Efficient generation of entangled multi-photon graph states from a single atom

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The central technological appeal of quantum science resides in exploiting quantum effects, such as entanglement, for a variety of applications including computing, communication and sensing [1]. The overarching challenge in these fields is to address, control and protect systems of many qubits against decoherence [2]. Against this backdrop, optical photons, naturally robust and easy to manipulate, represent ideal qubit carriers. However, the most successful technique to date for creating photonic entanglement [3] is inherently probabilistic and therefore subject to severe scalability limitations. Here we report the implementation of a deterministic protocol [4–6] for the creation of photonic entanglement with a single memory atom in a cavity [7]. We interleave controlled single-photon emissions with tailored atomic qubit rotations to efficiently grow Greenberger-Horne-Zeilinger states [8] of up to 14 photons and linear cluster states [9] of up to 12 photons with a fidelity lower bounded by 76(6)% and 56(4)%, respectively. Thanks to a source-to-detection efficiency of 43.18(7)% per photon we measure these large states about once every minute, orders of magnitude faster than in any previous experiment [3, 10–13].

In the future, this rate could be increased even further, the scheme could be extended to two atoms in a cavity [14, 15], or several sources could be quantum mechanically coupled [16], to generate higher-dimensional cluster states [17]. Overcoming the limitations encountered by probabilistic schemes for photonic entanglement generation, our results may offer a way towards scalable measurement-based quantum computation [18, 19] and communication [20, 21].

Entanglement plays a crucial role in quantum information science. For multi-qubit systems, many of the states considered, e.g. for entanglement purification, secret sharing, quantum error correction as well as interferometric measurements, belong to the family of graph states [9]. Two prominent examples are Greenberger-Horne-Zeilinger (GHZ) and cluster states, which are central ingredients for various measurement-based quantum information protocols [19–21]. One-way quantum computing [18], for instance, represents a conceptually appealing alternative to its circuit-based counterpart. Instead of carrying out unitary quantum-logic gates, computation relies on adaptive single-qubit measurements. This operational easing comes at the price that a multi-qubit entangled resource state, a cluster state, needs to be prepared in advance.

Although multi-qubit entanglement has been demonstrated on various platforms [3, 22–26], only small-scale implementations of measurement-based quantum computing (MBQC) have been realized so far [10, 27, 28]. Amongst these platforms, optical photons stand out as qubit carriers as these suffer negligible decoherence and benefit from crosstalk-free single-qubit addressability and measurement capabilities with off-the-shelf components. However, the most common sources for entangled photons are based on spontaneous parametric down conversion (SPDC). This scheme is inherently probabilistic and thus makes scaling up to larger states an increasingly difficult challenge, even for a moderate number of qubits.

To address this issue, deterministic schemes have been proposed [4–6]. These employ a single-spin memory qubit that mediates entanglement over a string of sequentially emitted photons. This approach is resource efficient as it permits the generation of in principle arbitrarily many entangled photons from a single device. First experiments along these lines have been performed with quantum dots [11, 12] demonstrating entanglement of up to three and four qubits, respectively, in a linear cluster state. Low photon generation and collection efficiencies, a noisy semiconductor environment, or the need for a probabilistic entangling gate were amongst the biggest obstacles for reaching higher photon numbers. Recent experiments with Rydberg superatoms [13, 29] demonstrated GHZ states of up to six photons. While the single-emitter strategy could in principle provide a stepping stone for photonic quantum computation, no implementation has demonstrated a performance that beats or even compares to the SPDC approach [3].

Here we produce large and high-fidelity photonic graph states of the GHZ and cluster type. Inspired by the proposals of ref. [4–6], which we adapt to our physical system, we use a cavity quantum electrodynamics (CQED) platform as an efficient photon source [30–34] and, for the first time, surpass the state-of-the-art SPDC platform. Arbitrary single-qubit rotations between photon emissions allow for the flexible preparation of different types of states in a programmable fashion. We generate and detect GHZ states of up to 14 photons and linear cluster states of up to 12 photons with genuine multipartite entanglement. In principle, higher dimensional cluster states can be created by coupling several sources [17], e.g. via optically mediated controlled NOT gates of the kind demonstrated recently [16]. By virtue of this feature, so far unique to the atomic CQED platform, our technique supports modular extension towards scalable architectures for one-way quantum computation [18, 19] as depicted in Fig. 1a.

**EXPERIMENTAL SETUP AND PROTOCOL**

Our apparatus is shown schematically in Fig. 1b. It consists of a single $^{87}$Rb atom at the center of a high-finesse optical cavity. A magnetic bias field oriented parallel to the cavity direction defines the quantization axis and gives rise to a Zeeman splitting with Larmor frequency $\omega_L = 2\pi \cdot 100$ kHz.
We first initialize the atom in the state \( |F = 2, m_F = 0 \rangle \) by optical pumping. Here, we write the atomic state as \( |F, m_F \rangle \) where \( F \) denotes the total angular momentum and \( m_F \) its projection along the quantization axis. Then, we apply a control pulse (1.5 \( \mu s \)) which induces the vSTIRAP process generating a photon (300 ns FWHM) entangled in polarization with the atomic state. This process can be written as \( |2, 0 \rangle \rightarrow \begin{pmatrix} 1, 1 \rangle |R \rangle - |1, -1 \rangle |L \rangle \end{pmatrix} / \sqrt{2} \), where \( |R/L \rangle \) denotes right/left circular polarization of the photon and \( \{1, 1\}, \{1, -1\} \) serves as our atomic qubit basis. We then perform a single qubit rotation \( R_\theta \) of angle \( \theta \) (1) on the atom. For cluster states, \( \theta = \pi / 2 \), for GHZ states no rotation is performed, i.e., \( \theta = 0 \). Afterwards, we transfer the qubit from \( [1, \pm 1] \) to \( [2, \pm 2] \) (2). Both steps (1) and (2) are realized via a series of Raman pulses with a 790 nm laser (Methods).

Finally, we induce the vSTIRAP process (3) by applying a control pulse which produces a photon (4) and takes the atom back to the states \( |1, \pm 1 \rangle \). Steps (2-4) can be summarized by writing \( |1, \pm 1 \rangle \rightarrow |1, \pm 1 \rangle |R/L \rangle \). One photon production cycle consisting of the steps (1-4) takes 200 \( \mu s \) (50 \( \mu s \)) for the cluster (GHZ) state sequence. It is repeated \( N - 2 \) times, each iteration adding another qubit to a growing chain of entangled photons. For the final photon however (closing), the atomic qubit is transferred from \( |1, \pm 1 \rangle \) to \( |2, \mp 1 \rangle \) (instead of \( |2, \pm 2 \rangle \)) such that in the subsequent emission process the atom ends up in \( |1, 0 \rangle \), which readily disentangles it from the photonic state. We note that for cluster states initializing as well as disentangling the atom are not strictly necessary as the same can be achieved by appropriate \( Z \) basis measurements of the first and...
For estimation of the fidelity it is sufficient to measure the non-zero entries on the diagonal and off-diagonal of the density matrix separately. The diagonal elements represent the populations \( P_N \) of the \(|R|^{\otimes N} \) and \(|L|^{\otimes N} \) components of the state and can be obtained by measuring all photons in the \( Z \) basis. The corresponding experimental data, shown in Fig. 2b in blue, agree well with the ideal GHZ state, for which \( P_N = 1 \), with only a weak dependence on \( N \). In order to demonstrate that the states \(|R|^{\otimes N} \) and \(|L|^{\otimes N} \) are in a coherent superposition, we set the measurement basis to \(|(R) \pm e^{i\phi}|L\rangle)/\sqrt{2} \) where \( \phi \in [0, \pi] \), thus spanning the full equator of the Bloch sphere. This allows us to measure the characteristic parity oscillations which behave as \( \cos(N\phi) \) (Methods), see Fig. 2a. The coherences \( C_N \) of the density matrix are extracted from the visibility of the oscillations for all photon numbers up to \( N = 10 \). For 14 photons the coincidence rate drops significantly due to the finite photon production efficiency. To acquire enough data we only measure the parity for \( \phi = 0 \) which is indicated by the yellow diamond in Fig. 2b. Eventually, the fidelity is calculated via the formula \( F_N = (P_N + C_N)/2 \) and is shown in Fig. 2b in red. As only a single measurement setting was used for \( C_{14} \), we additionally provide a lower bound for the fidelity based on an entanglement witness (Methods). With this we prove genuine 14-photon entanglement with a fidelity \( F_{14} \geq 76.6\% \), surpassing the 50% threshold by more than 4 standard deviations. To the best of our knowledge, this is the largest entangled state of photons to this day.

Within the measured range we observe that the decay of \( P_N \), \( C_N \) and \( F_N \) as a function of photon number is well captured by a linear model with a slope of 0.86(9)%, 1.3(2)% and 1.04(9)% per photon, respectively. By extrapolation of this trend we estimate that the fidelity will cross the 50% threshold at around 44 qubits. The remarkably slow decay in fidelity is particularly astonishing as we observe very little decoherence even when the sequence is deliberately chosen to exceed the intrinsic coherence time of the atomic qubit (~1 ms). This behaviour is explained by a dynamical decoupling effect built into the protocol, which arises from the opposite signs of the Zeeman splitting in the two hyperfine ground state manifolds. Hence, the qubit precession is reversed every time the atom is transferred from \(|F = 1\rangle\) to \(|F = 2\rangle\) or vice versa, which can be seen as two spin-echo pulses for every photon production cycle. While this mechanism contributes to the high-visibility fringes seen in Fig. 2a, no extra effort is needed to exploit it (Methods). We currently attribute the main source of infidelity to the vSTIRAP. This can be explained by the finite cooperativity that allows for unwanted paths in the emission process (Methods).

**GREENBERGER-HORNE-ZEILINGER STATES**

We start the experiment by producing GHZ states. In contrast to cluster states, GHZ states are more sensitive to noise and require a higher level of control in their preparation process. Regardless, since their density matrix contains only four non-zero entries, it is much easier to measure the fidelity \( F_N \) of an \( N \)-photon GHZ state [35] than for a cluster state, despite the large Hilbert space of dimension \( 2^N \). Therefore, the qualitative analysis of a multi-photon GHZ state, besides representing an interesting result by itself, provides a useful tool for benchmarking and gives insights into the inner dynamics of our system.

For estimation of the fidelity it is sufficient to measure the last photon [6]. In the case of GHZ states however, the protocol must be performed as described in Fig. 1c to obtain an \( N \)-photon state of the form \(|\text{GHZ}_N\rangle = (|R|^{\otimes N} + |L|^{\otimes N})/\sqrt{2}\).
The measured data the lower bound exceeds the threshold of 50% indicated by the grey dashed line. b. Measured stabilizers $S_k$ given by the expectation value $\langle Z_{k-1}X_kZ_{k+1} \rangle$ up to $k = 15$. All measured stabilizers are larger than 0.9, marked by the dashed line. Error bars represent 1 standard deviation.

Figure 4. Measured $N$-photon coincidence rate. The data (blue for GHZ and red for cluster states) represent the number of coincidences divided by the total measurement time. From an exponential fit to the data we extract a single photon generation and detection probability of $\eta = 43.18(7)\%$. The light colored lines represent the estimated coincidence rate assuming a loss-corrected efficiency of $\eta = 0.66$ (see also ref. [30]). Equivalent rates are plotted for state-of-the-art SPDC [3], quantum dot (QD) in purple [11] and orange [12], and Rydberg-based [13] (Ryd) systems. Error bars are smaller than the markers.

Figure 3. Clusters. a. Lower bound of the fidelity as provided by ref. [36] obtained from two measurement settings. For the measured data the lower bound exceeds the threshold of 50% indicated by the grey dashed line. b. Measured stabilizers $S_k$ given by the expectation value $\langle Z_{k-1}X_kZ_{k+1} \rangle$ up to $k = 15$. All measured stabilizers are larger than 0.9, marked by the dashed line. Error bars represent 1 standard deviation.

bound of the fidelity can be derived from $W$ requiring only two local measurement settings $XZXXZ...$ and $ZXZXX...$ (Methods). Compared to quantum state tomography, this has the advantage of a tremendous reduction in experimental overhead, but comes at the cost of a potentially significant underestimation of the true state fidelity. Nonetheless, the experimental results displayed in Fig. 3a exceed the 50% threshold for all measured points. Here, the data only includes events in which exactly $N$ photons are detected for a sequence of $N$ consecutive generation attempts. For the largest cluster state of 12 photons we find the fidelity to be lower bounded by 56(4)%. Comparing the results to the GHZ states in Fig. 2 we notice a significantly faster decay of the fidelity (3.6(2)% per photon). Besides the large number of Raman transfers in the protocol (5 transfers per cycle, see Methods), we attribute this mainly to the lower bound which by construction underestimates the fidelity. A tighter lower bound that was recently formulated [37] could provide a higher fidelity estimate in future experiments.

In addition to providing a lower bound for the fidelity, we now present the measured stabilizer operators defined as $S_k = Z_{k-1}X_kZ_{k+1}$ (Fig. 3b). Here $k \in \{1, 2, ..., N\}$, $Z_0 = Z_{N+1} = I$, and $X_k$ and $Z_k$ denote the respective Pauli matrices acting on the $k$th qubit. In this scenario events in which three consecutive photons, $k-1$, $k$ and $k+1$, are detected in the appropriate basis contribute to the stabilizer $S_k$. In principle arbitrarily many stabilizers could be measured by repeating the protocol for a corresponding number of cycles. Here however, we terminate the sequence at $k = 15$. We find an average of $\langle S_1 \rangle = 96.13(9)\%$ and $\langle S_k \rangle = 92(1)\%$ for $k \geq 2$, indicating a large overlap of the generated state with the target linear cluster state.

COINCIDENCE RATE

We emphasize that the ability of producing entanglement of up to 14 photons is based, on the one hand, on the excellent coherence properties of the atom, and on the other hand, the large photon generation and detection efficiencies. The latter is crucial as the success probability $p_s$ of detecting a coincidence of $N$ consecutive photons scales exponentially with the photon number, $p_s = \eta^N$. Here, $\eta$ denotes the probability to generate and detect a single photon for a given attempt. We can express $\eta$ as the product of the source efficiency $\eta_s$, i.e. the probability of producing a photon at the output of the cavity, and the detection efficiency $\eta_d$. It is clear that a low efficiency $\eta \ll 1$ poses a great obstacle to achieving large photonic states within reasonable measurement times.

Fig. 4 shows the raw rate of multi-photon coincidences as a function of photon number $N$ including post-selection and experimental duty cycle. The experimental sequence consists of 14 (12) consecutive photon generation attempts with all timing parameters identical to the GHZ (cluster) protocol and a new run starting every 1.1 ms (3.0 ms). The shown data (blue for GHZ and red for cluster states) is the coincidence count
rate of events in which $N$ consecutive photons were detected starting from the first attempt. For instance, for the largest state of 14 photons we recorded 151 coincidences in 7 hours of experimental runtime, equivalent to roughly one event every three minutes. From an exponential fit to the data we extract the overall single-photon generation and detection efficiency $\eta = 43.18(7)\%$. We estimate the intrinsic generation efficiency $\eta_0$ to be 66% mainly limited by the cooperativity and the escape efficiency (see ref. [30]). Both can be optimized by higher-quality mirrors and a smaller cavity-mode volume. The detection efficiency of $\eta_d = 0.7$ captures all the remaining loss contributions such as optical elements and detectors. These include free-space-to-fiber couplings (94% twice), propagation through optical fiber (97%), free-space optics (90%) and detector efficiency (90%). Correcting for the detection efficiency $\eta_d$, we infer an $N$-photon coincidence rate at the output of the cavity as given by the light blue line in Fig. 4. This represents the limit of our system with the current parameters. As a comparison, we also show the rate of the best available SPDC system as well as deterministic sources based on single quantum dots or Rydberg-blockaded atomic ensembles. Although the repetition rate for these systems is typically many orders of magnitude higher than in our protocol, our system outperforms previous implementations by far in terms of real time coincidence count rate as well as efficiency scaling.

**SUMMARY AND OUTLOOK**

To conclude, we have presented a scalable and freely-programmable source of entangled photons, demonstrating, to our knowledge, the largest entangled states of optical photons to this day. It is deterministic in the sense that no probabilistic entangling gates are required. This gives us a clear scaling advantage over previous schemes. Moreover, the ability to perform arbitrary single-qubit rotations on the emitter provides the flexibility to grow graph states of different types.

At this stage, our system faces mostly technical limitations, such as optical losses, finite cooperativity and imperfect Raman pulses. Even modest improvements in these respects would put us within reach of loss and fault tolerance thresholds for quantum error correction [19, 38–40]. Hence, a clear path towards one-way quantum computing architectures would be the generation of two-dimensional cluster states by entangling multiple photon sources [17]. For example, in a next step two of our systems could be coupled via remote quantum logic gates [16] to produce $2 \times N$ ‘ladder’ cluster states. Alternatively, entangling operations such as gates or Bell-state measurements could be performed on two (or more) individual atoms as single emitters in the same cavity [14, 15].

Similar strategies apply for the generation of tree graph states and one-way quantum repeaters [20, 21]. The present work thus opens up a new road for photonic quantum computation and communication.

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METHODS

Experimental setup

The central component of the setup used in this work is a high-finesse optical cavity with a $^{87}$Rb atom trapped at its center. The cavity consists of two highly reflective mirrors oriented parallel to each other at a distance of 500 µm with an optical mode waist of $w_0 = 30$ µm. The two mirrors have a transmissivity of $T_1 = 100$ ppm and $T_2 = 4$ ppm giving rise to a finesse of $F \approx 60,000$ such that photons populating the cavity mode are outcoupled predominantly through the low-reflective side. The cavity is tuned to the atomic D2 line with a detuning of $\Delta_c = -150$ MHz with respect to the transition $|F = 1\rangle \leftrightarrow |F' = 1\rangle$. The combined system of the atom and cavity is best described in the framework of cavity quantum electrodynamics with parameters $(g, \kappa, \gamma) = 2 \pi \cdot (c_{ge} \cdot 10.8, 2.7, 3.0)\text{MHz}$, $g$ being the atom-cavity coupling strength for the relevant transition, $\kappa$ the decay rate of the cavity field and $\gamma$ the free-space atomic decay rate associated with the D2 transition of $^{87}$Rb. $c_{ge}$ is the Clebsch-Gordan coefficient between the relevant excited state $(|e\rangle)$ coupling to the vSTIRAP control pulse and the final state $(|g\rangle)$ of the photon production process. The above parameters put our system in the intermediate to strong coupling regime with a cooperativity parameter defined as $C = g^2 / (2\kappa\gamma)$. Note that the specific value of $C$ depends on the transition path associated to a certain excited state. For example, for the emission from $|2, \pm 2\rangle$ as in the cycling step of the protocol we have $|g\rangle = |F = 1, m_F = \pm 1\rangle$ and $|e\rangle = |F' = 2, m_{F'} = \pm 2\rangle$. Hence, we get $c_{ge} = \sqrt{1/4}$, giving $C = 1.8$.

Atom loading

Atoms are transferred from a magneto-optical trap (MOT) to the center of the cavity where they are trapped by a two-dimensional optical lattice composed of two standing wave potentials, one at 772 nm oriented along the cavity axis and one at 1064 nm propagating perpendicular to the cavity axis. An EMCCD camera detects the atomic fluorescence which is collected via a high NA objective. A single atom is prepared quasi-deterministically by removing any excess atom with a resonant laser beam steered onto the atom via an acousto-optic deflector (AOD). The position of the atom is monitored during the experiment and controlled via appropriate feedback to the optical trapping potential.

Experimental duty cycle and post-selection

Because the atoms have a finite lifetime in the dipole trap, they have to be reloaded regularly. The average trapping time depends significantly on the type of conducted experiment (i.e. heating/cooling mechanisms). For our experiments, we achieved an average trapping time of roughly 20 seconds. The time required for loading and repositioning of the atom after occasional jumps to a different location reduces the experimental duty cycle. By counting the number of experimental runs carried out at a given repetition rate over a longer measurement interval, we evaluate the overall duty-cycle to be close to 50%.

Once a camera image shows that the atom has moved away from the target position, the corresponding data to that image is discarded via post-selection processing. The same applies to images with more than one atom near the cavity center.

Further post-selection is performed by processing the data collected by the single-photon detectors: an experimental run is considered successful when $N$ photons were detected in a row, each within predefined time windows (1 µs width in this work). Note that Fig. 4 shows the coincidence rate after applying post-selection.

Protocol

The full experimental sequence including timings of the optical pulses is shown in Extended Data Fig. 1. As described in the main text, it mainly consists of a repeating sequence of single-qubit rotations and photon emissions (cycling) with additional initialization and closing steps at the beginning and the end. The atom is initialized in the state $|2, 0\rangle$ by optical pumping (5 µs). A square-shaped control pulse (1.5 µs) produces the first photon, thus generating the atom-photon entangled state $|1, 1\rangle |R_1\rangle - |1, -1\rangle |L_1\rangle$ (up to normalization) where the index ‘1’ refers to the first photon. If no photon was detected, we immediately go back to the state preparation step and another photon attempt. We choose a maximum of seven attempts for the first photon in order to avoid excessive heating of the atom. After a successful first photon detection we start the cycling stage with the single-qubit gate, for which cluster states consist of a $\pi/2$ rotation contained in three Raman manipulations. First, the population in $|1, 1\rangle$ is transferred to $|2, 0\rangle$ with a $\pi$ pulse taking 53 µs. Then a $\pi/2$ pulse is applied to the transition $|1, -1\rangle \leftrightarrow |2, 0\rangle$ realizing the qubit rotation. Afterwards, the population in $|2, 0\rangle$ is transferred back to $|1, 1\rangle$ with another $\pi$ pulse. The above described operation transforms the basis states as follows: $|1, 1\rangle \rightarrow |1, 1\rangle + |1, -1\rangle$ and $|1, -1\rangle \rightarrow - |1, 1\rangle + |1, -1\rangle$. The whole pulse sequence for the single-qubit gate takes 132.5 µs. For GHZ states the required rotation angle is $\theta = 0$, which means that the qubit rotation can be skipped entirely. In order to produce the next photon we transfer the population from $|1, \pm 1\rangle$ to $|2, \pm 2\rangle$ via two sequential Raman $\pi$-pulses (790 nm) each taking 21 µs. We then apply a vSTIRAP control pulse leading to a photon emission. The atom-photon photon state then reads $(|1, 1\rangle |R_2\rangle + |1, -1\rangle |L_2\rangle) |R_1\rangle - (|1, +1\rangle |R_2\rangle + |1, -1\rangle |L_2\rangle) |L_1\rangle$ for cluster states, i.e. $\theta = \pi/2$. The index ‘2’ now refers to the second photon. The cycling step is repeated as many times as desired. In the very last cycle, the closing step is performed. Here, following the qubit rotation the atomic population is transferred from
where no could be eliminated by generating the photons on the D1 line,tering has occurred (Extended Data Fig. 4). In the future, this
photon arrival one can partly filter out events in which scat-
cohherent emission of the photon. By post-selecting on early
D2. This opens a decay channel which competes with the
improve the error rate.

Raman manipulations

The Raman transitions shown as orange and green arrows in
Extended Data Fig. 1 are performed with a 790 nm laser. The
duration of these transitions make up the most part of
the experimental sequence. In principle choosing a higher
Rabi frequency could drastically increase the repetition rate
of the protocol, but would lead to more crosstalk between the
transitions as they would start to overlap in frequency space.
As a consequence a compromise between repetition rate and
high-fidelity Raman manipulations has to be found. For our
choice of experimental parameters we estimate the infidelity
per single-qubit rotation to be smaller than 1%.
The Raman transfer in the closing step from $|F = 1\rangle$ to
$|F = 2\rangle$ is realized with a 795 nm Raman laser close to the
D1-line of Rubidium. For this specific Raman transition we
cannot choose a large detuning since this would lead to a de-
structive interference due to the Clebsch-Gordan coefficients.
As a consequence we have a chance of about 5% of sponta-
neous scattering, which reduces the fidelity. As mentioned in
the main text, alternatively the atom can also be disentangled
from the photonic state by measuring the most recently gener-
ated photon in the $Z$ basis. While this would slightly increase
the fidelity, the rate would drop as the detection of an addi-
tional photon is required.

Estimation of errors

For GHZ states we observe a total error rate of about 1% per photon. We attribute most of the infidelity to spontaneous
scattering during the photon production process, as the vSTI-
RAP control pulse couples to the $F' = 3$ excited state of the
D2. This opens a decay channel which competes with the coherent emission of the photon. By post-selecting on early
photon arrival one can partly filter out events in which scatter-
ing has occurred (Extended Data Fig. 4). In the future, this
could be eliminated by generating the photons on the D1 line, where no $F' = 3$ state is present. This should significantly
improve the error rate.
The same error mechanism applies in the case of clus-
ter states. Additionally, the single-qubit gate implemented
with Raman lasers introduces errors, which we estimate to be
smaller than 1%. These could be explained by finite frequency
resolution, pulse intensity fluctuations as well as drifts in op-
tical alignment. Increasing the Zeeman splitting for instance
would be a way to further optimize this process.

Minor sources of error include polarization alignment. For
setting the polarization detection basis we use a reference po-
larizer in front of the cavity and measure the polarization ex-
tinction to be on the order of 10,000:1. For switching the
detection basis we use a polarization electro-optic modula-
tor (EOM, QUBIG PC3R-NIR) with a switching time of 5 ns.
The extinction ratio is specified as $>1000:1$, whereas we mea-
sured values of around 5000:1.

The error rate for cluster states of 3.6% as given in the main
text is presumably overestimated due to the definition of the
fidelity lower bound. Taking into account the error sources
identified above, we estimate the true error rate to be smaller
than 2%. With the suggested improvements we expect a re-
duction well below 1% to be realistic.

Generation efficiency

The intrinsic source efficiency, i.e. the probability of ob-
taining a photon at the output of the cavity, is given by

$$\eta_0 = \frac{2C}{2C + 1} \eta_{esc},$$

(1)

where $C \approx 1.5$ is the cooperativity and $\eta_{esc} \approx 0.88$ denotes
the escape efficiency, i.e. the probability of a photon being
outcoupled from the output port [30]. Note that the above
formula is only valid in the case of a single excited state,
whereas the efficiency becomes a function of the detuning,
$\eta_0(\Delta)$, when multiple excited states are present.

The source efficiency could hence be improved by increas-
ing both the cooperativity and the escape efficiency. As the
two parameters are generally not independent, let us assume
for simplicity that we reduce the waist of the cavity mode by
a factor of 2. This increases the cooperativity by a factor of
4 without altering the escape efficiency. We would thereby
improve the source efficiency from 66% to 81%.

Furthermore, the efficiency of the detection setup could be
improved. For instance, by redesigning and optimization of the
setup one could replace a fiber-to-fiber coupling with a
fiber splice, eliminate a free-space-to-fiber coupling and re-
duce the losses from optical surfaces. In this scenario an
improvement of the detection efficiency $\eta_d$ from 0.7 to 0.85
seems feasible. Given these realistic improvements the com-
bined source-to-detection efficiency $\eta$ would reach the mark
of 2/3, an important threshold for linear optical quantum com-
putation [40].
GHZ state fidelity

In the mathematical formalism of spin 1/2 particles a GHZ state looks like

$$|\text{GHZ}_N\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\ldots\rangle + |\downarrow\downarrow\ldots\rangle),$$  \hspace{1cm} (2)

where in the photonic case $|\uparrow\rangle/|\downarrow\rangle$ corresponds to $|R\rangle/|L\rangle$. For measuring the diagonal elements of the density matrix, i.e., the populations $P_N$ of the $|\uparrow\rangle^\otimes N$ and $|\downarrow\rangle^\otimes N$ components it suffices to measure all particles in the $Z$ basis to obtain

$$P_N = (\langle \uparrow \otimes \uparrow \rangle^\otimes N + \langle \downarrow \otimes \downarrow \rangle^\otimes N).$$  \hspace{1cm} (3)

For the coherences we introduce the parity operator \cite{3, 35}

$$\mathcal{M}_\phi = \left(\begin{array}{cc} 0 & e^{-i\phi} \\ e^{i\phi} & 0 \end{array}\right)^\otimes N$$  \hspace{1cm} (4)

describing a measurement of all $N$ particles in the basis $(|\uparrow\rangle \pm e^{i\phi} |\downarrow\rangle) / \sqrt{2}$. Varying the parameter $\phi$ from 0 to $\pi$ corresponds to a continuous rotation of the measurement basis along the equator of the Bloch sphere. In the experiment this is achieved by scanning the angle of a half-wave plate in front of the PBS in the detection setup. It is straightforward to show that the expectation value of $\mathcal{M}_\phi$ for the ideal GHZ state is

$$\langle \text{GHZ}_N | \mathcal{M}_\phi | \text{GHZ}_N \rangle = \cos(N\phi).$$  \hspace{1cm} (5)

These characteristic parity oscillations are what can be seen in Fig. 2a of the main text. The amplitude of the oscillations as obtained from a cosine fit are a measure for the coherences of the density matrix. The fidelity is then obtained from the formula

$$\mathcal{F}_N^{(\text{GHZ})} = (P_N + C_N) / 2$$  \hspace{1cm} (6)

For the largest photon number of $N = 14$ we chose to measure an entanglement witness derived in ref. \cite{35} in order to obtain a fidelity lower bound. The witness is based on the stabilizer formalism, the stabilizing operators for GHZ states being

$$S^{(\text{GHZ})}_1 = X_1 \cdot X_2 \cdots X_N$$  \hspace{1cm} (7)
$$S^{(\text{GHZ})}_{k\geq2} = Z_{k-1} \cdot Z_k,$$  \hspace{1cm} (8)

where $k = 1, 2, \ldots, N$ and $Z_k, X_k$ are the Pauli matrices acting on the $k$th qubit. With this the fidelity is lower bounded by

$$\mathcal{F}_N^{(\text{GHZ})} \geq \frac{1 + S^{(\text{GHZ})}_1}{2} + \prod_{k\geq2} \frac{1 + S^{(\text{GHZ})}_k}{2} - 1.$$  \hspace{1cm} (9)

Witnessing cluster states entanglement

A lower bound for the fidelity can be derived in a similar fashion for 1D cluster states \cite{36}. With the set of stabilizers $S_k$ as defined in the main text the bound is given by the inequality

$$\mathcal{F}_N^{(C)} \geq \prod_{k \text{ even}} \frac{1 + S_k}{2} + \prod_{k \text{ odd}} \frac{1 + S_k}{2} - 1$$  \hspace{1cm} (10)

It is easy to verify by direct calculation that the terms for even and odd $k$ in Eq. 10 correspond to the local measurement settings $ZXZX\ldots$ and $XZX\ldots$, respectively. As an example, for a four qubit linear cluster state we have

$$\mathcal{F}_4^{(C)} \geq \frac{1}{4} (1 + Z_1X_2Z_3) (1 + Z_3X_4)$$
$$+ \frac{1}{4} (1 + X_1Z_2) (1 + Z_2X_3Z_4) - 1.$$  \hspace{1cm} (11)

Coherence and dynamical decoupling

In the main text we already introduced that our system benefits from a built-in dynamical decoupling mechanism due to the level structure of the atomic hyperfine ground states. A measurement of the intrinsic coherence time of the atom can be seen in Extended Data Fig. 3a. Here we look at the overlap between two photons both emitted from the atom with a variable time delay. The first photon is measured in the linear basis $(|H\rangle / |V\rangle)$ which projects the atom onto a superposition of the qubit states $|1, +1\rangle$ and $|1, -1\rangle$. The atomic state then precesses with twice the Lamor frequency. After a certain time $\tau$ the atomic qubit is read out by mapping it onto a photon which is then measured in the same basis as the first photon. The fidelity, which we define as the projection of the second photon on the first, shows oscillations damped by noise such as magnetic field fluctuations. After roughly 1.2 ms the envelope of the oscillations cross the classical threshold of 0.66 which defines the intrinsic coherence time of the atomic qubit. For the GHZ sequence however, we observe that the effect of decoherence is intrinsically reduced. We can show this by artificially extending the length of the sequence to 1.25 ms for a 6 photon GHZ state. In this case every photon production cycle takes 300 $\mu$s. The ratio of time the qubit resides in $|F = 1\rangle$ and $|F = 2\rangle$ can then be varied by scanning the delay $\tau$ between the hyperfine transfer from $|1, \pm1\rangle$ to $|2, \pm2\rangle$ and the vSTIRAP control pulse as illustrated in Extended Data Fig. 3b. For different values of $\tau$ we record the parity oscillations similar to Fig. 2a and infer the visibility. From the measured data we can see a clear dependence of the visibility as a function of $\tau$ with a rephasing appearing at around 85 $\mu$s. The maximum value is roughly equal to the 6 photon coherence displayed in Fig. 2 of the main text (shown as a dashed line for reference), for which the sequence length was only 250 $\mu$s. This is strong evidence that a large part of the decoherence is mitigated as an inherent feature of the protocol.
Extended Data Fig. 1. **Detailed experimental sequence.** As in Fig. 1c the sequence is divided into initialization, cycling and closing. After each run we perform several hundreds of microseconds of active power stabilization of laser pulses as well as atom cooling. The displayed sequence takes up to 3 ms depending on the number of photons and the type of photonic state (GHZ or cluster).
Extended Data Fig. 2. Parity oscillations. Complete data set for the GHZ coherence measurements for all measured photon numbers $N$. 
Extended Data Fig. 3. **Coherence properties of the emitter.** a, Intrinsic memory coherence measured as the overlap between two photons emitted from the atom with a variable delay. The inset shows a zoom of the oscillations due to the time evolution of the atomic qubit states. The arrow shows the sequence length for the data taken in b. b, Visibility of parity oscillations for a 6 photon GHZ state as a function of temporal delay $\tau$ between the hyperfine remapping and the vSTIRAP control pulse. As a model is difficult to obtain for an unknown noise spectrum, a gaussian fit to the data (solid line) provides a guide to the eye. A maximum of the visibility can be observed for around 85 $\mu$s.
Extended Data Fig. 4. **Inefficiency induced by vSTIRAP process.** Two photons are generated in subsequent cycles of the GHZ protocol and measured in the $R/L$ basis. Their correlation (red) is analyzed as a function of maximum permitted arrival time $t_{\text{max}}$ with respect to the beginning of the emission process. The relative efficiency (blue) displays the number of counts detected up to $t_{\text{max}}$ as opposed to the full photonic wave packet. The correlation decreases as a function of $t_{\text{max}}$, which we attribute to spontaneous scattering events induced by the vSTIRAP control pulse. The dashed line marks the value of $t_{\text{max}}$ used in this work.