Optical binding of cylinder photonic molecules in the near-field of partially coherent fluctuating Gaussian Schell model sources.
A coherent mode representation

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We present a theory and computation method of radiation pressure from partially coherent light by establishing a coherent mode representation of the radiation forces. This is illustrated with the near field emitted from a Gaussian Schell model source, mechanically acting on a single cylinder with magnetodielectric behavior, or on a photonic molecule constituted by a pair of such cylinders. Thus after studying the force produced by a single particle, we address the effects of the spatial coherence on the bonding and anti-bonding states of two particles. The coherence length manifests the critical limitation of the contribution of evanescent modes to the scattered fields, and hence to the nature and strength of the electromagnetic force, even when electric and/or magnetic partial wave resonances are excited.

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I. INTRODUCTION

The subject of radiation forces from partially coherent light is receiving increasing attention \cite{1, 11}. We recently put forward a systematic theory of photonic forces on small particles that characterized their magnitude by means of the diagonal elements of the cross-spectral density tensor $F$. A special aspect of this area has recently been recognized in connection with effects from thermal sources such as those due to vacuum fluctuations (Casimir, Van der Waals), and out of equilibrium forces \cite{6, 7}, whose analogy with random near field forces from partially coherent optical sources has been put forward \cite{8, 9}. In this connection, recent work deals with interaction between two dipoles in presence of random wavefields \cite{10}.

On the one hand, the subject of optical forces is especially present in studies where light assisted mechanical interaction and nanomanipulation of particles is of vital importance, (see Refs. \cite{11, 13} and references therein). On the other hand, the statistical properties of radiation introduces a new degree of freedom that plays a decisive role in optics \cite{13, 15}: like e.g. in scattering processes \cite{17, 19}, or speckle processing \cite{20}, of relevance for the mechanical action of radiation beyond coherent light approaches \cite{21}.

In this work we emphasize Schell model sources whose spectral degree of coherence and radiant intensity distribution are both Gaussian \cite{15} (GSMS). They constitute an extraordinary instance of partially coherent source that can be implemented in the laboratory without an excessive difficulty \cite{22}. Recently, the theory and consequences of the optical force generated by beams from this type of sources at far distances, or for general ABCD systems, where evanescent waves can be neglected, were reported \cite{2, 22, 23}. By contrast, in this work we address these forces in the near-field of the source.

For this purpose, we put forward a theory of forces based on the concept of coherent mode representation (CMR) of partially coherent fields, due to E. Wolf, (see Refs. \cite{25, 26}). This approach establishes that the cross spectral density of a system of any state of coherence may be expressed as the sum of contributions from spatially completely coherent elementary sources, and so are its consequences for the electromagnetic force. We shall use this CMR of optical forces not only on single particles, but also for studying radiation-induced forces between objects, usually referred to as optical binding \cite{11, 14}. Specifically, we shall address the forces due to GSMS light, acting between two cylinders. We will exploit the morphology dependent resonances (MDR) of these objects to form different types of bonds between them. It will also be shown how the spatial coherence of the source affects the attraction or repulsion of these bodies. Although dielectric, the particles here addressed are also magnetic, namely they respond to the incident wave magnetic vector via induced magnetic dipoles and multipoles. They recently have provoked much interest because their potential as exotic scatterers capable of introducing configurations with artificial magnetism \cite{27, 31}.

This paper is organized as follows: We briefly outline in Section II the theory of optical forces with partially coherent light emerging from a GSMS, with emphasis in the near-field. Then in Sections III and IV we develop the concept of stochastic forces from the point of view of the CMR. Later, in the subsequent subsections, this method is analyzed and implemented through calculations of increasingly complex configurations. In Sections
Where we can express the components of Eq. (1) as \( |8\)

(8) the first electric and magnetic Mie coefficients \[29, 31\].

An Appendix is added to show the confirmation of our discussion.

II. OPTICAL FORCES FROM GAUSSIAN SHELL SOURCES

We shall consider Mie dipolar particles, namely those whose scattering properties can be expressed in terms of the first electric and magnetic Mie coefficients \[24, 31\]. Then the ensemble-averaged force experienced by the object is decomposed into two contributions: a gradient (or conservative) force \( F^c \) and a non-conservative component \( F^n \), which in terms of the electric vector \( E(r, \omega) \) at frequency \( \omega \) reads \[5, 32, 33\]:

\[
\langle F_i (r, \omega) \rangle = \langle F_i^c (r, \omega) \rangle + \langle F_i^n (r, \omega) \rangle = \frac{1}{4} \text{Re} \alpha (E_{i}^* (r, \omega) E_j (r, \omega)) + \frac{1}{2} \text{Im} \alpha (E_{i}^* (r, \omega) \partial_j E_j (r, \omega)),
\]

where \((i, j) = (x, y, z)\), \( \langle \cdot \rangle \) denotes ensemble averaged and \( \alpha \) is the electric polarizability of the particle which characterizes the induced electric dipole: \( p(r, \omega) = \alpha(\omega)E(r, \omega) \) by the field emerging from the fluctuating source and impinging the particle.

We next make use of the angular spectrum of plane waves \( e(ks_\perp, \omega) \) \[15, 33, 37\]:

\[
E(r, \omega) = \int_{-\infty}^{\infty} e(ks_\perp, \omega) e^{i k s_\perp r} d^2 s_\perp.
\]

So that we can express the components of Eq. (11) as \(8\)

\[
\langle F_i^c (r, \omega) \rangle = -\frac{i}{2} \text{Re} \int_{-\infty}^{\infty} \text{Tr} A^{(e)}_{j k} (ks_\perp, \omega) \times (s^*_{i} - s^*_{l}) e^{-i k (s^* - s^*)} r d^2 s d^2 s',
\]

\[
\langle F_i^n (r, \omega) \rangle = \frac{1}{2} \text{Im} \int_{-\infty}^{\infty} \text{Tr} A^{(e)}_{j k} (ks_\perp, \omega) \times s^*_{i} e^{-i k (s^* - s^*)} r d^2 s d^2 s',
\]

where \( k = \omega/c \), \( c \) being the velocity of light in vacuum. Also

\[
e(ks_\perp, \omega) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} E(\rho, \omega) e^{-i k s_\perp \rho} d^2 \rho.
\]

\( E(\rho, \omega) \) is the field at the exit plane \( z = 0 \) of the source.

In these equations \( s = (s_1, s_2) \), \( s_\perp = (s_x, s_y) \), and \( s_z = \sqrt{1 - s^2_{\perp}} \), when \( |s^2_{\perp}| > 1 \); or \( s_z = \sqrt{s^2_{\perp} - 1} \) when \( |s^2_{\perp}| > 1 \), which correspond to homogeneous and evanescent waves, respectively. \( \text{Tr} \) denotes the trace of the electric angular correlation tensor: \( A^{(e)}_{j k} (ks_\perp, \omega) = \langle e^*_{j}(ks_\perp, \omega)e(ks'_{\perp}, \omega) \rangle \).

Now we address the specific case of a planar GSMS. This is characterized by a cross-spectral density tensor

\[
W_{ij}^{(0)} (\rho_1, \rho_2, \omega) = \langle E_i^* (\rho_1) E_j (\rho_2) \rangle \text{ at the plane } z = 0 \text{ of the source defined as } 13
\]

\[
W_{ij}^{(0)} (\rho_1, \rho_2, \omega) = \sqrt{S_i^{(0)} (\rho_1, \omega) S_j^{(0)} (\rho_2, \omega) \mu_{ij}^{(0)} (\rho_2 - \rho_1, \omega)},
\]

where \( S_i^{(0)} (\rho, \omega) = W_i^{(0)} (\rho, \rho, \omega) \) and \( \mu_{ij}^{(0)} (\rho_1, \rho_2, \omega) \) are the spectral density and the spectral degree of coherence of the source, respectively. In this model these quantities are both Gaussian, i.e.,

\[
S_i (\rho, \omega) = A_i \exp[-\rho^2/(2\sigma_{s,i}^2)]
\]

\[
\mu_{ij} (\rho_2 - \rho_1, \omega) = B_{ij} \exp[-(\rho_2 - \rho_1)^2/(2\sigma_{g,ij}^2)].
\]

\( A_i \) is a constant, (equal to 1 in this work). The parameters \( \sigma_{s,i} \) and \( \sigma_{g,ij} \) without the Cartesian subindex, understanding that they refer to the X-component of the electric vector.

A. Near Field Forces

Let us address the optical forces of fields from GSMSs on a small sphere, at distances from the source shorter than the wavelength. Whereas at larger distances, the trace of the angular correlation tensor can be approximated as \( \text{Tr} A_{ij} \simeq A_{xx} \), in the near-field, where the resolution of the system is beyond the diffraction limit: \( \lambda/2 \), the fluctuations on the \( Z \)-direction are as important as the rest of them \[33\]. It is well-known that this conveys a non-straightforward 3D generalization in the definition of the degree of polarization \( P(r, \omega) \) \[39, 42\].

Therefore, and in order to quantify the importance of these fluctuations we shall write \( c_{z} \) in terms of \( s_z \), i.e., \( c_z = -e_z s_x/s_z \), with the help of the divergence law: \( e(ks_\perp) \cdot s = 0 \). Hence, \( \text{Tr} A_{ij} = A_{xx} + A_{xz} \).

The forces are calculated from Eqs. (1)-(8) on writing \( s_x, s_y \), \( s_x, s_y \) = \( s(\cos \theta, \sin \theta) \). The azimuthal integrals are performed analytically, whereas the radial one is numerically done for \( \sigma_g \gg \sigma_z \), this corresponds to a globally spatially coherent source. In this limit, the four integrals of the calculation can be expressed as a product of two integrals. We shall first consider a test particle with a radius...
of the electric polarizability that conserves energy \( \sigma \), is \( \alpha = 4593 + i7 \, nm^3 \).

FIG. 1. (Color online). Mean forces. Conservative component \( F^{cons}_x \) (first row), and non-conservative component \( F^{nc}_x \) (second row), of \( F_x \) due to the contribution of \( \varepsilon_x \), versus the lateral displacement \( x \) of the sphere, (in wavelength units), for different spot sizes \( \sigma \). The third column displays the sum of the first and second columns. The distance of the particle to the source is \( z = 0.1 \lambda \).

FIG. 2. (Color online). The same as in Fig. 1 for \( F_z \).

III. COHERENT MODE REPRESENTATION

The coherent mode representation (CMR) establishes that a stationary optical field of any state of coherence may be represented as a superposition of coherent modes \( 26, 13 \), i.e.,

\[
W_{ij} (\mathbf{r}_1, \mathbf{r}_2, \omega) = \langle E^*_i (\mathbf{r}_1, \omega) E_j (\mathbf{r}_2, \omega) \rangle = \sum_q \lambda_q (\omega) \phi_{i,q} (\mathbf{r}_1, \omega) \phi_{j,q} (\mathbf{r}_2, \omega),
\]

where \( \lambda_q (\omega) \) are the eigenvalues and \( \phi_{i,q} \) denote the eigenfunctions which fulfill the equation \( 26 \):

\[
\int_D \phi_{i,q} (\mathbf{r}_1, \omega) W_{ij} (\mathbf{r}_1, \mathbf{r}_2, \omega) d^3 \mathbf{r}_1 = \lambda_q (\omega) \phi_{i,q} (\mathbf{r}_2, \omega).
\]

Let us consider an statistical ensemble of electromagnetic fields \( \{ \mathbf{E}(\mathbf{r}, \omega) \} \) where each realization can be ex-
pressed as a sum of individual eigenfunctions:

\[ E_q(r, \omega) = \sum_q a_q(\omega) \phi_{i,q}(r, \omega), \]  

(11)
a_q being a random coefficient. Substituting Eq. (11) into Eq. (9) we see that

\[ \langle a_q^*(\omega) a_{q'}(\omega) \rangle = \lambda_q(\omega) \delta_{qq'}, \]
\[ a_q(\omega) = \lambda_q^{1/2}(\omega) e^{i\alpha_q}, \]

(12)
where \( \alpha_q \) is a real random variable uniformly distributed in the interval \( 0 \leq \alpha_q < 2\pi \).

A. Coherent Mode representation of Optical forces

We can now write the ensemble-averaged force as a sum of coherent modes by using its expression from the momentum conservation law in terms of the Maxwell stress tensor (MST) [44–46]:

\[ \langle F(r, \omega) \rangle = \sum_q \int_\Sigma \frac{\varepsilon}{2} \text{Re} \left\{ \langle (E_q \cdot n) E_q^* \rangle - \frac{\varepsilon}{4} \langle E_q^* \cdot E_q \rangle \right\} n ds + \frac{\mu}{2} \text{Re} \left\{ \langle (H_q \cdot n) H_q^* \rangle - \frac{\mu}{4} \langle H_q^* \cdot H_q \rangle \right\} n ds. \]

(13)
\( \Sigma \) is a surface enclosing the object experiencing the force. \( n \) represents the outward unit normal. In our 2D calculations \( \Sigma \) will be a closed line. \( E_q, H_q \) and \( E_q^*, H_q^* \) are the \( q \)-modes and their complex conjugates. For brevity we have omitted the space and frequency dependence of the fields. \( \varepsilon \) and \( \mu \) are the permittivity and permeability of the surrounding medium embedding the particles, which in this work will be assumed to be vacuum. The sum of the partial forces from each propagated eigenmode renders the resulting force exerted on the particles by the total fields \( E \) and \( H \). Notice that Eq. (13) applies to any configuration, regardless of whether the source is spatially coherent (\( q = 0 \)) or partially coherent (\( q > 0 \)).

For dipolar particles the averaged total force Eq. (1) can now be expressed in terms of the coherent \( q \)-modes:

\[ \langle F_i(r, \omega) \rangle = \frac{1}{2} \sum_q \text{Re} \left\{ \alpha_i \mu_{i,q} \partial_i \phi_{i,q}^* \right\} = \frac{1}{2} \sum_q \lambda_q \text{Re} \left\{ \alpha_i \phi_{i,q} \partial_i \phi_{i,q}^* \right\}. \]

(14)

IV. CHARACTERIZATION OF THE FIELD Emitted By THE GSMS

Using the CMR, we shall follow the procedure put forward in [18] to characterize the fluctuating field from a GSMS. Then the problem is 2D so that plane of work will be \( XY \). \( y \) is the direction of propagation and the field fluctuates along 0Z (see Fig. 3). The GSMS plane is \( y = 0 \), thus the cross-spectral density function will be:

\[ W_{zz}^{(0)}(x_1, x_2, \omega) = A e^{\frac{-x_1^2-x_2^2}{2\sigma_y^2}} e^{-i\pi x_1 \sigma_y}, \]

(15)
For this case, the eigenfunctions and the eigenvalues have been determined previously [13, 47]:

\[ \phi_q(x, \omega) = \left( \frac{2e}{\pi} \right)^{1/4} \frac{1}{(2\pi q!)^{1/2}} H_q \left( x\sqrt{2c} \right) e^{-cx^2}, \]

(16)
\[ \lambda_q(\omega) = \left( \frac{\pi}{a+b+c} \right)^{1/2} \left( \frac{b}{a+b+c} \right)^q, \]

(17)
where \( H_q(x) \) is the Hermite polynomial of order \( q \), and

\[ a = \frac{1}{4\sigma_y^2}, \quad b = \frac{1}{2\sigma_y}, \quad c = (a^2+2ab)^{1/2}. \]

The angular amplitude \( \Phi(ks_x) \) of the eigenfunction \( \phi_q(x, \omega) \), is calculated by inverse Fourier transform of Eq. (10), (see [18]):

\[ \Phi(ks_x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi(x, \omega) e^{-iks_\omega x} dx \]
\[ = \frac{(-i)^q}{2\pi} \left( \frac{2\pi}{c} \right)^{1/4} \frac{1}{(2\pi q!)^{1/2}} e^{-\frac{x^2}{2c}} H_q \left( \frac{ks_x}{\sqrt{2c}} \right). \]

(19)

V. NUMERICAL SETUP

A pair of particles is illuminated by the GSMS wavefield whose mechanical action produces optical binding effects with characteristics of a photonic molecule. [18, 50]. 2D numerical calculations are done by means of a FEM (RF module of COMSOL 4.3a, http://www.comsol.com) and MATLAB. Aside from some depolarization effects, the main features of the physical process: light scattering, resonance excitation and binding, are analogous to those in 3D [51, 53].

Without loss of generality, a Si cylinder with \( \epsilon = 10.24 \) and radius \( r_0 = 0.3 \mu m \) [49] has been considered, due to its rich Mie resonance spectra in both the visible and near IR [54]. This will allow us to analyze the effects of spatial coherence on these resonances and their consequences for the induced optical forces on this pair, (see Section IV).

Following the scheme shown in Fig. 3(a), an incident wavefield with electric vector \( E_z \) perpendicular to the XY-plane, is launched upwards, propagating along OY. The choice of S-polarization (TE), in contrast with P-polarization (TM), excites whispering-gallery modes
molecular states associated to anti-symmetric field patterns with respect to the transversal axis of the molecule, [see insets $OE$ and $OO$ in Fig. 3(b)] are not destroyed, especially when the molecule is inclined with respect to the propagation direction of the beam. Notice that if both particles were simultaneously illuminated by the beam, only those WGMs related to symmetric field patterns with respect to the molecule axis would be excited, [see insets $EE$ and $EO$ in Fig. 3(b)].

The separation between the particles is $d_0 = 100nm$, which makes subwavelength the molecule dimensions, (compare the set size, $1.3\mu m$, to the range of wavelengths under study: $1.6\mu m - 8.0\mu m$). The center of the lower particle is $\approx 0.75\mu m$. We follow the nomenclature of [48, 50] for the molecular states, the classification being based on the $E(r)$ field symmetry with respect to the main directions defined by the molecule geometry, i.e., its longitudinal ($Y_{pm}$) and transversal ($X_{pm}$) axes, [see Fig. 3(b)]. As an example, we will examine the upper-left inset of Fig. 3(b). In this case, the upper lobe is opposite to the lower one, thus it is said that $E$ is even, (E), with respect to $OX$; however, $E$ is odd, (O), with respect to $OY$. Therefore the photonic state is even-odd (EO). If they mismatch one another, it would be $X_{pm}Y_{pm}/OO$, [cf. the upper-right inset in Fig. 3(b)]. These would be similar to a double bound in the molecule. In the case in which only one lobe of each particle interacts with the other (simple bound), the states will be $X_{pm}Y_{pm}/EE$ and $X_{pm}Y_{pm}/OE$, respectively.

In all cases the $E_z$ profile at frequency $\omega$ is that of a GSMS, described in Section IV. The field has an intensity $1W/m^2$ and $\sigma_s = 0.05 \times 1500nm$. The spatial coherence of the near field is gradually established as the ratio between the coherence length and the width of the beam $\sigma_g/\sigma_s = 100, 2, 0.5$ diminishes. The GSMS is placed in the lower boundary of the simulation window and is implemented as a discrete sum of modes $q$, [see Eq. (16)]. As explained in Section IV, the lower the ratio of $\sigma_g/\sigma_s$, the higher the value of $q$. (cf. Fig. 5.17 of [15]). An iterative process is followed in order to simulate the propagation of each of these $q$-modes through the calculation window. Subsequently, they are summed up to get the propagated total fields $E(r)$ and $H(r)$.

The next results show the time-averaged energy flow $\langle |S(x, y)| \rangle$, that shows light concentration in the probe cylinders. Because of their dielectric nature, we average $|\langle S(x, y) \rangle|$ in a circle which coincides with the geometrical cross section of the probe cylinder of radius $r_0$, [see Fig. 3(b)]. This stems from the fact that, if the particle is dielectric, the intensity of the light beam that couples to the particle WGM, is concentrated inside the cylinders, (see [54]), not outside them, (the latter occurs for plasmonic cylinders [57]). In all cases, these intensities are normalized to the maximum intensity of the incident Gaussian beam: $|\langle S_{max} \rangle| = 1W/m^2$.

The averaged force on the probe cylinders is calculated by employing the MST, Eq. (13). The line of integration $\Sigma$ surrounds each particle as seen in Fig. 3(b). In our 2D geometry, $\Sigma$ is the circumference of radius $r_{c}$, (see Fig. 3(b)). $\epsilon = \mu = 1$. Because of this 2D geometry,
our results are expressed as force per axial length unit, in $N/m$.

The COMSOL calculation with complex values of $\mathbf{E}(r)$ and $\mathbf{H}(r)$ as well as of the real physical fields: $\mathbf{E}^R(r, t) = \text{Re}\{\mathbf{E}(r) \exp(-i\omega t)\}$ and $\mathbf{H}^R(r, t) = \text{Re}\{\mathbf{H}(r) \exp(-i\omega t)\}$, is not straightforward. The details of the procedure have been given in [53]. The meshing used in the simulation has a maximum and a minimum element of $\lambda_{ref}/8$ and $2.7\,\text{nm}$, respectively. The reference wavelength being $\lambda_{ref} = 1620\,\text{nm}$. The maximum element growth rate, resolution of curvature, and resolution of narrow regions are $1.3, 0.3$, and $1$, respectively.

VI. A BI-PARTICLE MOLECULE ILLUMINATED BY A GSMS BEAM. EFFECTS OF PARTIAL COHERENCE IN THE “MOLECULAR” STATES

A. Localization of resonances of a single particle. Bi-particle set: Production of “molecular” states

In order to identify the resonant states of a photonic molecule, the spectral location of the resonances of the single particle is required. For the sake of accuracy needed in the calculations, and in order to deal with not too complex bonds between the particles, our study limits the search of resonances in each individual particle to those of low angular order. This suffices to illustrate the analysis in this work.

Hence, the chosen wavelength is the near infrared, (NIR), in which two multipolar peaks of field intensity localized in the cylinder, associated to its morphology dependent resonances (MDR) are found, [see Fig. 4(a)]. As the insets show, these are the WGMs $TE_{31}$ ($\lambda \approx 1205\,\text{nm}$) and a $TE_{21}$ ($\lambda \approx 1620\,\text{nm}$). At larger $\lambda$, as shown in Fig. 4(b) the MDRs $TE_{11}$ ($\lambda \approx 2610\,\text{nm}$) and $TE_{01}$ ($\lambda \approx 6710\,\text{nm}$) are excited, (cf. the insets of this figure). The $TE_{11}$ is interesting because, as shown in [27, 28, 51], the cylinder scattering cross section is dominated by the Mie coefficients $b_0$ and $b_1$ [51], associated to the electric and magnetic dipolar moments, $\mathbf{p}$ and $\mathbf{m}$, respectively, of the cylinder; therefore this particle behaves as magnetodielectric in this spectral range.

The concentration of intensity $|\langle \mathbf{S}(r) \rangle|$ inside each particle confining the photonic molecule is shown in Figs. 5(a) and 5(b). A comparison between the blue solid and the red dashed lines in Fig. 5(a) shows that the intensity $|\langle \mathbf{S}(r) \rangle|$ in the right particle is generally higher than that in the left one, [the same happens for the lower and the upper cylinders in 5(b)]. This happens because the particle directly illuminated by the beam concentrates more intensity $|\langle \mathbf{S}(r) \rangle|$.

The calculation is focused on the different non-degenerate collective states that can produce the mode $TE_{21}$ excited in both cylinders. Due to the disposition of the lobes of the resonance, ("even", $E$, or "odd", $O$ in the field $\mathbf{E}$ spatial distribution), for each particle with respect to the symmetry axes defined by the ensemble, which are longitudinal and transversal with respect to the molecule axis (hereafter denoted as $X_{pm}$ and $Y_{pm}$, respectively), such a resonance excited in this configuration can generate four “molecular” states [48, 53, 58]. The collective states $Y_{pm}/E$ can be obtained by illuminating the ensemble either in the direction parallel or transversal to the molecule axis.

The reason to select the configuration in which the $Y_{pm}$ axis appears inclined by an angle $\pi/2$ while the direction of the beam is parallel to the $Y$ axis of the calculation window, is explained in Subsection VI B. Figure 5(a) shows this geometrical configuration, which renders the molecular states $Y_{pm}/E$ as consequence of the splitting of the resonance $TE_{21}$ of the single particle into two new MDRs, associated to the disposition of the lobes with respect to the $X_{pm}$ axis, i.e. $X_{pm}Y_{pm}/OE$ and $X_{pm}Y_{pm}/EE$, at $\lambda = 1597\,\text{nm}$ and $\lambda = 1665\,\text{nm}$, respectively (see the insets) [49].

![FIG. 4. Spatially coherent illumination. (a) Spectral variation of the mean of the ensemble-averaged Poynting vector norm $|\langle \mathbf{S}(r) \rangle|$ (i.e. the light intensity), in a single cylinder illuminated by a totally coherent GSMS beam. The two magnetic multipole peaks are shown. (b) The same quantity in a range of higher $\lambda$ in which the Mie coefficients contributing to the scattering cross section are $b_0$, (electric dipole, $\lambda = 6 - 7\,\text{nm}$), and $b_1$, (magnetic dipole, $\lambda = 2.7\,\text{nm}$); hence the particle being magnetodielectric. The insets in (a) and (b) show the spatial distribution of $|\langle \mathbf{S}(r) \rangle|$ for WGMs: $TE_{31}/WGE_{21}$ and $TE_{11}/TE_{01}$, respectively.](image)
On the other hand, in order to reproduce the collective states $Y_{pm}/O$, the $Y_{pm}$ axis must be inclined by an angle of $\pi/4$ with respect to the propagation direction of the beam because of the number of intensity lobes for the resonance $TE_{21}$ in the single particle. This is seen in Fig. 5(b), where the molecular states $Y_{pm}/O$ arise as a new splitting of the resonance $TE_{21}$ of the single particle, i.e. $X_{pm}Y_{pm}/OO$ and $X_{pm}Y_{pm}/EO$, at $\lambda = 1582\,\text{nm}$ and $\lambda = 1693\,\text{nm}$, respectively, (see the detail in this figure).

All the non-degenerate states of this photonic molecule associated to the MDR $TE_{21}$ in each particle are shown by these two orientations of the ensemble. Both orientations present two collective resonances, the $X_{pm}/O$ and $X_{pm}/E$ being blue- and red-shifted, (i.e. more and less energetic, respectively). This can be explained by the insets of this figure: the $X_{pm}/O$ states concentrate relatively much more light intensity inside the cylinders than the $X_{pm}/E$ ones. Each set of orientation also reminds the formation of a simple, [Fig. 5(a)], or a double, [Fig. 5(b)], bond between the particles [49].

By increasing the wavelength $\lambda$ of illumination on this particle pair around the same range as in Fig. 6(b), the behavior of the collective resonances appears to be similar to that of Fig. 5(a) and Fig. 5(b) regarding the connection between their symmetry, ($X_{pm}/O$ and $X_{pm}/E$ lobes in $E$), and energy, (blue- and red-shifted peaks). These states being in this case originated by the $TE_{11}$ and $TE_{01}$ resonances excited in the single particle. Aiming to reproduce its $Y_{pm}/E$ and $Y_{pm}/O$ states, the $Y_{pm}$ axis is constrained to be either parallel, [see Fig. 6(a)], or perpendicular, [see Fig. 6(b)], to the direction of the light beam, respectively. The suppression, in both orientations, of the less energetic molecular state associated to the $TE_{01}$ WGM of the single particle, i.e the $X_{pm}Y_{pm}/EE$, is due to the fact that the illuminating wavelength $\lambda$ is much longer than the dimensions of the molecule, this latter now being almost invisible.
B. Effects of partially coherent illumination on the electromagnetic forces between the particles.
Bonding and anti-bonding “molecular” states

Next, we consider the cylinder pair illuminated by a GSMS with different coherence lengths $\sigma_g$. This allows us to understand its effect of the electromagnetic forces acting on its collective states. As previously remarked, for this S-polarization the fields associated to these states, although localized inside the particles, reach high intensity values in the area immediately outside them. Taking into account the calculation from the CMR of MST, Eq. 13, maximum forces are thus expected to appear when these states are excited. Two of the MDRs of the single particle: $TE_{21}$ and $TE_{11}$, are selected to study the electromagnetic forces acting in the optical binding between the two cylinders which conform the photonic molecule. As discussed in Section VI A each of these resonances splits into two collective states whose symmetry and electromagnetic forces acting in the optical binding between these states are excited. Two of the MDRs of the single particle: $TE_{21}$ and $TE_{11}$, are selected to study the electromagnetic forces acting in the optical binding between the two cylinders which conform the photonic molecule.

As discussed in Section VI A each of these resonances splits into two collective states whose symmetry and energy are related to each other. The $TE_{21}$ mode is chosen due to its possibility to generate states in the particle pair which remind those of a simple [Fig. 5(a)] and a double [Fig. 5(b)] bond in an atomic molecule. The $TE_{11}$ mode causes the particles to behave as magneto-dielectric, giving rise to an interaction not only between its electric dipoles, but also between its induced magnetic ones.

Figures 7(a)-(b) and 7(c)-(d) show the electromagnetic force between the two particles in the case of the collective states corresponding to the two first peaks of intensity $|\langle \mathbf{S} (\mathbf{r}) \rangle|$ in Fig. 5(a) and Fig. 5(b), respectively. They correspond to the splitting of the magnetic quadrupole $b_2$ of the single particle of Fig. 4(a). The reason to choose the orientation shown in Fig. 4(a) for the molecule is now clear since the total force on the particles has two contributions: the gradient force between the particles and that of scattering related to the radiation pressure of the incident beam along $OX$ and $OY$. On the other hand, the orientation used in Figs. 7(c)-(d) causes those two force components to mix with each other along $OY$, notwithstanding remaining possible to study the interaction between both particles by means of the force X-component.

Under completely coherent illumination, peaks of repulsive and attractive force between the two particles appear at $\lambda \approx 1597\text{nm}$ and $\lambda \approx 1665\text{nm}$, (cf. in Fig. 7(a) black lines with and without points for the X-component of the forces on particles 1 and 2, respectively). The same happens in Fig. 7(c) at $\lambda \approx 1582\text{nm}$ and $\lambda \approx 1693\text{nm}$. These results allow to identify the blue-shifted $X_{pm}Y_{pm}/OE$ and the red-shifted $X_{pm}Y_{pm}/EE$ in Fig. 5(a) collective states ($X_{pm}Y_{pm}/OO$ and $X_{pm}Y_{pm}/EO$ in Fig. 5(b)) as anti-bonding and bonding ones, respectively.

The forces in the vertical direction are higher for particle 1 (which is directly illuminated) in both orientations. In Fig. 7(a) this component, associated to the scattering force from the beam, is lower for the bonding molecular state at $\lambda \approx 1665\text{nm}$ than that for the antibonding one (at $\lambda \approx 1597\text{nm}$), since the former renders higher values of field intensity immediately outside the particles. For the orientation of Fig. 7(c), both collective states, the repulsive and the attractive one, at $\lambda \approx 1582\text{nm}$ and $\lambda \approx 1693\text{nm}$, suffer comparable Y-components of the total force because now in this direction the gradient force between the particles must also be taken into account.

When we decrease the coherence length of the source, (see red and blue lines standing for $\sigma_g = 2\sigma_s$ and $\sigma_g = 0.5\sigma_s$, respectively), both components of the force invariably diminish. Although the dimension of the molecule and its position with respect to the source, which is in the lower boundary of the calculation window), are subwavelength, these results are opposite to those of Fig. 4 in [32, 48–50, 58]. The interaction between the GSMS beam and the particles now being more complex due to the addition of the effect from the MDRs. In fact, the intensity pattern
of the interference process which renders the particle resonance decreases, i.e. the field lobes corresponding to the formation of the resonance in each particle loose contrast. This leads, taking into account the force calculation, to a decrement in the field intensity values reached outside the particles and hence in their optical attraction or repulsion.

![Graphs](image_url)

**FIG. 8.** (a)-(b) The same quantities as in Fig. [7](a)-(b) with the molecule oriented as in Fig. [6](a). (c)-(d) The same as in Fig. [6](a)-(b), the molecule now being oriented as in Fig. [6](b). The code of lines and colors is identical to that of Fig. [7].

Finally, the optical forces on the molecular states associated to the first two peaks of Fig. [6](a) and Fig. [6](b), [associated to the magnetic dipole of Fig. 4(b)], are shown in Figs. [8](a)-(b) and [8](c)-(d), respectively. The vertical orientation of the pair, although now mixing both contributions to the total force, (i.e. gradient component between the particles and scattering one due to radiation pressure of the light beam), renders its Y-component being the only significant one, and behaves just as expected at \( \lambda \approx 2230 \text{nm} \) and \( \lambda \approx 2860 \text{nm} \). Namely, repulsive and attractive forces arise acting on the blue-shifted, \( X_{pm}Y_{pm}/OE \), and the red-shifted, \( X_{pm}Y_{pm}/EE \), molecular states. As in the previous case, the loss of coherence in the light beam causes the decrement in the magnitude of both force components.

**VII. CONCLUSIONS**

We have presented a theory, illustrated by computer simulations, of optical binding of Mie dipolar dielectric particles with magnetodielectric behavior, in the near field of a partially coherent Gaussian-Schell model source. In connection with previous work [18], a straightforward representation has been chosen in the framework of the angular spectrum [24 55] and the coherent mode decomposition [15]. The excitation of the electric dipole and the magnetic dipole and multipoles confers to these systems a rich landscape of resonant forces. For adjusted parameters of the emitted wavefield, i.e. the spot size \( \sigma_s \) and the coherence length \( \sigma_g \), in contrast with to far-field effects, (see e.g. [1 3]), and confirming other near field results [8], as the coherence length \( \sigma_g \) decreases, the pulling force from the source on a single particle increases.

In addition, on extending the analysis to the dynamical interactions between the emitted light and a pair of cylinders forming a photonic molecule, we have shown the effects of the spatial coherence on the optical binding between the particles. This is linked to the symmetric and anti-symmetric molecular resonances, associated to bonding and anti-bonding states, respectively. The role of the interplay between the electric and magnetic induced dipoles when such Mie resonances are induced, has been shown to be important. Now the threshold of evanescent wave contribution to scattered field is critical. Namely, in addition to being at subwavelength distances from the source plane, the particles need to be practically in contact with each other for a substantial contribution of the inhomogeneous modes. As a consequence, as few evanescent modes are present, a decrease of the coherence length \( \sigma_g \) conveys lower bonding and antibonding forces.

All this confirms that the near field force strength is linked to evanescent waves and increases with a decrease of the near field coherence length, contrary to the effect in the far field where only propagating modes are present.

**APPENDIX**

We illustrate force calculations based on the coherent mode representation (CMR) of Section [IV]. We address a cylinder of radius \( \lambda/100 \), made of Silica glass \((\varepsilon_p = 2.1)\), illuminated by a GSMS placed at \( y = 0 \), (cf. Eq. [15]).
The distance between the source and the center of the particle is $\lambda/10$. The number of modes is determined by the ratio $\sigma_g/\sigma_s$, the first mode ($q = 0$) corresponding to the globally spatial coherent case studied in Section II. A. The scheme of the simulation window in which the beam propagates and is scattered by the particle, as well as the method to calculate the optical forces, is similar to that previously explained in Section II now for a single particle.

![Diagram](image_url)

**FIG. 9.** (Color online). Ensemble-averaged forces $F_x$ (first row), and $F_y$ (second row), from a partially coherent GSMS. The first column from the left pertains to the fully coherent source, $\sigma_g = 100\lambda \gg \sigma_s$, which would correspond to the case of Section II. For the center and right columns $\sigma_s = 0.3\lambda$ and $0.5\lambda$, respectively.

Fig. 9 displays the calculated force components. Here one cannot separate the conservative and non-conservative components of the force since in Eq. (13) the MST flow yields the total force. Each row of Fig. 9 represents the ensemble-averaged forces $\langle F_x \rangle$ and $\langle F_y \rangle$ for different values of $\sigma_g$ and $\sigma_s$ (see the legend of the figure). The first column, [Figs. 9(a), (d)], contains $\langle F_x \rangle$ and $\langle F_y \rangle$ for the same parameters of Figs. 1 and 2 (by inverting the color - line code). We see that for a fully coherent source, as we decrease the value of the spot size $\sigma_s$, the magnitude of the force increases, as stated in the main text. We also observe how $\langle F_y \rangle$ is negative, (i.e. the particle is pulled to the plane of the source), for $\sigma_s < 0.3\lambda$; this is due to the contribution of the evanescent waves. In the main text this fact is discussed.

The second column, [(Figs. 9(b), (e)], represents the force due to a partially coherent GSMS. We have fixed the spot size to $\sigma_s = 0.3\lambda$. Contrary to what one could expect, it is the most incoherent emitted field that which produces the maximum force. In the last column, although we can see a similar behavior, we also observe that for $\sigma_g > 0.5\lambda$ the force is positive, i.e., the particle is pushed by the source towards $y > 0$.

![Diagram](image_url)

**FIG. 10.** (Color online). The function exp$(-k^2 s_s^2/(4c^2))$ versus the transversal component $s_x$ for different values of the spot size $\sigma_s$ and coherence length $\sigma_s$. For $s_x > 1$ the evanescent waves are not negligible.

In order to explain all of these results, in Fig. 10 we show the exponential function exp$[-k^2 s_s^2/(4c^2)]$ of the angular spectrum, [cf. Eq. (19)], for different values of $\sigma_g$ and $\sigma_s$; this helps us to understand the behavior of the previous figures. The black and the blue point lines represent the width of the Gaussian function for two cases represented in the force in Figs. 9(a) and (d). One sees that for a fully coherent source, the Gaussian is broader for a lower value of the spot size, thus taking more evanescent modes of the angular spectrum. The red point-dashed line and the green-dashed lines represent two cases of Figs. 9(b) and (e). Now, for a partially coherent source, fixing the value of the spot size, the evanescent modes are more important as the coherence length of the source decreases. All this agrees with the results of Fig. 7 and Fig. 4 of 8.

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