Analysing an Optimal Angle in Basketball Free Throw

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Abstract. Basketball is a sport, played worldwide by people of all ages, from young to old. The most important skill that a football player should have is shooting. Shooting the ball into the hoop involves projectile motion. The ability of a player to shoot will determine the scores of his/her team. The angle and initial velocity taken during the shooting, plays a vital role, so that a perfect shooting could be achieved. This work has been conducted to determine the optimal throwing angle and initial velocity that a player should take in order to get the best shooting. The relationship of these factors were investigated: the throwing angle and player’s height, initial velocity with the player’s height, as well as the throwing angles versus time taken for the ball to reach the hoop. Our focus is to maximize the height of the ball, before it is thrown. Newton’s law and the concept of projectile motion were applied using a calculus-based model. Relationship between the player’s height with the initial velocity, optimal throwing angle and time taken for the ball to reach the hoop were discussed. Perfect optimal release angle were determined for thirty data of NBA players. It shows that the player’s height is inversely proportional to the initial velocity and the optimal throwing angle. The obtained results also concluded that the optimal throwing angle is directly proportional to the time taken for a ball to reach its maximum height.

Keywords: Basketball, free throw, projectile motion

1. Introduction

There are a lot of our daily life activities which involve projectile motions. However, it is most commonly encountered in sports, such as gymnastic and extreme sports, like skateboarding, snowboarding, football, BMX riding. One of the sports which involved numerous numbers of projectile motions is basketball, especially during shooting. The projectile motion will cause the ball to swish through the basket, regardless of the direction it is being shoot from forward or upward.

Basketball players usually face problems during free throw. Free throws or foul shots are throws awarded after a foul on the shooter by the opposing team. This is analogous to penalty shots in other team sports. Actually, it should be very easy for a player to score during free throw, as there is lesser pressure from the opposing team. However, many players still fail, and this is mostly caused by their failure in estimating the accurate throwing angle of the ball. Most of them assume that, the optimal throwing angle is 45 degrees, while it is actually not. Both optimal throwing angle and initial velocity are supposed to be different for different players, depending on their height [1]. This should be figured out by the players during their training.
The data used in this research were collected from international athletes (NBA) since they are more accessible and reliable. It will be later be applied to Malaysian basketball athletes to improve the qualities of their performances. The main purpose of this research is to determine the optimal throwing angle in the free throw. The objectives of this study are to determine the best release angle in the free throw, obtain the impact of angle due to the high basketball player, calculate the initial velocity and its relationship between the heights of the basketball players and also to find the effectiveness of the optimal release angle to the time taken for the ball to reach the basketball hoop and the maximum height of the ball can reach.

2. Literature Review
Projectile motion is the most frequent motion in many types of sports. Basketball is one of the sports, where this type of motion is very significant, especially in shooting. The shooting skill of a basketball player is measured according to many criteria, such as shooting technique, throwing zone, and also the number of scores. In basketball competition, the free throw problem was the most famous issue among players. Free throw was being considered as an important shot in the basketball where 20% from the point are mostly scored by the free throw [2]. Free throw shooting in basketball requires special skills, since the offensive player need to control every single thing. No movements to counteract, no reactions to the defence, and the distance of shooting is also fixed. The player will only need to constantly shoot the ball, for a certain period of time.

Shooting free throws in basketball is actually a simple element of the sport that anyone can practise to do and may perform as competent as professional athletes. This is because, for this shot, players will only need to throw a nine-inch wide ball weighing around twenty ounces into an eighteen-inch diameter rim. In many competitions, the points scored during the free throw may considerably change the results. The team which is able get a higher score during free throw usually will win the game. The shooting of the free throw actually should be easier since the player is working alone, without any defence or pressure from the opposing team [3]. Yet, the lack of skills especially in estimating the right throwing angle makes most of the players fail to score.

This main objective of our research is to find out the how these factors are related: the throwing angle and player’s height, initial velocity with the player’s height, as well as the throwing angles versus time taken for the ball to reach the hoop.

3. Methodology
Based on the NBA data, only the height of the players will be used to complete this research. It also will use the actual distance for the free throw and the height of the basketball hoop from the ground. All of the players always thought that the optimal release angle for free throw was 45 degrees where it was not the correct answers [1]. However, for this research, it will prove and calculate the actual optimal release angle form differences of the player’s height. To get the perfect optimal release angle, this research will focus on calculating the initial velocity. The time is taken for the ball to reach the basketball hoop and the maximum height of the ball can reach. All details about the research step will be explained in figure 1.

3.1. Determination of the projectile motion concept
The projectile motion happened when anything thrown with some initial velocity, it allowed movement under the action of gravity alone without using any engine or fuel to move it. The object moves in both horizontal and vertical directions simultaneously under the same acceleration, is also called as projectile motion [4, 5]. Projectile motion only happened when there is one force applied at the beginning of the trajectory, after which the only interface is gravity.

Projectile motion was first introduced by Galileo Galilei, a renown Italian astronomer, physicist, and engineer. He said that projectile motion has both components, horizontal and vertical. These components are independent to each other, occurred simultaneously and perpendicular to each other. Figure 3 below shows the component of the projectile motion [6].

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Figure 1. Flowchart of the research step

Step 1: Determine the projectile motion concept
Step 2: Collect the real data
Step 3: Construct computer programming code
Step 4: Calculate the initial velocity
Step 5: Calculate the perfect optimal release angle
Step 6: Calculate the time taken for the ball to reach a hoop
Step 7: Calculate the maximum height of the ball
Step 8: Result analysis and comparison

Figure 2. Trajectory of the ball from the player to the hoop

Figure 3. Horizontal and Vertical component

3.1.1. Horizontal Motion of Projectile. The horizontal motion of the projectile is the result of any object in the motion to remain in motion at the constant velocity. Without the horizontal forces, a projectile remains in motion with constants horizontal velocity. To keep a projectile moving horizontally, the
horizontal forces are not required. The only force acting to a projectile is gravity. The speed for the horizontal direction is and this speed will not change.

3.1.2. Vertical Motion of Projectile. Vertical motion of the projectile is controlled by the force of gravity. This will make unbalanced force acting on the ball then the ball will accelerate downwards with the acceleration of 9.8 m/s$^2$. Vertical motion of the projectile motion is referred as the movement for the object against the gravitational pull. This motion is perpendicular to the straight or flat surface. The motion also can be straight upward and downwards, but the perfect vertical motion is not usually in a straight-line path. When the object is in a vertical motion, it will have a certain amount of the velocity in it where it is depending upon the speed and position of the objects.

![Figure 4. The concept diagram of the vertical motion](image)

Figure 4 explains that when the sphere reached the maximum height, the motion of the sphere will be stopped, and it will start traveling in the downward path. The maximum height at the vertical motion of the sphere and the early height is usually zero. The time taken for the sphere to reach the maximum of the height is equal to the time taken for the sphere to fall downwards.

Based on Figure 2, it represents the concept of the free throw that will be used for this research. The calculation is based on the height of the athletes. Basketball players need to input their height in the GUI but the value of the distance from the hoop to the players and the height of the basketball hoop is set with suitable values in the free throw. Table 1 below shows the suitable value that is used in the free throw.

**Table 1.** List of variables used in this paper

| Variable                                           | Constant         |
|----------------------------------------------------|------------------|
| Distance the free throw line from the hoop to the players, $X$ | 4.572 meters     |
| The height of the basketball hoop, $H$              | 3.048 meters     |
| The acceleration due to gravity, $g$                | 9.81 meters /second$^2$ |

3.2. Collection of the real data

Based on data from National Basketball Association (NBA), the height of basketball players is listed as shown in table 2.
Table 2. List of basketball player with their corresponding heights

| Basketball players name's | Height of the player (meter) |
|---------------------------|-----------------------------|
| Isaiah Thomas             | 1.727                       |
| Kemba Walker              | 1.8288                      |
| Derrick Rose              | 1.854                       |
| Dion Waiters              | 1.905                       |
| Marcus Smart              | 1.905                       |
| Romeo Langford            | 1.93                        |
| Shake Milton              | 1.956                       |
| David Nwaba               | 1.956                       |
| Josh Richardson           | 1.956                       |
| Spencer Dinwiddie         | 1.956                       |
| Draymond Green            | 1.956                       |
| PJ Tucker                  | 1.981                       |
| Jaylen Brown              | 1.981                       |
| Caris LeVert              | 1.981                       |
| Justice Winslow           | 1.981                       |

| Basketball players name's | Height of the player (meter) |
|---------------------------|-----------------------------|
| Luka Doncic               | 1.981                       |
| Gordon Hayward            | 2.007                       |
| Tobias Harris             | 2.007                       |
| Jimmy Butler              | 2.007                       |
| De’Andre Hunter           | 2.032                       |
| Semi Ojeleye              | 2.032                       |
| Kevin Knox                | 2.032                       |
| John Collins              | 2.057                       |
| Dwight Howard             | 2.057                       |
| Enes Kanter               | 2.083                       |
| Kevin Durant              | 2.083                       |
| Henry Ellenson            | 2.083                       |
| Mitchell Robinson         | 2.136                       |
| Kristaps Porzingis        | 2.21                        |
| Boban Marjanovic          | 2.235                       |

3.3. The calculation of initial velocity
The equation for this initial velocity is described below:

Horizontal motion: $x = (v_0 \cos \theta) \cdot (t_1 + t_2)$

Vertical motion: $v_0 \sin \theta = gt_1$

To get the equation displacement of the ball in the vertical motion, it will use the equation (2):

$y = y_0 + v_{0,y} \cdot t - \frac{1}{2} gt^2$

$h’ = y - v_{0,y} \sin \theta (t_1 + t_2) - \frac{1}{2} g (t_1 + t_2)^2$

Then, $h’$ is simplified and reduced to (3) as below:

$h’ = \frac{g}{2} (t_1 + t_2) [2t_1 - (t_1 + t_2)]$

Substituting equation (1) and equation (2):

$h’ = \frac{g}{2} \left[ \frac{x}{v_0 \cos \theta} \right] \left[ 2t_1 - \frac{x}{v_0 \cos \theta} \right]$

$v_0 = \sqrt{\frac{gx^2 (\tan^2 \theta + 1)}{2 (x \tan \theta - h')}}$

To solve the problem, $v_0$ needs to be differentiated with respect to $\theta$. Simplify by using trigonometric identities $\tan^2 \theta + 1 = \sec^2 \theta$ and chain rule applied:

$df(u) = \frac{df}{du} \cdot \frac{du}{dx} \cdot \frac{dv_0}{d\theta} \left[ \frac{gx^2 \sec^2 \theta}{2x \tan \theta - 2h'} \right]^L = x \tan^2 \theta - 2h' \tan \theta - x$
The derivative equation will be equalled to zero:

$$x \tan^2 \theta - 2 h' \tan \theta - x = 0$$  \hspace{1cm} (5)

Solving using quadratic formula, then the optimal release angle as:

$$\theta = \tan^{-1} \left( \frac{h' + \sqrt{h'^2 + x^2}}{x} \right)$$  \hspace{1cm} (6)

Substituting the equation (6) back into equation (4), the result is:

$$v_0 = g \left( h' + \sqrt{h'^2 + x^2} \right)^{\frac{1}{2}}$$

where $h' = H - h$, $g = 9.81$ meters/second$^2$, $h'$ = height of the ball from the players to the hoop

3.4. The calculation of perfect optimal release angle
The equation as described in equation (6):

$$\theta = \tan^{-1} \left( \frac{h' + \sqrt{h'^2 + x^2}}{x} \right)$$

3.5. The calculation of time taken for the ball to reach the basket hoop
The calculation for this step is shown below:

$$x = \left( v_0 \cos \theta \right) (t_i + t)$$

$$t = \frac{x}{v_0 \cos \theta}$$

where $t = t_i + t_z$

3.6. The calculation of maximum height of the ball can reach
Maximum height of the ball can reach will be assumed as unknown $M$,

$$M = \frac{v_0^2 \sin^2 \theta}{2g}$$

4. Results and Discussion
The results of this study are the calculated value of initial velocity, optimal release angle, time taken for the ball to reach the basketball hoop and maximum height of the ball can reach. They are presented in table 3.

4.1. Relationship between the Heights of the Players with the Initial Velocity
The height of the player is inversely proportional to the initial velocity which means; the taller a player is; the lower the initial velocity will be [1]. It has been shown in table 3 and figure 5. This is because the taller the player is, their distance from the hoop will be shorter. This explains the decrease of their initial velocity, since as the motion begins the initial velocity start to be counted.
Table 3. Calculation of initial velocity, optimal release angle, time taken for the ball to reach the basketball hoop and maximum height of the ball can reach

| No | Basketball players name’s | Initial velocity (Meter/second)² | Optimal release angle (Degree) | Time taken for the ball to reach the basketball hoop (Second) | Maximum height of the ball can reach (Meters) |
|----|--------------------------|----------------------------------|---------------------------------|-------------------------------------------------|---------------------------------|
| 1  | Isaiah Thomas            | 1.321                            | 7.7230                          | 53.0579                                         | 0.9850                          | 1.9419                          |
| 2  | Kemba Walker             | 1.2192                           | 7.6406                          | 52.4657                                         | 0.9822                          | 1.8711                          |
| 3  | Derrick Rose             | 1.194                            | 7.6203                          | 52.3181                                         | 0.9815                          | 1.8538                          |
| 4  | Dion Waiters             | 1.143                            | 7.5792                          | 52.0181                                         | 0.9802                          | 1.8190                          |
| 5  | Marcus Smart             | 1.143                            | 7.5792                          | 52.0181                                         | 0.9802                          | 1.8190                          |
| 6  | Romeo Langford           | 1.118                            | 7.5591                          | 51.8705                                         | 0.9796                          | 1.8020                          |
| 7  | Shake Milton             | 1.092                            | 7.5383                          | 51.7166                                         | 0.9789                          | 1.7846                          |
| 8  | David Nwaba              | 1.092                            | 7.5383                          | 51.7166                                         | 0.9789                          | 1.7846                          |
| 9  | Josh Richardson          | 1.092                            | 7.5383                          | 51.7166                                         | 0.9789                          | 1.7846                          |
| 10 | Spencer Dinwiddie        | 1.092                            | 7.5383                          | 51.7166                                         | 0.9789                          | 1.7846                          |
| 11 | Draymond Green           | 1.092                            | 7.5383                          | 51.7166                                         | 0.9789                          | 1.7846                          |
| 12 | PJ Tucker                | 1.092                            | 7.5383                          | 51.7166                                         | 0.9789                          | 1.7846                          |
| 13 | Jaylen Brown             | 1.067                            | 7.5183                          | 51.5682                                         | 0.9783                          | 1.7679                          |
| 14 | Caris LeVert             | 1.067                            | 7.5183                          | 51.5682                                         | 0.9783                          | 1.7679                          |
| 15 | Justice Winslow          | 1.067                            | 7.5183                          | 51.5682                                         | 0.9783                          | 1.7679                          |
| 16 | Luka Doncic              | 1.067                            | 7.5183                          | 51.5682                                         | 0.9783                          | 1.7679                          |
| 17 | Gordon Hayward           | 1.041                            | 7.4974                          | 51.4135                                         | 0.9777                          | 1.7505                          |
| 18 | Tobias Harris            | 1.041                            | 7.4974                          | 51.4135                                         | 0.9777                          | 1.7505                          |
| 19 | Jimmy Butler             | 1.041                            | 7.4974                          | 51.4135                                         | 0.9777                          | 1.7505                          |
| 20 | De’ Andre Hunter         | 1.016                            | 7.4775                          | 51.2644                                         | 0.9772                          | 1.7340                          |
| 21 | Semi Ojeleye             | 1.016                            | 7.4775                          | 51.2644                                         | 0.9772                          | 1.7340                          |
| 22 | Kevin Knox               | 1.016                            | 7.4775                          | 51.2644                                         | 0.9772                          | 1.7340                          |
| 23 | John Collins             | 0.991                            | 7.4575                          | 51.1150                                         | 0.9766                          | 1.7175                          |
| 24 | Dwight Howard            | 0.991                            | 7.4575                          | 51.1150                                         | 0.9766                          | 1.7175                          |
| 25 | Enes Kanter              | 0.965                            | 7.4368                          | 50.9592                                         | 0.9760                          | 1.7005                          |
| 26 | Kevin Durant             | 0.965                            | 7.4368                          | 50.9592                                         | 0.9760                          | 1.7005                          |
| 27 | Henry Ellenson           | 0.965                            | 7.4368                          | 50.9592                                         | 0.9760                          | 1.7005                          |
| 28 | Mitchell Robinson        | 0.9144                           | 7.3966                          | 50.6550                                         | 0.9750                          | 1.6677                          |
| 29 | Kristaps Porzingis       | 0.838                            | 7.3362                          | 50.1932                                         | 0.9735                          | 1.6188                          |
| 30 | Boban Marjanovic         | 0.813                            | 7.3165                          | 50.0415                                         | 0.9730                          | 1.6030                          |
4.2. Relationship between the Heights of the Players with the Optimal Release Angle
Changjan and Mueanploy [1] described those 45 degrees is an optimal throwing angle which an object can travel from the origin to the maximum horizontal range with maximum velocity [1]. But our work proves that the optimal release angle was not 45 degree. Instead, this throwing angle will depend on the height of the player. The taller a player is, the lower the optimal throwing angle will be. This also means that the optimal throwing angles depend on the height of the ball before it is being shot. The relationship between the heights of the players with the optimal release angle is displayed in figure 6.

4.3. Relationship between the Optimal Release Angles with the Time Taken for the Ball to Reach the Basketball Hoop
As shown in figure 7, the higher the value of the optimal throwing angle, the lower the time taken for the ball to reach the hoop. Thus, the shorter players may have some advantage, since they will have a higher value of throwing angle, consequently they will need a shorter time to shoot the ball through the hoop.
Figure 7. Relationships between the optimal release angles with the time taken for the ball to reach the basketball hoop

4.4. Development of Graphical User Interface (GUI)

The main aim of inventing this calculator is to help the basketball players to score their game during the free throw. The players will just need to key in their height and the estimated values for each variable such as the height of the ball from the basketball player to the basketball hoop, initial velocity, optimal throwing angle, maximum height that the ball can reach and time taken for the ball to reach the hoop will be calculated. This obtained information may then be used to plan their playing strategies using GUI application as shown in figure 8.

Figure 8. Basketball free throw calculator

5. Conclusion

This research studies the projectile motion which plays a very important role for a perfect shooting in basketball. GUI was developed and the data used are based on the height of thirty NBA players. The results reveal the following relationships: Firstly, the player’s height is inversely proportional to the initial velocity and the optimal throwing angle. The taller players will have a lower initial velocity and lower optimal throwing angle and vice versa. Secondly, the optimal throwing angle is directly proportional to the time taken for a ball to reach its maximum height.

For future works, it should be improved by including the air resistance as one of the parameters to investigate. In this current work, the air resistance is assumed negligible. Other suggestion is to take into account the forces due to spinning motion of the ball and the kinetic energy that the players may use during the games. By taking these factors into consideration, the obtained results will be more reliable to be used as references for real life situations.
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