Higgs production at NNLOPS

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49th Rencontres de Moriond, QCD, La Thuile, 23 March 2014
Why going NNLO?

- sometimes NLO not enough:
  - large NLO/LO “K-factor”
    [perturbative expansion “not (yet) stable”]
  - very high precision needed
    \( \Rightarrow \) NNLO

- NNLO is the frontier:
  first \( 2 \rightarrow 2 \) NNLO computations in 2012-13!

- paramount example: Higgs production
  [corrections so large that NNNLO relevant - see next talk]

\[ \text{Anastasiou et al., '04-'05} \]
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  - very high precision needed  
    ⇒ NNLO

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aim: build an event generator that is NNLO accurate (NNLOPS)

- the approach presented here works for “$2 \rightarrow 1$” processes at the LHC.
- In 1309.0017 we used it for Higgs production.
1. $H+j @ NLO, H+jj @ LO \Rightarrow$ use $H+j @ NLOPS (POWHEG)$

$$d\sigma_{POWHEG} = d\Phi_n \tilde{B}_{NLO}(\Phi_n) \left\{ \Delta(\Phi_n; k_T^{\min}) + \Delta(\Phi_n; k_T) \frac{\alpha_s}{2\pi} \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} \right\} d\Phi_r$$

[+ $p_T$-vetoing subsequent emissions, to avoid double-counting]
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[+ p_T\text{-vetoing subsequent emissions, to avoid double-counting}] \\

\bar{B}_{\text{NLO}}(\Phi_n) d\Phi_n = \alpha_s^3(\mu_R) \left[ B + \alpha_s^{(\text{NLO})} V(\mu_R) + \alpha_s^{(\text{NLO})} \int d\Phi_r R \right] d\Phi_n \\

H+j is a 2-scales problem (\(\rightarrow\) choice of \(\mu\) not unique)
1. \( H+j @ NLO, H+j j @ LO \) \( \Rightarrow \) use \( H+j @ NLOPS (\text{POWHEG}) \)

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\]

\( H+j \) is a 2-scales problem (\( \rightarrow \) choice of \( \mu \) not unique)

\( \text{\textbullet} \) want to reach NNLO accuracy for e.g. \( y_H \), i.e. when fully inclusive over QCD radiation
- need to allow the 1st jet to become unresolved
- the above approach needs to be modified: as it stands, \( \tilde{B}(\Phi_n) \) is not finite when \( q_T \rightarrow 0 \)!
2. **integrate** over phase space regions where $H$ is produced with arbitrarily soft/collinear jet (i.e. finite results when integrating over all $q_T$ spectrum)

**MiNLO: Multiscale Improved NLO**

- original goal: method to **a-priori** choose scales in multijet NLO computation (where hierarchy among scales can spoil accuracy)
- how: correct weights of different NLO terms with CKKW-inspired approach:
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  - for all PS points, build the “more-likely” shower history that would have produced it (can be done by clustering kinematics with $k_T$-algo)
  - correct original NLO including $\alpha_S$ couplings evaluated at nodal scales and Sudakov FFs
  - make sure that **NLO accuracy is not spoiled**!
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\bar{B}_{\mathrm{NLO}} = \alpha_S^3(\mu_R) \left[ B + \alpha_S^{(\mathrm{NLO})} V(\mu_R) + \alpha_S^{(\mathrm{NLO})} \int d\Phi_R R \right]
\]

\[
\bar{B}_{\mathrm{MiNLO}} = \alpha_S^2(m_h) \alpha_S(q_T) \Delta_g^2(q_T, m_h) \left[ B \left( 1 - 2\Delta_g^{(1)}(q_T, m_h) \right) + \alpha_S^{(\mathrm{NLO})} V(\bar{\mu}_R) + \alpha_S^{(\mathrm{NLO})} \int d\Phi_R R \right]
\]

\[
\bar{\mu}_R = (m_h^2 q_T)^{1/3}
\]

\[
\log \Delta_f(q_T, m_h) = -\int_{q_T^2}^{m_h^2} \frac{dq^2}{q^2} \frac{\alpha_S(q^2)}{2\pi} \left[ A_f \log \frac{m_h^2}{q^2} + B_f \right]
\]

\[
\Delta_f^{(1)}(q_T, m_h) = -\frac{\alpha_S^{(\mathrm{NLO})}}{2\pi} \left[ \frac{1}{2} A_{1,f} \log^2 \frac{m_h^2}{q_T^2} + B_{1,f} \log \frac{m_h^2}{q_T^2} \right]
\]

\[
\mu_F = q_T
\]
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\bar{B}_{\text{MiNLO}} = \alpha_s^2(m_h) \alpha_s(q_T) \Delta^2_g(q_T, m_h) \left[ B \left( 1 - 2\Delta_g^{(1)}(q_T, m_h) \right) + \alpha_s^{(\text{NLO})} V(\mu_R) + \alpha_s^{(\text{NLO})} \int d\Phi_R R \right]
\]

- HJ-MiNLO yields finite results also when 1st jet is unresolved ($q_T \to 0$)
- $\bar{B}_{\text{MiNLO}}$ ideal to extend validity of $H+j$ POWHEG

Sudakov FF included on $H+j$ Born kinematics
accuracy of HJ-MiNLO for inclusive observables carefully investigated [Hamilton, Nason, Oleari, Zanderighi, 1212.4504]

HJ-MiNLO describes inclusive observables at order $\alpha_S$ (relative to inclusive H @ LO) to reach genuine NLO when inclusive, “spurious” terms must be of relative order $\alpha_S^2$, i.e.

$$O_{\text{HJ-MiNLO}} = O_{\text{H@NLO}} + O(\alpha_S^{b+2})$$

if $O$ is inclusive ($H@LO \sim \alpha_S^b$).

“Original MiNLO” contains ambiguous $O(\alpha_S^{b+3/2})$ terms.
“Improved” MiNLO & NLOPS merging

- accuracy of HJ–MiNLO for inclusive observables carefully investigated
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- HJ–MiNLO describes inclusive observables at order $\alpha_S$ (relative to inclusive H @ LO)
- to reach genuine NLO when inclusive, “spurious” terms must be of relative order $\alpha_S^2$, i.e.
  \[ O_{\text{HJ–MiNLO}} = O_{\text{H@NLO}} + O(\alpha_S^{b+2}) \]
  \[ (b = 2 \text{ for } gg \to H) \]
  if $O$ is inclusive ($H@LO \sim \alpha_S^b$).

- “Original MiNLO” contains ambiguous $O(\alpha_S^{b+3/2})$ terms.

- Possible to improve HJ–MiNLO such that $H @ NLO$ is recovered ($NLO^{(0)}$), without spoiling NLO accuracy of $H+j$ ($NLO^{(1)}$).
  - proof based on careful comparisons of MiNLO with general resummation formula
  - need to include $B_2$ in MiNLO-Sudakos
  - need to evaluate $\alpha_S^{(NLO)}$ in HJ–MiNLO at scale $q_T$, and $\mu F = q_T$

Effectively as if we merged NLO$^{(0)}$ and NLO$^{(1)}$ samples, without merging different samples (no merging scale used: there is just one sample).

Other NLOPS-merging approaches: [Hoeche,Krauss, et al.,1207.5030] [Frederix,Frixione,1209.6215]
[Lonnblad,Prestel,1211.7278 - Platzer,1211.5467] [Alioli,Bauer, et al.,1211.7049] [Hartgring,Laenen,Skands, 1303.4974]
**HJ-MiNLO** differential cross section \( \frac{d\sigma}{dy}_{HJ-MiNLO} \) is NLO accurate

\[
W(y) = \frac{\left( \frac{d\sigma}{dy} \right)_{NNLO}}{\left( \frac{d\sigma}{dy} \right)_{HJ-MiNLO}} = \frac{c_2 \alpha_S^2 + c_3 \alpha_S^3 + c_4 \alpha_S^4}{c_2 \alpha_S^2 + c_3 \alpha_S^3 + d_4 \alpha_S^4} \approx 1 + \frac{c_4 - d_4}{c_2} \alpha_S^2 + \mathcal{O}(\alpha_S^3)
\]

thus, reweighting each event with this factor, we get NNLO+PS
- obvious for \( y_H \), by construction
- \( \alpha_S^4 \) accuracy of \( HJ-MiNLO^* \) in 1-jet region not spoiled, because \( W(y) = 1 + \mathcal{O}(\alpha_S^2) \)
- if we had NLO\(^{(0)}\) + \( \mathcal{O}(\alpha_S^{2+3/2}) \), 1-jet region spoiled because

\[
[NLO^{(1)}]_{NNLOPS} = NLO^{(1)} + \mathcal{O}(\alpha_S^{4.5})
\]
HJ–MiNLO* differential cross section \((d\sigma/dy)_{\text{HJ–MiNLO}}\) is NLO accurate

\[
W(y) = \frac{(d\sigma/dy)_{\text{NNLO}}}{(d\sigma/dy)_{\text{HJ–MiNLO}}} = \frac{c_2\alpha_S^2 + c_3\alpha_S^3 + c_4\alpha_S^4}{c_2\alpha_S^2 + c_3\alpha_S^3 + d_4\alpha_S^4} \approx 1 + \frac{c_4 - d_4}{c_2}\alpha_S^2 + O(\alpha_S^3)
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\[
[NLO^{(1)}]_{\text{NNLOPS}} = \text{NLO}^{(1)} + O(\alpha_S^{4.5})
\]

* Variants for \(W\) are possible:

\[
W(y, p_T) = h(p_T) \int d\sigma_A^{\text{NNLO}} \delta(y - y(\Phi)) \int d\sigma_B^{\text{MiNLO}} \delta(y - y(\Phi)) + (1 - h(p_T))
\]

\[
d\sigma_A = d\sigma \ h(p_T), \quad d\sigma_B = d\sigma \ (1 - h(p_T)), \quad h = \frac{(\beta m_H)^2}{(\beta m_H)^2 + p_T^2}
\]

* \(h(p_T)\) controls where the NNLO/NLO K-factor is spread
* \(\beta\) cannot be too small, otherwise resummation spoiled
In 1309.0017 we used

\[ W(y, p_T) = h(p_T) \frac{\int d\sigma_{\text{NNLO}} \delta(y - y(\Phi)) - \int d\sigma_{\text{MiNLO}} \delta(y - y(\Phi))}{\int d\sigma_{\text{MiNLO}} \delta(y - y(\Phi))} + (1 - h(p_T)) \]

\[ d\sigma_A = d\sigma h(p_T), \quad d\sigma_B = d\sigma (1 - h(p_T)), \quad h = \frac{(\beta m_H)^2}{(\beta m_H)^2 + p_T^2} \]

- one gets exactly \((d\sigma/dy)_{\text{NNLOPS}} = (d\sigma/dy)_{\text{NNLO}}\) (no \(\alpha_S^5\) terms)
- we used \(h(p_{T1})\)

inputs for following plots:
- results are for 8 TeV LHC
- scale choices: NNLO input with \(\mu = m_H/2\), HJ-MiNLO “core scale” \(m_H\) (other powers are at \(q_T\))
- PDF: everywhere MSTW2008 NNLO
- NNLO always from HNNLO
- 6M events reweighted at the LH level
- plots after \(k_T\)-ordered PYTHIA 6 at the PS level (hadronization and MPI switched off)
- NNLO with $\mu = m_H / 2$, HJ-MiNLO “core scale” $m_H$
- $(7_Mi \times 3_{NN})$ pts scale var. in NNLOPS, 7pts in NNLO

Notice: band is 10% (at NLO would be $\sim$ 20-30%)

[Until and including $\mathcal{O}(\alpha_S^4)$, PS effects don’t affect $y_H$ (first 2 emissions controlled properly at $\mathcal{O}(\alpha_S^4)$ by MiNLO+POWHEG)]
\( \beta = \infty \) (W indep. of \( p_T \)) \hspace{1cm} \beta = 1/2

\[ \begin{array}{c}
\frac{d\sigma}{dp_T^H} [\text{pb/GeV}] \\
\end{array} \]

- **HQT**: NNLL+NNLO, \( \mu_R = \mu_F = m_H/2 \) [7pts], \( Q_{\text{res}} \equiv m_H/2 \) [HQT, Bozzi et al.]

- \( \beta = 1/2 \) & \( \infty \): uncertainty bands of HQT contain NNLOPS at low-/moderate \( p_T \)
- \( \beta = 1/2 \): HQT tail harder than NNLOPS tail (\( \mu_{HQT} < \mu_{\text{MiNLO}} \))
- \( \beta = 1/2 \): very good agreement with HQT resummation [“~ expected”, since \( Q_{\text{res}} \equiv m_H/2 \)]
\( \beta = \infty \) (W indep. of \( p_T \))

\[ \beta = 1/2 \]

- **HqT**: NNLL+NNLO, \( \mu_R = \mu_F = m_H/2 \) [7pts], \( Q_{res} \equiv m_H/2 \)
- \( \beta = 1/2 \): NNLOPS tail \( \rightarrow \) NLOPS tail [ \( W(y, p_T \gg m_H) \rightarrow 1 \) ]
  larger band (affected just marginally by NNLO, so it’s \( \sim \) genuine NLO band)
**NNLO+PS** \( (p_T^{j_1}) \)

\[
\varepsilon \left( p_T, \text{veto} \right) = \frac{\sum(p_T, \text{veto})}{\sigma_{\text{tot}}} = \frac{1}{\sigma_{\text{tot}}} \int d\sigma \theta \left( p_T, \text{veto} - p_T^{j_1} \right)
\]

- **JetVHeto**: NNLL resum, \( \mu_R = \mu_F = m_H/2 \) [7pts], \( Q_{\text{res}} \equiv m_H/2 \), (a)-scheme only
  
  [JetVHeto, Banfi et al.]

- nice agreement, differences never more than 5-6 %

Separation of \( H \to WW \) from \( t\bar{t} \) bkg: x-sec binned in \( N_{\text{jet}} \)

0-jet bin \( \Leftrightarrow \) jet-veto accurate predictions needed!
Conclusions

**NNLOPS:**
- **MiNLO-improved POWHEG** simulation allows to define a procedure to reach NNLOPS
- shown first results for Higgs production
- the code is public and can be found in the **POWHEG-BOX (V2)** repository

with this formalism NNLOPS doable for DY and $H+V$

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**MiNLO & improved MiNLO:**
- original motivation: assign scales and Sudakov FF in $B+n$ jets NLO computations
- ideal as starting point for **POWHEG**
- have shown results where MiNLO used on top of $pp \rightarrow H+j$. However procedure is more general (and indeed has been already used also on more complex cases)
- $B+j$ “improved” **MiNLO** allows to merge NLO$^{(0)}$ and NLO$^{(1)}$ samples, without the need of a merging scale
  - merging for higher multiplicity requires further study
Conclusions

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Thank you for your attention!
CKKW in a nutshell

- ME weight $B(\Phi_n) \Rightarrow \text{“most-likely” shower history (via } k_T\text{-algo): } Q > q_3 > q_2 > q_1 \equiv Q_0$

- New weight:

\[
\alpha^5_S(Q)B(\Phi_3) \rightarrow \alpha^2_S(Q)B(\Phi_3) \frac{\Delta_g(Q_0, Q)}{\Delta_g(Q_0, q_2)} \frac{\Delta_g(Q_0, Q)}{\Delta_g(Q_0, q_3)} \frac{\Delta_g(Q_0, q_3)}{\Delta_g(Q_0, q_1)} \\
\Delta_g(Q_0, q_2)\Delta_g(Q_0, q_2)\Delta_g(Q_0, q_3)\Delta_g(Q_0, q_1)\Delta_g(Q_0, q_1) \\
\alpha_S(q_1)\alpha_S(q_2)\alpha_S(q_3)
\]

where typically

\[
\log \Delta_f(q_T, Q) = -\int_{q_T^2}^{Q^2} \frac{dq^2}{q^2} \alpha_S(q^2) \left[ A_{1,f} \log \frac{Q^2}{q^2} + B_{1,f} \right]
\]

- Fill phase space below $Q_0$ with vetoed shower
Find “most-likely” shower history (via $k_T$-algo): $Q > q_3 > q_2 > q_1 \equiv Q_0$
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From CKKW to MiNLO

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- Evaluate $\alpha_S$ at nodal scales

$$\alpha^n_S(\mu_R) B(\Phi_n) \Rightarrow \alpha_S(q_1) \alpha_S(q_2) \ldots \alpha_S(q_n) B(\Phi_n)$$

* scale compensation requires $\mu^2_R = (q_1 q_2 \ldots q_n)^{2/n}$ in $V$
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- Sudakov FFs in internal and external lines of Born “skeleton”

$$B(\Phi_n) \Rightarrow B(\Phi_n) \times \{\Delta(Q_0, Q)\Delta(Q_0, q_i)\ldots\}$$

* Upon expansion, $O(\alpha_S^{n+1})$ (log) terms are introduced, and need to be removed

$$B(\Phi_n) \Rightarrow B(\Phi_n)\left(1 - \Delta^{(1)}(Q_0, Q) - \Delta^{(1)}(Q_0, q_i) + \ldots\right)$$

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$X + \text{jets cross-section finite without generation cuts}$

$\leftrightarrow \bar{B}$ with MiNLO prescription: ideal starting point for NLOPS (POWHEG) for $X + \text{jets}$
MiNLO: All $\alpha_S$ in Born term are chosen with CKKW (local) scales $q_1, \ldots, q_n$

$$\alpha_S^n(\mu_R)B \Rightarrow \alpha_S(q_1)\alpha_S(q_2)\ldots\alpha_S(q_n)B$$

- Normal NLO structure ($\mu = \mu_R$):

$$\sigma(\mu) = \underbrace{\alpha_S^n(\mu)B}_\text{Born} + \underbrace{\alpha_S^{n+1}(\mu)\left(C + nb_0 \log(\mu^2/Q^2)B\right)}_\text{Virtual} + \underbrace{\alpha_S^{n+1}(\mu)R}_\text{Real}$$

- Explicit $\mu$ dependence of virtual term as required by RG invariance:

$$\alpha_S^n(\mu')B = \left[\alpha_S(\mu) - nb_0\alpha_S^{n+1}(\mu)\log(\mu'^2/\mu^2)\right]B + \mathcal{O}(\alpha_S^{n+2})$$

$$\Rightarrow \text{Virtual}(\mu') = \text{Virtual}(\mu) + \alpha_S^{n+1}(\mu)nb_0 \log(\mu'^2/\mu^2)B + \mathcal{O}(\alpha_S^{n+2})$$

$$\Rightarrow \sigma(\mu') - \sigma(\mu) = \mathcal{O}(\alpha_S^{n+2})$$

- In MiNLO “scale compensation” kept if

$$\left(C + nb_0 \log(\mu_R^2/Q^2)B\right) \Rightarrow \left(C + nb_0 \log(\bar{\mu}_R^2/Q^2)B\right)$$

with $\bar{\mu}_R^2 = (q_1 q_2 \ldots q_n)^2/n$
“Improved” MiNLO & NLOPS merging

- Resummation formula

$$\frac{d\sigma}{dq_T^2 dy} = \sigma_0 \frac{d}{dq_T^2} \left\{ \left[ C_{ga} \otimes f_a \right](x_A, q_T) \times \left[ C_{gb} \otimes f_b \right](x_B, q_T) \times \exp S(q_T, Q) \right\} + R_f$$

$$S(q_T, Q) = -2 \int_{q_T^2}^{Q^2} \frac{dq^2}{q^2} \frac{\alpha_S(q^2)}{2\pi} \left[ A_f \log \frac{Q^2}{q^2} + B_f \right]$$

- If $C_{ij}^{(1)}$ included and $R_f$ is LO$^{(1)}$, then upon integration we get NLO$^{(0)}$

- Take derivative, then compare with MiNLO:

$$\sim \sigma_0 \frac{1}{q_T^2} \left[ \alpha_S, \alpha_S^2, \alpha_S^3, \alpha_S^4, \alpha_S L, \alpha_S^2 L, \alpha_S^3 L, \alpha_S^4 L \right] \exp S(q_T, Q) + R_f \quad L = \log(Q^2/q_T^2)$$

- Highlighted terms are needed to reach NLO$^{(0)}$:

$$\int_{q_T^2}^{Q^2} \frac{dq_T^2}{q_T^2} L^m \alpha_S^n(q_T) \exp S \sim \left( \alpha_S(Q^2) \right)^{n-(m+1)/2}$$

- If I don’t include $B_2$ in MiNLO $\Delta_g$, I miss a term $(1/q_T^2)\alpha_S^2 B_2 \exp S$

- Upon integration, violate NLO$^{(0)}$ by a term of relative $O(\alpha_S^{3/2})$

- “Wrong” scale in $\alpha_S^{(NLO)}$ in MiNLO produces again same error

Alternative proof also available in the paper.
MiNLO details

Few technicalities for original MiNLO:

- $\mu_F = Q_0$ (as in CKKW)
- Cluster with CKKW also $V$ and $R$ kinematics
  - Actual implementation uses FKS mapping for first cluster of $\Phi_{n+1}$
  - Ignore CKKW Sudakov for 1st clustering of $\Phi_{n+1}$ (inclusive on extra radiation)
- Some freedom in choice of $\alpha_S^{(\text{NLO})}$ (entering $V$, $R$ and $\Delta^{(1)}$):  
  * suggested average of LO $\alpha_S$
  * not free for “improved” MiNLO
- Used full NLL-improved Sudakovs $(A_1, B_1, A_2)$

Improved MiNLO: where are terms coming from when differentiating resum. formula?

- $1/q_T^2$, always from integration in Sudakov
- $\alpha_S$ from $C^{(0)} \times B_1$, ...
- $\alpha_S^2$ from $C^{(0)} \times B_2$, ...
- ...  
- $\alpha_S L$ from $A_1$ term in exponent
- $\alpha_S L^2$ from $A_2$ term in exponent
- ...
$p_T^H$ spectrum:

- “$\mu_{\text{HJ-MiNLO}} = m_H, m_H, p_T$”
- At high $p_T$, $\mu_{\text{HJ-MiNLO}} = p_T$
- If $\beta = 1/2$, NNLOPS $\rightarrow$ HJ-MiNLO at high $p_T$
- NNLO/NLO $\sim 1.5$, because HNNLO with $\mu = m_H/2$, $\mu_{\text{HJ-MiNLO,core}} = m_H$