Warning about Dark Matter Direct Detection in the Presence of LIGO-Motivated Primordial Black Holes

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We discuss formation of dark matter (DM) mini-halos around primordial black holes (PBHs) and its implication on DM direct detection experiments. Motivated by LIGO observations, we take \( f_{\text{DM}} \lesssim 0.01 \) as the fraction of DM in PBHs with masses \( 10M_\odot \sim 70M_\odot \). Under these conditions, previous computations show PBHs may capture more than half of the DM background at the time of the first galaxies. Our analytical estimates suggest that a significant part of the DM background ends up localized as mini-halos around PBHs inside galactic halos is plausible. High-resolution simulations are encouraged. If the proposed scenario is realized, DM direct detection experiments would be compromised.

INTRODUCTION

Nowadays, the idea of PBH dark matter has been strongly revitalized since the first detection of two merging black holes by the LIGO-Virgo collaboration [1]. This first detection (GW150914) corresponds to the merger of two black holes with masses \( \sim 30M_\odot \). Assuming all black hole binaries relevant to the LIGO observation share the same mass and other physical parameters, the estimation of dark matter in PBHs is estimated in \([14]\). The possibility of a clustered spatial distribution, the fraction of dark matter in PBHs is estimated in \([14]\). The merger of PBH binaries can be translated into an (potential) upper bound on \( f_{\text{DM}} \equiv \Omega_{\text{PBH}}/\Omega_{\text{DM}} \). This bound is stronger than most of current observational constraints \([3,10]\) for \( 10\sim 300M_\odot \) PBHs \([1]\). If PBHs comprise a significant fraction of dark matter, they would produce a merger rate larger than those observed by LIGO \([8,12,14]\).

Assuming an extended PBH mass function and the possibility of a clustered spatial distribution, the fraction of dark matter in PBHs is estimated in \([14]\). The merger of PBH binaries can explain the merger rate inferred by LIGO \([15]\) if \( f_{\text{DM}} = 4.5 \times 10^{-3} - 2.4 \times 10^{-2} \) for \( 30M_\odot \) PBHs and a lognormal mass function.

By considering tidal forces over the PBH binary coming from all remaining PBHs and standard large-scale adiabatic perturbations, potential upper bounds on \( f_{\text{DM}} \) as a function of PBH masses are discussed in \([13]\). LIGO O1 run \([16,18]\) would constrain \( \sim 10M_\odot - 300M_\odot \) PBHs for a fraction of dark matter no more than 1%. In particular, potential upper bounds for \( 10M_\odot \) and \( 300M_\odot \) PBHs are reported to \( f_{\text{DM}} \lesssim 8 \times 10^{-3} \) and \( 9 \times 10^{-3} \), respectively.

Generally speaking, the PBH merger rate may be affected by the cosmological evolution of binaries. After formation during the early Universe, some PBH binaries may be disrupted before merging. Analytical estimates suggest that most of the PBH binaries evolve without disruption until merger \([13]\). This is expected for the case \( f_{\text{DM}} \ll 1 \), but as the fraction of dark matter in PBHs increases, disruption of binaries should become more significant. A robust calculation of the binary merger rate needs to include a suitable suppression in the merger rate due to disruption coming from a third PBH close and/or dense PBH clusters. A small boost in the upper bound of \( f_{\text{DM}} \) is reported in \([19]\) after including a suppression factor on the binary merger rate coming from this effect.

Current constraints on \( f_{\text{DM}} \) derived from gravitational wave observations include several theoretical uncertainties. To obtain a robust conclusion, a complete set of numerical simulations covering several highly complex (but well-defined) astrophysical processes is needed. For our purposes, motivated by LIGO-Virgo detections and observational constraints, we take the conservative value \( f_{\text{DM}} \sim 0.01 \) for the characteristic fraction of dark matter in \( \sim 10M_\odot - 70M_\odot \) PBHs \([1]\).

To find the second dark matter component to accompany PBHs we need to look for beyond the Standard Model of particle physics. By considering shortcomings in this model, the weakly interacting massive particle (WIMP) and the axion \([20,23]\) are the strongest candidates. However, the WIMP and PBHs cannot coexist.

1 We mention that CMB constraints reported by Ricotti et al. (2008) \([11]\) overestimated the effects of PBHs on CMB observables by a factor of \( \mathcal{O}(10^2) \) as was discussed in \([10]\). Thus, we do not consider these constraints in our analysis.

2 Here we take into account CMB-anisotropy constraints from photoionization of the local gas from PBH radiation \([10]\).
unless the dark matter fraction in PBHs is highly suppressed \(^2\). Primordial black holes are expected to acquire dark mini-halos from the surrounding dark matter background. If this background is composed by WIMPs, then inner regions of these halos would undergo WIMPs self-annihilation becoming strong sources of gamma rays and neutrinos. This set up assumes self-annihilation of WIMPs during the late Universe into standard model particles; so there is no significant WIMP-antiWIMP asymmetry, or dominant annihilation into hidden sectors. If the LIGO events are due to PBHs, we can safely exclude the standard WIMP scenario\(^3\). So arguably, the most serious dark matter candidate to accompany PBH dark matter is the axion.

Even though we are primarily motivated by the axion in this work, we are not excluding other possible dark matter candidates. Since the formation of dark mini-halos around PBHs holds for any dark matter candidate, such as axion-like particles or generic dark matter, conclusions in this letter are extensive to any kind of candidate which forms the remaining 99% of dark matter. However, we expect accretion will not be efficient for ultra-light axions (or ultra-light scalar dark matter particles) when their De Broglie wavelength is comparable with or larger than the halo radius.

After PBHs formation in the early Universe, they will acquire a mini-halo from the dark matter background (a PBH with a mini-halo is sometimes called in the literature as dressed PBH). The theory of spherical gravitational collapse in an expanding Universe \(^2\) tells us that any overdensity within a sphere will seed the growth of a mini-halo. PBHs are local overdensities in the dark matter distribution and they act as seeds for the formation of dark matter structures. Analytical and numerical calculations \(^2\) show the dark mini-halo growth during the radiation-dominated era is only the order of the unity in units of the central PBH mass, \(M_{PBH}\). However, this growth can reach up to \(\sim 10^2 M_{PBH}\) during the matter-dominated era. During this era, the dark mini-halo mass and radius are parameterized in terms of the redshift as \(^1\)\(^\text{[11]}\)\(^\text{[27]}\)

\[
M_{\text{halo}}(z) = 3 \left( \frac{1000}{1 + z} \right) M_{PBH}, \quad (1)
\]

\[
R_{\text{halo}}(z) = 0.019 \text{pc} \left( \frac{M_{\text{halo}}}{M_\odot} \right)^{1/3} \left( \frac{1000}{1 + z} \right), \quad (2)
\]

where \(R_{\text{halo}}\) is about one-third of the turnaround radius. Equations \(^1\)\(^\text{[12]}\) assume a growth of the dark mini-halo in absence of external tidal forces. This assumption only holds before the epoch of formation of the first galaxies at redshift \(z \sim 30\). At that time, we expect dressed PBHs to begin to interact with non-linear structures. On the other hand, as dressed PBHs begin to form a significant part of the dark matter, the halo growth rate begins to decrease \(^2\). Thus, \(M_{\text{halo}}(z \sim 30)/M_{PBH} \simeq 10^2\) needs to be seen as an upper bound for the maximum mass of a PBH halo at that time.

From the above, we need to take seriously the possibility that nowadays a significant part of dark matter is part of dark mini-halos surrounding PBHs inside galactic halos. If this is the case, then the capability of earth based direct detection experiments, such as the ability of ADMX to detect axions \(^2\)\(^\text{[28]}\)\(^\text{[29]}\), would be compromised.

Even though we are mainly interested in LIGO-motivated PBHs with masses \(10 M_\odot - 70 M_\odot\), our warning is extensive to PBHs with larger, or smaller, masses. If we consider only collisional ionization of the background gas in CMB-anisotropy constraints, e.g. the radiation coming from PBHs is not able to ionize the local gas \(^1\), one may extend the mass range of PBHs of our interest to \(\sim 10 M_\odot - 300 M_\odot\) for \(f_{DM} \simeq 0.01\). In addition, primordial black holes which are formed with a mass \(M_{PBH} \geq 1.6 \times 10^{-17} M_\odot\) do not evaporate \(^4\) but begin to form compact dark-mini halos by accreting the surrounding dark matter. Since they may compose 1\% of dark matter without tension with observational constraints \(^3\)\(^\text{[30]}\)\(^\text{[34]}\), they may accrete in the form of mini-halos a significant fraction of dark matter background.

Previous studies have focused on analyzing the presence of Earth mass dark matter clumps (which are formed without the intervention of PBHs as seeds) inside Milky Way halo and their impact on signatures for direct and indirect detection of dark matter. Several cosmological simulations suggest that a significant number of them survive the effect of tidal forces \(^5\)\(^\text{[35]}\)\(^\text{[39]}\) with the subsequent potential effect on dark matter detection. Others suggest these dark matter substructures would have a negligible effect today on dark matter detection experiments \(^6\)\(^\text{[40]}\)\(^\text{[41]}\).

As far as we know, this is the first time that LIGO-motivated PBHs with dark mini-halos are proposed as a potential threat for direct detection of dark matter on the Earth.

**BINARY MERGER RATE IN THE PRESENCE OF DARK HALOS AROUND PBHS**

We expect that PBHs in binaries also acquire dark mini-halos before and after the binary formation. In this section, we will adopt notation used in \(^5\). Suppose that

\(^3\) WIMPs with masses \(\gtrsim 100\text{TeV}\) would evade this kind of constraint (private communication with S. Shirai).

\(^4\) Primordial black holes lighter than \(3 \times 10^{-19} M_\odot\) evaporate within the present age of the Universe.
\( \bar{x} \) is the mean physical separation of PBHs with mass \( M_{\text{PBH}} \) at the time of matter-radiation equality, \( t_{\text{eq}} \), defined as

\[
\bar{x} = \left( \frac{M_{\text{PBH}}}{\rho_{\text{PBH}}(t_{\text{eq}})} \right)^{1/3} = \frac{f_{\text{DM}}^{-1/3}}{(1 + z_{\text{eq}})} \left( \frac{M_{\text{PBH}}}{\Omega_{\text{DM}}\rho_{c,0}} \right)^{1/3},
\]

where we have assumed a monochromatic mass function for PBHs and \( \bar{x} \sim 3 \times 10^{15} \text{m} (M_{\text{PBH}}/M_{\odot})^{1/3}/(f_{\text{DM}})^{-1/3} \). For the mass range of our interest, binaries merging at the present time are formed at \( z \lesssim z_{\text{eq}} \) [13]. Suppose that two neighboring PBHs are separated by a physical distance \( x \) at \( t_{\text{eq}} \), where \( x \leq \bar{x} \). Define \( z_{\text{dec}} \) as the redshift at which the pair of PBHs decouples from the Hubble flow and form a gravitationally bound system. The binary is formed when the local energy density of the pair \( \rho_{\text{binary}}(z_{\text{dec}}) \) exceeds the radiation energy density \( \rho_{r}(z_{\text{dec}}) \), e.g.

\[
\frac{1}{1 + z_{\text{dec}}} \approx f_{\text{DM}} \left( \frac{\bar{x}}{x} \right)^3.
\]

The semi-major axis of the binary at the formation time can be written as \( a \sim x(1 + z_{\text{eq}})/(1 + z_{\text{dec}}) \). Using Eq. (1) we obtain \( a \sim x^4/(\bar{x}^3 f_{\text{DM}}) \). More detailed analytical and numerical results in [11] [13] suggest \( a \sim x^4/(\bar{x}^3 f_{\text{DM}}) \), where \( \alpha = 0.4 \). If the binary decouples at \( z_{\text{dec}} = z_{\text{eq}} \), we have \( x = f_{\text{DM}}^{1/3} \bar{x} \) at the decoupling time. Thus, the maximum semi-major axis of the PBH binary at the time of formation reads as

\[
a_{\text{max}} = \alpha f_{\text{DM}}^{1/3} \bar{x}.
\]

Including the formation of a dark mini-halo around PBHs will slightly modify this relation. From Eq. (1), we see that PBHs acquire a mini-halo between their formation and the formation of binaries. The energy density of the binary at redshift \( z \) will increase by a factor equal to \( (1 + M_{\text{halo}}(z)/M_{\text{PBH}}) \), where \( M_{\text{halo}}(z) \) is given by Eq. (1). Then, PBH binaries will decouple earlier during the radiation dominated era. We can rewrite the semi-major axis of the binary as \( a \sim \alpha x^4 \rho_{\text{eq}}/(2M_{\text{PBH}}) \), where \( \rho_{\text{eq}} \) is the energy density at the matter-radiation equality. Then, considering the mini-halo mass at the decoupling redshift, we have \( a_{\text{halo}} = a/[1 + M_{\text{halo}}(z_{\text{dec}})/M_{\text{PBH}}] \) for fixed \( x \). Here \( a_{\text{halo}} \) indicates the semi-major axis under the presence of a dark mini-halo around each PBH at the time of binary formation. Similarly, for the maximum semi-major axis of the binary, we have

\[
a_{\text{max, halo}} = \left[ 1 + \frac{M_{\text{halo}}(z_{\text{eq}})}{M_{\text{PBH}}} \right]^{1/3} a_{\text{max}} = 2^{1/3} a_{\text{max}},
\]

where we have used \( M_{\text{halo}}(z_{\text{eq}}) = M_{\text{PBH}} \) [27]. The maximum value for the semi-major axis of binaries is enhanced by \( \sim 26\% \) in the presence of dark mini-halos around PBHs.

The eccentricity of the binary orbit at the time of formation is directly related to the angular momentum per unit of reduced mass as \( j = \sqrt{1 - e^2} \). A simple estimate of this angular momentum is obtained by multiplying the torque over the binary exerted by the nearest third PBH and the free-fall time as \( j \sim (x/y)^2 \). Here \( y \) is the physical distance to the third PBH at \( t_{\text{eq}} \). A more detailed analysis is performed in [4] by including tidal forces from all surrounding PBHs suggesting \( j = \beta(x/y)^3 \), where \( \beta = 0.8 \). The presence of these tidal forces on the binary avoids a head-on collision between the two PBHs which form the binary. The addition of dark mini-halos around PBHs would not have a significant impact on the eccentricity at the time of binary formation. The torque exerted on the binary will increase as the binary formation occurs deeper in the early Universe. However, the formation of dark mini-halos is not significant at early times because there had not been enough time for dark matter accretion.

In agreement with estimates and numerical computations reported in [12], our estimates suggest the presence of dark mini-halos would have only a small effect on the parameter space defined by \( (a, e) \) at the time of binary formation. In addition, these authors performed N-body simulations of orbiting PBHs and their corresponding dark mini-halos after binary formation. Their main result is that the semi-major axis and the eccentricity of binaries will both decrease as a result of the dynamical friction and tidal forces exerted by dark mini-halos. This process occurs in a short time scale after binary formation, \( \mathcal{O}(10^4 \text{yr}) \) for binaries composed by \( 30M_{\odot} \) PBHs. After orbit parameters stabilize, the binary would decay in the usual way by gravitational radiation. Binary shrinking and orbit circularization tend to compensate each other with respect to their effects on the binary merger rate because, for an initial \( (q_0, e_0) \), the binary coalescence time through emission of gravitational waves reads as \( t_{\text{coal}} \sim a_0^3(1 - e_0^{2})^{7/2}/M_{\text{PBH}}^3 \) [13].

Limits on \( f_{\text{DM}} \) are about 2 times stronger than those derived by the case of naked PBHs in [13]. For the mass range 10 \( M_{\odot} \) – 300 \( M_{\odot} \), the presence of dark halos constrain \( f_{\text{DM}} \) to be no more than \( 4 \times 10^{-3} \) [12]. However, this constraint does not consider disruption of dressed PBH binaries after their formation coming from other isolated PBHs or clustered spatial distribution. This disruption should be enhanced in the presence of dark mini-halos.

Since all constraints on \( f_{\text{DM}} \) are subject to a wide spectrum of uncertainties and caveats, we consider that our conservative upper bound \( f_{\text{DM}} \approx 0.01 \) for \( 10M_{\odot} \) – 70 \( M_{\odot} \) PBHs still holds as a reasonable possibility.
EQUATION OF DRESSED PBHS

In our specific case, when PBHs correspond to an initial fraction of dark matter $f_{DM} \approx 0.01$, dressed PBHs will eventually begin to dominate the dark matter abundance during the accretion process. At that time, the accretion rate will become slower due to the decrease in the dark matter density background. In the PBH mass range of our interest, numerical simulations reported in [27] suggest the final mass of dark halos accreted by PBHs will decrease $\sim 50\% - 55\%$ of the original matter background at $z \sim 30$. At around this redshift, the dressed PBHs will begin to interact with non-linear systems to be finally incorporated (together to the remaining dark matter background) to galactic halos at $z \sim 6$ (for a review about cosmological structure formation see, for example, [44]). As galaxies evolve, further isolated or clustered dressed PBHs, as well as dark matter background, may be incorporated to galactic halos. Taking into consideration that clustering of dressed PBHs will enhance the accretion rate of dark matter, we expect that the accretion of the smooth dark matter background from PBHs inside galactic halos should keep going as a continuous process. On the other hand, we also expect dark mini-halos undergo different levels of disruption. The dominant disruption processes acting on dark matter substructures correspond to tidal forces coming from the mean-field potential of the Milky Way (tidal stripping) and the interaction with stars from the galactic disk (tidal shocking). Large dark matter substructures with masses greater than $10^7 M_\odot$ would be completely disrupted within a galactic radius of 30 kpc, as suggested by N-body simulations performed in [45]. By contrast, the fate of smaller and denser dark matter substructures is not totally clear and there is a possibility that a significant part of them survive until today in the solar neighborhood.

Even if we assume a narrow mass function for PBHs, we expect to have a broader mass distribution for mini-halos in Milky Way halo. Since we do not know the exact evolution of dressed PBHs between the time of the first galaxies and the time of galactic halos formation, we will assume the presence of dressed PBHs in Milky Way halo having halo mass and radius similar to those at $z \sim 30$ from Eqs. [12]. In the following analysis, we will normalize the halo mass and radius with respect to $M_{\text{max}}$ and $R_{\text{max}}$, where $M_{\text{max}} \equiv M_{\text{halo}}(R_{\text{max}}) \sim 10^2 M_{\text{PBH}}$.

The high density of dark mini-halos around PBHs offers them some level of protection against tidal stripping. In the point-like approximation, the tidal radius is calculated to be [46, 47]

$$r_{\text{tidal}} = \left( \frac{M_{\text{halo}}(R_{\text{halo}}) + M_{\text{PBH}}}{3 M_{\text{MW}}} \right)^{1/3} R_{p},$$

(7)

Here $R_{p}$ and $M_{\text{MW}} \sim 9 \times 10^{11} M_\odot$ correspond to the perigalactic distance of the dressed PBH and the Milky Way mass, respectively.

As a dressed PBH crosses through the galactic disk, high-speed interactions with stars may lead to mini-halo disruption. On average, we do not consider very close encounters which may immediately lead to a total mini-halo disruption. Using the distant-tide approximation, the disruption probability of a dark mini-halo crossing the stellar field of the disk is estimated as [40]

$$P_{\text{disrup}} \approx \frac{4 G N M_{\ast} n_{\ast} t_{\text{halo}} R_{\text{halo}}}{v_{\text{DM}}},$$

(8)

where $M_{\ast}$ is the typical star mass, $n_{\ast} \sim 0.1 \text{pc}^{-3}$ is the number density of stars in the galactic disk, and $v_{\text{DM}} \sim [G N (M_{\text{halo}}(R_{\text{halo}}) + M_{\text{PBH}})/R_{\text{halo}}]^{1/2}$ is the velocity of the dark matter particles. The dark mini-halo will be completely disrupted at $t_{\text{disrup}}$ when this probability is equal to the unity. Consider that the crossing time is estimated to be $t_{\text{cross}} \sim \text{Myr} \left( H/150 \text{pc} \right)/\left( V_{\odot}/300 \text{km/s} \right)$, where $H$ is the half-height of the disk and $V_{\odot}$ is the dressed PBH velocity perpendicular to the disk plane. We define $N_{\text{disrup}} \equiv t_{\text{disrup}}/t_{\text{cross}}$ as the number of disk crossings needed so that a dressed PBH is totally disrupted. Assuming an age of the galaxy $\sim 10 \text{Gyr}$, circulor orbits of dressed PBHs in the galactic frame, and the M11 Milky Way model [45], the number of disk crossings at the sun perigalactic distance is $N_{\text{cross}}(R_c) \sim 90$ [29].

By calculating $R_{p}$ at which $r_{\text{tidal}}/R_{\text{halo}} = 1$, Fig. 1 (top) shows dressed PBHs are relatively safe from tidal stripping at $R_c$. In addition, as we slowly move towards the galactic center, smaller dressed PBHs are more resistant under disruption. Figure 1 (bottom) shows that the number of crossing of dressed PBHs needed for total mini-halo disruption is the order of $N_{\text{cross}}(R_c)$. However, when we consider smaller dressed PBHs their resistance under tidal shocking significatively increases. Here we are not considering the internal structure of dressed PBHs, the angular distribution of their orbits, and the fact that PBHs should accrete dark matter between disk crossings. Since mini-halo density profile evolves as dressed PBHs cross the galactic disk, our result suggests that successive stellar encounters would be less effective to disrupt mini-halos (see [30, 31] for the case of dark matter clumps).

Even though both estimates from above are very rough, they give us some room to encourage further analysis. Since all astrophysical processes involved at low redshifts hold in a highly non-linear regime, a full set of high-resolution cosmological simulations is required to determine how much of the axion dark matter background would finally ends up incorporated to dark mini-halos around PBHs inside galactic halos. These set of simulations should include internal structure of dressed PBHs and their radial profile evolution under interactions, non-linear gravitational clustering, and continuous accretion.
in mini-halos around PBHs in the Milky Way halo, the number of encounters between the Earth and a dressed PBH per unit of time is given by

$$N_{\bigodot}^{PBH_d} = n_{PBH_d} \times \sigma_{\text{eff}} v_{\text{rel}}, \quad (9)$$

where $n_{PBH_d}$ is the number density of dressed PBHs, $\sigma_{\text{eff}}$ is the effective cross section for the encounter between the Earth and a dressed PBH, and $v_{\text{rel}} \approx 3 \times 10^5 \text{ km/s}$ is the relative velocity between both astrophysical objects. The number density of dressed PBHs in the Milky Way halo is given by $n_{PBH_d} = \rho_{DM,\text{local}}/[M_{\text{halo}}(z) + M_{PBH}]$. The effective cross section is given by the usual geometrical cross section enhanced by the gravitational focusing as $\sigma_{\text{eff}} = \pi (R_{\text{halo}} + R_\bigodot)^2 [1 + (v_{\text{esc}}/v_{\text{rel}})^2]$. Here $R_\bigodot$ is the Earth radius and $v_{\text{esc}} = (2GM_{\text{halo}}(z) + M_{PBH}/R_{\text{halo}})^{1/2}$ defines the escape velocity of the Earth from the gravitational pulling of a dressed PBH. Taking $M_{PBH} \sim 10^2 M_{\odot}$ and $R_{\text{halo}} \sim 3 \text{ pc} (M_{PBH}/M_\odot)$, we obtain $N_{\bigodot}^{PBH_d} \sim 0.6 \text{ Myr}^{-1}$. Hence the chances of direct detection of DM in these mini-halos is extremely small.

On the other hand, in the case that future numerical simulations report a significant clumpiness in dark mini-halos around PBHs in the Milky-Way halo, a positive direct detection in ground-based experiments, such as by ADMX for axions, would suggest that LIGO gravitational wave detections are not caused by the merger of PBH binaries (for possible explanations about the origin of LIGO black holes and the formation of their binaries see [16] and references therein).

**ACKNOWLEDGMENTS**

T. T. Y. is supported in part by the China Grant for Talent Scientific Start-Up Project and the JSPS Grant-in-Aid for Scientific Research No. 16H02176, No. 17H02878, and No. 19H05810 and by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. T. T. Y. thanks to Hamamatsu Photonics. M. P. H. is supported in part by National Science Foundation grant PHY-1720332.

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