Searching for lepton flavor violating decays $\tau \rightarrow P l$ in Minimal
R-symmetric Supersymmetric Standard Model

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Abstract

Considering the constraints from the experimental data on branching ratios of $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$, we analyze the lepton flavor violating decays $\tau \rightarrow P l$ ($P = \pi^0, \eta, \eta'; l = e, \mu$) in the scenario of the minimal R-symmetric supersymmetric standard model. The numerical results show that the theoretical predictions on branching ratios of $\tau \rightarrow P l$ can be enhanced close to the upper experimental bounds or future sensitivities, meanwhile the theoretical predictions on branching ratios of $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ satisfy the present experimental bounds.

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I. INTRODUCTION

Searching for Lepton Flavor Violating (LFV) decays are of great importance in probing New Physics (NP) beyond the Standard Model (SM) in which the theoretical predictions on those LFV are suppressed by small masses of neutrinos, and it is well known that neutrinoless semileptonic $\tau$ decays provide an ideal tool to search for NP. The present upper bounds on the LFV decays $\tau \rightarrow Pl (P = \pi^0, \eta, \eta^\prime; \ l = e, \mu)$ are shown in TABLE I. Assuming the integrated luminosity of 50 ab$^{-1}$, the future prospects of BR($\tau \rightarrow Pl$) in Belle II are extrapolated at the level of $O(10^{-9} - 10^{-10})$. In various extensions of the SM, the corrections to the branching ratios of LFV decays $\tau \rightarrow Pl$ are enhanced by new LFV sources. There are a few studies within models of non-SUSY, such as two Higgs doublet models [5, 6], 331 model [7], TC2 models [8], littlest Higgs model with T parity [9], leptoquark models [10, 11] and unparticle model [12]. Some models with heavy Dirac/Majorana neutrinos can have BR($\tau \rightarrow Pl$) close to the experimental sensitivity [13–16]. In Type III seesaw model, there are tree level flavor changing neutral currents in the lepton sector can enhance the predictions on BR($\tau \rightarrow Pl$) [17, 18]. There are also a few studies within models of SUSY, such as MSSM [19, 20], unconstrained MSSM [21], supersymmetric seesaw mechanism model [22], R-parity violating SUSY [23], the CMSSM-seesaw and NUHM-seesaw [24]. Within an effective field theory framework, the predictions on BR($\tau \rightarrow Pl$) or constraints from these LFV decays on the Wilson coefficients of LFV operators are studied [11, 25–34]. An overview on status of $\tau$ physics can be found in Ref. [35].

In this paper, we will study the LFV decays $\tau \rightarrow Pl$ in the Minimal R-symmetric Supersymmetric Standard Model (MRSSM) [36]. The MRSSM has an unbroken global $U(1)_R$ symmetry and provides a new solution to the supersymmetric flavor problem in MSSM. In
this model, R-symmetry forbids Majorana gaugino masses, $\mu$ term, $A$ terms and all left-right squark and slepton mass mixings. The $R$-charged Higgs $SU(2)_L$ doublets $\hat{R}_u$ and $\hat{R}_d$ are introduced in MRSSM to yield the Dirac mass terms of higgsinos. Additional superfields $\hat{S}$, $\hat{T}$ and $\hat{O}$ are introduced to yield Dirac mass terms of gauginos. Studies on phenomenology in MRSSM can be found in literatures [37–54]. Similar to the case in MSSM, the LFV decays mainly originate from the off-diagonal entries in slepton mass matrices $m^2_l$ and $m^2_r$. Taking account of the constraints from radiative decays $\tau \to l\gamma$ on the off-diagonal parameters, we explore $\tau \to Pl$ as a function of the off-diagonal parameters and other model parameters.

The paper is organized as follows. In Section [II] we provide a brief introduction on MRSSM. In Section [III] we present our notation and conventions for the operators and their corresponding Wilson coefficients. Then we present the Wilson coefficients for Feynman diagrams contributing to $\tau \to Pl$ in MRSSM in detail. The numerical results are presented in Section [IV] and the conclusion is drawn in Section [V].

II. MRSSM

In this section, we firstly provide a simple overview of MRSSM in order to fix the notations we use in this paper. The MRSSM has the same gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ as the SM and MSSM. The spectrum of fields in MRSSM contains the standard MSSM matter, Higgs and gauge superfields augmented by chiral adjoints $\hat{O}$, $\hat{T}$, $\hat{S}$ and two $R$-Higgs iso-doublets. The superfields with R-charge in MRSSM are given in TABLE II. The general

| Field       | Superfield | Boson | Fermion |
|-------------|------------|-------|---------|
| Gauge vector| $\hat{g}, \hat{W}, \hat{B}$ | 0     | $g, W, B$ | 0 | $\tilde{g}, \tilde{W}, \tilde{B}$ | +1 |
| Matter      | $\hat{l}, \hat{e}^c$ | +1 | $\hat{l}, \hat{e}^c$ | +1 | $l, e^c$ | 0 |
|            | $\hat{q}, \hat{d}, \hat{u}^c$ | +1 | $\hat{q}, \hat{d}, \hat{u}^c$ | +1 | $q, d, u^c$ | 0 |
| $H$-Higgs   | $\hat{H}_{d,u}$ | 0 | $H_{d,u}$ | 0 | $\tilde{H}_{d,u}$ | -1 |
| $R$-Higgs   | $\hat{R}_{d,u}$ | +2 | $R_{d,u}$ | +2 | $\tilde{R}_{d,u}$ | +1 |
| Adjoint chiral | $\hat{O}, \hat{T}, \hat{S}$ | 0 | $O, T, S$ | 0 | $\tilde{O}, \tilde{T}, \tilde{S}$ | -1 |
The form of the superpotential of the MRSSM is given by [37]

\[
W_{\text{MRSSM}} = \mu_d(\hat{R}_d\hat{H}_d) + \mu_u(\hat{R}_u\hat{H}_u) + \Lambda_d(\hat{R}_d\hat{T})\hat{H}_d + \Lambda_u(\hat{R}_u\hat{T})\hat{H}_u + \lambda_d\hat{S}(\hat{R}_d\hat{H}_d) \\
+ \lambda_u\hat{S}(\hat{R}_u\hat{H}_u) - Y_d\hat{q}\hat{H}_d - Y_e\hat{e}(\hat{l}\hat{H}_d) + Y_u\hat{u}(\hat{q}\hat{H}_u),
\]

where \(\hat{H}_u\) and \(\hat{H}_d\) are the MSSM-like Higgs weak iso-doublets, \(\hat{R}_u\) and \(\hat{R}_d\) are the \(R\)-charged Higgs \(SU(2)_L\) doublets and the corresponding Dirac higgsino mass parameters are denoted as \(\mu_u\) and \(\mu_d\). Although R-symmetry forbids the \(\mu\) terms of the MSSM, the bilinear combinations of the normal Higgs \(SU(2)_L\) doublets \(\hat{H}_u\) and \(\hat{H}_d\) with the Higgs \(SU(2)_L\) doublets \(\hat{R}_u\) and \(\hat{R}_d\) are allowed in Eq. (1). Parameters \(\lambda_u\), \(\lambda_d\), \(\Lambda_u\) and \(\Lambda_d\) are Yukawa-like trilinear terms involving the singlet \(\hat{S}\) and the triplet \(\hat{T}\).

For our phenomenological studies we take the soft-breaking terms involving scalar mass that have been considered in [39]

\[
V_{SB,S} = m^2_{\text{H}_d}(|H^0_d|^2 + |H^0_d|^2) + m^2_{\text{H}_u}(|H^0_u|^2 + |H^+_{\mu}|^2) + (B_\mu(H^-_dH^+_u - H^-_dH^0_u) + h.c.) \\
+ m^2_{\text{H}_d}(|R^0_d|^2 + |R^0_d|^2) + m^2_{\text{H}_u}(|R^0_u|^2 + |R^-_u|^2) + m^2_T(|T^0|^2 + |T^-|^2 + |T^+|^2) \\
+ m^2_S|S|^2 + m^2_O|O|^2 + d^2_{\text{L},i}m^2_{\text{L},ij}\tilde{d}_{\text{L},j} + d^2_{\text{R},i}m^2_{\text{R},ij}\tilde{d}_{\text{R},j} + \tilde{u}_{\text{L},i}m^2_{\text{L},ij}\tilde{u}_{\text{L},j} \\
+ \tilde{u}_{\text{R},i}m^2_{\text{R},ij}\tilde{u}_{\text{R},j} + \tilde{e}_{\text{L},i}m^2_{\text{L},ij}\tilde{e}_{\text{L},j} + \tilde{e}_{\text{R},i}m^2_{\text{R},ij}\tilde{e}_{\text{R},j} + \tilde{\nu}_{\text{L},i}m^2_{\text{L},ij}\tilde{\nu}_{\text{L},j} + \tilde{\nu}_{\text{R},i}m^2_{\text{R},ij}\tilde{\nu}_{\text{R},j}.
\]

All trilinear scalar couplings involving Higgs bosons to squarks and sleptons are forbidden in Eq. (2) because the sfermions have an R-charge and these terms are non R-invariant, and this relaxes the flavor problem of the MSSM [36]. The Dirac nature is a manifest feature of MRSSM fermions and the soft-breaking Dirac mass terms of the singlet \(\hat{S}\), triplet \(\hat{T}\) and octet \(\hat{O}\) take the form as

\[
V_{SB,DG} = M_B^2\hat{B}\hat{S} + M_W^W\hat{W}^a\hat{T}^a + M_D^O\hat{g}\hat{O} + h.c.,
\]

where \(\hat{B}, \hat{W}\) and \(\hat{g}\) are usually MSSM Weyl fermions. R-Higgs bosons do not develop vacuum expectation values (VEVs) since they carry R-charge 2. After electroweak symmetry breaking the singlet and triplet VEVs effectively modify the \(\mu_u\) and \(\mu_d\), and the modified \(\mu_i\) parameters are given by

\[
\mu_{\mu_d}^{\mu_d,+} = \frac{1}{2}\Lambda_d v_T + \frac{1}{\sqrt{2}}\lambda_d v_S + \mu_d, \quad \mu_{\mu_d}^{\mu_d,-} = -\frac{1}{2}\Lambda_u v_T + \frac{1}{\sqrt{2}}\lambda_u v_S + \mu_u.
\]

The \(v_T\) and \(v_S\) are vacuum expectation values of \(\hat{T}\) and \(\hat{S}\) which carry R-charge zero.

There are four complex neutral scalar fields and they can mix. Assuming the vacuum expectation values are real, the real and imaginary components in four complex neutral
scalar fields do not mix, and the mass-square matrix breaks into two 4 × 4 sub-matrices. In the scalar sector all fields mix and the SM-like Higgs boson is dominantly given by the up-type field. In the pseudo-scalar sector there is no mixing between MSSM-like states and singlet-triplet states, and the 4 × 4 mass-squared matrix breaks into two 2 × 2 submatrices. The number of neutralino degrees of freedom in MRSSM is doubled compared to MSSM as the neutralinos are Dirac-type. The number of chargino degrees of freedom in MRSSM is also doubled compared to MSSM and these charginos can be grouped to two separated chargino sectors according to their R-charge. The \( \chi^\pm \) -charginos sector has R-charge 1 electric charge; the \( \rho \)-charginos sector has R-charge -1 electric charge. Here, we don’t discuss the \( \rho \)-charginos sector in detail since it doesn’t contribute to the LFV decays. More information about the \( \rho \)-charginos can be found in Ref.\[39, 41, 43, 53\]. For convenience, we present the tree-level mass matrices for scalar and pseudo-scalar Higgs bosons, neutralinos, charginos and squarks of the MRSSM in Appendix A.

In MRSSM the LFV decays mainly originate from the potential misalignment in sleptons mass matrices. In the gauge eigenstate basis \( \tilde{\nu}_L \), the sneutrino mass matrix and the diagonalization procedure are

\[
m^2_{\tilde{\nu}} = m^2_\ell + \frac{1}{8}(g^2_1 + g^2_2)(v^2_d - v^2_u) + g_2 v_T M^W_D - g_1 v_S M^B_D, Z^V m^2_{\tilde{\nu}}(Z^V)^\dagger = m^2_{\tilde{\nu}}^{\text{diag}},
\]

(4)

where the last two terms are newly introduced by MRSSM. The slepton mass matrix and the diagonalization procedure are

\[
m^2_{\tilde{e}} = \begin{pmatrix} (m^2_{\tilde{e}})_{LL} & 0 \\ 0 & (m^2_{\tilde{e}})^{RR} \end{pmatrix}, Z^E m^2_{\tilde{e}}(Z^E)^\dagger = m^2_{\tilde{e}}^{\text{diag}},
\]

(5)

where

\[
(m^2_{\tilde{e}})_{LL} = m^2_\ell + \frac{1}{2} v^2_d |Y_e|^2 + \frac{1}{8}(g^2_1 - g^2_2)(v^2_d - v^2_u) - g_1 v_S M^B_D - g_2 v_T M^W_D,
\]

\[
(m^2_{\tilde{e}})^{RR} = m^2_r + \frac{1}{2} v^2_d |Y_e|^2 + \frac{1}{4} g^2_1 (v^2_u - v^2_d) + 2 g_1 v_S M^B_D.
\]

The sources of LFV are the off-diagonal entries of the 3 × 3 soft supersymmetry breaking matrices \( m^2_\ell \) and \( m^2_r \) in Eqs.(4, 5). From Eq.(5) we can see that the left-right slepton mass mixing is absent in MRSSM, whereas the \( A \) terms are present in MSSM.

Finally, the MRSSM has been implemented in the Mathematica package SARAH \[55–\]

57, and we use the Feynman rules generated with SARAH in our work.
III. DECAY WIDTH AND WILSON COEFFICIENTS

Using the effective Lagrangian method, we present analytical expressions for decay width of $\tau \to P l$. At the quark level, the interaction Lagrangian for $\tau \to P l$ can be written as

$$\mathcal{L}_{\tau \to P l} = \sum_{I=SV} C^I_{XY}(\bar{l}_\beta \Gamma_I P_X \tau)(\bar{d} \Gamma_I P_Y d) + C^I_{XY}(\bar{l}_\beta \Gamma_I P_X \tau)(\bar{u} \Gamma_I P_Y u) + h.c.,$$

(6)

where the index $\beta$ (=$1, 2$) denotes the generation of emitted lepton and $l_1(l_2) = e(\mu)$. Since only the axial-vector current contributes to $\tau \to P l$, the coefficients in Eq. (6) do not include photonic contributions but they include Z boson and scalar ones. Then the decay width for $\tau \to P l$ is given by

$$\Gamma(\tau \to P l) = \frac{\lambda^{1/2}(m_\tau, m_t, m_\pi^2)}{16\pi m_\tau^2} \sum_{i,f} |\mathcal{M}|^2,$$

(7)

where the averaged squared amplitude can be written as

$$\sum_{i,f} |\mathcal{M}|^2 = \sum_{I,J = SV} [2m_\tau m_l (a^I_P a^{I*}_P - b^I_P b^{I*}_P) + (m_\tau^2 + m_l^2 - m_{1P}^2)(a^I_P a^{I*}_P + b^I_P b^{I*}_P)].$$

(8)

The coefficients $a^S_P$ and $b^S_P$ are linear combinations of the Wilson coefficients in Eq. (6)

$$a^S_P = \frac{f_\pi}{2} \sum_{X = L,R} \left[ \frac{D^d_X(P)}{m_d} (B^S_{dX} + B^S_{RX}) + \frac{D^u_X(P)}{m_u} (C^S_{dX} + C^S_{RX}) \right],$$

$$b^S_P = \frac{f_\pi}{2} \sum_{X = L,R} \left[ \frac{D^d_X(P)}{m_d} (B^S_{RX} - B^S_{dX}) + \frac{D^u_X(P)}{m_u} (C^S_{RX} - C^S_{dX}) \right],$$

$$a^V_P = \frac{f_\pi}{4} C(P)(m_\tau - m_l)[-B^V_{LL} + B^V_{LR} - B^V_{RL} + B^V_{RR} + C^V_{LL} - C^V_{LR} + C^V_{RL} - C^V_{RR}],$$

$$b^V_P = \frac{f_\pi}{4} C(P)(m_\tau + m_l)[-B^V_{LL} + B^V_{LR} + B^V_{RL} - B^V_{RR} + C^V_{LL} - C^V_{LR} - C^V_{RL} + C^V_{RR}],$$

where $f_\pi$ is the pion decay constant. The expressions for coefficients $C(P), D^{d,u}_L(P)$ are listed in TABLE III [24]. Here, $m_\pi$ and $m_K$ denote the masses of the neutral pion and Kaon, and $\theta_\eta$ denote the $\eta - \eta'$ mixing angle. In addition, $D^{d,u}_R(P) = -(D^{d,u}_L(P))^*$. The contributions to Wilson coefficients $C^I_{XY}$ and $B^I_{XY}$ can be classified into Z penguins, Higgs penguins and box diagrams, shown in FIG.1, FIG.2 and FIG.3. Photon penguins are not included since only the axial-vector current contributes to $\tau \to P l$. In the following, we will calculate the Wilson coefficients separately.
### TABLE III: Coefficients for each pseudoscalar meson $P$

| $P = \pi^0$ | $P = \eta$ | $P = \eta'$ |
|-------------|-------------|-------------|
| $C(P)$ | 1 | $\frac{1}{\sqrt{6}}(\sin \theta \eta + \sqrt{2} \cos \theta \eta)$ | $\frac{1}{\sqrt{6}}(\sqrt{2} \sin \theta \eta - \cos \theta \eta)$ |
| $D_L^d(P)$ | $-\frac{m^2_\pi}{4}$ | $\frac{1}{4\sqrt{3}}[(3m^2_\pi - 4m^2_K) \cos \theta \eta - 2\sqrt{2}m^2_K \sin \theta \eta]$ | $\frac{1}{4\sqrt{3}}[(3m^2_\pi - 4m^2_K) \sin \theta \eta + 2\sqrt{2}m^2_K \cos \theta \eta]$ |
| $D_L^u(P)$ | $\frac{m^2_\pi}{4}$ | $\frac{1}{4\sqrt{3}}m^2_\pi(\cos \theta \eta - \sqrt{2} \sin \theta \eta)$ | $\frac{1}{4\sqrt{3}}m^2_\pi(\sin \theta \eta + \sqrt{2} \cos \theta \eta)$ |

#### Z boson contribution

The Z penguin diagrams contributing to $\tau \to Pl$ at one loop level in MRSSM are presented in FIG.1. Applying the high energy physics package Package-X [59], one can derive the Wilson coefficients $C^V_{XY}$ corresponding to FIG.1 (a-d) in terms of invariant Passarino-Veltman integrals [60] with the limit $m_e(m_\mu) \to 0$,\

$$C^V_{XY} = \sum_{i,j,k} \frac{-1}{m^2_Z} C^1_X C^4_Y (C^2_X C^3_X B_0 + (M^2_3 C^2_X C^3_X - M_1 M_2 C^2_X C^3_X - \kappa m_\tau M_1 C^2_X C^3_X) C_0$$

$$+ \kappa m_\tau (-M_1 C^2_X C^3_X + \kappa m_\tau C^2_X C^3_X + M_2 C^2_X C^3_X) C_1 - 2C^2_X C^3_X C_0), \quad (a,b)$$

$$C^V_{XY} = \sum_{i,j,k} \frac{-2}{m^2_Z} \kappa C^1_X C^2_X C^3_X C^4_Y C_0), \quad (c,d), \quad (9)$$

$$C^V_{XY} = \sum_{i,j,k} \frac{-1}{m^2_Z} C^1_X C^4_Y (C^2_X C^3_X B_0 + (M^2_3 C^2_X C^3_X - M_1 M_2 C^2_X C^3_X - \kappa m_\tau M_1 C^2_X C^3_X) C_0$$

$$+ \kappa m_\tau (-M_1 C^2_X C^3_X + \kappa m_\tau C^2_X C^3_X + M_2 C^2_X C^3_X) C_1 - 2C^2_X C^3_X C_0), \quad (a,b)$$

$$C^V_{XY} = \sum_{i,j,k} \frac{-2}{m^2_Z} \kappa C^1_X C^2_X C^3_X C^4_Y C_0), \quad (c,d), \quad (10)$$
where the subscript $X'$ is defined as

$$X' = \begin{cases} 
L, & \text{when } X = R \\
R, & \text{when } X = L 
\end{cases}$$

and so does $Y'$. The symbol $\kappa$ equals $-1$ if the loop lines contain $\chi^0$, and 1 otherwise. The symbols $M_1$, $M_2$ and $M_3$ stand for masses of particles in loop lines and the explicit expressions for FIG.1(a-d) are

$$M_1 = m^i_{\chi^0}, M_2 = m^j_{\chi^0}, M_3 = m^k_{\chi^0}, (a), M_1 = m^i_{\tilde{\chi}^0}, M_2 = m^j_{\tilde{\chi}^0}, M_3 = m^k_{\tilde{\chi}^0}, (b),$$

$$M_1 = m^i_{\tilde{\chi}^0}, M_2 = m^j_{\tilde{\chi}^0}, M_3 = m^k_{\tilde{\chi}^0}, (c), M_1 = m^i_{\tilde{\chi}^0}, M_2 = m^j_{\tilde{\chi}^0}, M_3 = m^k_{\tilde{\chi}^0}, (d). \quad (11)$$

The symbols $C^{1}_{X}$ and $C^{2}_{X}$ in Eqs.(9,10) stand for the left-handed or right-handed couplings of the interaction between leptons and sleptons. The symbol $C^{2}_{X}$ in Eq.(9) stands for the left-handed or right-handed coupling of the interaction between Z boson and neutralinos or charginos. The couplings $C^{1}_{X}$, $C^{2}_{X}$ and $C^{3}_{X}$ for FIG.1(a,b) are given by

$$C^{1}_{L} = iU_{13}Z_{k}\nu_{13}, C^{1}_{R} = -ig_2Z_{\nu_{13}}V_{13}, C^{2}_{L} = \frac{-i}{2}(2g_2c_wV_{13}V_{13}^{*} + (g_2c_w - g_1s_w)V_{13}^{*}V_{13}),$$

$$C^{2}_{R} = \frac{-i}{2}(2g_2c_wU_{13}U_{13}^{*} + (g_2c_w - g_1s_w)U_{13}^{*}U_{13}), C^{2}_{R} = -ig_2V_{13}^{*}Z_{\nu_{13}}, C^{3}_{R} = iY_{\nu_{13}}Z_{\nu_{13}}V_{13}, (a)$$

$$C^{1}_{L} = -i\sqrt{2}N_{13}^{*}Z_{k(3+\beta)}, C^{1}_{R} = -iY_{\beta}Z_{k(3+\beta)}N_{13}^{*}, C^{2}_{L} = \frac{i}{2}(g_1s_w + g_2c_w)(N_{13}^{*}N_{13} - N_{13}^{*}N_{13}),$$

$$C^{2}_{R} = \frac{i}{2}(g_1s_w + g_2c_w)(N_{13}^{*}N_{13} - N_{13}^{*}N_{13}), C^{3}_{L} = -iN_{j3}^{*}Y_{\tau}Z_{k6}, C^{3}_{R} = -i\sqrt{2}g_1Z_{k6}N_{j3}, (b, \chi^0)$$

$$C^{1}_{L} = -iN_{13}^{*}Z_{k}\nu_{13}, C^{1}_{R} = \frac{i}{2}Z_{k}\nu_{13}(g_1N_{13}^{*} + g_2N_{13}), C^{3}_{L} = \frac{i}{2}Z_{k}\nu_{13}(g_1N_{13}^{*} + g_2N_{13}),$$

$$C^{2}_{L} = \frac{i}{2}(g_1s_w + g_2c_w)(N_{13}^{*}N_{j3} - N_{13}^{*}N_{j3}), C^{2}_{R} = \frac{i}{2}(g_1s_w + g_2c_w)(N_{13}^{*}N_{j3} - N_{13}^{*}N_{j3}),$$

$$C^{3}_{R} = -iY_{\tau_{\alpha}Z_{k}}N_{j3}, (b, \chi^0). \quad (12)$$

By an interchange of sum indexes ($i \leftrightarrow k, j \leftrightarrow k$) of the couplings $C^{1}_{X}$ and $C^{3}_{X}$ for FIG.1(a) and FIG.1(b), one can get the expressions of the couplings $C^{1}_{X}$ and $C^{3}_{X}$ for FIG.1(c) and FIG.1(d) respectively. The symbol $C^{2}$ in Eq.(10) stands for the coupling of the interaction between Z boson and two sneutrinos or two sleptons, and it is noted worthwhile that the relevant Feynman rules in MRSSM are same with those in MSSM. The couplings $C^{2}$ for FIG.1(c,d) are given by

$$C^{2} = -\frac{i}{2}(g_1s_w + g_2c_w)\delta_{ij}, (c), C^{2} = \frac{ig_1}{2s_w}(\sum_{a=1,2,3}Z_{a1}Z_{a2}^{*} - 2s_w^{2}\delta_{ij}), (d). \quad (13)$$
The symbol $C^4_Y$ in Eqs. (9, 10) stands for the left-handed or right-handed coupling of the interaction between Z boson and two $u$ quarks, for which the relevant Feynman rules in MRSSM are same with those in SM. The couplings $C^4_Y$ are given by

$$C^4_L = -\frac{i}{6}(3g_2c_w - g_1s_w), \quad C^4_R = \frac{2i}{3}g_1s_w. \quad (14)$$

The symbols $B_0$, $C_0$, $C_1$ and $C_{00}$ denote the Passarino-Veltman integrals which take the form of

$$B_0 = \frac{i}{16\pi^2}B_0(0; M_2, M_1),$$

$$C_{0,1,00} = \frac{i}{16\pi^2}C_{0,1,00}(m_\tau^2, 0, 0; M_3, M_1, M_2).$$

These loop integrals are calculated by Mathematica package Package-X through a link to a fortran library Collier which is developed for the numerical evaluation of one-loop scalar and tensor integrals in perturbative relativistic quantum field theory [61]. The explicit expressions of these loop integrals are given in Refs. [62–64] and $\overline{\text{MS}}$ scheme is used to delete the infinite terms.

The Wilson coefficients $B^V_{XY}$ corresponding to FIG.1 (a–d) can be formulated by replacing the couplings of $u$ quark in Eq. (14) with the couplings of $d$ quark in Eq. (15).

$$C^4_L = -\frac{i}{6}(3g_2c_w + g_1s_w), \quad C^4_R = -\frac{1}{3}g_1s_w. \quad (15)$$

**Higgs boson contribution**

The Higgs penguin diagrams contributing to $\tau \to Pl$ at one loop level in MRSSM are presented in FIG.2. The Wilson coefficients $C^S_{XY}$ for FIG.2 (a–d) can be expressed as

$$C^S_{XY} = \sum_{i,j,k,l} \frac{1}{m_H^2}C^1_X^\ell C^4_Y(C^2_X C^3_X B_0 + (M_1 M_2 C^2_X C^3_X + \kappa m_\tau C^2_X C^3_X + M_3 C^2_X C^3_X)C_0 - \kappa m_\tau (M_1 C^2_X C^3_X + \kappa m_\tau C^2_X C^3_X + M_3 C^2_X C^3_X)C_1), \quad (a,b),$$

$$C^S_{XY} = \sum_{i,j,k,l} \frac{1}{m_H^2}C^1_X C^2_X C^4_Y (\kappa m_\tau C^3_X C_1 - M_3 C^3_X C_0), \quad (c,d). \quad (16)$$

The explicit expressions for $M_1$, $M_2$, $M_3$, $C^1_X$ and $C^3_X$ for FIG.2 (a), (b), (c) and (d) are same with those in FIG.1 (a), (b), (c) and (d) respectively. The symbol $C^2_X$ in Eq. (16) stands for the left-handed or right-handed coupling of the interaction between Higgs boson and two
neutralinos or two charginos, and can be expressed by

\[
C_L^2 = -\frac{i}{2} (U_{i1}^\dagger (2g_2 V_{j1}^1 Z_{i4}^h + \sqrt{2} \Lambda_d V_{j1}^2 Z_{i4}^h) + U_{i2}^\dagger (\sqrt{2} g_2 V_{j1}^1 Z_{i1}^h + V_{j2}^1 (\sqrt{2} \lambda_d Z_{i3}^h - \Lambda_d Z_{i4}^h)),
\]

\[
C_R^2 = -\frac{i}{2} (U_{j1}^1 (2g_2 V_{i1}^1 Z_{i4}^h + \sqrt{2} \Lambda_d V_{i2}^1 Z_{i4}^h) + U_{j2}^1 (\sqrt{2} g_2 V_{i1}^1 Z_{i1}^h + V_{i2}^1 (\sqrt{2} \lambda_d Z_{i3}^h - \Lambda_d Z_{i4}^h)), (a,h),
\]

\[
C_L^2 = -\frac{1}{2} (U_{i1}^1 (2g_2 V_{i1}^1 Z_{i4}^A + \sqrt{2} \Lambda_d V_{i2}^1 Z_{i4}^A) + U_{i2}^1 (\sqrt{2} g_2 V_{i1}^1 Z_{i1}^A - V_{i2}^1 (\sqrt{2} \lambda_d Z_{i3}^A + \Lambda_d Z_{i4}^A)), (a,0),
\]

\[
C_R^2 = \frac{i}{2} (N_{j1}^1 (\Lambda_d N_{i2}^2 Z_{j1}^h + N_{i3}^2 (\Lambda_d Z_{j1}^h + \sqrt{2} \lambda_d Z_{j3}^h) + \sqrt{2} \lambda_d N_{i2}^2 Z_{j1}^h) - g_2 N_{i2}^1 N_{i3}^2 Z_{j1}^h
\]

\[
+ g_1 N_{j1}^1 (N_{i2}^1 Z_{j1}^h - N_{i4}^1 Z_{j2}^h) - \sqrt{2} \lambda_u N_{j1}^1 N_{i4}^1 Z_{j3}^h + \Lambda_u N_{j4}^1 N_{i4}^2 Z_{j4}^h + g_2 N_{j2}^1 N_{i2}^2 Z_{j2}^h
\]

\[
- \sqrt{2} \lambda_u N_{j4}^1 N_{i4}^2 Z_{j2}^h + \Lambda_u N_{j4}^1 N_{i4}^2 Z_{j2}^h),
\]

\[
C_R^2 = \frac{i}{2} (\Lambda_d Z_{j1}^h N_{j2}^1 N_{j2}^1 + \Lambda_u Z_{j1}^h N_{j4}^1 N_{j4}^1 + g_1 Z_{j1}^h N_{j1}^1 N_{j3}^2 - g_2 Z_{j2}^h N_{j2}^1 N_{j3}^2 + \Lambda_d Z_{j4}^h N_{j4}^1 N_{j3}^2
\]

\[
+ \sqrt{2} \lambda_d N_{j1}^1 (Z_{j1}^h N_{j2}^2 + Z_{j3}^h N_{j3}^2 - g_1 Z_{j2}^h N_{j1}^1 N_{j4}^1 + g_2 Z_{j2}^h N_{j2}^1 N_{j4}^1 + \Lambda_u Z_{j4}^h N_{j4}^1 N_{j4}^1
\]

\[
- \sqrt{2} \lambda_u N_{j4}^1 (Z_{j2}^h N_{j1}^2 + Z_{j3}^h N_{j4}^1)), (b,0),
\]

\[
C_L^2 = \frac{1}{2} (-N_{j1}^1 (\Lambda_d N_{j2}^2 Z_{j1}^A + N_{j3}^2 (\Lambda_d Z_{j1}^A + \sqrt{2} \lambda_d Z_{j3}^A) + \sqrt{2} \lambda_d N_{j2}^2 Z_{j1}^A) - g_2 N_{j2}^1 N_{j3}^2 Z_{j1}^A
\]

\[
+ g_1 N_{j1}^1 (N_{j2}^1 Z_{j2}^2 - N_{j2}^2 Z_{j1}^2) + \sqrt{2} \lambda_u N_{j1}^1 N_{j4}^1 Z_{j3}^A - \Lambda_u N_{j4}^1 N_{j4}^2 Z_{j4}^A + g_2 N_{j2}^1 N_{j2}^2 Z_{j2}^A
\]

\[
+ \sqrt{2} \lambda_u N_{j4}^1 N_{j4}^2 Z_{j2}^A - \Lambda_u N_{j4}^1 N_{j4}^2 Z_{j2}^A),
\]

\[
C_R^2 = \frac{1}{2} (\Lambda_d Z_{j1}^A N_{j2}^1 N_{j2}^1 + \Lambda_u Z_{j1}^A N_{j4}^1 N_{j4}^1 - g_1 Z_{j1}^A N_{j1}^1 N_{j3}^2 + g_2 Z_{j2}^A N_{j2}^1 N_{j3}^2 + \Lambda_d Z_{j4}^A N_{j4}^1 N_{j3}^2
\]

\[
+ \sqrt{2} \lambda_d N_{j1}^1 (Z_{j1}^A N_{j2}^2 + Z_{j3}^A N_{j3}^2) + g_1 Z_{j2}^A N_{j1}^1 N_{j4}^1 - g_2 Z_{j2}^A N_{j2}^1 N_{j4}^1 + \Lambda_u Z_{j4}^A N_{j4}^1 N_{j4}^1
\]

\[
\text{FIG. 2: Higgs penguin diagrams contributing to } \tau \rightarrow Pl \text{ in MRSSM.}
\]
\[-\sqrt{2} \Lambda_u N_{i4}^u (Z_{i4}^A N_{j1}^u + Z_{i3}^A N_{j2}^u)), (b,A^0,\chi^0). \]  

(18)

The relevant couplings $C_X^2$ for FIG.2 (b) with $\chi^{0c}$ can be available by an interchange of the sum indexes $i \leftrightarrow j$ in $C_X^2$ for FIG.2 (b) with $\chi^0$. The symbol $C_1^2$ in Eq.(17) stands for coupling of interaction between Higgs boson and two sneutrinos or two sleptons, and can be expressed by

$$C_1^2 = \frac{i}{2} \delta_{ij} (4(g_1 M_B^D Z_{i3}^h - g_2 M_B^W Z_{i4}^h) - (g_1^2 + g_2^2)(v_d Z_{i1}^h - v_u Z_{i2}^h), (c,h),$$

$$C_1^2 = \frac{1}{2} \sum_{a=1,2,3} \left( 2(-2v_d Z_{j(3+a)}^E Y_a Y_d Z_{j(3+a)}^E Z_{i1}^h - 2v_d Z_{j(a)}^E Y_a Y_u Z_{j(a)}^E Z_{i1}^h + g_1 Z_{j(a)}^E + (g_2^2 - g_1^2)v_d Z_{i1}^h + (g_1^2 - g_2^2)v_u Z_{i2}^h) \right), (d,h).$$

It is noted worthwhile that, assuming both $M_B^W$ and $M_B^D$ are real numbers, the couplings $C_1^2$ for FIG.2 (c,d) with $A^0$ are zero and the relevant contribution can be neglected.

The symbol $C_1^4$ in Eqs.(16,17) stands for the left-handed or right-handed coupling of the interaction between Higgs boson and two $u$ quarks, for which the relevant Feynman rules in MRSSM are same with those in MSSM. The couplings $C_1^4$ for FIG.2 (a-d) are given by

$$C_1^4 = C_1^4 = \frac{i}{\sqrt{2}} Y_u Z_{i2}^h, (h), C_1^4 = -C_1^4 = \frac{i}{\sqrt{2}} Y_u Z_{i1}^h, (A^0).$$

(20)

The Wilson coefficients $B_X^{S,XY}$ corresponding to FIG.2 (a-d) can be formulated by replacing the couplings of $u$ quark in Eq.(20) with the couplings of $d$ quark in Eq.(21).

$$C_1^4 = C_1^4 = \frac{i}{\sqrt{2}} Y_d Z_{i1}^h, (h), C_1^4 = -C_1^4 = \frac{i}{\sqrt{2}} Y_d Z_{i2}^h, (A^0).$$

(21)

Box diagrams contribution

FIG. 3: Box diagrams contributing to $\tau \rightarrow Pl$ in MRSSM.

The Box diagrams contributing to $\tau \rightarrow Pl$ at one loop level in MRSSM are presented in FIG.3. To get the Wilson coefficients $C_X^{S,XY}$ and $C_X^{V,XY}$, the Dirac spinors in the amplitudes
should be rearranged. Before the rearrangement of spinors, the relevant coefficients $C^{S'}_{XY}$ and $C^{V'}_{XY}$ for FIG. 3 (a, b) can be expressed as

$$C^{S'}_{XY} = \sum_{i,j,k,l} M_1 C^1_{X} C^2_{Y} C^3_{X'}((M_3 C^4_1 + \kappa' m_\tau C^4_1) D_0 + \kappa' m_\tau C^4_1 (D_2 + D_1)), \tag{22}$$

$$C^{V'}_{XY} = \sum_{i,j,k,l} \kappa' \theta C^1_{X} C^2_{Y} C^3_{X'} C^4_{Y'} D_{00}. \tag{23}$$

The symbol $\kappa'$ equals $-1$ if there is one $\chi^{0c}$ connecting with the ingoing $\tau$ lepton, and 1 otherwise. The symbol $\theta$ equals $-1$ if there is one $\chi^{0c}$ connecting with the outgoing $e$ or $\mu$ lepton, and 1 otherwise. The explicit expressions for $M_1, M_2, M_3$ and $M_4$ are given by

$$M_1 = m^i_{\chi^0}, M_2 = m^l_{\chi^+}, M_3 = m^j_{\chi^0}, M_4 = m^k_{\overline{c}}, \quad (a),$$

$$M_1 = m^i_{\chi^0}, M_2 = m^l_{\chi}, M_3 = m^j_{\chi^0}, M_4 = m^k_{\overline{c}}, \quad (b),$$

and the Passarino-Veltman integrals $D_0, D_1, D_2$ and $D_{00}$ take the form of

$$D_{0,1,2,00} = \frac{i}{16\pi^2} D_{0,1,2,00}(0, 0, m^2_\tau; 0, 0, M_1, M_2, M_3, M_4).$$

Using the identities in Eq. (24), which are deduced from a generalized Fierz identities in chirality-diagonal and chirality-flipped cases \[65],

$$4[P_{L/R} \otimes P_{L/R}] \sim 2[P_{L/R} \otimes P_{L/R}] + \frac{1}{2} [\sigma^{\mu\nu} P_{L/R} \otimes \sigma_{\mu\nu} P_{L/R}],$$

$$4[\gamma^{\mu} P_{L/R} \otimes \gamma^{\mu} P_{L/R}] \sim -4[\gamma^{\mu} P_{L/R} \otimes \gamma^{\mu} P_{L/R}],$$

$$4[P_{L/R} \otimes P_{R/L}] \sim 2[\gamma^{\mu} P_{R/L} \otimes \gamma^{\mu} P_{L/R}],$$

$$4[\gamma^{\mu} P_{L/R} \otimes \gamma^{\mu} P_{R/L}] \sim 8[P_{R/L} \otimes P_{L/R}] - 2[\sigma^{\mu\nu} P_{R/L} \otimes \sigma_{\mu\nu} P_{L/R}],$$

one can obtain the relations between $C^{S(V)}_{XY}$ and $C^{S'(V')}_{XY}$ as

$$C^{S}_{XX} = \frac{1}{2} C^{S'}_{XX}, C^{S}_{XX'} = 2 C^{V'}_{XX}, C^{V}_{XX} = -C^{V'}_{XX'}, C^{V}_{XX'} = \frac{1}{2} C^{S'}_{XX'}.$$, \tag{25}$$

The symbols $C^1_X$ and $C^4_X$ in Eqs. (22, 23) stand for the couplings of interaction between leptons and sleptons, and the explicit expressions are same with those in FIG. 1 (a) and (b) respectively. The symbol $C^2_X$ in Eqs. (22, 23) stands for the couplings of interaction between anti-$u$ quark and squarks, and the symbol $C^3_X$ in Eqs. (22, 23) stands for the couplings of interaction between $u$ quark and squarks. The expressions of $C^2_X$ and $C^3_X$ are given by

$$C^2_L = i U_{12} Y_d Z_{14}^D C^2_R = 0, \quad (a),$$

$$C^2_L = -i N_{i4}^1 Y_u Z_{14}^U C^2_R = \frac{2\sqrt{3}}{3} g_1 Z_{14}^U N_{i4}^1, \quad (b, \chi^0),$$

$$C^2_L = -i N_{i4}^1 Z_{14}^U Y_u (3 g_2 N_{i4}^1 + g_1 N_{i4}^1), \quad (c),$$

$$C^3_L = 0, \quad C^3_R = i Z_{14}^D Y_d U_{12}^1, \quad (a),$$

$$C^3_L = 2 \frac{\sqrt{3}}{3} g_1 N_{j4}^1 Z_{14}^U C^3_R = -i N_{j4}^1 Y_u Z_{14}^U, \quad (b, \chi^0),$$

$$C^3_L = -i N_{j4}^1 Y_u Z_{14}^U, \quad C^3_R = -\frac{1}{3\sqrt{2}} Z_{14}^U (3 g_2 N_{j4}^1 + g_1 N_{j4}^1), \quad (b, \chi^0).$$, \tag{26}$$

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The Wilson coefficients \( B_{XY} \) and \( B_{XY}^T \) corresponding to FIG.3(a,b) can be formulated by replacing the couplings of \( u \) quark in Eq.(26) with the couplings of \( d \) quark in Eq.(27).

\[
\begin{align*}
C_L^2 &= -ig_2 V^{1*}_{i1} Z_{i1}^U, C_R^2 = iY_d Z^U_{i1} U^*_{i2}, \\
C_L^3 &= iY_d Z^U_{i1} U^*_{i1}, C_R^3 = -ig_2 V^{1*}_{i1} Z^U_{i1}, \\
C_L^2 &= -iN^3_{i3} Y_d Z_{i4}^D, C_R^2 = -\frac{i}{3\sqrt{2}} g_1 Z_{i4}^D N_{i1}, \\
C_L^3 &= -\frac{i}{3\sqrt{2}} g_1 N_{i1}^1 Z_{i4}^D, C_R^3 = -iN^3_{i3} Y_d Z_{i4}^D, \\
C_L^2 &= -iN^3_{i3} Y_d Z_{i4}^D, C_R^3 = -\frac{i}{3\sqrt{2}} Z_{i1}^D (-3g_2 N_{j2}^1 + g_1 N_{j1}^1), \\
C_L^3 &= -iN^3_{i3} Y_d Z_{i4}^D, C_R^3 = -\frac{i}{3\sqrt{2}} Z_{i1}^D (-3g_2 N_{j2}^1 + g_1 N_{j1}^1),
\end{align*}
\]

(27)

IV. NUMERICAL ANALYSIS

The experimental values of Higgs mass and \( W \) boson mass can impose stringent and nontrivial constraints on the model parameters. The one loop and leading two loop corrections to the lightest (SM-like) Higgs boson in MRSSM have been computed in Ref.[39] and the new fields and couplings can give large contributions to the Higgs mass even for stop masses of order 1 TeV and no top mixing. Meanwhile, the new fields and couplings can not give too large contribution to the \( W \) boson mass and muon decay in the same regions of parameter space. A better agreement with the latest experimental value for \( W \) boson mass has been investigated in Ref.[42]. It combines all numerically relevant contributions that are known in SM in a consistent way with all MRSSM one loop corrections. A set of updated benchmark point BMP1 is given in Ref.[42] and we display them in Eq.(28) where all mass parameters are in GeV or GeV^2.

\[
\begin{align*}
\tan \beta &= 3, B_\mu = 500^2, \lambda_d = 1.0, \lambda_u = -0.8, \Lambda_d = -1.2, \Lambda_u = -1.1, \\
M_D^B &= 550, M_D^W = 600, \mu_d = \mu_u = 500, v_S = 5.9, v_T = -0.33, \\
(m^2_{ij})_{11} &= (m^2_{ij})_{22} = (m^2_{ij})_{33} = (m^2_{ij})_{11} = (m^2_{ij})_{22} = (m^2_{ij})_{33} = 1000^2, \\
(m^2_{ij})_{11} &= (m^2_{ij})_{22} = (m^2_{ij})_{33} = (m^2_{ij})_{22} = (m^2_{ij})_{22} = (m^2_{ij})_{22} = 2500^2, \\
(m^2_{ij})_{33} &= (m^2_{ij})_{33} = (m^2_{ij})_{33} = 1000^2, m_T = 3000, m_S = 2000.
\end{align*}
\]

(28)

In the numerical analysis, the default values of the input parameters are set same with those in Eq.(28). The off-diagonal entries of squark mass matrices \( m^2_{ij} \), \( m^2_{ij} \), \( m^2_{ij} \) and slepton mass matrices \( m^2_{ij} \), \( m^2_{ij} \) in Eq.(28) are zero. The large value of \(|v_T|\) is excluded by measurement of \( W \) boson mass because the VEV \( v_T \) of the \( SU(2)_L \) triplet field \( T^0 \) gives a correction to \( W \) mass through \[37\]

\[
m^2_W = \frac{1}{4} g_2^2 (v_u^2 + v_d^2) + g_2^2 v_T^2.
\]

(29)
Similarly to most supersymmetry models, the LFV processes originate from the off-diagonal entries of the soft breaking terms $m_l^2$ and $m_r^2$ in MRSSM, which are parameterized by mass insertion

$$ (m_l^2)_{IJ} = \delta_l^{IJ} \sqrt{(m_l^2)_{II}(m_l^2)_{JJ}}, (m_r^2)_{IJ} = \delta_r^{IJ} \sqrt{(m_r^2)_{II}(m_r^2)_{JJ}}, $$  \hspace{1cm} (30) 

where $I, J = 1, 2, 3$. To decrease the number of free parameters involved in our calculation, we assume that the off-diagonal entries of $m_l^2$ and $m_r^2$ in Eq.(30) are equal, i.e., $\delta_l^{IJ} = \delta_r^{IJ} = \delta^{IJ}$. The experimental limits on LFV decays, such as radiative two body decays $l_2 \rightarrow l_1 \gamma$, leptonic three body decays $l_2 \rightarrow 3l_1$ and $\mu - e$ conversion in nuclei, can give strong constraints on the parameters $\delta^{IJ}$. In the following, we will use LFV decays $l_2 \rightarrow l_1 \gamma$ to constrain the parameters $\delta^{IJ}$ which are discussed in Ref. [54]. It is noted that $\delta^{12}$ has been set zero in following discussion since it has no effect on the predictions of $\text{BR}(\tau \rightarrow P l)$. Current limits of LFV decays $l_2 \rightarrow l_1 \gamma$ are listed in TABLE IV.

| Decay | Bound      | Experiment  | Decay | Bound      | Experiment |
|-------|------------|-------------|-------|------------|------------|
| $\mu \rightarrow e\gamma$ | $4.2 \times 10^{-13}$ | MEG(2016) [66] | $\tau \rightarrow e\gamma$ | $3.3 \times 10^{-8}$ | BABAR(2010) [67] |
| $\tau \rightarrow \mu\gamma$ | $4.4 \times 10^{-8}$ | BABAR(2010) [67] |

Taking $\delta^{13} = 0.1$, $\delta^{23} = 0$ and data in Eq.(28), we plot the theoretical predictions of $\text{BR}(\tau \rightarrow P e)$ from each diagram as a function of $\tan \beta$ in the left panel of FIG.4. Taking $\delta^{13} = 0$, $\delta^{23} = 0.1$ and data in Eq.(28), we plot the theoretical predictions of $\text{BR}(\tau \rightarrow P \mu)$ from each diagram as a function of $\tan \beta$ in the right panel of FIG.4. The lines corresponding to Higgs penguins, Z penguins and box diagrams indicate the values of $\text{BR}(\tau \rightarrow P e(\mu))$ given by only the listed contribution with all others set to zero. The total prediction for $\text{BR}(\tau \rightarrow P e(\mu))$ is also indicated. It shows Z penguins dominate the predictions on $\text{BR}(\tau \rightarrow P e(\mu))$, and the Higgs penguins contribution and box diagrams contribution are negligible, which is different from some SUSY models (e.g., [24], where the Higgs penguins contribution is dominant and Z penguins contribution is subdominant). The predictions on $\text{BR}(\tau \rightarrow P e, P \mu)$ from Higgs penguins increase as $\tan \beta$ varies from 3 to 40 while the total predictions and the predictions from Z penguins or box diagrams take a narrow band. The total predictions on $\text{BR}(\tau \rightarrow P e, P \mu)$ are one order or two orders of magnitude lower than...
the current experimental limits. Due to the existence of the transition from $d$-Higgsino to $u$-Higgsino in MSSM, which is governed by $\mu$-term, the well-known $\tan \beta$-enhancement is possible. A well-established way to understand the $\tan \beta$-enhancement is provided by mass-insertion diagrams involving insertions of the $\mu$-parameter and Majorana gaugino masses. However, the $\mu$-term and Majorana gaugino masses are forbidden in MRSSM and this leads to the result that $(\tau \to P l)$ are not enhanced by $\tan \beta$.

Taking $\delta^{23} = 0$ and data in Eq.(28), we plot the theoretical predictions of $\text{BR}(\tau \to Pe)$ versus $\log[\delta^{13}]$ in the left panel of FIG.5. Taking $\delta^{13} = 0$ and data in Eq.(28), we plot the theoretical predictions of $\text{BR}(\tau \to P \mu)$ versus $\log[\delta^{23}]$ in the right panel of FIG.5. A linear relationship in logarithmic scale is displayed between $\text{BR}(\tau \to Pe)$ and the flavor violating parameter $\delta^{13}(\delta^{23})$. At $\delta^{13}=0.1$, the prediction on $\text{BR}(\tau \to \pi e)$ is around $10^{-8}$ and this is very close to the current experimental limit. The predictions on $\text{BR}(\tau \to \eta e)$ and $\text{BR}(\tau \to \eta' e)$ are around $10^{-9}$ and these are about two orders of magnitude lower than the current experimental limits. At $\delta^{23}=0.1$, the predictions on $\text{BR}(\tau \to \pi \mu)$, $\text{BR}(\tau \to \eta \mu)$
FIG. 5: Left panel: Dependence of the branching ratios $\text{BR}(\tau \to Pe)$ on mass insertion $\delta^{13}$, where the solid blue line, dashed red line and dotted green line correspond to $\text{BR}(\tau \to \pi e)$, $\text{BR}(\tau \to \eta e)$ and $\text{BR}(\tau \to \eta' e)$, respectively. Right panel: Dependence of the branching ratios $\text{BR}(\tau \to P\mu)$ on mass insertion $\delta^{23}$, where the solid blue line, dashed red line and dotted green line correspond to $\text{BR}(\tau \to \pi \mu)$, $\text{BR}(\tau \to \eta \mu)$ and $\text{BR}(\tau \to \eta' \mu)$, respectively.

and $\text{BR}(\tau \to \eta' \mu)$ are around $10^{-8}$, $10^{-9}$ and $10^{-9}$ and these are one order or two orders of magnitude lower than the current experimental limits. In FIG.5 (and in following figures) it shows the following hierarchy, $\text{BR}(\tau \to \pi e) > \text{BR}(\tau \to \eta' e) > \text{BR}(\tau \to \eta e)$ and $\text{BR}(\tau \to \pi \mu) > \text{BR}(\tau \to \eta' \mu) > \text{BR}(\tau \to \eta \mu)$.

FIG. 6: Left panel: Dependence of $\text{BR}(\tau \to Pe)$ on $m_l$, where the solid blue line, dashed red line, dotted green line and dot dashed gray line correspond to $\text{BR}(\tau \to \pi e)$, $\text{BR}(\tau \to \eta e)$, $\text{BR}(\tau \to \eta' e)$ and $\text{BR}(\tau \to e\gamma)$, respectively. Right panel: Dependence of $\text{BR}(\tau \to P\mu)$ on $m_l$, where the solid blue line, dashed red line, dotted green line and dot dashed gray line correspond to $\text{BR}(\tau \to \pi \mu)$, $\text{BR}(\tau \to \eta \mu)$, $\text{BR}(\tau \to \eta' \mu)$ and $\text{BR}(\tau \to \mu \gamma)$, respectively.

Taking $\delta^{13} = 0.1$, $\delta^{23} = 0$ and data in Eq.(28), we plot the theoretical predictions of
$\text{BR}(\tau \rightarrow P e)$ as a function of the diagonal entries $m_l$ of the soft breaking term $m_l^2$ and $m_r^2$ in the left panel of FIG. 6. Taking $\delta^{13} = 0$, $\delta^{23} = 0.1$ and data in Eq. (28), we plot the theoretical predictions of $\text{BR}(\tau \rightarrow P \mu)$ as a function of the diagonal entries $m_l$ of the soft breaking term $m_l^2$ and $m_r^2$ in the right panel of FIG. 6. Here, $m_l = \sqrt{(m_l^2)_{11}} = \sqrt{(m_l^2)_{22}} = \sqrt{(m_l^2)_{33}} = \sqrt{(m_r^2)_{11}} = \sqrt{(m_r^2)_{22}} = \sqrt{(m_r^2)_{33}}$. The predictions on $\text{BR}(\tau \rightarrow P e, P \mu)$ in MRSSM increase as the slepton mass $m_l$ varies from 100 GeV to 1000 GeV, and the decoupling behaviour is obtained.

FIG. 7: Left panel: Dependence of $\text{BR}(\tau \rightarrow P e)$ on $M_D^W$, where the solid blue line, dashed red line, dotted green line and dot dashed gray line correspond to $\text{BR}(\tau \rightarrow \pi e)$, $\text{BR}(\tau \rightarrow \eta e)$, $\text{BR}(\tau \rightarrow \eta' e)$ and $\text{BR}(\tau \rightarrow e \gamma)$, respectively. Right panel: Dependence of $\text{BR}(\tau \rightarrow P \mu)$ on $M_D^W$, where the solid blue line, dashed red line, dotted green line and dot dashed gray line correspond to $\text{BR}(\tau \rightarrow \pi \mu)$, $\text{BR}(\tau \rightarrow \eta \mu)$, $\text{BR}(\tau \rightarrow \eta' \mu)$ and $\text{BR}(\tau \rightarrow \mu \gamma)$, respectively.

Taking $\delta^{13} = 0.1$, $\delta^{23} = 0$ and data in Eq. (28), we plot the theoretical predictions of $\text{BR}(\tau \rightarrow P e)$ as a function of the wino-triplino mass parameter $M_D^W$ in the left panel of FIG. 7. Taking $\delta^{13} = 0$, $\delta^{23} = 0.1$ and data in Eq. (28), we plot the theoretical predictions of $\text{BR}(\tau \rightarrow P \mu)$ as a function of the wino-triplino mass parameter $M_D^W$ in the right panel of FIG. 7. We clearly see that both the predictions for $\text{BR}(\tau \rightarrow P e)$ and $\text{BR}(\tau \rightarrow P \mu)$ show a weak dependence on $M_D^W$, and the predictions on $\text{BR}(\tau \rightarrow P e, P \mu)$ in MRSSM decrease slowly as $M_D^W$ varies from 100 GeV to 1000 GeV.

We are also interested to the effects from other parameters on the predictions of $\text{BR}(\tau \rightarrow$
$Pl$) in MRSSM. By scanning over these parameters, which are shown in Eq. (31),
\begin{align*}
-1.5 < &\lambda_d, \lambda_u, \Lambda_d, \Lambda_u < 1.5, \\
300 \text{ GeV} < &\mu_d, \mu_u, m_S, m_T, m_A < 3000 \text{ GeV}, \\
300 \text{ GeV} < & (m_{\tilde{q}})_{II}, (m_{\tilde{u}})_{II}, (m_{\tilde{d}})_{II} < 3000 \text{ GeV},
\end{align*}
the predictions are shown in relation to one input parameter (e.g. $m_T$ or others). The results show that varying those parameters in Eq. (31) have almost no effect on the predictions of $\text{BR}(\tau \rightarrow Pl)$ which take values along a narrow band.

V. CONCLUSIONS

In this work, taking account of the constraints from $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ on the parameter space, we analyze the LFV decays of $\tau \rightarrow Pl$ in the framework of the Minimal R-symmetric Supersymmetric Standard Model.

In MRSSM, the theoretical predictions on $\text{BR}(\tau \rightarrow Pe)$ and $\text{BR}(\tau \rightarrow P\mu)$ affected by the mass insertion $\delta^{13}$ and $\delta^{23}$, respectively. The predictions on $\text{BR}(\tau \rightarrow Pe)$ would be zero if $\delta^{13}=0$ is assumed, and so are the predictions on $\text{BR}(\tau \rightarrow P\mu)$ if $\delta^{23}=0$ is assumed. $Z$ penguins dominate the predictions on $\text{BR}(\tau \rightarrow Pe(\mu))$, and other contribution are negligible. Taking account of experimental bounds on radiative decays $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$, the values of $\delta^{13}$ and $\delta^{23}$ are constrained around 0.1. Assuming $\delta^{13} = 0.1$ and $\delta^{23} = 0.1$ and other parameter settings in Eq.(28), the predictions on $\text{BR}(\tau \rightarrow Pe)$ and $\text{BR}(\tau \rightarrow P\mu)$ are at the level of $O(10^{-8} - 10^{-9})$, which are one order or two orders of magnitude below the present experimental upper limits. The future prospects of $\text{BR}(\tau \rightarrow Pl)$ in Belle II are extrapolated at the level of $O(10^{-9} - 10^{-10})$ \cite{4} and very close to the predictions in MRSSM and other aforementioned models. Thus, the LFV decays $\tau \rightarrow Pl$ are very promising to be observed in near future experiment.

Appendix A: MRSSM mass matrices at tree level

In the weak basis $(\phi_d, \phi_u, \phi_S, \phi_T)$, the scalar Higgs boson mass matrix and the diagonalization procedure are
\begin{equation}
M_h = \begin{pmatrix}
M_{11} & M_{21}^T \\
M_{21} & M_{22}
\end{pmatrix}, \quad Z_h M_h (Z_h)^\dagger = M_h^{\text{diag}},
\end{equation}
where the submatrices \( (c_\beta = \cos \beta, \ s_\beta = \sin \beta) \) are

\[
M_{11} = \begin{pmatrix}
    m_Z^2 c_\beta^2 + m_A^2 s_\beta^2 & -(m_Z^2 + m_A^2) s_\beta c_\beta \\
    -(m_Z^2 + m_A^2) s_\beta c_\beta & m_Z^2 s_\beta^2 + m_A^2 c_\beta^2 
\end{pmatrix},
\]

\[
M_{21} = \begin{pmatrix}
    v_d (\sqrt{2} \lambda_d \mu_d^{eff} + g_1 M_B^D) & v_u (\sqrt{2} \lambda_u \mu_u^{eff} + g_1 M_B^D) \\
    v_d (\Lambda_d \mu_d^{eff} + g_2 M_W^Q) & -v_u (\Lambda_u \mu_u^{eff} + g_2 M_W^Q)
\end{pmatrix},
\]

\[
M_{22} = \begin{pmatrix}
    4(M_B^D)^2 + m_S^2 + \frac{\lambda_d^2 v_d^2 + \lambda_u^2 v_u^2}{2} & \frac{\lambda_d \lambda_d v_d^2 - \lambda_u \lambda_u v_u^2}{2 \sqrt{2}} \\
    \frac{\lambda_d \lambda_d v_d^2 - \lambda_u \lambda_u v_u^2}{2 \sqrt{2}} & 4(M_W^Q)^2 + m_T^2 + \frac{\lambda_d^2 v_d^2 + \lambda_u^2 v_u^2}{4}
\end{pmatrix}.
\]

In the weak basis \( (\sigma_d, \sigma_u, \sigma_S, \sigma_T) \), the pseudo-scalar Higgs boson mass matrix and the diagonalization procedure are

\[
M_{A^0} = \begin{pmatrix}
    B_{\mu} & B_{\tilde{\mu}} & 0 & 0 \\
    B_{\tilde{\mu}} & B_{\mu} & 0 & 0 \\
    0 & 0 & m_S^2 + \frac{\lambda_d^2 v_d^2 + \lambda_u^2 v_u^2}{2} & \frac{\lambda_d \lambda_d v_d^2 - \lambda_u \lambda_u v_u^2}{2 \sqrt{2}} \\
    0 & 0 & \frac{\lambda_d \lambda_d v_d^2 - \lambda_u \lambda_u v_u^2}{2 \sqrt{2}} & m_T^2 + \frac{\lambda_d^2 v_d^2 + \lambda_u^2 v_u^2}{4}
\end{pmatrix}, \quad Z^A M_{A^0}(Z^A) = M_{A^0}^{\text{diag}} \quad (\text{A2})
\]

In the weak basis of four neutral electroweak two-component fermions \( \xi_i = (\tilde{B}, \tilde{W}^0, \tilde{R}_d, \tilde{R}_u) \) with R-charge 1 and four neutral electroweak two-component fermions \( \varsigma_i = (\tilde{S}, \tilde{T}^0, \tilde{R}_d, \tilde{R}_u) \) with R-charge -1, the neutralino mass matrix and the diagonalization procedure are

\[
m_{\chi^0} = \begin{pmatrix}
    M_B^D & 0 & -\frac{1}{2} g_1 v_d & \frac{1}{2} g_1 v_u \\
    0 & M_W^Q & \frac{1}{2} g_2 v_d & -\frac{1}{2} g_2 v_u \\
    -\frac{1}{\sqrt{2}} \lambda_d v_d & -\frac{1}{2} \Lambda_d v_d & -\mu_{d}^{eff} & 0 \\
    \frac{1}{\sqrt{2}} \lambda_u v_u & -\frac{1}{2} \Lambda_u v_u & 0 & -\mu_{u}^{eff}
\end{pmatrix}, \quad (N^1)^* m_{\chi^0} (N^2)^\dagger = m_{\chi^0}^{\text{diag}}. \quad (\text{A3})
\]

The mass eigenstates \( \kappa_i \) and \( \varphi_i \), and physical four-component Dirac neutralinos are

\[
\xi_i = \sum_{j=1}^{4} (N^1_{ij})^* \kappa_j, \varsigma_i = \sum_{j=1}^{4} (N^2_{ij})^* \varphi_j, \chi^0_i = \begin{pmatrix} \kappa_i \\ \varphi_i^* \end{pmatrix}.
\]

In the basis \( \xi_i^+ = (\tilde{W}^+, \tilde{R}_d^+) \) and \( \varsigma_i^- = (\tilde{T}^-, \tilde{H}_d^-) \), the \( \chi^\pm \)-charginos mass matrix and the diagonalization procedure are

\[
m_{\chi^\pm} = \begin{pmatrix}
    g_2 v_T + M_B^W & \frac{1}{\sqrt{2}} \Lambda_d v_d \\
    \frac{1}{\sqrt{2}} g_2 v_d & -\frac{1}{2} \Lambda_d v_T + \frac{1}{\sqrt{2}} \Lambda_d v_S + \mu_d
\end{pmatrix}, \quad (U^1)^* m_{\chi^\pm} (V^1)^\dagger = m_{\chi^\pm}^{\text{diag}}. \quad (\text{A4})
\]
The mass eigenstates $\lambda_i^\pm$ and physical four-component Dirac charginos are

$$\xi_i^+ = \sum_{j=1}^{2} (V^1_{ij})^* \lambda_j^+, \quad \xi_i^- = \sum_{j=1}^{2} (U^1_{ij})^* \lambda_j^-, \quad \chi_i^\pm = \left( \begin{array}{c} \lambda_i^+ \\ \lambda_i^- \\ \end{array} \right).$$

The mass matrix for up squarks and down squarks, and the relevant diagonalization procedure are

$$m^2_u = \left( \begin{array}{cc} (m^2_u)_{LL} & 0 \\ 0 & (m^2_u)_{RR} \end{array} \right), \quad Z^U m^2_u (Z^U)^\dagger = m^{2,\text{diag}}_u,$$

$$m^2_d = \left( \begin{array}{cc} (m^2_d)_{LL} & 0 \\ 0 & (m^2_d)_{RR} \end{array} \right), \quad Z^D m^2_d (Z^D)^\dagger = m^{2,\text{diag}}_d,$$

where

$$(m^2_u)_{LL} = m^2_\tilde{q} + \frac{1}{2} v_u^2 |Y_u|^2 + \frac{1}{24} (g_1^2 - 3 g_2^2) (v_u^2 - v_d^2) + \frac{1}{3} g_1 v_S M^B_D + g_2 v_T M^W_D,$$

$$(m^2_u)_{RR} = m^2_\tilde{q} + \frac{1}{2} v_u^2 |Y_u|^2 + \frac{1}{6} g_2^2 (v_d^2 - v_u^2) - \frac{4}{3} g_1 v_S M^B_D,$$

$$(m^2_d)_{LL} = m^2_\tilde{q} + \frac{1}{2} v_u^2 |Y_d|^2 + \frac{1}{24} (g_1^2 + 3 g_2^2) (v_u^2 - v_d^2) + \frac{1}{3} g_1 v_S M^B_D - g_2 v_T M^W_D,$$

$$(m^2_d)_{RR} = m^2_\tilde{q} + \frac{1}{2} v_d^2 |Y_d|^2 + \frac{1}{12} g_2^2 (v_u^2 - v_d^2) + \frac{2}{3} g_1 v_S M^B_D.$$

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