Super-dispersive demultiplexer design using positive–negative refraction boundary and hetero-photonic crystals

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Abstract. In this paper, a systematic approach is employed to design a photonic crystal with a boundary between positive and negative refraction to boost the refractive properties of the crystal. Mathematical techniques are employed to turn the process of equi-frequency contours’ engineering into a nearly mechanical process free of trial-and-error steps. The designed demultiplexer’s operational characteristics are then further improved by adding a second photonic crystal with an oblique boundary. The refracted beams in the first crystal impinge on an oblique interface of the second photonic crystal to experience a change in Bloch wavenumbers and even greater refraction angles so that a novel super-dispersive two-step optical demultiplexer is made. A beam divergence of 161 degrees is obtained for an input spectrum of \( \lambda = [1474 \text{ nm}, 1550 \text{ nm}] \) which is quite superior than the design’s counterparts.

1 Introduction

The key to designing a high-performance optical demultiplexer is to improve the refractive properties of the demultiplexing medium. From the early studies of photonic crystals, their refractive properties have been in the spotlight [1] and their suitable refractive properties through the super-prism effect have been highlighted [2, 3]. Inspired by the potential of photonic crystals for demultiplexing, several groups tried to shed light on the problem of light refraction in photonic crystals [4–6] and the employment of super-prism effect in their structures [7–10] in order to produce high-performance demultiplexers which are a key component of communication systems at the optical wavelengths as well as other regions of the electromagnetic spectrum [11]. These studies led to the advent of the comprehensive theory of light refraction in photonic crystals and the role of equi-frequency contours (EFC) in determining the trajectory of incident light inside the crystal [12, 13]. Design of photonic crystals for the required EFC is essentially a procedure based on trial and error, but a mathematical design strategy has been proposed to turn this procedure into a step-by-step nearly mechanical process [14]. In this paper, by employing this strategy, a boundary between positive and negative refraction for the incident spectrum in the photonic crystal is created to enhance wavelength separation in the demultiplexer. The separated wavelengths then impinge on a second crystal with an oblique boundary to change their Bloch wavenumbers and increase the separation angles even further according to the technique described in [15].

The paper is organized as follows: The next section explains the theory of light refraction in photonic crystals and the mathematical tools and steps needed for EFC engineering. Designs of demultiplexers exhibiting positive–negative refraction boundaries are described in the third section. The fourth section includes the designs of two-step demultiplexers enjoying a positive–negative refraction boundary in the first step and a change of excited Bloch wavenumbers due to oblique interfaces of photonic crystals in the second step. The last section concludes the paper.

2 Theory

Light in photonic crystals can be directed to propagate in such a way as if the medium has a negative refractive index. This necessitates a thorough investigation of the rules of light refraction in photonic crystals if we are to use this phenomenon in our designs. This problem was solved for ordinary homogeneous and isotropic media a long time ago by Snell’s rule, but due to the periodic modulation of electric permittivity in photonic crystals, these structures don’t obey Snell’s rule.
Bloch-Floquet theory is the dominant theory for the explanation of light propagation problems inside photonic crystals, but it only determines the allowed propagation frequencies but not the light trajectory. Other types of periodic structures at other regions of electromagnetic spectrum can also have various applications and there are other approaches for their analysis [16]. Several theories were proposed by different groups with the goal of light trajectory prediction in photonic crystals but all of them proved to be insufficient [1, 6, 17–21]. Eventually, a paper by Notomi shed light on the subject by providing a theory that was in good agreement with experiments [12]. Refracted light trajectory in photonic crystals is determined with the help of the EFCs, a mathematical tool that is the heart of the theory of [12] and is the locus of all of the wavevectors in the reciprocal lattice which share a certain frequency of propagation as their eigenvalue.

To determine the propagation angles of the excited waves in a photonic crystal, the EFC of the incident light material should be drawn on top of the EFC of the photonic crystal. Figure 1 shows the EFC of an isotropic and homogeneous medium (top) and the schematic EFC of a 2D photonic crystal (bottom). For a homogeneous and isotropic medium, we have:

$$\omega = ck/n = c\sqrt{k_x^2 + k_y^2}/n$$  \hspace{1cm} (1)

where $\omega$ and $c$ are the angular frequency and the speed of light, respectively; $k$ is the wavenumber and $n$ is the refractive index of the medium. This equation shows that for a homogeneous and isotropic medium, the points in $k_x - k_y$ space that correspond to a certain frequency form a circle, which is the reason why the top EFC is circular in Fig. 1. The vertical line drawn from the $k_x$ point of the incident wave determines the value of $k_x$ in the photonic crystal because the tangential components of the electric field are the same in the two media to satisfy electromagnetic boundary conditions. Thus, The Bloch wavenumber of the excited wave in the photonic crystal is known. The propagation trajectory of the excited wave in the photonic crystal is in the direction of the group velocity vector which is the gradient of the angular frequency with respect to the wavevector $v = \text{grad}_k \omega$ which is a line perpendicular to the EFC.

The group velocity direction is obviously the direction of the gradient: if the EFCs are growing by frequency, the perpendicular line should be drawn outward, and if the contours are shrinking by increasing frequency, the line should be drawn inward. For the case depicted in Fig. 1, if the EFC of the photonic crystal has an inward gradient, the incident light propagates in a way as if the crystal has a negative refractive index. Therefore, in order to have a photonic crystal with negative refraction, we should design a crystal whose EFCs at the wavelength range of interest show an inward gradient.

The key idea toward EFC engineering is to use a specific kind of crystal that somehow eases the design procedure. One such crystal is a staircase profile for which closed-form relations exist between frequency and crystal parameters [22]. We consider a two-dimensional rectangular staircase (RSC) crystal whose permittivity function obeys [22]:

$$\varepsilon(x, y) = A(x) + B(y)$$  \hspace{1cm} (2)

where $A$ and $B$ are staircase profiles:

$$A(x) = \varepsilon_a + [u(x + t/2) - u(x - t/2)](\varepsilon_b - \varepsilon_a)$$  \hspace{1cm} (3a)

where $u$ is the unit step function and $X$ and $Y$ are the periods of the functions along the $x$ and $y$ directions. $m$ and $n$ are integers and $t$ and $s$ are positive constants for which we have: $t < X$ and $s < Y$. A schematic of such a crystal is shown in Fig. 2.

Two-dimensional wave equation for TE waves reads:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)E(x, y) + \frac{\omega^2}{c^2} \varepsilon(x, y)E(x, y) = 0$$  \hspace{1cm} (4)

where $E(x, y)$ is the electric field amplitude and $\omega$ and $c$ are the frequency and speed of light. Using the permittivity function of Eq. (2) and assuming $E(x, y) =$
Therefore, Eqs. 5a and 5b have Bloch solutions of the form
\[ \beta \text{ are periodic functions by periodicities of } X \text{ and } Y. \]

We can set \( X \) and \( Y \) to unity for the sake of normalization of \( c \) in these equations: \( \varepsilon_a, \varepsilon_b, \varepsilon_c, \varepsilon_d, t, s \). Thus, we can write a set of 6 simultaneous nonlinear equations to solve for the six unknown variables:

\[ \begin{align*}
1: & \quad \sin\left(\frac{\omega}{c} \sqrt{\varepsilon_a + \beta X - t}\right) \\
2: & \quad \sin\left(\frac{\omega}{c} \sqrt{\varepsilon_c + \beta (Y - s)}\right)
\end{align*} \]

\[ \begin{align*}
3: & \quad \cos\left(\frac{\omega}{c} \sqrt{\varepsilon_c + \beta (Y - s)}\right) \\
4: & \quad \cos\left(\frac{\omega}{c} \sqrt{\varepsilon_d + \beta s}\right)
\end{align*} \]

\[ \begin{align*}
5: & \quad \sin\left(\frac{\omega}{c} \sqrt{\varepsilon_a + \beta (X - t)}\right) \\
6: & \quad \sin\left(\frac{\omega}{c} \sqrt{\varepsilon_c + \beta (Y - s)}\right)
\end{align*} \]
Fig. 3 Schematic representation of a material showing a boundary between positive and negative refraction for the incident spectrum

$$(\kappa_i, \eta_i), i = 1, 2, 3$$ are points in the reciprocal lattice that we want to include in the EFCs. EFC engineering is done by a smart selection of these points to shape the EFCs for our desired purpose. We have employed numerical methods such as the Trust-Region-Dogleg method to solve the set of equations in (8) [23]. Note that due to the symmetries of the unit cell, the EFC profile can be shaped by shaping only a quarter of the $\eta$-$\kappa$ plane [24]. For example, for designing a circular EFC, it is adequate to design a 90° arc.

3 Design

Designing a crystal with a boundary between positive and negative refraction for the incident spectrum is a promising technique for a good demultiplexer design. As is schematically shown in Fig. 3, the change of refractive index is usually high in the boundary between positive and negative refraction and this produces a highly refractive material for the wavelength range of interest.

To design a crystal with a positive–negative refraction boundary, we choose our three points as follows:

$$(\kappa_1, \eta_1) = (0, 0.1) \text{ for } \omega = 0.78$$
$$(\kappa_2, \eta_2) = (0, 0.2) \text{ for } \omega = 0.74$$
$$(\kappa_3, \eta_3) = (0, 0.3) \text{ for } \omega = 0.76$$

These points are selected in a way to guarantee the boundary between positive and negative refraction: Moving from point 1 to 2, the EFCs are growing with decreasing frequency, i.e., we have negative refraction, while moving from point 2 to 3, the EFCs are growing with increasing frequency and we have positive refraction. Thus, there would be a boundary between positive and negative refraction.

Solving for the crystal parameters using the Trust-Region-Dogleg method, we have:

$$\varepsilon_a = 0.72, \varepsilon_b = 1.82, \varepsilon_c = 0.8,$$
$$\varepsilon_d = 1.97, s = 0.56, t = 0.62$$

EFCs of the designed RSC crystal are drawn with the help of the plane-wave expansion method and are shown in Fig. 4 [25]. As is clear in the figure, the selected points are successfully included in the EFCs.

The designed RSC crystal can be converted to standard pillar or air-hole structures which are easier to fabricate, with the help of mathematical techniques described in [14, 22] and. As described in [14, 22] and, the standard crystal parameters can be calculated in a way so that its first few Fourier coefficients are approximately equal to those of the designed RSC crystal. As a result of satisfying this condition, the two crystals show nearly identical EFCs [22]. Using this technique, the parameters of the pillar structure are calculated to be $r = 0.5, \varepsilon_a = 3.4$ and $\varepsilon_b = 1$ for the standard unit cell schematically shown in Fig. 5. EFCs of this crystal are drawn in Fig. 6, showing good conformity with those of the RSC crystal depicted in Fig. 4.

By reducing the unit cell dimensions from unity to $L = 0.14 \mu m$ and calculating the refraction angles of the incident spectrum according to principles outlined in [12], we reach the demultiplexer shown in Fig. 7 with $\lambda_1 = 1398 \text{ nm}, \lambda_2 = 1436 \text{ nm}, \lambda_3 = 1474 \text{ nm}, \lambda_4 = 1512 \text{ nm},$ and $\lambda_5 = 1550 \text{ nm}$. As seen in the figure, the designed demultiplexer produces 80-degree beam divergence for 152 nm deviation in incident wavelength showing excellent refractive performance.
Fig. 5 Unit cell of the standard photonic crystal

Fig. 6 EFCs of the standard crystal. The EFCs are similar to those of the designed RSC crystal drawn in Fig. 4

Fig. 7 Refractive properties of the designed demultiplexer in the wavelength range of interest. \( \lambda_1 = 1398 \text{ nm}, \lambda_2 = 1436 \text{ nm}, \lambda_3 = 1474 \text{ nm}, \lambda_4 = 1512 \text{ nm} \) and \( \lambda_5 = 1550 \text{ nm} \)

Fig. 8 EFCs of the standard crystal by choosing \((\kappa_1, \eta_1) = (0, 0.1)\) for \(\omega = 0.35\) and \((\kappa_1, \eta_1) = (0, 0.2)\) for \(\omega = 0.34\) and \((\kappa_1, \eta_1) = (0, 0.3)\) for \(\omega = 0.36\)

Fig. 9 Refractive properties of the designed demultiplexer in the wavelength range of interest. \(\lambda_1 = 1442 \text{ nm}, \lambda_2 = 1463 \text{ nm}, \lambda_3 = 1505 \text{ nm} \) and \(\lambda_4 = 1550 \text{ nm}\)

The aforementioned steps can be repeated to reach other demultiplexer designs. As an example, by choosing \((\kappa_1, \eta_1) = (0, 0.1)\) for \(\omega = 0.35\) and \((\kappa_1, \eta_1) = (0, 0.2)\) for \(\omega = 0.34\) and \((\kappa_1, \eta_1) = (0, 0.3)\) for \(\omega = 0.36\) and following the same steps, the parameters of the standard crystal are calculated to be \(r = 0.47\), \(\varepsilon_a = 20\) and \(\varepsilon_b = 5\) with the EFCs depicted in Fig. 8. By downscaling unit cell length to \(L = 527 \text{ nm}\), the designed demultiplexer would be able to produce 84-degree beam divergence for wavelength range \(\lambda = [1442 \text{ nm}, 1550 \text{ nm}]\). Figure 9 shows light trajectories inside the crystal for \(\lambda_1 = 1442 \text{ nm}, \lambda_2 = 1463 \text{ nm}, \lambda_3 = 1505 \text{ nm} \) and \(\lambda_4 = 1550 \text{ nm}\).
4 Two-step demultiplexing

The design technique described in the previous section can be combined with the technique reported in [15] to even further improve the demultiplexing properties of the design. As is clarified in [15], an oblique boundary between two photonic crystals can change the Bloch wavenumber of the propagating wave and change its traveling direction as shown in Fig. 10. The Bloch wavevector in the first crystal can be determined by drawing a vertical line from the initial wavevector. While determining the Bloch wavevector of the second crystal, we should have in mind that the projections of the wavevectors of the two media on the boundary should be equal. Therefore, we should draw a line perpendicular to the slope of the boundary as is shown in Fig. 10b. Schematic EFC of the second crystal together with the Bloch wavevector in the first crystal (κ₁) and also a line showing the slope of the boundary (α) are drawn in Fig. 10b. A dotted line perpendicular to the boundary is drawn from the end point of κ₁. The intersection of this line with the EFC determines the Bloch wavevector in the second crystal (κ₂). The slope of the boundary can be engineered to excite the desired wavevectors in the second crystal in order to produce a high angular separation between different wavelengths.

An example of employing this technique in demultiplexer design is described in Figs. 11 and 12. Figure 11 depicts the EFCs of the second photonic crystal. Points 1, 2 and 3 in this figure are the excited Bloch wavevectors in the first photonic crystal. As is shown in Fig. 11, a line perpendicular to the respective boundary (see Fig. 12) is drawn from the Bloch wavevector of each propagating wavelength. The intersections of these lines with the corresponding EFC determine the Bloch wavevectors and therefore the propagation directions in the second crystal. As is seen in Fig. 12, the two-step demultiplexer employing positive-negative refraction boundary in the first crystal and oblique boundaries with the second crystal, produces super-demultiplexing behavior, separating a spectrum of λ = [1474 nm, 1550 nm] by 161 degrees. This performance is excellent compared to other successful designs reported elsewhere and summarized in
### Table 1 Comparison between different successful designs

| Design | Wavelength separation |
|--------|-----------------------|
| [4]    | 30 degrees for 0.09 change in wavelength |
| [26]   | 21 degrees for 0.01 change in wavelength |
| [27]   | 75 degrees for 0.02 change in wavelength |
| [15]   | 133 degrees for 0.02 change in wavelength |
| This work | 161 degrees for 0.04 change in wavelength |

Table 1. This superiority is due to two facts that help us reach a more dispersive photonic crystal in the wavelength range of interest. Firstly, the presented novel design algorithm is effective in designing photonic crystals with positive–negative refraction boundaries which are inherently highly dispersive, by making the otherwise trial-and-error design processes into a mechanical process based on mathematical principles. Secondly, the novel demultiplexer structure with an oblique boundary between two photonic crystals changes the Bloch wavenumber of the propagating waves leading to further separation of the incident spectrum compared to the conventional designs.

### 5 Conclusion

In this paper, a systematic design algorithm was employed to create a positive–negative refraction boundary in a photonic crystal to boost the refractive properties of the optical demultiplexer. EFC engineering could be done by a smart selection of three points to be included in the EFC patterns of an RSC structure to guarantee the positive–negative refraction boundary. The RSC crystal was then transformed into a standard pillar structure with the help of mathematical methods based on Fourier analysis. The designed demultiplexer was able to produce 84-degree wavelength separation for the incident spectrum $\lambda = [1442 \text{ nm, } 1550 \text{ nm}]$.

The demultiplexer’s operational characteristics were even further improved by applying a two-step demultiplexing technique, in which the Bloch wavenumbers of the propagating waves in the first crystal could be changed after incidence on an oblique boundary of a second crystal. Using this technique, a wavelength separation of 163 degrees was obtained for the input wavelength range $\lambda = [1474 \text{ nm, } 1550 \text{ nm}]$.

### Data Availability Statement

This manuscript has no associated data or the data will not be deposited. [Authors’ comment: The data relating to the simulation results are completely reflected in the figures provided in the paper. The entirety of the work is theoretical and there is no experimental data to deposit.]
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