Fundamental study on the optimal path of centrifugal pendulum vibration absorbers in automatic transmissions

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Abstract. In recent years, high-power and low-cylinder engines have seen increasingly widespread use. These engines produce a strong torsional vibration due to engine combustion. Furthermore, automatic transmissions necessitate the use of a lock-up clutch system that directly links the engine and gear train when the engine rotational speed is low. Using a torsional spring called a lock-up damper in the torque converter is not sufficient to absorb the torsional vibration. Therefore, the centrifugal pendulum vibration absorber (CPVA) has been developed to suppress the torsional vibration. The natural frequency of a CPVA is proportional to the engine rotational speed, and it is expected to suppress the torsional vibration over a wide range of speeds. However, the natural frequency of the CPVA varies because of the nonlinearity when the vibration amplitude of the CPVA is large. In this study, the optimal path of the CPVA to suppress the torsional vibration of the engine was investigated.

1. Introduction
Automatic vehicle transmissions commonly use a torque converter to transmit the engine torque. When the difference between the rotational speeds at the input and output sides of the torque converter is small, a lock-up clutch begins to operate. With the operation of the lock-up clutch, the engine torque is transmitted directly to the gear train, which worsens the ride quality because of fluctuations in the torque due to engine combustion. To reduce the torsional vibration, a torsional damper called a lock-up damper can be installed behind the lock-up clutch. However, the torsional vibration cannot be sufficiently suppressed by this type of damper.

To address these problems, a centrifugal pendulum vibration absorber (CPVA) is often attached in the torque converter. The CPVA is a tuneable absorber that can tune its natural frequency according to the rotational speed. CPVAs are widely used for a long time. The natural frequency of a CPVA can be controlled by changing the path of the mass. Some researchers have examined the effect of the path on the vibration suppression performance [1]–[4]. However, the optimal path to suppress the torsional vibration has not yet been examined. In this study, the optimal path to suppress torsional vibration for automatic transmissions was designed using a genetic algorithm (GA).
2. Theoretical analysis

2.1. Analytical model

Figure 1 shows the analytical model of the automatic transmission power train, which is composed of an engine, a torque converter, an automatic transmission gear train, and a drive shaft. The power train is modelled as a four-degree-of-freedom system with inertias $J_1,\ldots,J_6$, torsional spring constants $K_1,\ldots,K_4$, and torsional damping constants $C_1,\ldots,C_4$. The angular displacements are given by $\theta_1,\ldots,\theta_4$. The fluctuation of the engine torque is described by $T \cos \omega t$ with amplitude $T$ and frequency $\omega$. The frequency is given by

$$\omega = \frac{N \Omega}{2},$$

where $N$ is the number of cylinders and $\Omega$ is the speed of rotation of the engine.

To suppress the torsional vibration, the CPVA is attached at point A, as shown in Figure 1. Modal analysis was applied to the vibration model in Figure 1, and the vibration mode was defined as a one-degree-of-freedom system for the target to suppress the torsional vibration. Figure 2(a) shows the analytical model applied to consider the suppressive effect of the CPVA. $J$ and $K$ are the modal mass (inertia) and the modal stiffness, respectively. The damping coefficient $C$ was defined to yield a modal damping ratio equal to that of the system in Figure 1. Figure 2(b) shows a CPVA with mass $m$ and length $r$ attached to the main system. The distance between the centre of rotation $O$ and the position $P$ at which the CPVA is attached is $R$.

![Figure 1. Analytical model.](image1)

![Figure 2. Centrifugal pendulum vibration absorber.](image2)
2.2. Equations of motion

The equations of motion of the system shown in Figure 2 are given by

\[
\begin{align*}
\{ J + m \left( R^2 + r^2 + 2Rr \cos \phi \right) \} \ddot{\theta} + m \left( r^2 + Rr \cos \phi \right) \dot{\phi} \\
- mRr \left( 2 \dot{\theta} + 2\omega + \dot{\phi} \right) \sin \phi + C \dot{\phi} + K \theta = T_a \cos \omega t \\
m \left( r^2 + Rr \cos \phi \right) \dddot{\phi} + mrr^2 \ddot{\phi} + mRr (\Omega + \dot{\theta})^2 \sin \phi = 0
\end{align*}
\]

(2)

where \( \theta \) is angular displacement from the steady rotation; \( \phi \) is the angular displacement of the CPVA in the rotational coordinate frame \( O - \xi\eta \), which is fixed to the main system; and \( T_a \) is the torque amplitude after the application of the modal analysis. The damping of the CPVA is neglected. In this case, the path of the CPVA is circular. Then the length of the pendulum \( r \) is constant of \( r_p \).

The equation of motion when only the CPVA is considered is given by

\[
m r^2 \ddot{\phi} + mRr \Omega^2 \sin \phi = 0.
\]

(3)

When the amplitude of \( \phi \) is small, the linearized natural frequency is given by

\[
\omega_p = \sqrt{\frac{R}{r} \Omega} = n \Omega.
\]

(4)

Equation (4) shows that the natural frequency of the CPVA is proportional to the rotational speed \( \Omega \). If the order \( n \) is set to \( N/2 \), the CPVA can suppress the torsional vibration. However, when the amplitude \( \phi \) is large, the natural frequency of the CPVA cannot maintain a constant value, worsening the suppressive effect.

To address this problem, the optimal path of the CPVA is examined. The path \( r \) is designed by the following polynomial equation of \( \phi \):

\[
r(\phi) = r_p \left( 1 + r_2 \phi^2 + r_4 \phi^4 + r_6 \phi^6 \right).
\]

(5)

The equations of motion are given by

\[
\begin{align*}
\left( J + mR^2 + mr^2 + 2mRr \cos \phi \right) \ddot{\theta} + (mr^2 + mRr \cos \phi + mR_{\phi} \sin \phi) \dot{\phi} \\
+ 2mrr_{\phi} (\Omega + \dot{\theta} + \dot{\phi} + r_{\phi} \cos \phi) + mR_{\phi}^{2} (r_{\phi} \sin \phi + r_{\phi} \cos \phi) + mR_{\phi} (2\Omega + 2\dot{\theta} + \dot{\phi})(r_{\phi} \cos \phi - r_{\phi} \sin \phi) \\
+ C \dot{\phi} + K \theta = T_a \cos \omega t \\
r^2 + Rr \cos \phi + Rr(\Omega - \dot{\theta})^2 + r_{\phi} \dot{\phi} + \dot{\phi} \ddot{\phi} + \frac{r_{\phi} r_{\phi} \ddot{\phi}^2}{r_{\phi}^2 - (\Omega + \dot{\theta})^2} + R(\Omega + \dot{\theta})^2 (r \sin \phi - r_{\phi} \cos \phi) = 0
\end{align*}
\]

(6)

where \( r = dr / dt \), \( r = d^2 r / dt^2 \).

The equation of motion when only the CPVA is considered is given by

\[
(r^2 + r_2^2) \ddot{\phi} + r_{\phi} \dot{\phi} \ddot{\phi} + r_{\phi} (\dot{\phi} - \Omega^2) + \Omega^2 (r \sin \phi - r_{\phi} \cos \phi) = 0,
\]

(7)

and the linearized natural frequency is given by

\[
\omega_p = \sqrt{\frac{R - 2r_2 (R + r_\phi)}{r_\phi}} \Omega.
\]

(8)

Equations (6) can be normalized as
(1 + μ_r + μ_r + 2μ_{Rr} cos φ)θ'' + (μ_r + μ_{Rr} cos φ + μ_{Rr} sin φ)θ'' + 2μ_{r1}θ'(z + θ' + φ')
+ φ''(μ_{Rr} sin φ + μ_{Rr} cos φ) + φ'(2z + 2θ' + φ')(μ_{Rr} sin φ) + θ + 2ζθ' = δ cos zτ
(μ_r + μ_{Rr} cos φ + μ_{Rr} sin φ)θ'' + (μ_r + μ_r)φ'' + μ_{r1}φ''
+ μ_{r2} \{(φ'' - (z + θ''))\} + (z + θ'')\{μ_{Rr} sin φ - μ_{Rr} cos φ\} = 0
\(\tag{9}\)

where
\[\mu_r = \frac{mR_r^2}{J}, \mu_r = \frac{mr^2}{J}, \mu_1 = \frac{mr_r^2}{J}, \mu_2 = \frac{mr_{Rr}^2}{J}, \mu_{r1} = \frac{mR_r}{J}, \mu_{r1} = \frac{mR_r}{J}, \mu_{r2} = \frac{mR_r}{J},\]
\[\mu_{r1} = \frac{mR_r}{J}, \mu_{r2} = \frac{mR_r}{J}, \delta = \frac{T_s}{K}, \zeta = \frac{C}{2\sqrt{JK}}, \omega_n^2 = \frac{K}{J}, z = \frac{\omega}{\omega_n}, \tau = \omega_l t\]

and the prime represents the derivative with respect to \(\tau\).

3. Numerical analysis

Table 1 gives the natural frequencies obtained from the vehicle model shown in figure 1. Table 1 also
gives the resonance rotational speeds of the three-cylinder engine \((N = 3)\). The target vibration mode
is the second mode. Table 2 lists the standard parameter values.
It is known that a CPVA with an epicycloidal path is tautochronic in a centrifugal field [5]. The
suppressing effect of such a CPVA was also examined here. In the calculation of the CPVA with an
epicycloidal path, the ratio between the base circles that form the epicycloidal path was defined such
that \(\omega_p = nΩ\).

To design the CPVA with the optimal path, a GA was used. The initial population was set to 100 with
16 bits, and the fitness function was defined as the reciprocal of the mean amplitude between 900 and
1400 rpm. Roulette wheel selection and two-point crossover were used to produce new generations.
Mutations were applied every five generations. The considered variables are \(r_r, r_s,\) and \(r_c\), and their
optimal values are given in table 3. \(r_z\) was set to zero to keep the linearized natural frequency in
Equation (8) constant.
The blue, green, and red lines in figure 3 show the characteristics of the free vibration of the CPVAs
with the circular, epicycloidal, and optimal paths, respectively. The abscissa represents the ratio of the
nonlinear natural frequency to \(\omega_p\) and the ordinate represents the pendulum amplitude. The natural
frequency of the CPVA with the epicycloidal path is independent of the amplitude of the pendulum.
The natural frequency of the CPVA with the optimal path is also close to an order of 1.5 when the
pendulum amplitude is small. The blue, green, and red lines in figure 4 show the frequency response
curves when CPVAs with the circular, epicycloidal, and optimal paths are attached, respectively. The
shooting method was used to calculate each frequency response curve. The frequency response curve
when the CPVA is not attached is also shown as a black line. The abscissa shows the rotational speed
of the engine, and the ordinate shows the peak-to-peak (pk-pk) amplitude of the angular displacement
of the main system. The amplitude was reduced relative to that of the baseline model by attaching the
CPVA with the circular path but was still large at low rotational speeds. The CPVA with the
epicycloidal path yielded greater amplitude reductions than the CPVA with the circular path at high
rotational speeds but very large amplitudes at low rotational speeds. Although the CPVA with the
epicycloidal path is tautochronic in a centrifugal field, the amplitude did not become zero. The CPVA
with the epicycloidal path is a linear system when the equation of motion is formulated as a function
of the arc length along the path. In this system, the equations of motion are nonlinearly coupled with
the angular displacements. This is the reason the amplitude of the main system does not become zero.
When the CPVA with the optimal path is attached, the amplitude of the main system is suppressed.
Figure 5 shows the circular, epicycloidal, and optimal paths. The ordinate in this plot is shown on a finer scale than the abscissa. Although the three paths are very similar, the precision of the path is important to efficiently suppress the torsional vibration.

Table 1. Natural frequencies and resonance rotating speed.

| Mode | 1st  | 2nd | 3rd | 4th |
|------|------|-----|-----|-----|
| Natural frequency [Hz] | 10   | 33  | 82  | 240 |
| Resonance rotating speed [rpm] | 400  | 1320| 3280| 9600|

Table 2. Standard parameters.

| Parameter | Value |
|-----------|-------|
| $\mu_1$   | 0.0561|
| $\mu_r$    | 0.0111|
| $\mu_{hr}$ | 0.0249|
| $\delta$  | 0.0159|
| $\zeta$    | 0.232 |
| $n$        | 1.5   |

Table 3. Optimal values.

| Parameter | Value |
|-----------|-------|
| $r_1$     | -0.02102|
| $r_2$     | -0.02962|
| $r_3$     | 0.03138 |

Figure 4. Frequency response curve.

4. Conclusions
In this study, the optimal path of a CPVA to suppress the torsional vibration caused by engine combustion was investigated. The optimal path was calculated using a GA. The frequency response curves of CPVAs with circular, epicycloidal, and optimal paths were evaluated. It was found that the CPVA with the optimal path can more effectively reduce the torsional vibration than the CPVAs with the circular and epicycloidal paths.

5. References
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