Electrostatic control of quantum dot entanglement induced by coupling to external reservoirs

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Abstract – We propose a quantum transport experiment to prepare and measure charge-entanglement between two electrostatically defined quantum dots. Coherent population trapping, as realized in cavity quantum electrodynamics, can be carried out by using a third quantum dot to play the role of the optical cavity. In our proposal, a pumping which is quantum mechanically indistinguishable for the quantum dots drives the system into a state with a high degree of entanglement. The whole effect can be switched on and off by means of a gate potential allowing both state preparation and entanglement detection by simply measuring the total current.

Introduction. – Experiments on transport through quantum dots (QDs) have recently experienced such a development that it is now possible to reproduce many of the phenomena that the field of quantum optics, involving atoms, has been exploring for many years [1, 2]. In particular, state preparation and manipulation of one or more QDs is nowadays feasible by controllable external means such as gate and bias potentials plus either continuous or AC electric and/or magnetic fields [3]. Each QD can be considered as a qubit with the lower state |0⟩ corresponding to a neutral QD and the upper state |1⟩ to having one extra electron in the QD. Due to Coulomb blockade, charging the QD with more than one electron requires an energy that can be considered infinite for any practical purpose, reducing the Hilbert space to that spanned by the two mentioned states. A key point for building up logical gates is the capability of producing entanglement between two qubits. Although entanglement effects in transport through double QDs have been extensively studied [2–9], quantum optics’s know-how can be helpful to control such a quantum feature.

The aim of this work is to make a proposal for experimentally preparing and measuring a charge-entangled state of two QDs. We start from the pioneering ideas of ref. [7], where the constraint of having no more than one electron in the whole system allows the population trapping in a dark-entangled state. The same configuration does not give the desired results within a regime more experimentally accessible (more than one electron in total) as we will show below. We obtain interesting results (in experimentally accessible conditions) when adding a new and crucial physical effect borrowed from cavity quantum electrodynamics (CQED): cross-terms in the incoherent pumping [10,11] of two QDs in a microcavity can produce entanglement by driving the system to a quasi-dark state (a state which is only weakly coupled to the cavity) [12]. However, it is difficult to detect this process by optical means. In the framework of transport, the role of the cavity can be played by a third QD and both incoherent pumping of the QDs and cavity photon emission find their counterpart for the transport realization in the tunneling processes produced by the application of a bias. We will show here that, apart from all the analogies, there is an important advantage in doing transport: entanglement could be easily detected in the same setup that prepares the equivalent to the quasi-dark (entangled) state.

The system and its dynamics. – Our system is presented in fig. 1(a). A two-dimensional electron gas is depleted in some regions by means of a series of gate potentials. A bias applied from the left to the right lead produces two tunneling processes: incoherent population of QDs A and B as well as electron current from QD C to the right lead. QDs A and B are coherently coupled to QD C (acting as the cavity in CQED). The gate-potentials $V_4$, $V_5$ and $V_7$ are designed to control the levels of the three qubits, $V_2$ and $V_8$ control the in- (Γ_p) and out- (Γ_x) tunneling rates while the gates $V_5$ and $V_6$ control
Wesimplify the plot by setting detunings to zero. of the dynamics in the Hilbert spaces spanned by $|n_A, n_B, n_C\rangle$ (depicted in fig. 1(b)) with $n_{A,B,C} = 0, 1$. Cross term effects that we are going to concentrate on only occur for electrons with the same spin. We therefore consider the system under the action of an in-plane magnetic field and neglect the spin. Both intra-dot Coulomb blockade and spin polarisation stand within an experimentally accessible regime. To reduce the Hilbert space to the lowest four states in fig. 1(b), as is done in [7], would require an extremely high inter-dot Coulomb repulsion, something unreasonable in a system such as the one shown in fig. 1(a). The coherent part of the dynamics is controlled by the Hamiltonian (we take $\hbar = 1$)

$$H_0 = \sum_{i=A,B} \Delta_i c_i^\dagger c_i + g_{AC} (c_i^\dagger c_C + c_C^\dagger c_i),$$

(1)

where $c_i$, $c_i^\dagger$ are annihilation and creation operators of an electron in QD $i$. We have taken the level of the QD $C$ as the origin of energies so that only the detunings $\Delta_A$, $\Delta_B$ and the couplings $g_{AC}$, $g_{BC}$ are relevant.

Following the methods of quantum optics [1,2,13], we obtain a master equation that describes the whole dynamics of the density matrix $\rho$ within the Born-Markov approximation [10–14]:

$$\frac{d\rho}{dt} = i[\rho, H_0] + ig_{AB}[\rho, (c_A^\dagger c_B + c_B^\dagger c_A)] + \frac{\Gamma_k}{2} \left( 2c_C^\dagger \rho c_C - c_C^\dagger c_C \rho - \rho c_C^\dagger c_C \right) + \frac{\gamma_d}{2} \sum_{i=1}^{8} (2P_i \rho P_i - P_i \rho P_i) + \sum_{i,j=A,B} \Gamma_p \delta_{ij} + \frac{\Gamma_{AB}}{2} (1 - \delta_{ij}) |2c_i^\dagger \rho c_j - c_j^\dagger c_i \rho - \rho c_j c_i^\dagger|,$$

(2)

where $P_i = |n_A, n_B, n_C\rangle \langle n_A, n_B, n_C|$. As is well known, the equation of motion consists of a coherent part and a dissipative part. The coherent part has the contribution of the Hamiltonian dynamics $H_0$ on the first line while the incoherent part features the in- ($\Gamma_p$) and out- ($\Gamma_k$) tunneling processes (on lines 2 and 4), and dephasing at rate $\gamma_d$ (line 3). There are also two extra ingredients which appear in eq. (2) from considering couplings to reservoirs up to the second order [10–14]. These two new terms are responsible for the new physics that appears in this problem.

The first term is an incoherent contribution to the dynamics given by a cross Lindblad term in the pumping part, on line 4, involving operators with different QD indexes. It appears when the QDs $A$ and $B$ are pumped from the same reservoir (left lead) in a complete

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**Fig. 1:** (Color online) (a) Scheme of the proposed setup, with a two-dimensional electron gas depleted by 8 gate potentials $V_1$, $V_3$, $V_4$ and $V_5$ control the levels of the QDs, $V_2$ and $V_6$ control $g_{AC}$ and $g_{BC}$. Switching $V_1$ from “on” to “off” tunes the pumping of $A$ and $B$ from distinguishable to indistinguishable quantum mechanically. The current is induced by a bias from left to right. (b) Scheme of the dynamics in the Hilbert space spanned by $|n_A, n_B, n_C\rangle$. We simplify the plot by setting detunings to zero.
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indistinguishable (quantum) way. In such a case, the corresponding rate of pumping is given by $\Gamma_{AB} = \Gamma_p$. This corresponds to switching off the gate $V_1$. By smoothly switching it on, the upper and lower parts of the left lead separate from each other. $\Gamma_{AB}$ is reduced down to the situation in which the two left reservoirs become completely independent and $\Gamma_{AB} = 0$. Cross terms in the pumping are therefore experimentally controllable.

The second term on the right-hand side of line 1, is a generalization of that contributing to the Lamb-shift of a single two-level system [10–14]. So, a common indistinguishable pumping provides an effective Förster-like coupling between QDs $A$ and $B$ with coupling parameter $g_{AB} = 2\Gamma_{AB}$ [14]. This coupling, which is thus coherent, contributes to enhancing the entanglement between QDs $A$ and $B$, although it stems from the incoherent pumping. Once again, one can experimentally control this mechanism from switching on $V_1$, which gives $g_{AB} = 0$, to switching it off, which gives $g_{AB} = 2\Gamma_p$.

Once the dynamics of the density matrix is known, one can compute the current flowing through the system, which is an easily measured experimental quantity. We use the input-output formulation of optical cavities [13], in which the role of the cavity is played by the QD $C$. The current is simply given by $I = \Gamma_c \langle c C^\dagger c C \rangle$. In the stationary limit, which is the case of greatest interest to us, the master equation (2) simplifies to a set of 64 linear equations, plus the normalization condition $\text{Tr}(\rho) = 1$. In order to correlate the current with the entanglement of $A$ and $B$, we quantify the latter by the tangle [15], that in our case reads

$$\tau = \left[ \text{max} \{ 0, 2 (|\langle 0| - \sqrt{\rho_{00} \rho_{11} \rho_{10} \rho_{11}} |1 \rangle^2) \} \right]^2.$$  \hspace{1cm} (3)

in terms of the matrix elements of $\rho = \text{Tr}_C(\rho)$ [12].

**Results.** – Let us discuss the results to be expected from the setup we propose. We consider that the QDs $A$ and $B$ are equal, $\Delta_A = \Delta_B = \Delta$, what can be achieved by adjusting independently the gates $V_3$ and $V_4$. The entanglement can be manipulated by having different couplings $g_{AC}$ and $g_{BC}$ as controlled by the gates $V_5$ and $V_6$. Hereafter, all the couplings and rates will be given in units of $g_{AC} = 1$.

First of all, we analyze the adequacy of truncating the Hilbert space basis to only four states [7]. For this purpose we consider the simple case of neglecting cross-terms and estimate the total mean number of excitations in the system. For a system that is pumped with a total rate $P_{\text{tot}}$, decays with $\kappa_{\text{tot}}$ and that has a saturation limit $S$, we find that the mean total number of excitations in the strong-coupling regime is given by the expression

$$\langle n \rangle = \frac{S P_{\text{tot}}}{P_{\text{tot}} + \kappa_{\text{tot}}}.$$  \hspace{1cm} (4)

In our case where $P_{\text{tot}} = 2\Gamma_p$ (two QDs each pumped at rate $\Gamma_p$), $\kappa_{\text{tot}} = \Gamma_\kappa$ and $S = 3$ (maximum of three electrons in the system), the general formula eq. (4) gives, in the symmetric case that we consider in this paper, $\Gamma_p = \Gamma_\kappa = \Gamma$:

$$\langle n \rangle = \frac{3(2\Gamma_p)}{2\Gamma_p + \Gamma_\kappa} = 2.$$  \hspace{1cm} (5)

This result, that agrees with numerical calculations, is the first indication of the inadequacy of truncating the Hilbert space to only one electron. Moreover, we find that the relevant magnitudes under study ($I$ and $\tau$) depend strongly on the truncation. Figure 2 shows $I$ and $\tau$ as a function of the detuning for two different dephasing rates, $\gamma_\kappa = 1$ and $\gamma_p = 0.001$, both with truncation (a maximum of one electron in the system) and without truncation (a maximum of three electrons in the system). When truncation is not imposed, $\tau$ is always zero so that entanglement is not achieved.

The approximation of keeping just one electron in the whole system forces the steady state of QDs $A$ and $B$ to be a singlet $|S\rangle = (|100\rangle - |010\rangle)/\sqrt{2}$ (with QD $C$ in the vacuum). This implies a tangle of one and no current passing through the system if the dephasing is negligible [7]. This is an example of a trapping mechanism. The pumping is populating both the singlet and its symmetric counterpart, the triplet state $|T\rangle = (|100\rangle + |010\rangle)/\sqrt{2}$. However, when the couplings are equal $g_{AC} = g_{BC}$, the singlet is dark, does not couple to other states and finally stores all the excitation of the system in the steady state. Therefore, when more than one electron is allowed,
this trapping mechanism breaks as also the states $|11n_C\rangle$ become pumped. In the absence of cross terms, the tangle drops to zero and there is current through the system. A negative result to be drawn from fig. 2 is that without cross terms, in the actual case of more than one electron, there is no entanglement to be expected experimentally.

Our main finding is the entanglement induced by cross terms in the dynamics, enhanced by the coherent coupling between $A$ and $B$. Hereafter we consider the general case, i.e., without truncation to only one electron. Figure 3 shows $I$ and $\tau$ as a function of $\Gamma_{AB}$ for the larger detuning $\Delta = 4$ and the lowest dephasing $\gamma_d = 0.001$ considered in fig. 2. An important fact is that now the couplings $g_{AC}$, $g_{BC}$ must be slightly different (for instance $g_{BC} = 0.7$) so that the singlet is not completely dark, but a quasi-dark state weakly coupled to the rest of the system (with a coupling given by $|g_{AC} - g_{BC}|/\sqrt{2}$). When the gate $V_1$ is completely switched on, $\Gamma_{AB} = 0$ and, as it happened in fig. 2, there is current larger than $I = 0.2$, implying no entanglement. Increasing $\Gamma_{AB}$ by quenching the gate $V_1$ does not affect the behavior of the system until the regime where cross terms apply fully is reached. Here, when $\Gamma_{AB}$ tends to $\Gamma_p$, adding cross terms in eq. (2) translates in pumping only the symmetric states (under QDs $A$, $B$ exchange). Therefore the incoherent pump with cross terms neither excites the singlet nor induces decoherence of it. This fact, together with the weak link between the singlet state and the other levels, results in a slow coherent transfer of population to the singlet $|S\rangle$, which can be described as a quasi-dark state free of decoherence. This novel trapping mechanism is enhanced strongly by the direct coupling $g_{AB}$, also induced by the cross pump. In this case, the tangle becomes close to its highest possible value of 1. The detectable manifestation is a sharp reduction of the current through the system, as QD $C$ is practically empty. This means a clear way of entangled state preparation between QDs $A$ and $B$ as well as a straightforward measurement associated to its occurrence (drop of the current).
Finally, we want to show how entanglement-induced by cross terms depends on the coherent part of the dynamics controlled by $H_0$. For this purpose, fig. 4 presents current $I$ (a) and tangle $\tau$ (b) as a function of the detuning $\Delta$ and the coherent coupling $g_{BC}$ (always in units of $g_{AC}$). The tangle plot shows that detuning is needed to generate a high degree of entanglement. As we explained, also slightly different couplings $g_{AC}$ and $g_{BC}$ are necessary to create the quasi-dark state. In fig. 3 we were giving results for the situation with highest tangle ($\tau = 0.85$), that is $g_{BC} = 0.7$ and $\Delta = 4$, corresponding also to lowest current ($I = 0.01$). On the other hand, for the symmetric case $g_{AC} = g_{BC}$, the singlet is completely dark and therefore there is no entanglement, as we also showed in fig. 2. In this case the current is nonzero. The correlation between high tangle and negligible current and vice versa is clear from fig. 4.

Conclusions. – In conclusion, we present a proposal of quantum transport experiment for preparing and measuring in the steady state a charge-entangled state of two non-interacting QDs: entanglement is produced by means of a quantum mechanically indistinguishable pumping when each QD is coherently coupled to a third one, playing the role of a cavity in CQED. Indistinguishable pumping produces cross Lindblad terms and a coherent direct coupling between the dots. Their combined effect results in a coherent trapping mechanism in an entangled state. This source of entanglement can be switched on and off by means of a gate potential. This allows both state preparation and entanglement detection by simply measuring the total current.

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Note added in proofs: We have recently become aware of another work on entanglement effects in transport through QDs published in *Europhys. Lett.* 76, 905 (2006) by S. Weiss, M. Thorwart and R. Egger.

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