Influence of geometric parameters of prismatic benchmarks with U-shaped grooves on the kind of their stress-strain state

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Abstract. Laboratory benchmarks for mechanical testing under conditions of static and cyclic loading are considered. The analysis of the stress-strain state (SSS) of prismatic benchmarks having stress concentrators in the form of U-shaped grooves is presented. The use of benchmarks of this type makes it possible to model the main features of SSS of various designs: the stress level and the type of SSS, characterized by the ratio of the principal stresses in the zone of the maximum stress level arising in the design. As a characteristic of the SSS type, the article uses the coefficient $P$, the ratio of the first invariant of the stress tensor to the second, introduced by the researcher Smirnov-Alyaev, and also the coefficient $K_\sigma$ – the coefficient of concentration of equivalent stresses that arise in the sample under load. The main regularities of the change in the type of SSS of the material in the working zone of the prismatic benchmark with the change in the values of its design parameters are revealed. These regularities make it possible to carry out a justified choice of the geometric parameters of the benchmarks of this type for the experimental study of the resistance to fatigue failure of various materials and structural components of various shapes made of them.

1. Introduction
Mechanic structure under the operating conditions typically undergo repetitive complex (cyclic) loads. Their SSS is characterized by a concentration of stresses near structural heterogeneities (holes, protrusions, grooves, etc.). The SSS that occurs near such a concentrator, as a rule, is two-dimensional or three-dimensional. It determines the service life of the structure under cyclic loading [1].

The type of SSS arising at some point of the structure under load (hereinafter referred to as the observation point) is characterized by the value of its stiffness coefficient $P$, equal to the ratio of the first invariant of the stress tensor to the second [2].

$$P = \frac{\sigma_1 + \sigma_2 + \sigma_3}{\sigma_i},$$

where $\sigma_1$, $\sigma_2$, $\sigma_3$ are the principal stresses arising at the observation point, $\sigma_i$ is equivalent stress defined by the formula:

$$\sigma_i = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}.$$ 

It is known [3 – 4] that an increase in the coefficient of rigidity of the SSS type in a certain zone of construction can lead to the fact that the center of destruction moves to this zone, despite the fact that the stress level in it is not the maximum for the structure as a whole. It is essential that such displacement can occur both in the case of cyclic loading of the structure [3] and in the case of its quasistatic destruction [4]. This circumstance must be taken into account at the design stage of both
the structure as a whole and its local amplifications that reduce the level of stresses in places of structural heterogeneity.

One of the problems arising from the experimental study of fatigue failure resistance in laboratory conditions is the difficulty in creating a biaxial SSS in the corresponding laboratory benchmarks, characterized by a different ratio of positive components of principal stresses \(1 < P < 2\), as the most dangerous from the point of view of nucleation and development of defects. Such SSS characterizes the deformation of a number of critical elements of highly loaded structures. In particular, choke points of capacitive equipment and parts of pipelines, carrying elements of vehicles, compressor and turbine disks, aircraft skinning and in a number of other cases. In this paper, the features of deformation of laboratory benchmarks of prismatic type [5] for fatigue mechanical tests before failure are considered, which make it possible to simplify the solution of this problem.

2. Description of the structure and conditions of deformation

The considered prismatic benchmark [5] is made in the form of a prismatic body 1 equipped with longitudinal projections 2 having an L-shaped cross-section (L-shaped projections) and bevels 3 at the ends of these protrusions (Figure 1). For simplicity of description, we will continue to assume that the directions "up", "down", "vertically" and "horizontally" correspond to Figure 1. On the side of the inner part of the prismatic benchmark, the longitudinal projection 2 is conjugated with the radial transition 9. The longitudinal L-shaped projection 2 has in cross section a rectilinear vertical part (bar) 10 adjacent to the body of the prism 1 and is conjugated to the horizontal part 11 by a 6. During the mechanical testing, the prismatic specimen is supported by its end parts on the surface of the end supports 5 over the area \(S_1\) (see Figure 1).

![Figure 1. Prismatic benchmark (a) and its design features (b) (type of quarter of the benchmark), 1 - prismatic body, 2 - L-shaped projections, 3 - support bevel, 4 - inclined support surface of prismatic support \(S_1\), 5 - support surface of end support \(S_2\), 6 - gland transition, 7 - test force, 8 - the loading surface, 9 - the inner radius transition, 10 - the vertical bar of the L-shaped projection, 11 - the horizontal bar of the L-shaped projection, \(\gamma\) - the slope angle of the reference surface \(S_1\), \(I\) - the region of the working area of the benchmark.](image-url)

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**Figure 1.** Prismatic benchmark (a) and its design features (b) (type of quarter of the benchmark), 1 - prismatic body, 2 - L-shaped projections, 3 - support bevel, 4 - inclined support surface of prismatic support \(S_1\), 5 - support surface of end support \(S_2\), 6 - gland transition, 7 - test force, 8 - the loading surface, 9 - the inner radius transition, 10 - the vertical bar of the L-shaped projection, 11 - the horizontal bar of the L-shaped projection, \(\gamma\) - the slope angle of the reference surface \(S_1\), \(I\) - the region of the working area of the benchmark.
In the central part of the prismatic benchmark, a transverse (in Figure 1 - vertical) test force is applied to its loading surface $8$. Under this force, the prismatic support beams $3$ slide along the inclined surface of the lateral support $4$. At the same time, protrusions of the prismatic pattern, a contact reaction perpendicular to the surface of the support bevel acts. The working zone of the sample is the lower surface of the L-shaped projection $2$ (horizontal bar $11$) in its middle part adjacent to transverse plane of symmetry, where a biaxial bending occurs.

In this paper, the values of the coefficients $K_\sigma$ and $P$ were considered as characteristics of the structure's SSS in evaluating their deformation features and subjected to numerical modelling on prismatic benchmarks. The value of the coefficient $P$ was determined by the formula (1), the value of $K_\sigma$ is the coefficient of equivalent stress concentration by the formula

$$K_\sigma = \frac{\sigma_{i_{\text{max}}}}{\sigma_{i_{\text{nom}}}}$$

where $\sigma_{i_{\text{max}}}$ is the calculated equivalent stress in the working area of the benchmark (Figure 2a), $\sigma_{i_{\text{nom}}}$ is calculated nominal equivalent stresses - intensity of stresses in the working zone of a benchmark of the corresponding (length, height and width) analogous to a loaded prismatic benchmark without concentric grooves on its lateral projections (Figure 2b).

It is assumed that when testing a laboratory benchmark, the SSS of the design being evaluated has been previously investigated using numerical simulation, and the corresponding values of the coefficients $K_\sigma$ and $P$ are known. The choice of these coefficients as criteria for the similarity of the laboratory benchmark and the design being evaluated is determined by the fact that the values of the coefficients $P$ and $K_\sigma$ are physically significant invariants of the stress fields arising in the simulated structure and determining its strength under conditions of static or cyclic loading.

3. Selection of the main variable geometric parameters of prismatic benchmark

The task of this stage of research is to study the features of elastic deformation of prismatic benchmark on the basis of variant computational experiments. This allows us, in evaluating the strength of the structure in the stress concentration region, to select the values of the geometric parameters of the prismatic benchmarks, which ensures the same level and type of SSS in the region of stress concentration of this part and in the corresponding benchmark.

The prismatic benchmark is characterized by a number of geometric parameters. In Figure 3 is the height of the main prism of benchmark $H_1$, the height of its vertical bar of L-shaped projections $H_2$, the radiuses of fillet transitions $r$, the angle of inclination of the support surfaces $\gamma$. The geometric parameters were chosen from the condition for modelling the value of $P$, characterized by the inequality $1<P<2$.

The following ranges of variation of the dimensionless geometric parameters of prismatic benchmarks were considered (Figure 3)
where $h_1$ – the relative thickness of the prismatic body, $h_2$ – the relative height of the vertical part of the L-shaped projection, $\rho$ – the relative radius of the fillet-concentrator, $\gamma$ – the relative angle of inclination of the reference surface, $S$ – the total thickness of the benchmark, $H_1$ – the height of the prismatic part of the benchmark, $H_2$ – the height of the vertical part of the L-shaped projection, $r$ – the radius of the fillet.

As unchanged in the process of variant studies, the following values were chosen:

- benchmark length $L = 220$ mm;
- width of the benchmark $S = 44$ mm;
- radius transition $R = 5$ mm;
- the width of the lateral projections $S_1 = 62$ mm;
- internal slot $S_2 = 36$ mm;
- the thickness of the vertical bar $S_3 = 4$ mm.

Variable parameters (4) contribute to the creation of a biaxial tension. The parameters $h_1$ and $h_2$ determine the degree of influence of the longitudinal bending stresses $\sigma_1$, $\rho$ and $\gamma$ – transverse bending stresses $\sigma_2$.

### 4. Development of discrete FE models, conditions for fixing and loading prismatic benchmarks

To minimize the error associated with the construction of computational models for the deformation of benchmarks, a detailed development of all components is performed, which makes it possible to realize the necessary conditions for a biaxial stretching in a prismatic benchmark and the loading scheme in accordance with Figure 1. Development of appropriate models of deformation is carried out with the help of numerical FE modelling and solving the contact problem of mechanics of a deformed body. To create tensile longitudinal stresses $\sigma_1$ caused by longitudinal bending of the prismatic benchmark, monolithic end supports have been created (Figure 4). To create transverse tensile stresses $\sigma_2$ caused by sliding of the support bevels of the L-shaped projections of the sample along the inclined support surface, a prismatic support is constructed. This support is a massive prism in the central part of which a longitudinal groove with inclined support surfaces is made, the angles of which are the same as those of the side projections of the test benchmark.

To carry out the testing of the benchmarks under consideration, a test force is applied to its loading surface (Figure 4), created by the pusher of the testing machine.
When creating FE models of a prismatic benchmark and supporting elements, a controlled algorithm for creating a FE grid in the most loaded zones and coupling of bodies using an isoparametric hexahedron was used. The principle of smoothness of the transition from large FE to smaller ones, in particular, near the contact surfaces of the parts, was also used.

Due to the symmetry of the benchmark and its loading scheme with respect to the vertical longitudinal and transverse planes, a fourth was considered for the computational modelling of its contact interaction with the support elements, with the specification of the boundary conditions for the kinematic fixation in the Cartesian coordinate system (Figure 4). The implementation of the boundary conditions for contact interaction is carried out using the software tool MSC Patran. In total, 4 pairs (taking into account a quarter) of contact surfaces are used in the deformation model under consideration (see Figure 4). By designation, these surfaces can be conditionally divided into two types:

1. Contact of support surfaces defining the longitudinal bending of the prismatic benchmark, which are perpendicular to the action of the test force. These include the following surfaces (Figure 4):
   - between the end bearing surfaces of the benchmark and the corresponding bearing surfaces of the end supports (surf. 1);
   - between the bearing surfaces of the end supports and the surface of the prismatic bearing groove (surf. 2);
   - between the load side of the benchmark and the pusher of the testing machine (surf. 3).

On these surfaces, a coupling scheme is implemented in the form of a contact without friction, with the possibility of lagging the contacting surfaces from each other.

**Figure 4.** Calculated finite element model of a prismatic benchmark for implementation of variant studies (type of quarter model)
2. The contact surfaces that determine the transverse bending of the prismatic benchmark and are located at an angle to the action of the test force-between the inclined support surfaces of the benchmark and the inclined support surfaces of the prismatic support (surf. 4 in Figure 4).

To estimate the computational modelling of the SSS of the benchmarks under consideration, the inclination angles of the reference surfaces $\gamma$ are assumed to be no more than 15 degrees, which makes it possible to carry out the FE analysis without taking friction forces into account. In the computational model, the boundary conditions for the kinematic fixation were applied to the base of the prismatic support in the form of a restriction of displacements and rotations in the direction of the $X, Y, Z$ axes (Figure 4). To determine the studied dependences of $P$ and $K_\sigma$, the loading of the benchmarks is carried out by unit pressure applied to the pusher.

The material of prismatic benchmarks in the computational experiment was the properties of the quenched spring-steel 50CrV4 steel – the modulus of elasticity $E = 218000$ MPa, the Poisson's ratio $\mu = 0.3$, the mass density $\rho = 7.85 \times 10^{-9}$ tons/mm$^3$. The material of the remaining elements of the prefabricated model - prismatic and end supports, as well as the pusher – was steel 30HGSA with the following properties: modulus of elasticity $E = 215,000$ MPa, Poisson's ratio $\mu = 0.3$ and mass density $\rho = 7.89 \times 10^{-9}$ tons/mm$^3$.

5. Results of numerical simulation of deformation of prismatic benchmarks
In the process of numerical simulation, about 80 design variants of prismatic type benchmarks were made, in accordance with Figure 3, for which the selected intermediate values of the geometric parameters $h_1$, $h_2$, $\rho$ and $\gamma$ were combined according to the scheme "each with each". The relative computational error in the numerical determination of the values of the principal and equivalent stresses in the working area did not exceed 5% [6]. The processing of the results of FE modelling is presented graphically in Figure 5 and Figure 6.

![Figure 5](image_url)

**Figure 5.** Dependence of the coefficient $P$ (a) and the concentration of equivalent stresses $K_\sigma$ (b) on the height of the vertical part of the L-shaped projection $h_2$ for $\gamma = 10^0$, $h_1 = 0.4$

![Figure 6](image_url)

**Figure 6.** Dependence of the coefficient $P$ (a) and the concentration of equivalent stresses $K_\sigma$ (b) on the height of the vertical part of the L-shaped projection $h_2$ for $\gamma = 15^0$, $h_1 = 0.2$
In Figure 5 shows the dependence of the coefficients \( P \) and \( K_e \) on the height of the vertical part of the L-shaped projection \( h_2 \) and the radius of the gantry transition \( \rho \) for fixed parameters of the angle of the abutment angle \( \gamma = \gamma_{\min} = 10^\circ \) and the thickness of the prismatic body \( h_1 = h_1^{\max} = 0.4 \). In Figure 6 shows the dependence of the coefficients \( P \) and \( K_e \) on the height of the vertical part of the L-shaped protuberance \( h_2 \) and the radius of the galvanic transition \( \rho \) for fixed parameters of the angle of the abutment angle \( \gamma = \gamma_{\max} = 15^\circ \) and the thickness of the prismatic body \( h_1 = h_1^{\min} = 0.25 \). From these figures it follows that an increase in the height of the vertical part of the L-shaped projection \( h_2 \) and the radius of the gantry transition \( \rho \) lead to an increase in \( P \) (Figure 5a and Figure 6a). At the same time, these parameters have the same effect on the creation of a biaxial stretching in the working zone, up to the maximum possible \( P = 2 \). The influence of these parameters on the level of stress intensity \( K_e \) in the working area (Figures 5b and Figures 6b) affects in different ways by increasing \( h_2 \), the coefficient \( K_e \) decreases monotonically, and the increase in \( \rho \) leads to an increase in \( K_e \), varying for both cases within \( 2.1 < K_e < 3.3 \).

Analyzing the results presented in Figure 5 and Figure 6, it can be noted that to increase \( P \) in the working zone of prismatic benchmarks, it is necessary to select the thickness of the prismatic body \( h_1 \) as small as possible from the range \( 1 < P < 2 \). A decrease in the parameter \( h_1 \) not only leads to a pronounced increase in \( P \), but also to a marked increase in the concentration of equivalent stresses in the working region of the benchmark. It can also be noted that the value of the coefficient \( P \) is significantly influenced by the angle of the inclined support surfaces \( \gamma \).

From Figure 5a and Figure 6a that in the case of an increase from \( \gamma = \gamma_{\min} = 10^\circ \) to \( \gamma = \gamma_{\max} = 15^\circ \), the value of \( P \) tends to increase and can be given any value in the range \( 1 < P < 2 \).

The results of variant studies of the design characteristics of prismatic benchmark SSS are given for fixed thicknesses of the prismatic body \( h_1 = 0.4 \) (Figure 5) and \( h_1 = 0.25 \) (Figure 6).

The revealed regularities in the volume in which they are shown in Figure 5 and Figure 6, are informative and sufficient when choosing the necessary sizes of prismatic benchmarks for modelling the required values of \( P \). Analysis of the dependencies (Figure 5 and Figure 6) showed that for any design SSS in the stress concentration region with a known value of \( P \) operating under conditions biaxial stretching, it is always possible to select a prismatic type model simulating the form of this SSS with the same value of \( P \). These data cover the entire range of biaxial stretching \( (1 < P < 2) \) with the possibility of adjusting the level \( q \) of the stresses \( (2.1 < K_e < 3.3) \).

6. Conclusions
It is established that for the proposed prismatic benchmark the variation of its geometric parameters allows modeling various types of biaxial stretching up to a SSS characterized by the maximum possible value of the coefficient \( P \left( P_{\max} = 2 \right) \). This makes it possible to apply the proposed benchmarks in evaluating the strength of structures of various shapes.

The increase in the height of the vertical part of the L-shaped projection \( h_2 \) and the radius of the galvanic transition \( \rho \) of prismatic benchmarks leads to an increase in the rigidity of the SSS (the coefficient \( P \)), and the stress intensity level \( K_e \) in the work zone affects differently - with increasing \( h_2 \), the coefficient \( K_e \) decreases monotonically, and an increase in \( \rho \) leads to an increase in \( K_e \), which allows for any design SSS in the stress concentration region with a known value of \( P \) operating under biaxial stretching conditions, to select an appropriate prismatic type benchmark, modeling view of this SSS with the same value of \( P \).

7. References
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