Comments on: Measurement of Parton Distributions of Strange Quarks in the Nucleon from Charged-Kaon Production in Deep-Inelastic Scattering on the Deuteron by the HERMES Collaboration

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Abstract

In this paper the author discusses the article by the HERMES Collaboration, Phys. Lett. B 666, 446 [1], where several important results concerning strange quark properties in the nucleon were presented. The strange sea distribution was found to be very different from the non-strange in the Bjorken x scaling variable. In addition, the magnitude of these two distributions at low x is similar, contrary to most of the available Parton Distribution Function sets. The strange quark helicity distribution was also extracted combining inclusive and semi-inclusive kaon asymmetries. The result of the first moment of the strange quarks helicity distribution in the measured range is slightly positive, while a rather significant negative value is expected from the world polarised inclusive Deep Inelastic Scattering measurements.

The author shows that a certain combination of fragmentation functions extracted from the preliminary HERMES kaon multiplicities presents a very strong $Q^2$ dependence. Such a strong dependence is not expected from the DGLAP evolution of the fragmentation functions. If a similar dependence was present in the data used in the analyses of the discussed article, their conclusion might have to be altered.

Keywords: strange quark, strange quark puzzle

1. Introduction

The information on the strange quark properties in the nucleon is rather scarce. The strange quark density is poorly known, the main information comes from the neutrino experiments NuTeV [2] and CCFR [3]. The analyses [4–7] suggest that the strange sea is suppressed with respect to the non-strange sea by a factor of about 2. This fact is not observed in the analysis of [8].

The HERMES results [1], brought a very important contribution to the understanding of the strange quark properties in the nucleon.
First of all, it was shown that the strange quark parton distribution function (PDF) has very different shape in the Bjorken $x$ scaling variable as compared to the distributions of $\bar{u}$ and $d$. It was also verified that for $x \approx 0.04$ the densities of the strange and non-strange sea are comparable. This result was recently confirmed by the ATLAS analysis [9], but the final uncertainty of the ATLAS result is still large.

Even more interesting results were obtained by HERMES in the polarised strange sector. From deep inelastic scattering measurements of the spin-dependent structure function $g_1$ it is known that the first moment of the strange quark distribution is negative $\Delta S = \int_0^1 \Delta s(x) + \Delta \bar{s}(x) dx = -0.09 \pm 0.01 \pm 0.01$, (see e.g. [10], [11]). However, HERMES analysis based on the combination of the inclusive asymmetry $A_{\text{incl}}^{||,d}$ and the semi-inclusive kaon asymmetry $A_{\text{KI}}^{||,d}$ concluded that in the measured range of $x$, the corresponding $\Delta S$ is consistent with zero and negative values are not preferred; $\int_{0.02}^{0.6} \Delta s(x) + \Delta \bar{s}(x) dx = 0.037 \pm 0.019 \pm 0.027$. These results are often called strange quark polarisation puzzle.

The HERMES observation lead to a sign changing solution of the polarised strange quarks distribution function in a PDF set by D. de Florian, R. Sassot, M. Stratmann and W. Vogelsang [12]. In the high $x$ region the strange quark polarisation is positive, changing sign for low values of $x$. As a result the semi-inclusive and the inclusive analyses of $\Delta S$ do no longer contradict each other.

The other possible solution of the puzzle was proposed by E. Leader, A. V. Sidorov and D. B. Stamenov [13]. They argued that the puzzle only appears if one uses the DSS set of fragmentation functions (FFs) [14], as done in [1]. Using another set, namely the HKNS [15] one, semi-inclusive data prefer negative strange polarisation in the whole $x$ range. Based on published materials in [1] it can be shown that the proposition of [13] has a big weakness. Namely the two discussed sets of FFs differ indeed a lot in the kaon sector. The strange quark fragmentation function into kaons is more than a factor two higher in DSS than in HKNS. As a result if the HKNS FF had been used for the extraction of the unpolarised strange quark PDF from the HERMES data, the resulting distribution would have been more than a factor two higher than extracted in [1], and in disagreement with all experiments.

However, the results of [1], published in 2008 results, are based on the analysis of the sum of kaon ($K^\pm$) multiplicities. The author analysed non-published but publicly available, preliminary HERMES results [16] presented in 2011, as well as results shown in PhD thesis of [17] based on HERMES data, from 2005. In these two the sources charged separated kaon multiplicities are available. It will be shown that a certain combination of FFs extracted from the difference of $K^\pm$ multiplicities has a very strong $Q^2(x)$ dependence not expected by DGLAP.

If such a dependence was present in the data used in [1], the explanation of E. Leader et al. could not be excluded, while conclusion of [1] might had to be changed.
2. HERMES analysis of unpolarised strange sea

The HERMES results \cite{1} on the strange quark PDF $S(x)$ are based on the analysis of sum of $K^\pm$ multiplicities $dN^K(x)/dN^{DIS}(x)$ in semi-inclusive deep inelastic scattering (SIDIS) of electrons on a deuteron target. As stated there:

$$S(x) \int D^K_S(z)dz \simeq Q(x) \left[ 5 \frac{d^2N^K(x)}{d^2N^{DIS}(x)} - \int D^K_Q(z)dz \right]$$

(1)

where $S(x) = s(x) + \bar{s}(x)$, $z$ is the ratio of energies of the hadron and the virtual photon in the target rest frame, $D^K_S(z) \equiv 4D^K_{u+\bar{u}}(z) + D^K_{d+d}(z)$ and $D^K_Q(z) \equiv 2D^K_{s+\bar{s}}(z)$.

As presented in Fig. 1 of \cite{1}, the kaon multiplicity is flat for high $x$, i.e. in the region where there is no strange quarks and it rises by about 50% for lower values of $x$. Without strange quarks the distribution should be almost flat (or even decreasing for lower $x$) therefore the large excess of kaon multiplicity in the low $x$ region is interpreted as a strong signature of the strange quarks presence. Their contribution was quantified using Eq. (1). The value of $\int D^K_S(z)dz$ was taken from DSS and $\int D^K_Q(z)dz$ was extracted directly from the HERMES data at high $x$. In the first order, the HERMES analysis neglected the possible negative four momentum transfer $Q^2$ dependence of the FF. However, in the later stage of the analysis the dependence as in DSS fragmentation function is taken into account. The expected $Q^2$ dependence is weak, of the order of 10-20%, in the measured range of $Q^2 \in (1-10) \text{(GeV/c)}^2$.

3. Tentative analysis of the HERMES publicly available data

The author extracted the $K^+$ and $K^-$ multiplicities on deuteron target from slide 15 of \cite{10}. These multiplicities cover the following kinematic ranges: $x \in (0.03 - 0.5)$ and $Q^2 \in (1 - 10) \text{(GeV/c)}^2$, and they were integrated in the $z$ range from 0.2 to 0.8. To be able to perform some quantitative analyses, the author of this paper assumes that in the strange sector there are three fragmentation functions: $D_{str} = D^K_s = D^K_\bar{s}$; $D_{fav} = D^K_u = D^K_{\bar{u}}$ and $D_{unf}$ for the remaining combinations. Thus: $D^K_Q \equiv 4D^K_{u+\bar{u}} + D^K_{d+d} = 4D^K_{fav} + 6D^K_{unf}$ and $D^K_S \equiv 2D^K_{s+\bar{s}} = 2D^K_{str} + 2D^K_{unf}$. Here for simplicity it was assumed that $D_i = \int_{0.2}^{0.8} D_i(z)dz$.

It turned out that a very important distribution to understand better the strange sector is the difference of $K^+$ and $K^-$ multiplicities, $dN^K_{diff}(x)/dN^{DIS}(x)$. First of all the multiplicity difference leads to a non-singlet distribution, whose evolution decouples from gluons, and is thus simpler. Secondly a lot of systematic uncertainties cancel in the multiplicity difference. Thirdly the disputed strange fragmentation function does not contribute to the difference. Finally using Eq. (2) one has a direct link to a certain combination of FF, namely $D^K_{fav} - D^K_{unf}$:

$$\frac{dN^K_{diff}(x)}{dN^{DIS}(x)} = \frac{4(u(x) + \bar{u}(x))}{5(u(x) + \bar{u}(x) + d(x) + \bar{d}(x)) + 4s(x)} (D^K_{fav}(x) - D^K_{unf}(x))$$

(2)
The presented equation is for the deuteron target. The unpolarised quark distributions of various flavors are denoted by $u(x)$, $\bar{u}(x)$, $d(x)$, $\bar{d}(x)$, $s(x)$. The denominator of the equation is closely related with the measured $1\gamma$ cross-section, while $u_v(x)$ and $d_v(x)$ are the valence distributions of the $u$ and $d$ quarks respectively. Therefore both these factors are known experimentally with a good precision. There is a strong correlation between $x$ and $Q^2$ in the HERMES experiment, thus for simplicity it was assumed here that FFs depend upon $x$, instead of $Q^2$.

The author evaluated $D_{fav}(x) - D_{unf}(x)$ using Eq. (2) and the CTEQ6L PDF set [18]. The systematic errors of $K^+$ and $K^-$ multiplicities are strongly correlated; but the correlation factors were not known to the author. He decided to scale the errors as given in [10] so that the fit of $(D_{fav} - D_{unf})(\log_{10}(x))$ by a straight line gives the $\chi^2$ value equal to the number of degrees of freedom. The scaling factor was found to be 2.5, thus quite large.

The resulting $D_{fav}(x) - D_{unf}(x)$ are presented in Fig. 1. The points are the $D_{fav}(x) - D_{unf}(x)$ values integrated in the measured range of $z \in (0.2 - 0.8)$, extracted by the author. The solid line is the straight line fit to these data. Finally the dashed-line is the DSS parametrisation, taking into account the average $Q^2$ values in the different $x$ intervals. The values of the multiplicities and PDF parameters used by the author are summarised in Tables 1 and 2.

Contrary to the expectation of [14], a very strong $x$ or $Q^2$ dependence of the evaluated FFs is observed. The author also verified that a similar, but a factor 2.5 weaker, dependence is also present in the results from the PhD thesis of [17]. However, for those results the multiplicities were evolved to the same $Q^2$ value, and in addition a region in $z$ smaller than in [1] was available. The author of [17] mentions that there is possibly a problem in the procedure of the multiplicity extraction, since the extracted $K^-$ multiplicities were sometimes negative.

![Figure 1: The distribution of $\int_{0.2}^{0.8} D_{fav} - D_{unf} \, dz$ as a function of $x$, obtained by the author from HERMES prel. data [16]. The continuous line is a straight line fit, while the dashed line corresponds to the DSS predictions related with $Q^2$ dependence of fragmentation functions.](image)
low $x$ or slightly changing it to take into account weak $Q^2$ evolution from DSS. However, this assumption is not supported by their preliminary data where $Q^2$ evolution of FFs can be studied. Since $D^0_Q = 4(D_{fav} - D_{unf}) + 10D_{unf}$, a strong $x$ or $Q^2$ dependence of $D_{fav} - D_{unf}$ influences $D^0_Q$. The possible impact of the observed $x$ or $Q^2$ dependence on the results obtained in [1] is discussed in two scenarios.

In the first scenario $D_{unf}$ is large at high $x$ and rapidly decreases for lower $x$ values. A typical $D_{unf}$ in DSS is of the order of 0.012-0.015; However, the observed change of $D_{fav}(x) - D_{unf}(x)$ is a factor about 5 larger. Thus introducing a strong $x$ or $Q^2$ dependence of $D_{unf}$ results in negative values in most of the phase space. At the same time the strange quark density is further increased.

In the second scenario $D_{fav}$, i.e. $D^K_u$ and $D^K_s$, increases for low $x$. In this case the increase of the sum of $K^+$, $K^-$ multiplicities for low $x$ observed in HERMES corresponds mainly to kaons originating from $u$, $\bar{u}$ instead of $s$, $\bar{s}$. As a result the strange quark PDF is lower than anticipated in [1]. It follows naturally that the non-negative $\Delta S$ obtained in the HERMES analysis could be explained by the fact that more than expected, positively polarised, $u$, $\bar{u}$ quarks contribute to a kaon sample from which the spin dependent asymmetry $A^K_{DSS}$ is extracted. As a consequence the obtained $\Delta S$ is biased towards positive values.

If the discussed data were used for the extraction of FFs, in the discussed scenario the resulting $D_{str}$ would be overestimated. As in previous cases part of the kaons originating from $u$ and $\bar{u}$ would be wrongly assigned as having $s$ or $\bar{s}$ origin. This possible overestimation of $D_{str}$ has further influence on the $\Delta S$ extraction, resulting in too optimistic statistical uncertainties and possibly further bias towards a positive value. An example of the correlation of the uncertainty and the value of $\Delta S$ for different ratios of $D_{str}/D_{fav}$ can be seen in COMPASS analysis [10].

The author tried to verify a possible impact of the observed $D_{fav}(x) - D_{unf}(x)$ dependence on the $D_{str}$. In order to do so a simple LO extraction of FFs from HERMES data was performed using as input the extracted integrated $K^+$ and $K^-$ multiplicities from [10] and the CTEQ6L PDF set.

Without any $Q^2$ dependence of FFs the fit $\chi^2/ndf$ was 75.4/15. The following $z$-integrated values of FFs were obtained: $D_{str} = 0.45 \pm 0.09$, $D_{fav} = 0.100 \pm 0.003$ and $D_{unf} = 0.017 \pm 0.002$. For comparison, the results of LO DSS analysis of the world data at $Q^2 = 2.5$ (GeV/c)$^2$ are presented: $D_{str} = 0.62$, $D_{fav} = 0.091$ and $D_{unf} = 0.012$. In the next step, an additional free parameter was added, modifying $D_{fav}(x) = D_{fav}(1) + \text{par} \cdot \log_{10}(x)$. The functional form of the $x$ dependence is selected taking into account on the empirical ob-

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1. Observe, that this conclusion is valid even in case one assumes only charge conjunction asymmetry of FFs, thus 6 independent FFs in the strange quark sector.
2. As a matter of fact some of the multiplicities obtained from HERMES data were used in the DSS FF fit, namely Information from [15] together with private communication with the author of that PhD thesis. However, detailed analysis of the impact of HERMES data and their unforeseen $D_{fav}(x) - D_{unf}(x)$ dependence on the full FF fit is far too complex to be done by the author in this paper.
The resulting $\chi^2$ of the fit improved by a factor 4 to 18.1/14. The slope was found to be about -0.061 as can be deduced from Fig. 1. However, at the same time the strange FF changed a lot, $D_{str} = -0.44 \pm 0.14$, i.e. the central value became unphysical. Adding an additional $Q^2$ dependence for $D_{unf}$ as expected from the DSS brings the central value of $D_{str}$ to -0.3 or -0.2 depending on if the $Q^2$ evolution of FF from DSS is applied as an additive or a multiplicative correction. Thus the allowed region is concentrated only for low values of $D_{str}$ i.e. as obtained in [15] and discussed in [13].

For completeness the author also verified that if the $D_{fav}$ is kept constant while $D_{unf}$ has a $x$ dependence of the form $D_{unf}(x) = D_{unf}(1) + \text{par} \cdot \log_{10}(x)$, the resulting $\chi^2/\text{ndf}$ is about 37.5/14, but $D_{unf}$ is negative for $x < 0.15$ and in addition $D_{str}/D_{fav} \approx 25$ thus not very reliable. A simultaneous fit of the $x$ dependence of both $D_{fav}$ and $D_{unf}$ did not improve the $\chi^2/\text{ndf}$ value significantly: 17.6/13 to be compared with 18.1/14 when only $D_{fav}(x)$ dependence was included. In addition the correlation between the fit parameters exceeds 99% in this case.

However, the most important observation is that indeed there is a very strong correlation between $D_{fav}(x) - D_{unf}(x)$ and $D_{str}$. Without a proper understanding of the $x$ dependence of $D_{fav} - D_{unf}$ it is very hard to obtain from SIDIS data any non-disputable results in the strange quark sector. Observe that a systematic effect giving rise to a 10% slope of $D_{fav}(x) - D_{unf}(x)$ results in a $D_{str}/D_{fav}$ bias of about 1. The author doesn’t speculate why such a strong $x$ or $Q^2$ dependencies are observed in the preliminary HERMES data sets. However, it is worth to mention that for the preliminary COMPASS multiplicities [20], the author extracted the $D_{fav}(x) - D_{unf}(x)$, which within uncertainties is flat in the whole measured $x$ region $x \in (0.004 - 0.7)$. The $Q^2$ range is between 1-30 (GeV/c)$^2$ for these data.

4. Summary

Based on publicly available HERMES data the author showed that the analysis of $K^+$ and $K^-$ multiplicities results in a not foreseen $x$ dependence of the $D_{fav}(x) - D_{unf}(x)$ fragmentation functions. This dependence can explain, in a certain scenario, the different than expected strange quark PDF obtained by HERMES as well as it will bias the strange quark helicity distribution towards a positive value. Therefore the key point is the verification by the HERMES collaboration that $D_{fav} - D_{unf}$ does not show any unforeseen $x$ dependence for the data used in [1]. The answer to this question is very important in order to understand the strange quark sector in the fixed target experiments domain.

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Appendix

In order to make the verification of the presented results possible, in Tables 1 and 2 the raw information used in this analysis is given.

Table 1: Mean values of $x$ and $Q^2$ in $(\text{GeV}/c)^2$ for the used data as well as values of the $x$-PDF for quark flavours from CTEQ6L. In the last column $\frac{4(u_c + d_c)}{5(u + \bar{u} + d + d) + 4s}$ is given.

| $x$  | $Q^2$ | $u$   | $\bar{u}$ | $d$   | $\bar{d}$ | $s$   | $\frac{4(u_c + d_c)}{5(u + \bar{u} + d + d) + 4s}$ |
|------|-------|-------|-----------|-------|-----------|-------|-------------------------------------------------|
| 0.033| 1.1   | 0.408 | 0.165     | 0.346 | 0.178     | 0.063 | 0.286                                           |
| 0.048| 1.4   | 0.450 | 0.153     | 0.359 | 0.178     | 0.062 | 0.322                                           |
| 0.065| 1.6   | 0.488 | 0.138     | 0.367 | 0.176     | 0.057 | 0.357                                           |
| 0.087| 1.7   | 0.527 | 0.119     | 0.372 | 0.169     | 0.050 | 0.399                                           |
| 0.120| 2.1   | 0.579 | 0.093     | 0.375 | 0.150     | 0.040 | 0.462                                           |
| 0.170| 3.1   | 0.624 | 0.064     | 0.358 | 0.108     | 0.027 | 0.551                                           |
| 0.240| 4.9   | 0.618 | 0.039     | 0.302 | 0.053     | 0.015 | 0.646                                           |
| 0.340| 7.4   | 0.513 | 0.019     | 0.202 | 0.014     | 0.006 | 0.727                                           |
| 0.450| 10.1  | 0.344 | 0.006     | 0.104 | 0.002     | 0.002 | 0.767                                           |

Table 2: $K^+ - K^-$ multiplicity differences extracted from protect [16] and their errors. Different intervals of $x$ and $z$ are shown. The final $z$-integrated multiplicity difference is presented in the last column.

| $x$  | $Q^2$ | $z \in (0.2 - 0.3)$ | $z \in (0.3 - 0.4)$ | $z \in (0.4 - 0.6)$ | $z \in (0.6 - 0.8)$ | $\frac{dN_{K^+}^f(x)}{dN_{K^-}^f(x)}$ |
|------|-------|----------------------|----------------------|----------------------|----------------------|---------------------------------|
| 0.033| 1.1   | 0.084 ± 0.026        | 0.080 ± 0.023        | 0.056 ± 0.009        | 0.031 ± 0.008        | 0.0338 ± 0.0042                |
| 0.048| 1.4   | 0.084 ± 0.012        | 0.095 ± 0.016        | 0.054 ± 0.006        | 0.032 ± 0.006        | 0.0352 ± 0.0026                |
| 0.065| 1.6   | 0.090 ± 0.023        | 0.084 ± 0.007        | 0.061 ± 0.007        | 0.035 ± 0.004        | 0.0367 ± 0.0029                |
| 0.087| 1.7   | 0.088 ± 0.035        | 0.081 ± 0.017        | 0.071 ± 0.005        | 0.039 ± 0.009        | 0.0391 ± 0.0044                |
| 0.120| 2.1   | 0.097 ± 0.041        | 0.082 ± 0.014        | 0.067 ± 0.006        | 0.040 ± 0.003        | 0.0395 ± 0.0046                |
| 0.170| 3.1   | 0.091 ± 0.052        | 0.091 ± 0.016        | 0.074 ± 0.005        | 0.037 ± 0.006        | 0.0404 ± 0.0057                |
| 0.240| 4.9   | 0.116 ± 0.027        | 0.096 ± 0.011        | 0.078 ± 0.006        | 0.041 ± 0.004        | 0.0451 ± 0.0033                |
| 0.340| 7.4   | 0.107 ± 0.045        | 0.108 ± 0.016        | 0.087 ± 0.008        | 0.040 ± 0.003        | 0.0469 ± 0.0051                |
| 0.450| 10.1  | 0.117 ± 0.088        | 0.028 ± 0.048        | 0.071 ± 0.008        | 0.027 ± 0.004        | 0.0341 ± 0.0102                |