Blind Curvelet-Based Denoising of Seismic Surveys in Coherent and Incoherent Noise Environments

Naveed Iqbal1 · Mohamed Deriche2 · Ghassan AlRegib3 · Sikandar Khan4

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Abstract
Distributed acoustic sensing (DAS) is a new seismic monitoring technology. DAS generates a large amount of data, necessitating the development of new technologies to allow for cost-effective processing and handling. The raw seismic data is noisy and must be processed. The curvelet transform is an excellent choice for processing seismic data due to its localized nature, as well as its frequency and dip characteristics. However, its capabilities are limited in case of noise other than white. This paper proposes a denoising method based on a combination of the curvelet transform and a whitening filter, as well as a procedure for estimating noise variance. The whitening filter is included to improve the performance of the curvelet transform in both coherent and incoherent noise cases, as well as to simplify the noise variance estimation method and make it easier to use standard threshold methodology without delving into the curvelet domain. Two data sets are used to validate the suggested technique. Pseudo-synthetic data set created by adding noise to the actual noise-free data collection from the Netherlands offshore F3 block and the on-site data set (with ground roll noise) from east Texas, USA. Experimental results demonstrate that the proposed algorithm achieves the best results under various types of noise.

Keywords Curvelet · Whitening · Noise · Seismic data

1 Introduction
Seismic surveys consist of large volumes of sensors. These sensors are laid on the earth’s surface to image the local geology for possible oil and gas reservoirs. Sound waves are produced using a source (vibrator). The waves bounce off various layers within the earth and reflected waves to the surface are captured by sensors. The recorded data is usually contaminated with unwanted energy or noise. The removal of the noise during the seismic data processing is essential in order to improve the signal-to-noise ratio (SNR) and also to avoid the wrong interpretation of seismic data [1–4], which can be detrimental. It is not possible to completely remove the noise during the seismic data processing; hence, the objective is to improve the SNR as much as possible. Improving the SNR will help in enhancing the image of the subsurface geology, which will improve the hydrocarbon exploration process. In Ulrych et al. [5], the signal of interest is defined as the signal energy due to the reason that it is coherent from trace to trace and desirable for interpretation from a geophysical perspective. Noise is caused due to unwanted seismic energy from various sources like, shot generation ground roll, weather (wind, rain), and human activities. Noise can also occur because of random occurrences within the earth. Broadly speaking, noises in seismic data are classified as random noise, coherent noise, and correlated noise [6, 7].
In short, noise in seismic surveys can be classified as follows:

- **Incoherent noise** It is independent of the seismic source signal and there is no noise correlation among the traces.
- **Correlated and uncorrelated noise** Incoherent noise can have correlation or no correlation among the noise samples within the trace and, hence, called correlated (Brownian noise [8], pink noise [9], and noise filtered through geophone [10]) or uncorrelated (or random or additive white noise [11]) noise, respectively.
- **Coherent noise** It is generated by the source and it is highly correlated with the source wavelet and across the traces. Figure 1 shows a case of coherent noise [12]. This noise creates more trouble because it can be highly correlated with the signal of interest [13].

Whatever the causes or types of seismic noise, these negatively impact the interpretation of results, such as the spectral and structural attributes, seismic imaging, and analysis [14–17].

In this paper, a novel and robust method based on the curvelet transform is proposed, with a data-whitening step capable of dealing with the most diverse and difficult types of noise. The pre-whitening stage, together with noise estimation from observation, is collectively called the variance blind curvelet-based denoising method. The method is independent of the type of noise and can be applied to a variety of data sets. Therefore, the term “blind” is used to indicate that noise type is not needed to be known and method can be applied blindly. Following are the main contributions of this work:

- The performance of the curvelet denoising is classified based on the types of images. For this purpose, seismic images and general-purpose images are compared.
- A method to estimate noise variance from the seismic images for the coherent and incoherent noise variance cases is presented based on the pattern of the images. This simple and straightforward method is in contrast with previous methods in which curvelets need to be identified and nullified to remove the noise.
- It is shown that the curvelet transform gives the best performance for the white noise (uncorrelated). For the colored (correlated) noise, there is still room for improvement.
- Inherently, the curvelet transform works well for uncorrelated noise. For performance enhancement, pre-whitening and whitening inverse steps are introduced for the correlated noise case.
- Finally, the method which includes pre-whitening, noise variance estimation from the image, and the whitening inverse is adopted for the highly coherent case of ground roll noise.
- Synthetic, pseudo-synthetic, and field data sets are utilized to depict the effectiveness of the proposed curvelet transform-based denoising method.

## 2 Related Work

In this section, some of the noise removal/reduction techniques for seismic data are briefly discussed.

Bandpass filtering and spectral filtering techniques are commonly utilized for improving the SNR in seismic applications. If the frequency of the signal and noise is the same then the bandpass filtering and spectral filtering techniques will not be able to fully remove the noise. Due to this reason, various other effective denoising techniques have been presented in the literature. A particle-filter-based denoising method is presented in Baziw [18] in order to improve the SNR. The main advantage of the particle filter is its effective removal of the broadband noise. During this denoising method, the characteristics of the signal and noise and the dynamics of the target system is directly modeled. In this denoising method, the filtering of the signal is directly related to the dynamics of the system, instead of performing a totally mathematical operation. In order to deal with abruptly varying seismic wavelet, Deng et al. [19] presented a time-frequency peak filtering approach that was based on an adaptive pre-processing methodology. For identifying the noisy signal as a signal, buffer, or noise, a threshold was designed. In conventional time-frequency peak filtering, the
use of the pseudo Wigner–Ville distribution (PWVD) for the estimation of the instantaneous frequency depicted a sensitivity to the interference of noise in masking the borderline between the signal and noise. A real-time Kalman filter was discussed in Baziw and Weir-Jones [20] for removing the background noise from the recorded seismic traces. However, in real conditions, the availability of statistics of noise is not possible. For transforming the seismic data from the time-space into the time-frequency-space (t-f-x) domain, Cai et al. [21] presented a generalized S-transform methodology. For suppressing the coherent and random noise, the empirical mode decomposition (EMD) is applied on each frequency slice. A wavelet-shrinkage-based algorithm was used in Ghael et al. [22] for designing a wavelet-domain Wiener filter. The shrinkage estimation provides an estimation of the signal subspace that facilitates the design of the filter. An adaptive singular-value decomposition (SVD) filter was presented in Lu [23] for enhancing the non-horizontal events. For this purpose, the direction of the seismic image texture was detected and then the estimated dip was aligned by data rotation. The texture direction was estimated by using the features derived from the co-occurrence matrix. This technique works for random noise; however, some artifacts may appear.

Wiener filtering has been used in the literature in order to obtain the optimum filter in the mean square sense. The direct use of the Wiener filter for denoising purposes in the seismic data requires information related to statistics of the seismic source signal or noise that is not normally available [24]. A possible way may be the assumption of a particular type of noise [7] or to model the noise as a sweep signal in order to eliminate ground roll noise [6].

In this work, we aim to develop a robust technique able to deal with diverse and most challenging types of noise, such as coherent noise (e.g., ground roll noise). It is worth mentioning that the denoising of ground roll noise is performed traditionally using bandpass filtering [25]. This increases the SNR by attenuating the ground roll noise, however, the seismic data is also affected by this frequency filtering. The reason is that the band that is excluded by the filtering process may contain important seismic events. Consequently, the vertical resolution of the seismic data is lowered. To alleviate this, Oliveira et al. [26] proposed a curvelet-based denoising method using hard thresholding. In this method, the authors identify the angular sections that contain the ground roll noise and erase the corresponding curvelet coefficients before reconstructing the seismic signal. The problem with this technique is that it requires visual inspection of the angular sections to identify the curvelet coefficients that are to be nullified for noise reduction. This process is demanding as it requires dedicated manpower to deal with the huge amount of data. A similar approach is also proposed in Neelamani and Baumstein [27]. Recently, Górszczyk et al. [28, 29] proposed the enhancement of 2D and 3D post-stack seismic data acquired in a hard-rock environment using 2D curvelet transform. In their work, authors tackle random and colored noise. Scale-adjusted thresholding is introduced, which is a modification of Starck et al. [30], i.e., three levels are used for thresholding curvelet coefficients instead of two.

### 3 Curvelet Transform

The transform-domain denoising methods are the most effective techniques in seismic applications. These methods achieve the goal of noise removal or reduction by utilizing the properties of sparseness and separateness present in the transform domain seismic data. Wavelet transforms are popular in scientific and engineering fields; however, they ignore the geometric properties of structures, regularity of edges, and curvature nature of edges. The curvelet transform [31] is a multi-resolution directional transform that allows sparse representation of objects with curved edges. In comparison with the Fourier transform (sparse in frequency-domain and use translation only) and the wavelet transform (sparse in time and frequency domains and use translations and dilation), the curvelet transform is sparse in time, frequency, and phase domains, and uses translations, dilation, and rotation.

The curvelet transform is a modified form of the wavelet transform that represents images at different scales and different angles. It overcomes the problem of missing directional selectivity of the wavelet transforms in 2D signals (images). Another transform, known as the Gabor transform, also uses different directions and scales like curvelet. However, curvelets completely cover the whole spectrum, whereas the frequency plan of the Gabor filters contains a large number of holes [32].

There are several other techniques of directional wavelet systems that have the same goal, i.e., optimally representing the directional features and a better analysis of signals in higher dimensions. These methods include steerable wavelets, Gabor wavelets, wedgelets, beamlets, bandlets, contourlets, shearlets, wave atoms, platelets, and surfacelets. However, none of these methods are as popular as the curvelet transform. A good comparison of the above-mentioned methods with curvelet can be found in [33].

In what follows, we will provide the mathematical background for the curvelet transform. Candès and Donoho [34] initially developed the curvelet transform that is widely used for seismic data processing [28, 29, 35–37]. A triple index, i.e., scale $j$, spatial location $k = (k_1, k_2)$, and orientation $l$, is used to represent the curvelets. A curvelet at any scale $j$ can be thought of as an oriented object that has a support in the rectangle having width and length of $2^{-j/4}$ and $2^{-j/2}$, respectively, and obeys the parabolic scaling rotation width $\approx \text{length}^2$ [31]. The parabolic dilation, translation, and rotation of
functions, and furthermore, these functions restrict the sup-
zero.

The rotation matrix $W$ is given as:

$$W = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos \frac{\pi}{2} & \sin \frac{\pi}{2} \\ \sin \frac{\pi}{2} & -\cos \frac{\pi}{2} \end{pmatrix}.$$  

The curvelet transform of a function $f$ is given by the convolution integral:

$$c_{j,l,k} = \int f(x) \Phi_{j,l,k}(x) dx.$$  

The coefficients of the curvelet transform, $c_{j,l,k}$, in the above equation are interpreted as the decomposition of function $f$ into the basis curvelet $\Phi_{j,l,k}$. The curvelet frequency domain view is shown in Fig. 2.

The filtering methodology works as follows. First, the image or 2D signal is transformed into the curvelet domain, then thresholding is applied, and finally, the signal is reconstructed after noise removal. For removing the noise from the image using the curvelet transform, we apply the standard methodology [26, 27, 30, 38, 39] which is outlined here for the sake of self-contentedness and clarity. Let us assume that the noisy data is represented as $x_{m,n} = I(m, n) + \sigma n_{m,n}$ where $I$ is the image of interest and $n$ is the white noise with mean zero and variance 1, i.e., $n_{m,n} \sim N(0, 1)$. Let $F$ denote the discrete curvelet transform matrix, then, we have $Fn \sim N(0, FF^T)$. Since the computation of the transform-domain variance $FF^T$ is prohibitively expensive, therefore, an approximate value noise variance $\hat{\sigma}^2$ (where $\lambda$ is the curvelet index) is used to calculate it. The curvelet transforms of some standard white noise images are used to estimate the diagonal elements of $FF^T$. Let $c_j$ be the curvelet coefficients ($c =Fx$) corresponding to noise. The following hard-thresholding rule is used for removing noise

$$\hat{c}_\lambda = \begin{cases} c_\lambda, & \text{if } |c_\lambda|/\sigma \geq k\hat{\sigma} \\ 0, & \text{if } |c_\lambda|/\sigma < k\hat{\sigma} \end{cases}$$  

In our experiments, similar to [30], we chose a value for $k$ which is scale-dependent; hence, we have $k = 4$ for the coarsest (first) scale ($j = 1$), whereas $k = 3$ for the other scales ($j > 1$).

### 4 Correlated Noise and Whitening Method

Curvelet transform-based denoising performs most effectively under the white noise scenario. The reason is that the threshold assumes white noise and that the noise samples are uncorrelated. For colored (correlated) noise, the performance is not as good as for uncorrelated noise (this is shown in the results section). In Oliveira et al. [26], the authors discussed the case of correlated noise, e.g., ground roll noise, however, they intuitively find the curvelet coefficients to be eliminated. To the best of our knowledge, this issue of threshold under the correlated noise case has not been addressed so far and previous works mainly used traditional methods.

Therefore, in this section, we introduce a simple yet effective technique to deal with this issue. We propose to use a whitening procedure to pre-whiten the data before curvelet-based processing in the case of colored noise. Here, we use the famous zero-phase component analysis (ZCA) for whitening. We start with principal component analysis (PCA). Suppose, $X$ is the image to be whitened. Note that, the columns of $X$ are correlated, hence, in the case of incoherent correlated noise columns represent traces, whereas, in the case of coherent noise rows represent traces. First, the mean of each column is subtracted from the respective column. An estimate of the covariance matrix of the image $X$ is then obtained as:

$$X^T X = UU^T$$  

The curvelet index is used to calculate it. The curvelet transforms of some standard white noise images are used to estimate the diagonal elements of $FF^T$. Let $c_j$ be the curvelet coefficients ($c =Fx$) corresponding to noise. The following hard-thresholding rule is used for removing noise

$$\hat{c}_\lambda = \begin{cases} c_\lambda, & \text{if } |c_\lambda|/\sigma \geq k\hat{\sigma} \\ 0, & \text{if } |c_\lambda|/\sigma < k\hat{\sigma} \end{cases}$$  

In our experiments, similar to [30], we chose a value for $k$ which is scale-dependent; hence, we have $k = 4$ for the coarsest (first) scale ($j = 1$), whereas $k = 3$ for the other scales ($j > 1$).
where $U$ is the square matrix whose $i^{th}$ column is the eigenvector $u_i$ of $X^TX$ and $\Lambda$ is the diagonal matrix whose diagonal elements are the corresponding eigenvalues. The whitening matrix $W_p$ then becomes

$$W_p = U \Lambda^{-1/2}$$

and the whitened image is:

$$X_w = XW_p = XU \Lambda^{-1/2}$$

The eigenmatrix $U$ of the covariance of $X$ is used to rotate the variables, which creates orthogonal components having different variances. The square root of the eigenvalues $\Lambda^{1/2}$ is used to whiten and scale the rotated variables. The most commonly used whitening technique is the PCA whitening technique. The PCA and ZCA whitening transformations are interrelated by a rotation matrix $U$ that helps in interpreting the ZCA whitening as a rotation followed by scaling followed by the rotation $U$ back to the original coordinate system. Here note that the whitening by PCA is not unique. Any rotation (multiplying with an orthogonal rotation matrix) will leave it whitened. Hence, taking, in particular, $U$ from the covariance matrix and multiplying it with the whitening matrix $W_p$ obtained from PCA, makes it unique. The new whitening matrix $W_z$ thus created a form basis for ZCA. Therefore, the ZCA whitening is given as

$$X_w = XW_z = XU \Lambda^{-1/2}U^T$$

$\Lambda^{-1/2}$ is a diagonal matrix with $1/(\sqrt{\lambda_i})$ on the diagonal. This is regularized to $1/(\sqrt{\lambda_i} + \epsilon)$, where $\epsilon = 0.0001$. The procedure for the case of correlated noise is as follows:

- The image is whitened by multiplying it with the whitening matrix $W_z$.
- Noise variance is estimated from the data (this will be discussed in the experimental results section)
- Then, the whitened data is denoised using the curvelet transform.
- Finally, the whitened denoised data is restored by multiplying with the pseudo-inverse of the whitening matrix, i.e., $(W_z^H W_z)^{-1} W_z^H$.

5 Experimental Results and Noise variance Estimation

In this section, we will perform some experiments to test the proposed curvelet transform method and present our noise variance estimation approach based on the pattern of seismic images.

5.1 Performance of Curvelet Transform for General-Purpose and Seismic Images

In the first experiment, we compare the curvelet transform denoising performance for general-purpose images and seismic images. In the second experiment, the method to estimate noise variance from the seismic images for incoherent noise (correlated or uncorrelated) is introduced. In the third experiment, the performance of curvelet transform is tested for correlated noise case, while pre-whitening/whitening inverse steps are introduced in the fourth experiment in order to enhance the performance of curvelet-based denoising under the correlated noise scenario. Finally, in the last experiment, the worst case of ground roll noise is tackled using the whitening/whitening inverse filters and noise variance estimation from the image.
5.2 Noise Variance Estimation

The value of the noise variance is needed for thresholding in the curvelet domain [27]. Previous works (mentioned before) about curvelet-based denoising rely on the a priori known value of the noise variance. However, in reality, the variance is not known, and hence, curvelet transform-based denoising needs to be performed using the hit-and-try method. This, in turn, adds to the complexity of the already heavy computational load of the curvelet transform. In previous experiments, the variance was assumed to be known. Here, we estimate the variance using the seismic image’s first $20 \times 20$ patch. Due to the pattern of the seismic image, the assumption here is that there is no signal of interest (seismic signal) in this patch, which is quite true. The assumption fits in the case of noise that is not generated by the source signal. The comparison of the PSNR for the known and estimated noise variance is shown in Fig. 5. The figure depicts that the curvelet-based denoising achieves almost the same performance with the known noise variance and the estimated one. These excellent results suggest to use this noise variance estimation for the rest of our experiments.

5.3 Correlated Noise

In the aforementioned experiments, white noise is used and curvelet-based denoising gives promising results. It is important to check its performance in correlated noise, which is likely to be present in the seismic data, e.g., ground roll noise. For the correlated noise, we generated noise with a $1/|f|^\alpha$ spectral characteristic over its entire frequency range. The most common values for $\alpha$ are $\alpha = 1$ (pink noise) and $\alpha = 2$ (Brownian noise). Brownian noise is considered to be the worst correlated noise in seismic applications. Note that $\alpha = 0$ refers to white noise. The autocorrelation and power spectrum density of various types of noises are shown in Fig. 6.

The comparison of the curvelet-based denoising under the correlated (color) and uncorrelated (white) noise reveals that the curvelet transform does not perform equally well in the latter case (Fig. 7). Hence, there is still room for performance improvement. One reason for sub-optimal performance is that the noise samples are correlated and the other reason is the threshold which assumes white noise. Furthermore, in the case of correlated noise, the noise level in each curvelet is different, so defining a global threshold does not give the best

![Graph showing performance of curvelet-based denoising for general-purpose images and seismic images.]

![Graph showing PSNR improvement for seismic images with known and estimated noise variance.]

![Graph comparing spectrum for various types of noise.]

![Graph comparing normalized amplitude for various types of noise.]

Marzouqi and AlRegib [40]. For general-purpose images, the authors assume that noise variance is known.
performance. One complex and difficult way is to identify the curvelets that contain noise and define a curvelet-dependent threshold based on the amount of noise in each curvelet. However, here we propose to include a pre-whitening filter that makes the noise distribute equally among all the curvelets and apply the noise variance estimation method described previously to get a similar performance as uncorrelated noise.

5.4 Whitening Filter

To further improve the performance of curvelet-based denoising in correlated noise case, a pre-whitening filter (explained previously) is used. The pre-whitening filter makes the image uncorrelated, and hence, curvelet transform-based denoising produces better results (close to the white noise case). However, using pre-whitening on the noisy image, some seismic signal information might be lost. For this purpose, after the denoising step using curvelet transform, the whitening inverse filter is applied to get the final denoised image. The noise variance estimation is performed as before by using the first \( 20 \times 20 \) patch from the pre-whitened seismic image. Figure 8 shows the autocorrelation matrix of the image before and after whitening using ZCA. It can be seen from the figure that the covariance matrix after denoising is diagonalized. The noisy and denoised (with and without whitening filter) is shown in Fig. 9. The performance difference of curvelet-based denoising with and without whitening can be noticed from the figure. From Fig. 10, it can be seen that the \( f - k \) magnitude spectra of the denoised seismic image closely matched with the \( f - k \) spectra of the noiseless seismic image. The PSNR plot for the various noise cases and whitening method is depicted in Fig. 11. The whitening method achieves close performance to that of the white noise case. The explicit PSNR values before and after denoising are shown in Table 1. Note here that the noise is estimated from the seismic image as before, which confirms that the noise variance estimation method gives the best performance for incoherent noise case (either correlated or uncorrelated).

In order to show the performance superiority of the proposed method, the technique is compared with wavelet decomposition and empirical mode decomposition. The “wden” function in the wavelet toolbox of Matlab is used for the wavelet decomposition-based denoising technique [41]. For the soft thresholding, the principle of Stein’s Unbiased Risk is used [42] and [43]. The comparison is shown in Table 2.

5.5 Field Data Set with Ground Roll Noise

To test the proposed method on field data with real noise (ground roll in this case), we used a pre-stacked 2D landline from east Texas, USA. The ground roll noise has low apparent velocity, low frequency, and high amplitude. For the field data set, the same procedure of denoising as before is used. First data is whitened using whitening filtering. Second, the curvelet-based denoising is used, i.e., data is transformed to the curvelet domain and thresholding is used for noise attenuation. Third, denoised pre-whitened data is obtained using inverse curvelet transform. Finally, data is recovered using the whitening inverse filter. Here, taking the first \( 20 \times 20 \) patch for noise variance estimation is not a good choice as in this patch there is no coherent noise. The reason is that the noise is generated by the source (i.e., it is not a background noise like incoherent noise). For noise variance estimation in the case of coherent noise, we proceed as follows: Considering the pattern of the ground roll noise in seismic image, the patch at the furthest location directly under the shot location is assumed to be the signal only and the patch beside this is assumed to contain the signal and noise. From these two patches, the noise can be estimated using subtraction. After subtraction, the variance of noise can be found for thresholding. The field data set, whitened data, denoised data, and the difference between denoised and field data set are shown in Fig. 12. The patches are shown in Fig. 12b by windows (signal-only part by black window, \( S \) and noise plus signal part by gray window, \( S_n \)). Using the windows, the variance is calculated as

\[
\sigma^2 = E \left[ (S_n(\cdot) - S(\cdot))^2 \right]
\]  

(8)

where, \( E[.\] represents the statistical expectation, and \( S_n(\cdot) \) and \( S(\cdot) \) represents the elements of the matrix \( S_n \) and \( S \), respectively, arranged in a vector. Note that for the real case noise, is a function of the source signal, hence, the assumption that noise is independent of the seismic signal is not valid in (8). The figure depicts the worst-case condition. The noise is very strong so the denoising method removes a significant portion of the noise leaving some residue.
In summary, the whitening filter provides a threefold advantage.

- First, noise variance estimation is carried out easily from the seismic image for coherent and incoherent cases. This is a simple and straightforward method without much human intervention when compared to estimating the noise in the curvelets domain by identifying the curvelets corresponding to noise and excluding them for seismic signal reconstruction (as done in Oliveira et al. [26]).
- Second, the performance of curvelet-based denoising is improved for correlated and coherent noise cases (after whitening procedure) and get close to the best denoising performance under the random noise case. It is known that denoising methods perform better in random noise case.
- Third, the conventional hard-thresholding method that assumes white noise becomes valid.

6 Discussion and Conclusion

Typical seismic data exhibit low SNR and correlated noise. The signal detection, seismogram composition studies, and the discrimination of local/regional seismic sources will be improved by removing noise. In this paper, a methodology based on the curvelet transform is proposed. The curvelet
Fig. 10 $f - k$ magnitude spectra: a Noise-less seismic data, b Noisy seismic data, $\alpha = 2$, PSNR before denoising = 5 dB. c Denoised seismic data

Table 1 PSNR before and after denoising for various types of noises

| PSNR before denoising (dB) | PSNR after denoising (dB) |
|---------------------------|---------------------------|
|                          | Without pre-whitening     | With pre-whitening |
|                          | $\alpha = 0$ | $\alpha = 1$ | $\alpha = 2$ | $\alpha = 0$ | $\alpha = 1$ | $\alpha = 2$ |
| $-4.93$                   | 18.32       | 0.956       | $-4.93$       | 18.32       | 18.00       | 17.64       |
| $-2.55$                   | 20.14       | 3.50        | $-1.90$       | 20.14       | 20.05       | 19.24       |
| 0.71                      | 22.26       | 6.62        | 1.15          | 22.26       | 22.15       | 20.81       |
| 6.02                      | 24.74       | 11.93       | 6.38          | 24.74       | 24.58       | 23.76       |
| 22.11                     | 30.16       | 26.24       | 22.32         | 30.16       | 28.01       | 28.01       |

Fig. 11 PSNR improvement for seismic images with various noises with pre-whitening

The transform is a generalized higher-order type of wavelet transform that can be used to present images at different scales and different angles. It basically overcomes the problem of missing directional selectivity of the wavelet transforms in images. Comparison of the curvelet transform denoising performance for general-purpose images and seismic images shows that it is better for seismic images. The reason is that the seismic data are more localized in time, frequency, and phase, which better matches the properties of curvelet transform. The noise variance needed for the threshold was estimated from the image patches themselves. For incoherent correlated (pink and brown noise) and coherent noise cases, a pre-whitening filter is introduced to enhance the per-

Table 2 Comparison of the proposed method with other denoising methods

| Method                        | PSNR (dB) |
|-------------------------------|-----------|
| Noisy data set                | 0.710     |
| Wavelet decomposition         | 11.366    |
| Empirical mode decomposition   | 10.524    |
| Proposed method               | 22.26     |
formance of curvelet denoising under colored noise. As a result, curvelet based denoising method performs similar to white noise case. Extensive testing of the proposed method on seismic images with incoherent (uncorrelated and correlated) and coherent noise shows very promising performance. Hence, suitable in the area of DAS processing and monitoring. In this work, we have used the 2D curvelet transform. This takes into consideration the correlation among the traces along the shot gathers. However, it will be interesting to use 3D curvelet transform to benefit from the correlation among the receiver gathers as well. Furthermore, other transforms like steerable wavelets, Gabor wavelets, wavelets, bandlets, contourlets, shearlets, wave atoms, platelets, and surfacelets can be used in a similar way. Although the addition of the whitening filter may increase complexity, the complexity requirement is relaxed due to offline processing.

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