Shedding light on the small-scale crisis with CMB spectral distortions

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The small-scale crisis, discrepancies between observations and N-body simulations, may imply suppressed matter fluctuations on subgalactic distance scales. Such a suppression could be caused by some early-universe mechanism (e.g., broken scale-invariance during inflation), leading to a modification of the primordial power spectrum at the onset of the radiation-dominating era. Alternatively, it may be due to nontrivial dark-matter properties (e.g., new dark-matter interactions or warm dark matter) that affect the matter power spectrum at late times, during radiation domination, after the perturbations re-enter the horizon. We show that early- and late-time suppression mechanisms can be distinguished by measurement of the distortion to the frequency spectrum of the cosmic microwave background. This is because the distortion is suppressed, if the power suppression is primordial, relative to the value expected from the dissipation of standard nearly-scale-invariant fluctuations. We emphasize that the standard prediction of the distortion remains unchanged in late-time scenarios even if the dark-matter effects occur before or during the era (redshifts $5 \times 10^4 \lesssim z \lesssim 2 \times 10^9$) at which distortions are generated.

I. INTRODUCTION

The canonical ΛCDM model of cosmic structure formation, in which structures grow from a nearly scale-invariant spectrum of primordial adiabatic perturbations, has achieved many successes. Still, there are discrepancies between observations on subgalactic scales and predictions from N-body simulations of structure and galaxy formation. We refer to these discrepancies collectively as the “small-scale crisis” (see Ref. 1 for a review), which includes the missing-satellite problem 2, the cusps-core problem 3, and the too-big-to-fail problem 4. While further elucidation of the relevant baryonic physics may help solve these problems 5–7, the small-scale crisis may also imply a suppression of density fluctuations below or around subgalactic scales 1.

Exotic mechanisms to suppress small-scale power can be classified into those that involve modification of the primordial power in the early Universe and those that suppress power at later times. Examples of primordial suppression are discussed in Refs. 8–12, and predict that subgalactic primordial adiabatic perturbations are suppressed at the onset of the radiation-dominated era. Examples of late-time suppression are those discussed, e.g., in Refs. 13–20; in such scenarios, subgalactic matter perturbations are present at the onset of the radiation-dominated era but then suppressed at later times, after the relevant scales re-enter the horizon. These mechanisms include free streaming of dark-matter (DM) particles (e.g., as in warm dark matter, WDM) or interactions between DM and standard-model or hidden particles.

There has been exploration of different astrophysical consequences of suppressed small-scale power, and constraints to models are already being derived. For example, WDM provides a particularly well-studied example of a late-time-suppression mechanism, and some recent work 21–28 suggests tensions between WDM solutions to the small-scale crisis and observations. Even if a WDM-only scenario for resolving the small-scale crisis is in tension with Lyman-α observations, a mixture of cold DM and WDM (mixed dark matter) can still evade Lyman-α constraints while helping to solve problems associated with the small-scale crisis 29–32. There is thus still good reason to consider solutions to the small-scale crisis based on power suppression.

For a primordial suppression, the shape of the suppressed spectrum on small scales depends on unknown and largely unconstrained early-universe physics. In the framework of single-field inflation, for instance, it is determined by the slope of the inflaton potential 5. Hence, even though a wide variety of late-time suppression mechanisms that predict different shapes for a suppressed power spectrum have been proposed, measurement of the shape of the late-time matter power spectrum on small scales cannot really distinguish whether the suppression is primordial or late-time.

Here we point out that primordial and late-time suppression mechanisms can be distinguished by the cosmic microwave background (CMB) distortion 33–38. Such distortions are produced by heating of the primordial plasma from dissipation of small-wavelength fluctuations. The Fourier modes that contribute to the distortions have comoving wavenumbers $50 \text{ Mpc} \lesssim k \lesssim 10^4 \text{ Mpc}$, and the distortion arises when these dissipate at redshifts $5 \times 10^4 \lesssim z \lesssim 2 \times 10^9$ (the $\mu$ era). In the standard scenario, where the nearly-scale-invariant spectrum of perturbations seen in the CMB 44 is extrapolated to smaller scales (as motivated by Occam’s razor and the simplest models of inflation), the induced distortion is $\mu \simeq 2 \times 10^{-5}$ 38–42.

For primordial suppression, the value of $\mu$ will be reduced relative to that expected from the standard almost-scale-invariant spectrum. If the suppression is strong enough, the $\mu$ parameter could even take a neg-
active value, \( \mu_{\text{BE}} \simeq -3 \times 10^{-9} \), due to the continuous extraction of energy from CMB photons by the nonrelativistic baryons to which they are coupled.

In contrast, for late-time suppression of small-scale perturbations, there are still primordial perturbations to be dissipated, and so the standard prediction is unmodified. The argument is not entirely trivial, as in many late-time scenarios, the suppression of the matter power spectrum occurs during the \( \mu \) era. For example, in the charged-particle-decay scenario, suppression of the matter power spectrum occurs until roughly 3.5 years after the Big Bang, at redshifts \( z \simeq 5 \times 10^5 \), right when the \( \mu \) distortion is being produced. This timescale is actually fairly generic, as this is the redshift at which subgalactic scales are entering the horizon. The crucial point is that the matter density is negligible compared with the radiation density during the \( \mu \) era. There can thus be dramatic smoothing of the matter distribution with little effect on the radiation-density perturbations. Similar arguments apply to the \( y \) distortion, which is created later at \( z \lesssim 5 \times 10^4 \). Here in addition, distortions are sourced by bulk flows at second order in the baryon velocity, \( v \). However, these contributions are subdominant relative to larger energy release from first stars and structure formation, as also recently discussed in Ref. \[50\]. For the same reasons the effects of primordial dark-matter isocurvature perturbations on spectral distortions are limited \[51\].

In the next Section, we calculate the value for \( \mu \) assuming a step-type suppression of primordial power below subgalactic scales, and we conclude in Section III.

II. PRIMORDIAL SUPPRESSION AND CMB \( \mu \) DISTORTION

We relate the primordial power suppression to the \( \mu \) distortion as follows. We employ the following description of primordial suppression in terms of the dimensionless primordial curvature power spectrum,

\[
P(k) = \mathcal{P}_s(k) \tilde{P}(k), \quad \mathcal{P}_s(k) = A \left( \frac{k}{k_p} \right)^n, \quad \tilde{P}(k) = \frac{1 + 10^{-\alpha}}{2} - \frac{1 - 10^{-\alpha}}{2} \tanh \left( \log \frac{k}{k_s} \right).
\]

That is, the power is suppressed by \( 10^{-\alpha} \) for \( k \gtrsim k_s \), relative to the standard spectrum \( \mathcal{P}_{s} \) with parameters \( A = 2.2 \times 10^{-9} \), \( k_p = 0.05 \text{Mpc}^{-1} \) and \( n_s = 0.97 \). Examples of the suppressed primordial spectra are shown in Fig. \[4\], where we take \( k_s = 1 \text{Mpc}^{-1}, 20 \text{Mpc}^{-1} \) and \( 35 \text{Mpc}^{-1} \), relevant for small-scale problems. The above step-type suppression would lead to a step-type suppression of the matter spectrum at low redshifts, similarly to mixed-DM scenarios \[29, 30\]. Hence, we can refer to those studies to gain insight into structure formation for the suppressed primordial spectrum we consider here. However, establishing a precise link between our spectrum and different aspects of the small-scale crisis is beyond the scope of this work, at least because of potentially important baryonic processes. Thus, we treat \( \alpha \) and \( k_s \) as free parameters and only illustrate that \( \mu \) can be significantly smaller than the expected standard value, \( \mu \approx 2 \times 10^{-8} \).

Additional information about the precise position of the transition scale might be accessible with future measurements of the exact spectral-distortion shape \[38, 40, 42\]. However, even if we were only to observe a significant suppression of \( \mu \), without additional information about the spectral-distortion shape, we could connect the small-scale crisis to primordial suppression. Ultimately, it will be instructive to investigate small-scale problems with simulation of structure formation with the various primordial spectra which are consistent with, e.g., simultaneous constraints from the Lyman-\( \alpha \) forest and \( \mu \) (and possibly taking into account baryonic processes).

The \( \mu \) distortion can be estimated \[52\] as \( \mu = \mu_{\text{ac}} + \mu_{\text{BE}} \), with \( \mu_{\text{BE}} \simeq -3 \times 10^{-9} \) and

\[
\mu_{\text{ac}} \simeq \int_{k_{\text{min}}}^{\infty} \frac{dk}{k} \mathcal{P}(k) W_{\mu}(k), \quad (2)
\]

with

\[
W_{\mu}(k) \simeq 2.8 A^2 \left[ \exp \left( -\frac{[\hat{k}/1360]^2}{1 + [\hat{k}/260]^{0.3} + \hat{k}/340} \right) - \exp \left( -\left[ \frac{\hat{k}}{32} \right]^2 \right) \right], \quad (3)
\]

where \( k_{\text{min}} \simeq 1 \text{Mpc}^{-1}, A \simeq 0.9 \) and \( \hat{k} = k \text{Mpc} \). This approximation is accurate at the \( \lesssim 20\% \) level and slightly underestimates the recovered value for \( \mu \) \[47\]. Hence, we renormalize the above window function \( W_{\mu} \) so that \( \mu_{\text{ac}} \simeq 2.3 \times 10^{-8} \) when \( \alpha = 0 \) (i.e. standard fluctuations), which is sufficient for our purposes.

The values of \( \mu \) as a function of \( \alpha \) are shown in Fig. \[2\]. When \( k_s \) is close to \( \sim 1 \text{Mpc}^{-1} \) and \( \alpha \) is sufficiently large, \( \mu \) becomes negative, approaching \( \mu_{\text{BE}} \). For \( k_s \simeq 35 \text{Mpc}^{-1} \), the asymptotic value is \( \mu \approx 0 \): that is, the energy injection due to the dissipation of sound waves and energy extraction due to interactions between photons and baryons are roughly balanced. If in the future \( \mu \) is constrained to be smaller than what is expected \( \mu \approx \mu_{\text{ac}} \simeq 2 \times 10^{-8} \), from the dissipation of the standard fluctuations, (in the Figure this corresponds to the

\[2\] Here we assume no other energy injection mechanisms in the early Universe exist, such as evaporating primordial black holes, decaying/annihilation particles, cosmic strings, primordial magnetic fields and axion-like particles (see Ref. \[42\] for an overview).
the small-scale crisis of ΛCDM may imply suppressed matter fluctuations on subgalactic scales. Such a suppression could result from some new physics that operates during inflation or could be the consequence of new dark-matter physics that operates at later times, after the relevant distance scales re-enter the horizon during radiation domination. Although the primordial and late-time suppression mechanisms are expected to impact structure formation in a similar fashion, we show here that they could be in principle distinguished by measurement of the μ distortion to the CMB frequency spectrum. This is because μ may be significantly reduced relative to the canonical value μ ≃ 2 × 10⁻⁸ if subgalactic power suppression is primordial. For power suppression sufficiently significant, μ could even become negative as a consequence of the transfer of energy from photons to baryons. On the other hand, for a late-time suppression, the CMB μ distortion would not be affected notably since it is mostly determined by primordial fluctuations rather than subhorizon dynamics of DM fluctuations during the radiation-dominated era. Thus, for a late-time suppression, μ is not expected to differ significantly from the standard positive value.

If μ is found to be unexpectedly small or negative by future high-sensitivity experiments measuring the energy spectrum of CMB photons, it may serve as a smoking gun for a primordial suppression. Note also that the negative contribution to μ can, in principle, be even smaller than μBE due to direct or indirect thermal coupling of non-relativistic DM with photons, since in this case more energy is extracted from photons to DM to maintain thermal equilibrium. If on the other hand the standard prediction for μ is verified, then it suggests that the small-scale crisis has to do with late-time physics. If we find μ to have the standard value, then another possibility, which we leave for future work, is that a matter-radiation isocurvature perturbation, correlated with the adiabatic perturbation, suppressed matter perturbations on small scales while preserving the primordial curvature (and thus radiation) perturbation on small scales.

In this paper, we emphasized that μ can be small for the primordial suppression scenario. However, ultimately it will be interesting to study the small-scale problems by N-body simulations for a variety of primordial spectra consistent with existing constraints from, e.g., Lyman-α observation, simultaneously calculating μ for each spectrum, possibly taking into account baryonic processes.

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