Enjoyable learning linear programming using mathematical modelling

E Yuliana¹, Y Hartono², D Wijoyo²
¹SMA N 2 Unggulan Tl.Ubi PALI, Palembang, Indonesia
²Universitas Sriwijaya, Palembang, Indonesia

*Corresponding author’s e-mail: yulianaeli31@yahoo.co.id, h.darmawijoyo@yahoo.com

Abstract. Linear programming is one of the important subjects in mathematics needed to understand more advanced subjects. The purpose of this study is to design enjoyable learning for linear programming using mathematical modelling. This study involved 38 high school students. Results show that mathematical modeling helps students do better in solving linear programming.

1. Introduction
Linear programming is a technique used to optimize an objective function in the form of a linear function by following the rules of linear equations and constraints in the form of linear inequalities. Linear programs are very useful in everyday life, namely when we plan an activity in the field of economics, production or other activities whose purpose is to get optimal results by utilizing the resources of activities effectively and efficiently [1]. Linear programming are operations research techniques that have been used extensively in various types of planning management problems [2].

Unfortunately, there are still many mistakes made by students in solving linear programming problems. Based on the research of Stebens & Palocsay [3] that in recent years students have experienced great difficulties in formulating models of linear program problems. This can be caused by several factors, according to Afgani's research, Darmawijoyo, & Purwoko [4] stating that there are still many mistakes made by students in solving linear program problems like there are still many students who are wrong in determining the settlement set area, this is because students do not understand the optimum value of objective functions such as the difficulty of interpreting the meaning of "the lowest possible cost or profit" in a linear program story problem. Other mistakes made by students were mistakes in writing down what was known and asked about the questions, making mathematical models, writing signs of inequality, manipulating algebra, difficulty in calculating and drawing conclusions. Based on the mistakes made by these students, according to Hidayat & Zanaton [5] research, these errors have an impact on students' errors in solving linear programming problems.

Based on this problem, an appropriate learning approach is needed. Learning approaches that make students master the material related to linear programs and also force them to become active in learning. One approach that is suitable with these characteristics is the Indonesian Realistic Mathematics Education Approach (PMRI). PMRI begins with things real or experienced by students,
emphasizes process skills, discussion, collaboration, arguing with fellow students, so students find themselves solving a problem with mathematics (Frudenthal) [6].

PMRI is an alternative that can be used by mathematics teachers in developing students' ability to think, reason logically, communicate and overcome problems in everyday life by Zulkardi [7]. Mathematics at PMRI is a human activity so PMRI uses real-world context as a starting point in learning. In PMRI, Simon & Tzur [8] introduced Hypothetical Learning Trajectory (HLT) which included learning objectives, descriptions of learning activities, and suspected learning trajectories to be carried out. Simon [9] also explained that the term hypothesis is based on the fact that the actual learning trajectory has not become the student’s knowledge. It means that the teacher can never be sure about what students think, what students will do and how they develop new interpretations, ideas and strategies to work on a problem (Fosnot & Dolk) [10].

Learning that is connected to the real world can increase students' attractiveness in learning mathematics as Powers, Kalder [11] teaches linear programs using real-world problems through cooperative learning. Stevens & Paloscay [3] teach linear programs by translating real-world problems into mathematical language through well-defined steps, from the steps of translating them students are able to make mathematical models and solve them.

Based on the background of the problem that linear programming material very important, so researchers are interested in designing linear program learning in the context of school fence design with Realistic Mathematics Learning Approach in Indonesian (PMRI).

2. Methods
The method of this study is a design research type validation study that aims to prove learning theories and develop Local Instruction Theory (LIT) in collaboration with researchers and teachers to improve the quality of learning (Van den Akker, Gravemeijer, mKenney, & Nieven, Gravemeijer & Cobb) [12,13]. Gravemeijer & Cobb [13] define three stages in research design, namely preparing for the experiment, the design experiment and retrospective analysis.

2.1. Preparing for the Experiments (research preparation)

a. Literature Review. On this step the researcher collected information in the form of studying material in mathematics textbooks on the Linear , PMRI Program and research design as the basis for formulating the initial strategies of students in the Linear Program learning.

b. Examine Students Initial Abilities. On this step the researcher conducted interviews with several students as a knowledge of how far the students' understanding of the material was the prerequisite for learning.

c. Design HLT. The researcher designed a series of learning activities that contain hypothetical learning trajectories. HLT is dynamic designed to form a cyclic process (cyclic process) which can change and evolve during the process of teaching experiment.

2.2. The Design Experiment (experimental design)

a. cycle 1 (pilot experiment). This cycle aims to pilot the initial HLT that has been designed. Six students with heterogeneous abilities (2 students with high abilities, 2 students with moderate abilities, and 2 students with low abilities) were involved in the first cycle (pilot experiment), at this stage researchers acted as teachers. The results of cycle 1 are used to revise the initial version of HLT for one class participating in cycle 2 (teaching experiment).
b. cycle 2 (teaching experiment). In cycle 2 students participate in this study. Learners are taught by their own teachers as model teachers and researchers act as observers of learning activities.

c. Retrospective analysis. On this step, the data obtained from the teaching experiment stage is analyzed to develop the design of the next learning activity. HLT is compared to the actual learning activities of students (Actual Learning Trajectory) to answer the formulation of research problems. The purpose of retrospective analysis in general is to develop Local Instruction Theory (LIT).

Data collection is done through several things including observation, making video recordings of events in class and group work, collecting students' work, giving initial tests and final tests, and interviewing students. HLT that has been designed is then compared with the actual learning trajectory of students during the implementation of learning to be retrospectively analyzed whether students learn or not learn from what has been designed in the learning disparities. Data analysis was followed by researchers and mentors to increase validity and reliability. Validity is done to see the quality of a set of data that influence the conclusion of this study. Reliability is described through a clear description of how data is collected so that conclusions can be drawn.

3. Results and Discussion

Learning designed for produce trajectory learn in learning Linear Program material uses context Fence School design with approach PMRI. Results research at the stage of teaching experiment involving 32 students in the second silk is as follows.

3.1 The first activity

The context of the school fence design is presented, namely the problem of the limited budget and the length of the fence that the school wants to build. From this problem students are explored to recognize sentences that contain the decision of variables, constraint functions, objective functions and non-negative functions. On initially student given chance for to know program linear problems in the life every day. Student answer the Question on student activity sheet. From the results answer student show that student have been able to write which sentence contains decision variable sentences, constraint functions, objective functions and non-negative functions. After student one able to identify variable decision sentences, constraint functions, objective functions and non-negative functions.

The context of the school fence design is presented, namely the problem of the limited budget and the length of the fence that the school wants to build. From this problem students are explored to recognize sentences that contain the decision of variables, constraint functions, objective functions and non-negative functions. Student given chance for to know program linear problems in life every day. Student answer the Question on student activity sheet. From the results answer student show that student have been able to write which sentence contains decision variable sentences, constraint functions, objective functions and non-negative functions. After student one able to identify variable decision sentences, constraint functions, objective functions and non-negative functions.

Process identification of sentences containing decision variables from the problem of school fence design could see on the following conversation.
Student 1: "Ma'am, what kind of decision variable is it like?"
Teacher: "Try to pay attention to the problem there ... what do you do school?"
Student 1: "For fence mam ... but there are two fences, a concrete fence and a railing fence"
Teacher: "Can the two fences be later compared with variables?"
Teacher: "Is the value asked ... where is the question asked is the maximum budget for painting right? what is it to paint?"
Student 2: "To paint the concrete fence and the railing fence"
Teacher: "Now, affect the budget not the number of fences that will be made? both railing and concrete fence ..."
Student 2: "Influencing mam ... the wage to paint is different between a concrete fence and a trellis fence."
Teacher: "So where is the decision variable?"

**Figure 1.** Activity dialogues identify linear program problems

On the figure 1, the students have known the sentence contains the variable decision. Next student identify sentences that contain constraints, objective functions and non-negative functions.

3.2 The second activity
Students’ asked for Model the decision variables sentences, objective functions, constraint functions and non-negative functions into the language of mathematics. the first step, the student specifies the decision of variable by using the letters x and y. The activity 2 can be seen at figure 2 and 3 below.

**Figure 2.** How to formulating a mathematical model

From figure 2 and 3 can be seen that students are able to make mathematical model of linear programming problem and students are able to find a general formula of the mathematical model of linear programming problem.

3.3 The thirth activity
Students asked to solve the problem of the school fence design with the graph method. Students strategy to solve linear program problems could seen on Figure 4.

**Figure 3.** Formulating a mathematical model

Teacher: "What do you see, what form of constraint function?"
Student: "The system of two variable linear inequality"
Student: "Linear inequality"
Teacher : “Yes ... how solve the system two variable linear inequality ”
Student : “With a graph. Than there will be a settlement area for you mam ”
Teacher : “After the settlement area was obtained?”
Student : “In the settlement set there are corner points ... those corner points are the x and y values mam ...?”
Teacher : “Yes ... if the corner point is on the x or y axis it is good ... can you see the x and y values directly ... if you leave the corner not on the coordinate axis what?”
Student : “Means that it must be solved by substitution or elimination of two linear inequalities of two intersecting variables”
Teacher : “Yes, how?”
Student : “It's like completing a two-variable linear equation system, bro?”
Teacher : “Yes, really ... can you not?”
Student : “Can you bu ...”

**Figure 4.** Formulating a mathematical model and completing it

Based on figure 4 show that student can download a strategy to solve the problem of linear programming that is designed to make fencing school. The results of student answers determine the set of settlement areas can be seen in the following figure.

**Figure 5.** Determine the corner point of the settlement set area

### 3.4 The fourth activity.
Students are asked to test the value of the decision variables obtained whether they have met all the requirements or not. The student's answer is shown in Figure 6.

### 3.5 The fifth activity
Students are asked to determine the optimum value of the objective function which in the problem of school fence design students are asked to determine the maximum budget that must be provided by the school. The student's answer to the fifth activity can be seen in Figure 7 below.
3.6 The sixth activity

Teacher remind student about How student understanding to complete linear program problems. From identifying sentence that contains variable decision, function, purpose, function of obstacles or non-negative function. Then formulate sentences into mathematical models, after that resolved with method chart with test point corner.

From the first, second, third, fourth, fifth and sixth activities can be concluded that it can help students to understand the concept of linear programming. Sallan, Lordan, Fernandez [14] In the linear programming the main thing that must be discovered first is what will be maximized or minimized in the objective function of the constraints with the following steps:

1. Identifying decision variables, objective functions, constraint functions and non-negative variables into a sentence.
2. Formulate a mathematical model after the first three steps are done, next step in sequence are:
   a. Declare the decision variable to a mathematical symbol eg x₁, x₂, ..., xₙ
   b. Express the objective function into a mathematical model
   c. Express the constraint function into the mathematical model
   d. Express non-negative functions
   e. Linear characteristics, which implies that all mathematical functions are linear
3. Determine the completion of a mathematical model
4. Finaly conclusion

Based on results analysis retrospective on activities 1, 2, 3, 4, 5 and 6 on cycle 2 (experiment teaching), learning corresponding with HLT designed and could concluded that student have understand concept linear program by using the context of school fence design. Series learning that is implemented that using PMRI approach shows how the characteristics of PMRI become the basis for the learning process of each activity. The five characteristics of PMRI which are the adoption of RME (Gravemeijer) [15] in the designed learning design will be explained as follows:

Use of contexts for phenomenologist exploration is the first characteristic in which learning activities begin with contextual problems that are often encountered by students as experience-based activities. In this study, the problem of design of school fence making was used as a contextual problem in designing linear program learning. Context in this study is an activity and things that can be imagined by students so that students can understand the problems given where this agrees, every activity in learning in cycle 1 or cycle 2, uses problems related to planning activities.
The second characteristic is use of models for mathematical concepts construction that the use of this model aims to connect students' understanding of abstract forms to realities which are commonly known as the transition from informal forms to formal forms (Gravemeijer) [15]. Here it is seen that students model decision variable sentences, objective functions, constraint functions and non-negative functions into formal mathematical sentences. The second characteristic emphasizes modelling activities.

The third characteristic is the use of students' creation and contribution. This third characteristic is seen during the learning process of a variable linear equation from a series of activities given. The teacher gives an appreciation of the contribution of students in the learning process both in group and individual activities. Students are given the freedom to express and answer questions using their respective strategies. Learning becomes more meaningful, one of which is due to variations in students’ answers in solving problems. In addition, students who have been able to find linear problem solving strategies can guide other students in the group during the discussion. This thing appears in every activity. In each activity the teacher acts as a facilitator and does not dominate learning so that students can be creative in accordance with their understanding.

Furthermore, the fourth characteristic is students activity and in the learning process. In learning using the discussion method in cycle 1 students look unfamiliar, making it difficult for them to work with group members. In cycle 2, interactivity between students, between teachers and students appears in every activity both in discussions and individuals. Students in this teaching experiment are very cooperative so learning can run smoothly.

The last characteristic is intertwining of mathematics concepts, aspects, and units. In this learning design is not separated from the connection with other materials, namely the concept of arithmetic operations. In addition, the relationship with other materials, especially linear functions, systems of equations and linear inequalities become indicators of the intertwining of other material learning topics.

4. Conclusion
Based on results research and discussion, can concluded that trajectory lessons learned consists of 6 activity, namely the activity of one, the students are able to identifying sentences that contain decision variables, constraints function, the objective function and the function of non-negative. Activity two, the formulate of a mathematical model of linear programming problem. Activity three, solving linear programming problems with graphical method and corner point test, activity four, testing values from the corner point of the settlement set, activity five, determining the optimum value of the objective function, the last activity is activity six, students are asked to recall how to move steps to be taken to solve linear program problems. In addition, the results of the study indicate that the use of the context of the school fence design could help interested to student to understand the concept of a linear program. The student more motivated and keen to resolve problems that are given. Using the context of the school fence design to find mathematical models and solve them can help students solve other linear program problems.

References
[1] Dantzig, G.B. (1963). Linear Programming and Extension. RAND Corporation
[2] Gaspersz, Vincent. 2005. Total Quality Management. Jakarta : PT Gramedia Pustaka Utama
[3] Stevens & Palocsay. 2004. Journal INFORMS Transactions on Education.Maryland : 38. USA
[4] Afgani, M. W., Darmawijoyo, & Purwoko. (2008). Jurnal Pendidikan Matematika, 2 (2), 47.
[5] Hidayat, R., & Zanaton. (2014). Misconception of Linear Programming in Senior High School. Malaysia: Universiti Kebangsaan Malaysia

[6] Zulkardi. (2002). Developing a Learning Environment on Realistic Mathematics Education for Indonesian Student Teachers. Enschede: University of Twente.

[7] Frudenthal, H. (1991). Revisiting Mathematics Education. Dordrecht. Kluwer Academic Publishers

[8] Simon, MA and Ron Tzur . (2004). Explicating the Role of Mathematical Tasks in Conceptual Learning: An Elaboration of the Hypothetical Learning Trajectory. Mathematical Thinking and Learning.

[9] Simon Hasanu. 1995. Social Forestry and Sustainable Forest Management. Cooperation between Perum Perhutani and The Faculty of Forestry. Yogyakarta: Gadjah Mada University.

[10] Fosnot, Catherine T. And Dolk, maarten L. (2001). Young Mathematicians at Work Constructing Multiplication and Division. Portsmouth, NH: Heinemann

[11] Powers, Kalder. (2006). Teaching Linear Programming To Low Achieving Mathematics Students Using Real World Problem. New Britian : Central Connecticut State University

[12] Akker, Jvd,Gravemeijer, K., McKenney, S., & Nieveen. (2006). Educational Design Research. London: Routledge Taylor and Francis Group

[13] . Gravemeijer, K. & Cobb, Paul.(2006). Design Research from a Learning Design Perspective. In Jan Van Den Akker, et.al. Educational Design Research. London:Routledge

[14] Sallan, Lordan, Fernandez. 2015. Modeling and Solving Linear Programming With R. Catalunya: Omnia Science

[15] Gravemeijer, K. (1994). Journal for Research in Mathematics Education, 25(5), 443