Abstract

It is shown that most of the unusual properties of the lowest lying scalar (and pseudoscalar) mesons can be understood, at the qualitative and quantitative level, on the basis of the breakdown of the $U_A(1)$ symmetry coupled to the vacuum expectation values of scalars by the spontaneous breaking of chiral symmetry.

I. INTRODUCTION

The light quark sector of QCD acquires a $U(N_f)_L \otimes U(N_f)_R$ symmetry and scale invariance in the limit of $N_f$ massless quarks. This limit is a reasonable starting point for constructing effective theories. The $u$ and $d$ quark masses are small enough compared to the naive confinement scale ($\Lambda_{QCD}$) to make the procedure reliable. Incorporating the $s$ quark makes sense once one realizes that the relevant scale is $\Lambda_\chi \approx 4\pi f_\pi$ [2]. The conventional approach to low energy QCD is to assume that the octet of lowest lying pseudoscalar mesons $\{\pi, K, \eta\}$ are approximate Goldstone Bosons (GB) associated with the spontaneous breakdown of the $SU(3)_L \otimes SU(3)_R$ symmetry. This belief is supported by the smallness of $\pi, K$ and $\eta$ meson masses as compared to the typical hadronic scale of 1GeV. The approach leads to a so far successful description of pseudoscalar meson physics where the relevant QCD Green functions are systematically expanded in powers of $\partial/\Lambda_\chi$, $m_q/\Lambda_\chi$, where $\Lambda_\chi$ stands for the chiral symmetry breaking scale, in the so-called Chiral Perturbation Theory ($\chi PT$) expansion [1]. The remaining pseudoscalar, the $\eta'$ meson, has a large mass which has been related to the breakdown of the $U_A(1)$ symmetry [3].

The symmetries of massless QCD are reduced to smaller symmetries in many ways. Both scale invariance and $U_A(1)$ symmetry are broken at the quantum level [3]. The residual $SU(3)_L \otimes SU(3)_R \otimes U(1)_V$ symmetry is spontaneously reduced down to $SU(3)_V$ (the vector $U(1)$ symmetry being trivial for mesons). Finally, if one introduces mass terms, $SU(3)_V$ reduces down to isospin, or completely breaks down, depending on the choice for the quark masses.

The most important contribution to the breakdown of the $U(1)_A$ symmetry comes from Euclidean classical field configurations with non-trivial topology (instantons). These effects are modulated by the $e^{-8\pi^2/g_s^2}$ factor, where $g_s$ denotes the strong coupling constant, thus being small at high energies but becoming important at low energies [3].

Physics beyond the low energy region (where $\chi PT$ is strictly valid) and up to the $\phi(1020)$ mass requires to introduce all the relevant degrees of freedom, namely, the $\eta'$ meson, the well
established $a_0(980)$ and $f_0(980)$ scalar mesons and the lowest lying vector meson nonet. The formulation of effective theories including the $\eta'$ and its relation to the $U(1)_A$ anomaly was firstly done in [4–7]. In some of these pioneering works [4–5] the starting point is a Lagrangian which exhibits chiral symmetry realized in a linear way. This amounts to consider scalar fields as the chiral partners of pseudoscalars. In particular in [8] the relation to the instanton induced quark interaction discovered by 't Hooft is established by expanding one of the terms in the Lagrangian. Scalar fields are then integrated out and authors focus on the properties of the vacuum and on the pseudoscalar phenomenology.

Over the past few years the problem of the scalar mesons has been under intense debate. The identification of $\bar{q}q$ scalar mesons is still awaiting for a final answer. This problem is related to the unique QCD prediction for the existence of bound states of gluons (glueballs) and the possibility for the existence of hybrid states. The well established low mass scalar mesons are: the $f_0(980)$, $f_0(1370)$ and the recently resurrected $f_0(400–1200)$ (or $\sigma$) in the $I = 0$ sector; the $a_0(980)$ and $a_0(1450)$ on the isovector side, and the isospinor $K^*_0(1430)$ [10]. In addition to these states there are claims for the existence of signals of other scalar mesons below 1 GeV [11–13]. In particular, the approach summarized in [13] which is based on a non-linear chiral Lagrangian including scalar degrees of freedom concludes the existence of a light isoscalar scalar $\sigma$ and an isovector scalar $\kappa$ with a mass around 850 MeV. This is consistent with results in [14] though these approaches differ in the mixing of scalars and its interpretation. The differences come from the fact that $U_A(1)$ anomaly effects are not taken into account in [13], while they are considered as fundamental in [14–16]. As we shall see below the picture arising in the later case is completely consistent and most of the controversial properties of scalar mesons find a natural and simple explanation in this framework.

Many other schemes have been put forth trying to understand the lowest lying scalar meson properties. A $\bar{q}q$ structure for the $a_0(980)$ and $f_0(980)$ was put on doubt since the corresponding quark models were not able to explain the tiny coupling of these mesons to two photons [17] and the quark content suggested by the mass spectrum was incompatible with the main decay channels [18]. More complicated models such as four quark states [18] and molecules [17] were advocated to explain these points. This is a very active field and no definite conclusion has been reached as to which states are to be considered as $\bar{q}q$, multi-quark, molecule, gluonia or hybrid states [14].

In this work we intend to shed some light on this problem. To this end let us first exhibit some of the properties of scalar (and pseudoscalar) mesons which seem to be in conflict with a $\bar{q}q$ interpretation.

On the base of a naive constituent quark model we expect the $^1S_0$ $\bar{q}q$ states ($q = u, d$) to have a mass $\approx 2m_q \approx 630$ MeV where $m_q$ stands for $u$ or $d$ constituent quark mass, which we consider as equal and evaluate to one third of the proton mass. In a similar way the $\bar{q}s$ ($ss$) states should have a mass $\approx 870$ MeV ($\approx 1010$ MeV) if we evaluate the constituent strange quark mass to $m_s \approx m_{\Omega}/3 \approx 555$ MeV. In this picture, a $^3S_1$ $\bar{q}q$ state would be slightly heavier than the $^1S_0$ state and a $^3P_0$ $\bar{q}q$ state should have an even larger mass.

The lowest lying vector mesons fulfill these naive expectations, it is in this sense a “well-behaved” sector. The pseudoscalar and scalar sectors, however, are not. The pions and kaons are lighter than expected from naive quark model considerations. The smallness of the pion and kaon masses is qualitatively understood on the base of spontaneous breaking of $\chi$-ral
symmetry. However, even with this explanation in mind, there exist at least one problem in this sector which is usually overlooked and which has to do with the quark content of pseudoscalars. Consider the isoscalar-pseudoscalar states with well defined quark content (flavor states): \( \eta_{ns} \equiv (\bar{u}u + \bar{d}d)/\sqrt{2} \) and \( \eta_s \equiv \bar{s}s \). Since they are not the physical states, they must be mixed somehow to yield the physical \( \eta \) and \( \eta' \) mesons. Independently of the mechanism which mixes the flavor isoscalar-pseudoscalar states, a straightforward analysis yields the relations

\[
m^2_{\eta_{ns}} = m^2_\eta \cos^2 \phi_P + m^2_{\eta'} \sin^2 \phi_P, \quad m^2_{\eta_s} = m^2_\eta \sin^2 \phi_P + m^2_{\eta'} \cos^2 \phi_P,
\]

where the information on the mixing mechanism is hidden in the mixing angle in the flavor basis \( \phi_P \). This angle has been estimated as \( \phi_P \approx 39.5^\circ \) \cite{23}, corresponding to an angle \( \theta_P \approx -15.2^\circ \) in the naive single-angle description of pseudoscalar mesons mixing in the usual singlet-octet basis. Introducing this information in Eq. (1) we obtain \( m_{\eta_{ns}} \approx 741 \text{ MeV} \) and \( m_{\eta_s} \approx 816 \text{ MeV} \). The obvious question here is: Why is the \( \eta_{ns} \) (a \( \bar{q}q \) state differing from the pion only in isospin quantum numbers) much heavier than the pion, even more massive than the \( \bar{q}s \) (\( K \)) and as heavy as the (purely strange) \( \eta_s \) state?

Concerning the scalar sector things are much more involved since even an undoubted identification of the corresponding nonet is still missing. Assume that the \( a_0(980) \) and \( f_0(980) \) mesons are the \( I = 1 \) and \( I = 0 \) members of the \( \bar{q}q \) scalar nonet. Their nearby degeneracy suggest that they are the scalar mesons analogous to the \( \rho(770) \) and \( \omega(780) \) vector mesons which are also almost degenerate in mass. On this identification grounds the \( f_0(980) \) should be predominantly non-strange but then: How can one explain its strong coupling to \( \bar{K}K \)? This could be qualitatively understood if the \( f_0(980) \) had a strong strange component but then one does not understand why this meson is almost degenerate in mass with the non-strange \( a_0(980) \). On the other hand the \( a_0(1450) \) and the \( f_0(1350) \) seem to be heavy enough to be out of the scope of the naive quark model considerations. The same can be said for the vector \( K^*_0(1430) \). Another possibility is the existence of light scalar mesons: \( \sigma(400 - 600) \) and \( \kappa(\approx 900) \). It seems that we have now a general consensus on the existence of a broad scalar structure in the low energy region \cite{11} though it is not clear yet what its nature is and which mechanism makes this meson so light. The \( \kappa(900) \) is a more controversial object \cite{12} and its existence has not been firmly established.

As mentioned above, the vector mesons spectrum is in qualitative agreement with naive quark model expectations. Thus, whatever the explanation for the scalar and pseudoscalar mesons spectrum may be, it should have no effect on the vector sector.

In this letter we suggest that all the points raised above can be understood by considering the instanton induced quark interaction which is also responsible for the \( U_A(1) \) symmetry breaking. The appealing property of this interaction is that it does not affect the vector sector since vector mesons consist of quark-anti-quark pairs that are either both left-handed or both right-handed and do not match the quantum numbers of the instanton induced interaction \cite{3,16}. This, and the fact that it is able to explain why the \( \eta' \) is not a Goldstone Boson make this interaction a natural candidate for explaining the unusual properties of pseudoscalar mesons and their chiral partners, the scalar mesons. Thus, we require a model considering both nonets in a chiral symmetric way and which takes into account t’Hooft interaction.

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II. THE MODEL

A model with the characteristics mentioned at the end of the previous section has been reconsidered recently [14–16]. The model, incorporates pseudoscalar and scalar degrees of freedom, exhibits chiral symmetry and incorporates $U_A(1)$ breaking in a phenomenological way. A $U(2) \otimes U(2)$ version of this model was formulated in connection with the explanation of the $U_A(1)$ anomaly [4]. Amazingly, similar models were put forward even before the birth of QCD [20]. The Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{SB}}$$

(2)

where $\mathcal{L}_{\text{sym}}$ denotes the $U(3) \times U(3)$ symmetric Lagrangian:

$$\mathcal{L}_{\text{sym}} = \left(\frac{1}{2} \partial_\mu M \left(\partial^\mu M^\dagger\right)\right) - \frac{\mu^2}{2} X (\sigma, P) - \frac{\lambda}{4} Y (\sigma, P) - \frac{\lambda'}{4} X^2 (\sigma, P)$$

(3)

with $M = \sigma + iP$, and $X, Y$ stand for the $U(3) \times U(3)$ chirally symmetric terms:

$$X (\sigma, P) = \langle MM^\dagger\rangle, \quad Y (\sigma, P) = \langle (MM^\dagger)^2\rangle.$$  

(4)

We closely follow the conventions in Ref. [14]. In particular, we use $F \equiv \frac{1}{\sqrt{2}} \lambda_i f_i$ with $F = \sigma, \ P; f_i = \sigma_i, \ p_i; \ i = 0..8$ and $\lambda_i$ denote Gell-Mann matrices. For further details the interested reader is referred to Ref. [14]. Chiral and $U_A(1)$ symmetries are explicitly broken by

$$\mathcal{L}_{\text{SB}} = \langle c\sigma \rangle - \beta Z (\sigma, P)$$

(5)

where $c \equiv \frac{1}{\sqrt{2}} \lambda_i c_i$, with $c_i$ constant and

$$Z (\sigma, P) = \det (M) + \det \left( M^\dagger \right)$$

(6)

The most general form of $c$ which preserves isospin and gives PCAC is such that the only non-vanishing coefficients are $c_0$ and $c_8$. The former gives, by hand, the pseudo-scalar nonet a common mass, while the later breaks the $SU(3)$ symmetry down to isospin. These parameters can be related to quark masses in QCD.

The $Z$ term in Eq. (5) corresponds to the instanton induced quark interaction in the case when the instanton angle $\theta_{\text{inst}}$ is aligned to zero. This interaction has the form of a determinant in flavor space and breaks $U(3)_L \otimes U(3)_R$ into $SU(3)_L \otimes SU(3)_R \otimes U(1)_V$ [13].

The linear $\sigma$ term in Eq. (4) induces $\sigma$-vacuum transitions which give to $\sigma$ fields a non-zero vacuum expectation value (hereafter denoted by $\{ \}$. Linear terms can be eliminated from the theory by performing a shift to a new scalar field $S = \sigma - V$ such that $\{ S \} = 0$, where $V \equiv \{ \sigma \}$. This shift generates new three-meson interactions and mass terms.

For the sake of simplicity let us write $V = \text{Diag}(a, a, b)$ where $a, b$ are related to $\{ \sigma \}$ through

$$a = \frac{1}{\sqrt{3}} \{ \sigma_0 \} + \frac{1}{\sqrt{6}} \{ \sigma_8 \}, \quad b = \frac{1}{\sqrt{3}} \{ \sigma_0 \} - \frac{2}{\sqrt{6}} \{ \sigma_8 \}$$

(7)

Meson masses generated by this procedure are:
i) Non-mixed sectors.

\[
m^2_\pi = \xi + 2\beta b + \lambda a^2, \quad m^2_K = \xi + 2\beta a + \lambda (a^2 - ab + b^2), \\
m^2_\eta = \xi - 2\beta b + 3\lambda a^2, \quad m^2_\kappa = \xi - 2\beta a + \lambda (a^2 + ab + b^2),
\]

where \( a \) and \( \kappa \) denote the scalar mesons analogous to \( \pi \) and \( K \) respectively and we used the shorthand notation \( \xi = \mu^2 + \lambda'(2a^2 + b^2) \).

ii) Mixed sectors.

\[
\begin{align*}
\mathcal{L}^P_2 &= -\frac{1}{2} \left( m^2_{0P} P^2_0 + m^2_{sP} P^2_s + 2m^2_{0sP} P_0 P_s \right), \\
\mathcal{L}^S_2 &= -\frac{1}{2} \left( m^2_{0S} P^2_0 + m^2_{sS} P^2_s + 2m^2_{0sS} P_0 P_s \right)
\end{align*}
\]

where

\[
\begin{align*}
m^2_{0P} &= \xi + \frac{1}{3}[\lambda (a^2 + 2b^2) + 2\beta (4a - b)], \\
m^2_{sP} &= \xi + \frac{1}{3}[\lambda (2a^2 + b^2) - 4\beta (2a + b)], \\
m^2_{0sP} &= \sqrt{\frac{2}{3}} (a - b) [\lambda (a + b) + 2\beta], \\
m^2_{sS} &= \xi + \frac{1}{3}[-2\beta (4a - b) + 3\lambda (a^2 + 2b^2) + 4\lambda'(a - b)^2], \\
m^2_{0S} &= \xi + \frac{1}{3}[4\beta (2a + b) + 3\lambda (2a^2 + b^2) + 2\lambda'(2a + b)^2], \\
m^2_{0sS} &= \sqrt{\frac{2}{3}} (a - b) [-2\beta + 3\lambda(2a + b) + 2\lambda'(2a + b)].
\end{align*}
\]

As we are interested in the quark content of fields it is convenient to analyze the mixed sector in the flavor \((|s>, |ns>)\) basis \([21]\) defined by:

\[
\eta_{ns} = \sqrt{\frac{1}{3}} P_s + \sqrt{\frac{2}{3}} P_0, \quad \eta_s = -\sqrt{\frac{2}{3}} P_s + \sqrt{\frac{1}{3}} P_0
\]

with analogous relations for the scalar \((\sigma_{ns}, \sigma_s)\) mixed fields. In this representation, the mass terms in the Lagrangian read

\[
\begin{align*}
\mathcal{L}^P_2 &= -\frac{1}{2} \left( m^2_{\eta_{ns}} \eta^2_{ns} + m^2_{\eta_s} \eta^2_s + 2m^2_{\eta_{ns-ss}} \eta_{ns} \eta_s \right), \\
\mathcal{L}^S_2 &= -\frac{1}{2} \left( m^2_{\sigma_{ns}} \sigma^2_{ns} + m^2_{\sigma_s} \sigma^2_s + 2m^2_{\sigma_{ns-ss}} \sigma_{ns} \sigma_s \right)
\end{align*}
\]

where

\[
\begin{align*}
m^2_{\eta_{ns}} &= \xi - 2\beta b + \lambda a^2, \quad m^2_{\eta_s} &= \xi + 2\beta b + 3\lambda a^2 + 4\lambda' a^2, \\
m^2_{\eta_{ns-ss}} &= \xi + \lambda b^2, \quad m^2_{\sigma_s} &= \xi + 3\lambda b^2 + 2\lambda' b^2, \\
m^2_{\eta_{ns-ss}} &= -2\sqrt{2}\beta a, \quad m^2_{\sigma_{ns-ss}} &= 2\sqrt{2}(\beta + \lambda' b) a.
\end{align*}
\]
Many of our initial concerns can be qualitatively understood from Eqs. (8,10,13). The first point to be noticed is that, in this model, the $U_A(1)$ anomaly gets coupled to the v.e.v.’s of scalar fields by the spontaneous breaking of chiral symmetry and contributes, via this effect, to the masses of all fields entering the theory except the strange fields. In the remaining of the paper we will call this the “anomaly-vacuum” effect. The mass terms arising as a consequence of this effect, have exactly the same origin as the mass terms coming from the $\phi^4$ terms in the $SU(2) \times SU(2)$ Linear Sigma Model of Gell-Mann and Levy. The novelty here is the existence of the three-meson determinantal instanton induced interaction which couples either two pseudoscalars to one scalar or three scalars (other possibilities being forbidden by parity). In the case when one of the scalars in the vertex has the same quantum numbers as the vacuum (the isospin singlets $\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$ or $\bar{s}s$) a mass term is generated by replacing this leg by the corresponding vacuum expectation value. In particular the vertex with a strange pseudoscalar a non-strange pseudoscalar and an isosinglet scalar field (which the determinantal structure dictates to be the non-strange isosinglet) gives rise to a mass term proportional to the strength of the anomaly, which mixes the strange and non-strange pseudoscalar fields as shown in Eq.(refmixed2). Similar phenomena occur in the scalar sector. The strength of the mixing due to the anomaly is exactly of the same size but opposite sign in the pseudoscalar and scalar sectors. In the latter case, there exist an additional contribution coming from the $\phi^4$ interaction in the Lagrangian Eq. (3) whose strength is measured by $\lambda$.

The extraction of the parameters of the model has been done in [14–16] using well known information on the pseudoscalar sector. In this concern, it is worth mentioning that the outcome of the model strongly depends on the input used. In Ref. [14] pseudoscalar meson masses ($m_\pi$, $m_\eta$, $m_{\eta'}$, $m_K$) and the pion decay constant ($f_\pi$) were used in order to fix the parameters entering the model ($\xi, \lambda, \beta, a, x = (b - a)/2a$). Ref. [14] uses $m_\pi, m_K, m_{\eta'}^2 + m_{\eta'}^2, f_K$ and $f_\pi$ as input while Ref. [16] uses the pseudoscalar mixing angle in the singlet-octet basis $\theta_P$ and $m_\pi, m_K, m_\eta, m_{\eta'}$. The outcome is different in all these three cases. In particular, the last approach yields a heavy scalar nonet. It must be pointed out, however, that according to recent work [22], the proper description of pseudoscalar mixing in the singlet-octet basis requires two mixing angles. In this concern, the use of the strange-non-strange basis is more appropriate since, in this basis, pseudoscalar mixing can be described using a single angle [23]. As we shall see below, the pseudoscalar spectrum is consistent, within the model, with a small mixing angle in the singlet-octet basis. An explanation for the physics behind this quantity requires the improvement of the model. The mixing angle of pseudoscalars in the flavor basis($\phi_P$) and its scalar analogous ($\phi_S$) can be extracted from (12) by diagonalizing the Lagrangian. A straightforward calculation yields

\[
\sin 2\phi_P = \frac{2m_{\eta-s}^2}{m_{\eta'}^2 - m_\eta^2}, \quad \sin 2\phi_S = \frac{2m_{\sigma-s}^2}{m_{\sigma'}^2 - m_\sigma^2}
\]

where $\sigma'$ and $\sigma$ denote the physical isoscalar-scalar mesons. Alternatively,

\[
\cos 2\phi_P = \frac{m_{\eta-s}^2 - m_{\eta-s}^2}{m_{\eta'}^2 - m_\eta^2}, \quad \cos 2\phi_S = \frac{m_{\sigma-s}^2 - m_{\sigma-s}^2}{m_{\sigma'}^2 - m_\sigma^2}
\]

The important point is that the parameter which measures the strength of the $U_A(1)$ breaking, $\beta$, turns out to be negative with the conventions in Eq.(3): $\beta \approx -1.5$ GeV. The value
of $a$ can be fixed from $a = f_K/\sqrt{2} = 65.7$ MeV, whereas the value of $b$, which we rewrite in terms of $x = (b - a)/2a$ can be extracted by the procedure used in \[14\] which yields $x = 0.22$ or from the input in \[14\] which gives $x = 0.39$. The values of $\lambda$ and $\lambda'$ are positive. Their actual values depend on the input used but in general $\lambda'$ turns out to be small \[14,15\]. Using the approach of Ref. \[14\], Eqs. (14,15) yield

\[
\sin 2\phi_P = 0.9202 \quad \cos 2\phi_P = -0.3911, \tag{16}
\]

which imply a value $\phi_P \approx 56.7^\circ$ for the pseudoscalar mixing angle in the $s - ns$ basis. This corresponds to a small angle in the singlet-octet basis ($\theta_P = \phi_P - 54.7^\circ \approx 2^\circ$) consistent with results in \[13\] ($\theta \approx -5^\circ$) \[1\]. The difference in these approaches comes from the use of $f_K$ as input in \[14\], instead of the combination $m_\eta'^2 - m_\eta^2$ used in \[14\].

The extraction of the scalar mixing angle requires to fix the coupling $\lambda'$ which enters the pseudoscalar and the unmixed scalar sectors only in the combination $\xi = \mu^2 + \lambda'(2a^2 + b^2)$ as can be seen from Eqs.(8,13). Thus, fixing this parameter necessarily requires to use information on the mixed scalar sector. In \[14\] the masses of physical scalar mesons where studied as a function of this parameter. This analysis leads to the identification of the isoscalar-scalar mesons with the $\sigma(400 - 600)$ and $f_0(980)$. This identification yield $\lambda' \approx 4$ which predicts a scalar mixing angle $\phi_S \approx -14^\circ$. This result is consistent with the analysis of $f_0 \to \gamma\gamma$ \[24\] and the recently measured $\phi \to \pi^0\pi^0\gamma$ \[25,26\]. The small scalar mixing angle is consistent with a mostly $\bar{u}u + \bar{d}d/\sqrt{2}$ for the sigma meson. The isovector and isospinor scalar fields are identified with the $a_0(980)$ and $\kappa(\approx 900)$ in the model ( the procedure followed in \[17\] identifies $\kappa$ with $K_0^*(1430)$, this is a consequence of using different input and to the high dependence of the outcome on the input scheme mentioned above).

### III. PSEUDOSCALAR SPECTRUM

The purpose of this letter is to remark the role played by the $U_A(1)$ anomaly in structuring the mass spectrum of scalar (and pseudoscalar) mesons. To this end, let us qualitatively analyze the splittings arising from from Eqs. (8,13) in order to disentangle the different contributions to the meson masses. In this concern, it is important to recall that the extraction of the parameters yields a negative sign for $\beta$ in Eq.(14) which quantify the strength of the anomaly. From Eqs. (8,13) we obtain the relations

\[
m_\eta^2 - m_\pi^2 = -4\beta b, \tag{17}
\]
\[
m_\eta^2 - m_K^2 = -2\beta (a + b) - \lambda b (b - a), \tag{18}
\]

\[1\]In Ref. \[14\] the pseudoscalar mixing angle in the $s - ns$ basis was extracted from the sine relation Eq.(14) and estimated as $\phi_P \approx 33.3^\circ$. A careful analysis of the diagonalization process shows that the correct value arises from the cosine relation and its actual value is $\phi_P \approx 56.7^\circ$. The problem when using the sine relation is that it does not distinguish between $\phi_P$ and $\pi/2 - \phi_P$ which is also a solution. We appreciate illuminating correspondence with Prof. G. 't Hooft which helped to clarify the input scheme dependence and lead us to reconsider the extraction of the pseudoscalar mixing angle.
which reveal that, in the absence of the anomaly, the pion and non-strange eta are degenerate and the kaon is heavier than both of them, which is consistent with the expectations of naive quark models. Thus, the $\eta_{ns} - \pi$ splitting is due to the “anomaly-vacuum” effect which pushes the pion and kaon masses down and the non-strange eta mass up as can be seen from Eqs. (8,13). As to the $\eta_{ns} - K$ splitting there is an additional contribution coming from the invariant in (3) whose strength is measured by the $\lambda$ coupling constant. From now on we will call such kind of contributions as the “normal” effects. They are “normal” in the sense that, in the absence of the anomaly, the splittings between pseudoscalar mesons are proportional to this coupling constant times the SU(3) symmetry breaking factor $(b - a)$ which can be related to the difference of strange and non-strange quark masses. In the case at hand, the “normal” effect tends to make $K$ heavier than $\eta_{ns}$, but the “anomaly-vacuum” effect goes in the opposite direction. The latter effect is stronger than the former thus rendering the non-strange eta heavier than the kaon.

Similar results are obtained for the $\eta_{s} - \pi$ and $\eta_{s} - K$ splittings as can be seen from the following relations

$$m_{\eta_{s}}^{2} - m_{\pi}^{2} = -2\beta b + \lambda (b + a) (b - a) \quad (19)$$

$$m_{\eta_{s}}^{2} - m_{K}^{2} = -2\beta a + \lambda a (b - a) . \quad (20)$$

In this case, both effects interfere constructively, reinforcing the corresponding splittings. The individual effects of the anomaly can be read from Eqs. (8,13). The anomaly leaves the strange eta untouched while, as noticed above pushes the pion and kaon masses down.

The next interesting effect in the pseudoscalar spectrum has to do with the $K - \pi$ splitting. From the relation

$$m_{K}^{2} - m_{\pi}^{2} = (-2\beta + \lambda b) (b - a) , \quad (21)$$

we see that $K - \pi$ mass splitting is proportional to the $SU(3)$ symmetry breaking. It must be noticed, however, that the “normal” effect is enlarged by the “anomaly-vacuum” effect thus making pions much lighter than kaons. Finally the $\eta_{s} - \eta_{ns}$ splitting is

$$m_{\eta_{s}}^{2} - m_{\eta_{ns}}^{2} = 2\beta b + \lambda (b + a) (b - a) . \quad (22)$$

In the absence of the anomaly the normal pattern appears, i.e. the strange field is heavier than the non-strange field. Turning on the anomaly modifies this pattern. The anomaly does not affect the strange pseudoscalar mass but it does push up the non-strange pseudoscalar. If we consider the mixing angle predicted by the model, the later turns out to be heavier than the former.

### IV. SCALAR SPECTRUM

Let us now turn to the scalar sector. The first point to be emphasized is that, the “anomaly-vacuum” contribution systematically has the opposite sign in the pseudoscalar and scalar sectors (we will call this the “sign” effect). The reader can be easily convinced of this by looking at the mass relations in Eqs. (8,13). There are two important consequences of the “sign” effect. The first one is that the $a_{0}$ and $\kappa$ masses are pushed up by the anomaly (in
contrast to the pseudoscalar analogous which are pushed down) and the non-strange sigma is pushed down (unlike the non-strange pseudoscalar which is pushed up). The second effect has to do with the mixing of strange and non-strange isoscalar scalar fields. The “anomaly-vacuum” effect in the mixing of scalars is exactly of the same size as in pseudoscalars but with the opposite sign. In this case, there exists an additional contribution coming from one of the chiral invariants in the Lagrangian, whose strength is measured by the $\lambda$' coupling. As discussed in [15] this term corresponds to disconnected quark diagrams, thus being suppressed by the Okubo-Zweig-Izuka rule. This term, however, becomes relevant to the mixing of scalars since it has the opposite sign to the anomaly and interferes destructively with it, thus rendering the scalars less strongly mixed than pseudoscalars.

The $\sigma_{ns} - a$ and $\sigma_{ns} - \kappa$ splittings are given by

\begin{align}
    m^2_{\sigma_{ns}} - m^2_a &= 4\beta b + 4\lambda a^2 \\
    m^2_{\sigma_{ns}} - m^2_\kappa &= 2\beta (a + b) - \lambda (2a + b)(b - a) + 4\lambda' a^2.
\end{align}

Notice that in the absence of the anomaly and the OZI forbidden $\lambda'$ coupling the $a$ and $\sigma$ fields have the same mass and the $\kappa$ meson is slightly heavier as expected from their constituent quark content. The individual effects of the anomaly mentioned above (the anomaly pushes the sigma mass down and the $a$ and kappa masses up) causes the $a - \sigma$ splitting making the $a$ field heavy and the sigma field light. In other words, the $\sigma$ meson is light as a consequence of the “anomaly-vacuum” and the “sign” effects. In the case of the $\kappa - \sigma$ splitting, these mesons are pushed in opposite directions by the anomaly thus reinforcing the “normal” pattern. In addition we have to take into account $\lambda'$ contribution in both cases.

The corresponding relations for $\sigma_s$ are

\begin{align}
    m^2_{\sigma_s} - m^2_a &= 2\beta b + 3\lambda (a + b)(b - a) + 2\lambda' b^2 \\
    m^2_{\sigma_s} - m^2_\kappa &= 2\beta a + \lambda (a + 2b)(b - a) + 2\lambda' b^2.
\end{align}

Turning off the anomaly (and the $\lambda'$ contribution) yields a pattern as expected from the naive constituent quark picture. The strange scalar is heavier than the non-strange one and the kappa meson. It worth noticing that the strange scalar field is not affected by the “anomaly-vacuum” effect (see Eqs. (13)), but the chiral partner of the pion, the $a$ field, is pushed up by the anomaly rendering it almost degenerate with the physical (mostly strange) $f_0$ meson. Thus the $a_0 - f_0$ degeneracy is accidental and has its origin in the instanton induced quark interaction. In both equations above, the “normal” effect and the $\lambda'$ contributions to the splittings have the same sign. These “normal” splittings are largely canceled by the “anomaly-vacuum” effect. This explains the $a_0 - f_0$ degeneracy and the close value of the $\kappa$ meson mass.

The $a_0 - \kappa$ splitting has the same structure as the corresponding pseudoscalar relation in Eq. (21), namely

\begin{align}
    m^2_\kappa - m^2_a &= (2\beta + \lambda (2a + b))(b - a)
\end{align}

but in this case the individual “anomaly-vacuum” effects go in the opposite direction due to the “sign” effect. Both meson masses are pushed up by the anomaly with different strengths as the anomaly is coupled to different v.e.v.’s. The result is a not so strong anomaly effect.
in the splitting which nevertheless is strong enough to largely cancel the “normal” splittings due to quark masses making these mesons roughly equally heavy.

Finally, the $\sigma_s - \sigma_{ns}$ splitting can be read from Eqs.(13) as

$$m_{\sigma_s}^2 - m_{\sigma_{ns}}^2 = -2\beta b + 3\lambda (b + a) (b - a) + 2\lambda' \left(b^2 - 2a^2\right).$$ (26)

In case of a vanishing anomaly, we obtain the “normal” pattern again. Switching on the anomaly affects the non-strange $\sigma$ mass only (see Eq.(13)) making this meson light. In other words, the “anomaly-vacuum” and “normal” contributions interfere constructively in the splitting. This makes the non-strange $\sigma$ much lighter than the strange sigma. The naively expected pattern is reinforced by the “anomaly-vacuum” effect in contrast to the pseudoscalar analogous case in Eq.(22) where the anomaly overcomes the “normal” effect.

The qualitative analysis above can be graphically described as in Figs. 1-2, where the splittings due to the “anomaly-vacuum” and $SU(3)$ breaking effects are shown.

![Diagram](image)

Fig.1. Effects of $U_A(1)$ symmetry breaking in the pseudoscalar sector.
Fig. 2. Effects of $U_A(1)$ symmetry breaking in the scalar sector.

V. SUMMARY

Summarizing, we have obtained a qualitative (and quantitative) understanding of the lowest lying scalar (and pseudoscalar) meson spectrum on the basis of a simple model which incorporates the most relevant properties of QCD in the corresponding energy region, namely, the appropriate degrees of freedom, spontaneous breaking of chiral symmetry and $U_A(1)$ symmetry breaking. Most of the unusual properties of both sectors are explained by the coupling of the $U_A(1)$ anomaly to the vacuum expectation values of scalars by the spontaneous breaking of chiral symmetry (“anomaly-vacuum” effect) which supplies mesons with not yet considered mass terms.

Certainly this is only a first step in the elucidation of the quark structure of scalar mesons, and we have considered the $U_A(1)$ anomaly as the only striking effect (beyond chiral symmetry breaking). It still remain to conciliate this point of view with the $1/N$ perspective [27] which is a main task. The fact that so many phenomena are explained by a single effect makes this model attractive as a starting point for more elaborate and systematic analysis. It is also reassuring that quantitative estimates of the model predictions for the scalar meson...
decays where intermediate vector mesons can not appear or their contribution is known
to be small are successful. Recently, it has been shown that the puzzling $a_0(980) \rightarrow \gamma \gamma$
and $f_0(980) \rightarrow \gamma \gamma$ decays are properly described within the model [24] and the recent
measurements for the $\phi \rightarrow \pi^0 \pi^0 \gamma$ decay [23] are consistent with model predictions [26].

The improvement of the model is necessary in order to reach a more complete description
of meson properties. This is shown by its failure to satisfactorily describe the pseudoscalar
mixing angle. Other effects such as the hadronic loops considered in [28] could remedy this.
Another possibility is to include degrees of freedom which could be relevant to the physics
in this energy region and so far have not been considered. In particular, glueballs degrees
of freedom could have some effects via their mixing with pure quarkonium states. A step
in this direction is the work in [29] where glueball degrees of freedom are considered in a
framework close to the one used here and its worthy to analyze its consequences in detail.
A different line of action which is worth exploring is the relation of the instanton induced
quark interaction with the recently raised group theoretical arguments in favor of the unitary
symmetry as an accidental symmetry due to the gluon anomaly [30].
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