Fine-grained uncertainty relation and biased non-local games in bipartite and tripartite systems

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The fine-grained uncertainty relation can be used to discriminate among classical, quantum and super-quantum correlations based on their strength of non-locality, as has been shown for bipartite and tripartite systems with unbiased measurement settings. Here we consider the situation when two and three parties, respectively, choose settings with bias for playing certain non-local games. We show analytically that while the fine-grained uncertainty principle is still able to distinguish classical, quantum and super-quantum correlations for biased settings corresponding to certain ranges of the biasing parameters, the above-mentioned discrimination is not manifested for all biasing.

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INTRODUCTION

Heisenberg uncertainty relation \[1\] infers the restriction inherently imposed by quantum mechanics that we cannot simultaneously predict the measurement outcomes of two non-commuting observables with certainty. This uncertainty relation was generalized for any two arbitrary observables by Schrödinger and Robertson \[2\]. In quantum information theory, it is more convenient to use the uncertainty relation in terms of entropy in stead of standard deviation. A lot of effort has been devoted towards improving entropic uncertainty relations \[3–5\], especially in terms of their practical applicability in several information processing scenarios such as quantum information locking and key generation \[6–8\]. Recently, a new fine-grained form of the uncertainty relation has been proposed \[9\] which is able to distinguish between uncertainties inherent in various possible measurement outcomes, and is linked with the degree of non-locality of the underlying theory.

Though the use of entanglement in information processing tasks is widely appreciated, quantum correlations may not be advantageous compared to classical ones in all types of situations. Since entanglement is a fragile resource, the question as to when precisely quantum non-locality following from entanglement is necessary for practical applications, is rather important. In this context, the application of the fine-grained uncertainty relation could be particularly relevant. The fine-grained uncertainty relation is able to discriminate between the degree of non-locality in classical, quantum and super-quantum correlations of bipartite systems, as was shown by Oppenheim and Wehner \[9\] in the context of a class of non-local retrieval games for which there exists only one answer for any of the two parties to come up with in order to win. The maximum probability of winning the retrieval game is equal to the upper bound of the uncertainty relation and this quantifies the degree of non-locality of the underlying physical theory. This upper bound could thus be used to discriminate among the degree of non-locality pertaining to various underlying theories such as classical theory, quantum theory and no-signaling theory with maximum non-locality for bipartite systems.

Further insight into the nature of difference between various types of correlations has been recently provided by the work of Lawson et al. \[10\]. For a class of Bell-CHSH \[11, 12\] games, they introduce the situation when the two parties decide to choose their measurements with bias. It has been shown that for certain range of the biasing parameters, quantum theory offers advantage and surprisingly for others, it does not provide a better result than classical mechanics. This leads towards the identification of situations when quantum entanglement is indeed essential for implementing a particular information processing task. A generalization for multipartite systems is also performed in which numerical results for the upper bound of the correlation function is presented when all parties measure with equal bias.

In this work we investigate the connection between the fine-grained uncertainty relation and non-locality in the context of biased non-local games for first bipartite and then tripartite systems. Our motivation is to utilize the fine-grained uncertainty relation in order to determine the nonlocal resources necessary for implementing this particular information processing task of winning a biased game played by two or three parties. Here we make use of the formalism developed by Oppenheim and Wehner \[9\] for bipartite systems, and its subsequent extension to the case of tripartite systems \[13\]. In case of bipartite systems, correlations are expressible in terms of Bell-CHSH \[11, 12\] form without ambiguity and can be used efficiently for above mentioned task of discrimination between classical, quantum and super-quantum theories. The scenario for the tripartite case is however, a bit different as there is an inherent non-uniqueness regarding the choice of correlations proposed by Svetlichny \[14\] and Mermin \[15\]. It has been shown \[13\] that the Svetlichny-type correlations can discriminate among the classical, quantum and no-signaling theory using the fine-grained
uncertainty principle, whereas the inequality extracted from the Mermin-type correlation is unable to perform the same task. In our present analysis we use the approach proposed by Bancal et al.\cite{16} in order to calculate the upper bound of the Svetlichny function analytically in case of the biased tripartite game. We are thus able to present without using numerical methods our results on the ranges of biasing parameters when quantum correlation are beneficial for non-local tasks. In what follows we will first present the description of biased nonlocal games as provided by Lawson et al.\cite{10}, using the terminology of fine-grained uncertainty relations \cite{9,13}. In the process, we will recount several results of Ref.\cite{11} for the bipartite game in the next section. The utility of our approach in deriving new analytical results will be clear in the section on tripartite games.

**FINE-GRAINED UNCERTAINTY AND BIASED NONLOCAL GAMES**

We begin with the description of the scheme of the game to be played within the bipartite system. The situation is such that the two parties, namely, Alice and Bob share a state $\rho_{AB}$ which is emitted and distributed by a source. Alice and Bob are spatially separated enough so that no signal can travel while experimenting. Alice performs either of her measurements $A_0$ and $A_1$ and Bob, either of $B_0$ and $B_1$ at a time. These measurements having the outcomes $+1$ and $-1$, can be chosen by Alice and Bob without depending on the choice made by the other. The CHSH inequality \cite{12}

\[
\frac{1}{4}[E(A_0B_0) + E(A_0B_1) + E(A_1B_0) - E(A_1B_1)] \leq \frac{1}{2} \tag{1}
\]

holds for any local hidden variable model and can be violated when measurements are done on quantum particles prepared in entangled states. Here $E(A_iB_j)$ are the averages of the product of measurement outcomes of Alice and Bob with $i,j = 0,1$. The above inequality refers to the scenario when the two parties have no bias towards choosing a particular measurement.

In the following picture, describing the biased game\cite{11}, the intention of Alice is to choose $A_0$ with probability $p(0 \leq p \leq 1)$ and $A_1$ with probability $(1-p)$. Bob intends to choose $B_0$ and $B_1$ with probabilities $q(0 \leq q \leq 1)$ and $(1-q)$, respectively. The measurements and their outcomes are coded into binary variables pertaining to an input-output process. Alice and Bob have binary input variables $s$ and $t$, respectively, and output variables $a$ and $b$, respectively. Input $s$ takes the values 0 and 1 when Alice measures $A_0$ and $A_1$, respectively. Output $a$ takes the values 0 and 1 when Alice gets the measurement outcomes $+1$ and $-1$, respectively. The identifications are similar for Bob’s variables $t$ and $b$. Now, the rule of the game is that Alice and Bob’s particles win (as a team) if their inputs and outputs satisfy

\[
a \oplus b = s \oplus t \tag{2}
\]

where $\oplus$ denotes addition modulo 2. Input questions $s$ and $t$ have the probability distribution $p(s,t)$ (for simplicity we take $p(s,t) = p(s)p(t)$ where $p(s = 0) = p$, $p(s = 1) = (1-p)$, $p(t = 0) = q$ and $p(t = 1) = (1-q)$ in our case).

The fine-grained uncertainty relation\cite{9} may be now invoked by noting that for every setting $s$ and the corresponding outcome $a$ of Alice one may formally denote a string $x_{s,a} = (x_{s,a}^1, x_{s,a}^2)$ determining the winning answer $b = x_{s,a}^t(\forall x \in (0,1))$ for Bob; $\{s\} \in S$ and $\{t\} \in T$, $S$ and $T$ being the set of Alice’s and Bob’s input settings, respectively. Alice and Bob receive the binary questions $s, t \in \{0,1\}$ (i.e. representing two different measurement settings on each side) with corresponding probabilities (for $p$ and $q \neq 0,1$ the game is non-local) and they win if their respective outcomes $a, b \in \{0,1\}$ satisfy the condition \cite{2}. Before starting the game (a biased CHSH-game), Alice and Bob communicate and discuss their strategy, i.e., choice of the bipartite state $\rho_{AB}$ they are sharing and their measurements. They are not allowed to communicate once the game starts. The probability of winning the game for a physical theory described by the bipartite state $\rho_{AB}$ is given by\cite{9},

\[
P^{\text{game}}(S,T,\rho_{AB}) = \sum_{s,t} p(s,t) \sum_a p(a,b = x_{s,a}^t|s,t)\rho_{AB} \tag{3}
\]

When $P^{\text{game}}(S,T,\rho_{AB})$ is less than 1, the outcome of the game is uncertain. The value of $P^{\text{game}}$ is bound by particular theories. For the unbiased case (i.e., $p(s,t) = p(s)p(t)$), the upper bounds of this value in classical, quantum and no-signaling theory are $\frac{3}{4}, \frac{2}{4} + \frac{1}{2\sqrt{2}}$ and 1 respectively. The form of $p(a,b = x_{s,a}^t|s,t)\rho_{AB}$ in terms of the measurements on the bipartite state $\rho_{AB}$ is given by,

\[
p(a,b = x_{s,a}^t|s,t)\rho_{AB} = \sum_b V(a,b|s,t)\langle (A^a_s \otimes B^b_t) \rangle_{\rho_{AB}} \tag{4}
\]

where, $A^a_s = \mathbb{I} + (-1)^s A_a$ is the measurement of the observable $A_a$ corresponding to the setting $s$ giving the outcome $a$ at Alice’s side; $B^b_t = \mathbb{I} + (-1)^s B_b$ is a measurement of the observable $B_b$ corresponding to the setting $t$ giving the outcome $b$ at Bob’s side and $V(a,b|s,t)$ filters the winning combination and is given by,

\[
V(a,b|s,t) = 1 \text{ if } a \oplus b = s \oplus t \text{ and } = 0 \text{ otherwise.} \tag{5}
\]

$P^{\text{game}}(S,T,\rho_{AB})$ can now be calculated using the Eqs.\cite{2,5} with the given probabilities of different measurements of Alice and Bob. For the bipartite state $\rho_{AB},$
the upper bound of the fine-grained uncertainty relation

to differentiate between classical and quantum correlations using
parameters are regulated in this region, we cannot differ-
gence. This result could be restated as follows. If the bias

correlation in performing the specified task in this re-
oneway where the values of all the observables are +1 giving

achieved by classical theory, and hence, quantum corre-

One sees that the upper bound is the same value as that

This reduces to the value $\frac{3}{4}$ for the unbiased case when

For considering the quantum strategy, Lawson et al.
divide the parameter space in two regions of $[p,q]$ space with the first region corresponding to $1 \geq p \geq (2q)^{-1} \geq \frac{1}{2}$ (region-1). In this region,

thus leading to

One sees that the upper bound is the same value as that
achieved by classical theory, and hence, quantum correlation (entanglement) offers no advantage over classical correlation in performing the specified task in this re-
gion. This result could be restated as follows. If the bias
parameters are regulated in this region, we can not differenti-
eat between classical and quantum correlations using
the upper bound of the fine grained uncertainty relation corresponding to the biased non-local game in context.

Now, let us consider the other region $1 \geq (2q)^{-1} > p \geq \frac{1}{2}$ (region-2), which gives the bound

This value is greater than the classical bound. So, the regulation of the biasing parameters in this region discrimi-

The upper bound of the fine-grained uncertainty relation (i.e., the maximum chance of winning the game) is in this case given by,

This also reduces to the unbiased value of $\left[\frac{1}{2} + \frac{1}{2\sqrt{2}}\right]$ for

We will now consider a biased non-local tripartite game with Alice, Bob and Charlie as players. Similar to the bipartite case, Alice, Bob and Charlie have their input binary variables (or questions) $s$, $t$ and $u$ ($s, u, t \in 0, 1$) corresponding to their respective two different measurements settings, and output binary variables (or answers) $a$, $b$ and $c$ ($a, b, c \in 0, 1$) corresponding to their respective outcomes of measurements. Given a rule (i.e., the winning condition) of the game, the maximum winning probability (having the established correspondence with the upper bound of the fine-grained uncertainty relation [13]) can be calculated by considering the various possibilities of outcomes (along with the measurements) satisfying the rule.

We consider a full-correlation box (namely, Svetlichny Box [14]) for which all one and two party correlations vanish [19]. The game is won if the answers satisfy

In this case, Alice intends to measure with her setting $A_0$ with probability $p$ (i.e., $p(s = 0) = p$) and $A_1$ with probability $(1 - p)$ (i.e., $p(s = 1) = (1 - p)$). Bob mea-

The winning probability is quantified as,

where $p(s, t, u) = p(s)p(t)p(u)$ is the probability of choosing the measurement settings $s$ by Alice, $t$ by Bob and
u by Charlie from their respective sets $S$, $T$ and $U$. $p(a, b, c|s, t, u)_{PABC}$ is the joint probability of getting outcomes, $a$, $b$ and $c$ for corresponding settings $s$, $t$ and $u$ given by,

$$p(a, b, c) = x_{s,t,a,b}^u|s, t, u)_{PABC}$$

$$= \sum_c V(a, b, c|s, t, u)\langle A^s_a \otimes B^b_t \otimes C^c_u \rangle_{PABC}$$

where $A^s_a$, $B^b_t$ and $C^c_u$ are the measurements (with the forms given in the treatment of bipartite system) corresponding to the setting $s$ and outcome $a$ at the Alice's side, setting $t$ and outcome $b$ at Bob’s side and setting $u$ and outcome $c$ at Charlie side. $V(a, b, c|s, t, u)$ equals 1 only when condition $|13|$ is satisfied; otherwise, 0. Using the condition $|13|$ and Eq.$|15|$, Eq.$|14|$ simplifies to

$$P_{game}(S, T, U, \rho_{ABC}, p, q, r) = \frac{1}{2}[1 + \langle S(p, q, r) \rangle_{PABC}]$$

where $S(p, q, r)$ is the Svetlichny function modified with the introduction of bias, given by

$$S(p, q, r) = pqrA_0 \otimes B_0 \otimes C_0 + pq(1 - r)A_0 \otimes B_0 \otimes C_1$$

$$+ p(1 - q)rA_0 \otimes B_1 \otimes C_0 + (1 - p)qrA_1 \otimes B_0 \otimes C_0$$

$$− p(1 - q)(1 - r)A_0 \otimes B_1 \otimes C_1$$

$$− (1 - p)(1 - q)rA_1 \otimes B_0 \otimes C_1$$

$$− (1 - p)(1 - q)(1 - r)A_1 \otimes B_1 \otimes C_1.$$  

(17)

To find the maximum probability of winning (which is the upper bound of fine-grained uncertainty relation as presented in Eq.$|14|$), we need to maximize $\langle S(p, q, r) \rangle_{PABC}$.

The case when all the three parties are quantum-correlated, has been handled only numerically in this context [10]. We will however, perform this maximization analytically using the scheme of bipartition modelling [16]. This method is based on the fact that maximal quantum violation for the tripartite Svetlichny inequality has been shown [16] even when the system does not feature genuine tripartite non-locality, i.e., only two of the three parties are correlated in a nonlocal way. Since this method of bipartition modelling will be useful for our subsequent analysis, we first recount here some of useful results obtained using it [16]. The Svetlichny function $S(p, q, r)$ can be rearranged as

$$S(p, q, r) = r[CHSH(p, q)] \otimes C_0 + (1 - r)[CHSH′(p, q)] \otimes C_1$$

(18)

where,

$$CHSH(p, q) = [pqA_0 \otimes B_0 + p(1 - q)A_0 \otimes B_1$$

$$+ (1 - p)qA_1 \otimes B_0 - (1 - p)(1 - q)A_1 \otimes B_1]$$

$$CHSH′(p, q) = [pqA_0 \otimes B_0 - p(1 - q)A_0 \otimes B_1$$

$$− (1 - p)qA_1 \otimes B_0 - (1 - p)(1 - q)A_1 \otimes B_1]$$

(19)

Here $CHSH(p, q)$ is the traditional form of CHSH-polynomial and $CHSH′(p, q)$ is an equivalent form when the mapping, $B_0 \rightarrow B_1$, $B_1 \rightarrow -B_0$, $q \rightarrow (1 - q)$ is applied.

Now, according to the form of $|18|$ let us temporarily change our point of view towards the game as following. The version of bipartite CHSH-game played by Alice and Bob is determined by Charlie’s input setting. Assume for a moment, that when Charlie’s input is $C_0$, Alice and Bob play the standard biased CHSH-game and when Charlie’s input is $C_1$, they play $CHSH′$. Alice and Bob are together (and separated from Charlie) and being unaware of Charlie’s measurements, produce any bipartite non-local probability distribution. Hence Alice and Bob are effectively playing the average game $r[⟨CHSH(p, q)⟩] + (1 - r)[⟨CHSH′(p, q)⟩]$. If Alice and Bob stay separated and any of them be with Charlie and knows about Charlie’s measurements, they will not be able to produce results better than the local bound. For the region $p, q, r \geq \frac{1}{2}$, the classical maximum is calculated to be,

$$⟨S⟩_{max} = 1 - 2(1 - p)(1 - q)$$

(20)

giving

$$P_{game}(S, T, U, \rho_{ABC})|_{maximunm} = 1 - (1 - p)(1 - q)$$

(21)

which reduces to the value $\frac{3}{4}$ for the unbiased game ($r$ is averaged off due to both the Bell functions possessing the same classical maximum).

In order to treat the quantum optimization we consider that the three parties share a three-qubit Greenberger-Horne-Zeilinger (GHZ) state $|\psi⟩ = \frac{1}{\sqrt{2}}|000⟩ + |111⟩$ (for the unbiased case, the maximum violation of the Svetlichny function occurs for the GHZ state [20]). In this process, generally, Charlie needs to choose two measurements in a way that he prepares two qubit entangled states (for Alice and Bob) which will maximize their corresponding CHSH-functions simultaneously. The purpose of this strategic choice of measurements by Charlie is to maximize the Svetlichny function and hence to improve the score of the non-local game to its best. Consider the choice being, $C_0 = σ_x$ and $C_1 = -σ_y$ which prepare the states $|φ_+⟩ = \frac{1}{\sqrt{2}}(|00⟩ ± |11⟩)$ and $|φ_−⟩ = \frac{1}{\sqrt{2}}(|00⟩ ± i|11⟩)$ respectively, for Alice and Bob. Note that

$$(I \otimes U_B)\rho_±(I \otimes U_B^†) = \rho_±$$

(22)
where \( \rho_{\pm} = |\phi_{\pm}\rangle\langle\phi_{\pm}| \) and \( \rho_{\pm} = |\phi_{\pm}\rangle\langle\phi_{\pm}| \) and \( U_B \) is a unitary rotation on the Bob’s qubit, given by
\[
U_B = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}
\] (23)
and consequently, the equivalence of optimizations of \( CHSH(p, q) \) and \( CHSH'(p, q) \) is realized as,
\[
|\langle \phi_{\pm}| CHSH(p, q) |\phi_{\pm}\rangle |
= |\langle \hat{\phi}_{\pm}|(I \otimes U_B^\dagger)CHSH'(p, q)(I \otimes U_B)|\hat{\phi}_{\pm}\rangle | (24)
\]
or simply,
\[
\langle CHSH(p, q)\rangle_{\rho_{\pm}} = \langle CHSH'(p, q)\rangle_{\rho_{\pm}} (25)
\]
The above equation is true provided the aforesaid mapping between the operators \( B_0 \) and \( B_1 \) and their probability distribution \( q \) (i.e., the mapping \( B_0 \rightarrow B_1, B_1 \rightarrow -B_0, q \rightarrow (1-q) \)) is considered. So, as we focus on achieving the best score for the present no-local game, we may now think of the situation (instead of Alice and Bob playing with two kinds of CHSH games) as only the standard CHSH-game being played that is averaged over the scenarios when Bob rotates unitarily his qubit before measurement and when he does not. The unitary rotation preserves the nonlocal property of the state causing no discrepancy. In the region \( p, q, r \geq \frac{1}{2} \), the maximum value of \( \langle S(p, q, r) \rangle \) is calculated (using a procedure for maximizing \( CHSH(p, q) \) similar to the bipartite case) to be,
\[
\langle S(p, q, r)\rangle_{\rho_{\pm}} = 1 - 2(1 - p)(1 - q) (26)
\]
for the region \( 1 \geq p \geq (2q)^{-1} \geq \frac{1}{2} \) which is the same as the classically achieved upper bound. Here \( \langle S \rangle \) is not a function of \( r \) because the nonlocal strength of Alice’s and Bob’s systems are identical for the two different measurements of Charlie. For the region \( 1 \geq (2q)^{-1} > p \geq \frac{1}{2} \), one obtains
\[
\langle S(p, q, r)\rangle_{\rho_{\pm}} = \sqrt{2q^2 + (1-q)^2} \sqrt{p^2 + (1-p)^2} (27)
\]
The bound \( (27) \) is greater than the bound \( (26) \), and hence, the quantum correlation dominates here. The expression for maximum winning probability in this case is given by
\[
p_{\text{same}}(S, T, U, \rho_{ABC})_{\text{maximum}} = \frac{1}{2} [1 + \sqrt{2} \sqrt{q^2 + (1-q)^2} \sqrt{p^2 + (1-p)^2}] (28)
\]
For every \( r \neq 0, 1 \) there is the same patch in the \( p - q \) space which separates the classical and quantum correlations in terms of their degree of non-locality.

It may be noted that the results for the tripartite system is quantitatively somewhat different from the numerical calculation provided by Lawson et.al. \[10\]. According to the latter if all the biasing parameters \( (p, q, r) \) are made equal, the no-quantum-advantage region is above \( p \approx 0.8406 \) which is slightly different from \( p \approx 0.7071 \) for our case. This deviation reflects the fact that the use of the bipartition model \[10\] does not, in general, capture all types of tripartite nonlocal correlations. Finally, for no-signaling theory the upper bound turns out to be 1, as expected. Note also, that in the other regions when all \( p, q \) and \( r \) or one or two of them are less than \( \frac{1}{2} \), the treatments are similar, as in the bipartite case.

**CONCLUSIONS**

In this work we have employed the fine-grained uncertainty relation \[9\] to distinguish between classical, quantum and super-quantum correlations based on their strength of nonlocality, in the context of biased games \[10\] involving two or three parties. Discrimination among the underlying theories with different degrees of nonlocality is possible for a particular range of the biasing parameters. This range of bias parameters turns out to be the region for which quantum correlations offer the advantage of winning the said nonlocal game over classical correlations. For the tripartite game in case of no bias, the Svetlichny inequality is able to discriminate \[13\] among classical, quantum and super-quantum correlations. But in the presence of bias, using a bipartition model \[10\] we observe here that there is a zone specified by the biasing parameters where even the Svetlichny inequality cannot perform this discrimination. The extent of non-locality that can be captured by the fine-grained uncertainty principle thus turns out to be regulated by the bias parameters. Our approach, in spite of featuring a narrower range of the biasing parameters providing quantum advantage, serves the purpose of developing an analytical approach to explore the connection between biased nonlocal retrieval games and the upper bound of fine-grained uncertainty capturing the nonlocal strengths of various correlations. Analytical generalizations to multiparty nonlocal games may indeed be feasible using this approach.

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