Dual Mode Pseudo-Range Differential Positioning Method

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Abstract. Use the Beidou/GPS dual-mode pseudo-range differential positioning method is used to perform high-precision positioning and navigation for the UAV. The error separation differential positioning method is proposed to solve the problem that the correction difference calculated by the traditional differential positioning method is affected by the change of the baseline length, the reference inter-satellite difference method eliminates the reference station receiver clock error and performs ultra-short baseline and short baseline positioning tests. The positioning accuracy is better than 0.4 meters in the horizontal direction and the elevation is better than 0.5 meters.

1. Introduction
Multi-rotor UAVs are widely used in environmental monitoring, fire rescue, power inspection, sports and shooting, short-distance drone transportation and other fields. Commonly used airborne navigation methods include global satellite navigation, integrated navigation, differential positioning navigation [1], etc. Use global satellite navigation or integrated navigation positioning accuracy are generally in the meter-level accuracy [2], using differential positioning the method can reach the sub-meter level to meet the demand for high-precision positioning of electric power inspection, landing and freight transportation in some special application backgrounds [3] [4]. The traditional position difference method uses the space-time correlation of the receiver to calculate the position correction amount, and the rover receives the data difference decomposition calculation. However, the method is sensitive to the baseline length and requires the satellite pseudo-range observation set used by the base station and the rover to be consistent [5]. The calculated position coordinates contain the same error term, and the obtained error correction number is larger, and the change of the differential correction number is solved. The rate does not reflect the actual change of the differential correction number. Therefore, the error separation differential positioning method is proposed to describe the error term more accurately to improve positioning resolution accuracy. And using the inter-satellite single difference, the virtual reference star is constructed, and the satellite clock difference and the receiver clock difference are eliminated so that the differential rate of change is not affected by the receiver clock hop, even if the differential correction number is not obtained for a long time. The correction result can be corrected using the correction number and the linear difference correction rate of the correction rate.

2. Error separation differential positioning method
The pseudo-range observation equation of the base station is as shown in equation (1) and (2):
\[ \rho_{r_i}^{Gi} = r(p_r, p_{s_i}) + c\delta t_r + c\tau_s + E_{eph} + T_{ion} + T_{tron} + MP_6 + n_r \]  
(1)

\[ \rho_{r_i}^{Bi} = r(p_r, p_{s_i}) + c\delta t_r + c\tau_s + E_{eph} + T_{ion} + T_{tron} + MP_6 + n_r \]  
(2)

\( \rho_{r_i}^{Gi}, \rho_{r_i}^{Bi} \) represents the pseudo-range observation of the i-th satellite of the base station to GPS and Beidou, \( r(p_r - p_{s_i}) \) and \( r(p_r - p_{s_i}) \) are the base stations for different systems respectively. The distance of the i-th satellite, \( p_r, p_s \) represents the position of the base station and the satellite, respectively. \( E_{eph} \) represents the ephemeris error, \( c\delta t_r, c\tau_s \) respectively represents the satellite clock difference and the clock residual, \( T_{ion}, T_{tron} \) represents ionospheric error and tropospheric error, MP represents multipath error, and \( n_r \) represents receiver internal error. The error can be reduced to two types, common error, and unique error. The common error is the error shared by all receivers in the smaller region, including \( c\delta t_r, E_{eph}, T_{ion}, T_{tron} \) can be eliminated by the difference method or reduce. Since the position of the base station is known, the satellite position can be calculated by the ephemeris, so that R and r can also be calculated, thereby obtaining the pseudo-range correction amount of the GPS and the Beidou satellite.

\[ \Delta \rho_{r_i}^{B / Gi} = \rho_{r_i}^{B / Gi} - r(p_r, p_{s_i}) \]  
(3)

\[ \Delta \Delta \rho_{r_i}^{B / Gi} = \Delta \rho_{r_i}^{B / Gi} - \Delta \rho_{r_i}^{B / Gi} \]  
(4)

The above formula calculates the required amount of differential correction. When the base station calculates it, it calculates the rate of change at the same time, and then broadcasts the differential correction number along with its rate of change to the surrounding mobile users [6]. After receiving the differential correction number broadcasted by the base station and its rate of change, the mobile user corrects the pseudo-range at the current time, and then uses the pseudo-range corrected pseudo-range to establish a positioning equation to perform position resolution using least squares. Next, we analyze the impact of the correction number of the broadcast satellite single difference on mobile users. The pseudo-range observation process of the mobile user is as shown in equation (5):

\[ \rho_{u_i}^{B / Gi} = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2 + c\delta t_r + c\tau_s + E_{eph} + T_{tron} + MP_6 + n_r} \]  
(5)

As shown in equation (5), except for the clock difference between the station star distance and the receiver on the right side of the equation, the error amount of the ephemeris error is left. In order to
eliminate this error amount, the subdivision difference correction number is deducted from both sides of the equation (5), that is, the equation (4) is subtracted, and the specific derivation process is as follows:

\[ \rho_a^{\beta / \alpha} - \Delta \rho_u^{\beta / \alpha} = r(p_a, p_\alpha^{\beta / \alpha}) + c\delta t_x + c\tau_x + E_{\text{eph}} + T_{\text{trop}} + M_P_{\beta / \alpha} + n_r \]

\[ - c\Delta t_x + c\tau_x + E_{\text{eph}} + T_{\text{ion}} + T_{\text{trop}} + M_P_{\beta / \alpha} + n_r \]

\[ = r(p_a, p_\alpha^{\beta / \alpha}) + c\delta t_x + E_{\text{eph}} - c\Delta t_x \quad (6) \]

When the pseudo-range is corrected by the differential correction value, the ephemeris error has been eliminated, but the corrected pseudo-range has an ephemeris error of the 1st star, for the ephemeris error (Actually, the ephemeris error and the small residual error are actually a small amount), the mobile station positioning equation is as follows:

\[
\begin{align*}
\rho_1^i &= \sqrt{(x_1 - x_a)^2 + (y_1 - y_a)^2 + (z_1 - z_a)^2} + c\delta t_a - c\Delta t_x \\
\rho_2^i &= \sqrt{(x_2 - x_a)^2 + (y_2 - y_a)^2 + (z_2 - z_a)^2} + c\delta t_a - c\Delta t_x \\
&\vdots \\
\rho_n^i &= \sqrt{(x_n - x_a)^2 + (y_n - y_a)^2 + (z_n - z_a)^2} + c\delta t_a - c\Delta t_x 
\end{align*}
\] \( (7) \)

For equation (10), satellite ephemeris error, ionosphere, troposphere, relativity and other errors have been eliminated at this time, and the carrier phase smoothing pseudo-range has been eliminated, eliminating most of the errors such as pseudo-range observation noise. Therefore, it has become a very "clean" observation at this time [7]. When the positioning equation is solved by least squares, R does not affect the positioning result. The solution error is actually the sum of the receiver clock error and the reference star calendar error. Therefore, the value of the inter-satellite single difference is used as the reference. The station's differential corrections are broadcast to the user and do not affect the rover's positioning results. Of course, if the mobile user needs to calibrate the local time with the positioning time, the difference method will have a certain influence on the timing, and the size of the influence is the reference star calendar error size.

3. virtual reference star construction method

Due to the high angular variation caused by satellite motion, the reference star is frequently changed, and the difficulty of understanding is increased. Therefore, the method of constructing a virtual star is introduced to solve the problem and eliminate the receiver clock difference [8]. Suppose a real satellite has the highest altitude angle. The pseudo-range observation is constructed by all satellite observations in the line of sight. The radial error of the virtual reference star is \( \Delta \rho_u^v \), which is equal to the radial error of the observation satellite within the line of sight. After the virtual reference star is constructed, calculate the differential correction.

\[ \Delta \rho_r^v = \sum_{i=1}^{n} \omega_i (E_{\text{eph}}^i + c\delta t_x) \]

\[ = E_{\text{eph}}^v + c\delta t_x \quad (8) \]

\[ \Delta \Delta \rho_r^i = \Delta \rho_r^i - \Delta \rho_r^v \]

\[ = E_{\text{eph}}^i - E_{\text{eph}}^v + c\Delta t_x \quad (9) \]
After receiving the differential correction amount, the mobile station first corrects the pseudo-range observation error according to the pseudo-range positioning principle, and uses the mathematical model to correct the ionosphere and tropospheric delay, and corrects the satellite clock error according to the broadcast parameters. The differential correction amount and the change rate are linearly calculated to obtain a differential correction number, and the pseudo-range value is corrected by the differential correction amount and the weighted least squares (WLS) solution is performed after subtracting the error [9].

Using the dual-mode pseudo-range differential positioning algorithm to locate the unmanned aerial vehicle in a static state within ten seconds, the positioning results are shown in Table 1. The data shown in the table are the positioning recording time, and the geocentric solid coordinate system is first. The three-dimensional coordinates, as well as the number of effective satellites and the difference decomposition time interval, observed when solving the positioning. From the NS data, the main advantages of dual-mode positioning are that more satellites can be observed under extreme conditions, ensuring stable and reliable positioning, and dual-system fusion positioning provides higher positioning accuracy for the UAV system.

| GPST   | X-ECEF       | Y-ECEF       | Z-ECEF       | NS | Δt |
|--------|--------------|--------------|--------------|----|----|
| 10:15:00 | -2182453.6925 | 5178813.1987 | 3006201.6767 | 16 | 0  |
| 10:15:01 | -2182453.6913 | 5178813.1932 | 3006201.6727 | 16 | 1  |
| 10:15:02 | -2182453.6959 | 5178813.2057 | 3006201.6814 | 16 | 2  |
| 10:15:03 | -2182453.6851 | 5178813.1933 | 3006201.6729 | 16 | 3  |
| 10:15:04 | -2182453.6788 | 5178813.1488 | 3006201.6524 | 16 | 4  |
| 10:15:05 | -2182453.6863 | 5178813.1594 | 3006201.6576 | 17 | 5  |
| 10:15:06 | -2182453.6982 | 5178813.1716 | 3006201.6660 | 17 | 6  |
| 10:15:07 | -2182453.6735 | 5178813.1404 | 3006201.6513 | 16 | 7  |
| 10:15:08 | -2182453.6761 | 5178813.1461 | 3006201.6539 | 17 | 8  |
| 10:15:09 | -2182453.6781 | 5178813.1451 | 3006201.6589 | 17 | 9  |

4. case analysis
In order to better analyze the differential positioning accuracy, the differential positioning result is compared with its precise position. Ultra-short baseline and short baseline verification were carried out, and the reference station was built on the roof of the building. The drones were respectively Flying at a distance of 10 meters around the building for ultra-short baseline acquisition and a baseline distance of 8.8 km. According to Fig. 1, it can be seen that in the case of a baseline of 10 m, the positioning result achieves an Accuracy of 0.4 m in the northeast direction and an accuracy of 0.5 m in the height direction. According to Fig. 2, it can be seen that in the case of the 8.8 km baseline, the positioning accuracy of 0.5 m is achieved in the northeast Direction, and the accuracy in the height direction is 0.8 m.
Figure 1. 10m baseline positioning accuracy

Figure 2. 8.8km baseline positioning accuracy

5. Conclusion
This paper proposes a method based on Beidou/GPS dual-mode airborne pseudo-range differential positioning. The base station adopts the error separation differential positioning method. When the rate of change of the differential correction number is solved, the actual variation of the differential correction number is better reflected, and the solution is obtained. The differential correction number is small in magnitude, and the virtual reference star is constructed by the inter-satellite single-difference method, which eliminates the clock difference of the base station receiver. The actual test shows that the drone receives the correction number to correct the local observation error and improves the positioning accuracy. Its positioning accuracy is decimeter level in real-time dynamic conditions, and static positioning can reach centimeter level in real time. It can effectively meet the demand of high-precision positioning of ordinary multi-rotor unmanned aerial vehicles.
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