Finite element analysis on vibration of a flexible single-link manipulator moved translationally

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Abstract. The objectives of this study are to formulate the equation of motion of a flexible single-link manipulator system and to develop computational codes by a finite-element method in order to perform dynamics simulation on vibration of the manipulator system. The system used in this research consist of an aluminium beam as a flexible link, a clamp-part to hold the link, a DC motor and a lead screw set to move the link translationally. Computational codes on time history responses and FFT (Fast Fourier Transform) processing were developed to calculate dynamic behaviour of the link. The simulation result show the dynamic behaviour of the system on free vibration and forced vibration by excitation force due to the translational motion of the single-link manipulator system.

Keywords: Finite-element method, flexible manipulator, translational motion.

1. Introduction

In industrial applications and robotics system, a single-link flexible manipulator is expected to achieve maximum workability during operations such as high speed with safe operation, improved positioning accuracy, lighter weight, and lower energy consumption. Problems that often arise in flexible-link manipulator are vibrations caused by its flexibility which causes deformations that interfere with the performance of the system.

In the last decade, a number of researchers have investigated dynamic behaviour of flexible manipulator. Muhammad et al investigated dynamic of flexible manipulator with rotational motion using finite-element method including vibration control [1]–[8]. Muhammad et al used two node element in the investigation.

This study aims to formulate the equation of motion of the system and to develop computational codes with finite-element methods to find out time history responses and FFT (Fast Fourier Transform) processing of the link. The results of the plot time history response and FFT generated the natural frequency of the system.

Grounded in the previous work on computer simulation of a flexible single-link manipulator [8] and a flexible two-link manipulator [3] which failed to include the translational motion, this study put an emphasis on the translational motions in which these motions are often used in industrial applications and robotics. Another previous study also reported that the result of numeric simulation of a two-link
flexible manipulator was used to verify the control scheme effectively [9]. In relation to the current research, some similarities were also identified such as the use of a kinetic energy, potential energy, and Lagrange equation to derive the equation of motion[10]. Beyond the similarities, Bien’s work focuses on translational and rotational joint of two-link manipulator while this piece of work focuses on translational motion with a clamp-part to hold the link.

Prior to the application of active-force control on vibration of a single-link manipulator [2], it is necessary to have a clear picture of the characteristic of the link without vibration control. Obtaining the value of the free vibration is necessary to find out the natural frequency of the link. Furthermore, the natural frequency is pivotal to determine the type of effective control.

2. Formulation by finite-element method

Grounded in a finite-element method [11], each partition of the link is partially calculated to find out the matrix of mass ($M$) and stiffness ($K$) which generated a natural frequency of system. The link with a cantilever structure with a length of 33 cm is divided into 6 elements and move translationally by a DC motor to determine the vibration characteristics of the link. The degrees of freedom of the finite-element are divided into two types namely the lateral deformation $v(t)$ and the rotational angle $\psi(t)$. The physical properties of the system consists of the length ($L_i$), the cross-sectional area ($S_i$), and the area moment of inertia ($I_i$). Each element of the mechanical properties (Young’s modulus and mass density) are denoted as $E_i$ and $\rho_i$.

2.1. Kinematics

Some text Figure 1 plots the directions of translational motion on the link into three types namely the displacement ($Y$), velocity ($\dot{Y}$), and acceleration ($\ddot{Y}$)

![Figure 1. Directions of translational motion on the link](image)

Figure 2 shows the position vector at point $P$ on the link in the global coordinate frame and translating coordinate frame. $O – XY$ is the global coordinate frame and $O – \text{yx}$ is the translational coordinate frame. When the link move translationally, the displacement at $X$ axis is $x_p$ and the displacement at $Y$ axis is $y_p$. The deflection due to the translational motion is denoted as $v_p$. The vector position $r(x, t)$ at point $P$ on the link at time $t = t$, in the global coordinate frame $O – XY$ shown in the figure 2 is given by:

$$r(x, t) = X_p(x, t)I + Y_p(x, t)J$$  \hspace{1cm} (1)
The position vector of $P$ in frame $O - XY$ is

$$X_p(x, t) = x_p$$
$$Y_p(x, t) = y + v_p$$

The velocity vector of $P$ in frame $O - xy$, given by

$$\mathbf{r}(x, t) = X_p(x, t)\mathbf{I} + Y_p(x, t)\mathbf{J}$$

2.2. **Finite-element method**

Figure 3. illustrates the partitions of the link which are divided into six elements. Each node has the lateral deformation $v(t)$ and the rotational angle $\Psi(t)$.
Then, figure 4 shows the element coordinate frame of the \(i\)-th element.

![Element coordinate frame](image-url)

**Figure 4.** Element coordinate frame of the \(i\)-th element

The nodal displacement vector is

\[
\delta_i = \{v_i \ \psi_i \ v_{i+1} \ \psi_{i+1}\}^T
\]

(5)

2.3. **Equation of motion**

The discretization generated from the previous section, the velocity square of \(P\) can be obtained

\[
\mathbf{r}^T \cdot \mathbf{r} = \dot{x}^2 + \dot{y}^2 + \dot{v}_p^2 + 2\dot{v} \cdot \dot{v}_p
\]

(6)

The kinetic energy \((T_i)\) of system can be given by

\[
T_i = \int_{V_i} \rho \cdot \mathbf{r}^T \cdot \mathbf{r} \cdot dV
\]

(7)

The kinetic energy \((T_i)\) of system can be expressed as

\[
T_i = \frac{1}{2} m_i \dot{x}_p^2 + \frac{1}{2} m_i \dot{y}^2 + \frac{1}{2} \delta_i^T \mathbf{M}_i \delta_i + \mathbf{y}^T \mathbf{f}_i \cdot \delta_i
\]

(8)

Where

\[
\mathbf{y}^T \mathbf{f}_i \cdot \delta_i = \frac{m_i}{12} [\[6 \ l_i \ 6 \ -l_i]]
\]

The potential energy of the system can be written as

\[
U_i = \frac{1}{2} \delta_i^T \mathbf{K}_i \delta_i
\]

(9)

The dynamic model is formulated using the Lagrange equation can be defined by

\[
\mathbf{M}_i \ddot{\delta}_i - \mathbf{y}(t) f_i^T + \mathbf{C}_i \dot{\delta}_i + \mathbf{K}_i \delta_i = 0
\]

(10)

The equation of motion of the \(i\)-th element is given by:

\[
\mathbf{M}_i \ddot{\delta}_i + \mathbf{C}_i \dot{\delta}_i + \mathbf{K}_i \delta_i = \mathbf{y}(t) f_i^T
\]

(11)
Where mass matrix \((M_i)\), damping matrix \((C_i)\), stiffness matrix \((K_i)\), and the excitation force of the translation motion \((\dot{y}(t)f_{ti}^T)\) are respectively represented in Equation (12), (13), (14), (15).

\[
M_i = \frac{\rho_i S L_i}{420} \begin{bmatrix}
156 & 22L_i & 54 & -13L_i \\
22L_i & 4L_i^2 & 13L_i & -3L_i^2 \\
54 & 13L_i & 156 & -22L_i \\
-13L_i - 3L_i^2 - 22L_i & 4L_i^2 & & \\
\end{bmatrix}
\]

\[
K_i = \frac{E_i I_z}{L_i^3} \begin{bmatrix}
12 & 6L_i & -12 & 6L_i \\
6L_i & 4L_i^2 & -6L_i & 2L_i^2 \\
-12 - 6L_i & 12 & -6L_i & \\
6L_i & 2L_i^2 & -16L_i & 4L_i^2 \\
\end{bmatrix}
\]

\[
C_i = \alpha_i K_i
\]

\[
f_{ti}^T = -\frac{\rho_i S L_i}{12} \{6, l_i, 6, -l_i\}
\]

The length of the \(i\)-th element, the length from element 1 to \(i\), and the Rayleigh damping factor are denoted by \(l_i\), \(l_{1:i}\), and \(\alpha\) respectively.

In the end, the equation of motion of the single-link with \(n\) element considering the boundary conditions is written by

\[
M_n \ddot{\delta}_n + C_n \dot{\delta}_n + K_n \delta_n = \ddot{y}(t)f_{tn}
\]

3. Dynamic behaviour of the system

3.1. Computational model

Figure 5 illustrates the system of a flexible single-link manipulator which has a track of the link, clamp as the holder of the link, a servo motor to rotate the link, and a DC motor to make a translation motion.

![Figure 5. Model system of flexible Single-link manipulator](image-url)
3.2. Physical parameters of dynamic models

Table 1. Physical parameters of the link

| Property                                           | Symbol | value          |
|----------------------------------------------------|--------|----------------|
| Total length (m)                                   | $L$    | $3.30 \times 10^1$ |
| Length of the link (m)                             | $l_i$  | $3.00 \times 10^{-1}$ |
| Breadth of cross section (m)                       | $b_i$  | $2.50 \times 10^{-2}$ |
| Height of cross section (m)                        | $h_i$  | $1.00 \times 10^{-3}$ |
| Cross section area of the link (m$^2$)             | $S_i$  | $1.95 \times 10^{-5}$ |
| Cross section area moment of inertia around i-axis of the link (m$^4$) | $I_i$ | $2.75 \times 10^{-12}$ |
| Young’s Modulus of the link (GPa)                  | $E_i$  | $7.00 \times 10^1$ |
| Density of the link (kg/m$^3$)                     | $\rho_i$ | $2.70 \times 10^3$ |
| Damping factor of the link                         | $\alpha$ | $0.10 \times 10^{-3}$ |

3.3. Time history response on free vibration

Time history response on free vibration simulation on free vibration was conducted using an impulse force as an external one. Figure 6 shows the simulation time history response of lateral deformation $v_p$ on the free vibration. Furthermore, the computational codes on time history response of single-link were developed. Figure 6 shows the calculated lateral deformation at Node 6 of the system under the impulse force.

![Figure 6. Time history response of the system](image)

3.4. FFT (Fast Fourier Transform) Processing.

The calculated result from time history response to lateral deformation with free vibration on the system was transferred by FFT processing to find its frequencies. Figure 7 shows the calculated natural frequencies of the flexible link manipulator. The first, second, and third natural frequency are 38.57 [Hz], 109.37 [Hz], and 174.32 [Hz].
Figure 7. Calculated natural frequencies of the link

3.5. Time history response of the system with excitation force.
The calculated result from time history response to lateral deformation with excitation force on the system is shown in the figure 8. Showing at 0.5 (s) in the timeline, the lateral deformation becomes large due to the excitation force ($f_t = 5$).

Figure 8. Time history response of the system with excitation force
4. Conclusions
The equation of motion on the single-link flexible manipulator moved translationally has been discretized using a finite-element method. The computational codes were developed to simulate the dynamic system using the Scilab application. The simulation and calculated result of time history response, natural frequencies, and vibration mode showed validated formulation, computational codes, and model system. Fast Fourier Transform (FFT) has also generated the first, second, and third natural frequencies values as (f1) 38.57 [Hz], (f2) 109.37 [Hz], and (f3) 174.32 [Hz].

Further research recommends to combine the translational and the rotational motion on the single-link flexible manipulator by finding the equation of translational and rotational motion. The further step will continue doing an experiment of the model.

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