I present a new formalism of the R-matrix theory where the formal parameters for the resonance energies and widths are identical to the observed values. By allowing the boundary condition parameters to vary from level to level, the freedom required to adjust the formal parameters for the pole positions to the observed values is obtained. The basis of the resulting theory becomes non-orthogonal, and I describe the procedure to construct a consistent R-matrix theory with such a non-orthogonal basis. And by adjusting the normalization of the states that form the basis, the formal parameters for the reduced decay widths also become the same as those observed, leaving no formal parameters that are different from the observed ones. A demonstration of the developed theory to the elastic $^{12}$C + $p$ scattering data is presented.

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complications can be removed.

In this paper, I address this issue by presenting a new R-matrix theory where there is no distinction between the two sets of parameters. The key steps of the developed formulation can be summarized as follows. When constructing the conventional R-matrix theory, the basis for the Hilbert space defined in the internal region of the theory is required to be orthogonal. Since the domain of the basis is not the whole configuration space but limited only to the internal region, the seemingly innocent orthogonality condition imposes quite a strict constraint on the states that form the basis: The boundary condition parameters that define the basis states should be level-independent in order to satisfy the orthogonality condition (see the text for more detailed explanation). This constraint is in fact responsible for the discrepancies between the formal parameters and the observed ones. However, it is possible to construct a consistent R-matrix theory with a non-orthogonal basis. It turns out that releasing the orthogonality condition provides the freedom to have level-dependent boundary conditions, with which the formal parameters can be equivalent to the observed values. To be more specific, if one adjusts the boundary condition parameters $B_{\lambda c}$ to the shift factor $S_c$ at $E_\lambda$ (the pole-position of level $\lambda$), the formal parameters for the pole positions coincide with the observed ones. Furthermore, there is additional freedom associated with the normalization of the basis states. By selecting the normalization given in Eq.(28), the formal parameters for reduced widths also become the same as observed. As a result, there are no formal parameters that are different from the observed values in the present theory, achieving the desired goal.

The new formalism shares many features in common with previous studies. For example, the very original work by Kapur and Peierls [2], has an energy-dependent boundary condition. Schemes with level-dependent boundary condition parameters were introduced by Barker [5] and by Azuma et al. [6]. Angulo and Descouvemont developed a formalism [7] where there are no level shifts, which is however applicable only to single-channel cases. In particular, this work is rather similar to the work of Brune [8], where an alternative parametrization was introduced to use the observed pole locations as inputs of the R-matrix theory, developing a transformation scheme between the formal and the observed parameters. In this work, the theory is formulated in such a way that all the formal parameters are directly equivalent to the observed values.

In Section II, I briefly review the R-matrix theory, explaining how the orthogonality
condition can be released in a consistent manner to allow the boundary condition parameters to be level-dependent. In Section III, I then describe the procedure to align the formal parameters to the observed values. In Section VI, a demonstration of the developed theory is made for the\textsuperscript{12}C + p elastic scattering reaction. Section V is devoted to discussions.

II. FORMALISM FOR THE EXTENDED BOUNDARY CONDITION

I begin with a brief review of the R-matrix theory described in Lane and Thomas (LT)\cite{3}. In the R-matrix theory, a nuclear system is described in terms of channels that consist of two subsystems, \( \alpha_1 = \{ Z_1, A_1, I_1 \} \) and \( \alpha_2 = \{ Z_2, A_2, I_2 \} \), where \( Z_i, A_i \) and \( I_i \) are the proton number, mass number and spin of the \( i \)-th subsystem, respectively. The quantum state of a channel may be denoted as \( c = \{ \alpha, (I_1, I_2)s, \ell; JJ_z \} \), where \( \alpha = \alpha_1 \otimes \alpha_2 \) is the partition index, \( s \) is the channel spin (\( \vec{s} = \vec{I}_1 + \vec{I}_2 \)), \( \ell \) is the relative orbital angular momentum, \( J \) is the total angular momentum (\( \vec{J} = \vec{s} + \vec{\ell} \)) and \( J_z \) is its projection\cite{4}. The Hamiltonian of the system in the center-of-mass frame then reads

\[
H = -\frac{\hbar^2}{2M_c} \nabla^2_{\vec{r}_c} + V_c(\vec{r}_c) + H_{\alpha_1} + H_{\alpha_2},
\]

(1)

where \( \vec{r}_c \) and \( M_c \) are the relative position vector and the reduced mass of channel \( c \), respectively, and \( H_{\alpha_i} \) is the Hamiltonian for the internal energy of the \( i \)-th subsystem. The wavefunction of the system that satisfies the Schrödinger equation at energy \( E \), \( H\Psi = E\Psi \) can be written as

\[
\Psi = \sum_c \varphi_c u_c(r_c),
\]

(2)

where \( u_c(r_c) \) is the radial function of channel \( c \). The spinors and the angular dependence of the subsystems are embodied in \( \varphi_c = \frac{1}{r_c} \left[ [\varphi_{\alpha_1} \otimes \varphi_{\alpha_2}] \otimes i^J Y_J(\hat{r}_c) \right]_{JJ_z} \). The radial space of each channel is divided into two parts: the internal region \( (r_c \leq a_c) \) and the external region \( (r_c > a_c) \), where \( a_c \) is the channel radius that defines the surface between the two regions. \( \varphi_c \) is assumed to be orthonormal when integrated on the channel surface,

\[
\int \varphi_c^* \varphi_{c'} dS = \delta_{cc'},
\]

(3)

where \( S \) is the surface defined by \( r_c = a_c \). Here and hereafter, unless stated otherwise, I follow the notation given in LT.
The wave function in the internal region is expanded as a linear combination of the basis states $X_\lambda$,
\[ \Psi = \sum \lambda C_\lambda X_\lambda, \quad (4) \]
where $C_\lambda$ are $E$-dependent coefficients. The basis states
\[ X_\lambda = \sum_c \varphi_c u_{\lambda c}(r_c) \quad (5) \]
are defined by the eigenvalue equation
\[ HX_\lambda = E_\lambda X_\lambda. \quad (6) \]
with a boundary condition imposed at the channel surface, which will be discussed soon.

Since the basis states are relevant only in the internal region, I define their inner products as
\[ J_{\lambda\lambda'} \equiv \int_\tau X_\lambda^* X_{\lambda'} d\tau, \quad (7) \]
where $\int_\tau d\tau$ denotes the volume integral limited only to the internal region. The orthogonality condition of the basis then corresponds to have $J_{\lambda\lambda'} = 0$ for $\lambda \neq \lambda'$. $J_{\lambda\lambda'}$ can be evaluated by the following steps: If one multiplies Eq.(6) by $X_{\lambda'}^*$ from the left and integrate it in the internal region, and then subtract it with interchanging $\lambda'$ and $\lambda$, one obtains
\[ (E_\lambda - E_{\lambda'}) \int_\tau X_\lambda^* X_{\lambda'} d\tau = \int_\tau [(HX_\lambda)^* X_{\lambda'} - X_\lambda^* (HX_{\lambda'})] d\tau \]
\[ = -\sum_c \frac{\hbar^2}{2M_c} \left( u_{\lambda c} \frac{du_{\lambda c}}{dr} - u_{\lambda c} \frac{du_{\lambda' c}}{dr} \right)_{r=a_c}, \quad (8) \]
where I have inserted the Hamilton given in Eq.(1) at the last step. Here and hereafter, I limit myself to the cases where the nuclear potential is hermitian and the radial functions are real, $u_{\lambda c}(r)^* = u_{\lambda c}(r)$. Dividing the above equation by $E_\lambda - E_{\lambda'}$, one is then led to
\[ J_{\lambda\lambda'} = -\frac{1}{E_\lambda - E_{\lambda'}} \sum_c \gamma_{\lambda c} (B_{\lambda c} - B_{\lambda' c}) \gamma_{\lambda' c}, \quad \text{for } \lambda \neq \lambda', \quad (9) \]

1 By making use of Eqs.(5) and (3), $J_{\lambda\lambda'}$ can also be represented as
\[ J_{\lambda\lambda'} = \sum_c \int_0^{a_c} dr_c u_{\lambda c}(r_c)^* u_{\lambda' c}(r_c). \]

2 Here I assume that for a given spin and parity, $X_\lambda$ is non-degenerate, and thus $\lambda \neq \lambda'$ implies $E_\lambda \neq E_{\lambda'}$. The degenerate levels with the same level-energy, if any, can be merged into a single level, as discussed in Ref. [8].
where
\[ \gamma_{\lambda c} \equiv \sqrt{\frac{\hbar^2}{2M_c a_c}} u_\lambda(a_c), \quad (10) \]
\[ B_{\lambda c} \equiv \frac{a_c}{u_{\lambda c}(a_c)} \left. \frac{d u_{\lambda c}(r)}{dr} \right|_{r=a_c}. \quad (11) \]

From Eq.(9), it is clear that the orthogonality condition for a general multi-level and multi-channel case can be guaranteed only when the boundary condition is level-independent, \( B_{\lambda c} = B_c \), as is demanded in the conventional R-matrix theory. However, the orthogonality is not a necessary condition for the basis of a consistent R-matrix theory. If one does not adhere to it, as I explain below, one is granted additional freedom to have level-dependent boundary condition parameters that can be used to remove the gap between the formal parameter set and the observed parameter set.

I now describe how a new R-matrix theory can be built with a non-orthogonal basis. From Eqs.(9) and (4), the coefficients \( C_\lambda \) read
\[ C_\lambda = \sum_{\lambda'} (J^{-1})_{\lambda' \lambda} \int_\tau X_\lambda \Psi d\tau, \quad (12) \]
and the integral in the above equation can be evaluated by inserting \( \Psi \) in place of \( X_{\lambda'} \) in Eq.(8),
\[ \int_\tau X_\lambda \Psi d\tau = \frac{1}{E_\lambda - E} \sum_{c} \gamma_{\lambda c} \sqrt{\frac{\hbar^2}{2M_c a_c}} \left( a_c \frac{d u_c(r)}{dr} - B_{\lambda c} u_c(a_c) \right)_{r=a_c}. \quad (13) \]
Insertion of the resulting coefficients into Eq.(4) gives the following equation,
\[ u_{\lambda'}(a_{\lambda'}) = \sum_{\lambda'} C_{\lambda'} u_{\lambda' \lambda'}(a_{\lambda'}) \]
\[ = \sum_{c} \sqrt{\frac{M_{\lambda' \lambda} \gamma_{\lambda' c}}{M_c}} \left( \mathcal{R}_{\lambda' \lambda} \frac{d u_c(r)}{dr} - \mathcal{R}_{\lambda' \lambda}^B u_c(a_c) \right)_{r=a_c}, \quad (14) \]
with
\[ \mathcal{R}_{\lambda' \lambda} \equiv \sum_{\lambda', \lambda} \gamma_{\lambda' \lambda'} (J^{-1})_{\lambda' \lambda} \frac{1}{E_\lambda - E} \gamma_{\lambda c}, \quad (15) \]
\[ \mathcal{R}_{\lambda' \lambda}^B \equiv \sum_{\lambda', \lambda} \gamma_{\lambda' \lambda'} (J^{-1})_{\lambda' \lambda} \frac{1}{E_\lambda - E} \gamma_{\lambda c} B_{\lambda c}. \quad (16) \]

On the other hand, the radial wave functions in the external region can be written analytically, because the channel radius \( a_c \) is assumed to be large enough so that all the nuclear
forces between the two subsystems vanish and only the Coulomb interaction remains,

\[ u_c(r) = \frac{1}{\sqrt{v_c}} \sum_c [I_c(r)\delta_{c'} - O_c(r)U_{c'c}] y_c, \quad r_c \geq a_c, \quad (17) \]

where \( y_c \) are coefficients, \( I_c(r) \) and \( O_c(r) \) are the incoming and outgoing radial wave functions, respectively, \( U \) is the collision matrix, and \( v_c = \sqrt{2|E_c|/M_c} \) are the relative velocities.

The collision matrix can be obtained by requiring that the logarithmic derivatives of the radial functions on the channel surface resulting from Eq.(17) should be equal to the derivatives derived from Eq.(14),

\[ U_{c'c} = \Omega_{c'} \left( \delta_{c'} + 2i\sqrt{P_c} \left[ 1 - \mathcal{R}(S_c + iP_c) + \mathcal{R}^B \right]^{-1} R \right)_{c'c} \sqrt{P_c} \Omega_c, \quad (18) \]

where \( \Omega_c = \sqrt{I_c/O_c} \), and the shift \( (S_c) \) and penetration \( (P_c) \) factors are the real and imaginary parts of the logarithmic derivative of the outgoing wavefunction on the channel surface, respectively,

\[ r(\partial O_c/\partial r)|_{r=a_c} = S_c + iP_c. \quad (19) \]

In this context, it is convenient to represent the collision matrix in terms of the so-called \( A \)-matrix, which is defined by

\[ \left[ 1 - \mathcal{R}(S + iP) + \mathcal{R}^B \right]^{-1} R \right)_{c'c} \sum_{\lambda',\lambda} \gamma_{\lambda'c'}A_{\lambda\lambda}\gamma_{c\lambda}, \quad (20) \]

or, equivalently,

\[ (A^{-1})_{\lambda\lambda'} = \mathcal{E}(E)_{\lambda\lambda'} - i \sum_c \gamma_{\lambda c}\gamma_{c\lambda'c}P_c(E), \quad (21) \]

where \( \mathcal{E}(E) \) is the real part of \( A(E)^{-1} \),

\[ \mathcal{E}(E)_{\lambda\lambda'} = (E\lambda - E)J_{\lambda\lambda'} - \sum_c \gamma_{\lambda c} [S_c(E) - B_{\lambda c}] \gamma_{c\lambda}. \quad (22) \]

The collision matrix with this \( A \)-matrix reads

\[ U_{c'c} = \Omega_{c'} \left( \delta_{c'} + 2i\sqrt{P_c} \sum_{\lambda',\lambda} \gamma_{\lambda'c'}A_{\lambda\lambda}\gamma_{c\lambda} \sqrt{P_c} \right) \Omega_c. \quad (23) \]

Using the above Eqs.(21,22,23), one can thus construct the collision matrix with the general inner products of the basis states given in Eq.(9), and the basis no longer needs to be orthogonal. The values of the level-dependent boundary condition parameters \( B_{\lambda c} \) and the diagonal elements \( J_{\lambda\lambda} \) should then be determined, which will be discussed in the next section.
III. DETERMINATION OF THE BOUNDARY CONDITION PARAMETERS

So far, I have shown that releasing the orthogonality condition of the basis states allows the boundary condition parameters $B_{\lambda c}$ to depend on the level. This section describes how to utilize this additional freedom associated with the level-dependence to make the formal parameters coincide with the observed ones.

Consider first the observed pole-positions of resonances, $E_{\lambda}^{\text{obs}}$. The precise definition of the pole-position may be ambiguous, and I adopt the convention of Ref. [8] where $E_{\lambda}^{\text{obs}}$ are defined as the zeroes of the determinant of the real part of the inverse of the $A$-matrix, or, equivalently, the solutions of the secular equation

$$\det \tilde{E}(E) = 0. \quad (24)$$

The aim of equalizing the observed pole positions with the formal parameters

$$E_{\lambda}^{\text{obs}} = E_{\lambda} \quad (25)$$

can be achieved if one sets the boundary condition parameters $B_{\lambda c}$ to be the shift factor at $E = E_{\lambda}$,

$$B_{\lambda c} = S_c(E_{\lambda}). \quad (26)$$

This can be seen by simply noting that $\tilde{E}(E_{\lambda})_{\lambda \lambda'}$ vanishes if one inserts Eq.(26) into Eq.(22). That is, for any $\lambda$, the entire $\lambda$-th row of the matrix $\tilde{E}(E_{\lambda})$ vanishes, which in turn makes $E_{\lambda}$ the solution of Eq.(24). This proves that the formal parameter $E_{\lambda}$ is equal to the observed $E_{\lambda}^{\text{obs}}$.

From Eqs.(21, 22, 23), it is not difficult to see that the collision matrix $U$ is invariant under the following transformation,

$$J_{\lambda \lambda} \rightarrow J_{\lambda \lambda}^{\text{new}},$$

$$\gamma_{\lambda c} \rightarrow \gamma_{\lambda c}^{\text{new}} = \sqrt{\frac{J_{\lambda \lambda}^{\text{new}}}{J_{\lambda \lambda}}} \gamma_{\lambda c}. \quad (27)$$

The normalization of $\gamma_{\lambda c}$ is thus determined by the values of $J_{\lambda \lambda}$, which are not yet determined. The off-diagonal elements of $J$ are given in Eq.(9). A natural extrapolation to the diagonal cases would be to take the limit $E_{\lambda'} \rightarrow E_{\lambda}$ of the equation, which results in, with Eq.(26),

$$J_{\lambda \lambda} = 1 - \sum_c \gamma_{\lambda c}^2 \frac{dS_c(E)}{dE} \bigg|_{E=E_{\lambda}}. \quad (28)$$
Insertion of the above equation into Eq.(22) yields

$$
\bar{\mathcal{E}}(E)_{\lambda\lambda'} = \begin{cases} 
E_\lambda - E - \sum_c \gamma_{\lambda c}^2 [S_c(E) - S_{\lambda c} + (E_\lambda - E)S'_{\lambda c}], & \text{for } \lambda' = \lambda, \\
- \sum_c \gamma_{\lambda c} \gamma_{\lambda' c} \left[ S_c(E) + \frac{(E_{\lambda' c} - E_{\lambda c})S_{\lambda c} - (E_{\lambda c} - E_{\lambda' c})S_{\lambda' c}}{E_{\lambda c} - E_{\lambda' c}} \right], & \text{for } \lambda' \neq \lambda,
\end{cases}
$$

(29)

where \( S_{\lambda c} \equiv S_c(E_\lambda) \).

I now consider the consequence of Eq.(28) on the observed widths of the resonances, which are usually defined with the “one-level approximation”. With this approximation, the collision matrix reads (LT)

$$
U_{c'c} \simeq \Omega_{c'} \left( \delta_{c'c} + i \frac{\sqrt{\Gamma_{\lambda c'}} \sqrt{\Gamma_{\lambda c}}}{E_\lambda - E - \frac{1}{2} \sum_{c''} \Gamma_{\lambda c''}} \right) \Omega_c.
$$

(30)

And the observed reduced widths \( \gamma_{\lambda c}^{\text{obs}} \) are defined by

$$
\Gamma_{\lambda c}(E_\lambda) = 2P_c(E_\lambda) \gamma_{\lambda c}^{\text{obs}}^2.
$$

(31)

On the other hand, if one inserts Eq.(29) into Eq.(23) and then takes the one-level approximation, the widths are given as

$$
\Gamma_{\lambda c}(E) = 2P_c(E) \gamma_{\lambda c}^2.
$$

(32)

Thus, the formal parameters for the reduced widths of the present formalism are the same as the observed ones,

$$
\gamma_{\lambda c}^{\text{obs}} = \gamma_{\lambda c}.
$$

(33)

This simple relation should be compared with

$$
\gamma_{\lambda c}^{\text{obs}} = \frac{\gamma_{\lambda c}}{\sqrt{1 + \sum_{c'} \hat{\gamma}_{\lambda c'}^2 S'_{\lambda c'}}},
$$

(34)

where \( \hat{\gamma}_{\lambda c} \) are the formal reduced width parameters in the conventional R-matrix theory.

I note that the alternate parametrization obtained by Brune [8] has the same off-diagonal terms of \( A^{-1} \) with Eq.(29). But the diagonal elements are different, and the values corresponding to \( J_{\lambda\lambda} \) used in Ref. [8] are 1 instead of those from Eq.(28). Consequently, Brune’s reduced width parameters are the same as those of the conventional R-matrix theory, \( \hat{\gamma}_{\lambda c} \), which are subject to the nonlinear relation described in Eq.(34).
IV. $^{12}$C + $p$ ELASTIC SCATTERING

To demonstrate the performance of the new R-matrix theory developed here, I developed a simple Mathematica code [9] which calculates the collision matrices and the differential cross-sections of nuclear reactions based on Eqs. (26,28,29). Since there is no need for conversions between the formal and the observed parameters, a substantial simplification could be achieved. The code is then applied to describe $^{12}$C + $p$ elastic scattering, for which accurate experimental data are available [10]. For a detailed discussion of the process in connection with the (conventional) R-matrix theory, see Ref. [4].

At low energy, this process is dominated by the three low-lying resonances of $^{13}$N: $J^\pi = 1/2^+$ at 0.421 MeV, $3/2^-$ at 1.559 MeV, and $5/2^+$ at 1.604 MeV, where the resonance energies are the center-of-mass energies of the $^{12}$C+$p$ system. I use the values of the observed parameters given in Ref. [4] for my formal parameters, which are listed in Table 1. There is little dependence on the channel radius, which was chosen as $a_c = 5$ fm for calculations.

For two particular center-of-mass angles, $\theta = 89.1^\circ$ and $146.9^\circ$, the resulting differential cross-sections with respect to the proton energy in the lab frame are drawn in Fig. 1. The figure shows that the code with the newly developed R-matrix theory reproduces nicely the experimental data [10].

In Table 2, the parameters of the conventional R-matrix theory are compared with those of the present theory for the first $J^\pi = 1/2^+$ resonance in $^{12}$C + $p$, varying the channel radius from 4 fm to 7 fm. The observed position and the width of the resonance are set to $E_R = 0.42$ MeV and $\Gamma_R = 32$ keV [4]. While the $E_1$ of the present theory is the same as the input value for the observed pole position $E_R$ by construction, the formal parameter

| $J^\pi$ | $E_\lambda$ [MeV] | $\Gamma_{\lambda c}$ [keV] | $\gamma_{\lambda c}^2$ [MeV] |
|---------|------------------|-----------------|-----------------|
| $1/2^+$ | 0.427            | 32.5            | 0.569           |
| $3/2^-$ | 1.559            | 51.4            | 0.0835          |
| $5/2^+$ | 1.604            | 48.1            | 0.414           |
FIG. 1: $^{12}$C + $p$ elastic scattering cross-sections (in mb/sr) with respect to the incident proton energy in the lab frame for the center-of-mass angle $\theta = 89.1^\circ$ (left panel) and $146.9^\circ$ (right panel). The experimental data are from Ref. [10] (closed circles).

TABLE II: R-matrix parameters for the first $1/2^+$ resonance ($E_R = 0.42$ MeV and $\Gamma_R = 32$ keV) in $^{12}$C + $p$ elastic scattering (in MeV). The parameters of the conventional R-matrix theory are from Table 10 of Ref. [4].

| $a_c$ [fm] | 4    | 5    | 6    | 7    |
|------------|------|------|------|------|
| $\gamma_1^2$ observed (Ref. [4]) | 1.089 | 0.592 | 0.353 | 0.227 |
| $\gamma_1^2$ formal (Ref. [4]) | 3.083 | 1.157 | 0.569 | 0.323 |
| $\gamma_1^2$ (this work) | 1.087 | 0.591 | 0.353 | 0.226 |
| $E_1$ formal (Ref. [4]) | $-2.152$ | $-0.614$ | $-0.110$ | $0.113$ |
| $E_1$ (this work) | 0.42 | 0.42 | 0.42 | 0.42 |

$E_1$, formal in the conventional theory has a strong dependence on the channel radius. The table also shows that $\gamma_1^2$ of the present theory agrees well with $\gamma_1^2$, observed in Ref. [4], which is the intended outcome of this work. The table also shows that these observed parameters are less dependent on $a_c$ than the formal parameters.

V. DISCUSSIONS

The conventional R-matrix theory has the problem of having formal parameters that are different from the observed ones, and thus requiring non-trivial conversions between the two sets of parameters. As discussed in the text, this drawback is a consequence of requiring orthogonality of the basis states for the Hilbert space in the internal region of the R-matrix.
theory. However, the orthogonality is not a necessary condition for the basis. By exploiting
the additional freedom that can be achieved when the orthogonality condition is released,
I have developed a new R-matrix theory that has no distinction between the two sets of
parameters.

In this theory, the boundary condition parameters are allowed to be level-dependent and
adjusted to make the formal parameters $E_\lambda$ identical to the observed pole positions. That
is, I assigned $B_{\lambda c}$ to the shift factor of channel $c$ at $E_\lambda$, see Eq. (26), which makes the secular
equation Eq. (24) vanish at that energy. Recalling that the observed pole-positions $E_{\lambda}^{\text{obs}}$
are defined to be the zeroes of the secular equation, one sees that the imposed boundary
condition leads to $E_\lambda = E_{\lambda}^{\text{obs}}$.

In addition, there is another freedom in the normalization of the diagonal elements $J_{\lambda\lambda}$,
which correspond to the square of the norm of the basis states. By selecting the normalization
factor of the basis states as given in Eq. (28), which can be viewed as a natural extrapolation
of the off-diagonal elements, I could derive the formal reduced width parameters to be the
same as the observed ones as well, $\gamma_{\lambda c}^{\text{obs}} = \gamma_{\lambda c}$. As a result, there are no formal parameters
which are different from the observed ones in the present formalism.

As a demonstration, I tested a computation code based on the developed R-matrix theory,
where the trial case was the elastic scattering of protons on $^{12}$C. The code required only the
resonance data as input and did not invoke any transformations of parameters. The code
was able to reproduce the experimental differential cross sections quite well.

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