Tensor-network Study of Correlation-spreading Dynamics in the Two-dimensional Bose-Hubbard Model

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Article

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Tensor-network study of correlation-spreading dynamics in the two-dimensional Bose-Hubbard model

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Abstract
Recent developments in analog quantum simulators based on cold atoms and trapped ions call for cross-validating the accuracy of quantum-simulation experiments with use of quantitative numerical methods; however, it is particularly challenging for dynamics of systems with more than one spatial dimension. Here we demonstrate that a tensor-network method running on classical computers is useful for this purpose. We specifically analyze real-time dynamics of the two-dimensional Bose-Hubbard model after a sudden quench starting from the Mott insulator by means of the infinite projected entangled pair state algorithm. Calculated single-particle correlation functions are found to be in good agreement with a recent experiment [Y. Takasu et al., Sci. Adv. 6, eaba9255 (2020)]. By estimating the phase and group velocities from the single-particle and density-density correlation functions, we predict how these velocities vary in the moderate interaction region, which serves as a quantitative benchmark for future experiments.

I. INTRODUCTION

State-of-art experimental platforms of cold atoms and trapped ions as analog quantum simulators have offered unique opportunities for studying far-from-equilibrium dynamics of isolated quantum many-body systems. Thanks to their high controllability and long coherence time, these platforms have already addressed a variety of intriguing phenomena that are in general difficult to simulate with classical computers, such as correlation spreading [1–3] and relaxation [4–6] after a quantum quench, many-body localization in a disorder potential [7–9], and quantum scar states [10, 11]. Nevertheless, accurate numerical methods using classical computers are highly demanded at the current stage of the studies of quantum many-body dynamics, since the classical computation still has complementary advantages over the quantum simulation in that it is free of noise and much more accessible owing to its wide dissemination. In this sense, it is important to cross-check the validity of quantum-simulation experiments and some numerical methods by comparing them with each other.

In particular, direct comparisons between experimental and numerical outputs have

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FIG. 1. (a) Schematic figure of iPEPS with a two-site unit cell. A rank-five tensor at each site is represented as a circle with four thin lines and one thick line. The former lines correspond to the virtual degrees of freedom with the bond dimension \( D \), while the latter line corresponds to the physical degrees of freedom with the dimension of the local Hilbert space \( D_{\text{phys}} \). The wave functions become more accurate as \( D \) increases. (b) Time dependence of the hopping \( J \) (a red solid line) and the onsite interaction \( U \) (a blue dashed line) with a finite-time quench. The parameter \( U/J \) is varied from \( \sim 99.4 \) to \( \sim 19.6 \) for \( -\tau_Q < t < 0 \). In the case of a sudden quench, we discard the region \( -\tau_Q < t < 0 \).

been made for dynamical spreading of two-point spatial correlations of the Bose-Hubbard model [1, 3, 12–15], which can be realized experimentally with ultracold bosons in optical lattices [16]. The correlation spreading has attracted much theoretical interest [12–15, 17–24]
in the sense that it is closely related to fundamental phenomena, including the propagation of quantum information and the thermalization. In one spatial dimension, quasi-exact numerical methods based on matrix product states (MPSs) have been used to validate the performance of the quantum simulators [1, 3, 12]. In two dimensions (2D), by contrast, accurate numerical simulations are challenging. Indeed, the comparisons with respect to a single-particle correlation have shown that a few types of the truncated Wigner approximation (TWA) fail to capture the real-time evolution accurately enough to extract the propagation velocity of the correlation [3, 14]. Moreover, while the propagation velocities obtained by a two-particle irreducible strong-coupling (2PISC) approach quantitatively agree with the experimental value, its applicability is limited to a strongly interacting regime, and it does not accurately give the value of the correlation itself [15].

In this paper, we present quantitative numerical analyses of the correlation-spreading dynamics of the 2D Bose-Hubbard model starting from a Mott insulating initial state with unit filling. To this end, we employ the tensor-network method based on the infinite projected entangled pair state (iPEPS) [25–32] or the tensor product state [33–37], which is an extension of MPS to 2D systems [see Fig.1(a)]. The iPEPS studies on real-time dynamics of isolated [38–46] and open [38, 39, 47–49] quantum systems in 2D have begun very recently. Previous simulations suggest that iPEPS can represent relatively low-entangled states in short-time dynamics for simple spin $1/2$ systems [38–41] and some itinerant electron systems [43]. This observation may be valid for real-time dynamics in Bose-Hubbard systems; however, little is known about it until now. We find that the single-particle correlation computed with iPEPS, as well as the estimated propagation velocity of the correlation front, agrees very well with the experimental result [3], demonstrating that iPEPS can be useful for actual quantum-simulation experiments. We also conduct numerical simulations in a moderate interaction region, which has not been addressed by the previous experiments [1, 3]. From the real-time evolution of the single-particle and density-density correlations, we show that the phase and group velocities approach each other when the interaction decreases.
II. RESULTS

A. Model

We consider the Bose-Hubbard model on a square lattice [50, 51]. The Hamiltonian is given as

$$\hat{H} = -J \sum_{\langle ij \rangle} (\hat{a}_i^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_i) + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \sum_i \hat{n}_i,$$  \hspace{1cm} (1)

where $\hat{a}_i^\dagger$ and $\hat{a}_i$ are the creation and annihilation operators at site $i$, $\hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$ is the number operator, $J$ is the strength of the hopping between nearest-neighbor sites, $U$ is the strength of the onsite interaction, and $\mu$ is the chemical potential. The notation $\langle ij \rangle$ indicates that sites $i$ and $j$ are nearest neighbors. For simplicity, we ignore the effects of the trap potential and the Gaussian envelopes of optical lattice lasers, which do not affect short-time dynamics. We set the lattice spacing $d_{\text{lat}}$ to be unity. The ground state at the commensurate filling is the Mott insulating (superfluid) state for $U \gg J$ ($U \ll J$). Hereafter, we will consider a sudden quench and a quench with a short time [see Fig. 1(b)].

B. Quench starting from the Mott insulator: Comparison with the exact diagonalization and the experiment

Let us first focus on the case of a sudden quench. We compare our results of iPEPS with those of the exact diagonalization (ED) method and obtain consistent results in a short time. In the ED simulations using the QuSpin library [52, 53], we choose the system sizes $L_x \times L_y$ up to $5 \times 4$ and use the periodic-periodic boundary condition. We examine to what extent the energy is conserved in the iPEPS simulations. The grand potential density $\langle \hat{H} \rangle$ at $T = 0$ starting from the Mott insulator $\otimes_i |n_i = 1\rangle$ should remain constant. They well converge for $D \geq 6$ and remain nearly constant up to $t \sim 0.4 \hbar / J$ [54]. We also investigate how the single-particle correlations converge with increasing bond dimensions. The equal-time single-particle correlation function at a distance $r = (x, y)$ for the system size $N_s$ is defined as

$$C_{r}^{sp}(t) = \frac{1}{2N_s} \sum_{i,j} \langle \hat{a}_i^\dagger(t) \hat{a}_j(t) + \hat{a}_j^\dagger(t) \hat{a}_i(t) \rangle.$$

(2)
FIG. 2. Comparison of the single-particle correlation functions in the case of a sudden quench between iPEPS (blue lines with symbols) and ED (gray lines). The correlations at distances (a) $|\mathbf{r}| = 1$ and (b) $|\mathbf{r}| = \sqrt{2}$ are shown. Both results overlap perfectly in a short time.

Here $\sum'_{i,j}$ denotes the summation over $(i,j)$ that satisfies $|x_j - x_i| = x$ and $|y_j - y_i| = y$. In the iPEPS simulations, $1/N_s \times \sum'_{i,j}$ is replaced by $1/2 \times \sum_{i=A,B} \sum'_{j}$ with $A$ and $B$ being sublattice sites because of the translational invariance. As shown in Fig. 2, $C_{|\mathbf{r}|=1}(t) := \frac{[C_{(1,0)}^{\text{sp}}(t) + C_{(0,1)}^{\text{sp}}(t)]}{2}$ exhibits a peak at $t \sim 0.15\hbar/J$ in both results, and they overlap perfectly in this short time. For $t \gtrsim 0.15\hbar/J$, the correlation functions of ED start to exhibit a significant finite-size effect, whereas those of iPEPS converge for $D \geq 6$. We observe similar behavior for $C_{|\mathbf{r}|=\sqrt{2}}^{\text{sp}}(t) := C_{(1,1)}^{\text{sp}}(t)$. The iPEPS results are better simulated up to a longer time.
FIG. 3. Comparison of the single-particle correlation functions in the case of a finite-time quench between iPEPS (blue lines with symbols) and the experiment (red circles). The correlations at distances (a) $|r| = 1$, (b) $|r| = 2$, and (c) $|r| = 3$ are shown. Both results are in good agreement.
Next, we compare the correlations of iPEPS with those of the experiment [3] for a finite quench time. Figures 3(a–c) show the time evolution of correlations at distances $|r| = 1, 2,$ and 3, respectively. Qualitative behavior is essentially equivalent to the case of the sudden quench, although the correlation function shifts to an earlier time. For $|r| = 1$, both data show a peak at $t \sim 0.12\hbar/J_{\text{final}}$. Similarly, the first-peak times are consistent with each other for $|r| = 2$ and 3, and they become longer with increasing distances. When the energy is approximately conserved (namely, for $t \lesssim 0.4\hbar/J_{\text{final}}$ [54]), the intensities of correlations also overlap very well. They are also consistent with those obtained by TWA [3, 13, 14], while the iPEPS simulations can deal with a slightly longer time and capture the correlation peaks more clearly [54].

C. Estimates of group and phase velocities in the moderate interaction region

Having confirmed the applicability of iPEPS simulations to real-time evolution of the Bose-Hubbard model, we study how information propagates by a sudden quench in the moderate interaction region. There are two kinds of velocity that are relevant to the correlation spreading. One is the group velocity $v_{gr}$, which corresponds to the propagation of the envelope of the wave packet and is a suitable quantity to characterize the spreading of correlations. In non-relativistic quantum many-body systems, $v_{gr}$ is bounded above, and the upper bound is known as the Lieb-Robinson bound [23, 24, 55, 56]. Notice that the Lieb-Robinson bound for the Bose-Hubbard model has not been rigorously derived with a few exceptions for limited situations [18, 23, 24]. The phase velocity $v_{ph}$ is the other characteristic quantity, which corresponds to the propagation of the first peak of the wave packet, and does not have to obey the Lieb-Robinson bound.

To estimate the group velocity from the single-particle correlations, long-time simulations are required in general. However, it is challenging in the iPEPS simulations. To circumvent the difficulty, we estimate the group velocity by the density-density correlation. It is known that the propagation velocity of the first peak of this correlation agrees very well with the group velocity [1, 12]. The equal-time density-density correlation function at a distance $\mathbf{r} = (x, y)$ for the system size $N_s$ is defined as

$$C_{\mathbf{r}}^{dd}(t) = \frac{1}{N_s} \sum_{i,j}' \langle \hat{n}_i(t) \hat{n}_j(t) \rangle_c,$$

(3)
FIG. 4. (a) Single-particle and (b) density-density correlation functions per bond at $U/J = 5$ for the bond dimension $D = 8$. The normalization factor at a distance $r = (x, y)$ is given as $N_{\text{bond}} = 2$ for $x \neq y$ ($|r| = 1, 2, \text{ and } 3$), while it is $N_{\text{bond}} = 4$ for $x = y$ ($|r| = \sqrt{2}$). The black symbol corresponds to the first peak in the correlation function obtained by cubic spline interpolation of data points. The propagation velocities along the horizontal or vertical axis are extracted by the data at $|r| = 1, 2, \text{ and } 3$. The velocity estimated from the density-density correlation functions is slower than that from the single-particle correlation functions.

where $\langle \cdots \rangle_c$ denotes a connected correlation function. In our simulations, $\langle \hat{n}_i(t)\hat{n}_j(t) \rangle_c = \langle \hat{n}_i(t)\hat{n}_j(t) \rangle - 1$ because $\langle \hat{n}_i(t) \rangle = 1$ for all sites and time steps. As in $C_{r}^{sp}(t)$, the summation is replaced by that within sublattice sites in the iPEPS simulations. The parity-parity correlation closely related to the density-density one can be measured in experiments by using the quantum-gas microscope techniques [1].
FIG. 5. Estimated group (red circles) and phase (blue squares) velocities from the density-density and single-particle correlation functions for the bond dimensions $D = 8$ and $D = 9$. The data for $D = 8$ and $D = 9$ overlap within the error bars. The velocities and their error bars are obtained by extrapolation of the distance dependence of the peak time. The results obtained by the 2PISC approach [15] (triangles), TWA [13] (diamonds), and the experiment [3] (a star) are also shown. Both velocities gradually merge with decreasing interaction.

We extract the propagation velocities from the first peak in both correlations for $|r| = 1$, 2, and 3. For simplicity, we consider the sudden quench hereafter. When the interaction becomes weaker, we have confirmed that the energy is conserved in a longer time frame; typically, $t \lesssim 0.9\hbar/J$ for $U/J \sim 5$ [54]. All the correlation peaks for $|r| \leq 3$ appear in this time frame (see Fig. 4). The first peak of the single-particle correlation appears at $t \sim 0.35\hbar/J$ for $|r| = 1$, while it appears at $t \sim 0.65\hbar/J$ for $|r| = 3$. By contrast, the first peak of the density-density correlation appears at $t \sim 0.25\hbar/J$ for $|r| = 1$, while it appears at $t \sim 0.7\hbar/J$ for $|r| = 3$. It takes a long time for propagation in the latter case. The first-peak time is almost a linear function of the distance [54], and the system exhibits the light-cone-like spreading of correlations.

We summarize the interaction dependence of the group and phase velocities in Fig. 5. In the weak interaction region, the estimated group velocities are $v_{gr} \sim 4J/\hbar$. They are similar to those obtained by the TWA at filling factor $\nu = 10$ [13]. They are also consistent with the
group velocity $v_{gr}(U = 0) = 4J/\hbar$ of a single particle [13]. In the strong interaction region, the estimated group velocity $v_{gr} \sim (8 \pm 2)J/\hbar$ at $U/J = 19.6$ coincides with that obtained by the 2PISC approach [15] within the error bar of extrapolation. It is also comparable to the group velocity $v_{gr}(U \gg J) = 6J/\hbar \times [1 + \mathcal{O}(J^2/U^2)]$ of a quasi-particle in the large $U$ limit [1, 3, 12]. Similarly, the estimated phase velocity agrees very well with the results of the 2PISC approach [15] and the experiment [3]. In the intermediate region, the estimated group velocity is closer to the single-particle group velocity in the superfluid region, whereas it is comparable to the 2PISC result near and above the critical point $U_c/J \sim 16.7$ [57–59]. Both group and phase velocities gradually converge to the same value as $U/J$ is decreased.

III. CONCLUSION

We have studied real-time dynamics of the 2D Bose-Hubbard model after a sudden quench starting from the Mott insulator with unit filling. We have employed the 2D tensor-network method based on the infinite projected entangled pair states, which are the 2D extension of the well-known matrix product states in one dimension. Calculated single-particle correlation functions reproduce the recent experimental results very well. The iPEPS algorithm can simulate real-time dynamics long enough for extracting the propagation velocities from correlations. We have also estimated the group and phase velocities in the moderate interaction region, in which the 2PISC approach and the TWA are not applicable. Our findings would be useful in the future analog quantum simulation of Bose-Hubbard systems. The ability of the tensor-network method that accurately calculates the real-time dynamics of 2D quantum many-body systems opens up the possibility of applying it to other quantum-simulation platforms, such as Rydberg atoms, trapped ions, and superconducting circuits.

IV. METHODS

A. Real-time evolution by infinite projected entangled pair states

We prepare iPEPS with a two-site unit cell [see Fig. 1(a)]. The symbols $D$ and $D_{\text{phys}}$ denote the virtual bond dimension and the dimension of the local Hilbert space, respectively. The former improves the accuracy of the wave function, whereas the latter corresponds to the maximum particle number $n_{\text{max}}$ as $D_{\text{phys}} = n_{\text{max}} + 1$. Although $n_{\text{max}}$ can take infinity in
Bose-Hubbard systems, it is practically bounded above in the presence of interaction [60, 61]. We can choose finite $D_{\text{phys}}$ in the simulations of real-time dynamics. In the case of a sudden quench to the Mott insulating region ($U/J > U_c/J \sim 16.7$ [57–59]), we set the dimension of the local Hilbert space as $D_{\text{phys}} = 3$ because the number of particles deviates only slightly from unity [60, 61]. For $U/J < U_c/J$, we choose $D_{\text{phys}} = 5$ so that the wave functions can further take into account the effect of particle fluctuations. When $U$ is close to zero (at $U/J = 2$ in our simulations), we use slightly larger $D_{\text{phys}} = 7$. The initial Mott insulating state $\otimes_i |n_i = 1\rangle$ can be represented with the bond dimension $D = 1$. As for static properties, the Bose-Hubbard model was investigated by finite PEPS or iPEPS, and the phase transition between the Mott insulating and superfluid phases was reproduced [62–70].

The wave function at each time $|\Psi(t)\rangle = e^{-idt\hat{H}/\hbar}|\Psi(0)\rangle$ is obtained by real-time evolving iPEPS [38–40]. The real-time evolution operator in a small time step $dt$ can be approximated by the Suzuki-Trotter decomposition [71–73] as $e^{-idt\hat{H}/\hbar} \sim \prod_{(ij)} e^{-idt\hat{H}_{ij}/\hbar}$, where

$$
\hat{H}_{ij} = -J(\hat{a}_i^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_i) + U[\hat{n}_i(\hat{n}_i - 1) + \hat{n}_j(\hat{n}_j - 1)]/(2z) - \mu(\hat{n}_i + \hat{n}_j)/z
$$

with the coordination number $z = 4$ is the local Hamiltonian satisfying $\hat{H} = \sum_{(ij)} \hat{H}_{ij}$. After applying the two-site gate $e^{-idt\hat{H}_{ij}/\hbar}$ to neighboring tensors, we approximate the local tensors by the singular value decomposition in such a way that the virtual bond dimension of iPEPS remains $D$. In the actual simulations, the second-order Suzuki-Trotter decomposition is used for this simple update algorithm [29, 74], and the tensor-network library TeNeS [75, 76] is adopted. The wave functions are optimized up to the bond dimension $D = 9$. As we will see later, qualitative behavior of correlation functions is nearly the same for $D \geq 6$. When extracting the propagation velocities, we mainly use the data for $D = 8$ and $D = 9$ to ensure sufficient convergence of physical quantities. We have numerically found that at these values of $D$, the number of particles is nearly conserved during the real-time evolution starting from the Mott insulator.

Physical quantities in the thermodynamic limit are calculated by the corner transfer matrix renormalization group (CTMRG) method [30–32, 34, 77–83]. The bond dimension of the environment tensors is chosen as $\chi = 2D^2$ to ensure that physical quantities are well converged.

To compare our results obtained by iPEPS with the experiment [3], we consider a quench with a short time $\tau_Q = 0.1\text{ms}$ [13, 14] [see Fig. 1(b)]. For $-\tau_Q < t < 0$, both $J$ and $U$ are controlled. The wave function is updated as $|\Psi(t + dt)\rangle \sim e^{-idt\hat{H}(t)/\hbar}|\Psi(t)\rangle$ with the time-
dependent Hamiltonian $\hat{H}(t)$ in this region. For $t > 0$, both parameters are fixed. We take $J_{\text{final}} = J(t = 0) \sim 0.0612 \hbar/\tau_Q$ as the unit of energy. The discrete time step for the real-time evolution is set to be $dt/(\hbar/J_{\text{final}}) = \tau_Q/(\hbar/J_{\text{final}})/15 \sim 0.00408$ for all $t$. To compare the iPEPS results with the exact real-time dynamics in finite-size systems, we also consider a sudden parameter change and set the time step as $dt/(\hbar/J) = 0.005$. We have checked that the simulations with doubled and halved $dt$ do not change the results significantly.

V. DATA AVAILABILITY

The data in this paper are available from the authors upon request.

VI. CODE AVAILABILITY

The codes in this paper are available from the authors upon request.

[1] M. Cheneau, P. Barmettler, D. Poletti, M. Endres, P. Schauß, T. Fukuahara, C. Gross, I. Bloch, C. Kollath, and S. Kuhr, Light-cone-like spreading of correlations in a quantum many-body system, Nature 481, 484 (2012).

[2] P. Jurcevic, B. P. Lanyon, P. Hauke, C. Hempel, P. Zoller, R. Blatt, and C. F. Roos, Quasiparticle engineering and entanglement propagation in a quantum many-body system, Nature 511, 202 (2014).

[3] Y. Takasu, T. Yagami, H. Asaka, Y. Fukushima, K. Nagao, S. Goto, I. Danshita, and Y. Takahashi, Energy redistribution and spatio-temporal evolution of correlations after a sudden quench of the bose-hubbard model, Sci. Adv. 6, eaba9255 (2020).

[4] S. Trotzky, Y.-A. Chen, A. Flesch, I. P. McCulloch, U. Schollwöck, J. Eisert, and I. Bloch, Probing the relaxation towards equilibrium in an isolated strongly correlated one-dimensional bose gas, Nat. Phys. 8, 325 (2012).

[5] T. Langen, S. Erne, R. Geiger, B. Rauer, T. Schweigler, M. Kuhnert, W. Rohringer, I. E. Mazets, T. Gasenzer, and J. Schmiedmayer, Experimental observation of a generalized gibbs ensemble, Science 348, 207 (2015).
[6] A. M. Kaufman, M. E. Tai, A. Lukin, M. Rispoli, R. Schittko, P. M. Preiss, and M. Greiner, 
Quantum thermalization through entanglement in an isolated many-body system, Science 353, 
794 (2016).

[7] M. Schreiber, S. S. Hodgman, P. Bordia, H. P. Lüschen, M. H. Fischer, R. Vosk, E. Altman, 
U. Schneider, and I. Bloch, Observation of many-body localization of interacting fermions in 
a quasirandom optical lattice, Science 349, 842 (2015).

[8] J.-y. Choi, S. Hild, J. Zeiher, P. Schauß, A. Rubio-Abadal, T. Yefsah, V. Khemani, D. A. Huse, 
I. Bloch, and C. Gross, Exploring the many-body localization transition in two dimensions, 
Science 352, 1547 (2016).

[9] J. Smith, A. Lee, P. Richerme, B. Neyenhuis, P. W. Hess, P. Hauke, M. Heyl, D. A. Huse, 
and C. Monroe, Many-body localization in a quantum simulator with programmable random 
disorder, Nat. Phys. 12, 907 (2016).

[10] H. Bernien, S. Schwartz, A. Keesling, H. Levine, A. Omran, H. Pichler, S. Choi, A. S. Zibrov, 
M. Endres, M. Greiner, V. Vuletić, and M. D. Lukin, Probing many-body dynamics on a 
51-atom quantum simulator, Nature 551, 579 (2017).

[11] C. J. Turner, A. A. Michailidis, D. A. Abanin, M. Serbyn, and Z. Papić, Weak ergodicity 
breaking from quantum many-body scars, Nat. Phys. 14, 745 (2018).

[12] P. Barmettler, D. Poletti, M. Cheneau, and C. Kollath, Propagation front of correlations in 
an interacting bose gas, Phys. Rev. A 85, 053625 (2012).

[13] K. Nagao, M. Kunimi, Y. Takasu, Y. Takahashi, and I. Danshita, Semiclassical quench dy-
namics of bose gases in optical lattices, Phys. Rev. A 99, 023622 (2019).

[14] K. Nagao, Y. Takasu, Y. Takahashi, and I. Danshita, Su(3) truncated wigner approximation 
for strongly interacting bose gases, arXiv:2008.09900.

[15] A. Mokhtari-Jazi, M. R. C. Fitzpatrick, and M. P. Kennett, Phase and group velocities for 
correlation spreading in the mott phase of the bose-hubbard model in dimensions greater than 
one, Phys. Rev. A 103, 023334 (2021).

[16] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Quantum phase transition 
from a superfluid to a mott insulator in a gas of ultracold atoms, Nature 415, 39 (2002).

[17] A. M. Läuchli and C. Kollath, Spreading of correlations and entanglement after a quench in 
the one-dimensional bose–hubbard model, J. Stat. Mech. Theory Exp. 2008, P05018 (2008).
[18] N. Schuch, S. K. Harrison, T. J. Osborne, and J. Eisert, Information propagation for interacting-particle systems, Phys. Rev. A 84, 032309 (2011).

[19] G. Carleo, F. Becca, L. Sanchez-Palencia, S. Sorella, and M. Fabrizio, Light-cone effect and supersonic correlations in one- and two-dimensional bosonic superfluids, Phys. Rev. A 89, 031602(R) (2014).

[20] L. Cevolani, J. Despres, G. Carleo, L. Tagliacozzo, and L. Sanchez-Palencia, Universal scaling laws for correlation spreading in quantum systems with short- and long-range interactions, Phys. Rev. B 98, 024302 (2018).

[21] M. R. C. Fitzpatrick and M. P. Kennett, Light-cone-like spreading of single-particle correlations in the bose-hubbard model after a quantum quench in the strong-coupling regime, Phys. Rev. A 98, 053618 (2018).

[22] J. Despres, L. Villa, and L. Sanchez-Palencia, Twofold correlation spreading in a strongly correlated lattice Bose gas, Sci. Rep. 9, 4135 (2019).

[23] Z. Wang and K. R. A. Hazzard, Tightening the lieb-robinson bound in locally interacting systems, PRX Quantum 1, 010303 (2020).

[24] T. Kuwahara and K. Saito, Lieb-robinson bound and almost-linear light cone in interacting boson systems, Phys. Rev. Lett. 127, 070403 (2021).

[25] M. A. Martín-Delgado, M. Roncaglia, and G. Sierra, Stripe ansätze from exactly solved models, Phys. Rev. B 64, 075117 (2001).

[26] F. Verstraete and J. I. Cirac, Renormalization algorithms for quantum-many body systems in two and higher dimensions, arXiv:cond-mat/0407066 .

[27] F. Verstraete and J. I. Cirac, Valence-bond states for quantum computation, Phys. Rev. A 70, 060302(R) (2004).

[28] F. Verstraete, V. Murg, and J. I. Cirac, Matrix product states, projected entangled pair states, and variational renormalization group methods for quantum spin systems, Adv. Phys. 57, 143 (2008).

[29] J. Jordan, R. Orús, G. Vidal, F. Verstraete, and J. I. Cirac, Classical simulation of infinite-size quantum lattice systems in two spatial dimensions, Phys. Rev. Lett. 101, 250602 (2008).

[30] H. N. Phien, J. A. Bengua, H. D. Tuan, P. Corboz, and R. Orús, Infinite projected entangled pair states algorithm improved: Fast full update and gauge fixing, Phys. Rev. B 92, 035142 (2015).
[31] R. Orús, *A practical introduction to tensor networks: Matrix product states and projected entangled pair states*, Ann. Phys. (N.Y.) **349**, 117 (2014).

[32] R. Orús, *Tensor networks for complex quantum systems*, Nat. Rev. Phys. **1**, 538 (2019).

[33] Y. Hieida, K. Okunishi, and Y. Akutsu, *Numerical renormalization approach to two-dimensional quantum antiferromagnets with valence-bond-solid type ground state*, New J. Phys. **1**, 7 (1999).

[34] K. Okunishi and T. Nishino, *Kramers-wannier approximation for the 3d ising model*, Prog. Theor. Phys. **103**, 541 (2000).

[35] T. Nishino, Y. Hieida, K. Okunishi, N. Maeshima, Y. Akutsu, and A. Gendiar, *Two-dimensional tensor product variational formulation*, Prog. Theor. Phys. **105**, 409 (2001).

[36] N. Maeshima, Y. Hieida, Y. Akutsu, T. Nishino, and K. Okunishi, *Vertical density matrix algorithm: A higher-dimensional numerical renormalization scheme based on the tensor product state ansatz*, Phys. Rev. E **64**, 016705 (2001).

[37] Y. Nishio, N. Maeshima, A. Gendiar, and T. Nishino, *Tensor product variational formulation for quantum systems*, arXiv:cond-mat/0401115.

[38] A. Kshetrimayum, H. Weimer, and R. Orús, *A simple tensor network algorithm for two-dimensional steady states*, Nat. Commun. **8**, 1 (2017).

[39] P. Czarnik, J. Dziarmaga, and P. Corboz, *Time evolution of an infinite projected entangled pair state: An efficient algorithm*, Phys. Rev. B **99**, 035115 (2019).

[40] C. Hubig and J. I. Cirac, *Time-dependent study of disordered models with infinite projected entangled pair states*, SciPost Phys. **6**, 31 (2019).

[41] A. Kshetrimayum, M. Goihl, and J. Eisert, *Time evolution of many-body localized systems in two spatial dimensions*, Phys. Rev. B **102**, 235132 (2020).

[42] A. Kshetrimayum, M. Goihl, D. M. Kennes, and J. Eisert, *Quantum time crystals with programmable disorder in higher dimensions*, Phys. Rev. B **103**, 224205 (2021).

[43] C. Hubig, A. Bohrdt, M. Knap, F. Grusdt, and J. I. Cirac, *Evaluation of time-dependent correlators after a local quench in ipeps: hole motion in the t-j model*, SciPost Phys. **8**, 21 (2020).

[44] Álvaro M. Alhambra and J. I. Cirac, *Locally accurate tensor networks for thermal states and time evolution*, arXiv:2106.00710.
[45] M. Schmitt, M. M. Rams, J. Dziarmaga, M. Heyl, and W. H. Zurek, *Quantum phase transition dynamics in the two-dimensional transverse-field ising model*, arXiv:2106.09046.

[46] J. Dziarmaga, *Time evolution of an infinite projected entangled pair state: Neighborhood tensor update*, Phys. Rev. B 104, 094411 (2021).

[47] H. Weimer, A. Kshetrimayum, and R. Orús, *Simulation methods for open quantum many-body systems*, Rev. Mod. Phys. 93, 015008 (2021).

[48] C. Mc Keever and M. H. Szynańska, *Stable ipepo tensor-network algorithm for dynamics of two-dimensional open quantum lattice models*, Phys. Rev. X 11, 021035 (2021).

[49] D. Kilda, A. Biella, M. Schiro, R. Fazio, and J. Keeling, *On the stability of the infinite Projected Entangled Pair Operator ansatz for driven-dissipative 2D lattices*, SciPost Phys. Core 4, 5 (2021).

[50] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, *Boson localization and the superfluid-insulator transition*, Phys. Rev. B 40, 546 (1989).

[51] D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, *Cold bosonic atoms in optical lattices*, Phys. Rev. Lett. 81, 3108 (1998).

[52] P. Weinberg and M. Bukov, *QuSpin: a Python Package for Dynamics and Exact Diagonalisation of Quantum Many Body Systems part I: spin chains*, SciPost Phys. 2, 003 (2017).

[53] P. Weinberg and M. Bukov, *QuSpin: a Python Package for Dynamics and Exact Diagonalisation of Quantum Many Body Systems. Part II: bosons, fermions and higher spins*, SciPost Phys. 7, 20 (2019).

[54] See Supplemental Materials.

[55] E. H. Lieb and D. W. Robinson, *The finite group velocity of quantum spin systems*, Commun. Math. Phys. 28, 251 (1972).

[56] M. B. Hastings, *Locality in quantum systems*, arXiv:1008.5137.

[57] N. Elstner and H. Monien, *Dynamics and thermodynamics of the bose-hubbard model*, Phys. Rev. B 59, 12184 (1999).

[58] B. Capogrosso-Sansone, Ş. G. Söyler, N. Prokof’ev, and B. Svistunov, *Monte carlo study of the two-dimensional bose-hubbard model*, Phys. Rev. A 77, 015602 (2008).

[59] K. V. Krutitsky, *Ultracold bosons with short-range interaction in regular optical lattices*, Phys. Rep. 607, 1 (2016).
[60] S. D. Huber, E. Altman, H. P. Büchler, and G. Blatter, *Dynamical properties of ultracold bosons in an optical lattice*, Phys. Rev. B 75, 085106 (2007).

[61] S. M. Davidson and A. Polkovnikov, *su(3) semiclassical representation of quantum dynamics of interacting spins*, Phys. Rev. Lett. 114, 045701 (2015).

[62] V. Murg, F. Verstraete, and J. I. Cirac, *Variational study of hard-core bosons in a two-dimensional optical lattice using projected entangled pair states*, Phys. Rev. A 75, 033605 (2007).

[63] J. Jordan, R. Orús, and G. Vidal, *Numerical study of the hard-core bose-hubbard model on an infinite square lattice*, Phys. Rev. B 79, 174515 (2009).

[64] A. Kshetrimayum, M. Rizzi, J. Eisert, and R. Orús, *Tensor network annealing algorithm for two-dimensional thermal states*, Phys. Rev. Lett. 122, 070502 (2019).

[65] S. S. Jahromi and R. Orús, *Universal tensor-network algorithm for any infinite lattice*, Phys. Rev. B 99, 195105 (2019).

[66] S. S. Jahromi and R. Orús, *Thermal bosons in 3d optical lattices via tensor networks*, Sci. Rep. 10, 19051 (2020).

[67] P. Schmoll, S. S. Jahromi, M. Hörmann, M. Mühlhauser, K. P. Schmidt, and R. Orús, *Fine grained tensor network methods*, Phys. Rev. Lett. 124, 200603 (2020).

[68] W.-L. Tu, H.-K. Wu, and T. Suzuki, *Frustration-induced supersolid phases of extended bose–hubbard model in the hard-core limit*, J. Phys.: Cond. Mat. 32, 455401 (2020).

[69] H.-K. Wu and W.-L. Tu, *Competing quantum phases of hard-core bosons with tilted dipole-dipole interaction*, Phys. Rev. A 102, 053306 (2020).

[70] P. C. G. Vlaar and P. Corboz, *Simulation of three-dimensional quantum systems with projected entangled-pair states*, Phys. Rev. B 103, 205137 (2021).

[71] H. F. Trotter, *On the product of semi-groups of operators*, Proc. Amer. Math. Soc. 10, 545 (1959).

[72] M. Suzuki, *Pair-product model of heisenberg ferromagnets*, J. Phys. Soc. Jpn. 21, 2274 (1966).

[73] M. Suzuki, *Relationship between d-dimensional quantal spin systems and (d+ 1)-dimensional ising systems: Equivalence, critical exponents and systematic approximants of the partition function and spin correlations*, Prog. Theor. Phys. 56, 1454 (1976).

[74] H. C. Jiang, Z. Y. Weng, and T. Xiang, *Accurate determination of tensor network state of quantum lattice models in two dimensions*, Phys. Rev. Lett. 101, 090603 (2008).
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VII. AUTHOR CONTRIBUTIONS

R.K. and I.D. designed and coordinated the studies. R.K. performed the numerical simulations. R.K. and I.D. contributed to the writing of the manuscript.
VIII. COMPETING INTERESTS

The authors declare no competing interests.

IX. ADDITIONAL INFORMATION

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