ASG version of integral sliding mode robust controller for AV nonholonomic 2D models avoiding obstacles

Hector Vargas · Jesús A. Meda · Alexander Poznyak

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Abstract In this paper, a new robust controller is developed and analyzed for an autonomous vehicle (AV) with nonholonomic dynamics driving in a 2D plane that can avoid colliding with a set of obstacles despite the presence of uncertainties in the mathematical model of the AV. The state variables and their velocities are considered to be measurable (two simple coordinates and three angles). The controller is based on the integral sliding mode (ISM) concept, which aims to minimize the current state’s convex (but not necessarily strongly convex) function. A cost function’s subgradient is likewise designed to be measured online. The averaged subgradient (ASG) technique is used to build and analyze an optimization algorithm. The major findings show that the intended regime (non-stationary analogue of sliding surface) can be reached from the start of the process and deriving an explicit upper bound for the cost function decrement, i.e., proving functional convergence and estimating the rate of convergence, thereby allowing for multiple obstacle avoidance. The proposed strategy is shown to perform effectively with a numerical example.

Keywords Car obstacle avoidance · Nonholonomic dynamic model · Integral sliding mode control · Averaged subgradient method

1 Introduction

1.1 Brief Survey

Due to its vast range of applications, such as industrial or military, autonomous vehicles have piqued the interest of the engineering community. In recent years, a variety of methods for modeling and managing AV have been developed. As a result of these studies, the idea of obstacle avoidance has emerged, along with a wide range of methodologies to tackle this problem. Artificial vision and intelligence algorithms have been widely used to determine an AV’s position and place it in a desired pose; such tools also enable the AV to identify forbidden zones. These methods are detailed in [1–4], where the problem has only been partially solved. There are, on the other hand, methods based on the dynamics of electromechanical system models which need measuring their location and velocity during the control moment. Furthermore, one must understand the environment in which the controlled system operates, which necessitates the use of a nonlinear control program. Online optimization, designed specially for dynamic systems and containing uncertainties in their models, is one of the forms of robust control. Article [5] studies the optimization problem with con-
constraints for linear and time invariant dynamic systems (LTI), where the dimension of control actions is smaller than the system state vector. The problem is to design a control signal capable to generate the system dynamics, avoiding some prohibited regions and minimizing the suggested cost function. The finite time convergence to a neighborhood of the equilibrium point is proven. In [6] an algorithm is developed, which is based on the convex optimization concept applied to a dynamic plant modeled using the Lagrangian approach, given by an ordinary differential equation of the second order and with unknown right-hand side, but with accessible states, as well as their velocities. Article [7] developed a finite-time optimal formation tracking for planar vehicles technique with holonomic dynamics, using Pontryagin’s maximum principle on a Lie group. In [8], the trajectory planning and tracking control of the autonomous bicycle robot are considered and solved despite the nonholonomic dynamics. The desired motion trajectory of the contact point of the bicycle’s rear wheel is constructed using a parameterized polynomial curve that can connect two given endpoints with associated tangent angles. The paper [9] works with a double-layer control framework proposed to construct the coordinated lateral and longitudinal motion control for over-actuated autonomous electric vehicles, which are independently driven by four in-wheel motors using sliding mode control. In that work a differential wheeled vehicle robot is aimed to reach any plane X-Y position in spite of the presence of obstacles in its way to the desired point. The vehicle robot is modeled following the Euler–Lagrange with nonholonomic constrains approach, which is not widely cover in the literature. This method leads to a set of differential equations which may contain an unknown right-hand side, and several uncertainties not only in their parameters, but also in their structure; nevertheless, as long as the states and their rates are accessible and measurable the approach can have practical application. Paper [10] deals with a fixed-time stabilization problem for a kind of nonholonomic systems in chained form with unmatched uncertainties and time-varying output constraints. A tan-type barrier Lyapunov function is used to solve the problem that with time-varying output constraints imply. A state feedback controller is designed using a power integrator technique and switching control strategy, and finally in an adaptive control strategy for hypersonic flight vehicles (HFVs) subject to parametric uncertainties is introduced. To achieve this, the disturbed control-oriented model of HFVs with uncertain parameters and faulty actuators is first rewritten into a parameterized form. By introducing a barrier Lyapunov functions to design procedures, specific tracking performances of velocity and altitude are guaranteed, while the other flight states of angle of attack, flight path angle, pitch angle and pitch angle rate can be kept within the prescribed ranges. The study in [11] deals with the tracking control problem of Euler–Lagrange systems in an environment with obstacles when there are external disturbances. An asymptotic tracking controller based on a special sliding manifold is proposed to assure that tracking errors converge to zero as time goes to infinity. Through introducing collision avoidance functions into the sliding manifolds, the controller can guarantee the obstacle avoidance. The control system provided in [12] enables for the maintenance of cruising conditions as well as the avoidance of probable traffic accidents, which is a first in comparison to previous alternatives. The suggested control system is implemented by vehicle supervisors, who, based on the data collected by the sensors, determine which current control mode is appropriate for each controlled vehicle. A feedback controller for a nonholonomic system with three states and two inputs is constructed using an artificial potential function with no local minima in [13], and the system’s stability of equilibria is investigated. When a potential function features saddle-type critical points, the saddles can be stable equilibria in addition to the stable equilibrium at the function’s minimum.

In this paper ISM approach together with the ASG technique (supposing that the current subgradient \( \partial F(x_t) \) of the convex function \( F(x_t) \) to be optimized is available online) is applied. The obstacles are included in the cost function as penalty terms. So with the specially selected parameters of the designed controller, the functional convergence of the convex (not obligatory strongly convex) loss function to its minimal value is proven. The major focus of this work is on developing a resilient nonlinear feedback system that allows the controlled system to achieve the intended behavior when the precise mathematical model is completely or partially unknown and cannot be determined for various physical reasons. This issue was largely ignored in the articles cited above (may be, because just a few factors were allowed to be uncertain, but not nonlinear functions in general such as friction, hysteresis, or Coriolis).
1.2 Main contribution

1. A variation of the known Lagrangian with nonholonomic constrains dynamic model is derived and analyzed.
2. A new concept of ASG version of ISM control technique is designed.
3. A complete analysis of the functional convergence of the controlled system (desired dynamics) is presented.
4. A high level of uncertainties (inertia tensor, forces non-potentials such as: friction, hysteresis or Coriolis) in the system model is admitted.
5. The effectiveness of the method is demonstrated theoretically and with a numerical simulation, where an Autonomous Vehicle (AV) avoids obstacles reaching a desired point.

1.3 Paper structure

The following is how the paper is structured. The first section of the paper contains an introduction to the work, which includes a brief survey as well as the main contributions of our research. Then, in Sect. 2 a full derivation of a nonholonomic AV model, which does not need to consider the so-called Lagrange multipliers, is developed. Section 3 is devoted to stating the main assumptions on which this research is founded as well as explaining the problem formulation. In the Sect. 4, an analysis of functional convergence is provided and explained by a concept that we named the desired dynamics and its properties. The main theorem of this paper, which is about the robust controller, is found in Sect. 5 and the reader can also find a full stability proof in this section. The numerical experiment is designed and depicted in Sect. 6, where we demonstrated that an AV can avoid three obstacles fixed in the XY plane, even when white noise is introduced as an extra disturbance. In addition, we present a set of graphical results that support the theoretical results. Finally, the conclusions of our work are stated in the seventh and last section.

2 Nonholonomic AV models

2.1 Nonholonomic Euler–Lagrange models

2.1.1 Coordinate systems

Two different coordinate systems (frames) need to be defined (see Fig. 1).

1. Inertial coordinate system This coordinate system is a global frame which is fixed in the environment or plane in which the AV moves in. Moreover, this frame is considered as the reference frame and is denoted as \( \{x_I, y_I\} \).

2. Relative (or proper) Coordinate System This coordinate system is a local frame attached to the considered AV, and thus, moving with it. This frame is denoted as \( \{x_r, y_r\} \). The origin of the relative frame is defined to be the mid-point \( A \) on the axis between the wheels. The center of mass \( C \) of the AV is assumed to be on the axis of symmetry, at a distance \( d \) from the origin \( A \).

As shown in Fig. 1, the robot position and orientation in the inertial frame can be defined as

\[
q^I := (x_a, y_a, \theta)^	op
\]  (1)
The important issue that needs to be explained at this stage is the mapping between these two frames [14]. The position of any point on the AV can be defined in the inertial frame and the relative frame as follows:

\( \Theta^I := (x^I, y^I, \theta^I)^T, \Theta^r := (x^r, y^r, \theta^r)^T \)  

(2)

Then, the two coordinates are related by the following transformation:

\( \Theta^I = R(\theta) \Theta^r \)  

(3)

where \( R(\theta) \) is the orthogonal (\( R^T(\theta) = R^{-1}(\theta) \)) rotation matrix

\[
R(\theta) = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(4)

This transformation enables also the handling of motion between frames.

2.1.2 Kinematic constraints of the differential-drive autonomous vehicle (AV)

Here, we will show that the motion of a differential-drive autonomous vehicle (AV) is characterized by two nonholonomic constraint equations, which are obtained by two main assumptions:

- **No lateral slip motion**: This constraint simply means that the AV can move only in a curved motion (forward and backward) but not sideward. In the relative frame, this condition means that the velocity of the center-point A is zero along the lateral axis, namely, for any time, \( t \geq 0 \)

\[
y^r = 0, \quad \dot{y}^r = 0
\]

(5)

which can be expressed as

\[-\dot{x}_a \sin \theta + \dot{y}_a \cos \theta = 0 \]

(6)

- **Pure rolling constrain**: The pure rolling constraint represents the fact that each wheel maintains a one contact point \( p \) with the ground. There is no slipping of the wheel in its longitudinal axis \((x^r)\) and no skidding in its orthogonal axis \((y^r)\).

The velocities of the contact points in the relative frame are related to the right \((r)\) and left \((l)\) wheel velocities by:

\[
v_p = R \dot{\varphi}_R, \quad v_l = R \dot{\varphi}_L
\]

(7)

In the inertial frame, these velocities can be calculated as a function of the velocities of the AV center-point \( A \):

\[
\begin{align*}
\dot{x}_{pR} &= \dot{x}_a + L \dot{\theta} \cos \theta \\
\dot{y}_{pR} &= \dot{y}_a + L \dot{\theta} \sin \theta
\end{align*}
\]

(8)

and

\[
\begin{align*}
\dot{x}_{pL} &= \dot{x}_a + L \dot{\theta} \cos \theta \\
\dot{y}_{pL} &= \dot{y}_a + L \dot{\theta} \sin \theta
\end{align*}
\]

(9)

Using the rotation matrix \( R(\theta) \) (4) and (7), the rolling constraint equations (6) are formulated as follows:

\[
\begin{align*}
\dot{x}_{pR} \cos \theta + \dot{y}_{pR} \sin \theta &= R \dot{\varphi}_R \\
\dot{x}_{pL} \cos \theta + \dot{y}_{pL} \sin \theta &= R \dot{\varphi}_L
\end{align*}
\]

(10)

Applying the contact points velocities equation (8)–(9) and the three constraint equations (6) and (10) can be rewritten in the following matrix form:

\[
\Lambda(q) \dot{q} = 0
\]

(11)

where \( q = (x_a, y_a, \theta, \varphi_R, \varphi_L)^T \) is the generalized position vector and \( \Lambda(q) \) is

\[
\Lambda(q) = \begin{bmatrix}
-\sin \theta \cos \theta & 0 & 0 & 0 \\
\cos \theta & \sin \theta & L & -R \\
\cos \theta & \sin \theta & -L & 0 \\
& & & -R
\end{bmatrix}
\]

(12)

2.1.3 Euler–Lagrange equations for nonholonomic dynamic models

To derive equations of motion for nonholonomic systems (with constrains depending on the derivatives \( \dot{q} \) of the generalized coordinate), we will apply the so-called Lagrangian approach using the Hamilton’s principle and the Euler–Lagrange equations, respectively. In case when we have no constrains for dynamic trajectories, according to the Hamiltonian principle [15, 16], the equations of motion for the considered system
provide the extremal value for the Hamiltonian action, that is,

$$\int_{t=a}^{b} L(q, \dot{q}, t) \, dt \rightarrow \text{extr} \quad q, \dot{q} \in \mathbb{R}^n$$

(13)

where \( L = T - \Pi \) is the Lagrange function with the kinetic \( T \) and potential \( \Pi \) energies, respectively. This corresponds to the condition

$$\delta \int_{t=a}^{b} L(q, \dot{q}, t) \, dt = \int_{t=a}^{b} \left( \frac{\partial T}{\partial q} \delta q + \frac{\partial T}{\partial \dot{q}} \delta \dot{q} \right) \, dt = 0 \quad \text{(14)}$$

Here, we choose virtual (under fixed \( t \)) variations \( \delta q_t \) of the curve \( q_t (t \in [a, b]) \) in such a way that \( \delta q_{t=a} = \delta q_{t=b} = 0 \). Integrating by parts with fixed endpoints, we get

$$\int_{t=a}^{b} \left( \frac{\partial T}{\partial q} - \frac{d}{dt} \frac{\partial T}{\partial \dot{q}} \right) \delta q dt = 0$$

which gives us, from the arbitrariness of \( \delta q \), the Euler–Lagrange equation (in the vector form)

$$\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = 0$$

(15)

Unfortunately, for the nonholonomic systems when there are constrains for the admissible trajectories we cannot conclude (15) since the admissible virtual variations \( \delta q \) are not arbitrary. They should satisfy (11) for all \( t \geq 0 \), or equivalently,

$$A(q) \, \delta q = 0$$

(16)

To apply correctly in our case the Hamiltonian principle, we need to take into account the relation (11) considering the following extremal problem:

$$\int_{t=a}^{b} L(q, \dot{q}, t) \, dt \rightarrow \text{extr} \quad q, \dot{q} \in \mathbb{R}^n$$

under constrains (16)

Using the Lagrange multipliers approach, we may conclude that the variation of the Hamiltonian action (13) around the admissible extremal curve \( q \), satisfying the constrains (16), corresponds to the condition (the extended version of D’Alembert’s principle)

$$\delta \int_{t=a}^{b} L(q, \dot{q}, t) \, dt + \int_{t=a}^{b} \lambda^T A(q) \, \delta q dt = 0 \quad \text{(18)}$$

where \( \lambda \) is the vector of Lagrange multipliers depending on \( t \) and the virtual variations now are arbitrary.

Following the analogous procedure as before, we conclude that (18) implies

$$\int_{t=a}^{b} \left( \frac{\partial T}{\partial q} \delta q + \frac{\partial T}{\partial \dot{q}} \delta \dot{q} + \lambda^T A(q) \, \delta q \right) \, dt = 0$$

and again, the integration by parts of the second term leads to

$$\int_{t=a}^{b} \left( \frac{\partial T}{\partial q} \delta q - \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}} \right) \delta q + \lambda^T A(q) \, \delta q \right) \, dt = 0$$

which, by the arbitrariness of \( \delta q \), is possible if and only if

$$\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} + A^T (q) \lambda = 0$$

or equivalently,

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = A^T (q) \lambda$$

(19)

In the presence of external nonpotential forces \( Q_{\text{nonpot}} \) the Euler–Lagrange equation for nonholonomic systems leads to

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = Q_{\text{nonpot}} + A^T (q) \lambda$$

(20)

The kinetic energy \( T \) for the considered AV contains three parts:

- the kinetic energy of the AV platform

$$T_c = \frac{1}{2} m_c v_c^2 + \frac{1}{2} I_c \dot{\theta}^2$$

- the kinetic energy of the right wheel

$$T_{wR} = \frac{1}{2} m_w v_{wR}^2 + \frac{1}{2} I_w \dot{\psi}_R^2$$

- the kinetic energy of the left wheel
where $m_c$ is the mass of the AV without the driving wheels and actuators (DC motors), $m_w$ is the mass of each driving wheel (with actuator), $I_c$ is the moment of inertia of the AV with respect to the vertical axis passing through the center of mass, $I_w$ is the moment of inertia of each driving wheel with a motor with respect to the wheel axis, and $I_m$ is the moment of inertia of each driving wheel with a motor with respect to the wheel diameter. Taking into account that all velocities may be expressed as a function of the generalized coordinates using the general velocity equation in the inertial frame, namely, as $v_i^2 = \dot{x}_i^2 + \dot{y}_i^2$, and in view of the relations

\[
x_c = x_a + d \cos \theta, \quad y_c = y_a + d \sin \theta \\
x_wR = x_a + L \sin \theta, \quad y_wR = y_a + L \cos \theta \\
x_wL = x_a - L \sin \theta, \quad y_wL = y_a + L \cos \theta
\]

we may conclude that

\[
T = T_c + T_wR + T_c = \frac{1}{2} m \left( \dot{x}_a^2 + \dot{y}_a^2 \right) \\
- m_c d \dot{\theta} (\dot{y}_a \cos \theta - \dot{x}_a \sin \theta) \\
+ \frac{1}{2} I_w \left( \dot{\phi}_R^2 + \dot{\phi}_L^2 \right) + \frac{1}{2} I \dot{\theta}^2
\]

where $m = m_c + 2m_w$ is the total mass of AV and

\[
I = I_c + m_c d^2 + 2m_w L^2 + 2I_m
\]

is the total equivalent moment of inertia. Since during 2D movements $\Pi = \text{const}$, the dynamic model of the nonholonomic Euler–Lagrange system (20) becomes

\[
\frac{d}{dt} \frac{\partial \lambda}{\partial \dot{q}} - \frac{\partial \lambda}{\partial q} = Q_{\text{nonpot}} + \Lambda^T(q) \lambda
\]

(21)

or, in the open format,

\[
M(q) \ddot{q} + V(q, \dot{q}) = B(q) \tau - \Lambda^T(q) \lambda
\]

(22)

where:

\[
M(q) = \begin{bmatrix}
  m & 0 & -md \sin \theta & 0 & 0 \\
  0 & m & -md \cos \theta & 0 & 0 \\
  -md \sin \theta & -md \cos \theta & I & 0 & 0 \\
  0 & 0 & 0 & I_w & 0 \\
  0 & 0 & 0 & 0 & I_w
\end{bmatrix}
\]

\[
V(q, \dot{q}) = \begin{bmatrix}
  0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
Q_{\text{nonpot}} = B \tau + F(q, \dot{q}) , B = \begin{bmatrix}
  1 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Here, $F(q, \dot{q})$ is the centripetal and Coriolis forces and $\tau$ is the control action vector (torques of the right and left motors).

Following [14], let us consider the kinematic relation

\[
\dot{\phi} = S \dot{\phi}, \quad \phi = \begin{bmatrix} \phi_R \\ \phi_L \end{bmatrix}
\]

(23)

where

\[
S(q) = \frac{1}{2} \begin{bmatrix}
  R \cos \theta & R \cos \theta \\
  R \sin \theta & R \sin \theta \\
  \frac{R}{L} & -\frac{R}{L} \\
  \frac{L}{2} & 0 \\
  0 & 2
\end{bmatrix}
\]

(24)

It can be verified that the matrix $S(q)$ (24) acts in the null space of the constraint matrix $\Lambda^T(q)$, that is,

\[
\Lambda(q) S(q) = 0, \quad S^T(q) \Lambda^T(q) = 0
\]

(25)

and therefore, differentiation of (23), leads to

\[
\ddot{\phi} = \dot{S}(q) \dot{\phi} + S(q) \ddot{\phi}
\]

(26)

Substitution (26) into (22) gives

\[
M(q) \ddot{q} + V(q, \dot{q}) = B(q) \tau - \Lambda^T(q) \lambda
\]

(27)
Multiplying both sides of (27) from left by $S^{T}(q)$ and using the property (25), we finally get

$$D(q)\dot{\varphi} + \dot{V}(q, \dot{q})\dot{\varphi} = \dot{B}\tau + \ddot{F}(q, \dot{q})$$

(28)

where

$$D(q) = S^{T}(q)M(q)S(q),$$

$$\dot{V}(q, \dot{q}) = S^{T}(q)\left[M(q)\dot{S}(q) + V(q, \dot{q})S(q)\right]$$

$$\dot{B}(q) = S^{T}(q)B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ddot{F}(q, \dot{q})=S^{T}(q)\mathcal{F}(q, \dot{q})$$

(29)

### 3 Assumptions and problem formulation

**Main assumptions**

Consider the following assumption that holds through the entire work:

1. Vectors $q_{t}, \dot{q}_{t}$ are physically measured for all $t \geq 0$.
2. Inertia Matrix $D(q_{t})$, which can contain unknown terms a priori, it is strictly positive defined and bounded, i.e.,

$$(0 < D_{-}I_{n \times n} \leq D(q) \leq D_{+}I_{n \times n}).$$

(30)

3. The functions $\mathcal{F}$ that define centripetal, Coriolis forces and may include the friction and hysteresis effects description are assumed to be unknown, but bounded:

$$\|\mathcal{F}(q_{t}, \dot{q}_{t})\| \leq F_{+}$$

(31)

($F_{+}$ is supposed to be known). The term $F$ evidently may include bounded noise affected the dynamics of the considered system.

**Remark 1** 1. The assumption on the physical measurements of the generalized states (two coordinates of the center of mass and one angle) seems to be natural in our consideration and may be easily measured by the application of standard sensors. As for their derivative (velocities), the availability online the corresponding signals is not always a trivial task. Here, two possibilities exist: one is to estimate them as the first back-Euler approximation $q_{t} \simeq q_{t-h} - \dot{q}_{t-h}$ with small $h > 0$, which works well when the controlled process are, respectively, slow (in our case we deal exactly with this case), and the second one is to design the velocity estimates $\dot{q}_{est,t}$ generated by a special observers of “super-twist” type (see, for example, Section 5.2.4 in Utkin et al. [17])

2. The inertia matrix $D(q_{t}) = S^{T}(q)M(q)S(q)$ cannot be known exactly because of the matrix $M(q)$ containing the elements $I$ (the joint mass) and $I_{w}$ (the moment of inertia of each driving wheel with a motor with respect to the wheel axis) which cannot be measured exactly and depend on a physical model realization. But some upper and lower estimates can be obtained before the control process.

3. The function $\mathcal{F}(q_{t}, \dot{q}_{t})$ cannot be known exactly because of several immeasurable factors, acting to the process. But its upper norm estimate may be adjusted by the, so-called, “try-to-test” method: if a current value of $F_{+}$ does not provides a good quality of the process, it may be augmented (or decreased) with the following test-experiment application.

#### 3.1 Cost function with penalty terms

Consider the distance $\delta_{t}$ between the current AV position $q_{t}$ and a desired final point $q^{*}$:

$$\delta_{t} := q_{t} - q^{*}$$

(32)

Let the cost functional be convex (not necessarily strictly convex) $F : \mathbb{R}^{n} \rightarrow \mathbb{R}$, which defines the quality of the actions of the control $\tau_{t}$. The following expression considers the class of convex functions to optimize:

$$F(\delta) = \sum_{i=1}^{n} |\delta_{i}| + \sum_{j=1}^{m} \frac{\mu_{j} g_{j}(\delta)}{[g_{j}(\delta)]_{+} + \epsilon_{j}}$$

(33)

where

- $g_{j}(\delta)$ describes the forbidden area

$$g_{j}(\delta) \leq 0,$$

(34)

- the operator $[\cdot]_{+} : \mathbb{R} \rightarrow \mathbb{R}$ is defined as

$$[z]_{+} := \begin{cases} z & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases},$$

- $\mu_{j}, \epsilon_{j}$ are positive parameters.
3.2 Problem formulation

**Problem 1** Under the assumptions 1–3 above, we need to find a control strategy \( \tau \) which provides the functional convergence of the cost \( F(\delta_t) \) to its minimum value \( F^* \), in the presence of uncertainties \( F(q_t, \dot{q}_t) \), that is,

\[
F(\delta_t) \rightarrow \inf_{t \rightarrow \infty} \delta \in \mathbb{R}^n: g_j(\delta) > 0, j=1,...,m \quad F(\delta) = F^* \tag{35}
\]

4 Desired dynamics and its properties

4.1 Auxiliary sliding variable \( s_t \)

Define the vector function \( s_t \in \mathbb{R}^n \), which from now on and throughout this work will be referred to as “sliding variable”:

\[
s_t = \delta_t + \frac{\delta_t + \eta}{t + \theta} + \tilde{G}_t, \eta \in \mathbb{R}^n
\]

\[
\tilde{G}_t := \frac{1}{t + \theta} \int_{\tau=t_0}^t a(\delta_\tau) \, d\tau, \theta > 0
\]

(36)

Here, \( \delta_t \in \mathbb{R}^n \) is defined in (32) and \( \tilde{G}_t \) is the averaged subgradient of the function \( F(q_t) \) (35).

**Remark 2** Note that the sliding variable \( s_t \) contains the integral term which is physically measurable.

4.1.1 Desired dynamic

The sliding variable \( s_t \) is associated with following system of ODE:

\[
\dot{\xi}_t = -a(\delta_t) \cdot \xi_t = 0, \delta_{t_0} \text{ is given } \tag{37}
\]

(\( t + \theta \)) \( \dot{\delta}_t + \delta_t + \eta = \xi_t, \quad \theta, \eta > 0, \)

\( t_0 \) is the moment when the desired dynamics may begin.

which is referred here to as a “desired dynamic.” It is evident that

\( (t + \theta) s_t = (t + \theta) \delta_t + \delta_t + \eta = \xi_t, \)

and therefore the desired dynamics (37) corresponds exactly to the situation when the sliding variable \( s_t \) is equal to zero for all \( t \geq t_0 \):

\[
s_t = \dot{s}_t = 0. \tag{38}
\]

Below we will show why the dynamic (37) is called a desired.

**Lemma 1** (Functional convergence in the desired regime.) For the variable \( \delta_t \) satisfying the ideal dynamics (38), with any \( \theta > 0 \) and \( \eta \), for all \( t \geq t_0 \geq 0 \) the following inequality is guaranteed:

\[
F(\delta_t) - F^* \leq \frac{\Phi_0}{t + \theta} \rightarrow 0 \quad \tag{39}
\]

where

\[
\Phi_0 = \Phi(\delta_{t_0}, \theta, \eta) := (t_0 + \theta) \left[ F(\delta_{t_0}) - F^* \right] + \frac{1}{t} \| \delta^* - \eta \|^2 \tag{40}
\]

**Proof** Defining \( \mu_t := t + \theta \) and

\[
\delta^* := \arg \inf_{s \in \mathbb{R}^n: g(s) > 0, j=1,...,m} F(s)
\]

we have

\[
\frac{d}{dt} \left[ \frac{1}{2} \| \xi_t \|^2 - \xi_t^T \delta^* \right] = \xi_t^T (\xi_t - \delta^*)
\]

\[
= -a^T(\delta_t) \left[ \mu_t \dot{\delta}_t + \delta_t + \eta - \delta^* \right]
\]

\[
= -a^T(\delta_t) (\delta_t - \delta^*) - a^T(\delta_t) (\mu_t \dot{\delta}_t + \eta).
\]

Using the inequality (see chapter 23 in [18])

\[
(\delta - \delta^*)^T a(\delta) \geq F(\delta) - F^*
\]

valid for convex (not obligatory strongly convex) functions in the first term on the right side and applying the identity

\[
a^T(\delta_t) \delta_t = \frac{d}{dt} \left[ F(\delta_t) - F^* \right],
\]

we get

\[
\frac{d}{dt} \left[ \frac{1}{2} \| \xi_t \|^2 - \xi_t^T \delta^* \right] \leq - \left[ F(\delta_t) - F^* \right]
\]

\[
- \mu_t \frac{d}{dt} \left[ F(\delta_t) - F^* \right] - a^T(\delta_t) \eta.
\]

Then, integrating the last inequality in the interval \([t_0, t]\) and applying the formula of integration by parts, we derive

\[
\int_{\tau=t_0}^t \left[ F(\delta_t) - F^* \right] d\tau \leq \frac{1}{2} \left( \| \xi_0 \|^2 - \| \xi_t \|^2 \right)
\]

\[
+ (\xi_t - \xi_0)^T \delta^* - (\mu_t \left[ F(\delta_t) - F^* \right])_{t_0}^t
\]

\[
+ \int_{\tau=t_0}^t \left[ F(\delta_t) - F^* \right] \mu_t d\tau - \left[ \int_{\tau=t_0}^t a^T(\delta_t) d\tau \right] \eta.
\]

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Since $\mu_t = 1$, the above inequality becomes

$$
\mu_t \left[ F (\delta_t - F^*) \right] \leq \mu_{t_0} \left[ F (\delta_{t_0} - F^*) \right] + \frac{1}{2} \left( \| \xi_{t_0} \|^2 - \| \xi_t \|^2 \right) + (\xi_t - \xi_{t_0})^T \delta^* + \xi_t^T \eta
$$

$$
= (t_0 + \theta) \left[ F (\delta_{t_0} - F^*) \right] + \left( \frac{1}{2} \| \xi_{t_0} \|^2 - \xi_{t_0}^T \delta^* \right)
+ \frac{1}{2} \| \delta^* - \eta \|^2
$$

$$
- \frac{1}{2} \left( \| \xi_t \|^2 - 2 \xi_t^T (\delta^* - \eta) \right)
+ \frac{1}{2} \| \delta^* - \eta \|^2
\leq (t_0 + \theta) \left[ F (\delta_{t_0} - F^*) \right] - \frac{1}{2} \| \xi_t - (\delta^* - \eta) \|^2
+ \left( \frac{1}{2} \| \xi_{t_0} \|^2 - \xi_{t_0}^T \delta^* \right) + \frac{1}{2} \| \delta^* - \eta \|^2 \leq \Phi_{t_0},
$$

(42)

from which we obtain (40). Lemma is proved. □

**Remark 3** The parameter $\eta$ will be chosen below in such a way that the desired optimization regime starts from the beginning of the process, when $t_0 = 0$.

5 ISM robust controller

5.1 Main theorem on robust controller

**Theorem 1** Under assumptions 1–3, the ISM robust controller

$$
\tau_t = D(q_t) S^T (q_t) \left[ S (q_t) S^T (q_t) + \rho I_{5 \times 5} \right]^{-1}
\left[ -k_t \text{SIGN} (s_t) + u_{\text{comp}, t} \right]
$$

$$
u_{\text{comp}, t} = -k_t \rho [S(q_t) S^T (q_t) + \rho I_{5 \times 5}]^{-1} \text{SIGN}(s_t)
- \left( I_{5 \times 5} + \rho [S(q_t) S^T (q_t) + \rho I_{5 \times 5}]^{-1} \right) p_{\text{reali}} \rho > 0
$$

$$k_t = \| S (q_t) D^{-1}(q_t) S^T (q_t) \| F^+ + \rho_0, \rho_0 > 0
$$

(43)

where

$$
p_{\text{reali}} := \dot{S} (q) \psi - S (q) D^{-1}(q) \tilde{V} (q, \dot{q}) \psi
+ \frac{1}{t + \theta} \left( \delta_t - \frac{\delta_t + \eta}{t + \theta} - \tilde{G}_t + a (\delta_t) \right)
$$

(44)

with

$$
\eta = -\theta \delta_{2,0} - \delta_{1,0}
$$

(45)

guarantees the functional convergence (39) from the beginning of the process ($t_0 = 0$). Here,

$$
\text{SIGN} (z) = \left[ \text{sign}(z_1), ..., \text{sign}(z_n) \right]^T, z \in \mathbb{R}^n,
$$

$$
\text{sign}(z_i) = \begin{cases}
1 & \text{if } z_i > 0 \\
-1 & \text{if } z_i < 0 \\
0 & \text{if } z_i = 0
\end{cases}, i \in [-1, 1]
$$

**Proof** In view of (29), we may conclude that the matrix $D(q)$ is positive definite and, hence, is invertible. Then, by the relations (26) and (28) we obtain

$$
\delta_t := q_t - q^*, \tilde{\delta}_t := \tilde{q}_t
$$

$$\begin{align*}
\tilde{\delta}_t & = \tilde{q}_t = \tilde{S} (q) \dot{\phi} + S (q) \ddot{\phi} \\
& = \tilde{S} (q) \dot{\phi} - S (q) D^{-1}(q) \tilde{V} (q, \dot{q}) \dot{\phi} \\
& + S (q) D^{-1}(q) \tau + S (q) D^{-1}(q) \tilde{F} (q, \dot{q}) + \frac{1}{t + \theta} \left( \delta_t - \frac{\delta_t + \eta}{t + \theta} - \tilde{G}_t + a (\delta_t) \right)
\end{align*}
$$

For the Lyapunov function $\dot{V} (s_t) = \frac{1}{2} s_t^T \dot{s}_t$, we have

$$
\dot{V} (s_t) = s_t^T \dot{s}_t = s_t^T \left( \tilde{\delta}_t + \frac{\delta_t}{t + \theta} - \frac{\delta_t + \eta}{t + \theta} \right)
- \frac{1}{t + \theta} \tilde{G}_t + \frac{1}{t + \theta} a (\delta_t)
$$

$$= s_t^T \left( \tilde{S} (q) \dot{\phi} - S (q) D^{-1}(q) \tilde{V} (q, \dot{q}) \dot{\phi} \\
+ S (q) D^{-1}(q) \tau + S (q) D^{-1}(q) \tilde{F} (q, \dot{q}) + \frac{1}{t + \theta} \left( \delta_t - \frac{\delta_t + \eta}{t + \theta} - \tilde{G}_t + a (\delta_t) \right) \right)
$$

$$= s_t^T \rho_0 + s_t^T S (q) D^{-1}(q) \tau
+ s_t^T S (q) D^{-1}(q) \tilde{F} (q, \dot{q})
$$

(46)

Selecting $\tau$ as in (43) for the second term in (46), we have

$$
s_t^T S (q) D^{-1}(q) \tau
= s_t^T S (q) D^{-1}(q) \left[ S (q) S^T (q) + \rho I_{5 \times 5} \right]^{-1}
\left[ -k_t \text{SIGN} (s_t) + u_{\text{comp}, t} \right]
$$

$$= s_t^T \left[ S (q) S^T (q) + \rho I_{5 \times 5} - \rho I_{5 \times 5} \right]
$$

$$= s_t^T \left[ S (q) S^T (q) + \rho I_{5 \times 5} - \rho I_{5 \times 5} \right]
$$

$$= s_t^T \left[ S (q) S^T (q) + \rho I_{5 \times 5} \right]^{-1} + s_t^T \rho \left[ S (q) S^T (q) + \rho I_{5 \times 5} \right]^{-1}
$$

$$+ s_t^T \rho \left[ S (q) S^T (q) + \rho I_{5 \times 5} \right]^{-1} \rho I_{5 \times 5}
$$

$$+ \rho I_{5 \times 5}
$$

(47)

$$
\text{Springer}
$$
Then, (46) becomes
\[
\dot{V} (s_t) = - k_s s_t^T \text{SIGN} (s_t) \\
+ s_t^T \left( I_{5 \times 5} + \rho \left[ S (q) S^T (q) + \rho I_{5 \times 5} \right] \right) u_{\text{comp}, t} \\
+ s_t^T p_t^\text{real} + s_t^T S (q) D^{-1} (q) \tilde{F} (q, \dot{q}) \\
\leq - k_s \sum_{i=1}^{n} |s_{i,t}| \\
+ s_t^T \left( I_{5 \times 5} + \rho \left[ S (q) S^T (q) + \rho I_{5 \times 5} \right] \right) u_{\text{comp}, t} \\
+ s_t^T p_t^\text{real} + \| s_t \| \| S (q) D^{-1} (q) S (q) \| \| \tilde{F} (q, \dot{q}) \| \\
\leq F^+ \tag{47}
\]

Taking into account that
\[
\sum_{i=1}^{n} |s_{i,t}| \geq \| s_t \|
\]
and selecting the compensating control \( u_{\text{comp}} \) and \( k_s \) as in (43) from (46), we derive
\[
\dot{V} (s_t) \leq - \rho_0 \| s_t \| = - \sqrt{2} \rho_0 \sqrt{V (s_t)}
\]

implying
\[
2 \left( \sqrt{V (s_t)} - \sqrt{V (s_{t_0})} \right) \leq - \sqrt{2} \rho_0 t
\]
and
\[
0 \leq \sqrt{V (s_t)} \leq \sqrt{V (s_{t_0})} - \frac{\rho_0 t}{\sqrt{2}}
\]

which leads to the conclusion that for all \( t \geq t_{\text{reach}} := \frac{1}{\rho_0} \sqrt{2V (s_{t_0})} = \frac{\| s_{0} \|}{\rho_0} \) we have that \( V (s_t) = 0 \) and \( s_t = 0 \). To make the reaching time \( t_{\text{reach}} = 0 \) it is sufficient to guarantee that \( s_{0,0} = 0 \). But since by (36)
\[
s_{0,0} = \delta_{0,0} + \frac{\delta_{0,0} + \eta}{\theta},
\]
we need to fulfill the condition \( s_{0,0} = 0 \)
\[
s_{0,0} = \delta_{0,0} + \frac{\delta_{0,0} + \eta}{\theta} = 0,
\]
that is possible taking \( \eta \) as in (45), providing
\[
t_{\text{reach}} = \frac{\| s_{0} \|}{\rho_0} = 0. \text{ Theorem is proven.} \quad \Box
\]

\section{2D car model avoiding obstacles}
To show the good performance of the method hereby suggested let us apply the controller (43) to the systems given by equation (22) and the main objective is that the system reaches a desired point \((x^*, y^*)\) in XY-plane despite the presence of several obstacles disposed in the same plane.

\subsection{1 Selected functions and parameters}

The dynamic equation (22) of the considered system can be represented as
\[
\ddot{\delta} = h_0 (q, \dot{q}) + B (q) \tau \tag{48}
\]

Select the cost function \( F (\delta) \), which includes also the penalty terms for 3 obstacles (two ellipsoids and one triangle)
\[
F (\delta) = |\delta_1| + |\delta_2| = \sum_{i=1}^{n=5} f_i |\delta_i| + \frac{\mu_1}{g_1 (\delta) + \epsilon_0} + \frac{\mu_2}{g_2 (\delta) + \epsilon_0} + \frac{\mu_3}{\chi (X_{\text{triangle}}) + \epsilon_0}, A \in \mathbb{R}^{2 \times 2}
\]

\[
f_1 = f_2 = 1, f_3 > 0, x_1^t = 9, y_1^t = 9, \chi = 0.01, \epsilon_0 = 0.5, \theta = 0.1 \]
\[
\tilde{F} (q, \dot{q}) = - \theta \left[ \begin{array}{ccc} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 0 & 2 \\ 2 & 0 & 2 \end{array} \right] \dot{q} + \tilde{\kappa} \xi = - \theta \dot{\phi} q + \tilde{\kappa} \xi
\]
\[
g_1 (\delta) = \left[ \begin{array}{ccc} \delta_1 + x_1^t - x_{01} \\ \delta_2 + y_1^t - y_{01} \end{array} \right] A_1 \left[ \begin{array}{ccc} \delta_1 + x_1^t - x_{01} \\ \delta_2 + y_1^t - y_{01} \end{array} \right] - 1 + \chi \]
\[
x_{01} = 0, y_{01} = 0, \mu_1 = 1, A_1 = \left[ \begin{array}{ccc} 0.3 & 0.4 \\ 0.4 & 0.9 \end{array} \right]
\]
\[
\tilde{\kappa} = 0.1
\]
\[
g_2 (\delta) = \left[ \begin{array}{ccc} \delta_1 + x_2^t - x_{02} \\ \delta_2 + y_2^t - y_{02} \end{array} \right] A_2 \left[ \begin{array}{ccc} \delta_1 + x_2^t - x_{02} \\ \delta_2 + y_2^t - y_{02} \end{array} \right] - 1 + \chi \]
\[
x_{02} = 5, y_{02} = -3, \mu_2 = 1, A_2 = \left[ \begin{array}{ccc} 0.3 & 0 \\ 0 & 0.9 \end{array} \right]
\]
\[
X_{\text{triangle}} = \left\{ \begin{array}{ll}
g_{\text{gen}, 1} (\delta) = - \delta_1 - x^* - \delta_2 - y^* \\ + \delta_1 + x^* - 4 \\ + \delta_2 + y^* \end{array} \right. + \chi \leq 0 \quad \&\quad g_{\text{gen}, 2} (\delta) = - \delta_1 + x^* - 4 \\ + \chi \leq 0 \quad \&\quad g_{\text{gen}, 3} (\delta) = - \delta_1 - x^* - \delta_2 - y^* \\ + \chi \leq 0
\]
\[ x(X_{\text{triangle}}) := \sum_{i=1}^{3} \left[ g_{\text{gen},i}(\delta) \right]^{2} \]

where \( \hat{\epsilon} \) is a white noise parameter and \( \xi \) is the “white noise” of the power 1 (generated by a MATLAB package). Finally, the subgradient has the components

\[
a(\delta) = \left( \begin{array}{c} \text{sign}(\delta_1) \text{ sign}(\delta_2) 0 0 0 \end{array} \right)^{T} \tag{50}
\]

\[
-2\mu_1 \frac{\hat{\epsilon}(\delta)}{(g(\delta)+\epsilon_0)^2} \begin{bmatrix}
A \left( \begin{array}{c}
\delta_1 + x_1^* - x_01 \\
\delta_2 + y_1^* - y_01 \\
\end{array} \right)
\end{bmatrix}
\]

\[
-\mu_2 \frac{\hat{\epsilon}(g_{\text{den}}(\delta))}{(g_{\text{gen}}(\delta) + \epsilon_0)^2} \begin{bmatrix}
\frac{\partial}{\partial \delta_1} g_{\text{gen}}(\delta) \\
\frac{\partial}{\partial \delta_2} g_{\text{gen}}(\delta) \\
0
\end{bmatrix}
\]

\[
(x_{\text{ini}}, y_{\text{ini}}) = \begin{bmatrix} -8 \\ -5 \end{bmatrix} \tag{51}
\]

\[
(x^{*}, y^{*}) = \begin{bmatrix} 9 \\ 4 \end{bmatrix} \tag{52}
\]

6.2 Simulation results and discussions

Finally, the simulation results are depicted in the following figures. The 2D robot trajectory in plane is shown in Fig. 2, which shows how the AV avoids obstacles in order to reach the desired point despite the presence of white noise.

In Fig. 3, we can see the behavior of the control signal.

Figure 4 shows the behavior of the Lyapunov function (46), which is successfully converging to zero as the time increases; a similar behavior can be seen in Fig. 5, because the \( s_i \) variables converge to zero, meaning the functional convergence is happening and therefore the desired dynamics are obtained, and finally, Fig. 6 depicts the cost function converges to its minimum.
7 Conclusions

A new robust controller is proposed and examined, which is based on the ASG-version of the convex optimization and includes obstacle avoidance. The system’s dynamics contains nonholonomic constraints and acknowledges the presence of some uncertainties. This robust controller is based on an integral sliding mode control approach combined with a subgradient strategy for cost function optimization that includes penalty terms, allowing the AV Nonholonomic 2D model to avoid forbidden areas. It is shown that by using such a structure of the robust controller we may guarantee the desired dynamics from the beginning of the process. Finally, the simulation results reveal that the proposed method, which merely measures the extended state vector and its derivatives, has strong workability, allowing it to be considered as a powerful and practical tool for engineering applications. Certainly, the noise effect in measurements may be considered in further works since it requires the construction of some-type of “state observers.”

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