A Computational Procedure Using Robust Ranking Method to Formulate and Solve Fuzzy DEA Models

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Abstract. In some real-world applications, performance assessment often has to be considered under uncertainty with fuzzy input and output. Data envelopment analysis (DEA) is a new and effective approach to measuring the relative efficiency of decision-making units concerned with performance evaluation. Recently, DEA has studied with fuzzy input and output. In this paper, we present a computational procedure using a robust ranking method to formulate and solve fuzzy DEA models. Based on this approach, we convert the fuzzy DEA input and output from fuzzy form to crisp. Also, a case study has been presented as in [6] to assess the performance of eight manufacturing enterprises with fuzzy data.

1. Introduction

The performance evaluation is one of the basic tools for measuring the quality of functional processes. Therefore, it has taken the attention of many researchers in quality and business affairs, because of its great importance in directing the organizational activity towards the horizon of success.

The performance appraisal was defined as "The method by which the actual performance level is measured to obtain sufficient information and analyze it, so it is a tool for measuring human effort at work and diagnosing the level of quality in this performance" [1].

The performance evaluation is a periodic procedure in which the performance of the employee is examined and measured, in order to identify his strengths and weaknesses and to find ways to improve his performance and raise the level of certification for him, and this is done through the use of many models for performance evaluation, which are quantifiable and give accurate indicators for performance [2].

Many performance evaluation researchers or organization owners differ in defining specific steps for the performance evaluation process, due to the difference in the nature of work, such as the organization's philosophy, environment, and type of work, or because of the size of job responsibility, which varies between one job and another. But most of these opinions agree that the performance evaluation passes the following steps [3]:

- Monitor performance problems.
- Examination and search for the causes of these problems.
- Determine the best way to overcome them.
- Follow up on the implementation of treatment methods.
- Business development, continuous improvement, and development.

The performance evaluation is the guiding guide that gives a complete perception about the strengths and weaknesses of the employees in their job positions that they have been assigned to accomplish, so
it is considered a tool for measuring the quality of the employee’s productivity during a certain period and indicating the extent of his contribution to achieving the goals of the organization. Therefore, the performance evaluation is characterized by the following features [4].

- It helps the boss to measure the employees' performance accurately
- It helps to optimize energies and resources.
- Helps reduce waste and wastes in work, which helps reduce labor costs
- It makes the outputs high quality and flawless.
- It is considered a guide to leadership in making appropriate decisions.
- It provides leadership with detailed and accurate performance reports in a record period.
- It reduces the frequency of the same mistakes and becomes a useful guide from previous experiences.

The methods of performance evaluation are continuously updating and continue developing as a result of the expansion of studies on this topic, and the organizations' desire to achieve efficiency in all parts of the organization, and to achieve competitive advantage by raising the level of the job performance of the human resource within the organization, so modern organizations rely on performance evaluation on standards Essential in the evaluation process such as effectiveness, efficiency, profits, competitive advantage, quality of job life, creativity, innovation and quality [5]. On the other hand, Data envelopment analysis (DEA) has been recently used to conduct the performance assessment under certainty and uncertainty. Many researchers studied different algorithms and solution methods for deterministic and fuzzy DEA model. For more details we refer to [6], [7],[8],[9],[10],[11].

In this paper, we present a computational approach to formulate and solve the fuzzy dea (FDEA) model using the robust ranking method.

2. Preliminaries

In this section, some basic concepts of fuzzy numbers are given. This material can be found in [12], [13], [14], [15] and [16].

2.1 Basic Definitions

Definition 1 "Let W be a classical set of objects. Then, $\tilde{B} = \{(x, \mu_{\tilde{B}}(x)) : x \in W\}$, where $\mu_{\tilde{B}} : W \rightarrow [0,1]$ is called a fuzzy set in W. Also, $\mu_{\tilde{B}}$ is known as the membership function".

Definition 2 "Let $\tilde{B}$ be a fuzzy set in W and $\beta \in [0,1], \beta \in \mathbb{R}$. Then, $B^\beta = \{x \in W : \mu_{\tilde{B}}(x) \geq \beta\}$ is called an $\beta$ -level set or $\beta$ -cut form of $\tilde{B}$".

2.2 Types of Fuzzy Numbers

Two well-known fuzzy numbers are described as follows.

2.2.1 Triangular Fuzzy Number

Definition 3 " $\tilde{B} = (b_1, b_2, b_3)$, is said to be a triangular fuzzy number if its membership function, $\mu_{\tilde{B}}(x)$, is given by"

$$
\mu_{\tilde{B}}(x) = \begin{cases} 
0 & , x < b_1 \\
\frac{x - b_1}{(b_2 - b_1)} & , b_1 \leq x \leq b_2 \\
\frac{b_3 - x}{(b_3 - b_2)} & , b_2 \leq x \leq b_3 \\
0 & , x > b_3 
\end{cases}
$$
Definition 4 "Let $\tilde{A} = (a, b, c)$ be a triangular fuzzy number”. “Then, its $\beta$-cut $A^\beta$ is defined as follows”:

$A^\beta = [a + (b - a)\beta, c - (c - b)\beta], 0 \leq \beta \leq 1.$

2.2.2 Trapezoidal Fuzzy Number

Definition 5 “$\tilde{B} = (b_1, b_2, b_3, b_4)$, is said to be a trapezoidal fuzzy number if its membership function, $\mu_{\tilde{B}}(x)$, is given by”

$$
\mu_{\tilde{B}}(x) = \begin{cases} 
\frac{x - b_1}{b_2 - b_1}, & b_1 \leq x < b_2 \\
1, & b_2 \leq x \leq b_3 \\
\frac{x - b_4}{b_3 - b_4}, & b_3 < x \leq b_4 \\
0, & \text{otherwise}
\end{cases}
$$

Definition 6 "Let $\tilde{A} = (a, b, c, d)$ be a trapezoidal fuzzy number. Then, its $\beta$-cut $A^\beta$ is as follows”:

$A^\beta = [a + (b - a)\beta, d - (d - c)\beta], 0 \leq \beta \leq 1$

2.3 Arithmetical Operation for Trapezoidal Fuzzy Numbers

i. "Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers. Then, $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$.”

ii. "Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers. Then, $\tilde{A}_1 \Theta \tilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$.”

iii. "Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers. Then, $\tilde{A}_1 \otimes \tilde{A}_2 = (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2)$.”

iv. "Let $\tilde{A} = (a_1, b_1, c_1, d_1)$. Then,

$$
y\tilde{A} = \begin{cases} 
(ya, yb, yc, yd) & y \geq 0 \\
(yd, yc, yb, ya) & y \leq 0
\end{cases}
$$

3. Robust Ranking Method

The robust ranking method is one of the important methods used to solve different types of optimization problems such as assignment problems, transportation problems, and linear programming problems, and so on. See for e.g. [13], [14], [15], [16], [17], and [18].

If $\tilde{x}$ be a fuzzy number, then $R(\tilde{x})$ represents the robust ranking approach for:

1. Trapezoidal case:

$$R(\tilde{x}) = \int_0^1 0.5(s^u_d, \tilde{s}^u_d)d\alpha;$$

where $(s^u_d, \tilde{s}^u_d) = [(x_2 - x_1)\alpha + x_1, x_4 - (x_4 - x_3)\alpha]$ represents the $\alpha$-cut for the trapezoidal fuzzy number $\tilde{x} = (x_1, x_2, x_3, x_4)$. Then $R(x_1, x_2, x_3, x_4) = \int_0^1 0.5[(x_2 - x_1)\alpha + x_1, x_4 - (x_4 - x_3)\alpha] d\alpha$.

2. Triangular case:

$$R(\tilde{x}) = \int_0^1 0.5(s^u_d, \tilde{s}^u_d)d\alpha;$$

where $(s^u_d, \tilde{s}^u_d) = [(x_2 - x_1)\alpha + x_1, x_3 - (x_3 - x_2)\alpha]$ represents the $\alpha$-cut for the trapezoidal fuzzy number $\tilde{x} = (x_1, x_2, x_3)$. Then $R(x_1, x_2, x_3) = \int_0^1 0.5[(x_2 - x_1)\alpha + x_1, x_3 - (x_3 - x_2)\alpha] d\alpha$.

4. The formulation of fuzzy data envelopment analysis (FDEA) model
The most well-known formulation of CCR model is based on linear programming formulation. For more information about new linear programming formulation we refer to [19],[20],[21],[22],[23] , and [24]. In this section, we will present the formulation of (FDEA) model as follows.

4.1 Mathematical model

(FDEA) model can be formulated in [6] and [7] as follows:

\[ \text{Max } Z(u,v) = \frac{v^T \tilde{y}_0}{u^T \tilde{x}_0} \]

s. t. \( V^T \tilde{y}_j \leq u^T \tilde{x}_j, j = 1,2, \ldots, n, \) and \( u \geq 0, \ v \geq 0 \)

This model is equivalent to the following linear model.

\[ \text{Max } Z(u,v) = V^T \tilde{y}_0 \]

s. t. \( V^T \tilde{y}_j \leq u^T \tilde{x}_j, j = 1,2, \ldots, n, \ u^T \tilde{x}_0 = 1, \) and \( u \geq 0, \ v \geq 0 \)

4.2 Computational procedure for solving (FDEA)

Now, we present the following computational procedure for solving (FDEA) model.

Step 1-Based on the input and the output of the model, form the fuzzy linear programming model.

Step 2- Transform all fuzzy coefficients in the fuzzy linear programming model into crisp coefficients using the robust ranking method.

Step 3- Form the crisp linear programming model.

Step 4- Find the optimal solution of the crisp model using Win QSB solver.

5. Case study

In this section, we present an example to demonstrate a robust ranking approach.

This example is taken from [6] "where eight manufacturing enterprises (DMUs) are to be evaluated in terms of two inputs and two outputs".

Solution:

Step 1. Form the linear programming model based on the data in [6] as follows:

For the first manufacturing enterprise:

\[ \text{Max } U(u,v) = (2120, 2170, 2210)u_1 + 1870u_2, \ S.t. \]

\[ \begin{align*}
(14500, 14790, 14860) & \cdot v_1 + (3.1, 4.1, 4.9) \cdot v_2 = 1 \\
(1420, 1460, 1500) & \cdot u_1 + 1340u_2 \\
(12470, 12720, 12790) & \cdot v_1 + (1.2, 2.1, 3.0) \cdot v_2 \leq 1 \\
(2510, 2570, 2610) & \cdot u_1 + 2360u_2 \\
(17900, 18260, 18400) & \cdot v_1 + (3.3, 4.3, 5.0) \cdot v_2 \leq 1 \\
(2300, 2350, 2400) & \cdot u_1 + 2020u_2 \\
(14970, 15270, 15400) & \cdot v_1 + (2.7, 3.7, 4.6) \cdot v_2 \leq 1 \\
(1480, 1520, 1560) & \cdot u_1 + 1550u_2 \\
(13980, 14260, 14330) & \cdot v_1 + (1.0, 1.8, 2.7) \cdot v_2 \leq 1 \\
(1990, 2030, 2100) & \cdot u_1 + 1760u_2 \\
(14030, 14310, 14400) & \cdot v_1 + (1.6, 2.6, 3.6) \cdot v_2 \leq 1 \\
(2200, 2260, 2300) & \cdot u_1 + 1980u_2 \\
(16540, 16870, 17000) & \cdot v_1 + (2.4, 3.4, 4.4) \cdot v_2 \leq 1 \\
(2400, 2460, 2520) & \cdot u_1 + 2250u_2 \\
(17600, 17960, 18100) & \cdot v_1 + (2.6, 3.6, 4.6) \cdot v_2 \leq 1 \\
u_1, \ u_2, \ v_1, \ v_2, & \geq 0
\end{align*} \]
Similarly, we do other seven manufacturing enterprises.

Step 2. Using the robust ranking method, the fuzzy coefficients become as follows:

| Enterprises | Inputs | Outputs |
|-------------|--------|---------|
| (DMUS)      | MC     | NOE     | GOV | PQ |
| A           | 2167.5 | 1870    | 14735 | 4.05 |
| B           | 1460   | 1340    | 12675 | 2.1 |
| C           | 2565   | 2360    | 18205 | 4.225 |
| D           | 2350   | 2020    | 15227.5 | 3.675 |
| E           | 1520   | 1550    | 14207.5 | 1.875 |
| F           | 2037.5 | 1760    | 14262.5 | 2.6 |
| G           | 2255   | 1980    | 16820 | 3.4 |
| H           | 2460   | 2250    | 17905 | 3.6 |

Step 3. Form the crisp linear programming model as follows.
Max \( U(u, v) = 2167.5u_1 + 1870u_2, \) s.t.

\[
\begin{align*}
14735v_1 + 4.05v_2 &= 1 \\
1460u_1 + 1340u_2 + 12675v_1 + 2.1v_2 &\leq 1 \\
2565u_1 + 2360u_2 + 18205v_1 + 4.225v_2 &\leq 1 \\
2350u_1 + 2020u_2 + 15227.5v_1 + 3.675v_2 &\leq 1 \\
1520u_1 + 1550u_2 + 14207.5v_1 + 1.875v_2 &\leq 1 \\
2037.5u_1 + 1760u_2 + 14262.5v_1 + 2.6v_2 &\leq 1 \\
2255u_1 + 1980u_2 + 16820v_1 + 3.4v_2 &\leq 1 \\
2460u_1 + 2250u_2 + 17905v_1 + 3.6v_2 &\leq 1 \\
u_1, u_2, v_1, v_2, &\geq 0
\end{align*}
\]

Step 4. The optimal solution of the crisp model using Win QSB solver is obtained as follows.

| No. | DMU | Score | Rank | Average | St Dev |
|-----|-----|-------|------|---------|--------|
| 1   | A   | 1     | 1    | Max     | 0.9522 |
| 2   | B   | 1     | 1    | Min     | 0.8829 |
| 3   | C   | 0.9551| 5    | St Dev  | 0.0466 |
| 4   | D   | 0.9076| 7    |         |        |
| 5   | E   | 1     | 1    |         |        |
To conduct performance assessments in fuzzy environments from different perspectives, a computational procedure has been presented for some ranking functions to formulate and solve fuzzy DEA models. Based on robust ranking formulas, we convert the fuzzy DEA input and output from fuzzy form to crisp. Furthermore, a case study has presented as in [6] to assess the performance of the given (DMUs) which are to be evaluated in terms of the two inputs and the two outputs. So that will facilitate to make accurate decisions for the performance evaluation and decreasing the period of decision-making procedures.

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| 6 | F | 0.8829 | 8 |
|---|---|--------|---|
| 7 | G | 0.9582 | 4 |
| 8 | H | 0.9136 | 6 |
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