Abstract

We review the recent progress achieved in the theoretical investigation of Quantum Chromodynamics in the high temperature regime, with a focus on results achieved by lattice QCD simulations. The discussion covers the structure of the phase diagram and the properties of the strongly interacting medium at finite $T$ and small baryon chemical potential.

Keywords: QCD Phase Diagram, Lattice QCD

1. Introduction

The theoretical study of QCD in the regime of high temperature and zero or small baryon chemical potential has been a field of steady and continuous progress in the last few years. Much progress has been obtained by means of lattice simulations, perturbation theory and effective model studies. Our discussion is divided in three parts: in Section 2 we review recent studies regarding the location and nature of the finite temperature transition as a function of the quark mass spectrum; Section 3 focuses on various properties of the thermal medium; finally, in Section 4 we review recent results regarding the introduction of finite chemical potentials or other relevant extensions of the QCD phase diagram.

2. The finite temperature QCD transition

Lattice QCD simulations have shown that, for physical quark masses, the large volume scaling around the temperature where chiral symmetry restoration takes place is consistent with the presence of a rapid, analytic change of thermodynamical properties (what is usually called a crossover) in place of a real phase transition [1]. The location of the chiral pseudo-critical temperature $T_c$ has been well established by independent lattice studies using different discretizations [2, 3]; a refinement has been presented at this conference, indicating $T_c = 156.5 \pm 1.5$ MeV [4].

It is interesting, at least from a theoretical point of view, to determine how the nature of the transition changes as a function of the quark mass spectrum. This information is summarized, for the theory with $N_f = 2 + 1$ flavours, in the so-called Columbia plot, which is sketched in Fig. 1. Most theoretical predictions are based on universality arguments [5], through the analysis of an effective model sharing with QCD only...
the degrees of freedom involved in the symmetry relevant to the transition. The standard argument applies to cases where QCD symmetries are exact, hence to the upper-right corner (quenched limit with exact center symmetry) and to the left border (chiral symmetry with massless up-down quarks): if the model does not show any fixed point, i.e. any continuous transition with diverging correlation length, then none is predicted for QCD as well and a first order transition is expected; on the contrary, if the model has a fixed point, then QCD could still have a first order transition, but a continuous transition in the same universality class of the effective model becomes the alternative possibility. This kind of analysis extends partially to the rest of the Columbia plot, since first order transitions are stable against small explicit breakings of the relevant symmetry, so a first order point implies a first order neighbourhood around it, separated from the crossover region by a second order line which is usually in the 3d-Ising ($Z_2$) universality class, because it delimits a region where two different phases emerge.

The quenched (pure gauge) limit is known to be first order, while the analysis of effective chiral models predicts a first order transition for $N_f \geq 3$ massless fermions (hence for the bottom-left corner) and a possible second order (in the $O(4)$ universality class) or a first order for $N_f = 2$ massless flavours; in the latter case, the effective strength of the axial $U_A(1)$ symmetry anomalous breaking around the transition could change the predicted universality class [6, 7] or make a second order transition unlikely.

However, checking these predictions has revealed to be a hard task. For instance, simulations of $N_f = 3$ and even $N_f = 4$ QCD have failed to determine a well defined continuum limit for the critical mass delimiting the chiral first order region [8, 9, 10, 11, 12]: the critical mass could approach zero in the continuum, therefore the question whether the first order region in the bottom-left corner exists at all is still open. One should consider, in this respect, that while making reference to an effective chiral model seems perfectly reasonable, there are various counter-examples of models in which the presence of gauge degrees of freedom can change the situation substantially [13, 14]. On the other hand, the possible presence of a second order critical point in the $O(4)$ universality class predicted in the chiral limit of $N_f = 2$ QCD (upper-left corner) has been challenged by studies which suggest a first order transition, at least on coarse lattices [15, 16, 17].

An update has been reported at this conference [18] regarding the analysis of the chiral limit of the $N_f = 2 + 1$ theory, i.e. leaving the strange quark mass at its physical value and approaching the massless limit for the up and down quarks, for QCD discretized via HISQ staggered quarks. A first order transition is absent down to pion masses as low as 80 MeV and the scaling with the quark mass of the chiral condensate and of its susceptibility, which is regulated by the critical index $\delta$, is consistent with a chiral transition in the $O(4)$ (or $O(2)$, the residual exact symmetry for staggered quarks) universality class, $\delta = 0.21$. That still does not exclude completely the possible presence of a $Z_2$ transition at some finite but very low critical mass, the exponent $\delta$ being practically indistinguishable in this case, even if, also in view of the status in the $N_f = 3$ bottom-left corner mentioned above, this is unlikely. Finally, a preliminary estimate has been provided for the critical temperature in the chiral and continuum limit, $T_c = 138(5)$ MeV [18].

3. Properties of the strongly interacting thermal medium

Continuum extrapolated results for the equation of state are available since a few years [19, 20] and have been obtained by lattice groups adopting different staggered discretizations: the good agreement shows that systematic effects are well under control. More recently, the computation has been extended to a region of significantly higher temperatures, of the order of few GeVs, both including [21] and not including [22] charm quarks: results reported at this conference for $N_f = 2 + 1$ QCD [23] are in agreement with results from 3-loop HTL perturbation theory [23] and show, by comparison with Ref. [21], that charm contributions become appreciable when $T \gtrsim 500$ MeV. A future challenge for lattice QCD is to bring computations with discretizations other than staggered at the same level of accuracy; promising approaches to improve on Wilson fermions are on the way, based on the imaginary moving frame approach [24, 25, 26], on the gradient flow [27, 28, 29], on non-equilibrium methods [30] and on twisted mass discretizations [31].

The analysis of the equation of state is not exhaustive of the many interesting changes occurring in the thermal medium across $T_c$. The fate of the confining properties is another issue of obvious theoretical and phenomenological interest. Deconfinement is clearly signalled by the suppression of color flux tubes across $T_c$ [32, 33, 34], even if structures similar to flux tubes still seem to form between heavy $Q\bar Q$ pairs.
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does first order survive
limit

$N_f = 3$

1st order

quenched limit

$O(4)?$

first order?

physical point

$N_f = 2$

Fig. 1. Sketch of the nature of the finite temperature transition for $N_f = 2 + 1$ QCD as a function of the quark masses (Columbia plot).

for temperatures up to $\sim 1.5 T_c$ [34], and have been related to the possible presence of chromomagnetic charges populating the Quark-Gluon Plasma (QGP) [35]. Precise determinations of the heavy $\bar{Q}Q$ potential at higher temperatures have been reported recently [36] (for $T$ up to $\sim 6$ GeV), showing that the potential is perturbative at short distances and screened for distances $r$ such that $r T \gtrsim 0.3$; results on the dependence of color screening on the baryon density have also been reported at this conference [37]. The investigation of $\bar{Q}Q$ interactions is essential to clarify the fate and suppression of heavy quark bound states within the thermal medium: many direct studies of such states have been reported recently [38, 39], and progress has been made also in the determination of a realistic in-medium potential [40], which is the correct quantity to be used in place of static $\bar{Q}Q$ free energies.

It is also interesting to ask what is the effect on the properties of the QGP of chiral symmetry restoration: this is visible, for instance, from the recovered degeneracy of various meson correlators. More recently, this has been clearly observed also in the baryon sector, where different parity doublets become degenerate as $T$ approaches $T_c$ from below [41, 42]. Also the axial $U_A(1)$ symmetry, even if always explicitly broken by the presence of the axial anomaly, is expected to undergo an effective restoration, because of the suppression of gauge configurations with non-zero winding number, and various studies show that this happens for $T \gtrsim 200$ MeV [43, 44, 45, 46].

Concerning the transport properties of the thermal medium, their study by non-perturbative lattice simulations is notoriously difficult, because they are related to non-equilibrium properties. In principle, Euclidean correlators give direct access to the relevant spectral functions and the transport coefficients can be obtained by solving appropriate integral equations [48], however that requires to have an extreme accuracy on the correlators, which is presently achievable only in the quenched theory: results for the heavy quark diffusion coefficient have been reported three years ago [49], while updated results for the bulk and shear viscosity have been discussed at this conference [50, 51]. Unfortunately, such high precision is not yet achievable in full QCD and success is limited to a few cases, like the computation of the electric conductivity [52, 53]; hopes for the future are linked to the development of efficient multilevel updating schemes [54]. Substantial progress has been reported at this conference on the computation of transport coefficients by perturbation
theory [55], or by model studies which take lattice data as an input [56].

4. Finite temperature QCD in the presence of external sources

When one tries to extend the exploration of the QCD phase diagram to the finite density case, one has to face the well known sign problem. One can investigate systematically only special cases, like that of matter with zero overall baryonic charge but with a finite density of isospin charge (e.g., a pion gas), for which recent results can be found in Ref. [57]. Instead, when one tries to switch on a finite baryonic chemical potential, the path integral measure becomes complex, thus hindering the exploitation of standard Monte-Carlo sampling techniques. While no definite solution has still been found, some partial solutions are known to work well in a regime of small values of $\mu_B/T$, namely Taylor expansion [58, 59, 60, 61], analytic continuation from an imaginary chemical potential [62, 63, 64].

A quantity for which these approximate techniques are well suited is the curvature of the pseudocritical line in the $T-\mu_B$ plane separating confined phase from the QGP phase, which is defined by

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left( \frac{\mu_B}{T_c} \right)^2 + O(\mu_B^4).$$

The curvature $\kappa$ can be determined by following explicitly how $T_c$ moves at imaginary $\mu_B$ and then continuing to real $\mu_B$, or by finding $dT_c/d\mu_B$ implicitly in terms of derivatives computed by lattice simulations at $\mu_B = 0$ (Taylor expansion). While these techniques have been applied since long, it is useful to mention some recent determinations, obtained with discretizations of QCD which are close to the physical point [65, 66, 67, 68, 69, 70, 71, 72, 73], and are summarized in Fig. 2; two of those determinations have been presented at this conference [4, 73, 74] and represent an important confirmation that, after systematic effects have been properly taken into account, Taylor expansion and analytic continuation lead to consistent and reliable results. The various determinations have been obtained for a variety of choices of the strange quark chemical potential: $\mu_s = 0$ [65, 66, 68, 69, 72], $\mu_s = \mu_i$ [67, 71] or tuned so as to guarantee strangeness neutrality [72, 74] as in heavy ion collisions; as a matter of fact, one finds that $\mu_s$ has a negligible effect on $\kappa$ [68, 69]. In the figure we report also an average (vertical dashed band) of the five most recent determinations obtained after continuum extrapolation (filled dots), this average is $\kappa = 0.014(2)$ and is quite stable when excluding one or more of the mentioned determinations; we have been conservative with the error estimate, since the errors of the various determinations include also systematic contributions, which are typically different for the different methods.

Other quantities which can be investigated systematically even with the available numerical approaches are the so-called generalized susceptibilities, i.e. higher order derivatives of the partition function with respect to the quark chemical potentials:

$$\frac{P}{T^4}(\mu_u, \mu_d, \mu_s) = \frac{P}{T^4}(0, 0, 0) + \sum_{i+j+k \geq 0} \chi_{ijk}(T) \left( \frac{1}{P_u} \frac{1}{P_d} \frac{1}{P_s} \right) ; \quad \chi_{ijk}(T) = \frac{1}{VT^4} \frac{\partial^{i+j+k} F(T, \mu)}{\partial \mu_u^i \partial \mu_d^j \partial \mu_s^k} \bigg|_{\mu_u=\mu_d=\mu_s=0}. $$

As for other quantities related to the dependence of the free energy density on the chemical potentials, the coefficients $\chi_{ijk}(T)$ can be determined as Taylor coefficients computed at zero chemical potential [58, 59, 60, 61, 75], or exploiting numerical simulations at imaginary $\mu$ and then analytic continuation [76, 77, 78, 79, 80]. Using the latter method one can reach up to the eighth order in the expansion [80]: new results along this line have been presented at this conference [81].

Generalized susceptibilities are important for various phenomenological and theoretical reasons. For instance, since they are directly related to fluctuations of conserved charges, they can be used to compare the QCD thermal medium with the experimental output from heavy ion collisions and obtain a model-independent determination of the freeze-out line [82, 83, 84]; an update of research along this line has been reported at this conference (posters by C. Ratti and C. Schmidt). They are also extremely useful as an input or benchmark for a variety of model approaches to the strongly interacting medium [85, 86, 87, 88, 89, 90].

Besides that, generalized susceptibilities can be used to investigate the properties of convergence of the Taylor series: this is useful since the possible presence of a critical point in the QCD phase diagram is
Fig. 2. Summary of recent determinations of the curvature of the pseudo-critical line, defined in Eq. (1), obtained at or close to the physical point. For each determination, the method adopted to extract $\kappa$ is indicated on the left, while the adopted discretization and the setup of chemical potentials are indicated on the right. Chiral and $O(2)$ refer to determinations obtained in the chiral limit assuming a second order critical scaling. Finally, filled circles indicate determinations obtained after continuum extrapolation. The vertical dashed band is an average of continuum extrapolated results obtained after 2015.

signalled by a finite radius of convergence. The radius can be estimated directly from the series expansion of the free energy density, or from those of its derivatives:

$$\rho_{n,m}^{(1)} = \left( \frac{\chi_{n}^{B}}{n!} \right) \left( \frac{\chi_{n}^{B}}{m!} \right) \left( \frac{\chi_{n}^{B}}{n(m-1)!} \right) \left( \frac{\chi_{n}^{B}}{m(m-1)!} \right), \quad \rho_{n,m}^{(2)} = \left( \frac{\chi_{n}^{B}}{n!} \right) \left( \frac{\chi_{n}^{B}}{m!} \right) \left( \frac{\chi_{n}^{B}}{n(m-2)!} \right) \left( \frac{\chi_{n}^{B}}{m(m-2)!} \right).$$

(3)

In presence of a finite radius of convergence both definitions should converge to a common value as $n, m \to \infty$; a correct estimate of the radius is thus linked to the possibility of accessing the asymptotic behaviour of the series: however, what “asymptotic” means is a priori not known, and estimates based on just the first 3-4 terms might be affected by unknown systematics. Various studies have exploited this method \cite{91,61,92}, including some studies appeared recently \cite{93,80,75}; new results have been reported at this conference as well \cite{94}. The overall outcome (see Ref. \cite{94} for more details) is that in some studies no convergence of the radius estimate is observed and that, even when this is observed by looking at just the first few ratios, one should be careful, since it could be fake. One should take into account the possibility that, even in the presence of a critical point, lattice data could not be sensitive to it, or we should be able access much higher orders in the series expansion, something which is currently unfeasible. In this respect, much insight could come from models which take lattice data as an input \cite{95,96}. The fact that no critical point is likely to be found close to $\mu_B = 0$ is also supported by the apparent lack of strong $\mu$-dependence of various susceptibilities (see Refs. \cite{4,69} and talk by S. Borsanyi at this conference).

There are other extensions of the QCD phase diagram which are of interest for heavy ion collisions. One of them is the introduction of a magnetic background field, which is relevant to at least the early stages of the collisions. Many things are well known, like the fact that the QGP behaves as a paramagnetic medium, with a magnetic susceptibility which strongly increases with the temperature \cite{97,98,99,100,101}. The pseudo-critical temperature decreases as a function of $B$ \cite{102}, and recent results suggest that this behaviour is qualitatively independent of the quark mass spectrum \cite{103}. Another interesting extension, of obvious interest to peripheral collisions, is the study of a rotating thermal medium; in this case the sign problem is back again, however preliminary studies exist which should be further pursued in the future \cite{104}. 

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Taylor +$O(2)$

Kaczmarek et al. PRD (2011)

Taylor

Endrodi et al. JHEP (2011)

Kaczmarek et al. PRD (2011)

Cea et al. PRD (2014)

Bonati et al. PRD (2014)

Bonati et al. PRD (2015)

Bellwied et al. PLB (2015)

Cea et al. PRD (2015)

Hegde et al. (Lattice 2015)

Taylor

Taylor

Taylor

Taylor

now new

new new

hotQCD @ QM2018

p4, $\mu = 0$

chiral

stout2, $\mu = 0$

HISQ, $\mu = \mu_t$

stout2, $\mu = 0$, $\mu_l$

stout2, $\mu = 0$, $\mu_t$

stout5, $S = 0$, $Q/B = 0.4$

HISQ, $\mu = \mu_t$

HISQ, $S = 0$

chiral

stout2, $\mu = 0$

HISQ, $S = 0$, $Q/B = 0.4$
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