Landau-Migdal vs. Skyrme

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Abstract

The magnitude and density-dependence of the non-spin dependent Landau-Migdal parameters are derived from Skyrme energy functionals and compared with the phenomenological ones. We perform RPA calculations with various approximations for the Landau-Migdal particle-hole interaction and compare them with the results obtained with the full Skyrme interaction. For the first time the next to leading order in the Landau-Migdal approach is considered in nuclear structure calculations.

Keywords: Skyrme forces, Landau parameters

1. Personal recollections by Josef Speth

I met Gerry for the first time 1972 in Osaka at a conference on magnetic moments. He invited me to Copenhagen and Stony Brook where I spent my first sabbatical in 1975. From that time on we met several times a year in Stony Brook, in Jülich and at various conferences. He spent the time of his Humboldt award in our institute in Jülich and became an expert for walking trails on the Sophien Höhe, the artificial moment near Jülich. Last but not least I was his tennis partner and instructor.

2. Introduction

Landau’s Theory of Fermi Liquids is a widely used and powerful approach to describe excitation properties of extended Fermion system\textsuperscript{[1]}. It has been generalized by Migdal to a Theory of Finite Fermi Systems\textsuperscript{[2]}. Since the early 1970 G.E. Brown was interested in the Landau-Migdal approach. In the famous article Landau,Brueckner-Bethe and Migdal Theories of Fermi Systems\textsuperscript{[3]} he reviewed and compared the most successful many-body approaches of that time. In connection with the pion condensation first discussed by Migdal, he pointed out\textsuperscript{[4]} that if one considers in Migdals calculation the spin-isospin dependent zero-range parameter $g'_0$ of the Landau-Migdal interaction the condensation disappears. In the same year Babu and Brown\textsuperscript{[5]} studied the quasi-particle interaction in $^3$He where they introduced the induced interaction, which allowed to satisfy the Pauli principle. Some years later he showed that in the spin-isospin channel of the nuclear particle-hole interaction one has to consider the pion and rho exchange contribution\textsuperscript{[6]}. This Stony-Brook Jülich interaction was successfully applied a large body of magnetic properties.
In the 1990’s, R. Shankar pointed out that the application of the renormalization group to rotationally invariant Fermi surfaces automatically leads to Landau’s Fermi-liquid theory as a fixed point of the renormalization group flow [7]. This observation was transferred from condensed matter physics to nuclear physics. Renormalization group techniques led to the derivation of the unique \( V_{\text{low}k} \) two-nucleon interaction in the vacuum. In a series of papers, Brown, Schwenk, and Friman applied renormalization group methods to derive the nuclear Fermi-liquid theory starting from the \( V_{\text{low}k} \) two-nucleon interaction and the Babu-Brown induced interaction [8, 9]. These studies opened a new approach first to neutron matter, and eventually to nuclear matter and finite nuclei. A summary of the present status of this field can be found in [10].

In the following, we want to concentrate on another extension of the Landau theory, based on effective interactions defined in a nuclear medium, the so-called Skyrme forces. These interactions have a simple mathematical structure which has facilitated extensions and applications of the theory beyond the mean-field approximation.

Landau-Migdal theory is based formally on a low-\( q \) (low momentum) expansion of the effective two-body interaction in the medium while the (few) model parameters are adjusted phenomenologically. The single-particle basis for the RPA (random-phase approximation) calculation with the Landau-Migdal interaction is taken from an empirical shell-model potential. The approach was taken up and further developed in detail for nuclear physics by the München-Jülich group [11]. Since then it has been applied extensively for a broad range of nuclei, for a review see [12]. At about the same time, another effective interaction from a low-\( q \) expansion appeared in nuclear physics, the Skyrme-Hartree-Fock (SHF) approach [13, 14, 15, 16]. It was constructed with different intention, predominantly as a self-consistent model for the nuclear ground state [17], but also applicable to compute excitation spectra within RPA [18]. For a review on SHF see [19]. The formal similarity of both approaches raises interest in a closer comparison. Thus the microscopic calculation of the phenomenological Landau-Migdal parameters from Skyrme energy functionals has a long history [20]. As self-consistent calculations are considered to be more fundamental then the Landau-Migdal approach, differences between the calculated and phenomenological parameters were assumed as a short coming of the Landau-Migdal theory. That this is not necessarily correct showed the former discussion on the incompressibility \( K \) and the excitation energy of the breathing mode in \(^{208}\text{Pb}\) which both are related to the parameter \( F_0 \). The value from Landau-Migdal theory of the München-Jülich group [11, 12] was nearly zero, which strongly deviated from values derived from the early SHF parametrizations [17]. The value of the incompressibility of the order of \( K \approx 250 \text{ MeV} \) and the predicted energy of the breathing mode deduced from the Landau-Migdal theory turned out to be close to data, whereas the old Skyrme value of the incompressibility \( K \approx 350 \text{MeV} \) was much too high. It was found in the next stage of SHF development that this deficiency was due to a too rigid modeling of the density dependence of the Skyrme interaction and that a more flexible density dependence can remove the discrepancy [21, 22]. This little historical example shows that a comparison of these two similar and yet different low-\( q \) models can be fruitful. It is the aim of this paper to continue those comparison at an up-to-date level of modeling.

In the present comparison, we will address two aspects: first, the prediction and density-dependence of Landau-Migdal parameters (LMP) modern SHF models, and second, the impact of kinetic terms in RPA calculations of nuclear giant resonances. In Landau’s theory, the interac-
tion parameters are constants. In Migdal’s extension, the force parameters are density-dependent to account for the finite size effects in the nuclei. SHF predicts a density dependence for the LMP.

We will compare the different density dependence and we will address the uncertainties in the SHF predictions on the basis of the techniques of error propagation in least-squares fits \[23, 24\]. For the studies of RPA excitations, we will employ a recently developed an RPA code which can perform Calculations within Landau-Migdal theory as well as with the full SHF residual interaction. This allows a direct comparison on the basis of the same numerical treatment. We use that for studying the impact of the kinetic terms and the Landau approximation.

The paper is outlines as follows: Section 3 provides a brief review of Landau-Migdal theory, section 4 discusses the SHF predictions of LMP, and section 5 contains a study of the impact of kinetic terms on RPA spectra.

3. Landau-Migdal Theory

The central object of the Landau-Migdal approach \[2\] is the response function \(L\) which is defined as:

\[
L(13, 24) = g(13, 24) - g(1, 2)g(3, 4).
\] (1)

where \(g(1, 2)\) and \(g(13, 24)\) are the one-body and two body Green functions. The response function obeys an integral equation:

\[
L(13, 24) = -g(1, 4)g(3, 2) - i \int d5d6d7d8 g(1, 5)K(57, 68) L(83, 74) g(6, 2)
\] (2)

where \(K\) is an effective two-body interaction. It is irreducible with respect to the particle-hole propagator. In Eq. (2) the kernel \(K\) as well as the one-particle Green functions are a priori unknown. The one-particle Green function is given by the Dyson equation:

\[
\frac{i}{2} \int d3 \{ S(1, 3) + \Sigma(1, 3) \} g(3, 2) = \delta(1, 2)
\] , \(3a\)

\[
S(1, 3) = \delta_{\nu_1 \nu_2} \delta(t_1 - t_3) \left[ \epsilon_{\nu_1} - \epsilon_{\nu_2} - i \frac{\delta}{\delta \xi_{\nu_1}} \right]
\] , \(3b\)

where \(\Sigma(1, 3)\) is the mass operator which is connected in a very complicated way with the two-body interaction of a given Hamiltonian \[12\]. The irreducible kernel \(K\) is defined by the functional derivative \[12\] :

\[
K = \frac{\delta \Sigma(1, 2)}{\delta g(3, 4)}.
\] (4)

In general, the effective interaction kernel is an involved four-point function. In momentum representation, it is a function of four momenta, however the energy- and density-independent part of \(K\) (in particular, the density-independent terms of the Skyrme interaction) depends actually only on three momenta due to the translation symmetry. For this part we have \(K = K(p, p', q)\) where \(p\) and \(p'\) are the momenta of the in-coming and out-going hole states and \(q\) is the transferred momentum. \(K\) is also energy dependent. Figure \[1\] illustrates these momenta for the case of a local interaction which is sufficient for our purposes because the Landau-Migdal as well as the Skyrme interaction are both local. All \(ph\) pairs carry net momentum \(q\). They differ by the other momentum \(p\), or \(p'\) respectively.
Following the quasi-particle concept of Landau, one separates the one-particle Green function into a singular quasi-particle term and a remainder. With this ansatz one can rewrite Eq. (2) and obtains after some analytical transformations (see Ref. [12]) the renormalized RPA equation for nuclei in terms of the excitation amplitudes $\chi$ in the single-particle configuration space [25]:

$$ (\epsilon_{v_1} - \epsilon_{v_2} - \Omega_m) \chi_{v_1v_2}^{(m)} = (n_{v_1} - n_{v_2}) \sum_{\nu_3\nu_4} F_{\text{ph}}^{\nu_1\nu_2\nu_3\nu_4} \chi_{\nu_3\nu_4}^{(m)} $$  

Eq. (5)

$F_{\text{ph}}$ is the ph-interaction, $\Omega_m$ are the excitation energies of the nucleus and $\chi^{(m)}$ the corresponding quasi-particle quasi-hole transition matrix elements. $F_{\text{ph}}$ is a complicated function of $K$ and the non-singular parts of the Green functions e.g. $K$ appears in the nominator as well as in the denominator [12]. Therefore $F_{\text{ph}}$ is a smooth function in momentum space and correspondingly of short-range in the $r$-space.

Moreover, in the Landau approach one considers the interaction on the Fermi surface and replaces the energies by the Fermi energy and the magnitude of the momenta by the Fermi momentum. Thus one can approximate $F_{\text{ph}}$ as a local contact (zero-range) interaction. This means that $F_{\text{ph}}$ is effectively independent on $q$ and $q'$, respectively. After all, $F_{\text{ph}}$ depends only on the angle between the ph-momenta $p$ and $p'$ before and after the collision; it reads

$$ F_{\text{ph}} \left( \frac{p \cdot p'}{p_F^2} \right) = C_0 \sum_{l=0}^{\infty} \left[ f_l + f_l' \tilde{\tau}_1 \cdot \tilde{\tau}_2 + g_l \tilde{\sigma}_1 \cdot \tilde{\sigma}_2 + g_l' \tilde{\sigma}_1 \cdot \tilde{\sigma}_2 \tilde{\tau}_1 \cdot \tilde{\tau}_2 \right] P_l \left( \frac{p \cdot p'}{p_F^2} \right) $$  

Eq. (6)

where $P_l(x)$ is the Legendre polynomial of order $l$ and the four terms containing different combinations of spin and isospin operators cover the typical four nuclear interaction channels [25]. Note the remarkable result that, by virtue of the Landau quasi-particle concept and the following renormalization, the whole information content of $K$ shrinks to a few model constants, the much celebrated Landau-Migdal parameters $f_l$, usually restricted to $l = 0$ and 1. These parameters are dimensionless and $C_0$ is defined as:

$$ C_0 = \frac{\pi^2 \hbar^2}{2m^* k_F} $$  

Eq. (7)

where $k_F = \left( \frac{3\pi^2 \rho_0}{2} \right)^{1/3}$ is the Fermi momentum. The scaling factor $C_0$ is proportional to the density of states at the Fermi surface. A typical value is $C_0 = 150 \text{ MeV fm}^3$ which is the standard
choice in phenomenological shell models where the effective mass is $m^*/m = 1$. It is to be noted that papers from the Landau-Migdal theory often use a factor which is twice as large [2]. The above scaling (in its flexible form with actual $m^*$ and $k_F$) is the standard in all SHF papers addressing Landau parameters. We will follow this option henceforth.

The Fourier transforms of the $(q$-independent) terms with $l = 0$ and $l = 1$ yield $\delta$-functions and derivatives of $\delta$-functions in coordinate space, of the above mentioned zero-range interactions.

In the Theory of Fermi Liquids the Landau parameters are constants. Migdal introduced in his Theory of Finite Fermi Systems density dependent parameters $f_l(\rho)$ in order to correct for the finite size of the nuclei. The form of the interaction in leading order ($l = 0$) in the $r$-space is thus written as:

$$F_{ph}^{(1, 2)}(r_1 - r_2) = C_0 \delta(r_1 - r_2) \cdot \left[ f_0(\rho) + f'_0(\rho) \sigma_1 \cdot \sigma_2 + g_0(\rho) \tau_1 \cdot \tau_2 + g'_0(\rho) \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 \right].$$

(8)

The density dependent Landau-Migdal parameters are parametrized in the following way [2]:

$$f(\rho) = f^{(ex)}(\rho) + (f^{(in)}(\rho) - f^{(ex)}(\rho)) \frac{\rho_0(r)}{\rho_0(0)}.$$  

(9)

where $f^{(ex)}$ stands for the exterior region of the nucleus and $f^{(in)}$ for the interior. So far only the leading-order contribution of the Landau-Migdal interaction has been considered in nuclear structure calculations. There is a similar expansion for the spin-spin part of the effective interaction leading to the Landau-Migdal parameters $g_l$, or $g'_l$ respectively for the spin response. We will not address spin response in this paper.

The question remains how to determine the model parameters. In the Landau-Migdal approach neither $g(1, 2)$ nor $K$ are derived microscopically from Eqs. (2–4) but are obtained by adjustment to phenomenological data. The parameters $f_0^{(in)}$ and $f'_0^{(in)}$ are related to the compressibility and symmetry energy and deduced from Eq.(11). The external parameters were adjusted to experimental data e.g. Ref. [26].

In order to solve the basic Landau Migdal equations one needs as input single particle-wave functions, single-particle energies and the ph-interaction. Migdal has designed his theory in close connection to Landau’s Theory of Fermi Liquids. Therefore one takes the input data from experiment or from models which reproduce the needed experimental data as good as possible. The single-particle wave functions are taken from an empirical single-particle model and the single-particle energies as far as possible from experiment.

4. Landau parameters from the Skyrme energy-density functional

The original formulation of the Skyrme-Hartree-Fock (SHF) method was based on the concept of an effective interaction, the Skyrme force [13]. It was observed long ago that the density dependence in the Skyrme force inhibits an interpretation as interaction [18]. The theoretically correct attitude is to see the SHF method as nuclear density functional approach. Many modern treatments of SHF thus start from a Skyrme energy-density functional, see e.g. the reviews [19, 27]. On the other hand, the Skyrme force, being a zero-range interaction, has a great formal similarity
to the Landau-Migdal force. We thus use it here as the generator of the Skyrme energy-density functional and write the functional as expectation value

$$E_{Sk} = E_{Sk,\text{dens}} + E_{Sk,\text{grad}} = \langle \Phi | \hat{V}_{Sk} | \Phi \rangle$$

$$E_{Sk,\text{dens}} = \langle \Phi | t_0 (1 + x_0 \hat{P}_\sigma) \delta (r_{12}) + \frac{t_3}{6} (1 + x_3 \hat{P}_\sigma) \rho^\sigma (r_1) \delta (r_{12}) | \Phi \rangle$$

$$E_{Sk,\text{grad}} = \langle \Phi | \frac{t_1}{2} (1 + x_1 \hat{P}_\sigma) \left( \delta (r_{12}) \hat{k}^2 + \hat{k}'^2 \delta (r_{12}) \right) + t_2 (1 + x_2 \hat{P}_\sigma) \hat{k}' \delta (r_{12}) \hat{k} | \Phi \rangle$$

(10a)

(10b)

(10c)

where $r_{12} = r_1 - r_2$ and $\hat{P}_\sigma = \frac{1}{2} (1 + \hat{\sigma}_1 \hat{\sigma}_2)$ is the spin-exchange operator. The momentum operators are $\hat{k} = -\frac{i}{2} \left( \nabla_1 - \nabla_2 \right)$ and $\hat{k}' = \frac{i}{2} \left( \nabla_1 - \nabla_2 \right)$ where $\hat{k}$ acts to the right and $\hat{k}'$ to the left.

Leading part is $E_{Sk,\text{dens}}$ for which the interpretation as energy-density functional is compulsory. It can be expressed in terms of local densities $\rho_p, \rho_n$ and spin densities $\rho_p^\sigma, \rho_n^\sigma$, for details see \[19, 27\]. The gradient part $E_{Sk,\text{grad}}$ involves additionally the gradients of density as well as spin-density and, as truly new ingredients, kinetic-energy densities and currents. We ignore here the spin-orbit and the tensor contributions to the Skyrme energy. They play no role for a comparison with Landau parameters.

Writing the SHF energy in terms of the Skyrme force, as done above, involves naturally the Skyrme parameters $t_i$ and $x_i$. Starting from an energy-density functional suggest other parameter combinations. To avoid confusion, it is preferable to express the SHF functional in terms of nuclear matter properties (NMP) which have one-to-one relation to the Skyrme parameters, except for the spin-orbit and tensor part which is anyway not discussed here. Consequently, it is most robust to express the Landau-Migdal parameters as derived from the SHF functional in terms of NMP. As the Landau-Migdal coefficients parametrize the response properties of a system, it is natural that response parameters from nuclear matter come into play, namely incompressibility $K$, symmetry energy $a_{\text{sym}}$, effective mass $m^*$, and isovector effective mass also characterized by the Thomas-Reiche-Kuhn sum rule enhancement $\kappa_{\text{TRK}}$, for details see e.g. [27]. The relations are

$$K (\rho) = \frac{h^2}{2m^*(\rho)} \frac{6k_F^2}{6k_F^2} (1 + f_0 (\rho)) \leftrightarrow f_0 (\rho) = \frac{2m^*(\rho)}{h^2} \frac{K (\rho)}{6k_F^2} - 1$$

(11a)

$$m^*(\rho) = \frac{1}{m} + \frac{f_1 (\rho)}{3} \leftrightarrow f_1 (\rho) = 3 \left( \frac{m^*(\rho)}{m} - 1 \right)$$

(11b)

$$a_{\text{sym}} (\rho) = \frac{1}{3} \frac{h^2 k_F^2}{2m^*(\rho)} (1 + f_0^* (\rho)) \leftrightarrow f_0^* (\rho) = \frac{2m^*(\rho)}{h^2} \frac{3}{k_F^2} a_{\text{sym}} (\rho) - 1$$

(11c)

$$\kappa_{\text{TRK}} (\rho) = \frac{m}{3m^*(\rho)} (f_1^* (\rho) - f_1 (\rho)) \leftrightarrow f_1^* (\rho) = 3 \left( \frac{m^*(\rho)}{m} - \kappa_{\text{TRK}} (\rho) - 1 \right)$$

(11d)

These relations refer to the dimensionless Landau-Migdal parameters as defined in Eq. (7). Note that the lowest order coefficients $f_0$ and $f_0^*$ acquire an involved density dependence due to the density dependence of the SHF functional $E_{Sk,\text{dens}}$. Its form differs from the simple linear interpolation Eq. (9).
The SHF functionals as such are usually adjusted to empirical data \[19, 27\]. However, once determined they constitute a universal parametrization aiming at describing all nuclei (except the smallest ones) as well as neutron and nuclear matter. They thus allow to “derive” Landau-Migdal parameters with Eqs. (11). This is different from Landau-Migdal theory where the Landau-Migdal parameters as such are adjusted phenomenologically. It is thus interesting to compare the parameters derived from SHF with those from pure Landau-Migdal theory. Scanning the SHF literature, one finds a puzzling variety of predictions for the Landau-Migdal parameters which is due to the fact that SHF fits with their weight on ground state properties determine some aspects of the dynamical nuclear response only loosely. A reliable protocol of these inherent uncertainties is achieved by the rules of error propagation in connection with least-squares fits \[28\]. Such systematic adjustment studies are becoming increasingly fashionable in SHF studies \[23, 24\]. We employ here two parametrizations, SV-min and SV-bas, from \[23\] to demonstrate the impact of fit data on the predictions of Landau-Migdal parameters. The parametrization SV-min stems from a straightforward fit exclusively to ground state properties (binding energies, radii, electro-magnetic formfactor) of a large pool of finite nuclei. The extrapolation uncertainties of SV-min are typical for all Skyrme forces fitted to ground state data. The parametrization SV-bas uses the same pool of data as SV-min, but includes additionally information on response properties of \(^{208}\)Pb, the isovector dipole polarizability and the peak energies of three giant resonances: isoscalar monopole, isovector dipole, and isoscalar quadrupole. These response data are similar to what is used in fitting Landau-Migdal parameters and it is interesting to their (indirect) impact through the SHF fits.

Figure 2 shows the dimensionless Landau-Migdal parameters \(f_0(\rho)\) and \(f'_0(\rho)\) for the SHF parametrizations SV-min and SV-bas from \[23\]. The extrapolation uncertainties on the Landau-Migdal parameters are also shown by error bars. The errors on \(f_0\) are very small. At larger densities, the errors increase with density and become visible for SV-min. SV-bas has generally smaller errors because it includes more data in the fit. Its errors remain below drawing precision for \(f_0\). The small errors for \(f_0\) are plausible because isoscalar properties are well determined by the the fits to known nuclei \[23, 27\]. The situation is much different for isovector properties and accordingly we see large uncertainties for the isovector parameter \(f'_0\), particularly for SV-min. There is a slight density dependence of its error. It is interesting to note that a minimum of uncertainty for SV-min is found in the inner surface region around densities \(\rho \approx 0.12\) fm\(^{-3}\). This density corresponds to the nuclear surface region to which the giant dipole resonance is most sensitive. The errors for \(f'_0\) with SV-bas are significantly smaller, particularly in the surface and volume region (\(\rho \approx 0.1–0.16\) fm\(^{-3}\)). This demonstrates very clearly the strong connection between response properties (which were included in the fit of SV-bas) and Landau-Migdal parameters. It is interesting to note that even for SV-bas the errors increase towards very low densities. This outer surface region is not well determined, neither by ground state data nor by resonance properties.

Figure 2 shows also the standard Landau-Migdal parameters for comparison. These look at first glance much different as their density dependence as given by Eq. (8) is linear in contrast to the much more involved density dependence of the SHF results. However, just in the dynamically most relevant region at surface densities \(\rho \approx 0.1\) fm\(^{-3}\) there emerges a nice agreement between SHF prediction and empirical parameters.

The \(l = 1\) parameters \(f_1\) and \(f'_1\) have a simple, linear density dependence also in SHF. Thus we spare a figure showing all trivial density dependence. We just quote the value at bulk equilibrium.
density $\rho = 0.16 \text{ fm}^{-3}$. We find for SV-min $f_1 = -0.144 \pm 0.215$ and $f'_1 = 0.071 \pm 0.781$, for SV-bas $f_1 = -0.302 \pm 0.000$ and $f'_1 = 0.778 \pm 0.010$. These parameters are surprisingly little determined by ground state data, but very well fixed by the response information from giant resonances. The empirical LMP are not well determined, only in Ref. [2] are numbers quoted: $|f_1| \approx 0.1-0.2$ and $|f'_1| \leq 0.1$.

5. Landau approximation versus exact Skyrme-RPA

The residual interaction for RPA calculations of nuclear excitation spectra can be deduced as first derivative of the mass operator, see Eq. (4), or directly as second functional derivative of the energy-density functional, $K \equiv F_{ph}^e \equiv \partial^2 E_{Sk} / \partial \rho_1 \partial \rho_2$. The purely density dependent part $E_{Sk,dens}$ yields a zero-range interaction in full compliance with the Landau-Migdal form (8). It is only the density dependence which differs from the simple linear ansatz (9) as we have already seen in figure 2. More critical is the kinetic term coming from the gradient functional $E_{Sk,grad}$. The correct but tedious way to determine the effective $ph$ interaction $F_{Sk,grad}^{ph}$ goes through the second
The functional derivative of $E_{\text{Sk,grad}}$. As this term corresponds to a pure two-body interaction, we can evaluate $F_{\text{Sk,grad}}^{\text{ph}}$ directly as the two-body matrix element between the four plane waves with momenta $p$, $(p + q)$, $p'$, and $(p' + q)$ as sketched in figure[1]. This yields

$$F_{\text{Sk,grad}}^{\text{ph}}(p, p', q) = [b_{00}^{(-)}1^\sigma 1^\tau + b_{00}^{(+)}(\sigma\sigma')1^\tau + b_{01}^{(-)}1^\sigma(\tau\tau') + b_{11}^{(-)}(\sigma\sigma')(\tau\tau')]q^2$$

$$+ [b_{00}^{(+)}1^\tau + b_{10}^{(+)}(\sigma\sigma')1^\tau + b_{01}^{(+)}1^\sigma(\tau\tau') + b_{11}^{(+)}(\sigma\sigma')(\tau\tau')]((p - p')^2) ,$$

(12a)

$$b_{00}^{(-)} = \frac{1}{16} [\pm (5 + 4x_2) t_2 + 3 t_1] ,$$

(12b)

$$b_{10}^{(-)} = \frac{1}{16} [\pm (1 + 2x_2) t_2 - (1 - 2x_1) t_1] ,$$

(12c)

$$b_{01}^{(-)} = \frac{1}{16} [\pm (1 + 2x_2) t_2 - (1 + 2x_1) t_1] ,$$

(12d)

$$b_{11}^{(-)} = \frac{1}{16} [\pm t_2 - t_1] .$$

(12e)

This is the full residual interaction as it must be taken into account in a consistent Skyrme-RPA calculation. The first line in Eq. (12a) corresponds to the direct ($D$) term of the interaction depicted in Fig. [1] while the second line corresponds to the exchange ($X$) term.

Let us check what happens if one applies the Landau approximation to this $F_{\text{Sk,grad}}^{\text{ph}}$. It reads in the limit of nuclear matter

$$q = 0 \quad , \quad p^2 = p'^2 = k_F^2.$$  

(13)

In this case, we have

$$q^2 = 2k_F^2 \left[1 - P_1(\cos \theta)\right] \quad , \quad \cos \theta = \frac{p \cdot p'}{k_F^2} ,$$

(14)

where $k_F^2 = (3\pi^2\rho/2)^{2/3}$. This wipes out totally the direct term $\propto q^2$. It remains (suppressing spin terms)

$$F_{\text{Land,grad}}^{\text{ph}} = 2k_F^2 [b_{00}^{(+)}1^\tau(\sigma\sigma')1^\tau + b_{10}^{(+)}1^\sigma(\tau\tau')] - 2k_F^2 [b_{00}^{(+)}1^\tau1^\sigma(\tau\tau') + b_{10}^{(+)}1^\tau1^\sigma(\tau\tau')] P_1(\cos \theta) \kappa_p .$$

(15)

The velocity dependent exchange terms contribute to the leading order of $F_{\text{ph}}^{\text{ff}}$ as well as to the next to leading order $F_{\text{ph}}^{\text{XX}}$. The correct result is recovered for $\kappa_p = 1$. This factor was introduced for the purpose of analysis. By varying $\kappa_p$ one can study the impact of the $p$-wave terms ($\propto P_1$) on the RPA results in finite nuclei.

To test the effect of the Landau approximation, we have computed the the strength functions of the isoscalar giant monopole resonance and of the electromagnetic $E1$ and $E2$ excitations in $^{208}\text{Pb}$ using the SHF parametrization SLy4 [29] and SkT6 [30]. The two parametrizations have very different momentum dependence, their effective masses are $m^*/m = 0.68$ and $m^*/m = 1.0$, respectively. The calculations were performed within RPA in which the single-particle continuum was discretized using a computational box of 18 fm radius. The space of the single-particle states was restricted to levels below 100 MeV. The full SHF residual interaction as deduced from the Skyrme energy functional was used in RPA. Although it is known that spin-orbit and Coulomb terms are crucial for a fully consistent Skyrme RPA calculation [31], we omit these terms here to
allow a more direct comparison with the residual interaction in the Landau approximation. The remaining part of the residual interaction, including velocity-dependent terms, was treated exactly in the calculations labeled by the symbol $D + X$ in the figures. The calculations indicated as $D = 0$ employed the Landau approximation (15) for the gradient term with different values of $\kappa_p$ (in this case the direct term in Eq. (12a) is completely omitted). The effect of $q$-dependence in the full residual interaction (12a) is seen by comparison with the Landau approximation (15) at $\kappa_p = 1$. The impact of the $f_1$ and $f'_1$ terms is seen by comparing different values of $\kappa_p$. In the following, we discuss three cases in detail.

Figure 3: Strength functions of the isoscalar $E_0$ and electromagnetic $E_1$ and $E_2$ excitations in $^{208}$Pb, calculated within the self-consistent DFT+DRPA approach based on the Skyrme forces SLy4 [29] (panels (a), (b) and (c)) and SkT6 [30] (panels (d), (e) and (f)). The discrete RPA spectra are folded with a Gaussian of width $\Delta = 500$ keV. The $E_0$ and $E_2$ strength functions are given in units $10^3$ fm$^4$/MeV, the $E_1$ strength is given in units fm$^2$/MeV. The solid (red) lines represent the Skyrme RPA results using the full $ph$ interaction (12a). The dashed (black), dotted (blue), and dashed-dotted (green) lines represent the results in Landau approximation ($D = 0$, Eq. (15)) for various values of the parameter $\kappa_p$ as indicated on the panel (a).

Fig. 3 shows that the $q$-dependence of the residual interaction is very important for the isoscalar resonances, see $E_0$ and $E_2$ distributions. The energy of the first $2^+$ state in $^{208}$Pb becomes even imaginary in the $D = 0$ calculations with SLy4 force at $\kappa_p = 1$ and $\kappa_p = 2$ and with SkT6 force at all $\kappa_p$. The isoscalar $q$-dependent part (parameter $b_{00}$ in Eq. (12a)) is well defined by the ground state fits and varies little between different parametrizations. The effect of the isoscalar
The $p$-wave from the kinetic terms differs very much between SLy4 and SkT6. SLy4 with the low $m'/m$ shows a large dependence. SkT6, on the other hand, reacts inert because $m'/m = 1$ means that there is no isoscalar kinetic contribution ($\varphi^{(s)}_{00} = 0$).

The isovector giant dipole resonance (GDR) shown in the panels (b) and (e) of Fig. 3 is generally more robust. The $q$-dependence and the impact of the $p$-wave terms are much smaller than for the isoscalar resonances. This relates to the fact that the isovector gradient terms are rather small for these two SHF parametrizations. The effect of varying the $p$-wave contribution on the GDR is smaller than for the isoscalar modes which is due to a moderate TRK sum rule enhancement factor $\kappa_{\text{TRK}} = 0.25$ for the SLy4 force. The effect is even much smaller for the GDR with SkT6 force because this parametrization has vanishing $\kappa_{\text{TRK}}$.

Altogether, we see that the Landau approximation can be disastrous in connection with Skyrme forces. It is not applicable in case of isoscalar modes for all relevant parametrizations. The case of isovector GDR is more forgiving. Parametrizations with high effective mass $m'/m \approx 1$ and low TRK sum rule enhancement $\kappa_{\text{TRK}}$ still allow to obtain acceptable results within the Landau approximation.

### 5.1. Landau-Migdal Interaction

What do we learn from the present investigation on the Landau-Migdal interaction. First of all one obtains in the isoscalar channel in next to leading order an attractive contribution depending on the magnitude of the effective mass. In heavy nuclei, where $m'/m \approx 1$ this contribution is negligible. In medium mass and light nuclei where $m'/m < 1$ the effect could be of the order of one to two MeV. The consequence would be that $f_1$ should be A-dependent. From Eq. (15), however, one notices that one obtains also a repulsive contribution to $f_0$ which makes the total effect of the exchange term slightly repulsive. This follows from the fact that for most of the known Skyrme parametrizations $b^{(\pm)}_{00} \gtrsim 0$. In this respect the leading order is a good approximation. The $q^2$-term which is neglected in the Landau approximation gives rise to a strong repulsion in the isoscalar channel, the magnitude of which is determined by the parameter $b^{(s)}_{00}$ which is of the same order for most of the Skyrme parametrizations. In particular, $b^{(+)}_{00} = 125$ and 110 MeV $\cdot$ fm$^2$ for SLy4 and SkT6 forces, respectively. This contribution is crucial in the self-consistent approach. In the phenomenological Landau-Migdal theory, however, this repulsion is included in the phenomenological parameters $f_0$.

### 6. Conclusion

The Landau-Migdal theory is a theory for the excitation modes of a many-Fermion system. It can be derived microscopically from first principles which leads eventually to an effective short-range interaction. For practical application, however, Landau’s quasi-particle concept is introduced which transforms the original microscopic theory into a phenomenological approach. The central quantity is the Landau-Migdal $ph$-interaction, which is parametrized in terms of the famous Landau-Migdal parameters. The Skyrme-Hartree-Fock (SHF) approach is a self-consistent theory for ground-state and dynamics of nuclei. It also based on zero-range effective interactions whose parameters are determined by a fit to empirical data, mostly from the nuclear ground state. In this paper, we have used the well calibrated SHF approach to “derive” the Landau-Migdal parameters.
up to first order. Within this "microscopic" model we thus can test the Landau approximation and compare it with the phenomenological results.

First, we have investigated the magnitude and density dependence of the leading order (non-spin dependent) parameters deduced from SHF. The density dependence differs from the one of the Landau-Migdal parameters. However, the crucial values at densities corresponding to the nuclear surface region are in good agreement with the phenomenological Landau-Migdal parameters. The SHF results do also supply uncertainties on the predicted parameters. The smallest uncertainties are found again in the crucial surface region. SHF models which included information on excitation properties in their fit deliver very small uncertainties.

Second, we were able to consider for the first time the next to leading order in the Landau-Migdal interaction. The attractive contribution in the isoscalar channel, generated by the first order parameter $f_1$, parameter, is compensated by a repulsive contribution to the leading order parameter $f_0$. Therefore the leading order is sufficient for a phenomenological adjustment. The gradient terms in the SHF functional produce in the $ph$-interaction a strongly repulsive term proportional to $q^2$ (where $q$ is the transferred momentum) which is neglected in the Landau approximation. This contribution is crucial in the Skyrme-Hartree-Fock approach. It is implicitly considered in the phenomenological $f_0$ parameters. The corresponding effects in the isovector channel are small and can be neglected. Therefore we conclude: all derivative terms need to be carefully included in RPA based on SHF, but there is no need to go beyond the leading order in the phenomenological Landau-Migdal interaction.

Acknowledgment: This work was supported by the Deutsche Forschungsgemeinschaft (grant RE322-13/1). N. L. and V. T. acknowledge financial support from the St. Petersburg State University under Grant No. 11.38.648.2013. J.S. acknowledges travel support by the Alexander von Humboldt foundation.

References

[1] L. D. Landau, E. M. Lifshitz, L. P. Pitaevski, Course of Theoretical Physics 9 – Statistical Physics, Pergamon press, Oxford, 1980.
[2] A.B. Migdal, Theory of Finite Fermi Systems and Application to Atomic Nuclei, Wiley, New York, 1967.
[3] G. Brown, Rev. Mod. Phys. 43 (1971) 1.
[4] S. Barshay, G. Brown, Phys.Lett 47B (1973) 107.
[5] S. Babu, G. Brown, Ann. Phys. 78 (1973) 1.
[6] J. Speth, V. Klemt, J. Wambach, G. E. Brown, Nucl. Phys. A 343 (1980) 382.
[7] R. Shankar, Rev.Mod.Phys. 66 (1994) 129–192.
[8] A. Schwenk, G. E. Brown, B. Friman, Nucl.Phys. A703 (2002) 745–769.
[9] A. Schwenk, B. Friman, G. E. Brown, Nucl.Phys. A713 (2003) 191–216.
[10] B. Friman, K. Hebeler, A. Schwenk, Lect.Notes Phys. 852 (2012) 245–285.
[11] J. Speth, L. Zamick, P. Ring, Nucl. Phys. A 232 (1974) 1.
[12] J. Speth, E. Werner, W. Wild, Phys.Rep. 33 (1977) 127.
[13] T. H. R. Skyrme, Nucl. Phys. 9 (1959) 615–634.
[14] J. W. Negele, D. Vautherin, Phys. Rev. C 5 (1972) 1472–1492.
[15] J. W. Negele, D. Vautherin, Phys. Rev. C 11 (1975) 1031–1041.
[16] K. F. Liu, G. E. Brown, Nucl. Phys. A 265 (1976) 385.
[17] D. Vautherin, D. Brink, Phys. Rev.C 5 (1972) 626.
[18] S. Krewald, V. Klemt, J. Speth, A. Faessler, Nucl. Phys. A281 (1977) 166–206.
[19] M. Bender, P.-H. Heenen, P.-G. Reinhard, Rev. Mod. Phys. 75 (2003) 121.
[20] S.-O. Bäckman, A. Jackson, J. Speth, Phys. Lett. B 56 (1975) 209.
[21] J. Bartel, P. Quentin, M. Brack, C. Guet, H.-B. Håkansson, Nucl. Phys. A A386 (1982) 79–100.
[22] M. Brack, C. Guet, H.-B. Håkansson, Phys. Rep. 123 (1985) 275–364.
[23] P. Klüpfel, P.-G. Reinhard, T. J. Bürvenich, J. A. Maruhn, Phys. Rev. C 79 (2009) 034310.
[24] M. Kortelainen, T. Lesinski, J. Moré, W. Nazarewicz, J. Sarich, N. Schunck, M. V. Stoitsov, S. Wild, Phys. Rev. C 82 (2010) 024313.
[25] P. Ring, P. Schuck, The Nuclear Many-Body Problem, Springer, New York, Heidelberg, Berlin, 1980.
[26] S. P. Kamerdzhiev, J. Speth, G. Tertychny, Phys. Rep. 393 (2004) 1.
[27] J. Erler, P. Klüpfel, P.-G. Reinhard, J. Phys. G 38 (2011) 033101.
[28] P. R. Bevington, D. K. Robinson, Data Reduction and Error Analysis for the Physical Sciences, McGraw-Hill, 2003.
[29] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, R. Schaeffer, Nucl. Phys. A 635 (1998) 231. Nucl. Phys. A643, 441(E).
[30] F. Tondeur, M. Brack, M. Farine, J. Pearson, Nucl. Phys. A 420 (1984) 297.
[31] T. Sil, S. Shlomo, B. K. Agrawal, P.-G. Reinhard, Phys. Rev. C 73 (2006) 034316.