PeerGAN: Generative Adversarial Networks with a Competing Peer Discriminator

A PREPRINT

Jiaheng Wei ∗
UC Santa Cruz
jiahengwei@ucsc.edu

Minghao Liu ∗
UC Santa Cruz
miu40@ucsc.edu

Jiahao Luo
UC Santa Cruz
jluo53@ucsc.edu

Qiutong Li
UC Santa Cruz
qli33@ucsc.edu

James Davis
UC Santa Cruz
davis@cs.ucsc.edu

Yang Liu
UC Santa Cruz
yangliu@ucsc.edu

ABSTRACT

In this paper, we introduce PeerGAN, a generative adversarial network (GAN) solution to improve the stability of the generated samples and to mitigate mode collapse. Built upon the Vanilla GAN’s two-player game between the discriminator $D_1$ and the generator $G$, we introduce a peer discriminator $D_2$ to the min-max game. Similar to previous work using two discriminators, the first role of both $D_1$, $D_2$ is to distinguish between generated samples and real ones, while the generator tries to generate high-quality samples which are able to fool both discriminators. Different from existing methods, we introduce another game between $D_1$ and $D_2$ to discourage their agreement and therefore increase the level of diversity of the generated samples. This property alleviates the issue of early mode collapse by preventing $D_1$ and $D_2$ from converging too fast. We provide theoretical analysis for the equilibrium of the min-max game formed among $G, D_1, D_2$. We offer convergence behavior of PeerGAN as well as stability of the min-max game. It’s worth mentioning that PeerGAN operates in the unsupervised setting, and the additional game between $D_1$ and $D_2$ does not need any label supervision. Experiments results on a synthetic dataset and on real-world image datasets (MNIST, Fashion MNIST, CIFAR-10, STL-10, CelebA, VGG) demonstrate that PeerGAN outperforms competitive baseline work in generating diverse and high-quality samples, while only introduces negligible computation cost.

1 Introduction

Vanilla GAN (Generative Adversarial Nets[14]) proposed a data generating framework through an adversarial process which has achieved great success in image generation [14, 20, 43, 13, 5, 41, 30, 39, 5, 10], image translation [82, 50, 9, 45], and other real-life applications [19, 27, 40, 5, 2, 47, 24, 34, 49, 25, 44]. However, training Vanilla GAN is usually accompanied with a number of common problems, for example, vanishing gradients, mode collapse and failure to converge. Unfortunately, none of these issues have been completely addressed. There is a large amount of follow up work on Vanilla GAN. Due to space limitations we only discuss the two most related lines of works.

Stable and diverse GAN training: Several stabilization techniques have been implemented in GAN variants. Modifying architectures is the most extensively explored category. Radford et al. [57] make use of convolutional and convolutional-transpose layer in training the discriminator and generator. Karras et al. [20] adopt a hierarchical architecture and trains the discriminator and generator with progressively increasing size. Huang et al. [18] proposed a generative model which consists of a top-down stack of GANs. Chen et al. [7] split the generator into the noise prior and also latent variables. The optimization task includes maximizing the mutual information between latent variables and the observation. Designing suitable loss functions is another favored technique. Successful designs include $f$-divergence based GAN [43, 28] (these two approaches replace loss functions of GAN by estimated variational $f$-divergence or least-square loss respectively), introducing auxiliary terms in the loss function [29] and integral probability metric based GAN ([41, 15, 21, 56]). A detailed survey of methods for stabilizing GANs exists [46].

∗Equal Contribution
Multi-player GANs: Multi-player GANs explore the situation where there are multiple generators or multiple discriminators. The first published work to introduce multiple discriminators to GANs is multi-adversarial networks, in which discriminators can range from an unfavorable adversary to a forgiving teacher [1]. Albuquerque et al. [11] formulate D2GAN, a three-player min-max game which utilizes a combination of Kullback-Leibler (KL) and reverse KL divergences in the objective function and is the most closely related to our work. Albuquerque et al. [11] show that training GAN variants with multiple discriminators is a practical approach even though extra capacity and computational cost are needed. Employing multiple generators and one discriminator to overcome the mode collapse issue and encourages diverse images has also been proposed [12, 13].

In contrast to the above existing work, we demonstrate the possibility of improving GAN training with a computationally light modification by adding only one competing discriminator. We introduce a competitive game among two discriminators and demonstrate the benefits of doing so in stabilizing and diversifying the training.

Our main contributions summarize as follows:

- We introduce a competition between two discriminators to encourage diverse predictions and avoid early failure. The intuition is that predictions with high consensus will be discouraged, and effectively both discriminators are rewarded for having diverse predictions. The introduced game between the two discriminators results in a different convergence pattern for the generator.
- We theoretically derive the equilibrium for discriminators and the generator. We show how PeerGAN alleviates the vanishing gradient issue and mode collapse. We derive evidence for how the peer discriminator helps the learning dynamics. In addition, we demonstrate that if the peer discriminator is better than a random guess classifier, the intermediate game and the objective function in PeerGAN are robust to corrupted/failed peer discriminator.
- Experimental results on a synthetic dataset validate that PeerGAN addresses mode collapse. Results on real datasets demonstrate that PeerGAN generates high-quality image samples compared with baseline work.

2 Background

We first review Vanilla GAN and D2GAN, which are the most relevant to understanding our proposed PeerGAN.

Vanilla GAN [14]: Let \( \{ x_i \}_{i=1}^n \subseteq \mathcal{X} \) denote the given training dataset drawn from the unknown distribution \( p_{\text{data}} \). Traditional GAN formulates a two-player game: a discriminator \( D \) and a generator \( G \). To learn the generator \( G \)'s distribution over \( \mathcal{X} \), \( G \) maps a prior noise distribution \( p_z(z) \) to the data space. \( \forall x \in \mathcal{X}, D(x) \) returns the probability that \( x \) belongs to \( p_{\text{data}} \) rather than \( p_g \), where \( p_g \) denotes the distribution of \( G(z) \) implicitly defined by \( G \). GAN trains \( D \) to maximize the probability of assigning the correct label to both training samples and those from the generator \( G \). Meanwhile, GAN trains \( G \) to minimize \( \log(1 - D(G(z))) \).

\[
\min_G \max_D V(D, G) = \mathbb{E}_{x \sim p_{\text{data}}} \log D(x) + \mathbb{E}_{z \sim p_z} \log \left( 1 - D(G(z)) \right)
\] (1)

D2GAN [32]: D2GAN is the most closely related method to PeerGAN. This three-player game aims to solve the mode collapse issue and the optimization task is equivalent to minimizing both KL divergence and Reverse-KL divergence between \( p_{\text{data}} \) and \( p_g \). The formulation of D2GAN comes as follows:

\[
\min_G \max_{D_1, D_2} V(D_1, D_2, G) = \alpha \cdot \mathbb{E}_{x \sim p_{\text{data}}} \log D_1(x) + \mathbb{E}_{z \sim p_z} [-D_1(G(z))] + \mathbb{E}_{x \sim p_{\text{data}}} [-D_2(x)] + \beta \cdot \mathbb{E}_{z \sim p_z} \log D_2(G(z))
\] (2)

Given a sample \( x \) in data space, \( D_1(x) \) rewards a high score if \( x \) is drawn from \( p_{\text{data}} \), and gives a low score if generated from the generator distribution \( p_g \). In contrast, \( D_2(x) \) returns a high score for \( x \) generated from \( p_g \) and gives a low score for a sample drawn from \( p_{\text{data}} \). Our work is similar to D2GAN in containing a pair of discriminators, however instead of discriminators with different goals, we use identical discriminators and introduce a competition between them.

3 PeerGAN

In this section, we first give the formulation and intuition of PeerGAN. Then we will present the equilibrium strategy of the generator and the discriminators.
Figure 1: Illustration of the proposed PeerGAN. Compared with Vanilla GAN, PeerGAN has one more identical discriminator and a competitive Peer Game between two discriminators. The introduced Peer Game induces diversified generated samples by discouraging the agreement between $D_1$ and $D_2$. In D2GAN, although both discriminators are trained with different loss functions, they do not interfere with each other in the training.

3.1 Formulation

Similar to related works, we assume that the data follows the distribution $p_{data}$, our ultimate goal is to achieve $p_g = p_{data}$ where $p_g$ is the generator’s distribution. PeerGAN formulates a three-player game which consists of two discriminators $D_1, D_2$ and one generator $G$. Denote by $p_{peer}$ an equal mixture of $\forall x, p_{data}$ and $p_g$: $p_{peer}(x) = \{p_{data}(x) + p_g(x)/2\}$. Recall that $p_z$ denotes the prior noise distribution. Now we are ready to formulate the min-max game of PeerGAN as follows:

$$\min_G \max_{D_1, D_2} \mathcal{L}(D_1, D_2, G) = \min_G \max_{D_1, D_2} E_{x \sim p_{data}}[\log D_1(x)] + E_{x \sim p_{data}}[\log D_2(x)]$$

$$+ \beta \cdot \text{Peer-D} + E_{z \sim p_z} \left[ \log \left(1 - D_1(G(z))\right) \right] + E_{z \sim p_z} \left[ \log \left(1 - D_2(G(z))\right) \right]$$

(3)

where Peer-D introduces the peer competition game among $D_1, D_2$, defined as:

$$\text{Peer-D} = E_{x \sim p_{peer}} \left[ \ell(D_1(x), I(D_2(x) > \frac{1}{2}) - \alpha \cdot \ell(D_1(x_{p_1}), I(D_2(x_{p_2}) > \frac{1}{2})) \right]$$

Term 1a

$$+ E_{x \sim p_{peer}} \left[ \ell(D_2(x), I(D_1(x) > \frac{1}{2})) - \alpha \cdot \ell(D_2(x_{p_1}), I(D_1(x_{p_2}) > \frac{1}{2})) \right]$$

Term 2a

(4)

In Peer-D, $x_{p_1}, x_{p_2}$ are drawn randomly from $p_{peer}$ and that $x, x_{p_1}, x_{p_2}$ are independent with each other. $I(\cdot)$ is the indicator function, $\alpha, \beta \in [0, 1]$ are hyper-parameters controlling the weight of different terms. $\ell$ is an evaluation function, for simplicity, we adopt $\ell = \log(\cdot)$, as commonly used in other terms in the min-max game. Thus, we have:

$$\ell(D_i(x), y) = \begin{cases} \log(D_i(x)) & \text{if } y = 1 \\ \log(1 - D_i(x)) & \text{if } y = 0 \end{cases}$$

To clarify the differences among Vanilla GAN [14], D2GAN [32] and PeerGAN, we use an workflow to illustrate in Figure[1]. The key differences in PeerGAN’s formulation can be summarized as follows:

- Compared with Vanilla GAN (see Eqn[1]), PeerGAN (see Eqn[3]) introduces a peer discriminator $D_2$ which has the same objective function as $D$ appeared in Eqn[1]. An intermediate competition game Peer-D is added which will be explained below.

- The difference between D2GAN (see Eqn[2]) and PeerGAN is highlighted with the underscores in red. Primarily, there is no interaction between discriminators in D2GAN, while our Peer-D term introduces another competition game between the discriminators, which we explain below. In addition to Peer-D, the objective function in PeerGAN encourages both discriminators to fit perfectly on both training samples and generated samples. While in D2GAN, one discriminator fits overly on training samples, the other fits overly on generated samples.
Competition introduced by Peer-D: Peer-D bridges $D_1$ and $D_2$ by introducing 4 terms specified in Eqn.4. Since we do not expect arbitrarily different discriminators, and both $D_1$s should play against the generator $G$, Term 1a and Term 2a encourage agreements between $D_1$ and $D_2$. With only these two terms, $D_1$ and $D_2$ will eventually be encouraged to converge to agree with each other. Mode collapse issue remains a possibility. Thus, PeerGAN introduces Term 1b and Term 2b to the objective function which punish $D_1$ and $D_2$ from overagreeing, especially at the early phase of training. Particularly, the Term 1b and 2b are evaluating the agreements of $D_1$ and $D_2$ on two entirely independent samples $x_{p_1}, x_{p_2}$. Because of the independence, the two discriminators’ predictions should not match with high probability! Therefore these two terms punish two discriminators from agreeing on $p_1, p_2$. Note that the calculation of Peer-D does not need label supervisions, which distinguishes our work from other works that introduces multiple discriminators but would require additional label supervisions [9].

We provide more details of our intuition as well as theoretical evidences of this property in Section 4.

3.2 The max game of discriminators

Denote the true label of $x$ as $y = 1$ if $x$ comes from $p_{data}$, otherwise, $y = 0$. For any given generator $G$, let us first analyze the best responding/optimal discriminator $D_{i,G}^*$, $i \in \{1, 2\}$. We define the following quantities:

$$r_{i,G}(x) := \mathbb{P}_{x \sim p_{data}}\left( 1(D_i(x) > \frac{1}{2}) = 1 \right), \quad p_{i,G} := \mathbb{E}_{x \sim p_{peeer}}\left( 1(D_i(x) > \frac{1}{2}) = 1 \right)$$

where $r_{i,G}(x)$ represents the probability/confidence of $x$ being categorized as the real data by $D_i$ and $p_{i,G}$ is the expectation of $r_{i,G}(x)$ for $x \sim p_{peeer}$. Let $r_{i,G}^*(x) := r_{i,G}(x) - \alpha \cdot p_{i,G}$. Given discriminator $D_i$, when there is no confusion, we use $D_j$ to denote the peer discriminator without telling $j \neq i$ in later sections.

**Proposition 1.** For $G$ fixed, let $w := \beta \cdot (1 - \alpha)$, the optimal discriminators $D_1, D_2$ are given by:

$$D_{i,G}^*(x) = \frac{p_{data}(x) + \beta \cdot r_{j,G}^*(x) \cdot p_{peeer}(x)}{p_{data}(x) + p_g(x) + w \cdot p_{peeer}(x)}, \quad i = 1, 2.$$

3.3 The min game of the generator

Remember that the training objective for $D_i$ can be interpreted as maximizing the log-likelihood for estimating the conditional probability $\mathbb{P}(Y = y|x)$ where $Y$ indicates whether $x$ comes from $p_{data}$ (with $y = 1$) or from $p_g$ (with $y = 0$). With the introduce of Peer Game, the distributions $p_{data}$ and $p_g$ in the Vanilla GAN got changed due to the appearance of $p_{peeer}$. Thus, we define the corresponding updated distributions in PeerGAN w.r.t. discriminator $D_i$ as $p_{data}$, and $p_g$, respectively. For a clean presentation, we defer the exact form of $p_{data}$, $p_g$ in Appendix (Eqn.6). Denote $C(G) := \max_{D} L(G, D_1, D_2)$, the inner-max game ($C(G)$) can be rewritten as (straightforward in the proof of Proposition[4]) which is available in the Appendix):

$$C(G) = \mathbb{E}_{x \sim p_{data}} \left[ \log D_{1,G}^*(x) \right] + \mathbb{E}_{x \sim p_{g_1}} \left[ \log (1 - D_{1,G}^*(x)) \right] + \mathbb{E}_{x \sim p_{g_2}} \left[ \log (1 - D_{2,G}^*(x)) \right]$$

**Theorem 1.** When $\alpha = 0$, $r_{j,G}(x) = \frac{1}{2}$, the global minimum of the virtual training criterion $C(G)$ is achieved if and only if $p_{data} = p_g$. At this point, $C(G)$ achieves the value of $- \log 16$.

When $r_{j,G}(x) = \frac{1}{2}$? Note that $r_{j,G}(x)$ is merely representing the probability that $D_j$ classifies $x$ to be real samples, $p_{j,G}$ is the probability that $D_j$ classifies a random sample as the real one. Without loss of generality, we assume real and generated samples are of uniform/equal prior. At the very beginning of the training process, the discriminator can do well in distinguishing real or generated samples, since the generator at this time generates low-quality samples. In this case, $r_{j,G}(x)$ is supposed to approach its max/min value, for example, $r_{j,G}(x) \rightarrow 0$ if $x$ is from generated samples, and otherwise, $r_{j,G}(x) \rightarrow 1$. During the training process, the generator progressively tries to mislead the predictions made by discriminators, which means the discriminator cannot decide whether the sample is being fake or real. Thus, $r_{j,G}(x) \rightarrow \frac{1}{2}$ At this time, for $\alpha = 0$, we have:

$$D_{i,G}^*(x) = \frac{p_{data}(x) + \beta \cdot r_{j,G}^*(x) \cdot p_{peeer}(x)}{p_{data}(x) + p_g(x) + \beta \cdot p_{peeer}(x)} \rightarrow \frac{p_{data}(x) + \beta \cdot p_{peeer}(x)}{p_{data}(x) + p_g(x) + \beta \cdot p_{peeer}(x)}, \quad i = 1, 2.$$
This allows us to rewrite $C(G)/2$ as:

$$
\mathbb{E}_{x \sim p_{data}} \left[ \log \frac{p_{data}(x) + \beta \cdot p_{peer}(x)}{p_{data}(x) + p_g(x) + \beta \cdot p_{peer}(x)} \right] + \mathbb{E}_{x \sim p_g} \left[ \log \frac{p_g(x) + \beta \cdot p_{peer}(x)}{p_{data}(x) + p_g(x) + \beta \cdot p_{peer}(x)} \right]
$$

Our subsequent proof is then based on the above reformulation. The formal algorithm for PeerGAN is stated in Algorithm 1.

**Algorithm 1 PeerGAN**

1: **Input:** two discriminators $D_1, D_2$, generator $G$, training samples $\{x_i\}_{i=1}^n$, weights $\alpha, \beta$.
2: **For** number of training iterations **do**
   
   **For** 1 to $k$ steps **do**
   
   • Sample mini-batch of $m$ noise samples $Z = \{z_1, \ldots, z_m\}$ from noise prior $p_z$.
   • Sample mini-batch of $m$ samples $X = \{x_1, \ldots, x_m\}$ from data generating distribution $p_{data}(x)$.
   • Combine two subsets $T := X \cup Z$, and denote by $T = \{t_1, \ldots, t_{2m}\}$.
   • Update discriminator $D_i (i \in \{1, 2\})$ by ascending the stochastic gradient:
   
   $$
   \nabla_{\theta_i} \frac{1}{m} \sum_{i=1}^{m} \left[ \log D_i(x_i) + \log \left( 1 - D_i(G(z_i)) \right) \right] + \beta \cdot \frac{1}{2m} \sum_{j=1}^{2m} \left[ \ell_{CE} \left( D_i(t_j), \mathbb{1} \left( D_j(t_j) > \frac{1}{2} \right) \right) - \alpha \cdot \ell_{CE} \left( D_i(t_{p_1}), \mathbb{1} \left( D_j(t_{p_2}) > \frac{1}{2} \right) \right) \right]
   $$
   
   where $t_{p_1}, t_{p_2}$ are randomly selected (with replacement) samples from $T$.

   • Update $G$ by descending its stochastic gradient:

   $$
   \nabla_{\theta} \frac{1}{m} \sum_{i=1}^{m} \left[ \log \left( 1 - D_i(G(z_i)) \right) \right] + \log \left( 1 - D_2(G(z)) \right)
   $$

In experiments, we train $G$ to minimize $\log(1 - D_i(G(z)))$ which is equivalent to maximizing $\log D_i(G(z))$.

## 4 Properties of PeerGAN

In this section, we first illustrate how PeerGAN solves common issues in GAN training, for example, the vanishing gradients issue and the mode collapse issue. Then we present properties of PeerGAN including its stability guarantee and converging behavior.

### 4.1 PeerGAN and common issues in GAN training

**Vanishing gradients issue:** In training GAN, discriminators might be too good for the generator to fool with and to improve progressively. When training with neural networks with back-propagation or gradient-based learning approaches, a vanishing small gradient only results in minor changes even with a large weight. As a result, the generator training may fail due to the vanishing gradients issue.

**Mode collapse issue:** Mode collapse refers to the phenomenon that generator will rotate through a small set of output types. For the given fixed discriminator, the generator over-optimizes in each iteration. Thus, the corresponding discriminator fails to learn its way out of the trap.

PeerGAN solves these two issues by preventing discriminators from “colluding” on its discriminating ability. In PeerGAN, for either discriminator $D_1, x_{p_1}, x_{p_2}$ are randomly drawn from $p_{peer}$ which are independent with each other,
the max game of $D_i$ given its peer discriminator $D_j$ is to perform the following optimization task:

$$
\max_{D_i} \mathcal{L}(D_i, G)|_{D_j} = \max_{D_i} \mathbb{E}_{x \sim p_{data}} [\log D_i(x)] + \mathbb{E}_{z \sim p_{z}} \left[ \log \left( 1 - D_i(G(z)) \right) \right] + \beta \cdot \mathbb{E}_{x \sim p_{fake}} \left[ \ell \left( D_i(x), \mathbb{I} \left( D_j(x) > \frac{1}{2} \right) \right) \right] - \alpha \cdot \ell \left( D_i(x_{p1}), \mathbb{I} \left( D_j(x_{p2}) > \frac{1}{2} \right) \right)
$$

(Clearly, Term $\ominus$ maximizes the probability of assigning the correct label to both real samples and generated samples. Term $\ominus$ maximizes the probability of matching predicted label with peer discriminator predicted ones. In other words, Term $\ominus$ controls the agreement level of $D_i$ with respect to its peer discriminator $D_j$.

However, note that Term $\ominus$ checks on the predictions of $D_i$ on two different tasks $x_{p1}, x_{p2}$, when $D_i$ agrees/fits overly on $D_j$, Term $\ominus$ receives a lower score if $D_j$’s predictions on these two different tasks are mismatching, mathematically, $\mathbb{I}(D_j(x_{p1}) > \frac{1}{2}) \neq \mathbb{I}(D_j(x_{p2}) > \frac{1}{2})$. And Term $\ominus$ will receive a high score if $D_j$’s predictions on these two different tasks are indeed different mathematically, $\mathbb{I}(D_j(x_{p1}) > \frac{1}{2}) \neq \mathbb{I}(D_j(x_{p2}) > \frac{1}{2})$. Moreover, the weight $\alpha$ controls this disagreement level compared with Term $\ominus$ by referring to the fact that a larger $\alpha$ encourages more disagreement/diverse predictions from discriminators.

In PeerGAN, when two discriminators are of a high disagreement level, there exists $X_{\text{data}}$ such that $\mathbb{I}(D_i(x) > \frac{1}{2}) \neq \mathbb{I}(D_j(x) > \frac{1}{2})$ for $x \in X_{\text{data}}$ and $X_{\text{data}}$ is non-negligible. Therefore, there exists at least one discriminator $D_i$ that can’t perfectly predict labels (real/generated) of given data samples. The generator will then be provided with enough information to make progress, for example, information or features that can be extracted from $X_{\text{data}}$. This property helps us address the vanishing gradients issue.

As for the mode collapse issue, suppose the over-optimized generator is able to find plausible outputs for both discriminators in the next generation. Note that optimization is implemented on mini-batches in practice, the randomly selected samples $x_{p1}, x_{p2}$ in Peer-D as well as the dynamically changing weights $\alpha, \beta$ can bring a certain degree of randomness in the next generation so that $D_i$ unlikely gets stuck in a local minimum. In section 4.2, we use synthetic experiments to show that PeerGAN addresses mode collapse issues.

4.2 Stability and convergence behavior

In section 4.1, we discussed the significant role of the introduced intermediate game. Now we discuss the potential downsides of introducing a second discriminator. Particularly, we are interested in understanding if the introduce of a peer discriminator $D_j$ will disrupt the training and make the competition game with $D_i$ unstable. Suppose $D_j$ diverges from the optimum in the max game, in other words, the diverged peer discriminator $\hat{D}_j$ fails to provide qualified verification label $Y^*_j$ (given by $D_{j,G}^*$), and provides $\hat{Y}_j$ instead. Mathematically, denote:

$$
e_{\text{data},j} := \mathbb{P}(\hat{Y}_j = 0|Y^*_j = 1), \quad e_{g,j} := \mathbb{P}(\hat{Y}_j = 1|Y^*_j = 0)
$$

For any peer discriminator $D_j$, for example, $D_j$ may be a diverged peer discriminator $\hat{D}_j$ or an optimal one $D_{j,G}^*$, we denote the Peer Game of $D_i$ given her peer discriminator $D_j$ as:

$$\text{Peer}(D_i)|D_j := \mathbb{E}_{x \sim p_{data}} \left[ \ell \left( D_i(x), \mathbb{I} \left( D_j(x) > \frac{1}{2} \right) \right) \right] - \alpha \cdot \ell \left( D_i(x_{p1}), \mathbb{I} \left( D_j(x_{p2}) > \frac{1}{2} \right) \right)$$

Theorem 2 explains the condition of stability (for $D_i$) when its peer discriminator in PeerGAN diverges from the corresponding optimum.

**Theorem 2.** Given $G$, if $D_i$ has enough capacity, and at one step of Algorithm 1, if $e_{\text{data},j} + e_{g,j} < 1$, $\alpha = 1$, the peer term of discriminator $D_i$ is robust with diverged peer discriminator $\hat{D}_j$. Mathematically,

$$
\max_{D_i} \text{Peer}(D_i)|\hat{D}_j \text{ is equivalent with } \max_{D_i} \text{Peer}(D_i)|D_{j,G}^*
$$
The above theorem implies that a diverging and degrading peer discriminator $D_j$ will not disrupt the training of $D_i$.

**Remark.** Note that assuming uniform prior of real and generated samples, the condition to be robust is merely requiring that the proportion of false/wrong $D_j$’s prediction is less than a half (random guessing). This condition can be easily satisfied in practice. Thus, Theorem 2 provides the robustness guarantee when the peer discriminator diverged from its optimum.

Build upon Theorem 1 with sufficiently small updates, Theorem 3 presents when $p_g$ converges to $p_{\text{data}}$.

**Theorem 3.** If $G$ and $D_i$s have enough capacity, and at each step of Algorithm 1, $D_i$s are allowed to reach its optimum given $G$, $D_i$ is updated so as to improve the criterion in Eqn.5 and $p_g$ is updated so as to improve:

$$C(G) = E_{x \sim p_{\text{data}}_1} [\log D^*_1, G(x)] + E_{x \sim p_{\text{data}}_2} [\log (1 - D^*_1, G(x))] + E_{x \sim p_{\text{data}}_2} [\log (1 - D^*_2, G(x))]$$

if $\beta = 0$, we have $D^*_1, G = D^*_2, G$, $p_g$ converges to $p_{\text{data}}$.

5 Experiments

In order to experimentally validate our theoretical findings, we implemented PeerGAN and evaluated the performance on both a synthetic task and several real world datasets. The synthetic task was specifically chosen to evaluate model diversity and mode collapse. Real world datasets include MNIST [23], FashionMNIST [48], CIFAR-10 [22], STL-10 [8], CelebA [26], and VGGFace2 [6]. These six datasets include content ranging from hand-written digits to human faces. PeerGAN is found to improve results when compared against existing methods.

5.1 Baseline methods

We compare with five existing baseline methods: DCGAN [37], D2GAN [32], WGAN [15], DRAGAN [21], LSGAN [36], MicroBatchGAN [31], and Dist-GAN [42].

WGAN and LSGAN aim to maximize the mutual information between latent variables and observation. WGAN replaces the loss function with an estimated Earth-Mover distance, LSGAN replaces the loss function with a least squares loss. DRAGAN is an integral probability metric based GAN method which studies the GAN training process as regret minimization. DCGAN proposes a novel generator and discriminator backbone for real-world image generation. WGAN, LSGAN, DRAGAN, and DCGAN have the same architecture as Vanilla GAN.

Dist-GAN introduced an additional auto encoder to apply distance constraints to guide generator during training. D2GAN and MicroBatchGAN are the most similar to our work, introducing additional discriminators to the Vanilla GAN architecture with the intention to overcome mode collapse. However they all lack an explicit term discouraging the discriminators from converging.

5.2 Synthetic example

We apply the experiment and model structures proposed in UnrolledGAN [29] to investigate whether the PeerGAN design can prevent mode collapse. This experiment aims to generate eight 2D Gaussian distributions with a covariance matrix $0.02I$, arranged around the same centroid with radius 2.0. Vanilla GAN fails on this example. D2GAN has been shown to outperform UnrolledGAN, so we include it as an alternate method which performs well.

Figure 2 shows symmetric KL-divergence, Wasserstein distance, and a visualization of results with Vanilla GAN, D2GAN, and PeerGAN. Knowing the target distribution $p_{\text{data}}$, we can employ symmetric KL divergence and Wasserstein distance, which calculate the distance between the true $p_{\text{data}}$ and the normalized histogram of 10,000 generated points. On the left of Figure 2 the plots for symmetric KL-divergence and Wasserstein distance show that PeerGAN has a much better score than Vanilla GAN and slightly better than D2GAN.

On the right side of Figure 2 is a visualization of 512 generated blue samples points, together with red data points drawn from the true distribution. Vanilla GAN generates data points around only a single valid mode of the data distribution. D2GAN and PeerGAN distribute data around all eight mixture components, demonstrating the ability to resolve modal collapse in this case.

5.3 Real image examples

We tested the proposed PeerGAN and baseline methods on MNIST [23], FashionMNIST [48], CIFAR-10 [22], STL-10 [8], CelebA [26], and VGGFace2 [6]. For quantitative evaluation, we adopt Fréchet Inception Distance (FID) [16].
Figure 2: Comparison of Vanilla GAN, D2GAN, and proposed PeerGAN on 2D synthesized data. The top-left graph shows the symmetric KL divergence over the training iterations, while the bottom left graph shows the Wasserstein distance. Both metrics compare the generated data points to data points drawn from the true target distribution. Note that PeerGAN has the best performance. The right side visualizes generated blue data points and true red data points. Note that Vanilla GAN has a clear mode collapse which both D2GAN and PeerGAN avoid.

and Inception score (IS) [38] as the evaluation metric. FID summarizes the distance between the Inception features of the generated images and the real images. A lower FID indicates both better accuracy and higher diversity, so that a batch of generated images with good accuracy but identical to each other will have a poor FID score. A higher IS score indicates a higher generated image quality.

We used the same generator and discriminator backbone for all the comparison methods in each dataset unless specified by the original author. We recorded the best performing checkpoints when evaluating each method.

Grey-scale images MNIST [23] and FashionMNIST [48] are small grey-scale image datasets including 60,000 training and 10,000 testing $28 \times 28$ gray-scale images of hand-written digits and clothing. Since they are of small-scale, we adopt the shallow version of the generator and discriminators to generate the grey-scale images. Table 2 first two columns show our method has the best FID score among all tested methods. Figure 3 (left) shows FashionMNIST image results.

Table 1: Experiment Inception score results of CIFAR-10 and STL-10.

| Method     | CIFAR10 | STL-10 |
|------------|---------|--------|
| WGAN [15]  | 3.82    | 3.97   |
| GAN [36]   | 2.61    | 2.17   |
| MicroBatchGAN [31] | 6.77   | 7.23   |
| DCGAN [37] | 6.40    | 5.87   |
| D2GAN [32] | 7.15    | 6.15   |
| PeerGAN (ours) | **7.45** | 6.22   |

Natural scene CIFAR-10 [22] and STL-10 [8] are natural scene RGB image datasets. CIFAR-10 includes 50,000 training and 10,000 testing $32 \times 32$ images with ten unique categories: airplane, automobile, bird, cat, deer, dog, frog, horse, ship, and truck. STL-10 is sub-sampled from ImageNet, and has more diverse samples than CIFAR-10, containing about 100,000 $96 \times 96$ images. We adopt the deep version of the generator and discriminator to generate $32 \times 32$ RGB images. Table 2 middle two columns show FID score results and Table 1 shows the inception score results. Note that the introduce of competitive Peer Game in two discriminator GAN setup, brings performance boost in all the experiments. Figure 3 (middle) shows STL-10 image results.

Human face Both CelebA [26] and VGGFace2 [6] are large-scale face datasets. CelebA includes 162,770 training and 19,962 testing images of celebrity faces. VGGFace2 includes more than 3.3 million face images of celebrities caught in the ‘wild’. There are different lighting conditions, emotions, and viewing angles. We randomly choose 200 categories in the VGGFace2 dataset and trained this reduced dataset. We adopt the deep version of the generator and discriminators to generate $32 \times 32$ RGB images on CelebA and $64 \times 64$ RGB images on VGGFace2. Table 2 last two columns show our method has the best FID score among tested methods. Figure 3 (right) shows CelebA image results.

5.4 Implementation details

Architecture: Our model architecture adopts the same generator and discriminator backbone as DCGAN [37]. In PeerGAN, the newly introduced discriminator is a duplicate of the first one.
Table 2: Experiment FID score results of grey-scale image dataset: MNIST and FashionMNIST; natural scene image dataset: CIFAR-10 and STL-10; human face image dataset: CelebA and VGGFace2. Baseline results denoted with (*) were extracted from the original paper report, not independently run in our experiments.

| Method             | MNIST | FashionMNIST | CIFAR10 | STL-10 | CelebA | VGG |
|--------------------|-------|--------------|---------|--------|--------|-----|
| WGAN[15]           | 14.07 | 28.24        | 35.37   | 60.21  | 15.23  | 39.24 |
| LSGAN[36]          | 23.80 | 43.00        | 51.42   | 70.37  | 15.35  | 55.96 |
| DRAGAN[21]         | 66.96 | 62.64        | 36.49   | 91.07  | 14.57  | 50.20 |
| MicroBatchGAN* [31]| 17.1  | -            | 77.7    | -      | 34.5   | -   |
| Dist-GAN* [42]     | -     | -            | 22.95   | 36.19  | 23.7   | -   |
| DCGAN [37]         | 19.86 | 24.78        | 27.45   | 59.79  | 17.38  | 49.99 |
| D2GAN [32]         | 22.20 | 29.33        | 27.38   | 54.12  | 17.30  | 20.67 |
| PeerGAN (ours)     | 7.87  | 21.73        | 21.55   | 51.37  | 13.95  | 19.05 |

Figure 3: Image results generated by proposed PeerGAN. Left: FashionMNIST, grey-scale clothing images; Middle: STL-10, natural scene images; Right: CelebA, large-scale celebrate face images.

For the smaller-scale MNIST datasets, we used a shallow version of generator and discriminator. Here we have three convolution layers in the generator and four layers in the discriminators. We use a deep version of generator and discriminator for natural scene and human face image generation, which have three convolution layers in the generator and seven layers in the discriminators.

Hyper-parameters: PeerGAN achieves low FID scores and high IS scores when $\alpha$ and $\beta$ are simply set to constant values. However we found that we could obtain an approximately 10% improvement through dynamic tuning. The parameter $\beta$ controls the overall weight of Peer-D, while $\alpha$ punishes the condition when $D_1$ over-agrees with $D_2$. In the early training phase when we have an unstable generator and discriminator, we set $\alpha$ and $\beta$ to 0. As training progresses, we gradually increase these parameters to a max value, which helps with vanishing gradients. After the midpoint of training we decrease these parameters to help the discriminators converge, until the parameters reach approximately 0 at the end of the training process. We use 0.3 as a max value for $\alpha$ and 0.5 as a max value for $\beta$. An ablation study of hyper-parameters tuning is shown in the Appendix.

6 Conclusion

We propose PeerGAN which introduces a peer discriminator to Vanilla GAN. The role of the peer discriminator is to allow an intermediate game between discriminators. Theoretical analysis demonstrates that the introduced intermediate game incentivizes incremental improvement, addresses vanishing gradients and mode collapse issues, punishes over-agreements among discriminators and is robust with diverged peer discriminator. Experimental results on a synthetic dataset and multiple real world datasets validate that PeerGAN produces high quality images, with lower error than competing techniques.
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Appendix

A  Omitted proofs

A.1  Proof of Proposition 1

We first introduce Lemma 1 which helps with the proof of Proposition 1.

**Lemma 1.** For any \((a, b) \in \mathbb{R}^2 \setminus \{0, 0\}\), the function \(y \rightarrow a \log(y) + b \log(1 - y)\) achieves its maximum in \([0, 1]\) at \(\frac{a}{a+b}\).

**Proof.** Denote by \(f(y) := a \log(y) + b \log(1 - y)\), clearly, when \(y = 0\) or \(y = 1\), \(f(y) = -\infty\). For \(y \in (0, 1)\):

\[
f'(y) = 0 \iff \frac{a}{y} - \frac{b}{1-y} = 0 \iff y = \frac{a}{a+b}
\]

Note that \(f'(y) > 0\) if \(0 < y < \frac{a}{a+b}\) and \(f'(y) < 0\) if \(1 > y > \frac{a}{a+b}\). Thus, the maximum of \(f(y)\) should be \(\max(f(a), f\left(\frac{a}{a+b}\right), f(b)) = f\left(\frac{a}{a+b}\right)\). And \(f(y)\) achieves its maximum in \([0, 1]\) at \(\frac{a}{a+b}\). \(\square\)

**Proof of Proposition 1**

**Proof.** The trainer criterion for the discriminator \(D_i\), given any generator \(G\), is to maximize the quantity \(\mathcal{L}(D_1, D_2, G)\):

\[
\mathcal{L}(D_1, D_2, G) = \int_x p_{\text{data}}(x) \left[ \log \left( D_1(x) \right) + \log \left( D_2(x) \right) \right] dx
\]

\[
+ \int_z p_z(z) \left[ \log \left( 1 - D_1(G(z)) \right) + \log \left( 1 - D_2(G(z)) \right) \right] dz
\]

\[
+ \beta \cdot \int_x p_{\text{peer}}(x) \left[ (r_{2,G}(x) - \alpha \cdot p_{2,G}) \cdot \log \left( D_1(x) \right) + (r_{1,G}(x) - \alpha \cdot p_{1,G}) \cdot \log \left( D_2(x) \right) \right] dx
\]

\[
+ \beta \cdot \int_x p_{\text{peer}}(x) \left( 1 - \alpha - r_{2,G}(x) + \alpha \cdot p_{2,G} \right) \cdot \log \left( 1 - D_1(x) \right) dx
\]

\[
+ \beta \cdot \int_x p_{\text{peer}}(x) \left( 1 - \alpha - r_{1,G}(x) + \alpha \cdot p_{1,G} \right) \cdot \log \left( 1 - D_2(x) \right) dx
\]

\[
= \int_x p_{\text{data}}(x) \left[ \log \left( D_1(x) \right) + \log \left( D_2(x) \right) \right] dx + \int_x p_y(x) \left[ \log \left( 1 - D_1(x) \right) + \log \left( 1 - D_2(x) \right) \right] dx
\]

\[
+ \beta \cdot \int_x p_{\text{peer}}(x) \left[ (r_{2,G}(x) - \alpha \cdot p_{2,G}) \cdot \log \left( D_1(x) \right) + (r_{1,G}(x) - \alpha \cdot p_{1,G}) \cdot \log \left( D_2(x) \right) \right] dx
\]

\[
+ \beta \cdot \int_x p_{\text{peer}}(x) \left( 1 - \alpha - r_{2,G}(x) + \alpha \cdot p_{2,G} \right) \cdot \log \left( 1 - D_1(x) \right) dx
\]

\[
+ \beta \cdot \int_x p_{\text{peer}}(x) \left( 1 - \alpha - r_{1,G}(x) + \alpha \cdot p_{1,G} \right) \cdot \log \left( 1 - D_2(x) \right) dx
\]

\[
= \int_x \left[ p_{\text{data}}(x) + \beta \cdot (r_{2,G}(x) - \alpha \cdot p_{2,G}) \cdot p_{\text{peer}}(x) \right] \cdot \log \left( D_1(x) \right) dx
\]

\[
+ \int_x \left[ p_y(x) + \beta \cdot (1 - \alpha - r_{2,G}(x) + \alpha \cdot p_{2,G}) \cdot p_{\text{peer}}(x) \right] \cdot \log \left( 1 - D_1(x) \right) dx
\]

\[
+ \int_x \left[ p_{\text{data}}(x) + \beta \cdot (r_{1,G}(x) - \alpha \cdot p_{1,G}) \cdot p_{\text{peer}}(x) \right] \cdot \log \left( D_2(x) \right) dx
\]

\[
+ \int_x \left[ p_y(x) + \beta \cdot (1 - \alpha - r_{1,G}(x) + \alpha \cdot p_{1,G}) \cdot p_{\text{peer}}(x) \right] \cdot \log \left( 1 - D_2(x) \right) dx
\]

For \(D_1, D_2\), the above objective function respectively achieves its maximum in \([0, 1], [0, 1]\) at:

\[
D^*_i,G(x) = \frac{p_{\text{data}}(x) + \beta \cdot (r_{2,G}(x) - \alpha \cdot p_{2,G}) \cdot p_{\text{peer}}(x)}{p_{\text{data}}(x) + p_y(x) + \beta \cdot (1 - \alpha) \cdot p_{\text{peer}}(x)}, \quad i \neq j.
\]


With the introduce of Peer Game, the distributions \( p_{\text{data}} \) and \( p_g \) in the Vanilla GAN got changed due to the appearance of \( p_{\text{peer}} \). Thus, we define the corresponding updated distributions in PeerGAN w.r.t. discriminator \( D_i \) as \( p_{\text{data}} \) and \( p_g \), respectively.

\[
\begin{align*}
 p_{\text{data}}(x) &:= \frac{p_{\text{data}}(x) + \beta \cdot \hat{r}_{j,G}^*(x) \cdot p_{\text{peer}}(x)}{\int_x p_{\text{data}}(x) + \beta \cdot \hat{r}_{j,G}^*(x) \cdot p_{\text{peer}}(x) \, dx} \\
p_g(x) &:= \frac{p_g(x) + \beta \cdot (1 - \hat{r}_{j,G}^*(x)) \cdot p_{\text{peer}}(x)}{\int_x p_g(x) + \beta \cdot (1 - \hat{r}_{j,G}^*(x)) \cdot p_{\text{peer}}(x) \, dx}
\end{align*}
\]

(A.2) Proof of Theorem 1

Proof. This is the subsequent proof which complements with the end of Section 3.3. When \( \alpha = 0 \), \( r_{j,G}(x) = \frac{1}{2} \).

\( \Leftarrow \) Given that \( p_{\text{data}} = p_g \), we have:

\[
C(G) = \max_D L(G, D_1, D_2)
\]

\[
= 2 \cdot \mathbb{E}_{x \sim p_{\text{data}}} \left[ \log \frac{p_{\text{data}}(x) + \beta \cdot p_{\text{peer}}}{p_{\text{data}}(x) + p_g(x) + \beta \cdot p_{\text{peer}}(x)} \right]
\]

\[+
2 \cdot \mathbb{E}_{x \sim p_g} \left[ \log \frac{p_g(x) + \beta \cdot p_{\text{peer}}}{p_{\text{data}}(x) + p_g(x) + \beta \cdot p_{\text{peer}}(x)} \right]
\]

\[= 2 \cdot \left( \log \frac{1}{2} + \log \frac{1}{2} \right) = - \log 16
\]

\( \Rightarrow \) Note that

\[2 \cdot \left( \mathbb{E}_{x \sim p_{\text{data}}} \left[ - \log 2 \right] + \mathbb{E}_{x \sim p_g} \left[ - \log 2 \right] \right) = - \log 16
\]

By subtracting this expression from \( C(G) \), we have:

\[
C(G) = - \log 16 + 2 \cdot KL\left( p_g + \frac{\beta}{2} \cdot p_{\text{peer}} \middle|\middle| \frac{p_{\text{data}} + p_g + \beta \cdot p_{\text{peer}}}{2} \right)
\]

\[+
2 \cdot KL\left( p_{\text{data}} + \frac{\beta}{2} \cdot p_{\text{peer}} \middle|\middle| \frac{p_{\text{data}} + p_g + \beta \cdot p_{\text{peer}}}{2} \right)
\]

where \( KL \) is the Kullback-Leibler divergence. Remember that:

\[
C(G) = - \log 16 + 2 \cdot JSD\left( p_{\text{data}} + \frac{\beta}{2} \cdot p_{\text{peer}} \middle|\middle| \frac{p_{\text{data}} + p_g + \beta \cdot p_{\text{peer}}}{2} \right)
\]

(7)

Since the Jensen-Shannon divergence between two distributions is always non-negative and zero only when they are equal, we have shown that \( C(G)^* = - \log 16 \) is the global minimum of \( C(G) \). Thus, we need

\[p_{\text{data}} + \frac{\beta}{2} \cdot p_{\text{peer}} = p_g + \frac{\beta}{2} \cdot p_{\text{peer}} \Leftrightarrow p_{\text{data}} = p_g
\]
A.3 Proof of Theorem 3

Proof. Ignoring the weight $\beta$, the peer term of discriminator $D_i$ w.r.t. its diverged peer discriminator $\tilde{D}_j$ comes:

$$
\text{Peer}(D_i)|_{\tilde{D}_j} := E_{x \sim p_{\text{data}}}[\ell(D_i(x), 1(\tilde{D}_j(x) > \frac{1}{2})) - \alpha \cdot \ell(D_i(x_{p1}), 1(\tilde{D}_j(x_{p2}) > \frac{1}{2}))]
$$

$$
= E_{x \sim p_{\text{data}}, Y_j^* = 1}[P(\tilde{Y}_j = 1|Y_j^* = 1) \cdot \ell(D_i(x), 1) + P(\tilde{Y}_j = 0|Y_j^* = 1) \cdot \ell(D_i(x), 0)]
+ E_{x \sim p_{\text{data}}, Y_j^* = 0}[P(\tilde{Y}_j = 1|Y_j^* = 0) \cdot \ell(D_i(x), 1) + P(\tilde{Y}_j = 0|Y_j^* = 0) \cdot \ell(D_i(x), 0)]
- \alpha \cdot E_{x \sim p_{\text{data}}}[P(\tilde{Y}_j = 1) \cdot \ell(D_i(x_{p1}), 1) + P(\tilde{Y}_j = 0) \cdot \ell(D_i(x_{p1}), 0)]
$$

Thus, 

$$
\text{Peer}(D_i)|_{\tilde{D}_j} = (1 - e_{\text{data},j} - e_{g,j}) \cdot \text{Peer}(D_i)|_{\tilde{D}_j,G} + (1 - \alpha) \cdot E_{x \sim p_{\text{data}}}[e_{\text{data},j} \cdot \ell(D_i(x), 0) + e_{g,j} \cdot \ell(D_i(x), 1)]
$$

Note that:

$$
\text{Bias} = (1 - \alpha) \cdot E_{x \sim p_{\text{data}}}[e_{\text{data},j} \cdot \log(1 - D_i(x)) + e_{g,j} \cdot \log(D_i(x))]
$$

Thus, given $\alpha = 1$, the Bias term is cancelled out. When $e_{\text{data},j} + e_{g,j} < 1$, we have:

$$
\max_{D_i} \text{Peer}(D_i)|_{\tilde{D}_j} = \max_{D_i} \text{Peer}(D_i)|_{\tilde{D}_j,G}
$$

A.4 Proof of Theorem 4

Proof. When $\beta = 0$, the overall min-max game becomes:

$$
\min_G \max_{D_1, D_2} \mathcal{L}(D_1, D_2, G)
= \min_G \max_{D_1, D_2} E_{x \sim p_{\text{data}}}[\log D_1(x)] + E_{x \sim p_2}[\log \left(1 - D_1(G(z))\right)]
+ E_{x \sim p_{\text{data}}}[\log D_2(x)] + E_{x \sim p_2}[\log \left(1 - D_2(G(z))\right)]
$$

Since we assume enough capacity, the inner max game is achieved if and only if: $D_1(x) = D_2(x) = \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + p_g(x)}$. To prove $p_g$ converges to $p_{\text{data}}$, only need to reproduce the proof of proposition 2 in [13]. We omit the details here. 

\qed
B Experiment details

B.1 Architecture comparison between GAN, D2GAN and PeerGAN

Figure 4 shows the architecture designs of single discriminator, dual discriminator, and our proposed PeerGAN. Compared with Vanilla GAN, PeerGAN has one more identical discriminator and a competitive Peer Game between two discriminators. The introduced Peer Game induces diversified generated samples by discouraging the agreement between \(D_1\) and \(D_2\). In D2GAN, although both discriminators are trained with different loss functions, they do not interfere with each other in the training.

![Figure 4: Architecture comparisons between GAN based method (first row), dual discriminators GAN based method (second row) and PeerGAN (third row).](image)

B.2 Ablation study of PeerGAN

During training, We initialize the \(\alpha\) and \(\beta\) as 0, and gradually increase to the set maximum value. We experimentally discover \(\alpha=0.3\) and \(\beta=0.5\) can achieve the best FID score in the datasets we tested on. Table 3 shows an thorough ablation of different hyper-parameter setting on STL-10 dataset. The bold text are the best \(\alpha\) setting when beta is fixed.

![Figure 5: The trend of \(\alpha\), \(\beta\) in the training.](image)

| \(\beta\) | \(\alpha=0.1\) | \(\alpha=0.3\) | \(\alpha=0.5\) | \(\alpha=0.7\) | \(\alpha=0.9\) |
|---------|--------------|--------------|--------------|--------------|--------------|
| \(\beta=0.25\) | 60.38        | 56.01        | 51.86        | 58.17        | 60.91        |
| \(\beta=0.50\) | 58.77        | 51.37        | 58.45        | 55.16        | 57.75        |
| \(\beta=0.75\) | **55.07**    | 59.58        | 58.58        | 58.22        | 57.75        |

![Table 3: Ablation study of max \(\alpha\) and max \(\beta\) value tuning on STL-10 dataset (evaluate with FID score).](image)