Alternative scheme for a two-qubit conditional phase gate by adiabatic passage under dissipation

Z J Deng\textsuperscript{1,2,3}, K L Gao\textsuperscript{1,2} and M Feng\textsuperscript{1,2}

\textsuperscript{1} State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan 430071, People’s Republic of China
\textsuperscript{2} Centre for Cold Atom Physics, Chinese Academy of Sciences, Wuhan 430071, People’s Republic of China
\textsuperscript{3} Graduate School of the Chinese Academy of Sciences, Beijing 100049, People’s Republic of China

E-mail: klgao@wipm.ac.cn and mangfeng1968@yahoo.com

Received 16 August 2006, in final form 5 December 2006
Published 10 January 2007
Online at stacks.iop.org/JPhysB/40/351

Abstract
We check a recent proposal (Goto and Ichimura 2004 Phys. Rev. A 70 012305) for a controlled phase gate through adiabatic passage under the influence of spontaneous emission and the cavity decay. We show a modification of the above proposal could be used to generate the necessary conditional phase gates in the two-qubit Grover search. Conditioned on no photon leakage either from the atomic excited state or from the cavity mode during the gating period, we numerically analyse the success probability and the fidelity of the two-qubit conditional phase gate by adiabatic passage. A comparison made between our proposed gating scheme and a previous one shows that Goto and Ichimura’s scheme is an alternative and feasible way in the optical cavity regime for two-qubit gates and could be generalized in principle to multi-qubit gates.

1. Introduction

Stimulated Raman adiabatic passage (STIRAP) has been extensively studied in coherent population transfer [1]. By partially overlapping pulses in the counterintuitive sequence, we have the population efficiently transferred between two quantum states while almost not populating the intermediate level. Thus the effect of spontaneous emission from the intermediate level is negligible. Moreover, the STIRAP is independent of the pulse shape, which makes it robust against moderate fluctuations of experimental parameters [2]. Because of the above-mentioned merits, many schemes concerning quantum information processing (QIP) are based on the technique of STIRAP, such as single qubit rotation [3], controlled NOT gates [4], controlled phase gates [5, 6], arbitrary state controlled-unitary gates [7], SWAP gates [8], generation of qubit entanglement [9] and so on.
All the above-mentioned QIP schemes are quite different from the dynamical ones [10], which need precise control of the Rabi frequency and the pulse duration. They are also quite different from the adiabatic geometric ones [11], which depend on a controllable loop in the parameter space. In [5], an attractive scheme to generate a multi-qubit controlled phase gate by only three steps in an optical cavity has been presented. As in [4, 7, 8], it utilizes the parameter space. In [5], an attractive scheme to generate a multi-qubit controlled phase gate different from the adiabatic geometric ones [11], which depend on a controllable loop in which need precise control of the Rabi frequency and the pulse duration. They are also quite different from the adiabatic geometric ones [11], which depend on a controllable loop in which need precise control of the Rabi frequency and the pulse duration. They are also quite different from the dynamical ones [10], which need precise control of the Rabi frequency and the pulse duration.

Therefore, it is natural for us to ask how well the quantum gating in [5] works under the influence of dissipation. In the present paper, we will show how to generate all the conditional phase gates in a two-qubit Grover search by the STIRAP technique, based on the main idea in [5]. To check the influence of spontaneous emission and the cavity decay, we will employ the quantum jump approach [12], which uses a wavefunction to describe the system subject to dissipation while the master equation employs a density matrix. Conditioned on no decay happening, the system’s evolution is governed by a non-Hermitian Hamiltonian. In some cases, the quantum jump approach could help us find analytical solutions for systems subject to dissipation [13]. But in the present case, due to complexity, we will have to numerically simulate the gating process and investigate the dependence of the success probability and the gate fidelity on the spontaneous emission rate $\Gamma$ and the cavity decay rate $\kappa$.

2. Conditional phase gate without decay

The conditional phase gate is used for labelling the target state in the Grover search algorithm [14, 15], which means an addition of a $\pi$ phase as a prefactor to the target state, but of nothing to other states. The target state labelling is a key step in the Grover search. For items represented by the computational states $|X\rangle$ with $X = 0, 1, \ldots, N - 1$, in a quantum register with $n$ qubits, we have $N = 2^n$ possible states. If the target state is $|t\rangle$, the conditional phase gate can be expressed as $I_t = I - 2|t\rangle\langle t|$, where $I$ is the $N \times N$ identity matrix. By redefining the energy levels in [5], we show that the second step in [5] to generate the two-qubit controlled phase gate is actually for a conditional phase gating to label the target state $|0\rangle_1|1\rangle_2$. Atoms are fixed in the optical cavity, as shown in figure 1, for an atom $j$ with five-level configuration, where levels $|0\rangle_j$ and $|\sigma\rangle_j$ are coupled to the excited state $|2\rangle_j$ by two lasers with Rabi frequencies $\Omega_{0,j}$ and $\Omega_{\sigma,j}$ respectively, while the level $|1\rangle_j$ is coupled to state $|2\rangle_j$ by the cavity mode with the coupling constant $g_j$. These three couplings are needed to construct a two-qubit or even a multi-qubit gate. The couplings of the levels $|0\rangle_j$, $|\sigma\rangle_j$, and $|1\rangle_j$ to another excited state $|3\rangle_j$ can be used to perform single qubit rotation as in [3].

We encode the qubits in the levels $|0\rangle_j$ and $|1\rangle_j$, and for simplicity we focus our discussion on the case of two atoms, although our case is extendable to multi-atom cases. So our task is to accomplish $I_{|0\rangle_j|0\rangle_2}$, i.e., adding a minus sign to the target states $|i\rangle_1|j\rangle_2$ where $i, j$ being 0, 1, respectively and the subscripts are for different atoms. Considering the pulses regarding $\Omega_{0,1}$ and $\Omega_{\sigma,2}$, the Hamiltonian is given by (assuming $\hbar = 1$)

$$H = \Omega_{0,1}|2\rangle_1\langle 0| + \Omega_{\sigma,2}|2\rangle_2\langle \sigma| + \sum_{m=1,2} g_m a|2\rangle_m\langle m| + H.c., \quad (1)$$
where $a$ is the annihilation operator for the cavity mode. As the cavity is initially in the vacuum state $|0\rangle_0|0\rangle_2|0\rangle_0$ and $|1\rangle_1|1\rangle_2|0\rangle_0$ are unaffected by the Hamiltonian, while $|0\rangle_1|0\rangle_2|0\rangle_0$ and $|0\rangle_1|1\rangle_2|0\rangle_0$ are associated with the Hamiltonian’s two dark states (i.e., eigenstates with zero eigenvalues) $|D_{01}\rangle \propto g_1|0\rangle_1|0\rangle_2|0\rangle - \Omega_0|1\rangle_1|0\rangle_2|1\rangle$, $|D_{10}\rangle \propto g_1\Omega_{a,2}|0\rangle_1|1\rangle_2|0\rangle + g_2\Omega_{a,1}|1\rangle_1|\sigma_2\rangle|0\rangle - \Omega_0|1\rangle_1|\sigma_2\rangle|1\rangle$ respectively. The procedure consists of two STIRAP processes: (i) in the first STIRAP, $\Omega_{a,2}$ precedes $\Omega_{a,1}$; (ii) the second STIRAP is a reverse process of the first one but with the phase regarding $\Omega_{a,2}$ added by $\pi$. In the adiabatic limit and on the condition that the Berry phase is equal to zero, we have [5] (assuming $\Omega_{a,1}$, $\Omega_{a,2}$ having the same phase in (i) and $g_1 = g_2 = g$)

\[
\begin{align*}
&|0\rangle_1|0\rangle_2|0\rangle \rightarrow (i) |0\rangle_1|0\rangle_2|0\rangle \rightarrow |0\rangle_1|0\rangle_2|0\rangle, \\
&|0\rangle_1|1\rangle_2|0\rangle \rightarrow (i) |1\rangle_1|\sigma_2\rangle|0\rangle \rightarrow -|0\rangle_1|1\rangle_2|0\rangle, \\
&|1\rangle_1|0\rangle_2|0\rangle \rightarrow (i) |1\rangle_1|0\rangle_2|0\rangle \rightarrow |1\rangle_1|0\rangle_2|0\rangle, \\
&|1\rangle_1|1\rangle_2|0\rangle \rightarrow (i) |1\rangle_1|1\rangle_2|0\rangle \rightarrow |1\rangle_1|1\rangle_2|0\rangle.
\end{align*}
\]

(2)

Thus the conditional phase gate for labelling the target state $|0\rangle_1|1\rangle_2$ is generated. All the other conditional phase gates can be generated by adding the NOT gate on both sides of the above two STIRAP processes. It is easy to see $I_{01}|0\rangle_0 = \sigma_{x,2}I_{01}|0\rangle_1\sigma_{x,2}$, $I_{10}|0\rangle_0 = \sigma_{x,1}\sigma_{x,2}I_{01}|0\rangle_1\sigma_{x,1}$, $I_{11}|0\rangle_0 = \sigma_{x,1}I_{01}|0\rangle_1$, where $\sigma_{x,i}$ ($i = 1, 2$) is the NOT gate acting on the $i$th atom, transforming states as $|0\rangle_i \rightarrow |1\rangle_i$, $|1\rangle_i \rightarrow |0\rangle_i$. The NOT gate $\sigma_{x,i}$ can be obtained by coupling $|0\rangle_i, |\sigma_i\rangle$ and $|1\rangle_i$ with the excited state $|3\rangle_i$, by three STIRAP processes: (1) $\Omega_{x,i}$ precedes $\Omega_{1,i}$; (2) $\Omega_{2,i}$ precedes $\Omega_{0,i}$; (3) $\Omega_{0,i}$ precedes $\Omega_{2,i}$. The dark states associated with the above three steps are $|D^{(1)}\rangle \propto \Omega_{x,i}|1\rangle_i = \Omega_{1,i}|\sigma_i\rangle$, $|D^{(2)}\rangle \propto \Omega_{1,i}|0\rangle_i - \Omega_{0,i}|1\rangle_i$, $|D^{(3)}\rangle \propto \Omega_{0,i}|\sigma_i\rangle - \Omega_{x,i}|0\rangle_i$, respectively. In the adiabatic limit and with the condition of zero Berry phase, we obtain in the case of the two pulses in each STIRAP with $\pi$ phase difference,

\[
\begin{align*}
&|0\rangle_i \rightarrow (i) |0\rangle_i \rightarrow |2\rangle_i \rightarrow |3\rangle_i, \\
&|1\rangle_i \rightarrow (i) |\sigma_i\rangle \rightarrow |2\rangle_i \rightarrow |3\rangle_i \rightarrow |0\rangle_i.
\end{align*}
\]

(3)

As a result, we realize the NOT gate. Alternatively, the NOT gate can also be reached by single qubit rotation as in [3], which also needs six pulses.

Figure 1. The level configuration for atom $j$, where the qubits are encoded in levels $|0\rangle_j$ and $|1\rangle_j$. Except that levels $|1\rangle_j$ and $|2\rangle_j$ are coupled resonantly by the cavity mode, all other arrows indicate resonant couplings by different lasers with the corresponding Rabi frequencies labelled nearby.
3. Conditional phase gate with decay

We introduce the spontaneous emission and the cavity decay from now on and will numerically analyse their effects on the conditional phase gate by the quantum jump approach. As long as there is no photon leakage either from the atomic excited state or from the cavity mode during the gating period, the Hamiltonian in equation (1) becomes

$$H_{\text{cond}} = \begin{bmatrix} 0 & \Omega_{\sigma,1}^* & g & 0 & 0 & 0 & 0 & 0 \\ \Omega_{\sigma,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ g^* & -i\frac{\Gamma}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Omega_{\sigma,2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega_{\sigma,2}^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Note that $H_{\text{cond}}$ is non-Hermitian, the norm of a state vector evolving under the corresponding Schrödinger equation decreases in general with time. For an arbitrary initial state $|\psi(t_i)\rangle$,

$$P_{\text{suc}}(t) = \langle \psi(t_i) | U_{\text{cond}}^\dagger(t, t_i) U_{\text{cond}}(t, t_i) | \psi(t_i) \rangle$$

defines the probability that no photon has been emitted at time $t$, where $U_{\text{cond}}$ is the time evolution operator for $H_{\text{cond}}$ and the corresponding normalized state vector is

$$|\psi(t)\rangle = U_{\text{cond}}(t, t_i) |\psi(t_i)\rangle / \sqrt{P_{\text{suc}}(t)}.$$

The gate fidelity is

$$F = |\langle \psi(\infty) | I_{|01\rangle|1\rangle} |\psi(t_i)\rangle|^2.$$

In our numerical simulation, we choose $\Omega_{0,1,\text{max}} = \Omega_{\sigma,2,\text{max}} = 0.16g$ in order to reduce the population of single-photon cavity states during the gating process, i.e., $|1\rangle_1|0\rangle_2|1\rangle$ and $|1\rangle_1|1\rangle_2|1\rangle$. Similar to [3, 5], we suppose that the pulses have the Gaussian shape $\exp[-(t - T/2 \pm t_0)^2/2\tau^2]$ with $T = 200/g$, $t_0 = 30/g$, and $\tau = 40/g$. In figure 2, we show the evolution of the initial state $\frac{1}{2}(|0\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2)|0\rangle$. Ideally, the final state would be $\frac{1}{2}(|0\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2)|0\rangle$. By our numerical calculation, with $k = 0.1g$ and $\Gamma = 0.1g$, the probability amplitudes for states $|0\rangle_1|0\rangle_2|0\rangle$, $|0\rangle_1|1\rangle_2|0\rangle$, $|1\rangle_1|0\rangle_2|0\rangle$, $|1\rangle_1|1\rangle_2|0\rangle$ are 0.4513, $-0.4523$, 0.5438, 0.5438 respectively, and the fidelity is 99.12% while the success probability is 84.55%. The reason that the amplitudes of the components $|1\rangle_1|0\rangle_2|0\rangle$ and $|1\rangle_1|1\rangle_2|0\rangle$ are increased compared with those in the initial state is that they did not participate in the evolution, while the other two
components dissipated in the evolution. So the relative weights of $|1\rangle_1|0\rangle_2$ and $|1\rangle_1|1\rangle_2$ are enlarged in the normalized final state.

In order to see how the success probability $P_{\text{succ}}$ and the gate fidelity $F$ depend on $\kappa$ and $\Gamma$, we plot $P_{\text{succ}}$ and $F$ in each of figures 3–5, for the evolution from the initial states $|0\rangle_1|1\rangle_2|0\rangle_0, |0\rangle_1|0\rangle_2|0\rangle_0, \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2 + |1\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2)|0\rangle_0$, respectively. From figure 3, we see that $P_{\text{succ}}$ is less affected by $\kappa$ than by $\Gamma$, while it is almost unaffected by $\Gamma$ in figure 4. Since, for the evolution of $|0\rangle_1|1\rangle_2|0\rangle_0$, the occupation probability of $|1\rangle_1|1\rangle_2|1\rangle$ is several times smaller than that of the excited atomic state, the spontaneous emission from the excited state $|2\rangle$ is more detrimental than cavity decay in figure 3. While for the evolution of $|0\rangle_1|0\rangle_2|0\rangle_0$ in figure 4, as the occupation in level $|2\rangle$ is negligible, the cavity decay is the main detrimental effect in this case. For an arbitrary initial state, $|\psi(t_i)\rangle = \alpha|0\rangle_1|1\rangle_2|0\rangle_0 + \beta|0\rangle_1|0\rangle_2|0\rangle_0 + \gamma|1\rangle_1|0\rangle_2|0\rangle_0 + \epsilon|1\rangle_1|1\rangle_2|0\rangle_0$, we have $U_{\text{cond}}(\infty, t_i)|\psi(t_i)\rangle = \alpha\sqrt{P_{\text{succ}1}}|\psi_{010}\rangle + \beta\sqrt{P_{\text{succ}2}}|\psi_{000}\rangle + \gamma|1\rangle_1|0\rangle_2|0\rangle_0 + \epsilon|1\rangle_1|1\rangle_2|0\rangle_0$, where $|\psi_{010}\rangle$ and $|\psi_{000}\rangle$ are, respectively, the final states after the time evolution of the component states $|0\rangle_1|1\rangle_2|0\rangle_0, |0\rangle_1|0\rangle_2|0\rangle_0$, and $P_{\text{succ}1}, P_{\text{succ}2}$ are their corresponding success probabilities. Thus the success probability for any initial state is $P_{\text{succ}} = |\alpha|^2 P_{\text{succ}1} + |\beta|^2 P_{\text{succ}2} + |\gamma|^2 + |\epsilon|^2$. Usually both $P_{\text{succ}1}$ and $P_{\text{succ}2}$ are smaller than 1. As a result, for the same $\Gamma$ and $\kappa$, the less the initial population in components $|0\rangle_1|1\rangle_2|0\rangle_0$ and $|0\rangle_1|0\rangle_2|0\rangle_0$, the higher the success probability for the gating process. Moreover, from figures 3 and 4, we know that the

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Time evolution with the initial state $\frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2)|0\rangle_0$, where $k = 0.1g$ and $\Gamma = 0.1g$. (a) Pulse shapes and sequences for $\Omega_{\text{R1}}$ and $\Omega_{\text{R2}}$. (b) Probability amplitudes for states $|0\rangle_1|1\rangle_2|0\rangle_0, |1\rangle_1|0\rangle_2|0\rangle_0, |1\rangle_1|0\rangle_2|0\rangle_0, |1\rangle_1|1\rangle_2|0\rangle_0$. (c) Probability of all the states containing an excited atomic state, i.e., $P_{\text{exc}} = P_{|0\rangle_1|1\rangle_2|0\rangle_0} + P_{|0\rangle_1|0\rangle_2|0\rangle_0} + P_{|0\rangle_1|0\rangle_2|0\rangle_0}$, and probability of all the states containing one photon, i.e., $P_{\text{ph}} = P_{|1\rangle_1|1\rangle_2|1\rangle_0} + P_{|1\rangle_1|0\rangle_2|1\rangle_0}$. (d) Success probability $P_{\text{succ}}$.}
\end{figure}
fidelity is very high for the initial state $|0\rangle_1|1\rangle_2|0\rangle$, which implies $|\psi_{00}\rangle \simeq -|0\rangle_1|1\rangle_2|0\rangle$, $|\psi_{000}\rangle \simeq |0\rangle_1|0\rangle_2|0\rangle$, while the relative weight ratio of the four component states $|0\rangle_1|1\rangle_2|0\rangle$, $|0\rangle_1|0\rangle_2|0\rangle$, $|1\rangle_1|0\rangle_2|0\rangle$, $|1\rangle_1|1\rangle_2|0\rangle$ is approximately $(-\alpha \sqrt{P_{\text{succ}1}}, \beta \sqrt{P_{\text{succ}2}}, \gamma, \epsilon)$, deviated from the ideal case $(-\alpha, \beta, \gamma, \epsilon)$. Obviously, the fidelity for the state $|\psi(t_i)\rangle$ would be very high in the following two cases: (1) $\gamma = \epsilon = 0$, $P_{\text{succ}1} \simeq P_{\text{succ}2}$; (2) $\gamma$, $\epsilon \neq 0$, $P_{\text{succ}1} \simeq P_{\text{succ}2} \simeq 1$. Therefore, it is easy to understand in figure 5 why $P_{\text{succ}}$ is increased in most regions while $F$ is decreased by comparing with those in figures 3 and 4 with the same $\Gamma$ and $\kappa$.

We also did the simulation for the NOT gate (not shown), which shows that with intense short pulses and the vacuum cavity state, the fidelity and the success probability are both near
Alternative scheme for a two-qubit conditional phase gate by adiabatic passage under dissipation

Figure 5. Dependence of the success probability $P_{\text{suc}}$ (upper subplot) and fidelity $F$ (lower subplot) on $\kappa$ and $\Gamma$ for the initial state $\frac{1}{2}(|0\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2)|0\rangle$.

unity even under large $\kappa$ and $\Gamma$. So the success probability and the fidelity for the other three conditional phase gates would have almost the same dependence on $\kappa$ and $\Gamma$ as in the above figures.

4. Discussion and conclusion

Combined with the single qubit Hadamard gate, which is achievable by a series of single qubit rotations as in [3], the performance of the two-qubit Grover search by conditional phase gates is straightforward [14, 15]. Our scheme has the following advantages compared with [15]. Atoms are fixed in the cavity, which is easier to control than in the case of atoms going through the cavity with a certain velocity. Moreover, all the operations are based on STIRAP, so they are very robust to fluctuation of experimental parameters.

We now briefly discuss the experimental feasibility. It has been experimentally reported that $g = 34 \times 2\pi$ MHz, $\kappa = 4.1 \times 2\pi$ MHz and $\Gamma = 2.6 \times 2\pi$ MHz [16]. If we adopt the above numbers, i.e., $\kappa/g \simeq 0.12$, $\Gamma/g \simeq 0.077$, the operation time is of the order of $10^{-6}$ s, as can be seen from figure 2(a). In this case, for the initial states $|0\rangle_1|1\rangle_2|0\rangle$, $|0\rangle_1|0\rangle_2|0\rangle$ and $\frac{1}{2}(|0\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2)|0\rangle$, we may obtain $P_{\text{suc}} \simeq 73.1\%$, 63.8\%, 84.2\%, and $F \simeq 99.9\%$, 100\%, 99.0\%, respectively. The fidelities are high enough, while the success probabilities need to be improved by reducing $\kappa/g$ and $\Gamma/g$ in future experiments.

We have noticed a previous smart scheme [17] for a two-qubit phase gate, based on the STIRAP technique, by almost the same configuration as in the present paper. Assisted by the Zeno effect, the state involved in adiabatic evolution in [17] is immune to both the cavity decay and spontaneous emission. From their simulation for a STIRAP process with fixed parameters $k = 0.1g$ and $\Gamma = 0.1g$, we see the variation of the maximum Rabi frequency and the detuning of the corresponding transitions of the atomic levels can lead to maximum success probability and high fidelity for the population transfer$^4$. In contrast, we did simulations in

$^4$ The simulation in [17] only involved one STIRAP process for population transfer. As a result, the optimal success probability and fidelity in the population transfer with fixed parameters $k = 0.1g$ and $\Gamma = 0.1g$ are 0.858, 0.999, respectively. For a complete controlled phase gate, however, there should be two successive STIRAP processes.
our work for successive two STIRAP processes without any change of the detuning and the pulse Rabi frequency. Given all the facts above and by comparing the simulation results, we consider that the proposal presented in [5] is not less effective than that in [17] in the presence of dissipation. Moreover, [5] is applicable to multi-qubit phase gates, which would be used in the multi-qubit Grover search algorithm. But with more qubits involved, the dark states containing components with more photons would be more sensitive to dissipation. However, with a very high Q cavity, [5] can be in principle feasible in the case of multi-qubit phase gates.

In summary, we have shown the generation of four conditional phase gates required by a two-qubit Grover search algorithm by means of the STIRAP technique, in the presence of dissipation. To meet the requirement of adiabatic conditions, the multiplication of the Rabi frequency and the implementation time should be much larger than unity. But the larger the Rabi frequency, the more the occupation probability in the cavity mode, yielding greater probability of the cavity decay. On the other hand, a lower Rabi frequency would require a longer implementation time, which would also enhance the probability of dissipation. In this sense, it is important to find suitable values of the above parameters. The numbers are fixed in our simulation, which would not be optimal for all the combinative values of $k/g$ and $\Gamma/g$. But we have tried to use numbers which could result in relatively large values of $P_{\text{succ}}$ and $F$ in the whole range of $k/g$ and $\Gamma/g$. We believe that our numerical investigation for the success probability and the fidelity of the two-qubit conditional phase gate subject to spontaneous emission and cavity decay would be useful for real experiments.

Acknowledgments

Z J Deng is grateful to Weibin Li, T Y Shi and X L Zhang for their warmhearted help. This work is partly supported by the National Natural Science Foundation of China under grant nos 10474118 and 60490280, by Hubei Provincial Funding for Distinguished Young Scholars, and partly by the National Fundamental Research Program of China under grant no 2005CB724502.

References

[1] Bergmann K, Theuer H and Shore B W 1998 Rev. Mod. Phys. 70 1003
[2] Carroll C E and Hioe F T 1990 Phys. Rev. A 42 1522
[3] Kis Z and Renzoni F 2002 Phys. Rev. A 65 032318
[4] Sangouard N, Lacour X, Guérin S and Jauslin H R 2006 Eur. Phys. J. D 37 451
[5] Goto H and Ichimura K 2004 Phys. Rev. A 70 012305
[6] Zheng S-B 2005 Phys. Rev. Lett. 95 080502
[7] Lacour X, Sangouard N, Guérin S and Jauslin H R 2006 Phys. Rev. A 73 042321
[8] Sangouard N, Lacour X, Guérin S and Jauslin H R 2005 Phys. Rev. A 72 062309
[9] Marr C, Beige A and Rempe G 2003 Phys. Rev. A 68 033817
Kis Z and Paspalakis E 2004 Phys. Rev. B 69 024510
Amniat-Talab M, Guérin S, Sangouard N and Jauslin H R 2005 Phys. Rev. A 71 023805
[10] Cirac J I and Zoller P 1995 Phys. Rev. Lett. 74 4091
Sørensen A and Mølmer K 1999 Phys. Rev. Lett. 82 1971
Juan E, Plenio M B and Jonathan D 2002 Phys. Rev. A 65 050302

For given values of $k/g$ and $\Gamma/g$, we may vary $\Omega_{0,1,\text{max}}/g$, $\Omega_{0,2,\text{max}}/g$, $g_{0}g_{\tau}$, the transition detunings to get the optimal values of $P_{\text{succ}}$ and $F$. However, our purpose is to illustrate how $P_{\text{succ}}$ and $F$ depend on $k/g$ and $\Gamma/g$. It is troublesome to adjust all the above-mentioned parameters to find the optimal values of $P_{\text{succ}}$ and $F$ for each combinative values of $k/g$ and $\Gamma/g$. On the contrary, with fixed parameters, we can analyse the underlying physics of why $P_{\text{succ}}$ and $F$ vary with $k/g$ and $\Gamma/g$ as plotted in figures 3–5. So we fix the detunings to be zero and all the parameters for the pulse configuration to be the ones as plotted in figure 2(a).
Alternative scheme for a two-qubit conditional phase gate by adiabatic passage under dissipation.

[11] Zanardi P and Rasetti M 1999 Phys. Lett. A 264 94  
Liu Y, Zhang P, Zanardi P and Sun C P 2004 Phys. Rev. A 70 032330  
Zhang P, Wang Z D, Sun J D and Sun C P 2005 Phys. Rev. A 71 042301

[12] Plenio M B and Knight P L 1998 Rev. Mod. Phys. 70 101

Scully M O and Zubairy M S 1997 Quantum Optics (Cambridge: Cambridge University Press)

[13] Deng Z J, Feng M and Gao K L 2006 Phys. Rev. A 73 014302  
Feng X L, Zhang M and Gao K L 2006 J. Phys. B: At. Mol. Opt. Phys. 39 3211  
Chen C Y, Zhang X L, Deng Z J, Gao K L and Feng M 2006 Phys. Rev. A 71 032328

[14] Grover L K 1997 Phys. Rev. Lett. 79 325

Feng M 2001 Phys. Rev. A 63 052308

[15] Yamaguchi F, Milman P, Brune M, Raimond J M and Haroche S 2002 Phys. Rev. A 66 010302  
Deng Z J, Feng M and Gao K L 2005 Phys. Rev. A 72 034306

[16] Boca A, Miller R, Birnbaum K M, Boozet A D, McKeever J and Kimble H J 2004 Phys. Rev. Lett. 93 233603

[17] Pachos J and Walther H 2002 Phys. Rev. Lett. 89 187903