FUZZY EVENT-TRIGGERED DISTURBANCE REJECTION
CONTROL OF NONLINEAR SYSTEMS

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Abstract. The problem of fuzzy based event-triggered disturbance rejection control for nonlinear systems is addressed in this paper. A new fuzzy event based anti-rejection controller is designed and a fuzzy reduced disturbance observer is constructed. Sufficient conditions for the closed loop system to be asymptotically stable under an $H_{\infty}$ performance index are derived. Based on these conditions, the design of a fuzzy event-triggered state feedback controller is formulated and solved. Numerical results are presented to demonstrate the correctness and effectiveness of our theoretical findings.

1. Introduction. To design an effective controller for a dynamic system, some knowledge of system model or structure needs to be known. It is well known that classic control theory is providing a large variety of methods for solving model-based controller design problems, particularly for those with linear structured systems. However, most of processes of practical significance are highly nonlinear and contain uncertain parameters so that conventional control theory is unable to solve them satisfactorily. To remedy this, the concept of fuzzy model based system has been introduced and applied to nonlinear systems successfully. In this approach, a system is assume to be fuzzy and controllers satisfying fuzzy rules are sought for the system. Clearly, this approach provides a practical controller design method even when only some rough knowledge of a process is available.

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Over the past several years, many successful applications of the aforementioned fuzzy approach have been obtained, especially for control problems using Takagi-Sugeno (T-S) [15] fuzzy model, and the fuzzy method has been successful for investigating nonlinear systems [13, 17, 5, 16, 14, 3, 24, 25, 1]. Furthermore, the method has also been applied to some complex biotechnological processes [23]. However, all these results are under the assumption that the signal of a controller is transmitted periodically. In practice, it turns out that even if the state has minor changes, the controller is still updated. Event triggered controller design is a better approach to this type of problem, in which comparison between the real state and the recent past one is carried out and the signal is transmitted only when the gap is sufficiently large, see [19, 20, 21].

On the other hand, noise or disturbance is still a big challenge for a system, especially when its distribution cannot be described exactly. Much work has been done to find methods for solving complex matrix equations [10, 11, 12, 22]. The disturbance observer based control (DOBC) is a good way to estimate the disturbance which occurs in the input channel. This disturbance will be rejected during controller design which will decrease its effect on the entire system. Some work has been done on disturbance rejection control problem. For instance, in [6], a nonlinear system is considered and disturbance rejection approach is addressed. In [4], some work for linear uncertain and time delay systems has been done, and extensively studies have been done in many areas, see [7, 8, 18].

In this paper, the fuzzy $H_\infty$ control problem with nonlinear systems in discrete-time domain is studied. The T-S fuzzy model is employed to describe the nonlinear system in terms of IF-THEN rules and the reduced order observer is constructed to estimate disturbance. The rest of the paper is organized as follows: Problem statement and preliminary results are presented in Section 2. In Section 3, stability analysis of the resulting closed loop fuzzy system is given. In Section 4, $H_\infty$ performance for the error dynamic system is analyzed and the fuzzy $H_\infty$ controller is designed such that the associated error dynamic system is asymptotically stable. A numerical example is given to illustrate the effectiveness of our approach in Section 5. Finally, some concluding remarks are made in Section 6.

Notation. Throughout the paper, $\mathbb{R}^n$ is used to denote the $n$-dimensional Euclidean space, $A^T$ denotes the transpose of the matrix $A$. A positive (negative) definite matrix $P$ is written as $P > 0$ ($P < 0$) and $*$ indicates a symmetric element in a symmetric matrix.

2. Problem statement and preliminaries. We consider a discrete-time nonlinear system which can be described by the following fuzzy model:

Plant rule $i$

IF $\theta_{ik}$ is $M_{i1}$, · · · , and $\theta_{gk}$ is $M_{ig}$

THEN

$$x(k + 1) = A_i x(k) + B_i (u(k) + d_1(k)) + H_i d_2(k),$$

(1)

where $i \in \{S\} = \{1, 2, 3, \ldots, v\}$, $M_{ij}$ is a given fuzzy set for any feasible $i$ and $j$, $v$ is the number of IF-THEN rules, $\theta_{ik}, \ldots, \theta_{gk}$ are the premise variables, $x(k) \in \mathbb{R}^l$ is the state vector of the system, $u(k) \in \mathbb{R}^l$ is the input vector of the system, $d_1(k) \in \mathbb{R}^p$ is the input disturbance of the system, $d_2(k) \in L_2^2 [0, \infty)$ is the external disturbance vector of the system, and $A_i$, $B_i$ and $H_i$ are constant matrices with appropriate dimensions at the working instant $k$. 

The input disturbance \(d_1(k)\) in system (1) is given as follows.

\[
\begin{align*}
    w(k+1) &= W_i w(k) + M_i d_3(k), \\
    d_1(k) &= V_i w(k),
\end{align*}
\]

where \(W_i, M_i\) and \(V_i\) are constant matrices with appropriate dimensions.

**Assumption 1.** For systems (1) and (2), it holds that 1) \((A_i, B_i)\) is controllable; and 2) \((W_i, B_i V_i)\) is observable.

The discrete time fuzzy system is inferred as follows:

\[
x(k + 1) = \frac{\sum_{i=1}^{n} \mu_i(\theta_k)[A_i x(k) + B_i (u(k) + d_1(k)) + H_i d_2(k)]}{\sum_{i=1}^{n} \mu_i(\theta_k)},
\]

where \(\theta_k = [\theta_{1k} \theta_{2k} \cdots \theta_{nk}]\), \(\mu_i(\theta_k) = \prod_{j=1}^{n} M_{ij} \theta_{jk}\), \(h_i(\theta_k) = \frac{\mu_i(\theta_k)}{\sum_{i=1}^{n} \mu_i(\theta_k)}\), and \(M_{ij} \theta_{jk}\) is the grade of membership of \(\theta_{jk}\) in \(M_{ij}\).

Assume that

\[
\mu_i(\theta_k) \geq 0 \quad \text{and} \quad \sum_{i=1}^{n} \mu_i(\theta_k) > 0.
\]

Then, we have:

\[
h_i(\theta_k) \geq 0 \quad \text{and} \quad \sum_{i=1}^{n} h_i(\theta_k) = 1.
\]

Therefore, system (1) is rewritten as:

\[
x_{k+1} = \sum_{i=1}^{v} h_i(\theta_k)[A_i x(k) + B_i (u(k) + d_1(k)) + H_i d_2(k)].
\]

Under the assumption that all of the system states are available, we need to estimate \(d_1(k)\). If \(\theta_{1k} = M_{i1}, \ldots, \theta_{nk} = M_{ig}\), then, a reduced-order observer is constructed below:

\[
\begin{align*}
    \dot{\hat{d}}_1(k) &= V_i \hat{\omega}(k), \\
    \dot{\hat{\omega}}(k) &= v(k) - L_i x(k), \\
    v(k+1) &= (W_i + L_i B_i V_i)(v(k) - L_i x(k)) + L_i(A_i x(k) + B_i u(k)),
\end{align*}
\]

where \(\hat{d}_1(k)\) and \(\hat{\omega}(k)\) are estimations of \(d_1(k)\) and \(w(k)\), respectively.

And the fuzzy reduced-order observer is obtained as:

\[
\begin{align*}
    \hat{d}_1(k) &= \sum_{i=1}^{n} h_i(\theta_k)[V_i \hat{\omega}(k)], \\
    \dot{\hat{\omega}}(k) &= \sum_{i=1}^{n} h_i(\theta_k)[v(k) - L_i x(k)], \\
    v(k+1) &= \sum_{i=1}^{n} h_i(\theta_k)[(W_i + L_i B_i V_i)(v(k) - L_i x(k)) + L_i(A_i x(k) + B_i u(k))].
\end{align*}
\]

To reduce the effects of disturbance, controller is constructed as

\[
u(k) = -\hat{d}_1(k) + K x(k).
\]

Let

\[
f(k) = w(k) - \hat{\omega}(k),
\]

\[
f(k+1) = \sum_{i=1}^{n} h_i(\theta_k) \{(W_i + L_i B_i V_i)f(k) + L_i H_i d_2(k) + M_i d_3(k)\}.
\]
To reduce network based transmission load, by event-triggered theory, let \( \hat{x}_k \) be a new signal applied to the controller in the time interval \((k, k+1]\) with
\[
\hat{x}(k) = \begin{cases} 
  x(k) & \text{if event condition is satisfied,} \\
  \hat{x}(k-1) & \text{if event condition is not satisfied,}
\end{cases}
\]
and \( \hat{x}(k) = 0 \) for \( k \leq 0 \) with the initial time \( k_0 = 0 \). Based on (8), let \( u(k) = -\hat{d}_1(k) + K\hat{x}(k) \). The following decision condition for signal transmission is given by the event generator:
\[
\|\hat{x}(k-1) - x(k)\| > \sigma \|x(k)\|,
\]
where \( \sigma > 0 \). Then, combing (8) and (9), we have an event-triggered based controller given below
\[
u(k) = \begin{cases} 
  -\hat{d}_1(k) + K_i x(k) & \text{if } \|\hat{x}(k-1) - x(k)\| > \sigma \|x(k)\|, \\
  -\hat{d}_1(k) + K_i \hat{x}(k-1) & \text{if } \|\hat{x}(k-1) - x(k)\| \leq \sigma \|x(k)\|.
\end{cases}
\]
Let \( \eta^T(k) = [x^T(k), f^T(k)] \), \( d^T(k) = [d_1^T(k), d_2^T(k)] \) and \( e(k) = \hat{x}(k) - x(k) \), combining systems (1), (2) and (5), we obtain a fuzzy based error estimation system:
\[
\eta(k+1) = \tilde{A}_{ij} \eta(k) + \tilde{B}_{ij} e(k) + \tilde{C}_{ij} d(k),
\]
where
\[
\tilde{A}_{ij} = \sum_{i=1}^{v} h_i \sum_{j=1}^{v} h_j \begin{bmatrix} A_i + B_i K_i & B_i V_j \\ 0 & W_j + L_j B_i V_j \end{bmatrix},
\]
\[
\tilde{B}_{ij} = \sum_{i=1}^{v} h_i \sum_{j=1}^{v} h_j \begin{bmatrix} B_i K_i \\ 0 \end{bmatrix},
\]
\[
\tilde{C}_{ij} = \sum_{i=1}^{v} h_i \sum_{j=1}^{v} h_j \begin{bmatrix} H_i \\ L_j H_i \end{bmatrix}.
\]

The reference output of system (11) is set as:
\[
z(k) = D_{ij} \eta(k),
\]
where \( D_{ij} = \sum_{i=1}^{v} h_i \sum_{j=1}^{v} h_j \begin{bmatrix} D_{1i} & D_{2j} \end{bmatrix} \).

To proceed further, some definitions are needed in developing our main results in the paper.

**Definition 2.1.** For a given initial state \( \eta(0) \), suppose
\[
\lim_{m \to \infty} \left\{ \sum_{k=0}^{m} \eta^T(k) \eta(k) | \eta(0) \right\} < \infty.
\]

Then, system (11) is said to be asymptotically stable and \( K_i \) is the gain matrix of the controller.

**Lemma 2.2.** [2] Let \( R > 0 \) be a given symmetric matrix, and let \( W_t, t = 1, 2, \ldots, h \) be matrices with appropriate dimensions, if \( 0 \leq \varepsilon_t \leq 1 \) and \( \sum_{i=1}^{h} \varepsilon_i = 1 \), then
\[
(\sum_{t=1}^{h} \varepsilon_t W_t)^T R (\sum_{t=1}^{h} \varepsilon_t W_t) \leq \sum_{t=1}^{h} \varepsilon_t W_t^T R W_t.
\]
Definition 2.3. For a given constant $\gamma > 0$, system (11) is said to be asymptotically stable and satisfy an $H_\infty$ performance index $\gamma$, if it is asymptotically stable and the following condition is satisfied:

$$
\sum_{k=0}^{\infty} z^T(k)z(k) \leq \gamma^2 \sum_{k=0}^{\infty} d^T(k)d(k).
$$

(13)

The aim of our work is to design an event trigger based fuzzy anti-disturbance controller to ensure that the error system (11) is asymptotically stable and an $H_\infty$ performance index is satisfied.

3. Stability analysis. In this section, sufficient conditions are given under which system (11) is asymptotically stable.

**Theorem 3.1.** Let $\sigma > 0$ be given. If there exist a positive definite symmetric matrix $P$, and a constant $\kappa$ such that

$$
\hat{\Gamma}_{ij} = \begin{bmatrix} -4P + \hat{A}_{ij}^T P \hat{A}_{ij} + \kappa S & \hat{A}_{ij}^T P \hat{B}_{ij} \\ \ast & \hat{B}_{ij}^T P \hat{B}_{ij} - \kappa I \end{bmatrix} \leq 0,
$$

(14)

then, system (11) is asymptotically stable.

**Proof.** We take into consideration of the following Lyapunov function

$$
V(\eta(k)) = \eta^T(k)P\eta(k).
$$

Taking the difference of the Lyapunov function along the trajectory of system (11) yields

$$
\Delta V(\eta(k)) = V(\eta(k+1)) - V(\eta(k)) = \eta^T(k+1)P\eta(k+1) - \eta^T(k)P\eta(k)
$$

$$
= \eta^T(k)\frac{1}{4} \sum_{i=1}^{v} \sum_{j=1}^{v} h_i h_j \hat{A}_{ij}^T P \sum_{i=1}^{v} \sum_{j=1}^{v} h_i h_j \hat{A}_{ij} \eta(k)
$$

$$
+ \eta^T(k)\frac{1}{4} \sum_{i=1}^{v} \sum_{j=1}^{v} h_i h_j \hat{A}_{ij}^T \hat{P} \sum_{i=1}^{v} \sum_{j=1}^{v} h_i h_j \hat{B}_{ij} e(k)
$$

$$
+ e^T(k)\frac{1}{4} \sum_{i=1}^{v} \sum_{j=1}^{v} h_i h_j \hat{B}_{ij}^T \hat{P} \sum_{i=1}^{v} \sum_{j=1}^{v} h_i h_j \hat{A}_{ij} \eta(k)
$$

$$
+ e^T(k)\frac{1}{4} \sum_{i=1}^{v} \sum_{j=1}^{v} h_i h_j \hat{B}_{ij}^T \hat{P} \sum_{i=1}^{v} \sum_{j=1}^{v} h_i h_j \hat{B}_{ij} e(k) - \eta^T(k)P\eta(k),
$$

where $\hat{A}_{ij} = \hat{A}_{ij} + \hat{A}_{ji}$, $\hat{B}_{ij} = \hat{B}_{ij} + \hat{B}_{ji}$. To ensure that $\Delta V(\eta(k)) \leq 0$, we need to have

$$
\hat{\Gamma}_{11ij} = \sum_{i=1}^{v} \sum_{j=1}^{v} h_i h_j \begin{bmatrix} -4P + \hat{A}_{ij}^T P \hat{A}_{ij} & \hat{A}_{ij}^T P \hat{B}_{ij} \\ \ast & \hat{B}_{ij}^T P \hat{B}_{ij} \end{bmatrix} \leq 0.
$$

(15)

Let

$$
\xi = \min_k \{\lambda_{\min}(-\hat{\Gamma}_{11ij})\},
$$

where $\lambda_{\min}(-\hat{\Gamma}_{11ij})$ is the minimal eigenvalue of $-\hat{\Gamma}_{11ij}$. Then,

$$
\Delta V(\eta(k)) \leq -\xi \eta^T(k)\eta(k),
$$
where
\[ \eta^T(k) = \begin{bmatrix} x(k) \\ f(k) \end{bmatrix}^T. \]

From this we have,
\[ \sum_{k=0}^{T} \Delta V(\eta(k)) = V(\eta(T+1)) - V(\eta(0)) \leq -\xi \sum_{k=0}^{T} \|\eta(k)\|^2. \]

This, in turn, implies that
\[ \sum_{k=0}^{T} \|\eta(k)\|^2 \leq \frac{1}{\xi} \{V(\eta(0)) - V(\eta(T+1))\} \leq \frac{1}{\xi} V(\eta(0)), \]

and hence
\[ \lim_{T \to \infty} E\{\sum_{k=0}^{T} \|\eta(k)\|^2\} \leq \frac{1}{\xi} V(\eta(0)). \]

By Definition 2.1, system (11) is asymptotically stable.

4. H_\infty performance analysis and controller design. Based on the conditions established in Theorem 3.1, we now consider performance analysis and controller design.

Theorem 4.1. Let \( \sigma \) be given. If there exist a positive definite symmetric matrix \( P \), and a constant \( \kappa \) such that
\[
\begin{bmatrix}
-4P + \kappa S & 0 & 0 & \hat{A}_{ij}^T & \hat{D}_{ij}^T \\
* & -\kappa I & 0 & \hat{B}_{ij}^T & 0 \\
* & * & -4\gamma^2 I & \hat{C}_{ij}^T & 0 \\
* & * & * & -P^{-1} & 0 \\
* & * & * & * & -I
\end{bmatrix} < 0,
\]

then system (11) is said to be asymptotically stable and an H_\infty performance index is satisfied.
Proof. To establish the $H_\infty$ performance for the system (11), the following cost function is introduced:

$$J(T) = \sum_{k=0}^{T} z^T(k)z(k) - \gamma^2 \sum_{k=0}^{T} d^T(k)d(k).$$

(19)

In the case of zero initial condition, $J(T)$ can be written as

$$J(T) \leq \sum_{k=0}^{T} \{z^T(k)z(k) - \gamma^2 d^T(k)d(k) + \Delta V(\eta(k))\}.$$  

(20)

Thus, we have

$$J(T) \leq \sum_{k=0}^{T} \{z^T(k)z(k) - \gamma^2 d^T(k)d(k) + \Delta V(\eta(k))\}$$

which leads to

$$\Theta_{ij} = \begin{bmatrix} -4P & 0 & 0 & \hat{A}^T_{ij} & \hat{D}^T_{ij} \\ * & 0 & 0 & \hat{B}^T_{ij} & 0 \\ * & * & -4\gamma^2 & \hat{C}^T_{ij} & 0 \\ * & * & * & -P^{-1} & 0 \\ * & * & * & * & -I \end{bmatrix} < 0.$$  

(21)

Then, for each fuzzy model, it follows that $J(T) \leq 0$ whenever $\Theta_{ij} < 0$.

Recalling condition (16) and following an argument similar to that in the proof of Theorem 3.1, we can show that system (11) is asymptotically stable and the following condition is also satisfied.

$$E \left\{ \sum_{k=0}^{\infty} \bar{z}_k^T \bar{z}_k \right\} \leq \gamma^2 E \left\{ \sum_{k=0}^{\infty} w_k^T w_k \right\}.$$  

(22)

Note that the inequality in Theorem 4.1 is unsolvable and we need to transform it to a solvable one.  

\hfill \Box
Theorem 4.2. Let \( \sigma \) be given. If there exist a positive definite symmetric matrix \( Q \), and a constant \( \kappa \) such that
\[
\begin{bmatrix}
-4G^T - 4G + 4Q & 0 & 0 & 0 & 0 \\
* & -8I + 4Q & 0 & 0 & 0 \\
* & * & -G^T - G + \frac{1}{\kappa} I & 0 & 0 \\
* & * & * & -4\gamma^2 & 0 \\
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
(\bar{A}_{ij} + \bar{B}_{ij}K_{ij})^T & 0 & 0 & 0 & 0 \\
(\bar{B}_{ij}V_j)^T & (W_j + L_jB_jV_j)^T & (D_{ij}G)^T & G^T \\
(\bar{H}_{ij})^T & (\bar{L}_{ij}\bar{H}_{ij})^T & 0 & 0 \\
-\bar{Q} & 0 & 0 & 0 \\
0 & -\bar{Q} & 0 & 0 \\
0 & 0 & -I & 0 \\
* & * & * & -\frac{1}{\kappa\sigma^2} \\
\end{bmatrix} < 0, \quad (23)
\]
then system (11) is asymptotically stable and satisfies an \( H_{\infty} \) performance index.

Proof. By the Schur complement, multiply from the left hand side and the right hand side of inequality (18) by \( \text{diag}\{G^T, I, G^T, I, I, I\} \) and \( \text{diag}\{G, I, G, I, I, I\} \), respectively, where \( G \in \mathbb{R}^{n \times n} \) is a positive definite diagonal matrix.

Noting that
\[
(P^{-1} - G)^T P(P^{-1} - G) \geq 0,
\]
or
\[
G^T PG \geq G^T + G - P^{-1}.
\]
Let \( Q = P^{-1}, \bar{A}_{ij} = A_i + A_j \) and \( \bar{K}_{ij} = K_{ij}G \). Then, Condition (23) is obtained which is linear in the variables \( G, Q \) and \( \bar{K}_{ij} \). Once a solution of (23) is obtained, the feedback gain can be calculated as \( K_{ij} = \bar{K}_{ij}G^{-1} \).

5. Numerical example. Consider a discrete-time fuzzy model based nonlinear system with the following system and other parameters:

\[
A_1 = \begin{bmatrix}
-1.02 & -0.1 & 0.8 \\
-0.1 & 0.2 & 0.2 \\
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
-0.2 \\
0.2 \\
\end{bmatrix},
\]
\[
W_1 = \begin{bmatrix}
0 & -1 \\
1 & 0 \\
\end{bmatrix}, \quad H_1 = \begin{bmatrix}
0.1 \\
-0.1 \\
\end{bmatrix},
\]
\[
M_1 = \begin{bmatrix}
0.1 \\
0.1 \\
\end{bmatrix}, \quad V_1 = \begin{bmatrix}
0.3 & 0.5 \\
\end{bmatrix},
\]
\[
D_{11} = \begin{bmatrix}
0.5 & 0.1 \end{bmatrix}, \quad D_{12} = \begin{bmatrix}
0.1 & 0 \end{bmatrix}.
\]
Our purpose is to design a fuzzy $H_{\infty}$ reduced order observer for system (1) such that the resulting error system (11) is asymptotically stable with an $H_{\infty}$ noise attenuation performance index.

Based on Theorem 4.2, let $\sigma = 0.01$, under event triggered condition, we have

$$L_1 = \begin{bmatrix} -2.1610 & 1.3701 \\ 1.3701 & -1.1195 \end{bmatrix}, \quad K_1 = \begin{bmatrix} -1.3097 & -0.4087 \end{bmatrix}.$$  

$$L_2 = \begin{bmatrix} -0.3125 & 1.8301 \\ 1.8301 & -0.6529 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 1.1065 & -0.3906 \end{bmatrix}.$$  

we obtain the following fuzzy based matrices of the observer and controller, under which the system is asymptotically stable with an $H_{\infty}$ performance index satisfied. For the case of $\sigma = 0.01$, the corresponding trajectories are shown in Figure 1. The disturbance $d_1(t)$, the estimation disturbance $\hat{d}_1(t)$, and the error disturbance $\check{d}_1(t) - \hat{d}_1(t)$ are illustrated in Figure 2. Obviously, the system concerned is asymptotically stable under such a controller.

6. Conclusion. In this paper, the problem of fuzzy event-based disturbance rejection control of nonlinear systems is studied. Sufficient conditions are given under which the closed-loop system is stable. Based on these conditions, fuzzy controller and rejection observer are formulated and solved. A numerical example illustrates the effectiveness of the proposed design procedure and the performance of the resulting closed-loop system.
Figure 2. Estimation of disturbance

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