Adaptive wavelet-based method for simulation of electronic circuits

Kai Bittner and Emira Dautbegovic

Abstract In this paper we present an algorithm for analog simulation of electronic circuits involving a spline Galerkin method with wavelet-based adaptive refinement. Numerical tests show that a first algorithm prototype, build within a productively used in-house circuit simulator, is completely able to meet and even surpass the accuracy requirements and has a performance close to classical time-domain simulation methods, with high potential for further improvement.

1 Introduction

Wavelet theory emerged during the 20th century from the study of Calderon-Zygmund operators in mathematics, the study of the theory of subband coding in engineering and the study of renormalization group theory in physics. The common foundation for the wavelet theory was laid down at the end of the 80’s and beginning of the 90’s by work of Daubechies [1,2], Morlet and Grossman [3], Donoho [4], Coifman [5], Meyer [6], Mallat [7] and others. Today wavelet-based algorithms are already in productive use in a broad range of applications [6–13], such as image and signal compression (JPEG2000 standard, FBI fingerprints database), speech recognition, numerical analysis (solving operator equations, boundary value problems), stochastics, smoothing/denoising data, physics (molecular dynamics, geophysics, turbulence), medicine (heart-rate and ECG analysis, DNA analysis) to name just a few. Recent approaches [14–18] to the problem of multirate envelope simulation indicate that wavelets could also be used to address the qualitative challenge by
a development of novel wavelet-based circuit simulation techniques capable of an efficient simulation of mixed analog-digital circuits [19].

The wavelet expansion of a function $f$ is given as

$$f = \sum_{k \in \mathcal{I}} c_k \phi_k + \sum_{j=0}^{\infty} \sum_{k \in \Lambda_j} d_{jk} \psi_{jk}. \quad (1)$$

Here, $j$ refers to a level of resolution, while $k$ describes the localization in time or space, i.e., $\psi_{jk}$ is essentially supported in the neighborhood of a point $x_{jk}$. The wavelet expansion can be seen as coarse scale approximation $\sum_{k \in \mathcal{I}} c_k \phi_k$ by the scaling functions $\phi_k$ complemented by detail information of increasing resolution $j$ in terms of the wavelets $\psi_{jk}$.

In the classical theory wavelets are generated as translation and dilations of a mother wavelet $\psi$, i.e., $\psi_{jk}(x) = \psi(2^{-j}x - k)$. However, more general approaches are often used, e.g., for the construction of wavelets on the interval [20] or wavelets for finite element spaces [21]. In particular, non-uniform spline wavelets [22] will be used in our wavelet-based circuit simulation.

Since a wavelet basis consist of an infinite number of wavelets, in practical computations one has to consider approximations of $f$ by partial sums of the wavelet expansion (1). A simple approach is to fix a maximal wavelet level $J$ and approximate $f$ by

$$f_J = \sum_{k \in \mathcal{I}} c_k \phi_k + \sum_{j=0}^{J} \sum_{k \in \Lambda_j} d_{jk} \psi_{jk}. \quad (2)$$

This approach is called linear approximation, since the approximation is determined in the linear space of wavelets with level less or equal $J$. For wavelets of sufficient regularity, one obtains error estimates of the form

$$\|f - f_J\|_{L_2} \leq C 2^{-Js} \|f\|_{W^s_2}, \quad (3)$$

with the Sobolev space $W^s_2$. However, approximation results as (3) hold also for other approximation methods, e.g., for Fourier sums (see [23]).

The real approximation power of wavelets is due to their locality, which implies that (3) holds also for small subintervals. Thus, a piecewise smooth function can be essentially approximated by some coarse scale approximation with wavelets added only at non-smooth parts to achieve a required accuracy. Doing this adaptively for any given signal leads to the notion of best $n$-term approximation, where the approximation is determined as linear combination of $n$ arbitrarily chosen wavelets. This results in an essentially improved approximation for a wide class of functions, e.g., piecewise smooth function with isolated singularities. For details about this adaptive, nonlinear approximation methods we refer to [23,24]. Usually it is not obvious which wavelets have to be chosen for optimal approximation results. In practice optimal wavelet representations can be determined by one of the two complementary strategies: coarsening or refinement.
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Coarsening is used if one has already a fine, highly accurate but expensive approximation, e.g., from measurements. The goal is to throw away as much information as possible, while introducing only a small error. For a wavelet representation this can be achieved quite easily by thresholding, which means that wavelets with small expansion coefficients are removed from the representations. Inherent stability properties of wavelets ensure that this elimination of terms with small coefficients do not add up to a significant error of the wavelet expansion. For wavelets with good localization and approximation properties, one has many small wavelet coefficients for piecewise smooth signals with few local singularities (e.g., sharp transients), which will result in an essential reduction of data for the coarsened signal. A disadvantage of coarsening is that it might be too costly to acquire the fine representation. In particular, for solving operator equations as in circuit simulation the reason for using adaptive wavelet techniques is the reduction of computational cost, which is thwarted by computing a non-adaptive solution in advance.

In contrast, the strategy of refinement is to start with coarse approximation and introduce successively more and more degrees of freedom (e.g., wavelets) in order to improve the approximation. However, since it is not known in advance, where refinements are necessary, one has to rely on rough estimates. Therefore it is reasonable to do the refinement in several steps. This allows to check the previous steps, while acquiring more information for later steps. This approach is in particular interesting for iterative methods, where the approximation is improved in each iteration step and the number of degrees of freedom can be increased accordingly.

2 An Adaptive Wavelet Galerkin Method

We consider circuit equations in the charge/flux oriented modified nodal analysis (MNA) formulation, which yields a mathematical model in the form of an initial-value problem of differential-algebraic equations (DAEs):

\[
\frac{d}{dt} q(x(t)) + f(x(t)) - s(t) = 0.
\]

(4)

Here \( x \) is the vector of node potentials and specific branch currents and \( q \) is the vector of charges and fluxes. Vector \( f \) comprises static contributions, while \( s \) contains the contributions of independent sources.

In our adaptive wavelet approach we first discretize the MNA equation (4) in terms of the wavelet basis functions, by expanding \( x \) as a linear combination of wavelets or related functions, i.e., \( x = \sum_{k=0}^{K} c_k \varphi_k \). For such \( x \) we integrate the circuit equations against test functions \( \theta_\ell \) and obtain the equations

\[
\int_0^T \left( \frac{d}{dt} q(x(t)) + f(x(t)) - s(t) \right) \theta_\ell \, dt = 0,
\]

(5)
for $\ell = 1, \ldots, n$. Together with the initial conditions $x(0) = x_0$, we have now $n + 1$ vector valued equations, which determine the coefficients $c_\ell$ provided that the test functions $\theta_\ell$ are chosen suitably to the basis functions $\phi_k$.

Due to the intrinsic properties of wavelets [19] nonlinear wavelet approximation can provide an efficient representation of functions with steep transients, which often appear in a mixed analog/digital electronic circuit. However, for an efficient circuit simulation we have to take into account further properties of a wavelet system. We consider spline wavelets to be the optimal choice since spline wavelets are the only wavelets with an explicit formulation. This permits the fast computation of function values, derivatives and integrals, which is essential for the efficient numerical solution of a nonlinear problem as given in (4) (see also [25, 26]). Spline wavelets have been already used for circuit simulation [27]. However, here we use a completely new approach based on spline wavelets from [22].

With a good initial guess, Newton’s method is known to converge quadratically. However, a good initial guess is usually not available. In practice we can often obtain convergence only with a slow converging initial phase of damped Newton steps, which will mainly contribute to the computational cost of the problem. On the other hand, to get a good approximation of the solution of (4), the space $X = \text{span}\{\phi_k : k = 0, \ldots, n\}$ has to be sufficiently large and the computational cost of each step depends on $n = \dim X$. Our approach is to use adaptive wavelet refinement during the Newton iteration, which leads to an efficient adaptive representation and essentially reduced computation time.

3 Interval Splitting Method

A prototype of the proposed adaptive wavelet algorithm is implemented within the framework of a productively used circuit simulator and tested on a variety of circuits. For tests on some typical RF circuits (amplifier, mixer, oscillator), we were able to reproduce the results from the transient analysis of the same circuit simulator up to high accuracy (see [28]). For all these examples, the wavelet method used a considerably smaller grid (i.e. larger stepsize) than the transient analysis, while the computation time was higher but still close to the standard method. This shows that there is a potential for wavelet methods in circuit simulation, if further optimization can be achieved.

However, in further tests with a Schmitt trigger circuit (Fig. 1, [29]) convergence could only be achieved with a highly accurate initial guess. This is of limited practical value, since we can usually not provide an initial guess of such quality. We identified inherent hysteresis of the Schmitt trigger as the main cause for this problem. In circuits exhibiting hysteresis, certain input voltages can result in different output, depending on the previous behaviour of the input signal. With an insufficient initial guess Newton’s method may approach locally the wrong result. This effect was observed in a Harmonic Balance simulation too, where the solution is also represented by a basis expansion over an entire period.
This convergence problem was successfully addressed by a further improvement of the basic wavelet method based on an interval splitting mechanism. Basically the wavelet method is applied to a series of smaller intervals when no convergence is detected. This is an analogous approach to the reduction of the step size in transient analysis if no convergence is encountered in the current time step. In order to preserve continuity, the initial value for each interval is obtained from the wavelet expansion of the solution on the previous interval. Furthermore, the interval size is adapted after each successful step, aiming to keep the problem size for the wavelet method in a nearly optimal range.

4 Numerical Tests

The interval splitting method was implemented as an enhancement to the basic wavelet algorithm and tested on a variety of circuits. For all examples we have compared the CPU time and the grid size (i.e., the number of spline knots or time steps) with the corresponding results from transient analysis of the underlying circuit simulator.

The error is estimated by comparison with well established high accuracy transient analysis. The estimate shown in the signal is the maximal absolute difference over all transient grid points, which gives a good approximation of the maximal error. That is, if we can obtain a small error for the wavelet analysis, this proves good agreement with the standard method. In particular, since we compare the solutions of two independent methods we have very good evidence that we approximate the solution of underlying DAE’s with the estimated error.

**Schmitt trigger.** The first test circuit is the Schmitt trigger [29]. As can be seen in Fig. 2 the output of the Schmitt trigger circuit signal jumps to a higher level if the input exceeds an upper threshold and jumps back to low if the input falls below
a lower threshold. However, due to capacitances present in the transistor model the jumps are slightly smoothed and delayed.

**Fig. 2** Input and output signal for the Schmitt trigger.

In both examples, the interval splitting wavelet method could produce the correct results. i.e., we achieve robustness for the wavelet-based approach. The performance is comparable to transient analysis, although the current implementation is not faster than the reference method. However, we see a big potential for the improvement of the implemented method.

**Inverter chain.** A further test circuit was an inverter chain consisting of 9 inverters. Therefore, the output signal represents the 9-times inverted digital input signal. However, we can observe a delay and a modification in the transition between high and low signal due to intrinsic properties of used technology. Similar to the hysteresis effect in the previous problem, the output depends strongly on the earlier behaviour of the input signal, which again requires the use of the interval splitting wavelet method to obtain the correct results.

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Fig. 4 Input and output signal of the inverter chain.

Fig. 5 Simulation results for the inverter chain. Computation time versus error (left), and grid size versus error (right) for transient analysis and adaptive wavelet analysis.

5 Conclusion

The results of the simulations indicate that the wavelet-based method is able to fulfill all accuracy requirements and may achieve the performance of the standard transient analysis. Since the relatively new wavelet approach has a large potential for optimization, we are optimistic that wavelet analysis will be a valuable tool for circuit simulation in the future. Therefore our activities on optimization and further development of the wavelet-based algorithm are continuing.

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