The Solar Neutrino Problem as Evidence of New Interaction

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Abstract—A new concept to solve the solar neutrino problem that is based on the hypothesis about the existence of a new interaction of electron neutrinos with nucleons mediated by massless pseudoscalar bosons is proposed. At each collision of a neutrino with nucleons of the Sun, its helicity changes from left- to right-handed and vice versa, and its energy decreases. The postulated hypothesis, having only one free parameter, provides good agreement between the calculated and experimental characteristics of all five observed processes with solar neutrinos.

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1. INTRODUCTION

This work is devoted to substantiating the fact that the discrepancies between the predictions of the standard solar model (SSM) for the rates of a number of processes caused by solar neutrinos and the results obtained in appropriate experiments testify to the existence of a new, fairly hidden interaction, which we call semi-weak. The postulated interaction extends at least to the electron neutrino and nucleons, but does not affect the electron (at the tree level). Its carrier is a massless pseudoscalar boson. The product of the coupling constants of this boson with the electron neutrino and nucleons is smaller than the electromagnetic and weak interaction constants, $\alpha$ and $g^2/4\pi$, by three orders of magnitude. At low energies typical for solar and reactor neutrinos, the semi-weak interaction of a neutrino with nucleons is characterized by cross sections of $\sim 10^{-36}$ cm$^2$, which is much larger than the cross sections of the standard $Z$-boson exchange interaction processes. Nevertheless, it is small enough to manifest itself in ordinary, nonunique situations. Its manifestation in the existing experiments with solar neutrinos is assisted by the fact that the Sun basically plays the role of part of the setup where each propagating neutrino undergoes about 10 collisions with nucleons. These collisions caused by the semi-weak interaction have the following three remarkable features.

First, at each collision with nucleons, the neutrino changes its helicity from left- to right-handed and vice versa. The ratio between the left- and right-handed electron neutrino fluxes at the Earth’s surface coincides with the ratio between the probabilities of even and odd numbers of collisions of a neutrino with nucleons before its exit from the Sun. Second, the total cross section for elastic scattering of solar neutrinos by nucleons is virtually independent of the neutrino energy. Therefore, all solar neutrinos, irrespective of their energies, may be considered to undergo, on average, the same number of collisions during their motion inside the Sun. Third, the relative fraction of the average energy loss of neutrinos as they collide with a nucleon of mass $M$ is proportional to their initial energy $\omega$:

$$\frac{\Delta \omega}{\omega} = \omega/M.$$ 

Therefore, the energy of the neutrinos from $p-p$ collisions remains virtually unchanged, and the main factor influencing the results of experiments on the $^{71}\text{Ga} \rightarrow ^{71}\text{Ge}$ transitions is the ratio of the left- and right-handed electron neutrino fluxes at the Earth’s surface. Since the cross sections for neutrino processes grow with increasing neutrino energy, a decrease in the energy of the tagged solar neutrinos due to their collisions with nucleons entails a decrease in the cross sections and observed rates of the processes involving solar neutrinos. The dependence of the cross sections on the neutrino energy is undoubtedly different for different processes, and, therefore, the difference in the effective solar neutrino fluxes from $^8\text{B}$, corresponding to the rates of elastic solar neutrino scattering by electrons and deuteron disintegration with the production of electrons, is natural.

Within the framework of the hypothesis about the existence of semi-weak interaction, the deuteron disintegration by neutral currents occupies a special place among the processes with solar neutrinos, because it is caused by two non-interfering sub-processes. One sub-process, involving only the left-handed neutrinos,
is a standard one and is caused by the Z-boson exchange. The other, unordinary sub-process involv-
ing both left- and right- handed neutrinos is caused by
the massless pseudoscalar boson exchange. Owing to
the fact that in the Lagrangian (1) (see Section 2
below) the coupling constants of the postulated boson
with the proton and the neutron are opposite, the
cross section of the second sub-process is character-
ized by the additional factor \((M_e - M_p)/M\)² and, for
this reason, it becomes comparable to the cross section
of the first, standard sub-process. It is quite natural
that the effective solar neutrino flux corresponding
the total theoretical rate of deuteron disintegration by
neutral currents turns out to be noticeably larger than
the effective flux associated with the disintegration
of deuterons by a charged current.

The effective number of solar neutrino collisions
with nucleons is taken as a single parameter of the
semi-weak interaction model describing the change in
the energy spectrum of solar neutrinos as they move
from the birthplace to the exit from the Sun. Knowing
this number, we estimate the neutrino—nucleon cou-
pling constant and use this estimate in calculating the
cross section for the deuteron disintegration by the
pseudoscalar neutral current of solar neutrinos.

Our logical, analytical, and numerical analysis
leads to the conclusion that the hypothesis about the existence of a new, semi-weak interaction is con-
firmed by good agreement between the calculated and experimental characteristics of all the observed pro-
cesses with solar neutrinos:

\[ ^{37}\text{Cl} \rightarrow ^{37}\text{Ar}, \quad ^{71}\text{Ga} \rightarrow ^{71}\text{Ge}, \quad \nu_e e^- \rightarrow \nu_e e^-, \]
\[ \nu_e D \rightarrow e^- \bar{p}p, \quad \nu_e D \rightarrow \nu_e np. \]

The probability that this agreement reflects not the true nature of the things referring to neutrinos, but a
play of chance seems to be negligibly small. It is easy to
verify that no model or hypothesis that, having only
one free parameter, would give good agreement with the
results of three or, a fortiori, four or five completely
different experiments has appeared in the last 40 years
in particle physics.

Using the solution of the solar neutrino problem
proposed in this paper, we should look more carefully
at the peculiarities of the interpretation of solar neu-
trino experiments based on the neutrino oscillation
hypothesis. Note only two of them. First, in addition
to the hypothesis about solar neutrino oscillations in a
vacuum based at least on two additional (massive)
neutrinos and two free parameters, the Wolfenstein—
Mikheev—Smirnov mechanism is used. Second,
despite hundreds of papers devoted to the solar neu-
trino oscillations, as yet there is no publication in the
world’s literature in which (1) an unambiguous proce-
dure for calculating the probability that the solar neu-
trino remains the electron one at the Earth surface,
depending on its energy, would be described, (2) all
free parameters of the procedure and their optimal
values would be specified, and (3), the most important
thing, the overall theoretical results for the rates of all
five observed processes with solar neutrinos would be
presented in comparison with the corresponding experi-
mental results.

It should be specially noted that the neutrino
experiments in which the possibility of neutrino oscil-
lation manifestations is investigated can be divided
into three groups, in accordance with three essentially
independent branches of the oscillation model used in
interpreting their results. Specifically (see the reviews
[1, 2]), the parameters \(\theta_{12}\) and \(\Delta m_{21}\) (with a large oscil-
lation length and a significant amplitude) are associ-
ated with the solar neutrino experiments, the param-
eters \(\theta_{13}\) and \(\Delta m_{31}\) (with a short oscillation length and a
small amplitude) are associated with the experiments
with antineutrinos from nearby reactors, and the
parameter \(\theta_{23}\) referring only to the muon neutrinos is
associated with the experiments with atmospheric and
accelerator neutrinos. Consequently, the absence of
manifestations of significant neutrino oscillations
(generated by the branch with the parameters \(\theta_{13}\)
and \(\Delta m_{31}\)) in solar neutrino experiments does not deny the
possibility of manifestations of small neutrino oscilla-
tions (generated by the branch with the parameters \(\theta_{13}\)
and \(\Delta m_{31}\)) and suitable oscillations in neutrino experi-
ments of other types and vice versa.

Only the KamLAND experiments with reactor
antineutrinos occupy a special place with respect to the
solar neutrino experiments, because the interpre-
tation of these and other experiments based on the
oscillation model gives values of the parameters \(\theta_{12}\)
and \(\Delta m_{21}\) with the same order of magnitude. The new
interaction, which provides an elegant solution to the
solar neutrino problem, bears no relation to the
appearance of discrepancies between the expected and
observed results in KamLAND. Recognizing the
power of probabilistic patterns, the cause of these dis-
crepancies should be sought primarily in the omissions
in setting up and processing the experiment. One
such possible omission noted in our paper [3] is related
to the attenuation of fluorescent light in the liquid
scintillator.

2. THE HYPOTHESIS ABOUT THE EXISTENCE
OF A MASSLESS PSEUDOSCALAR BOSON
AND ITS INTERACTION

In our solution of the solar neutrino problem, we
rely on the methods of the classical field theory.

We deem that the neutrino of each flavor is
described, like the electron, by the bispinor represen-
tation of the proper Lorentz group and its field obeys
the Dirac equation. Note that all solutions with a pos-
tive energy of the massless free Dirac equation, two of
which (left-handed and right-handed) may be taken as
the basic ones, describe the various states of the same
neutrino. If there is an external scalar or pseudoscalar field interacting with the neutrino, then both left and right spinors of the neutrino wave vector will have nonzero values.

Various kinds of hypothetical interactions involving neutrinos have been considered repeatedly. One of them proposed by us [4–6] was associated with a hypothetical massless axial photon. At present, we can assert with sufficient confidence, without going into details, that the solar neutrino problem cannot be solved using the axial photon as an interaction carrier. Concurrently, the interaction of a hypothetical massless scalar with Majorana neutrinos was hypothesized in [7–10]. Because of the standard long-range interaction, whereby the potential energy \( V(r) \) is proportional to \( r^{-1} \) at sufficiently large distances \( r \), only an extremely weak interaction of this scalar with others fermions was admitted, which exerted virtually no influence on both the results of Eötvös-type experiments and the electron magnetic moment and, thus, the solar neutrino spectrum.

Thus, we suppose that there exists a massless pseudoscalar boson \( \varphi_{ps} \) whose interaction with an electron neutrino, a proton, and a neutron is described by the Lagrangian

\[
\mathcal{L} = i g_{\nu_{ps}} \bar{\nu} \gamma^5 \nu \varphi_{ps} + i g_{\nu_{ps}} \bar{p} \gamma^5 p \varphi_{ps} - i g_{\nu_{ps}} \bar{n} \gamma^5 n \varphi_{ps},
\]

or a similar Lagrangian with \( u \)- and \( d \)-quarks instead of the proton \( p \) and neutron \( n \).

We intend neither to defend nor to deny the possibility of identifying the boson \( \varphi_{ps} \) with the Peccei–Quinn axion [11, 12], but we take into account the unsuccessfulness of an experimental search for the axion and, therefore, postulate, for the sake of simplicity, that the boson introduced by us is massless without ruling out a fairly small value of its mass. The interaction of the pseudoscalar boson \( \varphi_{ps} \) with the electron is deemed to be absent at the tree level, because it is impossible to simultaneously fulfill the following three conditions: the coupling constants of this boson with the electron neutrino and the electron should be of the same order of magnitude; the neutrino produced at the center of the Sun with an energy of \( \sim 1 \) MeV should undergo at least one collision with some electron of the Sun; the contribution of this boson to the electron magnetic moment should be within the uncertainty limits admissible by the standard theory and experiments [13]. The inequivalence of the electron neutrino and electron in the Lagrangian (1), similar to their inequivalence in the Yukawa relation with the neutral Higgs boson, does not affect the construction and consequences of the existing gauge models of the electroweak interaction (Salam–Weinberg, left–right symmetric, initially \( P \)-invariant models). We do not rule out the possibility that the interaction of the boson \( \varphi_{ps} \) with neutrinos of different flavors \( (\nu_e, \nu_{\mu}, \text{and } \nu_{\tau}) \) is nonuniversal, i.e., characterized by different coupling constants.

Note that the massless pseudoscalar field cannot manifest itself in Eötvös-type experiments, because the interaction of two nucleons mediated by it is similar to the magnetic interaction of spins or, more specifically [14],

\[
V(r) \propto r^{-3} [\sigma_i \sigma_j - 3 (\sigma_i \eta)(\sigma_j \eta)],
\]

where \( \sigma_i \) are the fermion spin matrices. Furthermore, due to the standard long-range interaction, the differential cross section for elastic scattering of two charged particles described by the Rutherford formula has a pole at zero in momentum transfer squared, while the total cross section for such scattering is infinite. In contrast to this, the differential cross section for elastic scattering of two fermions caused by the massless pseudoscalar boson exchange has no pole, while the total cross section for such scattering is finite (see Eqs. (2) and (4) below).

### 3. ELASTIC NEUTRINO–NUCLEON CROSS SECTIONS AND KINEMATICS

The differential cross section for elastic scattering of a left- or right-handed electron neutrino with an initial energy \( \omega_1 \) by a nucleon at rest with mass \( M \) derived from the Lagrangian (1) is given by expression

\[
d\sigma = \frac{(g_{\nu_{ps}} g_{\nu_{ps}})^2}{32 \pi M \omega_1} d\omega_2,
\]

where \( \omega_2 \) is the scattered neutrino energy, which, as follows from the energy-momentum conservation law and Eq. (2), can take uniformly distributed values in the interval

\[
\frac{\omega_1}{1 + 2 \omega_1/M} \leq \omega_2 \leq \omega_1.
\]

The total elastic \( v_{\nu} N \) scattering cross section that can be found from relations (2) and (3) is

\[
\sigma = \frac{(g_{\nu_{ps}} g_{\nu_{ps}})^2}{16 \pi M^2} \frac{1}{1 + 2 \omega_1/M}.
\]

It is virtually independent of the energy \( \omega_1 \), because its maximum value is 18.8 MeV [15].

The first consequence of the interaction (1) is that at each collision with a nucleon due to the pseudoscalar boson exchange, the neutrino changes its helicity from left- to right-handed and vice versa. For this reason, at the Earth’s surface one part of the solar neutrino flux has a left-handed helicity and the other part has a right-handed one. The contribution from the right-handed solar neutrinos to the processes with charged currents and the elastic scattering by electrons is extremely small, because intermediate bosons of the standard model are not coupled with such neutrinos, but only very heavy intermediate bosons of the left-
right symmetric model $W_R$ and $Z_{LR}$ can be coupled with them. An analysis of the nucleosynthesis in the early Universe [16] and an electroweak fit [17] give, respectively, the following estimates: $M_{W_R} > 3.3$ TeV and $M_{Z_{LR}} > 1.2$ TeV if $g_R = g_L$. The latter condition is automatically fulfilled in the initially $P$-invariant electroweak interaction model [18]. Remarkably, both solar neutrino helicities nevertheless manifest themselves, specifically, in the deuteron disintegration by neutral currents due to the massless pseudoscalar boson exchange.

Another consequence of the solar neutrino—nucleon interaction that follows from relations (2) and (3) is a decrease in the energy of the neutrino that underwent a collision compared to the initial energy, on average, by

$$\Delta \omega = \frac{\omega_0^2}{1 + 2\omega_0/M}.$$  \hspace{1cm} (5)

Formula (5) shows that the relative single change in energy $\Delta \omega_0/\omega_0$ for the solar neutrinos from $^8\text{B}$ (their average energy is 6.7 MeV) is higher than that for the neutrinos from $p-p$ (their maximum energy is 0.423 MeV) and $^7\text{Be}$ with energies 0.384 and 0.862 MeV by an order of magnitude. The neutrinos from $^8\text{B}$ play a major role in the $^{37}\text{Cl} \rightarrow ^{37}\text{Ar}$ transitions and a monopolistic role in the elastic scattering by electrons and the deuteron disintegration. At the same time, the neutrinos from $p-p$ and $^7\text{Be}$ make a dominant contribution to the $^{71}\text{Ga} \rightarrow ^{71}\text{Ge}$ transitions. Their energy changes very little after about ten neutrino—nucleon collisions and the discrepancy between the SSM and experimental results is determined almost completely by the ratio of the left- and right-handed neutrino fluxes. Since the experimental $^{71}\text{Ga} \rightarrow ^{71}\text{Ge}$ transition rates are slightly smaller than half the rate expected from the SSM, to a first approximation, we assume that the left- and right-handed neutrino fluxes near the Earth’s surface are equal.

Due to the collisions with nucleons, the solar neutrino from the time of its production until the exit from the Sun executes a Brownian motion in an inhomogeneous spherically symmetric medium. In principle, some distribution $P_\beta(n)$ in the number $n$ of solar neutrino—nucleon collisions dependent on the product of the coupling constants from the Lagrangian (1),

$$\beta \equiv \frac{g_{\nu,\nu} g_{nps}}{4\pi},$$

corresponds to this motion. Owing to relation (4), the distribution $P_\beta(n)$ may be deemed to be virtually independent of the initial neutrino energy. Since the solar neutrinos are produced in different places offset from the Sun’s center, on average, by a distance from 0.045 (from $^8\text{B}$) to 0.14 (from $\text{hep}$) of its radius [15], we should expect that the distribution $P_\beta(n)$ is sufficiently wide enough, i.e.,

$$|P_\beta(n+1) - P_\beta(n)| \ll 1.$$  \hspace{1cm} (6)

Hence we obtain an approximate equality of the probabilities of even and odd number of neutrino—nucleon collisions inside the Sun and an equality of the left- and right-handed neutrino fluxes at the Earth’s surface.

Being unable to accurately calculate the distribution $P_\beta(n)$, to a first approximation, we replace the exact rate of the process $\mathcal{A}$ with solar neutrinos, which includes all of the partial rates $v_\mathcal{A}(n)$ corresponding to the numbers of solar neutrino—nucleon collisions $n$ with probabilities $P_\beta(n)$, by the rate $v_\mathcal{A}(n_0)$ corresponding to a single number of neutrino—nucleon collisions $n_0$:

$$\sum_{n=0}^n P_\beta(n)v_\mathcal{A}(n) \rightarrow v_\mathcal{A}(n_0).$$  \hspace{1cm} (6)

The number $n_0$ from relation (6) undoubtedly depends on both constant $\beta$ and process $\mathcal{A}$. In the approximation under consideration, we necessarily assume that the number $n_0$ is the same for all the observed processes with solar neutrinos. A priori we cannot judge whether this assumption is good or bad. Fortunately, a comparison of the theoretical and experimental results shows that our assumption should be deemed good.

The integer number $n_0$ called the effective number of collisions serves as a single free parameter of the model under consideration. It is not related in any way to the ratio of the neutrino helicities at their exit from the Sun. The number $n_0$ describes the final energy distribution for a neutrino that had a fixed initial energy.

As has been noted above, after one elastic neutrino scattering by a nucleon at rest, the initial fixed neutrino energy turns into the uniformly distributed energy interval (3). After the second scattering, each value of the energy from this interval turns into its own interval like (3) and so on.

As regards the description of the energy distribution acquired by a neutrino with a fixed initial energy $\omega_i$ and $n_0$ collisions with nucleons acceptable for calculations (say, in FORTRAN, as we did this), we consider two variants.

In the first variant, one value of the energy equal to the mean value of the kinematic interval (3) is assigned to the neutrino after each collision with a nucleon, so that after 0, 1, ..., $n_0$ collisions we have sequentially

$$\omega_{0i} = \omega_i, \hspace{1cm} \omega_{1i} = \frac{\omega_i + \omega_{0i} / M}{1 + 2\omega_{0i} / M}, \hspace{1cm} ..., \hspace{1cm} \omega_{n_0i} = \frac{\omega_i + \omega_{n_0i-1} / M}{1 + 2\omega_{n_0i-1} / M}.$$  \hspace{1cm} (7)

In this variant, the final state of the neutrino is characterized by a single energy given by the last term of the sequence (7)
In the second variant, it is assumed that as a result of each collision with a nucleon, the neutrino energy takes one of the two boundary values of the interval (3) with an equal probability and, thus, after \( n_0 \) collisions the initial energy level \( \omega_i \) turns into a set of \( n_0 + 1 \) binomially distributed values whose elements are

\[
E_{i,j} = \omega_i, \quad E_{2j} = \frac{E_{i,j}}{1 + 2E_{i,j}/M}, \ldots, \\
E_{n_0+1,j} = \frac{E_{n_0,j}}{1 + 2E_{n_0,j}/M}.
\]  

Both variants yield close results. Thus, replacing the energy interval (3) by three, four, etc. equiprobable values seems inexpedient. We use everywhere only the second variant, which is logically more acceptable than the first one.

Let us now turn to specific experiments on the recording of solar neutrinos.

4. THE PROCESS \( \nu_e + ^{37}\text{Cl} \rightarrow e^- + ^{37}\text{Ar} \)

The first experiment of this kind [19] consisted in studying the process

\( \nu_e + ^{37}\text{Cl} \rightarrow e^- + ^{37}\text{Ar} \),

with a threshold energy of 0.814 MeV. At present, the experimental rate of such transitions is deemed to be 2.56 ± 0.16 ± 0.16 SNU (1 SNU = \( 10^{-36} \) interactions per target atom per second) [20]. At the same time, theoretical calculations based on the SSM give different, but significantly larger values, for example, 7.9 ± 2.6 SNU [15] and 8.5 ± 1.8 SNU [21].

We take the tabulated values of a number of quantities needed to calculate the rates of the \( ^{37}\text{Cl} \rightarrow ^{37}\text{Ar} \) and \( ^{71}\text{Ga} \rightarrow ^{71}\text{Ge} \) transitions induced by left-handed electron neutrinos from \[ ^{8}\text{B} \] and \[ ^{7}\text{Be} \] [15, 22, 23].

We use the dependence of the cross section for neutrino absorption by chlorine on the neutrino energy \( \omega \) given in Table IX from [15], including the information from Table VII, and specify a linear interpolation in each energy interval for the cross section. Note the fairly strong dependence of this cross section on energy \( \omega \) (expressed below in MeV) [24):

\[
\sigma^{\text{Cl}}(\omega) \sim \omega^{3.85}, \quad \text{if} \quad \omega \in [1,5],
\]

and

\[
\sigma^{\text{Cl}}(\omega) \sim \omega^{3.7}, \quad \text{if} \quad \omega \in [8,15].
\]

Therefore, we should expect that the decrease in the energy of solar neutrinos as a result of their collisions with nucleons affects the rate of the \( ^{37}\text{Cl} \rightarrow ^{37}\text{Ar} \) transitions fairly strongly.

The energies of the neutrinos from \( ^{8}\text{B} \), extending from 0 to about 16 MeV, are given in the table from [22] in the form of a set \( \omega_i^B = i\Delta^B \), where \( i = 1, \ldots, 160, \Delta^B = 0.1 \) MeV, and their distribution is expressed via the probability \( p(\omega_i^B) \) that the neutrinos possess an energy in the interval \( (\omega_i^B - \Delta^B/2, \omega_i^B + \Delta^B/2) \). Each of the energy distributions in the interval \([0, 1.73]\) MeV for the neutrinos from \( ^{15}\text{O} \) and in the interval \([0, 1.20]\) MeV for the neutrinos from \( ^{13}\text{N} \) is given in the tables from [23] for 84 points, while the distribution for the neutrinos from \( \text{hep} \) is given in the table from [15] for 42 energies in the interval \([0, 18.8]\) MeV. The energy spectrum has two lines, \( \omega_{^8\text{B}} = 0.862 \text{ MeV (89.7%)} \) and \( \omega_{^7\text{Be}} = 0.384 \text{ MeV (10.3%)} \), for the neutrinos from \( ^{7}\text{Be} \) and one line, \( \omega_{^8\text{B}} = 1.442 \text{ MeV} \), for the neutrinos from \( \text{hep} \).

For the solar neutrino fluxes at the Earth’s surface we take the values (in units of cm\(^{-2}\) s\(^{-1}\)) given in [21]:

\[
\Phi(^{8}\text{B}) = 5.79 \times 10^6 (1 \pm 0.23),
\]

\[
\Phi(^{7}\text{Be}) = 4.86 \times 10^9 (1 \pm 0.12),
\]

\[
\Phi(^{15}\text{O}) = 5.03 \times 10^8 (1.0^{+0.43}_{-0.39}),
\]

\[
\Phi(\text{hep}) = 1.40 \times 10^8 (1 \pm 0.05),
\]

\[
\Phi(^{13}\text{N}) = 5.71 \times 10^8 (1.0^{+0.35}_{-0.33}),
\]

\[
\Phi(\text{hep}) = 7.88 \times 10^7 (1 \pm 0.16).
\]

In our calculations we use only the central values of the fluxes without invoking an uncertainty in any estimations or judgments.

Assuming that the left-handed neutrino fluxes at the Earth’s surface are equal to half the above fluxes, the formulas for calculating the contributions to the rate of the \( ^{37}\text{Cl} \rightarrow ^{37}\text{Ar} \) transitions made by the neutrinos from \( ^{8}\text{B} \) and \( ^{7}\text{Be} \) can be presented in the form

\[
V(^{37}\text{Cl} \mid B) = 0.5\Phi(^{8}\text{B}) \sum_{i=1}^{160} \Delta^B p(\omega_i^B) X \sum_{n=1}^{n_0+1} \frac{n_1!}{2^n (n-1)!(n_0 + 1 - n)!} \sigma^{\text{Cl}}(\omega_{ni}^B),
\]

\[
V(^{37}\text{Cl} \mid \text{Be}) = 0.5 \cdot 0.897 \Phi(^{7}\text{Be}) X \sum_{n=1}^{n_0+1} \frac{n_1!}{2^n (n-1)!(n_0 + 1 - n)!} \sigma^{\text{Cl}}(\omega_{ni}^{\text{Be}}),
\]

where the energies \( \omega_{ni}^B \) and \( \omega_{ni}^{\text{Be}} \) are given by Eq. (8), in which the quantity \( \omega_i \) should be set equal to \( \omega_i^B \) and \( \omega_i^{\text{Be}} \), respectively. The contributions made by the neutrinos from \( ^{15}\text{O} \), \( ^{13}\text{N} \), and \( \text{hep} \) are calculated from a formula similar to (9), while the contributions from \( \text{hep} \) are calculated from a formula similar to (10).

Before performing our calculations using formulas like (9) and (10) with a nonzero number of neutrino–nucleon collisions \( n_0 \), we establish how much the
results obtained using the tabulated values and the interpolation for the cross section \( \sigma^{\text{Cl}}(\omega) \) under the condition of free neutrino motion in the Sun are close to those obtained in [15] based on more accurate calculations, though for slightly different spectra and fluxes. This comparison is reflected in Table 1.

The calculations pertaining to all of the processes considered below collectively give the best agreement with the experimental results at \( n_0 = 11 \). To display the dependence of the theoretical results on the integer number \( n_0 \), we present them in Table 1 and in what follows for two values of the number \( n_0, 10 \) and 11.

5. THE PROCESS \( \nu_e + 71\text{Ga} \rightarrow e^- + 71\text{Ge} \)

Let us now consider the process

\[
\nu_e + 71\text{Ga} \rightarrow e^- + 71\text{Ge},
\]

with a threshold energy of 0.233 MeV. The latest experiments have given the following values for the rate of this process: 65.4^{+2.6}_{-2.8} \text{ SNU} [25] and 62.9^{+6.0}_{-5.9} \text{ SNU} [26]. We note two of the theoretical results deserving attention: 132^{+20}_{-17} \text{ SNU} [15] and 131^{+12}_{-10} \text{ SNU} [21].

We use the neutrino energy dependence of the cross section for the process with gallium \( \sigma^{\text{Ga}}(\omega) \) presented in Table II from [23] and interpolate it within each interval by a linear function. In addition to the above information on the neutrino fluxes, data on the neutrinos from \( p^- p^- \) are also required. The tabulated energy spectrum of such neutrinos presented in [23] extends from 0 to 0.423 MeV and is specified by a set of 84 points. The solar neutrino flux from \( p^- p^- \) at the Earth’s surface is taken to be \( \Phi_{pp} = 5.94 \times 10^{10} \) (1 ± 0.01) cm\(^{-2}\) s\(^{-1}\) [21].

We calculate the contributions to the rate of the \( 71\text{Ga} \rightarrow 71\text{Ge} \) transitions made by the solar neutrinos from \( p^- p^-, 8\text{B}, 15\text{O}, 13\text{N}, \) and \( \text{hep} \) using a formula similar to (9) and the contributions from two \( 7\text{Be} \) lines and one \( \text{pep} \) line using a formula similar to (10). The results of our calculations are presented in Table 2.

The fact that the theoretical rate of the \( 71\text{Ga} \rightarrow 71\text{Ge} \) transitions at \( n_0 = 11 \) corresponds to the above experimental values is important evidence, first, for our assumption about an approximate equality of the left- and right-handed solar neutrino fluxes at the Earth’s surface and, second, for the consequence (5) following from the Lagrangian (1) about a decrease of the relative change in neutrino energy after a single collision with decreasing energy.

6. THE PROCESS \( \nu_e + e^- \rightarrow \nu_e + e^- \)

Let us consider the elastic scattering of solar neutrinos by electrons by taking into account the conditions and results of the Super-Kamiokande (SK) [27–29] and Sudbury Neutrino Observatory (SNO) [30–33] experiments.

The differential cross section for elastic scattering of a left-handed neutrino with an initial energy \( \omega \) by an electron at rest with a mass \( m \) is given by the formula (see, e.g., [34, 35])

\[
\frac{d\sigma_{\nu e}}{dE} = \frac{2G_F^2 m}{\pi} \left[ g_L^2 + g_R^2 \left( 1 - \frac{E - m}{\omega} \right)^2 \right] \equiv f_{\nu e}(\omega, E),
\]

where \( E \) is the recoil electron energy. For the electron neutrino scattering in the Weinberg–Salam electroweak interaction model we have

| \( n_0 = 10 \) | \( n_0 = 11 \) |
|-----------------|-----------------|
| 34.7            | 34.6            |
| 17.2            | 17.2            |
| 5.0             | 4.9             |
| 2.8             | 2.8             |
| 1.7             | 1.7             |
| 1.4             | 1.4             |

Table 1. Rates of the \( 37\text{Cl} \rightarrow 37\text{Ar} \) transitions in SNU

|     | \( ^8\text{B} \) | \( ^7\text{Be} \) | \( ^{15}\text{O} \) | \( \text{pep} \) | \( ^{13}\text{N} \) | \( \text{hep} \) | Total |
|-----|----------------|----------------|----------------|-------------|-------------|-------------|-------|
| SSM [15] | 6.1           | 1.1           | 0.3           | 0.2         | 0.1         | 0.03        | 7.9   |
| Interpolations, without interactions | 6.21          | 1.05          | 0.35          | 0.22        | 0.09        | 0.02        | 7.94  |
| Interaction (1), \( n_0 = 10 \) | 2.05          | 0.44          | 0.17          | 0.11        | 0.04        | 0.01        | 2.82  |
| Interaction (1), \( n_0 = 11 \) | 1.97          | 0.43          | 0.17          | 0.11        | 0.04        | 0.01        | 2.72  |

Table 2. Rates of the \( 71\text{Ga} \rightarrow 71\text{Ge} \) transitions in SNU

|     | \( p^- p^- \) | \( ^7\text{Be} \) | \( ^8\text{B} \) | \( ^{15}\text{O} \) | \( ^{13}\text{N} \) | \( \text{pep} \) | \( \text{hep} \) | Total |
|-----|---------------|----------------|----------------|-------------|-------------|-------------|-------------|-------|
| SSM [15] | 70.8          | 34.3          | 14.0          | 6.1         | 3.8         | 3.0         | 0.06        | 132   |
| Interpolations, without interactions | 69.8          | 34.9          | 14.0          | 5.7         | 3.4         | 2.9         | 0.05        | 130.7 |
| Interaction (1), \( n_0 = 10 \) | 34.7          | 17.2          | 5.0           | 2.8         | 1.7         | 1.4         | 0.02        | 62.8  |
| Interaction (1), \( n_0 = 11 \) | 34.6          | 17.2          | 4.9           | 2.8         | 1.7         | 1.4         | 0.02        | 62.6  |
\begin{equation}
g_L = \frac{1}{2} + \sin^2 \theta_W, \quad g_R = \sin^2 \theta_W, \quad (12)
\end{equation}

where we should set \( \sin^2 \theta_W = 0.231 \).

Based on the energy–momentum conservation law, we find that the recoil electron can acquire the energy \( E \) if the incident neutrino energy \( \omega \) satisfies the condition

\begin{equation}
\omega \geq E - m + \sqrt{E^2 - m^2} = h_\nu(E). \quad (13)
\end{equation}

When setting up an experiment on elastic neutrino–electron scattering, the difference between the true energy \( E \) of the recoil electron and its reconstructed (effective) energy \( E_{\text{eff}} \) is assumed to be given by a Gaussian probability density distribution:

\begin{equation}
P(E_{\text{eff}}, E) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{(E_{\text{eff}} - E)^2}{2 \sigma^2} \right], \quad (14)
\end{equation}

where the parameter \( \sigma \), being a function of the energy \( E \), depends on the peculiarities of the experimental setup.

Since the lower limit \( E_c \) is introduced for the reconstructed energy \( E_{\text{eff}} \) in all of the discussed experiments and it is no less than 3 MeV, the observed events are almost completely generated by the solar neutrinos from \( ^8 \text{B} \), while the contribution of the neutrinos from \( \text{hep} \) is very small. The contribution to the rate of the scattering of neutrinos from \( ^8 \text{B} \) by electrons with a reconstructed energy \( E_{\text{eff}} \) in the interval from \( E_k \) up to \( E_{k+1} \) (\( E_{k} \geq E_c \)) is calculated from the formula

\begin{equation}
V(\nu_e | B \parallel [E_k, E_{k+1}]) = 0.5 \Phi_{\text{eff}}^B \int_{E_k}^{E_{k+1}} dE_{\text{eff}}
\times \int_{16\text{MeV}}^{20\text{MeV}} \int_{1\text{MeV}}^{16\text{MeV}} dE \left[ P(E_{\text{eff}}, E) \sum_{n=1}^{160} \frac{\Delta^n p(\omega^n)}{2^n(n-1)! (n_0 + 1 - n)!} \right.
\left. \times f_{\nu_e}(\omega_{\nu, n}^B, E) \theta(\omega_{\nu, n}^B - h_\nu(E)) \right]. \quad (15)
\end{equation}

where \( \theta(x) \) is the Heaviside step function. The contribution of the neutrinos from \( \text{hep} \) is found using a similar formula. The result of a particular experiment and the theoretical calculation based on Eq. (15) are expressed via the effective (either observed or equivalent) flux of neutrinos from \(^\text{8B} \), \( \Phi_{\text{eff}}(^\text{8B}) \), which undergo no changes from the birthplace in the Sun to the experimental setup on Earth. The connection between such a result or calculation and the effective flux is given by the following relation:

\begin{equation}
V(\nu_e | B + \text{hep} \parallel [E_c, 20\text{MeV}]) = \Phi_{\nu_e}^B \int_{E_c}^{20\text{MeV}} dE \left[ P(E_{\text{eff}}, E) \right.
\times \int_{16\text{MeV}}^{20\text{MeV}} \int_{1\text{MeV}}^{16\text{MeV}} dE \left[ \sum_{n=1}^{160} \Delta^n p(\omega^n) f_{\nu_e}(\omega^n, E) \theta(\omega^n - h_\nu(E)) \right].
\end{equation}

Note here that the Gaussian distribution (14) affects significantly the distribution (15) of the rate of \( \nu e \) scattering in bins \([E_k, E_k + 0.5\text{MeV}]\) and has little effect on the effective neutrino flux found from equality (16).

Some details of the experiments and our calculations pertaining to the elastic scattering of solar neutrinos by electrons at rest are presented in Table 3, where \( T = E - m \).

7. DEUTERON DISINTEGRATION BY A CHARGED CURRENT
\( \nu_e + D \rightarrow \nu_e + p + p \)

Let us now consider the deuteron disintegration by solar neutrinos caused by a weak-interaction charged current,

\begin{equation}
\nu_e D \rightarrow \nu_e pp,
\end{equation}

We find the required differential cross section for this process,

\begin{equation}
\frac{d\sigma_{\nu_e}}{dE} = f_{\nu_e}(\omega, E),
\end{equation}

as a function of the energy \( \omega \) of the incident left-handed electron neutrino and the energy \( E \) of the produced electron in a tabulated form at the Web site [36] where the results of a field-theoretic analysis of the \( \nu D \) reaction in [37] are presented. In the tables from [36] the neutrino energy intervals are 0.2 MeV at \( \omega \leq 10 \text{MeV} \) and 0.5 MeV at \( \omega > 10 \text{MeV} \), while the intervals in \( E \) are varied in length. We resort to a linear extrapolation of the cross section in \( \omega \) and \( E \).

The kinematic condition for the neutrino energy that ensures the deuteron disintegration is (see, e.g., [38])

\begin{equation}
\omega \geq E + B + \delta \equiv h_\nu(E), \quad (17)
\end{equation}

where \( B = 2.2246 \text{MeV} \) is the deuteron binding energy and \( \delta = M_p - M_n = -1.2933 \text{MeV} \) is the difference of the proton and neutron masses.

We neglect the contribution of the neutrinos from \( \text{hep} \) to the deuteron disintegration and calculate the rate of the process

\begin{equation}
\nu_e + D \rightarrow \nu_e + p + p
\end{equation}

using a formula similar to (15), while we find the corresponding effective neutrino flux \( \Phi_{\nu_e}^{\text{rec}}(^\text{8B}) \) from a relation similar to (16).
The results of the SNO experiments and our calculations are presented in Table 4.

Noting good agreement between the results of our calculations and the results of the experiments regarding the effective neutrino fluxes, which correspond to the events of elastic $\nu_e$ scattering and the reaction $\nu_e D \rightarrow e^- p p$, we again draw attention to the fact that the difference in the theoretical values of the effective fluxes describing the two processes is achieved through a change in the shape of the spectrum of solar neutrinos due to their collisions with solar nucleons and through a different dependence of the cross sections for the processes on the neutrino energy.

8. DEUTERON DISINTEGRATION
BY NEUTRAL CURRENTS $\nu_e + D \rightarrow \nu_e + n + p$

Experiments on the deuteron disintegration into a neutron and a proton caused by neutral currents of solar neutrinos are deemed critically important for theoretical interpretations. The successive results of such experiments at SNO are expressed by the following values of the effective solar neutrino flux $\Phi_{\text{eff}}^{\nu_e}(8B)$ (in units of $10^6$ cm$^{-2}$ s$^{-1}$):

$$5.09^{+0.44}_{-0.35} \pm 0.46 \quad [30], \quad 5.21 \pm 0.27 \pm 0.38 \quad [31],$$

within the framework of our hypothesis about the existence of an interaction described by the Lagrangian (1), the deuteron disintegration into a neutron and a proton is described by two noninterfering sub-processes that differ by the neutrino helicities either in the initial state or in the final one.

The first sub-process caused by left-handed neutrinos is a standard one attributable to the $Z$-boson exchange. We use the tabulated values of the total cross section $\sigma^{\nu_e Z}(\omega)$ for this sub-process dependent on the incident neutrino energy $\omega$, which given in [37]. The rate of the deuteron disintegration sub-process due to the $Z$-boson exchange, $V(\nu_e Z) | B)$, is calculated using a formula similar to (9). The effective flux of solar neutrinos with their spectrum from the $8B$ decay, $V(\nu_e Z) | B)$, is calculated using the formula

$$V(\nu_e Z) | B) = \Phi_{\text{eff}}^{\nu_e}(8B) \sum_{i=1}^{160} \Delta B p(\omega_i) \sigma^{\nu_e Z}(\omega_i). \quad (18)$$

We have

$$\Phi_{\text{eff}}^{\nu_e}(8B) = \left[ \begin{array}{c} 2.16 \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}, \quad n_0 = 10, \\ 2.10 \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}, \quad n_0 = 11. \end{array} \right. \quad (19)$$

The results of the SNO experiments and our calculations are presented in Table 4.
The second sub-process of the deuteron disintegration into a neutron and a proton caused both left- and right-handed neutrinos is due to the massless pseudoscalar boson exchange. When choosing and implementing the procedure of calculating the cross section for this sub-process, we take into account a number of works on the deuteron disintegration by neutrinos, in particular, [37–43]. We do not aspire to the degree of carefulness in choosing the deuteron model and the accuracy of calculations that is achieved in [37, 39–41], but rely on a spherically symmetric approximation that is achieved in [42–45].

Let us divide the phase space of final states for the deuteron disintegration process into such pairs referring to the protons and neutrons that the pseudoscalar boson exchange. When choosing and implementing the procedure of calculating the cross section due to allowance for each set of final states from the phase space determined by the energy–momentum conservation law. This phase space reflecting the kinematics of the deuteron disintegration (20) as a 2 → 3 process is given by the following formula:

\[
d^3 I = \delta(\omega + M_D - \omega' - E_1 - E_2) \times \delta(k - k' - p_1 - p_2) d^3k' d^3p_1 d^3p_2.
\]

We use the nonrelativistic approximation

\[
E_i + E_2 = M_n + M_p + \frac{p_1^2}{2M} + \frac{p_2^2}{2M},
\]

substitute \(P^2\) and \(p\) for the variables \(p_1\) and \(p_2\) in Eq. (26), and perform integration over \(P\), removing the delta function of 3-moments. We obtain

\[
d^3 I = \delta \left( \omega - B - \omega' - \frac{(k - k')^2}{4M} - \frac{p^2}{M} \right) d^3k' d^3p.
\]

Based on relations (1), (25), and (23), we find the square of the absolute value of the matrix element \(|M|^2\) for the sub-process of the deuteron disintegration caused by the left- and right-handed neutrinos via the massless pseudoscalar boson exchange with nucleons. Substituting it into the formula for the differential cross section

\[
d\sigma = \frac{|M|^2}{16\omega \omega' M^2 (2\pi)^3} d^2 I,
\]

using equality (28) and the concept of relative energy \(E_r = p^2/M\), and performing a number of integrations, we obtain
Using the product of the coupling constants of the massless pseudoscalar boson with the electron neutrino and nucleons given below by equality (33) and the above values of other constants, we find the cross sections for the deuteron disintegration sub-process under discussion for a number of incident neutrino energies \( \omega = (2.2 + 0.2) \) MeV, \( j = 1, 2, ..., 70 \). Some of these values are given in Table 5. To make sure that the results of our calculations of the cross section \( \sigma^{nc(ps)} \) from Eq. (30) are satisfactory, we calculated the cross section \( \sigma^{nc(Z)} \) within an approach similar to that described above and placed the results in the “Control” columns of Table 5 near the fairly accurate results taken from [37].

We can now calculate the rate of the deuteron disintegration into a neutron and a proton, \( V(nc(ps)|B) \), caused by the neutrinos due to the massless pseudoscalar boson exchange with nucleons. We do this using a formula similar to (9), where the total flux of left- and right-handed neutrinos \( \Phi(B) \) should now be inserted instead of the flux of left-handed neutrinos \( 0.5\Phi(B) \). Subsequently, replacing the rate \( V(nc(Z)|B) \) by the rate \( V(nc(ps)|B) \) in equality (18) and reserving the status of the theoretical cross section calculated on the basis of the standard electroweak interaction model (as in [37]) for the cross section \( \sigma^{nc(Z)} \), we find the effective solar neutrino flux \( \Phi_{\text{eff}}^{nc(ps)}(8B) \) responsible for the deuteron disintegration rate \( V(nc(ps)|B) \):

\[
\Phi_{\text{eff}}^{nc(ps)}(8B) = \begin{cases} 
(2.90 \pm 0.36) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}, & n_0 = 10, \\
(2.87 \pm 0.36) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}, & n_0 = 11,
\end{cases}
\]

with the uncertainty in (31) being attributable only to the uncertainty in the coupling constant (33).

Thus, the effective solar neutrino flux corresponding to the total rate of the two sub-processes of the deuteron disintegration by neutral currents of solar neutrinos, \( V(nc(Z)|B) + V(nc(ps)|B) \), is

\[
\Phi_{\text{eff}}^{nc(ps)}(8B) = \begin{cases} 
(5.06 \pm 0.36) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}, & n_0 = 10, \\
(4.97 \pm 0.36) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}, & n_0 = 11,
\end{cases}
\]

which is in good agreement with the above-listed experimental SNO results.

### 9. COUPLING CONSTANTS

Let us find the coupling constant \( g_{\nu,ps}g_{Nps} \), based on the fact that the electron neutrinos undergo a small (~10) number of collisions with nucleons as they move in the Sun. Using the solar matter densities as a function of distance from the solar center tabulated in [15], we find that a tube with a cross section of 1 cm² stretching from the center to the periphery of the Sun contains \( 4.9 \times 10^{12} \) grams of matter and, consequently, \( 8.91 \times 10^{35} \) nucleons. Hence and based on relation (4) and the assumption that the neutrino traverses in the Sun from 0.7 to 0.9 (rectilinearly from 0.1 to 1.0) of the matter in this tube before its collision with a nucleon, we obtain

\[
\frac{g_{\nu,ps}g_{Nps}}{4\pi} = (3.2 \pm 0.2) \times 10^{-5}.
\]

A fairly accurate value of the coupling constant being discussed could be obtained if it were possible to perform extremely complex calculations of the short-term Brownian motion of solar neutrinos in an inhomogeneous, spherically symmetric medium from their birthplaces up to the exit from the Sun.

Thus, the product of the constants of the postulated interaction of a massless pseudoscalar boson with electron neutrinos and nucleons is smaller than the electromagnetic and weak interaction constants, \( \alpha \) and \( g^2/4\pi \), respectively, by several orders of magnitude. Therefore, the postulated interaction could be called superweak. However, since the total cross section for such a neutrino—nucleon interaction at a low energy, which is the case for solar neutrinos and reactor antineutrinos, is much larger than that for the stan-
standard weak interaction via the Z-boson exchange, we prefer to call the postulated interaction semi-weak.

It should be noted that after 10–11 solar neutrino–nucleon collisions, the neutrino energy output from the Sun decreases approximately by 0.3% compared to that expected in the SSM, which is less than the theoretical uncertainty for the neutrino flux, 1%. This suggests that the postulated interaction of stellar neutrinos with nucleons has very little effect on the evolution of a particular star.

Studying the possible manifestations of the described interaction in experiments on a direct search for dark matter particles is a separate interesting problem.

The answer to the question about the presence or absence of the interaction of a massless pseudoscalar boson with a muon neutrino should be sought in experiments with neutral currents of accelerator neutrinos at small values of the transferred momentum squared.

10. CONCLUSIONS

The range of phenomena that, possibly, can have a bearing on the manifestation of the postulated new interaction will expand with time. In our view, first of all close attention should be paid to setting up a number of experiments with reactor antineutrinos. We expect that in an experiment on the deuteron disintegration by neutral currents the contribution of the semi-weak interaction to the observed rate of events will be greater than the contribution of the electroweak interaction approximately by a factor of 3 [46]. A manifestation of the new interaction can be expected in observation that its contribution to the fission rate of a number of light stable nuclei (He-3, Li-7, Be-9, and F-19) by reactor antineutrinos will exceed the contribution of the electroweak interaction approximately by six orders of magnitude [47]. It also seems appropriate to elucidate the consequences of the interaction (1) in various astrophysical processes.

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