1 Introduction

In a recent paper [1], we have shown how the interpretation of quantum mechanics due to Landé [2−5] can be used to derive spin operators and their eigenvectors from first principles. This interpretation allows us to go further than has to date been done in the treatment of spin and ultimately enables us to obtain new generalized forms for the spin operators and their eigenvectors.

In [1], we applied this treatment to the case of spin 1/2 and derived the generalized formulas for $\sigma_x$, $\sigma_y$ and $\sigma_z$, the $x$, $y$ and $z$ components of spin, and for the eigenvectors of $\sigma_z$. The operators $\sigma_x$ and $\sigma_y$ were obtained through the ladder operators, which in turn were derived from their actions on the eigenvectors of $\sigma_z$. However, the generalized eigenvectors of $\sigma_x$ and $\sigma_y$ were not given. In this paper, we present these eigenvectors. Furthermore, we introduce a method for obtaining $\sigma_x$ and $\sigma_y$ and their eigenvectors without the agency of the ladder operators. This is very important because the method involving the ladder operators is quite tedious, as evidenced even for the simplest case spin 1/2. Finally, we prove that the generalized operator for the square of the spin is a unit matrix multiplied by the value of the square of the spin.

2 Basic Theory

The method we have devised and used to obtain the spin-1/2 quantities is founded on the Landé interpretation of quantum mechanics [2−5]. Let $A$, $B$ and $C$ be observables of a quantum system. The eigenvalues of $A$ are $A_1, A_2 ...$. The eigenvalues of $B$ are $B_1, B_2 ...$. Finally, the eigenvalues of $C$ are $C_1, C_2, ...$. Suppose that the initial state of the system corresponds to $A_i$. Starting from this state, let the probability amplitude for obtaining an eigenvalue $C_n$ upon measurement of $C$ be $\psi(A_i; C_n)$. Let the probability amplitude for obtaining the eigenvalue $B_j$ of $B$ be $\chi(A_i; B_j)$. Finally let the probability amplitude for obtaining the eigenvalue $C_n$ of $C$ when the system is initially in a state corresponding to $B_j$ be $\phi(B_j; C_n)$. Then the three sets of probability amplitudes are connected by the formula [2,3]

$$\psi(A_i; C_n) = \sum_j \chi(A_i; B_j)\phi(B_j; C_n).$$

(1)

These probability amplitudes obey two-way symmetry contained in the Hermiticity condition

$$\psi(A_i; C_n) = \psi^*(C_n; A_i),$$

etc. (2)

They also satisfy

$$\zeta(A_i; A_j) = \delta_{ij}, \quad \zeta(B_i; B_j) = \delta_{ij},$$

etc, (3)

which simply ensures the repeatability of measurement.
For spin, the three observables $A$, $B$ and $C$ correspond to projection measurements along three different quantization axes. This means that $\psi$, $\chi$ and $\phi$ are identical in form.

Suppose the quantity $R$ is a function of the final spin projection resulting from spin measurement. In matrix mechanics, $R$ is represented by the spin operator $[R]$. In [1], we derived the general expressions for the matrix elements of $[R]$ for the case of spin $1/2$. Thus, for this case spin $1/2$ let the initial spin projection be with respect to the unit vector $\hat{a}$ and let the final spin projection be with respect to the unit vector $\hat{c}$. Let the polar angles of $\hat{a}$ be ($\theta''$, $\phi''$) and the polar angles of $\hat{c}$ be ($\theta'$, $\phi'$). Let the probability amplitudes for measurements of spin from $\hat{a}$ to $\hat{c}$ be $\psi(m_1^{(a)}; m_f^{(c)})$, with $m_i, m_f = 1, 2$ and $m_1^{(a)} = m_1^{(c)} = +\frac{1}{2}$, while $m_2^{(a)} = m_2^{(c)} = -\frac{1}{2}$. For example, $\psi((+\frac{1}{2})^{(a)}; (-\frac{1}{2})^{(c)})$ is the probability amplitude for finding the spin projection to be down with respect to $\hat{c}$ when the system was initially in a state in which the spin projection was up with respect to $\hat{a}$.

Let the quantity $R$ take the values $r_1$ and $r_2$ when the spin projection is found up and down respectively with respect to the final direction. Then the elements of the matrix representation

$$[R] = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

(4)

of $R$ are [1]

$$R_{11} = \left| \phi((+\frac{1}{2})^{(b)}; (+\frac{1}{2})^{(c)}) \right|^2 r_1 + \left| \phi((+\frac{1}{2})^{(b)}; (-\frac{1}{2})^{(c)}) \right|^2 r_2$$

(5)

$$R_{12} = \phi^*((+\frac{1}{2})^{(b)}; (+\frac{1}{2})^{(c)})\phi((-\frac{1}{2})^{(b)}; (+\frac{1}{2})^{(c)})r_1 + \phi^*((+\frac{1}{2})^{(b)}; (-\frac{1}{2})^{(c)})\phi((-\frac{1}{2})^{(b)}; (-\frac{1}{2})^{(c)})r_2$$

(6)

$$R_{21} = \phi^*((-\frac{1}{2})^{(b)}; (+\frac{1}{2})^{(c)})\phi((-\frac{1}{2})^{(b)}; (+\frac{1}{2})^{(c)})r_1 + \phi^*((-\frac{1}{2})^{(b)}; (-\frac{1}{2})^{(c)})\phi((-\frac{1}{2})^{(b)}; (-\frac{1}{2})^{(c)})r_2$$

(7)

and

$$R_{22} = \left| \phi((-\frac{1}{2})^{(b)}; (+\frac{1}{2})^{(c)}) \right|^2 r_1 + \left| \phi((-\frac{1}{2})^{(b)}; (-\frac{1}{2})^{(c)}) \right|^2 r_2$$

(8)

Here $\phi(m_i^{(b)}; m_f^{(c)})$ are the probability amplitudes for the outcomes of spin projection measurements from the direction $\hat{b}$ to the direction $\hat{c}$, where $\hat{b}$ is an intermediate quantization direction defined by the angles ($\theta, \phi$).
3 Spin Operators and Eigenvectors

Suppose that the quantity \( R \) is the spin projection itself, which we measure in units of \( \hbar/2 \). If the initial spin state is characterized by the magnetic quantum number \( m_i \) defined with respect to the direction \( \hat{a} \), then the expectation value of the spin projection along the direction \( \hat{c} \) is

\[
\langle \sigma_{\hat{c}} \rangle = [\psi(m_i \hat{a})]^\dagger [\sigma_{\hat{c}}] [\psi(m_i \hat{a})],
\]

(9)

where

\[
[\sigma_{\hat{c}}] = \begin{pmatrix}
(\sigma_{\hat{c}})_{11} & (\sigma_{\hat{c}})_{12} \\
(\sigma_{\hat{c}})_{21} & (\sigma_{\hat{c}})_{22}
\end{pmatrix},
\]

(10)

and

\[
[\psi(m_i \hat{a})] = \begin{pmatrix}
\chi(m_i \hat{a} ; (+1/2) \hat{b}) \\
\chi(m_i \hat{a} ; (-1/2) \hat{b})
\end{pmatrix}
\]

(11)

is the vector state corresponding to the initial spin projection \( m_i \) along \( \hat{a} \).

The elements of \([\sigma_{\hat{c}}]\) are functions of the angles \((\theta, \phi)\) of \( \hat{b} \) and the angles \((\theta', \phi')\) of \( \hat{c} \). They are

\[
(\sigma_{\hat{c}})_{11} = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\phi - \phi'),
\]

(12)

\[
(\sigma_{\hat{c}})_{12} = \sin \theta \cos \theta' - \sin \theta' \cos \theta \cos(\phi - \phi') + i \sin(\phi - \phi'),
\]

(13)

\[
(\sigma_{\hat{c}})_{21} = \sin \theta \cos \theta' - \sin \theta' \cos \theta \cos(\phi - \phi') - i \sin(\phi - \phi'),
\]

(14)

and

\[
(\sigma_{\hat{c}})_{22} = -\cos \theta \cos \theta' - \sin \theta \sin \theta' \cos(\phi - \phi').
\]

(15)

This operator \([\sigma_{\hat{c}}]\) is the component of the spin along the direction \( \hat{c} \). This is evidently a more generalized form of this operator than

\[
[\sigma \cdot \hat{c}] = \begin{pmatrix}
\cos \theta' & \sin \theta' e^{-i\phi'} \\
\sin \theta' e^{i\phi'} & -\cos \theta'
\end{pmatrix},
\]

(16)

found in the literature. Here

\[
[\sigma] = \hat{\imath}[\sigma_x] + \hat{\jmath}[\sigma_y] + \hat{k}[\sigma_z]
\]

(17)

where \([\sigma_x] \), \([\sigma_y] \) and \([\sigma_z] \) are the Pauli spin matrices.

In the literature Eqn. (17) is presented as being the generalized form of the \( z \) component of the spin, since it refers to the arbitrary direction \( \hat{c} \). But this form can be obtained from the generalized form Eqs. (12) - (15) by setting \( \theta = \phi = 0 \); this corresponds to the vector \( \hat{b} \) pointing in the positive \( z \) direction. In the limit \( \theta = \theta' \) and \( \phi = \phi' \), we obtain from the generalized form the Pauli matrix

\[
[\sigma] = \hat{\imath}[\sigma_x] + \hat{\jmath}[\sigma_y] + \hat{k}[\sigma_z]
\]

(17)
\[
[\sigma_z] = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

Moreover, the generalized \(x\) and \(y\) components of spin which we obtained in [1], and which we shall derive in an alternative way below, change to the corresponding Pauli matrices in the same limit.

The spin state \([\psi_+]\) is [1]

\[
[\psi((+\frac{1}{2})\hat{a})] = \begin{pmatrix} \cos \theta'/2 \cos \theta/2 + e^{i(\phi''-\varphi)} \sin \theta''/2 \sin \theta/2 \\ \cos \theta'/2 \sin \theta/2 - e^{i(\phi''-\varphi)} \sin \theta''/2 \cos \theta/2 \end{pmatrix}.
\]

This state is evidently the most generalized form of the spin state.

If on the other hand the spin projection is initially down with respect to \(\hat{a}\), the vector \([\psi_+]\) is replaced by [1]

\[
[\psi((-\frac{1}{2})\hat{a})] = \begin{pmatrix} \sin \theta'/2 \cos \theta/2 - e^{i(\phi''-\varphi)} \cos \theta''/2 \sin \theta/2 \\ \sin \theta'/2 \sin \theta/2 + e^{i(\phi''-\varphi)} \cos \theta''/2 \cos \theta/2 \end{pmatrix}.
\]

in the expression Eq. (19) for the expectation value.

The eigenvalues of \([\sigma_z]\) are +1 with the eigenvector [1]

\[
[\xi^{(+)}_c] = \begin{pmatrix} \phi((+\frac{1}{2})\hat{c}); (+\frac{1}{2})\hat{b}) \\ \phi((+\frac{1}{2})\hat{c}); (-\frac{1}{2})\hat{b}) \end{pmatrix} = \begin{pmatrix} \cos \theta'/2 \cos \theta/2 + e^{i(\phi'-\varphi)} \sin \theta'/2 \sin \theta/2 \\ \cos \theta'/2 \sin \theta/2 - e^{i(\phi'-\varphi)} \sin \theta'/2 \cos \theta/2 \end{pmatrix}
\]

and −1 with the eigenvector [1]

\[
[\xi^{(-)}_c] = \begin{pmatrix} \phi((-\frac{1}{2})\hat{c}); (+\frac{1}{2})\hat{b}) \\ \phi((-\frac{1}{2})\hat{c}); (-\frac{1}{2})\hat{b}) \end{pmatrix} = \begin{pmatrix} \sin \theta'/2 \cos \theta/2 - e^{i(\phi'-\varphi)} \cos \theta'/2 \sin \theta/2 \\ \sin \theta'/2 \sin \theta/2 + e^{i(\phi'-\varphi)} \cos \theta'/2 \cos \theta/2 \end{pmatrix}.
\]

We also derived the generalized operators \([\sigma_x]\) and \([\sigma_y]\). These operators correspond to directions such that in the limit \(\theta = \theta'\) and \(\varphi = \varphi'\), they become the Pauli matrices \([\sigma_x]\) and \([\sigma_y]\). The elements of \([\sigma_x]\) are [1]

\[
(\sigma_x)_{11} = - \sin \theta \cos \theta' \cos (\varphi' - \varphi) + \sin \theta' \cos \theta,
\]

\[
(\sigma_x)_{12} = \cos \theta \cos \theta' \cos (\varphi' - \varphi) + \sin \theta \sin \theta' - i \cos \theta' \sin (\varphi' - \varphi),
\]

\[
(\sigma_x)_{21} = \cos \theta \cos \theta' \cos (\varphi' - \varphi) + \sin \theta \sin \theta' + i \cos \theta' \sin (\varphi' - \varphi),
\]

and

\[
(\sigma_x)_{22} = \sin \theta \cos \theta' \cos (\varphi' - \varphi) - \sin \theta' \cos \theta.
\]
while the elements of \([\sigma_y]\) are

\[
(\sigma_y)_{11} = \sin \theta \sin(\varphi' - \varphi), \tag{27}
\]

\[
(\sigma_y)_{12} = -\cos \theta \sin(\varphi' - \varphi) - i \cos(\varphi' - \varphi), \tag{28}
\]

\[
(\sigma_y)_{21} = -\cos \theta \sin(\varphi' - \varphi) + i \cos(\varphi' - \varphi) \tag{29}
\]

and

\[
(\sigma_y)_{22} = -\sin \theta \sin(\varphi' - \varphi). \tag{30}
\]

The eigenvectors of \([\sigma_x]\) and \([\sigma_y]\) were not given.

4 Alternative Derivation of the x and y Components of the Spin

We have found that we can obtain the \(x\) and \(y\) components of the spin operator by changing the arguments appropriately in the formula for the \(z\) component. This same change of argument applied to the eigenvectors of the \(z\) component yields the eigenvectors of the \(x\) and \(y\) components.

The change of argument \(\theta' \rightarrow \theta' - \pi/2\) in the formulas Eqs. 12 - 15 leads to elements of \([\sigma_x]\). Thus

\[
(\sigma^c_x)_{11} \rightarrow \cos \theta \sin \theta' - \sin \theta \cos \theta' \cos(\varphi - \varphi') = (\sigma_x)_{11} \tag{31}
\]

\[
(\sigma^c_x)_{12} \rightarrow \cos \theta \cos \theta' \cos(\varphi' - \varphi) + \sin \theta \sin \theta' - i \cos \theta' \sin(\varphi' - \varphi) = (\sigma_x)_{12}, \tag{32}
\]

\[
(\sigma^c_x)_{21} \rightarrow \cos \theta \cos \theta' \cos(\varphi' - \varphi) + \sin \theta \sin \theta' + i \cos \theta' \sin(\varphi' - \varphi) = (\sigma_x)_{21} \tag{33}
\]

and

\[
(\sigma^c_x)_{22} \rightarrow \sin \theta \cos \theta' \cos(\varphi' - \varphi) - \sin \theta' \cos \theta = (\sigma_x)_{22}. \tag{34}
\]

These are just the same expressions as Eq. 23 - 26. The same transformation should yield the eigenvectors of \([\sigma_x]\) if applied to those for \([\sigma^c_x]\). Under this change of argument, we obtain

\[
[\xi^{(+)}_c] \rightarrow [\xi^{(+)}_x] = \begin{pmatrix}
\frac{1}{\sqrt{2}} (\sin \theta'/2 + \cos \theta'/2) \cos \theta'/2 + e^{i(\varphi' - \varphi)} (\sin \theta'/2 - \cos \theta'/2) \sin \theta/2 \\
\frac{1}{\sqrt{2}} (\sin \theta'/2 - \cos \theta'/2) \sin \theta/2 - e^{i(\varphi' - \varphi)} (\sin \theta'/2 - \cos \theta'/2) \cos \theta/2
\end{pmatrix} \tag{35}
\]

and

\[
[\xi^{(-)}_c] \rightarrow [\xi^{(-)}_x] = \begin{pmatrix}
\frac{1}{\sqrt{2}} (\sin \theta'/2 - \cos \theta'/2) \cos \theta'/2 - e^{i(\varphi' - \varphi)} (\sin \theta'/2 + \cos \theta'/2) \sin \theta/2 \\
\frac{1}{\sqrt{2}} (\sin \theta'/2 - \cos \theta'/2) \sin \theta/2 + e^{i(\varphi' - \varphi)} (\sin \theta'/2 + \cos \theta'/2) \cos \theta/2
\end{pmatrix}. \tag{36}
\]
Direct calculation shows that

\[ [\sigma_x] [\xi^{(\pm)}_x] = (\pm 1) [\xi^{(\pm)}_x]. \]  

(37)

For the \( y \) component, the transformations required are \( \theta' = \pi/2 \) and \( \varphi' \rightarrow \varphi' - \pi/2 \). Under these transformations, we obtain

\[ (\sigma^c_y)_{11} \rightarrow \sin \theta \sin(\varphi' - \varphi) = (\sigma^c_y)_{11}, \]  

(38)

\[ (\sigma^c_y)_{12} \rightarrow \cos \theta \sin(\varphi' - \varphi) - i \cos(\varphi' - \varphi) = (\sigma^c_y)_{12}, \]  

(39)

\[ (\sigma^c_y)_{21} \rightarrow \cos \theta \sin(\varphi' - \varphi) + i \cos(\varphi' - \varphi) = (\sigma^c_y)_{21}, \]  

(40)

and

\[ (\sigma^c_y)_{22} \rightarrow -\sin \theta \sin(\varphi' - \varphi) = (\sigma^c_y)_{22}. \]  

(41)

The eigenvectors of \([\sigma^c_x]\) transform to

\[ [\xi^{(+)}_x] \rightarrow [\xi^{(+)}_y] = \left( \frac{1}{\sqrt{2}} \left[ \cos \theta/2 - ie^{i(\varphi' - \varphi)} \sin \theta/2 \right], \frac{1}{\sqrt{2}} \left[ \sin \theta/2 + ie^{i(\varphi' - \varphi)} \cos \theta/2 \right] \right). \]  

(42)

and

\[ [\xi^{(-)}_x] \rightarrow [\xi^{(-)}_y] = \left( \frac{1}{\sqrt{2}} \left[ \cos \theta/2 + ie^{i(\varphi' - \varphi)} \sin \theta/2 \right], \frac{1}{\sqrt{2}} \left[ \sin \theta/2 - ie^{i(\varphi' - \varphi)} \cos \theta/2 \right] \right). \]  

(43)

Again direct calculation proves that

\[ [\sigma_y][\xi^{(\pm)}_y] = (\pm 1)[\xi^{(\pm)}_y]. \]  

(44)

5 Interpretation of the Components of the Spin

The procedure for obtaining the "\( x \)" and "\( y \)" components of the spin operator from the "\( z \)" component may be summarized as follows. To obtain the \( x \) component, we change the final quantization direction in the expression for the operator from \( \hat{c} \) (whose polar angles are \( (\theta', \varphi') \)) to the direction \( \hat{c}_x \) defined by the polar angles \( (\theta' - \pi/2, \varphi') \). In order to obtain the \( y \) component we have to change the final quantization direction from \( \hat{c} \) to the vector \( \hat{c}_y \) whose angles are \( (\theta' = \pi/2, \varphi' - \pi/2) \). The three vectors are

\[ \hat{c} = (\sin \theta' \cos \varphi', \sin \theta' \sin \varphi, \cos \theta') \]  

(45)

\[ \hat{c}_x = (- \cos \theta' \cos \varphi', - \cos \theta' \sin \varphi', \sin \theta') \]  

(46)

and

\[ \hat{c}_y = (\cos \varphi', - \cos \varphi', 0). \]

They are mutually orthogonal:
\[ \hat{c} \cdot \hat{c}_x = \hat{c} \cdot \hat{c}_y = \hat{c}_y \cdot \hat{c} = 0. \]  

Moreover, their vector products are

\[ \hat{c}_x \times \hat{c}_y = \hat{c}; \quad \hat{c}_y \times \hat{c} = \hat{c}_x; \quad \hat{c} \times \hat{c}_x = \hat{c}_y \]  

Hence, these vectors form a new coordinate system whose z axis is defined by \( \hat{c} \), the x axis by \( \hat{c}_x \) and the y axis by \( \hat{c}_y \). These coordinate axes are rotated with respect to the original ones. We may use this information to characterize and interpret the components of spin.

Suppose the spin projection of a particle is known to be up or down along the quantization direction \( \hat{a} \). We then measure it along direction \( \hat{c} \). If we seek the expectation value of the spin projection for this situation, and convert the expression for this expectation value to matrix form by means of the expansion Eq. (47) using the intermediate quantization direction \( \hat{b} \), then the matrix operator that results is the z component of spin. The vector \( \hat{c} \) then defines the z axis of a new rotated coordinate system.

If instead, starting from the same situation, we make a measurement of the spin projection along the quantization direction defined by the positive x axis of the new rotated coordinate system, then the operator that occurs in the matrix expression for the expectation value is the x component of spin. The rotated x axis is defined by the unit vector \( \hat{c}_x \) whose angles are \( (\theta' - \frac{\pi}{2}, \varphi') \). It lies in the same plane as the vector \( \hat{c} \) and the original z axis.

The rotated coordinate system has a y axis defined by the angles \( \left( \frac{\pi}{2}, \varphi' - \frac{\pi}{2} \right) \). If we measure the spin projection along this direction and compute the expectation value by the matrix mechanics formula, using the intermediate quantization axis \( \hat{b} \), the operator that features is the y component of spin.

### 6 Generalized Form for the Square of the Spin

We obtain the elements of the matrix for the square of the spin by squaring the components of the spin and adding them. We now proceed to show that this labour is unnecessary, and that we can obtain these elements by a more general method.

Since the spin is being measured in units of \( \hbar/2 \), the eigenvalue of \( \sigma^2 \) is \( r_1 = r_2 = 3 \). Hence, using Eqs. (5) - (8), we obtain

\[ (\sigma^2)_{11} = 3 \left| \phi((+\frac{1}{2})(\hat{b}); (+\frac{1}{2})(\hat{c})) \right|^2 + \left| \phi((+\frac{1}{2})(\hat{b}); (-\frac{1}{2})(\hat{c})) \right|^2 \]  

\[ (\sigma^2)_{12} = 3 \left| \phi^*((+\frac{1}{2})(\hat{b}); (+\frac{1}{2})(\hat{c})) \phi((-\frac{1}{2})(\hat{b}); (+\frac{1}{2})(\hat{c})) \right|^2 + \left| \phi^*((+\frac{1}{2})(\hat{b}); (-\frac{1}{2})(\hat{c})) \phi((-\frac{1}{2})(\hat{b}); (-\frac{1}{2})(\hat{c})) \right|^2 \]

\[ \left| \phi((-\frac{1}{2})(\hat{b}); (+\frac{1}{2})(\hat{c})) \phi((-\frac{1}{2})(\hat{b}); (-\frac{1}{2})(\hat{c})) \right|^2 \]
\[
(\sigma^2)_{21} = 3[\phi^*((-\frac{1}{2})\hat{b}; (\frac{1}{2})\hat{c})\phi((\frac{1}{2})\hat{b}; (\frac{1}{2})\hat{c})] + [\phi^*((-\frac{1}{2})\hat{b}; (-\frac{1}{2})\hat{c})\phi((\frac{1}{2})\hat{b}; (-\frac{1}{2})\hat{c})]
\]

and

\[
(\sigma^2)_{22} = 3[|\phi((-\frac{1}{2})\hat{b}; (\frac{1}{2})\hat{c})|^2 + |\phi((-\frac{1}{2})\hat{b}; (-\frac{1}{2})\hat{c})|^2]
\]

Using the Landé expansion Eq. (1) and the Hermiticity condition Eq. (2), we see that

\[
(\sigma^2)_{11} = 3[\phi((\frac{1}{2})\hat{b}; (+\frac{1}{2})\hat{c})\phi((+\frac{1}{2})\hat{c}; (+\frac{1}{2})\hat{b})] + [\phi((\frac{1}{2})\hat{b}; (-\frac{1}{2})\hat{c})\phi((+\frac{1}{2})\hat{c}; (-\frac{1}{2})\hat{b})]
\]

\[
= 3\zeta((\frac{1}{2})\hat{b}; (+\frac{1}{2})\hat{b}) = 3
\]

where \(\zeta\) is the probability amplitude for measuring spin projections twice in a row along the same quantization axis. Similarly, we find that

\[
(\sigma^2)_{12} = 3[\phi((\frac{1}{2})\hat{b}; (+\frac{1}{2})\hat{c})\phi((\frac{1}{2})\hat{c}; (+\frac{1}{2})\hat{b})] + [\phi((\frac{1}{2})\hat{b}; (-\frac{1}{2})\hat{c})\phi((\frac{1}{2})\hat{c}; (-\frac{1}{2})\hat{b})]
\]

\[
= 3\zeta((-\frac{1}{2})\hat{b}; (+\frac{1}{2})\hat{b}) = 0
\]

Hence,

\[
(\sigma^2)_{21} = 0 \text{ and } (\sigma^2)_{22} = 3.
\]

7 Discussion and Conclusion

We have presented the eigenvectors of the generalized operators of the \(x\) and \(y\) components of spin. More importantly, we have shown how the components \([\sigma_x]\) and \([\sigma_y]\) can be obtained from the operator \([\sigma_c]\) by transformation of the angles.
We have obtained $[\sigma_x]$ and its eigenvectors by the transformation $\theta' \rightarrow \theta' - \pi/2$ applied to $[\sigma_z]$ and its eigenvectors. Similarly, we have obtained $[\sigma_y]$ and its eigenvectors by the transformations $\theta' = \pi/2$ and $\varphi' \rightarrow \varphi' - \pi/2$. This procedure for obtaining the $x$ and $y$ components of the spin operator from the $z$ component should be of general validity and should therefore save labour when the generalized operators and their eigenvectors for higher values of spin are sought.

We have also proved that the operator for the square of the spin is the unit matrix multiplied by the eigenvalue of the square of the spin. Every two-dimensional vector is an eigenvector of this operator, so that the requirement that the eigenvectors of $[\sigma_z]$ also be the eigenvectors of $[\sigma^2]$ is automatically satisfied.

We have now completed the task of deriving generalized operators and vectors for spin $1/2$ using the Landé interpretation of quantum mechanics. The next task is to extend this approach to spin $1$, both as a way of verifying the validity of the new approach, and to obtain the generalized operators and their eigenvectors for spin $1$.

8 References

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