Gauged $U(1)_{L_\mu-L_\tau}$ Symmetry and two-zero Textures of Inverse Neutrino Mass Matrix in light of Muon $(g-2)$

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Abstract

In the framework of anomaly free $U(1)_{L_\mu-L_\tau}$ model, charged scalar fields give rise to massive gauge boson ($Z_{\mu\tau}$) through spontaneous symmetry breaking. $Z_{\mu\tau}$ leads to one loop contribution to the muon anomalous magnetic moment. These scalar fields may, also, appear in the structure of right-handed neutrino mass matrix, thus, connecting the possible explanation of muon $(g-2)$ and low energy neutrino phenomenology through vevs associated with the scalar fields. In the present work, we consider textures of inverse neutrino mass matrix ($M_\nu^{-1}$) wherein any two elements of the mass matrix are zero. In this ansatz, with Dirac neutrino mass matrix diagonal, the zero(s) of right-handed Majorana neutrino mass matrix correspond to zero(s) in the low energy effective neutrino mass matrix (within Type-I seesaw). We have realized two such textures of $M_\nu^{-1}$ accommodating the muon $(g-2)$ and low energy neutrino phenomenology. The requirement of successful explanation of muon $(g-2)$, further, constrain the allowed parameter space of the model and results in sharp correlations amongst neutrino mixing angles, $CP$ invariants and effective Majorana mass ($M_\nu$). The model explains muon $(g-2)$ for $M_{Z_{\mu\tau}}$ in the range (0.035 GeV-0.100 GeV) and $g_{\mu\tau} \approx O(10^{-4})$ which is found to be consistent with constraints coming from the experiments like CCFR, COHERENT, BABAR, NA62 and NA64.

Keywords: Muon $(g-2)$; Phenomenology; Neutrino mass matrix; Texture zeros.
1 Introduction

The standard model (SM) of particle physics has been very successful in explaining interactions between fundamental particles and, at the same time, predicted a wide variety of phenomena. Despite immense success of the SM, it is facing a growing list of “anomalies” - a significant experimental divergence from theoretical predictions. For example, unsolved problems like origin of light neutrino masses, matter-antimatter asymmetry, dark matter, muon anomalous magnetic moment etc. find no explanation within the SM. The experimental observations of sub-eV scale neutrino masses and large mixing in the leptonic sector provide cardinal evidences propounding physics beyond the standard model (BSM) [1]. The neutrino flavor states ($\nu_\alpha$) are incoherent mixture of mass eigenstates ($\nu_i$). The magnitude of mixing is parameterized in terms of three mixing angles ($\theta_{ij}(i,j = 1,2,3; i < j)$) and one Dirac CP phase ($\delta$). The Dirac CP phase has not been observed experimentally, however, the recent measurements hint $\delta \approx -\pi/2$ [2]. Additionally, two more CP phases($\alpha, \beta$) appears for Majorana nature of neutrinos which have no influence on neutrino oscillations. Furthermore, the riddle of octant degeneracy ($\theta_{23}$ above or below 45°) and mass ordering (normal($m_1 < m_2 < m_3$) or inverted hierarchy($m_3 < m_1 < m_2$)) still remains unresolved. Furthermore, although absolute scale of neutrino mass is unknown, we have upper bound on sum of neutrino mass, $\sum m_i < 0.12$ eV from cosmological data [3,4]. In view of the above, the neutrino mass matrix, in general, contains more free parameters than one can measure experimentally so phenomenological ansatze are important to fully reconstruct the mass matrix in terms of less number of parameters to understand the underlying dynamics of neutrino mass generation.

The recent results from E989 experiment(Run I) [5] at FermiLab for the precise measurement of muon anomalous magnetic moment $a_\mu = (g - 2)/2$, shows a discrepancy with the theoretical prediction of the SM

$$a_\mu^{\text{FNAL}} = 116592040(54) \times 10^{-11}, \quad (1)$$

$$a_\mu^{\text{SM}} = 116591810(43) \times 10^{-11}, \quad (2)$$

which when amalgamated with the previous results of Brookhaven National Laboratory

$$a_\mu^{\text{BNL}} = 116592089(63) \times 10^{-11}, \quad (3)$$

raises the confidence level from 3.7$\sigma$ to 4.2$\sigma$ such that $\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = (251\pm59) \times 10^{-11}$, a compelling evidence of new physics.

The possible implication and interpretation of muon($g - 2$) anomaly have been discussed in different frameworks such as 2HDM [7–11], model with axion-like particles (ALP) [12], $A_4$ modular symmetry [13], vector-like leptons (VLL) [14,15], super-symmetric(SUSY) models [16,17] etc.. In general, $U(1)_L$ and $U(1)_B$ are accidental symmetries in the SM leading to lepton and baryon number conservation, respectively, but are anomalous. However, the symmetries originating from the difference of any two charged lepton flavors, i.e. $L_\alpha - L_\beta$, ($\alpha, \beta = e, \mu, \tau$) are anomaly free. Among these $U(1)_{L_\alpha - L_\beta}$ symmetries, $U(1)_{L_\mu - L_\tau}$ gauge
symmetry has been explored in different dimensions of neutrino mass model building sce-
narios \cite{18,20}. $L_\mu - L_\tau$ extension of SM in the framework of Type-I seesaw with one \cite{21} and two complex scalar singlets \cite{22} have been studied to explain the muon $(g - 2)$. In gen-
eral, spontaneous symmetry breaking of $U(1)_{L_\mu - L_\tau}$ symmetry manifest the $L_\mu - L_\tau$ massive
gauge boson ($Z_{\mu\tau}$). In the framework of $U(1)_{L_\mu - L_\tau}$ symmetry, $Z_{\mu\tau}$ do not interact with elec-
tron and quarks, evading constraints from LEP \cite{23,24} and LHC \cite{25}. It interacts only with $\mu$ and $\tau$ flavors which may contribute significantly to muon magnetic moment. Also, $Z_{\mu\tau}$ gauge boson contributes to the muon neutrino trident (MNT) process which constrains the mass of new gauge boson $Z_{\mu\tau} \leq 300$ MeV for the explanation of muon $(g - 2)$ \cite{26}. The new
developments in muon magnetic moment measurements compels to investigate theoretical
models accommodating explanation of muon $(g - 2)$.

The phenomenological ansatze like texture zeroes \cite{27-33}, hybrid textures \cite{34-37}, magic
symmetry \cite{38,41} lead to interesting predictions and correlation among low-energy observ-
ables. Also, texture zeroes in inverse neutrino mass matrices ($M_{\nu}^{-1}$) are imperative in the
sense, in diagonal charged lepton and Dirac mass basis, zeroes in right-handed neutrino
mass matrix corresponds to zeroes in $M_{\nu}^{-1}$. The phenomenological implications of inverse
neutrino mass textures have been studied in Refs. \cite{42-44}. Recently, the authors have in-
vestigated all possible two-zero texture inverse neutrino mass matrices ($M_{\nu}^{-1}$) in light of
large mixing angle(LMA) and $dark$-large mixing angle(DLMA) solutions of neutrino mixing
paradigm \cite{45}. Out of fifteen possible two-zero $M_{\nu}^{-1}$ textures only seven are found to be in
consonance with current neutrino oscillation data. In the present work, we have realized
two such textures $D_1$ and $E_1$ (for notation of textures see Ref. \cite{45}) accommodating
muon $(g - 2)$ anomaly and neutrino oscillation data, simultaneously. We have employed
$U(1)_{L_\mu - L_\tau}$ symmetry extending the SM with three right-handed neutrinos and three scalar
singlet fields. The gauge boson contributing to the possible explanation of muon anomalous
magnetic moment, further, constrain the allowed parameter space of these textures.

The rest of the paper is organised as follows. In Section 2, we have discussed the $U(1)_{L_\mu - L_\tau}$
model and corresponding charge assignments resulting in two-zero $M_{\nu}^{-1}$. The details of
numerical analysis and consequent discussion have been elaborated in Section 3. Finally, in
Section 4, we summarize our conclusions.

2. $U(1)_{L_\mu - L_\tau}$ Model

We have extended the SM field content with three heavy right-handed neutrinos ($N_e, N_\mu, N_\tau$)
having $L_\mu - L_\tau$ charges $(0, 1, -1)$, respectively, leading to Type-I seesaw origin of light
neutrino masses. In the scalar sector, three singlet scalar fields $\Phi_i$ ($i = 1, 2, 3$) with non-zero
$L_\mu - L_\tau$ charges have been employed. It is known that zeroes in $M_R$ are identical to zeroes in
$M_{\nu}^{-1}$ if Dirac and charged lepton mass matrices ($M_D$ and $M_\ell$) are diagonal. Therefore, the
charge assignments under $U(1)_{L_\mu - L_\tau}$ are chosen in such a way that $M_\ell$ and $M_D$ are diagonal,
in the model. $\Phi_i$ breaks the $U(1)_{L_\mu - L_\tau}$ symmetry by acquiring vacuum expectation values
$(vevs)$ $v_i$ ($i = 1, 2, 3$) consequently giving mass to the new $U(1)_{L_\mu - L_\tau}$ gauge boson $Z_{\mu\tau}$. Also,
the $Z_4$ symmetry have been used to constrain the structure of the Yukawa Lagrangian. The
The scalar potential is given by

\[ V(H, \Phi_i) = -\mu_i^2 (\Phi_i^\dagger \Phi_i)^2 + \lambda_{\Phi_i} (\Phi_i^\dagger \Phi_i)^2 + \lambda_{H\Phi_i} (H^\dagger H)(\Phi_i^\dagger \Phi_i) + \lambda_{\Phi_1\Phi_2}(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \]

\[ + [\mu_{12}\Phi_1^\dagger \Phi_2^\dagger + H.c.] + \lambda_{\Phi_2\Phi_3}(\Phi_1^\dagger \Phi_1)(\Phi_3^\dagger \Phi_3) + \lambda_{\Phi_2\Phi_3}(\Phi_2^\dagger \Phi_2)(\Phi_3^\dagger \Phi_3). \]

where \( i = 1, 2, 3 \).

The neutral component of SM Higgs(\( H \)) breaks the electroweak symmetry spontaneously whereas singlets \( \Phi_{1,2,3} \) breaks the \( L_\mu - L_\tau \) gauge symmetry after acquiring the vevs \( v_{1,2,3} \). The Yukawa Lagrangian for charged leptons is given by

\[ \mathcal{L}_\ell = -Y_{\ell e} \bar{L}_e H e_R - Y_{\ell_\mu} \bar{L}_\mu H \mu_R - Y_{\ell_\tau} \bar{L}_\tau H \tau_R + h.c., \]

which leads to charged lepton mass matrix diagonal, \( M_\ell = \frac{\mu}{\sqrt{2}} \text{diag}(Y_{\ell e}, Y_{\ell_\mu}, Y_{\ell_\tau}) \), where \( Y_{\ell_i} \) with \( i = e, \mu, \tau \) are the Yukawa couplings. The Lagrangian relevant for neutrino mass is given by

\[ \mathcal{L}_N = \bar{N}_\mu \gamma^\mu D_\mu N_\mu + \bar{N}_\tau \gamma^\tau D_\tau N_\tau - \frac{1}{2} M N_e N_e - Y_{e\tau} \Phi_1 N_e N_\tau - Y_{e\mu} \Phi_3 N_e N_\mu \]

\[ - Y_{\tau\tau} \Phi_2 N_\tau N_\tau - Y_{D_e} \bar{L}_e \tilde{H} N_e - Y_{D_\mu} \bar{L}_\mu \tilde{H} N_\mu - Y_{D_\tau} \bar{L}_\tau \tilde{H} N_\tau + h.c., \]

where \( \tilde{H} = i\sigma_2 H^* \) and \( M \) is a constant with dimension of mass.
| Symmetry  | $L_e$ | $L_\mu$ | $L_\tau$ | $e_R$ | $\mu_R$ | $\tau_R$ | $N_e$ | $N_\mu$ | $N_\tau$ | $H$ | $\Phi_1$ | $\Phi_2$ | $\Phi_3$ |
|-----------|------|--------|---------|-------|--------|--------|------|-------|--------|-----|--------|--------|--------|
| $SU(2)_L$ | 2    | 2      | 2       | 1     | 1      | 1      | 1    | 1     | 1      | 2   | 1      | 1      | 1      |
| $U(1)_Y$  | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | -1   | -1     | -1     | 0    | 0     | $\frac{1}{2}$ | 0   | 0      | 0      | 0      |
| $U(1)_{L_\mu-L_\tau}$ | 0    | 1      | -1      | 0     | -1     | 0      | 1    | -1    | 0      | 1   | 2      | -1     | -1     |
| $Z_4$     | -1   | 1      | i       | -1    | 1      | i      | -1   | 1     | i      | -1 | -1     | -1     | -1     |

Table 1: The field content of the model with respective charge assignments under $SU(2)_L \times U(1)_Y \times U(1)_{L_\mu-L_\tau} \times Z_4$.

After expanding the kinetic term in Eqn.(6), the mass of new $U(1)_{L_\mu-L_\tau}$ gauge boson can be found to be $M_{Z_{\mu\tau}} = g_{\mu\tau} \sqrt{v_1^2 + 4v_2^2 + v_3^2}$, where $g_{\mu\tau}$ is the $L_\mu - L_\tau$ gauge coupling.

Using Eqn.(10), the Dirac mass matrix is given by

$$M_D = \begin{pmatrix} d_e & 0 & 0 \\ 0 & d_\mu & 0 \\ 0 & 0 & d_\tau \end{pmatrix}, \quad (11)$$

where $d_\alpha = Y_{D\alpha} \frac{v}{\sqrt{2}}$ with $\alpha = e, \mu, \tau$. $Y_{D\alpha}$ are real Yukawa couplings and $\frac{v}{\sqrt{2}}$ is the vev of SM Higgs doublet, $H$. Using Eqn.(10), the right-handed Majorana mass matrix ($M_R$) is given by

$$M_R = \begin{pmatrix} M & Y_{e\mu}v_3 & Y_{e\tau}v_1e^{i\xi} \\ Y_{e\mu}v_3 & 0 & 0 \\ Y_{e\tau}v_1e^{i\xi} & 0 & Y_{\tau\tau}v_2 \end{pmatrix}, \quad (12)$$

where, in general, the elements of $M_R$ are complex. By redefinition of the fields, $\xi$ is the only remaining irremovable phase. Thus, $M_R$ depends on four real parameters $M$, $Y_{e\mu}$, $Y_{e\tau}$ and $Y_{\tau\tau}$ and a complex phase $\xi$. As a consequence of diagonal $M_D$ and $M_\ell$ the non-trivial neutrino mixing will arise from $M_R$.

Within the paradigm of Type-I seesaw, the inverse neutrino mass matrix can be written as

$$M_\nu^{-1} = -(M_D^T)^{-1}M_RM_D^{-1}. \quad (13)$$

Using $M_D$ and $M_R$ given in Eqns.(11) and (12), respectively, alongwith Eqn. (13), $M_\nu^{-1}$ is given by

$$M_\nu^{-1} = \begin{pmatrix} \frac{M}{d_e^2} & \frac{Y_{e\mu}v_3e^{i\xi}}{d_ed_\mu} & \frac{-Y_{e\tau}v_1}{d_ed_\tau} \\ \frac{Y_{e\mu}v_3e^{i\xi}}{d_ed_\mu} & 0 & 0 \\ \frac{-Y_{e\tau}v_1}{d_ed_\tau} & 0 & \frac{Y_{\tau\tau}v_2}{d_\tau^2} \end{pmatrix}, \quad (14)$$

which corresponds to $D_1$ texture of $M_\nu^{-1}$ studied in Ref. [45].

Also, if the charge assignment of $\Phi_2$ under $U(1)_{L_\mu-L_\tau}$ and $Z_4$ are replaced by $-2$ and $1$, respectively.
respectively, then we obtain, $M_R$, given by

$$
M_R = \begin{pmatrix}
M & Y_{e\mu}v_3 & Y_{e\tau}v_1 e^{i\xi} \\
Y_{e\mu}v_3 & Y_{\mu\mu} & 0 \\
Y_{e\tau}v_1 e^{i\xi} & 0 & 0
\end{pmatrix},
$$

while the mass matrices $M_\ell$ and $M_D$ remains diagonal. The two-zero texture of $M^{-1}_\nu$ obtained using Eqn.(15) corresponds to $E_1$ texture with zeroes at (2, 3) and (3, 3) place [45].

### 3 Numerical Analysis and Discussion

In this section, as a representative case, we perform the numerical analysis of texture $D_1$ obtained in Eqn.(14) in light of muon ($g - 2$) and neutrino oscillation data (Table 2). The $D_1$ texture defined in Eqn.(14) corresponds to two-zero texture $M_\nu$ with zeroes at (1, 1) and (1, 3). $M_\nu$ is numerically diagonalised by a unitary matrix $U$ such that $UM_\nu U^T = \text{diag}(m_1, m_2, m_3)$ and the neutrino mixing angles can be obtained using

$$
\sin^2 \theta_{13} = |U_{13}|^2, \quad \sin^2 \theta_{23} = \frac{|U_{23}|^2}{1 - |U_{13}|^2}, \quad \sin^2 \theta_{12} = \frac{|U_{12}|^2}{1 - |U_{13}|^2}.
$$

Also, the amount of $CP$ violation manifested in Jarlskog invariant ($J_{CP}$) [46][47] is defined as

$$
J_{CP} = \text{Im} [U_{11}U_{22}U_{12}^* U_{21}^*] = s_{23}s_{12}s_{13}c_{13}^2 \sin \delta,
$$

while other two rephasing invariants $I_1$ and $I_2$ are given by

$$
I_1 = \text{Im} [U_{11}^* U_{12}] = c_{12}s_{13}c_{13}^2 \sin \left(\frac{\alpha_1}{2}\right), \quad I_2 = \text{Im} [U_{11}^* U_{13}] = c_{12}s_{13}c_{13}\sin \left(\frac{\alpha_2}{2} - \delta\right),
$$

where $\alpha_1, \alpha_2$ are Majorana phases. Furthermore, the gyromagnetic ratio ($g$-factor) of the muon is the quantity which relates its spin ($\vec{s}$) to its magnetic moment ($\vec{\mu}$) as given by

$$
\vec{\mu} = g \left(\frac{q}{2m_\mu}\right) \vec{s},
$$

where, $q$ is muon charge and $m_\mu$ is muon mass. In Dirac’s theory of charged spin-half particles, the gyromagnetic ratio is $g = 2$. However, the recent developments at FermiLab hint towards non-trivial interactions of muon with BSM fields. The higher-order radiative corrections can generate additional contributions to magnetic moment of muon parameterised as

$$
g = 2_{\text{Dirac}}(1 + a_\mu) \quad \text{and} \quad a_\mu = \frac{1}{2} (g - 2).
$$

The correction $a_\mu$ to the Dirac’s predictions is called the anomalous magnetic moment. With in SM, the contribution to the anomalous magnetic moment of muon may comes from:
Figure 1: One loop Feynman diagram mediated by extra gauge boson $Z_{\mu\tau}$ contributing to muon $(g-2)$.

| Parameter | ±1σ range (NH) | ±1σ range (IH) | 3σ range (NH) | 3σ range (IH) |
|-----------|----------------|----------------|---------------|---------------|
| $\sin^2 \theta_{12}$ | 0.304$_{-0.012}^{+0.013}$ | 0.304$_{-0.012}^{+0.012}$ | 0.269-0.343 | 0.269-0.343 |
| $\sin^2 \theta_{13}$ | 0.0220$_{-0.00002}^{+0.000068}$ | 0.02238$_{-0.000062}^{+0.000063}$ | 0.02060-0.02435 | 0.02053-0.02434 |
| $\sin^2 \theta_{23}$ | 0.573$_{-0.023}^{+0.018}$ | 0.578$_{-0.021}^{+0.017}$ | 0.405-0.624 | 0.410-0.623 |
| $\Delta m_{12}^2$ | 7.42$_{-0.20}^{+0.21}$ | 7.42$_{-0.20}^{+0.21}$ | 6.82-8.04 | 6.62-8.04 |
| $\Delta m_{13}^2$ | 2.515$_{-0.028}^{+0.028}$ | 2.498$_{-0.029}^{+0.028}$ | 2.431-2.598 | -2.584- -2.413 |

Table 2: The neutrino oscillation data from global fit used in the numerical analysis [1].

(a) quantum electrodynamic (QED) contributions (b) electroweak (EW) contributions (c) hadronic vacuum polarisation contributions (d) hadronic light-by-light scattering contributions. As explained earlier, the SM is not consistent with the recent results on muon $(g-2)$ at FermiLab. Therefore, beyond standard model contribution is required to explain muon anomalous magnetic moment. In this model, the additional contribution to muon magnetic moment arises at one-loop (Fig.1) mediated by $U(1)_{L_{\mu}-L_{\tau}}$ gauge boson $Z_{\mu\tau}$ and is given by [48,49]

$$\Delta a_{\mu} = \frac{\alpha'}{2\pi} \int_0^1 dx \frac{2m_{\mu}^2x^2(1-x)}{x^2m_{\mu}^2 + (1-x)M_{Z_{\mu\tau}}},$$

where $\alpha' = \frac{g_{\mu\tau}^2}{4\pi}$ is the structure constant and $m_{\mu}$ is the mass of muon. The new gauge boson, $Z_{\mu\tau}$, gets mass after scalar singlet fields ($\Phi_i$) acquires vevs. Also, non-trivial neutrino mixing matrix is induced by vevs of scalar singlet fields (through $M_R$), thus, connecting explanation of muon $(g-2)$ to neutrino phenomenology.

There are eight free parameters in inverse neutrino mass matrix ($M_{\nu}^{-1}$), given as, $M_{ee}$, $V_1=Y_{e\mu}v_1$, $V_2=Y_{e\tau}v_2$, $V_3=Y_{\tau\tau}v_3$, $d_e$, $d_\mu$, $d_\tau$ and $\xi$. In order to obtain the predictions on neutrino oscillation parameters and muon $(g-2)$ anomaly for $D_1$ texture, we have randomly varied all the free parameters with uniform distribution in the ranges.
Figure 2: The model predictions of the neutrino mixing angles as correlations plots between sum of neutrino masses ($\sum m_i$) and mixing angles. The horizontal lines are 3$\sigma$ experimental bounds on the respective mixing angle. The shaded region is excluded by the cosmological bound on sum of neutrino masses.

Figure 3: The correlation plot between ($\sum m_i - J_{CP}$). The shaded region is excluded by the cosmological bound on sum of neutrino masses.

$$d_e, d_\mu, d_\tau = (10^{-5} - 10^{-3}) \text{ GeV,}$$
$$V_1, V_2, V_3 = (1 - 280) \text{ GeV,}$$
$$M = (1 - 10^4) \text{ GeV,}$$
$$\xi = (0 - 360)^\circ.$$  \hfill (22)

We have numerically diagonalized $M_\nu$ to obtain the neutrino mixing matrix $U$. The predictions for neutrino mixing angles obtained from Eqn. (16) are compared with 3$\sigma$ ranges given in Table 2 to ascertain the allowed parameter space of the model.

In Fig 2(a), we have depicted the correlation between sum of neutrino masses $\sum m_i$ versus $\sin^2 \theta_{12}$ and $\sin^2 \theta_{23}$ while Fig 2(b) shows the correlation between $\sum m_i$ versus $\sin^2 \theta_{13}$ at 3$\sigma$. It is evident that the model is consistent with the neutrino oscillation data on the mixing angles and predicts sum of neutrino masses $\sum m_i$ to be within the range $0.065 \lesssim \sum m_i (\text{eV}) \lesssim 0.075$. In Fig 3 we have given the correlation plot of ($\sum m_i - J_{CP}$). $J_{CP}$ lies
in the range $-0.03 \leq J_{CP} \leq 0.01$. The predictions for other two $CP$ rephasing invariants $I_1$ and $I_2$ are shown in Fig.4(a) and (b), respectively. It is evident from Fig.4(a) that $I_1 = 0$ is disallowed implying $D_1$ texture is necessarily $CP$ violating.

Using the Eqn.(21), we calculate the $Z_{\mu\tau}$ contribution to $\Delta a_\mu$ which has been shown in $M_{Z_{\mu\tau}} - g_{\mu\tau}$ plane in Fig.5. The gauge coupling is randomly varied in the range $10^{-4} - 10^{-3}$. It is evident from Fig.5 that the model accommodates the observed muon $(g-2)$ for $M_{Z_{\mu\tau}}$ in the range $(0.035 \text{ GeV}-0.100 \text{ GeV})$ and $g_{\mu\tau} \approx \mathcal{O}(10^{-4})$, which is consistent with constraints coming from experiments like COHERENT [50, 51], BABAR [52] and CCFR [53]. The sensitivities of future experiments NA62 [54] and NA64 [55, 56] are, also, shown in Fig.5. The upper left triangular region is excluded by the astrophysical bound from cooling of white dwarf (WD) [57].

**Benchmark point:** For the input parameters

\[
(d_\epsilon, d_\mu, d_\tau) \times 10^{-5} = (2.38, 1.57, 2.19) \text{GeV},
\]

\[
(V_1, V_2, V_3, M) = (65.6, 15.0, 37.1, 28.1) \text{GeV},
\]

\[
\xi = 309.54^\circ,
\]

\[
g_{\mu\tau} = 5.2 \times 10^{-4},
\]

the corresponding values of mass-squared differences, mixing angles, $M_{Z_{\mu\tau}}$ and $\Delta a_\mu$ are

\[
\Delta m^2_{23} = 2.45 \times 10^{-3} \text{eV}^2; \ \Delta m^2_{12} = 7.53 \times 10^{-5} \text{eV}^2,
\]

\[
\sin^2 \theta_{13} = 0.022; \ \sin^2 \theta_{12} = 0.32; \ \sin^2 \theta_{23} = 0.58,
\]

\[
M_{Z_{\mu\tau}} = 42.38 \text{MeV}; \ \Delta a_\mu = 3.42 \times 10^{-11}.
\]
Figure 5: The allowed parameter space of the model in \((M_{Z_{\mu\tau}} - g_{\mu\tau})\) plane accommodating muon \((g - 2)\) and neutrino oscillation data. The exclusion regions from various experiments are, also, shown.

4 Conclusions

In this work, we have realised two-zero textures of \(M_{\nu}^{-1}\) with anomaly free gauged \(U(1)_{L_{\mu} - L_{\tau}}\) extension of SM in light of the muon \((g - 2)\) anomaly. We have extended the SM field content by adding three scalar singlets \((\Phi_i)\) and three right-handed neutrinos \((N_e, N_{\mu}, N_{\tau})\). \(U(1)_{L_{\mu} - L_{\tau}}\) symmetry is broken as the new scalar singlets acquire vevs, thereby, giving mass to new \(U(1)_{L_{\mu} - L_{\tau}}\) gauge boson \(Z_{\mu\tau}\). The two-zeros in \(M_R\) corresponds to two-zeros in \(M_{\nu}^{-1}\) in the diagonal charged lepton and Dirac mass basis. Also, the non-trivial neutrino mixing depends on the structure of \(M_R\). Thus, the right-handed Majorana neutrino mass matrix connects the low energy neutrino phenomenology with \(M_{Z_{\mu\tau}}\) contributing to muon anomalous magnetic moment. We have scanned the model parameter space and have found the model consistent with the neutrino oscillation data within \(3\sigma\) ranges. The Jarlskog \(CP\) rephasing invariant, \(J_{CP}\), lies in the range \(-0.03 \leq J_{CP} \leq 0.01\). The texture is found to be necessarily \(CP\) violating as \(I_1 = 0\) is disallowed. The model predicts the mass of new gauge boson \((M_{Z_{\mu\tau}})\) in the range \(0.035\text{GeV} \leq M_{Z_{\mu\tau}} \leq 0.100\)GeV for gauge coupling \((g_{\mu\tau})\) between \(5 \times 10^{-4} \leq g_{\mu\tau} \leq 8 \times 10^{-4}\) which is consistent with constraints from experiments such as CCFR, COHERENT, BABAR, NA62 and NA64.
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