Structures generated by the Klein-like topology of space*

Vladimir Yershov
vny@mssl.ucl.ac.uk

Mullard Space Science Laboratory (University College London),
Holmbury St.Mary, Dorking RH5 6NT, United Kingdom

We hypothesise that properties of space could underly some patterns observed in nature. Let us explore the possibility that the observed variety of matter particles and the pattern of their properties are determined by the non-orientability of the manifold representing spacetime. That is, let us regard space as a moving medium with the topology of a 3-Klein bottle, whose motion is parameterised with the time coordinate. The “throat” of the Klein bottle (region II in Fig.1) could be viewed as a dislocation of space with vanishing radius, so that the unification of the “inner” (I) and “outer” (II) hyper-surfaces of the Klein bottle occur on a very small scale.

![Diagram of Klein-bottle](image)

Figure 1: One-dimensional scheme of the Klein-bottle. The “inner” (I) and the “outer” (II) hyper-surfaces are unified through the region II regarded here as a primitive particle.

The resulting structure resembles a microscopic black hole with its inner and outer hyperspaces identified. Typically, this could be visualised (as a point-like source) by reducing dimensionality and using a two-dimensional surface, as shown in Fig.2.

![Diagram of preon](image)

Figure 2: Two-dimensional visualisation of the preon II.

We shall regard this structure as a primitive particle (hereafter called “preon”) with its field extending to infinity. Since the inner space of this particle is identified with the rest of the system, such a model automatically incorporates Mach’s principle. This also means that preon’s properties are determined by

*Poster at the TH2002 conference, Paris, 22-26 July 2002
the parameters of this system (the universe) and that in multi-preon systems preons must be identical to each other, save their differences due to the torsional degrees of freedom of the medium.

In such a system different preons would interact with one another through pressure and tension of the moving medium. The torsional degrees of freedom would give rise to the SU(3)/SU(1)-symmetry of the fields (due to the tridimensionality of space). By analogy with quantum chromodynamics (dealing with these types of symmetry), we can endue preons with tripolar charges (colours), which are usually labelled as red, green, and blue. For simplicity let us use unit values for these charges, as well as for the preons’ masses.

Given two manifestations of space, we can resolve the preon’s field into two components, \( \phi_s \) and \( \phi_c \). To avoid singularities we shall assume that infinite energies are not accessible and hypothesise that the energy of both \( \phi_s \) and \( \phi_c \), after reaching the maximum, decays to zero at the origin. The simplest form for the split field that incorporates the requirements above is the following:

\[
F = \phi_s + \phi_c, \\
\phi_s = s \exp(-\rho^{-1}), \quad \phi_c = -\phi_s'(\rho),
\]

where the signature \( s = \pm 1 \) indicates the sense of the interaction (attraction or repulsion); the derivative of \( \phi_s \) is taken with respect to the radial coordinate \( \rho \). Far from the source, the second component of the field \( F \) mimics the Coulomb gauge, whereas the first component extends to infinity being almost constant (similarly to the strong field).

To formalise the calculation of masses, we shall represent the preon discharge with the use of auxiliary \( 3 \times 3 \) singular matrices \( \Pi \) containing the following elements:

\[
\pm \pi_{jk} = \pm \delta_j^i (-1)^{i+j},
\]

where \( \delta_j^i \) is the Kronecker delta-function; the \( \pm \)-signs correspond to the sign of the charge; and the index \( i \) stands for the colour \( (i = 1, 2, 3 \text{ or red, green and blue}) \). The diverging components of the field can be represented by reciprocal elements: \( \pi_{jk} = \pi_{jk}^{-1} \). Then, we can define the preon’s (unit) charges and masses by summation of these matrix elements:

\[
q_\Pi = u^\top \Pi u, \quad \hat{q}_\Pi = u^\top \hat{\Pi} u \\
m_\Pi = |u^\top \Pi u|, \quad \hat{m}_\Pi = |u^\top \hat{\Pi} u|
\]

(\( u \) is the diagonal of a unit matrix; \( \hat{q}_\Pi \) and \( \hat{m}_\Pi \) diverge). Let us assume that the field \( F(\rho) \) does not act instantaneously at a distance. Then, we can define the mass of a system containing, say, \( N \) preons, as proportional to the number of these particles, wherever their field flow rates are not cancelled. For this purpose, we shall regard the total field flow rate, \( v_N \), of such a system as a superposition of the individual volume flow rates of its \( N \) components. Then, the net mass of the system can be calculated (to a first order of approximation) as the number of particles, \( N \), times the normalised to unity (Lorentz-additive) field flow rate \( v_N \):

\[
m_N = |Nv_N|.
\]

Here \( v_N \) is computed recursively from:

\[
v_i = \frac{q_i + v_{i-1}}{1 + |q_i v_{i-1}|},
\]

with \( i = 2, \ldots, N \) and putting \( v_1 = q_1 \). The normalisation condition expresses the common fact that the superposition flow rate of, say, two antiparallel flows \( (\uparrow \downarrow) \) with equal rate magnitudes \( |v_\uparrow| = |v_\downarrow| = v \) vanishes \( (v_\uparrow \downarrow = 0) \), whereas, in the case of parallel flows \( (\uparrow \uparrow) \) it cannot exceed the magnitudes of the individual flow rates \( (v_\uparrow \uparrow \leq v) \).

Then, when two unlike-charged preons combine (say, red and antigreen), the magnitudes of their oppositely directed flow rates cancel each other (resulting in a neutral system). The corresponding acceleration also vanishes, which is implicit in (4). This formula implies the complete cancellation of masses in the system with vanishing electric fields (converted into the binding energy of the system), but this is only an approximation because in our case the preons are separated by a distance of equilibrium, whereas the complete cancellation of flows is possible only when the flow source centres coincide.

Making use of the known pattern of attraction and repulsion between colour charges (two like-charged but unlike-coloured particles are attracted, otherwise they repel) we can write the signature \( s_{ik} \) of the chromoelectric interaction between two preons with colours \( i \) and \( k \) as

\[
s_{ik} = -u^\top \Pi^k \Pi^i u.
\]
The corresponding pattern is shown in Fig. 3. It implies that the preons of different colours and charge polarities will cohere in structures, the simplest of which will be the charged and neutral colour-doublets (dipoles):

\[
q_{ik}^\pm = \Pi_i + \Pi_k, \\
q_{ik}^0 = \Pi_i + \Pi_k, \quad i, k = 1, 2, 3
\]
as well as the charged tripoles, denoted here as \( Y \) and \( \overline{Y} \) (for the opposite polarities):

\[
Y = \sum_{i=1}^{3} \Pi_i \quad \text{or} \quad \overline{Y} = \sum_{i=1}^{3} \Pi_i.
\]

According to (3), \( q(g_{ik}) = \pm 2, m(g_{ik}) = 2, q(g_{ik}) = 0, m(g_{ik}) = 0, q(Y) = \pm 3, m(Y) = 3 \). The tripole (\( Y \)-particle) will be colourless at infinity but colour-polarised nearby, which means that tripoles could combine in strings (pole-to-pole to each other) due to their residual chromaticism.

Pairs of like-charged tripoles \( Y \) will combine in short strings \( Y \& \), with the components 180°-rotated with respect to each other, Fig. 4(a), whereas unlike-charged tripoles \( (YY) \) would combine rotated by ±120°, Fig. 4(b).

The string of three like-charged tripoles will close in a loop (triplet) \( 3Y \) or \( 3\overline{Y} \), denoted here as \( e \) because, as we shall see, this structure by its properties can be identified with the electron. The triplet \( 3Y \) is charged, with the charge \( q_e = \pm 9 \) and mass \( m_e = 9 \) (expressed in units of preon’s charge and mass). The tripoles in this structure can be directed with their vertices towards or away from the centre of the loop, as shown in Fig. 5 and Fig. 6. However, it is seen that these configurations correspond to two different phases of the same structure, since the tripoles here have a rotational degree of freedom (around their common ring-closed axis). At the same time, the tripoles will orbit the centre of the structure moving along the ring-closed axis. The resulting currents have helical shapes with two possible helicity signs (clockwise or anticlockwise). These different helicities can be identified with two spins of the structure (\( e_\uparrow \) and \( e_\downarrow \)).

The pairs of unlike-charged tripoles can form longer strings: \( YY \&Y \&Y \ldots \) with the pattern of colour charges repeating after each six consecutive \( YY \)-pairs, which allows the closure of such a string in a loop (shown in Fig. 7 and denoted here as \( \nu_e = 6Y\overline{Y} \)). The structure \( \nu_e \) (formed of 36 preons) is electrically neutral and has a vanishing mass, according to (4), unless combined with a charged particle, say \( Y \) or \( 3Y \), which would restore the entire mass of the composite system.
The properties of the simplest preon structures are summarised in Table 1. Most of these structures are open strings, which are supposed to be unstable. The ring-closed structures, like $\nu_e$ and $e$, will be stable because their effective potentials are minimised.

The particles $Y$, $e$, and $\nu_e$ can combine with each other because of their residual chromaticism. The structure $Y_1=Y+\nu_e$ will have mass and be charged, with the charge $\pm 3$ units, corresponding to the charge of the $Y$-particle, and the mass of 39 preon mass units: $m_{Y_1} = n_{\nu_e} + m_Y = 36 + 3$. The charge of the configuration $e + \nu_e$ will correspond to the charge of the triplet $e$ ($\pm 9$ units). Its mass will be 45 units, if expressed in preon’s units of mass ($n_{\nu_e} + m_e = 36 + 9$).

Like-charged particles $Y_1$ of the same helicity signs would further combine (through an intermediate $\nu_{e'}$-particle with the opposite helicity) forming three-component strings. The string $Y_1 e \bar{\lambda}_1$ can be identified with the $u$-quark. Its charge will correspond to the charge of two $Y$-particles ($q_u = \pm 6$) and its mass will roughly be the sum of the masses of its two $Y_1$-components: $m_u = 2 \times 39 = 78$ (preon mass units). The positively charged $u$-quark (78 preon mass units) would be able to combine with the negatively charged particle $e - \nu_e$ (45 preon mass units) mass, forming the $d$-quark with a mass of 123 preon mass units, $m_d = m_u + m_{e\nu_e} = 78 + 45 = 123$, and with the charge derived from the charges of its constituents $q_d = q_u + q_e = +6 - 9 = -3$ units.

The process of structure formation involving the particles $3Y$ with $6Y\bar{Y}$ ($e$ and $\nu_e$) will be helicity-dependent. The configuration of colour charges of the structure $6Y\bar{Y}$ does not match that of the structure $3Y_4$, which would lead to the mutual repulsion of these particles. Only the structures $3Y_4 + 6Y\bar{Y}$ or $3Y_5$.
Table 1: Simple structures generated by the 3-Klein bottle topology of space

| Structure | Constituents of the structure | Number of colour charges in the structure | Charge (in preon charge units) | Mass (in preon mass units) |
|-----------|-------------------------------|------------------------------------------|-------------------------------|---------------------------|
|            |                               |                                          |                               |                           |
| The primitive particle (preon Π) |                               |                                          |                               |                           |
| Π          | Π                             | 1                                        | +1                            | 1                         |
|            |                               |                                          |                               |                           |
| First-order structures (combinations of preons) |                               |                                          |                               |                           |
| g          | 2Π                            | 2                                        | +2                            | 2                         |
| g'         | 2Π + 1Π                       | 2                                        | −1 + 1 = 0                    | ≈ 0                       |
| Y          | 3Π                            | 3                                        | +3                            | 3                         |
|            |                               |                                          |                               |                           |
| Second-order structures (combinations of tripoles Y) |                               |                                          |                               |                           |
| δ±         | 2Y                            | 6                                        | +6                            | 6                         |
| γ          | 1Y + 1Y                       | 6                                        | −3 + 3 = 0                    | ≈ 0                       |
| e−         | 3Y                            | 9                                        | −9                            | 9                         |
|            |                               |                                          |                               |                           |
| Third-order structures |                               |                                          |                               |                           |
| 2e−        | 3Y + 3Y                       | 9 + 9 = 18                               | −18                           | 18                        |
| e−e+       | 3Y + 3Y                       | 9 + 9 = 18                               | −9 + 9 = 0                    | ≈ 16                      |
| νe         | 6Y Y                         | 6 × (3 + 3) = 36                           | 6 × (−3 + 3) = 0              | ≈ 0                       |
| Y1         | νe + Y                       | 36 + 3 = 39                              | 0 − 3 = −3                   | 36 + 3 = 39               |
| W−         | νe + e−                      | 36 + 9 = 45                              | 0 − 9 = −9                   | 36 + 9 = 45               |
| u          | Y1 + νe , Y1                 | 39 + 36 + 39 + 114 = 144 +3 = +3 = −3 + 0 + 3 = +6 | 39 + 39 = 78               |
| νμ         | Y1 νe , Y1                 | 39 + 36 + 39 + 114 = 144 +3 = +3 = −3 + 0 + 3 = 0 | ≈ 0                       |
| d          | u + W−                      | 114 + 45 = 159                           | +6 = −9 = −3                 | 78 + 45 = 123            |
| μ−         | νμ , W−                     | 114 + 45 = 159                           | 0 − 9 = −9                   | 48 + 39 = 1872           |

and so on ...

*) two-component system

+ 6Y Y can be formed because the effective potential of these combinations implies attraction between the components. By contrast, if two structures of the same kind combine (e.g., e with e or νe with νe), their helicity signs must be opposite to create an attractive force between the components of the pair. This coheres with and probably explains the Pauli exclusion principle, which allows us to identify helicities of the structures in question with their spins.

Divided by nine, the charge of the 3Y-particle, gives us the conventional unit charge of the electron. Charges of the Y and δ-particles (fractions of the nine-unit electron charge) correspond to the quark fractional charges.

Very schematically the structures of the u- and d-quarks are shown in Fig.8 where for simplicity the particles νe, Y and e are visualised, respectively, as symbols O (in grey colour), Y and triple-Y (coloured in red or blue, depending on the polarities of their charges).

![Figure 8: Schemes of the u (left) and d (right) quarks.](image)

It is not unnatural to suppose that particles of the second and third generations of the fundamental fermions are formed of simpler structures belonging to the first generation. For example, the muon-neutrino can be a bound state of the, respectively, positively and negatively charged particles Y1 and Y1 (Fig.9 left):

νμ = Y 6Y Y 6Y Y Y = Y1 νe Y1.  

(7)
whereas the muon’s structure can be written as

$$\mu = (6\bar{Y}Y + 3\bar{Y}\bar{Y}) (6\bar{Y}Y, 6\bar{Y}\bar{Y}, \bar{Y}) = \overline{\nu_e} e^{-\nu_\mu},$$

(8)

which incidentally matches one of the muon’s decay modes (Fig. 9, right). The schemes of the rest of the fundamental fermions are shown in Fig. 10 to Fig. 12. One can hardly regard these structures as rigid bodies: they are rather oscillating clusters. In (8) we have enclosed the supposedly clustered components in parenthesis. The oscillatory energies are likely to contribute to the masses of these systems. Obtaining these masses is not a straightforward task, but, in principle, they are computable, which can be shown by a simple empirical formula, relating the oscillatory energy of the components to the sum $M$ of their masses $m_k$: 

$$M = \sum_k m_k,$$

(9)

multiplied by the sum $\tilde{M}$ of their reciprocal masses $1/\tilde{m}_k$:

$$\frac{1}{M} = \sum_k \frac{1}{\tilde{m}_k}.$$  

Then, by setting for simplicity the unit-conversion coefficient to unity, we can compute the masses of the combined structures as

$$M_{\text{total}} = M \tilde{M}$$

We shall abbreviate this summation rule by using the overlined notation:

$$M_{\text{total}} = \frac{m_1 + m_2 + \cdots + m_N}{\tilde{m}_1^{-1} + \tilde{m}_2^{-1} + \cdots + \tilde{m}_N^{-1}}.$$  

(10)

The masses of the fundamental fermions computed with the use of this rule are summarised in Table 2. As an example, let us compute the muon’s mass. The masses of its components, according to its structure, are: $m_1 = \tilde{m}_1 = 48$, $m_2 = \tilde{m}_2 = 39$, (in preon mass units) And the mass of the muon will be

$$m_\mu = \frac{m_1 + m_2}{\tilde{m}_1^{-1} + \tilde{m}_2^{-1}} = 1872 \text{ (preon mass units)}.$$  

For the $\tau$-lepton, according to its structure (Fig. 9), $m_1 = \tilde{m}_1 = 156$, $m_2 = \tilde{m}_2 = 201$, so that

$$m_\tau = \frac{m_1 + m_2}{\tilde{m}_1^{-1} + \tilde{m}_2^{-1}} = 31356 \text{ (preon mass units)}.$$  

For the proton, positively charged particle consisting of two $u$, and one $d$ quarks submerged in a cloud of gluons $g_{ij}$, the masses of its components are $m_u = \tilde{m}_u = 78$, $m_d = \tilde{m}_d = 123$, $m_g = 2m_u + m_d = 279$, $\tilde{m}_g = \infty$. The resulting proton mass is

$$m_p = \frac{m_u + m_u + m_d + m_g}{\tilde{m}_u^{-1} + \tilde{m}_u^{-1} + \tilde{m}_d^{-1} + \tilde{m}_g^{-1}} = 16523 \text{ (preon mass units)}.$$  

(11)

We can convert $m_\mu$ and other particle masses from preon mass units into proton mass units, $m_p$ by dividing these masses by the quantity (11). The results are given in the fourth column of Table 2. The experimental masses of the fundamental fermions (also expressed in units of $m_p$) are listed in the last
Figure 10: Schemes of the quarks c (left) and s (right).

Table 2: Computed and experimental masses of quarks and leptons. Values given in the third column can be converted into the proton mass units dividing them by \(m_p = 16523\).

| Particle and its structure (components) | Number of preons in the components | Predicted masses (preon mass units) | Predicted masses (in \(m_p\) units) | Experimental masses (in \(m_p\) units) |
|----------------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| \(\nu_e\) \(12Y^+\)                  | 36                                | \(~0\)                             | \(~0\)                             | \(<3 \times 10^{-3}\)             |
| \(e^-\) \(\frac{3\sqrt{2}}{}\)      | 9                                 | 9                                 | 0.0005447                          | 0.0005446170232                   |
| \(u\) \(Y_1\nu_e Y_1\)              | 78                                | 78                                | 0.00472                           | 0.0021 to 0.0058                   |
| \(d\) \(u\nu_e e^-\)                | 123                               | 123                               | 0.007443                          | 0.0058 to 0.0115                   |

| First generation                       |                                   |                                   |                                   |                                   |
|----------------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| \(\nu_\mu\) \(Y_1\nu_\mu Y_1\)     | \(114\)                           | \(~0\)                             | \(~0\)                             | \(<2 \times 10^{-4}\)             |
| \(\mu^-\) \(\nu_\mu + \nu_\mu e^-\)| \(48 + 39\)                       | 1872                              | 0.1133                            | 0.1126095173                      |
| \(c\) \(Y_2 + Y_2\)                  | \(165 + 165\)                     | 27225                             | 1.6477                            | 1.57 to 1.95                      |
| \(s\) \(c + e^-\)                    | \(165 + 165 + 9\)                 | 2751                              | 0.1665                            | 0.11 to 0.19                      |

| Second generation                      |                                   |                                   |                                   |                                   |
|----------------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| \(\nu_\tau\) \(u\nu_\tau\) \(\bar{\mu}\) | \(192\)                           | \(~0\)                             | \(~0\)                             | \(<2 \times 10^{-3}\)             |
| \(\tau^-\) \(\nu_\tau \nu_\mu \mu^-\)| \(156 + 201\)                     | 31356                             | 1.8977                            | 1.8939 \(\pm\) 0.0003            |
| \(t\) \(Y_3 + Y_3\)                  | \(1767 + 1767\)                   | 3122289                           | 188.96                            | 189.7 \(\pm\) 4.5               |
| \(b\) \(t + \mu^-\)                  | \(1767 + 1767 + 48 + 39\)        | 76061.5                           | 4.603                             | 4.3 to 4.7                        |

| Third generation                       |                                   |                                   |                                   |                                   |
|----------------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| \(\nu_e\) \(12Y^+\)                  | 36                                | \(~0\)                             | \(~0\)                             | \(<3 \times 10^{-3}\)             |
| \(e^-\) \(\frac{3\sqrt{2}}{}\)      | 9                                 | 9                                 | 0.0005447                          | 0.0005446170232                   |
| \(u\) \(Y_1\nu_e Y_1\)              | 78                                | 78                                | 0.00472                           | 0.0021 to 0.0058                   |
| \(d\) \(u\nu_e e^-\)                | 123                               | 123                               | 0.007443                          | 0.0058 to 0.0115                   |

Table 2 and Figures 8 to 12 illustrate family-to-family similarities in the particle structures. In each family, the \(d\)-like quark is a combination of the \(u\)-like quark, with a charged lepton belonging to the lighter family. Charged leptons appear as a combination of the neutrino from the same family with the neutrinos and charged leptons from the lighter family. It is conceivable that ring-closed strings, similar to that of the electron neutrino, may appear on higher structural levels of the hierarchy, which could be regarded as “heavy neutrinos”, \(\nu_h = 6Y_1\bar{Y}_1\). They can form part of “ultra-heavy” neutrinos \(\nu_{uh} = 3(Y_1\nu_h u)e^-\), and so on. With these neutral structures the components \(Y_2\) and \(Y_3\) of the \(c\)- and \(t\)-quarks could be written as \(Y_2 = u\nu_e u\nu_e e^-\) and \(Y_3 = \nu_{uh}Y\).

Conclusions

The scheme outlined here offers a reasonable explanation of the observed variety of matter particles. It generates the quantum numbers and masses of the fundamental fermions from first principles, without using free or experimental input parameters (in this sense our model is self-consistent). The computed masses agree with experiment to an accuracy of about 0.5% (the discrepancies are likely to be attributed to the assumed simplifications, e.g., we have neglected the binding energies of the structures, as well as the small residual masses of the electron neutrino, both contributing to the masses with opposite signs.
Figure 11: Schemes of the $\nu_\tau$ (left) and $\tau$ (right) leptons.

Figure 12: Schemes of the $t$ (left) and $b$ (right) quarks.