Differences in the calculation of reinforcement in reinforced concrete floor slabs, calculated in the LIRA program according to the theories of Wood R H and Karpenko N I

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Abstract. The calculation methodology presented in the program LIRA is based on the theory of academician N I Karpenko about reinforced concrete structures and their deformations. As an additional option for calculating and constructing elements in the program, the method of Wood's theory. At the same time, the results of reinforcement calculated according to these theories are significantly different, while the use of any of these theories is allowed by normative documents binding on the territory of Russia and the final choice of theory depends only on the experience of the engineer. The aim of this work is to identify the features of calculating the distribution of reinforcement according to the theories of Wood’s and Karpenko’s. The results of the study of reinforced concrete floor slabs loaded with a constant uniformly distributed load with articulated support on short sides are obtained. The study showed that reinforcement according to the theories of Karpenko’s and Wood's differs mainly by 10 and in environs of consolidation by 70%. The results were analyzed.

1. Introduction

One of the main works examining the behavior of reinforced concrete structures with cracks in a complex stress-strain state and setting forth methods for calculating such structures is the work of N.I.Karpenko [1].

Modern software systems for the automated calculation and design of reinforced concrete structures make it possible to perform computer and graphical calculations with a high accuracy as an integral model of a structure, its individual units and structures, as well as solving specific problems taking into account the specifics of specific cases. The LIRA computer-aided design system allows such calculations.

The methodology of calculations presented in the program is based on the theories of Karpenko's [1] and Wood's [2] about reinforced concrete structures and their deformations.

Figure 1 shows the difference between the approaches of the theories of Karpenko's and Wood's when accounting the plastic deformation of reinforcement in plates when cracks occur. As can be seen from the figure, Wood's theory suggests that in the cracks of floor slabs, when plastic deformation is achieved, the reinforcement becomes perpendicular to the plane of the crack. In Karpenko's theory, the angle of inclination of the reinforcement is calculated by the formula (1). In fairness, it should be
noted that the theory of the behavior of reinforcing bars in places of plastic bending in cracks of floor slabs was also considered in [3, 4] and others.

**Figure 1.** The difference in the nature of the plastic deformation of the reinforcement of floor slabs in cracks according to Karpenko’s theory (a, b) and Wood’s theory (c). 1 - reinforcement, 2 - line fluidity of reinforcement, 3 - compression zone, 4 - middle line of the plate.

For the reinforcement considered in figure 1, the components of the resistance tensor $M_{Tx}, M_{Ty}, M_{Txy} = 0$.

The components of the tensor of safety have the form

$$ A_x = M_{Tx} - M_x, \quad A_y = M_{Ty} - M_y, \quad A_{xy} = M_{Txy} - M_{xy} = -M_{xy} $$

From $A_x, A_y, A_{xy}$ here can determine the main components of the margin of safety. The condition of fluidity of the reinforcement material corresponds to the vanishing of the tensor of safety margin $A_{min} = 0$.

Using well-known expressions for the main components of the planar tensor, the yield condition is transformed to

$$ (M_{Tx} - N_x z_0 - M_x) (M_{Ty} - N_y z_0 - M_y) - (M_{xy} + N_{xy} z_0)^2 = 0. $$

Numerous experiments performed by various researchers have shown that conditions (1) take on the form (notation, see figure 1). The method of calculating the values $z_x, z_y$ proposed in [3, 4].

As an additional option for calculating and constructing elements in the program, was used the method of Wood’s which is based on the structural analysis of reinforced concrete structures [2]. The main difference between the methods is as follows.

The strength condition for reinforced concrete slabs reinforced with a rectangular mesh along the lower, stretched zone of the slab in accordance with Karpenko’s theory, first proposed by K Johansen has the form [1,5]:

$$ M_u = M_{Tx} \sin^2 \alpha_T + M_{Ty} \cos^2 \alpha_T, $$

(1)

where $M_u$ is the bending moment along the line of fracture (crack), $M_{Tx}, M_{Ty}$ are the ultimate moments of yield strength in the reinforcement, $\alpha_T$ is the angle between the reinforcement located along the x axis and the yield area passing along the fracture line.

K Johansen proposed to determine the angle $\alpha_T$ by experiment. A A Gvozdev [6], who developed the approach of K Johansen, derived a formula for determining the angle of orientation of the plasticity platform. The formula for determining the orientation of the yield plate is given in [1, 7]

$$ \tan \alpha_m = \pm \left( \frac{M_{yy} - M_y}{M_{xx} - M_x} \right)^{1/2}. $$
In the theory of Wood’s [2] applies the idea of a “complete bend” according to which the reinforcement, when flowing in cracks, bends so that it becomes normal to the crack. In this case, in formula (1), the values \( \sin^2 \alpha_x \cos^2 \alpha_x \) are replaced by \( \sin \alpha_x \cos \alpha_x \). This leads to an increase in the cross section of the reinforcement.

The derivation of the Wood equations follows from the fact that when bent moments are vectors, they are combined using the addition of vectors in a manner similar to Moor stress circle concept [8].

The resulting moments can be resolved at any angle of orientation. If as a result moments on the plane exceed the allowable capacity of the plate in this plane, then the plate can succumb to this plane. This theory is called the concept of equivalent moments. A detailed exposition of the foundations of this approach can be found in [9]. In theory of Karpenko’s the selection of longitudinal reinforcement is carried out with a minimum total consumption of reinforcement in the X and Y directions (plate plane), while observing the strength conditions and the requirements of the norms for limiting the width of the opening of normal cracks. The crack opening width is determined in accordance with [10].

2. Methods

The design task is to determine the required amount of reinforcement to ensure the bearing capacity of the structure. The problem is solved in relation to the spatial frame of a building with a foundation slab on an elastic foundation in the LIRA software package. Reinforced concrete slab 1.5 x 6 m in size, 0.2m thick, concrete class - B25, reinforcement class A500, protective layer thickness 0.03m, reinforcing bar pitch during selection 0.2 m. As the boundary conditions, the following is accepted (Figures 2 and 3) the short sides of the plate are freely supported along its entire length, the long sides of the plate are free. The calculation is made for the finite element mesh 0.5x0.5m the long sides of the slab is free. The LIRA program provides two options for solving the problem. The first option is based on the theory of academician Karpenko’s and domestic building regulations. In theory Karpenko’s the selection of longitudinal reinforcement is carried out with a minimum total consumption of reinforcement in the X and Y directions (plate plane), while observing the strength conditions and the requirements of the norms for limiting the width of the opening of normal cracks. The crack opening width is determined in accordance with [7]. The second option uses the theory of Wood’s.

The most important element in the design of structural elements is the selection of reinforcement. The selection of reinforcement in plate elements is carried out taking into account the operation of the reinforcement in the orthogonal direction. In this regard, during operation, the dependence of the selection of reinforcement on the order of supply of the calculated combinations of forces, calculated combinations of loads or forces was revealed. In order to minimize the selected reinforcement in two directions, the combinations are ordered in order of increasing stresses.

The sequence of solving the problem:

1. Creating a plate model
2. Setting the boundary conditions
3. Setting loading options
4. The task of stiffness parameters and materials to the elements of the plate
5. Task loads
6. Generation of tables of the calculated combination of efforts
7. Complete plate calculation
8 View and analyze the results of static calculation
9. View and analyze the results of reinforcement.

3. Results

The calculation results showed that the stress-strain state for the considered theories did not differ, and the reinforcement results had differences. Data on the applied loads and the results of reinforcing the lower face in the direction of the Y axis are given in tables 1 and 2 and figures 2 and 3.
Table 1. Floor loads.

| Name load                        | Standard value, t m⁻² | Overload coefficient, gᵢ | Estimated value, t m⁻² |
|----------------------------------|-----------------------|---------------------------|-----------------------|
| Permanent load (dead weight)     | 0.5                   | 1.1                       | 0.55                  |
| Temporary (useful) load          | 0.8                   | 1.2                       | 0.96                  |

Table 2. The results of the selection of reinforcing the lower face in the direction of the Y axis.

| No. | Parameter                                           | Karpenko's theory, cm² m⁻¹ | Wood's theory, cm² m⁻¹ | Δ, % |
|-----|-----------------------------------------------------|----------------------------|------------------------|------|
| 1   | Maximum area of reinforcement in the span, I-th ultimate state | 9.22                      | 11.1                   | 19   |
| 2   | Maximum area of reinforcement in the support zone, I-th limiting state |                         |                        |      |
| 2.1 | The upper right corner (Figures 2 and 3)            | 1                         | 1                      | 0    |
| 2.2 | Lower right corner                                  | 1.62                      | 2.8                    | 73   |
| 2.3 | Lower right corner                                  | 2.21                      | 3.8                    | 72   |
| 2.4 | Upper left corner                                   | 1.48                      | 2.52                   | 70   |

When choosing reinforcement according to Wood's theory, the reinforcement area in the span exceeds the similar data obtained in Karpenko's theory by an average of 10%. In the support zones, due to the arising tangential stresses, the selection of reinforcement according to Wood's theory overestimates the indicators by an average of 70%.

The use of any of these theories is permitted by documents binding on the territory of the Russian Federation [11 - 13] and the final choice of theory depends only on the experience of the engineer.

Figure 2. The selection of reinforcement according to Karpenko's theory.
4. Conclusions
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As can be seen from the study, the application of the theories considered with almost identical indicators of the stress-strain state has large differences in the parameters of the reinforcement used, therefore, studies in this direction require further consideration to eliminate the contradictions.

The use of any of these theories is permitted by documents binding on the territory of the Russian Federation [11-24] and the final choice of theory depends only on the experience of the engineer. Since both theories have legal circulation on the territory of the Russian Federation, and the application of Wood's theory requires more fittings, it is necessary to identify the reasons for this difference. Identification of the true cause of overuse of reinforcement or its insufficient use will lead to a significant reduction in the cost of construction using reinforced concrete structures or to an increase in the reliability of reinforced concrete building structures, which is undoubtedly important from the point of view of sufficient reliability of building structures, especially residential buildings.

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