Structural analysis and design of irregular shaped footings subjected to eccentric loading

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Abstract
This article presents a simplified analytical model for designing irregular shaped reinforced concrete (RC) footings supporting square column and subjected to eccentric loading, that is, axial load $P$ and biaxial moments, $M_x$ in $(x-x)$ axis and $M_y$ in $(y-y)$ axis, respectively. In this study, four design variations of footing sections are considered, namely, square, triangular, circular, and trapezoidal. Seven different footings (F-1 to F-7), each with a different loading condition, are used to analyze and design each of the selected irregular footings with the goal of getting the optimum footing section. The required reinforcing area of steel ($A_s$) is obtained using the SDM method in each selected footing which is then compared using finite element software (SAFE). The percentage difference of area of steel ($A_s$) for simplified method with the finite element software ranges within 1% to 13%. Moreover, the concrete volume results show that the circular and triangular footings prove to be the most economical footings followed by square and trapezoidal shaped footing sections. However, the results show that triangular shaped footings under heavy loads require a larger steel area ($A_s$) as in footing F-7, which is not economical for heavy loads.

KEYWORDS
eccentric loading, irregular footings, optimum footing section, steel area

1 | INTRODUCTION

Foundations are most important member of structure, which transmit the structural load to the soil. Foundations are classified as shallow or deep foundations, which defer in terms of geometry, soil behavior, and structure capability. Different types of shallow foundations are available according to their functionality; which are isolated, combined, strip, and mat footings.

Footing sizes are mostly governed by their loading parameters which are; axial load $P$, Biaxial moments $M_x$ and $M_y$, allowable soil pressure $Q_a$, unit weight of concrete $\gamma_c$, soil unit weight $\gamma_s$ and the depth of the footing base below the final grade $D_f$ as shown in Figure 1. Similarly, soil pressure distribution under a footing is normally a function of type of soil, relative rigidity of soil and footing, and depth of footing. For structural design purpose, it is quite common to assume the linearly distributed soil pressure to the footing surface.
Isolated footings are normally subjected to three different loading scenarios: (1) The footings subjected to axial load \( P \) only, (2) footing subjected to axial load \( P \) and unidirectional bending \( M_x \), moment in one direction only), and (3) footing subjected to axial load \( P \) and bi-directional bending \( M_x \) and \( M_y \), moments in both directions). \(^9\)

Square and rectangular footings are the most common shaped of the isolated RC footings in the construction industry, but other irregular shaped footings do exist such as circular, triangular and trapezoidal, depending on different scenarios in the construction field. Different mathematical models are presented in numerous studies \(^{10-17}\) to structurally analyze and design the irregular shaped footing sections under the provisions of ACI building code of design (ACI 318-14). \(^{18}\)

There are limited studies, exploring the detailed design for the irregular shaped footing sections. Stone et al\(^{19}\) studied the response of triangular footings subjected to centric and eccentric loading. Their study analyzed model tests with the derivation of equivalent rectangular section using the conventional bearing capacity theory. Huat et al\(^{20}\) studied the performance of triangular shell footings using finite element and field model test. The study concluded that the triangular shell is more efficient in carrying the load as compared to the traditional flat strip footing. Rojas\(^{21}\) proposed mathematical model of circular footing subjected to axial load and biaxial bending. He concluded that this new circular footing model is more economical and adjustable to actual soil conditions.

The previous research studies consisted of complicated mathematical models for the analysis and design of irregular shaped footing section. In most cases, the proposed models of footing sections do not consider the effect of bi-axial moments. They are analyzed and designed based on the axial load values only.

This article, however, presents simplified analytical model for designing irregular shaped reinforced concrete footings, supporting square column, and subjected to eccentric loading, that is, axial load \( P \) and biaxial moments; \( M_x \) in \((X-X)\) axis and \( M_y \) in \((Y-Y)\) axis, respectively. In this study, four design variations of footing sections are considered, that is, square, triangular, circular, and trapezoidal to be analyzed using the simplified method approach. Seven different footings \((F-1\) to \(F-7)\), each with a different loading condition, are used to analyze and design each of the selected irregular footings.

There are limited studies for the reinforced design of irregular shaped footings. This study will provide quick and easy approach to design such footings and will be useful for the students in their undergraduate and graduate courses as well as research related work.

Figure 2 includes the irregular shaped footing sections, which are studied in this research. Eccentric, shear and moment formulas are derived for each of these irregular shaped footings (square, circular, triangular, and trapezoidal). Mathcad software\(^{22}\) is used for all the necessary calculations needed for the simplified design method (SDM). The design results of this method are also compared with the computer aided software (SAFE software). The comparison mainly includes the footing dimensions, total steel rebar areas and concrete volume from safety and economic perspectives.

## 2 SHEAR AND FLEXURAL DESIGN FORMULAS

In general, the required footing area \( F_A \) is computed based on the axial load \( P \) and the effective soil pressure \( Q_e \). The equation is obtained from (ACI -318R-14).

\[
F_A = \frac{P}{Q_e} = \frac{DL + LL}{Q_e}.
\] (1)
FIGURE 2  Cross section of different shaped footings

Also,

\[ Qe = Qa - Wc - Ws \]  \hspace{1cm} (1a)

\[ Wc = \gamma_c \times h \]  \hspace{1cm} (1b)

\[ Ws = \gamma_c \times D_f \]  \hspace{1cm} (1c)

\[ h = d + d' \]  \hspace{1cm} (1d)

Where

- \( F_A \) = Footing area
- \( Qa \) = Allowable soil pressure
- \( Qe \) = Effective soil pressure
- \( Wc \) = Concrete weight
- \( Ws \) = Soil weight
- \( h \) = Total footing depth
- \( d \) = Effective depth
- \( d' \) = Cover to steel centroid.

2.1  Effective depth calculation

Both one-way and two-way shear are considered for estimating the footing’s effective depth. The critical section for one way and two-way shear to estimate the effective depth for each shape is shown in Figure 3. The ACI building design code (ACI-318R-14) formula is used for calculating one-way shear depth:

\[ d_{\text{one way}} = \frac{Vu}{\phi_s \times v_c \times b_w} \]  \hspace{1cm} (2)

Where

- \( Vu \) = Factored shear force,
- \( \phi_s \) = Shear reduction factor,
- \( v_c \) = Shear stress carried by the concrete,
- \( b_w \) = Footing width.
For two-way shear depth, the largest value is to be selected from the following ACI code (ACI-318-14) equations:

\[ d_{2w(1)} = \frac{6 \, Vu}{\varphi_s \left( 1 + \frac{8}{\beta_c} \right) \sqrt{f'_c} \, b_o} \]  
(3)

\[ d_{2w(2)} = \frac{12 \, Vu}{\varphi_s \left( 2 + \frac{\alpha_s}{b_o} \right) \sqrt{f'_c} \, b_o} \]  
(4)

\[ d_{2w(3)} = \frac{3 \, Vu}{\varphi_s \sqrt{f'_c} \, b_o} \]  
(5)

Where

- \( \beta_c \) = Ratio of long side of the column to the short of the column,
- \( f'_c \) = Specified compression strength of concrete,
- \( b_o \) = Perimeter around the punching area,
- \( \alpha_s \) = Ratio equals to 40, 30, and 20 for interior column, edge column, and corner column, respectively.
### 2.2 | Footing moments and reinforcement calculation

Footing bending moments \((Mu)\) in both axes are considered at the face of the column (Figure 4).

\[
Mu = \frac{L_p^2}{2} q_u b_w
\]  

(6)

and

\[
q_u = \frac{Pu}{FA} = \frac{DL \times DLF + LL \times LLF}{FA}
\]  

(7)

Where

- **\(Mu\)** = Fully factored bending moment,
- **\(L_p\)** = Maximum projected length,
- **\(q_u\)** = Bearing pressure for strength design,
- **\(DLF\)** = Dead load factor equal 1.2,
- **\(LLF\)** = Live load factor equal 1.6.

The reinforcement area \(As\) of the footing can be computed as:

\[
As = \frac{Mu}{\phi_b f_y \left( d - \frac{a}{2} \right)}
\]  

(8)

---

**FIGURE 4**  Tributary area for moments in different footings
Where

\( \varphi_b \) = Bending reduction factor,
\( f_y \) = Specified yield strength of non-prestressed reinforcing,
\( A_s \) = Area of tension steel,
\( d \) = Effective depth,
\( a \) = Depth of the compression block.

Also,

\[
\begin{align*}
d^L_B & \leq d \leq d^U_B \\
A_s^{\text{Mini}} & \leq A_s \leq A_s^{\text{Max}} \\
A_s^{\text{Max}} &= 0.75 \times \beta_1 \times \frac{f'_c}{f_y} \left(\frac{600}{600 + f_y}\right) bd \\
A_s^{\text{Mini}} &= \left(\frac{1.4}{f_y}\right) bd \\
\beta_1 &= 0.85 \text{ for } f'_c \leq 30 \text{ MPa} \\
\beta_1 &= 0.85 - 0.008 (f'_c - 30) \geq 0.65 \text{ for } f'_c > 30 \text{ MPa}
\end{align*}
\]

Where \( d^L_B \) and \( d^U_B \) are footing depth lower and upper bounds, and \( A_s^{\text{Mini}} \) and \( A_s^{\text{Max}} \) are footing steel reinforcement area lower and upper bounds, respectively.

The reinforcing bars must have the required length to provide enough strength. In other words, the bars must extend developmental length \( L_d \) from the face of the column (ACI 318-14).

\[
L_d < L_d^{\text{T Available}}
\]

Where

\( L_d \) = Required bar developmental length,
\( L_d^{\text{T Available}} \) = Available length in tension.

For the dowel bars under compression:

\[
A_{sdowels} \geq 0.005 A_{Column}
\]

\[
L_d^{\text{Comp}} < L_d^{\text{C Available}}
\]

\[
h > L_d^{\text{Comp}} + \text{Cover} + 2d_b
\]

Where

\( A_{sdowels} \) = Steel area of the dowels,
\( A_{Column} \) = Column area,
\( L_d^{\text{Comp}} \) = Required bar developmental length in compression,
\( L_d^{\text{C Available}} \) = Available length in compression,
\( h \) = Total footing depth,
\( \text{Cover} \) = Concrete cover thickness,
\( d_b \) = Bar diameter.

2.3 Eccentric formulation

Eccentric footing is the footing that is subjected to axial load \( P \) and biaxial moments \( M_x \) and \( M_y \) about x and y axes as shown in Figure 5A.
The soil pressure at the footing corners 1, 2, 3, and 4 is given by the following formula:

$$Q_{CORNERS} = -\frac{P}{A} \mp \frac{M_x C_y}{I_x} \pm \frac{M_y C_x}{I_y}$$  \hspace{1cm} (17)

Where

- $P$ = Axial Load,
- $M_x$ = Moment about the x axis ($P \times e_y$),
- $M_y$ = Moment about the y axis ($P \times e_x$),
- $A$ = Footing area,
- $I_x$ = Moment of inertia about the x axis,
- $I_y$ = Moment of inertia about the y axis,
- $C_x$ = Centroid coordinates on the x axis,
- $C_y$ = Centroid coordinates on the y axis.

Load $P$ acts at distance $e_x$ from the y axis and a distance $e_y$ from the x axis, therefore,

$$Q_{CORNERS} = -\frac{P}{A} \mp \frac{P \ e_y \ C_y}{I_x} \pm \frac{P \ e_x \ C_x}{I_y}.$$  \hspace{1cm} (18)

Substituting for $C_y$, $C_x$, $I_y$, and $I_x$ for rectangular footing in equation –18, the soil pressure equation becomes

$$Q_{CORNERS} = -\frac{P}{A} \left[ 1 \mp \frac{6 \ e_y}{L} \pm \frac{6 \ e_x}{B} \right].$$  \hspace{1cm} (19)
Similarly, for square footing

\[ Q_{\text{CORNERS}} = -\frac{P}{A} \left[ 1 \mp \frac{6 e_y}{B} \pm \frac{6 e_x}{B} \right]. \quad (20) \]

For circular footing

\[ Q_{\text{CORNERS}} = -\frac{P}{A} \left[ 1 \mp \frac{8 e_y}{D} \pm \frac{8 e_x}{D} \right]. \quad (21) \]

Also, the following two equations can also be used for circular footings

\[ Q_{\text{max}} = -\frac{P}{A} \left[ 1 - \frac{8 e}{D} \right] \quad (21a) \]

\[ Q_{\text{min}} = -\frac{P}{A} \left[ 1 + \frac{8 e}{D} \right] \quad (21b) \]

Where

\[ M = \sqrt{M_x^2 + M_y^2} \quad \text{and} \quad e = \frac{M}{P}. \]

For triangular footing with equal lengths (Figure 5B), the soil pressure equation at each corner is

\[ Q_{\text{CORNERS}}^{(1)} = -\frac{P}{A} \left[ 1 - \frac{12 e_y}{L} - 0 \right] \quad (22) \]

\[ Q_{\text{CORNERS}}^{(3)} = -\frac{P}{A} \left[ 1 + \frac{18 e_x}{a^2 - aB + B^2} + \frac{12 e_y}{L} \right] \quad (23) \]

\[ Q_{\text{CORNERS}}^{(4)} = -\frac{P}{A} \left[ 1 - \frac{18 e_x}{a^2 - aB + B^2} + \frac{12 e_y}{L} \right] \quad (24) \]

For trapezoidal footing (Figure 5C), the soil pressure equation at each corner is

\[ Q_{\text{CORNERS}}^{(1)} = -\frac{P}{A} \left[ 1 - \frac{e_y(12B)}{(b^2 + B^2)} - \frac{e_y(24b + 12B)(b + B)}{2L(b^2 + 4bB + B^2)} \right] \quad (25) \]

\[ Q_{\text{CORNERS}}^{(2)} = -\frac{P}{A} \left[ 1 - \frac{e_x(12B)}{(b^2 + B^2)} + \frac{e_y(24b + 12B)(b + B)}{2L(b^2 + 4bB + B^2)} \right] \quad (26) \]

\[ Q_{\text{CORNERS}}^{(3)} = -\frac{P}{A} \left[ 1 + \frac{e_x(12b)}{(b^2 + B^2)} + \frac{e_y(24B + 12b)(b + B)}{2L(b^2 + 4bB + B^2)} \right] \quad (27) \]

\[ Q_{\text{CORNERS}}^{(4)} = -\frac{P}{A} \left[ 1 + \frac{e_x(12b)}{(b^2 + B^2)} - \frac{e_y(24B + 12b)(b + B)}{2L(b^2 + 4bB + B^2)} \right]. \quad (28) \]

Moreover, the soil pressure at the corners must be in compression state and less than the effective soil pressure to determine the required footing size.

### 3 \ FOOTING DESIGN PROCEDURE

The following steps need to be followed in the SDM for an economical design of the eccentric footings.

**Step 1:** Determine the Effective Soil Pressure \( Q_e \) (Equation (1a)).

**Step 2:** Determine the initial footing dimensions based on the area of the footing \( F_A \) (Equation (1)).
### TABLE 1  Footing design loads

| Footing | $P_{DL}$ (kN) | $P_{LL}$ (kN) | $M_{ax}$ (kN-m) | $M_{ay}$ (kN-m) | Column size $(mm \times mm)$ |
|---------|---------------|---------------|-----------------|-----------------|-----------------------------|
| F1      | 200           | 100           | 60              | 40              | $300 \times 300$            |
| F2      | 120           | 70            | 30              | 50              | $270 \times 270$            |
| F3      | 1000          | 800           | 200             | 150             | $500 \times 500$            |
| F4      | 500           | 200           | 90              | 50              | $320 \times 320$            |
| F5      | 2000          | 1300          | 150             | 250             | $550 \times 550$            |
| F6      | 800           | 600           | 100             | 30              | $350 \times 350$            |
| F7      | 3000          | 2000          | 350             | 280             | $700 \times 700$            |

**Step 3:** Determine the final footing dimensions based on the appropriate eccentric formula (*Equations (18) to (28)*), based on the footing shape.

**Step 4:** Determine the required depth for one-way shear $d_{\text{one way}}$ (*Equation (2)*).

**Step 5:** Determine the required depth for two-way shear $d_{\text{two way}}$.

**Step 6:** Determine footing reinforcement $A_s$ (*Equation (8)*).

**Step 7:** Determine the required bar developmental length $L_d$ (*Equation (13)*).

**Step 8:** Determine the required bar developmental length for compression $L_{d_{\text{comp}}}$ (*Equations (14) to (16)*).

**Step 9:** Check if the total thickness $h$ satisfies Equation (16).

**Step 10:** Detailing of the footing.

### 4 | NUMERICAL ANALYSIS

The input values for all the selected seven footings (F-1 to F-7) are described in Table 1. The analysis and design of the seven footings for each shape are presented and the results obtained are compared with the finite element software (SAFE).

The common design parameter used for all footings are as follows:

- $f_y = 400 \text{ MPa}$
- $f'_c = 30 \text{ MPa}$
- $Qa = 200 \text{ kPa}$
- $\gamma_c = 25 \text{ kN/m}^3$
- $\gamma_s = 15 \text{ kN/m}^3$
- $d' = 75 \text{ mm}$
- $Df = 1 \text{ m}$.

### 5 | RESULTS AND DISCUSSION

The selected seven footings (F-1 to F-7) are designed individually as four different shaped footings (square, circular, triangular, and trapezoidal) using the simplified irregular design method approach. For each footing shape, they are further analyzed and designed by SAFE software. The results obtained are illustrated in Tables 2 to 5, respectively.

#### 5.1 | Square footing

In this section, the selected seven footings are designed and analyzed as eccentric square footing. This method gives safe and optimum dimensions for the assigned load. These footings are also analyzed using the computer software (SAFE) and the results obtained are presented in Table 2.

The reinforcement results obtained from the safe software show similarity with the SDM results regarding the area of steel required with a percentage difference of 2% to 13%. This shows the accuracy of proposed design formulas of square footing using SDM method. The required steel area results obtained are also displayed in bar chart (Figure 6). Moreover, the settlement contours obtained from the SAFE software are shown in Figure 7.
| Footing | SDM method | SAFE software |
|---------|-------------|---------------|
|         | Width (B) (m) | Thickness (h) (mm) | As (mm²) | Concrete volume (m³) | As (mm²) | Settlement (mm) |
| F1 | 2.2 | 350 | 2118 | 1.69 | 2121 | 5.32 |
| F2 | 2.8 | 350 | 2695 | 2.74 | 2670 | 2.98 |
| F3 | 3.7 | 550 | 6406 | 7.53 | 6390 | 12.98 |
| F4 | 2.5 | 360 | 2569 | 2.25 | 2547 | 10.24 |
| F5 | 4.7 | 730 | 11 560 | 16.13 | 9854 | 12.04 |
| F6 | 3.1 | 500 | 4611 | 4.81 | 4260 | 11.22 |
| F7 | 5.7 | 870 | 15 860 | 28.27 | 15 701 | 11.57 |

| Footing | SDM method | SAFE software |
|---------|-------------|---------------|
|         | Diameter (D) (m) | Thickness (h) (mm) | As (mm²) | Concrete volume (m³) | As (mm²) | Settlement (mm) |
| F1 | 2.1 | 350 | 2021 | 1 | 2042 | 6.20 |
| F2 | 2.6 | 350 | 2502 | 2.37 | 2513 | 2.91 |
| F3 | 4.1 | 530 | 6529 | 6.99 | 6521 | 9.88 |
| F4 | 2.7 | 350 | 2600 | 2.76 | 2650 | 8.71 |
| F5 | 5.2 | 730 | 11 920 | 15.5 | 11 938 | 11.47 |
| F6 | 3.5 | 510 | 5329 | 4.91 | 5654 | 10.40 |
| F7 | 6.4 | 870 | 17 810 | 27.9 | 17 907 | 11.83 |

| Footing | SDM method | SAFE software |
|---------|-------------|---------------|
|         | Width (B) (m) | Length (L) (m) | Thickness (h) (mm) | As (mm²) | Concrete volume (m³) | As (mm²) | Settlement (mm) |
| F1 | 2.8 | 2.8 | 350 | 2695 | 1.372 | 2748 | 4.92 |
| F2 | 2.5 | 2.5 | 350 | 2406 | 1.094 | 2434 | 3.87 |
| F3 | 5.2 | 5.2 | 530 | 11 290 | 7.166 | 10 681 | 9.77 |
| F4 | 3.6 | 3.6 | 530 | 4871 | 2.268 | 4398 | 7.67 |
| F5 | 6.5 | 6.5 | 720 | 18 330 | 15.21 | 16 366 | 11.62 |
| F6 | 4.4 | 4.4 | 500 | 8145 | 4.84 | 8042 | 10.37 |
| F7 | 8.1 | 8.1 | 870 | 28 020 | 28.54 | 24 504 | 11.97 |

| Footing | SDM method | SAFE software |
|---------|-------------|---------------|
|         | Width (B) (m) | Length (L) (m) | Least width (b) (mm) | Thickness (h) (mm) | As (mm²) | Concrete volume (m³) | As (mm²) | Settlement (mm) |
| F1 | 2.5 | 2.5 | 1.25 | 350 | 2406 | 1.64 | 2434 | 5.74 |
| F2 | 3.5 | 3.5 | 1.75 | 350 | 3369 | 3.22 | 3377 | 2.15 |
| F3 | 4.3 | 4.3 | 2.15 | 750 | 10 160 | 10.4 | 10 053 | 12.08 |
| F4 | 2.9 | 2.9 | 1.45 | 500 | 4314 | 3.15 | 4398 | 10.13 |
| F5 | 5.5 | 5.5 | 2.75 | 1000 | 17 810 | 22.7 | 17 907 | 13.74 |
| F6 | 3.6 | 3.6 | 1.8 | 700 | 7245 | 6.8 | 7854 | 12.98 |
| F7 | 6.6 | 6.6 | 3.3 | 800 | 16 750 | 26.14 | 18 477 | 13.30 |
In this section, the selected seven footings are designed and analyzed as eccentric circular footing. The diameter obtained for each circular footing gives safe and economical design against the applied load. These circular footings are analyzed using the computer software (SAFE) as well and results obtained are displayed in Table 3.

The reinforcement results obtained from the safe software are quite close to the ones obtained from the SDM approach with a percentage difference of 1% to 5%. This indicates the accuracy of the design formulas of circular footing using the SDM method. The required steel area results obtained are also displayed in bar chart (Figure 8). Moreover, the deflection contours obtained from the SAFE software are shown in Figure 9.

5.3 Triangular footing

This section includes the analysis and design of the selected seven footings as eccentric triangular footings. In this study, both sides of the triangle are kept equal for the design and analysis purposes. Also, the proposed formulas using the SDM
method work only for triangles with equal sides as this is the most common shaped triangular footing used in construction industry. For unequal legs, the formulas are needed to be changed accordingly. These triangular footings are also analyzed using the computer software (SAFE) and the results obtained are illustrated in Table 4.

The reinforcement results obtained from the safe software are corresponding to the ones obtained from the SDM approach with a percentage difference of 1% to 12%. This indicates the precision of the design formulas for equal leg triangular footing sections using the SDM method. The required steel area results obtained are also displayed in bar chart (Figure 10). Moreover, the deflection contours obtained from the SAFE software are shown in Figure 11.

5.4 | Trapezoidal footing

This section includes the analysis and design of the selected seven footings as eccentric trapezoidal footings using SDM. The width (B) and length (L) of the trapezoidal section as shown in Figure 2 are kept similar in this study. The triangular section is a variant of the trapezoidal section in which the least width dimension (b) of trapezoidal section is set as zero.
These selected footings are analyzed using the computer software (SAFE) as well and the results obtained are illustrated in Table 5.

The results obtained from the safe software and SDM are quite alike, with a percentage difference of 2% to 9%. The efficiency of design formulas of trapezoidal sections using the SDM method is thus further confirmed. The area of required steel results obtained is also displayed in bar chart (Figure 12). Moreover, the deflection contours obtained from the SAFE software are shown in Figure 13.

The results obtained from all of the selected shaped footing indicate that the SDM is an easy approach to design and analyze the irregular shaped footings (square, circular, triangular, and trapezoidal). Moreover, a cost comparison is also made to select the best optimum footing shape design against the applied loading.

The concrete cost is to be calculated against the volume of concrete required in each footing in addition to the required area of steel. The required concrete volume in each footing is calculated and mentioned in Tables 2 to 5 for square, circular, triangular, and trapezoidal shaped footing, respectively. The footing shape with higher concrete volume will have higher construction cost in comparison with the lower concrete volume footing. The concrete volume results obtained from each footing sections (Figure 14) reveal that the circular and triangular footings prove to be the most economical footings...
**Trapezoidal Footing**

![Graph showing area of steel comparison for trapezoidal footings](image1)

**FIGURE 12** Area of steel ($A_s$) comparison for trapezoidal footings

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**FIGURE 13** Settlement contours of trapezoidal footings

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**FIGURE 14** Concrete volume comparison for all footings

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**FIGURE 15**
followed by square and trapezoidal shaped footing sections. Also, triangular footings under heavy load tend to have a larger steel area (As) as in footing F-7 so it is not economical for such load.

Moreover, the area of steel (As) comparison for each footing shape section is displayed in Figure 15.

The bar chart in Figure 16 shows that all footings displayed an acceptable settlement value which is less than the value allowable according to the ACI code of design (ACI 318-14) indicating safe design.

6 | CONCLUSION

This article presents the irregular shaped reinforced concrete footings supporting square column subjected to eccentric loading, that is, axial load P and biaxial moments; $M_x$ in $(x-x)$ axis and $M_y$ in $(y-y)$ axis, respectively, by a simplified analytical model. Footings with four different shapes (square, circular, triangular, and trapezoidal) are studied in this research. The SDM is used to derive the formulas needed to analyze and design these footings.

Seven footings (F-1 to F-7) with different loading conditions are analyzed and designed individually as square, circular, triangular, and trapezoidal shaped footings, respectively, to get the optimum shaped design footing based on concrete volume and steel weight represented by steel area (As). The footings are analyzed and designed according to the ACI code of design (ACI 318R-14).

The area of steel required in each footing obtained from the SDM method showed promising results when compared with the finite element software (SAFE) with a percentage difference of 1% to 13%, respectively. Also, the concrete volume results obtained from each footing sections revealed that the circular and triangular footings prove to be the most economical footings followed by square and trapezoidal shaped footing sections.

However, triangular footings are not economical for heavy loads, as these require larger steel area as in footing F-7 under such loads. Even though square footing is not the most economical choice as far as concrete volume and steel weight (As) are concerned, yet it is used most commonly because it is easier to construct, saves time and labor work. All footings show a settlement value that is acceptable and less than the allowable limit indicating a secure and efficient design.
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CONFLICT OF INTEREST
The authors declare that there is no conflict of interest regarding the publication of this article.

AUTHOR CONTRIBUTIONS
Mohammed S. Al-Ansari: Data curation; formal analysis; investigation; methodology; supervision; validation; writing-original draft; writing-review and editing. Muhammad S. Afzal: Data curation; formal analysis; investigation; methodology; software; supervision; validation; writing-original draft; writing-review and editing.

NOTATIONS

| Symbol  | Description                                      |
|---------|--------------------------------------------------|
| $F_A$   | footing area                                     |
| $Q_a$   | allowable soil pressure                          |
| $Q_e$   | effective soil pressure                          |
| $W_c$   | concrete weight                                  |
| $W_s$   | soil weight                                      |
| $h$     | total footing depth                              |
| $d$     | effective depth                                  |
| $d'$    | cover to steel centroid                          |
| $V_u$   | factored shear force                             |
| $\varphi_s$ | shear reduction factor                         |
| $\nu_c$ | shear stress carried by the concrete             |
| $b_w$   | footing width                                    |
| $\beta_C$ | ratio of long side of the column to the short of the column |
| $f'_c$  | specified compression strength of concrete       |
| $b_0$   | perimeter around the punching area               |
| $\propto S$ | ratio equals 40, 30, and 20 for interior column, edge column, and corner column respectively |
| $Mu$    | fully factored bending moment                    |
| $L_p$   | maximum projected length                         |
| $q_u$   | bearing pressure for strength design             |
| $DLF$   | dead load factor equal 1.2                       |
| $LLF$   | live load factor equal 1.6                       |
| $\varphi_b$ | bending reduction factor                       |
| $f_y$   | specified yield strength of non-prestressed reinforcing |
| $A_s$   | area of tension steel                            |
| $d_b$   | bar diameter                                     |
| $a$     | depth of the compression block                   |
| $L_d$   | Required bar developmental length                |
| $L_{dT,Available}$ | available length in tension                |
| $A_{s,dowels}$ | steel area of the dowels                        |
| $A_{Column}$ | column area                                      |
| $L_{d,Comp}$ | required bar developmental length in compression |
| $L_{d,C,Available}$ | available length in compression                |

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