Study of mode localization in an umbrella antenna with tensile ropes

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Abstract. The umbrella antenna is a typical periodic structure in nature, and as its ribs are made of CFRP (carbon fibre reinforced plastic), the presence of irregularities in dimension, stiffness and mass is unavoidable during manufacturing process, which will result in mode localization of the antenna. Excessive mode localization will damage the reflector precision and endanger safety of the antenna, so how to suppress mode localization of the ribs is important for umbrella design. Combined with the development of an umbrella antenna, mode localization of the structure is studied. Firstly, dynamic equations of an umbrella antenna with tensile ropes are derived. Then, based on the ribs stiffness test results, further mode localization simulation and test are conducted. The final simulation and test results show that, when stiffness of the tensile ropes reaches to 3N/mm, the antenna structure will change from a weak coupling system to a strong coupling system and mode localization of the ribs are effectively suppressed.

1. Introduction
The umbrella antenna also named radial rib deployable antenna, is one of the mostly used types of large aperture antennas. The main structure of the umbrella antenna is composed of stiff ribs which distribute uniformly about the centre axis, the metal reflection mesh and tensile rope network. As a typical periodic structure, the deformation or vibration of local mesh will damage the regularity of the whole reflector, and reduce RF performance of the antenna. However, the presence of irregularities in dimension, stiffness and mass of the ribs is unavoidable during manufacturing process, which will further result in mode localization of the antenna. Mode localization of the ribs would cause local vibration of the mesh and damage surface stability of the antenna, and in the worst cases, if a single rib is in a continuous vibration status under in-orbit disturbance, reliability and safety of the antenna will be severely reduced.

As an unavoidable problem for periodic structure dynamic design, mode localization has been widely studied both in theory and practice [1-4]. Levine-West [5] has finished mode localization simulation and test of a twelve flexible beams structure, the research was focused on the influence of coupling stiffness of beams and excite forces on the mode localization, and the test results verified the conclusion that a weak coupling structure trends to induce mode localization. Bendiksen [6,7] conducted numerical simulation of a large space structure, and the simulation results shown that for the shape and dynamic control design of large spaces structure that were mode concentrated, the dispersal of subsystem should be carefully taken into consideration. Mode disorder of two systemically mounted solar panels were studied by S Zhang [8], and it was found that compared with difference of deployable mechanism stiffness, deviation of the solar panel length was much more probably to result in mode
disorder. XQ Liu\footnote{9,10} has studied mode localization of a wrapping rib antenna and carried out active vibration control of the antenna structure with or without mode localization.

Figure 1 shows sketch of a high precision umbrella antenna, which consists of sixteen periodically distributed ribs, metal reflector mesh and tensile ropes network. The ribs are made of CFRP, and the dimension is gradually changed for high stiffness consideration. The metal reflector mesh and tensile ropes network are pre-tensioned, and the surface precision could be modified by changing tensile forces of the ropes.

![Sketch of a high precision umbrella antenna](image1)

**Figure 1. Sketch of a high precision umbrella antenna**

2. **Structure dynamic modeling**

The first modal shape of the rib is bending flexions about Z axis in the plane, which corresponds to torsion mode of the reflector. The second modal shape of the rib is flexions about the deployment axis, which corresponds to folding mode of the reflector. As the rib has to resist pre-tension of tensile ropes which causes folding of the reflector, its second modal frequency is usually designed to be much higher than the first modal frequency. Figure 2 is the simplified sketch of the antenna structure, all the ribs can be simplified to be cantilever beams, and effect of tensile ropes on the ribs is equivalent to be a spring at free end of the beam.

![Simplified sketch of the antenna structure](image2)

**Figure 2. Simplified sketch of the antenna structure**

2.1. **Stiffness equivalence of tensile ropes**

Several loops of tensile ropes are assigned along the rib, and each loop of rope has its own stiffness, which is mainly determined by the length of the rope. For model simplification, stiffness of the whole rope loops is condensed to be stiffness of the most outer loop ropes with the method shown in Figure 3.

![Method for tensile ropes stiffness equivalence](image3)

**Figure 3. Method for tensile ropes stiffness equivalence**

Suppose the rib rotates $\theta$ under tension forces of the tensile ropes, the overall moment $M$ at the fixed end of the rib can be expressed as:

$$M = \sum_{j=1}^{n} k_j x_j L_j$$
Where $kj$, $xj$ and $Lj$ refer to stiffness of the jth loop tensile ropes, deflection at the jth connection point and distance from jth connection point to the fixed end of the rib.

The corresponding equivalent force at the free end of the rib is:

$$F = \frac{M}{L} = \sum_{j=1}^{n} \frac{k_j L_j^2 \theta}{L}$$

(2)

Deflection at the free end of the rib is $x = L \theta$, so the overall stiffness of whole rope loops can be simplified as:

$$k = \frac{F}{x} = \sum_{j=1}^{n} k_j \frac{L_j^2}{L^2}$$

(3)

### 2.2. Structure dynamic equation

For rib $B_k$, its displacement under vibration can be expressed as:

$$\Phi(x) = [\phi_1(x), \phi_2(x), ..., \phi_n(x)]$$

Where $\Phi(x) = [\phi_1(x), \phi_2(x), ..., \phi_n(x)]$ and $\xi_k(t) = [\xi_k1(t), \xi_k2(t), ..., \xi_kn(t)]$ represent modal shape vector and modal coordinate vector of the cantilever beam respectively.

Kinetic energy $T_{bk}$ and Strain energy $U_{bk}$ of rib $B_k$ can be expressed as:

$$T_{bk} = \frac{1}{2} \int_0^{L_k} \rho_k A_k \left( \frac{\partial^2 u_{bk}}{\partial t^2} \right)^2 dx$$

$$U_{bk} = \frac{1}{2} \int_0^{L_k} E_i I_k \left( \frac{\partial^2 u_{bk}}{\partial x^2} \right)^2 dx$$

(5)

(6)

Suppose the length of the adjacent ribs $B_k$ and $B_{k+1}$ is $L_k$ and $L_{k+1}$, and the free end coordinate at local coordinate system is $(x_k, y_k)$ and $(x_{k+1}, y_{k+1})$:

$$\begin{align*}
&x_k = L_k + u(L_k, t) = \Phi(L_k) \xi_k(t) \\
&x_{k+1} = L_{k+1} + u(L_{k+1}, t) = \Phi(L_{k+1}) \xi_{k+1}(t)
\end{align*}$$

(7)

By coordinate transformation:

$$\begin{align*}
X_k &= L_k \cos \phi \cdot \Phi(L_k) \xi_k(t) \sin \phi \\
Y_k &= L_k \sin \phi \cdot \Phi(L_k) \xi_k(t) \cos \phi \\
X_{k+1} &= L_{k+1} \cos (\phi + \theta) \cdot \Phi(L_{k+1}) \xi_{k+1}(t) \sin (\phi + \theta) \\
Y_{k+1} &= L_{k+1} \sin (\phi + \theta) \cdot \Phi(L_{k+1}) \xi_{k+1}(t) \cos (\phi + \theta)
\end{align*}$$

(8)

Tensile rope loops can be simplified to be a spring with stiffness $k$. As the ropes are very light, the kinetic of energy can be omitted, and the potential energy $U_{sk}$ under tension length $d$ is:

$$U_{sk} = \int_0^d k \cdot d \cdot dx = \frac{1}{2} kd^2$$

(9)

Where the tension length $d$ can be substituted by the free end coordinate $(x_k, y_k)$ and $(x_{k+1}, y_{k+1})$ as:

$$d = \sqrt{\left( X_k - X_{k+1} \right)^2 + \left( Y_k - Y_{k+1} \right)^2}$$

(10)

The potential energy $U_{sk}$ can be further expressed as:

$$U_{sk} = \frac{1}{2} k \left[ \ell_{k-1}^2 + 2 \ell_k^2 + \ell_{k+1}^2 - 2 \ell_k L_{k-1} \cos \theta - 2 L_{k-1} L_k \cos \theta \right]$$

$$+ k \left[ L_k \Phi(L_{k-1}) \sin \theta \cdot \xi_{k-1}(t) + \left[ L_k \Phi(L_k) \sin \theta \cdot L_{k-1} \Phi(L_k) \sin \theta \cdot \xi_k(t) - L_k \Phi(L_{k-1}) \sin \theta \cdot \xi_{k-1}(t) \right] \right]$$

$$+ \frac{1}{2} k \left[ 2 \Phi(L_{k-1}) \cdot \xi_{k-1}(t)^2 + 2 \Phi^2(L_k) \cdot \xi_k(t)^2 + 2 \Phi(L_{k+1}) \cdot \xi_{k+1}(t)^2 \right]$$

$$- 2 \Phi(L_{k-1}) \Phi(L_k) \cos \theta \cdot \xi_{k-1}(t) \xi_k(t) - 2 \Phi(L_k) \Phi(L_{k+1}) \cos \theta \cdot \xi_k(t) \xi_{k+1}(t)$$

(11)

The overall kinetic energy of the structure is:
The overall potential energy of the structure is:

$$U = \sum_{k=1}^{n}(U_{uk} + U_{ad})$$

By substitute T and U into the Lagrange equation:

$$\frac{d}{dt} \left[ \frac{\partial (T - U)}{\partial \xi^*_k} \right] - \frac{\partial (T - U)}{\partial \xi_k} = 0$$

After simplification, the equation comes to be:

$$m_k \ddot{\xi}_k + k_{kk} \xi_k + k_{k,1} \xi_{k+1} = 0$$

Let $\xi_k = \xi_k e^{i\omega t}$ and the equation becomes an eigenvalue problem:

$$K - \omega^2 M = 0$$

Elements of mass matrix M and stiffness matrix K can be defined as:

$$m_{ij} = 0 \quad i \neq j, \quad i, j = 1 \sim N$$

$$m_{ii} = \int \rho A \Phi^T(x) \Phi(x) dx \quad i = j, \quad i, j = 1 \sim N$$

$$k_{jj} = \int E J \Phi^T(x) \Phi(x) dx + k \left[ 2\Phi^2(L_j) - \Phi(L_j) \Phi(L_{j+1}) \cos \theta - \Phi(L_j) \Phi(L_{j+1}) \cos \theta \right] \quad j = 1 \sim N$$

$$k_{k,j} = \int \Phi^2(L_{j+1}) - \Phi(L_j) \Phi(L_{j+1}) \cos \theta \quad j = 1 \sim N$$

3. Stiffness test of ribs

During AIT (assembly, integration and test) period, stiffness test of the ribs is conducted. As the ribs have to resist pulling forces from tensile ropes at connection points which can be approximately treated as a uniform load, so a “uniformly-loaded but centrally-evaluated” method as shown in Figure 4 is adopted.

![Figure 4. Test plan of rib stiffness](image)

The rib is mounted on a stiff enough fixture, with concentrate mass $m_k$ at each connection point $k$, so the overall moment at the fixed end of the rib is:

$$M = \sum_{k=1}^{n} m_k g L_k$$

The corresponding equivalent force at the free end of the rib is:

$$F = \frac{M}{L} = \sum_{k=1}^{n} \frac{m_k g L_k}{L}$$

Then bending stiffness of the rib is calculated by the equation:
\[ K = \frac{F}{d} = \sum_{k=1}^{n} \frac{m_k g L_k}{L d} \]  

Table 1 summarizes the stiffness test results of the sixteen ribs used in the umbrella antenna, the stiffness distributes from 2.27N/mm to 2.88N/mm, and Figure 5 shows distribution map of rib stiffness, the distribution does not follow the normal distribution strictly, which may indicate that there still have possibilities for the manufacture process improvement.

Table 1. Rib stiffness test results

| Ribs ID | Bending Stiffness /Nmm\(^{-1}\) |
|---------|----------------------------------|
| 1       | 2.51                             |
| 2       | 2.27                             |
| 3       | 2.59                             |
| 4       | 2.68                             |
| 5       | 2.63                             |
| 6       | 2.39                             |
| 7       | 2.70                             |
| 8       | 2.65                             |
| 9       | 2.45                             |
| 10      | 2.64                             |
| 11      | 2.58                             |
| 12      | 2.69                             |
| 13      | 2.88                             |
| 14      | 2.62                             |
| 15      | 2.68                             |
| 16      | 2.76                             |

Figure 5. Distribution map of rib stiffness

4. Mode localization of the antenna

Figure 6 shows mode localization simulation results of the whole antenna structure in ideal status without tensile ropes, which means that all the ribs are the same in dimension (share the same length L, width B and height H) and material property (share the same elastic modulus E). Figure 6a gives distribution of the first modal frequencies, which shows that all the sixteen ribs share the same first modal frequency of 4.79Hz. Figure 6b illustrates distribution of the normalized modal amplitude at free end of the sixteen ribs, which shows that the first modal shape is totally the bending mode of No 1 rib, with zero amplitude at other ribs. As there is no tensile ropes in the antenna to pull the ribs together, so modal characteristics of the ribs are totally independent, and the structure is a no-coupling system.

Figure 7 shows mode localization simulation results of the whole antenna structure in discrete status without tensile ropes, which means that the dimension and material property are different for each rib. As shown in Figure 7a and Figure 7b, modal frequencies of the sixteen ribs are no longer the same and the first modal shape act as the bending mode of No 16 rib.
Figure 7. Modal characteristic of distributed status ribs without tensile ropes

Figure 8 compares the normalized modal amplitude at the free end of the sixteen ribs while the equivalent stiffness of tensile ropes is 0.1N/mm, 1N/mm and 3N/mm. As stiffness of tensile ropes increases, modal frequencies of the ribs also increases, and mode overlapping is suppressed, which is beneficial for the structural safety. When the tensile ropes stiffness reaches to 3N/mm, amplitude and phase of the first mode of the sixteen ribs tends to be the same, which mean that all the ribs vibrate to the same amplitude at the same time and mode localization is successfully suppressed. With the stiffness assistance of tensile ropes, the antenna structure changes from a weak coupling system to a strong coupling system.

Figure 8. Antenna modal characteristics with different tensile ropes stiffness

5. Modal test verification
Modal test of the sixteen ribs umbrella antenna with tensile rope stiffness 3N/mm is conducted to obtain the first modal frequencies and vibration amplitudes, which is used for further evaluation of the structural mode localization. Figure 9 shows assignment of the sensors for the test, where eight accelerometers are mounted on the antenna alternately to measure vibration amplitude and frequency of the ribs in the first in-plane bending mode.

Modal frequency response curves of the eight ribs are shown in Figure 7. All the curves reach peak values at 4.75Hz, which mean that the first modal frequencies of the eight ribs are the same. The frequency response amplitudes of the eight ribs fit well and no apparent mode localization was found.

Figure 9. Assignment of sensors    Figure 10. Modal test results
Table 2. Ratio of normalized FRF amplitudes

| Ribs ID | Normalized FRF Amplitude | Ribs ID | Normalized FRF Amplitude |
|---------|--------------------------|---------|--------------------------|
| 1       | 0.85                     | 9       | 0.81                     |
| 3       | 0.87                     | 11      | 0.87                     |
| 5       | 0.94                     | 13      | 0.90                     |
| 7       | 1                        | 15      | 0.82                     |

6. Conclusion

Mode Localization of a high precision umbrella antenna with tensile ropes is studied, dynamic equation of the antenna with tensile ropes is derived and further simulation and test are conducted based on ribs stiffness test results, and the simulation and test results show that:

a) As a result of the discreteness of composite materials, stiffness test results of the ribs do not converge very well, and the distribution do not follow the normal distribution strictly, which may indicate that there still have possibilities for the manufacture process improvement.

b) Mode localization of the whole antenna structure is largely affected by the equivalent stiffness of the tensile ropes, and if the stiffness reaches to 3N/mm, the structure will convert from a weak coupling system to be a strong coupling system.

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