Analysis of directed flow from three-particle correlations

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We present a new method for analysing directed flow, based on a three-particle azimuthal correlation. It is less biased by nonflow correlations than two-particle methods, and requires less statistics than four-particle methods. It is illustrated on NA49 data.

1. INTRODUCTION

The measurement of the successive harmonics of azimuthal correlations in a heavy ion collision \cite{1}, \( v_n \equiv \langle e^{in(\Phi - \Phi_R)} \rangle \), where \( \Phi_R \) is the impact parameter direction, is of paramount importance, since it yields information on the medium created in the collision. In particular, a novel behaviour has been predicted for the first harmonic \( v_1 \), the so-called directed flow, at ultrarelativistic energies \cite{2}.

However, the analysis of directed flow at these energies is highly nontrivial, because \( v_1 \) is very small. Thus, methods based on an analysis of two-particle azimuthal correlations \cite{3} are likely to be biased by “nonflow” two-particle correlations: quantum (HBT) correlations between identical particles and correlations due to global momentum conservation have been shown to be important at SPS \cite{4}, while correlations due to minijets may be large at RHIC \cite{5}. On the other hand, methods relying on four-particle correlations \cite{6}, which are free from this bias, are plagued by a lack of statistics due to the smallness of \( v_1 \), although they give good results for the analysis of elliptic flow \( v_2 \) (see Sec.\ 2). To remedy these shortcomings, we proposed a new method of \( v_1 \) analysis \cite{7}, based on the measurement of a mixed three-particle correlation, which involves both \( v_1 \) and \( v_2 \):

\[
\langle e^{i(\phi_a + \phi_b - 2\phi_c)} \rangle \simeq (v_1)^2 v_2,
\]

where \( \phi_a, \phi_b, \) and \( \phi_c \) denote the azimuths of three particles belonging to the same event, and the average runs over triplets of particles emitted in the collision, and over events. Once \( v_2 \) has been obtained from a separate analysis, this equation yields \( (v_1)^2 v_2 \), thus \( v_1 \).

Here, we apply this method to NA49 data on Pb-Pb collisions at 158 AGeV. Results obtained using the “standard” flow analysis are given in Ref. \cite{8}. In our method, the first step in the analysis is the measurement of a reference \( v_2 \). Then, an equation analogous to Eq. \cite{1} yields an estimate of the integrated \( v_1 \), i.e., its average value over some phase space region. Finally, restricting \( \phi_1 \) in Eq. \cite{1} to a small \((p_T, y)\) bin allows one to obtain

*Supported by “Actions de Recherche Concertées” of “Communauté Française de Belgique” and IISN-Belgium
more detailed, differential measurements of $v_1$ as a function of transverse momentum or rapidity.

2. ELLIPTIC FLOW FROM 2, 4, 6, 8-PARTICLE CORRELATIONS

As stated in the introduction, our method of analysis of directed flow $v_1$ requires the preliminary knowledge of an estimate of the elliptic flow $v_2$, integrated over some phase space region. Of course, this estimate must be obtained by analysing the same sample of events from which one wants to extract $v_1$.

In practice, the average over phase space is a weighted average:

$$\langle w_2 v_2 \rangle \equiv \left\langle w_2 e^{2i(\phi - \Phi_R)} \right\rangle,$$

where $w_2$ is the chosen weight. In order to reduce statistical fluctuations, $w_2$ must be larger for particles with stronger elliptic flow.

The value of $\langle w_2 v_2 \rangle$ is obtained using the cumulant method described in Ref. [6]: one can extract estimates of $\langle w_2 v_2 \rangle$ from cumulants of multi- (2-, 4-, 6-...) particle correlations. While two-particle methods are equivalent to the standard flow analysis, higher orders are essentially free from nonflow effects. They were first used in analysing data obtained by the STAR Collaboration at RHIC [9].

In Fig. 1, we present application of the method to NA49 data, and show the dimensionless quantity

$$v_2 \equiv \frac{\langle w_2 v_2 \rangle}{\sqrt{\langle (w_2)^2 \rangle}},$$

as a function of centrality for charged particles, where we have used $w_2 = p_T$. We display estimates using cumulants of two-, four-, six-, and even eight-particle correlations [6], as well as the corresponding quantity for charged pions obtained from the “standard” subevent method [3].

It is quite remarkable that the four-, six-, and eight-particle estimates all agree: this supports the idea that they are indeed free from nonflow effects, and correspond to a genuine collective motion in the direction of the impact parameter. Moreover, these multiparticle estimates show a slight discrepancy with the two-particle values, as expected if nonflow correlations are sizable [6]. Please note that the statistical uncertainties on high order cumulants remain reasonably small, especially for midcentral collisions. In the following, our reference elliptic flow value will preferably be the estimate from the four-particle cumulant, which is a priori the most reliable since it is free from nonflow correlations and has a smaller statistical error than higher order estimates.

3. INTEGRATED DIRECTED FLOW FROM 2, 3, 4-PARTICLE CORRELATIONS

The next step in the analysis is to determine the average value of directed flow over some phase space region. As in the case of elliptic flow, we perform a weighted average

$$\langle w_1 v_1 \rangle \equiv \left\langle w_1 e^{i(\phi - \Phi_R)} \right\rangle,$$
where stronger weight is given to particles with stronger directed flow. In the NA49 analysis, we used a rapidity dependent weight $w_1 = y - y_{CM}$, where $y_{CM}$ is the centre-of-mass rapidity.

This weighted average is obtained from the following three-particle correlation:

$$
\langle w_1(a)w_1(b)w_2(c)e^{i(\phi_a+\phi_b-2\phi_c)} \rangle = \langle w_1v_1 \rangle^2 \langle w_2v_2 \rangle, 
$$

(5)

where $w_2$ is the same as in Sec. 2. Using the value of $\langle w_2v_2 \rangle$ obtained in Sec. 2, we thus derive $\langle w_1v_1 \rangle$, up to a global sign.

In practice, the left-hand side of Eq. 5 is constructed using a generating function formalism detailed in Ref. [7]. This procedure is a very efficient way to sum over all possible triplets of particles, and also to remove automatically the effects of slight detector anisotropies.

The method was applied to NA49 data. In Fig. 2, we present as a function of centrality the dimensionless quantity $v_1 \equiv \langle w_1v_1 \rangle / \sqrt{\langle (w_1)^2 \rangle}$, analogous to Eq. 4, together with two- and four-particle estimates obtained with the method of Ref. [6]. It is worth noting that the statistical uncertainty on the three-particle estimate is barely larger than that on the two-particle value, while the systematic error due to nonflow correlations (not included in the plot) is a priori much smaller.

4. DIFFERENTIAL DIRECTED FLOW

Finally, we want to obtain detailed measurements of $v_1$, as a function of $p_T$ or $y$ and for each particle type. To derive $v_1(p_T, y)$, we use a three-particle correlation analogous to Eq. 5, where the particle labeled $a$ is restricted to a narrow $(p_T, y)$ bin:

$$
\langle w_1(b)w_2(c)e^{i(\phi_a+\phi_b-2\phi_c)} \rangle = \langle w_1v_1 \rangle \langle w_2v_2 \rangle v_1(p_T, y). 
$$

(6)

With the previously derived $\langle w_2v_2 \rangle$ and $\langle w_1v_1 \rangle$, this equation yields $v_1(p_T, y)$. We illustrate this method on the differential flow of charged pions in midcentral collisions, shown in

![Figure 1. Weighted elliptic flow $v_2$ of charged particles as a function of centrality (central left, peripheral right) for Pb-Pb collisions at 158 AGeV.](image1)

![Figure 2. Weighted directed flow $v_1$ of charged particles as a function of centrality (central left, peripheral right).](image2)
Figs. 3 and 4 together with two two-particle estimates, either uncorrected or corrected for the effect of momentum conservation [10].

At high $p_T$, $v_1$ from three-particle correlations is consistent with the two-particle value corrected for momentum conservation, but significantly lower than the uncorrected one. This shows that correlations from momentum conservation, which are large, are automatically removed in our method. This is also reflected by the behaviour at midrapidity, where $v_1$ vanishes, as it should by symmetry, while the uncorrected two-particle estimate does not. The three-particle estimate also vanishes more smoothly at $p_T = 0$ than two-particle estimates which may be biased by HBT correlations [4].

![Figure 3](image1.jpg)

Figure 3. Directed flow $v_1$ of charged pions as a function of transverse momentum for midcentral Pb-Pb collisions at 158 AGeV.

![Figure 4](image2.jpg)

Figure 4. Directed flow $v_1$ of charged pions as a function of $y - y_{CM}$ for midcentral collisions. Open points are reflected with respect to midrapidity.

ACKNOWLEDGEMENTS

We thank the NA49 Collaboration for permission to use their data.

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