A smooth polaron–molecule crossover in a Fermi system

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Abstract
The problem of a single down-spin particle interacting with a Fermi sea of up-spin particles is of current interest in the field of cold atoms. The Hubbard model, appropriate to atoms in an optical lattice potential, is considered in parallel with a gas model. As the strength of an attractive short-range interaction is increased there is a crossover from ‘polaron’ behaviour, in which the Fermi sea is weakly perturbed, to ‘molecule’ behaviour in which the down-spin particle is bound to a single up-spin particle. It is shown that this is a smooth crossover, not a sharp transition as claimed by many authors.

1. Introduction
The behaviour of a system in which a mobile particle couples to a fermion bath has been of interest for at least fifty years. A useful model of such a system is provided by a Hubbard model [1] with a single down-spin particle interacting with a Fermi sea of up-spin particles. The on-site interaction $U$ may be repulsive ($U > 0$) or attractive ($U < 0$). For low particle density the Hubbard lattice model maps onto a continuum gas model with short-range interactions and in this regime the theory of the two models with a single reversed spin can proceed in parallel. McGuire [2, 3] found exact solutions for the gas case in one dimension (1D) with both repulsive and attractive interactions. This pioneering work was a forerunner of Lieb and Wu’s exact Bethe ansatz solution of the 1D Hubbard model with arbitrary up- and down-spin densities [4]. Edwards [5] wrote McGuire’s exact wavefunction in a simple form which enabled Castella and Zotos [6] to calculate spectral properties. It is remarkable that the highly simplified case of one down-spin particle already contains the basic non-Fermi-liquid features of 1D conductors; in particular the down-spin quasiparticle weight vanishes in the thermodynamic limit.

The repulsive case in higher dimensions with one spin reversal was first studied in connection with the stability of ferromagnetism in the Hubbard model [7, 8]. In 1D the ferromagnetic state is never stable [4]. The use of increasingly sophisticated variational wavefunctions [9, 10] has provided strong evidence for the existence of a small region of the phase diagram (particle density $\rho$ versus $U/t$ where $t$ is the hopping parameter) of the Hubbard square lattice where a state of complete spin alignment is stable against reversing a spin.

The discovery of high-temperature superconductivity in CuO$_2$ planes launched a new wave of interest in the 2D Hubbard model. The unusual normal state in these materials led Anderson [11] to propose that non-Fermi-liquid behaviour, similar to that in 1D, might occur in the Hubbard square lattice. Since in 1D such behaviour is found already in states with a single spin reversal, Sorella [12] made accurate calculations of the down-spin quasiparticle weight $Z$ for this case in 2D. Except for the case of a half-filled up spin band, where the Fermi level is at a van Hove singularity, he found no evidence of a departure from Fermi liquid behaviour. $Z$ is found not to fall below 0.1 even for the strongly attractive case $U = -\infty$ with $\rho = 1/4$ where the down-spin particle might be expected to lose its fermionic character in forming a molecule with an up-spin particle.

The latest renaissance of the Hubbard model is associated with the experimental study of fermionic cold atoms in optical lattices. The related continuum gas model corresponds to harmonically trapped atoms where in theoretical work a uniform gas is often assumed. Strongly spin-polarized Fermi gases can be realized experimentally and there is an excellent review of the experimental and theoretical situation by Chevy and Mora [13]. They stress the importance of understanding the case of a single down-spin particle coupled to the up-spin Fermi sea by an attractive interaction. For
weak interaction the down-spin particle merely disturbs the Fermi sea by creating a few electron–hole pairs and may be described as a ‘polaron’. For very strong interaction the down-spin may bind to a single up spin to form a ‘molecule’. A number of authors claim that as the interaction strength is increased there is a sharp polaron–molecule transition with zero down-spin quasiparticle weight on the molecular side of the transition [14–20]. The first five of these references are concerned with the 3D system. The last two consider the 2D case, and their conclusions conflict with the work of Sorella discussed in the previous paragraph.

In this paper we argue that for the Hubbard and gas models, in both 2D and 3D, there is a smooth crossover between the polaron and molecule limits.

2. The exact wavefunction for one reversed spin

We consider the Hamiltonian

\[ H = \sum_{k\sigma} \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \frac{g}{V} \sum_{k, k', q} c_{k\uparrow}^\dagger c_{k'\downarrow}^\dagger c_{k'q\downarrow} c_{k'q\uparrow} + \text{H.c.} \]  

(1)

where \( c_{k\sigma} \) destroys a particle with wavevector \( k \) and spin \( \sigma \), and \( g/V \) is the interaction between particles of opposite spin where \( V \) is the volume of the system. In the Hubbard case

\[ \varepsilon_k = -t \sum_{\mathbf{R}} c^{\dagger}_{k\mathbf{R}} \]  

(2)

with summation over nearest neighbours \( \mathbf{R} \). Also \( g = Ud^2 \) where \( d \) is the dimensionality and \( a = |\mathbf{R}| \) is the lattice constant for the 1D, square and simple cubic lattices. In the gas case \( \varepsilon_k = \hbar^2 k^2/2m \), which corresponds to the long-wavelength limit of (2) with \( \hbar^2/2m = a^2t \). The wavevector summations are taken over the Brillouin zone in the Hubbard case and restricted by a suitable cutoff in the gas case (see e.g. [21]). We are concerned with the case of \( N_\uparrow \) spins and one \( \downarrow \) spin. The form of exact wavefunction for this case is [9]

\[ |\psi(\mathbf{K})\rangle = \sum_{Q} C(Q)c_{\mathbf{K}-\mathbf{Q}\downarrow}^\dagger |Q\rangle, \]  

(3)

where the summation is over all possible \( N_\uparrow \)-tuples \( Q \) of wavevectors \( k \) for the \( \uparrow \) spin particles. Here

\[ |Q\rangle = \prod_{k\in Q} c_{k\downarrow}^\dagger |0\rangle \quad Q = \sum_{k} \mathbf{k}, \]  

(4)

where \( |0\rangle \) is the vacuum state. The number of \( N_\uparrow \)-tuples is clearly \( \binom{N}{N_\uparrow} \) where \( N \) is the number of wavevectors in the zone, equal to the number of lattice sites.

The exact wavefunction may be written in an alternative form which emphasizes the polaronic nature of the \( \downarrow \) spin particle. This was done by Combescot and Giraud [22] for the gas case. The Hubbard case is essentially the same for \( N_\uparrow < N/2 \) and also for \( N_\uparrow > N/2 \) if we consider a gas of holes. We select a set of \( N_\uparrow \) wavevectors to form a \( \uparrow \) spin Fermi sea (FS). These will normally lie within the Fermi surface of the non-interacting system. By using FS as a new vacuum state we can express the exact wavefunction of equation (3) in terms of excitation of particle–hole pairs. The Fermi sea of \( N_\uparrow \) \( \uparrow \) spin particles is given by

\[ |\text{FS}(N_\uparrow)\rangle = \prod_{k\in\text{FS}(N_\uparrow)} c_{k\uparrow}^\dagger |0\rangle \]  

(5)

and we can rewrite (3) as

\[ |\psi(\mathbf{K})\rangle = \left[ \alpha c_{\mathbf{K}\uparrow}^\dagger + \sum_{k, q} \alpha_{kq} c_{\mathbf{K}-k+q\downarrow}^\dagger c_{k\uparrow}^\dagger c_{q\uparrow} \right. \]

\[ + \cdots + \left. \frac{1}{(n!)^2} \sum_{(k_1)(q_1)} \alpha_{(k_1)(q_1)} c_{p_1\uparrow}^\dagger \prod_{1\leq i\leq n} c_{k_i\uparrow}^\dagger c_{q_i\uparrow} + \cdots \right] |\text{FS}(N_\uparrow)\rangle, \]  

(6)

where states \( k, k_i \) are outside the FS and \( q, q_i \) are inside. The total wavevector \( \mathbf{K} \) is relative to the total wavevector of the assumed FS. In the general term, with \( n \) particle–hole pairs excited, the wavevector \( \mathbf{P} = \mathbf{K} - \sum_{1\leq i\leq n} \mathbf{k}_i + \sum_{1\leq j\leq n} \mathbf{q}_j \).

The coefficients \( \alpha_{(k_1)(q_1)} \) are antisymmetric with respect to the exchange of any of their arguments \( k_i \) or \( q_i \). In the Hubbard case the number of states with \( n \) particle–hole pairs is \( \binom{N}{n} \binom{N-N_\uparrow}{n} \) and the last term of the series corresponds to \( n = N_\uparrow \). Since

\[ \sum_{0\leq n\leq N_\uparrow} \binom{N_\uparrow}{n} \binom{N-N_\uparrow}{n} = \binom{N}{N_\uparrow}, \]  

(7)

the total number of independent \( \alpha \) coefficients in (6) is equal to the number of coefficients \( C(Q) \) in (3), as it should be. Approximate wavefunctions obtained by truncating the series (6) at a small number of terms have been used as variational forms by many authors in this field, commencing with Chevy [23]. We follow Parish and Levinsen [20] in denoting the wavefunction which ends with the term involving \( n \) particle–hole pairs by \( |P_{2n+1}(\mathbf{K})\rangle \).

Several authors [16, 19–21] have introduced another set of states which emphasize the molecular aspect of the problem. Again following the notation of [20] we write

\[ |M_{2n}(\mathbf{K})\rangle = \left[ \sum_{k} \tilde{\xi}_{kk\downarrow}^\dagger c_{k\uparrow}^\dagger \right. \]

\[ + \sum_{k, k_i, q_i} \tilde{\xi}_{kk_i q_i}^\dagger c_{k-k-k_i+q_i\downarrow}^\dagger c_{k\uparrow}^\dagger c_{q_i\uparrow} \]

\[ + \cdots + \frac{1}{(n!)^2} \sum_{(k_1)(q_1)} \tilde{\xi}_{(k_1)(q_1)}^\dagger c_{p_1\uparrow}^\dagger \prod_{1\leq i\leq n} c_{k_i\uparrow}^\dagger c_{q_i\uparrow} \]

\[ \times \prod_{1\leq i\leq n} c_{q_i\uparrow}^\dagger + \cdots \right] |\text{FS}(N_\uparrow - 1)\rangle, \]  

(8)

where \( |\text{FS}(N_\uparrow - 1)\rangle \) represents a FS containing \( N_\uparrow - 1 \) states and \( \mathbf{K} \) is the total wavevector relative to the total wavevector of this FS. Here \( \mathbf{P}' = \mathbf{K} - \sum \mathbf{k}_i + \sum \mathbf{q}_i \). If, as a special case,
we take $\xi_k = \alpha_0 \delta_{k,1} \delta_{k,2}$ and put $|FS(N_f)\rangle = e^{i k_1 \cdot r_1} |FS(N_f - 1)\rangle$, we see that $|M_{2n}(k)\rangle$ becomes identical to $|P_{2n-1}(k - k)\rangle$. We can include the total wavevector as a variational parameter and hence define $E_{2n}$ as the lowest energy, calculated variationally, for $|M_{2n}(k)\rangle$ and $E_{2n-1}$ as the lowest energy for $|P_{2n-1}(k)\rangle$. All energies are for a given particle number and interaction strength. We have shown that every $|P_{2n-1}\rangle$ state is a special case of a $|M_{2n}\rangle$ state and hence its energy is greater than that of this $|M_{2n}\rangle$ state and hence greater than $E_{2n}$, the lowest $M_{2n}$ energy. Hence $E_{2n-1} \geq E_{2n}$.

Similarly we can show that every $|M_{2n}\rangle$ state is a special case of a $|P_{2n+1}\rangle$ state so that $E_{2n} \geq E_{2n+1}$. Hence

$$E_{2n-1} \geq E_{2n} \geq E_{2n+1}.$$ (10)

Parish and Levinsen [20] have obtained a similar result, but they do not draw the following conclusions.

(i) Curves of $E_{2n-1}$ and $E_{2n}$ as functions of interaction strength, for a given particle number, should never cross. Apparent crossings found in 3D [14–18] and 2D [19, 20] have been interpreted as sharp transitions between a ‘polaron’ state $|P\rangle$ and a ‘molecule’ state $|M\rangle$. Sharp transitions of this type should not occur.

(ii) Since every $|M\rangle$ state is a special case of a $|P\rangle$ state there is really nothing to be gained by introducing $|M\rangle$ states. Combescot et al [24] have shown that the simple ansatz $|P_{2}(k)\rangle$ describes a smooth crossover from ‘polaron’ to ‘molecule’ behaviour quite accurately. The accuracy would be improved using $|P_{3}(k)\rangle$, $|P_{4}(k)\rangle$... although the numerical calculations rapidly become unmanageable [25]. An alternative approach, based on a variational approach which is exact in 1D but containing hardly more variational parameters than in $|P_{3}(k)\rangle$, is described in the next section. A variational treatment of the approximate ansatz $|P_{2n+1}(k)\rangle$ with up to $n$ electron–hole pairs excited leads to an equation for the $\downarrow$ spin exciton energy $\omega$ of the form [25]

$$\omega - \epsilon_{k} - \Sigma_{2n+1}(k,\omega) = 0.$$ (11)

The total energy of the state $|P_{2n+1}(k)\rangle$ is $E_0 + \omega$ where $E_0 = \sum_{k} \epsilon_{k}$ is the energy of the FS state given by equation (5). The self-energy $\Sigma_{2n+1}(k,\omega)$ corresponds diagrammatically to summing all diagrams with up to $2n + 1$ interaction lines and all possible line crossings linking the $\downarrow$ spin particle line with one of the $\uparrow$ spin particle(hole) lines. It is exact for the case $N_{f} = n$, for all Hubbard parameters $r$ and $U$, and hence gives the correct atomic limit ($t = 0$):

$$\Sigma_{at} = \alpha U n_{f} / [\omega - U (1 - n_{f})],$$ (12)

where $n_{f} = N_{f}/N$. This corresponds to the Green function

$$G_{at} = (\omega - \Sigma_{at})^{-1} = \frac{n_{f}}{\omega - U} + \frac{1 - n_{f}}{\omega}$$ (13)

as obtained by Hubbard [1]. Clearly $\Sigma_{at}(k,\omega)$ includes the simple second-order diagram which describes the weak interaction limit as well as sufficient diagrams to describe the atomic limit. This is why the state $|P_{3}(k)\rangle$ yields a smooth crossover from the weak coupling ‘polaron’ limit to the strong coupling limit with ‘molecule’ states forming a lower Hubbard band around $\omega = U (U < 0)$.

3. An alternative ansatz

Edwards [5, 9] introduced an ansatz which is equivalent to assuming that the function $C(Q)$ in the exact wavefunction of equation (3) is a determinant of one-particle orbitals $\phi_{s}(k), s = 1 \ldots N_{f}$. In the real-space (site) representation of the Hubbard model this becomes

$$\langle \chi(k) \rangle = \frac{1}{\sqrt{N}} \sum_{i} e^{i K \cdot x_{i}} c_{i}^{\dagger} \prod_{s} \left( \sum_{j} \phi_{s}(x_{j} - x_{i}) c_{j}^{\dagger} \right) |0\rangle$$ (14)

where $c_{i}^{\dagger}$ creates a particle of spin $\sigma$ at the lattice site $x_{i}$. This ansatz is exact in 1D [5] but contains only $NN_{f}$ variational parameters compared with $(N_{f}^{2} + N_{f})$ parameters in $|\phi(k)\rangle$ given by (3). This economy is due to the fact that in 1D with on-site interactions ($\delta$-function interaction in the gas case) particles entering a collision with wavevectors $k_{1}, k_{2}$ merge with the same or interchanged wavevectors. This is the basis of the Bethe ansatz. The same is true not in 2D or 3D but, certainly in 2D, $|\chi(k)\rangle$ remains a very good approximation in the Hubbard model [9, 12]. For $U > 0$ it has been tested by comparison with accurate results for small clusters [9].

In the gas case the wavefunction $|\chi(k)\rangle$ becomes

$$|\chi_{gas}(k)\rangle = (N_{f}! V)^{-1/2} e^{i K \cdot x_{0}} \det(\phi_{s}(x_{j} - x_{0}))$$ (15)

where $x_{0}$ is the position of the $\downarrow$ spin particle, $x_{j} (j = 1 \ldots N_{f})$ are positions of the $\uparrow$ spin particles, $\phi_{s} (s = 1 \ldots N_{f})$ are the orbitals with respect to which the energy of $|\chi_{gas}(k)\rangle$ is minimized and $V$ is the volume of the gas. The functions $\phi_{s}(x)$ are chosen to be orthonormal. For this gas case the Hamiltonian (1) may be written as

$$H_{gas} = - (h^2 / 2 m) \left( \nabla^{2} + \Sigma_{s \leq j \leq N_{f}} \nabla_{j}^{2} \right) + g \Sigma_{j} \delta(x_{j} - x_{0}).$$ (16)

with the $\delta$-function regularized in 2D and 3D by the momentum cutoff. Equations for the orthonormal functions $\phi_{s}$ are obtained by minimizing the expectation value $\langle \chi_{gas}(k) | H_{gas} | \chi_{gas}(k) \rangle$ with respect to variations in these functions. These take the form [5]

$$- \nabla_{j}^{2} \phi_{s} + i K \cdot \nabla \phi_{s} + \sum_{t} \int \phi^{*}_{t} \nabla \phi_{s} d^{3} r \cdot \nabla \phi_{t}$$
$$- \sum_{t} \int \phi^{*}_{t} \nabla \phi_{s} d^{3} r \cdot \nabla \phi_{t} + \frac{m}{h^{2}} [g \delta(x) + \lambda_{s}] \phi_{s}(x) = 0.$$ (17)

The volume integrals have been written as three-dimensional but the form of the equations holds in any number of dimensions. For $K = 0$ they take the form of Hartree–Fock equations for a system of particles of mass $m/2$ moving in a short-range potential $g \delta(x)$ with interactions $(p_{i}, p_{j})/m$, where
p_i is the momentum of particle i. Equations analogous to (17) have been obtained for the Hubbard case [5, 9] and it is not difficult to show that in the low density limit they reduce to (17) with $\hbar^2/2m = c^2 t$, $g = U a^d$ as expected. We now wish to consider qualitatively the crossover from the ‘polaron’ to ‘molecule’ regime from the present viewpoint. In 1D, where the present formalism is exact, one of the orbitals $\phi_i$ is a localized bound state for any negative interaction, and the degree of localization grows with increasing interaction strength. Furthermore, as $g$ decreases through 0, from positive values where there is no bound state, physical properties such as the ground state energy have been shown to vary continuously with a smooth crossover [3]. Since any attractive potential also binds in 2D we expect a bound orbital to develop much as in 1D. However, there is a crucial difference as regards the unbound orbitals. In 1D they are linear combinations of plane waves with wavevectors which are not the usual ones corresponding to periodic boundary conditions, although the complete orbitals do satisfy these conditions. A consequence of this structure is that there is no quasiparticle weight [6]. In the 2D Hubbard case Sorella [12] has made detailed numerical calculations using the present formalism to show convincingly that there is no transition to a state with zero quasiparticle weight for either attractive or repulsive interaction.

In 3D a bound state will first appear at a critical strength of the attractive interaction. This situation was considered by Kohn and Majumdar [26] for a static impurity. In the present context this corresponds to neglecting the Hartree–Fock terms in (17) which arise from the motion of the ↓ spin particle. Kohn and Majumdar showed that properties such as the particle density function and ground state energy are smooth (analytic) functions of the interaction strength, even at the critical value where a bound state appears. There seems little doubt that this continuity will still hold in the presence of weak interactions such as the momentum-dependent ones appearing in (17). We therefore expect a continuous crossover in 1D, 2D and 3D, in agreement with the conclusions of section 2.

4. Relation between the two approaches

In this section we show how the approximate wavefunctions discussed in section 2 can emerge from the ansatz of section 3. The first example we consider is the wavefunction $|\psi_q(\mathbf{K})\rangle$ which corresponds to (6) with only the first two terms retained on the right-hand side. This form is obtained from the ansatz (14) by taking each orbital $\phi_a$ as a plane wave $(1/\sqrt{N})e^{iQx}$ with a small correction, where $q$ lies within the FS and the label $s$ may be replaced by $q$. Thus

$$\phi_q(\mathbf{x}) = (1/\sqrt{N})e^{iQx} + \psi_q(\mathbf{x})$$

and (14) becomes

$$|\chi(\mathbf{K})\rangle = \frac{1}{\sqrt{N}} \sum_i \sum_j e^{i[K-\sum q_i x_i]c_{ij}^\dagger} \times \prod_q \left( c_{q_j}^\dagger + e^{iQx_j} \sum_j \psi_q(x_j) c_{q_j}^\dagger \right) |0\rangle.$$  (19)

On expanding the product, and retaining only terms up to first order in $\psi_q$, we obtain

$$|\chi(\mathbf{K})\rangle = \frac{1}{\sqrt{N}} \sum_i \sum_q \rho(q) e^{iQx_i} \times \prod_j \psi_q(x_j) c_{q_j}^\dagger |FS(N_1)\rangle$$  (20)

where the factor $\rho(q)$ takes values ±1. Here we have taken $K$ relative to the total wavevector of the FS, as in (6). On making a plane-wave expansion of $\psi_q$, we find

$$|\chi(\mathbf{K})\rangle = \left( c_{K\downarrow}^\dagger + \sum_{k, q} \alpha_{kq} e^{ikx_\downarrow} c_{k+q\downarrow}^\dagger c_{k\downarrow}^\dagger \right) |FS(N_1)\rangle$$  (21)

as required. In general, by expanding the product in (19) to order $n$ in $\psi_q$, we obtain a wavefunction of the form $|P_{2n+1}(\mathbf{K})\rangle$. However all coefficients in that general form are now expressed in terms of the comparatively small number of quantities $\alpha_{kq}$.

To obtain wavefunctions of the form $|M_{2n}(\mathbf{K})\rangle$ given by (9) from the ansatz of equation (14) we take one of the orbitals $\phi_a(x)$ to be a localized function representing a ‘molecule’. The remaining orbitals are treated as perturbed plane waves, just as before. Writing the localized orbital as

$$\phi(x) = (1/\sqrt{N}) \sum_k \xi_k e^{ikx},$$

and using (21) for the perturbation to the plane waves, we obtain a first-order expression having the form of $|M_4(\mathbf{K})\rangle$ defined by the first two terms on the right-hand side of (9). The coefficients in the second term are found to be given by $\xi_{k_1} \xi_{q_1} = \xi_k \alpha_{k, q_1}$.

We have shown that the ansatz $|\chi(\mathbf{K})\rangle$ given by (14) encompasses approximate wavefunctions of both the ‘polaron’ and ‘molecule’ type introduced by other authors. This emphasizes the point made in section 2 that these two types of state are not physically distinct and merge continuously into each other.

5. Conclusion

We consider a system with a single ↓ spin particle interacting with a Fermi sea of effectively non-interacting ↑ spin particles. It is shown that the transition from ‘polaron’ to ‘molecule’ behaviour is a smooth crossover, not a sharp transition. Consequently the ↓ spin quasiparticle remains fermionic, with non-zero quasiparticle weight, even for large interaction strength where it also behaves as part of a molecule.

This conclusion has been reached for the case when only a single ↓ spin particle is involved in interactions. The problem of a finite number $N_\downarrow$ of ↓ spin particles,
in particular the balanced spin case $N_\downarrow = N_\uparrow$, is the much more complex one of the full attractive Hubbard model. The considerations of this paper throw no light on the nature of the transition between the Bardeen–Cooper–Schrieffer (BCS) and the Bose–Einstein-condensation (BEC) limits of the superconducting state in this model.

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References

[1] Hubbard J 1963 Proc. R. Soc. A 276 238
[2] McGuire J B 1965 J. Math. Phys. 6 432
[3] McGuire J B 1966 J. Math. Phys. 7 123
[4] Lieb E and Wu F Y 1968 Phys. Rev. Lett. 20 1445
[5] Edwards D M 1990 Prog. Theor. Phys. Suppl. 101 453
[6] Castella H and Zotos X 1993 Phys. Rev. B 47 16186
[7] Nagaoka Y 1966 Phys. Rev. 147 392
[8] Roth L 1969 Phys. Rev. 186 428
[9] von der Linden W and Edwards D M 1991 J. Phys.: Condens. Matter 3 4917
[10] Wurth P, Uhrig G S and Müller-Hartmann E 1996 Ann. Phys., Lpz. 5 148
[11] Anderson P W 1990 Phys. Rev. Lett. 64 1839
[12] Anderson P W 1990 Phys. Rev. Lett. 65 2306
[13] Sorella S 1994 Phys. Rev. B 49 12373
[14] McGuire J B 1965 J. Math. Phys. 6 432
[15] von der Linden W and Edwards D M 1991 J. Phys.: Condens. Matter 3 4917
[16] McGuire J B 1966 J. Math. Phys. 7 123
[17] Chevy F and Mora C 2010 Rep. Prog. Phys. 73 112401
[18] Prokof’ev N and Svistunov B 2008 Phys. Rev. B 77 020408(R)
[19] Prokof’ev N and Svistunov B 2008 Phys. Rev. B 77 125101
[20] Punk M, Dumitrescu P T and Zwerger W 2009 Phys. Rev. A 80 053605
[21] Bruun G M and Massignan P 2010 Phys. Rev. Lett. 105 020403
[22] Trefzger C and Castin Y 2012 Phys. Rev. A 85 053612
[23] Parish M M 2011 Phys. Rev. A 83 051603(R)
[24] Parish M M and Levinsen J 2013 Phys. Rev. A 87 033616
[25] Zöllner S, Bruun G M and Pethick C J 2011 Phys. Rev. A 83 021603(R)
[26] Combescot R and Giraud S 2008 Phys. Rev. Lett. 101 050404
[27] Chevy F 2006 Phys. Rev. A 74 063628
[28] Combesco R, Recati A, Lobo C and Chevy F 2007 Phys. Rev. Lett. 98 180402
[29] Tan B W 1975 PhD Thesis London
[30] Kohn W and Majumdar C 1965 Phys. Rev. 138 A1617