Fourier single-pixel imaging based on edge attenuation technique

Yang-Di Hu\textsuperscript{1,a}, Zheng-Dong Cheng\textsuperscript{1,b}, Zhen-Yu Liang\textsuperscript{1,c}, Xiang-zhai\textsuperscript{1,d}, Zhe-Wang\textsuperscript{2}

\textsuperscript{1}National University of Defense Technology, State Key Laboratory of Pulsed Power Laser Technology, Hefei 230037, China
\textsuperscript{2}92664 Troops, China
\textsuperscript{a}huyangdi2014@163.com, \textsuperscript{b}chengzdmaths@163.com, \textsuperscript{c}liangzheyueei@163.com, \textsuperscript{d}njzhaixiang@gmail.com

Abstract: Fourier single-pixel imaging has been widely concerned with its real-time and high quality. However, DMD (Digital Micromirror Array) as an illumination source cannot produce a continuous gray level of the Fourier basis pattern. Therefore, the inevitable error is produced. This paper proposes a halftone technique based on edge attenuation, which can be applied to the Fourier basis pattern generation method on DMD without loss of resolution, and compared with the traditional methods, the invention solves the problem that the traditional methods has a single scanning direction and accumulated errors, finally; we improve the image quality.

1. Introduction

Single-pixel imaging is a new type of imaging technology that uses a bucket detector with no spatial resolution to receive light intensity for imaging. According to the spectral detection capability of the bucket detector, single-pixel imaging can even achieve invisible spectrum imaging such as infrared. Single-pixel imaging has attracted wide attention due to its low cost, high sensitivity and wide spectral characteristics, at the same time, it has the disadvantages of low imaging quality and slow speed. With the deepening of research, the research on single-pixel imaging is moving towards real-time and high-quality direction. Tao proposed the theory of compressed sensing, which broke the Nyquist sampling law and synchronizes data acquisition with data compression, avoiding the waste of resources caused by the traditional sampling and compression methods, and increased the rate. In 2006, RICE University applied the theory of compressed sensing to single-pixel imaging and successfully designed a single-pixel camera\cite{1}. In 2014, Zhang proposed the Fourier single-pixel imaging theory and applied it successfully\cite{2}. The binary grating pattern on the DMD was used as an illumination source to project on the object, and solved the Fourier spectral coefficient by the intensity value from the bucket detector, then made it through Fourier inverse operation, in 2017, Zhang used the spatial averaging effect to bind the pixels around the aimed pixel to sharing diffusion error\cite{3}, which improved the image quality, but the pixel binding caused the drop of spatial resolution, In 2019, Liang used Sierra-Lite error diffusion kernel to improve the accuracy of gray pattern based on DMD by improving the binary algorithm, and improved the quality of reconstruct images\cite{4}.

Images output devices are usually binary devices\cite{5}, compared to the grating image produced by the projector, the DMD can only output "0" and "1" by controlling the two mirror flip angles, but the high-speed mirror flip of the DMD provides the possibility of real-time measurement.
As is shown in Fig. 1, the DMD control mirror flip angle output "0" or "1", because the inevitably error caused in the binary illumination mode, therefore, there is plenty of room for improvement in this field by improving the projected grating binary pattern and making the pattern maintain better sinusoidal property to improve the measurement accuracy. To this end, this paper proposes an algorithm based on Liang's method, which improves the image quality by edge attenuation method[6], which compared with the traditional method for reducing the spatial resolution.

2. Principle

2.1. Four-step phase shift

In the 2D Fourier transformed domain, the image can be viewed as a weighted sum of 2D sinusoidal light patterns with different frequencies, so by solving each Fourier coefficient, the image can be reconstructed, each Fourier coefficient is calculated by measuring the intensity of light from four π/2 phase shifted grating lighting patterns.

\[ P(x, y, f_x, f_y, \phi) = a + b \cdot \cos(2\pi f_x x + 2\pi f_y y + \phi). \]  

\( P \) is a known illumination grating expression, \( a \) is the average light intensity, \( b \) is the contrast, and \( \phi \) is the initial phase.

Record \( R(x, y) \) as the reflectivity of the surface of the object, \( s \) is the illumination area, the grating pattern is projected onto the object, and the resulting reflection intensity \( I \) is:

\[ I(f_x, f_y) = \int \int R(x, y) \cdot P(x, y, f_x, f_y, \phi) dx dy. \]

Due to the presence of ambient light, the detector response can be expressed as \( D_\phi \):

\[ D_\phi(f_x, f_y) = D_n + k \cdot I_\phi(f_x, f_y). \]

\( D_n \) is noise from ambient light, and the k-factor is related to the size and position of the detector.

Four pairs of gratings with π/2 phase shift are sequentially projected, and the Fourier coefficient \( C \) is solved by the following formula:

\[ C(f_x, f_y) = [D_\phi(f_x, f_y) - D_\phi(f_{\pi/2}, f_y)] + j[D_\phi(f_x, f_{\pi/2}) - D_\phi(f_{\pi/2}, f_y)]. \]

In addition, with the sparsity of the natural image Fourier frequency, the image can be reconstructed with fewer samples.

2.2. Error diffusion method

Error diffusion is a popular algorithm for viewing continuous tone images on a binary display. It is known for its good reproduction grayscale and preservation ability[7], the technology was proposed by Floyd and Steinberg[8]. The core idea is to distribute the quantization error of the current pixel to the adjacent pixels with different weights, that is, multi-pixels share the excessive error caused by the single pixel judged by the threshold value, which can be represented by Fig. 2(a).
Fig. 2. Floyd-Steinberg error diffusion map (a) traditional scan path (b) "s" type scan path (c) "•" stands for the pixels that have been processed. The error caused by the binarization of the current pixel is assigned to the right, bottom right, bottom, and bottom left with a weight of 7:1:5:3. This method delivers the error to the surrounding pixels and reduces the visual error, traditional Floyd-Steinberg error diffusion is a popular halftone method. As is shown in Fig.2 (b), the single scanning mode of this method is easy to cause one-way error accumulation, inspired by Liang, the error diffusion method of "S" scan path is proposed as shown in Fig. 2 (c). The "S" scanning path adopts a different direction depending on odd and even line scanning, which can control the error accumulation caused by the traditional method.

2.3. Error diffusion based on edge attenuation technique

As a popular halftone technology, error diffusion technology can perfectly restore the original grayscale image. However, due to the "poor" nature of the gray level, the image and the original image still have different degrees of deviation. Cheng found that the process of two-dimensional image error diffusion actually spreads the low frequency part of the image to the high frequency part in the frequency domain[9], that is, in the error diffusion process, with the low frequency conversion, the image gradually deviates from the original image, and the error diffusion affects the spectrum. Therefore, in the process of generating a sinusoidal grating, if the loss of low frequency energy can be effectively suppressed, the sinusoidal can be more faithfully retained. As is shown in Fig. 3, taking the lena128×128 standard diagram as an example, Fig. 3(a) can clearly see that the image spectrum is mainly concentrated in the low-frequency component, and after the error diffusion processing Fig. 3 (b) the high-frequency component is gradually increased.

Fig. 3. Original image(a); Image after error diffusion (b); The normalized spatial spectrum of the original image of “a” (c); the normalized spatial spectrum of “b” (d)
In order to suppress the high frequency conversion in the error diffusion process, the edge attenuation technique is proposed. The core idea is to superimpose the image by itself before the error diffusion of the grating pattern and introduce the parameter $L$ to control the sharpness of the image\[10\]. The high and low frequency distribution is fitted by controlling the sharpening degree. The flow chart is shown in Fig. 4:

![Flow chart](image)

where $o(i,j)$ is the input signal, $e(i,j)$ is the error generated by the previous pixel, $h$ is the error filter, the distribution coefficient is controlled, $Q$ is the quantization function, determining the current pixel is black or white. $p(i,j)$ is the output signal, and the input signal $o(i,j)$ is superimposed with $L$ by itself after being diffused by the error of the previous pixel, and the output $P(x, y)$ is judged by the quantization function $Q$. Where, the error quantity $e(i,j)$ is passed to the next pixel. The process can be expressed by the following mathematical formula:

$$o'(i,j) = o(i,j) + e(i,j) \times h.$$  \hspace{1cm} (5)

$$o''(i,j) = o'(i,j) + o(i,j) \times L.$$ \hspace{1cm} (6)

$$p(i,j) = Q(o''(i,j)) = \begin{cases} 1, & o''(i,j) > 0.5 \\ 0, & o''(i,j) \leq 0.5 \end{cases}.$$  \hspace{1cm} (7)

$$e(i,j) = p(i,j) - o''(i,j).$$ \hspace{1cm} (8)

It should be noted that when $L$ is a positive value, the image information is reintroduced in the threshold to achieve the purpose of enhancing the edge of the image, and the high frequency component is increased in the frequency domain. When $L$ is a negative value, it is actually the edge information is attenuation and appears as a process of increasing the low-frequency component in the frequency domain. However, it is not as small as $L$ is as small as possible, because when the edge is sufficiently attenuation, numerous low frequency parts are concentrated on the fundamental frequency, and an increase in the lower-order harmonic part also causes image distortion. We traversed the parameter $L$ in the range of -1 to 1 with the step of 0.1, and got the result that when $L=-0.6$ was the best parameter. Here, a 256×256 grayscale grating image is taken as an example:
Fig. 5. Grating original image (a) original spectrum (b)spectrum when L is positive (c) spectrum when L is negative (d)

Fig. 5(a) shows the original image of the grating. The center of the spectrum is the DC component, which is represented by the illumination of the background light. The fundamental frequency component is conjugated symmetric, and the entire spectrum is clear and pure. (c) and (d) are positive and negative for L respectively. Compared with the original spectrum, we can clearly see from the graph that the high-frequency component away from the zero-frequency increases when L is positive, and the image details are enhanced on the image, and the edge is steep. The spectrum shows that the fundamental frequency component is attenuation, the sinusoidal contrast is reduced, the harmonic component is increased, the high-frequency component is suppressed when L is negative, and the low-frequency component is close to zero-frequency, which is smoother, more "pure" and closer to the original spectrum, so the edge attenuation technique is useful to maintain the better grayscale grating.

3. Experiment

In MATLAB program, we apply edge attenuation technique to error diffusion, using serpentine scanning type, compared with traditional Floyd-Steinberg method and Liang method, using Lena128×128 pixel image as the image to be tested, the sampling rates are 0.01, 0.05, 0.1, 0.2, with structural similarity (SSIM) and peak signal-to-noise ratio (PSNR) as image quality evaluation criteria. Table 1 gives the objective evaluation data of the imaging quality at each sampling rate, and Fig. 6 shows the comparison of the quality evaluation indicators under the different sampling rates of the three algorithms of the Lena image.
| Algorithm                  | SSIM%  | RMSE   | PSNR/dB |
|---------------------------|--------|--------|---------|
| Floyd-Steinberg method    | 42.72  | 0.1051 | 18.98   |
| Liang method              | 42.89  | 0.1047 | 19.11   |
| Edge attenuation method   | 43.14  | 0.1046 | 19.16   |

(b) Comparison of methods when the sampling rate is 0.05

| Algorithm                  | SSIM%  | RMSE   | PSNR/dB |
|---------------------------|--------|--------|---------|
| Floyd-Steinberg method    | 65.65  | 0.0706 | 22.70   |
| Liang method              | 66.45  | 0.0688 | 22.93   |
| Edge attenuation method   | 66.79  | 0.0681 | 22.94   |

(c) Comparison of methods when the sampling rate is 0.1

| Algorithm                  | SSIM%  | RMSE   | PSNR/dB |
|---------------------------|--------|--------|---------|
| Floyd-Steinberg method    | 74.58  | 0.0582 | 24.56   |
| Liang method              | 75.77  | 0.0551 | 24.95   |
| Edge attenuation method   | 76.36  | 0.0538 | 25.05   |

(d) Comparison of methods when the sampling rate is 0.2

| Algorithm                  | SSIM%  | RMSE   | PSNR/dB |
|---------------------------|--------|--------|---------|
| Floyd-Steinberg method    | 82.62  | 0.0466 | 26.93   |
| Liang method              | 83.60  | 0.0432 | 27.43   |
| Edge attenuation method   | 84.04  | 0.0411 | 27.68   |

It can be seen from Table.1 that compared with the other two methods, the SSIM% and PSNR of the proposed method are improved, and the RMSE is also reduced, in order to further compare the quality in different sampling rates. We plot Fig. 6 as follows:
(a) SSIM of reconstructed images by each method

(b) RMSE of reconstructed images by each method
Fig. 6 shows the change of the evaluation index of the image quality reconstructed by different methods under the conditions of each sampling rate. “—” represents the data obtained from edge attenuation method, “---” represents the data obtained from Liang method, “…” represents the data obtained from Floyd-Steinberg method. It can be seen from Fig. 6 that the Liang method is similar to the traditional Floyd-Steinberg method, and the quality of reconstructed image controlled by the negative parameter is slightly higher than the two, the SSIM% and the PSNR are better than Liang method. The average increase is 0.52, 0.08dB, and the RMSE is decreased by an average of 0.0013.

4. Conclusion
The error diffusion method based on edge attenuation technique can generate a higher quality of the sinusoidal grating. In this paper, we analyzed the frequency domain of the sinusoidal grating, and the negative parameter L was introduced to control the sharpness of a grating pattern to reduce the increments caused by error diffusion. The low-frequency components of the original grating were preserved to have better similarity with the original grating. Then, by traversing method, we found when L = -0.6 we could achieve the best effect, in addition, we could get better parameters by reducing the step size or selecting the appropriate filtering window[11]. Finally, we verified the rationality of the proposed method through simulation in MATLAB, we made a comparison of the traditional Floyd-Steinberg method and Liang method, the objective data of image quality evaluation shows that the proposed algorithm has an advantage than others.

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