DEEP CONVOLUTIONAL NEURAL NETWORKS FOR MULTI-TARGET TRACKING:
A TRANSFER LEARNING APPROACH

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ABSTRACT

Multi-target tracking (MTT) is a traditional signal processing task, where the goal is to estimate the states of an unknown number of moving targets from noisy sensor measurements. In this paper, we revisit MTT from a deep learning perspective and propose convolutional neural network (CNN) architectures to tackle it. We represent the target states and sensor measurements as images. Thereby we recast the problem as a image-to-image prediction task for which we train a fully convolutional model. This architecture is motivated by a novel theoretical bound on the transferability error of CNN. The proposed CNN architecture outperforms a GM-PHD filter on the MTT task with 10 targets. The CNN performance transfers without re-training to a larger MTT task with 250 targets with only a 13% increase in average OSPA. The code used for these numerical experiments is available on GitHub: [https://github.com/Damowerko/mtt](https://github.com/Damowerko/mtt)

Index Terms— multi-target tracking, convolutional neural networks, deep learning, transfer learning

1. INTRODUCTION

In multi-target tracking (MTT) the goal is to estimate the positions of multiple moving targets from noisy sensor data [1]. MTT is an important task. For instance, tracking airplanes using radar is vital in air traffic control. However, performing high-quality MTT is challenging [1] for several reasons. First, the sensor observations are unlabeled, meaning that we do not know which target caused a detection to occur. Instead, we must solve an implicit association problem of measurement to targets. Second, MTT suffers from false positive detections, also known as clutter. Third, the number of targets is not known either – new targets can be born at random, and existing targets may die.

There are three main approaches to MTT: joint probabilistic data association filters (JPDAF) [2], multiple hypothesis tracking (MHT) [3], and random finite set (RFS) [4,5] based approaches. JPDAF and MHT are established approaches, while RFS based filters are an area of active research and provide state of the art results. In JPDAF and MHT, the core methodology is to first solve the data association problem between sensor data and targets and then apply traditional filtering approaches [1]. Conversely, RFS approaches provide a holistic approach and solve both the tracking and association problem simultaneously [1].

Two canonical approaches that follow the RFS approach are the Gaussian mixture probability hypothesis density (GM-PHD) [6–8] and the generalized labeled multi-Bernoulli (GLMB) filter [9,10]. A GM-PHD represents the multi-target state as an unnormalized mixture of Gaussians which integrates to the expected number of targets. A GLMB represents the multi-target state as weighted mixture of hypotheses denoted which object could exist, for each of which there exists a corresponding probability distribution representing the uncertainty on the states. In general, GLMB filters provide state of the art results. However, widespread adoption of these filters is constrained by computational complexity despite recent improvements [11,12].

Recently, to overcome the computational complexity problems, a transformer based model was proposed [13,14]. The authors demonstrate that deep learning based approaches can rival the performance of RFS methods on simple problems. Moreover their approach outperforms state of the art methods on more complex tasks [14]. However, applying deep learning to large scale spatial problems is generally intractable due to the scale of the simulations used for training. Recent works have shown that CNNs trained on small scale spatial problems can transfer their performance to larger problems without retraining [15].

In this paper we propose a convolutional neural network (CNN) model for large-scale MTT. In particular, we recast the multi-target multi-sensor tracking problem as an image-to-image prediction problem. We argue that CNNs can be trained on windows of large signals and provide a bound on their generalization performance on the larger signals. Following our theoretical analysis we employ deep transfer learning to approach the MTT task and also overcome the scaling limitation. Our theoretical analysis is also supported by extensive numerical experiments that test this hypothesis on the MTT problem. In particular we show that training can be performed in 1km$^2$ window and the performance degrades by 13% on a 25km$^2$ window.

2. MULTI-TARGET TRACKING

In this section we formally define the multi-target tracking (MTT) problem. First, we introduce notation for the system state and how it evolves over time. Let $x_{n,i} \in X$ be a vector that represents the state of the $i$-th target at time $n$ in some state space $X$. Then, the multi-target state at time $k$ is a set $X_n = \{x_{n,1}, ..., x_{n,|X_n|}\}$, where $|X_n|$ is the cardinality of the multi-target state and may change over time. The multi-target state evolves over time according to a stochastic model with a transition probability density $P(X_{n+1}|X_n)$. Between each time step the multi-target state can change in three ways. First, the states of individual targets evolve according to a transition density $f(x_{n+1,i}|x_{n,i})$. Second, existing targets may die with probability $\lambda_{\text{death}}$. Third, new targets may be born according to a Poisson distribution with mean $\lambda_{\text{birth}}$.
Now we can describe the state, let’s focus on the description of the sensors. Assume there are $M$ sensors, which make observations in some space $Z$. At each time step a sensor $s \in \{1,...,M\}$ can detect each target $\mathbf{x} \in \mathcal{X}$ with probability $p_s^{\text{obs}}(\mathbf{x})$ or fail to detect the target with probability $1 - p_s^{\text{obs}}(\mathbf{x})$. The resulting observation $z^s \in Z$ is distributed according to a density $g_s(z^s|\mathbf{x}_n,i)$. The sensor may make false positive observations, called clutter. Clutter quantity is Poisson distributed with mean $\lambda_C$ and density function $g_C(z^c)$. Conventionally, clutter is uniformly distributed within the sensor range. Let $Z^s_n$ be the set of all true observations and clutter, made by the sensor at time $n$. Thus, $Z^s_n = \cup_{i \in \{1,...,M\}} Z^s_n$ is the set of observations for all sensors. Finally, we denote the sequence of past observations as $Z^1_n = (Z^1_n, ..., Z^n_n)$.

In MTT the underlying state $X_n$ is unknown. Instead we are given the observations $Z^1_n$ and are asked to estimate $X_n$. The relationship between the two is formalized by the posterior density function $p(X_n|Z^1_n)$. There are two common types of estimators: expected a posteriori (EAP) and maximum a posteriori (MAP). In this paper we consider the MAP estimator as defined by equation (1).

$$\hat{X}_n = \arg\max_{X_n} p(X_n|Z^1_n)$$

The MAP estimator minimizes the expected mean squared error between the target states and their estimates.

3. CONVOLUTIONAL NEURAL NETWORKS FOR MULTI-TARGET TRACKING

CNNs have been shown to be effective for a variety of spatial problems. CNNs are shift equivariant [17], which allows them to exploit symmetries present in such problems. Furthermore, we argue that under certain conditions convolutional neural networks scale well to processing large areas. Specifically, theorem [1] and equation (14) provide theoretical justification that we can train on a window of a larger signal.

In multi-target tracking, we represent the observations and the state by an image. The CNN learns a map from the observations image to the state image. If theorem [1] is to be applicable, we have to make certain assumptions about our images. Specifically, we consider multi-target tracking problems where the distribution of targets and sensors is uniformly random. We formally specify how we generate data with this property [5]. In simulations, the CNN is trained on a 1km x 1km window, and we show that tracking performance generalizes to larger windows.

We can represent the MTT multi-object state, $X_n$ by an intensity function. Specifically, we consider the superposition of Gaussian pulses with variance $\sigma^2$, centered at the position of each of the targets. Let $\mathbf{p}_{n,i} \in \mathbb{R}^2$ be the position of the $i$th target at time $n$. Equation (2) is the target intensity function at a point $\mathbf{p} \in \mathbb{R}^2$.

$$U(\mathbf{p}|X_n) = \sum_{i \in N_n} (2\pi\sigma^2)^{-1} \exp\left(-\frac{1}{2\sigma^2}\|\mathbf{p} - \mathbf{p}_{n,i}\|^2\right)$$

The target intensity function (2) encodes target states as a 2D image. A CNN predicting this intensity function $U$ is analogous to learning a PHD filter using a CNN.

Similarly, we can represent the set of sensor measurements $Z^s_n$ as an intensity function. The choice of the intensity function should be independent of the sensor measurement model. For each detection $z^s \in Z^s_n$, we consider the probability density function $p(z^s|x^s)$. This is also known as the posterior density [17] and can be derived using Bayes’ theorem. The general posterior is given by equation (3).

$$p(x|z^s) = \frac{g^s(z^s|x)p(x)}{\int g^s(z^s|q)p(q)dq}$$

The density function $p(x)$ is known as the prior [17] and depends on past observations. However, since we are interested in learning the filter using a CNN, the prior need only depend on the most recent observations. We assume the prior is uniform. [1] Equation (3) reduces to equation (4), since the prior is a constant.

$$p(\mathbf{p}|z^s) = \frac{g^s(z^s|\mathbf{p})}{\int g^s(z^s|u)d\mathbf{u}}$$

In equation (4) we intentionally abuse notation to consider the density on the position $\mathbf{p} \in \mathbb{R}^2$, instead of on a state $\mathbf{x} \in \mathcal{X}$. This is because we are interested in generating a 2D image. There is an implicit integral on all the other state dimensions.

Similarly to equation (2), the observation intensity function (5) is a superposition, for all sensors, of the posteriors of all true detections and clutter.

$$V(\mathbf{p}|Z_n) = \sum_{z^s \in Z_n} p(p|z^s)$$

In practice learning a CNN that maps $U$ to $V$ is intractable, as these functions are continuous. Instead we sample the functions at regular intervals. Let $\rho$, with units meters per pixel, be the spatial resolution at which we will sample. The spatial resolution has to be sufficiently small as to avoid aliasing [18]. The resulting images are matrices $U,V \in \mathbb{R}^{N \times N}$ which are $N = \lfloor w/\rho \rfloor$ pixels wide.

$$U_{ij}(X_n) = I(\rho [i,j] ; X_n)$$

$$V_{ij}(Z_n) = J(\rho [i,j] ; Z_n)$$

Equations (6) and (7) show how to sample the intensity function $U(p;X_n)$ and $V(p;Z_n)$ to obtain the images.

Hence we can describe the state $X_n$ and sensor observations $Z_n$ by images $U(X_n)$ and $V(Z_n)$, respectively. The role of the CNN is to learn a model $\Phi$ which, given a sequence of $K$ past observation images

$$\hat{V}(Z_{n-K+1}, ..., Z_n) = (V^{n-K+1}, ..., V^{n-1}, V^n).$$

estimates the target image, $U^n$. Figure [7] shows the target, sensor and CNN output images side by side.

This model depends on a set of parameters $\mathcal{H}$ which is the set of the CNN filter weights. Finding the parameters $\mathcal{H}$ is naturally formulated as an empirical risk minimization problem (ERM). We assume the existence of a dataset $\mathcal{T}$ of tuples of the form $(\hat{V}, U)$ where $V$ is a sequence of $K$ sensor images and $U$ is the corresponding target image.

$$\mathcal{H}^* = \arg\min_{\mathcal{H}} \frac{1}{|\mathcal{T}|} \sum_{(V,U) \in \mathcal{T}} ||U - \Phi(\hat{V}; \mathcal{H})||_2^2$$

Equation (8) is a common way to approximate the solution to the ERM problem by minimizing the mean squared error over a dataset.

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1The choice of a prior should be principled. For example, in an air traffic control scenario, the prior may be estimated from historical data about an airspace. In section [5] targets are born uniformly, so estimating the prior from data, would have given a uniform prior.
 Targets

 Observations

 CNN Output

Fig. 1. Example images as defined in equations (9), (7). Left from right: the target, observation, and CNN output image. The input to the CNN is the observation image and the output is an estimate of the target image.

4. TRANSFERABILITY PROPERTIES OF CONVOLUTIONAL NEURAL NETWORKS

In this section we focus on the transferability properties of a CNN. In particular, we study the output of a CNN to a periodic input and compare it to the output when the input is limited to a window that is smaller than the period of the signal. This will allow us to describe the behavior of a CNN that is trained on a time limited signal but tested on inputs with larger time support. Hence, let \( u(t) \) and \( v(t) \) be periodic signals on \( T \) that are related via:

\[
v(t) = f(u(t))
\]

Consider a CNN with \( L \) layers, \( \mathcal{H}(u) \). The output of the \( k \)-th layer of the CNN is described by the following recursive equation:

\[
z^k = \sigma(h_k + z^k)
\]

The input to the CNN is \( u(t) = z^0 \) and the output is \( \hat{v}(t) = z^L \).

Now let \( u_w(t) = \cap_w(t) \times u(t) \) be a windowed version of the original input signal \( u(t) \), where \( \cap_w(t) = 1(t \in [-w/2, w/2]) \). Equation (11) defines a CNN that processes windowed signals.

\[
z^w = \sigma(\cap_w(x^k + z^k))
\]

Similarly, the input to this CNN will be \( z^w_0 = u_w(t) \) and the output will be \( \hat{v}^w = \hat{v}_w \). To describe the transferability properties of the CNN we need to derive a bound on,

\[
\|v - \hat{v}\|^2 - \|v_w - \hat{v}_w\|^2
\]

where \( \|v\|^2 \) for a periodic \( v \) denotes the energy of one period. We assume the following to derive such a bound.

**Assumption 1** The window size is smaller than, but close to the period, \( w < T \) with \( T - w < \epsilon \).

**Assumption 2** The pointwise nonlinearity is normalized Lipschitz. That is,

\[
|\sigma(a) - \sigma(b)| \leq |a - b| \text{ for all } a, b \in \mathbb{R}
\]

and additionally \( \sigma(0) = 0 \).

**Assumption 3** The signals \( u, v \) are normalized. Specifically their absolute value is less than or equal to one \( |u(t)|, |v(t)| \leq 1 \).

**Assumption 4** The energy of the filters \( h_k \) is at most one \( \|h_k\|^2 \leq 1 \) and therefore they are BIBO stable.

**Theorem 1** Let \( v \) be the original infinite signal we wish to approximate via a CNN and \( v_w \) be a windowed version of it. Also let \( \hat{v} \) and \( \hat{v}_w \) be the CNN output that approximates \( v \) and \( v_w \) respectively. Then under Assumption 1, 2, 3, 4 it holds that:

\[
\|v - \hat{v}\|^2 - \|v_w - \hat{v}_w\|^2 \\
\leq 2\|v_w - \hat{v}_w\|^2 \\
+ \|v_w - \hat{v}_w\|(L + 2)(\sqrt{T} - w + \sqrt{K}) \\
+ (L + 2)^2(\sqrt{T} - w + \sqrt{K})^2
\]

Our main result is presented in Theorem 1. A proof for this theorem will be provided in a future journal article. We are interested in the case where the window size is close to the period of the signal. Specifically when \( T - w < \epsilon \) for a small \( \epsilon > 0 \).

\[
\|v - \hat{v}\|^2 - \|v_w - \hat{v}_w\|^2 \leq 2\|v_w - \hat{v}_w\|^2 + O(\sqrt{\epsilon} + \sqrt{K})
\]

Equation (14) shows that given an appropriate choice of window and kernel size, a CNN trained on windowed signals \( (u_w, v_w) \) can be used to filter a periodic signal \( (u, v) \). In the future we would like to extend this theory to random signals and provide a tighter bound. Note that the factor of \( \sqrt{K} \) is the error from zero padding the windowed signal at each layer of the network. This term in the bound could be made tighter by making assumptions about the smoothness or sparsity of the signal.

In the context of MTT, transferability of CNNs is an important property. Training a CNN over a large area with a high spatial resolution is intractable. Instead, we argue that a CNN can be trained on a small window with area \( 1 \text{ km}^2 \). If the CNN only has convolutional layers, then during inference the input image can be arbitrarily large. Thus, the CNN can be deployed to make predictions on arbitrarily large areas by leveraging shift equivariance and local computations.

5. NUMERICAL EXPERIMENTS

For training and testing of the CNN we simulate the multi-target tracking environment. The state of a target \( x_n \in X_n \) is a vector \( x_n \in \mathbb{R}^4 \) of positions and velocities. Specifically, the state is given by equation (15).

\[
x_n = [x_n \dot{x}_n y_n \dot{y}_n]
\]

where \( p_n = [x_n y_n] \in \mathbb{R}^2 \) is the position. The state \( x_n \) evolves according to the (nearly) constant velocity (CV) model [19][20] given by equation (16).

\[
x_{n+1} = x_n + T\dot{x}_n + \frac{T^2}{2}\eta_{x,n} \quad \dot{x}_{n+1} = \dot{x}_n + T\eta_{x,n} \\
y_{n+1} = y_n + T\dot{y}_n + \frac{T^2}{2}\eta_{y,n} \quad \dot{y}_{n+1} = \dot{y}_n + T\eta_{y,n}
\]

Above \( \eta_{x,n}, \eta_{y,n} \sim \mathcal{N}(0, \sigma_n^2) \) are Gaussian noise with \( \sigma_n^2 = 1 \) and the time step \( T = 1 \) seconds. Equation (16) is sometimes referred to as the white noise acceleration model.

Consider a square rendering window with width \( w \text{ km} \). From this window we render images \( U(X_n), V(Z_n) \) with \( \rho = \frac{128}{1000} \). Hence, if \( w = 1 \text{ km} \) the images are \( 128 \times 128 \) pixels. Notice that sensors can potentially be located outside this window, but still detect targets within the boundaries in the window. Therefore, we simulate the targets and sensors on a larger concentric square simulation region of width \( W = w + 2R \) where \( R \) is the maximum range of the sensors.
At the beginning of each simulation we initialize Poisson(100W^2) targets and Poisson(0.25W^2) sensors. Their positions are uniformly distributed within the simulation region. The velocity for each target is sampled from $\mathcal{N}(0, 5^2)$. The targets’ state evolves according to the CV model \cite{16}. New targets contain additive Gaussian noise sampled from $\mathcal{N}(0, 0.035^2)$ radians in bearing. The clutter density is $\lambda_C = 40$ events per sensor with the positions of the events uniformly distributed within the sensor detection radius.

Our CNN model has a fully convolutional encoder-decoder architecture similar to \cite{15}. The input to the model is a sequence of $K = 20$ observation images $V(z_{n-K:n}) \in \mathbb{R}^{w_p \times w_p \times K}$ – each image in the sequence is interpreted as a CNN channel. The encoder is a stack of three layers. Each layer is 2D convolution with a stride of 2, kernel size of 9 and 256 channels. Between the encoder and decoder there are two hidden layers. Each hidden layer is a 2D convolution with a kernel size of 1 and 2048 channels. The decoder uses 2D transpose convolutions with the same parameters. Therefore the output of the CNN is an image in $\mathbb{R}^{w_p \times w_p}$ which has the same dimensions as the CNN input, but only one channel. Throughout the CNN a leaky-ReLU activation with parameter 0.01 was used.

For training we ran 10,000 simulations with 100 steps each with a window width of $w = 1$km. We trained the model using AdamW \cite{21} with a batch size of 64 and a learning rate of 0.001 for . We test the trained model on several larger windows of width $w \in \{1, 2, 3, 4, 5\}$ km. For testing we ran 100 simulations when $w = 1$km but only 10 simulations for larger values due to computational constraints.

We benchmark the CNN performance using the and open source implementation of the GM-PHD filter \cite{17,22}. A merge threshold of 10 and a prune threshold of $10^{-3}$ was used in this filter. The range-bearing measurement model is non-linear. Therefore the GM-PHD uses an extended Kalman update \cite{23}. Due to the computational performance of the GM-PHD implementation we only evaluate the GM-PHD filter for $w = 1$.

The output of the GM-PHD filter is a Gaussian mixture. We assume that the sum of the weights of the Gaussian mixture is a good estimate of the number of targets, $|X_n|$. Then we take the $|X_n|$ largest components from the GM-PHD mixture. We similarly estimate positions from the CNN output image, but there is an additional prerequisite step. We first fit a Gaussian mixture to the image \cite{24}. Figure 3 shows the performance this methodology at estimating cardinality of the multi-target state.

As a metric to evaluate the performance of the filters we use OSPA \cite{25} with a cutoff of 500m. OSPA computes the mean squared error between to sets of positions by first finding the optimal assignment from one set to the other. Figure 4 shows the mean OSPA of the CNN tested with different window sizes. For $w = 1$km the CNN filter achieves an average OSPA of 238 ± 74m while the GM-PHD filter achieves 286 ± 71m over the 100 test simulations. Notably, OSPA for the CNN filter improves when tested for $w = 2$, 3km. This is probably because in the $w = 1$ case, the effects of zero padding during convolutions are significant. The CNN performance degrades at most to an OSPA of 268 ± 29m when evaluated on the 25km^2 area with $w = 5$.

6. CONCLUSIONS

In this paper we studied the problem of multi-target tracking (MTT) from a deep learning perspective. In particular, we model the actual target positions and the noisy sensor measurements as images/channels and train a convolutional neural network (CNN) to learn the MTT task. We also derive an analysis on the transferability of CNNs and show that our proposed approach is scalable and avoids the modeling inefficiencies of previous works. Our theoretical and experimental results indicate that CNNs allow MTT to be efficiently performed on larger areas, while trained in smaller ones, and offer new perspectives in a traditional signal processing problem.
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