Error analysis of the orthogonal parallel calibration device for six-axis heavy force sensor

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Abstract. An orthogonal parallel mechanism is employed to design the calibration device for six-axis heavy force sensor. The relationship between the input and output is proposed. The kinematics error of the orthogonal parallel mechanism is analyzed. The influence of the hinges and the electric cylinders on the calibration accuracy is calculated. The fixture positioning error is analyzed. All the results provide the theory evidence for the manufacture and assembling to reduce the calibration error.

1. Introduction
The calibration device for six-axis heavy force sensor influences the accuracy of the sensor. Then research on the kinematics error and assembling error of the calibration device is necessary.

Mechanism of the calibration device belongs to an orthogonal parallel mechanism. The error of the mechanism directly affects the final calibration error. Therefore, it is of great significance to study the error of the orthogonal parallel mechanism. Kumer[1] established the kinematic pose error model of parallel mechanism by matrix method and numerical method. Chiy[2] created the pose error model of 3-TPT parallel mechanism considering the hinge clearance factor by virtue of virtual displacement principle, and analyzed the pose error caused by the hinge clearance. Qstien[3] et al. created the kinematic pose error model of 3-TPR parallel robot based on interval analysis method, and analyzed the maximum pose error and position error of the mechanism. Wang[4] established the position and attitude input and output equation of the differential robot of parallel mechanism through coordinate transformation matrix, and built the error model. The model covers the inherent error of hinge, the error of actuator and the error of hinge positioning. After analyzing the processing, installation and motion errors of parallel robots, Wang[5] used mathematical differential method to study the error modeling of parallel robots. Lu[6] and others focused on the error probability algorithm. The pose error model of the end of the mechanism was established. Liu[7] established the error model of 3-RRR parallel mechanism by differential theory, and deduced that all errors in the structure changed with the increase of X-axis.

Here, we propose a six-axis heavy force sensor calibration device based on the orthogonal parallel mechanism. The relationship between the input and output is presented. The kinematics error of the orthogonal parallel mechanism is analyzed. The influence of the hinges and the electric cylinders on the calibration accuracy is calculated. The fixture positioning error is analyzed.

2. Principle of the orthogonal parallel mechanism
a 2-2-2-SPS(2-2-2-spherical hinge-prismatic joint-spherical hinge) orthogonal parallel mechanism is proposed by Jin\cite{8}. The orthogonal parallel mechanism is used in the calibration device. The force along the six rods are denoted as $f_i (i=1,2\ldots 6)$. The length of the branch in the orthogonal state is set as $l$. The relationship between the force acted on the center of the stage and the force of the six bars can be expressed as

$$
F_x = f_1 + f_2 \\
F_y = f_3 + f_4 \\
F_z = f_5 + f_6 \\
M_x = (f_1 - f_5)\frac{d}{2} \\
M_y = (f_2 - f_4)\frac{d}{2} \\
M_z = (f_3 - f_6)\frac{d}{2}
$$

According to equation(1), $F_i (i=x,y,z)$ and $M_i (i=x,y,z)$ determined by the force along or around the direction of them. Electric cylinder or hydraulic cylinder can be selected as the driving source of each branch. If electric cylinder or hydraulic cylinder with large rated load, the calibration device can be used to calibrate the six-axis force sensor with large measurement range. Three-jaw chuck is used to fix the lower end of sensor. The wedge expansion mechanism is used as the fixture of the upper part of the sensor.

3. The pose error analysis of the calibration device

Each mechanism branch can be regarded as a single open chain. The origin of fixed coordinate system is denoted as $o$, which is on the center of the stage. The origin of moving coordinate system moves with the stage, which is denoted as $o'$. According to the vector closed loop in figure 2, equation(2) can be obtained as,

$$
oB_i + B_i b_i = o o' + o'b_i'
$$

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Assembly drawing of calibration device}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{Vector closed loop}
\end{figure}
$B_b$ can be denoted as the product of the unit vector $e_i$ and actual length of the electric cylinder $l_i$. $o'b_i'$ can be expressed as the product of the vector $ob_i$ and the direction cosine matrix $R$. And $ob_i$ can be denoted $e_i$ for the convenient expression. Then equation (2) can be denoted as

$$oB_i + e_i l_i = ob_i = oo' + Rc_i$$

By differentiation of equation (3) with respect to time, we can obtained,

$$doB_i + de_i l_i + dl_i e_i = doo' + dRc_i + dc_i R$$

Multiplied by $e_i^T$, equation (4) can be expressed as follow,

$$e_i^T doB_i + e_i^T de_i l_i + e_i^T dl_i e_i = e_i^T doo' + e_i^T dRc_i + e_i^T dc_i R$$

$de_i$ can be expressed as differential form as the following[9]

$$de_i = \Delta \delta e_i = \begin{bmatrix} 0 & -\delta e_w & \delta e_v \\ \delta e_w & 0 & -\delta e_u \\ -\delta e_v & \delta e_u & 0 \end{bmatrix} e_i$$

So, $e_i^T de_i l_i$ is derived as,

$$e_i^T de_i l_i = e_i^T \begin{bmatrix} 0 & -\delta e_w & \delta e_v \\ \delta e_w & 0 & -\delta e_u \\ -\delta e_v & \delta e_u & 0 \end{bmatrix} e_i = 0$$

For $e_i^T e_i = 1$, we can obtain $e_i^T dl_i e_i$ ,

$$e_i^T dl_i e_i = dl_i$$

The differential of direction cosine matrix $R$ is denoted as $dR$, then $dR$ can be expressed as,

$$dR = \Delta \delta R = \begin{bmatrix} 0 & -\delta \theta_z & \delta \theta_y \\ \delta \theta_z & 0 & -\delta \theta_x \\ -\delta \theta_y & \delta \theta_x & 0 \end{bmatrix} R$$

Then we can obtain

$$e_i^T dRc_i = e_i^T \Delta \delta Rc_i$$

$\delta \theta = [\delta \theta_z, \delta \theta_y, \delta \theta_x]^T$ is denoted as the error vector of $R$. Set $e_i' = Rc_i$, then equation(10) can be expressed as

$$e_i^T dRc_i = e_i^T \cdot \delta \theta \times (Rc_i) = (c_i \times e_i')^T \cdot \delta \theta$$

Assumed $doB_i = \delta oB_i$, $doo' = \delta oo'$, $dc_i = \delta c_i$, $dl_i = \delta l_i$, then the equation (5) is simplified as,

$$e_i^T \delta oB_i + \delta l_i = e_i^T \delta oo' + (c_i \times e_i')^T \cdot \delta \theta + e_i^T \delta c_i R$$

Thus, the electric cylinder elongation error $\delta l_i$ is derived as,

$$\delta l_i = \left[ e_i^T (c_i \times e_i')^T \right] \cdot \left[ \delta oo' \quad \delta \theta \right]^T + \left[ e_i^T R - e_i^T \delta c_i \right] \cdot \left[ \delta oB_i \right]$$

Set $i$ of $\delta l_i$, $e_i'$, $e_i$, $c_i$ and $\delta oB_i$ from 1 to 6, and then the six electric cylinder elongation errors can be expressed as the following,

$$\begin{bmatrix} \delta l_1 \\ \delta l_2 \\ \delta l_3 \\ \delta l_4 \\ \delta l_5 \\ \delta l_6 \end{bmatrix} = \begin{bmatrix} e_1^T (c_1 \times e_1')^T \\ e_2^T (c_2 \times e_2')^T \\ e_3^T (c_3 \times e_3')^T \\ e_4^T (c_4 \times e_4')^T \\ e_5^T (c_5 \times e_5')^T \\ e_6^T (c_6 \times e_6')^T \end{bmatrix} \cdot \left[ \delta oo' \quad \delta \theta \right]^T + \begin{bmatrix} e_1^T R - e_1^T \delta c_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \left[ \delta oB_1 \right]$$
Set \( \delta L = [\delta l_1, \ldots, \delta l_6]^T \), \( \delta M = [\delta o o, \delta o \theta]^T \), \( K = \begin{bmatrix} e_i^T (e_i \times e_i)^T \\ e_i^T (e_i \times e_i)^T \\ \vdots \\ e_i^T (e_i \times e_i)^T \end{bmatrix} \), and \( N = \begin{bmatrix} \delta c_1 \\ \delta o B_e \\ \vdots \\ \delta c_6 \\ \delta o B_e \end{bmatrix} \).

Then the equation (14) can be simplified as follow,
\[
\delta L = K \delta M + K \delta N \tag{15}
\]

According to equation (15), the pose error of the calibration device can be deduced as follow,
\[
\delta L = K_1 \delta M + K_1 \delta N \tag{16}
\]

where, \( \delta M \) is the pose error of the calibration device, \( K = [K_1, -K_1 K_1] \) is the error transfer matrix. \( \delta L \) is the stretching length error of electric cylinder. \( \delta N \) is the error of all hinge points.

According to the pose error model in equation (16), the influence of the hinges and the electric cylinders on the calibration accuracy can be easily calculated. Equation (16) also provide the theory evidence for the manufacture and assembling to reduce the calibration error.

4. Conclusions
This paper presented a novel decoupled calibration device for six-axis heavy force sensor based on the orthogonal parallel mechanism. The mapping matrix of input force and output force and moment is proposed. And the influence of the hinges and the electric cylinders on the calibration accuracy is deduced. Then the error of the fixture positioning and clamping process is analyzed. All the analysis results will present important reference for design and manufacture of the six-axis heavy force sensor calibration device.

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