Nonthermal switching of charge order: dynamical slowing down and optimal control

Michael Schüler, Yuta Murakami, and Philipp Werner
Department of Physics, University of Fribourg, 1700 Fribourg, Switzerland

We investigate the laser-induced dynamics of electronically driven charge-density-wave order. A comprehensive mean-field analysis of the attractive Hubbard model in the weak-coupling regime reveals ultrafast switching and ultrafast melting of the order via a nonthermal pathway. The resulting nonequilibrium phase diagram exhibits multiple dynamical phase transitions with increasing field strength. Using an intuitive pseudospin picture, we show that the laser can be regarded as an external (pseudo) magnetic field, and that the distinct dynamical regimes can be connected to the spin precession angle. We furthermore study the effects of electron-electron interactions beyond mean-field to show that the main features of the phase diagram are robust against scattering or thermalization processes. For example, the nonthermal state with switched order is characterized by a particularly slow relaxation. We also demonstrate how these nonthermal phases can be stabilized by tailoring the pulse shape with optimal control theory.

The field of ultrafast coherent control of symmetry-broken phases in condensed matter experiences a surge of research activity. Important examples include the transient enhancement of superconductivity [1–9], light-controlled order of excitonic condensates [10–13] or the transient manipulation of spin [14, 15], orbital [16] and charge order [17–20]. Since charge-density-wave (CDW) order directly couples to external electric fields, it provides an ideal platform for inducing and tracing photoinduced phase transitions. Several recent experiments have addressed the mechanism behind the CDW-to-metal transition by ultrashort laser pulses. If the CDW originates from a lattice distortion, the light-induced melting is accompanied by coherent phonon excitations [14, 16–18, 21, 22]. In contrast, a CDW induced by electron-electron (e–e) correlations can be melted on the femtosecond timescale [10, 23–25], and nonthermal melting has been observed [10, 25]. These developments have stimulated intensive theoretical research of the laser-driven CDW dynamics [19, 26–31]. Furthermore, since CDW is typically competing with superconductivity [32], the suppression of such orders provides a route to light-induced superconductivity [33].

From a fundamental perspective, universal features in the time evolution of ordered states are attracting great interest. The dynamics after a generic excitation can exhibit qualitative changes – such as transitions from transiently trapped to vanishing order – beyond some critical excitation strength, which cannot be explained in terms of the total energy of the system within an equilibrium picture. Such dynamical phase transitions [34–36] have been studied for various kinds of ordered phases such as superconductors [37–39], excitonic insulators [13], antiferromagnetic [40–42], ferromagnetic [36, 43] and bosonic [44] systems.

Here we show that the e–e driven CDWs excited with short pulses can also exhibit such dynamical phase transitions. In contrast to the usual quench scenario, we identify multiple regimes of ultrafast nonthermal melting or switching depending on the pulse strength. We present a corresponding nonequilibrium phase diagram which reveals nonthermal slowing down and long-lived nonthermal states despite increased energy absorption. We provide an intuitive understanding of the switching and melting behavior based on time-dependent mean field (MF) theory and a pseudospin picture. In addition, by taking e–e scattering into account at the local second-Born level, we demonstrate that the qualitative features identified in the MF dynamics are also present in a more sophisticated description, which captures thermalization effects. To access these interesting transient states efficiently, we employ quantum optimal control theory (QOCT) [45–47], which provides optimized laser pulses for either switching or suppressing the CDW order. By minimizing energy absorption, we can suppress heating effects and stabilize the nonthermal states.

Model—In this work, we study the attractive Hubbard model at half filling

$$\hat{H} = -J_0 \sum_{\langle ij \rangle, \sigma} \hat{c}_{i \sigma}^\dagger \hat{c}_{j \sigma} + \frac{U}{2} \sum_i \left( \hat{n}_{i \uparrow} - \frac{1}{2} \right) \left( \hat{n}_{i \downarrow} - \frac{1}{2} \right),$$

where $\langle ij \rangle$ denotes nearest neighbors, $U$ is the on-site interaction and $\hat{n}_{i \sigma} = \hat{c}_{i \sigma}^\dagger \hat{c}_{i \sigma}$. This model gives rise to CDW order and superconductivity. Here we focus on the CDW state, which is characterized by the broken symmetry between neighboring sublattice sites. In what follows we focus on the weak coupling regime and treat a one-dimensional (1D) configuration to simplify the numerical calculations, having in mind a quasi 1D system embedded in higher dimensions [48].

![FIG. 1. (a) Schematic of the switching dynamics in the pseudospin picture (see text). A single-cycle field pulse shifts the band structure, inducing a $k$-dependent effective magnetic field $B^\alpha_k$. By tuning the precession time of the pseudospins, the contribution arising from the Fermi level can be switched completely. (b) MF dynamics of the order parameter induced by a single-cycle pulse as a function of its amplitude $F_0$.](attachment:fig1.png)
In momentum space, the kinetic term of the Hamiltonian (1) reads $\hat{H}_{\text{kin}} = \sum_{k, \sigma} \varepsilon_k \hat{c}^\dagger_{k \sigma} \hat{c}_{k \sigma}$, with the free band structure $\varepsilon_k = -2J_0 \cos(ka)$ (lattice constant $a$). The time-dependent external laser fields, described by the vector potential $A(t)$, are incorporated by the Peierls substitution $\varepsilon_k(t) = \varepsilon_{k'} - A_{k'0}$. To allow for CDW order, one introduces the nesting vector $Q = \pi/a$, which leads to nested bands and a reduced Brillouin zone. In what follows, we measure energies in units of $J_0$ and time in units of $J_0^{-1}$, while $k$ is given in units of $a^{-1}$.

We use the $e-e$ interaction $\mathcal{U} = -2$, which corresponds to the weak-coupling regime.

Results—First we discuss the MF dynamics which provides valuable insights into the short-time dynamics of the CDW system. To this end, we introduce the pseudospin representation by $\hat{S}^a_{k \sigma} = \hat{c}^\dagger_{k \sigma} \hat{\sigma}^a \hat{c}_{k \sigma}/2$ (Pauli matrices $\hat{\sigma}^a$) with $\hat{\varepsilon}_{k \sigma} = (\hat{c}_{k \sigma}, \hat{c}^\dagger_{k \sigma+Q, \sigma})$. This concept has provided an intuitive understanding of related problems [13, 42, 49–51]. The CDW order parameter describing the charge difference with respect to $(k, k + Q)$ (or, equivalently, sublattice sites) is given by $\Delta n = (1/N_k) \sum_{k, \sigma} \hat{\varepsilon}^\dagger_{k \sigma} \hat{\varepsilon}_{k \sigma} = (1/N_k) \sum_{\sigma, \sigma'} \langle \hat{S}^z_{k \sigma} \rangle$. The current between the sublattice sites is, on the other hand, related to $\langle \hat{S}^x_{k \sigma} \rangle$. CDW order corresponds to a finite total pseudospin projection in the $x$-direction, while the normal state in equilibrium consists of spins along the $z$-direction. With these definitions, the MF Hamiltonian and the corresponding equation of motion is cast into the simple form

$$\hat{H}^{\text{MF}}(t) = \sum_{k \sigma} B_k(t) \cdot \hat{S}_{k \sigma}, \quad \frac{d}{dt} \hat{S}_{k \sigma} = B_k(t) \times \hat{S}_{k \sigma},$$

where the momentum dependent pseudo-magnetic field is given by $B_k(t) = \mathcal{U} \Delta n(t)$. $B'_k = 0$ and $B''_k(t) = \varepsilon_k(t) - \varepsilon_{k+Q}(t)$. The pseudo-spin representation (2) provides an intuitive picture for the control of CDW configurations by external fields. For instance, the asymmetric electronic charge on the sublattice sites can be switched with a short single-cycle pulse (SCP) with a pulse duration $T_p$, see Fig. 1(a). During the pulse, the kinetic energy of the electrons with momentum $k$ is shifted by the vector potential to $\varepsilon_{k-Q}(t)$. In the pseudo-spin picture, this acts as a momentum-dependent magnetic field in the $z$ direction, whose maximum strength is half the band width. In the weak coupling regime, the major contribution to the CDW comes from the Fermi surface. Furthermore, $B_k(t)$ is smaller than the bandwidth and can be neglected during the pulse. Therefore, the CDW dynamics can be regarded as spin precession around the $z$ direction.

By tuning the pulse amplitude and $T_p$ we can control the pseudospins at the two Fermi points, which rotate in opposite directions (Fig. 1(a)). Optimizing the pulse to induce a precession by $(2n+1)\pi$ ($n \in \mathbb{N}$), the sign of the order is switched. Inducing a rotation by $2(n+1)\pi$ ($n \in \mathbb{N}$), on the other hand, the pseudospin projections cancel out, which leads to efficient destruction of the order. Note that the pseudospins at different $k$ process under a $k$-dependent pseudo-magnetic field. Hence the complete pseudospin dynamics is subject to dephasing, which can reduce the size of the order parameter after the switching. We now proceed to the numerical investigation of the pulse-induced CDW dynamics. We propagated the MF Hamiltonian (2) in the presence of a SCP described by the electric field $E(t) = F_0 \sin[\pi(t-t_0)/T_p] \sin(\omega(t-t_0))$ with $T_p = 2\pi/\omega$ (vector potential $A(t) = -\int_0^t dt' E(t')$). We found that $T_p = 13.6$ matches the typical time scale of the system and allows efficient switching as in the pseudospin picture.

In our MF study, we use the renormalized parameters $\tilde{J}_0 = 0.89J_0$ and $\tilde{U} = 0.625U$. For this choice, the MF band structure provides a good approximation of the renormalized bandstructure [52], and thus allows us to use the MF dynamics as a reference for the correlated treatment below. In both cases, one obtains an equilibrium order parameter of $\Delta n_{\text{eq}} \equiv -\Delta n_0 = -0.121$. The inverse temperature is fixed at $\beta = 40$.

The MF dynamics of $\Delta n$ for varying pulse strengths $F_0$ is presented in Fig. 1(b). Several regimes of qualitatively distinct dynamics can be identified (multiple dynamical phase transitions). For weak fields, the system exhibits persistent amplitude (AM) oscillations around a mean value $\bar{n}$. The corresponding frequency decreases with decreasing $|\Delta \bar{n}|$. The second regime corresponds to a rapid destruction of the CDW order. In particular, for $F_0 \approx 0.05$ the order is almost immediately destroyed after the pulse, which is due to dephasing of the pseudospins and does not correspond to a thermal melting process. We therefore refer to this regime as coherent destruction (CD). Increasing $F_0$ further, the order is switched, as expected from the pseudospin picture. One observes the emergence of AM oscillations around the switched value. However, a complete switching is prevented by the dephasing of the pseudospins. The order parameter reaches an average value of $\Delta \bar{n} \approx 0.66\Delta n_0$.

Let us also comment on the situation where an electron-phonon coupling (e–ph) contributes to the CDW. The analogous MF analysis of the Hubbard-Holstein model reveals [52] a similar nonequilibrium phase diagram in the case where the e–ph interactions dominate the CDW formation. For dominant e–ph coupling, in contrast, the dynamics is characterized by persistent coherent phonon excitations which inhibit the long-time stable switching or destruction of the CDW [48]. Hence, the distinct regimes in Fig. 1(b) are a clear indication of correlation-driven charge order.

For a quantitative discussion of the different regimes we analyze the dynamics by fitting the order parameter to the damped oscillating function $f(t) = c + [a \cos(\omega_0 t) + b \sin(\omega_0 t)]e^{-\gamma t}$ in the long-time limit. This form describes (i)
damped AM oscillations and (ii) non-oscillatory ($\omega_0 = 0$) decay of the order. The extracted damping constants $\gamma_0$ and oscillation frequencies $\omega_0$ are shown as a function of field amplitude in Fig. 2(a). The regime of AM oscillations is characterized by a finite $\omega_0$ and a very small $\gamma_0$. A closer inspection reveals a very slow algebraic decay. The CD of the order is found in the region $0.035 < F_0 < 0.05$. It is bounded by two special points of slowing down at nonthermal critical points [42], where oscillations of the order parameter are fully suppressed (Fig. 2(b)). In particular, at the right boundary $F_0 = 0.05$ the order is strongly suppressed already a short time after the pulse ("sweet spot", marked as line 3 in Fig. 2(b)). Within the CD regime, the system exhibits strongly damped AM oscillations after a rapid reduction of the mean value of $\Delta n$. Increasing the field strength further, the CDW system enters the switching regime. It is again characterized by AM oscillations, but around the switched value of the order parameter. For even larger pulse amplitude, the order is again destroyed and a second CD regime emerges.

Now we consider the effect of scattering and thermalization processes on the nonequilibrium phase diagram. In general, the absorbed pulse energy ($E_{abl}$) will lead to a thermal melting of the order in the long-time limit in most scenarios. These heating effects, which are often ignored in theoretical studies, deserve proper attention in the discussion of light-controlled order [7]. To address this, we switch to a description in terms of nonequilibrium Green’s functions [2, 54]. The single-particle Green’s function (GF) is defined as

$$G_z(z, z') = -i \langle \tau \hat{c}_z \hat{c}_{z'} \rangle \langle \hat{c}_{z'} \hat{c}_z \rangle$$

where the arguments $z, z'$ refer to the L-shaped Matsubara-Keldysh contour $C$ and $\tau$ denotes the contour ordering operator. Given some approximation to the self-energy $\Sigma_z(z, z')$, the Kadanoff-Baym equations (KBEs) yield the interacting time-dependent GF, from which the relevant observables can be extracted [52].

Generally, the self-energy is expressed as a diagrammatic series. Since the e–e interactions are weak in our setup, the second-order (second-Born, 2B) approximation to the self-energy yields an accurate description. In full generality, it incorporates momentum-dependent scattering processes. For low dimensional systems, however, the available phase space is strongly restricted and the e–e scattering is strongly suppressed. For numerical simplicity and to partially reflect that the system is embedded in high-dimensional systems we employ the local 2B (2Bloc) approximation $\Sigma_{ij,\sigma}(z, z') = \delta_{ij} U^2 G_{ii,\sigma}(z, z') G_{ii,\sigma}(z, z') G_{ii,\sigma}(z, z')$. The e–e scattering may thus be overestimated compared to an actual quasi-1D system. Hence, one can expect that the 2Bloc approximation provides an upper bound for thermalization effects.

Performing the analogous scan over the field strength $F_0$ with the same pulse shape as for the MF case yields the time-dependent order parameter and the nonequilibrium phase diagram presented in Fig. 3(a) and (b), respectively. Similarly to the MF case, the dynamics induced by the SCP exhibits qualitatively distinct regimes as $F_0$ is increased. For small amplitudes, the order parameter is subject to AM oscillations. Increasing the field strength, the total energy after the pump exceeds the energy of the equilibrium system at the critical temperature at the value $F_{\text{crit}} \approx 0.014$. Therefore, for $F > F_{\text{crit}}$, one expects complete melting of the CDW after thermalization. Nevertheless, for $F_{\text{crit}} < F_0 < 0.0275$ one finds long-lived AM oscillations with a small decay rate $\gamma_0$ (see Fig. 3(c)). Such a persistence of nonthermal behavior over a long time span has also been observed in the dynamics of antiferromagnetic order [41, 42].

To distinguish the effect of e–e scattering and thermalization from the effect of coherently precessing pseudospins, it is useful to introduce the fluctuation measure $F \equiv (1/\sqrt{N_k}) \sqrt{\sum_{k \sigma} \langle \hat{S}_{k \sigma}^\dagger \hat{S}_{k \sigma} \rangle}$. This quantity is related to the magnitude of the $k$-resolved current between the two sublattice sites and hence provides a convenient measure of a nonthermal state: $F = 0$ corresponds to thermal equilibrium, while $F > 0$ indicates a nonequilibrium state. Progressing thermalization can be tracked by the decay of $F(t)$. The nonthermal AM regime is thus characterized by an extremely slow thermalization, underpinned by the oscillating behavior of $F(t)$ around a nonzero value (Fig. 3(d)).

For larger field strength, the order is destroyed rapidly, similarly as in the MF case. The CD is bounded by two special points: (i) a point of critical slowing-down at $F_0 \approx 0.0275$ where the oscillation frequency approaches zero and (ii) the "sweet spot" $F_0 \approx 0.05$ where the CDW is rapidly destroyed. In contrast to the MF scenario, no (damped) AM oscillations occur; instead a non-oscillatory decay of $\Delta n$ is observed. The decay rate exhibits a maximum in the middle of the CD regime, similarly to the MF analysis. Since e–e collisions facilitate the suppression of the order, the CD regime is more extended. Note that the thermalization time is still rather long (Fig. 3(d)).

As in the MF analysis, a switching regime appears for larger field strength, though the order decays due to scattering processes, leading to additional damping (and almost completely suppressed AM oscillations). This is due to the larger energy.

![FIG. 3](image-url)
absorption, which acts against the CDW order. This is corroborated by the exponential decay of $P(t)$ quantified by the thermalization rate $\gamma_{\text{therm}}$ (shown in Fig. 3(b)). Nevertheless, there is a pronounced minimum of the decay rate $\gamma_0$ at $F_0 \approx 0.071$ in the middle of the switching regime. Here one encounters a nontrivial situation where, despite the increased $E_{\text{abs}}$, the relaxation of the order becomes slower (dynamical slowdown). As the monotonic dependence of $\gamma_{\text{therm}}$ shows, this slowdown is a nonthermal effect. It can be interpreted as a "trapping" in a state that is close to the MF transient state. A more detailed analysis also shows small oscillations on top of the exponential decay, which is reminiscent of the AM oscillations in the switching regime in Fig. 1(c).

Now, we show how QOCT can stabilize nonthermal states further. In an optimal switching protocol, the requirements of minimal energy absorption and switching efficiency need to be balanced. Using QOCT, we optimize the fields $A(t)$ to reach a target value of the order parameter, while simultaneously minimizing $E_{\text{abs}}$. Due to the substantial numerical cost, this procedure is applied only on the MF level. After identifying optimal pulses [56], we can then compare the MF and correlated dynamics. A result of this optimization procedure is shown in Fig. 4(b). The short-time 2Bloc dynamics is well described by the MF approximation. As compared to the simple SCP (Fig. 4(a)), the switching efficiency is bigger ($\Delta \bar{n} \approx 0.75 \Delta n_0$). In the MF results, the small amplitude of the AM oscillations indicates the stability of the state.

It is interesting to analyze the switching dynamics in the pseudospin picture. While the SCP rotates the spins close to the edge of the reduced Brillouin zone (Fermi points) by $\pi$ monotonically (Fig. 4(c)), the optimized pulse initially leads to a rotation in the opposite direction to the switching (Fig. 4(d)). As the snapshots of the $k$-dependent pseudospin configuration demonstrate, this procedure partially compensates the dephasing and thus results in a better switching.

Including the e–e scattering, $\Delta n(t)$ exhibits a damped behavior, whereas the short-time dynamics is well captured by the MF dynamics. Albeit $E_{\text{abs}}$ exceeds the energy difference to the unordered state, the order parameter decays very slowly. Hence, the nonthermal flipped ordered state discussed in Fig. 3 is due to minimizing $E_{\text{abs}}$ – much more pronounced as compared to switching by the SCP. In fact, $\Delta n(t)$ shows more prominent oscillations on top of the decay for shorter times. Especially the pronounced shoulder right after the pulse indicates that the dynamics is very close to the MF time evolution at short times. This is a clear signature of the switching regime in Fig. 1(c). Furthermore, the thermalization rate is reduced ($\gamma_{\text{therm}} = 0.0188$ for the SCP, $\gamma_{\text{therm}} = 0.0132$ for the optimized pulse).

Our pulse optimization can also be applied to the suppression of the CDW with minimal energy absorption. This yields a similarly efficient CD as for the SCP (cf. Fig. 3) but with reduced $E_{\text{abs}}$. Even though after complete thermalization the system ends up in the disordered phase, the fluctuations decay on the very long time scale of $\gamma_{\text{therm}}^{-1} \approx 170$.

In summary, we revealed the rich nonequilibrium phase diagram of correlation-driven CDWs after laser excitation, which includes nonthermal AM oscillations, a sweet spot of rapid destruction of the order, a dynamical slow-down of the decay in the switching regime and hence multiple dynamical phase transitions. These features are well explained by the MF treatment – which is rationalized by the pseudospin picture – and persist when including e–e scattering. The ability to switch the order or to completely suppress it is a clear fingerprint of correlation-driven CDWs. Purely phonon-driven CDW order, on the other hand, exhibits persistent oscillations after similar laser pulses. Therefore, the transient dynamics and nonthermal critical behavior allow us to identify the mechanism underlying the formation of a CDW. Furthermore, QOCT allows us to identify optimal pulses for switching or melting of the order. Minimizing the absorbed energy delays the thermalization and may enable the emergence of competing orders on intermediate timescales. Controlled access to long-lived transient states has already resulted in technological applications [57] and we believe that our approach to QOCT can be usefully applied to the manipulation of a broad range of electronic orders.

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In the pseudo-spin representation, effects of the phonon can be studied, see Supplemental Material [52].
regarded as a slowly changing magnetic field along the x direction. Therefore, \( B^\sigma_i \) does not change even when the pseudospin rotate by \((2n + 1)\pi \) (\( n \in \mathbb{N} \)), which makes the switched state unstable.

[54] K. Balzer and M. Bonitz, Nonequilibrium Green’s Functions Approach to Inhomogeneous Systems (Springer, 2012).

[55] G. Stefanucci and R. v. Leeuwen, Nonequilibrium Many-Body Theory of Quantum Systems: A Modern Introduction (Cambridge University Press, 2013).

[56] The pulses are represented by fourth-order B-splines \( A(t) = \sum c_i B_i(t) \) with given pulse duration \( T_p \). The coefficients \( c_i \) are optimized by a combination of a genetic algorithm and deterministic minimization close to the local minima. More details and a selection of optimized pulses are presented in the Supplemental Material.

[57] I. Vaskivskyi, J. Gospodaric, S. Brazovskii, D. Svetin, P. Sutar, E. Goreshnik, I. A. Mihailovic, T. Mertelj, and D. Mihailovic, Sci. Adv. I, e1500168 (2015).

Appendix A: Technical details and numerical considerations

1. Hamiltonian and basis representation

In order to describe charge-density wave (CDW) states in one dimension, we introduce an extended unit cell containing two lattice sites, as sketched in Fig. 5.

\[
\begin{array}{cccccc}
B & A & B & A & B & A \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{array}
\]

FIG. 5. Illustration of the choice of the non-primitive unit cell (lattice constant \( a' = 2a \)) in one dimension.

In what follows, let us denote the lattice sites of each unit cell by indices \( i, j, \ldots \), while the sites in each unit cell will be labelled by \( \alpha, \beta \). In general, CDW order is driven by two (possibly coexisting) mechanisms: electron-electron (e–e) interactions and electron-phonon (e–ph) interactions. In order to take both of them into account, we extend the Hubbard model considered in the main text to the Hubbard-Holstein model. Using the above convention, the Hamiltonian is expressed as

\[
\hat{H}(t) = \hat{H}_0(t) + \hat{H}_{e-e} + \hat{H}_{e-ph} + \hat{H}_{ph},
\]

where

\[
\hat{H}_0(t) = \sum_{\alpha} \sum_{i} \sum_{\sigma} \sum_{\beta} \sum_{\sigma} \hat{h}_{i\alpha\beta\sigma}(t) \hat{d}_{i\alpha\sigma}^\dagger \hat{d}_{j\beta\sigma}.
\]

denotes the time-dependent electron Hamiltonian. The e–e interaction is modeled by the Hubbard interaction

\[
\hat{H}_{e-e} = \frac{U}{2} \sum_{i,\alpha,\sigma} \left( \hat{d}_{i\alpha\sigma}^\dagger \hat{d}_{i\alpha\sigma} - \frac{1}{2} \right) \left( \hat{d}_{i\alpha\bar{\sigma}}^\dagger \hat{d}_{i\alpha\bar{\sigma}} - \frac{1}{2} \right)
\]

\[
= \frac{U}{2} \sum_{i,\alpha,\sigma} \left( \hat{d}_{i\alpha\sigma} - \frac{1}{2} \right) \left( \hat{d}_{i\alpha\bar{\sigma}} - \frac{1}{2} \right).
\]

As usual, \( \bar{\sigma} = \downarrow (\bar{\sigma} = \uparrow) \) for \( \sigma = \uparrow (\sigma = \downarrow) \). Furthermore, we account for Holstein-type phonons described by

\[
\hat{H}_{ph} = \omega_{ph} \sum_{\alpha} \hat{b}_{\alpha}^\dagger \hat{b}_{\alpha},
\]

while the electron-phonon interaction is of the form

\[
\hat{H}_{e-ph} = g \sum_{\alpha} \hat{\eta}_{\alpha\sigma} \hat{X}_{\alpha}.
\]

Here, \( \hat{X}_{\alpha} = [\hat{b}_{\alpha}^\dagger + \hat{b}_{\alpha}]/\sqrt{2} \) represents the phonon distortion. The two limiting cases of (i) purely correlation-driven and (ii) purely phonon-driven charge order can be obtained by (i) setting \( g = 0 \), or (ii) \( U = 0 \), respectively.

Transforming to momentum space via

\[
\hat{d}_{k\alpha\sigma} = \frac{1}{\sqrt{N}} \sum_{m} e^{-ik\alpha'} \hat{d}_{m\alpha\sigma},
\]

where \( N \to \infty \) denotes the number of cells, the Hamiltonian (A2) reads

\[
\hat{H}_0(t) = \sum_{k \in BZ} \sum_{\alpha} \sum_{\sigma} \sum_{\beta=0}^\infty \hat{h}_{\alpha\beta}(k; t) \hat{d}_{k\alpha\sigma}^\dagger \hat{d}_{k\beta\sigma}.
\]

Here, the one-body Hamiltonian (in matrix notation) is given by

\[
\hat{h}^{(0)}(k) = \begin{pmatrix}
-\mu & -J_0 [1 + \exp(-ik\alpha')] \\
-J_0 [1 + \exp(ik\alpha')] & -\mu
\end{pmatrix},
\]

which has eigenvalues \( \varepsilon \pm(k) = -\mu \pm 2J_0 \cos(ka) \). Here, \( A_F(t) \) denotes the electromagnetic vector potential along the chain direction. Note that the Brillouin zone refers to the extended unit cell, i.e. \( BZ = [-\pi/a', \pi/a'] \). We assume half filling (\( \mu = 0 \)).

The basis with respect to the sublattice sites (ss) in the extended unit cell is particularly convenient for introducing approximations to the e–e interaction, as discussed below. Therefore, all calculations have been performed in the sublattice representation. An equivalent basis – which has been used in the main text – is given by the momentum pair \((k, k + Q)\) with the nesting vector \( Q = \pi/a \). The momentum-pair (mp) representation is most useful for defining pseudospin operators, as introduced in the main text. Any \( k \)-dependent matrix in the ss representation \( (A_k) \) can be transformed to the mp basis by the unitary transformation

\[
A_k^{(mp)} = R_k A_k^{(ss)} R_k^+.
\]

with

\[
R_k = \frac{1}{\sqrt{2}} \begin{pmatrix}
e^{ika/2} & e^{-ika/2} \\
e^{-ika/2} & e^{ika/2}
\end{pmatrix}.
\]
2. Mean-field treatment

Within the mean-field (MF) approximation, the time-dependent Hamiltonian (A1) is replaced by

\[ \hat{H}^{MF}(t) = \sum_{k \in BZ} \sum_{\alpha} \sum_{\sigma} h^{MF}_{\alpha\beta}(k - A_F(t)) \hat{d}_{k\alpha\sigma} \hat{d}^\dagger_{k\beta\sigma}, \quad (A11) \]

where the one-body MF Hamiltonian reads

\[ h^{MF}(k) = h^{(0)}(k) + U \left( \langle \hat{n}_A \rangle - \frac{1}{2} \right) \left( \langle \hat{n}_B \rangle - \frac{1}{2} \right) + g \left( \langle \hat{X}_A \rangle \langle \hat{X}_B \rangle \right). \quad (A12) \]

Here, \( \langle \hat{n}_A \rangle \) (\( \langle \hat{n}_B \rangle \)) denotes the occupation on sublattice site A (B). We assume the paramagnetic case \( \langle \hat{n}_{A,B} \rangle = \langle \hat{n}_{A,B,\uparrow} \rangle \) and therefore drop the spin indices. \( \hat{X}_{A,B} \) measures the lattice distortion (corresponding to \( k = 0 \)) at the respective sublattice sites. We introduce the order parameter for CDW

\[ \Delta n = \langle \hat{n}_A \rangle - \langle \hat{n}_B \rangle \quad (A13) \]

and the distortion parameter \( \Delta X = \langle \hat{X}_A \rangle - \langle \hat{X}_B \rangle \). Expressing the MF Hamiltonian (A12) with these definitions and transforming to the mp basis then gives the MF Hamiltonian

\[ \hat{H}^{MF}(t) = \sum_{k \in BZ} \sum_{\alpha} \sum_{\sigma} \epsilon_{k\sigma} \hat{n}^{MF}_{\alpha\beta}(k, t) \hat{c}_{k\sigma} \quad (A14) \]

with \( \epsilon_{k\sigma} = \langle \hat{c}_{k\sigma} \hat{c}_{k+Q,\sigma} \rangle \) and

\[ \hat{n}^{MF}_{\alpha\beta}(k, t) = \left( \begin{array}{ccc} \epsilon_k(t) & \frac{\epsilon_k(t)}{2} \Delta X(t) + \frac{U}{2} \Delta n(t) & \epsilon_{k+Q}(t) \\ \frac{\epsilon_k(t)}{2} \Delta X(t) & \frac{\epsilon_k(t)}{2} \Delta n(t) & \Delta n(t) \\ \epsilon_{k+Q}(t) & \Delta n(t) & \epsilon_{k+Q}(t) \end{array} \right). \quad (A15) \]

Here, \( \epsilon_k(t) = \epsilon_k - A_F(t) \) with \( \epsilon_k = -2J_0 \cos(ka) \) denoting the free dispersion. The pseudospin representation can now be introduced in terms of the Pauli matrices \( \sigma^a \) by

\[ S_{k\sigma} = \frac{1}{2} \left( \begin{array}{cc} \epsilon_{k\sigma} & \epsilon_{k+Q,\sigma} \\ \epsilon_{k+Q,\sigma} & \epsilon_{k\sigma} \end{array} \right) \sigma^a \left( \begin{array}{c} \epsilon_{k\sigma} \\ \epsilon_{k+Q,\sigma} \end{array} \right). \quad (A16) \]

Exploiting \( \epsilon_{k+Q} = -\epsilon_k \), the Hamiltonian can be written as

\[ \hat{H}^{MF}(t) = \sum_{k \in BZ} \sum_{\alpha} \left( B_k^\alpha(t) \hat{S}_k^\alpha + B_k^\alpha(t) \hat{S}_k^\alpha \right), \quad (A17) \]

where the effective magnetic fields are given by

\[ B_k^\alpha(t) = g \Delta X(t) + U \Delta n(t), \quad B_k^\alpha(t) = \epsilon_k(t) - \epsilon_{k+Q}(t). \quad (A18) \]

The time-dependent MF equations are solved in two steps. First, the one-body density matrix in thermal equilibrium \( \rho_{eq}(k) \) is self-consistently computed by diagonalizing Eq. (A12) and calculating the order \( \Delta n \) and distortion \( \Delta X = -(2g/\omega_{ph}) \Delta n \) parameters, until convergence is reached. Using \( \rho(k, t = 0) = \rho_{eq}(k) \) as initial condition, the time evolution is determined by the time stepping

\[ \rho(k, t + \Delta t) = U_k(t + \Delta t, t) \rho(k, t) U_k^\dagger(t + \Delta t, t). \quad (A19) \]

Here, \( U_k(t + \Delta t, t) \) denotes the time evolution operator of the MF Hamiltonian, which is computed by fourth-order commutator-free matrix exponentials [1].

To determine the self-consistent MF Hamiltonian, the phonon amplitudes \( \langle \hat{X}_{A,B}(t) \rangle \) also need to be propagated. Combing their respective equations of motion, the distortion parameter is obtained from the equation

\[ \frac{1}{2\omega_{ph}} \left[ \frac{d^2}{dt^2} + \omega_{ph}^2 \right] \Delta X(t) = -g \Delta n(t), \quad (A20) \]

which we solve using the fourth-order Numerov method with the initial condition \( \frac{d}{dt} \Delta X(t) = 0 \) for \( t = 0 \).

Since the values \( \Delta X(t + \Delta t) \) and \( \Delta n(t + \Delta t) \) are needed to carry out the step from \( t \) to \( t + \Delta t \), the time propagation is performed in a predictor-corrector fashion.

3. Solution of the Kadanoff-Baym equations

Treating the electron-electron interactions up to second order in \( U \) is accomplished by the second-Born (2B) approximation to the self-energy. In the original lattice representation, the 2B self-energy reads

\[ \Sigma_{ij\sigma}(z, z') = U^2 G_{ij\sigma}(z, z') G_{ij\sigma}(z, z') G_{ij\sigma}(z', z). \quad (A21) \]

Here, \( z, z' \) denote the arguments on the Matsubara-Keldysh contour \( \mathcal{C} \), and \( G_{ij\sigma} \) stands for the one-body Green’s function

\[ G_{ij\sigma}(z, z') = -i \mathcal{T} \hat{d}_{i\sigma}(z) \hat{d}_{j\sigma}(z'), \quad (A22) \]

with \( \mathcal{T} \) the contour-ordering operator. More details on the formalism can be found, for instance, in Ref. [2].

As discussed in the main text, we employ the local second-Born (2Bloc) approximation, which is obtained from Eq. (A21) by replacing the index pair \( (ij) \) by the diagonal \( (ii) \). Switching to \( k \)-space, the 2Bloc is cast into a momentum-independent self-energy,

\[ \Sigma_{aa\sigma}(z, z') = U^2 G_{aa\sigma}(z, z') G_{aa\sigma}(z, z') G_{aa\sigma}(z', z), \quad (A23) \]

where the index \( a \) refers to the sublattice site basis and

\[ G_{a\beta\sigma}(z, z') = \frac{1}{|BZ|} \int_{BZ} dk G_{a\beta\sigma}(k; z, z'), \quad (A24) \]

defines the local Green’s function.

Switching to a matrix notation for the sublattice indices (and dropping the spin index), the equation of motion for the Green’s function assumes the standard form

\[ [i \partial_z - h^{MF}(z)] G(k; z, z') = \int_c dz'' \Sigma(z, z'') G(k; z'', z'). \quad (A25) \]
FIG. 6. Left panel: spectral function $A(k, \omega)$ (summed over bands) within the 2Bloc approximation. The dashed lines represent the fit by the MF model with renormalized parameters $\tilde{J}_0$ and $\tilde{U}$.  Right panel: band-resolved (green and purple filled curves) and total (gray filled curve) density of states within the 2Bloc approximation. The parameters are, as in the main text, $U = -2, \beta = 40$.

Projecting onto imaginary and real times and invoking the Langreth rules then yields the Kadanoff-Baym equations (KBEs). The KBEs are solved using an in-house massively parallel computer code based on a fourth-order implicit predictor-corrector algorithm. For the results presented in the main text, the time interval was split into $N_t = 3000$ equidistant points, while the imaginary branch (for the nonequilibrium calculations) was represented by $N_c = 800$ grid points. The Green’s function for every $k$-point has to be propagated simultaneously, which is accomplished by a distributed-memory layout. For obtaining converged results, we used $N_k = 256$ points in the Brillouin zone.

4. Equilibrium spectral function and mean-field fit

Before the KBEs can be solved (see subsec. A3), the equilibrium (Matsubara) Green’s function needs to be computed. To this end, we solve the corresponding Dyson equation

$$G(k; \tau) = g(k; \tau)$$

$$+ \int_0^\beta d\tau' \int_0^\beta d\tau'' g(k; \tau - \tau') \Sigma(\tau' - \tau'') G(k; \tau'') .$$

(A26)

Here, $g(k, \tau)$ denotes the MF Green’s function, while $\Sigma(\tau)$ is the self-energy in the 2Bloc approximation. The Dyson equation (A26) is solved by a combination of Fourier transformation and fifth-order fix-point iteration to improve the accuracy. A description of the method can be found in Ref. [3]. As for the nonequilibrium calculations, we use $N_k = 256$ $k$-points, whereas $N_c = 4096$ points on the Matsubara axis were needed for converging the results. For the nonequilibrium calculations, the Matsubara Green’s function is defined on the reduced imaginary grid by interpolation.

The spectral function $A(k, \omega)$ in real-frequency space is obtained by Padé analytic continuation as in Ref. [3]. The band-integrated spectral function $A(k, \omega) = \sum_i A_{\alpha\beta}(k; \omega)$ is shown in Fig. 6. In accordance with the Luttinger-Ward theorem, the broadening due to many-body effects is least pronounced in the vicinity of the chemical potential $\mu = 0$, while significant broadening is apparent at the band top and bottom. Since we are in the weak-coupling regime, the main effect of the electronic correlations is a renormalization of the bands.

In order to be able to directly compare the dynamics within the MF and 2Bloc approximation, the band renormalization is taken into account in the effective parameters of the MF Hamiltonian (A12), replacing $J_0 \to \tilde{J}_0$ and $U \to \tilde{U}$. These parameters are determined by fitting the MF band structure to the maximum (with respect to $\omega$) of $A(k, \omega)$, while requiring the order parameter to be identical (see Fig. 6). The result is $\tilde{J}_0 = 0.89J_0$ and $\tilde{U} = 0.625U$. The good quality of the fit is supported by the almost identical short-time dynamics within the MF and 2Bloc approximation, ensuring that applying the QOCT on the MF level provides optimal pulses for the correlated dynamics, as well.

5. Quantum optimal control

The optimization of the laser pulses by quantum optimal control theory (QOCT) is performed on the MF level. Since the MF dynamics is described by a nonlinear equation of motion for the one-body density matrix, the usual approach based on an (effective) Schrödinger equation (Krotov algorithm) is not applicable. In fact, one has to resort to gradient-free optimization methods because the derivative with respect to the driving field can not be obtained analytically.

One can expect that pulses containing a minimal amount of mean field energy $E_p = (\epsilon_0/T_p) \int dt |E_p(t)|^2 = (\epsilon_0/T_p) \int dt |A_F(t)|^2$ – as required to minimize heating effects – are relatively smooth functions without strong variations. On the other hand, the search space has to be large enough to find good approximations to the optimal fields. To fulfil these objectives, we parameterize the vector potential by

$$A_F(t) = \sum_{i=1}^{N_f} c_i B_i(t) ,$$

(A27)

where $B_i(t)$ are fourth-order B-splines with respect to the time interval $[t_0, t_0 + T_p]$. To ensure the corresponding electric field $E_p(t) = -A_F(t)$ is zero at the end points of the interval and make sure no momentum is transferred to the system ($A_F(t_0 + T_p) = A_F(t_0) = 0$), the boundary coefficients are fixed by $c_1 = c_2 = c_{N_f-1} = c_{N_f} = 0$.

For the switching scenario, we are interested in long-time stable dynamics of $\Delta n(t)$. As is known from the analysis with respect to the single-cycle pulses, amplitude mode oscillations are expected to be present around a switched value of the order parameter after time $t_f$. We thus perform a linear fit $\Delta n(t) = a(t - t_f) + b$ to the dynamics of $\Delta n(t)$ after the pulse. We then required that (i) the mean value of the order, encoded in $b$, is
maximal, while (ii) the average slope $|a|$ should be minimal. The condition (ii) is necessary for the long-time stability of the switched state to ensure no drift can occur at longer time scales. Similarly, for coherent destruction one requires $|b|$ to be minimal. Gathering the B-spline coefficients in the vector $e$, the target functional for switching the order from $\Delta n(t = 0) < 0$ is given by

$$J_{\text{switch}}[e] = -b + e_1|a| + e_2E_{\text{abs}}, \quad (A28)$$

while we use

$$J_{\text{CD}}[e] = |b| + e_1|a| + e_2E_{\text{abs}} \quad (A29)$$

for achieving coherent destruction. Here, $e_1$ is a penalty parameter for the slope, whereas $e_2$ denotes the penalty with respect to the absorbed energy $E_{\text{abs}}$. In order to evaluate the functionals (A28) or (A29), one has to perform the time propagation up to a sufficiently large time $T_{\text{max}}$ (we set $T_{\text{max}} = 500$), compute the fitting parameters $a$, $b$ and the absorbed energy. As mentioned above, the gradient with respect to $e$ cannot be calculated directly. Therefore, we minimize the functionals by a combination of the Pikaia genetic algorithm [4] for finding local minima and the NEWUOA algorithm [5] for finding the global minimum. This procedure depends on the parameters $N_b$, $e_1$, $e_2$ and the pulse duration $T_p$.

**Appendix B: Dynamics of phonon-driven charge order**

As discussed in the main text, the distinct regimes in the nonequilibrium phase diagram are tightly connected to the driving mechanism of the CDW. In order to corroborate this behavior and, moreover, test the robustness of nonequilibrium features with respect to lattice distortions, we included e-ph couplings. Both, the e–e and the e–ph coupling is responsible for the formation of the CDW. The relative contribution from either effect can be captured by the parameter

$$V_{\text{CDW}} \equiv V_{\text{ph}} + V_e = \frac{g^2}{\omega_{\text{ph}}} - \frac{U}{2}. \quad (B1)$$

The parameter (B1) is related to the interaction energy, as can be seen by computing the total energy in the MF approximation:

$$E_{\text{tot}} = 2 \sum_k \text{Tr} \left[ h^{(0)}(k)\rho(k) \right] + \sum_k \text{Tr} \left[ (h^{\text{MF}}(k) - h^{(0)}(k))\rho(k) \right]$$

$$\equiv E_0 + E_{\text{int}}.$$ 

Expressing the interaction energy by the order and distortion parameters in equilibrium, one finds $E_{\text{int}} = V_{\text{CDW}}\Delta n^2$. Note that an identical value of $V_{\text{CDW}}$ is a consequence of the individual contributions of the e–e or e–ph interactions, gives rise to the same value of $\Delta n$ and the gap size.

Fixing $V_{\text{CDW}} = 0.625$ (corresponding to the results of the main text) we now vary the ratio $V_{\text{ph}}/V_{\text{CDW}}$ and study how the increased contribution of e–ph interactions to the order affects the pulse-induced dynamics. Figure 7 shows the nonequilibrium phase diagram (MF approximation) analogous to Fig. 1(b) in the main text for $V_{\text{ph}}/V_{\text{CDW}} = 0.1$ (top), $V_{\text{ph}}/V_{\text{CDW}} = 0.2$ (middle) and $V_{\text{ph}}/V_{\text{CDW}} = 1$ (bottom). For a CDW dominated by e–e correlation effects, the different regimes of amplitude mode oscillations, coherent destruction and switching are qualitatively still present, but superimposed with coherent phonon oscillations. It is interesting to see that the lower boundary of the coherent destruction regime represents the fastest way to destroy the order, whereas the “sweet spot” is exhibiting more oscillations. In general, the amplitude of the phonon oscillations increases under stronger driving.

The qualitative behavior of the laser-driven nonequilibrium regimes is still present for $V_{\text{ph}}/V_{\text{CDW}} = 0.2$, albeit the boundaries are smeared out by the phonon oscillations. For an even larger contribution of the electron-phonon coupling, the dynamics is dominated by the phonons and thus displays the generic behavior of the purely phonon-driven case (bottom panel in Fig. 7). In this scenario, the persistent oscillations of $\Delta n$ with frequency $\omega_{\text{ph}}$ are the dominating feature. Neither destruction nor switching of the CDW is possible anymore.

We conclude that a qualitatively different dynamics of the order parameter for different pulse amplitudes is a clear signature of a predominantly correlation-driven CDW formation. Small e–ph coupling leads to small additional coherent phonon oscillations, but does not suppress the characteristic features discussed in the main text. A larger contribution of...
the phonons, on the other hand, suppresses any switching behavior.

Appendix C: Switching and coherent destruction by optimized pulses

As explained in subsec. A5, the pulse optimization with the aim of switching the CDW is performed on the MF level and depends on the number of B-spline coefficients $N_p$, the slope penalty $\epsilon_1$, the penalty with respect to the absorbed energy $\epsilon_2$ and the pulse duration $T_p$. We performed the pulse optimization for various combinations of these parameters and found that $N_p = 28$ is enough to find the optimal pulse shapes. Increasing $N_p$ yields essentially the same pulses with extra oscillations on top. Furthermore, the value of $\epsilon_1$ affects the pulses only weakly since most of the resulting pulses result in a vanishing average slope of $\Delta n(t)$. The pulse length $T_p$ was varied from $T_p = 5.0$ to $T_p = 20.0$; we select the best pulses in this range for a fixed value of $\epsilon_2$.

1. Switching dynamics

The energy penalty $\epsilon_2$ is the most crucial parameter. Choosing $\epsilon_2 = 0$ results in very strong pulses, leading to almost perfect switching on the MF level (Fig. 8(a)). However, within the 2Bloc approximation, the huge amount of absorbed energy rapidly destroys the order. Further analysis shows that the system thermalizes at a very high effective temperature shortly after the pulse. Gradually increasing $\epsilon_2$ decreases the switching efficiency (Fig. 8(b–e)) while reducing the energy absorption. This leads to a longer life time of the switched state within the 2Bloc dynamics. Interestingly, the shape of the vector potential $A_F(t)$ looks quite similar in Fig. 8(c–e). It corresponds to the minimization of dephasing which is explained in the main text. The best compromise between energy absorption and switching is provided by the pulse in Fig. 8(e). We found that applying a smoothening low-pass filter further reduces $E_{\text{abs}}$, while the short-time dynamics is not altered. This optimal pulse is the one presented and discussed in the main text. Note that increasing $\epsilon_2$ further leads to a suppression of switching, since the requirement to minimize the absorbed energy – which is zero if no pulse is applied – starts to dominate.

2. Coherent destruction dynamics

An analogous analysis was carried out for the coherent destruction of the CDW order. However, we found that the optimal pulse and the resulting dynamics is very robust against changes of $\epsilon_1$ and $\epsilon_2$. The pulse with the smallest $E_{\text{abs}}$ is shown in Fig. 9(b) and compared to the dynamics driven by the single-cycle pulse at the "sweet spot" (Fig. 9(a)) discussed in the main text. It is interesting to note that the simple single-cycle pulse results in perfect supression of the order while injecting only little energy into the system. Correspondingly, the optimized field $A_F(t)$ is qualitatively almost the same as the single-cycle pulse. However, the absorbed energy is reduced, such that the thermalization (Fig. 9(d)) is slower than for the single-cycle pulse (Fig. 9(c)).

Appendix D: Anisotropic two-dimensional lattice

In the main text, we consider a one-dimensional configuration of the lattice. Note that the local approximation to the self-energy leads to generic features of a higher-dimensional system, while the 1D character primarily enters via the free band structure. Most CDW orders observed in materials are,
in fact, two-dimensional (typically stripe or checker board order). In this section we confirm that our results based on the 1D system are also valid for the 2D case with anisotropic hopping. To be concrete, we consider a square lattice with hopping $J_0$ in the $x$-direction and $(1 - \delta)J_0$ in the $y$-direction (see Fig. 10(a)); $\delta = 0$ corresponds to the isotropic 2D system, while $\delta = 1$ recovers the 1D limit. The CDW forming in this configuration follows a checker-board order, corresponding to a nesting vector $\mathbf{Q} = (\pi/a, \pi/a)$.

The derivations in Section A are applicable to the 2D case, as well, after (i) replacing the 1D wave vector $k$ by a vector $\mathbf{k}$ from the reduced Brillouin zone shown in Fig. 10(b), and (ii) modifying the free Hamiltonian (mp basis) to

$$h^{(0)}(\mathbf{k}) = \begin{pmatrix} \epsilon(\mathbf{k}) & U/2 \Delta n \\ U/2 \Delta n & \epsilon(\mathbf{k} + \mathbf{Q}) \end{pmatrix}. \quad \text{(D1)}$$

Here,

$$\epsilon(\mathbf{k}) = -2J_0 \left( \cos(k_xa) + (1 - \delta) \cos(k_ya) \right) \quad \text{(D2)}$$

denotes the original free band structure.

We have performed equilibrium calculations with the 2Bloc approximation for different values of $\delta$ (see Fig. 10(c)). The order parameter $\Delta n$ deviates by less than 10% from the 1D value in the regime of anisotropy $\delta = 0.7 \ldots 1$. This relatively large span shows that small deviations from our 1D setup have almost no influence on the results discussed in the main text.

Furthermore, we have analyzed the pulse-induced dynamics in the 2D scenario. As an example, we show the dynamics of the order parameter at the “sweet spot” of coherent destruction within the MF approximation in Fig. 10(d). We applied the same pulse as for the 1D case (polarization along the $x$ direction). One observes similar behavior as for the equilibrium properties: for moderately strong anisotropy, the time evolution of $\Delta n$ is very close to the 1D case. Therefore, the different regimes of the nonequilibrium phase diagram discussed in the main text are also relevant for the anisotropic 2D system.

![Fig. 9. Dynamics on the 2Bloc level at the sweet spot of coherent destruction: (a) single-cycle pulse, (b) optimized pulse. The insets show the laser fields. The corresponding fluctuation measure $P(t)$ is shown in panels (c) and (d), respectively.](image)
FIG. 10. (a) Sketch of the checker-board CDW order in an anisotropic 2D square lattice. The shaded area shows the unit cell chosen for the calculations. (b) Full (while square) and reduced (gray shaded) Brillouin zone. (c) Dependence of the order parameter $\Delta n$ on the anisotropy $\delta$. (d) Dynamics induced by the single-cycle pulse at the point of coherent destruction for different values of $\delta$. 