THE PHASE DIAGRAM OF 2 FLAVOUR QCD WITH IMPROVED ACTIONS

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It has been proposed, that the chiral continuum limit of 2-flavour QCD with Wilson fermions is brought about by a phase in which flavour and parity symmetry are broken spontaneously at finite lattice spacing. At finite temperature this phase should retract from the weak coupling limit to form 5 cusps. This scenario is studied with tree level Symanzik improved actions for both gauge and fermion fields on lattices of size $8^3 \times 4$ and $12^2 \times 24 \times 4$.

1 Introduction and Motivation

The study of the finite temperature phase diagram of 2-flavour QCD with Wilson fermions has revealed a rather intricate picture. This picture is based on the idea of spontaneous breakdown of parity and flavour symmetry and has been investigated analytically as well as numerically. The main features of this phase diagram are:

- the critical line $\kappa_c(\beta)$ defined by a vanishing pion screening mass for finite temporal lattice size marks the phase boundary with a phase of spontaneously broken parity and flavour symmetry.
- for large enough $N_T$ five cusps moving towards weak coupling should develop, separating the 5 sets of doublers.
- the finite temperature phase transition line $\kappa_t(\beta)$ should not cross the critical line, but run past it towards larger values of the hopping parameter, presumably turning back toward weak coupling as one crosses the $m_q = 0$ line.

Because the Wilson term breaks all chiral symmetries of the naive fermion action, the critical line above cannot consistently be interpreted as the chiral limit of QCD at any finite lattice spacing. It has recently become clear that two scenarios are possible. Either there exists a second order phase transition to an Aoki phase (of width $a^3$) in which parity and flavour symmetry are broken...
and along its phase boundary the pion mass vanishes or the system exhibits a first order transition along the line of vanishing quark mass and the pion does not become massless. The analysis also suggests that which scenario is realized can change as one varies the action. One therefore has to check, that the phase diagram with improved Wilson fermions does exhibit an Aoki phase. As the Aoki phase retracts from the weak coupling limit as the temperature increases, it naturally explains the absence of any non-analyticities across the $m_q = 0$ line in the high temperature phase. In order to separate the high temperature side from the low temperature side of the phase diagram, the finite temperature transition line should run past the cusp of the Aoki phase and continue toward larger $\kappa$ values. The thermal line of the deconfinement phase transition therefore crosses the $m_q = 0$ line and should bounce back towards weaker coupling due to a symmetry under the change of sign of the mass term in the continuum theory. If the thermal line does not touch the tip of the cusp, there would be room for the second scenario mentioned above.

2 The Simulation

We have simulated 2 flavours of Wilson fermions on lattices of size $8^3 \times 4$ and $12^2 \times 24 \times 4$. We have employed tree level Symanzik improvement which for the fermion sector amounts to adding the so called clover term with coefficient one and for the gauge fields to adding a $2 \times 1$ loop. We have first mapped out the phase diagram on the smaller lattice and have then corroborated our results on the larger lattice for two $\beta$ values.
3 Quark mass and Pion screening mass

Our results for the quark and pion screening mass are shown in figure (1). The current quark mass is defined via the axial Ward identity:

\[ 2m_q \equiv \frac{\nabla_\mu \langle 0| A^\mu| \pi \rangle}{\langle 0| P| \pi \rangle} \tag{1} \]

For \( \beta = 2.8 \) we find some curvature for the quark mass as a function of \( 1/\kappa \), though a linear fit produces a reasonable \( \chi^2 \). For \( \beta = 3.1 \) we have also explored a region of hopping parameters where the quark mass becomes negative. There we find a rather peculiar behaviour. The quark mass first decreases as one lowers \( \kappa \) towards \( \kappa_c \) and only rises again very close to \( \kappa_c \). This results in different slopes of \( m_q \) as a function of \( 1/\kappa \) for positive and negative quark masses, which is in contrast to simulations with unimproved Wilson fermions that have shown the same behaviour for positive and negative quark masses.

We have also accurately measured the pion screening mass. For \( \beta = 2.8 \) the decrease of the pion mass is consistent with a linear behaviour \( m_\pi^2 \propto 1/\kappa - 1/\kappa_c \) down to small values of \( m_\pi^2 \). This also applies for \( \beta = 3.1 \) for \( \kappa \) values that correspond to positive quark masses. For negative quark masses the behaviour is quite different and inconsistent with a linear behaviour for the points measured. We will however argue below that for \( \beta = 3.1 \) the pion mass does not go to zero as the quark mass goes to zero, because one crosses the finite temperature transition line before the quark mass becomes zero.
4 Polyakov loop

Figure (2) shows our results for the Polyakov loop as a function of κ including data from both lattices. The vertical lines indicate the position of the extrapolated κc from the pion screening mass, except for β = 3.0 where the pion norm was used. This extrapolated κc decreases with increasing β. For β = 2.8 all our data for the pion mass lie in the confined region. We therefore conclude that the pion mass vanishes as the quark mass goes to zero, i.e. for β = 2.8 we hit the Aoki phase as we increase κ. For β = 3.0 and 3.1 this is no longer so clear. The Polyakov loop already shows a high temperature behaviour where the extrapolated pion mass would be small. For β = 3.0 the situation is less prominent and one could still argue that the pion becomes massless, but since the Polyakov loop is in the high temperature phase immediately after one crosses κc, the finite temperature line and the line of vanishing quark mass come very close together. For β = 3.1 it becomes evident, that one crosses the finite temperature transition line before the line of vanishing quark mass. The pion will therefore not become massless as the quark mass vanishes. Because the Polyakov loop continues to rise past the m_q = 0 line, one can exclude that the finite temperature line bounces back towards weaker coupling immediately.

5 Chiral order parameter

Because of the explicit breaking of chiral symmetry by the Wilson action, one has to define a properly subtracted chiral order parameter to obtain the correct continuum limit. Using axial Ward identities the order parameter is defined
as follows:\cite{5}:

\[ \langle \bar{\psi}\psi \rangle_{\text{sub}} = 2m_q \cdot Z \cdot \sum_x \langle \pi(x)\pi(0) \rangle \]  

(2)

Here \( Z \) is a renormalisation factor for which we take its tree level value \( Z = (2\kappa)^2 \). The sum over the pion correlation function is just the pion norm. Our results are shown in figure (3). For \( \beta = 2.8 \) the data extrapolate to a finite intercept at \( m_q = 0 \). For \( \beta = 3.1 \) the data show more curvature and we expect \( \langle \bar{\psi}\psi \rangle_{\text{sub}} \) to go to zero for vanishing quark mass.

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