Pilot Reuse Mode Based on Continuous Pilot Reuse Factors

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Massive multiple-input multiple-output (MIMO) techniques have attracted wide attention as one of the key technologies of 5G. Pilot reuse based on the same pilot sequence is necessary among different users due to limited pilot resources. In this study, a pilot reuse mode based on continuous pilot reuse factors is proposed to pursue a flexible pilot reuse mode with high spectral efficiency (SE). In this mode, users in every cell are grouped, and different groups use different pilot reuse factors. Therefore, any overall pilot reuse factor is achieved to increase the flexibility of pilot reuse considerably. A theoretical analysis proves that the proposed pilot reuse based on continuous pilot reuse factors is superior to the traditional pilot reuse based on single pilot reuse factor to some extent in terms of SE. A new method to search the optimal pilot reuse based on continuous pilot reuse factors is also introduced. Simulation results demonstrate that, in most cases, the optimal pilot reuse mode based on continuous pilot reuse factors is better than the traditional mode. Such superiority still exists under a limited number of antennas.

1. Introduction

The emerging mobile communication technology has driven the shift from traditional to mobile and has brought the Internet to every terminal [1, 2]. With the rapid development of the mobile Internet, the number of mobile terminals is rapidly increasing, and the communication system has an increasing demand for data transmission rates.

Massive multiple-input multiple-output (MIMO) technology [3] realizes the goal of decreasing noises and user interferences by increasing antennas in base stations. Bjornson et al. [4] stated that noises and interferences from users in a cell approach an inconsiderable size when the number of antennas is more than 10 times the quantity of user terminals. The authors disclosed that system performance is determined by interferences among users who use the same pilot sequence. Under this circumstance, a reasonable pilot reuse mode can improve the homogeneity and SE of the system.

System capacities under different pilot reuse factors (3, 4, and 7) have been compared in recent works. The optimal pilot reuse factor is 7 in dense urban deployment and 3 in suburb deployment [5]. Zhu et al. [6] proposed a soft pilot reuse (SPR). SPR divides users in a cell into central and marginal user groups. The former one uses the same pilot reuse, whereas the latter one uses mutually orthogonal pilot reuse, which relieves the pilot contamination of marginal users effectively. A hierarchical pilot reuse mode that applies different pilot reuse factors to users in different levels is proposed [7]. This mode relieves the pilot contamination and increases the net throughput capacity of the system. Zhu et al. [8] proposed a pilot allocation algorithm based on coalitional game theory to reduce channel estimation errors, which is far better than random pilot reuse. Chang et al. [9] designed the hazard- and secure-edge regions to manage the pilot reuse to suppress the increment of interference.

However, pilot reuse factors are limited within a group of specific integers \( U = \{1, 3, 4, 7, 9, 12, \ldots \} \) [10], which is attributed to the limitations of the traditional cell in honeycomb structure as a hexagon. Pilot reuse factors can neither be integers nor be nonintegers out of the range of the group \( U \), thereby restricting the flexibility of pilot reuse considerably. For this reason, a pilot reuse mode based on continuous pilot reuse factors is proposed. Pilot reuse factors of users are determined by a certain probability; hence, pilot
reuse mode based on any pilot reuse factors can be achieved theoretically. The optimal pilot reuse factor under this pilot reuse mode is disclosed.

The main contributions of our paper are summarized as follows:

1. Firstly, we propose a pilot reuse mode based on continuous pilot reuse factor, which reduces pilot pollution and improves spectral efficiency. The mode groups users and assigns different pilot reuse factors to different groups, so it achieves an arbitrary pilot reuse factor, which greatly improves the flexibility of pilot reuse.

2. Secondly, we carry out detailed modeling and analysis of pilot reuse schemes based on continuous pilot reuse factors and prove that this mode has advantages in terms of spectrum efficiency compared to traditional pilot multiplexing methods based on single pilot reuse factor.

3. Lastly, under the pilot reuse mode based on continuous pilot reuse factors, the optimal continuous pilot reuse factor is derived varying in the number of users and frame length. Particularly, the specific method of achieving the optimal solution is also described in detail.

2. System Introduction

A multiuser massive MIMO system with time-division duplex protocol comprises L cells, and each cell is equipped with one base station with M antennas in the cell center. All base stations are hypothesized to have the same performance, and each base station can serve up to K users the most. Each user is equipped with one antenna. Users in each cell are divided into central and marginal user groups. The former uses the same orthogonal pilot sequence, whereas the latter applies another group of orthogonal pilot sequence. The two pilot sequence sets allocated to the two groups are mutually orthogonal.

Channel state information (CSI) between the base station and users is expressed by the overlapping of large- and small-scale fading [11]. \( h_{jk} \) denotes the CSI between base station j and user k in cell l. \( h_{jk} \sim CN(0, \beta_{jk} I_M) \), where \( \beta_{jk} \) is the variance of channel fading and \( I_M \) is the M-order unit matrix:

\[
\beta_{jk} = \frac{C}{r_{jk}},
\]

where \( r_{jk} \) represents the distance between user k in cell l and base station j, \( y \) is the path loss exponent, and C is a fixed parameter:

\[
SE_k = \left(1 - \frac{B}{S}\right) \log_2(1 + SIR_k),
\]

where \( B > 0 \) out of the \( S \) symbols in each frame are reserved for uplink pilot signaling. The remaining \( S - B \) symbols are allocated for payload data. Then, the signal-to-interference ratio is given as

\[
SIR_k = \frac{\beta_{jk}^2}{\sum_{l \in \Phi_k} \beta_{jk}}.
\]

Equation (1) is integrated into equation (3), which yields

\[
SIR_k = \frac{1}{\sum_{l \in \Phi_k} (r_{jk}/r_{jk})^\gamma}.
\]

The distance between the users who produce the first layer of interference of user k with \( \lambda \) pilot reuse and user k can be calculated as \( r_{jk} = \sqrt{3\lambda} \times R \) [7]. For the target user k at the cell edges, \( r_{jk} = R \). Therefore, equation (4) can be rewritten as

\[
SIR_{k,\lambda} = \left(\frac{\sqrt{3\lambda}}{6}\right)^\gamma.
\]

where \( SIR_{k,\lambda} \) is the signal-to-interference ratio (SIR) produced when user k in the target cell applies \( \lambda \) pilot reuse factor.

As shown in Figure 1, the SIR calculated from equation (5) is lower than the actual value. This difference is caused by the hypothesis that users in the target cell are at the cell margins. Hence, equation (5) is a lower limit of \( SIR_{k,\lambda} \).

3. Pilot Reuse Mode Based on Continuous Pilot Reuse Factors

Traditional pilot reuse uses the same pilot sequence in different cells, and the single pilot reuse factor determines the degree of reuse of the pilot sequence. Such pilot reuse mode treats all uses in one cell equally and restricts the value of pilot reuse factor within a certain group of specific integers.

The pilot reuse factor is defined as the ratio between the number of orthogonal pilot sequences and the number of users in the unit cell:

\[
\lambda = \frac{N_{\text{pilot}}}{K} = \frac{B}{K}
\]

Different from the traditional pilot reuse mode, users in one cell are treated differently in pilot reuse and use different pilot reuse factors. Cell l has \( k_l \) users with a pilot reuse factor of \( \lambda \), which indicates that the reuse probability of \( \lambda \) is \( \theta_l = k_l/K \). From equation (6), we can see that \( N_{\text{pilot}} = K \times \lambda \) if we use a fixed pilot reuse factor. If different pilot reuse factors are used simultaneously, \( N_{\text{pilot}} = \sum_\lambda \lambda k_\lambda \) must exist. Under this circumstance, the pilot reuse factor of cell l that can be gained is

\[
\lambda_l = \frac{\sum_\lambda \lambda k_\lambda}{K} = \sum_\lambda \lambda \theta_l.
\]

Equation (7) implies that although the value of \( \lambda \) has a limited range, any pilot reuse factor \( \lambda_l \) can be gained by changing the value of \( \theta_l \). The SE of unit user under \( \lambda_l \) is

\[
SE = \left(1 - \frac{\lambda_l K}{S}\right) \sum_\lambda \theta_l \log_2(1 + SIR_{k,\lambda}).
\]
On the basis of the preceding systematic analysis of pilot reuse based on continuous pilot reuse factors, the manner in which the optimal pilot reuse is implemented under different K/S is introduced as follows. Accordingly, some definitions are proposed.

**Definition 1.** \( \Lambda = \{1, 3, 4, 7, 9, 12, 13, \ldots \} \) is the optional value of pilot reuse factor. \( [\lambda_i] \in \Lambda \) is defined as the maximum value smaller than \( \lambda_i \) and \( [\lambda_i]_\ast \in \Lambda \) is defined as the minimum value higher than \( \lambda_i \).

**Definition 2.** \( g(\lambda) = \log_2 (1 + (\sqrt{3\lambda})^y/6) \) and \( \omega(\lambda) = (1-\Lambda(K/S))\log_2 (1 + ((\sqrt{3\lambda})^y/6)) \) are important immediate functions. Notably, the value of \( \omega(\lambda) \) is equal to the SE of unit users when the single pilot reuse factor is applied; that is, \( \text{SE} = K\omega(\lambda) \).

**Definition 3.** \( A = K/S \). Given the constant path loss exponent, the ratio between the number of users and the coherent blocks is the only factor that determines the optimal pilot reuse strategy. This condition will be discussed in the subsequent analysis.

**Theorem 1.** If the function \( \omega(\lambda) = (1 - \lambda A)\log_2 (1 + ((\sqrt{3\lambda})^y/6)) \) reaches the maximum at \( \lambda = \lambda_i \), then the optimal solution of SE is composed of \( [\lambda_i]_\ast \) and \( [\lambda_i]_\ast \).

\[ (\theta_o, [\lambda_i]_\ast, 1 - \theta_o, [\lambda_i]_\ast) \text{ is the optimal pilot reuse mode based on continuous pilot reuse factors when } \theta_o = A([\lambda_i]_\ast - [\lambda_i]_\ast)g(([\lambda_i]_\ast) - (1 - A([\lambda_i]_\ast)(g([\lambda_i]_\ast) - g([\lambda_i]_\ast))/2A([\lambda_i]_\ast - [\lambda_i]_\ast)(g([\lambda_i]_\ast) - g([\lambda_i]_\ast))) \text{ and } \theta_o \in [0, 1]. \]

Then the optimal pilot reuse mode is based on the single pilot reuse factor, and \( \lambda = \arg \max \{\text{SE}(\lambda_i)_\ast, \text{SE}(\lambda_i)_\ast\} \).

Theorem 1 provides a theoretical basis for searching the optimal pilot reuse mode under different values of \( V \). This theorem implies that (1) the maximum point of function \( \omega(\lambda) \) may not be the optimal continuous pilot reuse factor; (2) the optimal continuous pilot reuse factor is between two adjacent optional values of \( \lambda \in \Lambda \) before and after the maximum point of function \( \omega(\lambda) \); and (3) the optimal continuous pilot reuse factor may be the same as the traditional pilot reuse factor, which is related to the specific values of \( A \) and \( \lambda_i \).

To prove the accuracy of Theorem 1, the following lemmas are defined.

**Lemma 1.** The domain of definition of function \( g(\lambda) \) is \( (\lambda \geq 1) \). Any three points \( \lambda_n < \lambda_s < \lambda_n \) exist in the domain of definition, and one \( \theta (0 < \theta < 1) \) exists to make \( \theta \times g(\lambda_n) + 1 - \theta g(\lambda_s) < g(\lambda_s) \) must exist.

**Proof.**

\[ g'(\lambda) = \frac{\sqrt{3}y}{12\ln 2} \times \frac{(\sqrt{3\lambda})^{y-1}}{\sqrt{\lambda}((1 + (\sqrt{3\lambda})^y/6))} > 0. \]

Let \( f(\lambda) = 1 + ((\sqrt{3\lambda})^y/6) \) and \( h(\lambda) = (\sqrt{3\lambda})^{y-1} \); then, we obtain the following equation:

\[ g''(\lambda) = \frac{\sqrt{3}y}{12\ln 2} \frac{(h'(\lambda)\sqrt{\lambda} f(\lambda) - h(\lambda)((1/2\sqrt{\lambda}) f(\lambda) + \sqrt{\lambda} f'(\lambda)))}{\lambda(f(\lambda))^2}. \]

Given \( g''(\lambda) > 0 \) and \( g''(\lambda) < 0 \) are constantly true, function \( g(\lambda) \) is a strict monotone increasing concave function. According to the definition of the concave function, \( \theta \times g(\lambda_n) + 1 - \theta g(\lambda_s) < g(\lambda_s) \) is true.

**Lemma 2.** Function \( g(\lambda) \) has four points \( \lambda_1 < \lambda_s < \lambda_2 < \lambda_3 \) in the domain of definition. For any \( (\theta_1, \theta_2) \) and \( (\theta_1', \theta_2') \) that make \( \theta_1 \lambda_1 + \theta_2 \lambda_2 = \lambda_3 \), \( \theta_1' \lambda_1 + \theta_2' \lambda_2 = \lambda_3 \) and \( \theta_1' + \theta_2' = 1 \), then \( \theta_1' g(\lambda_1) + \theta_2' g(\lambda_2) + \theta_2' g(\lambda_3) < \theta_1 g(\lambda_1) + \theta_2 g(\lambda_3) < g(\lambda_3) \) is true.
Lemma 3. Function $g(\lambda)$ has four points $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$ in the domain of definition. For any $(\hat{\theta}_1, \hat{\theta}_2)$ and $(\theta'_1, \theta'_2, \theta'_3)$ that make $\theta_1 \lambda_1 + \theta_2 \lambda_2 = \lambda_4$ and $\theta'_1 \lambda_1 + \theta'_2 \lambda_2 + \theta'_3 \lambda_3 = \lambda_4$, true,

then $\theta'_1 g(\lambda_1) + \theta'_2 g(\lambda_2) + \theta'_3 g(\lambda_3) < \theta_1 g(\lambda_2) + \theta_2 g(\lambda_3) < g(\lambda_4)$ is true.

Proof. One $\lambda_e (\lambda_2 < \lambda_e < \lambda_3)$ that makes $\theta'_2 \lambda_2 + \theta'_3 \lambda_3 = (\theta'_1 + \theta'_2) \lambda_e$ true must exist. According to Lemma 1, we have $(\theta'_1 \theta'_2 \theta'_3) g(\lambda_2) + (\theta'_1 \theta'_2) g(\lambda_3) < g(\lambda_4)$.

Multiply both sides by $(\theta'_1 + \theta'_2)$, and add $\theta'_1 g(\lambda_1)$ to both sides to get $\theta'_1 g(\lambda_1) + \theta'_2 g(\lambda_2) + \theta'_3 g(\lambda_3) < \theta_1 g(\lambda_2) + (\theta'_1 + \theta'_2) g(\lambda_3)$.

Let $\theta'_1 = \theta_1, \theta'_2 + \theta'_3 = \theta_2$. Notice that $g(\lambda_1)$ is a strict monotone increasing function; $g(\lambda_2) < g(\lambda_3)$ is true.

Therefore, $\theta'_1 g(\lambda_1) + \theta'_2 g(\lambda_2) + \theta'_3 g(\lambda_3) < \theta_1 g(\lambda_2) + \theta_2 g(\lambda_3) < g(\lambda_4)$ is true. □

Lemma 4. Function $g(\lambda)$ has four points $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$ in the domain of definition. For any $(\hat{\theta}_1, \hat{\theta}_2)$ and $(\theta'_1, \theta'_2, \theta'_3)$ that make $\theta_1 \lambda_1 + \theta_2 \lambda_2 = \lambda_3$ and $\theta'_1 \lambda_1 + \theta'_2 \lambda_2 + \theta'_3 \lambda_3 = \lambda_3$, true,

then $\theta'_1 g(\lambda_1) + \theta'_2 g(\lambda_2) + \theta'_3 g(\lambda_3) < \theta_1 g(\lambda_2) + \theta_2 g(\lambda_3) < g(\lambda_4)$ is true.

Proof. One $\lambda_e (\lambda_2 < \lambda_e < \lambda_3)$ that makes $\theta'_1 \lambda_1 + \theta'_2 \lambda_2 + \theta'_3 \lambda_3 = (\theta'_1 + \theta'_2) \lambda_e$ true must exist. According to Lemma 1, we have $(\theta'_1 \theta'_2 \theta'_3) g(\lambda_2) + (\theta'_1 \theta'_2) g(\lambda_3) < g(\lambda_4)$.

Multiply both sides by $(\theta'_1 + \theta'_2)$, and add $\theta'_1 g(\lambda_1)$ to both sides to get $\theta'_1 g(\lambda_1) + \theta'_2 g(\lambda_2) + \theta'_3 g(\lambda_3) < \theta_1 g(\lambda_2) + (\theta'_1 + \theta'_2) g(\lambda_3)$.

Let $\theta'_1 = \theta_1, \theta'_2 + \theta'_3 = \theta_2$. Notice that $g(\lambda_1)$ is a strict monotone increasing function; $g(\lambda_2) < g(\lambda_3)$ must be true.

Therefore, $\theta'_1 g(\lambda_1) + \theta'_2 g(\lambda_2) + \theta'_3 g(\lambda_3) < \theta_1 g(\lambda_2) + \theta_2 g(\lambda_3) < g(\lambda_4)$ is true. □

Hypothesis 1. A pilot reuse mode containing three pilot reuse factors $\lambda_e = \theta_1 \lambda_1 + \theta_2 \lambda_2 + \theta_3 \lambda_3 (\lambda_1 < \lambda_2 < \lambda_3)$ that makes $\text{SE}_e = K(1 - (\lambda_e, K/S)) (\theta_1 g(\lambda_1) + \theta_2 g(\lambda_2) + \theta_3 g(\lambda_3))$ reach the maximum exists.

If $\lambda_1 < \lambda_2 < \lambda_3$, then a group of $(\theta'_1, \theta'_2)$ must exist to achieve $(\theta'_1 + \theta'_2) \lambda_e = \lambda_e$. According to Lemma 2, $\theta'_1 g(\lambda_1) + \theta'_2 g(\lambda_2) + \theta_3 g(\lambda_3) < \theta'_1 g(\lambda_1) + \theta_3 g(\lambda_3)$.

Therefore, $\text{SE}_e < K(1 - (\lambda_e, K/S)) (\theta'_1 g(\lambda_1) + \theta'_2 g(\lambda_2))$.

In other words, the pilot reuse mode of $\theta'_1 \lambda_1 + \theta'_2 \lambda_2$ is better than that of $\theta_1 \lambda_1 + \theta_2 \lambda_2 + \theta_3 \lambda_3$, which disagrees with Hypothesis 1. Thus, Hypothesis 1 is false.

Clearly, if $\lambda_2 < \lambda_3$, then similar conclusions can be gained according to Lemma 3.

Similarly, any pilot reuse mode that contains more than three pilot reuse factors must be lower than at least one mode with two pilot reuse factors.

Hence, the optimal pilot reuse mode of $\text{SE}$ must be a continuous pilot reuse mode with only two pilot reuse factors.

Hypothesis 2. A continuous pilot reuse mode with two nonadjacent pilot reuse factors $\lambda_e = \theta_m \lambda_m + \theta_n \lambda_n, \lambda_m < \lambda_n$ is supposed to exist to achieve the maximum value of $\text{SE}_e = K(1 - (\lambda_e, K/S)) (\theta_m g(\lambda_m) + \theta_n g(\lambda_n))$, and $\lambda_m$ and $\lambda_n$ are not adjacent, and $\lambda_o$ exists between $\lambda_m$ and $\lambda_n$; that is, $\lambda_m < \lambda_o < \lambda_n$.

If $\lambda_m < \lambda_o < \lambda_n$, then, according to Lemma 4, we yield $\theta_m g(\lambda_m) + \theta_n g(\lambda_n) < \theta_m g(\lambda_m) + \theta_n g(\lambda_n)$.

Therefore, $\text{SE}_e < K(1 - (\lambda_e, K/S)) (\theta'_m g(\lambda_m) + \theta'_n g(\lambda_n))$, which is contradictory to Hypothesis 2.

If $\lambda_m < \lambda_o < \lambda_n$, then similar conclusion can be gained.

Therefore, the pilot reuse mode based on two adjacent pilot reuse factors is better than all modes with two pilot reuse factors.

In other words, the optimal pilot reuse mode of $\text{SE}$ must be a continuous pilot reuse mode with two adjacent $\lambda$.

Now, we prove that the optimal pilot reuse mode of $\text{SE}$ must be composed of $[\lambda_1]$ and $[\lambda_2]$.

Proof. If two adjacent $\lambda_m$ and $\lambda_n (\lambda_m < \lambda_n)$ exist to make the optimal solution of $\omega(\lambda)$ that comprises $\lambda_m$ and $\lambda_n$, then two situations, $[\lambda_1] < \lambda_m < \lambda_n$ and $\lambda_m < \lambda_n < [\lambda_1]$, occur.

For the first situation, $\text{SE} = (1 - \lambda_A) \theta_1 \log_2 (1 + ((\sqrt{3} \lambda_m)^{1/2})) + \theta_2 \log_2 (1 + ((\sqrt{3} \lambda_n)^{1/2})) < (1 - \lambda_A) \log_2 (1 + ((\sqrt{3} \lambda_m)^{1/2}) < (1 - [\lambda_1]) \lambda_A \log_2 (1 + ((\sqrt{3} [\lambda_1])^{1/2})$. Hence, the SE is lower than that of pilot reuse based on $[\lambda_1]$ alone.
Similarly, the SE in Situation 2 is lower than that based on \([\lambda_i]_1\) alone. Hence, the solution composed of \(\lambda_m\) and \(\lambda_r\) cannot be the optimal solution.

The effective value range of pilot reuse factor is \(1 \leq \lambda < (S/K)\). The physical importance of this value range is to prevent pilot interference among intracell users and prevent using all time-frequency resources to send pilot signals:

\[
\omega''(\lambda) = -2Ag'(\lambda) + (1 - A\lambda)g''(\lambda) < 0. \quad (12)
\]

Hence, function \(\omega(\lambda)\) has one extreme point at most in the domain of definition.

1. If function \(\omega(\lambda)\) has no extreme point in the domain of definition, then it must be a monotone decreasing function.

\[
SE = [1 - A[\lambda_i]_1 + A([\lambda_i]_1 - [\lambda_i]_1)\theta][g([\lambda_i]_1) + (g([\lambda_i]_1) - g([\lambda_i]_1))\theta],
\]

\[
\frac{dSE}{d\theta} = \frac{A([\lambda_i]_1 - [\lambda_i]_1\theta)(g([\lambda_i]_1) - g([\lambda_i]_1)) - (1 - A[\lambda_i]_1)(g([\lambda_i]_1) - g([\lambda_i]_1))}{2A([\lambda_i]_1 - [\lambda_i]_1\theta)(g([\lambda_i]_1) - g([\lambda_i]_1))}.
\]

Under this circumstance, \(\theta = \theta_o\). Notably, the value range of \(\theta\) should be \([0, 1]\). If \(\theta_o \in [0, 1]\), then \(\theta_o\) is the maximum point. If \(\theta_o \notin [0, 1]\), then the maximum is achieved at two ends; that is, \(\lambda = \arg\max SE([\lambda_i]_1), SE([\lambda_i]_1)\).

On the basis of the preceding analysis, the searching of the optimal pilot reuse mode based on continuous pilot reuse factors can be described via the following steps:

1. The values of the number of input users \((K)\) and the length of coherence block \((S)\) and integer \((\lambda)\) are determined.
2. The extreme point of function \(\omega(\lambda)\) is calculated, thus obtaining \(\lambda_i\).
3. If \(\lambda_i\) is not in the domain of definition \([1, S/K]\), then the optimal pilot reuse mode is the mode that all users use the same pilot reuse factor with a value of 1, that is, \(\lambda = 1\). The algorithm is ended.
4. If \(\lambda_i\) is in the domain of definition, then two integers are selected from the values of \(\lambda\). The interval between the two integers is the minimum interval that covers \(\lambda_i\). The end point is denoted as \([\lambda_i]_1, [\lambda_i]_1\).
5. \(\theta_o\) is calculated from equation (15).
6. If \(\theta_o \in (0, 1)\), then \((\theta_o [\lambda_i]_1, (1 - \theta_o) [\lambda_i]_1)\) is the optimal pilot reuse mode based on continuous pilot reuse factors. The algorithm is ended.
7. If \(\theta_o \notin (0, 1)\), then the optimal pilot reuse mode is \(\lambda = \arg\max SE([\lambda_i]_1), SE([\lambda_i]_2]\). The algorithm is ended.

Given \(\omega(1) = 1.184 \times (1 - A) > 0\) and \(\omega(S/K) = 0\), \(1 < (S/K)\) exists. Under this circumstance, the optimal pilot reuse mode is \(\lambda = 1\).

1. If the function has extremum in \([1, S/K]\), then this extreme point must be the maximum. According to Theorem 1, the optimal pilot reuse mode only contains \([\lambda_i]_1\) and \([\lambda_i]_2\).

For the continuous pilot reuse mode that contains \([\lambda_i]_1\) and \(\ldots\), equation (13) and the derivative of \(SE\) with respect to \(\theta\) expressed as equation (14) can be obtained. If we set \(dSE/d\theta\) equal to 0, we get the extremum expressed as equation (15):

4. Results

This section may be divided into subheadings. It should provide a concise and precise description of the experimental results and their interpretation as well as the experimental conclusions that can be drawn.

When function \(g(\lambda)\) is proven to be in the domain of definition \(\lambda \geq 1\), the situations at \(y = 1.6, 2.0, 3.7, 6.0\) are discussed. The values of \(y\) are consistent with the actual situation (Table 1).

As shown in Figure 2, the value of \(g(\lambda) = \log_2(1 + ((\sqrt{3} A) / 6))\) decreases with \(y\). The concavity of \(g(\lambda)\) can be observed regardless of how \(y\) changes.

From Figure 3, \(\omega(\lambda)\) always goes up and then it goes down with any value of \(K\). This is consistent with the above theoretical analysis. There is only one extreme value in the definition \([1, S/K]\), and \(\omega(\lambda)\) is equal to zero when \(\lambda = S/K\). Table 2 shows the extreme value of \(\lambda\) when \(K/S\) (part) is given. It is useful in situations where we do not have to find exact extremum. For example, in step (2) of the method searching of the optimal pilot reuse mode, the exact value of extremum is not needed, and only the two conventional pilot reuse factors between which extremum points are located need to be known.

In Figure 4, the advantages of pilot reuse based on continuous pilot reuse factors are mainly manifested at turning points of the optimal single pilot reuse mode.

Changes in the optimal continuous pilot reuse factors with \(K/S\) are shown in Figure 5. Specifically, the optimal solution of the single pilot reuse mode presents sudden
changes, whereas the curve of the continuous pilot reuse mode is continuous. The advantages of the pilot reuse strategy based on continuous pilot reuse factors are surrounding the sudden changes in the single pilot reuse mode.

According to the analysis from Figures 4 and 5, the advantages of the continuous pilot reuse mode can be perceived intuitively. However, the preceding simulation

| Mobile radio environment | Path loss exponent, $y$ |
|--------------------------|------------------------|
| Free space               | 2                      |
| Flat rural area          | 3                      |
| Typical urban area       | 2.7–3.5                |
| Building visual distance conditions | 1.6–1.8             |
| Building barrier conditions | 4–6                   |
| Typical mobile radio environment | 3.7–4               |

Table 1: (Partial) typical values of path loss exponent under different environments [12].

| $K/S$ | $\lambda$ |
|-------|------------|
| 0.05  | 6.5200     |
| 0.10  | 3.8355     |
| 0.15  | 2.8169     |
| 0.20  | 2.2606     |
| 0.25  | 1.9029     |
| 0.30  | 1.6506     |
| 0.35  | 1.4613     |
| 0.40  | 1.3133     |
| 0.45  | 1.1939     |
| 0.50  | 1.0952     |

Table 2: Extreme points (partial) of $\omega(\lambda)$ (solutions when $\omega'(\lambda) = 0$).

Figure 2: Variation curve of $g(\lambda)$ under different path losses $y$.

Figure 3: Variation curves of $\omega(\lambda)$ under different numbers of users $y = 3.7$, $S = 200$, and $K = 10:10:50$.

Figure 4: Maximum SE under different $K/S$ values.

Figure 5: Optimal pilot reuse factors under different $K/S$ values.
analysis hypothesizes that an infinite number of antennas exist. The situation under limited antennas is analyzed in the following text. The relevant simulation parameters are listed in Table 3.

Given limited antennas, the pilot reuse mode based on continuous pilot reuse factors is certainly superior to the single pilot reuse mode (Figure 6). Such superiority increases with the number of antennas.

### 5. Conclusions

In this study, a continuous pilot reuse mode is proposed. This mode divides users in a cell into two groups randomly. Different groups use different pilot reuse factors to gain any equivalent pilot reuse factors. A detailed theoretical analysis on the continuous pilot reuse mode is conducted, which proves its superiority to the traditional mode under certain K/S. A method for searching the optimal continuous pilot reuse mode is also introduced. The simulation analysis proves that the proposed pilot reuse mode is superior to the traditional mode to some extent under infinite and limited antennas.

### Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare no conflicts of interest.

### Authors’ Contributions

Dongsheng Han studied the synchronization problem of the target-tracking system. Kai Fu wrote the first draft of the manuscript. Dongsheng Han and Zhixiong Chen were involved in the discussion of the problem and revised the manuscript. All authors read and approved the final version of the manuscript.

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