High temperature limit of the Standard Model due to gauge groups contraction

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Abstract. The high temperature (high energy) limit of the Standard Model is developed with the help of contractions its gauge groups. The elementary particles evolution in the early Universe from Plank time up to several milliseconds is deduced from this limit theory. Particle properties at the infinite temperature look very unusual: all particles are massless, only neutral Z-bosons, u-quarks, neutrinos and photons are survived in this limit. The weak interactions become long-range and are mediated by neutral currents, quarks have only one color degree of freedom.

1. Introduction
The modern theory of elementary particles known as Standard Model consist of Electroweak Model, which unified electromagnetic and weak interactions, as well as Quantum Chromodynamics (QCD), describing their strong interactions. The Standard Model gives a good description of the experimental data and has been recently confirmed by the discover of Higgs boson at LHC. No new physics was observed in the experiments up to now. So today the Standard Model is the only theory of elementary particles and if we are interested in particle properties in the early Universe we need use its high temperature (high-energy) limit.

Standard Model is a gauge theory with the direct product of a simple groups $SU(3) \times SU(2) \times U(1)$ as a gauge group. The well known in physics operation of group contraction (or transition) [1] transforms initial group into some more simple limit group. For a symmetric physical system the contraction of its symmetry group means a transition to some limit state. We use this method in order to re-establish the evolution of elementary particles and their interactions in the early Universe. We are based on the modern knowledge of the particle world which is concentrated in Standard Model. For this we investigate the high-temperature (high-energy) limit of Standard Model generated by appropriate contractions of the gauge groups $SU(2)$ and $SU(3)$ [2–4].

2. High-temperature lagrangian of Electroweak Model
The contracted group $SU(2; \epsilon)$ is obtained [3] by the consistent rescaling of the fundamental representation of $SU(2)$ and the space $\mathbb{C}^2$ as follows

$$z'(\epsilon) = \left( \begin{array}{c} z'_1 \\ \epsilon z'_2 \end{array} \right) = \left( \begin{array}{cc} \alpha & \epsilon \beta \\ -\epsilon \beta & \bar{\alpha} \end{array} \right) \left( \begin{array}{c} z_1 \\ \epsilon z_2 \end{array} \right) = u(\epsilon)z(\epsilon),$$
the theory between the quarks. The QCD Lagrangian is taken in the form

\[ L = \sum_{ij} \delta^i_j \left( \partial_{\mu} A_{\mu}^i - i g_s \left( \frac{\lambda^a}{2} \right) A_{\mu}^i q^j \right), \] (6)

where \( D_{\mu} q \) are covariant derivatives of quark fields

\[ D_{\mu} q = \left( \partial_{\mu} - i g_s \left( \frac{\lambda^a}{2} \right) A_{\mu}^a \right) q, \] (7)

g_s is the strong coupling constant, \( t^a = \lambda^a / 2 \) are generators of \( SU(3) \), \( \lambda^a \) are Gell-Mann matrices. Gluon stress tensor is introduced as usual

\[ F_{\mu\nu}^a = \partial_{\mu} A_{\nu}^a - \partial_{\nu} A_{\mu}^a + g_s \delta^{\alpha\beta\gamma} A_{\mu}^\alpha A_{\nu}^\beta A_{\rho}^\gamma, \] (8)

where \( [t^a, t^b] = i f^{a\beta\gamma} t^\gamma, \) \( \alpha, \beta, \gamma = 1, \ldots, 8 \). Mass terms are not included as far as they are present in the electroweak Lagrangian.

3. Quantum Chromodynamics at high temperature

Like the Electroweak Model QCD is a gauge theory based on the local color degrees of freedom [6]. The QCD gauge group is \( SU(3) \), acting in three dimensional complex space of color quark states \( q^l = (q_1, q_2, q_3)^l \). The \( SU(3) \) gauge bosons (gluons) are the force carrier of the theory between the quarks. The QCD Lagrangian is taken in the form

The explicit form of intermediate parts of Lagrangian \( L_k \) was given in [3]. In particular the infinite temperature limit \( (\epsilon = 0) \) Lagrangian is equal to

\[ L_\infty = -\frac{1}{4} Z_{\mu\nu}^2 - \frac{1}{4} F_{\mu\nu}^2 + \nu_0^1 \partial_\mu \partial_\nu + u_0^1 \partial_\mu \partial_\nu + + \epsilon^4 \partial_\mu \partial_\nu + d^4 \partial_\mu \partial_\nu + u_0^1 \partial_\mu \partial_\nu + L_{\infty}(A_\mu, Z_\mu). \] (5)

The contraction parameter is monotonous function \( \epsilon(T) \) of the temperature of a system under consideration with the property \( \epsilon(T) \to 0 \) for \( T \to \infty \). Very high temperatures can exist in the early Universe just after its creation on the first stages of the Hot Big Bang [5] in pre-electroweak epoch.
The contracted special unitary group $SU(3; \kappa)$ is defined by the action

$$q'(\kappa) = \begin{pmatrix} q_1' \\ \kappa_1 q_2' \\ \kappa_1 \kappa_2 q_3' \end{pmatrix} = \begin{pmatrix} u_{11} & \kappa_1 u_{12} & \kappa_1 \kappa_2 u_{13} \\ \kappa_1 u_{21} & u_{22} & \kappa_2 u_{23} \\ \kappa_1 \kappa_2 u_{31} & \kappa_2 u_{32} & u_{33} \end{pmatrix} \begin{pmatrix} q_1 \\ \kappa_1 q_2 \\ \kappa_1 \kappa_2 q_3 \end{pmatrix} = U(\kappa)q(\kappa),$$

$$\det U(\kappa) = 1, \quad U(\kappa)U^\dagger(\kappa) = 1$$

on the complex space $\mathbb{C}_3(\kappa)$ in such a way that the hermitian form

$$q^\dagger(\kappa)q(\kappa) = |q_1|^2 + \kappa_1^2 (|q_2|^2 + \kappa_2^2 |q_3|^2)$$

remains invariant, when the contraction parameters tend to zero: $\kappa_1, \kappa_2 \to 0$. We take these parameters identical to the contraction parameter $\epsilon$ of the Electroweak Model: $\kappa_1 = \kappa_2 = \kappa = \epsilon$, so the limit $\kappa = \epsilon \to 0$ corresponds to the infinite temperature limit.

Transition from the classical group $SU(3)$ and space $\mathbb{C}_3$ to the group $SU(3; \epsilon)$ and space $\mathbb{C}_3(\epsilon)$ is given by the substitution

$$q_1 \to q_1, \quad q_2 \to \epsilon q_2, \quad q_3 \to \epsilon^2 q_3,$$

$$A^\mu_{GR} \to \epsilon A^\mu_{GR}, \quad A^\mu_{BG} \to \epsilon A^\mu_{BG}, \quad A^\mu_{BR} \to \epsilon^2 A^\mu_{BR},$$

and diagonal gauge fields $A_{\mu}^{RR}, A_{\mu}^{GG}, A_{\mu}^{BB}$ remain unchanged. Then from (6) we obtain the quark part of Lagrangian

$$\mathcal{L}_q(\epsilon) = \sum_q \left\{ i \bar{q}_q \gamma^\mu \partial_\mu q_1 + \frac{g_s}{2} |q_1|^2 \gamma^\mu A^\mu_{RR} + e^2 \left[ i \bar{q}_2 \gamma^\mu \partial_\mu q_2 + \frac{g_s}{2} \left( |q_2|^2 \gamma^\mu A^\mu_{GG} + q_1 \bar{q}_2 \gamma^\mu A^\mu_{GR} + \bar{q}_1 q_2 \gamma^\mu \bar{A}_{GR} \right) \right] + e^4 \left[ i \bar{q}_3 \gamma^\mu \partial_\mu q_3 + \frac{g_s}{2} \left( |q_3|^2 \gamma^\mu A^\mu_{BB} + q_1 \bar{q}_3 \gamma^\mu A^\mu_{BR} + \bar{q}_1 q_3 \gamma^\mu \bar{A}_{BR} \right) + g_2 q_2 \gamma^\mu A^\mu_{BG} + \bar{q}_2 q_3 \gamma^\mu \bar{A}_{BG} \right] \right\} = L_q^\infty + e^2 L_q^{(2)} + e^4 L_q^{(4)}.$$  

The gluon part of (6) is cumbersome, but can be represented as a sum of terms [4]

$$\mathcal{L}_{gl}(\epsilon) = L_{gl}^\infty + e^2 L_{gl}^{(2)} + e^4 L_{gl}^{(4)} + e^6 L_{gl}^{(6)} + e^8 L_{gl}^{(8)}.$$  

In the infinite temperature limit the most parts of gluon tensor components are equal to zero, so we can write out the QCD Lagrangian $\mathcal{L}_\infty$ explicitly

$$\mathcal{L}_\infty = L_q^\infty + L_{gl}^\infty = \sum_q \left\{ i \bar{q}_R \gamma^\mu \partial_\mu q_R + \frac{g_s}{2} |q_R|^2 \gamma^\mu A^\mu_{RR} \right\} - \frac{1}{4} (F_{\mu\nu}^{RR})^2 - \frac{1}{4} (F_{\mu\nu}^{GG})^2 - \frac{1}{4} F_{\mu\nu}^{RR} F_{\mu\nu}^{GG},$$  

$$L_{gl}^{(6)} = \frac{g_s^2}{16} \left( \left[ [4, 7] - [5, 6] \right]^2 \left( [4, 6] + [5, 7] \right)^2 + \right.$$
\[ +\left(\left[2,4\right] - \left[1,5\right]\right)^2 + \left(\left[1,4\right] + \left[2,5\right]\right)^2 + 4\left(\left[1,2\right] + \left[6,7\right]\right)\left[4,5\right]\] \\

\]

\[ L^{(8)}_{gl} = -\frac{g^2}{4}\left[4,5\right]^2. \tag{15} \]

Let us introduce the notations

\[ \partial A^k = \partial_{\mu} A^k_{\mu} - \partial_{\nu} A^k_{\nu}, \quad [k,m] = A^k_{\mu} A^m_{\nu} - A^m_{\mu} A^k_{\nu}, \tag{16} \]

The Standard Model passes in this limit through several stages, which are distinguished by the powers of contraction parameter, i.e. by powers of the temperature. Starting with \( L_q(\epsilon) \) (4), \( L_q(\epsilon) \) (12) and \( L_{gl}(\epsilon) \) (13) one can construct a number of intermediate models, namely, one model with explicit Lagrangian for each stage [4]. These models describe the restoration of particles properties in the Universe evolution.

4. Conclusion

The contraction of gauge group of the Standard Model gives an opportunity to order in time different stages of its development, but does not make it possible to bear their absolute dates.

**Figure 1.** History of the Universe

These dates can be estimated with the help of additional assumptions. We use the fact that the electroweak epoch starts at the temperature \( T_4 = 100 \) GeV (1 GeV=10\(^{13}\) K) whereas the QCD epoch begins at \( T_8 = 0.2 \) GeV (Figure 1), suppose that the contraction parameter inversely depends on temperature \( \epsilon(T) = AT^{-1} \) and use cutoff level \( \Delta \) for \( \epsilon^k L_k \), i.e. for \( \epsilon^k < \Delta \) all the terms \( L_k \) are negligible quantities in Lagrangian. Simple calculations [4] give the following
boundary values of the temperature in the early Universe (GeV): $T_1 = 10^{18}$, $T_2 = 10^7$, $T_3 = 10^3$, $T_4 = 10^2$, $T_6 = 1$, $T_8 = 2 \cdot 10^{-1}$. The obtained estimation for “infinity” temperature $T_1 \approx 10^{18}$ GeV is comparable with Planck energy $\approx 10^{19}$ GeV, where the gravitation effects are important. So the developed evolution of the elementary particles does not exceed the range of the problems described by electroweak and strong interactions.

At the infinite temperature limit ($T > 10^{18}$ GeV) all particles including vector bosons lose their masses and electroweak interactions become long-range. Monochromatic massless quarks exchange by only one sort of $R$-gluons. It follows from the explicit form of Lagrangians $L^\text{int}_\infty(A_\mu, Z_\mu)$ (5) and $L_\infty$ (14) that only the particles of the same sort interact with each other. Particles of different sorts do not interact. It looks like some stratification of leptons and quark-gluon plasma with only one sort of particles in each stratum.

At the level of classical gauge fields it is already possible to give some conclusions on the stages of elementary particles mass appearance in the Universe evolution. In particular we can conclude that half of quarks ($\approx \epsilon$, $10^{18}$ GeV $> T > 10^7$ GeV) first restore they mass. Then $Z$-bosons, electrons and other quarks become massive ($\approx \epsilon^2$, $10^7$ GeV $> T > 10^3$ GeV). Finally Higgs boson $\chi$ and charged $W^\pm$-bosons restore their masses ($\approx \epsilon^4$, $T \leq 10^2$ GeV).

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