Distributed economic dispatch for power generation with time-varying loads and external disturbances

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Abstract

The distributed economic dispatch problem for power generation in electricity grids is studied. A simple centralised algorithm is designed to obtain the solution of the dispatch problem without the external disturbances. Inspired by the centralised algorithm, a distributed algorithm is presented using the distributed average tracking algorithm. As far as we know, the proposed algorithm is the first one that is capable of time-varying loads and external disturbances while the tracking errors are asymptotic convergent. Simulations on the IEEE 14-BUS system are provided to validate the theoretical results.

1 | INTRODUCTION

The economic dispatch problem has been widely investigated in power systems [1–3]. The overall objective is to allocate the power generation while meeting the constraints of total load demand and minimising the total generation cost function. The traditional economic dispatch algorithms are centralised mainly based on the gradient search method [4–6] and the particle swarm optimisation [7]. For the centralised algorithms, a central unit is used to collect all the network information and calculate the optimal solution.

With the development of distributed control [8, 9] and future smart grids [10–14], the study of the distributed economic dispatch problem becomes increasingly attractive in the past few years. The problem is first solved by some discrete-time algorithms. In particular, the authors of [10] proposed an algorithm composed of a consensus term and an innovation term such that the total load require is satisfied. The authors of [11] presented an economic dispatch algorithm using a leader–follower consensus method where the leader collects the mismatch between the generation and load demands. Based on [11], a two-level consensus method was used in [12] to avoid the requirement of a leader. The case of the directed communication network was considered in [13] and [14]. In particular, the authors of [13] designed a ratio consensus-based algorithm with two linear iterations, and an estimation was presented in [14] to obtain the mismatch when all the agents are participated. The optimal coordination problem was considered in [15] for distributed energy resources, including distributed generators and energy storages. An algorithm based on the push-sum and gradient method was presented to solve the problem under the condition that the time-varying directed communication network is uniformly jointly strongly connected. Then the authors of [16] considered the general convex functions for the economic dispatch problem. On the other hand, the continuous-time algorithms were also studied. For example, an initialisation-free algorithm was presented in [17] based on a proportional-integral consensus and projection-based method.
The authors of [18] proposed a continue-time distributed algorithm to solve the problem via saddle point dynamics and consensus algorithms.

The distributed economic dispatch strategy has been widely used in many applications, for example, AC microgrids [19–21]. In [19], a novel control framework composed of a distributed economic dispatch module and a cooperative control module is presented to optimise the active power output of multiple generators in a distributed network and enhance the redundancy and the plug-and-play capability in microgrids. To fill the gap among the three control layers in the hierarchically controlled islanded microgrid, the authors of [20] proposed a distributed joint operation method to address the coordination problem and realise an optimal and stable operation of the system. Recently, a distributed event-triggered secondary control method is designed in [21] to deal with the economic dispatch and frequency restoration control for droop-controlled AC microgrids. The proposed control strategy can ensure economic dispatch and frequency restoration control at the same time.

Nonetheless, the results in the above literature focus on the time-invariant loads, but the time-varying loads are commonly encountered in many practical applications. There are few results on time-varying loads [22–24]. The authors of [22] presented a distributed coordination algorithm which was able to track time-varying loads. In [23], a finite-time economic dispatch algorithm was designed. The algorithm works properly with time-varying loads since the finite-time convergence to the optimal generation is achieved. Furthermore, a novel economic dispatch algorithm was developed in [24] based on the singular perturbation theory and a penalty function. Note that the algorithms in the above literature cannot guarantee the asymptotical convergence to the optimal value. For example, it was proven in [24] that the tracking error was uniformly ultimately bounded. The bounded error will result in power mismatch and limit the potential applications of the proposed algorithms.

In this paper, the distributed economic dispatch problem is considered under a connected undirected network. The main contributions of this paper are as follows. First, compared with the existing results [17–21], the presented distributed algorithm is designed based on a simple centralised algorithm to deal with time-varying load demand. In particular, the approaches introduced in [19–21] cannot be applied to the considered time-varying loads which are more applicable to practical applications and much more technically challenging. Moreover, we show that the presented centralised algorithm is irrelevant to the dual variable (see Remark 4 for more details). Second, in contrast to the literatures [22–24] with time-varying loads, the presented algorithm is capable of achieving asymptotically convergent other than a uniformly ultimately bound tracking result. Last but not the least, the presented algorithm is robust to bounded external disturbances which is not considered in [17–24].

The remaining is organised as follows. Section 2 provides the preliminaries and problem formulation. In Section 3, the main results are presented. Some discusses are given in Section 4. Some numerical simulations are given in Sections 5 and Section 6 concludes the paper.

## 2 | PRELIMINARIES AND PROBLEM FORMULATION

### 2.1 | Notation

For \( x_i \in \mathbb{R}^n \), \( i = 1, 2, ..., m \), \( \text{col}(x_1, ..., x_m) = [x_1^T, ..., x_m^T]^T \), where \( x_i^T \) is the transpose of \( x_i \). \( I_n \) is an \( n \times n \) identity matrix. \( \nabla_{x_i} f(x_i, t) \) and \( \nabla_{xx_i} f(x_i, t) \) denote the first and second partial derivatives of the cost function \( f(x_i, t) \) with respect to a vector \( x_i \), respectively. \( \nabla_{xx_i} f(x_i, t) \) denotes the first partial derivative of the function \( \nabla_{x_i} f(x_i, t) \) with respect to \( t \).

### 2.2 | Graph theory

Here, a graph \( G = (\mathcal{V}, \mathcal{E}) \) is used to describe the information exchange within a network system, where \( \mathcal{V} = \{1, 2, ..., N\} \) is the node set and \( \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \) is the edge set. \( (j, i) \in \mathcal{E} \) represents that player \( i \) can obtain information from player \( j \). For an undirected graph, if \( (j, i) \in \mathcal{E} \) holds, then there exists \((i, j) \in \mathcal{E}\). \( \mathcal{N}_i = \{j : (j, i) \in \mathcal{E}\} \) denotes the set of neighbours of node \( i \). A path is a sequence of edges of the form \((i, j), (j, k), ..., \). An undirected graph is connected if there is a path between each pair of nodes.

### 2.3 | Problem formulation

Consider a power grid with \( N \) buses and each bus contains one generator and one local load. Let \( P_i(t) \in \mathbb{R} \) and \( D_i(t) \in \mathbb{R} \) being the active power generated by the generator and the active power needed by the load in bus \( i \), respectively. An undirected \( \mathcal{G} = (\mathcal{V}, \mathcal{E}) \) is used to describe the information exchange within the buses. Moreover, suppose that each bus \( i \in \mathcal{V} \) is assigned with a local cost function

\[
\begin{equation}
    f_i(P_i) = a_i P_i^2 + b_i P_i + c_i,
\end{equation}
\]

where \( a_i > 0, b_i \geq 0 \) and \( c_i \geq 0 \). Then, consider the following distributed economic dispatch problem

\[
\begin{equation}
    \begin{aligned}
    &\text{minimise} \sum_{i=1}^{N} f_i(P_i) \\
    &\text{subject to} \sum_{i=1}^{N} P_i = \sum_{i=1}^{N} D_i(t)
    \end{aligned}
\end{equation}
\]

where \( P_i \) is only available to bus \( i \). The objective is to design distributed algorithms \( u_i \) for bus \( i \) with dynamics

\[
\dot{P}_i = u_i + d_i(t),
\]

such that \( P_i \to P^*_i(t) \) as \( t \to \infty \), where \( P^*_i(t) \) is the optimal solution to (2), and \( d_i(t) \) is the local disturbance.

**Assumption 1.** The undirected graph \( G \) is connected.

**Assumption 2.** The local disturbance \( d_i(t) \) is bounded.
Assumption 3. \( D_i, \bar{D}_i \) exist and are bounded.

Remark 1. Note that the assumption that each bus contains one generator and one local load is general. In particular, the non-dispatchable power sources, that is, PV power generation and wind power generation, can be regarded as the negative load. For the case that a bus contains local load \( D^p \) only without any generators, we assume that it is connected with a nearby bus \( i \) which contains one generator and one local load \( D_i \), and the local load of bus \( i \) is defined as \( D_i(t) + D^p(t) \) to deal with the only case load.

Remark 2. It follows from \( a_i > 0 \) that the global cost function \( \sum_{i=1}^{N} f_i(P_i) \) is convex and the solution to the optimisation problem (2) exists [25].

Remark 3. Assumption 1 is a common assumption in distributed time-varying optimisation problem [26, 27]. Assumption 2 is also a common condition for the solvability of the disturbances rejection. Assumption 3 can be satisfied in many situations since that the load demand is always with slowly changed velocity [22–24].

3 MAIN RESULTS

3.1 Centralised algorithm without external disturbances

Now, we first consider the following optimisation problem (2) without external disturbances rejection. Define the following Lagrangian function for the optimisation problem (2):

\[
L_1 = \sum_{i=1}^{N} f_i(P_i)+\lambda\left(\sum_{i=1}^{N} D_i(t) - \sum_{i=1}^{N} P_i\right),
\]

where \( \lambda \) is the Lagrange multipliers corresponding to the equality constraints. According to Proposition 1 in [28], a prediction-correction algorithm to the above Lagrangian functions is given as:

\[
\dot{x} = -\nabla_{\xi}^{-1} L_1 (\nabla_{\xi} L_1 + \nabla_{\xi} L_1),
\]

where \( \xi = \text{col}(P_1, \ldots, P_N, \lambda) \), \( S \) is a positive definite matrix satisfying \( S \geq \delta I_{N+1} \) for some \( \delta > 0 \). Let \( S = I_{N+1} \). Combining (4) and (5), it is not hard to have that

\[
\nabla_{\xi} L_1 + \nabla_{\xi} L_1 = \begin{bmatrix}
2a_1 P_1 + b_1 - \lambda \\
\vdots \\
2a_N P_N + b_N - \lambda \\
\sum_{i=1}^{N} D_i(t) - \sum_{i=1}^{N} P_i + \sum_{i=1}^{N} D_i(t)
\end{bmatrix}
\]

with \( \theta = \sum_{i=1}^{N} \frac{1}{2a_i} \). Therefore, it follows from (5) that

\[
\dot{P}_i = -P_i - \frac{b_i}{2a_i} + \frac{1}{2a_i} \sum_{i=1}^{N} \left( D_i(t) + \dot{D}_i(t) + \frac{b_i}{2a_i} \right),
\]

\[
\dot{\lambda} = -\lambda + \frac{1}{\theta} \sum_{i=1}^{N} \left( D_i(t) + \dot{D}_i(t) + \frac{b_i}{2a_i} \right).
\]

Lemma 1. Let \( P_i(t) \) be the solution of (6a). Then for any initial condition \( P_i(0) \), it follows that \( P_i(t) \to P_*^i(t) \) as \( t \to \infty \), where \( P_*^i(t) \) is the solution to (2).

Proof. Consider the Lagrangian function in (4). Let \( G_i(\lambda, t) = \min L_1(P_i, \lambda, t) \) being the dual function and \( \lambda^* = \argmax G_i(\lambda, t) \). Note that (4) is strong convex, and then the optimal primal-dual pair \( x^*(t) = (P_*^i, \lambda^*) \) is unique. According to Proposition 3.3.1 in [25], \( (P_*^i, \lambda^*) \) must satisfy the following Karush–Kuhn–Tucker (KKT) conditions:

\[
0 = \nabla_{\beta} L_1 (P_*^i, \lambda^*, t),
\]

\[
0 = \nabla_{\lambda} L_1 (P_*^i, \lambda^*, t).
\]

Since the algorithm (6) is obtained by (5) with \( S = I \), it follows from (5) that

\[
\frac{d}{dt} (\nabla_{\xi} L_1) = -\nabla_{\xi} L_1.
\]
Therefore, the solution of (6) satisfies
\[ \| \nabla L_1(\tilde{z}, t) \|_2 \leq e^{-t} \| \nabla L_1(\tilde{z}(0), 0) \|_2. \]
The KKT condition (7) means that \( \nabla L_1(\tilde{z}, t) = 0 \). According to the mean-value theorem, we have that
\[ \nabla L_1(\tilde{z}, t) = \nabla L_1(\eta(t), t)(\tilde{z} - \tilde{z}^*), \]
where \( \eta(t) \) is a convex combination of \( \tilde{z}(t) \) and \( \tilde{z}(t)^* \). Note that there exists constant \( \epsilon \) that satisfies \( \| \nabla L_1(\tilde{z}(0), 0) \|_2 \leq \epsilon \). It follows that
\[ \| \tilde{z} - \tilde{z}^* \| \leq \epsilon^{-1} \| \nabla L_1(\tilde{z}(0), 0) \|_2, \]
and the states of system (6) globally exponentially converges to \((P^*, \lambda^*)\).

**Remark 4.** Note that the dynamic \( P_i(t) \) in (6a) is irrelevant to \( \lambda(t) \). It follows that, using duality theory for the economic dispatch problem with a time-varying load, the presented centralised algorithm (6a) is irrelevant to the dual variable (or the Lagrange multiplier). Therefore, algorithm (6a) is very simple and suffice to obtain the optimal solution to (2).

### 3.2 Distributed algorithm with external disturbances

Inspired by (6a), consider the following algorithm:
\[ u_i = -\delta_i \text{sgn}(P_i - x_i), \quad (8a) \]
\[ \dot{x}_i = -x_i - \frac{b_i}{2a_i} + \frac{X_{i2}}{2a_i \Gamma_i(X_{i1})}, \quad (8b) \]
\[ \dot{\omega}_i = \sigma \sum_{j=1}^{N} a_{ij} \text{sgn}(x_i - x_j), \quad (8c) \]
\[ \dot{\chi}_i = \omega_i + \begin{pmatrix} \frac{1}{2a_i} \\ D_i(t) + \dot{D}_i(t) + \frac{b_i}{2a_i} \end{pmatrix}, \quad (8d) \]

where \( \Sigma_{i=1}^{N} \omega_i(0) = 0 \), \( \chi_i = \text{col}(x_i, \dot{x}_i, x_{i2}) \in \mathbb{R}^2 \), \( \Gamma_i(t) \) is the projection operator on set \( \left[ \frac{1}{2a_i}, +\infty \right) \), and \( \delta_i, \sigma \) are some constants to be determined.

**Remark 5.** Note that the dynamic \( x_i(t) \) in (8b) is designed according to (6a), and it is used to generate \( \lim_{t \to \infty} (x_i(t) - P_i^*(t)) = 0 \) where \( P_i^*(t) \) is the solution to (2). (8c) and (8d) is the distributed average tracking algorithm to estimate \( \sum_{i=1}^{N} \frac{1}{2a_i} \) and \( \sum_{i=1}^{N} (D_i(t) + \dot{D}_i(t) + \frac{b_i}{2a_i}) \). Moreover, algorithm (8a) is a sliding model tracking control law to deal with the local disturbance.

**Theorem 1.** Suppose that Assumptions 1, 2 and 3 hold. Let \( P_i \) be the solution of (8b). Then there exists \( \delta_i^* > 0 \) and \( \sigma^* > 0 \) such that, for any \( \delta \geq \delta_i^* \), and \( \sigma \geq \sigma^* \), \( \lim_{t \to \infty} (P_i(t) - P_i^*(t)) = 0 \) holds, where \( P_i^*(t) \) is the solution to (2).

**Proof.** Let \( \sigma^* = \max\{\|D_i(t) + \dot{D}_i(t)\|\} \), and
\[ e_{i1} = N\chi_{i1} - \theta, \]
\[ e_{i2} = N\chi_{i2} - \phi(t), \]
where \( \phi(t) = \sum_{j=1}^{N} (D_j(t) + \dot{D}_j(t) + \frac{b_j}{2a_j}) \). Based on Assumptions 1 and 3, according to Theorem 1 in [29], we have that \( \chi_{i1} \) and \( \chi_{i2} \) are bounded, and satisfy
\[ |N\chi_{i1}(t) - \theta| \to 0, \]
\[ |N\chi_{i2}(t) - \phi(t)| \to 0, \]
in finite time \( T_0 \) and the convergence time is upper bounded by \( T_0 \leq \left( \frac{\sigma^*-\sigma^*}{2} \right) \sum_{j=1}^{N} \| \chi_j(0) - \chi_j(0) \| \). Therefore, it follows that \( \lim_{t \to \infty} e_{i1}(t) = 0 \) and \( \lim_{t \to \infty} e_{i2}(t) = 0 \). Note that for \( t \geq T_0 \), \( \chi_{i1} \) is non-singular without projection since \( \theta \) is non-singular. Hence, the projection operation \( \Gamma_i(\chi_{i1}) \) simply returns \( \chi_{i1} \) itself.

Define the following Lagrangian function for the optimisation problem (2):
\[ L_2 = \sum_{i=1}^{N} f_i(x_i) + \lambda \left( \sum_{i=1}^{N} D_i(t) - \sum_{i=1}^{N} x_i \right), \]
where the Lagrange multipliers \( \lambda \) is defined in (6b). Let \( y = \text{col}(x_1, \ldots, x_N, \lambda) \). Combining (6b) and (8b), it is not hard to have that
\[ \nabla_y L_2 = \begin{pmatrix} 2a_1 x_1 + b_1 - \lambda \\ \vdots \\ 2a_N x_N + b_N - \lambda \end{pmatrix}, \]
\[ \nabla_y L_2 = \begin{pmatrix} 2a_1 x_1 + b_1 - \lambda \\ \vdots \\ 2a_N x_N + b_N - \lambda \end{pmatrix}. \]

It follows that
\[ \frac{d}{dt}(\nabla_y L_2) = -\nabla_y L_2 + (\nabla_y L_2)\gamma(t), \quad (10) \]
where $\nabla \dot{L}_2 = \nabla_{\infty} L_1$ and

$$
\gamma(t) = \left( \begin{array}{c}
\frac{\phi + e_{1,2}}{\Gamma_1 (\theta + e_{1,1})} - \frac{\phi}{\theta} \\
\vdots \\
\frac{\phi + e_{N,2}}{\Gamma_N (\theta + e_{N,1})} - \frac{\phi}{\theta} \\
0
\end{array} \right).
$$

According to (10), we have that

$$\| \nabla \dot{L}_2 (\zeta(t)) \|_2 \leq e^{\rho \tau} \| \nabla \dot{L}_2 (\zeta(t_0), h_0) \|_2 + \int_{t_0}^t e^{\rho \tau} \| (\nabla \dot{L}_2) \gamma(t) \|_2 dt.$$

Since $\lim_{t \to \infty} e_{i,1} = 0$ and $\lim_{t \to \infty} e_{i,2} = 0$ hold, then there exists $\lim_{t \to \infty} \nabla \dot{L}_2 = 0$. Using the same arguments on Lemma 1, we have that $\kappa_i(t)$ is bounded and $\lim_{t \to \infty} (\kappa_i(t) - P_i(t)) = 0$, where $P_i(t)$ is the solution to (2).

Let $e_i = P_i - \kappa_i$. It follows from (8a) that

$$\dot{e}_i = -\delta_i \sigma \text{sgn}(e_i) + (d_i(t) - \dot{\kappa}_i(t)).$$

Note that $\dot{\kappa}_i(t)$ is bounded since $\kappa_i(t)$ and $\chi_{i,2}$ are bounded. Let $\delta^*_i = \max\{\|d_i(t) - \dot{\kappa}_i(t)\|\}$ and $V_i = \frac{1}{2} e_i^2$. Then we have that $V_i \leq - (\delta_i - \delta^*_i) \| e_i \|$. According to Theorem 14.1 in [30], we have that $\lim_{t \to \infty} (P_i(t) - \chi_i(t)) = 0$, which implies that $\lim_{t \to \infty} (P_i(t) - \chi^*_i(t)) = 0$.

**Remark 6.** From the proof in Theorem 1, it is easy to obtain the distributed economic dispatch algorithm without considering the local disturbance. In particular, consider the following distributed algorithm:

$$u_i = -P_i - \frac{b_i}{2a_i} + \frac{\chi_{i,2}}{2a_i \Gamma_i (\chi_{i,1})},$$

$$\dot{\omega}_i = \sigma \sum_{j=1}^N a_{ij} \text{sgn}(\chi_i - \chi_j),$$

$$\chi_i = \omega_i + \left( \begin{array}{c}
\frac{1}{2a_i} \\
D_i(t) + D_i(t) + \frac{b_i}{2a_i}
\end{array} \right).$$

It follows that $P_i(t)$ is bounded and $\lim_{t \to \infty} (P_i(t) - \chi^*_i(t)) = 0$, where $P_i(t)$ is the solution to (2).

**Remark 7.** Note that the condition $\sigma \geq \sigma^* = \max\{\|D_i(t) + D_i(t)\|\}$ is required in algorithm (8). Hence, the present algorithm can satisfy the plug-and-play requirements of the generators if $\sigma$ is available to the generators.

## 4 | SWITCHING TOPOLOGY, TIME DELAYS, AND DISCRETE-TIME IMPLEMENTATION

### 4.1 | With switching topology

In the following, we consider the case of a switching topology to describe the situation that the information exchange among the buses are time varying.

**Theorem 2.** Suppose that Assumptions 2 and 3 hold, and the graph $G$ is switching, undirected, and connected at each time instant. Let $P_i$ be the solution of (8). Then there exists $\delta^*_i > 0$ and $\sigma^* > 0$ such that, for any $\delta \geq \delta^*_i$, and $\sigma \geq \sigma^*$, $\lim_{t \to \infty} (P_i(t) - \chi^*_i(t)) = 0$ holds, where $\chi^*_i(t)$ is the solution to (2).

**Proof.** Since Assumption 3 holds and the graph $G$ is switching, undirected, and connected at each time instant, according to Theorem 2 in [29], we have that $\chi_{i,1}(t)$ and $\chi_{i,2}(t)$ are bounded, and satisfy

$$| N\chi_{i,1}(t) - \theta | \to 0,$$

$$| N\chi_{i,2}(t) - \phi(t) | \to 0,$$

in finite time. Then the proof is similar to that of Theorem 1 and is hence omitted.

**Remark 8.** Theorem 2 implies that algorithm (8) is effective when the communication network is switching, undirected, and connected at each time instant. Hence, the algorithm is robust to the communication link fault when the undirected and connected condition holds. Note that algorithm (8) is not applicable for time-varying directed networks since the undirected and connected topology is necessary for the distributed average tracking algorithm (8c) and (8d).

### 4.2 | With time delays

Note that the algorithm (8) is composed of a distributed average tracking algorithm. It follows from [29] that, if the time delays occur, the system inevitably suffers from steady-state errors no matter which control algorithm is used for the distributed average tracking problem. Hence, algorithm (8) suffers from steady-state errors if the delays occur. In particular, assume that time delay $\tau_j$ is considered. Then algorithm (8) becomes

$$u_i = -\delta_i \text{sgn}(P_i - \chi_i),$$

$$\dot{x}_i = -x_i - \frac{b_i}{2a_i} + \frac{\chi_{i,2}}{2a_i \Gamma_i (\chi_{i,1})},$$

$$\dot{\omega}_i = \sigma \sum_{j=1}^N a_{ij} \text{sgn}(x_i - x_j).$$
FIGURE 1  The illustration of IEEE 14-BUS system

FIGURE 2  The communication topology of generators

\[ \chi_i = \omega_i + \left( \frac{1}{2a_i} D_i(t - \tau_i) + D_i(t - \tau_i) + \frac{b_i}{2a_i} \right) \]

It is trivial to show that

\[ \lim_{t \to \infty} \left( \sum_{i=1}^{N} P_i(t) - \sum_{i=1}^{N} D_i(t) \right) = 0. \]

4.3 | Discrete-time implementation

Now we present a discrete-time implementation of algorithm (8). Let \( T \) be the sampling period and \( k \) be the discrete-time index. The active power needed by the load in bus \( i \) is described by \( D_i(k + 1) = D_i(k) + T \Xi_i(k) \) and \( \Xi_i(k + 1) = \Xi_i(k) + T \Xi_i(k) + T \Pi_i(k) \).

TABLE 1  IEEE 14-BUS generator parameters

| Bus | \( a_i \) | \( b_i \) | \( c_i \) |
|-----|-----|-----|-----|
| 1   | 0.04 | 2.0 | 561 |
| 2   | 0.03 | 3.0 | 310 |
| 3   | 0.035| 4.0 | 78  |
| 6   | 0.03 | 4.0 | 561 |
| 8   | 0.04 | 2.5 | 78  |

\[ \Xi_i(k+1) = \Xi_i(k) + T \Xi_i(k) + T \Pi_i(k) \]

So the centralised algorithm (6a) is effective for the case with time-varying load requirements and without external disturbances.

In addition, Figures 5 and 6 show that the sum of the power outputs satisfies the load requires under algorithm (8). Thus, the proposed distributed algorithm (8) works properly with time-varying load requirements and local disturbances. Note that (8) includes a distributed average tracking algorithm which has a finite convergence time \( T_0 \) with \( T_0 \leq \frac{\sigma}{2} \sum_{i=1}^{N} \sum_{j \in N_i} | \chi_i(0) - \chi_j(0) | \). Therefore, the convergence time in Figure 6 is longer than that in Figure 4.

5 | EXAMPLES IN POWER GRIDS

In this section, we consider the distributed economic dispatch problem for the IEEE 14-BUS test system, which is shown in Figure 1. The communication graph for the generators is given in Figure 2. In Table 1, the generator parameters are given according to [10]. The loads and local disturbances are selected as \( D_i = 70 + 10i \sin(0.5t) \) and \( d_i = 2i \sin(0.5t) \). The initial generalised coordinates are chosen randomly in \([0,50]\). Figures 3 and 4 show that the sum of the power outputs satisfies the load requires under algorithm (6a). So the centralised algorithm (6a) is effective for the case with time-varying load requirements and without external disturbances.

By using a similar analysis to that in [29], we know that there exist steady-state errors in the discrete-time system which are related to \( \sigma \) and \( T \).

6 | CONCLUSIONS

Here, the time-varying load demand problem for the distributed economic dispatch problem is studied. We propose a distributed
The power generation meets the constraints of total load demand and the tracking error is shown to converge to zero. Moreover, the algorithm is feasible even when the external disturbances exist. Future works include the distributed economic dispatch control with generator constraints under time-varying directed networks.

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