We report new results in the study of CP violation in semileptonic top decays, in the context of the Weinberg Model.

1. Top decays in the Weinberg Model

Semileptonic top decays in the context of the Weinberg Model (WM) have been the focal point of extensive study. In the WM the new basic ingredient is the possibility of inducing CP violating effects in the leptonic sector, due to the presence of the additional charged Higgs sector. The way such CP violating effects arise can be seen from the relevant Lagrangian term

\[ \mathcal{L} = \frac{g_{mt}}{\sqrt{2}M} \bar{t}_R b_L c_1 c_3 e^{i\delta} H^+ - \frac{g_{m\tau}}{\sqrt{2}M} \bar{\nu}_L \tau_R c_1 c_2 s_3 e^{i\delta} H^+ + \text{h.c.}, \]

(1)

involving Yukawa couplings between the extra charged Higgs \( H^+ \) and the fermions (quarks and leptons). We note that the constants \( s_i, c_i \) and \( \delta \) appear in the CKM-like matrix operating in the charged Higgs sector and are not elements of the usual CKM matrix; \( M \) is the mass of the \( W \). The possibility for additional CP violating effects has been studied in the context of the decay mode \( t \to b\tau\nu \). The observable considered is the partial decay rate asymmetry (PRA), namely

\[ A = \frac{\Gamma(t \to b\tau^+\nu_\tau) - \Gamma(\bar{t} \to \bar{b}\tau^-\bar{\nu}_\tau)}{\Gamma(t \to b\tau^+\nu_\tau) + \Gamma(\bar{t} \to \bar{b}\tau^-\bar{\nu}_\tau)}. \]

(2)

At one loop the PRA receives contributions through interference terms between one-loop Standard Model (SM) graphs for the process \( t \to W^+b \to b\tau^+\nu_\tau \), and the tree-level WM graph for the process \( t \to H^+b \to b\tau^+\nu_\tau \). Consequently, the entire effect is proportional to \( m_t m_\tau \). Due to helicity mismatches only the longitudinal parts of the SM graphs contribute to the PRA. In addition, due to the fact that the Higgs couplings are complex numbers, it is only the imaginary parts of such
longitudinal contributions which is relevant. So, $\mathcal{A}$ is proportional to

$$\mathcal{A} \sim \int dq^2 f(q^2) \text{Im}(G_L),$$

(3)

where $f(q^2)$ is a phase space function and $G_L$ is the longitudinal component of any one-loop graph. In addition, it is important to notice the presence of the phase space integral, whose range extends from $m^2_\tau$ all the way up to $(m_t - m_b)^2$.

In computing the one-loop contribution to $\mathcal{A}$ the only graphs considered was the $W$ self-energy graphs, containing fermionic loops (although, as we will see in the next section, they are not the only graphs contributing to $\mathcal{A}$). The original motivation for singling out the $W$ propagator with fermionic loops was the expectation that due to the general resonant nature of such graphs, significant enhancement of the PRA might take place. As it was soon realized however this resonant behavior could not be exploited, because it is only the longitudinal parts of the self-energy graphs which contribute to the PRA, and it is only the transverse (but not the longitudinal) parts of the $W$ self-energy, which displays resonant behavior. So, when the $W$ propagator is decomposed in the form

$$G_{\mu\nu} = (g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2})G_T + \frac{q_{\mu}q_{\nu}}{q^2}G_L,$$

(4)

with

$$G_T = \frac{1}{q^2 - M^2 + i\epsilon_T},$$

(5)

and

$$G_L = \frac{1}{M^2 + i\epsilon_L},$$

(6)

where

$$\epsilon_T = \left(\frac{g^2}{32\pi}\right)\frac{(2q^2 + m_c^2)(q^2 - m_c^2)^2}{q^4},$$

(7)

and

$$\epsilon_L = \left(\frac{3g^2}{32\pi}\right)\frac{m_c^2(q^2 - m_c^2)^2}{q^4},$$

(8)

from Eq(5) and Eq(6) follows that

$$\text{Im}(G_T) = -\epsilon_T |G_T|^2, \quad \text{Im}(G_L) = -\epsilon_L |G_L|^2.$$  

(9)

We notice that due to rescattering the $\tau \nu$ loop should not contribute for CPT to be an exact symmetry, so that the next threshold is due to the $cs$ loop. Finally, when $\text{Im}G_L$ of Eq(6) is inserted in Eq(3) (instead of the resonant $\text{Im}G_T$ which does not contribute), the result is very small ($\mathcal{A} \sim 10^{-8}$).

In an attempt to exploit the resonant character of $\text{Im}G_T$, one then proceeded to compute two loop contributions to $\mathcal{A}$. In the two-loop calculation the helicity
mismatch argument operating at one-loop is not valid any more. Thus, the resonant $\text{Im} G_T$ starts contributing. So, in this calculation one hopes to compensate the suppression from the extra powers of the coupling constant (due to the second loop) with the resonant contributions now present, in such a way that the two-loop resonant contributions are effectively comparable to one-loop contributions. In estimating $\mathcal{A}$ the values of $s_i, c_i$, and $\delta$ have been maximized, subject to all experimental constraints. In particular, for $M_{H^+} = 200 \text{ GeV}$, $s_1 = 0.252$, $s_2 = 8.29 \times 10^{-3}$, $s_3 = 0.707$, and $\delta = \frac{\pi}{2}$, we have that $\mathcal{A} = -3.9 \times 10^{-5}$.

2. New one-loop contributions

As already indicated in the previous section, there is an entire class of graphs which contribute to $\mathcal{A}$ at one-loop, which have not been included in the original calculations. Such contributions originate from imaginary parts of self-energy, vertex, and box diagrams, which contain gauge boson loops instead of fermionic loops. The reason such graphs contribute to $\mathcal{A}$ is due to the fact that $\mathcal{A}$ receives contributions through the entire phase space integration range, from $m_\tau^2$ to $m_t^2$. There are two types of such thresholds:

i) bosonic thresholds, opening when $q^2 > M^2$, ($W \to W \gamma$); clearly, the imaginary parts of such graphs contribute in the phase space integration for $q^2 > M^2$.

ii) top thresholds, corresponding to $t \to W b$, from vertex and box (but not $W$ self-energy) graphs. The imaginary parts of such graphs are non-vanishing for every value of $q^2$, as long as $m_t^2 > M^2 + m_b^2$, which is of course true.

As before, only the longitudinal components contribute to $\mathcal{A}$ at one-loop. Moreover, such contributions are non-resonant, just as the longitudinal $W$ self-energy graphs containing fermion loops. However, since there is no suppression factor $\frac{m_\tau^2}{M^2}$ in this case, such graphs are in general expected to contribute significantly; as we will see shortly, this is indeed the case.

Having realized the relevance of the new thresholds, their computation is in principle straightforward. All one needs to do is isolate the longitudinal contributions and then compute their imaginary parts. It turns out that the process of isolating the longitudinal parts is significantly facilitated if one uses a particular type of gauges. So, instead of using the common choice of the renormalizable $R_\xi$ gauges, we will work in the context of the background field gauges (BFG) $\text{BFG}$, using appropriate Feynman rules. The reason for this choice is the fact that in the BFG framework, the self-energy and vertices satisfy the following set of naive, QED-like Ward identities:

$$q^\mu q^\nu \hat{\Pi}_{\mu\nu} - 2M q^\mu \hat{\Theta}_\mu + M^2 \hat{\Omega} = 0 \quad ,$$

$$q^\mu \hat{\Pi}^{\mu\nu} - M \hat{\Theta}_\nu = 0 \quad ,$$

$$q^\mu \hat{\Gamma}_\mu - M \hat{\Lambda} = 0 \quad .$$

(10)  

(11)  

(12)
where $\hat{\Pi}_{\mu\nu}$ is the $W^+W^-$ self-energy, $\hat{\Theta}_\mu$ is the $\phi^+W^-$ mixing term, $\hat{\Omega}$ the $\phi^+\phi^-$ self-energy, $\hat{\Gamma}_\mu$ is the $Wtb$ (or $W\tau\nu$) vertex and $\hat{\Lambda}$ is the $\phi tb$ (or $\phi\tau\nu$) vertex, all of them computed to one-loop, in the context of the BFG. $\phi^+$ is the charged would-be Goldstone boson. All the above quantities depend in general on the gauge-fixing parameter $\xi_Q$, used to gauge-fix the quantum field inside the loops. However, since the final answer is guaranteed to be $\xi_Q$-independent, provided all graphs are included, any choice for $\xi_Q$ is legitimate; in particular, we choose $\xi_Q = 1$.

Returning to the Ward identities, it is relatively straightforward to exploit them, in order to decompose the amplitude in transverse and longitudinal pieces, without detailed knowledge of the explicit closed expressions of the individual graphs. We define $\hat{\Gamma}_\mu^t = \hat{\Gamma}_\mu + q^\mu q^\nu M^2 \hat{\Lambda}$, (13) and $\hat{\Pi}_{\mu\nu}^t = \hat{\Pi}_{\mu\nu} - \frac{q^\mu q^\nu}{q^2} M^2 \hat{\Theta}$. (14) Both $\hat{\Gamma}_\mu^t$ are $\hat{\Pi}_{\mu\nu}^t$ are transverse, e.g.

\[ q^\mu \hat{\Gamma}_\mu^t = 0, \quad q^\mu \hat{\Pi}_{\mu\nu}^t = 0. \] (15)

Using the identity

\[ \frac{1}{M^2} = \frac{1}{q^2} + \frac{q^2 - M^2}{q^2 M^2}, \] (16)

we obtain for the propagator-like contribution $T_1$ of the $S$-matrix element

\[ T_1 = \Gamma_0^\mu \frac{1}{q^2 - M^2} \hat{\Gamma}_{\mu\nu}^t \frac{1}{q^2 - M^2} \Gamma_0^\nu + \Lambda_0 \frac{1}{q^2} \hat{\Omega} \frac{1}{q^2} \Lambda_0, \] (17)

and for the vertex-like piece $T_2$

\[ T_2 = \Gamma_0^\sigma \frac{g^\mu_{\sigma}}{q^2 - M^2} \hat{\Gamma}_\mu^t - \Lambda_0 \frac{1}{q^2} \hat{\Lambda}. \] (18)

It is important to notice that the longitudinal parts of Eq(17) and Eq(18) are multiplied by the kinematic factor $\frac{1}{q^2}$, instead of $\frac{1}{q^2 - M^2}$; they are therefore manifestly non-resonant, in the entire range of the phase space integration, even at $q^2 = M^2$.

3. Calculations and results

By virtue of this decomposition, we only need to calculate self-energy and vertex graphs with a charged $\phi$ (but not $W$) coming in; this represents a significant calculational simplification. On the other hand, since no such simple decomposition exists
for box-like parts of the $S$-matrix, we will compute the imaginary contributions of box diagrams directly, and then isolate their longitudinal parts. It turns out that graphs containing a $Z$ or a $\phi_z$ inside their loops are numerically suppressed. Since all such graphs form a gauge-invariant subset, their omission does not interfere with the gauge independence of the final answer.

The effect of these contributions is additionally enhanced due to the presence of large logarithms of the form $\ln\left(\frac{m_t^2}{m_b^2}\right)$, $\ln\left(\frac{m_t^2}{M^2}\right)$, and $\ln\left(\frac{m_t^2}{M^2}\right)$, which originate from vertex and box diagrams. After collecting all contributions and integrating over the available phase space, using the same values for the constants $s_i$, $c_i$ and $\delta$ and $M_{H^+}$ as before, we finally find $\mathcal{A} = -2.0 \times 10^{-5}$.

We notice that:

i) The result of these new threshold is comparable to the outcome of the two loop resonant calculation, and at least two orders of magnitude larger than the one-loop fermionic contributions.

ii) The new result comes with the same relative sign as the two-loop result; therefore, the entire effect is to further enhance the value of the PRA.

4. Conclusions

In this paper we addressed issues related to the calculation of the PRA in the WM. We focused on semileptonic top decays, on the dominant channel $t \rightarrow b\tau\nu$. We showed that due to the fact that the PRA receives contributions from the entire kinematically available phase space, new one loop contributions, not previously considered, arise. Such contributions are non-resonant and gauge-invariant. It turns out that the PRA so obtained is two orders of magnitude larger than the one calculated form the non-resonant fermionic contributions to the $W$ self-energy, and are comparable to the two loop result.

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6. References

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