The odd-even staggering in $^{122-124}$Xe and $^{124-128}$Ba nuclei

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Abstract. The $\gamma$-band energy staggering in low-spin, low-energy spectra of even-even $^{122-124}$Xe and $^{124-128}$Ba nuclei are discussed. The energy levels of ground and gamma band are calculated by the Soft Rotor Formula (SRF). The staggering is a function of spin has been analyzed in order to derive the information on the type of triaxiality present in $^{122-124}$Xe and $^{124-128}$Ba nuclei. The staggering indices of $\gamma$-soft and triaxial nuclei are also calculated by this SRF formula. It is found that these staggering indices have opposite signs and provide clear distinction between $\gamma$-soft and triaxial nuclei.

1. Introduction
The unity of the nucleon-nucleon interaction in the diversity of spectra of atomic nuclei is a major objective of nuclear theory. The nuclei around $A=130-200$ region have been studied by various workers [1, 2] using various models over the last four decades. The neutron deficient ($N<82$) light rare-earth nuclei like Xenon and Barium isotopes near $A \approx 130$ have the properties of rotational nuclei [3]. Nuclei near $A=130$ show the interesting phenomena of odd-even staggering (OES) in the $\gamma$-band at low spins. In some studies, the OES has been interpreted as a result of the interaction between the even levels of the $\gamma$-band and corresponding levels of $\beta$-band. This consideration has been addresses in the SU(3) limit of the Interacting Boson Model (IBM)[4]. Vector Boson Model scheme of Minkov et al., [5] provides a relevant way to study the interaction between the ground and $\beta$-band. Recently Rainovski et al., [6] have studied that $^{124}$Xe nucleus is close to the O(6) symmetry and excited states in $^{124}$Xe have been studied via the $^{12}$C ($^{124}$Xe, $^{124}$Xe$^*$) Coulomb excitation reaction. Bohr and Mottelson have commented that for $\gamma_0 \geq 24^\circ$ nuclei can no longer be considered deformed and the nucleus can be considered to take any shape including triaxial. A group of workers have studied in detail the contribution of the Coriolis interaction besides the band interaction effects while, Liao Ji - Zhi [7] illustrated the variation in OES with angular momentum $J$ of some well- deformed nuclei. Recentaly Singh et al., [8] have studied the yrast and $\gamma$-band for $^{120-130}$Xe nuclei using the Asymmetric Rotor Model (ARM) on employing the Lipas parameter and commented that the reason of OES is the splitting of $\gamma$-band in odd and even spin sequence. The structure of the $K^\pi = 2^+$ gamma vibrational bands and the quasi-gamma bands of even Z-even N nuclei is investigated on a global scale. The yrast band energies, OES in the $\gamma$-band for Xenon ($A=116-118$) and Cerium ($A=128,132$ and 134) chain have been done in the frame work of Asymmetric Rotor Model [9]. Recently McCutchan
et al. [10] studied the staggering in band energies and the transition between different structural symmetries in nuclei by using this expression

\[ S(J) = \frac{\{E(J) - E(J-1)\} - \{E(J-1) - E(J-2)\}}{E(2J+1)} \]  

(1)

The present work begins to search whether \(J(J+1)\) rule is obeyed in the three energy sequences of yrast and quasi \(\gamma\) band in Xe and Ba nuclei. For this we take the two parameter formula, suggested by von Brentano et al., [11] for testing these energies sequences.

**Calculation**

Two parameter von Brentano et al., [11] expression

\[ E = \frac{J(J+1)}{a(1+bJ)} \]  

(2)

This is also called the soft rotor formula (SRF). The values of \(a\) and \(b\) are calculated by fitting the \(2^+\) and \(4^+\) energies in even sequence and \(3^+\) and \(5^+\) energies in odd sequence. For these calculations, experimental data are taken from www.nndc.bnl.gov [12]

**Results and Discussion**

The level energies for the \(\gamma\)-band in \(^{122-124}\text{Xe}\) and \(^{124-128}\text{Ba}\) nuclei are plotted in Figure 1.

![Signature splitting S(J) is plotted versus spin in present work and experiment for \(^{122-124}\text{Xe}\) nuclei.](image)

**Figure 1.** Signature splitting \(S(J)\) is plotted versus spin in present work and experiment for \(^{122-124}\text{Xe}\) nuclei.

In Figure 1 we see the variation of staggering factor \(S(J)\) with spin \(J\) for \(^{122}\text{Xe}\) and \(^{124}\text{Xe}\) nuclei, the spacing between odd even spin levels in the present work are in close agreement with experimental values for \(S(4)\), \(S(5)\) and \(S(6)\). For higher values of \(J\) separation between evaluated and experimental values of \(S(J)\) increases, but in small amount.

In Figure 2 we see variation of \(S(J)\) with \(J\) for \(^{124}\text{Ba}\) and \(^{126}\text{Ba}\) nuclei. The calculated value of \(S(J)\) matches excellently with experiment for \(S(4)\) to \(S(10)\) factor while a small deviation in \(S(9)\) and \(S(11)\) is found for \(^{126}\text{Ba}\). Next we see the variation for \(^{128}\text{Ba}\) (see Figure 3), where
Figure 2. Signature splitting \( S(J) \) is plotted versus spin in present work and experiment for \(^{124-126}\text{Ba} \) nuclei.

Figure 3. Signature splitting \( S(J) \) is plotted versus spin in present work and experiment for \(^{128}\text{Ba} \) nuclei.

the calculated and experimental values of \( S(J) \) show excellent agreement up to \( S(10) \). Next in Figure 4 we see the comparison between the experimental and calculated values of the ground band and the gamma band energies. For the ground band the calculated energy shows good agreement up to \( J=8^+ \) after this there is a decrease in the calculated energies, whereas in the case of gamma band close agreement up to \( J=5^+ \) is observed after which there is fall in the gamma band energies. The same comparison is made for \(^{124-126}\text{Ba} \) nuclei in Figure 5. The ground band energy shows close agreement up to \( J = 8^+ \) similar as that of Xenon nuclei. But in the case of gamma bands the agreement is good up to \( J = 11^+ \) for \(^{126}\text{Ba} \) nuclei and \( J = 10^+ \) for \(^{124}\text{Ba} \) nuclei.

We took another test of triaxiality on the basis of energy relation \( \Delta E_1 = E(3_1^+) - \)
Figure 4. Comparison of experimental and calculated energy value of ground and gamma band for $^{122-124}\text{Xe}$ nuclei.

Figure 5. Comparison of experimental and calculated energy value of ground and gamma band for $^{124-126}\text{Ba}$ nuclei.

Table 1. The experimental and calculated difference $\Delta E_1$ and $\Delta E_2$

| Nuclei | $^{122}\text{Xe}$ | $^{124}\text{Xe}$ | $^{124}\text{Ba}$ | $^{126}\text{Ba}$ | $^{128}\text{Ba}$ |
|--------|------------------|------------------|------------------|------------------|------------------|
| Exp.$\Delta E_1$ | 40.04 | 46.1 | 60 | 106 | 157 |
| Exp.$\Delta E_2$ | 275.72 | 339.2 | 51 | 13 | 5 |
| Th.$\Delta E_1$ | 48.6 | 46.1 | 59.3 | 106.6 | 155.9 |
| Th.$\Delta E_2$ | 226.4 | 339.2 | 51 | 12.7 | 6.8 |

$\left[E(2^+_1) + E(2^+_2)\right]$ for triaxial nucleus given by [2] and $\Delta E_2 = E(3^+_1) - \left[2E(2^+_1) + E(4^+_1)\right]$ for $\gamma$-soft nucleus given by Wilets and Jean [13]. A large value of $\Delta E_1$ and small value of $\Delta E_2$.
reflects a γ-soft character. The difference $\Delta E_1$ is low while $\Delta E_2$ is large for $^{122-124}$Xe which reflects the triaxial nature. For $^{124-128}$Ba, $\Delta E_1$ is large and $\Delta E_2$ is small which shows the gamma soft character. The values of $\Delta E_1$ and $\Delta E_2$ are presented in Table 1.

**Conclusion**

In the present work we find that the odd-even staggering (OES) in the γ-bands help to distinguish between its rigid triaxial rotor and γ-soft nature of nuclei. We have used the SRF formula to identify the nature of the nuclei by considering the $^{122-124}$Xe and $^{124-128}$Ba isotopes. The $^{122-124}$Xe isotopes show the triaxiality nature and $^{124-128}$Ba isotopes show the γ-soft. Thus, the present study taken for $^{122-124}$Xe and $^{124-128}$Ba nuclei provides wide range for the applicability of the previous approach.

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