Description of space charge transport in oil-paper insulation using adaptive time-stepping transient upstream finite element method

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Abstract
The charge transport and accumulation in oil–paper can cause the insulation degradation. So far, the most widely used model to simulate space charge transport and accumulation is the bipolar charge transport (BCT) model, which can well simulate the space charge dynamics. However, there are two shortcomings in the algorithms for solving the BCT model. One is that there is almost no use of vectorisation technology, which may increase the complexity of the algorithm, the other is the usage of fixed step size which might bring extra computation cost. In view of this, an adaptive time-stepping transient upstream finite element method (FEM) is developed to solve the BCT model considering trapping/detrapping, as well as the recombination phenomenon under DC condition in this article. Then, a vectorisation technology is used in the method to optimise the algorithm. Moreover, the adaptive time-stepping method is introduced in simulation to reduce computation time and calculation amount. Simulation results are obtained by programing and later presented, which are basically consistent with the corresponding experimental results. Therefore, the proposed method is expected to promote the optimization design of the oil–paper insulation system.

1 | INTRODUCTION

The reliability of the Ultra High Voltage Direct Current (UHVDC) system can be directly affected by converter transformer, which is one of the main weak links in UHVDC transmission. The oil–paper insulation of converter transformers is subjected to high alternative and direct current field. In contrast to the alternative current field, the space charge phenomenon of oil–paper insulation in converter transformers under the direct current field is more obvious due to its accumulation trend and higher complexity of the field distribution. The accumulation and transport of space charges can cause the degradation or even breakdown of the oil–paper insulation. Many of methods for measuring space charge in dielectric materials have been proposed so far, such as the pulsed electro-acoustic (PEA), pressure wave propagation (PWP) [1, 2], etc. Meanwhile, the numeric calculation technique, as an important auxiliary technique of the experiment method, has been used for improving the optimal design of electrical insulation in computational high-voltage engineering. Currently, the numerical calculation has been widely applied to the space charge dynamic calculation of dielectric materials and electrical equipment insulation simulation [3, 4].

The simulation of charge transport is mainly based on the bipolar charge transport (BCT) model proposed by Alison and Hill [5]. The BCT model based on Poisson equation, current continuity equation and convection-reaction equation has been widely accepted to simulate space charge transport. Belgaroui presented a new method based on the high order Runge-Kutta method and the finite element method (FEM) to resolve a bipolar model of charge transport [6]. Roy found that combining a third order upwind scheme (QUICKEST) with a flux limiter (ULTIMATE) can effectively solve the transport equation in solid dielectrics [7]. Tian proposed a method to solve the Poisson equation and the convection-reaction equation using the boundary element method (BEM) and combined discontinuous
Galrkin method, respectively [8]. A differential weighted essentially non-oscillatory (WENO) method was proposed by Min to solve the charge advection-reaction equation, and acquired space charge behaviour of corona charged low-density polyethylene (LDPE) [9]. The upstream finite element method (UFEM) presented by Takuma was first employed for solving coupled equations, which was composed of Poisson equation and current continuity equation in ion flow field calculation [10]. The method was improved by Jin and applied to solve the transient field and dynamic behaviours of space charge [11].

These methods can be well used to solve the BCT model, however, almost no matrix operations are used to optimise algorithms and fix inefficient code. In this paper, the calculation model is expressed in the form of vectorisation, which makes the code concise and simplifies the complexity of the programme. Meanwhile, matrix operation reduces the number of loops used in the algorithm, thus the computation cost is reduced. In addition, the fixed time step method is used in space charge simulation, which may lead to multiple iterations and long computation time. Therefore, the adaptive time-stepping method is introduced to automatically adjust the step size. The benefits of using the adaptive time-stepping method are as follows [12, 13]: (1) The adaptive time-stepping method can automatically adjust the time step size at each time step; (2) Smart control of the step size ensures the precision of calculation and (3) It decreases calculation time by ensuring the time step increment is controlled within a specified range of truncation error.

Currently, most of the studies on space charge characteristics are focused on experiments. Combining experiments with numerical calculations can better explain experimental phenomena, and realize the visualization of physical processes. Therefore, the adaptive time-stepping transient upstream FEM is proposed to simulate space charge characteristics in this paper.

2 | ADAPTIVE TIME-STEPPING METHOD WITH IMPROVED TRANSIENT UPSTREAM FEM FOR SOLVING BCT MODEL

The traps attribute to defects and a certain amount of impurities in dielectric materials, can succeed in trapping space charges although some space charges may be able to escape from traps. The trapping and detrapping affect charge transport in dielectrics. Furthermore, an apparent recombination phenomenon will appear in transport processes of space charges under the high electric field. Therefore, the trapping/detrapping and recombination phenomenon should be considered when using the BCT model to simulate space charge transport and accumulation in dielectrics.

2.1 | Charge injection theory of BCT model

When the electrode is in contact with the dielectric, the free charges flow back and forth between the electrode and the dielectric until their energy levels are equal. In addition, an injection barrier \( \omega \) has to be overcome by charges. When the energy of carriers exceeds the barriers \( \omega \), they are emitted to vacuum or dielectric. The emission current density is related to temperature, which can be described by the Schottky injection model [14], as shown in Equations (1) and (2):

\[
J_e(t) = A T^2 \exp \left( \frac{\omega_e}{kT} \right) \exp \left( \frac{e}{kT} \sqrt{\frac{E_e(t)}{4\pi\varepsilon}} \right) \tag{1}
\]

\[
J_h(t) = A T^2 \exp \left( \frac{-\omega_h}{kT} \right) \exp \left( \frac{e}{kT} \sqrt{\frac{E_h(t)}{4\pi\varepsilon}} \right) \tag{2}
\]

where \( J_e(t) \) and \( J_h(t) \) stand for the current density of electrons and holes on the boundary. \( A \) refers to the Richardson constant that approximates to \( 1.2 \times 10^6 \text{ A/m}^2 \text{K} \), \( \omega_e \) and \( \omega_h \) refer to injection barriers (effective work function) of the two carriers. \( k \) stands for the Boltzmann constant that approximates to \( 1.38 \times 10^{-23} \text{ J/K} \). \( E_e(t) \) and \( E_h(t) \) stand for electric intensity at cathode and anode. \( e \) is the elementary charge that approximates to \( 1.6 \times 10^{-19} \text{ C} \), \( T \) is the temperature (K), \( \varepsilon \) is the relative dielectric constant (F/m).

2.2 | Charge transport theory of BCT model

The charge transport is described by a set of equations, which have been widely used in many literatures [11, 14]. Among the equations, Poisson equation, shown in Equation (3), is used to demonstrate the distribution of the electric field in dielectrics. Equation (4) is the current continuity equation. The Equation (5) is the convection–reaction equation, which depicts the variation of charge migration and density with time.

\[
\nabla^2 \varphi(t) = \frac{\rho(t)}{\varepsilon} \tag{3}
\]

\[
J_a(t) = -\mu_a \rho_a(t) E(t) \tag{4}
\]

\[
\frac{\partial \rho_a}{\partial t} + \nabla J_a(t) = S_a(t) \tag{5}
\]

where \( \varphi \) is the electric potential, \( \nabla \) is the nabla operator, \( \mu_a \) is the mobility of charges (m$^2$/V·s), \( J_a \) is the current density derived from charge transport (A/m$^2$), the subscript \( a \) is the type of space charge, \( \rho \) is the net charge density (C/m$^3$) and \( \rho_a \) is the density of four types of space charge, \( E \) is the electric intensity.

It is reported that a lot of traps in dielectrics which can capture electrons and holes, thus producing trapped electrons and trapped holes [15]. Free electrons, free holes, trapped electrons, and trapped holes co-exist in the dielectrics simultaneously, which are expressed by \( e \mu_e, h \mu_h, \varphi, \varrho \). The \( S_a(t) \) \( (a = e \mu_e, h \mu_h, \varphi, \varrho \) shown in Equation (5), is called source term that describes trapping, detrapping, and recombination process of four types of space charges in dielectrics (C/m$^3$·s). Four types of source terms are described as follows:
The net charge density $\rho$ in the dielectric is defined as:

$$\rho = \rho_{ht} + \rho_{h\mu} - \rho_{et} - \rho_{e\mu}$$  \hspace{1cm} (12)

The velocity vectors of charge density are defined as:

$$V_{\alpha}(t) = -\mu_{\alpha}E(t)$$  \hspace{1cm} (13)

Electric intensity is acquired by electric potential $\phi$, which can be depicted as:

$$E(t) = -V\phi(t)$$  \hspace{1cm} (14)

Combining Equation (3)–(5) and Equation (12)–(14), the rate of change for space charge densities can be acquired as [11, 15]:

$$\frac{\partial \rho_{\alpha}(t)}{\partial t} = -V_{\alpha}(t) \cdot V\rho_{\alpha}(t) - \frac{\mu_{\alpha}}{\varepsilon}\rho_{\alpha}(t)\rho(t) + S_{\alpha}(t)$$  \hspace{1cm} (15)

### 2.3 Improved Transient Upstream FEM for Solving the BCT Model

In order to obtain charge densities at each point in the sample, the whole region is divided into many triangular elements. Then, the upstream finite element method is used to acquire charge density at each node, which is important to determine upstream elements. The upstream element is shown in Figure 2. For each node, the upstream element is defined as a triangular element with a direction vector of carrier mobility towards that node. The space charge density in the upstream area of mobility direction vector is always larger than that of

![Space charge dynamics in dielectric material](Figure 1)
FiguRe 2  The determination of upstream element

the downstream, which is ensured by the upstream element. This phenomenon conforms to the actual situation [11]. According to Figure 2, the \( \mathbf{V}_i \) is the carrier mobility direction vector at node \( i \), and the triangle \( ijm \) is the upstream element of node \( i \).

According to the theory of the finite element, the node charge density can be shown as:
\[
\rho(x, y) = N_i\rho_i + N_j\rho_j + N_m\rho_m
\]  

(16)

Equation (15) is obtained. In addition, the first two items on the right-hand side of (15) can be expressed as Equations (18) and (19).
\[
\mathbf{V}_a \cdot \nabla \rho_a = \begin{bmatrix} V_{axi} & V_{ayi} \\ V_{axj} & V_{ayj} \\ V_{axm} & V_{aym} \end{bmatrix} \cdot \begin{bmatrix} b_i \\ b_j \\ b_m \end{bmatrix} + \frac{1}{2\Delta} \begin{bmatrix} \rho_{ai} \\ \rho_{aj} \\ \rho_{am} \end{bmatrix}
\]

(18)

\[
\mathbf{V}_a = \begin{bmatrix} V_{axi} \\ V_{ayi} \\ V_{axj} \\ V_{ayj} \\ V_{axm} \\ V_{aym} \end{bmatrix}^T, \quad \mathbf{V}_{ak} = \begin{bmatrix} \rho_{ai} \\ \rho_{aj} \\ \rho_{am} \end{bmatrix}
\]

where \( k = i, j, m \) is the charge density of four types of space charge at corresponding node.

\[
\begin{bmatrix} \rho_{ai} \\ \rho_{aj} \\ \rho_{am} \end{bmatrix} = \begin{bmatrix} S_{ai} \\ S_{aj} \\ S_{am} \end{bmatrix}
\]

(19)

The vectorisation form of the Equation (20) is obtained by the transforming form of the Equation (15).

The Equation (20) conforms to a differential equation, which can be shown as:
\[
\frac{\partial \mathbf{\rho}}{\partial t} + \mathbf{K}\mathbf{\rho} = \mathbf{S}
\]

(21)

where, the \( \mathbf{M} \) is a unit matrix, the \( \mathbf{\rho} \) is row vector of node charge density, the \( \mathbf{K} \) is a const coefficient matrix, the \( \mathbf{S} \) is a vector of node source term. Time-stepping method is the general approach for solving Equation (21). To acquire a fully
discrete scheme, the time domain discretisation is employed in
the differential equation [12], which can be shown as:

\[
(M + \theta \Delta t \cdot K)\rho_{n+1} = (M - (1 - \theta)\Delta t \cdot K)\rho_n + (1 - \theta)S_n + \theta S_{n+1}
\]  

(22)

The details of \(M + \theta \Delta t \cdot K\) and \(M - (1 - \theta)\Delta t \cdot K\) can be presented in Equations (23) and (24). Where \(\theta = 0, 0.5, 1\) is a constant corresponds to forward Euler method, Crank-Nicolson (C-N) method and backward Euler method. The advantages of the backward Euler method and Crank-Nicolson method include the unconditional stability. Compared with the first-order accuracy of the Euler’s method, the Crank-Nicolson’s method has the second order accuracy, which is a better choice. Time duration is divided into: 0, …, \(t_n\), \(t_{n+1}\), … and \(\Delta t\)

 Besides, the fixed step method cannot adjust the step size in the iteration, so it is necessary to use the adaptive time-stepping method. The schematic of fixed step size and adaptive time-stepping size are shown in Figure 3, which compares the small fixed step size (3a), the large fixed step size (3b) and the adaptive time-stepping size (3c). The circle in Figure 3 represents the contour line, and the function value is always the same on it, which is the constant \(C\).

Apparently, using small fixed step size may have good convergence property but multiple iterations and long computing time are inevitable. However, as shown in Figure 3b, selecting a large fixed step size in simulation may bring difficulties in convergence. The advantage of using the adaptive time-stepping size is that the adaptive step size will be changed with the third order derivative term. This fact indicates that the step size is large at the beginning and gradually decreases at the end.

\[
M + \theta \Delta t \cdot K = 
\begin{bmatrix}
\frac{\Delta t}{2} \left( V_{axi}b_i + V_{ayj}c_i - \frac{\mu_s}{\varepsilon} \rho_i \right) \\
\frac{\Delta t}{2} \left( V_{axi}b_i + V_{ayj}c_i \right) \\
\frac{\Delta t}{2} \left( V_{axm}b_i + V_{aym}c_i \right)
\end{bmatrix}
\]

\[
M - (1 - \theta)\Delta t \cdot K = 
\begin{bmatrix}
\frac{\Delta t}{2} \left( V_{axi}b_i + V_{ayj}c_i - \frac{\mu_s}{\varepsilon} \rho_i \right) \\
\frac{\Delta t}{2} \left( V_{axi}b_i + V_{ayj}c_i \right) \\
\frac{\Delta t}{2} \left( V_{axm}b_i + V_{aym}c_i \right)
\end{bmatrix}
\]

\[
\frac{\Delta t}{2} \left( V_{axi}b_i + V_{ayj}c_i \right)
\]

\[
\frac{\Delta t}{2} \left( V_{axi}b_i + V_{ayj}c_i \right)
\]

\[
\frac{\Delta t}{2} \left( V_{axi}b_i + V_{ayj}c_i \right)
\]

\[
\frac{\Delta t}{2} \left( V_{axi}b_i + V_{ayj}c_i \right)
\]

is the time interval, \(\rho_n, \rho_{n+1}, S_n, S_{n+1}\) are vector of node charge density and node source term at time \(t_n\) and \(t_{n+1}\), respectively.

2.4 | Adaptive time-stepping method of improved transient upstream FEM

Generally, the smaller the step size, the smaller the truncation error. However, with the decrease of step size, not only the calculation amount increases, but also the rounding error may accumulate seriously. Therefore, it is necessary to select the appropriate step size to reduce the amount of calculation as much as possible.

Besides, the fixed step method cannot adjust the step size in the iteration, so it is necessary to use the adaptive time-stepping method. The schematic of fixed step size and adaptive time-stepping size are shown in Figure 3, which compares the small fixed step size (3a), the large fixed step size (3b) and the adaptive time-stepping size (3c). The circle in Figure 3 represents the contour line, and the function value is always the same on it, which is the constant \(C\).

Apparently, using small fixed step size may have good convergence property but multiple iterations and long computing time are inevitable. However, as shown in Figure 3b, selecting a large fixed step size in simulation may bring difficulties in convergence. The advantage of using the adaptive time-stepping size is that the adaptive step size will be changed with the third order derivative term. This fact indicates that the step size is large at the beginning and gradually decreases at the end.

The step size of next time can be estimated by the local truncation error of the C-N method, which is derived from the Taylor expansion. The local truncation error is expressed as follows [17]:

\[
R_n = \frac{\Delta t^3}{12} \frac{d^3 \rho(\xi_n)}{d\xi^3} (t_{n-1} < \xi_n < t_n)
\]

\(\xi_n\) is any constant between \(t_{n-1}\) and \(t_n\). The third-order derivative term in Equation (25) is approximated by the difference quotient, which contains charge density \(\rho\) and step size \(\Delta t\). The difference quotient can be expressed as:
According to Equation (25), local truncation error of next time step can be shown as:

\[
R_{n+1} = \frac{\Delta t_{n+1}^3}{12} \frac{d^3 \rho(x_{n+1})}{dt^3}
\]

\[
= \frac{\lambda^3}{12} \Delta t_n^3 \frac{d^3 \rho(x_{n+1})}{dt^3}
\]

\[= \lambda R_n \]

(28)

\(\xi_{n+1}\) is any constant between \(t_n\) and \(t_{n+1}\). In order to prevent the local truncation error from exceeding the maximum permissible error \(R_{\text{max}}\), it required that \(|R_{n+1}| \leq R_{\text{max}}\), that is to say, \(\lambda^3 |R_n| \leq R_{\text{max}}\). By transforming the form of \(\lambda^3 |R_n| \leq R_{\text{max}}\), Equation (29) can be obtained as:

\[
\lambda \leq \sqrt{R_{\text{max}}/|R_n|}
\]

(29)

Since \(R_{\text{max}}\) is not a fixed value, a security coefficient \(K_{sc} = 0.8\) is set to acquire the next time step \(\Delta t_{n+1}\):

\[
\Delta t_{n+1} = K_{sc} \Delta t_n \sqrt{R_{\text{max}}/|R_n|}
\]

(30)

Automatic increment of the step size obtained by Equation (29) decreases calculation time within a given range of truncation error. In practice, the third derivative term would suddenly be changed sometimes. Thus, it is necessary to set an upper bound and a lower bound of step: \(\Delta t_{ib} \leq \Delta t \leq \Delta t_{ub}\).

The flow chart of space charge dynamic simulation in an oil impregnated insulation paper using the adaptive time-stepping transient upstream finite element method is shown in Figure 4.

### 3 | Algorithm Verification

#### 3.1 | Space Charge Distributions Under 2-dimension Plane Model

In order to verify the effectiveness of the adaptive time-stepping transient upstream finite element method for solving the BCT model under constant temperature and different electric fields, the experimental results in the literature [18] is used as an experimental basis to compare with the simulation results. In experiment [18], the thickness of the oil-paper sample is 170 μm. The anode is made of aluminium with the cathode covered by a 0.1-mm semiconductor. The voltage \((\phi)\) applied to the anode is changed from 1.7 kV to 5.1 kV and 10.2 kV and the cathode is grounded to investigate space charge dynamic under different electric fields \((E = 10 \text{ kV/mm}, 30 \text{ kV/mm}, 60 \text{ kV/mm})\). In this article, a two-dimensional plane model is constructed to denote the sample in simulation, with a model length of 170 μm and the width of 100 μm, respectively. The model is shown in Figure 5.
The parameters used for simulation are shown in Table 1. The mobility of holes and electrons is assumed as a constant according to [8, 11, 15]. Classical parameters used for describing the charge activities are selected based on existing researches [19–24]. Some parameters (Schottky injection barrier height $\omega_b$ and $\omega_e$, max trapped charge density $\rho_{hole}$ and $\rho_{el}$) in [18] are directly used. Then, the simulation is conducted, and the simulation results are shown in Figures 6–12. According to the Poisson equation, the distribution of electric potential under 10.2 kV is shown in Figure 6. The comparison between the experimental results [18] and the simulation results is depicted in Figure 7. The distribution of space charge density in the sample under 10.2 kV is shown in Figure 8. Space charge distributions under 10.2 kV at different times are shown in Figure 9. The simulation results of electric field can be seen in Figures 10–12.

As shown in Figure 7, the experimental data and the simulation data have similar trends in space charge density. The comparison between the experimental and the simulation curve in Figure 7 shows that the experimental results are basically consistent with the simulation results. The variation of curves shows that once a voltage is applied to electrodes, there is a bulk of homocharges accumulating near the two electrodes. This phenomenon is conformed to the Schottky injection. It seems that more homocharges are accumulated with the increase of applied voltage.

The experimental results from Figure 7 show that the positive charge density declines sharply within 30 μm from the anode whereas the negative charge density declines moderately within 50 μm from the cathode. Moreover, the amplitude of the homocharge at the anode area is higher than the homocharge at the cathode area. The phenomenon reflects that charge injection may be related to the material of electrode. A semiconductor electrode (cathode) has a greater injected capacity of negative charges, thus,
negative charges are enabled to move from the cathode into the sample, then the amplitude of charge density is reduced on the boundary. The situation for an aluminium electrode (anode) is on the contrary. In addition, it has been observed from Figures 7 and 8 that a great deal of homocharges are accumulated in the oil–paper under the electrical stresses. This is mainly due to that the number of trapped charges in oil–paper are increased with stressing duration [25–30].

The simulation of space charge distributions under 10.2 kV at \( t = 30 \) s, 300 s, 600 s, 1200 s, 1800 s is shown in Figure 9. It can be seen that with the increase of time, the space charge density increases, which indicates that more and more space charges inject into the oil–paper. When the time is close to 1800 s, the space charges in the oil–paper present the saturation phenomenon.

Figure 10 shows that the distribution of the electric field under 10.2 kV. It is observed that the distribution of the electric field is non-uniform, and the maximum electric field appeared in the middle of the sample due to the space charge effect. It can be seen from the colour bar that the electric field intensity in the middle area of the dielectric material is rather large, especially the field intensity in some high brightness areas exceeds 60 kV/mm, while it is below 60 kV/mm on the margin.

Figure 11 presents that electric field distributions under different electric stresses. It is shown that the electric field varies greatly under different electric stresses. It also can be seen that the electric field intense near the two electrodes is weakened, while the electric field strength in other areas is slightly increased. Figure 12 demonstrates the maximum electric fields of both simulation and experiments under different electric stresses. It is apparent that the field distortion becomes obvious with the increase of electric stress.
3.2 | Space Charge Distributions Under Different Temperature Gradients

In order to verify the effectiveness of the adaptive time-stepping transient upstream finite element method for solving the BCT model under different temperature gradients, the experimental data in the literature [18] is used to verify the simulation results. According to [18], anode is kept at a constant temperature (20°C) while cathode heats up to 40°C and 60°C. The temperature distributions in oil–paper are calculated by FEM. Temperature distribution at temperature gradient of 40°C is shown in Figure 13.

The relation of carrier mobility with temperature is as follows [11]:

$$\mu = \mu_0 \exp(-W_\mu/kT)$$  \hspace{1cm} (31)

where, $W_\mu = 0.485$ eV is the activation energy of mobility, $\mu_0 = 2.18 \times 10^{-6}$ is a coefficient and $k$ is the Boltzmann constant. Other parameters used in simulation are the same with the ones in Section 3.1, which is shown in Table 1. The experimental results and simulation results under different temperature gradients at $t = 1800$ s are shown in Figures 14 and 15, respectively. The experimental results are basically consistent with the simulation results.

It is observed that the temperature gradient significantly affects the behaviour of space charges. Positive and negative charges move from electrode into the interior of oil–paper under the applied voltage, while the accumulation of net negative charges is more significant. A bulk of net negative charges migrate from high temperature area to low temperature area of the oil–paper sample, and more heterocharges appear. This
phenomenon becomes more obvious with the increase of temperature gradient [20, 31–33].

3.3 | Comparison of Adaptive Time-stepping Method and Fixed Step Size Method

The FEM is used to partition the sample, which is divided into 674 elements with 299 nodes in this article. The adaptive time-stepping C-N method and fixed step size C-N method are compared in this paper, where six fixed step sizes are adopted in the simulation, namely $\Delta t = 0.01, 0.1, 0.5, 1, 5, 10$.

First, the adaptive time-stepping method is tested. A random net charge density of one point is selected, and the obtained simulation value is 0.5074 C m$^{-3}$. Then, a fixed step size programme is substituted with the adaptive step size programme. The comparison of computation time and calculation amount between fixed step size and adaptive step size is performed. Step size is set to range from 0.01 to 10. The comparison results are shown in Table 2. The NaN in Table 2 means that the calculation did not converge. According to Table 2, it is observed that once the fixed step size is between 0.01 and 1, the calculation results are almost the same with those gained through the adaptive time-stepping method. However, the adaptive method saved a third to half of time, and it also reduced the amount of calculation compared with $\Delta t = 0.1$ and $\Delta t = 0.01$.

4 | CONCLUSION

In order to reduce the complexity of the algorithm in space charge simulation and overcome the defects of the fixed step size method, an adaptive time-stepping transient upstream finite element method based on the upstream finite element method is proposed by introducing vectorisation technology and adaptive time-stepping method. Conclusions are summarised as follows:

(1) The matrix operation is introduced in this article to make the code concise and simplify the complexity of the program. Meanwhile, matrix operation reduces the number of loops used in the algorithm, which decreases the computation cost.

(2) The adaptive time-stepping method is applied to the algorithm which reduces computation time and quantities of computation. It also guarantees the convergence of the algorithm.

(3) The simulation results show that as the electric stress increases, more and more space charges are accumulated in the oil-paper and the electric field distortion becomes more obvious. Meanwhile, the simulation under different temperature gradients shows that the heterocharges increase as temperature gradient grows. The trend of the simulated results is consistent with the experimental results.

![Figure 14](image1.png) **Figure 14** Comparison of the experimental results [18] and the simulation results at the temperature gradient of 20°C

![Figure 15](image2.png) **Figure 15** Comparison of the experimental results [18] and the simulation results at the temperature gradient of 40°C

| Types of step size | Charge density (C m$^{-3}$) | Amount of calculation | Computation time(s) |
|--------------------|----------------------------|-----------------------|---------------------|
| Adaptive step size | $0.5074$                   | $2508$                | $116.338$           |
| Fixed step size    |                            |                       |                     |
| $\Delta t = 10$    | NaN                       | NaN                   | NaN                 |
| $\Delta t = 5$     | NaN                       | NaN                   | NaN                 |
| $\Delta t = 1$     | $0.5076$                  | $2507$                | $162.372$           |
| $\Delta t = 0.5$   | $0.5076$                  | $2507$                | $149.618$           |
| $\Delta t = 0.1$   | $0.5074$                  | $4008$                | $256.032$           |
| $\Delta t = 0.01$  | $0.5074$                  | $4882$                | $196.124$           |
Considering the behaviour of charges at the interface of oil–paper, future work will be focussed on the influence of interface charge on the charge transport in oil–paper.

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