Study of Elastic Sensing Elements for Vibration-Resistant Pressure Gauges

S P Pirogov¹, A Yu Chuba² and D A Cherentsov¹

¹ Industrial University of Tyumen, 38 Volodarskogo st., Tyumen, 625000, Russia
² State Agricultural University of Northern Zauralye, 7 Respubliky st., Tyumen, 625001, Russia

E-mail: cherentsovda@bk.ru

Abstract. The paper provides substantiation of the necessity to determine the natural oscillations frequencies of manometric tubular springs. A mathematical model of manometric tubular springs is overviewed. Based on the given model a computer program was developed that was incorporated into a software suit for manometric tubular spring calculations. The results of numerical experiments in the software suit allowed us to identify the impact of geometric parameters on the natural oscillation frequency.

1. Introduction
Pressure gauges often operate under vibration and tubular springs, primary elastic sensing elements, oscillate affecting the accuracy of gauges. Therefore, when designing manometric springs much attention should be given to dynamic calculations, resistance calculations in particular. Natural oscillation frequency is an important characteristic of vibration resistance, so key factors influencing this parameter must be identified. When designing manometric springs it is necessary to select such geometric parameters of tubes that will eventually ensure the required spring properties.

2. Manometric tubular spring designs
Manometric springs utilize the ability of a hollow thin-walled tube with a noncircular section to undergo deformation under pressure. A spring is a circular-bent tube with a noncircular cross-section that has one or two mutually perpendicular axes and is placed in such a way that one of its axes is a continuation of the curvature radius of the locus of the cross-sections’ centers of gravity (figure 1).

We will call this locus the central axis of the tube. One end of the spring is fixed in a holder with an opening through which pressure is transmitted into the inner cavity of the spring. The other end, called free, is tightly closed with a tip and is connected to the mechanism of the pressure gauge. Under the action of pressure the spring rebounds, and its free end performs a travel λ. The spring geometric parameters defining its dimensions are the curvature radius of the longitudinal axis – R, the central aperture angle – γ, the wall thickness – h, the major semi-axis – a and the minor semi-axis of the cross-section – b.
There are known spring designs with a variable cross-section along the longitudinal axis. Both the size and shape of the cross-section can be variable. Springs with varying geometric section parameters have better performance compared to springs with a constant cross-section. Despite their advantages, springs with a variable cross-section are not widely available. The main reason for holding back the introduction of spring designs with variable cross-section has been the lack of methods for determining the dynamic characteristics, including the frequency of natural oscillations. Therefore, an urgent task to determine the frequency of natural oscillations of manometric tubular springs was set and solved.

To improve the manometric springs performance a number of spring designs were proposed with a variable cross-section along the central axis and wall thickness. According to the patent [1], in order to increase the natural oscillation frequency a manometric spring was proposed with a variable section wall thickness decreasing from the base to its free end, with the cross-section dimensions reducing from the base to the free end as well (figure 2).
To increase the sensitivity, relative stiffness and strength of manometric springs operating in a force mode and under vibrations, a spring is proposed with a variable length section [2] gradually changing from elliptical at the free end to 8-shaped at the fixed one. A similar design with longitudinal ridges (corrugations) is proposed [3].

A manometric spring design is also proposed characterized by enhanced durability and reliable performance [4]. It features an asymmetrical cross-section with respect to its major axis which is formed smoothly varying along the longitudinal spring axis, при этом the outer contour of the cross section is formed smoothly varying from flat oval at the free end to 8-shaped at the fixed one, and the inner contour of the cross section is formed smoothly varying from elliptical at the free end of the spring to flat oval at the fixed end. To simplify the variable section designs and extend the functionality a spring is proposed [5] that has a section variable along the length and consisting of several interconnected tubes each of which has a constant wall thickness and semi-axes cross-section ratio. The wall thickness and size of the tube section semi-axes increase from the free end of the spring to the fixed one.

A composite spring design is also known made with a section variable along the length, characterized in that it consists of interconnected tubes each of which has a constant sectional shape along the length and wall thickness. As they approach the fixed end of the spring, the tubes sections that constitute the spring change from elliptical to flat oval and then from flat oval to 8-shaped, with the wall thickness of the tubes also increasing from the free end to the fixed one.

3. Developing a mathematical model

Let us present a dynamic model of the Bourdon tube in the form of a thin-walled curved bar oscillating in the central axis curvature plane. Figure 3 shows a curved rod and an infinitesimal element cut out of it. Travel of the center of gravity of the cross-section with the coordinate \( \varphi \) can be decomposed into radial \( w \) and circumferential \( u \) components.

![Figure 3. A manometric tubular spring bend.](image)

According to the principle of D'Alembert let us build the equations of motion of a tube element \( Rd\varphi \), projecting forces applied to the element (considering the inertia force) on the normal and tangent.
\[
\begin{align*}
- \frac{N}{R} - \frac{1}{R} \frac{\partial^2}{\partial \varphi^2} \left( \frac{M}{R} \right) &= m_i(\varphi) \frac{\partial^2 w}{\partial t^2}, \\
\frac{1}{R} \frac{\partial N}{\partial \varphi} - \frac{1}{R^2} \frac{\partial M}{\partial \varphi} &= m_i(\varphi) \frac{\partial^2 u}{\partial t^2},
\end{align*}
\]
where \( N \) – axial force; \( N = D \varepsilon \),
\( D \) – tensile sectional stiffness; \( D = \frac{ES(\varphi)}{1 - \mu^2} \);
\( \varepsilon \) – elongation of the longitudinal axis of \( \varepsilon = \frac{\partial u}{R \partial \varphi} + \frac{w}{R} = \frac{1}{R} \left( \frac{\partial u}{\partial \varphi} + w \right); \)
\( S(\varphi) \) – cross-sectional area of the tube, depending on the angular coordinate \( \varphi \) of this section;
\( \frac{\partial M}{\partial \varphi} = Q \) – shear force;
\( m_i(\varphi) \) – mass per unit length of the tube (cross-sectional mass with the coordinate \( \varphi \));
\( M \) – bending moment in the cross section of the tube \( M_\varphi = -B \chi = -\frac{B}{R^2} \left( \frac{\partial u}{\partial \varphi} - \frac{\partial^2 w}{\partial \varphi^2} \right) \);
\( B \) – tube bending stiffness; for the cross section of the manometric tube \( B = \frac{E \cdot J(\varphi) \cdot K_k(\varphi)}{(1 - \mu^2)} \);
\( E \) – elasticity modulus of the pipe material;
\( J(\varphi) \) – moment of inertia depending on the section angular coordinate \( \varphi \);
\( K_k(\varphi) \) – Karman constant, depending on the section angular coordinate \( \varphi \);
\( \mu \) – Poisson's ratio.

Boundary conditions. In the section of hard spring fixing (\( \varphi = 0 \)) tangent, normal movements and rotation angle of the tube cross-section are equal to zero, and at the opposite end (\( \varphi = \gamma \)) bending moment, shear, tensile forces become zero, which leads to the following boundary conditions:
\[
\begin{align*}
\text{At } \varphi = 0 : & \quad u(0) = 0; \quad w(0) = 0; \quad \frac{\partial w}{\partial \varphi}(0) = 0, \\
\text{At } \varphi = \gamma : & \quad M(\gamma) = 0; \quad Q(\gamma) = 0; \quad N(\gamma) = 0.
\end{align*}
\]
To solve the system of equations a Bubnov-Galerkin method was used.

4. A computer program for calculating the natural oscillation frequencies of manometric tubular springs
In order to make calculations automatic a program “PKRMTP” was compiled in the MATLAB environment that includes a subroutine to determine the Karman constant and allows one to calculate several natural oscillation frequencies of the manometric tubular spring. With this program, we calculated the natural oscillation frequencies. The calculation results allow us to estimate the influence of the geometric parameters of tubular springs on their natural oscillation frequencies.

5. Conclusions
1. It has been established that increasing the wall thickness and ratios of the lateral section curvature radius to the minor semi-axis increases the natural oscillations frequency, and increasing the central axis curvature radius, central angle and ratios of the minor semi-axis to the major one decreases the frequency.
2. It has been established that manometric springs with a variable section changing from 8-shaped to flat oval, as well as springs composed of conical tubes, have natural oscillation frequencies 20-40% higher than those of springs with a constant section, therefore, it is reasonable to use them in vibration-resistant pressure gauges.

References

[1] Patent 325522 USSR, Cl. G 01 L 7/04. Manometric tubular spring / SA Sibeykin, VA Zhukov, KG Shein. – №1237570/18 – 10; Filed 26.04.68; Published 07.01.72; Bulletin №3.

[2] Patent 2093805 RF, MPK 7, G 01 L 7/04. Manometric tubular spring / SP Pirogov, TG Ponomareva. – № 96105858/28; Filed 26.03.96; Published 20.10.97.

[3] Patent 2216001 RF, MPK 7, G 01 L 7/04. Manometric tubular spring (variants) / SP Pirogov, NP Mityagin, NN Ustinov, SS Samakalev. – № 2001122276/28; Filed 08.08.2001; Published 10.11.2003; Bulletin №31.

[4] Patent 2215274 RF, MPK 7, G 01 L 7/04. Manometric tubular spring / SS Samakalev, SP Pirogov. – № 2002127092/28; Filed 10.10.2002; Published 27.10.2003; Bulletin №30.

[5] Patent 2111465 RF, MPK 7, G 01 L 7/04. Manometric tubular spring / SP Pirogov, TG Ponomareva, AA Volzhakov. – № 96121675/28; Filed 10.11.96; Published 20.05.98.

[6] Zemenkov Yu D, Shalay V V and Zemenkova M Yu 2015 Expert systems of multivariable predictive control of oil and gas facilities reliability Procedia Engineering 113 312-5.

[7] Kurushina V and Zemenkov Y 2014 Innovative cyclical development of the Russian pipeline system WIT Transactions on Ecology and the Environment 190-2 881-8

[8] Zemenkov Yu D, Shalay V V and Zemenkova M Yu 2015 Immediate analyses and calculation of saturated steam pressure of gas condensates for transportation conditions Procedia Engineering 113 254-8.