Testing Time-Reversal: $\Lambda_b$ Decays into Polarized Resonances

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Abstract

Weak decays of beauty baryons like $\Lambda_b$ into $\Lambda V(J^P = 1^-)$, where the produced resonances are polarized, offer interesting opportunity to perform tests of Time-Reversal Invariance. This paper emphasizes the particular role of the resonance polarization-vectors and their physical properties by symmetry transformations. In particular, it is shown that the normal component of a polarization-vector, as defined in the Jackson’s frame, is Lorentz invariant and could get large values, notably in the case of $J/\psi$ production.

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1 Introduction

An important cornerstone of modern field theories describing the interactions among particles is the CPT theorem, of which various proofs have been provided by different authors, such as Pauli and Luders. This theorem asserts that the product of the three operators C (Charge Conjugation), P (Parity) and T (Time Reversal or TR), taken in any order, is a symmetry transformation of the physical laws of Nature. The validity of this theorem is assumed by almost all experiments in particle physics.

As an immediate consequence of this theorem, the violation product of two symmetrically operators, C and P, directly leads to a non conservation of the third symmetry operator, T. So, the common belief that T is violated was only based on indirect proof, until the experiments CP-LEAR and KTeV, respectively performed at CERN and Fermilab, shed light on the direct violation of the symmetry T in the $K^0\bar{K}^0$ system. This important fact opened the way to search for direct Time Reversal Violation (TRV) in other systems of particles: the beauty hadrons, mesons B or beauty baryons like $\Lambda_b$, which will be copiously produced with the forthcoming LHC machine.

However, direct search for TRV was already suggested by many people, just after the discovery of parity breakdown in 1957 \cite{1,2}. The main idea was to look for the transverse polarization of the emitted electrons in $\beta^-$ decay in neutron or hyperon (\Lambda, \Sigma...) decays. Transverse polarization of the electron according to the decay plane of its parent particle (n or \Lambda...) is defined as follows:

$$P_T = \langle \vec{s} \cdot \vec{n} \rangle \quad \text{with} \quad \vec{n} = \frac{\vec{p}_1 \times \vec{p}_2}{|\vec{p}_1 \times \vec{p}_2|},$$

where $\vec{p}_1$ and $\vec{p}_2$ are respectively the momenta of the daughter baryon and the emitted electron. $\vec{s}$ is the electron spin. It is easy to notice that $P_T$ changes sign under TR and, assuming TR as a good symmetry, it leads to $P_T = 0$. Therefore a nonzero $P_T$ could be a sign of TRV. In this case, $P_T$ is called a Time Odd (T-odd) observable. Usually, a T-odd observable is not sufficient to prove the violation of TR because of the Final State Interactions (FSI). It is also well known that the Final State Interactions during hadronization may modify the quantum state of the final particles and, thus could simulate a TRV process. Nevertheless, a T-odd physical quantity represents the point of departure to cross-check the Time Reversal symmetry, provided an estimation or (in an optimistic case) a complete calculation of the FSI are done. In the case where FSI are negligible, a T-odd variable would be a premise of direct TRV.

Since the spin of any particle is Parity-even and T-odd, studying its mean value or its polarization provides interesting tools to test both of these two symmetries, especially in weak decay processes where parity is known to be violated. It is worth noticing that checking TRV in a given decay, like $A \rightarrow a_1 + a_2$, or in its charge conjugate mode, $\bar{A} \rightarrow a_1 + \bar{a}_2$, is not necessarily related to the conservation or non-conservation of the CP symmetry in these two channels. The method outlined in this paper relies on the fact that direct check of TR symmetry is done without any theoretical assumption beyond the Standard Model.

Kinematic calculations are performed in the framework of the Jacob-Wick-Jackson (JWJ) formalism, while the decay dynamics relies on the OPE techniques supplemented
by the Heavy Quark Effective Theory (HQET), in order to do precise estimations of the transition form-factors which are involved in the evaluation of the hadronic matrix elements. We refer the reader to papers [4, 5, 6] for all calculation details.

The aim of the present paper is to stress the particular role of the polarization-vectors of both the hyperon Λ and the vector-meson $V(J^P = 1^-) = J/\psi, \rho^0, \omega$ coming from $\Lambda_b$ decays. Section II is devoted to set helicity states as well as cross-section in $\Lambda_b$ baryon decays. The component calculations and the behavior of the polarization-vectors according to the $\Lambda_b$ initial polarization density matrix (PDM) are analyzed in Section III. The use of polarization-vectors on search for TR violation is also discussed in this section.

In Section IV, emphasis is put on the choice of some special frames which help to clearly exhibit the T-odd observables related to the resonance polarizations. Finally, results are discussed and a few conclusions are drawn in Section V.

## 2 Production and decay of $\Lambda_b$ baryon

An essential point of our study is based on assuming that the $\Lambda_b$ baryons which are produced in $p-p$ collisions are transversely polarized like the ordinary hyperons ($\Lambda, \Sigma, ...$) in hadron-hadron or hadron-nucleus collisions. Because of the $\Lambda_b$ spin 1/2, its Polarization Density Matrix (PDM) is a $(2 \times 2)$ hermitian matrix whose trace, $Tr(\rho^{\Lambda_b})$, is normalized to one. The only complex elements are the non-diagonal ones, which verify $\rho^{\Lambda_b}_{12} = \rho^{\Lambda_b}_{21*}$.

No more assumptions are made concerning this density matrix. Moreover, the angular momentum states are constrained by the conservation of the total angular momentum: $s_\Lambda = s + s_\Lambda = 1/2$, $s_V = 1$ and $\bar{L}$ is the orbital angular momentum in the $\Lambda_b$ rest-frame. Taking the direction of $p_\Lambda = (p, \theta, \phi)$ as the quantization axis one, the contribution of the orbital angular momentum $\bar{L}$ will be suppressed and only four helicity states remain to describe the $\Lambda V$ system:

$$ (\lambda_\Lambda, \lambda_V) = (1/2, 0), (-1/2, -1), (1/2, 1), (-1/2, 0) , $$

to which correspond the hadronic matrix elements $A_{(\lambda_1, \lambda_2)}(\Lambda_b \rightarrow \Lambda V)$. Taking into account the $\Lambda_b$ PDM, the cross-section of the $\Lambda_b \rightarrow \Lambda V(1^-)$ decay is therefore given by the following relation:

$$ d\sigma \propto \sum_{M_\Lambda, M'_\Lambda} \sum_{\lambda_1, \lambda_2} \rho^{\Lambda_b}_{M_\Lambda M'_\Lambda} |A_{(\lambda_1, \lambda_2)}(\Lambda_b \rightarrow \Lambda V)|^2 d^{1/2}_{M_\Lambda} d^{1/2}_{M'_\Lambda} \exp i(M'_\Lambda - M_\Lambda) \phi . $$

Then, we introduce the helicity asymmetry parameter, $\alpha_{AS}$, which is the analogous of the $\Lambda$ asymmetry one in the standard decay $\Lambda \rightarrow p\pi^-$,

$$ \alpha_{AS} = \frac{|A_{\Lambda_b}(+)|^2 - |A_{\Lambda_b}(-)|^2}{|A_{\Lambda_b}(+)|^2 + |A_{\Lambda_b}(-)|^2} , $$

with the elements $A_{\Lambda_b}(\pm)$ defined by,

$$ |A_{\Lambda_b}(+)|^2 = |A_{(1/2, 0)}(\Lambda_b \rightarrow \Lambda V)|^2 + |A_{(-1/2, -1)}(\Lambda_b \rightarrow \Lambda V)|^2 , $$

$$ |A_{\Lambda_b}(-)|^2 = |A_{(-1/2, 0)}(\Lambda_b \rightarrow \Lambda V)|^2 + |A_{(1/2, 1)}(\Lambda_b \rightarrow \Lambda V)|^2 . $$

Therefore, the differential cross-section gets the following form:

\[
\frac{d\sigma}{d\Omega} \propto 1 + \alpha_{AS} \mathcal{P}^{\Lambda_b} \cos \theta + 2\alpha_{AS} \Re \left( \rho^{\Lambda_b} \exp i\phi \right) \sin \theta.
\]  

(4)

In Eq. (4), \( \mathcal{P}^{\Lambda_b} = \rho^{\Lambda_b}_{+} - \rho^{\Lambda_b}_{-} \) is the value of the \( \Lambda_b \) polarization. The above relations show the importance of the \( \Lambda_b \) polarization and its PDM in the angular distributions of the resonances, \( \Lambda \) or \( V \), in the \( \Lambda_b \) rest-frame.

3 Polarizations of the final resonances

Basic principles of Quantum Mechanics allow us to deduce the spin density matrix of the final \( \Lambda V \) system, which is an essential parameter to compute the polarization-vector of each resonance, \( \rho^f = T^\dagger \rho^{\Lambda_b} T \). \( T \) is the transition-matrix (the \( S \)-matrix being defined by \( S = 1 + iT \)) whose elements are explicitly given in Ref. [4]. The normalization of the matrix \( \rho^f \) is obtained by \( \text{Tr}(\rho^f) = \frac{d\sigma}{d\Omega} = NW(\theta, \phi) \), where \( \text{Tr} \) is the trace operator and \( N \) is a normalization constant. Consequently, the polarization-vector of any resonance \( R_i \) (\( R_1 = \Lambda, R_2 = V \)) is defined by

\[
\vec{P}_i = \langle \vec{S}_i \rangle = \frac{\text{Tr}(\rho^f_i \vec{S}_i)}{\text{Tr}(\rho^f_i)},
\]

(5)

where \( \rho^f_i \) is the spin density-matrix of the resonance, \( R_i \), deduced from \( \rho^f \). As the final state is a composite system made out of two particles with different spin (\( s_1 = 1/2, s_2 = 1 \)), each \( \rho^f_i \) will be obtained from the general expression of \( \rho^f \) by summing up over the degrees of freedom of the other resonance. Thanks to this method, we can obtain \( \rho^{\Lambda} \) and \( \rho^{V} \). See Ref. [4] for all analytical results.

3.1 Choice of particular frames

In the \( \Lambda_b \) rest-frame\(^4\), a specific frame according to the Jackson’s method is constructed for each resonance \( R_{(i)} \) (the index \( i \) will be dropped for convenience):

\[
\vec{e}_L = \frac{\vec{p}}{p}, \quad \vec{e}_T = \frac{\vec{e}_Z \times \vec{e}_L}{|\vec{e}_Z \times \vec{e}_L|}, \quad \vec{e}_N = \vec{e}_L \times \vec{e}_T, \quad \text{with} \quad \vec{e}_Z \parallel \vec{n}.
\]

(6)

where \( \vec{e}_Z \) is parallel to \( \vec{n} \). \( \vec{n} \) is initially defined as the normal unit-vector to the \( \Lambda_b \) production plane. Then, each polarization-vector can be expressed as:

\[
\vec{P} = P_L \vec{e}_L + P_N \vec{e}_N + P_T \vec{e}_T, \quad \text{with} \quad P_L, P_N \text{ and } P_T \text{ are respectively the longitudinal, normal and transverse components of } \vec{P}.
\]

(7)

It is worth noticing that the basis vectors \( \vec{e}_L, \vec{e}_T \) and \( \vec{e}_N \) have the following properties according to parity and TR: P-odd,T-odd; P-odd,T-odd and P-even, T-even respectively, while the polarization-vector \( \vec{P} \) is P-even and T-odd. So, any component of \( \vec{P} \) defined by the scalar product \( P_j = \vec{P} \cdot \vec{e}_j \) with \( j = L, N, T \) gets transformed as:

\[
P_L = P - \text{odd}, T - \text{even}, \quad P_T = P - \text{odd}, T - \text{even} \quad \text{and} \quad P_N = P - \text{even}, T - \text{odd}.
\]

\(^4\)The \( \Lambda_b \) rest frame used in our analysis is given in Figure 1.
Note that the longitudinal axis defined by $e^T_L$ is taken as the quantization axis. $e^-_N$ and $e^-_T$ are identified to $x$ and $y$ axis, respectively. The previous relation allows us to write down the formal expression of any $\vec{P}_i$ by expanding the trace operator over the different spin states\footnote{In order to perform these calculations, some simple and fundamental relations are used, whatever the spin is:}

\[
\mathcal{P}_i W(\theta, \phi) = N \sum_\lambda \left( \langle \theta, \phi, \lambda | \rho_i^f \vec{S} | \theta, \phi, \lambda \rangle \right)
\]

$W(\theta, \phi)$ being defined at the beginning of Section 3.

### 3.2 \(\Lambda\) and \(V(J^P = 1^-)\) polarization-vectors

The matrix elements given in the right-handed side of Eq. (8) can be explicitly calculated \footnote{Technical details of the computation of both $\mathcal{P}^\Lambda$ and $\mathcal{P}^\Lambda$ are given in Ref. [4].} and the three components of $\mathcal{P}^\Lambda$ get the following expressions:

\[
P^\Lambda_x W(\theta, \phi) \propto 2 \mathbb{R}e(\langle \theta, \phi, 1/2 | \rho^\Lambda | \theta, \phi, -1/2 \rangle),
\]

\[
P^\Lambda_y W(\theta, \phi) \propto -2 \mathbb{R}m(\langle \theta, \phi, 1/2 | \rho^\Lambda | \theta, \phi, -1/2 \rangle),
\]

\[
P^\Lambda_z W(\theta, \phi) \propto \bar{\omega}(+1/2) - \bar{\omega}(-1/2),
\]

where the $\bar{\omega}(\pm)$ are defined in [4]. The vector meson has spin $S_{(2)} = 1$ and therefore three helicity states. Based on

\[
\mathcal{P}^V W(\theta, \phi) = N \sum_{\lambda_2} \left( \sum_{\lambda_1} \langle \theta, \phi, \lambda_1, \lambda_2 | \rho^f \vec{S} | \theta, \phi, \lambda_1, \lambda_2 \rangle \right),
\]

the components of $\mathcal{P}^V$ are obtained in the same manner, although more tedious. One has

\[
P^V_x W(\theta, \phi) \propto \sqrt{2} \mathbb{R}e\left( \langle 0 | \rho^V | -1 \rangle + \langle 1 | \rho^V | 0 \rangle \right),
\]

\[
P^V_y W(\theta, \phi) \propto \sqrt{2} \mathbb{R}m\left( \langle 0 | \rho^V | -1 \rangle + \langle 1 | \rho^V | 0 \rangle \right),
\]

\[
P^V_z W(\theta, \phi) \propto \left( \langle 1 | \rho^V | 1 \rangle - \langle -1 | \rho^V | -1 \rangle \right).
\]

As it was expected, the helicity value $\lambda_V = 0$ does not contribute to the longitudinal polarization of the vector-meson. We also underline the importance of the initial $\Lambda_b$ polarization, $\mathcal{P}^{\Lambda_b}$, as well as the non-diagonal matrix element, $\rho^{\Lambda_b}_{+ -}$ in the components of $\mathcal{P}^V$. Details can be found in Ref. [4].

On the dynamical side, the Heavy Quark Effective Theory \footnote{Properties such as flavor and spin symmetries can be exploited in such a way that corrections of the order of $1/m_Q$ are systematically calculated within an effective field theory. Then, the hadronic amplitude of the weak decay is investigated by means of the effective Hamiltonian, $\Delta B = 1$, where the Operator Product Expansion formalism separates the soft and hard regimes. All results about transition form factors as well as hadronic matrix elements are given in Ref. [4].} (HQET) formalism is used to evaluate the hadronic form factors involved in $\Lambda_b$-decay. Weak transitions including heavy quarks can be safely described when the mass of a heavy quark is large enough compared to the QCD scale, $\Lambda_{QCD}$. Properties such as flavor and spin symmetries can be exploited in such a way that corrections of the order of $1/m_Q$ are systematically calculated within an effective field theory. Then, the hadronic amplitude of the weak decay is investigated by means of the effective Hamiltonian, $\Delta B = 1$, where the Operator Product Expansion formalism separates the soft and hard regimes. All results about transition form factors as well as hadronic matrix elements are given in Ref. [4].

\[
S_x | \lambda \rangle = (| \lambda + 1 \rangle + | \lambda - 1 \rangle) / \sqrt{2}, S_y | \lambda \rangle = i (| \lambda + 1 \rangle - | \lambda - 1 \rangle) / \sqrt{2}, S_z | \lambda \rangle = \lambda | \lambda \rangle.
\]

(Letters indicating other physical parameters are dropped for simplicity).
3.3 Numerical results

- In a first step, values for all input parameters are taken from [4]. The initial \(\Lambda_b\) polarization and \(\Lambda_b\) polarization density matrix element used in our numerical computations are \(P^{\Lambda_b} = 100\%\) and \(\Re e(\rho_{L-}^{\Lambda_b}) = 3m(\rho_{L-}^{\Lambda_b}) = \sqrt{2}/2\), respectively. The corresponding spectra of the three components of \(\vec{P}^L\) and \(\vec{P}^V\) are shown in Figures 2 and 3. Comments on longitudinal, transverse and normal components of these polarization vectors are the following: (i) the longitudinal components, \(P_L = P_z\), of both the two resonances are asymmetric because of parity violation in weak \(\Lambda_b\) decays; (ii) the spectra of the transverse components, \(P_T = P_y\), are quite symmetric and their asymmetries are \(\approx 1.0\%\); (iii) the normal components, \(P_N = P_x\), are clearly asymmetric. Their asymmetry values are respectively 23\% and \(-54\%\) for \(\Lambda\) and \(J/\psi\).

- In a second step, attempts to understand correlations between the \(\Lambda_b\) initial polarization and the physical properties of its decay products are made. All results are obtained with Monte-Carlo simulations by varying independently \(P^{\Lambda_b}\) and \(\rho_{+/-}^{\Lambda_b}\). Non-diagonal matrix elements being generally unknown, we set \(\Re e(\rho_{+/-}^{\Lambda_b}) = 3m(\rho_{+/-}^{\Lambda_b}) = 0\) and we let \(P^{\Lambda_b}\) vary between 100\% and 0\%. The resulting spectra of the normal components, \(P_N^\Lambda\) and \(P_N^{J/\psi}\) are usually sharp, while the transverse components, both \(P_T^\Lambda\) and \(P_T^{J/\psi}\), are always equal to zero. These two physical properties can be explained as direct consequences of \(\rho_{+/-}^{\Lambda_b} = 0\). In Table 1, mean values and asymmetry parameters of the normal component spectra for \(\Lambda\) and \(J/\psi\) are respectively listed. Interesting remarks can be drawn: (i) for \(P^{\Lambda_b} \neq 0\), the normal components are largely dominating and their asymmetries are nearly equal to minus one; (ii) for \(P^{\Lambda_b} = 0\), the \(J/\psi\) normal component is still dominating (\(\approx -0.8\)) while \(P_N^\Lambda\) is equal to zero, the \(\Lambda\) polarization being completely longitudinal (\(P_L^\Lambda = -100\%)\).

In order to understand the role of the \(\Lambda\) azimuthal distribution and its effects on the resonance polarization-vectors, a comparison between two series of \(P_N\) spectrum is performed. One series is obtained with \(\Re e(\rho_{+/-}^{\Lambda_b}) = 3m(\rho_{+/-}^{\Lambda_b}) = 0\), while the other one is obtained with the standard values, \(\Re e(\rho_{+/-}^{\Lambda_b}) = 3m(\rho_{+/-}^{\Lambda_b}) = \sqrt{2}/2\). In Figures 4 and 5, the spectra of \(P_N\) and \(P_T\) for \(\Lambda\) and \(J/\psi\) are respectively plotted. One notes that the spectra belonging to \(\Re e(\rho_{+/-}^{\Lambda_b}) = 3m(\rho_{+/-}^{\Lambda_b}) \neq 0\) are much broader than those belonging to \(\rho_{+/-}^{\Lambda_b} = 0\). Moreover, the absolute values of the asymmetry parameters decrease for both \(P_N^\Lambda\) and \(P_N^{J/\psi}\) when \(\rho_{+/-}^{\Lambda_b} \neq 0\), while the transverse components remain symmetric with a mean values around 0. Whatever the \(\Lambda_b\) PDM elements are, this exhaustive study indicates that the normal components of the polarization-vectors, which are T-odd observables, must be taken as serious candidates to cross-check TR symmetry.

4 Specific calculation of the normal component \(P_N\)

4.1 Relativistic form

In the previous paragraph, the resonance polarization-vectors are estimated by standard non-relativistic quantum mechanical methods in the \(\Lambda_b\) rest-frame. However, in order to measure \(\vec{P}^{(i)}\) for each resonance, \(R_{(i)}\), and its transformation by symmetry operations like Parity and Time-Reversal, a thorough examination of the \(R_{(i)}\) decay products by sophisticated methods (Byers, Dalitz) [10] must be done in the resonance rest-frame itself.
These calculations will not be developed in the present paper, but a rigorous method using the relativistic spin will be applied in order to understand the modification of $\vec{P}^{(i)}$ and, essentially its normal component $P_N$, when considering two different rest-frames: the $\Lambda_b$ one and the $R_{(i)}$ resonance one, which are related by a Lorentz transformation. Thus, we are led to study the spin of any particle and its polarization in their relativistic form. The latter is described by an axial four-vector $(S^\mu) = (S^0, \vec{S})$ which verifies the fundamental relation: $p_\mu S^\mu = 0$, $p_\mu$ being the 4-momentum of the particle. In what follows, we will drop the index $(i)$ which designates any resonance ($\Lambda$ or $J/\psi$) and we will assign a prime to the physical quantities defined in the $R_{(i)}$ rest-frame.

### 4.2 Different rest-frames

$P' = (0, \vec{P}')$ is the polarization 4-vector of the $\Lambda$ hyperon in its own rest-frame. Taking the $\Lambda$ hyperon at rest, the beauty baryon $\Lambda_b$ will move with a momentum $\vec{p}'$ (with respect to $\Lambda$) and an energy $E' = \sqrt{\vec{p}'^2 + M^2}$ where $M$ is the $\Lambda_b$ mass. In the following, we set $\cosh \alpha = E'/M$ and $\sinh \alpha = p'/M$. The unit-vector $\vec{e}_N$ previously defined in the $\Lambda_b$ rest-frame can be decomposed like:

$$\vec{e}_N = e_\parallel + e_\perp \text{ with } e_\parallel = \frac{(e_\parallel \cdot \vec{p})}{\vec{p}} \vec{p} \text{ and } e_\perp = e_N - e_\parallel,$$

where $\vec{p}$ is the $\Lambda$ momentum in the $\Lambda_b$ rest-frame. In order to get the transformed of $\vec{e}_N$ in the $\Lambda$ rest-frame, $\vec{e}'_N$, a new 4-vector must be defined in the $\Lambda_b$ rest-frame: $e_N = (0, e_\parallel, e_\perp)$. It is worth noting some important kinematic properties:

1. $e_\parallel$ is orthogonal to $\vec{p}_\perp = \vec{p}$ in the $\Lambda_b$ rest-frame, $e_\parallel = \vec{0}$, which implies $e_\perp = e_N$;
2. the momenta $\vec{p}$ and $\vec{p}'$ are parallel. Therefore the corresponding transformations of the 4-vector $e_N$ into $e'_N$ are the following: (a) $e'_{N\perp} = (0, e_{N\perp}) = (0, e_\perp)$, because the orthogonal component to the boost-direction is unmodified; (b) $e'_{N\parallel} = (\sinh \alpha \ |\vec{e}_N\|, \cosh \alpha \ e_\parallel)$ with $e_\parallel = 0$. The final expression of the 4-vector $e'_N$ will be given by

$$e'_N = e'_{N\parallel} + e'_{N\perp} = (0, e_\perp) = e_N .$$

Using the notations above, it can be noticed that, in each rest-frame, the normal component of the polarization-vector is given by:

$$\Lambda \text{ rest - frame } : P' \cdot e'_N = -\vec{P}' \cdot \vec{e}'_N = -P'_N ,$$

$$\Lambda_b \text{ rest - frame } : P \cdot e_N = -\vec{P} \cdot \vec{e}_N = -P_N . \quad (12)$$

By Lorentz-Invariance, one can easily deduce that $P \cdot e_N = P' \cdot e'_N$ which obviously gives $P_N = P'_N$. So, the normal-component of the polarization-vector as defined in the previous frame, $(\vec{e}_L, \vec{e}_T, \vec{e}_N)$, is a Lorentz Invariant. This interesting physical property allows us to cross-check TR symmetry either in the $\Lambda_b$ rest-frame or in any resonance rest-frame coming from $\Lambda_b$ decays.
5 Conclusion

Complete calculations based both on the helicity formalism (kinematics) and on the OPE techniques supplemented by HQET (dynamics) have been performed in a rigorous way for a precise determination of the physical properties of the $\Lambda_b \to \Lambda V(J^P = 1^-)$ decays. Resonances $\Lambda$ and $V(J^P = 1^-)$ being polarized, it is shown that the normal components of their polarization-vectors are T-odd observables. Furthermore, these components have large asymmetries and they are Lorentz-invariant. An exhaustive study of $P_{N}^{\Lambda_b,J/\psi}$ has been performed according to the $\Lambda_b$ polarization density matrix. Thanks to our analysis, it is confirmed that these observables are truly serious candidates to cross-check Time-Reversal symmetry, and we hope to detect these effects with the forthcoming LHC machine.

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Table 1: Mean values, $P_N^{\Lambda,J/\psi}$, and asymmetries, $A_s^{\Lambda,J/\psi}$, of the polarization-vector normal components of $\Lambda$ and $J/\psi$, respectively. Results are given as functions of the initial $\Lambda_b$ polarization varying from 100% to 0% and for the $\Lambda_b \rightarrow \Lambda J/\psi$ decay channel.

| $\mathcal{P}_{\Lambda_b}$ | $P_N^\Lambda$ | $A_s^\Lambda$ | $P_N^{J/\psi}$ | $A_s^{J/\psi}$ |
|---------------------------|---------------|---------------|----------------|---------------|
| 100%                      | -0.98         | -1.0          | -0.88          | -0.95         |
| 75%                       | -0.97         | -1.0          | -0.89          | -1.0          |
| 50%                       | -0.96         | -1.0          | -0.87          | -1.0          |
| 25%                       | -0.88         | -1.0          | -0.85          | -1.0          |
| 10%                       | -0.61         | -1.0          | -0.83          | -1.0          |
| 0%                        | 0.0           | 0.0           | -0.81          | -1.0          |

Figure 1: The $\vec{e}_x, \vec{e}_y, \vec{e}_z$ as well as the $\vec{e}_T, \vec{e}_N, \vec{e}_L$ frames in the $\Lambda_b$ rest-frame.
Figure 2: Spectra of the $\Lambda$ polarization-vector components: (from left to right) $P_L, P_N, P_T$, respectively in case of $\mathcal{P}_{h_b} = 100\%$ and $\Re(\rho_{+ -}^{\Lambda_b}) = \Im(\rho_{+ -}^{\Lambda_b}) = \sqrt{2}/2$.

Figure 3: Spectra of the $J/\psi$ polarization-vector components: (from left to right) $P_L, P_N, P_T$, respectively in case of $\mathcal{P}_{h_b} = 100\%$ and $\Re(\rho_{+ -}^{J/\psi}) = \Im(\rho_{+ -}^{J/\psi}) = \sqrt{2}/2$. 
Figure 4: Spectra of $P_N^\Lambda$ and $P_T^\Lambda$ with $P_{^3\!\!P}^b = 50\%$. Upper histograms correspond to the case of $\Re(\rho_{^3\!\!P}^b) = \Im(\rho_{^3\!\!P}^b) = 0$, while lower histograms correspond to $\sqrt{2}/2$.

Figure 5: Spectra of $P_N^{J/\psi}$ and $P_T^{J/\psi}$ with $P_{^3\!\!P}^b = 50\%$. Upper histograms correspond to the case of $\Re(\rho_{^3\!\!P}^b) = \Im(\rho_{^3\!\!P}^b) = 0$, while lower histograms correspond to $\sqrt{2}/2$. 