On mesoscopic forces and quantized conductance in model metallic nanowires

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Energetics and conductance in jellium modelled nanowires are investigated using the local-density-functional-based shell correction method. In analogy with studies of other finite-size fermion systems, e.g., simple-metal clusters or $^3$He clusters, we find that the energetics of the wire as a function of its radius (transverse reduced dimension) leads to formation of self-selecting magic wire configurations (MWC’s, i.e., discrete sequence of wire radii with enhanced stability), originating from quantization of the electronic spectrum, namely formation of subbands which are the analogs of electronic shells in clusters. These variations in the energy result in oscillations in the force required to affect a transition from one MWC of the nanowire to another, and are correlated directly with stepwise variations of the quantized conductance of the nanowire in units of $2e^2/h$.

I. INTRODUCTION

Identification and understanding of the physical origins and systematics underlying the variations of materials properties with size, form of aggregation, and dimensionality are some of the main challenges in modern materials research, of ever increasing importance in the face of the accelerated trend toward miniaturization of electronic and mechanical devices. While for over two decades studies of size-evolutionary patterns of materials have focused on atomic and molecular clusters
d in beams or embedded in inert matrices, more recent efforts concentrated on preparation, characterization, and understanding of finite solid-sate structures. These include nanometer-scale metal and semiconductor nanocrystals, surface-supported structures and quantum dots, and nanoscale junctions or wires.

Interestingly, it has emerged that concepts and methodologies developed in the context of isolated gas-phase clusters and atomic nuclei are often most useful for investigations of finite-size solid-state structures. In particular, it has been shown most recently through first-principles molecular dynamics simulations that as metallic (sodium) nanowires are stretched to just a few atoms in diameter, the reduced dimensions, increased surface-to-volume ratio, and impoverished atomic environment, lead to formation of structures, made of the metal atoms in the neck, which can be described in terms of those observed in small gas-phase sodium clusters; hence they were termed as supported cluster-derived structures (cds). The above prediction of the occurrence of “magic-number” cds’s in nanowires, due to characteristics of electronic cohesion and atomic bonding in such structures of reduced dimensions, are directly correlated with the energetics of metal clusters, where magic-number sequences of clusters sizes, shapes and structural motifs due to electronic and/or geometric shell effects, have been long predicted and observed.

Furthermore, these results lead one directly to conclude that other properties of nanowires, derived from their energetics, may be described using methodologies developed previously in the context of clusters. Indeed, in this paper, we show that certain aspects of the mechanical response (i.e., elongation force) and electronic transport (e.g., quantized conductance) in metallic nanowires can be analyzed using the local-density-approximation (LDA)-based shell correction method (SCM), developed and applied previously in studies of metal clusters. Specifically, we show that in a jellium-modelled, volume-conserving nanowire, variations of the total energy (particularly terms associated with electronic subband corrections) upon elongation of the wire lead to self-selection of a sequence of stable “magic” wire configurations (MWC’s, specified in our model by a sequence of the wire’s radii), with the force required to elongate the wire from one configuration to the next exhibiting an oscillatory behavior. Moreover, we show that due to the quantized nature of electronic states in such wires, the electronic conductance varies in a quantized step-wise manner (in units of the conductance quantum $g_0 = 2e^2/h$), correlated with the transitions between MWC’s and the above-mentioned force oscillations.

Prior to introducing the model studied in this paper, it is appropriate to briefly review certain previous theoretical and experimental investigations, which form the background and motivation for this study. Atomistic descriptions, based on realistic interatomic interactions, and/or first-principles modelling and simulations played an essential role in discovering the formation of nanowires, and in predicting and elucidating the microscopic mechanisms underlying their mechanical, spectral, electronic and transport properties.

Formation and mechanical properties of interfacial junctions (in the form of crystalline nanowires) have been predicted through early molecular-dynamics simulations, where the materials (gold) were modelled using semiempirical embedded-atom potentials. In these studies it has been shown that separation of the contact between materials leads to generation of a connective junction which elongates and narrows through a sequence of structural instabilities; at the early stages, elongation of the junction involves multiple slip events, while at the later stages, when the lateral dimension of the wire necks...
down to a diameter of about 15 Å, further elongation involves a succession of stress accumulation and fast relief stages associated with a sequence of order-disorder structural transformations localized to the neck region. These structural evolution patterns have been shown through the simulations to be portrayed in oscillations of the force required to elongate the wire, with a period approximately equal to the interlayer spacing. In addition, the “sawtoothed” character of the predicted force oscillations [see Fig. 3(b) in Ref. 8 and Fig. 3 in Ref. 22] reflects the stress accumulation and relief stages of the elongation mechanism. Moreover, the critical resolved yield stress of gold nanowires has been predicted to be \( \sim 4 \text{GPa} \), which is over an order of magnitude larger than that of the bulk, and is comparable to the theoretical value for Au (1.5 GPa) in the absence of dislocations.

These predictions, as well as anticipated electronic conductance properties, have been confirmed in a number of experiments using scanning tunneling and force microscopy, break junctions, and pin–plate techniques at ambient environments, as well as under ultrahigh vacuum and/or cryogenic conditions. Particularly, pertinent to our current study are experimental observations of the oscillatory behavior of the elongation forces and the correlations between the changes in the conductance and the force oscillations; see especially the simultaneous measurements of force and conductance in gold nanowires in Ref. 13, where in addition the predicted “ideal” value of the critical yield stress has also been measured (see also Ref. 24).

The jellium-based model introduced in this paper, which by construction is devoid of atomic crystallographic structure, does not address issues pertaining to nanowire formation methods, atomistic configurations, and mechanical response modes [e.g., plastic deformation mechanisms, interplanar slip, ordering and disordering mechanisms (see detailed descriptions in Refs. 5, 12, and 21), and a discussion of condensation dips in Ref. 12], defects, mechanical reversibility, and toughening of the wires’ morphology during elongation, nor does it consider the effects of the above on the electron spectrum, transport properties, and dynamics. Nevertheless, as shown below, the model offers a useful framework for linking investigations of solid-state structures of reduced dimensions (e.g., nanowires) with methodologies developed in cluster physics, as well as highlighting certain nanowire phenomena of mesoscopic origins and their analogies to clusters.

II. THE JELLIUM MODEL FOR METALLIC NANOWIRES: THEORETICAL METHOD AND RESULTS

Consider a cylindrical jellium wire of length \( L \), having a positive background with a circular cross section of radius \( R \ll L \). For simplicity, we restrict ourselves here to this symmetry of the wire cross section. Variations in the shape of the nanowire cross section \( \frac{2}{3} \) to affect the degeneracies of the electronic spectrum, without affecting our general conclusions. We also do not include here variations of the wire’s shape along its axis. Adiabatic variation of the wire’s axial shape introduces a certain amount of smearing of the conductance steps through tunneling \( \frac{1}{3} \), depending on the axial radius of curvature of the wire \( \frac{1}{3} \). Both the cross-sectional and axial shape of the wire can be included in our model in a rather straightforward manner.

The principal idea of the SCM is the separation of the total LDA energy \( E_T(R) \) as

\[
E_T(R) = \tilde{E}(R) + \Delta E_{\text{sh}}(R),
\]

where \( \tilde{E}(R) \) varies smoothly as a function of the system size, and \( \Delta E_{\text{sh}}(R) \) is an oscillatory term arising from the discrete quantized nature of the electronic levels. \( \Delta E_{\text{sh}}(R) \) is usually called a shell correction in the nuclear and cluster literature; we continue to use here the same terminology with the understanding that the electronic levels in the nanowire form subbands, which are the analog of electronic shells in clusters, where furthermore the size of the system is usually given by specifying the number of atoms \( N \). The SCM method, which has been shown to yield results in excellent agreement with experimental \( \frac{1}{3} \) and self-consistent LDA calculations \( \frac{1}{3} \) for a number of cluster systems, is equivalent to a Harris functional \( \frac{1}{3} \) approximation to the Kohn-Sham LDA with the input density obtained through variational minimization of an extended Thomas-Fermi (ETF) energy functional, \( E_{\text{ETF}}[\tilde{\rho}] \) (with the kinetic energy, \( T_{\text{ETF}}[\tilde{\rho}] \), given to 4th order gradients and the potential, \( V_{\text{ETF}} \), including the Hartree repulsion and exchange-correlation and positive-background attractions as in LDA). The smooth contribution in Eq. (1) is identified with \( E_{\text{ETF}}[\tilde{\rho}] \). The optimized density \( \tilde{\rho} \) at a given radius \( R \) is obtained under the normalization condition (charge neutrality)

\[
2\pi \int \tilde{\rho}(r) r dr = \rho_L^{(+)}(R),
\]

where \( \rho_L^{(+)}(R) = 3R^2/(4\pi^2) \) is the linear positive background density. Using the optimized \( \tilde{\rho} \), one solves for the eigenvalues \( \tilde{\epsilon}_i \) of the Hamiltonian \( H = -(\hbar^2/2m)\nabla^2 + V_{\text{ETF}}[\tilde{\rho}] \), and the shell correction is given by

\[
\Delta E_{\text{sh}} = E_{\text{harris}}[\tilde{\rho}] - E_{\text{ETF}}[\tilde{\rho}]
= \sum_{i=1}^{\text{occ}} \tilde{\epsilon}_i - \int \tilde{\rho}(r)V_{\text{ETF}}[\tilde{\rho}(r)] dr - T_{\text{ETF}}[\tilde{\rho}],
\]

where the summation extends over occupied levels, the dependence of all quantities on the pertinent size variable (i.e., the radius of the wire \( R \)) is not shown explicitly. Additionally, the index \( i \) can be both discrete and continuous, and in the latter case the summation is replaced by an integral.

Following the above procedure with a uniform background density of sodium (\( r_s = 4 \text{ a.u.} \)), a typical po-
potential \( V_{\text{ETF}}(r) \) for \( R = 12.7 \) a.u., where \( r \) is the radial coordinate in the transverse plane, is shown in Fig. 1, along with the transverse eigenvalues \( \bar{\epsilon}_{nm} \) and the Fermi level; to simplify the calculations of the electronic spectrum, we have assumed (as noted above) \( R \ll L \), which allows us to express the subband electronic spectrum as

\[
\bar{\epsilon}_{nm}(k_z; R) = \bar{\epsilon}_{nm}(R) + \frac{\hbar^2 k_z^2}{2m},
\]

(3)

where \( k_z \) is the electron wave number along the axis of the wire (z).

As indicated earlier, taking the wire to be charge neutral, the electronic linear density, \( \rho_L^{(+)}(R) \), must equal the linear positive background density, \( \rho_L^{(+)}(R) \). The chemical potential (at \( T = 0 \) the Fermi energy \( \epsilon_F \)) for a wire of radius \( R \) is determined by setting the expression for the electronic linear density derived from the subband spectra equal to \( \rho_L^{(+)}(R) \), i.e.,

\[
\frac{2}{\pi} \sum_{n,m} \sqrt{\frac{2m}{\hbar^2}} \left[ \epsilon_F(R) - \bar{\epsilon}_{nm}(R) \right] = \rho_L^{(+)}(R),
\]

(4)

where the factor of 2 on the left is due to the spin degeneracy. The summand defines the Fermi wave vector for each subband, \( k_{F,nm} \). The resulting variation of \( \epsilon_F(R) \) versus \( R \) is displayed in Fig. 2(a), showing cusps for values of the radius where a new subband drops below the Fermi level as \( R \) increases (or conversely as a subband moves above the Fermi level as \( R \) decreases upon elongation of the wire). Using the Landauer expression for the conductance \( G \) in the limit of no mode mixing and assuming unit transmission coefficients, \( G(R) = g_0 \sum_{n,m} \Theta[\epsilon_F(R) - \bar{\epsilon}_{nm}(R)] \), where \( \Theta \) is the Heaviside step function. The conductance of the nanowire, shown in Fig. 2(b), exhibits quantized step-wise behavior, with the step-rises coinciding with the locations of the cusps in \( \epsilon_F(R) \), and the height sequence of the steps is 1, 2, 4, 2, 0, 2, 4, 2, 0, ..., reflecting the circular symmetry of the cylindrical wires’ cross sections as observed for sodium nanowires [7]. Solving for \( \epsilon_F(R) \) [see Eq. (3)], the expression for the sum on the right-hand-side of Eq. (3) can be written as

\[
\sum_{n,m} \bar{\epsilon}_{nm} = \frac{2}{\pi} \sum_{n,m} \int_0^{k_{F,nm}} dk_z \bar{\epsilon}_{nm}(k_z; R) = \frac{2}{3\pi} \sum_{n,m} \left[ \epsilon_F(R) + 2\bar{\epsilon}_{nm}(R) \right] \sqrt{\frac{2m}{\hbar^2}} \left[ \epsilon_F(R) - \bar{\epsilon}_{nm}(R) \right],
\]

(5)

which allows one to evaluate \( \Delta \epsilon_{sh} \) [Eq. (3)] for each wire radius \( R \). Since the expression in Eq. (3) gives the energy per unit length, we also calculate \( E_{\text{ETF}}, T_{\text{ETF}} \), and the volume integral in the second line of Eq. (3) for cylindrical volumes of unit height. To convert to energies per unit volume [denoted as \( \gamma \), \( \bar{\gamma} \), and \( \Delta \epsilon_{sh} \)] all energies are further divided by the wire’s cross-sectional area, \( \pi R^2 \). The smooth contribution and the shell correction to the wire’s energy are shown respectively in Fig. 3(a) and Fig. 3(b). The smooth contribution decreases slowly towards the bulk value (~2.25 eV per atom). On the other hand, the shell corrections are much smaller in magnitude and exhibit an oscillatory behavior. This oscillatory behavior remains visible in the total energy [Fig. 3(e)] with the local energy minima occurring for values \( R_{\text{min}} \) corresponding to conductance plateaus. The sequence of \( R_{\text{min}} \) values defines the MWC’s, that is a sequence of wire configurations of enhanced stability.

From the expressions for the total energy of the wire [i.e., \( \Omega \epsilon_T(R) \), where \( \Omega = \pi R^2 L \) is the volume of the wire] and the smooth and shell (subband) contributions to it, we can calculate the “elongation force” (EF),

\[
F_T(R) = -\frac{d[\Omega \epsilon_T(R)]}{dL} = -\Omega \left\{ \frac{d\bar{\gamma}(R)}{dL} + \frac{d[\Delta \epsilon_{sh}(R)]}{dL} \right\} \equiv \bar{F}(R) + \Delta F_{sh}(R).
\]

Using the volume conservation, i.e., \( d(\pi R^2 L) = 0 \), these forces can be written as \( F_T(R) = (\pi R^2/2)d\epsilon_T(R)/dR \), \( \bar{F}(R) = (\pi R^3/2)d\bar{\gamma}(R)/dR \), and \( \Delta F_{sh}(R) = (\pi R^3/2)d[\Delta \epsilon_{sh}(R)]/dR \). \( \bar{F}(R) \) and \( F_T(R) \) are shown in Fig. 3(d,e). The oscillations in the force resulting from the shell-correction contributions dominate. In all cases, the radii corresponding to zeroes of the force situated on the left of the force maxima coincide with the minima in the potential energy curve of the wire, corresponding to the MWC’s. Consequently, these forces may be interpreted as guiding the self-evolution of the wire toward the MWC’s. Also, all the local maxima in the force occur at the locations of step-rises in the conductance [reproduced in Fig. 3(f)], signifying the sequential decrease in the number of subbands below the Fermi level (conducting channels) as the wire narrows (i.e., as it is being elongated). Finally the magnitude of the total forces is comparable to the measured ones (i.e., in the nN range).

III. CONCLUSIONS AND DISCUSSION

We investigated energetics, conductance, and mesoscopic forces in a jellium modelled nanowire (sodium) using the local-density-functional-based shell correction method. The results shown above, particularly, the oscillations in the total energy of the wire as a function of its radius (and consequently the oscillations in the EF), the corresponding discrete sequence of magic wire configurations, and the direct correlation between these oscillations and the step-wise quantized conductance of the nanowires, originate from quantization of the electronic states (i.e., formation of subbands) due to the reduced lateral (transverse) dimension of the nanowires. In fact such oscillatory behavior, as well as the appearance of “magic numbers” and “magic configurations” of enhanced stability, are a general characteristic of finite-size fermionic systems and are in direct analogy with...
those found in simple-metal clusters (as well as in \(^3\text{He}\) clusters and atomic nuclei), where electronic shell effects on the energetics (and most recently shape dynamics of jellium modelled clusters driven by forces obtained from shell-corrected energetics) have been studied for over a decade.

While these calculations provide a useful and instructive framework, we remark that they are not a substitute for theories where the atomistic nature and specific atomic arrangements are included. The focus in evaluation of the energetics (and dynamics) of these systems (see in particular Refs. 25,27, where first-principles molecular-dynamics simulations of electronic spectra, geometrical structure, atomic dynamics, electronic transport and fluctuations in sodium nanowires have been discussed).

Indeed, the atomistic structural characteristics of nanowires (including the occurrence of cluster-derived structures of particular geometries), which may be observed through the use of high resolution microscopy, influence the electronic spectrum and transport characteristics, as well as the energetics of nanowires and their mechanical properties and response mechanisms. In particular, the mechanical response of materials involves structural changes through displacement and discrete rearrangement of the atoms. The mechanisms, pathways, and rates of such structural transformations are dependent on the arrangements and coordinations of atoms, the magnitude of structural transformation barriers, and the local shape of the wire, as well as possible dependency on the history of the material and the conditions of the experiment (i.e., fast versus slow extensions). Further evidence for the discrete atomistic nature of the structural transformations is provided by the shape of the force variations (compare the calculated Fig. 3(b) in Ref. 8 and Fig. 3 in Ref. 22, with the measurements shown in Figs. 1 and 2 in Ref. 14, and the interlayer spacing period of the force oscillations when the wire narrows. While such issues are not addressed by our model, the mesoscopic (in a sense universal) phenomena described by it are of interest, and may guide further research in the area of finite-size systems in the nanoscale regime.

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25. (a) **Landman, U.; Barnett, R.N.; Luedtke, W.D. Z. Phys. D 1997, 40, 282.** (b) The original predictions pertaining to formation of \(cd\) and analogies to cluster phenomena (e.g., magic numbers) have been presented by one of us (U.L.) at ISSPIC8 [Copenhagen, July 1996, see Ref. 25(a)], and at the NATO ARW on Nanowires (Madrid, September 1996); see Proceeding paper by Landman, U.; Luedtke, W.D.; Barnett, R.N. in Nanowires; Serena, P.A., Garcia,
FIG. 1. Lower panel: The $V_{ETF}(r)$ potential for a sodium wire with a uniform jellium background of radius $R = 12.7$ a.u., plotted versus the transverse radial distance from the center of the wire, along with the locations of the bottoms of the subbands (namely the transverse eigenvalues $\tilde{\epsilon}_{nm}$; $n$ is the number of nodes in the radial direction plus one, and $m$ is the azimuthal quantum number of the angular momentum). The Fermi level is denoted by a dashed line. Top panel: The jellium background volume density (dashed line) and the electronic volume density $\tilde{\rho}(r)$ (solid line, exhibiting a characteristic spillout) normalized to bulk values are shown.

FIG. 2. Variation of the Fermi energy $\epsilon_F$ [shown in (a)] and of the conductance $G$ (shown in (b) in units of $g_0 = 2e^2/h$), plotted versus the radius $R$, for a sodium nanowire. Note the coincidence of the cusps in $\epsilon_F$ with the step-rises of the conductance. The heights of the steps in $G$ reflect the subband degeneracies due to the circular shape of the wire's cross section.

FIG. 3. (a-c): The smooth (a) and shell-correction (b) contributions to the total energy (c) per unit volume of the jellium-modelled sodium nanowire (in units of $u \equiv 10^{-4}$ eV/a.u.), plotted versus the radius of the wire (in a.u.). Note the smaller magnitude of the shell corrections relative to the smooth contribution. (d-e): The smooth contribution (d) to the total force and the total force (e), plotted in units of nN versus the wire’s radius. In (e), the zeroes of the force to the left of the force maxima occur at radii corresponding to the local minima of the energy of the wire (c). In (f), we reproduce the conductance of the wire (in units of $g_0 = 2e^2/h$), plotted versus $R$. Interestingly, calculations of the conductance for the MWC’s (i.e., the wire radii corresponding to the locations of the step-rises) through the Sharvin-Weyl formula [18], corrected for the finite height of the confining potential [26], yield results which approximate well the conductance values (i.e., the values at the bottom of the step-rises) shown in (f).
