Dissipative dynamics of the semiconductor-cavity QED with $q$-deformed bosons in the dispersive approximation

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Abstract

In this paper we give fully analytical description of the dynamics of a collection of $N$-Frenkel excitons in high density regime dispersively coupled to a single mode cavity field, in the presence of both exciton and cavity-field dissipations. By using excitonic operators as $q$-deformed bosonic operators for the system, we solve analytically the Liouville equation for the density operator at zero temperature and investigate the influence of the number of excitons and the effect of both dissipations on dynamical behavior of the system. We use the solution of master equation to explore the dissipative dynamics of non-classical properties such as, molecule-field entanglement, quadrature squeezing of the field, and molecular dipole squeezing. We find that the non-classical properties are strongly affected by the number of excitons and also by the existence of both dissipations.

1 Introduction

In quantum theory and cavity-quantum electrodynamics (Cavity-QED) of semiconductors where cavity is chosen as a bulk or confined systems such as quantum wells and quantum dots, we work in the low density regime of excitons, and treat these quasiparticles as ideal bosons to solve wide number of problems in quantum optics, cavity-QED and quantum information [1, 2]. Typically, these studies have been done under the condition that the influence of the environment is not taken into account. The environment which is represented by a thermal reservoir always exist, and affects the system under consideration. No matter how weak the coupling to such an environment, the evolution of quantum subsystems is eventually affected by non-unitary features such as decoherence, dissipation and heating. The initial information irreversibly leaks out the system into the very large number of uncontrollable degrees of freedom of the environment. Initially prepared pure states are typically corrupted on extremely short time scales due to quantum coherence loss that turns them into mixed states [3]. The dispersive effects caused by the energy exchange between the system and environment have been studied in the Jaynes-Cummings
model (JCM) [4], both analytically [5] and numerically [6]. In the last few years the JCM with phase damping, as applied to decoherence and entanglement [7], dissipative dynamics of JCM model [8], and nonlinear quantum dissipation effects on dynamical properties of the f-deformed JCM model [9] have been studied. In addition to the exact solvability of JCM within the rotating wave approximation, one of the most interesting aspects of its dynamics is the entanglement between atom and the field. Entanglement as a physical resource has been used in quantum information science such as quantum teleportation [10], super dense coding [11] and quantum cryptography [12]. It has been shown that due to the influence of the field dissipation in JCM under dispersive approximation that the amplitude of the entanglement between the field and the cavity-field decreases with the time and at least completely supresses [8, 9, 13]. Sub-Poissonian statistics and quadrature squeezing of the cavity-field are two remarkable non-classical effects have also been studied in different versions of generalized JCMs [9, 14].

Fascinating electronic and optical properties of spatially confined nanostructures like quantum wells and great potentially of such structures in semiconductor cavity-QED application has motivated permanent extension of their study. Among a variety of new results in this field, it is important to find out the influence of dissipation on non-classical properties, so in our model, we replace one atom in JCM with \( N \)-Frenkel excitons. However, if we want to replace two-level atoms in the cavity with high density approximation of excitons in semiconductor nanostructures, we can’t use the standard Dicke model because the excitons are not more ideal bosons. One way to deal with the problem is to replace ideal bosons with \( q \)-deformed ones in the Dicke model [15]. We also consider the influence of dissipation between excitons and their reservoir, visualized as a large number of harmonic oscillators, and also the influence of cavity-field damping due to its coupling to the environment, on the dynamics of the system. For this purpose, we use the Liouville equation for the density operator to study the dynamical behavior of a dissipative system composed of \( N \)-Frenkel excitons (\( N \)-two level molecules) interacting with a single mode cavity-field and we show the influence of dissipations and the number of molecules on quantum optical properties of system such as, entanglement between the cavity-field and molecules, quadrature field squeezing and also molecular dipole squeezing.

This paper is organized as follows, In Sec.2 we first introduce the Hamiltonian of the system without considering any dissipation in the presence of excitonic operators as \( q \)-deformed bosons. In Sec.3, we give an analytical solution for the master equation to derive the total density operator in the presence of both dissipations. In Sec.4 we employ the analytic results obtained in section.3 to investigate the influence of number of molecules and the effect of both reservoirs on the dynamical properties of molecules and cavity-field. Finally, we summarize our conclusion in Sec.5.

2 The Hamiltonian of the system

We consider our system as \( N \) identical two-level molecules of splitting \( \omega_{eg} \) interacting with a single-mode quantized cavity-field of frequency \( \omega_0 \). We assume that all molecules have equivalent mode position, so they interact with the cavity-field by the same coupling constant \( g \). In the case of relatively high density of molecules in the excited state the Hamiltonian of the system can be written in terms of the exciton operators \( b_q, b_q^\dagger \) (as
where $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators of the quantized cavity-field respectively, and the $q$-deformed bosonic operators $\hat{b}_q$ and $\hat{b}_q^\dagger$ satisfying the $q$-deformed commutation relation

$$\left[\hat{b}_q, \hat{b}_q^\dagger\right]_q = \hat{b}_q \hat{b}_q^\dagger - q \hat{b}_q^\dagger \hat{b}_q = 1, \quad q = 1 - \frac{2}{N},$$

are defined as $\hat{b}_q = \hat{b}_e + g \hat{b}_e \hat{b}_g / \sqrt{N}$, $\hat{b}_q^\dagger = \hat{b}_e \hat{b}_g + g / \sqrt{N}$, in which $\hat{b}_e, \hat{b}_e^\dagger (\hat{b}_g, \hat{b}_g^\dagger)$ are the annihilation and creation (creation and annihilation) operators of one molecule in the excited state and the ground state, respectively [15]. Following Peixoto et all, [8], in the large detuning approximation (dispersive limit), that is

$$|\delta| \gg \sqrt{n_{ph} + 1}, \quad \delta = \omega_{eg} - \omega_0,$$

for any relevant photon number $n_{ph}$, we obtain the following effective interaction Hamiltonian

$$H_{eff} = \omega A_{m,n} \left\{ |n, m\rangle \langle n, m| (\hat{a}^\dagger \hat{a} + 1) + \hat{a}^\dagger \hat{a} |n - 1, m + 1\rangle \langle n - 1, m + 1| \right\},$$

where we have defined $A_{m,n} \equiv n(m + 1)$, and $\omega \equiv g^2 / \delta$. The state $|n, m\rangle$ indicates that $n$ molecules ($m = N - n$ molecules) are in the excited state (ground state). Furthermore, we have used

$$\hat{b}_q |n, m\rangle = \frac{1}{\sqrt{N}} \hat{b}_e \hat{b}_g |n, m\rangle = \frac{\sqrt{n(m + 1)}}{\sqrt{N}} |n - 1, m + 1\rangle,$$

$$\hat{b}_q^\dagger |n, m\rangle = \frac{1}{\sqrt{N}} \hat{b}_e^\dagger \hat{b}_g |n, m\rangle = \frac{\sqrt{(n + 1)m}}{\sqrt{N}} |n + 1, m - 1\rangle.$$

The effective Hamiltonian (2.4) which does not cause any transition in the system creates an entanglement between the molecular and the field states.

### 3 The master equation and its analytical solution

We assume that our system is coupled with two types of reservoirs, one of them is coupled to the cavity-field and the other is coupled to molecules. The time evolution of the collection of $N$ two-level molecules interacting dispersively with a single mode cavity-field in the interaction picture can be described by the following master equation

$$\frac{d}{dt} \hat{\rho}(t) = -i \left[ H_{eff}, \hat{\rho}(t) \right] + \hat{D}_{molecules} \hat{\rho}(t) + \hat{D}_{field} \hat{\rho}(t), \quad \hbar = 1,$$
where \( \dot{\rho}(t) \) is the density operator of the system, and belongs to the set \( D(\mathcal{H}_M \otimes \mathcal{H}_F) \) of the trace class operators acting in the space corresponding to the direct product of the two Hilbert spaces \( \mathcal{H}_M \) and \( \mathcal{H}_F \) of the molecules and the field, \( H_{\text{eff}} \) is given by (2.4), and the damping of the cavity-field and molecular subsystems are phenomenologically represented by the superoperators \( \dot{D}_{\text{field}} \) and \( \dot{D}_{\text{molecules}} \). By using thermal reservoir at zero temperature for the field and one atom \([16, 17]\) and the same one for molecules, using \( q \)-deformed bosonic operators instead of atomic operators, we can write these superoperators as

\[
\dot{D}_{\text{field}} = k(2\hat{a}\cdot\hat{a}^\dagger - \hat{a}^\dagger\hat{a} - \cdots \hat{a}^\dagger\hat{a}),
\]

\[
\dot{D}_{\text{molecules}} = k'(2\hat{b}_q\cdot\hat{b}_q^\dagger - \hat{b}_q^\dagger\hat{b}_q - \cdots \hat{b}_q^\dagger\hat{b}_q),
\]

where \( k \) and \( k' \) are the corresponding dissipation constants. These superoperators are linear combinations of bosonic superoperators, and form a finite Lie algebra under commutation. The bosonic superoperators represent the action of creation and annihilation operators of the harmonic oscillator on an operator \( \hat{O} \):

\[
(\hat{a})\hat{O} \equiv a^\dagger\hat{O} \equiv \hat{a}\hat{O}, \quad (\hat{a}^\dagger)\hat{O} \equiv (a^\dagger)^\dagger\hat{O} \equiv \hat{a}^\dagger\hat{O},
\]

\[
(\hat{b}_q)\hat{O} \equiv b_q^\dagger\hat{O} \equiv \hat{b}_q\hat{O}, \quad (\hat{b}_q^\dagger)\hat{O} \equiv (b_q^\dagger)^\dagger\hat{O} \equiv \hat{b}_q^\dagger\hat{O}.
\]

Similarly, the action of \( q \)-deformed creation and annihilation operators \( \hat{b}_q^\dagger \) and \( \hat{b}_q \) on an operator \( \hat{O} \) can be defined as

\[
(\hat{b}_q)\hat{O} \equiv b_q^\dagger\hat{O} \equiv \hat{b}_q\hat{O}, \quad (\hat{b}_q^\dagger)\hat{O} \equiv (b_q^\dagger)^\dagger\hat{O} \equiv \hat{b}_q^\dagger\hat{O},
\]

To solve the master equation (3.7), we rewrite it in the basis of molecular states \( |n, m\rangle \) (note that there is just one mode of the cavity-field in the cavity which can interact with one molecule)

\[
\begin{pmatrix}
\dot{\rho}_{n,n} & \dot{\rho}_{n,n-1} \\
\dot{\rho}_{n-1,n} & \dot{\rho}_{n-1,n-1}
\end{pmatrix} =
\begin{pmatrix}
\hat{X}_{n,n} & \hat{X}_{n,n-1} \\
\hat{X}_{n-1,n} & \hat{X}_{n-1,n-1}
\end{pmatrix}. \tag{3.12}
\]

where \( \dot{\rho}(t) \equiv \frac{d}{dt}\rho(t) \), and we have defined

\[
\rho_{n,n}(t) \equiv \langle n, m | \rho(t) | n, m \rangle, \tag{3.13a}
\]

\[
\rho_{n,n-1}(t) \equiv \langle n, m | \rho(t) | n-1, m+1 \rangle, \tag{3.13b}
\]

\[
\rho_{n-1,n}(t) \equiv \langle n-1, m+1 | \rho(t) | n, m \rangle, \tag{3.13c}
\]

\[
\rho_{n-1,n-1}(t) \equiv \langle n-1, m+1 | \rho(t) | n-1, m+1 \rangle, \tag{3.13d}
\]

Furthermore

\[
\hat{X}_{n,n} = \{ iA_{n,n}\omega(P - M) - 2B_{n,n}k' + k(2J - M - P) \} \dot{\rho}_{n,n}(t), \tag{3.14a}
\]

\[
\hat{X}_{n,n-1} = \{-iA_{n,n}\omega(M + P + 1) - B_{n,n}k' + k(2J - M - P) \} \dot{\rho}_{n,n-1}(t), \tag{3.14b}
\]

\[
\hat{X}_{n-1,n} = \{ iA_{n,n}\omega(M + P + 1) - B_{n,n}k' + k(2J - M - P) \} \dot{\rho}_{n-1,n}(t), \tag{3.14c}
\]

\[
\hat{X}_{n-1,n-1} = \{ iA_{n,n}\omega(M - P) + k(2J - M - P) \} \dot{\rho}_{n-1,n-1}(t) + 2B_{n,n}^2k'\dot{\rho}_{n,n}(t), \tag{3.14d}
\]
are operators that act in $\mathcal{H}_F$ (remember that $n$ shows the number of excited molecules). In Eqs. (3.14), we have defined $B_{n,m} \equiv \sqrt{n(m+1)}/\sqrt{N} = \sqrt{A_{m,n}}/\sqrt{N}$, and also $M \equiv \hat{a}^\dagger \hat{a}$, $P \equiv -\hat{a}^\dagger \hat{a}$, $J \equiv \hat{a} \cdot \hat{a}^\dagger$, which satisfy the following commutation relations

$$[J, M] = J, \quad [J, P] = J, \quad [M, P] = 0.$$  

In order to find the total density operator we need to solve the four Liouvillian equations associated with each matrix element. For any initial state, the total density operator can be evaluated from the solution of Eq (3.12). We assume that the initial state of the interacting system is given by

$$|\psi_i\rangle = \frac{1}{\sqrt{2}} \{ |n, m\rangle + |n-1, m+1\rangle \} \otimes |\alpha\rangle,$$  

which means that at $t = 0$, molecules are in the superposition of two states $|n, m\rangle$ and $|n-1, m+1\rangle$, and the cavity-field is prepared in the Glauber coherent state. After some calculation we evaluate the matrix elements of the field density operator, using initial condition (3.10), as

$$\hat{\rho}_{n,n}(t) = \frac{1}{2} e^{-t B_{n,m}^2} \left| \alpha e^{-i t A_{m,n} \omega} \right\rangle \langle \alpha e^{-i t A_{m,n} \omega} \right|$$  

(3.17a)

$$\hat{\rho}_{n-1,n}(t) = e^{-t B_{n,m}^2} \left| \alpha e^{-i t A_{m,n} \omega} \right\rangle \langle \alpha e^{-i t A_{m,n} \omega} \right|$$  

(3.17b)

$$\hat{\rho}_{n-1,n-1}(t) = \frac{1}{2} \left| \alpha e^{-i t A_{m,n} \omega} \right\rangle \langle \alpha e^{-i t A_{m,n} \omega} \right| + \frac{1}{2} \left| e^{-t \alpha^2 e^{-2kt}} \right\rangle \langle e^{-t \alpha^2 e^{-2kt}} \right|$$

(3.17c)

$$\times \sum_{n,n'} \left( \frac{\alpha^2 e^{-i A_{m,n} \omega} e^{-i A_{m,n} \omega}}{\sqrt{n!} \sqrt{n'!}} |n\rangle \langle n'| \right) \frac{2 B_{n,m}^2 k'}{-2 B_{n,m}^2 k' + 2i A_{m,n} \omega (n' - n)}$$

(3.17d)

where the definitions

$$\Gamma(t) = -|\alpha|^2 \left( 1 - e^{-2kt} \right)$$

$$-\frac{|\alpha|^2 k}{k^2 + A_{m,n}^2 \omega^2} \left( e^{-2kt} (k \cos(2A_{m,n} \omega t) - A_{m,n} \omega \sin(2A_{m,n} \omega t)) - k \right),$$  

(3.18)

$$\Theta(t) = -A_{m,n} \omega t + \frac{|\alpha|^2 k}{k^2 + A_{m,n}^2 \omega^2}$$

$$\times \left( e^{-2kt} (k \sin(2A_{m,n} \omega t) + A_{m,n} \omega \cos(2A_{m,n} \omega t)) - A_{m,n} \omega \right)$$  

(3.19)

have been used. As a result, the total density operator of the interacting system can be written as

$$\hat{\rho}_{\text{total}}(t) = \hat{\rho}_{n,n}(t) \otimes |n, m\rangle \langle n, m| + \hat{\rho}_{n-1,n}(t) \otimes |n, m\rangle \langle n-1, m+1|$$

$$+ \hat{\rho}_{n-1,n-1}(t) \otimes |n-1, m+1\rangle \langle n, m| + \hat{\rho}_{n-1,n-1}(t) \otimes |n-1, m+1\rangle \langle n-1, m+1|.$$  

(3.20)
By taking the trace of the total density operator \( \rho_{\text{total}} \) on the molecular (field) variables, we get the reduced field (molecular) density operator: \( \hat{\rho}_f(t) = Tr_{\text{molecule}}(\hat{\rho}_{\text{total}}(t)) \) (\( \hat{\rho}_m(t) = Tr_{\text{field}}(\hat{\rho}_{\text{total}}(t)) \)). In the next two sections we are going to use the solution given by (3.20) to investigate the dynamical properties of the system.

### 4 Dynamical properties of the model

#### 4.1 Linear entropies and molecules-field entanglement

It is well-known that if the cavity-field and molecules are initially prepared in a pure state, then at \( t > 0 \) the molecules-field system evolves into an entangled state. In this entangled state the field and molecules are separately in mixed states. The stability of a pure state may be understood as the process where quantum coherence of the state is preserved along its time evolution. In this sense we say that an initial pure quantum state, described by the density operator \( \hat{\rho}(t) \) is stable if \( Tr\hat{\rho}^2(t) = 1 \), for all times. One way to measure the stability of an initial pure state is to use the linear entropy \[ s = 1 - Tr\hat{\rho}^2(t). \] (4.21)

The time evolution of the molecule(field) entropy reflects the time evolution of the degree of entanglement between the molecules and the field. The higher the entropy is, the greater the entanglement between the molecule and the field becomes. By using Eqs. (3.20) and (4.21) the linear entropy of the total system under consideration is obtained as follows

\[
s_{m-f}(t) = 1 - Tr \left( \hat{\rho}_{\text{total}}^2(t) \right) = \frac{3}{4} \left( 1 + e^{-4B_{m,m}^2k't} + 2e^{-2\Gamma t}e^{-2B_{m,m}^2k't} + 2e^{-2\Gamma t}e^{-2B_{m,m}^2k't} \right)
+ 2e^{-2|\alpha|^2} \sum_{n,n'} \frac{(|\alpha|^2e^{-2kt})^{n+n'}}{n!n'!} \frac{1}{(-2B_{m,m}^2k')^2 + 4A_{m,m}^2\omega^2(n-n')^2}
\times \left( (-2B_{m,m}^2k')^2 \left( e^{-2B_{m,m}^2k't} \cos(2A_{m,m}\omega(n-n')t) - 1 \right)
+ 4A_{m,m} \omega B_{m,m}^2 \left( n - n' \right) e^{-2B_{m,m}^2k't} \sin(2A_{m,m}\omega(n-n')t) \right)
+ e^{-2|\alpha|^2} \sum_{n,n'} \frac{(|\alpha|^2e^{-2kt})^{n+n'}}{n!n'!} \frac{1}{(-2B_{m,m}^2k')^2 + 4A_{m,m}^2\omega^2(n-n')^2}
\times \left\{ e^{-4B_{m,m}^2k't} + 2e^{-2B_{m,m}^2k't} \cos(2A_{m,m}\omega(n-n')t) + 1 \right\}. \] (4.22)

Furthermore, the linear entropy of the cavity-field is given by

\[
s_f(t) = 1 - Tr_f \left( \hat{\rho}_f^2(t) \right) = \frac{3}{4} \left( 1 + e^{-4B_{n,n}^2k't} + 2e^{-2\Gamma t}e^{-2B_{n,n}^2k't} + 2e^{-2\Gamma t}e^{-2B_{n,n}^2k't} \right)
+ 2e^{-2|\alpha|^2} \sum_{n,n'} \frac{(|\alpha|^2e^{-2kt})^{n+n'}}{n!n'!} \frac{1}{(2B_{n,n}^2k')^2 + 4A_{m,m}^2\omega^2(n-n')^2}
\times \left\{ e^{-4B_{n,n}^2k't} + 2e^{-2B_{n,n}^2k't} \cos(2A_{m,m}\omega(n-n')t) + 1 \right\}
+ 2e^{-2B_{n,n}^2k't} + 1 \right) e^{-2|\alpha|^2} \sum_{n,n'} \frac{(|\alpha|^2e^{-2kt})^{n+n'}}{n!n'!} \frac{1}{(2B_{n,n}^2k')^2 + 4A_{m,m}^2\omega^2(n-n')^2}
\times \left\{ (-2B_{n,n}^2k')^2 \left( e^{-2B_{n,n}^2k't} \cos(2A_{m,m}\omega(n-n')t) - 1 \right)
+ 4A_{m,m}^2 \omega B_{m,m}^2 \left( n - n' \right) e^{-2B_{m,m}^2k't} \sin(2A_{m,m}\omega(n-n')t) \right\}, \] (4.23)
and the molecular coherence loss will be measured by

\[ s_m(t) = 1 - Tr_m \left( \rho_m^2(t) \right) \]

\[ = \frac{1}{4} \left( e^{-4B_{n,m}^2k't} + 2e^{-2B_{n,m}^2k't} e^{\alpha} e^{-2k't(2\cos(2A_{n,m}\omega t) - 2)} + 4 + e^{-4B_{n,m}^2k't} - 4e^{-2B_{n,m}^2k't} \right). \]  

(4.24)

According to Eqs. (4.22)-(4.24), we plot the time evolution of the linear entropy of the total system, the cavity-field and the molecular subsystem as functions of scaled time \( \omega t \), in Figs. 1.

First, we discuss the coherence loss of the field. For all diagrams in Figs. 1, the field is initially prepared in the coherence state and \( s_f(t) = 0 \), but in a short stage \( s_f(t) \) increases and the coherence of the field loses (of course the equilibrium state of the field correspond to the vacuum, for which the linear entropy of the field is zero). The linear entropy of the field \( s_f \) shows local maxima and minima, corresponding to the entanglement and disentanglement between the field and excitons. In the presence of molecular dissipation (Figs. 1a, 1b), the field linear entropy still exhibits local maxima and minima, but minimum of disentanglement is not taking place at \( s_f = 0 \) and the field is not in a pure state. In the absence of molecular dissipation, in figure1c, when disentanglement happens periodically, the field is in a pure state \( (s_F = 0) \).

To verify the role of two types of dissipations on \( s_m \) and \( s_{m-f} \), we discuss the coherence loss of the molecules and of the system. We find that the linear entropy of the system and of molecules exhibit periodic behavior, and these periods coincide with the related periods of \( s_f \). Note that the field and molecules are not in pure state. In the course of time evolution, time, both \( s_m(t) \) and \( s_{m-f}(t) \), in the presence of molecular dissipation (Figs. 1a, 1b), find the maximum values (corresponding to maximum coherence loss), and then these linear entropies decrease smoothly to find their asymptotic values at zero (the equilibrium state of molecules corresponds to the state that all molecules are in the ground state and the linear entropies are zero). Furthermore, in the absence of molecular dissipation, these two entropies tend to an asymptotic non-zero value (Figs. 1c, 1d). We also show the linear entropies for one molecule in the absence of molecular dissipation (Fig. 1d), and find the same results as the usual JCM given in reference [8].

4.2 Quadrature squeezing of the cavity-field

In the past two decades, there has been major studies focused on the fluctuations in the quadrature amplitude of the electromagnetic field to produce squeezed light. This light is indicated by having less noise in one field quadrature than vacuum state with an excess of noise in the conjugate quadrature such that the product of canonically conjugate variances must satisfy the uncertainty relation. In the studies of quantum optics theory, this light occupies a wide area because of the various applications, e.g., in optical communication networks [19], in interferometric techniques [20], and in optical waveguide tap [21]. Furthermore, investigation of the squeezing properties of the radiation field is a central topic in quantum optics and noise squeezing can be measured by means of homodyne detection [17].

In order to investigate the quadrature squeezing of the model under consideration, we introduce the two slowly varying Hermition operators \( \hat{X}_{1a}(t) \) and \( \hat{X}_{2a}(t) \) defined by,
respectively,
\[
\hat{X}_{1a}(t) = \frac{1}{2} \left( \hat{a} e^{i\omega_0 t} + \hat{a}^\dagger e^{-i\omega_0 t} \right), \quad \hat{X}_{2a}(t) = \frac{1}{2} \left( \hat{a} e^{i\omega_0 t} - \hat{a}^\dagger e^{-i\omega_0 t} \right)
\]  
(4.25)

A state of the field said to be squeezed when one of the quadrature components \(\hat{X}_{1a}(t)\) and \(\hat{X}_{2a}(t)\) satisfies the relation
\[
\left\langle \left( \Delta \hat{X}_{ia}(t) \right)^2 \right\rangle < \frac{1}{4}, \quad (i = 1, 2).
\]  
(4.26)

The degree of squeezing can be measured by the squeezing parameter \(s_i(i = 1, 2)\) defined by
\[
s_i(t) = 4 \left\langle \left( \Delta \hat{X}_{ia}(t) \right)^2 \right\rangle - 1.
\]  
(4.27)

Therefore the condition for squeezing in the quadrature component can be simply written as \(s_i(t) < 0\). In Figs. 2, we have plotted the squeezing parameter \(s_1(t)\) given by
\[
s_1(t) = 2e^{-2|\alpha|^2} e^{-2kt} \sum_n \frac{(|\alpha|^2 e^{-2kt})^n}{n!} n + 2 \left\{ e^{-2B_{n,m}^2 k't} |\alpha|^2 e^{-2kt} \cos(2(\omega_0 - A_{m,n}\omega)t) \right. \\
+ |\alpha|^2 e^{-2kt} \cos(2(\omega_0 + A_{m,n}\omega)t) \left. \frac{1}{(2B_{n,m}^2 k')^2 + 4A_{m,n}\omega^2(-2)^2} \right\} \\
\times \left\{ - (2B_{n,m}^2 k')^2 \left( e^{-2B_{n,m}^2 k't} \cos(4A_{m,n}\omega t) - 1 \right) + 8A_{n,m}^2 \omega^2 B_{n,m}^2 k' e^{-2B_{n,m}^2 k't} \sin(4A_{m,n}\omega t) \right\} \\
+ |\alpha|^2 e^{-2kt} \sin(2(\omega_0 + A_{m,n}\omega)t) \left. \frac{1}{(2B_{n,m}^2 k')^2 + 4A_{m,n}\omega^2(-2)^2} \right\} \\
\times \left\{ - 8A_{m,n}^2 \omega^2 B_{n,m}^2 k' \left( e^{-2B_{n,m}^2 k't} \cos(4A_{m,n}\omega t) - 1 \right) - (2B_{n,m}^2 k')^2 e^{-2B_{n,m}^2 k't} \sin(4A_{m,n}\omega t) \right\} \\
+ 4 \left\{ e^{-2B_{n,m}^2 k't} \alpha e^{-kt} \cos((\omega_0 - A_{m,n}\omega)t) \right. \\
+ \alpha e^{-kt} \cos((\omega_0 + A_{m,n}\omega)t) \left. \frac{1}{(2B_{n,m}^2 k')^2 + 4A_{m,n}\omega^2} \right\} \\
\times \left\{ - (2B_{n,m}^2 k')^2 \left( e^{-2B_{n,m}^2 k't} \cos(2A_{m,n}\omega t) - 1 \right) + 4A_{n,m}^2 \omega^2 B_{n,m}^2 k' e^{-2B_{n,m}^2 k't} \sin(2A_{m,n}\omega t) \right\} \\
+ \alpha e^{-kt} \sin((\omega_0 + A_{m,n}\omega)t) \left. \frac{1}{(2B_{n,m}^2 k')^2 + 4A_{m,n}\omega^2} \right\} \\
\times \left\{ - 4A_{m,n}^2 \omega^2 B_{n,m}^2 k' \left( e^{-2B_{n,m}^2 k't} \cos(2A_{m,n}\omega t) - 1 \right) - (2B_{n,m}^2 k')^2 e^{-2B_{n,m}^2 k't} \sin(2A_{m,n}\omega t) \right\} \right\}^2,
\]  
(4.28)

versus the scaled time \(\omega t\) for the corresponding data used in Figs. 1. As it is seen, the squeezing parameter \(s_1(t)\) shows fast oscillations and the quadrature component \(\hat{X}_{1a}\) does not exhibit squeezing. In the presence of both dissipations (Fig. 2a), squeezing parameter \(s_1(t)\) present local maxima and minima due to the field interaction with molecules. In the presence of molecular dissipation (Figs. 2a, 2b), the field is no longer in a pure state, except initial time and equilibrium state (corresponding to the vacuum), and in the absence of molecular dissipation (Fig. 2c), periodically, the field can be find in pure state.

### 4.3 Molecular dipole squeezing

In this section we are going to discuss about the dynamical behavior of excitonic ensemble property that is molecular dipole squeezing.
To analyze the quantum fluctuation dipole variables, we consider $\hat{\sigma}_x$ and $\hat{\sigma}_y$ corresponding to the dispersive and absorptive components of the amplitude of molecular polarization\[22\]

$$
\hat{\sigma}_x(t) = \frac{1}{2}(\hat{b}_q^\dagger e^{-i\omega_{eg}t} + \hat{b}_q e^{i\omega_{eg}t}),
\hat{\sigma}_y(t) = \frac{1}{2i}(\hat{b}_q^\dagger e^{-i\omega_{eg}t} - \hat{b}_q e^{i\omega_{eg}t}).
$$

The fluctuations in the component $\hat{\sigma}_i (i = x \text{ or } y)$ are said to be squeezed if the variance in $\hat{\sigma}_i$ satisfies the condition

$$
\langle (\Delta \hat{\sigma}_x(t))^2 \rangle < \frac{1}{4} \langle |\hat{\sigma}_z(t)| \rangle (i = x \text{ or } y).
$$

Since $\langle \hat{\sigma}_i^2(t) \rangle = 1/4$ the condition of dipole squeezing may be written as

$$
F_i(t) = 1 - 4 \langle |\hat{\sigma}_i(t)|^2 \rangle - \langle |\hat{\sigma}_z(t)| \rangle < 0 \quad (i = x \text{ or } y) \tag{4.30}
$$

We find $F_y(t)$, corresponding to the squeezing of $\hat{\sigma}_y(t)$ for the system under consideration, as follows

$$
F_y(t) = 1 - 4 \left\{ \frac{1}{2} B_{n,m} e^{\Gamma t} e^{-B_{n,m}^2 kt} e^{-|\alpha|^2 e^{-2kt(\cos(2A_{m,n} \omega t)-1)}} \sin(\omega_{eg}t + \theta - |\alpha|^2 e^{-2kt} \sin(2A_{m,n} \omega t)) \right\}^2. \tag{4.31}
$$

To investigate the influence of both kinds of dissipations, we plot the time evolution of $F_y(t)$ in Figs. 3-6. As it is seen, in the course of time evolution, the function $F_y(t)$ shows rapid oscillations which are damped in the presence of dissipations, and under the condition that $B_{n,m} \neq 1$, dipole squeezing can occur. In Figs. 3, we show the influence of dissipations on $F_y(t)$, in the presence of both dissipations (figure 3a), in the absence of field dissipation (figure 3b), and in the absence of molecular dissipation (figure 3c).

We can see that with suppression of dissipations in Figs. 3b,3c, the rate of damping in oscillations decreases, and in the absence of molecular dissipation the oscillations continue to narrow band between two positive numbers. In Figs. 4a-4c, we show the influence of total number of excitons $N$ on $F_y(t)$. By increasing the number of excitons the squeezing can be much stronger and the oscillations damped faster.

Note that in all diagrams, under the condition $B_{n,m} \neq 1$, dipole squeezing appears and for any total number of excitons $N$, as it is clear in relation (1.31), for larger numbers of $B_{n,m}$, the effect of squeezing will be stronger. In Figs. 5a-5c, we show the influence of the parameter $B_{n,m}$ on $F_y(t)$. As we can see, with $B_{n,m} = 1$ there is no squeezing for $F_y(t)$. There is a an interesting instance for when molecular dissipation can be suppressed that by increasing the number of molecules, $F_y(t)$ shows squeezing all the time, periodically (figure.6a,6b), and by increasing the number of molecules it will be much stronger.

## 5 Summary and Conclusions

In this paper, we have studied theoretically the influence of the dissipations on non-classical properties of the system as $N$-Frenkel excitons interacting with a single mode cavity-field in the dispersive approximation. Using $q$-deformed bosonic operators for excitons, we obtained an effective interaction Hamiltonian and we used it to find the master equation containing molecular dissipation and cavity-field dissipation. By solving the master equation, the total density operator was obtained and some of non-classical properties were studied. In continue we summarize our conclusions:
First, we discussed the linear entropy to find the coherence loss of the cavity-field \( s_f(t) \), molecules \( s_m(t) \) and of the system \( s_{m-f}(t) \). We found that the linear entropies present local maxima and minima corresponding to entanglement and disentanglement due to the field interaction with excitons. As the time goes on, in the presence of both dissipations, all linear entropies decrease smoothly to find their asymptotic zero values.

Second, we discussed about quadrature squeezing of the field. We found that in the presence of dissipations, squeezing parameter \( s_1(t) \) shows fast oscillations with local maxima and minima due to the field interaction with molecules and the quadrature component \( \hat{X}_{1a} \) does not exhibit squeezing.

In the last section we discussed the influences of number of molecules and both dissipations on the temporal evolution of molecule dipole squeezing \( F_y(t) \). We found that \( F_y(t) \) shows rapid oscillations between positive and negative values that the minus values corresponding to the squeezing of \( \sigma_y(t) \) and squeezing can be occur when \( B_{n,m} \neq 1 \) (the parameter \( B_{n,m} \) has introduced in section 3, is related to the number of molecules which are in the excited and ground state), and so there is no squeezing when we have just one molecule in the cavity. We also found that by increasing the number of molecules, the squeezing can be much stronger and oscillations of \( F_y(t) \) damped faster.

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References

[1] H. Haug, S. W. Koch, Quantum Theory of The Optical and Electronic Properties of Semiconductors (World Scientific Publishing Co. Ltd, 1993).

[2] Y. Yamamoto, F. Tassone, and H. Cao, Semiconductor Cavity Quantum Electrodynamics (Springer-Verlag Berlin Heidelberg New York, 2000).

[3] R. Omnes, Phys. Rev. A 56, 3383 (1997); W. H. Zurek, Phys. Rev. D 24, 1516 (1981); H. Zurek, Phys. Today 44, 36 (1991).

[4] E. T. Jaynes, F.W. Cummings, IEEE 51, 89 (1963).

[5] S. M. Barnett, P. L. Knight, Phys. Rev. A 33, 2444 (1986); R. R. Puri, G. S. Agarwal, Phys. Rev. A 35, 3433 (1987); H. J. Briegel, B. G. Englert, Phys. Rev. A 47, 3311 (1993); A. J. Wouteren, Phys. Rev. A 56, 3116 (1997); A. Lindner, H. Freese, G. Quehl, D. Rei, K. Schiller, Eur. Phys. J. D 17, 99 (2001).

[6] J. Eiselt, H. Risken, Opt. Commun. 72, 351 (1989); J. Gea-Banachloce, Phys. Rev. A 47, 2221 (1993); B. G. Englert, N. Naraschevski, D. Schezl, Phys. Rev. A 50, 2667 (1994).

[7] L. M. Kuang, X. Cheng, G. H. Chen, M. L. Ge, Phys. Rev. A 56, 3139 (1997); H. A. Hessian, H. J. Ritsch, J. Phys. B: At. Mol. Opt. Phys. 35, 4619 (2002).

[8] J. G. Peixoto de Faria, and M. C. Nemes, Phys. Rev. A 59, 3918 (1999).
[9] M. H. Naderi and M. Soltanolkotabi, Eur. Phys. J. D 39, 471 (2006).
[10] D. S. Freitas, A. Vidiella-Barranco, J. A. Roversi, Phys. Lett. A249, 275 (1998).
[11] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. 70, 1895 (1993).
[12] C. H. Bennett, S. J. Wiesner, Phys. Rev. Lett. 69, 2881 (1992).
[13] L. Zhou, H. S. Song, and Y. X. Luo, J. Opt. B: Quantum Semiclass. Opt. 4, 103 (2002).
[14] V. V. Dodonov, W. D. José, and S. S. Mizrahi, J. Opt. B: Quantum Semiclass. Opt. 5, 567 (2003); Le-Man Kuang, Xin Chen, Guang-Hong Chen, and Mo-Lin Ge, Phys. Rev. A56, 3139 (1997); Le-Man Kuang, Xin Chen, and Mo-Lin Ge, Phys. Rev. A 52, 1857 (1995).
[15] Yu-Xi Liu, C. P. Sun, S. X. Yu, and D. L. Zhou, Phys. Rev. A 63, 023802 (2001).
[16] A. Royer, Phys. Rev. A43, 44 (1991); A. Royer, Phys. Rev. A45, 793 (1992); S. J. Wang, M. C. Nemes, A. N. Salguero, H. A. Weidenmuller, Phys. Rev. A66, 033608 (2002).
[17] M. O. Scully, M. S. Zubairy, Quantum Optics (Cambridge University Press, Cambridge, 1997).
[18] W. H. Zurek, S. Habib, J. P. Paz, Phys. Rev. Lett. 70, 1187 (1993); J. I. Kim, M. C. Nemes, A. F. R. de Toledo Piza, H. E. Borges, Phys. Rev. Lett. 77, 207 (1996); A. Isar, A. Sandulescu, W. Scheid, Phys. Rev. E60, 6371 (1999).
[19] H. P. Yuen, J. H. Shapiro, IEEE Trans. Inform. Theory IT-24, 657 (1978); J. H. Shapiro, H. P. Yuen, M. J. A. Machado, IEEE Trans. Inform. Theory IT-25, 179 (1979); H. P. Yuen, J. H. Shapiro, IEEE Trans. Inform. Theory IT-26, 78 (1980).
[20] C. M. Caves, B. L. Schumaker, Phys. Rev. A 31, 3068 (1985); B. L. Schumaker, C. M. Caves, Phys. Rev. A 31, 3093 (1985).
[21] J. H. Shapiro, Opt. Lett. 4, 351 (1980)
[22] S. M. Barnett, Opt. Commun. 61, 432 (1982); P. Zhou, J. S. Peng, Phys. Rev. A 44, 3331 (1991).
**Figure 1.** Time evolution of the linear entropy $s_{a-f}(t)$ (black solid curve), $s_f(t)$ (gray solid curve) and $s_a(t)$ (dashed curve) as functions of scaled time $\omega t$, for $\alpha = 1$, $N = 10$ and (Fig.1a) $k = k' = 0.05\omega$, $n = m = 5$ (Fig.1b); $k = 0$, $k' = 0.05\omega$ (Fig.1c); $k = 0.05\omega$, $k' = 0$, $A_{n,m} = B_{n,m} = 1$ (Fig.1d).

**Figure 2.** The time evolution of $s_1$ as a function of scaled time $\omega t$, for $N = 10$, $n = m = 5$ $\alpha = 1$, and for $k = k' = 0.05\omega$ (Fig.2a); $k = 0$, $k' = 0.05\omega$ (Fig.2b); $k = 0.05\omega$, $k' = 0$ (Fig.2c).

**Figure 3.** The time evolution of $F_y(t)$ as a function of scaled time $\omega t$, for $N = 10$, $m, n = 5$, $\alpha = 1$, and when $B_{n,m} \neq 1$: in the presence of both dissipations $k = k' = 0.05\omega$ (Fig.3a); in the absence of field dissipation $k = 0$, $k' = 0.05\omega$ (Fig.3b); in the absence of molecular dissipation $k = 0.05\omega$, $k' = 0$ (Fig.3c).

**Figure 4.** The time evolution of $F_y(t)$ as a function of scaled time $\omega t$ for three values of $N$, $k = k' = 0.05\omega$, $\alpha = 1$ when $B_{n,m} \neq 1$: $N = 20$, $m = 10$ (Fig.4a); $N = 50$, $m = 25$ (Fig.4b); $N = 100$, $m = 50$ (Fig.4c).

**Figure 5.** The time evolution of $F_y(t)$ as a function of scaled time $\omega t$ for $\alpha = 1$, when $B_{n,m} = 1$: $N = 1$ (Fig.5a); $N = 10$ (Fig.5b); $N = 100$ (Fig.5c).

**Figure 6.** The time evolution of $F_y(t)$ as a function of scaled time $\omega t$, for $\alpha = 1$, $k' = 0$, $k = 0.05\omega$, when $B_{n,m} \neq 1$: $N = 30$, $m = 15$ (Fig.6a); $N = 100$, $m = 50$ (Fig.6b).