Effects of in-medium vector meson masses on low-mass dileptons from SPS heavy-ion collisions

G. Q. Li\textsuperscript{a}, C. M. Ko\textsuperscript{a}, and G. E. Brown\textsuperscript{b}

\textsuperscript{a}Cyclotron Institute and Physics Department, Texas A\&M University, College Station, Texas 77843, USA
\textsuperscript{b}Department of Physics, State University of New York, Stony Brook, NY 11794, USA

Abstract

Using a relativistic transport model to describe the expansion of the fire-cylinder formed in the initial stage of heavy-ion collisions at SPS/CERN energies, we study the production of dileptons with mass below about 1 GeV from these collisions. The initial hadron abundance and their momentum distributions in the fire-cylinder are determined by following the general features of the results from microscopic models based on the string dynamics and further requiring that the final proton and pion spectra and rapidity distributions are in agreement with available experimental data. For dilepton production, we include the Dalitz decay of \(\pi^0\), \(\eta\), \(\eta'\), \(\omega\) and \(a_1\) mesons, the direct decay of primary \(\rho^0\), \(\omega\) and \(\phi\) mesons, and the pion-pion annihilation that proceeds through the \(\rho^0\) meson, the pion-rho annihilation that proceeds through the \(a_1\) meson, and the kaon-antikaon annihilation that proceeds through the \(\phi\) meson. We find that the modification of vector meson properties, especially the decrease of their mass due to the partial restoration of chiral symmetry, in hot and dense hadronic matter, provides a quantitative explanation of the recently observed enhancement of low-mass dileptons by the CERES collaboration in central S+Au collisions and by the HELIOS-3 collaboration.
in central S+W collisions.
I. INTRODUCTION

The study of the properties of vector mesons in hot dense matter formed in heavy-ion collisions is one of the most exciting problems in nuclear physics. Theoretically, lattice QCD simulations should provide us with the most reliable information about the temperature and/or density dependence of vector meson properties. Currently, these simulations can be carried out only for finite temperature systems with zero baryon chemical potential, using the quenched approximation. From the hadronic correlation functions at large spatial separations the screening masses of hadrons have been obtained in the lattice QCD [1–3]. In a recent paper, Boyd et al [3] have showed that, up to about 0.92Tc, there is no significant change of the rho-meson screening mass. They have also found that the quark condensate does not change very much until one is extremely close to the critical temperature. This is in contrast with the result from chiral perturbation theory which shows a decrease in the quark condensate [4].

Using effective lagrangians that include vector and axial-vector mesons as massive Yang-Mills fields of the chiral symmetry, Song [5] has also found that the rho meson mass at finite temperature remains unchanged at the leading order of the temperature. On the other hand, Pisarski [6] has shown using the gauged linear sigma model that the rho meson mass increases while the a1 meson mass decreases with temperature, and they become degenerate when chiral symmetry is restored.

To the contrary, based on the restoration of scale invariance of QCD for low momentum scales, Brown and Rho [7] and Adami and Brown [8] have shown that the mass of non-strange vector mesons should be reduced in dense matter. This is supported by studies using the QCD sum-rule approach [9,10], the effective hadronic model invoking $\bar{N}N$ polarization [11–13], and the quark-meson coupling model [14].

Experimentally, the properties of vector mesons in nuclear medium can be investigated by measuring the dileptons produced from heavy ion collisions. Since dileptons are not subject to the strong final-state interactions associated with hadronic observables, they are
the most promising probe of the properties of hot dense matter formed in the initial stage of high energy heavy ion collisions. Dileptons have been proposed as useful observables for studying the medium modification of the pion dispersion relation at both finite density and temperature [13,14], the in-medium properties of vector mesons [17–21], and the phase transition from hadrons to the quark-gluon plasma [22–33]. Indeed, our recent study using the relativistic transport model has shown that in heavy ion collisions at SIS/GSI energies significant differences exist between the dilepton spectra with and without medium modifications of the mass and width of vector mesons [21]. In particular, we have found, using the in-medium rho meson mass from the QCD sum-rule calculation, that the rho peak from pion-pion annihilation shifts to around 550 MeV, and its height increases by about a factor of four. A calculation similar to that of Ref. [21] has been carried out in the relativistic quantum molecular dynamics (RQMD) [34], and a shift in the rho meson peak in the dilepton invariant mass spectrum is also seen in heavy ion collisions at AGS/BNL energies.

Dileptons have already been measured at Bevalac/LBL by the DLS collaboration [35] in heavy-ion collisions at incident energies around 1 GeV/nucleon. Theoretical studies have shown that the observed dileptons with invariant masses above about 450 MeV are mainly from pion-pion annihilation [36,37]. Unfortunately, statistics are not good enough in the Bevalac experiments to give definite information on the in-medium vector meson properties. However, similar experiments with vastly improved statistics have been planned at SIS/GSI by the HADES collaboration [38].

For heavy ion collisions at the SPS/CERN energies, hot and dense matter is also formed in the initial stage of the collisions. We expect that medium effects will lead to a shift in vector meson peaks in the dilepton invariant mass spectra from these collisions as well. Experiments from both the HELIOS-3 [39] and the CERES [40] collaboration have shown that there is an excess of dileptons over those known and expected sources which cannot be explained by uncertainties and errors of the normalization procedures [41]. In particular, in the CERES experiment on central S+Au collisions at 200 GeV/nucleon, a significant enhancement of dileptons with invariant masses between 250 MeV to 1 GeV over that from
the proton-nucleus collision has indeed been found.

In a recent Letter [42], using the relativistic transport model for the expansion of the fire-cylinder formed in the initial stage of S+Au collisions, we have shown that the enhancement of low-mass dileptons can be quantitatively explained by the decrease of vector meson masses in hot and dense hadronic matter. In this paper we will expand and improve upon Ref. [42]. We will give the details of our model and the dynamical evolution of the fire-cylinder. The $\eta$, $\eta'$, $\phi$, and $a_1$ mesons that have been omitted in Ref. [42] will be included in our model. The medium modification of the $\eta$ and $\eta'$ mesons are obtained based on their light-quark content in free space, neglecting possible medium modifications of their mixing, while $a_1$ mesons are introduced based on the Weinberg sum rules, which relate the $a_1$ meson mass to that of its chiral partner, the rho meson [43,44]. These additional degrees of freedom are useful to reduce the initial temperature and pion chemical potential introduced in Ref. [42]. For a complete comparison with the experimental data for dileptons with invariant mass below 1 GeV, we will also evaluate the Dalitz decay of $\pi^0$, $\eta$, $\eta'$, $\omega$ and $a_1$ mesons, some of which contribute significantly to dileptons with mass below $2m_\pi$. We will also study the sensitivity of our results with respect to some of the parameters of our model, e.g., the initial abundance of mesons and their rapidity distributions. In Section II, we present the extended Walecka model in which we couple the light quarks of relevant hadrons to the scalar and vector fields in order to provide a consistent way to treat the medium modification of hadrons in the relativistic transport model. In Section III, we present the details of the relativistic transport model and the dynamical evolution of the fire-cylinder. We will also compare our results for particle spectra and rapidity distributions to the experimental data. The formalism for evaluating dilepton emission from the expanding fire-cylinder will be given in Section IV, together with results, discussions and comparisons with other theoretical calculations as well as with the experimental data from both the CERES and the HELIOS-3 collaboration. Section V is devoted to some general discussions. The paper ends with a brief summary in Section VI.
II. THE EXTENDED WALECKA MODEL

To study consistently the effects of dropping vector meson masses on the dilepton spectrum in heavy-ion collisions, we need a model for the in-medium vector meson masses that can be incorporated into the relativistic transport model to describe the dynamics of heavy-ion collisions. For this purpose we extend the Walecka model \footnote{45} from the coupling of nucleons to the scalar and vector fields to the coupling of light quarks to the scalar and vector fields, using the ideas of the meson-quark coupling model \footnote{14} and the constituent quark model. Recently, Gelmini and Ritzi \footnote{46}, Brown and Rho \footnote{47}, and Furnstahl et al. \footnote{48} have been able to obtain the Walecka mean field model from a chiral Lagrangian. Although the latter work appears to be quite different from the former two, the anomalous dimension of the loop correction is chosen in \footnote{48} to effectively linearize the Lagrangian as for Gelmini and Ritzi \footnote{46} and Brown and Rho \footnote{47}. We find that these works give us a firm basis for our formulation.

We consider a system of baryons (we take here the nucleon as an example), pseudoscalar mesons (\(\pi\) and \(\eta\) mesons), vector mesons (rho and omega mesons), and the axial-vector meson (\(a_1\)) at a temperature \(T\) and a baryon density \(\rho_B\). The nucleons couple to both the scalar and the vector field, while the nonstrange mesons other than the pion couple only to the scalar field, since in the quark model they are made of a quark and an antiquark which couple oppositely to the vector field. In the mean-field approximation, the thermodynamical potential is then given by \footnote{49}

\[
\Omega(T, \mu_B) = \frac{1}{2} (m_S^2 \langle S \rangle^2 - m_V^2 \langle V \rangle_0^2) - T \left\{ \frac{4}{(2\pi)^3} \int d^3 k \ln \left[ \exp\left(-\frac{E_N^* - \mu_B}{T}\right) + 1 \right] \right.
\]
\[
+ \frac{4}{(2\pi)^3} \int d^3 k \ln \left[ \exp\left(-\frac{E_N^* + \mu_B}{T}\right) + 1 \right] + \frac{3}{(2\pi)^3} \int d^3 k \ln \left[ \exp\left(-E_\pi/T\right) - 1 \right]
\]
\[
+ \frac{1}{(2\pi)^3} \int d^3 k \ln \left[ \exp\left(-E_\rho^* / T\right) - 1 \right] + \frac{9}{(2\pi)^3} \int d^3 k \ln \left[ \exp\left(-E_\rho / T\right) - 1 \right]
\]
\[
+ \frac{3}{(2\pi)^3} \int d^3 k \ln \left[ \exp\left(-E_{a_1}^* / T\right) - 1 \right] + \frac{9}{(2\pi)^3} \int d^3 k \ln \left[ \exp\left(-E_{a_1} / T\right) - 1 \right] \right\},
\]

(1)

where \(\langle S \rangle\) and \(\langle V_0 \rangle\) are the scalar and vector mean fields, respectively, and the baryon chemical potential \(\mu_B\) is determined by the baryon density
\[
\rho_B = -\frac{\partial \Omega}{\partial \mu_B} = \frac{4}{(2\pi)^2} \int d\mathbf{k} \left[ \frac{1}{\exp((E_N^*-\mu_B)/T) + 1} - \frac{1}{\exp((E_N^*+\mu_B)/T) + 1} \right].
\]

In the above expressions, we have \( E_N^* = \sqrt{k^2 + m_N^2} \) and similar expressions for \( E_\eta^*, E_\rho^*, E_\omega^* \) and \( E_{a_1}^* \). Since the medium modification of pions is neglected in this study, we have \( E_\pi = \sqrt{k^2 + m_\pi^2} \).

The scalar mean field is determined self-consistently from \( \partial \Omega / \partial \langle S \rangle = 0 \), i.e.,

\[
m^2_S(S) = \frac{4g_S}{(2\pi)^3} \int d\mathbf{k} \frac{m^*_{N}}{E^*_N} \frac{1}{\exp((E^*_N-\mu_B)/T) + 1} + \frac{1}{\exp((E^*_N+\mu_B)/T) + 1} \\
+ \frac{0.45g_S}{(2\pi)^3} \int d\mathbf{k} \frac{m^*_{\eta}}{E^*_\eta} \frac{1}{\exp(E^*_\eta/T) - 1} + \frac{6g_S}{(2\pi)^3} \int d\mathbf{k} \frac{m^*_{\rho}}{E^*_\rho} \frac{1}{\exp(E^*_\rho/T) - 1} \\
+ \frac{2g_S}{(2\pi)^3} \int d\mathbf{k} \frac{m^*_{\omega}}{E^*_\omega} \frac{1}{\exp(E^*_\omega/T) - 1} + \frac{6\sqrt{2}g_S}{(2\pi)^3} \int d\mathbf{k} \frac{m^*_{a_1}}{E^*_{a_1}} \frac{1}{\exp(E^*_{a_1}/T) - 1},
\]

where we have used the constituent quark model relations for nucleon and vector meson masses \[14\], i.e.,

\[
m^*_N \approx m_N - g_S(S), \quad m^*_\rho \approx m_\rho - (2/3)g_S(S), \quad m^*_\omega \approx m_\omega - (2/3)g_S(S),
\]

the quark structure of \( \eta \) meson \((\approx 0.58(u\bar{u} + d\bar{d}) - 0.57s\bar{s}) \) \[50\] in free space which leads to

\[
m^*_\eta \approx m_\eta - 0.45g_S(S),
\]

and the Weinberg sum rule relation \((m_{a_1} \approx \sqrt{2}m_\rho)\) between the rho-meson and \( a_1 \) meson masses \[13,14\], i.e,

\[
m^*_{a_1} \approx m_{a_1} - (2\sqrt{2}/3)g_S(S).
\]

We have thus neglected in the present model the possible change of \( \eta \) (also \( \eta' \)) properties at high temperature due to the Debye-type screening of instanton effects, which has recently been studied in Ref. \[51,52\].

The coupling constants \((g_V/m_V)\) and \((g_S/m_S)\) are determined by the nuclear matter ground state properties. We use here the values corresponding to the original Walecka model \[15\] and solve the self-consistent condition to determine baryon and meson masses at a given baryon density and temperature. The results for the effective nucleon and rho meson...
masses are shown in Fig. 1. It is seen that for a temperature around 165 MeV and a baryon
density of about 0.4 fm$^{-3}$ that correspond to the initial conditions of our fire-cylinder, the
rho meson mass is reduced to about 270 MeV. We would like to emphasize that the effects
of dropping vector meson masses on the dilepton spectrum do not depend very much on the
particular model one uses to describe the in-medium vector meson properties, as long as all
models give similar density and temperature dependence for the vector meson masses. For
example, Cassing et al. [53] have obtained similar results as ours [12] by assuming that the
vector meson masses in dense matter are given by that from the QCD sum-rule calculations
[9]. The choice of the present model is that it offers a consistent way to treat the change of
vector meson masses during heavy-ion collisions through the change of the scalar potential.
In this way, the total energy of the system is always conserved. Note that with a stronger
medium effect in the original Walecka model we have been able to lower the initial baryon
density substantially from that in Ref. [12], where the nonlinear Walecka model with scalar
meson self-interactions has been used. The original high baryon density in Ref. [12] gave an
unrealistically high density of hadrons.

The above formalism can be generalized to include strange hadrons, such as $K$, $\bar{K}$, and
hyperons that contain also light quarks. This is achieved by allowing the scalar and vector
fields to couple to the light quarks in these hadrons. In our model, the masses of these
particles are thus also reduced at finite temperature and density. Since the phi meson does
not consist of light quarks, its mass is not modified in the present study.

**III. THE EXPANDING FIRE-CYLINDER**

**A. Initial conditions**

Dileptons are produced in all stages of heavy-ion collisions at the SPS/CERN energies.
Since in this study we are chiefly concerned with dileptons with mass below 1 GeV, the
initial Drell-Yan processes, that contribute mainly to high-mass dileptons, can be safely
neglected. Furthermore, contributions from the quark-gluon-plasma, if it is formed in heavy-ion collisions at the SPS/CERN energies, to low-mass dileptons as well as low-momentum photons, have been found to be insignificant as compared to those from the hadronic phase \[54,55\]. We thus start our transport model from the expansion stage of heavy-ion collisions, which might be in the chiral restoration phase as indicated by the decrease of hadron masses. Very roughly speaking, this corresponds to the mixed phase in descriptions which do not employ medium-dependent masses. The initial hadron abundance and their momentum distributions in this expanding fire-cylinder are constrained by requiring that the final proton and pion spectra and rapidity distributions reproduce the measured ones. Furthermore, results from microscopic simulations based on the string dynamics will also be used to guide us in parametrizing the initial conditions \[56–58\]. We neglect in the present work the isospin asymmetry of the colliding nuclei and assume that all charged states of hadrons are equally populated.

1. **Baryon abundance**

Let us first discuss the initial baryon density in the fire-cylinder. For a central S+Au collision, simple geometrical estimates would give about 90 nucleons from the target nucleus if all projectile nucleons are participants. Of course, the number of participant nucleons from the target nucleus should be smaller in heavy-ion collisions at the SPS energy of 200 GeV/nucleon as a result of partial transparency and the not-exactly-zero impact parameter. From the experimental data on proton rapidity distributions in the mid- to projectile-rapidity region, which have been measured by the NA35 collaboration for S+Au collisions \[59\] and recently by the NA44 collaboration for S+Pb collisions \[60\], the total baryon number in the fire-cylinder can actually be determined. To describe reasonably the proton rapidity distribution, we find that the baryon number in the initial fire-cylinder is about 100 (32 from the projectile and 68 from the target). The center-of-mass rapidity of the fire-cylinder is then 2.65, and this is consistent with the pion rapidity distribution measured by the
NA35 collaboration which shows a broad peak around \( y = 2.6-2.7 \) \[61\]. We include all baryon resonances with masses below 1720 MeV and also the low-lying hyperons, i.e., \( \Lambda \), \( \Sigma \), \( \Lambda(1405) \) and \( \Sigma(1385) \). In our model, the hyperons couple to the scalar and vector fields with \( 2/3 \) of the strength of non-strange baryons, since they contain 2 light quarks instead of 3 in non-strange baryons. Their initial abundances are determined by assuming strangeness saturation as in Refs. \[62\]. Thus knowing the initial baryon density, these abundance can be uniquely determined.

Initially, all baryons are distributed in a fire-cylinder whose cross section is taken to be about 40 fm\(^2\), similar to the geometrical cross section of the projectile nucleus. If we further assume that the initial baryon density is about \( 2.5\rho_0 \) (\( \rho_0 = 0.16 \text{ fm}^{-3} \)), then the initial longitudinal length \( 2z_L \) of the fire-cylinder is found to be 6.2 fm. The initial volume of the fire-cylinder is thus about 250 fm\(^{-3}\).

2. Meson abundance

For mesons, we include \( \pi, \eta, \eta', \rho, \omega, \phi \), and \( a_1 \), as well as \( K \) and \( K^*(892) \). In the earlier stage of heavy-ion collisions preceding our initial fire-cylinder, mesons are expected to be copiously produced either from string fragmentation as in the RQMD \[57,58\] or from the hadronization of a quark-gluon plasma as in \[63\]. They can also be produced from processes like nucleon-antinucleon annihilation in a chirally restored phase \[64\]. The initial abundance of (non-strange) mesons (mainly \( \pi, \rho, \omega \) and \( a_1 \) that feed appreciably to the final pion multiplicity) in the fire-cylinder is determined by requiring that the final pion number agrees with the measured value and will be specified later.

For the meson properties in hot and dense hadronic matter, we will use two models. In the first model, all mesons are taken to have free masses, and their abundances are determined by assuming that they are in thermal and chemical equilibrium, i.e., the \( \pi, \rho, \omega \), and \( a_1 \) chemical potentials are related by \( \mu_\rho = 2\mu_\pi \) and \( \mu_{a_1} = \mu_\omega = 3\mu_\pi \). At a given temperature and pion chemical potential, we can determine the meson densities, and from which the meson
multiplicities are calculated using the initial volume of the fire-cylinder as determined above. Normally, one would expect that $\mu_\pi = 0$ so that pions are also in chemical equilibrium with baryons, which is indeed the case at the freeze-out as shown in Ref. [22]. However, we find that in order to reproduce the observed pion rapidity distribution and spectrum requires an initial pion chemical potential of about 130 MeV at an initial temperature of 165 MeV. This temperature is determined in our model by fitting the slopes of the proton and pion transverse momentum spectra after the full dynamical evolution of the system. Thus, in the initial stage of our simulation, mesons are out of chemical equilibrium with baryons, and they approach chemical equilibrium as the system expands. Microscopic simulations based on the RQMD show that in the central region of heavy-ion collisions the pion density after initial string fragmentations is about 0.4 fm$^{-3}$, and the pion temperature is about 170 MeV [65]. This density is nearly double the equilibrium one and indicates an appreciable pion chemical potential for those pions which come from string fragmentation. The initial $\pi$, $\rho$, $\omega$ and $a_1$ meson numbers are then determined to be 119, 56, 43, and 10, respectively.

Because of finite decay width, the vector meson mass is given the following distribution

$$P(M) \sim \frac{(m\Gamma)^2}{(M^2 - m^2)^2 + (m\Gamma)^2}, \quad (7)$$

with a proper normalization. In the above, $m$ denotes the centroid mass of the meson while $\Gamma$ is its width evaluated at a mass $M$.

For rho and phi mesons, their decay widths $\rho \rightarrow \pi \pi$ and $\phi \rightarrow K\bar{K}$ are given, respectively, by [21]

$$\Gamma_{\rho \rightarrow \pi \pi}(M) = \frac{g_{\rho\pi\pi}^2 (M^2 - 4m_\pi^2)^{3/2}}{4 \pi 12M^2}, \quad (8)$$

and

$$\Gamma_{\phi \rightarrow K\bar{K}}(M) = \frac{g_{\phi K\bar{K}}^2 (M^2 - 4m_K^2)^{3/2}}{4 \pi 6M^2}. \quad (9)$$

where $g_{\rho\pi\pi}^2/4\pi \approx 2.9$ and $g_{\phi K\bar{K}}^2/4\pi \approx 1.7$ are determined from the measured widths at $m_\rho \approx 770$ MeV and $m_\phi \approx 1020$ MeV, respectively.
The decay width for \( K^* \to K\pi \) has been derived in Ref. [82] to be
\[
\Gamma_{K^* \to K\pi}(M) = \frac{g_{K^*K\pi}^2}{4\pi} \frac{2k^3}{M^2},
\]
(10)
where \( k \) is the momentum of the pion in the center-of-mass frame of \( K^* \), and \( g_{K^*K\pi}^2/4\pi \approx 0.86 \) is determined from the measured \( K^* \) decay width at \( m_{K^*} \approx 892 \) MeV. For the decay width of \( a_1 \to \pi\rho \), we use the result of Ref. [78], i.e.,
\[
\Gamma_{a_1 \to \pi\rho} = \frac{G_{a_1\pi\rho}^2}{24\pi m_{a_1}^2} \left[ 2(p_\pi \cdot p_\rho)^2 + m_\rho^2(m_\pi^2 + k^2) \right],
\]
(11)
where \( k \) is the pion momentum in the rest frame of \( a_1 \), and \( G_{a_1\pi\rho} \approx 14.8 \) GeV\(^{-1} \) is determined from the \( a_1 \) decay width in free space using its centroid mass. There is, unfortunately, no simple expression for the decay width of \( \omega \to \pi^+\pi^-\pi^0 \). We use in this work the approximation that this width is directly proportional to the mass of the omega meson, which becomes exact in the chiral limit of \( m_\pi \to 0 \).

Since we will consider the \( \eta \) and \( \eta' \) Dalitz decay contributions to the dilepton yield, their initial multiplicities need to be determined. As for pions, the best way is to fit the experimental data after dynamical evolution of the system. There are published data from the WA80 collaboration for minimum-biased events and preliminary data for central events in S+Au collisions at 200 GeV/nucleon [67–69]. Both are given in terms of \( \eta/\pi^0 \) ratio as a function of the transverse momentum. We thus determine the initial \( \eta \) and \( \eta' \) number mainly by fitting the final \( \eta/\pi^0 \) ratio to the measured one. In the chemical equilibrium scenario for the meson chemical composition, both the \( \eta \) and \( \eta' \) chemical potentials are assumed to be about three times that of the pion. The large eta chemical potential is also consistent with that of RQMD calculations [65]. This gives us about 37 \( \eta \) mesons and 5 \( \eta' \) mesons in the initial fire-cylinder. The final \( \eta \) meson number at the freeze-out is about 32 (see Fig. 6 below). This includes the primary \( \eta \) mesons that have not been absorbed by nucleons or converted into pions through \( \eta\eta \to \pi\pi \), \( \eta \) mesons from \( N^*(1535) \) which decay into \( \eta N \) with a branching ratio of about 0.4 [70], \( \eta \) mesons from \( \eta' \) which decays into \( \pi\pi\eta \) with a branching ratio of about 2/3 [70], and \( \eta \) mesons produced from \( \pi\pi \to \eta\eta \). We find that the
experimental $\eta/\pi^0$ ratio is reasonably reproduced in this case (see Fig. 10 below). As we will show later, in CERES experiments the $\eta'$ contribution to dileptons is unimportant but that from $\eta$ dominates the dilepton spectrum at invariant mass between $150 < M < 350$ MeV.

In the second model for the meson properties, we take into account the effects of medium modifications according to that predicted by the extended Walecka model as discussed in the previous section. At an initial temperature of 165 MeV and an initial baryon density of $2.5\rho_0$, we find that the rho and omega meson masses reduce to about 270 MeV, while the mass of the $a_1$ meson reduces to about 550 MeV. As a result, their abundance in the initial fire-cylinder increases, and we need essentially a zero pion chemical potential in order to reproduce the experimental pion rapidity distribution and transverse momentum spectrum. We expect a similar trend in RQMD as a part of the energy is now converted to the field energy, so less energy is available for pion production, which is, however, compensated by an increase of rho, omega, and $a_1$ production due to reduced masses. The initial $\pi$, $\rho$, $\omega$, and $a_1$ meson numbers are then determined to be 44, 92, 33, and 26, respectively. The initial low-mass vector and axial-vector mesons thus act as a reservoir for the observed large pion yield. As in the free mass case, the mass distributions for these mesons are given by Eq. (7) with the centroid mass in the equation replaced by the in-medium one.

In the extended Walecka model, the masses of $\eta$ and $\eta'$ (using $\eta' \approx 0.40(\bar{u}u + \bar{d}d) + 0.82s\bar{s}$), we have $m_{\eta'}^* \approx m_{\eta'} - 0.2\langle S \rangle$ [50]) in the initial stage are reduced to about 210 and 810 MeV, respectively. Because of the large strange quark content in $\eta'$, the reduction of its mass is less significant than that of the $\eta$ meson mass as also noted in Ref. [51]. When the effective masses of $\eta$ and $\eta'$ are reduced in hot dense matter, either by the screening of instanton effects as in Refs. [51,52] or by the attractive scalar field as in our model, their initial chemical potentials can be significantly reduced. We are, however, unable to reduce them to zero as for the pion chemical potential due to the smaller contribution to $\eta$ production from higher meson resonances than to pion production. The inclusion of $U_A(1)$ restoration [51,52] and $a_0(980)$, that decays dominantly into $\eta\pi$, will modify this. In addition, since the
eta meson mass is reduced in hot dense matter, the process of eta production in pion-pion interaction $\pi\pi \rightarrow \eta\eta$ is favored over the reverse one. We find that in order to describe well the experimental $\eta/\pi^0$ ratio (see Fig. 10 below), we need initially an eta chemical potential of about 130 MeV. The initial $\eta$ and $\eta'$ numbers are then found to be 26 and 3, respectively, and the final $\eta$ number at the freeze-out is about 32.

3. Momentum distributions

The initial transverse momenta of all hadrons are assumed to be given by a thermal distribution, with a proper slope parameter that is loosely referred to as ‘temperature’ in the transverse direction. We find that an initial temperature of about 165 MeV is needed to reproduce the observed slopes of the transverse mass spectra for both protons and pions. This is somewhat lower than that used in Ref. [42] as a result of the inclusion of additional degrees of freedom. Introducing more degrees of freedom should lower it further. The longitudinal momentum distribution is determined by imposing a rapidity field as in the hydrodynamical model [71,72]. Specifically, we assume that the rapidity of a particle in the fire-cylinder frame is correlated to its longitudinal position via the following gaussian function, i.e.,

$$f(y, z) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y - y_L z/z_L)^2}{2\sigma^2}\right). \quad (12)$$

For mesons, RQMD simulations show that their initial rapidity distributions depend on the mass, with pions (the lightest) having the broadest rapidity distribution [65]. For simplicity, we will use the same $y_L$ and $\sigma$ in Eq. (12) for all mesons. We find that with $y_L = 0.6$ we have a good description of the experimental pion rapidity distribution, particularly in the mid-rapidity region (see Fig. 8 below), if the variance $\sigma$ is chosen to be 0.8 (also for baryons) so that there is substantial dispersion in the $y$-$z$ correlation. Thus, mesons are initially situated mostly around the mid-rapidity but becomes broader at the freeze-out due to interactions, leading to good agreements with the experimental data in the mid-rapidity region.
We note that the initial rapidity distributions in our model is consistent with a momentum distribution that is in local thermal equilibrium at a temperature of about 165 MeV. The introduction of a rapidity field, however, breaks the equilibrium between the longitudinal and transverse momentum distributions. This is again consistent with that from the RQMD [65].

Baryons, on the other hand, show two components in the initial rapidity distribution [73]. The lower component, with \( y_L = 1.0 \), is similar to the mesons, while the upper component, due to limiting fragmentation, has a larger \( y_L = 2.0 \). The initial baryon rapidity distribution is thus broader than that of mesons. This is necessary in order to fit the observed broad proton rapidity distribution, as the proton rapidity distribution is hardly changed by their interactions with other particles as a result of their large mass. In Fig. 2, we show the initial rapidity and transverse momentum distributions of protons and negative pions for the case of free masses.

B. The relativistic transport model

The expansion of the fire-cylinder is treated by the relativistic transport model [74,75]. In this model, hadrons (except for pions whose medium modifications are neglected in present work) are propagated in their mean-field potential. For a nucleon, this is

\[
\frac{dr}{dt} = \frac{k}{E_N^*}, \quad \frac{dk}{dt} = -\nabla_r (E_N^* + (g_V/m_V)^2 \rho_B),
\]

(13)

Similar equations are used for baryon resonances with their mean-field potentials taken to be the same as the nucleon one. Furthermore, particles also undergo stochastic two-body collisions. For baryon-baryon interactions, we include both elastic and inelastic scattering for nucleons, \( \Delta(1232) \), \( N(1440) \) and \( N(1535) \). Their cross sections are either taken from Refs. [37,76] or obtained using the standard detailed-balance procedure [77]. Lacking empirical information, we consider only elastic scattering for interactions involving higher baryon resonances and hyperons. This is a reasonable approximation as nucleon, \( \Delta(1232) \),
N(1440) and N(1535) account for about 80% of all baryons. Moreover, in our thermalized fire-cylinder, the energy of a colliding baryon pair is not very large, so higher resonance production from baryon-baryon interaction is unimportant. Moreover, their production cross sections in baryon-baryon interactions are about an order of magnitude smaller than those in meson-baryon interactions, which are included in our model through resonance formation and decay. For example, the interaction of a pion with a nucleon proceeds through the formation of Δ(1232), N(1440), · · · , N(1720). The formation cross sections are taken to be of the relativistic Breit-Wigner form, e.g., the isospin-averaged cross section

\[ \sigma_{\pi N \to \Delta(\sqrt{s})} = \frac{16\pi}{3k^2} \frac{(m_{\Delta} \Gamma_{\Delta})^2}{(s - m_{\Delta}^2)^2 + (m_{\Delta} \Gamma_{\Delta})^2}, \]

where \( k \) is the pion momentum in the center-of-mass frame of the Δ particle.

The meson-meson interactions are either formulated by the resonance formation and decay when the intermediate meson is explicitly included in our model or treated as a direct elastic scattering with a constant cross section estimated from various theoretical models.

For a pair of pions with a total invariant mass \( M \), a rho meson of this mass is formed with an isospin-averaged cross section given by the Breit-Wigner form [21],

\[ \sigma_{\pi\pi \to \rho(M)} = \frac{8\pi}{k^2} \frac{(m_{\rho} \Gamma_{\rho})^2}{(M^2 - m_{\rho}^2)^2 + (m_{\rho} \Gamma_{\rho})^2} \frac{M}{m_{\rho}^2}, \]

where \( k \) is the pion momentum in the center-of-mass frame of rho meson, and the multiplying factor \((M/m_{\rho})^2\) will be clarified later. Similarly, for \( \pi\rho \to a_1 \) and \( \pi K \to K^* \), the isospin-averaged cross sections [78,79] are, respectively,

\[ \sigma_{\pi\rho \to a_1(M)} = \frac{4\pi}{3k^2} \frac{(m_{a_1} \Gamma_{a_1})^2}{(M^2 - m_{a_1}^2)^2 + (m_{a_1} \Gamma_{a_1})^2}, \]

and

\[ \sigma_{\pi K \to K^*(M)} = \frac{4\pi}{k^2} \frac{(m_{K^*} \Gamma_{K^*})^2}{(M^2 - m_{K^*}^2)^2 + (m_{K^*} \Gamma_{K^*})^2}. \]

Interactions of pions with other mesons are treated through direct elastic scattering. The energy-averaged cross section for \( \pi\eta \to \pi\eta \), which is dominated by \( a_0(980) \) formation and
decay, is estimated from the amplitudes given in Ref. [51] and is about 20 mb, and the same is assumed for $\pi\eta' \rightarrow \pi\eta'$. The energy-averaged cross section for $\pi\omega \rightarrow \pi\omega$ is estimated from the omega collisional broadening width due to the pion-omega interaction in Ref. [80] and is about 15 mb, and the same is assumed for $\pi K^* \rightarrow \pi K^*$ and $\pi a_1 \rightarrow \pi a_1$. Finally, we have also included the following inelastic processes for pion-pion collisions, $\pi\pi \rightarrow \eta\eta$ and $\pi\pi \rightarrow K\bar{K}$. The cross section for the former is again obtained from Ref. [51] which is about 10 mb, while the cross section for the latter has been calculated in Ref. [81] based on the one-boson exchange model and is about 1-3 mb depending on the incident energy.

The energy-averaged cross section for $\eta\eta \rightarrow \eta\eta$ is estimated using amplitudes given in Ref. [51] and is about 12 mb. Interactions of an eta with other mesons are assumed to have similar cross sections. The inelastic cross section for $\eta\eta \rightarrow \pi\pi$ is obtained from $\pi\pi \rightarrow \eta\eta$ using detailed balance.

The collision of a kaon with a pion is treated through the $K^*(892)$ formation and decay as discussed earlier. The kaon-antikaon collision mainly proceeds through the formation and decay of the phi meson, and the isospin-averaged cross section is given by [21]

$$\sigma_{K\bar{K} \rightarrow \phi} = \frac{3\pi}{k^2} \frac{(m_\phi \Gamma_\phi)^2}{(M^2 - m_\phi^2)^2 + (m_\phi \Gamma_\phi)^2} \left( \frac{M}{m_\phi} \right)^2, \quad (18)$$

The cross section for the direct elastic scattering of a kaon and a rho meson is estimated to be about 30 mb from the rho meson collisional broadening width in Ref. [80]. This cross section is relatively large due to the presence of a broad $K^*(1270)$ resonance right above the $K\rho$ threshold. All other meson-meson collisions are treated as direct elastic processes with a constant cross section of 20 mb, which is similar to that used in Ref. [63].

The above discussions apply to both free and medium-dependent masses. In the latter, the centroid masses (e.g., $m_\rho$ in Eq. (13)) in these equations are replaced by the in-medium ones, while the decay widths (e.g. Eq. (8)) are evaluated with the physical mass $M$ of mesons.
C. Time evolution of the fire-cylinder

As an illustration, we show in Fig. 3 the baryon (left panel) and meson (right panel) distributions in the x-z plane at several time steps. Initially all particles are located in a compressed cylinder whose transverse length is slightly larger than the longitudinal one. Then the system expands longitudinally as well as transversely with corresponding flow velocities $\beta_z$ and $\beta_t$,

$$
\beta_z(z) = \frac{1}{N_z} \sum_{i=1}^{N_z} \frac{p_{iz}}{E_i}, \quad \beta_t(r_t) = \frac{1}{N_t} \sum_{i=1}^{N_t} \frac{p_{it}}{E_i},
$$

(19)

where the summation in the first expression is over all $N_z$ particles with longitudinal coordinates $(z - 0.5, z + 0.5)$ fm, and in the second expression it is over all $N_t$ particles with transverse coordinates $(r_t - 0.5, r_t + 0.5)$ fm ($r_t = \sqrt{x^2 + y^2}$). The results are shown in Fig. 4 at the freeze-out time $(t=20 \text{ fm/c})$. It is seen that the longitudinal flow velocity is essentially linear in $z$. For small $r_t$, the transverse flow velocity is also approximately linear in the transverse distance.

The time evolution of the baryon abundance is shown in Fig. 5. During initial expansion, we see an increase of the abundance of higher baryon resonances. This is mainly due to the fact that pions are initially out of chemical equilibrium with baryons, and higher baryon resonance formation is thus favored over the reverse process. As the system expands, baryon resonances eventually decay into pions and nucleons. We do not see much difference between the results using free meson masses (the left panel) and in-medium meson masses (the right panel) except that the increase of higher baryon resonances is less significant in the latter case. This is reasonable as in this case the initial pion density is smaller. A similar plot is shown in Fig. 6 for the time evolution of the meson abundance.

The time evolution of the central baryon density of the fire-cylinder is shown in Fig. 7, and it is seen that the system expands slower when in-medium meson masses are used. In this case, some of the energy goes into the field energy $(m_S^2/2)\langle S \rangle^2$. Since this energy is independent of baryon number, the associated pressure is negative and thus reduces the
kinetic pressure. This is not a large effect at SPS energies but will be very important at AGS energies, where the fraction of energy in the field is appreciable. We plan to work this out quantitatively for AGS energies, in the hope that dileptons can be measured at some time in the future in this energy domain. The freeze-out temperature of the system, extracted from the pion transverse mass spectrum by removing the effect from transverse flow, is about 130 MeV. The apparent slope parameters of the pion and proton transverse mass spectra are, however, larger as a result of transverse expansion.

The final proton and pion transverse momentum spectra are compared with the experimental data in Fig. 8. The results using free meson masses are shown by the dotted histograms, while those using in-medium masses are shown by the solid histograms. The pion transverse momentum spectra in both cases are seen to be in good agreement with the measured data from the CERES \cite{84} and the WA80 \cite{67} collaboration which have been presented with arbitrary normalizations. The inverse slope parameters of the pion and proton transverse mass spectra have been extracted from our calculation; they are, respectively, 150 and 220 MeV and are in good agreement with those extracted by the NA44 collaboration for central S+Pb collisions at 200 GeV/nucleon \cite{60}.

In Fig. 9, we compare the pion and proton rapidity distributions with the experimental data. In our calculation we have not removed the protons from the hyperon decay as in the NA44 collaboration \cite{60}. We have thus added the hyperons back to the proton rapidity distribution given in Ref. \cite{59} by the NA35 collaboration in order to make a meaningful comparison. In the mid-rapidity region covered by the CERES collaboration, our pion rapidity distribution is in good agreement with the data from the NA35 collaboration \cite{61}. Since we have not attempted to describe the physics around the target rapidity region, which does not affect our study of dilepton production in the mid- to projectile-rapidity region, our pion rapidity distribution in the small $y$ region should be compared with the open squares which are obtained by reflecting the experimental data at large $y$ with respect to $y_{cm} \approx 2.65$. In the mid-rapidity region our pion rapidity distribution also agrees with the preliminary data from the NA44 collaboration for central S+Pb collisions \cite{60}.
In Fig. 10, we compare the $\eta/\pi^0$ ratio obtained in our model with the experimental data from the WA80 collaboration \cite{68, 69}. For the minimum-biased events (solid circles) the data have been officially published \cite{69}, while for central collisions (solid squares) the data are very preliminary \cite{68}. Our $\eta/\pi^0$ ratio is larger than the minimum-biased data but slightly smaller than the central collision data, and this is especially so in the low transverse momentum region. We believe that this corresponds reasonably to the $\eta/\pi^0$ ratio in the CERES experiments. With an average charged-particle multiplicity of $dN_{ch}/d\eta \approx 125$ in the rapidity range $2.1 < \eta < 2.65$, the CERES experiments select a smaller centrality than the WA80 experiments which have an average charged-particle multiplicity of more than 160 in the same rapidity region \cite{68, 85}. This gives us the confidence that our results for the dilepton production from the $\eta$ and $\eta'$ Dalitz decay are quantitatively correct, since their contribution to the dilepton yield is essentially determined by their freeze-out multiplicities as a result of their much longer lifetime as compared to the lifetime of the fire-cylinder or the nucleus-nucleus interaction time (see discussions in next section and Fig. 18 below).

By fitting and extrapolating the measured $\eta$ and $\pi^0$ yields to the full phase space, an $\eta/\pi^0$ number ratio of $0.147\pm0.017$ (stat.)$\pm0.015$ (syst.) has been obtained for the minimum-biased S+Au collisions by the WA80 collaboration \cite{69}. This is to be compared with a ratio of about 0.08 in the proton-proton interaction at similar center-of-mass energies \cite{86}. On the other hand, it has been estimated in Ref. \cite{68} that for the 9% most central S+Au collisions this ratio could be twice that in minimum-biased collisions, implying an $\eta/\pi^0$ number ratio of about 0.29 in the 9% most central collisions. The $\eta/\pi^0$ number ratio in the CERES experiments should then be somewhere between these two limits, and our $\eta/\pi^0$ number ratio of about 0.2 is thus quite reasonable.

**IV. DILEPTON PRODUCTION**

The main contributions to dileptons with mass below 1 GeV are the Dalitz decay of $\pi^0$, $\eta$ and $\omega$, the direct leptonic decay of vector mesons such as $\rho^0$ and $\omega$, and the pion-pion
annihilation which proceeds through the $\rho^0$ meson by the vector dominance. In addition, we will also evaluate dilepton production from $\eta'$ and $a_1$ decay as well as from $\phi$ decay and kaon-antikaon annihilation.

A. Dalitz decay

The Dalitz decays of $\pi^0$, $\eta$, and $\omega$ contribute significantly to dileptons with mass below $2m_\pi$. The differential width of the Dalitz decay can be related to its radiative decay width [87]. For example, for $\pi^0 \rightarrow \gamma e^+ e^-$, we have [87]

$$\frac{d\Gamma(\pi^0 \rightarrow \gamma e^+ e^-)}{dM} = \frac{4\alpha}{3\pi} \frac{\Gamma(\pi^0 \rightarrow 2\gamma)}{M} \left(1 - \frac{4m_l^2}{M^2}\right)^{1/2} \times \left(1 + \frac{2m_l^2}{M^2}\right) \left(1 - \frac{M^2}{m_{\pi^0}^2}\right)^3 |F_{\pi^0}(M^2)|^2,$$

where $M$ is the mass of the produced dilepton, $\alpha$ is the fine structure constant, $\Gamma(\pi^0 \rightarrow 2\gamma) \approx 7.65$ eV is the neutral pion radiative decay width [70], and $m_l$ is the mass of the lepton. In the case of dielectron production, $m_l = m_e \approx 0.51$ MeV can be neglected. The electromagnetic form factor is parameterized as

$$F_{\pi^0}(M^2) = 1 + b_{\pi^0} M^2,$$

with $b_{\pi^0} = 5.5$ GeV$^{-2}$ as determined empirically [87]. For this process the form factor effects are small as the dilepton invariant mass is always small ($M < m_{\pi^0}$).

Similarly, for $\eta \rightarrow \gamma e^+ e^-$, we have

$$\frac{d\Gamma(\eta \rightarrow \gamma e^+ e^-)}{dM} = \frac{4\alpha}{3\pi} \frac{\Gamma(\eta \rightarrow 2\gamma)}{M} \left(1 - \frac{4m_l^2}{M^2}\right)^{1/2} \times \left(1 + \frac{2m_l^2}{M^2}\right) \left(1 - \frac{M^2}{m_\eta^2}\right)^3 |F_{\eta}(M^2)|^2,$$

where $\Gamma(\eta \rightarrow 2\gamma) \approx 0.463$ KeV is the radiative decay width of an eta meson [70]. The electromagnetic form factor of an eta meson is expressed in the pole approximation as

$$F_{\eta}(M^2) = \left(1 - \frac{M^2}{\Lambda_\eta^2}\right)^{-1},$$

(23)
with the cut-off parameter $\Lambda_\eta \approx 0.72$ GeV as determined experimentally from $\eta \rightarrow \gamma \mu^+ \mu^-$ [37]. This value is close to that predicted by the vector dominance model and used in Ref. [37]. In this case the form factor effect is important for dileptons with mass close to $m_\eta$.

For $\omega \rightarrow \pi^0 e^+ e^-$, the differential decay width is given by

$$\frac{d\Gamma(\omega \rightarrow \pi^0 e^+ e^-)}{dM} = \frac{2\alpha \Gamma(\omega \rightarrow \pi^0 \gamma)}{3\pi \frac{M^2}{m^2}} \left(1 - \frac{4m^2}{M^2}\right)^{1/2}\left(1 + \frac{2m^2}{M^2}\right)$$

$$\times \left[1 + \frac{M^2}{m^2 - m_{\pi^0}^2}\right] - \left(\frac{2m_\omega M}{m_{\pi^0}^2 - m_\pi^2}\right)^2 |F_{\omega \pi^0}(M^2)|^2,$$

(24)

where $\Gamma(\omega \rightarrow \pi^0 \gamma) = 0.717$ MeV is the omega meson radiative decay width [70]. The electromagnetic form factor is similar to Eq. (22) with $\Lambda_\eta$ replaced by $\Lambda_\omega$. In the vector dominance model, the cut-off parameter $\Lambda_\omega$ would have a value of about 0.77 GeV, the rho-meson mass. However, from the omega muonic decay [87] its value is found to be around 0.65 GeV, which will be used in the present work. The use of the vector dominance model prediction would underestimate the dileptons with mass close to $m_\omega - m_{\pi^0}$.

For $\eta' \rightarrow \gamma e^+ e^-$, we have a formula similar to Eq. (22). However, in this case the vector meson pole ($m_\rho$) occurs in the physical region allowed for the dilepton spectrum ($M < m_{\eta'}$), and the simple pole approximation is not valid. Instead we use the original vector-dominance form factor [21], which has been shown to reproduce the experimental data reasonably well for $\eta' \rightarrow \gamma \mu^+ \mu^-$ [87]. The treatment of the $a_1$ Dalitz decay is slightly different. As we have in our dynamical model explicitly the processes $a_1 \leftrightarrow \pi \rho$ and $\rho \rightarrow e^+ e^-$, the part of $a_1$ contribution to dileptons which proceeds through the physical $\rho$ meson as a two-step process has already been included. Thus, in evaluating the $a_1$ Dalitz decay ($a_1 \rightarrow \pi e^+ e^-$) contribution we need not introduce the vector-dominance model form factor. Otherwise, there would be double counting.

B. Vector meson decay into dileptons

The direct leptonic decay of vector mesons is an important source of dileptons. For the dilepton invariant mass region of interest to this work, we consider mainly the leptonic decay
of $\rho^0$, $\omega$ and $\phi$ mesons. The decay width for $\rho^0 \rightarrow e^+e^-$ is given by

$$\Gamma_{\rho^0 \rightarrow e^+e^-}(M) = \frac{g_{\rho\gamma}^2 e^2 M}{M^4} = \frac{\alpha^2}{(g_{\rho\pi\pi}^2/4\pi) 3M^3},$$  

(25)

where $M^4$ in the denominator arises from the virtual photon propagator, and $M$ in the numerator comes from the phase space integration. In obtaining the second expression, we have used the vector dominance relation $g_{\rho\gamma} = e m_{\rho}/g_{\rho\pi\pi}$ [88]. In the calculation, we use, however, $\Gamma_{\rho^0 \rightarrow e^+e^-}(M) = 8.814 \times 10^{-6} m_{\rho}^4/M^3$, with the coefficient determined from the measured width at $M = m_{\rho}$. Similarly, we have $\Gamma_{\omega \rightarrow e^+e^-}(M) = 0.767 \times 10^{-6} m_{\omega}^4/M^3$ and $\Gamma_{\phi \rightarrow e^+e^-}(M) = 1.344 \times 10^{-6} m_{\phi}^4/M^3$ for the omega and phi meson dieletron decay widths, respectively.

Combining Eqs. (8), (15), and (25), we obtain

$$\sigma_{\pi^+\pi^- \rightarrow \rho^0 \rightarrow e^+e^-} = \sigma_{\pi^+\pi^- \rightarrow \rho^0} \cdot \frac{\Gamma_{\rho^0 \rightarrow e^+e^-}}{\Gamma_{\rho^0}} = \frac{8\pi \alpha^2 k}{3M^3} |F_\pi(M)|^2,$$

(26)

with the pion electromagnetic form factor given by

$$|F_\pi(M)|^2 = \frac{m_{\rho}^4}{(M^2 - m_{\rho}^2)^2 + (m_{\rho} \Gamma_{\rho})^2}.$$

(27)

This is the dilepton production cross section from pion-pion annihilation commonly used in the form factor approach [15,21,36,37]. Thus, the dilepton yield from the pion-pion annihilation obtained in the dynamical approach with explicit rho meson formation agrees with that obtained in the form factor approach if medium effects on the intermediate rho meson are neglected [21]. This explains the multiplying factor $(M/m_{\rho})^2$ in Eq. (13) (and a similar factor for kaon-antikaon annihilation, Eq. (18)). In Refs. [21,42] we have used the rho meson dilepton decay width $\Gamma_{\rho^0 \rightarrow e^+e^-}(M) = 1/3\alpha^2 M(4\pi/g_{\rho\pi\pi}^2)$. In order for the relation Eq. (20) to hold, we thus need a somewhat complicated multiplying factor for the simple Breit-Wigner cross section as shown in Eqs. (17) and (27) of Ref. [21]. Neglecting the medium modification of intermediate vector mesons, the present way of separating $\sigma_{\pi^+\pi^- \rightarrow \rho^0}$ and $\Gamma_{\rho^0 \rightarrow e^+e^-}$ and the way used in Refs. [21,42] both give the same results as the form factor approach for the pion-pion (and similarly kaon-antikaon) contribution. For contributions
from the primary rho and omega mesons, different forms of $\Gamma_{\rho^0,\omega \rightarrow e^+e^-}$ as used here and in Ref. [42] do give rise to differences in dilepton yields with masses far away from the rho and omega centroid masses. Since low-mass dileptons are dominated by pion-pion annihilation and at high masses kaon-antikaon annihilation becomes significant, we find that the results of Ref. [42] are only slightly affected when the leptonic decay width with an explicit photon propagator is used.

When medium modifications of vector meson masses are included, the rho meson mass $m_\rho$ in Eqs. (8) and (25) is replaced by the in-medium mass $m^*_\rho$. In a static hadronic matter, one obtains from the relation $\sigma_{\pi^+\pi^- \rightarrow \rho^0 \rightarrow e^+e^-}(M) = \sigma_{\pi^+\pi^- \rightarrow \rho^0} \cdot \Gamma_{\rho^0 \rightarrow e^+e^-}/\Gamma_{\rho^0}$ again Eqs. (26) and (27) with $m_\rho$ replaced by $m^*_\rho$. We note that in obtaining this result the in-medium vector dominance relation, $g_{\rho\gamma}^* = e m_\rho^2 / g_{\rho\pi\pi}$, is assumed. With reduced rho meson mass in medium, the vector dominance is thus weakened. This effect has also been shown in studies based on the hidden gauge theory including loop corrections at finite temperature [89].

Another appreciable medium effect is the collisional broadening of vector meson widths in medium. Haglin [80] has shown that, at a temperature of 150 MeV, the collisional widths are about 50 MeV and 30 MeV for the rho and omega mesons, respectively. However, the collisional broadening will not reduce the total dilepton yield. Adding the collisional broadening width in the denominator of Eq. (27) requires the inclusion of dilepton production also from processes responsible for the collisional broadening. The net effect is thus not a reduction of the total dilepton yield but only a broadening of the invariant mass spectrum. Since the mass resolution in present experiments is comparable to the collisional broadening widths, the final results after being corrected by the experimental resolution will be similar whether we include the collisional broadening widths or not. We have thus neglected the vector meson collisional widths in the present study. We note that the collisional broadening widths of vector mesons are reduced when medium-dependent masses are used [20].
C. Dilepton emission from the expanding fire-cylinder

In our model, dileptons are emitted not only during the expansion of the fire-cylinder but also at the freeze-out. Let us illustrate the way dilepton production is treated in the transport model by considering rho meson decay. Denoting the differential multiplicity of neutral rho mesons at time $t$ by $dN_{\rho_0}(t)/dM$, then the differential dilepton production probability is given by

$$
\frac{dN_{e^+e^-}}{dM} = \int_0^{t_f} \frac{dN_{\rho_0}(t)}{dM} \frac{\Gamma_{\rho \rightarrow e^+e^-}(M)}{\Gamma_{\rho}(M)} dt + \frac{dN_{\rho_0}(t_f)}{dM} \frac{\Gamma_{\rho \rightarrow e^+e^-}(M)}{\Gamma_{\rho}(M)},
$$

(28)

where the freeze-out time $t_f$ is taken to be 20 fm when there is little interaction among the constituents of the fire-cylinder. For the rho meson, whose lifetime is about 1-2 fm/c, the contribution from the first term in Eq. (28), i.e, before the freeze-out, is much more significant than those from the second term, i.e., after the freeze-out. This remains to be so in the case of dropping rho meson mass. Although the rho meson lepton decay width is reduced when its mass decreases in medium (see Eq. (25)), the increase in the number of low-mass rho mesons in medium is more significant, as shown in Fig. 6, leading to an enhanced production of low-mass dileptons compared to the free mass case. The in-medium properties of rho meson can thus be studied from the shape of its associated dilepton spectrum.

Similar expressions as Eq. (28) are used for dilepton yields from other sources. For omega and phi mesons, whose lifetimes are more or less comparable to the lifetime of the fire-cylinder, the determination of their in-medium properties from the shape of their associated dilepton spectra becomes difficult (see Figs. 13 and 14 below and also Ref. [21]). One could, however, observe the interesting phenomenon of rho-omega splitting in the dilepton spectra if their masses are reduced in hot dense matter as also mentioned in Ref. [34].

Similarly, the $a_1$ Dalitz decay mainly contributes during the evolution of the system due to its large width, while the $\omega$ Dalitz decay has comparable contributions during the evolution and at the freeze-out. On the other hand, for the Dalitz decay of $\pi^0$, $\eta$, and $\eta'$, whose lifetimes are much longer than the lifetime of the fire-cylinder, contributions from the
first term in Eq. (28), i.e., before the freeze-out, are negligibly small (see Fig. 18 below). This immediately implies several important observations. First of all, there is almost no possibility to observe the medium modification of, e.g., $\eta$ and/or $\eta'$ meson, from the shapes of their associated dilepton spectra, as they decay into dileptons long after the disintegration of the hot dense matter, when their properties must have returned to those in free space. This has also been realized in Ref. [52]. Secondly, the dilepton yield from the Dalitz decay of $\pi^0$, $\eta$ and $\eta'$ is essentially determined by their abundance at the freeze-out. Thus, they can be accurately extracted once the abundance of these mesons are reliably determined experimentally from, e.g., two-photon reconstruction. Finally, any theoretical calculation (or speculation) of their contributions to dilepton production have to be constrained by their measured abundance by, e.g., the WA80 collaboration [67–69], as discussed at the end of last section. We will come back to this point later.

D. Results and comparison with the CERES data

In this subsection, we discuss dilepton production in central S+Au collisions as obtained in our calculation. We will show the dilepton invariant mass spectra with and without the medium modification of vector meson masses, and with and without the experimental acceptance correction. A mass resolution of about 10% of the dilepton invariant mass, in accordance with the CERES experiment, has always been included. In Fig. 11, the dilepton invariant mass spectra from the Dalitz decay of $\pi^0$, $\eta$, $\omega$, $\eta'$ and $a_1$, as well as the sum of these contributions (solid curve), are shown. It is seen that for dileptons with mass below 100 MeV $\pi^0$ Dalitz decay dominates. In the mass region from 150 to 400 MeV, the $\eta'$ Dalitz decay is the most important, while omega Dalitz decay dominates the Dalitz background from 500 to 650 MeV. The contributions from $\eta'$ and $a_1$ Dalitz decay become important only above 650 MeV, where the contributions from direct decay of vector mesons are much more significant. We do not see much difference between the results with and without the medium modification of meson properties. Although more mesons are present because of
reduced masses in the case where medium dependence is taken into account, the chemical potential enhances their numbers in the free mass case. The dilepton invariant mass spectra after applying the experimental acceptance cut are shown in Fig. 12. Note that both the \( \eta \) and \( \omega \) Dalitz decay contributions in our calculation are somewhat larger than those in the ‘cocktail’ of the CERES collaboration [40], as we have larger \( \eta/\pi^0 \) and \( \omega/\pi^0 \) ratios than what were assumed in Ref. [40] based on proton-proton and proton-nucleus data due to their enhanced production in heavy-ion collisions.

The dilepton invariant mass spectra from the decay of rho (including rho mesons formed from pion-pion annihilation, from the decay of \( a_1 \) mesons and higher baryon resonances, as well as primary rho mesons) and omega mesons are shown in Fig. 13. The dotted curves are obtained using free vector meson masses, while the solid curves are obtained using in-medium masses. In the low-mass region, the enhancement is about a factor of 3-5 and is similar to that found in Ref. [21] for heavy-ion collisions at SIS/GSI energies and in Ref. [34] at AGS/BNL energies. This is mainly from the decay of low-mass primary rho mesons in the initial hot dense fire-cylinder. We note that as the omega meson has a longer lifetime it decays mainly at the freeze-out where its mass has returned to that in free space, leading to the peak around \( m_\omega = 783 \) MeV in the dilepton invariant mass spectrum. A similar plot after the experimental acceptance cut is shown in Fig. 14.

The sum of dilepton invariant mass spectra from Dalitz decay and direct vector meson (including also phi meson) decay from Figs. 11 and 13 (without experimental acceptance cut) are shown in Fig. 15. The dotted curve is obtained with free meson masses, while the solid curve is obtained with in-medium masses. It is seen that in the mass region from \( 2m_\pi \) to about 550 MeV, there is an enhancement of about a factor of 3 when the in-medium vector meson masses are used. The enhancement at low masses is smaller than shown in Figs. 13 and 14 as at these masses the Dalitz decay contribution, which is very similar in the two cases, becomes important.

The comparison with the experimental data is shown in Fig. 16. The results using the free meson masses are shown with the dotted curve. It is seen that this model underestimates
the data from 250 to 550 MeV by about a factor of 3, while it slightly overestimates the data around \( m_{\rho,\omega} \). Most importantly, the shape of the dilepton spectrum is in clear disagreement with the experimental data. The results of our calculation using in-medium meson masses are shown in the figure by the solid curve. The agreement with the experimental data is greatly improved as a result of the factor of 3-4 enhancement in the yield of dileptons with masses from 250 MeV to about 550 MeV, and some reduction around \( m_{\rho,\omega} \).

**E. Comparison with other models**

In addition to our calculation using the relativistic transport model, several different dynamical models have recently been used to study dilepton production in heavy-ion collisions at CERN/SPS energies. In Ref. [55] a hydrodynamical model, with the assumption of quark-gluon-plasma formation and a first-order phase transition, has been used for the expansion of the fire-cylinder. The results for the dilepton invariant mass spectrum are reproduced in Fig. 17 by the dotted curve (the outstanding peak around 1 GeV would disappear if an experimental mass resolution of about 100 MeV were properly included in the calculation of Ref. [55]), together with our results using free meson masses (solid curve). In Ref. [53], the string dynamics for the initial stage of heavy-ion collisions has been combined with the hadronic transport model to study dilepton production. The results of Ref. [53] using free meson masses are reproduced in Fig. 17 by the dashed curve. It is interesting to see that the dilepton spectra obtained in all three calculations based on different dynamical models agree with each other within about 20\%, and all underestimate the data at the low-mass region by about a factor of 3 and overestimate the data around the rho-meson mass.

The failure of the three calculations using free vector meson masses and the quantitative explanation of the data with in-medium vector meson masses suggest that the observed enhancement of low-mass dileptons by the CERES collaboration might be the first direct experimental evidence for the decrease of vector-meson masses in hot and dense hadronic matter as a result of partial restoration of chiral symmetry. This is indeed a significant
conclusion and one may ask whether there are ‘conventional’ or ‘alternative’ mechanisms to explain the data.

Conventionally, one might think of the medium modification of the rho meson spectral function (or equivalently pion electromagnetic form factor) as a typical nuclear many-body effect. It has been shown in Refs. [91,92] that the rho-meson spectral function is modified in dense nuclear matter as a result of the softened pion dispersion relation due to delta-particle–nucleon-hole ($\Delta N^{-1}$) polarization. The form factor as determined in Ref. [92] has been used in Ref. [53] to study its effects on the dilepton spectrum. Indeed, one sees the enhancement of low-mass dileptons as compared to the case with the free-space rho-meson spectral function. A similar result has also been obtained in Ref. [93] in which not only the delta-particle–nucleon-hole polarization but also the nucleon-particle–nucleon-hole polarization in nuclear matter at finite temperature has been included. However, the effects found in Refs. [53,93], though considerably smaller than the effects from the decreasing rho meson mass, are most likely overestimated. Firstly, in heavy-ion collisions at SPS/CERN energies, the system is highly excited, so it consists not only of nucleons but also of baryon resonances, especially $\Delta(1232)$. The softening of the pion dispersion relation due to the delta-particle–nucleon-hole polarization is largely cancelled by the counter-effects from the nucleon-particle–delta-hole polarization. In this case the softening of the pion dispersion relation depends on an effective density $\rho_{eff} = \rho_N - \frac{1}{3}\rho_{\Delta}$ as shown in Ref. [94]. At an initial temperature of 165 MeV and initial baryon density of $2.5\rho_0$, we find that $\rho_N$ and $\rho_{\Delta}$ are about $0.8\rho_0$ and $0.9\rho_0$, respectively, thus the effective density $\rho_{eff} \approx 0.6\rho_0$. At this density the change of the rho spectral function is already insignificant [91,92]. Including the counter-effects from higher resonances such as $N^*(1440)$, the effective density will be further reduced. Moreover, pions outnumber nucleons by about a factor of 5-10 in heavy-ion collisions at CERN/SPS energies, so not all the pions can be converted into $(\Delta N^{-1})$’s. In the calculation of Refs. [91,92], the pion dispersion relation has been obtained always by putting only one pion into an infinite nuclear matter. Also, the mismatch in the pion and nucleon rapidity distributions in the initial stage when dileptons are produced would reduce
the effect of delta-hole polarization. For heavy-ion collisions at SPS/CERN energies, with a correct pion dispersion relation as a function of not only nucleon but also baryon resonance and pion densities as well as a proper consideration of the nucleon momentum distribution, this nuclear many-body effect is expected to be much smaller than that found in Ref. [53,93].

The pion dispersion relation is also softened at finite temperature due to coupling to the pion-rho loop [95]. The enhancement of low-mass dileptons from this effect is, however, largely offset by the reduction in the pion electromagnetic form factor at finite temperature [96].

Alternatively, in connection with the interesting possibility of $U(1)_A$ symmetry restoration that leads to medium modifications of the $\eta$ and/or $\eta'$ meson masses, several speculations have been put forward [51,52]. Looking at the Dalitz decay background, one might suggest that the medium modification of the $\eta$ and/or $\eta'$ meson masses might enhance low-meson dileptons. However, as mentioned previously, the Dalitz decays of the $\eta$ and $\eta'$ mesons occur dominantly long after the freeze-out of the system. This is quantified in Fig. 18 where the dilepton spectrum from the decay of eta mesons up to 20 fm/c is shown by the dotted curve, while their total contribution is shown by the solid curve. It is clearly seen that their contribution to the dilepton yield before the freeze-out, i.e., in hot and dense medium where $\eta$ and $\eta'$ masses might be modified, is negligible as compared to that after the freeze-out when their masses are the same as in free space. Therefore the shape of the dilepton spectra from the Dalitz decay of the $\eta$ and $\eta'$ mesons remains essentially the same, be there medium modifications or not. On the other hand, one might argue that the reduced $\eta$ and $\eta'$ meson masses increase their production cross section and thus increase overall the dileptons from their Dalitz decay. However, one needs to keep in mind an important constraint of the model as provided by the experimental data on the $\eta/\pi^0$ ratio [67,69]. Since we do reproduce the $\eta/\pi^0$ ratio reasonably well (see Fig. 9), we believe that our results for the Dalitz decay of the $\eta$ and $\eta'$ are quantitatively correct. As pointed out earlier, there is already some enhancement of Dalitz background with respect to the ‘cocktail’ of Ref. [10].

In Ref. [51], partial $U(1)_A$ restoration has been studied, and both the $\eta$ and $\eta'$ meson
masses are found to decrease at high temperature. It has been found in Ref. [51] that in order to describe quantitatively the $\eta/\pi^0$ ratio in central S+Au collisions as measured by the WA80 collaboration [67–69], one needs a finite $\eta$ chemical potential and a reduced $\eta$ meson mass. This is in agreement with our findings. We need either a large $\eta$ and $\eta'$ chemical potential, or a moderate chemical potential with reduced $\eta$ and $\eta'$ meson masses. The enhanced $\eta$ yield in central heavy-ion collisions as compared to that in proton-proton and proton-nucleus interaction are, however, not enough to explain the enhanced low-mass dilepton yield in the CERES experiments.

Similarly, $U(1)_A$ symmetry restoration in hot hadronic matter has been studied in Ref. [52], in which the medium modification of $\eta'$ mesons and their enhanced production are emphasized. By increasing the $\eta'$ contribution in the ‘cocktail’ of Ref. [40] by 16 times, the authors of Ref. [52] found that the Dalitz decays, together with hadron-hadron (especially pion-pion) contributions, might be able to explain the enhancement of low-mass dileptons in the CERES experiments. It should, however, be emphasized that the $\eta'$ contribution in the ‘cocktail’ of Ref. [40] has been obtained with an $\eta'/\pi^0$ ratio of about 0.04-0.05 [40,86]. Thus a 16-fold increase of $\eta'$ abundance implies an $\eta'/\pi^0$ ratio of about 0.7. Together with a similar $\eta/\pi^0$ ratio, this gives rise to a final $\eta/\pi^0$ ratio of greater than 1. This grossly overestimates the measured ratio in the most central S+Au collisions by the WA80 collaboration and is more so for less central CERES experiments.

F. Sensitivity of dilepton spectra to initial conditions

The main uncertainty in our model is the initial conditions for the meson chemical composition, the transverse momentum distributions of hadrons, and the rapidity field. Microscopic models based on string fragmentations can in principle provide the necessary information [65]. In the present work, only general features of the results from these models have been used. Values of the parameters in the model, particularly the initial meson compositions, are determined by fitting the measured proton and pion rapidity and trans-
verse momentum distributions. With free vector meson masses, these initial conditions fail to explain the observed enhancement of low-mass dileptons. One may wonder if different initial meson compositions can be used to increase the dilepton yield without invoking any medium effects. Fig. 17 has already demonstrated that using an initial meson composition from either string fragmentation or in chemical equilibrium can not lead to an enhanced production of low-mass dileptons. Here, we will show the results from a calculation with an initial meson chemical composition that is determined by the spin-isospin degeneracy as in Ref. [66], i.e., the initial ratio of the $\pi$, $\rho$, $\omega$, and $\alpha_1$ abundance is 1:3:1:3. To reproduce the observed pion multiplicity then requires that the numbers are 21, 63, 21, 63 for $\pi$, $\rho$, $\omega$, and $\alpha_1$, respectively. The dilepton invariant mass spectrum from this scenario is shown in Fig. 19 together with the results using the chemical equilibrium scenario. The dilepton spectra in both calculations are essentially the same, with the peak around $m_{\rho,\omega}$ slightly lowered and a small enhancement around 300 MeV to 600 MeV in the spin-isospin scenario.

We have also studied the dependence of the dilepton invariant mass spectrum on the parameters $y_L$ and $\sigma$, which characterize the initial meson rapidity distributions. By varying their values but still requiring that the final pion rapidity distribution agrees reasonably with the measured one, we have not found any appreciable change in the dilepton invariant mass spectra for both free and in-medium masses.

G. Comparison with the HELIOS-3 data

Enhancement of dilepton yields in heavy-ion collisions in both low-mass and intermediate-mass regions have also been observed by the HELIOS-3 collaboration which have measured dimuon invariant mass spectra in both S+W and p+W collisions at 200 GeV/nucleon [39]. The enhancement has been demonstrated in terms of the difference in dilepton yield per charged particle between S+W and p+W collisions as shown in Ref. [39]. The data then show that the enhancement is most pronounced around $M \approx 0.5 - 0.6$ GeV, which provides another possible indication, independent of the CERES data, that the vector
(rho) meson mass might be reduced in hot and dense matter.

To see whether our model, that has explained the enhancement of dilepton yield as observed by the CERES collaboration, can also explain the enhancement observed by the HELIOS-3 collaboration in the low-mass region, we have carried out two calculations for dimuon production. One assumes free meson masses and the other uses in-medium masses with the same parameters as specified earlier. Since the HELIOS-3 collaboration measures dimuons, with \( m_\mu \approx 0.106 \) GeV, we need to include a phase factor of \( (1 - (2m_\mu/M)^2)^{1/2}(1 + 2m_\mu^2/M^2) \) in Eq. (24) for vector meson decays. The results are shown in Fig. 20, and compared with the data from Ref. [39]. Note that the data are the difference between dilepton yields per charged particle in S+W and p+W collisions, i.e., \( (\Delta N_{\mu\mu}/N_{ch})_{SW} - (\Delta N_{\mu\mu}/N_{ch})_{pW} \), while our theoretical results include only dileptons from the decay of rho and phi mesons formed from pion-pion and kaon-antikaon annihilation, which are absent in p+W collisions. The y-axis, \( \Delta N_{\mu\mu}/N_{ch} (50\text{MeV}/c^2)^{-1} \), represents the total number of dimuon pairs in a mass bin of 50 MeV, including the experimental acceptance cuts and normalized to the total charged-particle multiplicity in the pseudorapidity region \( 3.7 < \eta < 5.2 \). It is seen that with free rho meson mass, the shape of the pion-pion contribution is in clear disagreement with the data; in the mass region below \( 650 \) MeV it underestimates the data by about a factor of 2-3, while around \( m_\rho \) it slightly overestimates the data. This is very similar to the case for the CERES data. Including decreasing rho-meson mass in hot and dense matter, the agreement with the data is again greatly improved. Specifically, we have about a factor of 2 enhancement in the mass region below about \( 600 \) MeV as compared with the case using the free rho-meson mass. There is also some reduction around \( m_\rho \) due to the shift of rho-meson peak to a lower mass. Overall, we have reasonably agreement with the experimental data up to about \( 800 \) MeV. In the mass region around \( 1 \) GeV, enhanced dilepton production in S+W collisions has also been observed which is most likely due to the kaon-antikaon annihilation that proceeds through the phi meson [21].

In both experiments [39–41] and our theoretical calculation, the enhancement of dilepton yield in the low-mass region in Fig. 20 is somewhat smaller than that in Fig. 16. There
are two possible reasons. Firstly, the CERES collaboration measures dileptons in the mid-
rapidity region where initially there are many low-mass rho mesons, while the HELIOS-3
collaboration measures the forward-rapidity region with a smaller number of low-mass rho
mesons. Secondly, in our model the dileptons measured in the CERES experiments are
produced early in the expansion when the baryon density is high, and the rho meson mass
is small. In the HELIOS-3 experiment, the dileptons measured in the forward rapidity are
produced later in the expansion when interactions lead to a broader rapidity distribution.
The baryon density at which large rapidity dileptons are produced is thus lower, and the
reduction of the rho meson mass is smaller.

V. GENERAL DISCUSSIONS

There have been discussions of processes which might fill in the low-mass dilepton spec-
trum. One example is the pionic bremsstrahlung process shown in Fig. 21. This process
would, indeed, contribute to low-mass dileptons. If the lower left $\rho$-meson is produced by a
$\gamma$-ray, one can see that this is just the electromagnetic polarizability of the pion, with the
final $\gamma$-ray going into an $e^+e^-$ pair. It is known, however, from the Das, Mathur and Okubo
relation \[97\] that the $\rho$-meson does not enter into the polarizability of the pion, although it
is most important in the pion electromagnetic form factor. A double-$\rho$-seagull enters with
equal magnitude and opposite sign in order to cancel the process of Fig. 21. This means
that the $\rho$ and $\pi$ couple uniquely through the $a_1$ as intermediate state \[98\], and this has
been employed in our study.

For systems with nonzero charge, the $0^{++}$ sigma meson can couple through $\pi^+\pi^-$ to a
single virtual photon and dileptons \[99\]. This bears some relation to the known possibility in
Walecka theory at finite density in nuclei that the sigma can couple to the time component
of the $\omega$-meson, which can then decay into dileptons. Thus, at finite density (and breaking
of Lorentz invariance) the sigma cannot decay into a single photon or dilepton pair does not
hold. Weldon \[99\] makes the argument that the sigma mass will drop with density and/or
temperature, so that dileptons from the sigma decay through a single virtual photon will fill up the low-mass region of dileptons. However, as long as the vector mesons are made in sufficient number, as we find for the process $\pi\pi \leftrightarrow \rho$, it does not matter how many additional processes are introduced. Very simply, an equilibrium distribution of vector mesons, with appropriate Boltzmann factors, is formed. If additional processes are added to make more vector mesons, then the reverse processes will remove it.

The goodness of our initial conditions, mainly the hadron momentum distributions, was found by Cassing et al. [53] and in the RQMD [65]. These authors began from initial distributions generated from string dynamics without the assumption of specific functional forms for the transverse momentum spectra and rapidity distributions. Yet it can be seen from our Fig. 17 (and a similar figure from Ref. [65]) that their results are similar to ours and to the hydrodynamical ones of Srivastava et al. [55]. Our somewhat larger contribution at low masses results from our finite pion and eta chemical potentials.

Finally, one can question our use of Walecka theory, although many people have used it at constituent quark level. It is true that the constituent quark effective masses in Walecka theory go to zero only as $\rho \to \infty$, whereas we believe Nambu–Jona-Lasinio theory to be more correct in letting the masses go to zero at finite (and not too high) densities. We have, however, started the evolution of our fire-cylinder when the constituent quark has about one-third of its on-shell mass, and the difference between Walecka theory and Nambu-Jona-Lasinio theory should not be too great here. It can be shown [100] that the field energy in the Walecka theory acts like a bag constant, and that the parameters in the Walecka linear sigma-omega model are such that if $m_Q^*$ is brought to zero, this effective bag constant faithfully reproduces that from lattice gauge calculations in QCD. The latter is obtained from changes in quark/gluon condensates as the temperature goes through the chiral transition [101]. In our expansion of the fire-cylinder it is important to have the correct value for the bag constant. The energy going into the bag constant takes away from the energy in expansion, providing negative pressure, so the expansion is slower as can be seen from our Fig. 6.
As noted above, RQMD-type string dynamics calculations have been and are being carried out to establish initial conditions. In our dropping mass scenario, at the chiral phase transition the vector meson masses are zero. Thus, there is a period between the phase transition and the time we begin evolution of the fire-cylinder in which the masses are smaller than those in our calculation. The decay of these very low mass vector mesons should not be very important, because of the small phase space and because they will chiefly give dileptons in the low-mass region, which is dominated by the background of Dalitz decays. Work is, however, underway to propagate the masses described in Nambu–Jona-Lasinio theory. This theory does bring the constituent quark masses smoothly to zero, and the bag constant can be arranged to match the decondensation energy obtained in lattice gauge calculations.

VI. SUMMARY

In conclusion, using the relativistic transport model for the expansion stage of the fire-cylinder formed in the initial stage of heavy-ion collisions at SPS/CERN energies, we have studied the enhancement of low-mass dileptons in central S+Au and S+W collisions as recently observed by the CERES and the HELIOS-3 collaboration. The initial conditions for the fire-cylinder are constrained by microscopic simulations based on string dynamics as well as experimentally measured proton and pion rapidity distributions and spectra. We have included the Dalitz decay of $\pi^0$, $\eta$, $\eta'$, $\omega$ and $a_1$ mesons, the direct decay of primary $\rho^0$, $\omega$ and $\phi$ mesons, and the pion-pion annihilation that proceeds through the $\rho^0$ meson, the pion-rho annihilation that proceeds through the $a_1$ meson, and the kaon-antikaon annihilation that proceeds through the $\phi$ meson. We have found that the modification of vector meson properties, especially the decrease of their mass due to the partial restoration of the chiral symmetry, in hot and dense hadronic matter, provides a quantitative explanation of the recently observed enhancement of low-mass dileptons by the CERES collaboration in central S+Au collisions and by the HELIOS-3 collaboration in central S+W collisions. In
comparison with the latter, we have, however, too few phi mesons. It has been shown in Ref. [102] that phi meson mass is considerably brought down by the substantial (positive) $\langle \bar{s}s \rangle$ present in the fire-cylinder. We are unable, as of yet, to include this in a thermodynamically consistent way in the transport model.

To continue this study, we have already started to carry out a calculation [65] in which the initial conditions are completely given by the string dynamics. Although a similar study has previously been carried out by Cassing et al. [53,56], our approach differs from theirs in the treatment of vector meson masses in hot dense matter as discussed in Section II. Preliminary results show that the results presented in the present paper remain essentially unchanged. This more microscopic approach allows us to make predictions regarding low-mass dilepton production from collisions between heavier nuclei, where the medium effects are expected to be more appreciable.

We are grateful to Peter Braun-Munzinger, Wolfgang Cassing, Kevin Haglin, Volker Koch, Zheng Huang, Mannque Rho, Bikash Sinha, Edward Shuryak, and Chungsik Song for useful discussions. Also, we thank Alex Drees, Itzhak Tserruya and Thomas Ullrich for communications about the CERES experiments, Ivan Kralik for communications about the HELIOS-3 experiments, Michael Murray for communications about the NA44 experiments, and Frank Plasil and Karl-Heinz Kampert for communications about the WA80 experiments. GQL and CMK are supported by the National Science Foundation under Grant No. PHY-9212209 and No. PHY-9509266, and GEB is supported by the Department of Energy under Grant No. DE-FG02-88ER40388.
REFERENCES

[1] C. DeTar and J. B. Kogut, Phys. Rev. D 36 (1987) 2828.

[2] T. Hashimoto, T. Nakamura, and I. O. Stamatescu, Nucl. Phys. B406 (1993) 325.

[3] G. Boyd, S. Gupta, F. Karsch, E. Laermann, B. Petersson, and K. Redlich, Phys. Lett. B 349 (1995) 170.

[4] J. Gasser and H. Leutwyler, Phys. Lett. B184 (1989) 83.

[5] C. S. Song, Phys. Rev. D, in press.

[6] R. D. Pisarski, Phys. Rev. D, in press.

[7] G. E. Brown and M. Rho, Phys. Rev. Lett. 66 (1991) 2720.

[8] C. Adami and G. E. Brown, Phys. Rep. 224 (1993) 1.

[9] T. Hatsuda and S. H. Lee, Phys. Rev. C 46 (1992) R34;
   T. Hatsuda, Nucl. Phys. A544 (1992) 27c.

[10] M. Asakawa and C. M. Ko, Phys. Rev. C 48 (1993) R526; Nucl. Phys. A560 (1993) 399.

[11] H. C. Jean, J. Piekarewicz, and A. G. Williams, Phys. Rev. C 49 (1994) 1891.

[12] H. Shiomi and T. Hatsuda, Phys. Lett. B 334 (1994) 281.

[13] C. Song, P. W. Xia, and C. M. Ko, Phys. Rev. C 52 (1995) 408.

[14] K. Saito and A. W. Thomas, Phys. Lett. B 327 (1994) 9; Phys. Rev. C 51 (1995) 2757.

[15] C. Gale and J. Kapusta, Phys. Rev. C 35 (1987) 2107; C 38 (1988) 2657.

[16] L. H. Xia, C. M. Ko, L. Xiong, and J. Q. Wu, Nucl. Phys. A485 (1988) 721.

[17] R. Pisarski, Phys. Lett. 110B (1982) 155.
[18] V. Koch, in *Proc. of Pittsburgh Workshop on Soft Lepton Pair and Photon Production*, ed. J. A. Thompson, (Nova Science Publishers, New York, 1992), p. 251.

[19] F. Karsch, K. Redlich and L. Turko, Z. Phys. C **60** (1993) 519.

[20] T. Hatsuda, Y. Koike and S. H. Lee, Nucl. Phys. **B394** (1993) 221.

[21] G. Q. Li and C. M. Ko, Nucl. Phys. **A582** (1995) 731.

[22] E. Feinberg, Nuovo Cimento, **A34** (1976) 39.

[23] E. V. Shuryak, Phys. Lett. B **78** (1978) 150; Phys. Rep. **67** (1980) 71.

[24] S. A. Chin, Phys. Lett. B **119** (1982) 51.

[25] G. Domokos, Phys. Rev. D **28** (1983) 123.

[26] L. D. McLerran and T. Tiomela, Phys. Rev. D **31** (1985) 545.

[27] K. Kajantie, J. Kapusta, L. McLerran, and A. Mekjian, Phys. Rev. D **34** (1986) 2746.

[28] K. Redlich, Phys. Rev. D **36** (1987) 3378.

[29] C. M. Ko and L. H. Xia, Phys. Rev. Lett. **62** (1989) 1595.

[30] J. Cleymans, K. Redlich and H. Satz, Z. Phys. C **52** (1991) 517.

[31] P. V. Russkanen, Nucl. Phys. **A544** (1992) 169c; in *Particle production in highly excited matter*, ed. H. H. Gutbrod (Plenum, New York, 1993).

[32] M. Asakawa, C. M. Ko, and P. Lévai, Phys. Rev. Lett. **70** (1993) 398.

[33] M. Asakawa and C. M. Ko, Phys. Lett. B **322** (1994) 33.

[34] M. Hoffman, *et al.*, Nucl. Phys. **A566** (1994) 15c.

[35] G. Roche *et al.*, Phys. Rev. Lett. **61** (1988) 1069; C. Naudet *et al.*, Phys. Rev. Lett. **62** (1988) 2652.
[36] L. Xiong, Z. G. Wu, C. M. Ko, and J. Q. Wu, Nucl. Phys. A552 (1990) 772.

[37] Gy. Wolf, W. Cassing, and U. Mosel, Nucl. Phys. A552 (1993) 549.

[38] W. Koenig, in Proc. Workshop on dilepton production in relativistic heavy-ion collisions, ed. H. Bokemeyer (GSI, Darmstadt, 1994).

[39] M. Masera for the HELIOS-3 Collaboration, Nucl. Phys. A590 (1995) 93c;
I. Králik for the HELIOS-3 Collaboration, in Proc. International Workshop XXIII on Gross Properties of Nuclei and Nuclear Excitations, ed. H. Feldmeier and W. Nörenberg, (GSI, Darmstadt, 1995) p. 143.

[40] G. Agakichiev et al., Phys. Rev. Lett. 75 (1995) 1272;
J. P. Wurm for the CERES Collaboration, Nucl. Phys. A590 (1995) 103c;
A. Drees for the CERES/NA44 Collaboration, in Proc. International Workshop XXIII on Gross Properties of Nuclei and Nuclear Excitations, ed. H. Feldmeier and W. Nörenberg, (GSI, Darmstadt, 1995) p. 151.

[41] I. Tserruya, Nucl. Phys. A590 (1995) 127c.

[42] G. Q. Li, C. M. Ko, and G. E. Brown, Phys. Rev. Lett. 75 (1995) 4007.

[43] S. Weinberg, Phys. Rev. Lett. 18 (1967) 507.

[44] J. I. Kapusta and E. V. Shuryak, Phys. Rev. D 49 (1994) 4694.

[45] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. 16 (1986) 1.

[46] G. Gelmini and B. Ritzi, Phys. Lett. B 357 (1995) 431.

[47] G. E. Brown and M. Rho, Nucl. Phys. A596 (1996) 503.

[48] R. J. Furnstahl, H. B. Tang, and B. D. Serot, Phys. Rev. C 52 (1995) 1368.

[49] R. J. Furnstahl and B. D. Serot, Phys. Rev. C41 (1990) 262.

[50] J. F. Donoghue, E. Golowich, and B. R. Holstein, Dynamics of the Standard Model,
(Cambridge University Press, Cambridge, 1992).

[51] Z. Huang and X. N. Wang, hep-ph/9507395.

[52] J. Kapusta, D. Kharzeev, and L. McLerran, hep-ph/9507343.

[53] W. Cassing, W. Ehehalt, and C. M. Ko, Phys. Lett. B 363 (1995) 35.

[54] D. K. Srivastava and B. Sinha, Phys. Rev. Lett. 73 (1994) 2421.

[55] D. K. Srivastava, B. Sinha, and C. Gale, Phys. Rev. C 53 (1996) R567.

[56] W. Ehehalt and W. Cassing, Nucl. Phys. A, in press.

[57] H. Sorge, H. Stöcker, and W. Greiner, Ann. Phys. 192 (1989) 266.

[58] H. Sorge, Phys. Rev. C 52 (1995) 3291.

[59] D. Rührlich for the NA35 collaboration, Nucl. Phys. A566 (1994) 35c.

[60] M. Murray for the NA44 Collaboration, in Proc. of Strangeness in Hadronic Matter, ed. J. Rafelski (AIP, New York, 1995), p. 162;
   B. V. Jacak for the NA44 collaboration, Nucl. Phys. A590 (1995) 215c

[61] M. Gazdzicki for the NA35 collaboration, Nucl. Phys. A590 (1995) 197c.

[62] P. Braun-Munzinger, J. Stachel, J. P. Wessels, and N. Xu, Phys. Lett. B 344 (1995) 43; Phys. Lett. B 365 (1996) 1.

[63] M. Herrmann and G. F. Bertsch, Phys. Rev. C 51 (1995) 328.

[64] G. E. Brown, A. D. Jackson, H. A. Bethe, and P. M. Pizzochero, Nucl. Phys. A560 (1993) 1035.

[65] G. Q. Li, C. M. Ko, H. Sorge and G. E. Brown, to be published.

[66] H. W. Barz, G. Bertsch, D. Kusnezov, and H. Schulz, Phys. Lett. B254 (1991) 332.

[67] R. Santo et al., Nucl. Phys. A566 (1994) 61c.
[68] A. Lebedev et al., Nucl. Phys. A566 (1994) 355c.

[69] R. Albrecht et al., Phys. Lett. B 361 (1995) 14.

[70] Particle Data Group, Review of Particle Properties, Phys. Rev. D 50 (1994) 1173.

[71] J. Bolz, U. Ornik, and R. M. Werner, Phys. Rev. C 46 (1992) 2047.

[72] R. Venugopalan, M. Prakash, M. Kataja, and P. V. Ruuskanen, Nucl. Phys. A566 (1994) 473c.

[73] E. Shuryak, private communications.

[74] C. M. Ko, Q. Li, and R. Wang, Phys. Rev. Lett. 59 (1987) 1084;
    C. M. Ko and Q. Li, Phys. Rev. C 37 (1988) 2270;
    Q. Li, J. Q. Wu and C. M. Ko, Phys. Rev. C 39 (1989) 849.

[75] G. Q. Li, C. M. Ko, X. S. Fang, and Y. M. Zheng, Phys. Rev. C 49 (1994) 1139.

[76] B. J. VerWest and R. A. Arndt, Phys. Rev. C 25 (1982) 1979.

[77] P. Danielewicz and G. F. Bertsch, Nucl. Phys. 533 (1991) 712.

[78] L. Xiong, E. Shuryak, and G. E. Brown, Phys. Rev D 46 (1992) 3789.

[79] C. M. Ko, Phys. Rev. C 23 (1981) 2760.

[80] K. Haglin, Nucl. Phys. A584 (1995) 719.

[81] G. E. Brown, C. M. Ko, Z. G. Wu, and L. H. Xia, Phys. Rev. C 43 (1991) 1881.

[82] C. M. Ko and D. Seibert, Phys. Rev. C 49 (1994) 2198.

[83] O. Kaymakcalan, S. Rajeev, and J. Schechter, Phys. Rev. D 30 (1984) 594.

[84] R. Bauer et al., Nucl. Phys. A566 (1994) 87c.

[85] R. Albrecht et al., Z. Phys. C55 (1992) 539.
[86] M. Aguilar-Benitez et al., Z. Phys. C 50 (1991) 405.

[87] L. G. Landberg, Phys. Rep. 128 (1985) 301.

[88] R. K. Bhaduri, Models of the Nucleon, (Addison-Wesley, Reading, MA, 1988).

[89] C. S. Song, S. H. Lee, and C. M. Ko, Phys. Rev. C52 (1995) R476.

[90] K. Haglin, private communications.

[91] M. Asakawa, C. M. Ko, P. Lévai, X. J. Qiu, Phys. Rev. C46 (1992) R1159.

[92] M. Herrmann, B. L. Friman, and W. Nörenberg, Nucl. Phys. A560 (1993) 411.

[93] R. Rapp, G. Chanfray, and J. Wambach, Phys. Rev. Lett. 76 (1996) 368.

[94] P. A. Henning and H. Umezawa, Nucl. Phys. A571 (1994) 617.

[95] C.S. Song, Phys. Rev. D49 (1994) 1556; Phys. Lett. B329 (1994) 312.

[96] C. S. Song, V. Koch, S. H. Lee, and C. M. Ko, Phys.Lett. B366 (1996) 379.

[97] T. Das, V. Mathur, and S. Okubo, Phys. Rev. Lett. 19 (1967) 859.

[98] B. R. Holstein, Comm. Nucl. Part. Phys. 19 (1990) 221.

[99] H. A. Weldon, Phys. Lett. B 274 (1992) 133.

[100] G. E. Brown, M. Buballa, and M. Rho, to be published.

[101] V. Koch and G. E. Brown, Nucl. Phys. A560 (1993) 345.

[102] M. Asakawa and C. M. Ko, Nucl. Phys. A572 (1994) 732.
Figure Captions

Fig. 1: In-medium mass of nucleon (left panel) and rho-meson (right panel) as a function of temperature at several baryon densities.

Fig. 2: Initial proton and pion transverse momentum and rapidity distributions.

Fig. 3: Baryon (left panel) and meson (right panel) distributions in x-z plane at several time steps.

Fig. 4: Longitudinal (left panel) and transverse (right panel) flow velocities at freeze-out.

Fig. 5: Time evolution of the nucleon, \(\Delta(1232)\) and higher baryon resonance abundance.

Fig. 6: Time evolution of the meson abundance.

Fig. 7: Time evolution of the central baryon density.

Fig. 8: Proton and pion transverse momentum spectra. Dotted and solid histograms are obtained from simulations based on the relativistic transport model with free and in-medium vector meson masses, respectively. Open and solid circles are experimental data from the CERES [84] and the WA80 [67] collaboration, respectively.

Fig. 9: Same as Fig. 8 for proton and pion rapidity distributions. The experimental data from the NA35 [61] are given by open circles, while open squares are obtained by reflecting the data with respect to \(y \approx 2.65\). Solid circles are the experimental data from the NA44 collaboration [60].

Fig. 10: \(\eta/\pi^0\) ratio. Open circles and squares are obtained from simulations based on the relativistic transport model with free and in-medium vector meson masses, respectively. The solid circles and squares are experimental data from the WA80 collaboration for the minimum-biased [69] and central [68] S+Au collisions, respectively.

Fig. 11: Dilepton invariant mass spectra from Dalitz decay using free masses (left panel) and in-medium masses (right panel).
Fig. 12: Same as Fig. 11 with the experimental acceptance cut.

Fig. 13: Dilepton invariant mass spectra from the direct decay of rho and omega mesons, using free masses (dotted curves) and in-medium masses (solid curves).

Fig. 14: Same as Fig. 13 with the experimental acceptance cut.

Fig. 15: Dilepton invariant mass spectra including both Dalitz decay and direct decay of vector mesons. The dotted and the solid curve are obtained using the free and in-medium masses, respectively.

Fig. 16: Same as Fig. 15 with the experimental acceptance cut. The experimental data from the CERES collaboration [40] are shown by the solid circles, with the statistical errors given by bars. The brackets represent the square root of the quadratic sums of the systematic and statistical errors. (We have followed the request of Itzhak Tserruya of the CERES collaboration to show the experimental errors in this way instead the linear sum used in the original publications [40], which were often misinterpreted.)

Fig. 17: Comparison of different calculations with free meson masses. The solid curve is from the present work, while the dotted and dashed curves are from Ref. [55] and Ref. [53], respectively.

Fig. 18: Dilepton invariant mass spectra from \( \eta \) Dalitz decay. The dotted curve gives the spectrum up to 20 fm/c, while the solid curve is the total.

Fig. 19: Dilepton invariant mass spectra obtained using free meson masses. The dotted and the solid curve are obtained, respectively, using the spin-isospin scenario and the chemical equilibrium scenario for the initial meson composition.

Fig. 20: Dimuon mass spectra from pion-pion annihilation. The solid and dotted histograms are obtained with in-medium and free meson masses, respectively. Solid circles are the experimental data for the difference in the dimuon yields per charged
particle between the S+W and p+W the collision from the HELIOS-3 collaboration.

Fig. 21: Pionic bremsstrahlung.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9608040v1
This figure "fig2-1.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9608040v1
This figure "fig3-1.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9608040v1
This figure "fig4-1.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9608040v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9608040v1
This figure "fig2-2.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9608040v1
This figure "fig3-2.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9608040v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9608040v1
This figure "fig2-3.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9608040v1
This figure "fig3-3.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9608040v1