Luttinger Liquid Physics in the Superconductor Vortex Core

Ashvin Vishwanath\textsuperscript{1,2} and T. Senthil\textsuperscript{2}

\textsuperscript{1}Physics Department, Princeton University, NJ 08544
\textsuperscript{2}Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106-4030

(Received May 13, 2018)

We study several aspects of the structure of vortices in conventional s-wave Type \textit{II} superconductors. It is well-known that there are low energy quasiparticles bound to the core of a vortex. We show that under certain conditions, these quasiparticles form a degenerate Fermi gas with a finite density of states at the Fermi energy. In three dimensional superconductors, the result is a one dimensional Fermi gas of quasiparticles bound to the core of a vortex line. As is usual in one dimensional systems, interactions between the quasiparticles lead to Luttinger liquid behaviour.

This may be probed through STM tunneling into the vortex core. We further suggest that a novel Peierls-type instability in the shape of the vortex line may develop due to interactions between the quasiparticle gas and fluctuations in the vortex line shape.

I. INTRODUCTION

The electronic properties of Type \textit{II} superconductors in the mixed phase are known to have several interesting features. In some pioneering work, Caroli et. al.\cite{Caroli} predicted theoretically the existence of low energy quasiparticle states bound to the core of a single isolated vortex in an s-wave superconductor. These low lying states are free to move parallel to the vortex, and so give rise to a series of one dimensional bands (minibands) as shown in Fig 1(a). The system is still gapped, though the energy gap to the lowest excitation (the minigap) is typically much smaller than the bulk gap. Subsequent theoretical work\cite{Vishwanath1, Vishwanath2} helped to put this prediction of low energy quasiparticle states bound to the vortex core on firm ground.

Striking experimental evidence for the existence of these bound states of the vortex was provided by the scanning tunneling microscopy (STM) studies of Hess et al.\cite{Hess}. These experiments were able to image the vortex quasiparticle states as a function of their energy, and agreed well with theory.

Most theoretical work on these low energy quasiparticle states in the vortex core has neglected the effect of the Zeeman coupling between the magnetic field and the spin of the quasiparticles. However as discussed in Ref.\cite{Caroli}, and as we argue below, the Zeeman coupling plays an important role in determining the low temperature properties of the system. The Zeeman coupling splits the quasiparticle energy levels, and the minigap is decreased since the energy of one spin species is lowered towards the Fermi energy. For sufficiently strong Zeeman splitting, the minibands of one spin species will be brought down below the Fermi energy, and a filled Fermi sea of spin polarised quasiparticles is formed. We show that the magnetic fields needed in typical materials to form this degenerate quasiparticle system are not large, and could be much smaller than $H_{c2}$.

We consider the effect of quasiparticle-quasiparticle interactions (ignored in the BCS mean field theory) on the low energy properties of the quasiparticles in the core of the vortex. The quasiparticles are bound in the direction perpendicular to the vortex line but are free to move along it, thus providing an interesting realization of a one dimensional system inside the superconductor. It is well-known in the theory of normal metals that interaction effects are dramatic in one dimension: the generic ground state of the interacting electron system is not a Fermi liquid but a different beast, the Luttinger liquid. We therefore focus attention primarily in the regime of magnetic fields well below $H_{c2}$ where the vortices may be treated in isolation. Are some of the striking properties of interacting 1D Fermi systems, such as Luttinger liquid physics, also present in the vortex quasiparticle system?

The answer is no, if the interactions and the Zeeman coupling are weak. The presence of the (min)gap implies that the T=0 state of this system is quite insensitive to weak interactions. However if the Zeeman energy is large enough to start filling the miniband, the system is gapless - interaction effects are then crucial, and we argue that the system is correctly described as a spin polarised Luttinger liquid. We then find that the interaction strength $g$ which controls the Luttinger liquid exponents is a function of the miniband filling, and in principle could even be negative. Luttinger liquids have several interesting properties\cite{Luttinger}: the fermion correlation functions are power law with anomalous exponents and the single particle density of states at the Fermi points vanishes as a power law. There have been a small number of experimental realizations of Luttinger liquid behaviour in systems such as 1D semiconductor wires\cite{Kohmoto}, fractional quantum hall edge states\cite{Kohmoto} and carbon nanotubes\cite{Kohmoto}. We show that a Luttinger liquid can be realised under appropriate conditions in the vortex core. Tunneling into a Luttinger liquid leads to a characteristic power law tunneling conductance, and we propose an experiment involving Scanning Tunneling Microscopy (STM) measurements on the vortex core as a probe of Luttinger liquid behaviour of the vortex quasiparticles.
The quasiparticles also interact with the collective modes of the vortex, which can have interesting consequences. In particular, if inter-vortex interactions (which become increasingly important as we increase the field towards \( H_c \)) are sufficiently strong, we find that a vortex analog of the Peierls effect could occur. The vortex line spontaneously undergoes a periodic modulation of its profile to lower the energy by opening up a gap in the quasiparticle spectrum.

The rest of this paper is organised as follows. In Section II, we discuss in a little more detail the core states of an isolated vortex, and the formation of the degenerate quasiparticle gas in the presence of the Zeeman splitting. In Section III we show that interactions between the quasiparticles can lead to them forming a Luttinger liquid, with a varying Luttinger liquid exponent and consider how this state may be observed in STM experiments. In Section IV, we couple the quasiparticles to the vortex collective modes and discuss in particular the possibility of a vortex Peierls transition when inter-vortex interactions are present. Section V discusses the validity of some of our approximations and highlights directions where more work needs to be done.

II. VORTEX QUASIPARTICLE STATES IN THE NON-INTERACTING LIMIT

In this section we review the spectrum of the vortex core states and discuss the effect of the Zeeman coupling on it. We begin with the BCS model Hamiltonian in the presence of an arbitrary magnetic field \( \vec{B} = \vec{\nabla} \times \vec{A} \) and pair potential \( \Delta(r) \) (we will include the Zeeman term later):

\[
H_{BCS} = \int d^4x \left[ \epsilon(x)c_\alpha^\dagger(x)c_\alpha(x) + \epsilon_\alpha(x)\Delta(x)c_\alpha^\dagger(x)c_\gamma^\dagger(x)c_\gamma(x) \right]
\]

where \( \epsilon[p] \) is the kinetic energy measured from the chemical potential. For simplicity, we will assume a quadratic energy dispersion \( \epsilon(p) = \frac{p^2}{2m} - E_F \). It is convenient to rewrite the Hamiltonian using the change of variables

\[
\begin{align*}
d_1(x) &= c_\gamma(x) \\
d_2(x) &= c_\alpha(x)
\end{align*}
\]

When written in these variables, there are no anomalous terms in the Hamiltonian:

\[
H_{BCS} = \int d^4x \left( \left( \begin{array}{c} d_1^\dagger(x) \\ d_2^\dagger(x) \end{array} \right) \right) \left( \begin{array}{c} d_1(x) \\ d_2(x) \end{array} \right) H \left( \begin{array}{c} d_1^\dagger(x) \\ d_2^\dagger(x) \end{array} \right)
\]

\[
H = \left[ \begin{array}{cc} \epsilon(-i\nabla - eA(x)) & \Delta^\dagger(x) \\ \Delta(x) & -\epsilon[i\nabla - eA(x)] \end{array} \right]
\]

Physically, the number of \( d \)-particles corresponds to the \( z \)-component of the electron spin. The absence of anomalous terms in the \( d \) representation thus reflects conservation of the \( z \)-component of the electron spin in the BCS Hamiltonian of Eqn. [1].

The Hamiltonian is diagonalised by going over to new quasiparticle operators that are defined as:

\[
\begin{align*}
\gamma_1^\dagger &= \int \left( u_\alpha(x)d_1^\dagger(x) + v_\alpha(x)d_2^\dagger(x) \right)dx \\
\gamma_2^\dagger &= \int \left( v_\alpha^*(x)d_1^\dagger(x) - u_\alpha^*(x)d_2^\dagger(x) \right)dx
\end{align*}
\]

The functions \( u_\alpha(x) \) and \( v_\alpha(x) \) are found by solving the following eigenvalue equation (with \( E_\alpha \geq 0 \)):

\[
H \left( \begin{array}{c} u_\alpha(x) \\ v_\alpha(x) \end{array} \right) = E_\alpha \left( \begin{array}{c} u_\alpha(x) \\ v_\alpha(x) \end{array} \right)
\]

This is just the Bogoliubov deGennes (BdG) equation for the quasiparticle states. The BdG Hamiltonian can be recast in a compact matrix notation as:

\[
H = \epsilon[-i\nabla \tau^x - eA(x)]\tau^x + \Delta^\dagger(x)\tau^y + \Delta(x)\tau^- \tag{4}
\]

where \( \tau^i \) are the 2 \times 2 Pauli matrices. In terms of the quasiparticle operators \( \gamma \), the Hamiltonian is simply:

\[
H_{BCS} = \sum_\alpha \left( \gamma_1^\dagger(\alpha) \gamma_2^\dagger(\alpha) \right) \left( \begin{array}{cc} E_\alpha & 0 \\ 0 & -E_\alpha \end{array} \right) \left( \begin{array}{c} \gamma_1(\alpha) \\ \gamma_2(\alpha) \end{array} \right)
\]

The ground state has all of the negative energy states filled so \( \gamma_1^\dagger(1) |0 \rangle = 0 \); all positive energy states are unoccupied \( \gamma_1^\dagger(1) |0 \rangle = 0 \). Note that a spin \( \uparrow \) excitation is created by \( \gamma_1^\dagger \) while a spin \( \downarrow \) excitation is created by \( \gamma_2 \) acting on the ground state.

After these generalities, let us specialise to the quasiparticle states bound to the vortex line in a superconductor. Consider an isolated, straight vortex oriented along the \( z \)-axis, sitting in a clean, conventional Type II superconductor. This is realised if the applied magnetic field is just above \( H_c \). The vortex bound state energy levels are obtained by solving Eqn. [1] with the appropriate gap profile \( \Delta(r) = |\Delta(r)|e^{i\phi} \), where \( \phi \) is the azimuthal angle about the vortex. It is useful to perform the singular gauge transformation that makes \( \Delta \) real everywhere (the London gauge):

\[
\left( \begin{array}{c} u' \\ v' \end{array} \right) = \left( \begin{array}{cc} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{array} \right) \left( \begin{array}{c} u \\ v \end{array} \right)
\]

\[
H' = \left[ \frac{1}{2m}(-i\nabla \tau^x - eA - \frac{\phi}{2})^2 - E_F \right]\tau^x + |\Delta(r)|\tau^- \tag{6}
\]

The transformed quasiparticle wavefunctions \( \left( \begin{array}{c} u' \\ v' \end{array} \right) \) obey antiperiodic boundary conditions to keep \( \left( \begin{array}{c} u \\ v \end{array} \right) \) single valued.
FIG. 1. Energy level structure of the states bound to the core of a vortex (a) Ignoring the effect of the magnetic field. The negative energy states are occupied and shown with the thick line. Energy is measured in units of the minigap energy ($\approx E_{\gamma2}$) and (b) In the presence of a sufficiently strong magnetic field. The Zeeman splitting causes the $\mu = \frac{1}{2}$ miniband to start being occupied and a degenerate Fermi sea is formed.

We first review the results of Ref. [1] on the structure of an isolated vortex. In Ref. [1] the effect of the magnetic field on the low energy quasiparticles is not included. This is only a good approximation for an isolated vortex in an extreme Type II superconductor; then the flux enclosed in the spatial region where the low energy states are bound is a small fraction of the flux quantum, and hence the magnetic field has little effect on these states. Subsequently we will see how these results need to be modified on including the effect of the magnetic field. The low energy spectrum of the vortex core states then is:

$$E^0(\mu, k_z) = \mu \left( \frac{c\Delta}{k_F\xi} \right) \left[ 1 - \frac{k_z^2}{k_F^2} \right]^{-\frac{1}{2}}$$  \hspace{1cm} (9)

and is shown in Fig 1(a). The corresponding quasiparticle wavefunctions in cylindrical coordinates take the form

$$\begin{align*}
(u_{\mu k}(r, \phi, z)) &= e^{i\mu\phi} e^{ik_z z} f_{\mu k_z}(r) \\
v_{\mu k}(r, \phi, z) &= e^{i\mu\phi} e^{ik_z z} g_{\mu k_z}(r)
\end{align*}$$

(10)

where $\mu \in \{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots\}$ appears as an angular momentum in the wavefunctions and is a half integer due to the anti-periodic boundary conditions, $k_z$ is the momentum along the vortex line, $c$ is a constant of order one and $\xi$ is the zero temperature coherence length. We have denoted by...
The value of the gap very far away from the vortex. The \( k_z \) dependence of the energy comes from the motion along the vortex line. The energy scale of the minigap is \( \Delta^2 / E^2 \) and is much smaller than the bulk gap \( \Delta_\infty \). The radial wave functions \( f \) and \( g \) are mainly confined to within a radius \( \xi \) (the coherence length) about the vortex for the low energy states \( (E(\mu, k_z) \ll \Delta_\infty) \).

Now consider the effect of the magnetic field \( \vec{B} = B\hat{z} \). We assume though that the field is much less than \( H_{c2} \) so that the vortices are well separated. Two effect occur: (a) the Zeeman term splits the energy levels and (b) there is an orbital effect. We consider these in turn, starting with the orbital effect.

For the magnetic fields of interest, the inter-vortex separation is typically smaller than the penetration depth so that the field in the superconductor is fairly uniform and equal to the external field (= \( B\hat{z} \), and we take \( \vec{A}(\vec{x}) = \frac{B}{2}(-y, x, 0) \)). In the presence of the vector potential additional terms are generated in the BdG Hamiltonian which in the London gauge take the form:

\[
\delta H = \delta H_1 + \delta H_2
\]

\[
\delta H_1 = \frac{|e|}{2m} (A(x) \cdot p + p \cdot A(x))
\]

\[
\delta H_2 = \frac{\langle eA(x)^2 \rangle}{2m} \tau^z + \frac{|e|B}{2m} \tau^x
\]

\( \delta H_2 \) may be neglected since the first term is small compared to the minigap for fields well below \( H_{c2} \) and the second term only makes a small shift of the Fermi energy. So we are left with \( \delta H_1 \) which can be written as:

\[
\delta H_1 = \frac{|e|}{2m} B \vec{r} \times \vec{p} = \frac{|e|}{2m} B L \vec{z}
\]

This does not affect the eigenstates [3] but shifts their energy by

\[
E(\mu, k_z, B) = E^0(\mu, k_z) + \frac{|e|B}{2m}\mu
\]

The orbital effect therefore increases the magnitude of energy of the quasiparticle states.

The Zeeman term takes the form

\[
H_Z = \frac{-g\mu_B B}{2} \int \left( c_i^\dagger(x) c_i(x) - c_i^\dagger(x) c_i(x) \right) d^d x
\]

\[
= \frac{-g\mu_B B}{2} \int \left( d_i^\dagger(x) d_i(x) + d_i^\dagger(x) d_i(x) \right) d^d x
\]

\[
= \frac{-g\mu_B B}{2} \sum \alpha (\gamma_1^\dagger \gamma_1 \alpha + \gamma_2^\dagger \gamma_2 \alpha)
\]

(In the second line, we have dropped an irrelevant constant and \( \mu_B = \frac{e\hbar}{2m} \) is the Bohr magneton). The Zeeman term thus behaves as a ‘chemical potential’ for the d-particles [1]. Increasing the magnetic field raises this chemical potential. Beyond a certain field it enters the first miniband and a degenerate Fermi sea of quasiparticles forms as shown in Fig 1(b). The condition for this is simply:

\[
E(\mu = \frac{1}{2}, k_z = 0, B) \leq \frac{g\mu_B B}{2}
\]

or equivalently

\[
E^0(\mu = \frac{1}{2}, k_z = 0) \leq \frac{(g - 1)}{2} \mu_B B
\]

Thus a degenerate gas of spin polarised fermions (in this case all spin \( \uparrow \)) can form due to the Zeeman coupling if the magnetic field exceeds \( H_{c2} = \frac{2}{\mu_B} E^0(\mu = \frac{1}{2}, k_z = 0) \) (assuming \( g = 2 \)). Note that it is only the first miniband that can be brought below the chemical potential. For \( \mu > \frac{1}{4} \), the orbital effect wins over the Zeeman splitting, and the levels do not cross the chemical potential.

The magnetic fields needed to begin filling the miniband states are very reasonable, since the minigap energy is typically small. To illustrate this point we consider the case of NbSe\(_2\) for which numerical calculations of the vortex quasiparticle spectrum are available [4]. NbSe\(_2\) has a superconducting transition temperature of \( T_c = 7.2K \), and numerical calculations find the minigap to be 0.4 Kelvin. The magnetic field required to close the minigap is \( [H_{c2}] \) is 1.3 Tesla (we have assumed \( g = 2 \)) while \( H_{c2} \) is larger at 3.2T for this material. Better candidate materials will have smaller ratios of \( \frac{H_{c2}}{H_{c2}} \) than NbSe\(_2\).

In passing we note that the Zeeman splitting gives rise to quasiparticles even at zero temperature, and in doing so ‘melts’ some of the condensate at the centre of the vortex. The new profile \( \Delta(r) \) can be calculated from the self consistency equation. This is analogous to the Kramer-Pesch effect [3], where temperature does the job of melting the condensate and expanding the vortex. We shall not take into account the effects of the changing gap profile on the quasiparticle states, as they are expected to be small.

### III. INTERACTION EFFECTS AND LUTTINGER LIQUID FORMATION

Interactions between fermions in clean 1D systems have a dramatic effect resulting in Luttinger liquid behaviour. Here we shall consider the effect of interactions between the quasiparticles in the vortex core that form the 1D Fermi gas. A few comments on the nature of these interactions [12] is in order. The underlying microscopic electronic system is characterized, at low energies, by three qualitatively different kinds of interactions: (a) the spin singlet density-density interaction in the particle-hole channel (b) the triplet spin density-spin density interaction in the particle-hole channel and (c) the interaction in the particle-particle channel. Conventional BCS superconductors arise when the interactions...
in the particle-particle channel are attractive. Indeed the electron gas is unstable toward the formation of Cooper pairs in the presence of such an attractive interaction. The BCS theory simply treats this attractive interaction in the particle-particle channel in a mean field approximation. One still however has to deal with the interactions in the particle-hole channel. (These are simply ignored in the so-called reduced BCS Hamiltonian). Inclusion of these leads to interactions between the quasiparticles in the superconductor. Note that the long-ranged Coulomb repulsion between the electrons in the singlet channel is screened out by the condensate in the superconducting phase. Thus the residual interactions between the quasiparticles are short-ranged. A further source of interactions between the quasiparticles comes from the inclusion of fluctuations of the mean field order parameter (again usually ignored in BCS theory).

Our general conclusions are insensitive to the precise form of this short-ranged interaction between the quasiparticles. We will therefore, for illustrative purposes, consider a particular model interaction. We add to the BCS Hamiltonian Eqn[4] the “interaction” term

$$H_{\text{int}} = \int \sum_{\alpha \sigma} V(|x - x'|) c_{\sigma}^\dagger(x') c_{\sigma}^\dagger(x') c_{\sigma}(x) d^3x d^3x'$$  \hspace{1cm} (21)

where $V(|x - x'|)$ is assumed to be short-ranged. To study the effect on the vortex core states, it is convenient to re-express this in terms of the $\gamma$ operators introduced in Section 4

$$H_{\text{int}} = \sum_{\alpha \beta \alpha' \beta'} \int [-V(|x - x'|)] : (u_\alpha^\dagger(x) \gamma_{1,\alpha} + v_\alpha^\dagger(r) \gamma_{2,\alpha})$$

$$\times (u_\beta(x) \gamma_{1,\beta} + v_\beta^\dagger(x') \gamma_{2,\beta})$$

$$\times (u_{\alpha'}^\dagger(x') \gamma_{1,\alpha'} + v_{\alpha'}^\dagger(r') \gamma_{2,\alpha'})$$

$$\times (v_{\beta'}(x') \gamma_{1,\beta'} - u_{\beta'}(x') \gamma_{2,\beta'}) : d^3x d^3x'$$

$$+ \ldots + \ldots$$

where the normal ordering is with respect to the superconductor vacuum, and we have written out only the first of the four terms present from different spin combinations.

Let us study the effect of interactions when we may treat the vortices as isolated and the lowest miniband is occupied with the two Fermi points at $\pm k_f$ (so $E_{\pm \phi, 0} > E_{\pm \phi, 0}, \dots$). Notice that the Fermi Sea consists of spin up quasiparticles only. The excitation spectrum for this case contains a low energy part of ‘particle-hole’ excitations in the vicinity of the Fermi points, and a high energy sector that involves either $\gamma_1$ quasiparticles hopping into higher minigap states, or the destruction of $\gamma_2$ quasiparticles from deep below the chemical potential. The latter two are gapped, with the gap being of order the minigap energy ($\approx \frac{\Delta}{kT}$). Since we are interested in the effects of weak interactions on the $T = 0$ state of the system, we confine ourselves to the low energy sector of the problem. Formally, we can achieve this by defining a projection operator $P$, that only retains states close to the Fermi points, i.e. $\{ \gamma_1 \text{ quasiparticles}, \mu = \frac{1}{2}, k_x \in (k_f - \epsilon, k_f + \epsilon) \text{ or } k_x \in (-k_f - \epsilon, -k_f + \epsilon); \epsilon \ll k_f \}$. Now, we can project the interaction term Eqn[22] down into this subspace, and it is easy to see that the only non-trivial term that remains is:

$$H_{\text{int}}^p = U \sum_{|q| < \epsilon} \rho_L(q) \rho_R(-q)$$  \hspace{1cm} (23)

where the left (and similarly the right) density operators $\rho_L$ and $\rho_R$ are constructed from the fermion operators in the usual way, $\rho_L$ for example is given by:

$$\rho_L = \sum_{|k + k_f| < \epsilon} \Gamma_k^\dagger \Gamma_k$$

and $\Gamma_k = \gamma_{k, \phi, 0}$. In general, doing a quantitative calculation of $U$ for a real material is not an easy task because a complete knowledge of the interaction potential and quasiparticle wavefunctions is needed. We simply note that $U$ will be a function of the filling of the miniband ($U = U(k_f)$), and in principle can even be negative (attractive interaction between the quasiparticles) [8].

For the particular interaction of Eqn. 21 we can express $U$ in terms of the interaction potential and the wavefunction of the states near the Fermi points:

$$U = \int_{x,y,x',y'} V(x - x', y - y', z) Q_{k_f}(x, y) Q_{k_f}(x', y')[1 - e^{2ik_f z}]$$  \hspace{1cm} (24)

Here the quasiparticle ‘charge’ $Q_{k_f}$ is defined by

$$Q_{k_f}(x, y) = |u_{\phi, k_f}(x, y)|^2 - |v_{\phi, k_f}(x, y)|^2$$  \hspace{1cm} (25)

We thus have a system of one-dimensional fermions with two Fermi points interacting through $H_{\text{int}}^p$. The fermions are spin-polarized (and hence, effectively, spinless). It is well-known that such a system is correctly described not as a Fermi liquid, but rather as a Luttinger liquid.

Luttinger liquids are characterised by power law correlations. The exponents are controlled by the dimensionless quantity $g = \frac{U}{2\pi v_f}$ where $v_f$ is the Fermi velocity of the one dimensional system. For example, $\langle \Gamma'(z) \Gamma'(z') \rangle \approx (1/|z - z'|^\nu)$ where $z$ is the position along the vortex line and the exponent $\nu = (1/\sqrt{1 - g^2})$. Relatedly, tunneling into the bulk of a Luttinger liquid is characterised by nonlinear I-V characteristics, in fact at low voltages the tunneling current-voltage relation is of the form $I \propto V^{\nu}$. This suggests that STM could be used to experimentally verify Luttinger liquid formation in the
vortex cores under the conditions described above. At sufficiently low temperatures when the Luttinger liquid behaviour is dominant, we expect the tunneling conductance close to the centre of the vortex to be given by:

$$\sigma(V, x) \propto |u_{\pm k_F}(x)|^2 |V|^{\nu_e - 1}$$

and $\nu_e$ is the exponent for tunneling into the edge of a Luttinger liquid $\nu_e = (1+g)\frac{1}{2} (1-g)^{-\frac{1}{2}}$ [12]. The temperatures at which such Luttinger liquid behaviour will be observed is necessarily small compared to the degeneracy temperature, which for the half filled miniband of NbSe$_2$ is $\sim 0.4$ Kelvin [13]. These effects may be observed at higher temperatures by using a superconductor with a larger minigap, but this would also require larger magnetic fields to close the minigap via the Zeeman splitting.

IV. COUPLING TO VORTEX COLLECTIVE MODES AND VORTEX PEIERLS EFFECT

So far we have only considered straight and rigid vortex lines, however in reality the vortex is a soft object and can undergo shape fluctuations. These collective modes of the vortex interact with the quasiparticles bound to the vortex line. As we show below the interacting quasiparticle - collective mode system is similar to the problem of interacting electrons and phonons in one dimension, which is known to undergo a Peierls transition (for commensurate filling) in which the lattice spontaneously distorts and a gap opens at the Fermi points. The gain in electronic energy by opening of the gap offsets the elastic energy cost of the lattice distortion. The electron scattering from the vicinity of one Fermi point to the other ($2k_F$ scattering) involves vanishing energy denominators and is responsible for this instability.

We can ask if an analogous effect occurs in the degenerate 1D quasiparticle system in the vortex core interacting with the shape fluctuations of the vortex - which can give rise to $2k_F$ scatterings [16 17]. Note that for an isolated vortex line, which is a one dimensional object it is not possible to spontaneously break the continuous symmetry of translations along the vortex line. However in the vortex lattice if inter vortex interactions are sufficiently strong, a vortex analog of the Peierls effect could occur at finite temperature. This issue was raised earlier by Bouchaud [15] who modelled the vortex core simply as a wire of normal electrons. However we believe this is not an adequate model to discuss this phenomena. A degenerate Fermi system is formed in the vortex only in the presence of the Zeeman splitting.

The collective modes of the vortex may be classified according to their angular momentum. Scalar modes scatter quasiparticles within the same $\mu$ miniband, while the lowest order coupling of quasiparticles to collective modes of higher angular momenta (say $l$) will scatter quasiparticles from one miniband ($\mu$) to a different one ($\mu \pm l$) by conservation of angular momentum. Since the low energy excitations all lie in a single $\mu = \frac{1}{2}$ miniband we only consider coupling of quasiparticles to the scalar collective modes of the vortex.

Scalar vortex modes modulate the radial profile of the vortex as one moves along the vortex line. For instance, a periodic modulation of the vortex radius (defined say as when the order parameter reaches half its asymptotic value) is an example of a scalar deformation of the vortex. If the scalar normal modes of the vortex are labelled by $n$, let $\sigma_n(q_z)$ create the $n^{th}$ radial mode with momentum $q_z$ along the vortex line. The coupling of the vortex modes to the quasiparticles (of the partially filled miniband) can be derived from the BdG equation, or simply from symmetry is found to be:

$$H^{scalar}_{int} = \sum_{q,k,n} g_{int}(k,q,n) \sigma_n(q) \Gamma_{k+q} \Gamma_k + h.c.$$  \hspace{1cm} (27)

The Hamiltonian for the scalar modes is:

$$H_{modes} = \sum_{n,q} E(n,q) \sigma_n^\dagger(q) \sigma_n(q)$$ \hspace{1cm} (28)

Thus, scalar modes of the vortex of momentum $2k_F$ will scatter quasiparticles from one Fermi point to the other. As we have noted previously, for an isolated vortex this does not lead to a gapped phase and the Luttinger liquid behaviour will persist. However if inter vortex interactions are sufficiently strong, then a finite temperature transition to a vortex Peierls phase can occur. Which of the modes $n_0$ has the highest transition temperature is a function of $g_{int}$, energy and inter vortex coupling for that mode. In the vortex Peierls phase, the quasiparticle spectrum is gapped, and we have a collective mode condensate i.e. $\langle \sigma_{n_0}(2k_F) \rangle \neq 0$ which implies a periodic modulation of the vortex profile.

We do not attempt to predict the precise conditions for the experimental realization of this phase. Increasing the vortex density, and hence increasing intervortex interactions can raise the transition temperature, as long as the core quasiparticles do not hop between vortices and destroy the one-dimensional nature of the system. Within a mean field approximation, the Fermi gas always undergoes the Peierls transition. The instability is enhanced for repulsive Luttinger liquids [13]. Thermal transport measurements along the vortex line are expected to be sensitive to the vortex Peierls transition.

The Luttinger liquid behaviour of the vortex quasiparticles would also be destroyed if pairing of quasiparticles were to occur. This would induce an additional (eg. $p$-wave) pairing amplitude in the vortex core, and could also lead to breaking of the rotational symmetry of the vortex line. This interesting situation was considered by Makhlin and Volovik in [21]. Once again, the transition
temperature for this instability is nonzero only in the presence of suitable inter-vortex interactions.

V. DISCUSSION

We have shown that a one-dimensional degenerate gas of quasiparticles can form in the vortex core, as a result of the Zeeman coupling and can serve as a laboratory for one dimensional physics. We showed that quasiparticle interactions drive the formation of a Luttinger liquid inside the vortex core. The interaction strength $U$ of the Luttinger liquid was shown to be a function of the filling, and could even be negative. Thus we have a 1D interacting Fermi system, where novel features arise due to the fact that the fermions involved are superconducting quasiparticles and the ‘wire’ confining them is a vortex.

We now examine in more detail some of the approximations that have been made, and the prospects for verifying the results we predict in experiments.

One of the main approximations that we have made is to treat each vortex as effectively isolated, which gives rise to the one dimensional nature of the vortex quasiparticle system. As the magnetic field is increased towards $H_{c2}$, the vortices get closer to each other and the wavefunctions of the vortex core states start to overlap. This leads to quasiparticle hopping between vortices that will eventually destroy the one dimensional nature of the system. To keep the hopping small, we need that the separation between vortices at the magnetic fields of interest is much larger than the coherence length. We therefore require that the field $H_{c2}$ at which levels start to fill satisfies $H_{c2} \ll H_{c1}$. However, the temperature scale associated with the field $H_{c2}$ should not be too small since the physics of interest occurs only below that temperature, suggesting materials with a relatively high $T_c$. A possible family of candidates that meet these criteria are the borocarbides [22]. Another possibility is to work with a material that has $H_{c2} < H_{c1}$. Then even at magnetic fields just above $H_{c1}$, when the vortices are very well isolated, a degenerate quasiparticle liquid will be formed in the vortex cores. Elemental Niobium, that has a relatively large $H_{c1} = 0.14$T is possibly such a system.

Throughout this work we have assumed that the superconductor is perfectly clean, but in any real system disorder is always present. The stability of the Luttinger liquid to weak disorder depends on the value of the interaction strength $g$ [22]. It is well-known that there exists a critical $g_c < 0$ such that for $g > g_c$, any disorder kills the Luttinger liquid behaviour leading instead to a phase with localized quasiparticle excitations. Still, for sufficiently clean systems there is a range of temperatures for which the properties of the system are controlled by the LL fixed point, even though the ultimate zero temperature state may be localised. In that case, the scanning tunneling conductance that we predict for the LL should be obtained in this crossover regime.

On the other hand, if $g < g_c$, the Luttinger liquid is, in fact, stable to weak disorder [22]. Since $g$ for the vortex core Luttinger liquid could be negative, the following interesting effect can occur. By varying the external magnetic field, $g$ can be varied and the system can be tuned through a one dimensional delocalization transition from the phase with localized quasiparticles (at small negative $g$) to the Luttinger liquid (at large negative $g$).

The formation of the degenerate quasiparticle gas in the vortex core can be probed by measurements of the low temperature specific heat - a linear temperature dependence with a field dependent coefficient should obtain at the lowest temperatures. Once signatures of the formation of this quasiparticle gas are observed and some of its physical parameters (e.g. the minigap and dispersion with $k_z$) measured, it would be of great interest to look for possible Luttinger liquid behaviour in, for instance, STM tunneling into the vortex core.

We also considered coupling the vortex quasiparticles to the collective modes of the vortex. We find that a vortex analog of the Peierls transition with a non-zero transition temperature could arise if the inter-vortex interactions are strong enough.

VI. ACKNOWLEDGEMENTS

We would like to thank K. Damle, C. Dasgupta, D. Huse, N.P. Ong, A. Melikidze, S. Sondhi and especially D. Haldane for useful discussions. One of us (A.V.) acknowledges support from grant NSF DMR-9809483. T.S was supported by NSF Grants DMR-97-04005, DMR95-28578 and PHY94-07194.

[1] C. Caroli, P.G. de Gennes, J. Matricon, Phys. Lett. 9,307 (1964)
[2] J. Bardeen et al, Phys. Rev. 187, 556 (1969)
[3] L. Kramer and W. Pesch, Z.Physik 269,59 (1974)
[4] F. Gygi and M. Schluter, Phys. Rev. B43, 7609(1991)
[5] H. F. Hess et al, Phys. Rev. Lett. 64,2711 (1990)
[6] E. Brun Hansen, Physics Letters 27A, 576 (1968)
[7] S. Tomonaga, Progr. Theor. Phys. 5, 544-569 (1950) J.M. Luttinger, J. Math. Phys. 4, 1154-1162 (1963) D.C. Mattis and E.H. Lieb, J. Math. Phys. 6, 304-312 (1965) F.D. Haldane, J. Phys. C14, 2585-2591 (1981)
[8] Tarucha, S. et al, Solid State Comm. 94, 413 (1995)
[9] Milliken, F.P. et al Solid State Comm. 97, 309 (1996).
[10] Bockrath, M. et al, cond-mat/9812233
[11] Equivalently, we can define spin up and spin down quasiparticle states with positive energies and these are then Zeeman split in the usual manner.

[12] See, for instance, the discussion in Appendix A of S. Vishveshwara, T. Senthil, and M.P.A. Fisher, cond-mat/9906238.

[13] Calculations with toy models of the interaction show that the dimensionless ratio $g/v_f$ that controls the non-trivial Luttinger liquid exponents can be quite large i.e. $O(1)$ which means the L.L. exponents would differ considerably from the free fermion values.

[14] C. Kane and M.P.A. Fisher, Phys Rev Lett 68, 1220 (1992)

[15] For comparison, the experiments of [5] were done at 0.3 Kelvin, but at low magnetic fields.

[16] A distortion of the crystal lattice will not be energetically favourable for this system due to the low density of quasiparticles forming the 1D gas in the vortex core.

[17] At some special wavevectors, when $4k_f = Q_z$ (a reciprocal lattice vector), it is possible that Umklapp scattering drives a density wave formation, but this requires fine tuning of the filling (Zeeman splitting).

[18] J.P. Bouchaud, cond-mat/9707117.

[19] J. Voit and H.J. Schulz, Phys Rev B 36, 968 (1987)

[20] Y.G. Makhlin and G.E. Volovik, JETP Lett. 62, 737 (1995)

[21] C. Mazumdar et al, Solid State Commun. 87, 413 (1993); H. Takagi, M. Nohara, R.J. Cava, Physica B 237-238, 292 (1997)

[22] W. Apel, J.Phys. C 15 1973 (1982); T. Giamarchi and H. Schulz, Phys. Rev. B 37, 325 (1988)