The $\mathcal{N} = 1 \ D = 3$ Lifshitz-Wess-Zumino model: A paradigm of reconciliation between Lifshitz-like operators and supersymmetry

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Abstract

By imposing the weighted renormalization condition and the (super)symmetry requirements, we construct a Lifshitz-like extension of the three-dimensional Wess-Zumino model, with dynamical critical exponent $z = 2$. In this context, the auxiliary field $F$ plays a key role by introducing the appropriate Lifshitz operator in the bosonic sector of the theory, avoiding so undesirable time-space mixing derivatives and inconsistencies concerning the critical $z$ exponent, as reported in the literature. The consistency of the proposed model is verified by building explicitly the susy algebra through the Noether method in the canonical formalism. This component-field Lifshitz-Wess-Zumino model is in addition rephrased in the Lifshitz superspace, a natural modification of the conventional one. Finally, the one-loop effective potential is computed to study the possibility of symmetry breaking. It is found that supersymmetry remains intact at one-loop order, while the $U(1)$ phase symmetry suffers a spontaneous breakdown above the critical value of the renormalization point. By renormalizing the one-loop effective potential within the cutoff regularization scheme, it is observed an improvement of the UV behavior of the theory compared with the relativistic Wess-Zumino model.

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I. INTRODUCTION

In the last few years, there has been an increasing interest in the study of quantum field theories with higher spatial derivative terms, known in the literature as Lifshitz-type quantum field theories. One of the reasons of this interest is the improved UV behavior of the propagators at high energies, without spoiling the unitarity of the theory, due to the introduction of higher spatial derivatives (Lifshitz-like operators) in the kinetic part of the Lagrangian. The renormalizability as well as the unitarity of this kind of theory are ensured by the so-called weighted renormalization condition \cite{1, 2}. This condition in turn requires an anisotropy (see Eq. 4) between space and time coordinates so that the Lorentz symmetry is lost in the UV region. It is believed, however, that this symmetry should emerge at low energies. This subtle issue was investigated in several papers, see for example \cite{3, 4}.

The simplest Lifshitz scalar theory with critical exponent $z = 2$ was proposed long ago with the intention of explaining the second-order phase transitions in condensed-matter systems \cite{5}. Since then several generalizations of this prototype have appeared in condensed-matter physics, high energy physics and gravity (see \cite{6} and references therein). In this last context, the Hořava-Lifshitz gravity proposed by Hořava \cite{7} is arguably one of the most important reincarnations of the Lifshitz’s ideas. The Hořava’s proposal is simply a quantum field theory of gravity of the Lifshitz type with critical exponent $z = 3$ which violates the Lorentz invariance, due to the introduction of Lifshitz-like operators, in favor of its renormalizability at high energies. This theory of course will make physical sense if the restoration of the Lorentz symmetry occurs in the IR region. At the present time this is still an open subject of investigation. (For a recent review of the Hořava-Lifshitz gravity, see \cite{8})

On the other hand, the implementation of Lifshitz-like operators in supersymmetric field theories is not a trivial task and so far there does not exist a natural method of doing it. This problem was faced in \cite{9–11} by employing the superfield formalism in four spacetime dimensions. Nevertheless, some inconsistencies concerning the ill-definedness of the critical exponent $z$ and the appearance of undesirable time-space mixing derivatives were observed in the Lifshitz-like constructions proposed in \cite{10}.

In order to see more clearly what is happening and eventually uncover the real roots of these inconsistencies, we tackle the problem of constructing a Lifshitz-like extension of the Wess-Zumino model in the three dimensional component formalism, i.e., without employing
the conventional superfield formalism. The three-dimensional framework constitutes an excellent theoretical laboratory for an in-depth study of these four-dimensional setbacks, since the notion of chirality does not exist in odd dimensions and so three-dimensional susy theories become simpler, conserving, however, the main features of their four-dimensional counterparts.

In this work, we show in detail that the insertion of Lifshitz-like operators according to the weighted renormalization condition is completely compatible with supersymmetry and the well-definedness of the critical exponent $z$. Furthermore, in the superspace reformulation of the proposed model (the Lifshitz-Wess-Zumino model), we show that the conventional superfield formalism is inappropriate for the formulation of supersymmetric theories of the Lifshitz type. It is not hard to notice that the conventional superfield formalism invented by Salam and Strathdee [12] for constructing relativistic susy theories does violate the weighted power-counting criterion. Indeed, since the susy-relativistic covariant derivative,

$$D_\alpha = \partial_\alpha + i (\gamma^\mu)_{\alpha\beta} \theta^\beta \partial_\mu = \partial_\alpha + i (\gamma^0)_{\alpha\beta} \theta^\beta \partial_0 + i (\gamma^i)_{\alpha\beta} \theta^\beta \partial_i,$$  

embraces the time and space derivatives with the same weight, this susy covariant object does not obey the anisotropic scaling rules (4), with $z > 1$. This fact is non-negotiable and illustrates the necessity of modifying the conventional superfield formalism before employing it in the construction of Lifshitz-like susy theories. This minor and necessary modification was carried out in Eq. (28) in order to express the component-field Lifshitz-Wess-Zumino model in terms of the superfield language (i.e., in the Lifshitz superspace as we call it).

In the same spirit as conventional (i.e., without susy) Lifshitz-type quantum field theories [1, 2], we attempted to split the five-dimensional superspace $\mathcal{SM}_5$ into the product of two disjoint submanifolds (supersectors): $\mathcal{SM}_t \times \mathcal{SM}_s$, where $\mathcal{SM}_t$ stand for the supertime manifold and $\mathcal{SM}_s$ the superspatial one. The goal of this separation is to create an environment more adequate and natural for constructing Lifshitz-type susy theories in which the time derivative $\partial_0$ and the spatial derivative $\partial_i$ could live (or act) in entirely distinct supersectors. As is shown in the Appendix, such a complete separation is unworkable without the introduction of extra Grassmannian coordinates. Needless to say, that this procedure would require a complete reformulation of the conventional superfield formalism. This issue perhaps deserves further attention, in particular, in the construction of gauge susy theories of the Lifshitz type. For the moment, this is beyond the scope of this paper.

3
Finally, the effective potential of the Lifshitz-Wess-Zumino model was computed at one-loop order. The purpose of this calculation is twofold: to investigate the possibility of susy breaking due to the Lifshitz-like operators implemented in the conventional Wess-Zumino model and to understand how the UV improvement occurs in susy theories of the Lifshitz type. By analyzing the stationary conditions of the one-loop effective potential, we show that supersymmetry remains intact, while the $U(1)$ phase symmetry suffers a spontaneous breakdown above the critical value of the renormalization point. On the other hand, the improved UV behavior of the theory becomes evident by introducing a two-dimensional cutoff ($\Lambda$) to regularize the one-loop effective potential. This result, nevertheless, depends on the exact cancellation of the quadratic divergences between the bosonic and fermionic contributions (see comments below Eq. (41)). In scalar field theories (without susy), the one-loop effective potential was computed in [13].

Our paper is organized as follows. In Sec. II we construct a Lifshitz-like extension of the three-dimensional Wess-Zumino model with critical exponent $z = 2$. This is done by imposing the weighted renormalization condition and the (super)symmetry requirements. In this section, the susy algebra is explicitly constructed by using the Noether method and the field-component model is rephrased in the Lifshitz superspace. In Sec. III the one-loop effective potential is calculated and their mimina analyzed. To investigate the structure of the UV divergences, the one-loop effective potential is regularized by using a two-dimensional cutoff ($\Lambda$). Finally, Sec. IV contains our main results.

II. THE $\mathcal{N} = 1 \mathcal{D} = 3$ LIFSHITZ-WESS-ZUMINO MODEL

In this section, we are going to construct a Lifshitz-like extension of the Wess-Zumino model in $(1 + 2)$ spacetime dimensions, by imposing the weighted renormalization condition and the (super)symmetry requirements. Since it is assumed that the Lorentz symmetry is explicitly broken at high energies, our starting point will be the action

$$S = \int dt dx^2 \left[ -\left( \partial^0 \bar{\phi} \partial_0 \phi + a^2 \bar{\phi} \partial_i \phi \right) + i \bar{\psi} \partial_0 \psi + ia \bar{\psi} \partial_i \psi + \bar{F} F + \mathcal{L}_{int} \right],$$

where $\phi$ is a complex scalar field, $\psi_\alpha$ a complex (Euclidean) spinor field, and $F$ a complex auxiliary field. Here $a$ is a dimensionless constant which measures the degree of the Lorentz violation in the isotropic case (notice that taking $a = 1$ the Lorentz symmetry is recovered).
From now on we shall adopt the notation of [14]. In particular, \(i \bar{\psi} \gamma_\mu \psi\) means \(i \bar{\psi} \gamma^\mu \gamma^0 \psi\).

Note that we maintain the residual Lorentz notation for the time and space derivatives, namely \(\partial^0 = -\partial_0\) and \(\partial^i = +\partial_i\). As will be seen later, the auxiliary field \(F\) plays a key role in our construction. In fact, in addition to its usual role of making susy manifest off-shell, \(F\) shall allow us to introduce a higher space derivative in the scalar sector of the theory without altering its susy algebra.

Switching off the interaction Lagrangian \(L_{\text{int}}\), i.e. taking \(L_{\text{int}} = 0\) in (2), it is easy to show that the resulting free action is invariant under the following susy transformations

\[
\begin{align*}
\delta \varphi &= -\epsilon \psi \\
\delta \psi &= \epsilon F - i\epsilon \gamma_0 \varphi - i\epsilon \gamma_i \varphi \\
\delta F &= -i\epsilon \gamma_0 \psi - i\epsilon \gamma_i \psi,
\end{align*}
\]  

(3)

where \(\epsilon\) is a Grassmann \(x\)-independent parameter.

Before proceeding with the construction of \(L_{\text{int}}\), it is necessary to state clearly the weighted renormalization condition (wrc) [1, 2]. If one writes \(L_{\text{int}}\) of a given theory as \(L_{\text{int}} = \sum_i g_i V_i\), where \(g_i\) label the coupling constants and \(V_i\) the interaction vertices, this condition simply says that the theory is renormalizable by weighted power counting iff the weighted scaling dimension, the weight for short, \([g_i]_w\) of each coupling constant \(g_i\) is greater or equal to zero, i.e., \([g_i]_w \geq 0\). Setting \([x^0 = t]_w = -z\) and \([x^i]_w = -1\), the weight \([O]_w\) of any object \(O\) is determined by enforcing the action \(S\) to be weightless. In terms of the vertices, since the weight of the Lagrangian \(L\) is \(d + z\), where \(d\) denotes the spatial dimensions, the wrc asserts that a vertex \(V_i\) is weighted renormalizable iff \([V_i]_w \leq d + z\).

It should be noted that the weight assignment in Lifshitz field theories is equivalent to demand the invariance of the action under the following anisotropic scale transformations,

\[
x^i \rightarrow \xi x^i \quad t \rightarrow \xi^z t,
\]  

(4)

where \(z\) is the well-known critical exponent which measures the degree of anisotropy between space and time. Moreover, notice that the weighted scaling dimension coincides with the usual mass one in natural units \((\hbar = 1 = c)\) when \(z = 1\).

The most general interaction Lagrangian \(L_{\text{int}}\) which satisfies the symmetry requirements and the wrc with \(z = 2\) can be cast in the form

\[
L_{\text{int}} = \bar{\psi} W_1 \psi + \left[ W_2 \bar{\psi}^2 + W_3 F + h.c. \right],
\]  

(5)
where $W_i$ depend on the fields $\varphi$ and $\bar{\varphi}$ and, perhaps, on the two-dimensional Laplace operator $\Delta = \partial^i \partial_i$. It is easy to see that the weights of the fields are $[\varphi]_w = (2 - z)/2$, $[\bar{\psi}_\alpha]_w = 1$, and $[F]_w = (2 + z)/2$, while the weight of the $a$ parameter is $[a]_w = z - 1$.

Here some comments are in order. First, due to the Hermiticity property of the action (2) as a whole, $W_1$ has to be a real operator, whereas $W_2$ and $W_3$ complex ones. Second, by demanding the invariance of the interaction action (5) under the susy transformations (3), it is feasible to verify that $W_3$ is the only independent operator. Indeed, to respect susy, $W_1$ and $W_2$ must be obtained from it by $\varphi-$differentiation,

$$W_1 = \frac{\partial W_3}{\partial \varphi}, \quad W_2 = \frac{\partial W_3}{\partial \bar{\varphi}}. \quad (6)$$

Finally, the form of $W_3$ is fully defined by employing the wrc with $z = 2$ and the reality of $W_1$. This last expressed mathematically as $\partial W_3/\partial \varphi = \overline{\partial W_3}/\partial \bar{\varphi}$. After doing these, we obtain

$$W_3 = m\varphi + b\Delta \varphi + g \varphi^p \bar{\varphi}^{p-1}, \quad (7)$$

where $p$ is an integer greater than 1. Since $b$ is a weightless parameter, $[b]_w = 0$, we should stress that $b\Delta \varphi$ is the only operator with higher space derivatives which can be implemented consistently in this kind of theory. Notice also that a scalar function $U(\varphi, \bar{\varphi})$ and a $4 -$spinor one $V(\varphi, \bar{\varphi}) \bar{\psi}^2 \psi^2$ were not included in (5), for these operators would break susy.

It is extremely important to expose the consistency of our model by setting explicitly up its superalgebra at classical level. Hence, in the balance of this section, we construct the Noether currents (and their respective charges) associated with each symmetry of the model under consideration and then we set up its superalgebra by using the canonical (anti-)commutation relations.

For simplicity and without loss of generality, we shall focus our attention in the model with $p = 2$. So, the interaction Lagrangian $L_{int}$ of our Lifshitz-Wess-Zumino (L-WZ) model, as we shall call it, reads

$$L_{int} = \bar{\psi} (m + b\Delta + 2g\bar{\varphi}\varphi) \psi + [g \varphi^2 \bar{\psi}^2 + (m\varphi + b\Delta \varphi + g \varphi^2 \bar{\varphi}) \bar{F} + h.c.] \quad (8)$$

Notice that setting $a \to 1$ and $b \to 0$ this theory reduces to the usual relativistic Wess-Zumino model [14, 15].

The L-WZ field equations which result from the principle of least action, $\delta S = 0$, are
given by

\[-\ddot{\phi} + a^2 \Delta \phi + mF + b \Delta F + g (\phi^2 \bar{F} + 2\bar{\phi} \phi F + 2\bar{\phi} \psi^2 + 2\phi \bar{\psi} \psi) = 0\]

\[(i\partial_t + i\alpha \partial_i + m + b \Delta) \psi + g \left(\bar{\phi} \psi + 2\bar{\phi} \psi\right) = 0\]  \hspace{1cm} (9)

\[F + m \phi + b \Delta \phi + g \bar{\phi} \phi^2 = 0.\]

In contrast with the relativistic Wess-Zumino model, we should realize that the auxiliary field equation is no longer an algebraic one, for it contains the extra space differential term $b \Delta \phi$. The auxiliary field $F$ and its complex conjugate $\bar{F}$, as mentioned earlier, play a leading role in the construction of our Lifshitz susy field theory, since they introduce naturally the right Lifshitz-operator, $\bar{\phi} \Delta^2 \phi$, in the bosonic sector of the theory. Indeed, after removing $F$ and $\bar{F}$ from (2-8) by means of their field equations, the bosonic part of the L-WZ Lagrangian reads

\[-L_{bos} = -\dot{\bar{\phi}} \dot{\phi} + (2mb - a^2) \bar{\phi} \Delta \phi + b^2 \bar{\phi} \Delta^2 \phi + m^2 \phi \phi + mg (\bar{\phi} \phi)^2 + bg \bar{\phi} \phi (\bar{\phi} \Delta \phi + \phi \Delta \bar{\phi}) + g^2 (\bar{\phi} \phi)^3.\]  \hspace{1cm} (10)

It is important to point out that, as far as we know, this is the first time this Lifshitz construction procedure in susy theories has been proposed. This method, in particular, avoids the glaring inconsistencies concerning the ill-definedness of the critical exponent $z$ as well as the unnatural time-space mixing derivatives observed in [10].

Coming back to the problem of building up the superalgebra, we first claim that it closes off-shell. Indeed, it is not hard to check that the commutator of two susy transformations yields once again a symmetry transformation, i.e. a linear asymmetric combination of time and space transformations,

\[\left[\delta_{\epsilon_2}, \delta_{\epsilon_1}\right] X = 2i \left(\epsilon_2 \gamma^0 \epsilon_1\right) \partial_0 X + 2ai \left(\epsilon_2 \gamma^i \epsilon_1\right) \partial_i X,\]  \hspace{1cm} (11)

where $X$ stands for the fields $\phi$, $\psi_\alpha$, and $F$. To prove (11) one has to make use of the Fierz identity: $\chi_\alpha (\xi \eta) = -\xi_\alpha (\chi \eta) - (\xi \chi) \eta_\alpha$. We should emphasize that this result and the others that we present in the rest of this section, in particular (22), are not only valid for the free theory, but also for the interaction one, i.e. including the interaction Lagrangian (8).

In other words, we show explicitly that the implementation of the Lifshitz-like operators $b\bar{\psi} \Delta \psi$ and $b^2 \bar{\phi} \Delta^2 \phi$, this last by means of the auxiliary field $F$, in the fermionic and bosonic
sectors, respectively, does not spoil the susy algebra of conventional (i.e. with $z = 1$) Lorentz-violating supersymmetric theories \[16\] at classical level.

According to the Noether theorem, it is not hard to show that the components of the supercurrent of the L-WZ model associated with the susy transformations \((3)\) are given by

\[- J^0_\alpha = \bar{\psi}_\alpha \dot{\varphi} + \psi_\alpha \dot{\varphi} + a (\psi \gamma^0 \partial_0)_\alpha \varphi + a (\bar{\psi} \gamma^0 \partial_0)_\alpha \varphi + i \bar{F} (\gamma^0 \psi)_\alpha + i F (\gamma^0 \bar{\psi})_\alpha \tag{12} \]

and

\[- J^i_\alpha = (\psi \mathcal{S}^i \mathcal{D})_\alpha \varphi + (\bar{\psi} \mathcal{S}^i \mathcal{D})_\alpha \varphi + i \bar{F} (\mathcal{S}^i \psi)_\alpha + i F (\mathcal{S}^i \bar{\psi})_\alpha, \tag{13} \]

where \(\mathcal{D}_{\alpha\beta} = (\gamma^0)_{\alpha\beta} \partial_0 + a (\gamma^i)_{\alpha\beta} \partial_i\) and \(\mathcal{S}^i_{\alpha\beta} = a (\gamma^i)_{\alpha\beta} - i b C_{\alpha\beta} \partial^i\). Using the field equations \((9)\), one can verify the conservation of the supercurrent \(J^\mu_\alpha\), namely

\[\partial^\mu J^\mu_\alpha = 0.\]

As in conventional supersymmetric theories, it follows that the conserved supercharge \(Q_\alpha = \int d^2x J^0_\alpha\) generates the susy transformations \((3)\). That is,

\[[\epsilon^\alpha Q_\alpha, X] = -i \delta X. \tag{14} \]

This relation may be checked out by using the classical field equations \((9)\) and the canonical (anti-)commutators for the fields,

\[[\varphi(x), \dot{\varphi}(y)] = i \delta_{x,y}, \quad [\bar{\varphi}(x), \dot{\varphi}(y)] = i \delta_{x,y}, \quad [\psi_\alpha(x), \bar{\psi}_\beta(y)] = (\gamma^0)_{\alpha\beta} \delta_{x,y}, \tag{15}\]

where we have omitted for simplicity the equal time variable \(t\) in the argument of the fields and used \(\delta_{x,y}\) to represent the two-dimensional Dirac delta function \(\delta^2(x - y)\).

In order to assemble the susy algebra we follow the same technique as in usual susy theories \[17\]. Namely, we employ the Jacobi identity,

\[[[A, B], C] = [A, [B, C]] - [B, [A, C]], \tag{16}\]

with \(A = \epsilon_2 Q, B = \epsilon_1 Q\) and \(C = X\). After simplifying this with the help of the formulas \((11)\) and \((14)\), one gets

\[[[\epsilon_2 Q, \epsilon_1 Q], X] = -2i (\epsilon_2 \gamma^0 \epsilon_1) \partial_0 X - 2ai (\epsilon_2 \gamma^i \epsilon_1) \partial i X. \tag{17}\]

At this point it is important to recognize that the L-WZ action \((2)\) is invariant under other symmetries. Invariance under time translations, \(\delta_\tau X = \tau \partial_0 X\), gives rise to a conserved current \(\bar{J}^\mu\) whose components are

\[\bar{J}^0 = \dot{\varphi} \dot{\bar{\varphi}} + a^2 \partial^i \varphi \partial_i \varphi - ia \bar{\psi} \partial_i \psi - m \bar{\psi} \psi - b \bar{\psi} \Delta \psi + FF - g \left( \varphi^2 \bar{\psi}^2 + \varphi^2 \bar{\psi}^2 + 2 \varphi \bar{\psi} \bar{\psi} \right) \tag{18} \]
and
\[ \bar{J}^i = -a^2 (\dot{\varphi} \partial^i \varphi + \dot{\varphi} \partial^i \varphi) + ia \bar{\psi} \gamma^i \psi + b \left( F^\dagger \partial^i \varphi + F^\dagger \partial^i \varphi + \bar{\psi} \partial^i \psi \right). \] (19)

Of course, in this case, the conserved charge
\[ H = \int d^2 x \bar{J}^0 \]
turns out to be the Hamiltonian of the Lifshitz-Wess-Zumino model. By using the canonical relations (15), one may show that \( H \) is in effect the generator of the time translations:
\[ [\tau H, X] = -i \delta_{\tau} X. \]

Invariance under spatial translations, \( \delta_{s} X = s^k \partial_k X \), gives rise to two conserved currents \( T^\mu_k \) (note that in planar physics there are only two spatial directions) whose components are
\[ T^0_k = \partial_k \dot{\varphi} \dot{\varphi} + \dot{\varphi} \partial_k \varphi + i \bar{\psi} \gamma^0 \partial_k \psi \] (20)
and
\[ T^i_k = -a^2 (\partial_k \dot{\varphi} \partial^i \varphi + \partial^i \dot{\varphi} \partial_k \varphi) + ia \bar{\psi} \gamma^i \partial_k \psi - b (\partial_k \varphi \partial^i \bar{F} + \partial^i \varphi \partial_k \bar{F} + \partial_k \bar{\psi} \partial^i \psi + \partial^i \bar{\psi} \partial_k \psi) - \delta^i_k \mathcal{L}. \] (21)

The two conserved charges \( P_k = \int d^2 x T^0_k \) define the momentum vector of the system. Once again it is easy to see that \( P_k \) are the generators of space translations:
\[ [s^k P_k, X] = -i \delta_{s} X. \]

With these results at hand, we eliminate the time and space derivatives from (17) in terms of the \( H \) and \( P_k \) commutators. In doing this, we finally obtain for the susy algebra
\[ \{Q_\alpha, Q_\beta\} = 2 \left( \gamma^0 \right)_{\alpha \beta} H + 2a \left( \gamma^i \right)_{\alpha \beta} P_i. \] (22)

This anticommutator of two supercharges is the three-dimensional version of that found in [16] regarding the violation of the Lorentz symmetry in the conventional four-dimensional Wess-Zumino model. Note however that in the present investigation we are considering Lorentz-violation operators with higher space derivatives, i.e. Lifshitz-like operators, a marked difference with respect to [16].

For completeness, we compute the Noether current \( J^\mu_R \) associated to the invariance of the L-WZ action (2) under the rotation group \( SO(2) \). The rotational transformations that leave the action invariant are given by
\[ \delta_{\theta} \varphi = -i \theta \dot{\varphi} \psi \quad \delta_{\theta} F = -i \theta \dot{\psi} \varphi \quad \delta_{\theta} \psi = -i \theta \dot{\psi} \varphi - i \theta \hat{\Sigma} \psi, \] (23)
where \( \hat{L} = i (x^2 \partial_1 - x^1 \partial_2) \) denotes the angular momentum generator and \( \Sigma = -i [\gamma^1, \gamma^2] / 4 \) the spinor generator. One can show by using the Noether’s method that the components of the conserved current \( J^\mu_R \) are
\[ J^0_R = x^2 T^0_1 - x^1 T^0_2 + \bar{\psi} \gamma^0 \Sigma \psi \] (24)
and
\[ J^i_R = x^2 T^i_1 - x^1 T^i_2 + a \bar{\psi} \gamma^i \Sigma \psi - ib \bar{\psi} \Sigma \partial^i \psi + ib \partial^i \bar{\psi} \Sigma \psi. \] (25)

The Noether charge \( J = \int d^2 x J^0_R \) corresponds to the angular momentum of the system and its conservation is guaranteed by \( \partial_\mu J^\mu_R = 0 \). Note that \( J \) is, as should be, the generator of the rotational transformations \((23)\), i.e., \([\theta J, X] = -i \delta_\theta X\).

We close this section by rephrasing our model in what we call the Lifshitz superspace, a natural modification of the conventional one to treat susy theories with Lifshitz-like operators. For this purpose, as in the usual case, we first compact the fields \( \varphi, \psi_\alpha \) and \( F \) into a scalar superfield \( \Phi \),
\[ \Phi (x^0, x^i, \theta) = \varphi + \theta^\alpha \psi_\alpha - \theta^2 F. \] (26)

Next, since the time and space coordinates are weighted differently in Lifshitz field theories, we split the usual susy covariant derivative \( D_\alpha = \partial_\alpha + i (\gamma^\mu)_{\alpha \beta} \theta^\beta \partial_\mu \) into a time 'covariant' derivative \( D_{t\alpha} \) and a space 'covariant' derivative \( D_{s\alpha} \):
\[ 2D_{t\alpha} = \partial_\alpha + 2i (\gamma^0)_{\alpha \beta} \theta^\beta \partial_0 \quad \text{and} \quad 2D_{s\alpha} = \partial_\alpha + 2ia (\gamma^i)_{\alpha \beta} \theta^\beta \partial_i. \] (27)

Here \( a \) is an essential parameter for counterbalancing the weight of the spatial \( \partial_i \) derivative compared with the weight of the time \( \partial_0 \) derivative. This parameter coincides with the one introduced in \((2)\) and weights \([a]_w = z-1 = 1\), while \([\theta_\alpha]_w = -z/2 = -1\). Within this Lifshitz superspace formulation, the term 'covariant' must be taken with great care, for these susy derivatives are covariant regarding the time and space supercharges \( Q_{t\alpha} \) and \( Q_{s\alpha} \) defined in the Appendix, but not with respect to the net supercharge \( Q_\alpha = Q_{t\alpha} + Q_{s\alpha} \) that realizes the susy algebra \((22)\). The net covariant derivative \( D_\alpha \) which anticommutes with \( Q_\alpha \) (see Appendix for details) is given by
\[ D_\alpha = D_{t\alpha} + D_{s\alpha} = \partial_\alpha + i (\gamma^0)_{\alpha \beta} \theta^\beta \partial_0 + ia (\gamma^i)_{\alpha \beta} \theta^\beta \partial_i. \] (28)

Note that this weighted covariant derivative \( D_\alpha \) becomes the usual one taking \( a \to 1 \).

In terms of these superobjects, the superfield counterpart of the L-WZ action can be cast in the form
\[ S = \int dt d^2 x d^2 \theta \left\{ -\frac{1}{2} D^\alpha \bar{\Phi} D_\alpha \Phi + \frac{4b}{a^2} D_0^2 \bar{\Phi} D_0^2 \Phi + m \bar{\Phi} \Phi + \frac{g}{2} (\bar{\Phi} \Phi)^2 \right\}. \] (29)

By carrying out explicitly the Grassmann integration or by doing this with the help of the projection techniques described in the Appendix, it is straightforward to show that this superaction reduces to the component one \((2)\) with \( \mathcal{L}_{\text{int}} \) given by \((8)\).
It is interesting to see that the higher space derivative term (apparently not covariant in the entire Lifshitz superspace by the presence of the space derivative $D_s$) is indeed covariant up to surface terms. To see this more closely, we integrate it by parts

$$\int dt^2 x \, D_s^2 \left[ \left( \frac{4b}{a^2} \right) D_s^2 \bar{\Phi} D_s^2 \Phi \right] = \int dt^2 x \left( \frac{8b}{a^2} \right) D_s^2 \left[ D_s^a \left( \bar{\Phi} \overleftrightarrow{D_s} D_s^2 \Phi \right) + 2 \bar{\Phi} (D_s^2)^2 \phi \right] \bigg|_{\Phi = \Phi_0} = \int dt^2 x \, D_s^2 \left[ D_s^a \left( \bar{\Phi} \overleftrightarrow{D_s} D_s^2 \Phi \right) + 2 \bar{\Phi} (D_s^2)^2 \phi \right] = \int dt^2 x d^2 \theta \left( b \bar{\Phi} \Delta \Phi \right),$$

(30)

where in the last line we have ignored the surface space term $D_s^2 D_s^a (\cdots) \sim \partial_i (\cdots)$ and used the identity $(D_s^2)^2 = a^2 \Delta / 4$. Since $\partial_i$ is a covariant derivative in the entire Lifshitz superspace, i.e., $[\partial_i, Q_\alpha] = 0$, the term $\bar{\Phi} \Delta \Phi$ is manifest covariant.

### III. THE EFFECTIVE POTENTIAL TO ONE-LOOP ORDER

This section is devoted to investigate the vacuum quantum effects of the Lifshitz operators implemented in the conventional Wess-Zumino model. For this purpose, we shall compute the one-loop effective potential of the L-WZ model (2) with $L_{int}$ defined in (8). Moreover, we shall take advantage of this calculation to examine the improvement of the ultraviolet behavior in this kind of theory. As is well known, up to a spacetime volume $v_3 = \int d^3 x$, the zero-loop potential $V_0$ is the negative of the classical action evaluated at the position-independent fields $\varphi(x) = \varphi_0 = (\sigma_1 + i \pi_1) / \sqrt{2}$, $F(x) = f_0 = (\sigma_2 + i \pi_2) / \sqrt{2}$ and $\psi_\alpha(x) = 0$. In doing this, one gets

$$V_0 = -\frac{1}{2} \left[ \sigma_2^2 + \pi_2^2 + 2m (\sigma_1 \sigma_2 + \pi_1 \pi_2) + g (\sigma_1^2 + \pi_1^2) (\sigma_1 \sigma_2 + \pi_1 \pi_2) \right]$$

$$= \frac{m^2}{2} (\sigma_1^2 + \pi_1^2) + \frac{mg}{2} (\sigma_1^2 + \pi_1^2)^2 + \frac{g^2}{8} (\sigma_1^2 + \pi_1^2)^3,$$

(31)

where in the last equality we have eliminated the real auxiliary fields $\sigma_2$ and $\pi_2$ by means of their field equations $\partial V_0 / \partial \sigma_2 = 0$ and $\partial V_0 / \partial \pi_2 = 0$. It is not hard to see that at classical level the theory exhibits two phases in regard to the spontaneous breaking of the global phase $U(1) \cong SO(2)$ symmetry group: $\sigma_i^i + i \pi_i^i = \exp(i \alpha) (\sigma_i + i \pi_i)$. These two phases are dictated by the sign of the order parameter $\xi = m/g$. In fact, by analyzing the minima of the classical potential (31), we conclude that whether $\xi \geq 0$, the $U(1)$ symmetry is preserved, since the vacuum state is unique and it corresponds to the trivial one $\sigma_1 = 0 = \pi_1$. On the other hand, if $\xi < 0$, the $U(1)$ symmetry is spontaneously broken, for in addition to the trivial
vacuum state, there is a manifold of non-trivial vacuum states defined by \( \sigma_1^2 + \pi_1^2 = 2|\xi| \). Note that supersymmetry in both phases remains intact due to the vanishing of the vacuum energy. In what follows, we confine our attention to the case \( \xi > 0 \), considering \( m > 0 \) and \( g > 0 \).

In order to compute the one-loop contribution for the effective potential we employ the steepest-descent method \[18\]. According to this method, the one-loop contribution becomes

\[
V_1 = -\frac{i}{2v_3} \ln \text{det} \left( Q^2 - 4\mathcal{R} \right) + \frac{i}{2v_3} \ln \text{det} \left( Q_{\alpha\beta}^2 - g^2 |\varphi_0|^4 C_{\alpha\beta} \right),
\]

where, defining the field-dependent mass \( M = m + 2g|\varphi_0|^2 \),

\[
Q = \partial_0^2 + (a^2 - 2bM) \Delta - b^2 \Delta^2 - M^2 + 2g \left( \varphi_0 \bar{f}_0 + \varphi_0 f_0 \right) - g^2 |\varphi_0|^4, 
\]

\[
\mathcal{R} = g\varphi_0 f_0 - gM\varphi_0^2 - gb\varphi_0^2 \Delta, 
\]

\[
Q_{\alpha\beta} = i \left( \gamma^0 \right)_{\alpha\beta} \partial_0 + i a \left( \gamma^i \right)_{\alpha\beta} \partial_i + C_{\alpha\beta} (M + b\Delta).
\]

Using the \( \zeta \)-functional method \[19\] for solving the functional determinants in (32), one gets

\[
V_1 = -\frac{i}{2} \int \frac{d^3k}{(2\pi)^3} \left\{ \ln \left[ -k_0^2 + (a^2 - 2b\mu_1) k^2 + b^2 k^4 + \mu_1^2 - g\sigma_1 \sigma_2 \right] + \ln \left[ -k_0^2 + (a^2 - 2b\mu_2) k^2 + b^2 k^4 + \mu_2^2 - 3g\sigma_1 \sigma_2 \right] \right\} + \frac{i}{2} \int \frac{d^3k}{(2\pi)^3} \left\{ \ln \left[ -k_0^2 + (a^2 - 2b\mu_1) k^2 + b^2 k^4 + \mu_1^2 \right] 
+ \ln \left[ -k_0^2 + (a^2 - 2b\mu_2) k^2 + b^2 k^4 + \mu_2^2 \right] \right\},
\]

where \( \mu_i \) are the field-dependent masses \( \mu_i = m + (2i - 1) g\sigma_i^2 / 2 \), with \( i = 1, 2 \). With respect to this result, some comments are pertinent. First, for simplicity and without loss of generality, we have set \( \pi_i = 0 \) in (32). This is always possible owing to the global \( U(1) \) symmetry of the effective potential. Second, in the analysis of the cancellation of infinities that will be carried out below, it is important to keep in mind that the first integral becomes from the bosonic determinant, whereas the second one (that with positive sign) from the fermionic one.

We compute the integrals in (36) by using the formula

\[
\int \frac{d^3k}{(2\pi)^3} \ln \left[ -k_0^2 + x^2 k^2 + y^2 k^4 + z^2 \right] = \frac{i}{32\pi y^3} \left[ 2y^2 z^2 - 2x^2 yz + (x^4 - 4y^2 z^2) \ln (x^2 + 2yz) \right] 
+ C_{1A} (y) z^2 + C_{2A} (x, y),
\]

where \( C_i \) are infinite constants given by

\[
C_1 (x, y) = \frac{i}{2} \int \frac{d^2k}{(2\pi)^2} \frac{1}{\sqrt{x^2 k^2 + y^2 k^4}} + \frac{i}{8\pi y} \ln x^2 \xrightarrow{\Lambda \to \infty} \frac{i}{4\pi y} \ln (2y\Lambda) \quad (38)
\]
\[ C_2(x, y) = i \int \frac{d^2 k}{(2\pi)^2} \sqrt{x^2 k^2 + y^2 k^4} - \frac{i x^4}{32\pi y^3} \ln x^2 \]

\[ \xrightarrow{\Lambda \to \infty} \frac{i}{64\pi y^3} \left[ x^4 (1 - 4 \ln (2y\Lambda)) + 8x^2y^2\Lambda^2 + 8y^4\Lambda^4 \right] \quad (39) \]

Here \( \Lambda \) represents an UV cutoff in the two-dimensional momentum space. Note also that the infinite constant \( C_1 \), for large \( \Lambda \), turns out to be independent of the \( x \)-parameter, yet \( C_1 \) is a function of \( x \) and \( y \) for finite values of \( \Lambda \). Adding the result of (36) to (31), with \( \pi i = 0 \), and then renormalizing it as described below, one gets the following expression for the renormalized 1-loop effective potential

\[ \frac{V_{\text{eff}}}{m^3} = -\frac{1}{2} \left( \sigma_2^2 + 2\sigma_1 \sigma_2 + g\sigma_1^2\sigma_2 \right) + \frac{1}{64\pi b^3} \left\{ 2b \left( a^2 - 2b\mu_1 \right) M_1 + 2b \left( a^2 - 2b\mu_2 \right) M_2 \\
+ a^2 \left( a^2 - 4b\mu_1 \right) \ln \left( 1 - \frac{2bM_1}{a^2} \right) + a^2 \left( a^2 - 4b\mu_2 \right) \ln \left( 1 - \frac{2bM_2}{a^2} \right) \\
+ 4gb^2\sigma_1 \sigma_2 \left[ \ln \left( 1 - \frac{2bM_1}{a^2} \right) + 3\ln \left( 1 - \frac{2bM_2}{a^2} \right) + 24\pi b\eta^2 - 2 \right] \right\} , \quad (40) \]

where \( M_i = \mu_i - \sqrt{\mu_i^2 - (2i - 1)g\sigma_1\sigma_2} \) and \( \eta \) is a renormalization point, defined by the equation

\[ \frac{1}{m^3} \left. \frac{\partial^2 V_{\text{eff}}}{\partial \sigma_1 \partial \sigma_2} \right|_{\sigma_1 = \eta, \sigma_2 = 0} = -1. \quad (41) \]

On the right-hand side of (40), we have made all quantities dimensionless by rescaling these in terms of the mass \( m \) parameter, i.e., \( \sigma_1 \to m^{1/2} \sigma_1, \sigma_2 \to m^{3/2} \sigma_2, b \to m^{-1}b \). Notice in particular that \( \mu_i \) in this equation stands for \( \mu_i = 1 + (2i - 1)g\sigma_1^2/2 \).

Before analyzing the minima of the effective potential, some remarks are in order in connection with the renormalization procedure used above. Note firstly that there was a complete cancellation of the \( C_2 \) infinities between the bosonic and fermionic contributions. This cancellation is essential for the UV improvement of the theory, since the conventional (i.e. without Lifshitz operators) three-dimensional Wess-Zumino model \[20\, 21\] contains only logarithmic and linear divergences at one-loop order and \( C_2 \) in (39) contains field-dependent quadratic divergences. The quartic divergences that might appear in the unrenormalized effective potential \( V_{\text{eff}} \) via (39) if this cancellation fails are of course matterless, for these are field-independent and might eventually be absorbed by introducing a ‘cosmological’ constant in the Lagrangian of the model. On the other hand, the cancellation of the \( C_1 \) infinities (logarithmic divergences) was partial. This residual susy divergence has been absorbed by
adding a mass-type counterterm, $A\sigma_1\sigma_2$, in the definition of the 1-loop effective potential: $m^{-3}V_{eff} = V_0 + V_1 + A\sigma_1\sigma_2$. By imposing the renormalization condition (11), it is easy to show that $A = \frac{g}{2\pi^2} \left[ 3\pi b\eta^2 + \ln \left( \frac{2b\Lambda}{\alpha} \right) \right]$.

Let us now examine the stationary conditions of the renormalized 1-loop effective potential (40). Defining $\mathcal{F}_{ab}(x) = \ln \left( 1 - 2bx/a^2 \right)$, they can be cast in the form

$$\frac{1}{m^3} \frac{\partial V_{eff}}{\partial \sigma_1} = -\sigma_2 - \frac{3}{2} g \left( \sigma_1^2 - \eta^2 \right) \sigma_2 - \frac{g\sigma_1}{8\pi b} (M_1 + 3M_2) - \frac{ga^2}{16\pi b^2} \left( \sigma_1 - \frac{b\sigma_2}{a^2} \right) \left[ \mathcal{F}_{ab}(M_1) + 3\mathcal{F}_{ab}(M_2) \right] = 0$$

and

$$\frac{1}{m^3} \frac{\partial V_{eff}}{\partial \sigma_2} = -\sigma_2 - \sigma_1 - \frac{g}{2} \sigma_1^3 + \frac{g\sigma_1}{16\pi b} \left[ \mathcal{F}_{ab}(M_1) + 3\mathcal{F}_{ab}(M_2) + 24\pi b\eta^2 \right] = 0.$$  

This pair of coupled equations, in principle, can be solved by first finding $\sigma_1 = \bar{\sigma}_1$ and $\sigma_2 = \bar{\sigma}_2 (\bar{\sigma}_1)$. This procedure, however, is impracticable due to the intricate form of the stationary equations and the field dependence of $M_i = M_i (\sigma_1, \sigma_2)$. Despite this fact, these equations provide relevant information for the study of spontaneous (super)symmetry breaking [22]. To see this, we must first observe that the condition

$$-\sigma_1 - \frac{g}{2} \sigma_1^3 + \frac{3}{2} g\eta^2 \sigma_1 = 0,$$

which becomes from (43) taking $\sigma_2 = 0$, has one real root at $\sigma_1 = 0$ for $\eta \leq \eta_c$ and three real roots at $\sigma_1 = 0$ and $\sigma_1 = \pm \sqrt{(3\eta^2 g - 2)/g}$ for $\eta > \eta_c$, where $\eta_c = \sqrt{2/3g}$ is a critical value of $\eta$. Next let us denote any of these roots by $\bar{\sigma}_1$ and note that $\sigma_2 (\bar{\sigma}_1) = 0$. Since (42) is also satisfied at $\sigma_2 = 0$, we conclude that the field configuration defined by $\sigma_1 = \bar{\sigma}_1$ and $\sigma_2 = 0$ is a stationary one. We notice, furthermore, that the effective potential $V_{eff}$ vanishes at this stationary point, i. e., $V_{eff} (\bar{\sigma}_1, \sigma_2 (\bar{\sigma}_1) = 0) = 0$. It follows from this fact and the positivity condition of the effective potential, $V_{eff} (\sigma_1, \sigma_2 (\sigma_1)) \geq 0$, that this stationary configuration is really an absolute minimum (with zero energy) and so susy remains unbroken at one-loop order. It is worthwhile to mention that the positivity condition of the energy in conventional susy theories holds in this kind of theory and is indeed assured by the susy algebra [22].
and noting that the second term in brackets is greater or equal to zero for all $\sigma_2$. Thus, $V_{\text{eff}}/m^3 \geq \sigma_2^2/2$. Finally, we observe that the $U(1)$ phase symmetry is preserved for $\eta \leq \eta_c$ and is spontaneously broken by radiative corrections for $\eta > \eta_c$.

IV. CONCLUSIONS

We deform the Wess-Zumino model by implementing higher space derivatives (i.e. Lifshitz-like operators) in the kinetic Lagrangian of it. This is done according to the weighted renormalization condition and the (super)symmetry requirements. In order to verify the consistency of the model, the susy algebra is explicitly constructed by using the Noether method in the canonical formalism. In addition, this model is rephrased in the Lifshitz superspace (a minor and necessary modification of the conventional one). By computing the one-loop effective potential and analyzing their minima, we conclude that the supersymmetry is preserved at one-loop order, while the $U(1)$ phase symmetry becomes spontaneously broken above a critical value of the renormalization point. To study the structure of the UV divergences, we regularized the one-loop effective potential by means of a two-dimensional cutoff ($\Lambda$). As expected, it is observed an improvement of the UV behavior of the theory. Indeed, the susy-Lifshitz residual divergence in the one-loop effective potential is logarithmic (and not linear as in the relativistic Wess-Zumino model). This residual divergence was removed by introducing a mass-type counterterm of the form $A\sigma_1\sigma_2$. At this point, it is important to point out, however, that the UV improvement depends on the exact cancellation of the “dangerous” quadratic divergences between the bosonic and fermionic contributions. It is not clear for us if this cancellation holds at higher orders of the perturbation expansion. So, a further study is necessary to clarify it. Finally, the construction of gauge susy theories of the Lifshitz type is still a challenge and an open field of research. In particular, it seems interesting to seek the Lifshitz-like version of the relativistic higher-derivative SQED$_3$ constructed in [23].

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Appendix: Lifshitz superspace in three dimensions

The Lifshitz superspace in its simplest way is parameterized, as in the standard case, by three bosonic coordinates $x^\mu$ and two fermionic (Grassmann) coordinates $\theta^\alpha$. Given that the time and space coordinates in Lifshitz field theories are weighted differently, we must split the Lifshitz superspace in two sectors, one generated by the time supercharge $Q_{t\alpha}$ and the other by the space supercharge $Q_{s\alpha}$:

$$2Q_{t\alpha} = i\partial_\alpha + (\gamma^0)_{\alpha\beta} \theta^\beta \partial_0$$
$$2Q_{s\alpha} = i\partial_\alpha + a (\gamma^i)_{\alpha\beta} \theta^\beta \partial_i. \tag{A.1}$$

These supercharges in turn allow us to introduce two derivatives $D_{t\alpha}$ and $D_{s\alpha}$ as in (27) by demanding the anticommutativity of these with the respective supercharges: $\{D_{t\alpha}, Q_{t\beta}\} = 0$ and $\{D_{s\alpha}, Q_{s\beta}\} = 0$. It is important to note that these supercharges do not realize the susy algebra (22), and so they are not covariant in the entire Lifshitz superspace. The supercharge $Q_\alpha$ that realizes (22) is

$$Q_\alpha = Q_{t\alpha} + Q_{s\alpha} = i\partial_\alpha + (\gamma^0)_{\alpha\beta} \theta^\beta \partial_0 + a (\gamma^i)_{\alpha\beta} \theta^\beta \partial_i, \tag{A.2}$$

and the covariant derivative $D_\alpha$ with regard to it is given by (28):

$$\{D_\alpha, Q_\beta\} = 0.$$  

The susy transformations (3) in superfield terms can be encapsulated in the equation

$$\delta \Phi = i\epsilon^\alpha Q_{t\alpha} \Phi + i\epsilon^\alpha Q_{s\alpha} \Phi = i\epsilon^\alpha Q_\alpha \Phi. \tag{A.3}$$

In this superspace formulation, the projection technique can be implemented in three completely equivalent ways. In fact, considering the scalar superfield $\Phi$ in (26), it is easy to show that

$$\varphi = \Phi \big|, \quad \psi_\alpha = 2D_{t,s\alpha} \Phi \big| = D_\alpha \Phi \big|, \quad F = 4D^2_{t,s} \Phi \big| = D^2 \Phi \big| \tag{A.4}$$

where the vertical bar $| \big|$ means evaluation at $\theta = 0$. Using the projection technique, one gets

$$\int dt^2 dx^2 \partial_0 \mathcal{L} = \int dt^2 dx \left(4D^2_{t,s} \mathcal{L}\right) \big| = \int dt^2 dx D^2 \mathcal{L} \big|. \tag{A.5}$$

The 'covariant' derivatives satisfy the following identities:

$$\{D_{t\alpha}, D_{t\beta}\} = i(\gamma^0)_{\alpha\beta} \partial_0 \quad \{D_{s\alpha}, D_{s\beta}\} = ia (\gamma^i)_{\alpha\beta} \partial_i.$$
\{D_{\alpha}, D_{\beta}\} = 2i (\gamma^0)_{\alpha\beta} \partial_0 + 2ai (\gamma^i)_{\alpha\beta} \partial_i

\begin{align*}
D_t^2 D_{\alpha} &= - D_{\alpha} D_t^2 = i (\gamma^0)_{\alpha\beta} \partial_0 D^\beta_t \\
D_s^2 D_{\alpha} &= - D_{\alpha} D_s^2 = \frac{a_i}{2} (\gamma^i)_{\alpha\beta} \partial_i D^\beta_s
\end{align*}

(A.6)

\begin{align*}
(D_t^2)^2 &= \frac{1}{4} \partial_0^2 \\
(D_s^2)^2 &= \frac{a^2}{4} \Delta \\
(D^2)^2 &= \partial_0^2 + a^2 \Delta,
\end{align*}

where \(\partial_0^2 = \partial^0 \partial_0\) and \(\Delta = \partial^i \partial_i\).

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