Low energy constraints on orbifold models

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Abstract

We review the low energy limits on Kaluza-Klein excitations in orbifold models. New vector-like quarks, as well as new $Z'$ gauge bosons, can be accommodated with masses observable at large colliders.

1 INTRODUCTION

Although the Standard Model (SM) is in agreement with experiment up to energies near the electroweak scale \cite{1}, it is believed to be an effective theory. Many of its completions, as for instance string theory, require extra dimensions which, if large enough, may manifest at large colliders.

Let $M_c$ be the compactification scale with $R \sim M_c^{-1}$ the size of the largest extra dimension. If the scale $M_s$ of the more fundamental theory is larger than $M_c$, there should be a range of energies where physics can be described by an effective theory in $d > 4$ dimensions. Such theories are interesting not only by their phenomenological implications but for giving new insights into more theoretical questions like the replica of families or the breaking of symmetries. All these questions have received a great deal of attention. Here we want to review the prospects for having new matter and/or interactions at the TeV scale, ignoring at this stage new gravitational effects.

Even in the case of one extra dimension, the simplest models allow for a rich phenomenology. Fields living in the fifth dimension, bulk fields, can be described by an infinite tower of Kaluza-Klein (KK) modes. The known particles corresponding to the zero modes and the heavy excitations to new exotic fermions and/or gauge bosons and scalars. Typically the heavy masses scale with $M_c$ which plays the role of the order parameter of the physics beyond the SM. Thus models can be classified according to the present experimental bounds on its value. In Table 1 we order the different classes of models with growing compactification scale limits.

As observed in Ref. \cite{2}, if there is conservation of the KK number, there are no couplings of SM fields with only one heavy mode. Hence the new contributions to SM observables come only at one loop, implying an extra suppression factor $\sim \alpha$. This translates into a relatively low $M_c$ limit, $\sim 300$ GeV, mainly resulting from the T parameter and the running of the heavy excitations around the loop. A simple realization of this scenario is a five-dimensional theory with the extra dimension parametrizing an orbifold $S^1/Z_2$: $-\pi R \leq y \leq \pi R$ with opposite points identified, and all fields in the bulk and without brane terms.

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Table 1: Classification of models with extra dimensions according to the present experimental bounds on the compactification scale $M_c$.

| Class of models   | $M_c$ bound | Constraining observable |
|-------------------|-------------|--------------------------|
| Universal         | $M_c \gtrsim 300$ GeV | T parameter [2]          |
| Brane interactions| $M_c \gtrsim 4$ TeV | Precision data [3, 4]   |
| Randall-Sundrum   | $M_c \gtrsim 10$ TeV | Precision data [5]      |
| Fermion spreading | $M_c \gtrsim 100$ TeV | FCNC [4, 6]             |
| Fermion multilocalization | $M_c \gtrsim 80$ TeV | $e^{-M/M_c}M_c \sim 300$ GeV |
|                   | $e^{-M/M_c}M_c \sim 300$ GeV | Precision and production data [7] |

The addition of brane interactions violates momentum conservation in the fifth dimension and introduces new vertices involving SM particles and only one heavy mode. Then, if standard matter is assumed to live only in the brane in order to avoid large Flavour Changing Neutral Currents (FCNC), the strongest constraint on the compactification scale results from the tree level exchange of the KK tower of gauge bosons, implying $M_c \gtrsim 4$ TeV to maintain the electroweak parameters within experimental limits [3, 4].

These simple models have a flat background but enough physics to share the main new effects. Nature can choose, however, a more general situation with a non-trivial background, as for instance in the case of the Randall-Sundrum model [8]. A detailed analysis gives in this case $M_c \gtrsim 10$ TeV [5].

A different scenario appears when the models are generalized to include fermions in the bulk in order to explain the observed pattern of fermion masses and mixings geometrically, split fermions [9]. Doing so the disalignment of the KK gauge couplings to SM fermions generates large FCNC [4, 6], which banish the gauge boson masses and $M_c$ to high energy, $\gtrsim 100$ TeV, making them unobservable at future colliders. This is only a particular case. When general five-dimensional masses and/or brane kinetic terms are introduced, the masses of the lightest KK excitations can decouple from $M_c$ due to their dependence on the size of the new terms, and become observable. Here we concentrate on this type of models and their manifestation at large colliders.

In the following we review the KK formulation of these models. Then we first introduce five-dimensional mass terms for the fermions and discuss simple models with multi-localization of the lightest KK fermion wave function in the extra dimension. In this way the observed pattern of fermions masses and mixings can be obtained with order one Yukawa couplings, together with relatively light exotic fermions [10] decoupled from a rather heavy compactification scale and observable at large colliders [7, 11]. Afterwards, we consider brane kinetic terms for gauge bosons [12]. For a choice of terms one can also reduce the mass of the lightest KK gauge boson excitation, making it eventually observable. This can also be done for fermions. In both cases the decoupling of the lightest heavy masses are not due to the multi-localization of the wave functions but to a sharper localization.

2 FOUR-DIMENSIONAL FIELDS, MASSES AND COUPLINGS

An effective five-dimensional theory whose Lagrangian reads

$$\mathcal{L}(\phi) = \mathcal{L}_{\text{kin}}(\phi) + \mathcal{L}_{\text{int}}(\phi),$$

(1)

can be described in terms of four-dimensional fields decomposing the corresponding five-dimensional fields in four-dimensional (KK) modes

$$\phi(x, y) = \sum_n \phi^{(n)}(x) f_n(y) \sqrt{2\pi R},$$

(2)
After integrating the extra dimension one is left with a four-dimensional Lagrangian with an infinite number of KK fields $\phi^{(n)}(x)$ with masses $m_n$, gauge couplings $g_n$... and Yukawa couplings $\lambda_n...$. In this procedure the wave functions $f_n(y)$ are fixed requiring orthonormality conditions, in order to guarantee canonical kinetic terms for $\phi^{(n)}$, and fulfilling the differential equations giving them diagonal masses. Obviously different $L_{\text{kin}}$ give different wave functions, thus modifying the gauge and Yukawa couplings, and the physics derived from them. For a bulk gauge boson $A_\mu$ and a fermion $\Psi$

$$L_{\text{gauge}} = - \int dy \, g_5 \bar{\Psi}\gamma^\mu A_\mu \Psi,$$

and after decomposing and integrating the fifth dimension

$$L_{\text{gauge}} = - \sum_{nmr} \left[ g_{nL}^{LLV} \bar{\Psi}_L^{(n)} \gamma^\mu A_\mu^{(r)} \Psi_L^{(m)} + (L \to R) \right],$$

with

$$g_{nL}^{LLV} = \frac{g_5}{\sqrt{2\pi R}} \int_{-\pi R}^{\pi R} dy \, f_n^{L} f_m^{L} f_r^{V},$$

the dimensionless four-dimensional gauge couplings. Similarly,

$$L_{\text{Yuk}} = - \int dy \left[ \lambda_5 \bar{\Psi} \chi \phi + h.c. \right],$$

$$= - \sum_{nmr} \left[ \lambda_{PS}^{SS} \bar{\Psi}_L^{(n)} \chi_R^{(m)} \phi^{(r)} + \lambda_{PS}^{SS} \bar{\Psi}_R^{(n)} \chi_L^{(m)} \phi^{(r)} + h.c. \right],$$

with

$$\lambda_{PS}^{SS} = \frac{\lambda_5}{\pi R} \int_{-\pi R}^{\pi R} dy \, f_n^{PS} f_m^{PS} f_r^{SS},$$

the dimensionless four-dimensional Yukawa couplings. Tree level estimates of the contribution of the KK excitations are derived in the properly normalized mass eigens tate basis using $g$ and $\lambda$ in Eqs. (5) and (7), respectively. For the particular case of a boundary Higgs (without scalar KK tower)

$$\lambda_{nm} = \frac{\tilde{\lambda}_5}{2\pi R} f_n^{PS}(0) f_m^{PS}(0),$$

where the tilde stands for the different dimension normalization.

3  FIVE-DIMENSIONAL FERMION MASSES AND MULTI-LOCALIZATION

Models with extra dimensions can explain the observed hierarchy of fermion masses and mixings relating them to the localization of the corresponding zero modes around different points in the extra dimensions. However, the lightest modes are often not localized but multi-localized if the five-dimensional fermion masses are non-trivial. Indeed, the addition of a step-function mass term in the $S^1/Z_2$ orbifold case allows for the bi-localization of the first two fermion modes. As shown in Ref. [7] for

$$L_{\text{kin}} = \int dy \, [i \gamma^5 \partial_N - M(y)] \Psi,$$

with $\gamma^N = \{ \gamma^\mu, i\gamma^5 \}$ and

$$M(y) = M\sigma(y)\sigma(\pi a - |y|), \quad \sigma(y) \equiv |y|/y,$$

one obtains the mass spectrum in Fig. 1 solving the corresponding normalization and eigenvalue equations. The corresponding wave functions for the zero mode and the lightest KK excitation are...
Figure 1: Values of the masses of the first KK modes as a function of the five-dimensional mass in units of $1/R$. We have fixed $a = R/2$ in Eq. (10).

Figure 2: Profiles of the massless zero mode $f_0^R$ and the first KK excitation $f_1^R$ with no multi-localization, $MR = -2$ (left), and with multi-localization, $MR = 2$ (right).

drawn in Fig. 2. (We choose the Right-Handed (RH) component even.) For $M = 0$ one obtains the usual equidistant spectrum, whereas for negative $M$ the four-dimensional masses get extra larger contributions but for positive $M$ the lightest KK mode can be as light as required. In this case the difference between the absolute values of the two wave functions in Fig. 2 are exponentially suppressed. Many (split fermion) models give a similar behaviour [13]. This type of models can accommodate in a natural way the observed spectrum of masses and mixings (starting with five-dimensional Yukawa couplings of order one), and at the same time an observable departure from the SM at large colliders [7, 11]. In Fig. 3 we plot the predicted mass of a new quark of charge $2/3$ and the third family charged current coupling as a function of the parameters of the model $^2$. The shaded region is excluded by electroweak precision data.

4 BRANE KINETIC TERMS AND LIGHT NEW PARTICLES

Brane kinetic terms are naturally present in orbifold theories [15]. They cannot be set to zero at all scales in general, since the breaking of translation invariance induces them at the quantum level. In this case the equations of motion are modified leading to a different spectrum which can include light KK states. In this section we review examples in which the inclusion of brane kinetic terms can induce a light first KK mode for gauge bosons and/or fermions. We do not intend to be exhaustive here but just show, with the very practical goal of getting light KK excitations, the effects of some of these terms and discuss the corresponding phenomenology. A more detailed study will be presented elsewhere [16].

Let us consider the Lagrangian for a five-dimensional ($U(1)$ for simplicity) gauge theory with

$^2$Multi-localization can also have interesting phenomenological implications for neutrino oscillations [14].
the following brane kinetic terms [12]

\[ \mathcal{L} = -\frac{1}{4} \int dy \left\{ F^{MN} F_{MN} + \left[ a_0 \delta_0 + a_\pi \delta_\pi \right] F^{\mu\nu} F_{\mu\nu} \right\}, \]  

(11)

where \( \delta_0 \equiv \delta(y) \) and \( \delta_\pi \equiv \delta(y - \pi R) \). Performing the KK expansion in the Lagrangian and requiring the quadratic terms to decouple, we arrive at the following orthogonality and differential equation conditions

\[ \int dy \left[ 1 + a_0 \delta_0 + a_\pi \delta_\pi \right] f^m f_n = \delta_{nm}, \]  

(12)

\[ \partial_y^2 f_n(y) = -m_n^2 \left( 1 + a_0 \delta_0 + a_\pi \delta_\pi \right) f_n(y), \]  

(13)

respectively. The corresponding spectrum consists on a massless flat zero mode plus a tower of massive modes with wave functions

\[ f_n(y) = A_n \left[ \cos(m_n y) - \frac{a_0 m_n}{2} \sin(m_n |y|) \right], \]  

(14)

where \( A_n \) is a normalization constant and \( m_n \) satisfies the eigenvalue equation

\[ (4 - a_0 a_\pi m_n^2) \tan(\pi m_n R) + 2(a_0 + a_\pi)m_n = 0. \]  

(15)

The main effects of these two brane kinetic terms are to decouple the heavy KK modes from the brane (it becomes less transparent to them) and to make the mass of the first KK mode light,

\[ m_1^2 \sim \frac{2}{\pi R} \frac{(a_0 + a_\pi)}{a_0 a_\pi}, \]  

for \( a_{0,\pi} \gg R. \)  

(16)

This first light KK mode does not completely decouple from the branes in the large \( a_0 \sim a_\pi \) limit but tends to a finite coupling, which seems necessary to correct the high energy behaviour of the zero mode coupling [12].

The phenomenological implications of brane kinetic terms strongly depend on the value of the coefficients. In the particular case that \( a_0 = a_\pi \gg R \) we have a single light KK excitation with coupling to the branes equal to that of the zero mode, the remaining modes being heavy \( (m_n \geq 2 \sim (n - 1)/R) \) and with very suppressed couplings to the branes. In this way we can get the first KK mode fulfilling the sequential \( Z' \) scenario (the case with no mixing corresponds to a Higgs in the bulk). Then all the experimental bounds on such a \( Z' \) apply, \( M_{Z'} > 1.5 \text{ TeV} \) [1]. This limit is saturated for large enough boundary kinetic terms [12].

A similar behaviour can be observed in the case of fermions [16]. Consider as an example the case in which the only relevant brane operator is a kinetic term for the even component (which we take to be the RH one),

\[ \mathcal{L} = \int dy \left\{ [1 + a_0 \delta_0 + a_\pi \delta_\pi] \bar{\Psi}_R \partial \Psi_R + \bar{\Psi}_L \partial \Psi_L + \bar{\Psi}_R \partial_\mu \Psi_L - \bar{\Psi}_L \partial_\mu \Psi_R \right\}. \]  

(17)
The orthonormality and differential equations, which determine the KK expansion, are in this case

\[ \int dy \left[ 1 + a_0 \delta_0 + a_\pi \delta_\pi \right] \frac{f_R^R f_R^n}{2\pi R} = \delta_{nm}, \]  

(18)

\[ \int dy \frac{f_L^m f_L^n}{2\pi R} = \delta_{nm}, \]  

(19)

and

\[ \partial_y f_R^n = m_n f_L^n, \]  

(20)

\[ \partial_y f_L^n = -m_n [1 + a_0 \delta_0 + a_\pi \delta_\pi] f_R^n, \]  

(21)

respectively. The two differential equations can then be iterated to get a second order differential equation for the even component, obtaining the same equation as the one for a gauge boson. In this case one also obtains a light KK fermion for large brane terms, and the allowed region in Fig. 3, in particular the cross-point with \( m_Q = 478 \) GeV and \( W_{tb}^L = 0.96 \), can be recovered for appropriate (large) values of \( a_{0,\pi} \).

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