Estimating Galaxy Redshift in Radio-Selected Datasets using Machine Learning

Kieran J. Luker\textsuperscript{a,b,*}, Ray P. Norris\textsuperscript{c}, Laurence A. F. Park\textsuperscript{c}, X. Rosalind Wang\textsuperscript{a,b}, Miroslav D. Filipović\textsuperscript{a}

\textsuperscript{a}Western Sydney University, Locked Bag 1797, Penrith, NSW 2751, Australia
\textsuperscript{b}CSIRO Astronomy and Space Sciences, Australia Telescope National Facility, PO Box 76, Epping, NSW 1710, Australia
\textsuperscript{c}CSIRO Data61, Epping, Australia

Abstract

All-sky radio surveys are set to revolutionise the field with new discoveries. However, the vast majority of the tens of millions of radio galaxies won’t have the spectroscopic redshift measurements required for a large number of science cases. Here, we evaluate techniques for estimating redshifts of galaxies from a radio-selected survey. Using a radio-selected sample with broadband photometry at infrared and optical wavelengths, we test the k-Nearest Neighbours (kNN) and Random Forest machine learning algorithms, testing them both in their regression and classification modes. Further, we test different distance metrics used by the kNN algorithm, including the standard Euclidean distance, the Mahalanobis distance and a learned distance metric for both the regression mode (the Metric Learning for Kernel Regression metric) and the classification mode (the Large Margin Nearest Neighbour metric). We find that all regression-based modes fail on galaxies at a redshift $z > 1$. However, below this range, the kNN algorithm using the Mahalanobis distance metric performs best, with an $\eta_{0.15}$ outlier rate of 5.85%. In the classification mode, the kNN algorithm using the Mahalanobis distance metric also performs best, with an $\eta_{0.15}$ outlier rate of 5.85%, correctly placing 74% of galaxies in the top $z > 1.02$ bin. Finally, we also tested the effect of training in one field and applying the trained algorithm to similar data from another field and found that variation across fields does not result in statistically significant differences in predicted redshifts. Importantly, we find that while we may not be able to predict a continuous value for high-redshift radio sources, we can identify the majority of them using the classification modes of existing techniques.

Keywords: methods: analytical; techniques: photometric; galaxies: photometry; galaxies: high-redshift

1. Introduction

New radio telescopes are set to revolutionise the radio astronomy regime, with the Evolutionary Map of the Universe (EMU) project to be completed on the Australian Square Kilometre Array Pathfinder (ASKAP; Johnston et al. 2007, 2008) telescope in particular set to increase the number of known radio sources from $\sim 2.5$ million to $\sim 70$ million (Norris et al. 2011). For most aspects of science, knowledge of an astronomical source’s redshift is an essential indicator of the distance and age of the source. Ideally, this redshift is measured directly using spectroscopy. However, even with modern advances — including Multi-Object Spectroscopy (MOS) which can allow hundreds to thousands of redshifts to be measured at once — deep spectroscopic surveys still fail to yield reliable redshifts from 30-60\% of measured spectra (Newman et al. 2015). Currently, the Sloan Digital Sky Survey (SDSS) has measured $\sim 4.8$ million spectroscopic redshifts — as of the 16\textsuperscript{th} data release (Ahumada et al. 2020) — and the 2dF Galaxy Redshift Survey (2dFGRS; Lewis et al. 2002) measured $\sim 250,000$ spectroscopic redshifts over its 5 year project.\textsuperscript{2} In the future, the Wide Area Vista Extragalactic Survey (WAVES) survey — expected to provide a further 2.5 million spectroscopic redshift measurements (Driver et al. 2016) — will increase the number of spectroscopically measured sources. However, this will still be significantly short of the expected $\sim 70$ million sources detected by the EMU project (even if all of these newly measured galaxies were exclusively selected from the EMU survey).

Photometric template fitting (Baum 1957) is able to estimate the redshift (hereafter $z_{\text{photo}}$) of a source, as well as ancillary data like galaxy classification. Photometric template fitting is completed by comparing the Spectral Energy Distribution (SED), measured across as many different wavelengths as possible, to templates constructed with astrophysical knowledge, or prior examples. Figure\textsuperscript{4} shows the 31 filter bands available in the COSMic evolUtion Survey (COSMOS) field and an example SED used by Ilbert et al. (2009) to achieve redshift accuracies of $\sigma_{\Delta z/(1+z_s)} = 0.012$.

Machine Learning (ML) has also been applied to the problem, with the k-Nearest Neighbours (kNN) algorithm

\textsuperscript{*}Corresponding author
Email address: k.luker@westernsydney.edu.au (Kieran J. Luker)

\textsuperscript{2}http://www.2dfgrs.net/Public/Survey/statusfinal.html

Preprint submitted to Astronomy and Computing March 1, 2022
Figure (1) The SED of an extragalactic source from the COSMOS survey (Object ID: 358521). The background shows the filter coverage of the 31 filters available in the COSMOS field (defined in Ilbert et al. (2009)). The “+” symbols in the foreground represent the galaxy’s photometric measurements in each band, and are coloured to match the corresponding filter in the background.

However, few of these solutions are appropriate for the large-scale radio surveys being conducted. While photometric template fitting provides a theoretically ideal solution, the quality of data required to make it highly accurate will not be available for the majority of sources in the all-sky EMU survey, since only all-sky photometric data will be available for most sources. An example of the broadband photometry used by this work is shown in Figure 2, which shows both a likely Active Galactic Nucleus (AGN; top), and Star Forming Galaxy (SFG; bottom).

Additionally, Norris et al. (2019) has shown that photometric template fitting performs poorly on radio-selected data sets (which are typically dominated by AGN), possibly because the majority of templates are unable to differentiate between emission from the AGN and emission from the galaxy itself (Salvato et al. 2018).

The ML based methods have mainly focused on optically-selected datasets, with most work drawing on the SDSS photometric and spectroscopic samples, often restricting the redshift range to $z < 1$, or using datasets containing only one type of object — e.g. the SDSS Galaxy or Quasi-Stellar Object (QSO) catalogs. While this kind of testing is entirely appropriate for non-radio selected samples, Norris et al. (2019) have shown that assumptions in the optical regime may not be valid in the radio regime, for two reasons. First, the redshift distribution of radio-selected sources is quite different from that of optically-selected sources, as shown in Figure 3. Second, radio-selected sources are often dominated by a radio-loud AGN which is poorly represented in optically-selected templates and training sets.

Finally, we should note that in support of the science goals of the EMU Project, we aim to minimise the number of estimations that catastrophically fail, rather than minimise for accuracy as most other works do.

The overall contributions of this study include:

- An in-depth study of the simple $k$-Nearest Neighbours algorithm, including the investigation of typically ignored distance metrics for the estimation of radio galaxy redshift;
- A comparison with the widely used Random Forest
Figure (2) The SED of the extragalactic source ATLAS3J033551.0283345C (top; likely AGN) and ATLAS3J033519.0273708C (bottom; likely SFG), in similar style to Figure 1. The background shows the filter coverage used by this work, with the “+” in the foreground representing the measured photometry at each band, taken from the data used in this study.
algorithm;

- An analysis of whether similarly observed fields can be used as training samples for alternate fields;
- An exploration into the effectiveness of regression vs classification modes in estimating high redshift galaxies, given the highly unbalanced nature of the training sample (noting that while classification modes might be able to identify high-redshift galaxies, it will not be able to estimate its actual redshift).

1.1. Formal Problem Statement

In this article, we investigate the problem of modelling the redshift of a source with respect to known measurements of the source. This can be a regression problem, where we attempt to model the function \( r_i = f(\vec{x}_i) \), such that the redshift of source \( i \), \( r_i \in \mathbb{R}^+ \) and \( \vec{x}_i \) the known measurements of source \( i \) are obtained from a catalogue with various domains. We can also model the redshift of a source as \( c_i = g(\vec{x}_i) \), where \( c_i \) is a class representing a specific domain of redshift. In the following section we describe the catalogues containing the known source information \( \vec{x}_i \) and the variables that are used for this representation.

2. Data

This work uses data from the Australia Telescope Large Area Survey (ATLAS; Norris et al. 2006; Franzen et al. 2015) radio continuum catalogue, providing a 1.4 GHz flux density measurement of 4780 unique sources. The ATLAS survey covers two regions of the sky – the extended Chandra Deep Field South (eCDFS) and the European Large Area ISO Survey–South 1 (ELAIS-S1), both to a depth of \( \sim 15\mu{\text{Jy}} \), and was completed as a first-look at what the EMU survey may provide. This survey was cross-matched by Swan (2018) with the Australian Dark Energy Survey (OzDES; Yuan et al. 2015; Childress et al. 2017; Lidman et al. 2020), Dark Energy Survey (DES; Dark Energy Survey Collaboration et al. 2016) and Spitzer Wide-Area Infrared Extragalactic Survey (SWIRE; Lonsdale et al. 2003) surveys, providing spectroscopic redshift measurements, \( g, r, i \) and \( z \) optical magnitudes, and 3.6, 4.5, 5.4 and 8.0 \( \mu{\text{m}} \) infrared flux measurements respectively.

This work used only those ATLAS sources with measured photometry in all provided optical and infrared bands, creating a final dataset containing 1311 sources with complete photometric coverage. Specifically, in both fields used in this work, we used the \( g, r, i \) and \( z \) optical magnitudes from DES, and 3.6, 4.5, 5.4 and 8.0 \( \mu{\text{m}} \) infrared flux measurements from the SWIRE survey, as shown in Figure 4. The redshift distribution of the final collated dataset is presented in Figure 5.

To prevent the different methods tested from being dominated by single features with wide variation, all features were standardised using Equation 1, setting the feature mean to 0, and the feature variance to unit variance.

\[
  z_i = \frac{x_i - \bar{x}}{s_x},
\]

where \( \bar{x} \) is the mean of the sample of variable \( x \) and \( s_x \) is its standard deviation.

During testing, we also examined the use of “colours” rather than optical magnitudes, and taking the log of the radio and infrared fluxes to better distribute the data. Colours are typically used instead of magnitudes to remove the brightness- and redshift-dependent nature of magnitudes, replacing them by the difference between the magnitudes that is dependent only on the SED.
2.1. Data Partitioning

All machine learning methods require the main dataset to be partitioned into multiple subsets, with the subsets set aside for training the model, validating hyperparameters, and testing the model. In this work, we split our data into two datasets – training and testing, with the hyperparameters being validated using k-fold cross validation on the training set. We partition our data differently for the three tests we complete:

- Typical random split, with 70% of the data split off as the training set, leaving the remaining 30% as the test set.
- The entire ELAIS-S1 field as the training set, leaving the eCDFS field as the test set.
- The entire eCDFS field as the training set, leaving the ELAIS-S1 field as the test set.

In addition to the above partitioning, in order to facilitate the use of the classification modes of our algorithms tested, we quantise the redshift values into 15 redshift bins (defined in Table 1), with equal numbers of sources in each in order to be able to predict a uniform distribution with using a matching distribution. The median of the spectroscopic redshifts within the bin is chosen as the redshift to predict.

3. Methods

3.1. kNN

The kNN algorithm (Cover and Hart, 1967) computes a similarity matrix between all sources based on the catalogue’s photometric measurements, and a given distance metric (the set of distance metrics tested in this work are explained in Subsections 3.1.1, 3.1.2, and 3.1.3). Once the similarity matrix is constructed, the kNN algorithm finds the k (Hereafter k_N) most similar sources with measured redshifts (where k_N is optimised using cross validation), and takes either the mean value (for regression) or the mode class (for classification) of the sources as the estimated redshift for each source. A simple illustration of the kNN algorithm is shown in Figure 6.

The value of k_N used in the kNN in this work is optimised using k-fold (where k is hereafter k_f and is set to 10 for this work) cross-validation. k_f-fold cross-validation randomly splits the data into k_f roughly even groups, iterating through using k_f – 1 groups as the training set and validating on the remaining group, until every group has been used in testing, with the average error used as the error for that value of k_f. This work tested all integer values of k_N for the kNN algorithm between 2 and 20 (k_N ∈ {2,...,20}).

The kNN algorithm requires a metric to determine which of the observations are neighbours to a given observation. In the next sections we present the set of metrics investigated in the experiment.

3.1.1. Euclidean Distance

Euclidean Distance is the simplest, and most widely used distance metric in literature, defined in Equation 2:

$$d(p, q) = \sqrt{(\vec{p} - \vec{q})^T(\vec{p} - \vec{q})},$$

where d(p, q) is the Euclidean distance between two feature vectors p and q. In this case the vectors contain the measured photometry of two galaxies.

3.1.2. Mahalanobis Distance

The Mahalanobis distance metric (Mahalanobis, 1936) normalises the variance and covariance of the input features by transforming the features using the inverse of the covariance matrix. For uncorrelated input features, the

![Figure (5) The redshift distribution of sources in the ATLAS dataset, broken down by field (ELAIS-S1 and eCDFS). The overall mean redshift is z = 0.47, ELAIS-S1 mean is z = 0.51 and eCDFS mean is z = 0.44.](image)

| Bin Number | Lower Bound | Upper Bound | Median Value |
|------------|-------------|-------------|--------------|
| 1          | 0           | 0.10        | < 0.1        |
| 2          | 0.10        | 0.15        | 0.12         |
| 3          | 0.15        | 0.19        | 0.17         |
| 4          | 0.19        | 0.22        | 0.21         |
| 5          | 0.22        | 0.26        | 0.24         |
| 6          | 0.26        | 0.29        | 0.27         |
| 7          | 0.29        | 0.32        | 0.31         |
| 8          | 0.32        | 0.35        | 0.34         |
| 9          | 0.35        | 0.41        | 0.38         |
| 10         | 0.41        | 0.50        | 0.46         |
| 11         | 0.50        | 0.58        | 0.54         |
| 12         | 0.58        | 0.66        | 0.62         |
| 13         | 0.66        | 0.80        | 0.73         |
| 14         | 0.80        | 1.02        | 0.91         |
| 15         | 1.02        | 4.33        | > 1.02       |

Table (1) Details of the bins used for the classifications tests, outlining the lower edge, upper edge and predicted value of each bin (where the predicted value is the median of the spectroscopic redshifts from within the bin).
Mahalanobis distance is equal to the scaled Euclidean distance, but for correlated features, it generalises the idea of Euclidean distance to take account of the covariance. The Mahalanobis distance is defined in Equation 3:

\[ d(\vec{p}, \vec{q}) = \sqrt{(\vec{p} - \vec{q})^T S^{-1} (\vec{p} - \vec{q})}, \]

where \( d(\vec{p}, \vec{q}) \) is the Mahalanobis distance between two feature vectors \( \vec{p} \) and \( \vec{q} \), and \( S \) is the covariance matrix.

### 3.1.3. Learned Distance Metrics

Conceptually, for best results the distance metric used should take into consideration the shape and structure of the data. Towards this end, we can generalise Equations 2 and 3 to Equation 4, noting that the \( M \) matrix can be any positive semi-definite matrix. The Identity matrix is used for Euclidean Distance, and the \( S \) matrix is used in the Mahalanobis. We can, however, go one step further, and attempt to learn an \( M \) matrix that can better warp the feature space so that observations with similar redshift are measured as close, while observations with different redshift are measured as distant.

\[ d(\vec{p}, \vec{q}) = \sqrt{(\vec{p} - \vec{q})^T M (\vec{p} - \vec{q})}, \]

where \( d(\vec{p}, \vec{q}) \) is the distance between two feature vectors \( \vec{p} \) and \( \vec{q} \).

For our regression tests, we used the Metric Learning for Kernel Regression (MLKR) distance metric, which performs a supervised Principle Component Analysis (Weinberger and Tesauro 2007). The MLKR distance metric begins by decomposing the \( M \) matrix from Equation 4 using Equation 5:

\[ M = A^T A \]

Using Equation 5, Equation 4 can be expressed as the modified Euclidean Distance metric in Equation 6:

\[ d(\vec{x}_i, \vec{x}_j) = ||A(\vec{x}_i - \vec{x}_j)||^2 \]

Matrix \( A \) is optimised using Gradient Descent, using Equation 7:

\[ \frac{\partial L}{\partial A} = 4A \sum_i (\hat{y}_i - y_i) \sum_j (\hat{y}_j - y_j) k_{ij} \vec{x}_i \vec{x}_j^T, \]

where \( \hat{y}_i \) is defined in Equation 8, \( k_{ij} \) is a Gaussian kernel defined in Equation 9, and \( \vec{x}_i = (y_i - \hat{y}_i)^2 \).

\[ \hat{y}_i = \frac{\sum_{j \neq k} y_j k_{ij}}{\sum_{j \neq k} k_{ij}} \]

\[ k_{ij} = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x_i - x_j)^2}{2\sigma^2}} \]

The loss function being minimised is a simple squared difference, defined in Equation 10:

\[ L = \sum_i (y_i - \hat{y}_i)^2 \]

For our Classification tests, the Large Margin Nearest Neighbour (LMNN; Weinberger et al., 2006) learned distance metric was used. The LMNN distance metric finds a transformation for the data that maximises the distance between different classes, and minimises the distance between similar classes. The loss function the LMNN algorithm optimises is defined in Equation 11:

\[ \epsilon(L) = \sum_{ij} \eta_{ij} ||L(\vec{x}_i - \vec{x}_j)||^2 + c \sum_{ijl} \eta_{jl} (1 - y_{il}) \left[ 1 + \frac{||L(\vec{x}_i - \vec{x}_j)||^2}{z_{il}^+} - \frac{||L(\vec{x}_i - \vec{x}_j)||^2}{z_{il}^-} \right] \]

where \( i, j, \) and \( l \) are individual galaxy feature vectors, \( \eta_{ij} \in [0, 1] \) describes whether \( \vec{x}_i \) is a target of \( \vec{x}_j \), \( c \) is a positive constant typically chosen through cross-validation and \( z_{il}^+ = \max(z, 0) \) - a hinge function.

An example of the transformation the LMNN algorithm attempts is in Figure 7 using three neighbours.
where the split, \( Q \) are the numbers of objects on the left and right sides of \( N \) best features and values to split the data on: at each node (defined in Equation 12) to determine the impurity explored using a tree, shown in Figure 8 (Morgan and Sonquist, 1963; Quinlan, 1987). Each DT finds the impurity maximised.

For Regression, the Mean Square Error is used (defined in Equation 13), whereas Classification often uses the Gini impurity (defined in Equation 14).

\[
H(X_m) = \frac{1}{N_m} \sum_{i \in N_m} (y_i - \left[ \frac{1}{N_m} \sum_{j \in N_m} y_j \right])^2, \tag{13}
\]

where \( N_m \) is the number of objects at node \( m \) and \( y_i \) and \( y_j \) are the response variables.

\[
H(X_m) = \sum_k p_{mk}(1 - p_{mk}), \tag{14}
\]

where \( p_{mk} \) is the proportion of split \( m \) that are class \( k \), defined formally in Equation 15

\[
p_{mk} = \frac{1}{N_m} \sum_{z_i \in N_m} I(y_i = k), \tag{15}
\]

where \( I \) is the indicator function, identifying the correct classifications.

### 3.3. Error Metrics

In order to evaluate each of the machine learning models, the prediction error of each model will be assessed. In this section, the set of statistics that measure the error of the models are presented. The primary error metric compared in this work is the \( \eta_{0.15} \) outlier rate, defined in Equation 16

\[
\eta_{0.15} = \frac{\text{count}(|\Delta z| > 0.15 \times (1 + z_{\text{spec}}))}{\text{Number Of Sources}}, \tag{16}
\]

where \( \Delta z = z_{\text{spec}} - z_{\text{photo}} \). The \( \eta_{0.15} \) outlier rate is a percentage representing the number of ‘catastrophic failures’ (the percentage of galaxies that have a residual greater than 0.15, scaled with redshift), and is commonly found in literature (Ilbert et al., 2009; Salvato et al., 2009, 2011; Cavuoti et al., 2012; Zitlau et al., 2016; Cavuoti et al., 2017; Jones and Singal, 2017; Moutrichas et al., 2017; Luken et al., 2019; Norris et al., 2019).

We also provide a secondary \( \eta_{2\sigma} \) outlier rate in order to provide a statistically sound comparison, defined in Equation 17

\[
\eta_{2\sigma} = \frac{\text{count}(|\Delta z| > 2\sigma)}{\text{Number Of Sources}} \times 100, \tag{17}
\]

where \( \Delta z = z_{\text{spec}} - z_{\text{photo}} \), and \( \sigma \) is the standard deviation of the estimated response, defined in Equation 18

\[
\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y})^2}, \tag{18}
\]

where \( N \) is the number of observations, \( y_i \) is an observation, and \( \hat{y} \) is the estimated observation.

As this work is presenting both regression and classification modes of the given algorithms, there are some error metrics that suit regression, and some that suit classification. These metrics are defined in Subsection 3.3.2 respectively.
3.3.1. Regression Error Metrics

For the regression tests, three additional error metrics were compared. The first is the Normalised Median Absolute Deviation (NMAD). The NMAD is a similar measure to the standard deviation, and is generally used for non-Gaussian distributions. It is more robust than the standard deviation, as it takes the median of the residuals, improving the resilience to outliers – an issue that can be prominent in redshift estimation due to the highly unbalanced data sets used, and is defined in Equation 19:

\[ \sigma_{\text{NMAD}} = 1.4826 \times (\text{median}(|X_i - \text{median}(X)|)) \]  

(19)

where NMAD is the Normalised Median Absolute Deviation, \(X_i\) is a vector of residuals from which \(X_1\) is taken.

The \(R^2\) Coefficient of Determination is the second regression-based error metric, and is the proportion of variance explained by the model. The \(R^2\) is defined in Equation 20:

\[ R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2} \]  

(20)

where \(y_i\) is a response variable, \(\hat{y}_i\) is the corresponding estimated response, and \(\bar{y}\) is the mean of the response variables.

Finally, the Mean Square Error (MSE) is a direct measure of the error produced by the model, with the lower the value the better, and is defined in Equation 21:

\[ \text{MSE} = \frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2 \]  

(21)

where \(N\) is the number of observations, \(y_i\) is the measured response, and \(\hat{y}_i\) is the estimated response.

3.3.2. Classification Error Metrics

Traditionally in ML classification settings, the Accuracy (defined in Equation 22), Precision, Recall and F1 score are reported. We also investigated the Normalised Mutual Information, which suggests how dependent one set of data is upon another. We do not report the Precision, Recall, F1 Score and Normalised Mutual Information, however, as we found they provided no additional information.

It is important to note that the main error metric being compared and minimised in both the regression and classification tests is the \(p_{0.15}\) outlier rate. As this outlier rate is explicitly accepting of a level of inaccuracy that scales with redshift, there is an inherent acceptance of some level of leakage between neighbouring classes during classification tests.

This acceptance of leakage means that models may present with high error rates due to rigid correct/incorrect classifications, and yet may still be acceptable models for this process.

\[ \text{Accuracy}(y, \hat{y}) = \frac{1}{n} \sum_{i=0}^{n-1} I(\hat{y}_i = y_i) \]  

(22)

where \(y\) is the measured response, \(\hat{y}\) is the predicted response, \(n\) is the number of samples, and \(I\) is an indicator function, indicating the cases where the predicted response matched the measured response.

3.4. Statistical Significance

Analysis of Variance (ANOVA) tests were used in this study to test for statistical significance. This allows us to test whether the changes in model correlate to a statistically significant change in estimated redshift, by testing to see if the means of two or more populations (in this case, experiments with different models) differ.

In all cases, the tests were run in a one-vs-many scenario, with the one being tested as our best performing metric in order to determine whether our best performing result is statistically significant.

3.5. Software

This work makes use of the Scikit-learn\[^1\] Python package [Pedregosa et al., 2011] for the implementation of the RF and kNN algorithms, as well as the Euclidean and Mahalanobis distance metrics. We made use of the PyLMNN\[^2\] package for the LMNN distance metric, and the metric-learn\[^3\] package for the MLKR distance metric. The code and data used in this work is available on Github\[^4\].

4. Results

Given the number of different algorithms, distance metrics and datasets tested in this work, we have assigned each combination a Test ID, defined in Table 2 for Regression-based tests, and Table 3 for Classification-based tests. Each test ID is made up of 4 characters, with the first character (R/C) representing whether the test is a regression- or classification-based test, the 2nd and 3rd characters (Eu/Ma/ML/Rf) representing the method used for estimation (Euclidean distance, Mahalanobis distance, a Learned distance metric, or Random Forest), and the final character (1/2/3) representing the training set used (1 = Random, 2 = ELAIS-S1, 3 = eCDFS).

As discussed in Section 2, we tested both statistical standardisation of input photometry, compared with the use of astrophysically derived “colours”, and taking the log of radio and infrared data to better distribute the data. In both cases, however, the result was very similar to the standardised dataset, and hence is the only value quoted.

\[^1\]https://scikit-learn.org/  
\[^2\]https://pypi.org/project/PyLMNN/  
\[^3\]http://contrib.scikit-learn.org/metric-learn/generated/metric_learn.MLKR.html  
\[^4\]https://github.com/kluken/Redshift-kNN-2021
Table (2) Description of the regression based tests conducted. The Test ID is used to reference the results in Table 4. The Method is the ML algorithm used in the test. The Distance Metric is the distance metric used in the kNN algorithm (and hence is empty for the RF algorithm). The Training Set is the method of choosing the training set for the ML algorithm.

| Test ID | Method | Distance Metric | Training Set |
|---------|--------|----------------|--------------|
| REu1    | kNN    | Euclidean      | Random       |
| REu2    | kNN    | Euclidean      | ELAIS-S1     |
| REu3    | kNN    | Euclidean      | eCDFS        |
| RMa1    | kNN    | Mahalanobis    | Random       |
| RMa2    | kNN    | Mahalanobis    | ELAIS-S1     |
| RMa3    | kNN    | Mahalanobis    | eCDFS        |
| RML1    | kNN    | MLKR           | Random       |
| RML2    | kNN    | MLKR           | ELAIS-S1     |
| RML3    | kNN    | MLKR           | eCDFS        |
| RRf1    | RF     | –              | Random       |
| RRf2    | RF     | –              | ELAIS-S1     |
| RRf3    | RF     | –              | eCDFS        |

Table (3) Description of the classification based tests conducted. The Test ID is used to reference the results in Table 5. The Method is the ML algorithm used in the test. The Distance Metric is the distance metric used in the kNN algorithm (and hence is empty for the RF algorithm). The Training Set is the method of choosing the training set for the ML algorithm.

| Test ID | Method | Distance Metric | Training Set |
|---------|--------|----------------|--------------|
| CEu1    | kNN    | Euclidean      | Random       |
| CEu2    | kNN    | Euclidean      | ELAIS-S1     |
| CEu3    | kNN    | Euclidean      | eCDFS        |
| CMa1    | kNN    | Mahalanobis    | Random       |
| CMa2    | kNN    | Mahalanobis    | ELAIS-S1     |
| CMa3    | kNN    | Mahalanobis    | eCDFS        |
| CML1    | kNN    | LMNN           | Random       |
| CML2    | kNN    | LMNN           | ELAIS-S1     |
| CML3    | kNN    | LMNN           | eCDFS        |
| CRf1    | RF     | –              | Random       |
| CRf2    | RF     | –              | ELAIS-S1     |
| CRf3    | RF     | –              | eCDFS        |

4.1. Regression

Based on the Regression tests outlined in Table 2, we present the results in a series of plots (Figures 9, 10, 11, and 12).

In each plot three subfigures are presented, with Subfigure A representing the random training set, Subfigure B representing the training set built using galaxies from the ELAIS-S1 field, and Subfigure C representing the training set built using galaxies from the eCDFS field. The subfigures are split into two plots – the top plot showing the measured spectroscopic redshift (x-axis) compared with the predicted redshift (y-axis), with a perfect 1:1 correlation (red dashed line) and the $\eta_{0.15}$ outlier rate boundaries (blue dashed lines) shown. The bottom plot shows the measured spectroscopic redshift (x-axis) against the residuals, again with the perfect 1:1 correlation (red dashed line) and the $\eta_{0.15}$ outlier rate boundaries (blue dashed lines) shown.

These results are summarised in Table 4. We show that the lowest $\eta_{0.15}$ outlier rate is achieved using the kNN algorithm paired with the Mahalanobis distance metric, and is statistically different from most other algorithms (kNN using Euclidean Distance: p value = 0.0183, and the RF algorithm: p value = 0.0183 compared with kNN using the Mahalanobis distance metric as a baseline). The kNN algorithm using the MLKR distance metric (a Mahalanobis-like distance metric) is not statistically significantly different (p value = 0.5750). However, all results (including the RF algorithm) suffer from the same issues of under-predicting high redshift ($z > 1$) galaxies.

The randomly selected training sets typically achieve lower $\eta_{0.15}$ outlier rates than the training sets built using one field only, however, neither field-based training set provides consistently better $\eta_{0.15}$ outlier rates than the other. This is confirmed statistically, with no statistically significant result measured (p value = 0.2072).
4.2. Classification

Based on the Classification tests outlined in Table 3, we show the classification-based results for the kNN algorithm – using Euclidean distance (Figure 14), Mahalanobis distance (Figure 15) and the LMNN learned distance metric (Figure 16) – and the RF algorithm (Figure 17), summarised in Table 5.

We present our classification-based results using scaled confusion matrices, where the x-axis shows the measured spectroscopic redshift, the y-axis shows the predicted redshift, the colour shows the density of the objects in that chosen bin, and the width of each bin is proportional to the range of redshift values represented by the bin. An example scatter plot (in the same style as Figure 9), demonstrating the effect of binning on the classification results when compared against the original spectroscopic measurements is shown in Figure 13.

The kNN algorithm paired with the Mahalanobis distance metric provides the lowest $\eta_{0.15}$ and $\eta_{2\sigma}$ outlier rates, as well as performing the best in terms of traditional ML classification metrics (accuracy, precision, recall and F1 score). However, while the results using the Mahalanobis distance metric are statistically significantly better than those using the Euclidean distance metric ($p = 0.00619$), they are not statistically different from the LMNN learned distance metric (a Mahalanobis-like distance metric; $p = 0.4276$).

Table (5) Results using the classification based algorithms. The Test column relates to the tests defined in Table 3. The best result for each metric is highlighted in bold.

| Test | k / $\eta_{0.15}$ | Acc  |
|------|------------------|------|
| CEu1 | 6 8.91 4.07 | 0.30 |
| CEu2 | 5 12.13 3.55 | 0.24 |
| CEu3 | 8 11.31 5.05 | 0.27 |
| CMa1 | 11 5.85 3.57 | 0.50 |
| CMa2 | 6 7.60 4.41 | 0.37 |
| CMa3 | 13 7.88 3.43 | 0.40 |
| CML1 | 7 6.36 5.85 | 0.40 |
| CML2 | 8 9.93 4.04 | 0.32 |
| CML3 | 7 9.90 4.04 | 0.40 |
| CRf1 | 53 7.12 3.82 | 0.36 |
| CRf2 | 39 10.54 4.41 | 0.27 |
| CRf3 | 32 10.91 4.24 | 0.33 |

Figure (9) Figures showing the results using the kNN Regression algorithm using Euclidean Distance as the Distance metric, varying the data used for training (with the numbers corresponding to the Test ID in Table 2). (REu1 – Left) uses a random training sample, (REu2 – centre) uses the ELAIS-S1 field as the training set, and (REu3 – right) uses the eCDFS field as the training set. The x-axes shows the measured spectroscopic redshift. The top-panel y-axes show the predicted redshift using the given model. The bottom-panel y-axes show the normalised residuals. The red-dashed line shows a perfect 1:1 prediction, and the blue-dashed lines show the decision boundaries based on the $\eta_{0.15}$ outlier rates.
or the RF algorithm \( (p = 0.8913) \). Again, we note that for most algorithms, the highest redshift galaxies remain a problem for estimation; however, the results can change significantly based on the distance metric used — for example, the \( z > 1.02 \) bin was correctly predicted \( \sim 50\% \) of the time using the kNN algorithm paired with Euclidean distance, however, using the Mahalanobis distance metric brought this up to \( \sim 74\% \).

As with the regression tests, the random training sample outperformed the training sets built from a single field in the primary \( \eta_{0.15} \) error metric, however, the results are statistically insignificant \( (p = 0.4397) \).

5. Discussion

Most previous work has treated the estimation of galaxy redshift as a regression task — estimating a continuous value for redshift. We have shown that the kNN algorithm — when using either a learned distance metric, or the Mahalanobis distance metric — outperforms the RF algorithm, particularly at a redshift of \( z < 1 \). At a redshift of \( z > 1 \), both the kNN and RF have systematic under-estimations, likely caused by the unbalanced training sample used, with the vast majority of samples at lower redshifts.

Here we try a different approach, by treating the problem of calculating point estimates of redshifts as a classification problem. Previous works have calculated probability density functions (PDFs) for galaxies by finely binning data and treating it as a classification problem \( (\text{Gerdes et al., 2010}; \text{Pasquet-Itam and Pasquet, 2018}; \text{Eriksen et al., 2020}) \). However, this is subtly different to this work as the primary purpose here is to identify the high-redshift galaxies — not to generate PDFs. We balance the data by binning the data into 15 bins with equal numbers of sources. By allowing this coarse mapping, we ensure that all redshift values we are attempting to predict are equally represented, and can therefore obtain better results at the higher redshift ranges. In particular, we have included a \( z > 1 \) bin, which in our best results (the kNN algorithm using the Mahalanobis distance metric) we predict in 74\% of cases (85\% of galaxies at \( z > 0.8 \) predicted in the highest two bins). Again, the kNN algorithm is able to outperform the RF algorithm.

In addition to the regression and classification tests, we tested using different training samples. The first training sample is made up of a random selection across both the ELAIS-S1 and eCDFS fields. The second and third samples were made up of only one of the fields, which was then used to predict the other. As expected, the random sample modelled the test sets better — though not significantly — than the training samples taken from a single field only. This is likely due to observational differences — while the

Figure (10) Figures showing the results using the kNN Regression algorithm using Mahalanobis Distance as the Distance metric, varying the data used for training (with the numbers corresponding to the Test ID in Table 2). (RMa1 – Left) uses a random training sample, (RMa2 – centre) uses the ELAIS-S1 field as the training set, and (RMa3 – right) uses the eCDFS field as the training set. The x-axes shows the measured spectroscopic redshift. The top-panel y-axes show the predicted redshift using the given model. The bottom-panel y-axes show the normalised residuals. The red-dashed line shows a perfect 1:1 prediction, and the blue-dashed lines show the decision boundaries based on the \( \eta_{0.15} \) outlier rates.
Figure (11) Figures showing the results using the kNN Regression algorithm using MLKR Distance as the Distance metric, varying the data used for training (with the numbers corresponding to the Test ID in Table 2). (RML1 – Left) uses a random training sample, (RML2 – centre) uses the ELAIS-S1 field as the training set, and (RML3 – right) uses the eCDFS field as the training set. The x-axes shows the measured spectroscopic redshift. The top-panel y-axes show the predicted redshift using the given model. The bottom-panel y-axes show the normalised residuals. The red-dashed line shows a perfect 1:1 prediction, and the blue-dashed lines show the decision boundaries based on the $\eta_{0.15}$ outlier rates.

Figure (12) Figures showing the results using the RF Regression algorithm, varying the data used for training (with the numbers corresponding to the Test ID in Table 2). (RRf1 – Left) uses a random training sample, (RRf2 – centre) uses the ELAIS-S1 field as the training set, and (RRf3 – right) uses the eCDFS field as the training set. The x-axes shows the measured spectroscopic redshift. The top-panel y-axes show the predicted redshift using the given model. The bottom-panel y-axes show the normalised residuals. The red-dashed line shows a perfect 1:1 prediction, and the blue-dashed lines show the decision boundaries based on the $\eta_{0.15}$ outlier rates.

surveys used in this work were designed to be as homogeneous as possible, there will always be slight differences.
Figure (13) An example scatter plot similar to Figure 9 for the CMa1 test, showing the effect of binning and classifying.

One such difference is the number of sources in each field. While there is not expected to be differences in the actual source counts between these fields, there is the potential issue of minor observational issues (for example, in the radio regime, there may have been more radio-frequency interference in one set of observations than another. In the optical regimes, it may have been that there was slightly different sky conditions affecting the observations) affecting the fields differently.

5.1. Comparison with Previous Works

Fair comparisons with previous works are often difficult due to different features being selected — often studies are conducted on well-observed, feature rich fields like the COSMOS field, providing a wealth of UV, NIR and X-ray data that is typically not available for the majority of the sky — and different selection methods — selecting sources from the SDSS Galaxy survey, where the redshift range is restricted towards higher redshift and typically don’t have a radio-counterpart, which are shown by Norris et al. (2019) to be a more difficult challenge. Where the data sets have been AGN selected, using similar features as in Dun-can et al. (2021) Lockman Hole AGN Experiments), we find our results are highly competitive, providing a lower outlier rate (∼ 6% compared with ∼ 22%, albeit with a higher scatter (σ = 0.12 compared with σ = 0.077), suggesting that while we are less accurate than others, we are having fewer estimates catastrophically fail. This optimisation of outlier rate (reducing the number of estimations that catastrophically fail) is motivated by the science goals of the EMU Project. We note that this comparison should be further tempered by the fact that the differing training

Figure (14) The confusion matrix of the kNN classification tests using the Euclidean distance metric. The x-axis of each plot shows the measured spectroscopic redshift, and the y-axis shows the estimated redshift. The size of the boxes is scaled based on the width of the bin used for classification. (CEu1 – Top) uses a random training sample, (CEu2 – Middle) uses a training sample consisting of the ELAIS-S1 field, and (CEu3 – Bottom) uses a training sample consisting of the cCDFS field.
Figure (15) The confusion matrix of the kNN classification tests using the Mahalanobis distance metric. The x-axis of each plot shows the measured spectroscopic redshift, and the y-axis shows the estimated redshift. The size of the boxes is scaled based on the width of the bin used for classification. (CMa1 – Top) uses a random training sample, (CMa2 – Middle) uses a training sample consisting of the ELAIS-S1 field, and (CMa3 – Bottom) uses a training sample consisting of the eCDFS field.

Figure (16) The confusion matrix of the kNN classification tests using the LMNN learned distance metric. The x-axis of each plot shows the measured spectroscopic redshift, and the y-axis shows the estimated redshift. The size of the boxes is scaled based on the width of the bin used for classification. (CML1 – Top) uses a random training sample, (CML2 – Middle) uses a training sample consisting of the ELAIS-S1 field, and (CML3 – Left) uses a training sample consisting of the eCDFS field.
Figure (17) The confusion matrix of the RF classification tests. The x-axis of each plot shows the measured spectroscopic redshift, and the y-axis shows the estimated redshift. The size of the boxes is scaled based on the width of the bin used for classification. (CRf1 – Top) uses a random training sample, (CRf2 – Middle) uses a training sample consisting of the ELAIS-S1 field, and (CRf3 – Bottom) uses a training sample consisting of the eCDFS field.

Figure (18) A comparison of the $\eta_{0.15}$ outlier rates presented in Section 4. The top panel compares the regression-based tests, with the bottom panel comparing the classification-based tests. The x-axis shows the different tests run, with the y-axis showing the $\eta_{0.15}$ outlier rate. The different colours/styled lines represent the different training data used – the purple solid line used a training sample randomly selected from both the ELAIS-S1 and eCDFS fields, the blue dotted line used a training sample comprised of galaxies in the ELAIS-S1 field, and the green dashed line used a training sample comprised of galaxies in the eCDFS field.

and test set sizes, and the redshift distribution of the overall datasets have a large impact on the overall statistics.

6. Conclusion

We have used a radio-selected point-source catalogue consisting of ATLAS radio data, DES optical photometry, SWIRE infrared data and OzDES spectroscopic redshifts to test the kNN and RF algorithms for estimating the redshift of galaxies.

We have used both regression and classification modes of these algorithms in order to balance highly unbalanced training sets, and have shown that using classification modes increases the effectiveness of the algorithms at a redshift of $z > 1$ (noting that while we are generally able to identify the galaxies at $z > 1$, we are unable to estimate their exact redshift). Given this increase in effectiveness at isolating the high-redshift galaxies using classification methods, a mix of classification- and regression-based methods would provide the best possible result over larger redshift ranges, allowing the higher redshift galaxies to be identified, while still being able to estimate the redshift of nearby galaxies to high accuracy.

In our tests we have shown that when using the classification modes of the algorithms, the kNN algorithm using the Mahalanobis distance metric performs better than the alternative methods tested. We particularly note that the $z > 1.02$ bin was correctly predicted in 74% of cases – far better than the regression regimes that fail consistently at that range.
When using the regression modes of the algorithms, the kNN algorithm using the Mahalanobis distance metric performed statistically significantly better than most of the alternate methods tested (kNN using Euclidean Distance: p value = 0.0183, and the RF algorithm 0.0183) – the kNN algorithm paired with the Mahalanobis-like MLKR distance metric was statistically insignificantly different (p = 0.5750). In both regression and classification methods, the kNN algorithm outperforms the much more widely used RF algorithm.

Finally, we tested whether there would be a significant difference in the $\eta_{0.15}$ outlier rate between a model trained and tested on randomly split data, and a model that is trained on one field of sky and tested on another. While the results were generally suggestive of the random training sample out-performing the field-specific training samples, the difference is not statistically significant (Regression p value = 0.2072 and classification p value = 0.4397). This suggests that for new fields being observed with similar strategies to previously observed fields, any differences in measured photometry should produce minimal effect on the estimated redshifts. It should be noted, however, that this does not take into consideration any differences in photometry caused by differences in telescopes. For example, if a galaxy was to have SkyMapper $g$, $r$, $i$ and $z$ photometry instead of DES photometry. To determine the impact of changes of telescopes on the data would require further testing, and will be the subject of further work utilising Transfer Learning.

7. Implications for Future Radio Surveys

Traditional photometric template fitting methods typically struggle to estimate the redshift of radio galaxies. In the near future, large area radio surveys like the EMU survey are set to revolutionise the field of Radio Astronomy, with the number of known radio galaxies set to increase by an order of magnitude. This work shows that broadband photometry at similar wavelengths to those available in present and near-complete all-sky surveys will be enough to estimate acceptable redshifts for ~95% of radio sources with coverage over those bands. Further, while continuous redshift values are particularly difficult to estimate for radio sources at high-redshift, they can still be identified as high redshift with the majority of sources correctly placed in the highest redshift bin using classification modes.

Acknowledgments

The Australia Telescope Compact Array is part of the Australia Telescope National Facility which is funded by the Australian Government for operation as a National Facility managed by CSIRO. We acknowledge the Gomeroi people as the traditional owners of the Observatory site.

Based in part on data acquired at the Anglo-Australian Telescope. We acknowledge the traditional owners of the land on which the AAT stands, the Gamilaroi people, and pay our respects to elders past and present.

This project used public archival data from the Dark Energy Survey (DES). Funding for the DES Projects has been provided by the U.S. Department of Energy, the U.S. National Science Foundation, the Ministry of Science and Education of Spain, the Science and Technology Facilities Council of the United Kingdom, the Higher Education Funding Council for England, the National Center for Supercomputing Applications at the University of Illinois at Urbana-Champaign, the Kavli Institute of Cosmological Physics at the University of Chicago, the Center for Cosmology and Astro-Particle Physics at the Ohio State University, the Mitchell Institute for Fundamental Physics and Astronomy at Texas A&M University, Financiadora de Estudos e Projetos, Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro, Conselho Nacional de Desenvolvimento Científico e Tecnológico and the Ministério da Ciência, Tecnologia e Inovação, the Deutsche Forschungsgemeinschaft, and the Collaborating Institutions in the Dark Energy Survey.

The Collaborating Institutions are Argonne National Laboratory, the University of California at Santa Cruz, the University of Cambridge, Centro de Investigaciones Energéticas, Medioambientales y Tecnológicas-Madrid, the University of Chicago, University College London, the DES-Brazil Consortium, the University of Edinburgh, the Eidgenössische Technische Hochschule (ETH) Zürich, Fermi National Accelerator Laboratory, the University of Illinois at Urbana-Champaign, the Instituto de Ciências de l’Espai (IEEC/CSIC), the Institut de Física d’Altes Energies, Lawrence Berkeley National Laboratory, the Ludwig-Maximilians Universität München and the associated Excellence Cluster Universe, the University of Michigan, the National Optical Astronomy Observatory, the University of Nottingham, The Ohio State University, the OzDES Membership Consortium, the University of Pennsylvania, the University of Portsmouth, SLAC National Accelerator Laboratory, Stanford University, the University of Sussex, and Texas A&M University.

Based in part on observations at Cerro Tololo Inter-American Observatory, National Optical Astronomy Observatory, which is operated by the Association of Universities for Research in Astronomy (AURA) under a cooperative agreement with the National Science Foundation.

This work is based on archival data obtained with the Spitzer Space Telescope, which was operated by the Jet Propulsion Laboratory, California Institute of Technology under a contract with NASA. Support for this work was provided by an award issued by JPL/Caltech.

References

Ahumada, R., Prieto, C.A., Almeida, A., Anders, F., Anderson, S.F., Andrews, B.H., Anguiano, B., Arcodia, R., Armengaud, E., Aubert, M., Avila, S., Avila-Reese, V., Badenes, C., Balland, C., Barger, K., Barrera-Ballesteros, J.K., Basu, S., Bautista, J., Beaton, R.L., Beers, T.C., Benavides, B.I.T., Bender, C.F.,
ioya, Y., Shopbell, P., Silverman, J., Smolčić, V., Surace, J., Taniguchi, Y., Thompson, D., Trump, J.R., Urry, M., Zamozjski, M., 2009. Photometric Redshift and Classification for the XMM-COSMOS Sources. Astrophysical Journal 690, 1250–1263. doi:10.1088/0004-637X/690/2/1250

Salvato, M., Ilbert, O., Hasinger, G., Rau, A., Civano, F., Zamorani, G., Brusa, M., Elvis, M., Vignali, C., Aussel, H., Comaastro, A., Fiore, F., Le Floc’h, E., Mainieri, V., Bardelli, S., Bolzonella, M., Bongiorno, A., Capak, P., Caputi, K., Cappelluti, N., Carollo, C.M., Contini, T., Garilli, B., Iovino, A., Fotopoulou, S., Fruchowie, A., Gilli, R., Halliday, C., Kneib, J.P., Kakazu, Y., Karataltepe, J.S., Koekemoer, A.M., Kovac, K., Ideue, Y., Ikeda, H., Impey, C.D., Le Fevre, O., Lamareille, F., Le Borgne, J.F., Le Brun, V., Lilly, S., Maier, C., Manohar, S., Masters, D., McCracken, H., Messias, H., Mignoli, M., Mobasher, B., Nagao, T., Pello, R., Puccetti, S., Perez-Montero, E., Renzini, A., Sargent, M., Sanders, D.B., Scodellio, M., Scoville, N., Shopbell, P., Silvermann, J., Taniguchi, Y., Tasca, L., Tresse, L., Trump, J.R., Zucca, E., 2011. Dissecting Photometric Redshift for Active Galactic Nucleus Using XMM- and Chandra-COSMOS Samples. Astrophysical Journal 742, 61. doi:10.1088/0004-637X/742/2/61

Salvato, M., Ilbert, O., Hoyle, B., 2018. The many flavours of photometric redshifts. Nature Astronomy doi:10.1038/s41550-018-0478-0

Swan, J.A., 2018. Multi-frequency matching, classification, and cosmic evolution of radio galaxy populations. Ph.D. thesis. University of Tasmania, Australia. URL: https://eprints.utas.edu.au/30014/.

Tagliaferri, R., Longo, G., Andreon, S., Capozziello, S., Donalek, C., Giordano, G., 2003. Neural Networks for Photometric Redshifts Evaluation. Lecture Notes in Computer Science 2859, 226–234. doi:10.1007/978-3-540-45216-4_26

Weinberger, K.Q., Blitzer, J., Saul, L.K., 2006. Distance Metric Learning for Large Margin Nearest Neighbor Classification, in: Weiss, Y., Schölkopf, B., Platt, J.C. (Eds.), Advances in Neural Information Processing Systems 18. MIT Press, pp. 1473–1480. URL: http://papers.nips.cc/paper/2795-distance-metric-learning-for-large-margin-nearest-neighbor-classification.pdf

Weinberger, K.Q., Tesauro, G., 2007. Metric Learning for Kernel Regression, in: Meila, M., Shen, X. (Eds.), Proceedings of the Eleventh International Conference on Artificial Intelligence and Statistics, PMLR, San Juan, Puerto Rico. pp. 612–619. URL: http://proceedings.mlr.press/v2/weinberger07a.html

Yuan, F., Lidman, C., Davis, T.M., Childress, M., Abdalla, F.B., Banerji, M., Buckley-Geer, E., Carnero Rosell, A., Carollo, D., Castander, F.J., D’Andrea, C.B., Diehl, H.T., Cunha, C.E., Foley, R.L., Friedman, J., Glazebrook, K., Gschwend, J., Hinton, S., Jouvel, S., Kessler, R., Kim, A.G., King, A.L., Kuehn, K., Kuhlmann, S., Lewis, G.F., Lin, H., Martini, P., McMahon, R.G., Mould, J., Nichol, R.C., Norris, R.P., O’Neill, C.R., Ostrovski, F., Papadopoulos, A., Parkinson, D., Reed, S., Romer, A.K., Rooney, P.J., Rozo, E., Rykoff, E.S., Sako, M., Scalzo, R., Schmidt, B.P., Soclnic, D., Seymour, N., Sharp, R., Soheira, F., Sullivan, M., Thomas, R.C., Tucker, D., Uddin, S.A., Wechsler, R.H., Wester, W., Wilcox, H., Zhang, B., Abbott, T., Allam, S., Bauer, A.H., Benoît-Lévy, A., Bertin, E., Brooks, D., Burke, D.L., Carrasco Kind, M., Covarrubias, R., Crocce, M., da Costa, L.N., DePoy, D.L., Desai, S., Doel, P., Eisen, T.F., Evrard, A.E., Fausti Neto, A., Flaugher, B., Fosalba, P., Gaztanaga, E., Gerdes, D., Gruen, D., Gruendl, R.A., Honscheid, K., James, D., Kuoapatkin, N., Lahav, O., Li, T.S., Maia, M.A.G., Makler, M., Marshall, J., Miller, C.J., Miquel, R., Ogando, R., Plazas, A.A., Roodman, A., Sanchez, E., Scarpine, V., Schubnell, M., Sevilla-Noarbe, I., Smith, R.C., Soares-Santos, M., Suchyta, E., Swanson, M.E.C., Tarle, G., Thaler, J., Walker, A.R., 2015. OzDES multibre spectroscopy for the Dark Energy Survey: first-year operation and results. Monthly Notices of the Royal Astronomical Society 452, 3047–3063. doi:10.1093/mnras/stv1507 arXiv:1504.03039

Zhang, Y., Ma, H., Peng, N., Zhao, Y., Wu, X.B., 2013. Estimating Photometric Redshifts of Quasars via the k-nearest Neighbor Approach Based on Large Survey Databases. The Astronomical Journal 146, 22. doi:10.1088/0004-6256/146/2/22

Zitlau, R., Hoyle, B., Pach, K., Weller, J., Rau, M.M., Seitz, S., 2016. Stacking for machine learning redshifts applied to SDSS galaxies. Monthly Notices of the Royal Astronomical Society 460, 3152–3162. doi:10.1093/mnras/stw1454.