Transconductance as a probe of nonlocality of Majorana fermions

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Each end of a Kitaev chain in topological phase hosts a Majorana fermion. Zero bias conductance peak is an evidence of Majorana fermion when the two Majorana fermions are decoupled. These two Majorana fermions are separated in space and this nonlocal aspect can be probed when the two are coupled. Crossed Andreev reflection is the evidence of the nonlocality of Majorana fermions. Nonlocality of Majorana fermions has been proposed to be probed by noise measurements since simple conductance measurements cannot probe it due to the almost cancellation of currents from electron tunneling and crossed Andreev reflection. Kitaev ladders on the other hand host subgap Andreev states which can be used to control the relative currents due to crossed Andreev reflection and electron tunneling. We propose to employ Kitaev ladder in series with Kitaev chain and show that the transconductance in this setup can be used as a probe of nonlocality of Majorana fermions by enhancing crossed Andreev reflection over electron tunneling.

I. INTRODUCTION

Majorana fermions (MFs) in condensed matter systems have been of immense interest in the last couple of decades due to the possibility of realization of topological quantum computation. MFs were predicted to exist at the ends of semiconductor quantum wires with spin-orbit coupling proximitized with singlet superconductor in presence of a Zeeman field. Such quantum wires are called topological quantum wires. Over subsequent years, many experiments have convincingly observed zero bias conductance peak in normal metal leads connected to topological quantum wires as predicted by the theory confirming their realization. The reason for zero bias conductance peak (with a theoretically predicted conductance of $2e^2/h$) is perfect Andreev reflection mediated by the single MF present at zero energy. There is one MF at each end of the topological quantum wire and the two MFs can be coupled to form a nonlocal Dirac fermion. In a setup consisting of two normal metal leads attached to a superconductor, an electron incident on the superconductor from one normal metal can do one of the four things: reflect back, reflect back as a hole, transmit through and exit at the other normal metal either as an electron or as a hole. These processes are called electron reflection (ER), Andreev reflection (AR), electron tunneling (ET) and crossed Andreev reflection (CAR) respectively. ET and CAR are nonlocal processes that are mediated by the nonlocal Dirac fermion formed by coupling between the two MFs. ET can happen even if the superconductor in between the two normal metals is replaced by a normal metal, but for CAR to happen the superconductivity is necessary. In AR, electron from one normal metal is absorbed resulting in hole at the same normal metal accompanied by a Cooper pair current flowing into the topological quantum wire and this process is mediated by the single MF present at the interface between the topological quantum wire and the normal metal. On the other hand, in CAR, an electron from the first normal metal is absorbed resulting in a hole at the second normal metal accompanied by a Cooper pair current flowing into the topological quantum wire and since the incident electron and the resultant hole flow in different normal metals this process is mediated by both the MFs. Hence, CAR is a probe for nonlocality of MFs. In an experiment, the local conductance - the ratio of differential current flowing in one normal metal to the differential voltage applied between the normal metal and the topological superconductor and the transconductance - the ratio of differential current flowing in the second normal metal to the differential voltage applied to the first normal metal maintaining the topological quantum wire and the second normal metal grounded can be measured. The transconductance gets a positive contribution from ET and a negative contribution from CAR. Therefore, a negative transconductance is a definite signature of nonlocality of MFs. The nonlocality of MFs has been proposed to be probed by noise measurements. The reason for resorting to noise measurements is that the transconductance in a setup consisting of two normal metal leads connected to a topological quantum wire is negative for certain parameters but the magnitude is very small despite nonlocal transport owing to the almost cancellation of electron and hole currents. Enhanced crossed Andreev reflection is a definite signature of nonlocality of MFs. In this paper, we propose a way to probe the nonlocality of MFs by conductance measurements by changing the setup slightly. In the alternate setup, we show that value of transconductance can be made large in magnitude and negative at the same time for some specific choices of parameters. We employ a Kitaev ladder in series with the Kitaev chain to realize a setup to achieve this.

To enhance CAR over ET is an important problem and there are many methods to achieve it. One among these methods is to employ superconducting ladder which consists of two superconductors differing by a superconducting phase difference forming a one-dimensional interface. Along the interface, subgap Andreev states (SAS) exist when a substantial phase difference between the two superconductors is maintained along with a sufficiently large coupling. SAS are extended along the interface and they mediate ET and CAR when the interface is con-
Eq. (1) can be written as:

\[ H_L = \sum_{n=-\infty}^{0} [-t(c_{n-1}^\dagger c_n + \text{h.c.}) - \mu c_n^\dagger c_n], \]

\[ H_{MF} = -t_0(c_{-1}^\dagger c_0 + \text{h.c.}) - \Delta_0(c_{L+1}^\dagger c_1 + \text{h.c.}) \]

\[ -\mu_0 \sum_{n=1,2} (c_{n,\sigma}^\dagger c_{n,\sigma} - \frac{1}{2}), \]

\[ H_{KL} = \sum_{\sigma=1,2} \sum_{n=1}^{N} [-t(c_{n+1,\sigma}^\dagger c_{n,\sigma} + \text{h.c.}) - \Delta(e^{i\phi_n} c_{n,\sigma}^\dagger c_{n+1,\sigma} + \text{h.c.})] \]

\[ -\mu \sum_{\sigma=1,2} \sum_{n=1}^{L} (c_{n,\sigma}^\dagger c_{n+1,\sigma} + \text{h.c.}) + t' \sum_{n=1}^{L} (c_{n,1}^\dagger c_{n,2} + \text{h.c.}), \]

\[ H_R = \sum_{n=L+1}^{\infty} [-t(c_{n+1}^\dagger c_n + \text{h.c.}) - \mu c_n^\dagger c_n], \]

\[ H_{LM} = -t_{LM}(c_{-2}^\dagger c_{-1} + \text{h.c.}), \]

\[ H_{MK} = -t_{MK}(c_{0,1}^\dagger + \text{h.c.}), \]

\[ H_{KR} = -t_{KR}(c_{L+1}^\dagger c_{L,1} + \text{h.c.}). \] (2)

Here the Kitaev chain is modeled with just two sites. In the limit \( t_0 = \pm \Delta_0 \) and \( \mu_0 = 0 \), the two MFs at the two ends of the chain are decoupled and two-site model is a good model to capture the effects of MFs in Kitaev chain. Two MFs are at two ends of the Kitaev chain and are nonlocal. To study the nonlocality of MFs, the two MFs at the two ends have to be coupled. This can be done by setting \( \Delta_0 \) close to \( t_0 \), but \( \Delta_0 \neq t_0 \). Then, the nonlocal fermions made from coupling of MFs are at energies \( \pm (t_0 - \Delta_0) \) and the quasiparticles belonging to the bulk band are at energies \( \pm (t_0 + \Delta_0) \). So, for sufficiently large values of \( t_0 \) \( (t_0 \gg \Delta) \) and \( \Delta_0 \sim t_0 \) the quasiparticles from the bulk band of the Kitaev chain are at energies very different from the subgap energies of the Kitaev ladder. Hence, only the MFs from the Kitaev chain participate in the nonlocal transport mediated by SAS of Kitaev ladder and a negative transconductance at bias energies equal to the energies of the nonlocal fermion formed in the Kitaev chain is a definite signature of the nonlocality of the MFs.

The wavefunction at site \( n \), an electron incident from \( N_L \) with energy \( E \) has a wavefunction:

\[ \psi_n^e = e^{ik_an} + r_n e^{-ik_an} \quad \text{for} \quad n \leq -2 \]

\[ = t_n e^{ik_an} \quad \text{for} \quad n \geq L + 1 \]

\[ \psi_n^h = r_h e^{ik_bn} \quad \text{for} \quad n \leq -2 \]

\[ = t_h e^{-ik_bn} \quad \text{for} \quad n \geq L + 1, \] (3)

where \( k_n a = \cos^{-1}[-(E+\mu)/2t] \) and \( k_b a = \cos^{-1}[(E-\mu)/2t] \) is the lattice spacing. The scattering coefficients \( r_n, t_c, r_h \) and \( t_h \) can be determined by writing down equation of motion using the full Hamiltonian (eq. (1)). The local (nonlocal) differential conductance \( G_{LL} \) \( (G_{RL}) \) defined as the ratio of differential change in current \( dI_L \) \( (dI_R) \) in

FIG. 1. Schematic diagram of the setup proposed. Normal metal \( (N_L) \) is connected to Kitaev chain \( (KC) \) which is then connected to the Kitaev ladder \( (KL) \) which in turn is connected to another normal metal \( (N_R) \). A bias voltage \( V \) is applied to \( N_L \) while grounding \( KC, KL \) and \( N_R \). Current meters denoted by \( I \) are attached through \( N_L \) and \( N_R \).
lead $N_L$ ($N_R$) to the differential change in applied voltage at lead $N_L$ can be calculated by the formulas\textsuperscript{10,22}:

$$G_{LL} = \frac{e^2}{h} \left[ 1 - |r_e|^2 + |r_h|^2 \sin k_h a \right]$$

$$G_{RL} = \frac{e^2}{h} \left[ |r_e|^2 - |r_h|^2 \sin k_e a \right]$$

(4)

We use the term transconductance instead of nonlocal differential conductance for $G_{RL}$.

The Kitaev ladder dispersion is

$$E = \nu_1 \sqrt{\epsilon_2^2 + t'^2 + \alpha_k^2 + \nu_2 \cdot 2t' \sqrt{\epsilon_k^2 + \alpha_k^2 \sin^2 \phi/2}} \quad (5)$$

where $\nu_1, \nu_2 = \pm 1$ represent bands formed by the hybridization of electron and hole excitations in the two legs of the ladder, $\phi = (\phi_1 - \phi_2)$, $\epsilon_k = -(2t \cos ka + \mu)$ and $\alpha_k = 2\Delta \sin ka$. This dispersion is typically gapped and the gap closes for $\phi = \pi$ when $t' > t'_c(\mu)$, where $t'_c(\mu) = \Delta/\sqrt{1 - \mu^2/(t'^2 - \Delta^2)}$.

### III. RESULTS AND ANALYSIS

We calculate the local conductance $G_{LL}$ and the transconductance $G_{RL}$ as functions of bias voltage $V$ and chemical potential $\mu$ for the choice of parameters: $t_0 = 10 t$, $\Delta_0 = 0.99\mu_0$, $\mu_0 = 0$, $t_{LM} = 0.3t$, $t_{MK} = 0.3t$, $t_{KR} = t$ and $L = 40$ in Fig. 2. Here, we choose $\Delta = 0.1t$, $t' = 3\Delta$, $\phi_1 = 0$, $\phi_2 = -\pi$ so that there are SAS in the ladder to mediate nonlocal transport in the full energy range. The Kitaev chain is weakly coupled to $N_L$ ($t_{LM} = 0.3t$) and to the Kitaev ladder ($t_{MK} = 0.3t$). The parameters for the Kitaev chain ($\mu_0$, $t_0$ and $\Delta_0$) are chosen so that only MFs participate in transport. The energy splitting between the MFs due to coupling is $2\Delta$ and the nonlocal fermion states are formed at energies $\pm \Delta$ for this choice of parameters. We see that the local transport is dominated by AR and the nonlocal transport is resonant at these energies ($eV = \pm \Delta$) from Fig. 3. The peaks in $G_{RL}$ have values close to $0.5e^2/h$ and the valleys have values close to $-0.5e^2/h$. The conductances have a periodic behavior as a function of $\mu$ due to Fabry-Pérot interference of the SAS\textsuperscript{10,12,14,23}. The periodic behavior in the transconductance $G_{RL}$ with negative values as a function of $\mu$ at $eV = \pm (t_0 - \Delta_0)$ is a definite signature of nonlocality of the MFs. The Fabry-Pérot interference condition ($k_{i+1} - k_i)aL = \pi$ determines the spacing between consecutive peaks $\mu_{i+1} - \mu_i$. We now make two changes to the parameters of Fig. 2 by choosing $\phi_1 = \pi$ and $\phi_2 = 0$ and plot the results in Fig. 3. The broad features due to Fabry-Pérot interference remain the same except for a change in the details. The reason for difference in details is that the Kitaev chain is connected only to the upper leg of the ladder. The Kitaev chain which has zero phase forms a Josephson junction with the upper leg of the ladder which has a phase of $\pi$ for parameters in Fig. 3 whereas for the choice of parameters in Fig. 2 Kitaev chain does not form a Josephson junction with the upper leg of the ladder.

Now we turn to the dependence of the two conductances on the bias and the superconducting phase difference $\phi = (\phi_1 - \phi_2)$, keeping $\phi_1$ constant. We fix $\mu = 0$ and keep other parameters same as earlier. We see that the transconductance is enhanced in magnitude near $\phi = \pi$ in Fig. 4. The peak at zero bias in local conductance $G_{LL}$ for $\phi = 0, 2\pi$ is due to the MFs in the ladder. As $\phi$ increases from $0$ to $\pi$, the MFs belonging to the two legs of the ladder hybridize and split in energy. This split in energy can be seen in the thick red lines originating at $(eV, \phi) = (0, 0)$. As $\phi$ changes from $0$ to $\pi$, the bulk states of the two legs of the ladder enter the gap $(-2\Delta, 2\Delta)$ and form SAS. These are responsible for enhanced ET and enhanced CAR in the regions close to $\phi = \pi$. Now, we change $\phi_1 = \pi$ and $\phi_2 = -\pi$ and plot the dependence of the two conductances as functions of the bias $eV$ and the phase difference $\phi$ in Fig. 5. If we compare the local conductance plots of Fig. 4 and Fig. 5, we can see a clear contrast. Near zero $\phi$, there is a peak in $G_{LL}$ at $eV = \pm \Delta$ for $\phi_1 = \pi$ while it is absent for $\phi_1 = 0$. Also, zero bias peak in $G_{LL}$ at $\phi = 0$ is absent for $\phi_1 = \pi$ unlike the case $\phi_1 = 0$. This is an effect of interference between the MFs in the Kitaev ladder and the MF in Kitaev chain. At $\phi = 0$, even the Kitaev ladder hosts MFs. When $\phi_1 = 0$, the MF of the Kitaev ladder at zero energy is responsible for the zero bias peak. The peaks in local conductance at energies $\pm \Delta$ due to the fermion formed in the Kitaev chain are absent since the ladder allows for finite transmission through the evanescent modes. The peaks in local conductance at energies $\pm 2\Delta$ are because the band bottom/top of the gapped SAS dispersion lies close to $\pm 2\Delta$ and these modes cannot carry any current across due to zero group velocity. But when $\phi_1 = \pi$ and $\phi = 0$, the Kitaev ladder and the Kitaev chain form exactly a $\pi$-Josephson junction and the subgap transport across a $\pi$-Josephson junction happens primarily by local Andreev reflection when the length of the Kitaev ladder is long$^{24}$. It is interesting to see that for both the cases, the transconductance has peaks and valleys near $\phi = \pi$. Motivated by the change in results when $\phi_1$ is changed, we study the dependence of the two conductances as a function of the overall phase keeping the phase difference the same. In Fig. 6 we plot the two conductances as functions of the bias and the overall phase of the ladder $\phi_0$ defined as $\phi_0 = \phi_1 - \phi_2 - \pi$. Here, we maintain the phase difference between the two legs of the ladder to be $\pi$ since the ladder dispersion becomes gapless for this choice of $(\phi_1 - \phi_2)$. We see from this figure that the peaks in $G_{LL}$ at $eV \sim \pm \Delta$ have a slight variation as a function of $\phi_0$, while the transconductance has thick regions of enhanced ET and enhanced CAR.
FIG. 2. $G_{LL}$ (left panel) and $G_{RL}$ (right panel) in units of $e^2/h$ for the choice of parameters: $t_0 = 10t$, $\Delta_0 = 0.99t_0$, $\mu_0 = 0$, $\Delta = 0.1t$, $t' = 3\Delta$, $\phi_1 = 0$, $\phi_2 = -\pi$, $t_{LM} = 0.3t$, $t_{MK} = 0.3t$, $t_{KR} = t$ and $L = 40$.

FIG. 3. $G_{LL}$ (left panel) and $G_{RL}$ (right panel) in units of $e^2/h$ for the choice of parameters: $t_0 = 10t$, $\Delta_0 = 0.99t_0$, $\Delta = 0.1t$, $t' = 3\Delta$, $\phi_1 = \pi$, $\phi_2 = 0$, $t_{LM} = 0.3t$, $t_{MK} = 0.3t$, $t_{KR} = t$ and $L = 40$.

FIG. 4. $G_{LL}$ (left panel) and $G_{RL}$ (right panel) in units of $e^2/h$ versus bias $eV$ and the phase difference $\phi = \phi_1 - \phi_2$ with $\phi_1 = 0$ for the choice of parameters: $t_0 = 10t$, $\Delta_0 = 0.99t_0$, $\Delta = 0.1t$, $t' = 3\Delta$, $\mu = 0$, $t_{LM} = 0.3t$, $t_{MK} = 0.3t$, $t_{KR} = t$ and $L = 40$.

FIG. 5. $G_{LL}$ (left panel) and $G_{RL}$ (right panel) in units of $e^2/h$ versus bias $eV$ and the phase difference $\phi = \phi_1 - \phi_2$ with $\phi_1 = \pi$ for the choice of parameters: $t_0 = 10t$, $\Delta_0 = 0.99t_0$, $\Delta = 0.1t$, $t' = 3\Delta$, $\mu = 0$, $t_{LM} = 0.3t$, $t_{MK} = 0.3t$, $t_{KR} = t$ and $L = 40$.

IV. DISCUSSION AND CONCLUSION

An interesting question that pops up in the analysis of the setup proposed is- 'what happens when the Kitaev chain is in the non-topological phase?' One way to take this limit is to set $t_0 = 10t$, $\Delta_0 = 0.99t_0$ and $\mu_0 = 3t_0$. Transport problem is solved in this limit taking other parameters same as that for Fig. 2. In this limit, the local conductance shows peaks for some values of the chemi-
becomes a normal metal when $\mu = 0$ and $\phi = \pi + \phi_0$ for the choice of parameters: $\ell_0 = 10t$, $\Delta_0 = 0.99t_0$, $\Delta = 0.1t$, $t = 3\Delta$, $\mu = 0$, $t_{LM} = 0.3t$, $t_{MK} = 0.3t$, $t_{KR} = t$ and $L = 40$.

Potential $\mu$ and the bias $eV$, but the magnitude of the local conductance value is very small ($\sim 10^{-6}$). The transconductance shows negative values for some range of chemical potential $\mu$ and the bias $eV$ but the magnitude is very small ($\sim 10^{-7}$). This shows that electron reflection dominates for all values of the parameters of the Kitaev ladder. This is because the Kitaev chain acts like an insulator due to its discrete energy levels which are away from the band of the Kitaev ladder. Another way to take the limit of non-topological phase of the Kitaev chain is to set $\Delta_0 = 0$. In this limit, the Kitaev chain becomes a normal metal when $\mu_0 = 0$ and $t_0 = t$. The entire setup then becomes a Kitaev ladder connected to two normal metal leads with a quantum dot in between the left normal metal and the Kitaev ladder which has been essentially analyzed in Ref. 16. This shows negative values of transconductance for a range of chemical potential $\mu$ and $\Delta_0 = 0$. The presence of MF is detected by peak in local conductance at the energy of the nonlocal fermion state formed (i.e., at $eV = \pm (t_0 - \Delta_0)$ with a value $2e^2/h$) when Kitaev chain is connected to a normal metal lead while a negative value of transconductance (high in magnitude at the energies of the nonlocal fermion formed in the Kitaev chain) in the setup proposed here is the evidence of the nonlocality of Majorana fermions once the presence of MFs is detected.

To conclude, we have seen that when a Kitaev ladder hosting SAS is connected to Kitaev chain hosting MFs, the transconductance shows large positive and negative values for some choices of the parameters. Further, the transconductance shows periodic behavior which can be explained by Fabry-Pérot interference condition. Negative values of transconductance indicate enhanced crossed Andreev reflection. We have thus shown that transconductance in the proposed setup can be used as a probe of the nonlocality of Majorana fermions.

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