A LATE-TIME TRANSITION IN THE EQUATION OF STATE VERSUS $\Lambda$ CDM

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We study a model of the dark energy which exhibits a rapid change in its equation of state $w(z)$, such as occurs in vacuum metamorphosis. We compare the model predictions with CMB, large scale structure and supernova data and show that a late-time transition is marginally preferred over standard $\Lambda$CDM.

1 Introduction

By now there are a number of experiments which establish at a high confidence that the universe is currently undergoing a phase of accelerated expansion. The luminosity distance estimated from type Ia supernovae$^1$ favours recent acceleration while the Cosmic Microwave Background (CMB) data$^2$, $^3$, $^4$ suggest that the universe has almost zero spatial curvature (assuming a FLRW background). This, combined with clustering estimates$^5$ of the cosmic energy budget giving $\Omega_m \sim 0.3$ provides strong evidence for a dominant, unclustered, universal element; a conclusion supported by the height of the first, and the position of the second acoustic peak in the CMB$^6$.

There are several ways to explain such an acceleration, but none of them is compelling. The oldest idea is that of a cosmological constant $\Lambda \neq 0$. Conventional wisdom implies that it is very difficult to generate the required tiny scale $\Lambda \sim (10^{-3}{\text{eV}})^4$ from the Planck scale $M_{pl} \sim 10^{19}$ GeV. The best-studied alternative explanation is quintessence$^7$, $^8$, a very light scalar field $Q$, whose effective potential $V(Q)$ leads to a violation of the strong energy condition and hence to acceleration in the late universe. However, quintessence suffers from extreme fine-tuning since not only must one set the cosmological constant to zero but one must arrange for the quintessence field to dominate at very late times only.

Another possibility is that quantum effects have become important at low redshifts and have stimulated the universe to begin accelerating. Examples are vacuum metamorphosis, put
forward recently by Parker & Raval (PR)\textsuperscript{14} and the work of Sahni and Habib\textsuperscript{12}. In particular, PR consider a massive scalar field in a flat, FLRW background and compute the effective action non-perturbatively to all orders in the Ricci scalar, $R$. They show that the trace of the semi-classical Einstein equations contains quantum corrections, some of which are proportional to

$$\frac{\hbar G m^4 R}{m^2 + (\xi - 1/6) R} [1 + \mathcal{O}(R)]$$

which diverges when $R \to -m^2/(\xi - 1/6)$ signalling significant quantum contributions to the equation of state of the scalar field. Here $\xi \neq 1/6$ is the non-minimal coupling constant. At early times the equation of state is dust on average and then makes a transition from dust to a cosmological constant plus radiation\textsuperscript{9}.

To explain the supernova Type Ia (SN1a) data the scalar field is forced to be extremely light, $m^2/(\xi - 1/6) \sim 10^{-33}$ eV. Vacuum metamorphosis therefore suffers from the same fine-tuning problems as quintessence. The idea of a sudden phase transition is very attractive however and is more general than just the example of vacuum metamorphosis. In fact the idea of late-time phase transitions is rather old, dating back at least as far as 1989\textsuperscript{12,13}.

We therefore choose a phenomenological model which captures the basic features of a transition in the equation of state, but which is not strictly linked to any specific model. We then ask whether current CMB and large scale structure (LSS) data rule out such a transition, or indeed, favour it over the now standard $\Lambda$CDM model.

### 2 The Phenomenological Model

In addition to baryons, neutrinos and cold dark matter our model is characterized by a scalar field $Q$ with a redshift dependent equation of state $p_Q = w(z) \rho_Q$. The functional form of $w(z)$ is specified in a way that largely includes the model of PR. In particular, we choose $w(z)$ to have the following form

$$w(z) = w_0 + \frac{(w_f - w_0)}{1 + \exp\left(\frac{z - z_t}{\Delta}\right)}$$

(2)

In this analysis, we shall restrict ourselves to the case where the initial equation of state is $w_0 = 0$ (i.e. pressure-free matter). We also fix the transition width, which is controlled through the parameter $\Delta$. We choose $z_T/\Delta = 30$, which allows our simulation to resolve the transition while ensuring that $w = w_1$ at $z = 0$. This represents the case of a rapid transition, and our results start to change only for significantly larger widths. The scalar field dynamics is therefore described by three free parameters: the final equation of state (given by $w_f$), the redshift $z_t$ of the transition and the energy density of the scalar field in units of the critical energy density, $\Omega_Q = \rho_Q / \rho_c$.

We also assume that any coupling of the scalar field with other matter components is negligible. In this case the energy density $\rho_Q$ is determined from energy conservation, $\dot{\rho}_Q = -3H \rho_Q (1 + w(z))$, which can be explicitly integrated if $w(z)$ is known. Using Eq. (2) for $w(z)$ and specifying initial conditions for the scalar field, one obtains the scalar field potential $V(Q)$ and its derivatives along the “background” trajectory $Q(t)$.

### 3 The Data Analysis

Due to computational restrictions, we decided to fix all cosmic parameters not directly linked to the scalar field $Q$. We choose a flat universe, $\Omega_{\text{tot}} = 1$, with a baryon content of $\Omega_b = 0.05$. We do not include any tensor perturbations (gravitational waves), set the reionisation optical depth to zero and assume a scale invariant initial spectrum for the scalar perturbations ($n_s = 1$). We
also fix the Hubble constant, $H_0 = 65 \text{ km/s/Mpc}$. We would like to emphasize that fixing $\Omega_b$ and $H_0$ can produce artificially narrow likelihood curves, especially for $\Omega_Q$. Our results should therefore not be interpreted as a measurement of $\Omega_Q$, but as a comparison between the more “standard” $\Lambda$CDM model and the possibility of a late transition. Since the assumed parameters are close to those of the best-fit $\Lambda$CDM model this is not a restriction – at worst we overlook a much better fitting metamorphosis model.

For the CMB data analysis, we consider the COBE DMR, BOOMERanG, MAXIMA and DASI data sets. The large scale structure data is represented by the 2dF redshift survey (analysis of $\Omega_Q$), IRAS PSCz 0.6 Jy, and the Abell/ACO cluster survey. The connection between the large scale structure and the CMB is given through bias limits, $b \in (1/5, 5)$ for 2dF and PSCz, and $b \in (1/9, 9)$ for Abell/ACO. For the supernovae, we use the redshift-binned data from which includes the HZT and SCP data.

4 Results

In order to keep this contribution short, we omit a detailed discussion of the imprints from the transition onto the different observables. This can be found in the full publication. Here we present only the final results for the combined data in figure 1. This shows our main results through the marginalised 1-d and 2-d likelihoods for $(z_t, w_f, \Omega_Q)$.

![Figure 1: The marginalised 1-d likelihood plots for our variables $(w_f, z_t, \Omega_Q)$ for the combination of CMB, LSS and SN1a data.](image)

The $\chi^2$ values of the overall best fit model (with $w_f = -1.0$, $z_t = 1.5$ and $\Omega_Q = 0.73$) are 33 (CMB) + 36 (LSS) + 4 (SN1a), in total 73. On the same parameter grid, the best fit $\Lambda$CDM model has $\Omega_Q = 0.73$ as well. Its $\chi^2$ values are 40 (CMB), 34 (LSS) and 4 (SN1a), in total 78. In total we have approximately (neglecting correlations within the experiments as well as between them) 93 degrees of freedom for our model, and 95 dof for the $\Lambda$CDM models.

We can see that both groups of models are perfectly consistent with current data. Given the error bars of the data sets, the family of $\Lambda$CDM models is included in our phenomenological models for $w = -1$ and large $z_t$. The figure shows that current data slightly prefers a low-$z$ phase transition, which is still true when taking into account that we have to add two degrees of freedom for pure $\Lambda$CDM models. On the other hand, the difference is too small to speak of a detection; assuming Gaussian errors and 3 “parameters of interest” ($\Omega_Q, w_f, z_t$), models with a $\Delta\chi^2$ of 5 above the best-fit would formally be excluded at about 83%, hence less than 2 $\sigma$.

5 Conclusions

We have studied a phenomenological model in which the dark energy of the universe is described by a scalar field $Q$ whose equation of state $w$ undergoes a sudden transition (metamorphosis) from $w_0 = 0$ (dust) to $w_f < -0.3$ at a specific redshift $z_t$. While similar to the quintessence paradigm in practical respects, the underlying philosophy is very different since we are interested in the
possibility of detecting radical new physics in the dark energy, such as the vacuum metamorphosis model (PR). We use the current CMB, large-scale structure (LSS) and supernovae (SN1a) data to constrain our phenomenological parameter space variables ($\Omega_Q$, $z_t$, $w_f$).

The CMB and SN1a data are sensitive to a transition if it occurs at low redshifts ($z_t < 3$) due to the delay in the epoch at which cosmic acceleration can begin, relative to the standard $\Lambda$CDM models. We found that the global best-fit to the current data occurs for $z_t = 1.5$, $w_f = -1.0$ and $\Omega_Q = 0.73$. This model is consistent with the data and is a marginally better fit than the best $\Lambda$CDM model.

Finally, an intriguing possibility is that the rapid transition studied here may provide a solution to the current impasse for quintessence models in explaining the varying-$\alpha$ data: quintessence models can explain the apparent variation of $\alpha$ around $z \sim 1 - 3$ but cannot simultaneously match the null-results of the Okun natural reactor at $z \sim 0$. [15]

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