Crossing of the phantom divided barrier with Lorentz invariance violating fields

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Abstract - We study the possible crossing of the phantom divided barrier in a Lorentz invariance violating dark-energy model. Lorentz invariance violation, which is achieved by introducing a vector field in the action, incorporates directly in the dynamics of the scalar field and equation of state. This interesting feature allows us to study phantom divided barrier crossing in the context of Lorentz invariance violation. We show that for a suitable choice of the parameter space, the equation of state can cross the phantom divided barrier just by one scalar field and a Lorentz violating vector field controls this crossing.

Motivation. – Recently, Lorentz invariance violation (LIV) has been studied in the context of scalar-vector-tensor theories [1]. It has been shown that Lorentz violating vector fields affect the dynamics of the inflationary models. One of the interesting feature of this scenario is that the exact Lorentz violating inflationary solutions are related to the absence of the inflaton potential. In this case, the inflation is completely associated with the Lorentz violation and depends on the value of the coupling parameter [2]. Standard cosmology with a pressureless fluid as matter content of the universe, predicts a universe either expanding forever or recollapsing eventually depending on the spatial geometry. Recent evidences from supernova searches data [3,4], cosmic microwave background (CMB) results [5-7] and also Wilkinson microwave anisotropy probe (WMAP) data [8,9], indicate an accelerating phase of cosmological expansion today and this feature shows that the simple picture of the universe consisting of pressureless fluid is not enough; the universe may contain some sort of additional negative-pressure dark energy. The analysis of the three year WMAP data [10-12] shows that there is no indication for any significant deviations from Gaussianity and adiabaticity of the CMB power spectrum and therefore suggests that the universe is spatially flat to within the limits of observational accuracy. Further, the combined analysis of the three-year WMAP data with the supernova Legacy survey (SNLS) [10], constrains the equation of state \( w_{de} \), corresponding to almost 74% contribution of dark energy in the currently accelerating universe, to be very close to that of the cosmological constant value. Moreover, observations appear to favour a dark-energy equation of state, \( w_{de} < -1 \) [13]. Therefore a viable cosmological model should admit a dynamical equation of state that might have crossed the value \( w_{de} = -1 \), in the recent epoch of cosmological evolution. Various aspects of this crossing have been studied extensively (see, for instance, [14] and reference therein). However, a possible impact between LIV and the phantom divided barrier crossing has not been studied yet. Since there are some traces of Lorentz invariance violation in the high-energy regime [15,16], it is interesting to study the possible implication of this symmetry breaking on the dynamics of the equation of state and especially the crossing of the phantom divided barrier. The purpose of this letter is to take a small step in this direction.

A Lorentz violating cosmology. – In this section, following [1,2], we summarize the cosmological dynamics of Lorentz invariance violating fields. Our goal is to find a relation between the Lorentz invariance violation parameter and the dynamics of the scalar field. This
relation will affect the equation of state of the scalar field which is the central object of the following sections.

We start with the following action for a typical scalar-vector-tensor theory which admits Lorentz invariance violation

\[ S = S_g + S_u + S_\phi, \]

where the actions for the tensor field \( S_g \), the vector field \( S_u \), and the scalar field \( S_\phi \) are defined as follows:

\[ S_g = \int d^4x \sqrt{-g} \frac{1}{16\pi G} R, \]

\[ S_u = \int d^4x \sqrt{-g} \left[ -\beta_1 \nabla^\mu u^\nu \nabla_\mu u_\nu - \beta_2 \nabla^\mu u^\nu \nabla_\nu u_\mu \right. \]
\[ - \left. \beta_3 \left( \nabla_\mu u^\mu \right)^2 - \beta_4 \nabla^\mu u^\nu \nabla_\nu u^\alpha + \lambda (u^\mu u_\mu + 1) \right], \]

\[ S_\phi = \int d^4x \sqrt{-g} \mathcal{L}_\phi. \]

This action is allowed to contain any non-gravitational degrees of freedom in the framework of the Lorentz violating scalar-tensor-vector theory of gravity. As usual, we assume \( u^\mu u_\mu = -1 \) and that the expectation value of the vector field \( u^\mu \) is \( \langle 0 | u^\mu u_\mu | 0 \rangle = 0 \) [17]. \( \beta_i(\phi) (i = 1, 2, 3, 4) \) are arbitrary parameters with the dimension of the mass squared and \( \mathcal{L}_\phi \) is the Lagrangian density for the scalar field. Note that \( \beta_3 \) are mass scales of the Lorentz symmetry breakdown [1, 17]. The detailed cosmological consequences of this action are studied in ref. [1].

Assuming a homogeneous and isotropic universe, we describe the universe with the following metric:

\[ ds^2 = -\mathcal{N}^2(t) dt^2 + e^{2\alpha(t)} \delta_{ij} dx^i dx^j, \]

where \( \mathcal{N} \) is a lapse function and the scale of the universe is determined by \( \alpha \) [1, 2]. By variation of the action with respect to metric and choosing a suitable gauge, one obtains the following field equations:

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \]

where \( T_{\mu\nu} = T_{\mu\nu}^{(s)} + T_{\mu\nu}^{(\phi)} \) is the total energy momentum tensor, \( T_{\mu\nu}^{(s)} \) and \( T_{\mu\nu}^{(\phi)} \) are the energy momentum tensors of vector and scalar fields, respectively. The time and space components of the total energy momentum tensor are given by [2]

\[ T_0^0 = -\rho_u - \rho_\phi, \quad T_i^i = p_u + p_\phi, \]

where the energy density and pressure of the vector field are calculated as follows:

\[ \rho_u = -3\beta H^2, \]

\[ p_u = \left( 3 + 2 \frac{H'}{H} + 2 \frac{\beta'}{\beta} \right) \beta H^2, \]

\[ \beta \equiv \beta_1 + 3\beta_2 + \beta_3. \]

The prime denotes the derivative of any quantity with respect to \( \alpha \) and \( H \equiv \mathcal{N}'/\mathcal{N} = \dot{\alpha} \) is the Hubble parameter. One can see that \( \beta_i \) does not contribute to the background dynamics [1, 2]. The energy equations for the vector field \( u \) and scalar field \( \phi \) are as follows:

\[ \rho'_u + 3(\rho_u + p_u) = +3H^2 \beta', \]
\[ \rho'_\phi + 3(\rho_\phi + p_\phi) = -3H^2 \beta', \]

respectively. So, the total energy equation in the presence of both the vector and the scalar fields reads

\[ \rho' + 3(\rho + p) = 0, \quad (\rho = \rho_u + \rho_\phi). \]

With these preliminaries, the dynamics of the model is described by the following Friedmann equations [1, 2]:

\[
\left(1 + \frac{1}{8\pi G\beta}\right) H^2 = \frac{1}{3\beta} \rho_\phi, \\
\left(1 + \frac{1}{8\pi G\beta}\right) (H H' + H^2) = -\frac{1}{6} \left( \frac{\rho_\phi}{\beta} + \frac{3p_\phi}{\beta} \right) - H^2 \frac{\beta'}{\beta},
\]

In the absence of the vector field, that is, when all \( \beta_i = 0 \), one recovers the standard equations of dynamics. For the scalar sector of our model we assume the following Lagrangian:

\[ \mathcal{L}_\phi = -\frac{\eta}{2} (\nabla \phi)^2 - V(\phi), \]

where \( (\nabla \phi)^2 = g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \). Ordinary scalar fields correspond to \( \eta = 1 \), while \( \eta = -1 \) describes phantom fields. For the homogeneous scalar field, the density \( \rho_\phi \) and pressure \( p_\phi \) are given as follows:

\[ \rho_\phi = \frac{\eta}{2} H^2 \phi'^2 + V(\phi), \]
\[ p_\phi = \frac{\eta}{2} H^2 \phi'^2 - V(\phi). \]

The corresponding equation-of-state parameter is

\[ \omega_\phi = \frac{p_\phi}{\rho_\phi} = -1 - \eta \frac{H^2 \phi'^2}{2V}. \]

Now, the Friedmann equation takes the following form [2]:

\[ H^2 = \frac{1}{3\beta} \left( \frac{\eta}{2} H^2 \phi'^2 + V(\phi) \right), \]

where \( \bar{\beta} = \beta + \frac{1}{8\pi G} \). Using this equation we can show that

\[ \phi' = -2\eta \bar{\beta} \left( \frac{H_\phi}{H} + \frac{\bar{\beta}_\phi}{\bar{\beta}} \right). \]

Substituting this equation into the Friedmann equation, the potential of the scalar field can be written as [2]

\[ V = 3\bar{\beta} H^2 \left[ 1 - 2 \frac{2}{3} \eta \bar{\beta} \left( \frac{\bar{\beta}_\phi}{\bar{\beta}} + \frac{H_\phi}{H} \right)^2 \right]. \]
Note that in the above equations the Hubble parameter $H$ has been expressed as a function of $\phi$, $H = H(\phi(t))$. One can show that the equation of state has the following form:

$$\omega_\phi = -1 + \frac{4}{3} \eta \beta \left( \frac{H_\phi}{H} + \frac{\beta_\phi}{\beta} \right)^2$$

$$= -1 + \frac{1}{3} \frac{\phi'^2}{\eta \beta}.$$  \hspace{1cm} (23)

Equations (21) and (23) are essential in forthcoming discussions. Note that the violation of the Lorentz invariance, which has been introduced by the existence of a vector field in the action, now is incorporated in the dynamics of the scalar field and equation of state via the existence of $\beta$. This interesting feature allows us to study phantom divided barrier crossing in the context of Lorentz invariance violation. We need to solve these two equations, (21) and (23), to find the dynamics of the scalar field $\phi$ and the equation of state $\omega_\phi$.

This will be achieved only if the Hubble parameter $H(\phi)$ and the vector field coupling $\beta(\phi)$, are known. In what follows, our strategy is to choose some different cases of the Hubble parameter $H(\phi)$ and the vector field coupling $\beta(\phi)$ and then investigating a possible crossing of the phantom divided barrier in this context. We concentrate on suitable domains of the parameter space which admit such a crossing.

**LIV and crossing of the phantom divided barrier.**

- To investigate phantom divided barrier crossing in this context, we have to solve eqs. (21) and (23) for four unknowns: $H$, $\omega_\phi$, $\beta$, and $V$. This is impossible unless two of these unknowns be specified a priori. A large class of equations of state for scalar field has been studied in [18] and [19] and some classes of potentials allowing for the scalar field equation of state were described. Also some authors have used vector field models for the coincidence of dark energy [20–23]. The main point of these studies, which is crucial for our subsequent discussions, is the fact that to have a successful model with phantom divided barrier crossing within a minimally coupled scalar field scenario, one should consider both quintessence and phantom fields. However, with non-minimally coupled scalar fields, one can achieve the phantom divided barrier crossing just with one of these fields [14]. As we will show, in the presence of Lorentz invariance violating fields, it is possible to cross phantom divided barrier just by considering one field, i.e., the quintessence field. This may reflect the fact that LIV has something to do with the non-minimal coupling of scalar field and gravity. In other words, our basic action defined in eqs. (1) and (2) are actually minimally coupled, but our results for the crossing of the phantom divided barrier are very similar to the non-minimal model results presented in [14]. So, it seems that there is a close relation between LIV and the non-minimal scenario in the present context. In what follows, based on above argument, we consider just a quintessence scalar field with $\eta = 1$. We study the possible crossing of the phantom divided barrier in some model universes with concentration on a suitable range of the parameter space.

**Case 1.** In the first step we consider a model universe with the following simple choices:

$$H = H_0, \quad \beta(\phi) = m \phi^2,$$

where $H_0$ is a positive constant. With these choices, eqs. (21) and (23) lead us to the following relations:

$$\phi(t) = \phi_0 \exp \left[ -4 \eta m H_0 (t - t_0) \right],$$

$$\omega_\phi = -1 + \frac{16}{3} m, \hspace{1cm} (26)$$

where $\phi(t = t_0) = \phi_0$ is a constant. This model shows that cosmic evolution starts from a constant value of the scale factor and then grows exponentially, $a(t) = a_0 \exp[\eta t (t - t_0)]$. Equation (26) shows that the equation of state $\omega_\phi$ has no dynamics since it only depends on the value of the vector field coupling parameter $m$. Since an accelerated expansion occurs for $\omega_\phi < -1/3$ [2], then we should have $m < 1/8$ for a typical quintessence field. However, the present data of the universe seems to tell us that $\omega_\phi$ might be less than $-1$. Thus, the value of $m$ may be chosen to fit with the present observable constraint on the equation-of-state parameter. If we use the values of $m$ obtained in [2], we can calculate $\omega_\phi$ vs. $m$. This is shown in fig. 1. This figure shows that for the crossing of the phantom divided line, $\omega_\phi = -1$, the lower value of $m$ should be negative. However, this is impossible since $m$ is related to $\sqrt{\beta}$ as mass scales of the Lorentz symmetry breakdown. So, in this model universe, the crossing of the phantom divided barrier with one scalar field is impossible. This is not
surprising since the equation of state in this case has no dynamics and there is no reasonable fine tuning. However, it is simple to show that the existence of two scalar fields, one quintessence and the other phantom field, as usual can lead to the possibility of cosmological constant equation-of-state crossing. Note that in ref. [2], the authors have found that $m$ should be restricted to the interval $1/6 < m < 3/8$ to provide a suitable inflation model. In summary, this model universe provides no consistent scenario for a dynamical equation of state containing phantom divided barrier crossing.

Case 2. As a second case, we suppose $H(\phi) = H_0 \phi^\xi$ and $\beta(\phi) = m \phi^2$. With these choices, we find the following equation of state:

$$\omega_\phi = -1 + \frac{4}{3} m(\xi + 2)^2,$$

(27)

which evidently has no dynamics. The condition for the acceleration of the universe, $H'/H > -1$, yields

$$m < \frac{1}{2\xi(\xi + 2)}.$$  

(28)

The scalar field evolve as follows:

$$\phi(t) = \phi_0 \left(1 + \frac{H_0 \phi_0^\xi}{p}(t - t_0)\right)^{-1/\xi}.$$  

(29)

The variation of $\omega_\phi$ vs. $m$ and $\xi$ is shown in fig. 2. This figure shows that only for positive values of $m$ and all possible values of $\xi$, the equation of state satisfies the condition $\omega_\phi > -1$. For other positive values of $\xi$ and negative values of $m$, the equation of state has a phantom divided line since $\omega_\phi < -1$. But we should stress that since $m$ cannot attain negative values, we must restrict $m$ to the interval $0 < m < 0.166$. As the figure shows, with this restriction it is impossible to cross the phantom divided line with one scalar field.

In these two examples, the equation of state has no dynamics. Only with variation of parameters $m$ and $\xi$ one can formally attain a phantom divided barrier crossing. The case with a dynamical equation of state is more reasonable. Now, we turn to the situation where the equation of state is dynamical. For this purpose, we generalize the vector field coupling to achieve the value of $\beta(\phi) = m \phi^n$, for $n > 2$.

Case 3. To have a dynamical equation of state, we consider a model universe where the vector field coupling parameter is written as a power of the scalar field

$$H = H_0, \quad \beta(\phi) = m \phi^n,$$

(30)

where $H_0$ and $n$ are constant positive parameters. Following the above procedure, the scalar field $\phi$, the vector field coupling $\beta$ and equation of state $\omega_\phi$ have the following dynamics [2]:

$$\phi(t) = \frac{\phi_0}{\left[1 + 2mnH_0(n - 2)\phi_0^{n-2}(t - t_0)\right]^{1/2}},$$

(31)

$$\beta(t) = \frac{m \phi_0^n}{\left[1 + 2mnH_0(n - 2)\phi_0^{n-2}(t - t_0)\right]^{1/2}},$$

(32)

$$\omega_\phi(t) = -1 + \frac{4mn^2\phi_0^{n-2}3}{1 + 2mnH_0(n - 2)\phi_0^{n-2}(t - t_0)}.$$  

(33)

Remember that $\beta(t)$ plays the role of Lorentz invariance violation in this setup. The equation of dynamics for $\beta(t)$ implicitly has an important meaning: by a suitable fine tuning one can construct a Lorentz violating cosmology consistent with observational data. In other words, this setup provides an important basis for testing LIV in the cosmological context.

Although many different models can also lead to phantom divided barrier crossing, our model is special in this respect since it contains only one scalar field and the presence of a Lorentz violating vector field controls the crossing. In this sense, fine tuning of the parameters’ space based on observational data restricts the value that $\beta(t)$ can attain. Any non-vanishing value of $\beta$ in our model shows violation of the Lorentz symmetry in this cosmological setup. If the dynamics of $\beta(t)$, which is described by eq. (32), can be detected and constraint by observational data, this will be a manifestation of LIV in the cosmological context. Lorentz invariance violating inflation models constraint by WMAP and other observational data may provide...
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\[ \phi(t) = \frac{1}{\left[ H_0(t-t_0)(-4 \xi m + 4 \xi mn + 2 \xi^2 m - 4 mn + 2 mn^2) + \phi_0 \right]^\left(\frac{n-2}{n+2}\right)} \]  

\[ \omega_\phi(t) = -1 + \frac{4}{3} m \left[ H_0(t-t_0)(-4 \xi m + 4 \xi mn + 2 \xi^2 m - 4 mn + 2 mn^2) + \phi_0 \right]^\left(\frac{n-2}{n+2}\right) \]  

other test of LIV in the cosmological setup. To see the possible detection of Lorentz violating fields in cosmology, see [24].

In this model, the cosmic evolution starts from a constant value of the scale factor and then grows exponentially, \( a(t) = a_0 e^{H_0(t-t_0)} \), while the vector field coupling, \( \beta \), starts from a constant value of the scalar field, \( m \phi_0^n \).

The equation of state \( \omega_\phi \) is dynamical. Figure 3 shows the variation of the equation of state vs. \( m \) and \( t \). This figure clearly demonstrates a crossing of the phantom divided line. We stress that such results are essentially model dependent. In principle, these results should be translated to a red-shift language which provides a better framework to compare with observational data. While this is essentially important, the purpose of this letter is to show that LIV may help us to achieve the phantom divided line crossing with only one scalar field. In other words in the presence of LIV, just one scalar field is enough to achieve the phantom divided barrier crossing and the existence of the vector field controls the situation.

A more general case. In this section we consider a more general case where both the vector field coupling and the Hubble parameter are functions of \( \phi \) defined as follows:

\[ H = H_0 \phi^\xi, \quad \beta(\phi) = m \phi^n, \quad n > 2. \]  

Using eq. (21), for this case we obtain

see eq. (35) above

and using eq. (23) we find

\[ \omega_\phi(t) = -1 + \frac{4}{3} m \phi^{n-2}(t)(\xi + n)^2. \]  

The condition for the acceleration of the universe, that is, \( H'/H > -1 \), yields

\[ m^2 < \frac{1}{4(-1)^n \phi^{n-2}(t)(\xi + n)^2}, \quad n > 2. \]  

With \( \phi \) defined as (35), the equation of state takes the following form:

see eq. (38) above

which explicitly has a dynamical behavior. This model allows us to choose a suitable parameter space to cross the phantom divided barrier. This parameter space should be checked by observational data to have a reasonable model. The most important aspect of the present model is the fact that in principle, LIV fields provide a situation that one scalar field and another vector field together can lead us to describe the phantom divided barrier crossing.

Figure 4 shows the crossing of the phantom divided barrier for a dynamical equation of state. Figure 4 may be used to explain why we are living in an epoch of \( \omega < -1 \) since in late time we see that \( \omega < -1 \). This is the second cosmological coincidence problem.

Two point should be stressed here: firstly, as figs. 3 and 4 show, there are some sudden jumps of the equation of state. In many existing models whose equation of state can cross the phantom divided barrier, \( \omega \) undulates around \(-1\) randomly ([25] and references therein). These jumps are actually a manifestation of this undulation which may be a signature of the chaotic behavior of the equation of state. Secondly, as these figures show, the crossing of the phantom divided barrier can occur at late time. This fact, as the second cosmological coincidence problem, needs additional fine-tuning in model parameters and the trigger mechanism, for instance, can be used to alleviate this coincidence.

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Conclusions. – As a new mechanism for the crossing of the phantom divided barrier via equation of state, in this paper we have incorporated a possible violation of Lorentz invariance in a cosmological setup. We have shown that by a suitable choice of the parameter space, it is possible to have a phantom divided barrier crossing in a Lorentz invariance violating context just by a single scalar field. This is an important result since in the absence of LIV, as previous studies have shown, it is impossible to cross the phantom divided barrier by just one scalar field minimally coupled to gravity. In this regard, the existence of a Lorentz invariance violating vector field provides a framework for crossing the phantom divided barrier with one scalar field. On the other hand, this model provides a possible framework for testing Lorentz invariance violation in a cosmological context. Using observational data and by a suitable fine tuning it is possible to construct a reliable Lorentz violating cosmological model. A similar strategy has been applied for inflation in ref. [2]. Another aspect of our model is the fact that it contains several crossings of the phantom divided barrier, a phenomena which has been seen in other scenarios. As we have shown, the equation of state takes a different form by different choices of the parameter space.

In this framework it is possible to use the “trigger mechanism” to explain the dynamical equation of state. This means that we assume the scalar-vector-tensor theory containing Lorentz invariance violation which acts like the hybrid inflation models. In this situation, vector and scalar fields play the role of the inflaton and the “waterfall” field, respectively. This conjecture is under investigation [26].

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