Supermassive black holes or boson stars? 
Hair counting with gravitational wave detectors

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Abstract
The evidence for supermassive Kerr black holes in galactic centers is strong and growing, but only the detection of gravitational waves will convincingly rule out other possibilities to explain the observations. The Kerr spacetime is completely specified by the first two multipole moments: mass and angular momentum. This is usually referred to as the “no-hair theorem”, but it is really a “two-hair” theorem. If general relativity is the correct theory of gravity, the most plausible alternative to a supermassive Kerr black hole is a rotating boson star. Numerical calculations indicate that the spacetime of rotating boson stars is determined by the first three multipole moments (“three-hair theorem”). The Laser Interferometer Space Antenna (LISA) could accurately measure the oscillation frequencies of these supermassive objects. We propose to use these measurements to “count their hair”, unambiguously determining their nature and properties.
The present observational evidence for the existence of astrophysical black holes (BHs) at the centers of galaxies is strong and growing [1]. The most convincing case comes from observations of stellar proper motion at the center of our own Galaxy, indicating the presence of a “dark object” of mass $M \approx (3.7 \pm 0.2) \times 10^6 M_\odot$ [2] and size smaller than about one astronomical unit [3]. A Schwarzschild BH of the given mass has radius $R = 2GM/c^2 \approx 0.073$ astronomical units, compatible with the observations.

The massive BH picture has become a paradigm. All scientific paradigms are dangerous, since it is easy to believe that we have proved what we expect to find. To argue convincingly that galactic centers contain BHs we must rule out all possible alternatives. Some of these alternatives are already incompatible with observations. For example, the central mass in our Galaxy (and several other galactic nuclei) cannot be modelled by any distribution of individual objects: such a distribution would be gravitationally unstable [4]. A possible exotic alternative to BHs (fermion balls) is also incompatible with observations of the supermassive object at the Galactic center [5].

Assessing the BH nature of these massive objects by electromagnetic observations is a hard task. Despite recent claims that (under plausible assumptions) an event horizon is required to explain observations of the massive object at the center of our own Galaxy [6], some hold the view that an observational proof of the existence of event horizons based on electromagnetic observations is fundamentally impossible [7].

A conclusive test of the nature of these supermassive objects will come from observations of gravitational radiation with the Laser Interferometer Space Antenna (LISA). Elaborating on ideas proposed by Ryan [8], Kesden et al. [9] showed that if the central object has no horizon (their particular model assumes it is a soliton star) this will leave a characteristic imprint on the gravitational radiation emitted by the inspiral of solar-mass compact objects. Ryan’s proposal to map spacetimes using inspiral waveforms is promising, but the data analysis task is plagued by a “confusion problem”: the possibility of misinterpreting truly non-Kerr waveforms by Kerr waveforms with different orbital parameters [10].

Here we propose a different idea, based on measuring the oscillation frequencies (“ringdown waves”) of isolated supermassive objects. In Ref. [11] we provided a general criterion (valid for any interferometric gravitational wave detector) on the signal-to-noise ratio required to test the BH nature of the source from a detection of ringdown waves. In the next section we explain the basic idea behind this test, summarizing our analysis in [11].

Gravitational wave astronomy will look at the Universe from a new perspective, and is likely to present us with surprises. For this reason in the rest of the paper we speculate on possible alternatives to the BH hypothesis, under the “conservative” assumption that general relativity is the correct theory of gravity. We focus on the most promising (and fascinating) family of particle-physics candidates which have not been ruled out by observations: boson stars [12, 13].
Black hole ringdown and tests of the no-hair (two-hair) theorem

Newly formed BHs are expected to emit characteristic radiation (ringdown waves) whose amplitude can be expressed as a linear superposition of a discrete set of damped oscillations, known as quasi-normal modes (QNMs):

\[
h = h_+ + i h_\times = \frac{M}{r} \sum_{l,m,n} A_{l,m,n} e^{i(\omega_{l,m,n} t + \phi_{l,m,n})} S_{l,m,n}. \tag{1}
\]

The functions \( S_{l,m} \) are spin-weighted spheroidal harmonics, used to separate the angular dependence of the perturbations and characterized by angular quantum numbers \((l, m)\). The amplitudes \( A_{l,m,n} \) and phases \( \phi_{l,m,n} \) of the various modes are determined by the specific process that formed the final hole. The complex mode frequencies \( \omega_{l,m,n} = 2\pi f_{l,m,n} + i/\tau_{l,m,n} \) \((f_{l,m,n} \text{ is the oscillation frequency and } \tau_{l,m,n} \text{ is the damping time of the oscillation})\) are labelled by an integer \( n \) that sorts them by the magnitude of their imaginary part. Large imaginary part means short damping time, so low-\( n \) modes are (in principle) easier to observe. According to the standard lore, \( l = m = 2 \) modes (having the longest damping time and corresponding to bar-shaped deformations) should dominate the signal [14]. The QNM frequencies \( \omega_{l,m,n} \) are universal, depending only on the mass \( M \) and specific angular momentum \( a = J/M \) of the BH. This universality is a consequence of the so-called “no-hair” theorem, which is really a “two-hair” theorem: the spacetime of a Kerr BH (as determined by the mass multipole moments \( M_n \) and current multipole moments \( S_n \)) is completely specified once we know the first two moments \( M_0 = M \) and \( S_1 = J = Ma \), since \( M_n + i S_n = M(ia)^n \).

LISA should be able to detect ringdown waves from oscillating supermassive BHs throughout the observable universe in the frequency range \(10^{-5} \text{ - } 10^{-1} \) Hz, with maximum sensitivity around \( 10^{-2} \) Hz. The signal-to-noise ratio (SNR) depends on the BH’s distance, mass, angular momentum and (more importantly) on the ringdown efficiency \( \epsilon_{rd} \) (fraction of mass radiated in ringdown waves). Usually the SNR is quite large, because ringdown waves from supermassive BHs are emitted in the maximum sensitivity range of LISA: the fundamental QNM of a Schwarzschild BH has frequency and damping time

\[
f_{200} = 1.2 \cdot 10^{-2}(10^6 M_\odot/M) \text{ Hz}, \quad \tau_{200} = 55 \left( M/10^6 M_\odot \right) \text{ s}. \tag{2}
\]

For example, a BH with \( a/M = 0.8 \) and mass \( \sim 4 \times 10^6 \ M_\odot \) that radiates 1 % of its mass at luminosity distance \( D_L = 3 \) Gpc will be detected by LISA with SNR \( \sim 10^4 \) (see Fig. 8 of [11]). Since measurement errors are inversely proportional to the SNR, typical measurements of QNM frequencies should be exquisitely precise.

Detection of a single QNM provides us with two observables: \( f_{l,m,n} \) and \( \tau_{l,m,n} \), which are functions of \( M \) and \( a \) only. Inverting these (known) functional relations we can determine both the mass and the angular momentum of the BH, if we know which mode we are detecting. In order to test the no-hair (“two-hair”) theorem, however, it is necessary (though not sufficient) to resolve two QNMs. Roughly speaking, one mode is used to measure \( M \) and \( a \), the other to test consistency with the GR prediction.
In [11], using a simple extension of the Rayleigh criterion for resolving sinusoids, we estimated the SNR required to resolve the frequencies and/or damping times of various pairs of modes, as a function of $a/M$. Under the plausible assumption that the amplitude of the first overtone ($n = 1$) is $1/10$ that of the fundamental mode ($n = 0$) we found that tests of the no-hair theorem should be feasible, even under rather pessimistic assumptions on $\epsilon_{rd}$, as long as the first overtone radiates a fraction $\sim 10^{-2}$ of the total ringdown energy. However, resolving both frequencies and damping times typically requires a SNR greater than about $10^3$. This is only possible under rather optimistic assumptions about the radiative efficiency, and it can be impossible if the dominant mode has $l = m = 2$ and the BH is rapidly spinning. Requiring SNRs at least as large as $10^2$ implies that resolving QNMs will be impossible for redshifts larger than about 10.

The most probable outcome of the proposed no-hair test is a verification of the BH hypothesis, combined with an accurate measurement of the BH’s mass and spin. This would be an exciting experimental test of general relativity in the strong field regime. However, we must leave the door open for surprises.

What if LISA observations yield QNM frequencies that are not compatible with the BH hypothesis? There are two possibilities: either general relativity is not the correct theory of gravity, or the compact object we are observing is not a black hole. If we conservatively exclude the first hypothesis, the most plausible, theoretically motivated alternative to explain the observations of galactic centers is a boson star [12]. In the following we present a bird’s eye view on the theoretical properties of boson stars, and propose a general-purpose test to “count the hair” of supermassive objects. We also suggest that the hypothetical detection of such a “hairy” object could provide interesting information on particle physics and cosmology.

**Exotic strawmen: boson star hair counting**

Scalar particles such as the Higgs boson, the axion and the dilaton play an important role in early Universe and high-energy physics. Fundamental scalar fields have not been detected yet, but if they exist they could condense to form boson stars: localized solutions of the Einstein-Klein-Gordon system which are a natural generalization of Wheeler’s geons [15].

Boson stars are macroscopic quantum states prevented from complete gravitational collapse by Heisenberg’s uncertainty principle. In this sense they are “gravitational atoms” that can have macroscopic size and large masses. The main difference between boson star models consists in the scalar self-interaction potential [13]. To simplify, we can distinguish between three broad classes of boson star models:

1) *Miniboson stars*. If the scalar field is non-interacting [15], the maximum boson star mass $M_{\text{max}} \simeq 0.633m_{\text{Planck}}^2/m$ is much smaller than the Chandrasekhar mass for fermion stars $M_{\text{Ch}} \sim m_{\text{Planck}}^3/m^2$ (hence the name). To support supermassive objects we need an ultralight boson of mass $m = 8.45 \times 10^{-26}\text{GeV} \,(10^6M_\odot/M_{\text{max}})$.

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3More general self-interaction potentials have been considered in [16]. There are also non-singular, time-dependent equilibrium configurations of self-gravitating real scalar fields known as oscillatons [17].
2) Boson stars. The requirement of ultralight bosons to have astrophysical-size objects can be lifted if we consider self-interacting scalar fields with a quartic self-interaction potential of the form \( \lambda |\phi|^4/4 \) [18]. As long as the coupling constant \( \lambda \gg (m/m_{\text{Planck}})^2 \) the maximum boson star mass can be of the order of the Chandrasekhar mass or larger, \( M_{\text{max}} \simeq 0.062 \lambda^{1/2} m_{\text{Planck}}^3/m^2 \). Supermassive objects can exist if the boson mass and coupling constant \( \lambda \) are such that \( m = 3.2 \times 10^{-4} \) GeV \( \lambda^{1/4} (10^6 M_{\odot}/M_{\text{max}})^{1/2} \).

3) Nontopological soliton stars. If the self-interaction takes the form \( U = m^2 |\phi|^2 (1 - |\phi|^2/\phi_0^2)^2 \) (where \( \phi_0 \) is a constant) we can have nondispersive solutions with a finite mass, confined to a finite region of space, even in the absence of gravity [19]. The critical mass \( M_{\text{max}} \simeq 0.0198 m_{\text{Planck}}^4/(m\phi_0^2) \). One usually assumes \( \phi_0 \sim m \), so that a \( \sim 10^6 M_{\odot} \) boson star corresponds to a relatively heavy boson with \( m \sim 500 \) GeV.

Boson stars are good “strawmen” for supermassive BHs. They are indistinguishable from BHs in the Newtonian regime. Boson stars being very compact, deviations in the properties of orbiting objects occur close to the Schwarzschild radius and are very hard to detect electromagnetically [12]. If the scalar field interacts only gravitationally with matter compact objects could safely inspiral inside the boson star, the only basic difference with a BH being the absence of an event horizon [9]. Boson masses yielding supermassive boson star candidates differ by so many orders of magnitude that even if we could set independent bounds on \( m \) (by, say, high-energy particle collisions) some candidates would still survive.

The main point we wish to make is that ringdown waves could provide a way to unambiguously rule out boson stars as supermassive BH “strawmen”. If instead (serendipitously) gravitational wave observations should turn out to be compatible with boson stars, we could learn something about scalar field masses and their interaction with matter, perhaps shedding some light on the low-energy limit of string theories.

Any credible alternative to astrophysical Kerr BHs must include the effects of rotation. Stable rotating boson star solutions with \( \lambda \gg (m/m_{\text{Planck}})^2 \) were obtained by Ryan [23]. Considering a pure state for the scalar field of the form \( \phi = \Phi(r, \theta) e^{i(s\varphi - \Omega t)} \), Ryan showed that the structure of a rotating boson star is determined by three independent parameters: 1) a scaling factor \( \lambda^{1/2}/m^2 \), 2) \( \Omega = \Omega/m \) and 3) \( s = s/\lambda^{1/2}/m \). Ryan’s numerical structure calculations indicate that these three parameters are in one-to-one correspondence with the first three multipole moments of the star: mass \( M \), spin \( S_1 \) and quadrupole moment \( M_2 \).

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4 A sum over pure states would yield time-dependent stress-energy tensors, so it’s reasonable to expect gravitational wave emission to lead the star to a pure state on short timescales. The resulting configurations are torus-shaped, so one should rather talk about “boson doughnuts”. Notice also that the uniqueness of the scalar field under a 2π-rotation requires \( s \) to take only integer values, i.e. the angular momentum is quantized. Despite this Ryan considers \( s \) as a continuous parameter, since the spacing of allowed values is extremely small.

5 Interestingly, for a relatively slowly spinning boson star \( (S_1/M^2 \lesssim 0.2) \) the dimensionless multipole moments are almost constant, and much larger than the value of unity corresponding to a Kerr BH: 10 \( \lesssim -M_2/M S_1^2 \lesssim 150, 20 \lesssim -S_3/M S_1^3 \lesssim 200 \). Low masses, corresponding to less compact stars, have larger multipole moments. This weak variability as a function of the rotation rate could be a useful feature for our “hair counting” proposal.
Given a measurement of these moments one can infer the three boson star parameters. A fourth measurement can then be used to test the boson star hypothesis, which is really a "three-hair" theorem (in the same way as the BH no-hair test is really a "two-hair" test).

The extension of the Kerr BH "two-hair test" to a "hair counting test" for rotating supermassive objects requires the knowledge of QNM frequencies for rotating boson stars with self-interaction of the type considered by Ryan. Unfortunately this calculation has not been performed yet, but some insight on the feasibility of the test can be obtained from the QNM frequencies of nonrotating miniboson stars, which are known \[21\].

For the dominant quadrupole perturbations \(l = 2\) the resulting QNM spectrum is very different from the QNM spectrum of Kerr BHs\(^6\). The ringdown signal from boson stars should be a superposition of many modes with comparable damping. For all modes the imaginary part is about an order of magnitude smaller than the real part (for Kerr BHs this happens for modes with \(m > 0\) in the limit \(a/M \to 1\)). The SNR for different modes should be comparable, requiring a multi-mode data analysis similar to the one developed in \[11\].

These expectations are confirmed by recent three-dimensional time-evolutions of perturbed boson stars. Fig. 4c of Ref. \[22\] shows that many modes are indeed present in the signal, that the \(l = 2\) component dominates the energy emission, and that all modes with \(n < 12\) are necessary to fit the waveform (modes with \(n = 5 - 10\) being dominant).

To assess detectability by LISA, let us consider the modes with \(n = 7 - 8\) (the most excited in the numerical simulations) as representative. These modes have a dimensionless frequency \(\sigma = \omega/m \simeq 2.3 + 0.13i\) \[21\], yielding

\[
f \simeq 4.7 \times 10^{-2} (10^6 M_\odot/M_{\text{max}}) \text{ Hz}, \quad \tau \simeq 60 \left( M_{\text{max}}/10^6 M_\odot \right) \text{ s}.
\]

(3)

These numbers should be compared with the typical supermassive BH ringdown frequency \[2\]. Our simple estimate shows that simply choosing the boson mass \(m\) to match the mass of a galactic BH also brings the QNM frequencies in the interesting range for LISA.

Gravitational wave emission from oscillating boson stars with self-interaction has not been studied so far, but some general features can be anticipated from existing calculations of the radial pulsation frequencies. Fig. 4a in \[20\] shows that radial frequencies for \(\lambda \neq 0\) have the same order of magnitude as in the non-interacting case, generally decreasing with \(\lambda\) and scaling as \(\lambda^{-1/2}\) for large \(\lambda\). Eventually an explicit calculation for rotating, self-interacting boson stars of the kind considered by Ryan (as well as for other models, including soliton stars and oscillatons) will be necessary to perform our proposed test\(^7\).

If boson stars exist and ringdown waves from these object will be detected, the payoff could be great. In Ryan’s model, the measurement of two QNMs (four observables) should be enough to determine all three boson star parameters and to assess the boson star nature

\(^6\)In fact, the structure of the spectrum is reminiscent of the \(w\)-modes of relativistic stars.

\(^7\)We mention in passing that Ref. \[24\] provides Newtonian estimates for the gravitational wave emission from the decay of excited states of a newly formed boson star into the (stable) ground state. Their mechanism is quite different from the nonradial oscillations of a boson star in the ground state, but it is interesting to note that their estimate for the gravitational wave frequencies lies in the LISA range.
of the object. In particular, a measurement of $\lambda^{1/2}/m^2$ could be important for high-energy physics (e.g. to test the low-energy limit of string theories). If instead ringdown frequencies are consistent with a black hole, we would have a striking confirmation of Einstein’s general relativity in the strong-field regime.

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