Defining the parameters of a power transmission line equivalent circuit on the basis of phasor measurements

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Abstract. Power system control is based on employing its computational model, the backbone of which is an equivalent circuit. The equivalent circuit is involved in solving the problems of steady-state computation and analysis, state estimation, transient analysis etc. Its elements parameters are generally defined by the corresponding equipment reference data or datasheets. Although considered to be constant, these parameters depend upon the element actual load, weather conditions and other factors. Consequently, the results of the outlined problems may be subject to significant bias due to the difference between the reference and the actual elements parameters. Hence the task of identifying the actual equivalent circuit parameters is of paramount importance. In terms of power system control the actual measurements are to be used in order to provide the relevant information on the considered power system component state. As for the transmission lines, the state measurements must include the currents and voltages at both ends of the line regarding the Π-shaped equivalent circuit. That said, at present time the equivalent circuit parameters might be defined involving modern systems of phasor measurements (WAMS) on a real-time basis. The method of defining the equivalent circuit parameters based on phasor measurements along with general relations between power system state parameters is proposed. It should be noted as well that while dealing with the actual WAMS data obtained from power system the measurement errors influence the results quite substantially.

1. Introduction

Most aspects of power system control rely on its computational model. The model, in turn, comprises the equivalent circuit for most power system components, parameters of which are traditionally defined according to reference values or datasheets and are generally assumed to be constant. Nevertheless, they depend upon actual operating and weather conditions, may be influenced by the adjacent equipment, may vary with equipment deterioration etc. As for the power transmission lines, the corresponding investigation [1] shows that the errors of defining their equivalent circuit parameters (longitudinal impedance \( Z = R + jX \) and shunt conductance \( Y = G + jB \) ) according to reference data is subject to significant errors, the results of the research are presented in table 1.

The transmission line equivalent circuit is involved in solving the problems of steady-state computation [2] and analysis, state estimation, transient analysis [3] and numerous other applications [4, 5]. One of the steady-state operation criteria is a static stability margin [6]. The stability limit of the line is generally derived from its operating conditions on the basis of its equivalent circuit. Excessive stability margins are undesirable due to several reasons: limited transmission capacity may result in extra capital
costs for additional generation or transmission capacity. Moreover, in case of power deficiency its price may significantly increase in certain area. At the same time insufficient stability margin limits the stability and reliability of the power system. Refining the equivalent circuit parameters allows to re-determine the stability margins [7] while ensuring the proper levels of stability and reliability.

Refining the equivalent circuit parameters has positive effect on electric power losses calculation since the process is based on the computational model as well.

Table 1. The errors of transmission line equivalent circuit parameters reference values.

| Parameter | Reference value | Reference value error range | Reference value error cause |
|-----------|-----------------|-----------------------------|----------------------------|
|          |                 |                             |                            |
| $R$       |                 | $-24…+8\%$                 | Neglecting weather conditions and skin effect |
| $X$       |                 | $-10…+8\%$                 | Uncertainty of equivalent dimensions; multiply grounded lightning protection cables and parallel lines |
| $G$       |                 | $25…30\%$                  | Neglecting wire sag and grounded lightning protection cable effects, grounding impedance and air humidity |
| $B$       |                 | $50…200\%$                 | Construction design, operating and weather conditions |

Precise parameters of transmission lines equivalent circuit have positive impact on transient analysis as the more accurate the model is – the more accurate results may be obtained by utilizing it.

In addition to the outlined fields the actual transmission lines parameters may find wide application for wires temperature control, fault location [8], defining automated protection systems thresholds [9] etc. Hence the problem of defining the valid equivalent circuit parameters is of crucial importance in terms of power system controllability, stability and reliability.

2. Defining the parameters of a transmission line equivalent circuit

Various researchers have investigated the problem at hand [7–11]. Solving the stated problem implies following one of the two approaches. The first of them is based on utilizing simultaneously the measurements of voltages and currents obtained from both ends of the line in two different system states [10]. Measurements in two states are involved since the line is considered as a quadrilateral regarding its Π-shaped equivalent circuit. The second approach presumes investigating the measurements of a single system state. Utilizing synchronized phasor measurements allows developing the algorithm for defining the transmission line parameters that relies on the fundamental relations between the voltages and currents at the ends of the transmission line. Moreover, various algorithms are being developed for the purpose of identifying the power system equipment parameters based on instantaneous electrical state measurements arrays [12–14] collected from, for example, digital fault recorders (DFRs) installed at power system entities.

3. The proposed method

Wide-area monitoring systems implementing the concept of synchronized phasor measurements enable obtaining the direct measurements of voltages and currents from both ends of the transmission line and also the derived values of active and reactive power. These parameters correspond to the set used in steady-state analysis software etc. so the actual measurements are convenient to compare with the computational models. Although it may be useful in particular applications, the three-phase equivalent circuit is rarely employed so the basic one-line equivalent circuit is considered from this point on, with few exceptions. The traditional Π-shaped equivalent circuit comprises a longitudinal impedance $Z = R + jX$ and shunt conductance $Y = G + jB$ and the measurements are distributed as shown in figure 1.

The following is correct for the equivalent circuit:
\[ I_{12} = I_1 - U_1 \cdot Y = I_2 + U_2 \cdot Y. \] (1)

Consequently,

\[ Y = \frac{I_1 - I_2}{U_1 + U_2}, \] (2)

and the equivalent circuit shown in figure 2 may be put in correspondence with equation (2).

That said, the shunt conductance \( Y = G + jB \) is obtained according to equation (2) and hence the longitudinal impedance \( Z \) is derived as

\[ Z = R + jX = \frac{U_1 - U_2}{I_{12}} = \frac{U_1 - U_2}{I_1 - U_1 \cdot Y}. \] (3)

The proposed method has been validated involving a set of simulated data and employed in order to obtain the actual parameters of a transmission line in operation, as described in the next section.

4. Method validation
The developed method validation includes two stages: validation involving simulated data and validation involving actual data collected from the WAMS system installed in the Unified Power System of Russia.

4.1. Validation based on a set of simulated data
The simulated data under consideration has been obtained by means of MATLAB software package (version R2009b) and Simulink module. The 500 kV transmission line connecting the buses Bus 1 and Bus 2 with \( Z = 5 + j59.66 \Omega \) and \( Y = 2 - j880 \mu S \) has been modelled. Two different operation states have been considered as well: the low-load and the high-load states with the corresponding power flows from Bus 1 to Bus 2 of \( \text{MVA}_1 = 500 \) and \( \text{MVA}_2 = 100MVA \). The simulation results are presented in table 2. As the line and the load are symmetrical all the measurements of voltages and currents are listed for phase A.

Similar to the simulation results, the equivalent circuit parameters are defined for a single phase. The derived parameters are \( Z = 5.021 + j59.679 \Omega \), \( Y = 2.112 - j879.665 \mu S \) and \( Z = 5.022 + j59.603 \Omega \), \( Y = 2.182 - j877.926 \mu S \) for two states, correspondingly. The errors appear to be less than 1 % for all of the parameters with the exception of \( \approx 10 \% \) for active conductance.

It can be seen that the equivalent circuit parameters may be obtained directly from phasor measurements with practically appropriate accuracy which is a major step forward compared to the reference data.
### Table 2. Simulation results.

|                  | Voltage, V      | Current, A       |
|------------------|-----------------|------------------|
| **Low-load state (60 + j500 MVA)** |                 |                  |
| Bus 1            | 293876.14∠0.06° | 296.75∠−77.79°   |
| Bus 2            | 268538.90∠−0.29° | 540.93∠−83.45°   |
| **High-load state (500 + j100 MVA)** |                 |                  |
| Bus 1            | 293362.63∠−0.01° | 580.64∠7.13°     |
| Bus 2            | 289111.05∠−6.79° | 589.67∠−18.10°   |

#### 4.2. Validation based on actual WAMS data

The next stage of the developed algorithm validation involves actual data collected from the WAMS system deployed in the Unified Power System of Russia. The test cases of a single transmission line and two adjacent transmission lines were considered.

**4.2.1. Single transmission line.** First, the equivalent circuit parameters have been defined for the 500 kV transmission line of roughly 200 km length with 993 MW power flow. The line is equipped with the phasor measurement units (PMUs) at both ends. The considered measurements include voltages and currents phasors of three phases. The scheme of the PMUs installation is shown in figure 3. The results (for phases A, B and C) are shown in table 3. There are expected values (EV) and standard deviations (STD) for the time span of 10 minutes.

It can be seen that the situation is close to the one with the simulated data. The relative errors for the parameters are quite significant for active conductance and longitudinal resistance while reactive parameters are defined rather accurately. The significant errors are associated with the nature of the active conductance, as outlined in the Introduction, and the initial measurement errors. At this point it is worth noting that the measurement errors could not be corrected without measurements excessiveness and this leads to the next section where the two adjacent transmission lines are considered.

**4.2.2. Two adjacent transmission lines.** WAMS development allows to extend the algorithm of defining the transmission lines equivalent circuit parameters based on synchronized phasor measurements obtained not from the single line but from the group of lines. The measurements excessiveness is utilized in order to achieve higher precision of the defined parameters.

Two adjacent 750 kV transmission lines are considered in this section. The PMUs are installed at both ends of each line as shown in figure 4, the total of four 10-minute data sets was collected in order to run the algorithm. The results are shown in table 4. There are relative errors for the defined parameters as well.
Table 3. The defined equivalent circuit parameters.

| Parameter (reference value) | EV and STD (for three phases) |
|-----------------------------|------------------------------|
| $R, \Omega$ (5.42)         | A        | B        | C        |
| $X, \Omega$ (68.30)        | 6.001    | 1.351    | 1.272    |
|                            | 0.102    | 0.0855   | 0.101    |
| $G, S$ (7×10⁻⁴)           | 63.476   | 65.050   | 64.186   |
|                            | 0.0519   | 0.0503   | 0.0508   |
| $B, S$ (−9.830×10⁻⁴)      | 1.205×10⁻⁴ | 1.498×10⁻⁴ | 1.143×10⁻⁴ |
|                            | 1.870×10⁻⁶ | 1.429×10⁻⁶ | 1.366×10⁻⁶ |
|                            | -9.445×10⁻⁴ | -9.595×10⁻⁴ | -9.132×10⁻⁴ |
|                            | 4.370×10⁻⁶ | 4.374×10⁻⁶ | 4.497×10⁻⁶ |

Table 4. The defined equivalent circuit parameters.

| Parameter (reference value) | The defined values for two lines (relative error, %) |
|-----------------------------|-----------------------------------------------------|
| $R, \Omega$ (9.8, 2.66)    | Line 1-2                                           |
|                            | 487,555 (4875 %)                                   |
|                            | Line 2-3                                           |
|                            | 2,741 (3 %)                                        |
| $X, \Omega$ (112, 35.34)   | 162,661 (45 %)                                     |
|                            | 78,693 (123 %)                                     |
| $G, \mu S$ (0, 0)          | 11552                                             |
|                            | 49                                                |
| $B, \mu S$ (−480, −477)    | -1187 (-19 %)                                      |
|                            | -10956 (-2197 %)                                   |

Table 5. Initial measurements errors correction.

| Parameter (reference value) | EV and STD (for three phases) |
|-----------------------------|-------------------------------|
|                            | PMU 1-2 | PMU 2-1 | PMU 2-3 | PMU 3-2 |
| Voltage magnitude, %        | 0.50    | -0.43   | 0.50    | -0.46   |
| Voltage angle, degrees      | -0.48   | 0.45    | 0.45    | 0.50    |
| Current magnitude, %        | 0.31    | 0.50    | 0.50    | 0.35    |
| Current angle, degrees      | 0.50    | -0.40   | -0.39   | 0.50    |

Table 6. The defined equivalent circuit parameters.

| Parameter (reference value) | The defined values for two lines (relative error, %) |
|-----------------------------|-----------------------------------------------------|
|                            | Line 1-2                                           |
| $R, \Omega$ (9.8, 2.66)    | 35,211 (259 %)                                      |
|                            | Line 2-3                                           |
| $X, \Omega$ (112, 35.34)   | 181,320 (62 %)                                      |
| $G, \mu S$ (0, 0)          | 1287                                             |
| $B, \mu S$ (−480, −477)    | -1885 (27 %)                                       |

- Table 3: The defined equivalent circuit parameters.
- Table 4: The defined equivalent circuit parameters.
- Table 5: Initial measurements errors correction.
- Table 6: The defined equivalent circuit parameters.
According to the relative error values, only a few parameters may be considered correct. The relative errors significantly exceed the deviations of the actual parameters from their references listed in table 1. This allows for the conclusion that the result cannot be treated as the parameters refinement.

Although the results are quite poor, the measurements excessiveness enables automatic correction of initial data systematic errors. The criterion function of the parameters errors sum is minimized employing the nonlinear optimization approach. At that, the imposed limitations correspond to the instrument transformers rated accuracy of 0.5 class [15, 16]. The optimization results are shown in table 5. The equivalent circuit parameters defined after the initial data correction (“re-defined”) are presented in table 6.

One can see that the relative errors have been significantly decreased and now they are close to the limits outlined in table 1. We also have to keep in mind that limiting the error by instrument transformer class 0.5 is quite optimistic since the actual errors may exceed the rated values [17, 18]; in case the error is not limited in that way the accuracy of defining the equivalent circuit parameters reaches 10 %, which indicates the result may be treated as the reference parameters refinement.

5. Conclusion
The problem of defining the actual parameters of a transmission line equivalent circuit is relevant. The computational algorithm of appropriate accuracy has been developed and validated. The proposed algorithm may find extensive industrial application in various electrical engineering problems.

Evolution and expansion of modern measurement devices based on the concept of synchronized phasor measurements offer the potential to investigate the power system equipment and performance on a deeper level with the outcome of a more efficient and economical operation. The field of application of modern measurement systems includes but is not limited to defining the actual equipment parameters. Although the results of the proposed algorithm validation hold much promise, there are barriers, such as measurement errors and insufficient quantity of measurement devices, hindering the development in the outlined direction.

With this in mind the general suggestion for the PMUs to be installed is utilizing the instrument transformers of higher accuracy, which would allow to implement the algorithm of defining the transmission line equivalent circuit parameters. The secondary benefit would be the possibility to detect and correct the systematic measurement errors by solely software means.

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Acknowledgments
The work was supported by Act 211 Government of the Russian Federation, contract № 02.A03.21.0006 and the Ministry of Education and Science of Russian Federation (in the framework of state assignment, № 13.1928.2014/K (project № 1928)).