Particle-hole symmetry and transport properties of the flux state in underdoped cuprates

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Transport properties, i.e., conductivities $\sigma_{\mu\nu}$, Hall constant $R_H$, and thermopower $S$ are studied for the flux state with the gauge flux $\phi$ per plaquette, which may model the underdoped cuprates, with the emphasis on the particle-hole and parity/chiral symmetries. This model is reduced to the Dirac fermions in (2+1)D with a mass gap introduced by the antiferromagnetic (AF) long range order and/or the stripe formation. Without the mass gap, the Hall constant $R_H$ and the thermopower $S$ obey the two-parameter scaling laws, $R_H \approx \frac{a^2}{|t|} f_{R_H} \left( \frac{\sqrt{x}}{a_B}, \frac{b}{a_B} \right)$ and $S \approx \frac{b}{|t|} f_S \left( \frac{\sqrt{x}}{a_B}, \frac{b}{a_B} \right)$, with $a$ being the lattice constant, $x$ the hole concentration, $\tau$ the transport lifetime. The $R_H$ and $S$ show the strong temperature dependence due to the recovery of the particle-hole symmetry at high temperature. The $x$-dependences of $\sigma_{\mu\nu}(\sqrt{x})$ and $\sigma_{x\tau}$ (independent of $x$) are in a sharp contradiction with the experiments. Therefore there is no signature of the particle-hole symmetry or the massless Dirac fermions in the underdoped cuprates even above the Neel temperature $T_N$. With the mass gap introduced by the AF order, there occurs the parity anomaly for each of the Dirac fermions. However the contributions from different valleys and spins cancel with each other to result in no spontaneous Hall effect even if the time-reversal symmetry is broken with $\phi \neq \pi$. The effects of the stripes are also studied. The diagonal and vertical (horizontal) stripes have quite different influence on the transport properties. The suppression of $R_H$ occurs at low temperature only when (i) both the AF order and the vertical (horizontal) stripe coexist, and (ii) the average over the in-plane direction is taken. Discussions on the recent experiments are given from the viewpoint of these theoretical results.

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I. INTRODUCTION

Since the discovery of high-Tc cuprates, intensive studies have been done on the two-dimensional antiferromagnets. It is now established that the ground state of the 2D Heisenberg antiferromagnet with the nearest neighbor interaction on the square lattice shows the antiferromagnetic (AF) long range ordering at zero temperature\cite{I}, and the low energy spin excitation can be described in terms of the spin wave theory. However this does not mean that the electronic state in the antiferromagnets is fully understood. Compared with the triplet channel of the two-particle correlation functions, the single particle properties such as the angle resolved photo-emission spectra (ARPES)\cite{II}, and the singlet channel correlation functions such as the charge transport properties\cite{II} still remain controversial. In fact, there are two different pictures for it. One is the conventional spin density wave (SDW) picture\cite{IV} with the wavenumber $Q = (\pi, \pi)$ where both the weak and strong coupling regions can be smoothly connected. The other picture is the $\pi$-flux state\cite{III} originated from the resonating valence bond (RVB) idea\cite{V}. At half filling, the $\pi$-flux state is equivalent to the d-wave singlet paring state\cite{VI, VII} due to the particle-hole SU(2) symmetry\cite{VIII}. The Gutzwiller projected wavefunction with the d-wave paring and AF orders gives a better energy compared with that only with the AF order\cite{IX}. Also the higher energy continuum of the neutron scattering spectra and Raman scattering spectra has been analyzed in terms of this $\pi$-flux state\cite{X}.

The dispersion of a single hole put into this antiferromagnet is an important issue studied intensively in terms of self-consistent Born approximation\cite{XI}, exact diagonalization\cite{XII}, spinon-holon bound state picture\cite{XIII}, and variational method\cite{XIV}. With the $t$-$J$ model, all the analyses give the maximum of the hole dispersion at $\vec{k} = (\pm \pi/2, \pm \pi/2)$. This dispersion can be understood in terms of the $\pi$-flux picture\cite{XIII, XIV}, which introduces the nodal Fermi points of the spinons at $\vec{k} = (\pm \pi/2, \pm \pi/2)$ with the dispersion similar to the d-wave superconductors. This fits the ARPES experiments in the undoped cuprates\cite{IV}.

At finite doping, the slave-boson mean-field theory of the $t$-$J$ model predicts the state with both the singlet RVB and AF orders for small $x$\cite{XV}. The SU(2) symmetry has been employed to represent the constraint and the underdoped pseudogap region is characterized as the staggered flux state with spin-charge separation\cite{XVI}, which can be regarded as the fluctuating state between the d-wave paring state and the current order state. On

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the other hand, the staggered flux state with the electron coordinates with the real current ordering and periodicity doubling has been proposed for the underdoped cuprates.\textsuperscript{22}

Recently there appeared several experiments on the Hall coefficient $R_H$ and the thermopower $S$ in the heavily underdoped cuprates, which raised the issue of particle-hole symmetry\textsuperscript{6, 7, 8}. The $R_H$ as well as the $S$ show the strong suppression below the Neel temperature $T_N$ in some YBCO samples\textsuperscript{6} while it is not the case for other samples\textsuperscript{7}. This strongly suggests that the transport properties below $T_N$ is sensitive to the oxygen chain ordering and/or the self-organization of electrons such as the stripes, which depends on the annealing procedure in the sample preparation. Another interesting clue is that in LSCO the suppression of the Hall effect is observed only in the vertical (horizontal) stripe state while is not in the diagonal stripe state for $x < 0.05$\textsuperscript{23, 24}. Above $T_N$, $R_H$ is a decreasing function of temperature, which remains one of the most puzzling features in the normal state properties\textsuperscript{3, 4, 5, 6, 25}.

In the SDW picture, the large metallic Fermi surface enclosing the area of $1 - x$ ($x$: hole concentration) is recovered above $T_N$. Therefore we expect the drastic change of the Hall constant from $R_H = -a^2/(1 - x)$ to $R_H = a^2/x$ when the temperature $T$ is lowered across $T_N$. (Here $a$ is the lattice constant.) Correspondingly the resistivity should be affected by the onset of the AF order. This is in sharp contradiction with the experiments. Even above $T_N$, the system behaves as the doped Mott insulator with the small number of holes\textsuperscript{3, 4, 6}, and even a slight change of the resistivity is not observed at $T_N$\textsuperscript{4, 6}.

Considering all these clues, it is worthwhile to study the transport properties of the slightly doped $\pi$-flux state both above and below $T_N$, which we undertake in this paper. We will neglect the interactions between electrons and the disorder potentials. The former is justified because the quasi-particle number is small at low temperature due to the reduced density of states and the el-el interaction is irrelevant. The latter becomes important at low temperature for small $x$ where the resistivity shows an upturn, but is irrelevant for the temperature and $x$-range of our interest. We also study the effect of the stripe formation on the Hall constant and the thermopower, and show that oxygen chain ordering is crucial for these quantities. The deviation of the flux from $\pi$, which breaks the time-reversal symmetry and produces current ordering, is also studied. This issue is closely related to the parity anomaly\textsuperscript{24} in (2+1)D because the two species of the Dirac fermions acquires the mass gap due to the AF ordering. Therefore the undoped and underdoped cuprates offers an interesting laboratory to study the transport properties of the Dirac fermions with nontrivial topological nature influenced by the AF order and/or the stripe formation. The Dirac fermion has been studied also in the context of the nodal quasi-particle in the d-wave superconductors\textsuperscript{27}. In this case the Fermi energy is always at $E = 0$ and the particles-hole symmetry remains. Under the external magnetic field $H$, the formation of the vortex lattice is crucial, which introduces the magnetic length scale $\ell_H \sim H^{-1/2}$. In the present case, on the other hand, there occurs no Meissner effect, and one can see the response of the Dirac fermions in the uniform state, while the particle-hole symmetry is broken due to the shift of the Fermi energy from $E = 0$. This introduces the length scale $\ell_x \sim x^{-1/2}$, i.e., the inter-hole distance, and we can have the new kind of scaling when neither AF order nor the stripe formation occur.

The plan of this paper follows. In sec. II, our model Hamiltonian is introduced and its spectrum under the external magnetic field is reviewed. In sec. III, the electromagnetic and thermal responses are studied for the Dirac fermions without stripes. The effects of the stripes are studied in sec. IV, and sec. V is devoted to discussions in comparison with experiments.

II. HAMILTONIAN

We start with the most generic Hamiltonian in the staggered flux state with anti-ferromagnetic and stripe (quasi-)order which are treated as on-site potentials in the mean field theory. Hereafter, we take the units in which $\hbar = c = 1$.

\begin{equation}
H = - \sum_{\langle \vec{r}, \vec{r} \rangle, \sigma} t_{\vec{r}, \vec{r}} c^\dagger_{\vec{r} \sigma} c_{\vec{r} \sigma} - \sum_{\vec{r}, \pi} \left[ \mu + u_{\pi} \cos(\vec{Q} \cdot \vec{r}) + v \cos(\vec{Q} \cdot \vec{r}) \right] c^\dagger_{\vec{r} \sigma} c_{\vec{r} \sigma} \tag{1}
\end{equation}

The transfer integral $t_{\vec{r} \pm \vec{x}, \vec{r}} = t_{\vec{r} \pm \vec{y}, \vec{r}} = t e^{i \frac{\pi}{\sqrt{2}}} \delta_{r_x + r_y}$ (with $r_x + r_y$ even) represents the staggered flux $\phi$ for each plaquette. In the RVB picture, $\phi$ is generated by the superexchange interaction $J$ and equals $\pi$ in the undoped case. Therefore, in our representation, the transfer integral $t$ is estimated as $t \cong 0.8J$, and should not be confused with $t$ in the $t$-$J$ model\textsuperscript{18}. In the second term, $u_{\pi}$ is the mean field potential for the $\pi$ spin electrons with the wavenumber $\vec{Q} = (\pi, \pi)$, which originates from the AF order $m_{AF}$ and/or the diagonal stripe formation $m_{\text{diag}}$ as $u_{\pi} = \sigma m_{AF} + m_{\text{diag}}$, while $v$ is that from the vertical stripe formation with $\vec{Q}_v = (\pi, 0)$. This is the result of the higher order process in the stripe potential with the wavenumber $\vec{Q}_{\text{stripe}} = (\pi/M, \pi/M)$ (diagonal case) or $\vec{Q}_{\text{stripe}} = (\pi/M, 0)$ (vertical case). The chemical potential $\mu$ is zero at the half-filling. The eigenvalues for each $\vec{k}$ are
\[
\epsilon_\sigma(\vec{k}) = \pm \left[ 4t^2(\cos^2 k_x + \cos^2 k_y) + u_\sigma^2 + v^2 \pm 2 \sqrt{\left( 4t^2 \cos \frac{\phi}{2} \cos k_y \right)^2 + v^2 \left( 4t^2 \cos^2 k_y + u_\sigma^2 \right)} \right]^{\frac{1}{2}}.
\]

(2)

First, we consider the case without the vertical stripe. In this case, the low energy electronic states are described in terms of the two Dirac fermions at \( \vec{k}_1 = (\pi/2, \pi/2) \) and \( \vec{k}_2 = (\pi/2, -\pi/2) \), which we call 1 and 2 sector, respectively. Measuring the momenta from these \( \vec{k}_1,2 \)-points, and in the continuum approximation the Hamiltonian is given as

\[
H_{\text{eff}} = \sum_{i,\sigma} \int d^2 r \, \psi_{i\sigma}^\dagger(\vec{r}) \hat{H}_{i\sigma} \psi_{i\sigma}(\vec{r}),
\]

(3)

where \( \psi_{i\sigma}^\dagger(\vec{r}) \) is the annihilation (creation) operator of \( i \)-sector \((i = 1, 2)\) with \( \sigma \)-spin,

\[
\hat{H}_{1\sigma} = \begin{bmatrix}
    u_\sigma - \mu & 2ta(\hat{p}_x + e^{-i\frac{\phi}{2}} \hat{p}_y) \\
    2ta(\hat{p}_x + e^{i\frac{\phi}{2}} \hat{p}_y) & -u_\sigma - \mu
\end{bmatrix},
\]

(4)

and \( \hat{H}_{2\sigma} = \hat{H}_{1\sigma \mid \sigma \rightarrow -\sigma} \). Here \( \hat{p}_\mu = -i\partial_\mu \) and \( a \) is the lattice constant. As is evident from the above Hamiltonian, each Dirac fermion has the mass term with positive or negative sign, and shows the parity anomaly [21]. When the external electromagnetic (EM) field \( \mathbf{A}_\mu \) is coupled to each Dirac fermion, the Chern-Simons term \( \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \) is generated. Therefore the Dirac fermion with positive (negative) \( u_\sigma \) has the right (left) chirality. Because there are two right (R) and left (L) Dirac fermions, there occurs the cancellation of the Chern-Simons terms, and no spontaneous Hall effect results.

This remains true even when the flux \( \phi \) is different from \( \pi \) and the time-reversal symmetry is broken to produce the current order, as long as diagonal stripes are absent, i.e. \( m_{\text{diag}} = 0 \). It is because the current pattern is staggered, and does not affect the uniform response in an essential way.

Now we briefly review on the Dirac fermions in the presence of the external uniform magnetic field \( B \), which can be analytically solved [22, 23]. In the effective theory represented by eq. (3), the first-quantized Hamiltonian eq. (2) is rewritten by the replacement, \( \hat{p}_\mu \rightarrow \hat{\pi}_\mu = \hat{p}_\mu - eA_\mu(\vec{r}) \), where \( [\nabla \times \mathbf{A}(\vec{r})]_z = B \). Then we define the following bosonic operators as

\[
\hat{a}_\phi = \frac{1}{\sqrt{2|eB_\phi|}} \left[ \hat{\pi}_x + e^{i\text{sgn}(eB_\phi)\frac{\pi}{2}} \hat{\pi}_y \right],
\]

(5)

\[
\hat{a}_\phi^\dagger = \frac{1}{\sqrt{2|eB_\phi|}} \left[ \hat{\pi}_x + e^{-i\text{sgn}(eB_\phi)\frac{\pi}{2}} \hat{\pi}_y \right].
\]

(6)

These operators satisfy the commutation relation \([\hat{a}_\phi, \hat{a}_\phi^\dagger] = 1\). Using these operators and the Pauli matrices \( \tau_i, (i = 1, 2, 3) \), eq. (4) is rewritten as

\[
\hat{\mathcal{H}}_{1\sigma} = \frac{K_{eB_\phi}}{2} \left[ (\hat{a}_\phi + \hat{a}_\phi^\dagger) \tau_1 - i \text{sgn}(eB_\phi) (\hat{a}_\phi - \hat{a}_\phi^\dagger) \tau_2 \right] + u_\sigma \tau_3 - \mu I.
\]

(7)

The eigenvector \((n \geq 1)\) is

\[
|n \sigma \pm\rangle = \frac{1}{\sqrt{2 \epsilon_n}} \left[ \sqrt{\epsilon_n \pm \text{sgn}(eB_\phi) u_\sigma} |n\rangle \otimes |\uparrow\rangle \pm \sqrt{\epsilon_n \mp \text{sgn}(eB_\phi) u_\sigma} |n-1\rangle \otimes |\downarrow\rangle \right] \otimes |\sigma\rangle,
\]

(8)

with the eigenvalue \( \pm \epsilon_{n\sigma} \) being given by

\[
\epsilon_{n\sigma} = \sqrt{K_{eB_\phi}^2 n + u_\sigma^2},
\]

(9)

\[
K_{eB_\phi} = \sqrt{8t^2a^2|eB_\phi|}, \quad B_\phi = B \sin \frac{\phi}{2}.
\]

(10)

Here \(|n\rangle\) is the eigenvector of \( \hat{a}_\phi^\dagger \hat{a}_\phi \) with the eigenvalue \( n \), \(|\uparrow\rangle\) (\(|\downarrow\rangle\)) is the eigenvector of \( \text{sgn}(eB_\phi) \tau_1 \) with the eigenvalue \(+1(-1)\), and the quantum index for the intra-level orbitals is omitted. \(|\uparrow\rangle\) (\(|\downarrow\rangle\)) should not be confused with a real-spin eigenvector \(|\sigma\rangle\), but it comes from the two-component nature of a Dirac fermion. Especially, the zero mode is given by

\[
|0 \sigma \pm\rangle = \frac{1}{2} \left[ 1 \pm \text{sgn}(u_\sigma eB_\phi) \right] |0\rangle \otimes |\uparrow\rangle \otimes |\sigma\rangle.
\]

(11)

Therefore, there exists only one zero-mode \(|0 \sigma +\rangle\) or \(|0 \sigma -\rangle\) for each sector and its energy is \( \epsilon_{0\sigma} = |u_\sigma| \) or \(-|u_\sigma| \) depending on the direction of the external magnetic field \( B \) and the chirality of Dirac fermions. We can also define effective current-operators as follows, \( J_x = 2ta \tau_1 \) and \( J_y = 2ta(\cos \frac{\phi}{2} \tau_1 + \sin \frac{\phi}{2} \tau_2) \).
III. ELECTROMAGNETIC AND THERMAL RESPONSES

The electromagnetic and thermal linear response functions are obtained by the Kubo formula. We put the above solutions into the Kubo formula for $\sigma_{\mu\nu}(\omega)$ and approximate the effect of the relaxation by replacing $\omega$ by $\omega + i/\tau$ with the lifetime $\tau$. This approximation reproduces the Drude formula for $\sigma_{\mu\nu}(\omega)$ in the simplest case. The contribution from 1-sector with $\sigma$-spin is presented in the appendix A. The total conductivity is given by summing up the contribution from all sectors. This procedure makes anomalous terms, which come from the inter-band effect, cancel out with each other, and lead to the consistent result in the AF ordered state, i.e. no inter-band effect, cancel out with each other, and lead to the consistent result in the AF ordered state, i.e. no parity symmetry breaking in the limit $B \to 0$.

The eqs. (A5) and (A4) are valid for general cases including both the quantum limit ($\tau \to \infty$ with finite $B$) and the semi-classical limit ($K_{\mu\nu}, \tau \ll |\mu|\tau$). In the former case, eqs. (A5) and (A6) represent the integer quantum Hall effect, while in the latter case they correspond to the usual Hall effect. In the latter case, we obtain the same formula with that given by the Boltzmann equation. For example, with only the AF order and at low temperature, eqs. (A5) and (A6) lead to

$$\sigma_{xx} \cong \frac{e^2}{2\pi} \frac{8\pi x t^2 \tau}{\sqrt{4\pi x t^2 \sin \frac{\phi}{2} + m_{AF}^2}},$$

(12)

$$\sigma_{xy} \cong \frac{e^2}{2\pi} \frac{(8\pi x t^2 \tau)^2}{4\pi x t^2 \sin \frac{\phi}{2} + m_{AF}^2} \frac{a^2 |e| B}{2\pi x} \sin^2 \frac{\phi}{2}$$

(13)

and the Hall coefficient $R_H$ is given by

$$R_H \cong \frac{\sigma_{xy}}{B \sigma_{xx}} \cong \frac{a^2}{|e| x} \sin^2 \frac{\phi}{2}.$$  

(14)

independent of the presence/absence of the AF order. This is reduced to the conventional result $R_H = a^2 / (|e| x)$ for the hole system with concentration $x$ when we put $\phi = \pi$. However the $x$-dependences of $\sigma_{xx}$ and $\sigma_{xy}$ are peculiar in the absence of the AF order, namely $\sigma_{xx} \propto \sqrt{x}$ and $\sigma_{xy}$ being independent of $x$ when we neglect the $x$-dependence of $\tau$. These are understood as follows. The $\sigma_{xx}$ is proportional to the density of states at the Fermi energy, which is proportional to $\sqrt{x}$ for the massless Dirac fermion. As for $\sigma_{xy}$, on the other hand, the geometric interpretation is useful [23]. The Hall conductivity is determined by the scattering path length, $l(\k) = \tau \sqrt{\epsilon(\k)}$, which is independent of the magnitude $|\k|$ for linear dispersion and hence $\sigma_{xy}$ is independent of $x$. In eq. (A3), this behavior is the consequence of the dominant contribution from the zero mode of the Dirac fermions to $\sigma_{xy}$. On the other hand, when $\sqrt{4\pi x t} \ll |m_{AF}|$, the usual dispersion $\epsilon(\k) \propto \k^2$ is relevant and the usual $x$-dependences, $\sigma_{xx} \propto x$ and $\sigma_{xy} \propto x$ result. The thermopower $S$ at sufficiently low temperature is also obtained in a similar way as

$$S \cong \frac{k_B}{|e|} \frac{\pi^2}{3} k_B T \frac{4\pi x t^2 \sin \frac{\phi}{2} + 2m_{AF}^2}{4\pi x t^2 \sin \frac{\phi}{2} \left[4\pi x t^2 \sin \frac{\phi}{2} + m_{AF}^2 \right]^2}.$$  

(15)

The $x$-dependence of $S$ is also peculiar in the absence of the AF order, i.e., $S \propto 1/\sqrt{x}$.

Next we consider the temperature dependences of $R_H$ and $S$, which originates from the Fermi distribution function $f_F(T, \epsilon, \mu(T))$. Here the energy is averaged over $\sim k_B T$, and both the particle and hole branches contribute with the opposite signs to $R_H$ and $S$ when $k_B T > |\mu(T)|$. Furthermore, $|\mu(T)|$ is a decreasing function of temperature as presented in the appendix B. For example, when $m_{AF} = 0$, it behaves at $k_B T \ll O(t\sqrt{x})$

$$|\mu(T)| \cong \sqrt{4\pi x t^2 \sin \frac{\phi}{2}} - \frac{\pi^2}{3} (k_B T)^2,$$

(16)

and at $k_B T \gg O(t\sqrt{x})$

$$|\mu(T)| \cong \frac{\pi x t^2}{\ln 2} \sin \frac{\phi}{2} \frac{1}{k_B T}.$$  

(17)

Therefore, as long as the flux order is present, the particle-hole symmetry is approximately restored at high temperature even in the doped system, and both $R_H$ and $S$ are reduced as shown in Figs. (a) and (b), which are the results of the original lattice-model. The peak in $R_H$ occurs because the suppression of $\sigma_{xx}$ as increasing $T$ dominates at low temperature. After all, the low carrier density and the narrow gap result in these features, i.e., having a peak at moderate temperature and decreasing at high temperature. It is worthwhile to note that the temperature dependence of $S$ resembles those of rare-earth compounds, such as YBCuAl, which have low carrier densities and narrow gaps.

In the case of $m_{AF} = 0$ and the limit $B \to 0$, the following scaling laws are expected by the continuum model

$$R_H = \frac{a^2}{|e| x} f_{R_H} \left( \frac{t\sqrt{x}}{k_B T} \frac{1}{\tau k_B T} \right),$$

(18)

$$S = \frac{k_B}{|e|} f_S \left( \frac{t\sqrt{x}}{k_B T} \frac{1}{\tau k_B T} \right),$$  

(19)

where $f_{R_H}$ and $f_S$ are dimensionless functions. The details are given in the appendix C. The dependence on $t\sqrt{x}/(k_B T)$ is because of $\mu/(k_B T)$ being a dimensionless function of $t\sqrt{x}/(k_B T)$, and the dependence on $\tau k_B T$ comes from the transition between the highest valence band and the lowest conduction band. The latter is neglected in the Boltzmann transport theory. In a physical sense, the contribution from transitions between two bands would be very small when the condition $1 \ll |\mu|\tau$ is satisfied, i.e. in the semi-classical limit. In this case,
we get the one-parameter scaling lows,
\[ R_H \cong \frac{a^2}{|e|x} f_R \left( \frac{t \sqrt{x}}{k_B T} \right), \quad S \cong \frac{k_B}{|e|} f_S \left( \frac{t \sqrt{x}}{k_B T} \right). \] (20)

Therefore, both \( xR_H \) and \( S \) are expected to scale as functions of \( t \sqrt{x}/(k_B T) \). Figs. 3(a) and 3(b) show the scaling behaviors of \( R_H \) and \( S \) given in Figs. 2(a) and 2(b). We can see that the single-parameter scaling law of \( R_H \) works fairly well in broad temperature range. On the other hand, for \( S \), it works only at low temperature. This is because \( S \) contains the energy integral which has one more energy-dimension than the integral for \( R_H \), and \( S \) is more sensitive to the higher energy region where the lattice structure is relevant.

In Figs. 3(a) and 3(b), the temperature dependences of \( R_H \) and \( S \) are shown in the presence of the AF order and the vertical (horizontal) stripe. These results are obtained for the lattice model in terms of the Boltzmann transport theory where the inter-band effect is neglected. We can observe that the AF order (without the stripe formation) results in the enhancement of \( R_H \) and \( S \). One reason of this enhancement is that the AF order suppresses the recovery of the particle-hole symmetry at high temperature. (The details are given in the appendix B.) Another reason is that the heavier mass \( m_{AF} \) makes the conductivity smaller as seen in eqs. (12) and (13). The former reason is crucial both for \( R_H \) and \( S \). On the other hand, the latter would be crucial only
for $S$ as expected from the comparison of eqs. (14) and (13). This is because the explicit dependence of $\sigma_{xx}$ and $\sigma_{yy}$ on $m_{AF}$ nearly cancel each other in $R_H$. Therefore, the enhancement of $S$ is more drastic than that of $R_H$. As for the effect of the stripe, we will consider it closely in the next section.

IV. EFFECT OF STRIPE FORMATION

The quasi-one dimensional spin/charge ordering occurs in some cuprates\[2, 23\], and consequently affects the transport properties\[1, S\]. Therefore it is worthwhile to study the effect of the stripe formation on the flux state. Some works have been done assuming the one-dimensionality, which corresponds the limit of strong stripe potential\[23\]. In this section, we give an alternative and complemental study starting from the 2D flux state. It is easy to see that the effects of the stripes are essentially different between the diagonal and vertical (horizontal) ones, because the vertical (horizontal) stripe introduces the off-diagonal matrix elements between the two Dirac fermions around $\vec{k}_1$ and $\vec{k}_2$, while the diagonal one does not. Therefore the effect of the diagonal stripe is similar to that of the AF order and we do not expect any drastic change of $R_H$ in the case of the diagonal stripe, although it makes the mass gaps of different spins unbalanced and leads to the modification of $S$. It is noted that the change of the mass gap modifies $S$ as in eq. (13). (The unbalance of the mass gaps of different spins also leads to the ferrimagnetism.) After all, the drastic modifications of both $R_H$ and $S$ are expected only when the vertical (horizontal) stripe formation occurs. We will focus on this case below.

In the presence of the stripe, there appears the anisotropy in the plane. A naive expectation is that the 1D nature of the transport along the stripe reduces $R_H$. However, this is not the case, because the stripe does suppress not only $\sigma_{xy}$ but also one of $\sigma_{xx}$ and $\sigma_{yy}$ with $R_H \approx \sigma_{xy}/(B\sigma_{xx}\sigma_{yy})$ being unchanged. This is the case also in our explicit calculation showing that $R_H = a^2/(4|x|)$ is a robust feature at low temperature. At this point, we must consider the configuration of the stripe in cuprates. When there are stripes, their direction would be different in each CuO$_2$ layer as in LSCO, or there are domains of stripes with different directions in a layer as in twinned YBCO. In the former system, we simply assume that the vertical and horizontal types occurs alternately. Then it is reasonable to average the contributions from different layers, because they can be regarded as a parallel circuit, i.e. we should take an average of $\sigma$ not of $\sigma_{xx}$ and $\sigma_{yy}$.

Then the observed $R_H$ is suppressed because only $\sigma_{xy}$ is reduced considerably while $\sigma = (\sigma_{xx} + \sigma_{yy})/2$ is not, and $R_H \approx \sigma_{xy}/(B\sigma^2)$ is suppressed.

Before discussing general cases, we consider some special cases where $R_H$ and $S$ can be analytically evaluated at low temperature by the Boltzmann equation. The first limiting condition is that $|m_{AF}|$ is sufficiently larger than the kinetic energy $O(t_+/\beta)$. The second condition is $|v| \ll |m_{AF}|$ or $\sqrt{4t^2 + m_{AF}^2} \ll |v|$. With these conditions, the low-energy physics would be approximately described by quasi-particles of linear dispersions at $\vec{k} = (\pi/2, \pi/2)$ and $(\pi/2, 0)$, respectively. It is noted here that these quasi-particles are not simple two-component Dirac fermions, because the vertical stripe formation mixes the two-component Dirac fermions with different chiralities as mentioned above. For the case $|v| < |v_c| \approx 2\pi xt^2/m_{AF}$, the analysis is complicated because we should consider two bands doped in different way. It is noted here that there are two upper bands and two lower bands for each spin degree of freedom when $m_{AF} \neq 0$ and $v \neq 0$. The analysis of $R_H$ in this case is given in the appendix. When $|v|$ is larger than the critical value $|v_c|$, only the second band is doped. Therefore, for $|v_c| < |v| \ll |m_{AF}|$ or $\sqrt{4t^2 + m_{AF}^2} \ll |v|$, the result is expressed as

$$R_H \approx \frac{a^2}{|v_c|} \cdot \frac{4t^2 + 2m_{AF}^2}{(t^2 + \beta^2)^2},$$

(21)

where $\tilde{t} = t\sqrt{1 - |v/m_{AF}|}$ for $|v_c| < |v| \ll |m_{AF}|$, or $\tilde{t} = t\sqrt{|v|/\sqrt{4t^2 + m_{AF}^2} - 1}$ for $\sqrt{4t^2 + m_{AF}^2} \ll |v|$. On the other hand, the thermopower $S_\mu \approx \beta\langle J^p_{\mu}J^p_{\mu} \rangle^{-1}\langle \tilde{J}^p \rangle$, where $\tilde{J}^p$ is the electric current density and $\tilde{J}^p$ the heat current density, along the direction $\mu = x, y$ remains almost isotropic because the anisotropy of the correlation functions $\langle J^p_{\mu} \rangle$ and $\langle J^p_{\mu}J^p_{\mu} \rangle$ cancels each other in the numerator and denominator, respectively. However, $S$ is rather sensitive to the change of the gap and the electronic dispersion, because $\langle J^p_{\mu}J^p_{\mu} \rangle$ contains additional dimension of energy and is suppressed by the coexistence of the AF order and vertical (horizontal) stripe. For the case $|v_c| < |v| \ll |m_{AF}|$ or $\sqrt{4t^2 + m_{AF}^2} \ll |v|$, it is given at low temperature by

$$S \approx \frac{k_B}{|v_c|} \cdot \frac{\pi^2}{3} \frac{k_B T}{\tilde{t}} \cdot \frac{8\pi x \tilde{t}^2 + \tilde{m}^2}{8\pi x \tilde{t} \[8\pi x \tilde{t} + \tilde{m}^2\]^{3/2}}.$$

(22)

where $\tilde{t} = t\sqrt{1 - |v/m_{AF}|}$ and $\tilde{m} = |m_{AF}| - |v|$ for $|v_c| < |v| \ll |m_{AF}|$, or $\tilde{t} = t\sqrt{|v|/\sqrt{4t^2 + m_{AF}^2} - 1}$ and $\tilde{m} = |v| - \sqrt{4t^2 + m_{AF}^2}$ for $\sqrt{4t^2 + m_{AF}^2} \ll |v|$. Comparing eq. (13) in the case $\phi = \pi$ and eq.(22), we can see that the weak stripe, i.e. $|v| \sim |v_c|$, strongly suppresses the thermopower with sufficiently strong AF order, i.e., large $|m_{AF}|$. Therefore a crucial test of this scenario is to measure the thermopower $S$ in the untwined sample where $R_H$ is not suppressed.

Now let us turn to the results for $R_H$ and $S$ in general cases. Although the analytic results for $R_H$ (eq. (21)) and $S$ (eq. (22)) are valid only in the limiting cases, they
show what are crucial for each quantity. As for $R_H$, the anisotropy of the transfer integral, i.e., the anisotropy of the velocity, is crucial with the help of the averaging of the conductivity tensor. On the other hand, as for $S$, the mass reduction, i.e., the reduction of the band gap, is also crucial as well as the anisotropy of the transfer integral. Therefore, interesting results are expected in the nontrivial case $m_{AF} \sim |v| \sim \sqrt{4t^2 + m_{AF}^2}$, because the energy dispersion is highly anisotropic and there is no band gap between the two middle bands. We employ the Boltzmann transport theory and numerically evaluated the correlation functions. Figs. 4(a) and 4(b) shows $R_H$ and $S$ as a function of the strength of the AF order $m_{AF}$ and the vertical stripe $v$, respectively. The temperature is fixed at $0.02t/k_B$. As mentioned above, the region near the left axis in Fig. 4(b) shows that $S$ is once suppressed by the weak stripe, i.e., $|v| \sim |v_c|$. Then both are remarkably suppressed when $m_{AF}$ and $v$ are comparable with each other, i.e., near the diagonal line. Figs. 5(I), 5(II), and 5(III) show the equal-energy contours and Fermi surfaces for the 3 points in Figs. 4(a) and 4(b). Especially, Fig. 5(II) belongs to the nontrivial case and shows the peculiar shape of the Fermi surface. According to ref. [30], $\sigma_{xy}$ could be reduced in the nontrivial Fermi surface compared to that of the circular Fermi surface surrounding the same size of area in $\vec{k}$-space. Especially when the Fermi surface have both parts of positive convexity and negative one, this reduction would be very effective as
Finally, the effects of the AF order and the vertical (horizontal) stripe on the temperature dependences of $R_H$ and $S$ are shown in Figs. 5(a) and 5(b). The AF order enhances the both quantities at the almost all temperature range except for the region near the peak of $R_H$, and shift the peaks to higher temperature. This is because the AF order introduces the additional energy scale, i.e., the band gap. Then the vertical (horizontal) stripe remarkably suppresses both of $R_H$ and $S$ in the whole temperature range when $v$ is comparable with $m_{AF}$. These results are parallel to the above results where the temperature is fixed.

V. DISCUSSION

Now we discuss the above results in comparison with the experiments. The most important issue is whether the high temperature phase above $T_N$ and the pseudo-gap region can be described in terms of the massless Dirac fermions by putting $m_{AF} = 0$. The temperature and $x$-dependences of $R_H$ and $S$ appear to be qualitatively consistent with the experiments. However, when one looks at the $x$-dependence of the conductivity $\sigma_{xx}$ at $T = 300K$ above $T_N$, it fits better with $\propto x^{3/2}$ rather than $\propto x$ due to the $x$-dependence of the hole mobility. On the other hand, $\sigma_{xx} \propto \sqrt{x}$ and $\sigma_{xy} \propto x^0$ in the our model, which is contradicting with the above experiments. Furthermore, the onset of the AF order at $T_N$ does not affect the conductivity $\sigma_{xx}$ in the experiment, which is more difficult to understand from any mean field picture. This is related to the interpretation of the insulating gap at $x = 0$. In the mean field picture, it is due to the AF ordering while there remains a large gap even without the AF order in the Mott insulator picture. If the large gap disappears and crossing of the two bands occurs above $T_N$, the upper band becomes relevant at around $T_x \equiv J_\pi(0)$ and particle-hole symmetry will be recovered above $T_x$. This massless Dirac spectrum also gives the novel $x$-dependence as described above. Therefore it seems that the $x$-dependence observed in experiments suggests that the Mott insulating picture is more appropriate for the high temperature phase. In other words, the Dirac fermion without the AF gap is never relevant to the underdoped cuprates. This is also consistent with the asymmetry of ARPES between hole-doped and electron-doped cuprates. Recent experiments on NCCO strongly suggest that the minima of the electron dispersion are at $\tilde{k} = (\pi, 0)$ and $(0, \pi)$, and the particle-hole symmetry is broken. This appears to be consistent with the SDW picture with appropriate longer range hopping integrals $t'$ and $t''$. However, the sign of $t'$ and $t''$ is reversed when one consider the $t$-$t'$-$t''$-$J$ model for electron doped case, and the minimum at $(\pi, 0)$ is recovered by self-consistent Born approximation, and its results can be interpreted as the flux state with the AF order. Therefore, it makes sense to consider the flux state together with the AF order, but does not without it.

The AF ordered state, on the other hand, can be well described in terms of the mean field state with the flux order. In this AF state, the conventional behavior of the doped carriers are expected without the stripe formation. Therefore the effects of the stripes are the most interesting issue. It is found that $R_H = a^2/(|e|x)$ is a rather robust feature at low temperature. The suppression occurs only when the AF order and the vertical (horizontal) stripe coexist, and the directional average is taken within the plane. The thermopower is also suppressed by the stripe formation, and does not need the directional average. The difference between ref. 1 and ref. 2 seems to be due to the sample preparation. The longer time annealing has been done for the former case, while the sample is quenched in the latter case. Therefore it is expected that the chain is more ordered in the former.

---

( I ) $\phi = \pi$, $m_{AF} = 2.5 t$, $v = 0.5 t$
( II ) $\phi = \pi$, $m_{AF} = 2.5 t$, $v = 2.7 t$
( III ) $\phi = \pi$, $m_{AF} = 0.5 t$, $v = 3.0 t$

FIG. 5: Schematic view of the energy contours for three parameter sets. Thick lines in each figure represent the Fermi surface for $x = 0.02$. The reduced Brillouin zone is the square with the corners $[\pm \pi/2, \pm \pi/2]$. Copies are also drawn over the original Brillouin zone for $\phi = m_{AF} = v = 0$.
case, which might help the stripe formation. The 4-fold
symmetry observed in the magnetoresistance[36] is inter-
preted to be due to the induced stripe in terms of the
in-plane external magnetic field.

In conclusions, we have studied the transport proper-
ties of the flux state as a model for underdoped cuprates.
This model shows several remarkable features such as
parity anomaly, scaling laws for \( R_H \) and \( S \), the recovery
of particle-hole symmetry at high temperature. Com-
pared with the existing experimental data, in particularly
the \( x \)-dependence of \( \sigma_{xx} \), it is unlikely that this model
describes the underdoped cuprates without the AF or-
der. However, the flux state with the AF order, which
gives the mass gap to the Dirac fermions, describes well
the ordered state. In this case, the stripe order affects
the transport properties in a nontrivial way, and we have
discussed the experiments from this view point.

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APPENDIX A: CONDUCTIVITY IN THE
CONTINUUM MODEL

This appendix is devoted to present the uniform con-
ductivity \( \sigma_{\mu\nu}(\Omega) \) of the continuum effective theory con-
structed in sec. II. When the effect of the relaxation is
approximated by replacing \( \omega \) by \( \Omega = \omega + i/\tau \) with the
life time \( \tau \) and we can diagonalize a given Hamiltonian,
the linear response theory generally gives the following
representation for the uniform conductivity.

\[
\sigma_{\mu\nu}(\Omega) = \frac{i e^2}{\pi} \sum_{\alpha,\beta} \frac{f_F(\epsilon_\beta - \mu) - f_F(\epsilon_\alpha - \mu)}{\epsilon_\alpha - \epsilon_\beta} \times \langle \alpha | J_\nu | \beta \rangle \langle \beta | J_\mu | \alpha \rangle \frac{\Omega + \epsilon_\beta - \epsilon_\alpha}{\Omega + \epsilon_\beta - \epsilon_\alpha},
\]

(A1)

where \( \alpha \) and \( \beta \) are the quantum indices of eigen states,
\( J_\mu \) represents \( \mu \)-component of the current density, and
\( f_F(\epsilon - \mu) \) is the Fermi distribution function. We apply
the above formula to the effective theory in sec. II, i.e.
\((2+1)D\) Dirac fermions in the external magnetic field.
For example, in the case of the contribution from 1-sector
with \( \sigma \)-spin, the following replacements are sufficient.

\[
\sum_{\alpha,\beta} \to \frac{|eB|}{2\pi} \sum_{n,n'=0} \sum_{s,s'=\pm}, |\alpha\rangle \to |n \sigma \pm\rangle, \epsilon_\alpha \to \pm \epsilon_{n\sigma},
J_x \to \hat{j}_x = 2t_a \tau_1,
J_y \to \hat{j}_y = 2t_a (\cos \phi \tau_1 + \sin \phi \tau_2).
\]

(A2)

Here \( |n \sigma \pm\rangle \) and \( \epsilon_\alpha \) are defined by eqs. (8)-(11) in sec. II.
\( \tau_i \) are the Pauli matrices in the space of \( |\uparrow(\downarrow)\rangle. \) [See
eq (8).] Then the contribution from 1-sector with \( \sigma \)-spin
is represented by
\[ \sigma_{\mu \nu}^{1\sigma}(\Omega) = i e^2 |eB| \sum_{n,n'=0, s,s'=\pm} \infty \sum \frac{f_F(s'\epsilon_{n'\sigma} - \mu) - f_F(s\epsilon_{n\sigma} - \mu)}{s\epsilon_{n'\sigma} - s'\epsilon_{n'\sigma}} \langle n \sigma s | \hat{j}_\nu | n' \sigma s' \rangle \langle n' \sigma s' | \hat{j}_\mu | n \sigma s \rangle \]
\[ = i e^2 |eB| \sum_{n,n'=0, s,s'=\pm} \infty \sum \left[ \frac{1}{\epsilon_{n'\sigma} + \epsilon_{n\sigma}} \frac{\langle n \sigma s | \hat{j}_\nu | n' \sigma - s \rangle \langle n' \sigma - s | \hat{j}_\mu | n \sigma s \rangle}{\Omega - s(\epsilon_{n'\sigma} + \epsilon_{n\sigma})} \right. \]
\[ + \frac{f_F(\epsilon_{n'\sigma} - s\mu) - f_F(\epsilon_{n\sigma} - s\mu)}{\epsilon_{n\sigma} - \epsilon_{n'\sigma}} \frac{\langle n \sigma s | \hat{j}_\nu | n' \sigma s \rangle \langle n' \sigma s | \hat{j}_\mu | n \sigma s \rangle}{\Omega + s(\epsilon_{n'\sigma} - \epsilon_{n\sigma})} \]
\[ \left. - \frac{f_F(\epsilon_{n'\sigma} + s\mu) - f_F(\epsilon_{n\sigma} - s\mu)}{\epsilon_{n\sigma} + \epsilon_{n'\sigma}} \frac{\langle n \sigma s | \hat{j}_\nu | n' \sigma s \rangle \langle n' \sigma s | \hat{j}_\mu | n \sigma s \rangle}{\Omega - s(\epsilon_{n'\sigma} + \epsilon_{n\sigma})} \right] \]
\[ = i e^2 |eB| \sum_{n,n'=0, s,s'=\pm} \sum \left[ \frac{2 f_F(\epsilon_{n\sigma} - s\mu) - 1}{\Omega^2 - (\epsilon_{n'\sigma} + \epsilon_{n\sigma})^2} \right. \]
\[ \times \left\{ \frac{(-i\Omega) \Re \left[ \langle n \sigma s | \hat{j}_\nu | n' \sigma - s \rangle \langle n' \sigma - s | \hat{j}_\mu | n \sigma s \rangle \right]}{\epsilon_{n'\sigma} + \epsilon_{n\sigma}} + s \Im \left[ \langle n \sigma s | \hat{j}_\nu | n' \sigma - s \rangle \langle n' \sigma - s | \hat{j}_\mu | n \sigma s \rangle \right] \right\} \]
\[ + \frac{2 f_F(\epsilon_{n\sigma} - s\mu)}{\Omega^2 - (\epsilon_{n'\sigma} - \epsilon_{n\sigma})^2} \]
\[ \times \left\{ \frac{(-i\Omega) \Re \left[ \langle n \sigma s | \hat{j}_\nu | n' \sigma s \rangle \langle n' \sigma s | \hat{j}_\mu | n \sigma s \rangle \right]}{\epsilon_{n\sigma} - \epsilon_{n'\sigma}} + s \Im \left[ \langle n \sigma s | \hat{j}_\nu | n' \sigma s \rangle \langle n' \sigma s | \hat{j}_\mu | n \sigma s \rangle \right] \right\} \] (A3)

It is noted that all distribution functions in the first line represent those of electrons. In the second line, we have done the transformation, \( f_F(-\epsilon - \mu) \rightarrow 1 - f_F(\epsilon + \mu) \), for negative energy modes in order to change the distribution function of the valence band to that of the quasi-hole. In the third line, the difference of distribution functions is pulled apart to each distribution function. This procedure is justified because \( \langle n \sigma s | \hat{j}_\mu | n' \sigma s' \rangle \neq 0 \) only for \( n' = n \pm 1 \) as we will see below and the distribution function \( f_F(\epsilon_{n\sigma} \pm \mu) \) decreases sufficiently fast as \( n \) increases.

The final procedure is to substitute the matrix elements by the following explicit forms,

\[ \langle n \sigma \pm | \tau_1 | n' \sigma \pm \rangle = \pm \frac{1}{2} \left[ \sqrt{\frac{E_{n'\sigma \pm + E_{n+1\sigma \pm \epsilon_{n\sigma} \epsilon_{n+1\sigma}}}{\epsilon_{n+1\sigma} \epsilon_{n\sigma}}} \delta_{n+1,n'} + \sqrt{\frac{E_{n'\sigma \pm + E_{n'\sigma \pm \epsilon_{n'\sigma} \epsilon_{n\sigma}}}{\epsilon_{n'\sigma} \epsilon_{n\sigma}}} \delta_{n,n'+1} \right], \]
\[ \langle n \sigma \pm | \tau_1 | n' \sigma \mp \rangle = \pm \frac{1}{2} \left[ \frac{E_{n'\sigma \pm + E_{n+1\sigma \pm \epsilon_{n\sigma} \epsilon_{n+1\sigma}}}{\epsilon_{n+1\sigma} \epsilon_{n\sigma}} \delta_{n+1,n'} - \frac{E_{n'\sigma \pm + E_{n'\sigma \pm \epsilon_{n'\sigma} \epsilon_{n\sigma}}}{\epsilon_{n'\sigma} \epsilon_{n\sigma}} \delta_{n,n'+1} \right], \]
\[ \langle n \sigma \pm | \tau_2 | n' \sigma \pm \rangle = \mp i \text{sgn}(eB_\sigma) \frac{1}{2} \left[ \frac{E_{n'\sigma \pm + E_{n+1\sigma \pm \epsilon_{n\sigma} \epsilon_{n+1\sigma}}}{\epsilon_{n+1\sigma} \epsilon_{n\sigma}} \delta_{n+1,n'} + \frac{E_{n'\sigma \pm + E_{n'\sigma \pm \epsilon_{n'\sigma} \epsilon_{n\sigma}}}{\epsilon_{n'\sigma} \epsilon_{n\sigma}} \delta_{n,n'+1} \right], \]
\[ \langle n \sigma \pm | \tau_2 | n' \sigma \mp \rangle = \pm i \text{sgn}(eB_\sigma) \frac{1}{2} \left[ \frac{E_{n'\sigma \pm + E_{n+1\sigma \pm \epsilon_{n\sigma} \epsilon_{n+1\sigma}}}{\epsilon_{n+1\sigma} \epsilon_{n\sigma}} \delta_{n+1,n'} - \frac{E_{n'\sigma \pm + E_{n'\sigma \pm \epsilon_{n'\sigma} \epsilon_{n\sigma}}}{\epsilon_{n'\sigma} \epsilon_{n\sigma}} \delta_{n,n'+1} \right] \] (A4)

where \( E_{n\sigma \pm} = \epsilon_{n\sigma} \pm \text{sgn}(eB_\sigma) u_\sigma \). (The absence of one of zero modes is appropriately represented by \( E_{n\sigma \pm} \).) As for the dc conductivity, the contribution from 1-sector with \( \sigma \)-spin is given by
\[
\sigma_{xx}^{1\sigma} = \frac{e^2}{2\pi} \tau^{-1} K_{cB}^2 \sum_{s=\pm} f_F(|u_\sigma| - s\mu) \frac{|u_\sigma| [1 + \text{sgn}(su_\sigma eB_\phi)]}{(K_{cB_\phi}^2 + \tau^{-2})^2 + 4\tau^{-2}u_\sigma^2} \\
+ \sum_{n=1} f_F(\epsilon_n - s\mu) \left[ (K_{cB_\phi}^4 - \tau^{-4})^2 + 8(K_{cB_\phi}^4 + \tau^{-4})\tau^{-2}\epsilon_n^2 + (4\tau^{-2}\epsilon_n^2)^2 \right]^{-1} \\
\times \left[ \tau^{-2}(\tau^{-2} + 4\epsilon_n^2)(\epsilon_n + u_\sigma^2\epsilon_n^2) + K_{cB_\phi}^4(3\epsilon_n - u_\sigma^2\epsilon_n^2) - 4K_{cB_\phi}^2\tau^{-2}|u_\sigma|\text{sgn}(su_\sigma eB_\phi) \right] \\
+ \frac{e^2}{2\pi} \tau^{-1} K_{cB}^2 \sum_{n=0}^{\infty} \frac{1}{(\epsilon_n + \epsilon_{n+1})[\tau^{-2} + (\epsilon_n + \epsilon_{n+1})^2]} \left( 1 + \frac{u_\sigma^2}{\epsilon_n \epsilon_{n+1}} \right), \\
(A5)
\]
\[
\sigma_{xy}^{1\sigma} = \frac{e^2}{2\pi} \text{sgn}(eB) K_{cB_\phi}^2 \sum_{s=\pm} \frac{s}{2} f_F(|u_\sigma| - s\mu) \frac{(K_{cB_\phi}^2 + \tau^{-2})[1 + \text{sgn}(su_\sigma eB_\phi)]}{(K_{cB_\phi}^2 + \tau^{-2})^2 + 4\tau^{-2}u_\sigma^2} \\
+ \sum_{n=1} s f_F(\epsilon_n - s\mu) \left[ (K_{cB_\phi}^4 - \tau^{-4})^2 + 8(K_{cB_\phi}^4 + \tau^{-4})\tau^{-2}\epsilon_n^2 + (4\tau^{-2}\epsilon_n^2)^2 \right]^{-1} \\
\times \left[ K_{cB_\phi}^2(K_{cB_\phi}^4 - \tau^{-4} + 4\tau^{-2}u_\sigma^2) + \tau^{-2}(K_{cB_\phi}^4 - \tau^{-4} + 4\tau^{-2}\epsilon_n^2)|u_\sigma|\epsilon_n^{-1}\text{sgn}(su_\sigma eB_\phi) \right] \\
- \frac{e^2}{2\pi} \text{sgn}(\sin\phi) \frac{K_{cB_\phi}^2}{2} \sum_{n=0}^{\infty} \frac{u_\sigma(\epsilon_n + \epsilon_{n+1})}{\epsilon_n \epsilon_{n+1} [\tau^{-2} + (\epsilon_n + \epsilon_{n+1})^2]} \\
+ \sigma_{xx}^{1\sigma} \cos\frac{\phi}{2}, \\
(A6)
\]

The terms without the distribution function represent the parts of the inter-band effect which remain even in the case \(\mu = 0\) and \(k_B T = 0\). As for \(\sigma_{xy}^{1\sigma}\), the contribution from the inter-band effect is negligible in the semi-classical limit (\(K_{cB_\phi} \ll 1 \ll |\mu|/\tau\)) compared to the contribution from the Fermi level. In this case, the Boltzmann theory is good approximation. (The inter-band effect is neglected in the Boltzmann theory.) On the other hand, as for \(\sigma_{xy}^{1\sigma}\), the last two terms, which include the inter-band effect remaining in the case \(\mu = 0\) and \(k_B T = 0\), cancel out after summing up the contribution from all sectors. In other words, when the parity symmetry is breaking, i.e. the numbers of right and left Dirac fermions are unbalanced, we cannot neglect the remaining inter-band effect. Especially in the case \(\tau = \infty\) and \(B \to 0\), the last line but one in \(\sigma_{xy}^{1\sigma}\) gives a contribution \(\pm 1/2\) in the unit \(e^2/h\) as long as \(u_\sigma \neq 0\), where the sign depends on \(u_\sigma\) and \(\phi\), and \(h\) is Planck’s constant. However, it is noted that the total \(\sigma_{xy}^{1\sigma}\) should take a integer value in the unit \(e^2/h\) when it is quantized. Therefore, in the case where this inter-band effect is crucial, we must seriously consider the contribution from the bottom of valence bands and the top of conduction bands, which are out of range of the continuum model.

APPENDIX B: TEMPERATURE DEPENDENCE OF THE CHEMICAL POTENTIAL

Here we consider the temperature dependence of the chemical potential \(\mu\) in the limit \(B \to 0\). When there is no stripe formation, the doping parameter of the continuum model is given by

\[
-x = d_s \sum_{\epsilon = \pm} \int \frac{d^2p}{(2\pi)^2} s f_F(\epsilon, \vec{p}) - s\mu = \frac{d_f}{8\pi t^2 |\sin \frac{\phi}{2}|} \int_{|m_{AF}|}^{\infty} dc s f_F(\epsilon - s\mu)(B1)
\]

Here \(d_s\) is the number of spin degrees of freedom, i.e. \(d_s = 2, d_f\) is the total number of inner degrees of freedom, i.e. \(d_f = (\text{left} + \text{right}) \times d_s = 4\), and

\[
\epsilon(\vec{p}) = \sqrt{(2ta)^2 \left( p^2 \pm 2\cos \frac{\phi}{2} p_x p_y \right) + m_{AF}^2}. \quad (B2)
\]

The sign of \(x\) is taken as it is positive when \(\mu < 0\).

In the low-temperature approximation, \(k_B T \ll (|\mu| - |m_{AF}|)\), we can estimate the right hand side of eq. (B1) by using the sharpness of the Fermi distribution function.
as follows,

\[-x = \frac{d_f}{16\pi t^2} \text{sgn}(\mu) \theta(|\mu| - |m_{AF}|) \times \left[ \mu^2 - m_{AF}^2 + \frac{\pi^2}{3}(k_B T)^2 + \cdots \right].\]  

(B3)

Then, in the hole doping case, i.e. \( \mu < -|m_{AF}| \), the temperature dependence of \( \mu \) is given by

\[|\mu| \approx \sqrt{4\pi t^2 \sin^2 \frac{\phi}{2} + m_{AF}^2 - \frac{\pi^2}{3}(k_B T)^2},\]  

(B4)

where we have used \( d_f = 4 \).

On the other hand, in the high-temperature approximation, \( k_B T \gg |\mu| \), we can estimate the right hand side of eq. (B3) as follows

\[-x = \frac{d_f}{8\pi t^2} \frac{\mu}{\sin^2 \frac{\phi}{2}} \sum_{s=\pm} \frac{1}{\beta} \int_{|\beta m_{AF}|}^{\infty} dy \frac{s y}{1 + e^{y - s\beta \mu}},
\]

\[\approx \frac{d_f}{4\pi t^2} \frac{\mu}{\sin^2 \frac{\phi}{2}} \left[ \beta |m_{AF}| \right. \left. \frac{1}{1 + e^{\beta |m_{AF}|}} + \ln(1 + e^{-\beta |m_{AF}|}) + \cdots \right],\]  

(B5)

where \( \beta = 1/(k_B T) \) and we have expanded the integrand in \( \beta \mu \). Finally, in the hole doping case, i.e. \( \mu < 0 \), the temperature dependence of \( \mu \) is given by

\[|\mu| \approx \frac{\pi t^2 \beta}{1 + e^{\beta |m_{AF}|}} \sin \phi \frac{\mu}{2} + \ln(1 + e^{-\beta |m_{AF}|}),\]  

(B6)

where \( d_f = 4 \) is substituted. It is noted that, fixing the doping parameter and the temperature, the above formula is a increasing function of \( |m_{AF}| \). Therefore, the AF order \( m_{AF} \) suppresses the recovery of the particle-hole symmetry.

**APPENDIX C: SCALING LAWS OF \( R_H \) AND \( S \)**

In this appendix, we present the derivation of the scaling laws of \( R_H \) and \( S \) in the case of \( m_{AF} = m_{diag} = v = 0 \) and in the limit \( B \to 0 \). In order to see the temperature dependence of the Fermi distribution function \( f_F(\xi) \), we introduce the function \( f(\beta \xi) = f_F(\xi) \) where \( \beta = 1/(k_B T) \). Then, from eq. (B1) with \( m_{AF} = 0 \), the doping parameter \( x \) is given by the following equation,

\[-x = \frac{1}{2\pi t^2} \frac{\mu}{\sin^2 \frac{\phi}{2}} \sum_{s=\pm} \int_{0}^{\infty} dy \frac{s y}{1 + e^{y - s\beta \mu}} \int_{0}^{\infty} d\epsilon \epsilon s f(\beta(\epsilon - s\mu)),\]

\[= \frac{1}{2\pi t^2} \frac{\mu}{\sin^2 \frac{\phi}{2}} \sum_{s=\pm} \int_{0}^{\infty} dy \frac{s y}{1 + e^{y - s\beta \mu}} s f(y - s\beta \mu),\]  

(C1)

It is easy to see that \( t\sqrt{x}/(k_B T) \) is represented by a function of \( \mu/(k_B T) \). Therefore, when we can consider the inverse of the function, \( \mu/(k_B T) \) is represented by a function of \( t\sqrt{x}/(k_B T) \).

In the same way, by making the energy integral dimensionless, the response functions are represented as

\[\sigma_{xx} \approx \frac{e^2}{2\pi} \frac{(2\tau)}{\sin^2 \frac{\phi}{2}} \sum_{s=\pm} \int_{0}^{\infty} dy \left[ -y \frac{df}{dy}(y - s\beta \mu) + \left( \frac{1}{2} - f(y - s\beta \mu) \right) \frac{1}{1 + \left( \frac{2\tau}{y} \right)^2} \right],\]  

(C2)

\[\sigma_{xy} \approx \frac{e^2}{2\pi} \frac{(2\tau)}{\sin^2 \frac{\phi}{2}} \frac{2a^2 eB}{\sqrt{x}} (\beta t \sqrt{x})^2 \frac{(2\tau)^2}{\beta} \sum_{s=\pm} \int_{0}^{\infty} dy \frac{df}{dy}(y - s\beta \mu) \left[ \frac{1}{1 + \left( \frac{2\tau}{y} \right)^2} \right],\]  

(C3)

\[\beta \sigma_{xx}^Q \approx \frac{e^2}{2\pi} \frac{(2\tau)}{\sin^2 \frac{\phi}{2}} \sum_{s=\pm} \int_{0}^{\infty} dy \left[ -s y (y - s\beta \mu) \frac{df}{dy}(y - s\beta \mu) - \left( \frac{1}{2} - f(y - s\beta \mu) \right) \frac{\beta \mu}{1 + \left( \frac{2\tau}{y} \right)^2} \right],\]  

(C4)

where \( \sigma_{\mu\nu}^Q \) is the response function given by the correlation function of the electric current density \( e\vec{J} \) and the heat current density \( J^Q \). In the first-quantized representation, the heat current density for the 1-sector with \( \sigma \)-spin is given by

\[\hat{j}_x^Q = (2ta)^2 \left( \hat{\rho}_x + \cos \frac{\phi}{2} \hat{\rho}_y \right) - \mu \hat{j}_x,\]  

(C5)

\[\hat{j}_y^Q = (2ta)^2 \left( \hat{\rho}_y + \cos \frac{\phi}{2} \hat{\rho}_x \right) - \mu \hat{j}_y.\]  

(C6)
This is the continuum version of the heat current density derived in the lattice model, which is presented in the appendix [D]. The right hand side of each response function is a function of $t/\sqrt{(k_BT)}$ and $\tau k_BT$, because $\mu/(k_BT)$ is a function of $t/\sqrt{(k_BT)}$. Therefore, $x R_H \equiv x \sigma_{xy}/(B \sigma_{xx})$ and $S \equiv \sqrt{x}/(T \sigma_{xx})$ is functions of $t/\sqrt{(k_BT)}$ and $\tau k_BT$. In the semi-classical limit, the dependence of $\tau k_BT$ is negligible, and $x R_H$ and $S$ scale as functions of $t/\sqrt{(k_BT)}$.

**APPENDIX D: HEAT CURRENT DENSITY IN THE LATTICE MODEL**

The definition of the heat current density is not straightforward as against that of the electric current density. Here we present the heat current density derived in the lattice model, which is presented in eq. (1). The Hamiltonian is rewritten as $H = \sum_{\sigma} h_{\vec{r}}$ where

$$
h_{\vec{r}} = \frac{1}{2} \sum_{\delta, \sigma} \left[ -t_{\vec{r}+\delta, \vec{r}} c_{\vec{r}+\delta, \sigma}^\dagger c_{\vec{r}, \sigma} - t_{\vec{r}, \vec{r}+\delta} c_{\vec{r}, \sigma}^\dagger c_{\vec{r}+\delta, \sigma} \right] + \sum_{\sigma} \left( V_{\vec{r}\sigma} - \mu \right) c_{\vec{r}\sigma}^\dagger c_{\vec{r}\sigma}, \quad (D1)
$$

$$
V_{\vec{r}\sigma} = \left[ u_\sigma \cos(\vec{Q} \cdot \vec{r}) + v \cos(\vec{Q}_x \cdot \vec{r}) \right], \quad (D2)
$$

and the sum of the unit vector $\hat{\delta}$ runs $\pm \hat{x}$ and $\pm \hat{y}$. It is clear that $h_{\vec{r}}$ is interpreted as a local heat density. The heat current density would be conceptually defined by the averaging the product of the local velocity and $h_{\vec{r}}$. However, it is difficult to define the local velocity, especially in the second quantized formalism. Here we define the heat current density from the analogy with the conceptual definition. First, using $h_{\vec{r}}$, we introduce the quantity $\vec{R}$ as follows, $\vec{R} = V^{-1} \sum_{\vec{r}} h_{\vec{r}}$, where $V$ is the volume (area) of the system. This quantity has the dimension of heat density multiplied by length. Then, we define the heat current density as a time derivative of $\vec{R}$, i.e., $\vec{J}^Q = \partial_t [H, \vec{R}]$. From the dimension of $\vec{R}$, $\vec{J}^Q$ correctly has the dimension of heat current density. Finally, the explicit form of $\vec{J}^Q$ is given by

$$
\vec{J}^Q = \vec{J}^E - \mu \vec{J}, \quad (D3)
$$

$$
\vec{J}^E = \frac{1}{V} \sum_{\vec{r}} \left[ -\frac{i}{2} \sum_{\delta, \delta', \sigma} (\hat{\delta} + \hat{\delta}') t_{\vec{r}+\delta, \vec{r}'} t_{\vec{r}+\delta', \vec{r}} c_{\vec{r}+\delta', \sigma}^\dagger c_{\vec{r}+\delta, \sigma} + \frac{i}{2} \sum_{\delta, \sigma} \left( V_{\vec{r}+\delta} + V_{\vec{r}} \right) t_{\vec{r}+\delta, \vec{r}} c_{\vec{r}+\delta, \sigma}^\dagger c_{\vec{r}, \sigma} \right],
$$

$$
= \frac{1}{V} \sum_{k, \sigma} \left\{ \sum_{\hat{\mu} = \pm \hat{x}, \hat{y}} \left( -4t^2 \right) \hat{\mu} \sin k_{\hat{\mu}} \left\{ \cos k_{\mu} + \cos \frac{\phi}{2} \cos (\epsilon_{\mu \nu} k_{\nu}) \right\} c_{k, \sigma}^\dagger \delta, \sigma \right\} + 2vt \hat{y} \sin k_{\hat{y}} \left\{ -\cos \frac{\phi}{4} \delta, \sigma \right\} \sin k_{\hat{y}} \left\{ -\cos \frac{\phi}{4} \delta, \sigma \right\} \sin k_{\hat{y}} \left\{ \cos \frac{\phi}{4} \delta, \sigma \right\} \sin k_{\hat{y}} \left\{ \cos \frac{\phi}{4} \delta, \sigma \right\}, \quad (D4)
$$

$$
\vec{J} = \frac{i}{V} \sum_{\vec{r}} \sum_{\delta, \sigma} \hat{\delta} t_{\vec{r}+\delta, \vec{r}} c_{\vec{r}+\delta, \sigma}^\dagger c_{\vec{r}, \sigma} = \frac{1}{V} \sum_{k, \sigma} \sum_{\hat{\mu} = \pm \hat{x}, \hat{y}} 2\hat{\mu} \sin k_{\hat{\mu}} \left\{ \cos \frac{\phi}{4} \delta, \sigma \right\} \sin k_{\hat{y}} \left\{ \cos \frac{\phi}{4} \delta, \sigma \right\} \sin k_{\hat{y}} \left\{ \cos \frac{\phi}{4} \delta, \sigma \right\} \sin k_{\hat{y}} \left\{ \cos \frac{\phi}{4} \delta, \sigma \right\}, \quad (D5)
$$

where $\vec{Q} = (\pi, \pi), \vec{Q}_x = (\pi, 0)$, and $\vec{Q}_y = (0, \pi)$. The symbol $(-1)^\hat{\mu}$ means that $(-1)^\hat{\mu} = 1$ and $(-1)^\hat{\mu} = -1$. In a physical sense, $\vec{J}^E$ is the energy current density, and $\vec{J}$ is the current density.

As for the continuum limit, the following transformation of bases are performed before taking the limit.

$$
\begin{bmatrix}
    c_{\vec{k} \sigma} \\
    c_{\vec{k}+\vec{Q}_x \sigma} \\
    c_{\vec{k}+\vec{Q}_y \sigma} \\
    c_{\vec{k}+\vec{Q}_x+\vec{Q}_y \sigma}
\end{bmatrix} =
\begin{bmatrix}
    U_+ & O \\
    O & U_-
\end{bmatrix}
\begin{bmatrix}
    \psi_{\vec{k}, 1\sigma} \\
    \psi_{\vec{k}, 2\sigma}
\end{bmatrix}, \quad (D6)
$$

where $\psi_{\vec{k}, i\sigma}$ ($i = 1, 2$) is a two-component operator respectively, and

$$
U_{\pm} = \frac{1}{\sqrt{2}} (\tau_3 \pm \tau_1) \left( \cos \frac{\phi}{8} \tau_1 \pm \sin \frac{\phi}{8} \tau_2 \right). \quad (D7)
$$

When there is no stripe, $\psi_{\vec{k}, i\sigma}$ with $\vec{k} \sim (\pi/2, \pi/2)$ corresponds to the Dirac fermion of $i$-sector ($i = 1, 2$) with $\sigma$-spin.
APPENDIX E: \( R_H \) IN THE CASE \(|v| < |v_c|\)

Here we present the analysis of \( R_H \) when two lower bands are doped. Around the tops of these two bands, their dispersions are approximated as

\[
\epsilon_\pm (\vec{p}) = -\sqrt{2t_\pm a^2 p_x^2 + (2\tilde{t}_\pm a^2 p_y^2 + \tilde{m}_\pm^2)}, \tag{E1}
\]

where \( \tilde{t}_\pm = t/\sqrt{1 \pm |v/m_{AF}|} \) and \( \tilde{m}_\pm = |m_{AF}| \pm |v| \). In this case, by considering the two bands separately and then adding the contributions at the stage of conductivity, we can get the Hall coefficient \( R_H \approx a^2/(|e|x) \) at low temperature, where \( x \) is the total doping parameter. However, when there are domains of stripes with different directions, we should take an average of conductivity tensor. Then, we obtain the following result.

\[
R_H \approx \frac{a^2}{|e|} \left[ \frac{4 \left[ x - \left\vert \frac{v}{m_{AF}} \right\vert (x_+ - x_-) \right]}{2x - \left\vert \frac{v}{m_{AF}} \right\vert (x_+ - x_-)} \right]^2. \tag{E2}
\]

It is noted here that \( x_\pm \) includes the contribution of both spin degrees of freedom. When \(|v|\) is larger than the critical value \(|v_c|\), i.e.,

\[
|x_+ - x_-| = |m_{AF}| \left( \frac{\sqrt{2\pi x t}}{m_{AF}} \right)^2 \left[ \left[ 1 + \frac{1}{4} \left( \frac{\sqrt{2\pi x t}}{m_{AF}} \right)^4 - \frac{1}{2} \left( \frac{\sqrt{2\pi x t}}{m_{AF}} \right)^2 \right] \right], \tag{E5}
\]

only the second band is doped, i.e., \( x_+ = 0 \) and \( x_- = x \).

We can also analyze the thermopower \( S \) by the similar way. However, its expression is more complicated than that of \( R_H \) and is not suggestive. Therefore, we do not give the explicit analysis, but pointed out that, in the region \( 0 < |v| < |v_c| \), \( S \) is highly reduced when \(|v|\) increases as shown in Fig. 4(b).

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