Chapter 1

Spin waves in thin films and magnonic crystals with Dzyaloshinskii-Moriya interactions

Rodolfo Gallardo\textsuperscript{a,b}, David Cortés-Ortuño\textsuperscript{c}, Roberto Troncoso\textsuperscript{d} and Pedro Landeros\textsuperscript{a,b}

\textsuperscript{a}Departamento de Física, Universidad Técnica Federico Santa María, Avenida España 1680, Valparaíso, Chile
\textsuperscript{b}Center for the Development of Nanoscience and Nanotechnology (CEDENNA), 917-0124 Santiago, Chile
\textsuperscript{c}Faculty of Engineering and Physical Sciences, University of Southampton, Southampton SO17 1BJ, United Kingdom
\textsuperscript{d}Center for Quantum Spintronics, Department of Physics, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway

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1.1 Introduction

In the late fifties of the past century, Dzyaloshinskii proposed in his seminal paper a phenomenological theory of anti-symmetric exchange coupling between spins to explain the phenomenon of weak ferromagnetism in antiferromagnetic compounds [1]. Two years later, Moriya derived this interaction as a spin-orbit coupling between electrons within the framework of superexchange theory [2–4]. It was then shown that this anisotropic exchange interaction arises in materials that lack of inversion symmetry and where strong spin-orbit coupling effects are present. Nowadays, this anti-symmetric exchange coupling is known as the Dzyaloshinskii-Moriya interaction (DMI), which has been a key ingredient for the explanation of the magnetic properties of a variety of compounds with broken symmetry [5–10]. These include non-centrosymmetric bulk ferromagnets, multiferroics, perovskites, cuprates, and ferromagnetic thin films [11–16], among others. The study of materials hosting DMIs has been pursued with high interest because it has been thoroughly established, both by theory and experiment, that DMIs induce chiral, topological and non-reciprocal features. A primary consequence is the occurrence of chiral spin textures such as magnetic helices, skyrmions, skyrmion lattices and chiral domain walls in ferromagnetic materials [17–42].

Several theories have been developed to understand the origin of the interfacial DMI in ultrathin ferromagnetic/heavy-metal interfaces. Originally, Levy and Fert [43–46] developed a theory for disordered magnetic alloys with heavy-metal impurities that involves an additional contribution to the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction [47, 48]. This extra term is of DMI type and arises from the spin-orbit scattering of the conduction electron gas with the heavy metal impurities [45]. Recently, it has been proposed that the origin of the interfacial DMI is closely related to the proximity-induced magnetic moment in the heavy metal [49], while ab-initio calculations reveal that proximity-induced magnetic moment in the heavy metal has no direct correlation with the DMI in Co/Pt interfaces [50]. Using a density functional approach, Belabbes et al. [51] demonstrated that the sign and magnitude of DMI are related to the degree of 3d-5d orbital hybridization around the Fermi level. The temperature dependence of the DMI and its interplay with the anisotropy of the orbital magnetic moment and the magnetic dipole moment in Pt/Co/MgO trilayers was studied experimentally and theoretically by Kim et al. [52], where interfacial DMI originates from the asymmetric charge distribution caused by the breaking of inversion symmetry. According to these studies, the underlying physical mechanism to fully understand the DMI in ferromagnetic ultrathin films is not completely understood and is still a topic under discussion.

Among the different kinds of elementary excitations in condensed matter physics, spin waves play a significant role because of the shorter wavelengths that can be attained at GHz frequencies and also due to the reduced losses by heating. Spin waves have been observed from millime-
Spin waves in thin films with bulk DMI

In this section, a theoretical background is given for the spin wave dynamics in thin films with bulk DMI. It is known that two counter-propagating spin waves show a frequency difference in magnetic films with bulk DMI, but in the backward volume geometry [95], that is with the magnetization and wave vector both parallel to each other and along the film’s plane. This
Table 1.1. The table illustrates the DM energy density and the corresponding asymmetry in the spin wave dispersion relation for different symmetry classes. The case of interfacial DMI corresponds to the $C_{nv}$ symmetry group. Table adapted from Ref. [95].

| Symmetry class | Energy density $\omega_{DM}$ | Asymmetry $\Delta f$ |
|---------------|-----------------------------|---------------------|
| $T$           | $(D/M_s^2) \left( \mathcal{L}_{xy}^{(x)} + \mathcal{L}_{zy}^{(z)} + \mathcal{L}_{zx}^{(x)} \right)$ | $2\gamma D(\pi M_s)^{-1}k \cos \phi_k \cos \phi_m$ |
| $C_{nv}$      | $(D/M_s^2) \left( \mathcal{L}_{xy}^{(x)} + \mathcal{L}_{zy}^{(z)} \right)$ | $-2\gamma D(\pi M_s)^{-1}k \sin \phi_k \cos \phi_m$ |
| $D_{2d}$      | $(D/M_s^2) \left( \mathcal{L}_{xy}^{(x)} + \mathcal{L}_{zy}^{(z)} \right)$ | $-2\gamma D(\pi M_s)^{-1}k \cos \phi_k \cos \phi_m$ |
| $D_n$         | $(D^{(1)}/M_s^2) \left( \mathcal{L}_{xy}^{(x)} - \mathcal{L}_{zy}^{(z)} \right) + (D^{(2)}/M_s^2) \left( \mathcal{L}_{zx}^{(y)} \right)$ | $2\gamma D^{(1)}(\pi M_s)^{-1}k \cos \phi_k \cos \phi_m$ |
| $C_n$         | $(D^{(1)}/M_s^2) \left( \mathcal{L}_{xy}^{(x)} + \mathcal{L}_{zy}^{(y)} \right)$ | $-2\gamma (\pi M_s)^{-1}k |D^{(1)}| \sin \phi_k$ |
|               | $+ (D^{(2)}/M_s^2) \left( \mathcal{L}_{xy}^{(x)} - \mathcal{L}_{zy}^{(z)} \right)$ | $-D^{(2)} \cos \phi_k \cos \phi_m$ |
Figure 1.1. Geometry and notation for the systems studied in Sec. 1.2 and 1.3. Vectors $\mathbf{M}_0$, $\mathbf{H}_0$ correspond, respectively, to the equilibrium magnetization and DC applied field, while $d$ denotes the thickness of the magnetic film. The coordinate system $(x,y,z)$ is fixed in the film, while the coordinates $(X,Y,Z)$ are defined according to the equilibrium magnetization, such that $Z$ is always parallel to $\mathbf{M}_0$, which makes an angle $\phi_m$ with the film. Spin waves propagate in the plane at an angle $\phi_k$ with the $z$ axis.

behavior has been measured in some chiral lattice ferromagnets $[101-104,111]$ and also in the non-centrosymmetric antiferromagnet $\alpha$Cu$_2$V$_2$O$_7$ $[112]$. The dynamics of the magnetization is described theoretically by following Refs. $[95,113,114]$, where the magnetization is composed by a static and a dynamic part: $\mathbf{M}(\mathbf{r};t) = M_Z \hat{Z} + \mathbf{m}(\mathbf{r};t)$. The static part points always along the $Z$-axis, which is chosen along the equilibrium magnetization orientation (see Fig. 1.1.). The dynamic magnetization can be written as $\mathbf{m}(\mathbf{r};t) = \mathbf{m}(\mathbf{r})e^{i\omega t}$, so that its spatial part is $\mathbf{m}(\mathbf{r}) = m_X(\mathbf{r})\hat{X} + m_Y(\mathbf{r})\hat{Y}$. For small deviations of the magnetization from the equilibrium position, it can be assumed that $m_{X,Y} \ll M_s$ and then $M_Z \approx M_s - (m_X^2 + m_Y^2)/(2M_s)$ $[113]$. The dynamic magnetization components are represented in Fourier form in terms of the wave vector $\mathbf{k}$ as

$$m_{\alpha,\beta}(\mathbf{r}) = \frac{1}{\sqrt{L^2 d}} \sum_{\mathbf{k}} m_{\alpha,\beta}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}},$$  

(1.1)
where \( \mathbf{r} \) and \( \mathbf{k} \) lies in the plane of the film of area \( L^2 \) and thickness \( d \), and \( \alpha, \beta = X, Y \). The spin wave Hamiltonian can be expressed as \([113, 114]\)

\[
\mathcal{H} = \frac{1}{2M_s} \sum_{\mathbf{k}} \sum_{\alpha\beta} \mathcal{W}_{\alpha\beta}(\mathbf{k}) m_\alpha^*(\mathbf{k}) m_\beta(\mathbf{k}),
\]

(1.2)

where \( \mathcal{W}_{\alpha\beta}(\mathbf{k}) \) are the stiffness fields, which contain contributions from dipolar energy, Zeeman energy, exchange interaction, anisotropies, and the DMI. These energies were calculated in Refs. \([113, 114]\), while the DMI was studied in Refs. \([95, 109]\) for several Lifshitz invariants representing different crystal symmetries \([12, 17–19, 115]\).

In the micromagnetic limit, the DMI Hamiltonian can be written as

\[
\mathcal{H}_{DM}(\mathbf{r}) = \int_V w_{DM} dV,
\]

(1.3)

where the energy density \( w_{DM} \) involves combinations of terms known as Lifshitz invariants \([12, 17–19]\), which are defined as

\[
\mathcal{L}^{(k)}_{ij} = \frac{M_i}{\mu_0} \frac{\partial M_j}{\partial x_k} - \frac{M_j}{\mu_0} \frac{\partial M_i}{\partial x_k}.
\]

(1.4)

Depending on the the symmetry of the non-centrosymmetric crystal, \( w_{DM} \) assumes different forms. The most common sum of invariants has the structure (see page 18 in Ref. \([115]\))

\[
\mathcal{L} = \mathcal{L}^{(z)}_{yx} + \mathcal{L}^{(y)}_{xz} + \mathcal{L}^{(x)}_{zy} = \mathbf{M} \cdot (\nabla \times \mathbf{M})
\]

(1.5)

that is characteristic of non-centrosymmetric cubic crystals, for instance MnSi, FeGe, MnGe or Cu$_2$OSeO$_3$ \([109, 115]\). The DM energy density is given by \( w_{DM} = \mu_0 \lambda_{DM} \mathcal{L} \), where \( \lambda_{DM} = D/\mu_0 M_s^2 \) has units of length, and is proportional to the usual DMI strength \( D \) in units of Joule/meter$^2$. Then, for the cubic crystals, the DM energy reads

\[
\mathcal{H}_{DM}(\mathbf{r}) = \frac{D}{M_s^2} \int_V \mathbf{M} \cdot (\nabla \times \mathbf{M}) dV.
\]

(1.6)

For different crystal symmetries, one can write the DMI in terms of other combinations of Lifshitz invariants, as shown in Table 1.1, which was adapted from Ref. \([95]\), where cgs units were used with the strength of the DMI represented by the length \( \lambda_{DM} \). In Table 1.1, as in the rest of the chapter, SI units are used, while the strength of the DMI is represented by the parameter \( D \) \([96, 109]\). Following Refs. \([95, 114]\), the magnetization components in the fixed coordinate system \( xyz \) (see Fig. 1.1.1) are specified as

\[
M_x = m_X(\mathbf{r})
\]

(1.7a)

\[
M_y = M_Z \sin \phi_m + m_Y(\mathbf{r}) \cos \phi_m
\]

(1.7b)

\[
M_z = M_Z \cos \phi_m - m_Y(\mathbf{r}) \sin \phi_m.
\]

(1.7c)
where $\phi_m$ is the angle between the equilibrium magnetization and the plane of the film. By expanding the dynamic components of the magnetization up to second order, the sum of invariants (see Eq. (1.5)) are expressed as

$$
\mathcal{L} \approx M_s \frac{\partial m_Y}{\partial x} + M_s \sin \phi_m \frac{\partial m_X}{\partial z} + m_Y \cos \phi_m \frac{\partial m_X}{\partial z} - m_X \cos \phi_m \frac{\partial m_Y}{\partial z}.
$$

(1.8)

In the thin film limit, the dynamic magnetization components do not depend on the normal coordinate $y$, so that $m_{X,Y}(r) = m_{X,Y}(x, z)$, and then it is possible to integrate along the thickness to obtain

$$
\mathcal{H}_{DM} = \frac{Dd \cos \phi_m}{M_s^2} \int \left[ m_Y \frac{\partial m_X}{\partial z} - m_X \frac{\partial m_Y}{\partial z} \right] dx dz,
$$

(1.9)

where the linear terms have been disregarded, since they cancel out due to the equilibrium condition. Using the Fourier representation (see Eq. (1.1)) the Hamiltonian associated to the bulk DMI is given by

$$
\mathcal{H}_{DM} = \frac{1}{2M_s} \sum_k \left\{ \mathcal{W}_{XY}^{DM}(k)m_X^*(k; t)m_Y(k; t) + \mathcal{W}_{YY}^{DM}(k)m_Y^*(k; t)m_X(k; t) \right\},
$$

(1.10)

in which the DMI stiffness fields are given by

$$
\mathcal{W}_{XY}^{DM}(k) = -\frac{2D}{M_s} i k \cos \phi_k \cos \phi_m
$$

(1.11)

and $\mathcal{W}_{YY}^{DM}(k) = -\mathcal{W}_{XY}^{DM}(k)$. Here, $\phi_k$ is the angle between the wave vector and the projection of the equilibrium magnetization onto the plane, as shown in Fig. 1.1. Using the other energetic contributions (see Ref. [95, 114] for details) that add to the total Hamiltonian (Eq. (1.2)), which include the DM term (Eq. (1.10)), the stiffness fields are calculated as

$$
\mathcal{W}_{XX}(k) = W_{XX}(0) + \mu_0 M_s F(kd) \sin^2 \phi_k + D_{ex} k^2 \\
\mathcal{W}_{YY}(k) = W_{YY}(0) - \mu_0 M_s F(kd) \cos(2\phi_m) \\
+ \sin^2 \phi_k \sin^2 \phi_m + D_{ex} k^2 \\
\mathcal{W}_{XY}(k) = -\mu_0 M_s F(kd) \sin \phi_k \cos \phi_m \sin \phi_m \\
- \frac{2D}{M_s} i k \cos \phi_k \cos \phi_m \\
\mathcal{W}_{YX}(k) = -\mu_0 M_s F(kd) \sin \phi_k \cos \phi_k \sin \phi_m \\
+ \frac{2D}{M_s} i k \cos \phi_k \cos \phi_m
$$

(1.12a)

(1.12b)

(1.12c)

(1.12d)
where \( D_{\text{ex}} = 2A/M_s \) is the exchange stiffness, and \( F(x) = 1 - (1 - e^{-x})/x \). Additionally,

\[
W_{XX}(0) = \mu_0 H_0 \cos(\phi_h - \phi_m) - \mu_0 M_{\text{eff}} \sin^2 \phi_m \tag{1.13a}
\]

\[
W_{YY}(0) = \mu_0 H_0 \cos(\phi_h - \phi_m) + \mu_0 M_{\text{eff}} \cos(2\phi_m). \tag{1.13b}
\]

In Eq. (1.13), \( H_0 \) is a dc magnetic field applied at an angle \( \phi_h \) respect to the plane, and \( M_{\text{eff}} = M_s + H_s \) is the effective magnetization, with \( H_s \) an easy-plane surface anisotropy. The angle \( \phi_m \) can be calculated from a simple equilibrium condition \([95,114]\).

To calculate the spin wave dispersion relation, the dynamic magnetization components are promoted to operators, with well-known commutation rules \([113,116]\). The temporal evolution of the dynamic magnetization is given by

\[
i \hbar \dot{m}_{X,Y}(k; t) = [m_{X,Y}(k; t), \mathcal{H}(t)]. \tag{1.14}
\]

Then, using \( m_{X,Y}(-k; t) = m^*_{X,Y}(k; t) \), the Hamiltonian given by Eq. (1.2) and (1.12), the commutation rules from Ref. \([113,116]\) and the definition of the absolute value of the gyromagnetic ratio \( \gamma = -\mu h^{-1} \), with \( \mu \) as the the magnetic moment of a magnetic ion in the film, it is possible to obtain a pair of first order differential equations that can be written in matrix form,

\[
\begin{pmatrix}
\dot{m}_X \\
\dot{m}_Y
\end{pmatrix}
= \begin{pmatrix}
0 & \gamma W_{XY} \\
\gamma W_{XX} & 0
\end{pmatrix}
\begin{pmatrix}
m_X \\
m_Y
\end{pmatrix}. \tag{1.15}
\]

The dispersion relation of the spin waves can now be computed by assuming harmonic solutions for the dynamic magnetization components, and reads

\[
\omega_{\pm}(k) = \gamma \left\{ -3 |W_{XY}| \pm \sqrt{W_{XX}W_{YY} - \Re[W_{XY}]^2} \right\}. \tag{1.16}
\]

Here, \( \Im[W_{XY}] \) and \( \Re[W_{XY}] \) denote the imaginary and the real part of the function \( W_{XY} \). Notice that the above formula is valid for any symmetry class, provided that \( W_{XY}(k) = W^*_{YX}(k) \) \([95]\). For a system with bulk DMI, the dispersion relation becomes

\[
\omega(k) = \frac{2\gamma D}{M_s} k \cos \phi_k \cos \phi_m + \gamma |W_{XX}(k)W_{YY}(k)|
- \left( \frac{\mu_0 M_s}{2} kd \sin \phi_k \cos \phi_m \sin \phi_m \right)^{1/2}. \tag{1.17}
\]

The DMI appears only in the linear term proportional to \( D \). For the Lifshitz invariants describing chiral lattice ferromagnets (\( T \) symmetry class in Table 1.1), the influence of the DMI on the spectrum is strongest when the magnetization lies in the plane \( (\phi_m = 0) \) and when wave vectors are parallel to the equilibrium magnetization \( (\phi_k = 0) \), which are known as backward volume magnetostatic spin waves. Under wave vector inversion \( (k \to -k) \), where \( k \) rotates \( \pi \) radians, one must change \( \phi_k \) by \( \phi_k \pm \pi \). In this case, the DM term in the dispersion relation
changes its sign, since \( \cos(\phi_k \pm \pi) = -\cos \phi_k \), and therefore, because of this asymmetry, the relation is non-reciprocal \([95, 101, 103, 109]\). According to this, the frequency \( f(k) = \omega(k)/(2\pi) \) can be used to define a frequency asymmetry given by

\[
\Delta f = f(k) - f(-k). \quad (1.18)
\]

In the case of the \( T \) symmetry class, the non-reciprocity in the spin wave dispersion relation is linear with respect to the wave vector and reads

\[
\Delta f = \frac{2\gamma D}{\pi M_s} k \cos \phi_k \cos \phi_m. \quad (1.19)
\]

For this particular crystallographic class and in the backward-volume configuration (\( \phi_k = 0 \)), non-reciprocal spin waves have been observed recently in different magnetic compounds. These include the bulk non-centrosymmetric ferromagnet \( \text{LiFe}_5\text{O}_8 \) \([101]\), \( \text{Cu}_2\text{OSeO}_3 \) \([103]\), MnSi \([102, 104, 111]\), and the non-centrosymmetric antiferromagnet \( \alpha\text{Cu}_2\text{V}_2\text{O}_7 \) \([112]\).

Similar results are also obtained for the other allowed combinations of Lifshitz invariants, which are governed by the symmetry of the crystallographic classes. The \( \Delta f \) frequency expressions of the different invariants are summarised in Table 1.1. They follow a common structure because in the thin film limit there is no dependence of the dynamic magnetization on the \( y \)-coordinate, and the second order terms share the same structure. Furthermore, as Eq. (1.19) dictates, all the asymmetries vanish if the system is saturated perpendicularly to the film plane (\( \phi_m = \pi/2 \)), and \( \Delta f \) is largest when \( M \) is parallel to the film plane. The differences in the frequency asymmetries are the dependence on the angle \( \phi_k \) between \( M \) and \( k \), and the number of DMI constants involved.

1.3 Spin waves in thin films with interfacial DMI

A different mechanism for the presence of DMI in ferromagnets is the spin-orbit coupling arising at the interface of a thin film ferromagnet adjacent to heavy metal layer. Under the presence of interfacial DMI, spin waves exhibit similar effects than the ones in bulk DMI systems. These effects have been theoretically studied by several groups \([86, 88–100]\). Furthermore, in these film samples, of particular interest is the dependence of the interfacial DMI strength \([86]\) on the heavy-metal thickness, which requires the evaluation of the DMI vector. Both the thickness dependence and spin wave effects in interfacial DMI films are discussed in this section.

Starting with the analysis of the heavy-metal thickness, the theoretical approach used here is based on the microscopic DMI Hamiltonian, which reads

\[
\mathcal{H}_{\text{DMI}} = \sum_{\langle ij \rangle} \mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j). \quad (1.20)
\]
The DMI couples any pair of neighboring atomic spins $S_i$ and $S_j$ in the interfacial layer of the ferromagnetic film through a third heavy metal site. The indexes $i$ and $j$ label a pair of interacting ferromagnetic spins at the interface. The DM vector $D_{ij}$ links ferromagnetic spins at sites $i$ and $j$ with a third heavy metal site, labeled by a lattice vector $l$ in the heavy metal, that is perpendicular to the triangle described by the three sites $11, 12, 13$. This theory follows the formalism developed by Fert and Levy for disordered magnetic alloys with heavy metal impurities [45, 46]. The existence of a DMI at the ferromagnet/heavy metal interface is a well-established phenomena, in which the DMI strength decreases with the thickness $d$ of the ferromagnetic layer, due to the interface nature of the DMI [75], and increases up to a saturation value with the thickness of the heavy metal layer [86, 87]. This behavior suggests that there is a significant number of heavy metal atoms contributing to the strength of the interfacial DMI. Therefore, to calculate the DM vector, the thickness and lattice structure of the heavy metal need to be considered. The DM vector that links a pair of ferromagnetic spins includes contributions from a group of neighboring heavy metal atoms, and can be evaluated by summing over several heavy metal lattice vectors $l$, see Fig. 1 in Ref. [86]), this is

$$D_{ij} = \sum_{l} V_1 \frac{\hat{R}_i^l \cdot \hat{R}_j^l}{R_i^l R_j^l R_{ij}} \mathbf{R}_i^l \times \mathbf{R}_j^l \sin[k_F (R_i^l + R_j^l + R_{ij}) + \pi Z_d/10],$$

where $V_1 = \frac{135\pi}{32} \frac{\lambda d^2}{E_F^2} \sin(\pi Z_d/10)$ has units of Joule meter$^3$, and depends on the SOC constant $\lambda_d$, the Fermi energy $E_F$ and Fermi wave vector $k_F$ of the conduction electrons, the coupling constant $\Gamma$ between ferromagnetic spins, and the number of incomplete sub-shell electrons $Z_d$ (see Eqs. (5) and (6) in Ref. [45], and Eq. (26) in Ref. [46] for details). The position vectors $\mathbf{R}_i^l$ and $\mathbf{R}_j^l$, join the ferromagnetic sites $S_i$ and $S_j$ to the heavy metal sites. The DM vector between spins at sites $i$ and $i + x$ can be written as $D_{i,j;i+i+x} = -D \hat{z}$, while the corresponding DM vector between spins at $i$ and $i + z$ reads $D_{i,j;i+i+z} = D \hat{x}$ [86]. The $y$-component of $D_{ij}$ vanishes due to the assumed perfectly flat interface.

The calculation of the spin wave dispersion relation and the frequency non-reciprocity follows the same approach of the previous section, but with a different combination of Lifshitz invariants, namely $\mathcal{L} = \mathcal{L}_{zy}^{(2)} + \mathcal{L}_{xz}^{(2)}$. After replacing the magnetization components $M_x, M_y$ and $M_z$ (see Eq. Eq. (1.7)) up to second order into $\mathcal{L}$, it is obtained

$$\mathcal{L} \approx M_s \frac{\partial m_Y}{\partial z} - M_s \sin \phi_m \frac{\partial m_X}{\partial x} - \cos \phi_m m_Y \frac{\partial m_X}{\partial x},$$

In the limit of a thin film the DMI Hamiltonian becomes

$$\mathcal{H}_{DM} = \frac{D d \cos \phi_m}{M_s^2} \int \left[ m_Y \frac{\partial m_X}{\partial z} - m_X \frac{\partial m_Y}{\partial z} \right] dx \, dz.$$
Within a Fourier representation of the dynamic magnetization, the frequency dispersion of the spin waves is separated into two contributions, \( f(k) = f_r(k) + f_{DM}(k) \), a reciprocal part and a non-reciprocal one associated to the DMI, which is given by

\[
\frac{f_{DM}(k)}{f_r(k)} = \frac{\gamma D(d_{HM})}{\pi M_s} |k| \sin \phi_k \cos \phi_m. \tag{1.24}
\]

In this expression \( D(d_{HM}) \equiv \frac{s^2}{\sum_{ijkl} |D_{ijkl}|} \) is the volume averaged DMI strength, \( \gamma \) is the gyromagnetic ratio, \( a_0 \) is the separation between nearest-neighbor ferromagnetic spins and \( n \) is an integer number introduced to consider first, second or even third neighbors [86]. The reciprocal part \( f_r(k) \) includes exchange, dipolar, anisotropy, and Zeeman contributions [95]. The frequency difference of oppositely propagating spin waves, \( \Delta f = f(k) - f(-k) \), is then calculated as

\[
\Delta f(k, d_{HM}) = \frac{2\gamma D(d_{HM})}{\pi M_s} |k| \sin \phi_k \cos \phi_m. \tag{1.25}
\]

Most of the parameters involved in the frequency difference can be controlled by the experimental setup. For example, \( \phi_m \) is controlled with the orientation of the applied field, \( \phi_k \) and \( k \) are controlled with the excitation mechanism, while \( \gamma \) and \( M_s \) can be measured by conventional magnetometry or ferromagnetic resonance (FMR). Then, by measuring \( \Delta f \) the strength of the interfacial DMI can be directly measured, as was first theoretically proposed in Ref. [95], and then measured by several groups, mainly using BLS [72–87], as summarized in Table 1.2 and 1.3. It is worth to mention that the estimation of the DMI strength through measurement of the frequency difference between counter-propagating spin waves in the field polarized case, is the most reliable method, since there is no assumptions about the possible magnetic texture. Moreover, there is no need to neglect the dipolar interactions, as in the case of domain-wall based methods.

The theoretical model developed previously has been applied to analyze experimental observations of the strength of the DM constant in Pt\((d_{HM})/\)CoFeB films. Specifically, Tacchi et al. [86] studied the influence of the heavy metal thickness \((d_{HM})\) on the induced DMI by means of BLS measurements of ultrathin CoFeB films. As a result, it was found that the strength of the interfacial DMI increases with heavy metal thickness, reaching a saturation value for \(d_{HM}\) larger than a few nanometers [86]. In the BLS spectra measurements reported in Ref. [86], the Stokes and anti-Stokes peaks, corresponding to spin waves propagating in opposite directions, are simultaneously observed with comparable intensity. These peaks are characterized by a frequency shift that increases with heavy metal thickness. The frequency of both peaks interchanges when reversing the direction of the applied magnetic field, due to the reversal of the propagation direction of the spin waves. It was also found that the frequency asymmetry exhibits a linear dependence as a function of \( k \), and it becomes more pronounced...
when increasing the thickness of the heavy metal. The in-plane angular dependence of the frequency shift was also measured with BLS \cite{73,75,86,87}, which evidences a clear sinusoidal dependence with $\phi_k$, in agreement with Eq. (1.25), as predicted theoretically in Ref. \cite{95}.

1.4 Micromagnetic simulations of spin waves with interfacial DMI

Micromagnetic simulations are a powerful numerical method for the analysis of spin dynamics in magnetic samples of a few microns. In particular, this numerical technique can be used to calculate the spectrum of spin waves and, when a DMI is present, it reveals different properties characteristic of these systems, such as the spin wave asymmetry. In this context, simulations have successfully supported theoretical formulations \cite{80,95,96,98,131–136} and experimental measurements \cite{72–87,137} on thin film systems with an homogeneous DMI.

A micromagnetic simulation is based on numerically discretizing the continuum description of the magnetic system into a mesh of magnetic moments whose arrangement depends on the discretization method. The two most commonly used methods are finite differences and finite elements. For the former, the sample is divided into a regular grid of cuboids which each represents a magnetic moment (see Fig. 1.2. (a)), and the expressions of the magnetic interactions are approximated according to this numerical method. The spacing between cuboids is usually chosen with a magnitude smaller than the exchange length. To simulate the dynamics of the magnetization, the Landau-Lifshitz-Gilbert equation of motion is numerically integrated for every magnetic moment. Three publicly available finite difference micromagnetic simulation codes, these are OOMMF \cite{138}, MuMax3 \cite{139} and Fidimag \cite{140}.

The method for the simulation of spin waves propagating in a single direction has been discussed in detail in Refs. \cite{141,142} and in Ref. \cite{143} for the case of a thin film with DMI. Following the techniques of the aforementioned studies, a similar system has been simulated (see Fig. 1.2. (a)) using the parameters shown in Table 1.4. To simulate the infinite film it is customary to specify a sufficiently long waveguide along the spin wave propagation direction, in this case the $x$-direction. Consequently, the sample is saturated with a bias field $\mathbf{H}$. To excite the spin waves, a weak periodic magnetic field $\mathbf{h}_{\text{exc}}$ is applied in a region at the center of the waveguide, in order to generate waves propagating in the two opposite directions, and it is applied perpendicular to the bias field. Experimentally, the spin wave pulses can be generated using an antenna \cite{144} (see Fig. 1.2. (a)). The amplitude of the simulated periodic field is significantly smaller than the bias field. It is suggested to use a sinc function dependent on time for the field, delayed by a time $t_0$, in order to excite all possible wave modes within a specific range of frequencies given by a cut-off frequency $f_0$ in the sinc function argument. A similar method can be applied to the spatial coordinates to restrict the range of wave vector magnitudes \cite{141}. When exciting the system, the magnetization components are saved for
Table 1.2. Reported magnitudes for the DM coupling in various heavy metal/ferromagnet interfacial systems. Experimental measurements include BLS, electrical and domain wall (DW) based methods, micromagnetic simulations and microscopy techniques. The thicknesses of each layer comprising the structure is measured in nanometers (nm).

| Material                        | $|D|$ (mJ/m$^2$) | Method        |
|---------------------------------|---------------|---------------|
| Pt(3)/CoFe(0.6)/MgO [57]       | 0.5           | DW            |
| Pt(5)/Co(0.7)/Ir(0.46)/Pt(3) [117] | 0.12          | DW            |
| Pt(4)/Co(1.6)/Ni [72, 73]      | 0.44          | BLS           |
| Pt(2)/CoFeB(0.8)/MgO [74]      | 1.0           | BLS           |
| Pt(4)/CoFeB(1-1.0)/AlO$_x$ [75] | 0.8 – 0.4     | BLS           |
| Pt(4)/Co(1-2)/AlO$_x$ [75]     | 1.2 – 0.9     | BLS           |
| Pt(6)/NiFe(1.3-10)/SiN [76]    | 0.15 – 0.025  | BLS           |
| Pt(6)/NiFe(4)/Si [77]          | 1.2           | BLS           |
| Pt(3)/Co(0.6-1.2)/AlO$_x$ [78] | 2.7 – 1.57    | BLS           |
| Ta/Pt(4)/Co(1.35)/AlO$_x$ [79] | 1.65          | BLS           |
| Pd/Fe/Ir [118]                 | 3.9           | SP-STM/DW     |
| (Ir(1)/Co(0.6)/Pt(1)) [41]     | 1.6 – 1.9     | STXM/DW       |
| Ir(4)/Co(1.2-3)/AlO$_x$ [80]   | 0.3 – 0.7     | BLS           |
| W(2-3)/CoFeB(1)/MgO [81]       | 0.25 – 0.27   | BLS           |
| W(2-3)/CoFeB(1)/MgO [81]       | 0.23 – 0.12   | DW            |
| TaN(1)/CoFeB(1)/MgO [81]       | 0.31          | BLS           |
| TaN(1)/CoFeB(1)/MgO [81]       | 0.05          | DW            |
| Hf(1)/CoFeB(1)/MgO [81]        | 0.15          | BLS           |
| Hf(1)/CoFeB(1)/MgO [81]        | 0.01          | DW            |
| Pt(3)/Co(16)/MgO [119]         | 0.25          | Electrical    |
Table 1.3. Continuation of Table 1.2. The magnitudes reported in the section at the bottom of this table, refer to DMI values from atomistic parameters or spatial units.

| Material | $|D|$ (mJ/m$^2$) | Method |
|----------|----------------|--------|
| Pt(3)/CoFe(1-1.6)/MgO(1.8) [82] | 1.4 – 2.3 | BLS |
| MgO/CoFe(0.8-1.6)/Pt(4) [82] | 0.9 – 1.5 | BLS |
| W/CoFeB(0.85)/SiO$_2$ [83] | 0.25 | BLS |
| Ta(5)/CoFeB(1)/Pt(0.15)/MgO [84] | 0.056 | BLS |
| Ta(5)/Co$_{20}$Fe$_{60}$B$_{20}$(0.8)/MgO(2) [120] | 0.057 | Kerr/DW |
| Pt(0.4-6)/CoFeB(2)/Cu [86] | 0 – 0.45 | BLS |
| Co(1.4)/Pt [121] | > 0.5 | DW |
| Pt(5.4)/Co(2.5)/Au,Ir(0-2.5)/Pt(2.6) [122] | 0.6 | BLS |
| Ta/Co-Fe-B/TaO$_x$ [87] | 0.22 | BLS |
| IrCo$_2$FeAl(0.9) [123] | 0.34 | BLS |
| MgO(5)/Fe(3)/Pt(4) [124] | 0.349 | BLS |
| SiO$_2$/Fe(3)/Pt(4) [124] | 0.225 | BLS |
| Pt(6)/MgO(367)/Fe(3)/Pt(4) [124] | 0.335 | BLS |
| Pt(6)/MgO(367)/Fe(3)/Al(5)/SiO$_2$(5) [124] | 0.1025 | BLS |
| Pt(6)/Co(3) [125] | 0.33 – 0.43 | BLS |
| Pt/Cu(0.8)/Ir(0-2)/Ta [126] | 0.75 – 1.68 | BLS |
| Pt/Cu(0.8)/Ir(0-2)/Ta [126] | 0.48 – 1.25 | DW |
| Graphene/Ni$_8$Fe$_{20}$(3)/Ta(2) [127] | 67 $\times$ 10$^{-3}$ | BLS |
| FeFe/W [128] | 7.5 $\text{meV/atom}$ | SP-STM |
| Simulation |
| FeFe/W [70] | 0.5 – 0.9meV | SPEELS |
| $(\text{Co/Nil})_n$/Ir/Pt [35] | 0.12 $\text{meV/atom}$ | BLS |
| $(\text{Ni/Co})_n$/Ir [129] | 0.36 $\text{meV/atom}$ | BLS |
| Graphene/Co/Ru [130] | 0.11 $\text{meV/atom}$ | SPLEEM/DW |
Micromagnetic simulations of spin waves with interfacial DMI

Figure 1.2. (a) Thin film waveguide system with DMI, where a ferromagnetic film is in contact with a heavy metal surface and the sample is saturated with a bias field. An antenna at the center of the sample can be used to excite counter-propagating spin waves (characterized by a spin wave vector \( \mathbf{k} \)) across the waveguide. Micromagnetic simulations are specified by a regular grid of cuboids representing magnetic moments. (b) Magnonic crystal specified by a regular pattern of heavy metal stripes.

every magnetic moment in regularly spaced time steps \( \Delta \tau \). Finally, after a sufficiently long time \( \tau \), a two-dimensional Fourier transform in space and time variables is performed in order to obtain the spin wave spectrum. To avoid noise in the spectrum caused by the reflection of the spin waves at the sample edges, it has been shown that using a strong damping towards the sample edges minimizes this effect \[145\]. Alternatively, it is also possible to use periodic boundary conditions.

The result of the simulated spin wave spectrum in a thin film of permalloy, specified in Table 1.4, is depicted in Fig. 1.3, where the spin waves propagate perpendicular to the saturation field. The excited modes in this case are known as Damon-Eshbach modes. This simulation was obtained with the MuMax3 code using periodic boundary conditions along the waveguide width. The spin wave spectrum is shown as the intensity of the two-dimensional Fourier trans-
Table 1.4. Parameters for the simulation of spin waves propagating in a thin film with DMI.

| Thin film waveguide with periodicity along the y-direction |
|----------------------------------------------------------|
| Dimensions | 2000 nm × 128 nm × 1 nm |
| Magnetic parameters | | |
| $A$ | Exchange | 11.1 pJ m$^{-1}$ |
| $D$ | DMI | 3.0 mJ m$^{-2}$ |
| $M_s$ | Magnetization | 0.658 MA m$^{-1}$ |
| $\mu_0 H$ | External field | (0.0, 0.25, 0.0) T |
| $\mu_0 H_{exc}$ | Excitation field | (0.025, 0.0, 0.0) T |
| $f_0$ | Cut-off frequency | 60 GHz |
| $t_0$ | Excitation delay | 50 ps |
| $\gamma$ | Gyromagnetic ratio | $1.76 \times 10^{11}$ Hz T$^{-1}$ |
| $\alpha$ | Damping | 0.01 |
| $\tau$ | Excitation time | 4 ns |
| $\Delta \tau$ | Saving time step | 0.5 ps |

form of the $x$-component of the magnetization, which is scaled logarithmically, and is directly compared with the theory [95,96]. A test for the accuracy of the simulation is the estimation of the DMI from the spin wave asymmetry. Accordingly, a second order polynomial is fitted using the data from $k = -0.0785$ nm$^{-1}$ up to $k = 0.2985$ nm$^{-1}$, and the asymmetry predicts a DMI magnitude of 2.89 mJ m$^{-2}$, in good agreement with the original value of 3 mJ m$^{-2}$. It is possible to extend the lower limit of the data for the curve fit, however the polynomial becomes less accurate in reproducing the intensity peaks away from the minimum, which can be improved by setting a larger simulation time. Moreover, since a cut-off frequency was set to 60 GHz, the spectrum is approximately valid down to $k \approx -0.1$ nm$^{-1}$.

From the result discussed previously, it can be concluded that micromagnetic simulations are highly accurate to reproduce the theoretical predictions. It is now straightforward to extend the simulations to more complex scenarios. For instance, in Fig. 1.3.(b) the same permalloy system is simulated by setting a periodic DMI along the long axis of the waveguide. This can be obtained by patterning the top or bottom surface of the ferromagnetic system with regularly spaced heavy metal wires [146]. The simulated system of Fig. 1.3.(b) was specified by alternating regions of material with and without DMI of 50 nm width. In this case, the periodic DMI induces flat bands and indirect band gaps, which are not characteristic of a magnonic system [146]. These effects are thoroughly discussed in Sec. 1.5.
Figure 1.3. Simulations of spin wave propagation in permalloy thin films with (a) homogeneous DMI from a heavy metal film on top of the ferromagnet, and (b) periodic DMI from regularly patterned heavy metal stripes. Simulations were obtained with the MuMax3 code. Details of simulation (a) is specified in Table 1.4. Simulation (b) is similar but the DMI is set in regions of 50 nm width with a periodicity of 100 nm.
1.5 Spin waves in thin films with a periodic DMI

Spin waves propagating in a periodic magnetic media, such as a magnonic crystal [147] - the magnetic counterpart of photonic crystals - exhibit magnonic band gaps induced by Bragg scattering, which means there are frequencies where propagation is forbidden. These band gaps appear at the boundaries of the Brillouin zones (BZs), and are induced when waves with wave vector \( k = n\pi/a \), being \( a \) the lattice parameter and \( n \) an integer number, satisfy the Bragg condition. On the other hand, the Bragg condition is modified by the breaking of symmetry induced by DM interactions, since two counter propagating waves at the same frequency have different wave lengths, therefore neither magnonic band gaps nor the standing waves appear at the BZ edges anymore. According to this, magnonic crystals with a periodic chiral interaction exhibit indirect band gaps, where band edges lie outside the borders of the Brillouin zones. Different cases of magnonic crystals with chiral properties have been studied recently [99,100], reporting the observation of indirect band gaps. Topological properties were also found in a theoretical study of periodic arrays of magnetic nano-islands on top of a heavy metal layer [148]. In addition, it has been shown, by means of micromagnetic simulations, how the DMI induces a non-trivial temporal evolution of standing spin waves in a geometrically confined structure [149]. Recently, it has been shown how a periodic DMI can produce all these effects in a magnonic thin film system [146], namely (i) indirect band gaps, (ii) low-frequency flat bands, and (iii) an unconventional temporal evolution of the standing waves at the band gap edges. Following this latter work, this section is dedicated to the theoretical background of the role of a periodic DMI.

An overview of a chiral magnonic crystal is shown in Fig. 1.2.(b), where a periodic array of heavy metal stripes is put in contact with a ferromagnetic layer. Owing to the high SOC of the heavy metal and the ultrathin thickness of the ferromagnetic layer, a periodic DMI is expected to occur in the system. In the continuum limit, effective fields associated to the different interactions are required to study the dynamic of the MC by means of both the Landau-Lifshitz equation and the plane-wave method. In the latter approach, the periodic quantities are expanded into Fourier Series so that the equation of motion is treated as an eigenvalue problem [146]. While the effective fields associated to anisotropies, dipolar, and exchange interactions are very well known, the effective field due to DMI \( \mathcal{H}_{DM} \) becomes non-trivial, because the DMI strength (\( D \)) is now a space-dependent periodic parameter. To derive the DM effective field, it is necessary to start from the atomic Hamiltonian \( \mathcal{H}_{DM} = \sum_{(ij)} D_{ij} \cdot (S_i \times S_j) \). Since the DMI is short ranged, one can see that only the terms \( D_{i-1,i} \cdot (S_{i-1} \times S_i) + D_{i,i+1} \cdot (S_i \times S_{i+1}) = S_i \cdot (D_{i-1,i} \times S_{i-1} - D_{i,i+1} \times S_{i+1}) \) are involved in \( \mathcal{H}_{DM} \), thus

\[
\mathcal{H}^{DM} = \sum_i S_i \cdot (D_{i-1,i} \times S_{i-1} - D_{i,i+1} \times S_{i+1}) \tag{1.26}
\]
Here, $S_{i\pm 1}$ represents the neighbor spin of $S_i$, then the following effective field for the spin at site $i$ is obtained,

$$h_{i}^{\text{DM}} = D_{i-1,i} \times S_{i-1} - D_{i,i+1} \times S_{i+1}. \quad (1.27)$$

In a very extended array of spins, the variation of the spin orientation between neighboring sites is very smooth, therefore, in the continuum approximation, the elements located around site $i$ can be written as

$$S_{i\pm 1} \simeq S_i \pm \frac{\partial S_i}{\partial z} \delta z, \quad (1.28)$$

$$D_{i,i+1} \simeq D_{i-1,i} + \frac{\partial D_{i-1,i}}{\partial z} \delta z. \quad (1.29)$$

Notice that it has been assumed that the DM coupling only varies along the $z$-axis (see Fig. 1.2(b)), thus the effective field in Eq. (1.27) becomes

$$h_{i}^{\text{DM}} \simeq \left( -2D_{i-1,i} \times \frac{\partial S_i}{\partial z} - \frac{\partial D_{i-1,i}}{\partial z} \times S_i \right) \delta z. \quad (1.30)$$

Taking into account that $D_{i-1,i} \perp (r_i - r_{i\pm 1})$, the effective DMI field in the continuum limit is

$$h^{\text{DM}} = \frac{1}{\mu_0 M_s^2(r)} \left\{ [2D(r)\partial_z m_z(r) + m_z(r) \partial_z D(r)] \hat{y} - [2D(r)\partial_z m_y(r) + m_y(r) \partial_z D(r)] \hat{z} \right\}, \quad (1.31)$$

where $D(r)$ is the DM strength that is a periodic function of $z$, and can be expanded as $D(r) = \sum G D(G)e^{iG \cdot r}$, with $G = (2\pi n/a)\hat{z}$ a reciprocal lattice vector. By including all the energy contributions into the Landau Lifshitz equation of motion [150], the following eigenvalue problem is obtained

$$\hat{T} m_G = i \frac{\omega}{\mu_0 \gamma} m_G \quad (1.32)$$

where $m_G^T = [m_z(G_1), ..., m_z(G_N), m_y(G_1), ..., m_y(G_N)]$ is the eigenvector and $\hat{T}$ is given by

$$\hat{T} = \begin{pmatrix} \hat{T}_{zz} & \hat{T}_{zy} \\ \hat{T}_{yz} & \hat{T}_{yy} \end{pmatrix}. \quad (1.33)$$

The matrix elements $T^{\eta\eta'}_{G,G'}$ of the submatrices $T^{\eta\eta'}$ (with $\eta = z, y$), are defined as follows

$$T^{zz}_{G,G'} = T^{yy}_{G,G'} = -\frac{2D(G - G')}{\mu_0 M_s} \left( \frac{G + G'}{2} + k \right) \cdot \hat{z},$$

$$T^{zy}_{G,G'} = -\left[ M_s (G + k)^2 \lambda_{ex}^2 + H + M_F G, k \right] \delta_{G,G'},$$

$$T^{yz}_{G,G'} = \left[ M_s (G + k)^2 \lambda_{ex}^2 + H + M_s (1 - F_G, k) \right] \delta_{G,G'}.$$

In Eq. (1.34), $F_G, k = e^{-|G + k|/d/2}$, $\lambda_{ex}$ is the exchange length, and $d$ is the thickness of the ferromagnet.
Figure 1.4. (a,c) Dispersion relation for a permalloy film covered with a heavy metal-wire array, where \( a = 100 \text{ nm} \), \( D = 0 \), \( D = 1.5 \) and \( 3 \text{ mJ/m}^2 \). The lines correspond to the theory and the color code to simulations with the OOMMF code, where darker (lighter) color represents an intensity maximum (minimum). Figure (d) depicts the wave-vector shift \( \Delta k \), while (e) illustrates the modes I and II (see (b)) as a function of \( D \). In figure (f) the 1st band gap as a function of the Dzyaloshinskii-Moriya strength \( D \) is shown. The shaded areas correspond to three ranges of \( D \)-values where different behaviors are predicted. Insets in (f) show the first band gap behavior during the transition between the shaded areas [146].
1.5.1 Spin wave dynamics under periodic DMI

To describe the dynamic properties of the system shown in Fig. 1.2(b), where the equilibrium magnetization is pointing along $x$ and the spin wave propagation is along $z$, typical magnetic parameters of permalloy are used (see Table 1.4). Under zero DMI, the dispersion is completely reciprocal, as shown in Fig. 1.4(a). When a periodic DMI is present, indirect band gaps open, which are generated by the non-reciprocal character of the DMI. Interestingly, when the DMI strength is increased the low frequency branches decrease in frequency and become flat (see Fig. 1.4(c,d)). One particular feature of the spin wave dynamics with a periodic DMI is that the transition from reciprocal dispersion (see Fig. 1.4(a)) up to the emergence of low-frequency flat bands, is not monotonic, as illustrated in Fig. 1.4(d,f). While the parameter $D$ is increased, the indirect character of the spin waves also increases. Then, the mode II [minimum of the band-gap edge shown in Fig. 1.4(b)] reaches the second Brillouin zone (2BZ) and it remains pinned under the increasing of $D$, as shown by the triangles in Fig. 1.4(e). At this point, mode I is still shifting to high $k$ values, so that when mode I also reaches the 2BZ the flat region is reached and the mode shift to lower frequencies. If $D$ further increases, the upper-frequency branches also become flat. The emergence of these flat bands is enhanced if the lattice parameter $a$ increases (not shown), and it can be manipulated by changing the geometrical properties of the heavy-metal stripes [146]. These flat bands possibly have interesting consequences, such as the Bose-Einstein condensation [151] predicted in bosonic systems hosting flat bands [152], and frustrated quantum magnets [153,154]. Topological properties of higher-dimensional periodic DMI structures are still to be explored.

It is important to remark that the influence of a continuous heavy metal film in contact with a ferromagnetic layer is not observed in the FMR response [95]. The inclusion of a periodic DMI modifies the band structure in a way that additional modes can be now observed in FMR experiments [100,146,149]. A difficulty arises, however, when trying to understand the temporal evolution of these waves, because the standing waves around the gaps originate outside the BZs. Indeed, while a typical standing behavior is observed in the zones without DMI, the spin waves have a non-zero phase velocity in the areas in contact with the heavy metal stripes. This time dependence can be understood by taking into account the boundary conditions induced by DMI at the interfaces, since the DMI produces canted states of the dynamic magnetization inducing, thus, a non trivial temporal evolution of the spin waves along the whole system (see Ref. [146] for details).

It can be shown that the inclusion of damping does not modify the band structure obtained in Fig. 1.4. notoriously. It is well known that if a ferromagnet is coupled to a heavy metal the damping increases, which is not favorable for applications. Hence, other kinds of nonreciprocal sources could be of interest, as for instance (i) an electric current using the adiabatic spin transfer torque [155,156], (ii) metallized MCs [157,158] (iii) a bicomponent MC composed by
a normal ferromagnet and a bulk periodic DMI material, as well as (iv) periodic ferromagnetic bilayers, where non-reciprocity is induced by dynamic dipolar interaction [?,62,68,159–162], and (v) films with graded magnetization along the thickness [?].

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