Boosting Parallel Influence-Maximization Kernels for Undirected Networks with Fusing and Vectorization

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Abstract—Influence maximization (IM) is the problem of finding a seed vertex set which is expected to incur the maximum influence spread on a graph. It has various applications in practice such as devising an effective and efficient approach to disseminate information, news or ad within a social network. The problem is shown to be NP-hard and approximation algorithms with provable quality guarantees exist in the literature. However, these algorithms are computationally expensive even for medium-sized graphs. Furthermore, graph algorithms usually suffer from spatial and temporal irregularities during memory accesses, and this adds an extra cost on top of the already expensive IM kernels. In this work, we leverage fused sampling, memoization, and vectorization to restructure, parallelize and boost their performance on undirected networks. The proposed approach employs a pseudo-random function and performs multiple Monte-Carlo simulations in parallel to exploit the SIMD lanes effectively and efficiently. Besides, it significantly reduces the number of edge traversals, hence the amount of data brought from the memory, which is critical for almost all memory-bound graph kernels. We apply the proposed approach to the traditional MIXGREEDY algorithm and propose INFUSER-MG which is more than 3000× faster than the traditional greedy approaches and can run on large graphs that have been considered as too large in the literature. For instance, the new algorithm runs in 2.09, 0.08, 0.36 seconds on graphs Amazon, NetHEP, NetPhy with 16 threads where the sequential baseline takes 141.3, 259.1 and 1725.2 seconds, respectively. To compare INFUSER-MG with the state-of-the-art approximation algorithms, we conduct a thorough experimental analysis with various influence settings. The results on real-life, undirected networks show that on 16 threads, INFUSER-MG is 2.3×-173.8× faster than state-of-the-art while being superior in terms of influence scores, and using a comparable amount of memory.

1 INTRODUCTION

With their rapid growth, the study of effective information diffusion in networks becomes a fruitful area of research with several applications from many fields such as viral marketing [1], [2], social media analysis [3], [4], and recommendation systems [5]. Since these networks have been used for educational, political, economical, and social purposes, the diffused information can have various importance levels. Furthermore, the diffusion can be a time-critical process, but it can be costly to increase its speed and coverage by other means. Hence, novel approaches to find good vertex sets which effectively spreads information are vital in practice.

The Influence Maximization (IM) problem is introduced by Kempe et al. [6]. Formally, it focuses on finding the most promising seed (vertex) set with a given cardinality that increases the expected number of influenced vertices. IM is proven to be NP-hard [6] and there are various simplifications and heuristics proposed in the literature [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17]. It has also been shown that a greedy Monte-Carlo approach provides a constant approximation for the optimal solution [6]. For a graph with \( n \) vertices, the expected complexity of this greedy algorithm, estimating an influence score \( \sigma \), running \( R \) simulations and selecting \( K \) seed vertices is \( O(KRn\sigma) \). Hence, for real-life networks with hundreds of thousands of vertices, the approach is expensive. However, these simulation-based, greedy algorithms provide the best possible approximation guarantees. Therefore they are considered as the gold standard for IM.

Performing the simulations of a greedy algorithm in parallel is an immediate and straightforward remedy to reduce the execution time of IM kernels and make them scalable for large-scale networks. However, restructuring the kernels to leverage instruction-level parallelism have not been investigated before. Although modern compilers can efficiently and automatically utilize instruction-level parallelism for applications with regular memory access patterns, it is not a straightforward task for graph processing kernels due to their irregular memory accesses. Furthermore, vectorization attempts on such kernels usually fail to provide significant performance improvements. In this work;

- We propose INFUSER-MG, an ultra-fast and high-quality Influence Maximization algorithm for undirected networks. Unlike the traditional greedy approach, the proposed approach samples the edges as they are being traversed in multiple simulations. Hence, for a single simulation, sampling and diffusion processes are fused.
- By running concurrent simulations at once, we reduce the amount of connectivity information read from the memory. Hence, the proposed approach reduces the pressure on the memory sub-system. Furthermore, we utilize vectorized instructions almost with full efficiency for the cascade model to regularize the memory-accesses.
- INFUSER-MG can be around 200000× faster compared

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to the traditional greedy approaches. It is usable on large graphs that have been considered as too large in the literature. For instance, the new algorithm runs in 2.09, 0.08, 0.36 seconds on networks Amazon, NetHEP, NetPhy with 16 threads where the sequential baseline takes 141.3, 259.1 and 1725.2 seconds, respectively. In fact, with a 302,400 seconds (3.5 days) timeout, the sequential baseline can process only the above-mentioned 3 (out of 12) real-life graphs, having 1.2M, 58.9K and 231.5K edges. On the other graphs, the original algorithm cannot complete the simulations within the time limit. However, INFUSE-RMG completes all of 12 graphs around 1200 seconds in total, where the maximum is 654 seconds for the Orkut network having 3.1M vertices and 117.2M edges.

To better position the performance of INFUSE-RMG in the IM literature, we compare the performance, memory usage and influence score with a state-of-the-art approximation algorithm IMM [18]. The experiments show that INFUSE-RMG is $2.3 \times 13.8 \times$ faster than state-of-the-art while always being (marginally) superior in terms of influence scores, and using a comparable amount of memory. To be fair, we want to emphasize that the state-of-the-art tool can also work with directed graphs where INFUSE-RMG only supports undirected graphs.

The paper is organized as follows: In Section 2 we present the background on IM and introduce the mathematical notation. Section 3 describes the proposed approach in detail. In Section 4 a thorough performance comparison over the traditional algorithms is provided by conducting experiments on various real-world datasets and influence settings. Besides, a comparison with the state-of-the-art from the literature is given. Section 5 presents a comparative overview of the existing work. Finally, Section 6 discusses future work and concludes the paper.

2 Notation and Background

Let $G = (V,E)$ be an undirected graph where the $n$ vertices in $V$ correspond the agents, and $m$ edges in $E$ correspond the relations between the agents in $V$. The neighborhood of a vertex $u \in V$ is denoted as $\Gamma_G(u) = \{ v: \{u,v\} \in E \}$. Each edge $\{u,v\} \in G$ has a weight $w_{u,v}$ associated with the diffusion probability from $u$ to $v$. In practice, $w_{u,v}$ can be determined by the strength of $u$ and $v$’s relationship. Although the graph is undirected, to emphasize the direction of diffusion, we will use tuples instead of sets to denote edges. That is an edge $\{u,v\} \in E$ can be encountered either in form $(u,v)$ ($v$ influences $u$) or $(v,u)$ ($u$ influences $v$).

A graph $G' = (V',E')$ is a subgraph of $G$ if $V' \subseteq V$ and $E' \subseteq E$. If all the vertices in $G'$ are connected $G'$ is called as a connected subgraph of $G$. If the subgraph is a maximally connected subgraph of $G$ it is called a connected component (CC) of $G$.

2.1 Influence Maximization

Given a graph, Influence Maximization aims to find a seed set $S \subseteq V$ among all possible size $K$ subsets of $V$ that maximizes an influence spread function $\sigma$ when the diffusion process is initiated from $S$. Although we focus on undirected graphs, for IM, the graph can be directed or undirected depending on the initial construction. Figure 1 shows a (Fig. 1a) and directed (Fig. 1b) graph for which the weights on the edges are diffusion/influence probabilities.

The influence spread function $\sigma_{G,M}(\cdot)$ computes the expected number of agents/nodes/vertices influenced (activated) through a diffusion model $M$. For the sake of simplicity, we drop $M$ from the notation; in the rest of the text, $\sigma_G$ refers to $\sigma_{G,M}$. Some of the popular diffusion models for IM in the literature are independent and weighted cascade (IC and WC), and linear threshold (LT) [6].

- **Cascade** model runs in rounds and activates a vertex $v$ if one of its (incoming) edges, $(u,v)$, is used during the diffusion process which happens with probability $w_{u,v}$ given that $v$ has already been activated in the previous rounds. In the independent variant, which we focus on in this work, activation probabilities are independent (from each other and previous activations) as in Figure 1a. The weighted variant of the cascade model uses a directed graph $G$ as in Figure 1b (even when the original graph is undirected). A classical approach for assigning the edge weights, as performed in [7], is setting $w_{u,v} = 1/d_v$ where $d_v$ is the number of incoming edges of $v$ (which is equal to $\Gamma_G(v)$ in the original graph). Hence, if $v$ has $\ell$ neighbours activated after the last round, its activation probability in the current round is $1 - (1 - 1/d_v)^\ell$.

- **Linear threshold** generalizes the cascade models and activates a vertex $v$ if the total activation coming from $v$’s neighbors surpasses a threshold $\theta_v$. Throughout the

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### Table 1: Table of Notations

| Variable | Definition |
|----------|------------|
| $G = (V,E)$ | Graph $G$ with vertices $V$ and edges $E$ |
| $\Gamma_G(v)$ | Neighborhood of vertex $v$ in graph $G$ |
| $w_{u,v}$ | Probability of $u$ directly influencing $v$ |
| $R_G(v)$ | Reachability set of vertex $v$ on graph $G$ |
| $S$ | Seed set to maximize influence |
| $K$ | Size of the seed set |
| $R$ | Number of Monte-Carlo simulations performed |
| $\sigma_G(S)$ | Influence score of $S$ in $G$, i.e., expected number of vertices reached from $S$ in $G$ |
| $\sigma_G(S,v)$ | Marginal influence gain by adding vertex $v$ to seed set $S$ |
| $h(u,v)$ | Hash function for edge $(u,v)$ |
| $h_{max}$ | Maximum value hash function $h$ can return |
| $B[a,\ldots,a]_B$ | Batch size, number of simultaneous simulations ran. |
| $\sigma$ | Vector of size $B$, contains all $a$ |

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Fig. 1. (a) IC and (b) WC graphs for which the weights on the edges are diffusion/influence probabilities.
process, all the \( \{u, v\} \) (or \( (u, v) \) in a directed graph) edges with active \( u \) vertices are taken into account. When the sum of these edge weights exceeds \( \theta_{uv} \), \( v \) is activated \[6\].

In this paper, we focus on the independent cascade model, but the proposed techniques are also applicable to the other models in the literature for undirected graphs.

### 2.2 Existing approaches for Influence Maximization

There exist simulation-based \[6\], \[7\], sketch-based \[19\], and proxy-based \[7\], \[14\] approaches in the literature to find a seed set \( S \) that maximizes the influence spread in a graph. Simulation-based approaches run Monte-Carlo simulations whereas sketch-based ones utilize approximate data structures. On the other hand, proxy-based approaches simplify the IM problem and utilize simpler heuristics.

As stated before, the IM problem is NP-hard under the cascade and linear threshold models. The influence function is monotone and submodular, which means that adding a single vertex to the current seed set can only increase the overall influence and decreases the marginal influence scores for the remaining vertices that are not in the set. Due to these properties, the influence score of a greedy solution which always adds the most promising vertex with the highest marginal gain to a seed set of final size \( K \) is at least \( 1 - (1 - 1/K)^K \geq 63\% \) of the optimal solution \[20\].

Kempe et al. \[6\] proposed the greedy Monte-Carlo-based algorithm using the above-mentioned approach and set the foundations. At each step, the greedy algorithm finds the vertex that increases the influence the most. As Feige’s optimal inapproximability result shows \[21\], the guaranteed approximation ratio is the gold standard for problems with a non-trivial size, both asymptotically and practically. On the contrary, the other, sketch- and proxy-based approaches do not guarantee this approximation ratio. This is why we target the greedy, simulation-based algorithms that use the proposed techniques to boost their performance. To the best of our knowledge, these algorithms have experimented only on small-scale graphs in the literature.

Since IM is an expensive problem, there exist studies in the literature focusing on improving the algorithmic complexity. Instead of trying all the vertices at each step, the Cost-effective Lazy Forward (CELF) algorithm of Leskovec et al. \[22\] keeps the vertices in a priority queue w.r.t. their marginal influence gains. Due to the submodularity property of the influence spread function, these values set upper bounds for the current marginal gains. When a vertex is visited, its current marginal gain is updated, i.e., its exact value is computed, and the vertex is replaced further down in the queue. When a vertex is seen twice, the remaining vertices are guaranteed to have smaller marginal gains. Hence, the greedy decision can be immediately taken. The bottleneck of CELF is in its initialization; the (marginal) influence scores for all the vertices must be computed, which is the most time-consuming part and makes the approach expensive for large-scale graphs. This approach is improved by Goyal et al. \[23\] by further exploiting the submodularity of the influence spread function.

Chen et al. improve CELF with MixGreedy \[7\]. Instead of running Monte-Carlo simulations from each vertex to find the initial marginal gains, MixGreedy uses one iteration of another IM algorithm NewGreedy whose pseudocode is given in Algorithm 1. The algorithm greedily chooses \( K \) vertices to form the seed set \( S \). To choose each seed vertex, \( R \) graph samples are used. For each iteration (lines \[6\]–\[12\]), the algorithm samples a subgraph \( G' \) from \( G \). The pseudocode of the sampling algorithm, SAMPLE, is given in Algorithm 2 where each edge \( \{u, v\} \) is included with probability \( w_{uv} \). Then the marginal gains of \( G' \)’s vertices are computed by using the reachability sets.

**Algorithm 1 NewGreedy**

**Input:** \( G = (V, E) \): the influence graph

- \( K \): number of seed vertices
- \( R \): number of MC simulations per seed vertex

**Output:** \( S \): a seed set that maximizes influence on \( G \)

**mg:** marginal influence scores

1. \( S \leftarrow \emptyset \)
2. \( \text{for } k = 1 \ldots K \) do
3. \( \text{for } v \in V \) do
4. \( \text{mg}_v \leftarrow 0 \)
5. \( \text{for } r = 1 \ldots R \) do
6. \( G' = (V, E') \leftarrow \text{SAMPLE}(G) \)
7. \( \text{Compute } R_{G'}(S) \)
8. \( \text{Compute } |R_{G'}(v)| \text{ for all } v \in V \)
9. \( \text{for } v \in V \setminus S \) do
10. \( \text{if } v \notin R_{G'}(S) \) then
11. \( \sigma_{G'}(S, v) \leftarrow |R_{G'}(v)| \)
12. \( \text{mg}_v \leftarrow \text{mg}_v + \sigma_{G'}(S, v) \)
13. \( \text{mg}_v \leftarrow \frac{\text{mg}_v}{R} \text{ for all } v \in V \setminus S \)
14. \( S \leftarrow S \cup \{ \text{argmax}_{v \in V} \{ \text{mg}_v \} \} \)
15. \text{return } S, \text{mg}

**Algorithm 2 Sample(G)**

**Input:** \( G = (V, E) \): the original graph

**Output:** \( G' = (V, E') \): a subgraph of \( G \)

1. \( E' \leftarrow \emptyset \)
2. \( \text{for each } \{u, v\} \text{ in } E \) do
3. \( \text{Randomly choose } r \in [0, 1] \text{ from a uniform dist.} \)
4. \( \text{if } r \leq w_{uv} \) then
5. \( E' \leftarrow E' \cup \{u, v\} \)
6. \( \text{Construct } G' = (V, E') \)
7. \text{return } G'

The pseudocode of MixGreedy is given in Algorithm 3. Note that MixGreedy uses only a single iteration of NewGreedy with parameters \( (G, 1, R) \). Even though NewGreedy can be used to find each of the \( K \) vertices in \( S \) one by one, Chen et al.’s experiments revealed that NewGreedy is only faster in the initialization stage. For consequent vertices, the experiments show that performing the CELF approach and adding a vertex to the seed set in case of a revisit in the queue is faster.

In this work, we propose INFUSER-MG, the fused and restructured form of MixGreedy. The memory accesses and floating-point operations performed by the existing algorithm are restructured to reduce the memory pressure for the marginal gain computations. This enables fused-sampling and vectorization. Furthermore, memoization is
Algorithm 3 \textsc{MixGreedy}(G, K, R)

\textbf{Input:} \(G = (V, E)\): the influence graph  
\(K\): number of seed vertices  
\(R\): number of MC simulations per seed vertex  

\textbf{Output:} \(S\): a seed set that maximizes influence on \(G\)  
1: \(S, mg \leftarrow \text{NewGreedy}(G, 1, R)\)  
2: \(\sigma_G(S) \leftarrow \max_{v \in V} \{mg_v\} \triangleright mg_v = \sigma_G(\emptyset, v)\)  
3: \(Q \leftarrow \text{PriorityQueue()}\)  
4: \textbf{for} \(v \in V \setminus S\) \textbf{do}  
5: \(Q\).enqueue\((v, \text{priority}=mg_v)\)  
6: \(\text{iter}_v \leftarrow 0, \forall u \in V\)  
7: \textbf{while} \(|S| < K\) \textbf{do}  
8: \(u \leftarrow Q\).top()  
9: \textbf{if} \(\text{iter}_u = |S|\) \textbf{then}  
10: \(S \leftarrow S \cup \{u\}\)  
11: \(Q\).enqueue\((u, \text{priority}=mg_u)\)  
12: \(\sigma_G(S) \leftarrow \sigma_G(S) + mg_u\)  
13: \textbf{else}  
14: \(mg_u \leftarrow \text{RandCas}(G, S \cup \{u\}, R) - \sigma_G(S)\)  
15: \(\text{iter}_u \leftarrow |S|\)  
16: \(Q\).updatePriority\((u, \text{priority}=mg_u)\)  
17: \textbf{return} \(S\)

Algorithm 4 \textsc{RandCas}(G, S, R)

\textbf{Input:} \(G = (V, E)\): the influence graph  
\(S\): the seed set  

\textbf{Output:} \(\sigma_G(S)\): influence score of seed set \(S\) on \(G\)  
1: \(\sigma_S \leftarrow 0\)  
2: \textbf{for} \(r = 1 \ldots R\) \textbf{do}  
3: \(G' = (V, E') \leftarrow \text{Sample}(G)\)  
4: \(R_{G'}(S)\)  
5: \(\sigma_G(S) \leftarrow \sigma_G(S) + \frac{|R_{G'}(S)|}{R}\)  
6: \textbf{return} \(\sigma_G(S)\)

applied to reduce the cost of the CELF phase. The proposed techniques in this paper can be adopted by other probabilistic graph algorithms, as well as other IM kernels, to boost their performance. Although they are not focusing on probabilistic algorithms and fusing, SIMD-based alterations of graph kernels to regularize memory accesses have been studied before, e.g., to compute centrality metrics [24, 25].

2.3 Single instruction multiple data (SIMD)

\textit{Single Instruction-Multiple Data} architectures allow parallelism at the instruction level. Initially started with 128-bit MMX vector extensions, many enhancements have been implemented in modern processors. In this work, we employed Advanced Vector Extensions (AVX2) instruction set. AVX2 works on 256-bit registers in many packed forms including 1x256, 2x128, 4x64, and 8x32 storage patterns. We added these vector instructions manually to the code since, even though compilers translate and optimized most of the loops to vectorized forms, compare and move-mask operations were not recognized by auto-vectorization in our preliminary experiments. For completeness, the intrinsics explicitly used in this paper are described in Table 2.

### 3 Boosting Influence Maximization

Classical Monte-Carlo based IM algorithms first sample a sub-graph and then perform a single simulation. Such an approach is amenable to thread-level, coarse-grain parallelization since the simulations are independent of each other. However, this requires the graph to be read from the memory for every simulation. The state-of-the-art implementations use this \textit{one-sample-per-simulation} approach and build a unique graph for every sample to find the marginal influence scores [2]. With coarse-grain parallelization, this makes the IM kernels inefficient in terms of performance since the graphs are sparse (and samples are sparser), memory accesses are irregular, and performing a single simulation per graph traversal increases the already hindering pressure on the memory subsystem and makes the IM process further memory bound. As mentioned before, to make the IM computations faster, heuristics, sketches, and proxy models have been proposed in the literature. Unlike these, INFUSER-MG exploits the properties of the greedy Monte-Carlo algorithm. It is tuned for the undirected graphs and the Independent Cascade model. However, the techniques such as fusing can be adopted by the other models or Monte-Carlo graph algorithms using sampling. INFUSER-MG leverages three techniques to achieve its goals.

- Instead of explicitly constructing a data structure for each subgraph, the proposed approach uses direction-oblivious pseudo-random numbers throughout the edge-based simulation to fuse the sampling with the computation of influence scores.
- To reduce the memory subsystem pressure, INFUSER-MG leverages batched simulations and instruction-level parallelism and when possible, utilizes each edge access for multiple simulations.
- To reduce the number of operations performed, the component IDs for each vertex and sampled subgraph are memoized which can then be used while computing the marginal gains during the CELF stage.

On top of these, multi-core parallelism is applied to further increase the performance by running multiple threads and assigning each batch to a different thread.

#### 3.1 Direction oblivious hash-based sampling

Traditionally, the cascade model requires a new sample, i.e., a subgraph, from \(G = (V, E)\) to simulate the diffusion pro-
cess. State-of-the-art implementations sample edges from $E$ and add them to a set along with reversely oriented edges to make the subgraph, which is constructed from this sampled edge set, undirected. INFUSER-MG does not explicitly sample. Whenever an edge with a certain orientation is read from the memory, it is sampled or skipped depending on the outcome of direction-oblivious sampling that assigns the same sampling probability for both directions, $(u, v)$ and $(v, u)$. We utilize a hash function $h(u, v) = h(v, u)$ to get the same probability for forward and backward directions within the same simulation. The hash function used is

$$h(u, v) = \text{MURMUR3}(\min(u, v) || \max(u, v))$$  \hspace{1cm} (1)

where $||$ is the concatenation operator. To avoid the cost of hashing during simulations, all possible hash values are pre-computed. Although there exist $\frac{n(n-1)}{2}$ possible vertex pairs, we only need the vertex pairs having an edge in between, i.e., only $m$ hash values are pre-computed. We have tried a few other hash algorithms as well; we chose MURMUR due to its simplicity and good avalanche behavior with maximum bias 0.5%.

Although the above-mentioned approach generates a unique hash value for each edge, and hence a unique sampling probability, different simulations require different probabilities. To achieve this, we use a random number $X_r$ for each simulation $r$. To compute the sampling probability of $\{u, v\}$ during simulation $r$, $h(u, v)$ is first XOR’ed with a uniformly randomly chosen $X_r \in [0, h_{max}]$ and the outcome is divided to the maximum possible hash value $h_{max}$. Let $\rho(u,v)_r$ denote this sampling probability for $\{u, v\}$ in simulation $r$. Formally,

$$\rho(u,v)_r = \frac{X_r \oplus h(u,v)}{h_{max}}.$$ \hspace{1cm} (2)

The edge $\{u, v\}$ is verified to be in the sample if $\rho(u,v)_r$ is smaller than or equal to the threshold $w_{u,v}$. With the proposed approach, sampling an edge reduces to an XOR and compare-greater-than operation. The branching on the latter can be removed to enable SIMD instructions as explained later in this section.

MURMUR3 guarantees a change on the 50% of the bits when a single bit of the input changes. Furthermore, all bits independently change when the input is changed. These properties allow us to generate good pseudo-random values to simulate the process. For practical considerations, we stored all the $\rho(u,v)_r$’s generated for various real-life networks and plotted the Cumulative Distribution Function (CDF) of these values. For a given graph $G = (V,E)$, the CDF of a sampling probability $x$ is computed as $Pr(x \leq \rho(u,v)_r)$ for all $\{u,v\} \in E$ and $0 \leq r < R$. Figure 2 shows the CDFs for 12 real-life networks. The sampling probability distribution with hash-based computation is almost identical with the uniform distribution which is required to simulate the diffusion process.

In INFUSER-MG, the diffusion is performed on a subgraph which is never constructed; in fact, each diffusion is simulated on $G$. Thus the overhead of generating and storing a sample and reading it back from the memory is avoided. However, for each visit of $\{u,v\}$, since INFUSER-MG does not know if the edge is in the sample or not, $\rho(u,v)_r$ is recomputed. Another immediate benefit of fusing is traversing only the vertices that contribute to influence score and their neighbors. On the other hand, a non-fused implementation would traverse all edges for all simulations. Often, the total influence is a very small fraction of the total number of vertices and hence, fusing is vital to have a scalable IM kernel.

### 3.2 Vectorized Monte-Carlo graph traversal

In MixGREEDY (Algorithm 3), both the NEWGREEDY step and marginal gain computations utilize graph sampling. By leveraging vectorization, a single thread in INFUSER-MG can process a batch of $B$ samples/simulations at once. A high-level visualization of how the samples are batched is given in Figure 3. In a perfect, fused, and batched execution, the edges (of the original graph) flow from the memory to the cores and they are consumed by carefully structured SIMD kernels. Once an edge is visited, all $R$ simulations are taken into account by batches of $B$ simulations. Although fusing and vectorization can incur redundant computations, as the experiments will show, the proposed approach significantly boosts the performance.

![Image](image_url)

**Fig. 2.** Cumulative distribution function of hash-based sampling probabilities on various real-life networks.

![Image](image_url)

**Fig. 3.** (a) Two sampled subgraphs of the toy graph from Figure 1a with 5 vertices and 10 edges. (b) The simulations are performed in a way to be fused with sampling. Each edge is labeled with the corresponding sample/simulation IDs.

### 3.2.1 A vectorized NEWGREEDY step

For an undirected graph, the NEWGREEDY step of MixGREEDY needs to identify the reachability sets $R_G(v)$ for all $v \in G$. Traditional IM implementations work on a single subgraph and initiate many graph traversals until
all vertices are visited. The time complexity of this process is linear in terms of the number of vertices and edges. However, its memory access pattern tends to be irregular; many random memory accesses are required which results in low CPU utilization. Instead of graph traversal, e.g., Breadth-First Search, the connected components within a sampled subgraph can be found via label propagation, which starts by assigning unique labels to each vertex. Then at each iteration, the edges are visited and the labels of both endpoints are set to the minimum of the two. This process continues until convergence; i.e., no label is changed within a single iteration. The total amount of work performed by this algorithm is superlinear since each edge is touched at each iteration. To reduce the time complexity, one can mark the (live) vertices whose labels are updated in the current step, and only process their edges in the next step. Although this does not guarantee a linear-time algorithm, it significantly reduces the number of edge accesses.

INFUSE-MG runs the above-mentioned, label-propagation-based approach in a fused and batched manner. For all R samples, the propagation is simulated on the original graph G by taking only the sampled edges into account. The simulations are processed on batches of B = 8 samples which are never constructed. To do that, the existence of the edge in these samples is rechecked every time it is being processed. All the live vertices within a single iteration are processed in parallel by multiple threads. Further parallelization at this stage comes from the existence of the edge in these samples. The simulations are processed on batches of mg samples which are never constructed. To do that, mg = 8 is taken as the multiple of B = 8 simulations is given in Figure 4 (continuing from Figure 3).

Algorithm 5 NEWGREEDYSTEP-VEC(G, R)

Input: G = (V,E): the influence graph
R: number of MC simulations per seed vertex
Output: mg: marginal influence scores
l: connected component labels
1: for v ∈ V do
2: l_v ← [v, ..., v]_R
3: L ← V
4: while L is not empty do
5: L′ ← ∅
6: for u ∈ L in parallel do
7: for v ∈ Γ_u do
8: r ← 0
9: while r < R do
10: for r′ = r . . . r + 7 do
11: if r′ ≤ 7 then
12: minllabel ← min(l_u[r], l_v[r′])
13: if minl ≠ l_v[r′] then
14: l_v[r′] ← minl
15: L′ ← L′ ∪ {v}
16: r ← r + 8
17: L ← L′
18: for v ∈ V in parallel do
19: mg_v ← 0
20: for r = 1 . . . R do
21: mg_v ← mg_v + |{u : l_u[r] = l_v[r]}|
22: return mg, l

Fig. 4. (a) The initial state on a toy graph for label propagation; all vertices are labeled with their ids. (b) First, the edges of A are processed; the edge to C is in both samples. C’s labels are updated. (c) B’s edges are processed. The edge to C exists in the second sample. C’s second label is smaller, hence no update is performed. (d) C’s edges are being processed. It has edges to A, B, D, and E in the samples. The labels i_A, A_2 are propagated to D and E since the edges are in both samples. Besides, B’s second label is updated because only sample 2 contains the corresponding (C, B) edge. (e)(f) D and E edges in the samples. However, the are no updates.

Algorithm 5 describes the fused and vectorized NEWGREEDYSTEP-VEC. The algorithm takes two inputs G, the original graph, and R, the number of simulations. It works along the same lines with the original NEWGREEDY with additional operations for label propagation. The labels for each vertex are initially set as the vertex IDs (lines 1–2). The outer while loop checks if there exist any more live vertices. Here, a vertex is said to be live if at least one of its R labels is changed during the previous iteration. The first inner for at line 6 loops over the live vertices in a multi-threaded fashion. A single thread runs the next for loop at line 7 to visit the edges of the live vertex being processed. The operations corresponding to each of the R simulations are performed for a visited edge (u, v) in batches of 8.

For each 0 ≤ r < R, where r is a multiple of B = 8, the vectorized steps that perform the operations in simulations r to r + 7 are given between lines 10–15. These steps are per-
Algorithm \texttt{veclabel} ($r, u, v, X_r, l_u, l_v, r$)

\textbf{Input:} $r$: ID of the first simulation in the current batch  \\
$u$: source vertex  \\
$v$: target vertex  \\
$X_r$: random number for simulations $r$ to $r+7$  \\
l$_u$: vector of component labels of $u$  \\
l$_v$: vector of component labels of $v$  

\textbf{Output:} $l_v$: labels of vertex $v$ after traversing edge $(u, v)$  \\
live$_v$: a boolean which state if $v$ is live  

1: mask $\leftarrow$ \texttt{mm256 cmpgt_epi32}(l$_u$[r], l$_v$[r])  
2: labels $\leftarrow$ \texttt{mm256 blendv_epi18}(l$_u$[r], l$_v$[r], mask)  
3: hashes $\leftarrow$ \texttt{mm256 set1_epi32}(hash(u, v))  
4: probs $\leftarrow$ \texttt{mm256 xor_si256}(hashes, X$_r$)  
5: w$_{vec}$ $\leftarrow$ \texttt{mm256 set1_epi32}([w$_{uv}$ $\times$ INT$_{MAX}$])  
6: select $\leftarrow$ \texttt{mm256 cmpgt_epi32}(w$_{vec}$, probs)  
7: l$_v$[r] $\leftarrow$ \texttt{mm256 blendv_epi18}(w$_v$, labels, select)  
8: live$_v$ $\leftarrow$ \texttt{mm256 movemask_ps}  
9: return l$_v$, live$_v$

formed as described in Algorithm 6 \texttt{veclabel}. The algorithm first compares the labels using element-wise compare intrinsic \texttt{mm256 cmpgt_epi32} which returns all 1’s $(2^{32} - 1)$ when the first value is larger, and 0 otherwise. Then, pairwise minimum of the labels from the two vectors can be selected by \texttt{mm256 blendv_epi18} that employs the mask entries generated by the previous step. This intrinsic selects the bytes from the first vector if the corresponding mask entry is not zero. Otherwise, it selects the bytes from the second vector. Hence for an edge $(u, v) \in E$, the resulting vector, \texttt{labels}, contains the smaller of the endpoints’, i.e., $u$’s and $v$’s labels, for each simulation. The edge $(u, v)$ may not have been sampled by all simulations. To find the simulations it is sampled, the algorithm generates the sampling probabilities by XORing the corresponding hash, $h(u, v)$, and the random values, $X_r$. Being computed in the preprocessing step, the hash is promoted to a vector, hashes, by the intrinsic \texttt{mm256 set1_epi32}. The XOR operations are performed in a SIMD fashion with the intrinsic \texttt{mm256 xor_si256}. We then promote w$_{uv}$, to a vector w$_v$ by first multiplying it with INT$_{MAX}$ using the \texttt{mm256 set1_epi32} intrinsic. Then, this vector is element-wise compared to the vector probs by using \texttt{mm256 cmpgt_epi32}. The result of this operation is the select vector containing the selection masks for simulations. Blending labels (from line 2) with v’s current labels based on the select entries produces v’s final labels for the corresponding simulations $r$ to $r+7$ by using the intrinsic \texttt{mm256 blendv_epi18}.

After the new labels are computed, we check if any of v’s labels are modified to verify whether the process is converged or not. To do this, we first perform bitwise-and operations for the elements in mask and select by using the intrinsic \texttt{mm256 and_si256}. Then, the first bits of the 32-bit elements are extracted in a compact 8-bit format by using the \texttt{mm256 movemask_epi18} intrinsic. This intrinsic eliminates 8 comparison branches and produces a \texttt{bool}enue variable live$_v$. As mentioned above, at each iteration, the algorithm only processes the vertices whose labels are changed in the previous iteration. Initially, all the vertices are considered live. Each thread uses these live$_v$ values to keep track of the set $L$ of live vertices. To do this, we use an array of size $n$ in which the $v$th entry is marked if $v$ is live. After an iteration is finished, $L$ is updated. This approach allows us only to process live vertices.

3.3 Finding marginal gains with memoization During the label propagation stage in \texttt{newgreedystepvec}, \texttt{infuser-mg} computes and stores all the component labels $l$ (obtained by concatenating each $l_v$ for all $v \in V$) that can be considered as a two-dimensional $n \times R$ array. The first seed vertex is indeed the one having the largest expected (average) component size. Instead of resampling, this information can be utilized during the CELF stage while computing marginal gains and finding the remaining $K - 1$ seed vertices. The marginal gain for a vertex $u$, i.e., $mg_u$, can be found by computing the average number of vertices (over all the $R$ samples) that belong to $u$’s connected component but do not belong to the components of the seed vertices. This is equal to the expected number of additional vertices that will be influenced by inserting $u$ to the seed set $S$.

While computing $mg_u$, for all simulations, one can compare $u$’s label to all the component labels of the seed vertices in respective simulation. In our implementation, the data structure $l$ is stored as a single large memory block where the $R$ labels of a single vertex are stored consecutively for a better spatial locality. The component sizes are also stored in similar two-dimensional $n \times R$ array where rows correspond to component labels and columns correspond to simulations. Labels that do not map to a component are wasted for fast access while keeping the asymptotic space complexity the same (as $l$’s space complexity). This process is equivalent to using \texttt{randcas} over existing $R$ samples for finding marginal gains, except, no graph traversal or sampling is performed. Compared to the original approach, the memory accesses are more regular and the cache is better utilized. Furthermore, this operation can be efficiently parallelized as shown in the pseudocode of INFUSER-MG, Algorithm 7 [lines 15-16].

3.4 Implementation Details All the algorithms use the Compressed Sparse Row (CSR) graph data structure. In CSR, an array, $xadj$, holds the starting indices of each vertices neighbors, other vector, $adj$, holds neighbors of each vertex consecutively. So, to reach neighbors of vertex $i$, first we visit $xadj[i]$ and $index[i + 1]$ to find start and end positions in data, then scan starting from $adj[index[i]]$ until $adj[index[i + 1]]$ position.

4 Experimental Results All the experiments are performed on a server equipped with two 8-core Intel Xeon CPU E5-2620v4 sockets running on 2.10GHz and 192GB memory. Hence, there exist 16 cores in total. The OS running on the server is Ubuntu 16.04.2 LTS with Linux 4.4.0-66 generic kernel. The algorithms are implemented in C++ and compiled with gcc 8.2.0 with -Ofast as the optimization flag. Multi-threaded CPU parallelization is obtained with OpenMP pragmas. We have manually utilized AVX2 instructions available on the CPUs by using compiler intrinsics to implement the algorithms.
Algorithm 7 INFUSER-MG($G, K, R$)

Input: $G$: Graph
$K$: size of the seed set
$R$: number of MC simulations to perform

Output: $S$: a seed set that maximizes influence

1. $mg, l \leftarrow \text{NEWGREEDYSTEP-VEC}(G, R)$
2. $S \leftarrow \{\emptyset\}$
3. $q \leftarrow \text{PriorityQueue}()$
4. $iter_u \leftarrow 0, \forall u \in V$
5. for $v \in V$ do
6.   $q.\text{enqueue}(v, \text{priority}=mg_v)$
7. $R_G(v) \leftarrow 0, \forall v \in V$
8. while $|S| < K$ do
9.   $u \leftarrow q.\text{dequeue}()$
10. if $iter_u = |S|$ then
11.   $R_G(S) \leftarrow R_G(S \cup \{u\}) \triangleright \text{Append} \ l_u \to R_G(S)$
12.   $S \leftarrow S \cup \{u\} \triangleright \text{Commit} \ u \to S$
13. else
14.   $mg_u \leftarrow 0$
15. for $r = 1$ to $R$ in parallel reduce($mg_u$) do
16.   $mg_u \leftarrow mg_u + |l_u[r] \in \{1 \mid R_G(S)\}|$
17. $iter_u \leftarrow |S|
18. $q.\text{enqueue}(u, \text{priority}=mg_u)$
19. return $S$

### TABLE 3

| Dataset                  | No. of Vertices | No. of Edges | Avg. Weight | Avg. Degree |
|-------------------------|-----------------|--------------|-------------|-------------|
| Amazon                  | 262,113         | 1,234,978    | 1.00        | 4.74       |
| DBLP                    | 317,081         | 1,049,867    | 1.00        | 3.31       |
| NetHEP                  | 15,235          | 58,692       | 1.85        | 3.87       |
| NetPhy                  | 37,151          | 231,508      | 1.28        | 6.23       |
| Orkut                   | 3,072,441       | 117,185,083  | 1.00        | 38.14      |
| Youtube                 | 1,134,891       | 2,987,625    | 1.00        | 2.63       |
| Undirected              |                 |              |             |             |
| Directed                |                 |              |             |             |
| Epinions                | 75,880          | 508,838      | 1.00        | 6.71       |
| LiveJournal             | 4,847,571       | 68,993,773   | 1.00        | 14.23      |
| Pokec                   | 1,632,803       | 30,622,564   | 1.00        | 18.75      |
| Slashdot0911            | 77,360          | 905,468      | 1.00        | 11.70      |
| Slashdot0902            | 82,168          | 948,464      | 1.00        | 11.54      |
| Twitter                 | 81,306          | 2,420,766    | 1.37        | 29.77      |

#### 4.1 Network datasets used in the experiments

The experiments are performed on twelve graphs (six undirected, six directed) that have been frequently used for Influence Maximization. For directed datasets, the reverse edges are added to obtain undirected variants. The datasets are Amazon co-purchase network [27], DBLP collaboration network [27], Epinions consumer review trust network, LiveJournal [27], NetHEP citation network [27], NetPhy citation network [27], Orkut [27], Pokec Slovakian poker game site friend network [27], Slashdot friend-foe networks (08-11, 09-11) [27], Twitter list co-occurrence network [27], and Youtube friendship network [27]. The properties of these datasets are given in Table 3.

For a thorough experimental evaluation, four influence settings are simulated; for each network, we use

1. constant edge weights $p = 0.01$ (as in [6] and [7]),
2. constant edge weights $p = 0.1$ (as in [6]),
3. uniformly distributed weights from the interval $[0, 0.1]$,
4. normally distributed weights with mean 0.05 and std. deviation 0.025 so that 95% of the weights lie in $[0, 0.1]$.

#### 4.2 Metrics used to evaluate the performance

Following the literature, we employ three metrics to evaluate an algorithm: (i) the influence score, i.e., the expected number of vertices that are influenced (ii) the execution time, (iii) maximum memory size. There is an interplay among these metrics; it is trivial to devise an ultra-fast IM algorithm with a bad influence score. Similarly, using more memory can make an algorithm avoid computations. We present these metrics for each algorithm on all graphs.

When the algorithms run on the same machine, the reported execution times and memory usages of different algorithms are comparable. However, the reported influence scores can be misleading since the algorithms may be using different approaches to estimate the influence score. To find the expected number of vertices, we requested and used the original implementation from Chen et al. [7] as an oracle with minor modifications; i.e., without logging and using heap memory instead of stack memory to handle large-scale graphs. The random values in the oracle are generated by C++’s Mersenne Twister 32-bit pseudo-random generator mt19937, with a state size of 19937 bits.

#### 4.3 Algorithms evaluated in the experiments

The algorithms that are evaluated can be classified into three groups. The first class contains MIXGREEDY, obtained from Chen et al. [7], which is also used as the oracle to compute the influence scores. The second class contains two variants from the current state-of-the-art, Minutoli et al.’s IMM [18]. IMM is a fast algorithm robustly producing high-quality seed sets which can influence a large number of vertices. In the original paper, the variant with $\epsilon = 0.13$, a user-defined hyper-parameter controlling the approximation boundaries, is suggested. We use this variant along with a much faster one with $\epsilon = 0.5$, which is also experimented in [18].

The third class of algorithms contains two INFUSER-MG variants. To show the speedup breakdown, we consider each variant as a separate algorithm. The first variant is FUSEDSAMPLING which only integrates the sampling step by generating probabilities on the fly without any algorithmic improvements or edge traversal savings. This variant performs the simulations one-by-one as in MIXGREEDY. The second variant is the proposed approach INFUSER-MG employing vectorization and memoization. Both of these variants employ CELF and use the queue-based vertex processing as the base algorithm MIXGREEDY.

In this section, we first compare the INFUSER-MG variants with MIXGREEDY to present the speedups over the baseline with fusing and vectorization. We then compare INFUSER-MG employing the state-of-the-art to better position the proposed approach in the literature. Last, we evaluate the multi-threading performance of INFUSER-MG with $\tau \in \{1, 2, 4, 8, 16\}$ threads. In all experiments, we use a time-limit of 302,400 seconds (3.5 days).

#### 4.4 Comparing INFUSER-MG with MIXGREEDY

Table 4 shows the execution times (columns 2–5), memory usages (columns 6–8), and influence scores (columns 9–11) of the baseline algorithm and INFUSER-MG variants. MIXGREEDY runs with a single thread and finishes only in
three graphs Amazon, NetHEP, and NetPhy in 141.3, 259.1 and 1725.2 seconds, respectively. In fact, with a 302,400 seconds (3.5 days) timeout, these are the only three (out of 12) real-life graphs (with 1.2M, 58.9K, and 231.5K edges) that can be processed by MIXGREEDY. For the others, the original algorithm cannot find a seed set of K = 50 vertices within the time limit. However, INFUSER-MG with τ = 16 threads completes all the 12 graphs around 1200 seconds in total, where the maximum runtime is 654.5 seconds for the Orkut network having 3.1M vertices and 117.2M edges. The shortest execution time of INFUSER-MG on a graph that cannot be completed by MIXGREEDY is 1.5 seconds. Hence, INFUSER-MG with τ = 16 threads is up to 200,000× faster than the baseline. Only by looking at the sequential execution times of FUSEDAMPLING on three graphs, we can conclude that 3×–21× of this speedup comes from fusing.

The fifth column of Table 4 presents the execution times of INFUSER-MG to find the first seed vertex which is simply Algorithm 2 where the while loop is executed only once, which is equivalent to the setting with K = 1. Comparing these values with the ones in the previous column, we can argue that the benefits of the memoization are more for large K values such as 500 or 1000, since most of the time is spent on the NEWGREEDYSTEP-VEC. For instance, for large graphs, adding the next 49 seeds only takes 10%–20% of the overall execution time. The actual value depends on the number of the CELF stage; for Amazon, to add the remaining seed vertices, INFUSER-MG needs only 79 vertex visits. This is why the cost of the CELF stage is negligible.

Although it is extremely useful, memoization is also the reason of high memory usage. The values for NetHEP and NetPhy seem to be relatively lower compared to the baseline. However, these two graphs have only 15K and 37K vertices, much lower than the other graphs. In fact, FUSEDAMPLING can be a more efficient implementation of MIXGREEDY memory-wise. Comparing the memory use of FUSEDAMPLING with that of INFUSER-MG reveals the overhead of memoization more clearly. However, even with this overhead, the proposed approach stays practical and extremely efficient on a single server.

Overall, INFUSER-MG is a practical algorithm, and unlike MIXGREEDY, it can be used on undirected graphs that have been considered too large in the literature. On the comparable instances, it runs in 2.1, 0.1, 0.4 seconds where MIXGREEDY takes 141.3, 259.1, and 1725.2 seconds, respectively. Furthermore, as the last three columns of Table 4 show, the influence scores of the proposed approach are comparable with those of MIXGREEDY.

### 4.5 Comparing INFUSER-MG with State-of-the-Art

To better position INFUSER-MG within the literature, we compare the performance, memory usage, and influence score with a fast, state-of-the-art approximation algorithm IMM [18] which can produce high-quality seed sets that influences a large number of vertices for both directed and undirected graphs. We also run IMM by setting the undirected graph parameter.

Tables 5 and 6 show the execution times (in secs.) and memory use (in GBs), respectively, of INFUSER-MG and two IMM variants for 12 graphs and 4 simulation settings.
given in Section 4.1. The experiments show that INFUSER-MG is 2.5×–173.8× faster than state-of-the-art while always being (marginally) superior in terms of influence scores, and using a comparable amount of memory. As expected, the memory usage of IMM is increasing with smaller ε values. In addition, it also increases when the edge weights are larger, i.e., when the samples are denser. For instance, with p = 0.01, IMM(ε = 0.5) uses 20GBs for Orkut. However, when p = 0.1, the memory usage increases to 72GBs. Furthermore, IMM(ε = 0.13) cannot run on LiveJournal, Orkut, and Pokec networks due to insufficient memory. On the other hand, INFUSER-MG’s memory usage does not change with different values since it never explicitly creates and stores the samples thanks to fusing. Last, as shown in Table 7, the influence scores of the proposed approach and IMM(ε = 0.13) are comparable. Figure 5 shows the speedup values of INFUSER-MG with respect to IMM(ε = 0.13).

### 4.6 Scalability with multi-threaded parallelism

Figure 6 shows the speedup values obtained via OpenMP parallelization. Since most of time is spent by NEWGREEDYSTEP-VEC, the parallelization efficiency at line 6 of Algorithm 3 has a significant impact on the performance. In our implementation, the parallel processing of live (as source) vertices seems to be necessary to reduce the number of visited edges. However, since a (target) vertex can be a target for multiple sources, the update operation at line 14 of this push-based approach is a potential source of race conditions. For denser samples, e.g., for p = 0.1, this happens more frequently. Hence, larger influence probabilities may increase (1) the false sharing probability and (2) the number of iterations due to vectorized updates. We argue that these are the reasons for 3×–5× speedup with τ = 16 threads. Still, although the push-based approach seems necessary, we will investigate pull-based and hybrid, i.e., pull/push-based approaches in the future.

### 5 Related Work

Although they can be inferior in terms of influence score, recent IMM algorithms are shown to be just fast enough compared to conventional simulation-based approaches such as MIXGREEDY. However, in this work, we show that INFUSER-MG, which is a conventional algorithm, can be orders of magnitude faster than MIXGREEDY while keeping the quality of the seed vertices the same. Techniques such as using GPUs, sketches for finding set intersections, reverse sampling to estimate influence from a small subset of vertices, and estimating the necessary number of simulation-samples required for each step greatly reduces asymptotic boundaries of execution time [18], [19], [22], [28], [29], [30].

**TABLE 6** Memory use (in GBs) of the algorithms on the networks with K = 50 seeds in different simulation settings.

| Dataset      | IMM (ε = 0.13) | IMM (ε = 0.5) | INFUSER-MG (ε = 0.13) | INFUSER-MG (ε = 0.5) | p ∈ [0.005, 0.025] | IMM (ε = 0.13) | IMM (ε = 0.5) | INFUSER-MG (ε = 0.13) | INFUSER-MG (ε = 0.5) | p ∈ [0.001] |
|--------------|----------------|---------------|------------------------|----------------------|---------------------|----------------|---------------|------------------------|----------------------|----------------|
| Amazon       |                |               |                        |                      |                     |                |               |                        |                      |                |
| DBLP         |                |               |                        |                      |                     |                |               |                        |                      |                |
| Epinions     |                |               |                        |                      |                     |                |               |                        |                      |                |
| LiveJournal  |                |               |                        |                      |                     |                |               |                        |                      |                |
| NetHEP       |                |               |                        |                      |                     |                |               |                        |                      |                |
| NetPHY       |                |               |                        |                      |                     |                |               |                        |                      |                |
| Slashdot0811 |                |               |                        |                      |                     |                |               |                        |                      |                |
| Slashdot0902 |                |               |                        |                      |                     |                |               |                        |                      |                |
| Orkut        |                |               |                        |                      |                     |                |               |                        |                      |                |
| Pokec        |                |               |                        |                      |                     |                |               |                        |                      |                |
| Twitter      |                |               |                        |                      |                     |                |               |                        |                      |                |
| Youtube      |                |               |                        |                      |                     |                |               |                        |                      |                |

**TABLE 7** Influence scores of the algorithms on the networks with K = 50 seeds in different simulation settings.

| Dataset      | IMM (ε = 0.13) | IMM (ε = 0.5) | INFUSER-MG (ε = 0.13) | INFUSER-MG (ε = 0.5) | p ∈ [0.005, 0.025] | IMM (ε = 0.13) | IMM (ε = 0.5) | INFUSER-MG (ε = 0.13) | INFUSER-MG (ε = 0.5) | p ∈ [0.001] |
|--------------|----------------|---------------|------------------------|----------------------|---------------------|----------------|---------------|------------------------|----------------------|----------------|
| Amazon       |                |               |                        |                      |                     |                |               |                        |                      |                |
| DBLP         |                |               |                        |                      |                     |                |               |                        |                      |                |
| Epinions     |                |               |                        |                      |                     |                |               |                        |                      |                |
| LiveJournal  |                |               |                        |                      |                     |                |               |                        |                      |                |
| NetHEP       |                |               |                        |                      |                     |                |               |                        |                      |                |
| NetPHY       |                |               |                        |                      |                     |                |               |                        |                      |                |
| Slashdot0811 |                |               |                        |                      |                     |                |               |                        |                      |                |
| Slashdot0902 |                |               |                        |                      |                     |                |               |                        |                      |                |
| Orkut        |                |               |                        |                      |                     |                |               |                        |                      |                |
| Pokec        |                |               |                        |                      |                     |                |               |                        |                      |                |
| Twitter      |                |               |                        |                      |                     |                |               |                        |                      |                |
| Youtube      |                |               |                        |                      |                     |                |               |                        |                      |                |
part of the network and scalable on only sparse networks. Liu et al. proposed IMGPU [30], an IM estimation method by utilizing a bottom-up traversal algorithm. It performs a single Monte-Carlo simulation on many GPU threads to find the reachability of the seed set. It is $5.1 \times$ faster than MIXGREEDY on a CPU. The GPU implementation is up to $60 \times$ faster with an average speedup of $24.8 \times$.

Borgs et al. [28] proposed Reverse Influence Sampling (RIS) which samples a fraction of all random reverse reachable sets. Then it computes a set of $K$ seeds that covers the maximum number of those. The number of samples is calculated with respect to the number of visited vertices. The algorithm has an approximation guarantee of $(1 - 1/e - \epsilon)$. Minutoli et al. improved RIS and proposed IMM that works on multi-threaded and distributed architectures [18]. Recently, the authors extended the algorithm to work on GPUs [31].

6 CONCLUSION AND FUTURE WORK

In this work, we proposed fusing and vectorization for IM computations. Better utilization of the CPU cores is achieved by running concurrent simulations at the same time. A comparison with a conventional MC-based algorithm MIXGREEDY and a high-quality, state-of-the-art IM algorithm is presented on real-world datasets and simulation settings. With the proposed techniques, INFUSER-MG can be up to $200000 \times$ faster than MIXGREEDY and $2.3 \times - 173.8 \times$ faster than state-of-the-art on undirected graphs.

A natural extension of this work is adapting INFUSER-MG to directed graphs. Due to the parallel nature of the simulations, INFUSER-MG can benefit from GPUs if the device memory can be used effectively and efficiently. Also as IMM [18], the proposed algorithm can work on larger, massive-scale networks on distributed architectures. In the future, we are planning to pursue these research avenues.
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