}{T} - \sum_{i=1}^{n} C m_{i} \sum_{r=1}^{R} D_{ir} - \sum_{j=1}^{J} \sum_{s=1}^{S} F_{js} v_{js} \\
&- \frac{\sum_{i=1}^{n} A s_{i}}{T} - \frac{T}{2} \sum_{i=1}^{n} h_{ir} \sum_{r=1}^{R} D_{ir} u_{ir}^{-1}
\end{align*}
$$

(23)
Since the equivalent single level formulation is a nonlinear programing model we can use the nonlinear optimization solvers to obtain the equilibrium values.

5.1. The existence and uniqueness of the equilibrium solution. In a similar manner that we discussed in the section 4, SNGC formulation can be transformed to a single level optimization problem. Hence, to explore the existence and uniqueness properties of the equilibrium, we need to show that the leader’s objective function is strongly pseudo concave with respect to its decision variables. Furthermore, the convexity of the solution space should be explored.

Lemma 4. The objective function of SNGC is strongly pseudo concave with respect to the retail price variable.
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Variables Retailer Variables Retailer
(SNG) 1 2 3 (SNGC) 1 2 3
\[ p^*_{ir} \]
\[ 358.1 \] 443.1 413.7 \[ 232.7 \] 284.7 265.9
\[ 342.7 \] 486.5 486.5 \[ 190.3 \] 271.8 167.4
\[ 330.2 \] 518.7 373.6 \[ 205.4 \] 321.4 234.0
\[ 345.6 \] 420.3 369.1 \[ 239.8 \] 292.7 255.6

\[ ad^*_{ir} \]
\[ 29.8 \] 52.1 25.9 \[ 19.4 \] 34.0 16.8
\[ 72.2 \] 91.2 35.7 \[ 40.1 \] 51.0 19.7
\[ 76.8 \] 172.9 101.9 \[ 47.8 \] 107.1 63.9

\[ D^*_{ir} \]
\[ 31.6 \] 55.9 72.0 \[ 64.3 \] 106.3 137.5
\[ 14.8 \] 29.3 15.8 \[ 32.4 \] 57.3 33.4
\[ 15.1 \] 16.7 27.7 \[ 28.0 \] 29.7 49.8

Table 1. The retailers' equilibrium strategies

Proof. See appendix D.

Lemma 5. The objective function of SNGC is strongly pseudo concave with respect to the advertising expenditure variable.

Proof. See appendix E.

Lemma 6. The objective function of SNGC is concave with respect to the shared production cycle variable.

Proof. See appendix F.

Lemma 7. The solution space of SNGC is a convex set.

Proof. See appendix G.

Bringing together all we discussed in Lemmas 4 to 7, it is inferred that the objective function in SNGC is strongly pseudo concave with regard to its decision variables and the solution space is a convex set. Therefore, the equilibrium of the proposed game uniquely exists.

6. Numerical results. In this section, we present a numerical analysis to illustrate how the proposed models and solution algorithms work and to evaluate the results.

A comprehensive sensitivity analysis is also conducted on the main parameters of the models. For comparison purposes, we bring up the same example given by Naimi Sadigh et al. [36] which considers a multi-product three-echelon supply chain consisting of one manufacturer, four suppliers and three retailers, which produce four products using five raw materials.

The equilibrium solutions of SNG and SNGC scenarios have been achieved through the equivalent single level formulations described in section 4, and section 5, respectively. The models have been coded in GAMS and solved using CONOPT solver [16]. Tables 1, and 2 compare the equilibrium values of the retailers and suppliers in SNG and SNGC, respectively. Furthermore, the manufacturer chooses \( T^* = 0.8700 \) in SNGC scenario and \( \psi_1^* = 128.27, \psi_2^* = 88.14, \psi_3^* = 96.75, \psi_4^* = 120.97, \) and \( T^* = 1.0263 \) in SNG scenario.

As can be seen in Table 1, the retailers choose higher prices and spend more on advertisement but capture less demand in SNG setting comparing with SNGC. On the other side, Table 2 shows that the suppliers can charge the manufacturer more for the required raw materials and also gain further raw materials demand in SNGC. Therefore, the suppliers acquire the higher benefit under SNGC scenario.
Variables Raw Material
(SNG) 1 2 3 4 (SNGC) 1 2 3 4
\( F^*_{js} \)
8.1 8.6 10.8 7.5 15.5 15.8 16.4 15.4
11.6 7.6 12.6 8.1 22.9 15.0 24.7 15.9
6.9 9.0 10.9 8.3 14.1 18.5 22.3 16.9
4.0 4.1 4.2 3.5 8.4 8.6 9.0 7.5
\( v^*_{js} \)
43.7 43.7 61.8 41.5 82.3 85.7 117.4 76.4
39.3 46.2 49.9 42.2 86.5 97.2 105.3 90.9
61.4 29.0 62.8 35.8 119.6 56.9 123.3 70.1
23.3 45.7 55.0 38.0 47.4 92.1 111.2 77.9
29.7 27.0 33.4 27.9 62.0 59.9 69.6 55.5

Table 2. The suppliers’ equilibrium strategies

R1 R2 R3 R4 M T
Nash 19449.7 48501.5 50858.4 15378.8 6214.9 5542.3 10155.1 4804.2 163613.9
SNG 14880.6 42892.2 40079.2 37803.0 1548.4 1363.0 2459.6 1238.7 142264.7
SNGC 150431.6 6388.9 5818.4 10079.9 5195.4 177914.2

Table 3. Comparisons among different game settings

6.1. Discussion. Table 3 illustrates the objectives of the supply chain members as well as the whole supply chain (SC) benefit in different game settings including Nash game \[36\], SNG, and SNGC.

It can be inferred from Table 3 that:

- Comparing SNG equilibrium with the Nash game equilibrium, where all the players decide simultaneously on their actions, the manufacturer can make more benefit when he acts as the leader of the channel. But both the suppliers and the retailers gain less benefit when they follow the manufacturer’s decisions. Therefore, the SNG equilibrium can be stable only if the manufacturer has the dominant power in the market and consequently is able to impose his decisions to the other supply chain members. However, this conflict of interests in the SNG setting leads to the minimum total benefit for the whole supply chain compared to the Nash and SNGC setting.

- When the retailers and the manufacturer enter a cooperation to boost sales, their total profit grows considerably (about 12.1% and 10.9% in comparison with the Nash game and SNG, respectively). Hence, the proposed cooperative setting can be preferable for the retailers and for the manufacturer even when he has the dominant power of the supply chain. This surplus profit can be then divided among the members according to their market power.

- On the other side, the suppliers never prefer to play SNG as they gain less benefit in this game setting. In fact, if the power structure allows, they choose to decide simultaneously with the other members rather than SNG setting. It is noteworthy that the SNGC setting is the best scenario for the suppliers as they can earn more benefit even though they act as the followers. The main reason is the higher demand achieved by the whole supply chain under cooperative scenario.

- The cooperation between the retailers and the manufacturer not only makes more profit for the coalition members but also leads to the maximum benefit for the whole supply chain as well as the suppliers. In fact, the proposed SNGC setting can boost the total benefit of the supply chain about 25.1% compared to SNG setting and 8.7.6% compared to the Nash game setting. However, if cooperation does not occur, then the supply chain will achieve
more benefit when players possess the same power and choose their decisions simultaneously.

6.2. **Sensitivity analysis.** Here, we carry out a sensitivity analysis on the main parameters of the proposed models to illustrate the behavior of the models in SNG and SNGC settings. In order to perform a sensitivity analysis of the SNG and SNGC equilibrium, we examined the rate of change in the supply chain members’ prices with respect to the retail price elasticity, advertising expenditure elasticity, and raw material price elasticity. The results are illustrated in Figures 3, 4, and 5, respectively.

**Figure 3.** The effect of retail price elasticity

Increasing the retail price elasticity ($\alpha$) makes the market more competitive and as seen in figure 3 it leads to a remarkable decrease in the retail, wholesale, and raw material prices throughout the supply chain echelons. Besides, as it can be seen in Figure 3 (c), and (d) the SNGC setting is less sensitive to changes in ($\alpha$). In fact,
SNGC faces less reduction in retail prices under more competitive environment. However, the sensitivity of raw materials’ prices to changes in \((\alpha)\) is almost same in both SNG and SNGC (Figure 3 (a), and (b)).

Figure 4. The effect of advertising expenditure elasticity

The growth of advertising expenditure elasticity \((\beta)\) implies that the end consumers are much sensitive to the advertisement. In such a market, as illustrated in Figure 4, retailers try to spend more on advertising plans and consequently they are able to boost the retail prices. Moreover, the retail price increment is more sensitive to \((\beta)\) in SNG compared to SNGC. The raw materials’ prices are not sensitive to advertising expenditure in SNG, but they are rarely sensitive in SNGC.

As Figure 5 shows when the raw material price elasticity \((\eta)\) grows, the suppliers are necessitated to decrease the raw material prices to keep their market share. This reduction in raw material prices decreases the final prices and boosts the demand. In addition, the rate of decrement in retail and raw materials prices is almost same for
both the retailers and the suppliers, however SNGC is less sensitive to \( \eta \) variations compared to SNG.

Eventually, it is worth mentioning that the rate of price variations in SNGC setting is always lower than the observed rate in SNG setting. As a matter of fact, the proposed cooperative structure can empower the supply chain against any alterations in the market situations.

According to Table 4, as \( \alpha \) increases, the market becomes more competitive and the supply chain achieves lower benefit due to the price reduction. On the other side, the superiority of SNGC over SNG and Nash game grows considerably when \( \alpha \) raises. In the other words, SNGC setting is strongly recommended in more competitive market environment. Any growth in \( \beta \) allows the supply chain to offer higher prices to the end customers. This price increment enhances the profit for the manufacturer and even more for the retailers. However, SNGC advantages reduces for the products whose demands are more sensitive to advertisement. When \( \eta \) rises, the suppliers and consequently the manufacturer and the retailers choose lower

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**Figure 5.** The effect of raw materials’ price elasticity
prices and gain more demand. Therefore, the whole supply chain makes further benefit. Albeit, changes in \( \eta \) do not have significant impact on SNGC preference.

7. **Concluding remarks.** In this paper, we developed Stackelberg-Nash game models for multi-echelon supply chains with horizontal and vertical competition that can determine the optimal coordinated price and advertising decisions at the retailing level, the most profitable production-inventory strategy at the manufacturing level and the equilibrium raw material prices at the upstream level. A nonlinear advertising and price sensitive demand was considered in the models which usually can provide a better fit to the data in many applications. Aligning the global profitability of the supply chain with the most proper strategies for individual members which they may choose in reaction to the horizontal and vertical competing partners’ actions is critical for the companies especially in the presence of a dominant power member. Two different power structures were discussed to compare profitability of supply chain as well as individual members and to illustrate the stability of the decisions.

Analyzing the computational results reveals some interesting insights: The proposed SNGC setting can achieve the most global profit for the supply chain as well as the individual members. The decisions chosen by individual members and the profit they gained are less sensitive to any changes in market conditions under SNGC scenario. However, Both the suppliers and the retailers gain less profit and the conflict of the members’ interests leads to the minimum total profit for the whole supply chain under SNG scenario. The more price-elasticity of consumer’s demand leads to the less profit for the supply chain and the more advertising-elasticity of demand results in higher retail prices together with the higher supply chain profit. The price competition among the suppliers can reduce the retail prices and enhance the supply chain demand. The models proposed in this paper is applicable for supply chains where manufacturers possess the dominant power such as automobile

| Parameter | SC benefit | Test problem No. |
|-----------|------------|------------------|
| \( \alpha \) | | |
| Nash | 644055 | 251381 | 163614 | 107806 | 71085 | 46138 | 28418 |
| SNG | 553476 | 213389 | 142265 | 98018 | 50194 | 37006 |
| SNGC to Nash improvement | 1% | 2.8% | 4.8% | 8.7% | 14.7% | 24% | 38.9% | 67% |
| SNGC to SNG improvement | 17.5% | 21.7% | 23.5% | 25.1% | 26.2% | 27% | 27.7% | 28.2% |
| \( \beta \) | | |
| Nash | 151669 | 177914 | 184970 | 192807 | 201560 | 211394 |
| SNG | 146552 | 151931 | 158314 | 165743 | 173021 | 186900 |
| SNGC to Nash improvement | 9.5% | 9.3% | 9% | 8.7% | 8.5% | 8.2% | 7.8% | 7.5% |
| SNGC to SNG improvement | 25.6% | 25.5% | 25.3% | 25.1% | 24.8% | 24.5% | 24.2% | 23.8% |
| \( \eta \) | | |
| Nash | 133047 | 142265 | 148411 | 154815 | 162263 | 170702 |
| SNG | 127737 | 146931 | 154815 | 162263 | 170702 |
| SNGC to Nash improvement | 9.9% | 9.5% | 9.1% | 8.7% | 8.5% | 8.2% | 8% | 7.8% |
| SNGC to SNG improvement | 25.1% | 25.1% | 25.1% | 25.1% | 25.1% | 25.1% | 25.1% |

**Table 4.** Sensitivity of the whole supply chain benefit with respect to the main parameters
industry and electronic product industry. As future studies, it would be interesting to consider the mutual interactions between multiple manufacturers or different substitutable products.

**Appendix A.** Definition 1: If \( f(x_2) \leq f(x_1) \) and \( \nabla f(x_1). (x_2 - x_1) < 0 \) holds, then \( f \) is strongly pseudo convex.

Definition 2: If \( f \) is strongly pseudo convex, then \(-f\) is strongly pseudo concave.

According to definitions 1 and 2, first, it is assumed that \( \Pi_M(\psi_1) \leq \Pi_M(\psi_2) \).

Therefore, we have:

\[-D_2 (\psi_2 - Cm - \frac{Thm}{2}) \leq -D_1 (\psi_1 - Cm - \frac{Thm}{2}) \quad (A.1)\]

By replacing the demand variable \( D \) from Eq (16), and Eq. (17), the inequality (A.1) is changed as follows:

\[-f \left( \frac{\alpha (\psi_2 + \frac{Thm}{2})}{\alpha - \beta - 1} \right)^{-\alpha} * \left( \frac{\beta (\psi_2 + \frac{Thm}{2})}{\alpha - \beta - 1} \right)^{\beta} (\psi_2 - Cm - \frac{Thm}{2}) \leq -f \left( \frac{\alpha (\psi_1 + \frac{Thm}{2})}{\alpha - \beta - 1} \right)^{-\alpha} * \left( \frac{\beta (\psi_1 + \frac{Thm}{2})}{\alpha - \beta - 1} \right)^{\beta} (\psi_1 - Cm - \frac{Thm}{2}) \quad (A.2)\]

Simplifying the inequality (A.2) leads to:

\[ (\psi_2 + \frac{Thm}{2})^{-\alpha + \beta} (\psi_2 - Cm - \frac{Thm}{2}) \geq (\psi_1 + \frac{Thm}{2})^{-\alpha + \beta} (\psi_1 - Cm - \frac{Thm}{2}) \quad (A.3)\]

Now, calculating the first derivative of the manufacturer’s objective with respect to \( \psi_1 \) results in:

\[ \frac{\partial \Pi_M(\psi_1)}{\partial \psi_1} = -f \left( \frac{\alpha}{\alpha - \beta - 1} \right)^{-\alpha} * \left( \frac{\beta}{\alpha - \beta - 1} \right)^{\beta} (\psi_1 + \frac{Thm}{2})^{-\alpha + \beta - 1} \]

\[ ((-\alpha + \beta + 1) \psi_1 - (-\alpha + \beta) Cm - (-\alpha + \beta - 1) hm) \quad (A.4)\]

Then, the two following different states can be considered to determine the sign of \( \frac{\partial \Pi_M(\psi_1)}{\partial \psi_1} \):

\[ \psi_1 \geq \frac{(-\alpha + \beta) cm + (-\alpha + \beta - 1) hm}{-\alpha + \beta + 1} \rightarrow \frac{\partial \Pi_M(\psi_1)}{\partial \psi_1} > 0 \quad (A.5)\]

\[ \psi_1 \leq \frac{(-\alpha + \beta) cm + (-\alpha + \beta - 1) hm}{-\alpha + \beta + 1} \rightarrow \frac{\partial \Pi_M(\psi_1)}{\partial \psi_1} < 0 \quad (A.6)\]

First, let Eq. (A.5) hold. In order to meet Eq. (A.2), it is necessary to put \( \psi_2 \) inside the boundary below:

\[ \psi_1 \geq \psi_2 \geq \frac{(-\alpha + \beta) cm + (-\alpha + \beta - 1) hm}{-\alpha + \beta + 1} \quad (A.7)\]

Thereby, it is concluded that:

\[ \psi_1 \geq \psi_2 \rightarrow \frac{\partial \Pi_M(\psi_1)}{\partial \psi_1} (\psi_2 - \psi_1) < 0. \quad (A.8)\]

Second, let Eq. (A.6) hold. Since Eq. (A.2) is an ascending function, \( \psi_2 \) cannot be less than \( \psi_1 \). Then, it is concluded that:

\[ \psi_1 \leq \psi_2 \rightarrow \frac{\partial \Pi_M(\psi_1)}{\partial \psi_1} (\psi_2 - \psi_1) < 0. \quad (A.9)\]

Therefore, the inequality \( \frac{\partial \Pi_M(\psi_1)}{\partial \psi_1} (\psi_2 - \psi_1) < 0 \) is true for both possible states of \( \psi_1 \) and consequently the manufacturer’s objective function in the upper level is strongly pseudo concave with respect to the wholesale price variable. \( \square \)
Appendix B. According to definitions 1 and 2 in appendix A, we first consider that $\Pi_M(T_2) \leq \Pi_M(T_1)$, thus:

$$- D_2 \left( \psi - Cm - \frac{T_2hm}{2} \right) \leq -D_1 \left( \psi - Cm - \frac{T_1hm}{2} \right). \quad (B.1)$$

By replacing $D$ in Eq. (B.1), we have:

$$\left( \psi + \frac{T_2hm}{2} \right)^{-\alpha + \beta} \left( \psi - Cm - \frac{T_2hm}{2} \right) \geq \left( \psi + \frac{T_1hm}{2} \right)^{-\alpha + \beta} \left( \psi - Cm - \frac{T_1hm}{2} \right). \quad (B.2)$$

Calculating the derivative of the manufacturer’s objective function with respect to $T_1$ leads to:

$$\frac{\partial \Pi_M(T_1)}{\partial T_1} = \frac{fh}{2} \left( \frac{\alpha}{\alpha - \beta - 1} \right) - \alpha \left( \frac{\beta}{\alpha - \beta - 1} \right) + \left( \psi + \frac{T_1hm}{2} \right)^{-\alpha + \beta - 1} \left( \alpha - \beta \right) \left( \psi - Cm - \frac{T_1hm}{2} \right) + \psi + \frac{T_1hm}{2}. \quad (B.3)$$

According to Eq. (B.2), the sign of the derivative can be negative or positive regarding to the two states below:

$$T_1 \geq \frac{2 \left[ (\alpha - \beta + 1) \psi - (\alpha - \beta) Cm \right]}{(\alpha - \beta - 1) hm} \rightarrow \frac{\partial \Pi_M(T_1)}{\partial T_1} < 0 \quad (B.4)$$

$$T_1 \leq \frac{2 \left[ (\alpha - \beta + 1) \psi - (\alpha - \beta) Cm \right]}{(\alpha - \beta - 1) hm} \rightarrow \frac{\partial \Pi_M(T_1)}{\partial T_1} > 0 \quad (B.5)$$

Firstly, let Eq. (B.4) hold, then due to the fact that Eq. (B.3) is an ascending function, it is inferred that:

$$T_2 \geq T_1 \rightarrow \frac{\partial \Pi_M(T_2)}{\partial T_1} (T_2 - T_1) < 0 \quad (B.6)$$

Secondly, let Eq. (B.5) hold, then due to the fact that Eq. (B.2) is a descending function, we have:

$$T_2 \leq T_1 \rightarrow \frac{\partial \Pi_M(T_2)}{\partial T_1} (T_2 - T_1) < 0 \quad (B.7)$$

Therefore, the inequality $\frac{\partial \Pi_M(T_1)}{\partial T_1} (T_2 - T_1) < 0$ has been proved for both possible states of $T_1$ and the manufacturer’s objective function in the upper level is strongly pseudo concave with respect to the shared production cycle variable.

Appendix C. Since all the equality constraints in the SNG single level model are linear, they are convex. Here, it is just necessary to investigate the convexity of the nonlinear inequality constraints including:

$$T \sum_{i=1}^{n} \sum_{r=1}^{R} Cm_{ir} D_{ir} - B_m \leq 0 \quad (C.1)$$

$$u_{ji} \left( \sum_{r=1}^{R} D_{ir} \right) - Q_j \leq 0 \quad \text{for } j = 1, 2, \ldots, J \quad (C.2)$$

Since both the nonlinear constraints are less than or equal inequalities and the only nonlinear term of them is the demand function, it is sufficient to prove the convexity of the demand function regarding to the manufacturer’s decision variables. In other words, it is necessary to show that the Hessian matrix of $D_{ir}$ is positive semi definite. By placing $p_{ir}^*$ and $a_{ir}^*$ in the demand function, we have:

$$D_{ir} = f_{ir} \left( \frac{\alpha_{ir}}{\alpha_{ir} - \beta_{ir} - 1} \right)^{-\alpha_{ir}} \left( \frac{\beta_{ir}}{\alpha_{ir} - \beta_{ir} - 1} \right)^{\beta_{ir}} \left( \psi_{ir} + \frac{T_{hir}}{2} \right)^{-\alpha_{ir} + \beta_{ir}} \quad (C.3)$$

Then, the Gradient of $D_{ir}$ can be obtained as follows:

$$\nabla = \left[ \frac{\partial D_{ir}}{\partial \beta} \right] = \left[ \frac{2 \left( -\alpha + \beta \right) \left( \frac{\alpha}{\alpha - \beta - 1} \right)^{-\alpha} \left( \frac{\beta}{\alpha - \beta - 1} \right)^{\beta} \left( \psi + \frac{T_{hir}}{2} \right)^{-\alpha + \beta - 1}}{\left( -\alpha + \beta \right) \left( \frac{\alpha}{\alpha - \beta - 1} \right)^{-\alpha} \left( \frac{\beta}{\alpha - \beta - 1} \right)^{\beta} \left( \psi + \frac{T_{hir}}{2} \right)^{-\alpha + \beta - 1}} \right] \quad (C.4)$$
For simplicity, we set $A = f(-\alpha + \beta)\left(\frac{-\alpha}{\alpha-\beta-1}\right)^{-\alpha} \left(\frac{\beta}{\alpha-\beta-1}\right)^{\beta}$ where $A$ is negative, then the Hessian can be calculated as follows:

$$
H = \begin{bmatrix}
\frac{\partial^2 D}{\partial p^2} & \frac{\partial^2 D}{\partial p \partial T} \\
\frac{\partial^2 D}{\partial p \partial T} & \frac{\partial^2 D}{\partial T^2}
\end{bmatrix} = 
\begin{bmatrix}
A(-\alpha + \beta - 1)\left(\psi + \frac{T_h}{2}\right)^{-\alpha+\beta-2} & \frac{Ah}{4}(-\alpha + \beta - 1)\left(\psi + \frac{T_h}{2}\right)^{-\alpha+\beta-2} \\
\frac{Ah}{4}(-\alpha + \beta - 1)\left(\psi + \frac{T_h}{2}\right)^{-\alpha+\beta-2} & \frac{A^2}{4}(-\alpha + \beta - 1)\left(\psi + \frac{T_h}{2}\right)^{-\alpha+\beta-2}
\end{bmatrix}
$$

(C.5)

It can be easily observed that $A(-\alpha + \beta - 1)\left(\psi + \frac{T_h}{2}\right)^{-\alpha+\beta-2} < 0$ and $\frac{Ah}{4}(-\alpha + \beta - 1)\left(\psi + \frac{T_h}{2}\right)^{-\alpha+\beta-2} < 0$, so the main diagonal elements are both negative. In addition, the determinant of $H$ is zero. Therefore, it can be concluded that the Hessian matrix is positive semi definite and the proof is complete. □

**Appendix D.** Without loss of generality, suppose that all the retailers have the same production and holding costs. In fact, these costs are presumed to be equally distributed among the retailers. Regarding to the definitions 1 and 2 in appendix A, we suppose that $\Pi_{MR}(p_2) \leq \Pi_{MR}(p_1)$, thus we have:

$$
-D_2\left(p_2 - ad - \frac{T_h}{2} - Cm - \frac{Thm}{2}\right) \leq -D_1\left(p_1 - ad - \frac{T_h}{2} - Cm - \frac{Thm}{2}\right) 
$$

(D.1)

Replacing the demand function $D$ in Eq. (D.1) results in

$$
p_2^{\alpha}\left(p_2 - ad - \frac{T_h}{2} - Cm - \frac{Thm}{2}\right) \geq p_1^{\alpha}\left(p_1 - ad - \frac{T_h}{2} - Cm - \frac{Thm}{2}\right) 
$$

(D.2)

Differentiating the objective function respecting to $p_1$ leads to

$$
\frac{\partial \Pi_{MR}(p_1)}{\partial p_1} = -f_1^{p_1-1}a^d\delta^*\left((1-\alpha)p_1 + \alpha\left(ad + \frac{T_h}{2} + Cm + \frac{Thm}{2}\right)\right) 
$$

(D.3)

The sign of the derivative can be positive or negative regarding to two possible states of $p_1$ as follows:

$$
p_1 \geq \frac{\alpha\left(ad + \frac{T_h}{2} + Cm + \frac{Thm}{2}\right)}{\alpha - 1} \rightarrow \frac{\partial \Pi_{MR}(p_1)}{\partial p_1} > 0 
$$

(p.4)

$$
p_1 \leq \frac{\alpha\left(ad + \frac{T_h}{2} + Cm + \frac{Thm}{2}\right)}{\alpha - 1} \rightarrow \frac{\partial \Pi_{MR}(p_1)}{\partial p_1} < 0 
$$

(p.5)

Firstly, let Eq. (D.4) hold, then to meet Eq. (D.2), we need that $p_2$ satisfies the following conditions:

$$
\frac{\alpha\left(ad + \frac{T_h}{2} + Cm + \frac{Thm}{2}\right)}{\alpha - 1} \leq p_2 \leq p_1 
$$

(D.6)

which causes to

$$
p_2 \leq p_1 \rightarrow \frac{\partial \Pi_{MR}(p_1)}{\partial p_1} (p_2 - p_1) < 0 
$$

(D.7)

Secondly, let Eq. (D.5) hold, to meet the conditions of Eq. (D.2) it is necessary that

$$
p_1 \leq p_2 \leq \frac{\alpha\left(ad + \frac{T_h}{2} + Cm + \frac{Thm}{2}\right)}{\alpha - 1} 
$$

(D.8)

which results in

$$
p_2 \geq p_1 \rightarrow \frac{\partial \Pi_{MR}(p_1)}{\partial p_1} (p_2 - p_1) < 0 
$$

(D.9)

Hence, we prove that $\frac{\partial \Pi_{MR}(p_1)}{\partial p_1} (p_2 - p_1)$ is always negative and the proof is complete. □
Appendix E. Regarding to the definitions 1 and 2 in appendix A, assume that $\Pi_{MR}(a_2) \leq \Pi_{MR}(a_1)$, it means

$$-D_2 \left(p - ad_2 - \frac{Th}{2} - Cm - \frac{Thm}{2}\right) \leq -D_1 \left(p - ad_1 - \frac{Th}{2} - Cm - \frac{Thm}{2}\right) \quad (E.1)$$

Replacing $D$ with the demand function results in

$$ad_2^\beta \left(p - ad_2 - \frac{Th}{2} - Cm - \frac{Thm}{2}\right) \geq ad_1^\beta \left(p - ad_1 - \frac{Th}{2} - Cm - \frac{Thm}{2}\right) \quad (E.2)$$

Differentiating the objective with respect to the advertising expenditure leads to

$$\frac{\partial \Pi_{MR}(ad_1)}{\partial ad_1} = fp - \alpha^* ad_2 - \beta^* \left(1 + \beta\right) ad - \beta \left(p - Th - Cm - Thm\right) \quad (E.3)$$

According to Eq. (E.3), the sign of the derivative can be defined based on the two possible cases below:

$$ad_1 \geq \frac{\beta \left(p - Th - Cm - Thm\right)}{\beta + 1} \rightarrow \frac{\partial \Pi_{MR}(ad_1)}{\partial ad_1} > 0 \quad (E.4)$$

$$ad_1 \leq \frac{\beta \left(p - Th - Cm - Thm\right)}{\beta + 1} \rightarrow \frac{\partial \Pi_{MR}(ad_1)}{\partial ad_1} < 0 \quad (E.5)$$

Firstly, we consider Eq. (E.4) holds, then to meet the conditions of Eq. (E.2), we need to put $a_2$ inside the boundary below:

$$\frac{\beta \left(p - Th - Cm - Thm\right)}{\beta + 1} \leq ad_2 \leq ad_1 \quad (E.6)$$

Therefore, we have

$$ad_2 \leq ad_1 \rightarrow \frac{\partial \Pi_{MR}(ad_1)}{\partial ad_1} \left(ad_2 - ad_1\right) < 0 \quad (E.7)$$

Secondly, suppose that Eq. (E.5) holds, and then to meet the conditions in Eq. (E.2), it is obtained that

$$ad_1 \leq ad_2 \leq \frac{\beta \left(p - Th - Cm - Thm\right)}{\beta + 1} \quad (E.8)$$

Thus, we have

$$ad_2 \geq ad_1 \rightarrow \frac{\partial \Pi_{MR}(ad_1)}{\partial ad_1} \left(ad_2 - ad_1\right) < 0 \quad (E.9)$$

Hence, the negativity of $\frac{\partial \Pi_{MR}(p_2)}{\partial p_1} (p_2 - p_1)$ is proved and the proof is complete. □

Appendix F. The second derivative of the objective function with respect to $T$, can be written as

$$\frac{\partial^2 \Pi_{MR}(T)}{\partial T^2} = 2\sum_{i=1}^n A_s_i + 2\sum_{r=1}^R A_{tr} > 0 \quad (F.1)$$

Since the second derivative is positive, the objective function is concave with regard to $T$ and the proof is complete. □
Appendix G. Since all the linear constraints are convex, we need just to examine the convexity of the nonlinear constraints in the model. Here, without loss of generality, we can replace the equality (3) with the below inequality

\[ u_{ji} \left( \sum_{r=1}^{R} D_{ir} \right) - Q_{j} \leq 0 \quad \text{for} \ j = 1, 2, \ldots, J \quad (G.1) \]

Insomuch as the cost of purchasing raw materials appeared with a negative sign in the objective function, the inequality (G.1) always holds in the form of equality. Since all the nonlinear constraints are less than or equal inequalities, it is sufficient to prove the convexity of the demand function (Eq. (4)) regarding to decision variables \( p \) and \( a \). In other words, it is necessary to show that the Hessian matrix is positive semi definite. The Gradient can be written as follows:

\[ \nabla = \left[ \frac{\partial \mathbf{u}}{\partial p} \right] = \left[ \begin{array}{c} f(-\alpha) (p)^{\alpha-1} \ast (ad)^{\beta} \\ f(\beta) (p)^{-\alpha} \ast (ad)^{\beta-1} \end{array} \right] \quad (G.2) \]

And the Hessian matrix can be calculated as:

\[ \mathbf{H} = \left[ \frac{\partial^2 \mathbf{u}}{\partial p^2} \right] = \left[ \begin{array}{cc} f(-\alpha) (-\alpha - 1) * p^{-\alpha - 2}ad^\beta & f(-\alpha) (\beta) \ast p^{-\alpha - 1}ad^{\beta - 1} \\ f(-\alpha) (\beta) \ast p^{-\alpha - 1}ad^{\beta - 1} & f(\beta) (\beta - 1) (p)^{-\alpha} \ast (ad)^{\beta - 2} \end{array} \right] \quad (G.3) \]

Now if \([ p \ a ] [ \begin{array}{cc} \frac{\partial^2 D}{\partial p \partial a} & \frac{\partial^2 D}{\partial a \partial d} \\ \frac{\partial^2 D}{\partial a \partial d} & \frac{\partial^2 D}{\partial a^2} \end{array} ] [ \begin{array}{c} p \\ ad \end{array} ] > 0 \) holds, the Hessian matrix will be positive definite.

\[ p^{-\alpha}ad^\beta (\alpha^2 + \alpha - 2\alpha\beta + \beta^2 - \beta) = (\alpha - \beta) (\alpha - \beta + 1) p^{-\alpha}ad^\beta \quad (G.4) \]

Regarding to the problem assumptions, we have \( (\alpha > 1 + \beta) \). Thus Eq. (G.4) is always positive and the Hessian matrix is positive semi definite. Therefore, all the nonlinear constraints in the solution space are convex and the proof is complete. □

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E-mail address: naimi@irandoc.ac.ir
E-mail address: skch@modares.ac.ir
E-mail address: m_mozafari@iauec.ac.ir