Changes in linear and nonlinear elastic properties of aluminium alloy under static deformation

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Abstract. Mechanical tests and ultrasonic measurements of an aluminium alloy sample were carried out. As a result, the dependencies of velocities of shear and longitudinal waves as well as acoustoelastic coefficients for waves’ propagation times on the sample elongation were received. The linear and nonlinear elastic constants were calculated. It is shown that the effective elastic properties significantly change under plastic deformation. In addition, it presents the results of the determination of the acoustic nonlinearity parameter and acoustoelastic coefficients for different values of the sample deformation.

1. Introduction
The study of elastic properties of solids with mesoscale defects and inhomogeneities is of interest for the development of ultrasonic nondestructive evaluation techniques. The effective elastic properties of metal alloys and, accordingly, the velocities of elastic waves significantly depend on the applied stresses, the accumulation of microdamage, the evolution of the dislocation structure, the change in the texture and the allocation of new phases that is the basis of the various ultrasonic material evaluation techniques, for example [1-8].

A plastic deformation of polycrystalline metal leads to the accumulation of structural nonlinearity, which affects the effective elastic properties. The study of the influence of plastic deformation on linear and nonlinear elastic properties of metal alloys is relevant for the further development of ultrasonic evaluation techniques.

It was previously noted [9] that acoustoelastic coefficients determined by combinations of linear and nonlinear elastic constants are sensitive to microstructural changes.

The purpose of this paper is to evaluate changes in the linear and nonlinear elastic properties of aluminium alloy under static plastic deformation using the acoustoelastic effect.

2. Theoretical background
The linear elastic properties of isotropic solid are fully described by two independent second order elastic constants, for example, Lame constants \( \lambda \) and \( \mu \). The nonlinear elastic properties of an isotropic solid are fully described by three independent third order elastic constants, for example, Murnaghan constants \( l, m \) and \( n \).

The ultrasonic velocities and the elastic constants are related by the Christoffel equation. In the absence of stress, the relationships between the velocities of shear and longitudinal waves and Lame constants are given by.
\[ \rho V_{0i}^2 = \mu, \]  
(1) 
\[ \rho V_{0i}^2 = \lambda + 2\mu. \]  
(2)

In the presence of stress, for ultrasonic waves propagating perpendicular to the direction of applied stress, the velocities and the elastic constants are related by [10]

\[ \rho V_{s1}^2 = \mu + \frac{\sigma}{3\lambda + 2\mu} \left( m + \frac{\lambda n}{4\mu} + \lambda + 2\mu \right), \]  
(3) 
\[ \rho V_{s2}^2 = \mu + \frac{\sigma}{3\lambda + 2\mu} \left( m - \frac{\lambda + \mu}{2\mu} n - 2\lambda \right), \]  
(4) 
\[ \rho V_{l}^2 = \lambda + 2\mu + \frac{\sigma}{3\lambda + 2\mu} \left( 2l - \frac{2\lambda}{\mu} (m + \lambda + 2\mu) \right). \]  
(5)

Here \( \sigma \) is the applied uniaxial stress, \( V_s \) and \( V_l \) are the velocities of shear and longitudinal waves respectively, \( \rho \) is the density. The subscripts \( s1 \) and \( s2 \) refer to the shear wave polarization parallel and perpendicular to the direction of stress respectively. The subscript 0 refers to zero stress.

The equations (3), (4) and (5) can be linearized and, considering equations (1) and (2), brought to the form

\[ \frac{\Delta V_{s1}}{V_{0s1}} = \frac{\sigma}{2\mu(3\lambda + 2\mu)} \left( m + \frac{\lambda n}{4\mu} + \lambda + 2\mu \right), \]  
(6) 
\[ \frac{\Delta V_{s2}}{V_{0s2}} = \frac{\sigma}{2\mu(3\lambda + 2\mu)} \left( m - \frac{\lambda + \mu}{2\mu} n - 2\lambda \right), \]  
(7) 
\[ \frac{\Delta V_{l}}{V_{0l}} = \frac{\sigma}{2(\lambda + 2\mu)(3\lambda + 2\mu)} \left( 2l - \frac{2\lambda}{\mu} (m + \lambda + 2\mu) \right), \]  
(8)

where \( \Delta \) denotes a stress-induced change in the ultrasonic velocity.

In practice, the relative change in the propagation time of ultrasonic wave depending on the applied stress is easier to measure experimentally than the relative change in the velocity of ultrasonic wave.

The linear dependences of the relative changes in the propagation times of ultrasonic waves on the applied uniaxial stress can be presented by

\[ \frac{\Delta t_{s1}}{t_{0s1}} = k_{s1} \sigma, \]  
(9) 
\[ \frac{\Delta t_{s2}}{t_{0s2}} = k_{s2} \sigma, \]  
(10) 
\[ \frac{\Delta t_{l}}{t_{0l}} = k_{l} \sigma. \]  
(11)

Here \( k_{s1}, k_{s2} \) and \( k_{l} \) are the acoustoelastic coefficients for stress-induced relative changes in the propagation times of ultrasonic waves. In the first approximation, the relative change in the propagation time is related to relative change in velocity and relative change in the path length by

\[ \frac{\Delta t_{l}}{t_{0l}} = \frac{\Delta h}{h_0} - \frac{\Delta V_{l}}{V_{0l}}. \]  
(12)
Here \( t \) is the propagation time of ultrasonic wave, \( h \) is the path length. The subscript \( i \) refers to wave type.

The relative change in the path length can be calculated from

\[
\frac{\Delta h}{h_0} = -\frac{\lambda}{2\mu(3\lambda + 2\mu)} \sigma.
\]  

(13)

Taking into account equations since (6) to (13), the acoustoeelastic coefficients can be expressed by

\[
k_{s1} = -\frac{1}{2\mu(3\lambda + 2\mu)} \left( 2\lambda + 2\mu + \frac{\lambda n}{4\mu} + m \right),
\]  

(14)

\[
k_{s2} = -\frac{1}{2\mu(3\lambda + 2\mu)} \left( m - \lambda - \frac{\lambda + \mu}{2\mu} n \right),
\]  

(15)

\[
k_l = -\frac{\lambda}{2\mu(3\lambda + 2\mu)} - \frac{1}{2(\lambda + 2\mu)(3\lambda + 2\mu)} \left( 2l - \frac{2\lambda}{\mu}(\lambda + 2\mu) \right).
\]  

(16)

Finally, the nonlinear elastic constants \( l, m \) and \( n \) can be calculated from equations (14), (15) and (16) by

\[
n = 8\mu^2(k_{s2} - k_{s1}) - 4\mu,
\]  

(17)

\[
m = -\lambda - 2\mu - 4\mu(\lambda + \mu)k_{s1} - 2\lambda \mu k_{s2},
\]  

(18)

\[
l = -\lambda - \frac{\lambda^2}{2\mu} - 4\lambda(\lambda + \mu)k_{s1} - 2\lambda^2 k_{s2} - (\lambda + 2\mu)(3\lambda + 2\mu)k_l.
\]  

(19)

The linear elastic constants can be found from equations (1) and (2) by

\[
\mu = \rho V_{0s}^2,
\]  

(20)

\[
\lambda = \rho V_{0s}^2 - 2\rho V_{0s}^2.
\]  

(21)

The velocities of ultrasonic waves can be obtained by

\[
V_{0s} = \frac{2h_0}{t_{0s1} + t_{0s2}}.
\]  

(22)

\[
V_{0l} = \frac{h_0}{t_{0l}}.
\]  

(23)

The determination of the linear and nonlinear elastic constants allows characterizing the damage of structural elements. The measurement of the acoustic nonlinearity parameter \( \beta \) is highly sensitive ultrasonic technique, which is capable of monitoring microstructural changes in metals. The acoustic nonlinearity parameter \( \beta \) is a function of second and third order elastic constants. For the isotropic material, it may be shown from [8] that

\[
\beta = -3 - \frac{2l + 4m}{\lambda + 2\mu}.
\]  

(24)

Another vital application for the ultrasound, in which the determination of the linear and nonlinear elastic constants is of great importance, is residual stress evaluation based on the acoustoeelastic effect.
There are two the most promising ultrasonic techniques for residual stress evaluation. The first is based on the measurement of shear wave’s birefringence $B$ \[ B = B_0 + M\sigma, \] (25)

and the second is based on measurement of time of flight of a critically refracted longitudinal wave $t_{LCR}$ \[ \sigma = \frac{E}{L} \frac{\Delta t_{LCR}}{t_{0,LCR}}. \] (26)

Here $M$ is the acoustoelastic coefficient for birefringence of shear waves, $L$ is the acoustoelastic coefficient for longitudinal wave propagated along direction of applied stress, $E$ is the Young modulus.

The acoustoelastic coefficient $M$ can be written as \[ M = \frac{1}{2\mu} \left( 1 + \frac{n}{4\mu} \right). \] (27)

The acoustoelastic coefficient $L$ can be expressed from \[ L = 2 + \frac{(3\lambda + 2\mu)\mu + 4(\lambda + \mu)m + 2\mu l}{2(\lambda + \mu)(\lambda + 2\mu)}. \] (28)

3. Experimental procedures
For the present study, the deformable aluminium alloy 93% Al and 6% Mg widely used for arctic and aerospace applications was chosen. The initial dimensions of investigated working area of sample were 60×20×6 mm. The sample was plastically deformed until the destruction in stages to elongation for 5, 9, 15 and 19% to produce various degrees of damage. At each stage, the sample was subjected to uniaxial static elastic tension, and ultrasonic measurements were carried out at the same time. All experiments were conducted at room temperature.

For the mechanical tests, the testing machine Tinious Olsen H100KU was used. The loading rate was 2 mm/min. The stress was measured with an accuracy of no less than 1 MPa.

The ultrasonic measurements were performed in three points on the sample. Data in the form of dependencies $t_i(\sigma)$ were obtained by a specially designed experimental device shown in figure 1. The wide band piezoelectric transducers V156 and V110 Panametrics-NDT were used for generation and reception of shear and longitudinal ultrasonic waves propagated perpendicular to the surface of the material. The central frequency was 5 MHz, the diameter was 6 mm. The ultrasonic flaw detector A1212 MASTER ACS was used to generate electrical pulses. The digital oscilloscope LA-n10USB with ADCLab software was used to obtain the amplitude-time diagram of signals from the piezoelectric transducer on the PC. The sampling frequency of the digital oscilloscope was 100 MHz. The times of flight between echo pulses reflected from the reverse surface were measured.

At each stage, the acoustoelastic coefficients $k_i$ and propagation times at zero stress $t_{0,i}$ were found as a result of linear approximation of data.

To determine the path length $h_0$, the thickness of the sample free from stress was measured using a micrometer in three points, where the ultrasonic measurements were performed. Then the ultrasonic velocities $V_0$ and $V_0$ were obtained using equations (22) and (23). In initial state and after destruction of the sample, the density $\rho$ was measured by a hydrostatic weighing method. After that the elastic constants $\lambda$, $\mu$, $l$, $m$ and $n$ were calculated using equations since (17) to (21). Also the acoustic nonlinearity parameter $\beta$ was calculated using equation (24), and the acoustoelastic coefficients $M$ and $L$ were calculated using equations (27) and (28).
4. Results and discussion
The results of mechanical tests and ultrasonic measurements are presented in the table 1. The parameters calculated using corresponding equations are presented in the table 2. The dependencies of all parameters on the sample elongation $\varepsilon$ are shown in figures below. For all parameters their average values and errors are given.

**Table 1.** Values determined experimentally.

| $\varepsilon$, % | $\rho$, kg\times m$^{-3}$ | $V_0 s$, m/s | $V_0 l$, m/s | $k_{s1}$, TPa$^{-1}$ | $k_{s2}$, TPa$^{-1}$ | $k_l$, TPa$^{-1}$ |
|------------------|--------------------------|--------------|--------------|---------------------|---------------------|------------------|
| 0                | 2647±1                   | 3204±2       | 6559±12      | 25.7±1.1            | -13.5±0.8           | -12.4±1.9        |
| 5                | 2641*                    | 3202±3       | 6494±5       | 28.5±0.6            | -12.5±0.6           | -13.3±0.7        |
| 9                | 2636*                    | 3197±4       | 6550±22      | 28.8±0.8            | -11.6±0.5           | -15.2±3.3        |
| 15               | 2629*                    | 3194±4       | 6585±23      | 29.9±0.5            | -12.4±1.3           | -16.4±2.8        |
| 19               | 2624±1                   | 3188±4       | 6562±16      | 32.6±0.4            | -12.5±0.5           | -16.9±3.8        |

In asterisks indicate values of the density calculated in the linear approximation.

**Table 2.** Values calculated using equations.

| $\mu$, GPa | $\lambda$, GPa | $n$, GPa | $m$, GPa | $l$, GPa | $\beta$ | $M$, TPa$^{-1}$ | $L$ |
|------------|----------------|----------|----------|----------|---------|----------------|-----|
| 27.17±0.04 | 59.5±0.3       | -341±8   | -312±10  | -230±54  | 12±1    | -39.3±1.3      | -3.8±0.2 |
| 27.09±0.05 | 57.2±0.1       | -349±5   | -333±6   | -252±22  | 13±1    | -41.1±0.8      | -4.4±0.1 |
| 26.94±0.06 | 59.2±0.6       | -343±6   | -344±7   | -234±89  | 13±2    | -40.5±0.9      | -4.4±0.3 |
| 26.83±0.06 | 60.4±0.7       | -351±8   | -354±6   | -229±76  | 13±1    | -42.3±1.4      | -4.5±0.2 |
| 26.67±0.06 | 59.7±0.4       | -364±4   | -373±4   | -264±101 | 15±2    | -45.1±0.7      | -5.0±0.3 |

As it can be seen in figure 2, the velocity of shear wave monotonically decreases during plastic deformation unlike the velocity of longitudinal wave. The changes in Lame constants during plastic deformation shown in figure 3 are similar to the changes in the velocities of ultrasonic waves. The maximum relative changes in Lame constants $\lambda$ and $\mu$ were 4% and 2% respectively.
The dependencies of acoustoelastic coefficients for waves’ propagation times on the sample elongation shown in figure 4 are well described by a cubic polynomial. The same is true for Murnaghan constants \( l \), \( m \) and \( n \) shown in figure 5, the acoustic nonlinearity parameter \( \beta \) shown in figure 6 and the acoustoelastic coefficients \( M \) and \( L \) shown in figure 7.

As it can be seen in figure 5, \( m \) changes most strongly among Murnaghan constants. The maximum relative changes in Murnaghan constants \( m \) and \( n \) were 16% and 7% respectively. Taking into account the significant errors, \( l \) remains virtually constant. Such significant errors in the determination of \( l \) are associated with a strong variation of acoustoelastic coefficients for longitudinal wave’s propagation time across three zones.

If it is talked about the acoustic nonlinearity parameter \( \beta \) and the acoustoelastic coefficients \( M \) and \( L \), it can be noted that in fact the changes occur only at the initial and final stages of plastic deformation.

**Figure 2.** The dependencies of velocities of ultrasonic waves on the sample elongation.

**Figure 3.** The dependencies of Lame constants on the sample elongation.

**Figure 4.** The dependencies of acoustoelastic coefficients for waves’ propagation times on the sample elongation.

**Figure 5.** The dependencies of Murnaghan constants on the sample elongation.
It should be noted that this study does not take into account the effect of anisotropy of the material due to plastic deformation. In the initial state, the shear wave velocities differ by an average of 0.36% and by an average of 0.75% after 19% sample elongation. In this study, it is assumed that the observed changes in the elastic properties are caused primarily by accumulation of structural damage. The maximum decompaction of the material as a result of plastic deformation was 0.9%.

5. Conclusion
The results of mechanical tests and ultrasonic measurements of aluminium alloy sample show that the effective elastic properties significantly change under static plastic deformation. This may be due to the accumulation of structural damage in the material. Thus, the structural state of the material should be taken into account when using ultrasonic applications such as the evaluation of residual stresses. In addition, the results of this work indirectly confirm the high sensitivity and effectiveness of the nonlinear ultrasonic nondestructive evaluation techniques.

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