Ω-dibaryon production with hadron interaction potential from the lattice QCD in relativistic heavy-ion collisions

S. Zhang(张松) and Yu-Gang Ma(马余刚)∗
Key Laboratory of Nuclear Physics and Ion-beam Application (MOE),
Institute of Modern Physics, Fudan University, Shanghai 200433, China
(Dated: August 7, 2020)

Recently HAL QCD Collaboration reported the Ω−Ω and N−Ω interaction potentials by Lattice QCD algorithm. Based on these results, NΩ (1S2) and ΩΩ (1S0) bound state were proposed with binding energy about a few MeV and N−Ω HBT correlation were also calculated and measured by the STAR and the ALICE Collaboration. These results provided dynamical information if Ω-dibaryons exist from the interaction aspects. Another necessary point is the experimental environment where the bound states can produce and survive in the system or not, such as in relativistic heavy-ion collisions. So there are at least two necessary conditions to constrain the production probability of Ω-dibaryons, i.e. the short-range attractive interaction to form bound states and experimental environment to provide abundant enough strangeness and multiplicity of nucleons. In this work the Ω−Ω and Ω−nucleon interaction potentials by the lattice QCD were employed to obtain ΩΩ (1S0) and NΩ (1S2) wave functions, and the production of Ω-dibaryons was estimated by use of a dynamical coalescence mechanism with considering the environment in relativistic heavy-ion collisions at \( \sqrt{s_{NN}} = 200 \) GeV and 2.76 TeV.

PACS numbers: 25.75.Gz, 12.38.Mh, 24.85.+p

I. INTRODUCTION

Strangeness dibaryon has been investigated in theory and experiments for a long period since \( H−dibaryon was predicted by Jaffe \[^{[1]}\]. However to date there is only one stable dibaryon measured as deuteron (d). Ω−dibaryons, namely NΩ (1S2) and ΩΩ (1S0), were proposed in several theoretical works and considered as the most promising candidates of strangeness dibaryons \[^{[2, 3]}\]. Goldman et al. \[^{[4]}\] predicted the strangeness-3 dibaryons by use of two different quark models of hadrons. In the framework of the quark delocalization color screening model and the chiral quark model, the NΩ dibaryon was further studied and the binding energy was estimated from a few MeV to hundred MeV within different configurations \[^{[3, 4]}\]. A baryon-baryon interaction model with meson exchanges also calculated the N−Ω two-body system and suggested a quasibound state with binding energy 0.1 MeV \[^{[6]}\]. Lattice QCD reported the results of the spin-2 NΩ dibaryon with large binding energy 18.9 MeV by the HAL QCD collaboration by using pion mass \( m_π = 875 \) MeV and kaon mass \( m_K = 916 \) MeV \[^{[7]}\]. A further work from the lattice QCD near the physical point suggested that the binding energy of NΩ dibaryon is 2.46 MeV and 1.54 MeV, respectively, with and without Coulomb attraction \[^{[8]}\].

ΩΩ dibaryon with strong Ω−Ω attraction was predicted by a chiral quark model \[^{[10, 12]}\], while a weak repulsion Ω−Ω interaction was suggested by other models \[^{[3, 13]}\]. Based on the possible production channel \[^{[14, 15]}\], an extended version of a multi-phase transport (AMPT) model \[^{[16]}\] estimated the production probability of ΩΩ dibaryon in Au+Au collisions at \( \sqrt{s_{NN}} = 130 \) GeV. The HAL QCD collaboration’s previous work \[^{[17]}\] showed a moderate attraction with \( m_π = 700 \) MeV and recently the HAL QCD method with \( m_π \) close to the physical point presented ΩΩ binding energy 1.6 MeV or 0.7 MeV with the Coulomb repulsion \[^{[18]}\].

The momentum correlation functions of hadron pairs can reflect the hadron-hadron interaction \[^{[19, 20]}\] which can provide the information if the pairs can form a bound state. Based on the lattice QCD simulations of N−Ω interaction \[^{[8]}\], the momentum correlation functions of N−Ω was calculated in Ref. \[^{[21]}\] for probing NΩ dibaryons. The STAR collaboration conducted this measurement \[^{[22]}\] in Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV and the results favored the p−Ω bound state with binding energy 27 MeV. By use of the interaction potential from recent lattice QCD calculations at nearly physical quark masses \[^{[3, 13]}\], the p−Ω and Ω−Ω momentum correlation functions were updated \[^{[23]}\]. Recently the ALICE collaboration reported the measurement of p−Ω− and p−Ξ− correlations \[^{[24]}\] in pp collision at \( \sqrt{s} = 13 \) TeV and the results of p−Ξ− was in agreement with the predicted functions by the HAL QCD results \[^{[23]}\], and p−Ω− correlation should be investigated further in nucleus-nucleus collisions in theory and experiment aspects.

In this work, we reported the production of NΩ (1S2) and ΩΩ (1S0) dibaryons calculated by a dynamical coalescence model with considering N−Ω and Ω−Ω interaction potential from the HAL QCD results \[^{[3, 13]}\] in Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV and Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV. It was found that the production probabilities of NΩ (1S2) \( \sim 10^{-3} \) and ΩΩ (1S0) \( \sim 10^{-6} \), respectively.
II. A BRIEF INTRODUCTION TO ALGORITHM

Dynamical coalescence model is able to describe hadron and light nuclei production in heavy-ion collisions \cite{23,29}, and the component interaction was reflected in the overlap wave function. For two-body clustered object, the multiplicity of the object can be obtained by \cite{23,29},

\[ N_{2b} = g_2 \int \left( d^4 x_1 S_1(x_1, p_1) d^3 p_1 \right) \times \left( d^4 x_2 S_2(x_2, p_2) d^3 p_2 \right) \times \phi^W_2(x_1, x_2; p_1, p_2), \]

where \( \phi^W_2(x_1, x_2; p_1, p_2) \) is the Wigner density function which gives the coalescence probability, \( g_2 \) is the coalescence factor \cite{30}, 3/8 for deuteron, 5/8 for \( N\Omega \) and 1/4 for \( \Omega\Omega \), \( S(x, p) \) is the phase space distribution of the components at coordinate-momentum space \((x, p)\) with energy \( E \). Note that the phase space distribution of neutron \((n)\) was assumed the same as that of proton \((p)\) if a certain cluster contains neutron in this work.

The phase-space distribution can be expressed by a blast-wave model \cite{26,29,31,32},

\[ S(x, p) d^4 x = m_T \cosh(\eta_s - y_T) f(x, p) J(\tau) \times \tau d\tau d\eta_s d\mathbf{r} d\phi_s, \]

where \( \eta_s \) and \( m_T \) are respectively the rapidity and transverse mass of the hadron, \( \tau \), \( \eta_s \), \( \phi_s \) are the polar coordinates, proper time, pseudorapidity and azimuthal angle in coordinate space. A Gaussian distribution for the freeze-out proper time is given by, \( J(\tau) = \frac{\tau_0}{\sqrt{2\pi\tau}} \exp\left(-\frac{(\tau_0 - \tau)^2}{2\tau}\right) \), where \( \tau_0 \) and \( \Delta \tau \) are the mean value and the dispersion of the \( \tau \) distribution. The statistical distribution function \cite{33} \( f(x, p) \) is defined by \( f(x, p) = \frac{2^{s+1}}{(2\pi)^{s+1}} \exp\left(\frac{m_T^2 p^2}{2m_T^2}\right) \left(1\right)^{-1} \), where \( s \) is the spin of the particle, \( u_\mu = e^{-i\eta_s x_\mu + i\eta_s p_\mu} \) is the four-velocity of a fluid element in the fireball of the emission source, and \( T_{kin} \) is the kinetic freeze-out temperature. The energy in the local rest frame of the fluid can be written as, \( p^\mu u_\mu = m_T \cosh \rho \cos(\eta_s - y_T) - p_T \sinh \rho \cos(\phi_s - \phi_p) \), where \( \phi_p \) is azimuthal angle in momentum space, \( \rho \) is the transverse flow rapidity distribution of the fluid element in the fireball with a transverse radius \( R_0 \), defined as \( \rho = \frac{\rho_0 R_0}{R} \) without considering the anisotropic part \cite{28,31,32}.

Fixing the parameters of \( \tau_0, \Delta \tau, \rho_0, R_0, T_{kin} \), one can obtain the transverse momentum distribution of the hadrons by,

\[ \frac{dN}{2\pi p_T dp_T dy_T} = \int S(x, p) d^4 x. \]

By using transport model to obtain the component phase-space, the dynamical coalescence model for two-body clustered object can be written as \cite{26,27},

\[ N_{2b} = g_2 \int d^3 p_1 d^3 p_2 \times (\phi^W_2(x_1, x_2; p_1, p_2)), \]

where \((\ldots)\) denotes event averaging. In this work a multi-phase transport (AMPT) model \cite{34} with version 2.26t7b are employed to provide the phase-space of neutrons, protons and \( \Omega \)’s. AMPT simulates the relativistic heavy-ion collisions dynamically in a framework of multi phases, in which the initial state is given by the Heavy Ion Jet Interaction Generator (HIJING) model \cite{32,39}, and the melted partons from the HIJING interact with each other by the Zhang’s Parton Cascade (ZPC) model \cite{37}, and finally the interacting-ceased partons converts to hadrons by a simple quark coalescence model or the Lund string fragmentation followed by a relativistic transport model performing the hadron rescattering \cite{38}. AMPT can well describe physics in relativistic heavy-ion collisions at the RHIC \cite{34} and LHC \cite{39} energies, including pion-HBT correlations \cite{40}, di-hadron azimuthal correlations \cite{41,42}, collective flow \cite{43,44} and strangeness production \cite{45,46} and so on.

The Wigner phase-space densities of the objects \( \rho^W_2(x_1, x_2; p_1, p_2) \) can be obtained from,

\[ \rho^W_2(\vec{r}, \vec{q}) = \int J(\vec{r} + \vec{R}) \phi^*(\vec{r} - \vec{R}) \times \exp(-i\vec{q} \cdot \vec{R}) d\vec{R}, \]

where \( \vec{q} = (m_2 p_2 - m_1 p_1)/(m_1 + m_2) \) and \( \vec{r} = (\vec{r}_1 - \vec{r}_2) \) are the relative momentum and relative coordinate, respectively, \( \phi(\vec{r}) \) is the overlap wave function of the two components. In our previous work for three components and other calculations for two-body, the overlap wave function always were taken to be a spherical harmonic oscillator. In this work, the wave function and binding energy \( E_B \) for \( d \) \( \left(^3S_1\right) \), \( N\Omega \) and \( \Omega\Omega \) at assumed bound state \( ^3S_2 \) and \( ^1S_0 \) will be obtained by solving the radial Schrödinger equation with the potential for \( n - p \) \cite{47}, \( N - \Omega \) \cite{48}, \( \Omega - \Omega \) \cite{49}, respectively,

\[ V_{np}(r) = \sum_{i=1}^{2} C_i e^{-\mu_ir}, \]

\[ V_{N\Omega}(r) = b_1 e^{-b_2 r^2} + b_3 \left(1 - e^{-b_2 r^2}\right) \left(\frac{e^{-m_\pi r}}{r}\right)^2, \]

\[ V_{\Omega\Omega}(r) = \sum_{i=1}^{3} C_i e^{-(r/d_i)^2}, \]

here the parameters for the potential are listed in Table II.

In the calculation the Coulomb interaction was also taken into account for the charged pairs by adding \( \pm \alpha/r \) with \( \alpha = e^2/4\pi \) to the potential in equation (6). Figure II showed the numerical results of the wave function \( \phi(r) \) for \( d \left(^3S_1\right) \), \( N\Omega \left(^3S_2\right) \) and \( \Omega\Omega \left(^1S_0\right) \). The Hulthen wave function \cite{26,48} for \( d \) which is also presented in Fig. II is higher than the calculated wave function by using potential \( V_{np}(r) \) in short relative distance region, which is because of the repulsion core of the potential for \( n - p \) interaction and this pattern was also found in \( \Omega\Omega \)
TABLE I: The parameters for the potential, $V_{np}(r)$ [27], $V_N\Omega(r)$ [3] and $V_N^b(r)$ [13].

| $V_{np}(r)$ | $C_1$ (MeV) | $C_2$ (MeV) | $\mu_1$ (fm$^{-1}$) | $\mu_2$ (fm$^{-1}$) |
|------------|-------------|-------------|---------------------|---------------------|
|            | 626.885     | 1438.72     | 1.55                | 3.11                |
| $V_N\Omega(r)$ | $b_1$ (MeV) | $b_2$ (fm$^{-2}$) | $b_3$ (MeV$\cdot$fm$^2$) | $b_4$ (fm$^{-2}$) |
|            | -313        | -292        | 85                  |                     |
| $V_N^b(r)$ | $C_1$ (MeV) | $C_2$ (MeV) | $C_3$ (MeV) |
|            | 914         | 305         | -112                |
|            | d$_1$ (fm) | d$_2$ (fm) | d$_3$ (fm) |
|            | 0.143       | 0.305       | 0.949               |

FIG. 1: Calculated wave function of d (n-p), N$\Omega$ and $\Omega$, and the Hulthén wave function [24, 48] for d. (color online)

The $N - \Omega$ attractive potential resulted from experiments by using equation (1). In Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, the data were collected for NN at 0-10% centrality, 5% for $p$, 0-10% for $\Omega$ and $d$. The fitting to the experimental data was shown in Fig. 2 and the extracted parameters were $R_0 = 12$ fm, $r_0 = 9$ fm/c, $\Delta \tau = 3.5$ fm/c, $T_{kin} = 111.6$ MeV, and $\rho_0 = 0.98$ for proton and 0.9 for $\Omega$ in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and $R_0 = 19.7$ fm, $r_0 = 15.5$ fm/c, $\Delta \tau = 1$ fm/c, $T_{kin} = 122$ MeV, and $\rho_0 = 1.2$ for proton and 1.07 for $\Omega$ in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.

By using the above configured blast-wave model and coalescence model (BLWC) as in equation (1), the transverse momentum $p_{T}\sigma$ spectra of $n\Omega$, $p\Omega$ and $\Omega\Omega$ dibaryons were calculated and shown in Fig. 3 in which panel (a) is for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and panel (b) for Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. The $p_T$ integrated yields $dN/dy$ of objects at midrapidity were given in Table II and the calculated $dN/dy$ of $p$, $\Omega$ and $d$ were comparable with those from experimental results from the RHIC data [49, 51] as well as from the ALICE data [52, 54]. The predicted $dN/dy$ of $p\Omega$, $n\Omega$ and $\Omega\Omega$ were 7.51$\times$10$^{-4}$, 7.39$\times$10$^{-4}$, 1.24$\times$10$^{-6}$ for the RHIC top energy and 1.31$\times$10$^{-5}$, 1.27$\times$10$^{-3}$, 3.15$\times$10$^{-6}$ for the ALICE at 2.76 TeV, respectively, in nucleus-nucleus collisions. It is seen that the production of $N\Omega$ and $\Omega\Omega$ at the ALICE energy was about 2 times of those at the RHIC top energy. And these calculated results were similar to the previous work by using naive coalescence model [55] or analytical coalescence model [29] as well as using the AMPT model with $\Omega\Omega$ production channel [10].

The production of $p\Omega$, $n\Omega$ and $\Omega\Omega$ bound states were also calculated by using phase-space data from the AMPT model [34] via dynamical coalescence mechanism equation (1) (AMPTC). To fit proton spectra, some parameters defined in the AMPT model [34, 39] were adjusted as, $(a, b) = (0.55, 0.1)$ for the RHIC energy and $(0.21, 0.075)$ for the LHC energy, and $p_{T}$ (MeV) and $a$ and $b$ are the Lund string fragmentation parameters defined in reference [34]. And the coalescence mechanism for $\Omega$ was also developed as in Ref. [56] to fit $\Omega$ spectra. Figure 4 presented the fitted $p_T$ spectra for proton and $\Omega$ as well as the coalesced $p_T$ spectra of $p\Omega$, $n\Omega$ and $\Omega\Omega$ bound states in Au+Au central collisions at $\sqrt{s_{NN}} = 200$ GeV.

TABLE II: The calculated binding energy together with the collected values from other published results [6, 18, 47].

|       | this work | value/Reference |
|-------|-----------|-----------------|
| $E_\rho$ (MeV) | 2.23 | 2.2307 [47] |
| $E_{p\Omega}$ (MeV) | 3.09 | 3.00 [9] |
| $E_{n\Omega}$ (MeV) | 1.38 | 1.54 [9] |
| $E_{\Omega\Omega}$ (MeV) | 0.6 | 0.7 [18] |

TABLE III: $dN/dy$ of $p\Omega$, $n\Omega$, $\Omega\Omega$ at mid-rapidity.

|       | $p\Omega$ | $n\Omega$ | $\Omega\Omega$ |
|-------|-----------|-----------|---------------|
| $200$ GeV | 7.51$\times$10$^{-4}$ | 7.39$\times$10$^{-4}$ | 1.24$\times$10$^{-6}$ |
| BLWC    | 9.5$\times$10$^{-4}$ | 9.5$\times$10$^{-4}$ | 3.23$\times$10$^{-6}$ |
| AMPTC   | $\pm 7.9\times 10^{-5}$ | $\pm 7.9\times 10^{-5}$ | $\pm 4.6\times 10^{-6}$ |

from the PHENIX experiment [43], for $\Omega$ and $d$ from the STAR experiments [50, 51] (centrality, 5% for $p$, 0-10% for $\Omega$ and $d$). In Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, the data of $p$, $\Omega$ and $d$ were taken from the ALICE experiments [52, 54] (centrality, 5% for $p$, 0-10% for $\Omega$ and $d$). The fitting to the experimental data was shown in Fig. 2 and the extracted parameters were $R_0 = 12$ fm, $r_0 = 9$ fm/c, $\Delta \tau = 3.5$ fm/c, $T_{kin} = 111.6$ MeV, and $\rho_0 = 0.98$ for proton and 0.9 for $\Omega$ in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and $R_0 = 19.7$ fm, $r_0 = 15.5$ fm/c, $\Delta \tau = 1$ fm/c, $T_{kin} = 122$ MeV, and $\rho_0 = 1.2$ for proton and 1.07 for $\Omega$ in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.

III. RESULTS AND DISCUSSION

In blast-wave model [28, 29, 31, 32], the parameters of $T_{kin}$, $\rho_0$, $r_0$, $\Delta \tau$ and $R_0$ can be obtained by fitting experimental transverse momentum $p_T\sigma$ spectra of $p$ and $\Omega$ by using equation [3] and the parameters of $r_0$, $\Delta \tau$ and $R_0$ can be further fixed by fitting deuteron’s $p_T$ spectra from experiments by using equation [3]. In Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, the data were collected for $p$...
FIG. 2: Transverse momentum $p_T$ spectra of $p$, $d$, $\Omega$, $n\Omega$, $p\Omega$ and $\Omega\Omega$ in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV (a), and in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV (b), respectively. Lines: blast-wave model coupled with dynamical coalescence model (BLWC); Markers: data from the RHIC [49–51] and the ALICE [52–54]. (color online)

FIG. 3: Transverse momentum $p_T$ spectra of $p$, $d$, $\Omega$, $n\Omega$, $p\Omega$ and $\Omega\Omega$ in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV (a), and in Pb+Pb central collisions at $\sqrt{s_{NN}} = 2.76$ TeV (b), respectively. Open Marker: AMPT coupled with dynamical coalescence model (AMPTC); Solid Markers: data from the RHIC [49,51] and the ALICE [52,54]. (color online)

(a) and in Pb+Pb central collisions at $\sqrt{s_{NN}} = 2.76$ TeV (b), respectively. Based on the adjusted AMPT parameters and the developed coalescence mechanism for $\Omega$, the $p_T$ spectra of $p$ and $\Omega$ could be described well and the $p_T$ spectra of $p\Omega$, $n\Omega$ and $\Omega\Omega$ were similar to those via BLWC shown in Fig. 2. Yields of $dN/dy$ of $p\Omega$, $n\Omega$ and $\Omega\Omega$ were also listed in Table III and agreed with those from the BLWC.

In this work the particle interaction potential from the lattice QCD was taken into account and the overlap wave function was assumed to be the $S$–wave function calculated by solving Schrödinger equation. By using the phase-space information from the blast-wave model as well as the transport model (AMPT), the coalesced $p\Omega$, $n\Omega$, $\Omega\Omega$ gave the similar results and agreed with the previous predictions [10,28,55]. These consistent results implied that $N\Omega$ and $\Omega\Omega$ could be bounded in $S$–wave state and produced via coalescence mechanism at final stage in relativistic heavy ion collisions. And it also illustrated that the hyperon-hyperon ($Y Y$) and hyperon-nucleon ($Y N$) interactions from first principle calculation, such as the lattice QCD, could be examined by investigated the production of $\Omega$-dibaryons.

IV. SUMMARY

The overlap $S$–wave function and Wigner density function are calculated through solving Schrödinger equation with the $N–\Omega$ and $\Omega–\Omega$ potential from the lattice QCD which was recently published by the HAL QCD collaboration and the calculated binding energy is consistent with the published results. In the coalescence mechanism frame, the blast-wave model and AMPT model...
are employed to provide the phase-space information to coalesce $N\bar{N}$ and $\Omega\bar{\Omega}$ bound states by using the calculated Wigner density function, and the production rate of the bound states agree with other model predicted results. This dynamical coalescence calculation of the $\Omega$-dibaryons sheds light on the experiment searching for the (most)-strangeness dibaryon bound states at the STAR and the ALICE experiments which can help us to understand the $YY$ and $YN$ interactions.

**Acknowledgments:** This work was supported in part by the National Natural Science Foundation of China under contract Nos. 11875066, 11421505, 119809714, 11925502, 11961141003, National Key R&D Program of China under Grant No. 2016YFE0100900 and 2018YFE0104600, the Key Research Program of Frontier Sciences of the CAS under Grant No. QYZDJ-SSW-SLH002, and the Key Research Program of the CAS under Grant No. XDPB09.
[51] J. Adams et al. (STAR Collaboration), Phys. Rev. Lett. 98, 062301 (2007).
[52] B. Abelev et al. (ALICE Collaboration), Phys. Rev. C 88, 044910 (2013).
[53] J. Adam et al. (ALICE Collaboration), Phys. Rev. C 93, 024917 (2016).
[54] B. Abelev et al. (ALICE Collaboration), Physics Letters B 728, 216 (2014).
[55] N. Shah, Y. G. Ma, J. Chen, and S. Zhang, Physics Letters B 754, 6 (2016).
[56] F.-T. Wang and J. Xu, Phys. Rev. C 100, 064909 (2019).