Contextuality lays at the heart of quantum mechanics. In the prevailing opinion it is considered as a signature of “quantumness” that classical theories lack. However, this assertion is only partially justified. Although contextuality is certainly true of quantum mechanics, it can not be taken by itself as discriminating against classical theories. Here we consider a representative example of contextual behaviour, the so-called Mermin-Peres square, and present a simple discrete model which faithfully reproduces quantum predictions that lead to contradiction with the assumption of non-contextuality. This illustrates that limited information gain and measurement disturbance provide enough means for reconstruction of quantum-like contextual effects in classical systems too.

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I. INTRODUCTION

Weirdness of quantum mechanics is usually presented by way of contradiction with simple classical intuitions that we hold about the world. Many of these paradoxes are often raised in the debates on the interpretation of the theory, and in particular the possibility of its hidden variable account. There are two major no-go theorems to the effect that such a description is subject to severe constraints: the Bell’s theorem shows violation of locality [1, 2], and the Kochen-Specker theorem contradicts the premise of non-contextuality [3, 4]. Clearly, this presents a challenge to our intuitive understanding of quantum theory and sets the bar high for hidden variable models [5, 6]. In this work, we are concerned with quantum contextuality which, in a nutshell, says that there is no consistent assignment of values to quantum mechanical observables and whose objective existence is independent of the context of other observables that are being simultaneously measured. This surprising result does not quite fit in with our naive conception of the act of measurement revealing value of the observable irrespective of whether or not other quantities are also being observed. Are we then bound to draw the conclusion after Asher Peres that “unperformed experiments have no results”? If so, how to answer the David Mermin’s dramatic question: “Is the moon there when nobody looks?”? It is not clear what is a good way out of this conundrum and, in particular, what form acceptable hidden variable models could take to that effect. Certainly, one should seriously reflect on the John Bell’s dictum "... what is proved by impossibility proofs is lack of imagination". It might suggest that perhaps revision of the relation between the concept of observable and measurement is required for better understanding of the theory. In this article, we explore this possibility and show that careful distinction between these concepts opens the way for endorsement of contextual effects in classical systems too.

Difficulty in making sense of contextuality in classical terms often prompts to consider it as a signature distinguishing between quantum and classical realms. Indeed, the possibility of contextual hidden variable models aiming at reconstruction of quantum predictions is hardly explored. On the other hand, the hypothesis of non-contextual hidden variable models has been thoroughly investigated and proved to be directly testable [7, 8]. In particular, many state-independent quantum-contextuality experiments have been recently performed e.g. with trapped ions [9], photons [10, 11] and magnetic-resonance systems [12]. Essentially all of them boil down to checking of the pattern in the so called Mermin-Peres square [13, 14]. Certainly, these results provide compelling evidence for contextual behaviour in these experimental setups, thus pushing the project of hidden variable models to the less explored contextual camp. In this work we present a simple probabilistic model which reproduces the pattern of quantum observables considered in the Mermin-Peres square and demonstrates quantum-like contextuality in a classical bipartite system. One immediate consequence of the model is state-independent violation of contextuality inequality [7] which coincides with quantum mechanical predictions.

Many results suggest that quantum states can be understood as states of knowledge. Strong evidence in favour of this view is given, in particular, by concrete models providing classical analogues of various phenomena typically associated with strictly quantum mechanical characteristics [15–24]. Most notable in this respect is the Spekkens’ toy model [15] reproducing a surprisingly large array of effects in a simple discrete system. This work has recently sparked a lot of interest and hope for ψ-epistemic reconstructions of quantum theory, in which quantum state is essentially understood as a state of knowledge about some ontic reality subject to epistemic restrictions [25] (see also [26, 31] for discussion of various properties and structural constraints to be satisfied by such reconstructions). However, we should note that none of these models treats the problem of contextuality or non-locality in a straightforward and uncontrived manner. These features hold the stage presenting

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whether it is possible to think of an observable as having a definite (but unknown) value before it is measured and being revealed only in experiment? A theory is said to be non-contextual if such an assignment of values to all observables is possible irrespective of the context of other observables that are simultaneously measured (i.e. details of the experimental setup). For example, in the case of the Mermin-Peres square this would mean that measurement procedures \( \mathcal{R}_1 \) and \( \mathcal{C}_3 \) should reveal the same value of the phase bit \( \sigma_x \otimes \sigma_x \). In general, if quantum theory were non-contextual then it would admit assignment of values ±1 to all observables in Fig. 1. However, this is not possible if the assigned values are to preserve the relations between simultaneously measurable (commuting) observables. To reach the contradiction in the Mermin-Peres square it is enough to observe that the product of observables in each row and column is equal ±1. This leads to logical inconsistency since the product of all nine values in the square calculated row by row equals equal to +1. This means that, quantum theory is contextual. This means that, the value of a quantum-mechanical observable nontrivially depends on what commuting set it is actually being measured with (i.e. what is the experimental setup).

Contradiction attained in the Mermin-Peres square rests upon the assumption of the so-called counterfactual definiteness, which ascribes values (or properties) to observables independent of whether or not the measurement actually takes place. It rules out the possibility of reconstructing quantum mechanical predictions in terms of non-contextual hidden variables, but remains salient about less explored contextual models that are immunised against arguments of the Kochen-Specker type.

FIG. 1. The Mermin-Peres square. Demonstration of fundamental inconsistency in assignment of non-contextual values to nine observables in a bipartite system of two qubits.
III. CLASSICAL ANALOGUE OF THE MERMIN-PERES SQUARE

Having discussed the Mermin-Peres square, and how it shows contextuality of quantum observables, we proceed to the main result of this paper which is demonstration of a purely classical system that exhibits just the same behaviour. More specifically, we will describe discrete bipartite system and measurement procedures that will follow the pattern of (non)commutativity and (non)locality captured in Fig. 1. In other words, the classical model will show the same character of contextuality as the quantum case described in the Mermin-Peres square.

From now on we switch to the classical realm and conventional probabilistic setup where a system is described by probabilities (epistemic states) on a well specified sample space of ontic states (hidden variables) \[15, 25\]. We begin with definition of a bipartite system as composed of two separable components. Then, we explicitly describe local and non-local measurement procedures and pay special attention to post-measurement disturbance. This will allow for discussion of sequential measurements, compatible observables, and the meaning of measurement context. Along the way, we will demonstrate that the structure of so defined (classical) observables coincides with the Mermin-Peres square.

A. Definition of the system

Let us imagine elementary system as a cube whose vertices represent ontic states, i.e., the sample space consists of eight states \( \Omega = \{ \omega_1, ..., \omega_8 \} \). For future convenience, we will think of the cube as fixed in the cartesian reference frame with vertices at points \( x, y, z = \pm 1 \) and adopt the convention in which states are labeled by triples \( \omega = (x, y, z) \), see Fig. 2. Standard description of such a single elementary system consists in specifying its epistemic state which is a probability distribution over (ontic) states in \( \Omega \), i.e., a vector \( p \) in the probability simplex \( \Delta = \{ (p_1, ..., p_8) : \sum_{\mu=1}^8 p_\mu = 1, p_\mu \geq 0 \} \).

Now, we are in position to define a bipartite system as composed of two elementary ones, called system (1) and system (2). Then the sample space is simply the cartesian product \( \Omega^{(1 \& 2)} = \Omega^{(1)} \times \Omega^{(2)} \). It can be represented by two cubes with the joint ontic state \((\omega^{(1)}, \omega^{(2)})\) being completely specified by the ontic states \(\omega^{(i)} = (x_i, y_i, z_i)\) of the individual elementary systems, see Fig. 2. Note that it is quite appropriate to think of them as two distinct classical particles with internal degrees of freedom that can be (spatially) separated. Epistemic state of the system is described by a vector \( p \) in the probability simplex \( \Delta^{(1 \& 2)} = \Delta^{(1)} \otimes \Delta^{(2)} \). In other words, \( p = \sum_{\mu, \nu} p_{\mu \nu} p_\mu \otimes p_\nu \) is a probability distribution \( (p_{\mu \nu} \geq 0, \sum_{\mu, \nu} p_{\mu \nu} = 1) \) over the ontic states in \( \Omega^{(1 \& 2)} \), and \( p_\mu \otimes p_\nu \) corresponds to both systems being definitely in the respective ontic states \( \omega^{(1)}_\mu \) and \( \omega^{(2)}_\nu \). Note that from the construction such an account allows only for classical correlations between elementary systems.

The concept of measurement is subtle as its role is two-fold. First of all, it reveals information about the system, and secondly it effects a change leaving the system in some post-measurement state. Complete description of a measurement procedure should give account of both these aspects. Although peripheral for a single-shot experiment, post-measurement state disturbance is crucial in the analysis of sequential measurements. As we will see, it will turn out responsible for quantum-like behaviour of the probabilistic model that we discuss below.
B. Local measurements

We begin by defining three kinds of measurements on the elementary system which tell on which of the chosen pair of opposite faces of the cube the system resides:

Measurement X: front (x = +1) vs. back (x = −1),
Measurement Y: right (y = +1) vs. left (y = −1),
Measurement Z: up (z = +1) vs. down (z = −1).

In other words, a measurement reveals one bit of information about one of the coordinates of the system’s ontic state ω = (x, y, z). To complete the definition we assume that after the measurement the system is left with equal probability in one of the four compatible ontic states. See Fig. 8 for schematic illustration. More precisely, for measurement X depending on the result x = ±1 (front/back) the measurement effects the change ω → p_{x±} ∈ Δ, where

\[ p_{x+} : P(\omega = (x, y, z)) = \frac{1}{4}, \quad P(\omega = (x, y, -z)) = 0, \]
\[ p_{x-} : P(\omega = (x, y, z)) = 0, \quad P(\omega = (x, y, -z)) = \frac{1}{4}. \]

Similarly, upon measuring Y = ±1 (right/left) the system gets disturbed as follows ω → p_{y±} ∈ Δ, where

\[ p_{y+} : P(\omega = (x, y, 1)) = \frac{1}{4}, \quad P(\omega = (x, y, -1)) = 0, \]
\[ p_{y-} : P(\omega = (x, y, 1)) = 0, \quad P(\omega = (x, y, -1)) = \frac{1}{4}. \]

Finally, in result of measuring \(Z = \pm 1\) (up/down) we are left with ω → p_{z±} ∈ Δ, where

\[ p_{z+} : P(\omega = (x, y, 1)) = \frac{1}{4}, \quad P(\omega = (x, y, -1)) = 0, \]
\[ p_{z-} : P(\omega = (x, y, 1)) = 0, \quad P(\omega = (x, y, -1)) = \frac{1}{4}. \]

In short, a measurement reveals one of the coordinates and completely randomises the remaining ones. Note that since post-measurement states are also eigenstates of the respective measurement procedures this definition guarantees repeatability of measurements of the same kind. Furthermore, due to measurement disturbance no further information gain about the system is possible. As an aside we remark that so described system can be shown equivalent to a constrained version of a qubit restricted to the convex hull of stabilizer states, Clifford transformations and Pauli observables; see [22] for a complete account.

For the bipartite system above defined measurements may be performed independently on each component. Since these procedures do not affect the (possibly distant) system we call them local measurements. In total, we have \(3 \times 3 = 9\) possible measurement arrangements that will be denoted by \(A_i \otimes B_j\), where \(A, B = X, Y, Z\). Each such measurement reveals two bits of information \((a_i, b_j)\) about the joint ontic state of the system \(\omega = (\omega^{(1)}, \omega^{(2)})\), where \(\omega^{(i)} = (x_i, y_i, z_i)\) are ontic states of individual subsystems \((i = 1, 2)\), and leaves the the joint system in the uncorrelated product state \(\omega \rightarrow p_{a_i} \otimes p_{b_j} \in \Delta^{(1)} \otimes \Delta^{(2)}\). See Fig. 4 (on the left) for schematic illustration. Note that due to disturbance no further information can be inferred about the system, and hence this sort of measurement can be thought of as maximal in this restricted setting.

C. Non-local measurements

Now, we proceed to measurements which test correlations between components of the bipartite system. They will be called non-local measurements since a straightforward realisation requires both systems brought together.

Let us imagine a device with two knobs which can be set to \(A_1B_2 \& C_1D_2\), where the choice of \(A, B, C, D\) ranges over \(X, Y, Z\). We assume that measurement \(A_1B_2 \& C_1D_2\) reveals two bits of information \((a_1b_2, c_1d_2)\) about the joint (ontic) state of the system \(\omega = (\omega^{(1)}, \omega^{(2)})\) with \(\omega^{(i)} = (x_i, y_i, z_i)\). For example, measurement \(X_1X_2 \& Y_1Y_2\) tests the joint property of the bipartite system answering the following two questions: (i) are both systems together on the front or back face of the cube \((x_1x_2 = ±1)\), and (ii) are both systems together on the right or left face of the cube \((y_1y_2 = ±1)\); e.g. the result \((+1, -1)\) means that both systems occupy the same face as regards the \(\hat{x}\) direction and different faces in the \(\hat{y}\) direction.

In order to complete description of the measurement procedure we need to specify how it subsequently affects the system. We will assume that the measurement effects the change \(\omega \rightarrow p \in \Delta^{(1)} \otimes \Delta^{(2)}\) leaving the system in equiprobable mixture of eight appropriately chosen ontic states. For sake of simplicity in the following we will only consider measurement settings listed in Table 1 and Table 2 (Fig. 5); it is enough for the purpose at hand. Accordingly, for measurement \(X_1X_2 \& Y_1Y_2\) depending on the outcome, i.e. \((+1, +1), (+1, -1), (-1, +1), (-1, -1)\), let the resulting state be respectively:

\[ p_{φ+} : P(φ) = \frac{1}{8} \quad \text{if} \quad x_1 = x_2, \quad y_1 = y_2, \quad z_1 = z_2, \quad \]
\[ p_{φ−} : P(φ) = \frac{1}{8} \quad \text{if} \quad x_1 = x_2, \quad y_1 \neq y_2, \quad z_1 = z_2, \quad \]
\[ p_{φ−} : P(φ) = \frac{1}{8} \quad \text{if} \quad x_1 \neq x_2, \quad y_1 = y_2, \quad z_1 = z_2, \quad \]
\[ p_{φ−} : P(φ) = \frac{1}{8} \quad \text{if} \quad x_1 \neq x_2, \quad y_1 \neq y_2, \quad z_1 \neq z_2, \quad \]
\[ P(φ) = 0 \quad \text{otherwise.} \]

The pattern of correlations behind these definitions is best illustrated and analysed on pictures, see Fig. 5 (at the top). In particular, one readily verifies that such defined measurement procedure is repeatable (with states \(p_{φ±}, p_{φ±}\) being the respective eigenstates) and maximal (i.e. post-measurement disturbance prevents further information gain about the system). For measurements \(X_1X_2 \& Z_1Z_2\) and \(Z_1Z_2 \& Y_1Y_2\) the assignment of post-measurement states to measurement outcomes follow the pattern of Table 1 (Fig. 3). In a similar manner we define post-measurement states for non-local measurements \(X_1Y_2 \& Y_1X_2, Y_1X_2 \& Z_1Z_2\) and \(X_1Y_2 \& Z_1Z_2\) which are listed in Table 2 (Fig. 6). Note that in this case we use another set of epistemic states
FIG. 4. **Local vs. non-local measurement.** Two types of measurement procedures performed on the bipartite system (in the middle). On the left, measurement $X_1 \& Y_2$ which consists in simultaneous local measurements of observables $X_1$ and $Y_2$ reveals that system (1) was on the front ($x_1 = +1$) and system (2) on the left ($y_2 = -1$) face of the respective cube, and leaves the systems in the uncorrelated product state $p_{+ \otimes -}$. On the right, systems are brought together and undergo non-local measurement $X_1 X_2 \& Y_1 Y_2$ which reveals joint information, described by observables $X_1 X_2$ and $Y_1 Y_2$, that systems occupied opposite faces in the $\hat{x}$ and $\hat{y}$ directions (i.e. $x_1 x_2 = -1$ and $y_1 y_2 = -1$), and subsequently leaves both systems in the correlated state $p_{\psi^\pm}$.

$p_{\psi^\pm}$ defined as follows:

- $p_{\psi^+} : P(\omega) = \frac{1}{8}$ if $x_1 = y_2, \qquad y_1 = x_2, \qquad z_1 = z_2$,
- $p_{\psi^-} : P(\omega) = \frac{1}{8}$ if $x_1 = y_2, \qquad y_1 \neq x_2, \qquad z_1 \neq z_2$,
- $p_{\phi^+} : P(\omega) = \frac{1}{8}$ if $x_1 \neq y_2, \qquad y_1 = x_2, \qquad z_1 \neq z_2$,
- $p_{\phi^-} : P(\omega) = \frac{1}{8}$ if $x_1 \neq y_2, \qquad y_1 \neq x_2, \qquad z_1 = z_2$,

and $P(\omega) = 0$ otherwise. See Fig. 5 (at the bottom) for graphical illustration. A quick check shows that all measurements defined above are repeatable and maximal.

This completes description of the choice of measurement procedures considered in the model. Now, we proceed to discussion of observables and simultaneous measurability to show contextuality of the model and recover the pattern captured in the Mermin-Peres square (Fig. 1).

**D. Compatible observables and context**

The primary role of measurement is to reveal information about some property of the system which is called an observable. In the model we have direct access to $3 + 3 + 3 \times 3 = 15$ non-trivial observables of the form $A_1, B_2$ and $A_1 B_2$. However, post-measurement state disturbance imposes limitations on how much information is actually accessible to the agent probing the system. Upon performing a measurement the scope of available information about the system is reduced to a certain subset of observables and information about the remaining ones is irrevocably lost. Of course, assignment of values in subsequent measurements of the same kind is repeatable since post-measurement states are also eigenstates of the respective procedures. Accordingly, each local measurement $A_1 \& B_2$ gives access to observables $A_1, B_2$, as well as their functions (e.g. $A_1 B_2$ is calculated as product of the respective values). Similarly, non-local measurements of the kind $A_1 B_2 \& C_1 D_2$ provide means to learn $A_1 B_2, C_1 D_2$, and functions thereof.

Note that some of the observables can be measured in several ways, e.g. $X_1 X_2$ can be measured locally by $X_1 \& X_2$ or via non-local procedures $X_1 X_2 \& Y_1 Y_2$ or $X_1 X_2 \& Z_1 Z_2$. Clearly, learning information about an observable happens in a broader measurement context. The latter is crucial for determining the set of simultaneously measurable observables. For example, local procedures $X_1 \& X_2, Y_1 \& Y_2, X_1 \& Y_2, Y_1 \& X_2$ provide measurement contexts for simultaneous (single-shot) measurement of observables listed in rows $R_1, R_2$ and columns $C_1, C_2$ respectively (Fig. 1). We observe that in each case product of revealed values is equal to $+1$ as required for reconstruction of the Mermin-Peres square (e.g. for column $C_1$ we have $x_1 \cdot y_2 \cdot x_1 y_2 = (x_1)^2 (y_2)^2 = +1$).

Sequential measurement is a sequence of single-shot measurements performed one after another on the same system. The resulting series of outcomes assigns values to the measured observables. If the assignment is repeatable, i.e. subsequent measurements repeat outcomes for the same observables no matter what is their order in the sequence, then such a collection of observables is said to be compatible and considered to be simultaneously measurable. In mathematical terms, the neces-
Correlations in post-measurement states. At the top, illustration of states $p_{\phi^\pm}, p_{\psi^\pm}$ being equiprobable mixtures of eight (ontic) states. Each cube represents both systems at the same time with the following convention defining the set of allowed joint ontic states: systems (1) and (2) may occupy only vertices at the opposite ends of the solid lines drawn on the pictures. Note that individually systems can take any of the eight vertices, but their joint position is highly correlated (e.g. for state $p_{\psi^-}$ systems are bound to occupy opposite corners of the cubes). At the bottom, illustration of states $p_{\psi^\pm_i}, p_{\phi^\pm_i}$. Here, again, states are equiprobable mixtures of eight (ontic) states of the joint system. For states $p_{\psi^\pm_i}$ systems either occupy the same vertex chosen from the four vertices depicted in the upper cubes, or occupy opposite ends of the solid lines drawn on the lower cubes. If we depict system system (1) as green and system (2) as red, then for states $p_{\psi^\pm_i}$ systems may occupy only the respective ends of the solid lines drawn on the cubes. Like before, individually systems can take any of the eight vertices, but their joint position is highly correlated.

Observe that we use the term simultaneous measurability as it is understood in quantum theory. That is, in spite the fact that the measurements can not be realised all at the same time, one relaxes the condition of simultaneity to include sequential measurements of a set of observables when they are repeatable. This boils down to the notion of compatible observables defined through the property of having a common eigenbasis. In the quantum version of the Mermin-Peres square (Fig. 1) the respective bases are product states $|\pm x\rangle \otimes |\pm x\rangle, |\pm y\rangle \otimes |\pm y\rangle, |\pm x\rangle \otimes |\pm y\rangle, |\pm y\rangle \otimes |\pm x\rangle$ for $R_1, R_2, C_1, C_2$, and entangled states $|\psi^\pm\rangle, |\phi^\pm\rangle$ for $R_3$ and $|\psi^\pm_i\rangle, |\phi^\pm_i\rangle$ for $R_3$.

Extension of the notion of simultaneous measurability to compatible observables requires further qualification of the measurement context when sequential measurements are taken into account. Observe that for a sequence of measurements only the first one is bound to disclose the

necessary and sufficient condition for compatibility of a collection of observables is existence of the corresponding set of measurements all of which share the same post-measurement states. Of course, observables measurable in a single-shot measurement furnish a simple example of compatible set, e.g. observables in rows $R_1, R_2,$ and columns $C_1, C_2$ which correspond to the choice of post-measurement states $p_{\psi^\pm} \otimes p_{\psi^\pm}, p_{\psi^\pm} \otimes p_{\psi^\pm}, p_{\psi^\pm} \otimes p_{\psi^\pm}, p_{\psi^\pm} \otimes p_{\psi^\pm}$ respectively. For a less trivial example take column $C_3$ which also constitutes a compatible set in the model. Measurement of these observables requires a sequence of at least two different non-local measurements chosen from Table 1 (Fig. 1), all of which share the same set of post-measurement states $p_{\psi^\pm}, p_{\psi^\pm}$. The same applies to observables in row $R_3$ with the corresponding measurements in Table 2 (Fig. 1) and the choice of post-measurement states $p_{\psi^\pm_i}, p_{\psi^\pm_i}$. 
responding to the respective outcomes (local measurements and their post-measurement states corresponding classical analogues of column C).

### Table 1

|        | $(+, +)$ | $(+, -)$ | $(-, +)$ | $(-, -)$ |
|--------|----------|----------|----------|----------|
| $X_1X_2 \& Y_1Y_2$ | $p_{\phi^+}$ | $p_{\phi^+}$ | $p_{\phi^-}$ | $p_{\phi^-}$ |
| $X_1X_2 \& Z_1Z_2$ | $p_{\psi^+}$ | $p_{\psi^+}$ | $p_{\psi^-}$ | $p_{\psi^-}$ |
| $Z_1Z_2 \& Y_1Y_2$ | $p_{\phi^-}$ | $p_{\phi^+}$ | $p_{\phi^+}$ | $p_{\phi^-}$ |

### Table 2

|        | $(+, +)$ | $(+, -)$ | $(-, +)$ | $(-, -)$ |
|--------|----------|----------|----------|----------|
| $X_1Y_2 \& Y_1X_2$ | $p_{\phi^+_i}$ | $p_{\psi^-_i}$ | $p_{\psi^+_i}$ | $p_{\phi^-_i}$ |
| $Y_1X_2 \& Z_1Z_2$ | $p_{\psi^+_i}$ | $p_{\phi^+}$ | $p_{\psi^-}$ | $p_{\phi^-}$ |
| $X_1Y_2 \& Z_1Z_2$ | $p_{\phi^+_i}$ | $p_{\psi^-_i}$ | $p_{\psi^+_i}$ | $p_{\phi^-_i}$ |

**FIG. 6. Assignment of post-measurement states.** Non-local measurements and their post-measurement states corresponding to the respective outcomes ($\pm 1, \pm 1$). Each table contains a set of simultaneously measurable observables providing classical analogues of column $C_3$ (Table 1) and row $R_3$ (Table 2) in the Mermin-Peres square (Fig. 1).

actual value of the observable, whereas values revealed in the subsequent measurements of compatible observables can be affected by post-measurement state disturbance and hence may no longer reflect the initial state of affairs. How it manifests depends on details of the model and is only constrained by the repeatability condition. In the presented model this is taken to the extreme since the maximal information that can be learned about the system is limited to two bits that are decided already upon the first measurement. In other words, once the first measurement is performed, providing the context, values of the remaining compatible observables revealed in subsequent measurements are fixed in the appropriate manner. For example, take three compatible observables in column $C_3$, measurement of which requires at least two different non-local measurements from Table 1 (Fig. 1). If we begin with $X_1X_2 \& Y_1Y_2$ and obtain outcome $(+, -)$ then post-measurement state is $p_{\phi^+}$ which dictates outcomes for all subsequent measurements, e.g. measurement of $X_1X_2 \& Z_1Z_2$ will certainly yield outcome $(+, +)$ and again reproduce $p_{\phi^+}$. In this case assignment of values to compatible observables $X_1X_2$, $Y_1Y_2$ and $Z_1Z_2$ reads $+1$, $-1$ and $+1$. Observe that the value taken by $Z_1Z_2$ is arbitrarily decided and might have been different if measurements were performed in different order. It is the first measurement which reveals ‘true’ values and provides the context in which all the rest is fixed. A closer look at definition of states $p_{\psi^+}$, $p_{\phi^+}$ and Table 1 (Fig. 1) shows that for any sequential measurement of compatible observables in column $C_3$ one always ends up with the product of values equal to $x_1x_2 \cdot y_1y_2 \cdot z_1z_2 = -1$. Similarly, sequential measurements of compatible observables in row $R_3$ are bound to produce $x_1y_2 \cdot y_1x_2 \cdot z_1z_2 = +1$, see definition of states $p_{\phi^+_i}$, $p_{\phi^-_i}$ and Table 2 (Fig. 1).

This completes reconstruction of the structure of observables captured in the Mermin-Peres square.

**IV. DISCUSSION**

In summary, we have presented classical model of the bipartite system and described two kinds of measurement procedures which take as a pattern the structure of quantum observables in the Mermin-Peres square. Observables discussed in the model have the same (non)local character, are sensitive to the measurement context, preserve simultaneous measurability along rows and columns, as well as uphold the curious property held by the product of values. Given all this we have faithfully reconstructed within our simple model all features of the Mermin-Peres square relevant to the discussion of contextuality. Accordingly, quantum arguments translate verbatim to our case and hence the model contradicts the assumption of non-contextuality precisely in the same manner as quantum observables considered in the Mermin-Peres square. Note that in constructing the model we proceeded in close parallel to the operational framework of quantum mechanics. In effect, on top of simulating quantum contextuality, we obtained a simple classical analogue which in a natural way reflects the structure and behaviour of a bipartite quantum system probed by measurements in the restricted setting of the Mermin-Peres square. In particular, the model violates contextuality-inequality in the same manner as quantum mechanics does; the violation is state-independent and insensitive to order of measurements.

This clearly demonstrates that classical systems can be contextual too, and indeed in a very characteristic quantum-like fashion. Hence the conclusion that contextuality by itself is not enough to be taken as a signature of "quantumness". In fact, the model shows that contextual behaviour may be simply an effect of limited information gain and post-measurement state disturbance, both being a plausible scenario in the classical realm too. It can be classified as a contextual hidden variable model with the ontic states $\Omega^{(1 \& 2)} \equiv \Omega^{(1)} \times \Omega^{(2)}$ playing the role of hidden variables, and contextuality deriving from subtle distinction between the concept of simultaneous measurability, compatible observables and actual measurement procedures providing the context. Let us point out that our construction differs from the existing proposals which can be immediately observed from the structure of the ontic state space (cf. 18, 20, 22, 23). Moreover, it presents a different ontology which in the model builds upon the classical picture of a bipartite system being composed of two separate components (with all effects resulting from classical correlations).

If contextuality by itself is not a token of non-classicality, then what makes quantum theory so different? Or more generally, which conceptual features distinguish quantum mechanics from classical theories. This sort of questions occupy a profound place in quantum
foundations. As a result of recent research, considerable progress has been made in separating quantum from classical effects by means of toy models (see e.g. [15, [21]) and study of ψ-epistemic reconstructions [25]. In this paper, we aimed at demystifying the concept of contextuality by showing that it manifests in the classical regime too. The presented model in a straightforward manner contributes to an often debated topic whether or not contextuality is a ‘true’ signature of non-classicality. Clearly, the opinion that it is typically quantum effect is not fully justified and requires further qualification (e.g. by bringing separability and non-locality to the spotlight [6 [32]).

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