A new model of open end yarn twist using torus coordinate based on dynamical mechanics

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Abstract. Yarn structure and its relationship with the properties have attracted many researchers and scientist in textile science. The importance of this subject has increased due to the need of yarn with best possible quality. In this study, we have established a new equation to relate open end (OE) yarn count number in metric count ($N_m$) toward yarn twist on OE spinning. Based on this research, twist has been determined and formulated by analyzing the movement of fibre inside yarn using tensor calculus on torus coordinate based on dynamical mechanics.

Keywords: Yarn Twist, Torus Coordinate, Fibre Movement

1. Introduction

Yarn structure and its relationship with the properties have attracted many researchers and scientist in textile science. The importance of this subject has increased due to the need of yarn with best possible quality. Fibre migration or fibre movement inside of OE yarn is the change in the range of a fibre (along it’s length) from the axis of a yarn, which happens during the movement of fibre inside OE yarn. The research of yarn and fibre equation as well as its influence of characteristics of yarn have been studied by some scientists both experimentally as well as theoretically [1-16]. According to Putra, et al [1-3] the fibre movement of yarn can be analyzed by classical mechanics in a certain coordinate. As stated by Lawrence [7], the characteristic of spun yarn can be explained by the fibre movement inside yarn in certain coordinate and also yarn structure as shown in Figure 1 below.

Figure 1. OE Yarn Properties [7]

As reported by Backer, et al [5], the tenacity or strength of amount of fibre is determined by the rate of twist and the relation can be exposed as the lower of twist, the higher is the tenacity or strength of yarn per tex and vice versa. Some researchers, such as Rohlena [11] reported that breakage rate of yarn is affected by the yarn twist. The lower the twist, the lower is the breakage rate. According to Musa and Ayse [8], Prendzova [10] and Penava & Oreskovic [9], the yarn diameter influenced the yarn strength and yarn count number in indirect system (Nm). Musa and Ayse [8] reported that the wider yarn radius, the stronger is the tenacity or strength of yarn. Putra, et al.[1] composed the relationship between twist...
and yarn count number in metric count as Eq.(1) which is related by twist multiplier \((a_m)\). Rohlena [11] and Backer, et al [5] formulated yarn twist as a function of yarn count number as Equation (2) and Equation (3), whereas Lawrence [7] and Putra et al. [3] composed the relationship between twist and yarn delivery speed, \(v_d\), with rotor diameter, \(d_{rotor}\), as well as rotor speed, \(n_{rotor}\), as Equation (4) and Equation (5).

\[
T_{solenoid} = \frac{\sqrt{F_0}}{n_r \pi^2 d_{yarn} d} \sqrt{N_m}, \quad (1)
\]
\[
T = \frac{\tan \theta}{2 \pi r_{yarn}} \quad (2)
\]
\[
T = \alpha_m \sqrt{N_m} \quad (3)
\]
\[
T = \frac{n_{rotor}}{v_d} \quad (4)
\]
\[
T = \frac{n_{rotor}}{v_d} + \frac{1}{\pi d_{rotor}} \quad (5)
\]

In this study, the relationship of yarn twist to yarn count number would be derived by looking the movement of fibre inside yarn using torus coordinate. Tensor calculus has a main role to solve the movement of fibre inside yarn and to get the relationship of it. According to many researchers [15-19] mathematical models have their basis in applied physics and can be used to determine the motion of object. Since the subject of mechanics deals with the motion of objects in space, fibre equation inside of yarn and its characteristic of analytical mechanics can be formulated better with a certain coordinate, such as torus coordinate. An n-dimensional Euclidean space is defined to consist of vectors \(S\) whose components are \(r, u, v\), now assumed to be real. In this study, we used geometric mechanics to measure and to derive the equation of fibre in curve linear coordinate on torus coordinate.

2. Research Method

Development of Fibre Equation on Torus

Fibre movement on torus coordinate (caused by the external force \((F_o)\) and the internal force) can be obtained using analytical mechanics model. By knowing the certain coordinate system, the fibre movement inside a certain yarn can be formulated. In particular, a fibre inside a yarn with certain length (\(dS\)) travels in a certain coordinate, whose length is \(du\) during a time (\(dt\)) and radius of yarn cross section (\(dv\)). A yarn can be pretended to be formed in torus coordinate, whose radius of yarn \((r)\) and the length of gap \((b)\), are shown in Figure 2.

![Figure 2. Model of OE yarn on torus coordinate](image)

A coordinate system, such as a torus, is kind of the curvilinear type. Consider the three quantities of \(V_m\) (\(m = 1,2,3\)) which relate to the rectangular coordinates and the three transformed quantities (torus coordinate) of \(\bar{V}_\mu\) (\(\mu = 1,2,3\)) related to \(V_m\), as:

\[
S = (x, y, z) = (r, u, v) \quad (6)
\]
\begin{equation}
S = (\rho + r\cos \nu)\cos u, (\rho + r\cos \nu)\sin u, rsin \nu.
\end{equation}

Equation (7) can be used to get the fibre movement on torus coordinate, the square of line element \((dS^2)\) can be given below

\begin{equation}
dS^2 = (b + r\cos \nu)^2 du^2 + r^2 dv^2 + dr^2,
\end{equation}

The metric element \((g_{mn})\) can be demonstrated as shown below:

\begin{equation}
g_{mn} = \begin{pmatrix}
(b + r\cos \nu) & 0 & 0 \\
0 & r^2 & 0 \\
0 & 0 & 1
\end{pmatrix}.
\end{equation}

The components of a unit vector can be written in matrix format, as illustrated below

\begin{pmatrix}
\hat{u} \\
\hat{v} \\
\hat{r}
\end{pmatrix}
= \begin{pmatrix}
-sin \ u & \cos \ u & 0 \\
-sin \ \nu \cos \ u & -sin \ \nu \sin \ u & \cos \ \nu \\
\cos \ \nu \cos \ u & \cos \ \nu \sin \ u & \sin \ \nu
\end{pmatrix}
\begin{pmatrix}
i \\
j \\
k
\end{pmatrix}.

The acceleration of fibre on each axis \((a^u, a^v, a^r)\) on torus coordinate can be written as (after some calculations):

\begin{align}
\frac{d^2x^1}{dt^2} + \Gamma_{a^u}^1 \frac{dx^a}{dt} \frac{dx^b}{dt} &= \frac{d^2u}{dt^2} - \frac{2rsin \ \nu}{b+rcos \ \nu} \frac{du}{dt} \frac{dv}{dt} + \frac{2cos \ \nu}{b+rcos \ \nu} \frac{du}{dt} \frac{dr}{dt} = a^u, \\
\frac{d^2x^2}{dt^2} + \Gamma_{a^v}^2 \frac{dx^a}{dt} \frac{dx^b}{dt} &= \frac{d^2v}{dt^2} + \frac{1}{r} \sin \nu (b + r\cos \nu) \frac{du}{dt} \frac{dv}{dt} + \frac{2}{r} \frac{dv}{dt} \frac{dr}{dt} = a^v, \\
\frac{d^2x^3}{dt^2} + \Gamma_{a^r}^3 \frac{dx^a}{dt} \frac{dx^b}{dt} &= \frac{d^2r}{dt^2} - \cos \nu (b + r\cos \nu) \frac{du}{dt} \frac{dr}{dt} = a^r.
\end{align}

The general acceleration \((\mathbf{a}^i)\) for a certain mass \((M)\), including the stress tensor \((\sigma^{ij})\) and the external force \((F^i)\), can be written as shown below:

\begin{equation}
\sigma^{ij} = \frac{\partial F^i}{\partial x^j} = Ma^i
\end{equation}

to find the general equation of fibre movement influenced by the stress tensor as traction tensor \((T^r, T^u, T^v)\) caused by the external force, hence we can substitute Equation (11), Equation (12) and Equation (13) to Equation (14) and after some calculations, it can be shown as below:

\begin{equation}
\frac{\partial T^r}{\partial r} + \frac{1}{b+rcos \ \nu} \frac{\partial T^u}{\partial u} + \frac{1}{r} \frac{\partial T^v}{\partial v} + \frac{cos \nu T^r - sin \nu T^v}{b+rcos \ \nu} + T^r \frac{dr}{r} + \ddot{F} = M \ddot{a}.
\end{equation}
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Equation (15) can be derived as

\[
\frac{\partial \sigma^{rr}}{\partial r} \hat{p} + \frac{\partial \sigma^{rr}}{\partial r} \hat{u} = \frac{\partial \sigma^{rr}}{\partial v} \hat{u} + \frac{1}{b + r \cos \alpha} \frac{\partial \sigma^{rr}}{\partial u} \hat{u} + \frac{1}{r} \frac{\partial \sigma^{rr}}{\partial v} \hat{v} + \frac{1}{b + r \cos \alpha} \frac{\partial \sigma^{rr}}{\partial u} \hat{v}
\]

\[
\frac{\partial \alpha^{uv}}{\partial u} \hat{p} + \frac{\partial \alpha^{uv}}{\partial v} \hat{u} = \frac{\partial \alpha^{uv}}{\partial v} \hat{v} + \frac{1}{b + r \cos \alpha} \frac{\partial \alpha^{uv}}{\partial u} \hat{v} + \frac{1}{r} \frac{\partial \alpha^{uv}}{\partial v} \hat{v} + \frac{1}{b + r \cos \alpha} \frac{\partial \alpha^{uv}}{\partial u} \hat{v}
\]

\[
\cos(\sigma^{rr} \hat{p} + \sigma^{uu} \hat{u} + \sigma^{vv} \hat{v} - \sin(\sigma^{rr} \hat{p} + \sigma^{uu} \hat{u} + \sigma^{vv} \hat{v})) + \frac{(\sigma^{rr} \hat{p} + \sigma^{uu} \hat{u} + \sigma^{vv} \hat{v})}{r}
\]  

(16)

If we take \( \hat{r} \) axis and analyze the fibre movement inside yarn which is influenced by the external force from machine \( \mathbf{F}_o = \mathbf{F}_{\text{take-off}} \) as well as internal force (caused by deformation force), also substitute Equation (13) to Equation(16), therefore we get Equation (17)

\[
\frac{\partial \sigma^{rr}}{\partial r} + \frac{1}{b + r \cos \alpha} \frac{\partial \sigma^{rr}}{\partial u} + \frac{1}{r} \frac{\partial \sigma^{rr}}{\partial v} + \frac{\sigma^{rr} \cos \alpha - \sigma^{rr} \sin \alpha}{b + r \cos \alpha} + \frac{\sigma^{rr}}{r} = \frac{\sigma^{rr}}{r} - F_o = m \left( \frac{d^2 r}{dt^2} - \cos \nu (b + r \cos \nu) u^2 - r \dot{v}^2 \right).
\]  

(17)

If \( \mathbf{v} \) is kept constant and the speed of \( \mathbf{u} \approx \dot{v} \) and there’s no fibre migration on \( \hat{r} \) axis, \( \frac{d^2 r}{dt^2} = 0 \), hence it gives

\[
\frac{\partial \sigma^{rr}}{\partial r} + \frac{1}{b + r \cos \alpha} \frac{\partial \sigma^{rr}}{\partial u} + \frac{1}{r} \frac{\partial \sigma^{rr}}{\partial v} + \frac{\sigma^{rr} \cos \alpha - \sigma^{rr} \sin \alpha}{b + r \cos \alpha} + \frac{\sigma^{rr}}{r} = \frac{\sigma^{rr}}{r} - F_o = - M \cos \nu (b + r \cos \nu) u^2 - M r \dot{v}^2 \approx - M u^2 (\cos \nu (b + r \cos \nu) + r),
\]  

(18)

In case \( r \ll b \) and \( \dot{u} \approx \dot{v} \) or \( \mathbf{n}_x \approx \mathbf{n}_d \), hence

\[
\frac{\partial \sigma^{rr}}{\partial r} + \frac{1}{b + r \cos \alpha} \frac{\partial \sigma^{rr}}{\partial u} + \frac{1}{r} \frac{\partial \sigma^{rr}}{\partial v} + \frac{\sigma^{rr} \cos \alpha - \sigma^{rr} \sin \alpha}{b + r \cos \alpha} + \frac{\sigma^{rr}}{r} = \frac{\sigma^{rr}}{r} - F_o = - M \cos \nu (b + r \cos \nu) u^2,
\]  

(19)

For a case, if twist angle, \( \theta \), is kept constant, \( \mathbf{\alpha}^{\mu} \text{ is constant} \) value and \( r \ll b, \mathbf{\sigma}^{\mu} = 0 \), we can formulate

\[
F_o = \sigma^{rr} \left( \frac{\cos \alpha}{b + r \cos \alpha} + \frac{1}{r} \right) = M \sin \theta (b + r \sin \theta) u^2,
\]  

(20)

Now by Equation (20) , we have

\[
F_o = \sigma^{rr} \left( \frac{2 r \cos \nu + b}{b + r \cos \alpha} \right) = F_o - \frac{\sigma^{rr}}{r} \approx M \sin \theta b \dot{u}^2.
\]  

(21)

\[
M (R_t \sin^2 \theta) \dot{u}^2 = F_o - \frac{\sigma^{rr}}{r},
\]  

(22)

After some calculations, we have

\[
\tan \theta = 0.7 \frac{F_{\text{take-off}}}{n_t n_d \sqrt{N_m}}.
\]  

(23)
Substitute Equation (23) to Equation (2), hence

\[ T = \frac{\tan \theta}{2\pi r_{yarn}} = \frac{0.7}{2\pi r_{yarn}} \frac{\sqrt{F_{\text{take-off}}}}{n_r \pi d} \sqrt{N_m}, \]

(24)

According to Backer et al. [5] and Putra et al. [1-2], the relation among yarn angle, twist and yarn count number can be related by Equation (24) as shown below:

\[ T = \frac{0.7}{2\pi r_{yarn}} \frac{\sqrt{F_{\text{take-off}}}}{n_r \pi d} \sqrt{N_m} = \alpha_m \sqrt{N_m} \]

(25)

Equation (25) can be drawn the relationship of twist and yarn count in metric as Figure 3 below (for a special case that the yarn constant, \( \alpha_m = 136 \))

Figure 3. Relationship of Twist and Yarn Count in Metric

According to the experimental result in Industry by following data below (Table 1)

| rotor speed (rpm) | tenacity of yarn (cN(tex)) | \( a_m \) | \( N_m \) yarn count | \( Ti \) (tex) | \( v_d \) (m/min) | \( T_{\text{machine}} \) (tpm) |
|-------------------|-----------------------------|----------|----------------------|----------------|----------------|-------------------|
| 72000             | 15.51                       | 136      | 67                   | 14.79          | 77.7           | 1118.18           |
| 72000             | 15.13                       | 136      | 83                   | 11.83          | 69.2           | 1250              |
| 72000             | 14.56                       | 136      | 100                  | 9.86           | 63.1           | 1369.31           |

By using Table 1, the relationship of twist and yarn count in metric \( N_m \) as shown in Figure 4 below can be created
3. Result and Discussion

The equation of fibre movement traveling inside of yarn has been established and derived using torus coordinate occurred on spinning machine. According to this theory, the relationship of yarn count number correlated by twist of yarn has been found.

In this research, the connection between twist and yarn count number has been established. According to Rohlena [11], the relationship between yarn count number and yarn twist is correspondence with the movement of fibre inside the yarn, furthermore it can be formulated on torus coordinate as we have established in this research. Based on this study, The higher of yarn count number, the higher is the twist of yarn. Besides yarn fineness or yarn count number in indirect system, twist also influences the strength of yarn. Lawrence.[7] proposed that the tenacity of rotor open end spinning yarn is influenced by the rate of twist and the relation can be shown as: the higher the strength of yarn per tex and vice versa the lower is the yarn twist. As stated by Some researchers, [1-13] yarn strength decreases when yarn twist and yarn count number increase.

the strength of yarn per tex is affected by the rate of yarn twist has been reported by Becker, et al [5]. Based on this study, the relation of yarn count number to twist has been predicted as Equation (25).

Based on the Equation (25) as well as the data from experimental result, it can be established that the correlation of yarn twist to yarn number (i.e. yarn count number) can be formulated by using following equation:

\[ T = \frac{0.7}{2\pi r_{\text{yarn}}} \sqrt{\frac{F_{\text{take-off}}}{n_{rtd}}} \sqrt{N_{m}}. \]  

(26)

4. Conclusion

Fiber movement on yarn has been analyzed to present and to predict the relationship of yarn count in metric to yarn twist. In this study, it was found that yarn twist is affected by yarn count number in metric on torus coordinate. It has been established that the formula to relate the relationship is

\[ T = \frac{0.7}{2\pi r_{\text{yarn}}} \sqrt{\frac{F_{\text{take-off}}}{n_{rtd}}} \sqrt{N_{m}}. \]

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