The semi-leptonic baryonic $b \rightarrow s$ decay, $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$, has been studied and new angular observables and asymmetries have been proposed which can test the presence of new physics beyond the standard model.

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I. INTRODUCTION

Rare decays of the b-quark offer a unique possibility to study the weak interactions operating at the fundamental level governing the decays in conjunction with the strong forces responsible for keeping the constituents bound in various colourless hadronic states. The large mass of the b-quark compared to the typical QCD scale ensures that the perturbative hard part can be factorized from the long distance hadronic dynamics. It is in fact the long distance hadronic dynamics which is at the heart of the problem of obtaining accurate and reliable results. After factorizing the short and long distance pieces, the hadronic matrix elements (to be defined below in detail for the relevant case) are expressed in terms of the form factors which carry the non-trivial $q^2$ dependence, where $q$ generically denotes the momentum transfer for the process in question (see [1] for a review). Being of non-perturbative origin, these form factors need to be calculated via methods like lattice methods or QCD sum rules. On the perturbative side, one can hope to compute the short distance pieces (called Wilson coefficients) to higher order and ensure better accuracy and stability of the results.

Semi-leptonic decays mediated by the quark level transition $b \rightarrow s \ell^+ \ell^-$ offer cleaner probes compared to non-leptonic exclusive hadronic decays. In the latter case, theoretical calculations are more difficult in general and they are also marred with issues related to the QCD effects, both perturbative and non-perturbative, in a bigger way. Semi-leptonic decays on the other hand are somewhat easier at the theoretical level as the leptonic sub-system factorizes as far as the QCD effects between the final state sub-systems go. Further, since LHCb hints at deviations from the standard model (SM) predictions in observables related to $B \rightarrow K(\ast)\mu^+\mu^-$ channels (see [2] for anomalies in $K^*$ channel and [3] for hints of lepton universality violation in $K$ channel), which proceed at the quark level by the same $b \rightarrow s$ semi-leptonic decay, it is of utmost importance to study any other such semi-leptonic decay modes to clarify the situation and pin point the source of these deviations. Since the hadronic effects bring along large uncertainties, the above said hints can not be conclusively taken as evidence for new physics, which is part of the short distance structure. However, if similar pattern emerges for decays with different hadronic particles but governed by the same $b \rightarrow s \ell^+ \ell^-$ quark level transition, then that would amount to unambiguous signal for physics beyond SM. The baryonic decay $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ satisfies all these requirements and therefore it is useful to study it in detail. This decay has been studied theoretically in the past [4] but the emphasis has been somewhat different from that in this paper. This decay mode, like $B \rightarrow K^*\ell^+\ell^-$ has many angular observables to offer as probes. This fact was utilized to some extent in [5]. Here we take it further and also construct some new angular observables which are theoretically clean and do not depend sensitively on the hadronic form factors. On the experimental side, this decay was observed at the Tevatron [6]. Recently, LHCb has measured the branching fraction along with some angular coefficients [7], [8]. The errors are still quite large but one hopes to have better results in near future.

II. EFFECTIVE HAMILTONIAN AND THE DECAY $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$

The framework to study such rare decays is that of effective Hamiltonian which is obtained after integrating out the heavy degrees of freedom. The rare decay $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ is governed by effective Hamiltonian

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}^* V_{ts} \sum_i C_i(\mu) O_i + \text{h.c.}$$

(1)
where contribution of the term $\propto \frac{V_{ub}V_{cb}^*}{V_{tb}V_{tb}^*}$ is neglected. $O_i$ are the effective local operators and $C_i(\mu)$ are called Wilson coefficients evaluated at scale $\mu$. The factorization scale $\mu$ distinguishes between short distance physics (above scale $\mu$) and long distance physics (below scale $\mu$). Wilson coefficients encode information about heavy degrees of freedom which have been integrated out while matrix elements of local operators $O_i$ dictate the low energy dynamics (for a review see [1]).

The operators contributing significantly to the process $b \rightarrow s \ell^+ \ell^-$ in SM are

$$O_7 = \frac{e}{16\pi^2} m_b (s_\alpha \sigma_{\mu\nu} R b_\alpha) F^{\mu\nu},$$
$$O_9 = \frac{e^2}{16\pi^2} (s_\alpha \gamma^\mu L b_\alpha) (\bar{l} i \gamma_\mu l),$$
$$O_{10} = \frac{e^2}{16\pi^2} (s_\alpha \gamma^\mu L b_\alpha) (\bar{l} i \gamma_\mu \gamma_5 l),$$

where, $\alpha, \beta$ are the color indices, $L, R = \frac{1}{2} (1 \mp \gamma_5)$ represent chiral projections, $T^{a}$ are the SU(3) color charges and $m_b$ is the $b$-quark mass. All the information about short distance physics and possible new physics effects is contained in the Wilson coefficients which are computed in perturbation theory as a series in $\alpha_s$ and evaluated at the scale $\mu$ using the renormalization group equations. Beyond SM, there could be new operators with flipped helicity structure or different tensorial structure like scalar-pseudoscalar operators.

At first sight the decay $\Lambda_b \rightarrow M^+ M^-$ may seem not to be too useful owing to larger uncertainties in the transition form factors involved, when compared to the mesonic counterpart $B \rightarrow K^+ K^-$. However, this decay offers a larger number of observables. For example, in contrast to $K^+ \rightarrow K \pi$ decay in the mesonic counterpart which is parity conserving, $\Lambda \rightarrow N \pi$ is a parity violating decay and hence brings along the possibility of measuring forward-backward asymmetry in the hadronic system as well. This decay has been studied theoretically, but the emphasis in most of those studies was mainly on the lepton forward-backward asymmetry and/or lepton polarization asymmetry. Since the decay was observed at Tevatron, there has been some activity, both on the form factors [9] as well as exploiting the angular observables [5]. In the present work, we extend the analysis of [5] and also propose new observables and asymmetries which are theoretically clean and can be used with the limited data expected in near future.

The four body differential decay $\Lambda_b(p) \rightarrow \Lambda(k)[\rightarrow N(k_1)\pi(k_2)]\ell^+ \ell^- (q_1 \cdot q_2)$ can be conveniently written in terms of the variables: invariant mass squared of the lepton system $q^2 = (p - k)^2$, helicity angles $\theta_\Lambda$ and $\theta_\ell$ of the hadronic and leptonic sub-systems respectively, and the azimuthal angle $\phi$ between the hadronic and leptonic planes. Taking into account the polarizations of $\Lambda_b$ and $\Lambda$, there are a host of form factors that enter the calculations (see [5] for details).

The four body differential decay rate can be written as

$$\frac{d^4\Gamma}{dq^2 \sin \theta_\ell \sin \theta_\Lambda d\phi} = \frac{3}{8\pi} K(q^2, \cos \theta_\ell, \cos \theta_\Lambda, \phi)$$

where

$$K(q^2, \cos \theta_\ell, \cos \theta_\Lambda, \phi) = K_{1ss} \sin^2 \theta_\ell + K_{1cc} \cos^2 \theta_\ell + K_{1c} \cos \theta_\ell$$
$$+ (K_{2ss} \sin^2 \theta_\ell + K_{2cc} \cos^2 \theta_\ell + K_{2c} \cos \theta_\ell) \cos \theta_\Lambda$$
$$+ (K_{3c} \sin \theta_\ell \cos \theta_\Lambda + K_{3s} \sin \theta_\Lambda) \sin \theta_\ell \cos \phi$$
$$+ (K_{4c} \sin \theta_\ell \cos \theta_\Lambda + K_{4s} \sin \theta_\Lambda) \sin \theta_\ell \sin \phi$$

The angular coefficients $K_i$ depend only on the dilepton invariant mass, $q^2$, and carry the hadronic information. They are in turn expressed in terms of the transversity amplitudes. These transversity amplitudes are written as combinations of Wilson coefficients and baryonic form factors. A typical form factor is denoted as $H(s_\Lambda, s_\Lambda)$, where we have suppressed the indices $V, T, A$ signifying the type of operator sandwiched between the external hadronic states but have explicitly shown the two spin projection vectors which take values $\pm 1/2$.

In SM, the transversity amplitudes read

$$A_{1,1}^{L(R)} = \sqrt{2} N \left( C_{9,10}^{L(R)} H_{1}^{V}(-1/2, 1/2) - \frac{2m_b C_7}{q^2} H_{1}^{T}(-1/2, 1/2) \right)$$
$$A_{1,1}^{L(R)} = -\sqrt{2} N \left( C_{9,10}^{L(R)} H_{1}^{A}(-1/2, 1/2) + \frac{2m_b C_7}{q^2} H_{1}^{T5}(-1/2, 1/2) \right)$$
$$A_{1,0}^{L(R)} = \sqrt{2} N \left( C_{9,10}^{L(R)} H_{0}^{V}(1/2, 1/2) - \frac{2m_b C_7}{q^2} H_{0}^{T}(1/2, 1/2) \right)$$
$$A_{1,0}^{L(R)} = -\sqrt{2} N \left( C_{9,10}^{L(R)} H_{0}^{A}(1/2, 1/2) + \frac{2m_b C_7}{q^2} H_{0}^{T5}(1/2, 1/2) \right)$$

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FIG. 1: The schematic diagram of the angular distribution of \( \Lambda_b \to \Lambda(\to N\pi)\ell^+\ell^- \) decay with the description of the angles \( \theta_\ell, \theta_\Lambda \) and \( \phi \).

where \( C_{9,10}^{(L/R)} = (C_9 \mp C_{10}) \) and \( N = G_F V_{tb} V_{ts}^* \alpha_c \sqrt{\frac{q^2}{3m_{\Lambda_b}^2} \lambda} \) is the normalization factor.

In terms of these transversity amplitudes, the angular coefficients appearing in the fully differential decay rate read

\[
K_{1ss} = \frac{1}{4} \left[ |A_{\perp1}^R|^2 + |A_{\parallel1}^R|^2 + 2|A_{\perp0}^R|^2 + 2|A_{\parallel0}^R|^2 + (R \leftrightarrow L) \right]
\]

\[
K_{1cc} = \frac{1}{2} \left[ |A_{\perp1}^R|^2 - |A_{\parallel1}^R|^2 + (R \leftrightarrow L) \right] - \text{Re} \left\{ A_{\perp1}^R A_{\parallel1}^R - (R \leftrightarrow L) \right\}
\]

\[
K_{2ss} = \frac{\alpha}{2} \text{Re} \left\{ A_{\perp1}^R A_{\perp1}^{*R} + 2A_{\perp0}^R A_{\parallel0}^{*R} + (R \leftrightarrow L) \right\}
\]

\[
K_{2cc} = \alpha \text{Re} \left\{ A_{\perp1}^R A_{\parallel1}^{*R} + (R \leftrightarrow L) \right\}
\]

\[
K_{2c} = -\frac{\alpha}{2} \left[ |A_{\perp1}^R|^2 - |A_{\parallel1}^R|^2 - (R \leftrightarrow L) \right]
\]

\[
K_{3sc} = \frac{\alpha}{\sqrt{2}} \text{Im} \left\{ A_{\perp1}^R A_{\parallel0}^{*R} - A_{\parallel1}^R A_{\perp0}^{*R} + (R \leftrightarrow L) \right\}
\]

\[
K_{3s} = \frac{\alpha}{\sqrt{2}} \text{Im} \left\{ A_{\perp1}^R A_{\parallel0}^{*R} - A_{\parallel1}^R A_{\perp0}^{*R} - (R \leftrightarrow L) \right\}
\]

\[
K_{4sc} = \frac{\alpha}{\sqrt{2}} \text{Re} \left\{ A_{\perp1}^R A_{\parallel0}^{*R} - A_{\parallel1}^R A_{\perp0}^{*R} + (R \leftrightarrow L) \right\}
\]

\[
K_{4s} = \frac{\alpha}{\sqrt{2}} \text{Re} \left\{ A_{\perp1}^R A_{\parallel0}^{*R} - A_{\parallel1}^R A_{\perp0}^{*R} - (R \leftrightarrow L) \right\}
\]

where the parameter \( \alpha \) is the parity violating parameter in the \( \Lambda \to N\pi \) decay.

The task then is to experimentally determine these angular coefficients. In principle, once there is sufficient data, a full angular fit would end up determining these coefficients (up to discrete ambiguities). One could proceed by studying angular asymmetries allowing for the extraction of specific angular coefficients and/or some combinations of those. In [5], the authors considered the following observables which provide a handle on a select few angular coefficients:

(i) Decay rate as a function of \( q^2 \)

\[
\frac{d\Gamma}{dq^2} = 2K_{1ss} + K_{1cc} \tag{7}
\]

(ii) Transverse (and therefore longitudinal) polarization fraction

\[
F_L = 1 - F_T = \frac{2K_{1ss} - K_{1cc}}{2K_{1ss} + K_{1cc}} \tag{8}
\]
(iii) Forward-backward asymmetries in the leptonic, hadronic and mixed sub-systems:

\[ A_F^\ell = \frac{3}{2} \frac{K_{1c}}{2K_{1ss} + K_{1cc}} \]

\[ A_A^B = \frac{1}{2} \frac{K_{2c}}{2K_{1ss} + K_{1cc}} \]

\[ A_F^A = \frac{3}{4} \frac{K_{2c}}{2K_{1ss} + K_{1cc}} \]  \hspace{1cm} \text{(9)}

Analogous to the lepton forward-backward asymmetry in \( B \to K^* e^+ e^- \), \( A_F^\ell \) and \( A_F^A \) have a zero crossing, which essentially depends on the short distance parameters only (in the approximation when the form factor dependence more or less cancels) and its value is same as in the the mesonic case, scaled by the \( \Lambda_b \) mass instead of the B-meson mass.

### III. MORE ASYMMETRIES AND NEW OBSERVABLES

We extend the previous work by constructing asymmetries such that all the angular coefficients can be extracted. To this end, we construct the following observables:

\[ Y_2 = \frac{\int_0^{2\pi} d\phi \left[ \int_0^1 \frac{d}{2\pi} \frac{d \cos \theta_A \left[ \int_{-1/2}^{1/2} - \int_{-1/2}^{1/2} + \int_{1/2}^{1/2} \right] d \cos \theta \pi K(q^2, \theta, \theta_A, \phi) \right]}{\int_{-1/2}^{1/2} d \cos \theta \pi K(q^2, \theta, \theta_A, \phi)} \]  \hspace{1cm} \text{(10)}

\[ Y_{3s} = \frac{\left[ \int_0^{2\pi} d\phi \int_{-1}^{1} d \cos \theta_A \int_{-1}^{1} d \cos \theta \pi K(q^2, \theta, \theta_A, \phi) \right]}{\int_{-1}^{1} d \cos \theta \pi K(q^2, \theta, \theta_A, \phi)} \]  \hspace{1cm} \text{(11)}

\[ Y_{3sc} = \frac{\left[ \int_0^{2\pi} d\phi \int_{-1}^{1} d \cos \theta_A \int_{-1}^{1} d \cos \theta \pi K(q^2, \theta, \theta_A, \phi) \right]}{\int_{-1}^{1} d \cos \theta \pi K(q^2, \theta, \theta_A, \phi)} \]  \hspace{1cm} \text{(12)}

\[ Y_4s = \frac{\left[ \int_0^{2\pi} d\phi \int_{-1}^{1} d \cos \theta_A \int_{-1}^{1} d \cos \theta \pi K(q^2, \theta, \theta_A, \phi) \right]}{\int_{-1}^{1} d \cos \theta \pi K(q^2, \theta, \theta_A, \phi)} \]  \hspace{1cm} \text{(13)}

and

\[ Y_{4sc} = \frac{\left[ \int_0^{2\pi} d\phi \int_{-1}^{1} d \cos \theta_A \int_{-1}^{1} d \cos \theta \pi K(q^2, \theta, \theta_A, \phi) \right]}{\int_{-1}^{1} d \cos \theta \pi K(q^2, \theta, \theta_A, \phi)} \]  \hspace{1cm} \text{(14)}

Clearly, Eq.\(^{(10)}\)-Eq.\(^{(14)}\) along with the other equations above determine all the angular coefficients. Although in principle true, in practice any such determination will be severely hampered by the poorly known transition form
factors. In the baryonic case, the form factors are rather poorly known when one compares the situation with
the mesonic counterparts. In the latter, there has been lot of progress in having reliable set of form factors. But even
there, hadronic uncertainties prevent from making any sound claim of new physics when encountering deviations from
SM.

The kinematic region can be divided into the large and small \( q^2 \) or equivalently the low and large recoil regions. In
each of the regions, one can make suitable approximations which allow a smaller set of form factors to be employed,
and there are certain relations that emerge between various form factors. A typical matrix element one is interested in
is of the form: \( \langle \Lambda(k, s_\Lambda) | s \Gamma b | \Lambda_b(p, s_{\Lambda_b}) \rangle \), where \( s_{\Lambda_b} \) are the spin vectors associated with the baryons. In full generality,
there are a large number of form factors that would contribute to the physical decay rate. If, however, one makes
use of the heavy quark symmetry (working systematically in heavy quark effective theory (HQET)), the number of
independent form factors reduces to just two. Further, in the large recoil limit, there is only one independent form
factor. There exist several estimates of the form factors [10]. However, only limited information is available on form
factors computed directly in QCD using lattice techniques [9]. Employing HQET, the two relevant form factors appear
in the hadronic matrix elements as:

\[
\langle \Lambda(k, s_\Lambda) | s \Gamma b | \Lambda_b(p, s_{\Lambda_b}) \rangle = \bar{u}(k, s_\Lambda) [F_1(k, v) + \rho F_2(k, v)] \Gamma U(v, s_{\Lambda_b})
\]  

(15)

where \( v \) is the velocity of \( \Lambda_b \) and the two form factors depend only on the invariant \( k \cdot v \), the energy of \( \Lambda \) in the rest
frame of \( \Lambda_b \). The spinors satisfy the relations

\[
\sum_{s=1,2} \bar{u}(p, s) u(p, s) = m_\Lambda + \not p, \quad \sum_{s=1,2} \bar{U}(v, s) U(v, s) = 1 + \not p
\]  

(16)

It turns out that the two linear combinations \( F_1 = F_1 \pm F_2 \) are more useful and one therefore prefers to work with
them. At present, the lattice calculations provide a reliable estimate of the form factors \( F_1 \) (or \( F_1,2 \)) only in the
region \( q^2 \geq 13 \) GeV². Below this \( q^2 \) value, only model dependent extrapolations are to be made and relied on. But
it is still reassuring to observe that the lattice results extrapolated over the whole \( q^2 \) range tend to give a reasonable
fit to the available experimental data. From the results of the lattice calculations [9], one finds that over the whole
kinematic range, the ratio of the two form factors \( F_2 \) and \( F_1 \) takes the value

\[
-\frac{F_2}{F_1} \sim 0.21
\]  

(17)

with a gentle variation as the baryon energy is varied: the ratio lies in the interval \([0.19, 0.23]\). Further, using the
relations between \( F_1 \) and \( F_1,2 \), one finds that \( F_+ \sim 0.8F_1 \) and \( F_- \sim 1.2F_1 \). One therefore finds that in the heavy
quark limit (strictly valid only when \( m_b \to \infty \)), there is essentially one form factor, especially in the large \( q^2 \) region.
Incidentally, the recent LHCb measurements of the branching ratio and the simplest angular asymmetries are also in
the large \( q^2 \) region.

Although at the present level of accuracy, only one form factor seems to be enough (since due to the lattice
determination of the ratio \( F_2/F_1 \), which does not show much variation, essentially one is dealing with only one form
factor), it is clear that the other form factor will be needed to validate SM itself. To this end it is worthwhile and
important to construct observables which are as free of the hadronic inputs as possible. With this spirit we propose
the following observables, all of which have a zero crossing point in the large \( q^2 \) region. This will allow to have
a meaningful comparison with the experimental data as lattice results can be trusted in this kinematic region and
accurate numerical predictions can be obtained.

\[
T_1 = \frac{A_{R+}^{R+} A_{R+}^{R+} - A_{R-0}^{R+} A_{R-0}^{R+} - (R \to L)}{A_{R+}^{R+} A_{R+}^{R+} + A_{R-0}^{R+} A_{R-0}^{R+} + (R \to L)}
\]  

(18)

\[
T_2 = \frac{A_{R+}^{R+} A_{R+}^{R+} + A_{R+}^{R-0} A_{R+}^{R-0} - (A_{R+}^{R+} A_{R-0}^{R-0} - A_{R+}^{R-0} A_{R-0}^{R-0})}{A_{R+}^{R+} A_{R+}^{R+} + A_{R-0}^{R+} A_{R-0}^{R+} + (A_{R+}^{R+} A_{R-0}^{R-0} + A_{R+}^{R-0} A_{R-0}^{R-0})}
\]  

(19)

and

\[
T_3 = \frac{|A_{R+}^{R+}|^2 - |A_{R-0}^{R+}|^2 + (R \to L)}{|A_{R+}^{R+}|^2 + |A_{R-0}^{R+}|^2 + (R \to L)}
\]  

(20)
The zero crossing points for these observables, particularly $T_1, T_2$ are completely free of the form factors. They read (neglecting the small imaginary part in the Wilson coefficient $C_9$, i.e., assuming it to be real for illustration):

$$s_0(T_1) = \frac{M_{\Lambda_b}(C_9 M_{\Lambda_b} F_+ - 2C_7 m_\ell F_-)}{2C_9 F_+ F_-} = \frac{M_{\Lambda_b}(C_9 M_{\Lambda_b} - 2C_7 m_\ell)}{2C_9}$$  \hspace{1cm} (21)

$$= \frac{M_{\Lambda_b}^2}{2} - \frac{M_{\Lambda_b} m_\ell C_7}{C_9} \approx \frac{M_{\Lambda_b}^2}{2} + \frac{s_0}{2} \sim 17.52 \text{ GeV}^2$$

where in the penultimate step we used the approximate relation of the leptonic forward-backward zero crossing. However, it should be immediately clear that even without making use of this relation, $s_0(T_1)$ is a genuinely short distance quantity and therefore has a precise value within SM that can be unambiguously compared with the experimental determination. Next consider the zero crossing point of $T_2$:

$$s_0(T_2) = -\frac{M_{\Lambda_b}^2 C_{10}^2 + 8M_{\Lambda_b} m_\ell C_9 C_7 + (M_{\Lambda_b} C_9 + 2C_7 m_\ell)^2}{4C_9 C_{10}} \sim 17.63 \text{ GeV}^2$$  \hspace{1cm} (22)

This is again a pure short distance quantity and like $s_0(T_1)$ turns out to be very clean. The zero crossing point of the third observable works out to be:

$$s_0(T_3) = \frac{1}{4(C_{10}^2 + C_9^2)} \left[ M_{\Lambda_b}^2 (C_{10}^2 + C_9^2) - 4M_{\Lambda_b} m_\ell C_9 C_7 + 4m_\ell^2 C_7^2 ight]$$

$$+ \sqrt{(M_{\Lambda_b}^2 C_{10}^2 + (M_{\Lambda_b} C_9 - 2C_7 m_\ell)^2) - 64C_7^2 M_{\Lambda_b}^2 m_\ell^2 (C_{10}^2 + C_9^2)}$$  \hspace{1cm} (23)

$$\sim 17.24 \text{ GeV}^2$$

In the last observable, the replacement $\perp \rightarrow \parallel$ generates another observable which has the same zero crossing point. One could simply combine these two into linear combinations and study the profiles. It is quite evident that all the three zero crossings lie well in the large $q^2$ region where at present there is more control theoretically. Recent LHCb results on the angular analysis of $B \rightarrow K^+ \ell^+ \ell^-$ have shown deviations from SM expectations, especially for the observable $P_5^\ell$. Many possible solutions have been suggested, among which the minimal solution that gives a reasonably good fit is the solution where the SM operator basis is employed and the only deviation is in $C_9$: $\delta C_9 \sim -1.5$, while there are practically no deviations in the other two Wilson coefficients. Assuming this scenario, it is clear that the above observables, in particular the zero crossings can, very effectively and in a robust manner, test this hypothesis. In fact, extension to an extended operator basis is straightforward. We thus immediately see the immense potential of these asymmetries and zero crossing points which are very clean. The other advantage of these zero crossing points lies in the fact that they lie in the high $q^2$ region, in sharp contrast to the zero crossings of the observables in $B \rightarrow K^* \ell^+ \ell^-$. This additional feature will also help in understanding possible $q^2$ dependence and differentiate between possibly overlooked hadronic contribution from genuine new physics contribution which by definition should be $q^2$ independent.

These zero crossing points, along with the zeroes of the leptonic and hadronic forward-backward asymmetries can be simultaneously used to not only test SM but also to infer more about the form factors. Without making any assumptions, these quantities are functions of various form factors (actually ratios of various form factors). Measurement of these quantities, along with the profiles of various observables will allow us to extract some of these ratios at specific points. This information can then be utilized to cross-check the consistency of the form factors that one has employed. At present, this may appear as a daunting task but with more data available and more observables measured precisely, a simultaneous fit will provide this valuable information. We hasten to add that there are several other angular observables that can be constructed like:

$$O_1 = \frac{|A_{11}^R|^2 + |A_{10}^R|^2 - (R \rightarrow L)}{|A_{11}^R|^2 + |A_{10}^R|^2 + (R \rightarrow L)}$$  \hspace{1cm} (24)

and

$$O_2 = \frac{|A_{11}^R|^2 + |A_{10}^R|^2 - (R \rightarrow L)}{|A_{11}^R|^2 + |A_{10}^R|^2 + (R \rightarrow L)}$$  \hspace{1cm} (25)

These are also interesting and measurable quantities and have zero crossing points, which lie in the low $q^2$ region. Since at present both the form factor availability and experimental results are in the high $q^2$ region, we do not pursue
them further. A detailed numerical study, taking into account all the asymmetries and observables discussed above will be presented elsewhere. One can systematically study the effects of other operators and the impact they have on various observables, in particular zero crossings.

All what has been discussed so far has been in purview of form factors obtained with heavy quark limit in mind. An immediate improvement would be to start with the HQET relations and try to include the first sub-leading terms. In the absence of a lattice calculation along these lines, such an endeavour would be phenomenological to some extent but would still be worthwhile.

IV. DISCUSSION AND CONCLUSIONS

The standard model of particle physics has stood various experimental tests and there has not been any evidence of new physics till now. However, the fact that experiments have unambiguously established that neutrinos have mass, that there is dark matter in the universe and the cosmological observations clearly showing that the energy budget of the universe is dominated by dark energy already point to the fact that there is definitely something beyond SM. Further, the observed baryon asymmetry of the universe cannot be explained within SM, which falls short by orders of magnitude. All these experimental evidences call for some extension of SM. Though not conclusive at the moment, there are several anomalies in the flavour sector. Most recent ones are related to $b \rightarrow s$ semi-leptonic decay modes. In this vain, it is important to study different modes and channels which are mediated by the same $b \rightarrow s$ fundamental interactions. Recent times have seen a lot of theoretical and experimental effort in exploiting $B \rightarrow K^{(*)}\ell^+\ell^-$ modes to their full potential. The corresponding baryonic mode $\Lambda_b \rightarrow \Lambda^+\ell^-\ell^-$ has started to be studied experimentally. Since the baryonic counterpart now involves a completely different set of hadronic inputs, this becomes a very useful playground to cross-check the anomalies seen in the mesonic channels. Only recently, a more systematic approach to fully exploit the host of angular observables this mode has to offer has been initiated. In the present work we have extended that effort and listed all the angular asymmetries that pin down the complete set of angular coefficients. We have also proposed several other angular observables which should be easy to access experimentally. At present, this theoretical effort is limited by our knowledge of the baryonic form factors. The observables suggested in the present work are not affected by this limitation and in fact are best suited to work in the region of validity of the available form factors. These new observables are theoretically clean and therefore probe the genuine short distance content of the underlying theory. Moreover, they have a zero crossing point in the large $q^2$ region, the region where reliable form factors are available from lattice calculations. Therefore, this baryonic decay has an immense potential to test SM precisely and even with limited amount of data available in near future, there may be a hope to have a good indication of any new physics, if it is really there at the TeV scale.

Before closing, we would like to mention some of the possible improvements and future directions. As is obvious, theoretical calculations are presently limited by the form factors. The available form factors from lattice have been obtained in the strict heavy quark limit. One may attempt to improve those by trying to include sub-leading terms, even if in somewhat approximate manner. The other direction is to include other operators beyond SM and study the proposed observables within the extended operator basis. This would shed some light on the (ir)relevance of some operators. When combined with similar studies on the mesonic counterparts, this could limit the beyond SM contributions significantly. In particular, a detailed numerical investigation of the baryonic mode with inputs and recent hints of possible new physics from $B \rightarrow K^{(*)}\ell^+\ell^-$ would be very useful.

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