Radiative Properties of the Stueckelberg Mechanism

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We examine the mechanism for generating a mass for a U(1) vector field introduced by Stueckelberg. First, it is shown that renormalization of the vector mass is identical to the renormalization of the vector field on account of gauge invariance. We then consider how the vector mass affects the effective potential in scalar quantum electrodynamics at one-loop order. The possibility of extending this mechanism to couple, in a gauge invariant way, a charged vector field to the photon is discussed.

Keywords: Stueckelberg; renormalization; effective action.

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1. Introduction

The masslessness of the photon is well established; indeed experiment shows that $m_\gamma < 3 \times 10^{-27}$eV. However, the vector Bosons associated with the weak interactions must be massive. It has been shown how the non-Abelian gauge Bosons associated with the electroweak interactions can be given this mass in a way that retains gauge invariance through the Higgs mechanism, thereby ensuring the renormalizability of the model.

There is, however, a way of providing a mass to a $U(1)$ vector Boson that retains renormalizability and unitarity without the use of the Higgs mechanism. This “Stueckelberg mechanism” does not involve the presence of an extra degree of freedom in the physical spectrum, in contrast to the Higgs mechanism.

In Ref. 3 and 4, the possibility of the vector Boson associated with the $U(1)$ sector of the Standard Model acquiring a mass through the Stueckelberg mechanism was examined. There it was demonstrated that the mass matrix associated with the vector Boson in the full $SU(2) \times U(1)$ electroweak model does not have a vanishing eigenvalue if there is only a single $U(1)$ vector and a single Higgs doublet and consequently there is no massless photon. This means that if the Stueckelberg mass is non-zero in addition to the usual Standard Model parameters, the photon cannot be massless. There is no reason why the possibility of having a non-vanishing
photon mass should be discarded within the content of the usual Standard Model; a non-vanishing Stueckelberg mass is consistent with renormalizability and unitarity. When dealing with the usual Standard Model, there is no more reason for setting the photon mass equal to zero than there is for setting the cosmological constant equal to zero. This forces one to consider extensions of the Standard Model to reconcile it with the observed masslessness of the photon, if indeed “what is not forbidden must be allowed.”

The Stueckelberg mechanism can be used in conjunction with other models involving gauge symmetries. Indeed, its extensions to supersymmetric gauge models\textsuperscript{5,6} and to spin-two models\textsuperscript{7,8} have been considered. It also arises naturally in effective actions generated by string models\textsuperscript{9}.

In Ref. 4 and 9, the Standard Model is extended so as to accommodate a massless photon when the Stueckelberg mechanism occurs by including a second $U(1)$ sector. This results in the mass matrix for the vectors having a vanishing eigenvalue, so that a massless photon can be incorporated into the model even when a Stueckelberg mass arises. Another extension of the Standard Model that ensures that a massless photon occurs is to embed the $U(1)$ symmetry of the Standard Model into a larger non-Abelian gauge group such as in the Grand Unified $SU(5)$ model. Since one cannot generalize the Stueckelberg mechanism so as to accommodate a non-Abelian gauge symmetry, the $U(1)$ sector of the Standard Model cannot be associated with the Stueckelberg mechanism when this $U(1)$ gauge symmetry is just a remnant of the larger non-Abelian symmetry once its symmetry is broken.

With the Stueckelberg mechanism having possible application in the Standard Model, it is relevant to consider some of its field theoretical consequences. We first examine how the Stueckelberg mass is renormalized, showing that on account of gauge invariance, this mass renormalization is dictated by the renormalization of the photon wave function and consequently of the $U(1)$ gauge coupling constant. Unlike other masses in the Standard Model it is not renormalized independently of other quantities that occur.

Next we demonstrate how in scalar electrodynamics, the presence of a Stueckelberg mass for the vector field considerably alters the form of the radiatively generated effective potential in the model.

Finally, it is shown how the Stueckelberg field can be used not only as to permit one to introduce a mass for a $U(1)$ vector field, but also to couple a complex vector field to a photon in a way that preserves gauge invariance for the complex vector field. Unfortunately, the resulting model, through gauge invariant, is not renormalizable. It thus appears that only through the Higgs mechanism can a massive charged vector field arise if one is only accepting of renormalizable models.
2. Renormalization of the Stueckelberg Mass

The usual Maxwell Lagrangian can be supplemented by a gauge invariant mass term to yield the Stueckelberg Lagrangian

\[ \mathcal{L}_s = -\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{1}{2} m_s^2 \left( A_\mu + \frac{1}{m_s} \partial_\mu \sigma \right)^2; \]  

(1)

this possess the gauge invariance

\[ A_\mu \rightarrow A_\mu + \partial_\mu \Omega \]
\[ \sigma \rightarrow \sigma - m_s \Omega. \]  

(2)

If Eq. (1) is supplemented by the gauge fixing term,

\[ \mathcal{L}_{gf} = -\frac{1}{2\xi} (\partial \cdot A - \xi m_s \sigma)^2 \]  

(3)

then \( A_\mu \) and \( \sigma \) decouple in \( \mathcal{L}_s + \mathcal{L}_{gf} \).

The propagator for the field \( A_\mu \) is

\[ \langle A_\mu A_\nu \rangle = -i \frac{k^\mu k^\nu - (1 - \xi) k_\mu k_\nu}{k^2 - \xi m_s^2} \pi(k^2). \]  

(4)

Renormalizability is manifest if \( \xi = 1 \), when \( \xi \rightarrow \infty \) we recover the usual propagator for a massive vector, and when \( \xi = 0 \) we are in a “unitary” gauge in which only the transverse degrees of freedom of \( A_\mu \) contribute.

If we now couple \( A_\mu \) to a spinor field \( \psi \) so that we have in addition to \( \mathcal{L}_s \) and \( \mathcal{L}_{gf} \)

\[ \mathcal{L}_\psi = \bar{\psi} [(i \partial_\mu - eA_\mu) \gamma^\mu - m] \psi \]  

(5)

then the gauge transformations (2) are accompanied by

\[ \psi \rightarrow e^{-ie\Omega} \psi. \]  

(6)

On account of the gauge invariance of \( \mathcal{L}_s + \mathcal{L}_\psi \), the usual Ward-Takahashi-Slavnov-Taylor (WTST) identities of quantum electrodynamics (QED) persist even when \( m_s^2 \neq 0 \). As a result, the regulated one particle irreducible two point function \( \langle A_\mu A_\nu \rangle \) in momentum space, \( \pi_{\mu\nu}(k) \), is of the form

\[ i\pi_{\mu\nu}(k) = i(g_{\mu\nu} k^2 - k_\mu k_\nu) \pi(k^2) \equiv ig_{\mu\nu} T(k^2) \pi(k^2). \]  

(7)

Working in the gauge \( \xi = 0 \), iteration of this contribution to the two point function leads to

\[ \frac{-ig_{\mu\nu} T}{k^2 - m_s^2} + \frac{-ig_{\mu\lambda} T}{k^2 - m_s^2} (i\pi_{\lambda\sigma}) \frac{-ig_{\nu\sigma} T}{k^2 - m_s^2} + \frac{-ig_{\mu\rho} T}{k^2 - m_s^2} (i\pi_{\rho\kappa}) \frac{-ig_{\nu\kappa} T}{k^2 - m_s^2} + \ldots \]

\[ = \frac{-ig_{\mu\nu} T(1 - \pi)}{k^2 - m_s^2 \left( \frac{1}{1 - \pi} \right)}. \]  

(8)
From Eq. (8) we see that divergences that appear in $\pi(k^2)$ when the regulating parameter approaches its limiting value appear in two places; those in the numerator of Eq. (8) serve to renormalize the external wave function, while those in the denominator renormalize $m^2_s$. The WTST identity that relates the vertex wave function to the spinor self energy, relates this renormalization of the wave function to the renormalization of the coupling constant $\epsilon^2$. In fact, renormalization of the electric charge is given by,

$$e_R^2 = e^2(1 + \pi)^{-1}_{\text{div}}$$

and also by Eq. (8),

$$(m^2_s)_R = m^2_s(1 + \pi)^{-1}_{\text{div}}$$

where $(1 + \pi)^{-1}_{\text{div}}$ indicates the divergent contribution to $(1 + \pi)^{-1}$ that arises when the regulating parameter approaches its limiting value. The finite renormalized coupling and mass are $e_R^2$ and $(m^2_s)_R$ respectively. It follows from Eqs. (8) and (9) that the renormalization group functions that dictate how $e_R^2$ and $(m^2_s)_R$ vary with the renormalization scale are identical.

**3. The Effective Action**

The one-loop radiative corrections to the effective action in scalar electrodynamics have been considered in Ref. 10. We here consider how the inclusion of a Stueckelberg mass into the model affects this calculation.

The Lagrangian $L_s$ of (1) is supplemented with a Lagrangian which couples $A_\mu$ to a complex scalar field $\phi(x)$,

$$L_\phi = (\partial_\mu + i\epsilon A_\mu)\phi^* (\partial^\mu - i\epsilon A^\mu)\phi - \kappa^2 \phi^* \phi - \lambda(\phi^* \phi)^2$$

(11)

If we assume that $\phi(x)$ has a constant background component $f$, taken to be real, then

$$\sqrt{2} \phi(x) = f + h_1(x) + ih_2(x)$$

(12)

and we find that the most convenient gauge fixing Lagrangian is no longer Eq. (3) but rather,

$$L_{gf} = -\frac{1}{2\xi} [\partial \cdot A - \xi(m_s\sigma - e\phi h_2)]^2.$$  

(13)

The terms in $L_s + L_\phi + L_{gf}$ that are bilinear in the quantum fields in the gauge where $\xi = 1$ are,

$$L^{(2)} = \frac{1}{2} (h_1, h_2, \sigma, A_\mu) H \begin{pmatrix} h_2 \\ h_1 \\ \sigma \\ A_\nu \end{pmatrix}$$

(14)
where
\[
H = \begin{pmatrix}
p^2 - (\kappa^2 + 3\lambda f^2) & 0 & 0 & 0 \\
0 & p^2 - (\kappa^2 + \lambda f^2 + e^2 f^2) & \frac{1}{2} m_s e f & 0 \\
0 & \frac{1}{2} m_s e f & p^2 - m_s^2 & 0 \\
0 & 0 & 0 & g^{\mu\nu} (-p^2 + m_s^2 + e^2 f^2) \\
\end{pmatrix}
\]

The one-loop effective potential is now given by \(^{11}\)
\[
V^{(1)} = -\ln(\det H)^{-\frac{1}{2}},
\]
where \(H\) is the functional matrix appearing in Eq. (14). We note that on account of the form of the gauge fixing term in Eq. (13) the Stueckelberg field \(\sigma\) does not decouple from the other fields. This gauge fixing does, however, ensure that \(A_\mu\) decouples in \(H\).

Diagonalizing the matrix \(H\), we find that,
\[
V^{(1)} = -\ln \left[ \det \begin{pmatrix}
p^2 - (\kappa^2 + 3\lambda f^2) & 0 & 0 & 0 \\
0 & g_{\mu\nu} (p^2 - m_s^2 - e^2 f^2) & 0 & 0 \\
0 & 0 & p^2 - m_+^2 & 0 \\
0 & 0 & 0 & p^2 - m_-^2 \\
\end{pmatrix} \right]^{-\frac{1}{2}}
\]
where,
\[
2m_{\pm} = [m_s^2 + \kappa^2 + (\lambda + e^2) f^2] \pm \sqrt{[\kappa^2 + (\lambda + e^2) f^2 - m_s^2]^2 + e^2 m_s^2 f^2}.
\]

The functional determinant in Eq. (16) can be evaluated using operator regularization, a variant of \(\zeta\)-function regularization \(^{12}\), which preserves symmetries and circumvents all explicit divergences. With this we find that
\[
V^{(1)} = -\frac{1}{2} \lim_{s \to 0} \frac{d}{ds} \frac{\mu^{2s}}{\Gamma(s)} \left[ \operatorname{tr} (it)^{s-3} e^{-im^2 t} \right] \Gamma(s-2) (m^2)^{2-s}
\]
where \(H^0\) is the diagonal matrix in Eq. (16).

Since
\[
\operatorname{tr} (e^{i p^2 t}) = \int \frac{d^4 p}{(2\pi)^4} e^{i p^2 t} = \frac{i}{(4\pi t)^2}
\]
and
\[
\int_0^\infty d(it)(it)^{s-3} e^{-im^2 t} = \Gamma(s-2) (m^2)^{2-s}
\]
Eq. (18) reduces to,
\[
V^{(1)} = \frac{1}{32\pi^2} \lim_{s \to 0} \frac{d}{ds} \frac{\mu^{2s} \Gamma(s-2)}{\Gamma(s)} \left[ (\kappa^2 + 3\lambda f^2)^{2-s} + 4(m_s^2 + e^2 f^2)^{2-s} + (m_+^2)^{2-s} + (m_-^2)^{2-s} \right]
\]

\[
eq \frac{1}{64\pi^2} \left( (\kappa^2 + 3\lambda f^2)^2 \left( \ln \left( \frac{\kappa^2 + 3\lambda f^2}{\mu^2} \right) - \frac{3}{2} \right) + 4(m_s^2 + e^2 f^2)^2 \left( \ln \left( \frac{m_s^2 + e^2 f^2}{\mu^2} \right) - \frac{3}{2} \right) \right)
\]
\[ \frac{m_2^2}{\mu^2} \left( \ln \left( \frac{m_2^2}{\mu^2} \right) - \frac{3}{2} \right) + m_2^2 \left( \ln \left( \frac{m_2^2}{\mu^2} \right) - \frac{3}{2} \right) \]. \quad (21) \]

Supplementing \( V^{(1)} \) with \( V^{(0)} = \frac{M^2}{4} \) to form the effective potential \( V(f) = V^{(0)} + V^{(1)} \) leads to rather complicated dependence of \( V \) on \( f \), especially when \( m_s^2 \neq 0 \). Minimizing \( V \) at \( f = v \) leads to a vacuum expectation value of \( \phi(x) \).

### 4. Charged Vector Field

A complex vector field \( W_\mu \) with action,
\[
\mathcal{L}_W = -\frac{1}{2} (\partial_\mu W^*_\nu - \partial_\nu W^*_\mu)(\partial^\mu W^\nu - \partial^\nu W^\mu) + m_W^2 \left( W^*_\mu + \frac{1}{m_W} \partial_\mu \Sigma^* \right) \left( W^\mu + \frac{1}{m_W} \partial^\mu \Sigma \right), \quad (22)
\]
possesses the gauge invariance
\[
W_\mu \to W_\mu + \partial_\mu \omega \\
\Sigma \to \Sigma - m_W \omega \quad (23)
\]
where \( \Sigma \) is a complex scalar and \( \omega \) is a complex gauge function. Coupling this vector field to a massive photon through replacement of the ordinary derivative by a covariant derivative leads to [13]
\[
\mathcal{L}_{WA} = -\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2
\]
\[
-\frac{1}{2} \left[ (\partial_\mu + i e A_\mu) W^*_\nu - (\partial_\nu + i e A_\nu) W^*_\mu \right] \left[ (\partial^\mu - i e A^\mu) W^\nu - (\partial^\nu - i e A^\nu) W^\mu \right]
\]
\[
+ \frac{m_A^2}{2} \left( A_\mu + \frac{1}{m_A} \partial_\mu \sigma \right)^2 + m_W^2 \left( W^*_\mu + \frac{1}{m_W} \partial_\mu \Sigma^* \right) \left( W^\mu + \frac{1}{m_W} \partial^\mu \Sigma \right) \quad (24)
\]
This is invariant the gauge transformations
\[
\sigma \to \sigma - m_A \theta \\
W_\mu \to e^{i e \theta} W_\mu \\
A_\mu \to A_\mu + \partial_\mu \theta \quad (25)
\]
Regrettably the gauge transformation of Eq. (23) is broken. However, the Lagrangian,
\[
\mathcal{L} = -\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2} \left[ (\partial_\mu + i e A_\mu) W^*_\nu - (\partial_\nu + i e A_\nu) W^*_\mu + \frac{i e}{m_W} (\partial_\mu A_\nu - \partial_\nu A_\mu) \Sigma^* \right]
\]
\[
\left[ (\partial^\mu - i e A^\mu) W^\nu - (\partial^\nu - i e A^\nu) W^\mu - \frac{i e}{m_W} (\partial^\mu A^\nu - \partial^\nu A^\mu) \Sigma \right]
\]
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\[ + m_W^2 \left[ W_\mu^* + \frac{1}{m_W} (\partial_\mu + ieA_\mu) \Sigma^* \right] \left[ W^\mu + \frac{1}{m_W} (\partial^\mu - ieA^\mu) \Sigma \right] \]

\[ + \frac{m_A^2}{2} (A_\mu + \frac{1}{m_A} \partial_\mu \sigma)^2 \]

(26)

does in fact possess the gauge invariance

\[ W_\mu \rightarrow W_\mu + (\partial_\mu - ieA_\mu) \omega \]

\[ A_\mu \rightarrow A_\mu \]

\[ \Sigma \rightarrow \Sigma - m_W \omega \]

(27)
as well as that of Eq. (25).

An obvious generalization of the gauge fixing Lagrangian (3) is,

\[ L_{gf} = -\frac{1}{2\xi} (\partial \cdot A - \xi m_A \sigma)^2 \]

\[ - \frac{1}{\xi} \left( (\partial + ieA) \cdot W^* - \zeta m_W \Sigma^* \right) \left[ (\partial - ieA) \cdot W - \zeta m_W \Sigma \right]. \]

(28)

Unfortunately, this gauge fixing Lagrangian does not serve to completely decouple the Stueckelberg field \( \Sigma \). There still remains the coupling

\[ L_I = -\frac{e^2}{2m_W^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \Sigma^* \Sigma \]

(29)

which destroys renormalizability. This results in a counter term proportional to \( [(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)]^2 \) being required, even at one loop order. It appears that in order to couple a photon \( A_\mu \) to a massive vector field \( W_\mu \), one requires an \( O(3) \) Yang Mills interaction, with the Higgs mechanism used to provide a mass to the field \( W_\mu \), if one is to retain renormalizability (two of the components of the \( O(3) \) gauge field are used to compose \( W_\mu \) and \( W_\mu^* \); the third component is identified with \( A_\mu \)).

5. Conclusion

We have examined several aspects of the Stueckelberg mechanism for generating a mass for a \( U(1) \) vector field. First of all, we have demonstrated that renormalization of the vector field is proportional to that of the Stueckelberg mass. Next we have shown how the presence of a Stueckelberg mass affects the one-loop effective potential in scalar electrodynamics. Finally we have attempted (unsuccessfully) to use an extension of the Stueckelberg mechanism to formulate a renormalizable model for a charged massive vector field.

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