Freehand compatibility analysis of building structures

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Abstract. This paper proposes a sufficiently precise graphic method of obtaining the stresses in certain models with low static hyperstaticity widely found in buildings, so as to facilitate the manual analysis at the early phases of the design. The considered models are certain types formed by flat, one-level porticoes that could serve to cover large surface areas. Their stresses are obtained graphically using the consistent deflection method, considering manual field hypotheses and disregarding torsional stresses. The procedure is a modification of an earlier method (Lacort, 2016) which worked out the deflection graphically based on a graph representing the stiffness matrix. In both cases, making sketches freehand greatly reduces the application time but the operations continue to be highly precise, given that the sketches made on a mesh formed by squares with sides measuring one unit in length, which prevents any major errors from being committed. Moreover, the size of the mesh does not seem to have any great influence on the precision of the results. Based on this work, possible lines of the research are suggested for developing further graphic methods that can be used to analyse other types of the structure accurately.

1. Introduction
This paper proposes a manual method for graphically but accurately determining stresses in certain spatial structures widely found in building construction, formed by porticoes with stiff nodes. The procedure supplements [1] in proposing the classic consistent deflection method for analysing models, this has been observed to be perhaps more suitable than other equilibrium methods depending on the stiffness of the bars involved, given that the number of unknowns could clearly be lower and the equations could be distributed over more systems. This suggests that the procedure could be used in the early stages of design. The paper considers manual hypotheses, disregards torsional stiffness, and the exterior actions applied to the nodes can come from the layout in figure 1.

![Figure 1. Decomposition of the stresses in a portico](image)

[Image: Figure 1. Decomposition of the stresses in a portico]
The study begins with a description of the type of structure, the redundant stresses selected and the resulting isostatic model. It then gives a qualitative description of the three types of equation systems derived. The flexibility matrix of each one is represented via a graph as in [2], and the graphs by an area suggested in [1], which is used to graphically resolve the equation system using the Gauss method. As in the earlier procedure, choosing the order of elimination of unknown is suitably so as to simplify sketching. Other areas are also included which serve to obtain the independent coefficients of the systems in the form of segments. Finally, some simple rules are suggested for determining the shear stress diagrams based on the redundant stresses and a model is analysed graphically.

2. Type

The type selected is suitable for covering large surface areas, and is formed with porticoes that are parallel to the YOZ plane as per the reference in figure 2d, connected crosswise via articulated bars. Their beams are equal and all their pillars have the same cross section. The ground is flat in the OX direction but may vary in OZ, so the pillar lengths on the planes of the porticoes may be different.

![Figure 2. Model for analysis and redundant stresses: a), b), c) transfer of momentums in nodes; d) reference system; e) model for analysis & f) isostatic model; redundant momentums (g) in crosswise analysis and (h) on the plane of a portico; i), j) details of supports](image)

Assuming the structure to be subject fundamentally to gravity, the inertia of the beams on OX \( (I_{xb}) \) is the greatest in the model, and is considered to be at least eight times that of the pillar \( (I_{xp}) \). This, together with the disregarding of the torsional stiffness of the bars, means that the momentums of the internal nodes are transferred basically as shown in figures 2a, b, c. As a result, in the model for analysis (figure 2e) the bending momentums of the pillars are not permitted to transfer to the beams.
The redundant stresses in the model are chosen so as to minimise their influence and facilitate the preparation of the compatibility equations. The redundant stresses in question are the bending momentums on OX and OY of the beams at their junction to the pillars, and the momentums on OX and OZ of certain pillars at their junction to the ground. The isostatic model in figure 2f is thus obtained, horizontally stabilised with the pillars on the planes \( A_1-A_1-A_2 \) and \( \pi_1 \). The pillars on the first plane are joined to the ground via nodes (figure 2j) that behave differently depending on their orientation in space; the pillars on the second plane have the same junctions but their orientation is different (figure 2i). Finally, the pillar on the corner \( A_1 \) is embedded in all directions.

The compatibility equations are expressed as a function of some coefficients shown in table 1 (\( a \) & \( b \) as a function of \( 1/\text{EI}_b \) and \( c \) & \( d \) as a function of \( \text{EI}_b \)) and are grouped into independent systems.

There are two systems (1 & 2) for each portico which determine the bending momentums on OX, and a third system that results from analysing the model transversely (Figure 2g).

**Table 1. Model coefficients**

|   | \( a \) | \( b \) | \( c \) | \( d \) |
|---|---|---|---|---|
|   | \( L_b \) & \( m_1 \) | \( 1/\text{EI}_b \) | \( 3/\text{EI}_b \) | \( L_b^3 \) & \( K_A \) |

\( L_b \) = length of beam; \( m_1 = F_b/I_b \); \( m_2 = F_p/I_p \); \( m_3 = F_p/I_p \); \( F_b, F_p \) = inertia of beam; \( F_p, F_p \) = inertia of pillar \( L_A \) = length of pillar \( A \); \( K_A \) = number of pillars identical to pillar \( A \) in the portico

3. **Type 1 systems**

Type 1 systems depend on the redundant stresses of the beams (figure 2h). (1) shows the system associated with a portico with six spans. The values \( \Delta \theta \) depend on the actions \( M'N \) at the nodes \( N \), and \( X'x_N \) (\( = c/2 \)) is the value of the redundant stress \( Xx_N \) to make the rotation at \( N \) compatible when \( \Delta \theta_N \) is one, using the positive criteria in figure 2h for rotations and bending momentums.

\[
\begin{bmatrix}
\frac{1}{X''_g} & \frac{1}{2c} & 0 & 0 & 0 \\
\frac{1}{2c} & \frac{1}{X''_c} & \frac{1}{2c} & 0 & 0 \\
0 & \frac{1}{2c} & \frac{1}{X''_d} & \frac{1}{2c} & 0 \\
0 & 0 & \frac{1}{2c} & \frac{1}{X''_e} & \frac{1}{2c} \\
0 & 0 & 0 & \frac{1}{2c} & \frac{1}{X''_f}
\end{bmatrix}
\begin{bmatrix}
\Delta \theta_g \\
\Delta \theta_c \\
\Delta \theta_d \\
\Delta \theta_e \\
\Delta \theta_f
\end{bmatrix}
= -\frac{1}{\text{EI}_b}
\begin{bmatrix}
X'_g \\
X'_c \\
X'_d \\
X'_e \\
X'_f
\end{bmatrix}
\]

The flexibility matrix of (1) can be expressed as per [2] via a branch-shaped graph (figure 3b) whose vertices and arcs have the values of the matrix coefficients. Figure 3c shows those related to \( X''_d \), which result in the deformation in figure 3a. The graph can be shown as per [1] via an area formed by squares measuring one unit per side in which the values of the vertices and arcs are set out in the form of segments. When those segments are greater than one, the values should be modified as suggested in [3] to keep them within the squares. Figure 3d shows an auxiliary area that represents the branch in figure 3b where the segments provided by \( X''_d \) are located. Its squares are arranged differently from the way suggested in [1] so that there are no empty squares. The segments in this area are modified according to mnemonic rules that reproduce the process of transformation of the flexibility matrix when the system is resolved using the Gauss method, with unknowns eliminated in the order shown in figure 3b. This results in two areas (figures 3h,i) which are used later.
Figure 3. System 1, graphic resolution: a) deflection by redundant stress; b) graph and c) detail; d) area represented by the graph; $\Delta \theta$ produced by $MxD$; e) location, f) graphic calculation as a function of $(1/EIxb)$ and g) systematic calculation of $\Delta \theta$; h, i) obtaining of the redundant momentums

The $\Delta \theta$ coefficients of (1) are produced by the nodal momentums $Mx$, and are obtained by assuming that each $MxN$ applied in $N$ deflects the beam located to the left of $N$, producing two nodal $\Delta \theta$ such as those in figure 3e. Their values can be obtained as a function of $1/EIxb$ in the form of segments using an auxiliary area such as that of node $D$ (figure 3f), formed with squares measuring one unit per side, based on a vertical line drawn from segment $MxD$. By joining the auxiliary areas for all the nodes, another area is obtained (figure 3g) which serves to obtain the $\Delta \theta$ considering any combination of nodal momentums. Finally, these segments $\Delta \theta$ are transformed with the areas in figures 3h,i as per [1] to obtain the values of $Xx$.

4. Type 2 systems

Type 2 systems are more complex, as each redundant stress $Yx$ in each system (figure 2h) influences the other redundant stresses in that system by contributing $\Delta \theta$ to all the articulated supports (figure 4a). The flexibility matrix is represented via a complete graph (figure 4b), which can be simplified when there are identical pillars in the portico (figure 4e). In this case, each group $G$ of $K_N$ identical articulated pillars is replaced by another articulated pillar of equal length with an inertia equivalent to the sum of the inertias of the pillars in the group (figure 4f). Figure 4c shows the new graph derived from figure 4b and the information provided by the new redundant stress $H''D$. The area that represents it (figure 4d) is staggered in shape, and each new redundant stress $H''$ divided by $K$ has the value of the redundant stress of one pillar in the group.

The rotations $\Delta \theta$ caused by the actions at $A$ depend on the lengths of the pillars (figure 4g) and can be calculated systematically from the line $a-b-c$ drawn on the area in figure 4h.
Figure 4. System 2, graphic resolution: a) deflection from a redundant momentum; graph b) general and c) modified; d) auxiliary area; e) complete portico and f) simplified; \(\Delta \theta\) produced by \(F_Z^A \& M_X^A\); g) location and f) graphic calculation as a function of \((1/EI_b)\)

Figure 5. System 3, graphic resolution: a) deformation from a redundant momentum; graph b) and c) detail; d) auxiliary area; \(\Delta \theta\) produced by \(F_Z^D \& M_Z^D\); e) location, f) graphic calculation as a function of \((1/EI_b)\) and g) systematic calculation
5. System 3
The model can be analysed transversely by using a portico with a bending stiffness equivalent to the sum of the stiffnesses of all the porticoes involved. The flexibility matrix is represented by the graph in figure 5b, and figure 5c shows a detail that is identified with figure 5a. The area represented by the graph (figure 5d) has the form necessary to ensure that there are no empty squares.

The rotations $\Delta \theta$ from $F'_D$ and $M'_D$ (figure 5e) only affect the adjacent nodes and can be obtained graphically in a way similar to that used in system 1, using the area in figure 5f, and systematically via that in figure 5g.

6. Stress diagrams
The diagram of the momentums of a pillar from $F_A$ (figure 6a) can be obtained based on the sketch in figure 6b by drawing the broken line $a-b-c-d$ (figure 6c) starting from segment $a-e$, which is $F_A$. Moreover, the shear stress diagram $c_1-d_1-c_2-d_2$ (figure 6f) of a beam (figure 6d) can be obtained by drawing $a_1-b_1-c_1-d_1-a_2-b_2-c_2-d_2$ from $X_1$, and the axial stresses are determined consistently with this diagram.

![Figure 6](image_url)

**Figure 6.** Stress diagrams: momentums of a pillar: a) model, b) sketch and c) obtaining ($f-e = 1$); shear stress in a beam: d) model, e) sketch and f) obtaining

7. Example and discussion
The idea is to determine the redundant momentums for the first portico in the model in figure 7a subject to the general actions in figures 7b,g,k, assuming that the links between the inertias of the bars are $I_b = I_b = 10I_b = 3,3I_b$. The redundant momentums in system 1 are determined with the areas in figures 7e,f, derived from the transformation of the area in figure 7d formed with squares measuring 2.5 cm on a side, that represents the graph in figure 7c. System 2 is transformed into another system with 3 unknowns by turning one portico into another, simpler one (figure 7g) given that the first is formed by four groups of pillars. The information provided by the resulting graph is simplified so that the segments of its auxiliary area (figure 7h) are inside the squares. The graph for system 3 (figure 7m) is modified for the same reason: the vertices are doubled and the arcs are reduced by the same proportion. This means that the arcs in the dashed line are disregarded and the auxiliary area (figure 7n) takes the form of the area in system 1. This area was also drawn up considering squares measuring 1 cm per side, and similar results were obtained (figure 7o).
Figure 7. Example: a) general measurements; system 1: b) actions $M$, c) graph, d) auxiliary area and e, f) calculation areas; system 2: g) action, h) auxiliary area and calculation areas i, j); system 3: k) actions, graph m), auxiliary area when squares measure 2.5 cm per side and n, o) when they measure 1 cm.
The results obtained with the areas measuring 2.5 cm per side are compared with the exact values obtained numerically as per [2] (table 2). The difference in accuracy between the figures for systems 1 and 3 may not be significant considering that those in system 3 are very small. However, those for system 2, in general, seem accurate enough, though in some cases errors occur, probably due to the inaccuracies in the drawing.

|                | $X_B^x$ | $X_C^x$ | $X_D^x$ | $H_B^y$ | $H_D^y$ | $H_F^y$ | $X_B^y$ | $X_C^y$ | $X_F^y$ |
|----------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| **Exact result** | -0.38M  | -0.5M   | -0.58M  | 1.21F   | 0.9F    | 0.62F   | -0.122F | -0.01F  | -0.06F  |
| **Result as per graph** | -0.36M  | -0.48M  | -0.56M  | 0.8F    | 0.8F    | 0.7F    | -0.08F  | -0.02F  | -0.08F  |
| **Error %**     | 6       | 4       | 5       | 33      | 11      | 13      | 34      | 50      | 33      |

8. Conclusions

Based on this research, the possibility emerges of calculating certain hyperstatic structures, commonly found in the building, graphically and almost exactly by freehand.

Possible lines of research are also pointed to in the developing of further graphic methods that can analyse other types of the structure directly and accurately.

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