Phase diagram of the isovalent phosphorous-substituted 122-type iron pnictides

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Recent experiments demonstrated that isovalent doping system gives the similar phase diagram as the heterovalent doped cases. For example, with the phosphorous (P)-doping, the magnetic order in BaFe2(As1−xP)x2 compound is first suppressed, then the superconductivity dome emerges to an extended doping region but eventually it disappears at large x. With the help of a minimal two-orbital model for both BaFe2As2 and BaFe2P2, together with the self-consistent lattice Bogoliubov-de Gennes (BdG) equation, we calculate the phase diagram against the P content x in which the doped isovalent P-atoms are treated as impurities. We show that our numerical results can qualitatively compare with the experimental measurements.

I. INTRODUCTION

Recent intensive studies on the 122 family of iron-pnictide XFe2As2 (where X = Ba, Sr or Ca) compounds found that the superconductivity (SC) can be induced by different means1. In all these cases, the resulting phase diagrams are quite similar. Starting from a co-linear spin-density wave (SDW) metal, a SC dome emerges with doping in the parent compound, while the SDW gets suppressed. In the electron-doped and hole-doped materials, the emergence of SC is due to the imbalance between the electron and hole carrier densities in the system. However, isovalent substitution of As with phosphorous (P) in BaFe2As2 shows similar phase diagram5–8 without introducing additional net charge carriers. This raises the question about the underlying mechanism for the phase transition from the SDW to SC, and eventually to normal metal as the doping parameter x in the system increases, which is the issue we wish to address in the present work.

Since P anion is smaller than As anion, it has been suggested that superconductivity in this isovalent system is induced by a chemical pressure10–13, or it is correlated with the distinct role of lattice parameters, e.g., the As-Fe-As bond angle14 and bond length of Fe-As and Fe-As15. Besides, it has also been suggested that the uniaxial pressure14 similar to the electron-doped Ba(Fe1−xCo,x)2As2 system16,17 and charge inhomogeneity18 plays an important role in the emergence of superconductivity. On the other hand, Rotter and co-authors19 compared structure data determined by high-resolution x-ray powder and single-crystal diffraction, with theoretical models obtained by DFT calculations, and emphasized that even subtle details of the crystal structures are crucial to magnetism and superconductivity.

In this work, we want to answer the outstanding question that “What is the essential ingredient to cause the phase diagram of BaFe2(As1−xP)x2?”. Before we dive into the detail of our minimal effective Hamiltonian, we need to address several points for how we choose the tight-binding model. First, in order to understand the electronic structure of the compound BaFe2(As1−xP)x2, the scatterings of charge carriers by the randomly doped P-atoms or sites should be carefully addressed. Since the impurity (or P-atom) concentration are taking from 0 to 100% in the calculation, it is essential to construct a large real-space lattice to capture reasonable statistical ensemble. Historically, several microscopic multi-orbital models have been developed20–22 and we adopt a minimal two-dimensional (2D) and two-orbital model from Tai and his co-author22. This model has been tested for capturing several important features of the 122 BaFe2As2 compounds, such as the paramagnetic band structure and Fermi surface topology according to the dzz(dzy) orbital ordering, the entire hole- and electron-doped phase diagram23,24 the evolution of impurity quasiparticle states25 and the Fermi surface evolution for the co-linear SDW phase26. All these features are in agreement with experiments. More importantly, our effective two-orbital model helps reducing the degree of freedom from the complexity associated with the multi-orbital models and enables us to construct a large lattice Hamiltonian to perform a cost-effective calculation in the presence of disorders and to calculate the phase diagram as a function of the P-doping. It is almost impossible to apply the three-orbital (dxx, dyy and dzz), the five d-orbital and the eight-orbital (Fe-d+As-p) models to perform the same type of calculation27. Afterall, the essential features of the magnetic and SC orders at arbitrary doping could be accounted for by dzz and dzy. The other orbitals only make minor and quantitative modifications.

II. MODEL CONSTRUCTION

We begin with BaFe2As2, a prototype parent compound of superconductors, showing a co-linear SDW antiferromagnetic ground state28. The Fe atoms in BaFe2As2 form a square lattice, while the As atoms sit in the center of each square plaquette of the Fe lattice and are displaced alternatively above and below the Fe-Fe plane, which leads to two sublattices of Fe atoms denoted by sublattice A and B. In Ref. 29 the authors proposed a minimal 2-orbital model with these two Fe atoms per unit-cell through considering the orbital ordering physics of Fe-3dzz and Fe-3dzy orbitals, and later on it has been proven that one could also use a gauge transform to represent this model within 1-Fe atom per unit-cell30.

Here we start with the kinetic term of the lattice Hamiltonian for the mixed compound, BaFe2(As1−xP)x2, written as,
\[ H_i = H_\tilde{t} + H_{\text{int}} \]  

Where \( H_\tilde{t} \) is the hopping term in real space and the tilde symbol of \( \tilde{t} \) is presenting either for pure BaFe\(_2\)As\(_2\), pure BaFe\(_2\)P\(_2\), or the different ones caused by the As-P mixture.

\[
H_i = \sum_{i,j,\alpha,\beta,\sigma} \tilde{t}^{\alpha\beta}_{ij} \psi_{i\alpha\sigma}^{\dagger} \psi_{j\beta\sigma} - \sum_{i,\alpha,\sigma} \mu_i \psi_{i\alpha\sigma}^{\dagger} \psi_{i\alpha\sigma} \tag{1}
\]

where \( c_{i \alpha \sigma}^\dagger \) and \( c_{i \alpha \sigma} \) are respectively the creation and annihilation operators for an electron with spin \( \sigma \) in the orbitals \( \alpha = 1 \) or \( 2 \) on the \( i \)-th lattice site, \( \mu \) is the chemical potential which is adjusted to give a fixed filling factor, and \( \tilde{t}^{\alpha\beta}_{ij} \) are the hopping integrals. We choose the nonvanishing hopping elements as [32,33].

Now, we are ready to write down the full Hamiltonian with interaction terms, \( H_{\text{int}} \) and \( H_\Delta \). Where \( H_{\text{int}} \) contains an on-site Hubbard, \( U \), and Hund’s coupling, \( J_H \), which is responsible for the co-linear SDW order.

\[
H_{\text{int}} = U \sum_{i,\alpha,\sigma} \langle \tilde{n}_{i\alpha\sigma} \rangle \tilde{n}_{i\alpha\sigma} + U' \sum_{i,\alpha,\beta,\sigma,\sigma' \neq \sigma} \langle \tilde{n}_{i\alpha\sigma} \rangle \tilde{n}_{i\beta\sigma} \\
+ (U' - J_H) \sum_{i,\alpha,\beta,\sigma,\sigma' \neq \sigma} \langle \tilde{n}_{i\alpha\sigma} \rangle \tilde{n}_{i\beta\sigma} \tag{2}
\]

\[ H_\Delta = \sum_{i,j,\alpha} V_{ij} \left( \langle \epsilon_{i\alpha} \epsilon_{j\alpha} \rangle \epsilon_{i\alpha}^{\dagger} \epsilon_{j\alpha}^{\dagger} + H.c. \right) \tag{5} \]

where \( \tilde{n}_{i\alpha\sigma} = c_{i\alpha\sigma}^{\dagger} c_{i\alpha\sigma} \). The orbital rotation symmetry imposes the constraint \( U = U' + 2 J_H \).

Here our pairing interaction is represented by \( H_\Delta = -\sum_{i,j} V_{ij} n_{i\alpha} n_{j\alpha} \) with \( V_{ij} > 0 \). The mean-field decoupling of \( H_\Delta \) can be written in the following form,

\[
H_\Delta = \sum_{i,j,\alpha} V_{ij} \left( \langle \epsilon_{i\alpha} \epsilon_{j\alpha} \rangle \epsilon_{i\alpha}^{\dagger} \epsilon_{j\alpha}^{\dagger} + H.c. \right) \tag{5} \]

\[ V_{ij} \langle \epsilon_{i\alpha} \epsilon_{j\alpha} \rangle = \Delta^{\alpha}_{ij} \] is the SC bond pairing order parameter between site \( i \) and site \( j \). In principle, such an ‘attractive’ interaction \(( -V_{ij} \rangle \) between electrons in real space could be generated via the on-site Hubbard-U interaction according to the spin-fluctuation theory in real-space, \( V_{ij} \sim \chi_{ij} \), as described in Ref. [33] which makes a consistent picture as one considers the case of spin-fluctuation theorem in k-space [32].

In this paper, we want to emphasis on the effect of the mixed hopping, \( t \), and we do not address the full real-space RPA calculation of \( V_{ij} \)[33]. The nearest neighboring pairing interaction \( V_{NN} \) is known to give rise to the d-wave pairing, while the next-nearest neighboring \( V_{N_{NN}} \) would be responsible for the \( s_\pm \)-pairing symmetry that clearly gaining experimental supports for the electron and hole-doped BaFe\(_2\)As\(_2\) compounds.

We believe that the d-wave component of the SC order parameter must be completely suppressed in the compound. In this paper, we assume that SC of BaFe\(_2\)As\(_2\) has the \( s_\pm \)-pairing symmetry in the Fe-plane. Thus we only need to consider \( V_{N_{NN}} \) for the pairing interaction. The interplay between \( V_{NN} \) and \( V_{N_{NN}} \) on the pairing symmetry of the electron-doped BaFe\(_2\)As\(_2\) has also been recently studied by our group [33]. It was demonstrated that the \( d \)-wave component of the SC order parameter could easily be suppressed by the \( s_\pm \)-wave pairing component, as well as by the disordered scatterings due to the randomly distributed dopants in the Fe-planes. Therefore, we only take the NNN pairing interaction \( V_{N_{NN}} \) account and treated it effectively as a constant, and this interaction would generate a SC with the \( s_\pm \)-wave pairing symmetry. In our model calculation, we did not find that the charge-density wave state is stable [32].

We write down the matrix form of Eq. [5] with basis \( \psi_{i\alpha} = (c_{i\alpha1}, c_{i\alpha2})^T \) \( H = \sum_{i,j,\alpha,\beta} H_{ij} \psi_{i\alpha}^\dagger \psi_{j\beta} \), and calculate the eigenvalue and eigenvector of \( H_{\text{ddg}} \).

\[
\left( \begin{array}{c}
H_{ij}^\alpha \\
\Delta_{ij}^\alpha \\
\Delta_{ij}^\alpha \\
-H_{ij}^{\alpha} \\
\epsilon_{ij}^\beta \\
\epsilon_{ij}^\beta \\
\end{array} \right) \left( \begin{array}{c}
u_{ij}^\alpha \\
\nu_{ij}^\alpha \\
\nu_{ij}^\alpha \\
\nu_{ij}^\beta \\
\nu_{ij}^\beta \\
\nu_{ij}^\beta \\
\end{array} \right) = E_i \left( \begin{array}{c}
u_{ij}^\alpha \\
\nu_{ij}^\alpha \\
\nu_{ij}^\alpha \\
\nu_{ij}^\beta \\
\nu_{ij}^\beta \\
\nu_{ij}^\beta \\
\end{array} \right) \tag{6}
\]

where, \( H_{ij}^{\alpha} = [H_0 + H_{\text{int}}]_{ij}^{\alpha\beta} \), is the matrix-element for the single-particle Hamiltonian. We solve the mean-field order parameters \( \langle \tilde{n}_{i\alpha} \rangle = \sum_n |u_{i\alpha}^n|^2 f(E_n) \), \( \langle \tilde{n}_{i\alpha} \rangle = \sum_n |u_{i\alpha}^n|^2 [1 - f(E_n)] \) and

\[
\Delta_{ij}^\alpha = \frac{V_{ij}}{4} \sum_n (u_{i\alpha}^n u_{j\alpha}^{n*} + u_{j\alpha}^n u_{i\alpha}^{n*}) \tanh \left( \frac{E_n}{2k_BT} \right) \tag{7}
\]

self-consistently with Eq. [6] where \( f(E_n) \) is the Fermi-Dirac distribution function. As described above, we only consider the NNN intra-orbital pairing with pairing potential \( V_{ij} = \)
also compare these features through experiments and LDA calculations from the literature. Due to the periodicity, the real-space kinetic term without spin indices, \( \sum_{i,j} \phi_{i,j}^\dagger \phi_{j,i} + \sum_{i,\alpha} V_{\text{inter}} \phi_{i,\alpha} \phi_{i,\alpha} \), can be easily Fourier transformed to the k-space, \( H_t = \frac{1}{N} \sum_k M_k \phi_k \), with the 1-Fe per unit cell basis. Here \( \phi_k = (c_{1,k}, c_{2,k})^T \) and

\[
M_k = \left( \begin{array}{cc}
\epsilon_s + \zeta_+ - \mu & \epsilon_o + V_{\text{inter}} \\
\epsilon_o + V_{\text{inter}} & \epsilon_s - \zeta_- - \mu
\end{array} \right)
\]

where,

\[
\epsilon_s = 2 \tilde{t}_1 (\cos k_x + \cos k_y) + 2 \tilde{t}_0 (\cos 2k_x + \cos 2k_y),
\]

\[
\epsilon_o = 2 \tilde{t}_0 (\cos k_x + \cos k_y) + 4 \tilde{t}_4 \cos k_x \cos k_y.
\]

We diagonalize Eq. 8 to obtain the electronic structures of pure \( \text{BaFe}_2\text{As}_2 (t = t', V_{\text{inter}} = 0) \) and pure \( \text{BaFe}_2\text{P}_2 (t = t'; V_{\text{inter}} = 0.031) \) compounds with the parameters given in Eq. 2. The band structures and Fermi surfaces of these two systems are shown in Fig. 1 for the 1-Fe atom per unit cell Brillouin zone (BZ). Through the literature study, we found that the Fermi surface for \( \text{BaFe}_2\text{As}_2 \) and \( \text{BaFe}_2\text{P}_2 \) share very similar signature, e.g., the de Haas-van Alphen experiment \cite{11315} and the angle-resolved photoemission spectroscopy (ARPES) experiment \cite{39}. Here, from our model parameter, it can be seen from Fig. 1 that the band structures and Fermi surfaces of these two compounds are similar to each other. However, the hole pocket around the \( \Gamma \) point becomes more expanded in \( \text{BaFe}_2\text{P}_2 \) than that in \( \text{BaFe}_2\text{As}_2 \), which is also in good agreement with experiments \cite{13}. Moreover, the band width of \( \text{BaFe}_2\text{P}_2 \) is wider than that of \( \text{BaFe}_2\text{As}_2 \), which is consistent with recent ARPES experiment \cite{39}. More detailed band structures and Fermi surfaces based on the folded BZ of 2-Fe per unit cell are shown in the Appendix B.

Now, we study the static spin susceptibility bubble for the tight-binding model without electron-electron interaction. The static spin susceptibility can be obtained as,

\[
\chi_s(q, i\Omega) = -\frac{T}{2N} \sum_{k,\omega_n} \text{Tr} [G(k + q, i\omega_n + i\Omega)G(k, i\omega_n)],
\]

\[
= -\frac{1}{2N} \sum_{k,\nu} \frac{|(k + q, \nu)|^2}{i\Omega + E_{\nu,k} + q - E_{\nu',k}} \times \left( f(E_{\nu,k} + q) - f(E_{\nu',k}) \right),
\]

where \( E_{\nu,k} \) and \( |k, \nu\rangle \) is the \( \nu \)-th eigenvalue and corresponding eigenvector given by Eq. 8.

Fig. 2 shows the static spin susceptibility \( \chi_s(q, 0) \) versus \( q \) for pure \( \text{BaFe}_2\text{As}_2 \) and \( \text{BaFe}_2\text{P}_2 \), respectively. It shows the largest values of static spin susceptibilities in both of these systems occur around \( q = Q \in \{(\pm \pi, 0); (0, \pm \pi)\} \), which is responsible for the scattering vector of the co-linear SDW instability \cite{13}. At a first glance on Fig. 2, we know that the spin fluctuations in \( \text{BaFe}_2\text{As}_2 \) is much stronger than \( \text{BaFe}_2\text{P}_2 \) from

\[
\begin{align*}
V_{\text{NNN}} = V & \quad \text{to serve the } s^\pm \text{ pairing symmetry. To facilitate the discussion of physical observables and generating of the phase diagram, we define respectively the staggered lattice magnetization and the } s^-\text{ wave projection of SC order parameter at each site } i, \text{ as: } m_i = \frac{1}{2} \sum_{\alpha} \langle \hat{n}_{i,\alpha} \rangle - \langle \hat{n}_{i,\alpha}^\dagger \rangle, \\
\Delta_i = \frac{1}{8} \sum_{\delta,\alpha} \Delta^\alpha_{i+\delta}, \text{ where } \delta \in \{\pm x, \pm y\}. \text{ The neighbors of site } i \text{ are reached by } \delta. \text{ Besides, we also calculate the averaged values } \langle |M_i| \rangle = \frac{1}{N} \sum_i |m_i| \text{ and } \langle |\Delta_i| \rangle = \frac{1}{N} \sum_i |\Delta_i|, \text{ to investigate the phase diagram. } N \text{ is the number of Fe sites in the real-space lattice.}
\end{align*}

Throughout this paper, we choose fixed interaction parameters \( (U, J_H, V) = (3.2, 0.6, 1.05) \), no matter whether there are substituted-P atoms or not. In principal, \( U, J_H \text{ and } V \) should be changed somewhat due to the substitution of P. If we do so, our result could be better fitted to the experiment. However, in the present study, we would like to focus our attention only to the hopping effect on the phase diagram without changing the interaction terms.

III. BAND STRUCTURE AND STATIC SPIN SUSCEPTIBILITY

![Fig. 1](image-url) (Color online) The model calculated band structure of two-orbital model on the unfolded (1-Fe atom per unit cell) Brillouin Zone (BZ) for (a) \( \text{BaFe}_2\text{As}_2(x = 0) \), (b) \( \text{BaFe}_2\text{P}_2(x = 1) \); and Fermi surface on the unfolded BZ for (c) \( \text{BaFe}_2\text{As}_2(x = 0) \), (d) \( \text{BaFe}_2\text{P}_2(x = 1) \). Here the fermi energy is shifted to zero for 1/2 filling.

In this section, we systematically study the band structure and Fermi surface for pure \( \text{BaFe}_2\text{As}_2 \) and \( \text{BaFe}_2\text{P}_2 \). We will also compare these features through experiments and LDA
to calculate the temperature dependence on Fig. 3(b). The order parameters, \( \langle |M| \rangle \) and \( \langle |\Delta_s| \rangle \), are site-averaged through the site-averaged order parameters, \( \langle |M| \rangle \) and \( \langle |\Delta_s| \rangle \), as shown in Fig. 3(a). Then, we gradually increase the temperature dependence on Fig. 3(b). The phase diagrams exhibited in Fig. 3 are results after making averages over 25 impurity-configurations. Above or beyond the SDW or SC transition temperature, the relevant averaged order parameters are less than 2% of those magnitudes at \( T = 0 \). Here the temperature \( T \) is in units of the hopping term, \( |t_5| \).

In Fig. 3 the pure BaFe\(_2\)As\(_2\) is a co-linear SDW state, and the pure BaFeP\(_2\) is a paramagnetic metal. The calculated SC order in doped BaFe\(_2\)As\(_{1-x}\)P\(_x\) is much reduced as if we only consider \( H_f \) for the kinetic term, and it has been further suppressed if we also take \( H_{P_{\text{inter}}} \) into account[20]. This is not so surprising that the mean-field results of these two pure compounds are expected from our static spin susceptibility study of previous section. In the region, \( 0 < x < 0.37 \),
the SDW order of BaFe$_2$As$_2$ is gradually suppressed by the randomly distributed P-substitution. The SC with the $s$±-pairing symmetry emerges as a competing order where it has a co-existing region with the SDW order in the region $0.37 < x < 0.43$ and the SDW order is suddenly dropped to zero at $x = 0.43$ as indicated in Fig. 3(a). At $x \approx 0.45$, the SC order parameter reaches a maximum value, and gradually decreases for $x > 0.45$. Finally, the SC order becomes completely suppressed after $x > 0.73$. The overall trend of Fig. 3 is in good agreement with many experiments.

In order to understand the SDW-SC competing effect and together with the As-P mixing picture, we present the spatial images of the local magnetic, $m$, and the SC, $\Delta$, orders respectively in Fig. 4 and Fig. 5. Below each of the graphs of Fig. 4 and Fig. 5 is the averaged value, $\langle |M| \rangle$ and $\langle \Delta \rangle$, respectively. We show several different $x$ at $T = 0$ with a specific configuration of the P-impurities marked by red× symbols.

In Fig. 4(a) the magnetic order at $x = 0$ corresponds to a perfect co-linear SDW phase. These graphs, Fig. 4(a)-4(d) and Fig. 5(a)-5(d) demonstrate how the magnetic order weakens as the doping changes for $x = (0, 0.2, 0.4, 0.45)$ and the behavior of the SC dome for $x = (0.4, 0.45, 0.55, 0.65)$. For $x = 0.2$, we find that the SDW order is a bit suppressed around the P-impurities as shown in Fig. 4(b). For $x = 0.4$, the SDW order is further suppressed as in Fig. 4(c), moreover, the SC order now appears as in Fig. 5(a). By comparing Fig. 4(c) and Fig. 5(a) ($x = 0.4$), we learn that the SDW and SC orders form domains that the stronger region of SC is separated from the stronger region of SDW. For $x = 0.45$, the long range co-linear SDW entirely disappears but there are still short range magnetic order with extremely weak strength as shown in Fig. 4(d). On the other hand, Fig. 5(b) shows the strongest SC on average, we observe that the high intensity spots of the SC order are most likely setting around the As-P boundaries. For $x = 0.55$, the SC order is further suppressed and Fig. 5(c) shows that the high-intensity spots are localized. Finally, when $x$ reaches 0.65 as shown in Fig. 5(d), the SC order is further localized and its high intensity spots are mostly setting on the P-site free region which tells us that the SC pairing intensity in BaFe$_2$As$_2$ is stronger than that in BaFe$_2$P$_2$.

V. CONCLUSION

In conclusion, we have obtained the phase diagrams in the isovalent substituted system BaFe$_2$(As$_{1-x}$P$_x$)$_2$ with an effective model for the first time. The resulting phase diagrams are in agreement with experiments. Here we suppose that the hopping integrals between the Fe sites around the substituted-P are altered. Besides, we assume that there exists a very weak onsite-interorbital scattering on the adjacent four Fe site as well. The suppress of the SDW order in this compound as $x$ increases is caused by the incoherent scattering effect due to the randomly distributed P atoms. The competing SC order starts to show up only when the SDW order becomes significantly weakened. In our calculation, we put the pairing interaction $V_{ij}$ unchanged. However, due to the spin fluctuations, $V_{ij}$ may become smaller as the concentration of substituted-P atoms increases. Unfortunately, it is unclear how to treat this issue in the disordered region. Since $U$ is fixed in our system, as a first order approximation, we can use $t$-$J_1$ and $-J_2$ model to study the superconductivity in perfect BaFe$_2$As$_2$ and BaFe$_2$P$_2$ systems. Here $V_{ij} \sim J_2$ might be weak, but it should not be zero in BaFe$_2$P$_2$. Although we keep the value of $V_{ij}$ around the P-sites unchanged, we also assume a weak onsite inter-orbital scattering-terms near these sites in our calculation. The choice of a smaller $V_{ij}$ around substituted-P would not change the main results of the present paper. Beside, we recognize that there may exist other impurity models for this compound. In fact we have tried several different formalism, and it appears that only the model described in the Appendix A is able to qualitatively account for the experimental phase diagram. With the theoretically obtained phase diagram for BaFe$_2$(As$_{1-x}$P$_x$)$_2$ system, we should be able to calculate and understand the electronic and thermodynamic properties of this compound at any doping level $x$ (from 0 to 1).

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VI. APPENDICES

A. The mixing hopping integrals

Here, we give the details about how the hopping integrals, \( \tilde{t} \), mixing with the existence of the substituted-P atoms in the \( \text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2 \) system. The six hopping integrals are \( t_{1-6} \) for pure \( \text{BaFe}_2\text{As}_2 \) (see Refs. 26 and 31) and \( t'_{1-6} \) for pure \( \text{BaFe}_2\text{P}_2 \). The details of \( t_{1-6} \) and \( t'_{1-6} \) are given in the present manuscript. We group the mixing hopping integrals into two categories: i) NN hopping integrals, ii) NNN hopping integrals, as shown in Fig. 6. In Fig. 6, the green-solid circle presents for the Fe atoms, the open circle presents for either As or substituted P atoms, the orange-dotted line is the NN hopping terms (\( \tilde{t}_{1,5} \)), the red-dashed line is the NNN hopping terms (\( \tilde{t}_{2,3,4} \)); we use numbers (1 ... 6) to locate the atoms which affect the nearby NN hopping term (orange-solid line) and alphabets (A ... G) to locate the atoms which affect the nearby NNN hopping term (red-solid line).

\[ \tilde{t}_{1,5} = \begin{cases} 
1) & \text{Both As atoms in 2 and 5 are substituted by P:} \\
& t'_{1,5} \\
2) & \text{Only one As atom in 2 or 5 is substituted by P:} \\
& a t_{1,5} + b t'_{1,5} \\
3) & \text{At least one As in 1, 3, 4 or 6 is substituted by P:} \\
& a' t_{1,5} + b' t'_{1,5} \\
4) & \text{All of them (1 to 6) are As atoms:} \\
& t_{1,5} 
\end{cases} \] (10)

where \( a = 1/4, b = 3/4 \) and \( a' = 1/2, b' = 1/2 \).

ii) NNN hopping integrals (the red-solid line in Fig. 6):

\[ \tilde{t}_{2,3,4} = \begin{cases} 
1) & \text{the As atoms in A are substituted by P:} \\
& t'_{2,3,4} \\
2) & \text{At least one As atom in B-G is substituted by P:} \\
& t_{2,3,4} + b t'_{2,3,4} \\
3) & \text{All of them (A to G) are As atoms:} \\
& t_{2,3,4} 
\end{cases} \] (11)

where \( a = 1/4, b = 3/4 \). Note that there are two directions of the NN (NNN) hopping integrals which corresponds to two different configurations of the labeling, 1 to 6 (A to G); here we only show one of it.

In our previous discussions, we suppose there exist different hopping integrals between all and some certain substituted-P atoms. The ratios of \( a \) \((b)\) and \( a' \) \((b')\) are chosen arbitrarily, originally, if we choose \( a = 1/2, \) and \( a' = 1, \) the SDW will emerge at a big value of \( x \approx 0.5 \). We have tried different sets of ratios, and these values, chosen in the paper, could give qualitatively comparable experimental phase diagram.

B. Band structure and Fermi surface of the 2-Fe per unit cell Brillouin Zone

In Fig. 7, we show the band structures and Fermi surfaces of the pure \( \text{BaFe}_2\text{As}_2 \) and \( \text{BaFe}_2\text{P}_2 \) systems in the BZ of 2-Fe atoms per unit cell respectively.

FIG. 6. Schematic picture for the concept of mixing NN and NNN hopping terms for \( \text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2 \).
FIG. 7. (Color online) The model calculated band structure of two-orbital model on the 2-Fe per unit cell Brillouin Zone (BZ) for (a) BaFe$_2$As$_2$ ($x = 0$), (b) BaFe$_2$P$_2$ ($x = 1$); and Fermi surface on the 2-Fe per unit cell BZ for (c) BaFe$_2$As$_2$ ($x = 0$), (d) BaFe$_2$P$_2$ ($x = 1$). Here the fermi energy is shifted to zero for 1/2 filling.

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Aside from the simplicity of our two-orbital model for BaFe$_2$As$_2$, it could not account for any physics involved with $d_{xy}$ orbital, for example, the Fermi surface due to this orbital observed in the hole doped BaFe$_2$As$_2$. Since our model is essentially two-dimensional, it could not account for any physics involved with three-dimension which is associated with the $d$-orbital along z-direction. We believe that the essential feature of the SC and the magnetic orders is originated in the $d_{xz}$- and $d_{yz}$- orbitals, and many electronic and thermodynamic quantities calculated from the two-orbital models could be qualitatively compared with the experiments.

In Ref. 35, mathematically, we can also find another mean-field decoupled scheme for the interaction $H_{\Delta}$, which is $\sum_{ij} \langle n_i \rangle n_j$. This mean-field decoupled order is usually responsible for a charge-density-wave (CDW), however, we did not find that such an order is stable.

Our assumption of the existence of a weak “on-site interorbital scattering term $H_{P_{\text{inter}}}$” is also consistent with the view point of Ciechan and Wysokinski\cite{32} that such an inter-orbital scattering term is worth being considered in a complex system.