Residential Demand Response-Based Load-Shifting Scheme to Increase Hosting Capacity in Distribution System

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ABSTRACT Increasing the use of solar photovoltaic (PV) generation in order to decarbonize the electric energy system results in many challenges. Overvoltage is one of the most common problems in distribution systems with high penetration of solar PV. Utilizing demand-side resources such as residential demand response (RDR) have the potential to alleviate this problem. To increase the solar PV hosting capacity, we propose an RDR based load-shifting scheme that utilizes the interaction between the distribution system operator (DSO) and demand-side resources. We first model a customer utility that consists of the cost of purchasing power, revenue from the subsidy, and discomfort due to load shifting. When an overvoltage problem is expected, DSO issues a local subsidy, and customers in the distribution system move their load in response. An optimization framework that minimizes the additional cost due to the subsidy while keeping the voltages in a prescribed range is proposed. Because of the non-linearity of the power flow analysis, we propose a sub-optimal algorithm to obtain a subsidy, prove the performance gap between the optimal subsidy and the subsidy obtained by the algorithm. A case study shows that the proposed RDR scheme increases the hosting capacity to almost its theoretical limit at a lower cost than the curtailment method.

INDEX TERMS Residential demand response, hosting capacity, distribution system operator, renewable energy.

NOMENCLATURE

N Set of buses.
D Set of PV installed buses.
n Index of bus.
i Index of customer.
Mn Set of customers in bus n.
m Index of customer in bus Mn.
T Set of time for one day.
t Index of period.
p the electricity purchasing price at t.
Vn Phasor voltage at bus n at t.
Pnetn Net real power at bus n at t.
Qnetn Net reactive power at bus n at t.
Pn Real power of generator at bus n at t.
Qn Reactive power of generator at bus n at t.
Ploadn Real power of load at bus n at t.
Qloadn Reactive power of load at bus n at t.
HCn PV hosting capacity at bus n.
ρn Efficiency of PV generation at bus n at t.
Vn Voltage magnitude at bus n at t.
δn Phase angle at bus n at t.
Ybus Admittance matrix.
Ynk Admittance between buses n and k.
Gnk Conductance between buses n and k.
Bnk Susceptance between buses n and k.
Vmin Lower bound of the voltage regulation.
Vmax Upper bound of the voltage regulation.
p the Subsidy at t.
I. INTRODUCTION

Climate change caused by greenhouse gases is one of the biggest challenges facing the world today. Therefore, many countries such as the USA, EU, and Korea have made commitments to net-zero emissions by 2050 and beyond. As a large component of these efforts, many countries are setting aggressive goals for renewable electric energy and electrification in all sectors [1]. Specifically, greater use of renewable sources such as wind and solar photovoltaic (PV) generation is the leading strategy to help decarbonize the electric energy power sector [2]. For example, in 2019, more than 70 % of the newly installed generators used renewable energy [3], and renewable energy (including hydro) accounted for 27 % of the total electrical energy. Solar PV has the largest share in newly added renewable energy. In 2019, solar PV contributed 115 GW of the total renewable capacity of 200 GW [4]. While other renewable generators, such as wind and hydro, are largely restricted to centralized, utility scale deployment, solar PV is more versatile and amenable to deployment in distribution systems [5]. However, the addition of distributed solar PV sources beyond the distribution system hosting capacity (HC) [1] causes severe problems in the distribution system as it is designed for a unidirectional power flow. Among these, overvoltage [7] is the most common power quality problem. Therefore, if DSO can alleviate the overvoltage problem, the distribution system can install more solar PV, i.e., an increase in HC [8].

Conventionally, DSOs have used their resources to solve the overvoltage problem. For example, voltage and reactive power control methods using legacy devices such as on-load tap changer (OLTC), capacitor banks, and static VAR compensator (SVC). However, with only legacy devices, voltage violations in the distribution system might occur for a short period because of their delayed response [9]. Therefore, advanced voltage control methods are required that promptly react for the voltage deviation. These new methods include using smart inverters installed at solar PV and energy storage systems (ESS). Recently, another approach to improve HC, which uses customer resources, has been in the spotlight. The recent rapid development in information and communication technologies (ICT) makes it possible to engage customers in grid operations such as the residential demand response (RDR) [10]. As a result, DSOs can reduce or shift customers’ load for a reliable operation of the distribution system [11], [12].

In this work, we propose an RDR based load-shifting scheme to increase HC in distribution systems. The proposed RDR scheme issues a subsidy to shift load only for times that the overvoltage is expected. In response to the subsidy, customers in the distribution system move their load, thereby resolving the overvoltage problem, thus increasing HC. The main features of the proposed RDR scheme are 1) DSO-customer interaction, 2) customer behavior analysis with respect to subsidy, and 3) a simple algorithm to solve overvoltage while minimizing additional cost. Also, we compare the proposed RDR scheme to the direct load control (DLC) scheme which can be regarded as the optimal HC improvement using customer resources. The contributions of this work are summarized as follows:

1) In the setting we propose, the DSO is responsible for a stable operation of the distribution system, and it can communicate with its customers. To suppress the overvoltage, we assume that the DSO issues a subsidy that promotes customer load shift. Unlike a general demand response program that affects all the utility company customers, this subsidy works only in a particular distribution system. It is because the overvoltage from solar PV in the distribution system is a local problem. Therefore, we utilize an interaction between DSO and customers. 2) To design an RDR program, we model a cost function of customers, which consists of the cost of purchasing power, revenue from the subsidy, and discomfort due to load shifting. Then, we derive a closed-form solution that minimizes the customer cost according to the baseline price of power and the subsidy. 3) Because of the non-linearity of the power flow analysis, we propose a sub-optimal algorithm to obtain a subsidy that solves the overvoltage while minimizing the additional cost from the subsidy. Furthermore, we prove the performance gap between the optimal subsidy and the subsidy obtained by the algorithm depends on the step of the proposed algorithm. 4) We compare the HC improvement of the proposed RDR scheme with the DLC scheme. We formulate an optimization framework for the DLC scheme, so the HC improvement of the DLC scheme is the maximum improvement with customer resources. A case study shows that the HC improvement of the proposed is almost the same as that of the DLC scheme.

The remainder of this paper is organized as follows. We first review related works in Section II, and then describe our system model, including the distribution system and customer in Section III. In Section IV, the two RDR based load-shifting schemes as an optimization framework and a sub-optimal algorithm are presented. After demonstrating the proposed scheme’s performance in Section V, this paper is concluded in Section VI.

\[ I^t \] Original load of customer \( i \) at \( t \).
\[ X^t_i \] Adjusted load of customer \( i \) at \( t \).
\[ \mu_i \] Discomfort coefficient for customer \( i \).
\[ \alpha \] Ratio of shiftable load.
\[ G_i(\cdot) \] Gain from the subsidy for customer \( i \).
\[ D_i(\cdot) \] Discomfort of customer \( i \).
\[ C^t_i(\cdot) \] Total cost of customer \( i \).
\[ v^t_i \] Lagrangian multiplier of customer \( i \).
II. RELATED Work

Solar PV HC improvement methods can be divided into two categories: using grid resources and customer resources [13].

One of the fundamental solutions for HC improvement is grid reinforcement [14]. In addition, DSOs use voltage or reactive power control devices such as OLTC [15], [16], capacitor bank [17], and SVR [18]. Although these approaches can effectively improve solar PV HC, they are very costly in terms of both money and time. Smart inverter installed at PV generator is a powerful device that can control reactive power. Coordinated operation of smart inverters and SVCs can improve HC on distribution systems [18]. In [19], the authors investigated a framework to obtain the optimal sizing and location of ESS on medium-voltage (MV) feeders in Germany. Electric vehicles (EVs) can impact distribution systems negatively unless there is a coordination by DSO. However, leveraging energy storage of EV batteries, EVs and EV chargers can help improve HC [20].

Other HC improvement methods are based on customer-side resources. Curtailing the solar PV output is the simplest method in this approach. Su et al. [21] proposed an integrated solar PV inverter reactive power control and real power curtailment method to improve HC. Although curtailing the solar PV output is a simple and powerful method, it wastes solar energy, which is undesirable and self-defeating. EVs can be a good solution to reduce the amount of solar PV curtailment. To this end, a shift of EV charging demand while managing the distribution system has been explored in [22], [23]. They modeled the EV charging scheduling problem as an optimization framework and auction mechanism in [23] and [22], respectively. However, these works do not extend their scope to hosting capacity maximization. The use of RDR to increase HC can be classified into direct and indirect load controls. The DLC scheme allows full control of the customers’ load who make a contract with DSO. In contrast, the indirect load control scheme, which generally uses price or incentive, can indirectly shift customer loads with a carefully designed pricing scheme [24].

In [25], the authors used demand resources to increase HC, but they simply assumed that DSO could change customers’ load patterns with proper incentives. Ren et al. [26] proposed a joint scheduling and voltage regulation strategy that uses customer loads and tap changes of a voltage regulator. However, this work does not solve the voltage violation completely, so it is not related to increasing HC. In [27], the authors proposed a DR program that uses a water heater to improve solar PV HC. Rahman et al. [15] showed that a control scheme using DR and OLTC efficiently improves HC in suburban LV networks in Australia. In [28], the authors proposed a distributed load management scheme for HC improvement using heating, ventilation, and air conditioning (HVAC) loads, electric water heater, and two-way communication. All previous RDR studies for HC improvement use the DLC scheme. That is, DSO has full control power of customers’ load based on an assumption of the contract between customers and DSO. However, such contracts raise significant privacy concerns. In addition, the DLC scheme has a scalability issue.

In this paper, we propose a load-shifting scheme based on indirect RDR for HC improvement. We compare the proposed scheme with the DLC scheme. Even though the DLC scheme has full control of shiftable loads of customers, a case study shows that the proposed RDR scheme increases HC almost similar to the DLC scheme. Because the proposed scheme controls customer load using subsidy, it is scalable and free from privacy issues. We note that our proposed HC improvement method can be used in addition to other methods such as OLTC and smart inverters.

III. SYSTEM MODEL

A. DISTRIBUTION SYSTEM

We consider a radial distribution system with $N$ buses as shown in Fig. 1. For each bus $n$, $M_n$ denotes the number of customers connected to it. We assume that bus 1 is at a substation, and some buses have installed solar PV generators, and those solar PV generators operate by a maximum power point tracking (MPPT) controller, so they cannot change the reactive power independently. Each day is divided into $T$ periods: $T = 1, 2, \ldots, T$ and $t$ is used to denote the time index. It is assumed that the electricity purchasing price to customers at $t$, $P^T_t$, follows the time-of-use (ToU) rate.

At each bus $n$, let $V^t_n$, $P^t_n$, and $Q^t_n$ denote the phasor voltage, net real power, and net reactive power at time $t$, respectively. Net power includes generator and load terms. That is,

$$
P^t_n = P^t_{Gn} - P^t_{Ln}
$$

$$
Q^t_n = Q^t_{Gn} - Q^t_{Ln},
$$

where $P^t_{Gn}$, $P^t_{Ln}$, $Q^t_{Gn}$, and $Q^t_{Ln}$ respectively denote generator and load real and reactive powers at bus $n$. The phasor voltage can be expressed as $V^t_n = V^* ne^{j\delta^t_n}$, where $V^t_n$ and $\delta^t_n$ are the voltage magnitude and phase angle respectively at bus $n$.

The admittance between buses $n$ and $k$ is denoted by $Y_{nk}$ and consists of conductance and susceptance, that is, $Y_{nk} = G_{nk} + jB_{nk}$. The admittance matrix is denoted by $Y_{bus}$. Then, we have the power flow equations at bus $n$:

$$
P_n = \sum_{k=1}^{N} V_k [G_{kn} \cos(\delta_n - \delta_k) + B_{kn} \sin(\delta_n - \delta_k)],
$$

$$
Q_n = \sum_{k=1}^{N} V_k [G_{kn} \sin(\delta_n - \delta_k) - B_{kn} \cos(\delta_n - \delta_k)].
$$

\begin{figure}[htb]
\centering
\includegraphics[width=0.5\textwidth]{fig1}
\caption{An example of a distribution system.}
\end{figure}
A DSO accounts for the stable and reliable operation of the distribution system, such as voltage stability and outage management [29]. More specifically, in each distribution system, a voltage regulation range exists to supply an agreed quality of power. Operating the voltage within the range is crucial for the DSO. Let \( V_{\text{min}} \) and \( V_{\text{max}} \) denote the lower and upper bounds of the voltage regulation range, respectively. Then, the voltage regulation constraint can be written as follows:

\[
V_{\text{min}} \leq V_n(t) \leq V_{\text{max}}.
\]  

(5)

**B. DSO AND CUSTOMER INTERACTION**

We assume that customers have installed a smart meter and home energy management system (HEMS).\(^3\) The number of customers who have the smart meter and HEMS is denoted as \( M \). Through HEMS, customers can re-schedule their controllable loads such as HVAC, water heater and batteries. Fig. 2 shows this interaction. To alleviate the overvoltage problem, the DSO announces a subsidy \( (p'_s) \) in units of $/kWh at the overvoltage time \( t \) to customers who have HEMS. Let \( l_i^s \) and \( x_i^t \) denote the original and adjusted loads of customer \( i \) at time \( t \), respectively.\(^4\) Then, the gain from the subsidy for customer \( i \) is given as follows:

\[
G_i(x_i^t) = p'_s(x_i^t - l_i^s).
\]  

(6)

**IV. RDR BASED LOAD-SHIFTING SCHEME**

We propose a method using customer loads to resolve the overvoltage problem. In this section, both indirect and direct load control schemes are presented. The proposed RDR based load-shifting scheme is an indirect load control scheme. To show the proposed scheme’s performance, we also present a DLC scheme that assumes DSO has full control power for the customer loads. Because DLC allows for complete control of customer load, in principle, it should lead to maximum achievable HC improvement from load shifting. Note that the DLC scheme in this work is similar to the schemes proposed by [15], [26].

**A. INDIRECT LOAD CONTROL SCHEME**

1) **ANALYSIS OF THE CUSTOMER SIDE**

A customer with HEMS minimizes the overall cost while supplying the load given the power purchasing price \( p' \). We model the cost of customer \( i \) over a day as a summation of the cost of power purchased, revenue from the subsidy \( p'_s \), and cost for discomfort due to load shifting. It can be expressed as

\[
C_i(x_i^t) = \sum_{t \in T} (p' x_i^t - G_i(x_i^t) + D_i(x_i^t)).
\]  

(8)

Now, we can define the cost minimization problem for customer \( i \) as follows:

\[
\begin{align}
\text{min}_{\{x_i^t\}_{t \in T}} & \quad \sum_{t \in T} C_i(x_i^t) \\
\text{subject to} & \quad x_i^t \geq (1 - \alpha) l_i^s, \quad \forall t \in T \\
& \quad \sum_{t \in T} x_i^t = \sum_{t \in T} l_i^s
\end{align}
\]  

(9a) \hspace{1cm} (9b) \hspace{1cm} (9c)

where \( \alpha \) denotes the ratio of the shiftable load to total load. In this optimization framework, the control variable is the customer \( i \)'s load at time \( t \). The first constraint means that the shifted load at each time \( t \) is bound by the maximum shiftable load. The other constraint means that the total amount of adjusted loads in a day is the same as that of the original loads.

As the objective function is a quadratic function and the constraints are linear functions, the problem (C) is a convex optimization problem. Also, this problem has a feasible

\(^3\)According to the Federal Energy Regulatory Commission, more than half of customers had a smart meter in the US in 2018, [30] and 100% penetration of smart meters in Italy.

\(^4\)We assume that the DSO knows the original load of each customer. In the DR program, this is called baseline estimation. The customers who participate in DR programs have an incentive to inflate their baseline, so baseline estimation is an important research topic. Recent research [33] has designed a DR program that requires a self-reported baseline for each customer.

\(^5\)It is assumed that customers’ loads and solar PV’s power output are forecast with reasonable accuracy [32], [34].
region that contains an interior point. For example, a solution \( x_{i}^{*} = l_{i}^{*} \) strictly satisfies all the constraints. This is Slater’s condition, which is a sufficient condition for strong duality. We obtain a solution \( x_{i}^{*} \) as

\[
x_{i}^{*} = \begin{cases} (1 - \alpha)l_{i}^{*}, & t \in T_{1} \\ l_{i}^{*} + \frac{1}{2\mu_{i}} (-p_{t}^{i} + p_{s}^{i} - v_{t}^{s}), & t \in T_{2} \end{cases}
\]

(10)

where \( T_{1} \) and \( T_{2} \) are the time periods in which the boundary condition equation (9b) meets and does not meet, respectively. Further, we can obtain \( v_{i}^{s} \) as follows:

\[
v_{i}^{s} = \frac{1}{T_{1}} \sum_{t \in T_{1}} (-p_{t}^{i} + p_{s}^{i}) - \frac{2\mu_{i}\alpha}{T_{1}} \sum_{t \in T_{2}} l_{i}^{*}.
\]

(11)

The detailed procedures to obtain this solution are presented in Appendix A.\(^{6}\)

2) ANALYSIS OF THE DSO SIDE

DSOs take responsibility for the stable and reliable operation of distribution systems. Further, they want to minimize their operational costs. When the voltages across each bus are within the reference voltage ranges in the proposed structure, there is no additional cost incurred for stabilizing the distribution network. However, when an overvoltage occurs, the DSO issues a subsidy to suppress the overvoltage. It is assumed that DSO can estimate each customer’s reaction according to \( p_{i}^{s} \) through the historically collected data of each customer.\(^{7}\)

The DSO needs to ensure all the bus voltages are in a normal range while minimizing additional costs due to subsidy. It is mathematically formulated as

\[
(\text{U}) \min_{p_{s}^{i}} \sum_{t \in T} \sum_{n \in N} \sum_{m \in M_{n}} p_{s}^{i} (x_{m,n}^{i} - l_{m,n}^{i})
\]

subject to \( V_{\min} \leq V_{n}^{t} \leq V_{\max}, \forall n \in N, \forall t \in T \)

(12b)

where \( t, n, \) and \( m \) denote the indexes of time, bus, and customer in a bus, respectively. Note that, in Section IV-A1, the customer index was \( i \), while it is \( [m, n] \) in this section, which means the \( m \)th customer in bus \( n \).

Unlike the customer side problem (C), the problem (U) is not a convex problem because the voltage and power consumption have a nonlinear relation \([36]\). Therefore, we propose an algorithm to find a sub-optimal solution. The sub-optimal algorithm adds a small price \( \Delta p \) to the subsidy when the overvoltage problem occurs.

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\(^{6}\)To get a closed-form solution, the customer load modeling in this work is simple compared to previous RDR research \([12, 26]\). If we model the customer load in more detail, a closed-form solution cannot be obtained. We perform another simulation by adding a practical constraint of the load shifting range. The general tendency of the result is the same, but HC reduced about 17% compared to that without the new constraint.

\(^{7}\)Although the estimation from the DSO is not perfectly correct, the proposed scheme can still apply to the overvoltage problem because the load-shifting and subsidy are positively co-related \([35]\). However, DSO does not know the exact response, so it can occasionally cause minor overvoltage problems can happen sometimes.

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### Algorithm 1: Subsidy Exploration Algorithm

**Input:** topology, \( l_{e}^{i}, e_{n}^{i}, \) and \( p_{t}^{i}, \forall n, t \)

**Output:** \( \tilde{p}_{s}^{i}, \forall t \)

**Initialization**

1: \( p_{s}^{i} = 0, \forall t \)

**Power flow calculation**

2: Obtaining \( V_{n}^{t}, \delta_{n}^{t} \forall n, t \)

3: while \( V_{n}^{t} > V_{\max} \forall n, t \) do

**Overvoltage**

4: for \( t = 1 \) to \( T \) do

5: if \( V_{n}^{t} > V_{\max} \) then

6: \( p_{s}^{i} = p_{s}^{i} + \Delta p \)

7: end if

8: end for

9: Calculate customer load shift

10: Update \( x_{i}^{s} \)

11: Power flow calculation

12: while

13: return \( \tilde{p}_{s}^{i} = p_{s}^{i}, \forall t \)

---

Although this algorithm simply increases the subsidy price during an overvoltage occurrence, it provides a good sub-optimal subsidy.

**Proposition 1:** Suppose that \( p_{s}^{i} \) is the optimal solution of the problem (U) in a radial distribution system. Then the Algorithm 1 converges to \( p_{s}^{i} \) such that the following inequality holds:

\[
\tilde{p}_{s}^{i} - p_{s}^{i} < \Delta p.
\]

(13)

**Proof:** The proof will show two properties at bus \( n \):

(i) \( p_{s}^{i} \) results in \( x_{i}^{s} \) means an increase in power load at the bus, i.e., \( P_{L}^{i}, Q_{L}^{i} \uparrow \), and (ii) \( P_{L}^{i}, Q_{L}^{i} \uparrow \) results in \( V_{n}^{t} \downarrow \). Therefore, \( V_{n}^{t} \) monotonically decreases with \( p_{s}^{i} \).

(i) We will show that the first derivative of \( x_{i}^{s} \) with respect to \( p_{s}^{i} \) is positive, that is, \( \frac{\partial x_{i}^{s}}{\partial p_{s}^{i}} > 0 \). From Eq. (10), it is because

\[
\frac{\partial x_{i}^{s}}{\partial p_{s}^{i}} = \begin{cases} 0, & t \in T_{1} \\ \frac{1}{2\mu_{i}}, & t \in T_{2} \end{cases}
\]

(14)

Therefore, when a subsidy price \( p_{s}^{i} \) increases with \( t \) for \( \forall t \in T_{2} \), the power consumption at time \( x_{i}^{s} \) increases.\(^{8}\)

(ii) Assuming that a typical voltage phase angle difference in a distribution feeder is 0.1° per mile \([37]\), the power flow

\(^{8}\)For \( t \in T_{1} \), customers do not change their load with a change of \( p_{s}^{i} \). Therefore, when an overvoltage problem occurs at \( t \in T_{1} \), there is no feasible solution for the problem (U) using \( p_{s}^{i} \). Because this proof assumes that the problem (U) has a solution, we do not consider this case.

\(^{9}\)Given the large variety of systems such as urban, suburban, and rural with heavily and lightly loaded, the assumption of 0.1° per mile might not be enough. Even if we relaxed this assumption as 2-3° per mile, the error between the original equation and approximated one is less than 0.5%. Therefore, we can use this approximation.
equation (3) can be approximated by
\[
P'_n \simeq V'_n \sum_{k \in N} V'_k G_{kn} \tag{15}
\]
\[
= (V'_n)^2 G_{nn} + V'_n \sum_{k \in N, k \neq n} V'_k G_{kn} \tag{16}
\]
\[
Q'_n \simeq V'_n \sum_{k \in N} V'_k (-B_{kn}) \tag{17}
\]
\[
= - \left( (V'_n)^2 B_{nn} + V'_n \sum_{k \in N, k \neq n} V'_k B_{kn} \right) \tag{18}
\]

Then, differentiating both sides with respect to \( V'_n \), and rearranging terms results in
\[
\frac{\partial P'_n}{\partial V'_n} = \sum_{k \in N, k \neq n} (V'_k - 2V'_n) G_{kn} \tag{19}
\]
\[
\frac{\partial Q'_n}{\partial V'_n} = - \sum_{k \in N, k \neq n} (V'_k - 2V'_n) B_{kn}. \tag{20}
\]

In power systems, \( G_{kn} < 0 \) and \( B_{kn} > 0 \) for \( \forall k \neq n \), and \( V'_k \simeq 1 \) for \( \forall k \) [36]. Therefore, \( \frac{\partial P'_n}{\partial V'_n} > 0 \) and \( \frac{\partial Q'_n}{\partial V'_n} > 0 \). Assuming that the DSO cannot control generator power, i.e., \( P'_n \) and \( Q'_n \) are constant, \( \frac{\partial P'_n}{\partial V'_n} < 0 \) and \( \frac{\partial Q'_n}{\partial V'_n} < 0 \) from (1). Then, \( \frac{\partial V'_n}{\partial V'_n} = 0 \). This means that an increase [decrease] of load at a bus results in a decrease [increase] of the voltage magnitude at the bus.

Note that we assume that the voltage relationship at neighboring buses is negligible in this proof. That is, \( \frac{\partial V'_n}{\partial V'_n} = 0 \).

With \( \frac{\partial V'_n}{\partial V'_n} \neq 0 \), the same result can be derived by solving simultaneous equations.

Owing to (i) and (ii), an increase in \( P'_n \) always results in a decrease in \( V'_n \). Therefore, if we keep raising \( P'_n \), the overvoltage problem is solved. As the Algorithm 1 increases \( P'_n \) in steps of \( \Delta p \), the difference between the solution of this algorithm \( P'_n \) and the optimal \( P'_n^{\star} \) is at most \( \Delta p \), that is
\[
\tilde{P}'_n - P'_n^{\star} < \Delta p. \tag{21}
\]

From the Proposition 1, the Algorithm 1 can always find a solution if there is one.

Corollary 1: Suppose that the Algorithm 1 cannot find any solution to the problem (U). Then the feasible set of the problem (U) is empty.

Proof: For a proof by contradiction, suppose that the Algorithm 1 cannot find a solution of the problem (U), and the feasible set is nonempty. Let \( \tilde{P}'_n > 0 \) be a solution of the problem. That means \( V'_n \leq V_{\text{max}} \) with \( \tilde{P}'_n \). The Algorithm 1 increases \( P'_n \) as \( \Delta p \) in each step when \( V'_n > V_{\text{max}} \), and \( V'_n \) monotonically decreases with \( P'_n \). Therefore, \( P'_n \) will be greater than or equal to \( P'_n \) by the Algorithm. In other words, the Algorithm 1 finds a solution. Since we have a contradiction, it must be that the feasible set is nonempty.

\[\Box\]

B. DIRECT LOAD CONTROL SCHEME

In this section, we model an optimization framework for the HC maximization problem using DLC. It is assumed that the DSO has already made a contract with each customer who wants to participate demand response program. Therefore, the DSO can control the shiftable load of its customers. The objective function of this optimization framework is a sum of solar PV capacities in the distribution network. The HC maximization problem is defined as:

\[
(D) \quad \max_{HC_n \in x_{n,a}^D, a \in D} \sum_{n \in D} HC_n \tag{22}
\]

subject to (9b), (9c), and (12b),

where \( HC_n \) and \( x_{n,a}^D \) denote the maximum of PV capacity at bus \( n \) with no voltage violation and the customer load, respectively. They are the two two control variables for the optimization framework. Three constraints of this problem come from the problems (C) and (U): two customer load constraints and one nominal voltage range constraint. The problem (D) is not a convex optimization problem due to the quadratic relation between \( P \) and \( V \). Therefore, we propose an iterative algorithm that uses linearization of the quadratic equation.

To see the voltage violation part clearly, we change the two customer constraints (9b) and (9c) to the bus \( n \) point of view, that is

\[
P_{xn} \geq (1 - \alpha)P_{L,n}, \quad \forall t \in T \tag{23}
\]

\[
\sum_{t \in T} P_{xn} = \sum_{t \in T} P_{L,n} \tag{24}
\]

where \( P_{xn} = \sum_{m \in Mx} x_{m,n}^t \) and \( P_{L,n} = \sum_{m \in Mx} \dot{P}_{L,n}^t \). Accordingly, the customer load control variable is also changed to \( P_{xn} \).

The proposed iterative algorithm relaxes the quadratic relation to linear using the voltage difference. The voltage at \( j \)th iteration is defined by

\[
V_n^{j+1} = V_n^{j+1} + \Delta V_n^{j+1} \tag{25}
\]

where \( \Delta V_n^{j+1} \) is the voltage difference at \( j \)th iteration which comes from active and reactive power change. Then, the voltage regulation constraint (12b) can be written as

\[
V_{\text{min}} - V_n^{j+1} \leq \Delta V_n^{j+1} \leq V_{\text{max}} - V_n^{j+1}. \tag{26}
\]

The voltage difference at \( j \)th iteration can be obtained by using the voltage sensitivity matrix \( J^{-1} \) which is the inverse form of the Jacobian matrix in the current operating condition [38]. The voltage sensitivity matrix is given as

\[
J^{-1} = \begin{bmatrix}
\frac{\partial \theta}{\partial P} & \frac{\partial \dot{V}}{\partial P} \\
\frac{\partial \dot{V}}{\partial V} & \frac{\partial \dot{V}}{\partial Q}
\end{bmatrix} \tag{27}
\]

where \( \dot{\theta}, \dot{V}, \ddot{P} \) and \( \dot{Q} \) denote vectors of voltage angle, voltage magnitude, active power and reactive power, respectively.
The voltage difference at $j$th iteration $\Delta V_n^{t,j}$ is obtained by
\[
\Delta V_n^{t,j} = \sum_{n \in D, n \neq 1} \frac{\partial V_n^{t,j-1}}{\partial P_n^{t,j}} \Delta P_n^{t,j} + \sum_{n \in N, n \neq 1} \frac{\partial V_n^{t,j-1}}{\partial Q_n^{t,j}} \Delta Q_n^{t,j}
\]  
(28)
where $\Delta P_n^{t,j}$ and $\Delta Q_n^{t,j}$ are real and reactive power difference at $j$th iteration, respectively. They are defined as
\[
\Delta P_n^{t,j} = P_n^{t,j} - P_n^{t,j-1}
\]  
(29)
\[
\Delta Q_n^{t,j} = Q_n^{t,j} - Q_n^{t,j-1}
\]  
(30)
By Eqs. (28), (29), and (30), the updated voltage regulation constraint (26) linearized as Eqs. (31) and (32), as shown at the bottom of the page.

Note that all the terms are constant except $P_n^{t,j}$ and $Q_n^{t,j}$ in Eqs. (31) and (32), so they are linear equations. We can easily obtain $P_n^{t,j}$ and $Q_n^{t,j}$ using $P_{\text{Gen}}^{t,j}, P_{\text{sh}}^{t,j}$, and power factor.

The linearized HC maximization problem is defined as:
\[
\begin{align*}
(D') & \quad \max_{H C_n, P_{L_n}^{t,j}, n \in D} \sum_{n \in N} H C_n \\
& \text{subject to (23), (24), (31) and (32)} \quad (33)
\end{align*}
\]

**Algorithm 2: Direct Load Control Algorithm**

**Input:** topology and $P_{L_n}^{t,j}, \forall m, n$  
**Output:** $H C_n, P_{L_n}^{t,j}, \forall m, n, t$

**Initialization**

1: $j = 0$, $H C_n = 0$, $P_{L_n}^{t,j} = P_{L_n}^{0}$, $\Delta = \epsilon + 1$, $\forall m, n, t$
2: while $\Delta < \epsilon$
3: \hspace{1cm} $j \leftarrow j + 1$
4: \hspace{1cm} for $t = 1$ to $T$
5: \hspace{2cm} Power flow calculation
6: \hspace{2cm} Obtain $\frac{\partial V_n^{t,j}}{\partial P_n^{t,j}}, \frac{\partial V_n^{t,j}}{\partial Q_n^{t,j}}, \forall n$ from $J^{-1}$
7: \hspace{2cm} end for
8: \hspace{1cm} Solve $(D')$, i.e., obtain $H C_n, P_{L_n}^{t,j}$
9: \hspace{1cm} Update $P_{n}^{t,j}, Q_{n}^{t,j}$
10: \hspace{1cm} $\Delta = H C_n - H C_{n-1}$
11: end while
12: return $H C_n, P_{L_n}^{t,j}, \forall m, n, t$

The linearization method to obtain the voltage difference might have a high error when $\Delta P_n^{t,j}$ and $\Delta Q_n^{t,j}$ are high. Therefore, we propose an iterative algorithm to obtain an accurate solution of the problem $(D')$ as shown in Algorithm 2.

**V. EVALUATION**

In this section, we evaluate the proposed RDR based load-shifting scheme in terms of cost and HC.

**TABLE 1. PG&E ToU pricing.**

| Period       | Time          | Price ($) |
|--------------|---------------|-----------|
| Peak         | 1 p.m. - 7 p.m. | 0.39      |
| Partial-peak | 7 p.m. - 9 p.m. | 0.27      |
| Off-peak     | 9 p.m. - 11 p.m. | 0.20      |

**A. SIMULATION SETTINGS**

For the customer loads and output power of solar PV generators, we use the Pecan Street data set of August 2019 [39], which consists of hourly data. For the power purchasing price from the main grid $p^J$, the ToU price of Pacific Gas and Electric Company (PG&E) in summer 2019 is used as shown in Table 1.

To determine the number of shiftable loads in households, we assume that 10% of controllable loads are the total amounts of shiftable loads. Examples of controllable loads are HVAC load, water heater, and refrigerator. According to EIA’s survey in 2015, we set the ratio of shiftable load $\alpha$ to 5%, that is, $\alpha = 0.05$. Note that as this setting is an example, any $\alpha$ can be applied to the proposed scheme.

The proposed load-shifting scheme is tested on the modified IEEE 11-kV, 15-bus distribution system as shown in Fig. 3. This distribution system is a radial network, and the substation is at bus 1. The line impedance and load data are given in Table 2. In each bus, it is assumed that 140 customers

\[\sum_{n \in D, n \neq 1} \frac{\partial V_n^{t,j-1}}{\partial P_n^{t,j}} P_n^{t,j} + \sum_{n \in N, n \neq 1} \frac{\partial V_n^{t,j-1}}{\partial Q_n^{t,j}} Q_n^{t,j} \geq V_{\text{min}} - V_n^{t,j-1} + \sum_{n \in D, n \neq 1} \frac{\partial V_n^{t,j-1}}{\partial P_n^{t,j}} P_n^{t,j-1} + \sum_{n \in D, n \neq 1} \frac{\partial V_n^{t,j-1}}{\partial Q_n^{t,j}} Q_n^{t,j-1} \]  
(31)
\[\sum_{n \in D, n \neq 1} \frac{\partial V_n^{t,j-1}}{\partial P_n^{t,j}} P_n^{t,j} + \sum_{n \in N, n \neq 1} \frac{\partial V_n^{t,j-1}}{\partial Q_n^{t,j}} Q_n^{t,j} \leq V_{\text{max}} - V_n^{t,j-1} + \sum_{n \in D, n \neq 1} \frac{\partial V_n^{t,j-1}}{\partial P_n^{t,j}} P_n^{t,j-1} + \sum_{n \in D, n \neq 1} \frac{\partial V_n^{t,j-1}}{\partial Q_n^{t,j}} Q_n^{t,j-1} \]  
(32)

\[18550 \quad \text{VOLUME 10, 2022}\]
TABLE 2. Parameters of the modified IEEE 15-bus system.

| Bus Number | Send. end | Recv. end | Line Impedance R (p.u.) | Line Impedance X (p.u.) | Bus Load of Recv. end P (kW) | Bus Load of Recv. end Q (kVAR) |
|------------|----------|-----------|-------------------------|-------------------------|-----------------------------|-----------------------------|
| 1          | 2        | 0.112     | 0.109                   | 441                     | 450                         |
| 2          | 3        | 0.097     | 0.095                   | 701                     | 714                         |
| 3          | 4        | 0.070     | 0.068                   | 1400                    | 1428                        |
| 4          | 5        | 0.126     | 0.085                   | 441                     | 450                         |
| 5          | 6        | 0.211     | 0.143                   | 700                     | 714                         |
| 6          | 7        | 0.090     | 0.061                   | 441                     | 450                         |
| 7          | 8        | 0.103     | 0.070                   | 1400                    | 1428                        |
| 8          | 9        | 0.166     | 0.110                   | 1400                    | 1428                        |
| 9          | 10       | 0.139     | 0.094                   | 700                     | 714                         |
| 10         | 11       | 0.148     | 0.100                   | 1400                    | 1428                        |
| 11         | 12       | 0.202     | 0.136                   | 700                     | 714                         |
| 12         | 13       | 0.166     | 0.112                   | 441                     | 450                         |
| 13         | 14       | 0.184     | 0.124                   | 700                     | 714                         |
| 14         | 15       | 0.163     | 0.067                   | 1400                    | 1428                        |

are connected to the distribution system (\(M_n = 140\)). In this case study, PV solar generators can be installed on buses 4, 5, 6, 7, 10, and 13. We analyze a case of installing solar PV on bus 13, which is the most vulnerable position to cause the overvoltage problem if there is no specific explanation on PV solar generators. The nominal voltage range is set to [0.91, 1.04] per unit, based on the South Korean standard [40].

We use a Gaussian distribution to model the discomfort coefficient for each customer \(\mu\). According to reference works [31], [41] and the minimum price of the ToU pricing, the mean and standard deviation of this distribution are 0.2 and 0.063, respectively. The unit of the discomfort coefficient is $/kWh^2$ because it is the product of \(\mu\) and the square of the power as shown in Eq. (7).

B. EFFECT OF \(\mu\)

In Eq. (7), \(\mu_i\) represents the degree of discomfort for customers. A customer with high [low] \(\mu_i\) will move little [much] load shift as shown in Fig. 4. It is observed that when the DSO issued a subsidy, the power consumption during the subsidized time increased, and the increased load was drawn from the other times. In this example, \(p^{ts}_{ts} = 0.12\) at \(t = 12\) p.m., which is the solution of Algorithm 1 for a 6 MW solar PV installed on bus 13. The total shifted load in Figs. 4a and 4b are 0.199 kWh and 1.24 kWh, respectively, and their percentages of a total load of a day are 0.51 % and 3.66 %, respectively.

C. OVERVOLTAGE PROBLEM

With no solar PV, no overvoltage problem occurred as shown in Fig. 5a. The horizontal blue line with a value of 1.01 is the voltage of the substation (bus 1). In this case study, the bus containing the solar PV (bus 13) is the most vulnerable. Therefore, we increase the solar PV capacity installed on bus 13 to see the overvoltage problem. Fig. 5b shows the per-unit voltage of bus 13 with the solar PV capacity ranging up to 7.5 MW. The first overvoltage problem occurs with 5.5 MW solar PV at 12 p.m. because of the high output of the solar PV at noon.

D. CASE WITH 6 MW SOLAR PV GENERATOR

With 6 MW solar PV, the overvoltage problem occurs at bus 13. In the proposed RDR scheme, the DSO issued a subsidy when the overvoltage occurred (12 p.m.). Algorithm 1 with \(\Delta p = 0.01\) finds \(p^{ts}_{ts} = 0.12\) at \(t = 12\) p.m., and \(p^{ts}_{ts} = 0\) otherwise. Fig. 6 shows the sum of the daily load with and without the proposed RDR scheme. Because of the subsidy, customers perform a load shift to the subsidized period, resulting in a decrease in voltage. Therefore, the overvoltage problem is settled as shown in Fig. 5c. The large fluctuations from 11 to 15 hours come from the subsidy to suppress overvoltages. In case of 6 MW solar PV, the total energy shifted by the subsidy is 1137 kWh. The voltages at bus 13 without and with the RDR were 1.0468 and 1.0396, respectively. The additional cost incurred by the utility company due to the provision of subsidy is $65.56 per day.

Another solution to the overvoltage problem is curtailment. In the case study, the total amount of curtailed energy to make the voltage of the bus 13 below 1.04 p.u. is 539 kWh, which is about 1.2 % of the total energy of the solar PV in a day. If the ToU price of PG&E is the standard charge, the value of the curtailed energy of the solar PV is $145.6, which is about 2.2 times higher than the cost incurred by the proposed RDR scheme.
E. HOSTING CAPACITY OF ONE SOLAR PV AT BUS 13

The solar PV capacity installed on bus 13 is increased. As shown in Fig. 5b, the overvoltage problem occurs from the solar PV capacity of 5.5 MW. However, using the proposed RDR scheme, no overvoltage issue occurs even when the solar PV capacity is 7.5 MW as shown in Fig. 5c. Beyond 7.5 MW, the proposed RDR scheme cannot solve the overvoltage problem, even though all shiftable loads are moved. Therefore, the HC of this distribution system with the proposed RDR scheme is 7.5 MW.

However, this solution comes with a cost. Through the $p_s$ subsidy issued, each customer could move the load to the subsidized period. Figs. 7 and 6 show the subsidy issued to solve the overvoltage problem and the moved load because of the subsidy. The subsidy also increases with high solar

11Open Meteorological Data Portal (Korean), https://data.kma.go.kr/
12This cost comes from a Korean case study of grid reinforcement [42].
PV capacity, which is approximately 0.1 $/kWh at 6 MW capacity and 2 $/kWh at 7.5 MW capacity. The horizontal dashed line in Fig. 7 represents the ToU price of PG&E, and the price is written at the top of the figure. At 7.5 MW capacity, the subsidy is higher than the selling price to the customer, which means that customers earn money when they use power during the subsidy period, i.e., so-called minus pricing. As this adds to the financial burden of the DSO, the effective HC with the proposed RDR scheme is set to 7 MW without any curtailment, which is a 28.2 % increase from the baseline HC of 5.46 MW.

We compared the HC of the proposed RDR scheme with the curtailment method. Fig. 8 shows the energy and cost incurred by the two methods. In terms of energy, the amount of curtailed energy is approximately half of the energy moved by the proposed RDR scheme. It is because the curtailment method directly controls the bus that the overvoltage problem occurred, while the RDR indirectly solves the problem by shifting all customers’ loads in the distribution system. From the cost point of view, the value of the curtailed energy is higher than the cost to move energy using the proposed method with 5.5 MW, 6 MW, 6.5 MW, and 7 MW capacity. However, it is the opposite with a 7.5 MW capacity. It is because the value of the curtailed energy increases linearly. On the other hand, the cost of the RDR scheme increases quadratically because the discomfort function $D(\cdot)$ in Eq. (7) is a quadratic function. Therefore, the total cost for the shifting load also increases quadratically. In addition to cost, Fig. 8 shows customer benefit from the proposed RDR scheme. The benefit of customers consists of revenue from the subsidy and discomfort due to load shifting. The revenue of customers is the same as “Cost to Move the Energy” of the DSO, so customers’ revenue also increases quadratically. Also, the discomfort that negatively affects the customer benefit increases as solar PV capacity increases.

Note that the HC of the DLC scheme is a little higher than that of the proposed RDR scheme. An analysis of the DLC scheme is in the following section.

F. SCENARIOS WITH THE ESTIMATION ERRORS

So far, all the simulation results are based on an assumption of a perfect estimation of the discomfort parameters $\mu_i$ for all customers. However, this is not a practical assumption. The DSO cannot perfectly estimate the discomfort parameters. Therefore, we model the estimation error for $\mu_i$ as a Gaussian random variable $\epsilon_i$ with zero mean and variance $\sigma_i$. Then, the discomfort of customer $i$ is expressed as

$$\mu_i^e = \mu_i + \epsilon_i,$$

where $\mu_i^e$ and $\mu_i$ denote the actual discomfort and the estimated discomfort of customer $i$, respectively. When the DSO finds a subsidy to solve overvoltage problems, it uses $\mu_i$. However, each customer $i$ actually reacts to the subsidy according to $\mu_i^e$.

We simulate the cases with a solar PV capacity of 5.5 MW, 6 MW, 6.5 MW, 7 MW, and 7.5 MW at bus 13 and the estimation error. We generate 100 scenarios for each case. Although the standard deviation of $\epsilon_i$ is set to 30 %, there is no voltage violation for the case of 5.5 MW, 6 MW, and 6.5 MW solar PV. In the case of 7.0 MW and the same standard deviation, only one scenario shows a minor voltage rise over 1.04 p.u., i.e., 0.002 %. However, a slight error as the standard deviation of 1 % causes voltage deviation for one-third of the total scenarios in the case of 7.5 MW.

We use the term value for the curtailed energy because this is not the cost of the utility company.

Note that the HC of the DLC scheme is a little higher than that of the proposed RDR scheme. An analysis of the DLC scheme is in the following section.

G. HOSTING CAPACITY FOR VARIOUS CASES

Table 3 shows the HCs of legacy, the proposed indirect RDR, and DLC schemes. The legacy scheme means distribution system operation without RDR and DLC schemes. The numbers in parenthesis indicate the buses installed solar PV generators. As shown in Table 3, the difference between the HCs of the proposed and DLC schemes is 0.69 %. Because the HC of the DLC scheme is the theoretical HC improvement limit, it is confirmed that the proposed RDR scheme can almost increase HC to the maximum value. That little performance gap comes from the sub-optimal subsidy obtained by Algorithm 1. The difference between $\tilde{p}_t^s$ and $p_t^*$ is at most $\Delta P = 0.01$ as shown in Proposition1.

The proposed RDR scheme improves HC by 33.6 % compared to the legacy scheme. However, as we discussed in
Section V-E, this improvement comes at a cost, and an economic HC limit is about mid-20%. In the case of one solar PV installed on bus 13, we simulate with different $\alpha$. As $\alpha$ increases, i.e., more shiftable loads, the HC improvement also increases.

We also simulate more cases that solar PV generators are installed on various bus positions. Again, it is confirmed that the HC improvement of the proposed RDR and DLC schemes are almost the same. With the number of buses installed solar PV, HC in the distribution system increases. It is because power flow is distributed due to the solar PV generators in various bus positions. Among the cases of three solar PV generators, the case of (7, 10, 13) shows the minimum HC because all the buses are located at the end of feeders. On the other hand, the case of (4, 10, 13) shows the maximum HC. Because bus 4 is close to the substation, it can install a high-capacity solar PV generator without violating the voltage limit.

It shows the maximum capacity of each solar PV generator. The solar PV generator installed on bus 13 has the lowest capacity solar PV generator without violating the voltage limit.

FIGURE 9. Hosting capacity improvement of the proposed RDR scheme. "1st, 2nd, and 3rd" in legend stands for each bar graph in parenthesis. Legacy scheme is written as "w/o RDR."

Fig. 9 shows detailed information on HC for various cases. It shows the maximum capacity of each solar PV generator. The solar PV generator installed on bus 13 has the lowest capacity of any bus combination because its position is the most vulnerable. On the other hand, the solar PV generator installed near the substation, such as bus 4, can have a larger HC. With the proposed RDR scheme, all buses increase their HC compared to the legacy scheme.

VI. CONCLUSION

A renewable energy-based power system is the need of the hour. Distribution systems with high solar PV suffer from overvoltage problems. The customer-engaged approach is suggested as a solution to increase the HC of solar PV in the distribution system without grid reinforcement. This paper proposes an RDR based load-shifting scheme to increase HC. Under this program, the DSO issues subsidies for a certain period to solve the overvoltage problem. Because customers move their load to the subsidized time to reduce cost, the overvoltage problem is resolved. The proposed RDR scheme is mathematically formulated, and a sub-optimal algorithm to obtain a solution is proposed. It is proved that the sub-optimal algorithm successfully finds an optimal solution with a small error. Therefore, the proposed RDR scheme performs almost similar to that of the DLC scheme which can be regarded as the maximum HC. Using the modified IEEE 15-bus distribution system, the case study shows an average of 33.6% increases in HC at diverse solar PV generator positions.

APPENDIX A

DETAILED PROCEDURES TO SOLVE (C)

The solution of the problem (C) is derived in this section. We omit the customer index $i$ for simplicity, and change the constraints into a standard convex optimization form, that is

$$(C) \min_{\{x^t\}} \sum_{t \in T} \left( p^t x^t - p^t_0 (x^t - l^t) + \mu (x^t - l^t)^2 \right)$$

subject to $-x^t + (1 - \alpha)l^t \leq 0, \ \forall t \in T$ \ (35b)

$$\sum_{t \in T} (x^t - l^t) = 0.$$  \ (35c)

The Lagrangian is

$L(x^t, \lambda^t, \nu) = \sum_t \left( \mu (x^t)^2 + (p^t - p^t_0 - 2\mu l^t)x^t + \mu (l^t)^2 + p^t_0 l^t \right) + \sum_{t} \lambda^t (-x^t + (1 - \alpha)l^t) + \nu \sum_t (x^t - l^t)$, \ (36)

where $\lambda^t$ and $\nu$ are the Lagrangian multipliers. The optimal solution $x^{ts}$ should satisfy Karush-Kuhn-Tucker (KKT) conditions [43], that is

$x^{ts} \geq (1 - \alpha)l^t, \ \forall t \in T$ \ (37)

$$\sum_{t \in T} (x^{ts} - l^t) = 0,$$ \ (38)

$\lambda^t_{s} \geq 0, \ \forall t$, \ (39)

$\lambda^t_{s} \left( -x^{ts} + (1 - \alpha)l^t \right) = 0, \ \forall t \in T$, \ (40)

$2\mu x^{ts} + (p^t - p^t_0 - 2\mu l^t) - \lambda^t_{s} + \nu s = 0$. \ (41)

Three possible cases can be considered to satisfy the slackness condition Eq. (40):

1) $\lambda^t_{s} = 0, \ \forall t$: Then, $-x^{ts} + (1 - \alpha)l^t \leq 0, \ \forall t \in T$. Using Eq. (38) and Eq. (41), the solution is obtained as

$$x^{ts} = l^t + \frac{1}{2\mu} \left( -p^t + p^t_0 - \frac{1}{T} \sum_{t \in T} (-p^t + p^t_0) \right)$$ \ (42)

2) $\lambda^t_{s} > 0, \ \forall t$: In this case, $x^{ts} = (1 - \alpha)l^t, \ \forall t$ because of Eq. (40). However, this solution cannot satisfy a constraint of Eq. (9c), so there is no solution.

3) $\lambda^t_{s} = 0, \ \forall t \in T_1$, and $\lambda^t_{s} > 0, \ \forall t \in T_2$: Then, $x^{ts} = l^t + \frac{1}{2\mu} \left( -p^t + p^t_0 - \nu s \right), \ \forall t \in T_1$, and $x^{ts} = (1 - \alpha)l^t$, for $t \in T_2$. Using Eq. (9c) and Eq. (41), we can obtain the Lagrangian multiplier as

$$\nu s = \frac{1}{T_1} \sum_{t \in T_1} (p^t - p^t_0) - \frac{2\mu \alpha}{T_1} \sum_{t \in T_2} l^t.$$ \ (43)
And, the solution is

\[ x^* = \begin{cases} 
(1 - \alpha)^{\frac{1}{t_1}}, & t \in T_1 \\
\frac{1 + \frac{1}{2\mu}(-p^d + p^l - v^d)}, & t \in T_2
\end{cases} \] (44)

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