Is noncommutative eternal inflation possible?

Yi-Fu Cai\textsuperscript{1} and Yi Wang\textsuperscript{2,3}

\textsuperscript{1} Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, People’s Republic of China
\textsuperscript{2} Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, People’s Republic of China
\textsuperscript{3} The Interdisciplinary Center for Theoretical Study of China (USTC), Hefei, Anhui 230027, People’s Republic of China

E-mail: caiyf@mail.ihep.ac.cn and wangyi@itp.ac.cn

Received 16 May 2007
Accepted 5 June 2007
Published 21 June 2007

Abstract. We investigate the condition for eternal inflation to take place in the noncommutative spacetime. We find that the possibility for eternal inflation’s happening is greatly suppressed in this case. If eternal inflation cannot happen in the low energy region where the noncommutativity is very weak (the UV region), it will never happen during the whole inflationary history. Based on these conclusions, we argue that an initial condition for eternal inflation is available from the property of spacetime noncommutativity.

Keywords: string theory and cosmology, inflation, physics of the early universe
Is noncommutative eternal inflation possible?

Contents

1. Introduction
2. Fluctuations generated outside the horizon
3. Conclusion and discussions
   Acknowledgments
   References

1. Introduction

Inflation has been widely considered as a remarkably successful theory in explaining many problems in the very early universe, such as the flatness, the horizon and the monopole problem [1]–[4]. It predicts that the quantum fluctuations of the inflaton field were generated to form today’s large scale structure [5]–[9], and these fluctuations fit with current observations of the cosmic microwave background very well [10,11].

It is a common viewpoint that eternal inflation [12]–[14] can take place in a variety of inflation models. In particular, with the monomial chaotic inflaton potential, inflation generally becomes eternal in the high energy regime. Eternal inflation is not only important in concept, but also provides a possible realization for the string theory landscape [15]–[18]. There have been an increasing number of papers investigating whether we can measure eternal inflation and calculate probabilities in the multiverse, see [19]–[24].

Usually, eternal inflation happens when the energy scale of the universe is extremely high. Thus we would like to take into consideration on more fundamental theories in logic, namely, string theory. In this paper, we focus on a universal property of string theory, i.e. noncommutativity of spacetime. By considering the effects of spacetime noncommutativity, we study the condition for eternal inflation and its implications. As required by the stringy spacetime uncertainty relation [25]–[27], the physical time $t_p$ and the physical length $x_p$ should satisfy

$$\Delta t_p \Delta x_p \geq l_s^2,$$

where $l_s$ is a length scale given by the string theory.

There have been a lot of attempts to apply this uncertainty relation into inflationary cosmology, dubbed noncommutative inflation, see [31]–[33]. More detailed work has been investigated in a number of papers [34]–[40]. Here we briefly review the concept of noncommutative inflation proposed by Brandenberger and Ho [33], and then examine whether eternal inflation takes place with such noncommutativity. Note that here we have slightly generalized the discussion in [33] from power law inflation into a generally quasi-exponential inflationary scenario.

In order to introduce the noncommutativity into the four-dimensional Friedmann–Robertson–Walker universe, we would like to define another time coordinate $\tau$,

$$ds^2 = dt^2 - a^2(t) dx^2 = a^{-2}(\tau) d\tau^2 - a^2(\tau) d\vec{x}^2,$$
where \( a \) is the scale factor and we have assumed a spatially flat universe \( (K = 0) \). For the quasi-exponential expansion where we have imposed the usual slow roll approximation self-consistently, we have

\[
d\tau = a \, dt \simeq a_0 e^{Ht} \, dt, \quad a \simeq H \tau,
\]

where \( H = \dot{a}/a \) is the Hubble parameter with the dot represents the derivative with respect to the cosmic time \( t \) and \( a_0 \) is an arbitrary parameter for the scale factor at a fixed time. Then the spacetime uncertainty relation takes the form

\[
\Delta \tau \Delta x \geq l_s^2,
\]

where \( x \) is the comoving radial coordinate. This can be realized by the commutation relation of spacetime

\[
[\tau, x]_* = i l_s^2,
\]

where the \(*\)-product is defined as

\[
(f * g)(x, \tau) = \exp \left( -\frac{i}{2} l_s^2 (\partial_x \partial_{\tau'} - \partial_{\tau} \partial_y) \right) f(x, \tau) g(y, \tau') \bigg|_{y=x, \tau'=\tau}.
\]

(6)

Based on these considerations, for the fluctuation of the inflaton field \( \delta_q \varphi_k \), we can derive that the canonical normalized perturbation variable \( u_k \simeq a \delta_q \varphi_k \) satisfies the equation of motion

\[
u_k'' + \left( k^2 - \frac{z_k''}{z_k} \right) u_k = 0,
\]

where the prime denotes the derivative with respect to \( \tilde{\eta} \) defined as \( d\tilde{\eta} \equiv a_{\text{eff}}^{-2} \, d\tau \). The parameter \( a_{\text{eff}} \) is an effective scale factor appearing in the dispersion relation between a mode \( k \) and its energy defined with respect to \( \tau \),

\[
a_{\text{eff}}^2 \equiv \left( \beta_k^+ \beta_k^- \right)^{1/2} = \frac{k}{E_k}, \quad \beta_k^+(\tau) \equiv \frac{1}{2} [a^2(\tau - l_s^2 k) + a^2(\tau + l_s^2 k)],
\]

(8)

and \( z_k \) is defined as

\[
z_k^2 \equiv (\beta_k^- \beta_k^+) \frac{1}{2} z^2, \quad z \equiv a \frac{\dot{\varphi}}{H}.
\]

(9)

As the scale factor is expanding nearly exponentially, when \( l_s^2 k \) is not too small compared with \( \tau \) we can take the approximate form of \( \beta_k^+ \) as

\[
\beta_k^+ \simeq \frac{1}{2} a^2(\tau + l_s^2 k) \simeq \frac{1}{2} \left[ H(\tau + l_s^2 k) \right]^2,
\]

(10)

\[
\beta_k^- \simeq \frac{1}{2} a^{-2}(\tau - l_s^2 k) \simeq \frac{1}{2} \left[ H(\tau - l_s^2 k) \right]^{-2}.
\]

(11)

Moreover, because of the relations \( \Delta x \sim 1/k, \Delta \tau \sim 1/E_k \) and the spacetime uncertainty (4), there is an initial time \( \tau_k \) for the perturbations to be generated,

\[
a_{\text{eff}}(\tau) \geq a_{\text{eff}}(\tau_k) = l_s k,
\]

(12)

and the fluctuations are not allowed to exist before \( \tau_k \).
Is noncommutative eternal inflation possible?

Due to this initial time for the fluctuations, the quantum fluctuations of inflaton in a noncommutative environment can be generated inside or outside the Hubble radius. These fluctuations are called UV modes and IR modes, respectively. In the UV mode limit, the effects of spacetime noncommutativity become very weak and hence the evolution of the fluctuations is similar to the commutative case. Therefore the spectrum of UV modes is roughly the same as in the commutative case. While in the IR region, noncommutativity dominates the behaviour of the perturbations, and some familiar pictures will be totally modified. In this paper, we shall use the analysis of noncommutativity mentioned above (especially the IR region) to study eternal inflation.

This paper is organized as follows. In section 2, we calculate the fluctuations generated outside the horizon and provide the constraint for eternal inflation to exist. In section 3, we draw a conclusion and discuss some other possibilities.

2. Fluctuations generated outside the horizon

In this section, we investigate the quantum fluctuations generated in the noncommutative inflationary era and discuss the condition of eternal inflation. Since in the UV case the noncommutative property only contributes a few corrections to the usual perturbation theory [33, 34], the condition for eternal inflation to happen is basically the same as the commutative one. Therefore, we focus our consideration on the IR case that the quantum fluctuations are generated outside the horizon. At the time the fluctuations start to be generated, the effective scale factor is

\[ a_{\text{eff}}(\tau_k) = l_s k. \]  

(13)

Making use of equations (10) and (11), and the fact that the Hubble scale is larger than the noncommutativity scale in this case, we get the initial time \( \tau_k \) and the initial scale factor \( a_k \) as

\[ \tau_k = \sqrt{l_s^2 k^2 \left( 1 + \frac{1}{l_s^2 H^2} \right)} \simeq l_s^2 k, \quad a_k \simeq H l_s^2 k. \]  

(14)

Since the IR modes are generated outside the horizon, it is required that \( k < a_k H \). By using the second relation in equation (14), we can see that

\[ H > l_s^{-1} \]  

(15)

in the IR case. This verifies the physical picture that \( H \) should be larger than the noncommutativity scale.

Now we calculate the quantum fluctuation in the momentum space \( \delta_q \varphi_k \). \( \delta_q \varphi_k \) is linked to the canonical perturbation \( u_k \) by \( u_k \simeq a \delta_q \varphi_k \), and when the perturbation begins to be generated the initial \( u_k \) was canonically normalized as \( u_k \simeq 1/\sqrt{2k} \). Consequently, when \( \delta_q \varphi_k \) is born its amplitude can be given by

\[ \delta_q \varphi_k \simeq \frac{1}{\sqrt{2k}} \frac{1}{H l_s^2 k}. \]  

(16)
Is noncommutative eternal inflation possible?

After that, the fluctuations outside the horizon are nearly frozen. By transforming to the coordinate space, we obtain the relation

\[
\langle \delta_q \phi^2 \rangle = \int_{k = aH}^{k = (e \times a)H} \frac{dk}{k} \frac{k^3}{2 \pi^2} \delta_q \phi_k \delta_q \phi_{-k} \approx \left( \frac{1}{2 \pi} \frac{1}{H_0^2} \right)^2
\]

during one Hubble time, and correspondingly the IR quantum fluctuation \( \delta_q \phi \) per e-fold in the noncommutative spacetime is given by

\[
\delta_q \phi \approx \frac{1}{2 \pi} \frac{1}{H_0^2},
\]

Note that this result is strongly different from the commutative one \( \delta_q \phi \approx H/2\pi \). Due to this, the physics of eternal inflation is greatly modified in the noncommutative case and the condition for eternal inflation to happen needs to be reconsidered.

As usual, the classical motion of the inflaton during one Hubble time takes the form

\[
\delta_c \phi \approx \dot{\phi} H^{-1} \sim \frac{V_\phi}{H^2},
\]

where \( V_\phi \) denotes \( dV(\phi)/d\phi \). As usual, the condition that inflation becomes eternal is roughly \( \delta_q \phi > \delta_c \phi \), or

\[
H > V_\phi l_s^2.
\]

Compared with the eternal inflation condition in the commutative case \( H^3 > V_\phi \), the noncommutative eternal inflation is more unlikely to happen. It is because the quantum fluctuation is generally smaller due to the suppression by the Hubble parameter. In the following, we shall consider two explicit examples with the chaotic potentials.

Firstly, let us consider the model \( V = (1/2)m^2 \phi^2 \). From the condition (15) we can see that the inflation is in the IR region when \( \phi > M_p l_s m \) while in the UV region when \( \phi < M_p l_s m \). Consequently, the condition that inflation has become eternal in the UV region is

\[
\varphi_{\text{IR}} \equiv \frac{M_p}{l_s m} > \sqrt{\frac{M_p^2}{m}}, \quad m < \frac{1}{M_p l_s^2}.
\]

Interestingly, we note that from (20) the eternal inflation will nicely continue in the IR region if

\[
m < \frac{1}{M_p l_s^2}.
\]

To substitute the well-known value \( m \sim 10^{-6} M_p \) [41] into (22), we obtain the result that eternal inflation is allowed to happen only if \( M_s \simeq l_s^{-1} > 10^{-3} M_p \).

So we conclude that, if inflation has become eternal before entering the IR region, it can continue to be eternal in the IR region. But the amplitude of fluctuation cannot grow to larger values because \( \delta_q \phi \) is now suppressed by a factor of \( 1/H \). This is very different from the commutative eternal inflation and may constrain the initial condition space for the eternal inflation. On the other hand, if inflation has not become eternal before entering the IR region, then the inequality (22) does not hold. In this case inflation will never become eternal because the quantum fluctuation cannot be large enough.
Is noncommutative eternal inflation possible?

Figure 1. Evolution of the chaotic inflation field with $V(\varphi) = (1/2)m^2\varphi^2$ in the noncommutative spacetime. Here we have assumed $M_s \simeq l_s^{-1}$ to be large enough so that eternal inflation can take place in the UV region. In this case, inflation is eternal in part of the noncommutative UV region and the whole IR region.

As a second example, consider $V = \lambda M_p^{4-p}\varphi^p$ with $p > 2$. Then similarly, the condition for entering the IR region $H > 1/l_s$ requires

$$\varphi > \lambda^{-1/p} l_s^{-2/p} M_p^{(p-2)/p}. \tag{23}$$

From the inequality (20), we obtain the condition for inflation to be eternal in the IR region as follows:

$$\varphi < p^{-2/(p-2)} \lambda^{-1/(p-2)} l_s^{-4/(p-2)} M_p^{p-6/(p-2)}. \tag{24}$$

In order that (23) and (24) has overlap and inflation can be eternal in the IR region, we need

$$\lambda < p^{-p} \left( \frac{1}{l_s M_p} \right)^{p+2}. \tag{25}$$

If (25) is satisfied, the noncommutative eternal inflation is allowed to exist in the IR region, and there is an upper bound for the energy density of eternal inflation which arises from (24). Since the inflaton field cannot increase higher than the bound (24) with large probability, this provides a possible initial condition for eternal inflation.

Now let us examine whether there is eternal inflation at all in the $p > 2$ model above. For eternal inflation to take place in the UV region, we again obtain the relation

$$\lambda < p^{-p} \left( \frac{1}{l_s M_p} \right)^{p+2}, \tag{26}$$

which is the same as the IR region eternal condition (25).

So we get the similar conclusion with the $(1/2)m^2\varphi^2$ ($p = 2$) model that, if eternal inflation takes place in the UV region, it is allowed to extend into the IR region. On the other hand, if inflation cannot be eternal in the UV region, unfortunately there will not be any eternal inflation in the whole noncommutative inflationary history. Besides, there is an interesting difference between the $p > 2$ and $p = 2$ cases in that, in the $p > 2$...
Is noncommutative eternal inflation possible?

Figure 2. Evolution of the chaotic inflation field with $V(\phi) = \lambda \phi^4$ in the noncommutative spacetime. Here we have assumed $M_s \simeq l^{-1}_s$ to be large enough so that eternal inflation can take place in the UV region. In this case, inflation is eternal in part of the noncommutative UV region and part of the IR region. This figure is different from figure 1 in that eternal inflation cannot happen in the green part of the IR region where the energy scale is extremely high.

case, noncommutative eternal inflation requires an upper bound on the inflaton while this bound does not exist in the $p = 2$ case.

For example, in the $\lambda \phi^4$ model, if we expect eternal inflation to take place, then we need the noncommutative scale $l_s^{-1} > \lambda^{1/6} M_p$. To apply the data $\lambda \sim 10^{-14}$, we obtain that this scale is around $10^{-2} M_p$. If $l_s^{-1}$ is just above this scale, the upper bound (24) on $\phi$ is around $10^3 M_p$ and the corresponding energy density is about $10^{-2} M_p^4$. Consequently, we conclude that the limitation from noncommutativity can be much stronger than that from the Planck density. Such a string scale can be realized easily if we do certain compactification on a large manifold, which should be common and have a large prior probability in the string landscape.

On the other hand, such a scale $l_s^{-1} \sim 10^{-2} M_p$ or $l_s^{-1} \sim 10^{-3} M_p$ is considerably higher than the scale for the final 60 e-folds inflation. So if we can observe the noncommutativity in future CMB experiments [39], then eternal inflation is not expected to have happened.

3. Conclusion and discussions

Spacetime noncommutativity, predicted by string theory, has become a fundamental principle and been studied in a number of works (see, e.g., [28]–[30]). This principle brings new physics when applied to inflation theory. In this paper, we have seen that, for the chaotic inflationary potential, the scenario of eternal inflation in noncommutative spacetime is remarkably different from the usual one. If inflation does not become eternal in the UV region, then it can never have happened.

We have also discussed that, if eternal inflation happens both in the UV region and the IR region, an initial condition for eternal inflation can be provided or constrained. From the derivation in this paper, we can see that $\delta_q^2 \phi$ becomes smaller and smaller along the potential when eternal inflation enters the IR region. Therefore, for the $p = 2$ model
the initial condition space is greatly reduced; while for the $p > 2$ model there is an upper bound for $\varphi$ explicitly. Eternal inflation cannot increase into higher energy regions than this bound. This provides an initial condition for eternal inflation. As is discussed in [20, 23], initial conditions may be essential for predictions in the multiverse. The initial condition discussed in this paper provides a possible solution for the initial condition problem and can be used in calculating the eternal inflationary probabilities.

In the derivation made in section 2, we have used the standard method according to a number of calculations for noncommutative and non-eternal inflation in the literature. However, generally there are several possibilities which may change the results we obtain in this paper.

The simplest possibility is that there is no spacetime noncommutativity at all or the scale of noncommutativity is of the same order as the Planck scale. Therefore, the noncommutativity will not alter the picture of the usual eternal inflation.

As another possibility, the noncommutative field theory may not be precise enough to describe the generation of the quantum fluctuations. This saturation is somewhat similar to that inflationary fluctuations described by the common quantum field theory suffer from the transPlanckian problems [42]. To see this problem in the noncommutative field theory, we note that we have used the relation $\Delta x \sim 1/k$ in the calculations of noncommutative inflation. However, this relation does not hold in theories with certain UV–IR relations while UV–IR relations commonly arise together with noncommutativity. So there is the possibility that the perturbations are generated even if $k > a_{\text{eff}}/l_s$, but it is still an open issue for us to fully understand the physics in this region.

Finally, the background geometry may be affected considerably by noncommutativity in the IR region. However, up to now this case has not been carefully studied even in the non-eternal inflationary regime.

Acknowledgments

We thank Bin Chen, Miao Li, Yun-Song Piao and Xinmin Zhang for helpful discussions. This work is supported in part by the National Natural Science Foundation of China under grant nos. 90303004, 10533010, 19925523 and 10405029, and by the Chinese Academy of Science under grant no. KJCX3-SYW-N2. YW gratefully acknowledges grants from the NSFC.

References

[1] Guth A H, 1981 Phys. Rev. D 23 347 [SPIRES]
[2] Linde A D, 1982 Phys. Lett. B 108 389 [SPIRES]
[3] Albrecht A and Steinhardt P J, 1982 Phys. Rev. Lett. 48 1220 [SPIRES]
[4] For earlier attempts on an inflationary model, see Starobinsky A A, 1979 JETP Lett. 30 682
   Starobinsky A A, 1979 Pis. Zh. Eksp. Teor. Fiz. 30 719
   Starobinsky A A, 1980 Phys. Lett. B 91 99 [SPIRES]
[5] Mukhanov V and Chibisov G, 1981 JETP 33 549
[6] Guth A H and Pi S-Y, 1982 Phys. Rev. Lett. 49 1110 [SPIRES]
[7] Hawking S W, 1982 Phys. Lett. B 115 295 [SPIRES]
[8] Starobinsky A A, 1982 Phys. Lett. B 117 175 [SPIRES]
[9] Bardeen J M, Steinhardt P J and Turner M S, 1983 Phys. Rev. D 28 679 [SPIRES]
[10] Miller A D et al, 1999 Astrophys. J. 524 L1 [SPIRES] [astro-ph/9906421]
   de Bernardis P et al, 2000 Nature 404 955 [SPIRES] [astro-ph/0004040]
   Hanany S et al, 2000 Astrophys. J. 524 L5 [SPIRES] [astro-ph/0005123]
   Halverson N W et al, 2002 Astrophys. J. 568 38 [SPIRES] [astro-ph/0104489]
Is noncommutative eternal inflation possible?

Mason B S et al, 2003 Astrophys. J. 591 540 [SPIRES] [astro-ph/0205384]
Benoit A et al, 2003 Astron. Astrophys. 399 L25 [SPIRES] [astro-ph/0210306]
Goldstein J H et al, 2003 Astrophys. J. 599 773 [SPIRES] [astro-ph/0212517]

Spergel D N et al, 2003 Astrophys. J. Suppl. 149 175 [astro-ph/0302209]
Spergel D N et al, 2006 Preprint astro-ph/0603449

Steinhardt P J, 1982 The Very Early Universe Proc. p 251

Vilenkin A, 1983 Phys. Rev. D 27 2848 [SPIRES]

Linde A D, 1986 Phys. Rev. Lett. A 1 81 [SPIRES]

Vilenkin A, 2003 Phys. Rev. D 68 046005 [SPIRES] [hep-th/0301240]

Douglas M R and Kachru S, 2000 J. Cosmol. Astropart. Phys. JCAP01(2000)017 [SPIRES] [ hep-th/0003187]

Douglas M R and Kachru S, 2006 Preprint hep-th/0608207

Vilenkin A, 2003 J. Cosmol. Astropart. Phys. JCAP06(2003)010 [SPIRES] [ hep-th/0203134]

Vilenkin A, 2006 J. Cosmol. Astropart. Phys. JCAP01(2007)022 [SPIRES] [hep-th/0611043]

Linde A, 2007 Preprint 0705.1160 [hep-th]

Podolsky D and Enqvist K, 2007 Preprint 0704.0144 [hep-th]

Li M and Wang Y, 2007 Preprint 0704.1026 [hep-th]

Li M and Yoneya T, 1997 Phys. Rev. Lett. 78 1219 [SPIRES] [hep-th/9611072]

Yoneya T, 2000 Prog. Theor. Phys. 103 1081 [SPIRES] [hep-th/0004074]

Connes A, Douglas M and Schwars A, 1998 J. High Energy Phys. JHEP02(1998)003 [SPIRES] [ hep-th/9711162]

Seiberg N and Witten E, 1999 J. High Energy Phys. JHEP09(1999)032 [SPIRES] [hep-th/9908142]

Bigatti D and Susskind L, 2000 Phys. Rev. D 62 066004 [SPIRES] [hep-th/9908056]

Seiberg N, Susskind L and Tounbas N, 2000 J. Cosmol. Astropart. Phys. JHEP06(2000)021 [SPIRES] [hep-th/0005040]

Lue A Y, Recknagel A and Schomerus V, 2000 J. High Energy Phys. JHEP05(2000)010 [SPIRES] [ hep-th/0003187]

Lue C S and Ho P M, 2002 Nucl. Phys. B 636 141 [SPIRES] [ hep-th/0203186]

Lue C S, Greene B R and Shin G, 2001 Mod. Phys. Lett. A 16 2231 [SPIRES] [hep-th/0011241]

Lizzi F, Mangano G, Miele G and Peloso M, 2002 J. High Energy Phys. JHEP06(2002)049 [SPIRES] [ hep-th/0203099]

Brandenberger R H and Ho P M, 2002 Phys. Rev. D 66 023517 [SPIRES] [hep-th/0203119]

Huang Q-G and Li M, 2003 J. High Energy Phys. JHEP06(2003)014 [SPIRES] [hep-th/0304203]

Huang Q-G and Li M, 2003 J. Cosmol. Astropart. Phys. JCAP11(2003)001 [SPIRES] [astro-ph/0308458]

Huang Q-G and Li M, 2004 Nucl. Phys. B 713 219 [SPIRES] [astro-ph/0311378]

Tsujikawa S, Maartens R and Brandenberger R, 2003 Phys. Lett. B 574 141 [SPIRES] [astro-ph/0308169]

Koh S and Brandenberger R H, 2007 Preprint hep-th/0702217

Brandenberger R H, 2007 Preprint hep-th/0703173

Cai R-G, 2004 Phys. Lett. B 593 1 [SPIRES] [hep-th/0403134]

Huang Q-G and Li M, 2006 Nucl. Phys. B 755 286 [SPIRES] [astro-ph/0603782]

Zhang X and Wu F Q, 2006 Phys. Lett. B 633 396 [SPIRES] [astro-ph/0604195]

Huang Q-G, 2006 Phys. Rev. D 74 063513 [SPIRES] [astro-ph/0605442]

Zhang X, 2006 J. Cosmol. Astropart. Phys. JCAP12(2006)002 [SPIRES] [hep-th/0608207]
Is noncommutative eternal inflation possible?

Huang Q G, 2006 J. Cosmol. Astropart. Phys. JCAP11(2006)004 [SPIRES] [astro-ph/0610389]

Kim H S, Lee G S and Myung Y S, 2005 Mod. Phys. Lett. A 20 271 [SPIRES] [hep-th/0402018]

Calcagni G, 2004 Phys. Rev. D 70 103525 [SPIRES] [hep-th/0406006]
Calcagni G, 2005 Phys. Lett. B 606 177 [SPIRES] [hep-ph/0406057]
Calcagni G and Tsujikawa S, 2004 Phys. Rev. D 70 103514 [SPIRES] [ astro-ph/0407543]

Cai Y-F and Piao Y-S, 2007 Preprint gr-qc/0701114
Xue W, Chen B and Wang Y, 2007 Preprint 0706.1843 [hep-th]
Linde A, 2007 Preprint 0705.0164 [hep-th]
Martin J and Brandenberger R H, 2001 Phys. Rev. D 63 123501 [SPIRES] [hep-th/0005209]