Computer aided calculus of the radial bridge kinetic energy of a sedimentation tank

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Abstract. The paper presents energy related aspects of a circular sedimentation tank’s radial bridge in order to assess the effectiveness of the equipment and to propose optimisations. In steady state conditions $E_{\text{total}} = E_{\text{Kinetic}} + E_{\text{Losses}}$, where $E_{\text{total}}$ is provided by an electric motor of power $P$ and the kinetic energy is $E_{\text{Kinetic}} = \frac{1}{2} J \cdot \omega^2$, where $J = \sum m_i \cdot r_i^2$. Several methods to assess the value of the $J$ moment of inertia were explored. Finally, the moment of inertia is computed using an analytical method, by dividing the radial bridge in $N_C$ ‘simple shape’ components and by the use of an original C++ software consisting of >1700 computer code lines. Being a flexible and heavily parameterized original software, an optimised version of the radial bridge geometry was considered, for which $J_{\text{opt}} = 83.57\% \cdot J_{\text{init}}$. We know the time in which a complete rotation is performed, $T$, therefore we compute $\omega$ and the kinetic energy. It results $E_{\text{Losses}}$ and we noticed that, because $\omega$ has a very small value, most of the $E_{\text{total}}$ energy is spent to cover the ‘losses’. The computation of the $J$ moment of inertia allowed us to conclude regarding the effectiveness of the equipment and to extent the library of C++ header files to be reused in other software projects for the development of the hybrid models in mechanical engineering.

1. Introduction
Technical features of a radial bridge belonging to a sedimentation tank from a wastewater treatment plant were extensively investigated. After a thorough analysis, due to the complexity of the phenomena, we concluded that a hybrid model must be conceived, [1]. In this way there were created several theoretical studies: computer aided analytical models, [2, 3], computer aided design model in NX, finite element models in Femap/Nastran, [4], input data generators and software interfaces in Visual Basic used to import the data in Femap/Nastran, [5]. Hydrodynamic phenomena were also taken into account, [6]. The flexibility of the original software allowed us to study the structure’s strength in various load scenarios, [7]. Experimental studies were also conceived in order to verify the results of the theoretical models, [8]. However, the technical studies have a common aspect, that is to conceive an optimal solution from an economic standpoint, i.e. to minimise the manufacturing costs and the exploitation expenses. This idea is also used in the context of the wastewater treatment power plant in order to minimise the costs. Regarding the sedimentation tank, the aim is to minimise the expenses for the electric energy spent to process the wastewater. The wide experience acquired in
hybrid modelling based on the development of the various types of software components, [9], allowed us to conceive a computer method useful to study the energy of this equipment.

2. Theoretical background
The equipment which supports the scrapping arms of the radial bridge is a structure in rotational motion having a constant speed, \( \omega, \left[ s^{-1} \right] \). Its outer end is supported on a wheel which rolls on the concrete boundary of the basin, figure 1, [8]. A complete rotation of the wheel around the basin’s boundary is done in \( T = 57 \) minutes.

![Figure 1. CAD model of the radial bridge in rotational constant motion around the central vertical axis.](image)

An extensive documentation regarding the mechanical aspects of the problem was done, being investigated the “solid rigid in rotational motion with constant speed” topic, [10-15]. The acceleration being null, the energy of the system is

\[
E = E_k + E_L,
\]

where \( E_k \) is the kinetic energy and \( E_L \) is the summation of all types of energy ‘consumed’ in the system to overcome friction, drag etc., so to say ‘losses’ is the system.

For a physical body in constant motion, the kinetic energy is

\[
E_k = \frac{1}{2} \cdot m \cdot v^2.
\]

The shape of the radial bridge is very complex, therefore, according to Descartes's method, we may use rule #2, "To divide each of the difficulties under examination into as many parts as possible, and as might be necessary for its adequate solution.\text{"}, [16]. According to this suggestion, the radial bridge is divided in \( NC \) so-to-say ‘simple’ components, and the kinetic energy may be expressed as

\[
E_k = \sum_{i=1}^{NC} \left( \frac{1}{2} \cdot m_i \cdot v_i^2 \right),
\]

where \( m_i \) is the mass of the \( i \) component, and \( v_i \) is its speed.
By expressing the linear speed with respect to the rotational speed, i.e. $v_i = \omega \cdot r_i$, the previous relation becomes

$$E_K = \frac{1}{2} \sum_{i=1}^{NC} \left( m_i \cdot r_i^2 \right) \omega^2 = \frac{1}{2} \cdot J \cdot \omega^2 ,$$

where

$$J = \sum_{i=1}^{NC} (m_i \cdot r_i^2)$$

is the moment of inertia defined using the integral

$$J = \int r^2 \cdot dm .$$

In order to compute the moment of inertia, $J$, using (5), we must compute the $m_i$ masses together with the according locations of their centres of gravity, $\left( x_{Gi}, y_{Gi}, z_{Gi} \right)$, which allows us to compute the $r_i$ radii with respect to the central vertical axis of rotation,

$$r_i = \sqrt{x_{Gi}^2 + y_{Gi}^2} .$$

Once we know $J$, we can compute the angular speed, $\omega$, using the time spent for a complete rotation, $T$, and, further on, the $E_K$ kinetic energy, using (4).

Figure 2. Propulsion system of the radial bridge, consisting of an electric motor, gear reducer and wheel.

The electric motor used to move to radial bridge has the power $P$, therefore we may compute the energy spent for a complete rotation,

$$E = P \cdot T .$$

Once we compute $E_K$, it results

$$E_L = E - E_K .$$

Preserving the same steady state conditions of the equipment, the energy of the system, $E$, may be reduced either by reducing $E_L$, or $E_K$. The $E_L$ energy is related to the phenomena inside the tank, therefore we consider the reducing the $E_K$ energy using an optimised geometry of the bridge for which

$$m_i^{opt} \leq m_i \text{ and } r_i^{opt} \approx r_i ,$$

this means to use a smaller amount of material without injuring the phenomena, the according optimised masses being located in the same positions as the initial ones.
In this way we compute the optimised kinetic energy, $E_{K}^{\text{opt}}$, and the energy of the optimised radial bridge,

$$E_{K}^{\text{opt}} = E_{K}^{\text{opt}} + E_{L}.$$

Finally, we compare the amount of energy which is saved by the optimized structure, i.e.

$$\Delta E = \frac{E_{\text{Total}} - E_{\text{Total}}^{\text{opt}}}{E_{\text{Total}}} \times 100\%.$$

The accuracy of the results depends on the accuracy of the method used to compute the $E_{K}$, kinetic energy.

3. Original computer based method
In order to have an accurate value of the $E_{K}$, therefore of the $J$ moment of inertia, we must have an accurate method to compute the $m_{i}$ masses of the ‘simple-shape’ components, together with the coordinates of their centres of gravity, $(x_{GI}, y_{GI}, z_{GI})$. 

![Figure 3](image-url)
We remark that the radial bridge may be divided in major components, such as: scrapping systems (inner, middle and outer), horizontal pipes (one for each scrapping system), upside down ‘drain trap’ (one for each horizontal pipe), in-tension beams, cantilevers along the main beam, main beam, access bridge, central-rectangular-parallelepiped and motor-gearbox-wheel unit.

Being a large number of components which require detailed modelling of the geometry, an original software was conceived in order to perform the huge amount of calculi.

All the three scrapping systems have a similar shape, being possible to parameterise the dimensions. In this way, the same computing module may be used to compute the moment of inertia of all of them. The input data for one of the scrapping systems is presented in figure 3.

![Figure 4](image)

**Figure 4.** Data used to define the horizontal pipe of the inner scrapping system and the principle used for the division of the upside-down ‘drain trap’.

The data used to define the horizontal pipes and the idea used to define the upside-down ‘drain trap’ are presented in figure 4. The larger number of arcs we consider, the more accurate results we have regarding the moment of inertia of the upside-down ‘drain trap’.

![Figure 5](image)

**Figure 5.** In-tension vertical beams of the inner scrapping system.

![Figure 6](image)

**Figure 6.** Cantilevers along the main beam which support the pipes and the scrapping systems.

The data used to the in-tension vertical beams of the scrapping systems are presented in figure 5. The cantilevers along the main beam are presented in figure 6. The cantilevers support the in-tension vertical beams, which support the scrapping systems and their horizontal pipes.

To perform the calculi a software application was developed in C++ and it consists of >1700 computer code lines. The application is not hard-coded, the necessary data being read from input CSV files. In this way new optimisation versions of the radial bridge may be considered, by modifying a small amount of data.
Figure 7. Configuration file is itself, a CSV file.

The structure of the configuration file is presented in figure 7, the configuration file being itself a CSV file. The readily use of the CSV files is allowed by the simple inclusion of the "load_csv_file.h" and "load_csv_of_strings.h" original header files in the computer code. As it can be noticed, the contents of each input data file is explained in the configuration file.

Figure 8. An example regarding the inner scrapping system dimensions.

Each input file has data which represent either dimensions, or coordinates of the points used to define the 'simple' shape. Figure 8 presents an example regarding the dimensions of the inner scrapping system. Colours are assigned to each set of values in order to easily identify a certain group of data related to a subsystem. The values are explained, the units are also presented, the order of multiplicity is specified and the index of a calculus scheme presented in a distinct figure is also mentioned.

All these detailed information is useful to have a clear explanation of the input data. Once the data sheet is filled in, it is saved as a CSV file which becomes an input file for the original software.
The computer code was structured in several functions. Each function is used to compute the moment of inertia of a given major component. The overall moment of inertia is initialized with zero and the according values of the moment of inertia computed for each major component are added during the data processing.

![Image of Eclipse IDE](image_url)

**Figure 9.** The computer code in Eclipse environment.

The application may be run without any tracing message, or using tracing messages in a given function, in order to closely watch the variables’ values in running conditions.

The results are stored in a report text file, figure 10. As it can be noticed, in the file’s designation was inserted a time stamp, in order to keep track of all the versions of the radial bridge dimensions.

![Image of Notepad](image_url)

**Figure 10.** Contents of the output data report for one of the optimisation scenarios.

4. **Discussion**

For the initial version of the radial bridge, the moment of inertia is

$$J_{ini} = 3571170.61 \text{ kg} \cdot \text{m}^2.$$  \hspace{1cm} (13)

For the optimised version of the radial bridge it results

$$J_{opt} = 2984523.159 \text{ kg} \cdot \text{m}^2,$$  \hspace{1cm} (14)
which is
\[ J_{\text{opt}} = 83.57\% \cdot J_{\text{init}}, \] (15)

therefore
\[ \Delta E_K = 16.43\% \cdot E_{K,\text{init}}. \] (16)

Starting from \( \omega \cdot T = 2 \cdot \pi \), for \( T = 57 \text{ min} = 3420 \text{ s} \), it results the angular speed
\[ \frac{\omega}{T} = \frac{2 \cdot \pi}{3420} = 0.0018372 \text{ s}^{-1}, \] (17)

which is a very small value.

The kinetic energy of the initial system is
\[ E_K = \frac{1}{2} \cdot J \cdot \omega^2 = \frac{1}{2} \cdot 3.571170.61 \cdot (0.0018372)^2 = 6.0268 \text{ J}, \] (18)

which is a very small value due to the small rotational speed, (17).

According to the technical data of the sedimentation tank, the power of the electric motor of the motor-gear reducer-wheel assembly, figure 2, is
\[ P = 0.55 \text{ kW} = 550 \text{ W}. \] (19)

The energy spent in a complete rotation of the radial bridge, in \( T = 57 \text{ min} = 3420 \text{ s} \) is
\[ E = P \cdot T = 550 \cdot 3420 = 1881000 \text{ J} = 1881 \text{ kJ}. \] (20)

The energy used to cover the ‘losses’ in the system is:
\[ E_L = E - E_K = 1881000 - 6.0238 = 1880993.9762 \text{ J}, \] (21)

which, in comparison with the energy produced by the electric motor is
\[ E_L = 99.999679\% \cdot E. \] (22)

For an optimised system we have (14), i.e.
\[ E_{K,\text{opt}} = \frac{1}{2} \cdot J \cdot \omega_{\text{opt}}^2 = \frac{1}{2} \cdot 2984523.159 \cdot (0.0018372)^2 = 5.0368 \text{ J}. \] (23)

the necessary energy to be produced by the electric motor being
\[ E_{\text{opt}} = E_L + E_{K,\text{opt}} = 1880993.9762 + 5.0368 = 1880999.013 \text{ J}, \] (24)

and the according power being
\[ P_{\text{opt}} = \frac{E_{\text{opt}}}{T} = \frac{1880999.013}{3420} = 549.9997 \text{ W} \geq 0.55 \text{ kW}. \] (25)

This means that the initial system was appropriately dimensioned according to our assumptions that the maximum efficiency is reached in the initial steady state conditions, which may be or may be not the most effective steady state conditions.

5. Conclusions
The effectiveness of the equipment used in the sedimentation tank is revealed using the results of this study. As it can be noticed, most of the energy produced by the electric motor is used to cover the so called ‘losses’ in the system. The power of the electric motor is very low, 550 W, the according expenses being also low.

The study is based on an original computer based method which was implemented in a C++ software consisting of >1700 computer code lines. This software instrument was used to compute the moment of inertia of the radial bridge. Several other ideas to evaluate the moment of inertia were considered, [18-20], but the high complexity of the structure made us consider the aforementioned analytical method that has a simple computing principle.

The original software instrument allowed us to conceive optimisation scenarios which minimise the kinetic energy which is however small in comparison with the overall energy.

The development of the computer code was an opportunity to conceive new header files which will be reused at a later stage, in other software projects for hybrid models in mechanical engineering. Moreover, the experience gained in the development of this software is useful to upgrade/generalise the programs.
already developed for the study of the sedimentation tank’s radial bridge: input data generator, analytical model implementation, data converter to Femap/Nastran and analytical model of the hydrodynamic loads.

6. References

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