Verification of Time-Aware Business Processes using Constrained Horn Clauses

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Abstract. We present a method for verifying properties of time-aware business processes, that is, business process where time constraints on the activities are explicitly taken into account. Business processes are specified using an extension of the Business Process Modeling Notation (BPMN) and durations are defined by constraints over integer numbers. The definition of the operational semantics is given by a set $\text{OpSem}$ of constrained Horn clauses (CHCs). Our verification method consists of two steps. (Step 1) We specialize $\text{OpSem}$ with respect to a given business process and a given temporal property to be verified, whereby getting a set of CHCs whose satisfiability is equivalent to the validity of the given property. (Step 2) We use state-of-the-art solvers for CHCs to check the satisfiability of such sets of clauses. We have implemented our verification method using the VeriMAP transformation system, and the Eldarica and Z3 solvers for CHCs.

1 Introduction

A business process, or BP for short, consists of a set of activities, performed in coordination within a single organization, which realize a business goal [28,31]. The Business Process Model and Notation, or BPMN for short, is one of the most popular graphical languages proposed for visualizing business processes [24]. The primary goal of BPMN is to provide a standard notation that can be understood by all business stakeholders, which include the business analysts who define and modify the processes, the technical developers in charge of their implementation, and the business managers who monitor and manage them.

A BPMN model is a procedural, semi-formal description of the order of execution of the activities of a given process and how these activities must coordinate, abstracting away from many other aspects of the process itself, such as the manipulation of data and the duration of the activities. However, for many analysis tasks these aspects are very significant in practice. In particular, the duration of the activities is critical, when we want to reason about time constraints (e.g., deadlines) that should be satisfied by process executions.
Various approaches for BP modeling with duration and time constraints have been proposed in the literature (see [6] for a recent survey). Some of these approaches define the semantics of time-aware BPMN models by means of formalisms such as time Petri nets [22], timed automata [29], and process algebras [32]. Properties of these models can then be verified by using very effective reasoning tools available for those formalisms [11, 13, 20].

However, the above mentioned formalisms and tools may not be adequate if we want to complement time-based reasoning with general purpose logical reasoning, which is often needed if we take into account more complex aspects of knowledge manipulation activities relative to business processes. For instance, some verification approaches make use of ontology-based reasoning about the business domain where processes are executed [27, 30], while others combine reasoning on the finite-state process behavior with reasoning on the manipulation of data objects of an infinite type, such as databases or integers [8, 2, 26].

Thus, in view of an integration of various reasoning tasks needed to analyze business processes from different perspectives, we propose a logic-based approach to modeling and verifying time-aware business processes.

The main contributions of the paper are the following. We present a logic-based language to specify time-aware BPMN models, where time and duration of activities are explicitly represented. Then we define an operational semantics of time-aware BPMN models by means of deduction rules that allow us to infer the time intervals when a particular activity is in execution or ‘enacting’ (using the BPMN terminology). Next, in order to prove properties of time-aware BPMN models, we follow a transformational approach similar to the one proposed in [10] for the verification of imperative programs. First, we consider an encoding $OpSem$ of the operational semantics into Constrained Horn Clauses (CHCs) [5] (or, equivalently, Constraint Logic Programs [18]). Then, we specialize $OpSem$ to the time-aware BPMN model under consideration and temporal property of interest, thereby deriving a new set of CHCs whose satisfiability is equivalent to (and thus implies) the validity of the property. Finally, we use state-of-the-art solvers for CHCs (in particular, ELDarica [17] and Z3 [11]) to check the satisfiability of such set of clauses.

Since the CHCs are generated in an automatic way by the CHC specializer from the formal definition of the semantics of the BPMN models, and the CHC solvers are general purpose reasoning systems, our approach is, to a large extent, parametric with respect to other extensions of BP models one may want to consider in the future. Moreover, recent advances in the field of CHC solving can be exploited to get very effective reasoning tools for verifying properties of business processes.

The paper is structured as follows. In Section 2 we recall some basic notions about Constrained Horn Clauses over integer numbers and BPMN. In Section 3 we present our logic-based language for specifying time-aware BPMN models and the operational semantics of the language. In Section 4 we present the CHC encoding of the semantics and the transformation techniques for specializing $OpSem$ with respect to a given time-aware BPMN model and a given property.
In Section 5 we report on the implementation of the verification technique we have made using the VeriMAP transformation and verification system [9], and the CHC solvers Eldarica and Z3. Finally, in Section 6 we discuss related work in the field of BP verification.

2 Preliminaries

In the next two subsections we recall some basic notions concerning constrained Horn clauses and the Business Process Model and Notation. We consider discrete time and we model the time line as the set of integers. However, our approach applies directly to dense or continuous time.

2.1 Constrained Horn Clauses over Integers

First we need the following notions about constraints, constrained Horn clauses, and constraint logic programming. For related notions not familiar to the reader, we refer to [18,21].

Constraints are defined as follows. Let \( \text{RelOp} \) be the set of predicate symbols \( \{ =, \neq, \leq, \geq, <, > \} \). If \( p_1 \) and \( p_2 \) are linear polynomials with integer variables and coefficients, then \( p_1Rp_2 \), with \( R \in \text{RelOp} \), is an atomic constraint. A constraint \( c \) is a (possibly empty) conjunction of atomic constraints. An atom is a formula of the form \( p(t_1, \ldots, t_m) \), where \( p \) is a predicate symbol not in \( \text{RelOp} \) and \( t_1, \ldots, t_m \) are terms constructed as usual from variables, constants, and function symbols. In particular, we assume that there are the two predicate symbols \( \text{true} \) and \( \text{false} \) of arity 0, and a predicate symbol \( \text{eq} \) denoting identity. A constrained Horn clause (or simply, a clause) is an implication of the form \( A \leftarrow c, G \), where the conclusion (or head) \( A \) is an atom, and the premise (or body) \( c \) is a constraint, and \( G \) is a (possibly empty) conjunction of atoms. The empty conjunction is identified with \( \text{true} \). A constrained goal (or simply, a goal) is a clause of the form \( \text{false} \leftarrow c, G \).

An \( \mathbb{Z} \)-interpretation of a set \( \mathcal{P} \) of CHCs is defined to be an interpretation \( I \) of \( \mathcal{P} \) such that: (i) \( \text{true} \) holds in \( I \), (ii) \( \text{false} \) does not hold in \( I \), (iii) \( I \) is the usual interpretation over the set of the integer numbers \( \mathbb{Z} \) for the constraints, and (iv) \( I \) is the Herbrand interpretation for predicate and function symbols not in \( \text{RelOp} \cup \{ \text{true}, \text{false}, +, \times \} \) (in particular, \( \text{eq}(x, y) \) holds if and only if \( x \) and \( y \) are identical terms in the Herbrand universe). An \( \mathbb{Z} \)-model of \( \mathcal{P} \) is an \( \mathbb{Z} \)-interpretation \( M \) such that every clause of \( \mathcal{P} \) holds in \( M \). A set of CHCs is satisfiable if it has an \( \mathbb{Z} \)-model. (Note that a set of CHCs may be unsatisfiable if it contains goals.) Every satisfiable set \( \mathcal{P} \) of CHCs has a unique least \( \mathbb{Z} \)-model, denoted \( M(\mathcal{P}) \) [13].

2.2 Business Processes Model and Notation

A BPMN model is defined through a diagram drawn by using graphical constructs representing flow objects and sequence flows (sequence flows will also be
called *flows* for short). That diagram can be extended, if so desired, to include information about data flow, resource allocation (i.e., how the work to be done is assigned to the participants in the process), and exception handling (i.e., how erroneous behaviors should be handled).

For reasons of simplicity, in this paper we will only consider a subset of the flow objects and sequence flows that can occur in a BPMN model, but our approach can easily be extended to full BPMN. The flow objects we will consider are of three kinds only: either (i) *tasks*, denoted by rounded rectangles, or (ii) *events*, denoted by circles, or (iii) *gateways*, denoted by diamonds. Tasks represent atomic units of work performed within the process. Events denote something that ‘happens’ during the enactment of a business process. We will only consider *start events* and *end events*, which start and end the process enactment, respectively. Gateways model the branching and merging of activities. There are several types of gateways in BPMN, each of which can be a *branch* gateway if it has multiple outgoing flows and a single incoming flow, or a *merge* gateway if it has multiple incoming flows and a single outgoing flow. We will consider the following gateways: (i) the *parallel branch* gateway that concurrently activates all the outgoing flows, (ii) the *parallel merge* gateway that activates the outgoing flow when all the incoming flows have been activated (that is, the parallel merge synchronizes the incoming flows) (iii) the *exclusive branch* gateway that (non-deterministically) activates exactly one out of many outgoing flows, and (iv) the *exclusive merge* gateway that activates the single outgoing flow upon activation of one of the incoming flows. The diamonds representing parallel and exclusive gateways are labeled by ‘+’ and ‘×’, respectively. A sequence flow, denoted by an arrow, links two flow objects and denotes a control flow relation, i.e., it states that the control flow can pass from the source to the target object. If there is a sequence flow from $x$ to $y$, then $x$ is a *predecessor* of $y$ and $y$ is a *successor* of $x$. A *path* in a BPMN model is a sequence of flow objects such that every pair of consecutive objects is connected by a sequence flow.

We assume that BPMN models are *well-formed*, that is, they satisfy the following properties: (1) every process contains a unique start event and a unique end event, (2) every flow object occurs on a path from the start event to the end event, (3) start events have exactly one successor and no predecessor, (4) end events have exactly one predecessor and no successor, (5) branch gateways have exactly one predecessor and at least one successor, while merge gateways have at least one predecessor and exactly one successor, (6) tasks have exactly one predecessor and one successor, and (7) on every cyclic path there is at least one occurrence of a task (i.e., no cycles through gateways only are allowed).

In Figure 1 we show the BPMN model of a purchase order process $PO$, describing a typical interaction pattern between an e-commerce vendor and a customer.

At the beginning of the purchase order process the customer adds one or more items to the shopping cart. Subsequently, the customer pays for the items then the vendor (i) issues the invoice then sends it to the customer, and (ii) prepares the order then ships it using either a standard or an express delivery method.
Fig. 1. The BPMN model of the purchase order process \(PO\). The italicized labels are not part of the model and are only used to denote the corresponding flow objects.

The process terminates when the invoice has been sent and the order has been delivered.

3 Specification and Semantics of Business Processes

In this section we introduce the notion of a Business Process Specification (BPS), which is a way of formally representing a business process by means of CHCs. Then we define the operational semantics of a BPS.

3.1 Specifying Business Processes through CHCs

A BPS \(\mathcal{B}\) contains a set of ground facts of the form \(p(c_1, \ldots, c_n)\), where \(c_1, \ldots, c_n\) are constants denoting flow objects (that is, either tasks, or events, or gateways) and \(p\) is a predicate symbol. We will make use of the following predicates:
- \(\text{flow.object}(x)\): \(x\) is either a task, or an event, or a gateway;
- \(\text{task}(x)\): \(x\) is a task;
- \(\text{start}(e)\) and \(\text{end}(e)\): \(e\) is a start event and an end event, respectively;
- \(\text{seq}(x, y)\): there is a sequence flow from \(x\) to \(y\);
- \(\text{par.branch}(g)\) and \(\text{par.merge}(g)\): \(g\) is a parallel branch and a parallel merge gateway, respectively;
- \(\text{exc.branch}(g)\) and \(\text{exc.merge}(g)\): \(g\) is an exclusive branch and exclusive merge gateway, respectively;
- \(\text{duration}(x, d)\): the enactment of the flow object \(x\) takes \(d\) units of time to be completed.

We assume that: (i) for every task \(x\) there exists in \(\mathcal{B}\) a single clause of the form \(\text{duration}(x, d) \leftarrow d_{\min} \leq d \leq d_{\max}\), where \(d_{\min}\) and \(d_{\max}\) are positive integer constants representing the minimal and the maximal time duration of \(x\), respectively, and (ii) for every event and gateway \(x\) there exists in \(\mathcal{B}\) a single clause of the form \(\text{duration}(x, d) \leftarrow d = 0\) (that is, events and gateways are instantaneous).
The CHC specification of the BPMN process $PO$ of Figure 1 is shown in Table 3.1. In our $PO$ example we will use the standard Prolog syntax for clauses.

\[
\begin{align*}
\text{start}(\text{start}). & \quad \text{end}(\text{end}). \\
\text{exc\_merge}(\text{g1}). & \quad \text{exc\_branch}(\text{g2}). \quad \text{exc\_branch}(\text{g4}). \quad \text{exc\_merge}(\text{g5}). \\
\text{par\_branch}(\text{g3}). & \quad \text{par\_merge}(\text{g6}). \\
\text{seq}(\text{start}, \text{g1}). & \quad \text{seq}(\text{g2}, \text{g1}). \quad \text{seq}(\text{g3}, \text{i}). \quad \text{seq}(\text{g3}, \text{o}). \quad \text{seq}(\text{g4}, \text{sd}). \quad \text{seq}(\text{ed}, \text{g5}). \\
\text{seq}(\text{g1}, \text{a}). & \quad \text{seq}(\text{g2}, \text{p}). \quad \text{seq}(\text{i}, \text{s}). \quad \text{seq}(\text{o}, \text{g4}). \quad \text{seq}(\text{g4}, \text{ed}). \quad \text{seq}(\text{g5}, \text{g6}). \\
\text{seq}(\text{a}, \text{g2}). & \quad \text{seq}(\text{p}, \text{g3}). \quad \text{seq}(\text{s}, \text{g6}). \quad \text{seq}(\text{sd}, \text{g5}). \quad \text{seq}(\text{g6}, \text{end}). \\
\text{task}(\text{a}). & \quad \text{duration}(\text{a}, D):- D\geq 1, D\leq 6. \quad \% \text{add item} \\
\text{task}(\text{p}). & \quad \text{duration}(\text{p}, D):- D\geq 1, D\leq 2. \quad \% \text{pay} \\
\text{task}(\text{i}). & \quad \text{duration}(\text{i}, D):- D\geq 1, D\leq 2. \quad \% \text{issue invoice} \\
\text{task}(\text{o}). & \quad \text{duration}(\text{o}, D):- D\geq 1, D\leq 3. \quad \% \text{send invoice} \\
\text{task}(\text{o}). & \quad \text{duration}(\text{o}, D):- D\geq 3, D\leq 5. \quad \% \text{prepare order} \\
\text{task}(\text{sd}). & \quad \text{duration}(\text{sd}, D):- D\geq 2, D\leq 4. \quad \% \text{deliver order (standard)} \\
\text{task}(\text{ed}). & \quad \text{duration}(\text{ed}, D):- D\geq 1, D\leq 3. \quad \% \text{deliver order (express)} \\
& \quad \text{duration}(X, D):- \text{not\_task}(X), D=0. \quad \% \text{gateways and events}
\end{align*}
\]

Table 3.1. The set of facts encoding the schema of the purchase order process $PO$ of Figure 1

Note that a BPS is always satisfiable (note that, in particular, it contains no goals), and hence it has a least $Z$-model.

Our formalization also includes a set $W$ of clauses that represent the meta-model of the BPS, defining: (i) disjointness relationships among sets of elements, for instance, $false \leftarrow \text{task}(X), \text{par\_branch}(X)$, (ii) properties of the BPS corresponding to Conditions 1–7 of Section 2.2, which define the well-formedness of a BPMN model. These properties are expressed as CHCs as follows:

\[
\begin{align*}
(1) \quad & \text{eq}(X, Y) \leftarrow \text{start}(X), \text{start}(Y) \quad \text{and} \quad \text{eq}(X, Y) \leftarrow \text{end}(X), \text{end}(Y); \\
(2) \quad & \text{seq}^*(S, X) \leftarrow \text{start}(S), \text{flow\_object}(X) \quad \text{and} \quad \text{seq}^*(X, E) \leftarrow \text{flow\_object}(X), \text{end}(E) \quad \text{where seq}^* \text{ is the reflexive, transitive closure of seq;}
\end{align*}
\]

\[
\begin{align*}
(3) \quad & \text{eq}(Y, Z) \leftarrow \text{start}(S), \text{seq}(S, Y), \text{seq}(S, Z) \quad \text{and} \quad false \leftarrow \text{start}(S), \text{seq}(Y, S); \\
(4) \quad & \text{eq}(Y, Z) \leftarrow \text{end}(E), \text{seq}(Y, E), \text{seq}(Z, E) \quad \text{and} \quad false \leftarrow \text{end}(E), \text{seq}(E, Y); \\
(5) \quad & \text{eq}(Y, Z) \leftarrow \text{par\_branch}(X), \text{seq}(Y, X), \text{seq}(Z, X) \quad \text{and} \quad \text{eq}(Y, Z) \leftarrow \text{par\_merge}(X), \text{seq}(X, Y), \text{seq}(X, Z) \\
& \quad \text{and, similarly, for the exc\_branch and exc\_merge gateways;}
\end{align*}
\]

\[
\begin{align*}
(6) \quad & \text{eq}(Y, Z) \leftarrow \text{task}(X), \text{seq}(X, Y), \text{seq}(X, Z) \quad \text{and} \quad \text{eq}(Y, Z) \leftarrow \text{task}(X), \text{seq}(Y, X), \text{seq}(Z, X); \\
(7) \quad & \text{false} \leftarrow \text{gateway\_path}(X, X) \quad \text{where gateway\_path}(X, Y) \text{ is a predicate that holds iff there is a path from } X \text{ to } Y \text{ with gateways only.}
\end{align*}
\]

Note that the existence of at least one predecessor and at least one successor for any task or gateway (required by Conditions 5 and 6 of Section 2.2) is enforced by the clauses at Point 2.

A BPS $B$ is well-formed if all clauses in $W$ hold in the least $Z$-model of $B$. 

3.2 Operational Semantics

We start off by introducing the notion of a *state*, which is represented by a set of properties, called *fluents*, that hold at a given time point. A state \( s \in \text{States} \) is a pair \((F, t)\), where \( F \) is a set of fluents and \( t \) is a time point in \( \mathbb{Z} \).

A fluent is a term of one of the following forms: (i) \( \text{begins}(x) \), which represents the beginning of the enactment (or execution) of the flow object \( x \), (ii) \( \text{completes}(x) \), which represents that \( x \) has completed its execution, and (iii) \( \text{enables}(x, y) \), which represents that the flow object \( x \) has completed its execution and it enables the execution of its successor \( y \), and (iv) \( \text{enacting}(x, r) \), which represents that the enactment of \( x \) requires \( r \) units of time to completion (for this reason \( r \) is also called the *residual time of* \( x \)). Thus, \( \text{begins}(x) \) is equivalent to \( \text{enacting}(x, r) \), where \( r \) is the duration of \( x \), and \( \text{completes}(e) \) is equivalent to \( \text{enacting}(x, 0) \). (This redundancy of representation allows us to write simpler rules for the operational semantics below.)

The operational semantics is defined by a binary transition relation \( \rightarrow \) which is a subset of \( \text{States} \times \text{States} \). The *initial state*, denoted \( \text{init} \), is the pair \( \langle \{ \text{begins}(\text{start}) \}, 0 \rangle \). In the rules below, which define \( \rightarrow \), we also use the following predicates, besides the ones introduced in Section 3.1: (i) \( \text{not.par_branch}(x) \), which holds if \( x \) is not a parallel branch, and (ii) \( \text{not.par_merge}(x) \), which holds if \( x \) is not a parallel merge.

\[\begin{align*}
(S_1) & \quad \text{begins}(x) \in F \quad \text{duration}(x, d) \\
& \quad \langle F, t \rangle \rightarrow \langle \langle F \setminus \{\text{begins}(x)\} \rangle \cup \{\text{enacting}(x, d)\}, t \rangle \\
(S_2) & \quad \text{completes}(x) \in F \quad \text{par_branch}(x) \\
& \quad \langle F, t \rangle \rightarrow \langle \langle F \setminus \{\text{completes}(x)\} \rangle \cup \{\text{enables}(x, s) \mid \text{seq}(x, s)\}, t \rangle \\
(S_3) & \quad \text{completes}(x) \in F \quad \text{not.par_branch}(x) \quad \text{seq}(x, s) \\
& \quad \langle F, t \rangle \rightarrow \langle \langle F \setminus \{\text{completes}(x)\} \rangle \cup \{\text{enables}(x, s)\}, t \rangle \\
(S_4) & \quad \forall p \; \text{seq}(p, x) \rightarrow \text{enables}(p, x) \in F \quad \text{par_merge}(x) \\
& \quad \langle F, t \rangle \rightarrow \langle \langle F \setminus \{\text{enables}(p, x) \mid \text{enables}(p, x) \in F\} \rangle \cup \{\text{begins}\}, t \rangle \\
(S_5) & \quad \text{enables}(p, x) \in F \quad \text{not.par_merge}(x) \\
& \quad \langle F, t \rangle \rightarrow \langle \langle F \setminus \{\text{enables}(p, x)\} \rangle \cup \{\text{begins}\}, t \rangle \\
(S_6) & \quad \text{enacting}(x, 0) \in F \\
& \quad \langle F, t \rangle \rightarrow \langle \langle F \setminus \{\text{enacting}(x, 0)\} \rangle \cup \{\text{completes}\}, t \rangle \\
(S_7) & \quad \text{no_other_premises}(F) \quad \exists x \exists r \; \text{enacting}(x, r) \in F \quad m > 0 \\
& \quad \langle F, t \rangle \rightarrow \langle \langle F \setminus \{\text{enacting}(x, r) \mid \text{enacting}(x, r) \in F\} \rangle \\
& \quad \cup \{\text{enacting}(x, r-m) \mid \text{enacting}(x, r) \in F\}, t+m \rangle
\end{align*}\]
where: (i) \(\text{no\_other\_premises}(F)\) holds iff none of the rules \(S_1\)–\(S_6\) has its premise true, and (ii) \(m = \min\{r \mid \text{enacting}(x, r) \in F\}\).

Let us first observe that \(S_7\) is the only rule that formalizes the flow of time, as it infers transitions of the form \(\langle F, t \rangle \rightarrow \langle F', t+m \rangle\), with \(m > 0\). In contrast, rules \(S_1\)–\(S_6\) infer instantaneous state transitions, that is, transitions of the form \(\langle F, t \rangle \rightarrow \langle F', t \rangle\).

Rules \(S_1\)–\(S_7\) have the following meaning.

\((S_1)\) If the execution of a flow element \(x\) begins at time \(t\), then, at the same time \(t\), \(x\) is enacting and its residual time is the duration \(d\) of \(x\);

\((S_2)\) If the execution of the parallel branch \(x\) completes at time \(t\), then \(x\) enables all its successors at time \(t\);

\((S_3)\) If the execution of \(x\) completes at time \(t\) and \(x\) is not a parallel branch, then \(x\) enables precisely one of its successors at time \(t\) (in particular, this case occurs when \(x\) is a task);

\((S_4)\) If all the predecessors of \(x\) have enabled the parallel merge \(x\) at time \(t\), then the execution of \(x\) begins at time \(t\);

\((S_5)\) If at least one predecessor \(p\) of \(x\) enables \(x\) at time \(t\) and \(x\) is not a parallel merge, then the execution of \(x\) begins at time \(t\) (in particular, this case occurs when \(x\) is a task);

\((S_6)\) If a flow object \(x\) is enacting at time \(t\) with residual time \(0\), then the execution of \(x\) completes at time \(t\);

\((S_7)\) Suppose that: (i) none of rules \(S_1\)–\(S_6\) is applicable to infer the successor state of \(\langle F, t \rangle\), (ii) at time \(t\) at least one task is enacting with positive residual time (note that flow objects different from tasks cannot have positive residual time), and (iii) \(m\) is the least among the residual times of all the tasks enacting at time \(t\). Then every task \(x\) that is enacting at time \(t\) with residual time \(r\), is enacting at time \(t+m\) with residual time \(r-m\).

We say that state \(\langle F', t' \rangle\) is reachable from state \(\langle F, t \rangle\), if \(\langle F, t \rangle \rightarrow^* \langle F', t' \rangle\), where \(\rightarrow^*\) denotes the reflexive, transitive closure of the transition relation \(\rightarrow\).

4 Encoding Time-Dependent Properties of Business Processes into CHCs

In this section we show the CHC interpreter that encodes the operational semantics and the property to be verified. We also present two transformation techniques: (1) a technique for removing the interpreter and deriving a set of clauses that is amenable to automatic satisfiability checking, and (2) a technique for reducing the size of sets of CHC clauses by using a suitable notion of predicate equivalence.

4.1 Encoding the Operational Semantics in CHCs

A state \(\langle F, t \rangle\) of the operational semantics is encoded by a term of the form \(s(F,T)\), where \(F\) is a list encoding the set \(F\) of fluents and \(T\) encodes the time
point $t$ at which the fluents in the set $F$ hold. The transition relation $\rightarrow$ between states and its reflexive, transitive closure $\rightarrow^*$ are encoded by the binary predicates $\text{tr}$ and $\text{reach}$, respectively, whose defining clauses are shown in Table 4.2. In the body of the clauses, the atoms that encode the premises of the rules of the operational semantics have been underlined.

The predicate $\text{member}(X,L)$ selects an element $X$ from the list $L$. The predicate $\text{update}(F,R,A,FU)$ holds iff $FU$ is the list obtained from the list $F$ by removing all the elements of $R$ and adding all the elements of $A$. The predicate $\text{no\_other\_premises}(F)$ holds iff the premise of every rule in $\{S1,\ldots,S6\}$ is false. The predicate $\text{mintime}(\text{Enacts},M)$ holds iff $\text{Enacts}$ is a list of terms of the form $\text{enacting}(X,R)$ and $M$ is the minimum value of $R$ for the elements of $\text{Enacts}$. The predicate $\text{decrease\_residual\_times}(\text{Enacts},M,\text{EnactsU})$ holds iff $\text{EnactsU}$ is the list of terms obtained by replacing every element of $\text{Enacts}$, of the form $\text{enacting}(X,R)$, with the term $\text{enacting}(X,RU)$ where $RU=R-M$. The predicates $\text{sublist}(S,L)$ and $\text{findall}(X,G,L)$ have the usual meaning.

S1. $\text{tr}(s(F,T), s(FU,T)) :- \text{member}(\text{begins}(X), F), \text{duration}(X,D),$ 
$\text{update}(F, [\text{begins}(X)], [\text{enacting}(X,D)], FU).$

S2. $\text{tr}(s(F,T), s(FU,T)) :- \text{member}(\text{completes}(X), F), \text{par\_branch}(X),$ 
$\text{findall}(\text{enables}(X,S), (\text{seq}(X,S)), \text{Enbls}),$ 
$\text{update}(F, [\text{completes}(X)], \text{Enbls}, FU).$

S3. $\text{tr}(s(F,T), s(FU,T)) :- \text{member}(\text{completes}(X), F), \text{not\_par\_branch}(X), \text{seq}(X,S),$ 
$\text{update}(F, [\text{completes}(X)], [\text{enables}(X,S)], FU).$

S4. $\text{tr}(s(F,T), s(FU,T)) :- \text{member}(\text{enables}(P,X), F), \text{par\_merge}(X),$ 
$\text{findall}(\text{enables}(P,X), (\text{seq}(P,X)), \text{Enbls}),$ 
$\text{sublist}(\text{Enbls}, F), \text{update}(F, \text{Enbls}, [\text{begins}(X)], FU).$

S5. $\text{tr}(s(F,T), s(FU,T)) :- \text{member}(\text{enacting}(X,R), F), R=0,$ 
$\text{update}(F, [\text{enacting}(X,R)], [\text{completes}(X)], FU).$

S6. $\text{tr}(s(F,T), s(FU,T)) :- \text{no\_other\_premises}(F), \text{member}(\text{enacting}(X,R), F),$ 
$\text{findall}(Y, (Y=\text{enacting}(X,R), \text{member}(Y,F)), \text{Enacts}),$ 
$\text{mintime}(\text{Enacts}, M), M>0,$ 
$\text{decrease\_residual\_times}(\text{Enacts}, M, \text{EnactsU}),$ 
$\text{update}(F, \text{Enacts}, \text{EnactsU}, FU), \text{TU}=T+M.$

R1. $\text{reach}(S,S).$ R2. $\text{reach}(S,S2) :- \text{tr}(S,S1), \text{reach}(S1,S2).$

Table 4.2. The CHC interpreter for the operational semantics of time-aware business processes.

4.2 Encoding Time-Dependent Properties

By using the $\text{reach}$ predicate and integer constraints, we can specify many interesting time-dependent properties. In particular, we can specify safety properties (stating that ‘no unsafe state can be reached’), schedulability properties (stating that a process will be completed within a given deadline), response properties (stating that, whenever a task is executed, another task will be executed within a given time), and many other quantitative temporal properties.
In order to see how we encode time-dependent properties of business processes, we consider a property of the PO process stating that, whenever the customer pays and the process completes, then completion occurs within 9 time units from payment. By using the reachability relation, this property can be written as follows:

\[ prop: \text{if } \text{init} \rightarrow \ast \langle \{\text{completes}(p)\}, t_p \rangle \rightarrow \ast \langle \{\text{completes}(e)\}, t_e \rangle, \text{ then } t_e \leq t_p + 9. \]

The reader can check that \( prop \) holds for the PO process because, in the worst case, the time needed for preparing and delivering the order is actually 9 time units and this time is greater than the time needed for issuing and sending the invoice, which is 5 time units. The property \( prop \) is encoded by the following goal:

\[
\text{NP. false :- } T_s=0, T_p>T_s, T_e>T_p+9, \\
\text{reach}(s([\text{begins(start)},T_s], s([\text{completes(p)},T_p]), \\
\text{reach}(s([\text{completes(p)},T_p], s([\text{completes(e)},T_e])).}
\]

The clauses S1-S7,R1,R2,NP, together with the clauses encoding the PO process, will be collectively referred to as the interpreter \( I \). We have that the property \( prop \) is valid for the PO process iff the set \( I \) of CHCs is satisfiable.

Despite several tools have been developed for checking the satisfiability of constrained Horn clauses, none of them can effectively be leveraged in our example. Constraint logic programming systems [18] are focused on proving the unsatisfiability of sets of clauses, rather than their satisfiability, and they fail to terminate for the given set \( I \) because of recursive reach clause (note, in particular that the add_item task can be executed an unbounded number of times). State-of-the-art CHC solvers [11,17] also fail because the predicates in \( I \) are defined over lists and structured terms (not just integers) and they depend on the findall predicate, which is not available in those solvers.

In order to be able to effectively use off-the-shelf CHC solvers for checking the validity of time-dependent properties, we apply the so-called removal of the interpreter transformation [10,25], a program specialization strategy based on unfold/fold transformation rules, which takes the program \( I \) as input and produces as output a program \( I_{sp} \) that is equivalent to \( I \) with respect to satisfiability. Indeed, by the correctness of the unfold/fold transformation rules [12], we have that \( I \) is satisfiable iff \( I_{sp} \) is satisfiable.

A notable effect of applying the removal of the interpreter is that the program \( I_{sp} \) contains no occurrences of the predicates and terms used for encoding the operational semantics and the PO process. Indeed, the clauses of \( I_{sp} \) will be of the form \( A \leftarrow c, B \), where the arguments of the atoms are variables and \( c \) is a constraint. For instance, in the PO example, the goal expressing the property \( prop \) is transformed into the goal:

\[
\text{false :- } A=0, B=2, C=6, D=5, E=0, F-E=9, B=1, C=1, D=3, \\
\text{new1}(C,A,E), \text{new2}(B,D,E,F).
\]

The new predicates \text{new1} and \text{new2} have been introduced by the definition rule, and the extra constraints have been derived by the unfolding rule. We refer to [10] for the details of the transformation. The whole set of clauses derived
by the removal of the interpreter is listed in Appendix A.1. The satisfiability of
this set of clauses can be proved in a fully automatic way by using either the
ELDARICA or the Z3 solver, as it will be demonstrated in Section 5.

4.3 Predicate Equivalence

Now we introduce a transformation that allows us to reduce the size of a set of
CHC clauses when suitable equivalences between predicates hold. Since predicate
equivalence is undecidable in general, we consider a decidable notion of predicate
equivalence based on predicate names and constraint equivalence.

We assume, without loss of generality, that all clauses are in pure form, that
is, of the form \( p(X) \leftarrow B \), where \( X \) is a tuple of distinct variables. Let \( P \) be a set
of CHCs. By \( Pred(P) \) we denote the set of predicate symbols occurring in \( P \). A
predicate renaming for \( P \) is a, possibly not injective, mapping \( \pi: Pred(P) \rightarrow Q \),
where \( Q \) is a set of predicate symbols. Given a set \( S \) of formulas with predicates
in \( Pred(P) \), \( \pi(S) \) is a new set of formulas obtained by replacing, for all predicates
\( p \in Pred(P) \), every occurrence of \( p \) in \( S \) by \( \pi(p) \).

For every \( k \)-ary predicate \( p \in Pred(P) \) we assume that all clauses for \( p \) have
head \( p(X) \), where \( X \) is a \( k \)-tuple of distinct variables. We define \( Bodies(p(X), P) \)
to be the set \{ \( B \mid p(X) \leftarrow B \) is a clause in \( P \) \}. We write \( Bodies(p(X), P) \equiv
Bodies(q(X), P) \) if there exists a bijection \( \eta: Bodies(p(X), P) \rightarrow Bodies(q(X), P) \)
such that, for every \( B \in Bodies(p(X), P) \), \( \exists Y B \) and \( \exists Z \eta(B) \) are equivalent
modulo constraints, where \( Y \) is the tuple of variables occurring in \( B \) and not in \( X \),
and \( Z \) is the tuple of variables occurring in \( \eta(B) \) and not in \( X \).

**Definition 1 (Predicate Equivalence).** Let \( P \) be a set of clauses in pure
form, and \( E = \{ P_1, \ldots, P_n \} \) be a partition of \( Pred(P) \). For \( i = 1, \ldots, n \), let \( e_i \)
be a predicate symbol in \( P_i \), and \( \pi: Pred(P) \rightarrow \{ e_1, \ldots, e_n \} \) be a predicate renaming
for \( P \) such that, for \( i = 1, \ldots, n \), \( \pi(p) = e_i \) iff \( p \in P_i \).

The partition \( E \) is a cp-equivalence on \( P \) if, for \( i = 1, \ldots, n \), given any two
predicates \( p, q \) in \( P_i \), \( p \) and \( q \) have the same arity \( k \) and, for any \( k \)-tuple \( X \) of
new, distinct variables, \( \pi(Bodies(p(X), P)) \equiv \pi(Bodies(q(X), P)) \).

Note that one can compute the coarsest cp-equivalence on \( P \) by a greatest
fixpoint construction starting from the partition where all predicate symbols
belong to the same equivalence class.

Given a cp-equivalence \( E \) on \( P \) together with the predicate renaming \( \pi \)
considered in Definition 1 we can transform \( P \) into a set \( \pi(P, E) \) of clauses in two
steps: (i) we remove from \( P \) all clauses whose head predicate does not appear in
the range of \( \pi \), and (ii) we apply \( \pi \) to the remaining clauses.

**Theorem 1.** For any cp-equivalence \( E \) on a set \( P \) of clauses, \( P \) is satisfiable
iff \( \pi(P, E) \) is satisfiable.

To see an example of cp-equivalence, let us consider the following subset of
the clauses derived by the removal of the interpreter in the \( PO \) example:
The following partition of the set of predicates occurring in the above clauses is a cp-equivalence:

\[ E = \{ \{ \text{new5}, \text{new4} \}, \{ \text{new7}, \text{new6} \}, \{ \text{new21} \}, \{ \text{new10} \} \} \]

associated with the following predicate renaming:

\[ \pi(\text{new5}) = \pi(\text{new4}) = \text{new4} \]
\[ \pi(\text{new7}) = \pi(\text{new6}) = \text{new6} \]
\[ \pi(\text{new21}) = \text{new21} \]
\[ \pi(\text{new10}) = \text{new10} \]

By the transformation \( \bar{\pi} \), the clauses for \text{new5} are removed and all occurrences of \text{new5} are replaced by \text{new4}. In Appendix A.2, we show the effect of this transformation on the whole set of clauses derived by the removal of the interpreter.

## 5 Automated Verification

We have implemented the transformation strategies presented in Section 4.2 (Removal of the Interpreter) and Section 4.3 (Predicate Equivalence) by using the VeriMAP transformation and verification system [9]. Then, we have used the SMT solvers Eldarica\(^4\) and Z3\(^5\) for checking the satisfiability of the CHCs generated by VeriMAP. The satisfiability check requires the following two steps:

(i) a preliminary Translate step, in which VeriMAP translates the CHCs into the SMT-LIB language, and
(ii) the Verify step, in which an SMT solver is invoked for checking satisfiability.

Now we report on the results obtained by using our prototype implementation on the Purchase Order business process shown in Figure 1. The experiments have been performed on an Intel Core i5-2467M 1.60GHz processor with 4GB of memory under GNU/Linux OS. The removal of the interpreter (see Table 4.2), that is, its specialization with respect to the facts encoding the business process (see Table 3.1) and the temporal property (see clause NP) requires 0.42 seconds and generates a set \( RI \) of 51 clauses. The transformation of the clauses \( RI \) based on predicate equivalence requires 0.02 seconds and generates a set \( PE \) of

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\(^4\) v1.2-rc in client-server mode with options -horn -hsmt -princess

\(^5\) v4.4.2, master branch as of 2016-02-18, with the Duality fixed-point engine. See: [http://research.microsoft.com/en-us/projects/duality/default.aspx](http://research.microsoft.com/en-us/projects/duality/default.aspx)
33 clauses. Running the SMT solvers on the clauses $RI$ requires: (i) 1.28 seconds using $Eldarica$ (0.11 seconds for $Translate$ and 1.17 seconds for $Verify$) and (ii) 1.09 seconds using $Z3$ (0.12 seconds for $Translate$ and 0.97 seconds for $Verify$).

Running the SMT solvers on $PE$ requires: (i) 0.81 seconds using $Eldarica$ (0.11 seconds for $Translate$ and 0.70 seconds for $Verify$) and (ii) 0.68 seconds using $Z3$ (0.11 seconds for $Translate$ and 0.57 seconds for $Verify$). We have that both SMT solvers $Eldarica$ and $Z3$ are able to prove the satisfiability of $RI$ and $PE$.

We may observe that the transformation times are negligible and, in particular, that the transformation based on predicate equivalence, by reducing the sizes of the sets of CHCs clauses, allows solvers to improve their performance. Indeed, for both solvers, the difference between the $Verify$ time taken on $RI$ and the one taken on $PE$ (that is, before and after the application of the transformation) is much higher than the time taken for applying the transformation itself.

6 Related Work

Several papers have proposed approaches to model business processes with time constraints and, in particular, duration [17,14,15,32] (see also [6] for a recent survey).

The approach of Arbab et al. [1] provides a translation of BPMN into the coordination language REO. Due to REO’s Constraint Automata semantics, in principle this translation permits formal reasoning about BPMN processes depending on time and resources. However, the paper does not provide any formalized verification technique.

The workflow conceptual model proposed in [7] enables the specification and analysis of time constraints in business processes. The paper proposes temporal constructs to express duration, delays, relative, absolute, and periodic constraints. They also introduce the concept of controllability for workflow schemata and its evaluation at process design time. Controllability refers to the capability of executing a workflow for any possible duration of tasks, where the minimum and the maximum durations for each task are known. Their algorithms for testing controllability enumerate the possible choices, and therefore suffers from memory growth.

Gonzalez del Foyo and Silva consider in [14] workflow diagrams extended with task durations and the latest execution deadline of each task. They provide a translation into Time Petri Nets [3], where clocks are associated with each transition in the net, and use the tool TINA [4] to answer schedulability questions.

The approach described in [15] enables the specification of temporal constraints (such as ‘As Soon as Possible’) and temporal dependencies. However, unlike the approach presented here, no automated verification mechanism of time-dependent properties is provided.

The approach presented in [32] uses a timed semantic function which takes a diagram describing a collaboration, and returns a CSP process [16] that models the timed behavior of that diagram, by using the notion of a relative time in the
form of delays chosen non-deterministically within given intervals. Properties are then verified by using the FDR system [13]. Due to some intricacy of CSP, some behavioral properties of business processes, may not be easy to express for BP developers.

Some proposals, such as [29] and others surveyed in [6], make use of timed automata to model business processes with time constraints, and use the UP-PAAL tool [20] for the automated verification of some of their properties. As already mentioned, these proposals, as well as the ones cited above, may not be adequate when taking into consideration properties of business processes that require more advanced logical reasoning.

Finally, we would like to mention work on modeling and analyzing business processes with explicit time representation based on the Event Calculus [19] (see, for instance, [28]). However, the Event Calculus lacks a simple translation into constrained Horn clauses (in particular, it makes use of negation), which has been proposed in this paper as a means to enable the use of very effective automated verification systems.

7 Conclusions

We have presented a logic-based language to specify BPMN models where time and duration of activities are explicitly represented. The language enables the specification of timing constraints, given in the form of lower and upper bounds associated with the duration of tasks. These are useful features with an intuitive meaning that enable the specifier to annotate activities with timing restrictions. The language supports the specification of a wide range of time-dependent properties, such as the schedulability and response time.

The main advantage of our approach is that it allows us to automatically generate constrained Horn clauses from the formal definition of the semantics of the BPMN models and the time-dependent properties of interest. Then, by exploiting recent advances in the field of CHC solving, we get very effective reasoning tools for verifying properties of business processes. Finally, the fact that our approach is parametric with respect to the semantics of the process modeling languages we consider, allows us to take into account future extensions of those languages with very little effort.

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A Output of transformations

A.1 Removal of the interpreter (Section 4.2)

new44(A,B,C) :- A=0, B=C.
new44(A,B,C) :- D=0, E=A+B, A>0, new44(D,E,C).
new37(A,B,C) :- A=0, D=<3, D>=1, new17(D,B,C).
new37(A,B,C) :- A=0, D=<4, D>=2, new11(D,B,C).
new37(A,B,C) :- D=0, E=A+B, A>0, new37(D,E,C).
new21(A,B,C) :- A=0, D=<3, D>=1, new10(D,B,C).
new21(A,B,C) :- D=0, E=A+B, A>0, new21(D,E,C).
new17(A,B,C) :- A=0, B=C.
new17(A,B,C) :- D=0, E=A+B, A>0, new17(D,E,C).
new11(A,B,C) :- A=0, B=C.
new11(A,B,C) :- D=0, E=A+B, A>0, new11(D,E,C).
new10(A,B,C) :- A=0, B=C.
new10(A,B,C) :- D=0, E=A+B, A>0, new10(D,E,C).

new7(A,B,C,D) :- B=0, A=0, C=D.
new7(A,B,C,D) :- A=0, new10(B,C,D).
new7(A,B,C,D) :- B=0, new11(A,C,D).
new7(A,B,C,D) :- E=0, F=A-B, G=A+C, A-B=<0, A>0, new7(E,F,G,D).
new7(A,B,C,D) :- E=0, F=A-B, G=B+C, B>0, A-B>=0, new7(F,E,G,D).
new6(A,B,C,D) :- B=0, A=0, D=C.
new6(A,B,C,D) :- A=0, new10(B,C,D).
new6(A,B,C,D) :- B=0, new17(A,C,D).
new6(A,B,C,D) :- E=0, F=-A+B, G=A+C, A-B=<0, A>0, new6(E,F,G,D).
new6(A,B,C,D) :- E=0, F=A-B, G=B+C, B>0, A-B>=0, new6(F,E,G,D).
new5(A,B,C,D) :- B=0, A=0, E=<3, E>=1, new10(E,C,D).
new5(A,B,C,D) :- B=0, E=<3, E>=1, new7(A,E,C,D).
new5(A,B,C,D) :- E=0, F=-A+B, G=A+C, A-B=<0, A>0, new5(E,F,G,D).
new5(A,B,C,D) :- E=0, F=A-B, G=B+C, B>0, A-B>=0, new5(F,E,G,D).
new4(A,B,C,D) :- A=0, E=<3, E>=1, new6(E,B,C,D).
new4(A,B,C,D) :- A=0, E=<4, E>=2, new7(E,B,C,D).
new3(A,B,C,D) :- A=0, B=0, E=<3, E>=1, new17(E,C,D).
new3(A,B,C,D) :- A=0, B=0, E=<4, E>=2, new11(E,C,D).
new3(A,B,C,D) :- B=0, new37(A,C,D).
new3(A,B,C,D) :- E=0, F=-A+B, G=A+C, A-B=<0, A>0, new3(E,F,G,D).
new3(A,B,C,D) :- E=0, F=A-B, G=B+C, B>0, A-B>=0, new3(F,E,G,D).
new2(A,B,C,D) :- A=0, E=<3, E>=1, new3(E,B,C,D).
new2(A,B,C,D) :- B=0, E=<3, E>=1, new4(E,A,C,D).
new2(A,B,C,D) :- B=0, E=<4, E>=2, new5(E,A,C,D).
new2(A,B,C,D) :- A=0, B=0, E=<3, F=<3, E>=1, F>=1, new6(F,E,C,D).
new2(A,B,C,D) :- A=0, B=0, E=<3, F=<4, E>=1, F>=2, new7(F,E,C,D).
new2(A,B,C,D) :- E=0, F=-A+B, G=A+C, A-B=<0, A>0, new2(E,F,G,D).
new2(A,B,C,D) :- E=0, F=A-B, G=B+C, B>0, A-B>=0, new2(F,E,G,D).
new1(A,B,C) :- A=0, D=< 6, D>=1, new1(D,B,C).
new1(A,B,C) :- A=0, D=<2, D>= 1, new4(D,B,C).
new1(A,B,C) :- D=0, E=A+B, A>0, new1(D,E,C).
false :- A=0, B=<2, C=<6, D=<5, E>0, F-E>9, B>=1, C>=1, D>=3, new1(C,A,E),
         new2(B,D,E,F).

A.2 Transformation Based on Predicate Equivalence (Section 4.3)

The following partition of the set of predicates occurring in the clauses shown in A.1 is a cp-equivalence:

\[ E = \{ \{\text{new44, new17, new11, new10}\}, \{\text{new7, new6}\}, \{\text{new5, new4}\}, \{\text{new37}\}, \{\text{new21}\}, \{\text{new3}\}, \{\text{new2}\}, \{\text{new1}\} \} \]

associated with the following predicate renaming:

\[ \pi(\text{new44}) = \pi(\text{new17}) = \pi(\text{new11}) = \pi(\text{new10}) = \pi(\text{new6}) \]
\[ \pi(\text{new5}) = \pi(\text{new4}) = \pi(\text{new2}) \]

and \( \pi(p) = p \) for all other predicate symbols.

By applying \( \tilde{\pi} \), the clauses in A.1 are transformed into the following set:

new37(A,B,C) :- A=0, D=<3, D>=1, new10(D,B,C).
new37(A,B,C) :- A=0, D=<4, D>=2, new10(D,B,C).
new21(A,B,C) :- A=0, D=<3, D>=1, new10(D,B,C).
new21(A,B,C) :- D=0, E=A+B, A>0, new21(D,E,C).
new10(A,B,C) :- A=0, B>=C.
new10(A,B,C) :- D=0, E=A+B, A>0, new10(D,E,C).
new6(A,B,C,D) :- B=0, A=0, D=C.
new6(A,B,C,D) :- A=0, new10(B,C,D).
new6(A,B,C,D) :- B=0, new10(A,C,D).
new6(A,B,C,D) :- E=0, F=-A+B, G=A+C, A-B=<0, A>0, new6(E,F,G,D).
new6(A,B,C,D) :- E=0, F=A-B, G=B+C, B>0, A-B>=0, new6(F,E,G,D).
new4(A,B,C,D) :- A=0, E=<3, E>=1, new10(E,C,D).
new4(A,B,C,D) :- B=0, E=<3, E>=1, new6(E,A,C,D).
new4(A,B,C,D) :- E=0, F=-A+B, G=A+C, A-B=<0, A>0, new4(E,F,G,D).
new4(A,B,C,D) :- E=0, F=A-B, G=B+C, B>0, A-B>=0, new4(F,E,G,D).
new3(A,B,C,D) :- A=0, E=<3, E>=1, new6(E,B,C,D).
new3(A,B,C,D) :- A=0, E=<4, E>=2, new6(E,B,C,D).
new3(A,B,C,D) :- A=0, B=0, E=<3, E>=1, new10(E,C,D).
new3(A,B,C,D) :- A=0, B=0, E=<4, E>=2, new10(E,C,D).
new3(A,B,C,D) :- B=0, new37(A,C,D).
new3(A,B,C,D) :- E=0, F=-A+B, G=A+C, A-B=<0, A>0, new3(E,F,G,D).
new3(A,B,C,D) :- E=0, F=A-B, G=B+C, B>0, A-B>=0, new3(F,E,G,D).
new2(A,B,C,D) :- A=0, E=<3, E>=1, new3(B,E,C,D).
new2(A,B,C,D) :- B=0, E=<3, E>=1, new4(E,A,C,D).
new2(A,B,C,D) :- B=0, E=<4, E>=2, new4(E,A,C,D).
new2(A,B,C,D) :- A=0, B=0, E=<3, F=<3, E>=1, F>=1, new6(F,E,C,D).
new2(A,B,C,D) :- A=0, B=0, E=<3, F=<4, E>=1, F>=2, new6(F,E,C,D).
new2(A,B,C,D) :- E=0, F=-A+B, G=A+C, A-B=<0, A>0, new2(E,F,G,D).
new2(A,B,C,D) :- E=0, F=A-B, G=B+C, B>0, A-B>=0, new2(F,E,G,D).
new1(A,B,C) :- A=0, D=<6, D>=1, new1(D,B,C).
new1(A,B,C) :- A=0, D=<2, D>=1, new10(D,B,C).
new1(A,B,C) :- D=0, E=A+B, A>0, new1(D,E,C).
false :- A=0, B=<2, D=<5, E>0, F-E>9, B>=1, C>=1, D>=3, C=<6, G=E, new1(C,A,G),
         new2(B,D,E,F).