ABSTRACT

Flow over arrays of cubes is an extensively studied model problem for rough wall turbulent boundary layers. Not only are cube elements notionally representative of the surface roughness encountered in real engineering applications such as urban canopies and heat exchangers, but a great deal of variation in drag and heat transfer can be realized through simple measures such as changing inter-cube spacing and orientation. While considerable research has been performed in computationally investigating these topologies using DNS and LES, the ability of sublayer-resolved RANS to predict the bulk flow phenomena of these systems is relatively unexplored, especially at high packing densities. Here, RANS simulations are conducted on six different packing densities of cubes in aligned and staggered configurations. The packing densities investigated span from what would classically be defined as isolated, up to those in the $d$-type roughness regime. Three different sublayer-resolved turbulence closure models were tested for each case; a low Reynolds number k-epsilon model, the Menter k-omega SST model, and a full Reynolds stress model. Comparisons of the velocity fields, secondary flow features, and drag coefficients are made between the RANS results and existing LES and DNS results from the literature. There is a high degree of variability in the performance of the various RANS models across all metrics of comparison. The Reynolds stress model however demonstrated the best predictive value in terms of the mean velocity profile as well as drag partition across the range of packing densities.

Keywords: Roughness, DNS, LES, RANS, Cubes, CFD.

1 INTRODUCTION

Rough wall turbulent boundary layers are a common feature in many real engineering applications. At large scales, atmospheric flows interact with buildings to create highly complex turbulent structures around urban canopies [1]. At smaller scales, the performance of internal cooling channels in turbine blades and heat exchangers is directly impacted by the surface roughness, which depends on material properties, wear, and other environmental factors as described in Bons et al. [2]. In order to accurately predict pressure drop and heat transfer in open channel and internal flows, understanding the effects that surface roughness has on the underlying physics is essential.

Investigation of this class of turbulent boundary layer problem was long restricted to experimental observations. Early studies of friction loss in sand-grained roughened pipes by Nikuradse [3] and Moody [4] spawned seventy years of experiments covering a wide range of surface roughness morphologies [5, 6]. These experimental results have allowed for the development of a host of correlations that attempt to predict the effects of deterministic and irregular roughness on the mean flow as a function of the surface statistics [7, 8].

In addition, with the growth of additive manufacturing as a viable alternative to conventional metallurgy, there is active research in the characterization of how the variable and complex roughness of an AM surface affects friction and heat transfer; see Stimpson et al. [9], Kirsch and Thole [10], and Hanson et al. [11].

However, due to the high overhead costs and limited bandwidth of experimental investigation, using CFD as a predictive method for rough wall turbulent boundary layer problems has been turned to as an alternative, although there are still several
shortcomings with this approach. Scale resolving methods such as Direct Numerical Simulation (DNS) or Large-Eddy Simulation (LES) are very computationally expensive at the Reynolds numbers encountered in many real engineering applications, and become more prohibitively so for real rough surfaces. That being the case, simpler geometries are often relied upon.

Cube arrays are a common surrogate for more complex roughness morphologies. Not only are the elements notionally representative of the surface roughness encountered in many real applications, but a great deal of variation in drag and heat transfer can be realized through simple measures such as changing inter-cube spacing or orientation. Cubes also have the added benefit of having been extensively studied in their isolated form, in both experimental and numerical contexts [12–15]. The single-cube data provides a benchmark for the lower limit of packing density. In addition, due to their comparative geometric simplicity and periodicity, these cube arrays have been extensively investigated using DNS and LES.

Experimental results of flow over cubic roughness elements can be found in Cheng et al. [16], Castro et al. [17] and Perret et al. [18] and span low to moderate surface packing densities. Coceal et al. [19] performed DNS on both staggered and aligned cube arrays at 25% packing density, and those results served as a benchmark for many of the following computational investigations. DNS data for a wider variety of packing densities was further reported in Leonardi and Castro [20]. LES results can be found in Inagaki et al. [21], Kono et al. [22], Kanda et al. [23] and Yang et al. [24–26].

While much of the investigation of these roughness arrays has been through the use of turbulence resolving CFD (DNS and LES), the question of whether accurate prediction of the interactions between turbulent fluctuations and the mean flow is necessary to capture bulk performance remains. In order to explore this, Reynolds Averaged Navier-Stokes (RANS) simulations, which do not resolve any turbulence, can be used.

Xie and Castro [27], Cheng et al. [28] and Santiago et al. [29] have explored using RANS on cube geometries with varying success depending upon the packing density and RANS turbulence closure model used. However, existing RANS studies have relied almost exclusively upon wall-modeled forms of the turbulence closure equations, and little attention has been paid to very low packing densities, and very high packing densities.

Here, sublayer-resolved RANS simulations are carried out on a range of 6 packing densities with two different configurations; aligned and staggered, using three well established turbulence models. By comparing steady RANS results to high pedigree LES and DNS, the accuracy and drawbacks of this tool can be assessed, and it can be determined whether prediction of certain mean or secondary flow features require scale resolving tools. If not, the significantly reduced computational cost of RANS compared to LES and DNS could be of benefit for fluid dynamics practitioners, and allow more computationally efficient investigation of internal rough wall flow without sacrificing fidelity.

This paper is organized as follows. First, the different geometric configurations investigated are delineated, and the computational procedures are reported. Then results from a series of numerical studies on multiple arrays of cubic roughness elements at various packing densities are reported, and comparisons are made to existing data. Finally, an assessment of the capabilities and drawbacks of RANS is given and recommendations for further investigation are made.

2 APPROACH

All of the geometric configurations considered in this study are of wall mounted cubes in a half-channel. Two different configurations are considered; Aligned and Staggered. Figures 1 and 2 show top down views of these arrangements respectively. X is the streamwise coordinate, Y is the spanwise coordinate, and Z is the wall normal coordinate. In the aligned configuration of Fig. 1, cubes are each separated in the streamwise and spanwise directions by equivalent distances \( L_x \) and \( L_y \), with the front face perpendicular to the flow direction, which is from left to right. For cubes in the staggered arrangement, as seen in Fig. 2, every other column of cubes is shifted in the spanwise direction by half of the separation distance \( L_y \). This causes the separation distance between cubes with the same spanwise coordinate to have a separation distance of \( 2L_y \). While there are multiple combinations of streamwise and spanwise separation distances that can yield the same packing density, this pairing is the one most commonly used in the literature. The half-channel height is \( L_z \), and can be seen from the isometric view of the topology in Fig. 3.

![FIGURE 1: Top view of Aligned geometry with flow from left to right. Dashed line encloses one period of the array.](image)

By varying the magnitude of the cube separation distances, different packing or surface coverage densities, \( \lambda_p \), are achieved. Surface coverage density is defined as the ratio of area obstructed...
by cubes to the total ground area of the channel. For each configuration, six different packing densities are investigated, 0.08%, 1.0%, 4.4%, 25%, 50% and 70%. For the sake of brevity, this paper often refers to the 0.08%, 1.0%, and 4.4% cases collectively as Low Packing Density cases (LPD), and 25%, 50% and 70% together as the High Packing Density cases (HPD).

At very low packing density, \( \lambda_p < O(3\%) \), wall mounted cubes can be classified as isolated roughness [25]. In this regime, the cubes are separated enough that they have a limited effects on one another, and thus act as isolated or single roughness elements. As packing density increases to more moderate values, cube arrays begin to exhibit k-type roughness behavior. In this regime, the wake of the upstream roughness elements directly affects those downstream, a phenomena which has been termed sheltering [26,30,31]. In this study, the staggered 25% case is in the k-type regime. The aligned 25% case and the rest of the HPD configurations studied are d-type in nature. Here, the packing density is high enough that there is little active momentum exchange between the outer flow above the roughness height, and the flow in the region below and between the elements [6]. In effect, the flow tends to skim over the top of the cubes without significant interaction with the bottom wall.

The height of the wall mounted cube is \( h \) is set equal to 1.0 for all configurations, and the half channel height, \( L_z \), is 3.5\( h \) for the three low packing density cases, and 4.0\( h \) for the three higher packing densities, consistent with Yang et al. [25], Coceal et al. [19], and Xu et al. [32]. These studies have shown that the roughness sublayer, or the region of the flow affected by the presence of the wall mounted roughness is relatively thin, and that the vertical height of the domain in comparison to the roughness height is sufficient to capture the flow.

While there are multiple methods of defining the friction velocity in a rough wall setting, this paper uses the definition given by Eqn. 1.

\[
\tau_f = \sqrt{\frac{f_b L_z}{\rho}}
\]

FIGURE 2: Top view of Staggered geometry with flow from left to right. Dashed line encloses one period of the array.

Here, \( f_b \) is the volumetric body force. The attendant friction Reynolds number is defined as

\[
Re_\tau = \frac{u_\tau L_z}{\nu}
\]

(2)

For the sparse packed cubes, \( Re_\tau = 4735 \), consistent with Yang et al. [25], and \( Re_\tau = 500 \) for all of the high packing density cases, consistent with Coceal et al. [19] and Xu et al. [32]. At these Reynolds numbers, the flow can be considered to be fully-rough at most intermediate packing densities, and becomes transitionally rough at either end of the packing density distribution. Table 1 lists spacing information for all cases. Cases are identified by their case ID, with A for Aligned and S for Staggered, followed by integers indicating packing density.

The RANS simulations were conducted using STAR-CCM+ (v2019.2) [33], a finite-volume CFD package that is widely used in industry and research settings. All cases employed second order spatial discretization to solve the incompressible RANS equations. Flow is driven using a constant volumetric body force \( f_b \), chosen to achieve the desired \( Re_\tau \). In this study, we focus on the effects of employing sublayer resolved forms of the RANS turbulence closure equations. All cases are simulated using three different models. We apply two different widely used two-equation eddy viscosity models; the Menter-k-\( \omega \) SST model [34], and the Lien k-\( \varepsilon \) model [35]. In addition, we apply a sublayer resolved Reynolds Stress Transport (RST) model based on Launder and Shima [36]. RSTs are more computationally expensive than eddy viscosity models as they solve modelled transport equations for the six individual Reynolds stresses themselves (plus a length scale providing dissipation scalar). However they are also more accurate for many flows, in particular where stress anisotropy significantly impacts the mean-flow evolution.
For the pressure strain term, we rely on a two-layer formulation based on Gibson and Launder [37] and Rodi [38]. All three turbulence models are available in STARCCM+, and no changes were made to any of the standard model constants.

One period of the cube array is simulated for each configuration, where each of the four sides of the computational domain is a cyclic boundary condition, and the top of the domain at the half-channel height is a symmetry boundary. The dashed lines in Fig. 1 and Fig. 2 enclose the periodic domain. The cube surface and substrate are treated as no-slip walls. Because the domain is periodic, no specific inflow or far-field boundary conditions are needed for the RANS and turbulence transport equations, which reduces the variability in turbulence model performance uncertainty. A structured wall resolved mesh is used for all cases, with a $y^+$ less than unity in conformance with the low Reynolds number forms of the turbulence models used. Figure 4 shows a cross-section of the A440 grid for reference. The cell count ranges from 200,000 for the A70 case where the packing density is highest, to 6.0 million cells for the S008 case at the other end of the test matrix. A grid study was performed leading to certainty of grid independence in the presented results.

3 RESULTS
3.1 VELOCITY PROFILES

First we examine the velocity profiles for the low packing density arrays. For rough wall turbulent boundary layers, either comprehensive spatial averaging (CSA) or intrinsic spatial averaging (ISA) can be used to represent averaged flow quantities [39]. All averages used herein will be CSA, i.e., with the roughness occupied space included in the spatial average with zero velocity. $U_{xy}$ denotes the streamwise velocity comprehensively averaged in both the streamwise and spanwise directions. Figures 5 and 6 show the CSA of streamwise velocity, normalized by the friction velocity for the three sparse packing densities in aligned and staggered configurations respectively. The LES data from Yang et al. [25] for these cases is also plotted for comparison.

For four of the cases A440, S440, A100 and S100, all three RANS models overestimate the velocity at the channel centerline, although better agreement is observed below the height of the roughness elements ($Z=h=1$). The SST model for these four cases diverges the most from the LES data, with both $k$-$\varepsilon$ and RST more closely reflecting the LES profile. RST performs the best for both A100 and S100, while $k$-$\varepsilon$ performs the best for both A440 and S440.

In the LES of results of the aforementioned four cases, a flattening of the velocity profile above the roughness element is observed. This is caused by strong secondary flow which transports low velocity fluid from below the roughness height to the center of the channel, causing a nearly flat averaged velocity profile above the cube height. While this secondary flow phenomena will be discussed further in the next section, these features have a pronounced impact upon the average profiles, especially above the cube height, with the difference especially striking for A100 case.
For both the aligned and staggered .08% cases, which approach the limit of a smooth channel flow, the SST model results provide the best agreement with the LES data, while both k-ε and RST predict that the velocity deficit caused by the roughness to be greater than what is expected, although the differences between the three turbulence models is the smallest among the low packing density cases. The Log-Law line in Fig. 5 and Fig. 6 is that of a smooth channel.

We now move to the HPD cases. Figure 7 shows the CSA for the S25 RANS results, with DNS from Coceal et al. [19] for comparison. In addition, wall modeled RANS data produced by Santiago et al. [29] is included. Here again, we observe that SST produces a significant center-line velocity overestimation, while both k-ε and RST provide a much more accurate prediction, and outperform the existing wall-modeled results.

Figure 8 depicts the local velocity profiles at a point directly between the cubes (location depicted by the red X included in the sketch). For comparison, both LES and wall modeled RANS data from Xie and Castro [27] are shown as well. Locally, both the wall resolved k-ε and RST velocity profiles conform well to the LES data, and are almost identical to their wall modeled counterparts. Once again, SST performs poorly both above and below the cube height. Turning to the aligned case and Fig. 9, there is a greater spread in the performance of the three models for A25 compared to S25. While SST remains inaccurate and k-ε produces a small over-prediction, RST demonstrates excellent agreement with the DNS results reported in Xu et al. [32].

Figure 10 details the velocity profiles for the other d-type cases. For case A50, the RST performs significantly better than its other RANS counterparts when compared to the DNS of Xu et al. [32]. For both A70 and S70, all three RANS models give very
similar predictions for the profiles above the cubes, although all overestimate the velocity compared to the DNS in the case of A70.

Based on these results, it is clear that the predictive capabilities of RANS is highly sensitive to the turbulence model used. The RST performs the best on average across the case matrix, with SST performing the worst. This tends to support the hypothesis that capturing the effects of turbulence anisotropy is necessary to predict the flow fields produced by these cubes. For the low packing density cases, RANS provides stronger predictive capability below the height of the roughness, with performance deteriorating with increasing height above the roughness. Performance is similar to that of some of the wall-modeled results previously published. In the low packing density region, the LES results show that there is very little difference between the aligned and staggered profiles for a given packing density. This trend is also generally reflected in the RANS results.

In addition, there is no clear correlation between the packing density and the performance of a given turbulence model. The RST for example produces the best results for A25, with accuracy decreasing as it moves to higher or lower packing densities.

### 3.2 SECONDARY FLOW FEATURES

Also of interest in this work is the question of the predictive capability of RANS in the context of secondary flows. Here $U_x$, $V_x$ and $W_x$ are the three components of velocity comprehensively averaged in the streamwise direction. Figures 11, 12, and 13 show streamwise averaged streamwise velocity contours with in plane streamwise averaged velocity vectors for the A100 geometry (velocity vectors are shown on only half the domain for clarity of visualization). The scaling of the in-plane velocity is shown on the left half of the figure. Here again, we observe more evidence of the wide variation in performance between the turbulence models. While the k-$\varepsilon$ model predicts very little in the way of secondary flow, the RST model predicts the presence of strong counter-rotating vortices on either side of the cube. These vortices transfer low momentum fluid from below the cube height to the upper regions of the boundary layer, causing spanwise variation in the streamwise velocity. This also leads to the comparatively lower center-line mean velocity of the RST case observed earlier when compared to the 2-equation models. The maximum in-plane velocity for the RST case is on the order of ten percent of the center-line velocity, which is stronger than either the k-$\varepsilon$ or SST results, and is consistent with the results of Yang et al. [25]. Of added interest here is that while the SST model better reflects the correct secondary flow behavior, this does not reflect in an accurate mean velocity profile.

Further evidence of the difference in the flow fields can be seen in Figure 14, 15, and 16, which depict contours of streamwise velocity on a horizontal plane $3h$ from the wall for A100. Both the SST and RST models predict the presence of a high momentum pathway above the roughness strip ($y=0$), and low momentum pathways between the roughness strips, with the magnitude of the pathways being comparatively larger in the RST case. This is the appropriate trend as evidenced by the LES data (see Yang et al. [25] for corresponding figures), but the opposite is observed for the k-$\varepsilon$ model.

To further examine the relationship between a strong mean flow prediction and the accuracy of secondary flows, we turn to the HPD case A50 RST. Streamwise velocity contours at $X=0$ (the cube centerline) are shown in Fig. 17, with RANS results on the left and DNS from Xu et al. [32] on the right. The black line represents the contour line passing through $Z/h=1.2$ at $Y=0$. There is a weak concavity to this line, suggesting the presence of a weak high momentum pathway above the cube. RANS here slightly overestimates the size of this feature, as the DNS show
more homogeneity in the spanwise direction above the cube. Focusing in on the region below the height of the cube, Fig. 18 depicts contours of streamwise velocity as well as in-plane streamlines at Z/h=.8 for A50 RST. RANS is able to correctly predict the presence of the arch-vortex in the wake of the cubes, but marginally over-predict the streamwise velocity in the trench to either side of the cubes compared to the DNS.

3.3 REYNOLDS AND DISPERSIVE STRESSES

Another area of interest in this work is how accurately the RANS RST model predicts the various components of the Reynolds stress. The XZ components of the comprehensively averaged Reynolds and dispersive stress tensors for A100 RST are plotted in Fig. 19, along with the corresponding LES data from Yang et al. [25]. The RANS data shows excellent agreement with the LES data above the height of the roughness, but less so within the roughness layer itself. Indeed, the RANS dispersive stress is more consistent with that of a higher packing density than one percent, which have positive values below the cube height. Additionally, for much of the domain, the sum of the Reynolds and dispersive stresses (τ^τ+T) very closely matches the theoretical line τ^τ+T = -1 + Z/Lz, which is plotted as grey dashes in Fig. 19.

In the case of a higher packing density arrangement, such as A50, the RST model provides a somewhat different picture. In Fig. 20, we see that RANS over-predicts the normal components of the Reynolds stress compared to the DNS data in Xu et al. [32], although there is excellent agreement with the XZ component. The error in the normal components is especially evident at the height of the cube.
3.4 DRAG PARTITION

The drag partition and drag coefficients are also examined for comparison, as these are often quantities of interest in engineering applications. The drag coefficient is evaluated using Equation 3, where $F$ is the drag force on one wall mounted cube, which includes both the pressure and viscous drag, and $U_h$ is the comprehensive spatial average of $U$ evaluated at the cube height.

$$C_d = \frac{F}{\rho U_h^2 h^2} \quad (3)$$

Figure 21 shows the drag coefficient for all of the sparse cube cases, once again with LES data from Yang et al. [25] used for comparison. Here there are two clear patterns. First, across all cases and turbulence models, RANS generally gives an underestimation of the drag coefficient. This is an artifact of the over-prediction of the velocity profiles, leading to a larger than expected value of $U_h$. The second is that the RST model provides the better prediction for the A/S008 and A/S100 cases, but not for A/S440.

Figure 22 details the ratio of drag force on the channel substrate $F_S$ to the drag force on the cubes $F_D$ as a function of packing density for the aligned HPD cases, with Xu et al. [32] DNS data used for comparison. With increasing packing density, this ratio decreases as the substrate area shrinks and is increasingly sheltered from the outer flow. Here, RST exhibits good agreement with the DNS.

4 CONCLUSION

RANS studies have been performed for a suite of twelve cube roughness array configurations for which DNS and LES data are available. The purpose of these studies was to assess the performance of various wall-resolved closure models, and determine whether their reduced computational cost could be leveraged without sacrificing fidelity. Overall, there was a high degree of performance variability between models and across packing densities for the various performance metrics. While the trends in most cases were reasonable, all of the models were incapable of capturing the wide swath of physics across the test matrix. The RANS RST model had the most consistent performance across the roughness configurations, especially in terms of mean velocity and drag partition. The periodic nature of these geometries...
and lack of dependence on free-stream conditions makes direct comparison of the model performances considerably easier.

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NOMENCLATURE

\[ C_D \] Drag Coefficient
\[ \rho \] Density
\[ D_{ij} \] Dispersive stress tensors ij component
\[ \lambda_p \] Packing density
\[ f_b \] Volumetric Body Force
\[ L_x \] Streamwise Domain Length
\[ L_y \] Spanwise Domain Length
\[ L_z \] Wall-Normal Domain Height
\[ \nu \] Kinematic viscosity
\[ R_{ij} \] Reynolds stress tensors ij component
\[ \tau_{ij} \] Sum of Reynolds and dispersive stress
\[ U_h \] Mean velocity at cube height
\[ U_{xy} \] Comprehensive spatial average in the x-y plane
\[ u_t \] Friction Velocity

REFERENCES

[1] Lien, F. S., and Yee, E., 2004, “Numerical Modelling of the Turbulent Flow Developing Within and Over a 3D Building Array, Part I: A High-Resolution Reynolds-Averaged Navier-Stokes Approach,” Bound-Layer Meteorol, 112(3), pp. 427–466.
[2] Bons, J. P., Taylor, R. P., McClain, S., and Rivir, R., 2001, “The Many Faces of Turbine Surface Roughness,” ASME J. Turbomach., 123(4), pp. 739–748.
[3] Nikuradse, J., 1937, “Law of Flow in Rough Pipes,” NACA, Washington, Technical Memorandum 1292.
[4] Moody, L. F., 1944, “Friction Factors for Pipe Flow,” Trans. ASME, 66, pp. 671–681.
[5] Schultz, M. P., and Flack, K. A., 2008, “Turbulent Boundary Layers on a Systematically Varied Rough Wall,” Physics of Fluids, 21, pp. 015104/1–9.
[6] Jimenez, J., 2004, “Turbulent Flows Over Rough Walls,” Annual Review of Fluid Mechanics, 36, pp. 173–196.
[7] Bons, J. P., 2010, “A Review of Surface Roughness Effects in Gas Turbines,” ASME J. Turbomach., 132(2), pp. 021004/1–16.
[8] Flack, K. A., and Schultz, M. P., 2010, “Review of Hydraulic Roughness Scales in the Fully Rough Regime,” ASME Journal of Fluids Engineering, 132(4), pp. 041203/1–10.
[9] Stimpson, C. K., Snyder, J. C., Thole, K. A., and Mongillo, D., 2016, “Roughness Effects on Flow and Heat Transfer for Additively Manufactured Channels,” ASME J. Turbomach., 138(5), pp. 051208/1–10.
[10] Kirsch, K. L., and Thole, K. A., 2018, “Numerical Optimization, Characterization, and Experimental Investigation of Additively Manufactured Communicating Microchannels,” ASME J. Turbomach., 140(11), pp. 111003/1–11.
[11] Hanson, D. R., McClain, S. T., Snyder, J., Kunz, R., and Thole, K., 2019 “Flow in a scaled turbine coolant channel with roughness due to additive manufacturing” Proceedings of ASME Turbo Expo 2019, Phoenix, Arizona, pp. 1–12.
[12] Macdonald, R. W., Griffiths, R. F., and Hall, D. J., 1998, “An Improved Method for the Estimation of Surface Roughness of Obstacle Arrays,” Atmospheric Environment, 32(11), pp. 1857–1864.
[13] Lim, H., Thomas, T., and Castro, I., 2009, “Flow around a cube in a turbulent boundary layer: LES and experiment,” Journal of Wind Engineering and Industrial Aerodynamics, 97(2), pp. 96–109.
[14] Rodi, W., 1997, “Comparison of LES and RANS calculations of the flow around bluff bodies,” Journal of Wind Engineering and Industrial Aerodynamics, 69-71, pp. 55–75.
[15] Tominaga, Y., and Stathopoulos, T., 2010, “Numerical simulation of dispersion around an isolated cubic building: Model evaluation of RANS and LES,” Building and Environment, 45(10), pp. 2231–2239.
[16] Cheng, H., Hayden, P., Robins, A. G., and Castro, I. P., 2007, “Flow over cube arrays of different packing densities,” Journal of Wind Engineering and Industrial Aerody-
namics, 95(8), pp. 715–740.

[17] Castro, I., Cheng, H., and Reynolds, R., 2006, “Turbulence Over Urban-Type Roughness: Deductions From Wind Tunnel Measurements,” Boundary-Layer Meteorol., 118, pp. 109–131.

[18] Perret, L., Piquet, T., Basley, J., and Mathis, R., 2017, “Effects of plan area densities of cubical roughness elements on turbulent boundary layers,” In ScienceConf, CFM.

[19] Coceal, O., Thomas, T. G., Castro, I. P., and Belcher, S. E., 2006, “Mean flow and turbulence statistics over groups of urban-like cubical obstacles,” Boundary-Layer Meteorol., 121, pp. 491–519.

[20] Leonardi, S., and Castro, I. P., 2010, “Channel flow over large cube roughness: a direct numerical simulation study,” J. Fluid Mech., 651, pp. 519–539.

[21] Inagaki, A., Castillo, M., Yamashita, Y., Kanda, M., and Takimoto, H., 2012, “Large-Eddy Simulation of Coherent Flow Structures within a Cubical Canopy,” Boundary-Layer Meteorol., 142, pp. 207–222.

[22] Kono, T., Tamura, T., and Ashie, Y., 2010, “Numerical Investigations of Mean Winds Within Canopies of Regularly Arrayed Cubical Buildings Under Neutral Stability Conditions,” Boundary-Layer Meteorol., 134, p. 131–155.

[23] Kanda, M., Moriwaki, R., and Kasamatsu, F., 2004, “Large-Eddy Simulation of Turbulent Organized Structure Within and Above Explicitly Resolved Cube Arrays,” Boundary-Layer Meteorol., 112, p. 343–368.

[24] Yang, X. I., and Meneveau, C., 2016, “Large eddy simulations and parameterisation of roughness element orientation and flow direction effects in rough wall boundary layers,” Journal of Turbulence, 17(11), pp. 1072–1085.

[25] Yang, X. I., Xu, H., Huang, X., and Ge, M., 2019, “Drag force on sparsely packed cube arrays,” J. Fluid Mech., 880, pp. 992–1019.

[26] Yang, X. I., Sadique, J., Mittal, M., and Meneveau, C., 2016, “Exponential roughness layer and analytical model for turbulent boundary layer flow over rectangular-prism roughness elements,” J. Fluid Mech., 789, p. 127–165.

[27] Xie, Z., and Castro, I. P., 2006, “LES and RANS for Turbulent flow over arrays of wall-mounted obstacles,” Flow Turbulence Combust., 76, pp. 291–312.

[28] Cheng, Y., Lien, F. S., Yee, E., and Sinclair, R., 2003, “A comparison of large Eddy simulations with a standard k–ε Reynolds-averaged Navier–Stokes model for the prediction of a fully developed turbulent flow over a matrix of cubes,” Journal of Wind Engineering and Industrial Aerodynamics, 91(11), pp. 1301–1328.

[29] Santiago, J. L., Coceal, O., Martilli, A., and Belcher, S., 2008, “Variation of the Sectional Drag Coefficient of a Group of Buildings with Packing Density,” Boundary-Layer Meteorol., 128, p. 445–457.

[30] Raupach, M., 1992 “Drag and drag partition on rough surfaces,” Boundary-Layer Meteorol. 60, pp. 1–25.

[31] Yang, X. I., and Ge, M., 2021, “Revisiting Raupach’s Flow-Sheltering Paradigm,” Boundary-Layer Meteorol., pp. 1–11.

[32] Xu, H., Altland, S., Yang, X., and Kunz, K., 2020, “Flow over closely packed cubical roughness” J. Fluid Mech.

[33] Siemons, 2019. Star CCM+ v 2019.2 User Manual.

[34] Menter, F. R., 1994, “Two-Equation Eddy-Viscosity Turbulence Models for Engineering Applications,” AIAA. 32(8), pp. 1598–1605.

[35] Lien, F., Chen, W., and Leschziner, M., 1996, “Low Reynolds-number eddy-viscosity modelling based on non-linear stress-strain/vorticity relations,” Engineering Turbulence Modelling and Experiments, 3, pp. 91–100.

[36] Launder, B. E., and Shima, N., 1989, “Second-Moment Closure for the Near-Wall Sublayer: Development and Application,” AIAA, 27(10), pp. 1319–1325.

[37] Gibson, M., and Launder, B., 1978, “Ground effects on pressure fluctuations in the atmospheric boundary layer,” J. Fluid Mech., 86, pp. 491–511.

[38] Rodi, W., 1991, “Experience with Two-Layer Models Combining the k-ε Model with a One-Equation Model Near the Wall,” AIAA, 29th Aerospace Sciences Meeting.

[39] Xie, Z., and T., Fuka, V., 2018, “A Note on Spatial Averaging and Shear Stresses Within Urban Canopies,” Boundary-Layer Meteorol., 167, pp. 171–179.