Symmetrised dot pattern technique for fault diagnosis in a spur gear assembly using vibration signals

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Abstract. Rotating machinery is widely used in industrial applications and it is important to ensure its smooth running in order to prevent system failure. Therefore, gear health monitoring becomes vital and gears can fail due to various faults like shaft misalignment, crack, missing tooth, wear etc. In this paper, condition monitoring of a spur gear assembly is done with normal gear and also with faulty gear with cracked tooth. The dynamic simulation of crack propagation at the root level of a single tooth of the spur gear is done with Abaqus finite element analysis software using the contour integral method. The gear transmission is subjected to different input torque and also run at different speeds and loads. Vibration amplitude obtained from the simulation in terms of velocity is used for fault diagnosis. The velocity data captured is passed to the Symmetrized Dot Pattern (SDP) algorithm developed in MATLAB that generates visual polar plot to identify the different gear faults easily. This plot is a unique signature for the given gear health condition. Using the developed gear fault diagnosis technique, we are able to successfully identify the health of the gear assembly, including the faulty condition i.e. crack in the gear. The computational time required to run this algorithm is also less compared with the other fault diagnosis techniques.

1. Introduction
Gearboxes are widely used in many applications like automotive, aerospace, manufacturing machines, etc. and come under the category of rotating machinery. They are produced in different geometries and sizes depending on the power requirements. When the gearboxes are run continuously over a long period of time, they tend to wear and produce faults like: cracks, shaft misalignment, spall, etc. Out of the many gear faults, gear crack is of much interest. Failure of gearboxes can prove to be detrimental to the system or the plant and can lead to large downtime, thereby affecting the productivity. Hence, it will be highly advantageous to monitor the health of the gears in real time to predict their failure in advance to do the preventive maintenance. Condition monitoring is a technique by which health of the machinery is monitored by instrumenting the machine with appropriate sensors and diagnostic software run by specially built electronic control unit (ECU) (or) computers. By monitoring the sensor signal(s), the system is capable of detecting any defects that develop during the use of machine and warns the machine operator with current health of the system. Vibration signal is the most widely used data in gear health monitoring system. Accelerometer sensor is placed on the bearing cover of the gear box, which measure the vibration data.

Various methods based on vibration signal analysis have been developed by researchers in order to diagnose the gear crack. Kurtosis, RMS and power spectrum are conventional methods which have proved to be effective in fault diagnosis [1,2,3]. The crack fault vibration mode [4] is studied on the basis of a Parametrical-Laplace wavelet method, which is designed as a health indicator to detect the crack fault. The above methods are used for gear damage detection, by observing the difference intime-domain amplitude or frequency domain spectrum of vibration signals. However, these methods fail, when the signals caused by the damaged elements are much smaller in their amplitudes compared with the background noise (i.e. low signal-to-noise (S/N) ratio). Resonance based signal sparse
decomposition and comb filter (RSSD-CF) [5] method can be used to effectively extract the weak fault feature signals and make them more prominent, thereby able to detect gear faults. However, all the above methods are computationally complex and requires more processing time.

In the current study, it is proposed to use the Symmetrised Dot Pattern (SDP) technique for diagnosing gear cracks. This signal processing technique has been previously used by Pickover [6] for speech waveform analysis, by Shibata [7] for fault diagnosis of bearings and Jian-Da Wu for internal combustion engines [8]. In the present work, dynamic simulation of the spur gear assembly is performed in Abaqus for both faultless and faulty conditions. Crack propagation on a single tooth along the root of the flank is simulated. Here, the health of a spur gear assembly is monitored using the SDP technique and the SDP algorithm has been developed using MATLAB code. Simulated vibration signals of the healthy as well as the cracked gear obtained under different torques and rpm conditions are used as input to the SDP algorithm. The pattern generated is unique and is successful in mapping the gear to its current running condition.

2. Methodology
2.1 Symmetrised dot pattern (SDP) technique
SDP is a technique used for visual representation of acoustic and vibration signals to quickly identify any faulty condition of the system. This technique transforms the time waveform into a scatter plot space consisting of the radial component $P$ and two angles $\alpha$ and $\beta$. The vibration signal is visualized as a pattern of six-fold symmetry with snowflake-shape. This method can easily differentiate even smaller changes in the amplitude and frequency of input signal (i.e. vibration signal).

2.1.1 SDP- Mathematical formulation
SDP algorithm maps a point in the time waveform into a radial component, while the neighboring point is mapped to an angular component as shown in figure 1 below.

![Figure 1](image)

(a)

**Figure 1.** Schematic of time waveform and SDP principle.

Initially, a set of data $(A(t), A(t + n))$ is plotted about the initial line (datum) of the polar coordinate graphs such that it is linearly symmetrical. A figure symmetrical about a point is plotted by repeatedly
rotating the axis of linear symmetry as shown in figure 1(a). \(A_j\) denotes the amplitude of the discrete signal in figure 1(a). The time-axis signal at time \(t\) and its value at \((t+n)\) is converted to polar coordinates \((P_j, \alpha_j, \beta_j)\). Here, \(n\) is the number of points in the waveform window. \(A_{\text{max}}\) and \(A_{\text{min}}\) are the highest and lowest values of the waveform respectively. From literature survey, it is seen that the angle of rotation set to 60° (i.e. six-fold symmetry) conveys the characteristics of the signal with good distinguishability. \(\xi\) is the gain and is taken as 50. It is advised to choose \(\xi\) such that \(\xi \leq (360°/m)\) where the number of mirror planes \((m)\) is taken as six. The description of all the variables is given in table 1 and the equations are given below.

\[
P_j = \left(\frac{A_j - A_{\text{min}}}{A_{\text{max}} - A_{\text{min}}}\right) \xi \quad (1)
\]

\[
\alpha_j = \alpha' + \left(\frac{A_{j+1} - A_{\text{min}}}{A_{\text{max}} - A_{\text{min}}}\right) \xi \quad (2)
\]

\[
\beta_j = \alpha' - \left(\frac{A_{j+1} - A_{\text{min}}}{A_{\text{max}} - A_{\text{min}}}\right) \xi \quad (3)
\]

where \(j = 1, 2, 3, 4, \ldots, n-1\) and \(\alpha' = (360°/m) \times i\) and \(i = 1, 2, 3, \ldots, m\)

| Variable | Description |
|----------|-------------|
| \(P_j\) | SDP radius |
| \(\alpha_j\) | Angle of dot in SDP |
| \(\beta_j\) | Negative angle of dot in SDP |
| \(A_j\) | Vibration signal |
| \(A_{\text{max}}\) | Maximum value of waveform |
| \(A_{\text{min}}\) | Minimum value of waveform |
| \(\alpha\) | Initial line rotation angle |
| \(n\) | Number of dots in the waveform |
| \(\xi\) | Gain used to normalize the waveform |

2.2 Simulation in Abaqus

The finite element analysis of the spur gear assembly was performed in Abaqus software. The analysis was done on both healthy and cracked gears. The crack is developed along the flank of a single tooth. The tooth of a gear behaves like a cantilever beam, which experiences bending and thus causes the crack to originate and further propagate. Important parameters like contact pressure (shown in figure 2) of the meshing teeth are obtained in the simulation and are different for both faultless and cracked gears. The details of the pinion and the gear are shown in table 2.

| Parameter | Pinion | Gear |
|-----------|--------|------|
| Young’s Modulus | \(2 \times 10^7\) N/mm² | \(2 \times 10^7\) N/mm² |
| Poisson’s ratio | 0.3 | 0.3 |
| Pressure angle | 14.5° | 14.5° |
| Pitch diameter | 76.2 mm | 152.4 mm |
| Number of teeth | 18 | 36 |
| Density | 7850 kg/m³ | 7850 kg/m³ |
| Mass | 0.7375 kg | 3.1171 kg |
The moment is applied to the pinion, which is the driving gear. Surface to surface sliding is chosen for the contact analysis. The driven gear is chosen as the master surface (since bigger in size) and the pinion is chosen as the slave. The friction coefficient is set to 0.25. A 4 node linear tetrahedron element is used for meshing the gears. The contact stresses are obtained through the simulation and is shown in figure 2.

Figure 2. Simulation of healthy gear.  
Figure 3. Crack front and its normal.

2.2.2 Crack propagation simulation of gears
A crack severely affects the meshing stiffness of the teeth and also changes its vibration characteristics. Stress concentration increases around the crack region, due to the discontinuity. In the simulation, it is assumed that the crack propagates in a straight line along the root of the flank. For modelling the crack propagation, contour integral method has been used. The J-Integral is suitable for evaluating responses of linear materials (obeying Hooke’s law); hence chosen for performing the contour integral.

Contour integral method requires the mesh to conform to the cracked geometry. The crack front is defined and the crack extension direction is specified normal to the crack plane as shown in figure 3 above. Five contour integrals are performed at each and every node along the crack line. A 8-node linear brick element is used for meshing the gears.

3. Simulation Analyses
Dynamic simulation of the spur gear assembly was performed in Abaqus under different conditions of input torque and rpm. It is assumed that the power input to the driving gear (in this case the pinion gear) is constant and there is no slip. The analysis is done for both faultless and cracked gears. The details of the gear parameters and the running conditions are given in table 3.

| Condition | Pinion torque (Nm) | Gear torque (Nm) | Pinion velocity (rad/s) | Gear velocity (rad/s) |
|-----------|-------------------|------------------|-------------------------|-----------------------|
| 1         | 2                 | 4                | 200                     | 100                   |
| 2         | 4                 | 8                | 100                     | 50                    |

Table 3. Running conditions of the gear.

Figure 4. Dynamic simulation of gear assembly (Healthy gears).  
Figure 5. Velocity Vs Time waveform (Healthy gears).
The velocity amplitude is obtained for all the above running conditions. Full-Newton solving technique is chosen for the dynamic simulation since it gives better convergence rate for non-linear finite element analysis. The dynamic simulation of the gear assembly is done using Abaqus software and the velocity is shown in the figures 4 and 5 respectively.

4. SDP analyses

The SDP algorithm as discussed in section 2.1.1 is developed using MATLAB software. The output obtained from the dynamic simulation in Abaqus is given as the input to the SDP algorithm and it generates visual pattern. The symmetrised dot patterns for all the running conditions (at different speeds) are shown in figure 6.

![Figure 6](image)

(a) Speed: 200 rad/s, (healthy gears).(b) Speed: 200 rad/s, (pinion cracked).

(c) Speed: 100 rad/s (healthy gears).(d) Speed: 100 rad/s (pinion cracked)

Figure 6. SDP output the gear assembly at different pinion speeds.

It can be seen that each SDP pattern is unique and is a characteristic of the running condition. The patterns become easier to distinguish due to symmetrisation. The difference in patterns of the faultless and faulty conditions can be easily observed, as circular patterns are generated at the centre in the case of faulty gear. For healthy conditions it can be seen that the boundaries are clear and the density of dots is higher at larger radii. Figure 6(a) and 6(c) are SDPs of the faultless conditions that can be distinguished on the basis of the intersection points. It can be seen from figure 6(a) that the concentration of dots at the region of intersection is higher at higher pinion speed. For the faulty conditions (figure 6(b) and 6(d)) the patterns appear to be similar for human eyes, but they are different with minute variations. However, image processing algorithms can be used to distinguish between patterns, which are not visually distinguishable.

In the next set of simulations, the speed of operation is kept at constant value and in order to study the effect of load, the output gear is subjected to different torque loads and the SDPs generated are shown in the figure 7(a) to 7(c). It can be seen that the dot patterns generated for the cracked conditions of
the gear with and without load (also for different load conditions) are visually distinguishable.

![Figure 7. SDP of cracked gears with and without loads.](image)

5. Conclusions and future work
In this work, health monitoring of a spur gear assembly is done using the Symmetrized Dot Pattern (SDP) algorithm to identify the normal healthy gear and a faulty gear with cracked tooth. The dynamic simulation of crack propagation at the root level of a single tooth of the spur gear is done with Abaqus finite element analysis software using the contour integral method. The gear transmission is subjected to different input torque and also run at different speeds and loads. Vibration amplitude obtained from the simulation in terms of velocity is used for fault diagnosis. The velocity data captured is passed to the Symmetrized Dot Pattern (SDP) algorithm developed in MATLAB that generated visual polar plots for gear different load and speed conditions. This plot is a unique signature for the given gear health condition. Using the developed gear fault diagnosis technique, we are able to successfully identify the health of the gear assembly, including the faulty condition i.e. crack in the gear. The computational time required to run this algorithm is also found to be less compared with the other fault diagnosis techniques. A simple database can be made consisting of all the patterns for different running conditions and the system running condition can be matched to the closest pattern with image processing algorithms for real time automated condition monitoring. This method can be used for various other faults like wear, spall, shaft misalignment etc. and can tested with experiments where the data is captured through sensors like accelerometers or acoustic emission sensors.

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