Landau-Lifshitz and Weinberg
Energy-Momentum Complexes for 2 + 1
Dimensional Black Hole Solutions

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Abstract
The aim of this paper is to evaluate the energy distribution of some 2 + 1 black hole solutions applying the Landau-Lifshitz and Weinberg definitions. The metrics under consideration describe the charged black hole, the solution coupling to a static scalar field and the static and circularly symmetric exact solution of the Einstein-massless scalar equation. Further, we compare the expressions for energy with those obtained using the Einstein and Møller prescriptions and give a discussion of the results.

Keywords: Landau-Lifshitz energy-momentum complex, Weinberg energy-momentum complex, 2 + 1 dimensional black hole solutions
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1 Introduction
The subject of energy-momentum localization lacks of a definite answer and continues to be one of the most interesting and challenging problem of gen-
eral relativity. Since Einstein [1] has given his important expression for the energy-momentum complex [1], the localization of energy is also connected to the use of other energy-momentum complexes, including the prescriptions of Landau-Lifshitz [2], Papapetrou [3], Bergmann-Thomson [4], Weinberg and Qadir-Sharif [5] and Møller [6]. These prescriptions have been criticized because of their drawback, they are coordinate dependent. Except the Møller energy-momentum complex which enables one to calculate the energy distribution in any coordinate system, the other prescriptions give meaningful results only if the calculations are restricted to quasi-Cartesian coordinates.

In the recent years the issue of the energy-momentum localization by using the energy-momentum complexes was re-opened. Many researchers studied different space-times and obtained interesting results, which demonstrate that these definitions are powerful concepts for energy-momentum localization. In 1990 Bondi [7] gave his opinion that “a nonlocalizable form of energy is not admissible in general relativity, because any form of energy contributes to gravitation and so its location can in principle be found”. Misner et al [8] sustained that to look for a local energy-momentum means that is looking for the right answer to the wrong question. They also concluded that the energy is localizable only for spherical systems. On the other hand, Cooperstock and Sarracino [9] demonstrated that if the energy is localizable in spherical systems then it is also localizable in any space-times. In this context, of great importance is the Cooperstock hypothesis [10] which states that energy and momentum are confined to the regions of non-vanishing energy-momentum tensor for the matter and all non-gravitational fields.

The problem of energy-momentum localization by applying the energy-momentum complexes was revived at the beginning of the last decade and many interesting results have been obtained [11]. Virbhadra [11] showed that the Einstein, Landau-Lifshitz, Papapetrou and Weinberg energy-momentum complexes give the same energy density as the Penrose definition for a general non-static spherically symmetric metric of the Kerr-Schild class. Furthermore, important works were done with the energy-momentum complexes in 2- and 3-dimensional space-times [12]. Considerable investigations have been performed in elaborating interesting papers, which demonstrate that energy-momentum complexes yield the same results as their tele-parallel versions for a given space-time [13]. All these considerations point out the significance of these prescriptions and stress the usefulness of energy-momentum complexes for energy-momentum localization.

In our paper we evaluate the energy distribution of three 2+1 dimensional
black hole solutions applying the Landau-Lifshitz and Weinberg definitions. The paper is organized as follows. In Sec. 2 we briefly present the 2 + 1 dimensional black hole solutions that we study. In Sec. 3 we give a description of the Landau-Lifshitz and Weinberg energy-momentum complexes and we compute the energy distributions for the three 2 + 1 dimensional black hole solutions, and also make a comparison with the values of energy obtained in the Einstein and Møller prescriptions. Finally, in Sec. 4 we make a summary of the obtained results and some concluding remarks. The metrics under consideration describe the charged black hole [14], the solution coupling to a static scalar field [14] and the static and circularly symmetric exact solution of the Einstein-massless scalar equation [15]. Through the paper we follow the convention that Latin indices run from 0 to 2 and the Greek indices run from 1 to 2.

2 Black Hole Solutions in 2 + 1 Dimensions

In recent years the Einstein theory of gravity in 2 + 1 dimensions has gained considerable attention [16]. Due to an expectation is that the study of 2+1 dimensional theories would provide relevant information about the corresponding theory in 3+1 dimensions. In 2+1 dimensions the number of independent components of the Riemann curvature tensor and the Einstein tensor are the same, consequently the imposition of Einstein’s equations in vacuum implies that the curvature tensor also vanishes and there are no gravitational waves and no interactions between masses. Therefore, the space-time described by the vacuum solutions to Einstein’s equations in 2 + 1 dimensions allows that the existence of the black holes would be prevented [17]. However, Baiadoss, Teitelboim and Zanelli (BTZ) [18] have discovered a black hole solution to the EM equations (with a negative cosmological constant) in 2 + 1 dimensions, which is characterized by mass, angular momentum and charge parameters.

Because the energy-momentum complexes are fundamental quantities used for energy-momentum localization, we study the energy distributions of some black hole solutions in 2 + 1 dimensions applying the Landau-Lifshitz and Weinberg prescriptions.

The metrics under consideration in our paper are:

i) Charged black hole [14], which is expressed by the line element
\[ ds^2 = -(\Lambda r^2 - M - 2Q^2 \ln(\frac{r}{r_+}))dt^2 + (\Lambda r^2 - M - 2Q^2 \ln(\frac{r}{r_+}))^{-1} dr^2 + r^2 d\phi^2, \]

where \( r_+ = \sqrt{\frac{M}{\Lambda}} \).

ii) Solution coupling to a static scalar field [14] which is described by the metric

\[ ds^2 = -\left(\frac{r - 2B}{r} (B + r)^2 \Lambda \right) dt^2 + \left(\frac{r - 2B}{r} (B + r)^2 \Lambda \right)^{-1} dr^2 + r^2 d\phi^2, \]

where \( B \) is a constant and the scalar field is given by \( \Phi = \sqrt{\frac{B}{\pi(r+B)}} \).

iii) The static and circularly symmetric exact solution of the Einstein-massless scalar equation [15] which is given by

\[ ds^2 = -B dt^2 + B^{-1} dr^2 + r^2 d\phi^2, \]

where \( B = (1 - q)R^q \), \( R = \frac{r}{r_0} \), \( q \) is the scalar charge and the scalar field is given by \( \Phi = \sqrt{\frac{q}{r}} \ln R \). The case \( q = 0 \) corresponds to the flat space-time in \( 2+1 \) dimensions.

For these three \( 2+1 \) dimensional black hole solutions we compute the energy distributions using the Landau-Lifshitz and Weinberg prescriptions.

### 3 Landau-Lifshitz and Weinberg Prescriptions for 2 + 1 Dimensional Black Hole Solutions

The Landau-Lifshitz energy-momentum complex [2] is given by

\[ L^k_i = \frac{1}{2k} S^{ijklm} s_{lm}, \]

where \( k \) is the coupling gravitational constant and

\[ S^{ijklm} = -g^{ik} g^{lm} - g^{il} g^{km}. \]

\( L^{00} \) and \( L^{a0} \) are the energy and momentum density components, respectively.
The Landau-Lifshitz energy-momentum complex satisfies the local conservation law

$$\frac{\partial L^{ik}}{\partial x^k} = 0.$$  \hspace{1cm} (6)

Using Gauss’s theorem, the energy and momentum components are

$$P^i = \frac{1}{2} \int \int S^{i0\alpha m} n_\alpha dS,$$  \hspace{1cm} (7)

where \( n_\alpha = \left( \frac{\hat{z}}{r}, \frac{\hat{y}}{r}, \frac{\hat{z}}{r} \right) \) are the components of a normal vector over an infinitesimal surface element.

The energy and momentum for a three dimensional background are given by

$$P^i = \int \int L^i dx^1 dx^2.$$  \hspace{1cm} (8)

The Weinberg energy-momentum complex \[5\] is given by

$$W^{ik} = \frac{1}{2} D^{ik \lambda \lambda},$$  \hspace{1cm} (9)

where

$$D^{ik \lambda \lambda} = \frac{\partial h^i_{\alpha}}{\partial x_l} \eta^{\lambda k} - \frac{\partial h^i_{\alpha}}{\partial x_l} \eta^{\lambda k} - \frac{\partial h^i_{\alpha}}{\partial x_l} \eta^{\lambda k} + \frac{\partial h^i_{\alpha}}{\partial x_l} \eta^{\lambda k} + \frac{\partial h^i_{\alpha}}{\partial x_l} \eta^{\lambda k},$$  \hspace{1cm} (10)

with

$$h_{ik} = g_{ik} - \eta_{ik},$$  \hspace{1cm} (11)

where \( \eta^{ik} = \text{diag}(-1, 1, 1) \) and \( W^{00} \) and \( W^{\alpha 0} \) are the energy and the momentum density components, respectively.

The Weinberg energy-momentum complex satisfies the local conservation law

$$\frac{\partial W^{ik}}{\partial x^k} = 0.$$  \hspace{1cm} (12)

Applying Gauss’s theorem, the energy and momentum components are
\[ P^i = \frac{1}{2k} \int \int D^{i0\alpha} n_\alpha dS, \]  
(13)

where \( n_\alpha = \left( \frac{x}{r}, \frac{y}{r}, \frac{z}{r} \right) \) are the components of a normal vector over an infinitesimal surface element.

The energy and momentum in the Weinberg prescription for a three-dimensional background are given by

\[ P^i = \int \int W^{i0} dx^1 dx^2. \]  
(14)

For carrying out the calculations with the Landau-Lifshitz and Weinberg energy-momentum complexes we transform the general metric given by

\[ ds^2 = -v(r) dt^2 + w(r) dr^2 + r^2 d\varphi^2, \]  
(15)

to quasi-Cartesian coordinates \( t, x, y \) by using \( x = r \cos \varphi, \ y = r \sin \varphi \) and we obtain

\[ ds^2 = -v dt^2 + (dx^2 + dy^2) + \frac{w - 1}{x^2 + y^2}(xdx + ydy)^2. \]  
(16)

The determinant of the metric (16) is \( g = -vw \) and the covariant components of the metric tensor are given by

\[ g_{ik} = \begin{vmatrix} \frac{x^2 w + y^2}{x^2 + y^2} & \frac{xy(w-1)}{x^2 + y^2} & 0 \\ \frac{xy(w-1)}{x^2 + y^2} & \frac{y^2 w + x^2}{x^2 + y^2} & 0 \\ 0 & 0 & -v \end{vmatrix}. \]  
(17)

For the contravariant components of the metric tensor we obtain

\[ g^{ik} = \begin{vmatrix} \frac{y^2 w + x^2}{(x^2 + y^2)w} & -\frac{xy(w-1)}{(x^2 + y^2)w} & 0 \\ -\frac{xy(w-1)}{(x^2 + y^2)w} & \frac{y^2 w + x^2}{(x^2 + y^2)w} & 0 \\ 0 & 0 & -\frac{1}{v} \end{vmatrix}. \]  
(18)

The required nonvanishing components \( S^{i0\alpha} \) of the Landau-Lifshitz energy-momentum complex are

\[ S^{001} = \frac{x(1 - w)}{x^2 + y^2}, \ S^{002} = \frac{y(1 - w)}{x^2 + y^2}. \]  
(19)
For the Weinberg prescription, the required nonvanishing components $D^{0 \alpha}$ are given by

$$D^{01} = \frac{x(1-w)}{x^2 + y^2}, \quad D^{02} = \frac{y(1-w)}{x^2 + y^2}.$$  \hfill (20)

Using (8), (14), (18), (19), (20), applying Gauss’s theorem and after performing the calculations we obtain that the energy within a circle with radius $r$ in the Landau-Lifshitz and Weinberg prescriptions is given by

$$E_{LL} = E_W = \frac{1}{2k} \int (1-w) d\varphi.$$  \hfill (21)

We obtain that the expression of energy in the Landau-Lifshitz prescription is the same as in the Weinberg prescription.

For the aforementioned black hole solutions in 2+1 dimensions we obtain the next results in the Landau-Lifshitz and Weinberg prescriptions:

i) For the charged black hole $v = \Lambda r^2 - M - 2Q^2 \ln\left(\frac{r}{r_c}\right)$, $w = \left(\Lambda r^2 - M - 2Q^2 \ln\left(\frac{r}{r_c}\right)\right)^{-1}$ and the energy distribution computed with the Landau-Lifshitz and Weinberg prescriptions is given by

$$E_{LL} = E_W = \frac{\pi}{k} \left[ \frac{\Lambda r^2 - M - 2Q^2 \ln\left(\frac{r}{r_c}\right) - 1}{\Lambda r^2 - M - 2Q^2 \ln\left(\frac{r}{r_c}\right)} \right].$$  \hfill (22)

ii) In the case of the solution coupling to a static scalar field $v = \frac{(r-2B)(B+r)^2 \Lambda}{r}$, $w = \left(\frac{(r-2B)(B+r)^2 \Lambda}{r}\right)^{-1}$ and for the energy we obtain

$$E_{LL} = E_W = \frac{\pi}{k} \left[ \frac{\Lambda(r-2B)(B+r)^2 - r}{\Lambda(r-2B)(B+r)^2} \right].$$  \hfill (23)

iii) For the static and circularly symmetric exact solution of the Einstein-massless scalar equation $v = (1-q)R^q$, $w = ((1-q)R^q)^{-1}$ and the Landau-Lifshitz and Weinberg energy-momentum complexes yield for the energy distribution the expression

$$E_{LL} = E_W = \frac{\pi}{k} \left[ \frac{(1-q)R^q - 1}{(1-q)R^q} \right].$$  \hfill (24)

Some remarks are needed. In a previous work [19] we computed the energy distributions of these three 2+1 dimensional black hole solutions applying the Einstein and Møller prescriptions and we obtained
\[ E_E = \frac{1}{2k} \oint \frac{v}{\sqrt{vw}} (1 - w) d\varphi \]  
\[ \text{and} \]
\[ E_M = -\frac{1}{k} \oint \frac{r}{\sqrt{vw}} \frac{\partial v}{\partial r} d\varphi, \]

respectively.

We make a comparison with the values of energy obtained using the Landau-Lifshitz and Weinberg energy-momentum complexes. In the case of the all three 2 + 1 dimensional black hole solutions we conclude that between the Einstein, Landau-Lifshitz and Weinberg prescriptions there is a relationship given by

\[ E_{LL} = E_W = w E_E. \]  

Møller’s energy-momentum complex yields different results for the energy distribution of the aforementioned 2+1 dimensional black hole solutions than the Einstein, Landau-Lifshitz and Weinberg prescriptions.

It is important that the expression for the energy obtained in the Landau-Lifshitz prescription exactly matches with that computed applying the Weinberg prescription. Even these definitions of Einstein, Landau-Lifshitz, Weinberg and Møller do not provide the same result for the energy distribution, we point out that the connections between the expressions for energy obtained in the Einstein, Landau-Lifshitz and Weinberg prescriptions are similar to the 3 + 1 dimensional case [20] (see therein eqs. 39-42, for \( A = B^{-1}, D = 1 \) and \( F = 0 \)), when the calculations are done in Schwarzschild-Cartesian coordinates.

4 Discussion

For solving the problem of energy and momentum localization, many attempts have been made in the past but this remains an important issue to be settled. The difficulty relies in the lack of a generally accepted expression for the energy density. Even the energy-momentum complexes ”seem” to be useful for the localization of energy, there are doubts that these prescriptions could give acceptable results for a given space-time. The results obtained
by several authors [11]-[13] demonstrated that the energy-momentum complexes are good tools for evaluating the energy and momentum in general relativity, and working with them we can obtain acceptable expressions for the energy associated with a given space-time. Chang, Nester and Chen [21] showed that the energy-momentum complexes are actually quasi-local and legitimate expression for the energy-momentum. They concluded that there exist a direct relationship between energy-momentum complexes and quasi-local expressions because every energy-momentum complexes is associated with a legitimate Hamiltonian boundary term. Their idea supports the energy-momentum complexes and the role which these are playing in energy-momentum localization. Furthermore, important studies have been done about the new idea of quasi-local approach for energy-momentum complexes [21]-[22] and a large class of new pseudotensors connected to the positivity in small regions have been studied and constructed [23]. In this light, the quasi-local quantities are associated with a closed 2-surface (L. B. Szabados, [22] and [http://relativity.livingreviews.org/Articles/lrr-2004-4/]). The Hamiltonian boundary term determines the quasi-local quantities for finite regions and the special quasi-local energy-momentum boundary term expressions correspond each of them to a physically distinct and geometrically clear boundary condition [24].

In this paper we continue the investigations concerning the energy of some 2 + 1 dimensional black hole solutions. We evaluate the energy distribution for three 2 + 1 dimensional black hole solutions using the Landau-Lifshitz and Weinberg prescriptions. Furthermore, we compare our result with those obtained in the Einstein and Møller prescriptions and investigate the connections between the expressions for the energy obtained with these energy-momentum complexes. The Landau-Lifshitz and Weinberg prescriptions yield the same expressions for the energy distribution of the aforementioned 2 + 1 dimensional black hole solutions, sustaining the viewpoint that different energy-momentum complexes can give the same result for a given space-time. The connection between the expressions for the energy distribution obtained in these three prescriptions, Einstein, Landau-Lifshitz and Weinberg is given by the relationship $E_{LL} = E_W = wE_F$. It is important to notice that the connections between the expressions for energy obtained in the Einstein, Landau-Lifshitz and Weinberg prescriptions are similar to the 3 + 1 dimensional case [20] (see therein eqs. 39-42, for $A = B^{-1}$, $D = 1$ and $F = 0$), when the calculations are done in Schwarzschild-Cartesian coordinates.
As $r$ becomes larger, the energy distributions of the three $2 + 1$ aforementioned dimensional black hole solutions become finite. Furthermore, in the case $r \to \infty$ the energy distributions of these solutions do not diverge. These energy distributions do not diverge because Einstein’s and Möller’s energy complexes are covariant, but Landau-Lifshitz and Weinberg’s energy complexes are contravariant. These three black hole solutions are not asymptotically flat, so the covariant energy complex will be divergent. However, the contravariant energy complex will be not divergent.

Our paper extends a previous study [19] about the energy of $2 + 1$ dimensional black hole solutions and sustains the viewpoint that the energy-momentum complexes are important concepts for energy-momentum localization. Furthermore, our work also supports a) the opinion that different energy-momentum complexes can yield the same expression for the energy in a given space-time and b) the connection between the values of energy obtained applying the Einstein, Landau-Lifshitz and Weinberg prescriptions are similar to the $3 + 1$ dimensions when the calculations are done in Schwarzschild-Cartesian coordinates. An open question remains, why these prescriptions (ELLPW) and Möller do not allow obtaining the same expression for the energy distribution. We conclude that we obtained different results applying the definitions of Einstein, Landau-Lifshitz and Weinberg because these energy-momentum complexes are pseudotensors and are non-covariant, coordinate dependent expressions [21]-[24] and this agrees with the equivalence principle which states that gravity cannot be detected at a point.

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