Efficiency of Covid-19 Containment by Measuring Time Dependent Doubling Time

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Abstract

While containment policies of the Covid-19 epidemic have been applied in different countries, a comparative quantitative measure of the efficiency of these policies is not available. We show here that the time evolution of the pandemic doubling time $T_d(t)$ is a reliable measure of the efficiency of the "Lockdown, case Finding, mobile Tracing" (LFT) policy using modern technologies versus the traditional “Lockdown Stop and Go” (LSG) policy. This goal has been achieved to verify by extracting the time evolution of the doubling time $T_d(t)$. The results provide a quantitative measure and make possible predictions on the time evolution of epidemic for different containment policy. Where the lowest $s$ factor, the characteristic time of the exponential growing $T_d(t)$ in the arrested regime, was reached by LFT policy also the lowest number of deaths and the shorter time width of the epidemic dome was reached. The record for the largest number of deaths is correlated with the largest $s$ factor in the near threshold regime where no containment measures were taken in time. LFT policy has been able to reduce both the intensity and time width of the Covid-19 pandemic dome achieving both a low number of deaths and low economic losses.

The diffusion of Covid-19 is a transnational phenomenon that involves all continents reaching a million of positive cases and thousands of deaths at early April, 2020. Following the threshold of the very fast Covid-19 epidemic in January 2020 in Wuhan [1-2] it was found that a very small characteristic time (about 2 days) of the exponential growth and the characteristic number $R_0$ of infected humans by one infected case was about $R_0=2.3$, well larger than the critical value of 1, pointing to an explosion of the pandemic with possible millions of infected humans in few weeks [3].

In the absence of a Covid-19 vaccine, scientists informed the policy makers for the urgent need of actions for controlling the pandemic [4] to reduce the expected number of hundreds of thousands of deaths in the short time of pandemic peak. The textbook epidemic control measures are addressed to reduce the number of daily new cases $N(t)$ to avoid an
unacceptable load to the health care systems. The traditional method to reduce the number of deaths was the lockdown. It expands the time lapse of the virus diffusion by stretching the exponent of its exponential growth. Therefore while the number of cases will decrease at the top, the time extent of the epidemic will become longer with an increased negative impact to economy.

China first focused on an unconventional Covid-19 policy to reduce both the number of deaths and the time extent of the epidemic [1]. This unconventional policy called "Lockdown, Case Finding, Mobile Tracing" (LFT) was based on the combination of measures. This policy looked to reduce both the number of new daily cases at epidemic peak and to reduce the time width of the pandemic peak. Achieving the reduction of both maximum and width of the diffusion peak this policy obtained the drastic reduction of the total number of deaths. Today after about 80 days from the epidemic threshold, on April 7th 2020 the long-term lockdown was “stopped” in China with less the 3400 fatality. It has been an “experiment” to test the efficiency of the new LFT policy in the history of the epidemiology, which takes advantage of both mass search of positive cases and the tracing of infected cases by a mobile phone application. A similar approach has been considered by other countries, all taking advantage of advanced methods of new mobile phone technologies and the treatments of Big Data developed in the last years. The success of this Covid-19 policy was followed by Israel, South Korea, Singapore and Taiwan. [5]

An alternative policy has been considered by other countries such as USA and UK and was called “Lockdown stop and go” (LSC), a type of “herd immunity”, after criticisms to the contact tracing approach chosen by the P.R. of China [6]. The LCS approach is made of combination of advices for the population to keep “physical distancing” to protect others, to stay at home only for positive cases, household quarantine of their family members and to reduce traveling. Intermittent physical distancing measures are planned to be temporarily relaxed in short time windows, and reintroduced when case numbers rebound. [7] The media informed the population about the actual numbers of epidemic cases and the advices were diffused only weeks after the Covid-19 pandemic onset.

Other countries, e.g., Italy, France and Spain, have followed in March 2020 the “Mandatory Full Lockdown” (MFL) policy with few days delay. This traditional epidemiologic confinement approach was strongly enforced. These unprecedented measures of imposition by law all over the country consist of physical distancing, ordering enforced closure of schools, universities, all national manufactures, ban of mass gatherings and public events, and confinement at home of the entire population. This approach had the key target of reducing the number of infected cases day per day paying attention not to overcome the maximum number of person requiring critical care (invasive mechanical ventilation or ECMO) that can be hospitalized in any region/country. In fact, the MFL policy gives priority to the health care system respect to the economic demands.

Academic epidemiology analysis is usually made at the conclusion of the epidemic event. On the contrary, in the case of the Covid-19 epidemic some countries immediately shared verified data and made them available in public repositories. This opportunity gives scientists the possibility to shed light on the new physics of epidemics with containment measures [8-18] a growing field of high interest for population, health systems, and economics [19] where there is need to develop interdisciplinary research activity at the
intersections between biological physics, advanced statistical physics of complex systems, and the most recent big data and fast data analysis methods. The early results have shown that also Covid-19 follows in the early days the fundamental laws of epidemic spreading. However, in spite of the relevance of the question, none was able to measure in a quantitative way the relative efficiency of the different containment measures based on verified Covid-19 data released by official institutions and health agencies, a key information need to stop the advances of this world-wide epidemic. Only two weeks after the start of the MFL policy, [20,21] it was pointed out that the slowing down of the pandemic diffusion was much less effective for the MFL that for the LFT approach. This data analysis approach was focusing on the quantitative determination of the doubling time (T_d) of the Covid-19 pandemic spread calculated averaging day by day data over a 5 days interval. Moreover it was possible to identify in the LFT controlled Covid-19 epidemic growth monitored in China and South Korea two well separated regimes: the first described by a stretching exponential with a slowly increasing stretched characteristic time followed by a second phase, the arrested or frustrated growth process following the Ostwald growth over the course of time, where one phase transforms into another metastable phase, but with a similar free energy [22-24]. This mechanism has been observed in the diffusion of oxygen interstitials diffusion in quantum complex matter [25-27] and in the crystallization of complex molecules [28] and proteins [29].

![Figure 1. Panel a.](image)

**Figure 1. Panel a.** Cumulative number of positive cases N_c(t) vs. time in South Korea (orange), France (blue), China (light blue), Italy (green), Spain (magenta) and USA (red). The time scale for each curve of each country starts on the threshold day t_0. The curves overlap in the near threshold regime while later separate. The curves of the cumulative number of cases of China and South Korea become flat after about 30 days, when the pandemic is arrested. Panels (b,c) the theoretical predictions for the number of needed beds (open symbols) from [8] for an uncontrolled diffusion (b) and for severe containment measures (c). We show in panel (b) our calculation of the exponential variation of the doubling time T_d(t). The time dependent doubling time curves calculated in this work for uncontrolled (black curve in panel b) and controlled epidemic (blue curve in panel c). Our calculations predict a different exponential increase of the doubling time in the near threshold regime (blue area) and in the arrested regime (yellow area). Moreover we predict the stop of the epidemic where T_d(t) reaches the value of 50 days.

The different efficacy of the containment policies can be recognized in Figure 1a where the cumulative number of cases N_c(t) in different countries is plotted in the time scale with the zero sets at the first day t_0 of the exponential growth. As it can be seen in the panel (a) of Figure 1 the diffusion rate for the different policies in various countries is similar in the near threshold regime. Indeed, the reported curves of the cumulative number of cases N_c(T) overlap in the near threshold regime during the first days while they strongly diverge in the
arrested regime. The epidemic time scale of the epidemic in UK and USA using the standard individual-based simulation model developed to support pandemic influenza planning was reported in [8,9] are plotted in panels (b) and (c). The calculated time dependent number of needed beds for the case of wild uncontrolled epidemic in panel (b) will imply the breakdown of UK and USA health care systems. To avoid this breakdown it was proposed to reduce the peak intensity in panel (b) by expanding the epidemic time scale by a factor 1.5 using non-pharmaceutical interventions (including case isolation, home quarantine and social distancing) shown in panel (c).

In this work in order to measure the time evolution of the epidemic growth, we propose the use as the key physical parameter, the time dependent doubling time \(T_d(t)\)

\[
T_d(t) = 
\frac{\ln(2)}{\frac{d}{dt} \ln[N(t)]}
\]

where \(N(t)\) is the cumulative number of cases and the derivative at each time \(t\) is obtained by fitting the \(N(t)\) curve over a period of five days.

The efficacy of the containment policies is probed by the increase of \(T_d(t)\) from its minimum value \(T_{d0}=2\) days, at the threshold time \(t_0\) to the value of \(T_{d}=50\) days, which is the average time of the lifetime of the infected case, i.e., when the epidemic spread is expected to stop. Therefore we have calculated the curves \(T_d(t)\) in panels (b) and (c) of Figure 1 which verifies the empirical exponential increase of \(T_d(t)\) in the time range of the epidemic dome found in ref. [20,21]. A kink in \(T_d(t)\) separates two different exponential increasing regimes: a the first near threshold regime (shaded blue region) and in the arrested regime (shaded yellow region) separated by the transition regime around the peak of the pandemic curve \(N(t)\) of the number of new daily cases. The theoretical curves of \(T_d(t)\) for the wild uncontrolled (black filled dots in panel b) and for the strong lockdown policy (blue filled dots) show relevant changes on the time evolution behavior, which depends on the containment policy. The doubling time in the near threshold regime follows a first exponential growth (red line in the semi-log scale) with the characteristic time \(s_1\)

\[
T_{d1} = C_1 e^{t/s_1}
\]

and in the arrested regime follows a second exponential growth (blue line in the semi-log scale) with the characteristic time \(s_2\)

\[
T_{d2} = C_2 e^{t/s_2}
\]

where the theory predicts that \(s_2\) is much smaller than the characteristic time \(s_1\). Moreover it is possible to measure the average \(<s>\) factor by fitting the full \(T_d(t)\) curve in the range \(2<T_d<50\) introduced in ref. [20,21] to provide the quantitative measure of the efficiency and the effectiveness in term of time of the containment policy. The extraction of the parameters \(<s>\), \(s_1\), and \(s_2\) factors provides a straightforward quantitative evaluation and, in addition, a quantitative comparison of the different containment policies adopted to control the epidemic. The calculations show that the factor \(s_1\) decreases in the near threshold regime from the wild regime value \(s_1=140\) days in panel (b) to the lockdown regime \(s_1=30\) days in panel (c). The factor \(s_2\) in the arrested regime increases from the wild regime value \(s_2=8.2\) days in panel (b) to the lockdown regime \(s_2=14.2\) days in panel (c) in the standard influenza control approach. The stop of the wild uncontrolled pandemic (defined as the day where the doubling time \(T_d\) becomes 50) is predicted after 70 days while the stop in the most severe lockdown policy is predicted after 110 days, i.e., the pandemic is predicted to be about 1.6
times longer in the lockdown regime in agreement with the ratio of the half width values at half maximum (HWHM).

The recent fast explosion of the Covid-19 pandemic has seen a fast response of the scientific community proposing models which can be quickly verified or falsified by open experimental data banks which is possible today by fast computer codes for analysis of the Covid-19 data focusing to test the multiple proposed containment measures [10-18].

Here in Figure 2 we report the experimental doubling time $T_d(t)$ as a function of time, extracted by verified data vs. time with the zero sets at the threshold time $t_0$ in several countries where different Covid-19 epidemic policies have been enforced.

The time evolution of the experimental doubling time $T_d(t)$ for China and South Korea where the "Lockdown, Case Finding, Mobile Tracing" LFT policy was applied is plotted in Panel (a). The epidemic dome in both countries is shown by the curve of verified data of the number N(t) of Daily New Cases.

The experimental doubling time $T_d(t)$ where the time unit is one day, is calculated using equation (1), and it does not need any normalization. The full lifetime $A$ of the experimental epidemic dome is directly measured by the day where $T_d$ assumes the value of 50 days, the lifetime of infected cases spreading the virus. We have found $A= 27$ days for China and $A= 24$ days for South Korea. Introducing a normalized scale, i.e., dividing the time in the x axis by $A$, both the normalized domes of N(t) and the experimental $T_d(t)$ curves for China and South Korea fully overlap providing evidence for a characteristic behavior associated with this policy. A key result of the data analysis is that the kink in the $T_d(t)$ curves, which separates the near threshold regime from the arrested regime, occurs at $t/A=0.5$, i.e., in the range of 13-14 days. The quite different value of the $s_f$ factor in the near threshold regime is due to the immediate activation of the LSF policy in South Korea respect to China.

The overlapping epidemic dome in both countries shows also that both the epidemic peak and its full width at half maximum have been both strongly reduced. In table 1 we report all the key parameters obtained by fit of data to unveil the physics of in the time evolution of pandemic in the investigated seven different countries.

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**Figure 2. Panel a)** Evolution of the experimental doubling time $T_d(t)$ in the countries where the LFT policy was enforced: South Korea (filled orange dots) and China (filled blue dots). The open circles are the known curves of the number of daily new cases $N(t)$ which are used here to track the time evolution pandemic domes. **Panel b)** the experimental time dependent doubling time $T_d(t)$ in the countries Spain (filled orange dots) and France (filled blue dots) and Italy, (filled blue dots) where the "Mandatory Full Lockdown" MFL policy was applied. **Panel c)** the evolution of the doubling time in countries USA (filled red dots) and UK (filled blue dots) applying the policy called “Lockdown stop and go” (LSC). The N(t) curves are plotted in semi-log scale in panel b and c. In panel (a) and (b) the numbers in the N(t) curves are normalized to overlap all curves in the near threshold region. In panel (c) the N(t) curves are not normalized. The near threshold time regime in panel (b) and panel (c) identified in the range between the threshold and the kink in the $T_d(t)$ is 27 days in panel (b) and similar in panel (c).
Table 1: Time measured in days, \( t_0 \): day of the year of epidemic threshold, \( T_d(3d)\) :Doubling time on the third day, \( s_1 \): s-factor in the near threshold regime, \( s_2 \): s-factor in the arrested regime, \(<s>\) average s factor in the range \( 2.5<T_d<50 \)

| Country | \( t_0 \) (day of year) | \( T_d(3d) \) (3rd day) | \( s_1 \) (days) | \( s_2 \) (days) | \(<s>\) (days) | Deaths (at 102th DOY) | Deaths (/102-t_0) |
|---------|-----------------|---------------------|-------------|-------------|----------|--------------------|------------------|
| USA     | 65              | 3                   | 388         | 12.7        | 63       | 20,577            | 541.50           |
| Italy   | 52              | 2.5                 | 30          | 11          | 20       | 19,468            | 381.73           |
| Spain   | 56              | 2.7                 | 56.5        | 10          | 30       | 16,606            | 353.32           |
| France  | 55              | 2.6                 | 29.5        | 11.3        | 12       | 13,822            | 288.17           |
| UK      | 65              | 3                   | 84          | 13.8        | 60       | 9,875             | 259.87           |
| China   | 22              | 2                   | 9.4         | 7           | 10       | 3,339             | 41.22            |
| South Korea | 48          | 2.3                 | 43          | 4.8         | 12       | 214               | 3.89             |

In the arrested regime the \( s_2 \) factor for China and South Korea, where LFT policy was applied, are much smaller for other countries using the LSG policy as shown in panel (a) of Fig. 3. We have plotted in panel a) of figure 3 the number of Covid-19 deaths at the 102th day of the year divided by the number of days of the epidemic from its threshold at \( t_0 \). This plot shows that where the \( s_2 \) factor is small i.e., where the doubling time rapidly increase in the arrested regime (a typical behavior of the LTF) the number of deaths is smaller and the time duration of the lockdown is smaller. Therefore it shows that LTF is the most efficient control policy. Panel b of Fig.2 shows that the time dependent doubling time \( T_d(t) \) in Italy, Spain, France (filled blue dots) where “Mandatory Full Lockdown” MFL policy was applied fully overlap. In the near-threshold regime the values of the doubling time are very similar and the three curves show the same kink probing the transition from the near-threshold to the arrested regime at the same time i.e., 27 days, which has been used to normalize the time scale.

Figure 3. The number of deaths per day in each country occurring in the time interval, between 4-10-2020 and the day of the epidemic threshold \( t_0 \), as function of the \( s_2 \) factor.

In the arrested regime which has started only recently in the three countries the s factors are similar. From the extrapolation of the line in the semi-log scale (determined by the exponential curve of \( T_d(t) \) in the arrested regime) plotted in panel (b) of Fig.2, it is possible to predict the time needed to reach the stop of the epidemic, which is predicted to occur at 54 days from the explosion day \( t_0 \) in the three countries where \( T_d=50 \) days. However we cannot predict if at this saturation time the number of daily new cases will go to zero as in China or it will continue to increase with a linear rate, as it occurred in South Korea.

The time dependent doubling time is plotted in panel (c) of Fig.2 for two countries USA and UK following the “Lockdown stop and go” LSG policy. While scientists asked their governments to activate as soon as possible containment measures [5] the USA government
choice was “doing nothing”. This policy has been clearly well realized as indicated by the flat curve in the near-threshold regime which gives a very large s1 factor due to a constant low time dependent doubling time for USA in panel (c).

In panel (b) of Fig.3 we have plotted the normalized number of deaths as a function of the s1 factor measured in the near threshold regime in the five considered countries which enforced the LSG policy. Clearly the record of the number of deaths in USA is associated with the largest value of the s1 factor. The huge number of positive cases and the number of fatalities over the full epidemic dome in USA is due to the lack of containment measures during the short critical time near threshold and corresponds to the very high s1 factor in USA shown in panel b of figure 3. A similar choice seems was made in Lombardy region in Italy in the first two weeks of the epidemic while in other regions the lockdown was applied early after the explosion time with a very small number of deaths.

It is interesting to remark that recently, the doubling time line for USA exhibits a kink at 27 days after the explosion time at t0 we can use the predictions of our study of the evolution of the pandemic to predict the stop time of epidemic in USA which could be of help for policy makers in the next coming days. If these predictions are correct our data analysis shows that the time width of the epidemic dome is about 54 days on all five cases of countries which enforced the LSG policy.

In conclusion we have verified the theoretical predictions of the time evolution of the doubling time Td(t) by data analysis of confirmed data extracted from available data banks. The analysis clearly shows that countries that selected advanced technologies, i.e., the containment policy "Lockdown, Case Finding, Mobile Tracing” (LFT) have been able to reduce both the intensity of peak in the curve of the daily new cases of the Covid-19 pandemic dome and its length compressing the dome only to a short time lapse of 27 days with a minimum number of deaths shown in panel (a) of Fig.3. Therefore they have achieved both a huge reduction of the fatalities in their populations as well as the impact on the economy keeping manufacture lockdown as short as possible which follows the epidemic as in the 1918 influenza [19].

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