Theory of friction: contribution from fluctuating electromagnetic field

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Abstract

We calculate the friction force between two semi-infinite solids in relative parallel motion (velocity $V$), and separated by a vacuum gap of width $d$. The friction force result from coupling via a fluctuating electromagnetic field, and can be considered as the dissipative part of the van der Waals interaction. We consider the dependence of the friction force on the temperature $T$, and present a detailed discussion of the limiting cases of small and large $V$ and $d$.

1 Introduction

Because of its great practical importance and because of the development of new experimental techniques, sliding friction has become a topic of increasing attention [1]. In this paper we consider the friction force between two solids in relative motion, separated by a vacuum gap of width $d$. This “vacuum” friction is in most cases of no direct practical importance since the main contribution to the friction force when a body is slid on another body comes from the area of real (atomic) contact [1]. Thus, the frictional stress between two semi-infinite metallic (e.g., copper) bodies, moving parallel to each other with the relative velocity $V = 1$ m/s, and separated by the distance $d = 10$ Å,
is only (see Ref. 2 and below) $\sim 10^{-6} \text{N/m}^2$. This stress is extremely small compared with the typical frictional stress $\sim 10^8 \text{N/m}^2$ occurring in the area of atomic contact even for (boundary) lubricated surfaces. Nevertheless, vacuum friction is important in some special cases (see Ref. 2), and determines the ultimate limit to which friction can be reduced. Quantum and thermal fluctuations of the polarization and the magnetization of solids give rise a fluctuating electromagnetic field. For two stationary solids the interaction mediated by this field result in the well-known attractive van der Waals force. For two solids in relative motion this interaction will also give rise to a friction force between the bodies. The static aspect of the van der Waals interaction is well understood but there are still controversial results concerning the dynamical part. Different author’s have recently studied the van der Waals friction using different approaches, and obtained results which are in sharp contradiction to each other. The first calculation of van der Waals friction was done by Teodorovitch[3]. Schaich and Harris[4], and Pendry[5] argue that Teodorovitch calculation is in error. For two metallic bodies Schaich and Harris found that the friction force is independent of any metal property, in contrast to the results of other authors. The friction forces calculated by Levitov[6], Polevoi[7] and Mkrtchiau[8] vanish in the non-retarded limit (formally obtained when the light velocity $c \to \infty$). This result is very surprising (and in our opinion incorrect), since neglecting retardation is a good approximation at short separations $d$ between the surfaces, in which case one would expect the friction force to be particularly large. Even at large separation, where the non-retarded contribution is negligible, our result differs from those of Levitov, Polevoi and Mkrtchiau. Pendry[5] considered only the case of zero temperature, and Persson and Zhang[2] the case of small sliding velocities, and both groups neglected retardation effects. To clarify the situation we present a straightforward calculation of the van der Waals friction based on the general theory of the fluctuating field developed by Rytov[9] and applied by Lifshitz[10] for studying the conservative part of the van der Waals interaction. In this approach the interaction between the two bodies is mediated by the fluctuating electromagnetic field which is always present in the vicinity of any collection of atoms. Beyond the boundaries of a solid this field consist partly of traveling waves and partly of evanescent waves which are damped exponentially with the distance away from the surface of the body. The method we use for calculating the interaction forces is quite general, and is applicable to any body at arbitrary temperature. It also takes into account retardation effects, which become important for
large enough separation between the bodies. A similar approach was used by Polevoi[7] but he obtained a nonzero friction force only in the relativistic limit, in contrast to the present calculations and the earlier calculations of Persson and Zhang[2], and Pendry[5]. Polevoi did not give enough details of his calculation to compare his theory with the present calculation, but we believe that he overlooked effects related to the change in the reflectivity of electromagnetic waves from moving bodies, which occur even in the nonrelativistic limit. In the nonretarded limit and for zero temperature the present calculation agree with the results of Pendry. Similarly, in the nonretarded limit and for low sliding velocities, we agree with the study of Persson and Zhang.

2 Calculation of the fluctuating electromagnetic field

We consider two semi-infinite solids having flat parallel surfaces separated by a distance $d$ and moving with velocity $V$ relative to each other, see Fig. 1. We introduce the two coordinate systems $K$ and $K'$ with coordinate axes $xyz$ and $x'y'z'$. In the $K$ system body 1 is at rest while body 2 is moving with velocity $V$ along the $x$-axis ($xy$- and $x'y'$-planes are in the surface of body 1, $x$- and $x'$-axes have the same direction and the $z$- and $z'$-axes pointing toward body 2). In the $K'$ system body 2 is at rest while body 1 is moving with velocity $-V$ along the $x$-axis. Following to Lifshitz to calculate the fluctuating field in the interior of the two bodies, we shall use the general theory which is due to Rytov and is described in detail in his book[9]. This method is based on the introduction into the Maxwell equations of a "random" field (just as, for example, one introduce a "random" force in the theory of Brownian motion). In the $K$ system in a dielectric, nonmagnetic body 1 for a monochromatic field (time factor $e^{-i\omega t}$) these equations are

$$\nabla \times \mathbf{E}_1 = i\frac{\omega}{c} \mathbf{B}_1,$$

$$\nabla \times \mathbf{B}_1 = -i\frac{\omega}{c} \varepsilon_1(\omega) \mathbf{E}_1 - i\frac{\omega}{c} \mathbf{F}_1,$$

(1)

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, $\varepsilon_1(\omega)$ is the complex dielectric constant for body 1, and $\mathbf{F}$ is the random field. Accordingly to Rytov the correlation function of the latter, determining the average value
of the product of components of $\mathbf{F}$ at two different points in space, is given by the formula

$$\langle F_i(x, y, z)F_k(x', y', z') \rangle = 4\hbar \left( \frac{1}{2} + n(\omega) \right) \varepsilon''(\omega) \delta_{ik} \delta(x - x') \delta(y - y') \delta(z - z'), \quad (2)$$

$$n(\omega) = \frac{1}{e^{E/\beta} - 1},$$

where $T$ is the temperature and $\varepsilon''$ is the imaginary part of $\varepsilon = \varepsilon' + \varepsilon''$. The function $F(x, y, z)$ can be represented in the form of a Fourier integral, which can be written for the half space $z < 0$ in the form

$$F(x, y, z) = \int_{-\infty}^{+\infty} g(k) e^{ikr} d^3k, \quad (3)$$

where a two dimensional vector $\mathbf{q}$ is parallel to the surface, $k^2 = k_z^2 + q^2$, and $r$ is the radius vector in the $x - y$ plane. For the Fourier components $g(k)$, the correlation function corresponding to the spatial correlation (3) is

$$\langle g_i(k, \omega)g^*_k(k', \omega) \rangle = \frac{\hbar \left( \frac{1}{2} + n(\omega) \right) \varepsilon''(\omega)}{\pi^3} \delta_{ik} \delta(k - k'). \quad (4)$$

In body 1 ($z < 0$) the fields $\mathbf{E}$ and $\mathbf{B}$ can be written in the form [10]

$$\mathbf{E}_1 = \int_{-\infty}^{+\infty} \left\{ a_1(k) \cos k_z z + i b_1(k) \sin k_z z \right\} e^{ikr} d^3k + \int_{-\infty}^{+\infty} u_1(q) e^{iqr - i\omega_1 z} d^2q, \quad (5)$$

$$\mathbf{B}_1 = \frac{c}{\omega} \int_{-\infty}^{+\infty} \left\{ (\mathbf{q} \times \mathbf{a}_1) + k_z [\mathbf{e}_z \times \mathbf{b}_1] \right\} \cos k_z z d^3k + i \left\{ (\mathbf{q} \times \mathbf{b}_1) + k_z [\mathbf{e}_z \times \mathbf{a}_1] \right\} \sin k_z z d^3k + \frac{c}{\omega} \int_{-\infty}^{+\infty} \left\{ [\mathbf{q} \times \mathbf{u}_1] - s_1 [\mathbf{e}_z \times \mathbf{u}_1] \right\} e^{iqr - i\omega_1 z} d^2q, \quad (6)$$

where $\mathbf{e}_z$ is a unit vector in the direction of the $z$ axis, and

$$s_1 = \sqrt{\frac{\omega^2}{c^2} - q^2}, \quad (7)$$
where the sign of the root is to be chosen so that the imaginary part of \( s \) will be positive.

The first terms in the expressions (5) and (6) represent a solution of the inhomogeneous equations (1). Substituting them in the second equation of (1) and writing \( F \) in the form (3), one can find the following relations, expressing \( a_1 \) and \( b_1 \) in terms of the Fourier components \( g_1 \) of the random field

\[
a_1 = \frac{1}{\varepsilon_1(k^2 - \omega^2 \varepsilon_1/c^2)} \left[ \frac{\omega^2}{c^2} \varepsilon_1 g_1 - q(q \cdot g_1) - k_z^2 g_{1z} e_z \right],
\]

\[
b_1 = -\frac{k_z}{\varepsilon_1(k^2 - \omega^2 \varepsilon_1/c^2)} \left[ e_z(q \cdot g_1) + q g_{1z} \right].
\]

The second integrals in (5,6) represent the solution of the homogeneous equations (1) (i.e. the equations with \( F \) omitted), and describe the plane wave field reflected from the boundary of body. The condition for transversality of these waves is

\[
u_1 \cdot q - s_1 u_{1z} = 0.
\]

In the space between bodies (vacuum) \( \varepsilon = 0, F = 0 \) and the field in the \( K \) system is given by the general solution of the homogeneous equations, which can be written in the form

\[
E_3 = \int_{-\infty}^{+\infty} \left\{ v(q,\omega)e^{ipz} + w(q,\omega)e^{-ipz} \right\} e^{iq \cdot r} d^2 q,
\]

\[
B_3 = \frac{c}{\omega} \int_{-\infty}^{+\infty} \left\{ ((q \times v) + p[e_z \times v]) e^{ipz} + ([q \times w] - p[e_z \times w]) e^{-ipz} \right\} e^{i q \cdot r} d^2 q,
\]

where

\[
p = \sqrt{\frac{\omega^2}{c^2} - q^2},
\]

and \( v \) and \( w \) satisfy the transversality conditions

\[
v \cdot q + pv_z = 0, \ w \cdot q - pw_z = 0.
\]
it is convenient to write the corresponding equations for components of the fields along the vectors 

\[ \mathbf{e}_q = q / q \] and 

\[ \mathbf{e}_n = [\mathbf{e}_z \times \mathbf{e}_q] \], this gives the following equations

\[
\int_{-\infty}^{+\infty} a_{1q}dk_z + u_{1q} = v_q + w_q, \\
\int_{-\infty}^{+\infty} a_{1n}dk_z + u_{1n} = v_n + w_n, \\
\int_{-\infty}^{+\infty} (qa_{1z} - k_z b_{1q})dk_z + qu_{1z} + s_1 u_{1q} \\
= q(v_z + w_z) - p(v_q - w_q), \\
\int_{-\infty}^{+\infty} -k_z b_{1n}dk_z + s_1 u_{1n} = -p(v_n - w_n),
\]

(15)

where \( a_{1q} = \mathbf{e}_q \cdot \mathbf{a}_1 \), \( a_{1n} = \mathbf{e}_n \cdot \mathbf{a}_1 \) and so on. In what follow we shall need only the field between two media. Using the transversality conditions (10) and (14) and the expressions (8,9) from the equations (15) we can obtain the following equations

\[
ps_1 \int_{-\infty}^{\infty} (g_{1z}(q,k_z,\omega) - s_1 g_{1q}(q,k_z,\omega)) \frac{1}{k_z^2 - s_1^2} dk_z \\
= -(s_1 + p \varepsilon_1) v_q(q,\omega) - (p \varepsilon_1 - s_1) w_q(q,\omega),
\]

(16)

\[
 s_1 (\frac{\omega}{c})^2 \int_{-\infty}^{\infty} \frac{g_{1n}(q,k_z,\omega)}{k_z^2 - s_1^2} dk_z \\
= (p + s_1) v_n(q,\omega) + (s_1 - p) w_n(q,\omega).
\]

(17)

In the \( K' \) system the Maxwell equations have the same form (1) and in the second medium ( the half space \( z > d \)), the field \( \mathbf{E}_2', \mathbf{B}_2' \) is given by the same formulas (5-9) with \( x \)- coordinate changed to \( x' \), the index 1 changed to 2, \( \cos k_z z, \sin k_z z \) replaced by \( \cos k_z(z - d), \sin k_z(z - d) \) and change in the sign of \( s \) ( the "reflected" waves now propagate along the positive \( z \) direction). In the space between media in the \( K' \) system the field is given by the same formulas (11-13) with \( x \) changed to \( x' \), \( \mathbf{v}, \mathbf{w} \) replaced by \( \mathbf{v}', \mathbf{w}' \). The relations between the field in the \( K \) and \( K' \) systems are determined by
Lorentz transformation. Neglecting by the terms of the order \((V/c)^2\) these relations are given by

\[
v'(q', \omega') = v(q, \omega) + \frac{V}{\omega} [e_x [k \times v(q, \omega)]],
\]

\[
w'(q', \omega') = w(q, \omega) + \frac{V}{\omega} [e_x [\bar{k} \times w(q, \omega)]],
\]

where \(k = (q, p)\), \(\bar{k} = (q, -p)\), \(\omega' = \omega - q_x V\), \(q' = q - (V\omega/c^2)e_x\).

In the \(K'\) system the boundary conditions at the surface of body 2 at \(z = d\) give the equations

\[
p s_2^- \int_{-\infty}^{\infty} \left( q_y g_{2z}(q', k_z, \omega') + s_2^- g_{2q}(q', k_z, \omega') \right) \frac{1}{k_z^2 - s_2^-} dk_z
\]

\[= (p \varepsilon_2^- - s_2^-) v'_q(q', \omega') e^{ipd} + (s_2^- + p \varepsilon_2^-) w'_q(q', \omega') e^{-ipd},
\]

\[
 s_2^- \left( \frac{\omega'}{c} \right)^2 \int_{-\infty}^{\infty} \frac{g_{2n'}(q', k_z, \omega')}{k_z^2 - s_2^-} dk_z = (s_2^- - p) v'_{n'}(q', \omega') e^{ipd}
\]

\[+ (s_2^- + p) w'_{n'}(q', \omega') e^{-ipd},
\]

where \(\varepsilon_2^- = \varepsilon_2(\omega - q_x V)\),

\[
s_2^- = \sqrt{(\omega'/c^2\varepsilon_2(\omega') - q'^2) = \sqrt{\frac{(\omega - q_x V)^2}{c^2}(\varepsilon_2(\omega - q_x V) - 1) + p^2},
\]

\(p\) is invariant under Lorentz transformation. Now from the equations (18-19) with accuracy to the terms of the first order in \(V/c\) we have

\[
v'_q(q', \omega') = (v' \cdot e_{q'}) \approx v_q(q, \omega) + \frac{q_y p^2 V}{\omega q^2} v_n(q, \omega),
\]

\[
v'_{n'} = (v' \cdot e_{n'}) \approx \frac{\omega'}{\omega} v_n - \frac{\omega q_y V}{c^2 q^2} v_q.
\]

The similar equations can be written for \(w'_{q'}, w'_{n'}\). After substituting (23) and (24) into the equations (16-17) and (20-21) we get the system of four equations. These equations can be solved considering the second terms in
the equations (23,24) as a small perturbation. In zero order we neglect by
the second terms. The zero order solution has the form:

\[
v^0_q = \int_{-\infty}^{\infty} \frac{p\Delta}{k_z^2 - s_1^2} \left\{ s_1 e^{-ip\Delta} (s_2^- + s_2^- \rho) g_{12}(q,k_z,\omega) - s_1 g_{1q}(q,k_z,\omega) + s_2^- (\varepsilon_1 p - s_1) \frac{q' g_{2z}(q',k_z,\omega') + s_2^- g_{2q}(q',k_z,\omega')}{k_z^2 - s_2^2} \right\} dk_z, \tag{25}
\]

\[
w^0_q = \int_{-\infty}^{\infty} \frac{p\Delta}{k_z^2 - s_1^2} \left\{ -s_1 e^{ip\Delta} (s_2^- - s_2^- \rho) g_{12}(q,k_z,\omega) - s_1 g_{1q}(q,k_z,\omega) - s_2^- (\varepsilon_1 p + s_1) \frac{q' g_{2z}(q',k_z,\omega') + s_2^- g_{2q}(q',k_z,\omega')}{k_z^2 - s_2^2} \right\} dk_z, \tag{26}
\]

\[
v^0_n = \int_{-\infty}^{\infty} \frac{\omega}{c^2 \Delta'} \left\{ -\omega s_1 e^{-ip\Delta} (s_2^- + p) g_{1n}(q,k_z,\omega) + \omega s_2^- (s_1 - p) \frac{g_{2n}(q',k_z,\omega')}{k_z^2 - s_2^2} \right\} dk_z, \tag{27}
\]

\[
w^0_n = \int_{-\infty}^{\infty} \frac{\omega}{c^2 \Delta'} \left\{ \omega s_1 e^{ip\Delta} (s_2^- - p) g_{1n}(q,k_z,\omega) - \omega s_2^- (s_1 + p) \frac{g_{2n}(q',k_z,\omega')}{k_z^2 - s_2^2} \right\} dk_z, \tag{28}
\]

\[
v_z = -\frac{qv_q}{p}, \quad w_z = \frac{qw_q}{p}, \tag{29}
\]

where we have introduced the notation:

\[
\Delta = e^{ip\Delta}(\varepsilon_1 p - s_1)(\varepsilon_2 p - s_2) - e^{-ip\Delta}(\varepsilon_1 p + s_1)(\varepsilon_2 p + s_2),
\]

\[
\Delta' = e^{ip\Delta}(s_1 - p)(s_2^- - p) - e^{-ip\Delta}(p + s_1)(p + s_2^-).
\]

The first order solution has the form:

\[
v^1_q = \frac{(p \varepsilon_1 - s_1) \Lambda}{\Delta}, \quad w^1_q = -\frac{(p \varepsilon_1 + s_1) \Lambda}{\Delta}, \tag{30}
\]

\[
v^1_n = \frac{(s_1 - p) \Lambda'}{\Delta'}, \quad w^1_n = -\frac{(p + s_1) \Lambda'}{\Delta'}, \tag{31}
\]
where
\[
\Lambda = -\frac{q_y p^2 V}{\omega q^2} \left[ (p\varepsilon_2 - s_2) v_n^0 e^{ipd} + (p\varepsilon_2 + s_2) w_n^0 e^{-ipd} \right],
\]
\[
\Lambda' = \frac{\omega' q_y V}{c^2 q} \left[ (s\varepsilon_2 - p) v_q^0 e^{ipd} + (s\varepsilon_2 + p) w_q^0 e^{-ipd} \right].
\]

3 Calculation of the force of friction

The frictional stresses \(\sigma\) and \(-\sigma\) which act on the surfaces of the two bodies can be obtained from the \(xz\)-component of the Maxwell stress tensor \(\sigma_{ij}\), evaluated at \(z = 0\):

\[
\sigma = \frac{1}{8\pi} \int_{-\infty}^{+\infty} d\omega \left[ \langle E_{3z} E_{3x}^* \rangle + \langle E_{3z} E_{3x}^* \rangle + \langle B_{3z} B_{3x}^* \rangle + \langle B_{3z} B_{3x}^* \rangle \right]_{z=0} \tag{32}
\]

Here \(\langle . . \rangle\) denote a statistical average over the random field. The averaging is carried out with the aid of equation (4). Note that the components of the random field \(g_1\) and \(g_2\) referring to different media are statistically independent, so the average of their product gives zero. Writing the squares of the integrals (11-12) in the usual way as double integrals, and carrying out one integration over the \(\delta\)-function, we obtain

\[
\sigma = \frac{1}{8\pi} \int d\omega d^2q \left\{ \langle E_{3z}(q,\omega) E_{3x}(q,\omega) \rangle + \langle E_{3z}(q,\omega) E_{3x}(q,\omega) \rangle + \langle B_{3z}(q,\omega) B_{3x}(q,\omega) \rangle + \langle B_{3z}(q,\omega) B_{3x}(q,\omega) \rangle \right\}_{z=0} \tag{33}
\]

where one must substitute in place of \(E_3\) and \(B_3\), the expressions in the integrands (11-12) determined by the formulas (25-31), and the average product \(\langle g_i(k,\omega) g_k(k,\omega) \rangle\) is to be taken as \((1/2 + n(\omega))\varepsilon''(\omega)\delta_{ik}/\pi^3\). For a given value of \(q\) it is convenient to express the components \(E_x\) and \(B_x\) in terms of the components along the two vectors \(e_q\) and \(e_n\)

\[
E_x = \frac{(q_x/q)}{q} E_q - \frac{(q_y/q)}{q} E_n,
\]
\[
B_x = \frac{(q_x/q)}{q} B_q - \frac{(q_y/q)}{q} B_n.
\]

Thus we can write

\[
\sigma = \frac{1}{8\pi} \int d\omega d^2q \left\{ \frac{q_x}{q} \left[ \langle E_z(q,\omega) E_q^*(q,\omega) \rangle + \langle E_z^*(q,\omega) E_q(q,\omega) \rangle \right] \right\}
\]

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Accordingly to eq. (11-12)

\[ E_z = (v_z + w_z) = (v_q + w_q) = (q/p)(w_q - v_q), \]

\[ E_q = v_q + w_q, \]

\[ E_n = v_n + w_n, \]

\[ B_z = (cq/\omega)(v_n + w_n), \]

\[ B_q = (cp/\omega)(w_n - v_n), \]

\[ B_n = (\omega p^*/c|p|^2)(v_q - w_q). \]

(34)

After substituting these expressions into formula (34) one can see that the second term with \( q_y \) is identically equal to zero. From equations (25-31) it follows that the zero and first order solutions are statistically independent, then, neglecting by the terms of the order \((v/c)^2\), from (34-35) we obtain

\[
\sigma = \frac{1}{4\pi} \int_0^{+\infty} d\omega \int d^2q_x \left( \left[ \frac{1}{p^2} (p + p^*)(\langle |w_q^0|^2 \rangle - \langle |v_q^0|^2 \rangle) \right.ight.
\]

\[
+ (p - p^*)(\langle v_q^0 w_q^0 - v_q^0 w_q^0 \rangle) \left. \right]
\]

\[
+ \left( \frac{c}{\omega} \right)^2 \left[ (p + p^*)(\langle |w_n^0|^2 \rangle - \langle |v_n^0|^2 \rangle) - (p - p^*)(\langle v_n^0 w_n^0 - v_n^0 w_n^0 \rangle) \right],
\]

(36)

where we change the integration over \( d\omega \) between the limits \(-\infty\) and \(+\infty\) on the integration only over positive values of \( \omega \) what gives the extra factor two.

Taking into account that \( p = p^* \) for \( q < \omega/c \), \( p = -p^* \) for \( q > \omega/c \), and carrying out the integration over \( dk_z \) with the help of the formula

\[
\int_{-\infty}^{\infty} \frac{dk_z}{|k_z^2 - s^2|^2} = \frac{i\pi}{|s|^2(s-s^*)},
\]

after substituting (25-31) into (36) we obtain

\[
\sigma = \frac{\hbar}{8\pi^3} \int_0^{+\infty} d\omega \int_{q<\omega/c}^{+\infty} d^2q_x
\]

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\[
\times \left\{ \frac{(1 - |R_{1p}|^2)(1 - |R_{2p}|^2)}{1 - e^{2ipd}R_{1p}R_{2p}^{-1}} (n(\omega - q_x V) - n(\omega)) + [R_p \to R_s] \right\}
\]

\[+
\frac{\hbar}{2\pi^3} \int_0^\infty d\omega \int_{q > \omega/c} d^2 q x e^{-2|p|d/d}
\]

\[\times \left\{ \frac{\text{Im} R_{1p} \text{Im} R_{2p}^{-1}}{1 - e^{-2|p|d}R_{1p}R_{2p}^{-1}^2} (n(\omega - q_x V) - n(\omega)) + [R_p \to R_s] \right\}
\]

where

\[R_{ip} = \frac{\varepsilon_i p - s_i}{\varepsilon_i p + s_i}, \quad R_{is} = \frac{s_i - p}{s_i + p},\]

\[R_{ip}^\pm = \frac{\varepsilon_i^\pm p - s_i^\pm}{\varepsilon_i^\pm p + s_i^\pm}, \quad R_{is}^\pm = \frac{s_i^\pm - p}{s_i^\pm + p},\]

\[\varepsilon_i^\pm(\omega) = \varepsilon_i(\omega \pm q_x V) \quad \text{and} \quad s_i^\pm(\omega) = s_i(\omega \pm q_x V), \quad i = 1, 2.\]

Note that \(R_{ip}\) and \(R_{is}\) are the electromagnetic reflection factors for \(p\)-polarized and \(s\)-polarized light, respectively. \((p\)-polarized light has the electric field vector in the plane of incidence while the electric field vector is perpendicular to this plane for \(s\)-polarized light.\) The first term in (37) is the contribution to the friction force from the propagating (radiating) electromagnetic field, i.e., the black body radiation. This term includes only the thermal radiation and is equal to zero at \(T = 0\). The second term is derived from the evanescent field, i.e., from the component of the electromagnetic field which decay exponentially with the distances away from the surfaces of the bodies. This term does not vanish even at \(T = 0K\) because of quantum fluctuations in the charge density in the solids.

### 4 Some limiting cases

Let us first consider distances \(d << c/\omega_p\), where \(\omega_p\) is the plasma frequency of the metals. For typical metals, \(c/\omega_p \approx 200 \text{ Å}\). In this case the main contribution comes from \(q >> \omega_p/c\), and we have \(s_1 \approx s_2 \approx p \approx iq\), \(R_s \approx 0\) and

\[R_p \approx \frac{\varepsilon - 1}{\varepsilon + 1}\]

In this approximation the integration over \(d^2 q\) can be extended on the all \(q\)-plane. Using these approximations, the second term in (11) can be written as

\[\sigma = \frac{\hbar}{4\pi^3} \int_0^\infty d\omega \int d^2 q x e^{-2|p|d/d}\]
\begin{equation}
\times \left\{ \frac{\text{Im} R_{1p} \text{Im} R_{2p}}{1 - e^{-2q|q|d R_{1p} R_{2p}^-}} + (1 \leftrightarrow 2) \right\}
(n(\omega - q_x V) - n(\omega)) + [R_p \rightarrow R_s]
\end{equation}

\begin{equation}
= \frac{\hbar}{2\pi^3} \int_{-\infty}^{\infty} dq_y \int_{0}^{\infty} dq_x q_x e^{-2qd} \left\{ \int_{0}^{\infty} d\omega n(\omega) - n(\omega + q_x v) \right\}
\times \left\{ \frac{\text{Im} R_{1p}^+ \text{Im} R_{2p}^-}{1 - e^{-2q|q|d R_{1p}^+ R_{2p}^-}} + (1 \leftrightarrow 2) \right\}
\end{equation}

\begin{equation}
- \int_{0}^{q_x v} d\omega [n(\omega) + 1/2] \left\{ \frac{\text{Im} R_{1p}^+ \text{Im} R_{2p}^-}{1 - e^{-2q|q|d R_{1p}^+ R_{2p}^-}} + (1 \leftrightarrow 2) \right\}
\end{equation}

(38)

where we have used the relation \( n(-\omega) = -n(\omega) - 1 \). At zero temperature the Bose-Einstein factor \( n(\omega) = 0 \) and only the second term in (38) will contribute to the sliding friction; in this limit our expression for the friction force for two identical solids is in agreement with Pendry [his result is, however, not symmetric with respect to \( 1 \leftrightarrow 2 \) which must be the case because of symmetry in the nonretarded limit]. In appendix A we show that the zero-temperature result can be generalized to include nonlocal optics effects, by replacing the reflection factor \( R_p(\omega) \) in (38) by the surface response function \( g(q, \omega) \). Next, let us consider the limiting case of low sliding velocity or high temperature namely, \( V \ll cd/d_W \), where \( d_W = \frac{ch}{k_B T} \) is the Wien length (typically \( d_W \approx 10^5 \text{Å} \)). In this case

\begin{equation}
n(\omega) - n(\omega + q_x V) \approx -q_x V \frac{dn}{d\omega} = \frac{e^{\hbar\omega/k_B T}}{(e^{\hbar\omega/k_B T} - 1)^2} \frac{hq_x V}{k_B T}
\end{equation}

and in the second term in (38) we can put

\begin{equation}
n(\omega) \approx k_B T/\hbar\omega
\end{equation}

Substituting these results in (38) gives

\begin{equation}
\sigma = \frac{\hbar V}{2\pi^2} \int_{0}^{\infty} dq_y \int_{0}^{\infty} dq_x q_x e^{-2qd} \int_{0}^{\infty} d\omega \left( -\frac{dn}{d\omega} \right) \frac{\text{Im} R_{1p}^+ \text{Im} R_{2p}^-}{1 - e^{-2q|q|d R_{1p}^+ R_{2p}^-}}
\end{equation}

\begin{equation}
+ \frac{2}{\pi^3 k_B T} \int_{-\infty}^{\infty} dq_y \int_{0}^{\infty} dq_x q_x e^{-2qd} \int_{0}^{q_x V} d\omega \frac{\text{Im} R_{1p}^+ \text{Im} R_{2p}^-}{\omega} \frac{1}{1 - e^{-2q|q|d R_{1p}^+ R_{2p}^-}}
\end{equation}

(39)

The second term in this expression is proportional to \( \sim V^2 \) as \( V \to 0 \) (see below) and can be neglected in the limit of small \( V \). The first term is \( \sim V \) and is in agreement with the result obtained by Persson and Zhang if one
assumes local optics (which implies replacing \( g(q, \omega) \rightarrow R_p(\omega) \) in [2]). For free-electron like metals the local optics is accurate if \( d \gg l \), where \( l \) is the electron mean free path in the metal. If this condition is not satisfied, the general formula of Persson and Zhang must be used.

Let us consider two identical metals described by the dielectric function

\[
\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i/\tau)},
\]

where \( \tau \) is the relaxation time and \( \omega_p \) the plasma frequency. Thus, for small frequencies

\[
\text{Im} R_p \approx \frac{2\omega}{\omega_p^2 \tau}, \quad \text{Re} R_p \approx 1.
\]

and if we neglect the imaginary part of \( R_p \) in the denominator of the integrand in (13) we obtain

\[
\sigma = \xi \left( \frac{k_B T}{\hbar \omega_p} \right)^2 \frac{1}{(\omega_p \tau)^2} \frac{\hbar V}{d^4} + \frac{2\pi k_B T}{45 \hbar \omega_p} \frac{1}{(\omega_p \tau)^2} \frac{\hbar V^2}{\omega_p d^6},
\]

where

\[
\xi = \frac{1}{8} \int_0^\infty \frac{dx x^2}{e^x - 1} \approx 0.5986.
\]

In deriving (41) we have used the following standard integrals

\[
\int_0^\infty \frac{dx}{e^x - 1} = \frac{\pi^2}{6}, \quad \int_0^\infty \frac{dx x^3}{e^x - 1} = \frac{\pi^4}{15}.
\]

The ratio between the second and first term in (41) equals \( \approx (V/c)(dW/d) \), and in deriving (41) we have assumed that this quantity is much smaller than unity. As an example, if \( d = 10\,\text{Å} \) and \( V = 1\,\text{m/s} \) then for typical metals at room temperature \( (k_B T \approx 0.025 \,\text{eV}, \omega_p \tau \approx 100, \hbar \omega_p \approx 10 \,\text{eV}) \) the first and the second terms in (41) give \( \sigma \approx 10^{-8} \) and \( \approx 10^{-13} \,\text{N/m}^2 \), respectively.

On the other hand, if \( (V/c)(dW/d) >> 1 \) we get

\[
\sigma = \frac{\xi}{2} \left( \frac{k_B T}{\hbar \omega_p} \right)^2 \frac{1}{(\omega_p \tau)^2} \frac{\hbar V}{d^4} + \zeta \frac{1}{(\omega_p \tau)^2} \frac{\hbar V}{d^4} \left( \frac{V}{d\omega_p} \right)^2,
\]

where

\[
\zeta = \frac{5}{29\pi^2} \int_0^\infty \frac{dx x^4}{e^x - 1} = 0.024610.
\]
The ratio of the second and first term in (42) equals \( \sim 0.1(V/c)^2(d_W/d)^2 \). It is clear that at low temperature or high velocities, the second term in (42) will dominate.

Next, let us consider the sliding friction to linear order in the sliding velocity when \( c/\omega_p << d << d_W \). There will be two contributions associated with \( R_p \) and \( R_s \). As shown in Appendix B, the contribution from \( R_p \) is

\[
\sigma_p \approx \frac{3\xi hV}{\pi^2 d^4} \left( \frac{d}{d_W} \right)^2 \frac{k_BT}{\hbar \omega_p \omega_p \tau} \left( 1 + \frac{1}{e} + \ln \frac{d_W}{d} \right).
\] (43)

The contribution \( \sigma_s \) from the term involving \( R_s \) is given by (see Appendix B)

\[
\sigma_s \approx C \omega_p \tau \frac{V h \omega_p}{c d^2 d_W},
\] (44)

where \( C \approx 0.394 \). Comparing (43) with (44) we obtain

\[
\sigma_s/\sigma_p \approx (\omega_p \tau)^2 (h \omega_p/k_B T)^2.
\]

For typical metals at room temperature, \( h \omega_p/k_B T \sim 10^3 \) and \( \omega_p \tau \sim 100 \) so that \( \sigma_s/\sigma_p \sim 10^{10} \), i.e., the main contribution comes from the term involving \( R_s \). As an illustration, if \( d = 10^4 \AA \) and \( V = 1 \text{m/s} \) then for metals at room temperature, characterized by the same parameter values as used above, one get \( \sigma_s \approx 10^{-8} \text{N/m}^2 \).

Now, let us consider the radiative contribution to the friction force, which is given by the first term in (38). In linear order in the sliding velocity we get

\[
\sigma_{rad} = \frac{hV}{8\pi^3} \int_0^\infty d\omega \int_{\omega/c}^{\omega/c} d^2q d^2q' \times \left\{ \frac{(1 - |R_{1p}|^2)(1 - |R_{2p}|^2)}{|1 - e^{2ipd}R_{1p}R_{2p}|^2} \left( - \partial n(\omega) / \partial \omega \right) + [R_p \rightarrow R_s] \right\}.
\] (45)

For separations \( d \) much smaller than the Wien wavelength \( d_W = c\hbar/k_BT \) we can put \( \exp(ipd) \approx 1 \). In this case and for the small frequencies, when \( \omega \leq k_BT/\hbar < < 1/\tau \), we get for identical metals described by the dielectric function (40)

\[
\frac{(1 - |R_p|^2)^2}{|1 - R_p^2|^2} \approx \frac{1}{2} + \frac{(\varepsilon s)^2 + (\varepsilon s^*)^2}{4 |\varepsilon s|^2} \approx \frac{1}{2} + \frac{\varepsilon^* + \varepsilon}{4 |\varepsilon|} \approx \frac{1}{2} \left( 1 + \frac{\omega \tau}{2} \right) \approx \frac{1}{2}. \] (46)
and the same result is obtained when $R_p$ in (46) is replaced by $R_s$. The final result for the radiative friction force has the form.

$$
\sigma_{\text{rad}} = \frac{\hbar V}{8\pi^2 c^4} \int_0^\infty d\omega \omega^3 n(\omega) = \frac{\pi^2}{120} \hbar V \left( \frac{k_B T}{\hbar c} \right)^4.
$$

(47)

Note that the radiative stress does not depend on the separation and is proportional to $T^4$. The latter result is, of course, only valid as long as $d$ is small compared with the lateral extend (or linear size) $L$ of the bodies. When $d$ becomes comparable with or larger than $L$, the friction force between the two bodies will decrease monotonically with increasing $d$. At room temperature and at the sliding velocity $V = 1 m/s$ one get $\sigma_{\text{rad}} \approx 10^{-15} N/m^2$. The ratio of this contribution to $\sigma_s$ from (44) is

$$
\frac{\sigma_{\text{rad}}}{\sigma_s} = 0.1 \left( \frac{d}{d_W} \right)^2 \frac{k_B T}{\hbar \omega_p} \frac{1}{\omega_p \tau} \sim 10^{-6} \left( \frac{d}{d_W} \right)^2.
$$

Thus for $d \sim d_W$ (which is of order $\sim 10^6$ for typical metals at room temperature) the nonradiative part dominate over the radiative contribution by a factor $\sim 10^6$. However, for large enough distances the radiative part dominate as this contribution is finite for arbitrary separations $d$.

5 Summary and conclusion

We have calculated the friction force between two arbitrary bodies with flat surfaces separated by a vacuum slab of thickness $d$, and moving with a relative velocity $V$. The separation $d$ is assumed to be so large that the only interaction between the bodies is via the electromagnetic field associated with thermal or quantum fluctuations in the solids. A general formula for the friction force has been obtained, which is valid for arbitrary velocity $V$, separation $d$ and temperature $T$, and applicable to any bodies. At low sliding velocity only thermal fluctuations give a contribution to the friction force, linearly proportional to the velocity $V$. Quantum fluctuations give a nonlinear (in $V$) contribution to the friction force and is usually negligible compared with the thermal contribution. [There is also a contribution from quantum fluctuations which is proportional to $V$, resulting from higher order electron-photon processes than considered in our work. However, this contribution decays as $\sim exp(-2Gd)$ (where $G = 2\pi/a$ is the smallest reciprocal
lattice vector) and is negligible small already for $d = 10 \, \text{Å}$. See \[11, 2\]. We have studied the detailed distance dependence of the friction force from short distances, where retardation effects can be neglected, to large distances where retardation effects and black body radiation are important. In most practical cases, involving sliding of a block on a substrate, the van der Waals friction makes a negligible contribution to the friction force (the main part of the friction arises from the regions of real contact between the solids). However, in some special cases the van der Waals friction is very important\[2\]. For example, quantum fluctuations contribute in an important manner to the friction force acting on thin physisorbed layers of atoms sliding on metallic surfaces \[12\] [In this case there is an overlap of the wavefunctions of the sliding layer and those of the metal, which result in second contribution to the friction force, derived from the repulsive “contact interaction” (Pauli repulsion) between the sliding layer and the substrate]. In addition, the contribution from thermal fluctuations gives the dominating drag force in some experiments involving parallel 2D-electron systems \[13\].

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In this appendix we derive an expression for the (nonlinear) sliding friction using a nonlocal optics description of the metals. For simplicity, we focus on zero temperature $T = 0 \, \text{K}$ and assume that $d$ is so short that retardation effects can be neglected (only in this “short-distance” region will nonlocal effects be important). The calculation is based on the formalism developed in \[14, 2\]. Let us first define the linear response function $g(q, \omega)$ which is needed below. Assume that a semi-infinite metal occupy the half space $z \leq 0$. A charge distribution in the half space $z > d$ gives rise to an (external) potential which must satisfy Laplace equation for $z > d$ and which therefore can be written as a sum of evanescent plane waves of the form

$$\phi_{\text{ext}} = \phi_0 e^{iq_z z} e^{i(q_x x - \omega t)}$$

where $q = (q_x, q_y)$ is a 2D-wavevector. This potential will induce a charge distribution in the solid (occupying $z < 0$) which in turn gives rise to an electric potential which must satisfy the Laplace equation for $z > 0$, and
which therefore can be expanded into evanescent plane waves which decay with increasing $z > 0$. Thus the total potential for $0 < z < d$ can be expanded in functions of the form

$$\phi_{\text{ext}} = \phi_0 \left( e^{qz} - ge^{-qz} \right) e^{i \mathbf{q} \cdot \mathbf{x} - i \omega t}$$

where the reflection factor $g = g(q, \omega)$. For the present purposes, we can treat the low-energy electron-hole pair excitations in the metals as bosons.

As shown in Ref. [14], the Hamiltonian for the total system can be written as

$$H = \sum_{\mathbf{q} \alpha_1} \hbar \omega_{\mathbf{q} \alpha_1} b_{\mathbf{q} \alpha_1}^+ b_{\mathbf{q} \alpha_1} + \sum_{\mathbf{q} \alpha_2} \hbar \omega_{\mathbf{q} \alpha_2} b_{\mathbf{q} \alpha_2}^+ b_{\mathbf{q} \alpha_2} + \hbar \omega b^+ b$$

$$+ \sum_{\mathbf{q} \alpha_1 \alpha_2} C_{\mathbf{q} \alpha_1} e^{-q z n} \left( b_{\mathbf{q} \alpha_1} e^{i \mathbf{q} \cdot (\mathbf{x}_n + \mathbf{V} t)} + h.c. \right). \tag{48}$$

Here $\omega_{\mathbf{q} \alpha_1}$, $b_{\mathbf{q} \alpha_1}^+$ and $b_{\mathbf{q} \alpha_1}$ are the angular frequency and creation and annihilation operators for the bosons (of solid 1) with the quantum numbers $(\mathbf{q}, \alpha_1)$, and $C_{\mathbf{q} \alpha_1}$ parameters determining the coupling between the boson excitations in solid 1 with the electrons in solid 2. Similarly, $b_{\mathbf{q} \alpha_2}^+$ and $b_{\mathbf{q} \alpha_2}$ are creation and annihilation operators for bosons in solid 2, and $(\mathbf{x}_n, z_n)$ is the position operator of electron $n$ in solid 2, which in principle could be expressed in terms of the operators $b_{\mathbf{q} \alpha_2}^+$ and $b_{\mathbf{q} \alpha_2}$, but for the present purpose this is not necessary. As shown in [14], $C_{\mathbf{q} \alpha_1}$ can be related to $\text{Im} g_1(q, \omega)$ via

$$\sum_{\alpha_1} |C_{\mathbf{q} \alpha_1}|^2 \delta(\omega - \omega_{\mathbf{q} \alpha_1}) = \frac{2e^2 \hbar}{A q} \text{Im} g_1(q, \omega) \tag{49}$$

We can write the interaction Hamiltonian between solid 1 and 2 as

$$H' = \sum_{\mathbf{q}} \left( \hat{V}_{\mathbf{q}} e^{i \mathbf{q} \cdot \mathbf{V} t} + h.c. \right)$$

Using time-dependent perturbation theory (with $H'$ as the perturbation) we can calculate the energy transfer from the translational motion (kinetic energy) to internal excitations in the solids (boson excitations $\omega_{\mathbf{q} \alpha_1}$ and $\omega_{\mathbf{q} \alpha_2}$ in solid 1 and 2, respectively):

$$P = \frac{2\pi}{\hbar^2} \sum_{\mathbf{q} \alpha_1 \alpha_2} \hbar \omega_{\mathbf{q} \alpha_1} \delta(\omega_{\mathbf{q} \alpha_1} - \omega_{\mathbf{q} \alpha_2} - \omega_{\mathbf{q} \alpha_1}) |C_{\mathbf{q} \alpha_1}|^2$$

$$\times e^{-2qd} |\langle n_{\mathbf{q} \alpha_1} = 1, n_{\mathbf{q} \alpha_2} = 1 | \sum_n e^{-q(z_n - d)} e^{-i \mathbf{q} \cdot \mathbf{x}_n} b_{\mathbf{q} \alpha_1}^+ | 0, 0 \rangle|^2 \tag{50}$$
where \( \omega_q = |q \cdot V| \). To simplify (A3), let us write
\[
\delta(\omega_q + \omega_{q\alpha_2} - \omega_{q\alpha_1}) = \int d\omega' \delta(\omega' - \omega_{q\alpha_1}) \delta(\omega_q - \omega' - \omega_{q\alpha_2})
\] (51)
Substituting (A4) in (A3) and using (A2) gives
\[
P = \frac{4\pi e^2}{A} \sum_q \frac{\omega_q}{q} e^{-2qd} \int d\omega' \text{Im} g_1(q, \omega') M_q(\omega_q - \omega')
\] (52)
where
\[
M_q(\omega) = \sum_{\alpha_2} \delta(\omega - \omega_{q\alpha_2}) |\langle n_{q\alpha_2} = 1 | \sum_n e^{-q(z_n - d)} e^{-iq \cdot x_n} | 0 \rangle|^2
\]
But it has been shown elsewhere that \[15\]
\[
\frac{A \hbar q}{2\pi^2 e^2} \text{Im} g_2(q, \omega) = \sum_{\alpha_2} \delta(\omega - \omega_{q\alpha_2}) |\langle n_{q\alpha_2} = 1 | \sum_n e^{-q(z_n - d)} e^{-iq \cdot x_n} | 0 \rangle|^2
\]
so that
\[
M_q(\omega) = \frac{A \hbar q}{2\pi^2 e^2} \text{Im} g_2(q, \omega)
\] (53)
Substituting this result in (A5) gives
\[
P = \frac{2\hbar}{\pi} \sum_q \omega_q e^{-2qd} \int d\omega' \text{Im} g_1(q, \omega') \text{Im} g_2(q, \omega_q - \omega')
\] (54)
Finally replacing
\[
\sum_q \rightarrow \frac{A}{4\pi^2} \int d^2q
\]
and using the relation \( P = \sigma AV \) between the power \( P \) and the shear stress \( \sigma \) gives
\[
\sigma = \frac{\hbar}{2\pi^3} \int d^2q \left| q_x \right| e^{-2qd} \int_0^{|q_x|V} d\omega' \text{Im} g_1(q, \omega') \text{Im} g_2(q, |q_x|V - \omega')
\] (55)
where we have used that \( \text{Im} g(q, \omega) = 0 \) for \( \omega < 0 \). The coupling \( H' \) does not only give rise to real excitations but also to screening (image charge effects). To take these into account one must go to higher order in perturbation theory. Following Ref.\[2\] this gives a modification of (A8)
\[
\sigma = \frac{\hbar}{2\pi^3} \int d^2q \left| q_x \right| e^{-2qd} \left| 1 - g_1(q, 0)g_2(q, 0)e^{-2qd} \right|^2
\]
where we have assumed that the small frequencies involved in the real excitations are screened in an adiabatic manner so that \( g_1 \) and \( g_2 \) can be evaluated at zero frequency in the screening factor. In the case of local optics, this expression for \( \sigma \) agree with the last term in (38) evaluated at zero temperature. At finite temperature \( (T > 0) \) an extra factor of \([1 + 2n(\omega')]\) must be inserted in the frequency integral in (A9) to take into account the enhanced probability for excitation of bosons at finite temperature. For \( T > 0 \) one must, in addition to the process considered above, also include scattering processes where a thermally excited boson is annihilate either in solid 1 or in solid 2, namely \((n_{q_\alpha_1} = 0, n_{q_\alpha_2} = 1) \to (1, 0)\) and \((1, 0) \to (0, 1)\). These processes was considered in Ref. \[2\] and give, in the local optics case, the frictional stress corresponding to the first term in (38).

In this appendix we calculate the sliding friction to linear order in the sliding velocity when \( c/\omega_p << d << d_W \) (where \( d_W = c\hbar/k_B T \)). In this case the main contribution \( \sigma \) comes from the first term in (39) to which we must add the similar term involving \( R_s \). In this integral we replace the integration variable \( q \) with \( \bar{p} = 2dq \). The integral over \( \bar{p} \) is divided into two parts: the integral over \((0, \bar{p}_0)\) and over \((\bar{p}_0, \infty)\), where \( p_0 \sim k_B T/c\hbar << 1 \). In the integral over \((\bar{p}_0, \infty)\) and for \( \omega > \omega_0 \), where

\[ \omega_0 \sim \left( \frac{c}{\omega_p d} \right)^2 \frac{1}{\tau} \]

we can put

\[ Im R_p \approx \frac{2}{\bar{p}} \left( \frac{2\omega d}{c} \right) \left( \frac{\omega}{2\omega_p^2 \tau} \right)^{1/2} \]

\[ Im R_s \approx -\frac{2\bar{p}c}{\omega_p d} \left( \frac{1}{\omega \tau} \right)^{1/2} \]

Let us first consider the contribution \( \sigma_p \) to (39) from terms involving \( R_p \). We then obtain

\[ \sigma_p = \frac{\hbar v}{\pi^2} \left( \frac{1}{2d} \right)^4 \int_{\bar{p}_0}^{\infty} d\bar{p} \frac{d}{d\bar{p}} \left( -\frac{1}{\bar{e}^\bar{p} - 1} \right) \int_0^\infty d\omega \omega \frac{\omega}{2\omega_p^2 \tau} \left( \frac{2\omega d}{c} \right)^2 \left( -\frac{dn}{d\omega} \right) \]

\[ \approx \frac{3}{8\pi^2} \frac{\hbar v}{d^4} \left( 1 + \int_{\bar{p}_0}^{1} d\bar{p} \frac{1}{\bar{e}^\bar{p} - 1} + \int_1^\infty d\bar{p} e^{-\bar{p}} \right) \left( \frac{k_B T d}{\hbar c} \right)^2 \frac{k_B T}{\hbar \omega_p \omega \tau} \int_0^\infty \frac{dx x^2}{e^x - 1} \]
\[
\frac{3\xi}{\pi^2 d^4} \left( \frac{k_B T d}{h c} \right)^2 \frac{k_B T}{\hbar \omega_p \omega_p \tau} \left( 1 + \frac{1}{e} + \ln \frac{d_w}{d} \right)
\]

where \( \xi = 0.5986 \) (see Sec. 3). The integral over \( 0 < \bar{p} < \bar{p}_0 \) can be shown to give a negligible contribution to the linear (in the sliding velocity) friction force. This follows from the following equation

\[
\frac{d}{d\omega} \frac{(\text{Im} R_p)^2}{|1 - e^{-\bar{p} R_p R_p}|^2} \approx \frac{d}{d\omega} \frac{(\text{Im}(s/\epsilon))^2}{|s/\epsilon|^2} \approx 0
\]

where we have used that \( \text{Im}(s/\epsilon)/|s/\epsilon| \) is approximately independent of frequency. Next, let us consider the contribution \( \sigma_s \) to (39) from the term involving \( R_s \). We get

\[
\sigma_s \approx \frac{5}{16\pi^2 d^4} \frac{\hbar}{k_B T} \left( \frac{c}{\omega_p d} \right)^2 \int_0^\infty \frac{d\bar{p}^4}{e^{\bar{p}} - 1} \int_{x_0}^\infty \frac{dx}{x} \frac{d \omega}{\omega_p \tau} \left( -\frac{1}{e^x - 1} \right)
\]

where

\[
C = \frac{5}{32\pi^2} \int_0^\infty \frac{d\bar{p}^4}{e^{\bar{p}} - 1} = 0.394
\]

\[
x_0 = \frac{\hbar \omega_0}{k_B T} \approx \frac{\hbar}{k_B T \tau} \left( \frac{c}{\omega_p d} \right)^2 << 1
\]

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