Gluon production in the Lipatov effective action formalism.

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Abstract

Gluon production on two scattering centers is studied in the formalism of reggeized gluons. Different contributions to the inclusive cross-section are derived with the help of the Lipatov effective action. The AGK relations between these contributions are established. The found inclusive cross-section is compared to the one in the dipole picture and demonstrated to be the same.

1 Introduction

One of the basic processes in high-energy collisions is the inclusive gluon production off heavy nuclear targets. At high energies, in the QCD with a large number of colours $N_c \to \infty$ it can be studied either in the interacting reggeized gluon approach [1, 2, 3] or, alternatively, in the framework of the dipole picture (or equivalent JIMWLK approach with certain approximations) in which the interacting hadrons are presented in terms of colour dipoles with their density evolving in rapidity [4]. The two approaches seem to be based on different pictures and approximations and it is vitally important to understand if they are completely equivalent or have some significant differences. It is well-known that the total cross-section for the dipole-dipole scattering turns out to be identical in the reggeized gluons and evolving dipoles pictures, both approaches leading to the same BFKL equation, although in different spaces, momentum or coordinate ones. The situation with the inclusive gluon production turned out to be more complicated. In particular in [3] it was advocated that this cross-section off two centers found in the framework of the dipole approach in [4] is not complete and has to be supplemented by new terms involving states composed of three and four reggeized gluons (the so-called BKP states [5, 6]). On the other hand in [7] it was shown that, at least in lowest orders, contributions from such states in fact cancel, so that one is left with exactly the cross-section obtained in the dipole approach. It should be stressed however that this conclusion was found in the purely transversal approach in which the validity of the standard AGK relations [8] for different cuts of the scattering amplitude was assumed without proof. To finally compare the reggeized gluon and dipole approaches one has to study contributions from all different cuts and check the validity of the mentioned AGK rules. This cannot be done in the purely transversal approach used in [1, 2, 7] (nor in the dipole approach) but requires knowledge of the amplitude as a function of its longitudinal momenta. This information is trivial for production on a single scattering center in the target. But it becomes considerably more complicated when the target involves two or more centers.

To study the inclusive gluon production taking into account the dependence of the relevant amplitudes on the longitudinal momenta we use the Lipatov effective action [9], which provides a powerful and constructive technique for the calculation of all Feynman diagrams in the Regge kinematics.

To understand our derivation it is instructive to first visualize the elastic amplitude for the scattering off the nuclear target in the reggeized gluon technique, which, as mentioned is completely equivalent to the dipole picture. The bulk of the amplitude can then be presented in terms of pomeron propagating from the projectile to many nucleons of the target each one splitting in two with a certain
known triple pomeron vertex (pomeron fan diagrams). These diagrams are to be supplemented by simpler ones with less pomerons, which contribution corresponds to the glauberized initial condition for the dipole-nucleus amplitude in the dipole formalism. These two parts are shown in Fig. 1 for the scattering on two centers.

To calculate the inclusive cross-section one has to fix a real intermediate gluon either inside the initial pomeron or inside the triple pomeron vertex. The contribution from the real gluons inside the pomerons after splitting is absent due to trivial AGK cancellations. So the inclusive cross-section actually consists of three parts: emission from the initial pomeron before all splittings, emission from the triple pomeron vertex (Fig. 1 the first diagram) and emission from the pomeron directly coupled to the targets (Fig. 1 the second diagram). These three parts are illustrated in Fig. 2. Emission from the pomeron chain is well understood and has the same form in the reggeized gluon picture and the dipole model. Thus the comparison of both approaches is reduced to calculation of the contribution from the triple pomeron vertex. Since the latter does not involve evolution and so contains only one intermediate gluon this comparison can be made in the lowest order in the coupling constant and moreover for any choice of the projectile and targets. This allows to simplify the problem and reduce it to calculation of the inclusive cross-section on only two centers in the lowest order with the projectile and two targets chosen as quarks. This calculation constitutes the subject of our paper.

Figure 1: Diagrams for the total cross-section on two centers.

Figure 2: Diagrams for the inclusive cross-section on two centers.

Note that one can additionally draw diagrams with consecutive splitting of the initial pomeron into first three and afterwards four reggeized gluons, as shown in Fig. 3. As was shown long ago,
their contribution in fact reduces to that of the diagrams in Fig. 1 with the effective 3-pomeron vertex \[10\]. However, there remained a question whether such a reduction worked for the inclusive cross-sections. Precisely this problem was discussed in \[3\]. As mentioned above in \[7\] in the lowest non-trivial (next-to-leading) order it was shown that diagrams of the type depicted in Fig. 3 give no new contributions. In the dipole approach this result is valid at all orders \[4\]. For this reason we do not consider such diagrams.

As mentioned the important tool in our calculation is the Lipatov effective action which allows to calculate the vertices for transition of a reggeized gluon ("reggeon" (R)) into one, two or three reggeons with emission of a real gluon ("particle" (P)), that is transitions R→RP, R→RRP, R→RRRP. While the R→RP ("Lipatov") vertex has been known since long ago, the required R→RRP and R→RRRP vertices were recently calculated in papers \[11\] and \[12\] respectively. A remarkable result derived in \[13\] for the R→RRP vertex, and in \[12\] for the R→RRRP is that the 4 dimensional amplitudes calculated with the help of the effective action can be obtained from the purely transverse picture with real intermediate particles (quarks and gluons) carrying the standard Feynman propagators (see \[14\] for details). This technique will be the main instrument in our calculations of the inclusive cross-section.

We have also to mention that a part of the inclusive cross-section corresponding to the intermediate states with real quarks from both of the targets (see Fig. 4) has already been calculated in \[11\]. So in this paper we calculate the rest of the contributions: the diffractive one (D) and that with only one of the target quarks appearing as real in the intermediate states ("single cut" (SC)). After summing all the contributions we compare the result with the dipole picture. Our conclusion is that the inclusive cross-section for gluon production is the same in both approaches.

The paper is organized as follows. Sections 2 and 3 are devoted to the calculations of SC and D contributions, respectively. In Section 4 we sum all the contribution to find the final expression for
the inclusive cross-section and compare our result with the one in the dipole model in the same order to establish their identity. Finally we present our conclusions in the last section. Our appendices serve to establish correspondence of our diagrams with the cross-section and recall the derivation of the inclusive cross-section for gluon production from the BFKL chain.

2 "Single cut” contribution

2.1 General notation

![](image)

Figure 5: Diagrams of the "single cut” contribution.

We consider the imaginary part of the amplitude for production of a gluon \( p \) in the collision of the projectile quark with equal initial and final momenta \( k \) with two target quarks with initial momenta \( l_1 = l_2 = l \) and final momenta \( l'_1, l'_2, l'_1 + l'_2 = l_1 + l_2 \) (see Fig. 5.1). We denote \( k' = k - q \) the intermediate quark momentum. The momentum transferred to the targets is \( \lambda = l'_1 - l_1 = l_2 - l'_2 \). To the total result one has to add the complex conjugated contribution corresponding to Fig. 5.2 and a similar amplitude with the exchanged targets \( l_1 \) and \( l_2 \). The latter gives the same contribution and so can be dropped for the imaginary part which is one half of the discontinuity.

The general Regge kinematics with the emitted gluon momentum \( p \) in the central region is defined by the following relations:

\[
\begin{align*}
  k_+ &> p_+ \approx q_+ >> q_{1,2,3+} \sim l_{1,2+} \sim l'_{1,2+}, \\
  l_{1,2-} &\sim l'_{1,2-} >> q_{1,2,3-} \sim p_- >> q_- \sim k_-, 
\end{align*}
\]

whereas all transverse components are implied to be of the same order much smaller than the largest longitudinal components. As shown in Appendix 1, the leading contribution to the production on two targets from the heavy nucleus is determined only by the delta-functional singularity at \( 2(k\lambda) = \sqrt{s}\lambda_0 = 0 \) with \( \lambda_+ = 0, \lambda_- = 0 \). We work in the c.m. system where

\[
\begin{align*}
  k_+ &= \sqrt{s}, \quad k_- = 0, \quad l_+ = \sqrt{s}, \quad l_- = 0, \quad k_+ = l_+ = 0; \quad s = (k + l)^2.
\end{align*}
\]

In the Regge approximation one have to neglect all transverse components of momenta compared to large longitudinal ones. The relative order of components of different momenta was considered in [14] and it was found that one has to neglect also the component \( p_- \) (which equals \(-p_+^2/p_+\) for real gluon) compared to \( \lambda_- \) before separating out the terms singular in the point \( \lambda_- = 0 \). These rules are equivalent to taking the limit \( k_+, p_+ \rightarrow \infty, k_+ >> p_+ \).

The gluon production amplitude with three reggeons attached to the targets (the filled blob on the right side of the cut in Fig. 5.1) was found in [12] as a sum of three terms:

\[
\begin{align*}
  -g^4 \gamma_+ f^{db_3a} &\left[ \frac{k_+^2 \cdot T^{d_1} T^{b_2} T^{b_1}}{(k - q_1 - q_2)^2 + i0)((k - q_1)^2 + i0)} + \frac{k_+^2 \cdot T^{b_1} T^{d_3} T^{b_2}}{((k' - q_1)^2 + i0)((k - q_2)^2 + i0)} \right]
\end{align*}
\]
The conjugated amplitude on the left side of the cut in Fig. 51 is

\[-ig^3[\bar{u}(k)\gamma_+ u(k')] \cdot [\bar{u}(l_2^+) \gamma_- u(l_2^+)] f^{b_4 a T b'} L(p, q_2) \frac{q_2^{2,\perp}}{q_2^{2,\perp}} \cdot \]

Diagrams for "single cut" contribution are shown in Figs. 68. The notation of momenta is as follows. The target interacting with two reggeons (uncut line) has the momentum $l_1$, the correspondent reggeon momentum flowing into the right interaction vertex is denoted $q_1$ for all cases and $q_3$ is the momentum flowing into the left vertex, so that $q_1 + q_3 = l_1' - l_1 = \lambda$. The other target (cut line) has the momentum $l_2$, it interacts with the reggeon $q_2$ on the right side of the cut and with the reggeon $q_2' = q - p$ on the left side (Diagram Fig. 51 presents the momentum flow). From the conservation law $q_2' = q - p = q_2 + l_2 - l_2' = q_2 + \lambda$ the relation $q_2^{2,\perp} = q_2^{2,\perp}$ follows (used in 68). Reggeons with momenta $q_1$, $q_2$, $q_3$ are labeled as 1,2,3 in Figs. 68 the correspondent color indices are denoted as $b_1$, $b_2$, $b_3$, respectively.

To invoke the overall colorless exchange we assume that the color structure of the diagrams corresponds to interaction with quark loops. Closing of quark loops also implied for the spinor structure, although we do not perform loop momentum integration. Thus for all diagrams we have the color factor for the target quark lines (both cut and uncut) $tr(T^{b_1} T^{b_2})/N_c = \delta^{bb'}/2N_c$. Here factor $1/N_c$ compensates the sum over $N_c$ colors. The rest color factors are different and will be calculated separately for each diagram.

For the spinor factors we sum over spins of intermediate quark states and take an average over spins of the initial and the final quark states. So for the projectile quark we have (here and below we use the Regge approximation neglecting small components of momenta)

\[ i \frac{1}{2} \sum_{\sigma} \bar{u}_\sigma(k) \gamma_+ \cdot (\hat{k} - \hat{q}) \cdot \gamma_+ u_\sigma(k) = \frac{1}{2} tr(\gamma_+ (\hat{k} - \hat{q}) \gamma_+ \hat{k}) \approx 4k_+^2 \]

for the uncut target quark line:

\[ i \frac{1}{2} \sum_{\sigma} \bar{u}_\sigma(l_1) \gamma_- \cdot \frac{i}{l_1 + q_1} \cdot \gamma_- u_\sigma(l_1) \approx -l_1 = -l_- \]

and for the cut one:

\[ i \frac{1}{2} \sum_{\sigma} \bar{u}_\sigma(l_2) \gamma_- \cdot (\hat{l}_2 + \hat{q}_2) \cdot \gamma_- u_\sigma(l_2) \approx 2i l_-^2 = 2i l_-^2 . \]

These factors are common for all diagrams of this Section. The common order in the coupling constant is $g^{10}$. 

\[ + \left( \frac{k_+^2 \cdot T^{b_1} T^{b_2} T^d}{((k' + q_2)^2 + i0)((k' + q_1 + q_2)^2 + i0)} \right) L(p, q_3) + P \]

\[ + \frac{k_+^2 \cdot T^{b_1} T^{b_2} T^d}{((k' + q_2)^2 + i0)((k' + q_1 + q_2)^2 + i0)} \right) L(p, q_3) + P \]

\[ + \frac{k_+^2 \cdot T^{b_1} T^{b_2} T^d}{((k' + q_2)^2 + i0)((k' + q_1 + q_2)^2 + i0)} \right) L(p, q_3) + P \]

\[ + \frac{k_+^2 \cdot T^{b_1} T^{b_2} T^d}{((k' + q_2)^2 + i0)((k' + q_1 + q_2)^2 + i0)} \right) L(p, q_3) + P \]

\[ + \frac{k_+^2 \cdot T^{b_1} T^{b_2} T^d}{((k' + q_2)^2 + i0)((k' + q_1 + q_2)^2 + i0)} \right) L(p, q_3) + P \]
2.2 Calculation of integrals over longitudinal variables

With our notation, momenta $q_1$ and $q_2$ are independent and can be chosen as variables of integration. Using the conservation law $q = p + q_2 + \lambda$ and mass-shell conditions for cut quark lines we can integrate over two of four longitudinal components (factors $1/2$ are from our normalization of longitudinal components $q_\pm = q_0 \pm q_z$):

$$\int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} 2\pi \delta((k-q)^2) \cdot 2\pi \delta((l_2+q_2)^2)$$

$$= \frac{1}{2k_+l_2} \int \frac{dq_{1+}dq_{1-}}{2(2\pi)^2} \int \frac{dq_{2+}dq_{2-}}{(2\pi)^4}.$$  \hspace{1cm} (12)

Due to the Regge kinematics \(\Pi\) in what follows we have to neglect momentum components

$$q_- = p_- + q_{2-} + \lambda_- = - \frac{q_2^2}{k_+-q_+}$$  \hspace{1cm} (13)

and

$$q_{2+} = -l_{2+} - \frac{q_{2\perp}^2}{l_{2-}+q_{2-}}.$$  \hspace{1cm} (14)

Calculation of integrals over the remaining longitudinal components can be carried out in a general form. For all diagrams considered in this and the following sections with our notation of reggeon momenta the momentum of the virtual target quark interacting with two reggeons is always $l_1 + q_1$. As argued in \[13\], the propagator of the fast moving target quark in the Lipatov effective theory is not

$$i \frac{l_1 + \hat{q}_1}{(l_1 + q_1)^2 + i0},$$  \hspace{1cm} (15)

but is reduced to its delta-functional part

$$i(l_1 + \hat{q}_1) (-i\pi)\delta((l_1 + q_1)^2) = \frac{l_1 + \hat{q}_1}{l_{1-} + q_{1-}} \pi\delta(q_{1+})$$  \hspace{1cm} (16)

where we used $l_{1+} = 0$ and $l_{1-} >> |q_{1\perp}|$. Then the common integral with two propagators has the form

$$I_2(p_1) = \int \frac{dq_{1+}dq_{1-}}{2(2\pi)^2} \frac{1}{(p_1 - q_1)^2 + i0} \pi\delta(q_{1+})$$

$$= \frac{1}{4} \int \frac{dq_{1-}}{2\pi} \frac{1}{(p_{1-} - q_{1-})p_{1+} + (p_1 - q_1)^2 + i0}.$$  \hspace{1cm} (17)

Neglecting the transverse part and splitting the integrand into the principal value part and $\delta$-function we get

$$I_2(p_1) \approx \frac{-i}{8|p_{1+}|}.$$  \hspace{1cm} (18)

The common integral with three propagators has the general form

$$I_3(p_1, p_2) = \int \frac{dq_{1-}dq_{1+}}{2(2\pi)^2} \frac{1}{((p_1 - q_1)^2 + i0)((p_2 + q_1)^2 + i0)} \pi\delta(q_{1+})$$

$$\approx \frac{-i}{4p_{1+}p_{2+}} \cdot \frac{\text{sign}(p_{1+})}{p_{1-} + p_{2-} + i0 \cdot \text{sign}(p_{1+})},$$  \hspace{1cm} (19)

when $p_{1+}p_{2+} > 0$, and turns to zero, when $p_{1+}p_{2+} < 0$.

After integration over longitudinal components we are left with the integral over transverse components

$$\int \frac{d^2q_{1\perp}d^2q_{2\perp}}{(2\pi)^4} \frac{1}{(q_{1\perp}^2q_{2\perp}^2)^2}.$$  \hspace{1cm} (20)
This integration is always implied in final results and will be suppressed in the following together with the common factor

\[ (-i)8g^{10}\frac{4k^2_+(-l_1-2i\bar{\lambda})}{2k_+l_2}L(p,q_2) = -32g^{10}k_1l_2L(p,q_2) = -32g^{10}\sqrt{s}L(p,q_2). \]  

(21)

2.3 Calculation of diagrams from Fig. 6

Diagrams with gluon production from the R→RP (Lipatov) vertex, corresponding to the part of the amplitude, are shown in Fig. 6.

For the momentum factor of the diagram 1 from Fig. 6 we get

\[ \frac{k^2_+ I_2(k)L(p,q_2)}{(k-q_1-q_3)^2 + i0} = i \frac{L(p,q_2)}{8(\lambda_- + i0)}. \]  

(22)

The momentum factor for diagram 2 from Fig. 6 is

\[ \frac{k^2_+ I_2(\lambda-k)L(p,q_2)}{(k-\lambda)^2 + i0} = i \frac{L(p,q_2)}{8(\lambda_- + i0)}. \]  

(23)

The color structure of diagram 1 from Fig. 6 is

\[ \frac{1}{4N^3_c}f^{b_2a}\cdot \text{tr}(T^{b_2}T^{d_3}T^{b_1})f^{d_2a} = \frac{1}{4N^3_c}\text{tr}(T^{b_2}T^{b_1}T^{b_1}) = \frac{(N^2_c - 1)^2}{16N^3_c}. \]  

Diagram 2 differs from diagram 1 only by the exchange of reggeons 1 and 3. Since these reggeons with color indices \( b_1 \) and \( b_3 \) form a symmetrical colorless state, the color factor for Diagram 2 from Fig. 6 is the same as for diagram 1.

For the rest diagrams we present a list of results where diagrams are arranged in pairs having a common color factor.

5.6. The momentum factor for diagram 5 from Fig. 6 is

\[ \frac{k^2_+ I_2(k-q+\lambda)L(p,q_2)}{(k-q+\lambda)^2 + i0} = -i \frac{L(p,q_2)}{8(\lambda_- + i0)} \]

and for diagram 6 is

\[ \frac{k^2_+ I_2(q-k)L(p,q_2)}{(k-q+\lambda)^2 + i0} = -i \frac{L(p,q_2)}{8(\lambda_- + i0)}, \]

the common color factor is

\[ \frac{1}{4N^3_c}f^{b_2a}\cdot \text{tr}(T^{b_2}T^{b_1}T^{d_3})f^{d_2a} = \frac{(N^2_c - 1)^2}{16N^3_c}. \]  

9.10. The momentum factor for diagram 9 from Fig. 6 is

\[ \frac{k^2_+ I_2(k-q+\lambda)L(p,q_1)}{(k-q+p+\lambda)^2 + i0} = -i \frac{L(p,q_1)}{8(\lambda_- + i0)} \]

and for diagram 10 is

\[ \frac{k^2_+ I_2(q-k)L(p,q_3)}{(k-q+p+\lambda)^2 + i0} = -i \frac{L(p,q_3)}{8(\lambda_- + i0)}, \]

the common color factor is

\[ \frac{1}{4N^3_c}f^{b_2a}\cdot \text{tr}(T^{b_2}T^{b_1}T^{d_3})f^{d_2a} = -\frac{N^2_c - 1}{32N_c}. \]
Figure 6: Diagrams with R→RP vertices.

11,12. The momentum factor for diagram 11 from Fig. 6 is

\[ \frac{k_+^2 I_2 (k-p) L(p, q_1)}{(k-p-\lambda)^2 + i0} = i \frac{L(p, q_1)}{8(\lambda_+ - i0)} \]

and for diagram 12 is

\[ \frac{k_+^2 I_2 (p-k+\lambda) L(p, q_3)}{(k-p-\lambda)^2 + i0} = i \frac{L(p, q_3)}{8(\lambda_+ - i0)} , \]

the common color factor is

\[ \frac{1}{4N_c^3} f_{\lambda b_2 a} \cdot \text{tr}(T^{b_2} T^{b_1} T^d) f_{db_1 a} = \frac{N_c^2 - 1}{32N_c} . \]

13,14. The momentum factor for diagram 13 from Fig. 6 is

\[ \frac{k_+^2 I_2 (k-q+p+\lambda) L(p, q_3)}{(k-q+p+\lambda)^2 + i0} = -i \frac{L(p, q_3)}{8(\lambda_+ + i0)} \]

and for diagram 14 is

\[ \frac{k_+^2 I_2 (q-k-p) L(p, q_1)}{(k-q+p+\lambda)^2 + i0} = -i \frac{L(p, q_1)}{8(\lambda_+ + i0)} , \]

the common color factor is

\[ \frac{1}{4N_c^3} f_{\lambda b_2 a} \cdot \text{tr}(T^{b_2} T^d T^{b_1} T^d) f_{db_1 a} = \frac{N_c^2 - 1}{32N_c} . \]
The momentum factor for diagram 15 from Fig. 6 is

$$\frac{k^2_+ I_2(k) L(p, q_3)}{(k - p - \lambda)^2 + i0} = i \frac{L(p, q_3)}{8(\lambda - i0)}$$

and for diagram 16 is

$$\frac{k^2_+ I_2(\lambda - k) L(p, q_1)}{(k - p - \lambda)^2 + i0} = i \frac{L(p, q_1)}{8(\lambda - i0)},$$

the common color factor is

$$\frac{1}{4N_c^3} f^{b_2 a} \cdot \text{tr}(T^{b'_2} T^{b_2} T^{d} T^{b_1}) f^{d b_1 a} = \frac{N_c^2 - 1}{32N_c}.$$ 

The momentum factor for diagram 3 from Fig. 6 is

$$k^2_+ I_3(k, q - k - \lambda) L(p, q_2) = 0,$$

because the sign of $k_+(q - k - \lambda)_+ = -k^2_+$ is negative. For the same reason the momentum factors are zero for diagrams 4,7,8,17,18 from Fig. 6.

Taking into account the common ‘-’ sign from (3) we find the total contribution from diagrams 1-6 from Fig. 6

$$\frac{(N_c^2 - 1)^2}{16N_c^3} \left( 2i \frac{L(p, q_2)}{8(\lambda - i0)} - 2i \frac{L(p, q_2)}{8(\lambda + i0)} \right) = \frac{(N_c^2 - 1)^2}{32N_c^3} L(p, q_2) \pi \delta(\lambda -)$$

$$= \frac{(N_c^2 - 1)^2}{64N_c^3} L(p, q_2) \sqrt{s} 2\pi \delta(2k\lambda),$$

whereas contributions from diagrams 7-18 cancel completely. Taking into account the complex conjugate term doubles this result.

The “double cut” contribution from diagrams such as in Fig. 4 to the coefficient $F$ of the singularity $F\delta(\lambda_z)$ of the discontinuity, where $\lambda_z$ is the $z$-component in the lab. system, was found in [14] as a sum of four terms $F = F_1 + F_2 + F_3 + F_4$. Now we prefer to define the singular part as (see Eq. (80) in Appendix 1)

$$2\pi \delta(2k\lambda) F = \frac{2\pi}{k_+} \delta(\lambda_{z.m.}) F = \frac{2\pi m}{s} \delta(\lambda^{ab,sys.}) F,$$

therefore we have to divide the old result by $2\pi m/s$. Also the overall coefficient has to be taken two times smaller than in [14] since the discontinuity is twice the imaginary part. Then the previously found contributions can be presented in the following form:

$$F_1 = s^2 g^{10} N_c^2 - 1 B^2(p, q_2, q_1), \quad F_2 = s^2 g^{10} \frac{(N_c^2 - 1)^2}{N_c^3} L^2(p, q_1),$$

$$F_3 = -\frac{1}{2} s^2 g^{10} \frac{N_c^2 - 1}{N_c} L(p, q_1)L(p, q_2), \quad F_4 = -\frac{1}{2} s^2 g^{10} \frac{N_c^2 - 1}{N_c} B(p, q_2, q_1) \left( L(p, q_1) - 2L(p, q_2) \right),$$

where the transverse integration (20) is implied.

Analogously, restoring the common factor (21) we find the new contribution to the coefficient $F$ of the imaginary part:

$$F_5 = -s^2 g^{10} \frac{(N_c^2 - 1)^2}{N_c^3} L^2(p, q_2).$$

(28)
Figure 7: Diagrams with R→RRP vertices.
2.4 Calculation of diagrams from Fig. 7

Diagrams with gluon production from R→RRP (Bartels) vertex, corresponding to the part 4 of the amplitude, are shown in Fig. 7. Color factors are also coincide for each pair of diagrams which differ only by the exchange of reggeons $1$ and $3$.

1,2. The momentum factor for diagram 1 from Fig. 7 is

$$\frac{k_+ q_+ I_2(k) B(p, q_2, q_3)}{(q - \lambda)^2 + i0} = \frac{i B(p, q_2, q_3)}{8(\lambda_+ - i0)}$$

and for diagram 2 is

$$\frac{k_+ q_+ I_2(\lambda - k) B(p, q_2, q_1)}{(q - \lambda)^2 + i0} = \frac{i B(p, q_2, q_1)}{8(\lambda_+ - i0)} ,$$

the common color factor is

$$\frac{i}{4N_c^3} f^{b_2a} \cdot \text{tr}(T^{b'_1 T^{b_1}}) f^{c b_1 d} f^{d b_2 a} = - \frac{N_c^2 - 1}{16N_c} .$$

3,4. The momentum factor for diagram 3 from Fig. 7 is

$$\frac{k_+ q_+ I_2(q - k) B(p, q_2, q_3)}{(q - \lambda)^2 + i0} = \frac{i B(p, q_2, q_3)}{8(\lambda_+ - i0)}$$

and for diagram 4 is

$$\frac{k_+ q_+ I_2(k - q + \lambda) B(p, q_2, q_1)}{(q - \lambda)^2 + i0} = \frac{i B(p, q_2, q_1)}{8(\lambda_+ - i0)} ,$$

the common color factor is

$$\frac{i}{4N_c^3} f^{b_2a} \cdot \text{tr}(T^{b'_1 T^{b_1}}) f^{c b_1 d} f^{d b_2 a} = \frac{N_c^2 - 1}{16N_c} .$$

5,6. The momentum factor for diagram 5 from Fig. 7 is

$$k_+ q_+ I_3(k, -p - \lambda) B(p, q_3, q_2) = 0 ,$$

because the sign of $k_+(-p - \lambda)_+ = -k_+ p_+$ is negative, and also for Diagram 6 is

$$k_+ q_+ I_3(\lambda - k, p) B(p, q_1, q_2) = 0 .$$

7,8. The momentum factor for diagram 7 from Fig. 7 is

$$k_+ q_+ I_3(q - k, -p - \lambda) B(p, q_3, q_2) = -i \frac{B(p, q_3, q_2)}{4(\lambda_+ + i0)}$$

where we have taken into account that $k_+ >> p_+$. For diagram 8 the momentum factor is

$$k_+ q_+ I_3(k - q + \lambda, p) B(p, q_1, q_2) = -i \frac{B(p, q_1, q_2)}{4(\lambda_+ + i0)} .$$

The common color factor is

$$\frac{i}{4N_c^3} f^{b_2a} \cdot \text{tr}(T^{b'_1 T^{b_1}}) f^{c b_1 d} f^{d b_2 a} = \frac{N_c^2 - 1}{32N_c} .$$

9,10. The momentum factor for diagram 9 from Fig. 7 is

$$\frac{k_+ q_+ I_2(p + \lambda) B(p, q_3, q_1)}{(k - q + p + \lambda)^2 + i0} = -i \frac{B(p, q_3, q_1)}{8(\lambda_+ + i0)}$$
and for diagram 10 is
\[
\frac{k_+ q_+ I_2(-p) B(p, q_1, q_3)}{(k - q + p + \lambda)^2 + i0} = -i \frac{B(p, q_1, q_3)}{8(\lambda_+ + i0)},
\]

the common color factor is
\[
\frac{i}{4N_c^3} f^{b_2 a} f^{b_1 a} \cdot \text{tr}\left( T^{b_2} T^{b_2} T^{b_2} \right) f^{c b_1 d} f^{d b_1 a} = -\frac{N_c^2 - 1}{16N_c}.
\]

11, 12. The momentum factor for diagram 11 from Fig. 7 is
\[
\frac{k_+ q_+ I_2(p + \lambda) B(p, q_3, q_1)}{(k - p - \lambda)^2 + i0} = i \frac{B(p, q_3, q_1)}{8(\lambda_+ - i0)},
\]

and for diagram 12 is
\[
\frac{k_+ q_+ I_2(-p) B(p, q_1, q_3)}{(k - p - \lambda)^2 + i0} = -i \frac{B(p, q_1, q_3)}{8(\lambda_+ - i0)},
\]

the common color factor is
\[
\frac{i}{4N_c^3} f^{b_2 a} f^{b_1 a} \cdot \text{tr}\left( T^{b_2} T^{b_2} T^{c} \right) f^{c b_1 d} f^{d b_1 a} = -\frac{N_c^2 - 1}{16N_c}.
\]

The total sum is
\[
\frac{i}{8} \frac{N_c^2 - 1}{16N_c} \left( -\frac{B(p, q_3, q_2)}{\lambda_+ + i0} - \frac{B(p, q_1, q_2)}{\lambda_+ + i0} + \frac{B(p, q_3, q_1)}{\lambda_+ + i0} + \frac{B(p, q_1, q_3)}{\lambda_+ + i0} + \frac{B(p, q_3, q_1)}{\lambda_+ - i0} + \frac{B(p, q_1, q_3)}{\lambda_+ - i0} \right)
\]
\[
= \frac{i}{8} \frac{N_c^2 - 1}{16N_c} \left( -\frac{B(p, q_3, q_2)}{\lambda_+ + i0} + \frac{B(p, q_1, q_2)}{\lambda_+ + i0} + 2 \left( B(p, q_3, q_1) + B(p, q_1, q_3) \right) \cdot P \frac{1}{\lambda_+} \right),
\]

where only diagrams 7-12 contribute. Taking into account the complex conjugate term leaves us with
\[
-\frac{N_c^2 - 1}{32N_c} \cdot \frac{B(p, q_3, q_2) + B(p, q_1, q_2)}{2} \pi\delta(\lambda_-) = -\frac{N_c^2 - 1}{64N_c} \cdot \frac{B(p, q_3, q_2)}{2} \pi\delta(2k\lambda).
\]

Using the invariance of the transverse integration \(29\) with respect to changing the variable \(q_1 \perp \rightarrow q_3 \perp = \lambda_\perp - q_1 \perp = -q_1 \perp\) and restoring the common factor we find the contribution of diagrams from Fig. 7 to the coefficient \(F\):
\[
F_6 = \frac{1}{2} s^2 g^{10} \frac{N_c^2 - 1}{N_c} L(p, q_2) B(p, q_1, q_2).
\]

2.5 Calculation of diagrams from Fig. 8

Diagrams with gluon production from \(R \rightarrow \text{RRRP}\) vertex, corresponding to the part \(5\) of the amplitude, are shown in Fig. 8.

1, 2. The momentum factor for diagram 1 from Fig. 8 is
\[
\frac{q_+^2 I_2(p + \lambda) B(p, q_3 + q_1, q_2)}{(p + \lambda)^2 + i0} = -i \frac{B(p, q_3 + q_1, q_2)}{8(\lambda_+ + i0)}.
\]

and for diagram 2 is
\[
\frac{q_+^2 I_2(-p) B(p, q_1 + q_3, q_2)}{(p + \lambda)^2 + i0} = -i \frac{B(p, q_3 + q_1, q_2)}{8(\lambda_+ + i0)}.
\]

The common color factor is
\[
\frac{1}{4N_c^3} f^{b_2 a} f^{b_1 a} \cdot \text{tr}\left( T^{b_2} T^{b_2} T^{b_2} \right) f^{c b_1 d} f^{d b_1 a} = -\frac{N_c^2 - 1}{8N_c}.
\]
3.4. The momentum factor for diagram 3 from Fig. 8 is
\[ q_3 I_3(q, -p - \lambda)B(p, q_3 + q_2, q_1) = 0, \]
because the sign of \( q_+(-p - \lambda)_+ = -q_+^2 \) is negative. Also for diagram 4 the momentum factor is zero:
\[ q_+^2 I_3(\lambda - q, p)B(p, q_1 + q_2, q_3) = 0. \]

5.6. The momentum factor for diagram 5 from Fig. 8 is
\[ \frac{q_+^2 I_2(p + \lambda)B(p, q_2 + q_3, q_1)}{(q - \lambda)^2 + i0} = \frac{iB(p, q_2 + q_3, q_1)}{8(\lambda_+ - i0)} \]
and for diagram 6 is
\[ \frac{q_+^2 I_2(-p)B(p, q_2 + q_1, q_3)}{(p + \lambda)^2 + i0} = \frac{iB(p, q_2 + q_1, q_3)}{8(\lambda_- - i0)} \].

The common color factor is
\[ \frac{1}{4N_c^2} f^{b' b_2 a} \cdot \text{tr}(T^{b' b} T^{b b_1 c} f^{cb_1 d} f^{d b_2 a}) = -\frac{N_c^2 - 1}{8N_c}. \]

To calculate the total contribution we take into account that
\[ B(p, q_3 + q_1, q_2) = \frac{(p + q_3 + q_1, \epsilon_+)}{(p + q_3 + q_1)^2} - \frac{(p + q_3 + q_1 + q_2, \epsilon_+)}{(p + q_3 + q_1 + q_2)^2} \equiv L(p, q_2) \tag{32} \]
and

\[ B(p, q_2 + q_3, q_3) = \frac{(p + q_2 + q_1, \epsilon_\perp)}{(p + q_2 + q_1)^2_\perp} - \frac{(p + q_2 + q_1 + q_3, \epsilon_\perp)}{(p + q_2 + q_1 + q_3)^2_\perp} \]

\[ = \frac{(p + q_2 + q_1, \epsilon_\perp)}{(p + q_2 + q_1)^2_\perp} - \frac{(p + q_2, \epsilon_\perp)}{(p + q_2)^2_\perp} = -B(p, q_2, q_1), \]

\[ B(p, q_2 + q_3, q_1) \equiv -B(p, q_2, q_3), \]

since \( q_1^\perp + q_3^\perp = \lambda^\perp = 0 \). Then we get together with the complex conjugate term

\[ -\frac{N_c^2 - 1}{8N_c} \left( -2i \frac{L(p, q_2)}{8(\lambda^- + i0)} + i \frac{-B(p, q_2, q_1) - B(p, q_2, q_3)}{8(\lambda^- - i0)} \right) + c.c. \]

\[ = \frac{N_c^2 - 1}{16N_c} (L(p, q_2) - B(p, q_2, q_1)) \pi \delta(\lambda^-) = \frac{N_c^2 - 1}{32N_c} (L(p, q_2) - B(p, q_2, q_1)) \sqrt{2} \pi \delta(2k\lambda), \]

where the invariance with respect to the change \( q_1^\perp \leftrightarrow q_3^\perp \) under the sign of integration (20) is used. Restoring the common factor (21) we find the contribution of diagrams from Fig. 8 to the coefficient \( F \) as the sum of two terms:

\[ F_7 = -s^2 g^{10} \frac{N_c^2 - 1}{N_c} L^2(p, q_2) \quad \text{and} \]

\[ F_8 = s^2 g^{10} \frac{N_c^2 - 1}{N_c} L(p, q_2) B(p, q_2, q_1). \]

Figure 9: Production amplitude in the diffractive configuration.

3 Diffractive contribution

In this section we calculate the contribution from the diffractive configuration. The corresponding diagrams are shown in Fig 9. Three more diagrams with exchanged reggeons of momenta \( q_1 \) and \( q_3 \) have to be added. Here we denote reggeon momenta \( q_1 \) and \( q_3 \) to preserve the kinematical relation \( q_1 + q_3 = \lambda \) with \( q_1^\perp + q_3^\perp = 0 \).

The initial formulae for the calculation of the amplitude we re obtained in [11, 13]. For all diagrams from Fig. 9 the propagator (16) of the target quark multiplied by two reggeon propagators and two reggeon-quark vertices is

\[ \frac{i\gamma^-}{2} \left[ i\hat{l}_1 + \hat{q}_1 (\mp i\pi) \delta((l_1 + q_1)^2) \right] \frac{i\gamma^-}{2} \cdot \frac{-2i}{q_1^2} \cdot \frac{-2i}{q_3^2} \cdot \frac{\gamma^-}{(q_{1\perp}^2)\gamma^-} \cdot 2\pi \delta(q_{1+}). \]

It produces a spinor factor \( \gamma^- \) and a momentum factor which allows us to represent the integral over longitudinal components of the loop momentum \( q_1 \) in terms of \( 2l_2 \) (see (17)).

1. The spinor factor for the projectile quark for diagram 1 from Fig. 9 is

\[ \frac{i\gamma^+}{2} (ik) \frac{i\gamma^+}{2} \cdot (-2i) = -k^+_+ \cdot \gamma^+. \]

(37)
where the factor \((-2i)\) is from the reggeon propagator. The momentum factor for diagram 1 is
\[-2I_2(k-p)L(p,q_1) = \frac{iL(p,q_1)}{4k_+}\]
and for the diagram with exchanged reggeons is
\[-2I_2(\lambda + p - k)L(p,q_3) = \frac{iL(p,q_3)}{4k_+} .\]
The common color factor is
\[\frac{1}{2N_c} f^{cb_1a} T^{b_1} T^c = -\frac{i}{4} T^a .\]

2. The spinor factor for the projectile quark for diagram 2 from Fig. 9 is the same as (37). The momentum factor for diagram 2 is
\[-2I_2(k)L(p,q_3) = \frac{iL(p,q_3)}{4k_+}\]
and for the diagram with exchanged reggeons is
\[-2I_2(\lambda - k)L(p,q_1) = \frac{iL(p,q_1)}{4k_+} .\]
The common color factor is
\[\frac{1}{2N_c} f^{cb_1a} T^{b_1} T^c = \frac{i}{4} T^a .\]

3. The spinor factor for the projectile quark for diagram 3 from Fig. 9 is
\[\frac{i\gamma_+}{2} \cdot (-2i) = \gamma_+ .\]
The momentum factor for diagram 3 is
\[q_+ 2I_2(q) B(p,q_3,q_1) = -\frac{iB(p,q_3,q_1)}{4}\]
and for the diagram with exchanged reggeons is
\[q_+ 2I_2(\lambda - q) B(p,q_1,q_3) = -\frac{iB(p,q_1,q_3)}{4} .\]
The common color factor is
\[-\frac{i}{2N_c} f^{db_1d} f^a b_1 T^b = \frac{i}{2} T^a .\]

From the relation \(q_{1\perp} + q_{3\perp} = 0\) the identities
\[B(p,q_1,q_3) = \frac{(p + q_{1\perp}, \epsilon_{1\perp})}{(p + q_{1\perp})^2} - \frac{(p + q_{1\perp} + q_{3\perp}, \epsilon_{1\perp})}{(p + q_{1\perp} + q_{3\perp})^2} = -L(p,q_1)\]
and
\[B(p,q_3,q_1) = -L(p,q_3) = -L(p,-q_1)\]
follow. As a result the total amplitude is found to be proportional to
\[\frac{T^a}{8} (L(p,-q_1) + L(p,q_1)) ,\]
where the contribution of diagrams 1, 2 and analogous ones with exchanged reggeons cancels. Collecting all factors we get for the diffractive amplitude
\[ig^5(\gamma_+ \otimes \gamma_-) \frac{T^a}{4} \int \frac{d^2q_{1\perp} L(p,q_1)}{(2\pi)^2 (q_{1\perp}^2)^2} .\]
The cut line of the projectile quark with momentum \( k' = k - (p + \lambda) \) contributes factor
\[
2\pi\delta((k - p - \lambda)^2) = 2\pi\delta(-k_+\lambda_-) = 2\pi\delta(2k\lambda),
\] (43)
so the correspondent contribution to \( F \) equals to just the squared modulus of the amplitude. The color factor is
\[
\frac{1}{N_c} \text{tr}(T^a T^a) = \frac{N_c^2 - 1}{32N_c},
\] (44)
the spinor structure for the projectile quark is
\[
\frac{1}{2} \sum_\sigma \bar{u}_\sigma(k) \gamma_+ \cdot \hat{k}' \cdot \gamma_+ u_\sigma(k) \approx 4k_+^2 = 4s.
\] (45)

One have to consider the two targets in the amplitude and its complex conjugated as different, so for both target quarks we have
\[
\frac{1}{2} \sum_\sigma \bar{u}_\sigma(l) \gamma_- u_\sigma(l) = \frac{1}{2} \text{tr}(\hat{l}_-) = 2l_- = 2\sqrt{s}.
\] (46)

If we denote \( q_2 \) the integration variable in the conjugated amplitude, we find the total diffractive contribution as
\[
F_9 = \frac{1}{2} s^2 g^{10} \frac{N_c^2 - 1}{N_c} L(p, q_1) L(p, q_2),
\] (47)
where the transverse integration (20) is implied.

4 Inclusive cross-sections

4.1 The total high-energy part \( F \)

Collecting all results one can observe essential cancellations between different contributions. Since
\[
F_2 + F_5 = 0, \quad F_3 + F_9 = 0, \quad F_4 + F_6 = F_8,
\] (48)
the total coefficient is
\[
F = F_1 + F_7 + 2F_8 = s^2 g^{10} \frac{N_c^2 - 1}{N_c} \left[ B^2(p, q_2, q_1) - L^2(p, q_2) + 2L(p, q_2)B(p, q_2, q_1) \right].
\] (49)

Note that the obvious symmetry of the transverse integral (20) with respect to the exchange \( q_1 \leftrightarrow q_2 \) is used when needed. Using the definition of vertices we get the relation
\[
B(p, q_2, q_1) + L(p, q_2) = L(p, q_12), \quad \text{where} \; q_{12} = q_1 + q_2,
\] (50)
which allows us to simplify the expression
\[
F = s^2 g^{10} \frac{N_c^2 - 1}{N_c} \left[ L^2(p, q_{12}) - 2L^2(p, q_2) \right].
\] (51)

To calculate the cross-section one have to sum over polarizations of the emitted gluon. Note that the light-cone gauge \( \epsilon_\ell = 0 \) (equivalent to \( \epsilon_+ = 0 \)) was used in [11]-[14] and is implied in (6) and (7). For this gauge the spin sum
\[
\sum_\lambda \epsilon^\lambda_\mu(p) \epsilon^\lambda_{\mu'}(p) = -g_{\mu\nu} + \frac{p\mu l_\nu + p\nu l_\mu}{(pl)},
\] (52)
turns to \(-g_{\mu\nu}\) for purely transverse components. Then the contribution of the term \( L^2(p, q) \) is
\[
- \frac{1}{p_+^2} - \frac{1}{(p + q)_+^2} + 2 \frac{(p, p + q)_+}{p_+^2(p + q)_+^2} = -\frac{q_+^2}{p_+^2(p + q)_+^2},
\] (53)
so from (51) we get

\[ F = -s^2 g^{10} N_c^2 - \frac{1}{N_c} \left[ \frac{q_{12}^2}{p_{12}^2 (p + q_{12})^2} - 2 \frac{q_{2}^2}{p_{2}^2 (p + q_{2})^2} \right]. \] (54)

The second term in the brackets depends on only \(q_2\). Such contributions involve the pomeron at zero distance between the reggeized gluons and vanish. This brings us to the final result

\[ F = -s^2 g^{10} N_c \frac{q_{2}^2}{p_{2}^2 (p - q_{2})^2}, \] (55)

where we change the sign of integration variables \(q_1, q_2\) and pass to the large \(N_c\) limit to compare the expression with one of the dipole picture.

Note that the same expression was obtained in [2] within the purely transverse picture assuming the validity of the AGK rules for the relation of the diffractive, single cut and double cut contributions as 1:-1:2. Our present derivation thus demonstrates the validity of these rules.

### 4.2 Impulse approximation

We define the inclusive cross-section as

\[ J(p) = \frac{(2\pi)^2 d\sigma}{d^2 pdy} \] (56)

leaving the additional factor \(1/4\pi\) for the diagram.

We start from the impulse approximation, in which

\[ J_A^{(1)} = AJ \] (57)

and \(J\) is the cross-section on the nucleon (\(J_A^{(1)}\) corresponds to the cut diagram shown in Appendix 1 in Fig. [11]). As a diagram it carries momentum factor \(4(kl)^2\) and colour factor \(N_c^3\). The cut couplings to the projectile and target give two \(\delta\)-functions

\[(2\pi)^2 \delta(k+q-) \delta(l+q+)\]

which allow to do the longitudinal integrations and produce factor \(1/4(kt)\). Each emission vertex gives \(2L(p,q)\) and we get

\[ \text{Im}A = 2(kl) N_c^3 g^{6} \frac{4\pi}{4\pi} \int \frac{d^2q}{(2\pi)^2} \frac{L^2(p,q)}{q^4} \]

(alternative \(1/2\) comes because we take the imaginary part and not twice it, the latter corresponding to unitarity). Including factor \(2g^2 N_c/4\pi\) into the BFKL interaction and dividing by \(s\) we find

\[ J_A^{(1)} = \frac{N_c^2 g^{4} \alpha_s N_c}{p^2} \int \frac{d^2q}{(2\pi)^2} \frac{1}{q^4 (p - q)^2}, \] (58)

where \(p\) and \(q\) denote transverse momenta in the Euclidean metric that is with \(p^2 \geq 0\).

To interpret this expression we note that

\[ \frac{g^2 N_c}{2q^4} \equiv P_y^{(0)}(q) \]

give the lowest approximation for the pomeron \(P_y(q)\) attached to the target. A similar lower order pomeron attached to the projectile is

\[ P_{Y-y}^{(0)}(p-q) = \frac{g^2 N_c}{2(p-q)^4} \]
where $Y$ is the overall rapidity $\ln s$. So we can rewrite

$$J_A^{(1)} = A \frac{4\alpha_s N_c}{p^2} \int \frac{d^2 q}{(2\pi)^2} (p - q)^2 F^{(0)}_{Y-y}(p - q) q^2 P_y^{(0)}(q)$$

or in the coordinate space

$$J_A^{(1)} = A \frac{4\alpha_s N_c}{p^2} \int d^2 x e^{ipx} \left( \Delta P_{Y-y}^{(0)}(r) \right) \left( \Delta P_y^{(0)}(r) \right).$$

In higher orders we obtain

$$J_A^{(1)} = A \frac{4\alpha_s N_c}{p^2} \int d^2 x e^{ipx} \left( \Delta P_{Y-y}^{(0)}(r) \right) \left( \Delta P_y(r) \right)$$

which is the standard expression for the gluon emission from the BFKL ladder.

### 4.3 Double scattering

Now we have

$$J^{(2)} = \frac{1}{s^2} F \left( \frac{1}{2} A(A - 1) \int d^2 b T_A^2(b) \right).$$

Restoring the coefficients and transverse integration we have in the Euclidean metric

$$F = 4(kt)^2 N_c^3 g^2 \frac{\alpha_s N_c}{p^2} \int \frac{d^2 q_1 d^2 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (p - q_1)(p - q_2)}.$$

We interpret

$$\frac{g^2 N_c}{2q_1^2} = P_y^{(0)}(q_1), \quad \frac{g^2 N_c}{2q_2^2} = P_y^{(0)}(q_2)$$

and as before introduce

$$P_{Y-y}^{(0)}(p - q_{12}) = \frac{g^2 N_c}{2(p - q_{12})^4}.$$

Then we find

$$F = 2s^2 g^2 \frac{4\alpha_s N_c}{p^2} \int \frac{d^2 q_1 d^2 q_2}{(2\pi)^4} q_{12}(q_{12} - p)^2 P_{Y-y}^{(0)}(p - q_{12}) P_y^{(0)}(q_1) P_y^{(0)}(q_2).$$

We transform this expression to the coordinate space. Passing to integration variables $q_{12}$ and $q_1$ with $q_2 = q_1 - q_{12}$ we have the integral over $q_1$

$$\int \frac{d^2 q_1}{(2\pi)^2} \int d^2 r_1 d^2 r_2 e^{iq_1\cdot r_1 + i(q_1 - q_{12})\cdot r_2} P_y^{(0)}(r_1) P_y^{(0)}(r_2) = \int d^2 r e^{iq_{12}\cdot r} P_y^{(0)}(r) P_y^{(0)}(-r).$$

Multiplying this by $q_{12}^2$ we get

$$- \int d^2 r e^{iq_{12}\cdot r} \Delta \left( P_y^{(0)}(r) P_y^{(0)}(-r) \right).$$

As a result

$$F = 2s^2 g^2 \frac{4\alpha_s N_c}{p^2} \int \frac{d^2 q_1 d^2 q_2}{(2\pi)^4} \frac{d^2 r d^2 r'}{(2\pi)^2} e^{iq_{12}\cdot r'} \Delta \left( P_y^{(0)}(r') e^{iq_{12}\cdot r} \Delta \left( P_y^{(0)}(r) P_y^{(0)}(-r) \right) \right)$$

$$= s^2 g^2 \frac{4\alpha_s N_c}{p^2} \int d^2 r \Delta \left( P_y^{(0)}(r) \right) \Delta \left( P_y^{(0)}(r) P_y^{(0)}(-r) \right).$$

Dividing by $s^2$ we get the inclusive cross section

$$J_A^{(2)} = 2g^2 \frac{4\alpha_s N_c}{p^2} \rho^{(2)} \int d^2 r \Delta \left( P_y^{(0)}(r) \right) \Delta \left( P_y^{(0)}(r) P_y^{(0)}(-r) \right)$$

$$= g^2 \frac{4\alpha_s N_c}{p^2} A(A - 1) \int d^2 r \Delta \left( P_y^{(0)}(r) \right) \Delta \left( P_y^{(0)}(r) P_y^{(0)}(-r) \right) \int d^2 b T_A^2(b).$$
4.4 Comparison with the dipole picture

We have obtained in the lowest approximation the total contribution

\[ J_A = J_A^{(1)} + J_A^{(2)} = \frac{4\alpha_s N_c}{p^2} \int d^2 b \int d^2 re^{ipr} \left( \Delta P^{(0)}_{Y-y}(r) \right) \Delta \left[ P_y^{(0)}(r) T_A(b) + g^2 \left( P_y^{(0)}(r) T_A(b) \right)^2 \right]. \]  \( (66) \)

In the dipole picture the inclusive cross-section has the form (with some trivial redefinitions)

\[ J_A = \frac{2\alpha_s N_c}{p^2} \int d^2 b \int d^2 re^{ipr} \left( \Delta Q_y Y - y(r) \Delta \left[ 2N_y(r) - N_y^2(r,b) \right] \right) \]  \( (67) \)

where

\[ Q_y = -\frac{1}{g^2} P^{(0)} T_A(b) \]

and \( N(r,b) \) is the solution of the Balitsky-Kovchegov equation \([15, 16]\) with the initial condition

\[ N^{(0)}(r,b) = 1 - e^{g^2 P^{(0)}(r) T_A(b)} \]  \( (68) \)

so that in the two lowest orders in \( g^2 \) (evolution starts from terms of the order \( \alpha_s N^2 \sim g^6 \))

\[ N(r,b) = -g^2 P^{(0)}(r) T_A(b) - g^4 \left( P^{(0)}(r) T_A(b) \right)^2 \]  \( (69) \)

and

\[ 2N^{(0)} - N^2 = -2g^2 \left[ P^{(0)}(r) T_A(b) + g^2 \left( P^{(0)}(r) T_A(b) \right)^2 \right]. \]  \( (70) \)

One observes that in the lowest approximation the dipole cross-section \((67)\) coincides with the one found in our picture.

5 Conclusions

The main result of our paper is demonstration of the validity of the AGK rules for the inclusive gluon production, namely that the diffractive, single and double cut contribution are related as \( 1 : -1 : 2 \). Taking this into account we have found that in the lowest approximation the inclusive cross-section found in the reggeized gluon technique is identical with the one obtained in the dipole picture. The decisive instrument in this derivation has been the effective QCD action in the Regge kinematics \([9]\). As stated in the Introduction this is sufficient to establish equivalence of the dipole inclusive cross-section with the one determined within the BFKL-Bartels picture in all orders provided the inclusive cross-section is derived from the intermediate states present in the forward elastic scattering amplitude in Fig. 1.

We have to note that precisely this derivation has been questioned in \([3]\). Its authors argue that since the structure in Fig. 1 is obtained from Feynman diagrams with the use of the so-called bootstrap relation, the inclusive cross-section found directly from Feynman diagrams may be different from the one found from Fig. 1 because probably the bootstrap does not work for inclusive diagrams. In other terms the inclusive cross-section may contain additional terms which do not change the total cross-section (integrate to zero over the gluon momentum). According to \([3]\) these additional terms are to appear in the next-to-leading order in the inclusive cross-section. However direct calculation of the next-to-leading order performed in \([7]\) demonstrated that such terms do not appear and the cross-section in the interacting reggeized gluon framework continues to coincide with the dipole picture. This makes us believe that the procedure to seek the emitted intermediate gluon directly in Fig. 1 is correct, so that the equivalence of both approaches is true.

6 Acknowledgements

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Appendix 1. Heavy nucleus in the Glauber approach

7.1 Relativistic $AN^A$ vertex and the nucleus wave function

To formulate the Glauber approach to collisions with a heavy nucleus in the diagrammatic technique we first have to relate the relativistic $AN^A$ vertex $\Gamma$ with the nucleus wave function. To this end we study the baryonic form-factor of the nucleus illustrated in Fig. 10 where vertices $\Gamma$ are shown with blobs. In the lab. system and at zero momentum transfer it is equal to $2AM$ where $M = A(m - \epsilon)$ is the nucleus mass. So we get the normalization condition

\[ A \int \prod_{j=2}^{A} d^4l_j \frac{2l_{10}\Gamma^2(l_j)}{(2\pi)^4 i (m^2 - l_j^2 - i0) (m^2 - l_1^2 - i0)^2} = 2AM. \]  

(71)

Here $\sum_{1}^{A} l_j = Al$ and $A^2l^2 = M^2$. For simplicity we neglect spins and consider all particles as scalar. In the nucleus rest system we have $l_0 = m - \epsilon$, and $l = 0$, so that putting $l_j = l + \lambda_j$, $j = 2, ... A$ we find

\[ m^2 - l_j^2 = 2m\epsilon - 2m\lambda_j0 - \lambda_{j,\perp}^2, \quad m^2 - l_1^2 = 2m\epsilon + 2m \sum_{2}^{A} \lambda_{0j} - \lambda_{1,\perp}^2, \]

where we used the orders of magnitude $\lambda_0 \sim \epsilon, |\lambda_\perp| \sim \sqrt{m\epsilon}$. Integrations over $\lambda_{0j}, \lambda_j, j = 2, ... A$ transform (71) into

\[ \left(\frac{1}{2m(2\pi)^3}\right)^{A-1} \int \prod_{j=2}^{A} d^3l_2 \frac{\Gamma^2(l_2, ... l_A)}{(2Am\epsilon + \sum_{1}^{A} l_j^2)^2} = A. \]

(72)

Comparing with the standard normalization of the nucleus wave function $\psi_A(l_j)$ we find the desired relation

\[ \psi_A(l_j) = \frac{1}{\sqrt{A}} \left(\frac{1}{2m(2\pi)^3}\right)^{(A-1)/2} \frac{\Gamma(l_j)}{2Am\epsilon + \sum_{1}^{A} l_j}, \]

(73)

which allows to relate the relativistic $AN^A$ vertex with the nucleus wave function in the momentum space.

To see how this works consider scattering on the nucleus in the impulse approximation, Fig. 11.

The corresponding amplitude is given by

\[ A^{imp} = Aa_N \int \prod_{j=2}^{A} d^4l_j \frac{\Gamma^2(l_2, ... l_A)}{(2\pi)^4 i (m^2 - l_j^2 - i0) (m^2 - l_1^2 - i0)^2}, \]

(74)
where $a_N$ is the forward scattering amplitude off the nucleon. Using (71) we find that the integral is equal to $A$ and we get $A^{\text{imp}} = A^2 a_N$. But the relativistic flux on the nucleus is $A$ times that on the proton, so that we get $\sigma_A = A \sigma_N$, as expected.

### 7.2 Double scattering on the nucleus

Now consider double scattering on the nucleus illustrated in Fig. 12. Now the amplitude is given by

$$ A = A(A - 1) \int \left( \prod_{j=3}^{A} \frac{d^4 l_j}{(2\pi)^4 i (m^2 - l_j^2 - i0)} \right) \frac{d^4 l_2}{(2\pi)^4 i} \frac{d^4 l_2'}{(2\pi)^4 i} H(l_2 - l_2') $$

$$ \times \frac{\Gamma(l_2, l_3, \ldots l_A)}{(m^2 - l_1^2 - i0)(m^2 - l_2^2 - i0)} \frac{\Gamma(l_2', l_3, \ldots l_A)}{(m^2 - l_1'^2 - i0)(m^2 - l_2'^2 - i0)} . \tag{75} $$

Here $H$ is the high-energy part and it is taken into account that it can only depend on the $z$-component of the transferred momentum, since it is the only of the spatial components which enters multiplied by the high projectile momentum. Factor $A(A - 1)$ combines factor 2 from two different ways in which $H$ can be coupled to the nucleus, corresponding to the change $l_1' \leftrightarrow l_2'$ and the choice of the active pair, which contributes $(1/2)A(A - 1)$. 

Figure 11: Scattering on the nucleus in the impulse approximation.

Figure 12: Double scattering on the nucleus.
Integrations over zero components of the nuclear momenta factorize and taking into account (73) we get

\[ A = \frac{A^2(A - 1)}{2m(2\pi)^3} \int d^3l_2 d^3l_1' \prod_{j=3}^{A} d^3l_j H(l_{2z} - l_{2z}') \psi_A(l_2, l_3, \ldots l_A) \psi_A(l_2', l_3, \ldots l_A). \tag{76} \]

We rewrite

\[ \int d^3l_2 d^3l_1' \prod_{j=3}^{A} d^3l_j H(l_{2z} - l_{2z}') \psi_A(l_2, l_3, \ldots l_A) \psi_A(l_2', l_3, \ldots l_A) \]

\[ = \int d^3l_1 d^3l_1' d^3l_2 d^3l_2' \delta^3(l_1 + l_2 - l_1' - l_2') \prod_{j=3}^{A-1} d^3l_j \psi_A(l_1, l_2, l_3, \ldots l_{A-1}) \psi_A(l_1', l_2', l_3, \ldots l_{A-1}) \]

\[ = \int d^3l_1 d^3l_1' d^3l_2 d^3l_2' \delta^3(l_1 + l_2 - l_1' - l_2') \rho_A(l_1, l_2 | l_1', l_2') H(l_{2z} - l_{2z}'). \tag{77} \]

where now \( l_A = \sum_{j=1}^{A-1} l_j \) and \((1/2)A(A - 1)\rho_A\) is the density matrix for the two active nucleons in the nucleus with \( A \) nucleons. So the amplitude becomes

\[ A = \frac{A}{m(2\pi)^3} \int d^3l_1 d^3l_1' d^3l_2 d^3l_2' \delta^3(l_1 + l_2 - l_1' - l_2') \rho_A(l_1, l_2 | l_1', l_2') H(l_{2z} - l_{2z}'). \tag{77} \]

To do the integrations over the transverse momenta we present

\[ \delta^2 \frac{1}{2} (l_1 + l_2 - l_1' - l_2') = \frac{1}{(2\pi)^2} \int d^2b e^{-ib(l_1 + l_2 - l_1' - l_2')} \]

and

\[ \rho_A(l_1, l_2 | l_1', l_2') = \frac{1}{(2\pi)^4} \int d^2b_1 d^2b_2 d^2b_1' d^2b_2' \rho_A(1_{\perp}, l_1; 1_{\perp}, l_2; 1_{\perp}', l_1'; 1_{\perp}', l_2') \]

\[ e^{-ib_1 l_{1\perp} - ib_1 l_{2\perp} + ib_1' l_{1\perp}' + ib_1' l_{2\perp}'}. \]

Then after transversal integrations we get

\[ A = \frac{A}{2m\pi} \int dl_{1z} dl_{1z}' dl_{2z} dl_{2z}' \delta(l_{1z} + l_{2z} - l_{1z}' - l_{2z}') d^2b H(l_{2z} - l_{2z}') \rho_A(b, l_{1z}; b, l_{2z}; b, l_{1z}', b, l_{2z}'). \tag{78} \]

Now we pass to spatial \( z \) components and introduce the \( z \)-component of the transferred momentum \( \lambda_z = l_{2z} - l_{2z}' \). Presenting

\[ \int dl_{1z} dl_{1z}' dl_{2z} dl_{2z}' \delta(l_{1z} + l_{2z} - l_{1z}' - l_{2z}') = \int dl_{1z} dl_{2z} d\lambda_z \]

we finally find

\[ A = \frac{A}{2\pi m} \int dz_1 dz_2 d\lambda_z e^{-i\lambda_z(z_1 - z_2)} H(\lambda_z) e^{-i\lambda_z} \rho_A(b, z_1; b, z_2| b, z_1, b, z_2). \tag{79} \]

The Glauber approximation follows if \( H(\lambda_z) \) has a singularity at \( \lambda_z = 0 \). Typically

\[ \Im H(\lambda_z) = 2\pi \delta(2(k\lambda)) F = F \frac{\pi}{k_0} \delta(\lambda_z). \tag{80} \]

Here we use \( \lambda_0 << |\lambda_z| \) and \( k_0 = k_z > 0 \). In this case we get the Glauber approximation for the imaginary part of the double scattering amplitude

\[ \Im A = \frac{A}{s} F \rho^{(2)} \tag{81} \]
Figure 13: Double elastic collision on the nucleus.

where

$$\rho^{(2)} = \frac{1}{2} A(A - 1) \int d^2 b T_A^2(b).$$  (82)

Dividing by the flux $A$s we get the total cross-section

$$\sigma_A = \frac{1}{s^2} F \rho^{(2)}. \quad (83)$$

To see how this formula works consider again the simplest case of the double scattering corresponding to double elastic collision shown in Fig. 13. In this case the direct and crossed diagrams in Fig. 13 give

$$H(\lambda_z) = 2\pi i \delta(2k_0\lambda_z)a_N^2,$$  (84)

so that $F = ia_N^2$ and

$$A = \frac{i}{2s} \frac{A^2(A - 1)}{a_N^2} \int d^2 b T_A^2(b).$$  (85)

The flux is $A$ times greater than off the nucleon. Dividing by $A$s we find

$$\sigma_A^{\text{double}} = \frac{1}{2} A(A - 1)\sigma_N^2 \int d^2 b T_A^2(b)$$  (86)

which is the standard expression for the double scattering in the Glauber approximation.

The same derivation remains valid for the inclusive cross-section with the obvious result

$$\frac{(2\pi)^3 d\sigma_A}{d^3p} = \frac{1}{s^2} F \rho^{(2)}$$  (87)

where of course $F$ refers to the diagrams with the external emitted gluon of momentum $p$.

8 Appendix 2. Inclusive cross-section from the BFKL chain

Consider the imaginary part $D$ of the forward amplitude for quark-quark scattering with $n$ real gluons in the intermediate state corresponding to the diagram shown in Fig. 14. $D$. It is given by the square modulus of the amplitude for the production of $n$ gluons $F$ shown in Fig. 14.$F$. 
Figure 14: Scattering amplitude $D$ and production amplitude $F$ with $n$ real gluons.

Explicitly $2\text{Im} \ D_n$ is given by the expression

$$2\text{Im} \ D_n = g^2 N_c \left( \frac{2}{4\pi} \right)^2 \prod_{i=0}^{n+1} \frac{dp_i^+}{4\pi p_i^+} \left( 2\pi \right)^2 \delta \left( \sum_{i=0}^{n+1} p_i^+ - k^+ \right) \delta \left( \sum_{i=0}^{n+1} p_i^+ - k^+ \right) \int \prod_{i=1}^{n+1} \frac{d^2 q_{i+}^\perp}{\left( 2\pi \right)^2} \left| F_n \right|^2.$$ 

The production amplitude $F_n$ is given by

$$F_n = 2s \frac{1}{t_{n+1}} \prod_{i=1}^{n} \frac{2L(p_i, q_{i+1})}{t_i},$$

where

$$\frac{1}{t_i} = \frac{s_{i,i-1}^{\omega(t_i)}}{t_i}$$

with $t_i = -\left| q_i^2 \right|$ and $L$ is the Lipatov vertex.

Apart from factor $1/(2\pi)^2$ associated with the transverse integration, each intermediate gluon line carries factor $g^2 N_c / \pi$. Summing over polarizations with the factor $2g^2 N_c / 4\pi = 2\alpha_s$ converts $4L^2(p_i, q_{i+1})$ into the standard BFKL interaction

$$-V(-q_{i+1}, q_{i+1} - q_i + p_1, q_i + p).$$

We are left with the additional factor $1/q_i^4$ for the last reggeon and factors $s_{i,i-1}^{\omega(t_i)}$ in all reggeon propagators. Note that all factors $q_i^4$ except the last are included in $V$'s. We denote $\left| F_n \right|^2 = 4s^2 T_n$ where $T_n$ is just the expression which corresponds to the purely transverse picture.

The imaginary part of the amplitude $D$ takes the form

$$2\text{Im} \ D_n = \left( \frac{g^2 N_c}{4\pi} \right)^2 \left( 2\pi \right)^2 4s^2 \prod_{i=0}^{n+1} \frac{dp_i^+}{p_i^+} \delta \left( \sum_{i=0}^{n+1} p_i^+ - k^+ \right) \delta \left( \sum_{i=0}^{n+1} p_i^+ - k^+ \right) T_D \equiv (g^2 N_c s)^2 \mathcal{L}_n T_n,$$

where $\mathcal{L}_n$ is the longitudinal part

$$\mathcal{L}_n = \prod_{i=0}^{n+1} \frac{dp_i^+}{p_i^+} \delta \left( \sum_{i=0}^{n+1} p_i^+ - k^+ \right) \delta \left( \sum_{i=0}^{n+1} p_i^+ - k^+ \right).$$

In the Regge kinematics

$$p_0^+ \gg p_1^+ \gg \cdots \gg p_n^+ \gg p_{n+1}^+,$$
we can transform the $\delta$-functions into

$$\delta(p_{0+} - k_+)\delta(p_{n+1,-} - l_-)$$

and do the integrations over $p_{0+}$ and $p_{n+1,-}$. We obtain

$$\mathcal{L}_n = \frac{2}{s} \int \prod_{i=1}^{n} \frac{dp_i}{p_i} . \quad (88)$$

Next step is to transform the remaining longitudinal phase volume to the variables introduced by Lipatov $s_{i,i-1} = 2p_{i-1,i,+}$ to do the integrations by means of the two $\delta$-functions.

We arrange integrations as

$$\int d\tau_n \equiv \int \frac{dp_{1,-}}{p_{1,-}} \int \frac{dp_{2,-}}{p_{2,-}} \ldots \int \frac{dp_{n,-}}{p_{n,-}} .$$

Then we rewrite the integral as

$$\int d\tau_n = \int \frac{d(2p_{0+}p_{1-})}{2p_{0+}p_{1+}} \int \frac{d(2p_{1+}p_{2-})}{2p_{1+}p_{2-}} \ldots \int \frac{d(2p_{n-1,+}p_{n-})}{2p_{n-1,+}p_{n,-}} = \int \prod_{i=1}^{n} ds_{i,i-1}.$$ Now we use

$$\prod_{i=1}^{n+1} s_{i,i-1} = s \prod_{i=1}^{n} |p_{i\perp}|^2$$

and

$$\int ds_{n+1,n} \delta(\prod_{i=1}^{n+1} s_{i,i-1} - s \prod_{i=1}^{n} |p_{i\perp}|^2) = \frac{1}{\prod_{i=1}^{n} s_{i,i-1}}$$

to write the final expression for (SS) in the Lipatov form

$$\mathcal{L}_n = \frac{2}{s} \int \prod_{i=1}^{n+1} ds_{i,i-1} \delta(\prod_{i=1}^{n+1} s_{i,i-1} - s \prod_{i=1}^{n} |p_{i\perp}|^2) . \quad (89)$$

So we find

$$2\text{Im} D_n = 2s(g^2 N_c)^2 T_n \int \prod_{i=1}^{n+1} ds_{i,i-1} \delta(\prod_{i=1}^{n+1} s_{i,i-1} - s \prod_{i=1}^{n} |p_{i\perp}|^2) . \quad (90)$$

To pass to the inclusive cross-section we select some particular gluon with momentum $p_m, 1 \leq m \leq n,$ and split the integrands in both $T_n$ and $\mathcal{L}_n$ in two parts above and below the selected real gluon. For the transverse part it is trivial

$$T_n(q_1, \ldots q_n) = \int \frac{d^2 q_m}{(2\pi)^2} \frac{d^2 q_{m+1}}{(2\pi)^2} \frac{d^2 q_{m-1}}{(2\pi)^2} T_{m-1}(q_1, \ldots q_{m-1}) q_m^4 V(-q_{m+1}q_{m+1} - q_mq_m) T_{n-m}(q_{m+1}, \ldots q_n) ,$$

where $q_{m+1} = q_m - p$. Factor $q_m^4$ eliminates the redundant denominator $1/q_m^4$ which is contained in both $T_m$ and $V$. To factorize the longitudinal part we use

$$\delta(\prod_{i=1}^{n+1} s_{i,i-1} - s \prod_{i=1}^{n} |p_{i\perp}|^2) = \delta(\prod_{i=1}^{n+1} s_{i,i-1} - s \prod_{i=1}^{n} |p_{i\perp}|^2) \delta(\prod_{i=m+1}^{n+1} s_{i,i-1} - s \prod_{i=m+1}^{n} |p_{i\perp}|^2).$$

As a result we present $2\text{Im} D_n$ in the factorized form

$$2\text{Im} D_n = 2s(g^2 N_c)^2 \int ds_1 ds_2 (s_1 s_2 - s p_m^2) \frac{d^2 q_m}{(2\pi)^2} \frac{d^2 q_{m+1}}{(2\pi)^2} \frac{d^2 q_{m-1}}{(2\pi)^2} .$$
\[
\left[q^4_m T_{m-1}(q_1, \ldots q_{m-1}) \prod_{i=1}^{m} ds_{i,i-1} \delta(\prod_{i=1}^{m} s_{i,i-1} - s_1 \prod_{i=1}^{m-1} \|p_{i\perp}\|^2) \right] \
\left[-V(-q_{m+1}\bar{q}_{m+1} - q_m + p_m, q_{m+1} + p)\right] \\
\left[T_{n-m}(q_{m+1}, \ldots q_n) \prod_{i=m+1}^{n+1} ds_{i,i-1} \delta(\prod_{i=m+1}^{n+1} s_{i,i-1} - s \prod_{i=m+1}^{n} \|p_{i\perp}\|^2) \right].
\]

Fixing \(p_m = p\) and summing over both \(m\) and \(n\) we finally find

\[
2 \text{Im} D_n = -2s(g^2 N_c)^2 \int ds_1 ds_2 (s_1 s_2 - s p_m) \frac{d^2 p}{(2\pi)^2} \frac{d^2 q}{(2\pi)^2} P_{s_1}(p + q) q^4 P_{s_2}(q) V(-q, q| - q - p, q + p),
\]

where

\[
P_{s_1}(p + q) = g^2 N_c \sum_m \left[T_{m-1}(q_1, \ldots q_{m-1}) \prod_{i=1}^{m} ds_{i,i-1} \delta(\prod_{i=1}^{m} s_{i,i-1} - s_1 \prod_{i=1}^{m-1} \|p_{i\perp}\|^2) \right] \tag{91}
\]

with \(q_m = p + q\) has the meaning of the pomeron with energy \(\sqrt{s_1}\) attached to the projectile and similarly \(P_{s_2}(q)\) is the pomeron attached to the target.

The inclusive cross-section will be given by

\[
\frac{(2\pi)^2 d\sigma}{d^2 p dy} = - \int \frac{d^2 q}{(2\pi)^2} V(-q, q| - q - p, q + p)(p + q)^4 P_{Y-y}(p + q) P_y(q),
\]

or using the explicit expression for the BFKL interaction

\[
\frac{(2\pi)^2 d\sigma}{d^2 p dy} = \frac{4 \alpha_s N_c}{p^2} \int \frac{d^2 q}{(2\pi)^2} P_{Y-y}(p + q) q^2 (p + q)^2 P_y(q) \tag{92}
\]

which is the standard expression.

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