Sensor networks

Quantum Mechanical Approach to Modeling Reliability of Sensor Reports

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Abstract—Dempster-Shafer (D-S) evidence theory is widely applied in multi-sensor data fusion. However, lots of uncertainty and interference exist in practical situations, especially on the battlefield. It is still an open issue to model the reliability of sensor reports. Many existing methods are proposed based on the relationship among collected data. In this letter, we proposed a quantum mechanical approach to evaluate the reliability of sensor reports, which is based on the properties of a sensor itself. The proposed method is used to modify the combining of evidences. A numerical example is used to show the effectiveness and efficiency of our method.

Index Terms—Sensor networks, sensor report reliability, quantum mechanical approach, Dempster-Shafer (D-S) evidence theory, multi-sensor data fusion.

I. INTRODUCTION

Data fusion has been widely studied in the past few decades, especially its military applications. Multi-sensor data fusion (MSDF) technology plays an increasingly significant role during field military maneuvers. How to fuse the sensor data is still an open issue [1]–[4]. Due to the powerful ability of handling uncertain information, Dempster-Shafer (D-S) evidence theory is widely used in MSDF [5]–[8]. However, much interference exists in the complex practical situation. The information provided by a sensor report is likely to be disturbed and incorrect. In this case, strong conflicts may exist among evidences leading to a wrong fusion result. Handling conflicts is crucial in data fusion [9]–[13]. To address it, many approaches have been proposed [14]–[16].

To deal with conflictive information, most existing methods handle evidences based on the relationship among the data collected by sensors [17]–[20]. In this letter, however, a method which bases on the properties of a sensor itself is proposed. To evaluate the reliability of sensor reports, a confidence coefficient curve is determined based on the relationship among the data collected by sensors [17]–[20]. In this letter, we proposed a quantum mechanical approach to evaluate the reliability of sensor reports, which is based on the properties of a sensor itself. The proposed method is used to modify the combining of evidences. A numerical example is used to show the effectiveness and efficiency of our method.

II. PRELIMINARIES

D-S evidence theory was proposed by Dempster in 1967 [25] and modified by Shafer in 1978 [26]. In evidence theory, the basic set \( \Theta \), called the frame of discernment, consists of a set of \( N \) mutually exclusive and exhaustive hypotheses, symbolized by \( \Theta = \{X_1, X_2, \ldots, X_N\} \). Let \( P(\Theta) \) denote the power set composed of \( 2^N \) elements of \( \Theta \):

\[
P(\Theta) = \{\emptyset, \{X_1\}, \{X_2\}, \ldots, \{X_N\}, \ldots, \{X_1 \cup X_2\}, \{X_1 \cup X_3\}, \ldots, \Theta\}.
\]

The basic probability assignment (BPA) is a mapping from \( P(\Theta) \) to \([0, 1]\), defined by

\[
m : P(\Theta) \rightarrow [0, 1],
\]

satisfying the following conditions:

\[
\sum_{A \in 2^N} m(A) = 1,
\]

\[
m(\emptyset) = 0
\]

where the mass function \( m \) represents a supporting degree to the element \( A \). The elements of \( P(\Theta) \) that have a non-zero mass are called focal elements. A body of evidence (BOE) is the set of all the focal elements [27]:

\[
(R, m) = \left\{ [A, m(A)] : A \in P(\Theta), m(A) > 0 \right\}
\]

Fig. 1. Particle location in classical and quantum mechanics.
where \( R \) is a subset of \( P(\Theta) \), and each of \( A \in P(\Theta) \) has a fixed value. The classical Dempster’s combining rule of two BPAs \( m_1 \) and \( m_2 \) is defined as follows:

\[
m(A) = \frac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1 - K}
\]

(4)

where \( A, B, \) and \( C \) are the elements, and \( K \) is called the conflict coefficient measuring the conflict between two BPAs:

\[
K = \sum_{B \cap C = \emptyset} m_1(B) m_2(C).
\]

(5)

### III. QUANTUM MECHANICAL MODELLING OF THE SENSOR RELIABILITY IN DATA FUSION

Radar plays an important role in the modern battlefield. Usually, to obtain the overall information, data from several radars need to be fused. The comprehensive process of data fusion based on D-S evidence theory is shown as Fig. 2.

Aiming to do a more reasonable fusion, we propose a method based on quantum mechanics to determine the confidence coefficient curve of radar sensor reports. We assume that the reliability of sensor reports relates to the distance between an object and sensors in some degree. For each distance \( x \), the sensor has an according confidence coefficient whose maximum value is 1. Hence, the confidence coefficient curve \( \mu(x) \) is defined as a function to describe this relation.

For a radar detection system, \( P_t \) denotes the transmit power of the object; \( G_t \) denotes the antenna gain of the object; \( G_r \) denotes the antenna gain of the reconnaissance radar; \( x \) denotes the distance between the object and a radar; and \( P_r \) denotes the signal power received by a radar:

\[
P_r = \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi x)^2}
\]

(6)

where \( \lambda \) is the wavelength and \( \sigma \) is the radar cross-section which is the product of the geometric cross-section, the reflection coefficient and the direction coefficient. The sensitivity of a radar is denoted as \( P_{\text{min}} \). Then, the maximal reconnaissance distance \( x_r \) is calculated as

\[
x_r = \left[ \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^2 P_{\text{min}}} \right]^{\frac{1}{2}}.
\]

(7)

If an object is far beyond this distance, it will not be effectively reconnoitred. According to the quantum-mechanical rules of quantification, we should write an operator \( H \) which corresponds to the received signal power

\[
H = -c^2 \frac{\partial^2}{\partial x^2} - V(x)
\]

(8)

where \( c \) is a scale factor, and \( V(x) \) is a quasi-potential function to model the received signal power

\[
V(x) = \begin{cases} 
\frac{\gamma}{x} & 0 < x \leq x_r \\
\infty & x \leq 0, x > x_r 
\end{cases}
\]

(9)

where \( \gamma \propto \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^2} \) corresponds to the parameters in (6). The quasi-potential function \( V(x) \) is roughly illustrated as Fig. 3.

Based on quantum-mechanical rules, a quasi time-independent Schrodinger equation can be derived as

\[
H \psi(x) = L \psi(x)
\]

(10)

where \( \psi(x) \) is a quasi-amplitude distribution and \( L \propto -P_{\text{min}}(d B m W) \), where \( P_{\text{min}}(d B m W) \) is the decibels of the radar sensitivity relative to \( 1 m W \)

\[
P_{\text{min}}(d B m W) = 10 \log \left( \frac{P_{\text{min}}(m W)}{10^{-3}} \right) dB.
\]

(11)

When the object is within the maximal reconnaissance distance \( x_r \), the solution of (10) is as follows:

\[
\psi(x) \propto \sqrt{x} \left[ J_{\alpha} \left( \frac{\sqrt{T}}{c} \right) + Y_{\alpha} \left( \frac{\sqrt{T}}{c} \right) \right]
\]

(12)

where \( J_{\alpha} \) and \( Y_{\alpha} \) are Bessel functions of the first kind and the second kind, respectively [28], and \( \alpha \) is their order:

\[
\alpha = \frac{1}{2} \sqrt{c^2 - 4 \gamma}
\]

(13)

where \( c \) and \( \gamma \) are the parameters in (8) and (9), respectively. Then, let us consider the other situation, when the object is beyond \( x_r \), the value of \( V(x) \) is infinite. According to quantum mechanics, it is impossible for a particle to penetrate the well wall if it is within an infinite well potential. Hence, we can conclude that \( \psi(x) = 0 \) in this case. Then, the probability distribution \( P(x) \) can be derived as

\[
P(x) = |\psi(x)|^2 \propto x \left[ J_{\alpha} \left( \frac{\sqrt{T}}{c} \right) + Y_{\alpha} \left( \frac{\sqrt{T}}{c} \right) \right]^2
\]

(14)

which is illustrated graphically in Fig. 4(a).

By amplifying (14), the confidence coefficient curve \( \mu(x) \) can be derived. As can be seen from Fig. 4(b), the curve rises rapidly when \( x \) is smaller than \( x_0 \) and comes to its maximum at \( x_0 \). Then it declines
slowly until \( x \) comes to \( x_r \), which is reasonable. In a practical situation, due to the precision and some other intricate issues, a radar does not work well when it is too close to the object. There exists an optimal distance \( x_0 \) for a radar to work. Then, the performance of a radar becomes poorer as it is located further. When the distance is further than the maximal reconnaissance distance \( x_r \), the object cannot be reconnoitred effectively. With the basis of this curve, we can evaluate the reliability of radar reports effectively. For different types of radars, their according confidence coefficient curves can be obtained as Fig. 5. The parameters of these curves are shown in Table I. Generally, \( c \) is used to adjust the width of a curve. For a fixed \( c \), a larger product of \( L \) and \( r \) corresponds to a larger \( x_r \). The optimal distance \( x_0 \) has a nonlinear relation to parameters \( L \) and \( r \). Based on the above, the curves describing the sensor reliabilities with the basis of the properties of sensors themselves are obtained.

In the following, the curves are used in combining evidences. Assume that we have \( k \) pieces of BPA\( s: m_1, m_2, \ldots, m_k \), collected from \( k \) radar sensors. According to the location and confidence coefficient curve of a sensor, each evidence corresponds to one confidence coefficient: \( \mu_1, \mu_2, \ldots, \mu_k \). The credibility degree \( \text{Crd}_i \) of \( m_i \) is defined as

\[
\text{Crd}_i = \frac{\mu_i}{\sum_{i=1}^{k} \mu_i}, \quad (15)
\]

It is easy to find that \( \sum_{i=1}^{k} \text{Crd}_i = 1 \). Hence, the credibility degree reveals the relative importance of the collected evidence. After determining the credibility of each evidence, we do a modified average for all \( k \) pieces of evidences to obtain a new evidence \( m' \):

\[
m' = \sum_{i=1}^{k} \text{Crd}_i \times m_i. \quad (16)
\]

Then, we can use the classical combining rule (4) to combine \( m' \) for \( k - 1 \) times, which is the same as Murphy’s approach [19]. Obviously, if a BOE is collected from a sensor with relatively high reliability, it will have more effect on the final combination results. On the contrary, if a BOE is collected from a sensor with relatively low reliability, it will matter less in the final combination results. In sum, the flow chart of our method is shown as Fig. 6.

### IV. NUMERICAL EXAMPLE

In this section, a numerical example is illustrated to show the effectiveness of our method. In a target recognition system, the spacial locations are illustrated in Fig. 7. As the object is closer to the radars, radar sensors have collected five pieces of BOEs for 5 times shown as follows (to simplify, assume the collected BOEs keep the same for each time):

\[
(R_1, m_1) = [[(A), 0.6], [(B), 0.15], [(A, C), 0.25]]
\]
\[
(R_2, m_2) = [[(A), 0.5], [(B), 0.3], [(C), 0.2]]
\]
\[
(R_3, m_3) = [[(B), 0.95], [(C), 0.05]]
\]
\[
(R_4, m_4) = [[(A), 0.55], [(B), 0.25], [(A, C), 0.2]]
\]
\[
(R_5, m_5) = [[(A), 0.6], [(B), 0.3], [(B, C), 0.1]]
\]
TABLE 2.  Confidence Coefficients of Radars at Different Time.

|       | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|-------|-----|-----|-----|-----|-----|
| Radar 1 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| Radar 2 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| Radar 3 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| Radar 4 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| Radar 5 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |

TABLE 3. Fusion Results and Comparison.

|       | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|-------|-----|-----|-----|-----|-----|
| Our method at t1 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| Our method at t2 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| Our method at t3 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| Our method at t4 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| Our method at t5 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| Classical rule | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
| Murphy’s approach | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |

Four evidences prefer to recognizing the target as A. Hence, data from radar 3 is probable to be interfered and incorrect. The reliabilities of these sensor reports are shown in Table II five times, which are obtained based on their confidence coefficient curves.

As can be seen from Table III, the classical rule will lead to a totally incorrect result as strong conflict exists (m3(A) = 0). The relation among the data of evidences is considered in Murphy’s approach. Each evidence is deemed to weight averagely, which modifies the fusion results heavily. However, by handling the evidences in a new framework based on the properties of a sensor itself, we do a modified average for the evidences. The fusion results of our method are even better during the whole process that the object flies closer to radars. Especially, the earlier the BOEs are collected, the better the result is, which matters a lot as it is always better for us to detect the object earlier in the practical. Hence, the target can be effectively recognized with our method. Admittedly, in some cases (like radar 2 is hugely disturbed), the fusion result will be impacted negatively in our method. To address it, we will consider both the sensor properties and the relation among the collected data (like the evidence differing a lot will be weighted less) to evaluate the reliability in later works.

V. CONCLUSION

In summary, we propose a new method to model the reliability of sensor reports. Unlike existing methods, we focus on the properties of a sensor itself. The confidence coefficient curve of a radar sensor is derived by solving a quasi time-independent Schrodinger equation. The method is used in combining of evidences. The result shows the efficiency of our method. In later works, we will focus on the combination of the new method and existing methods.

ACKNOWLEDGMENT

This work was supported in part by the National Natural Science Foundation of China under Grant 61671384, in part by the Natural Science Basic Research Plan in Shaanxi Province of China under Grant 2016JM6018, and in part by the Aviation Science Foundation under Grant 20165553036.

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