INTRODUCTION

During coalbed methane (CBM) exploration and development, the reservoir petrophysical properties, particularly the permeability, are crucial for potential and productivity evaluation of CBM resources. The low-permeability characteristic of coal reservoirs in China requires reservoir stimulation during CBM exploitation. The hydraulic fracturing is currently the most widely used and effective stimulation technique. It can propagate the fractures, improve the permeability of coal reservoir, increase the area of desorption and seepage, and thus improve the productivity of CBM wells. Therefore, the reservoir parameters (especially the effect of fracture propagating and the permeability prediction) are critical for the productivity evaluation of CBM wells.

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increasing) of postfracturing should be paid more attention in the study of productivity forecast for CBM wells.

The morphology of hydraulic fractures is very complex. Fractures are the dominant pathways for fluid flow and mass transport.\textsuperscript{11-13} The conventional fracturing models include two-dimensional model, the pseudo-three-dimensional model and the true three-dimensional model. The two-dimensional model assumes that the hydraulic fracture height is always a constant controlled by the upper and lower layers, characterized by PKN, KGD, and Radial model. PKN model considered the hydraulic fracture width is proportional to the height.\textsuperscript{14,15} Khristianovic and Zheltov proposed the KGD model, and it has been developed and tested by a number of scholars.\textsuperscript{16} It assumes that the hydraulic fracture width is proportional to the length.\textsuperscript{17,18} The Radial model is a horizontal fracture model, and it assumes that the hydraulic fracture is discoid.\textsuperscript{19} Palmer and Luiskutty proposed a relatively complete pseudo-three-dimensional model which considers the difference in the horizontal geostress.\textsuperscript{20} However, the elastic modulus and the toughness of the interbedded rocks are neglected in this model. Bouteca first proposed a fully three-dimensional analytical model of hydraulic fracture propagation which could be used to determine both the least in-situ stress and the fracture height if the pressure was recorded as a function of time.\textsuperscript{21} Various numerical models have their own characteristics, and the two-dimensional models (PKN, KGD models) are the most widely used methods in the simulation of hydraulic fracture propagation. Generally, the PKN model is suitable for the simulation where the fracture length is much more than the height, while the KGD model is more suitable for the condition where the fracture length-height ratio is small.\textsuperscript{22,23}

During the hydraulic fracturing operations, the fracturing curves and pressure decline curves contain a lot of reservoir information. Nolte pioneered an analysis of pressure decline after fracturing to gain information on fluid loss characteristics, fracture dimensions, fluid efficiency, and fracture closure time.\textsuperscript{24} Since then, this method has received ever increasing attention in the petroleum industry. Based on the fracturing pressure decline data, scholars have improved the calculation methods for fracture dimensions, leakoff coefficients, spurt loss, and other fracturing parameters.\textsuperscript{19,25-31} This pressure decline analysis method has gradually become the classical theory for hydraulic fracturing.

The theoretical basis and technique of hydraulic fracturing for CBM wells are similar to those for conventional oil and gas wells. The fracture propagation theories developed from conventional reservoirs are also applicable to the study of coal reservoirs. In this paper, the morphology of hydraulic fractures was identified on the basis of in-situ stress analysis in the Zhengzhuang block of the Qinshui Basin, and then, the corresponding propagation model was selected. A simple method to obtain the fitting pressure was proposed, and the fluid loss coefficient was corrected according to the stress sensitivity of the coal reservoir. Subsequently, a prediction method of postfracturing permeability for the coal reservoir was given on the basis of the injection well testing theory. Based on the data from hydraulic fracturing curves, this paper establishes a methodology to calculate the fitting pressure and permeability after fracturing, which could provide data support for further productivity prediction of CBM wells.

2 | GEOLOGIC SETTING

The Zhengzhuang block is located in the south of the Qinshui Basin (Figure 1), which is the most important development area and first commercial exploitation area of coalbed methane in China at present. The block belongs to the Jincheng administrative district of Shanxi Province; it is on the west side of the Fanzhuang block, bounded by the Sitou fault.\textsuperscript{32} The Zhengzhuang block is a NW trending monocline as a whole. Structures in this area are relatively simple with a few internal secondary folds and small-scale normal faults.\textsuperscript{33} The wide and gentle folds are commonly developed in the research area, mainly extending to NNE and SN.\textsuperscript{34}

The strata in the Zhengzhuang field is composed of the Cambrian (Є) deposits, the Ordovician Fengfeng (O₂f) formation, the Pennsylvanian Benxi (C₂b), and diachronous stratigraphic Taiyuan (C₂t-f-P₁f) formations, the Permian Shanxi (P₁s), Xiashihezi (P₁t), Shangshihezi (P₂s) and Shiqianfeng (P₂sh) formations, and the Quaternary (Q) deposits. The main coal-bearing strata in the research area are the Taiyuan and Shanxi formations. The Shanxi formation primarily includes dark gray mudstone, quartz sandstone, and siltstone.\textsuperscript{35} The main minable No. 3 coal seam of the Lower Permian Shanxi Formation is the main production layer for CBM, and it has a relatively stable structure. The No. 3 coal seam was deposited in swamp environments between distributary channels in a delta plain. The maximum vitrinite reflectance ($R_o$, m) ranges from 3.51% to 3.69%, which belong to anthracites.\textsuperscript{33} The most undeformed, cataclastic coals, and less granulated coals in the research area mainly comprise bright and semi-bright coals. The thickness of the No. 3 coal seam ranges from 2.40 to 7.35 m, with an average of 5.39 m. The burial depth of the No. 3 coal seam of the Shanxi Formation decreases from north to south, ranging from 383.05 to 1336.9 m. The gas content of the No. 3 coal seam in the Zhengzhuang block is relatively high, ranging from 1.49 to 31.44 m$^3$/t, with an average of 19.44 m$^3$/t. Generally, the sealing capability of the No. 3 coal seam is good for a gas reservoir.

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3 | MORPHOLOGY OF HYDRAULIC FRACTURES AND PREDICTION MODEL OF POSTFRACTURING PERMEABILITY

3.1 | Morphology of hydraulic fractures in coal reservoir

In-situ stress has great influence on the original cleats orientation and permeability of coal reservoir, as well as hydraulic fractures of CBM wells. If the coal is considered to be homogeneous, continuous, and isotropic, the hydraulic fractures will first crack in the direction that is perpendicular to the minimum horizontal principal stress. Vertical fractures will be generated when the horizontal stress is at its minimum, and horizontal fractures will be generated when the vertical stress is at its minimum. However, due to the effects of cleats, macro petrographic composition and anisotropy, the mechanical properties of coal along the vertical bedding direction differ greatly from those along the parallel bedding direction. In addition, the elasticity modulus, Poisson's ratio, and compressive strength of the coal along the vertical bedding direction are greater than those along the parallel bedding direction, whereas the opposite is true of the tensile strength of the coal. As analyzed by geological mechanics, hydraulic fractures are generated at the locations with minimum strength and minimum external force resistance. If the tensile strengths along the vertical bedding direction and the parallel bedding direction are expressed by $\sigma_{bt\perp}$ and $\sigma_{bt\parallel}$, respectively, then the conditions for the generation of vertical and horizontal fractures are as follows:

Vertical fractures:

$$\{\sigma_h, \sigma_H\}_{\min} + \sigma_{bt\parallel} < \sigma_v + \sigma_{bt\perp}$$  \hspace{1cm} (1)

Horizontal fractures:

$$\sigma_v + \sigma_{bt\perp} < \{\sigma_h, \sigma_H\}_{\min} + \sigma_{bt\parallel}$$  \hspace{1cm} (2)

In most cases, $\sigma_{bt\perp}$ is smaller than $\sigma_{bt\parallel}$. However, the tensile strengths of coal are relatively low along the two directions and their difference is typically less than 0.5 MPa, and the difference between in-situ stress $\{\sigma_h, \sigma_H\}_{\min}$ and $\sigma_v$ is typically greater than 1 MPa, so the formation of vertical and horizontal fractures is mainly determined by the status of in-situ stress. During hydraulic fracturing, several fractures are initially generated near the wellbore, and these fractures change direction or intertwine with each other at a certain distance from the wellbore. With the propagation of the hydraulic fractures, a main fracture is formed perpendicularly to the direction of the minimum principal stress. Moreover, the greater the horizontal principal stress difference is, the more easily the main straight hydraulic fracture will form. The results of in-situ stress tests of
the No. 3 coal seam in the study area indicate that the change in reservoir pressure with the burial depth is quite linear, and the in-situ stress increases as the burial depth increases. The maximum horizontal principal stress is within the range of 11.01-45.26 MPa, with an average of 25.01 MPa, and the minimum horizontal principal stress ranges from 6.43 to 29.16 MPa, with an average of 16.54 MPa. The horizontal stress is smaller than the vertical stress when the burial depth is shallower than 650 m. As the burial depth increases, the maximum horizontal principal stress becomes greater than the vertical stress, but the minimum horizontal principal stress is smaller than the vertical stress at all times (Figure 2). The $\sigma_H > \sigma_v > \sigma_h$ is the dominant condition and the stress regime is of strike-slip type in the study area. Thus, the hydraulic fractures in the study area are mainly vertical, which is also verified by microseismic monitoring data (Table 1). In addition, the present-day stress field in the study area is distributed along the ENE direction and the hydraulic fractures propagate mainly along the NE-E direction, indicating that the hydraulic fractures mainly propagate along the direction of the maximum principal stress (Figure 3).

### 3.2 | Analysis model of hydraulic fracture in coal reservoir

#### 3.2.1 | Model selection

Due to the vertical fracture generation during the hydraulic fracturing in Zhengzhuang block, a two-dimensional model for vertical fracture calculation was selected to describe their propagation characteristics. It is generally recognized that PKN model is more suitable for hydraulic fracture analysis when the fracture length is much greater than the height. The microseismic monitoring results of the 17 CBM wells show that the fracture lengths are all much greater than fracture heights (Table 1), so the PKN model was selected to study the hydraulic fracture characteristics in the research area (Figure 4). Concentrating on vertical fracture propagation, the PKN model has the following basic assumptions: (a) the fracture height is constant, and the cross-section perpendicular to the propagation direction is elliptical; (b) the fracturing fluid is laminar flow in the direction of fracture propagation; (c) the pressure drop in the $x$ direction is determined by the flow resistance in a narrow elliptically shaped flow channel; (d) the maximum width $w(x, t)$ is proportional to the net pressure in the fracture; (e) the injection displacement remains constant.

#### 3.2.2 | Calculation model of leakoff coefficient

The fracturing fluid leakoff is controlled by filtrate viscosity, formation fluid compression, and wall building property. The filter loss can be characterized by a leakoff coefficient. For CBM wells, the filter cake is difficult to be built, because clear fracturing fluid is usually using in the hydraulic fracturing. Thus, the calculation of leakoff coefficient for CBM wells only considers the influence of filtrate viscosity and formation fluid compression.

According to the classical leakoff theory, the leakoff coefficients can be calculated by Equations (3)-(5):

\[
C_1 = 0.17 \sqrt{K_1 \Delta P' \varphi / \mu_1} \quad (3)
\]

\[
C_2 = 0.136 \Delta P' \sqrt{K_2 C_1 \varphi / \mu_2} \quad (4)
\]

\[
C_0 = 2C_1 C_2 / \left( C_1 + \sqrt{C_1^2} \right) \quad (5)
\]

where $C_1$ and $C_2$ are viscous fluid loss control coefficient and fluid compression control coefficient, respectively, m/min$^{0.5}$; $C_0$ is the initial overall leakoff coefficient for coal reservoir, m/min0.5; $K_1$ and $K_2$ are permeability for fracturing fluid and permeability for formation fluid, respectively, $\mu$m$^2$; $\Delta P'$ is the difference between the pressure in the fracture and the initial reservoir pressure, MPa; $\varphi$ is reservoir porosity, %; $\mu_1$ and $\mu_2$ are viscosity of fracturing fluid and formation fluid, respectively, MPa-s; $C_1$ is total reservoir compressibility, MPa$^{-1}$.

The computation of the overall leakoff coefficient is often very different from the real situation, because the fluid pressure is increased by the injection of fracturing fluid, and the effective stress has been changed in the coal reservoir. The dynamic permeability can be expressed as a function of effective stress as in Equation (6): \[K = K_0 e^{3C_1 (P - P_0)} = K_0 e^{3C_1 \Delta P}\]
where $K_0$ is initial reservoir permeability, $\mu$m$^2$; $P$ is flowing bottomhole pressure, MPa; $P_0$ is reservoir pressure, MPa; $C_f$ is pore compressibility, MPa$^{-1}$.

The relationship between the pore volume and pore pressure can be described by pore compressibility:

$$C_f = \varphi^{-1} \frac{d\varphi}{dP}$$

(7)

Equation (8) can be obtained from the integration of Equation (7):

$$\varphi = \varphi_0 e^{C_f \Delta P}$$

(8)

where $\varphi_0$ is initial reservoir porosity, %.

Combining Equations (3)-(8), the dynamic overall leakoff coefficient can be calculated as follows:

$$C = C_0 e^{2C_f \Delta P}$$

(9)

**TABLE 1** Geometric parameters of hydraulic fracturing monitoring with microseismics in research area

| Well No. | Depth (m) | Azimuth (°) | Fracture length (m) | Fracture length of west wing (m) | Fracture length of east wing (m) | Fracture height (m) | Inclination (°) | Morphology |
|----------|-----------|-------------|---------------------|---------------------------------|---------------------------------|---------------------|----------------|------------|
| ZS30     | 638.5     | NE56.2      | 152                 | 90.9                            | 61.1                            | 6.7                 | 0              | Vertical   |
| ZS38     | 659.95    | NE52.8      | 218.7               | 109.4                           | 109.3                           | 8                   | 1              | Vertical   |
| ZS54     | 1038.5    | NW56.5      | 221.3               | 106.2                           | 115.1                           | 6.7                 | 0              | Vertical   |
| ZS64     | 1242.4    | NE81.8      | 170.7               | 92.2                            | 78.5                            | 6.7                 | 1              | Vertical   |
| ZS73     | 894.65    | NE70        | 197.71              | 105.26                          | 92.45                           | 6.56                | 1              | Vertical   |
| ZS76     | 515.4     | NE56.1      | 187.8               | 89.2                            | 98.6                            | 7.42                | 1              | Vertical   |
| ZS78     | 702.2     | NE64.7      | 228.59              | 112.36                          | 116.23                          | 6.78                | 1              | Vertical   |
| ZS80     | 752.1     | NE49.6      | 216.74              | 106.26                          | 110.48                          | 6.83                | 0              | Vertical   |
| ZS82     | 798.6     | NE81.5      | 192                 | 97.15                           | 94.85                           | 7.46                | 0              | Vertical   |
| ZS83     | 925.8     | NE64.5      | 195.05              | 114.53                          | 80.52                           | 7.45                | 1              | Vertical   |
| ZS85     | 474.55    | NE85.3      | 218.04              | 110.02                          | 108.02                          | 4.39                | 0              | Vertical   |
| ZS86     | 423       | NE54.5      | 202.71              | 88.35                           | 114.36                          | 6.5                 | 0              | Vertical   |
| ZS91     | 1072.85   | NE60.2      | 181.43              | 79.59                           | 101.84                          | 7.49                | 1              | Vertical   |
| ZS93     | 613       | NE66.5      | 193.71              | 82.9                            | 110.81                          | 5.77                | 0              | Vertical   |
| ZS94     | 687.8     | NE62.4      | 212.36              | 94.5                            | 117.86                          | 9.04                | 0              | Vertical   |
| ZS97     | 1259.15   | NE53.7      | 220.93              | 110.01                          | 110.92                          | 5.69                | 0              | Vertical   |
| ZS100    | 938       | NE78.2      | 177.97              | 86.25                           | 91.72                           | 6.68                | 0              | Vertical   |

**FIGURE 3** The azimuth of principal hydraulic fractures in research area

**FIGURE 4** The geometrical model of PKN$^{22,43}$
During hydraulic fracturing, the effective stress decreases with the continued injection of fracturing fluid, and the dynamic overall leakoff coefficient increases exponentially.

### 3.2.3 Analysis of the pressure decline

The cross-section of hydraulic fracture is assumed as an ellipse, and the sectional area is:

\[ A(x,t) = aW(x,t) \quad (10) \]

These cross-sections are elliptical in shape with a maximum width:

\[ W(x,t) = \frac{2H}{E'} \Delta P(x) \quad (11) \]

\[ \Delta P(x) = (P_f(0) - P_c) \sqrt{1 - \left(\frac{x}{L}\right)^2} \quad (12) \]

where \( \Delta P(x) \) is the net pressure of hydraulic fracture propagating to \( x \) position.\(^{45}\)

During the hydraulic fracturing, the fracture length has the following relationship with time:

\[ L = a_1 t^m \quad (13) \]

where \( a_1 \) is constant; \( m \) is propagation rate, Nordgren analyzed the lower limit of \( m \) is 0.5 in the case of high filtration.\(^{15}\) In this paper, \( m = 0.5 \) is selected for the following study.

According to material balance, part of the total fracturing fluid propagates in the fractures, and the other part filter into the reservoir:\(^{45}\)

\[ Q_p \cdot t_p = \frac{4aH}{E'} P_p \int_0^{t_p} \sqrt{1 - \left(\frac{x}{L}\right)^2} \, dx + 4C_0 H_L \int_0^{t_p} \frac{1}{\sqrt{t-t\left(\frac{x}{L}\right)^2}} \, dx \quad (14) \]

accordingly

\[ Q_p \cdot t_p = FP_p + D_m C_0 H_L t_p \sqrt{t_p} \quad (15) \]

where \( P_p \) is the difference between average pump pressure and closure pressure, MPa; \( L_0 \) is the fracture length at the end of injection, m; \( F = \frac{2H \Gamma(0.5) \Gamma(1.5)}{E'T(2)} \), \( D_m = \frac{0.89581 \Gamma(1.5)}{\Gamma(2)} \) = 6.2816, \( \Gamma \) is gamma function.

After shut-in, the geometrical shape of the hydraulic fracture is assumed to be invariant, and the filter loss between two shut-in times can be expressed as:

\[ \Delta V(t_1,t_2) = V(t_1,t_2) - V(t_1,t_1) \]

\[ = \int_{t_1}^{t_2} \left[ \frac{4C_0}{\sqrt{t-t_p(x^2/L_p^2)}} H_L \right] \, dx \]

\[ (16) \]

\( \Delta t \) is the reference time since shut-in, \( \delta = \frac{\Delta t}{t_p} \), and this provides:

\[ V_{LS}(t_1,t_2) = 2\pi C_0 H_L \sqrt{t_p} G(\delta_2,\delta_1) \quad (17) \]

where

\[ G(\delta_2,\delta_1) = \frac{4}{\pi} \left[ g(\delta_2) - g(\delta_1) \right] \]

During closure, the pressure difference between two dimensionless delta times can be obtained by:

\[ P_1 - P_2 = \frac{2C_0 H L E'}{aH} \sqrt{1 - g^2} \quad (18) \]

Thus, the match pressure is equivalent to:

\[ P^* = \frac{P_1 - P_2}{G(\delta_2,\delta_1)} = \frac{2C_0 H L E'}{aH} \sqrt{1 - g^2} \quad (19) \]

Then, the overall leakoff coefficient can be calculated as follows:

\[ C_0 = \frac{P^* aH}{2 H L E' \sqrt{1 - g^2}} \quad (20) \]

Combining Equations (19), (20), (9) and (15), the fracture length can be obtained by:

\[ L_p = \frac{Q_p}{FP_p + 0.3927D_m CH_L \sqrt{t_p}} \quad (21) \]

### 3.3 Prediction model of postfracturing permeability for a CBM well

After hydraulic fracturing, it is assumed that the fractures do not connect to the fault or other tectonic anomaly. Based on the injection well testing theory,\(^{46,47}\) the relationship between the pressure decline and time can be expressed as follows:

\[ P(t) = P_0 - 2.121 \times 10^{-3} \frac{Q \mu B}{\Delta Kh} \left[ \lg t + \lg \left( \frac{K}{\varphi \mu^2 \rho_c C_l} \right) + 0.9077 + 0.8686 \right] \quad (22) \]

where \( Q \) is filter loss after shut-in, \( m^3; \) \( \mu \) is fluid viscosity, MPa·s; \( B \) is formation volume factor, dimensionless; \( h \) is reservoir thickness, m; \( t \) is test time after shut-in, min; \( r_w \) is wellbore radius, m; \( S \) is skin coefficient, dimensionless.

Assuming that

\[ a = -2.121 \times 10^{-3} \frac{Q \mu B}{\Delta Kh} \]

and

\[ b = P_0 - 2.121 \times 10^{-3} \frac{Q \mu B}{\Delta Kh} \left[ \lg \left( \frac{K}{\varphi \mu^2 \rho_c C_l} \right) + 0.9077 + 0.8686 \right] \]

the Equation (22) becomes:

\[ P(t) = a \lg t + b \quad (23) \]
According to the pressure decline data, the semilogarithmic curve between time and pressure decline can be plotted, where the value of $a$ will be the slope of the straight line. Therefore, the postfracturing permeability can be calculated by:

$$K = \frac{-2.121 \times 10^{-3} V_{LS} \mu B}{ah\Delta t} \tag{24}$$

where $V_{LS}$ is the filter loss in $\Delta t$, which can be obtained by Equation (17), $\text{m}^3$.

4 | MODEL ANALYSIS AND APPLICATION

4.1 | Classification of fracturing curve in study area

Based on the characteristics of the fracturing curves and detailed operation reports on the CBM wells in the study area, the fracturing curves from 47 CBM wells can be classified into different types. The classification was mainly based on the shapes of the pressure curves obtained at the normal sand-carrying stage, and the curves obtained during the stop stage of sand-carrying due to structural or formation conditions were discarded. The pressure curves with less fluctuation throughout the entire course of fracturing were considered to be the stable curves. At the same time, attention was paid to any frequent fluctuations of displacement curves and pressure curves during hydraulic fracturing. The fracturing curves collected from 47 CBM wells in the study area have been classified into five types, namely, stable curves, stably fluctuating curves, descending curves, ascending curves, and fluctuating curves.

For stable fracturing curves, both the pressure curve and displacement curve remain stable throughout the entire course of hydraulic fracturing, and the sand ratio curve does not show any interruptions. The stable curves have two cases: (a) As a large number of natural fractures have occurred in the coal reservoir near the wellbore with a relatively low cracking pressure, the fracturing fluid infiltrates steadily along the fractures and the pressure remains stable and low (Figure 5A); (b) There are not many natural fractures existing in the coal reservoir, and new hydraulic fractures are created in the hydraulic fracturing. The hydraulic fractures propagate in a relatively simple and steady path (Figure 5B).

For stably fluctuating curves, the pressure curve is fluctuating with small amplitude throughout the whole process. The displacement curve also shows a small amplitude fluctuation, and the sand ratio curve may exhibit interruptions with the variation of pressure (Figure 6). The stably fluctuating curve indicates that natural fractures have been developed in the coal reservoir and that there is filtration in the formation. In addition, hydraulic fractures are created, propagating, and connecting with natural fractures continuously.

For descending fracturing curves, the displacement curve remains stable at the normal sand fracturing stage. The sand ratio curve increases, and the pressure curve shows a decreasing trend (Figure 7). During hydraulic fracturing, the cracks are created in the coal reservoir and the fracturing fluid infiltrates along the main fracture. The main fracture will connect with natural fractures or create some small branches during its propagation, resulting in small fluctuations during the pressure decline.

For ascending fracturing curves, the displacement curve remains stable in the hydraulic fracturing, and the pressure shows a trend of increase as the sand ratio increases (Figure 8). This mainly indicates that there is no obvious main fracture in the reservoir due to the influence of in-situ stress and inhomogeneous fractures. In addition, a large number of microfractures are formed continuously, resulting in radial and reticulated fractures.

For fluctuating fracturing curves, the pressure is extremely unstable and difficult to control. The sand ratio curve shows interruptions during the hydraulic fracturing, and the displacement curve also fluctuates with large amplitudes (Figure...
This phenomenon indicates that the fracturing fluid mainly infiltrates along the natural fractures in the coal, and the “T” shaped and “I” shaped fracture may form in the interface between the coal reservoir and its roof or floor. The hydraulic fracturing effect is worse, the length of fracture propagation is limited, and many complex branches are formed.

There are 19 wells with stable fracturing curves, 5 wells with stably fluctuating curves, 11 wells with descending fracturing curves.
curves, 6 wells with ascending curves, and 6 wells with fluctuating curves in the study area. The hydraulic fracturing effect is affected by geologic, engineering, and other factors. The fracture extension grad can reflect the characteristics of different fracturing curves to a certain extent. The fracture extension grad is a concentrated expression of the minimum horizontal principal stress, the reservoir pressure, and the morphology of hydraulic fractures, and it can be expressed as:

\[ F_G = \frac{P_F}{H} \]  

(25)

\[ P_F = P_S + P_H \]  

(26)

where \( F_G \) is fracture extension grad, MPa/m; \( P_F \) is fracture extension pressure, MPa; \( H \) is burial depth of coal reservoir, m; \( P_S \) is instantaneous shut-in pressure, MPa; \( P_H \) is hydrostatic fluid column pressure, MPa.

The fracture extension grad has been calculated for 27 CBM wells in the study area. In general, the stable and descending curves feature small fracture extension grads, the stably fluctuating and ascending curves feature medium fracture extension grads, and the fluctuating curves feature large fracture extension grads (Figure 10). It is shown that the stable and descending fracturing curves represent a better fracturing effect and that the fractures propagate easily, and the fluctuating fracturing curves represent a poorer fracturing effect and that the fractures propagate with greater difficulty.

4.2 Calculation of fracture length and postfracturing permeability

Fitting pressure is very critical for the analysis of pressure decline, because its accuracy directly affects the reliability of estimated parameters such as fluid loss coefficient for the fracturing fluid and geometric dimensions of fractures. The traditional method to solve the fitting pressure employs type-curve matching, which requires complicated operations and a longer period of pressure decline. Castillo modified Nolte’s plot by introducing the G function, which makes the pressure decline vs G function a straight and the
slope of this straight line is the fitting pressure. However, how to select the reference time, in other words, how to select \( \delta_1, \delta_2 \) in formula (18), is still the key to determining the slope of the straight line. The value of \( \delta_p \) (\( \delta_p = \text{pressure decline time/pump time} \)) is within the range of 0.33–1.01, and it is generally shorter than that for the hydraulic fracturing of conventional reservoirs. To analyze the characteristics of the G function under different conditions corresponding to \( \delta_p \), the fracturing curves of seven CBM wells with complete data of well testing, hydraulic fracturing, and microseismic monitoring have been selected to analyze the calculation methods of the fitting pressure and postfracturing permeability. The values of \( \delta_p \) for the seven wells range from 0.33 to 1.01. The following illustrates the acquisition method of the fitting pressure with ZS36 and ZS78 wells data.

The formula (18) can be transformed as:

\[
\Delta P (\delta_p, \delta_0) = P^* \cdot G (\delta_p, \delta_0)
\]  

(27)

where \( \delta_0 \) is initial dimensionless delta time for calculating G function; \( G (\delta, \delta_0) \) is G function corresponding to \( \delta_0; \) \( \Delta P (\delta, \delta_0) \) is the pressure decline corresponding to \( \delta_0 \). Plotting the scatter diagram of \( \Delta P (\delta_p, \delta_0) \) vs G Function (Figure 11), the slope of the straight line is the fitting pressure.

It can be observed from Figure 11 that \( \Delta P (\delta_p, \delta_0) \) is positively linear with \( G(\delta_p, \delta_0) \) (\( R^2 > 0.95 \)) when different values of \( \delta_0 \) are chosen, indicating that it is credible to calculate the fitting pressure using formula (27). However, the linear relationship of the initial is negative, and the main reason is that the initial stage is interpreted as fracture extension. Theoretically, the calculation of fitting pressure first needs to choose the appropriate \( \delta_0 \) value to eliminate the influence of fracture extension in the stage of pressure decline. Different \( \delta_0 \) values correspond to different slopes of the fitting straight line (Figure 11), but it is found that the slopes of the linear segments of the G function curves with different \( \delta_0 \) values are consistent (Figure 12). With the increase in the \( \delta_0 \) values, the intercepts of the linear segments of the G function curves die out to zero, and it becomes to match the function (27) (Figure 12). Thus, the fitting pressure can be obtained by the slope of the linear segment of the G(\( \delta, 0 \)) curve. It is not necessary to determine the specific \( \delta_0 \) value, which solves the calculation problem of fitting pressure.
After the fitting pressure has been obtained (Figure 13), the dynamic overall leakoff coefficient, filter loss, and postfracturing permeability of seven CBM wells are calculated by formulas (9), (17) and (24). The specific calculation parameters and results are summarized in Tables 2 and 3. The predicted postfracturing permeabilities (1.55–8.83 × 10^{-3} μm^2) of the reservoir have typically increased 5.10–294.33 times compared with the prefracturing permeability, and the gas production displays a positive correlation with the postfracturing permeability (Figure 14), indicating that hydraulic fracturing substantially improves the petrophysical properties of coal reservoirs.

**TABLE 3** Calculation results of the fracture length and permeability

| Well No. | \( P^* \) (MPa) | \( C_0 \) (m/min^{0.5}) | \( C \) (m/min^{0.5}) | \( V_{LA} \) (m^3) | \( a \) | \( 10^{-3} \mu m^2 \) | \( K_0 (10^{-3} \mu m^2) \) | \( K/K_0 \) |
|----------|----------------|------------------------|----------------|----------------|----------|------------------|-----------------|----------------|
| ZS27     | 3.5942         | 0.00191                | 0.00981         | 29.78         | –2.26    | 8.83             | 0.03            | 294.33         |
| ZS30     | 8.2613         | 0.00191                | 0.10853         | 26.92         | –10.18   | 3.48             | 0.29            | 12             |
| ZS36     | 2.3179         | 0.00068                | 0.00072         | 145.85        | –3.95    | 5.41             | -              | -              |
| ZS64     | 0.2321         | 0.00008                | 0.00011         | 12.53         | –0.30    | 3.99             | 0.047           | 84.89          |
| ZS69     | 1.6371         | 0.00008                | 0.00044         | 67.89         | –1.84    | 3.09             | 0.153           | 20.20          |
| ZS76     | 2.3641         | 0.00060                | 0.00456         | 65.16         | –3.14    | 1.55             | 0.011           | 140.91         |
| ZS78     | 1.8042         | 0.00046                | 0.00173         | 50.73         | –3.18    | 1.73             | 0.339           | 5.10           |

5 | CONCLUSIONS

The morphology of hydraulic fractures of CBM wells is controlled by in-situ stress. The vertical fractures are generated in the Zhengzhuang block and propagate along the direction of the maximum principal stress. The fracturing curves can be classified into stable curves, stably fluctuating curves, descending curves, ascending curves, and fluctuating curves. The fracturing effects of stable and descending fracturing curves are better than those of fluctuating fracturing curves.
The fitting pressure can be obtained by the slope of the linear segment of the $G(\delta,0)$ curve using the classical analysis method of pressure decline. The injection well testing theory has been introduced to the analysis of the fracturing pressure decline, and a simple prediction method of postfracturing permeability has been established. The predicted postfracturing permeabilities range from $1.55 \times 10^{-3}$ to $8.83 \times 10^{-3} \, \mu m^2$, indicating that hydraulic fracturing substantially improves the petrophysical properties of coal reservoirs.

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**NOMENCLATURE**

- $\sigma_h$ minimum principal stress
- $\sigma_H$ maximum principal stress
- $\sigma_v$ vertical stress
- $C$ dynamic overall leakoff coefficient
- $A$ cross-sectional area of hydraulic fracture
- $W$ fracture width at any time
- $x$ coordinate in fracture-propagation
- $t$ time
- $a$ cross-sectional coefficient of hydraulic fracture
- $H$ fracture height
- $E'$ plane-strain modulus
- $E$ Young's Modulus
- $v$ Poisson's ratio
- $P_i(0)$ pressure at some point in fracture
- $P_c$ closure pressure
- $L_p$ fracture half-length
- $\Gamma$ Gamma Function
- $V_{LS}$ filter loss
- $\Delta t$ reference time since shut-in
- $\delta$ dimensionless delta time
- $G(\delta_2,\delta_1)$ dimensionless difference function
- $g(\delta)$ dimensionless loss-volume function
- $P^*$ fitting pressure
- $ISIP$ instantaneous shut-in pressure
- $Q_p$ pump rate

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