Implication of BaBar’s new data on the $D_{s1}(2710)$ and $D_{sJ}(2860)$

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Abstract

The strong decays of the $D_{s1}(2710)$ and $D_{sJ}(2860)$ are investigated in the framework of the $^3P_0$ model. Its decay properties newly reported by the BaBar Collaboration can be reasonably accounted for in the presence of the $D_{s1}(2710)$ being a mixture of the $D_s(2^3S_1)$ and $D_s(1^3D_1)$. The orthogonal partner of the $D_{s1}(2710)$ is expected to have a mass of about 2.66 $\sim$ 2.9 GeV in quark models and a width of about 40 $\sim$ 60 MeV in the $^3P_0$ model. The predicted decay properties turn out to be consistent with the BaBar’s new data in both the orthogonal partner of the $D_{s1}(2710)$ and the $D_s(1^3D_3)$ interpretations for the $D_{sJ}(2860)$. The available experimental information is not enough to distinguish these two possibilities. The $E1$ radiative transitions of the $D_{s1}(2710)$ and $D_{sJ}(2860)$ are also studied. We tend to conclude that the $D_{s1}(2710)$ can be identified as a mixture of the $D_s(2^3S_1)$ and $D_s(1^3D_1)$, and the $D_{sJ}(2860)$ could be either the orthogonal partner of the $D_{s1}(2710)$ or the $D_s(1^3D_3)$. Further experimental information on the $D_{sJ}(2860)$ in the $D_s\eta$, $D_s^*\eta$, and $DK^*$ channels is needed.

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I. Introduction

In 2006, two new charm-strange states $D_{sJ}(2860)$ [Mass: $2856.6 \pm 1.5 \pm 5.0$ Mev, Width: $48\pm7\pm10$ MeV] and $D_{sJ}(2688)$ [Mass: $2688\pm4\pm3$ Mev, Width: $112\pm7\pm36$ MeV] were observed by the BaBar Collaboration in the $D\bar{K}$ channel[1]. There is no evidence for the $D_{sJ}(2688)$ and $D_{sJ}(2860)$ in the $D^*\bar{K}$ or $D_s\eta$ mode and hence their possible $J^P$ quantum numbers can be $0^+, 1^-, 2^+, 3^-$, etc. Subsequently, the Belle Collaboration observed a vector state $D_{s1}(2710)$ [Mass: $2708\pm9\pm11\pm10$ MeV, Width: $108\pm23\pm36\pm27$ MeV] in the $D\bar{K}$ channel[2]. Since their reported masses and widths are consistent with each other, the $D_{sJ}(2688)$ and $D_{s1}(2710)$ are believed to refer to a single state with a mass of $2690\pm7$ MeV and a width of $110\pm27$ MeV[3].

More recently, the $D_{sJ}(2860)$ and $D_{s1}(2710)$ were found again by BaBar Collaboration in both $D\bar{K}$ and $D^*\bar{K}$ channels[4]. The resulting masses and widths of these two states are

$$M(D_{sJ}(2860)^+) = 2862\pm2^{+5}_{-2}$$
$$\Gamma(D_{sJ}(2860)^+) = 48\pm3\pm6$$ MeV,

(1)

$$M(D_{s1}(2710)^+) = 2710\pm2^{+12}_{-7}$$
$$\Gamma(D_{s1}(2710)^+) = 149\pm7^{+39}_{-52}$$ MeV,

(2)

and the following ratios of branching fractions were also obtained:

$$\frac{B(D_{s1}(2710)^+ \rightarrow D^*\bar{K})}{B(D_{s1}(2710)^+ \rightarrow D\bar{K})} = 0.91\pm0.13\pm0.12,$$

(3)

$$\frac{B(D_{sJ}(2860)^+ \rightarrow D^*\bar{K})}{B(D_{sJ}(2860)^+ \rightarrow D\bar{K})} = 1.10\pm0.15\pm0.19.$$  

(4)

The observation of the $D_{sJ}(2860)$ in both $D\bar{K}$ and $D^*\bar{K}$ channels rules out it to be the $0^+$ state, since a $^3P_0$ $c\bar{s}$ state is forbidden to decay into $D^*\bar{K}$. Its possible spin-parity should be $1^-, 2^+, 3^-$, etc, if the structure at 2.86 GeV observed by the BaBar Collaboration in the $D\bar{K}$ and $D^*\bar{K}$ channels refer to a single resonance \(^{1}\).

Apart from the $D_{s1}(2710)$ and $D_{sJ}(2860)$, in the $D^*\bar{K}$ channel the BaBar Collaboration also found the evidence for the $D_{sJ}(3040)$ whose mass and decay properties have been discussed recently in Refs.[6, 7, 8]. Here, we shall focus on the implications of the BaBar’s new data on the $D_{s1}(2710)$ and $D_{sJ}(2860)$. Our main purpose is to check whether the experimental data for these two states can be reasonably accounted for in a simple $c\bar{s}$ picture or not. Therefore, the

\(^{1}\)The two largely overlapping resonances, namely a pair of radially excited tensor and scalar $c\bar{s}$ states, might also exist at about 2.86 GeV, as proposed by Van Beveren and Rupp [5].
more complex pictures such as multiquark configuration[9, 10] and two-state structure[5, 8] are not adopted.

Systematic studies on the heavy-light meson spectra in quark models show that the expected $D_s$ masses are about $2.66 \sim 2.8$ GeV for $1^-[D_s(2^3S_1)]$, $2.7 \sim 2.9$ GeV for $1^-[D_s(1^3D_1)]$, $2.8 \sim 3.0$ for $3^-[D_s(1^3D_3)]$, $3.0 \sim 3.2$ GeV for $2^+[D_s(2^3P_2)]$, and $3.1 \sim 3.3$ GeV for $2^+[D_s(1^3F_2)]$, respectively[11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. Only from mass, the $J^P = 1^-$ assignment for the $D_{s1}(2710)$ is strongly favored, consistent with the experiment. It therefore would be most plausible that the $D_{sJ}(2860)$ is assigned to be either a $1^-$ or a $3^-$ state. In the pure S-wave or D-wave $c\bar{s}$ picture, the decay properties of the $D_{s1}(2710)$ and $D_{sJ}(2860)$ have been studied for several different quantum numbers using various approaches[20, 21, 22, 23, 24, 25, 26]. Further theoretical efforts are still required in order to explain all the data for decays of the $D_{s1}(2710)$ and $D_{sJ}(2860)$ satisfactorily[27].

As mentioned above, the pure $D_s(2^3S_1)$ and $D_s(1^3D_1)$ have the same $J^P$ and similar masses, in general, they can mix to produce two physical $1^-$ states lying in the mass region of about $2.66 \sim 2.9$ GeV. Therefore, the observed $D_{s1}(2710)$ is most likely a mixture of the $D_s(2^3S_1)$ and $D_s(1^3D_1)$[20, 28]. If this picture is reasonable, a natural question is whether the $D_{sJ}(2860)$ can be described as the orthogonal partner of the $D_{s1}(2710)$ or not, since in the mass region of about $2.66 \sim 2.9$ GeV, only the $D_{sJ}(2860)$ is a plausible $1^-$ charm-strange candidate at the present time. The masses of the $D_s(2^3P_2)$ and $D_s(1^3F_2)$ are much higher than 2.86 GeV, we therefore will only focus on the $1^-$ and $3^-$ assignments for the $D_{sJ}(2860)$. In the present work we try to clarify (i) the possibility of the $D_{s1}(2710)$ and the $D_{sJ}(2860)$ being in fact the mixtures of the $D_s(2^3S_1)$ and $D_s(1^3D_1)$ and (ii) whether the $D_{sJ}(2860)$ can be assigned to be a $D_s(1^3D_3)$ or not by comparing the $3^P_0$ model predictions for their strong decays with the available experimental data.

The organization of this paper is as follows. In Sec. II, we discuss the strong decays of the $D_{s1}(2710)$ and $D_{sJ}(2860)$ for different possible assignments. The radiative transitions of these

\footnote{In 2004, the $D_{sJ}(2632)$ was reported by the SELEX Collaboration in the final states $D_s^+\eta$ and $D^0K^+$[29]. Various possible interpretations for the $D_{sJ}(2632)$ turn out to be very unlikely[30]. It was even regarded to be an experimental artefact[30, 31]. Therefore, we don’t consider the possibility of the $D_{sJ}(2632)$ being the $1^- c\bar{s}$[32], although its mass is close to the quark model prediction of about 2.66 GeV for the $D_s(2^3S_1)$[17].}
two states are given in Sec. III. The summary and conclusion are given in Sec. IV.

II. Strong decays of the $D_{s1}(2710)$ and $D_{sJ}(2860)$

In the scenario of the $D_{s1}(2710)$ being the mixture of the pure $D_s(2^3S_1)$ and $D_s(1^3D_1)$, the eigenvectors of $D_{s1}(2710)$ and its orthogonal partner $D_{s1}(M_X)$ can be written as

$$|D_{s1}(2710)⟩ = \cos \theta |2^3S_1⟩ - \sin \theta |1^3D_1⟩,$$

(5)

$$|D_{s1}(M_X)⟩ = \sin \theta |2^3S_1⟩ + \cos \theta |1^3D_1⟩,$$

(6)

where the $\theta$ is the mixing angle and $M_X$ denotes the mass of the physical state $D_{s1}(M_X)$. We apply Eqs.(5) and (6) to $c\bar{s}$ rather than $s\bar{c}$ states. When one applies these equations to $s\bar{c}$ states, the mixing angle would be opposite in sign to our $\theta$.

In this work, we shall employ the $^3P_0$ model to evaluate the tow-body open-flavor strong decays of the initial state. The $^3P_0$ model, also known as the quark pair creation model, has been extensively applied to evaluate the strong decays of mesons from light $q\bar{q}$ to heavy $c\bar{b}$, since it gives a considerably good description of many of the observed decay amplitudes and partial widths of hadrons. Some detailed reviews on the $^3P_0$ model can be found in Refs.[33, 34, 35, 36]. Also, the simple harmonic oscillator (SHO) approximation for the meson space wave functions is used in the strong decays computations. This is typical of strong decay calculations and it has been demonstrated that using the more realistic wave functions, such as those obtained from Coulomb, plus the linear potential model, does not change the results significantly[37, 38, 39].

In the $^3P_0$ model, the partial widths of the $D_{s1}(2710)$ and $D_{s1}(M_X)$ can be given by

$$\Gamma(D_{s1}(2710) \to BC) = \frac{\pi P}{4M^2_{D_{s1}(2710)}} \sum_{LS} |\cos \theta \mathcal{M}^{LS}_{D_s(2^3S_1) \to BC} - \sin \theta \mathcal{M}^{LS}_{D_s(1^3D_1) \to BC}|^2,$$

(7)

$$\Gamma(D_{s1}(M_X) \to BC) = \frac{\pi P}{4M^2_{D_{s1}(M_X)}} \sum_{LS} |\sin \theta \mathcal{M}^{LS}_{D_s(2^3S_1) \to BC} + \cos \theta \mathcal{M}^{LS}_{D_s(1^3D_1) \to BC}|^2,$$

(8)

where $B$ and $C$ denote the final state mesons, $P$ is the final state momentum, and $\mathcal{M}^{LS}$ is the partial amplitude. According to the explicit expression for $\mathcal{M}^{LS}$ given by our previous work[40, 41, 42], the input parameters include the light $q\bar{q}$ production strength $\gamma$, the SHO wave function scalar parameters $\beta$, and the constituent quarks masses. The meson effective $\beta$ values used in this work are shown in Table 1. These effective SHO $\beta$ values were obtained
by equating the root mean square radius of the SHO wave function to that obtained from the simple nonrelativistic potential model proposed by Lakhina and Swanson\[^3\]. In this potential model, the zeroth-order Hamiltonian is

$$H_0 = \frac{P^2}{M_r} - \frac{4}{3} \frac{\alpha_s}{r} + 2br + \frac{32\alpha_s\sigma^3 e^{-\sigma^2 r^2}}{\sqrt{\pi}m_q m_\bar{q}} \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}},$$

(9)

where $M_r = 2m_q m_\bar{q}/(m_q + m_\bar{q})$; $m_q$ and $\mathbf{S}_q$ ($m_\bar{q}$ and $\mathbf{S}_{\bar{q}}$) are the mass and spin of the constituent quark $q$ (antiquark $\bar{q}$), respectively; The parameters were chosen to reproduce reasonably the masses of the low lying charm-strange states $D_s$, $D_s^*$, $D_{s1}(2459)$, $D_{s1}(2535)$, $D_{s0}(2317)$, and $D_{s2}(2573)$ and are $\alpha_s = 0.53$, $b = 0.135$ GeV$^2$, $\sigma = 1.13$ GeV. The constituent quarks masses are taken to be $m_u = m_d = 0.33$ GeV, $m_s = 0.55$ GeV, and $m_c = 1.45$ GeV. We take $\gamma = 6.25$ by fitting to the decay $D_{s2}(2573) \rightarrow DK + D^*K + D_s\eta$ \[^4\]. The meson masses used to determine the phase space and final state momenta are $M_K = 496$ MeV, $M_\eta = 548$ MeV, $M_{K^*} = 894$ MeV, $M_D = 1867$ MeV, $M_{D_s} = 1969$ MeV, $M_{D^*} = 2009$ MeV, $M_{D_s^*} = 2112$ MeV, $M_{D_{s2}(2573)} = 2573$ MeV, $M_{D_{s1}(2710)} = 2710$ MeV, and $M_{D_{sJ}(2860)} = 2862$ MeV. The meson flavor functions follow the conventions of Ref.[40], for example, $D_s^+ = -c\bar{s}$, $D^0 = c\bar{u}$, $K^+ = -u\bar{s}$, and $\eta = (u\bar{u} + d\bar{d})/2 - s\bar{s}/\sqrt{2}$.

Table 1: The meson effective $\beta$ values in MeV.

| $n$ $^{2S+1}L_J$ | $u\bar{u}$ | $u\bar{s}$ | $s\bar{s}$ | $c\bar{u}$ | $c\bar{s}$ |
|-----------------|------------|------------|------------|------------|------------|
| $^1S_0$         | 618        | 456        | 476        | 476        | 410        |
| $^3S_1$         | 267        | 296        | 337        | 340        | 408        |
| $^2S_1$         | 226        | 247        | 276        | 276        | 323        |
| $^1P_J$         | 248        | 269        | 299        | 297        | 346        |
| $^3D_J$         | 232        | 251        | 278        | 278        | 318        |

The numerical results for the partial widths of the $D_{s1}(2710)$ based on (7) are listed in Table 2. The variation of these partial widths and the ratio of $D^*K$ to $DK$ widths with the mixing angle $\theta$ is shown in Fig. 1. From the experimental result (3), the mixing angle $\theta$ is found to approximately satisfy (see Fig. 1 (a))

$$1.12 \leq \theta \leq 1.38 \text{ radians.} \quad (10)$$

\[^3\]In the procedure, the Mathematica program\[^4\] is used.

\[^4\]Model: $\Gamma = 20$ MeV, $\Gamma(D^*K)/\Gamma(DK) = 0.11$; PDG[3]: $\Gamma = 20 \pm 5$ MeV, $\Gamma(D^*K)/\Gamma(DK) < 0.33$. 

Table 2: Partial widths of the $D_{s1}(2710)$ and $D_{sJ}(2860)$ as the $D_s(23S_1)$ and $D_s(13D_1)$ mixtures in MeV ( $c \equiv \cos \theta$, $s \equiv \sin \theta$).

| Mode | $D_{s1}(2710)$ | $D_{sJ}(2860)$ |
|------|----------------|----------------|
| $DK$ | $4.4c^2 - 39.1cs + 86.8s^2$ | $63.3c^2 - 8.1cs + 0.3s^2$ |
| $D_s\eta$ | $0.8c^2 - 6.2cs + 12.7s^2$ | $20.2c^2 + 3.9cs + 0.2s^2$ |
| $D^*K$ | $34.9c^2 + 72.1cs + 37.2s^2$ | $37.7c^2 - 45.1cs + 13.5s^2$ |
| $D_s^*\eta$ | $1.4c^2 + 2.9cs + 1.5s^2$ | $8.5c^2 - 12.9cs + 4.9s^2$ |
| $DK^*$ | $37.9c^2 - 108.1cs + 77.1s^2$ | |

$\Gamma_{\text{total}} = 41.4c^2 + 29.6cs + 138.2s^2$ $\Gamma_{\text{total}} = 167.5c^2 - 170.2cs + 96.0s^2$

Figure 1: Partial widths and $\Gamma(D^*K)/\Gamma(DK)$ for the $D_{s1}(2710)$ and $D_{sJ}(2860)$ as $J^P = 1^-$ versus $\theta$. The horizontal dashed lines indicate the upper and lower limits of the BaBar’s data[4].

It is obvious from Fig. 1(b) the total width of the $D_{s1}(2710)$ can be reasonably reproduced in the presence of $1.12 \leq \theta \leq 1.38$ radians, which therefore suggests that the picture of the $D_{s1}(2170)$ being in fact a mixture of the $D_s(23S_1)$ and $D_s(13D_1)$ appears reasonable. The quantitative calculations in a constituent quark model with effective Lagrangians[8] also support this picture.

In the presence of the $D_{s1}(2710)$ corresponding to one physical state in the $D_s(23S_1)$-$D_s(13D_1)$ mixing scenario, from condition (10) we can learn some decay information on another physical state $D_{s1}(M_X)$. Based on (8), the predicted total width and $\Gamma(D^*K)/\Gamma(DK)$ for the $D_{s1}(M_X)$ are shown in Fig. 2 as functions of the initial state mass $M_X$ and the mixing angle $\theta$. The $D_{s1}(M_X)$ mass is restricted to be about $2.66 \sim 2.9$ GeV expected by the quark models.
From Fig. 2, one can see that with the variations of the initial state mass and the mixing angle, the total width of the $D_{s1}(M_X)$ varies from about 40 to 60 MeV. The total width depends weakly on $\theta$ while the ratio of $D^*K$ to $DK$ widths varies dramatically with $\theta$. The predicted width would be helpful to search for and confirm another $1^-$ charm-strange state in about 2.66 $\sim$ 2.9 GeV experimentally.

![Figure 2: Total width and $\Gamma(D^*K)/\Gamma(DK)$ for the $D_{s1}(M_X)$ versus $M_X$ and the mixing angle $\theta$.](image)

We now turn to the possible assignments for the $D_{sJ}(2860)$. If it is a $1^-$ state, as mentioned in Sec. I, most likely it is the orthogonal partner of the $D_{s1}(2710)$. Under this picture, the numerical results for partial widths of the $D_{sJ}(2860)$ are listed in Table 2 and the dependence of these partial widths as well as $\Gamma(D^*K)/\Gamma(DK)$ on the mixing angle $\theta$ is shown in Fig. 1. It is clear from Figs. 1(c) and 1(d) that both the total width and $\Gamma(D^*K)/\Gamma(DK)$ for the $D_{sJ}(2860)$ can be properly reproduced with $1.26 \leq \theta \leq 1.31$ radians, just lying on the range of $1.12 \leq \theta \leq 1.38$ radians. Therefore, in the $^3P_0$ model, the possibility of the $D_{sJ}(2860)$ being in fact the orthogonal partner of the $D_{s1}(2710)$ does exist, if the $D_{sJ}(2860)$ is indeed a $J^P = 1^-$ $c\bar{s}$ state.

On the other hand, if the $D_{sJ}(2860)$ is a $3^-$ state, it would be a natural candidate for the $D_s(1^3D_3)$ based on its mass. In this case, the predicted partial widths are listed in Table 3. The total width and the decay branching ratio fraction between $D^*K$ and $DK$ modes are

$$\Gamma \simeq 67 \text{ MeV}, \quad \frac{\Gamma(D^*K)}{\Gamma(DK)} \simeq 0.8. \quad (11)$$

The predicted ratio of $D^*K$ to $DK$ widths is in agreement with the experiment result (4), and
the total width is also roughly consistent with the datum of $48 \pm 7 \pm 10$ MeV within errors. Therefore, the $D_s(1^3D_3)$ interpretation for the $D_{sJ}(2860)$ also seems acceptable. The recent lattice QCD study also favors this assignment[45].

Table 3: Partial widths of the $D_{sJ}(2860)$ as the $D_s(1^3D_3)$ in MeV.

| $\Gamma(DK)$ | $\Gamma(D_s\eta)$ | $\Gamma(D^*K)$ | $\Gamma(D^*_s\eta)$ | $\Gamma(DK^*)$ | $\Gamma_{\text{total}}$ |
|------------|------------------|----------------|------------------|--------------|------------------|
| 35.6       | 1.6              | 26.8           | 0.6              | 2.7          | 67               |

The available experimental information on the $D_{sJ}(2860)$ is not enough to distinguish the $D_s(1^3D_3)$ assignment from the orthogonal partner of the $D_{s1}(2710)$ interpretation for the $D_{sJ}(2860)$. However, the decay patterns for these two assignments are very different. For example, for the $D_s(1^3D_3)$, $\Gamma(D^*_s\eta)/\Gamma(DK) \simeq 0.02$, $\Gamma(D_s\eta)/\Gamma(DK) \simeq 0.05$, and $\Gamma(DK^*)/\Gamma(DK) \simeq 0.08$, while for the orthogonal partner of the $D_{s1}(2710)$, $\Gamma(D^*_s\eta)/\Gamma(DK) \simeq 0.5$, $\Gamma(D_s\eta)/\Gamma(DK) \simeq 0.9$, and $\Gamma(DK^*)/\Gamma(DK) \simeq 13$. The further experimental search of the $D_{sJ}(2860)$ in the $D_s\eta$, $D^*_s\eta$, and $DK^*$ channels would be crucial to distinguish the above two possible assignments.

III. Radiative Transitions

It is well known that radiative transitions can probe the internal charge structure of hadrons, and therefore they will likely play an important role in determining the quantum numbers and hadronic structures of the $D_{s1}(2710)$ and $D_{sJ}(2860)$. In this section, we shall evaluate the $E1$ transitions widths of the $D_{s1}(2710)$ and $D_{sJ}(2860)$.

The partial width for the $E1$ transitions between the $n^2S+1L_J$ and $n''^2S'+1L'_J$ $c\bar{s}$ states in the nonrelativistic quark model is given by[46, 47]

$$\Gamma_{E1}(n^2S+1L_J \to n''^2S'+1L'_J + \gamma) = \frac{4}{3} \alpha e_Q^2 C_{fi} \delta_{SS'} \left| \langle n''^2S'+1L'_J | r | n^2S+1L_J \rangle \right|^2 \frac{E_\gamma^3 E_f}{M_i},$$

where $e_Q = \frac{2m_c-m_s}{3(m_c+m_s)}$, $\alpha = \frac{1}{137}$ is the fine-structure constant, $E_\gamma$ is the final photon energy, $E_f$ is the energy of the final state $n''^2S'+1L'_J$, $M_i$ is the initial state mass, and the angular matrix element $C_{fi}$ is

$$C_{fi} = \text{Max}(L, L')(2J' + 1) \left\{ \begin{array}{ccc} L' & J' & S \\ J & L & 1 \end{array} \right\}^2.$$
The wave functions used to evaluate the matrix element \( \langle n'2S'+1L'J'|n\bara 2S+1L_J\rangle \) are obtained from the simple nonrelativistic quark model (9). Masses of the final states are \( M_{D_{s1}(2573)} = 2573 \) MeV, \( M_{D_{s0}(2317)} = 2317 \) MeV, \( M_{D_{s1}(2460)} = 2459 \) MeV, and \( M_{D_{s1}(2536)} = 2535 \) MeV. The eigenvectors of the physical states \( D_{s1}(2460) \) and \( D_{s1}(2536) \) are taken to be

\[
\begin{align*}
|D_{s1}(2459)\rangle &= \cos \phi |P_1\rangle + \sin \phi |P_1\rangle, \\
|D_{s1}(2535)\rangle &= -\sin \phi |P_1\rangle + \cos \phi |P_1\rangle,
\end{align*}
\]

where the mixing angle \( \phi = -54.7^\circ \) as Refs.[43, 47]. The resulting \( E1 \) transitions widths of the \( D_{s1}(2710) \) and \( D_{sJ}(2860) \) together with the photon energies are given in Table 4. Without doubt, the experimental studies on the processes such as \( D_{sJ}(2860) \to D_{s0}(2317)\gamma \), \( D_{s1}(2459)\gamma \) and \( D_{s1}(2535)\gamma \) will be helpful to distinguish the \( D_s(1^3D_3) \) assignment from the orthogonal partner of the \( D_{s1}(2710) \) interpretation for the \( D_{sJ}(2860) \), since the decay modes of \( D_s(3P_0)\gamma \), \( D_s(3P_1)\gamma \), and \( D_s(1P_1)\gamma \) are forbidden if the \( D_{sJ}(2860) \) is the \( D_s(1^3D_3) \) while they are allowable if the \( D_{sJ}(2860) \) is the mixture of the \( D_{s1}(2^3S_1) \) and \( D_{s1}(1^3D_1) \).

### Table 4: \( E1 \) transitions widths of the \( D_{s1}(2710) \) and \( D_{sJ}(2860) \). (\( E_\gamma \) in MeV, \( \Gamma \) in keV, \( c \equiv \cos \theta \), and \( s \equiv \sin \theta \).

| Final meson | \( D_{s1}(2710) \text{[5-D mixing]} \) | \( D_{sJ}(2860) \text{[5-D mixing]} \) | \( D_{sJ}(2860) \text{[1^3D_3]} \) |
|-------------|---------------------------------|---------------------------------|---------------------------------|
| \( D_{s2}(2573) \) | \( 0.50c^2 + 0.19cs + 0.02s^2 \) | \( 0.69 - 0.12 \) | \( 0.14c^2 - 1.53cs - 4.13s^2 \) | \( 3.31 - 3.48 \) |
| \( D_{s0}(2317) \) | \( 0.36c^2 - 1.68cs + 0.70s^2 \) | \( 7.80 - 6.07 \) | \( 0.23c^2 - 5.12cs + 3.93s^2 \) | \( 6.52 - 1.04 \) |
| \( D_{s2}(2460) \) | \( 1.10c^2 + 2.05cs + 0.15s^2 \) | \( 1.47 - 1.56 \) | \( 3.48c^2 - 7.50cs + 4.03s^2 \) | \( 1.80 - 2.13 \) |
| \( D_{s1}(2536) \) | \( 0.20c^2 + 0.37cs + 0.17s^2 \) | \( 0.27 - 0.29 \) | \( 0.99c^2 - 2.15cs + 1.16s^2 \) | \( 0.52 - 0.61 \) |

### IV. Summary and conclusion

The \( D_{s1}(2710) \) has the definite \( J^P = 1^- \) and its mass is similar to the quark models expectations for the masses of the pure \( 2^3S_1 \) and \( 1^3D_1 \) \( c\bar{s} \). Therefore, the \( D_{s1}(2710) \) is most likely a mixture of the pure \( 2^3S_1 \) and \( 1^3D_1 \) \( c\bar{s} \) states. We first check this possibility by studying its strong decay properties in the \( 3P_0 \) model. Our calculations do support this possibility. We

\footnote{According to the PDG[3], the well established \( c\bar{s} \) states include the \( D_s(1668), D_s^*(2112), D_{s0}(2317), D_{s1}(2460), D_{s1}(2536), \) and \( D_{s2}(2573) \). The \( 2^3S_1, 1^3D_1, \) and \( 1^3D_3 \) \( c\bar{s} \) are forbidden to decay into \( D_s(3^3S_1)\gamma \) and \( D_s(1^3S_0)\gamma \) based on Eq. (13). Therefore, we only consider the processes where the final states contain the \( D_{s0}(2317), D_{s1}(2460), D_{s1}(2536), \) and \( D_{s2}(2573) \).}
also find the $1^3D_1$ component of the $D_{s1}(2710)$ is large, in agreement with the recent lattice QCD study[45]. The orthogonal partner of the $D_{s1}(2710)$ is predicted to have a width of about 40 $\sim$ 60 MeV. This small decay width maybe result from the node structure in its wave function where the large $2^3S_1$ component is expected to exist. Also, the predicted width of about 40 $\sim$ 60 MeV would be helpful to search for and confirm another 1$^-$ charm-strange state in the mass region of about 2.66 $\sim$ 2.9 GeV experimentally.

The observation of the $D_{sJ}(2860)$ in the $D^*K$ and $DK$ channels makes that it should have $J^P = 1^-, 2^+, 3^-, \cdots$. The constituent quark models predictions for the spectra of higher $c\bar{s}$ states strongly favor that the $D_{sJ}(2860)$ is either a 1$^-$ or a 3$^-$ state.

If $D_{sJ}(2860)$ is a 1$^-$ $c\bar{s}$, its most plausible assignment would be the orthogonal partner of the $D_{s1}(2710)$ based on its mass. In this picture, we evaluate the strong decay pattern of the $D_{sJ}(2860)$. Our results indicate that all the available data on its decays can be well explained. If the $D_{sJ}(2860)$ is a 3$^-$ $c\bar{s}$, from its mass, it would be a good $1^3D_3$ candidate. The predicted decay pattern turn out to be also consistent with the data. Therefore, both of these assignments appear likely in the $^3P_0$ model.

The decay patterns of the above two assignments for the $D_{sJ}(2860)$ are very different. The information on the partial widths of $\Gamma(D_{s\eta})$, $\Gamma(D^*_s\eta)$, and $\Gamma(DK^*)$ is crucial to distinguish these two possibilities. If further measurements refute these two possible assignments for the $D_{sJ}(2860)$, the more complex interpretations such as the tetraquark state or two-state structure would be really necessary. Therefore, the further experimental search of the $D_{sJ}(2860)$ in the $D_s\eta$, $D^*_s\eta$, and $DK^*$ channels is strongly called for.

We don’t consider the possibility of the $D_{sJ}(2860)$ being a $2^3P_2$ or $1^3F_2$ $c\bar{s}$, since its mass is much lower than the quark models predictions for the $2P$ and $1F$ charm-strange mesons. The results from a constituent quark with effective Lagrangians[8] don’t yet favor this possibility.

We also investigate the $E1$ radiative transitions of the $D_{s1}(2710)$ and $D_{sJ}(2860)$. The experimental studies on the $E1$ transitions between $D_{sJ}(2860)$ and $1P$ charm-strange mesons will also be helpful in determining the quantum numbers of the $D_{sJ}(2860)$.

Comparing the predicted masses from the quark models and strong decay properties from
the $^3P_0$ model with the BaBar’s new data on the $D_{s1}(2710)$ and $D_{sJ}(2860)$, we tend to conclude that the $D_{s1}(2710)$ can be identified as a mixture of the $D_s(2^3S_1)$ and $D_s(1^3D_1)$, and the $D_{sJ}(2860)$ could be either the orthogonal partner of the $D_{s1}(2710)$ or the $D_s(1^3D_3)$.

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