NEUTRINOS FROM THE NEXT GALACTIC SUPERNova

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The next core-collapse supernova in our Galaxy will be a spectacular event, with some $10^4$ neutrino detections in total expected among several detectors. This data will allow unprecedented tests of neutrino properties and new opportunities in astrophysics. In this paper, I focus on two main topics: (1) Measurement of the $\nu_\mu$ and $\nu_\tau$ masses by time-of-flight, with an emphasis on introducing as little supernova model dependence as possible, and (2) Methods for locating a supernova by its neutrinos in advance of the light, which may allow improved astronomical observations. In the latter, I also discuss the recent result that the positrons from $\bar{\nu}_e + p \rightarrow e^+ + n$ are not isotropically emitted, as commonly thought.

1 Supernova Neutrino Mass Measurements

1.1 Current Knowledge About Neutrino Masses

What is known about the neutrino masses, in terms of direct experimental evidence? After several decades of experiments on the tritium beta spectrum, the published limit on the mass of the $\bar{\nu}_e$ (and hence also $\nu_e$) is now about 3 eV. These results are still attended by some controversy, due to apparent systematic errors near the endpoint (discussed in detail at a recent workshop at the Institute for Nuclear Theory in Seattle). Much less is known about the $\nu_\mu$ mass ($< 170$ keV from $\pi$ decay) and the $\nu_\tau$ mass ($< 18$ MeV from $\tau$ decay). It would be a great advance if the latter two limits could be improved to the eV or tens of eV level, and it seems unlikely that any terrestrial experiment could do that.

There are two indirect arguments which may restrict the $\nu_\mu$ and $\nu_\tau$ masses. The first says that the predicted background of relic neutrinos ($\approx 100$/cm$^3$) should not overclose the universe. This leads to a limit on the sum of the neutrino masses of about 100 eV in the most conservative case, and about 10 eV if reasonable values for the non-baryonic dark matter density and the Hubble constant are used. The second is based on the recent evidence for neutrino oscillations (sensitive only to differences of neutrino masses). For three neutrino flavors, there are two independent mass differences, which can be chosen to explain the solar and atmospheric neutrino data. The overall scale can be fixed by using the limit from tritium beta decay, and since the observed mass differences are small, then all masses are below a few eV.
These indirect arguments sound compelling, but they are not hard to evade. The cosmological bound doesn’t apply if the neutrinos are allowed to decay. There are many such models for heavy \( \nu_\tau \) masses, motivated by particle physics or astrophysics. The oscillation bound doesn’t apply if there are more free mass differences than there are positive signals of neutrino oscillations. For example, suppose the atmospheric neutrinos are undergoing maximal \( \nu_\mu \leftrightarrow \nu_\tau \) mixing, LSND is ruled out, and the solar neutrinos are undergoing \( \nu_e \leftrightarrow \nu_{\text{sterile}} \) mixing. Then nothing prevents the \( \nu_2 \) and \( \nu_3 \) masses from having a tiny splitting near 50 or even 500 eV, and leaving the \( \nu_1 \) mass below a few eV.

It may be possible in the future to make new astrophysical tests of the neutrino masses, using data on large-scale structure, the cosmic microwave background, weak lensing, and the Lyman-alpha forest. While these techniques claim sensitivities of order 1 - 10 eV for the sum of the neutrino masses, none are based on direct detection of neutrinos, and hence may be vulnerable to uncertainties in their assumptions.

Thus while indirect evidence will be valuable, there is still a need for a direct measurement of the \( \nu_\mu \) and \( \nu_\tau \) masses (or, in the presence of mixing, the masses of the heavy mass eigenstates).

1.2 Supernova Neutrino Emission

When the core of a large star (\( M \geq 8M_\odot \)) runs out of nuclear fuel, it collapses, with a change in the gravitational binding energy of about \( 3 \times 10^{53} \) ergs. This huge amount of energy must be removed from the proto-neutron star, but the high density prevents the escape of any radiation. Inside of the hot proto-neutron star, neutrino-antineutrino pairs of all flavors are produced. Despite their weak interactions, even the neutrinos are trapped and diffuse out over several seconds, in the end carrying away about 99% of the supernova energy. (I have neglected the flux of \( \nu_e \) neutrinos necessary to change the core from \( N \simeq Z \) nuclei into a neutron star, because the total energy release of this flux is of order 1% of the pair emission phase).

When the neutrinos are about one mean free path from the edge, they escape freely, with a thermal spectrum (approximately Fermi-Dirac) characteristic of the surface of last scattering. Because different flavors have different interactions with the matter, and because the neutron star temperature is decreasing with increasing radius, the neutrino decoupling temperatures are different. The \( \nu_\mu \) and \( \nu_\tau \) neutrinos and their antiparticles have a temperature of about 8 MeV, the \( \bar{\nu}_e \) neutrinos about 5 MeV, and the \( \nu_e \) neutrinos about 3.5 MeV. Equivalently, the average energies are about \( \langle E \rangle \simeq 25 \) MeV, \( \simeq 16 \) MeV,
and \( \simeq 11 \) MeV, respectively. The luminosities of the different neutrino flavors are approximately equal at all times. The neutrino luminosity rises quickly over 0.1 s or less, and then falls off over several seconds. The SN1987A data can be reasonably fit to a decaying exponential with time constant \( \tau = 3 \) s. The detailed form of the neutrino luminosity used below is less important than the general shape features and their characteristic durations.

The estimated core-collapse supernova rate in the Galaxy is about 3 times per century\(^{[4]}\). The present neutrino detectors can easily observe a supernova anywhere in the Galaxy or its immediate companions (e.g., the Magellanic Clouds). Unfortunately, the present detectors do not have large enough volumes to observe a supernova in even the nearest galaxy (Andromeda, about 700 kpc away).

### 1.3 Time-of-Flight Concept

Even a tiny neutrino mass will make the velocity slightly less than for a massless neutrino, and over the large distance to a supernova will cause a measurable delay in the arrival time. A neutrino with a mass \( m \) (in eV) and energy \( E \) (in MeV) will experience an energy-dependent delay (in s) relative to a massless neutrino in traveling over a distance \( D \) (in 10 kpc, approximately the distance to the Galactic center) of

\[
\Delta t(E) = 0.515 \left( \frac{m}{E} \right)^2 D, \tag{1}
\]

where only the lowest order in the small mass has been kept. A distance of 10 kpc corresponds to a travel time of about \( 10^{12} \) s, and the delays of interest are less than 1 s.

If the neutrino mass is nonzero, lower-energy neutrinos will arrive later, leading to a correlation between neutrino energy and arrival time. This is exploitable in some of the charged-current detection reactions for \( \nu_e \) and \( \bar{\nu}_e \) since the neutrino energy can be reasonably measured. Using this idea, the next supernova will allow sensitivity to a \( \bar{\nu}_e \) mass down to about 3 eV\(^{[4]}\). However, the terrestrial experiments now claim\(^{[4]}\) to rule out a mass that large, so I ignore the \( \nu_e \) mass in describing the supernova neutrino signal.

For measuring the masses of the \( \nu_\mu \) and \( \nu_\tau \) neutrinos, one must realize that only neutral-current detection reactions are possible (the charged-current thresholds are too high). Since the neutrino energy is not measured in neutral-current interactions, an energy-time correlation technique cannot be used (the incoming neutrino energy is not determined since a complete kinematic reconstruction of the reaction products is not possible).
Instead, the strategy for measuring the $\nu_\tau$ mass is to look at the difference in time-of-flight between the neutral-current events (mostly $\nu_\mu, \nu_\tau, \bar{\nu}_\mu$, and $\bar{\nu}_\tau$, because of the higher temperature of those flavors) and the charged-current events (just $\nu_e$ and $\bar{\nu}_e$). We assume that the $\nu_\mu$ is massless and will ask what limit can be placed on the $\nu_\tau$ mass (other cases will be discussed below). There are three major complications to a simple application of Eq. (1): (i) The neutrino energies are not fixed, but are characterized by spectra; (ii) The neutrino pulse has a long intrinsic duration of about 10 s, as observed for SN1987A; and (iii) The statistics are finite. In some early work, the duration of the pulse was thought to be of order 100 milliseconds, nearly a delta-function. Here (considering the case of the smallest detectable mass), the delay is much less than the width of the pulse, which is what makes the problem much more difficult (the large-mass case is covered elsewhere).

Given that the energy-time correlation method cannot be used, we must turn to considerations of the shape of the event rate as a function of time.

### 1.4 Neutrino Scattering Rate

For a time-independent spectrum $f(E)$ and luminosity $L(t_i)$, the double-differential number distribution at the source is

$$\frac{d^2 N_\nu}{dE_\nu dt_i} = f(E_\nu) \frac{L(t_i)}{\langle E_\nu \rangle}. \quad (2)$$

The neutrino spectrum and average energy $\langle E_\nu \rangle$ are different for each flavor, but the luminosities $L(t_i)$ of the different flavors are roughly equal, so that each flavor carries away about $1/6$ of the total binding energy $E_B$. For a massive neutrino, the double-differential number distribution at the detector is

$$\frac{d^2 N_\nu}{dE_\nu dt} = f(E_\nu) \frac{L(t - \Delta t(E_\nu))}{\langle E_\nu \rangle} \quad (3)$$

and the scattering rate for a given reaction is

$$\frac{dN_{sc}}{dt} = C \int dE_\nu f(E_\nu) \left[ \frac{\sigma(E_\nu)}{10^{-42} \text{cm}^2} \right] \left[ \frac{L(t - \Delta t(E_\nu))}{E_B/6} \right]. \quad (4)$$

The overall constant for an H$_2$O target is

$$C = 9.21 \left[ \frac{E_B}{10^{53} \text{ ergs}} \right] \left[ \frac{1 \text{ MeV}}{T} \right] \left[ \frac{10 \text{ kpc}}{D} \right]^2 \left[ \frac{\text{det. mass}}{1 \text{ kton}} \right] n, \quad (5)$$

where $n$ is the number of targets per molecule for the given reaction. For a D$_2$O target, the leading factor becomes 8.28 instead of 9.21.
Table 1: Calculated numbers of events expected in SK (32 kton H$_2$O) with a 5 MeV threshold and a supernova at 10 kpc. The other parameters (e.g., neutrino spectrum temperatures) are given in the text. In rows with two reactions listed, the number of events is the total for both. The second row is a subset of the first row that is an irreducible background to the reactions in the third and fourth rows. The detected final-state particles are $e^+, e^-, \gamma$.

| reaction | number of events |
|----------|------------------|
| $\bar{\nu}_e + p \rightarrow e^+ + \gamma$ | 8300 |
| $\bar{\nu}_e + p \rightarrow e^+ + n$ ($E_{e^+} \leq 10$ MeV) | 530 |
| $\nu_\mu + ^{16}$O $\rightarrow \nu_\mu + \gamma + X$ | 355 |
| $\bar{\nu}_\mu + ^{16}$O $\rightarrow \bar{\nu}_\mu + \gamma + X$ | 355 |
| $\nu_\tau + ^{16}$O $\rightarrow \nu_\tau + \gamma + X$ | 355 |
| $\bar{\nu}_\tau + ^{16}$O $\rightarrow \bar{\nu}_\tau + \gamma + X$ | 355 |
| $\nu_e + e^- \rightarrow \nu_e + e^-$ | 200 |
| $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$ | 60 |
| $\nu_\mu + e^- \rightarrow \nu_\mu + e^-$ | 60 |
| $\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$ | 60 |
| $\nu_\tau + e^- \rightarrow \nu_\tau + e^-$ | 60 |
| $\bar{\nu}_\tau + e^- \rightarrow \bar{\nu}_\tau + e^-$ | 60 |

For a massless neutrino, $\Delta t(E_\nu) = 0$, so the luminosity comes outside of the integral as $L(t)$ and completely specifies the time dependence. For a massive neutrino, the time dependence from the luminosity is modified in an energy-dependent way by the mass effects, as above.

1.5 Separation of the Massive Signal

The scattering rates of the massless neutrinos ($\nu_e$ or $\bar{\nu}_e$) can then be used to measure the shape of the luminosity as a function of time. The scattering rate for the massive neutrinos ($\nu_\tau$) can be tested for additional time dependence due to a mass. We define two rates: a Reference $R(t)$ containing only massless events, and a Signal $S(t)$ containing some fraction of massive events.

The Reference $R(t)$ can be formed in various ways, for example from the charged-current reaction $\bar{\nu}_e + p \rightarrow e^+ + n$ in the light water of SK or SNO. (The numbers of events expected for all reactions are given in Table I for SK and Table II for SNO).

The Signal $S(t)$ can be based on various neutral-current reactions, though here I will focus on the results for SNO. Thus the primary component of the Signal $S(t)$ are the 485 neutral-current events on deuterons. With the hierarchy of temperatures assumed here, these events are 18% ($\nu_e + \bar{\nu}_e$), 41%
Table 2: Calculated numbers of events expected in SNO for a supernova at 10 kpc. The other parameters (e.g., neutrino spectrum temperatures) are given in the text. In rows with two reactions listed, the number of events is the total for both. The middle column indicates the detected particle(s). The notation $\nu$ indicates the sum of $\nu_e$, $\nu_\mu$, and $\nu_\tau$, though they do not contribute equally to a given reaction, and $X$ indicates either $n+^{15}$O or $p+^{15}$N.

| Events in 1 kton $D_2O$ |  |
|-------------------------|---|
| $\nu + d \rightarrow \nu + p + n$ | $n$ |
| $\bar{\nu} + d \rightarrow \bar{\nu} + p + n$ | $n$ |
| $\nu_e + d \rightarrow e^- + p + p$ | $e^-$ |
| $\bar{\nu}_e + d \rightarrow \bar{\nu} + e^+ + n + n$ | $e^+ nn$ |
| $\nu + ^{16}O \rightarrow \nu + \gamma + X$ | $\gamma, \gamma n$ |
| $\bar{\nu} + ^{16}O \rightarrow \bar{\nu} + \gamma + X$ | $\gamma, \gamma n$ |
| $\nu + ^{16}O \rightarrow \nu + n + ^{15}O$ | $n$ |
| $\bar{\nu} + ^{16}O \rightarrow \bar{\nu} + n + ^{15}O$ | $n$ |
| $\nu + e^- \rightarrow \nu + e^-$ | $e^-$ |
| $\bar{\nu} + e^- \rightarrow \bar{\nu} + e^-$ | $e^-$ |

| Events in 1.4 kton $H_2O$ |  |
|-------------------------|---|
| $\nu_e + p \rightarrow e^+ + n$ | $e^+$ |
| $\nu + ^{16}O \rightarrow \nu + \gamma + X$ | $\gamma$ |
| $\bar{\nu} + ^{16}O \rightarrow \bar{\nu} + \gamma + X$ | $\gamma$ |
| $\nu + e^- \rightarrow \nu + e^-$ | $e^-$ |
| $\bar{\nu} + e^- \rightarrow \bar{\nu} + e^-$ | $e^-$ |

$(\nu_\mu + \bar{\nu}_\mu)$, and 41% $(\nu_\tau + \bar{\nu}_\tau)$. The flavors of the neutral-current events of course cannot be distinguished. Under our assumption that only $\nu_\tau$ is massive, there is already some unavoidable dilution of $S(t)$.

In Fig. 1, $S(t)$ is shown under different assumptions about the $\nu_\tau$ mass. The shape of $R(t)$ is exactly that of $S(t)$ when $m_{\nu_\tau} = 0$, though the number of events in $R(t)$ will be different. The rates $R(t)$ and $S(t)$ will be measured with finite statistics, so it is possible for statistical fluctuations to obscure the effects of a mass when there is one, or to fake the effects when there is not. We determine the mass sensitivity in the presence of the statistical fluctuations by Monte Carlo modeling. We use the Monte Carlo to generate representative statistical instances of the theoretical forms of $R(t)$ and $S(t)$, so that each run represents one supernova as seen in SNO. The best model-independent test of a $\nu_\tau$ mass seems to be a test of the average arrival time $\langle t \rangle$. Any massive component in $S(t)$ will always increase $\langle t \rangle$, up to statistical fluctuations.
Figure 1: The expected event rate for the Signal $S(t)$ at SNO in the absence of fluctuations for different $\nu_\tau$ masses, as follows: solid line, 0 eV; dashed lines, in order of decreasing height: 20, 40, 60, 80, 100 eV. Of 535 total events, 100 are massless ($\nu_e + \bar{\nu}_e$), 217.5 are massless ($\nu_\mu + \bar{\nu}_\mu$), and 217.5 are massive ($\nu_\tau + \bar{\nu}_\tau$). These totals count events at all times; in the figure, only those with $t \leq 9$ s are shown.
1.6 \( \langle t \rangle \) Analysis

Given the Reference \( R(t) \) (i.e., the charged-current events), the average arrival time is defined as

\[
\langle t \rangle_R = \frac{\sum_k t_k}{\sum_k 1},
\]

where the sum is over events in the Reference. The effect of the finite number of counts \( N_R \) in \( R(t) \) is to give \( \langle t \rangle_R \) a statistical error:

\[
\delta (\langle t \rangle_R) = \frac{\sqrt{\langle t^2 \rangle_R - \langle t \rangle_R^2}}{\sqrt{N_R}}.
\]

For a purely exponential luminosity, \( \langle t \rangle_R = \sqrt{\langle t^2 \rangle_R - \langle t \rangle_R^2} = \tau \simeq 3 \text{ s} \). Given the Signal \( S(t) \) (i.e., the neutral-current events), the average arrival time \( \langle t \rangle_S \) and its error \( \delta (\langle t \rangle_S) \) are defined similarly. The signal of a mass is that the measured value of \( \langle t \rangle_S - \langle t \rangle_R \) is greater than zero with statistical significance.

Using the Monte Carlo, we analyzed \( 10^4 \) simulated supernova data sets for a range of \( \nu_\tau \) masses. For each data set, \( \langle t \rangle_S - \langle t \rangle_R \) was calculated and its value histogrammed. These histograms are shown in the upper panel of Fig. 2 for a few representative masses. (Note that the number of Monte Carlo runs only affects how smoothly these histograms are filled out, and not their width or placement.) These distributions are characterized by their central point and their width, using the 10%, 50%, and 90% confidence levels. That is, for each mass we determined the values of \( \langle t \rangle_S - \langle t \rangle_R \) such that a given percentage of the Monte Carlo runs yielded a value of \( \langle t \rangle_S - \langle t \rangle_R \) less than that value. With these three numbers, we can characterize the results of complete runs with many masses much more compactly, as shown in the lower panel of Fig. 2. Given an experimentally determined value of \( \langle t \rangle_S - \langle t \rangle_R \), one can read off the range of masses that would have been likely (at these confidence levels) to have given such a value of \( \langle t \rangle_S - \langle t \rangle_R \) in one experiment. From the lower panel of Fig. 2, we see that SNO is sensitive to a \( \nu_\tau \) mass down to about 30 eV if the SK \( R(t) \) is used, and down to about 35 eV if the SNO \( R(t) \) is used.

We also investigated the dispersion of the event rate in time as a measure of the mass. A mass alone causes a delay, but a mass and an energy spectrum also causes dispersion (as does the separation of the massive and massless portions of the Signal). We defined the dispersion as the change in the width \( \sqrt{\langle t^2 \rangle_S - \langle t \rangle_S^2} - \sqrt{\langle t^2 \rangle_R - \langle t \rangle_R^2} \). We found that the dispersion was not statistically significant until the mass was of order 80 eV or so, i.e., a plot very similar to Fig. 2 can be formed for the width, and the error band is much wider. Since the dispersion is irrelevant, the average delay is well-characterized by a single energy, which for SNO is \( E_c \simeq 32 \text{ MeV} \).
Figure 2: The results of the $\langle t \rangle$ analysis for a massive $\nu_\tau$, using the Signal $S(t)$ from SNO defined in the text. In the upper panel, the relative frequencies of various $\langle t \rangle_S - \langle t \rangle_R$ values are shown for a few example masses. The solid line is for the results using the SK Reference $R(t)$, and the dotted line for the results using the SNO $R(t)$. In the lower panel, the range of masses corresponding to a given $\langle t \rangle_S - \langle t \rangle_R$ is shown. The dashed line is the 50% confidence level. The upper and lower solid lines are the 10% and 90% confidence levels, respectively, for the results with the SK $R(t)$. The dotted lines are the same for the results with the SNO $R(t)$.
1.7 Analytic Results

Another nice feature of the proposed \( \langle t \rangle \) analysis, besides its simplicity, is that good estimates can be made analytically. The characteristic delay is

\[
\langle t \rangle_S - \langle t \rangle_R \simeq \text{frac}(m > 0) \times 0.515 \left( \frac{m}{E_c} \right)^2 D, \tag{8}
\]

where \( \text{frac}(m > 0) \) is the fraction (about 1/2) of massive events in the neutral-current Signal. The above numerical analysis shows that the mass effects occur most significantly in the delay and not the width (are characterized with a single energy, and negligible dispersion). That characteristic energy \( E_c \) is roughly at the peak of \( f(E)\sigma(E) \) (see Fig. 3). This formula for the first moment is always true; the point of the Monte Carlo analysis was to show that the other moments are not changed, so this completely describes the data.

If the cross section \( \sigma(E_\nu) \) depends on energy as \( E_\nu^\alpha \) (\( \alpha \sim 2 \) for \( \nu + d \)), then the characteristic energy \( E_c \sim (2 + \alpha)T \) and the thermally-averaged cross section is proportional to \( T^\alpha \), where \( T \) is the \( \nu_\mu \) and \( \nu_\tau \) temperature. For \( \alpha = 2 \), and a neutron detection efficiency \( \epsilon_n \), the following scaling relations hold:

\[
\langle t \rangle_S - \langle t \rangle_R \sim \left( \frac{m}{T} \right)^2 D. \tag{9}
\]

\[
N_S \sim \frac{1}{T D^2} T^2 \epsilon_n \sim \frac{T \epsilon_n}{D^2}, \tag{10}
\]

\[
\delta (\langle t \rangle_S - \langle t \rangle_R) \sim \frac{\tau}{\sqrt{N_S}} \sim \frac{\tau D}{\sqrt{T \epsilon_n}}. \tag{11}
\]

For a non-exponential luminosity, \( \tau \) is more generally the width of the pulse. If zero delay is measured, then the mass limit is determined by the largest positive delay that could have fluctuated to zero, and thus

\[
m_{\text{limit}} \sim T^{3/4} \tau^{1/2} \epsilon_n^{-1/4}. \tag{12}
\]

Note that is independent of \( D \) for any supernova distance in the Galaxy (i.e., as long as there is a reasonable number of counts). Note that the mass limit scales with the fourth root of the number of events (via the efficiency \( \epsilon_n \)).

Besides the scaling, the value of \( m_{\text{limit}} \) at fixed values of these quantities can also be found easily. The analytic results above are in excellent agreement with the full numerical results. Over reasonable ranges of the input parameters, the results hardly change. However, some estimates of the \( \nu_e \) mass sensitivity for proposed detectors assumed a very short duration of the supernova (\( \tau \simeq 0.5 \) s). If such a short duration were valid, then the SNO sensitivity would also be of order 10 eV.
Figure 3: The shapes of response functions for different reactions. The solid line is a Fermi-Dirac distribution \( f(E_\nu) \) with temperature 8 MeV. The dashed lines are products of that with the various neutral-current cross sections, i.e., \( f(E_\nu)\sigma(E_\nu) \). The normalizations were chosen so that all curves would have the same height. From left to right, the cross sections are \( \nu^- + e^- \), \( \nu^- + d \), \( \nu^- + ^{12}\text{C} \), \( \nu^- + ^{16}\text{O} \). For \(^{12}\text{C}\), only the level at 15.11 MeV was used. For \(^{16}\text{O}\), a fit to the full calculation was used. In a given experiment, only the integral of one of these curves is measured, but each samples the underlying \( f(E_\nu) \) differently. The peak positions are what is called \( E_c \) in the text.

1.8 How Model-Dependent are the Results?

Above, I discussed the proposed \( \langle t \rangle \) technique for analyzing the supernova neutrino data for a \( \nu_\tau \) mass. This technique was designed to be as model-independent as possible. One underlying theme is to calibrate the assumed models as much as possible from the data.

While is is probably a good approximation that the luminosities of the different flavors are equal as a function of time, the normalization of the luminosity drops out in the definition of \( \langle t \rangle \), and hence the results depend on the weaker assumption that the shapes of the luminosities are approximately equal as a function of time. The differences among the different flavors at the very early stages (for example, the higher \( \nu_e \) flux) persist over a duration much less
than 10 s, and hence have no effect on the analysis.

Another very important point is that the details of the shape of the luminosity as a function of time are never used, because the delay is measured just from the $\langle t \rangle$ of the neutral-current events minus the same for the charged-current events. Why is this enough? In general, one could describe the scattering rates of charged-current and neutral-current events by their normalizations and a series of all moments ($\langle t \rangle$, $\langle t^2 \rangle$, etc.). While this would likely require many terms, it is efficient to consider the difference in the moments between the two. The mass effects do not change the normalizations (the numbers of events), and we have shown that the higher moments (e.g., the width) are not significantly changed by a mass. That is, the only statistically significant effects of a mass on the event rate difference are in the average arrival time $\langle t \rangle$. If the details of the supernova model are not known from theory, then $\langle t \rangle$ is what is called a sufficient statistic for the mass.

The only aspect of the shape of the luminosity $L(t)$ that was used in the analysis was its duration, and this appears only in the error on the delay. Thus the particular assumption of using an exponential luminosity completely disappears from the problem. Any other reasonable model of similar duration would give a very similar result.

One also needs to know the $\nu_\tau$ temperature to determine the mass limit. Since none of the neutral-current reactions measure the incoming neutrino energy, there is no direct spectral information. The $\nu_\tau$ temperature must be determined from the number of events. Since different neutral-current reactions have different energy dependence, one can crudely reconstruct the underlying spectrum (see Fig. 3). The moments analysis of the rates reveals that only the average delay is significant, and thus that just a single energy (the characteristic energy $E_c$) is relevant, instead of the fine details of the spectral shape.

The final results for the mass sensitivity are given in Table III.
2 Supernova Location by Neutrinos

2.1 Overview

There has been great interest recently in the question of whether or not a supernova can be located by its neutrinos. If so, this may offer an opportunity to give an early warning to the astronomical community, so that the supernova light curves can be observed from the earliest possible time. An international supernova early alert network has been formed for this purpose, and the details of its implementation were the subject of a recent workshop. One of the primary motivations for such a network is to greatly reduce the false signal rate by demanding a coincidence between several different detectors. The second motivation is to locate the supernova by its neutrino signal.

The interest in the latter is driven by two considerations. First, the neutrinos leave the core at the time of collapse, while the electromagnetic radiation leaves the envelope some hours later, depending on the stellar mass. Thus the neutrino observations can give the astronomers a head start. Second, the next Galactic supernova will likely be in the disk, and hence obscured by dust, and perhaps never visible optically.

There are two classes of techniques to locate a supernova by its neutrinos. The first class of techniques is based on angular distributions of the neutrino reaction products, which can be correlated with the neutrino direction. In this case, a single experiment can independently announce a direction and its error. The second method of supernova location, triangulation, is based on the arrival-time differences of the supernova pulse at two or more widely-separated detectors. This technique would require significant and immediate data sharing among the different experiments.

2.2 Angular Distributions

Neutrino-Electron Scattering: Electron Angular Distribution

In neutrino-electron scattering, the scattered electrons have a very forward angular distribution, due to the small electron mass. At supernova neutrino energies, the angle between the incoming neutrino and the outgoing electron is about $10^\circ$, depending somewhat on neutrino energy and flavor. However, multiple scattering of the struck electron smears out its Čerenkov cone, washing out the dependence on energy and flavor, and introducing an angular resolution of about $25^\circ$.

Naively, if the one-sigma width of the electron angular distribution is $25^\circ$, then the precision with which its center (i.e., the average) can be defined given
\[ \delta \theta \simeq \frac{25^\circ}{\sqrt{N_S}}, \]  

(13)

where \( N_S \) is the number of events. For SK, \( N_S \approx 320 \), so the cone center could be defined to within about 1.5°. For SNO (using both the light and heavy water), \( N_S \approx 25 \), so the cone center could be defined to about 5°. The equivalent error on the cosine is \( \delta(\cos \theta) \simeq (\delta \theta)^2 / 2 \), i.e., \( 3 \times 10^{-4} \) and \( 4 \times 10^{-3} \), respectively.

However, the neutrino-electron scattering events are indistinguishable on an event-by-event basis from other reactions with only a single electron or positron detected. The primary background in SK or the light water in SNO is thus \( \bar{\nu}_e + p \rightarrow e^+ + n \) (in the heavy water in SNO, there are several background reactions).

This problem is similar to the SK solar neutrino studies, in which the neutrino-electron scattering events have to be separated from the intrinsic detector background (due to radioactivity, etc). The SK results are presented as an angular distribution in the cosine along the known direction to the Sun, and a clear bump in the solar direction is seen, with signal/noise \( \simeq 1/10 \). Though the concept is the same, for the supernova events the signal/noise is only \( \simeq 1/30 \). Taking this into account makes the centroiding error larger by a correction factor of \( \simeq 4 \) for both SK and SNO. With some cuts on energy, it may be possible to reduce this correction factor to \( \simeq 2 - 3 \).

**Neutrino-Nucleus Scattering: Lepton Angular Distributions**

In the charged-current reactions of \( \nu_e \) and \( \bar{\nu}_e \) on nucleons or nuclei, the outgoing electrons or positrons typically have angular distributions of the form

\[ \frac{d\sigma}{d \cos \theta} \simeq 1 + v_e a(E_\nu) \cos \theta, \]  

(14)

where \( \theta \) is the angle between the neutrino and electron (or antineutrino and positron) directions in the laboratory (where the nuclear target is assumed to be at rest) and \( v_e \) is the lepton velocity in \( c = 1 \) units. At energies higher than for supernova neutrinos, terms proportional to higher powers of \( \cos \theta \) appear, and at the highest energies, all reaction products are strongly forward simply by kinematics. Just above threshold, where the lepton velocity is small, the angular distribution obviously becomes isotropic. At supernova neutrino energies, \( v_e = 1 \).

Typically, the asymmetry coefficient \( a(E_\nu) \) is not large, so the angular distribution is weak, but the number of events may be large (for \( \bar{\nu}_e + p \rightarrow e^+ + n \),
there are nearly $10^4$ events expected in SK). Thus it may be possible to use these events to locate the supernova. We first consider how well one could localize the supernova assuming that $a$ is known and constant.

Given a sample of events, one can attempt to find the axis defined by the neutrino direction. Along this axis, the distribution should be flat in the azimuthal angle $\phi$ and should have the form in Eq. (14) in $\cos \theta$. Along any other axis, the distribution will be a complicated function of both the altitude and azimuthal angles. We assume that the axis has been found numerically, and ask how well the statistics allow the axis to be defined. A convenient way to assess that is to define the forward-backward asymmetry as

$$A_{FB} = \frac{N_F - N_B}{N_F + N_B},$$

(15)

where $N_F$ and $N_B$ are the numbers of events in the forward and backward hemispheres. The total number of events is $N = N_F + N_B$. Note that $A_{FB}$ will assume an extremal value $A_{FB}^{\text{extr}}$ along the correct neutrino direction. It can be shown that

$$\delta A_{FB} = \frac{1}{\sqrt{N}} \sqrt{1 - \left(\frac{a}{2}\right)^2} \simeq \frac{1}{\sqrt{N}},$$

(16)

where the error is nearly independent of $a$ for small $|a|$, which is the case for the reactions under consideration.

In the above, the coordinate system axis was considered to be correctly aligned with the neutrino direction. Now consider what would happen if the coordinate system were misaligned. While in general, all three Euler angles would be needed to specify an arbitrary change in the coordinate system, symmetry considerations dictate that the computed value of $A_{FB}$ depends only upon one – the angle $\theta$ between the true and the supposed neutrino axis. Thus $A_{FB}$ is some function of $\theta$ if the axis is misaligned. Using a Legendre expansion, one can show that

$$A_{FB}(\theta) = \frac{a}{2} \cos \theta.$$

(17)

The error on the alignment is then

$$\delta(\cos \theta) = \frac{2}{|a|} \delta A_{FB} \simeq \frac{2}{|a|} \frac{1}{\sqrt{N}}.$$

(18)

Treating the nucleons as infinitely heavy, the coefficient $a$ in Eq. (14) is related to the competition of the Fermi (no spin flip) and Gamow-Teller (spin
flip) parts of the matrix element squared:

\[
a = \frac{|M_F|^2 - |M_{GT}|^2}{|M_F|^2 + 3|M_{GT}|^2}.
\]  \tag{19}

Naive use of this formula for \(\bar{\nu}_e + p \to e^+ + n\) gives \(a = -0.10\) for \((|M_{GT}/M_F| = 1.26)\). One expects \(10^4\) events in SK, so that \(\delta(\cos \theta) \simeq 0.2\). Even though the asymmetry parameter \(a\) is quite small, the number of events is large enough that this technique in SK could give a reasonable pointing error (SNO only has 400 of these events, so \(\delta(\cos \theta) \simeq 1.0\), which is too large to be useful).

For the charged-current reactions on deuterons, \(\bar{\nu}_e + d \to e^+ + n + n\) and \(\nu_e + d \to e^- + p + p\), one would have \(a = -1/3\) \((M_F/M_{GT} \simeq 0)\). If these channels could be combined (160 events in total), then \(\delta(\cos \theta) \simeq 0.5\), which is again rather large.

In general, the coefficient \(a(E_\nu)\) has energy dependence coming from recoil and weak magnetism corrections, each of order \(1/M\), where \(M\) is the nucleon mass (such terms also change the total cross section \([23,31]\), which has some important applications \([23]\)). For the reaction \(\bar{\nu}_e + p \to e^+ + n\), these corrections have a dramatic effect on the angular distribution, making it backward at low energies, isotropic at about 15 MeV, and forward at higher energies. Averaged over the expected \(\bar{\nu}_e\) spectrum from a supernova, one obtains \(\langle \cos \theta \rangle = +0.08\), i.e., a forward distribution. Using the common assumption that the angular distribution is backward (with \(a = -0.10\)) would lead one to infer exactly the wrong direction to the supernova.

It is convenient to describe the angular distribution by the average cosine, weighted by the differential cross section. In the limit that Eq. \([14]\) holds,

\[
\langle \cos \theta \rangle = \frac{1}{3} \nu_e a(E_\nu).
\]  \tag{20}

For \(\bar{\nu}_e + p \to e^+ + n\), the variation of the average cosine can be written as (keeping only the largest terms of the full expression \([31]\)):

\[
\langle \cos \theta \rangle \simeq -0.034 \nu_e + 2.4 \frac{E_\nu}{M}.
\]  \tag{21}

The factor 2.4 is actually 1.0 + 1.4, the first term from recoil, which always makes the angular distribution more forward, and the second from weak magnetism (which would change sign for the reaction \(\nu_e + n \to e^- + p\), if there were free neutron targets). The variation of \(\langle \cos \theta \rangle\) with \(E_\nu\) is shown in Fig. 4.

Since the deuteron is weakly bound, the recoil and weak magnetism corrections to \(\langle \cos \theta \rangle\) can be reasonably estimated \([31]\). The effects of recoil and weak magnetism add in the \(\bar{\nu}_e + d\) channel, and partially cancel in the \(\nu_e + d\) channel, as seen in Fig. 5.
Inverse Beta Decay: Positron-Neutron Separation Vector

The results above were based on lepton angular distributions as observed in water-Čerenkov detectors (light or heavy water). Some of the same reactions occur in scintillator detectors, but the lepton angular distributions are not observable due to the isotropic character of scintillation light.

Despite that, there is a way to get directional information with a scintillator detector. The idea makes use of the fact that scintillator detectors can measure final particle positions by the relative timing between different phototubes. For $\bar{\nu}_e + p \rightarrow e^+ + n$, the positron is detected nearly at the point of creation. The neutron, however, will be detected (by its capture gamma rays), on average a few centimeters forward of the point of creation. The initially forward motion of the neutron is just a consequence of kinematics. Thus the
Figure 5: The average lepton cosine for the charged-current deuteron reactions, versus the neutrino or antineutrino energy, using Kubodera’s calculations. We plot only from 1 MeV above threshold, so the $\nu_e$-dependence at low energies is not shown. The small jitters are due to the coarse integration grid.

positron-neutron separation vector points in the incoming antineutrino direction.

However, before the neutron can be captured, it must be thermalized. On the first several scatterings (about half of the kinetic energy is lost on each step), the forward motion tends to be preserved. In the remaining scatterings until thermal energy is reached, the scattering is isotropic. In the last phase, the neutron wanders randomly, undergoing further scatterings at thermal energy, until it is captured. The neutron wander has the effect of degrading the significance of the initially forward motion. The average displacement and its fluctuations can be calculated by Monte Carlo simulation\(^{31,34}\).

At the energies of reactor antineutrinos, the positron-neutron separation is about 1.5 cm and the statistical fluctuation several cm, depending on the neutron capture time in the scintillator\(^31\). There is an additional uncertainty in the position due to the gamma-ray localization error. For reactor antineutrinos, the source direction is known, so a measurement of the average positron-
neutron separation vector can be used to make a background measurement, since background events will degrade the significance of the separation vector from the expectation. The precision with which the reactor can be located was evaluated by the Chooz collaboration and found to be about 18°. Given the small displacement and the large error in localization, this is an impressive measurement.

At the higher energies of supernova neutrinos, the neutron kinetic energy is larger and the elastic scattering cross section smaller, so the displacement is larger. The Chooz collaboration estimates that if they were to scale their detector to the size of SK (32 kton), that they would be able to locate a supernova to within about 9°, not so much worse than the neutrino-electron scattering result of about 5° in SK. The positron-neutron displacement technique is not possible in SK since neutrons are not detected, and not possible in SNO because the neutron wanders over several meters.

2.3 Triangulation

Finally, there is the technique of triangulation by arrival-time differences. The idea is very simple – for two detectors, the cosine of the angle $\theta$ between the axis connecting the two detectors (with separation $d$) and the supernova direction is determined by the arrival-time difference $\Delta t$ by

$$\cos \theta = \frac{\Delta t}{d}. \quad (22)$$

For detectors on opposite sides of the Earth, $d = 40$ ms, and for SK and SNO, $d = 30$ ms. Two detectors thus define a cone of allowed directions to the supernova. Since the arrival time difference has an error $\delta(\Delta t)$, the cone will have an angular thickness

$$\delta(\cos \theta) = \frac{\delta(\Delta t)}{d}. \quad (23)$$

The timing error is obviously the crucial point, and in order to have $\delta(\cos \theta) = 0.1$ (comparable to the neutrino-electron scattering result in SNO), one would need $\delta(\Delta t) = 3$ ms for the SK-SNO difference. This is much smaller than the duration of the supernova pulse (10 s) or even the risetime (assumed 100 ms).

In order to assess the best that triangulation could do, consider the scenario where SK and the light water portion of SNO are each perfect detectors, identical except for size. (The shape of the event rate in the heavy water in SNO will be different, due to the different reactions, and it is not clear how to correct for that). Then the scattering rates observed in SK and the light
water in SNO will have the same underlying distribution, which depends on unknown details of the supernova models.

The observed scattering rates in SK and the light water of SNO can then differ only in normalization, statistical fluctuations, and a possible delay. The normalizations matter only in that they determine the scale of the statistical fluctuations. SK will have about $10^4$ events and the light water in SNO about 400 events, so the statistical error will be dominated by the timing error in SNO. That is, the underlying distribution will be measured in SK and this template will be used in SNO to test for a delay.

In order to make estimates of the timing error, we need the shape of the event rate. We assume a short rise (in the current supernova models, this is somewhere between 0 and 200 ms, so we will very conservatively use 100 ms), followed by a long decay (10 s, as for SN1987A).

For the normalized event rate, we take

$$f(t, t_0) = \alpha_1 \times \frac{1}{\tau_1} \exp \left[ \frac{t - t_0}{\tau_1} \right], \quad t < t_0$$

(24)

$$f(t, t_0) = \alpha_2 \times \frac{1}{\tau_2} \exp \left[ -\frac{(t - t_0)}{\tau_2} \right], \quad t > t_0$$

(25)

where

$$\alpha_1 = \frac{\tau_1}{\tau_1 + \tau_2}, \quad \alpha_2 = \frac{\tau_2}{\tau_1 + \tau_2}.$$  

(26)

Then $f(t, t_0)$ is a normalized probability density function built out of two exponentials, and joined continuously at $t = t_0$. In what follows, we assume that this form of $f(t, t_0)$ is known to be correct and that $\tau_1$ and $\tau_2$ are known. The shape of the event rate is shown in Fig. 6.

The event rate in SNO will consist of $N$ events sampled from $f(t, t_0)$, and the event rate in SK will consist of $N'$ events sampled from $f(t, t'_0)$. Then $\Delta t = t_0 - t'_0$, and $\delta(\Delta t) \approx \delta t_0$, since the SNO error dominates. We consider only the statistical error determined by the number of counts. As noted, we want to determine the minimal error on the triangulation. This model, while simple, contains the essential timescales and an adjustable offset.

These considerations lead to a well-posed statistical problem: If $N$ events are sampled from a known distribution $f(t, t_0)$, how well can $t_0$ be determined? The Rao-Cramer theorem provides an answer to this question (with a possible subtlety\textsuperscript{[3]}). This theorem allows one to calculate the minimum possible variance on the determination of a parameter (here $t_0$), by any technique whatsoever. This minimum variance can be achieved when all of the data are used as “efficiently” as possible, which is frequently possible in practice.
Figure 6: The schematic form of the normalized event rate $f(t, t_0)$ is shown. To the left of $t_0$ there is an exponential rise with time constant $\tau_1$. To the right of $t_0$ there is an exponential decay with time constant $\tau_2$. The tick marks on the $t$-axis are in units of the respective time constants $\tau_1$ and $\tau_2$. For clarity of display, we used $\tau_1/\tau_2 \simeq 10^{-1}$ in the figure, instead of the $\tau_1/\tau_2 \simeq 10^{-2}$ assumed in the analysis.

One requirement of the theorem is that the domain of positive probability must be independent of the parameter to be determined. This condition is obviously not met for a zero risetime, since then the domain is $(-\infty, \infty)$. For a nonzero risetime, the domain is technically $(-\infty, \infty)$, independent of $t_0$, and so the theorem applies. The minimum possible variance on the determination of $t_0$ is:

$$\frac{1}{(\delta t_0)^2_{\text{min}}} = N \times \int dt f(t, t_0) \left[ \frac{\partial \ln f(t, t_0)}{\partial t_0} \right]^2.$$  \hspace{1cm} (27)

This is the general form for an arbitrary parameter $t_0$. When $t_0$ is a translation parameter, i.e., $f(t, t_0)$ depends only on $t - t_0$, this reduces to

$$\frac{1}{(\delta t_0)^2_{\text{min}}} = N \times \int dt f(t, t_0) \left[ \frac{\partial \ln f(t, t_0)}{\partial t} \right]^2.$$  \hspace{1cm} (28)
\[ N \times \int dt \frac{\partial f(t, t_0)/\partial t}{f(t, t_0)}^2. \]  

(29)

For the particular choice of \( f(t, t_0) \) above, this reduces to

\[ \frac{1}{(\delta t_0)_{\text{min}}^2} = N \times (\alpha_1/\tau_1^2 + \alpha_2/\tau_2^2), \]  

(30)

and so the minimum error is

\[ (\delta t_0)_{\text{min}} = \frac{\sqrt{\tau_1/\tau_2}}{\sqrt{N}} = \frac{\tau_1}{\sqrt{N_1}}. \]  

(31)

Note that \( N_1 = N(\tau_1/\tau_2) \) is approximately the number of events in the rising part of the pulse. Since the rise is the sharpest feature in \( f(t, t_0) \), it is unsurprising that it contains almost all of the information about \( t_0 \). The total number of events \( N \) is fixed by the supernova binding energy release, so a change in the assumed total duration of the pulse, i.e., \( \tau_2 \), would affect the peak event rate and hence the fraction of events in the leading edge. For a more general \( f(t, t_0) \), one would replace \( \tau_1/\tau_2 \) by this fraction computed directly.

For SNO, \( N_1 \simeq 10^{-2} \times 400 \simeq 4 \), so \( \delta(t_0) \simeq 30 \text{ ms}/\sqrt{4} \simeq 15 \text{ ms} \). Since SK has about 25 times more events, the corresponding error would be about 3 ms. Therefore, the error on the delay is \( \delta(\Delta t) \simeq 15 \text{ ms} \) and \( \delta(\cos \theta) \simeq 0.50 \) at one sigma. We have not specified the method for extracting \( t_0 \) and hence \( \Delta t \) from the data. That is exactly the point of the Rao-Cramer theorem – that one can determine the minimum possible error without having to try all possible methods.

If the risetime were zero, which seems to be unrealistic, then one can show by application of order statistics that the error becomes

\[ \delta(t_0) = \frac{\tau_2}{N}. \]  

(32)

Here \( \tau_2/N \) is simply the spacing between events near the peak. For a more general \( f(t, t_0) \), but still with a sharp edge at \( t_0 \), one would simply replace \( \tau_2 \) by \( 1/f(t_0) \). In fact, the shape of \( f(t, t_0) \) is irrelevant except for its effect on the peak rate, i.e., \( f(t_0) \). So long as \( f(t, t_0) \) has a sharp edge and the right total duration, allowing a more general time dependence would therefore not change the results significantly.

For the two cases, zero and nonzero risetime, we used different mathematical techniques. This may seem like an artificial distinction, and that these two cases do not naturally limit to each other. In particular, it may seem incompatible that in the first case the error \( \sim 1/\sqrt{N} \), while in the second the error
Further, one obviously cannot take \( \tau_1 \to 0 \) in the first result to obtain the second. However, it can be shown that the two techniques have disjoint regions of applicability (as a function of \( \tau_1 \)) and that the formulas match numerically at the boundary between the two. The boundary is the value of \( \tau_1 \) such that the number of events in the rise is of order 1, i.e., the edge appears sharp for this or smaller \( \tau_1 \).

The final results for the pointing errors are given in Table IV.

### 3 Concluding Remarks

The next Galactic supernova should be a bonanza for neutrino physics and astrophysics. Should, that is, provided that we can make sense of the data. The difficulty is that there are many unknown aspects of both the particle physics properties of neutrinos and the astrophysics of the expected signal. Since the expected core-collapse supernova rate in the Galaxy is about 3 per century, and since there are no neutrino detectors sensitive to supernovae in distant galaxies, we will not have the luxury of many observations. Of course, there is a worldwide effort to improve the numerical models of core-collapse supernovae. And besides neutrinos, there is one other direct probe of the supernova core, and that is gravitational radiation. LIGO may be sensitive to supernovae out to the Virgo Cluster (at rates of order 1 per year), and its observations may improve the understanding of stellar collapse.

Since there are uncertainties in the supernova models, the \( \langle t \rangle \) test for the \( \nu_\mu \) and \( \nu_\tau \) masses was designed to be as model-independent as possible. The resulting sensitivities for either \( \nu_\mu \) or \( \nu_\tau \) are in Table III. If the \( \nu_\mu \) and \( \nu_\tau \) are maximally mixed with a small mass difference, then the mass test is really on \( m_2 = m_3 \), and the limits in Table III improve by a factor of about \( \sqrt{2} \).

| detector | neutral-current signal | mass limit |
|----------|------------------------|------------|
| SNO      | \( \nu + d \)          | 30 eV      |
| SK       | \( \nu + ^{16}O \)     | 45 eV      |
| SK       | \( \nu + e^- \)        | 50 eV      |
| Kamland  | \( \nu + ^{12}C \)     | 55 eV      |
| Borexino | \( \nu + ^{12}C \)     | 75 eV      |

Table 3: Mass sensitivity for various detectors and neutral-current reactions. The mass quoted is the limit which would be placed on either the \( \nu_\mu \) or \( \nu_\tau \) mass at 90% CL if no delay were seen (equivalently, it is the smallest mass which would cause an unambiguous delay). If the \( \nu_\mu \) and \( \nu_\tau \) masses are nearly equal, then the sensitivity is improved by \( \sqrt{2} \). See also [21] for the \( ^{12}C \) results.
There are also many possible signals of supernova neutrino oscillations\cite{38}. Ideally, the current and near-term terrestrial neutrino oscillation experiments can answer some of the key questions before the next supernova. Particularly crucial is the question of whether there are sterile neutrinos (or several flavors thereof). SNO\cite{39} may tell us whether the solar $\nu_e$ neutrinos are oscillating into sterile or active flavors (or at all). Similarly for SK\cite{40}, K2K\cite{41}, and MINOS\cite{42} and the atmospheric $\nu_\mu$ neutrinos. And MiniBoone\cite{43} may decide whether the LSND\cite{44} anomaly is correct (and hence whether sterile neutrinos are necessary). These questions and more are discussed thoroughly in several recent reviews\cite{45}.

Since the neutrinos leave the supernova core a few hours before the light leaves the envelope, it should be possible for the neutrino experiments to give astronomers an advance warning of a Galactic core-collapse supernova, and also to provide some guidance as to where in the sky to look. Neutrino-electron scattering in SK seems to be the best technique, with an error of order 5°, depending on distance and the suppression of backgrounds from other reactions. While 5° may sound large to an astronomer, it is about 0.1% of the sky, which will make subsequent searches with small telescopes much easier (the role of amateur astronomers is discussed in a recent article\cite{46}). Supernova location by triangulation seems to be rather difficult at present (see Table IV).

Table 4: One-sigma errors on how well the direction to the supernova is defined by various techniques, at $D = 10$ kpc. The other parameters used are noted in the text. For neutrino-electron scattering, the most pessimistic background assumptions were used.

| technique                                               | error                        |
|--------------------------------------------------------|------------------------------|
| $\nu + e^-$ forward scattering (SK)                    | $\delta \theta \simeq 5^\circ, \delta (\cos \theta) \simeq 4 \times 10^{-3}$ |
| $\nu + e^-$ forward scattering (SNO)                   | $\delta \theta \simeq 20^\circ, \delta (\cos \theta) \simeq 6 \times 10^{-2}$ |
| $\bar{\nu}_e + p$ angular distribution (SK)           | $\delta (\cos \theta) \simeq 0.2$ |
| $\bar{\nu}_e + p$ angular distribution (SNO)          | $\delta (\cos \theta) \simeq 1.0$ |
| $\nu_e + d, \bar{\nu}_e + d$ angular distributions (SNO) | $\delta (\cos \theta) \simeq 0.5$ |
| triangulation (SK and SNO)                             | $\delta (\cos \theta) \simeq 0.5$ |
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