The effective potential and universality in GUT inspired gauge-Higgs unification

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Abstract

The effective potential for the Aharonov-Bohm phase $\theta_H$ in the fifth dimension in GUT inspired $SO(5) \times U(1) \times SU(3)$ gauge-Higgs unification is evaluated to show that dynamical electroweak symmetry breaking takes place with $\theta_H \neq 0$, the 4D Higgs boson mass 125 GeV being generated at the quantum level. The cubic and quartic self-couplings ($\lambda_3, \lambda_4$) of the Higgs boson are found to satisfy universal relations, i.e. they are determined, to high accuracy, solely by $\theta_H$, irrespective of values of other parameters in the model. For $\theta_H = 0.1$ (0.15), $\lambda_3$ and $\lambda_4$ are smaller than those in the standard model by 7.7% (8.1%) and 30% (32%), respectively.
1 Introduction

The Higgs boson is responsible for the electroweak symmetry breaking in the standard model (SM). The Higgs potential is arranged such that the Higgs field spontaneously develops a nonvanishing vacuum expectation value. Its couplings to quarks and leptons (Yukawa couplings) are determined such that the observed quark-lepton mass spectrum is reproduced. Although the SM seems consistent with almost all experimental data so far obtained, it is yet to be seen whether or not the Higgs boson is exactly what is postulated in the SM. Unlike the gauge sector in the SM, the Higgs sector lacks a principle, which leaves arbitrariness in the theory. The Higgs boson mass acquires large quantum corrections which must be cancelled by fine-tuning of parameters in the model.

One approach to overcome these difficulties is gauge-Higgs unification in which the 4D Higgs boson is identified with the 4D fluctuation mode of an Aharonov-Bohm (AB) phase in the fifth dimension. The 4D Higgs field is contained in the extra-dimensional component of gauge potentials. As an AB phase the Higgs boson is massless at the tree level, but acquires a finite mass at the quantum level, independent of a cutoff scale and regularization method. The gauge hierarchy problem is naturally solved.

Recently substantial advances have been made in gauge-Higgs unification. Realistic models have been constructed which yield nearly the same phenomenology as the SM at low energies and give many predictions to be explored at LHC and ILC. Most of gauge-Higgs unification models are constructed on orbifolds such as $M^4 \times (S^1/Z_2)$ and the Randall-Sundrum (RS) warped space. Chiral fermions naturally emerge on orbifolds. The $SU(2)_L$ doublet Higgs field must appear as a zero mode of the fifth-dimensional component of gauge fields. This condition leads to gauge groups such as $SU(3) \times U(1)_X \times SU(3)_C$ or $SO(5) \times U(1)_X \times SU(3)_C$, among which the latter accommodates the custodial symmetry in the Higgs sector. Quark-lepton multiplets are introduced such that with orbifold conditions specified zero modes appear precisely for quarks and leptons, but not for exotic light fermions. They must have observed couplings to $W$ and $Z$ bosons, and their masses must be reproduced. Further the effective potential for the AB phase $\theta_H$ must have a global minimum at $\theta_H \neq 0$ so that the electroweak gauge symmetry is dynamically broken to $U(1)_{EM}$. As a model satisfying these conditions $SO(5) \times U(1)_X \times SU(3)_C$ gauge-Higgs unification is formulated in the RS space.

In the RS space, which is an AdS spacetime sandwiched by UV and IR branes, wave functions of dominant components of $W$ and $Z$ bosons are almost constant in the bulk region so that gauge couplings of quarks and leptons turn out nearly the same as those
in the SM. The hierarchy between the Kaluza-Klein (KK) mass scale ($\sim 10 \text{ TeV}$) and the weak scale ($\sim 100 \text{ GeV}$) naturally emerges. Two typical ways of introducing fermions have been investigated. In one type of the models (the A model) quarks and leptons are introduced in the vector representation of $SO(5)$. The model predicts large parity violation in the $Z'$ couplings of quarks and leptons, which can be checked in the early stage of the ILC experiments with polarized electron and positron beams.[14, 17, 21, 22]

It has been noticed, however, that there arises a difficulty in promoting the A model to grand unification.[23]-[27] The natural extension of the $SO(5) \times U(1)_X \times SU(3)_C$ model is $SO(11)$ gauge-Higgs grand unification.[24] Up-type quarks are contained in the spinor representation of $SO(11)$, but not in the vector representation so that up-type quarks in the A model do not appear from the $SO(11)$ gauge-Higgs unification. A new way of introducing fermion multiplets has been found which can be embedded into the $SO(11)$ gauge-Higgs grand unification.[16] In this GUT inspired model, or the B model, quarks and leptons are introduced in the spinor and singlet representations of $SO(5)$. It has been shown that quarks and leptons have correct gauge couplings. Furthermore the flavor mixing is nicely incorporated with gauge-invariant brane interactions in the B model. The CKM matrix is obtained, and remarkably flavor changing neutral current (FCNC) interactions are naturally suppressed.[20]

In this paper we evaluate the effective potential $V_{\text{eff}}(\theta_H)$ in GUT inspired $SO(5) \times U(1)_X \times SU(3)_C$ gauge-Higgs unification. It will be shown that with appropriate choice of parameters $V_{\text{eff}}(\theta_H)$ has global minimum at $\theta_H \neq 0$ and the Higgs boson mass $m_H = 125 \text{ GeV}$ is obtained. The cubic and quartic self-couplings of the Higgs boson are determined from $V_{\text{eff}}(\theta_H)$. We shall show that those cubic and quartic self-couplings are, to high accuracy, determined as functions of $\theta_H$ only. They do not depend on other parameters of the theory. It will be explained how this universality results in the model.

The effective potential $V_{\text{eff}}(\theta_H)$ is important in discussing phase transitions at finite temperature as well. Recently a possibility of having first-order phase transitions in gauge-Higgs unification has been argued.[28] At the moment the nature of phase transitions at finite temperature in $SO(5) \times U(1) \times SU(3)$ gauge-Higgs unification remains unclear.

The Higgs boson as an AB phase in gauge-Higgs unification has similarity to that in composite Higgs models in which the Higgs boson appears as a pseudo-Nambu-Goldstone boson.[9, 29] In both scenarios the Higgs boson field has a character of a phase, but has a quite different mechanism for acquiring its mass. In gauge-Higgs unification the Higgs boson mass is generated by gauge-invariant dynamics of the AB phase, whereas it results from ungauged part of global symmetry in composite Higgs models. Further in
gauge-Higgs unification left-handed and right-handed components of quarks and leptons are normally localized in opposite branes; if left-handed components are localized near UV (IR) brane, then right-handed components are localized near IR (UV) brane. In composite Higgs models all light quarks and leptons are assumed to be localized near UV brane. This leads to big difference in phenomenology associated with $Z'$ or techni-rho bosons. In gauge-Higgs unification in RS space there appears large parity violation in $Z'$ couplings of quarks and leptons,\[14, 30] whereas such asymmetry is absent in composite Higgs models. Gauge-Higgs unification is strictly regulated by gauge principle.

The paper is organized as follows. In Section 2 the model is introduced. In Section 3 the effective potential $V_{\text{eff}}(\theta_H)$ is evaluated. We show that dynamical EW symmetry breaking takes place. The cubic and quartic self-couplings, $\lambda_3$ and $\lambda_4$, of the Higgs boson are evaluated from $V_{\text{eff}}(\theta_H)$. It is observed there that $\lambda_3$ and $\lambda_4$ are determined to high accuracy as functions of $\theta_H$, irrespective of other parameters in the model. The origin of the $\theta_H$ universality in the RS space is clarified in Section 4. In Section 5 the spectrum of dark fermions is evaluated. Section 6 is devoted to summary. Mass spectra of all fields in the model are summarized in Appendix A. Functions used for the evaluation of $V_{\text{eff}}(\theta_H)$ are summarized in Appendix B.

2 Model

The GUT inspired $SO(5) \times U(1)_X \times SU(3)_C$ gauge-Higgs unification has been introduced in refs.\[16, 20]. It is defined in the RS warped space with metric given by\[31]

$$ds^2 = g_{MN}dx^M dx^N = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (2.1)$$

where $M, N = 0, 1, 2, 3, 5$, $\mu, \nu = 0, 1, 2, 3$, $y = x^5$, $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$, $\sigma(y) = \sigma(y + 2L) = \sigma(-y)$, and $\sigma(y) = ky$ for $0 \leq y \leq L$. In terms of the conformal coordinate $z = e^{ky}$ ($1 \leq z \leq z_L = e^{KL}$) in the region $0 \leq y \leq L$

$$ds^2 = \frac{1}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + \frac{dz^2}{k^2} \right). \quad (2.2)$$

The bulk region $0 < y < L$ ($1 < z < z_L$) is anti-de Sitter (AdS) spacetime with a cosmological constant $\Lambda = -6k^2$, which is sandwiched by the UV brane at $y = 0$ ($z = 1$) and the IR brane at $y = L$ ($z = z_L$). The KK mass scale is $m_{KK} = \pi k/(z_L - 1) \simeq \pi k z_L^{-1}$ for $z_L \gg 1$. 


In addition to gauge fields $A_{M}^{SU(3)_C}$, $A_{M}^{SO(5)}$ and $A_{M}^{U(1)_X}$ of $SU(3)_C$, $SO(5)$, and $U(1)_X$, we introduce matter fields listed in Table [I]. Fields defined in the bulk satisfy orbifold boundary conditions. Each gauge field satisfies

$$
\begin{pmatrix}
A_\mu \\
A_y
\end{pmatrix}(x, y_j - y) = P_j \begin{pmatrix}
A_\mu \\
A_y
\end{pmatrix}(x, y_j + y) P_j^{-1}
$$

(2.3)

where $(y_0, y_1) = (0, L)$. $P_0 = P_1 = I_3$ for $A_{M}^{SU(3)_C}$ and $P_0 = P_1 = 1$ for $A_{M}^{U(1)_X}$. $P_0 = P_1 = P_{5}^{SO(5)} = \text{diag} (I_4, -1)$ for $A_{M}^{SO(5)}$ in the vector representation and $P_0 = P_1 = P_{4}^{SO(5)} = \text{diag} (I_2, -I_2)$ in the spinor representation, respectively. Quark and lepton multiplets satisfy

$$
\Psi^\alpha_{(3,4)}(x, y_j - y) = -P_{4}^{SO(5)} \gamma^5 \Psi^\alpha_{(3,4)}(x, y_j + y), \\
\Psi^{\pm\alpha}_{(3,1)}(x, y_j - y) = \mp \gamma^5 \Psi^{\pm\alpha}_{(3,1)}(x, y_j + y), \\
\Psi^\alpha_{(1,4)}(x, y_j - y) = -P_{4}^{SO(5)} \gamma^5 \Psi^\alpha_{(1,4)}(x, y_j + y), \quad (2.4)
$$

where $\alpha = 1 \sim 3$. Dark fermion multiplets satisfy

$$
\Psi^\beta_F(x, y_j - y) = (-1)^j P_{4}^{SO(5)} \gamma^5 \Psi^\beta_F(x, y_j + y), \\
\Psi^{\pm\gamma}_{(1,5)}(x, y_j - y) = \pm P_{5}^{SO(5)} \gamma^5 \Psi^{\pm\gamma}_{(1,5)}(x, y_j + y), \quad (2.5)
$$

where $\beta = 1 \sim N_F$ and $\gamma = 1 \sim N_V$.

Table 1: $SU(3)_C \times SO(5) \times U(1)_X$ content of matter fields is shown in the GUT inspired B model and previous A model. In the A model only $SU(3)_C \times SO(4) \times U(1)_X$ symmetry is preserved on the UV brane so that the $SU(2)_L \times SU(2)_R$ content is shown for brane fields. The B model is analyzed in the present paper.

| B model | A model |
|---------|---------|
| quark   | $\Psi^\alpha_{(3,4)} : (3, 4)_{\frac{1}{5}}, \Psi^{\pm\alpha}_{(3,1)} : (3, 1)^{\pm}_{-\frac{1}{4}}$ | $\Psi^\alpha_{\frac{1}{2}} : (3, 5)_{\frac{1}{2}}, \Psi^\phi_{\frac{1}{2}} : (3, 5)_{-\frac{1}{2}}$ |
| lepton  | $\Psi^\alpha_{(1,4)} : (1, 4)_{-\frac{1}{4}}$ | $\Psi^\alpha_{\frac{1}{2}} : (1, 5)_{-1}, \Psi^\phi_{\frac{1}{2}} : (1, 5)_0$ |
| dark fermion | $\Psi^\beta_F : (3, 4)_{\frac{1}{2}}, \Psi^{\pm\gamma}_{(1,5)} : (1, 5)^{\pm}_{0}$ | $\Psi^\phi_{\frac{1}{2}} : (1, 4)_{\frac{1}{4}}$ |
| brane fermion | $\chi^\alpha : (1, 1)_0$ | $\chi^q_{1,2,3} : (3, [2, 1])_{\frac{2}{5}, \frac{1}{5}, -\frac{2}{5}}$ |
| brane scalar | $\Phi_{(1,4)} : (1, 4)_{\frac{1}{4}}$ | $\Phi_{(1, [2, 1])_{\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}}}$ |
| symmetry of brane interactions | $SU(3)_C \times SO(5) \times U(1)_X$ | $SU(3)_C \times SO(4) \times U(1)_X$ |

The bulk action of each gauge field, $A_{M}^{SU(3)_C}$, $A_{M}^{SO(5)}$, or $A_{M}^{U(1)_X}$, is given by

$$
S_{\text{gauge}}^{\text{bulk}} = \int d^5 x \sqrt{-\det G} \left[ -\text{tr} \left( \frac{1}{4} F^{MN} F_{MN} + \frac{1}{2 g^2} (f_{g})^2 + \mathcal{L}_{gh} \right) \right], \quad (2.6)
$$
where \( \sqrt{-\det G} = 1/kz^5 \) and \( F_{MN} = \partial_M A_N - \partial_N A_M - ig[A_M, A_N] \) with each 5D gauge coupling constant \( g \). The gauge fixing \( f_{gf} \) and ghost terms \( L_{gh} \) have been specified in ref. [16]. Each fermion multiplet \( \Psi(x, y) \) in the bulk has its own bulk-mass parameter \( c \). [32]

The covariant derivative is given by

\[
D(c) = \gamma^A e_A^M \left( D_M + \frac{1}{8} \omega_{MBC}[\gamma^B, \gamma^C] \right) - c\sigma'(y),
\]

\[
D_M = \partial_M - ig A^M_A - i g^A M^{SO(5)} - i g A^M A^{U(1)}.
\]

(2.7)

Here \( \sigma' = d\sigma(y)/dy \) and \( \sigma'(y) = k \) for \( 0 < y < L \). \( g_A, g_B \) are \( SU(3)_C, SO(5), U(1)_X \) gauge coupling constants. The bulk part of the action for the fermion multiplets are given, with \( \overline{\Psi} = i\Psi^\dagger \gamma^0 \), by

\[
S^\text{fermion}_{\text{bulk}} = \int d^5x \sqrt{-\det G} \left\{ \sum J \overline{\Psi}^J D(c_J) \Psi^J \right. \\
- \sum_{\alpha} \left( m_{D\alpha} \overline{\Psi}_{(3,1)}^{+\alpha} \Psi_{(3,1)}^{-\alpha} + \text{H.c.} \right) + \sum_{\gamma} \left( m_{\psi} \overline{\Psi}_{(1,5)}^{+\gamma} \Psi_{(1,5)}^{-\gamma} + \text{H.c.} \right) \left\},
\]

(2.8)

where the sum \( \sum_J \) extends over \( \Psi^J = \Psi_{(3,4)}, \Psi_{(1,4)}^{\pm\alpha}, \Psi_{(3,1)}^{\alpha}, \Psi_{(1,5)}^{\beta} \) and \( \Psi_{(1,5)}^{\pm\gamma} \).

The action for the brane scalar field \( \Phi_{(1,4)}(x) \) is given by

\[
S^\Phi_{\text{brane}} = \int d^5x \sqrt{-\det G} \delta(y) \\
\times \left\{ - (D_{\mu} \Phi_{(1,4)})^\dagger D^\mu \Phi_{(1,4)} - \lambda_{\Phi_{(1,4)}} \left( \Phi_{(1,4)}^\dagger \Phi_{(1,4)} - |w|^2 \right)^2 \right\},
\]

(2.9)

where \( D_{\mu} = \partial_{\mu} - ig A^M_A - i g^A M^{SO(5)} - i g B A^{U(1)}_{\mu} \). The action for the gauge-singlet brane fermion \( \chi^\alpha(x) \) is

\[
S^\chi_{\text{brane}} = \int d^5x \sqrt{-\det G} \delta(y) \left\{ \frac{1}{2} \chi^\alpha \gamma^\mu \partial_{\mu} \chi^\alpha - \frac{1}{2} M^{\alpha\beta} \chi^\alpha \chi^\beta \right\}.
\]

(2.10)

\( \chi^\alpha(x) \) satisfies the Majorana condition \( \chi^c = \chi \);

\[
\chi = \begin{pmatrix} \xi \\ \eta \end{pmatrix}, \quad \chi^c = \begin{pmatrix} +\eta^c \\ -\xi^c \end{pmatrix} = e^{i\delta_C} \begin{pmatrix} +\sigma^2\eta^* \\ -\sigma^2\xi^* \end{pmatrix}.
\]

(2.11)

On the UV brane there are \( SU(3)_C \times SO(5) \times U(1)_X \)-invariant brane interactions among the bulk fermion, brane fermion, and brane scalar fields. Relevant parts of the brane interactions are given by

\[
S^\text{int}_{\text{brane}} = - \int d^5x \sqrt{-\det G} \delta(y) \times
\]

On page 6.
\[
\left\{ \kappa^{\alpha\beta} \bar{\Psi}^\alpha_{(3,4)} \Phi_{(1,4)} \cdot \Psi^{+\beta}_{(3,1)} + \bar{\kappa}^{\alpha\beta} \chi^{\beta} \bar{\Phi}^\dagger_{(1,4)} \Psi^\alpha_{(1,4)} + \text{H.c.} \right\} \quad (2.12)
\]

where \( \kappa \)'s and \( \bar{\kappa} \)'s are coupling constants and

\[
\Phi_{(1,4)} = \begin{pmatrix} \Phi_{[2,1]} \\ \Phi_{[1,2]} \end{pmatrix}, \quad \bar{\Phi}_{(1,4)} = \begin{pmatrix} i\sigma^2 \Phi^*_{[2,1]} \\ -i\sigma^2 \Phi^*_{[1,2]} \end{pmatrix} \quad (2.13)
\]

When \( \langle \Phi_{(1,4)} \rangle = (0, 0, 0, w)^t \), (2.12) generates additional mass terms

\[
\int d^5x \sqrt{-\det G} \delta(y) \left\{ 2\mu^{\alpha\beta} d^\alpha_R D_L^{+\beta} + \text{H.c.} \right\}, \quad \mu^{\alpha\beta} = \frac{\kappa^{\alpha\beta} w}{\sqrt{2}} \quad (2.14)
\]

in the down-type quark sector and

\[
-\int d^5x \sqrt{-\det G} \delta(y) \frac{m_B^{\alpha\beta}}{\sqrt{k}} (\bar{\chi}^\beta \nu^\alpha_R + \bar{\nu}^\alpha_R \chi^\beta), \quad m_B^{\alpha\beta} = \bar{\kappa}^{\alpha\beta} w \sqrt{k} \quad (2.15)
\]

in the neutrino sector. With the Majorana masses in (2.10), the mass term (2.15) induces inverse seesaw mechanism in the neutrino sector.\[26\] Further \( \langle \Phi_{(1,4)} \rangle \neq 0 \) breaks \( SO(4) \times U(1)_X \) down to \( SU(2)_L \times U(1)_Y \). We assume that \( w \gg m_{\text{KK}} \). The 4D \( SU(2)_L \) gauge coupling is given by \( g_w = g_A/\sqrt{L} \). The 5D gauge coupling \( g_5^Y \) of \( U(1)_Y \) and the 4D bare Weinberg angle at the tree level, \( \theta_W^0 \), are given by

\[
g_5^Y = \frac{g_{AB}}{\sqrt{g_A^2 + g_B^2}}, \quad \sin \theta_W^0 = \frac{s_\phi}{\sqrt{1 + s_\phi^2}}, \quad s_\phi = \frac{g_B}{\sqrt{g_A^2 + g_B^2}} \quad (2.16)
\]

The bare Weinberg angle \( \theta_W^0 \) with a given \( \theta_H \) is determined to fit the LEP1 data for \( e^+e^- \to \mu^+\mu^- \) at \( \sqrt{s} = m_Z \).\[33\] Approximately \( \sin^2 \theta_W^0 \approx 0.1140 + 0.1186 \cos \theta_H - 0.0014 \cos 2\theta_H \). Evaluated gauge couplings turn out very close to those in the SM with \( \sin^2 \theta_W = 0.2312 \).\[20\]

The 4D Higgs boson \( \Phi_H(x) \) is contained in the \( SO(5)/SO(4) \) part of \( A_y^{SO(5)} \). In the \( z \) coordinate \( A_z = (kz)^{-1} A_y \) (1 \( \leq z \leq z_L \)), and

\[
A_z^{(5)}(x, z) = \frac{1}{\sqrt{k}} \phi_j(x) u_H(z) + \cdots,
\]

\[
u_H(z) = \sqrt{\frac{2}{z_L^2 - 1}} z,
\]

\[
\Phi_H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{pmatrix} \quad (2.17)
\]
At the quantum level $\Phi_H$ develops a nonvanishing expectation value. Without loss of generality we assume $\langle \phi_1 \rangle, \langle \phi_2 \rangle, \langle \phi_3 \rangle = 0$ and $\langle \phi_4 \rangle \neq 0$, which is related to the Aharonov-Bohm (AB) phase $\theta_H$ in the fifth dimension. Eigenvalues of
\[
\hat{W} = P \exp \left\{ i g_A \int_{-L}^{L} dy A_y \right\} \cdot P_1 P_0 \quad (2.18)
\]
are gauge invariant. For $A_y = (2k)^{-1/2} \phi_4(x)v_H(y)T^{(45)}$, where $v_H(y) = k e^{ky} u_H(z)$ for $0 \leq y \leq L$ and $v_H(-y) = v_H(y) = v_H(y + 2L)$, one finds
\[
\hat{W} = \exp \left\{ i f_H^{-1} \phi_4(x) \cdot 2T^{(45)} \right\},
\]
\[
f_H = \frac{2}{g_A} \sqrt{\frac{k}{z_L^2 - 1}} = \frac{2}{g_w} \sqrt{\frac{k}{L(z_L^2 - 1)}},
\]
\[
\theta_H = \frac{\langle \phi_4 \rangle}{f_H}. \quad (2.19)
\]
Note
\[
A^{(45)}_i(x, z) = \frac{1}{\sqrt{k}} \left\{ \theta_H f_H + H(x) \right\} u_H(z) + \cdots \quad (2.20)
\]
where $H(x)$ is the neutral Higgs boson field. There is a large gauge transformation which shifts $\theta_H$ by $2\pi$, preserving the boundary conditions. Physics is invariant under $\theta_H \to \theta_H + 2\pi$. We shall evaluate the effective potential $V_{\text{eff}}(\theta_H)$ in the next section.

3 Effective potential

The effective potential $V_{\text{eff}}(\theta_H)$ at the one-loop level is evaluated from the mass spectra of all fields which depend on $\theta_H$. After the Wick rotation into the Euclidean signature it is expressed as
\[
V_{\text{eff}}(\theta_H) = \sum \pm \frac{1}{2} \int \frac{d^4 p_E}{(2\pi)^4} \sum_n \ln \left\{ p_E^2 + m_n(\theta_H)^2 \right\}, \quad (3.1)
\]
where the sign $+$ ($-$) corresponds to bosons (fermions). When the Kaluza-Klein (KK) spectrum $\{m_n(\theta_H)\}$ is determined by zeros of a function $\rho(z; \theta_H)$, namely by
\[
\rho(m_n; \theta_H) = 0 \quad (n = 1, 2, 3, \cdots), \quad (3.2)
\]
then $V_{\text{eff}}(\theta_H)$ is given by
\[
V_{\text{eff}}(\theta_H) = \sum \pm \frac{1}{(4\pi)^2} \int_0^\infty dy y^3 \ln \rho(iy; \theta_H). \quad (3.3)
\]
The $\theta_H$-dependent part of $V_{\text{eff}}^{1\text{loop}}(\theta_H)$ is finite, independent of a cutoff and regularization method employed.

The spectrum-determining functions $\rho(z; \theta_H)$ for all fields in the model have been given in ref. [16]. They are summarized in Appendix A for convenience. Relevant contributions come from $W$ and $Z$ gauge fields, top-bottom quark multiplets, and dark fermions in the spinor and vector representations. Contributions from light quarks and leptons are negligible. To avoid unnecessary confusion in the following argument, we denote the effective potential as $V_{\text{eff}}(\theta)$. Physical value $\theta_H$ corresponds to the global minimum of $V_{\text{eff}}(\theta)$, namely $dV_{\text{eff}}/d\theta|_{\theta=\theta_H} = 0$. One finds

$$V_{\text{eff}}(\theta) = 2(3 - \xi^2)A_W(\theta) + (3 - \xi^2)A_Z(\theta) + 3\xi^2A_S(\theta) - 12A_{\text{top}}(\theta) - 12A_{\text{bottom}}(\theta) - 12n_FA_F(\theta) - 8n_VA_V(\theta) ,$$

$$A_p(\theta) \equiv \left(\frac{kz_L^{-1}}{4\pi}\right)^4 \int_{0}^{\infty} dq q^3 \ln \left\{ 1 + \sum_{n=1}^{2} Q_p^{(n)}(q) \cos(n\theta) \right\} , \quad (3.4)$$

where $n_F$ and $n_V$ are the number of $\Psi_F$ and $\Psi^{\pm}_{(1,5)}$, and $\xi$ is a gauge parameter in the generalized $R_{\xi}$ gauge. The integration variable has been changed from $y$ in (3.3) to $q = k^{-1}z_Ly$. In the following we take $z_L = e^{kL} = 10^{10}$ and $\xi = 0$. The contributions from $W$, $Z$ towers and Goldstone boson tower are given by

$$Q^{(1)}_W(q) = Q^{(1)}_Z(q) = Q^{(1)}_S(q) = 0 ,$$

$$Q^{(2)}_W(q) = -\frac{1}{4iz_Lq^{-1}\hat{C}'(q)\hat{S}(q) + 1} ,$$

$$Q^{(2)}_Z(q) = -\frac{1 + s^2_\phi}{4iz_Lq^{-1}\hat{C}'(q)\hat{S}(q) + 1 + s^2_\phi} ,$$

$$Q^{(2)}_S(q) = -\frac{1}{-2iz_Lq^{-1}\hat{C}(q)\hat{S}(q) + 1} . \quad (3.5)$$

$\hat{C}(q)$, $\hat{S}(q)$ etc. in the expressions above and $\hat{C}_L(q, c)$, $\hat{S}_L(q, c)$ etc. in the expressions below are given in Appendix B. They are expressed in terms of modified Bessel functions.

Top and bottom quark contributions are given by

$$Q^{(2)}_{\text{top}}(q) = Q^{(2)}_{\text{bottom}}(q) = 0 ,$$

$$Q^{(1)}_{\text{top}}(q) = -\frac{1}{2\hat{S}_L(q; c_i)\hat{S}_R(q; c_i) + 1} ,$$

$$Q^{(1)}_{\text{bottom}}(q) = -\frac{\hat{S}_L\hat{S}_R}{2\hat{S}_L(q; c_i)\hat{S}_R(q; c_i)\hat{S}_L\hat{S}_R + 2|\mu|^2\hat{C}_R(q; c_i)\hat{S}_R(q; c_i)\hat{C}_L\hat{S}_L + 1} ,$$

where
\[
\begin{align*}
\tilde{S}_L \tilde{S}_R &= \hat{S}_L(q; c_{D_\alpha} + \bar{m}_{D_\alpha}) \hat{S}_R(q; c_{D_\alpha} - \bar{m}_{D_\alpha}) + \hat{S}_L(q; c_{D_\alpha} - \bar{m}_{D_\alpha}) \hat{S}_R(q; c_{D_\alpha} + \bar{m}_{D_\alpha}) \\
+ \hat{C}_L(q; c_{D_\alpha} + \bar{m}_{D_\alpha}) \hat{C}_R(q; c_{D_\alpha} - \bar{m}_{D_\alpha}) + \hat{C}_L(q; c_{D_\alpha} - \bar{m}_{D_\alpha}) \hat{C}_R(q; c_{D_\alpha} + \bar{m}_{D_\alpha}) - 2 ,
\end{align*}
\]
\[
\tilde{C}_L \tilde{S}_L = \hat{C}_L(q; c_{D_\alpha} + \bar{m}_{D_\alpha}) \hat{S}_L(q; c_{D_\alpha} - \bar{m}_{D_\alpha}) + \hat{C}_L(q; c_{D_\alpha} - \bar{m}_{D_\alpha}) \hat{S}_L(q; c_{D_\alpha} + \bar{m}_{D_\alpha}) . \tag{3.6}
\]

In the above expressions we have assumed that the brane interaction term \((2.12)\) is diagonal in generation space. \(c_{D_\alpha}\) is the bulk mass parameter of \(\Psi_{(3,1)}^{\pm}\) and \(\bar{m}_{D_\alpha} = m_{D_\alpha}/k\). Numerically the contribution of bottom quark is very small and may be ignored. There are two kinds of dark fermions (\(\Psi_F^\beta\) and \(\Psi_{(1,5)}^{\pm\gamma}\)). Their contributions are given by

\[
\begin{align*}
Q_F^{(1)}(q) &= \frac{1}{2S_L(q; c_F)S_R(q; c_F) + 1} , \\
Q_F^{(2)}(q) &= 0 , \\
Q_V^{(1)}(q) &= 0 , \\
Q_V^{(2)}(q) &= -\frac{2}{B_0(q; c_V, \bar{m}_V)} , \\
\hat{B}_0(q; c_V, \bar{m}_V) &= \hat{C}_L(q; c_V + \bar{m}_V) \hat{C}_R(q; c_V - \bar{m}_V) + \hat{C}_L(q; c_V - \bar{m}_V) \hat{C}_R(q; c_V + \bar{m}_V) \\
+ \hat{S}_L(q; c_V + \bar{m}_V) \hat{S}_R(q; c_V - \bar{m}_V) + \hat{S}_L(q; c_V - \bar{m}_V) \hat{S}_R(q; c_V + \bar{m}_V) . \tag{3.7}
\end{align*}
\]

For the sake of simplicity we set degenerate bulk mass parameters \(c_F\) for \(\Psi_F^\beta\), and degenerate masses \(m_V = k\bar{m}_V\) and bulk mass parameters \(c_V\) for \(\Psi_{(1,5)}^{\pm\gamma}\). We note that contributions from gauge bosons and \(\Psi_{(1,5)}^{\pm\gamma}\) fields to \(V_{\text{eff}}(\theta)\) are periodic in \(\theta\) with a period \(\pi\), whereas those from top-bottom quarks and \(\Psi_F^\beta\) fields are periodic with a period \(2\pi\).

The parameters of the model are determined in the following steps. (i) We pick the value of \(\theta_H\). In other words we are going to adjust the parameters of the model such that \(V_{\text{eff}}(\theta)\) has a global minimum at \(\theta = \theta_H\). (ii) We take \(z_L = 10^{10}\). Then \(k\) is determined for \(m_Z\) to be reproduced, and the KK mass scale \(m_{\text{KK}} = \pi k(z_L - 1)^{-1}\) is fixed. (iii) The bulk mass parameters of \(\Psi_{(3,4)}^{\alpha}\) and \(\Psi_{(1,4)}^{\alpha}\) are fixed from the masses of up-type quarks and charged leptons. In particular, \(\alpha\) is determined by \(m_t\). (iv) The bulk mass parameters \(c_{D_\alpha}\) of \(\Psi_{(3,1)}^{\pm\alpha}\) and brane interaction coefficients \(\mu^{\alpha\beta}\) are determined so as to reproduce the masses of down-type quarks and CKM matrix. Similarly the Majorana mass terms \(M^{\alpha\beta}\) and brane interactions \(k^{\alpha\beta}\) are determined so as to reproduce neutrino masses and PMNS matrix. As remarked above, these parameters are numerically irrelevant for \(V_{\text{eff}}(\theta)\). (v) At
this stage there remain five parameters to be determined; \((n_F, c_F)\) of \(\Psi^\beta_F\) and \((n_V, c_V, \tilde{m}_V)\) of \(\Psi^\pm_{(1,5)}\). There are two conditions to be satisfied;

\[
(a) : \quad \frac{dV_{\text{eff}}}{d\theta} \bigg|_{\theta=\theta_H} = 0 ,
\]

\[
(b) : \quad m_H^2 = \frac{1}{f_H^2} \frac{d^2V_{\text{eff}}}{d\theta^2} \bigg|_{\theta=\theta_H} ,
\]

where \(m_H = 125.1\text{ GeV}\). The second condition for the Higgs boson mass \(m_H\) follows from the fact that the effective potential for the 4D Higgs field \(H(x)\) is given by \(V_{\text{eff}}(\theta_H + f_H^{-1}H)\) as inferred from \((2.20)\). The conditions \((3.8)\) give two constraints to be satisfied among the five parameters \((n_F, c_F, n_V, c_V, \tilde{m}_V)\). We first fix, for instance, \((n_F, n_V, c_V)\) and determine \((c_F, \tilde{m}_V)\) by \((3.8)\).

One may wonder whether the arbitrary choice of the parameters in the last step diminishes prediction power of the model. Quite surprisingly many of the physical quantities do not depend on such details in the parameter choice, being determined solely by \(\theta_H\). There appears the \(\theta_H\)-universality which will be explained in the next section.

We give some examples. The parameters fixed in the steps (i) to (iv) above are tabulated in Table 2. In Fig. 1 the effective potential for \(\theta_H = 0.1, n_F = n_V = 2\) and \(c_V = 0\) is displayed. \(c_F = 0.319\) and \(\tilde{m}_V = 0.0806\) are chosen to satisfy \((3.8)\). One observes that the electroweak symmetry is dynamically broken. In Fig. 2 contributions of relevant fields to the effective potential \(V_{\text{eff}}(\theta)\) are displayed. There is a lower bound for \(\theta_H\) in order to reproduce the top quark mass. \(\theta_H \geq \theta_{c1}\) where \(\theta_{c1} \sim 0.015\) for \(z_L = 10^{10}\). Similarly there is a constraint for the warp factor. For \(\theta_H = 0.1, n_F = n_V = 2\), the top quark mass is reproduced only if \(z_L \geq z_{L1} \sim 10^{8.1}\) and dynamical electroweak symmetry breaking is achieved only if \(z_L \leq z_{L2} \sim 10^{15.5}\).

Table 2: Parameters determined for \(z_L = 10^{10}\) and \(\theta_H = 0.05, 0.1, 0.15\). We have set \(\mu^{\alpha\beta} = \mu_\alpha \delta_{\alpha\beta}\), and have taken \(c_{D_b} = 1.04\). \(\mu_b\) is determined so as to reproduce \(m_b\).

| \(\theta_H\) | \(k [10^{13} \text{ GeV}]\) | \(m_{KK} [\text{TeV}]\) | \(c_t\) | \(c_{D_b}\) | \(\mu_b\) |
|------|----------------|----------------|------|-------|------|
| 0.05 | 7.68           | 24.1           | -0.226 | 1.04  | 0.106 |
| 0.10 | 3.84           | 12.1           | -0.227 | 1.04  | 0.104 |
| 0.15 | 2.57           | 8.07           | -0.230 | 1.04  | 0.0990 |

The effective potential \(V_{\text{eff}}(\theta)\) has more information. By expanding \(V_{\text{eff}}(\theta_H + H/f_H)\), one finds Higgs self-couplings \(\lambda_n H^n\). The \(n\)-th self-coupling \(\lambda_n\) is given by

\[
\lambda_n \equiv \frac{1}{n! f_H^n} \frac{d^n V_{\text{eff}}}{d\theta^n} \bigg|_{\theta=\theta_H} .
\]
Figure 1: The effective potential for $\theta_H = 0.1$, $n_F = 2$, $n_V = 2$ and $c_V = 0$. The global minimum is located at $\theta = \theta_H$.

Figure 2: Contributions of relevant fields to the effective potential for $\theta_H = 0.1$, $n_F = 2$, $n_V = 2$ and $c_V = 0$ are displayed.

These couplings $\lambda_3$ and $\lambda_4$ are plotted in Fig. 3 and Fig. 4 as functions of $\theta_H$ for $c_V = 0.2$, $n_F = n_V = 2$. The fitting curves are given by

$$B \text{ model: } \frac{\lambda_3}{\text{GeV}} = 39.6 \cos \theta_H - 5.21(1 + \cos 2\theta_H) - 0.00911 \cos 3\theta_H ,$$

$$\lambda_4 = -0.0695 + 0.0852 \cos \theta_H + 0.00725 \cos 2\theta_H . \quad (3.10)$$

In Figs. 3 and 4 $\lambda_3$ and $\lambda_4$ in the A model are also plotted, for which the fitting curves are given by

$$A \text{ model: } \frac{\lambda_3}{\text{GeV}} = 32.4 \cos \theta_H - 2.26(1 + \cos 2\theta_H) - 1.1 \cos 3\theta_H ,$$

$$\lambda_4 = -0.00264 - 0.0129 \cos \theta_H + 0.0363 \cos 2\theta_H . \quad (3.11)$$

Note that $\lambda_3$ vanishes at $\theta_H = \frac{1}{2} \pi$ as a consequence of the $H$ parity in GHU models.\cite{[35]} For $\theta_H \geq 0.6$, $\lambda_4$ becomes negative, which, however, does not mean the instability. The $\theta$-dependent part of $V_{\text{eff}}(\theta)$ is finite, bounded from below. In gauge-Higgs unification there does not arise the vacuum instability problem which afflicts most of 4D field theories.
From the experimental constraints from the LEP1, LEP2 data and from the LHC data for the nonobservation of $Z'$ events it is inferred that $\theta_H \lesssim 0.11$. For $\theta_H \sim 0.1 \ (0.15)$, $\lambda_3$ and $\lambda_4$ are smaller than those in the SM by 7.7% (8.1%) and 30% (32%), respectively.

**Figure 3:** The cubic coupling $\lambda_3$ of the Higgs boson. The fitting curves are given by $\lambda_3/\text{GeV} = 39.6 \cos \theta_H - 5.21(1 + \cos 2\theta_H) - 0.00911 \cos 3\theta_H$ for the B-model and $\lambda_3/\text{GeV} = 32.4 \cos \theta_H - 2.26(1 + \cos 2\theta_H) - 1.1 \cos 3\theta_H$ for the A-model. The SM value is $\lambda_{3,SM} = 31.5 \text{GeV}$.  

**Figure 4:** The quartic coupling $\lambda_4$ of the Higgs boson. The fitting curves are given by $\lambda_4 = -0.0695 + 0.0852 \cos \theta_H + 0.00725 \cos 2\theta_H$ for the B-model and $\lambda_4 = -0.00264 - 0.0129 \cos \theta_H + 0.0363 \cos 2\theta_H$ for the A-model. The SM value is $\lambda_{4,SM} = 0.0320$.

## 4 $\theta_H$ universality

As remarked in the previous section there remains the arbitrariness in the choice of the parameters in the model. Among the five parameters $(n_F, c_F, n_V, c_V, \bar{m}_V)$ there are only two conditions in (3.8) to be obeyed. In the examples given in the previous section we first fixed $(n_F, n_V, c_V)$ and determined $(c_F, \bar{m}_V)$ by (3.8). The Higgs cubic and quartic couplings, $\lambda_3$ and $\lambda_4$, are evaluated with this choice. One might wonder how $\lambda_3$ and $\lambda_4$ depend on the choice of the parameters $(n_F, n_V, c_V)$.

In this section we shall show that $\lambda_3$ and $\lambda_4$ are determined, to high accuracy, as functions of $\theta_H$ only, but do not depend on the details of the parameter choice. It has
Table 3: $\theta_H$ universality in $\lambda_3$ and $\lambda_4$ for $\theta_H = 0.1$ and $z_L = 10^{10}$. With given $(n_F, n_V, c_V)$, $c_F$ and $\tilde{m}_V$ are determined to satisfy the condition (3.8), and $\lambda_3$ and $\lambda_4$ are evaluated by (3.9).

| $n_F$ | $n_V$ | $c_V$ | $c_F$ | $\tilde{m}_V$ | $\lambda_3$(GeV) | $\lambda_4$ |
|-------|-------|-------|-------|---------------|-----------------|-----------|
| 2     | 2     | 0.2   | 0.319 | 0.0777        | 29.03           | 0.02083   |
| 2     | 2     | 0.5   | 0.322 | -0.0371       | 29.02           | 0.02078   |
| 4     | 2     | 0.2   | 0.425 | 0.0794        | 29.02           | 0.02082   |
| 4     | 2     | 0.5   | 0.425 | -0.0350       | 29.01           | 0.02076   |
| 2     | 4     | 0.2   | 0.318 | 0.0964        | 29.03           | 0.02084   |
| 2     | 4     | 0.5   | 0.318 | 0.0937        | 29.03           | 0.02084   |
| 2     | 4     | 0.2   | 0.319 | 0.0615        | 29.03           | 0.02083   |

been known that the 3-point Higgs couplings to $W$, $Z$, quarks and leptons also have the same property.\[36\] These physical quantities are determined by $\theta_H$ to high accuracy. It may be called as the $\theta_H$ universality. The $\theta_H$ universality leads to profound power for predictions. Once the value of $\theta_H$ is determined by one of the physical quantities, then the values of other physical quantities are predicted.

In Table 3 evaluated values of $(\lambda_3, \lambda_4)$ for $\theta_H = 0.1$ are shown with various choices of $(n_F, n_V, c_V)$. Although the values of determined $c_F$ and $\tilde{m}_V$ depend on the choice of $(n_F, n_V, c_V)$, the evaluated values of $\lambda_3$ and $\lambda_4$ are universal to high accuracy. $\lambda_3$ and $\lambda_4$ are determined as functions of $\theta_H$ only.

There is a reason for the $\theta_H$ universality. We first examine global behavior of $V_{\text{eff}}(\theta)$ with a given $\theta_H$. Notice that the function $A_p(\theta)$ in (3.4) is expanded as

\[
A_p(\theta) = \frac{(kz^{-1}_L)^4}{(4\pi)^2} \sum_{\ell=1}^\infty \sum_{n=1}^2 \alpha_p^{(n,\ell)} \cos^\ell n\theta,
\]

\[
\alpha_p^{(n,\ell)} = (-1)^{\ell+1} \int_0^\infty dq q^3 \left(Q_p^{(n)}(q)\right)^\ell.
\]

(4.1)

As either $Q_p^{(1)}(q)$ or $Q_p^{(2)}(q)$ with given $p$ vanishes in (3.4), $\alpha_p^{(1,\ell)}$ or $\alpha_p^{(2,\ell)} = 0$ for each $p$ in (4.1).

To understand qualitative behavior of $V_{\text{eff}}(\theta)$, let us approximate $A_p(\theta)$ in (3.4) by

\[
\frac{(4\pi)^2}{(kz^{-1}_L)^4} A_p(\theta) = \begin{cases} 
\alpha_p^{(2,1)} \cos 2\theta & \text{for } p = W, Z, S, V, \\
\alpha_p^{(1,1)} \cos \theta + \alpha_p^{(1,2)} \cos^2 \theta & \text{for } p = \text{top, } F.
\end{cases}
\]

(4.2)
Note that $|\alpha_p^{(n,2)}/\alpha_p^{(n,1)}| < 0.05$. As the contributions from $p = \text{top}, F$ are one order of
magnitude larger than those from $p = W, Z, S, V$, the $\cos^2 \theta$ terms have been retained for
top and $F$. $V_{\text{eff}}(\theta)$ in this approximation, denoted as $V_{\text{app}}(\theta)$, is given by

$$V_{\text{app}}(\theta) = \frac{(kz_L^{-1})^4}{(4\pi)^2} \left(-B_1 \cos \theta + B_2 \cos 2\theta\right),$$

$$B_1 = 12\alpha_{\text{top}}^{(1,1)} + 12n_F\alpha_F^{(1,1)},$$

$$B_2 = \alpha_{\text{gauge}} - 8n_V\alpha_V^{(2,1)} - 6\alpha_{\text{top}}^{(1,2)} - 6n_F\alpha_F^{(1,2)},$$

$$\alpha_{\text{gauge}} = 2(3 - \xi^2)\alpha_W^{(2,1)} + (3 - \xi^2)\alpha_Z^{(2,1)} + 3\xi^2\alpha_S^{(2,1)}.$$  \hfill (4.3)

The condition $(a)$ in (3.8) leads to $B_1 = 4B_2 \cos \theta_H$. Then the condition $(b)$ in (3.8)
implies that

$$B_2 \sim \frac{16\pi^2}{g_w^2(kL)^2} \left(\frac{m_H}{m_W}\right)^2$$

\hfill (4.4)

where the relations $m_{\text{KK}} \sim \pi k z_L^{-1}$, $m_W \sim (k/L)^{1/2} z_L^{-1} \sin \theta_H$, $f_H \sim 2m_W/g_w \sin \theta_H$ have been made use of. It follows that

$$V_{\text{app}}(\theta) = V_0 u(\theta),$$

$$u(\theta) = -4 \cos \theta \cos \theta + \cos 2\theta,$$

$$V_0 = \frac{m_W^2m_H^2}{g_w^2 \sin^4 \theta_H}.$$  \hfill (4.5)

The cubic and quartic Higgs self-couplings are given by

$$\lambda_3^{\text{app}} \sim \frac{g_w m_H^2}{4m_W} \cos \theta_H,$$

$$\lambda_4^{\text{app}} \sim \frac{g_w^2 m_H^2}{96m_W^2} (7 \cos^2 \theta_H - 4).$$  \hfill (4.6)

The approximate formulas (4.5) and (4.6) represent qualitative behavior of the effective potential $V_{\text{eff}}(\theta)$, but exhibit slight deviation from the values in Table 3 and the fitting curves (3.10) and (3.11). We first note that the form of $V_{\text{app}}(\theta)$ is fixed, once one makes an Ansatz that $V_{\text{eff}}(\theta)$ is expressed in terms of two functions $\cos \theta$ and $\cos 2\theta$. The relevant quantities are $B_1$ and $B_2$, but not detailed values of the parameters in the models considered. In the A- or B-model the same universality relations (4.6) result in this approximation. It is easy to confirm that the formulas (4.6) reproduce the SM values at $\theta_H = 0$:

$$\lambda_3^{\text{app}}|_{\theta_H=0} = \lambda_{3,\text{SM}}, \quad \lambda_4^{\text{app}}|_{\theta_H=0} = \lambda_{4,\text{SM}}.$$  \hfill (4.7)
We also note that $u(\pi) - u(0) = 8 \cos \theta_H$ and $u(\theta_H) - u(0) = -2(1 - \cos \theta_H)^2$. For small $\theta_H$, $u(\pi) - u(0) \sim 8$ and $u(\theta_H) - u(0) \sim -\frac{1}{2} \theta_H^2$, which explains the behavior of $V_{\text{eff}}(\theta)$ for $\theta_H = 0.1$ seen in fig. 1.

To understand the $\theta_H$ universality demonstrated in the previous section, refinement of the arguments is necessary. The universality was first found in the A-model of $SO(5) \times U(1)$ gauge-Higgs unification. The mechanism for yielding the $\theta_H$ universality has been explained in ref. 37. We generalize the argument for the current B-model. The important observation is that $\lambda_3$ and $\lambda_4$ are determined by the local behavior of the effective potential $V_{\text{eff}}(\theta_H)$ in the vicinity of the global minimum at $\theta = \theta_H$, and the universality reflects the local, but not global behavior of $V_{\text{eff}}(\theta_H)$.

The effective potential $V_{\text{eff}}(\theta)$, (3.4), is decomposed into three parts:

$$V_{\text{eff}}(\theta) = \frac{(k z_L^{-1})^4}{(4\pi)^2} \left\{ h_0(\theta) + n_F h_F(\theta; c_F, z_L) + n_V h_V(\theta; c_V, \bar{m}_V, z_L) \right\} \quad (4.8)$$

where $h_0(\theta)$ represents the contributions from gauge and top quark fields. With $\theta_H$, $z_L$ and $\xi$ specified, $k$ is determined from $m_Z$ and $c_L$ is subsequently determined by $m_t$ so that $h_0(\theta)$ is fixed. All other parameters associated with quarks and leptons are irrelevant for $V_{\text{eff}}(\theta)$. There remain five parameters $(n_F, c_F, n_V, c_V, \bar{m}_V)$ to be specified in (4.8). They must be adjusted such that the two conditions in (3.8) are satisfied. The important feature in the RS space with $z_L \gg 1$ is that the $\theta$ dependence of $h_F(\theta; c_F, z_L)$ and $h_V(\theta; c_V, \bar{m}_V, z_L)$ factorizes near $\theta = \theta_H$

$$h_F(\theta; c_F, z_L) \simeq \alpha_F(c_F, z_L) \tilde{h}_F(\theta),$$
$$h_V(\theta; c_V, \bar{m}_V, z_L) \simeq \alpha_V(c_V, \bar{m}_V, z_L) \tilde{h}_V(\theta) \quad (4.9)$$

to very high accuracy. This can be confirmed numerically from the formula for $A_p(\theta)$ in (3.4). The relation (4.9) implies, for instance, that the ratio $h_F(\theta; c_F^{(1)}, z_L)/h_F(\theta; c_F^{(2)}, z_L)$ is $\theta$-independent near $\theta_H$. For $\theta_H = 0.1$, $z_L = 10^{10}$, and $(c_F^{(1)}, c_F^{(2)}) = (0.3, 0.4)$, the ratio varies from 1.7916 to 1.7896 in the range $0.09 \leq \theta \leq 0.11$. The variation is only 0.1%. We stress that this factorization formulas are valid only locally, namely near $\theta = \theta_H$, and $\tilde{h}_F(\theta)$ and $\tilde{h}_V(\theta)$ depend on $\theta_H$. The $z_L$ dependence of $h_F(\theta; c_F, z_L)$ and $h_V(\theta; c_V, \bar{m}_V, z_L)$ is also tiny in the range $10^8 \lesssim z_L \lesssim 10^{15}$.

Let us pick a set of values $(n_F, n_V, c_V)$ and determine $(c_F, \bar{m}_V)$ by (3.8). Making use of (4.9), one finds

$$\left[ \frac{dh_0}{d\theta} + n_F \alpha_F(c_F, z_L) \frac{d\tilde{h}_F}{d\theta} + n_V \alpha_V(c_V, \bar{m}_V, z_L) \frac{d\tilde{h}_V}{d\theta} \right]_{\theta=\theta_H} = 0,$$

16
\[
\left[ \frac{d^2 h_0}{d\theta^2} + n_F \alpha_F (c_L, z_L) \frac{d^2 \tilde{h}_F}{d\theta^2} + n_V \alpha_V (c_V, \bar{m}_V, z_L) \frac{d^2 \tilde{h}_V}{d\theta^2} \right]_{\theta = \theta_H} = \frac{(4\pi)^2 m_H^2 f_H^2}{(kz_L^{-1})^4}. \tag{4.10}
\]

We do this procedure for two sets; \((n_F, n_V, c_V) = (n_F^{(1)}, n_V^{(1)}, c_V^{(1)})\) and \((n_F^{(2)}, n_V^{(2)}, c_V^{(2)})\). Then (4.10) implies that
\[
\begin{align*}
n_F^{(1)} \alpha_F (c_F^{(1)}, z_L) &= n_F^{(2)} \alpha_F (c_F^{(2)}, z_L) \equiv \beta_F, \\
n_V^{(1)} \alpha_V (c_V^{(1)}, \bar{m}_V^{(1)}, z_L) &= n_V^{(2)} \alpha_V (c_V^{(2)}, \bar{m}_V^{(2)}, z_L) \equiv \beta_V. \tag{4.11}
\end{align*}
\]

Although values of \((c_F, \bar{m}_V)\) depend on the choice of \((n_F, n_V, c_V)\), \(\beta_F = n_F \alpha_F (c_F, z_L)\) and \(\beta_V = n_V \alpha_V (c_V, \bar{m}_V, z_L)\) are universal, provided solutions exist. Consequently one obtains
\[
V_{\text{eff}} (\theta) \simeq \frac{(kz_L^{-1})^4}{(4\pi)^2} \tilde{h} (\theta), \quad \tilde{h} (\theta) = h_0 (\theta) + \beta_F \tilde{h}_F (\theta) + \beta_V \tilde{h}_V (\theta). \tag{4.12}
\]

It immediately follows that
\[
\begin{align*}
\lambda_3 (\theta_H) &= \frac{g_w m_H^2 \sin \theta_H}{12m_W} \frac{\tilde{h}^{(3)} (\theta_H)}{\tilde{h}^{(2)} (\theta_H)}, \\
\lambda_4 (\theta_H) &= \frac{g_w^2 m_H^2 \sin^2 \theta_H}{96m_W^2} \frac{\tilde{h}^{(4)} (\theta_H)}{\tilde{h}^{(2)} (\theta_H)}, \tag{4.13}
\end{align*}
\]
which explains the \(\theta_H\) universality observed in the previous section. The relevant quantities for \(\lambda_3\) and \(\lambda_4\) are \(\beta_F (\theta_H)\) and \(\beta_V (\theta_H)\), but not \((n_F, n_V, c_V)\). As mentioned above, the \(z_L\)-dependence of \(h_F (\theta; c_F, z_L)\) and \(h_V (\theta; c_V, \bar{m}_V, z_L)\) is weak. The \(\theta_H\) universality stays valid to good approximation even for varying \(z_L\). For instance, for \(\theta_H = 0.1\) and \((n_F, n_V, c_V) = (2, 2, 0)\), the resultant \((\lambda_3, \lambda_4)\) is \((28.93 \text{ GeV}, 0.02042)\) for \(z_L = 1.237 \times 10^8\), which should be compared to \((29.03 \text{ GeV}, 0.02083)\) for \(z_L = 10^{10}\).

The \(\theta_H\) universality is observed in other physical quantities. The Higgs boson couplings \(g_{WWH}\) and \(g_{ZZH}\) to \(W, Z\), and Yukawa couplings \(y_f\) to quarks and leptons are given, to good approximation, by [20, 36]
\[
\begin{align*}
g_{WWH} &= g_w m_W \cos \theta_H, \\
g_{ZZH} &= \frac{g_w m_Z}{\cos \theta_W} \cos \theta_H, \\
y_f &= \begin{cases} 
\frac{m_f}{v_{\text{SM}}} \cos \theta_H & \text{in the A model} \\
\frac{m_f}{v_{\text{SM}}} \cos^2 \frac{1}{2} \theta_H & \text{in the B model}
\end{cases} \tag{4.14}
\end{align*}
\]
where \(v_{\text{SM}} = f_H \sin \theta_H = 2m_W / g_w\). For small \(\theta_H\), deviation in the Higgs couplings in (4.14) is small, whereas deviation in \(\lambda_3\) and \(\lambda_4\) becomes substantial.
5 Dark Fermions

Although the $\theta_H$ universality holds for various couplings associated with the Higgs boson, masses of dark fermions $\Psi_F$ and $\Psi^\pm_{(1,5)}$, for instance, sensitively depend on the choice of the parameters $(n_F, n_V, c_V)$. They are determined by (A.12) for $\Psi_F$, and by (A.13) and (A.14) for charged and neutral components of $\Psi^\pm_{(1,5)}$. In Table 4 their masses are tabulated for various $\theta_H$ with $n_F = n_V = 2$. Dark fermions have relatively small masses compared with the KK mass scale $m_{KK}$. The lightest neutral component of the dark fermions can be a candidate for dark matter.

Table 4: KK mass and dark fermion masses are shown in the unit of TeV for various $\theta_H$ with $z_L = 10^{10}$, $n_F = n_V = 2$ and $c_V = 0.2$. Charged and neutral components of $\Psi^\pm_{(1,5)}$ have nearly the same masses.

| $\theta_H$ | $m_{KK}$ | $\Psi_F$ | $\Psi^\pm_{(1,5)}$ |
|------------|----------|----------|---------------------|
| 0.05       | 24.1     | 6.30     | 5.60                |
| 0.10       | 12.1     | 3.42     | 2.84                |
| 0.15       | 8.07     | 2.38     | 1.91                |
| 0.20       | 6.08     | 1.82     | 1.45                |

In Fig. 5 the mass of $\Psi_F$ is plotted as a function of $\theta_H$ for several $n_F$. The mass decreases as $n_F$ increases. Similar behavior is obtained for $\Psi^\pm_{(1,5)}$ as $n_V$ is varied with $c_V$ fixed.

Figure 5: $\theta_H$ and $n_F$ dependence of the mass of dark fermion $\Psi_F$. $z_L = 10^{10}$, $n_V = 2$, $c_V = 0.2$. 

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6 Summary

In this paper we have examined the effective potential \( V_{\text{eff}}(\theta_H) \) in GUT inspired \( SO(5) \times U(1) \times SU(3) \) gauge-Higgs unification to confirm that electroweak symmetry breaking is dynamically induced by the Hosotani mechanism. From \( V_{\text{eff}}(\theta_H) \) the cubic and quartic self-couplings, \( \lambda_3 \) and \( \lambda_4 \), of the Higgs boson are determined. We have shown the \( \theta_H \) universality of these couplings, i.e. they are determined as functions of \( \theta_H \) to high accuracy, irrespective of the details of other parameters in the theory. For \( \theta_H = 0.1 \) (0.15), \( \lambda_3 \) and \( \lambda_4 \) are smaller than those in the standard model by 7.7% (8.1%) and 30% (32%), respectively. The \( \theta_H \) universality in \( \lambda_3 \) and \( \lambda_4 \) is understood as a result of the factorization property of each component in the contributions to the effective potential, which is valid to high accuracy in the Randall-Sundrum warped space with \( z_L \gg 1 \).

The \( \theta_H \) universality gives the model great prediction power. Once the value of \( \theta_H \) is determined by one of the experimental data, then many other physical quantities such as masses and couplings of various particles are predicted. It has been known that gauge-Higgs unification models in the RS space predict large parity violation in the couplings of quarks and leptons to \( Z' \) particles (KK modes of \( \gamma, Z \) and \( Z_R \)). Its effect can be clearly seen in electron-positron collision experiments with polarized electron/positron beams in which \( \theta_H \) is the most important parameter. \( Z' \) particles can be directly produced at LHC, and parity-violating couplings would manifest in the rapidity distribution in \( t\bar{t} \) production. CKM mixing with natural FCNC suppression is also incorporated in the GUT inspired gauge-Higgs unification. It is curious to pin down the behavior of the model at finite temperature and implications to cosmology. \( SO(5) \times U(1) \times SU(3) \) gauge-Higgs unification is one of the most promising scenarios beyond the standard model. We shall come back to these issues in future.

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A Mass spectrum

In evaluating the effective potential \( V_{\text{eff}}(\theta_H) \) in Section 3, one needs to know the mass spectrum of each KK tower of the fields in the model. It is sufficient to know the form of functions whose zeros determine the mass spectrum. These functions have been given in ref. [16]. We summarize them in this appendix for the convenience.

We first introduce

\[ F_{\alpha,\beta}(u, v) \equiv J_\alpha(u)Y_\beta(v) - Y_\alpha(u)J_\beta(v), \quad (A.1) \]

where \( J_\alpha(u) \) and \( Y_\alpha(u) \) are the first and second kind Bessel functions. For gauge fields we define

\[ C(z; \lambda) = \frac{\pi}{2} \lambda zz_L F_{1,0}(\lambda z, \lambda z_L), \]
\[ S(z; \lambda) = -\frac{\pi}{2} \lambda z F_{1,1}(\lambda z, \lambda z_L), \]
\[ C'(z; \lambda) = \frac{\pi}{2} \lambda^2 zz_L F_{0,0}(\lambda z, \lambda z_L), \]
\[ S'(z; \lambda) = -\frac{\pi}{2} \lambda^2 z F_{1,1}(\lambda z, \lambda z_L). \quad (A.2) \]

For fermion fields with a bulk mass parameter \( c \), we define

\[
\begin{pmatrix} C_L \\ S_L \end{pmatrix}(z; \lambda, c) = \pm \frac{\pi}{2} \lambda \sqrt{zz_L} F_{\frac{1}{2} c + \frac{1}{2} \pm \frac{1}{2}}(\lambda z, \lambda z_L), \\
\begin{pmatrix} C_R \\ S_R \end{pmatrix}(z; \lambda, c) = \pm \frac{\pi}{2} \lambda \sqrt{zz_L} F_{\frac{1}{2} c - \frac{1}{2} \pm \frac{1}{2}}(\lambda z, \lambda z_L), \quad (A.3)
\]

and

\[
\begin{align*}
C_{R1}(z; \lambda, c, \tilde{m}) &= C_R(z; \lambda, c + \tilde{m}) + C_R(z; \lambda, c - \tilde{m}), \\
C_{R2}(z; \lambda, c, \tilde{m}) &= S_R(z; \lambda, c + \tilde{m}) - S_R(z; \lambda, c - \tilde{m}), \\
S_{L1}(z; \lambda, c, \tilde{m}) &= S_L(z; \lambda, c + \tilde{m}) + S_L(z; \lambda, c - \tilde{m}), \\
S_{L2}(z; \lambda, c, \tilde{m}) &= C_L(z; \lambda, c + \tilde{m}) - C_L(z; \lambda, c - \tilde{m}), \\
C_{L1}(z; \lambda, c, \tilde{m}) &= C_L(z; \lambda, c + \tilde{m}) + C_L(z; \lambda, c - \tilde{m}), \\
C_{L2}(z; \lambda, c, \tilde{m}) &= S_L(z; \lambda, c + \tilde{m}) - S_L(z; \lambda, c - \tilde{m}), \\
S_{R1}(z; \lambda, c, \tilde{m}) &= S_R(z; \lambda, c + \tilde{m}) + S_R(z; \lambda, c - \tilde{m}), \\
S_{R2}(z; \lambda, c, \tilde{m}) &= C_R(z; \lambda, c + \tilde{m}) - C_R(z; \lambda, c - \tilde{m}). \quad (A.4)
\end{align*}
\]
A.1 Gauge bosons

The mass spectrum \( \{m_n = k\lambda_n\} \) of \( W \) and \( W_R \) towers is determined by

\[
W \text{ tower} : \quad 2S(1; \lambda)C'(1; \lambda) + \lambda \sin^2 \theta_H = 0,
\]
\[
W_R \text{ tower} : \quad C(1; \lambda) = 0. \tag{A.5}
\]

The spectrum of \( \gamma, Z, Z_R \) and \( A_z \) towers is determined by

\[
\gamma \text{ tower} : \quad C'(1; \lambda) = 0,
\]
\[
Z \text{ tower} : \quad 2S(1; \lambda)C'(1; \lambda) + (1 + s_\beta^2)\lambda \sin^2 \theta_H = 0,
\]
\[
Z_R \text{ tower} : \quad C(1; \lambda) = 0,
\]
\[
A_z \text{ tower} : \quad S(1; \lambda)C'(1; \lambda) + \lambda \sin^2 \theta_H = 0. \tag{A.6}
\]

A.2 Fermions

With given up-type quark masses \( m_Q = (m_u, m_c, m_t) \) the bulk mass parameter \( c_Q = (c_u, c_c, c_t) \) of up-type quark multiplets is fixed by

\[
S_L(1; \lambda, c_Q)S_R(1; \lambda, c_Q) + \sin^2 \frac{\theta_H}{2} = 0, \tag{A.7}
\]

where \( \lambda = \lambda_Q = m_Q/k \). Then the spectrum of up-type quark towers is determined by \( \text{(A.7)} \). In the down-type quark sector there are brane interactions which mix \( d^\alpha \) and \( D_+^\beta \) through \( (2.14) \). When brane interactions are diagonal in the generation space, \( \mu_1^\alpha = \delta_\alpha^\beta \mu_\alpha \), the spectrum of down-type quark tower is determined by

\[
\left( S_L^Q S_R^Q + \sin^2 \frac{\theta_H}{2}\right) \left( S_L^D S_R^D - S_L^D S_R^D \right) + |\mu_1|^2 C_Q S_R^Q \left( S_L^D C_L^D - S_L^D C_L^D \right) = 0,
\]

\[
S_L^Q = S_L(1; \lambda, c_Q), \quad S_L^D = S_L(1; \lambda, c_D, \tilde{m}_D), \quad \text{etc.} \tag{A.8}
\]

where \( \mu_1 = (\mu_d, \mu_s, \mu_b) \), \( c_D = (c_{D_d}, c_{D_s}, c_{D_b}) \) and \( \tilde{m}_D = (\tilde{m}_{D_d}, \tilde{m}_{D_s}, \tilde{m}_{D_b}) \). The parameters \( \mu_1, c_D, \tilde{m}_D \) are determined such that \( \lambda = (\lambda_d, \lambda_s, \lambda_b) = k^{-1}(m_d, m_s, m_b) \) solves \( \text{(A.8)} \) in each generation. For the third generation, for instance, we take \( (\mu_b, c_{D_b}, \tilde{m}_{D_d}) = (0.1, 1.044, 1.0) \). Only top quark multiplet among quark multiplets gives a relevant contribution to \( V_{\text{eff}}(\theta_H) \). By considering general \( \mu_1^\alpha \) the CKM mixing is incorporated with natural FCNC suppression.[20]

With given charged lepton masses \( m_L = (m_e, m_\mu, m_\tau) \) the bulk mass parameter \( c_L = (c_e, c_\mu, c_\tau) \) of charged lepton multiplets is fixed by

\[
S_L(1; \lambda, c_L)S_R(1; \lambda, c_L) + \sin^2 \frac{\theta_H}{2} = 0, \tag{A.9}
\]

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where \( \lambda = \lambda_L = m_L/k \). Then the spectrum of charged lepton towers is determined by (A.9). In the neutrino sector brane interactions mix \( \nu^\alpha, \nu'^\alpha, \chi^\alpha \). When both \( M^{\alpha\beta} = \delta^{\alpha\beta}M_{\alpha} \) in (2.10) and \( m^{\alpha\beta}_B = \delta^{\alpha\beta}m_B^\alpha \) in (2.15) are diagonal, the spectrum of neutrino tower is determined by

\[
(k\lambda - M)\left(S_L^LS_R^L + \sin^2\frac{\theta_H}{2}\right) + \frac{m^2_B}{k}S_R^LC_R^L = 0 ,
\]

\( S_R^L = S_R(1; \lambda, c_L), \) etc. (A.10)

where \( M = (M_1, M_2, M_3) \) and \( m_B = (m^1_B, m^2_B, m^3_B) \). With \( c_L < -\frac{1}{2} \) the light neutrino mass is given by

\[
m_{\nu} \sim \frac{m^2_L M}{(2|c_L| - 1)m^2_B} \quad \text{(A.11)}
\]
in each generation. Contributions from lepton multiplets to \( V_{\text{eff}}(\theta_H) \) are negligible.

The spectrum of dark fermion \( \Psi_F \) tower is determined by

\[
S_L^L(1; \lambda, c_F)S_R^L(1; \lambda, c_F) + \cos^2\frac{\theta_H}{2} = 0 .
\]

(A.12)

The spectrum of charged components of dark fermions \( \Psi_{(1,5)}^\pm \) tower is determined by

\[
S_{L1}(1; \lambda, c_V)S_{R1}(1; \lambda, c_V) - S_{L2}(1; \lambda, c_V)S_{R2}(1; \lambda, c_V) = 0 ,
\]

(A.13)

whereas the spectrum of neutral component tower is determined by

\[
\begin{aligned}
\{ B_0(\lambda, c_V, \bar{m}_V) - 2\cos 2\theta_H \}^2 &= 0 , \\
B_0(\lambda, c, \bar{m}) &= C_L(1; \lambda, c + \bar{m})C_R(1; \lambda, c - \bar{m}) + C_L(1; \lambda, c - \bar{m})C_R(1; \lambda, c + \bar{m}) \\
+ S_L(1; \lambda, c + \bar{m})S_R(1; \lambda, c - \bar{m}) + S_L(1; \lambda, c - \bar{m})S_R(1; \lambda, c + \bar{m}).
\end{aligned}
\]

(A.14)

There are two degenerate towers.\(^1\)

## B Useful functions

As shown in the formula (3.3), a mass-determining function \( \rho(m; \theta_H) \) is analytically continued to \( \rho(iy; \theta_H) \). We summarize functions used in the evaluation of \( V_{\text{eff}}(\theta_H) \) in Section 3. We introduce

\[
\hat{F}_{\alpha,\beta}(u, v) \equiv I_{\alpha}(u)K_{\beta}(v) - e^{-i(\alpha-\beta)\pi}K_{\alpha}(u)I_{\beta}(v) ,
\]

(B.1)

\(^1\)There was a typo in (D.16) of ref. [16]. The last term in the second line, \( s_H^2 c_H^2 (c_{L1}^V c_{L2}^Y - c_{L2}^V c_{L1}^Y)^2 \), should be \( s_H^2 c_H^2 (c_{L1}^V c_{L2}^V - c_{L2}^V c_{L1}^V)^2 \). With this correction (D.16) of ref. [16] coincides with (A.14) in the current paper.

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where \( I_\alpha(u) \) and \( K_\alpha(u) \) are first and second kind modified Bessel functions. In terms of \( \hat{F}_{\alpha,\beta}(u,v) \) we define

\[
\begin{align*}
\hat{C}(q) &= q \hat{F}_{1,0}(qz_L^{-1}, q), \\
\hat{S}(q) &= iqz_L^{-1} \hat{F}_{1,1}(qz_L^{-1}, q), \\
\hat{C}'(q) &= q^2 z_L^{-1} \hat{F}_{0,0}(qz_L^{-1}, q), \\
\hat{S}'(q) &= -iq^2 z_L^{-2} \hat{F}_{0,1}(qz_L^{-1}, q)
\end{align*}
\] (B.2)

for gauge fields. For fermion fields with \( c > 0 \) we define

\[
\begin{align*}
\hat{C}_L(q; c) &= q z_L^{-1/2} \hat{F}_{c+\frac{1}{2},c-\frac{1}{2}}(qz_L^{-1}, q), \\
\hat{S}_L(q; c) &= iq z_L^{-1/2} \hat{F}_{c+\frac{1}{2},c+\frac{1}{2}}(qz_L^{-1}, q), \\
\hat{C}_R(q; c) &= q z_L^{-1/2} \hat{F}_{c-\frac{1}{2},c+\frac{1}{2}}(qz_L^{-1}, q), \\
\hat{S}_R(q; c) &= -iq z_L^{-1/2} \hat{F}_{c-\frac{1}{2},c-\frac{1}{2}}(qz_L^{-1}, q)
\end{align*}
\] (B.3)

For \( c < 0 \), we use the the relations

\[
\hat{C}_L(q; -c) = \hat{C}_R(q; c), \quad \hat{S}_L(q; -c) = -\hat{S}_R(q; c)
\] (B.4)

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