Non-parametric spatial curvature inference using late-universe cosmological probes

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ABSTRACT
Inferring high-fidelity constraints on the spatial curvature parameter, $\Omega_K$, under as few assumptions as possible, is of fundamental importance in cosmology. We propose a method to non-parametrically infer $\Omega_K$ from late-Universe probes alone. Using Gaussian Processes (GP) to reconstruct the expansion history, we combine Cosmic Chronometers (CC) and Type Ia Supernovae (SNe Ia) data to infer constraints on curvature, marginalized over the expansion history, calibration of the CC and SNe Ia data, and the GP hyper-parameters. The obtained constraints on $\Omega_K$ are free from parametric model assumptions for the expansion history, and are insensitive to the overall calibration of both the CC and SNe Ia data (being sensitive only to relative distances and expansion rates). Applying this method to Pantheon SNe Ia and the latest compilation of CCs, we find $\Omega_K = -0.03 \pm 0.26$, consistent with spatial flatness at the $\mathcal{O}(10^{-1})$ level, and independent of any early-Universe probes. Applying our methodology to future Baryon Acoustic Oscillations and SNe Ia data from upcoming Stage IV surveys, we forecast the ability to constrain $\Omega_K$ at the $\mathcal{O}(10^{-2})$ level.

Key words: cosmological parameters – cosmology:observations – distance scale

1 INTRODUCTION
Measuring the spatial curvature of the Universe is of longstanding interest in cosmology. Determining the sign and value of the curvature parameter $\Omega_K$ would be of great significance for fundamental physics, given its implications for both early- and late-Universe physics in relation to the inflationary paradigm and the ultimate fate of the Universe respectively. This question has received significant, renewed interest in the past years, particularly in light of the Planck Cosmic Microwave Background (CMB) legacy data release (Aghanim et al. 2020), whose temperature and polarization data, taken on their own, could appear at face value to suggest that the Universe might not be spatially flat.

The ability of primary CMB data to constrain $\Omega_K$ is, ultimately, limited by the geometrical degeneracy (Bond et al. 1997; Efstathiou & Bond 1999; Zaldarriaga et al. 1997), which can be broken by including “external” data, such as Baryon Acoustic Oscillation (BAO) measurements. While this usually results in the inference of the Universe being spatially flat to within $\approx 1\sigma$ (see e.g. Alam et al. 2017; Aghanim et al. 2020; Efstathiou & Gratton 2020; Vagnozzi et al. 2020; Vagnozzi et al. 2021), doubts have been cast on the soundness of such combinations, suggesting the possibility of there being a “curvature tension” in current data (see e.g. Handley 2021; Di Valentino et al. 2019, 2021).

Without going into the details of the previous discussions, it is clearly of considerable interest to obtain high-fidelity measurements of $\Omega_K$.1 Moreover, many of the constraints on $\Omega_K$ available in the literature depend on the assumed parametric form of the late-time expansion rate. It is therefore also desirable to obtain non-parametric, purely geometrical constraints on $\Omega_K$ from late-Universe cosmological probes alone, independent of and complementary to the early-Universe CMB constraints.

In this Letter, we propose a non-parametric approach for inferring $\Omega_K$ using only late-Universe distance and ex-

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1 For earlier works examining constraints and forecasted constraints on $\Omega_K$, see for instance Vardanyan et al. (2009); Takada & Dore (2015); Leonard et al. (2016); Yu & Wang (2016); Witzemann et al. (2018); Denissenya et al. (2018); Li et al. (2020); Park & Ratra (2020); Khadka & Ratra (2020); Nunes & Bernui (2020); Benisty & Staicova (2021); Cao et al. (2021).
expansion rate indicators, in the form of Type Ia Supernovae (SNe Ia) and cosmic chronometers (CC). Imposing a Gaussian Process (GP) smoothness prior on the expansion history, we use the CC and SNe Ia data to jointly infer the curvature parameter, (non-parametric) expansion history, calibration of the CC and SNe Ia data, and GP hyperparameters. The resulting marginal constraints on $\Omega_K$ are free from parametric model assumptions about the expansion history. Moreover, being only sensitive to relative distances and expansion rates from the SNe Ia and CC measurements respectively, our constraints on $\Omega_K$ are insensitive to the overall calibration (and associated systematics) in either dataset. Applying our methodology to current SNe Ia data from the Pantheon compilation (Scolnic et al. 2018) and the latest compilation of CC measurements, we find $\Omega_K = -0.03 \pm 0.26$, consistent with flatness at the $\sim O(10^{-3})$ level. Applying our methodology to future Stage-IV dark energy missions, we forecast that these data will be able to deliver $\sim O(10^{-2})$-level constraints on $\Omega_K$, competitive with the current CMB-only constraints from Planck (Aghanim et al. 2020) and ACT+WMAP (Aiola et al. 2020).

2 DATA AND METHODOLOGY

Over the past two decades, Type Ia Supernovae (SNe Ia) and cosmic chronometers (CC) have emerged as important probes of the late-time expansion history. SNe Ia are excellent late-time distance indicators, and have been used among the other things to infer the existence of cosmic acceleration, to measure the properties of the dark energy (DE) component responsible for cosmic acceleration (Riess et al. 1998; Perlmutter et al. 1999; Scolnic et al. 2018), and (once calibrated with nearby distances to their host galaxies) to infer the present day value of the expansion rate, i.e. the Hubble constant $H_0$ (Riess et al. 2019; Freedman et al. 2019). SNe Ia are sensitive to the luminosity distance $d_L(z)$ via the distance modulus:

$$\mu = 5 \log(d_L) + 25, \quad \text{(1)}$$

where the luminosity distance is given by:

$$d_L = \frac{cz}{H_0}\sqrt{|\Omega_K|} \sinh \left( \sqrt{\Omega_K} \int_0^z \frac{dz'}{E(z')} \right), \quad \text{(2)}$$

with $sinn$ indicating either sin or sinh depending on whether $\Omega_K < 0$ or $\Omega_K > 0$, and where $H(z)$ is the Hubble expansion rate, $E(z) \equiv H(z)/H_0$ is the normalized expansion rate, and $M$ is the absolute SNe Ia calibration. While an absolute SNe Ia luminosity calibration (achieved for instance through Cepheids or the Tip of the Red Giant Branch) is necessary to determine absolute distances and hence $H_0$, for the purpose of inferring $\Omega_K$ only relative distances are required. Therefore, inferences on the curvature parameter are expected to be insensitive to the overall SNe Ia calibration, as we will explicitly demonstrate later.

CCs are tracers of the evolution of the differential Universe as a function of redshift, from which the Hubble expansion rate $H(z)$ can be inferred directly, essentially by inverting the age-redshift relation (Jimenez & Loeb 2002). Massive, early, passively-evolving galaxies have been found to be very good tracers in this sense (see e.g. Cimatti et al. 2004; Thomas et al. 2010; Morecco 2015; Morecco et al. 2018, 2020, for important works in this direction), and have been used extensively over the past two decades to measure $H(z)$ up to $z \approx 2$ (see e.g. Jimenez et al. 2003; Simon et al. 2005; Stern et al. 2010; Morecco et al. 2012; Morecco 2015; Morecco et al. 2016; Ratsimbazafy et al. 2017, whose measurements we use). When combining direct measurements of $H(z)$ from CCs with SNe Ia to constrain curvature, the information added by CCs only influences our $\Omega_K$ inference via $E(z) = H(z)/H_0$. Therefore, as for SNe Ia, inferences on the curvature parameter are expected to be insensitive to the absolute calibration of the CC measurements.

In this work we combine SNe Ia and CC data to constrain the spatial curvature parameter $\Omega_K$, assuming a non-parametric model for the expansion history $H(z)$, i.e. independent of any fixed parametric cosmological model for the expansion rate. In this way, we are able to infer constraints on spatial curvature that are independent of $a)$ any assumed cosmological model for the late-time expansion, $b)$ the absolute calibration of either the SNe Ia or CC measurements, and $c)$ early-Universe measurements.

2.1 Data and likelihoods

We use the latest compilation of SNe Ia distance moduli measurements from the Pantheon compilation (Scolnic et al. 2018), combined with 31 CC $H(z)$ measurements in the range $0.07 < z < 1.965$. The CC measurements have been compiled in Jimenez et al. (2003); Simon et al. (2005); Stern et al. (2010); Morecco et al. (2012); Morecco (2015); Morecco et al. (2016); Ratsimbazafy et al. (2017), and are summarized in Tab. I of Vagnozzi et al. (2021).

The SNe Ia data comprise measurements of the distance moduli as a function of redshift, $\mu(z)$. We take the binned Pantheon data vector $d_{SN}$ and systematics-marginalized covariance matrix $C_{SN}$ from Scolnic et al. (2018), with log-likelihood given by (up to an additive constant):

$$\ln P(d_{SN}|\Omega_K, H(z), M) = -\frac{1}{2} [\mu - \mu(\Omega_K, H(z), M)]^T C_{SN}^{-1} [\mu - \mu(\Omega_K, H(z), M)], \quad \text{(3)}$$

where $\mu$ is the data vector of measured distance moduli, and $\mu_i = \mu(z_i, \Omega_K, H(z), M)$ are the predicted distance moduli for redshift bins $\{z_i\}$, see Eqs. (1,2).

For the CC data, with data vector given by $d_{CC}$, we assume independent Gaussian uncertainties, and hence a log-likelihood given by (up to an additive constant):

$$\ln P(d_{CC}|H(z)) = -\frac{1}{2} \sum_{j} (\hat{H}_j - H(z_j))^2/\sigma_{CC,j}^2, \quad \text{(4)}$$

where the CC data consists of measurements of $\{\hat{H}_j\}$ at redshifts $\{z_j\}$, with uncertainties $\sigma_{CC,j}$ respectively.

2.2 Priors

We set (improper) uniform priors on the spatial curvature parameter $\Omega_K$ and the absolute SNe Ia calibration $M$. With regards to the expansion history $H(z)$, we proceed non-parametrically, assuming solely that $H(z)$ is a smooth function of $z$. The “smoothness” of $H(z)$ is controlled by a set of hyper-parameters $\eta$, which are included as free parameters...
to be eventually marginalized over. To this end, we impose a Gaussian process (GP) prior on \( H(z) \): \[
P(H(z)|\eta) = \mathcal{N}(H(z)|m(z), K_\eta),
\]
where \( m(z) \) is the prior mean. On the other hand, the prior covariance between the values of \( H(z) \) at two redshifts \( z \) and \( z' \) is specified by the kernel function \( K_\eta \): \[
K_\eta \equiv K_\eta(z, z') = \langle (H(z) - m(z))(H(z') - m(z')) \rangle.
\]
The properties of the chosen kernel function (governed by its functional form and hyper-parameters \( \eta \)) characterize the prior assumptions on the smoothness of the expansion history as a function of redshift.

In this work, we adopt two common choices for the Gaussian process kernel functions. As our baseline, we use the squared-exponential kernel, given by:
\[
K_\eta(z, z') = a^2 \exp\left(-\frac{|z - z'|^2}{2\ell^2}\right),
\]
where the hyper-parameters \( \eta \equiv (a, \ell) \) control the amplitude and length-scale of the prior covariance respectively. We set flat (positive) priors on the kernel hyper-parameters for both the squared-exponential and Matérn-3/2 kernels, given by:
\[
K_\eta(z, z') = a^2 \left(1 + \frac{\sqrt{3}|z - z'|}{\ell} \right) \exp\left(-\frac{\sqrt{3}|z - z'|}{\ell}\right),
\]
where again the hyper-parameters \( \eta \equiv (a, \ell) \) control the amplitude and length-scale of the prior covariance. We set flat (positive) priors on the kernel hyper-parameters for both the squared-exponential and Matérn-3/2 kernels, and take the GP prior mean to be \( m(z) = 100 \) (setting an appropriate scale for \( H(z) \) over the redshift range of interest).

2.3 Joint posterior and inference strategy

In practice, in order to sample from the joint posterior for \( \Omega_K, M \) and \( H(z) \) including the unknown “function” \( H(z) \), one has to discretize \( H(z) \rightarrow H \) at some redshifts \( \{z_k\} \), and include those nodes as free parameters (to be marginalized over). The Gaussian process prior on \( H(z) \) translates into a simple multivariate Gaussian prior on \( H \):
\[
P(H|\eta) = \frac{1}{\sqrt{|2\pi K_\eta|}} \exp\left[ -\frac{1}{2} (H - m)^\top K_\eta^{-1} (H - m) \right]
\]
with mean \( m_k = m(z_k) \) and covariance (specified by the Gaussian process kernel function) \( K_{\eta, kl} = K_\eta(z_k, z_l) \). We choose redshift nodes for \( H(z) \) that are dense enough in redshift so that the luminosity distance integrals [Eq. (2)] in the SNe Ia likelihood can be performed numerically, and include additional nodes at the CC redshift values (as required for evaluating the CC likelihood).

2 Note that in many common use cases for Gaussian processes under Gaussian likelihoods, the latent function can be marginalized over analytically. However, in this case since the SNe Ia likelihood is non-Gaussian in \( H(z) \), the function needs to be discretized and explicitly sampled over in the inference pipeline.

3 This ensures that the resulting distance integrals are accurate enough, i.e. with errors much smaller than the (root) diagonals of the SNe Ia data covariance.

3.1 Current constraints

Applying the methodology described above to the Pantheon SNe Ia and current CC data, we infer \( \Omega_K = -0.03 \pm 0.26 \) (68% credible region), after marginalizing over all other parameters (including the GP hyper-parameters). Therefore, with current data we are able to reach a precision at the \( \sim O(10^{-5}) \) level, approximately one order of magnitude weaker than the parametric constraints from Planck primary CMB data alone (\( \Omega_K = -0.044_{-0.015}^{+0.018} \), under a 7-parameter \( \Lambda \)CDM+\( \Omega_K \) model (Aghanim et al. 2020)). However, we stress that our results rely on late-Universe measurements alone, and do not assume any parametric form for the late-time expansion history (see Figure 1 for the GP reconstruction of the Hubble parameter as a function of redshift). Within the achieved precision, we observed no deviations from spatial flatness, as shown in Figure 2.

We note that other earlier works, including Cai et al. (2016); Wei & Wu (2017); Yang & Gong (2020); Liu et al. (2020), derived non-parametric constraints on \( \Omega_K \) from similar (albeit in some cases older) dataset combinations. Our work is the first to perform a self-consistent joint inference of
the cosmological, calibration, and GP hyper-parameters. In the earlier works, the GP regression step of the analysis was instead treated separately from the cosmological parameter inference, which could cause the uncertainties to be underestimated. Some of these earlier works depend on null tests to verify whether the data is consistent with $\Omega_K = 0$, rather than inferring $\Omega_K$ itself. This process requires taking the derivative of noisy data, and is naturally avoided in our methodology. Finally, unlike some of these earlier works, our method does not require knowledge of the absolute $H(z)$ and SNe Ia calibration, which are naturally marginalized over.

Other studies in the literature have used a cosmographic expansion to describe the Hubble parameter (and related quantities such as the deceleration parameter) in the late universe, and constrain $\Omega_K$ using low redshift cosmological probes. For example Collett et al. (2019) adopt strong lensing time-delays and SNe Ia data to constrain $\Omega_K$ at the $\sim 2-3 \times 10^{-1}$ level (similar to our constraints). Jesus et al. (2020) adopts a similar dataset combination, using a cosmokinetic parametrization of the Hubble parameter, deceleration parameter, and comoving distance as a function of redshift, and finds constraints on $\Omega_K$ at the $\sim 2-3 \times 10^{-1}$ level. Compared to these earlier works, our method makes fewer assumptions on the late-time expansion history, while finding a similar precision on the inferred value of $\Omega_K$.

### 3.2 Forecasts

A number of planned Stage IV missions will start taking data in the coming decade. These surveys will make use of multiple, complementary probes, such as BAO, SNe Ia, and cosmic shear. Since they are all calibrated to the same value of the sound horizon $r_s$, even without knowing $r_s$ (which would require making assumptions about pre-recombination physics), radial BAO measurements probe the evolution of the dimensionless Hubble rate, much as CC data does. Since mock future CC data are not yet available, we forecast the ability of our method to constrain $\Omega_K$ from a combination of future SNe Ia and radial BAO measurements.

Surveys such as the Vera C. Rubin Observatory and the Nancy Grace Roman Space Telescope (NGRST) are expected to increase the yield of high-z SNe Ia by an order of magnitude compared to current samples, with an improved control over systematic uncertainties. Concerning BAO measurements, the Dark Energy Spectroscopic Instrument (DESI, DESI Collaboration et al. 2016), Euclid (Laureijs et al. 2011), and NGRST (Green et al. 2012) are designed to extend the redshift range of BAO measurements to $z \gg 1$. This will result in significant overlap with the redshift range probed by the SNe Ia magnitude-redshift relation, with considerable benefits for our method.

We use the forecasts provided for the NGRST SNe Ia magnitude-redshift relation by Houssell et al. (2018). The inputs for our analysis include the binned redshift distribution along with the statistical errors on the distance and the systematics covariance matrix, in a similar way to the Pantheon compilation described above. Concerning future probes of $E(z)$, we consider mock radial BAO data in the redshift range $0.1 \lesssim z \lesssim 2.0$, matching the expected sensitivity and instrumental specifications of DESI (DESI Collaboration et al. 2016), Euclid (Refregier et al. 2010), and NGRST (Green et al. 2012). Mock data to forecast the constraining power of our method is generated assuming a fiducial $\Lambda$CDM cosmology with $\Omega_M = 0.3$ and $\Omega_K = 0$.

We find that our method in combination with future SNe Ia and BAO data considered above will be able to constrain $\Omega_K$ with a 1σ uncertainty of 0.026 – an order of magnitude improvement over current Pantheon SNe Ia and CC data (blue shaded area in Figure 2). This uncertainty is competitive with the current uncertainty coming from Planck primary CMB data alone. Therefore, these future measurements will be able to provide a useful and independent test of whether the apparent preference for a closed Universe from Planck primary CMB data alone is “real”, or a statistical fluctuation (as suggested for instance in Efstathiou & Gratton (2019)). We verified that our results are insensitive to the overall calibration of either the SNe Ia or CC data by re-scaling those data by (different) arbitrary constants, obtaining identical marginal constraints on $\Omega_K$.

### 4 DISCUSSION AND CONCLUSION

In this Letter, we have presented a novel, self-consistent, non-parametric approach for constraining spatial curvature from late-time cosmological probes alone. After marginalizing over the calibration parameters, GP hyper-parameters, and expansion history $H(z)$ at intermediate redshifts, we infer a value of $\Omega_K = -0.03 \pm 0.26$ from current data. This result is consistent with spatial flatness, albeit with an error bar too large to weigh in on the apparent preference for a closed Universe from Planck primary CMB data alone.
Applying our method to BAO and SNe Ia data matching the expected sensitivity of upcoming Stage IV missions, we forecast an order of magnitude improvement in constraints on $\Omega_K$, which will reach a $\sim O(10^{-2})$ precision (competitive with the current Planck primary CMB constraints).

Our method does not depend on the absolute calibration of $H(z)$, and is thus immune to concerns pertaining, for instance, to the Hubble tension (to the extent that those issues are related to absolute calibration). Moreover, relying exclusively on late-Universe probes, our approach is completely independent of the CMB constraints on $\Omega_K$. While parametrized approaches towards constraining spatial curvature will likely remain the standard, our method provides an important complement to parametric constraints.

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DATA AVAILABILITY

The data underlying this article will be shared upon request to the corresponding author(s). The associated repository is: https://github.com/sdhawan21/Curvature_GP_LateTime.

REFERENCES

Aghanim N., et al., 2020, Astron. Astrophys., 641, A6
Aiola S., et al., 2020, JCAP, 12, 047
Alam S., et al., 2017, Mon. Not. Roy. Astron. Soc., 470, 2617
Bellini E., Zumalacarregui M., 2015, Phys. Rev. D, 92, 063522
Benisty D., Staicova D., 2021, Astron. Astrophys., 647, A38
Bond J. R., Efstathiou G., Tegmark M., 1997, Mon. Not. Roy. Astron. Soc., 291, L33
Cai R.-G., Guo Z.-K., Yang T., 2016, Phys. Rev. D, 93, 043517
Cao S., Ryan J., Ratra B., 2021, arXiv e-prints, arXiv:2101.08817
Cimatti A., et al., 2004, Nature, 430, 184
Collett T., Montanari F., Räsänen S., 2019, Phys. Rev. Lett., 123, 231301
DESI Collaboration et al., 2016, arXiv e-prints, arXiv:1611.00036,
Denissenya M., Linder E. V., Shafieloo A., 2018, JCAP, 03, 041
Di Valen.tino E., Melchiorri A., Silk J., 2019, Nature Astron., 4, 196
Di Valen.tino E., Melchiorri A., Silk J., 2021, Astrophys. J. Lett., 908, L9
Efstathiou G., Bond J. R., 1999, Mon. Not. Roy. Astron. Soc., 304, 75
Efstathiou G., Gratton S., 2019, arXiv e-prints, arXiv:1910.00483,
Efstathiou G., Gratton S., 2020, Mon. Not. Roy. Astron. Soc., 496, L91
Freedman W. L., et al., 2019, Astrophys. J., 882, 34
Green J., et al., 2012, arXiv e-prints, arXiv:1208.4012,