Measurement of Aspheric Surfaces Using an Arcuate Region Scanning Method by 2D Laser Displacement Sensor

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Abstract. This study proposes an effective measurement of aspheric surfaces using arcuate region scanning method by 2D laser displacement sensor. In a global coordinate system, the point cloud data of each arcuate region in aspheric surface are obtained successively, coordinated with the rotation of the rotating platform and the transverse movement of the 2D laser displacement sensor in a fixed step size. When scanning different areas, it is necessary to calculate the scanning angle gap and included angle according to different scanning length of arc. Then the aspheric surface error could be calculated through the reconstruction consequence. It can be used for in-situ measurement in the processing stage of aspheric mirrors. We present an experiment compared with the measurement results of PGI to verify the validity of this measurement method.

1. Introduction
With the development of science and technology, optical systems have been widely used in aerospace, weapons, ultraviolet lithography, laser nuclear fusion and remote observation. The inherent defects of traditional spherical mirror in focusing imaging make it difficult to meet the needs of engineering application [1]. Compared with the traditional plane and spherical mirror, aspheric mirrors have more design degrees of freedom. It has the advantages of reducing system weight, simplifying system structure and expanding system function. At the same time, it can not only improve the resolution and improve the working distance of the system, but correct the system aberration [2-3].

In aspheric surface testing, some aspheric surface precision measuring instruments have been successfully developed, such as Taylor Hobson PGI dimension profilometer. The measuring range can be up to 300mm with a resolution of 0.125μm. The Leitz Infinity CMM is mainly used for the measurement of high-precision optical lens and curved surfaces, which has the measurement range of (1200×1000×700) mm and the accuracy of (0.3+L/100) μm. However, these instruments are mainly used in the off-line measurement of optical elements. In the processing process of large-diameter aspheric surface, the installation and adjustment of elements are complex, and the off-line measurement is easy to introduce new errors. Therefore, in-situ measurement technology is proposed and applied to the actual machining process according to the contour scanning method [4-5]. At present, in-situ measurement error compensation technology includes contact and non-contact sampling methods, which are most widely used in compensation processing stage [6-7]. The research...
team of Xi’an Jiaotong University has also carried out research on large-scale aspheric surface testing, including contact and non-contact measurements in the processing stage of large-diameter aspheric surfaces. The high-precision in-situ measurement and compensation of large-diameter aspheric surface are realized by contact measurement method. After four times of compensation, the contour error of 900 mm diameter aspheric surface can be reduced from 40 to 5μm [8-10], as shown in the figure 1.

![Figure 1](image1.png)

Figure 1. The diagrams of contact and non-contact measurement system for large-diameter aspheric of Xi’an Jiaotong University. (a) contact measurement system. (b) non-contact measurement system.

In this paper, an effective measurement of aspheric surfaces using arcuate region scanning method by 2D laser displacement sensor is presented. In a global coordinate system, the point cloud data of each arcuate region of aspheric surface are obtained successively, coordinated with the rotation of the rotating platform and the transverse movement of the 2D laser displacement sensor in a fixed step size. When scanning different areas, it is necessary to calculate the scanning angle gap and included angle according to different scanning length of arc. Then the aspheric surface error could be calculated through the reconstruction consequence. It can be used for in-situ measurement in the processing stage of aspheric mirrors, especially in the slow tool servo machining of off-axis aspheric surface. We use an aspheric to verify the reliability of the method. This paper is structured as follows. In section 2, the measurement principle and method of arcuate region scanning based on line laser scanning sensor are introduced. In order to verify the validity of the proposed method, we study the applicability of the proposed method and carry out experimental verification on an aspheric mirror with a diameter of 50mm in Section 3. The conclusion is given in section 4.

2. Measurement Principle and Methods

2.1. Principle of the Arcuate Region Scanning Method

This measurement method is described in the following three steps:

Step 1: As shown in the figure 2 and figure 3, in a global coordinate system, during the rotation of the rotating platform, the 2D laser displacement sensor is traversed in fixed steps to scan multiple annular areas of the aspheric surface successively. The scanning time of the 2D laser displacement sensor passing through each arc ring area of the aspheric mirror is calculated, and the coordinates (x, y) of each sampling point of the 2D laser displacement sensor are calculated.

![Figure 2](image2.png)

Figure 2. Principle of the arcuate region scanning method.

![Figure 3](image3.png)

Figure 3. Sketch of arcuate regions.
Then, the scanning times of the 2D laser displacement sensor in the process of scanning a single arcuate region are calculated, which is called $M$.

$$M=t \times f = \frac{L \times f}{v}$$  \hspace{1cm} (1)

Where $L$ is the scanning length of arc, $v$ is the speed of rotating platform, the time of 2D laser displacement sensor passing through the scanning area is $t = \frac{L}{v}$, $f$ is the frequency of 2D laser displacement sensor. The other parameters that need to be calculated or known: the width of scanning line of laser scanning sensor is $s$, the number of sampling points is $N$, the distance between the sampling point and the center of the coordinate system is $d_i = d_0 + \Delta d \times i (i = 0, 1, 2, \ldots, N)$, the distance between the front scanning point of the 2D laser displacement sensor and the center of the global coordinate system is $d_0$, the distance between adjacent sampling points is $\Delta d = s / N$. These parameters are prepared for the calculation in Chapter 2.2.

Step 2: The data of the 2D laser displacement sensor is corresponding to the coordinates $(x, y)$ of the sampling points on each arcuate region of the aspheric, then the three dimensional coordinates $(x, y, z)$ of point cloud of each arcuate regions domain are obtained;

Step 3: Then the aspheric surface is reconstructed, and the aspheric surface error is calculated by the point cloud data of aspheric. The deviation of all point cloud data $(x, y, z)$ in Z direction of the i-th measuring point relative to the ideal surface is calculated, called $Z_i (i = 0, 1, 2, \ldots, N)$, the number of measuring points is $N$. The maximum value is $Z_{\text{max}}$, and the minimum value is $Z_{\text{min}}$. Then the value of Peak to Valley $PV = Z_{\text{max}} - Z_{\text{min}}$, and the value of root mean square is $RMS$.

2.2. Coordinate Calculation Method of Sampling Points in Different Arcuate Region

In the process of rotary scanning of circular elements, the scanning time is calculated by the scanning length of arc, which is decided by three different ring arc lengths: front, middle and rear, to avoid losing data and affecting the measurement accuracy, as shown in the figure 4. The following three situations will be introduced respectively:

![Figure 4. Three kinds of scanning length of arc. (a)Situation 1. (b)Situation 2. (c)Situation 3.](image)

Situation 1: The distance between the front scanning point of the 2D laser displacement sensor and the center of the coordinate system is $d_0 \leq \sqrt{\delta^2 + (\phi/2)^2} - s$. The rear arc length of the arcuate region is calculated according to the scanning arc length, as shown in figure 3 (a).

The half of the included angle($\theta$)between the scanning area and the center of the turntable is calculated:

$$\cos \theta = \frac{(d_0 + s)^2 + \delta^2 - (\phi/2)^2}{2\delta(d_0 + s)}$$  \hspace{1cm} (2)
The included angle ($\alpha_j$) between the j-th scanning line and the x-axis is calculated as:

$$\alpha_j = \Delta \alpha \times j - \theta = \frac{2\theta}{M} \times j - \theta = \frac{2\theta v}{L \times f} \times j - \theta = \frac{180v_j}{\pi (d_0 + s)} - \theta$$

(3)

Where, $\Delta \alpha = \frac{2\theta}{M}$, $L = \frac{2\theta \pi (d_0 + s)}{180}$;

The coordinates of the j-th scanning line and the i-th scanning point are calculated as $(x_j, y_j)$:

$$x_j = (d_0 + \Delta d \times i) \cos \alpha_j = (d_0 + \Delta d \times i) \cos \left( \frac{180v_j}{\pi (d_0 + s)} - \theta \right)$$

$$y_j = (d_0 + \Delta d \times i) \sin \alpha_j = (d_0 + \Delta d \times i) \sin \left( \frac{180v_j}{\pi (d_0 + s)} - \theta \right)$$

(4)

Situation 2: The distance between the front scanning point of the 2D laser displacement sensor and the center of the coordinate system is $\sqrt{\delta_i^2 + (\phi/2)^2} < d_0 < \sqrt{\delta_m^2 + (\phi/2)^2}$. The middle arc length of the arcuate region is calculated according to the scanning arc length, as shown in figure 3 (b). The calculation method in situation 1 is adopted. The coordinates of the j-th scanning line and the i-th scanning point are calculated as $(x_j, y_j)$:

$$x_j = (d_0 + \Delta d \times i) \cos \alpha_j = (d_0 + \Delta d \times i) \cos \left( \frac{180v_j}{\pi \sqrt{\delta_i^2 + (\phi/2)^2}} - \theta \right)$$

$$y_j = (d_0 + \Delta d \times i) \sin \alpha_j = (d_0 + \Delta d \times i) \sin \left( \frac{180v_j}{\pi \sqrt{\delta_i^2 + (\phi/2)^2}} - \theta \right)$$

(5)

Situation 3: The distance between the front scanning point of the 2D laser displacement sensor and the center of the coordinate system is $d_0 \geq \sqrt{\delta_m^2 + (\phi/2)^2}$. The front arc length of the arcuate region is calculated according to the scanning arc length, as shown in figure 3 (c). The calculation method in situation 1 is adopted. The coordinates of the j-th scanning line and the i-th scanning point are calculated as $(x_j, y_j)$:

$$x_j = (d_0 + \Delta d \times i) \cos \alpha_j = (d_0 + \Delta d \times i) \cos \left( \frac{180v_j}{\pi d_0 f} - \theta \right)$$

$$y_j = (d_0 + \Delta d \times i) \sin \alpha_j = (d_0 + \Delta d \times i) \sin \left( \frac{180v_j}{\pi d_0 f} - \theta \right)$$

(6)

3. Experiment Verification

In the aspheric processing, especially off-axis aspheric, this method can be realized by carrying an X-direction transverse platform outside the machine and cooperating with the turntable of the machine. The purpose of the experiment is to verify the accuracy of the aspheric surface error calculation of this method. Thus, the experiment is carried out on a mobile platform in this paper. The parameters of aspheric mirror are: $r = 130$mm, $k = -0.01$, aperture $\phi = 50$mm.

The experimental setup is shown in figure 5, which is composed of a mobile platform, a rotating platform and a 2D laser displacement sensor. The mobile platform includes three vertical coordinates (X-axis, Y-axis, Z-axis), but only the X direction of motion is required in this experiment. In addition, the rotating platform is equipped to meet the experimental requirements. Each axis is operated by the servo motor control and the movement along the precision ball track. The spatial positioning error of
the whole system is less than 2 μm, which can meet the requirements of experiment. The employed 2D laser displacement sensor consists of a controller and a LJ-V7076 LDS, which are all produced by Panasonic (Aichi-ken, Japan). The LDS uses a row of blue(405nm) semiconductor laser as the light source. The scanning line consists of 800 points in maximum, and the width of it is 16mm.

Figure 5. The experiment setup.

According to the aperture of the tested element and the length of light knife based on the 2D laser displacement sensor, four areas have been scanned. Through the method introduced in the second chapter of this paper, the surface point cloud of each ring domain is obtained, and we select the data of a cross-section compared with the ideal surface to obtain the error curve, as shown in the figure 6. Otherwise, we measured a cross-section of the tested element by Taylor Hobson PGI 3D, and the measurement results and experiments of PGI are shown in the figure 7.

Figure 6. Point cloud diagram and the error curve of a cross-section. (a) Arcuate region 1. (b) Arcuate region 2. (c) Arcuate region 3. (d) Arcuate region 4. (e) Integrated surface. (f) The error curve of a cross-section of (e).
**Figure 7.** The measurement results and experiment by Taylor Hobson PGI. (a) Measurement result diagram of a cross-section. (b) The measurement experiment using PGI.

Due to the high accuracy and high recognition of Taylor Hobson PGI 3D, this paper compares the measurement results to verify the accuracy of this method. The results are shown in Table 1. We can conclude that the deviation of PV is 7.261 μm and the deviation of RMS is 2.435 μm compared with PGI. It meets the requirements of aspheric surface shape error in rough processing stage, but there is still some way to go compared with the PGI. The calibration of sensor position error and the point cloud splicing between arcuate regions are the later work to improve the measurement accuracy.

| Parameters      | PV (Peak-to-valley value /μm) | RMS (Root Mean Square /μm) |
|-----------------|-------------------------------|----------------------------|
| The paper method| 7.85                          | 2.48                       |
| PGI             | 0.589                         | 0.045                      |

4. Conclusions and Discussion
This study proposes an effective measurement of aspheric surfaces using arcuate region scanning method by 2D laser displacement sensor. According to the experiment, the deviation of PV is 7.261 μm and the deviation of RMS is 2.435 μm compared with PGI. It meets the requirements of aspheric surface shape error in rough processing stage and has high application and popularization value in engineering. It can be used for in-situ measurement in the processing stage of aspheric mirrors, especially in the slow tool servo machining of off-axis aspheric surface. In-situ measurement of off-axis aspheric surface and the improvement of measurement accuracy will be conducted in the future work.

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