A remark concerning the standard approach to $CP$ violation in a system of $K^o$ mesons

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Abstract

Within the standard approach to $CP$ violation in a system of $K^o$ mesons, the normalization factor in the expression for the transition probability $|K^o_1|^2$ contains the $CP$ violation phase. A normalization multiplier for the transition probability can obviously not contain a phase term. In this work two simple methods are proposed for resolving this issue.

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1 Introduction

Parity $P$ was previously supposed to be a good number; however, after theoretical [1] and experimental [2] works it has become clear that $P$ parity is violated in weak interactions. Then, in ref. [3] an assumption was put forward that, although $P$ parity is not conserved in weak interactions, $CP$ parity is conserved. Ref. [4] reported that with a probability of about 0.2% there exists in $K_L$ decays a two $\pi$ decay mode which actually points to $CP$ parity violation.

In all textbooks and monographs, where the problem of $CP$ violation in a system of $K^o$ mesons is dealt with, the primary $K^o$ and $\bar{K}^o$ mesons are, first, considered to transform under the violation of strangeness $S$, into a superposition of $K^o_1$ and $K^o_2$ mesons and, then, under $CP$ violation these $K^o_1$ and $K^o_2$ mesons transform into a superposition of $K_S$, $K_L$ mesons [5]

\begin{align}
K_S &= \frac{1}{\sqrt{1+|\varepsilon|^2}}(K^o_1 + \varepsilon K^o_2), \\
K_L &= \frac{1}{\sqrt{1+|\varepsilon|^2}}(\varepsilon K^o_1 + K^o_2).
\end{align}
where \( \epsilon \) is complex value. Inverse transformation of expressions (1) results in

\[
K_1^o = \sqrt{\frac{1 + |\epsilon|^2}{1 - |\epsilon|^2}}(K_S - \epsilon K_L), \\
K_2^o = \sqrt{\frac{1 + |\epsilon|^2}{1 - |\epsilon|^2}}(-\epsilon K_S + K_L).
\]  

(2)

We can rewrite \( \epsilon \) as \( \epsilon = |\epsilon|e^{-i\delta} \). Then, taking into account that \( K_S(t) = e^{(-iE_S - \Gamma_S/2)t}K_S(0) \), \( K_L(t) = e^{(-iE_L - \Gamma_L/2)t}K_L(0) \), we obtain

\[
|K_1^o(t)|^2 = \frac{1 + |\epsilon|^2}{|1 - \epsilon^2|^2}[e^{\Gamma st} + |\epsilon|^2e^{\Gamma lt} - 2|\epsilon|e^{\frac{\Gamma_s + \Gamma_L}{2}t}\cos((E_L - E_S)t)] = \frac{1 + |\epsilon|^2}{(1 + |\epsilon|^4 - 2|\epsilon|^2\cos\delta)[e^{\Gamma st} + |\epsilon|^2e^{\Gamma lt} - 2|\epsilon|e^{\frac{\Gamma_s + \Gamma_L}{2}t}\cos((E_L - E_S)t + \delta)].
\]

(3)

\[
|K_2^o(t)|^2 = \frac{1 + |\epsilon|^2}{(1 + |\epsilon|^4 - 2|\epsilon|^2\cos\delta)[|\epsilon|^2e^{\Gamma st} + e^{\Gamma lt} - 2|\epsilon|e^{\frac{\Gamma_s + \Gamma_L}{2}t}\cos((E_L - E_S)t - \delta)].
\]

(3’)

We see that the normalization factor in (3) contains the phase term \( \cos\delta \). Evidently, this phase term is not related to the normalization of states. Therefore, we have to get rid of this term. We shall further deal with such an approach.

2 \textit{CP violation without phase term in normalization factor in expression for transition probability}

To avoid the presence of a phase term in the normalization factor for the transition probability we replace \( \epsilon \) in the second term of (1) by \( \epsilon^* = |\epsilon|e^{i\delta} \):

\[
K_S = \frac{1}{\sqrt{1 + |\epsilon|^2}}(K_1^o + \epsilon K_2^o) \equiv \frac{1}{\sqrt{1 + |\epsilon|^2}}(K_1^o + |\epsilon|e^{-i\delta} K_2^o) \\
K_L = \frac{1}{\sqrt{1 + |\epsilon^*|^2}}(\epsilon^* K_1^o + K_2^o) \equiv \frac{1}{\sqrt{1 + |\epsilon|^2}}(|\epsilon|e^{i\delta} K_1^o + K_2^o).
\]  

(4)

By inverse transformation of expression (4) we obtain

\[
K_1^o = \sqrt{\frac{1 + |\epsilon|^2}{1 - |\epsilon|^2}}(-K_S + |\epsilon|e^{-i\delta} K_L), \\
K_2^o = \sqrt{\frac{1 + |\epsilon|^2}{1 - |\epsilon|^2}}(|\epsilon|e^{i\delta} K_S - K_L).
\]  

(5)
Then, for $|K_1^o(t)|^2$ and $|K_2^o(t)|^2$ we get

$$|K_1^o(t)|^2 = \frac{1 + |\varepsilon|^2}{1 - |\varepsilon|^2} [e^{\Gamma_S t} + |\varepsilon|^2 e^{\Gamma_L t} - 2|\varepsilon|e^{\frac{\Gamma_S + \Gamma_L}{2} t}\cos((E_L - E_S) t + \delta)].$$

(6)

$$|K_2^o(t)|^2 = \frac{1 + |\varepsilon|^2}{1 - |\varepsilon|^2} [|\varepsilon|^2 e^{\Gamma_S t} + e^{\Gamma_L t} - 2|\varepsilon|e^{\frac{\Gamma_S + \Gamma_L}{2} t}\cos((E_L - E_S) t + \delta)].$$

(6')

We can go farther and use the following expressions for the $K_S, K_L$ states:

$$K_S = \frac{1}{\sqrt{1 + |\varepsilon|^2}} (K_1^o + \varepsilon K_2^o) \equiv \frac{1}{\sqrt{1 + |\varepsilon|^2}} (K_1^o + |\varepsilon|e^{-i\delta} K_2^o)$$

$$K_L = \frac{1}{\sqrt{1 + |\varepsilon|^2}} (-\varepsilon^* K_1^o + K_2^o) \equiv \frac{1}{\sqrt{1 + |\varepsilon|^2}} (-|\varepsilon|e^{i\delta} K_1^o + K_2^o).$$

(7)

where $-\varepsilon^*$ is substituted for $\varepsilon$ in the second term of expressions (1).

By inverse transformation we obtain

$$K_1^o = (K_S - |\varepsilon|e^{-i\delta} K_L),$$

$$K_2^o = (|\varepsilon|e^{i\delta} K_S + K_L).$$

(8)

Then, for $|K_1^o(t)|^2$ we get

$$|K_1^o(t)|^2 = [e^{\Gamma_S t} + |\varepsilon|^2 e^{\Gamma_L t} - 2|\varepsilon|e^{\frac{\Gamma_S + \Gamma_L}{2} t}\cos((E_L - E_S) t + \delta)].$$

(9)

3 Conclusion

Within the standard approach to $CP$ violation in a system of $K^o$ mesons, the normalization factor in the expression for the transition probability $|K_1^o|^2$ contains a $CP$ violation phase. A normalization multiplier for the transition probability can obviously not contain a phase term. In the present work, two simple methods are put forward for resolving this issue. To this end two approaches are applied. In the first approach $\varepsilon^*$ is substituted for the term $\varepsilon$ in the expression for $K_L$, (1), while in the second $-\varepsilon^*$ is substituted for $\varepsilon$ in expression (1) for $K_L$. Then, the renormalization factor in the expression for $|K_1^o|^2$ no longer contains any phase term, i.e. no $CP$ violation phase term is present in the normalization factor of the expression for the transition probability $|K_1^o|^2$. 

So we see that in expressions for transition probabilities $|K_1^0(t)|^2$ and $|K_2^0(t)|^2$ the phase term $\delta$ has different signs (see expr. (3) and (3')) while these probabilities have the same sign in our approach (see expr. (6) and (6')). Besides difference between old and our normalization factor is $\Delta N = 1 + |\varepsilon|^2 - 2|\varepsilon|^2 \cos\delta - 1 + |\varepsilon|^2 \approx 2|\varepsilon|^2 \cos\delta$ (where $|\varepsilon| = 2.23 \cdot 10^{-3}$ (see ref. [5]) and it is very small value).

It is necessary to remark that in experiment with high precision we can fulfill examination of normalization factor and sign of phase factor $\alpha$ in order to determine which of the above two approaches is realized indeed.

References

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