Quantum mechanical motion of classical particles

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Abstract. The Schrödinger equation for the particle wave function introduced via its action was derived from Newton equation for a point-like particle moving under effect of both potential force and fluctuation-dissipative environment, provided considering only stable motion of the particle. The model considered assumes existence of wave function as a physical field rather than just a mathematical abstraction.

1. Introduction
Copenhagen (orthodox) interpretation of Quantum Mechanics (QM) rejects existence of real wave function associated with a QM particle and suggests dealing with an “empty” QM wave function, the squared modulus of which gives probability of finding the particle at a given point of space at certain time (Max Born). At the same time, this interpretation leaves outside the QM theory a consideration of possible causes and origin of such probabilistic behaviour of the particles with mass. Nevertheless, many attempts to clarify “physical nature” in interpretations of QM have never stopped. Moreover, the interest in explanations of the physical nature of wave-particle dualism and indeterminism in QM has grown significantly in the last decade. There are several interpretations of QM, which in the authors opinion explain the essence of the things from the viewpoints much closer to the classical physics rather than to the abstract mathematical constructions of QM. It is accepted that a QM particle is governed by Schrödinger equation which describes its wave properties and enables to evaluate probability of its finding at a given coordinate at the given time instance.

In this short paper we show the way of derivation of Schrödinger equation from Newton equation for a point-like particle moving in both a potential force and fluctuation-dissipative environment. Application of the Chetaev motion stability theorem for a particle moving under dissipative forces in a potential field is an essential point of this derivation [1]. In this way we have shown that quantum potential which provides compensation of the environment action onto the particle motion has a fluctuation-dissipative nature and defines stable motion of the particles. The second objective of this short paper is verification of the fluctuation-dissipative interpretation of a quantum potential.

2. Schrödinger equation as a condition for the stability of the particle motion in the presence of dissipative forces
Consider a non-relativistic particle motion under both a potential force and fluctuation-dissipative environment, including the forces generated by the quantum potential.
The main assumptions of the developed theory are as follows: (1) non-relativistic particle with mass $m$ governed by the Newton motion equation, in the right side of which the forces generated by external potentials, including the fluctuation-dissipation potential due to the fluctuations of the vacuum are taken into account; (2) in addition, the particle is affected by the forces produced by the quantum potential, whose existence is postulated; (3) an essentially new proposition of the theory is the assumption on the dissipative nature of particle interaction with the environment (fluctuating vacuum), on the one hand, and assumption of the existence of stable trajectories of the particle due to exact compensation of its fluctuations by the quantum potential (QP).

Equation of one-dimensional motion of a classical particle under the action of potential and dissipative forces, characterized by the potentials $U(x,t)$ and $Q(x,t)$, respectively, may be written in terms of its action and energy as follows:

$$\frac{\partial S(x, \nu, t)}{\partial t} = T(\nu, t) + U(x, t) + Q(x, t) \quad (1)$$

where $T(\nu, t) = \frac{1}{2} m \nu^2$ is the particle kinetic energy, $S(x, \nu, t) = \int m \nu dx$ is the particle action.

The dissipative potential force, which leads to the destruction of the motion integral of the corresponding conservative system has been introduced in Newton equation and as a potential $Q(x,t)$ in Eq.(1) in contrast to the Jacobi equation, which relates the kinetic energy of a particle with its action and potential for conservative systems. If we neglect dissipation term in Eq. (1) the latter will be identical to the Jacobi equation. On the other hand, for only stable trajectories of the particle under consideration, the equations describing such motions turn out to be conservative again, i.e. the system under consideration is Hamiltonian one! Now, if we introduce wave function $\psi$ in standard way $\psi = A \exp(\ii \hbar S)$ and apply Chetaev condition for stable motion [1] we may readily derive Schrödinger equation with respect to the function $\psi$:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \psi + U \psi \quad (2)$$

along with probability continuity equation and relation between quantum potential and wave function amplitude, respectively:

$$\frac{\partial A}{\partial t} = -\frac{A}{2m} \sum \frac{\partial^2 S}{\partial x_i^2} - \sum \frac{\partial A}{\partial x_i} \frac{p_i}{m} \quad \text{and} \quad Q = -\frac{\hbar^2}{2m} \Delta A \quad (3)$$

We have shown that QP which provides compensation of the environment action onto the particle motion has a fluctuation-dissipative nature and defines stable motion of the particles [2]. QP calculated with the help of wave function for the stochastic Schrödinger equation describing quantum oscillator with dissipation, produces the dissipative forces and fluctuations with zero mean value, which leads us to quantum Langevin equation. The particle wave properties may be tied with its local complicated (but periodical) motion around a mass center and may be described in terms of known “zitterbewegung” models [3]. Deriving the Fischer information for the QP and using Cramer-Rao inequality we directly arrive at the Heisenberg uncertainty principle for moving particles. In this way description of QM particle is derived from classical equations of motion for a point-like particle in potential field and fluctuation-dissipative environment as its stable motion provided compensation of the particle fluctuations by the QP.

References

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