Threshold resummation in rapidity for colorless particle production at LHC

Goutam Das
Deutsches Elektronen-Synchroton

Loops and Legs in Quantum Field Theory
St. Goar, May 1, 2018

with V. Ravindran, P. Banerjee, P. K. Dhani

Based on arXiv: 1708.05706 and 1805.XXXX (Preprint: DESY 18-067)
Prologue: Higgs & Drell-Yan Production

Higgs production:

• Precise prediction is needed to understand the Higgs properties.

• Higgs production is important in the context of BSM search.

DY production:

• Extraction of parton distribution functions (PDFs).

• Important tool to discovery of new physics (BSM).

• Important for electro-weak precision measurement.
  Measurement of W mass and properties.
Inclusive Higgs production in gluon fusion is known upto NNNLO.

Inclusive production of Drell-Yan known upto NNNLO SV.

Rapidity distribution@NNLO

Going beyond 2-loop is extremely challenging!

Soft+Virtual correction to rapidity distribution

Resummation of soft-gluons improves the fixed order result.
Prologue: Resummation of Rapidity

- **Catani and Trentadue approach** for $x_F$ distribution:
  - Threshold limit is taken by $z_1 \to 1$ and $z_2 \to 1$ simultaneously
  - Resums Delta and Distributions in both $z_1$ and $z_2$.

  ▶ Resummation for lepton-pair at NLL.  
  Westmark & Owens (’17)

- **Laenen and Sterman approach** for rapidity:
  - Threshold limit is taken by $z \to 1$.
  - Resums Delta and Distributions in $z$,
    only Delta piece in partonic $y$.

  ▶ Resummation for $W^\pm$ production.  
  Mukherjee & Vogelsang (’06)

  ▶ Resummation for DY production.  
  Bolzoni (’06), Bonvini, Forte & Ridolfi (’12)
Plan

- Formalism for the differential soft-virtual cross-section.
  - Form factor
  - Soft function
- Soft gluon resummation
- Numerical analysis: Higgs and DY
- Summary
Rapidity Distribution

- Rapidity distribution for colorless particle:

\[
\frac{d\sigma^I}{dy} = \hat{\sigma}_B^I \sum_{ab=q,\bar{q},g} \int_{x_1^0}^{1} \frac{dz_1}{z_1} \int_{x_2^0}^{1} \frac{dz_2}{z_2} \hat{H}_{ab}^I \left( \frac{x_1^0}{z_1}, \frac{x_2^0}{z_2}, \mu^2 \right) \hat{\Delta}_{d,ab}^I (z_1, z_2, q^2, \mu^2)
\]

Drell-Yan production:

\[
\sigma^I = \frac{d\sigma^q (\tau, q^2, y)}{dq^2}.
\]

Higgs production (gluon/bottom):

\[
\sigma^I = \sigma^{g(b)} (\tau, q^2, y).
\]

- Hadronic rapidity

\[
y = \frac{1}{2} \ln \left( \frac{p_2 \cdot q}{p_1 \cdot q} \right) = \frac{1}{2} \ln \left( \frac{x_1^0}{x_2^0} \right) \quad \text{and} \quad \tau = \frac{q^2}{S} = x_1^0 x_2^0
\]

- Partonic scaling variable

\[
z_i = \frac{x_i^0}{x_i}
\]
• Perturbative expansion of coefficient function:

\[
\Delta^I_d = \delta(1 - z_1)\delta(1 - z_2) + a_s \left\{ c_1^{(1)} \delta(1 - z_1)\delta(1 - z_2) + c_2^{(1)} \left( \frac{\ln(1 - z_1)}{1 - z_1} \right) + R_1(z_1, z_2) + z_1 \leftrightarrow z_2 \right\} + O(a_s^2)
\]

Virtual, Soft radiation

Soft Radiation

Regular terms (Virtual + Real)
Soft-Virtual contribution

• Expansion around $z_j \rightarrow 1$

$$\Delta I(z_i) = \Delta I,sing(z_i) + \Delta I,hard(z_i)$$

$$\Delta I,sing(z_j) \equiv \Delta SV(z_j) = \Delta SV \delta(1-z_j) + \sum_{i=0}^{2n-1} \Delta_i SV D_i$$

$$D_i = \left( \frac{\ln^i(1-z_j)}{1-z_j} \right)$$

• Significant contributions from singular terms when $z_j \rightarrow 1$
Soft-Virtual Exponentiation

- Form-Factor square normalised SV cross-section

\[ \hat{\sigma}_{d}^{SV}(z_i, q^2) = Z^I(\hat{\alpha}_s, \mu_R^2, \mu_F^2, \epsilon)^2 |\hat{F}^I(\hat{\alpha}_s, Q^2, \mu^2)|^2 \exp(2 \Phi_d^I(\hat{\alpha}_s, q^2, \mu^2, z_i)) \]

with \( C_{\epsilon f(z_i)} = \delta(1 - z_i) + \frac{f(z_i)}{1!} + \frac{1}{2!} f(z_i) \otimes f(z_i) + \cdots \)

- Mass factorisation:

\[ \hat{\sigma}_{d,I}^{SV}(z_i, q^2, \mu_F^2) = \Gamma_{II}^{T}(z_i, \mu_F^2) \otimes \Delta_{d,I}^{SV}(z_i, \mu_F^2) \otimes \Gamma_{II}(z_i, \mu_F^2) \]
SV Cross-section

- Finite Soft+Virtual cross-section:

\[
\Delta^{SV}_{d,l} = C \exp \left( \Psi^I_d (q^2, \mu_R^2, \mu_F^2, \bar{z}_1, \bar{z}_2, \epsilon) \right) \bigg|_{\epsilon = 0}
\]

with

\[
\Psi^I_d = ( \ln(Z^I(\hat{a}_s, \mu_R^2, \mu^2, \epsilon))^2 \\
+ \ln |\hat{F}^I(\hat{a}_s, Q^2, \mu^2, \epsilon)|^2 \delta(\bar{z}_1) \delta(\bar{z}_2) \\
+ 2\Phi^I_d(\hat{a}_s, q^2, \mu^2, \bar{z}_1, \bar{z}_2, \epsilon) \\
- C( \ln \Gamma_{II}(\hat{a}_s, \mu^2, \mu_F^2, \bar{z}_1, \epsilon) \delta(\bar{z}_2) + (\bar{z}_1 \leftrightarrow \bar{z}_2) )
\]

- Total sum has to be **UV and IR finite**!
Form Factor

- Onshell Matrix element of composite operators.

\[
\langle g(p') | G^a_{\mu\nu} G^{\mu\nu,a}(p) | g(p) \rangle
\]

Gluon Form factor

\[
\langle q(p') | \bar{\psi} \gamma^\mu \psi | q(p) \rangle
\]

Quark Form factor

Higgs production

DY production
Form Factor

• Form-factor satisfies K-G equation:

\[
\frac{d \ln \hat{F}^I(\hat{a}_s, Q^2, \mu^2, \epsilon)}{d \ln Q^2} = \frac{1}{2} \left[ K^I(\hat{a}_s, \epsilon, \mu^2_R, \mu^2) + G^I(\hat{a}_s, \epsilon, Q^2, \mu^2_R, \mu^2) \right]
\]

- Poles in $\epsilon$ Q-independent
- Finite in $\epsilon \to 0$ Q-dependent

• RG invariance of the Form-factor:

\[
\mu^2_R \frac{d}{d \mu^2_R} K^I \left( \frac{\hat{a}_s}{\mu^2}, \frac{\mu^2_R}{\mu^2}, \epsilon \right) = -\mu^2_R \frac{d}{d \mu^2_R} G^I \left( \frac{\hat{a}_s}{\mu^2}, \frac{Q^2}{\mu^2_R}, \frac{\mu^2_R}{\mu^2}, \epsilon \right) = -A^I(\hat{a}_s(\mu^2_R))
\]

• Casimir duality

\[
\frac{A_q}{A_g} = \frac{C_F}{C_A}
\]

Valid upto 3-loops

[Sen, Mueller, Collins, Magnea]
Form Factor

- Solve $K^I, G^I$: Expand in powers in $a_s(\mu_R^2)$

\[
K^I(\hat{a}_s, \mu_R^2, \mu^2, \epsilon) = \hat{a}_s \left( \frac{\mu_R^2}{\mu^2} \right)^{\epsilon/2} S_\epsilon \left( -\frac{2A^I_1}{\epsilon} \right) + \hat{a}_s^2 \left( \frac{\mu_R^2}{\mu^2} \right)^{\epsilon} S_\epsilon^2 \left( \frac{2\beta_0 A^I_1}{\epsilon^2} - \frac{A^I_2}{\epsilon} \right) \\
+ \hat{a}_s^3 \left( \frac{\mu_R^2}{\mu^2} \right)^{3\epsilon/2} S_\epsilon^3 \left( -\frac{8\beta_0^2 A^I_1}{3\epsilon^3} + \frac{2\beta_1 A^I_1 + 8\beta_0 A^I_2}{3\epsilon^2} - \frac{2A^I_3}{3\epsilon} \right)
\]

Found from lower order (RG)

\[
G^I(\hat{a}_s, Q^2, \mu_R^2, \mu^2, \epsilon) = \sum_{i} a_s^i(Q^2)G^I_i(\epsilon) + \sum_{i=1}^{\infty} \hat{a}_s^i \left( \frac{\mu_R^2}{\mu^2} \right)^{i\epsilon/2} \left[ \left( \frac{Q^2}{\mu_R^2} \right)^{i\epsilon/2} - 1 \right] S_\epsilon^i K^{I,(i)}(\epsilon)
\]

New at this order

Soft Ano. Dim.

\[
G_1^I = 2(B_1^I - \delta_{I,g} \beta_0) + f_1^I + \sum_{j=1}^{\infty} \epsilon^j g_1^{I,j}
\]

\[
G_2^I = 2(B_2^I - 2\delta_{I,g} \beta_1) + f_2^I - 2\beta_0 g_1^{I,1} + \sum_{j=1}^{\infty} \epsilon^j g_2^{I,j}
\]

Coll. Ano. Dim.  UV Ano. Dim.  From $Z(a_s(\mu_R^2))$  Explicit computation

\[
\frac{f_q}{f_g} = \frac{C_F}{C_A}
\]

upto 3-loop
Form Factor

- Solution upto 3-loops:

\[
\ln \hat{F}^I(\hat{a}_s, Q^2, \mu^2, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left( \frac{Q^2}{\mu^2} \right)^{i \frac{\epsilon}{2}} S^i \hat{L}^I_{F,i}(\epsilon)
\]

- All terms can be found, only computation of finite piece is needed.
SV Cross-section

- Soft+Virtual cross-section:

\[ \Delta_{d,l}^{SV} = C \exp \left( \Psi_d^I \left( q^2, \mu_R^2, \mu_F^2, \bar{z}_1, \bar{z}_2, \epsilon \right) \right) \bigg|_{\epsilon=0} \]

with

\[ \Psi_d^I = \left( \ln \left( Z_d^I (\hat{\alpha}_s, \mu_R^2, \mu^2, \epsilon) \right)^2 \right) + \ln \left| \hat{F}_d^I (\hat{\alpha}_s, Q^2, \mu^2, \epsilon) \right|^2 \delta(\bar{z}_1) \delta(\bar{z}_2) + 2\Phi_d^I (\hat{\alpha}_s, q^2, \mu^2, \bar{z}_1, \bar{z}_2, \epsilon) \]

- Form factor
- Overall operator renormalisation
- Soft distribution function
- DGLAP kernel
Collinear RGE

- RGE for DGLAP kernel $\Gamma$:

$$\mu_F^2 \frac{d}{d\mu_F^2} \Gamma(z_i, \mu_F^2, \epsilon) = \frac{1}{2} P(z_i, \mu_F^2) \otimes \Gamma(z_i, \mu_F^2, \epsilon)$$

- Altarelli-Parisi Splitting functions

- In SV cross-section, diagonal part contributes

$$P_{II}^i(z_j) = 2 \left[ B_{i+1}^I \delta(1 - z_j) + A_{i+1}^I D_0(z_j) \right] + P_{\text{reg,II}}^i(z_j)$$

$\Gamma$ is known upto $\mathcal{O}(a_s^4)$

[Moch, Vogt, Vermaseren]
• RGE for overall operator renormalisation constant $Z$:

$$\mu^2_R \frac{d}{d\mu^2_R} \ln Z^I(\hat{a}_s, \mu^2_R, \mu^2, \epsilon) = \sum_{i=1}^{\infty} a^i_s(\mu^2_R) \gamma^I_{i-1}$$

• $\gamma^g, \gamma^b$ are known upto $\mathcal{O}(a_s^3)$

• For Drell-Yan: $\gamma^q = 0$ \Rightarrow $Z^q(\hat{a}_s, \mu^2_R, \mu^2, \epsilon) = 1$
SV Cross-section

- Soft+Virtual cross-section:

\[ \Delta_{d,I}^{SV} = C \exp \left( \Psi_d^I(q^2, \mu_R^2, \mu_F^2, \bar{z}_1, \bar{z}_2, \epsilon) \right)_{\epsilon=0} \]

\[ \Psi_d^I = (\ln(Z_d^I(\hat{a}_s, \mu_R^2, \mu_F^2, \epsilon))^2 + \ln|\hat{F}_d^I(\hat{a}_s, Q^2, \mu_F^2, \epsilon)|^2 \delta(\bar{z}_1) \delta(\bar{z}_2) + 2\Phi_d^I(\hat{a}_s, q^2, \mu_F^2, \bar{z}_1, \bar{z}_2, \epsilon) - C(\ln \Gamma_d^I(\hat{a}_s, \mu_F^2, \mu_R^2, \bar{z}_1, \epsilon) \delta(\bar{z}_2) + (\bar{z}_1 \leftrightarrow \bar{z}_2)) \]

Form factor

DGLAP kernel

Overall operator renormalisation

Soft distribution function

Threshold resummation
Soft Function

- **RGE** for the Soft function:

\[
\mu_R^2 \frac{d}{d \mu_R^2} \Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \epsilon) = 0
\]

- Demand finiteness in \( \lim_{\epsilon \to 0} \Psi_d^I(z_i, q^2, \epsilon) \)

K-G type equation for Soft function

\[
q^2 \frac{d}{dq^2} \Phi_d^I = \frac{1}{2} \left[ K_d^I \left( \hat{a}_s, \frac{\mu_R^2}{\mu^2}, z_1, z_2, \epsilon \right) + G_d^I \left( \hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z_1, z_2, \epsilon \right) \right]
\]

Poles in \( \epsilon \)

Finite in \( \epsilon \to 0 \)

[Ravindran ('06, '07)]
Soft Function

• One possible ansatz:

\[
\Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left( \frac{q^2 (1 - z_1) (1 - z_2)}{\mu^2} \right)^{i \frac{\epsilon}{2}} S^i_\epsilon \left( \frac{(i\epsilon)^2}{4(1 - z_1)(1 - z_2)} \right) \hat{\phi}^I_d(i) (\epsilon),
\]

[Ravindran, Smith, van Neerven]

• Solution:

\[
\hat{\phi}^I_d(i) (\epsilon) = \mathcal{L}_F^I(i) (\epsilon) \left( A^I \to -A^I, G^I (\epsilon) \to \overline{G}_d^I (\epsilon) \right)
\]

\[
\overline{G}^I_{d,1}(\epsilon) = -f_1^I + \sum_{k=1}^{\infty} \epsilon^k \overline{G}^I_{d,1,k}
\]

\[
\overline{G}^I_{d,2}(\epsilon) = -f_2^I - 2\beta_0 \overline{G}^I_{d,1} + \sum_{k=1}^{\infty} \epsilon^k \overline{G}^I_{d,2,k}
\]

\[
\overline{G}^I_{d,3}(\epsilon) = -f_3^I - 2\beta_1 \overline{G}^I_{d,1} - 2\beta_0 \left( \overline{G}^I_{d,2} + 2\beta_0 \overline{G}^I_{d,1} \right) + \sum_{k=1}^{\infty} \epsilon^k \overline{G}^I_{d,3,k}
\]

Explicitly compute
• Find differential soft function from inclusive one:

\[ \int dy \frac{d\sigma^I}{dy}(y, \tau) = \sigma^I(\tau) \]

Easy to work in Mellin space

\[ \int_0^1 dx_1^0 \int_0^1 dx_2^0 (x_1^0 x_2^0)^{N-1} \frac{d\sigma^I}{dy} = \int_0^1 d\tau \, \tau^{N-1} \sigma^I \]

Need Inclusive soft function !!

\[ \text{Inclusive} \quad \Phi^I(q^2, z, \epsilon) \]

\[ \text{Rapidity} \quad \Phi^I_d(q^2, z_1, z_2, \epsilon) \]

\[ \frac{\bar{G}^q_{d,i}}{\bar{G}^{g,k}_{d,i}} = \frac{C_F}{C_A} \quad \Rightarrow \quad \frac{\phi^q_d}{\phi^g_d} = \frac{C_F}{C_A} \quad \text{upto } O(\alpha_s^3) \]

Soft distributions are universal

[Smith, van Neerven, Ahmed, Mandal, Rana, Ravindran]
• Soft-virtual cross-section:

\[ \Delta_{d,l}^{SV} = C \exp \left( \Psi_d^I (q^2, \mu_R^2, \mu_F^2, \bar{z}_1, \bar{z}_2, \epsilon) \right) \bigg|_{\epsilon=0} \]

\[ \Psi_d^I = \left( \ln \left( Z^I (\hat{a}_s, \mu^2_R, \mu^2, \epsilon) \right) \right)^2 \]

\[ + \ln |\hat{F}_d^I (\hat{a}_s, Q^2, \mu^2, \epsilon)|^2 \delta(\bar{z}_1) \delta(\bar{z}_2) \]

\[ + 2 \Phi_d^I (\hat{a}_s, q^2, \mu^2, \bar{z}_1, \bar{z}_2, \epsilon) \]

\[ - C \left( \ln \Gamma_{II} (\hat{a}_s, \mu^2, \mu_F^2, \bar{z}_1, \epsilon) \delta(\bar{z}_2) + (\bar{z}_1 \leftrightarrow \bar{z}_2) \right) \]
Soft-gluon resummation

\[ \Delta_{d,l}^{SV} = C \exp \left( \Psi_d^I(q^2, \mu_R^2, \mu_F^2, \bar{z}_1, \bar{z}_2, \epsilon) \right) \bigg|_{\epsilon=0} \]

\[ \Psi_d^I = \delta(\bar{z}_2) \left( \frac{1}{\bar{z}_1} \left\{ \int_{\mu_F^2}^{q^2 \bar{z}_1} \frac{d\lambda^2}{\lambda^2} A_I(a_s(\lambda^2)) + D_d^I(a_s(q^2 \bar{z}_1)) \right\} + \frac{1}{2} \left( \frac{1}{\bar{z}_1 \bar{z}_2} \left\{ A_I(a_s(\bar{z}_{12})) + \frac{dD_d^I(a_s(\bar{z}_{12}))}{d \ln \bar{z}_{12}} \right\} \right) + \frac{1}{2} \delta(\bar{z}_1) \delta(\bar{z}_2) \ln \left( g_{d,0}^I(a_s(\mu_F^2)) \right) \bigg) + \bar{z}_1 \leftrightarrow \bar{z}_2 \]

Related to \( \tilde{G}_d^I \)

\[ a_s(\bar{z}_{12}) = a_s(q^2 \bar{z}_1 \bar{z}_2) \]
Soft-gluon resummation

- $\Delta_{d}^{I,SV}$
  - Distributions $\left(\frac{\ln(1-z_i)}{(1-z_i)}\right)$ and Delta $\delta(1-z_i)$

- Threshold limits: $z_1 \rightarrow 1, z_2 \rightarrow 1$

- Sum all threshold large logs to all orders in perturbation theory

**Threshold resummation**
- **Sudakov type exponentiation** of soft-function.

- Resummation in conjugate space! **Amplitude factorises** both in z-space and N-space. Phase space factorises only in N-space.

- Logarithmic enhanced contribution in N-space also **contains subleading terms** when transformed into z-space.
Soft-gluon resummation

- Double Mellin transformation:

\[
\tilde{\Delta}_{d}^{I,SV}(\omega) = \int_{0}^{1} dz_1 z_1^{N_1-1} \int_{0}^{1} dz_2 z_2^{N_2-1} \Delta_{d}^{I,SV}(z_1, z_2)
\]

\[
\omega = a_s \beta_0 \ln \left( \frac{1}{N_1 N_2} \right)
\]

\[
\bar{N}_i = e^{\gamma_E} N_i
\]

- SV limits:

\[
\delta(1 - z_i) \rightarrow 1
\]

\[
\frac{\ln^i (1 - z_j)}{1 - z_j} \rightarrow \ln^{i+1} \bar{N}_j
\]

\[
z_1 \rightarrow 1 \quad \bar{N}_1 \rightarrow \infty
\]

\[
z_2 \rightarrow 1 \quad \bar{N}_2 \rightarrow \infty
\]

- Resummed rapidity distribution:

\[
\tilde{\Delta}_{d}^{SV,I}(\omega) = \tilde{g}_{d,0}^{I}(a_s) \exp \left( g_{d}^{I}(a_s, \omega) \right)
\]

\[
\bar{N}_i \text{ independent}
\]

Logarithmically enhanced
For any colorless particle,

\[
\bar{g}_{d,1}^I = \frac{1}{\omega} \left( \omega + (1 - \omega) \ln(1 - \omega) \right),
\]

\[
\bar{g}_{d,2}^I = \omega \left( \bar{A}_{1}^I \bar{\beta}_1 - \bar{A}_{2}^I \right) + \ln(1 - \omega) \left( \bar{A}_{1}^I \bar{\beta}_1 + \bar{D}_{d,1}^I - \bar{A}_{2}^I \right) + \frac{1}{2} \ln^2(1 - \omega) \bar{A}_{1}^I \bar{\beta}_1 + L_{qr} \ln(1 - \omega) \bar{A}_{1}^I + L_{fr} \omega \bar{A}_{1}^I,
\]

\[
\bar{g}_{d,3}^I = -\frac{\omega}{2} A_{3}^I - \frac{\omega}{2(1 - \omega)} \left( -A_{3}^I + (2 + \omega) \bar{A}_{1}^I A_{2}^I + \left( (\omega - 2) \bar{\beta}_2 - \omega \bar{\beta}_1^2 - 2 \zeta_2 \right) A_{1}^I + 2 \bar{D}_{d,2}^I - 2 \bar{\beta}_1 \bar{D}_{d,1}^I \right)
\]

\[
- \ln(1 - \omega) \left( \frac{\bar{\beta}_1}{(1 - \omega)} \left( A_{2}^I - \bar{D}_{d,1}^I - \frac{A_{1}^I \bar{\beta}_1}{\omega} \right) - \frac{\bar{A}_{1}^I \beta_2}{(1 - \omega)} \right) + \frac{\ln^2(1 - \omega) A_{1}^I \beta_2}{2(1 - \omega)} + L_{fr} A_{2}^I \omega - \frac{1}{2} L_{fr}^2 A_{1}^I \omega
\]

\[
-L_{qr} \frac{1}{(1 - \omega)} \left( \left( A_{2}^I - \bar{D}_{d,1}^I \right) \omega - \bar{A}_{1}^I \beta_1 (\omega + \ln(1 - \omega)) \right) + \frac{1}{2} L_{qr}^2 \frac{\omega}{(1 - \omega) A_{1}^I}.
\]

where \( \bar{g}_{d,1}^I = g_{d,1}^I, \bar{g}_{d,i+2}^I = g_{d,i+2}^I / \beta_0^i, \bar{A}_{i}^I = A_{i}^I / \beta_0^i, \bar{D}_{d,i}^I = D_{d,i}^I / \beta_0^i, \)

\( \bar{\beta}_i = \beta_i / \beta_0^{i+1}, L_{fr} = \ln \left( \mu_F^2 / \mu_R^2 \right), L_{qr} = \ln \left( q^2 / \mu_R^2 \right). \)

Also,

\[
\ln(g_{d,0}^I) = \sum_{i=0}^{\infty} \alpha_s^i I_{g_0}^I,i,
\]
Soft-gluon resummation

- **Resummed coefficient:**

\[
g_d^I(a_s, \omega) = g_{d,1}^I(\omega) \ln(\bar{N}_1 \bar{N}_2) + \sum_{i=0}^{\infty} a_s^i g_{d,i+2}^I(\omega)
\]

| Order \(O(a_s)\) | Resummed Logs | Resummed Coeff. |
|-------------------|---------------|----------------|
| \(O(a_s)\)       | \(\ln^2(\bar{N}_1 \bar{N}_2)\) | \(g_{d,1}^I \ln(\bar{N}_1 \bar{N}_2)\) |
| \(O(a_s^2)\)     | \(\ln^3(\bar{N}_1 \bar{N}_2)\) | \(a_s^m \ln^{m+1}(\bar{N}_1 \bar{N}_2)\) |
| \(O(a_s^3)\)     | \(\ln^4(\bar{N}_1 \bar{N}_2)\) | \(a_s^m \ln^m(\bar{N}_1 \bar{N}_2)\) |
| \(\vdots\)       | \(\vdots\)   | \(\vdots\)    |
| \(\text{LL}\)    | \(\text{NLL}\) | \(\text{NNLL}\) |

\(a_s \ln a_s \ln(\bar{N}_1 \bar{N}_2)\)
Matching: DY rapidity

- Matching Fixed order (NNLO) with resum (NNLL) for DY:

\[
\frac{d^2 \sigma_{q,\text{res}}}{dq^2 dy} = \frac{d^2 \sigma_{q,f.o}}{dq^2 dy} + \sigma_B^q \int_{c_1-i\infty}^{c_1+i\infty} \frac{dN_1}{2\pi i} \int_{c_2-i\infty}^{c_2+i\infty} \frac{dN_2}{2\pi i} e^{y(N_2-N_1)} (\sqrt{T})^{2-N_1-N_2} \tilde{f}_q(N_1) \tilde{f}_q(N_2) \left( \tilde{\Delta}_{d,q}^SV - \tilde{\Delta}_{d,q}^SV \right)_{\text{trunc}}.
\]

- Inverse Mellin transformation by Minimal Prescription scheme.
  
  \[
  \tilde{\Delta}_{d,q}^SV - \tilde{\Delta}_{d,q}^SV \bigg|_{\text{trunc}} = \tilde{\Delta}_{d,q}^SV \bigg| \geq \mathcal{O}(a_s^{n+1})
  \]

- For NNLO+NNLL prediction:

\[
\frac{d^2 \sigma_{q,f.o}}{dq^2 dy} : \text{Upto NNLO} \quad \tilde{\Delta}_{d,q}^SV : \text{Upto NNLL}
\]
\[ \sqrt{S} = 13 \, \text{TeV} \]

\[ M_H = 125 \, \text{GeV} \]

\[ m_t = 173 \, \text{GeV} \]

\[ n_f = 5 \]

\[ \text{PDF} = \text{MMHT 2014} \]

\[ \frac{d\sigma^{g,f_0}}{dy} \text{ from FEHIP} \]

[Anastasiou, Melnikov, Petriello (’05)]
Higgs rapidity

- Better perturbative convergence.
- Magnitude and sign of resummed contribution varies depending on the order, $y$ and scale.
  
  Banerjee, GD, Dhani, Ravindran ('17)

- Resummed prediction is stable with choice of central scale.
• **F.O. and resummed results** for few benchmark values of \( y \)

| \( y \) | LO       | LO + LL  | NLO      | NLO + NLL | NNLO     | NNLO + NNLL | NNLO + NNNLL |
|-------|----------|----------|----------|-----------|----------|-------------|--------------|
| 0.0   | 4.435 ± 1.145 | 6.231 ± 1.950 | 8.255 ± 1.684 | 9.632 ± 2.286 | 10.329 ± 1.088 | 10.938 ± 1.050 | 10.517 ± 0.820 |
| 0.8   | 4.134 ± 1.067 | 5.833 ± 1.831 | 7.517 ± 1.530 | 8.820 ± 2.124 | 9.407 ± 0.988 | 9.992 ± 1.025 | 9.641 ± 0.718  |
| 1.6   | 3.189 ± 0.819 | 4.630 ± 1.468 | 5.522 ± 1.117 | 6.611 ± 1.676 | 6.877 ± 0.744 | 7.380 ± 0.849 | 7.045 ± 0.563  |
| 2.4   | 1.904 ± 0.492 | 2.887 ± 0.942 | 2.985 ± 0.597 | 3.715 ± 0.998 | 3.683 ± 0.410 | 4.040 ± 0.501 | 3.821 ± 0.305  |

• **Corrections from LL** varies between **40%** to **50%**.

• **At NLL** it is **17%** to **24%**; at **NNLL** **6%** to **10%**.

• We also predict **NNLO+NNNLL** which is however within uncertainty band of **NNLO+NNLL**.

---

**Banerjee, GD, Dhani, Ravindran (’17)**
\sqrt{S} = 14 \text{ TeV}

Q = M_Z

\frac{d^2 \sigma^{q,f,o}}{dq^2 dy}

From Vrap

[Anastasiou, Dixon, Melnikov, Petriello]
### Comparison Table

| $y$ | $\left( \frac{p_R}{M_Z}, \frac{p_F}{M_Z} \right)$ | LO   | LL$_{M-F}$ | LL$_{M-M}$ | NLO   | NLL$_{M-F}$ | NLL$_{M-M}$ | NNLO | NNLL$_{M-F}$ | NNLL$_{M-M}$ |
|-----|---------------------------------|------|------------|------------|------|-------------|-------------|------|-------------|--------------|
| 0.0 | (2, 2)                          | 72.626 | +0.988     | +3.219     | 73.450 | +1.639      | +1.796      | 70.894 | +0.630      | +0.646       |
| 0.0 | (2, 1)                          | 63.197 | +0.768     | +2.595     | 70.625 | +0.761      | +1.017      | 70.360 | +0.292      | +0.317       |
| 0.0 | (1, 2)                          | 72.626 | +1.095     | +3.577     | 73.535 | +1.912      | +1.760      | 70.509 | +0.510      | +0.395       |
| 0.0 | (1, 1)                          | 63.197 | +0.851     | +2.887     | 71.395 | +0.858      | +0.901      | 70.537 | +0.248      | +0.167       |
| 0.0 | (1, 1/2)                        | 53.241 | +0.621     | +2.216     | 67.581 | +0.156      | +0.140      | 69.834 | -0.001      | -0.094       |
| 0.0 | (1/2, 1)                        | 63.197 | +0.953     | +3.278     | 72.355 | +0.945      | +0.681      | 70.266 | +0.091      | -0.015       |
| 0.0 | (1/2, 1/2)                      | 53.241 | +0.695     | +2.504     | 69.259 | +0.102      | -0.154      | 70.283 | -0.039      | -0.146       |

- **Input:** Same PDF at all orders.
- **Significant change** at LO+LL level. At NLO+NLL, NNLO+NNLL non-trivial change at coefficient level.

Banerjee, GD, Dhani, Ravindran (to be online)
Rapidity Distribution

• Central scale:

\[(\mu_r^C, \mu_f^C) = (1, 1)m_Z\]

• Scale uncertainty:

\[(\mu_r, \mu_f) = (\kappa_1, \kappa_2) \otimes (\mu_r^C, \mu_f^C)\]

\[1/2 \leq \frac{\kappa_1}{\kappa_2} \leq 2\]

• Scale uncertainty at NNLO: 1.96%

• Scale uncertainty at NNLO+NNLL: 4.03%

Better perturbative convergence!!

Expected better scale uncertainty?
Central Scale Choice

- Scale variation around the central scale by:

\[(\mu_T, \mu_f) = (\kappa_1, \kappa_2) \otimes (\mu_T^C, \mu_f^C)\]

with

\[1/2 \leq \kappa_1/\kappa_2 \leq 2\]

Best prediction

\[(\mu_T^C, \mu_f^C) = (m_Z/2, m_Z)\]

SCET resummation

\[(\mu_T^C, \mu_f^C) = (1, 1/2)m_Z\]  

[Ebert, Michel, Tackmann]
Best Prediction: FO vs RESUM

• Scale uncertainty at NNLO \( (\mu_r^C, \mu_f^C) = (1, 1)m_Z \): \(1.96\%\)

• Scale uncertainty at NNLO+NNLL \( (\mu_r^C, \mu_f^C) = (1/2, 1)m_Z \): \(3.06\%\)
Renorm. scale dependence

• Only resums large Distributions arising in $q\bar{q}$ channel.

  • NNLO ($q\bar{q}$): 2.36%
  • NNLO+NNLL ($q\bar{q}$): 1.53%

• Strong cancellation of scale dependence among different channels at NNLO.

Accidental?
• PDF uncertainties are within 2%. 

Includes latest NOMAD data
Rapidity Distribution at LHC

- Electro-weak corrections are important at this accuracy.

- Both $e^+e^-$ and $\mu^+\mu^-$ give negative contribution at NLO EW level.

- Integrated Z-peak region

$$60 < q < 120 \text{ GeV}$$

- In $G_\mu$ scheme, EW in $e^+e^-$ : -3.5\% (NLO)

EW in $\mu^+\mu^-$ : -1.8\% (NLO)

From Horace [Carloni Calame, Montagna, Nicrosini, Vicini]
\[ \sqrt{S} = 1.8 \text{ TeV} \]

\[ 66 < q < 116 \text{ GeV} \]

- Scale uncertainty
  - ABMP16 \sim 1.64\%
  - NNPDF31 \sim 1.68\%
Summary

• We have developed a systematic way of resuming threshold logs for rapidity distribution for colorless particle production.

• We exploited the factorisation, RG invariance and K+G equations to find different pieces.

• Following Catani-Trentadue approach we find an all order resummed formula in rapidity distribution for Higgs and DY production in 2-D Mellin space.

• Rapidity distribution is presented with NNLO+NNLL accuracy. The resummed prediction changes the fixed order predictions significant way and improves the reliability of the perturbative prediction.

• We have presented a detailed study on scale choice, scale variation, pdf variation also along with available EW results which will be useful at the LHC in near future.
Thanks to S. Moch, S. Alekhin and J. Blumlein for many fruitful discussion.

Thanks to organisers of Loops and Legs 2018 for their endless effort behind such conference.
thank you!