In this issue, Karl Crary and Robert Harper respond to the critique of higher-order abstract syntax appearing in Logic Column 14, “Nominal Logic and Abstract Syntax” (SIGACT News 36(4), December 2005).

I am always looking for contributions. If you have any suggestion concerning the content of the Logic Column, or if you would like to contribute by writing a column yourself, feel free to get in touch with me.

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Higher-Order Abstract Syntax: Setting the Record Straight

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A recent SIGACT News Logic column, guest-written by James Cheney, discussed nominal logic [1], an approach to abstract syntax with binding structure. In addition to providing a worthy tutorial on nominal logic, that column leveled five criticisms at higher-order abstract syntax, an alternative approach for dealing with binding structure in abstract syntax. We argue below that three of those criticisms are factually inaccurate, and the other two are misguided.

Since higher-order abstract syntax (HOAS) is a general concept, not a specific formalism, it is difficult to make precise statements about it. Therefore, we will consider the column’s claims in the context of the Logical Framework (LF) [2], the best-known and best-developed realization of higher-order abstract syntax.

1. The column claims that HOAS is based on complicated semantic and algorithmic foundations, and mentions explicitly higher-order logic, recursive domain equations, and higher-order unification. None of the three are necessary to understand LF:

   • All the metatheory of LF can be (and typically is) carried out in first-order logic; higher-order logic is not necessary.

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• The metatheory of LF is carried out using entirely syntactic means. There is no need to resort to any denotational semantics, including recursive domain equations.

• While higher-order unification is used in the Twelf implementation [3] of LF to support type inference and proof search, it is not at all relevant to the methodology of encoding abstract syntax in LF. Moreover, even in regards to the Twelf implementation, experience suggests that understanding the higher-order unification algorithm is not essential; students who do are not familiar with it nevertheless have little difficulty in using Twelf.

2. The column claims that structural properties of the meta-language must be inherited by the object language. This is simply untrue. For example, linear logic can be adequately represented in ordinary LF using a linearity judgement; one need not resort to a linear variant of LF.

3. The column states that variable names in LF “have no existence or meaning outside their scope.” This is true, in the sense that variable names alpha-vary, as is standard practice in logic and programming languages. The fact that alpha-equivalence is built-in is a strength of HOAS (and of de Bruijn representation) not enjoyed by first-order abstract syntax or by nominal logic. One might wish to argue that the usual practice of considering terms modulo alpha-equivalence is counter-productive, but if so, the objection is with the field in general, not with HOAS.

Moreover, taking alpha-equivalence as built-in does not preclude the isolation of variables in LF (which can be done easily using a hypothetical judgement). Consequently, it is not difficult to encode logics that deal with variables specially or that compare free variables for equality.

The real contrast between HOAS and nominal logic lies in their philosophy toward binding structure. Both HOAS and nominal logic provide a sophisticated treatment of binding structure not enjoyed by first-order accounts, but ones of very different character. The philosophy of LF is that binding and alpha-conversion are fundamental concepts, and thus builds them in at the lowest level. Consequently, name management in LF is ordinarily a non-issue. In contrast, nominal logic promotes name management as an object of study. It makes binding into a sophisticated artifact, rather than a primitive concept as in LF.

4. A fundamental part of the LF methodology is the notion of an adequate encoding. Syntactic terms and derivations of judgements are represented in LF as certain canonical forms, and the encoding is said to be adequate if there exists an isomorphism (in a sense that can be made precise) between the terms/derivations on the object language and the canonical forms that represent them. This provides a clear criterion for whether an encoding is correct or not.

The column notes that inadequate encodings have appeared in papers, and takes that fact as evidence that it is too hard to develop adequate encodings. To the contrary, we see the existence of incorrect encodings as merely the inevitable consequence of the existence of a correctness criterion.

To the best of our knowledge, nominal logic does not currently enjoy a precise criterion for correctness. Consequently, it is hard formally to justify the statement that anything at all has been encoded in nominal logic. When nominal logic is better understood, we expect that a correctness criterion will be developed for it as well. Once that happens, we expect that that criterion too will be used to identify incorrect encodings.
5. The column claims that HOAS cannot address ML-style let-polymorphism due to difficulties with open terms. This is not so. There are at least two perfectly satisfactory formulations of let-polymorphism in LF. The simplest is to re-typecheck the let-bound variable at each reference:

\[
\text{of-let} \quad : \quad \text{of (let } E_1 ([x] E_2 x) ) T \\
\quad \quad \quad <- \quad \text{of } E_1 T' \\
\quad \quad \quad <- \quad \text{of (} E_2 E_1 \text{) } T.
\]

If one prefers a formulation with an explicit account of generalization, that is also possible:

\[
\begin{align*}
\text{polytp} & : \text{type.} \\
\text{mono} & : \text{tp} \to \text{polytp}. \\
\text{forall} & : (\text{tp} \to \text{polytp}) \to \text{polytp}. \\
\text{of} & : \text{term} \to \text{tp} \to \text{type.} \\
\text{polyof} & : \text{term} \to \text{polytp} \to \text{type.} \\
\text{assm} & : \text{term} \to \text{polytp} \to \text{type.} \\
\text{polyof-mono} & : \text{polyof } E \text{ (mono } T) \\
\quad \quad \quad <- \quad \text{of } E \ T. \\
\text{polyof-forall} & : \text{polyof } E \text{ (forall ([t] S t))} \\
\quad \quad \quad <- \quad (t: \text{tp} \text{ polyof } E \text{ (S t))}. \\
\text{of-let} & : \text{of (} \text{let } E_1 ([x] E_2 x) \text{) } T \\
\quad \quad \quad <- \quad \text{polyof } E_1 S \\
\quad \quad \quad <- \quad (x: \text{term} \text{ assm } x \ S \to \text{of (} E_2 x \text{) } T). \\
\end{align*}
\]

In the alternative formulation alluded to by the column, the let-bound term is given a type (possibly including free variables) which is then forcibly generalized by a polymorphic closure operator. The inherent clunkiness of such a formulation does makes it awkward to encode in LF. However, the difficulty does not result from its use of open terms, which are easily expressible:

\[
\begin{align*}
\text{openterm} & : \text{type.} \\
\text{trm} & : \text{term} \to \text{openterm.} \\
\text{close} & : (\text{term} \to \text{openterm}) \to \text{openterm.}
\end{align*}
\]

LF does lends itself particularly to languages enjoying hygienic scoping disciplines. Although nothing prevents one from employing first-order representations in LF, to do so can sacrifice much of the advantage afforded by LF. Consequently, nominal logic looks very attractive for languages with more novel notions of scope. However, for languages with largely conventional scoping, LF provides an extraordinary tool that is being used by a growing number of researchers on a daily basis to formalize their metatheoretic results.
References

[1] M. J. Gabbay and A. M. Pitts. A new approach to abstract syntax with variable binding. *Formal Aspects of Computing*, 13:341–363, 2002.

[2] Robert Harper, Furio Honsell, and Gordon Plotkin. A framework for defining logics. *Journal of the ACM*, 40(1):143–184, January 1993.

[3] Frank Pfenning and Carsten Schürmann. *Twelf User’s Guide, Version 1.4*, 2002. Available electronically at http://www.cs.cmu.edu/~twelf.