Magnetization of Relativistic Current-Carrying Jets with Radial Velocity Shear

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Astrophysical jets at different scales

- Blanford-Znajek mechanism
  - Magnetically dominated rapidly expanding, mildly relativistic jet
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Blanford-Znajek mechanism

Magnetically dominated rapidly expanding, mildly relativistic jet

Collimation and acceleration

Due to magnetic tension and magnetic pressure gradient.

Matter dominated jet already at blazar scale.

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Furter propagation

Poloidal magnetic field components is expected to drop much faster than toroidal component in an expanding jet. For example in a free expanding, conical jet poloidal components ($B_r, B_z$) drop as $z^{-2}$ and toroidal ($B_\phi$) as $z^{-1}$

At large distances from the jet lunching region we expect almost cyrindrical jet with the dominant toroidal component of the magnetic field.
Jet model assumptions

Geometry and profiles:
• Cylindrical axisymmetric geometry.
• Radial velocity shear.
• Non-rotating.
• Magnetic fields has only toroidal component.

Physical constrains:
• Magnetohydrostatic equilibrium within the jet, pressure equilibrium with the external medium.

Equation of state:
• Ultra-relativistic equation of state.
How does it translate to equations?
Any spatial changes in the magnetic pressure across the jet must be counterbalanced by the changes in the particle pressure and the magnetic tension.
MHD force equation

\[ \rho \left[ \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right] = -\nabla P + \mathbf{j} \times \mathbf{B} \]

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Parametrisation

Magnetic field and pressure normalization:

\[ x \equiv \frac{r}{R_j} \]

For a given magnetic field and velocity profile we can calculate pressure profile.

\[ J' \propto \nabla \times B' \]

\[ \partial_r P = -\frac{1}{8\pi r^2} \partial_r \left( \frac{r^2 B_{\phi}^2}{\Gamma^2} \right) \]

\[ b(x) \equiv \frac{B_{\phi}(x)}{B_{\phi}(1)} : \quad b(0) = 0, \quad b(1) \equiv 1 \]

\[ f(x) \equiv \frac{b^2(x)}{\Gamma^2(x)} \]

\[ \mu(x) \equiv \frac{P(x)}{P(1)} : \quad \mu(0) > 0, \quad \mu(1) \equiv 1 \]

Normalized rest-frame magnetic pressure.
Magnetization

Energy flux associated with jet particles.

\[ L_p = 8\pi c \int dr \, r \, \beta \Gamma^2 P \]

Poynting flux.

\[ L_B = \frac{1}{2} c \int dr \, r \, \beta \, B_{\phi}^2 \]

Magnetisation.

\[ \sigma \equiv \frac{L_B}{L_p} \]
Magnetisation effect on particles acceleration mechanism

Magnetic reconnection

Increasing magnetisation increases efficiency of particles acceleration.

Shocks

Increasing magnetisation causes decrease of the shock compression rate, what makes shock acceleration much less efficient.

Energy spectrum of accelerated electrons through magnetic reconnection for different magnetisations. Sironi & Spitkovsky 2014

Fraction of ions (red) or electrons (blue) in the nonthermal tail; (d) fraction of energy in the ion (red) or electron (blue) nonthermal tail, with respect to the total kinetic energy of that specie. Sironi & Spitkovsky 2011
Definition of the model

We define:

• Magnetic field profile
• Velocity profile
• q parameter value

We calculate:

• Pressure profile.
• Magnetization.
• \( \sigma \) and \( \beta_{pl} \) profiles

Calculations:

\[
p(x) = 1 + q - q f(x) + 2q \int_{x}^{1} ds \frac{f(s)}{s}
\]

\[
\sigma = \frac{q}{2} \frac{\int_{0}^{1} dx x \beta(x) \Gamma^2(x) f^2(x)}{\int_{0}^{1} dx x \beta(x) \Gamma^2(x) p(x)}
\]

\[
\beta_{pl}^{-1}(x) \equiv \frac{P_B(x)}{P(x)} = \frac{q f(x)}{p(x)}
\]
Motivations

Observation of Lisanti & Blandford (2007): for the jet model setup as considered here, numerical solutions to the jet magnetization parameter always return \( \sigma < 1 \).

We wanted to:

- Find an analytical proof for this.
- Explore local changes of magnetisation and beta parameter.
Proof of low magnetization for a class of models

• Assumptions on $f(x)$ - comoving magnetic field square:
  • $f(x)$ is continuous.
  • $f(x)$ maximal value is reached on the jet boundary: $f(1) = 1$.

• Result:
  • For every profile magnetisation parameter is:

$$\sigma < \frac{1}{2}$$

Details: Król et al. 2022, ApJ
Other profiles

1. Follows the assumptions of proof.
Other profiles

1. Follows the assumptions of proof.

2. Does not follow the assumptions of proof, with one maximum of $f(x)$.

3. Does not follow the assumptions of proof, with multiple maxima of $f(x)$. 
Magnetization as a function of q parameter
Stratification in magnetisation

Magnetization has a crucial influence on the properties of the shocks and magnetic reconnection as well as on accelerated particles energy spectrum.

Due to stratification in magnetisation, depending on the layer different mechanism may dominate.

In this very simple jet model, we are able to obtain stratification in local magnetization and in the value of the beta parameter.
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Due to velocity stratification Doppler boosting effect will be the strongest for different layers of the jet depending on viewing angle.

Depending on the viewing angle we may observe regions with different magnetizations and therefore different dominating acceleration mechanism and its properties.
Is it possible to get magnetically dominated jet within the framework of this simple model?
Yes.

**BUT** at the expense of:

- Huge pressure gradients are required (\(\rightarrow\) discontinuity)
- No velocity shear
- Maximum magnetizations at most of the order \(\sim\) few

To obtain \(\sigma \sim 5\) we need a pressure drop of the order of \(10^7\)
Assumptions on jet model: cylindrical axisymmetric geometry, radial velocity shear, non-rotating. MHD equilibrium within the jet, ultra-relativistic equation of state.

We analytically proved that jet is always matter-dominated if the maximal value of the normalized rest-frame magnetic pressure is at the jet boundary.

For jet to be Poynting flux dominated we need huge gradient of the gas pressure (or discontinuity) and magnetisation parameter is still < O(10).

For relatively simple profiles we obtain stratification in magnetisation parameter may result in much different properties of the jet depending on layer. Together with stratification in velocity and Doppler effect it may result in heavy dependence on observed characteristics on viewing angle.

Plans for future:
- Checking stability of the stratification in magnetic field with RMHD simulations.
- We study radiation and polarisation of this kind of jet.
Backup slides
Stability and poloidal magnetic field

In the absence of velocity shear, jets with purely toroidal magnetic field are known to be susceptible to current-driven instability. Adding Bz component may stabilize the jet.

How adding Bz component affects our results?

If Bz is uniform across the jet (\( \partial_r B_z = \partial_z B_z = \partial_\phi B_z = 0 \)) the results are unchanged since no additional current component appears (\( J' \propto \nabla \times B' \)) and therefore magnetohydrostatic equilibrium condition does not change (\( \nabla' P \propto J' \times B' \)). The Poynting flux on the other hand is increased only in \( \phi \) direction (so it does not add to Poynting flux in along the jet).

In the presence of radial gradient (\( \partial_r B_z \neq 0 \)) the situation changes. Additional current component appears and therefore pressure profile changes. However, configurations with narrow jet spine tend to be in equipartition (Lyubarsky 2019). Moreover, configurations with poloidal jet spine and dynamically important toroidal magnetic field in the outer part of the jet are unstable (Mizuno et al. 2012).
Magnetohydrodynamics approximation

- Fluid approximation:
  - We can describe plasma as a fluid (with local thermodynamical quantities)
  - Time scale of the change of thermodynamical parameters $>>$ timescale of processes at kinetic scale
- Plasma is electrically neutral: length scale $>>$ Debye length
- Ideal MHD: plasma is perfectly conductive, all dissipative processes are neglected.
Introducing parametrization

Total magnetization

$$\sigma = \frac{q}{2} \frac{\int_0^1 dxx \beta(x) \Gamma^2(x) f^2(x)}{\int_0^1 dxx \beta(x) \Gamma^2(x) p(x)}$$

Local magnetization

$$\sigma(x \pm \Delta) \equiv \frac{\langle f(x) \rangle_\Delta}{\langle p(x) \rangle_\Delta} \equiv \frac{q}{2} \frac{\int_{x-\Delta}^{x+\Delta} dy h(y) f(y)}{\int_{x-\Delta}^{x+\Delta} dy h(y) p(y)}$$
Ideal MHD equations (non-relativistic)

- **Continuity equation**
  \[
  \frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{V}) = 0,
  \]
  Conservation of mass.

- **Euler equation**
  \[
  \rho \left[ \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \nabla) \mathbf{V} \right] = -\nabla P + \mathbf{j} \times \mathbf{B}
  \]
  "Forces" = pressure gradient + magnetic tension and gradient of magnetic pressure.

- **Entropy equation**
  \[
  \frac{d}{dt} \left( \frac{P}{\rho^\gamma} \right) = 0
  \]
  Entropy is conserved along the stream line, \(d/dt\) - material derivative.

- **Induction equation**
  \[
  \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B})
  \]
  The magnetic field is frozen into the fluid and has to move along with it.