Toward coupling of groundwater drawdown and pumping time in a constant discharge

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Abstract
Groundwater is considered an essential water resource in arid and semi-arid regions such as Iran. This study used a copula-based approach to analyze the joint frequency of groundwater level and the duration of groundwater pumping with a constant discharge. In particular, this study examines the correlation between the pumping time and groundwater drawdown variables for two cases of 26.6 and 28.8 l/s constant discharges and a pumping time of 220 min. In addition, the Weibull probability distribution and Galambos copula were used for these two tests. To estimate the groundwater drawdown at different pumping times with different probabilities, the obtained typical curves by providing the contour curves of the cumulative groundwater drawdown probability and the pumping time in both tests were obtained. For example, for 150 min of pumping, the groundwater drawdown for pumping discharge of 26.64 and 28.8 l/s with a 60% probability is about 7.4 and 8 m, respectively. The results of the joint-occurrence frequency analysis in the study area showed that for each unit of increase in pumping discharge in the pumping well, a drawdown of 0.32 m is imaginable in the observation well. In the next step, the groundwater drawdown got analyzed in both tests simultaneously. Since the pumping time is the same, the effect of increasing the pumping discharge in the study area is observable in the joint-occurrence probability curve.

Keywords Aquifer · Copula functions · Frequency analysis · Probability of occurrence

Introduction
Hydrodynamic coefficients are among the most basic required data for optimum utilization of groundwater resources. Hydraulic conductivity values are effective criteria in determining the hydraulic relationship between surface water and groundwater, designing drainage systems, estimating leakage from open canals or reservoirs, and predicting and eliminating groundwater pollution, and should be estimated close to reality. Using pumping constant discharge from the pumping well, the values of aqueous layers hydrodynamic coefficients, based on the drawdown in piezometric level in the observation well, are calculable with Theis, Chow, and Cooper–Jacob equations. All these calculations require pumping testing, which is costly and time-consuming. If there is a pumping test by frequency analyzing and simulating, it may be possible to avoid further tests and tests with different discharges. One of the most important tools in this regard, which has been extensively studied, is copula (Sklar 1959). The use of copula functions in various studies in various fields such as simulation, modeling, frequency analysis, and prediction of different variables in different dimensions has flourished (De Michele and Salvadori 2003; De Michele et al. 2005, 2007; Khozeimeh-Nezhad and Tahroudi 2019; Tahroudi et al. 2020a, b; Nazeri-Tahroudi et al. 2021a, b; Khashei-Siuki et al. 2021).

Ma et al. (2013) investigated drought in the Weihe River Basin using Student Gaussian copula to model the joint distribution between drought, intensity, and peak time. The properties of different copula models for data simulation and extreme value prediction were also discussed. Xu et al. (2015) developed a model of regional drought frequency analysis based on 3 copula with respect to temporal–spatial changes in drought. The results show that drought frequency analysis should fully consider all three parameters of duration, affected area, and severity that reflect the spatial–temporal variability of drought. Hao et al. (2017) used

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k-means and t-copula classification methods to show the probability of regional drought and return period based on 3 drought features, for example, drought duration, severity, and peak. The results showed that appropriate classification is important for regional drought frequency analysis and copulas are useful tools in examining the relationship between drought characteristics and drought frequency analysis. Tahroudi et al. (2020b) developed an approach based on the conditional density of copulas, and presented a new method for frequency analysis, leading to an equation for predicting different parameters. Nazeri-Tahroudi et al. (2021a) used a copula-based approach to analyze the frequency and predict flood hydrographs and present a flood routing approach. The results of comparing the proposed method with other conventional methods in flood routing showed that the copula-based approach had higher accuracy and certainty. Saghafran and Sanginabadi (2020) performed multivariate analysis of groundwater drought using copulas. Standardized GW Index (SGI) was used in this study. While investigating different copulas, frequency-based analysis was performed. Finally, comparing the probabilities of drought and experimental groundwater drought using statistical tests proved the adequate accuracy of the copula-based model in multivariate drought analysis. They stated that multivariate drought analysis addressed the main features of the copula-based model in drought monitoring and managing the groundwater network in each region, in this study, it has been attempted to investigate a copula-based approach to the frequency analysis of groundwater drought with respect to pumping time. According to the above, the main objective of this study was to provide a copula-based approach to the analysis of the parameters involved in the pumping test with constant discharge. In this regard, pair-variables (a: groundwater level-pumping time and b: groundwater level–groundwater level in two pumping tests) have been considered to copula-based frequency analysis.

**Materials and methods**

The study area is Bazargan plain in West Azerbaijan Province located in northwestern Iran. In this area, an exploratory well has been drilled at AgBelakh site with a depth of 71.3 m and a diameter of 6 inches and a piezometric well at a distance of 10.3 m with a diameter of 6 inches and a depth of 35.3 m. In this area, two pumping tests with constant discharge of 26.64 and 28.8 l/s have been performed. The results showed that the most important effect on groundwater drawdown was related to the utilization of groundwater in the region. Runoff and irrigation were both inversely related to water surface depth due to groundwater recharge. The results of their research provided solutions on regulating irrigation and groundwater utilization for effective management of groundwater resources.

Copulas in the discussion of simulation of various parameters have also been developed in recent studies. For example, Khashei-Siuki et al. (2021) used a copula-based approach to simulate potential evapotranspiration. The results showed the acceptable accuracy of a copula-based model in simulating the mentioned values. Guo et al. (2022) investigated the effect of extreme climate and simultaneous events on the growth of essential crops in the Songliao Plain during the statistical period 1981–2016 using copulas. They used the index of crop surplus and water shortage, extreme growth of degree-day, and intensity of heat stress at different stages of maize growth. Finally, the study results led to the presentation of the relationship between simultaneous events and maize yield, indicating different degrees of water shortage and warming process during each growing season. Ma and Zhang (2022) developed multivariate modeling of ocean data by developing a truncated copula-based model. The proposed model was based on physical constraints to study asymmetric-dependent ocean data. Finally, by comparing the advantages and disadvantages of different copula-based models, the features of different copula-based models for data simulation and extreme value prediction were presented. Singh et al. (2022) to simulate the discharge using Ganges River discharge data during the 25-year statistical period (1949–1973) used various copulas such as Frank, Clayton, Gumbel–Hougaard, and Ali–Mikhail–Haq. Compared to ARMA model, they showed that copula performed better than ARMA for each series.

According to the literature review, the use of copulas in different fields has been investigated and its accuracy has been confirmed in all cases. This method, considering the use of marginal distributions and the involvement of effective parameters in modeling and simulations, has presented regional results that are specific to the study area and closer to reality. For this reason, to present a proposed approach for monitoring and managing the groundwater network in each region, in this study, it has been attempted to investigate a copula-based approach to the frequency analysis of groundwater drawdown with respect to pumping time. According to the above, the main objective of this study was to provide a copula-based approach to the analysis of the parameters involved in the pumping test with constant discharge. In this regard, pair-variables (a: groundwater level-pumping time with constant discharge and b: groundwater level–groundwater level in two pumping tests) have been considered to copula-based frequency analysis.
Kendall’s tau is equivalent to Spearman’s rho in terms of underlying hypotheses. However, Kendall’s tau and Spearman’s rho are not identical regarding the size, because their basic logic and computational formulas are quite different. The main advantage of using Kendall’s tau over Spearman’s rho is that Kendall’s tau can interpret a value as a direct measurement of coordinated and contrasting pairs. The disadvantage of the rank correlation coefficient is data loss when reduced to zero. With normally distributed data, the power of the Pearson correlation coefficient gets reduced (Gautier 2001). In addition, they are not applicable for identifying dependence when involved with more than two variables. See Kruskal (1958) and Nazeri-Tehroudi et al. (2021c) for further explanation and relationships of Kendall’s tau test.

Copulas

The use of copulas in this study plays a key role in the bivariate analysis of groundwater drawdown with pumping time. Copulas link CDF of marginal distributions of studied pair-variables (in the present study, pair-variables of groundwater drawdown-pumping time with constant discharge of 26.64 and 28.8 L per second and pair-variables of groundwater drawdown with discharge of 26.64 L per second and groundwater drawdown with pumping discharge of 28.8 L per second in the studied well). These marginal distributions are the same as continuous univariate probabilistic distributions and include values between zero and one. Hence, copulas can provide a multivariate distribution based on univariate margin distributions regardless of their type or shape. Introduction of copulas is attributed to Sklar (1959), who described in a theory how one-dimensional distribution functions can be combined into multivariate distributions. For the continuous N-dimensional random variables $X_1, X_2, \ldots, X_N$ with marginal distributions $F_{X_i}(x_i) = P_{X_i}(X_i \leq x_i)$, the joint distribution of the variable X can be defined as follows:

$$H_{X_1,\ldots,X_N}(x_1, x_2, \ldots, x_N) = P[X_1 \leq x_1, X_2 \leq x_2, \ldots, X_N \leq x_N].$$

(1)

A copula is a function that combines univariate margin distribution functions to form a two- or multivariate distribution function. Thus, Sklar showed that the probabilistic multivariate distribution of H using the marginal distributions and the dependence structure can be expressed by the copula C

$$C(F_{X_1}(x_1), F_{X_2}(x_2), \ldots, F_{X_N}(x_N)) = H_{X_1,\ldots,X_N}(x_1, x_2, \ldots, x_N),$$

(2)

where $F_{X_i}(x_i)$ is the $i$th continuous margin distribution and $H_{X_1,\ldots,X_N}$ is the joint cumulative distribution of $X_1, X_2, \ldots, X_N$. Given that for continuous random variables, the function of the cumulative distribution of margins from zero to one is non-decreasing, the copula C can be considered as a conversion of $H_{X_1,\ldots,X_N}$ from $[-\infty, +\infty]^N$ to $[0, 1]^N$. This conversion separates the marginal distributions, and as a result, the copula C is only related to the relationship between the variables.
and a complete description of the dependence structure is obtained (Nelson, 2007). For two-dimensional copulas, $H$ is assumed to be a joint distribution of variables $X_1$ and $X_2$ with cumulative distributions of $u_1 = F_{X_1}(x_1)$ and $u_2 = F_{X_2}(x_2)$. In this case, a two-dimensional copula exists in the set of real numbers and is expressed as Eq. (3)

$$H(x_1, x_2) = C(u_1, u_2) = C(F_{X_1}(x_1), F_{X_2}(x_2)).$$  \tag{3}$$

In two-dimensional copulas, there are properties such as $C(u_1, 0) = C(0, u_2) = 0$ and $C(u_1, 1) = C(1, u_2) = 1$ for $u_1$ and $u_2$ in $[0, 1]$ (De Michele et al. 2005; Tahroudi et al. 2021b).

The inference functions for margins method also used to estimate the copula-dependent parameter. This method is more efficient mathematically than the method of moments, maximum-likelihood, and canonical maximum-likelihood methods for estimating the copula-dependent parameter (Tahroudi et al. 2020b).

**Empirical copulas**

Given that the complexity of the copulas increases by increasing dimensions of the problem, so most researchers use the empirical copulas for multivariate analysis. The concept of empirical copulas is in fact similar to the concept of plotting position formulas used for univariate statistical analyses (for example Weibull formula). In fact, these copulas are rank cumulative probability criteria (Nelson 2007). For a sample of $n$, the following $d$-dimensional empirical copula $C_n$ is defined as follows:

$$C_n(x_1, x_2, \ldots, x_d) = \frac{1}{n} \sum_{i=1}^{n} I\left(\frac{R_{i1}}{n + 1} \leq u_{i1}, \ldots, \frac{R_{id}}{n + 1} \leq u_{id}\right),$$  \tag{4}$$

where $n$ is the number of observations and $I(A)$ is the indicator variable of $A$ logical expression. If $A$ expression is correct, it takes the value of one, and if it is incorrect, it takes the value of zero. $R_{i1}, \ldots, R_{id}$ is the rank of observational data (or $u_{i1}, \ldots, u_{id}$) and the value of the cumulative distribution function is related to the data studied.

Based on copulas, the distribution function $(K_C)$ can be defined, which is a probabilistic cumulative criterion for

$$\left\{ (u_1, \ldots, u_d) \in [0, 1]^d | C_{U_1, \ldots, U_d}(u_1, \ldots, u_d) \leq b \right\}.$$  

The distribution function is defined as Eq. (6)

$$K_C(t) = P[C_{U_1, \ldots, U_d}(u_1, \ldots, u_d) \leq b],$$  \tag{6}$$

where $P$ is the value and $b$ is threshold of probability. For Archimedean copula, the distribution function can be defined based on the generator function $(\phi)$

$$K_C(b) = b - \frac{\phi(b)}{\phi'(b)}, \quad b \in [0, 1].$$  \tag{7}$$

For some non-Archimedean copulas, it may not be possible to obtain a specific expression for the calculation of $K_C$. In this case, other methods such as Monte Carlo simulation are used for this purpose. Another method is to use the empirical distribution function, which is presented in accordance with the empirical copula as follows (Genest and Rivest 1993):

$$K_{C_n}(t) = \frac{1}{n},$$  \tag{8}$$

where $t$ is equal to the number of samples $\{x_1, \ldots, x_d\}$ that apply to the condition $C_n(k_1/n, \ldots, k_d/n) \leq 1/n$. Equation (8) can also be rewritten as follows:

$$K_{C_n}(t) = \frac{1}{n} \sum_{j=1}^{n} I(e_{jm} \leq t),$$  \tag{9}$$

where $e_{jm}$ is obtained from Eq. (10)

$$e_{jm} = \frac{1}{n - 1} \sum_{k=1}^{n} I(x_{1(j)}, \ldots, x_{d(j)}),$$  \tag{10}$$

where $I(A)$ like Eq. (5) is the indicator variable of $A$ logical expression, and if $A$ expression is correct, the value is one, and if it is incorrect, the value of zero is assigned to it. The empirical copula $(C_n)$ and empirical distribution function $(K_{C_n})$ can be used to validate and evaluate the performance of theoretical copulas in multivariate modeling of the studied phenomena. In this study, to analyze the mentioned variables, seven different copulas of Clayton, Gumbel–Hougaard, Frank, Ali–Mikhail–Haq, Galambos, Farlie Gumbel Morgenstern, and Plackett were used. The equations for each copula are presented in Table 1.

**Rotated version of copula**

Since that some copulas cannot showed that the negative tail dependencies and dependencies in all direction, such as Clayton and Gumbel and these copulas, will not fit.
Table 1  Copula used (Nelson 2007)

| Copula                  | Probability density function and cumulative distribution of the copula function | θ   |
|-------------------------|---------------------------------------------------------------------------------|-----|
| Ali–Mikhail–Haq (AMH)   | $C(u, v) = \exp \left\{ \left( u - \theta (1 - u) v \right) \right\}$ | $-1 \leq \theta \leq 1$ |
|                         | $c(u, v) = \frac{1 - \theta (1 - u) v}{1 - \theta (1 - u) (1 - v)} \exp \left\{ \left( u - \theta (1 - u) v \right) \right\}$ | |
| Clayton                 | $C(u, v) = \max \left\{ \left( u^\theta + v^\theta - 1 \right)^{-1/\theta}, 0 \right\}$ | $\theta \geq 0$ |
|                         | $c(u, v) = \left( \theta + 1 \right) \left( u^\theta + v^\theta - 1 \right)^{-1/(\theta - 2)} (uv)^{-\theta - 1}$ | |
| Frank                   | $C(u, v) = -\frac{1}{\theta} \ln \left\{ 1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{\theta} \right\}$ | $\Theta \neq 0$ |
|                         | $c(u, v) = -\frac{1}{\theta} \ln \left\{ 1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{\theta} \right\}$ | |
| Galambos                | $C(u, v) = uv \exp \left\{ \left( -\ln u \right)^\theta + \left( -\ln v \right)^\theta \right\}$ | $\theta \geq 0$ |
|                         | $c(u, v) = \frac{C(u,v)}{uv} \left\{ 1 - \left[ \left( -\ln u \right)^\theta + \left( -\ln v \right)^\theta \right]^{\frac{1}{\theta - 1}} \left[ \left( -\ln u \right)^{\theta - 1} + \left( -\ln v \right)^{\theta - 1} \right]^{-\frac{1}{\theta - 1}} \right\}$ | |
| Gumbel–Hougaard (GH)    | $C(u, v) = \exp \left\{ \left( -\ln u \right)^\theta + \left( -\ln v \right)^\theta \right\}$ | $\theta \geq 1$ |
|                         | $c(u, v) = C(u,v) \left[ \left( -\ln u \right)^\theta + \left( -\ln v \right)^\theta \right]^{\frac{1}{\theta - 1}} \times \left( \left( -\ln u \right)^{\theta - 1} + \left( -\ln v \right)^{\theta - 1} \right)^{-\frac{1}{\theta - 1}} + 1 \right\}$ | |
| Farlie–Gumbel–Morgenstern (FGM) | $C(u, v) = uv[1 + \theta(1 - u)(1 - v)]$ | $-1 \leq \theta \leq 1$ |
|                         | $c(u, v) = 1 + \theta(1 - 2u)(1 - 2v)$ | |
| Plackett                | $C(u, v) = \frac{1}{\theta - 1} \left\{ 1 + (\theta - 1)(u + v) - \theta(\theta - 1)(u + v)^2 \right\}^{\frac{1}{\theta - 1}}$ | $\theta \geq 0$ |
|                         | $c(u, v) = \left[ \left( 1 + (\theta - 1)(u + v) \right)^2 - 4\theta(\theta - 1)uv \right]^{\frac{1}{\theta - 1}}$ \(\theta[1 + (\theta - 1)(u + v - 2uv)]\) | |

Because random variable has negative dependence. To overcome this problem, copulas may then be “rotated” and re-applied (Cech, 2006; Luo, 2011; Sriboonchitta et al. 2013).

The rotating version of copulas at different angles such as 90°, 180°, and 270° can be derived from a simple copula. Rotate version of copula 180° is defined as
\( \bar{u} = 1 - u \)
\( \bar{v} = 1 - v, \)

which the copula of \( \bar{u} \) and \( \bar{v} \) defined as (Cech, 2006)

\[ C^{--}(u, v) = u + v - 1 + C(1 - u, 1 - v) \]
and
\[ c^{--}(u, v) = c(1 - u, 1 - v), \]

where \( c^{--} \) is the density of copula \( C^{--} \). And for a 90° rotated copula

\[ C^{++}(u, v) = u - C(1 - u, v) \]
and
\[ c^{++}(u, v) = c(1 - u, v). \]

Also, for a 270° rotated copula

\[ C^{+-}(u, v) = u - C(u, 1 - v) \]
and
\[ c^{+-}(u, v) = c(u, 1 - v). \]

**Joint-occurrence probability**

The joint-occurrence probability makes it possible to calculate the probability distribution of variable \( X \) if the variable \( Y \) exceeds a certain threshold (Eq. 15) or the probability distribution of the variable \( Y \) if the variable \( X \) exceeds a certain threshold (Eq. 16) (Shiau, 2006)

\[ P(X \leq x|Y \geq y) = \frac{F_X(x) - F_Y(y)}{1 - F_Y(y)} = \frac{F_X(x) - C(F_Y(y).F_X(x))}{1 - F_Y(y)} \]

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**Fig. 3** The flowchart of research methods used in this study

1. Preparation of groundwater drawdown data and pumping time in two constant discharge modes of 26.6 and 28.8 liters per second
2. Investigation of the correlation between the pair-variable of pumping time and groundwater drawdown in two constant discharge modes of 26.6 and 28.8 liters per second
3. Selection of marginal distribution of the studied data based on Kolmogorov-Smirnov and Anderson-Darling tests in two modes of constant discharge
4. Selection of copula function appropriate to the pair-variable of pumping time and groundwater drawdown for two modes of constant discharge
5. Evaluation the cumulative probability of both groundwater drawdown and pumping time for pumping test with discharges of 26.6 and 28.8 liters per second
6. Repeat the mentioned steps for the pair of groundwater drawdown with discharge of 26.6-groundwater drawdown with discharge of 28.8
where the values of $x$ and $y$ are the threshold for random variables, and $C$ is the best copula function. $F(x)$ and $F(y)$ are the values of the best margin distribution.

**Performance evaluation**

Akaike information criterion (AIC), Nash–Sutcliffe efficiency (NSE), root-mean-square error (RMSE), and mean absolute error (MAE) evaluation criteria were used to evaluate the calculations performed in this study (Nash and Sutcliffe 1970; Zhang and Singh 2006; Ma et al. 2013; Akbarpour et al. 2020)

\[
P(Y \leq y | X \geq x) = \frac{F_Y(y) - F_{XY}(x,y)}{1 - F_X(x)} = \frac{F_Y(y) - C(F_X(x), F_Y(y))}{1 - F_X(x)},
\]

(16)

where the values of $x$ and $y$ are the threshold for random variables, and $C$ is the best copula function. $F(x)$ and $F(y)$ are the values of the best margin distribution.

**Table 2** Results of marginal distributions of values of groundwater drawdown and pumping time with a discharge of 26.64 l/s

| Probability distribution | NSE  | NRMSE | Time (Min) | Drawdown (m) | NSE  | NRMSE | Time (Min) | Drawdown (m) |
|--------------------------|------|-------|------------|--------------|------|-------|------------|--------------|
| NOR                      | 0.82 | 0.96  | 12.32      | 5.47         | 0.82 | 0.96  | 12.32      | 5.47         |
| LN                       | **0.99** | 0.95  | **2.32**   | **6.75**     | **0.99** | 0.95  | **2.32**   | **6.75**     |
| EXP                      | 0.95 | 0.01  | 6.44       | 28.87        | 0.95 | 0.01  | 6.44       | 28.87        |
| GAM                      | 0.98 | 0.95  | 3.77       | 6.32         | 0.98 | 0.95  | 3.77       | 6.32         |
| GEV                      | 0.99 | 0.93  | 3.46       | 7.71         | 0.99 | 0.93  | 3.46       | 7.71         |
| LOG                      | 0.90 | 0.98  | 9.00       | 4.25         | 0.90 | 0.98  | 9.00       | 4.25         |
| GLOG                     | 0.99 | 0.97  | 2.61       | 4.70         | 0.99 | 0.97  | 2.61       | 4.70         |
| Rayleigh                 | 0.13 | 0.26  | 27.17      | 25.05        | 0.13 | 0.26  | 27.17      | 25.05        |
| Nakagami                 | 0.95 | 0.96  | 6.61       | 5.95         | 0.95 | 0.96  | 6.61       | 5.95         |
| GPA                      | 0.99 | −0.10 | 2.62       | 30.57        | 0.99 | −0.10 | 2.62       | 30.57        |
| WEI                      | 0.99 | **0.98** | 2.86     | **4.14**     | 0.99 | **0.98** | 2.86     | **4.14**     |
where, $Q_i$, $\bar{Q}_i$, and $\bar{Q}$ are the measured, simulated, and mean data, respectively, and $N$ and $m$ are the number of data and number of parameters, respectively. The steps of the research method were presented in the form of a flowchart in Fig. 3.

$$AIC = 2m + N \ln(MSE)$$  \hspace{1cm} (20)$$

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (\bar{Q}_i - Q_i)^2,$$  \hspace{1cm} (21)$$

Table 3 Results of marginal distributions of values of groundwater drawdown and pumping time with a discharge of 28.8 l/s

| Probability distribution | NSE  | NRMSE | Drawdown (m) | NSE  | NRMSE | Drawdown (m) |
|--------------------------|------|-------|--------------|------|-------|--------------|
| NOR                      | 0.72 | 0.99  | 15.30        | 3.51 |       |               |
| LN                       | 0.99 | 0.97  | 2.73         | 4.84 |       |               |
| EXP                      | 0.69 | 0.06  | 16.10        | 28.15|       |               |
| GAM                      | 0.95 | 0.98  | 6.27         | 4.36 |       |               |
| GEV                      | 0.99 | 0.91  | 3.39         | 8.64 |       |               |
| LOG                      | 0.81 | 0.99  | 12.67        | 2.96 |       |               |
| GLOG                     | 0.99 | 0.99  | 3.15         | 3.30 |       |               |
| Rayleigh                 | -0.32| 0.31  | 33.28        | 24.01|       |               |
| Nakagami                 | 0.92 | 0.98  | 8.27         | 3.96 |       |               |
| GPA                      | 0.98 | -0.15 | 3.80         | 31.09|       |               |
| WEI                      | 0.98 | 0.99  | 4.58         | 2.90 |       |               |

**Results and discussion**

To investigate the behavior of groundwater drawdown and pumping time in the study area, a joint frequency analysis was used, which is based on copulas. According to the relationships presented in the physics of copulas, the existence of correlations between the studied variables is one of the fundamental conditions. This study uses two tests of constant pumping with a discharge of 26.64 and 28.8 l/s. Figures 4 and 5 show the study results of the correlation between values of groundwater drawdown and pumping time in the two tests. An acceptable correlation is observable in the figures between the mentioned values. In various studies in the field of frequency analysis and copula-based simulations, different correlations such as 0.4 and above were reported to be acceptable (De Michele et al. 2007; Salvadori et al. 2007; Tahroudi et al. 2020a, b; Nazeri-Tahroudi et al. 2022).

To fully cover the data scatter in all directions due to the data scatter in the northern and northeastern regions of scattering curves, this study uses rotated states of copulas. These copulas can cover correlations in all directions. After confirming the correlation between the studied data, the study investigates the marginal distribution functions appropriate to the groundwater drawdown and pumping time data. Tables 2 and 3 show the results of these investigations.

According to the results shown in Tables 2 and 3, the log-normal distribution, as regards fitting with the pumping time data in both discharges of 26.64 and 28.8 l/s, has
superiority. The fit of the Weibull probability distribution also had a better fit for both mentioned discharges according to the NSE and NRMSE statistics. By selecting two log-normal and Weibull marginal distributions for pumping time and groundwater drawdown values, respectively, different copulas, including Clayton (Cly), Ali-Mikhail-Haq (AMH), Farlie–Gumbel–Morgenstern (FGM), Frank (Fra), Gumbel–Hougaard (GH), Galambos (Gal), and Plackett (Pla), were fitted to the data. Then, the best copula was selected based on NSE and RMSE criteria. Figures 6 and 7 show the results of the mentioned statistics for choosing the best copula for the pair of variables of groundwater drawdown and pumping time.

According to NSE and RMSE evaluation statistics for selecting the best copula, results showed that with both discharges of 26.64 and 28.8 l/s, Galambos had the lowest RMSE value (0.06 and 0.05 for 26.64 and 28.8 l/s, respectively) and the highest NSE value (0.96 for both discharges). According to the proposed criteria, Galambos was selected as the best copula for both variables of groundwater drawdown and pumping time. Also, the $\theta$ coefficient of selected copula between the pair-variables...

**Fig. 8** Contour lines with a cumulative probability of groundwater drawdown and pumping time for pumping test with a discharge of 26.64 l/s

**Fig. 9** Contour lines with a cumulative probability of both groundwater drawdown and pumping time for pumping test with a discharge of 28.8 l/s
of groundwater drawdown and pumping time for 26.64 and 28.8 l/s were estimated to be 4.38 and 9.05, respectively. Finally, this study analyzed the joint frequency of the groundwater drawdown and pumping time pair-variables with selected margin distributions and Galambos copula. Figures 8 and 9 show the chosen copula results on the mentioned values for estimating the cumulative probability with a discharge of 26.64 l/s and 28.8 l/s, respectively.

According to the contour lines shown in Figs. 8 and 9, it is easy to estimate the probability of groundwater drawdown at different pumping times. For example, as shown in Fig. 8, pumping with a constant discharge of 26.64 l/s for 200 min, with a probability of joint-occurrence of 10 and 70%, the groundwater drawdown in the study area will be about 6.1 and 7.6 m, respectively. With increasing the constant pumping flow to 28.8 l/s in the same period in the pumping well, the drawdown in groundwater level in the study area with a probability of 10 and 70% (Fig. 9) will be about 6.5 and 8.3 m, respectively. In this way, the groundwater drawdown is estimable with different probabilities. These contour lines are functional as typical curves for the study area. Increasing 2.16 pumping units in the pumping well in this area, and an 80% probability, the groundwater drawdown has escalated by about 0.7 m. For each unit increase in pumping discharge in the pumping well, a drawdown of 0.32 m is observable in the observation well. Using copulas in various studies leads to distinguishing results and provides supportive data in the water resources management field. The reason for acquiring these results is marginal distribution and involving other effective parameters in many analyzes. According to the presented curves, the pumping in the area is controllable, and the groundwater drawdown under critical conditions is preventable. The curves in the area play a prominent role in monitoring the groundwater level. The study analyzes two time series of groundwater drawdowns due to the discharge of 26.64 and 28.8 l/s to more accurately investigate the groundwater drawdown in the study area under different pumping conditions. Marginal functions proportional to the values of groundwater drawdown affected by pumping with a constant discharge of 26.64 and 28.8 l/s are the Weibull distribution. The results presented in Figs. 8 and 9 can be used specifically for hydraulic conductivity studies. Because the amount of groundwater drop can be estimated at different pumping times. Another application of these curves is planning for aquifer harvest management and groundwater monitoring. In addition, Fig. 10 shows the correlation between studied pair-variables. According to this figure, an acceptable correlation was between the mentioned values.

Different copulas fitted according to the confirmation of the correlation between the groundwater drawdown values in the study area. Table 4 shows the results of this fitting. According to different statistics (AIC, MAE, NSE, and RMSE), the Galambos copula was selected as the best one. The results of Clayton and Gumbel–Hougaard were almost identical to those of Galambos. The copula coefficient for the best copula was also estimated to be 19.44. Galambos achieved 98% efficiency and an error rate (RMSE) of 0.04 m, better than other copulas. In this study, the frequency analysis of the variables of groundwater drawdown due to pumping with a constant discharge of 26.64 and 28.8 l/s was performed using the best copula (Galambos) and its coefficient equal to 19.44. Figure 11 shows the results of this analysis. This figure shows the changes in groundwater level caused by pumping 26.64 l/s if the discharge increases by 28.8 l/s.

![Fig. 10](image_url) Kendall’s tau results for the correlation between the values of groundwater drawdown due to pumping with a constant discharge of 26.64 and 28.8 l/s

| Copula  | AIC     | MAE     | NSE     | RMSE |
|---------|---------|---------|---------|------|
| Clayton | − 6.48  | 0.03    | 0.98    | 0.04 |
| AMH     | − 5.50  | 0.11    | 0.84    | 0.11 |
| FGM     | − 2.72  | 0.17    | 0.60    | 0.18 |
| Frank   | − 6.14  | 0.04    | 0.97    | 0.05 |
| Gal     | − 6.53  | 0.03    | 0.98    | 0.04 |
| GH      | − 6.52  | 0.03    | 0.98    | 0.04 |
| Placket | − 5.86  | 0.07    | 0.94    | 0.07 |
As shown in Fig. 11 with different probabilities, the groundwater drawdown in the study area can be estimated due to pumping discharge of 28.8 l/s, given by the pumping discharge of 26.64. For example, with the drawdown of 7.6 m with the constant pumping discharge of 26.64 l/s, and the probability of 60, the drawdown due to 28.8 l/s pumping discharge will be about 7.7 m. Similarly, with the 6.4 m groundwater drawdown with 26.64 l/s pumping discharge and a 10% probability, if pumping discharge increases to 28.8 l/s, it will be 6.6 m. Since the pumping time is the same, the effect of increasing the pumping discharge in the study area can be well observed in the joint-occurrence probability curve. Using the above curves in the study area and performing a pumping test, we can estimate the drawdown due to the increase in pumping. Pumping tests have been performed in each region to determine hydraulic conductivity, which is costly and time-consuming. In addition to managing and monitoring groundwater resources and managing utilization of the aquifer, these curves can also be effective in reducing the costs of performing pumping tests and preparing pumping sheets.

**Conclusion**

This study used two pumping tests with constant discharge (26.64 and 28.8 l/s) in the Bazargan plain in northwestern Iran and performed the copula-based joint frequency analysis for pair-variables of groundwater-level drawdown-pumping time and pair-variables of groundwater-level drawdown-groundwater-level drawdown. To this end, this study used copula-based bivariate frequency analysis. By investigating the best marginal distribution functions regarding the studied pair-variables, the log-normal and Weibull distributions were selected as marginal distributions. While confirming the appropriate correlation between the studied pair-variables using Kendall’s tau, joint frequency analysis and estimation of joint-occurrence probability in the two mentioned cases were performed. According to the evaluation statistics, the results showed that the Galambos copula is best in all cases. Using the Nash–Sutcliffe statistic, the efficiency of higher than 96% for Galambos copula in all cases got confirmed. The first step was to simultaneously analyze and investigate the groundwater-level drawdown and pumping time at each discharge (26.64 and 28.8 l/s). These results provide typical contour lines curves of the joint probability of groundwater drawdown and pumping time. According to the provided curves in both constant discharge cases, the results showed that with the provided curves, it is easy to estimate the groundwater drawdown due to pumping discharge at different times and with different probabilities. For example, for 100 min of pumping, the groundwater drawdown for pumping discharge rate of 26.64 and 28.8 l/s with a 50% probability is about 7.4 and 7.8 m, respectively. With different probabilities, the drawdown is estimable at numerous pumping times. In the next step, the bivariate analysis of groundwater drawdown of two pumping cases with different discharges was investigated. Due to the same pumping time in the two tests, using copula functions in the subject of joint analysis and the possibility of joint occurrence, with different probabilities, the groundwater drawdown is estimable if the discharge increases. While estimating the groundwater drawdown, there can be restrictions on the amount of withdrawals as a groundwater-level monitoring system. The copula functions approach can estimate the pair-variables with high accuracy and certainty by joint frequency analysis. The proposed method can
be used as a management tool to evaluate changes in groundwater depth as a function of pumping (utilization in various fields). This study provided a successful approach for monitoring and joint frequency analyzing the groundwater drawdown and the corresponding pumping time in an aquifer. This approach acts as a practical, theoretical, and technical approach for the sustainable management of groundwater resources in a basin.

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Declarations

Conflict of interest No conflicts of interest involved.

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