Generalized two-point tree-level amplitude $j f \to j' f'$ in a magnetized medium (extended version)

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(Dated: September 2, 2014)

The tree-level two-point amplitudes for the transitions $j f \to j' f'$, where $f$ is a fermion and $j$ is a generalized current, in a constant uniform magnetic field of an arbitrary strength and in charged fermion plasma, for the $j f$ interaction vertices of the scalar, pseudoscalar, vector and axial-vector types have been investigated. The particular cases of a very strong magnetic field, and of the coherent scattering off the real fermions without change of their states (the “forward” scattering) have been analysed.

PACS numbers: 12.20.Ds, 14.60.Cd, 97.10.Ld, 94.30.-d

I. INTRODUCTION

Nowadays, there exists rather keen interest to astrophysical objects with the scale of the magnetic field strength near the critical value of $B_c = m^2 / e \approx 4.41 \times 10^{13} \text{G}$ \footnote{We use natural units $c = \hbar = k_B = 1$, $m_f$ is the fermion mass, and $e_f$ is the fermion charge.}. This group of objects includes the radio pulsars and the so-called magnetars, which are the neutron stars featuring the magnetic field strengths from $10^{12}$ G (radio pulsars) to $4 \times 10^{14}$ G (magnetars) \footnote{In a pure constant uniform magnetic field}. The spectra analysis of these objects also provides an evidence for the presence of electron-positron plasma in the radio pulsars and magnetars environment, with the minimum magnetospheric plasma density being of the order of the Goldreich-Julian density $\mathbb{E}$:

$$n_{GJ} \approx 3 \cdot 10^{13} \text{cm}^{-3} \left( \frac{B}{100 B_c} \right) \left( \frac{10 \text{s}}{P} \right).$$

It is well-known that strong magnetic field and/or plasma could have an essential influence on various quantum processes, because the external active medium catalyses the processes, by changing their kinematics and inducing new interactions. Therefore, the effects of magnetized plasma on microscopic physics should be incorporated in the magnetosphere models of strongly magnetized neutron stars. In the present paper we consider the two-point processes, because such reactions can have possible resonant behavior, and therefore they are very interesting for astrophysical applications \footnote{We use natural units $c = \hbar = k_B = 1$, $m_f$ is the fermion mass, and $e_f$ is the fermion charge.}.

The investigation of the two-point processes in an external active medium (electromagnetic field and/or plasma) has a rather long history. The most general expression for a two-vertex loop amplitude of the form $j \to j f \to j'$ in a pure constant uniform magnetic field and in a crossed field was obtained previously in Ref. \footnote{We use natural units $c = \hbar = k_B = 1$, $m_f$ is the fermion mass, and $e_f$ is the fermion charge.}, where all possible combinations of scalar, pseudoscalar, vector, and pseudovector interactions of the generalized currents $j$ and $j'$ with fermions were considered.

The typical example of a tree-level process with two vector vertices in the presence of magnetized plasma is the Compton scattering as a possible channel of the radiation spectra formation. This process was studied in a number of papers, see e.g. \footnote{We use natural units $c = \hbar = k_B = 1$, $m_f$ is the fermion mass, and $e_f$ is the fermion charge.}, but the results were presented there in the form without taking account of the photon dispersion properties. In the recent paper \footnote{We use natural units $c = \hbar = k_B = 1$, $m_f$ is the fermion mass, and $e_f$ is the fermion charge.} this neglect was corrected. The expression for the Compton scattering amplitude, with the initial and final electrons being on the lowest Landau level was presented in Ref. \footnote{We use natural units $c = \hbar = k_B = 1$, $m_f$ is the fermion mass, and $e_f$ is the fermion charge.} in the explicit Lorentzian level and for generalized current types. All the amplitudes are presented in the explicit Lorentzian invariant form. The other example of the Compton like process with the vector and axial-vector vertices, the photon transition into the neutrino pair, $\gamma \to \nu \bar{\nu}$, in the presence of magnetized plasma, was studied in Ref. \footnote{We use natural units $c = \hbar = k_B = 1$, $m_f$ is the fermion mass, and $e_f$ is the fermion charge.}. However, the results in that paper were presented in rather cumbersome form with an implicit covariance. Those results would be poorly applicable for an analysis of the other photon-fermion scattering processes with the production of exotic particles, such as axion, neutralino, etc.

Thus, it is interesting to consider the tree-level two-point amplitude for the transition of the type $j f \to j' f'$ in a constant uniform magnetic field and charged fermion plasma, for different combinations of the vertices that were used in the paper \footnote{We use natural units $c = \hbar = k_B = 1$, $m_f$ is the fermion mass, and $e_f$ is the fermion charge.}. Particularly, we generalize the results, obtained in Ref. \footnote{We use natural units $c = \hbar = k_B = 1$, $m_f$ is the fermion mass, and $e_f$ is the fermion charge.} to the case of magnetized plasma, since such a situation looks the most realistic for astrophysical objects. Such a generalization was performed in part in Ref. \footnote{We use natural units $c = \hbar = k_B = 1$, $m_f$ is the fermion mass, and $e_f$ is the fermion charge.} for the case of the photon polarization operator in a magnetized electron-positron plasma. The paper is organized as follows. In Sec. \footnote{We use natural units $c = \hbar = k_B = 1$, $m_f$ is the fermion mass, and $e_f$ is the fermion charge.} we calculate the scattering amplitudes for different spin states of the initial and final fermions and for generalized vertices of the scalar, pseudoscalar, vector or axial vector types. All the amplitudes are presented in the explicit Lorentzian and gauge invariant forms. In Sec. \footnote{We use natural units $c = \hbar = k_B = 1$, $m_f$ is the fermion mass, and $e_f$ is the fermion charge.} we consider the particular case, when all the fermions occupy the ground Landau level (the strong field limit). A coherent scattering of neutral particles off the real fermions
without change of their states ("forward" scattering) is analysed in Sec. XIV. Final comments and discussion of the obtained results and possible astrophysical applications are given in Sec. XV.

II. THE SET OF EXPRESSIONS FOR THE AMPLITUDES

The generalized amplitude of the transition \( jf \rightarrow j'f' \) will be analyzed by using the effective Lagrangian for the interaction of the current \( j \) with fermions in the form

\[
\mathcal{L}(x) = \sum_k g_k \bar{\psi}(f) \Gamma_k \psi(x) j_k(x),
\]

where the generalized index \( k = S, P, V, A \) numbers the matrices \( \Gamma_k, \Gamma_S = 1, \Gamma_P = \gamma_5, \Gamma_V = \gamma_\alpha, \Gamma_A = \gamma_\alpha \gamma_5 \); \( j_k(x) \) are the generalized currents \( (j_S, j_P, j_V, j_A) \) or the photon polarization vectors, \( g_k \) are the coupling constants, and \( \psi(f) \) are the fermion wave functions. The \( \gamma_5 \) matrix is defined as \( \gamma_5 = i \gamma_0 \gamma_1 \gamma_2 \gamma_3 \).

The S-matrix element in the tree approximation is described by the Feynman diagrams shown in Fig. I and has the form

\[
S^{s^*}_{j' k} = -g_k g_{k'} \int d^4x d^4y j_k(x) j_{k'}(Y) \times \left[ \bar{\psi}^{s^*}_{p, k'}(Y) \Gamma_{k'} \bar{s}(Y, X) \Gamma_k \psi_{p, e}(X) \right].
\]

Here, \( p^\mu = (E_l, p) \) and \( p'^\mu = (E'_l, p') \) are the four-momenta of the initial and final fermions correspondingly, \( \psi_{p, n}(X) \) are the fermion wave functions in the presence of external magnetic field, \( X^\mu = (X_0, X_1, X_2, X_3) \).

There exist several descriptions of a procedure of obtaining the fermion wave functions in the presence of external magnetic field by solving the Dirac equation, see e.g. \([21, 27]\) and also \([8, 3]\). In the most cases, the solutions are presented in the form with the upper two components of the bispinor corresponding to the fermion states with the spin projections \( 1/2 \) and \(-1/2 \) on the magnetic field direction. In this approach, we use a representation of the fermion wave functions as the eigenstates of the covariant operator \( \hat{\mu}_z = m_f \Sigma_z - i \gamma_0 \gamma_5 [\Sigma \times \mathbf{P}]_z \). Here, \( \hat{\mathbf{P}} = -i \nabla - e_f \mathbf{A} \) is the generalized momentum operator. We take the frame where the field is directed along the \( z \) axis, and the Landau gauge where the four-potential is: \( A^\mu = (0, 0, x B, 0) \).

Our choice of the Dirac equation solutions as the eigenfunctions of the operator \( \hat{\mu}_z \) is caused by the following arguments. Calculations of the process widths with two or more vertices in an external magnetic field by the standard method, including the squaring the amplitude with all the Feynman diagrams and with summation or averaging over the fermion polarization states, contain significant computational difficulties. In this case, it is convenient to calculate partial contributions to the amplitude from the channels with different fermion polarization states and for each diagram separately, by direct multiplication of the bispinors and the Dirac matrices. The result, up to a total for both diagrams non-invariant phase will have an explicit Lorentz invariant structure. On the contrary, the amplitudes obtained with using the solutions for a fixed direction of the spin, do not have Lorentz invariant structure. Only the amplitude squared, summed over the fermion polarization states, is manifestly Lorentz-invariant.

The fermion wave functions having the form

\[
\Psi_{p, n}^s(X) = \frac{e^{-i(E_n X_0 - p_n X_2 - p_n X_3)}}{\sqrt{4E_n M_n (E_n + M_n)(M_n + m_f)L_y L_z}} U^s_n(\xi),
\]

where

\[
E_n = \sqrt{M_n^2 + p_z^2}, \quad M_n = \sqrt{m_f^2 + 2\beta n},
\]

are the solutions of the equation

\[
\hat{\mu}_z \Psi_{p, n}^s(X) = s M_n \Psi_{p, n}^s(X), \quad s = \pm 1.
\]

It is convenient to present the bispinors \( U^s_n(\xi) \) in the form of decomposition over the solutions for negative and positive fermion charge, \( U^s_{n, q}(\xi) \):

\[
U^s_n(\xi) = \frac{1 - \eta}{2} U^s_{n, -}(\xi) + \frac{1 + \eta}{2} U^s_{n, +}(\xi),
\]

where

\[
U^s_{n, -}(\xi) = \begin{pmatrix}
-i \sqrt{2\beta n} p_z V_{n-1}(\xi) \\
(E_n + M_n)(M_n + m_f) V_n(\xi)
\end{pmatrix},
\]

\[
U^s_{n, +}(\xi) = \begin{pmatrix}
-i \sqrt{2\beta n} p_z V_{n-1}(\xi) \\
p_z(M_n + m_f) V_n(\xi)
\end{pmatrix},
\]
\[ U_{n,+}^{-}(\xi) = \begin{pmatrix} i\sqrt{2\beta n} p_x V_n(\xi) \\ i\sqrt{2\beta n}(E_n + M_n) V_n(\xi) \\ -p_z (M_n + m_f) V_n^{-1}(\xi) \end{pmatrix}, \tag{10} \]
\[ U_{n,+}^{+}(\xi) = \begin{pmatrix} (E_n + M_n)(M_n + m_f) V_n(\xi) \\ i\sqrt{2\beta n}(E_n + M_n) V_n(\xi) \\ p_z (M_n + m_f) V_n^{-1}(\xi) \end{pmatrix}, \tag{11} \]

\[ V_n(\xi) = \frac{\beta^{1/4} e^{-\xi^2/2}}{\sqrt{2^n n! \sqrt{\pi}}} H_n(\xi), \tag{12} \]

\[ \xi = \sqrt{\beta} \left( X_1 - \eta \frac{p_y}{\beta} \right). \tag{13} \]

The currents \( j_k \) in Eq. 3 can be expressed through the amplitudes in the momentum space:

\[ j_k(X) = e^{-i(qX)} \frac{\sqrt{2\beta}}{2q_0 V} j_k(q). \tag{14} \]

We use the fermion propagator in the form of the sum over the Landau levels [3, 28]:

\[ S(X, X') = \sum_{n=0}^{\infty} S_n(X, X'), \tag{15} \]

\[ S_n(X, X') = \frac{i}{2^n n!} \sqrt{\frac{\beta}{\pi}} \exp \left( -\beta X_1^2 + X_1'^2 \right) \times \int \frac{dp_0 dp_y dp_z}{(2\pi)^3} \frac{e^{-i(p(x-x'))}}{p_0^2 - m^2 - 2\beta n + i\varepsilon} \times \exp \left\{ -\frac{p_y^2}{\beta} - p_y [X_1 + X_1' - i(X_2 - X_2')] \right\} \times \left\{ \left[ (p_0^2 + m^2) \right] [H_n(\xi) H_n(\xi')] + \Pi_+ H_n(-1)(\xi) H_n(-1)(\xi') + i2n \sqrt{\beta} \gamma^1 [H_n(-1)(\xi) H_n(-1)(\xi')] \right\}, \tag{16} \]

where \( \xi \) and \( \xi' \) are defined similarly to Eq. 13.

Hereafter we use the following notations: four-vectors with the indices \( \perp \) and \( \parallel \) belong to the Euclidean \( \{1, 2\} \) subspace and the Minkowski \( \{0, 3\} \) subspace correspondingly. Then for arbitrary 4-vectors \( A_\mu, B_\mu \) one has

\[ A_\mu^a = (0, A_1, A_2, 0), \quad A_\mu^a = (A_0, 0, 0, A_3), \]
\[ (AB)_\perp = (AAB)_1 = A_1 B_1 + A_2 B_2, \]
\[ (AB)_\parallel = (A\Lambda B)_0 = A_0 B_0 - A_3 B_3, \]

where the matrices \( \Lambda_{\mu\nu} = (\varphi \varphi)_{\mu\nu}, \quad \Lambda_{\mu\nu} = (\varphi \varphi)_{\mu\nu} \) are constructed with the dimensionless tensor of the external magnetic field, \( \varphi_{\mu\nu} = F_{\mu\nu}/B \), and the dual tensor, \( \varphi_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \varphi_{\rho\sigma} \). The matrices \( \Lambda_{\mu\nu} \) and \( \Lambda_{\mu\nu} \) are connected by the relation \( \Lambda_{\mu\nu} - \Lambda_{\mu\nu} = g_{\mu\nu} = \text{diag}(1, -1, -1, -1) \), and play the role of the metric tensors in the perpendicular (\( \perp \)) and the parallel (\( \parallel \)) subspaces respectively.

After integration in Eq. 3 over \( d^4X \) and \( d^4Y \) we obtain

\[ S_{k,k}^{\epsilon'} = \frac{i(2\pi)^3\delta^{(3)}(P - p' - q')}{\sqrt{2q_0 V q_0 2E_{\perp}L_{\perp} L_{\perp} L_{\perp} L_{\perp}} \sqrt{2\beta}} M_{k,k}^{\epsilon' \epsilon} \tag{17} \]

where \( \delta^{(3)}(P - p' - q') = \delta(p_0 - E_{\perp}' - q_0)\delta(p_y - p_y' - q_y)\delta(p_z - p_z' - q_z) \), \( P_\alpha = (p + q)_\alpha, \alpha = 0, 2, 3, \) and the partial amplitudes \( M_{k,k}^{\epsilon' \epsilon} \) can be presented in the following form:

\[ M_{k,k}^{\epsilon' \epsilon} = \frac{\exp \left\{ -i\theta \right\} \times \left\{ \exp \left[ \frac{i(q\varphi'^{\prime} - i(q\varphi^\prime))}{2\beta} \right] \right\} \times \sum_{n=0}^{\infty} \left( \frac{(q\Lambda q') - i(q\varphi'^{\prime})}{\sqrt{q_0'^2 q_1'^2}} \right)^n \frac{R_{k,k}^{\epsilon' \epsilon}}{P_{\perp} - m^2 - 2\beta n} \right\} \times \left\{ \exp \left[ -\frac{i(q\varphi'^{\prime} - i(q\varphi^\prime))}{2\beta} \right] \right\} \times \sum_{n=0}^{\infty} \left( \frac{R_{k,k}^{\epsilon' \epsilon}}{P_{\perp} - m^2 - 2\beta n} \right), \tag{18} \]

where \( \theta = (q_{\ell} - q_{\ell}')(p_y + p_y')/(2\beta) \) is the general phase for both diagrams in Fig. 1.

The main part of the problem is to calculate the values \( R_{k,k}^{\epsilon' \epsilon} \) which are expressed via the following Lorentz covariants in the \{0, 3\}-subspace

\[ \mathcal{K}_{1a} = \sqrt{\frac{2}{(p\Lambda p') + M_\ell M_{\ell'}}} \times \left\{ M_\ell (\Lambda p')_a + M_{\ell'} (\Lambda p')_a \right\}, \tag{19} \]
\[ \mathcal{K}_{2a} = \sqrt{\frac{2}{(p\Lambda p') + M_\ell M_{\ell'}}} \times \left\{ M_\ell (\varphi p')_a + M_{\ell'} (\varphi p')_a \right\}, \tag{20} \]
The following integrals appear in the calculations:

\[
\mathcal{K}_3 = \sqrt{2 \left( (p \bar{\alpha} p') + M_\ell M_\nu \right)},
\]

\[
\mathcal{K}_4 = -\sqrt{\frac{2}{(p \bar{\alpha} p') + M_\ell M_\nu}} (p \bar{\alpha} p').
\]

The results for \( R_{k,k}^{j,j'} \) are presented below. Hereafter we use the following definitions: \( P_\alpha = (p + q)_\alpha, P_\alpha' = (p - q')_\alpha \), \( I_{n,\ell} \equiv I_{n,\ell} (q_\ell^2/(2\beta)) \) and \( I_{n,\ell}^{j,j'} \equiv I_{n,\ell} (q_\ell^2/(2\beta)) \).

For definiteness, we further consider the fermion with a negative charge, \( \eta = -1 \).

1. In the case when \( j \) and \( j' \) are scalar currents \( (k, k' = S) \) the calculation yields

\[
R_{SS}^{+} = g_s g_s' j_s j_s' \left\{ 2 \sqrt{\ell \ell'} \left[ (K_1 P) - m_f K_3 \right] I_{n,\ell} I_{n,\ell} + (M_\ell + m_f)(M_\nu + m_f) \left[ (K_1 P) + m_f K_3 \right] \right\};
\]

\[
\times I_{n-1,\ell-1} I_{n-1,\ell-1} - 2 \sqrt{\beta n K_4} \left[ \sqrt{\ell} (M_\ell + m_f) I_{n-1,\ell} I_{n,\ell} + \sqrt{\ell'} (M_\nu + m_f) I_{n,\ell} I_{n,\ell-1} \right] \right\};
\]

\[
R_{SS}^{-} = -i g_s g_s' j_s j_s' \left\{ 2 \sqrt{\beta \ell} (M_\ell + m_f) \left[ (K_2 P) + m_f K_4 \right] I_{n,\ell} I_{n,\ell} - \sqrt{2 \beta \ell'} (M_\nu + m_f) \left[ (K_2 P) - m_f K_4 \right] \right\};
\]

\[
\times I_{n-1,\ell-1} I_{n-1,\ell-1} - 2 \sqrt{\beta n K_4} \left[ (M_\ell + m_f)(M_\nu + m_f) I_{n-1,\ell} I_{n,\ell} - M_\ell + m_f I_{n,\ell} I_{n,\ell-1} \right] \right\};
\]

\[
R_{SS}^{++} = g_s g_s' j_s j_s' \left\{ (M_\ell + m_f)(M_\nu + m_f) \left[ (K_1 P) + m_f K_3 \right] I_{n,\ell} I_{n,\ell} + 2 \sqrt{\beta \ell \ell'} \left[ (K_1 P) - m_f K_3 \right] \right\};
\]

\[
\times I_{n-1,\ell-1} I_{n-1,\ell-1} - 2 \sqrt{\beta \ell} (M_\ell + m_f) I_{n-1,\ell} I_{n,\ell} + \sqrt{\ell} (M_\ell + m_f) I_{n,\ell} I_{n,\ell-1} \right\}.
\]

2. In the case when \( j \) is scalar current and \( j' \) is pseudoscalar current \( (k = S, k' = P) \) we obtain

\[
R_{PS}^{++} = g_s g_s' j_s' j_s \left\{ 2 \sqrt{\beta \ell} \left[ (K_2 P) + m_f K_4 \right] I_{n,\ell} I_{n,\ell} - (M_\ell + m_f)(M_\nu + m_f) \left[ (K_2 P) - m_f K_4 \right] \right\};
\]

\[
\times I_{n-1,\ell-1} I_{n-1,\ell-1} - 2 \sqrt{\beta n K_4} \left[ \sqrt{\ell} (M_\ell + m_f) I_{n-1,\ell} I_{n,\ell} - \sqrt{\ell'} (M_\ell + m_f) I_{n,\ell} I_{n,\ell-1} \right] \right\}.
\]
3. In the case where \( j \) is scalar current and \( j' \) is a vector current \((k = S, k' = V)\) we obtain
\[ R_{SV}^{\pm} = g_s g'_s \left\{ -2\beta \sqrt{\ell \ell'} \left[ (P \Lambda j') K_3 + (P \tilde{j}' j') K_4 - m_f (K_1 j') \right] I_{n,\ell} I_{n,\ell}' + (M_\ell + m_f) (M_\ell' + m_f) \left[ (P \Lambda j') K_3 + (P \tilde{j}' j') K_4 + m_f (K_1 j') \right] I_{n-1,\ell-1} I_{n-1,\ell-1}' \right. \]
\[ -2\beta \sqrt{n} (K_1 j') \left[ \sqrt{\ell} (M_\ell + m_f) I_{n-1,\ell-1} I_{n,\ell} - \sqrt{\ell'} (M_\ell + m_f) I_{n,\ell} I_{n,\ell}' \right] \left. \right\} ; \tag{37} \]
\[ R_{SV}^{\pm} = g_s g'_s \left\{ -2\beta \sqrt{\ell \ell'} \left[ (P' \Lambda j') K_3 - (P' \tilde{j}' j') K_4 - m_f (K_1 j') \right] I_{n,\ell} I_{n,\ell}' + (M_\ell + m_f) (M_\ell' + m_f) \left[ (P' \Lambda j') K_3 - (P' \tilde{j}' j') K_4 + m_f (K_1 j') \right] I_{n-1,\ell-1} I_{n-1,\ell-1}' \right. \]
\[ -2\beta \sqrt{n} (K_1 j') \left[ \sqrt{\ell} (M_\ell + m_f) I_{n-1,\ell-1} I_{n,\ell} - \sqrt{\ell'} (M_\ell + m_f) I_{n,\ell} I_{n,\ell}' \right] \left. \right\} ; \tag{38} \]
\[ R_{VS}^{\pm} = -ig_s g'_s \left\{ \sqrt{2\beta \ell} (M_\ell + m_f) \left[ (P \Lambda j') K_4 + (P \tilde{j}' j') K_3 - m_f (K_2 j') \right] I_{n,\ell} I_{n,\ell}' + \sqrt{2\beta \ell} (M_\ell' + m_f) \left[ (P \Lambda j') K_4 + (P \tilde{j}' j') K_3 + m_f (K_2 j') \right] I_{n-1,\ell-1} I_{n-1,\ell-1}' \right. \]
\[ + \sqrt{2\beta n} (K_2 j') \left[ (M_\ell + m_f) (M_\ell' + m_f) I_{n-1,\ell-1} I_{n,\ell} + 2\beta \sqrt{\ell \ell'} I_{n,\ell} I_{n,\ell}' \right] - \frac{(q' \Lambda j') + i(q' \phi j')}{\sqrt{q'^2}} \left. \right\} \left[ (M_\ell + m_f) \left[ (K_2 P) - m_f K_4 \right] I_{n-1,\ell-1} I_{n,\ell} + 2\beta \sqrt{n} K_4 I_{n,\ell} I_{n,\ell}' \right] \left. \right\} \times \left( M_\ell + m_f \right) \left[ (M_\ell + m_f) \left[ (K_2 P) - m_f K_4 \right] I_{n,\ell} I_{n,\ell}' \right] \left. \right\} ; \tag{39} \]
\[ R_{S}^{\pm} = i g_s g_{\lambda_{j}} \left\{ \sqrt{2} \beta \ell (M_\ell + m_f) \left[ (P' \bar{p} j') K_3 - (P' \bar{\lambda} j') K_4 + m_f (K_2 j') \right] I_{n,\ell} I'_{n,\ell} \right\} \]

\[ + \sqrt{2} \beta \ell (M_\ell + m_f) \left[ (P' \bar{p} j') K_3 - (P' \bar{\lambda} j') K_4 - m_f (K_2 j') \right] I_{n-1,\ell-1} I'_{n-1,\ell-1} \]

\[ + \sqrt{2} \beta n (K_2 j') \left[ (M_\ell + m_f) (M_\ell + m_f) I_{n-1,\ell-1} I'_{n-1,\ell} + 2 \sqrt{\ell} \beta \ell' I_{n,\ell} I'_{n,\ell-1} \right] + \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q'^2}} \]

\[ \times (M_\ell + m_f) \left[ (K_2 P') + m_f K_4 \right] I_{n-1,\ell-1} I'_{n-1,\ell-1} - 2 \beta \sqrt{n \ell} K_4 I_{n,\ell} I'_{n-1,\ell-1} \]

\[ + 2 \beta \sqrt{\ell} \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q'^2}} \left[ \sqrt{\ell'} ((K_2 P') - m_f K_4) I_{n-1,\ell} I'_{n-1,\ell-1} - \sqrt{n} (M_\ell + m_f) K_4 I_{n,\ell} I'_{n-1,\ell-1} \right] \]
\[ R_{V_S}^-= g_s g' j_s \left\{ (M_\ell + m_f)(M_\ell + m_f) \left[ (P \tilde{\lambda} j') K_3 + (P \tilde{\varphi} j') K_4 + m_f (K_1 j') \right] I_{n,\ell} I_n \right\} (43) \]

\[ -2 \beta \sqrt{\ell \ell'} \left[ (P \tilde{\lambda} j') K_3 + (P \tilde{\varphi} j') K_4 - m_f (K_1 j') \right] I_{n-1,\ell-1} \]

\[ +2 \beta \sqrt{n} (K_1 j') \left[ \sqrt{\ell'} (M_\ell + m_f) I_{n-1,\ell-1} I_n \right] \]

\[ - \sqrt{\frac{2 \beta \ell'}{q_\perp^2}} \left[ (q' \Lambda j') + i(q' \varphi j') \right] \left[ \sqrt{\ell} (M_\ell + m_f) I_{n,\ell} I_{n-1,\ell-1} - 2 \beta \sqrt{n} \right] \]

\[ + \sqrt{\frac{2 \beta}{q_\perp^2} (M_\ell + m_f) [(q' \Lambda j') - i(q' \varphi j')] \left[ \sqrt{\ell} (M_\ell + m_f) I_{n,\ell} I_{n-1,\ell-1} - 2 \beta \sqrt{n} \right] \]
4. In the case where $j$ is scalar current and $j'$ is a pseudovector current ($k = S$, $k' = A$) we obtain

$$ R_{AS}^{++} = -g_s g' S \left\{ 2\beta \sqrt{\ell \ell'} \left[ (P \bar{\lambda} j') K_4 + (P \bar{\phi} j') K_3 + m_f (K_{2j'}) \right] I_{n',t} I_{n,t} \right. $$

\begin{align*}
+ & (M_e + m_f) (M_e + m_f) \left[ (P \bar{\lambda} j') K_4 + (P \bar{\phi} j') K_3 - m_f (K_{2j'}) \right] I_{n-1,e-1} I_{n-1,t-1} \\
+ & 2\beta \sqrt{n} (K_{2j'}) \left[ \sqrt{\ell} (M_e + m_f) I_{n-1,e-1} I_{n,t} + \sqrt{\ell'} (M_e + m_f) I_{n',e} I_{n-1,t-1} \right] \\
- & \frac{\sqrt{2\beta}}{q^2} (M_e + m_f) [(q' A j') + i(q' \varphi j')] \left[ \sqrt{\ell} [(K_2 P) + m_f K_4] I_{n-1,e-1} I_{n,t} + \sqrt{n} (M_e + m_f) K_4 I_{n',e-1} I_{n-1,t-1} \right] \\
- & \frac{\sqrt{2\beta}}{q^2} [(q' A j') - i(q' \varphi j')] \left[ (M_e + m_f) [(K_2 P) - m_f K_4] I_{n-1,e-1} I_{n-1,t-1} \right. \\
& \left. + 2\beta \sqrt{n} K_4 I_{n',e-1} I_{n,t} \right) \right\}; \tag{45}
\end{align*}

$$ R_{SA}^{++} = -g_s g' A \left\{ 2\beta \sqrt{\ell \ell'} \left[ (P' \bar{\lambda} j') K_4 - (P' \bar{\phi} j') K_4 + m_f (K_{2j'}) \right] I_{n,e} I_{n',t} \right. $$

\begin{align*}
+ & (M_e + m_f) (M_e + m_f) \left[ (P' \bar{\lambda} j') K_4 - (P' \bar{\phi} j') K_4 - m_f (K_{2j'}) \right] I_{n-1,e-1} I_{n',t-1} \\
+ & 2\beta \sqrt{n} (K_{2j'}) \left[ \sqrt{\ell} (M_e + m_f) I_{n-1,e-1} I_{n',t} + \sqrt{\ell'} (M_e + m_f) I_{n,e} I_{n-1,t-1} \right] \\
+ & \frac{\sqrt{2\beta}}{q^2} [(q' A j') + i(q' \varphi j')] \left[ (M_e + m_f) [(K_2 P') + m_f K_4] I_{n-1,e-1} I_{n,t-1} - 2\beta \sqrt{n} K_4 I_{n,e} I_{n-1,t-1} \right] \\
+ & \frac{\sqrt{2\beta}}{q^2} (M_e + m_f) [(q' A j') - i(q' \varphi j')] \left[ \sqrt{\ell'} [(K_2 P') - m_f K_4] I_{n-1,e-1} I_{n',t-1} - \sqrt{n} (M_e + m_f) K_4 I_{n-1,e-1} I_{n',t-1} \right) \right\}; \tag{46}
\end{align*}

$$ R_{AS}^{-+} = \sqrt{2\beta} \sqrt{q^2} \left\{ (M_e + m_f) \left[ (P \bar{\lambda} j') K_3 + (P \bar{\phi} j') K_4 + m_f (K_{1j'}) \right] I_{n',e} I_{n,t} \right. $$

\begin{align*}
- & \frac{\sqrt{\ell}}{q^2} (M_e + m_f) \left[ (P \bar{\lambda} j') K_3 + (P \bar{\phi} j') K_4 - m_f (K_{1j'}) \right] I_{n-1,e-1} I_{n-1,t-1} \\
+ & \sqrt{2\beta} \sqrt{n} (K_{1j'}) \left[ (M_e + m_f) (M_e + m_f) I_{n-1,e-1} I_{n',t} - 2\beta \sqrt{\ell \ell'} I_{n',e} I_{n-1,t-1} \right] \\
- & \frac{(q' A j') + i(q' \varphi j')}{\sqrt{q^2}} \left[ (M_e + m_f) [(K_1 P) + m_f K_3] I_{n-1,e-1} I_{n,t} - 2\beta \sqrt{n} K_3 I_{n',e-1} I_{n-1,t-1} \right] \\
& \left. + 2\beta \sqrt{\ell'} \frac{(q' A j') - i(q' \varphi j')}{\sqrt{q^2}} \left[ \sqrt{\ell} [(K_1 P) - m_f K_3] I_{n-1,e-1} I_{n-1,t-1} - \sqrt{n} (M_e + m_f) K_3 I_{n-1,e-1} I_{n',t-1} \right) \right\}; \tag{47}
\end{align*}
\[ R_{-A}^+ = \imath g_s g'_s j_s \left\{ \sqrt{2\beta\ell} (M_e + m_f) \left[ (P'\hat{\Lambda}j')K_3 - (P'\hat{\varphi}j')K_4 - m_f (K_1 j') \right] I_{n,e} I'_{n,e} \right. \]
\[ - \sqrt{2\beta\ell} (M_e + m_f) \left[ (P'\hat{\Lambda}j')K_3 - (P'\hat{\varphi}j')K_4 - m_f (K_1 j') \right] I_{n-1,e-1} I'_{n-1,e-1} \]
\[ - \sqrt{2\beta n} (K_1 j') \left[ (M_e + m_f) (M_e + m_f) I_{n-1,e-1} I'_{n-1,e-1} - 2\beta\sqrt{\ell} I_{n,e} I'_{n,e} \right] \]
\[ \left. + \frac{(q'\Lambda j') + \imath (q'\varphi j')}{\sqrt{q_2^*}} (M_e + m_f) \left[ (M_e + m_f) [(K_1 P') + m_f K_3] I_{n-1,e-1} I'_{n-1,e-1} - 2\beta\sqrt{n} K_3 I_{n-1,e} I'_{n-1,e} \right] \right\}; \]

\[ R_{+A}^- = -\imath g_s g'_s j_s \left\{ \sqrt{2\beta\ell} (M_e + m_f) \left[ (P'\hat{\Lambda}j')K_3 - (P'\hat{\varphi}j')K_4 - m_f (K_1 j') \right] I_{n,e} I'_{n,e} \right. \]
\[ - \sqrt{2\beta\ell} (M_e + m_f) \left[ (P'\hat{\Lambda}j')K_3 - (P'\hat{\varphi}j')K_4 - m_f (K_1 j') \right] I_{n-1,e-1} I'_{n-1,e-1} \]
\[ + \sqrt{2\beta n} (K_1 j') \left[ 2\sqrt{\ell} I_{n-1,e-1} I_{n,e} - (M_e + m_f)(M_e + m_f) I_{n,e} I'_{n-1,e-1} \right] \]
\[ -2\beta\sqrt{\ell} \frac{(q'\Lambda j') - \imath (q'\varphi j')}{\sqrt{q_2^*}} (M_e + m_f) \left[ (M_e + m_f) [(K_1 P') + m_f K_3] I_{n-1,e-1} I'_{n-1,e-1} + 2\beta\sqrt{n} K_3 I_{n-1,e} I'_{n-1,e} \right] \right\}; \]

\[ R_{+A}^+ = \imath g_s g'_s j_s \left\{ \sqrt{2\beta\ell} (M_e + m_f) \left[ (P'\hat{\Lambda}j')K_3 - (P'\hat{\varphi}j')K_4 - m_f (K_1 j') \right] I_{n,e} I'_{n,e} \right. \]
\[ - \sqrt{2\beta\ell} (M_e + m_f) \left[ (P'\hat{\Lambda}j')K_3 - (P'\hat{\varphi}j')K_4 - m_f (K_1 j') \right] I_{n-1,e-1} I'_{n-1,e-1} \]
\[ - \sqrt{2\beta n} (K_1 j') \left[ 2\sqrt{\ell} I_{n-1,e-1} I_{n,e} - (M_e + m_f)(M_e + m_f) I_{n,e} I'_{n-1,e-1} \right] \]
\[ -2\beta\sqrt{\ell} \frac{(q'\Lambda j') + \imath (q'\varphi j')}{\sqrt{q_2^*}} (M_e + m_f) \left[ (M_e + m_f) [(K_1 P') + m_f K_3] I_{n-1,e-1} I'_{n-1,e-1} - \sqrt{n} (M_e + m_f) K_3 I_{n-1,e} I'_{n-1,e} \right] \right\}; \]
5. In the case where $j$ and $j'$ are pseudoscalar currents ($k = k' = P$) we obtain

\[
\mathcal{R}_{\ell,\ell'}^{++} = -g_p g_{j'j} \left\{ - (M_\ell + m_f) (M_\ell + m_f) \left[ (P_\ell \hat{j}') K_4 + (P \hat{j}'') K_3 - m_f (K_2 j') \right] I_{n,\ell} I_{n,\ell} \right. \\

- 2\beta \sqrt{\ell \ell'} \left[ (P_\ell \hat{j}') K_4 + (P \hat{j}'') K_3 - m_f (K_2 j') \right] I_{n-1,\ell-1} I_{n-1,\ell-1} \\

- 2\beta \sqrt{\ell \ell'} \left[ \sqrt{\ell} (M_\ell + m_f) I_{n-1,\ell-1} I_{n,\ell} + \sqrt{\ell} (M_\ell + m_f) I_{n,\ell} I_{n-1,\ell-1} \right] \\

+ \sqrt{\frac{2\beta \ell'}{q_{\perp}^2}} \left[ (q' \Lambda j') + i(q' \phi j') \right] \left[ (M_\ell + m_f) [(K_2 P) - m_f K_4] I_{n-1,\ell-1} I_{n-1,\ell-1} + 2\beta \sqrt{\ell n} K_4 I_{n,\ell} I_{n-1,\ell-1} \right] \\

+ \sqrt{\frac{2\beta \ell'}{q_{\perp}^2}} \left[ (M_\ell + m_f) [(q' \Lambda j') - i(q' \phi j')] \sqrt{\ell} [(K_2 P) + m_f K_4] I_{n-1,\ell-1} I_{n-1,\ell-1} \right] \\

- \sqrt{\frac{2\beta \ell'}{q_{\perp}^2}} \left[ (q' \Lambda j') - i(q' \phi j') \right] \left[ (M_\ell + m_f) [(K_2 P) - m_f K_4] I_{n,\ell} I_{n-1,\ell-1} - 2\beta \sqrt{\ell n} K_4 I_{n,\ell} I_{n-1,\ell-1} \right] \right\}; \\

(51)

\[
\mathcal{R}_{\ell,\ell'}^{-+} = -g_p g_{j'j} \left\{ - (M_\ell + m_f) (M_\ell + m_f) \left[ (P_\ell j') K_3 - (P' \hat{j}) K_4 + m_f (K_2 j') \right] I_{n,\ell} I_{n,\ell} \right. \\

- 2\beta \sqrt{\ell \ell'} \left[ (P_\ell j') K_3 - (P' \hat{j}) K_4 + m_f (K_2 j') \right] I_{n-1,\ell-1} I_{n-1,\ell-1} \\

- 2\beta \sqrt{\ell \ell'} \left[ \sqrt{\ell} (M_\ell + m_f) I_{n-1,\ell-1} I_{n,\ell} + \sqrt{\ell} (M_\ell + m_f) I_{n,\ell} I_{n-1,\ell-1} \right] \\

- \sqrt{\frac{2\beta \ell'}{q_{\perp}^2}} \left[ (q' \Lambda j') + i(q' \phi j') \right] \left[ \sqrt{\ell} [(K_2 P) + m_f K_4] I_{n-1,\ell-1} I_{n-1,\ell-1} - \sqrt{\ell n} (M_\ell + m_f) K_4 I_{n,\ell} I_{n-1,\ell-1} \right] \\

+ \sqrt{\frac{2\beta \ell'}{q_{\perp}^2}} \left[ (M_\ell + m_f) [(q' \Lambda j') - i(q' \phi j')] \left[ \sqrt{\ell} [(K_2 P) - m_f K_4] I_{n-1,\ell-1} I_{n-1,\ell-1} - 2\beta \sqrt{\ell n} K_4 I_{n,\ell} I_{n-1,\ell-1} \right] \right\}; \\

(52)

5. In the case where $j$ and $j'$ are pseudoscalar currents ($k = k' = P$) we obtain

\[
\mathcal{R}_{++}^{++} = -g_p g_{j'j} \left\{ 2\beta \sqrt{\ell \ell'} [(K_1 P) + m_f K_3] I_{n,\ell} I_{n,\ell} + (M_\ell + m_f) (M_\ell + m_f) [(K_1 P) - m_f K_3] \right. \times \\

I_{n-1,\ell-1} I_{n-1,\ell-1} - 2\sqrt{\ell} n K_3 \left[ \sqrt{\ell} (M_\ell + m_f) I_{n-1,\ell-1} I_{n,\ell} + \sqrt{\ell} (M_\ell + m_f) I_{n,\ell} I_{n-1,\ell-1} \right] \right\}; \\

(53)

\[
\mathcal{R}_{++}^{-+} = -ig_p g_{j'j} \left\{ \sqrt{2\beta \ell} (M_\ell + m_f) [(K_2 P) + m_f K_4] I_{n,\ell} I_{n,\ell} - \sqrt{2\beta \ell} (M_\ell + m_f) [(K_2 P) - m_f K_4] \right. \times \\

I_{n-1,\ell-1} I_{n-1,\ell-1} - \sqrt{2\beta n} K_4 \left[ (M_\ell + m_f) (M_\ell + m_f) I_{n-1,\ell-1} I_{n,\ell} - 2\beta \sqrt{\ell \ell'} I_{n,\ell} I_{n-1,\ell-1} \right] \right\}; \\

(54)

\[
\mathcal{R}_{++}^{--} = ig_p g_{j'j} \left\{ \sqrt{2\beta \ell} (M_\ell + m_f) [(K_2 P) - m_f K_4] I_{n,\ell} I_{n,\ell} - \sqrt{2\beta \ell} (M_\ell + m_f) [(K_2 P) + m_f K_4] \right. \times \\

I_{n-1,\ell-1} I_{n-1,\ell-1} - \sqrt{2\beta n} K_4 \left[ 2\beta \sqrt{\ell \ell'} I_{n-1,\ell-1} I_{n,\ell} - (M_\ell + m_f) (M_\ell + m_f) I_{n,\ell} I_{n-1,\ell-1} \right] \right\}; \\

(55)

\[
\mathcal{R}_{pp}^{++} = -g_p g_{j'j} \left\{ (M_\ell + m_f) (M_\ell + m_f) [(K_1 P) - m_f K_3] I_{n,\ell} I_{n,\ell} + 2\beta \sqrt{\ell \ell'} [(K_1 P) + m_f K_3] \right. \times \\

I_{n-1,\ell-1} I_{n-1,\ell-1} - 2\beta \sqrt{\ell n} K_3 \left[ \sqrt{\ell} (M_\ell + m_f) I_{n-1,\ell-1} I_{n,\ell} + \sqrt{\ell} (M_\ell + m_f) I_{n,\ell} I_{n-1,\ell-1} \right] \right\}. \\

(56)
For second diagram we have the following replacement $P_a \rightarrow P'_a', \mathcal{I}_{m,n} \leftrightarrow \mathcal{I}'_{m,n}$.

6. In the case where $j$ is pseudoscalar current and $j'$ is a vector current ($k = P, k' = V$) we obtain

\[
\mathcal{R}^{++}_{+} = g_p g'_V j_p \left\{ 2\beta \sqrt{\ell \ell'} \left[ (P \tilde{A} j') K_4 + (P \tilde{\phi} j') K_3 - m_f (K_2 j') \right] I'_{n,\ell} \right\} I_{n,\ell}
\]

\[
+ (M_{\ell} + m_f)(M_{\ell'} + m_f) \left\{ (P \tilde{A} j') K_4 + (P \tilde{\phi} j') K_3 + m_f (K_2 j') \right\} I'_{n-1,\ell-1} I_{n-1,\ell-1}
\]

\[
+ 2\beta \sqrt{n} (K_2 j') \left[ \sqrt{\ell}(M_{\ell'} + m_f) I'_{n-1,\ell-1} I_{n,\ell} + \sqrt{\ell'}(M_{\ell} + m_f) I'_{n,\ell} I_{n-1,\ell-1} \right]
\]

\[
- \sqrt{\frac{2\beta}{q^2}} \left[ (q' \tilde{\lambda} j') + i(q' \phi j') \right] \left[ \sqrt{\ell} \left( [K_2 P] - m_f K_4 I'_{n-1,\ell-1} I_{n,\ell} + \sqrt{n}(M_{\ell} + m_f) K_4 I'_{n,\ell} I_{n-1,\ell-1} \right) \right]
\]

\[
- \sqrt{\frac{2\beta}{q^2}} \left[ (q' \tilde{\lambda} j') - i(q' \phi j') \right] \left[ (M_{\ell} + m_f) \left( [K_2 P] + m_f K_4 I'_{n-1,\ell} I_{n-1,\ell-1} + 2\beta \sqrt{n} K_4 I'_{n,\ell} I_{n-1,\ell} \right) \right]
\]
\[
\mathcal{R}_{PV} = i g_p g_{\alpha} \left\{ \sqrt{2 \beta \ell} (M_e + m_f) \left[ (P' \tilde{\Lambda} j') K_3 - (P' \tilde{\varphi} j') K_4 - m_f K_1 j' \right] I_n, e I_{n', \ell} + \sqrt{2 \beta \ell} (M_e + m_f) \left[ (P' \tilde{\Lambda} j') K_3 - (P' \tilde{\varphi} j') K_4 + m_f K_1 j' \right] I_{n-1, e-1} I_{n', \ell}, I_{n', \ell-1} - \sqrt{2 \beta \ell} (K_1 j') \left[ (M_e + m_f)(M_e + m_f) I_{n-1, e-1} I_{n', \ell} - 2 \beta \sqrt{\ell \ell'} I_{n, e} I_{n', \ell} \right] \right\} + i \sqrt{2 \beta \ell} (q' \Lambda j' + \bar{q} (q' \varphi j')) (M_e + m_f) \left[ (M_e + m_f) (K_1 P') - m_f K_3 I_{n-1, e-1} I_{n', \ell-1} - 2 \beta \sqrt{\ell \ell'} K_3 I_{n-1, e} I_{n', \ell} \right] + 2 \beta \sqrt{\ell} \left( q' \Lambda j' + \bar{q} (q' \varphi j') \right) \left[ \sqrt{\ell} \left[ (K_1 P') - m_f K_3 I_{n, e} I_{n', \ell-1} - \sqrt{\ell} (M_e + m_f) K_3 I_{n, e} I_{n', \ell} \right] \right] \};
\]

\[
\mathcal{R}_{PV} = i g_p g_{\alpha} \left\{ \sqrt{2 \beta \ell} (M_e + m_f) \left[ (P' \tilde{\Lambda} j') K_3 + (P' \tilde{\varphi} j') K_4 + m_f K_1 j' \right] I_{n, e} I_{n', \ell} + \sqrt{2 \beta \ell} (M_e + m_f) \left[ (P' \tilde{\Lambda} j') K_3 + (P' \tilde{\varphi} j') K_4 - m_f K_1 j' \right] I_{n-1, e-1} I_{n', \ell-1} - \sqrt{2 \beta \ell} (K_1 j') \left[ (M_e + m_f)(M_e + m_f) I_{n, e} I_{n', \ell} - 2 \beta \sqrt{\ell \ell'} I_{n, e} I_{n', \ell} \right] \right\} - 2 \beta \sqrt{\ell} \left( q' \Lambda j' + \bar{q} (q' \varphi j') \right) \left[ \sqrt{\ell} \left[ (K_1 P') + m_f K_3 I_{n, e} I_{n', \ell-1} - \sqrt{\ell} (M_e + m_f) K_3 I_{n, e} I_{n', \ell} \right] \right] + \frac{(q' \Lambda j' - \bar{q} (q' \varphi j'))}{\sqrt{q_+^2}} \left( M_e + m_f \right) \left[ (M_e + m_f) \left[ (K_1 P') - m_f K_3 I_{n, e} I_{n', \ell-1} + 2 \beta \sqrt{\ell \ell'} K_3 I_{n, e} I_{n', \ell} \right] \right] \};
\]

\[
\mathcal{R}_{PV} = i g_p g_{\alpha} \left\{ \sqrt{2 \beta \ell} (M_e + m_f) \left[ (P' \tilde{\Lambda} j') K_3 - (P' \tilde{\varphi} j') K_4 + m_f K_1 j' \right] I_{n, e} I_{n', \ell} + \sqrt{2 \beta \ell} (M_e + m_f) \left[ (P' \tilde{\Lambda} j') K_3 - (P' \tilde{\varphi} j') K_4 - m_f K_1 j' \right] I_{n-1, e-1} I_{n', \ell-1} - \sqrt{2 \beta \ell} (K_1 j') \left[ (M_e + m_f)(M_e + m_f) I_{n, e} I_{n', \ell} - 2 \beta \sqrt{\ell \ell'} I_{n, e} I_{n', \ell} \right] \right\} + 2 \beta \sqrt{\ell} \left( q' \Lambda j' + \bar{q} (q' \varphi j') \right) \left[ \sqrt{\ell} \left[ (K_1 P') + m_f K_3 I_{n-1, e-1} I_{n', \ell-1} - \sqrt{\ell} (M_e + m_f) K_3 I_{n-1, e} I_{n', \ell} \right] \right] + \frac{(q' \Lambda j' + \bar{q} (q' \varphi j'))}{\sqrt{q_+^2}} \left( M_e + m_f \right) \left[ (M_e + m_f) \left[ (K_1 P') - m_f K_3 I_{n-1, e-1} I_{n', \ell-1} + 2 \beta \sqrt{\ell \ell'} K_3 I_{n-1, e} I_{n', \ell} \right] \right] \};
\]
\[ \mathcal{R}_{V^c} = g_p g_p' \left\{ - (M_\ell + m_f)(M_\ell + m_f) \left[ (P \tilde{\Lambda} j') K_4 + (P \tilde{\varphi} j') K_3 + m_f (K_2 j') \right] I_{n,e} I_{n,l} + 2\beta \sqrt{\ell'\ell} \left[ (P \tilde{\Lambda} j') K_4 + (P \tilde{\varphi} j') K_3 - m_f (K_2 j') \right] I_{n-1,e} I_{n-1,l} - 2\beta \sqrt{n} (K_2 j') \left[ \sqrt{\ell'} (M_\ell + m_f) I_{n-1,e} I_{n,l} + \sqrt{\ell} (M_\ell + m_f) I_{n,e} I_{n-1,l} \right] \right\} \]  

(63)

\[ \mathcal{R}_{F^c} = -g_p g_p' \left\{ - (M_\ell + m_f)(M_\ell + m_f) \left[ (P' \tilde{\varphi} j') K_3 - (P' \tilde{\Lambda} j') K_4 + m_f (K_2 j') \right] I_{n,e} I_{n,l} + 2\beta \sqrt{\ell'\ell} \left[ (P' \tilde{\varphi} j') K_3 - (P' \tilde{\Lambda} j') K_4 - m_f (K_2 j') \right] I_{n-1,e} I_{n-1,l} - 2\beta \sqrt{n} (K_2 j') \left[ \sqrt{\ell'} (M_\ell + m_f) I_{n-1,e} I_{n,l} + \sqrt{\ell} (M_\ell + m_f) I_{n,e} I_{n-1,l} \right] - \sqrt{\frac{2\beta}{q_{2}} (M_\ell + m_f) [(q' \Lambda j') + i(q' \varphi j') \left[ \sqrt{\ell'} [(K_2 P') + m_f K_4] I_{n-1,e} I_{n-1,l} - \sqrt{n} (M_\ell + m_f) K_4 I_{n,e} I_{n-1,l} \right] \right\}; \]  

(64)

7. In the case where \( j \) is pseudoscalar current and \( j' \) is a pseudovector current (\( k = P, k' = A \)) we obtain

\[ \mathcal{R}_{AP} = -g_p g_p' \left\{ - 2\beta \sqrt{\ell'\ell} \left[ (P \tilde{\Lambda} j') K_3 + (P \tilde{\varphi} j') K_4 + m_f (K_1 j') \right] I_{n,e} I_{n,l} + (M_\ell + m_f)(M_\ell + m_f) \left[ (P \tilde{\Lambda} j') K_3 + (P \tilde{\varphi} j') K_4 - m_f (K_1 j') \right] I_{n-1,e} I_{n-1,l} - 2\beta \sqrt{n} (K_1 j') \left[ \sqrt{\ell'} (M_\ell + m_f) I_{n-1,e} I_{n,l} - \sqrt{\ell} (M_\ell + m_f) I_{n,e} I_{n-1,l} \right] \right\}; \]  

(65)
\[
R_{P,j}^{+} = \int g_{p} g_{a} \beta \sqrt{\ell} \left\{ -2 \beta \sqrt{\ell} [P'(\Lambda j')] K_{3} - (P' \bar{\varphi} j') K_{4} + m_{f}(K_{1} j') \right\} \mathcal{I}_{n,e} \mathcal{I}_{n',e} \\
+ (M_{\ell} + m_{f})(M_{e} + m_{f}) [P'(\Lambda j') K_{3} - (P' \bar{\varphi} j') K_{4} - m_{f}(K_{1} j')] \mathcal{I}_{n-1,e-1} \mathcal{I}_{n',e-1} \\
- 2 \beta \sqrt{\ell} \left( (K_{1} j') \right) \left[ \sqrt{\ell} (M_{e} + m_{f}) \mathcal{I}_{n-1,e-1} \mathcal{I}_{n',e} - \sqrt{\ell} (M_{e} + m_{f}) \mathcal{I}_{n,e} \mathcal{I}_{n',e-1} \right] \\
+ \sqrt{\frac{2 \beta \ell}{q_{\ell}^{2}}} (q' \Lambda j') + i(q' \varphi j') \left[ (M_{e} + m_{f}) \left[ (K_{1} P') - m_{f} K_{3} \right] \mathcal{I}_{n-1,e-1} \mathcal{I}_{n',e-1} - 2 \beta \sqrt{\ell} n K_{3} \mathcal{I}_{n,e} \mathcal{I}_{n',e-1} \right] \\
- \sqrt{\frac{2 \beta}{q_{\ell}^{2}}} (M_{e} + m_{f}) \left[ (q' \Lambda j') - i(q' \varphi j') \right] \left[ \sqrt{\ell} \left( (K_{1} P') + m_{f} K_{3} \right) \mathcal{I}_{n,e} \mathcal{I}_{n',e} - \sqrt{n}(M_{e} + m_{f}) K_{3} \mathcal{I}_{n-1,e} \mathcal{I}_{n',e-1} \right] \right\}; \\

R_{A,P}^{+} = \int g_{p} g_{a} \beta \sqrt{\ell} \left\{ \sqrt{2 \beta \ell} (M_{e} + m_{f}) \left[ (P' \Lambda j') K_{4} + (P' \bar{\varphi} j') K_{3} + m_{f}(K_{2} j') \right] \mathcal{I}_{n,e} \mathcal{I}_{n',e} \\
+ \sqrt{2 \beta \ell} (M_{e} + m_{f}) \left[ (P' \Lambda j') K_{4} + (P' \bar{\varphi} j') K_{3} - m_{f}(K_{2} j') \right] \mathcal{I}_{n-1,e-1} \mathcal{I}_{n',e-1} \\
+ \sqrt{2 \beta} n (K_{2} j') \left[ (M_{e} + m_{f})(M_{e} + m_{f}) \mathcal{I}_{n-1,e-1} \mathcal{I}_{n',e-1} + 2 \beta \sqrt{\ell} \mathcal{I}_{n,e} \mathcal{I}_{n',e-1} \right] - \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_{\ell}^{2}}} \\
\times (M_{e} + m_{f}) \left[ (M_{e} + m_{f}) \left[ (K_{2} P') - m_{f} K_{4} \right] \mathcal{I}_{n-1,e} \mathcal{I}_{n',e-1} + \sqrt{n} (M_{e} + m_{f}) K_{4} \mathcal{I}_{n-1,e} \mathcal{I}_{n',e} \right] \right\}; \\

R_{A,P}^{+} = \int g_{p} g_{a} \beta \sqrt{\ell} \left\{ \sqrt{2 \beta \ell} (M_{e} + m_{f}) \left[ (P' \Lambda j') K_{4} - (P' \bar{\varphi} j') K_{3} - m_{f}(K_{2} j') \right] \mathcal{I}_{n,e} \mathcal{I}_{n',e} \\
+ \sqrt{2 \beta \ell} (M_{e} + m_{f}) \left[ (P' \Lambda j') K_{4} - (P' \bar{\varphi} j') K_{3} + m_{f}(K_{2} j') \right] \mathcal{I}_{n-1,e-1} \mathcal{I}_{n',e-1} \\
+ \sqrt{2 \beta} n (K_{2} j') \left[ (M_{e} + m_{f})(M_{e} + m_{f}) \mathcal{I}_{n-1,e-1} \mathcal{I}_{n',e-1} + 2 \beta \sqrt{\ell} \mathcal{I}_{n,e} \mathcal{I}_{n',e-1} \right] + \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_{\ell}^{2}}} \\
\times (M_{e} + m_{f}) \left[ (M_{e} + m_{f}) \left[ (K_{2} P') - m_{f} K_{4} \right] \mathcal{I}_{n-1,e} \mathcal{I}_{n',e-1} - \sqrt{n} (M_{e} + m_{f}) K_{4} \mathcal{I}_{n-1,e} \mathcal{I}_{n',e} \right] \right\}; \\

R_{A,P}^{+} = \int g_{p} g_{a} \beta \sqrt{\ell} \left\{ \sqrt{2 \beta \ell} (M_{e} + m_{f}) \left[ (P' \Lambda j') K_{4} + (P' \bar{\varphi} j') K_{3} + m_{f}(K_{2} j') \right] \mathcal{I}_{n,e} \mathcal{I}_{n',e} \\
+ \sqrt{2 \beta \ell} (M_{e} + m_{f}) \left[ (P' \Lambda j') K_{4} + (P' \bar{\varphi} j') K_{3} - m_{f}(K_{2} j') \right] \mathcal{I}_{n-1,e-1} \mathcal{I}_{n',e-1} \\
+ \sqrt{2 \beta} n (K_{2} j') \left[ (M_{e} + m_{f})(M_{e} + m_{f}) \mathcal{I}_{n-1,e-1} \mathcal{I}_{n',e-1} + 2 \beta \sqrt{\ell} \mathcal{I}_{n,e} \mathcal{I}_{n',e-1} \right] - \frac{(q' \Lambda j') - i(q' \varphi j')}{\sqrt{q_{\ell}^{2}}} \\
\times (M_{e} + m_{f}) \left[ (M_{e} + m_{f}) \left[ (K_{2} P') + m_{f} K_{4} \right] \mathcal{I}_{n-1,e} \mathcal{I}_{n',e-1} + \sqrt{n} (M_{e} + m_{f}) K_{4} \mathcal{I}_{n-1,e} \mathcal{I}_{n',e} \right] \right\}; \\

(R_{A,P}^{+})}
\[ R_{PA}^+ = g_p g'_a p_j \left\{ \sqrt{2 \beta \ell} (M + m_f) \left[ (P' \hat{\varphi}_j') K_3 - (P' \hat{\Lambda} j') K_4 + m_f (K_2 j') \right] \mathcal{I}_{n, \ell} \mathcal{I}_{n, \ell}' \right. \\
+ \sqrt{2 \beta \ell} (M + m_f) \left[ (P' \hat{\varphi}_j') K_3 - (P' \hat{\Lambda} j') K_4 - m_f (K_2 j') \right] \mathcal{I}_{n-1, \ell-1} \mathcal{I}_{n-1, \ell-1}' \\
+ \sqrt{2 \beta n} (K_2 j') \left[ 2 \beta \sqrt{\ell} \mathcal{I}_{n-1, \ell-1} \mathcal{I}_{n, \ell}' + (M + m_f)(M + m_f) \mathcal{I}_{n, \ell} \mathcal{I}_{n, \ell}' \mathcal{I}_{n-1, \ell-1}' \right] \\
+ 2 \beta \sqrt{\ell} \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{|q|^2}} \left[ \sqrt{\mathcal{V}} \left[ (K_2 P') + m_f K_4 \mathcal{I}_{n-1, \ell-1} \mathcal{I}_{n-1, \ell-1}' - \sqrt{n} (M + m_f) K_4 \mathcal{I}_{n-1, \ell} \mathcal{I}_{n-1, \ell}' \right] \\
+ \frac{(q' \Lambda j') - i(q' \varphi j')}{\sqrt{|q|^2}} (M + m_f) \left[ (M + m_f) [(K_2 P') - m_f K_4 \mathcal{I}_{n, \ell} \mathcal{I}_{n, \ell}' \mathcal{I}_{n-1, \ell-1}' - 2 \beta \sqrt{n} K_4 \mathcal{I}_{n, \ell-1} \mathcal{I}_{n-1, \ell-1}] \right] \right\} \]

\[ R_{AP}^- = -g_p g'_a p_j \left\{ (M + m_f)(M + m_f) \left[ (P \hat{\varphi}_j') K_3 + (P \hat{\Lambda} j') K_4 - m_f (K_1 j') \right] \mathcal{I}_{n, \ell} \mathcal{I}_{n, \ell}' \right. \\
- 2 \beta \sqrt{\ell} \left[ (P \hat{\varphi}_j') K_3 + (P \hat{\Lambda} j') K_4 + m_f (K_1 j') \right] \mathcal{I}_{n-1, \ell-1} \mathcal{I}_{n-1, \ell-1}' \\
+ 2 \beta \sqrt{n} (K_1 j') \left[ \sqrt{\mathcal{V}} (M + m_f) \mathcal{I}_{n-1, \ell-1} \mathcal{I}_{n, \ell} - \sqrt{\mathcal{V}} (M + m_f) \mathcal{I}_{n, \ell} \mathcal{I}_{n-1, \ell-1}' \right] \\
- \sqrt{\frac{2 \beta \ell}{|q|^2}} [(q' \Lambda j') + i(q' \varphi j')] \left[ (M + m_f) [(K_1 P') - m_f K_3 \mathcal{I}_{n-1, \ell-1} \mathcal{I}_{n-1, \ell-1}' - 2 \beta \sqrt{n} K_3 \mathcal{I}_{n-1, \ell} \mathcal{I}_{n-1, \ell}'] \right] \\
+ \sqrt{\frac{2 \beta}{|q|^2}} (M + m_f) [(q' \Lambda j') - i(q' \varphi j')] \left[ \sqrt{\mathcal{V}} [(K_1 P') + m_f K_3 \mathcal{I}_{n-1, \ell-1} \mathcal{I}_{n-1, \ell-1}' - \sqrt{n} (M + m_f) K_3 \mathcal{I}_{n, \ell} \mathcal{I}_{n-1, \ell}'] \right] \right\} \]

\[ R_{PA}^- = g_p g'_a p_j \left\{ (M + m_f)(M + m_f) \left[ (P' \hat{\varphi}_j') K_3 - (P' \hat{\Lambda} j') K_4 - m_f (K_1 j') \right] \mathcal{I}_{n, \ell} \mathcal{I}_{n, \ell}' \right. \\
- 2 \beta \sqrt{\ell} \left[ (P' \hat{\varphi}_j') K_3 - (P' \hat{\Lambda} j') K_4 + m_f (K_1 j') \right] \mathcal{I}_{n-1, \ell-1} \mathcal{I}_{n-1, \ell-1}' \\
+ 2 \beta \sqrt{n} (K_1 j') \left[ \sqrt{\mathcal{V}} (M + m_f) \mathcal{I}_{n-1, \ell-1} \mathcal{I}_{n, \ell} - \sqrt{\mathcal{V}} (M + m_f) \mathcal{I}_{n, \ell} \mathcal{I}_{n-1, \ell-1}' \right] \\
- \sqrt{\frac{2 \beta \ell}{|q|^2}} [(q' \Lambda j') + i(q' \varphi j')] \left[ (M + m_f) [(K_1 P') + m_f K_3 \mathcal{I}_{n-1, \ell-1} \mathcal{I}_{n-1, \ell-1}' - \sqrt{n} (M + m_f) K_3 \mathcal{I}_{n, \ell} \mathcal{I}_{n-1, \ell}'] \right] \\
+ \sqrt{\frac{2 \beta}{|q|^2}} [(q' \Lambda j') - i(q' \varphi j')] \left[ (M + m_f) [(K_1 P') - m_f K_3 \mathcal{I}_{n-1, \ell-1} \mathcal{I}_{n-1, \ell-1}' - 2 \beta \sqrt{n} K_3 \mathcal{I}_{n-1, \ell} \mathcal{I}_{n-1, \ell}'] \right] \right\} \]
8. Both vertices are leaves \((k = k' = V)\):

\[
R_{VV}^{++} = g_v g_v' \left\{ 2 \beta \sqrt{\ell' \ell} \left[ (P\Lambda j')(K_{1j}) + (P\Lambda j)(K_{1j'}) - (j\Lambda j')(K_1 P) - m_f [(j\Lambda j')K_3 + (j\varphi j')K_4] \right] I_{n,\ell} I_{n,\ell'} \right. \\
+ (M_\ell + m_f)(M_{\ell'} + m_f) \left[ (P\Lambda j')(K_{1j}) + (P\Lambda j)(K_{1j'}) - (j\Lambda j')(K_1 P) + m_f [(j\Lambda j')K_3 + (j\varphi j')K_4] \right] \\
\times I_{n-1,\ell-1} I_{n-1,\ell-1} + 2\beta \sqrt{m} [(j\Lambda j')K_3 + (j\varphi j')K_4] \left[ \sqrt{\ell'} (M_\ell + m_f) I_{n-1,\ell-1} I_{n,\ell} + \sqrt{\ell} (M_\ell + m_f) I_{n,\ell} I_{n-1,\ell-1} \right] \\
- \frac{2 \beta \ell'}{q^2} (M_\ell + m_f) [(q\Lambda j') + \left( q\varphi j' \right)] [(P\Lambda j')K_3 - (P\varphi j)K_4 - m_f (K_{1j})] I_{n,\ell-1} I_{n,\ell-1} \\
- \frac{2 \beta \ell'}{q^2} (M_\ell + m_f) [(q\Lambda j') + \left( q\varphi j' \right)] [(P\Lambda j')K_3 + (P\varphi j')K_4 - m_f (K_{1j'})] I_{n,\ell-1} I_{n,\ell-1} \\
- \frac{2 \beta \ell'}{q^2} (M_\ell + m_f) [(q\Lambda j') + \left( q\varphi j' \right)] [(P\Lambda j')K_3 - (P\varphi j)K_4 + m_f (K_{1j})] I_{n-1,\ell} I_{n,\ell} \\
+ (M_\ell + m_f)(M_{\ell'} + m_f) [(j\Lambda j') + \left( j\varphi j' \right)] [(K_1 P) - m_f K_3] \left( \frac{(q\Lambda q') - \left( q\varphi q' \right)}{\sqrt{q_1^2 q_2^2}} \right) I_{n,\ell-1} I_{n,\ell-1} \\
+ 2\beta \sqrt{\ell'} [(j\Lambda j') + \left( j\varphi j' \right)] [(K_1 P) + m_f K_3] \left( \frac{(q\Lambda q') + \left( q\varphi q' \right)}{\sqrt{q_1^2 q_2^2}} \right) I_{n,\ell-1} I_{n,\ell-1} \\
- \sqrt{2 \beta n} (K_{1j}) \left[ 2 \beta \sqrt{\ell' \ell} \frac{(q\Lambda j') - \left( q\varphi j' \right)}{\sqrt{q_1^2}} I_{n-1,\ell} I_{n,\ell} + (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j') + \left( q\varphi j' \right)}{\sqrt{q_1^2}} I_{n,\ell-1} I_{n-1,\ell-1} \right] \\
- \sqrt{2 \beta n} (K_{1j'}) \left[ (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) - \left( q\varphi j \right)}{\sqrt{q_1^2}} I_{n-1,\ell-1} I_{n,\ell-1} + 2\beta \sqrt{\ell' \ell} \frac{(q\Lambda j) + \left( q\varphi j \right)}{\sqrt{q_1^2}} I_{n,\ell} I_{n-1,\ell} \right] \\
+ 2\beta \sqrt{n} K_3 \left[ \sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) + \left( q\varphi j \right)}{\sqrt{q_1^2}} \frac{(q\Lambda j') + \left( q\varphi j' \right)}{\sqrt{q_1^2}} I_{n,\ell-1} I_{n,\ell-1} \right].
\]
\( R_{VV}^{-} = i g_v g_s \left\{ \sqrt{2 \beta \ell'} (M_\ell + m_f) \left[ (P \tilde{A}_j)(K_{2j}) + (P \tilde{A}_j)(K_{2j})' - (j \tilde{A}_j)(K_2P) \right] \right. \\
- m_f [(j \tilde{A}_j')K_4 + (j \tilde{A}_j')K_3] \right. \left. I_{n,e}^I_{n,\ell} - \sqrt{2 \beta \ell'} (M_\ell + m_f) \left[ (P \tilde{A}_j')(K_{2j}) + (P \tilde{A}_j')(K_{2j})' \right] \right. \\
- (j \tilde{A}_j')(K_2P) + m_f [(j \tilde{A}_j')K_4 + (j \tilde{A}_j')K_3] \right. \left. I_{n-1,e}^I_{n-1,\ell} \right. \\
+ \sqrt{2 \beta n} [(j \tilde{A}_j')K_4 + (j \tilde{A}_j')K_3] [(M_\ell + m_f)(M_\ell + m_f)I_{n-1,e}^I_{n-1,\ell} - 2 \beta \sqrt{\ell' l'_{\ell}} I_{n,e}^I_{n,\ell}I_{n-1,e}^I_{n-1,\ell-1} \\
- (M_\ell + m_f)(M_\ell + m_f) \left( \frac{q' \tilde{A}_j'}{\sqrt{q_{\ell}^2}} + i \frac{q' \phi j}{\sqrt{q_{\ell}^2}} \right) (P \tilde{A}_j)K_4 - (P \tilde{A}_j)K_3 - m_f(K_{2j}) I_{n,e}^I_{n,\ell} \\
+ \frac{2 \beta \sqrt{\ell' l'}}{\sqrt{q_{\ell}^2}} [(q\tilde{A}_j - i(q\phi j)) [((P \tilde{A}_j')(K_{2j}) + (P \tilde{A}_j')(K_{2j})'] I_{n,e}^I_{n,\ell} - 2 \beta \sqrt{\ell' l'_{\ell}} I_{n,e}^I_{n,\ell}I_{n-1,e}^I_{n-1,\ell-1} \\
+ \frac{2 \beta \sqrt{\ell' l'}}{\sqrt{q_{\ell}^2}} [(q\tilde{A}_j - i(q\phi j')) [((P \tilde{A}_j')(K_{2j}) + (P \tilde{A}_j')(K_{2j})'] I_{n-1,e}^I_{n-1,\ell} \\
- (M_\ell + m_f)(M_\ell + m_f) \left( \frac{q' \tilde{A}_j}{\sqrt{q_{\ell}^2}} + i \frac{q' \phi j}{\sqrt{q_{\ell}^2}} \right) (P \tilde{A}_j)K_4 - (P \tilde{A}_j)K_3 + m_f(K_{2j}) I_{n-1,e}^I_{n-1,\ell} \\
- \sqrt{2 \beta l'} (M_\ell + m_f)[(j \tilde{A}_j') + i(j \phi j')] [(K_{2j}P) - m_fK_4] \left( \frac{q \tilde{A}_j - i(q \phi j)}{\sqrt{q_{\ell}^2 q_{\ell}'}^2} \right) I_{n-1,e}^I_{n,\ell} \\
+ \sqrt{2 \beta l'} (M_\ell + m_f) [(j \tilde{A}_j') - i(j \phi j')] [(K_{2j}P) + m_fK_4] \left( \frac{q \tilde{A}_j + i(q \phi j)}{\sqrt{q_{\ell}^2 q_{\ell}'}^2} \right) I_{n-1,e}^I_{n-1,\ell} \\
- 2 \beta \sqrt{n} (K_{2j}) \left[ \sqrt{\ell'} (M_\ell + m_f) \left( \frac{q' \tilde{A}_j}{\sqrt{q_{\ell}^2}} - i \frac{q' \phi j}{\sqrt{q_{\ell}^2}} \right) I_{n-1,e}^I_{n,\ell} - \sqrt{l'} (M_\ell + m_f) \left( \frac{q' \tilde{A}_j}{\sqrt{q_{\ell}^2}} + i \frac{q' \phi j}{\sqrt{q_{\ell}^2}} \right) I_{n,e}^I_{n,\ell}I_{n-1,e}^I_{n-1,\ell-1} \right] \\
+ 2 \beta \sqrt{n} (K_{2j}) \left[ \sqrt{\ell'} (M_\ell + m_f) \left( \frac{q \tilde{A}_j - i(q \phi j)}{\sqrt{q_{\ell}^2}} \right) I_{n-1,\ell}^I_{n,\ell} - \sqrt{l'} (M_\ell + m_f) \left( \frac{q \tilde{A}_j + i(q \phi j)}{\sqrt{q_{\ell}^2}} \right) I_{n,e}^I_{n,\ell}I_{n-1,e}^I_{n-1,\ell} \right] \\
- \sqrt{2 \beta \ell} (K_{2j}) \left[ \sqrt{\ell'} (M_\ell + m_f) \left( \frac{q \tilde{A}_j}{\sqrt{q_{\ell}^2}} - i \frac{q \phi j}{\sqrt{q_{\ell}^2}} \right) I_{n-1,\ell}^I_{n,\ell} - \sqrt{l'} (M_\ell + m_f) \left( \frac{q \tilde{A}_j}{\sqrt{q_{\ell}^2}} + i \frac{q \phi j}{\sqrt{q_{\ell}^2}} \right) I_{n,e}^I_{n,\ell}I_{n-1,e}^I_{n-1,\ell} \right] \\
- (M_\ell + m_f)(M_\ell + m_f) \left( \frac{q \tilde{A}_j + i(q \phi j)}{\sqrt{q_{\ell}^2}} \right) (P \tilde{A}_j)K_4 - (P \tilde{A}_j)K_3 - m_f(K_{2j}) I_{n,e}^I_{n,\ell} \right\} \right. 
\)
\[
\mathcal{R}_{\psi_1} = i\epsilon_\nu g_\nu' \left\{ \sqrt{2\beta\ell} (M_\ell + m_f') \left[ (P\Lambda j')(K_{2j}) + (P\Lambda j')(K_{2j}') - (j\Lambda j')(K_{2P}) \right] + m_f[(j\Lambda j')(K_4) + (j\Lambda j')(K_{3j})] \mathcal{I}_{n,\ell',\ell} + \sqrt{2\beta\ell'} (M_\ell + m_f) \left[ (P\Lambda j')(K_{2j}) + (P\Lambda j')(K_{2j}') \right] \\
- (j\Lambda j')(K_{2P}) - m_f[(j\Lambda j')(K_4) + (j\Lambda j')(K_{3j})] \mathcal{I}_{n-1,\ell-1,\ell-1} \\
- \sqrt{2\beta n} [(j\Lambda j')(K_4) + (j\Lambda j')(K_{3j})] \left[ 2\beta \sqrt{\ell' \ell} I_{n-1,\ell-1,\ell} - (M_\ell + m_f)(M_\ell + m_f) \mathcal{I}_{n,\ell',\ell} \right] \right\}
\]
\[ R_{VV} = g_\alpha g'_\alpha \left\{ (M_\ell + m_f)(M_\ell' + m_f) \left[ (P\partial_j')(K_{1,j}) + (P\partial_j')(K_{1,j}') - (j\partial_j')(K_{1,P}) \right] + m_f[(j\partial_j')(K_3) + (j\partial_j')(K_4)] T_{n,\ell}^n T_{n,\ell} + 2\beta \sqrt{\ell' \ell} \left[ (P\partial_j')(K_{1,j}) + (P\partial_j')(K_{1,j}') \right] - (j\partial_j')(K_{1,P}) - m_f[(j\partial_j')(K_3) + (j\partial_j')(K_4)] T_{n-1,\ell-1}^n T_{n-1,\ell-1} + 2\beta \sqrt{n}(j\partial_j')(K_3) + (j\partial_j')(K_4)[\sqrt{\ell}(M_\ell + m_f)T_{n-1,\ell-1}^n T_{n,\ell} + \sqrt{\ell}(M_\ell' + m_f)T_{n,\ell}^n T_{n-1,\ell-1} - \sqrt{\ell' \ell} (M_\ell + m_f)[(q'\Lambda_j') + i(q'\varphi j')][(P\partial_j')K_3 - (P\partial_j')K_4 + m_f(K_{1,j})]T_{n,\ell}^n T_{n,\ell-1} - \sqrt{\ell' \ell} (M_\ell + m_f)[(q'\Lambda_j') - i(q'\varphi j')][(P\partial_j')K_3 - (P\partial_j')K_4 - m_f(K_{1,j})]T_{n-1,\ell}^n T_{n,\ell-1} - \sqrt{\ell' \ell} (M_\ell + m_f)[(q'\Lambda_j') + i(q'\varphi j')][(P\partial_j')K_3 + (P\partial_j')K_4 - m_f(K_{1,j})]T_{n-1,\ell-1}^n T_{n-1,\ell} - \sqrt{\ell' \ell} (M_\ell + m_f)[(q'\Lambda_j') - i(q'\varphi j')] [(K_j') + m_fK_3] \frac{(q'\Lambda j') - i(q'\varphi j')}{{\sqrt{q_2^2}}} T_{n,\ell-1}^n T_{n,\ell-1} + (M_\ell + m_f)(M_\ell' + m_f) [(j\Lambda_j') - i(j\varphi j')][(K_j') - m_fK_3] \frac{(q'\Lambda j') + i(q'\varphi j')}{{\sqrt{q_2^2}}} T_{n,\ell}^n T_{n-1,\ell-1} - \sqrt{\ell' \ell} (K_{1,j}) [(M_\ell + m_f)(M_\ell' + m_f) \frac{(q'\Lambda j') - i(q'\varphi j')}{{\sqrt{q_2^2}}} T_{n-1,\ell}^n T_{n,\ell} + 2\beta \sqrt{\ell' \ell} \frac{(q'\Lambda j') + i(q'\varphi j')}{{\sqrt{q_2^2}}} T_{n,\ell-1}^n T_{n-1,\ell-1} - \sqrt{\ell' \ell} (K_{1,j'}) [2\beta \sqrt{\ell' \ell} \frac{(q\Lambda j) - i(q\varphi j)}{q_2^2} T_{n-1,\ell}^n T_{n,\ell-1} + (M_\ell + m_f)(M_\ell' + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{q_2^2} T_{n,\ell}^n T_{n-1,\ell}] + 2\beta \sqrt{n} K_3 [\sqrt{\ell} (M_\ell + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{q_2^2} (q'\Lambda j') - i(q'\varphi j')] \frac{T_{n,\ell}}{q_2^2} T_{n-1,\ell}^n T_{n-1,\ell-1} + \sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{q_2^2} (q'\Lambda j') + i(q'\varphi j')] \frac{T_{n,\ell}^n T_{n-1,\ell}}{q_2^2} T_{n,\ell-1}^n T_{n-1,\ell-1}]\right\}.

For second diagram we have the following replacement \( P_\alpha \to P'_\alpha, q_\alpha \to -q'_\alpha, j_\alpha \to j'_\alpha, I_{m,n} \leftrightarrow I_{n,m} \).
9. In the case where $j$ is a vector current and $j'$ is a pseudovector current ($k = V$, $k' = A$) we obtain

$$
\mathcal{R}^{++}_{AV} = g_{\rho\sigma} \Theta_{\rho\sigma} \left\{ 2\beta\sqrt{\ell\ell'} \left[ (P\hat{A}j')(K_{2j}) + (P\hat{A}j)(K_{2j'}) - (j\hat{A}j')(K_{2P}) \right] 
- M_\ell [ (j\hat{A}j')K_4 + (j\hat{A}j)K_3 ] I_{n_1,\ell} I_{n_\ell} - (M_\ell + m_f)(M_{\ell'} + m_f) \left[ (P\hat{A}j')(K_{2j}) + (P\hat{A}j)(K_{2j'}) \right] 
- (j\hat{A}j')(K_{2P}) - m_f [ (j\hat{A}j')K_4 + (j\hat{A}j)K_3 ] I_{n_{-1},\ell'-1} I_{n_{-1},\ell-1} 
+ 2\beta\sqrt{\ell} [ (j\hat{A}j')K_4 + (j\hat{A}j)K_3 ] [ \sqrt{\ell} (M_\ell + m_f) I_{n_{-1},\ell'-1} I_{n_\ell} ] 
- \sqrt{\ell} (M_\ell + m_f) I_{n_1,\ell} I_{n_\ell} 
+ (M_\ell + m_f) [ q\Lambda j - i(q\varphi j)] \left[ (P\hat{A}j')K_4 + (P\hat{A}j)K_3 - m_f (K_{2j'})I_{n_{-1},\ell} I_{n_{-1},\ell-1} \right] 
+ (M_\ell + m_f) [ q\Lambda j - i(q\varphi j)] \left[ (P\hat{A}j')K_4 + (P\hat{A}j)K_3 - m_f (K_{2j'})I_{n_{-1},\ell} I_{n_{-1},\ell-1} \right] 
- (M_\ell + m_f) [ q\Lambda j - i(q\varphi j)] \left[ (P\hat{A}j')K_4 + (P\hat{A}j)K_3 - m_f (K_{2j'})I_{n_{-1},\ell} I_{n_{-1},\ell-1} \right] 
+ 2\beta\sqrt{\ell\ell'} \left[ (j\hat{A}j') - i(j\hat{A}j') \right] [ (K_{2P}) - m_f K_4 ] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_j^2 q_{\ell'}^2}} I_{n_{-1},\ell} I_{n_{-1},\ell-1} 
- \sqrt{2\beta n} K_{1j} \left[ 2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q_j^2}} I_{n_{-1},\ell} I_{n_{-1},\ell} - (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q_j^2}} I_{n_{-1},\ell'-1} I_{n_{-1},\ell-1} \right] 
+ \sqrt{2\beta n} K_{1j'} \left[ (M_\ell + m_f)(M_{\ell'} + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_j^2}} I_{n_{-1},\ell} I_{n_{-1},\ell} - 2\beta\sqrt{\ell\ell'} \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_j^2}} I_{n_{-1},\ell} I_{n_{-1},\ell} \right] 
- 2\beta\sqrt{\ell} K_4 \left[ \sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_j^2}} \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q_{\ell'}^2}} I_{n_{-1},\ell} I_{n_{-1},\ell} \right] 
- \sqrt{\ell} (M_\ell + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_j^2}} \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q_{\ell'}^2}} I_{n_{-1},\ell} I_{n_{-1},\ell} \right\} ;
$$
\[
\mathcal{R}_{\nu\Lambda}^{\nu\Lambda} = \frac{g_0 g'}{\sqrt{\ell'}} \left\{ 2\beta \sqrt{\ell'} \left[ (P'\tilde{\Lambda}j')(K_{2j}) + (P'\tilde{\Lambda}j')(K_{2j'}) - (j\tilde{\Lambda}j')(K_{2j'}) \right] 
- m_f[(j\tilde{\Lambda}j')K_4 - (j\tilde{\varphi}j')K_3] I_{n,\ell} I_{n',\ell} - (M_\ell + m_f)(M_\ell + m_f) \left[ (P'\tilde{\Lambda}j')(K_{2j}) + (P'\tilde{\Lambda}j')(K_{2j'}) \right] 
- (j\tilde{\Lambda}j')(K_{2j'}) + m_f[(j\tilde{\Lambda}j')K_4 - (j\tilde{\varphi}j')K_3] I_{n-1,\ell-1} I_{n',\ell-1} 
\right. 
\left. + 2\beta \sqrt{n}(j\tilde{\Lambda}j')K_4 - (j\tilde{\varphi}j')K_3 \left[ \sqrt{\ell'}(M_\ell + m_f)I_{n-1,\ell-1} I_{n',\ell} - \sqrt{\ell'}(M_\ell + m_f)I_{n,\ell} I_{n',\ell-1} \right] 
+ \frac{2\beta \ell}{q_\perp^2} (M_\ell + m_f)[(q\Lambda j) + i(q\varphi j)] \frac{(P'\tilde{\Lambda}j')(K_{2j}) - (P'\tilde{\Lambda}j')(K_{2j'}) - m_f(K_{2j'})I_{n,\ell} I_{n',\ell}}{\sqrt{\ell'}(M_\ell + m_f)I_{n,\ell} I_{n',\ell-1}} 
- \frac{2\beta \ell'}{q_\perp^2} (M_\ell + m_f)[(q' \Lambda j') - i(q' \varphi j')] [(P'\tilde{\Lambda}j')(K_{2j}) - (P'\tilde{\Lambda}j')(K_{2j'}) - m_f(K_{2j'})I_{n-1,\ell} I_{n',\ell-1}] 
- \frac{2\beta \ell'}{q_\perp^2} (M_\ell + m_f)[(q' \Lambda j') - i(q' \varphi j')] [(P'\tilde{\Lambda}j')(K_{2j}) - (P'\tilde{\Lambda}j')(K_{2j'}) - m_f(K_{2j'})I_{n-1,\ell} I_{n',\ell-1}] 
+ \frac{2\beta \ell}{q_\perp^2} (M_\ell + m_f)[(q\Lambda j) + i(q\varphi j)] \frac{(P'\tilde{\Lambda}j')(K_{2j}) + (P'\tilde{\Lambda}j')(K_{2j'}) - m_f(K_{2j'})I_{n,\ell} I_{n',\ell-1}}{\sqrt{\ell'}(M_\ell + m_f)I_{n,\ell} I_{n',\ell-1}} 
- (M_\ell + m_f)(M_\ell + m_f)[(q\Lambda j) - i(q\varphi j)] \frac{(P'\tilde{\Lambda}j')(K_{2j}) - (P'\tilde{\Lambda}j')(K_{2j'}) - m_f(K_{2j'})I_{n,\ell} I_{n',\ell-1}}{\sqrt{\ell'}(M_\ell + m_f)I_{n,\ell} I_{n',\ell-1}} 
+ 2\beta \sqrt{\ell'} [(j\tilde{\Lambda}j') + i(j\tilde{\varphi}j')][(K_{2j'}) + m_fK_4] \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{\ell'}(M_\ell + m_f)I_{n,\ell} I_{n',\ell-1}] 
+ \sqrt{2\beta n} (K_{2j'}) \left[ 2\beta \sqrt{\ell'} \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} I_{n-1,\ell} I_{n',\ell} - (M_\ell + m_f)(M_\ell + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{q_\perp^2} I_{n,\ell} I_{n',\ell-1} \right] 
- \sqrt{2\beta n} (K_{2j}) \left[ (M_\ell + m_f)(M_\ell + m_f) \frac{(q' \Lambda j') - i(q' \varphi j')}{q_\perp^2} I_{n-1,\ell-1} I_{n',\ell-1} - 2\beta \sqrt{\ell'} \frac{(q' \Lambda j') + i(q' \varphi j')}{q_\perp^2} I_{n,\ell} I_{n',\ell-1} \right] 
- 2\beta \sqrt{n} K_4 \left[ \sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q' \Lambda j') - i(q' \varphi j')}{q_\perp^2} I_{n-1,\ell} I_{n',\ell-1} \right] 
- \sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_\perp^2}} \frac{(q' \Lambda j') + i(q' \varphi j')}{q_\perp^2} I_{n,\ell} I_{n',\ell-1} \right\};
\]
\[ R^\mu_{\lambda\nu} = i g q^j \left\{ -\sqrt{2\beta\ell'} (M^\mu + m_f) \left[ (P\tilde{\Lambda} j')(K_j 1j) + (P\tilde{\Lambda} j')(K_j 1j') - (j\tilde{\Lambda} j')(K_1 P) \right] \\
- m_f \left[(j\tilde{\Lambda} j')K_3 + (j\tilde{\phi} j')K_4 \right] I_{\nu,\ell} I_{n,\ell} + \sqrt{2\beta\ell} (M^\mu + m_f) \left[ (P\tilde{\Lambda} j')(K_j 1j) + (P\tilde{\Lambda} j')(K_j 1j') \right] \\
- (j\tilde{\Lambda} j')(K_1 P) - m_f \left[(j\tilde{\Lambda} j')K_3 + (j\tilde{\phi} j')K_4 \right] I_{\nu,1,\ell-1} I_{n,1,\ell-1} \right\} \\
+ 2\sqrt{\beta n} \left[(j\tilde{\Lambda} j')K_3 + (j\tilde{\phi} j')K_4 \left[(M^\mu + m_f)I_{\nu,\ell} I_{n,\ell} + 2\beta\ell\ell' I_{\nu,\ell} I_{n,\ell-1} \right] \right] \\
- (M^\mu + m_f)(M^\mu + m_f) \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp^2}} \left[ (P\tilde{\Lambda} j')K_3 - (P\tilde{\phi} j')K_4 + m_f(K_1 j') \right] I_{\nu,1,\ell} I_{n,\ell} \right\} \\
- \frac{2\sqrt{\beta\ell'}}{\sqrt{q_\perp^2}} \left[(q\Lambda j') - i(q\varphi j') \right] \left[ (P\tilde{\Lambda} j')K_3 + (P\tilde{\phi} j')K_4 + m_f(K_1 j') \right] I_{\nu,\ell} I_{n,\ell-1} \right\} \\
- \frac{2\beta\ell'}{\sqrt{q_\perp^2}} \left[(q\Lambda j') - i(q\varphi j') \right] \left[ (P\tilde{\Lambda} j')K_3 - (P\tilde{\phi} j')K_4 - m_f(K_1 j') \right] I_{\nu,\ell} I_{n,\ell-1} \right\} \\
- \frac{2\beta\ell'(M^\mu + m_f)\left[(j\Lambda j') + i(j\varphi j') \right](K_1 P) + m_fK_3 \right\} \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_\perp^2} q_\perp^2} \left[ I_{\nu,1,\ell} I_{n,\ell-1} \right] \right\} \\
+ \frac{2\beta\ell'(M^\mu + m_f)\left[(j\Lambda j') + i(j\varphi j') \right](K_1 P) - m_fK_3 \right\} \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_\perp^2} q_\perp^2} \left[ I_{\nu,1,\ell} I_{n,\ell-1} \right] \right\} \\
- 2\sqrt{\beta} n \left[(K_1 j') \right] \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q_\perp^2} q_\perp^2} \left[ I_{\nu,\ell,1} I_{n,\ell-1} + \sqrt{\beta} \right] \frac{(q\Lambda j') + i(q\varphi j')}{\sqrt{q_\perp^2} q_\perp^2} \left[ I_{\nu,\ell} I_{n,\ell-1} \right] \right\} \\
- 2\sqrt{\beta} n \left[(K_1 j') \right] \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q_\perp^2} q_\perp^2} \left[ I_{\nu,1,\ell-1} I_{n,\ell-1} \right] \right\} \\
+ 2\beta nK_3 \left[ 2\beta\ell\ell' \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q_\perp^2} q_\perp^2} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_\perp^2} q_\perp^2} \right] I_{\nu,\ell} I_{n,\ell-1} \right\} \\
+(M^\mu + m_f)(M^\mu + m_f) \frac{(q\Lambda j') + i(q\varphi j')}{\sqrt{q_\perp^2} q_\perp^2} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_\perp^2} q_\perp^2} \left[ I_{\nu,1,\ell} I_{n,\ell-1} \right] \right\} ; \]
\[
\mathcal{R}_{\hat{A}}^+ = i g_n g_a \left\{ - \sqrt{2 \beta \ell'} (M_\ell + m_f) \left[ (P'\hat{\Lambda}_j')(K_{1j}) + (P'\hat{\Lambda}_j)(K_{1j'}) - (j\hat{\Lambda}_j')(K_{1P'}) + m_f [(j\hat{\Lambda}_j')K_3 - (j\hat{\phi}_j')K_4] \mathcal{I}_{n,\ell'\ell} + \sqrt{2 \beta \ell} (M_\ell + m_f) \left[ (P'\hat{\Lambda}_j')(K_{1j}) + (P'\hat{\Lambda}_j)(K_{1j'}) - (j\hat{\Lambda}_j')(K_{1P'}) + m_f [(j\hat{\Lambda}_j')K_3 - (j\hat{\phi}_j')K_4] \mathcal{I}_{n-1,\ell'\ell} - 1 \mathcal{I}_{n-1,\ell'\ell} - 1 \right] + \sqrt{2 \beta n} [(j\hat{\Lambda}_j')K_3 - (j\hat{\phi}_j')K_4] [(M_\ell + m_f)(M_\ell + m_f) \mathcal{I}_{n-1,\ell'\ell} + 2 \beta \sqrt{\ell' \ell n} \mathcal{I}_{n,\ell'\ell} \mathcal{I}_{n-1,\ell'\ell} - 1] + (M_\ell + m_f)(M_\ell + m_f) \frac{(q\Lambda_j') + i(q\varphi j')}{\sqrt{q^2_\perp}} [(P'\hat{\Lambda}_j')K_3 - (P'\hat{\phi}_j')K_4 - m_f(K_{1j'})] \mathcal{I}_{n,\ell' \ell} - 1 \mathcal{I}_{n,\ell} + \frac{2 \beta \sqrt{\ell' \ell}}{\sqrt{q^2_\perp}} [(q\Lambda_j') - i(q\varphi j')] [(P'\hat{\Lambda}_j')K_3 - (P'\hat{\phi}_j')K_4 - m_f(K_{1j'})] \mathcal{I}_{n,\ell' \ell} - 1 \mathcal{I}_{n,\ell} + (M_\ell + m_f)(M_\ell + m_f) \frac{(q\Lambda_j') + i(q\varphi j')}{\sqrt{q^2_\perp}} [(P'\hat{\Lambda}_j')K_3 - (P'\hat{\phi}_j')K_4 + m_f(K_{1j'})] \mathcal{I}_{n-1,\ell' \ell} - 1 \mathcal{I}_{n-1,\ell} + \sqrt{2 \beta \ell} (M_\ell + m_f) [(j\Lambda_j') - i(j\varphi j')][(K_{1P'}) - m_fK_3] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q^2_\perp q^2_\perp}} \mathcal{I}_{n-1,\ell' \ell} - 1 \mathcal{I}_{n-1,\ell} + \sqrt{2 \beta \ell} (M_\ell + m_f) [(j\Lambda_j') + i(j\varphi j')][(K_{1P'}) + m_fK_3] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q^2_\perp q^2_\perp}} \mathcal{I}_{n-1,\ell' \ell} - 1 \mathcal{I}_{n-1,\ell} + 2 \beta \sqrt{n} (K_{1j'}) \left[ \sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q^2_\perp}} \mathcal{I}_{n-1,\ell' \ell} + \sqrt{\ell} (M_\ell + m_f) \frac{(q\Lambda j') + i(q\varphi j')}{\sqrt{q^2_\perp}} \mathcal{I}_{n,\ell' \ell} - 1 \mathcal{I}_{n-1,\ell} \right] + 2 \beta \sqrt{n} (K_{1j}) \left[ \sqrt{\ell'} (M_\ell + m_f) \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q^2_\perp}} \mathcal{I}_{n-1,\ell' \ell} + \sqrt{\ell} (M_\ell + m_f) \frac{(q\Lambda j') + i(q\varphi j')}{\sqrt{q^2_\perp}} \mathcal{I}_{n,\ell' \ell} - 1 \mathcal{I}_{n-1,\ell} \right] + \sqrt{2 \beta n} K_3 \left[ 2 \beta \ell' \ell n \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q^2_\perp}} \mathcal{I}_{n-1,\ell' \ell} - 1 \mathcal{I}_{n,\ell} + (M_\ell + m_f)(M_\ell + m_f) \frac{(q\Lambda j') + i(q\varphi j')}{\sqrt{q^2_\perp}} \mathcal{I}_{n,\ell' \ell} - 1 \mathcal{I}_{n-1,\ell} \right] \right\} ;
\]

\[ (80) \]
\( \mathcal{R}_{AV} = i g_{\mu \nu} \left\{ -\sqrt{2\beta} \ell (M_{\ell} + m_f) \left[ (P\tilde{A}j')(K_{1j}) + (P\tilde{A}j)(K_{1j}') - (j\tilde{A}j')(K_{1P}) \right] \\
- m_f [(j\tilde{A}j')(K_{3} + (j\tilde{A}j')(K_{3})] \right\} \left[ I_{n,e}^t I_{n,t} - \sqrt{2\beta} \ell (M_{\ell} + m_f) \right] \left[ (P\tilde{A}j')(K_{1j}) + (P\tilde{A}j)(K_{1j}') \right] \\
- (j\tilde{A}j')(K_{1P}) + m_f [(j\tilde{A}j')(K_{3} + (j\tilde{A}j')(K_{3})] \right\} \right] T_{n-1,e}^t T_{n-1,t}^e \\
- \sqrt{2\beta} \ell [(j\tilde{A}j')(K_{3} + (j\tilde{A}j')(K_{3})] \left[ 2\sqrt{\ell} \ell' T_{n-1,e}^t T_{n-1,t}^e + (M_{\ell} + m_f)(M_{\ell'} + m_f) I_{n,e}^t I_{n,t} \right] \\
+ \frac{2\beta \sqrt{\ell} \ell'}{\sqrt{q_{1z}^2}} \left[ (q'\Lambda j') + i(q'\varphi j') \right] \left[ (P\tilde{A}j)(K_{3} - (P\tilde{A}j)(K_{3})\right] I_{n-1,e}^t I_{n-1,t}^e \\
+ \frac{(M_{\ell} + m_f)(M_{\ell'} + m_f)}{\sqrt{q_{1z}^2}} \left[ (q\Lambda j') - i(q\varphi j') \right] \left[ (P\tilde{A}j)(K_{3} + (P\tilde{A}j)(K_{3})\right] I_{n-1,e}^t I_{n-1,t}^e \\
+ \frac{2\beta \sqrt{\ell} \ell'}{\sqrt{q_{1z}^2}} \left[ (q'\Lambda j') + i(q'\varphi j') \right] \left[ (P\tilde{A}j)(K_{3} + (P\tilde{A}j)(K_{3})\right] I_{n-1,e}^t I_{n-1,t}^e \\
- \sqrt{2\beta} \ell [(j\Lambda j') + i(j\varphi j')] \left[ (K_{1P}) - m_f K_{3} \right] \left[ \frac{q\Lambda q'}{\sqrt{q_{1z}^2}} - i(q\varphi q') \right] I_{n-1,e}^t I_{n-1,t}^e \\
- \sqrt{2\beta} \ell [(j\Lambda j') + i(j\varphi j')] \left[ (K_{1P}) + m_f K_{3} \right] \left[ \frac{q\Lambda q'}{\sqrt{q_{1z}^2}} + i(q\varphi q') \right] I_{n-1,e}^t I_{n-1,t}^e \\
+ 2\beta \sqrt{\ell} K_{1j} \left[ \sqrt{\ell} (M_{\ell'} + m_f) \left[ \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q_{1z}^2}} \right] I_{n-1,e}^t I_{n-1,t}^e + \sqrt{\ell} (M_{\ell} + m_f) \left[ \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_{1z}^2}} \right] I_{n-1,e}^t I_{n-1,t}^e \right] \\
+ 2\beta \sqrt{\ell} K_{1j'} \left[ \sqrt{\ell} (M_{\ell'} + m_f) \left[ \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q_{1z}^2}} \right] I_{n-1,e}^t I_{n-1,t}^e + \sqrt{\ell} (M_{\ell} + m_f) \left[ \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_{1z}^2}} \right] I_{n-1,e}^t I_{n-1,t}^e \right] \\
- \sqrt{2\beta} n K_{3} \left[ (M_{\ell} + m_f)(M_{\ell'} + m_f) \left[ \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q_{1z}^2}} \right] I_{n-1,e}^t I_{n-1,t}^e \right] \\
+ 2\beta \sqrt{\ell} \ell' \left[ \frac{(q\Lambda j') + i(q\varphi j')}{\sqrt{q_{1z}^2}} \right] I_{n-1,e}^t I_{n-1,t}^e \right\} ;
\[ R_{\pm} = \frac{ig_{\alpha}}{2} \left\{ \sqrt{2\beta} \left( M_{\ell} + m_{\ell}' \right) \left[ (P'\bar{\Lambda}j')(K_{11}) + (P'\bar{\Lambda}j)(K_{11}j') - (j\bar{\Lambda}j')(K_{1}P') \right] \\
+ m_{f}[(j\bar{\Lambda}j')(K_{3}) - (j\bar{\Lambda}j')\mathcal{K}_{2}] I_{n,\ell}I_{n',\ell} - \sqrt{2\beta'/(M_{\ell} + m_{f})} \left[ (P'\bar{\Lambda}j')(K_{11}) + (P'\bar{\Lambda}j)(K_{11}j') - (j\bar{\Lambda}j')(K_{1}P') \right] \\
- \frac{\sqrt{2\beta} \left( m_{f} \left( (j\bar{\Lambda}j')(K_{3}) - (j\bar{\Lambda}j')(K_{4}) \right) I_{n,\ell}I_{n',\ell} \right)}{\sqrt{\beta}} \right\}; \]
\[ \mathcal{R}_{A} = g_{v} g'_{a} \left\{ (M_{v} + m_{f})(M_{v} + m_{f}) \left[ (P \hat{A} j')(K_{2 j}) + (P \hat{A} j')(K_{2 j'}) - (j \hat{A} j')(K_{2 P}) \right] - m_{f}[(j \hat{A} j')(K_{4}) + (j \hat{A} j')(K_{3})] T_{n_{e}, n_{e} - 1} - 2 \beta \sqrt{\ell' \ell} \left[ (P \hat{A} j')(K_{2 j}) + (P \hat{A} j')(K_{2 j'}) \right] - (j \hat{A} j')(K_{2 P}) + m_{f}[(j \hat{A} j')(K_{4}) + (j \hat{A} j')(K_{3})] T_{n_{e} - 1, n_{e} - 1} + 2 \beta \sqrt{\ell'(j \hat{A} j')(K_{4}) + (j \hat{A} j')(K_{3})} \times [\sqrt{\ell}(M_{v} + m_{f})I_{n_{e}, n_{e} - 1} - \sqrt{\ell}(M_{v} + m_{f})I_{n_{e}, n_{e} - 1, n_{e} - 1}] \right. \\
\left. - \sqrt{\frac{2 \beta \ell'}{q_{i}^{2}}} (M_{v} + m_{f})[(q' \Lambda j') + i(q' \varphi j')] [(P \hat{A} j')K_{4} - (P \hat{A} j')K_{3} - m_{f}(K_{2 j})] I_{n_{e}, n_{e} - 1} \right. \\
\left. + \sqrt{\frac{2 \beta \ell}{q_{i}^{2}}} (M_{v} + m_{f})[(q \Lambda j) - i(q \varphi j')] [(P \hat{A} j')K_{4} + (P \hat{A} j')K_{3} - m_{f}(K_{2 j'})] I_{n_{e}, n_{e} - 1} \right. \\
\left. + \sqrt{\frac{2 \beta \ell}{q_{i}^{2}}} (M_{v} + m_{f})[(q \Lambda j) - i(q \varphi j')] [(P \hat{A} j')K_{4} - (P \hat{A} j')K_{3} - m_{f}(K_{2 j})] I_{n_{e}, n_{e} - 1} \right. \\
\left. - 2 \beta \sqrt{\ell'}[(j \Lambda j') + i(j \varphi j')] [(K_{2 P}) - m_{f}K_{4}] \frac{(q\Lambda q') - i(q \varphi q')}{\sqrt{q_{i}^{2} q_{i}^{2}}} I_{n_{e}, n_{e} - 1} I_{n_{e}, n_{e} - 1} \right. \\
\left. + (M_{v} + m_{f})(M_{v} + m_{f}) [(j \Lambda j') - i(j \varphi j')] [(K_{2 P}) + m_{f}K_{4}] \frac{(q\Lambda q') + i(q \varphi q')}{\sqrt{q_{i}^{2} q_{i}^{2}}} I_{n_{e}, n_{e} - 1} I_{n_{e}, n_{e} - 1} \right. \\
\left. - \sqrt{\frac{2 \beta \ell}{q_{i}^{2}}} (K_{2 j}) [(M_{v} + m_{f})(M_{v} + m_{f}) \frac{(q \Lambda j') - i(q \varphi j')}{\sqrt{q_{i}^{2}}} I_{n_{e} - 1, n_{e} - 1} - 2 \beta \sqrt{\ell' \ell} \frac{(q \Lambda j') + i(q \varphi j')}{\sqrt{q_{i}^{2}}} I_{n_{e}, n_{e} - 1} I_{n_{e}, n_{e} - 1}] \right. \\
\left. + \sqrt{\frac{2 \beta \ell}{q_{i}^{2}}} (K_{2 j'}) [(2 \beta \sqrt{\ell' \ell} \frac{(q \Lambda j) - i(q \varphi j)}{\sqrt{q_{i}^{2}}} I_{n_{e} - 1, n_{e} - 1} - (M_{v} + m_{f})(M_{v} + m_{f}) \frac{(q \Lambda j) + i(q \varphi j)}{\sqrt{q_{i}^{2}}} I_{n_{e}, n_{e} - 1} I_{n_{e}, n_{e} - 1}] \right. \\
\left. - 2 \beta \sqrt{\ell} K_{4} \left[ \frac{1}{\sqrt{\ell}} (M_{v} + m_{f}) \frac{(q \Lambda j) - i(q \varphi j)}{\sqrt{q_{i}^{2}}} \frac{(q \Lambda j') - i(q \varphi j')}{\sqrt{q_{i}^{2}}} I_{n_{e}, n_{e} - 1} I_{n_{e}, n_{e} - 1} \right. \right. \\
\left. - \frac{1}{\sqrt{\ell}} (M_{v} + m_{f}) \frac{(q \Lambda j) + i(q \varphi j)}{\sqrt{q_{i}^{2}}} \frac{(q \Lambda j') + i(q \varphi j')}{\sqrt{q_{i}^{2}}} I_{n_{e}, n_{e} - 1} I_{n_{e}, n_{e} - 1} \right] ; \]
\[
\mathcal{R}_{\tilde{V}A} = \text{geg}_a \left\{ (M_\ell + m_f) (M_\ell + m_f) \left[ (P' \hat{\Lambda} j')(K_{2j}) + (P' \hat{\Lambda} j)(K_{2j'}) - (j\hat{\Lambda} j')(K_{2P'}) \right] + m_f [(j\hat{\Lambda} j')(K_4 - (j\hat{\varphi} j')(K_3)] I_{n,e} I'_{n,e} - 2\beta \sqrt{\ell'\ell} \left[ (P' \hat{\Lambda} j')(K_{2j}) + (P' \hat{\Lambda} j)(K_{2j'}) \right] - (j\hat{\Lambda} j')(K_{2P'}) - m_f [(j\hat{\Lambda} j')(K_4 - (j\hat{\varphi} j')(K_3)] I_{n-1,e-1} I'_{n-1,e-1} + 2\beta \sqrt{n} [(j\hat{\Lambda} j')(K_4 - (j\hat{\varphi} j')(K_3)]
\times \left[ \sqrt{\ell'} (M_\ell + m_f) I_{n-1,e-1} I'_{n,e} - \sqrt{\ell} (M_\ell + m_f) I_{n,e} I'_{n-1,e-1} \right]
\] 
\] 
\[+ \sqrt{\frac{2\beta \ell'}{q_2^2}} (M_\ell + m_f) [(q\Lambda j') + i(q\varphi j')] [(P' \hat{\Lambda} j')(K_4 + (P' \hat{\varphi} j') \Lambda q + m_f (K_{2j'})] I_{n,e} I'_{n,e} - 2\beta \sqrt{\ell'\ell} [(P' \hat{\Lambda} j')(K_4 - (P' \hat{\varphi} j') \Lambda q + m_f (K_{2j'})] I_{n-1,e-1} I'_{n-1,e-1}
\] 
\[+ \sqrt{\frac{2\beta \ell'}{q_2^2}} (M_\ell + m_f) [(q\Lambda j') + i(q\varphi j')] [(P' \hat{\Lambda} j')K_3 + (P' \hat{\varphi} j') \Lambda q + m_f (K_{2j'})] I_{n-1,e-1} I'_{n-1,e-1}
\] 
\[-2\beta \sqrt{\ell'\ell} [(j\Lambda j') - i(j\varphi j')] [(K_{2P'}) + m_f (K_4)] \frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_2^2 q_2^2}} I_{n,e-1} I'_{n,e-1}
\] 
\[+ (M_\ell + m_f) [(M_\ell + m_f) [(j\Lambda j') + i(j\varphi j')] [(K_{2P'})] - m_f (K_4)] \frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_2^2 q_2^2}} I_{n-1,e-1} I'_{n-1,e-1}
\] 
\[+ \sqrt{2\beta n} (K_{2j'}) \left[ (M_\ell + m_f)(M_\ell + m_f) \left( \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q_2^2}} I_{n-1,e-1} I'_{n,e} - 2\beta \sqrt{\ell'\ell} \left( \frac{(q\Lambda j') + i(q\varphi j')}{\sqrt{q_2^2}} I_{n,e-1} I'_{n-1,e-1} \right) \right) \right]
\] 
\[-\sqrt{2\beta n} (K_{2j'}) [2\beta \sqrt{\ell'\ell} \left( \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_2^2}} I_{n-1,e-1} I'_{n,e-1} - (M_\ell + m_f)(M_\ell + m_f) \left( \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_2^2}} I_{n,e-1} I'_{n-1,e-1} \right) \right)
\] 
\[-2\beta \sqrt{\ell} (M_\ell + m_f) \frac{(q\Lambda j') + i(q\varphi j')} {\sqrt{q_2^2}_q} \frac{(q'\Lambda j') + i(q'\varphi j')} {\sqrt{q_2^2}} I_{n,e-1} I'_{n-1,e-1} \right] \right\}.
\]
10. In the case where \( j \) and \( j' \) are pseudovector currents \( (k = k' = A) \) we obtain

\[
\mathcal{R}^{±+}_{±A} = g_a g'_a \left\{ 2\beta \sqrt{\ell' \ell} \left[ (P \tilde{A} j')(K_1, j) + (P \tilde{A} j)(K_1, j') - (j \tilde{A} j')(K_1, P) + m_f [(j \tilde{A} j')K_3 + (j \tilde{A} j')K_4] \right] \right.
\]

\[
\times \mathcal{I}_{n-1, \ell-1} I_{n-1, \ell} + \sqrt{\ell'}(M_\ell + m_f) \mathcal{I}_{n-1, \ell-1, \ell} I_{n-1, \ell} + \sqrt{\ell'}(M_\ell + m_f) \mathcal{I}_{n, \ell-1} I_{n-1, \ell-1} \bigg[ \sqrt{\ell}(M_\ell + m_f) \mathcal{I}_{n-1, \ell-1} I_{n, \ell} 
\]

\[
- \frac{2\beta \ell}{q_+^2} (M_\ell + m_f) [(q' \Lambda j') - i(q' \varphi j')][(P \tilde{A} j')K_3 - (P \tilde{A} j')K_4 + m_f(K_1, j')(I_{n, \ell-1} I_{n, \ell}) \bigg]
\]

\[
- \frac{2\beta \ell}{q_+^2} (M_\ell + m_f) [(q' \Lambda j') + i(q' \varphi j')][(P \tilde{A} j')K_3 - (P \tilde{A} j')K_4 + m_f(K_1, j')I_{n-1, \ell} I_{n-1, \ell} - 1] \bigg]
\]

\[
+ \left( M_\ell + m_f \right) \left( M_f + m_f \right) [(j \tilde{A} j') + i(j \varphi j')][(K_1, P) + m_f K_3] \left( \frac{(q \Lambda q') - i(q' \varphi q')}{\sqrt{q_+^2 q_+^2}} \right) \mathcal{I}_{n, \ell-1} I_{n, \ell}
\]

\[
+ 2\beta \sqrt{\ell'} [(j \tilde{A} j') - i(j \varphi j')][(K_1, P) + m_f K_3] \left( \frac{(q \Lambda q') + i(q' \varphi q')}{\sqrt{q_+^2 q_+^2}} \right) \mathcal{I}_{n-1, \ell} I_{n-1, \ell}
\]

\[
- \frac{2\beta n}{(K_1, j')} [2\beta \sqrt{\ell' \ell} \left( \frac{(q' \Lambda j') - i(q' \varphi j')}{\sqrt{q_+^2}} \mathcal{I}_{n-1, \ell} I_{n, \ell} + (M_\ell + m_f)(M_\ell + m_f) \right) \left( \frac{(q' \Lambda j') + i(q' \varphi j')}{\sqrt{q_+^2}} \mathcal{I}_{n, \ell-1} I_{n-1, \ell} - 1 \right]
\]

\[
- \frac{2\beta n}{(K_1, j')} [(M_\ell + m_f)(M_\ell + m_f) \left( \frac{(q \Lambda j) - i(q \varphi j)}{\sqrt{q_+^2}} \mathcal{I}_{n, \ell-1} I_{n-1, \ell} - 1 \right] + 2\beta \sqrt{\ell'} \left( \frac{(q \Lambda j)}{\sqrt{q_+^2}} \mathcal{I}_{n, \ell} I_{n-1, \ell} - 1 \right)
\]

\[
+ 2\beta \sqrt{\ell' \ell} \left( \frac{(q \Lambda j) + i(q \varphi j)}{\sqrt{q_+^2}} \mathcal{I}_{n, \ell-1} I_{n-1, \ell} \right) \right\};
\]
\[ R_{AA}^+ = i g_\alpha g_\beta \left\{ \sqrt{2 \beta l}(M_\xi + m_f) \left[ (P\bar{\Lambda}j')(K_j) + (P\bar{\Lambda}j)(K_j') - (j\bar{\Lambda}j')(K_2P) \right] \\
+ m_f[(j\bar{\Lambda}j')\mathcal{K}_4 + (j\bar{\Lambda}j')\mathcal{K}_3] \mathcal{I}_{n,e}^\prime \mathcal{I}_{n,e} - \sqrt{2 \beta l}(M_\xi + m_f) \left[ (P\bar{\Lambda}j')(K_j) + (P\bar{\Lambda}j)(K_j') \right] \\
- (j\bar{\Lambda}j')(K_2P) - m_f[(j\bar{\Lambda}j')\mathcal{K}_4 + (j\bar{\Lambda}j')\mathcal{K}_3] \mathcal{I}_{n-1,e-1}^\prime \mathcal{I}_{n-1,e-1} + \sqrt{2 \beta n}[(j\bar{\Lambda}j')\mathcal{K}_4 + (j\bar{\Lambda}j')\mathcal{K}_3] \mathcal{I}_{n-1,e-1}^\prime \mathcal{I}_{n-1,e-1} \\
\times [(M_\xi + m_f)(M_\xi + m_f)\mathcal{I}_{n-1,e-1}^\prime \mathcal{I}_{n,e} - 2 \beta \sqrt{\ell l^\prime} \mathcal{I}_{n,e}^\prime \mathcal{I}_{n-1,e-1}^\prime] \\
- (M_\xi + m_f)(M_\xi + m_f)\left( \frac{g^\prime \Lambda j'}{\sqrt{q_{1\prime}}} + i g^\prime \varphi j' \right) \left[ (P\bar{\Lambda}j)\mathcal{K}_4 - (P\bar{\Lambda}j)\mathcal{K}_3 + m_f(K_2j')\mathcal{I}_{n,e}^\prime \mathcal{I}_{n,e} \\
+ \frac{2 \beta \sqrt{\ell l^\prime}}{\sqrt{q_{1\prime}}} [(q\Lambda j) - i(q\varphi j)][(P\bar{\Lambda}j)\mathcal{K}_4 + (P\bar{\Lambda}j)\mathcal{K}_3 + m_f(K_2j')\mathcal{I}_{n,e}^\prime \mathcal{I}_{n,e}] \\
+ \frac{2 \beta \sqrt{\ell l^\prime}}{\sqrt{q_{1\prime}}} [(q\Lambda j') - i(q\varphi j')][P\bar{\Lambda}j)\mathcal{K}_4 - (P\bar{\Lambda}j)\mathcal{K}_3 - m_f(K_2j')\mathcal{I}_{n,e}^\prime \mathcal{I}_{n,e}] \\
- (M_\xi + m_f)(M_\xi + m_f)\left( \frac{g^\prime \Lambda j'}{\sqrt{q_{1\prime}}} + i g^\prime \varphi j' \right) \left[ (P\bar{\Lambda}j)\mathcal{K}_4 - (P\bar{\Lambda}j)\mathcal{K}_3 + m_f(K_2j')\mathcal{I}_{n,e}^\prime \mathcal{I}_{n,e} \\
- \sqrt{2 \beta l}(M_\xi + m_f)[(j\bar{\Lambda}j') + i(j\varphi j')][K_2P] + m_f\mathcal{K}_4[\frac{g\Lambda q'}{\sqrt{q_{1\prime}}} + i(q\varphi q') \mathcal{I}_{n,e}^\prime \mathcal{I}_{n,e} \\
+ \sqrt{2 \beta l}(M_\xi + m_f)[(j\bar{\Lambda}j') - i(j\varphi j')][K_2P] - m_f\mathcal{K}_4[\frac{g\Lambda q'}{\sqrt{q_{1\prime}}} + i(q\varphi q') \mathcal{I}_{n,e}^\prime \mathcal{I}_{n,e} \\
+ \sqrt{2 \beta l}(M_\xi + m_f)[(j\bar{\Lambda}j') - i(j\varphi j')][K_2P] + m_f\mathcal{K}_4[\frac{g\Lambda q'}{\sqrt{q_{1\prime}}} + i(q\varphi q') \mathcal{I}_{n,e}^\prime \mathcal{I}_{n,e} \\
- \sqrt{2 \beta n}(K_2j)[\sqrt{\ell l}(M_\xi + m_f)[g\Lambda j') - i(g\varphi j') \mathcal{I}_{n-1,e}^\prime \mathcal{I}_{n,e} - \sqrt{\ell l}(M_\xi + m_f)[g\Lambda j') + i(g\varphi j') \mathcal{I}_{n-1,e}^\prime \mathcal{I}_{n-1,e}] \\
+ 2 \beta \sqrt{\ell l}(M_\xi + m_f)[(g\Lambda j') - i(g\varphi j') \mathcal{I}_{n-1,e}^\prime \mathcal{I}_{n-1,e} - \sqrt{\ell l}(M_\xi + m_f)[g\Lambda j') + i(g\varphi j') \mathcal{I}_{n-1,e}^\prime \mathcal{I}_{n-1,e}] \\
- \sqrt{2 \beta n}\mathcal{K}_4[2 \beta \sqrt{\ell l} \frac{g\Lambda j) - i(g\varphi j)}{\sqrt{q_{1\prime}}} \mathcal{I}_{n-1,e}^\prime \mathcal{I}_{n-1,e} - \sqrt{\ell l}(M_\xi + m_f)[(g\Lambda j) + i(g\varphi j) \mathcal{I}_{n-1,e}^\prime \mathcal{I}_{n-1,e} \\
- \sqrt{2 \beta n}\mathcal{K}_4[2 \beta \sqrt{\ell l} \frac{(g\Lambda j) - i(g\varphi j)}{\sqrt{q_{1\prime}}} \mathcal{I}_{n-1,e}^\prime \mathcal{I}_{n-1,e} - \sqrt{\ell l}(M_\xi + m_f)[(g\Lambda j) + i(g\varphi j) \mathcal{I}_{n-1,e}^\prime \mathcal{I}_{n-1,e} \\
- (M_\xi + m_f)(M_\xi + m_f)\left( \frac{g\Lambda j) + i(g\varphi j)}{\sqrt{q_{1\prime}}} \mathcal{I}_{n-1,e}^\prime \mathcal{I}_{n-1,e}] \right) \right\} : \]
\[ R_{\hat{A}A} = ig_q \left\{ -\sqrt{2\beta\ell} (M_\ell + m_f) \left[ (\hat{P}\hat{A} j') (K_{2j}) + (\hat{P}\hat{A} j') (K_{2j'}) - (j\hat{A} j') (K_{2P}) \right] 
- m_f [(j\hat{A} j') K_4 + (j\hat{\varphi} j') K_3] I_{n,\ell} I_{n,\ell} + \sqrt{2\beta\ell'} (M_\ell + m_f) \left[ (\hat{P}\hat{A} j') (K_{2j}) + (\hat{P}\hat{A} j') (K_{2j'}) \right] 
- (j\hat{A} j') (K_{2P}) + m_f [(j\hat{A} j') K_4 + (j\hat{\varphi} j') K_3] I_{n-1,\ell-1} I_{n-1,\ell-1} \right\} (\hat{P}\hat{A} j') (K_{2j}) + (\hat{P}\hat{A} j') (K_{2j'}) \right] 
- (j\hat{A} j') (K_{2P}) + m_f [(j\hat{A} j') K_4 + (j\hat{\varphi} j') K_3] I_{n-1,\ell-1} I_{n-1,\ell-1} \right\} I_{n-1,\ell-1} I_{n-1,\ell-1} \right] \right) 
+ \frac{2\beta\sqrt{\ell\ell'}}{q^2} \left[ (q'\Lambda j') + i(q'\varphi j') \right] [(P\hat{A}j) K_4 - (P\hat{\varphi}j) K_3 - m_f (K_{2j'})] I_{n,\ell} I_{n,\ell} 
- (M_\ell + m_f) (M_\ell + m_f) \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q^2 q'^2}} \left[(P\hat{A} j') K_4 + (P\hat{\varphi} j') K_3 - m_f (K_{2j'}) \right] I_{n,\ell} I_{n,\ell} 
- (M_\ell + m_f) (M_\ell + m_f) \frac{(q\Lambda j') - i(q\varphi j')}{\sqrt{q^2 q'^2}} \left[(P\hat{A} j') K_4 + (P\hat{\varphi} j') K_3 - m_f (K_{2j'}) \right] I_{n-1,\ell-1} I_{n-1,\ell-1} 
+ \frac{2\beta\sqrt{\ell\ell'}}{q^2} \left[ (q'\Lambda j') + i(q'\varphi j') \right] [(P\hat{A} j') K_4 + (P\hat{\varphi} j') K_3 + m_f (K_{2j'})] I_{n-1,\ell-1} I_{n-1,\ell} \n+ \sqrt{2\beta\ell'} (M_\ell + m_f) [(j A j') + i(j \varphi j')] \left[(K_{2P}) - m_f K_4 \right] \frac{(q\Lambda q') - i(q q')}{\sqrt{q^2 q'^2}} I_{n,\ell} I_{n,\ell} 
- \frac{2\beta\ell'}{q^2} (M_\ell + m_f) [(j A j') + i(j \varphi j')] \left[(K_{2P}) + m_f K_4 \right] \frac{(q\Lambda q') + i(q q')}{\sqrt{q^2 q'^2}} I_{n-1,\ell} I_{n-1,\ell} 
+ 2\beta\sqrt{\ell} (M_\ell + m_f) \frac{(q A j') - i(q \varphi j')}{\sqrt{q^2}} I_{n-1,\ell} I_{n,\ell} - \sqrt{\ell'} (M_\ell + m_f) \frac{(q A j') + i(q \varphi j')}{\sqrt{q'^2}} I_{n-1,\ell-1} I_{n,\ell-1} \right\} 
- 2\beta\sqrt{\ell} (M_\ell + m_f) \frac{(q A j') - i(q \varphi j')}{\sqrt{q^2}} I_{n-1,\ell} I_{n,\ell-1} - \sqrt{\ell'} (M_\ell + m_f) \frac{(q A j') + i(q \varphi j')}{\sqrt{q'^2}} I_{n-1,\ell} I_{n-1,\ell} \right\} 
+ \frac{2\beta}{\sqrt{\ell}} (M_\ell + m_f) \frac{(q A j') + i(q \varphi j')}{\sqrt{q^2}} \left[(K_{2j'}) + (K_{2j'}) \right] I_{n-1,\ell} I_{n-1,\ell} \right\} 
- 2\sqrt{\ell} (M_\ell + m_f) \frac{(q A j') + i(q \varphi j')}{\sqrt{q^2}} \left[(K_{2j'}) + (K_{2j'}) \right] I_{n-1,\ell} I_{n-1,\ell} \right\} \right) ; \]
\[ R_{AA} = g_a g_a^* \left\{ (M_e + m_f)(M_{e'} + m_f) \left[ (P\tilde{A} j')(\mathcal{K}_1 j) + (P\tilde{A} j)(\mathcal{K}_1 j') - (j\tilde{A} j')(\mathcal{K}_1 P) \right] - m_f[(j\tilde{A} j')(\mathcal{K}_3 + (j\tilde{A} j')(\mathcal{K}_4)] I_{n',e} I_{n,e} + 2\sqrt{\ell\ell'} \left[ (P\tilde{A} j)(\mathcal{K}_1 j) + (P\tilde{A} j)(\mathcal{K}_1 j') \right] - (j\tilde{A} j')(\mathcal{K}_1 P) + m_f[(j\tilde{A} j')(\mathcal{K}_3 + (j\tilde{A} j')(\mathcal{K}_4)] \right\} I_{n-1,e' -1} I_{n-1,e}
\]

\[ + 2\sqrt{\ell\ell'}[(j\tilde{A} j')(\mathcal{K}_3 + (j\tilde{A} j')(\mathcal{K}_4)] \left[ \sqrt{\ell'}(M_e + m_f) I_{n-1,e' -1} I_{n,e} + \sqrt{\ell}(M_{e'} + m_f) I_{n,e} I_{n-1,e} -1 \right] \]

\[ - \sqrt{\frac{2\beta\ell\ell'}{q_{12}^2}} (M_e + m_f)[(q'\Lambda j') + i(q'\varphi j')]\left[(P\tilde{A} j)\mathcal{K}_3 - (P\tilde{A} j)\mathcal{K}_4 - m_f(\mathcal{K}_1 j)] I_{n',e} I_{n,e} \]

\[ - \sqrt{\frac{2\beta\ell'}{q_{12}^2}} (M_{e'} + m_f)[(q\Lambda j) - i(q\varphi j)]\left[(P\tilde{A} j')\mathcal{K}_3 + (P\tilde{A} j')\mathcal{K}_4 - m_f(\mathcal{K}_1 j') I_{n',e} I_{n-1,e} \]

\[ - \sqrt{\frac{2\beta\ell'}{q_{12}^2}} (M_{e'} + m_f)[(q\Lambda j') - i(q\varphi j')]\left[(P\tilde{A} j')\mathcal{K}_3 + (P\tilde{A} j')\mathcal{K}_4 + m_f(\mathcal{K}_1 j') I_{n-1,e' -1} I_{n-1,e} \]

\[ + 2\beta\sqrt{\ell\ell'}[(j\tilde{A} j') + i(j\varphi j')][(\mathcal{K}_1 P) - m_f\mathcal{K}_3]\frac{(q\Lambda q') - i(q\varphi q')}{\sqrt{q_{12}^2}} I_{n',e' -1} I_{n,e} \]

\[ + (M_e + m_f)(M_{e'} + m_f)[(j\tilde{A} j') - i(j\varphi j')][(\mathcal{K}_1 P) + m_f\mathcal{K}_3]\frac{(q\Lambda q') + i(q\varphi q')}{\sqrt{q_{12}^2}} I_{n-1,e' -1} I_{n-1,e} \]

\[ - \sqrt{2\beta n}(\mathcal{K}_1 j)[(M_e + m_f)(M_{e'} + m_f)\frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_{12}^2}} I_{n-1,e' -1} I_{n,e} + 2\beta\sqrt{\ell\ell'}\frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_{12}^2}} I_{n',e' -1} I_{n-1,e} \]

\[ + \sqrt{2\beta n}(\mathcal{K}_1 j')\left[ 2\beta\sqrt{\ell\ell'}\frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_{12}^2}} I_{n-1,e' -1} I_{n,e} - (M_e + m_f)(M_{e'} + m_f)\frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_{12}^2}} I_{n,e} I_{n-1,e} \right] \]

\[ + 2\beta\sqrt{\mathcal{K}_3}\left[ \sqrt{\ell}(M_e + m_f)\frac{(q\Lambda j) - i(q\varphi j)}{\sqrt{q_{12}^2}} \frac{(q'\Lambda j') - i(q'\varphi j')}{\sqrt{q_{12}^2}} I_{n',e' -1} I_{n-1,e} \right] \]

\[ \sqrt{\ell} (M_e + m_f)\frac{(q\Lambda j) + i(q\varphi j)}{\sqrt{q_{12}^2}} \frac{(q'\Lambda j') + i(q'\varphi j')}{\sqrt{q_{12}^2}} I_{n',e' -1} I_{n-1,e} \right] \]

For second diagram we have the following replacement: \[ P_{\alpha} \rightarrow P_{\alpha}', q_{\alpha} \leftrightarrow -q_{\alpha}', j_{\alpha} \leftrightarrow j_{\alpha}' I_{m,n} \leftrightarrow I_{m,n}'. \]

### III. Ground Landau Level

The obtained results can be essentially simplified in several special cases. In the present section we consider the strong field limit, where the magnetic field strength \( B \) is the maximal physical parameter, namely, \( \sqrt{\ell B} \gg \omega, E, \) etc. In this case \( n, \ell, \ell' = 0, M_e = M_{e'} = m_f \) and we obtain the following expressions for the amplitudes,
where

\[ R_{0SS}^{(1)} = g_s g'_s j_s j'_s (\langle K_1 P \rangle + m_f \Lambda_3) \quad (90) \]
\[ R_{0SS}^{(2)} = g_s g'_s j_s j'_s (\langle K_1 P' \rangle + m_f \Lambda_3) \quad (91) \]
\[ R_{0PS}^{(1)} = g_s g'_p j_s j'_p (\langle K_2 P \rangle - m_f \Lambda_4) \quad (92) \]
\[ R_{0PS}^{(2)} = -g_s g'_p j_s j'_p (\langle K_2 P' \rangle + m_f \Lambda_4) \quad (93) \]
\[ R_{0SV}^{(1)} = g_s g'_s j_s (\langle P \Lambda j' \rangle \Lambda_3 + (P \bar{\phi} j') \Lambda_4 + m_f (\Lambda_4 j')) \quad (94) \]
\[ R_{0SV}^{(2)} = g_s g'_s j_s (\langle P \bar{\Lambda} j' \rangle \Lambda_3 - (P' \bar{\phi} j') \Lambda_4 + m_f (\Lambda_4 j')) \quad (95) \]
\[ R_{0SA}^{(1)} = g_s g'_s j_s (\langle P \Lambda j' \rangle \Lambda_4 - (P' \bar{\phi} j') \Lambda_3 + m_f (\Lambda_4 j')) \quad (96) \]
\[ R_{0SA}^{(2)} = g_s g'_s j_s (\langle P \bar{\Lambda} j' \rangle \Lambda_4 + (P' \phi j') \Lambda_3 - m_f (\Lambda_4 j')) \quad (97) \]
\[ R_{0AP}^{(1)} = -g_p g'_a j_p j'_p (\langle K_1 P \rangle - m_f \Lambda_3) \quad (98) \]
\[ R_{0AP}^{(2)} = -g_p g'_a j_p j'_p (\langle K_1 P' \rangle - m_f \Lambda_3) \quad (99) \]
\[ R_{0PV}^{(1)} = -g_p g'_a j_p j'_p (\langle P \Lambda j' \rangle \Lambda_4 + (P \bar{\phi} j') \Lambda_3 + m_f (\Lambda_4 j')) \quad (100) \]
\[ R_{0PV}^{(2)} = g_p g'_a j_p j'_p (\langle P \bar{\Lambda} j' \rangle \Lambda_4 - (P' \phi j') \Lambda_3 - m_f (\Lambda_4 j')) \quad (101) \]
\[ R_{0AP}^{(1)} = -g_p g'_a j_p (\langle P \Lambda j' \rangle \Lambda_3 + (P \bar{\phi} j') \Lambda_4 - m_f (\Lambda_4 j')) \quad (102) \]
\[ R_{0AP}^{(2)} = -g_p g'_a j_p (\langle P \bar{\Lambda} j' \rangle \Lambda_3 - (P' \phi j') \Lambda_4 + m_f (\Lambda_4 j')) \quad (103) \]
\[ R_{0VV}^{(1)} = g_v g'_v \left\{ (P \Lambda j')(\Lambda_1 j) + (P \bar{\Lambda} j)(\Lambda_1 j') \right\} + m_f (\Lambda_3 + (j \bar{\phi} j') \Lambda_4) \quad (104) \]
\[ R_{0VV}^{(2)} = g_v g'_v \left\{ (P' \bar{\Lambda} j')(\Lambda_1 j) + (P' \Lambda j)(\Lambda_1 j') \right\} - m_f (\Lambda_3 + (j \bar{\phi} j') \Lambda_4) \quad (105) \]
\[ R_{0VV}^{(1)} = g_v g'_v \left\{ (P \Lambda j')(\Lambda_1 j) + (P \bar{\Lambda} j)(\Lambda_1 j') \right\} + m_f (\Lambda_3 + (j \bar{\phi} j') \Lambda_4) \quad (106) \]
\[ R_{0VV}^{(2)} = g_v g'_v \left\{ (P' \bar{\Lambda} j')(\Lambda_1 j) + (P' \Lambda j)(\Lambda_1 j') \right\} - m_f (\Lambda_3 + (j \bar{\phi} j') \Lambda_4) \quad (107) \]
\[ R_{0VV}^{(1)} = g_v g'_v \left\{ (P \Lambda j')(\Lambda_1 j) + (P \bar{\Lambda} j)(\Lambda_1 j') \right\} + m_f (\Lambda_3 + (j \bar{\phi} j') \Lambda_4) \quad (108) \]
\[ R_{0VV}^{(2)} = g_v g'_v \left\{ (P' \bar{\Lambda} j')(\Lambda_1 j) + (P' \Lambda j)(\Lambda_1 j') \right\} - m_f (\Lambda_3 + (j \bar{\phi} j') \Lambda_4) \quad (109) \]

We note that the obtained results for \( M_{\bar{\nu}V} \) and \( M_{\nu A} \) exactly coincide with the amplitude of the photo-neutrino process from Ref. [30] (see also [18]) after taking account of the second diagram and of the momentum conservation law.

**IV. FORWARD SCATTERING**

For generalization of the results obtained in Ref. [11], to the case of magnetized plasma we consider the process of a coherent scattering of the generalized current \( j \) off the real fermions without change of their states (the “forward” scattering). In this case, under the generalized current \( j \) in the initial state we mean only the field operator of a single particle, while the generalized current \( j' \) in the final state could be both the field operator of a single particle and e.g. the neutrino current. In this case: \( s = s', q^\mu = q'^\nu, p^\mu = p'^\nu, \chi_{\alpha \beta} = 2(p \Lambda), K_{2 \alpha} = 2(\bar{\phi} p)^\alpha, K_3 = 2M, K_4 = 0 \). We obtain the following results for the amplitudes:

\[ M_{k \rightarrow k} = -\frac{\beta}{2\pi^2} \sum_{\ell, n=0}^\infty \int \frac{dp_q}{E_f} f_f(E_f) \]
\[ \times \left\{ \frac{\mathcal{P}_{k \rightarrow k}^{(1)}(p + q)^2 - m_f^2 - 2\beta n}{(p + q)^2 - m_f^2 - 2\beta n} + \frac{\mathcal{P}_{k \rightarrow k}^{(2)}(p - q)^2 - m_f^2 - 2\beta n}{(p - q)^2 - m_f^2 - 2\beta n} \right\} \]

where \( f_f(E_f) = [1 + \exp(E_f - \mu_f)/T]^{-1} \) is the fermion distribution function, \( T \) and \( \mu_f \) are the temperature and
the chemical potential of plasma correspondingly,

\[ D_{SS}^{(1)} = g_s g_{\rho j' j} \left\{ [(q \tilde{\lambda} p) + 2\beta \ell + 2m_s^2] \times (\mathcal{I}_{n,\ell}^2 + \mathcal{I}_{n-1,\ell-1}^2) - 4\beta \sqrt{n \ell} \mathcal{I}_{n,\ell} \mathcal{I}_{n-1,\ell-1} \right\}; \]

\[ D_{SS}^{(2)} = D_{SS}^{(1)}(q \rightarrow -q); \]

\[ D_{SP}^{(1)} = g_s g_{\rho j' j p} (q \tilde{\varphi} p) \left[ \mathcal{I}_{n,\ell}^2 - \mathcal{I}_{n-1,\ell-1}^2 \right]; \]

\[ D_{SP}^{(2)} = D_{SP}^{(1)}(q \rightarrow -q); \]

\[ D_{VS}^{(1)} = g_s g_{\rho j' j} m_f \left\{ 2(p \tilde{\varphi} j') + (q \tilde{\lambda} j') \right\} \left[ \mathcal{I}_{n,\ell}^2 + \mathcal{I}_{n-1,\ell-1}^2 \right]; \]

\[ D_{VS}^{(2)} = D_{VS}^{(1)}(q \rightarrow -q); \]

\[ D_{AP}^{(1)} = -g_p g_{\rho j' j p} \left\{ [(q \tilde{\lambda} p) + 2\beta \ell] \left[ \mathcal{I}_{n,\ell}^2 + \mathcal{I}_{n-1,\ell-1}^2 \right] - 4\beta \sqrt{n \ell} \mathcal{I}_{n,\ell} \mathcal{I}_{n-1,\ell-1} \right\}; \]

\[ D_{AP}^{(2)} = D_{AP}^{(1)}(q \rightarrow -q); \]

\[ D_{PA}^{(1)} = -g_p g_{\rho j' j p} m_f \left\{ (q \tilde{\lambda} j') \left[ \mathcal{I}_{n,\ell}^2 + \mathcal{I}_{n-1,\ell-1}^2 \right] \right\}; \]

\[ D_{PA}^{(2)} = D_{PA}^{(1)}(q \rightarrow -q); \]
\[ D_{VV}^{(1)} = g_v g' \left\{ [(p\bar{\Lambda}j)(P\bar{\Lambda}j') + (P\bar{\Lambda}j)(p\bar{\Lambda}j') - (j\bar{\Lambda}j')(q\bar{\Lambda}j') - m_0^2 (j\bar{\Lambda}j')] [I_{2,n,\ell}^2 - I_{2,n-1,\ell-1}^2] \right\} \]

\[ + 4\sqrt{\beta \ell} (j\bar{\Lambda}j')I_{n,\ell}I_{n-1,\ell-1} + \frac{2\beta \ell}{q_2^2} \left[ [(P\bar{\Lambda}j)(q\Lambda j') + i(q\varphi j') + (P\bar{\Lambda}j')(q\Lambda j) - i(q\varphi j)] \right] I_{n,\ell}I_{n-1,\ell} \]

\[ - \frac{2\beta \ell}{q_2^2} \left[ [(P\bar{\Lambda}j)(q\Lambda j') - i(q\varphi j') + (P\bar{\Lambda}j')(q\Lambda j) + i(q\varphi j)] \right] I_{n-1,\ell-1}I_{n-1,\ell} \]

\[ - \frac{2\beta n}{q_2^2} \left[ [(p\bar{\Lambda}j)(q\Lambda j') - i(q\varphi j') + (p\bar{\Lambda}j')(q\Lambda j) - i(q\varphi j)] \right] I_{n,\ell}I_{n-1,\ell} \]

\[ - \frac{2\beta n}{q_2^2} \left[ [(p\bar{\Lambda}j)(q\Lambda j') + i(q\varphi j') + (p\bar{\Lambda}j')(q\Lambda j) - i(q\varphi j)] \right] I_{n-1,\ell-1}I_{n-1,\ell} + [2\beta \ell + (p\bar{\Lambda}q)] \]

\[ \times [(j\Lambda j') + i(j\varphi j')]I_{2,n-1}^2 + [(j\Lambda j') - i(j\varphi j')]I_{2,n-1,\ell}^2 + \frac{4\sqrt{\beta \ell}}{q_2^2} [(q\Lambda j)(q\Lambda j') - (q\varphi j)(q\varphi j')]I_{n,\ell}I_{n-1,\ell} \}

\[ D_{VV}^{(2)} = D_{VV}^{(1)} (q \rightarrow -q, j \leftrightarrow j'); \]

\[ D_{AV}^{(1)} = g_v g'_n \left\{ [(P\bar{\Lambda}j)(j'\bar{\varphi}p) + (P\bar{\Lambda}j')(j'\bar{\varphi}p) - (j\bar{\Lambda}j')(q\bar{\varphi}p) - m_0^2 (j\bar{\Lambda}j')] [I_{2,n,\ell}^2 - I_{2,n-1,\ell-1}^2] \right\} \]

\[ + \frac{2\beta \ell}{q_2^2} \left[ [(P\bar{\varphi}j)(q\Lambda j') + i(q\varphi j') + (P\bar{\varphi}j')(q\Lambda j) - i(q\varphi j)] \right] I_{n,\ell}I_{n-1,\ell-1} \]

\[ - \frac{2\beta \ell}{q_2^2} \left[ [(P\bar{\varphi}j)(q\Lambda j') - i(q\varphi j') + (P\bar{\varphi}j')(q\Lambda j) + i(q\varphi j)] \right] I_{n-1,\ell-1}I_{n-1,\ell} \]

\[ + \frac{2\beta n}{q_2^2} \left[ [(p\bar{\varphi}j)(q\Lambda j') - i(q\varphi j') + (p\bar{\varphi}j')(q\Lambda j) - i(q\varphi j)] \right] I_{n,\ell}I_{n-1,\ell} \]

\[ - \frac{2\beta n}{q_2^2} \left[ [(p\bar{\varphi}j)(q\Lambda j') + i(q\varphi j') + (p\bar{\varphi}j')(q\Lambda j) - i(q\varphi j)] \right] I_{n-1,\ell-1}I_{n-1,\ell} \]

\[ + (p\bar{\varphi}q) [(j\Lambda j') + i(j\varphi j')]I_{2,n-1}^2 - [(j\Lambda j') - i(j\varphi j')]I_{2,n-1,\ell}^2 \} ; \]
\[ \mathcal{D}^{(2)}_{V_A} = g_\nu g^\nu_a \left\{ (P^* \bar{\Lambda} j)(j' \bar{\varphi} p) + (P^* \bar{\Lambda} j')(j \bar{\varphi} p) + (j \bar{\Lambda} j')(q \bar{\varphi} p) - m_f^2 (j \bar{\varphi} j') \right\} \left[ \mathcal{I}^2_{n,\ell} - \mathcal{I}^2_{n-1,\ell-1} \right] \]

\[ - \sqrt{\frac{2\beta}{q^2_\perp}} \left[ (P^* \bar{\varphi} j')(q\Lambda j) + i(q\varphi j) + (P^* \bar{\varphi} j)[(q\Lambda j') - i(q\varphi j')] \right] \mathcal{I}_{n,\ell} \mathcal{I}_{n,\ell-1} \]

\[ + \sqrt{\frac{2\beta}{q^2_\perp}} \left[ (P^* \bar{\varphi} j'(q\Lambda j) - i(q\varphi j) + (P^* \bar{\varphi} j)[(q\Lambda j') + i(q\varphi j')] \right] \mathcal{I}_{n-1,\ell-1} \mathcal{I}_{n,\ell} \]

\[ - \sqrt{\frac{2\beta n}{q^2_\perp}} \left[ (p \bar{\varphi} j'(q\Lambda j) - i(q\varphi j)) + (p \bar{\varphi} j')[(q\Lambda j) + i(q\varphi j)] \right] \mathcal{I}_{n,\ell} \mathcal{I}_{n-1,\ell} \]

\[ + \sqrt{\frac{2\beta n}{q^2_\perp}} \left[ (p \bar{\varphi} j')[(q\Lambda j') - i(q\varphi j']) + (p \bar{\varphi} j) [(q\Lambda j) + i(q\varphi j)] \right] \mathcal{I}_{n-1,\ell-1} \mathcal{I}_{n,\ell-1} \]

\[ - (p \bar{\varphi} q)[(j \Lambda j') - i(j \varphi j') \mathcal{I}^2_{n,\ell-1} - [(j \Lambda j') + i(j \varphi j') \mathcal{I}^2_{n-1,\ell}] \} ; \]

\[ \mathcal{D}^{(1)}_{AA} = g_n g^a \left\{ (P\bar{\Lambda} j)(p\Lambda j') + (p\Lambda j)(P\bar{\Lambda} j') - (j \bar{\Lambda} j')(M_f^2 + m^2 + (p\bar{\Lambda} q))[\mathcal{I}^2_{n,\ell} + \mathcal{I}^2_{n-1,\ell-1}] \right\} \]

\[ + 4\sqrt{\beta n} \mathcal{I}_{n,\ell} \mathcal{I}_{n-1,\ell-1} - \sqrt{\frac{2\beta}{q^2_\perp}} \left[ (P\bar{\Lambda} j)[(q\Lambda j') + i(q\varphi j')] + (P\bar{\Lambda} j')[(q\Lambda j) - i(q\varphi j)] \right] \mathcal{I}_{n,\ell} \mathcal{I}_{n,\ell-1} \]

\[ - \sqrt{\frac{2\beta n}{q^2_\perp}} \left[ (p\Lambda j')[(q\Lambda j') - i(q\varphi j')] + (p\Lambda j)'[(q\Lambda j) + i(q\varphi j)] \right] \mathcal{I}_{n,\ell} \mathcal{I}_{n-1,\ell} \]

\[ - \sqrt{\frac{2\beta n}{q^2_\perp}} \left[ (p\Lambda j)[(q\Lambda j') + i(q\varphi j')] + (p\Lambda j')[q\Lambda j) - i(q\varphi j)] \right] \mathcal{I}_{n-1,\ell-1} \mathcal{I}_{n,\ell-1} \]

\[ + (M_f^2 + m^2 + (p\bar{\Lambda} q)) \]

\[ \times [(j \Lambda j') + i(j \varphi j') \mathcal{I}^2_{n,\ell-1} + [(j \Lambda j') - i(j \varphi j') \mathcal{I}^2_{n-1,\ell}] + \frac{4\sqrt{\beta n}}{q^2_\perp} [(q\Lambda j)(q\Lambda j') - (q\varphi j)(q\varphi j')] \mathcal{I}_{n,\ell-1} \mathcal{I}_{n-1,\ell} \} ; \]

\[ \mathcal{D}^{(2)}_{AA} = \mathcal{D}^{(1)}_{AA}(q \rightarrow -q, j \leftrightarrow j') . \]

We notice, that the expressions for amplitudes \( \mathcal{M}_{VS}, \mathcal{M}_{VP}, \mathcal{M}_{VV} \) and \( \mathcal{M}_{AV} \) are manifestly gauge invariant.

\[ V. \ \text{DISCUSSION} \]

In this paper, we have calculated the tree-level two-point amplitudes for the transitions \( jf \rightarrow j'f' \) in a constant uniform magnetic field of an arbitrary strength, and in charged fermion plasma, for generalized verticals of the scalar, pseudoscalar, vector or axial types. It is remarkable, that all the amplitudes obtained are manifestly Lorentz invariant, due to the choice of the Dirac equation solutions as the eigenfunctions of the covariant operator \( \mu_\gamma \). In this case, partial contributions to the amplitude from the channels with different fermion polarization states are calculated separately, by direct multiplication of the bispinors and the Dirac matrices. This approach is an alternative to the method where the amplitudes squared are calculated, summed over the fermion polarization states, see, e.g. \[31, 32\].

**Acknowledgements**

The study was supported in part by the Russian Foun-
dation for Basic Research (Project No. 11-02-00394-a).

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