Generalized Measures of Fault Tolerance in Exchanged Hypercubes*

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Abstract

The exchanged hypercube $EH(s, t)$, proposed by Loh et al. [The exchanged hypercube, IEEE Transactions on Parallel and Distributed Systems 16 (9) (2005) 866-874], is obtained by removing edges from a hypercube $Q_{s+t+1}$. This paper considers a kind of generalized measures $\kappa(h)$ and $\lambda(h)$ of fault tolerance in $EH(s, t)$ with $1 \leq s \leq t$ and determines $\kappa(h)(EH(s, t)) = \lambda(h)(EH(s, t)) = 2^h(s + 1 - h)$ for any $h$ with $0 \leq h \leq s$. The results show that at least $2^h(s + 1 - h)$ vertices (resp. $2^h(s + 1 - h)$ edges) of $EH(s, t)$ have to be removed to get a disconnected graph that contains no vertices of degree less than $h$, and generalizes some known results.

Keywords: Combinatorics, networks, fault-tolerant analysis, exchanged hypercube, connectivity, super connectivity

1 Introduction

It is well known that interconnection networks play an important role in parallel computing/communication systems. An interconnection network can be modeled by a graph $G = (V, E)$, where $V$ is the set of processors and $E$ is the set of communication links in the network. For graph terminology and notation not defined here we follow [15].

A subset $S \subset V(G)$ (resp. $F \subset E(G)$) of a connected graph $G$ is called a vertex-cut (resp. edge-cut) if $G - S$ (resp. $G - F$) is disconnected. The connectivity $\kappa(G)$ (resp. edge-connectivity $\lambda(G)$) of $G$ is defined as the minimum cardinality over all vertex-cuts (resp. edge-cuts) of $G$. The connectivity $\kappa(G)$ and edge-connectivity $\lambda(G)$ of a graph $G$ are two important measurements for fault tolerance of the network since the larger $\kappa(G)$ or $\lambda(G)$ is, the more reliable the network is.

Because the connectivity has some shortcomings, Esfahanian [11] proposed the concept of restricted connectivity, Latifi et al. [3] generalized it to restricted $h$-connectivity
which can measure fault tolerance of an interconnection network more accurately than the classical connectivity. The concepts stated here are slightly different from theirs.

A subset $S \subset V(G)$ (resp. $F \subset E(G)$) of a connected graph $G$, if any, is called an $h$-vertex-cut (resp. edge-cut), if $G - S$ (resp. $G - F$) is disconnected and has the minimum degree at least $h$. The $h$-connectivity (resp. edge-connectivity) of $G$, denoted by $\kappa(h)(G)$ (resp. $\lambda(h)(G)$), is defined as the minimum cardinality over all $h$-vertex-cuts (resp. $h$-edge-cuts) of $G$. It is clear that, for $h \geq 1$, if $\kappa(h)(G)$ and $\lambda(h)(G)$ exists, then $\kappa(h-1)(G) \leq \kappa(h)(G)$ and $\lambda(h-1)(G) \leq \lambda(h)(G)$. For any graph $G$ and any integer $h$, determining $\kappa(h)(G)$ and $\lambda(h)(G)$ is quite difficult. In fact, the existence of $\kappa(h)(G)$ and $\lambda(h)(G)$ is an open problem so far when $h \geq 1$. Only a little knowledge of results have been known on $\kappa(h)$ and $\lambda(h)$ for particular classes of graphs and small $h$’s, such as [2, 4, 5, 8, 10, 14, 16, 17, 19, 20].

It is widely known that the hypercube $Q_n$ has been one of the most popular interconnection networks for parallel computer/communication systems. Xu [14] determined $\lambda(h)(Q_n) = 2^h(n-h)$ for $h \leq n-1$, and Oh et al. [11] and Wu et al. [13] independently determined $\kappa(h)(Q_n) = 2^h(n-h)$ for $h \leq n-2$.

This paper is concerned about the exchanged hypercubes $E H(s, t)$, proposed by Loh et al. [2]. As a variant of the hypercube, $E H(s, t)$ is a graph obtained by removing edges from a hypercube $Q_{s+t+1}$. It not only keeps numerous desirable properties of the hypercube, but also reduced the interconnection complexity. Very recently, Ma et al. [10] have determined $\kappa(1)(E H(s, t)) = \lambda(1)(E H(s, t)) = 2s$. We, in this paper, will generalize this result by proving that $\kappa(h)(E H(s, t)) = \lambda(h)(E H(s, t)) = 2^h(s + 1 - h)$ for any $h$ with $0 \leq h \leq s$.

The proof of this result is in Section 3. In Section 2, we recall the structure of $E H(s, t)$ and some lemmas used in our proofs.

## 2 Definitions and lemmas

For a given position integer $n$, let $I_n = \{1, 2, \ldots, n\}$. The sequence $x_n x_{n-1} \cdots x_1$ is said a binary string of length $n$ if $x_r \in \{0, 1\}$ for each $r \in I_n$. Let $x = x_n x_{n-1} \cdots x_1$ and $y = y_n y_{n-1} \cdots y_1$ be two distinct binary string of length $n$. Hamming distance between $x$ and $y$, denoted by $H(x, y)$, is the number of $r$’s for which $|x_r - y_r| = 1$ for $r \in I_n$.

For a binary string $u = u_n u_{n-1} \cdots u_1 u_0$ of length $n + 1$, we call $u_r$ the $r$-th bit of $u$ for $r \in I_n$, and $u_0$ the last bit of $u$, denote sub-sequence $u_j u_{j-1} \cdots u_{i+1} u_i$ of $u$ by $u[j : i]$, i.e., $u[j, i] = u_j u_{j-1} \cdots u_{i+1} u_i$. Let

$$V(s, t) = \{u_{s+t} \cdots u_{t+1} u_t \cdots u_1 u_0 | \ u_0, u_i \in \{0, 1\}, \ i \in I_{s+t}\}.$$ 

**Definition 2.1** The exchanged hypercube is an undirected graph $E H(s, t) = (V, E)$, where $s \geq 1$ and $t \geq 1$ are integers. The set of vertices $V$ is $V(s, t)$, and the set of edges $E$ is composed of three disjoint types $E_1, E_2$ and $E_3$.

$$E_1 = \{uv \in V \times V | u[s + t : 1] = v[s + t : 1], \ u_0 \neq v_0\},$$

$$E_2 = \{uv \in V \times V | u[s + t : t + 1] = v[s + t : t + 1],$$

$$H(u[t : 1], v[t : 1]) = 1, u_0 = v_0 = 1\},$$

$$E_3 = \{uv \in V \times V | u[t : 1] = v[t : 1],$$

$$H(u[s + t : t + 1], v[s + t : t + 1]) = 1, u_0 = v_0 = 0\}.$$
Now we give an alternative definition of \(EH(s, t)\).

**Definition 2.2** An exchanged hypercube \(EH(s, t)\) consists of the vertex-set \(V(s, t)\) and the edge-set \(E\), two vertex \(u = u_{s+t} \cdots u_{t+1} u_t \cdots u_1 u_0\) and \(v = v_{s+t} \cdots v_{t+1} v_t \cdots v_1 v_0\) linked by an edge, called \(r\)-dimensional edge, if and only if the following conditions are satisfied:

a). \(u\) and \(v\) differ exactly in one bit on the \(r\)-th bit or on the last bit.

b). if \(r \in I_t\), then \(u_0 = v_0 = 1\).

c). if \(r \in I_{s+t} - I_t\), then \(u_0 = v_0 = 0\).

The exchanged hypercubes \(EH(1, 1)\) and \(EH(1, 2)\) are shown in Figure 1.

![Figure 1: Two exchanged hypercubes EH(1, 1) and EH(1, 2)](image)

From Definition 2.2 it is easy to see that \(EH(s, t)\) can be obtained from a hypercube \(Q_{s+t+1}\) with vertex-set \(V(s, t)\) by removing all \(r\)-dimensional edges that link two vertices with the last bit 0 if \(r \in I_t\) and two vertices with the last bit 1 if \(r \in I_{s+t} - I_t\). Thus, \(EH(s, t)\) is a bipartite graph with minimum degree \(\min\{s, t\} + 1\) and maximum degree \(\max\{s, t\} + 1\). The following three lemmas obtained by Loh et al. [7] and Ma [8] are very useful for our proofs.

**Lemma 2.3** (Loh et al. [7]) \(EH(s, t)\) is isomorphic to \(EH(t, s)\).

By Lemma 2.3, without loss of generality, we can assume \(s \leq t\) in the following discussion, and so \(EH(s, t)\) has the minimum degree \(s + 1\). For fixed \(r \in I_{s+t}\) and \(i \in \{0, 1\}\), let \(H_r^i\) denote a subgraph of \(EH(s, t)\) induced by all vertices whose the \(r\)-th bit are \(i\).

**Lemma 2.4** (Loh et al. [7]) For a fixed \(r \in I_{s+t}\), \(EH(s, t)\) can be decomposed into 2 isomorphic subgraphs \(H_r^0\) and \(H_r^1\), which are isomorphic to \(EH(s, t - 1)\) if \(r \in I_t\) and \(t \geq 2\), and isomorphic to \(EH(s - 1, t)\) if \(r \in I_{s+t} - I_t\) and \(s \geq 2\). Moreover, there are \(2^{s+t-1}\) independent edges between \(H_r^0\) and \(H_r^1\).

**Lemma 2.5** (Ma [8]) \(\kappa(EH(s, t)) = \lambda(EH(s, t)) = s + 1\) for any \(s\) and \(t\) with \(1 \leq s \leq t\).
3 Main results

In this section, we present our main results, that is, we determine the $h$-connectivity and $h$-edge-connectivity of the exchanged hypercube $EH(s, t)$.

Lemma 3.1 $\kappa(h)(EH(s, t)) \leq 2^h(s + 1 - h)$ and $\lambda(h)(EH(s, t)) \leq 2^h(s + 1 - h)$ for $h \leq s$.

Proof. Let $X$ be a subset of vertices in $EH(s, t)$ whose the rightmost $s + t + 1 - h$ bits are zeros and the leftmost $h$ bits do not care, denoted by

$$X = \{*^h0^{s+t+1-h}| \ast \in \{0, 1\}\}.$$  

Then the subgraph of $EH(s, t)$ induced by $X$ is a hypercube $Q_h$. Let $S$ be the neighborhood of $X$ in $EH(s, t) - X$ and $F$ the edge-sets between $X$ and $S$. By Definition 2.2, $S$ has the form

$$S = \{*^h0^p10^{s-h-p-1}0^{t+1}| 0 \leq p \leq s - h - 1, \ast \in \{0, 1\}\} \cup \{*^h0^{s+t-h}1\}.$$  

On the one hand, since every vertex of $X$ has degree $s + 1$ in $EH(s, t)$ and $h$ neighbors in $X$, it has exactly $s - h + 1$ neighbors in $S$. On the other hand, every vertex of $S$ has exactly one neighbor in $X$. It follows that

$$|S| = |F| = 2^h(s + 1 - h).$$

We show that $S$ is an $h$-vertex-cut of $EH(s, t)$. Clearly, $S$ is a vertex-cut of $EH(s, t)$ since $|X \cup S| = 2^h(s + 2 - h) < 2^{s+t+1}$. Let $Y = EH(s, t) - (X \cup S)$ and $v$ be any vertex in $Y$. We only need to show that the vertex $v$ has degree at least $h$ in $Y$. In fact, it is easy to see from the formal definition of $S$ that if $v$ is adjacent to some vertex in $S$ then it has only the form

$$v = *^h0^p10^{s-h-p-1}0^{t+1}_s$$  
or

$$v = *^h0^{s-h}0^p10^{t-r-1}_t$$  
or

$$v = *^h0^p10^q10^{s-h-p-q-2}0^{t+1}_s.$$  

If $v$ has the former two forms, then $v$ has one neighbor in $S$, thus $v$ has at least $(s+1-1 = s \geq h)$ neighbors in $Y$. If $v$ has the last form, then $s - h \geq 2$ and $v$ has two neighbors in $S$. Thus, $v$ has at least $(s+1-2 = s - 1 > h)$ neighbors in $Y$.

By the arbitrariness of $v \in Y$, $S$ is an $h$-vertex-cut of $EH(s, t)$, and so

$$\kappa(h)(EH(s, t)) \leq |S| = 2^h(s + 1 - h)$$

as required.

We now show that $F$ is an $h$-edge-cut of $EH(s, t)$. Since every vertex $v$ in $EH(s, t) - X$ has at most one neighbor in $X$, then $v$ has at least $(s+1-1 = s \geq h)$ neighbors in $EH(s, t) - X$. By the arbitrariness of $v \in EH(s, t) - X$, $F$ is an $h$-edge-cut of $EH(s, t)$, and so

$$\lambda(h)(EH(s, t)) \leq |F| = 2^h(s + 1 - h)$$

The lemma follows.
Corollary 3.2 \( \kappa^{(1)}(EH(1, t)) = \lambda^{(1)}(EH(1, t)) = 2 \) for \( t \geq 1 \).

Proof. On the one hand, \( \kappa^{(h)}(EH(1, t)) \leq 2 \) and \( \lambda^{(h)}(EH(1, t)) \leq 2 \) by Lemma 3.1 when \( s = 1 \). On the other hand, by Lemma 2.3, \( \kappa(EH(1, t)) = \lambda(EH(1, t)) = 2 \), thus \( \kappa^{(h)}(EH(1, t)) \geq \kappa(EH(1, t)) = 2 \) and \( \lambda^{(h)}(EH(1, t)) \geq \lambda(EH(1, t)) = 2 \). The results hold.

Theorem 3.3 For \( 1 \leq s \leq t \) and any \( h \) with \( 0 \leq h \leq s \),
\[
\kappa^{(h)}(EH(s, t)) = \lambda^{(h)}(EH(s, t)) = 2^h(s + 1 - h).
\]

Proof. By Lemma 3.1, we only need to prove that,
\[
\kappa^{(h)}(EH(s, t)) = \lambda^{(h)}(EH(s, t)) \geq 2^h(s + 1 - h).
\]

We proceed by induction on \( h \geq 0 \). The theorem holds for \( h = 0 \) by Lemma 2.3. Assume the induction hypothesis for \( h - 1 \) with \( h \geq 1 \), that is,
\[
\kappa^{(h-1)}(EH(s, t)) = \lambda^{(h-1)}(EH(s, t)) \geq 2^{h-1}(s + 2 - h). \quad (3.1)
\]

Note \( h = 1 \) if \( s = 1 \). By Corollary 3.2, \( \kappa^{(1)}(EH(1, t)) = \lambda^{(1)}(EH(1, t)) = 2 \) for any \( t \geq 1 \), the theorem is true for \( s = 1 \). Thus, we assume \( s \geq 2 \) below.

Let \( S \) be a minimum \( h \)-vertex-cut (or \( h \)-edge-cut) of \( EH(s, t) \) and \( X \) be the vertex-set of a minimum connected component of \( EH(s, t) - S \). Then
\[
|S| = \begin{cases} 
\kappa^{(h)}(EH(s, t)) & \text{if } S \text{ is a vertex cut;} \\
\lambda^{(h)}(EH(s, t)) & \text{if } S \text{ is an edge cut.}
\end{cases}
\]

Thus, we only need to prove that
\[
|S| \geq 2^h(s + 1 - h). \quad (3.2)
\]

To the end, let \( Y \) be the set of vertices in \( EH(s, t) - S \) not in \( X \), and for a fixed \( r \in I_{s+t} \) and each \( i = 0, 1 \), let
\[
X_i = X \cap H_i^r, \\
Y_i = Y \cap H_i^r \text{ and} \\
S_i = S \cap H_i^r,
\]

Let \( J = \{i \in \{0, 1\} \mid X_i \neq \emptyset\} \) and \( J' = \{i \in J \mid Y_i \neq \emptyset\} \). Clearly, \( 0 \leq |J'| \leq |J| \leq 2 \) and \(|J'| = 0 \) only when \(|J| = 1 \). We choose \( r \in I_{s+t} \) such that \(|J|\) is as large as possible. For each \( i \in \{0, 1\} \), we write \( H_i \) for \( H_i^r \) for short. We first prove the following inequality.
\[
|S_i| \geq 2^{h-1}(s + 1 - h) \text{ if } X_i \neq \emptyset \text{ and } Y_i \neq \emptyset \text{ for } i \in \{0, 1\}. \quad (3.3)
\]

In fact, for some \( i \in \{0, 1\} \), if \( X_i \neq \emptyset \) and \( Y_i \neq \emptyset \), then \( S_i \) is a vertex-cut (or an edge-cut) of \( H_i \). Let \( u \) be any vertex in \( X_i \cup Y_i \). Since \( S \) is an \( h \)-vertex-cut (or \( h \)-edge-cut) of \( EH(s, t) \), \( u \) has degree at least \( h \) in \( EH(s, t) - S \). By Lemma 2.4, \( u \) has at most one neighbor in \( H_j \), where \( j \neq i \). Thus, \( u \) has degree at least \( h - 1 \) in \( H_i \), which implies that \( S_i \) is an \( (h - 1) \)-vertex-cut (or edge-cut) of \( H_i \), that is,
\[
|S_i| \geq \kappa^{(h-1)}(H_i) \quad \text{or} \quad |S_i| \geq \lambda^{(h-1)}(H_i)). \quad (3.4)
\]
If $r \in I_{s+t} - I_t$, then $H_i \cong EH(s-1, t)$ by Lemma 2.4. By the induction hypothesis (3.1), $\kappa(h^{-1})(H_i) = \lambda(h^{-1})(H_i) \geq 2^{h-1}(s+1-h)$, from which and (3.4), we have that $|S_i| \geq 2^{h-1}(s+1-h)$.

If $r \in I_t$, then $H_i \cong EH(s, t-1)$ by Lemma 2.3. If $t \geq s+1$, by the induction hypothesis (3.1),

$$\kappa(h^{-1})(H_i) = \lambda(h^{-1})(H_i) \geq 2^{h-1}(s+1-h),$$

from which and (3.4), we have that $|S_i| \geq 2^{h-1}(s+1-h)$.

If $t = s$, then $EH(s, t-1) \cong EH(s-1, t)$ by Lemma 2.3. By the induction hypothesis (3.1),

$$\kappa(h^{-1})(H_i) = \lambda(h^{-1})(H_i) \geq 2^{h-1}(s+1-h),$$

from which and (3.4), we have that $|S_i| \geq 2^{h-1}(s+1-h)$. The inequality (3.3) follows.

We now prove the inequality in (3.2).

If $|J| = 1$ then, by the choice of $J$, no matter what $r \in I_{s+t}$ is chosen, the $r$-th bits of all vertices in $X$ are the same. In other words, the $r$-th bits of all vertices in $X$ are the same for any $r \in I_{s+t}$, and possible different in the last bit. Thus $|X| \leq 2$ and $h \leq 1$. By the hypothesis of $h \geq 1$, we have $h = 1$ and $|X| = 2$. The subgraph of $EH(s, t)$ induced by $X$ is an edge in $E_1$, thus

$$|S| = s + t \geq 2s = 2^h(s+1-h),$$

as required. Assume $|J| = 2$ below, that is, $X_i \neq \emptyset$ for each $i = 0, 1$. In this case, $|J'| \geq 1$.

If $|J'| = 2$ then, for each $i = 0, 1$, since $X_i \neq \emptyset$ and $Y_i \neq \emptyset$, we have that $|S_i| \geq 2^{h-1}(s+1-h)$ by (3.3). Note that $|S| = |S_0| + |S_1|$ if $S$ is an $h$-vertex-cut and $|S| \geq |S_0| + |S_1|$ if $S$ is an $h$-edge-cut. It follows that

$$|S| \geq |S_0| + |S_1| \geq 2 \times 2^{h-1}(s+1-h) = 2^h(s+1-h),$$

as required.

If $|J'| = 1$, then one of $Y_0$ and $Y_1$ must be empty. Without loss of generality, assume $Y_1 = \emptyset$ and $Y_0 \neq \emptyset$.

Clearly, $S$ is not an $h$-edge-cut, otherwise, $|Y| < |H_0| < |X|$, a contradiction with the minimality of $X$. Thus, $S$ is an $h$-vertex-cut. By (3.3), $|S_0| \geq 2^{h-1}(s+1-h)$. Since $Y_1 = \emptyset$, we have

$$|X_1| = |H_1| - |S_1| \quad \text{and} \quad |Y| = |H_0| - |X_0| - |S_0|. \quad (3.5)$$

If $|S_1| < |S_0|$, then by (3.5), we obtain that $|Y| < |X_1| < |X|$, which contradicts to the minimality of $X$. Thus, $|S_1| \geq |S_0|$, from which and (3.3) we have that

$$|S| = |S_0| + |S_1| \geq 2|S_0| \geq 2 \times 2^{h-1}(s+1-h) = 2^h(s+1-h),$$

as required. Thus, the inequality in (3.2) holds, and so the theorem follows.
**Corollary 3.4** (Ma and Zhu [10]) If $1 \leq s \leq t$, then $\kappa^{(1)}(EH(s, t)) = \lambda^{(1)}(EH(s, t)) = 2s$.

A dual-cube $DC(n)$, proposed by Li and Peng [6] constructed from hypercubes, preserves the main desired properties of the hypercube. Very recently, Yang and Zhou [18] have determined that $\kappa^{(h)}(DC(n)) = 2^n(n + 1 - h)$ for each $h = 0, 1, 2$. Since $EH(n, n)$ is isomorphic to $DC(n)$, the following result is obtained immediately.

**Corollary 3.5** For dual-cube $DC(n)$, $\kappa^{(h)}(DC(n)) = \lambda^{(h)}(DC(n)) = 2^n(n + 1 - h)$ for any $h$ with $0 \leq h \leq n$.

4 Conclusions

In this paper, we consider the generalized measures of fault tolerance for a network, called the $h$-connectivity $\kappa^h$ and the $h$-edge-connectivity $\lambda^h$. For the exchanged hypercube $EH(s, t)$, which has about half edges of the hypercube $Q_{s+t+1}$, we prove that $\kappa^{(h)} = \lambda^{(h)} = 2^h(s+1-h)$ for any $h$ with $0 \leq h \leq s$ and $s \leq t$. The results show that at least $2^h(s+1-h)$ vertices (resp. $2^h(s+1-h)$ edges) of $EH(s, t)$ have to be removed to get a disconnected graph that contains no vertices of degree less than $h$. Thus, when the exchanged hypercube is used to model the topological structure of a large-scale parallel processing system, these results can provide more accurate measurements for fault tolerance of the system.

Otherwise, Ma and Liu [9] investigated bipancyclicity of $EH(s, t)$. However, there are many interesting combinatorial and topological problems, e.g., wide-diameter, fault-diameter, pancyclicity, spanning-connectivity, which are still open for the exchanged hypercube network.

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