On the possibility of a discontinuous quark-mass dependence of baryon octet and decuplet masses

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Abstract

We compute the quark-mass dependence of the baryon octet and decuplet masses within the \( \chi \)-MS scheme at the one-loop level. The results are confronted with recent lattice QCD simulations of the MILC collaboration. A fair reproduction of the quark-mass dependence as suggested by the lattice simulations is obtained up to pion masses of about 700 MeV. Based on the chiral one-loop results we predict that the dependence of the baryon octet and decuplet masses on the quark-masses is discontinuous. Typically the pion-mass dependence is smooth up to about 300 MeV only. This is a consequence of self consistency imposed on the partial summation, i.e. the masses used in the loop functions are identical to those obtained from the baryon self energies. The 'mysterious' quark-mass dependence of the \( \Xi \) mass predicted by the MILC collaboration is recovered in terms of a discontinuous chiral extrapolation.

1 Introduction

The present-day interpretation of QCD lattice simulations requires a profound understanding of the dependence of observable quantities on the light quark masses. A powerful tool to derive such dependencies is the chiral Lagrangian, an effective field theory based on the chiral properties of QCD. The application of strict chiral perturbation theory to the SU(3) flavor sector of QCD is plagued by poor convergence properties for processes involving baryons [12,13,14]. Thus it is important to establish partial summation schemes that enjoy improved convergence properties and that are better suited for chiral extrapolations of lattice simulations. It was suggested by Donoghue and Holstein [15] to improve the convergence properties by introducing a finite cutoff into the heavy-baryon effective field theory of Jenkins and Manohar [9]. Another scheme was suggested in [10,11], the construction of which was guided by keeping manifestly covariance, analyticity and causality. The computation of
the baryon octet and decuplet masses in this scheme was worked out recently indicating convincing convergence properties of the chiral loop expansion [12].

For studies of chiral extrapolations of the nucleon mass within the chiral SU(2) Lagrangian we refer to the work by Procura, Hemmert and Weise [16]. The latter study applied the partial summation scheme of Becher and Leutwyler [17]. It was based on simulations of the CP-PACS collaboration that used dynamical u- and d-quarks only [18]. The application of the scheme by Becher and Leutwyler [17] to the chiral SU(3) Lagrangian is questioned by poor convergence properties as was demonstrated by Ellis and Torikoshi at hand of the baryon octet masses [4]. The recent work of Pascalutsa and Vanderhagen [19] suggested to take seriously the partial summation implied by the extended on-mass shell scheme (EOMS) introduced by Gegelia and Japaridze [20]. Their computation addressed so far the chiral extrapolation of the nucleon and isobar mass only.

It is the purpose of the present work to confront the results of [12] with recent lattice QCD simulation of the MILC collaboration [13,14], that use dynamical u-,d- and s-quarks in the staggered approximation. We do not aim at a quantitative extrapolation of the lattice simulation, which would require a continuum limit extrapolation and a quantitative control of systematic errors. Rather, we want to use the available partial lattice results to make predictions for future QCD lattice simulations and possibly obtain rough constraints on some chiral parameters. An analysis of the recent MILC results, similar in spirit, was undertaken by Frink and Meißner [15] based on the cutoff scheme of Donoghue and Holstein.

Adjusting the values of the $Q^2$ counter terms to the physical masses we predict a discontinuous dependence of the baryon masses on the pion mass. This is a consequence of self consistency imposed on the partial summation approach, i.e. the masses used in the loop function are identical to those obtained from the baryon self energy. The latter is a crucial requirement since the loop functions depend sensitively on the precise values of the baryon masses. Our results may explain the mysterious quark-mass dependence observed by the MILC collaboration for the $\Xi$ mass [13,14].

2 Chiral interaction terms

We collect the terms of the chiral Lagrangian that determine the leading orders of baryon octet and decuplet self energies [21,22]. Up to chiral order $Q^2$ the baryon propagators follow from
\[ L = \text{tr} \left( \bar{B} \left[ i \bar{\varphi} - \overset{\circ}{M}_{[8]} \right] B \right) \]
\[ - \text{tr} \left( \Delta_{\mu} \cdot \left[ \left[ i \bar{\varphi} - \overset{\circ}{M}_{[10]} \right] g^{\mu\nu} - i (\gamma^\mu \partial^\nu + \gamma^\nu \partial^\mu) + \gamma^\mu \left[ i \bar{\varphi} + \overset{\circ}{M}_{[10]} \right] \gamma^\nu \right] \Delta_{\nu} \right) \]
\[ - 2 d_0 \text{tr} \left( \Delta_{\mu} \cdot \Delta_{\nu} \right) \text{tr} \left( \chi_0 \right) - 2 d_D \text{tr} \left( \left( \Delta_{\mu} \cdot \Delta_{\nu} \right) \chi_0 \right) \]
\[ + 2 b_0 \text{tr} \left( \bar{B} B \right) \text{tr} \left( \chi_0 \right) + 2 b_F \text{tr} \left( \bar{B} \left[ \chi_0, B \right] \right) + 2 b_D \text{tr} \left( \bar{B} \{ \chi_0, B \} \right), \]

\[ \chi_0 = \begin{pmatrix} m^2_\pi & 0 & 0 \\ 0 & m^2_\tau & 0 \\ 0 & 0 & 2m^2_K - m^2_\pi \end{pmatrix}. \quad (1) \]

We assume perfect isospin symmetry through out this work. The fields are decomposed into isospin multiplets

\[ \Phi = \tau \cdot \pi + \alpha^\dagger \cdot K + K^\dagger \cdot \alpha + \eta \lambda_8, \]
\[ \sqrt{2} B = \alpha^\dagger \cdot N + \lambda_8 \Lambda + \tau \cdot \Sigma + \Xi^\dagger i \sigma_2 \cdot \alpha, \quad (2) \]

with the Gell-Mann matrices, \( \lambda_i \), and the isospin doublet fields \( K = (K^+, K^0)^t \) and \( \Xi = (\Xi^0, \Xi^-)^t \). The isospin Pauli matrices \( \sigma = (\sigma_1, \sigma_2, \sigma_3) \) act exclusively in the space of isospin doublet fields \( (K, N, \Xi) \) and the matrix valued isospin doublet \( \alpha, \alpha^\dagger = \frac{1}{\sqrt{2}} (\lambda_4 + i \lambda_5, \lambda_6 + i \lambda_7), \quad \tau = (\lambda_1, \lambda_2, \lambda_3). \quad (3) \]

The evaluation of the baryon self energies to order \( Q^3 \) probes the meson-baryon vertices

\[ L = \frac{F}{2f} \text{tr} \left( \bar{B} \gamma_5 \gamma^\mu \left[ \partial_\mu \Phi, B \right] \right) + \frac{D}{2f} \text{Tr} \left( \bar{B} \gamma_5 \gamma^\mu \left\{ \partial_\mu \Phi, B \right\} \right) \]
\[ - \frac{C}{2f} \text{tr} \left( \Delta_{\mu} \cdot (\partial_\nu \Phi) \left[ g^{\mu\nu} - \frac{1}{2} Z \gamma^\mu \gamma^\nu \right] B + \text{h.c.} \right) \]
\[ - \frac{H}{2f} \text{tr} \left( \left[ \Delta_{\mu} \cdot \gamma_5 \gamma_\nu \Delta_{\mu} \right] (\partial^\nu \Phi) \right), \quad (4) \]

where we apply the notations of \[11\]. We use \( f = 92.4 \text{ MeV} \) in this work. The values of the coupling constants \( F, D, C \) and \( H \) may be correlated by a large-\( N_c \) operator analysis \[23,24,25\]. At leading order the coupling constants can be expressed in terms of \( F \) and \( D \) only. We employ the values for \( F \) and \( D \) as suggested in \[26,11\]. All together we use

\[ F = 0.45, \quad D = 0.80, \quad H = 9F - 3D, \quad C = 2D, \quad (5) \]
in this work. We take the parameter $Z = 0.72$ from a detailed coupled-channel study of meson-baryon scattering that was based on the chiral SU(3) Lagrangian \[11\]. The latter parameter is an observable quantity within the chiral SU(3) approach: it contributes at order $Q^2$ to the meson-baryon scattering amplitudes but cannot be absorbed into the available $Q^2$ counter terms \[11\].

3 Chiral loop expansion at the one-loop order

We briefly recall the results of \[12\]. A summation approach was defined by performing a chiral loop expansion rather than a strict chiral expansion: for a given truncation of the relativistic chiral Lagrangian we take the loop expansion that is defined in terms of the approximated Lagrangian seriously. The number of loops we would consider is correlated with the chiral order to which the Lagrangian is constructed. A renormalization based on the Passarino-Veltman reduction was devised that installs the correct minimal chiral power of a given loop function. The residual dependence on the renormalization scales is used to monitor the convergence properties of the expansion and therewith estimate the error encountered at a given truncation.

The loop contribution to the baryon octet and decuplet masses read:

\[
\begin{align*}
\Delta M_{B \in [8]}^{\text{loop}} &= \sum_{Q \in [8], R \in [8]} \left( \frac{G_{QR}^{(B)}}{2 f} \right)^2 \left\{ \frac{M_R^2 - M_B^2}{2 M_B} I_Q \right. \\
&\quad - \frac{(M_B + M_R)^2}{E_R + M_R} p_{QR}^2 \left( I_{QR} + \frac{I_Q}{M_R^2 - m_Q^2} \right) \left. \right\} \\
&\quad + \sum_{Q \in [8], R \in [10]} \left( \frac{G_{QR}^{(B)}}{2 f} \right)^2 \left\{ \frac{(M_R - M_B)(M_R + M_B)^3 + m_Q^4}{12 M_B M_R^2} \right. \\
&\quad - \frac{(Z (Z + 2) - 5) M_B^2 + 2 (2 Z (Z - 1) - 3) M_R M_B + 2 M_R^2}{12 M_B M_R^2} m_Q^2 \left. \right\} \bar{I}_Q \\
&\quad - \frac{2}{3} \frac{M_B^2}{M_R^2} (E_R + M_R) p_{QR}^2 \left( I_{QR} + \frac{I_Q}{M_R^2 - m_Q^2} \right) ,
\end{align*}
\]

and

\[
\begin{align*}
\Delta M_{B \in [10]}^{\text{loop}} &= \sum_{Q \in [8], R \in [8]} \left( \frac{G_{QR}^{(B)}}{2 f} \right)^2 \left\{ \frac{(M_R - M_B)(M_R + M_B)^3 + m_Q^4}{24 M_B^3} \right. \\
&\quad - \frac{3 M_B^2 + 2 M_R M_B + 2 M_R^2}{24 M_B^3} m_Q^2 \left. \right\} \bar{I}_Q \\
&\quad - \frac{3 M_B^2}{24 M_B^3} \left( E_R + M_R [\ldots] \right) ,
\end{align*}
\]
\[- \frac{1}{3} (E_R + M_R) p_{QR}^2 \left( \frac{\bar{I}_Q}{M_R^2 - m_Q^2} + \frac{\bar{I}_Q}{M_R^2} \right) \]

\[ + \sum_{Q \in [8], R \in [10]} \left( \frac{G_{QB}^{(B)}}{2 f} \right)^2 \left\{ \left( \frac{(M_B + M_R)^2 m_Q^4}{36 M_B^2 M_R^2} \right) \left( \frac{M_B^2 + M_R^2}{M_R^2} \right) \bar{I}_Q \right. \]

\[ + \frac{3 M_B^2 - 2 M_B^2 M_R + 3 M_B^2 M_R^2 - 2 M_B^2 m_Q^2}{36 M_B^2 M_R^2} \left( \frac{M_B^4 + M_R^4 + 12 M_B^2 M_R^2 - 2 M_B^2 M_R (M_B^2 + M_R^2)}{36 M_B^2 M_R^2} \right) \bar{I}_Q \]

\[- \frac{(M_B + M_R)^2}{9 M_B^2} \frac{2 E_R (E_R - M_R)}{E_R + M_R} \frac{1}{p_{QR}^2} \left( \frac{\bar{I}_Q (M_B^2) + \bar{I}_Q}{M_R^2 - m_Q^2} \right) \right\},

where

\[ \bar{I}_Q = \frac{m_Q^2}{(4 \pi)^2} \ln \left( \frac{m_Q^2}{\mu_{UV}^2} \right), \]

\[ \bar{I}_{QR} = \frac{1}{16 \pi^2} \left\{ \frac{2 \pi \mu_{IR}}{M_R} + \left( \frac{1}{2} \frac{m_Q^2 + M_R^2}{m_Q^2 - M_B^2} \right) \ln \left( \frac{m_Q^2}{M_R^2} \right) \right. \]

\[ + \frac{p_{QR}}{M_B} \left( \ln \left( 1 - \frac{M_B^2 - 2 p_{QR} M_B}{m_Q^2 + M_B^2} \right) \right) \left. - \ln \left( 1 - \frac{M_B^2 + 2 p_{QR} M_B}{m_Q^2 + M_B^2} \right) \right\}, \]

\[ p_{QR}^2 = \frac{M_B^2}{4} - \frac{M_R^2 + m_Q^2}{2} + \frac{(M_R^2 - m_Q^2)^2}{4 M_B^2}, \quad E_R = M_R^2 + p_{QR}^2. \quad (8) \]

The sums in (6, 7) extend over the intermediate Goldstone bosons (Q \in [8]) baryon octet (R \in [8]) and decuplet states (R \in [10]). The various coupling constants \(G_{QB}^{(B)}\) are determined by the parameters \(F, D, C, H\). They are listed in [12]. We emphasize that (6, 7) depend on the physical meson and baryon masses \(m_Q\) and \(M_R\). This defines a self consistent summation since the masses of the intermediate baryon states in (6, 7) should match the total masses. At order \(Q^3\) the latter are the sum of the tree-level contributions linear in the parameter \(b_0, b_D, b_F\) or \(d_0, d_D\) and the loop contribution (6, 7).

The mesonic tadpole \(\bar{I}_Q\) enjoys a logarithmic dependence on the ultraviolet renormalization scale \(\mu_{UV}\) and the one-loop master function \(\bar{I}_{QR}\) a linear dependence on the infrared renormalization scale \(\mu_{IR}\). By construction the results (6, 7) are necessarily consistent with all chiral Ward identities as discussed in [12]. As emphasized by the authors the parameters \(b_0, b_D, b_F\) and \(d_0, d_D\) are strongly dependent on the infrared scale \(\mu_{IR}\). Applying a further chiral expansion to (6, 7) it was demonstrated that the physical masses are scale independent as they should be. However, the convergence properties reflect the choice of the renormalization scales. Good convergence properties can only be expected for natural values thereof. For the infrared scale \(\mu_{IR} \sim Q\) a natural window \(350 \text{ MeV} < \mu_{IR} < 550 \text{ MeV}\) was suggested in [12]. Keeping
the partial summation as defined by (6, 7) a residual dependence on the renormalization scales remains. This is analogous to the residual cutoff dependence of the scheme of Donoghue and Holstein [6]. As long as such dependencies are small and decreasing as higher order terms are included they do not pose a problem, rather, they offer a convenient way to estimate the error encountered at a given truncation.

4 Quark-mass dependence of the baryon masses

We discuss the implications of (6, 7) for chiral extrapolations of lattice simulations of the baryon masses [13,14]. Since we are not aiming at a chiral extrapolation of the lattice simulations down to physical quark masses we adjust part of the parameters to empirical data directly. The goal of the present study is a qualitative understanding of the quark-mass dependence of the baryon masses.

For given values of the infrared and ultraviolet renormalization scales the parameters $b_D, b_F$ and $d_D$ are fitted to the mass differences of the octet states and decuplet states. The absolute mass scale of the octet and decuplet states can be reproduced by appropriate values of the bare baryon masses. This procedure leaves undetermined the two parameters $b_0$ and $d_0$. Good representations of the physical baryon masses can be obtained for a wide range of the latter parameters. The parameter $b_0$ may be used to dial a given pion-nucleon sigma term at physical pion masses. Similarly the unknown parameter $d_0$ may be determined as to reproduce a given pion-delta sigma term.

In this work $b_0$ and $d_0$ are adjusted as to reproduce the baryon octet and decuplet masses in the SU(3) limit at $m_\pi \simeq 690$ MeV. The MILC simulations suggest the values $M_{[8]} \simeq 1575$ MeV and $M_{[10]} \simeq 1710$ MeV. This procedure is biased to the extent that it assumes that the chiral one-loop results will be applicable at such high quarks masses. On the other hand as long as there are no continuum limit results of the MILC collaboration available this is an economical way to minimize the influence of lattice size effects. The latter are expected to be smaller at large quark masses. The procedure may be justified in retrospect if it turns out that the extrapolation recovers the behavior predicted by the lattice simulation.

The parameters used in this work are collected in Tab. 1 together with the implied masses of the baryon octet and decuplet states. A fair representation of the physical baryon masses is obtained. For simplicity the values quoted in Tab. 1 correspond to a computation where the intermediate baryon masses are put to their empirical values. Since the resulting masses are very close to the physical masses this is well justified. As discussed in detail in [12] the
The parameters are fitted so that Table 1 reproduces the baryon masses at physical pion masses as well as the SU(3) limit values $M_{[8]} \simeq 1575$ MeV and $M_{[10]} \simeq 1710$ MeV at $m_\pi \simeq 690$ MeV. We use $\mu_{UV} = 800$ MeV. The masses are decomposed into their chiral moments.

parameters $b_{0,D,F}$ and $d_{0,D}$ show a strong dependence on the infrared scale $\mu_{IR}$. In contrast the physical baryon masses suffer from a weak dependence only. For natural values of the infrared scale the chiral expansion appears well converging as indicated by the decomposition of the baryon masses into their

|          | $\mu_{IR} = 350$ MeV | $\mu_{IR} = 450$ MeV | $\mu_{IR} = 550$ MeV |
|----------|----------------------|----------------------|----------------------|
| $b_0$ [GeV$^{-1}$] | -0.89                | -0.63                | -0.38                |
| $b_D$ [GeV$^{-1}$]  | +0.29                | +0.19                | +0.10                |
| $b_F$ [GeV$^{-1}$]  | -0.34                | -0.25                | -0.15                |
| $d_0$ [GeV$^{-1}$]  | -0.22                | -0.15                | -0.08                |
| $d_D$ [GeV$^{-1}$]  | -0.35                | -0.30                | -0.24                |
| $M_N$ [MeV]        | 750 + 310 − 121      | 813 + 232 − 105      | 875 + 153 − 89       |
| $M_A$ [MeV]        | 750 + 536 − 150      | 813 + 398 − 79       | 875 + 260 − 9        |
| $M_\Sigma$ [MeV]   | 750 + 882 − 425      | 813 + 630 − 239      | 875 + 379 − 54       |
| $M_\Xi$ [MeV]      | 750 + 934 − 361      | 813 + 680 − 171      | 875 + 427 + 19       |
| $M_\Delta$ [MeV]   | 1082 + 241 − 91      | 1108 + 164 − 39      | 1133 + 87 + 12       |
| $M_\Omega$ [MeV]   | 1082 + 347 − 49      | 1108 + 253 + 17      | 1133 + 160 + 83      |
| $M_\Sigma$ [MeV]   | 1082 + 453 − 4       | 1108 + 343 + 78      | 1133 + 233 + 160     |
| $M_\Omega$ [MeV]   | 1082 + 558 + 34      | 1108 + 432 + 134     | 1133 + 305 + 235     |
| $\sigma_{\pi N}$ [MeV] | 52.4              | 53.9                  | 55.7                 |
| $\sigma_{K^- p}$ [MeV] | 384.0             | 380.1                | 386.3                |
| $\sigma_{K^- n}$ [MeV] | 359.4             | 354.8                | 361.5                |

Table 1
Fig. 1. The pion mass dependence of baryon octet and decuplet masses predicted by the chiral loop expansion taking the parameters of Tab. 1. The lines represent the masses for the infrared scale put at $\mu_{IR} = 450$ MeV. The solid squares are the simulation points of the MILC collaboration.

We emphasize that the values of the pion-nucleon sigma term shown in Tab. 1 are an immediate consequence of the MILC simulation of the SU(3) symmetric point defined by $m_\pi \simeq 690$ MeV as discussed above. No further results of the MILC simulation are used. Taking the residual scale dependence of the pion-nucleon sigma term as a naive error estimate we obtain $\sigma_{\pi N} = 54 \pm 2$ MeV. The latter value is somewhat larger than the canonical value of Gasser, Leutwyler and Sainio $\sigma_{\pi N} \simeq 45$ MeV [27]. Our value is, however, quite compatible with a recent analyses of Frink and Meissner who suggest $\sigma_{\pi N} \simeq 52$ MeV [15]. Also Pascalutsa and Vanderhagen [19] arrive at a somewhat larger value $\sigma_{\pi N} \simeq 57$ MeV based on a chiral SU(2) analysis of the MILC simulation points.

We turn to the pion-mass dependence of the baryon octet and decuplet masses. Our results are collected in Fig. 1 based on the parameters of Tab. 1. We show results for the particular choice $\mu_{IR} = 450$ MeV. It should be emphasized that the baryon masses are a solution of a set of coupled and non-linear equations in the present scheme. This is a consequence of self consistency imposed on the
partial summation approach. The latter is a crucial requirement since the loop functions depend sensitively on the precise values of the baryon masses. As a consequence of the non-linearity for a given parameter set there is no guarantee for a unique solution to exist, nor that solutions found are continuous in the quark masses. Indeed as illustrated by Fig. 1 the pion-mass dependence predicted by the chiral loop expansion is quite non-trivial exhibiting various discontinuities. Upon inspecting the energy dependence of the baryon self energies this is readily understood: the baryon self energies, as implied by (6, 7), allow in certain cases for multiple solutions of the mass equation,

\[ \hat{\rho} = \hat{\rho} + \Sigma_B(\hat{\rho}) , \quad (9) \]
even though the self consistency requirement singles out a particular solution at \( \hat{\rho} = \rho_B \). Such a scenario appears inconsistent at first: the meson baryon loop functions have to be evaluated with all states provided by (9). In contrast the computation of the meson-baryon loop functions (6, 7) takes the existence of uniquely defined baryon states for granted. In addition it is assumed that the mass poles of the baryon propagators have residuum one. Thus our computation considers implicitly additional counter terms of the form:

\[ \Sigma^\text{wave-function}_B(\hat{\rho}) = \zeta_B (\hat{\rho} - \rho_B) , \quad (10) \]

where \( \rho_B \) denotes the baryon masses that solve the self consistent system. If the term (10) is added to the self energies it does not affect the solutions of the self consistent system. However, it does affect the energy dependence of the final baryon self energy. By adjusting the parameters \( \zeta_B \), it is possible to arrive at uniquely defined baryon states.

Within the scheme described above it is intuitive that for a given set of parameters multiple solutions of the self consistent system may exist. A priori it is not evident which of the possible solutions one should select. Our strategy is to continuously evolve the solution established at physical pion masses to larger pion masses. Similarly we evolve down to smaller pion masses the solution that reproduces the lattice simulations at the SU(3) symmetric point defined by \( m_\pi \simeq 690 \text{ MeV} \). Both solutions reach end points beyond which we do not find any continuation. Mathematically the endpoints are characterized by the divergence of the sigma term. In the region of intermediate pion masses we, however, find another solution, suggesting a discontinuous dependence of the baryon masses on the quark masses of QCD. One may expect that the physical solutions are those of lowest mass always. The three branches discussed above are shown in Fig. 1 and confronted with the simulation points of the MILC collaboration [13,14].

It is striking to see that we reproduce the 'mysterious' pion-mass dependence
of the Ξ mass, i.e. the quite flat behavior which does not seem to smoothly approach the physical mass. Given the present uncertainties from finite lattice spacing, the staggered approximation and the theoretical uncertainties implied by higher order contributions, we would argue that we arrive at a fair representation of the lattice simulation points for all baryons with some reservation concerning the nucleon. We note that the parameter set used before in [12] leads to results qualitative similar to those shown in Fig. 1 for the baryon octet masses. While the description of the nucleon is improved showing no discontinuity at large pion masses, the sizeable jump of the Ξ mass at small pion masses remains. However, the MILC simulation points of the isobar mass disfavor the latter choice. An overestimate of the lattice simulations of the isobar mass at larger quark masses would be the consequence. We checked that reasonable variations of the parameter set does not change the results qualitatively. We could not find parameters that lead to a smooth chiral extrapolation.

Incorporating the many $Q^4$ counter terms offered by the chiral Lagrangian it is reasonable to expect that the latter will further improve the picture. However, as long as there is no detailed analysis available that performs the continuum limit and estimates the uncertainty from the staggered fermion approximation there is not much point considering the $Q^4$ counter terms.

5 Summary

We evaluated the baryon octet and decuplet self energies at the one-loop level applying the relativistic chiral Lagrangian. Adjusting the $Q^2$ counter terms an excellent representation of the physical baryon octet and decuplet masses is obtained. The size of the pion-nucleon sigma term was estimated by including in the fit the baryon octet and decuplet masses at the SU(3) symmetric point defined by $m_\pi \simeq 690$ MeV. For the latter the MILC collaboration suggests the values $M_{[8]} \simeq 1575$ MeV and $M_{[10]} \simeq 1710$ MeV. As a result we predict $\sigma_{\pi N} \simeq 54$ MeV. The kaon-nucleon sigma terms are $\sigma_{K^- p} \simeq 380$ MeV and $\sigma_{K^- n} \simeq 355$ MeV.

Given this scenario the pion-mass dependence of the baryon octet and decuplet masses were evaluated. The latter are a solution of a set of coupled and non-linear algebraic equations. This is a direct consequence of self consistency imposed on the partial summation, i.e. the masses used in the loop functions are identical to those obtained from the baryon self energies. As a striking consequence we predict a discontinuous dependence of the baryon masses on the pion mass. Typically the baryon masses jump at pion masses as low as 300 MeV. Most spectacular is the behavior of the Ξ mass. At small pion masses it decreases with increasing pion masses. At a critical pion mass
of about 300-400 MeV it jumps up to a value amazingly close to the prediction of the MILC collaboration. For all baryon masses for which the MILC collaboration published simulation points our results are reasonably close to the lattice estimates, given the present uncertainties from finite lattice spacing, the staggered approximation and the theoretical uncertainties implied by higher order contributions to the baryon self energies.

It is interesting to speculate on the physics behind such an unexpected quark-mass dependence of the baryon masses. A discontinuous behavior may signal a new type of phase transition - or perhaps only some so-far unknown instability of QCD for certain parameter choices. One may argue that in the intermediate quark-mass region a ghost state appears causing an instability. At present we can not exclude the possibility that our results merely indicate that the chiral extrapolation stops making sense at quite small quark masses. On the other hand further chiral correction terms may connect the various branches in a smooth manner providing a quite non-liner quark mass dependence of the baryon masses.

The answer to these questions is outside the scope of the present work. However, we do not see any argument that this can not be and therefore take our results as a quest for further detailed studies.

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