Magneto shot noise in noncollinear diffusive spin-valves

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We develop a semiclassical Boltzmann-Langevin theory of the spin polarized shot noise in a diffusive normal metal spin-valve connected by tunnel contacts to ferromagnetic reservoirs with noncollinear magnetizations. We obtain basic equations for correlations of the fluctuating spin-charge distribution and current density matrices by taking into account the spin-flip processes and precession of the spin accumulation vector in the normal metal. Applying the developed theory to a two terminal FNF structure, we find that for a small spin-flip strength and a substantial polarization of the terminals the shot noise has a nonmonotonic variation with the angle between magnetization vectors. While the shot noise is almost unchanged from the normal structure value for parallel configuration and increases well above the normal value for antiparallel configuration, it suppresses substantially at an intermediate angle depending on the ratio of the conductances of the N metal and the tunnel contacts. We also demonstrated pronounced effects of the polarization and the spin-flip scattering on the shot noise which reveals the interplay between relaxation and precession of the spin accumulation vector in the N metal.

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I. INTRODUCTION

Spin-polarized transport in magnetoelectronic structures has recently attracted an intense interest largely due to their important technological applications such as non-volatile magnetic random access memories (MRAMs), read heads for mass data storage and high sensitive magnetic sensors. The main effect is the (giant) magnetoresistance in magnetic multilayers and spin-valves, i.e. the remarkable decrease in the resistance when the orientation of the magnetization vectors of the ferromagnetic regions change from antiparallel to parallel. A spin-valve consists of two ferromagnetic leads as the spin injector and detector, connected through a normal metal spacer which serves for the spin accumulation. Recent interest in transport with noncollinear (neither parallel nor antiparallel) magnetizations was stimulated by the spin-transfer magnetization torque, which is essential for the new devices such as the spin-flip and spin torque transistors.

The most appropriate theoretical formalism for noncollinear ferromagnet-normal metal structures is the semiclassical circuit theory which is based on dividing of the structure into the reservoirs, nodes and the junctions and expressing the currents in terms of the scattering matrices of the junctions and the isotropic electronic distributions inside the nodes and reservoirs. For the noncollinear spin-transport the spin-charge current and the corresponding distributions inside the nodes and reservoirs, as well as the scattering matrices acquire the $2 \times 2$ matrix characteristics of the Pauli spin space. The semiclassical Boltzmann equations provide the standard method to calculate the noncollinear distribution matrix in a continuous bulk medium in the diffusive limit. Magnetoelectronic circuit theory complemented by the Boltzmann diffusion equations have been widely used to study different magnetoelectronic DC effects. In spite of this there has been little attention toward the fluctuations of the noncollinear spin-polarized current. Time-dependent current fluctuations at low temperatures, the so called shot noise, provides valuable information about the transport process which are not extractable from the average current. In magnetoelectronic structures, in which the spin of electrons plays an essential role, the shot noise is expected to contain spin-resolved information, including spin-dependent correlations and spin accumulation and relaxation. This together with the importance of the noise in magnetoelectronic devices in view of applications motivate studying of the spin-polarized current fluctuations.

The theory for current fluctuations in disorder conducting systems can be formulated by including the Langevin sources of fluctuations due to the random scattering from disorders and the fluctuations of the distribution in the semiclassical Boltzmann equation. The resulting Boltzmann-Langevin (BL) formalism has been used to calculate the shot noise in many mesoscopic structures and its results are widely believed to be identical with an ensemble average of the exact quantum results. One important example is the universal one-third suppression of the shot noise in a diffusive normal metal, which was correctly predicted within the BL approach. Recently spin-polarized shot noise has been studied theoretically in normal metals connected by the ferromagnetic terminals with collinear magnetizations. In Ref. a semiclassical BL theory of the collinear spin-polarized current fluctuations in a diffusive normal metal was developed. It was found that in a multi-terminal spin-valve structure the shot noise and the cross correlations measured between currents of two different ferromagnetic terminals can deviate substantially from the unpolarized values, depending on the spin accumulation and the spin-flip scattering strength. On the other hand only a few works have...
have been devoted to shot noise in noncollinear systems. Very recently Braun et al. showed that the frequency-dependent shot noise in a quantum dot spin-valve with noncollinear magnetizations of ferromagnets in the presence of an external magnetic field can be used to detect single-spin dynamics in the quantum dot.

Tserkovnyak and Brataas extended the circuit theory to obtain the Landauer–Büttiker (LB) formula for the shot noise of a FN contact with noncollinear magnetization. They analyzed the angular dependence of the shot noise in a N metal node connected to two F reservoirs for different types of junctions between the reservoirs and the node. However, the effect of the spin-flip scattering inside the N metal as well as the spatial variation of the noncollinear spin accumulation vector, which are essential in diffusive spin-valves with wire geometry, are disregarded so far. These effects require an extension of the BL method to include the diffusion and the relaxation of the noncollinear spin accumulation inside the diffusive N metal. To our knowledge there is no BL theory for shot noise of a noncollinear spin-polarized transport. The aim of the present work is to develop such a theory.

In this paper we develop a semiclassical BL equation for the noncollinear spin-polarized current fluctuations in the presence of the spin-flip scattering. We obtain the basic diffusion equations for the fluctuating spin-charge distribution and current matrices which allows to calculate the mean current and the correlations of the corresponding fluctuations in a diffusive normal metal connected by tunnel contacts to several F reservoirs. The developed BL equations are supplemented by the the generalized LB formula for the shot noise at the contact points.

To illustrated the main behavior of the noncollinear shot noise in magnetoelectronic systems we apply the developed BL equations to calculate the Fano factor, defined as the ratio of the noise power to average current, in a two terminal FNF structure. The noncollinear orientation of the magnetizations causes a precession of the spin accumulation vector through the N metal, which in presence of the spin-flip scattering is associated with a damping as a function of the distance from the F reservoirs. For a small spin-flip intensity and a finite polarization of the tunnel contacts the precession of the spin accumulation results in a nonmonotonic angular dependence of the Fano factor. For a parallel configuration Fano factor is almost the same as the the normal state value but for antiparallel configuration it increases well above this value due to a large spin accumulation in the N metal. We find a substantial decrease of the Fano factor at an intermediate orientation determined from the conductances of the N metal and the tunnel contacts. Introducing the spin-flip scattering as well as decreasing the spin polarization diminish the nonmonotonic behavior. We present a full analysis of the magneto shot noise which demonstrates the effects of the spin-flip induced relaxation and the precession imposed by noncollinear magnetizations.

The paper is organized as follows. Section II devotes to introduce the BL equation for noncollinear transport and derive diffusion equations. In section III we calculate all possible correlations of intrinsic fluctuations and Langevin sources. Section IV devotes to calculate fluctuations and average of the current in double barrier FNF system. We show results for charge current shot noise in section V and finally give some conclusions in section VI.

II. BOLTZMANN-LANGEVIN EQUATIONS FOR NONCOLLINEAR TRANSPORT

In this section we develop a semiclassical BL formalism for fluctuations of the noncollinear spin-polarized current in a diffusive normal metal contacted through tunnel contacts to a number of ferromagnetic reservoirs. When the magnetization vectors of the reservoirs have a noncollinear orientation, the spin accumulation vector in the normal metal will also have a noncollinear direction with respect to the quantization axis ($z$). The semiclassical electronic distribution is then determined by a $2 \times 2$ matrix in the Pauli spin space of the form

$$
\hat{f}(\mathbf{k}, \mathbf{r}, t) = \begin{pmatrix} f^{\uparrow \uparrow}(\mathbf{k}, \mathbf{r}, t) & f^{\uparrow \downarrow}(\mathbf{k}, \mathbf{r}, t) \\ f^{\downarrow \uparrow}(\mathbf{k}, \mathbf{r}, t) & f^{\downarrow \downarrow}(\mathbf{k}, \mathbf{r}, t) \end{pmatrix}.
$$

(2.1)

The fluctuating matrix distribution function $\hat{f}(\mathbf{k}, \mathbf{r}, t) = \hat{f}(\mathbf{k}, \mathbf{r}) + \delta \hat{f}(\mathbf{k}, \mathbf{r}, t)$, depends on the momentum $\mathbf{k}$, the coordinate $\mathbf{r}$ and time $t$. It is convenient to expand $\hat{f}$ into the charge and the spin vector distributions in terms of the $2 \times 2$ unit matrix and the Pauli matrices $\{1, \sigma \equiv (\sigma_x, \sigma_y, \sigma_z)\}$. The matrices for two distributions whose spin vector distributions are pointed in opposite directions take the form

$$
\hat{f}^\pm(\mathbf{k}, \mathbf{r}, t) = f_c(\mathbf{k}, \mathbf{r}, t) \hat{1} \pm \hat{\sigma} \cdot \hat{f}_{\sigma}(\mathbf{k}, \mathbf{r}, t).
$$

(2.2)

Here $f_c = (f^{\uparrow \uparrow} + f^{\downarrow \downarrow})/2$ is the charge or the spin independent part of the distribution matrix and $\hat{f}_\sigma$ the spin distribution vector whose $z$-component $f_{sz} = (f^{\uparrow \uparrow} - f^{\downarrow \downarrow})/2$ is spin polarization along the quantization axis and the other two components $f_{sx}$, $f_{sy}$ describe the spin-polarization oriented perpendicular to the quantization axis. Note that the two distributions $\hat{f}^\pm$ are transformed to each other by making the replacement $\hat{f}_\sigma \rightarrow -\hat{f}_\sigma$.

The BL equation for the noncollinear distribution matrix is written as

$$
\frac{d}{dt} \hat{f}^+ = \hat{f}^{\text{imp}}[\hat{f}^+] + \hat{f}^{\text{sf}}[\hat{f}^+, \hat{f}^-] + \hat{\xi}^{\text{imp}} + \hat{\xi}^{\text{sf}},
$$

(2.3)

where $\hat{f}^{\text{imp,sf}}$ is the collision integral for normal spin-independent impurity (spin-flip) scattering and $\hat{\xi}^{\text{imp,sf}}$ the corresponding Langevin source of the current fluctuations. The matrix collision integrals are expressed as the following

$$
\hat{f}^{\text{imp}}[\hat{f}^+] = \int d\mathbf{k}' W^{\text{imp}}(\mathbf{k}, \mathbf{k}') [\hat{f}^+(\mathbf{k}') - \hat{f}^+(\mathbf{k})],
$$

(2.4)

$$
\hat{f}^{\text{sf}}[\hat{f}^+, \hat{f}^-] = \int d\mathbf{k}' W^{\text{sf}}(\mathbf{k}, \mathbf{k}') [\hat{f}^-(\mathbf{k}') - \hat{f}^+(\mathbf{k})],
$$

(2.5)
where \( W^{\text{imp}}(k, k') \) is the rate of the impurity scattering in which an electron scatters from the state with momentum \( k \) into \( k' \) without changing its spin state. The spin-flip scattering rate \( W^{sf}(k, k') \) describes the transition from the state with momentum \( k \) and spin state \( |\hat{s}, \pm\rangle \) to \( k' \) and the flipped spin state \( |\hat{s}, \mp\rangle \), where \( |\hat{s}, \mp\rangle \) are the up and down spin eigen states in the quantization axis parallel to the local spin polarization vector \( \vec{f}_s = |f_s|\hat{s} \). In writing the expressions \( \hat{J}_r \) and \( \hat{J}_f \), we assumed that \( W^{\text{imp}(sf)}(k, k') = W^{\text{imp}(sf)}(k', k) \), which follows from the detail balance principle. We also ignored dependence of the scattering rates on the spin state of electron.

The Langevin sources are expressed in terms of the fluctuating part of the matrix distribution as follows

\[
\hat{\xi}^{\text{imp}} = \int \! dk' W^{\text{imp}}(k, k') [\delta \hat{f}^+(k') - \delta \hat{f}^+(k)], \quad (2.6)
\]
\[
\hat{\xi}^{\text{sf}} = \int \! dk' W^{sf}(k, k') [\delta \hat{f}^-(k') - \delta \hat{f}^+(k)]. \quad (2.7)
\]

For a diffusive normal metal we apply the standard diffusive approximation. Assuming that all quantities are sharply peaked around Fermi level, the momentum vector \( k \) is expressed in terms of the energy \( \varepsilon \) and the direction of the Fermi momentum \( n \). Then the following relations are hold for the elastic scattering of electrons by the normal impurities and spin-flip disorders:

\[
W^{\text{imp}(sf)}(k, k') = \frac{2}{N_0} \delta(\varepsilon - \varepsilon') W^{\text{imp}(sf)}(n, n'), \quad (2.8)
\]

where \( N_0 \) is the density of states in the Fermi level. In the diffusive regime the electronic distribution is weakly anisotropic in the momentum space, and the distribution matrix can be expanded up to the linear term in \( n \):

\[
\hat{f}(n, \varepsilon, r, t) = \hat{f}_0(\varepsilon, r, t) + \mathbf{n} \hat{f}_1(\varepsilon, r, t). \quad (2.9)
\]

Here the anisotropic part of the matrix distribution is determined by the vector \( \hat{f}_1 \) whose components are \( 2 \times 2 \) matrices in the spin space. The matrix current density is expressed as

\[
\hat{J}(r, t) = \hat{\mathbf{J}}(r) + \delta \hat{J}(r, t) = (eN_0 v_F / 6) \int d\varepsilon d\hat{f}_1, \quad (2.10)
\]

where \( v_F \) is the Fermi velocity. The isotropic part \( \hat{f}_0 \) determines the fluctuating electrochemical potential:

\[
\hat{\varphi}(r, t) = \hat{\varphi}(r) + \delta \hat{\varphi}(r, t) = (1/e) \int d\varepsilon d\hat{f}_0. \quad (2.11)
\]

Replacing expansion \( \hat{f}(n, \varepsilon, r, t) = \hat{f}_0(\varepsilon, r, t) + \mathbf{n} \hat{f}_1(\varepsilon, r, t) \) in Eq. \( (2.11) \) we obtain the diffusion equations for the current density and the potential matrices, which read

\[
\nabla^2 \hat{\varphi}(r) = \frac{1}{\tau_{sf}^2} \left( \hat{\varphi} - \frac{Tr(\hat{\varphi})}{2} \mathbf{1} \right), \quad (2.12)
\]
\[
\nabla \cdot \mathbf{j} = \frac{e^2 N_0}{2 \tau_{sf}} \left( \hat{\varphi} - \frac{Tr(\hat{\varphi})}{2} \mathbf{1} \right) + \hat{\sigma} \cdot \mathbf{\zeta}, \quad (2.13)
\]
\[
\mathbf{j} = -\sigma \nabla \hat{\varphi} + \hat{\eta}. \quad (2.14)
\]

In these equations \( \sigma = e^2(N_0/2)D \) is the conductivity and \( \tau_{sf} = \sqrt{D\tau_{sf}} \) is the spin-flip scattering length, where the diffusion constant is given by \( D = v_F^2 \pi / 3 \). The impurity and the spin-flip relaxation times are defined by the following relations

\[
\frac{n}{\tau_{imp}} = \int d\varepsilon' w^{\text{imp}}(n, n')(n - n'), \quad (2.15)
\]
\[
\frac{n}{\tau_{sf}} = 2 \int d\varepsilon' w^{sf}(n, n')(n - n'), \quad (2.16)
\]
\[
\frac{1}{\tau} = \frac{1}{\tau_{imp}} + \frac{1}{\tau_{sf}}. \quad (2.17)
\]

The matrix Langevin current fluctuations in Eq. \( (2.14) \) has the form

\[
\hat{\eta} = e v_F (N_0/2) \tau \int (\hat{\xi}^{\text{imp}} + \hat{\xi}^{\text{sf}}) n d\varepsilon. \quad (2.18)
\]

Also,

\[
\hat{\sigma} \cdot \mathbf{\zeta}(r, t) = (eN_0/2) \int d\varepsilon d\mathbf{n} \left( \hat{\xi}^{\text{sf}} - \frac{Tr(\hat{\xi}^{\text{sf}})}{2} \mathbf{1} \right), \quad (2.19)
\]

is the matrix Langevin divergence term due to the non-conserved nature of the spin-flip scattering. These two fluctuating Langevin terms have zero average values.

Note that we disregarded the explicit dependence of \( \hat{f} \) on time, given by the term \( \partial / \partial t \) of \( d / dt \) in the left hand side of the BL equation, since we are interested in the zero frequency noise power. To obtain these equations we focused on the realistic limit when \( \tau_{imp} \ll \tau_{sf} \) and ignored the effect of spin-flip scattering on the conductivity of the diffusive normal metal.

### III. CORRELATIONS OF CURRENT FLUCTUATIONS

#### A. Correlations of the Langevin sources of current fluctuations

To calculate correlations between components of the fluctuating Langevin terms \( \hat{\eta} \) and \( \mathbf{\zeta} \) we refer again to the BL equation \( (2.14) \). But, now we choose the spin quantization axis to be parallel to the local mean spin distribution vector \( \vec{f}_s \). Denoting the projection matrices along \( \hat{s} \) and two perpendicular unit vectors \( \bar{s}_1, \bar{s}_2(i = 1, 2) \) by \( \bar{v}_s \) (= \( \hat{\sigma} \cdot \hat{s} \)) and \( \bar{v}_{s_\perp} \), respectively, the matrix distribution function can be expanded as

\[
\hat{f}^\pm = \frac{1}{2} (f_{s_+} + f_{s_-}) \mathbf{1} \pm \frac{1}{2} (f_{s_+} - f_{s_-}) \bar{v}_s \pm \frac{1}{2} \sum_i (\delta f_{s_{\perp i}+} - \delta f_{s_{\perp i}-}) \bar{v}_{s_{\perp i}}, \quad (3.1)
\]

where the fluctuating transverse components of the spin polarization vector have zero mean values.
We substitute the fluctuating parts of the matrices \( \tilde{\xi}^{\text{imp}(sf)} \) into the Langevin source terms in Eq. (2.24) and obtain the result

\[
\tilde{\xi}^{\text{imp}(sf)} = \frac{1}{2} \tilde{\xi}^{\text{imp}(sf)+} + \frac{1}{2} \tilde{\xi}^{\text{imp}(sf)-} + \frac{1}{2} (\tilde{\xi}_{\bar{s},+} - \tilde{\xi}_{\bar{s},-}) \hat{\nu}_{\bar{s}},
\]

where

\[
\tilde{\xi}^{\text{imp}(sf)+}_s = \int d\mathbf{k}{[\delta J^{\text{imp}(sf)}_{\bar{s},(-)}(\mathbf{k}', \mathbf{k}) - \delta J^{\text{imp}(sf)}_{\bar{s},(+)}(\mathbf{k}, \mathbf{k}')]},
\]
\[
\delta J^{\text{imp}(sf)}_{\bar{s},(+)}(\mathbf{k}, \mathbf{k}') = J^{\text{imp}(sf)}_{\bar{s},(+)}(\mathbf{k}, \mathbf{k}') - J^{\text{imp}(sf)}_{\bar{s},(+)}(\mathbf{k}', \mathbf{k}),
\]

and we have introduced the current in individual impurity (spin-flip) scattering as \( J^{\text{imp}(sf)}_{\bar{s},(+)}(\mathbf{k}, \mathbf{k}') = W^{\text{imp}(sf)}(\mathbf{k}, \mathbf{k}') f_{\bar{s},+}(\mathbf{k}) [1 - f_{\bar{s},-}(\mathbf{k}')] \) for component of spin in direction of \( \bar{s} \).

Now we apply the central assumption of the BL approach and assume that all scattering events are independent elementary processes and thus the correlations of the associated currents fluctuate obey the Poissonian relation:

\[
\langle \delta J^{\text{imp}(sf)}_{\bar{s},+} \delta J^{\text{imp}(sf)}_{\bar{s},-} \rangle = \delta_{ij} \delta_{\alpha \alpha'} \delta_{\alpha \beta} \delta_{\bar{s}, \bar{s}} \delta(\mathbf{k}_1 - \mathbf{k}_2) \delta(\mathbf{k}_3 - \mathbf{k}_4) \delta(\mathbf{k}_1' - \mathbf{k}_2') \delta(\mathbf{k}_3' - \mathbf{k}_4') \delta(t - t') \delta(t' - t) \langle \tilde{\xi}^{\text{imp}(sf)+}_{\bar{s},+} \tilde{\xi}^{\text{imp}(sf)-}_{\bar{s},-} \rangle,
\]

and

\[
\langle \delta J^{\text{imp}(sf)}_{\bar{s},+} \delta J^{\text{imp}(sf)}_{\bar{s},-} \rangle = 0.
\]

Using these relations we can calculate the correlations between matrix elements of the Langevin sources \( \langle \tilde{\xi}^{\text{imp}(sf)} \rangle \), from that we obtain all the possible correlations between components of the vector matrices \( \hat{\eta} \) and \( \zeta \). The results read

\[
\langle \hat{\eta}_{\alpha \beta}^{\text{imp}(sf)}(\mathbf{r}, t) \hat{\eta}_{\alpha' \beta'}^{\text{imp}(sf)}(\mathbf{r}', t') \rangle = \frac{1}{2} \delta_{\alpha \alpha'} \delta_{\beta \beta'} \delta(\mathbf{r} - \mathbf{r}') \delta(t - t') \sigma
\]

\[
\times \left[ \delta^{\text{imp}(sf)}_{\bar{s},+}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{r}, t) \delta^{\text{imp}(sf)}_{\bar{s},-}(\mathbf{k}_3, \mathbf{k}_4, \mathbf{r}', t') \right],
\]

\[
\langle \zeta^{\text{imp}(sf)}(\mathbf{r}, t) \zeta^{\text{imp}(sf)}(\mathbf{r}', t') \rangle = \delta(\mathbf{r} - \mathbf{r}') \delta(t - t') \sigma
\]

\[
\times \frac{1}{D_{\text{imp}}} \tilde{\eta}_{\alpha \beta}^{\text{imp}(sf)} \tilde{\eta}_{\alpha' \beta'}^{\text{imp}(sf)} \sum_{\nu} \Pi_{\bar{s},\nu \nu}(\mathbf{r}),
\]

where

\[
\Pi_{\bar{s},\nu \nu}(\mathbf{r}) = \int d\mathbf{\xi} f_{\bar{s},\nu}(\mathbf{\xi}, \mathbf{r}) [1 - f_{\bar{s},\nu}(\mathbf{\xi}, \mathbf{r})].
\]

Here \( f_{\bar{s},\nu}(\mathbf{\xi}, \mathbf{r}) \) is the isotropic part of the up and down spin components of the mean distribution function with respect to the quantization axis parallel to \( \bar{s} \). So, if we know the mean matrix distribution function we can easily calculate these correlations. It is a remarkable result that, just the longitudinal fluctuations of the distribution function has non-vanishing contribution to correlations and hence in shot noise. But, the transverse fluctuations of distribution function, actually make no contribution in the shot noise.

\[\text{B. Boundary conditions and correlations of intrinsic fluctuations}\]

The diffusion equations and Eqs. (2.24-2.26) are the extension of the BL equations obtained in Ref. 21 for the collinear spin-polarized shot noise to the non-collinear magnetizations. These equations are a complete set of equations, which have to be implemented by the appropriate boundary conditions at the contacts of the normal metal to the ferromagnetic terminals. For a F/N junction with a noncollinear magnetization the expression for the matrix current reads

\[
e\tilde{I}^F = C_{\text{up}} \hat{u}^\dagger \left( \tilde{f}_F - \tilde{f}_N \right) \hat{u} + C_{\text{down}} \hat{u}^\dagger \left( \tilde{f}_F - \tilde{f}_N \right) \hat{u} - C_{\text{trans}} \hat{u}^\dagger \tilde{f}_N \hat{u} + (C_{\text{trans}})^* \hat{u}^\dagger \tilde{f}_N \hat{u}^*,
\]
metal and/or the adjacent reservoirs are fluctuating. For a contact between the N metal with fluctuating part of the matrix distribution \( \delta f_n \) and a F reservoir held at equilibrium, the expression for the matrix current fluctuations has the form

\[
e\Delta \hat{I}^C = \delta \hat{I} - G^{\uparrow\uparrow} \hat{u}^\dagger \delta \hat{f}_N (r) \hat{u}^\uparrow - G^{\downarrow\downarrow} \hat{u}^\dagger \delta \hat{f}_N (r) \hat{u}^\downarrow
- G^{\uparrow\downarrow} \hat{u}^\dagger \delta \hat{f}_N (r) \hat{u}^\downarrow - (G^{\uparrow\downarrow} + G^{\downarrow\uparrow}) \hat{u}^\dagger \delta \hat{f}_N (r) \hat{u}^\uparrow. \tag{3.10}
\]

In this equation \( \hat{G}^{s_{1}s_{2}} = [M - \sum_{n,m} r_{nm}^{s_{1}s_{2}}(r_{nm}^{s_{1}s_{2}})^*] \) and \( \hat{S}^{s_{1}s_{2}s_{3}s_{4}} = \frac{e^2}{N} [M - \sum_{nmn'm'} r_{nm}^{s_{1}s_{2}}(r_{nm}^{s_{1}s_{2}})^* r_{n'm'}^{s_{3}s_{4}}(r_{n'm'}^{s_{3}s_{4}})^*] \) are elements of the conductance matrix \( \hat{G} \) and the shot noise matrix \( \hat{S} \) respectively. The general expressions of these matrices are considerably simplified for a tunnel barrier, for which an expansion in terms of the small transmission coefficients \( \delta r_{nm} = 1 - r_{nm} \), leads to the following results\textsuperscript{24}

\[
\hat{G} = \frac{G}{2} \left( \begin{array}{cccc}
1 + P & 1 & 1 & 1 - P \\
1 & 1 & 1 & 1 - P \\
2 + P & 2 + P & 2 + P & 2 \\
2 + P & 2 + P & 2 + P & 2 \\
2 & 2 & 2 & 2 - P \\
2 & 2 & 2 & 2 - P \\
2 & 2 & 2 & 2 - P
\end{array} \right), \tag{3.12}
\]

\[
\hat{S} = \frac{G}{2} \left( \begin{array}{cccc}
2 + P & 2 + P & 2 + P & 2 \\
2 + P & 2 + P & 2 + P & 2 \\
2 & 2 & 2 & 2 - P \\
2 & 2 & 2 & 2 - P \\
2 & 2 & 2 & 2 - P
\end{array} \right). \tag{3.13}
\]

Where we have the total conductance \( G = G^{\uparrow\uparrow} + G^{\downarrow\downarrow} \), the polarization \( P = (G^{\uparrow\uparrow} - G^{\downarrow\downarrow})/G \), \( \text{Re}G^{\uparrow\downarrow} = G/2 \) and \( \text{Im}G^{\uparrow\downarrow} = 0 \) for the tunnel contact. Thus by knowing the average distribution function matrices inside the connected N and F metals, we can calculate all possible correlations between the intrinsic current fluctuations.

Now we have all requirements to calculate the shot noise in a ferromagnetic multi-terminal diffusive structure. The first step is to solve the diffusion equations\textsuperscript{22,23,24} implemented by the boundary conditions which are temporal current conservation laws in the contacts to obtain the mean distribution matrix in N metal and the charge and spin currents fluctuations in the end points of N metal in terms of the Langevin current fluctuations and the intrinsic current fluctuations at these points. Now, knowing the correlations between these fluctuations from Eqs. (3.5-3.7) and (3.11), enable us to calculate the correlations of the charge and spin currents fluctuations. In the next section we demonstrate the above developed formalism by studying the noncollinear shot noise in a two terminal spin-valve.

IV. ANGULAR SHOT NOISE OF A DOUBLE BARRIER F-N-F SYSTEM

We consider a two terminal diffusive spin-valve system as shown in Fig. 1. Two ferromagnetic reservoirs F\(_1\) and F\(_2\) with a noncollinear orientation \( \theta \) of the magnetization vectors are connected through tunnel contacts to a diffusive normal metal wire (N) of length \( L \). The reservoirs F\(_1\), F\(_2\) are held at 0 and \( V \) potentials, respectively. We consider a symmetric structure for which both of the tunnel contacts have the same conductance and shot noise matrices given by Eqs. (3.12,3.13).

The mean distribution function matrix inside the N metal is obtained by solving the diffusion equation (2.12).
The general solution of this equation can be written as the energy integral of the following relation:
\[
\tilde{f}_N(x) = \left[ \tilde{f}_1 + (\tilde{f}_2 - \tilde{f}_1)(a + b \frac{x}{L}) \right] \delta + \sigma \cdot \left[ \tilde{c} \sinh(\lambda x/L) + \tilde{d} \cosh(\lambda x/L) \right] (\tilde{f}_2 - \tilde{f}_1), \tag{4.1}
\]
where the matrix distribution function at the terminals \( F_{1,2} \) are determined by the Fermi distribution functions via \( \tilde{f}_1 = f_{FD}(\varepsilon) \) and \( \tilde{f}_2 = f_{FD}(\varepsilon - eV) \), and \( \lambda = L/\ell_d \) is a dimensionless parameter which measures the intensity of the spin-flip scattering. The scalars \( a, b \) and components of the vectors \( \tilde{c} = (c_1, c_2, c_3) \) and \( \tilde{d} = (d_1, d_2, d_3) \) are coefficients which have to be determined by the boundary conditions. The boundary conditions are the spin-charge current conservation rules which in the absence of spin-flip process at the tunnel contact \( i \) (\( i = 1, 2 \), read
\[
\tilde{T}_C^i + \tilde{T}_N^i = 0, \tag{4.2}
\]
where \( \tilde{T}_C^i \) and \( \tilde{T}_N^i \) are expressions for the mean matrix current through the ith contact which are calculated by relations \( 3.3 \) and the diffusion equation \( 2.1.1 \), respectively. The mean charge current is determined by the coefficient \( b \) only:
\[
\tilde{I}_c = Tr(\tilde{T}) = -2G_N b, \tag{4.3}
\]
where the normal conductance of the N wire is given by \( G_N = \sigma A / L \), \( A \) is the cross section area of the wire.

To obtain correlations of the Langevin sources in Eqs. \( 3.5 \) and Eq. \( 3.10 \) we need the matrix distribution function in the quantization axis parallel to the mean spin distribution vector. This is achieved by diagonalizing the distribution function matrix to get \( \tilde{f}_{s,\pm}(x) \). The result is
\[
\tilde{f}_{s,\pm}(x) = f_1 + (f_2 - f_1) \left[ (a + b \frac{x}{L}) \right] \delta + \tilde{c} \sinh(\lambda x/L) + \tilde{d} \cosh(\lambda x/L)], \tag{4.4}
\]
where \( f_{1,2} \) are the corresponding Fermi distributions in the terminals. Now we calculate the fluctuations of the matrix current \( \Delta \tilde{f}_N \) at the contacts \( x = 0, L \) inside N wire. Using the equation for \( \Delta \tilde{J} \) \( 2.1.1 \) and Eq. \( 2.1.9 \) we obtain
\[
\Delta \tilde{f}_N(0, L) = -\sigma \int ds \cdot \nabla \phi_{s0}(L)(x) \frac{Tr(\delta \tilde{\phi})}{2} \delta + \nabla \phi_{s0}(L)(x) \left( \delta \tilde{\phi} - \frac{Tr(\delta \tilde{\phi})}{2} \right) + \delta \tilde{I}(0, L), \tag{4.5}
\]
where integration is over the surface of the N metal and
\[
\delta \tilde{I}(0, L) = \int dr \left[ \frac{1}{2} (Tr(\tilde{\eta}) \cdot \nabla) \phi_{s0}(L)(x) \right] + \left( \sigma \cdot \zeta + (\tilde{\eta} - \frac{Tr(\tilde{\eta})}{2}) \cdot \nabla \right) \phi_{s0}(L)(x). \tag{4.6}
\]
Here the potential functions are defined as
\[
\phi_{s0}(x) = 1 - \frac{x}{L}, \tag{4.7}
\]
\[
\phi_{sL}(x) = \frac{x}{L}, \tag{4.8}
\]
\[
\phi_{s0}(x) = \frac{\sinh[\lambda(1 - x/L)]}{\sinh(\lambda)}, \tag{4.9}
\]
\[
\phi_{sL}(x) = \frac{\sinh(\lambda x/L)}{\sinh(\lambda)}. \tag{4.10}
\]
Note that as a result of the spin-flip scattering the spin current and hence, its fluctuations are not conserved through the wire.

Now we can apply the current conservation law for the fluctuations of the matrix currents at two contacts:
\[
\Delta \tilde{I}_C^i + \Delta \tilde{I}_N^i = 0 \tag{4.11}
\]
Replacing the expressions of the current fluctuations from Eqs. \( 4.5 \) \( 14 \) and Eq. \( 3.10 \) in Eq. \( 4.11 \) we obtain a system of eight linear equations whose solutions give the fluctuations of the matrix chemical potential \( \delta \hat{\phi}(0, L) \) at the points \( x = 0, L \) in terms of the intrinsic matrix current fluctuations \( \delta \tilde{I}_{1,2} \) and the Langevin matrix current fluctuations \( \delta \hat{I}(0, L) \). Replacing this result into Eq. \( 3.10 \) the matrix current fluctuations are expressed in terms of the fluctuating Langevin sources of currents and the intrinsic current matrices as
\[
\Delta \tilde{J}^{\alpha \beta} = \sum_{\alpha' \beta'} \left[ A_{\alpha' \beta'}^{\alpha \beta} \delta \tilde{I}^{\alpha' \beta'}_1 + B_{\alpha' \beta'}^{\alpha \beta} \delta \tilde{I}^{\alpha' \beta'}_2 + C_{\alpha' \beta'}^{\alpha \beta} \delta \hat{I}^{\alpha' \beta'}(0) + D_{\alpha' \beta'}^{\alpha \beta} \delta \hat{I}^{\alpha' \beta'}(L) \right], \tag{4.12}
\]
where \( A_{\alpha' \beta'}^{\alpha \beta}, B_{\alpha' \beta'}^{\alpha \beta}, C_{\alpha' \beta'}^{\alpha \beta}, \) and \( D_{\alpha' \beta'}^{\alpha \beta} \) are the coefficients which depend on \( G/G_N, P, \lambda, \) and \( \theta \). From this result we can calculate the correlations of the charge current \( S = 2 \int \delta \tilde{I} \delta \tilde{I} \) and the corresponding Fano factor \( F = S/2e \tilde{I}_c \). The resulting expressions are too lengthy to be written down here. In the next section we analyze the magneto shot noise, the dependence of the \( F \) on the the relative angle \( \theta \), for different values of the involved parameters.

V. RESULTS AND DISCUSSION

Let us start our analysis of the shot noise with the limiting case of negligible spin-flip scattering, i.e. when \( \lambda \to 0 \), and highly resistive tunnel contacts \( G/G_N \to 0 \). In this limit we retrieve the results of Tserkovnyak and Brataas for charge current shot noise, that is, a monotonic variation of \( F \) as a function of the relative angle of the magnetization vectors \( \theta \):
\[
F = \frac{1}{2} \left[ 1 + P^2 \sin^2(\theta/2) \right]. \tag{5.1}
\]
For a finite $G/G_N$ the angular dependence of Fano factor deviates from the above simple monotonic behavior. This is illustrated in Fig. 2 where we have plotted $F(\theta)$ for different values of $G/G_N$ in the limit $\lambda \to 0$ and the polarization $P = 0.99$. By increasing $G/G_N$, $F$ decreases below the value given by Eq. (5.1) and finds a nonmonotonic dependence on $\theta$. The nonmonotonic behavior is more pronounced for a value of $G/G_N$ which depends on the polarization. The Fano factor develops a minimum at a finite angle $\theta \neq 0$, which depends on $P$ and $G/G_N$. In the limit of $G/G_N \gg 1$, $F$ returns back to the constant normal state value $1/3$, as it is expected. This holds for not perfectly polarized terminals $P \neq 1$. We found that for $P = 1$, Fano factor retains its nonmonotonic angular dependence in the limit $G/G_N \gg 1$. In this case $F$ has a minimum well below $1/3$ at a finite $\theta$, a sharp peak at $\theta = \pi$ where it reaches one and a smooth peak at $\theta = 0$ with the value $1/3$. The corresponding behavior in the conductance of the system was studied in Ref. [12].

Next we study the effect of spin-flip scattering in the normal metal. In Fig. 3 we show dependence of Fano factor on $\lambda$ for different angles, when $G = 1$ and $P = 1$. As it can be seen from Fig. 3 a strong spin-flip scattering destroys any spin polarization and thus suppresses the dependence of the Fano factor on $\theta$. In this limit ($\lambda \gg 1$) $F$ reduces to the normal state value independent of $\theta$

$$F = \frac{112 + 12G/G_N + 6(G/G_N)^2 + (G/G_N)^3}{(2 + G/G_N)^3}.$$  \hspace{1cm} (5.2)

At a finite $\lambda$ the shot noise of different $\theta$s is separated. The most strong variations happens for $\lambda \to 0$. Increasing $\lambda$ from zero, $F(\lambda)$ passes through a minimum or maximum, depending on $\theta$, before reaching the normal state value at large $\lambda$s. In the case of Fig. 3 the minimum and maximum happen for $\theta < \pi/2$ and $\theta > \pi/2$, respectively. While the minimum and maximum values of $F$ are located at $\lambda = 0$ for $\theta = 0$ and $\theta = \pi$, they can happen at finite $\lambda$s for intermediate angles.

To show the effect of contacts polarization $P$, we have plotted in Fig. 4 the Fano factor vs. $P$ in the limit $\lambda \to 0$, for different $\theta$s. For $P \to 0$ the Fano factor takes the normal state value, irrespective of the value of $\theta$. Increasing $P$ the dependence of $F$ on $\theta$ is appeared. The Fano factor increases (decreases) monotonically with $P$ for $\theta = \pi$ ($\theta = 0$) and reaches a maximum (minimum) at $P = 1$. However for an intermediate angles $F(P)$ can have a nonmonotonic variation with a maximum at $P < 1$. This is in contrast to the limit of the normal metal node geometry ($G/G_N \to 0$), where a monotonic dependence on $P$ was predicted (Eq. (5.1)).

VI. CONCLUSIONS

In this paper we have presented a semiclassical theory of the spin-polarized current fluctuations in a diffusive normal metal which is connected by tunnel contacts to ferromagnetic terminals with noncollinear magnetizations. Based on the Boltzmann-Langevin approach, we have developed diffusion equations which allow for calculation of the charge-spin distribution and current density matrices and the correlations of the corresponding fluctuations in noncollinear multiterminal systems. Our theory takes into account the precession of the noncollinear spin accumulation as well as the spin-flip induced relaxation

![Fig. 2: Fano factor versus the relative angle $\cos(\theta)$ for different values of $G/G_N$ ($G_N = 1$) in the limit of $\lambda \to 0$ and for $P = 0.99$.](image)

![Fig. 3: Fano factor versus the spin-flip scattering intensity $\lambda = L/t_{sf}$ for different values of relative angle $\theta$ for a system with $G = 1$ and $P = 1$.](image)
in the normal metal. Applying the developed theory to a two terminal FNF structure we have found that the Fano factor has a nonmonotonic dependence on the magnetizations angle, provided that the spin-flip intensity is small and the tunnel contacts have an appreciable polarization. For antiparallel orientation the Fano factor is found to increases well above the unpolarized value due to a large spin accumulation in the N metal. In contrast, for the parallel orientation the shot noise is almost identical with its normal state (unpolarized) value because of the spin accumulation suppression. At the intermediate angles we have found that the interplay between the spin accumulation precession and the suppression with magnetizations angle causes that the Fano factor develops a minimum. The minimum shot noise can be substantially below the normal value depending on the relative conductances of the N metal and tunnel contacts $G/G_N$ and the polarization $P$. We further have shown that the spin-flip scattering diminishes the amplitude of the nonmonotonic behavior as well as the polarization effects.

In spite of a few early and recent experimental studies devoted to spin-polarized shot noise in magnetic tunnel junctions between ferromagnets, to our knowledge, there have not been reports on shot noise measurements in metallic spin-valve systems. We expect that such experiments will be performed in near future, which will reveal the effects predicted in the present paper.

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