Performance-Guaranteed Switching Adaptive Control for Nonlinear Systems With Multiple Unknown Control Directions

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ABSTRACT In this paper, an adaptive control strategy with novel performance-guaranteed switching mechanism is proposed for a class of nonlinear parametric strict-feedback systems with unknown parameters and multiple unknown control directions. Firstly, adaptive controller is designed by combining tuning function design and barrier Lyapunov function (BLF) techniques, to handle parameter uncertainties and avoid over-parametrization phenomenon. The unknown control directions are estimated online with the proposed novel switching criterion composed of a BLF-type monitoring function and a threshold function. It is shown that the correct directions can be identified rapidly with only finite times of switching and mis-switching problem is avoided. At switching moments, a novel reset mechanism is triggered to adjust the value of integrator to avoid failure of BLF method. Consequently, tracking error of reference signal converges to zero asymptotically and all closed-loop signals are proved to be bounded. In addition, the tracking errors can be restricted into prescribed intervals without information of control directions. Numerical results demonstrate the effectiveness and performance of the proposed method.

INDEX TERMS Adaptive control, barrier Lyapunov function, switching controller, unknown control directions.

I. INTRODUCTION Recent years have seen intensively studies on control problems of uncertain nonlinear systems [1] due to practical application demands [2], [3] and theoretical challenges [4]–[6]. To cope with the significant nonlinearities and uncertainties, various control methods like adaptive control [7]–[9], fuzzy mathematics and control [10]–[13] were put forward and fruitful results have been obtained. However, the aforementioned adaptive controllers were designed assuming that the signs of the control gains are known in a priori. In practical case, there is a need to solve tracking problems in the absence of knowledge of the sign of the high-frequency gain, i.e., the control direction.

Generally speaking, there are two common techniques to solve the so-called unknown control direction problems. One is the famous Nussbaum-type function [14], which has been widely adopted to solve the problem with unknown signs of the control gains [15]–[17]. Reference [15] studied the case with unknown actuator failures with Nussbaum function and high order Lyapunov functions. In [16], to solve the problem of unknown time-varying control coefficients and unknown time-varying delays, Nussbaum gain technique and Lyapunov-Krasovskii functionals (LKFs) were integrated. The challenges encountered when using Nussbaum gain technique are that only practical stability can be derived when multiple unknown control directions exist [18], [19] and large overshoot may appear. While some achievements have been made toward these problems, like in [20], there remain issues like overparameterization cannot be removed in the framework of Nussbaum method [21].

The other method to solve unknown control directions resorts to switching-type controller. For switched systems, switching control law are designed to stabilize multiple subsystems [22]–[26]. While for systems with unknown control signs, switching of control parameters are implemented to

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approximate the correct directions based on a switching logic and once reached, stability is achieved. The switching rule is essential in the construction of switching adaptive controller and recent studies using the upper bound of integral-type Lyapunov function [27], Lyapunov functions [28]–[30] to construct the switching criteria. Specifically, Lyapunov-based logic switching rule was developed in [29] to tune the control directions and finite-time stability of the closed-loop system with multiple unknown control directions was guaranteed. Reference [31] constructed a monitoring function with the output information to determine switching time and asymptotic stability was achieved for linear time-invariant systems. Reference [32] utilized the upper limit of an integral-type Lyapunov function to construct the switching logic, based on which the finite-time stabilization of a class of uncertain nonlinear systems was realized. To solve the problem of over-parameterization in adaptive control, tuning function design was combined with a novel logic-based switching criterion in [21]. The proposed incremental-error switching technique accelerated the estimation process of the correct directions and the number of estimators was largely reduced.

Although stability can be ensured, like in [21], [33], [34], beneficial results concerning prescribed tracking performance cannot be derived with the previous control schemes. It is well-known that various control techniques have been developed for constrained control problems [35]–[37]. Among these solutions, prescribed performance control (PPC) and barrier Lyapunov function (BLF) methods [38]–[41] are two effective approaches regarding to restricting transient tracking performance. The combination of these two methods with fuzzy systems [22], [36], [42], [43] and neural networks [44]–[46] have been intensively studied recently.

According to the previous discussions, a naturally challenging topic is to design a performance-guaranteed switching controller for uncertain nonlinear systems with multiple unknown control directions. We will address this problem by resorting to BLF method and designing a novel BLF-type switching logic, which has not been studied yet. The combination of BLF method and switching controller is not straightforward. As the sudden change of virtual signal values at switching instant may violate the preconditions of BLF design and lead to divergence. The major contributions of our paper can be summarized as follows:

Asymptotic output tracking performance is achieved for uncertain parametric strict feedback (PSF) systems with multiple unknown control directions under the obtained controllers. Moreover, tracking performance is guaranteed in the sense that tracking errors are constrained in prescribed intervals. The most relevant work was presented in [47], where Nussbaum gains and barrier Lyapunov function are first studied jointly, while only practical stability can be proved.

A novel reset mechanism is designed. The integrator states are reset to specific values whenever switching is triggered. In this way, possible violations of preconditions of BLF design are avoided. Furthermore, we proposed a BLF-type switching logic composed of a monitoring function and a threshold function. Then, by selecting reasonable small performance bounds, the process of identifying correct directions is accelerated. Also, mis-switching which may arise in previous logic-based switching mechanism [21], is avoided. It follows that only a finite number of switching which is bounded by a constant relating to the system order, is needed.

Notation: Throughout this paper, we use the following standard notation. \( \log(\cdot) \) stands for natural logarithm function. \( \text{sign}(\cdot) \) denote the signum function. \( \lambda_{\max}(\cdot) \) gives the largest eigenvalue of a matrix. \( \mathbb{R} \) denotes the set of real numbers and \( \mathbb{R}^n \) is the \( n \)-dimensional real vector space. For any \( x \in \mathbb{R}^n \), \( ||x|| \) denotes its Euclidean norm. \( \text{diag}(\cdot) \) refers to a diagonal matrix.

II. PROBLEM FORMULATION

The model for the problem of global adaptive control of the PSF system with unknown control directions is formulated as [18]:

\[
\begin{align*}
\dot{x}_i &= \mu_i x_{i+1} + \theta^T \phi_i(x_1, \ldots, x_i) \quad i = 1, \ldots, n - 1, \\
\dot{x}_n &= \mu_n u + \theta^T \phi_n(x), \\
y &= x_1,
\end{align*}
\]

where \( x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n \) is the state vector. \( u \in \mathbb{R} \) and \( y \in \mathbb{R} \) denote the control input and output, respectively. The problem of multiple unknown control directions appears as the signs of \( \mu_i \neq 0 \), \( i = 1, \ldots, n \) are unknown. \( \theta = [\theta_1, \ldots, \theta_q, \mu_1, \ldots, \mu_{n-1}]^T \in \mathbb{R}^{q+n-1} \) represents the unknown parameter vector to be estimated adaptively. It should be noted that \( \mu_n \) need not to be estimated here. \( \phi_i \in \mathbb{R}^{q+n-1}, i = 1, \ldots, n \) are known smooth functions.

Reference signal \( r_0 \) is sufficient smooth and its high-order derivatives are denoted by

\[
\dot{r}_i = r_{i+1}, \quad i = 0, \ldots, n - 1.
\]

Tracking control problem will be explored with backstepping, so we denote \( z_i, i = 0, \ldots, n \) as the tracking errors whose analytical expressions will be given later.

The control objective is to force the output asymptotically tracking the reference signal \( r_0(t) \) while ensuring that the performance requirements on tracking errors are not violated, i.e., \( |z_i| < k_0 \) with \( k_0 \) being positive design constants, even in the presence of unknown control directions and parameter uncertainties.

Following assumptions are made.

Assumption 1: The unknown parameter vector \( \theta \in \mathbb{R}^{q+n-1} \) lies in a bounded convex set and \( ||\theta|| \leq \theta_{\max} \) with \( \theta_{\max} \in \mathbb{R}^+ \).

Assumption 2: There exist known positive constants \( \mu_{\min, i} \) and \( \mu_{\max, i} \), such that \( 0 < \mu_{\min, i} \leq |\mu_i| \leq \mu_{\max, i} < \infty \) for \( i = 1, 2, \ldots, n \).

Remark 1: Assumption 1 is commonly adopted in related studies and a similar but more restrictive one can be found in [48]. In Assumption 2, the control coefficients are assumed...
to be bounded by known constants which is reasonable in practical application. All these bounds will not be used in controller design, but in the construction of switching rule. The bounds need not to be accurate since the switching logic is tracking error based and the discrepancies can be compensated with additional parameters.

III. PERFORMANCE GUARANTEED SWITCHING ADAPTIVE CONTROLLER DESIGN

A. BLF BASED ADAPTIVE CONTROLLER DESIGN

Backstepping technique is exploited to address the triangular structure system (1). The tracking errors are defined as

\[ z_1 = x_1 - r_0, \]
\[ z_i = x_i - a_{i-1}, \quad i = 2, \ldots, n, \]  

(3)

where the functions \( a_i \in \mathbb{R}, i = 0, \ldots, n-1 \) are intermediate control signals. In fact, the final control input \( u \) is obtained with the same recursive procedure applied to \( a_i \) [49]. Thus, the \( n \)th virtual function \( a_n \) is well-defined and \( u = a_n \). For \( i = 1, \ldots, n \), \( a_i \) is designed as

\[ a_i = K_i \rho_i \hat{\rho}_i \]  

(4)

where \( K_i \in \{1, -1\} \) is switching signal used to approximate the sign of control gain and determined by the switching index \( k \in \mathbb{Z} \cap [0, k_f] \) where \( k_f \) denotes the final switching index. \( \rho_i \) and \( \hat{\rho}_i \) are control functions to be designed. The corresponding switching moments are described with a monotone increasing sequence \( 0 = T_0 < T_1 < \cdots < T_{k_f} \). Though \( a_i \) is discontinuous at the switching moments, it is smooth over time interval \([T_j, T_{j+1})\), \( j = 0, \ldots, k_f - 1 \). The controller design process is then presented as follows for any fixed \( k \).

**step 1:** Define

\[ \dot{V}_1(t) = \frac{1}{2} \dot{\mu}_1 K_1 \left( \rho_1 + \frac{K_1}{\mu_1} \right)^2 + \frac{1}{2} \mu_1 K_1 \left( \rho_1 + \frac{K_1}{\mu_1} \right)^2, \]

(5)



where \( \hat{\theta} = \theta - \hat{\theta} \) and \( \hat{\theta} \in \mathbb{R}^{q+n-1} \) is the estimate of \( \theta \), \( \sigma \in \mathbb{R} \) is a positive parameter and \( \Gamma \in \mathbb{R}^{(q+n-1) \times (q+n-1)} \) is a symmetric positive matrix. Taking the time derivative of \( V_1 \) except for the switching moments yields

\[ \dot{V}_1(t) = \frac{z_1}{k_{b_1}^2 - z_1^2} \left[ \mu_1 \left( a_1 + z_2 \right) + \left( \hat{\theta}^T + \hat{\theta}^T \right) \phi_1 + \rho_0 \right] + \frac{1}{\sigma} \left( \mu_1 K_1 \rho_1 \hat{\rho}_1 + \hat{\theta}^T \right) + \hat{\theta}^T \Gamma^{-1} \left( \hat{\theta} - \hat{\theta} \right). \]

(6)

Substituting (4) into (6) it has

\[ \dot{V}_1(t) = \mu_1 K_1 \rho_1 \left( \frac{z_1}{k_{b_1}^2 - z_1^2} \hat{\rho}_1 + \frac{1}{\sigma} \rho_1 \right) + \frac{z_1}{k_{b_1}^2 - z_1^2} \left( \hat{\theta}^T \phi_1 + \rho_0 \right) + \frac{1}{\sigma} \hat{\rho}_1 + \hat{\theta}^T \Gamma^{-1} \left( \Gamma \frac{z_1}{k_{b_1}^2 - z_1^2} \hat{\rho}_1 + \frac{1}{\sigma} \rho_1 \right) + \mu_1 z_2 \phi_2. \]

(7)

Further defining

\[ \hat{\rho}_1 = -\frac{z_1}{k_{b_1}^2 - z_1^2} \sigma \hat{\rho}_1, \]
\[ \hat{\rho}_1 = \left( k_{b_1}^2 - z_1^2 \right) c_1 z_1 - r_1 + \hat{\theta}^T \phi_1, \]
\[ \tau_1 = \Gamma \frac{z_1}{k_{b_1}^2 - z_1^2} \phi_1, \]

(8)  

(9)  

(10)

where \( c_1 > 0 \) is a design constant, it can be derived that

\[ \dot{V}_1(t) = -c_1 z_1^2 - \hat{\theta}^T \Gamma^{-1} \left( \hat{\theta} - \tau_1 \right) + \mu_1 z_1 z_2 \phi_2. \]

(11)

**step 2:** Define

\[ V_2(t) = V_1 + \frac{1}{2} \log \frac{k_{b_2}^2}{k_{b_2}^2 - z_2^2} + \frac{1}{2} \mu_2 K_2 \left( \rho_2 + K_2 \mu_2 \right)^2. \]

(12)

Similar to the procedure in **step 1**, we compute the time derivative of \( V_2(t) \). Noting that \( a_1 \) is a function of \( x_1, \rho_1, \hat{\theta} \) and \( r_1 \), it has

\[ \dot{V}_2 = -c_1 z_1^2 - \hat{\theta}^T \Gamma^{-1} \left( \hat{\theta} - \tau_1 \right) + \mu_2 z_2 z_3 \phi_2 + \frac{1}{\sigma} \mu_2 K_2 \rho_2 \hat{\rho}_2 + \frac{1}{\sigma} \rho_2 + \mu_1 z_2 \phi_2 \]

\[ + \left( \mu_2 \phi_2 - \frac{\partial \mu_2}{\partial a_1} \right) \phi_2 + \frac{1}{\sigma} \mu_2 K_2 \rho_2 \hat{\rho}_2 + \frac{1}{\sigma} \rho_2 + \mu_1 z_2 \phi_2 \]

\[ + \frac{1}{\sigma} \mu_2 K_2 \rho_2 \hat{\rho}_2 + \frac{1}{\sigma} \rho_2 + \mu_1 z_2 \phi_2 \]

\[ + \mu_2 \phi_2. \]

(13)

where \( e_j \in \mathbb{R}^{n+q-1} \) is a vector whose elements are zeros except that its \( j \)th entry equals to one. Similar to **step 1**, recalling (4) and we define

\[ \hat{\rho}_2 = -\frac{z_2}{k_{b_2}^2 - z_2^2} \sigma \hat{\rho}_2, \]
\[ \hat{\rho}_2 = \left( k_{b_2}^2 - z_2^2 \right) c_2 z_2 \phi_2 \]
\[ + \frac{\partial a_1}{\partial \rho_1} \frac{z_1}{k_{b_1}^2 - z_1^2} \frac{z_2}{k_{b_2}^2 - z_2^2} \phi_2 - \frac{\partial a_1}{\partial r_1} r_1 + \frac{\partial a_1}{\partial r_1} r_2 - \frac{\partial a_1}{\partial \hat{\theta}} \phi_2 \]
\[ + \mu_2 \phi_2. \]

(14)  

(15)
\( \tau_2 = \tau_1 + \Gamma \frac{z_2}{k_{b_2} - \frac{z_2}{c_2}} \left[ \frac{k_{b_2} - \frac{z_2}{c_2}}{k_{b_1} - \frac{z_1}{c_1}} e_{q+1} + \phi_2 \right] - \frac{\partial \alpha_1}{\partial x_1} \left( x_2 e_{q+1} + \phi_1 \right), \) (16)

where \( c_2 > 0 \) is a design parameter. Substituting (14)-(16) into (13), we can obtain

\[
\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 - \left( \hat{b}^T \Gamma^{-1} + \frac{z_2}{k_{b_2} - \frac{z_2}{c_2}} \frac{\partial \alpha_1}{\partial \hat{\theta}} \right) \left( \hat{\theta} - \tau_2 \right) \\
+ \mu_2 \frac{z_2}{k_{b_2} - \frac{z_2}{c_2}} + \sum_{i=3}^{n} \left( \hat{b}^T \Gamma^{-1} + \sum_{j=1}^{i-2} \frac{z_j + 1}{k_{b_{j+1}} - \frac{z_j + 1}{c_{j+1}}} \frac{\partial \alpha_j}{\partial \hat{\theta}} \right) \left( \hat{\theta} - \tau_{i-1} \right), \quad (17)
\]

step i, \( i = 3, \ldots, n \): Using induction, suppose

\[
\dot{V}_{i-1} = -\sum_{j=1}^{i-1} c_j z_j^2 + \mu_{i-1} \frac{z_{i-1} - z_i}{k_{b_{i-1}} - \frac{z_{i-1}}{c_{i-1}}} \\
- \left( \hat{b}^T \Gamma^{-1} + \sum_{j=1}^{i-2} \frac{z_j + 1}{k_{b_{j+1}} - \frac{z_j + 1}{c_{j+1}}} \frac{\partial \alpha_j}{\partial \hat{\theta}} \right) \left( \hat{\theta} - \tau_{i-1} \right) \\
+ \frac{z_i}{k_{b_i} - \frac{z_i}{c_i}} \left( \mu_i x_{i+1} + \hat{\theta}^T \phi_i - \hat{\alpha}_{i-1} \right), \quad (20)
\]

Taking the derivative of \( V_{i}(t) \) w.r.t time gives

\[
\dot{V}_{i}(t) = -\sum_{j=1}^{i-1} c_j z_j^2 + \frac{1}{\sigma} \left( \mu_{i} K_{i} \rho_{i} + \rho_{i} \right) + \mu_{i-1} \frac{z_{i-1} - z_i}{k_{b_{i-1}} - \frac{z_{i-1}}{c_{i-1}}} \\
- \left( \hat{b}^T \Gamma^{-1} + \sum_{j=1}^{i-2} \frac{z_j + 1}{k_{b_{j+1}} - \frac{z_j + 1}{c_{j+1}}} \frac{\partial \alpha_j}{\partial \hat{\theta}} \right) \left( \hat{\theta} - \tau_{i-1} \right) \\
+ \frac{z_i}{k_{b_i} - \frac{z_i}{c_i}} \left( \mu_i x_{i+1} + \hat{\theta}^T \phi_i - \hat{\alpha}_{i-1} \right), \quad (20)
\]

Combining \( \mu_{i-1} = \hat{\theta}^T e_{q+i-1} \) and (4), (20) can be simplified as

\[
\dot{V}_{i}(t) = -\sum_{j=1}^{i-1} c_j z_j^2 + \mu_{i} K_{i} \rho_{i} + \rho_{i} \frac{\partial \alpha_1}{\partial \hat{\theta}} + \frac{1}{\sigma} \hat{\theta}_{i} \\
- \left( \hat{b}^T \Gamma^{-1} + \sum_{j=1}^{i-2} \frac{z_j + 1}{k_{b_{j+1}} - \frac{z_j + 1}{c_{j+1}}} \frac{\partial \alpha_j}{\partial \hat{\theta}} \right) \left( \hat{\theta} - \tau_{i-1} \right) \\
+ \frac{z_i}{k_{b_i} - \frac{z_i}{c_i}} \left[ \hat{b}^T e_{q+i-1} + \frac{k_{b_i} - \frac{z_i}{c_i}}{k_{b_{i-1}} - \frac{z_{i-1}}{c_{i-1}}} z_{i-1} + \hat{\theta}^T \phi_i - \hat{\alpha}_{i-1} \right] \\
+ \mu_{i} \frac{z_i z_{i+1}}{k_{b_i} - \frac{z_i}{c_i}}, \quad (21)
\]

Noting that

\[
\hat{\alpha}_{i-1} = \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \hat{x}_j - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \rho_j} \frac{\partial \rho_j}{\partial \hat{\theta}} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \hat{\theta} + \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial r_j} r_{j+1}, \quad (22)
\]

Further define

\[
\tau_i = \tau_{i-1} + \Gamma \frac{z_i}{k_{b_i} - \frac{z_i}{c_i}} \left[ e_{q+i-1} + \frac{k_{b_i} - \frac{z_i}{c_i}}{k_{b_{i-1}} - \frac{z_{i-1}}{c_{i-1}}} z_{i-1} + \phi_i \right] \\
- \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left( e_{q+i} x_{j+1} + \phi_j \right), \quad (23)
\]

\[
\hat{\rho}_i = -\frac{z_i}{k_{b_i} - \frac{z_i}{c_i}} \sigma \hat{\rho}_i, \quad (24)
\]

\[
\hat{\rho}_i = \left( k_{b_i} - \frac{z_i}{c_i} \right) c_i z_i - \left[ \sum_{j=1}^{i-2} \frac{z_{j+1}}{k_{b_{j+1}} - \frac{z_{j+1}}{c_{j+1}}} \frac{\partial \alpha_j}{\partial \hat{\theta}} \right] \Gamma - \hat{\theta}^T \left[ e_{q+i-1} + \frac{k_{b_i} - \frac{z_i}{c_i}}{k_{b_{i-1}} - \frac{z_{i-1}}{c_{i-1}}} z_{i-1} + \phi_i \right] \\
- \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left( e_{q+i} x_{j+1} + \phi_j \right) - \frac{\partial \alpha_{i-1}}{\partial \theta} \tau_i \]

\[
+ \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial r_j} r_{j+1}, \quad (25)
\]

where \( c_i \) is a positive design constant. Directly substituting (22)-(25) into (21), after tedious calculations, \( \dot{V}_{i}(t) \) can be rewritten as

\[
\dot{V}_{i}(t) = -\sum_{j=1}^{i} c_j z_j^2 - \left( \hat{b}^T \Gamma^{-1} + \sum_{j=1}^{i-2} \frac{z_{j+1}}{k_{b_{j+1}} - \frac{z_{j+1}}{c_{j+1}}} \frac{\partial \alpha_j}{\partial \hat{\theta}} \right) \left( \hat{\theta} - \tau_{i} \right) \\
+ \frac{z_i}{k_{b_i} - \frac{z_i}{c_i}} \left[ \hat{b}^T e_{q+i} + \frac{k_{b_i} - \frac{z_i}{c_i}}{k_{b_{i-1}} - \frac{z_{i-1}}{c_{i-1}}} z_{i-1} + \hat{\theta}^T \phi_i - \hat{\alpha}_{i-1} \right] \\
+ \mu_{i} \frac{z_i z_{i+1}}{k_{b_i} - \frac{z_i}{c_i}}, \quad (26)
\]

At step \( n \), letting

\[
\hat{u} = \alpha_n, \quad (27)
\]

\[
\hat{\theta} = \tau_n, \quad (28)
\]

finally, we have

\[
\dot{V}_n(t) = -\sum_{j=1}^{n} c_j z_j^2, \quad (29)
\]

The controller design process is completed, what remains to be determined is the switching criterion. It should be noted that \( \dot{V}_n(t) \) is semi-negative definite regardless of the values of \( K_i \).
B. SWITCHING MECHANISM DESIGN

In this subsection, we introduce the three-fold switching mechanism composed of a switching rule, a mapping between direction vector and switching index, and a resetting mechanism. The switching logic is used to decide the switching moment and outputs switching index \( k \). Switching vector \( \mathbf{K} = [K_1, \ldots, K_n]^T \) is characterized by function \( v(k) \). Reset mechanism is developed to avoid possible violation of barrier constraints because of switching.

1) SWITCHING CRITERION

Letting \( \mu_i K_i = |\mu_i| \), for \( i = 1, 2, \ldots, n \), then from (19) it has

\[
V_n = \frac{1}{2} \sum_{i=1}^{n} \log \frac{k_{bh}^2}{k_{bh}^2 - \zeta_i^2(t)} + \frac{1}{2} \gamma^T(t) \Gamma^{-1} \gamma(t) + \frac{1}{2} \sum_{i=1}^{n} |\mu_i| \left( \rho_i(t) + \frac{1}{|\mu_i|} \right)^2 .
\]

Define

\[
P(t) = \frac{1}{2} \sum_{i=1}^{n} \log \frac{k_{bh}^2}{k_{bh}^2 - \zeta_i^2(t)} + \frac{1}{2} \sum_{i=1}^{n} P_{\mu,i}(t) + \sum_{i=1}^{n} \rho_i(t) + \frac{1}{|\mu_i|} \zeta_i(t, |\mu_i|). \tag{30}
\]

Expanding the second term of \( P(t) \), it has

\[
\frac{1}{2} |\mu_i| \left( \rho_i(t) + \frac{1}{|\mu_i|} \right)^2 \leq |\mu_i| \rho_i^2(t) + \frac{1}{|\mu_i|} \zeta_i(t, |\mu_i|). \tag{33}
\]

Taking the second derivative of \( \zeta_i(t, |\mu_i|) \) with respect to \( |\mu_i| \), it has \( \tilde{\rho}^2 \zeta_i(t, |\mu_i|)/|\mu_i|^3 = 2/|\mu_i|^3 > 0 \) which indicates that \( \zeta_i(t, |\mu_i|) \) is concave function on closed interval \([\mu_{min,i}, \mu_{max,i}]\), then it can be concluded that

\[
\frac{1}{2} |\mu_i| \left( \rho_i(t) + \frac{1}{|\mu_i|} \right)^2 \leq P_{\mu,i} .
\]

Further noticing that

\[
\tilde{\theta}^T(t) \theta(t) \leq \left( \theta_{max} + \left\| \tilde{\theta}(t) \right\| \right)^2 .
\]

it can be obtained that \( 0 \leq V_n(t) < P(t) \). Considering

\[
\frac{1}{2} |\mu_i| \left( \rho_i(t) + \frac{1}{|\mu_i|} \right)^2 = \frac{1}{2} \left( \rho_i(t) \sqrt{|\mu_i|} + \frac{1}{\sqrt{|\mu_i|}} \right)^2 \geq |\rho_i| ,
\]

it has

\[
V_n(t) \leq \frac{1}{2} \sum_{i=1}^{n} \log \frac{k_{bh}^2}{k_{bh}^2 - \zeta_i^2(t)} + \frac{1}{\sigma} \sum_{i=1}^{n} |P_{\mu,i}(t)| .
\]

Integrating (29) over time interval \([T_k, t], k = 0, 1, \ldots, k_f \), yields

\[
V_n(t) = V_n(T_k) - \int_{T_k}^{t} \sum_{i=1}^{n} c_i z_i^2 dt. \tag{38}
\]

Then define BLF-type monitoring function

\[
M = \sum_{i=1}^{n} \frac{1}{\sigma} \log \frac{k_{bh}^2}{k_{bh}^2 - \zeta_i^2(t)}. \tag{39}
\]

and threshold function

\[
\tilde{P}(t, T_k) = P(T_k) - \int_{T_k}^{t} \sum_{i=1}^{n} c_i z_i^2 dt - \frac{1}{\sigma} \sum_{i=1}^{n} |\rho_i|. \tag{40}
\]

The switching criterion is formulated as

\[
M < \tilde{P}(t, T_k). \tag{41}
\]

For any \( t > T_k \), if (41) is violated at time \( t \), then the next switching moment \( T_{k+1} = t \) and switching index \( k+k \). We start from \( k = 0 \) and switch to \( k = 1 \) and so on.

Solving (41) yields

\[
|z_i(t)| < \delta(t, T_k) k_{bh}, \quad i = 1, 2, \ldots, n, \quad \delta(t, T_k) = \sqrt{1 - e^{-2P(T_k)}}, \tag{42}
\]

where \( \delta(t, T_k) k_{bh} \) is the time-varying performance bound for \( |z_i(t)|, i = 1, 2, \ldots, n, \) for \( t \in [T_k, T_{k+1}] \).

2) DIRECTION VECTOR

the direction vector \( K = v(k) \) is formulated by

\[
v(k) = \text{ind}_n \left( \text{mod}_{2^n} (k) \right) \tag{43}
\]

where \( \text{mod}_{2^n} (p) \) is the remainder of \( p \) divided by \( 2^n \), function \( \text{ind}_n : \mathbb{N} \rightarrow \mathbb{R}^n \) is given as

\[
\text{ind}_n(m) = [1 - 2B_0 (m), 1 - 2B_{n-1}(m), \ldots, 1 - 2B_1 (m)]^T. \tag{44}
\]

with \( B_i(k) \) being the \( i \)th lowest bit of switching index \( k \) in binary format.

3) RESET MECHANISM

Whenever switching is triggered, controller states \( \rho_{i-1}, i = 2, 3, \ldots, n \) are reset to

\[
\rho_i(t^+) = \frac{x_i(t)}{K_i - 1} \frac{t^+}{t^+}. \tag{45}
\]

where \( t^+ \) refers to the time right after switching. In this way, it can be verified that the initial conditions for BLF design, that is, \( |z_i| < k_{bh} \) will not be violated due to the discontinuity caused by switching.

Remark 2: To improve the sensitivity of the proposed criteria, \( \tilde{P}(t, T_k) \) should be small. To this end, a relatively large \( \Gamma \) should be chosen. In addition, increasing \( c_i \) can accelerate the rate of decreasing of \( \tilde{P}(t, T_k) \), which is also beneficial to early detection of wrong direction. In this way, the proposed
criterion is still efficient even bounds of uncertain parameters in Assumption 1-2 are conservative. In fact, from the expression of (39), $M \rightarrow \infty$ as tracking errors approach the bounds. Therefore, inaccurate $\theta_{\text{max}}$ and $\mu_{\text{max},i}$ will not affect the validity of the switching logic.

**Remark 3:** From (31) and (40), $\dot{P}(T_k, T_k) > 0$. According to (42) when $\dot{P}(t, T_k) \rightarrow 0$, $\delta(t, T_k) \rightarrow 0$ over time interval $[T_k, T_{k+1})$. Further considering (41), switching will be triggered before $\dot{P}(t, T_k)$ goes negative. It is concluded that $\dot{P}(t, T_k) > 0$ for any $t$.

**Remark 4:** Function $v(k)$ is a periodic function borrowed from [16]. In each cycle, it traverses all possible control directions of an $n$-order nonlinear system. In the following subsection, it will be seen that with our method, the switching function need not to be periodic. Adopting (43) is for comparison purpose.

**Remark 5:** The idea of resetting integrator state when meeting specific conditions is also formed in the so-called reset control [50] to improve the transient performance of linear systems. In this paper, unlike reset control, the integrator states are not reset to zeros and the motivation behind is different.

#### C. MAIN RESULTS

**Theorem 1:** Consider the nonlinear strict-feedback system (1) under Assumptions 1-2. Suppose that the initial conditions $|z_i(0)| < k_{h_i}$, $i = 1, 2, \ldots, n$ are satisfied. If control input is designed as (27), adaptive law is given as (28) and the switching mechanism is presented as (41), (43) and (45), then the following properties hold:

1. All signals in the closed-loop system are bounded on time interval $[0, \infty)$.
2. Total switching times is no more than $2^n$ times for $n$-order systems.
3. The signals $z_i$, $i = 1, 2, \ldots, n$ always satisfy the prescribed performance requirements $|z_i(t)| < k_{h_i}$.
4. The output tracking error $z_1$ converges to zero asymptotically.

**Proof:** We first prove that if only finite switching occurs, all signals are bounded on $[0, \infty)$. Then we show the number of switching times is indeed finite and bounded by $2^n$.

Suppose that $k_f$ is the final switching index and $T_{k_f}$ is the corresponding time instant. The maximal solution exists on $[0, t_f)$. When $k = 0$, $t \in [0, T_1)$, from initial conditions it can be derived that $V_n(0)$ and $\dot{P}(0, T_0)$ are bounded. Then from (38) and (42), $z_i$ and $V_n$ are bounded. Subsequently, $\rho_i$, $\dot{\rho}_i$ with $i = 1, 2, \ldots, n$ and $\dot{\theta}$ are bounded. $\alpha_i$ is a smooth function of $\rho_i$ and $\dot{\rho}_i$, thus is bounded. Then according to (3), $x_i$ is bounded. Since $\dot{\theta} = \theta - \dot{\theta}$, $\rho_i, z_i, x_i$ are all in some closed compact sets for $t \in [0, T_1)$. According to Proposition C.3.6 in [51], the solution can be extended to $[0, T_1)$, hence $\dot{\theta}(T_1)$ and $\rho_i(T_1)$ are bounded, then $V_n(T_1)$ and $\dot{P}(T_1)$ are bounded. For $t \in [T_k, T_{k+1})$, $k = 1, 2, \ldots, k_f - 1$, given that $V_n(T_k)$ and $\dot{P}(T_k, T_k)$ are bounded, with similar analysis, the boundedness of $\rho_i, \dot{\rho}_i, \dot{\theta}, \theta, z_i, \alpha_i, x_i$ can be proved. When $k = k_f$ and $t \in [T_{k_f}, t_f)$, analogously, it can be deduced that all closed-loop signals belong to closed compact set. Then according to Proposition C.3.6 in [51], no finite-time escape phenomenon may occur and $t_f = \infty$.

Then we prove $k_f \leq 2^n$. Obviously for an $n$-order system, there are total $2^n$ possible control directions, which will all be traversed once with the proposed periodical function (43) in one cycle. Therefore, a sufficient condition of the original proposition is that when correct directions are reached, mis-switching will not happen. Without loss of generality, we suppose that when $k = 2^n, K_i = \text{sign}(\mu_i)$ for $i = 1, 2, \ldots, n$. According to (38) and the boundedness of $V(T_k)$, $V_n(t)$ is positive definite and bounded. Recalling $V_n(t) < P(t, T_k)$ and (37), then the inequality (41) can be always satisfied. Consequently, the total switching times will be no more than $2^n$.

From the analysis above, the performance bounds (42) are always satisfied. Then obviously the constraints $|z_i(t)| < k_{h_i}$, $i = 1, 2, \ldots, n$ are met.

According to (38), the integral of $c_i z_i^2$ over $[T_{k_f}, \infty)$ can be calculated as

$$\int_{T_{k_f}}^{\infty} \sum_{i=1}^{n} c_i z_i^2 dt = V_n(T_{k_f}) - V_n(\infty) < \infty. \quad (46)$$

Direct application of Barbalat’s lemma in [52], we have $\lim_{t \rightarrow \infty} |z_1| = \lim_{t \rightarrow \infty} |y - r_0| \rightarrow 0$. This completes the proof. □

**Remark 6:** In previous logic-based switching references, e.g. [21], meeting the switching conditions does not necessarily mean the estimate of direction is wrong. It may also be induced by improper parameters. Unfortunately, once mis-switching phenomenon happens, the time of estimation process is greatly increased, and the tracking performance is thus significantly reduced. The proposed method solves this problem successfully with the novel BLF based switching logic.

**Remark 7:** The complexity of the proposed algorithm is low, which can be concluded with the following observations. First of all, compared with traditional adaptive design method [18], the use of tuning function design greatly reduces the number of estimators. Relevant discussions can be found in [49]. Then recalling the switching criterion (31), (39), (40) and (41), the value of $P(T_k)$ is updated if only (41) is violated. This event triggered mechanism reduces the computation burden. Moreover, the calculation of $P(t, T_k)$ can be easily done with only one integrator.

#### IV. SIMULATION STUDIES

**First Case:** In this case, we demonstrate the effectiveness and performance of the proposed method by comparing its simulation results with those of the switching adaptive controller in [21]. The benchmark example is also taken from [21] and formulated by

$$\dot{x}_1 = \mu_1 x_2 + \theta_1 x_1,$$
$$\dot{x}_2 = \mu_2 x_3 + \theta_2 x_1 x_2.$$
where \( x = [x_1, x_2, x_3]^T \) is the state vector whose initial values are all zeros, \( \theta_1 = 0.2, \theta_2 = 0.5, \mu_1 = -1, \mu_2 = 1, \mu_3 = -1 \) are uncertain parameters whose estimate is represented as \( \hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \hat{\mu}_1, \hat{\mu}_2]^T \). Reference signal is chosen as \( r_0(t) = \sin(t/4) \).

All initial conditions of adaptive variables are set to be zeros. Control parameters are chosen as \( k_{b_1} = 0.1, k_{b_2} = 0.3, k_{b_3} = 20, \sigma = 20, c_1 = 10, c_2 = 1c_3 = 5, c = 0.001 \) and \( \Gamma = \text{diag}(3, 10, 10, 5) \). For \( i = 1, 2, 3 \), \( \mu_{\text{min},i} = 0.8 \) and \( \mu_{\text{max},i} = 1.2, \theta_{\text{max}} = 1.8 \). To verify the performance-guaranteed feature as well as the rapid estimation process of the proposed method, we compare it with the incremental-based switching technique in [21]. Simulation results are presented in Figs. 1-9.

In Fig.1, it can be seen that the proposed method has faster response and higher tracking precision. In view of Figs. 2-4, the prescribed performance bounds, that is \( |z_1| < 0.1, |z_2| < 0.3 \) and \( |z_3| < 20 \) are strictly met with our proposed method.

Fig. 5 is given to show that the correct directions can be identified in a shorter time period by choosing appropriate parameters. Fig. 6 shows the evolutions of switching signals \( K_1, K_2 \) and \( K_3 \). The trajectories of adaptive variables are shown in Fig. 7. Suppose that at time \( t = 0.15s \), switching is triggered and the sign of \( \alpha_2 \) changes. If without reset mechanism, as seen in Fig. 8, \( |z_2| = |x_2 - [-\alpha_1(0.15)]| > k_{b_2} = 0.3 \)
and BLF method will lose efficiency. Fig. 9 demonstrates the effectiveness of the reset mechanism. $|z_2|$ can be always adjusted to zero at the switching moment.

Second Case: In this case, we compare our method with the BLF based Nussbaum function control scheme in [47]. The studied nonlinear system is presented as

$$
\begin{align*}
\dot{x}_1 &= \theta_1 x_1^2 + \mu_1 x_2 \\
\dot{x}_2 &= \theta_2 x_1 x_2 + \theta_3 x_1 + \mu_2 u \\
y &= x_1
\end{align*}
$$

(48)

where $\theta_1 = 0.1$, $\theta_2 = 0.2$, $\theta_3 = 1$, $\mu_1 = -1$ and $\mu_2 = 2$. Initial values of states are $x_1(0) = 0.6$ and $x_2(0) = -0.1$. 

\[
\begin{align*}
\dot{\alpha}_1 - \alpha_1 z_2 &= 0 \\
\dot{\alpha}_2 &= 0
\end{align*}
\]
The reference signal is given by $r_0 = 0.5 \cos(0.5t)$. The tracking errors should satisfy $|z_1| < 0.5$ and $|z_2| < 1.252$. For controller design, $c_1 = 1.5$, $c_2 = 2$, $c = 0.001$, $\theta_{\text{max}} = 1.5$, $\mu_{\text{max},1} = 1.2$, $\mu_{\text{min},1} = 0.8$, $\mu_{\text{max},2} = 2.4$, $\mu_{\text{min},2} = 1.6$, $k_{b_1} = 0.5$, $k_{b_2} = 1.252$, $\sigma = 20$, $\Gamma = \text{diag}(3, 10, 10, 5)$, $\hat{\theta}(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$, $\rho_1(0) = 0$ and $\rho_2(0) = 0$. Simulation results are given in Figs. 10-14.

The simulation results are displayed in Figs. 10-14. Fig. 10 depicts the reference signal and the system outputs under both two methods. Nussbaum function method and the proposed switching controller can achieve good tracking performance. Figs. 11 and 12 show tracking error trajectories under the two control methods. Though both two methods can satisfy predefined performance constraints, the propose method provides higher tracking accuracy in the long run. The result verifies the conclusion of Theorem 1 in terms of asymptotic stability. As observed in Figs. 13 and 14, two times of switching is needed and no more than 1s, the correct directions can be identified, that is, $K_1 = \text{sign}(\mu_1) = -1$ and $K_2 = \text{sign}(\mu_2) = 1$.

V. CONCLUSION

In this paper, we propose a performance-guaranteed switching adaptive controller for PSF nonlinear systems in the presence of multiple unknown control directions. The controller is developed by combining BLF method and tuning function design. Asymptotically tracking performance is achieved and tracking errors can be strictly limited into predefined regions even without information of control gains. A novel switching logic which is composed of a BLF-type monitoring function and a threshold function is designed. When compared with existing studies, the proposed scheme can identify the correct directions faster and only finite times of switching are required. Meanwhile, undesirable mis-switching phenomenon is avoided. Furthermore, the proposed reset mechanism guarantees that the preconditions of BLF method can be always satisfied at switching moments. Simulation results verify the effectiveness of the proposed method. Based on the obtained results, we will study the switching adaptive controller design for a more general class of nonlinear systems with external disturbances and actuator faults.

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