Contour Error Control of X-Y Platform Based on Nominal Model in Polar Coordinate System

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Abstract. In the case of the x-y linear motor platform, the contour error model of the nonlinear dynamic and curve trajectory is relatively complicated, and the traditional control system does not have the problem of systematic adjustment of parameters, and the precision of the contour is affected when machining. In this paper, the dynamic model of the system is modeled under the polar coordinate system, and the polar coordinate contour error nonlinear model of the linear motor x-y platform is obtained. At the same time, the nonlinear model application is based on the theory of sliding mode control, which makes the error of the system close to zero and can have good robustness. The theoretical derivation and simulation platform verification results show that the control system designed in this paper can improve the precision of the contour machining of the linear motor x-y platform.

1. Introduce
NC machine tools are developing to the direction of precision, high speed and compounding. The X-Y platform of linear servo motor has been widely used in high-speed machining equipment, such as fast response, accurate positioning and high reliability.

Many scientists have applied a large number of modern control theories, such as robust control, variable structure control, model reference adaptive control, neural network control and genetic algorithm, to the X-Y platform of linear motor and achieved good results. However, these traditional control methods are based on the Cartesian coordinate system. These methods have some shortcomings. Firstly, the feed-forward, feedback and cross-coupling controllers consider the case of each axis separately. The coupling relationship between the two axes is not clear enough, and there is no systematic method to design a comprehensive controller. Secondly, for complex paths, cross-coupling is used\cite{1}. The linear approximation method is used to reduce the order of the controller, and the traditional method based on the linear model expressed by the transfer function is difficult to be applied to nonlinear systems.
In view of the above shortcomings, Shyh-Leh Chen and others put forward the idea of contour control for two-axis system in polar coordinate system[2]. This paper adopts this idea and designs a sliding mode controller based on nominal model to enhance the robustness of the system[2]. The theoretical derivation and simulation results show that the control system designed in this paper can effectively improve the contour machining accuracy of linear motor X-Y platform.

2. Nonlinear Model of Polar Coordinate Contour Error for X-Y Platform

2.1. Mathematical Model of X-Y platform

The dynamic equations of the two-axis linear motor system are defined as follows:

\[ M \ddot{q} + V(q, \dot{q}) + \Delta(q, \dot{q}) + dt = T \]  

Where M is the positive definite mass matrix of 2x2; q is the matrix of 2x1, representing the displacement of X-Y axis; V(q, \dot{q}) is viscosity friction coefficient; \Delta(q, \dot{q}) is the uncertain part of modeling; dt is the external interference to the system; T is the input of the control signal.

2.2. The X-Y platform Model of Linear Motor based on Polar coordinates

For X-Y biaxial systems, the traditional control methods are the contour errors defined in the Cartesian coordinate system, as shown in figure 1, which is the path distance from the actual position to the desired position:

\[ \varepsilon(t) = \sqrt{x(t)^2 + y(t)^2 - r_d} \]  

(2)

The main cause of contour error is the dynamic mismatch of two axes. From formula (2) we can see that the contour error originates from the real-time dynamic position of XY axis, so the controller for contour error should not be designed independently. The controller is designed based on the definition of contour error and the real time XY axis dynamic position information. The method to solve this problem is to model the contour error in polar coordinate system.

In polar coordinate system, the contour error of trajectory can be expressed as radial error \( \varepsilon_r(t) \).

\[ \varepsilon_r(t) = r(t) - r_d(t) \]  

(3)

Where \( r(t) \) is the radius of the actual trajectory circle; \( r_d(t) \) is the radius of the orbit circle in it.

\[ \text{Figure 1. Tracking error and Contour error.} \]

However, for the curve of the non-positive circle trajectory, the angle error \( \varepsilon_\theta(t) \) is also needed to represent the contour error completely.

\[ \varepsilon_\theta(t) = \theta(t) - \theta_d(t) \]  

(4)
Where \( \theta(t) \) is the angle of the actual track circle; \( r_d(t) \) is the angle of the orbit circle.

In this paper, taking ellipse as an example, we discuss the case of machining nonlinear trajectory of linear motor X-Y platform.

\[
\begin{align*}
\theta(t) &= r_d(t) \\
\end{align*}
\]

Figure 2. Model of the X positioning stage.

The expression of the trajectory converted to polar coordinate system is: \( x(t) = r \cos \theta, y(t) = r \sin \theta \).

Owing to:

\[
q = \begin{bmatrix}
x(t) \\
y(t)
\end{bmatrix} = \begin{bmatrix}
r \cos \theta \\
r \sin \theta
\end{bmatrix}
\]

So:

\[
\begin{align*}
\dot{q} &= \begin{bmatrix}
\cos \theta & -r \sin \theta \\
\sin \theta & r \cos \theta
\end{bmatrix} \begin{bmatrix}
\dot{r} \\
\dot{\theta}
\end{bmatrix} \\
\end{align*}
\]

Order:

\[
R = \begin{bmatrix}
\cos \theta & -r \sin \theta \\
\sin \theta & r \cos \theta
\end{bmatrix}, \quad \dot{p} = \begin{bmatrix}
\dot{r} \\
\dot{\theta}
\end{bmatrix}, \quad \ddot{p} = \begin{bmatrix}
\ddot{r} \\
\ddot{\theta}
\end{bmatrix}
\]

So there is

\[
\ddot{q} = R \ddot{p} + \dot{\theta} S \dot{p}
\]

Order: \( A = \dot{\theta} MR^{-1} M^{-1} S + MR^{-1} M^{-1} VR; B = MR^{-1} M^{-1} \)

From the formula (1), (5), (6) and (7), the dynamical equations of the system in polar coordinate system can be obtained:

\[
M \ddot{p} + A \dot{p} + B dt = B T
\]

3. Controller Design of Contour Error of X-Y Platform

3.1. The structure of control system

As can be seen from figure 3, the control system consists of two controllers, one of which is the sliding mode controller of the actual system, and the other is the controller for the nominal model, which is implemented in polar coordinates:

\[
r \rightarrow r_d, \theta \rightarrow \theta_d
\]

Thus, In the World Coordinate System:

\[
x(t) \rightarrow x_d(t), y(t) \rightarrow y_d(t)
\]
3.2. Design of Contour error Controller

From the kinetic equation (9), the following results can be obtained:

\[ M\ddot{P} + A\dot{P} = T + w \]  \hspace{1cm} (11)

Where \( T = BT, w = -Bdt \).

\( P_d \) is taken as input instruction and \( e = P_d - P \) is used as error signal. The sliding surface is designed as follows:

\[ S = \dot{e} + Ce, C = \text{diag}(c_1, c_2, \ldots c_n), c_i > 0 \]  \hspace{1cm} (12)

Defining Lyapunov function:

\[ V = \frac{1}{2} S^T MS \]  \hspace{1cm} (13)

There will be: \( \dot{V} = \frac{1}{2} S^T \dot{MS} + S^T M \dot{S} = \frac{1}{2} S^T (\dot{M} - 2A)S + S^T AS + S^T M \dot{S} \)

Because \( M \) is a constant positive definite matrix, therefore \( \dot{M} = 0 \) and:

\[ \dot{V} = S^T M \dot{S} \]  \hspace{1cm} (14)

From the expression (13):

\[ \dot{S} = \dot{\varepsilon} + C\dot{\varepsilon} \]  \hspace{1cm} (15)

At the same time:

\[ \dot{\varepsilon} = \dot{P}_d - \dot{P} \]  \hspace{1cm} (16)

Bring (12) (16) (17) into (15), we can get : \( \dot{V} = S^T M \dot{S} = S^T [M(\ddot{P}_d + Ce) + A\dot{P} - T - w] \).

Accordingly, the control law is designed as:

\[ T = M_o(\ddot{P}_d + Ce) + A_o\dot{P} - w_o + \Gamma \text{sgn}(S) \]  \hspace{1cm} (17)

Where \( M_o, A_o \) and \( w_o \) are nominal values of \( M, A \) and \( w \) respectively:

\( \Delta M = M - M_o, \Delta A = A - A_o, \Delta w = w - w_o \)

Thereupon: \( \dot{V} = S^T [\Delta M(\ddot{P}_d + Ce) + \Delta A\dot{P} - \Delta w] - \Gamma |S| \)

Where \( \Gamma = \text{diag}(\gamma_1, \gamma_2, \ldots \gamma_n), (\gamma_i > 0) \).

Taking

\[ \gamma_i |\Delta M|_{\text{max}} (\ddot{P}_d + Ce) + |\Delta A|_{\text{max}} \dot{P} - |\Delta w|_{\text{max}} \]  \hspace{1cm} (18)

So, we can get \( \dot{V} \leq 0 \). When and only as \( S = 0, \dot{V} = 0 \)
When \( t \to \infty \), \( S \to 0 \).

![Figure 4. Linear motor platform control block diagram](image)

### 4. Controller Simulation And Analysis

In this paper, the linear motor platform of Huimusen BJSM-III series is simulated. The parameters are as follows table 1.

| Parameters                        | Symbol | X-Axis | Y-Axis |
|-----------------------------------|--------|--------|--------|
| Full Thrust (N)                   | \( F_p \) | 240.0  | 625    |
| Sustained Thrust (N)              | \( F_c \) | 76     | 276    |
| Constant of the machine (\( \sqrt{M} \)) | \( K_m \) | 7.8    | 32.0   |
| Maximum Speed (m/s)               | \( v \) | 10     | 6.6    |
| Maximal Acceleration (m/s²)       | \( a \) | 580    | 150    |
| Reverse Potential Constant(Vpeak/m/s) | \( K_r \) | 27.5   | 51.4   |
| Thrust Constant (N/Arms)          | \( K_f \) | 33.7   | 59.8   |
| Mover mass (kg)                   | \( M \) | 3.7    | 8.0    |

The instruction path is an "8" glyph, and the two-axis input instruction is:

\[
x_{d1} = \sin \theta \cos \left( \cos \theta \right), x_{d2} = \sin \theta \sin \left( \cos \theta \right)
\]

Interference is set to \( \lambda \text{ sgn}(s) \), \( \lambda = 10 \).

The sliding mode coefficient is \( c_1 = 20; c_2 = 30; M_0 = 0.8M, A_0 = 0.8A, w_0 = 0.8w \).

The response curves of each axis are obtained by matlab simulation as follows:

![Figure 5. Actual output and Ideal input of X Y](image)
As can be seen from figure 5, the actual signal quickly overlaps with the ideal signal. From figure 6, we can see that the contour error of the X-axis in polar coordinate system is basically maintained in the range of $\pm 1.6 \times 10^{-5}$.

By calculating the mean value of contour error, it is obtained that the average contour error between actual trajectory and instruction trajectory is $3.7 \times 10^{-6}$ m, which is compared with that in reference [8-9] as follows:

| Control Method | Contour control based on Cross-Coupling Control based on Contour error Vector estimation | Contour control based on coordinate transformation | Contour error Control of X-Y platform based on nominal Model in Polar coordinate system |
|----------------|------------------------------------------------------------------------------------------|-----------------------------------------------|----------------------------------------------------------------------------------|
| Input          | 100mm                                                                                     | 100mm                                         | 100mm                                                                           |
| Average Error  | $5 \times 10^{-3}$                                                                         | $4 \times 10^{-4}$                           | $3.7 \times 10^{-6}$                                                            |
Figure 8. Instruction Trajectory and Actual Trajectory

From figure 9, we can see that actual trajectory and instruction trajectory is mainly coincidence.

The simulation results show that the contour error control of the X-Y platform based on nominal model in the polar coordinate system can not only effectively guarantee the robustness of the servo system, but also can effectively improve the precision of the contour error, and the design of the controller is relatively simple.

5. CONCLUDING

In this paper, the contour error control system of X-Y platform in polar coordinate system is used to realize the synchronous tracking control of the two-axis linear motor platform, and the sliding mode control based on the nominal model is adopted, which not only can effectively guarantee the robustness of the servo system. At the same time, it can improve the precision of contour error effectively, and the controller design is relatively simple. Simulation results show that the designed controller can improve the contour control accuracy of linear motor.

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