Δκ′_V and ΔQ_V (V = γ, Z) form factors in the Georgi-Machacek model

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Abstract. The CP-even static form factors Δκ′_V and ΔQ_V (V = γ, Z) associated with the WWV vertex are studied in the context of the Georgi-Machacek model (GMM), which predicts nine new scalar bosons accommodated in a singlet, a triplet and a fiveplet. General expressions for the one-loop contributions to Δκ′_V and ΔQ_V arising from neutral, singly and doubly charged scalar bosons are obtained in terms of both parametric integrals and Passarino-Veltman scalar functions, which can be numerically evaluated. It is found that the GMM yields 15 (28) distinct contributions to Δκ′_γ and ΔQ_γ (Δκ′_Z and ΔQ_Z), though several of them are naturally suppressed.

1. Introduction
Models with scalar triplet representations have attracted considerable attention due to their appealing features, such as the possibility of implementing the see-saw mechanism to endow the neutrinos with naturally light Majorana masses (the so called type-II see-saw), the appearance of the H^+W^-Z coupling at the tree level, and the presence of doubly charged scalar particles. In this respect, the Georgi-Machacek model (GMM) [1, 2] is one of the most attractive Higgs triplet models as it preserves the relationship ρ = 1 at the tree level via an SU(2) custodial symmetry. The physical scalar spectrum of the GMM is given by the SM-like Higgs boson h and one extra CP-even singlet H, one scalar triplet H_3 (H_3^0, H_3^±), and one scalar fiveplet H_5 (H_5^0, H_5^{±±}, H_5^{±}). All of these multiplets are mass degenerate as a result of the custodial symmetry. Even if there is not enough energy available to produce the new scalar particles predicted by the GMM, one can search for their virtual effects through some observables. Particular interest has been put on the radiative corrections to the WWV (V = γ, Z) vertex, which represents a very sensitive scenario to search for any NP effects and test the gauge sector of the SM.

The on-shell WWV vertex can be written in terms of four form factors that define the CP-even and CP-odd static properties of the W boson. The two CP-odd form factors Δκ′_V and ΔQ_V are absent up to the one-loop level in the SM and are thus expected to be negligibly small. As far as the CP-even form factors Δκ′_V and ΔQ_V are concerned, they arise at the one-loop level in the SM and any other renormalizable theory, thereby being highly sensitive to NP effects.

The vertex function that determines the WWV CP-conserving coupling can be written as

\[ \Gamma_V^{\mu\alpha\beta} = ig_V \left[ A \left( 2p_\mu g^{\alpha\beta} + 4 \left( Q_\beta g^{\mu\alpha} - Q_\alpha g^{\mu\beta} \right) \right) \right] \]
2. \( \Delta \kappa_V^I \) and \( \Delta Q_V \) form factors in the GMM

The new scalar one-loop contributions from the GMM to the \( \Delta \kappa_V^I \) and \( \Delta Q_V \) form factors arise from generic triangle diagrams (the bubble diagrams do not contribute) that can be classified according to the number of distinct particles circulating into the loop. Each type of diagram involves trilinear vertices between gauge bosons only or between scalar bosons and gauge bosons. In Fig. 2 we show a set of Feynman diagrams that contribute to both the \( WW\gamma \) and \( WWZ \) vertices.

Contrary to the couplings of the photon to a pair of charged scalar bosons, which can only be of diagonal type due to electromagnetic gauge invariance, the \( Z \) boson can have nondiagonal couplings to a pair of neutral or charged scalar bosons. Therefore, in addition to the diagrams of Fig. 2, the \( \Delta \kappa_Z^I \) and \( \Delta Q_Z \) form factors can receive extra contributions from the Feynman diagrams shown in Fig. 3, which have three distinct particles circulating into the loop. Below we will present the contributions to the \( \Delta \kappa_V^I \) and \( \Delta Q_V \) form factors for all these type of diagrams.

Before presenting our results, some remarks about our calculation are in order:

- The Feynman diagrams were evaluated via the unitary gauge. In order to make a cross check of our results we used both, the Feynman parametrization technique and the Passarino-Veltman method to solve the loop integrals.
Figure 3. Extra contributions to the $\Delta \kappa_Z'$ and $\Delta Q_Z$ form factors from nondiagonal couplings.

- We verified that all the contributions of bubble diagrams to the $\Delta \kappa_V'$ and $\Delta Q_V$ form factors involving quartic vertices with two scalar bosons and two gauge bosons vanish, and thus the only contributions arise from triangle diagrams.

- The mass shell and transversality conditions for the gauge bosons enabled us to make the following replacements

$$Q^2 = m_V^2, \quad p \cdot Q = 0, \quad p^2 = m_W^2 - m_V^2,$$

and

$$p_\alpha \rightarrow Q_\alpha, \quad p_\beta \rightarrow -Q_\beta, \quad p_\mu \rightarrow 0,$$

which results in a considerable simplification of the calculation.

- Instead of dealing with the calculation of the $WW\gamma$ and $WWZ$ vertices separately, we performed instead the calculation of the general $WWV$ vertex, with $V$ a massive neutral gauge boson. We have exploited the fact that there are only three generic trilinear vertices involved in the one-loop contributions to the $WWV$ vertex and thus a model independent calculation was done using the generic Feynman rules of Fig. 4. The result for the contribution of each type of Feynman diagram will be presented in terms of loop functions, given as parametric integrals and also in terms of Passarino-Veltman scalar integrals, times a factor involving all the generic coupling constants associated with each vertex participating in the particular diagram. The contribution to the form factors of the $WW\gamma$ and $WWZ$ vertices follow easily from our general expressions after taking the appropriate mass limits and substituting the corresponding coupling constants of the GMM or any other extension model.

- We corroborated all the contributions to the $\Delta \kappa_V'$ and $\Delta Q_V$ form factors are free of ultraviolet divergences.

Once the amplitude for each Feynman diagram is written down, the Feynman parametrization technique and the Passarino-Veltman method can be applied straightforwardly, along with some cumbersome algebra. Thereafter one can express the contributions to the $\Delta \kappa_V'$ and $\Delta Q_V$ form factors for each type of Feynman diagram of Fig. 2 as follows

$$\Delta \kappa_V^{\alpha} = -\frac{C_V^i}{16\pi^2} F^{-i}(x_A, x_B, x_V),$$

$$\Delta Q_V^{\alpha} = -\frac{C_Q^i}{16\pi^2} F^{-i}(x_A, x_B, x_V),$$

where $C_V^i$ and $C_Q^i$ are the generic coupling constants associated with the vertices $V$ and $Q$, respectively.
\[ \Gamma_{\mu\alpha\beta} = g_{\mu\alpha}(p_1 - p_3)\beta + g_{\alpha\beta}(p_2 - p_1)\mu + g_{\beta\mu}(p_3 - p_2)\alpha. \]

\[ V = \gamma, Z, \phi_I (I = A, B, C) \text{ denote a neutral singly, or doubly charged scalar boson, and } X_J (J = A, B) \text{ stands for a neutral or charged gauge boson.} \]

\[ \Delta Q^i_V = -\frac{C^i_V}{16\pi^2} I^{-i}_Q(x_A, x_B, x_V), \]

for \( V = Z, \gamma \) and \( i = a, b, c \). We have introduced the scaled variable \( x_I = m_I^2/m_V^2 \) \((I = A, B)\), with \( m_A \) and \( m_B \) denoting the masses of the particles circulating into each type of diagram. A word of caution is in order here as \( m_A \) and \( m_B \), and thereby \( x_A \) and \( x_B \), are distinct for each type of contribution, which in turn is denoted by the superscript \( i \) \((i = a, b, c)\). As for the loop functions \( I^{-i}_k \) and \( I^{-i}_Q \), they are in terms of parametric integrals and Passarino-Veltman scalar integrals and can be found in [5], while the \( C^i_V \) factors are given in term of the coupling constants of the vertices involved in each Feynman diagram.

As explained above, the \( \Delta \kappa^i_Z \) and \( \Delta Q^i_Z \) form factors can be obtained from the general expressions (4) -(5) by taking the \( m_V \rightarrow 0 \) limit. We have verified that these expressions are in agreement with the results presented in Ref. [4], where the \( WW\gamma \) vertex was studied in the context of little Higgs models.

As far as the Feynman diagrams of Fig. 3 are concerned, they only contribute to the \( WWZ \) vertex and the respective form factors depend now on three distinct internal masses. They can be written as follows

\[ \Delta \kappa^i_Z = -\frac{C^i_Z}{16\pi^2} I^{-i}_k(x_A, x_B, x_C, x_Z), \]

\[ \Delta Q^i_Z = -\frac{C^i_Z}{16\pi^2} I^{-i}_Q(x_A, x_B, x_C, x_Z). \]

This time the superscript \( i = d, e, f \) stands for the whole contributions of diagrams \( i_1 \) and \( i_2 \), and again the loop functions \( I^{-i}_k \) and \( I^{-i}_Q \) can be found in [5].

Once the general expressions for the different kinds of contributions are obtained, we can compute the total contribution of the scalar sector of a given model by simple adding up all the partial contributions. We will present below a numerical analysis of the contributions of the GMM. For the numerical evaluation we computed the parametric integrals via the Mathematica numerical routines. A cross check was done using the results obtained by evaluating the results given in terms of Passarino-Veltman scalar functions with the help of the LoopTools routines.

2.1. \( \Delta \kappa^i_Z \) and \( \Delta Q^i_Z \) form factors

Excluding the pure SM contributions, in the GMM, the \( \Delta \kappa^i_Z \) and \( \Delta Q^i_Z \) form factors receive 10 contributions of the type-(a) diagrams, 3 of the type-(b) diagrams, and 2 of the type-(c) diagrams. Notice that all the new scalar bosons participate in the type-(a) diagrams, whereas the type-(b) diagrams only receive contributions from the singlet and the fiveplet scalar bosons,
and the type-(c) diagrams from the fiveplet scalar bosons only. We examine the general behavior of \( \Delta \kappa'_Z \) and \( \Delta Q_Z \) as functions of the masses of the scalar bosons. For the type-(b) and type-(c) contributions we show in Fig. 5 the form factors as a function of the mass of the scalar boson circulating into the loop, whereas for type-(a) diagram we consider two scenarios: when both scalar bosons are degenerate and when one scalar boson mass is fixed and the other one is variable.

![Figure 5](image_url)

**Figure 5.** Behavior of the \( \Delta \kappa'_Z \) and \( \Delta Q_Z \) form factors as functions of the masses of the scalar bosons circulating into the loops of each type of contribution divided by the \( C_i \) coefficient and in units of \( a = g^2/(96\pi^2) \). While type-(a) contribution depends on two scalar boson masses \( m_{S_1} \) and \( m_{S_2} \), type-(b) and type-(c) diagrams depends on only one scalar boson mass \( m_{S_1} \).

### 2.2. \( \Delta \kappa'_Z \) and \( \Delta Q_Z \) form factors.

We will now analyze the \( \Delta \kappa'_Z \) and \( \Delta Q_Z \) form factors, for which we will follow a similar approach to that used above. We thus start by studying the general behavior of the distinct types of contributions. Apart from the diagrams of Fig. 2, there is additional contributions due to the diagrams of Fig. 3. As for the contributions of type (a), (b) and (c), their behavior is quite similar to that observed in Fig. 5, so here we will thus focus on the analysis of the extra contributions, whose behavior will turn out to be rather similar to that of contributions type (a), (b) and (c). There are 7 contributions of type (d), 4 of type (e), and 3 of type (f) to the \( \Delta \kappa'_Z \) and \( \Delta Q_Z \) form factors. Although our general results allow us to calculate type-(d) contributions with three distinct scalar boson masses \( m_{S_1}, m_{S_2} \) and \( m_{S_3} \), in the GMM all the masses of the same multiplet are degenerate. It means that type-(d) contributions arise only from diagrams with at least two degenerate scalar bosons. Also, although type-(e) contribution arise from diagrams that can have two distinct scalar bosons, their masses are degenerate and there is dependence on one mass only, and this is also true for type-(f) contributions. Therefore, we expect that type-(d) contributions will be dominant as long as there is a large mass splitting between the scalar boson masses, whereas type-(e) and type-(f) contributions will only be important for a relatively light scalar boson mass. This is depicted in Fig. 6, where we show the behavior of the \( \Delta \kappa'_Z \) and \( \Delta Q_Z \) form factors for all the scenarios allowed in the GMM. For type-(d) contributions we consider three scenarios: \( m_{S_1} \) fixed and \( m_{S_2} = m_{S_1} \) variables, \( m_{S_1} = m_{S_2} \) fixed and \( m_{S_3} \) variable, and the three scalar boson masses degenerate \( m_{S_1} = m_{S_2} = m_{S_3} \). On the other hand, for type-(e) contributions we only consider the case when the two scalar bosons are degenerate. In Fig. 6 we observe that \( \Delta \kappa'_Z \) and \( \Delta Q_Z \) have a similar behavior to that of the \( \Delta \kappa'_\gamma \) and \( \Delta Q_{\gamma} \) form factors. In particular, the largest contributions to \( \Delta \kappa'_\gamma \) are reached when there is a large mass splitting or when all the scalar bosons masses circulating into each loop are relatively light.
However, the decrease of $\Delta \kappa'_{\gamma}$ for large $m_{S_1}$ is now less quick than in the case of $\Delta \kappa'_{\gamma}$. Again, the $C'_Z$ factor is proportional to $v$, so the values shown in the plots for type-(e) and type-(f) contributions will increase by two orders of magnitude. As for $\Delta Q_Z$, it will reach its large value for the smallest allowed scalar boson masses as in the case of $\Delta Q_\gamma$. When the scalar bosons are very heavy, they will be approximately degenerate, in which case $\Delta Q_Z$ will decrease significantly. Extra suppression for both form factors can arise from the $C'_Z$ coefficients and from potential cancellations between the distinct contributions.

![Figure 6. Behavior of the $\Delta \kappa'_{V}$ and $\Delta Q_{V}$ form factors as a function of the masses of the scalar bosons circulating into the loops of each type of contribution divided by the $C'_Z$ coefficient. We only consider the possible scenarios arising in the GMM.](image_url)

3. Conclusions

A model independent calculation was done via both the Feynman parameter technique and the Passarino-Veltman reduction scheme. Our general results are expressed in terms of a six generic contributions to $\Delta \kappa'_{V}$ and $\Delta Q_{V}$ from scalar bosons that can be used to calculate the corrections arising from models with an extended scalar sector predicting new neutral, singly, and doubly charged scalar bosons. For the numerical analysis we have focused on the GMM, this model predicts 9 new scalar bosons accommodated in a singlet, a triplet and a fiveplet, which yield 15 new contributions to $\Delta \kappa'_{\gamma}$ and $\Delta Q_{\gamma}$, whereas $\Delta \kappa'_{Z}$ and $\Delta Q_{Z}$ receive 28 distinct contributions. The general behavior of the $\Delta \kappa'_{V}$ and $\Delta Q_{V}$ form factors was analyzed and it was found that $\Delta \kappa'_{V}$ reaches values of the order of $a = g^2/(96\pi^2)$, with the largest values arising from the diagrams with two nondegenerate scalar bosons provided that there is a large splitting between their masses. On the other hand $\Delta Q_{V}$ reaches values of the order of $10^{-2}a$, with the largest contributions arising from diagrams with relatively light degenerate scalar bosons. Both form factors decrease rapidly when all the scalar boson masses are heavy. The values for $\Delta \kappa'_{V}$ and $\Delta Q_{V}$ predicted by the GMM are competitive with the ones predicted by other weakly coupled SM extensions, but a very high precision still would be necessary to detect such effects.

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