$J/\psi$ electromagnetic production associated with light hadrons at $B$ factories

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Abstract

The electromagnetic productions of $J/\psi$ associated with light hadrons (LH) and leptonic pairs ($\mu^+\mu^-, \tau^+\tau^-$) at $B$ factories are studied. We find that the direct electromagnetic production cross sections of $J/\psi$($\psi(2S)$) associated with light hadrons is about $0.10\pm0.04$ pb. The direct production cross sections of $J/\psi$($\psi(2S)$) associated with $\mu^+\mu^-$ is about $0.056\pm0.020$ pb. If we include the contributions from $\psi(2S)$ decay, we can get the prompt cross section $\sigma[e^+e^- \rightarrow J/\psi + \mu^+\mu^-] = (0.068 \pm 0.002)$ pb, about $(16\pm5)$\% of the Belle data $\sigma[e^+e^- \rightarrow J/\psi + X_{non-c\bar{c}}] = (0.43 \pm 0.09 \pm 0.09)$ pb, meanwhile the $e^+e^- \rightarrow J/\psi + \tau^+\tau^-$ process only contributes 2\%. The prompt cross section $\sigma[e^+e^- \rightarrow J/\psi + \text{Light Hadrons}] = (0.121 \pm 0.005)$ pb is about $(28\pm8)$\% of the Belle data.

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I. INTRODUCTION

Heavy quarkonium physics is an important ground to test quantum chromodynamics (QCD) both perturbatively and non-perturbatively. Non-relativistic quantum chromodynamics (NRQCD) factorization approach\[1\] has achieved a series of successes in describing heavy quarkonium production and annihilation decay. However, there are still some predictions which are less satisfactory. For more details, see a concise review\[2\]. Among the problematic comparisons with experiment, the large discrepancy between theoretical predictions and experimental data on the charmonium production in $e^+e^-$ annihilation has drawn much attention recently.

In recent years, the B factories has provided systematic measurements on charmonium production. Some results are puzzling, because of the large gap between the measurements and the theoretical predictions. For example, the cross section $\sigma(e^+ + e^- \rightarrow J/\psi + \eta_c)$, measured by Belle Collaboration\[3\] and Babar Collaboration\[4\], is almost one order-of-magnitude larger than the leading-order(LO) predictions\[5–7\]. By introducing the QCD perturbative correction\[8, 9\], and in combination with relativistic correction\[10, 11\], this discrepancy was largely resolved. Besides the challenges in the exclusive process, the large ratio of $J/\psi$ production associated with charmed hadrons is also confusing, which was measured by Belle\[3\]

$$R_{cc} = \frac{\sigma[e^+e^- \rightarrow J/\psi + c\bar{c}]}{\sigma[e^+e^- \rightarrow J/\psi + X]} = 0.59^{+0.15}_{-0.13} \pm 0.12. \quad (1)$$

In contrast to the leading-order NRQCD predictions, this ratio is only about 0.1\[12–14\]. The next-leading-order(NLO) QCD corrections were also introduced in $J/\psi$ inclusive production to resolve the discrepancy between experimental measurements and LO calculations. The NLO QCD corrections to $e^+e^- \rightarrow J/\psi c\bar{c}$ process enhance the cross section with a $K$ factor of about 1.8\[15, 16\], and only about 20 percent for $e^+e^- \rightarrow J/\psi gg$ process\[17, 18\]. So the discrepancy is greatly alleviated. To check what role does the color-octet process play, the NLO QCD corrections to color-octet $J/\psi$ inclusive production was calculated\[19\]. Combining the relativistic corrections to $e^+e^- \rightarrow J/\psi gg$\[20\], it may imply that the values of color-octet matrix elements are much smaller than the expected ones which are estimated by using the naive velocity scaling rules.

Another interesting topic is the $e^+e^- \rightarrow J/\psi + X_{\text{non-cc}}$ production. Most recently, the prompt $J/\psi$ production in association with charmed and non-charmed final particles was
measured\footnote{21}

\[ \sigma(e^+e^- \to J/\psi + X) = (1.17 \pm 0.02 \pm 0.07) \text{pb}, \]
\[ \sigma(e^+e^- \to J/\psi + c\bar{c}) = (0.74 \pm 0.08) \text{pb}, \]
\[ \sigma(e^+e^- \to J/\psi + X_{\text{non-cc}}) = (0.43 \pm 0.09 \pm 0.09) \text{pb}. \]  \hfill (2)

These processes are investigated in Ref.\footnote{20, 22}, the results show that including both the \( O(\alpha_s) \) radiative correction and the \( O(\nu^2) \) relativistic correction, the color-singlet contribution to \( e^+e^- \to J/\psi gg \) has saturated the latest observed cross section \( e^+e^- \to J/\psi + X_{\text{non-cc}} \) measured by Belle.

Aside from the above QCD process, the pure QED process should also be considered\footnote{23}. Especially, in our paper, we calculate virtual-photon-associated production \( \sigma(e^+e^- \to J/\psi \gamma) \times B(\gamma^* \to \ell\bar{\ell}(or \ Light \ Hadrons)) \). The rest of the paper is organized as follows. In Section II, we will give the formulations of \( e^+e^- \to J/\psi + \mu^+\mu^- \). In Section III, the QED production of \( e^+e^- \to J/\psi + LH \) is discussed. In section IV, we will give the numerical results and discussion. Finally we summarize our results in section V.

II. THE FORMULATIONS OF \( e^+e^- \to J/\psi + \mu^+\mu^- \)

In NRQCD factorization scheme, the cross section of \( e^+e^- \to J/\psi + \mu^+\mu^- \) can be described as follows

\[ \mathcal{A}(e^+(k_1)e^-(k_2) \to J/\psi(2p) + \mu^+(p_1) + \mu^-(p_2)) = \sqrt{C_S} \sum_{LsS_z} \sum_{s_1s_2} \sum_{jk} \langle \frac{1}{2}s_1; \frac{1}{2}s_2 \mid SS_z \rangle \langle LL_z; SS_z \mid JJ_z \rangle \langle 3j; 3k \mid 1 \rangle \times \]
\[ \mathcal{A}(e^+e^- \to c_j^+(p) + \bar{c}_k^+(p) + \mu^+(p_1) + \mu^-(p_2)) \]  \hfill (3)

where \( \langle 3j; 3k \mid 1 \rangle = 1/\sqrt{3} \), \( \langle s_1; s_2 \mid SS_z \rangle \), \( \langle LL_z; SS_z \mid JJ_z \rangle \) are respectively the color-SU(3), spin-SU(2), and angular momentum Clebsch-Gordan coefficients for \( c\bar{c} \) pairs projecting out appropriate bound states. \( \mathcal{A}(e^+e^- \to c_j(p) + \bar{c}_k(p) + \mu^+(p_1) + \mu^-(p_2)) \) is the scattering amplitude for \( c\bar{c} \) production. The coefficient \( C_S \) can be related to the radial wave function of the bound state and reads

\[ C_S = \frac{1}{4\pi} \mid R_S(0) \mid^2. \]  \hfill (4)
We introduce the spin projection operators $P_{SS_z}(p, q)$ as
\[ P_{SS_z}(p, q) \equiv \sum_{s_1 s_2} (s_1; s_2 | SS_z) v(p - q; s_1) \bar{u}(p + q; s_2). \] (5)

Expanding $P_{SS_z}(p, q)$ in terms of the relative momentum $q$, we get the projection operators at leading term of $q$, which will be used in our calculation, as follows
\[ P_{1S_z}(p, 0) = \frac{1}{\sqrt{2}} \epsilon^* (S_z)(p + M/2). \] (6)

where $M$ is the mass of the charmonium. It is two times of charm quark mass $m$ in the non-relativistic approximation. The polarized cross section can be calculated by defining the longitudinal polarization vector as follows
\[ \epsilon^\mu_L(p) = \frac{2p^\mu}{M} - \frac{M n^\mu}{2 n \cdot p}, \] (7)

where $4p^2 = M^2$ and $n^\mu = (1, -\vec{p}/|\vec{p}|)$.

The Feynman diagrams of $e^+e^- \rightarrow J/\psi \mu^+ \mu^-$ are shown in Fig. 1. The process of final states with plus charge parity is depicted in diagram Fig.1(a, b) and denoted as $C = +$. The process of final states with minus charge parity is depicted in diagram Fig.1(c, d) and denoted as $C = -$. The $C = +$ process is dominant, and the $C = -$ process is suppressed by a factor of
\[ f \sim \ln \left( \frac{s}{4m_{\mu}^2} \right) \frac{s}{M^2}, \] (8)

here the logarithm term come from quasi-collinear divergence with $m_{\mu}^2 \ll s$. The $s/M^2$ term come the photon propagator. One can get $f \sim 10^{-2}$ for $e^+e^- \rightarrow J/\psi \mu^+ \mu^-$ at $\sqrt{s} = 10.6$ GeV for $B$ factories.

FIG. 1. The Feynman diagrams of $e^+e^- \rightarrow J/\psi \mu^+ \mu^-$. 
III. THE QED PRODUCTION OF $e^+e^- \rightarrow J/\psi + LH$

Similar to the $e^+e^- \rightarrow J/\psi \mu^+\mu^-$ process, the fragment process is also dominant in $e^+e^- \rightarrow J/\psi + Light\ Hadrons$ process. The fragment process can be considered as $e^+e^- \rightarrow \gamma^*\gamma^*$, then the virtual photons fragment into $J/\psi$ and light hadrons respectively. By using the approach of the calculation of the hadronic part of the muon $g-2[24]$, this process can be described as

$$\frac{d\sigma^{QED}[e^+e^- \rightarrow J/\psi + LH]}{dm_{LH}^2} \sim \frac{d\sigma[e^+e^- \rightarrow J/\psi + \mu^+\mu^-]}{dm_{\mu^+\mu^-}^2} \times R^{had}(m_{\mu^+\mu^-}^2)\bigg|_{m_{\mu^+\mu^-}=m_{LH}}^{},$$

(9)

where

$$R^{had}(\Lambda^2) = \frac{\sigma[e^+e^- \rightarrow hadrons]}{\sigma[e^+e^- \rightarrow \mu^+\mu^-]}\bigg|_{m_{e^+e^-}^2=\Lambda^2^2},$$

(10)

because of the contribution of the $C=-$ process is negligible, after subtracting the effect of $\gamma^* \rightarrow c\bar{c}$ from $R^{had}(\Lambda^2)$ by a naive factor $4/3\Theta(\Lambda - M_{c\bar{c} - The})$, we get

$$\frac{d\sigma^{QED}[e^+e^- \rightarrow J/\psi + LH]}{dm_{LH}^2} = \frac{d\sigma[e^+e^- \rightarrow J/\psi + \mu^+\mu^-]}{dm_{\mu^+\mu^-}^2} \times \left[R^{had}(m_{\mu^+\mu^-}^2) - \frac{4}{3}\Theta(m_{\mu^+\mu^-}^2 - M_{c\bar{c} - The}^2)\right]\bigg|_{m_{\mu^+\mu^-}=m_{LH}}^{},$$

(11)

where $\Theta$ is step function, $M_{c\bar{c} - The}$ should be correspond to the $c\bar{c}$ threshold of $M_{J/\psi}$, $2M_D$, etc. The uncertainties should be discussed in the next section.

IV. NUMERICAL RESULT

In numerical calculations, the parameters are selected as [25]:

$$M_\mu = 0.1057GeV, \quad M_{J/\psi} = 3.0969GeV, \quad \sqrt{s} = 10.6GeV,$$

$$M_\tau = 1.7768GeV, \quad M_{\psi(2S)} = 3.686GeV, \quad \alpha = 1/132.33$$

(12)

The table of $R^{had}(\Lambda^2)$ have been given in Ref.[24]. We construct an interpolation of $R^{had}(\Lambda^2)$ corresponding to the table with first-order interpolation and setting $R^{had}(\Lambda^2) = 0$ when $\Lambda$ is less than $\lambda_{min}$ in the table.

The wave function at the origin can be extracted from the leptonic width $\Gamma(V \rightarrow l^+l^-)$

$$|R_S(0)|^2 = \frac{m_V^2}{4e^2\alpha^2}\Gamma[V \rightarrow e^+e^-].$$

(13)
The leptonic width of charmonium decays into $e^+e^-$ has been given in Ref.\[25\]
\[
\Gamma[J/\psi \rightarrow e^+e^-] = 5.55 \pm 0.14 keV, \\
\Gamma[\psi(2S) \rightarrow e^+e^-] = 2.38 \pm 0.04 keV. 
\] (14)

When we calculate the prompt production cross sections of $J/\psi$, we take into account the feeddown contribution from $\psi(2S)$ by $B[\psi(2S) \rightarrow J/\psi + X] = (57.4 \pm 0.9)\%$ \[25\] and ignore the contribution of the other charmonium. Then we can get the direct production cross section of $J/\psi(\psi(2S))$ associated with $\tau^+\tau^-$ and $\mu^+\mu^-$ at $B$ factories as
\[
\sigma^{\text{direct}}[e^+e^- \rightarrow J/\psi + \mu^+\mu^-] = 56 \pm 2 \text{ fb} \\
\sigma^{\text{direct}}[e^+e^- \rightarrow J/\psi + \tau^+\tau^-] = 6.4 \pm 0.2 \text{ fb}
\] (15)

and
\[
\sigma^{\text{direct}}[e^+e^- \rightarrow \psi(2S) + \mu^+\mu^-] = 20 \pm 1 \text{ fb} \\
\sigma^{\text{direct}}[e^+e^- \rightarrow \psi(2S) + \tau^+\tau^-] = 1.8 \pm 0.1 \text{ fb}
\] (16)

Most of the uncertainties come from the uncertainty of leptonic width in Eq.(14). The others come from the effect of fine structure constant $\alpha$ and higher order QED corrections and so on. The QCD corrections have been taken into account in the leptonic width of $J/\psi(\psi(2S))$.

The cross sections for $C = -$ process is only 1.6%(1.0%) of that for $C = +$ process in $J/\psi(\psi(2S))$ production associated with $\mu^+\mu^-$. And the ratio is about 6.0%(3.9%) in $J/\psi(\psi(2S))$ production associated with $\tau^+\tau^-$. These results are in agreement with the estimation in Eq.(8). So the contribution of $C = -$ process can be ignored in the calculation of electromagnetic $J/\psi(\psi(2S))$ production associated with light hadrons. Finally, we get the direct production cross section of $J/\psi(\psi(2S))$ associated with light hadrons at $B$ factories as
\[
\sigma^{\text{direct}}_{\text{QED}}[e^+e^- \rightarrow J/\psi + LH] = 100 \pm 5 \text{ fb} \\
\sigma^{\text{direct}}_{\text{QED}}[e^+e^- \rightarrow \psi(2S) + LH] = 36 \pm 1 \text{ fb}
\] (17)

here we choose $M_{c\bar{c}-\text{Th}} = 2M_D$ in Eq.(11). If we choose $M_{c\bar{c}-\text{Th}} = M_{J/\psi}$, there is a difference of $-1 fb$. So the uncertainties from $M_{c\bar{c}-\text{Th}}$ can be ignored. Most of the uncertainties come from $R^{\text{had}}$ and the leptonic decay width.
FIG. 2. The $J/\psi$ and $\psi(2S)$ energy spectra of direct production processes $e^+e^- \rightarrow V + \mu^+\mu^-$ ($V = J/\psi, \psi(2S)$).

The energy distributions of direct $J/\psi(\psi(2S))$ production from the processes $e^+e^- \rightarrow J/\psi(\psi(2S))l\bar{l}$ and $e^+e^- \rightarrow J/\psi(\psi(2S))LH$ are shown in Fig.2 and Fig.3, respectively. Unfortunately, the endpoint peak was not measured in Ref. [21]. The polarization of $J/\psi(\psi(2S))$ direct production for $e^+e^- \rightarrow V + \mu^+\mu^-$ ($V = J/\psi, \psi(2S)$) process as a function of the energy of $J/\psi$ is shown in Fig.4. The polarization of $J/\psi$ production associated with light hadrons is similar to the results of $e^+e^- \rightarrow V + \mu^+\mu^-$ process. The angular distributions of direct $J/\psi(\psi(2S))$ production from the processes $e^+e^- \rightarrow V + \mu^+\mu^- (V = J/\psi, \psi(2S))$ and $e^+e^- \rightarrow 2\gamma^* \rightarrow V + LH$ ($V = J/\psi, \psi(2S)$) are shown in Fig.5 and Fig.6, respectively.

Now we give the prompt production cross sections of electromagnetic $J/\psi$ production associated with leptonic pairs and light hadrons as

$$
\sigma_{QED}^{\text{prompt}} [e^+e^- \rightarrow J/\psi + \mu^+\mu^- + X] = 68 \pm 2 \text{ fb}
$$

$$
\sigma_{QED}^{\text{prompt}} [e^+e^- \rightarrow J/\psi + \tau^+\tau^- + X] = 7.4 \pm 0.1 \text{ fb}
$$

$$
\sigma_{QED}^{\text{prompt}} [e^+e^- \rightarrow J/\psi + LH] = 121 \pm 5 \text{ fb}.
$$

(18)
FIG. 3. The $J/\psi$ and $\psi(2S)$ energy spectra of direct production processes $e^+e^- \rightarrow 2\gamma \rightarrow V + LH$ ($V = J/\psi, \psi(2S)$).

FIG. 4. The helicities of $J/\psi$ and $\psi(2S)$ direct production processes $e^+e^- \rightarrow V + \mu^+\mu^-$ ($V = J/\psi, \psi(2S)$) as a function of the energy of $J/\psi$ and $\psi(2S)$.

V. SUMMARY

In summary, the electromagnetic productions of $J/\psi$ associated with light hadrons (LH) and leptonic pairs ($\mu^+\mu^-, \tau^+\tau^-$) at $B$ factories are studied. We find that the direct electromagnetic production cross sections of $J/\psi(\psi(2S))$ associated with light hadrons is about $0.10(0.04)$ pb. The direct production cross section of $J/\psi(\psi(2S))$ associated with $\mu^+\mu^-$
FIG. 5. The angular distributions of direct production processes \( e^+e^- \rightarrow V + \mu^+\mu^- (V = J/\psi, \psi(2S)) \). Here \( \theta \) is the angle between \( J/\psi(\psi(2S)) \) momentum and beam.

FIG. 6. The angular distributions of direct production processes \( e^+e^- \rightarrow 2\gamma \rightarrow V + LH \) (\( V = J/\psi, \psi(2S) \)). Here \( \theta \) is the angle between \( J/\psi(\psi(2S)) \) and beam.

is about 0.056(0.020) pb. If we include the contribution from \( \psi(2S) \) decay, we can get the prompt cross section \( \sigma[e^+e^- \rightarrow J/\psi + \mu^+\mu^- + X] = (68 \pm 2) \) pb, about (16\( \pm \)5)% of the Belle data \( \sigma[e^+e^- \rightarrow J/\psi + X_{non-ce}] = (0.43 \pm 0.09 \pm 0.09) \) pb, meanwhile the \( e^+e^- \rightarrow J/\psi + \tau^+\tau^- \) process only contributes 2%. The prompt cross section \( \sigma[e^+e^- \rightarrow J/\psi + Light Hadrons] = (0.121 \pm 5) \) fb is about (28 \( \pm \)8)% of the Belle data. Unfortunately, the endpoint peak of energy distribution for \( J/\psi \) electromagnetic production associated with leptonic pairs and
light hadrons was not measured in Ref. [21]. The polarization of $J/\psi$ electromagnetic production associated with light hadrons is transversal, while the polarization of $J/\psi$ inclusive production associated with light hadrons from QCD process is longitudinal [22]. We also notify that the charge parity of final states is plus for the QED process calculated in our paper. And it is minus for color singlet process $e^+e^- \rightarrow J/\psi + gg$ and color octet process $e^+e^- \rightarrow J/\psi + g$.

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