Quark Confinement and Deconfinement in QCD
from the Viewpoint of
Abelian-Projected Effective Gauge Theory

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Abstract

We give another derivation of quark confinement in QCD from the viewpoint of the low-energy effective Abelian gauge theory of QCD obtained via Abelian projection. It is based on the recently discovered Berezinskii-Kosterlitz-Thouless phase transition in the four-dimensional Abelian gauge theory. Moreover, we show that there exists a critical gauge coupling constant in QCD, above which confinement sets in and below which there is no confinement. In a SU(N) gauge theory with $N_f$ flavored fermions, we argue that this leads to a critical value of fermion flavors $N_f^c$ for the confinement as well as the chiral symmetry breaking, which separates the deconfining and chiral symmetric phase from the confining and chiral symmetry breaking phase. A finite-temperature deconfinement transition is also discussed briefly.

Key words: quark confinement, topological field theory, magnetic monopole, chiral symmetry breaking

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Proving quark confinement in quantum chromodynamics (QCD) is a long-standing problem in theoretical particle physics. In a series of papers [1, 2, 3, 4], the author claimed that a novel reformulation of Yang-Mills (YM) theory leads to quark confinement in QCD. The reformulation is based on the identification of the YM theory as a perturbative deformation of a novel topological quantum field theory (TQFT). The TQFT is suggested from the gauge-fixing and the Faddeev-Popov ghost terms in the maximal Abelian gauge (MAG) [5]. The MAG is a choice of partial gauge fixing which realizes in the context of quantized gauge field theory an idea of Abelian projection proposed by ’t Hooft [6]. After the Abelian projection, the original non-Abelian gauge theory is rewritten into the Abelian gauge theory with magnetic monopoles and adjoint matter fields. The TQFT in question is a field theory which describes the topologically non-trivial (background) gauge field configurations leading to non-vanishing magnetic monopole current loop (due to topological conservation law) in four space-time dimensions. As demonstrated in [2], such topological field configurations in the functional integral give the dominant contribution to the string tension $\sigma$ (or area decay of the Wilson loop), while the remaining part is identified with the perturbative contribution around such topological configurations (leading to at most perimeter decay). This result supports the dual superconductor picture of QCD vacuum [7].

The term ‘Abelian projection’ is somewhat misleading, since it sounds as if off-diagonal parts are neglected a priori. Indeed, such a standpoint was taken in the early stage of investigation on quark confinement [8, 9]. It is a trivial statement that the Abelian gauge theory is obtained from a non-Abelian gauge theory by neglecting the off-diagonal gauge fields. Note that the Abelian projection here takes fully into account the off-diagonal parts by integration (in the sense of Wilsonian renormalization group), which reduces to the renormalization of the effective Abelian gauge theory as shown in [1].

We adopt as a criterion of quark confinement the area law of the large (non self-
intersecting) Wilson loop (large compared with the typical size of QCD). What we have shown in the previous papers and what we are going to show in this paper are concerned with the physics in the low-energy or long-distance region of QCD. Therefore, we don’t say anything novel about the high-energy or short-distance region of QCD where the asymptotic freedom is essential. This way of thought constitutes the core of Abelian dominance in low-energy physics of QCD [8]. Actually, it turns out that Abelian dominance is lost in the high-energy or short-distance region of QCD.

The above reformulation does not necessarily guarantee the non-existence of additional non-perturbative effects (coming from the remaining gluon self-interactions) which might invalidate the proof claimed above, although the validity of the identification is checked by numerical simulations. In this paper, in order to get rid of this doubt, we give another derivation of quark confinement by combining previous works [1] and [2, 3, 4].

First, we consider a non-Abelian gauge theory described by the Yang-Mills (YM) theory with a compact (semi-simple) non-Abelian gauge group $G$. Then we decompose the gauge field $A_\mu$ into the diagonal $a_\mu$ and off-diagonal $A_\mu$ pieces (Cartan decomposition). The diagonal gauge field belongs to the maximal torus group $H$ of $G$, while the off-diagonal gauge field to the coset $G/H$. The Abelian projection is realized as a partial gauge fixing which fixes the gauge degrees of freedom corresponding to $G/H$. Under the residual gauge transformation of $H$, the diagonal gluon fields transform as Abelian gauge fields, whereas the off-diagonal gluons transform as adjoint matter fields. Second, we obtain the Abelian gauge theory with a gauge group $H$ by integrating out all the off-diagonal gauge fields belonging to $G/H$ [10]. This is possible by introducing the auxiliary antisymmetric tensor field $B_{\mu\nu}$. We called

\footnote{The main idea is still the dual superconductor picture of QCD vacuum where the magnetic monopole plays the crucial role. However, this does not imply that the instantons and other topological non-trivial configurations such as vortex have nothing to do with quark confinement. Exact separation of monopole from other configuration is meaningless, since they are overlapped with each other.}
the Abelian gauge theory obtained in this way the Abelian-projected effective gauge theory (APEGT) [1].

A quite important observation is that the APEGT obtained in this way is considered as the low-energy effective gauge theory (LEEGT) of the original non-Abelian gauge theory, e.g. QCD. This is because the off-diagonal gluons become massive after the MAG, as shown by direct numerical simulations [11] and an analytical consideration [2]. Therefore, if the integration over the coset components performed above is identified with the integration over the high-energy mode in the sense of Wilsonian renormalization group, the resulting Abelian gauge theory is regarded as the LEEGT which is meaningful in the low energy-momentum region \( p < O(m_A) \) or in the length scale \( \ell \) larger than the inverse of the off-diagonal gluon mass \( m_A (\ell > O(m_A^{-1})) \). This is another important point which is sometimes misunderstood by the literatures.

The APEGT of the YM theory with a gauge group \( G \) obtained according to the above procedures has the action [1] (in 3+1 space-time dimensions),

\[
S_{\text{APEGT}} = \int d^4x \left[ -\frac{1 + z_f}{4g^2} f_{\mu\nu} f^{\mu\nu} - \frac{1 + z_B}{4} g^2 B_{\mu\nu} B^{\mu\nu} + \frac{1}{4} z_k B_{\mu\nu} \epsilon_{\mu\nu\rho\sigma} f^{\rho\sigma} + \cdots \right],
\]

where \( f_{\mu\nu} \) is the Abelian field strength \( f_{\mu\nu} := \partial_\mu a_\nu - \partial_\nu a_\mu \), \( B_{\mu\nu} \) is the rank 2 anti-symmetric tensor field and the dots denote ghost self-interaction terms and higher derivative terms [1]. The renormalization factors are given by

\[
\begin{align*}
z_f &= -\frac{22}{3} C_2(G) \frac{g^2}{16\pi^2} \ln \mu, \\
z_B &= +2C_2(G) \frac{g^2}{16\pi^2} \ln \mu, \\
z_k &= +4C_2(G) \frac{g^2}{16\pi^2} \ln \mu,
\end{align*}
\]

where \( \mu \) is the relevant scale of the theory and \( C_2(G) \) is the quadratic Casimir operator, \( C_2(G) \delta^{AB} = f^{ACD} f^{BDC} \). The APEGT [1] is an Abelian gauge theory with the gauge group \( H_e \times H_m \) where \( H_e = U(1)^{N-1} \) denotes the maximal torus subgroup of \( G \) (the subscript e is an abbreviation of electric) and \( H_m \) is the same group as

\(^2\) Although, in the paper [1], the actual calculation has been presented only for \( G = SU(2) \), the extension to \( G = SU(N) \) is straightforward, at least in the one-loop level, as demonstrated in [12].
$H_e$ but identified with the gauge group for the dual gauge field $b_\mu$ (the subscript m is an abbreviation of magnetic). Here the dual gauge field $b_\mu$ is obtained from the Hodge decomposition of $B_{\mu\nu}$ in 3+1 dimensions, $B_{\mu\nu} = b_{\mu\nu} + \tilde{\chi}_{\mu\nu}$, $b_{\mu\nu} := \partial_\mu b_\nu - \partial_\nu b_\mu$, $\tilde{\chi}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} (\partial_\alpha \chi_\beta - \partial_\beta \chi_\alpha)$. Consequently, the dual gauge field gets coupled with the monopole current $k_\mu$ as

$$\int \! d^4 x \, B_{\mu\nu} \tilde{f}^{\mu\nu} \equiv \int \! d^4 x \, \tilde{B}_{\mu\nu} f^{\mu\nu} = \int \! d^4 x \, b_\mu k_\mu + \ldots, \quad (5)$$

where the non-vanishing magnetic monopole current is deduced as violation of Bianchi identity for the Abelian part, $k_\mu := \partial^\nu \tilde{f}_{\mu\nu}$, where $\tilde{f}_{\mu\nu} := \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} f^{\rho\sigma}$.

The APEGT can give dual descriptions of the low-energy physics in the following sense. After eliminating $a_\mu$ field by integration, the APEGT yields the dual Ginzburg-Landau theory (DGL) with an action $S_{DGL}[b]$ which is written in terms of the dual gauge field $b_\mu$ alone. On the other hand, the APEGT is rewritten into another Abelian gauge theory with an action $S_{ZSM}[a]$ written in terms of the $a_\mu$ field alone after integrating out the $b_\mu$ field. In this sense APEGT is an interpolating theory between two theories just described (We can also write the monopole current action $S_m[k]$).

The duality of the APEGT is explicit in the action (1), since at the level of field

3 The theory was called the Zwanziger–Suzuki-Maeda (ZSM) theory, since it was obtained by Suzuki for $G = SU(2)$ and Maedan-Suzuki for $G = SU(3)$ [9] using the Zwanziger formalism [13]. However, the derivation of ZS theory was based on the ad hoc treatment of the off-diagonal parts under the assumption of Abelian dominance [8] and assumption of the monopole condensation which leads to the mass for the dual gauge field as a result of spontaneous breakdown of the dual gauge symmetry $H_m$ (dual Meissner effect or dual Higgs mechanism). These points are improved in [4].
equation, $B^{\mu\nu} = \tilde{f}^{\mu\nu}$, or $f^{\mu\nu} = \tilde{B}^{\mu\nu}$, up to a multiplicative constant (renormalization factor).

It is possible to discuss the confinement through the DGL theory which is derived from APEGT as a LEEGT of QCD. However, in order to conclude quark confinement in this scenario (dual superconductivity [7]), it is necessary to show that monopole condensation really takes place. The analytical studies on quark confinement so far have been based on the assumption of monopole condensation in QCD. Although it is actually shown to occur at least in the strong coupling region based on the energy (action)-entropy argument in the lattice regularization [1], we will adopt another path in this paper. In (1), $g$ denotes the bare coupling constant and hence the prefactors $z_f, z_B$ imply the renormalization effect. The calculated prefactor $z_B$ shows that the DGL theory $S_{DGL}[b]$ does not exhibit asymptotic freedom, as expected.

On the other hand, eliminating the field $B_{\mu\nu}$ from the interpolating theory (1) by a Gaussian integration, we obtain up to one-loop level

$$S = -\int d^4x \frac{1}{4g(\mu)^2} f_{\mu\nu} f^{\mu\nu} + \cdots,$$  \hspace{1cm} (6)

which is nothing but the Abelian gauge theory with a gauge group $H = U(1)^{N-1}$. However, a remarkable difference from the true Abelian gauge theory is that the coupling constant $g(\mu)$ runs according to the same renormalization group beta function as the original YM theory (see eq. (2)),

$$\beta(g) := \mu \frac{dg(\mu)}{d\mu} = -\frac{b_0}{16\pi^2} g(\mu)^3, \hspace{0.5cm} b_0 = \frac{11C_2(G)}{3} > 0.$$  \hspace{1cm} (7)

In this sense, this LEEGT reflects correctly the asymptotic freedom of the original YM theory. Indeed, the beta function (4) leads to the running coupling,

$$g(\mu)^2 = \frac{g(\mu_0)^2}{1 + \frac{b_0}{\pi^2} g(\mu_0)^2 \ln \frac{\mu}{\mu_0}},$$  \hspace{1cm} (8)

Note that this action is valid just before the monopole condensation occurs. After the monopole condensation, the action should be modified. We consider approaching the confinement phase from the deconfinement phase.
As the length scale is larger and larger, the running gauge coupling becomes larger and eventually goes to infinity in the one-loop level, whereas, at two-loop level, it converges to the infrared fixed point $\alpha = \alpha_* \quad [14]$ for $\alpha := \frac{g^2}{4\pi}$.

The purpose of this paper is to derive quark confinement in QCD based on the effective Abelian gauge theory \[6\] derived from the APEGT without assuming monopole condensation. We avoid to use the Zwanziger formalism which introduces an arbitrary unit four vector $n_\mu$. Hence we do not need to worry about the ambiguity how to choose the vector $n_\mu$. The same effect as monopole condensation is supplied with the vortex condensation based on the reformulation of the Abelian gauge theory \[3\] as a deformation of another TQFT \[3\]. This approach can greatly simplifies the proof (e.g., we can avoid the tedious instanton calculations) and avoid the ambiguity just mentioned. In this strategy, we are dealing with the Abelian gauge theory alone. Hence the doubt on the identification of the perturbative sector does not occur, since the Maxwell theory has no self-interaction among the gauge fields. This is another advantage of this approach. Moreover, this view gives an insight into other interesting problems, e.g. quark deconfinement transition and chiral symmetry breaking (CSB) discussed below. Technical details will be given elsewhere \[13\].

Now we show that the quark confinement is realized in the non-Abelian gauge theory in the sense that the expectation value of the Wilson loop obeys the area law for sufficiently large (non self-intersecting) Wilson loop. In other words, the dominant contribution of the static quark potential between a quark and an anti-quark is given by the linear potential in the large separation (In addition, there is the Coulomb potential and a constant part which expresses the self-energy contribution). \[5\] The information on the high-energy physics is incorporated into the APEGT through the running coupling constant \[8\]. This is quite important to prove quark confinement based on the effective Abelian gauge theory.

\[5\] It should be remarked that we do not conclude anything new about the short distance behavior of the Wilson loop or static potential, because the following argument is based on the APEGT which is meaningful only in the large distance or low-energy region as explained above.
In fact, if we regard the above theory (6) with the LEEGT of the YM theory, it turns out that the quark confinement in QCD follows immediately from the result of the previous paper [3], i.e. the existence of a confining phase in QED. In [3] it was shown that the four-dimensional Abelian gauge theory has a confining phase in the strong coupling region $g > g_c$ where the fractional charge is confined by the linear static potential. The steps are as follows. The theory (6) has the residual Abelian gauge invariance $H$. In order to quantize this theory, we must fix also the residual Abelian gauge degree of freedom. We adopt a manifestly covariant gauge fixing of Lorentz type. For a choice of gauge-fixing parameter $\beta = -2$ [3], the dimensional reduction occurs. Then the critical coupling $g_c$ is determined from the Berezinskii-Kosterlitz-Thouless (BKT) transition temperature $T_{BKT} = 8\pi$ of the two-dimensional $O(2)$ nonlinear sigma model (NLSM): $g_c = \pi$. The weak coupling region $g < g_c$ is the conformal (Coulomb) phase. The above steps are shown schematically as follows.

Furthermore, the evaluation of the Wilson loop is performed using the equivalence of $O(2)$ NLSM with (neutral) Coulomb gas, sine-Gordon model and the massive Thirring model in two dimensions. This fact will shed light on the confining string picture [16] in the low-energy region of QCD.

Note that the Abelian gauge theory (6) is different from the usual perturbative QED in the following points. First, the Abelian group $H$ is compact, since it was embedded in the compact (non-Abelian) group $G$, while the $U(1)$ gauge group in the perturbative QED is considered to be non-compact. Due to compactness or peri-
odicity, the Abelian gauge field can have the topologically non-trivial configuration (vortices). It is the periodicity why the fractional charge is confined while the integral charge is not confined. Second, the **true** Abelian gauge theory cannot be asymptotic free. That is to say, in the infrared limit $p \downarrow 0$ or long distance limit $\ell \uparrow \infty$, the effective charge $\alpha(p)$ goes to zero. Therefore, QED inevitably goes into the Coulomb phase. On the contrary, the running gauge coupling (8) of (6) increases monotonically in the length scale $\ell$ and eventually becomes larger than the critical coupling $\alpha_c$ with increasing $\ell$. Thus, in the low-energy region $\mu < \mu_c$ (or $\ell > \ell_c$) compatible with $\mu < O(m_A)$ (or $\ell > O(m_A^{-1})$), the theory (6) lies in the confining phase. The two critical scales $m_A, \mu_c$ are expected to be the same order of magnitude ($\sim 1\text{GeV}$). Thus the critical scale $\mu_c$ where the confinement sets in is given by

$$\alpha(\mu_c) = \alpha_c = \pi/4. \quad (9)$$

The above consideration should be compared with the previous work [2] where quark confinement has been derived by directly studying the non-Abelian gauge theory without considering the LEEGT. The above consideration supplements the previous treatment [2]. By the existence of the scale $\mu_c$, the separation of the gauge variable $A_\mu$ into $\Omega_\mu$ and $V_\mu$ performed in [2] has the following meaning. The background piece $\Omega_\mu$ represents the low-energy (or long-distance) mode $\mu < \mu_c$ while the perturbative piece $V_\mu$ the high-energy (or short distance) mode $\mu > \mu_c$ in QCD.

From the result of [3] on the Abelian Wilson loop, therefore, the above consideration gives another proof of quark confinement in the sense of area law of the diagonal Wilson loop claimed in [2]. Finally, the area law of the non-Abelian Wilson loop is shown as follows. The non-Abelian Wilson loop is rewritten as a path integral of the Abelian Wilson loop via a non-Abelian Stokes theorem as exemplified in the paper [4]. Therefore, the area law of the non-Abelian Wilson loop follows from that of the Abelian-projected Wilson loop which is written in terms of the Abelian (or diagonal) part alone, see [15].

In what follows, we consider a few applications of the above result. First, we
consider a SU(N) gauge theory with $N_f$ flavored fermions. If the massless fermions are present in the above consideration, the Wilson loop loses its meaning as a criterion of quark confinement. This is because the virtual pairs of a fermion and an antifermion are produced from the vacuum and they break the string of inter-quark flux leading to the perimeter decay of the Wilson loop. Then the static potential is saturated to be a constant in the asymptotic region. However, if the fermion mass $m_f$ is non-zero (even if small), the above criterion is still meaningful. The reason is as follows. Once the dynamical fermion mass $m_f$ is produced, the fermions decouple below this scale, leaving the pure gauge theory behind. Therefore, the linear potential $V(R) = \sigma R$ survives the inclusion of light quarks for the length scale $m_A^{-1} < \ell < m_f \sigma^{-1}$ ($m_f^{-1} \sigma < p < m_A$). \footnote{Here we have implicitly assumed that the string tension does not change substantially despite the inclusion of dynamical fermions in the relevant region.} If the theory with fermions is asymptotic free and hence the running coupling constant is monotonically increasing toward the infrared region, a criterion of quark confinement is given by $\alpha(\mu) > \alpha_c = \frac{\pi}{4}$ as long as the fermion is massive ($m_f \neq 0$).

On the other hand, it is known that the chiral symmetry is spontaneously broken above the critical coupling $\alpha > \alpha'_c$. For example, the SU(N) gauge theory has the critical coupling for the chiral symmetry breaking (CSB), $\alpha'_c = \frac{\pi}{3(N^2 - 1)}$, according to the naive Schwinger-Dyson (SD) equation \footnote{In the region $\alpha'_c < \alpha < \alpha_c$ (which is expected to occur for large $N$), the fermion is massive and the chiral symmetry is broken, but quark is not confined.}. This implies that the chiral symmetry is broken for not too large $N_f$ below a critical value $N_f^c$, $N_f < N_f^c$. This leads to a critical value of fermion flavors $N_f^c$ for the confinement as well as the CSB, which separates the deconfinement and chiral symmetric phase as follows. For $N = 2, 3$, the critical coupling $\alpha'_c$ of CSB is greater than or equal to the critical coupling $\alpha_c$ of quark confinement, $\alpha'_c \geq \alpha_c$ (In fact, for $N = 2$, $\alpha'_c = 4\pi/9$ and for $N = 3$ $\alpha'_c = \pi/4$ which happens to coincide with the critical coupling $\alpha_c$). In the region $\alpha_c < \alpha_* < \alpha'_c$ ($\alpha_*$: the infrared fixed point), although the fermion remains massless in the naive treatment without topological contribution, the vortex plasma
(monopole condensation) can induce the CSB which causes quark confinement. It is also known that monopole condensation enhances the CSB \cite{17}. If this observation is correct, the CSB and confinement can occur simultaneously.

We can estimate the critical number of flavors $N_f^c$ for the deconfinement/confinement transition as follows. For SU(N) QCD with $N_f$ fermions in the fundamental representation, the renormalization group beta function is given by

$$\beta(\alpha) := \mu \frac{\partial \alpha}{\partial \mu} = -b \alpha^2(\mu) - c \alpha^3(\mu) \cdots,$$

with the first two coefficients,

$$b = \frac{1}{6\pi} (11N - 2N_f), \quad c = \frac{1}{24\pi^2} \left( 34N^2 - 10NN_f - 3\frac{N^2-1}{N}N_f \right).$$

Note that, above the value $N_f = (11/2)N$ ($b < 0, c < 0$) ($N_f = 11$ for $N = 2$ and $N_f = 16\frac{1}{2}$ for $N = 3$), the theory is not asymptotic free, i.e., the beta function is positive for any $\alpha$, $\beta(\alpha) > 0$. Below this value ($b > 0, c < 0$), the infrared fixed point $\alpha_*$ exists where the fixed point is given by $\alpha_* = -b/c > 0$. From this relation, we have $\alpha_* = \frac{4\pi(11N^2-2NN_f)}{(13N^2-3)N_f-34N^2}$, so that the above relation is meaningful only when $N_f > 34N^3/(13N^2 - 3)$, since $\alpha_* > 0$ only if this condition is satisfied (otherwise, $\alpha_* < 0$). Note that $\alpha_*$ increases monotonically as the flavor $N_f$ is decreasing and $\alpha_* \uparrow +\infty$ as $N_f \downarrow 34N^3/(13N^2 - 3)$. For large $N$, $N_f$ approaches $(34/13)N$. For $N_f < 34N^3/(13N^2 - 3)$, $\beta(\alpha) < 0$ and the theory is asymptotic free (neglecting higher orders $O(\alpha^5)$). This argument gives the lower bounds, $N_f^c \geq 5.55$ for $N = 2$ and $N_f^c \geq 8.05$ for $N = 3$. The relationship between the critical flavor and the critical coupling for the deconfinement/confinement transition is obtained by putting $\alpha_* = \alpha_c$ as

$$N_f^c = \frac{2(22\pi + 17\alpha_c N)N^2}{8\pi N + \alpha_c(13N^2 - 3)}.$$

The critical coupling $\alpha_c = \pi/4$ leads to $N_f^c = 8.64$ for $N = 2$ and $N_f^c = 11.9$ for $N = 3$, under the assumption that the CSB has already occurred. Therefore, the critical number of flavors is estimated: $8 \geq N_f^c \geq 6$ for $N = 2$ and $12 \geq N_f^c \geq 8$ for $N = 3$. These results are consistent with the results of lattice simulations \cite{18}.
Here we have used the result of SD gap equation for the CSB \[14\]. However, in order to really understand the relationship between confinement and chiral symmetry breaking, it is desirable to derive both of them on equal footing from QCD. This will be given in a forthcoming paper \[15\].

Next, we consider a finite-temperature deconfinement transition briefly. The effective coupling at finite temperature behaves as \(\alpha_{\text{EFF}}(T) \sim 1/(b_0 \log(T/\Lambda))\). If the temperature \(T\) is sufficiently high so that \(\alpha_{\text{EFF}}(T) < \alpha_c\), quark confinement can not occur based on the above arguments. Due to monotonicity of \(\alpha_{\text{EFF}}(T)\), therefore, the confinement–deconfinement transition point is determined by the equation, \(\alpha_{\text{EFF}}(T) = \alpha_c = \pi/4\).

The above strategy for proving quark confinement can be applied to the non-Abelian gauge theory defined in the \(D = d + 1\) dimensional space time for arbitrary \(D > 2\). For example, if we apply the above method to the 2+1 dimensional non-Abelian gauge theory, the 2+1 dimensional TQFT is, after the dimensional reduction, reduced to the 0+1 dimensional NLSM which is not a model of the quantum field theory, but that of quantum mechanics. It is nothing but the plane rotator model in quantum mechanics. In quantum mechanics, the phase transition does not occur and the theory lives in one phase. The phase should correspond to the confining phase of the 2+1 dimensional non-Abelian gauge theory. However, the Wilson loop can not be defined in 0+1 dimensional spacetime. We will need other criterion of quark confinement.

Obviously, it is important to extend our argument to the two-loop level by deriving the APEGT of QCD at two-loop level \[13, 12\]. Furthermore, it is desirable to extend the arguments given for \(G = SU(2)\) in this paper to \(G = SU(N), N > 2\). In a subsequent paper \[15\] we will deal with the generalized BKT transition corresponding to \(G = SU(N)\). Moreover, the area law of the non-Abelian Wilson loop will be derived based on fermionization of generalized sine-Gordon model \[15\].

In \[2\], quark confinement has been explained as a result of instantons in two-
dimensional NLSM. However, this is not inconsistent with the vortex picture for the effective Abelian gauge theory given in this paper, since it turns out that an instanton is composed of a vortex and an anti-vortex. The Abelian and non-Abelian quark confinement will be understood in a unified treatment [13].

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