Spacetime topology change and black hole information

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Topology change – the creation of a disconnected baby universe – due to black hole collapse may resolve the information loss paradox. Evolution from an early time Cauchy surface to a final surface which includes a slice of the disconnected region can be unitary and consistent with conventional quantum mechanics. We discuss the issue of cluster decomposition, showing that any violations thereof are likely to be unobservably small. Topology change is similar to the black hole remnant scenario and only requires assumptions about the behavior of quantum gravity in Planckian regimes. It does not require non-locality or any modification of low-energy physics.

INTRODUCTION: BLACK HOLE INFORMATION LOSS

What is the ultimate fate of something that falls into a black hole? Is it crushed out of existence at a singularity, or does it end up “somewhere else”? The answer to this question is central to the black hole information loss paradox \cite{1}. We will examine the possibility that black hole formation leads to spacetime topology change, and that matter that falls through the horizon ultimately reaches some topologically disconnected component of the universe, referred to here as a baby universe. This scenario leads to a resolution of the paradox without non-locality or modifications of low energy physics.

There are numerous excellent reviews \cite{2}–\cite{6} of the black hole information problem, so we give only a brief overview here. Figure 1 depicts the spacetime of an initially large, but subsequently evaporating, black hole. The dashed line (Cauchy slice 1) indicates a Cauchy surface on which the Schrodinger wavefunction and its derivatives, describing the matter and gravitational fields, fully specify future evolution (we assume the universe is in a pure quantum state at early times). Some of the information (on the part of slice 1 behind the event horizon) will never reach future infinity $B$. In particular, the Hawking radiation from the hole is spacelike separated from the information behind the horizon, so there is no mechanism for its escape which does not violate causality and locality. (Some interesting mechanisms for such non-locality have been proposed in string theory \cite{6} and quantum gravity \cite{7}.) A description of physics on slice 2 is necessarily a mixed state if we are required to trace over the lost information that falls into the hole. Dire physical consequences related to energy non-conservation have been deduced for systems in which pure states evolve into mixed states \cite{8}.

There are two main objections against topology change as a solution to the information problem, which we list here and address later in this paper.

\textbf{Objection I.} An effective description of the evolution of a pure state on past infinity (region $A$ in Figure 1) to a mixed state on future infinity (region $B$ in Figure 2) must be sick, perhaps exhibiting energy non-conservation, due to the arguments of \cite{8}.

\textbf{Objection II.} Processes which produce topologically disconnected universes generally lead to violation of cluster decomposition (locality) \cite{9}.

Earlier work on topology change and its relation to black hole information includes an unpublished preprint by Dyson \cite{10}, papers by Strominger \cite{11} and Polchinski and Strominger \cite{12}, and work by Jacobson \cite{5,13} and Easson and Brandenberger \cite{14}. Also of interest is the work of Ashtekar and Bojowald \cite{15}, in which quantization of the classical singularity region of a black hole allows evolution of a large spacetime. The authors of \cite{10}, \cite{11}, \cite{12}, \cite{13} and \cite{14} all state that new universe creation might alleviate the black hole information problem, although the specific objections I and II on which we focus are not fully addressed. In \cite{12} the scheme of third quantization (originally developed for spacetime wormholes \cite{16}) is used, which leads to a peculiar kind of indeterminacy. We do not assume the framework of the wormhole calculus and third quantization here, although we use the term baby universe to describe the disconnected universes. For related work on baby universes from black hole interiors – specifically, dynamical mechanisms by which black hole formation might lead to new universe creation – see \cite{17}. For further analysis of evolution of pure to mixed states, see \cite{18}, and for some early discussions of spacetime topology change in string theory, see \cite{19}.

TOPOLOGY CHANGE

Below we describe the specific assumptions of our scenario, which concern the dynamics quantum gravity (QG), but do not modify physics at large distances or low energies.

Gravitational collapse leads to a region of Planckian densities and curvature, where QG effects (fluctuations of the metric) are large. The size of this QG-dominated region increases with the size of the black hole, and it likely resolves the singular collapse endpoint found in classical relativity. It seems plausible, and we assume, that QG tunneling in this high curvature region can lead to topol-
FIG. 1: Penrose diagram of black hole evaporation. Two spacelike slices are indicated. Slice 1 is a Cauchy surface, while slice 2 is not Cauchy surface for the entire universe (parent plus baby) if black hole formation leads to topology change as in Figure 2.

FIG. 2: Topology change due to black hole formation. The creation of the baby universe proceeds via quantum gravitational effects and may lead to a rich internal structure. After formation, a complete Cauchy surface for the entire universe (combined spacetimes) must include a slice $B'$ of the baby universe.

An observer falling into the hole hits the QG region after a finite proper time and her constituents tunnel into the baby universe. Banks [3] has emphasized that the classical geometry of the black hole interior resembles that of a wormhole with a long throat. We assume that QG effects cause this throat to pinch off into a baby universe which is disconnected from the spacetime of the parent universe. That is, there is no smooth, semiclassical path which connects the interior of the parent to the interior of the baby universe.

Any locally conserved quantities such as mass, angular momentum or gauge charge that would prevent tunneling are still manifest in the original connected spacetime as black hole hair, and can eventually be radiated away. Because the Bondi mass (as measured at asymptotic distance) of the hole does not change, the universe that pinches off must have zero total energy, and similar arguments apply to its gauge charge and angular momentum. However, because of negative gravitational binding energy, this does not preclude a rich internal structure in the baby universe. One example of how a complex internal structure might arise is the creation of an inflationary universe from a finite vacuum bubble [20]. Indeed, in [20] the vacuum bubble appears to exterior observers as a black hole, and the creation of the sub-universe is caused by quantum tunneling.

A complete specification of the state of the (now topologically nontrivial) universe requires the wavefunction on $B \cup B'$. Occupants of $B$, without access to $B'$, have incomplete information about the universe as a whole, but time evolution $A \rightarrow B \cup B'$ is completely unitary. (See further discussion in next section.)

This scenario is similar to that of a remnant, except that in place of the remnant there is a disconnected region of spacetime which contains the information that crossed the horizon. We discuss the relation between the two scenarios below.

It is possible, but not necessary, that information return via QG tunneling after some long timescale (e.g., exponential in the black hole mass or area). The energy limitations of the type placed on black hole evaporation do not apply, as the tunneling modes can have very long wavelength. If enough (space)time is available, an unlimited amount of information can be emitted in arbitrarily long wavelength modes. The returned information emerges from the vertical segment connected to $B'$ in Figure 1, and eventually reaches future null infinity $B$. In this case evolution from $A$ to $B$ is completely unitary, and $B'$ ceases to exist. This case is similar to that of a decaying, but invisible, remnant.

**EVOLUTION FROM PURE TO MIXED STATES**

Objection I, given above, relates to the evolution of pure to mixed states. Here we explain that this is only a
problem if the initial and final surfaces are both complete Cauchy surfaces.

In quantum field theory, any subset of a Cauchy surface is generically described by a mixed state, even if the entire Cauchy surface is in a pure state. It is therefore not surprising if the Hawking radiation which remains after black hole evaporation is in a mixed state, since all of it crosses the incomplete slice 2 in Figure 1.

Black hole formation and evaporation from an initial pure state can be consistent with ordinary quantum mechanics if the final state extends beyond future infinity in some way. In our case the information resides in the topologically disconnected baby universe.

Banks, Peskin and Susskind [8] identified problems for local dynamics (i.e., generalizations of Schrödinger evolution) which evolve a pure state into a mixed state. In the topology change scenario, there is no local dynamics which causes evolution from pure to mixed states. Mixed states only enter the description if one discards (traces over) some degrees of freedom of the final state, i.e., by considering only a subset of a final Cauchy surface.

For example, consider two entangled particles, initially in a pure state. Suppose one of the two falls into the hole and ends up in the baby universe. The wavefunction describing the two particles evolves unitarily and is still a pure state even after the hole evaporates, although the Hilbert space describing the entire universe (parent plus baby) is then of the form $H = H_{\text{baby}} \otimes H_{\text{parent}}$. The state of the two particles is at all times a vector in $H$, and not a density matrix. A mixed state is only obtained if, in order to obtain a description of the system in terms of degrees of freedom remaining in the parent universe, one traces over those degrees of freedom which fall past the horizon (i.e., we trace over $H_{\text{baby}}$ to obtain a density matrix valued only on $H_{\text{parent}}$).

Two examples illustrate why there is no problem. In both cases it is the incompleteness of the final region of description – not dynamical evolution – that leads to a mixed state and no pathological phenomena, such as energy non-conservation [8], are expected.

1) Wald hyperboloid (Figure 3). Wald [21] gave the example of evolution a massless scalar field from a complete Cauchy surface (e.g., any spacelike slice in flat spacetime) to a surface such as a hyperboloid which is entirely contained in the interior of a future lightcone. The state of the scalar field on the hyperboloid will be mixed even if it was originally pure on the initial Cauchy surface, and evolution is unitary. This is simply due to the incompleteness of the hyperboloid as a Cauchy surface. If correlations exist between modes which intersect the lightcone but which do not (because they travel on the lightcone), then tracing over these lightcone modes leaves a mixed state on the hyperboloid.

2) Inflation (Figure 4). In this example two entangled qubits experience an inflationary expansion which leaves them many horizon distances apart by the end of the expansion. A local description of either qubit requires tracing over the other qubit degrees of freedom, leading to a mixed state even if evolution is unitary. An observer in a causal patch near one qubit will see the other qubit exponentially redshifted as it reaches the de Sitter horizon, so that access to its quantum information is clearly lost. The horizon of an individual qubit is of course not a complete Cauchy surface.

**CLUSTER DECOMPOSITION**

Topology change by black holes can lead to violation of cluster decomposition, which is objection II above. This point was emphasized in [9], and led Polchinski and Strominger [12] to propose a third quantization scenario for baby universes, with consequent loss of predictability similar to that due to spacetime wormholes.
Here, we note that the violations of cluster decomposition are likely to be unobservably small, even in hypothetical gedanken experiments. (Although, as stressed by Susskind [1], they remain vexing as a question of principle in the definition of probabilities for outcomes of processes involving black hole evaporation.)

Baby universes are distinguishable only by their internal state. By translation invariance (or general covariance) they carry no memory of the coordinates of the black hole collapse that led to their creation. Consider two black hole collapses at spacelike separated points $x_1$ and $x_2$. If the internal states $|B_1\rangle$ and $|B_2\rangle$ of baby universes $B_1$ and $B_2$ are identical (or at least have nonzero overlap), the additional “crossed” amplitude (see Figure 5) where $B_2$ is created by the collapse at $x_1$ and $B_1$ created by the collapse at $x_2$ must be included, which violates cluster decomposition (i.e., independence of spacelike separated events). To obtain the density matrix describing the state of the parent universe after both black holes have evaporated, one traces over lost degrees of freedom. This trace receives crossed contributions if the overlap $\langle B_1|B_2\rangle$ is nonzero, and can violate clustering.

However, it is essentially impossible to realize such interference processes with macroscopic black holes. The interference is suppressed if the overlap of internal states $|B_1\rangle$ and $|B_2\rangle$ is small. For example, the charges, masses and angular momenta of the initial states from which the black holes form must be equal to within semiclassical uncertainties, or else the Hilbert spaces describing the two interiors will be different. Even if the two internal Hilbert spaces coincide, they are necessarily of very high dimension for a macroscopic collapse (i.e., dimension equal to the number of degrees of freedom, for example Avogadro’s number). As the dimensionality of a space becomes large, the inner product (overlap) of any two normalized vectors chosen at random goes to zero – i.e., most vectors are nearly orthogonal. This means that even a small deviation between the initial configurations which form $B_1$ and $B_2$ will suppress the interference effect.

An experimenter trying to realize the effect does not, for example, have control over vacuum fluctuations near or within the horizon of each hole, and consequently cannot produce identical internal states, even if the initial collapsing matter configurations are identical. The simplest way to see this is to note that even if the initial configurations are identical the observed Hawking radiation from each hole will be slightly different, which means the internal states of $B_{1,2}$ are not the same. We expect small perturbations of otherwise identical initial conditions to produce very different internal states $|B_{1,2}\rangle$, because the infalling matter experiences strong dynamics at Planck densities. Although the average (semiclassical) properties of $|B_{1,2}\rangle$ may be similar, because of the large number of degrees of freedom the overlap $\langle B_1|B_2\rangle$ will be extremely small, and hence any interference effect unobservable.

As a concrete example of the mechanism described above, we can imagine that interactions are strong enough that the infalling matter has thermalized by the time it reaches Planck densities and pinches off to form a baby universe. Then, the internal state of the baby universe will be a particular Hilbert space vector on a constant energy surface (microcanonical ensemble). Such vectors have similar coarse grained or statistical properties (average energy, particle distribution, etc.), but any two chosen at random will be nearly orthogonal: their overlap squared decreases as the number of dimensions of the constant energy surface, which is very large. Under strong dynamics, two almost identical initial matter configurations will evolve into very different (essentially, random) state vectors on the constant energy surface, so it is impossible (highly improbable) to choose initial conditions that guarantee a large overlap between the internal states $|B_{1,2}\rangle$.

The suppression of cluster decomposition violation discussed above applies to baby universes with large numbers of internal degrees of freedom. Presumably, the area result for black hole entropy precludes small black holes (i.e., with masses of order the Planck energy, and therefore only a few bits of entropy) from producing complex baby universes, so topology change on Planck scales would still lead to cluster decomposition problems. However, it may be that only large holes produce baby universes, and those always have rich internal structure, for example due to a mechanism analogous to that in [20]. Smaller black holes would then behave as long lived, but unstable, remnants, without the usual problems (see next section) since there are only a finite number of species. Such remnants ultimately return all information back to the parent universe.

For example, the probability of baby universe formation could be strongly dependent on the black hole size, varying from close to zero for small holes to unity for
large semiclassical holes. In this hybrid scenario any virtual effects due to information loss from the parent universe would be suppressed by a factor exponential in $M_*$, the mass scale at which a black hole can be considered semiclassical (e.g., a large number times the Planck mass).

**RELATION TO REMNANTS**

There are obvious parallels between our topology change scenario and that of remnants. The potentially enormous amount of information stored in a remnant instead disappears into a baby universe. In the remnant case one imagines that the throat of the distorted spacetime region connecting the black hole horizon to the classical singularity is somehow stabilized, rather than pinching off.

The main problem with remnant scenarios is that there must be a distinct, long-lived, roughly Planck mass species of remnant for each black hole which could possibly be formed. With such a large number of species, it is hard to see why virtual processes involving remnants would be suppressed – the multiplicity factor is enough to overcome even an exponentially small coupling. This concern is alleviated in our case, as the baby universes do not manifest themselves directly in the parent universe, and any long lived remnants arising from small black holes are finite in number.

In the remnant language our proposal can be summarized as follows, with $M_*$ some scale an order or magnitude or so larger than the Planck mass. (1) small black holes ($M < M_*$) lead to somewhat long lived (but not eternal) remnants, whose evaporation is unitary. (2) large black holes ($M > M_*$) evaporate as well, but internally cause topology change and loss of information from the parent universe. However, the consequences of this apparent non-unitarity are, for the reasons discussed, small and not excluded by experiment. Any virtual effects of large black holes are suppressed exponentially in $M_*$ (as $e^{-M_*^2}$, for example). If information somehow tunnels back from the baby universes to the parent universe, this tunneling is manifested in ordinary (but long wavelength) degrees of freedom, and there are only a finite number of remnant species in our scenario.

**CONCLUSIONS**

We discussed a solution of the black hole information paradox which depends entirely on details of Planckian physics – no modifications of low-energy physics, such as non-locality, are required.

The main assumptions are that the interior evolution of large black holes produces topology change, and that the quantum gravitational dynamics of pinching off are strongly coupled. Thus, small perturbations to the initial state of a black hole lead to different internal state vectors describing the resulting baby universe, even if the semiclassical properties are only slightly changed. Under this assumption, any violation of cluster decomposition will be practically unobservable.

If this scenario is correct, there is no violation of causality or locality at the semiclassical black hole horizon, and no stable Planck mass remnant of black hole evaporation. Instead, much as Hawking first proposed, information is lost: to a baby universe, from which it may or may not someday emerge via tunneling. If the information emerges again, evolution within the parent universe is unitary. If information remains in the baby universe, the parent universe appears to evolve from a pure to mixed state, but the evolution of parent and baby together is unitary. There are no dire consequences, such as energy non-conservation.

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