Free Boson Representation of $U_q(\hat{sl}_3)$

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ABSTRACT

A representation of the quantum affine algebra $U_q(\hat{sl}_3)$ of an arbitrary level $k$ is constructed in the Fock module of eight boson fields. This realization reduces the Wakimoto representation in the $q \to 1$ limit. The analogues of the screening currents are also obtained. They commute with the action of $U_q(\hat{sl}_3)$ modulo total differences of some fields.

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1. Introduction

Recently the anti-ferroelectric spin-1/2 XXZ-Hamiltonian was exactly diagonalized [1] by using the technique of $q$-vertex operators [2]. Further, an integral formula for correlation functions of local operators was found in the case of spin 1/2 [3] with the help of a boson representation of $U_q(\widehat{sl}_2)$ of level 1 [4]. The technique of vertex operators was also been applied to the case of higher-spin XXZ models [5]. Construction of the free boson representation of $U_q(\widehat{sl}_2)$ of higher levels is crucial to obtain the integral formula for the correlation functions in this case. In fact the bosonization of $U_q(\widehat{sl}_2)$ of an arbitrary level was given [6][7] using an analogue of the currents defined by the Drinfeld realization [8] of $U_q(\widehat{sl}_2)$. The Drinfeld’s generators are expressed in terms of three boson fields.

To study the higher rank version of the XXZ model, that is, the vertex model associated with the $R$ matrix of $U_q(\widehat{sl}_n)$, we are interested in a bosonization of $U_q(\widehat{sl}_n)$. In this article we construct a bosonization of the quantum affine algebra $U_q(\widehat{sl}_3)$ as the first step toward this direction. In the $q \to 1$ limit, this new representation reduces to the Wakimoto representation with bosonized $\beta - \gamma$ system [9][10][11]. Further, this Wakimoto representation has a connection with the difference operator representation of $U_q(\widehat{sl}_3)$ obtained in [12].

We obtain two analogues of the screening currents in terms of boson fields. These operators have the property that they commute with the currents modulo total differences of some operators. Hence a suitable Jackson integral of the screening currents should commute exactly with the currents. A Jackson integral formula for the solution to $q$-deformed Knizhnik-Zamolodchikov equation [4] was found by Matsuo [13] and Reshetikhin [14]. We think that there is a deep connection between the existence of our screening currents and these Jackson integral formulas.

2. Free boson fields $a^i, b^\mu,$ and $c^\mu$ ($i = 1, 2, \mu = 1, 2, 3$)

In this article we consider bosonization of the Drinfeld realization of $U_q(\widehat{sl}_3)$. We construct Drinfeld’s generators in terms of eight free boson fields. Hereafter let $q$ be a generic complex number such that $|q| < 1$. We will frequently use the following standard notation:

$$[m] = \frac{q^m - q^{-m}}{q - q^{-1}},$$

for $m \in \mathbb{Z}$. 
Let $k$ be a complex number. Let $\{a_n^i, b_n^\mu, c_n^\nu, Q_a^\mu, Q_b^\nu, Q_c^\mu | n \in \mathbb{Z}, i = 1, 2, \mu = 1, 2, 3\}$ be the set of operators satisfying the following commutation relations:

\[
[a_n^i, a_m^j] = \delta_{n+m,0} \frac{[a_{i,j}^0]}{[k+3]^n}, \quad [a_0^i, Q_a^j] = a_{i,j}(k+3),
\]

\[
[b_n^\mu, b_m^\nu] = -\delta_{\mu,\nu} \delta_{n+m,0} \frac{[n]}{n}, \quad [b_0^\mu, Q_b^\nu] = -\delta_{\mu,\nu},
\]

\[
[c_n^\nu, c_m^\nu] = \delta_{\mu,\nu} \delta_{n+m,0} \frac{[n]}{n}, \quad [c_0^\nu, Q_c^\nu] = \delta_{\mu,\nu},
\]

where $(a_{i,j})_{i,j=1}^2$ is the Cartan matrix of of type $A_2$, i.e. $a_{11} = a_{22} = 2, a_{12} = a_{21} = -1$. The remaining commutators vanish.

Let us introduce eight free boson fields $a^i, b^\mu, c^\nu$ ($i = 1, 2, \mu = 1, 2, 3$) carrying parameters $M, N \in \mathbb{Z}_{>0}, \alpha \in \mathbb{R}$. Define $a^i(M; N | z; \alpha)$ ($i = 1, 2$) by

\[
a^i(M; N | z; \alpha) = -\sum_{n \neq 0} \frac{[Mn]}{[Nn]} \frac{a_n^i}{[n]} z^{-n_q^n} \frac{q^{n\alpha}}{N} \log z + \frac{Ma_0^i}{N}. \quad (2)
\]

Note that this definition is slightly different from the one given in [4]. We define $b^\mu(M; N | z; \alpha), c^\nu(M; N | z; \alpha)$ ($\mu = 1, 2, 3$) in the same way. In the case $M = N$ we write

\[
a^i(z; \alpha) = a^i(M; M | z; \alpha),
\]

and likewise for $b^\mu(z; \alpha), c^\nu(z; \alpha)$. Furthermore we introduce $a^i_{\pm}(M; N | z)$ ($i = 1, 2$) as

\[
a^i_{\pm}(M; N | z) = \pm \left( (q-q^{-1}) \sum_{n > 0} \frac{[Mn]}{[Nn]} \frac{a_n^i}{n} z^{-n} \frac{Ma_0^i}{N} \log q \right). \quad (3)
\]

The fields $b^\mu_{\pm}(M; N | z), c^\nu_{\pm}(M; N | z)$ ($\mu = 1, 2, 3$) are defined in the same way. We note that $a^i_{\pm}(M; N | z)$ can be expressed using $a^i(1; N | z; \alpha)$ as

\[
a^i_{\pm}(M; N | z) = a^i(1; N | q^{\pm \alpha} z; \alpha - M) - a^i(1; N | q^{\pm (\alpha-M)} z; \alpha), \quad (4)
\]

where $\alpha$ is any real number. We note that the newly introduced fields $a^i_{\pm}(M; N | z), b^\mu_{\pm}(M; N | z), c^\nu_{\pm}(M; N | z)$ are thus not independent of the fields $a^i(M; N | z; \alpha), b^\mu(M; N | z; \alpha), c^\nu(M; N | z; \alpha)$. In the following sections, we will find it convenient to use this new notation. It will also be convenient to use the following shorthand notation:

\[
(Ma^i_{N})(z; \alpha) \equiv a^i(M; N | z; \alpha), \quad (5)
\]

\[
(Ma^i_{N})(z) \equiv a^i_{\pm}(M; N | z), \quad (6)
\]

\[
(b+c)^\mu(z; \alpha) \equiv b^\mu(z; \alpha) + c^\nu(z; \alpha). \quad (7)
\]
3. *q*-difference operator and the Jackson integral

Following [3], we define the *q*-difference operator with a parameter \( n \in \mathbb{Z}_{>0} \):

\[
\partial_z f(z) = \frac{f(q^n z) - f(q^{-n} z)}{(q - q^{-1}) z}.
\]

Let \( p \) be a complex number such that \(|p| < 1\) and \( s \in \mathbb{C}^\times \). We define the Jackson integral by

\[
\int_0^{s \infty} f(t) dp = s(1-p) \sum_{m=-\infty}^{\infty} f(sp^m)p^m,
\]

whenever it is convergent. If the integrand \( f(t) \) is a total difference of some function \( F(t) \):

\[
f(t) = n\partial_z F(t),
\]

then by taking \( p = q^n \), we have

\[
\int_0^{s \infty} f(t) dp = 0.
\]

4. Fock module and Wick’s Theorem

First we define the Fock module. Let \(|0\rangle\) be the vector having the following properties

\[
a_i^n |0\rangle = b_i^n |0\rangle = c_i^n |0\rangle = 0 \quad i = 1,2 \quad \mu = 1,2,3 \quad n \geq 0.
\]

Define the vectors

\[
|n\rangle = \exp \left\{ \sum_{i,j=1}^{2} r_i a_{ij}^{-1} Q_{ij}^{\mu} \frac{Q_{ij}}{k + 3} + \sum_{\mu=1}^{3} s_{\mu}(Q_{\mu}^{b} + Q_{\mu}^{c}) \right\} |0\rangle,
\]

where \( a_{ij}^{-1} \) is the inverse of the Cartan matrix \( a_{ij} \), and \( r_i, s_{\mu} \in \mathbb{Z} \) \((i = 1,2, \mu = 1,2,3)\). Let \( F \) be a free \( \mathbb{Q}(q) \) module generated by \( \{a_i^n, b_i^n, c_i^n | n \in \mathbb{Z}_{>0}, i = 1,2, \mu = 1,2,3 \} \). Now we define the Fock modules \( F_{r_1,r_2;s_1,s_2,s_3} \) by

\[
F_{r_1,r_2;s_1,s_2,s_3} = F |r_1, r_2; s_1, s_2, s_3\rangle.
\]

Further we write the total Fock module \( \mathcal{F} \) as

\[
\mathcal{F} = \bigoplus_{r_1,r_2,s_1,s_2,s_3 \in \mathbb{Z}} F_{r_1,r_2;s_1,s_2,s_3}.
\]

We regard \( \{a_i^n, b_i^n, c_i^n | n \in \mathbb{Z}_{>0}, i = 1,2, \mu = 1,2,3 \} \) as the set of annihilation operators, and \( \{a_i^n, b_i^n, c_i^n, Q_a^n, Q_b^n, Q_c^n | n \in \mathbb{Z}_{<0}, i = 1,2, \mu = 1,2,3 \} \) that of creation operators. We denote by \( : \ldots : \) the corresponding normal ordering of operators. For example,

\[
:exp\left\{b^\mu(z;\alpha)\right\} := \exp\left\{-\sum_{n<0} \frac{b^n}{[n]} z^{-n} q^{|n|} \alpha\right\} \exp\left\{-\sum_{n>0} \frac{b^n}{[n]} z^{-n} q^{|n|} \alpha\right\} e^{Q_a^n z^{-n} Q_a^n}.
\]
Note that such normal ordered operators are not well defined by themselves, because they have no meaning as a formal power series. If, however, we regard these operators as ones acting on the Fock module $\mathcal{F}$, they have a well defined meaning.

The propagators of the boson fields $a^i$ read as follows:

$$
\langle a^i(M; N|z; \alpha) a^j(M'; N'|w; \beta) \rangle
= -\sum_{n>0} \frac{[Mn][M'n]}{[Nn][N'n]} \begin{pmatrix} a^i_n, a^j_{-n} \end{pmatrix} \left( \frac{w}{z} \right)^n q^{(\alpha+\beta)n} + \frac{MM'[a^i_0, Q^j_d]}{NN'} \log z
\tag{8}
$$

The formal power series in $w/z$ is convergent if $|w/z| << 1$. We introduce the propagators for boson fields $b^\mu$ and $c^\mu$ in the same manner. One can rewrite them simply by using the logarithm. For example,

$$
\langle b^\mu(z; \alpha) b^\nu(w; \beta) \rangle = -\delta_{\mu,\nu} \log(z - q^{\alpha+\beta}w), \quad |z| > |q^{\alpha+\beta}w|.
\tag{9}
$$

Using these propagators, we obtain Wick’s Theorem in the following form:

**Proposition 1 (Wick’s Theorem)**

$$
: \exp \left\{ a^i(M; N|z; \alpha) \right\} : : \exp \left\{ a^j(M'; N'|w; \beta) \right\} :
= \exp \left\{ \langle a^i(M; N|z; \alpha) a^j(M'; N'|w; \beta) \rangle \right\} : \exp \left\{ a^i(M; N|z; \alpha) + a^j(M'; N'|w; \beta) \right\} :.
$$

There are similar formulas for $b^\mu$ and $c^\mu$.

5. Current algebra In this section we construct the $U_q(sl_3)$ currents $J^\pm_i(z)$, $\psi_i(z)$ and $\varphi_i(z)$ ($i = 1, 2$). Let us define the fields $J^\pm_i(z)$ as follows:

$$
J^+_1(z) = -: 1 \partial_z \exp \left\{ -a^1(z; 0) \right\} : \exp \left\{ -b^1(z; 1) \right\} :;
$$

$$
J^+_2(z) = -: 1 \partial_z \exp \left\{ -c^2(qz; 0) \right\} : \exp \left\{ -b^2(qz; 1) \right\} \exp \left\{ b^+_2(z) - b^1_+(qz) \right\} :;
$$

$$
-: 1 \partial_z \exp \left\{ -c^3(z; 0) \right\} : \exp \left\{ -b^3(z; 1) \right\} \exp \left\{ (b + c)^1(z; 0) \right\} :;
$$

$$
J^-_1(z) = : k+3 \partial_z \exp \left\{ \frac{1}{k+3} a^1(z; -\frac{k+3}{2}) + \frac{k+2}{k+3} b^1(z; 1) \right\} (z; 1) + \frac{k+1}{k+3} c^1(z; 0) \right\} :.
$$
Define further the fields $\psi_i(z), \varphi_i(z)$ as

$$
\psi_1(z) = \exp\left\{ a_+^1 \left( q^\frac{k+3}{2} z \right) + \left( \frac{3}{2} b_1 \right)^1 \left( q^\frac{k}{2} + 1 \right) z \right\} b_2^1 \left( q^\frac{k}{2} + 2 \right) z + b_3^1 \left( q^\frac{k}{2} + 3 \right) z \right\}, \\
\varphi_1(z) = \exp\left\{ a_-^1 \left( q^{-\frac{3}{2}} z \right) + \left( \frac{3}{2} b_1 \right)_-^1 \left( q^{-\frac{k}{2}} - 1 \right) z \right\} b_2^1 \left( q^{-\frac{k}{2}} - 2 \right) z + b_3^1 \left( q^{-\frac{k}{2}} - 3 \right) z \right\}, \\
\psi_2(z) = \exp\left\{ a_+^2 \left( q^{\frac{3}{2}} z \right) - b_1^1 \left( q^{\frac{k}{2}} + 1 \right) z \right\} b_2^2 \left( q^{\frac{k}{2}} + 2 \right) z + b_3^2 \left( q^{\frac{k}{2}} z \right) \right\}, \\
\varphi_2(z) = \exp\left\{ a_-^2 \left( q^{-\frac{3}{2}} z \right) - b_1^1 \left( q^{-\frac{k}{2}} - 1 \right) z \right\} b_2^2 \left( q^{-\frac{k}{2}} - 2 \right) z + b_3^2 \left( q^{-\frac{k}{2}} z \right) \right\}.
$$

The formulas (10) are not so useful for OPE calculation, because they contain difference operators, and bosons in these formulas are somewhat complicated. By the definition of the boson fields $a^1, b^\mu, c^\mu, a_\pm^1, b_\pm^1, c_\pm^1$, and the $q$-difference operator, we can recast the fields $J_i^\pm(z)$ as

$$
J_1^+(z) = \frac{-1}{(q - q^{-1}) z} : \left( \exp\left\{ b_1^1(z) - (b + c)^1(qz; 0) \right\} - \exp\left\{ b_1^1(z) - (b + c)^1(q^{-1}z; 0) \right\} \right) :, \\
J_2^+(z) = \frac{-1}{(q - q^{-1}) z} : \left( \exp\left\{ -b_1^1(qz) + b_2^2(qz) + b_3^2(z) - (b + c)^2(q^2z; 0) \right\} \right).
$$
Our main purpose is to state the OPE algebra of these currents. To this end, let us

\[
J_1^-(z) = \frac{1}{(q - q^{-1})z} : \exp \left\{ a_+^1(q^{k+3}/2z) + b_+^1(q^{k+2}z) - (b + c)^2(q^{k+3}z) + (b + c)^3(q^{k+2}z; 0) \right\} : 
\]

\[
+ \exp \left\{ a_+^1(q^{-k+3}/2z) + b_+^1(q^{-k+2}z) - (b + c)^2(q^{-k+3}z; 0) \right\} : 
\]

\[
- \exp \left\{ a_-^1(q^{k+3}/2z) + b_-^1(q^{k+2}z) - (b + c)^2(q^{k+3}z; 0) \right\} : 
\]

\[
+ \exp \left\{ a_-^1(q^{-k+3}/2z) + b_-^1(q^{-k+2}z) - (b + c)^2(q^{-k+3}z; 0) \right\} : 
\]

\[
+ \exp \left\{ a_+^3(q^{k+3}z) + b_+^3(q^{k+2}z) - (b + c)^2(q^{k+3}z; 0) \right\} : 
\]

\[
- \exp \left\{ a_+^3(q^{k+3}z) - b_+^3(q^{k+2}z) - (b + c)^2(q^{k+3}z; 0) \right\} : 
\]

\[
J_2^-(z) = \frac{1}{(q - q^{-1})z} : \exp \left\{ a_+^2(q^{k+3}/2z) + b_+^2(q^{k+2}z) - (b + c)^2(q^{k+2}z; 0) \right\} : 
\]

\[
- \exp \left\{ a_-^2(q^{-k+3}/2z) + b_-^2(q^{-k+2}z) - (b + c)^2(q^{-k+2}z; 0) \right\} : 
\]

\[
+ \exp \left\{ a_-^2(q^{-k+3}/2z) - b_-^2(q^{-k+2}z) - (b + c)^2(q^{-k+2}z; 0) \right\} : 
\]

\[
- \exp \left\{ a_-^2(q^{-k+3}/2z) - b_-^2(q^{-k+2}z) + (2b)^2(q^{-k+2}z) \right\} : 
\]

\[
- \exp \left\{ a_-^2(q^{-k+3}/2z) + b_-^2(q^{-k+2}z) + (2b)^2(q^{-k+2}z) \right\} : 
\]

Our main purpose is to state the OPE algebra of these currents. To this end, let us
Further define \( g_{ij}(z) \) as the following formal power series
\[
g_{ij}(z) = (q^{-a_{ij}} - z) \times \sum_{n \geq 0} (q^{-a_{ij}} z)^n,
\]
and its inverse
\[
g_{ij}(z)^{-1} = (q^{a_{ij}} - z) \times \sum_{n \geq 0} (q^{a_{ij}} z)^n.
\]
Further define \( \delta(z) \) by
\[
\delta(z) = \sum_{n \in \mathbb{Z}} z^n. \tag{12}
\]
Using Wick’s Theorem, we get the following formulas.

**Proposition 2** Let \( J^\pm_i(z), \psi_i(z) \) and \( \varphi_i(z) \) \((i = 1, 2)\) be the fields defined as above, and let \( \gamma = q^k \), then the following relations hold in the sense of a formal power series:

\[
[\varphi_i(z), \varphi_j(w)] = 0,
\]
\[
[\psi_i(z), \psi_j(w)] = 0,
\]
\[
\varphi_i(z)\psi_j(w) = g_{ij}(zw^{-1}\gamma^{-1})g_{ij}(zw^{-1}\gamma)^{-1}\psi_j(w)\varphi_i(z),
\]
\[
\varphi_i(z)J^\pm_j(w) = g_{ij}(zw^{-1}\gamma^{\mp 1/2}q^{\pm 1})J^\pm_j(w)\varphi_i(z),
\]
\[
\psi_i(z)J^\pm_j(w) = g_{ij}(z^{-1}w\gamma^{\mp 1/2})q^{\pm 1}J^\pm_j(w)\psi_i(z),
\]
\[
\left[J^+_i(z), J^-_j(w)\right] = \frac{\delta_{ij}}{(q - q^{-1})zw} \left( (z w^{-1} \gamma^{-1}) \psi_i(\gamma^{1/2} w^{-1} \gamma - \gamma w^{-1} \gamma) \varphi_i(\gamma^{-1/2} w^{-1}) - (z w^{-1} \gamma) \varphi_i(\gamma^{1/2} w^{-1}) \right),
\]
\[
(z - wq^{a_{ij}})J^\pm_i(z)J^\pm_j(w) = (zq^{a_{ij}} - w)J^\pm_i(w)J^\pm_j(z),
\]
\[
\left\{ J^\pm_i(z_1)J^\pm_j(z_2)J^\pm_j(w) - (q + q^{-1})J^\pm_i(z_1)J^\pm_j(w)J^\pm_j(z_2) + J^\pm_j(w)J^\pm_j(z_1)J^\pm_j(z_2) \right\}
+ \left\{ z_1 \leftrightarrow z_2 \right\} = 0 \quad \text{for } a_{ij} = -1.
\]

Before we consider the mode expansions of the fields \( J^\pm_i(z), \psi_i(z), \) and \( \varphi_i(z) \), let us introduce the operators \( J^3_i \) \((n \in \mathbb{Z})\) and \( K^\pm_i \) as

\[
J^3_i = a_1^n q^{-\frac{3}{2} |n|} + \frac{[2n]}{[n]} b_1^n (-k_1 + 1)^{|n|} - b_1^n q^{-\frac{k_1}{2} |n|} + b_2^n q^{-\frac{k_2}{2} |n|},
\]
\[
J^3_i = a_2^n q^{-\frac{3}{2} |n|} - b_1^n q^{-\frac{k_1}{2} |n|} + \frac{[2n]}{[n]} b_1^n (-k_1 + 1)^{|n|} + b_2^n q^{-\frac{k_2}{2} |n|},
\]
\[
K^\pm_i = q^{\pm a_0 + \sum_{j=1}^{2} a_{ij} b_{0}^j + b_0^j}.
\]
Then we can write $\psi_i(z)$ and $\varphi_i(z)$ as follows

$$\psi_i(z) = K_i \exp \left\{ (q - q^{-1}) \sum_{n>0} J^3_{in} z^{-n} \right\},$$

$$\varphi_i(z) = K^{-1}_i \exp \left\{ -(q - q^{-1}) \sum_{n<0} J^3_{in} z^{-n} \right\}.$$

Since the currents $J^\pm_i(z), \psi_i(z)$, and $\varphi_i(z)$ are well defined operators acting on the Fock module $\mathcal{F}$, then we can consider the following mode expansions

$$\sum_{n \in \mathbb{Z}} J^\pm_{in} z^{-n-1} = J^\pm_i(z), \quad \sum_{n \in \mathbb{Z}} \psi_{in} z^{-n} = \psi_i(z), \quad \sum_{n \in \mathbb{Z}} \varphi_{in} z^{-n} = \varphi_i(z). \quad (13)$$

Now we are ready to state our main proposition:

**Proposition 3** The operators $\{J^3_{in} | n \in \mathbb{Z}, i = 1, 2\}, \{J^\pm_{in} | n \in \mathbb{Z}, i = 1, 2\}, K_i (i = 1, 2)$ and $\gamma^{\pm1/2}$ acting on the Fock module $\mathcal{F}$, satisfy the following relations.

$$\gamma^{\pm1/2} \in \text{the center of the algebra,}$$

$$[J^3_{in}, J^3_{jm}] = \delta_{n+m,0} \frac{1}{n} [a_{ij} n] \frac{\gamma^n - \gamma^{-n}}{q - q^{-1}},$$

$$[J^3_{in}, K_j] = 0,$$

$$K_i J^\pm_{jn} K^{-1}_i = q^{\pm a_{ij}} J^\pm_{jn},$$

$$[J^3_{in}, J^\pm_{jm}] = \pm \frac{1}{n} [a_{ij} n] \gamma^{\pm |n|/2} J^\pm_{jn+m},$$

$$J^\pm_{in+1} J^\pm_{jm} = q^{\pm a_{ij}} J^\pm_{jm} J^\pm_{in+1} = q^{\pm a_{ij}} J^\pm_{jm+1} J^\pm_{in} - J^\pm_{jm+1} J^\pm_{in},$$

$$\left[ J^+_m, J^-_{jm} \right] = \frac{\delta_{i,j}}{q - q^{-1}} \left( \gamma^{(n-m)/2} \psi_{in+m} - \gamma^{(m-n)/2} \varphi_{in+m} \right),$$

$$\left\{ J^\pm_{il} J^\pm_{jm} - (q + q^{-1}) J^\pm_{in} J^\pm_{jm} + J^\pm_{jn} J^\pm_{im} \right\}$$

$$+ \left\{ l \leftrightarrow m \right\} = 0 \quad \text{for } a_{ij} = -1.$$

These are exactly the relations of the Drinfeld realization of $U_q(\mathfrak{sl}_3)$ for level $k$. Thus (10) and (11) yields the required bosonization. Since the vectors $|l_1, l_2; 0, 0, 0 \rangle (l_1, l_2 \in \mathbb{Z})$ have the following properties

$$K_i |l_1, l_2; 0, 0, 0 \rangle = q^i |l_1, l_2; 0, 0, 0 \rangle \quad i = 1, 2,$$

$$J^+_m |l_1, l_2; 0, 0, 0 \rangle = 0 \quad i = 1, 2 \quad n \geq 0,$$

$$J^-_{in} |l_1, l_2; 0, 0, 0 \rangle = 0 \quad i = 1, 2 \quad m > 0,$$

$$\psi_{im} |l_1, l_2; 0, 0, 0 \rangle = 0 \quad i = 1, 2 \quad m > 0,$$
we have the highest weight representations of $U_q(\hat{sl}_3)$ in the Fock module $F$. One can immediately find that this representation reduces to the Wakimoto representation in the $q \to 1$ limit. Note that we have a nice relation between this Wakimoto representation and the difference operator representation of $U_q(\hat{sl}_3)$ given in ref.[12].

6. screening currents

Let us define the screening currents $S_i(z)\ (i = 1, 2)$ as follows:

$$S_1(z) = -: \left[ \left[ 1 \partial_z \exp \left\{-c^1(qz; 0)\right\} \right] \exp \left\{-b^1(qz; -1)\right\} \exp \left\{-b^3(z) + b^2(qz)\right\} \right.$$ 

$$+ \left[ 1 \partial_z \exp \left\{-c^3(z; 0)\right\} \right] \exp \left\{-b^3(z; -1)\right\} \exp \left\{(b + c)^2(z; 0)\right\} \right] \exp \left\{ -\left(\frac{1}{k+3}a\right)^1(z; -\frac{k+3}{2}) \right\} :.$$ 

$$S_2(z) = -: \left[ 1 \partial_z \exp \left\{-c^2(z; 0)\right\} \right] \exp \left\{-b^2(z; -1)\right\} \exp \left\{ -\left(\frac{1}{k+3}a\right)^2(z; -\frac{k+3}{2})\right\} : .$$

Then we get the following proposition and its corollary.

**Proposition 4** The following commutation relations hold.

$$[\psi_i(z), S_j(w)] = 0,$$

$$[\varphi_i(z), S_j(w)] = 0,$$

$$[J^+_i(z), S_j(w)] = 0,$$

$$[J^-_i(z), S_j(w)] = \delta_{ij} \frac{1}{k+3} \partial_w \left( \delta(w/z) : \exp \left\{-\left(\frac{1}{k+3}a\right)^i(w; \frac{k+3}{2})\right\} : \right).$$

**Corollary 5** If the Jackson integral of the screening currents

$$\int_0^{s_\infty} S_i(t) d_pt, \quad p = q^{2(k+3)}$$

are convergent, then they commute with the action of $U_q(\hat{sl}_3)$ exactly.

Results obtained in this article can be extended to higher rank algebra $U_q(\hat{sl}_n)$ [15].

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