Online Appendices to “The Role of Expectations in Changed Inflation Dynamics”*

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These online appendices accompany the paper “The Role of Expectations in Changed Inflation Dynamics” and present results for a number of robustness checks.

Appendix A. Robustness Exercises

A.1. Alternative Processes for Inflation Expectations

In this section, we provide some sensitivity with respect to the inflation expectations process by relaxing Equation (5) in the paper and allowing a more general process of expectation formation:

\[ E_t \Delta p_{t+1} = \mu M_t \Delta p_{t+1} + \lambda E_{t-1} \Delta p_t + \theta_3 \Delta p_{t-1} + \theta_4 y_t + v_t. \] (A.1)

This process for expectations allows the key features of a reduced-form Phillips-curve relationship—the output gap and lagged inflation—to enter the specification directly. Equation (A.1) thus also nests additional processes for expectation formation, such as naive expectations and adaptive expectations. Under the benchmark model-consistent expectations (MCE) model, \( \theta_3 = \theta_4 = 0 \) in addition to \( \lambda = 0 \) and \( \mu = 1 \). Table A.1 presents the results.

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### Table A.1. Alternative Process for Inflation Expectations

|       | (1) Michigan 1978–1996 | (2) Michigan 1997–2015 | (3) SPF CPI 1978–1996 | (4) SPF CPI 1997–2015 |
|-------|------------------------|------------------------|-----------------------|-----------------------|
| $\gamma$ | 0.102 | 0.527 | 0.421 | 0.312 |
| $\kappa$ | [-0.117, 0.313] | [0.102, 0.928] | [-0.057, 0.782] | [-0.049, 0.658] |
| $\sigma_\varepsilon$ | 1.527 | 0.794 | 2.108 | 0.829 |
| $\lambda$ | [1.096, 1.949] | [0.522, 1.055] | [1.238, 2.744] | [0.469, 0.896] |
| $\mu$ | [0.871, 1.659] | [0.066, 1.091] | [0.480, 1.301] | [0.075, 0.562] |
| $\theta_3$ | [-0.819, 0.061] | [-1.066, 0.047] | [-0.290, 0.344] | [-0.518, 0.105] |
| $\theta_4$ | [0.114, 0.101] | [0.134, 0.025] | [0.050, 0.207] | [0.040, 0.132] |
| $\sigma_\nu$ | [0.052, 0.382] | [0.010, 0.252] | [0.303] | [0.113] |
| $\rho$ | [0.290, 0.763] | [0.129, 0.496] | [0.154, 0.446] | [0.053, 0.170] |
| $\sigma_\omega$ | [0.308, 0.000] | [0.516, 0.996] | [0.118, 0.999] | [0.000, 0.608] |
| $\phi_1$ | [1.207] | [1.142, 1.483] | [1.033, 1.395] | [1.132, 1.479] |
| $\phi_2$ | [0.255] | [0.324] | [-0.264] | [-0.319] |
| $\phi_3$ | [-0.436, -0.075] | [-0.503, -0.151] | [-0.445, -0.082] | [-0.501, -0.142] |
| $\phi_4$ | [-0.168, -0.024] | [-0.255, 0.177] | [-0.167, -0.025] | [-0.262, 0.171] |
| $\sigma_{\eta}$ | [0.658] | [0.581] | [0.659] | [0.582] |

**Note:** Estimated using Bayesian methods; the priors are described in appendix B.
A.2. Additional Survey Measures

Table A.2 includes results for two additional survey measures: the Livingston survey of forecasters and the SPF measure of forecasts for inflation as measured by the GDP deflator. For convenience, the table also reproduces the results for the Michigan survey and SPF/CPI from Table 1 in the paper. The Livingston survey is a semiannual survey of economists from industry, government, banking, and academia. An advantage of the Livingston survey is that it is available on a consistent basis over a long sample (indeed, it has been collected since the 1940s). Similarly, the SPF for GDP prices has been collected on a consistent basis over our sample. One peculiarity of the Livingston survey is that it is collected semiannually. In this section, we estimate our model for the Livingston survey using semiannual data; in the next section, we introduce the Livingston survey into a model using quarterly data and allow the Kalman filter to fill in missing information.\(^1\) Because the frequency is semiannual, the coefficient magnitudes will be different for the Livingston survey. Also, when we measure expectations using the SPF for GDP prices, we use the corresponding measure of inflation.

Looking first at the results for the Phillips curve in the upper panel, the estimates of $\kappa$ with the Livingston survey are similar to those from the Michigan survey and SPF/CPI—in particular, the estimates of $\kappa$ drop by about the same amount as for SPF/CPI. The results for the SPF/PGDP are different, however: The point estimate of $\kappa$ is lower than for the other measures in the earlier period and higher in the latter period and, as a consequence, does not fall as much.

Turning to the results for the model of inflation expectations in the middle panels, the results for the SPF/PGDP are similar to those for the Michigan survey: The point estimate for $\mu$ falls substantially, with the point estimates in each period lying outside the confidence intervals in the complementary period. This may help account for the relatively small decline in the Phillips-curve slope $\kappa$ with this

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\(^1\)For the output gap, we average the two quarterly values. For inflation, we average the two quarterly inflation rates, making the inflation measure effectively the (annualized) percent change between the final quarters of each half year.
Table A.2. Additional Survey Measures

|                | (1) Livingston 1978–1996 | (2) Livingston 1997–2015 | (3) Michigan 1978–1996 | (4) Michigan 1997–2015 | (5) SPF CPI 1978–1996 | (6) SPF CPI 1997–2015 | (7) SPF PGDP 1978–1996 | (8) SPF PGDP 1997–2015 |
|----------------|---------------------------|---------------------------|------------------------|------------------------|-----------------------|-----------------------|------------------------|------------------------|
| $\gamma$      | 0.839                     | 0.819                     | 0.139                  | 0.566                  | 0.432                 | 0.211                 | 1.188                  | 0.603                  |
| $\kappa$      | 0.384                     | 0.101                     | 0.271                  | 0.133                  | 0.143                 | 0.331                 | 0.077                  | 0.239                  |
| $\sigma_\varepsilon$ | 1.920                     | 0.716                     | 1.568                  | 0.528                  | 0.796                 | 2.159                 | 0.467                  | 1.544                  |
| $\lambda$     | 0.610                     | 0.062                     | 0.786                  | 0.390                  | [0.013, 0.802]        | 0.778                 | 0.454                  | 0.713                  |
| $\mu$         | 0.257                     | 0.414                     | 0.198                  | 0.159                  | [0.159, 0.728]        | 0.159                 | 0.336                  | 0.183                  |
| $\sigma_\nu$  | 0.394                     | 0.183                     | 0.322                  | 0.218                  | [0.087, 0.303]        | 0.163                 | 0.190                  | 0.085                  |
| $\rho$        | 0.538                     | 0.000                     | 0.879                  | 0.640                  | [0.075, 0.352]        | 0.081                 | [0.021, 0.104]         | 0.019                  |
| $\sigma_\omega$ | 0.321                    | 0.270                     | 0.319                  | 0.175                  | 0.452                  | 0.496                 | 0.005                  | 0.015                  |
| $\phi_1$      | 1.391                     | 1.402                     | 1.214                  | 1.220                  | [1.039, 1.394]        | 1.142                 | 1.161                 | 1.122                  |
| $\phi_2$      | -0.491                    | -0.437                    | -0.260                 | -0.277                 | [-0.717, -0.270]      | -0.510                | -0.517                | -0.522                  |
| $\phi_3$      | -0.351                    | -0.351                    | -0.099                 | -0.101                 | [-0.680, -0.199]      | -0.510                | -0.517                | -0.522                  |
| $\phi_4$      | 0.211                     | -0.67                     | -0.46                  | -0.120                 | [-0.902, 0.201]       | -0.168                | -0.296                | -0.241                  |
| $\sigma_\eta$ | 0.823                     | 0.832                     | 0.659                  | 0.661                  | [-0.596, 0.457]       | [-0.393, 0.032]       | [-0.119, 0.042]        | [-0.335, 0.101]        |

Note: Estimated using Bayesian methods; the priors are described in appendix B.
measure of expectations. For the Livingston survey, the results are broadly similar to those for the SPF/CPI, with $\mu$ rising and $\lambda$ falling. In all four cases, the shock to the structural expectations equation, $\sigma$, fell across the two samples.

The point estimates in each sample are similar for the two sets of SPF results, and are smaller than for the other two surveys, which are broadly similar to each other.

A.3. Pooling the Information from the Surveys

In this exercise, we take the position that three of the measures of expectations of consumer prices—from the SPF, the Livingston survey, and the Michigan survey—are noisy indicators of the same underlying process. Thus, we assume that the same “true” measure of expectations is a common factor driving all three surveys, as in Equation (5), reproduced here for convenience:

$$E_t \Delta p_{t+1} = \mu M_t \Delta p_{t+1} + \lambda E_{t-1} \Delta p_t + v_t. \quad (A.2)$$

Each survey, however, has its individual measurement error:

$$S^i_t \Delta p_{t+1} = E_t \Delta p_{t+1} + u^i_t. \quad (A.3)
\nu^i_t = \rho^i_t \nu^i_{t-1} + \omega^i_t. \quad (A.4)$$

This specification has the advantage of bringing more information to bear on the estimation of Equation (5) at the risk of potential misspecification if the behavior of the fundamental component of expectations in fact differs across survey measures.

Table A.3 shows results for this joint estimation. For the structural Phillips curve, the results suggest that the slope parameter is smaller in the later sample, by a factor of four, similar to the results for the SPF/CPI in Table 1. The estimated degree of indexation ($\gamma$) is about the same in the two periods. The model of inflation expectations, which is common across the three expectations measures, also shows some shift between the two sample periods. The parameter

\footnote{It is somewhat surprising that the results for the SPF/PGDP should more closely resemble those for the Michigan survey than those for the SPF/CPI, since the respondents are the same for the two SPF series. This issue bears further investigation.}
Table A.3. Results for the Joint Estimation

|               | (1) Combination/Equal 1978–1996 | (2) Combination/Equal 1997–2015 | (3) Combination/Factors 1978–1996 | (4) Combination/Factors 1997–2015 |
|---------------|---------------------------------|---------------------------------|-----------------------------------|-----------------------------------|
| $\gamma$     | 0.295                           | 0.139                           | 0.158                             | 0.076                             |
|               | [0.047, 0.532]                  | [-0.069, 0.340]                 | [-0.056, 0.366]                   | [-0.062, 0.202]                   |
| $\kappa$     | 0.268                           | 0.069                           | 0.217                             | 0.062                             |
|               | [0.119, 0.413]                  | [0.028, 0.108]                  | [0.092, 0.341]                    | [0.028, 0.098]                    |
| $\sigma_\varepsilon$ | 1.881                        | 0.589                           | 1.626                             | 0.556                             |
| $\lambda$    | 0.787                           | 0.419                           | 0.758                             | 0.364                             |
|               | [1.451, 2.280]                  | [0.451, 0.711]                  | [1.256, 1.979]                    | [0.458, 0.647]                    |
| $\mu$        | 0.169                           | 0.371                           | 0.216                             | 0.421                             |
|               | [0.072, 0.872]                  | [0.277, 0.545]                  | [0.664, 0.850]                    | [0.274, 0.463]                    |
| $\sigma_\nu$ | 0.270                           | 0.117                           | 0.324                             | 0.115                             |
|               | [0.087, 0.249]                  | [0.231, 0.525]                  | [0.119, 0.320]                    | [0.276, 0.550]                    |
| $\Lambda_{SPF}$ | —                              | —                              | 0.751                             | 1.026                             |
| $\Lambda_{Liv}$ | —                              | —                              | [0.644, 0.859]                    | [0.729, 1.295]                    |
| $\rho_{Mich}$ | 0.855                           | 0.771                           | 0.819                             | 0.772                             |
|               | [0.749, 0.968]                  | [0.631, 0.906]                  | [0.693, 0.959]                    | [0.634, 0.927]                    |
| $\sigma_{\omega 1}$ | 0.335                        | 0.385                           | 0.304                             | 0.382                             |
|               | [0.283, 0.385]                  | [0.328, 0.438]                  | [0.246, 0.360]                    | [0.327, 0.436]                    |
| $\rho_{SPF}$  | 0.463                           | 0.366                           | 0.001                             | -0.366                            |
|               | [0.001, 0.837]                  | [-0.366, -0.366]                | [0.000, 0.001]                    | [-0.366, -0.366]                  |
| $\sigma_{\omega 2}$ | 0.066                        | 0.062                           | 0.074                             | 0.061                             |
|               | [0.022, 0.108]                  | [0.033, 0.091]                  | [0.031, 0.113]                    | [0.033, 0.089]                    |
| $\rho_{Liv}$  | 0.360                           | 0.006                           | 0.421                             | 0.029                             |
|               | [0.001, 0.625]                  | [-0.320, 0.295]                 | [0.046, 0.728]                    | [-0.298, 0.334]                   |
| $\sigma_{\omega 3}$ | 0.269                        | 0.242                           | 0.285                             | 0.243                             |
|               | [0.200, 0.333]                  | [0.191, 0.291]                  | [0.213, 0.356]                    | [0.189, 0.294]                    |
| $\phi_1$     | 1.215                           | 1.355                           | 1.216                             | 1.334                             |
|               | [1.041, 1.393]                  | [1.166, 1.514]                  | [1.042, 1.392]                    | [1.169, 1.505]                    |
| $\phi_2$     | -0.264                          | -0.346                          | -0.263                            | -0.345                            |
|               | [-0.441, -0.087]                | [-0.522, -0.166]                | [-0.445, -0.089]                  | [-0.524, -0.173]                  |
| $\phi_3$     | -0.099                          | -0.104                          | -0.100                            | -0.112                            |
|               | [-0.173, -0.029]                | [-0.326, 0.114]                 | [-0.171, -0.029]                  | [-0.327, 0.084]                   |
| $\phi_4$     | -0.048                          | -0.103                          | -0.046                            | 0.091                             |
|               | [-0.126, 0.029]                 | [-0.316, 0.120]                 | [-0.127, 0.032]                   | [-0.326, 0.111]                   |
| $\sigma_\eta$ | 0.658                           | 0.583                           | 0.659                             | 0.582                             |
|               | [0.568, 0.747]                  | [0.504, 0.664]                  | [0.564, 0.749]                    | [0.499, 0.660]                    |

Note: Estimated using Bayesian methods; the priors are described in appendix B.
\( \lambda \) falls somewhat, while the parameter \( \mu \) rises. The sum of the two coefficients declines, from 0.96 to 0.79. The results also suggest that structural shocks to inflation expectations are less important in the later sample: The estimated standard deviation of the parameter \( \sigma_\nu \) falls by a factor of two, and the shock is propagated forward less because \( \lambda \) is smaller.

We also estimate an alternative variant of pooling, where we allow for different loading factors, \( \Lambda_{j,t} \), for different survey measures of expectation formation. These results are shown in columns 3 and 4 of the table.

Overall, in these pooled results, the results more closely line up with those from the SPF than from the Michigan survey. In particular, the sharp reduction in \( \mu \) for the Michigan survey is not evident here.

### A.4. Oil-Price Shocks

In this section, we provide some sensitivity with respect to the inflation expectations process by further relaxing Equation (A.1) and allowing for an effect of oil-price shocks:

\[
E_t \Delta p_{t+1} = \mu M_t \Delta p_{t+1} + \lambda E_{t-1} \Delta p_t + \theta_3 \Delta p_{t-1} + \theta_4 y_t + \theta_5 oil_t + v_t. \tag{A.5}
\]
Table A.4. Results with Oil Price Shocks

|        | (1) Michigan 1978–1996 | (2) Michigan 1997–2015 | (3) SPF CPI 1978–1996 | (4) SPF CPI 1997–2015 |
|--------|------------------------|------------------------|----------------------|----------------------|
| $\gamma$ | 0.105                  | 0.533                  | 0.414                | 0.320                |
|         | [-0.108, 0.319]        | [0.130, 0.929]         | [-0.083, 0.785]      | [-0.045, 0.679]      |
| $\kappa$ | 0.171                  | 0.045                  | 0.299                | 0.039                |
|         | [-0.015, 0.357]        | [-0.108, 0.198]        | [0.096, 0.503]       | [-0.017, 0.094]      |
| $\sigma_\varepsilon$ | 1.536                 | 0.800                  | 2.099                | 0.692                |
|         | [1.118, 1.964]         | [0.532, 1.049]         | [1.188, 2.757]       | [0.471, 0.902]       |
| $\lambda$ | 1.257                 | 0.535                  | 0.843                | 0.316                |
|         | [0.881, 1.635]         | [0.100, 1.028]         | [0.502, 1.362]       | [0.077, 0.559]       |
| $\mu$   | -0.371                 | -0.579                 | 0.042                | -0.196               |
|         | [-0.774, 0.052]        | [-1.133, -0.004]       | [-0.362, 0.325]      | [-0.516, 0.102]      |
| $\theta_3$ | -0.003                | -0.019                 | 0.006                | 0.117                |
|         | [-0.108, 0.103]        | [-0.223, 0.190]        | [-0.058, 0.076]      | [0.055, 0.182]       |
| $\theta_4$ | 0.211                 | 0.144                  | 0.054                | 0.086                |
|         | [0.057, 0.370]         | [0.025, 0.266]         | [0.079, 0.231]       | [0.040, 0.131]       |
| $\theta_5$ | 0.017                 | 0.008                  | 0.197                | -0.090               |
|         | [-0.564, 0.597]        | [-0.448, 0.461]        | [-0.586, 1.014]      | [-0.828, 0.640]      |
| $\sigma_\nu$ | 0.461                | 0.302                  | 0.307                | 0.113                |
|         | [0.215, 0.700]         | [0.114, 0.481]         | [0.148, 0.458]       | [0.054, 0.168]       |
| $\rho$  | 0.808                  | 0.770                  | 0.608                | 0.311                |
|         | [0.444, 1.000]         | [0.546, 0.999]         | [0.119, 1.000]       | [0.000, 0.606]       |
| $\sigma_\omega$ | 0.300                | 0.315                  | 0.275                | 0.097                |
|         | [0.157, 0.441]         | [0.175, 0.444]         | [0.136, 0.412]       | [0.043, 0.146]       |
| $\phi_1$ | 1.206                  | 1.313                  | 1.208                | 1.307                |
|         | [1.028, 1.383]         | [1.142, 1.492]         | [1.027, 1.386]       | [1.138, 1.482]       |
| $\phi_2$ | -0.254                 | -0.324                 | -0.259               | -0.319               |
|         | [-0.427, -0.068]       | [-0.500, -0.141]       | [-0.439, -0.078]     | [-0.498, -0.143]     |
| $\phi_3$ | -0.097                 | -0.041                 | -0.097               | -0.040               |
|         | [-0.169, -0.026]       | [-0.253, 0.184]        | [-0.168, -0.027]     | [-0.257, 0.177]      |
| $\phi_4$ | -0.050                 | -0.181                 | -0.052               | -0.174               |
|         | [-0.132, 0.026]        | [-0.397, 0.034]        | [-0.128, 0.027]      | [-0.392, 0.045]      |
| $\sigma_\eta$ | 0.659                | 0.582                  | 0.658                | 0.582                |
|         | [0.568, 0.748]         | [0.499, 0.661]         | [0.566, 0.749]       | [0.503, 0.661]       |

Note: Estimated using Bayesian methods; the priors are described in appendix B.
A.5. Inflation Expectations of High-Income and College Graduates

We also check the robustness of our results for the Michigan survey by considering only high-income households or college graduates.

Table A.5. Results with Inflation Expectations of High-Income Households (HI) and College Graduates (CG)

|       | (1) Michigan: CG 1978–1996 | (2) Michigan: CG 1997–2015 | (3) Michigan: HI 1978–1996 | (4) Michigan: HI 1997–2015 |
|-------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| $\gamma$ | 0.158 [-0.060, 0.371] | 0.565 [0.136, 0.991] | 0.156 [-0.096, 0.407] | 0.546 [0.119, 0.972] |
| $\kappa$ | 0.205 [0.082, 0.334] | 0.141 [0.062, 0.219] | 0.232 [0.075, 0.435] | 0.121 [0.045, 0.195] |
| $\sigma_e$ | 1.640 [1.276, 2.001] | 0.799 [0.515, 1.068] | 1.610 [1.146, 2.047] | 0.758 [0.482, 1.025] |
| $\lambda$ | 0.740 [0.630, 0.834] | 0.277 [-0.179, 0.759] | 0.792 [0.635, 0.884] | 0.267 [0.166, 0.721] |
| $\mu$ | 0.231 [0.128, 0.350] | -0.055 [-0.389, 0.282] | 0.182 [0.095, 0.353] | 0.053 [-0.332, 0.443] |
| $\sigma_{\nu}$ | 0.350 [0.239, 0.458] | 0.220 [0.073, 0.362] | 0.354 [0.184, 0.491] | 0.260 [0.085, 0.425] |
| $\rho$ | 0.000 [0.000, 0.000] | 0.486 [0.280, 0.686] | 0.729 [0.271, 1.000] | 0.481 [0.275, 0.690] |
| $\sigma_{\omega}$ | 0.399 [0.299, 0.501] | 0.542 [0.443, 0.642] | 0.346 [0.191, 0.508] | 0.546 [0.438, 0.659] |
| $\phi_1$ | 1.221 [1.042, 1.398] | 1.315 [1.147, 1.490] | 1.227 [1.045, 1.406] | 1.316 [1.146, 1.491] |
| $\phi_2$ | -0.269 [-0.450, 0.090] | -0.328 [-0.503, 0.151] | -0.274 [-0.454, -0.089] | -0.329 [-0.509, -0.154] |
| $\phi_3$ | -0.101 [-0.173, 0.030] | -0.037 [-0.257, 0.180] | -0.100 [-0.171, -0.028] | -0.036 [-0.255, 0.184] |
| $\phi_4$ | -0.046 [-0.125, 0.034] | -0.177 [-0.395, 0.036] | -0.042 [-0.119, 0.038] | -0.176 [-0.391, 0.039] |
| $\sigma_\eta$ | 0.659 [0.567, 0.749] | 0.581 [0.501, 0.662] | 0.661 [0.502, 0.750] | 0.582 [0.502, 0.662] |

Note: Estimated using Bayesian methods; the priors are described in appendix B.
A.6. A More General Process for the Output Gap

In Table A.6, we present results assuming the output gap follows a more general process:

\[ y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 \Delta p_{t-1} + \phi_4 \Delta p_{t-2} + \phi_5 y_{t-3} \]
\[ + \phi_6 y_{t-4} + \phi_7 \Delta p_{t-3} + \phi_8 \Delta p_{t-4} + \eta_t. \]  

(A.6)

Table A.6. Results with a More General Process for Output Gap

|                | (1) Michigan 1978–1996 | (2) Michigan 1997–2015 | (3) SPF CPI 1978–1996 | (4) SPF CPI 1997–2015 |
|----------------|------------------------|------------------------|-----------------------|-----------------------|
| \( \gamma \)   | 0.136                  | 0.556                  | 0.433                 | 0.214                 |
|                | [–0.103, 0.367]        | [0.114, 0.963]         | [0.142, 0.742]        | [0.007, 0.414]        |
| \( \kappa \)   | 0.265                  | 0.143                  | 0.309                 | 0.076                 |
|                | [0.113, 0.417]         | [0.065, 0.218]         | [0.126, 0.482]        | [0.033, 0.118]        |
| \( \sigma_\varepsilon \) | 1.554            | 0.788                  | 2.384                 | 0.602                 |
|                | [1.148, 1.974]         | [0.523, 1.062]         | [1.811, 2.947]        | [0.476, 0.729]        |
| \( \lambda \)  | 0.777                  | 0.380                  | 0.750                 | 0.470                 |
|                | [0.670, 0.875]         | [–0.015, 0.797]        | [0.646, 0.858]        | [0.321, 0.625]        |
| \( \mu \)     | 0.202                  | –0.032                 | 0.192                 | 0.292                 |
|                | [0.104, 0.311]         | [–0.318, 0.270]        | [0.094, 0.253]        | [0.196, 0.459]        |
| \( \sigma_\nu \) | 0.317              | 0.212                  | 0.308                 | 0.062                 |
|                | [0.196, 0.431]         | [0.072, 0.341]         | [0.102, 0.283]        | [0.021, 0.102]        |
| \( \rho \)    | 0.900                  | 0.646                  | 0.641                 | 0.250                 |
|                | [0.803, 1.000]         | [0.438, 0.853]         | [0.195, 1.000]        | [0.000, 0.460]        |
| \( \sigma_\omega \) | 0.323               | 0.382                  | 0.182                 | 0.135                 |
|                | [0.186, 0.463]         | [0.291, 0.474]         | [0.077, 0.285]        | [0.101, 0.171]        |
| \( \phi_1 \)  | 1.367                  | 1.273                  | 1.340                 | 1.297                 |
|                | [1.181, 1.547]         | [1.091, 1.457]         | [1.129, 1.547]        | [1.116, 1.482]        |
| \( \phi_2 \)  | –0.211                 | –0.158                 | –0.299                | –0.163                |
|                | [–0.497, 0.083]        | [–0.448, 0.130]        | [–0.614, 0.028]       | [–0.452, 0.127]       |
| \( \phi_3 \)  | –0.137                 | –0.031                 | –0.093                | –0.087                |
|                | [–0.211, –0.061]       | [–0.248, 0.195]        | [–0.191, 0.001]       | [–0.319, 0.136]       |
| \( \phi_4 \)  | –0.098                 | –0.122                 | –0.107                | –0.073                |
|                | [–0.174, –0.022]       | [–0.351, 0.109]        | [–0.204, –0.012]      | [–0.307, 0.153]       |
| \( \phi_5 \)  | 0.227                  | –0.010                 | 0.223                 | 0.040                 |
|                | [0.150, 0.305]         | [–0.230, 0.223]        | [0.123, 0.322]        | [–0.193, 0.272]       |
| \( \phi_6 \)  | –0.072                 | –0.150                 | –0.134                | –0.132                |
|                | [–0.160, 0.022]        | [–0.367, 0.079]        | [–0.257, –0.007]      | [–0.358, 0.083]       |
| \( \phi_7 \)  | –0.313                 | –0.183                 | –0.266                | –0.218                |
|                | [–0.581, –0.052]       | [–0.489, 0.118]        | [–0.587, 0.058]       | [–0.522, 0.081]       |
| \( \phi_8 \)  | 0.119                  | 0.063                  | 0.161                 | 0.075                 |
|                | [–0.062, 0.291]        | [–0.131, 0.256]        | [–0.071, 0.385]       | [–0.123, 0.264]       |
| \( \sigma_\eta \) | 0.583              | 0.582                  | 0.731                 | 0.583                 |
|                | [0.498, 0.664]         | [0.500, 0.665]         | [0.614, 0.848]        | [0.500, 0.666]        |

Note: Estimated using Bayesian methods; the priors are described in appendix B.
A.7. Robustness: Unemployment Gap instead of Output Gap

In the results shown in Table A.7, we replace the output gap with unemployment gap in Equation (9).

| (1) Michigan 1978–1996 | (2) Michigan 1997–2015 | (3) SPF CPI 1978–1996 | (4) SPF CPI 1997–2015 |
|------------------------|------------------------|------------------------|------------------------|
| \( \gamma \) | 0.125 | 0.522 | 0.444 | 0.155 |
| | [-0.098, 0.352] | [0.093, 0.936] | [0.197, 0.701] | [-0.053, 0.353] |
| \( \kappa \) | -0.430 | -0.220 | -0.555 | -0.109 |
| | [-0.688, -0.178] | [-0.349, -0.097] | [-0.844, -0.254] | [-0.171, -0.049] |
| \( \sigma_\varepsilon \) | 1.556 | 0.775 | 2.326 | 0.577 |
| | [1.137, 1.962] | [0.505, 1.048] | [1.867, 2.798] | [0.451, 0.695] |
| \( \lambda \) | 0.747 | 0.461 | 0.670 | 0.385 |
| | [0.636, 0.860] | [0.066, 0.877] | [0.553, 0.789] | [0.163, 0.599] |
| \( \mu \) | 0.223 | -0.109 | 0.205 | 0.372 |
| | [0.118, 0.338] | [-0.418, 0.205] | [0.119, 0.288] | [0.197, 0.541] |
| \( \sigma_\nu \) | 0.297 | 0.240 | 0.255 | 0.073 |
| | [0.179, 0.414] | [0.085, 0.384] | [0.130, 0.381] | [0.028, 0.119] |
| \( \rho \) | 0.879 | 0.631 | 0.325 | 0.308 |
| | [0.761, 1.000] | [0.419, 0.847] | [0.000, 0.655] | [0.000, 0.549] |
| \( \sigma_\omega \) | 0.326 | 0.370 | 0.296 | 0.133 |
| | [0.186, 0.461] | [0.271, 0.471] | [0.170, 0.427] | [0.090, 0.175] |
| \( \phi_1 \) | 1.635 | 1.640 | 1.654 | 1.650 |
| | [1.494, 1.771] | [1.512, 1.772] | [1.502, 1.802] | [1.523, 1.782] |
| \( \phi_2 \) | -0.661 | -0.668 | -0.680 | -0.677 |
| | [-0.803, -0.523] | [-0.798, -0.536] | [-0.834, -0.531] | [-0.809, -0.549] |
| \( \phi_3 \) | 0.073 | -0.085 | 0.074 | -0.069 |
| | [-0.165, -0.004] | [-0.049, 0.097] | [-0.038 | -0.151, 0.013 |
| \( \phi_4 \) | -0.040 | 0.084 | -0.038 | 0.008 |
| | [-0.066, -0.014] | [-0.065, -0.010] | [-0.014, 0.150] | 0.214 |
| \( \sigma_\eta \) | 0.213 | 0.214 | 0.225 | 0.183, 0.243 |
| | [0.183, 0.241] | [0.183, 0.243] | [0.190, 0.260] | [0.183, 0.243] |

Note: Estimated using Bayesian methods; the priors are described in appendix B.
A.8. Allowing for Correlated Shocks

Tables A.8 and A.9 summarize the results where we allow for correlation between shocks in our model.

### Table A.8. Results with Correlated Shocks

|                | (1) Michigan 1978–1996 | (2) Michigan 1997–2015 | (3) SPF CPI 1978–1996 | (4) SPF CPI 1997–2015 |
|----------------|------------------------|------------------------|-----------------------|------------------------|
| \( \gamma \)   | 0.158                  | 0.540                  | 0.455                 | 0.199                  |
|                | [-0.096, 0.405]        | [0.113, 0.955]         | [0.158, 0.749]        | [-0.009, 0.401]        |
| \( \kappa \)   | 0.256                  | 0.150                  | 0.288                 | 0.078                  |
|                | [0.102, 0.400]         | [0.071, 0.232]         | [0.115, 0.462]        | [0.034, 0.118]         |
| \( \sigma_\varepsilon \) | 1.602                  | 0.789                  | 2.415                 | 0.600                  |
|                | [1.148, 2.042]         | [0.518, 1.054]         | [1.866, 2.979]        | [0.473, 0.723]         |
| \( \lambda \)  | 0.770                  | 0.388                  | 0.746                 | 0.438                  |
|                | [0.668, 0.866]         | [-0.017, 0.807]        | [0.639, 0.857]        | [0.273, 0.614]         |
| \( \mu \)     | 0.213                  | -0.047                 | 0.175                 | 0.344                  |
|                | [0.111, 0.315]         | [-0.341, 0.251]        | [0.095, 0.254]        | [0.206, 0.484]         |
| \( \sigma_\nu \) | 0.310                  | 0.217                  | 0.186                 | 0.066                  |
|                | [0.192, 0.429]         | [0.074, 0.346]         | [0.075, 0.291]        | [0.023, 0.107]         |
| \( \rho \)    | 0.866                  | 0.644                  | 0.654                 | 0.260                  |
|                | [0.727, 1.000]         | [0.451, 0.840]         | [0.211, 0.999]        | [0.000, 0.475]         |
| \( \sigma_\omega \) | 0.325                  | 0.380                  | 0.186                 | 0.133                  |
|                | [0.182, 0.463]         | [0.287, 0.472]         | [0.075, 0.291]        | [0.095, 0.170]         |
| \( \phi_1 \)  | 1.210                  | 1.302                  | 1.161                 | 1.320                  |
|                | [1.031, 1.388]         | [1.129, 1.475]         | [0.963, 1.354]        | [1.150, 1.490]         |
| \( \phi_2 \)  | -0.257                 | -0.314                 | -0.232                | -0.334                 |
|                | [-0.436, -0.076]       | [-0.493, -0.136]       | [-0.427, -0.037]      | [-0.512, -0.161]       |
| \( \phi_3 \)  | -0.097                 | -0.058                 | -0.073                | -0.095                 |
|                | [-0.166, -0.024]       | [-0.276, -0.171]       | [-0.159, 0.015]       | [-0.310, 0.131]        |
| \( \phi_4 \)  | -0.051                 | -0.164                 | -0.089                | -0.102                 |
|                | [-0.129, 0.029]        | [-0.386, 0.046]        | [-0.184, 0.004]       | [-0.318, 0.113]        |
| \( \sigma_\eta \) | 0.664                  | 0.585                  | 0.813                 | 0.587                  |
|                | [0.571, 0.756]         | [0.503, 0.665]         | [0.687, 0.933]        | [0.507, 0.671]         |
| \( corr(\sigma_\eta, \sigma_\varepsilon) \) | 0.085                  | -0.108                 | 0.184                 | -0.134                 |
|                | [-0.119, 0.287]        | [-0.314, 0.099]        | [-0.020, 0.386]       | [-0.335, 0.055]        |

**Note:** Estimated using Bayesian methods; the priors are described in appendix B.
Table A.9. Results with Correlated Shocks: Alternative Specification

|                | (1) Michigan 1978–1996 | (2) Michigan 1997–2015 | (3) SPF CPI 1978–1996 | (4) SPF CPI 1997–2015 |
|----------------|------------------------|------------------------|-----------------------|-----------------------|
| $\gamma$      | 0.193                  | 0.519                  | 0.504                 | 0.659                 |
|                | [-0.096, 0.475]        | [0.106, 0.923]         | [0.204, 0.817]        | [0.354, 0.981]        |
| $\kappa$      | 0.242                  | 0.146                  | 0.239                 | 0.078                 |
|                | [0.092, 0.383]         | [0.063, 0.223]         | [0.063, 0.410]        | [0.030, 0.123]        |
| $\sigma_e$    | 1.676                  | 0.785                  | 2.517                 | 0.883                 |
|                | [1.151, 2.182]         | [0.524, 1.042]         | [1.931, 3.086]        | [0.664, 1.102]        |
| $\lambda$     | 0.757                  | 0.371                  | 0.715                 | 0.282                 |
|                | [0.649, 0.865]         | [-0.027, 0.797]        | [0.580, 0.852]        | [0.098, 0.473]        |
| $\mu$         | 0.225                  | 0.017                  | 0.210                 | 0.439                 |
|                | [0.117, 0.333]         | [-0.347, 0.320]        | [0.102, 0.315]        | [0.281, 0.593]        |
| $\sigma_\nu$  | 0.308                  | 0.214                  | 0.193                 | 0.132                 |
|                | [0.183, 0.427]         | [0.066, 0.358]         | [0.081, 0.285]        | [0.085, 0.180]        |
| $\rho$        | 0.832                  | 0.645                  | 0.660                 | 0.278                 |
|                | [0.580, 1.000]         | [0.440, 0.854]         | [0.229, 1.000]        | [0.0-00, 0.547]       |
| $\sigma_\omega$| 0.323                 | 0.377                  | 0.189                 | 0.106                 |
|                | [0.171, 0.475]         | [0.271, 0.486]         | [0.081, 0.294]        | [0.054, 0.153]        |
| $\phi_1$      | 1.208                  | 1.305                  | 1.151                 | 1.326                 |
|                | [1.031, 1.385]         | [1.131, 1.480]         | [0.954, 1.342]        | [1.158, 1.497]        |
| $\phi_2$      | 0.255                  | 0.317                  | 0.226                 | 0.340                 |
|                | [-0.434, 0.074]        | [-0.504, 0.142]        | [-0.423, 0.036]       | [-0.511, 0.159]       |
| $\phi_3$      | 0.095                  | 0.059                  | 0.069                 | 0.048                 |
|                | [-0.166, 0.022]        | [-0.278, 0.164]        | [-0.153, 0.018]       | [-0.273, 0.166]       |
| $\phi_4$      | 0.054                  | 0.163                  | 0.094                 | 0.143                 |
|                | [-0.135, 0.025]        | [-0.382, 0.050]        | [-0.185, 0.001]       | [-0.351, 0.085]       |
| $\sigma_\eta$ | 0.663                  | 0.585                  | 0.811                 | 0.588                 |
|                | [0.571, 0.754]         | [0.504, 0.667]         | [0.686, 0.932]        | [0.508, 0.674]        |
| $corr(\sigma_\eta, \sigma_e)$ | 0.093 | -0.103 | 0.212 | -0.009 |
|                | [-0.100, 0.295]        | [-0.314, 0.094]        | [0.011, 0.401]        | [-0.190, 0.155]       |
| $corr(\sigma_\nu, \sigma_e)$ | 0.071 | -0.039 | -0.296 | -0.797 |
|                | [-0.416, 0.272]        | [-0.520, 0.442]        | [-0.830, 0.188]       | [-0.997, 0.628]       |

*Note:* Estimated using Bayesian methods; the priors are described in appendix B.
A.9. Robustness with Respect to Detrending

Table A.10 presents the results when we do not detrend inflation and inflation expectations using a measure of long-run inflation expectations.

Table A.10. Results without Detrending with PTR

|       | (1) Michigan 1978–1996 | (2) Michigan 1997–2015 | (3) SPF CPI 1978–1996 | (4) SPF CPI 1997–2015 |
|-------|------------------------|------------------------|-----------------------|-----------------------|
| γ     | 0.182                  | 0.599                  | 0.552                 | 0.334                 |
|       | [–0.101, 0.461]        | [0.155, 1.018]         | [0.184, 0.886]        | [–0.005, 0.696]       |
| φ     | 0.599                  | 0.816                  | 2.905                 | 0.677                 |
| σε    | 1.655                  | 0.552                  | 0.334                 | 0.083                 |
|       | [0.117, 0.422]         | [0.083, 0.886]         | [0.031, 0.134]        | [/0.667]              |
| λ     | 0.668                  | 0.488                  | 0.203                 | 0.566                 |
| μ     | 0.219                  | 0.115                  | 0.184                 | 0.382                 |
|       | [0.117, 0.632]         | [0.054, 0.845]         | [0.102, 0.406]        | [0.249]               |
| σν    | 0.362                  | 0.232                  | 0.292                 | 0.078                 |
|       | [0.117, 0.327]         | [0.076, 0.376]         | [0.128, 0.276]        | [0.102, 0.406]        |
| ρ     | 0.790                  | 0.642                  | 0.187                 | 0.078                 |
|       | [0.117, 0.632]         | [0.076, 0.376]         | [0.128, 0.276]        | [0.102, 0.406]        |
| σω    | 0.390                  | 0.370                  | 0.077                 | 0.445                 |
|       | [0.135, 0.437]         | [0.370, 0.859]         | [0.158, 0.999]        | [0.102, 0.789]        |
| φ1    | 1.312                  | 1.319                  | 1.324                 | 1.333                 |
|       | [0.143, 0.437]         | [0.271, 0.470]         | [0.132, 0.396]        | [0.100, 0.186]        |
| φ2    | 1.135                  | 1.145                  | 1.147                 | 1.161                 |
|       | [0.583, 0.324]         | [0.538, 0.800]         | [0.125, 0.500]        | [0.554, 0.211]        |
| φ3    | 0.407                  | 0.370                  | 0.426                 | 0.382                 |
|       | [0.130, 0.015]         | [0.118, 0.334]         | [0.061, 0.255]        | [0.158, 0.202]        |
| φ4    | 0.057                  | 0.056                  | 0.056                 | 0.025                 |
|       | [0.050, 0.101]         | [0.087, 0.068]         | [0.128, 0.017]        | [0.158, 0.202]        |
| ση    | 0.701                  | 0.580                  | 0.023                 | 0.068                 |
|       | [0.606, 0.797]         | [0.501, 0.660]         | [0.050, 0.101]        | [0.248, 0.115]        |

Note: Estimated using Bayesian methods; the priors are described in appendix B.
Table A.11. Results with a Different Break Date

|       | (1) Michigan 1978–1992:Q2 | (2) Michigan 1992:Q3–2015 | (3) SPF CPI 1978–1992:Q2 | (4) SPF CPI 1992:Q3–2015 |
|-------|---------------------------|---------------------------|--------------------------|--------------------------|
| \( \gamma \) | 0.130 | 0.519 | 0.400 | 0.177 |
|       | [-0.105, 0.356] | [0.130, 0.901] | [0.090, 0.709] | [-0.039, 0.384] |
| \( \kappa \) | 0.216 | 0.135 | 0.315 | 0.075 |
|       | [0.068, 0.364] | [0.061, 0.209] | [0.131, 0.489] | [0.029, 0.118] |
| \( \sigma_\varepsilon \) | 1.768 | 0.723 | 2.355 | 0.555 |
|       | [1.315, 2.219] | [0.495, 0.954] | [1.752, 2.948] | [0.434, 0.666] |
| \( \lambda \) | 0.744 | 0.324 | 0.760 | 0.481 |
|       | [0.626, 0.859] | [0.067, 0.706] | [0.662, 0.864] | [0.292, 0.687] |
| \( \mu \) | 0.232 | 0.007 | 0.172 | 0.306 |
|       | [0.113, 0.359] | [-0.297, 0.314] | [0.093, 0.250] | [0.144, 0.469] |
| \( \sigma_\nu \) | 0.364 | 0.180 | 0.191 | 0.070 |
|       | [0.220, 0.507] | [0.059, 0.293] | [0.095, 0.280] | [0.029, 0.111] |
| \( \rho \) | 0.575 | 0.796 | 0.654 | 0.455 |
|       | [0.135, 0.992] | [0.660, 0.940] | [0.220, 1.000] | [0.107, 0.761] |
| \( \sigma_\omega \) | 0.318 | 0.359 | 0.187 | 0.132 |
|       | [0.144, 0.493] | [0.276, 0.437] | [0.075, 0.291] | [0.093, 0.169] |
| \( \phi_1 \) | 1.235 | 1.296 | 1.249 | 1.305 |
|       | [1.038, 1.435] | [1.137, 1.455] | [1.050, 1.450] | [1.148, 1.462] |
| \( \phi_2 \) | -0.288 | -0.307 | -0.305 | -0.315 |
|       | [-0.489, -0.087] | [-0.469, -0.142] | [-0.508, -0.104] | [-0.477, -0.153] |
| \( \phi_3 \) | -0.098 | -0.060 | -0.100 | -0.098 |
|       | [-0.178, -0.018] | [-0.253, 0.136] | [-0.180, -0.020] | [-0.300, 0.096] |
| \( \phi_4 \) | -0.044 | -0.162 | -0.038 | -0.117 |
|       | [-0.132, 0.043] | [-0.358, 0.029] | [-0.127, 0.048] | [-0.313, 0.079] |
| \( \sigma_\eta \) | 0.728 | 0.553 | 0.729 | 0.553 |
|       | [0.610, 0.845] | [0.484, 0.619] | [0.609, 0.842] | [0.487, 0.621] |

Note: Estimated using Bayesian methods; the priors are described in appendix B.
### A.11. ATSIX 12-Month-Ahead CPI Inflation Expectations

In table A.12 we present a robustness analysis for our late-sample SPF CPI inflation expectations.

#### Table A.12. Estimates with ATSIX 12-Month-Ahead Expectations and Comparison with Late Sample SPF CPI

|         | (1) ATSIX 1997–2015 | (2) SPF CPI 1997–2015 |
|---------|----------------------|------------------------|
| γ       | 0.253                | 0.211                  |
|         | [0.002, 0.488]       | [-0.008, 0.422]        |
| κ       | 0.089                | 0.077                  |
|         | [0.039, 0.132]       | [0.033, 0.120]         |
| σε      | 0.625                | 0.600                  |
|         | [0.475, 0.775]       | [0.467, 0.722]         |
| λ       | 0.492                | 0.454                  |
|         | [0.338, 0.655]       | [0.299, 0.621]         |
| μ       | 0.280                | 0.336                  |
|         | [0.148, 0.407]       | [0.196, 0.472]         |
| σν      | 0.064                | 0.064                  |
|         | [0.028, 0.100]       | [0.021, 0.104]         |
| ρ       | 0.509                | 0.256                  |
|         | [0.193, 0.835]       | [0.000, 0.470]         |
| σω      | 0.094                | 0.135                  |
|         | [0.054, 0.130]       | [0.099, 0.171]         |
| φ₁      | 1.334                | 1.331                  |
|         | [1.161, 1.506]       | [1.161, 1.502]         |
| φ₂      | -0.347               | -0.343                 |
|         | [-0.522, -0.166]     | [-0.517, -0.165]       |
| φ₃      | -0.068               | -0.082                 |
|         | [-0.283, 0.151]      | [-0.296, 0.138]        |
| φ₄      | -0.128               | -0.120                 |
|         | [-0.347, 0.086]      | [-0.335, 0.101]        |
| ση      | 0.582                | 0.581                  |
|         | [0.499, 0.659]       | [0.500, 0.660]         |
Appendix B. Priors

Table B.1 summarizes the priors we use for the Bayesian estimation of the model we introduce in Section 2.1 in the paper.

Table B.1. Priors for Bayesian Estimation

|     | Mean | SD  | Distribution | Min. | Max. |
|-----|------|-----|--------------|------|------|
| $\gamma$ | 0.5  | 0.5 | Normal       | −1   | 3    |
| $\kappa$ | 0.1  | 0.5 | Normal       | −10  | 10   |
| $\sigma_\varepsilon$ | 1    | 0.5 | Beta         | 0    | 5    |
| $\lambda$ | 0.75 | 0.5 | Normal       | −2   | 2    |
| $\mu$ | 0.25 | 0.5 | Normal       | −10  | 10   |
| $\theta_3$ | 0    | 0.5 | Normal       | −10  | 10   |
| $\theta_4$ | 0    | 0.5 | Normal       | −10  | 10   |
| $\sigma_\nu$ | 1    | 0.5 | Beta         | 0    | 5    |
| $\rho$ | 0.5  | 0.5 | Normal       | 0    | 1    |
| $\sigma_\omega$ | 1    | 0.5 | Beta         | 0    | 5    |
| $c_1$ | 0.1  | 2   | Normal       | −10  | 10   |
| $c_2$ | 0.1  | 2   | Normal       | −10  | 10   |
| $\phi_1$ | 1.3  | 0.5 | Normal       | −10  | 10   |
| $\phi_2$ | −0.5 | 0.5 | Normal       | −10  | 10   |
| $\phi_3$ | 0.1  | 1   | Normal       | −10  | 10   |
| $\phi_4$ | 0.1  | 1   | Normal       | −10  | 10   |
| $\sigma_\eta$ | 1    | 0.5 | Beta         | 0    | 5    |
| $\text{corr}(\sigma_\eta, \sigma_\varepsilon)$ | 0    | 0.5 | Normal       | −1   | 1    |
| $\text{corr}(\sigma_\nu, \sigma_\varepsilon)$ | 0    | 0.5 | Normal       | −1   | 1    |
| $\phi_5$ | 0    | 1   | Normal       | −10  | 10   |
| $\phi_6$ | 0    | 1   | Normal       | −10  | 10   |
| $\phi_7$ | 0    | 1   | Normal       | −10  | 10   |
| $\phi_8$ | 0    | 1   | Normal       | −10  | 10   |
| $c_p$ | 0.1  | 2   | Normal       | −10  | 10   |
| $c_y$ | 0.1  | 2   | Normal       | −10  | 10   |