Experimental demonstration of entanglement-enabled universal quantum cloning in a circuit

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No-cloning theorem forbids perfect cloning of an unknown quantum state. A universal quantum cloning machine (UQCM), capable of producing two copies of any input qubit with the optimal fidelity, is of fundamental interest and has applications in quantum information processing. This is enabled by delicately tailored nonclassical correlations between the input qubit and the copying qubits, which distinguish the UQCM from a classical counterpart, but whose experimental demonstrations are still lacking. We here implement the UQCM in a superconducting circuit and investigate these correlations. The measured entanglements well agree with our theoretical prediction that they are independent of the input state and thus constitute a universal quantum behavior of the UQCM that was not previously revealed. Another feature of our experiment is the realization of deterministic and individual cloning, in contrast to previously demonstrated UQCMs, which either were probabilistic or did not constitute true cloning of individual qubits.

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INTRODUCTION

An unknown quantum state cannot be cloned perfectly due to the linearity associated with the unitary transformation of quantum mechanics. This feature, discovered by Wooters and Zurek in 1982 and known as the no-cloning theorem, represents one of the fundamental differences between quantum information and classical information. In particular, it ensures the security of quantum cryptography schemes.

Because of the impossibility of perfect quantum cloning, much attention has been paid to the possibility of producing copies close to the original states. In the seminal paper by Buzek and Hillery, a universal quantum cloning machine (UQCM) was proposed, which produces two identical approximate copies via controllably entangling them with the original qubit. The output state of each of these two copy qubits has a fidelity of 5/6 to the input state, which is independent of the input state and was proven to be optimal. Besides fundamental interest, quantum cloning can be used to improve the performance of some quantum computational tasks, to distribute quantum information, and to realize minimal disturbance measurements. The UQCM has been reported in nuclear magnetic resonance systems, but the true cloning of individual quantum systems cannot be achieved due to the ensemble aspect. Huang et al. presented a proof-of-principle demonstration in an optical system, where only a single photon was involved; its polarization state was copied onto one path freedom degree.

Several optical experiments have been reported, where the state of a photon was copied onto another photon, but the cloning processes are probabilistic for lack of a deterministic two-qubit controlled gate between different photons in these experiments.

Besides the limitation of ensemble aspect or probabilistic nature, previous experiments did not reveal the nonclassical correlations between the original input qubit and the copying qubits. These correlations enable the information carried by the input state to be equally imprinted on the clones with the optimal fidelity and represent the most fundamental difference between the UQCM and a classical cloning machine. Quantitative characterization of these correlations is important for revealing the true quantum behavior of the UQCM, which is closely related to the universality and optimality of the copying operation.

We here adapt a scheme proposed in the context of cavity quantum electrodynamics (QED) to a superconducting circuit involving Xmon qubits controllably coupled to a bus resonator. The high degree of control over the qubit–qubit interactions enables realizations of all gate operations required for approximating the cloning of each qubit in a deterministic way. We indicate the universality of the implemented UQCM and quantitatively characterize the entanglement between the input qubit and each of the copy qubits. The results confirm our theoretical prediction that this entanglement is also input state independent and represents a universal quantum feature of the UQCM. The entanglement between the two copy qubits is also measured.

RESULTS

Implementation of UQCM

The sample used to perform the experiment involves five Xmon qubits, three of which are employed in our experiment and labeled from \( Q_1 \) to \( Q_3 \); these qubits are almost symmetrically coupled to a central bus resonator, as sketched in Fig. 1a. The resonator has a fixed frequency of \( \omega_c / 2\pi = 5.588 \text{ GHz} \), while the frequencies of the qubits are individually adjustable, which enables us to tailor the system dynamics to accomplish the
copying task. The Hamiltonian for the total system is
\[
H = \hbar \left[ \omega a^\dagger a + \sum_{i=1}^{3} \omega_{q_{i}} |1_{i}\rangle \langle 1_{i}| + \sum_{j=1}^{3} g_{j} (a_{j} S_{j}^{-} + a_{j}^{\dagger} S_{j}^{+}) \right].
\]

where \(a^\dagger\) and \(a\) are the photonic creation and annihilation operators for the resonator, respectively, \(S_{j}^{-} = |1_{i}\rangle \langle 0_{j}|\) and \(S_{j}^{+} = |0_{j}\rangle \langle 1_{i}|\) are the flip operators for \(q_{i}\), with \(|0_{j}\rangle\) and \(|1_{i}\rangle\) being its ground and first excited states separated by an energy gap \(\hbar \omega_{q_{i}}\). \(g_{j}\) are the corresponding qubit–resonator coupling strengths, and \(\hbar\) is the reduced Planck constant. In our sample, these coupling strengths are almost identical, e.g., \(g_{j} \approx g \approx 2n \times 20\) MHz. The system parameters are detailed in Supplementary Note 3. The qubit frequency tunability makes the system dynamics programmable.

When two or more qubits are detuned from the resonator by the same amount much larger than \(g\), they are coupled by virtual photon exchange. In our experiment, \(Q_{1}\) acts as the original qubit whose state is to be cloned, and \(Q_{2}\) and \(Q_{3}\) are used as the copying qubits.

The experimental sequence for realizing the UQCM with our setup is shown in Fig. 1b. The experiment starts with initializing the resonator to the vacuum state \(|0\rangle\) and the qubits to their ground state \(|0_{1}, 0_{2}, 0_{3}\rangle\) at their idle frequencies. These idle frequencies are highly detuned from the resonator frequency and off-resonant with each other, ensuring each qubit to be effectively decoupled from the resonator and other qubits when staying at its idle frequency. After the initialization, a suitable rotation is applied to \(Q_{1}\) to prepare it in the state to be cloned
\[
|\psi_{m}\rangle = a |0_{1}\rangle + \beta |1_{1}\rangle,
\]
where \(a\) and \(\beta\) are complex numbers, satisfying \(|a|^{2} + |\beta|^{2} = 1\).

Prior to the copying operation, we have to prepare \(Q_{2}\) and \(Q_{3}\) in the entangled state \(|\psi_{2,3}\rangle = (|0_{2}\rangle + |0_{3}\rangle)/\sqrt{2}\). To prepare this state, we first transform \(Q_{2}\) to the excited state \(|1_{2}\rangle\) by a \(\pi\) rotation \(X_{e}\), and then tune \(Q_{2}\) and \(Q_{3}\) to the working frequency \(\omega_{\text{res}}/2\pi = 5.44\) GHz. With this setting, the resonator will remain in the ground state during this process and can be discarded in the description of the system dynamics. In the interaction picture, the state evolution of the qubits is governed by the effective Hamiltonian
\[
H_{e} = -\lambda \sum_{j=1}^{3} S_{j}^{+} S_{j}^{-}, j \neq k.
\]

After the production of \(|\psi_{2,3}\rangle\), \(Q_{2}\) and \(Q_{3}\) are tuned on resonance with \(Q_{1}\) at the working frequency, where these qubits are red-detuned from the resonator by the same amount \(\Delta = 2n \times 148\) MHz. With this setting, the resonator does not exchange photons with the qubits due to the large detuning but can mediate a coupling of strength \(\lambda = \sqrt{d^{2}/\Delta}\) between any two of these qubits. The resonator will remain in the ground state during this process and can be discarded in the description of the system dynamics. In the interaction picture, the state evolution of the qubits is governed by the effective Hamiltonian
\[
H_{e} = -\lambda \sum_{j=1}^{3} S_{j}^{+} S_{j}^{-}, j \neq k.
\]

Under this Hamiltonian, \(Q_{2}\) and \(Q_{3}\) symmetrically interact with \(Q_{1}\) through excitation exchange, with the number of the total excitations being conserved. After an interaction time \(\tau = 2\pi n/9\lambda\), the three-qubit coupling \(C_{1,2,3}\) evolves \(Q_{1}\), \(Q_{2}\), and \(Q_{3}\) to the entangled state
\[
\alpha \left( \sqrt{\frac{2}{3}} |0_{1}\rangle |0_{2}\rangle |0_{3}\rangle + \sqrt{\frac{1}{3}} e^{-i\theta_{q}} |1_{1}\rangle |0_{2}\rangle |0_{3}\rangle \right) + \beta \left( \sqrt{\frac{2}{3}} |0_{1}\rangle |1_{2}\rangle |1_{3}\rangle + \sqrt{\frac{1}{3}} e^{-i\theta_{q}} |1_{1}\rangle |0_{2}\rangle |1_{3}\rangle \right).
\]

Then \(Q_{1}\) is tuned back to its idle frequency of 5.367 GHz and decoupled from \(Q_{2}\) and \(Q_{3}\), which remain at the working frequency and continue to interact with each other. The state components \(|0_{2}\rangle |0_{3}\rangle |1_{1}\rangle\) and \(|1_{2}\rangle |1_{3}\rangle |1_{1}\rangle\) are eigenstates of the two-qubit interaction Hamiltonian \(H_{e} = -\lambda \left( S_{2}^{+} S_{2}^{-} + S_{3}^{+} S_{3}^{-} \right)\) with the zero eigenvalue, while \(|\psi_{2,3}\rangle\) is an eigenstate of \(H_{e}^{'\prime}\) with the eigenvalue of \(-\lambda\). As a result, this swapping interaction does not affect \(|0_{2}\rangle |0_{3}\rangle |1_{1}\rangle\) and \(|1_{2}\rangle |1_{3}\rangle |1_{1}\rangle\) but produces a phase shift \(\lambda \tau \) to \(|\psi_{2,3}\rangle\), with \(\tau\) being the interaction time. With the choice \(\tau = \pi n/3\lambda\), the two-qubit coupling \(C_{2,3}\) cancels the phase factor \(e^{-i\theta_{q}}\) associated with \(|\psi_{2,3}\rangle\), evolving the three qubits to
\[
\alpha \left( \sqrt{\frac{2}{3}} e^{i\phi} |1_{1}\rangle |0_{2}\rangle |0_{3}\rangle + \sqrt{\frac{1}{3}} e^{-i\theta_{q}} |1_{1}\rangle |0_{2}\rangle |0_{3}\rangle \right) + \beta \left( \sqrt{\frac{2}{3}} |1_{1}\rangle |0_{2}\rangle |1_{3}\rangle + \sqrt{\frac{1}{3}} e^{-i\theta_{q}} |1_{1}\rangle |0_{2}\rangle |1_{3}\rangle \right),
\]
where the phase \(\phi\) is due to the frequency shift of \(Q_{1}\) during the \(Q_{2}–Q_{3}\) interaction, which does not affect the reduced density matrices for both \(Q_{2}\) and \(Q_{3}\), each of which in the basis \(|0_{i}\rangle, |1_{i}\rangle\) is
different input states \{01, 10\}}. We characterize the performance of the UQCM by preparing their idle frequencies of 5.223 and 5.311 GHz, respectively. States according to the qubits performed to correct the measured probabilities for the qubit tomography in our experiment, the readout calibration is tomography (see Supplementary Note 7). Note that, for the corresponding output states of \( \rho \), the optimal value \( 5/6 \), corresponding output clone. Each of these corresponding matrix elements of the output states yielded by the perfect UQCM.

\[
(\frac{3}{8}|a|^2 + \frac{3}{8}|b|^2 \pm \frac{3}{8}ab^*),
\]

For the perfect UQCM, the fidelity of these two output copiers with respect to the input state \( |\psi_i\rangle \) is 5/6, irrespective of the probability amplitudes \( a \) and \( b \) associated with the components \( |0\rangle \) and \( |1\rangle \). Due to the nonuniform qubit–resonator couplings and the existence of the direct but also nonuniform qubit–qubit couplings in our device\(^{28–30} \), each qubit is asymmetrically coupled to the other two qubits with the effective coupling strengths slightly different from \( \lambda \). In order to produce the optimal outcome, the coupling operations \( C_{123} \) and \( C_{23} \) are calibrated simultaneously, and consequently, the optimal coupling times \( \tau \) and \( \tau' \) that deviate from the values for the ideal case are 40.8 and 69.5 ns, respectively. After the copy process, \( Q_2 \) and \( Q_3 \) are tuned back to their idle frequencies of 5.223 and 5.311 GHz, respectively.

Characterization of performance

We characterize the performance of the UQCM by preparing different input states \( \{|0\rangle, (|0\rangle + i|1\rangle)/\sqrt{2}, (|0\rangle - i|1\rangle)/\sqrt{2}, (|0\rangle + i|1\rangle)/\sqrt{2}, (|0\rangle - i|1\rangle)/\sqrt{2}, |1\rangle \} \) and measuring the corresponding output states of \( Q_2 \) and \( Q_3 \) through quantum state tomography (see Supplementary Note 7). Note that, for the tomography in our experiment, the readout calibration is performed to correct the measured probabilities for the qubit states according to the qubits’ \( |0\rangle \) and \( |1\rangle \) state measurement fidelities (see Supplementary Table 1 and Note 6). The measured density matrices for the clones of the above-mentioned six input states are respectively displayed in Fig. 2a–f, where the upper and lower panels denote the measured output density matrices of \( Q_2 \) and \( Q_3 \), respectively. The fidelities of the output states of \( Q_2 \) \((Q_3)\) to these six ideal input states, defined as \( F = \langle \phi_i | P_{\text{out}} | \phi_i \rangle \), are, respectively, 0.784 ± 0.002, 0.784 ± 0.002, 0.784 ± 0.002, 0.784 ± 0.002, 0.784 ± 0.002, 0.784 ± 0.002, 0.784 ± 0.002, 0.784 ± 0.002, 0.784 ± 0.002, 0.784 ± 0.002, 0.784 ± 0.002, 0.784 ± 0.002, where \( P_{\text{out}} \) denotes the measured density matrix for the corresponding output clone. Each of these fidelities is close to the optimal value 5/6, confirming that the performance of the UQCM is independent of the input state. The slight difference between the output states of the two copy qubits is mainly due to direct qubit–qubit couplings. These nonuniform couplings also make the qualities of the output states slightly depend on the input state. We note that, for each of the six input states, the output state of \( Q_3 \) has a fidelity very close to the theoretical upper bound. This is partly due to the asymmetry between the two clones. The other reason is that the qubit–qubit couplings during the copy process partly protect the qubits from dephasing, so that the real \( T_2 \) times of the qubits coupled at the working frequency are longer than the corresponding results listed in Supplementary Table 1, which are measured without qubit–qubit couplings\(^{27,28} \).

To further examine the performance of the UQCM, we perform the quantum process tomography (see Supplementary Note 7), achieved by preparing the above-mentioned six distinct input states, and measuring them and the corresponding output states of \( Q_2 \) and \( Q_3 \) through quantum state tomography. The measured process matrices associated with the output states of \( Q_2 \) and \( Q_3 \), \( \chi_{\text{meas},2} \) and \( \chi_{\text{meas},3} \), are respectively measured in Fig. 3a, b, respectively. The fidelities of \( \chi_{\text{meas},2} \) and \( \chi_{\text{meas},3} \) with respect to the ideal cloning process \( \chi_{\text{id}} \), defined as \( F = \text{Tr}(\chi_{\text{meas}}|\chi_{\text{id}}) \), are 0.679 ± 0.001 and 0.743 ± 0.002, respectively. These process fidelities are close to the result of the perfect UQCM, 0.75, demonstrating a good quantum control over the multiqubit–resonator system.

Demonstration of universal entanglement behavior

The nonclassical correlations between the original input qubit and the clones play an essential role in implementation of the UQCM and represents one of the most fundamental differences between universal quantum and classical cloning but have not been quantitatively investigated. Characterization of these correlations is important for understanding the quantum behavior of the UQCM. We find that the degree of the entanglement between each output clone and the original input qubit, quantified by concurrence\(^{31} \), is 2/3 for an ideal UQCM, which is independent of the input state (see Supplementary Note 1). To detect these nonclassical correlations, we respectively measure the joint \( Q_1 – Q_2 \) and \( Q_1 – Q_3 \) output density matrices. The results for the six input states \( \{|0\rangle, (|0\rangle + i|1\rangle)/\sqrt{2}, (|0\rangle - i|1\rangle)/\sqrt{2}, (|0\rangle + i|1\rangle)/\sqrt{2}, (|0\rangle - i|1\rangle)/\sqrt{2}, |1\rangle \} \) are displayed in Fig. 4a–f, where the upper
are deterministically realized. We characterize the performance of quantum operations necessary for constructing a UQCM network. We have demonstrated universal cloning of an arbitrary state of a genuine three-particle entangled state, revealing the fundamental entanglement. The output state tomography. The reconstructed joint density matrices produced by the perfect UQCM.

**DISCUSSION**

We have demonstrated universal cloning of an arbitrary state of an individual qubit with a circuit QED set-up, where all the quantum operations necessary for constructing a UQCM network are deterministically realized. We characterize the performance of the UQCM by quantum state tomography, confirming the universality of the copying process. We measure the entanglement between each copy qubit and the original qubit, with the results being in well agreement with the theoretical prediction that this entanglement is input state independent and represents a universal quantum behavior of the UQCM. We further measure the entanglement between the two clones, verifying the existence of true three-particle entanglement at the output. These results underline the fact that the universal entanglement behavior underlies the performance of the UQCM.

**DATA AVAILABILITY**

All data needed to evaluate the conclusions in the paper are present in the paper and/or the Supplementary Materials. Additional data related to this paper may be requested from the authors.

**CODE AVAILABILITY**

All codes used in the paper are available from the corresponding authors upon reasonable request.

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**AUTHOR CONTRIBUTIONS**

S.-B.Z. conceived the experiment. Z.-B.Y., P.-R.H., X.-J.H. and K.X. performed the experiment and analyzed the data with the assistance of W.N.H. and D.Z. provided the devices used for the experiment. S.-B.Z., Z.-B.Y., K.X., and H.F. wrote the manuscript with feedbacks from all the authors.

**COMPETING INTERESTS**

The authors declare no competing interests.

**ADDITIONAL INFORMATION**

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