Decoherence enhanced quantum measurement of a quantum dot spin qubit

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We study the effect of phonons on a proposed scheme for the direct measurement of two-electron spin states in a double quantum dot by monitoring the noise of the current flowing through a quantum point contact coupled to one of the dots. We show that although the effect of phonons is damaging to the procedure at extremely low temperatures characteristic of spin-in-quantum-dots experiments, and may even be fatal, increasing the temperature leads to a revival of the schemes usefulness. Furthermore, at higher, but still reasonably low temperatures phonon effects become advantageous to the measurement scheme, and lead to the enhancement of the spin-singlet noise without disturbing the low spin-triplet noise. Hence, the uncontrollable interaction of the measured system with the open bosonic environment, can be harnessed to increase the distinguishability between the measured states.

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Measurement processes have attracted a lot of attention recently, due to the growing need to stretch the attainable measurement precision, as well as the necessity to measure system properties which are hard to access. This led to the development of a new field in quantum information science, namely, quantum metrology [1, 2], the purpose of which is to enhance measurement capabilities by making the measurement device quantum itself. For the moment, quantum metrology requires pure states, and the proposed schemes are expected to react very badly to decoherence [3, 4]. It has been shown that the most popular entanglement based scheme of quantum enhanced metrology suffers from an unavoidable drawback due to quantum noise which reduces (for long chains of quantum systems) the quantum (Heisenberg) metrology limit to the classical shot-noise limit [5]. Since in solid state systems decoherence is unavoidable, a different approach is needed. It turns out that in some situations, decoherence can be helpful (e.g. to facilitate electron transport in quantum dot (QD) chains [6, 10]), and may even be utilized for measurement purposes [11, 12]. Here, we discuss a system for which a strong interaction with the environment can lead to the enhancement of the distinguishability between the measured states, and be beneficial for the measurement.

The system to be measured consists of two QDs, each containing an electron. If the electrons are in a spin-triplet configuration, electron transitions between the dots are forbidden due to the Pauli principle, unless they can occupy one of the excited QD levels. Since QDs are zero dimensional structures, all QD realizations are very small with dimensions varying between a few and a few tens of nanometers in each direction. It follows that the first excited state of a QD is typically very distant energetically from its ground state. The energy difference ranges from 10 meV in larger QDs to a 100 meV in small dots, hence, considering the other energy scales characteristic for a double QD (DQD) system, excited QD states can be neglected in almost all situations.

In the case, when the DQD electrons are in a spin-singlet configuration, electron transitions between the dots are allowed, and both electrons can occupy a single dot. This is hindered by the Coulomb interaction between the electrons which is much larger when both electrons occupy a single dot than for singly occupied dots. This repulsion is of the order of 1 meV, and the energy required to enable such a process can be supplied, e.g., by unavoidable interactions with the solid state environments of the QD. Hence, depending on the two-electron spin configuration, the occupation of the QDs will either be constant (for the spin-triplet), or will fluctuate between zero, one and two electrons localized in each dot (for the spin-singlet).

This difference in charge fluctuations is the basis for the direct quantum point contact (QPC) measurement of the two-electron spin configurations which was proposed in Ref. [17]. The idea is, that since the current flowing through a QPC located near a QD depends on the occupation of the QD via the Coulomb interaction between electrons in the QD and the electrons in the QPC, the current fluctuations will be strongly affected by the fluctuations of the QD occupation. Since triplet occupation cannot fluctuate, the current flowing through a QPC coupled to one of the dots forming the DQD is Poissonian, as expected. Yet, if the DQD is in a singlet configuration, the interaction between the DQD and the QPC can induce inter-dot transitions and the resulting QD occupation fluctuations in the coupled QD lead to QPC current fluctuations and subsequently the current noise becomes strongly super-Poissonian. Therefore, by measuring the QPC current noise one is able to distinguish between the spin-triplet and spin-singlet states.

The weakness of this measurement scheme lies in the fact that the QDs are embedded in a solid state structure and the QD electrons are subject to an unavoidable interaction with vibrations of the crystal lattice surrounding the dots (phonons) [14, 16]. This interaction can also induce transitions between singly and doubly occupied QD states while it conserves two-electron spin symmetry [17] (in fact, it causes exactly the same transitions as the DQD-QPC interaction [18]). As shown in Ref. [11], at very low temperatures which are characteristic for exper-
iments performed on electron-spin states confined in QDs
(strongly sub-Kelvin) [20–26] the electron-phonon inter-
action counteracts transitions to the high-energy doubly
occupied states. This hinders the distinguishability of
the measured states, which relies on charge fluctuations.
In the extreme case, when the electron-phonon inter-
action is strong enough relative to the DQD-QPC coupling
strength, the electron-phonon interaction can lead to the
situation when the singlet state is primarily composed
only of the low energy configuration (with one electron
in each dot), which cannot be distinguished from the triplet
state by the QPC.

In this paper we analyse the measurement scenario
and the interaction with phonons further and find, that
the damaging effect can be overcome by increasing the
temperature. Furthermore, for sufficiently high tem-
peratures the electron-phonon interaction can lead to an
enhancement of the singlet noise, increasing the
singlet-triplet distinguishability instead of damaging it.
Thus, we demonstrate a temperature-driven transition
from the regime of decoherence-induced suppression to
decoherence-assisted enhancement of the measurement in
this setup.

Let us first set the framework for the theoretical de-
scription of the system (following Refs. [13,19]) by defin-
ing its Hamiltonian and finding the equations of motion
for its density matrix, which will then allow us to analyze
the QPC current noise. We do not include any interac-
tions that could induce singlet-triplet transitions. If the
excited QD states are energetically unavailable, there is
only one triplet configuration and the evolution in the
triplet sector is trivial. Hence, in the following the focus
is laid on the singlet subspace of the Hilbert space.

The Hamiltonian of the DQD system with two elec-
trons is given by

\[ H_{\text{DQD}} = \Delta \sum_{\sigma = \uparrow, \downarrow} (a_{i\sigma}^{\dagger} a_{L\sigma} + a_{L\sigma}^{\dagger} a_{i\sigma}) + U \sum_{i=R,L} n_{i\uparrow} n_{i\downarrow}, \]

where \( \Delta \) is the inter-dot tunnelling amplitude, \( a_{i\sigma}, a_{i\sigma}^{\dagger} \)
are the annihilation and creation operators of an elec-
tron in dot \( i = R,L \) (right, left) with spin \( \sigma = \uparrow, \downarrow \),
\( n_{i\sigma} = a_{i\sigma}^{\dagger} a_{i\sigma} \) gives the number of electrons with spin \( \sigma \) in
dot \( i \), and \( U \) is the Coulomb charging energy for adding
a second electron to a QD. The eigenstates of this Hamil-
tonian are easily found and in the singlet subspace are
given by

\begin{align}
|s_0\rangle &= \xi'(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) - \xi(|d_L\rangle + |d_R\rangle), \\
|s_1\rangle &= \xi(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) + \xi'(|d_L\rangle + |d_R\rangle), \\
|s_2\rangle &= \frac{1}{\sqrt{2}}(|d_L\rangle - |d_R\rangle),
\end{align}

where \( |\sigma\sigma'\rangle = a_{L\sigma}^{\dagger} a_{R\sigma'}^{\dagger}|0\rangle \) denote singly occupied states,
and \( |d_i\rangle = a_{i\uparrow}^{\dagger} a_{i\downarrow}^{\dagger}|0\rangle, i = L, R \), are doubly occupied states.

The parameters are equal to

\[ \xi = \frac{1}{\sqrt{2}} \sin(\theta/2), \]
\[ \xi' = \frac{1}{\sqrt{2}} \cos(\theta/2), \]

where \( \theta = \text{atan}(4\Delta/U) \). The corresponding eigenenergies
are equal to \(-J, U, U + J\), respectively, where

\[ J = \frac{1}{2}(\sqrt{U^2 + 16\Delta^2} - U) \]

is the energy difference between the lowest energy singlet
and the triplet states. The triplet energy is taken equal to
zero, and serves as a reference for the calculated energies
of the singlet states.

The QPC is situated near one of the QDs in such a
way that it is sensitive to the occupation of only this one
(right) dot. The DQD-QPC interaction is described by
the Hamiltonian

\[ H_{\text{tun}} = \sum_{p,q,\sigma} (T_{pq} + \chi_{pq} n_R) a_{Sp\sigma}^{\dagger} a_{Dp\sigma} + \text{H.c.}, \]

which accounts for the tunnelling of electrons through
the QPC and contains a factor dependent on the occu-
pation of the right dot, \( n_R \). Hence, electron tunnelling
consists of a robust part, independent of the QD occu-
pation described by the constants \( T_{pq} \), and a Coulomb
interaction induced enhancement \( \chi_{pq} \). The tunnelling
constants are assumed to be slowly varying over the en-
ergy range where tunnelling is allowed [15,27] and are
taken constant. Here, \( a_{p\sigma}^{\dagger}, a_{p\sigma} \) are the QPC electron
annihilation and creation operators corresponding to an
electron in lead \( n = S,D \) (source, drain) and in mode \( p \),
with the distinction of spin \( \sigma \) which is constant through-
out the tunnelling. In this paper, we study a QPC which
operates in the high bias regime, that is, in the situation
when the chemical potential offset between the leads is
large enough to induce transitions to doubly excited states [13].

The electron-phonon interaction Hamiltonian is given by

\[ H_{\text{e-\text{ph}}} = \sum_{\sigma,i} \sum_{k,\lambda} F^{(\lambda)}(k)a_{i\sigma}^{\dagger} a_{i\sigma}(b_{k,\lambda} + b_{k,\lambda}^{\dagger}), \]

where \( b_{k,\lambda} \) and \( b_{k,\lambda}^{\dagger} \) are phonon annihilation and creation
operators for a phonon from branch \( \lambda \) with wave vector
\( k \). \( F^{(\lambda)}(k) = F^{(\lambda)}(k)e^{\pm ikz_d/2} \) are electron-phonon
coupling constants, and \( d \) is the inter-dot distance. The
coupling constants depend on material parameters, the types
of phonons and electron-phonon couplings taken into account,
and the electron wavefunction. Their explicit form
can be found in Ref. [19].

Following Refs. [13,19], we find the quantum master
equation in the Lindblad form for the DQD interact-
ing with both environments (the bosonic phonon envi-
ronment and the fermionic QPC environment) assuming
that the environments are not correlated with each other,

$$\dot{\rho}(t) = \mathcal{L}(\rho) = -\frac{i}{\hbar}[H_{\text{DQD}}, \rho] + \frac{1}{\hbar^2} \left( \sum_{i} \left( \frac{1}{2} C_i \rho C_i^\dagger + \rho C_i^\dagger C_i \right) + \frac{3}{2} C_i \rho C_i^\dagger \right) + \frac{1}{\hbar^2} \left( \sum_{i} \left( \frac{1}{2} B_i^\dagger B_i \rho + \rho B_i^\dagger B_i + \frac{4}{2} B_i \rho B_i^\dagger \right) \right).$$  

\[ (4) \]

The Lindblad operators corresponding to the DQD-QPC interaction are given by [13]

$$C_1 = \nu \sqrt{\frac{V - (U + J)}{\hbar}} \sin \frac{\theta}{2} |s_2\rangle \langle s_0|,$$

$$C_2 = \nu \sqrt{\frac{V + (U + J)}{\hbar}} \sin \frac{\theta}{2} |s_0\rangle \langle s_2|,$$

$$C_3 = \sqrt{\frac{V}{\hbar}} \left( (\mathcal{T} + \nu) \mathbb{I} + \nu \cos \frac{\theta}{2} (|s_1\rangle \langle s_2| + |s_2\rangle \langle s_1|) \right),$$

where $V = (\mu_S - \mu_D)$ is the QPC bias, $\mathcal{T} = \sqrt{4\pi g_S g_D} T$ is the unconditional tunnelling constant, with $T = T_{pq}$, and $\nu = \sqrt{4\pi g_S g_D}$ is the tunnelling constant conditioned on the occupation of the right QD, with $\chi = \chi_{pq}$. Here, $g_i$ is the density of states of the $i$-th QPC lead ($i = S, D$).

The Lindblad operators for the electron-phonon interaction are of the standard form, $B_{\nu} = \sqrt{\gamma_\nu} |s_i\rangle \langle s_j|$. Two operators correspond to transitions between $|s_0\rangle$ and $|s_1\rangle$ when $i = 0$, $j = 1$ and $i = 1$, $j = 0$, while the other two describe transitions between $|s_1\rangle$ and $|s_2\rangle$ when $i = 1$, $j = 2$ and $i = 2$, $j = 1$. The transition rates $\gamma_{ij}$ are calculated using the Fermi golden rule based on the Hamiltonian in eq. [3], with the usual temperature dependence $\gamma_{ij}(T) = \gamma_{ij}(T = 0)[n(\omega_j - \omega_i, T) + 1]$, where $n(\omega, T)$ is the Bose distribution and $\hbar \omega_i$ are the energies corresponding to a given singlet state.

The time evolution of the DQD system is found by solving the above generalized master equation. Specifically, the singlet steady state, found from the condition $\dot{\rho}(t) = 0$, is

$$\rho^{(s)}_\infty = \frac{1}{N} \left( (A_0^2 + \gamma_{02})(A_0^2 + \gamma_{21}) |s_0\rangle \langle s_0| + (A_0^2 + \gamma_{12})(A_0^2 + \gamma_{20}) |s_1\rangle \langle s_1| + (A_0^2 + \gamma_{20})(A_0^2 + \gamma_{21}) |s_2\rangle \langle s_2| \right),$$

\[ (5) \]

where

$$N = (A_0^2 + \gamma_{02})(A_0^2 + \gamma_{21}) + (A_0^2 + \gamma_{12})(A_0^2 + \gamma_{20}) + (A_0^2 + \gamma_{20})(A_0^2 + \gamma_{21}),$$

$$A_\pm = \nu \sqrt{(V \pm J \pm U)/\hbar \sin(\theta/2)},$$

$$A_0 = \nu \sqrt{V/\hbar \cos(\theta/2)}.$$

The singlet steady state depends explicitly on the strengths of the phonon-couplings relative to the QPC coupling strengths.

The interaction between the DQD and the QPC eventually leads to the localization of any two-electron initial state in either the singlet or the triplet subspace [13]. This process is not disturbed by the presence of phonons, and localization occurs on timescales of tens of microseconds [13]. Since the excited QD states are not taken into account, the QPC current noise for a triplet state remains Poissonian regardless of temperature due to the Pauli exclusion principle, and the triplet Fano factor is always equal to one (see Ref. [13, 19] for details). Hence, to quantify the distinguishability of the singlet and triplet states, it is sufficient to study the QPC current noise for the singlet steady state. To this end we will examine the Fano factor for the current noise corresponding to state (5) and compare it to unity, which is the Fano factor value for Poissonian noise, characteristic for the triplet states.

The Fano factor is defined as [22, 23] $F = S(0)/2e \bar{I}$, where $\bar{I}$ is the mean current. The QPC noise power spectrum for steady states is given by

$$S(\omega) = 2 \int_0^\infty d\tau G(\tau)e^{-i\omega \tau}.$$  

\[ (6) \]

The current-current correlation function, given by

$$G(t, t') = \langle I(t')I(t) \rangle - \langle I(t') \rangle \langle I(t) \rangle,$$

depends only on $\tau = t' - t$ in the long-time limit. Following Refs. [13, 20] we can find the correlation function for the singlet steady state,

$$G(\tau) = e^2 \left[ \text{Tr} \left\{ \sum_{i, j} C_i [e^{\mathcal{L}(\tau - \Delta \tau)} C_i \rho^{(s)}_\infty C_i^\dagger] \right\} - \text{Tr} \left\{ \sum_i C_i \rho^{(s)}_\infty C_i^\dagger \right\}^2 + \text{Tr} \left\{ \sum_i C_i \rho^{(s)}_\infty C_i^\dagger \right\} \delta(\tau) \right].$$

The results presented in this paper are obtained using parameters corresponding to GaAs structures [13, 21]. The QPC energies are taken $U = 1$ meV and $J = 0.1$ meV, and the QPC bias is taken to be $V = 2$ meV. The zero-temperature phonon transition rates are $\gamma_{02}(T = 0) = 1.15 \times 10^{-3}$ ns$^{-1}$ and $\gamma_{21}(T = 0) = 6.01 \times 10^{-8}$ ns$^{-1}$. The scaling parameter $\alpha = T_0 = T_0' = 0.1$ and $\nu_0 = 2.25 \times 10^{-3}$, is chosen in such way that $\alpha = 1$ corresponds to the situation when the interaction with phonons is roughly the same strength as the interaction with the QPC, meaning that $\sqrt{\gamma_{02}} = \nu_0 \sqrt{V/\hbar}$. Then phonons dominate at large $\alpha$ (small QPC current), while the DQD-QPC interaction dominates for small $\alpha$.

Fig. [11] shows the dependence of the singlet Fano factor as a function of temperature for relatively strong electron-phonon interaction at $\alpha = 3$; Fano factor equal to one corresponding to the triplet state is marked by the dashed grey line. Although at $T = 0$ phonons of such strength completely destroy the distinguishability of the singlet and triplet state via QPC current noise (both singlet and triplet Fano factors are equal to one), this is
FIG. 1: The Fano factor for a relatively strong electron
phonon-interaction, $\alpha = 3$, as a function of temperature (red,
solid line); no-phonon Fano factor (blue, dotted line). Inset:
The Fano factor as a function of the strength of the electron-
phonon interaction relative to the DQD-QPC interaction, $\alpha$,
for different temperatures. In both plots, the dashed grey li-
ne is set at Fano factor equal to one (Poissonian noise).

remedied already by a slight increase of the temperature. At $T = 8.5$ K, the redistribution of the different singlet
occupations is strong enough to restore the Fano factor to the no-phonon value found (the no-phonon value of
the Fano factor is marked by the blue, dotted line). This is
due to the rising importance of phonon-induced trans-
sitions to higher energy, doubly occupied states which
compensate for the transitions to the lower energy, singly
occupied state. Further increasing the temperature leads
to an enhancement of the Fano factor beyond the no-
phonon value. The effect of temperature is strong be-
tween 0 and 20 K, and then starts to slowly saturate,
yielding already ample enhancement of the Fano factor
by 15 K, long before the excited QD states need to be
taken into account.

This is further quantified in the inset of the figure
where the Fano factor is plotted as a function of the
scaling parameter $\alpha$, which is inversely proportional to
the strength of the DQD-QPC interaction, at different
temperatures. Regardless of the temperature, phonon ef-
fects are negligible when the DQD-QPC interaction dom-
inates, near $\alpha = 0$. Their importance rises quickly, when
the interactions become comparable, and then saturates
slowly after the interaction with phonons becomes twice
as strong as the interaction with the QPC. Hence, the
interaction with phonons is relevant when the QPC is
coupled weakly to the DQD, and may serve to increase
the effectiveness of the measurement in this situation.

At high enough temperatures, the electron-phonon in-
teraction will cause transitions to excited QD states.
This will lead to double occupations in the triplet sub-
space in the situation when one of the electrons is in the
QD ground state, and the other is in the QD ex-
cited state. The occurrence of transitions to such states
will lead to fluctuations of the QPC current noise, which
will also become super-Poissonian. Although at not ex-
tremely high temperatures this super-Poissonian char-
acter will be much weaker than in the singlet case, it
could complicate data interpretation. However, since the
dependence on temperature is strong for relatively
small temperatures, the high temperature range is of no
experimental interest.

We have studied the temperature dependence of a pro-
posed scheme for the direct measurement of the DQD
spin-singlet and spin-triplet states using the noise of the
current flowing through a QPC coupled to one of the
QDs. We show, that although at low temperatures the
interaction with phonons has a destructive effect on the
measurement scheme, the situation may be inverted by a
small increase of the temperature. Furthermore, beyond
$T = 8.5$ K phonon-induced transitions actually help the
distinguishability of the singlet and triplet states, lead-
ing to an enhancement rather than destruction of measure-
ment capability. Already at 15 K, the Fano factor, which
is a measure of noise power relative to Poissonian noise
power, can be raised by a factor of 1.32, and by a factor
of 1.58 at 40 K, meaning that the effect occurs already at
temperatures far below the limit when the excited states
of the QDs need to be taken into account. Hence, we have
shown that the unavoidable phonon-induced decoh-
rence mechanism can be used to facilitate measurement.

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