Quantum Brownian representation for the quantum field modes

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When analyzing the particle-like excitations in quantum field theory it is natural to regard
the field mode corresponding to the particle momentum as an open quantum system, together
with the opposite momentum mode. Provided that the state of the field is stationary,
homogeneous and isotropic, this scalar two-mode system can be equivalently represented in
terms of a pair of quantum Brownian oscillators under a Gaussian approximation. In other
words, the two-mode system behaves as if it were interacting linearly with some effective
environment. In this paper we build the details of the effective linear coupling and the
effective environment, and argue that this quantum Brownian representation provides a
simple, universal and non-perturbative characterization of any single particle-like excitation.
As immediate applications of the equivalence, we reanalyse the interpretation of the self-
energy in terms of decay rates in a general background state, and present the master equation
for the field mode corresponding to the particle momentum.

I. INTRODUCTION AND MOTIVATION

Quantum fields can be regarded from the viewpoint of open quantum systems [1, 2, 3, 4].
The degrees of freedom of the field which are relevant for the physical problem in question
constitute the reduced subsystem and the rest form the environment. If there are several
fields in interaction, and the object of interest is a particular field, it is natural to trace
over the other environment fields. For instance, in electrodynamics one can study the so-
called Euler-Heisenberg effective action for the photons [5], considering the electrons as the
environment, or the complementary case, in which the electrons are taken as the system
of interest and the photons are integrated out [6, 7, 8, 9]. Similarly, in stochastic gravity
[10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21] the system of interest is the gravitational
field and the matter fields are integrated out. In many other circumstances it is natural
to consider as the reduced system the modes of the quantum field which are below some
ultraviolet cutoff, with the ultraviolet modes constituting the environment. This approach
has been used, for instance, studying bubble nucleation [22, 23], analyzing decoherence in
field theory [24, 25, 26] and in inflationary cosmology [27, 28, 29, 30]. As these references
illustrate, the open quantum system point of view has often provided new tools and insights
to different field theory problems, specially when dealing with states different than the
Minkowski vacuum.

The propagation of a particle-like excitation over a given field background is another
situation in which there is a natural system–environment separation: the field mode corre-
sponding to the particle momentum is the object of interest, and the rest of modes of the
field, along with any other field in interaction, form the environment. In the Minkowski

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vacuum, the Källén-Lehmann representation of the propagator [3, 31, 32, 33] provides a complete description of single particle excitations from the field theory perspective, but there was no equivalent analysis in thermal backgrounds [34, 35, 36, 37, 38], or in curved spacetimes [39, 40, 41, 42]. The open quantum system approach proves a useful tool when reanalising the propagation of particle-like excitations in non-vacuum or non-Minkowski backgrounds [43, 44].

Therefore, in this paper we elaborate on the open quantum description of the field mode corresponding to the particle momentum, with the goal of developing useful techniques for the analysis of particle and quasiparticle excitations. We concentrate on the basic methods and tools, and give example applications; the general analysis of the particle-like excitations in non-vacuum or non-flat backgrounds is left for separate publications [43, 44].

We would like to consider the field mode as the system and the remaining modes as the environment. However, it proves difficult to implement this system–environment division directly, since the mode-decomposed field $\phi_p$ is not real, but is a complex quantity obeying the constraint $\phi_p = \phi_p^*$ and acting on both the Hilbert space sector with momentum $p$ and the sector with momentum $-p$. Instead of focusing on a single mode, given that the field naturally links modes with opposite momentum, we shall choose as the system of interest any two modes with a given opposite momentum, and as the environment the remaining modes of the field, as well as the modes of any other field in interaction. Namely, given a single scalar field $\phi$, the system degrees of freedom are the two modes $\phi_p$ and $\phi_{-p}$, and the other modes $\phi_q$, with $q \neq \pm p$, form the environment. A similar system–environment division has been used in Refs. [45, 46, 47] in an inflationary context.

The main goal of this paper is to show that this two-mode system behaves, under certain assumptions, as if it was an open quantum system interacting linearly with some effective environment, even if the original field interaction is non-linear. The paradigm of linear open quantum system is the quantum Brownian motion (QBM) model, whose system of interest is a non-relativistic particle interacting linearly with an infinite bath of harmonic oscillators. This model has had many applications in different contexts, among which one may mention the quantum to classical transition [48, 49], the escape from a potential well [50, 51, 52, 53], the Unruh effect [54, 55] or quantum optics [3, 56]. In an influential paper Caldeira and Leggett [57] applied the influence functional model of Feynman and Vernon [58, 59] to the QBM model. The QBM model can be generalised to encompass the general class of linearly interacting open quantum systems [60, 61].

The results of this paper can be seen as an explicit implementation of the statement by Hu and Matacz [55] that the motion of a Brownian particle can be used to depict the behaviour of a single quantum field mode.

We will allow for stationary and isotropic, but otherwise arbitrary, states for the field (a Gaussian approximation will be also assumed later on). Field theory with arbitrary field states can be studied within the closed time path (CTP), or in-in, method, which was originally proposed by Schwinger [62] and Keldysh [63]. The most characteristic feature of the CTP method, in contrast to the conventional in-out method, is the doubling of the number of degrees of freedom.

The paper is organised as follows. In Sect. II the relevant system–environment separation is discussed. In Sect. III the central section of this paper, we present and analyse the QBM representation for the field modes, given the structure of the generating functional and the
two-point propagators. In Sect. [IV] we illustrate the utility of the QBM representation with two example applications. First, we rederive the interpretation of the imaginary part of the self-energy. Second, we build and analyze the master equation for the field modes corresponding to the particle momentum. The main body of the paper ends with Sect. [V] where we summarise and discuss the main results. In order not to break the continuity of the exposition, some details of the analysis are left for the appendices, which also provide background reference material which fixes the notation and makes the paper relatively self-contained. In appendix [A] we briefly present the CTP approach to field theory and apply it to the analysis of the structure of the two-point propagators, and in appendix [B] we introduce the theory of linear open quantum systems, focusing also in the structure of the two-point propagators. The analysis of the propagators done in the appendices is important for the discussion in Sect. [III].

Throughout the paper we work with a system of natural units with $\hbar = c = 1$, denote quantum mechanical operators with a hat, and use a volume-dependent normalisation in the definition of the field modes [see Eq. (1) below]. The same symbol will be used for a quantity and its Fourier transform whenever there is no danger of confusion.

II. FIELD MODES REGARDED AS OPEN QUANTUM SYSTEMS

Let us now present the relevant system–environment separation. For concreteness, we consider a self-interacting field theory model consisting of a single scalar field $\phi$, although results can be straightforwardly extended to any number of fields. The field $\phi$ can be decomposed in modes according to

$$\phi_p = \frac{1}{\sqrt{V}} \int d^3x \ e^{-ip\cdot x} \phi(x), \quad (1)$$

where $V$ is the volume of the space, a formally infinite quantity. The factor $V^{-1/2}$ in the definition of $\phi_p$ is chosen so that the propagators verify:

$$G_\pm(t, t'; p) = \int d^3x \ e^{-ip\cdot x} (0)\hat{\phi}(t, x)\hat{\phi}(t', 0)|0\rangle = \langle 0|\hat{\phi}_p(t)\hat{\phi}_{-p}(t')|0\rangle.$$ 

As stated in the introduction, given a particular momentum $p \neq 0$, the system is composed by the two modes $\phi_p$ and $\phi_{-p}$, and the environment is composed by the other modes of the field, $\phi_q$, with $q \neq \pm p$. Should there be other fields in interaction of any arbitrary spin, the modes of these additional fields would also form part of the environment.

The Hilbert space can be decomposed as $\mathcal{H} = \mathcal{H}_{\text{sys}} \otimes \mathcal{H}_{\text{env}}$, where in turn $\mathcal{H}_{\text{sys}} = \mathcal{H}_k \otimes \mathcal{H}_{-k}$. Notice that this separation does not correspond to the Fock space decomposition. The entire system is in a state $\hat{\rho}$; the state of the reduced system is $\hat{\rho}_s = \text{Tr}_{\text{env}} \hat{\rho}$, and the state of the environment is $\hat{\rho}_e = \text{Tr}_{\text{sys}} \hat{\rho}$. Generally speaking, the state for the entire system is not a factorised product state (i.e., $\hat{\rho} \neq \hat{\rho}_s \otimes \hat{\rho}_e$).

The action can be decomposed as $S = S_{\text{sys}} + S_{\text{count}} + S_{\text{env}} + S_{\text{int}}$, where $S_{\text{sys}}$ is the renormalized system action,

$$S_{\text{sys}} = \int dt \left( \hat{\phi}_p \hat{\phi}_{-p} - E_p^2 \hat{\phi}_p \hat{\phi}_{-p} \right), \quad (2a)$$
$S_{\text{count}}$ is the appropriate counterterm action,

$$S_{\text{count}} = \int dt \left\{ (Z_p - 1)\dot{\phi}_p \dot{\phi}_{-p} - [Z_p(p^2 + m_0^2) - E_p^2] \phi_p \phi_{-p} \right\},$$

(2b)

(with $m_0$ being the bare mass of the field), and $S_{\text{env}}$ and $S_{\text{int}}$ are the environment and interaction actions, respectively, which depend on the particular field theory model. Notice that we have allowed for an arbitrary rescaling of the field $\phi \rightarrow \phi / Z_p^{1/2}$ and for an arbitrary frequency of the two-mode system $E_p$ [which needs not be necessarily of the form $(p^2 + m^2)^{1/2}$].

Let us draw our attention on the field rescaling and the frequency renormalisation. Since it is always possible to freely move finite terms from the system to the counterterm action and vice versa, both the field rescaling and the frequency renormalisation should be taken into account even if no infinities appeared in the perturbative calculations. A physical criterion needs to be chosen in order to fix the values of these two parameters. In the vacuum, such criterion is provided by the on-shell renormalisation scheme; in non-vacuum situations, it is investigated in Ref. [43], and will be briefly discussed in Sect. IV. Notice that the form of $Z_p$ and $E_p$ is not necessarily dictated by the Lorentz symmetry: even if the counterterms which remove the infinities from the vacuum theory also remove the infinities in general field states, there can be finite Lorentz-breaking contributions. Anyway for the purposes of most of this paper the values of $Z_p$ and $E_p$ are not be relevant and will be left unspecified (some comments will be made in Sect. IV B though).

The system variables $\phi_p$ and $\phi_{-p}$ are complex quantities verifying $\phi_{-p}^* = \phi_p$. We can construct real degrees of freedom by introducing the following change of variables:

$$\phi_\Sigma = \frac{1}{\sqrt{2}}(\phi_p + \phi_{-p}), \quad \phi_\Delta = \frac{-i}{\sqrt{2}}(\phi_p - \phi_{-p}).$$

(3)

In terms of the real variables, the system action can be reexpressed as

$$S_{\text{sys}} = \frac{1}{2} \int dt \left( \dot{\phi}_\Sigma^2 - E_p^2 \phi_\Sigma^2 + \dot{\phi}_\Delta^2 - E_p^2 \phi_\Delta^2 \right).$$

(4)

We could alternatively have directly obtained these two real degrees of freedom by working with the sine and cosine Fourier transform [55]. However we prefer to work with the exponential Fourier transform to make manifest the momentum conservation properties.

Most information on the reduced quantum system can be extracted from the set of correlation functions, or equivalently from the CTP generating functional for the reduced system, which can be written as

$$Z[j_a,\alpha] = \exp \left\{ -\frac{1}{2!} \int dt \int dt' j_a^a(t) j_b^b(t') G_{ab}^{\alpha \beta} (t, t') ight. 
- \frac{1}{4!} \int dt \int dt' \int dt'' \int dt''' j_a^a(t) j_b^b(t') j_c^c(t'') j_d^d(t''') G^{(C)}_{abcd}^{\alpha \beta \gamma \delta} (t, t', t'', t''') + \cdots \right\}. 
(5)$$

This expression is somewhat cumbersome and needs some clarification. Latin indices $a, b, c \ldots$ are CTP indices and take the values 1 and 2, indicating respectively the forward and backwards time branches characteristic of the CTP formalism (see appendix A). Greek
indices $\alpha, \beta, \gamma, \ldots$ take the two values $+p$ and $-p$, and make reference to the two field modes $\phi_p$ and $\phi_{-p}$. An Einstein summation convention is used both for Latin and Greek indices. The propagator $G_{ab}(t, t')$ is the 2-point propagator connecting CTP indices $a$ and $b$, whose external legs correspond to particles with momenta $\alpha$ and $\beta$. When the state is translation-invariant and isotropic, momentum conservation imposes:

$$G_{ab}(+p)(-p)(t, t') = G_{ab}(-p)(+p)(t, t') = G_{ab}(t, t').$$  (6)

In turn, $G_{abcd}^{(C)}(\alpha\beta\gamma\delta)(t, t', t'', t''')$ is the connected part of the four-point correlation function having external legs with momenta $\alpha$, $\beta$, $\gamma$ and $\delta$. For translation-invariant states momentum conservation implies that only when momentum is balanced (i.e., two incoming and two outgoing external legs) the correlation function is non-vanishing. Terms with a higher number of external legs behave similarly.

The open quantum system is non-linear, and a systematic treatment of the generating functional can be done by using the tools of non-linear open quantum systems. We follow a different path in the next section.

### III. QUANTUM BROWNIAN MOTION ANALOGY

Let us reconsider the generating functional, Eq. (5). It depends on the $n$-point correlation functions, with $n$ being arbitrarily large. However, in many situations one is interested in properties which only depend on the two-point correlation functions. In other cases one is doing a perturbative expansion of the generating functional, and connected higher order correlation functions are usually also of higher order in the expansion parameter. Finally there are situations in which one only has access to the two-point correlation functions, and expects (or simply hopes) that connected higher order correlation functions are subdominant. In any of this situations one can be tempted to approximate the generating functional by the following Gaussian expression:

$$Z[j_a, \Sigma] \approx \exp \left[ -\frac{1}{2!} \int dt \, dt' \, j_a(t) j_b(t') G_{ab}(t, t') \right].$$  (7)

This Gaussian approximation can be controlled in the framework or the large-$N$ expansion (where $N$ is the number of scalar fields) [64, 65, 66]. We should emphasise that it does not necessarily imply any perturbative expansion in the coupling parameter, nor any free field approximation.

The Gaussian approximation of the generating functional can be expanded to

$$Z[j_a, \Sigma, j_b, \Delta] = \exp \left\{ -\frac{1}{2} \int dt \, dt' \, \left[ j_a^a(t) j_b^b(t') G_{ab}(t, t') + j_a^a(t) j_b^b(t) G_{ab}(t', t) \right] \right\},$$  (8)

or equivalently, in terms of the real variables $\phi_{\Sigma}$ and $\phi_{\Delta}$ and its corresponding classical sources,

$$Z[j_a, \Sigma, j_o, \Delta] = \exp \left\{ -\frac{1}{2} \int dt \, dt' \, \left[ j_a^a(t) j_b^b(t') G_{ab}(t, t') + j_a^a(t) j_b^b(t') G_{ab}(t', t) \right] \right\}. $$  (9)
TABLE I: Detail of the equivalent linear QBM system for a $\lambda\phi^4$ quantum field theory. The $\lambda\phi^4$ model has been chosen for concreteness, but the correspondence would be analogous for any other field theory model. The symbol (=) indicates that the original and equivalent quantities are indeed identical despite the name change.

| System d.o.f. | Original system | Equivalent linear QBM |
|---------------|-----------------|------------------------|
| System d.o.f. | $\phi_p, \phi_{-p}$ | 2 copies of $q$ |
| Environment   | other modes 3-d field | 2 1-d field |
| Env. d.o.f.   | $\phi_q, q \neq \pm p$ | $\varphi_p$ |
| Frequency     | $E_p$ | $\Omega (=)$ |
| Coupling      | $\frac{1}{3!} \sum_{qq'} \phi_{\pm p} \phi_q \phi_{\pm q'} \phi_{q'} \phi_{-q}$ | $g^2 \sum_p \hat{I}(p) \hat{q} \varphi_p$ |
| 2-point function | $G_{ab}(t,t'; p)$ | $G_{ab}(t,t') (=)$ |

Eq. (10) shows that for translation-invariant states within the Gaussian approximation the reduced two-mode state effectively behaves as two decoupled quantum mechanical degrees of freedom.

The Gaussian approximation implies that the expression of the generating functional of a reduced two-mode in terms of the two-point propagators coincides with that of a QBM model, provided the system is isotropic and translation-invariant: compare Eq. (B24) with Eq. (10). Moreover, by comparing Eqs. (B23) with Eqs. (A19), we also realise that the structure of the two-point propagators is identical in both cases. Notice that this latter fact is independent of the Gaussian approximation.

Therefore, we conclude that, assuming homogeneity, isotropy and stationarity, there is an equivalent QBM system for every scalar two-mode pair treated under the Gaussian approximation. In other words, within the Gaussian approximation, every two-mode of a given quantum field theory can be described in terms of a pair of quantum Brownian particles interacting linearly with some effective environment. We must stress that, similarly as the linear interaction does not coincide with the real coupling, the effective environment does not coincide with the real environment. The precise details of the equivalence are summarised in Table I for the particular case of the $\lambda\phi^4$ theory. We will see below that both the strength of the linear coupling and the state of the equivalent environment depend on the details of the original environment.

Let us investigate on this correspondence. On the one hand, as is is shown in appendix B, the effect of the environment in the QBM system is fully encoded in two kernels, the dissipation kernel $D(t,t')$ —or its closely related counterpart $H(t,t')$, see Eq. (B11b) — and the noise kernel $N(t,t')$. The dynamics of the quantum Brownian particle can be determined once the frequency and the noise and dissipation kernels are known. On the other hand, as shown in appendix A in a quantum field theory the two-point correlation functions are fully characterised by the frequency $E_p$ and the retarded and Hadamard self-energies, $\Sigma_R(t,t')$ and $\Sigma^{(1)}(t,t')$ respectively. By comparing again Eqs. (B22) and (B23) with Eq. (A19) we
realise that the precise analogy goes as follows:

\[ \Omega = E_p, \]  
\[ H(t, t') = p^2 + m^2 - E_p + \Sigma_R(t, t'; p), \]  
\[ D(t, t') = -i \text{Im} \Sigma_R(t, t'; p), \]  
\[ N(t, t') = \frac{i}{2} \Sigma^{(1)}(t, t'; p), \]

where the quantities on the right hand side correspond to the original field theory system and the quantities on the left hand side correspond to the equivalent QBM model. The correspondence is valid no matter the renormalisation scheme chosen to fix \( Z_p \) and \( R_p \).

This representation provides thus a first rough interpretation for the retarded and Hadamard self-energies, \( \Sigma_R(t, t'; p) \) and \( \Sigma^{(1)}(t, t'; p) \) respectively. The retarded self-energy corresponds to the dissipation kernel, so that it determines the dissipative properties of the system, and it is independent of the state of the equivalent environment (though not independent of the state of the original environment). The Hadamard self-energy corresponds to the noise kernel, and thus it is basically related to fluctuations.

Although the description in terms of the noise and dissipation kernels is often sufficient, the equivalent QBM system can be alternatively described in terms of the linear coupling constant to the effective environment, \( g \), the distribution of frequencies of the environment \( I(\omega) \) (see appendix B), and the occupation number of the modes of the effective environment \( n(p) = \text{Tr}[\hat{\rho}_e^{(\text{eff})} \hat{a}_p^\dagger \hat{a}_p] \). The product \( g^2 I(\omega) \), which determines the coupling strength to the \( \omega \)-mode of the effective environment, can be obtained from the imaginary part of the self-energy:

\[ \text{Im} \Sigma_R(\omega; p) = iD(\omega) = -\frac{g^2}{2} \omega I(\omega). \]  

This last equation implies that the equivalent coupling depends on the state of the real environment, since the retarded self-energy is state-dependent in general. The occupation numbers of the effective environment \( n(p) \) can be reproduced from the Hadamard self-energy:

\[ \Sigma^{(1)}(\omega; p) = -2iN(\omega) = -2ig^2 |\omega| I(\omega) \left[ \frac{1}{2} + n(|\omega|) \right], \]

The knowledge of the occupation numbers fully determine a Gaussian stationary state for the equivalent environment. These results follow from Eqs. (B12) and (B14).

Note that the analogy depends only on the Gaussian approximation, so that it can be extended to all orders in perturbation theory. Notice also that the results derived from the QBM interpretation are exact for all those properties which depend only on the two-point correlation functions; for the properties which depend on higher order correlation functions, it is a correct approximation depending on the validity of the Gaussian approximation, i.e., depending on the relative importance of the connected parts of the correlation functions with respect to the disconnected parts.
IV. EXAMPLE APPLICATIONS

A. Interpretation of the self-energy

As an example application, let us make use of the QBM correspondence to analyse the physical significance of the self-energy in general backgrounds. Our findings will coincide with the result by Weldon [67] (which nowadays is a textbook result [37]) as far as the imaginary part of the self-energy is concerned. However, while Weldon’s original analysis was only valid to first order in perturbation theory, our technique is valid to all orders. Moreover, our analysis will not be tied to any field theory model. Additionally, we will also obtain an interpretation for the other components of the self-energy. In this case the QBM analogy is exact since no four-point correlation functions are involved.

To start we consider the probability that an excitation of energy $\omega$ decays into the one-dimensional environment, in the equivalent QBM system. The probability $\Gamma_-$ that an excitation of the Brownian particle with positive energy $\omega$ decays into the environment is given by (see e.g. Ref. [37])

$$\Gamma_-(\omega) = \frac{1}{2\omega} \int \frac{dk}{2\pi^2|k|} 2\pi \delta(\omega - |k|) |\mathcal{M}|^2 [1 + n(|k|)],$$

(13)

where $\mathcal{M}$ is the amplitude of the transition and $n(|k|)$ is the occupation number of the environment states with energy $|k|$. The factor $1 + n(|k|)$, which is due to the Bose-Einstein statistics, enhances the decay probability to those states which are already occupied. Since the equivalent QBM system is linear, the squared decay amplitude is simply given to first order in the linear coupling constant $g$ by

$$|\mathcal{M}|^2 = g^2 \mathcal{I}(\omega) \omega^2,$$

(14)

where $\mathcal{I}(\omega)$ is the distribution of frequencies of the effective environment. The factor $\omega^2$ is a consequence of the derivative coupling in the QBM model. The decay probability is therefore:

$$\Gamma_-(\omega) = \frac{1}{2} g^2 \mathcal{I}(\omega)[1 + n(\omega)].$$

(15)

Likewise, the probability that an excitation of positive energy $\omega$ is created spontaneously from the environment is given by

$$\Gamma_+(\omega) = \frac{1}{2\omega} \int \frac{dk}{2\pi^2|k|} 2\pi \delta(\omega - |k|) |\mathcal{M}|^2 n(|k|) = \frac{1}{2} g^2 \mathcal{I}(\omega)n(\omega).$$

(16)

In the original system, $\Gamma_-$ can be interpreted as the probability that a (possibly off-shell) excitation with energy $\omega$ decays into the environment, and $\Gamma_+$ can be interpreted as the probability that an environment spontaneously creates an excitation with energy $\omega$.

Notice that the notion of decay rate in quantum mechanics is meaningful only when the excitations are long-lived, or, to put it differently, when the product $g^2 \mathcal{I}(\omega)$ is very small. There is an inherent uncertainty in the concept of decay rate, which can be traced to the time-energy uncertainty principle. Therefore it is sufficient to present results to first order.
in $g$: the inherent uncertainty to the notion of decay rate in quantum mechanics is of the same order as the error done by neglecting higher powers of $g$. In any case, this does not mean that we are doing any perturbative expansion in the original system: the decay rate can be computed to any desired order in the original perturbative coupling constant (e.g., $\lambda$ in the case of the $\lambda \phi^4$ theory).

We next analyse the self-energy components in the equivalent QBM mode. Given that $\text{Im} \Sigma_R(\omega) = (i/2)[\Sigma^{21}(\omega) - \Sigma^{12}(\omega)]$ (see appendix [A]), we start by analyzing $\Sigma^{21}(\omega)$. Applying CTP Feynman rules we get

$$-i \Sigma^{21}(t, t') = -(ig)^2 \partial_t \partial_{t'} \int \frac{dp}{2\pi} \mathcal{I}(p) \text{Tr}_{\text{env}} \left[ \hat{\rho}_e \hat{\varphi}_{I_p}(t) \hat{\varphi}_{I(-p)}(t') \right],$$

(17)

where $\hat{\varphi}_{I_p}$ is $p$-mode of the environment field in the interaction picture. We have exploited that first order perturbation theory yields exact results for the self-energy in linear systems. Introducing two resolutions of the identity in the basis of eigenstates of the environment Hamiltonian it is a simple exercise [68] to show that the above equation can be developed to

$$\Sigma^{21}(t, t') = ig^2 \partial_t \partial_{t'} \int_0^\infty dp \mathcal{I}(p) \sum_n \rho_{p,n} \left[ (n+1) e^{-ip(t-t')} + n e^{ip(t-t')} \right],$$

where $\rho_{p,n}$ is the diagonal value of the reduced density matrix of the $p$-mode of the equivalent environment: $\text{Tr}_{q\neq p} \langle n_p | \hat{\rho}_e | m_p \rangle = \rho_{p,n} \delta_{nm}$. Introducing the Fourier transform we get

$$\Sigma^{21}(\omega) = ig^2 \omega \mathcal{I}(\omega) \sum_n \rho_{\omega,n} [(n+1) \delta(\omega - p) + n \delta(\omega + p)],$$

and restricting to positive energies,

$$\Sigma^{21}(\omega) = -ig^2 \omega \mathcal{I}(\omega) \sum_n \rho_{\omega,n} (n+1) = ig^2 \omega \mathcal{I}(\omega) [1 + n(\omega)], \quad \omega > 0,$$

(18)

where we recall that $n(\omega) = \text{Tr}[\hat{\rho}_e^{(\text{eff})} \hat{a}_\omega \hat{a}_\omega] = \sum_n \rho_{\omega,n} n$ is the occupation number the $\omega$-mode of the environment. By comparing with Eq. (15), we thus see that $i$ proportional to the decay rate:

$$\Sigma^{21}(\omega) = 2i\omega \Gamma_-(\omega), \quad \omega > 0.$$

(19)

Repeating the calculation for $\Sigma^{21}(\omega)$ we similarly find

$$\Sigma^{12}(\omega) = ig^2 \omega \mathcal{I}(\omega) \sum_n \rho_{\omega,n} n = ig^2 \omega \mathcal{I}(\omega) n(\omega), \quad \omega > 0,$$

(20)

having the corresponding interpretation in terms of the creation rate,

$$\Sigma^{12}(\omega) = 2i\omega \Gamma_+(\omega), \quad \omega > 0.$$

(21)

When the energies are negative $\Sigma^{21}(\omega)$ and $\Sigma^{12}(\omega)$ exchange roles.
The imaginary part of the retarded self-energy is therefore given by

$$\text{Im} \Sigma_R(\omega) = \frac{i}{2} [\Sigma^{21}(\omega) - \Sigma^{12}(\omega)] = -\frac{1}{2} g^2 \omega I(\omega),$$

(22)

and can be interpreted as the net decay rate for an excitation of energy $\omega$ —i.e., decay rate minus creation rate:

$$\text{Im} \Sigma_R(\omega) = -\omega [\Gamma_- (\omega) - \Gamma_+ (\omega)].$$

(23)

We therefore recover Weldon’s result [67].

We can additionally get an interpretation for the Hadamard self-energy. It is given by

$$\Sigma^{(1)}(\omega) = -\Sigma^{21}(\omega) - \Sigma^{12}(\omega) = -ig^2 |\omega I(\omega)| [1 + 2n(|\omega|)],$$

(24)

and is proportional to the probability of decay plus the probability of creation,

$$\Sigma^{(1)}(\omega) = -2|\omega| [\Gamma_- (\omega) + \Gamma_+ (\omega)].$$

(25)

B. Master equation for relativistic quasiparticles

As another example application of the correspondence, let us study the master equation for the second-quantized relativistic quasiparticles. A quasiparticle an elementary excitation propagating in some background, characterized by a momentum $p$, an energy $E_p$, and a decay rate $\gamma_p$ verifying $\gamma_p \ll E_p$ (so that the excitation is long-lived). The basic properties characterizing the long-time evolution of the relativistic quasiparticles can be extracted from the analysis of the retarded propagator corresponding to the field mode in a model-independent way [43] (differently to the short-time behaviour, which is model-dependent). In particular, the real and imaginary parts of the self-energy, when evaluated on shell, correspond to the physical energy and the decay rate of the quasiparticle, respectively:

$$E_p^2 = m^2 + p^2 + \text{Re} \Sigma_R(E_p, p), \quad \gamma_p = -\frac{1}{E_p} \text{Im} \Sigma_R(E_p, p),$$

(26)

Using the QBM correspondence, these two equations can be equivalently written as:

$$\text{Re} H(\Omega) = 0, \quad \gamma_p = -\frac{i}{\Omega} D(\Omega),$$

(27)

The value of $E_p$ (or equivalently $\Omega$) is fixed by requiring that $E_p$ represents the physical energy of the quantum mode as would be measured by a particle detector. We refer to Ref. [43] (which makes use of the QBM correspondence) for further details on the field theory analysis of the quasiparticle excitations.

Thus, let us study the master equation (B27) for $q = \{\phi_\Sigma, \phi_\Delta\}$. To this end it will prove useful to express the on-shell values of the noise and dissipation kernels in terms of the energy $\Omega = E_p$ and the decay rate $\gamma = \gamma_p$:

$$D(\Omega) = -H(\Omega) = i\gamma, \quad N(\Omega) = \gamma \Omega \left(\frac{1}{2} + n\right),$$

(28)
where \( n = n(\Omega) \) is the equilibrium occupation number of the mode in question. Notice that since the decay rate is necessarily small, the coupling constant to the effective environment is also small. Therefore the mode can be considered to be weakly coupled to the effective environment.

In the weak coupling regime, the master equation coefficients are given by Eqs. \([B28]\), and are usually divergent for short times \([60]\). These divergences are associated to the fact that the assumption of factorized initial conditions for the initial state is unphysical, as commented in appendix \([B]\). Since we are interested in the long-time behaviour of the quasiparticles, we study the asymptotic value of these coefficients, which is free from the short-time divergences.

It is easy to show that the frequency shift vanishes:

\[
\delta \Omega^2 = -2 \int_{t_i}^{\infty} D(s, t_i) \cos \Omega(s - t_i) \, ds = \frac{1}{2} \text{Re} \int_{-\infty}^{\infty} H(s, t_i) e^{i\Omega(s-t_i)} \, ds = \text{Re} \, H(\Omega) = 0. 
\]  
\[(29a)\]

Taking into account that the dissipation kernel is antisymmetric, the evaluation of the dissipative factor is also straightforward:

\[
\Gamma = \frac{1}{\Omega} \int_{t_i}^{\infty} D(s, t_i) \sin \Omega(s - t_i) \, ds = \frac{1}{2\Omega} \int_{-\infty}^{\infty} D(s, t_i) e^{i\Omega(s-t_i)} \, ds 
\]

\[
= \frac{1}{2\Omega} iD(\omega) = \frac{\gamma_p}{2}. 
\]  
\[(29b)\]

The first diffusion factor is also easily computed, this time recalling that the noise kernel is symmetric:

\[
\Gamma h = \int_{t_i}^{\infty} N(s, t_i) \cos \Omega(s - t_i) \, ds = \frac{1}{2} \int_{-\infty}^{\infty} N(s, t_i) e^{i\Omega(s-t_i)} \, ds 
\]

\[
= \frac{N(\Omega)}{2} = \Omega \gamma \left( \frac{1}{2} + n \right). 
\]  
\[(29c)\]

The second diffusion factor requires some extra work:

\[
\Gamma f = \frac{1}{\Omega} \int_{t_i}^{\infty} N(s, t_i) \sin \Omega(s - t_i) \, ds = \frac{1}{\Omega} \text{Im} \int_{-\infty}^{\infty} N(s, t_i) \theta(s - t_i) e^{i\Omega(s-t_i)} \, ds 
\]

\[
= \frac{1}{\Omega} \text{Im} \int \frac{d\omega}{2\pi} \frac{iN(\omega)}{\Omega - \omega + i\epsilon} = -\frac{1}{2\Omega} \int \frac{d\omega \, iD(\omega) \text{sign}(\omega)}{2\pi} \left[ 1 + 2n(|\omega|) \right]. 
\]

The integrand in the last equality is only significantly different from zero when \( \omega \sim \Omega \). Therefore as a first approximation we may write:

\[
\Gamma f \approx -\frac{1}{2\Omega} \int \frac{d\omega \, iD(\omega)}{2\pi} \frac{1}{\Omega - \omega} \left[ 1 + 2n(|\omega|) \right] = \frac{1}{2\Omega} \text{Re} \, H(\Omega)(1 + 2n) = 0. 
\]  
\[(29d)\]

Thus the master equation can be written as:

\[
\frac{i}{\partial t} \rho_s(q, q', t) = \left[ -\frac{1}{2} \left( \frac{\partial^2}{\partial q^2} - \frac{\partial^2}{\partial q'^2} \right) + \frac{1}{2} \Omega^2 (q^2 - q'^2) 
\right. 

- \frac{i}{2} \gamma (q - q') \left( \frac{\partial}{\partial q} - \frac{\partial}{\partial q'} \right) - i \Omega \gamma \left( \frac{1}{2} + n \right) (q - q')^2 \left. \right] \rho_s(q, q', t). 
\]  
\[(30)\]
or, equivalently, in terms of the Wigner function, as

\[
\frac{\partial W_s}{\partial t} = -p \frac{\partial W_s}{\partial q} + \Omega^2 \frac{\partial W_s}{\partial p} + \gamma \frac{\partial p W_s}{\partial p} + \frac{\Omega \gamma}{4} (1 + 2n) \frac{\partial^2 W_s}{\partial p^2}.
\]

This is identical to the master equation found by Caldeira et al. [69], corresponding to weak couplings.\(^2\)

The reduced density matrix and the Wigner function are to be interpreted in terms of the field modes, and not directly in terms of particles or quasiparticles themselves. Qualitatively, for Gaussian initial conditions the solution of the master equation can be described in terms of the cumulants of a Gaussian distribution in the following way [48, 70]. The expectation value of the field \(\langle q \rangle\) follows the trajectory of a classical underdamped harmonic oscillator \((\ddot{q} + \gamma \dot{q} + \Omega^2 q = 0)\), namely, it oscillates and slowly decays towards the origin at a rate \(\gamma/2\). However, for single quasiparticle excitations the expectation value of the field is always vanishing. Therefore, it is more appealing to consider the dynamics of the second order cumulants, and in particular the dynamics of the energy of the mode, \(\langle \frac{1}{2} \dot{q}^2 + \Omega^2 q^2 \rangle\). When perturbed by the introduction of a quasiparticle, the energy of the mode slowly decays at a rate \(\gamma\) towards its equilibrium value \(\Omega (1/2 + n)\), with \(n\) being the original occupation number of the mode [43].

V. SUMMARY AND DISCUSSION

In this paper we have explored the open quantum system viewpoint for a pair of field modes of opposite momentum, which are the relevant degrees of freedom for the analysis of particle-like excitations in field theory. The main results have been, on the one hand, showing that in any interacting field theory, assuming homogeneity, stationarity and Gaussianity, this open quantum system can be equivalently represented by two identical quantum Brownian particles interacting linearly with an effective environment, and, on the other hand, exploring the details of the equivalence, which are expressed in Eqs. (11) and (12) and in table I.

The Brownian motion equivalence is based on three simple well-known observations. First, the fact that the structure of the two-point correlation functions, which we reexamined in the appendices, is identical regardless of the nature of the interactions. A byproduct of this fact has been establishing a link between the self-energy and the noise and dissipation kernels, thus connecting the open quantum system and field theory notation and languages. Second, the recognition that a Gaussian truncation leads to a generating functional which coincides with that of a linear theory. Finally, the observation that when assuming homogeneity, stationarity and isotropy the two-mode pair behave as a two copies of a single degree of freedom.

---

1 Notice that in this equation \(p\) is the canonical momentum associated to the variable \(q\), and has nothing to do with the physical quasiparticle momentum \(p\).

2 Fleming et al. [70] found logarithmically divergent results for the master equation coefficients in the ohmic dissipation case, even in the asymptotic regime and in the weak coupling limit. This divergences are associated to the unphysical ultraviolet behaviour of the ohmic dissipation model. Our approximate treatment overlooks the possible divergences associated to the high energy limit of the noise and dissipation kernel.
If the system is Gaussian, stationary and homogeneous, the equivalence is exact. If the system is non-Gaussian, the analogy is also exact as far as two-point correlation functions are concerned, but only approximate for higher order correlation functions. If the system is non-stationary or non-homogeneous, there will be corrections to the results of the order of $L^p$, where $L$ is the characteristic inhomogeneity time or length scale, and $p$ is the relevant energy or momentum scale. Therefore the analogy is perfectly valid in non-homogeneous backgrounds as long as we consider modes whose characteristic wavelengths are much smaller than the inhomogeneity scale.

It must be noted that for the equivalence to be useful one still needs the field theoretic computation of the two-point correlation functions (or, equivalently, the self-energy). The correspondence does not help in this calculation, but must be regarded as a tool useful for interpreting and analyzing the dynamics of the two-mode system. In this sense, there are three basic characteristics which make the QBM equivalence appealing.

First, the equivalence is universal, in the sense that it provides the most general description of the dynamics of the scalar two-mode pair within the Gaussian approximation. The description in terms of a linear open quantum system allows a unified description of many different quantum field theory systems: the details of the quantum field theory are unimportant once the equivalent noise and dissipation kernels are known. Different field theory models can be thus classified in the same equivalence class if they lead to the same QBM equivalent model.

Second, the equivalence provides a simple characterization of the interaction, given that an arbitrarily complicated coupling with any number of fields is reduced to a linear interaction with a one-dimensional field. Linearly interacting systems have been thoroughly studied in the literature (see e.g. references given in the introduction and appendix B), and exhibit many interesting properties, among which one can cite the fact that they are exactly solvable. Therefore, within the regime of validity of the Gaussian approximation, all the methods developed in the literature for linear quantum systems can be applied to analyse the dynamics of a two-mode pair.

Finally, the equivalence is non-perturbative, since it does not rely on any perturbative development in the coupling constant of the original field theory. It can be therefore extended to all orders in perturbation theory and applied to strongly interacting systems.

The QBM correspondence, which has also been implicitly used in Refs. [71, 72], proves a useful tool when analyzing the properties of particle-like excitations in general backgrounds from a field theory perspective [43, 44]. The utility of the QBM analogy is already highlighted by the first two immediate applications presented in this paper.

As a first application, we reexplored the well-known result that the imaginary part of the retarded self-energy corresponds to the net decay rate of the particle excitations. This was a rather simple application which however improved the conventional textbook derivation by taking profit of the three properties we have remarked above: in the first place, the universality of the QBM analogy, given that the derivation presented in this paper was not tied to any specific field theory model; in the second place, its simplicity, since the calculation essentially reduced to doing trivial perturbative expansions in the equivalent linear system, and, finally, its non-perturbative character, since the derivation avoided any perturbative expansion in the original system.

As a second application, we presented the relevant master equation for the dynamics
of the modes corresponding to the quasiparticle momentum. This was a slightly more involved application, relying on the results of Ref. [43] (and also complementing them), which highlighted the fact that the linear open quantum system machinery can readily be exported to quantum field theory. The presentation and analysis of the master equation, as done in the paper must be understood as a first approximation to the problem, which might be sufficiently interesting by itself to deserve further work.

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APPENDIX A: THE CLOSED TIME PATH METHOD AND THE TWO-POINT PROPAGATORS IN FIELD THEORY

In this appendix we give a brief introduction to the closed time path (CTP) method (also called in-in method, in contrast to the conventional in-out method), stressing those aspects relevant for this paper, and apply it to the analysis of the two-point propagators. We address the reader to Refs. [73, 74, 75, 76, 77] for further details on the CTP method, and to Refs. [68, 78, 79, 80, 81] for further details on the structure of the two point functions. For the purposes of this appendix we shall consider a free or an interacting scalar field $\phi$, although results also apply for a single quantum mechanical degree of freedom.

The path-ordered generating functional $Z_C[j]$ is defined as

$$ Z_C[j] = \text{Tr} \left( \hat{\rho}_C e^{i \int_C d^3 x \hat{\phi}(x) j(x)} \right), \quad (A1) $$

where $\hat{\phi}(x)$ is the field operator in the Heisenberg picture, $C$ is a certain path in the complex $t$ plane, $T_C$ means time ordering along this path and $j(x)$ is a classical external source. By functional differentiation of the generating functional with respect to $\phi$, path-ordered correlation functions can be obtained:

$$ G_C(x, x') = \text{Tr} \left[ \hat{\rho}_C \hat{\phi}(x) \hat{\phi}(x') \right] = - \left. \frac{\delta^2 Z_C}{\delta j(x) \delta j(x')} \right|_{j=0}. \quad (A2) $$

Introducing a complete basis of eigenstates of the field operator in the Heisenberg picture, $\hat{\phi}(t, x)|\phi, t\rangle = \phi(t, x)|\phi, t\rangle$, as a representation of the identity, the generating functional can be expressed as:

$$ Z_C[j] = \int \tilde{d}\phi \tilde{d}\phi' \langle \phi, t_i | \hat{\rho} | \phi', t_i \rangle \langle \phi', t_i | T_C e^{i \int_C d^3 x \hat{\phi}(x) j(x)} | \phi, t_i \rangle \quad (A3) $$

The functional measures $\tilde{d}\phi$ and $\tilde{d}\phi'$ go over all field configurations of the fields at fixed initial time $t$. If the path $C$ begins and ends at the same point $t_i$, then the transition element
of the evolution operator can be computed via a path integral:

$$Z_C[j] = \int \widetilde{d}\phi \widetilde{d}\phi' \langle \phi, t_i | \hat{\rho} | \phi', t_i \rangle \int_{\varphi(t_i,x)=\phi(x)}^{\varphi(t_f,x)=\phi'(x)} D\varphi \, e^{i \int_{t_i}^{t_f} dt \int d^3x \{ L[\varphi]+\varphi(x)j(x) \}}, \tag{A4}$$

where $L[\phi]$ is the Lagrangian density of the scalar field.

Let us consider the time path shown in Fig. 1. If we define $\varphi_{1,2}(t,x) = \varphi(t,x)$ and $j_{1,2}(t,x) = j(t,x)$ for $t \in C_{1,2}$, then the generating functional can be reexpressed as:

$$Z[j_1,j_2] = \int \widetilde{d}\phi \widetilde{d}\phi' \widetilde{d}\phi'' \langle \phi, t_i | \hat{\rho} | \phi', t_i \rangle \times \int_{\varphi_1(t_i,x)=\phi(x)}^{\varphi_1(t_f,x)=\phi''(x)} D\varphi_1 \, e^{i \int_{t_i}^{t_f} dt \int d^3x \{ L[\varphi_1]+\varphi_1(x)j_1(x) \}} \times \int_{\varphi_2(t_i,x)=\phi(x)}^{\varphi_2(t_f,x)=\phi''(x)} D\varphi_2 \, e^{-i \int_{t_i}^{t_f} dt \int d^3x \{ L[\varphi_2]+\varphi_2(x)j_2(x) \}}. \tag{A5}$$

In the following it will prove useful to use a condensed notation where neither the boundary conditions of the path integral nor the integrals over the initial and final states are explicit. With this simplified notation the above equation becomes

$$Z[j_1,j_2] = \int D\phi_1 D\phi_2 \langle \phi, t_i | \hat{\rho} | \phi', t \rangle \, e^{i \int d^4x \{ L[\varphi_1]-L[\varphi_2]+\varphi_1(x)j_1(x)-\varphi_2(x)j_2(x) \}} \tag{A6}$$

An operator representation is also possible:

$$Z[j_1,j_2] = \text{Tr} \left( \hat{\rho} \, \mathcal{T} e^{-i \int_{t_i}^{t_f} dt \int d^3x \hat{\phi}(x)j_2(x)} \mathcal{T} e^{i \int_{t_i}^{t_f} dt \int d^3x \hat{\phi}(x)j_1(x)} \right). \tag{A7}$$

By functionally differentiating the generating functional, the Feynman and Dyson prop-
agators and the Whightman functions can be obtained:

\[ G_{11}(x, x') = G_F(x, x') = \frac{\delta^2 Z}{\delta j_1(x) \delta j_1(x')} \bigg|_{j_1 = j_2 = 0}, \]

\[ G_{21}(x, x') = G_+(x, x') = \frac{\delta^2 Z}{\delta j_2(x) \delta j_1(x')} \bigg|_{j_1 = j_2 = 0}, \]

\[ G_{12}(x, x') = G_-(x, x') = \frac{\delta^2 Z}{\delta j_1(x) \delta j_2(x')} \bigg|_{j_1 = j_2 = 0}, \]

\[ G_{22}(x, x') = G_D(x, x') = \frac{\delta^2 Z}{\delta j_2(x) \delta j_2(x')} \bigg|_{j_1 = j_2 = 0}. \]

These four propagators can be conveniently organised in a 2 \times 2 matrix, the so-called direct matrix.

\[
G_{ab}(x, x') = \begin{pmatrix}
G_F(x, x') & G_-(x, x') \\
G_+(x, x') & G_D(x, x')
\end{pmatrix}
\]  

(A9)

Lowercase roman indices may acquire the values 1 and 2 are raised and lowered with the “CTP metric” \( c_{ab} = \text{diag}(1, -1) \). Higher order correlation functions can be obtained in a similar way.

We may also consider the Pauli-Jordan or commutator propagator,

\[ G(x, x') = \text{Tr} \left( \hat{\rho} [\hat{\phi}(x), \hat{\phi}(x')] \right), \]  

(A10a)

and the Hadamard or anticommutator function

\[ G^{(1)}(x, x') = \text{Tr} \left( \hat{\rho} \{\hat{\phi}(x), \hat{\phi}(x')\} \right). \]  

(A10b)

For linear systems\(^3\) the Pauli-Jordan propagator is independent of the state and carries information about the system dynamics. Finally, one can also consider the retarded and advanced propagators

\[ G_R(x, x') = \theta(x^0 - x'^0)G(x, x') = \theta(x^0 - x'^0) \text{Tr} \left( \hat{\rho} [\hat{\phi}(x), \hat{\phi}(x')] \right), \]  

(A11a)

\[ G_A(x, x') = \theta(x'^0 - x^0)G(x, x') = \theta(x'^0 - x^0) \text{Tr} \left( \hat{\rho} [\hat{\phi}(x), \hat{\phi}(x')] \right), \]  

(A11b)

which also do not depend on the the state for linear systems. The retarded and advanced propagators and the Hadamard function can be used as an alternative basis to (A9) in the so-called physical or Keldysh basis.

It is also useful to introduce the correlation functions in momentum space, which are defined as the Fourier transform of the spacetime correlators with respect to the difference variable \( \Delta = x - x' \) keeping constant the central point \( X = (x + x')/2 \):

\[ G_{ab}(\omega, p; X) = \int d^4 \Delta \ e^{i\omega \Delta^0 - ip \Delta} G_{ab}(X + \Delta/2, X - \Delta/2). \]  

(A12)

\(^3\) By linear systems we mean systems whose Heisenberg equations of motion are linear. These correspond either to non-interacting systems or to linearly coupled systems.
Mixed time-momentum representations of the propagator, $G_{ab}(t, t'; p; X)$, can be similarly introduced. For homogeneous and static backgrounds the Fourier transformed propagator does not depend on the mid point $X$. The canonical example of static and homogeneous background is the thermal background, in which the state of the field is $\hat{\rho} = e^{-\beta H}/\text{Tr} (e^{-\beta H})$. Thermal field theory can be thus treated as a particular example of field theory over an arbitrary background. This viewpoint corresponds to the so-called real time approach to field theory [37, 78, 82].

For interacting theories the self-energy can be introduced similarly to the vacuum case. Interaction theory mixes the two CTP branches and therefore the self-energy has matrix structure and is implicitly defined through the equation

$$G_{ab}(x, x') = G_{ab}^{(0)}(x, x') + \int d^4y d^4y' G_{ac}^{(0)}(x, y)[-i\Sigma^{cd}(y, y')]G_{db}(y', z), \quad (A13)$$

where $G_{ab}^{(0)}(x, x')$ are the propagators of the free theory and $G_{ab}(x, x')$ are the propagators of the full interacting theory. Notice that Eq. (A13) is matrix equation relating the four components of the self-energy with the four components of the propagator. Therefore there is no diagonal relation between $G_{11}(x, x')$ and $\Sigma_{11}(y, y')$ as in the vacuum case. The $ab$ component of the self-energy can be computed, similarly to the vacuum case, as the sum of all one-particle irreducible diagrams with amputated external legs that begin and end with type $a$ and type $b$ vertices, respectively. CTP Feynman rules are completed with the prescription of adding one minus sign for every type 2 vertex.

A particularly useful combination is the retarded self-energy, defined as $\Sigma^R(x, x') = \Sigma^{11}(x, x') + \Sigma^{12}(x, x')$. It is related to the retarded propagator through

$$G_{R}(x, x') = G_{R}^{(0)}(x, x') + \int d^4y d^4y' G_{R}^{(0)}(x, y)[-i\Sigma_R(y, y')][G_{R}(y', z)], \quad (A14)$$

This equation can be regarded as a consequence of the causality properties of the retarded propagator. A similar relation holds between the advanced propagator $G_{A}(x, x')$ and the advanced self-energy $\Sigma^A(x, x') = \Sigma^{11}(x, x') + \Sigma^{22}(x, x')$. Another useful combination is the Hadamard self-energy, which is defined as $\Sigma^{(1)}(x, x') = \Sigma^{11}(x, x') + \Sigma^{22}(x, x')$ [or equivalently as $\Sigma^{(1)}(x, x') = -\Sigma^{12}(x, x') - \Sigma^{21}(x, x')$] and which is related to the Hadamard propagator through

$$G^{(1)}(x, x') = -i \int d^4y d^4y' G_{R}(x, y)[\Sigma^{(1)}(y, y')][G_{A}(y', x')]. \quad (A15)$$

All self-energy combinations can be determined from the knowledge of the Hadamard self-energy and the imaginary part of the retarded self-energy. This latter quantity can be obtained from the following cutting rule:

$$\text{Im} \Sigma^R(x, y) = \frac{i}{2} \left[ \Sigma^{21}(x, y) - \Sigma^{12}(x, y) \right]. \quad (A16)$$

So far, all expressions in this appendix refer to arbitrary background states $\hat{\rho}$. For static and homogeneous backgrounds, Eq. (A14) can be solved for the retarded propagator by going to the momentum representation:

$$G_{R}(\omega, p) = \frac{-i}{-\omega^2 + m^2 + p^2 + \Sigma^R(\omega, p) - p^0 \omega}. \quad (A17)$$
We have considered that the free propagators of the mode $p$ are those corresponding to the action (2a). Notice that in general the self-energy is a separate function of the energy $\omega$ and the 3-momentum $p$, and not only a function of the scalar $p^2$, as in the vacuum case. The Hadamard function admits the following expression [which can be derived from Eq. (A15)]:

$$G^{(1)}(\omega, p) = i |G_R(\omega, p)|^2 \Sigma^{(1)}(\omega, p) = \frac{i \Sigma^{(1)}(\omega, p)}{-\omega^2 + m^2 + p^2 + \text{Re} \Sigma_R(\omega, p)^2 + |\text{Im} \Sigma_R(\omega, p)|^2}.$$  

(A18)

From the retarded propagator and the Hadamard function we can derive:

$$G_F(\omega, p) = -i \frac{\left[-\omega^2 + E_p^2 + \text{Re} \Sigma_R(\omega, p)\right]}{-\omega^2 + m^2 + p^2 + \text{Re} \Sigma_R(\omega, p)^2 + |\text{Im} \Sigma_R(\omega, p)|^2},$$  

(A19a)

$$G_D(\omega, p) = i \frac{\left[-\omega^2 + E_p^2 + \text{Re} \Sigma_R(\omega, p)\right]}{-\omega^2 + m^2 + p^2 + \text{Re} \Sigma_R(\omega, p)^2 + |\text{Im} \Sigma_R(\omega, p)|^2},$$  

(A19b)

$$G_-(\omega, p) = \frac{i \Sigma^{(1)}(\omega, p)}{-\omega^2 + m^2 + p^2 + \text{Re} \Sigma_R(\omega, p)^2 + |\text{Im} \Sigma_R(\omega, p)|^2},$$  

(A19c)

$$G_+(\omega, p) = \frac{i \Sigma^{(1)}(\omega, p)}{-\omega^2 + m^2 + p^2 + \text{Re} \Sigma_R(\omega, p)^2 + |\text{Im} \Sigma_R(\omega, p)|^2}. $$  

(A19d)

When the field state is not exactly homogeneous, the above expressions are still correct up to order $Lp$, where $L$ is the relevant inhomogeneity time or length scale.

**APPENDIX B: LINEAR OPEN QUANTUM SYSTEMS**

We present those aspects of the theory of linear open quantum systems relevant in this paper, focusing on the propagators of the Brownian particle. For a more complete presentation check Refs. [1, 2, 3, 4, 50, 83].

We shall consider a quantum Brownian motion (QBM) model: an open quantum system composed of a harmonic oscillator $q(t)$, which will be the subsystem under study, linearly coupled to a free massless field $\phi(t, x)$, which will act as environment or reservoir. The action for the full system can be decomposed in the action of the harmonic oscillator, the action of the scalar field and the interaction term as

$$S_{\text{sys}}[q] = \int dt \left[\frac{1}{2} \dot{q}^2 - \frac{1}{2} \Omega^2 q^2\right],$$  

(B1a)

$$S_{\text{env}}[\phi] = \int dt dx \left[\frac{1}{2} (\partial_t \phi)^2 - \frac{1}{2} (\partial_x \phi)^2\right],$$  

(B1b)

$$S_{\text{int}}[q, \phi] = g \int dt dx \delta(x) \dot{q} \phi,$$  

(B1c)

with $\Omega$ being the frequency of the harmonic oscillator and $g$ being the coupling constant. The oscillator is taken to have unit mass. A counterterm action including a frequency shift could also be considered.

We use a one-dimensional free field as the environment, following the treatment of Ref. [48]. This is equivalent to the alternative representation in which the environment
is modelled by a large ensemble of harmonic oscillators \[^{57}\]. This equivalence can be seen performing a mode decomposition in the interaction term \([B1c]\),

\[
S_{\text{int}}[q, \varphi] = \sqrt{L} \int dt \frac{dp}{2\pi} g \dot{q} \varphi_p, \tag{B2}
\]

where \(\varphi_p(t)\) is proportional to the spatial Fourier transform of the scalar field,

\[
\varphi_p(t) = \frac{1}{\sqrt{L}} \int dx \ e^{-ipx} \varphi(t, x),
\]

where \(L\) is the length of the real axis (formally infinite).

The standard model, also called ohmic model, can be generalised by replacing the delta interaction of equation \((B1c)\) by a function \(f(x)\). In this case the interaction term is,

\[
S_{\text{int}}[q, \varphi] = \int dt \int dx \ f(x) g \dot{q}(t) \varphi(t, x). \tag{B3a}
\]

or equivalently in the Fourier space,

\[
S_{\text{int}}[q, \varphi] = \sqrt{L} \int dt \frac{dp}{2\pi} \tilde{f}(-p) g \dot{q}(t) \varphi_p(t). \tag{B3b}
\]

The real even function \(\mathcal{I}(p) = \tilde{f}(p) \tilde{f}(-p)\) is called the distribution of frequencies.\(^4\) The product \(g^2\mathcal{I}(\omega)\) characterises the properties of the coupling with the environment at a given energy \(\omega\). The QBM model this way generalised encompasses the entire class of linearly coupled environments.

When the system and the environment are initially uncorrelated, \(i.e.,\) when the initial density matrix factorises —\(\hat{\rho}(t_i) = \hat{\rho}_s(t_i) \otimes \hat{\rho}_e(t_i)\), where \(\hat{\rho}_s(t_i)\) and \(\hat{\rho}_e(t_i)\) mean, respectively, the density matrix operators of the system and the environment at the initial time—, the evolution for the reduced density matrix can be written as

\[
\rho_s(q, q', t_f) = \int dq_i dq'_i J(q_f, q'_f; q_i, q'_i, t_i) \rho_s(q_i, q'_i, t_i), \tag{B4}
\]

where the propagator \(J\) is defined in a path integral representation by

\[
J(q_f, q'_f; q_i, q'_i, t_i) = \int \mathcal{D}q \int \mathcal{D}q' e^{i(S[q] - S[q'] + S_{\text{IF}}[q, q'])}, \tag{B5}
\]

with \(S_{\text{IF}}[q, q']\) being the influence action, which is related to the the influence functional introduced by Feynman and Vernon \[^{58, 59}\] through \(F[q, q'] = e^{iS_{\text{IF}}[q, q']}\). In turn, the influence functional can be expressed in the following way:

\[
F[q, q'] = \int \mathcal{D}\varphi \mathcal{D}\varphi' \rho_e(|\varphi|, |\varphi'|, t_i) \exp \left[ i (S[\varphi] - S[\varphi'] + S_{\text{int}}[q, \varphi] - S_{\text{int}}[q', \varphi']) \right]. \tag{B6}
\]

\(^4\) In the literature the distribution of frequencies is frequently defined as \(\omega \mathcal{I}(\omega)\).
The path integral has the boundary conditions \( \varphi(x, t_i) = \varphi_i(x) \), \( \varphi'(x, t_i) = \varphi'_i(x) \), \( \varphi(x, t_f) = \varphi_f(x) \); there is also an implicit sum over initial and final states, \( \varphi_i(x) \), \( \varphi'_i(x) \) and \( \varphi_f(x) \).

Considering a factorized initial state is a rather unphysical hypothesis that leads to surprising results in many circumstances (see for instance Ref. [60]). The methods presented in this appendix can be generalized to more natural initial density matrices by the use of the so-called preparation functions \([4, 84]\). However the preparation function method does not completely solve all the problems because it is based in a sudden change of the density matrix. A more physical approach involves a continuous preparation of the system \([83]\). In any case, these techniques are increasingly more involved, and we shall be only interested in studying the dynamics much after the typical decoherence time. In this case the system and environment have had enough time to interact and become entangled, and the precise form of the initial state becomes unimportant.

When the initial density matrix of the environment \( \rho_e([\varphi], [\varphi'], t_i) \) is Gaussian, the path integrals can be exactly performed and one obtains \([50, 58, 86]\):

\[
S_{IF}[q, q'] = -2 \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} dt' \Delta(t) D(t, t') \hat{Q}(t') + i \frac{1}{2} \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} dt' \Delta(t) N(t, t') \Delta(t'), \tag{B7}
\]

where \( \Delta(t) = q(t) - q'(t) \) and \( Q(t) = |q(t) + q'(t)|/2 \).

The kernels can be computed as

\[
D(t, t') = \frac{ig^2}{2} \int \frac{dp}{2\pi} I(p) \text{Tr} \left( \hat{\rho} \left[ \varphi_{(-p)}(t), \varphi_{ip}(t') \right] \right), \tag{B8a}
\]

\[
N(t, t') = \frac{g^2}{2} \int \frac{dp}{2\pi} I(p) \text{Tr} \left( \hat{\rho} \left\{ \varphi_{(-p)}(t), \varphi_{ip}(t') \right\} \right), \tag{B8b}
\]

where \( \varphi_{i}(x, t) \) is the field operator in the interaction picture and \( \varphi_{ip}(t) \) is the \( p \)-mode of the same field operator. By integration by parts, the influence action can also be expressed as

\[
S_{IF}[q, q'] = \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} dt' \Delta(t) H(t, t') Q(t') + i \frac{1}{2} \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} dt' \Delta(t) N(t, t') \Delta(t'), \tag{B9}
\]

or as

\[
S_{IF}[q, q'] = -2 \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} dt' \Delta(t) D(t, t') Q(t') + \int_{t_i}^{t_f} dt \delta \Omega^2 \Delta(t) Q(t) + \frac{i}{2} \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} dt' \Delta(t) N(t, t') \Delta(t'), \tag{B10}
\]

where the different kernels are defined as

\[
H(t, t') = -2 \frac{\partial}{\partial t'} \frac{\partial}{\partial t} \left[ \theta(t - t') D(t, t') \right] \tag{B11a}
\]

\[
= -2 \theta(t - t')D(t - t') + \delta \Omega^2 \delta(t - t'), \tag{B11b}
\]

\[
D(t, t') = \frac{\partial}{\partial t'} \frac{\partial}{\partial t} D(t, t'), \tag{B11c}
\]

\[
N(t, t') = \frac{\partial}{\partial t'} \frac{\partial}{\partial t} N(t, t'). \tag{B11d}
\]
The kernels $D(t, t')$ and $N(t, t')$ are called respectively dissipation and noise kernels. The frequency shift $\delta \tilde{\Omega}^2$ is a formally divergent quantity given by

$$\delta \tilde{\Omega}^2 = 2 \lim_{t \to t'} \frac{\partial D(t, t')}{\partial t}. \quad (B11e)$$

The dissipation and noise kernels can be computed following Eqs. (B8) and (B11). The value of the dissipation kernel in the frequency space is [68]:

$$D(\omega) = \frac{i \omega g^2}{2} I(\omega). \quad (B12)$$

The dissipation kernel is closely related to the kernel $H(\omega)$ [see Eqs. (B11a) and (B11b)], which is also state-independent and given by:

$$H(\omega) = g^2 \int \frac{dp}{2\pi} \frac{\omega I(\omega)}{\omega - \omega' + i \epsilon} + \delta \tilde{\Omega}^2. \quad (B13)$$

In contrast, the noise kernel [B11d] is state-dependent. For a general Gaussian stationary environments, characterised by the occupation numbers $n(p) = Tr(\hat{\rho}_e \hat{a}_p \hat{a}_p^\dagger)$, the noise kernel in Fourier space is given by

$$N(\omega) = g^2 |\omega| I(\omega) \left[ \frac{1}{2} + n(|\omega|) \right]. \quad (B14)$$

The dissipation and noise kernels are related through:

$$N(\omega) = -i \text{sign}(\omega) \left[ \frac{1}{2} + n(|\omega|) \right] D(\omega). \quad (B15)$$

For the particular case of an environment in thermal equilibrium at a temperature $T$ the occupation numbers are given by $n(|\omega|) = 1/(e^{\omega/T} - 1)$ and the above equation becomes the fluctuation-dissipation theorem:

$$N(\omega) = -i \text{sign}(\omega) \coth \left( \frac{|\omega|}{2T} \right) D(\omega). \quad (B16)$$

By considering an arbitrary distribution of frequencies $I(\omega)$ and an arbitrary Gaussian state for the environment $\hat{\rho}_e$, the dissipation and noise kernels may adopt almost any value. In the rest of the appendix we shall try to express all results in terms of the dissipation and noise kernels. To this end, it will prove useful to reexpress Eq. (B11b) in Fourier space:

$$H(\omega) = -2 \int \frac{d\omega'}{2\pi} \frac{i D(\omega')}{\omega - \omega' + i \epsilon} + \delta \tilde{\Omega}^2. \quad (B17a)$$

The kernel $H(\omega)$ can be decomposed in its real and imaginary parts as:

$$H_R(\omega) = \text{Re} \ H(\omega) = -2 P \int \frac{d\omega'}{2\pi} \frac{i D(\omega')}{\omega - \omega'} + \delta \tilde{\Omega}^2, \quad (B17b)$$

$$H_I(\omega) = \text{Im} \ H(\omega) = i D(\omega). \quad (B17c)$$
We have used the property $1/(x + i\epsilon) = P(1/x) - i\pi\delta(x)$. Notice that $H(-\omega) = H^*(\omega) = H_R(\omega) - iH_1(\omega)$. The real and imaginary parts of the kernel in frequency space correspond, respectively, to the even and odd parts in time space. Notice also the Kramers-Kronig relation between the real and imaginary parts of the kernel $H(\omega)$:

$$H_R(\omega) = -2\pi\int \frac{d\omega'}{2\pi} \frac{H_1(\omega')}{\omega - \omega'} + \delta\tilde{\Omega}^2.$$  \hspace{1cm} (B18)

The frequency shift can be always absorbed in $\tilde{\Omega}$. From now on, and in the main body of the paper, we will assume that such absorption has been carried out.

For a Gaussian environment and asymptotic initial boundary conditions the generating functional can be expressed as\(^5\) \[83\]

$$Z[j_1, j_2] = e^{\frac{1}{2} \int dt_1 \int dt_2 \int dt_3 \int dt_4 j_\Delta(t_1) G_R(t_1, t_2) N(t_2, t_3) j_\Delta(t_4) G_R(t_4, t_3) - \int dt_1 \int dt_2 j_\Delta(t_1) G_R(t_1, t_2) j_\Sigma(t_2),}$$  \hspace{1cm} (B19)

where $j_\Sigma(t) = [j_1(t) + j_2(t)]/2$, $j_\Delta(t) = j_1(t) - j_2(t)$ and $G_R(t, t')$ is the retarded propagator of the kernel

$$L(t, t') = \left( \frac{d^2}{dt^2} + \Omega^2 \right) \delta(t - t') + H(t, t'),$$  \hspace{1cm} (B20)

\textit{i.e.}, is the kernel which verifies

$$\int ds G_R(t, s)L(s, t') = -i\delta(t - t'), \quad G_R(t, t') = 0 \quad \text{if} \quad t < t'.$$  \hspace{1cm} (B21)

Explicit expressions are most easily obtained in the Fourier space, in which the retarded propagator adopts the form:

$$G_R(\omega) = \frac{-i}{L(\omega)} = \frac{-i}{\omega^2 + \Omega^2 + H(\omega)}.$$  \hspace{1cm} (B22)

It can be checked that $G_R(t, t')$, besides being the retarded propagator of the kernel $L(t, t')$, in the sense of Eq. (B21), is also the retarded propagator of the quantum mechanical system, in the sense of Eq. (A11a).

Differentiating the CTP generating functional, according to Eqs. (A8), we obtain the different correlation functions in terms of the noise and dissipation kernels:

$$G_F(\omega) = \frac{-i}{\left[ -\omega^2 + \Omega^2 + H_R(\omega) \right]} + N(\omega) \left[ -\omega^2 + \Omega^2 + H_R(\omega) \right]^2 + [H_1(\omega)]^2, \hspace{1cm} (B23a)$$

$$G_D(\omega) = \frac{i}{\left[ -\omega^2 + \Omega^2 + H_R(\omega) \right]} + N(\omega) \left[ -\omega^2 + \Omega^2 + H_R(\omega) \right]^2 + [H_1(\omega)]^2, \hspace{1cm} (B23b)$$

$$G_- (\omega) = \frac{N(\omega) + H_1(\omega)}{\left[ -\omega^2 + \Omega^2 + H_R(\omega) \right]^2 + [H_1(\omega)]^2}, \hspace{1cm} (B23c)$$

$$G_+ (\omega) = \frac{N(\omega) - H_1(\omega)}{\left[ -\omega^2 + \Omega^2 + H_R(\omega) \right]^2 + [H_1(\omega)]^2}. \hspace{1cm} (B23d)$$

\(^5\) In the general case there would be a prefactor taking into account the initial conditions for the system. However, since the system has a dissipative dynamics and the initial conditions are given in the remote past, initial conditions for the system turn out to be completely irrelevant.
The generating functional can be alternatively expressed in terms of these correlation functions as
\[ Z[j_1, j_2] = N e^{-\frac{1}{2} \int dt dt' j^a(t) G_{ab}(t, t') j^b(t')} . \]  
(B24)

If desired, the dynamics of the Brownian oscillator can be analyzed in terms of a Langevin equation,
\[ \ddot{q}(t) + \Omega^2 q(t) + \int_{t_i}^\infty dt' H(t, t') q(t') = \xi(t), \]  
(B25)

where \( \xi(t) \) is a stochastic Gaussian field defined by the correlation functions
\[ \langle \xi(t) \rangle = 0, \quad \langle \xi(t) \xi(t') \rangle = N(t, t'), \]  
(B26)

with \( \langle \cdots \rangle \) meaning stochastic average. The stochastic correlation functions derived from the Langevin equation correspond to a subset of the quantum correlation functions [3, 83]. It is also possible to study the master equation for the reduced density matrix of the system \( \rho_s \), which is given by [60, 83, 87]:
\[ i \frac{\partial}{\partial t} \rho_s(q, q', t) = \left[ \frac{1}{2} \left( \frac{\partial^2}{\partial q^2} - \frac{\partial^2}{\partial q'^2} \right) + \frac{1}{2} \left( \Omega^2 + \delta \Omega_0^2(t) \right) (q^2 - q'^2) 
- i \Gamma(t)(q - q') \left( \frac{\partial}{\partial q} - \frac{\partial}{\partial q'} \right) - i \Gamma(t) h(t)(q - q')^2 
+ \Gamma(t) f(t)(q - q') \left( \frac{\partial}{\partial q} + \frac{\partial}{\partial q'} \right) \right] \rho_s(q, q', t). \]  
(B27)

where \( \delta \Omega_0^2(t), \Gamma(t), h(t) \) and \( f(t) \) are a frequency shift, a dissipative factor and two dispersive factors, respectively, which in the weak coupling limit are given by [60]:
\[ \delta \Omega_0^2(t) = -2 \int_{t_i}^t D(s, t_i) \cos \Omega(s - t_i) \, ds, \]  
(B28a)
\[ \Gamma(t) = \frac{1}{\Omega} \int_{t_i}^t D(s, t_i) \sin \Omega(s - t_i) \, ds, \]  
(B28b)
\[ \Gamma(t) h(t) = \int_{t_i}^t N(s, t_i) \cos \Omega(s - t_i) \, ds, \]  
(B28c)
\[ \Gamma(t) f(t) = \frac{1}{\Omega} \int_{t_i}^t N(t_i, s) \sin \Omega(s - t_i) \, ds. \]  
(B28d)

The corresponding expressions valid for arbitrary couplings can be found in Refs. [60, 83]. Recall that the time \( t_i \) is the time at which the density matrix is assumed to factorize.

Alternatively, it is also possible to introduce the reduced Wigner function,
\[ W_s(q, p, t) = \frac{1}{2\pi} \int d\Delta e^{i\Delta p} \rho_s(q - \Delta/2, q + \Delta/2, t), \]  
(B29)
in terms of which the master equation adopts a Fokker-Planck form:
\[ \frac{\partial W_s}{\partial t} = -p \frac{\partial W_s}{\partial q} + [\Omega^2 + \delta \Omega_0^2(t)] q \frac{\partial W_s}{\partial p} + 2\Gamma(t) h(t) \frac{\partial^2 W_s}{\partial p^2} + \Gamma(t) f(t) \frac{\partial^2 W_s}{\partial p \partial q}. \]  
(B30)
The Wigner function has some similarities with a classical distribution function, although it cannot be interpreted as a true probability density, among other reasons, because it can adopt negative values [3].

[1] E. B. Davies, *Quantum theory of open systems* (Academic Press, London, 1976).
[2] H. P. Breuer and F. Petruccione, *The theory of open quantum systems* (Oxford University Press, Oxford, 2002).
[3] C. W. Gardiner and P. Zoller, *Quantum Noise* (Springer, Berlin, 2000).
[4] U. Weiss, *Quantum Dissipative Systems* (World Scientific, 1999).
[5] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).
[6] P. M. V. B. Barone and A. O. Caldeira, Phys. Rev. A 43, 57 (1991).
[7] C. Anastopoulos, Phys. Rev. D 56, 6702 (1997), hep-th/9611155.
[8] C. Anastopoulos and A. Zoupas, Phys. Rev. D 58, 105006 (1998), hep-th/9709223.
[9] J. A. Wheeler and R. P. Feynman, Rev. Mod. Phys. 21, 425 (1949).
[10] E. Calzetta and B. L. Hu, Phys. Rev. D 49, 6636 (1994), gr-qc/9312036.
[11] B. L. Hu and A. Matacz, Phys. Rev. D 51, 1577 (1995), gr-qc/9403043.
[12] B. L. Hu and S. Sinha, Phys. Rev. D 51, 1857 (1995).
[13] A. Campos and E. Verdaguer, Phys. Rev. D 53, 1927 (1996).
[14] R. Martín and E. Verdaguer, Phys. Rev. D 61, 124024 (2000), gr-qc/0001098.
[15] E. Calzetta, B. L. Hu, and F. D. Mazzitelli, Phys. Rept. 352, 459 (2001), hep-th/0102199.
[16] B. L. Hu and E. Verdaguer, Class. Quant. Grav. 20, R1 (2003), gr-qc/0211090.
[17] B. L. Hu and E. Verdaguer, Living Reviews in Relativity 7, 3 (2004), gr-qc/0307032.
[18] E. Calzetta, A. Campos, and E. Verdaguer, Phys. Rev. D 56, 2163 (1997), gr-qc/9704010.
[19] F. C. Lombardo and F. D. Mazzitelli, Phys. Rev. D 55, 3889 (1997), gr-qc/9609073.
[20] R. Martín and E. Verdaguer, Phys. Rev. D 60, 084008 (1999), gr-qc/9904021.
[21] R. Martín and E. Verdaguer, Phys. Lett. B 465, 113 (1999), gr-qc/9811070.
[22] E. Calzetta, A. Routra, and E. Verdaguer, Phys. Rev. D 64, 105008 (2001), hep-th/0106091.
[23] E. Calzetta, A. Routra, and E. Verdaguer, Phys. Rev. Lett. 88, 010403 (2002), hep-ph/0101052.
[24] F. Lombardo and F. D. Mazzitelli, Phys. Rev. D 53, 2001 (1996), hep-th/9508052.
[25] C. Greiner and B. Müller, Phys. Rev. D 55, 1026 (1997).
[26] F. C. Lombardo, F. D. Mazzitelli, and R. J. Rivers, Nucl. Phys. B 672, 462 (2003).
[27] A. Matacz, Phys. Rev. D 56, R1836 (1997).
[28] T. Tanaka and M. Sakagami, Prog. Theor. Phys. 100, 547 (1998), gr-qc/9705054.
[29] F. C. Lombardo and D. López Nacir, Phys. Rev. D 72, 063506 (2005).
[30] J. Zanella and E. Calzetta (2006), hep-ph/0611321.
[31] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, Mass., 1998).
[32] L. S. Brown, *Quantum Field Theory* (Cambridge University Press, Cambridge, England, 1992).
[33] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, 1995).
[34] J. I. Latore, P. Pascual, and R. Tarrach, Nucl. Phys. B 437, 60 (1995), hep-th/9408016.
[35] D. Arteaga, R. Parentani, and E. Verdaguer, Phys. Rev. D 70, 044019 (2004), gr-qc/0311065.
[36] D. Arteaga, R. Parentani, and E. Verdaguer, Int. J. Theor. Phys. 44, 1665 (2005).
[37] M. le Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, England, 1996).
[38] J. Bros, H. Epstein, and U. Moschella (2006), hep-th/0612184.
[39] I. T. Drummond and S. J. Hathrell, Phys. Rev. D 22, 343 (1980).
[40] G. M. Shore, Contemp. Phys. 44, 503 (2003), gr-qc/0304059.
[41] D. Arteaga, J. Phys. A 40, 6901 (2007), 0704.0456 [gr-qc].
[42] D. Arteaga, R. Parentani, and E. Verdaguer, Int. J. Theor. Phys. 46, 227 (2007).
[43] D. Arteaga, Ann. Phys. 324, 920 (2009), 0801.4324 [hep-ph].
[44] D. Arteaga, Clas. Quant. Grav. 25 (2008), 0806.1332 [gr-qc].
[45] D. Campo and R. Parentani, Phys. Rev. D 74, 025001 (2006), astro-ph/0505376.
[46] D. Campo and R. Parentani, Phys. Rev. D 72, 045015 (2005), astro-ph/0505379.
[47] D. Campo and R. Parentani, Phys. Rev. D 70, 105020 (2004), gr-qc/0312055.
[48] W. G. Unruh and W. H. Zurek, Phys. Rev. D 40, 1071 (1989).
[49] L. D. Romero and J. P. Paz, Phys. Rev. A 55, 4070 (1997).
[50] A. O. Caldeira and A. J. Leggett, Ann. Phys. (NY) 149, 374 (1983).
[51] D. Arteaga, E. Calzetta, A. Roura, and E. Verdaguer, Int. J. Theor. Phys. 42, 1257 (2003).
[52] E. Calzetta and E. Verdaguer, J. Phys. A39, 9503 (2006), quant-ph/0603047.
[53] A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. 46, 211 (1981).
[54] S. Massar, R. Parentani, and R. Brout, Class. Quant. Grav. 10, 385 (1993).
[55] B. L. Hu and A. Matacz, Phys. Rev. D 49, 6612 (1994), gr-qc/9312035.
[56] D. F. Walls and G. J. Milburn, Quantum Optics (Springer, Berlin, 1994).
[57] A. O. Caldeira and A. J. Leggett, Physica 121A, 587 (1983).
[58] R. P. Feynman and F. L. Vernon, Ann. Phys. (NY) 24, 118 (1963).
[59] R. P. Feynman and A. R. Hibbs, Quantum Mechanics and Path Integrals (McGraw-Hill, New York, 1965).

[60] B. L. Hu, J. P. Paz, and Y. Zhang, Phys. Rev. D 45, 2843 (1992).
[61] B. L. Hu, J. P. Paz, and Y. Zhang, Phys. Rev. D 47, 1576 (1993).
[62] J. S. Schwinger, J. Math. Phys. 2, 407 (1961).
[63] L. V. Keldysh, Zh. Eksp. Teor. Fiz 47, 1515 (1965), [Sov. Phys. JEPT 20, 1018 (1965)].
[64] E. Tomboulis, Phys. Lett. B 70, 361 (1977).
[65] F. Cooper and E. Mottola, Phys. Rev. D 36, 3114 (1987).
[66] F. Cooper, S. Habib, Y. Kluger, E. Mottola, and J. P. Paz, Phys. Rev. D 50, 2848 (1994).
[67] H. A. Weldon, Phys. Rev. D 28, 2007 (1983).
[68] D. Arteaga, Ph.D. thesis, Universitat de Barcelona (2007), arXiv:0707.3899 [hep-ph].
[69] A. O. Caldeira, H. A. Cerdeira, and R. Ramaswamy, Phys. Rev. A 40, 3438 (1989).
[70] C. Fleming, B. Hu, and A. Roura (2007), 0705.2766 [quant-ph].
[71] R. Parentani (2007), 0709.3943 [hep-th].
[72] R. Parentani (2007), 0710.4664 [hep-th].
[73] K.-C. Chou, Z.-B. Su, B.-L. Hao, and L. Yu, Phys. Rept. 118, 1 (1985).
[74] E. Calzetta and B. L. Hu, Phys. Rev. D 35, 495 (1987).
[75] A. Campos and B. L. Hu, Phys. Rev. D 58, 125021 (1998).
[76] A. Campos and E. Verdaguer, Phys. Rev. D 49, 1861 (1994).
[77] S. Weinberg, Phys. Rev. D 72, 043514 (2005).
[78] A. Das, Finite temperature field theory (World Scientific, Singapore, 1997).
[79] I. Hardman, H. Umezawa, and Y. Yamanaka, J. Math. Phys. 28, 2925 (1987).
[80] I. D. Lawrie, Phys. Rev. D 40, 3330 (1989).
[81] I. D. Lawrie and D. B. McKernan, Phys. Rev. D55, 2290 (1996), hep-th/9610175.
[82] N. P. Landsman and C. G. van Weert, Phys. Rep. 145, 141 (1987).
[83] E. Calzetta, A. Roura, and E. Verdaguer, Physica 319A, 188 (2003), quant-ph/0011097.
[84] H. Grabert, P. Schramm, , and G.-L. Ingold, Phys. Rep. 168, 115 (1988).
[85] J. R. Anglin, J. P. Paz, and W. H. Zurek (1996), quant-ph/9611045.
[86] A. Roura, Ph.D. thesis, Universitat de Barcelona (2001).
[87] J. J. Halliwell and T. Yu, Phys. Rev. D 53, 2012 (1996), quant-ph/9508004.