Suppressing the ferroelectric switching barrier in hybrid improper ferroelectrics

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INTRODUCTION
Since the discovery of ferroelectricity in BaTiO3, perovskite oxides have been heavily studied and utilized in applications as ferroelectric materials. Versatility of the perovskite structure allows the large number of complex oxides to be synthesized, but among those, only a small fraction are ferroelectrics. A major breakthrough in perovskite-related ferroelectrics is the discovery of hybrid improper ferroelectricity (HIF) as a materials design route in 2011, which led to an explosion in the predictions of novel ferroelectric oxides. Among those, the list of examples that are experimentally verified includes A2B2O7 HIFs (Ca,Sr)3Ti2O7 (Sr, Ca)3Sn2O7, Sr3Zr2O7, as well as a weak ferromagnetic (Ca0.69Sr0.46Tb1.85Fe2O7)7.

Despite the prediction of ferroelectricity and observation of a polar crystal structure in many compounds, experimentally observing the switching of polarization is challenging. For example, the original HIF Ca3Ti2O7 was reported to have a polar structure 20 years before the idea of HIFs was introduced9, but the direct evidence of polarization switching was not observed until 20153. The reason behind the absence of switching in these materials was initially believed to be large intrinsic coercive fields, or defects in the materials, which typically increase the coercive field10. The high experimental coercive field is not surprising, because the energy scale that needs to be overcome for switching is considered to be determined by the octahedral rotations, which often have an energy scale significantly higher than that of the ferroelectric distortions in typical perovskite oxides. Switching was observed in other HIF materials with coercive fields ranging from 120 to 200 kV cm−1, and very recently, the smallest coercive field of 39 kV cm−1 was observed in single crystals of Sr3Sn2O711. Though these coercive fields are comparable to values suitable for integration to silicon chips (Ec ≈ 50 kV cm−1), applications such as high-power actuators and low-voltage logic and memory elements ask for ferroelectrics with robust polarizations that can be switched by a lower coercive field12,13. Ultra-low coercive fields as low as 5 kV cm−1 were observed in pulsed laser deposition grown Ca3Ti2O7 thin films, but the reason behind this reduction (and whether it is an intrinsic or an extrinsic effect) is not clarified yet14.

Understanding the intrinsic mechanisms that affect the coercive field of HIF materials, and finding new design strategies to reduce these fields are important for their applications. In this paper, we illustrate that strain can be an effective means to achieve this. Epitaxial strain, obtained by growing thin films on lattice-mismatched substrates, has been used extensively as a way to tune the ferroelectric and dielectric properties of perovskites15,16. Both the octahedral rotations, and the proper ferroelectric order parameter are strongly coupled with the biaxial strain in most materials, and strain is shown to change the switching energy barrier of ferroelectrics as well17. HIFs are shown to undergo interesting structural phase transitions under strain as well18, but there is no detailed study of the switching behavior of HIFs under biaxial strain. The original study on HIFs showed that the lowest energy switching path and energy (which is correlated with the coercive field) is strain dependent, but the recent works that illustrate the richness of possible switching paths make it necessary to re-evaluate the polarization switching behavior of strained HIFs19,20.

In this study, we perform density functional theory (DFT) calculations on 13 different A2B2O7 Ruddlesden–Popper compounds to map out the strain-tolerance factor phase diagram, and show that the strain-induced non-polar or anti-polar phases emerge in compounds with a finite range of tolerance factors. We then show, by performing nudged elastic band (NEB) calculations, that the intrinsic coherent polarization switching energy barrier decreases as the compounds get closer to phase boundaries by biaxial strain. This suppression of switching barrier is not always accompanied with a decrease in the polarization, which makes strain tuning of HIF Ruddlesden–Poppers a viable tool to obtain low coercive field ferroelectrics with a robust polarization. We also show that the tensile and compressive strains favor different switching pathways, which can be intuitively understood in terms of which octahedral rotations or tilts are favored by strain.

This paper is organized as follows: We start by explaining the crystal structures and important normal modes. We then present and discuss the strain-tolerance factor phase diagram of HIF RPs. Then, we present the trends of the intrinsic switching barrier as a function of strain. We conclude with a brief summary and conclusions.

RESULTS
Review of crystal structures
The A2B2O7 compounds considered in this study are the n = 2 members of the Ruddlesden–Popper series21,22. They can be

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While there is no direct group $Ruddlesden$ Brillouin zone in the ABO$_3$ perovskites can fold back onto the same various structural instabilities that are at different points of the effects, the different periodicity of the Ruddlesden smaller Brillouin zone than ABO$_3$ perovskites. The equivalents of along the layering direction (Fig. 1a). The extra AO layers cause a shift by ($a/2, a/2, 0$) on the $ab$ plane, and hence the structure becomes body-centered tetragonal with space group $I4/mmm$ ($#139$). This shift also breaks the connectivity of the oxygen octahedra, and the AO double layer is held together by mostly ionic bonds between the A-site cations and O anions. The resulting dimensional reduction has important consequences on the electronic structure and lattice response (for example, refs $25-28$). Apart from the dimensional effects, the different periodicity of the Ruddlesden–Popper phases along the layering direction (c-axis, or the [001] direction) leads to a smaller Brillouin zone than ABO$_3$ perovskites. The equivalents of various structural instabilities that are at different points of the Brillouin zone in the ABO$_3$ perovskites can fold back onto the same point in A$_2$B$_2$O$_7$. Ruddlesden–Poppers, which leads to interesting couplings between them as discussed below. (This point can be qualitatively understood in analogy to a subduction problem, where a zone boundary mode of the parent group corresponds to a zone center mode of the subgroup. For example, when the unit cell of a cubic perovskite is doubled along the [001] axis as a result of cation order, the space group becomes $P4/mmm$ and the zone boundary $X'_5$ mode splits into $\Gamma_5 \oplus X'_5 \oplus X'_5$, where $\Gamma_5$ is polar. While there is no direct group–subgroup relationship between the Ruddlesden–Popper and perovskite structures, the $n = 2$ Ruddlesden–Poppers have two perovskite blocks in their unit cells, and it is thus possible to recognize some phonon modes folded onto the $k_z = 0$ plane).

By far the most common structural distortions that decrease the symmetry of oxide perovskites are the oxygen octahedral rotations: About 90% of all oxide perovskites have this type of distortions in their crystal structures, which reduce the symmetry of the parent Pm$3m$ phase$^{27}$. These distortions can be described in terms of symmetry-adapted-modes, which can be classified by irreducible representations (irreps) of the parent space group Pm$3m$. The phonon modes that correspond to these distortions are the $M$ point mode $M'_5$, which is an in-phase rotation of octahedra around one axis, and the $R$ point mode $R'_5$, which is an out-of-phase rotation of octahedra around one axis. The former is denoted by a ‘$+$’ superscript in the Glazer notation, such as $d^0d^0c^+$, and the latter is denoted by a ‘$-$’ superscript, such as $a^+a^-a^-$. The most common rotation pattern that more than half of all oxide perovskites have is $a^+a^-a^+$, which leads to the space group Pn$ma$ (#62)$^{31}$. Another distortion that is often significant in the Pn$ma$ structure is the $X'_5$ out-of-phase A-site displacement. Unlike the $M'_5$ and $R'_5$, the $X'_5$ often does not show up as an unstable phonon mode in the high symmetry (Pm$3m$) phase. Rather, it is an improper order parameter, which attains a nonzero magnitude only because of a trilinear coupling in the Landau free energy

$$F_{\text{trilinear}} = \gamma M'_5 R'_5 X'_5.$$  

(1)

The presence of $F_{\text{trilinear}}$ in the free energy expansion, which is imposed by group theory, guarantees a nonzero $X'_5$ distortion whenever the octahedral rotations $M'_5$ and $R'_5$ are present, no matter the sign of the coupling $\gamma$.

Instabilities in the A$_2$B$_2$O$_7$ Ruddlesden–Poppers that are similar to the $M'_5$ and $R'_5$ normal modes in the ABO$_3$ perovskites give rise to a wider range of different combinations and resultant symmetries. (For simplicity, we follow the convention to refer
octahedral rotations around the out-of-plane (c) axis as ‘rotations’ (OOR), and the rotations around the in-plane axes as ‘tilts’ (OOT)."

One reason for this is that there is a new degree of freedom, since the body-centered primitive cell now contains two oxygen octahedra. Also, the double AO layers break the connectivity of oxygen octahedra, and hence the relative phase of neighboring octahedra on either side of the double layer is not fixed. As an example, we consider the modes relevant to the $A_2\alpha m$ phase observed in $Ca_2Ti_2O_7$ and many other HIF Ruddlesden–Popper compounds in Fig. 2. In ABO$_3$ perovskites, there are two possible rotation patterns around, for example, the c axis: in-phase ($M_{22}$, $\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\alpha^0\α
modes couple with different $M$ modes, such as $M_{1}^{p}$ or $M_{2}^{p}$, and give rise to nonpolar phases, where the dipole moments of each atomic layer cancel each other within each perovskite slab between to double AO layers (Fig. 3c and d). (We refer to phases where dipole moments of each slab are zero as ‘nonpolar’) Many of these phases are observed to emerge in various $A_{2}B_{2}O_{7}$ oxides under biaxial strain or equivalent doping, and are also shown to be important as intermediate states in the coherent switching of polarization.\textsuperscript{5,16,20–22,33} This is in addition to single-tile systems observed, for example, at finite temperature.\textsuperscript{7} In the next subsection, we draw the strain-tolerance factor phase diagram of these compounds to identify regions where these antipolar and nonpolar multi-tilt phases emerge.

Strain phase diagram

Most—more than half—of oxide perovskites have a tolerance factor of $\tau < 1$, and attain the space group $Pnma$ at low temperatures.\textsuperscript{29}

The corresponding octahedral rotation pattern $a^{+} a^{-} c^{+}$ is also common in $A_{2}B_{2}O_{7}$ Ruddlesden–Popplers, and gives rise to the polar space group $A2_{1}am$ observed in HIFs. In addition to the polar phase, strain phase diagrams of these compounds often abound with transitions to nonpolar phases introduced in the preceding subsection. As an example, in Fig. 4a, we present the energy of three lowest energy structures for $Sr_{3}Sn_{2}O_{7}$ as a function of biaxial strain.\textsuperscript{24} The zero temperature DFT calculations reproduce the experimentally observed room temperature phase $A2_{1}am$ in the unstrained compound. Both tensile and compressive strain decrease the energy difference between this phase and the next lowest energy state, and there are phase transitions to nonpolar phases for strain $> 2.5\%$ on either direction. Similar strain-driven transitions have been predicted for $Sr_{3}Zr_{2}O_{7}$ and $Ca_{3}Ti_{2}O_{7}$ HIF compounds previously, and the pattern of octahedral rotations often change under strain in the ABO$_{3}$ compounds as well. A common trend in $A^{2+}B^{4+}$O$_{3}$ perovskites is that tensile biaxial strain suppresses OOR around the out-of-plane axis, whereas compressive strain enhances it. $Sr_{3}Sn_{2}O_{7}$ follows a similar trend: the transition under tensile strain is to the $P4_{2}/mmnm$ phase, which has only $X_{3}^{-}$ tilts, whereas the transition under compressive strain is to the $Aeaa$ phase, which has only the $X_{1}^{+}$ rotations around the $c$ axis. The transition to these nonpolar phases is not a result of a continuous suppression of polarization by strain: the magnitudes of polarization in the $A2_{1}am$ phase on both phase boundaries are sizable, and is even enhanced under tensile strain, as shown in Fig. 4b.

In order to elucidate the behavior of different HIF compounds under strain, in Fig. 5 we map out the strain-tolerance factor phase diagram by considering 11 different $A_{2}B_{2}O_{7}$ compounds. (We do not include two compounds with larger tolerance factors, since they do not display any OOR or OOT. Most of these compounds have been studied from first principles before, but to the best of our knowledge, this is the first time that this information is compiled to display all compounds together. We consider a strain range of $\pm 4\%$, which covers the experimentally feasible range. For most of the compounds with $\tau < 1$ that we consider, the lowest energy unstrained structure is $A2_{1}am$, which corresponds to the HIF phase. For $0.92 \leq \tau \leq 1$, nonpolar structures emerge under both tensile and compressive strain. We observe three different nonpolar structures: $Pnma$ and $P4_{2}/mmnm$ under tensile strain, and $Pnma$ and $Aeaa$ under compressive strain. They correspond to the following changes in the octahedral rotations and tilts:

The trilinear couplings are obtained using the ‘Invariants’ tool in the Isotropy Software Suite.\textsuperscript{65} While we performed DFT calculations for the energies of all of these phases, only the ones that are close to the lowest energy are shown in the plots.

![Fig. 3](image)

**Fig. 3** Possible low-energy stable and metastable structures of RP-phase perovskites $A_{3}B_{2}O_{7}$ with more than one oxygen octahedral rotation modes. Analysis of these modes are presented in Table 1.
Fig. 4 Effect of strain on Sr$_3$Sn$_2$O$_7$. a The energy of different metastable phases vary with biaxial strain. Transitions to nonpolar phases are observed on both tensile and compressive strain. b The polarization strength of the polar phase as a function of strain. The background colors indicate different ground state structures.

Fig. 5 Phase diagram of HIF A$_3$B$_2$O$_7$ compounds under biaxial strain. Red color represents ferroelectric (HIF) phase, the others are all non-polar structures. Results for Ba$_3$Ti$_2$O$_7$ ($t = 1.06$) and Ba$_3$Ge$_2$O$_7$ ($t = 1.10$), which do not display any rotation or tilting, are not shown here. Proper ferroelectric phases of large tolerance factor compounds, such as the one in Sr$_3$Ti$_2$O$_7$ under large tensile strain $^{20,46}$, are not displayed either.

- Compressive strain-induced OOT suppression (leads to Aeeaa): This is observed in Sr$_3$Sn$_2$O$_7$ and Ca$_3$Ge$_2$O$_7$. The OOT mode amplitude drops to zero and OOR mode phase changes under compressive strain as shown in Fig. 6c, d.
- Tensile strain-induced OOR suppression (leads to P4$_4$/mmn): This is observed in Sr$_3$Zr$_2$O$_7$ and Sr$_3$Sn$_2$O$_7$. Similar to the first situation, but the OOR mode drops to zero under tensile strain instead of OOT mode, as shown in Fig. 6b, c.
- Tensile/compressive strain-induced OOR phase change (leads to Pnab): This is observed in Ca$_3$Ti$_2$O$_7$ under both tensile and compressive strain, in Sr$_3$Zr$_2$O$_7$ under compressive strain, or in Cd$_3$Ti$_2$O$_7$ under small tensile as well as compressive strains (Fig. 6a, b). Amplitudes of both the OOR and OOT mode retain non-zero, but the in-phase OOR mode changes into out-of-phase manner. This structure is shown in Fig. 3b. The A-site cations around two interfaces move in the opposite direction, which cancels the polarization in bulk.

Some of these transitions are explained by local measures such as the global instability index (GII), which is known to predict the octahedral rotation patterns and angles in ABO$_3$ perovskites successfully$^{29,35}$. For example, the transition to P4$_4$/mmn in Sr$_3$Sn$_2$O$_7$ is coincident with the strain value above which the GII of this phase is the smallest as shown in Supplementary Fig. 6. However, the GII by itself does not explain why the polar A$_2$am structure is preferred over the Aeeaa one, for these two phases have very similar GII values under compressive strain. It is possible that the interplay of GII with the long-range Coulomb interaction (which is an important factor in stabilizing the polarization in proper ferroelectrics such as BaTiO$_3$ $^{36}$) is responsible of the transition to the Aeeaa phase.

The transition to a single-tile system can be explained phenomenologically by the cross-term between OOR and OOT—a large OOR might suppress OOT and vice versa. All compounds in the A$_2$am follow the same aforementioned trend as many ABO$_3$ perovskites that compressive strain enhances OOR, whereas tensile strain enhances OOT (Fig. 6a–d) (for example, see refs. 20,37,38). This trend is likely the result of the strain reducing particular B–O bond lengths, which can be increased by the OOT or OOR distortions. The lowest order cross-term between the OOR and OOT in the free energy is $F = \beta R T^2$ (where we denote the amplitudes of rotations and tilts by $R$ and $T$, respectively). For fixed value of $R$, this term renormalizes the coefficient of the $T^2$ term $-\alpha T^2$ as $-(\alpha + \beta R^2)$, and hence for large OOR $R^2 > -\alpha \beta$, the tilting instability is suppressed, and it becomes energetically favorable to have no tilts, as is the case in compressively strained Sr$_3$Sn$_2$O$_7$ in the Aeeaa phase.

A phenomenological explanation of the strain-induced transition to nonpolar Pnab structure requires not only the biquadratic terms between the OOR and the OOT modes, but also various trilinear terms that couple these modes to other antiferrodistortive displacements$^{21}$. It is particularly interesting that in Ca$_3$Ti$_2$O$_7$, this transition is re-entrant in the sense that it happens under both tensile and compressive strains. The GII does not have an obvious trend that explains this transition, and the electrostatic interaction between the O ions on different layers is possibly important. We leave the microscopic explanation of this transition to a future study.

Strain tuning of the ferroelectric switching barrier
Enhanced susceptibilities near second-order phase transitions can be exploited to design materials with large responses, for example, magnetic permeability or dielectric constants. While no such enhancement of linear susceptibility is mandated near first-order transitions, it is nevertheless possible to obtain large response near a first-order phase boundary if the external field can induce the transition. Examples of demonstrations of this approach include Terfenol, Pb(Zr,Ti)O$_3$, and BiFeO$_3$ $^{39–41}$. The phase boundaries of structural transitions depend on strain very sensitively, and as a result, this approach is a promising means to enhance the response of materials via strain.

The question we focus on in this subsection is whether the ferroelectric polarization switching barrier is affected when strain is used to tune the materials to the vicinity of the polar–nonpolar phase transitions. In order to answer this question, we use the minimum energy barrier for coherent polarization switching as a proxy to the coercive electric field. While in an actual experiment defects, domain structure, as well as size and shape effects significantly alter the coercive field, trends of coherent switching barrier can be used as a first principles proxy to the trends of the coercive field$^{42}$ as explicitly shown in HfO$_2$ $^{19}$. (Finite element methods which take into account the domain structure provide much lower switching barriers$^{43}$). In practice, the coherent switching field calculated from the first principles energy barrier by assuming that the dipole moment in every unit cell of an infinite crystal switches at the same time is a gross overestimate. As a result, we do not report the electric field required for switching, but instead report only the energy barriers.
Since in the hybrid improper ferroelectric $\text{A}_3\text{B}_2\text{O}_7$ compounds, the polarization emerges as an improper order parameter through a trilinear coupling with rotation and tilting modes, switching one of these two modes is necessary to switch the polarization. It was recognized as early on as in the first HIF paper that this makes different switching pathways possible, and that the corresponding energy barriers can be tuned by strain. Later, the work of Nowadnick and Fennie analyzed the possible roles of different switching pathways, and Munro et al. used the idea of distortion symmetry groups to identify other switching pathways. Since then, the energetics of switching in various HIF compounds have been studied, for example in ref. 45. However, to the best of our knowledge, a comparison of different compounds and their strain dependence have not been performed yet.

In Fig. 7a–d, we show four possible polarization switching pathways. We follow the convention of the distortion symmetry groups to name these pathways. This process involves identifying not only the symmetry operations shared by all images on the pathway, but also those operations that reverse the distortion, which is the polarization in this case. The latter are referred to as distortion reversal symmetries, and are denoted by a * superscript. For example, $\text{Pnab}^*$ means that each image along the switching path has two glide planes with translations along $a$ and $b$ axes; and the glide plane $n$ reverses the distortion. Three of the switching paths we consider ($\text{Pnab}, \text{Pbam},$ and $\text{Pnam}$) have a similar name as their intermediate phase (up to the asterisks), because the spatial symmetry elements of the intermediate phase either remain unchanged or become reversal symmetry operation for other images. But this is not the case for $\text{Pn}^*\text{2}\text{m}$. All of the four are so-called two-step switching pathways, where there exists a local minimum of energy on the switching path, as seen from Fig. 8a, and they are the lowest ones among such paths for the three compounds we considered. They each have distinct intermediate states, but the same initial and final states. Since the Ruddlesden–Popper structure consists of weakly bound perovskite blocks separated by an interface between two rock-salt AO layers, it is possible to consider supercells extended along the [001] direction, and polarization being switched in one perovskite block at a time. This, in principle, gives rise to an infinite number of different switching pathways, the barrier energy per formula unit can be arbitrarily small (since only one block out of arbitrarily many switches at each step). This has been observed in refs. 22,44, where typically the four-step switching paths have lower (but comparable) barriers than the two-step ones, which in turn have lower barriers than the single-step paths. (The path with a very large number of steps can be considered to be a simple model of domain wall motion along the [001] direction.) However, this does not necessarily imply that the pathway with the highest number of steps determines the coercive field, because what is more important for the switching under an electric field is the slope of the energy vs. polarization curve. For simplicity, as well as computational manageability, we focus only on two-step switching pathways.

Each of the four pathways can be reproduced within the same doubled conventional cell as the polar structure. The $\text{Pn}^*\text{ab}$ and $\text{Pn}^*\text{am}$ pathways (Fig. 7a, b) involve changes in the direction of the OOR mode, and both of them have nonpolar intermediate structures, with space groups $\text{Pnab}$ and $\text{Pnam}$, respectively. The out-of-phase displacements of the A-site cations are along the polar axis in both of these intermediate structures. The $\text{Pbam}$ pathway involves switching the direction of the OOT mode ($X_3^*$), whereas in the $\text{Pn}^*\text{2}\text{m}$ both OOR and OOT change directions, as shown in Fig. 7c, d. Mode decompositions of these switching pathways are given in the Supplementary information.

In Fig. 8a, we plot the energy as a function of the reaction coordinate for these four switching pathways in unstrained $\text{Sr}_3\text{Sn}_2\text{O}_7$. The energy barriers are comparable and the lowest one is for the $\text{Pn}^*\text{am}$ pathway. Results presented in Fig. 8b show how the energetics of this path behaves under tensile strain: Tensile strain monotonically decreases the $\text{Pn}^*\text{am}$ switching barrier, thus lowering the expected coercive field required for switching. This is not a surprising result, since the OOR’s weaken under tensile strain, as shown in Fig. 6c and the $\text{Pn}^*\text{am}$ pathway involves a change in the OOR character. What is interesting, and important for applications, is that this reduction in the switching barrier is not accompanied with a lower polarization under tensile.
Thus, strain can be used as a means to lower the coercive field of hybrid improper ferroelectrics. The strong strain dependence of the switching barrier is not specific only to Sr$_3$Sn$_2$O$_7$, or the Pn*am pathway. In Fig. 9a, b, we show the barrier for different switching paths of Sr$_3$Sn$_2$O$_7$ and Ca$_3$Ti$_2$O$_7$ as a function of strain throughout the strain range that the HIF phase is stable. While the error bars in the energy barriers from the NEB calculations cause the curves to be rather rugged, two trends are evident: (i) under tensile strain, the barriers for pathways that involve changing the direction of the OOR mode (Pn*am and Pn*ab) are lowered, and (ii) under compressive strain, the barrier for the pathway that only involve changing the direction of the OOT mode (Pb*nm) is lowered. These are consistent with the tendencies towards OOR and OOT distortions becoming weaker under tensile and compressive strain as discussed earlier. Near 0% strain, the lowest barrier pathway switches from Pb*nm or Pn*21m to either Pn*ab or Pn*am, and either strain direction leads to a lower coherent switching energy barrier. The lowest barriers are obtained near the phase boundaries between the polar and nonpolar phases, and the maximum suppression is about 50% in both compounds. Ca$_3$Sn$_2$O$_7$ has a lower tolerance factor than Sr$_3$Sn$_2$O$_7$ and Ca$_3$Ti$_2$O$_7$, and it does not display a strain-induced phase transition in the strain range we considered. It does not show a strain-induced change in the switching pathway, or a significant decrease in the switching barrier either (Fig. 9c). This is likely because this compound is very far from the phase boundaries, and

**Fig. 7** Four possible polarization switching pathways. **a** Pn*ab, **b** Pn*am, **c** Pb*nm, and **d** Pn*21m. The octahedra that remain in their original rotation direction are shown in blue, whereas those that switch their rotation direction are shown in yellow.

**Fig. 8** Energy barriers for polarization switching in Sr$_3$Sn$_2$O$_7$. **a** The energy barriers of different pathways for unstrained Sr$_3$Sn$_2$O$_7$. The horizontal axis is the "reaction coordinate" that parametrizes the switching path. Arrows indicate the barrier heights. **b** The energy of the Pn am pathway in Sr$_3$Sn$_2$O$_7$, as a function of tensile biaxial strain.
with its small tolerance factor, it has such large OOR and OOT that the strain-induced changes in the instabilities are inconsequential.

**DISCUSSION**

Since its discovery about a decade ago, HIF have provided fertile ground for first principles materials by design approaches. Experiments have also been catching up rapidly, verifying theoretical predictions. Multiple hybrid improper ferroelectric Ruddlesden–Popper phases have already been synthesized using bulk methods (for example, see refs [4][5][11]). Although thin film growth of Ruddlesden–Popper phases, especially for thermodynamically unstable compositions and at large strain values, is usually challenging because of the required stoichiometry control, there has been successful demonstration of switchable HIF in PLD grown films[15], and both hybrid and conventional oxide molecular beam epitaxy have been used to synthesize phases that are not thermodynamically stable[46,47]. Current efforts focus on understanding more than the emergence of ferroelectricity, and to find ways to optimize properties such as the coercive field required for polarization switching.

In this study, we used first principles calculations to shed light on the strain-tolerance factor phase diagram of \( n = 2 \) Ruddlesden-Popper HIF’s, and to come up with a design strategy for obtaining lower coherent switching energy barriers. This quantity, which we used as a proxy for the coercive field, decreases significantly when strain is used to tune the HIF’s to the nonpolar phase boundaries, because of the weakening of one of the rotation or tilt modes. We further showed that this weakening, and the resulting decrease in the switching barrier, is not always accompanied with a decrease in the polarization magnitude, for example in \( \text{Sr}_3\text{Sn}_2\text{O}_7 \), verifying the point made early on in ref. [6] that a lower barrier does not necessarily mean a lower polarization. Our results thus show that biaxial strain, which has historically been used to induce ferroelectricity in many oxides, can also be used as a means to tune the coercive field of hybrid improper ferroelectrics.

**METHODS**

First principles and other calculations

DFT calculations are performed using the projector-augmented wave approach[48] as implemented in the Vienna ab-initio simulation package (VASP[49,50], and using the PBEsol generalized gradient approximation[51]. All calculations are done in a 48-atom (four formula unit) supercell, which can be viewed as a \( \sqrt{2} \times \sqrt{2} \times 2 \) multiple of the primitive cell of the reference 1\( \bar{4} \)mm structure. A F-centered 6 \( \times \) 6 \( \times \) 2 grid of k-points is used for the Brillouin zone integrals.

We consider all \( A_2\text{B}_6\text{O}_{19} \) compounds with \( A = \text{Ca}, \text{Sr}, \text{Ba} \) and \( B = \text{Ti}, \text{Zr}, \text{Sn}, \text{Ge} \), as well as \( \text{Ca}_3\text{Ti}_2\text{O}_7 \)[4,33,52–55]. These compounds are all band insulators with sizable gaps, so using the PBEsol generalized gradient approximation is expected to reproduce the crystal structures with reasonable accuracy. Biaxial strain boundary conditions are simulated by fixing the in-plane lattice constants, and allowing the out of plane component, as well as internal atomic positions, to relax with an force threshold of 2 meV/A. The zero strain is defined for each compound by the \( \alpha \) lattice constant obtained by completely relaxing the structure in the reference high-symmetry structure 1\( \bar{4} \)mmm. The Goldschmidt tolerance factor[48], which is used as a simple measure to predict tendency towards octahedral rotations, and is originally defined in terms of the ratio \( r \) using

\[
\tau = \frac{r_A + r_O}{\sqrt{2} (r_B + r_O)} \quad (2)
\]

is instead calculated using the bond lengths for 12 coordinated A-site (\( d_{AO} \)) and 6 coordinated B-site (\( d_{BO} \)) ions from the bond valence model as

\[
\tau = \frac{d_{AO}}{\sqrt{2} \cdot d_{BO}} \quad (3)
\]

(This approach is following ref. [29].)

In order to calculate the minimum energy barrier for polarization switching, climbing-image nudged elastic band (CI-NEB) method was used to further relax linearly interpolated switching paths to the minimum energy path[57]. The spring constant was set to 5 eV/Å\(^2\), and a convergence criterion of 1 meV per supercell was used. Distortion symmetry groups[44,45] are used to enumerate and name the possible initial pathways following ref. [22] with the help of the DiSPy package[58]. All the switching pathways reported in the text retain their symmetry for all values of the reaction coordinate under NEB calculation.

As various points in this paper, symmetry, and group theory-related arguments are built using the Isotropy Software Package[56] and the Bilbao Crystallographic Server[59–61]. VESTA software was used for visualization of crystal structures[62].

**DATA AVAILABILITY**

Data for the phase diagram and the switching paths are available at the Data Repository for University of Minnesota at https://doi.org/10.13020/hvr3-bg02.

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![Fig. 9](Image)
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AUTHOR CONTRIBUTIONS
Both authors contributed to the planning of the project, and writing of the manuscript. First principles calculations were performed by S.L.

COMPETING INTERESTS
The authors declare no competing interests.

ADDITIONAL INFORMATION
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