QUANTUM VACUUM AND ACCELERATED EXPANSION

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Abstract. A new approach to extraction of quantum vacuum energy, in the context of the accelerated expansion, is proposed, and it is shown that experimentally realistic orders of values can be derived. The idea has been implemented in the framework of the Friedmann–Lemaître–Robertson–Walker geometry in the language of the effective action in the relativistic formalism of Schwinger’s proper time and Seeley–DeWitt’s heat kernel expansion.

1 Introduction

The following three, well-known problems of modern physics and cosmology, accelerated expansion of the Universe (Riess et al. 1998; Perlmutter et al. 1999), very small but non-vanishing cosmological constant or dark energy (Weinberg 1989), (Carroll 2001; Padmanabhan 2003 2006), and theoretically extraordinarily huge quantum vacuum energy density (Zel’dovich 1967), (Volovik 2005 2006) can be treated as mutually related or as independent problems. An old approach to the issue of the cosmological constant \( \Lambda \) utilizes quantum vacuum energy as a solution of this issue, but unfortunately it does not work properly. Namely, it appears that directly calculated, Casimir-like value of quantum vacuum energy is more than one hundred orders greater than expected. Such a huge value of quantum vacuum energy is a serious theoretical problem in itself. Lowering the UV cutoff scale down from the planckian to the supersymmetric one is a symbolic improvement (roughly, it cuts the order by two (Weinberg 1989)). A more radical reduction of the cutoff could cure the situation but it would create new problems. Sometimes, it is claimed that vacuum energy for one or another reason does not influence gravitational field.

In this paper, following ideas presented in (Broda et al. 2008), we show in what sense quantum vacuum energy influences gravitational field, and in what

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sense it does not. Actually, we propose a reasonable derivation of a contribution of quantum vacuum energy which influences gravitational field, and which assumes an experimentally reasonable value.

2 Quantum vacuum energy

The standard approach (Weinberg 1989) to estimate the quantum vacuum energy density $\varrho_{\text{vac}}$ in the spirit of Casimir energy yields for a single bosonic scalar mode

$$\varrho_{\text{vac}} = \frac{1}{2} \int_0^{\Lambda_{\text{UV}}} \frac{4\pi c}{(2\pi\hbar)^3} \sqrt{mc^2 + k^2} \, k^2 \, dk,$$

(2.1)

where $m$ is the mass of the mode. For the planckian UV cutoff, $\Lambda_{\text{UV}} = \Lambda_{p} = \sqrt{\hbar c^3 / G} \approx 6.5 \, \text{kg m/s}$, we obtain $\varrho_{\text{vac}} \approx 3.1 \cdot 10^{111} \, \text{J/m}^3$, whereas the experimentally estimated value is of the order of the critical density of the Universe, $\varrho_{\text{crit}} = 3(H_0c)^2 / 8\pi G \approx 10^{-9} \, \text{J/m}^3$.

In our opinion, the explicitly absurd result follows from an erroneous approach. Namely, in classical as well as in quantum theory interactions are being mediated by fields or particles. In Eq. (2.1) no explicit or implicit coupling to gravitational field appears on any stage. Therefore, by construction, we assume that gravitation does not couple (is insensitive) to the term (2.1). As there is no coupling to (2.1), its huge value is isolated from the outer world and therefore invisible (non-existent). What we have just said is, so to say, a negative part of our reasoning. In the positive part we should cure the situation somehow proposing a reasonable solution. In (Broda et al. 2008) we have sketched our idea and proposed an estimation of the quantum vacuum energy. Actually, it is possible to allow another interpretation of our calculations. For example, in our opinion, the idea of “the rearrangement” of vacuum motivated by thermodynamics and condensed-matter physics advocated in (Volovik 2005, 2006) could be implemented just this way.

Anyway, our original calculus (Broda et al. 2008) consists in careful considering only contributions coming from attached classical external lines. More precisely, in the first step, we should estimate quantum vacuum fluctuations of a matter field in the background of an external classical gravitational field. In the next step we should retain the most divergent part and subtract the term without gravitational field.

3 The estimation

We will work in the formalism of the effective action, throughout. A euclidean version of our approach has been given in (Broda et al. 2008), and here we present a relativistic one. Full quantum contribution to the effective action coming from a single (non-self-interacting) mode is (DeWitt 1975, 2003)

$$S_{\text{eff}} = \pm \frac{i\hbar}{2} \log \det D,$$

(3.1)
where $D$ is a second-order differential operator, in general, with classical external fields, and the upper (plus) sign corresponds to a boson, whereas the lower (minus) one corresponds to a fermion, respectively. Proper-time UV regularized version of (3.1) in Schwinger’s formalism assumes the form (Birrell & Davies 1982, (DeWitt 1975, 2003))

$$ S_{\text{eff}}^e = \mp \frac{i \hbar}{2} \int_\varepsilon^{\infty} \frac{id s}{is} \text{Tr} e^{-isD}. $$

(3.2)

Next, we apply the Seeley–DeWitt “heat-kernel” expansion in four dimensions (Birrell & Davies 1982, (DeWitt 1975, 2003), (Ball 1989)),

$$ \langle x | e^{-isD} | x \rangle = i(4\pi)^{-2} \sum_{n=0}^{\infty} a_n(x)(is)^{n-2}. $$

(3.3)

The contribution coming from the first term, we are interested, i.e. $a_0(x)$, is

$$ S_{\text{vac}} = \mp \frac{1}{2} \frac{1}{2e^2 (4\pi)^2} \frac{1}{\hbar^2} \text{Tr} a_0(x). $$

(3.4)

Since $a_0(x) = 1$, and for planckian UV cutoff $\varepsilon = \frac{\hbar G}{c^2}$, we obtain

$$ S_{\text{vac}} = \mp \frac{1}{4} \frac{c^7}{(4\pi)^2 \hbar^2} \int \sqrt{-g} \, d^3 x \, d t. $$

(3.5)

For simplicity, we confine ourselves to the spatially flat Friedmann–Lemaître–Robertson–Walker metric with the scale factor $a(t)$. To ease our calculus further, we set the present coordinate time $t = 0$, and normalize the coordinates to unity, i.e. $a(0) = 1$. Expanding $a(t)$ around $t = 0$ we have

$$ a(t) = 1 + H_0 t - \frac{1}{2} q_0 H_0^2 t^2 + O(t^3), $$

(3.6)

where $H_0$ is the present day Hubble expansion rate, and $q_0$ is the present day deceleration parameter. Hence

$$ \sqrt{-g} = [a^2(t)]^{3/2} = [1 + 2H_0 t + (1 - q_0) H_0^2 t^2 + O(t^3)]^{3/2}. $$

(3.7)

Now, one can easily show that the infinitesimal gauge transformation of the metric,

$$ \delta g_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, $$

(3.8)

with the gauge parameter

$$ \xi_\mu = \left( \frac{1}{2} H_0 x^2, -H_0 t x^i \right), $$

(3.9)
cancels the linear in $t$ part in (3.7). There are also general arguments supporting this cancellation given in (Shapiro 2007). Therefore
\begin{equation}
\sqrt{-g} \approx 1 + \frac{3}{2} (1 - q_0) H_0^2 t^2,
\end{equation}
and
\begin{equation}
S_{\text{vac}} \approx \mp \frac{1}{4} \frac{c^7}{(4\pi)^2 \hbar G^2} \int \left[ 1 + \frac{3}{2} (1 - q_0) H_0^2 t^2 \right] dt \int d^3 x.
\end{equation}
The number one in the bracket corresponds to the term uncoupled to gravitational field, and it should be subtracted. By the way, such a subtraction is a standard procedure in quantum field theory. As we are interested in a density rather than in a total value we should get rid off all integrals. Since the integrand is only time-dependent we can simply discard the spatial volume $\int d^3 x$. As far as the time integrand is concerned we should take into account that our calculus is perturbative in $t$ and valid only in the vicinity of $t = 0$. Therefore, we have to take the limit of “infinitesimal” time. From the point of view of quantum field theory the “infinitesimal” time is the Planck time $T_p = \sqrt{\hbar G/c^5}$. So, our density is a time average, i.e. $T_p^{-1} \int_0^{T_p} dt \cdot$, and assumes the form
\begin{equation}
\varrho \approx \mp \frac{1}{4} \frac{c^7}{(4\pi)^2 \hbar G^2} \frac{1}{2} (1 - q_0) H_0^2 T_p^2,
\end{equation}
or finally
\begin{equation}
\varrho \approx \mp \frac{1}{48\pi} (1 - q_0) \varrho_{\text{crit}},
\end{equation}
where we have used the relation: $H_0^2 = \frac{8\pi G}{3} \varrho_{\text{crit}}$. For, e.g. $q_0 = -0.7$ (Virey et al. 2004), we get
\begin{equation}
\varrho \approx \mp 0.01 \varrho_{\text{crit}},
\end{equation}
a very promising result.

4 Conclusions

In the framework of standard quantum field theory, without any additional more or less exotic assumptions we are able to derive an experimentally reasonable result (3.14). This numeric value corresponds to only a single mode. Therefore in the real world it should be multiplied by a small natural number.

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