Tip-vortex instabilities of two in-line wind turbines

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Abstract. The hydrodynamic stability of a vortex system behind two in-line wind turbines operating at low tip-speed ratios is investigated using the actuator-line method in conjunction with the spectral-element flow solver Nek5000. To this end, a simplified setup with two identical wind turbine geometries rotating at the same tip-speed ratio is simulated and compared with a single turbine wake. Using the rotating frame of reference, a steady solution is obtained, which serves as a base state to study the growth mechanisms of induced perturbations to the system. It is shown that, already in the steady state, the tip vortices of the two turbines interact with each other, exhibiting the so-called overtaking phenomenon. Hereby, the tip vortices of the upstream turbine overtake those of the downstream turbine repeatedly. By applying targeted harmonic excitations at the upstream turbine’s blade tips a variety of modes are excited and grow with downstream distance. Dynamic mode decomposition of this perturbed flow field showed that the unstable out-of-phase mode is dominant, both with and without the presence of the second turbine. The perturbations of the upstream turbine’s helical vortex system led to the destabilization of the tip vortices shed by the downstream turbine. Two distinct mechanisms were observed: for certain frequencies the downstream turbine’s vortices oscillate in phase with the vortex system of the upstream turbine while for other frequencies a clear out-of-phase behaviour is observed. Further, short-wave instabilities were shown to grow in the numerical simulations, similar to existing experimental studies \cite{1}.

1. Introduction

Nowadays, wind turbines of highly clustered wind farms often operate in the wake of upstream wind turbines. In some cases, wind turbines may even be subjected to the wake of multiple upstream turbines. A wind turbine operating inside a full wake experiences reduced power production due to the incoming velocity deficit and may be subjected to higher fatigue loads caused by the unsteadiness of the approaching flow. The flow characteristics, such as the mean wind profile and the turbulence statistics, inside a wind farm are different compared to atmospheric turbulence \cite{2}. One of the factors influencing these characteristics is the evolution of coherent flow structures, such as the helical tip vortices.

The stability of wind-turbine tip vortices in uniform flow has been extensively analysed in theoretical \cite{3, 4, 5, 6}, experimental \cite{7, 1, 8, 9} and numerical \cite{10, 11} studies. These investigations show that for higher tip-speed ratios the mutual induction of adjacent vortex filaments promotes instabilities of the helical vortex system. Low-level disturbances excite these unstable modes, leading to their exponential growth and the subsequent breakdown of the tip vortices. For low tip-speed ratios, the tip vortices are more stable, as the distance between neighboring helix loops...
is greater. Previous numerical studies have confirmed this behavior [12]. When comparing the flow of an isolated wind turbine with tip-speed ratios $\lambda = 3$, $\lambda = 6$ (optimal operating condition) and $\lambda = 10$, structured tip vortices can be clearly identified many diameters downstream for the lower tip speed ratio, even with turbulent inflow, as opposed to the high tip-speed ratio case, where the vortex system immediately breaks down to turbulence. These results indicate that coherent structures may travel farther downstream for turbines operating at some off-design conditions. However, a wake system generated by multiple turbines might be unstable even if the turbines are operating at low tip-speed ratios, as the tip vortices and wakes generated by several turbines may interact with each other. Since coherent flow structures may increase fatigue loads, it is relevant to understand the evolution and breakdown of the vortex system generated by multiple wind turbines.

In this work, we investigate the wake interaction of two in-line turbines, focusing on the hydrodynamic stability of the helical vortex system generated by the two turbines, which has previously not been studied. This stability analysis will generate a deeper understanding of the detailed interaction of in-line wind turbines. By simulating the flow in the rotating frame of reference, without free-stream turbulence, a steady solution is obtained, which is then used as a base state to study the growth mechanisms of induced perturbations to the system. The tip vortices are perturbed harmonically, following the methodology applied in previous studies [10, 13], to identify the dominant modes contributing to the growth of instabilities which lead to a breakdown of the vortex system. We include perturbations only at the blade tips of the upstream turbine, in order to investigate how the wake and vortex system of the upstream turbine influences and destabilizes that of the downstream wind turbine.

2. Methods

2.1. Numerical Solver

The three-dimensional incompressible Navier-Stokes equations are solved with the spectral-element method Nek5000. The spectral-element method (SEM) exhibits very little numerical dispersion and dissipation [14], which is relevant for stability calculations. In each spectral element seventh-order Legendre polynomials on Gauss-Legendre-Lobatto quadrature points are used for spatial discretization and a third-order implicit/explicit scheme is applied for temporal discretization. The non-linear terms are treated explicitly by third-order extrapolation while the viscous terms are discretized implicitly using a third-order backward differentiation scheme. Filtering is applied to the highest modes to stabilize the numerical simulation [15].

The simulations are conducted in the rotating frame of reference, in order to obtain a steady-state solution. Therefore, centrifugal and Coriolis forces are added to the momentum equations.

2.2. Domain and Boundary Conditions

All quantities are non-dimensionalized by the wind-turbine radius $R$ and the free-stream velocity $W_\infty$. As seen in figure 1(a), the upstream turbine $T_1$ is positioned at the origin of the cylindrical domain which has radius $R_{rad} = 10$ and extends 25 radii. The streamwise spectral-element discretization is uniform between $-0.52 < z < 14$, with a constant spacing of $dz_E = 0.13$, beyond which each element is stretched by 3% w.r.t. the previous one. The radial spectral-element discretization is shown in figure 1(b). Constant uniform velocity is imposed at the inflow and lateral boundaries. A natural outflow boundary condition is applied at the outlet.

2.3. Wind Turbine Modelling

The wind turbines are modelled using the actuator line method [16]. In this method, the blades are represented by body forces calculated from airfoil data and local velocity. Experimental data, obtained by [13], provide the two-dimensional lift and drag coefficients of the 14% thick NREL S826 airfoil. Then, considering the local velocity and the chord and twist distributions, the
Figure 1. (a) Schematic view of the computational domain. The upstream turbine ($T_1$) is located at $(x_1, y_1, z_1) = (0, 0, 0)$, while the downstream turbine ($T_2$) is located at $(x_2, y_2, z_2) = (0, 0, 6)$. $R_{rad} = 10$, $z_{in} = 7$, $\Delta z = 6$ and $z_{out} = 12$. All distances are non-dimensionalized by the radius. (b) Mesh at the center of the domain, showing the spectral elements.

normal and tangential forces are calculated for $N_{ACL}$ points ($x_n$) along the blades. The discrete two-dimensional force vector ($F_{2D}$) is projected on the domain ($x$) using the convolution of the force with a three-dimensional Gaussian kernel:

$$F(x, t) = F_{2D}(x_n, t) * \eta(|x - x_n|),$$

where $\eta$ is a smearing parameter. Following parametric studies [17], $\varepsilon \approx 2.5\Delta r$ (where $\Delta r$ is the averaged grid spacing) and each blade is discretized with 70 points.

In order to compare the interaction of the tip vortices of the two-turbine system with the evolution of a single wind-turbine wake, an additional case with only one turbine ($T_1$) is also simulated. For all simulations the radius-based Reynolds number used is $Re_R = 50000$.

### 2.4. Perturbation at blade tip

Perturbations are included as streamwise forcing slightly upstream of the blade tips of the upstream turbine ($T_1$) as the sum of discrete harmonic functions

$$F_z(t) = \sum_{n=1}^{N_f} A_n \sin(2\pi \cdot St_n \cdot t + \phi_n),$$

where the amplitude of each frequency is defined as $A_n = A_{total}/N_f = 2.5 \cdot 10^{-3}$, the discrete Strouhal numbers are $St_n = St_{max} \cdot n/N_f$, with $St_{max} = 5 \lambda/(2\pi)$ and $N_f = 20$ ($St$ and $t$ are
nondimensionalized using $R$ and $W_{\infty}$). The phase is randomly generated in order to create a spread signal. The forcing is then distributed using a three-dimensional Gaussian,

$$F_z(x,t) = F_z(t) \frac{1}{\varepsilon_f \pi^{3/2}} \exp \left[ -\left( \frac{|x-x_0|}{\varepsilon_f} \right)^2 \right], \quad (3)$$

with $F_z(x,t)$ being the streamwise body force per unit mass term [14], $\varepsilon_f = 0.1$ as a smearing parameter and $x_0$, the center of the Gaussian, being 0.025 upstream of the blade tips.

2.5. Dynamic Mode Decomposition

In order to extract the most important modes and frequencies associated with the vortex system destabilization, dynamic mode decomposition (DMD) [21, 22] is employed. The discretized flow field $u(x_i,t_j)$ is decomposed in modes $\phi_k(x_i)$ associated with a complex frequency $i\omega_k$:

$$u(x_i,t_j) = \sum_{k=0}^{m-1} \phi_k(x_i)e^{i\omega_k j \Delta t}, \quad (4)$$

using a finite sequence of $m$ snapshots. DMD has already been shown to capture the dominant frequencies and structures that lead to the instability of wind-turbine tip vortices, consistent with Fourier analysis [11].

Dynamic modes were computed separately for the intervals $0.5 \leq z \leq 3$ (downstream of $T_1$) and $6.5 \leq z \leq 9$ (downstream of $T_2$) in the region of the tip vortices ($0.7 \leq r \leq 1.3$). The sampling interval is $\Delta t = 1/50$ and the number of snapshots $m = 838$ is chosen to cover two periods of the lowest excited frequency.

3. Results

3.1. Steady solution

Without perturbations, a steady solution is obtained for both cases. Since the turbines are operating in a non-optimal condition, changes in the attached circulation lead to the release of vorticity along the entire actuator line and into the wake, as shown in figures 2 and 3. An optimally operating wind turbine would typically exhibit near-constant circulation along the blades, resulting in strong tip and root vortices with little vorticity shed in-between. Besides the vortex sheet emitted because of the lift distribution, additional vorticity is created by the actuator lines due to high values of drag in this case. The rolling-up of the trailing-edge vorticity around the tip vortices creates additional weak counter-rotating vortices in the vicinity of the tip vortices, as can be noticed in figure 3. The velocity deficit downstream of the turbines and the effect of the pressure gradient on the streamwise velocity upstream of the turbines is shown in figure 2.

The downstream turbine modifies the position of the tip vortices, as can be seen in figure 3(c). The tip vortices of $T_1$ expands upstream of $T_2$ and pass outside of the downstream turbine, which sheds a system of three helical vortices that are inside the outer vortex system. This double triple-helix vortex system exhibits a pattern similar to the global pairing of helices observed in the experiments of [9] and numerical simulations of [23] for two helicoidal vortices of different radius. In the current study, the vortices are steady in the rotating frame of reference. However, the pairing can be better compared with experiments by thinking of the vortex cores shown in figure 3 as the evolution in time of a vortex in a fixed frame of reference. In this frame of reference, an vortex emitted by $T_1$ initially increases its radial position and, due to its larger streamwise convection velocity, catches up with an internal vortex shed by $T_2$. When the two vortex cores are roughly at the same streamwise position, the radial position of the external
Figure 2. Streamwise velocity along the wake for the steady solution. Iso-surfaces of vorticity magnitude ($|\omega| = 10$) are shown in grey. (a) Single turbine. (b) Two turbines.

Figure 3. (a) Vorticity magnitude for two-turbine steady solution. (b) Close-up of vortex pairing. Due to interaction, vortex cores elongate in the direction of the line connecting the vortex centers (red dashed line in the figure). Low-intensity counter-rotating vortex can be seen in dark blue, highlighted by a white circle. (c) Contours of constant $y$-vorticity ($\omega_y = -5$) showing the position of the tip vortices. Vortices generated by $T_1$ are represented by black lines and $T_2$ by red lines. The vortices of single turbine simulation are represented by grey lines, for reference.

Vortex decreases while the radial position of the internal vortex increases. After a region of minimum radial distance, the outer vortex pairs with the subsequent inner vortex, restarting the cycle. This phenomenon, in which there is no exchange of roles between the internal and external helical vortex is called “overtaking” by [23], as opposed to the “leapfrogging” effect, where the inner helix takes the role of the outer helix in the subsequent cycle.
Figure 4. Selected dynamic modes depicted using positive and negative iso-contours of the real part of streamwise component of the mode. Iso-surfaces of vorticity magnitude are shown in grey. Upper row: Downstream of $T_1$ ($0.5 \leq z \leq 3$). Lower row: Downstream of $T_2$ ($6.5 \leq z \leq 9$). (a) and (c) $St_k = 1.5$; (b) and (d) $St_k = 3$.

The interaction also modifies the shape of the tip vortex cores. In figure 3, it can be seen that the vortices downstream of the second wind turbine have more elliptical cross-sections than the vortices of a single turbine. This elongation of vortices in the direction of the line connecting the vortex centers for a corotating pair of vortices is due to the combination of rotation and strain induced by the vortices [24].

3.2. Unsteady solution
The stability of the system is investigated numerically by introducing the perturbations and evaluating the response of the flow using dynamic mode decomposition. For each frequency extracted through DMD, the corresponding travelling wave is represented by a pair of phase-shifted modes (real and imaginary part). Using the normalization $St_k = St \cdot (2\pi) / \lambda$, the frequency of the waves travelling along the helix can be directly related to the azimuthal wavenumber, $k = 2\pi r/l$ where $r$ is the radial position and $l$ is the wavelength. Since the excited waves are mainly advected by the flow (also verified by simulations in the fixed frame of reference [25]), the phase speed of the waves in the rotating frame of reference is $c \approx \Omega r$, giving $k \approx St_k$. This assertion can be observed in figure 4, where selected DMD modes computed in the intervals $0.5 \leq z \leq 3$ (downstream of $T_1$) and $6.5 \leq z \leq 9$ (downstream of $T_2$), are shown.

For $St_k = 1.5$ downstream of $T_1$ in figure 4(a) a clear out-of-phase behaviour of adjacent spirals can be observed. Previous studies have shown that these out-of-phase dynamics, corresponding to wavenumbers $k = N(j + 1/2)$, where $N$ is the number of blades and $j$ is an integer, lead to a maximum dimensionless growth rate in the helical vortex system [10, 11]. The mode with $St_k = 3$ corresponds to a stable, in-phase disturbance for a single turbine, where all wavenumbers $k = Nj$ have zero growth rate.

DMD modes extracted in the region downstream of $T_2$ show that the perturbed vortices of the upstream turbine excite those of the downstream turbine. For $St_k = 1.5$, the tip vortex of
Figure 5. Schematic representation of the in-phase and out-of-phase tip vortex dynamics of the modes shown in figure 4. The tip vortices of $T_1$ are depicted in blue, those of $T_2$ are red. Wavenumber of (a) and (c) $k = 1.5$; (b) and (d) $k = 3$. (a) Out-of-phase single-turbine vortices; (b) In-phase single-turbine vortices; (c) $T_1$ vortices are out-of-phase with each other but in-phase with the nearest vortices of $T_2$; (d) $T_1$ vortices are in-phase with each other, but out-of-phase with nearest vortices of $T_2$.

the downstream turbine oscillate in-phase with the nearest vortex, as can be seen in figure 4(c). For $St_k = 3$, the in-phase displacement of the upstream turbine’s vortices induces out-of-phase dynamics for the downstream turbines vortices, as can be seen in figure 4(d). Figure 5 represents schematically the dynamics observed for $St_k = 1.5$ and $St_k = 3$.

We hypothesize that the two different mechanisms seen for $St_k = 1.5$ and $St_k = 3$ can both be understood by the dominance of out-of-phase perturbations. For $St_k = 1.5$ the mode shown in figure 4(a) has already experienced a significant growth due to its higher growth rate. Hence, the downstream turbines tip vortices are induced to adopt the same displacement, thus retaining the out-of-phase behaviour. For $St_k = 3$ the growth of the mode is close to zero upstream of $T_2$ [11] as the vortices are displaced in phase. For this stable configuration, the unstable out-of-phase displacement of the vortices of $T_2$ is favored since it has the highest growth rate. Similar dynamics to the modes corresponding to $St_k = 1.5$ and $St_k = 3$ are observed for other modes with different frequencies. Some of the modes that clearly exhibit in-phase induced dynamics are in the region $1 \leq St_k \leq 2$, which also have the highest amplitudes (figures 6) downstream of $T_2$. We therefore assume that the dominance of either the in-phase or the out-of-phase perturbation of the tip vortices of $T_2$ w.r.t. $T_1$ is dependent on the amplitudes and growth rate of the mode behind the upstream turbine. However, further studies are needed to understand this phenomenon.

As expected, the results of the dynamic mode decomposition for the single turbine simulation in the interval $0.5 \leq z \leq 3$ do not show any relevant differences to the double turbine simulation. Figure 6 shows that the energy of the modes extracted for the region closer to $T_1$ is spread along the excited frequencies ($0.25 \leq St_k \leq 5$). However, farther from $T_1$, the out-of-phase mode, $St_k = 1.5$, tends to predominate for the single wind turbine, as it has the highest theoretical
Figure 6. Amplitude of the dynamic modes normalized by the highest amplitude downstream of $T_1$ ($0.5 \leq z \leq 3$). (a) Single-turbine simulation; (b) Two-turbine simulation.

growth rate [3]. The presence of the second turbine does not influence this effect, as can be observed by comparing the amplitudes of both simulations (figure 6).

The modes related to the excited frequencies ($St_k \leq 5$) show the pattern of displacement modes imposed by the streamwise forcing: an increase or reduction of the streamwise velocity along the helix which displaces the vortex filament. However, beyond the Strouhal number associated with the imposed excitations ($St_k > 5$), nonlinear interaction triggers different mode structures: negative and positive lobes at the same position along the helix, as can be observed in figures 7. These structures are also represented by phase-shifted DMD modes, indicating travelling waves. As seen in figure 8, the modes show a modulation of the vortex core, which is associated with short-wave instabilities, such as the elliptical instability, which is known to occur in strain fields [26], and the curvature instability [6]. Short-wave instabilities have already been observed experimentally for low pitch helical vortices [1]. In our perturbed simulation, short-wave oscillations are observed before the vortices destabilize. Figure 9 shows a comparison between the short wavelength oscillations seen in the numerical simulations with experimental observations made in [1].

Figure 7. Selected dynamic modes downstream of $T_1$ ($0.5 \leq z \leq 3$). (a) $St_k = 5.5$; (b) $St_k = 7$.

4. Conclusions
The hydrodynamic stability of the vortex system of two in-line wind turbines operating at tip-speed ratio $\lambda = 3$ is investigated using the actuator-line method in conjunction with a spectral-element flow solver. The simulations are conducted in a rotating frame of reference to obtain a steady base state. The results are then compared to a simulation of a single turbine flow field. Already in the steady solution, interaction between the tip vortices of the two turbines is observed. The tip vortices of the upstream turbine pair globally with the vortices of the
downstream turbine, similar to the “overtaking” phenomenon described by [23]. This steady two-turbine interaction modifies the position and shape of the vortex cores compared to the single-turbine solution.

By perturbing the flow close to the tip of the upstream turbine blades, unstable modes are excited. Dynamic mode decomposition shows that the mode consisting of out-of-phase displacement of adjacent vortex spirals of the upstream turbine, with a wavenumber $k = 1.5$, has the highest amplitude in both simulations.

The perturbation of the tip vortices of $T_1$ also excite the tip vortices of $T_2$. Two distinct dynamics are observed: for certain frequencies, the inner vortices of $T_2$ move in phase with the outer vortices of $T_1$ (e.g. for $St_k = 1.5$), while for other frequencies (e.g. for $St_k = 3$), they move out of phase w.r.t. the vortices of $T_1$. We intend to further investigate these effects by single-frequency disturbances and investigating the isolated response to those specific frequencies.

DMD modes with significant amplitudes were found for higher frequencies, which were not explicitly excited at the upstream turbine. While the DMD modes of the excited frequencies indicate displacement of the helical vortex filament, these short-wave unexcited modes indicate waves that distort the vortex core. These modes show some of the characteristics of short-wave instabilities. Future studies will investigate these modes, in order to distinguish the main mechanisms for their growth.
Acknowledgments

The computations were performed on resources provided by the Swedish National Infrastructure for Computing (SNIC) at PDC. This work was conducted within StandUp for Wind. Further, we gratefully acknowledge the funding provided by the Swedish Energy Agency.

References

[1] Leweke T, Quaranta H, Bolnott H, Blasco- Rodríguez F and Le Dizès S. Long-and short-wave instabilities in helical vortices. In: Journal of Physics: Conference Series. vol. 524. IOP Publishing; 2014. p. 012154.

[2] Frandsen ST. Turbulence and and turbulence-generated structural loading in wind turbine clusters. Technical University of Denmark; 2007. PhD thesis.

[3] Gupta B and Loewy R. Theoretical analysis of the aerodynamic stability of multiple, interdigitated helical vortices. 1974. AIAA journal; 12(10):1381–1387.

[4] Okulov V. On the stability of multiple helical vortices. 2004. Journal of Fluid Mechanics; 521:319–342.

[5] Okulov V and Sørensen JN. Stability of helical tip vortices in a rotor far wake. 2007. Journal of Fluid Mechanics; 576:1–25.

[6] Hattori Y and Fukumoto Y. Short-wavelength stability analysis of a helical vortex tube. 2009. Physics of fluids; 21(1):014104.

[7] Felli M, Camussi R and Di Felice F. Mechanisms of evolution of the propeller wake in the transition and far fields. 2011. Journal of Fluid Mechanics; 682:5–53.

[8] Quaranta HU, Bolnott H and Leweke T. Long-wave instability of a helical vortex. 2015. Journal of Fluid Mechanics; 780:687–716.

[9] Quaranta HU, Brynjell-Rahkola M, Leweke T and Henningson DS. Local and global pairing instabilities of two interlaced helical vortices. 2019. Journal of Fluid Mechanics; 863:927–955.

[10] Ivanell S, Mikkelsen R, Sørensen JN and Henningson DS. Stability analysis of the tip vortices of a wind turbine. 2010. Wind Energy; 13(8):705–715.

[11] Sarmast S, Dafdar R, Mikkelsen RF, Schlatter P, Ivanell S, Sørensen JN and Henningson DS. Mutual inductance instability of the tip vortices behind a wind turbine. 2014. Journal of Fluid Mechanics; 755:705–731.

[12] Kleusberg E, Mikkelsen RF, Schlatter P, Ivanell S and Henningson DS. High-order numerical simulations of wind turbine wakes. In: Journal of Physics: Conference Series. vol. 854. IOP Publishing; 2017. p. 012025.

[13] Sarmast S and Mikkelsen RF. The experimental results of the NREL S826 airfoil at low Reynolds numbers. KTH Royal Institute of Technology; 2012. Technical Report.

[14] Fischer P, Lottes J, Kerkemeier S, Marin O, Heisey K, Obabko A, Merzari E and Peet Y. Nek5000: User’s manual; 2015. Technical Report ANL/MCS-TM-351, Argonne National Laboratory.

[15] Negi P, Schlatter P and Henningson D. A re-examination of filter-based stabilization for spectral-element methods. KTH Royal Institute of Technology; 2017. Technical Report.

[16] Sorensen JN and Shen WZ. Numerical modeling of wind turbine wakes. 2002. Journal of fluids engineering; 124(2):393–399.

[17] Kleusberg E. Wind turbine simulations using spectral elements; 2017. Licentiate thesis.

[18] Pierella F, Krogstad PÅ and Sætran L. Blind Test 2 calculations for two in-line model wind turbines where the downstream turbine operates at various rotational speeds. 2014. Renewable Energy; 70:62–77.

[19] Krogstad PÅ and Eriksen PE. Blind test calculations of the performance and wake development for a model wind turbine. 2013. Renewable energy; 50:325–333.

[20] Mühlke, Schottler J, Bartl J, Putrzynski R, Evans S, Bernini L, Schito P, Draper M, Guggeri A, Kleusberg E and et al. Blind test comparison on the wake behind a yawed wind turbine. 2018. Wind Energy Science; 3(2):883–903.

[21] Rowley CW, Mezić I, Bagheri S, Schlatter P and Henningson DS. Spectral analysis of nonlinear flows. 2009. Journal of fluid mechanics; 641:115–127.

[22] Schmid PJ. Dynamic mode decomposition of numerical and experimental data. 2010. Journal of fluid mechanics; 656:5–28.

[23] Selyuk SC. Numerical study of helical vortices and their instabilities. Université Pierre et Marie Curie; 2016. PhD thesis.

[24] Leweke T, Le Dizes S and Williamson CH. Dynamics and instabilities of vortex pairs. 2016. Annual Review of Fluid Mechanics; 48:507–541.

[25] Kleusberg E, Benard S and Henningson D. Tip-vortex breakdown of wind turbines subject to sheared inflow. In: 71st Annual Meeting of the APS Division of Fluid Dynamics. Bulletin of the American Physical Society; 2018.

[26] Kerswell RR. Elliptical instability. 2002. Annual review of fluid mechanics; 34(1):83–113.