An Analysis of the Performance of the “1/N” Naïve Portfolio Strategy in Korean Stock Markets

Sang-Hoon Lee⁴, Jihun Kim⁵

⁴Assistant Manager, KB Asset Management, Seoul, Republic of Korea
⁵Research Fellow, KB Research, KB Financial Group, Seoul, Republic of Korea

Abstract

In this study, we compare three measurements of 12 asset-allocation strategies to a “1/N” portfolio using historical data of Korean stock markets, which is a representative emerging market, from January 2000 to December 2015. We find that in Korean stock markets the asset allocation strategies with short sale constraint have better performance than the “1/N” portfolio (i.e., high Sharpe ratios and high certainty-equivalent returns). While these optimal asset-allocation models have higher turnover than the “1/N” portfolio, the Sharpe ratio of short sale constraint is still higher than the “1/N” portfolio when we consider transaction costs. The results are surprising because no asset-allocation strategies are consistently dominant on “1/N” portfolio using US stock market data.

Keywords: Asset-Allocation, Korean Stock Market, Portfolio Performance, “1/N” Portfolio, Short Sale Constraint Portfolio

I. Introduction

After Markowitz (1952), various kinds of asset allocation models have been developed, in order to solve the problem that the out-of-sample performance of the mean-variance model is worse than the in-sample performance due to the estimation error. However, DeMiguel et al. (2009) found that no optimal asset-allocation strategy gives better performance than the “1/N” strategy from the U.S. stock markets. They argue that “1/N” strategy has a dominant advantage over the other asset allocation strategies in that it is free of estimation processes and therefore is not affected by estimation error. To confirm that their argument holds up in Korean stock markets, we investigate the effectiveness of various portfolio optimizing using data of the Korean stock markets. In emerging market there are relatively more constraints than matured market. Therefore, our study is meaningful in that the Korean financial markets are representative emerging market (Baek et al, 2018). In this study, we compare three measurements of 12 asset-allocation strategies to a “1/N” portfolio using historical data generated by the Korean stock market mainly from January 2000 to December 2015.

In this study, we investigate portfolio optimizing strategies using following DeMiguel et al. (2009). We consider the Bayesian approach to estimation error as it applies to a shrinkage estimator (Jobson et al. 1979; Jobson and Korkie, 1980; Jorison, 1985,
1986), and prior asset-pricing models which investors continue to believe to be useful (Pastor, 2000; Pastor and Stambaugh, 2000). We also examine non-Bayesian approaches such as the minimum-variance portfolio, the value-weighted market portfolio, and the missing-factor model (MaKinlay and Pastor 2000). These portfolio strategies apply moment restrictions to minimize estimation error. We further study optimal combinations of portfolios (Kan and Zhou, 2007; DeMiguel et al. 2009) and portfolios with short sale-constraints (Frost and Savarino, 1988; Chopra, 1933; Jagannathan and Ma, 2003). We finally choose the “1/N” portfolio strategy as a benchmark strategy because it is well known that “1/N” portfolio strategy is not strictly defeated by other asset allocation strategy in the most matured financial market, the US market.  

However, it is not assured that the same story is supported empirically in the emerging market, in which there is frictional cost, such as short-sale constraint, taxes, fees, and etc.  

In this study we show that, unlike stock markets in the U.S., in Korean stock market, optimal portfolio strategies with short-sale constraint give better performances than “1/N” portfolio in all six datasets consistently.  

We confirm these results based on three different portfolio-performance measures: Sharpe ratio, CEQ, and turnover. Our results show that the mean-variance portfolio with short sale constraint, the Bayes-Stein portfolio with short sale constraint, and the minimum-variance portfolio with short sale constraint have higher Sharpe ratios than equal-weight portfolios, although the differences are not statistically significant across all datasets. In addition, the mean-variance portfolios with short sale constraint and Bayes-Stein portfolios with short sale constraints have a higher CEQ than is obtained by employing the “1/N” strategy. For the robustness, we compute the Sharpe ratios with transaction cost as 30bp again. Although here we consider transaction cost, we obtain consistent results that the mean-variance portfolio with short sale constraint and the Bayes-Stein strategy with short sale constraint have higher Sharpe ratios than equal-weight portfolios.  

The contribution of our study is the following. Firstly, we compare 14 portfolio strategies using Korean stock data. There are few studies of empirical test of portfolio in Korean stock markets. Secondly, from our investigation, we find that the short sale constraint strategies are better in performance than the “1/N” strategy. The results are inconsistent with the results using U.S data.  

The remainders are organized as follows. We present our six datasets and various asset-allocation strategies in Section 2. In Section 3, we introduce measurement methodology for optimal portfolio performance. We show empirical results of diverse asset-allocation strategies, compared across six datasets in Section 4. Our conclusions are summarized in Section 5.

II. Information on data and asset-allocation strategy

In this study, we investigate performance of portfolio optimizing strategies which is designed to reduce estimation error in Korean stock markets. We consider 12 asset allocation strategies following DeMiguel et al. (2009). We consider the Bayesian approach to estimation error as it applies to a shrinkage estimator (Jobson et al. 1979; Jobson and Korkie, 1980; Jorison, 1985, 1986), and prior asset-pricing models which investors continue to believe to be useful (Pastor, 2000; Pastor and Stambaugh, 2000). We also examine non-Bayesian approaches such as the minimum-variance portfolio, the value-weighted market portfolio, and the missing-factor model (MaKinlay and Pastor 2000). These portfolio strategies apply moment restrictions

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1) This approach is used generally until recently. Wang and Lee (2017) use the equal weighted portfolio strategy as a benchmark in their study. Incidentally, although there is not found asset allocation strategies that is always dominant on “1/N” portfolio strategy in terms of performance, it does not mean “1/N” portfolio strategy performs best in any sample sets.  

2) However, in the special case where the number of asset is much larger than number of observation, e “1/N” strategy is not good (Lee, at al. 2018).
to minimize estimation error. We further study optimal combinations of portfolios (Kan and Zhou, 2007; DeMiguel et al. 2009) and portfolios with short sale-constraints (Frost and Savarino, 1988; Chopra, 1993; Jagannathan and Ma, 2003). Finally, we choose “1/N” portfolio strategy as a benchmark.

This section covers various asset-allocation strategies and the notations we considered. We refer to the notation in DeMiguel et al. (2009). We use $\mathbf{\mu}_t$ as a $N$ risky asset’s expected returns at time $t$, $\mathbf{\Sigma}_t$ as an $N \times N$ variance-covariance matrix of excess returns. $\mathbf{\Sigma}_t$ and $\mathbf{\mu}_t$ are sample moments of $\mathbf{\mu}_t$ and $\mathbf{\Sigma}_t$ respectively. $M$ is a time period of the estimation window for moment estimations and $T$ is the whole period of the data series. $\mathbf{1}$ is a size $N \times 1$ one vector and $\mathbf{I}$ is $N \times N$ identity matrix. $w_t$ is the portfolio weights vector of investing $N$ risky assets, and we invest $1 - \mathbf{1}^\prime \mathbf{1} w_t$ in a risk-free asset. Hence, we invest only risky asset weight $w_t$ as following:

$$w_t = \frac{x_t}{\mathbf{1}_N^\prime x_t} \quad (1)$$

To compare various asset-allocation models easily, we assume that investors’ preferences can be expressed by the mean-variance of excess returns from selected portfolio weight $x_t$. At time $t$, the investor chooses $x_t$ to maximize the following expected utility function:

$$\max_{x_t} x_t^\prime \mathbf{\mu}_t - \frac{\gamma}{2} x_t^\prime \mathbf{\Sigma}_t x_t \quad (2)$$

where $\gamma$ is the risk aversion coefficient of investor. The optimization solution of portfolio weights for the above equation is $x_t = \frac{1}{\gamma} \mathbf{\Sigma}_t^{-1} \mathbf{\mu}_t$. So, the portfolio weight at time $t$ is

$$w_t = \frac{\mathbf{1}_N^\prime \mathbf{\mu}_t}{\mathbf{1}_N^\prime \mathbf{\Sigma}_t^{-1} \mathbf{\mu}_t} \quad (3)$$

The weights of asset-allocation strategies that we considered are computed from equation (3), substituting $\mathbf{\mu}_t$ and $\mathbf{\Sigma}_t$ to estimated moment $\hat{\mathbf{\mu}}_t$ and $\hat{\mathbf{\Sigma}}_t$ respectively.

### A. Asset Allocation Strategies

We consider the naïve portfolio strategy as a benchmark portfolio, which rebalances $1/N$ weights to risky assets at every month. We name the naïve portfolio “ew” or “1/N” in this paper. This strategy does not require estimation and optimization. In this section, we move the complex equations of asset allocation strategies because explaining the equation in detail is our of the purpose of this study.

In Markowitz (1952), the investors select their portfolios to optimize the portfolio return in mean-variance space. We called this model “mv” in this thesis; we find mean-variance portfolio weights replaced equation (3) by sample $\hat{\mathbf{\mu}}_t$ and $\hat{\mathbf{\Sigma}}_t$ respectively.

$$\mu_t = \frac{1}{M} \sum_{s=1-M}^{t} R_s \quad (4)$$

$$\hat{\Sigma}_t = \frac{1}{M-N-2} \sum_{s=1-M}^{t} (R_s - \hat{\mu}_t)(R_s - \hat{\mu}_t) \quad (5)$$

The Bayesian approach estimates $\mathbf{\mu}$ and $\mathbf{\Sigma}$, applying asset returns’ predictive distribution. This distribution consists of a conditional likelihood $f(R | \mathbf{\mu}, \mathbf{\Sigma})$ and a subjective prior $p(\mathbf{\mu}, \mathbf{\Sigma})$. The Bayes-stein portfolio “bs” represents the exercise conducted by James and Stein (1961) exploring shrinkage estimation. We calculate the “bs” portfolio weights using the sample shrinkage estimator $\hat{\mu}_t$ and $\hat{\Sigma}_t$ as in equation (3), which is a convex combination of sample mean $\hat{\mu}_t$ and common grand means $\bar{\mu}$. We consider the shrinkage estimator of expected return and covariance matrix proposed by Jorison (1986).

3) In this field, it is called the estimation based on optimal weighted average between two estimation with different characteristics as shrinkage estimation.
According to the CAPM (Capital Asset Pricing Model), the value-weighted (“vw”) market portfolio emerged as the optimal portfolio. Hence, we consider the value weighted market portfolio as one of asset allocation models.

Mackinlay and Pastor (2000) assume an estimation error in covariance matrix from unobservable factors. Missing risk factors lead to mispricing, and this problem is linked to the covariance matrix. In consequence, the optimal weights have to be proportional to the estimated expected return, implementing an identity matrix as a covariance matrix. We denote this allocation strategy as “mp.” As we consider one factor model, the return can be explained by following equation:

\[ z_t = \beta_h^* \tilde{e}_{1t} + u_t \] (13)

Where \( z_t \) is return vector of N assets in time t, \( \beta_h \) is N \times 1 factor loading matrix, \( \tilde{e}_{1t} \) is 1-factor portfolio returns in time t.

Likelihood function:

\[
L(\alpha, \theta, \sigma^2 | z_1, \ldots, z_T) = \left[\alpha \theta \sigma + \sigma^2 I_N \right]^{-\frac{T}{2}} \exp \left\{ -\frac{1}{2} \sum_{t=1}^{T} \left( z_t - \alpha \theta \right)^{\top} \left( \alpha \theta \sigma + \sigma^2 I_N \right)^{-1} \left( z_t - \alpha \right) \right\}
\] (15)

The maximum likelihood estimators are \( \hat{\alpha}, \hat{\theta}, \sigma^2 > 0 \), which maximize the joint log-likelihood function. There is no closed-form solution to maximize the likelihood function, so we use a quasi-Newton numerical method to solve equation (15).

In reality, the short sale constraint is very usual to individual investors. We impose the short sale constraint on the three models that we introduced in the previous section: the mean-variance-constraint portfolio (“mv-c”), the Bayes-Stein-constraint portfolio (“bs-c”), and the minimum-variance-constraint portfolio (“min-c”). In addition, DeMiguel et al. (2009) propose
Table 1. List of various asset-allocation models considered
This table is from Victor DeMiguel, Lorenzo Garlappi, Raman Uppal (2009) page 1917.

| #   | Model                                                                 | Abbreviation |
|-----|----------------------------------------------------------------------|--------------|
| 0   | Naïve 1/N with rebalancing(benchmark strategy)                       | 1/N          |
| 1   | Classical approach that ignores estimation error                    | mv           |
| 2   | Bayesian approach to estimation error                                | bs           |
| 3   | Bayesian Data-and-Model Moment restrictions                          | dm           |
| 4   | Minimum-variance                                                     | min          |
| 5   | Value-weighted market portfolio                                      | vw           |
| 6   | MacKinlay and Pastor's(2000) missing-factor model                   | mp           |
| 7   | Sample-based mean-variance with short sale constraint                | mv-c         |
| 8   | Bayes-Stein with short sale constraints                              | bs-c         |
| 9   | Minimum-variance with short sale constraints                         | min-c        |
| 10  | Minimum-variance with generalized constraints                        | g-min-c      |
| 11  | Kan and Zhou's(2007) “Three-fund”model                              | mv-min       |
| 12  | Mixture of minimum-variance and 1/N                                  | ew-min       |

the shrinkage weights from the minimum-variance-constraint portfolio and equally-weighted portfolio. In this model, all weights have to be higher than 1/2N by restriction. We denote this model as “g-min-c”.

To improve Bayes-Stein Shrinkage estimators, Kan and Zhou (2007) introduce the “three-fund” (“mv-min”) portfolio. In this portfolio, they deal with estimation error using tangent portfolio and risk-free asset combination only. To diminish estimation error, investors have to hold another risky-asset portfolio. Kan and Zhou (2007) find this three-fund portfolio using combined mean-variance portfolios and minimum-variance portfolios.

DeMiguel et al. (2009) propose an “ew-min” portfolio, which is an idea from “mv-min” strategy. We summarize the notation of strategies in Table 1.

B. Data description

To compare portfolio strategies, we use portfolio dataset mainly from the Dataguide, which is a major provider of Korean stock markets data. The source and brief description of datasets are listed in Table 2. Our datasets are corresponding to the datasets used in DeMiguel et al. (2009). We use over the 2-year Monetary Stabilization bond as a proxy of risk free rate to calculate monthly excess returns of datasets. The main data sets are at monthly frequency and our sample period is from January 2000 to December 2015. We denote proxy of market excess return as “MKT”, which is the excess return of value-weighted portfolio in Korean two main stock exchange market (i.e., KOSPI and KOSDAQ). We obtain the factor mimicking portfolios, “SMB” and “HML” from Fama and French (1993) and refer to Wang and Nguyen (2015) paper for a better understanding, “RMW” and “CMA” from Fama and French (2015), and “UMD”
from Carhart (1997). We construct monthly excess returns of 25 size and book-to-market portfolios sorted by book to market and size following Fama and French (1993). As Wang (2005) pointed out, “MKT, SMB and HML are almost a linear combination of the 25 Fama and French portfolios”, we choose 20 size and book-to-market portfolios from the 25 portfolios, excepting the five largest portfolios in size. The “MKF Sector” dataset contains monthly excess returns on ten value-weighted portfolios. The ten sectors are composed of Energy, Material, Consumer-Staples, Consumer-Discretionary, Medical, Financials, Information-Technology, Telecommunications, and Utilities securities from July 2002 to June 2015.

C. Methodology

In this section, we introduce methodology to compare the performance of variety asset-allocation models. The methodologies we consider correspond to DeMiguel et al. (2009). We implement a rolling out-of-sample test to assess optimal strategies’ performances. We set estimation windows at M=60 months for estimation of parameters and weights, and we use the estimation to access the performance of the portfolio at time t=M+1. The number of total time series observations is T, so we have T-M months out-of-sample time series returns.

We use three measures to assess performance of diverse optimal portfolio strategies. The first measure is an out-of-sample Sharpe ratio of each asset-allocation strategy k:

\[ \widehat{SR}_k = \frac{\mu_k}{\sigma_k} \]  \tag{16} 

We compute p-value to check how each strategies’ Sharpe ratios are statistically distinguishable from the “1/N” portfolio suggested by Jobson and Korkie (1981) and modified by Memmel (2003), so the null hypothesis is that the Sharpe ratio of each asset-allocation strategy i is equal to that of “1/N” portfolio, as shown in equation (36). The statistical distribution is as follow in equation (37)
Also we compute the in-sample Sharpe ratio of mean-variance portfolio to gauge how the negative effect of estimation errors affect portfolio strategies' performance. We use total time series of excess returns to estimate parameters to gain an in-sample Sharpe ratio (i.e., M=T). The mean-variance strategy in-sample Sharpe ratio is as follows in equation (39):

\[
\text{Sharpe Ratio} = \frac{\left( \hat{\mu}_k \right)_k - \bar{\mu}}{\sqrt{\sum_k \left( \hat{\sigma}_k^2 \right)_k}}
\]

(20)

Where \( \hat{\mu}_k \) is the in-sample mean, \( \hat{\sigma}_k^2 \) is the in-sample covariance matrix; \( \hat{\sigma}_k \) is portfolio weight using \( \hat{\mu}_k \), and \( \bar{\mu} \). Secondly, we compute the certainty-equivalent return (CEQ). CEQ represents the risk-free rate investors select rather than portfolio strategies. The equation of CEQ for each strategy in-sample Sharpe ratio is statistically distinguishable from "1/N" portfolio as suggested by Greene (2002).

\[
\text{CEQ}_k = \hat{\mu}_k - \frac{\gamma}{2} \hat{\sigma}_k^2
\]

(21)

Where \( \hat{\mu}_k \) is mean of out-of-sample excess returns, \( \hat{\sigma}_k^2 \) is variance of out-of-sample excess returns and \( \gamma \) is the risk aversion coefficient. We choose risk aversion coefficient \( \gamma = 1 \) because we follow DeMiguel et al. (2009)\(^4\). We compute p-value to check how each strategy's CEQ is statistically distinguishable from "1/N" portfolio by Greene (2002).

\[
v : = \left( \mu_i; \mu_n, \sigma_i^2, \sigma_n^2 \right)
\]

(22)

\[
f(v) : = \left( \mu_i - \frac{\gamma}{2} \sigma_i^2 \right) - \left( \mu_n - \frac{\gamma}{2} \sigma_n^2 \right)
\]

(23)

\[
\sqrt{T}f(v) \sim N\left(0, \frac{\partial f}{\partial v} \theta \frac{\partial f}{\partial v} \right)
\]

(24)

Where \( \theta = \left( \begin{array}{ccc} \sigma_i^2 & \sigma_{i,n} & 0 \\ \sigma_{i,n} & \sigma_n^2 & 0 \\ 0 & 0 & 2 \sigma_i^2 \\ 0 & 0 & 2 \sigma_{i,n}^2 \\ 0 & 0 & 2 \sigma_n^2 \\ 0 & 0 & 2 \sigma_i^2 \end{array} \right)\)

(25)

The third quantity is the average sum of the absolute turnover to measure each strategy's transaction. When the investor invests in N assets during T-M periods, the turnover is as follows:

\[
\text{Turnover} = \frac{1}{T-M} \sum_{t=1}^{T-M} \sum_{j=1}^{N} |w_{k,j,t+1} - w_{k,j,t}|)
\]

(26)

Where \( w_{k,j,t+1} \) is the weight of strategy k on asset j at time t, \( w_{k,j,t} \) is the weight of strategy k on asset j at time t+1 before rebalancing. And we calculate the return-loss of each strategy against the “1/N” strategy. The return-loss is defined as “the additional return needed for strategy k to perform as well as the 1/N strategy in terms of the Sharpe ratio.” (DeMiguel et al. (2009)).

\[
\text{Return-losse} = \frac{\mu_{cw}}{\sigma_{cw}} \cdot \sigma_k - \mu_k
\]

(27)

III. Empirical Results

A. Sharpe ratios

In Table 3, the first row shows the Sharpe ratio of our benchmark strategy “1/N” for the six different datasets. First of all, we note that when we apply estimation of moments with estimation error to mean-variance portfolio in out-of-sample, then this out-of-sample mean-variance portfolio will perform poorly compared to the “1/N” portfolio. The second
Table 3. Sharpe ratio of six empirical data without cost

This table describes the monthly Sharpe ratio of all datasets listed in Table 2, excluding transaction cost and tax. The p-value from differences between 1/N portfolio’s Sharpe ratio and each portfolio’s Sharpe ratio is calculated using Jobson and Korkie (1981) methodology.

| Sharpe Ratio (no cost) | MKT/SMB/HML N=3 | MKT/SMB/HML/CMA/RMW N=5 | MKF Sectors N=11 | FF 1-factor N=21 | FF 3-factor N=23 | FF 4-factor N=24 |
|------------------------|-----------------|--------------------------|-----------------|-----------------|-----------------|-----------------|
| 1/N                    | 0.2130          | 0.3099                   | 0.0539          | 0.1373          | 0.1426          | 0.1467          |
| mv(Insample)           | 0.2837          | 0.4681                   | 0.3504          | 0.6259          | 0.6388          | 0.6411          |
| mv                     | 0.2144          | 0.3597                   | -0.0929         | 0.1254          | 0.0646          | 0.1144          |
| bs                     | (0.99)          | (0.75)                   | (0.21)          | (0.92)          | (0.54)          | (0.80)          |
| dm                     | (0.47)          | (0.58)                   | (0.25)          | (0.93)          | (0.64)          | (0.85)          |
| min                    | 0.1134          | 0.0404                   | 0.0043          | 0.2249          | 0.2284          | 0.2355          |
| (0.16)                 | (0.23)          | (0.68)                   | (0.49)          | (0.50)          | (0.49)          |
| vw                     | 0.2414          | 0.4096                   | 0.1716          | 0.1979          | -0.2304         | -0.2041         |
| (0.44)                 | (0.48)          | (0.18)                   | (0.46)          | (0.00)          | (0.00)          |
| mp                     | 0.0766          | 0.2239                   | 0.0884          | 0.0684          | 0.0407          | 0.0841          |
| (0.26)                 | (0.65)          | (0.78)                   | (0.56)          | (0.51)          | (0.57)          |
| mv-c                   | 0.2601          | 0.3738                   | 0.0906          | 0.2724          | 0.3339          | 0.3393          |
| (0.56)                 | (0.68)          | (0.49)                   | (0.00)          | (0.02)          | (0.03)          |
| bs-c                   | 0.2793          | 0.3934                   | 0.0784          | 0.2776          | 0.3397          | 0.3356          |
| (0.31)                 | (0.56)          | (0.63)                   | (0.00)          | (0.02)          | (0.03)          |
| min-c                  | 0.2414          | 0.4227                   | 0.0886          | 0.1508          | 0.2415          | 0.2477          |
| (0.44)                 | (0.41)          | (0.57)                   | (0.75)          | (0.12)          | (0.15)          |
| g-min-c                | 0.2437          | 0.4138                   | 0.0426          | 0.1426          | 0.2375          | 0.2293          |
| (0.32)                 | (0.25)          | (0.75)                   | (0.81)          | (0.01)          | (0.02)          |
| mv-min                 | 0.2655          | 0.3898                   | -0.0367         | -0.0389         | 0.1760          | 0.1028          |
| (0.51)                 | (0.58)          | (0.41)                   | (0.17)          | (0.00)          | (0.72)          |
| ew-min                 | 0.2079          | 0.4083                   | 0.1353          | 0.1296          | 0.2464          | 0.1813          |
| (0.43)                 | (0.47)          | (0.29)                   | (0.93)          | (0.38)          | (0.76)          |

row of the Table 3 gives the Sharpe ratio of the in-sample Markowitz mean-variance strategy “mv(in-sample)”; this strategy’s Sharpe ratio should be highest of all the strategies considered if there are no estimation errors. We can check the loss from estimation error from the difference between the out-of-sample mean-variance strategy “mv” and the in-sample mean-variance strategy “mv(in-sample).” In six datasets, the differences are serious. For example, the first column “MKT/SMB/HML” in-sample mean-variance portfolio has a monthly Sharpe ratio 0.2837, but the Sharpe ratio of the equally weighted portfolio “1/N” is 0.2130. Similarly, in the last column “FF-4-Factor,” the in-sample mean variance portfolio has a monthly Sharpe ratio 0.6411, while the 1/N portfolio is only 0.1467.

However, the performance of “mv” strategy in out-of-sample is very poor in the Korean market. This result is consistent with results of Kim (2014). The third row in Table 3 shows the out-of-sample portfolio’s Sharpe ratio, from which we can infer a huge estimation error because the out-of-sample mean-variance strategy has a much lower Sharpe ratio than the in-sample mean-variance strategy in all six datasets. Moreover, except the two datasets “MKT/SMB/HML” and “MKT/SMB/HML/CMA/
The other four datasets have lower Sharpe ratios than the "1/N" portfolio. Also, out-of-sample Sharpe ratios from "MKT/SMB/HML" and "MKT/SMB/HML/CMA/RMW" are not statistically significant. Hence, because the estimation error is too large, less gain is realized from optimal diversification. For example, the dataset "MKF sectors," in-sample mean-variance portfolio has a monthly Sharpe ratio 0.3504; the out-of-sample mean-variance portfolio’s Sharpe ratio drops -0.0929, while we obtain 0.0539 for the 1/N portfolio. Also, in dataset "FF-4-Factor," the in-sample mean-variance portfolio has a Sharpe ratio 0.1144, while the out-of-sample mean-variance portfolio has a Sharpe ratio 0.0646, and 0.1467 in the 1/N portfolio. By comparing the Sharpe ratios between the "1/N" portfolios and the out-of-sample "mv" portfolios, we confirm that using the classical sample estimation of moments of asset returns in accordance with Markowitz’s mean-variance model performs poorly in the Korean market.

We further observe that existing Bayesian strategies demonstrate poor performance in the Korean market. We find the application of Bayesian strategy does not reduce estimation error well when estimate covariance is in evidence. Table 3 shows that the Bayes-Stein portfolio "bs" has a lower Sharpe ratio than the "1/N" portfolio in all datasets except "MKT/SMB/HML" and "MKT/SMB/HML/CMA/RMW," and even the p-value of differences from the "1/N" portfolio are not statistically significant in "MKT/SMB/HML" and "MKT/SMB/HML/CMA/RMW." Also, the Sharpe ratio of the "bs" model is higher than in the mean-variance portfolio in only two datasets "MKT/SMB/HML" and "MKT/SMB/HML/CMA/RMW," and the difference from "1/N" portfolio is not statistically significant (p-value is 0.47 and 0.58 respectively). In the other four datasets, the "bs" portfolio has a poorer Sharpe ratio than the "mv" portfolio. The Data-and-Model portfolio "dm" based on investor’s prior CAPM model’s mispricing α has a higher Sharpe ratio than the "bs" portfolio in four datasets ("MKF Sectors," "FF-1-factor," "FF-3-factor," "FF-4-factor"), but the differences are not statistically significant (each p-value 0.68, 0.49, 0.50 and 0.49 respectively). Also the "dm" model has a higher Sharpe ratio than the "1/N" portfolio in "FF-1-factor," "FF-3-factor," and "FF-4-factor," but the differences from the "1/N" portfolio are not statistically significant (each p-value 0.49, 0.50 and 0.49 respectively).

Next, we observe the asset-allocation strategy with restriction on estimations of return moments. The minimum-variance portfolio "min" in Table 3 does not require estimating mean of returns, but only estimating the covariance matrix of returns. This "min" portfolio gains a higher Sharpe ratio than the "mv" portfolio in four datasets except the "FF-3-factor," and the "FF-4-factor" (but the p-value of four datasets is not significant). Also, the "min" portfolio has a higher Sharpe ratio than the "1/N" portfolio in four datasets except the "FF-3-factor," and the "FF-4-factor," but the difference remains insignificant. The value-weighted portfolio "vw" has a lower Sharpe ratio than the "1/N" portfolio in all datasets, partly because of small firm effect. The "mp" portfolio suggested by MacKinlay and Pastor (2000) has a poorer Sharpe ratio than the "1/N" portfolio except in the "MKF Sector" (though the p-value is 0.78).

Our final observation is that, the asset-allocation strategies with short sale-constraint performs well in the Korean stock market. The minimum-variance-constrained portfolio "min-c" outperforms the "1/N" portfolio in all datasets (but all in six the difference is not statistically significant). The mean-variance-constrained portfolio "mv-c" and Bayes-Stein-constrained portfolio "bs-c" have higher Sharpe ratios in all datasets. The difference is statistically significant in the 95% confidence interval in the "FF-1-factor," the "FF-3-factor," and the "FF-4-factor." These results are inconsistent with other previous research. For example, DeMiguel et al. (2009) found that no asset-allocation strategy outperforms the "1/N" strategy in all datasets from the U.S. equity market. However, we can’t conclude that no asset-allocation strategy outperforms the "1/N" strategy in the Korean equity market.
Table 4. Certainty-equivalent returns of six empirical data

This table describes the certainty-equivalent returns of all datasets listed in Table 2, excluding transaction cost and tax. The p-value from differences between 1/N portfolio’s CEQ and each portfolio’s CEQ is calculated using Greene (2002) methodology.

| CEQ No cost | MKT/SMB/HML N=3 | MKT/SMB/HML/CMA/RMW N=5 | MKF Sectors N=11 | FF 1-factor N=21 | FF 3-factor N=23 | FF 4-factor N=24 |
|-------------|-----------------|-------------------------|-----------------|-----------------|-----------------|-----------------|
| l/N         | 0.0002          | 0.0001                  | 0.0004          | 0.0007          | 0.0007          | 0.0007          |
| mv(in sample)| 0.0008          | 0.0102                  | 0.0278          | 0.1669          | 0.0194          | 0.0198          |
| mv          | 0.0078          | 0.0079                  | -0.9390         | -3.4115         | 0.0014          | -0.0090         |
|             | (0.21)          | (0.20)                  | (0.00)          | (0.00)          | (0.53)          | (0.46)          |
| bs          | 0.0074          | 0.0073                  | -0.2466         | -0.8785         | -0.0068         | -0.0125         |
|             | (0.19)          | (0.26)                  | (0.00)          | (0.00)          | (0.02)          | (0.33)          |
| dm          | 0.0050          | -0.0054                 | -0.0543         | -0.3543         | -0.3898         | -0.3032         |
|             | (0.95)          | (0.48)                  | (0.03)          | (0.00)          | (0.00)          | (0.00)          |
| min         | 0.0054          | 0.0065                  | 0.0060          | 0.0115          | -0.0018         | -0.0017         |
|             | (0.82)          | (0.47)                  | (0.13)          | (0.36)          | (0.05)          | (0.05)          |
| Vw          | 0.0036          | 0.0004                  | 0.0006          | 0.0036          | 0.0036          | 0.0036          |
|             | (0.58)          | (0.18)                  | (0.41)          | (0.20)          | (0.17)          | (0.15)          |
| mp          | 0.0018          | 0.0092                  | -0.1149         | -1.4969         | -0.1450         | -0.0437         |
|             | (0.71)          | (0.12)                  | (0.01)          | (0.00)          | (0.00)          | (0.07)          |
| mv-c        | 0.0073          | 0.0075                  | 0.0036          | 0.0175          | 0.0096          | 0.0094          |
|             | (0.24)          | (0.24)                  | (0.37)          | (0.00)          | (0.54)          | (0.58)          |
| bs-c        | 0.0070          | 0.0073                  | 0.0028          | 0.0178          | 0.0103          | 0.0097          |
|             | (0.19)          | (0.24)                  | (0.53)          | (0.00)          | (0.43)          | (0.52)          |
| min-c       | 0.0054          | 0.0066                  | 0.0026          | 0.0074          | 0.0054          | 0.0052          |
|             | (0.82)          | (0.40)                  | (0.62)          | (0.91)          | (0.62)          | (0.57)          |
| g-min-c     | 0.0054          | 0.0064                  | 0.0009          | 0.0071          | 0.0081          | 0.0076          |
|             | (0.73)          | (0.32)                  | (0.70)          | (0.94)          | (0.70)          | (0.80)          |
| mv-min      | 0.0076          | 0.0072                  | -0.0962         | -1.5332         | 0.0087          | 0.0014          |
|             | (0.18)          | (0.26)                  | (0.00)          | (0.00)          | (0.00)          | (0.69)          |
| ew-min      | 0.0051          | 0.0064                  | 0.0054          | 0.0084          | 0.0020          | 0.0017          |
|             | (0.83)          | (0.48)                  | (0.18)          | (0.88)          | (0.27)          | (0.22)          |

B. Certainty equivalent returns

Table 4 shows various portfolios’ CEQ to check robustness of a portfolio’s performance. The performance of CEQ is consistent with the performance of the Sharpe ratio. Similar to the above empirical results, the in-sample mean-variance portfolio has higher CEQ in all datasets. This table also indicates the important difference between the result from the U.S. equity market and the Korean equity market, that the out-of-sample mean-variance-constraint portfolio (“mv-c”) and Bayes-Stein-constraint portfolio (“bs-c”) have higher CEQ than the “1/N” portfolio in all datasets, although in all six, p-value are not significant because of the limited number of samples.

C. Portfolio turnover

The “1/N” portfolio is commonly known as a good portfolio with low turnover compared to other portfolio strategies. In Table 5, Panel A compares various strategies’ turnover and Panel B compares the loss realized from equation (43).

In Panel A of table 5, all optimized portfolios have higher turnover than the “1/N” portfolio except the “vw” portfolio (“vw” has zero turnover because of its definition).Datasets “MKT/SMB/HML” and
Table 5. Turnover and loss of six empirical data
This table describes the turnover and loss of all datasets listed in Table 2, excluding transaction cost and tax.

Panel A Turnover

|                  | MKT/SMB/HML | MKT/SMB/HML/CMA/RMW | MKF Sectors | FF 1-factor | FF 3-factor | FF 4-factor |
|------------------|-------------|---------------------|-------------|-------------|-------------|-------------|
|                  | N=3         | N=5                 | N=11        | N=21        | N=23        | N=24        |
| 1/N              | 0.0271      | 0.0269              | 0.0360      | 0.0242      | 0.0262      | 0.0276      |
| mv               | 0.2010      | 0.0883              | 189.5880    | 2913.5      | 3.6488      | 13.0080     |
| bs               | 0.1185      | 0.0585              | 72.2362     | 297.3884    | 2.5991      | 9.8728      |
| dm               | 0.8972      | 4.4225              | 10.9569     | 89.5589     | 78.0503     | 74.6814     |
| min              | 0.0336      | 0.0397              | 0.4537      | 1.4218      | 0.2358      | 0.2438      |
| vw               | 0           | 0                   | 0           | 0           | 0           | 0           |
| mp               | 3.2477      | 0.3537              | 17.77       | 90.4765     | 15.8862     | 29.5266     |
| mv-c             | 0.0581      | 0.0694              | 0.2135      | 0.1753      | 0.0881      | 0.0959      |
| bs-c             | 0.0598      | 0.0544              | 0.2136      | 0.1660      | 0.1098      | 0.1380      |
| min-c            | 0.0336      | 0.0370              | 0.0756      | 0.0915      | 0.0344      | 0.0458      |
| g-min-c          | 0.0312      | 0.0326              | 0.0567      | 0.0451      | 0.0374      | 0.0581      |
| mv-min           | 0.1298      | 0.0575              | 44.3648     | 164.5771    | 7.6764      | 14.2886     |
| ew-min           | 0.0350      | 0.0391              | 1.1799      | 9.0478      | 5.4008      | 5.3440      |

Panel B Loss

|                  | MKT/SMB/HML | MKT/SMB/HML/CMA/RMW | MKF Sectors | FF 1-factor | FF 3-factor | FF 4-factor |
|------------------|-------------|---------------------|-------------|-------------|-------------|-------------|
|                  | N=3         | N=5                 | N=11        | N=21        | N=23        | N=24        |
| 1/N              | 0           | 0                   | 0           | 0           | 0           | 0           |
| mv               | -0.00006    | -0.0011             | 0.1880      | 0.0325      | 0.0080      | 0.0094      |
| bs               | -0.0016     | -0.0015             | 0.0829      | 0.0287      | 0.0115      | 0.0158      |
| dm               | 0.0059      | 0.0410              | 0.0166      | -0.1120     | -0.0979     | -0.0931     |
| min              | -0.0007     | -0.0016             | -0.0046     | -0.0043     | 0.0029      | 0.0028      |
| vw               | 0.0066      | 0.0168              | 0.0008      | 0.0024      | 0.0027      | 0.0029      |
| mp               | 0.0168      | 0.00002             | -0.0119     | 0.1240      | 0.0585      | 0.0244      |
| mv-c             | -0.0014     | -0.0013             | -0.0021     | -0.0101     | -0.0057     | -0.0055     |
| bs-c             | -0.0017     | -0.0016             | -0.0013     | -0.0104     | -0.0062     | -0.0057     |
| min-c            | -0.0007     | -0.0018             | -0.0013     | -0.0008     | -0.0023     | -0.0022     |
| g-min-c          | -0.0007     | -0.0016             | -0.0004     | -0.0003     | -0.0035     | -0.0030     |
| mv-min           | -0.0016     | -0.0015             | 0.0366      | 0.3017      | -0.0020     | 0.0084      |
| ew-min           | -0.0005     | -0.0016             | -0.0040     | -0.0010     | -0.0008     | -0.0003     |

“MKT/SMB/HML/CMA/RMW” have lower turnover than other datasets because “HML,” “SMB,” “CMA,” and “RMW” are already managed well, and these datasets have small components N=3, N=5.

The first observation in Table 5 is that mean-variance portfolio “mv” has drastically higher turnover in almost all datasets. For example, “mv” portfolio turnover is 2913.5 in “FF-1-factor” although “1/N” portfolio turnover is only 0.0242. These results are consistent with results of DeMiguel et al. (2009). For instance, “mv” portfolio turnover is 606594.36 in the “Industry portfolio,” although “1/N” portfolio turnover is only 0.0216. We can identify how the Bayes-Stein portfolio “bs” and the Data-and-Model
Table 6. Sharpe ratio of six empirical data with cost

This table describes the monthly Sharpe ratio of all datasets listed in Table 2, including transaction cost and tax. The p-value from differences between 1/N portfolio’s Sharpe ratio and each portfolio’s Sharpe ratio is calculated using Jobson and Korkie (1981) methodology.

| Sharpe Ratio (Cost) | MKT/SMB/HML N=3 | MKT/SMB/HML/CMA/RMW N=5 | MKF Sectors N=11 | FF 1-factor N=21 | FF 3-factor N=23 | FF 4-factor N=24 |
|---------------------|------------------|--------------------------|------------------|------------------|------------------|------------------|
| 1/N                 | 0.2099           | 0.3055                   | 0.0516           | 0.1363           | 0.1414           | 0.1454           |
| mv                  | 0.1997           | 0.3483                   | -0.2258          | -0.0943          | -0.0547          | -0.0330          |
| (0.93)              | (0.78)           | (0.10)                   | (0.17)           | (0.26)           | (0.31)           |
| bs                  | 0.2563           | 0.3806                   | -0.1769          | -0.1770          | -0.0661          | -0.0426          |
| (0.65)              | (0.60)           | (0.19)                   | (0.04)           | (0.04)           | (0.28)           |
| dm                  | 0.0689           | -0.0514                  | -0.0949          | 0.0131           | 0.0225           | 0.0221           |
| (0.13)              | (0.11)           | (0.38)                   | (0.47)           | (0.47)           | (0.47)           |
| min                 | 0.2372           | 0.4026                   | 0.1372           | 0.1377           | -0.3207          | -0.2944          |
| (0.58)              | (0.49)           | (0.48)                   | (0.98)           | (0.00)           | (0.01)           |
| vw                  | 0.0937           | 0.0121                   | 0.0379           | 0.0937           | 0.0937           | 0.0937           |
| (0.04)              | (0.04)           | (0.47)                   | (0.37)           | (0.33)           | (0.30)           |
| mp                  | -0.0617          | 0.1851                   | 0.0784           | -0.0685          | -0.0959          | 0.0426           |
| (0.10)              | (0.49)           | (0.88)                   | (0.18)           | (0.12)           | (0.52)           |
| mv-c                | 0.2543           | 0.3641                   | 0.0794           | 0.2652           | 0.3252           | 0.3285           |
| (0.68)              | (0.70)           | (0.70)                   | (0.01)           | (0.11)           | (0.12)           |
| bs-c                | 0.2726           | 0.3854                   | 0.0664           | 0.2708           | 0.3373           | 0.3328           |
| (0.48)              | (0.58)           | (0.83)                   | (0.01)           | (0.09)           | (0.11)           |
| min-c               | 0.2372           | 0.4162                   | 0.0823           | 0.1463           | 0.2372           | 0.2414           |
| (0.58)              | (0.42)           | (0.73)                   | (0.85)           | (0.27)           | (0.31)           |
| g-min-c             | 0.2335           | 0.4078                   | 0.0383           | 0.1405           | 0.2345           | 0.2245           |
| (0.47)              | (0.23)           | (0.80)                   | (0.89)           | (0.08)           | (0.11)           |
| mv-min              | 0.2531           | 0.3813                   | -0.1681          | -0.1888          | -0.2903          | -0.1398          |
| (0.60)              | (0.59)           | (0.19)                   | (0.06)           | (0.01)           | (0.09)           |
| ew-min              | 0.2035           | 0.4014                   | 0.0646           | -0.0707          | -1.5751          | -1.3784          |
| (0.38)              | (0.48)           | (0.90)                   | (0.10)           | (0.00)           | (0.00)           |

Our second observation is that, a portfolio with short sale-constraint has lower turnover than portfolios without short sale-constraint. For example, “mv,” “bs,” and “min” have turnover “2913.5,” “214.6818,” and “1.4218” respectively in the “FF-1-factor; “mv-c,” “bs-c” and “min-c” have turnover “0.1753,” “0.1660,” and “0.0915” respectively. The “min-c,” “mv-c,” and “bs-c” portfolios have lower turnover similar to the “1/N” portfolio. This means that the “min-c,” “mv-c,” and “bs-c” portfolio weights at every time t are similar to weights at time t-1. Hence, the “min-c,” “mv-c,” and “bs-c” portfolio estimates parameter at time t with small estimation error.

Furthermore, we compare the performance between short sale constraint strategies (i.e., “mv-c,” and “bs-c”) and “1/N” with transaction cost. We assume 30bp cost per a transaction, which reflects that in Korean stock markets investors are imposed on transaction tax when selling. Table 6 shows the Sharpe ratios with transaction cost. Almost all the Sharpe ratios of the “mv” portfolios with relatively high turnover fall to negative. However, “min-c,” “mv-c,” and “bs-c” still have higher Sharpe ratios than the “1/N” portfolio. Although the differences between
“1/N” portfolio and “min-c” portfolio are not significant in all datasets, “mv-c” and “bs-c” have statistically significant higher performance than “1/N” in the “FF-1-factor.”

From the above analysis, the “min-c,” “mv-c,” and “bs-c” show better performance in term of Sharpe ratio and CEQ than the “1/N” portfolio. These results are not consistent with the previous results from U.S. stock market. DeMiguel et al. (2009) shows that there is no portfolio strategy which outperforms the “1/N” portfolio from the dataset based on US stock market. We show that “mv-c,” and “bs-c” have higher Sharpe ratios and CEQ than the “1/N” portfolio. Also “mv-c,” and “bs-c” have lower turnover relative to other strategies; thus we consider these portfolio strategies as competitive candidates to the “1/N” portfolio in Korean stock markets. In next section, we look for the factors which cause short-sale constraint portfolio strategies to perform best.

V. Concluding Remarks

We have applied DeMiguel et al. (2009)’s methodology regarding the U.S. equity market to the Korean stock market. We compared the performance of a naïve portfolio and the other different 13 asset-allocation portfolios to six datasets as they did. As a result, we obtained some similar results and some results that differed from the previous advanced research. We found that, in conformance with the U.S. equity market, the Sharpe ratio of an in-sample mean-variance portfolio is higher than all 13 different asset-allocation model’s Sharpe ratios and that the out-of-sample mean-variance portfolio did not outperform the “1/N” portfolio in statistically significant ways in all datasets. Also there are no strategies that have lower turnover than the “1/N” portfolio. In contrast to the earlier research, without transaction cost, the Sharpe ratios of “mv-c,” “min-c,” and “bs-c” portfolios outperform the “1/N” portfolio in all six datasets, although p-value is not statistically significant. So we additionally computed the Sharpe ratio with transaction cost 30bp. Still, the Sharpe ratio of “mv-c,” “min-c,” and “bs-c” portfolios outperformed the “1/N” portfolio in all datasets, but the “mv-c” and “bs-c” portfolio have only one statistically significant p-value in the “FF-1-Factor” dataset and “min-c” has no statistically significant p-value in all datasets. This means we can’t say that there are no strategies that outperform the “1/N” portfolio in all datasets in the Korean equity market same as DeMiguel et al. (2009) asserted in their paper about the markets in the United States.

Unlike the U.S. equity market, short sale constraint strategies have better performance than a “1/N” portfolio in Korean equity markets. We additionally calibrate the Korean equity market using advanced research methodology to explain the relationship between the length of estimation windows and estimation error regarding mean-variance model performance. We find asset-allocation strategies combined short sale constraint such as “mv-c” and “bs”-c” have poor performance when estimation windows become longer. Hence, the poor performance of the out-of-sample mean-variance portfolio and good performance of “mv-c” and “bs-c” in our empirical results can account for the short estimation

5) In our test, the short estimation is when it is estimated with 60 month observations.
We use conventional three indicators such as Sharpe ratio, CEQ, and turnover for evaluating asset allocation performance. There are also challenging measures for asset allocation performance such as maximum drawdown, information ratio, and downside risk, and employing these measures for emerging stock market can be next research in this field.

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Appendix A The equation of asset allocation models

A.1 Pastor (2000), and Pastor and Stambaugh (2000) Bayesian Data-and-Model

$$\mu^{DM} = \omega \left( \bar{\mu}_j + (1 - \omega) \frac{\mu_a}{\mu_j} \right)$$  \hspace{1cm} (A.1)

$$\Sigma^{DM} = \left( \Sigma_{\mu a}^{DM}(a) \right)^T \Sigma_j^{-1} \left( \Sigma_{\mu a}^{DM}(a) \right)$$  \hspace{1cm} (A.2)

$$\hat{S} = \mu_j \Sigma_{jj}^{-1} \mu_j$$  \hspace{1cm} (A.3)

$$\omega = \frac{1}{1 + M \bar{S}^2}$$  \hspace{1cm} (A.4)

$$\Sigma_{\mu a}^{DM}(a) = b (a \bar{\beta} + (1 - \omega) \bar{\beta}) \Sigma_{jj} (a \bar{\beta} + (1 - \omega) \bar{\beta}) + h (a \bar{\delta} + (1 - \omega) \bar{\delta}) (a \bar{\Omega} + (1 - \omega) \bar{\Omega})$$  \hspace{1cm} (A.5)

$$\Sigma_{\mu j}^{DM}(a) = b (a \bar{\beta} + (1 - \omega) \bar{\beta}) \Sigma_{jj}$$  \hspace{1cm} (A.6)

$$\Sigma_{\mu j}^{DM}(a) = b \Sigma_{jj}$$  \hspace{1cm} (A.7)

$$\bar{\delta} = \frac{MM - 2 + 1}{MM - 1 - 2} \frac{1 + 3 \hat{S}^2}{MM - 1 - 2 + \hat{S}^2}$$  \hspace{1cm} (A.8)

$$\bar{\delta} = \frac{MM - 2}{MM - 1 - 2} \frac{M + 1}{M - (N - 1) - 1 - 1}$$  \hspace{1cm} (A.9)

$$b = \frac{M + 1}{M - 1 - 2}$$  \hspace{1cm} (A.10)

$$h = \frac{M}{M - (N - 1) - 1 - 1}$$  \hspace{1cm} (A.11)

A.2 Kan and Zhou (2007) “Three-fund” model

$$\omega^{DM} = \frac{(M - 1)M - (N - 1)}{\Sigma^{DM}(a)} \left[ \omega \left( \frac{1}{\Sigma_j} \mu_j \right)^T \Sigma_j^{-1} \left( \frac{1}{\Sigma_j} \mu_j \right) \right]$$  \hspace{1cm} (A.12)

$$\hat{S} = \frac{(M - 1)M - (N - 1)}{\Sigma} \left( \frac{1}{\Sigma_j} \mu_j \right)^T \Sigma_j^{-1}$$  \hspace{1cm} (A.13)

$$B_{\omega}(a,b) = \frac{1}{b^\omega - 1} (1 - y)^{b-1} dy$$  \hspace{1cm} (A.14)

$$B_{\omega}(a,b) = \frac{1}{b^\omega - 1} (1 - y)^{b-1} dy$$  \hspace{1cm} (A.15)

A.3 DeMiguel et al. (2009) Mixture of minimum-variance and 1/N

$$\omega^{\omega \omega} = \frac{(M - 1)M - (N - 1)}{\Sigma^{DM}(a)} \left[ \omega \left( \frac{1}{\Sigma_j} \mu_j \right)^T \Sigma_j^{-1} \left( \frac{1}{\Sigma_j} \mu_j \right) \right]$$  \hspace{1cm} (A.16)

$$\hat{S} = \frac{(M - 1)M - (N - 1)}{\Sigma} \left( \frac{1}{\Sigma_j} \mu_j \right)^T \Sigma_j^{-1}$$  \hspace{1cm} (A.17)

$$B_{\omega}(a,b) = \frac{1}{b^\omega - 1} (1 - y)^{b-1} dy$$  \hspace{1cm} (A.18)

$$B_{\omega}(a,b) = \frac{1}{b^\omega - 1} (1 - y)^{b-1} dy$$  \hspace{1cm} (A.19)