Foot trajectory analysis of a robotic leg

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Abstract. When it comes to walking robots, foot trajectory is a crucial element that can significantly influence the efficiency of the walking robot. This paper analyses the various foot return trajectories, which can provide higher step length while consuming less power. It is done through mathematical analysis and verified using simulations in software such as MSC Adams and Solidworks. This paper also discusses the kinematic and dynamic analysis of the two degrees of freedom leg using theoretical approaches in MATLAB and verifies the results using the simulation in MSC Adams.

1. Introduction

The first documented walking mechanism appeared in 1870. Since then, researchers had been trying to develop the technology of walking machines. There has been history about several walking machines evolved in the past. Teresa Zielinska [1] studied the history of walking machines, which can influence present machines' design and give new ideas. The first documented walking mechanism appeared in 1870, and the first computer-controlled legged robot appeared around the mid-1970s. The four-legged machine designed by Hutchinson and Smith in 1940 was an essential milestone in walking technology [1]. However, the complexity of walking technology and low efficiency made it challenging to progress in this field substantially. Due to improvements in computing power, finding solutions to complex mechanical problems has been made easy [14]. This massive leap in technology has enabled more and more researchers to venture into the world of walking robotics. The advantages of walking machines over wheeled vehicles have also played a role in the popularity of walking robots [15]. Legged machines are preferred in space exploration and military purposes because they are suitable for uneven terrains, confined spaces and suitable for climbing stairs [18]. Therefore, the development of walking machines for space exploration, disabled people is of great interest.

Efficiency is an essential factor when designing any machine. It is no different in the case of a walking robot. Factors that affect the efficiency of a walking robot include the foot trajectory [10], power consumption, step length, ground clearance [15]. D McCloy [2] studied the power consumption of 2-DOF robotic legs. In the swing phase and stance phase, maximum power is demanded from the hip actuator and knee actuator, respectively. The energy required to swing the leg varies with a different trajectory. D McCloy [3] studied the effect of foot trajectories on the power consumption required. Ballistic, rectangular, and smoothed rectangular trajectories were studied. Ballistic trajectory proved to be the most efficient one. However, the smoothed rectangular trajectory was the practical solution.
Step length plays a vital role in deciding the efficiency of a robotic leg. D McCloy [4] derived a formula for the step length in limb length and ground clearance. Finding new trajectories or optimizing the existing trajectories can be done to achieve the required efficiency without compromising the step length and exceeding the power consumption. Finding a trajectory that can emulate a living creature's performance to tackle problems such as stair climbing was very complex. There have been many reports of robots with remarkable mobility on rough terrains. However, the research of walking machines in artificial terrains such as stairs was less. Chih-Chung Ko et al. [5] developed an algorithm for walking machines' trajectories to climb stairs using geometrical interactions between the robotic leg and stairs.

The use of serial and parallel driven manipulators influences power consumption. D McCloy [6] conducted experiments and found out that parallel operated manipulators are superior to serial driven manipulators. The same applies to robotic legs. Actuators, be it for a robotic leg or an arm can be of different types like electric, pneumatic [11]-[14] etc. Alain Segundo Potts et al. [7] examined the kinematic model of a complex quadruped robot. The primary purpose of the robot was to deal with a flat surface and an inclined surface. The position of the robot's platform and the foot of each leg at the climbing surface cannot be arbitrary but must satisfy constraints based on the geometry of the trajectory. High-speed running robots are among the most exciting and promising systems in robotic applications. For these robots, the actuation system needs to produce notably high output forces at high speed. It should be able to do it with high accuracy and fast response time. The actuation system must have high power capacity, back-drivability, and good controllability. Byeonghun Na et al. [8] designed an actuation system for a high-speed quadruped robot known as Cheetaroid-I. The introduction of a simple actuation mechanism that had an application for the high-speed robot produces tremendous advantages, improving mobility, back-drivability, and responsiveness. Nature was an inspiration that leads to the development of walking machines. Garcia-Lopez et al. [9] observed the locomotion of animals in nature, and inspired by nature, the foot trajectories of animals were considered. Accordingly, kinematic analysis for trajectory generation was done for one leg of a hexapod robot.

2. Kinematic analysis

2.1. Step length
Step length is a necessary element before moving onto further analysis. The basic geometry of the leg is shown in figure 1. The $\theta_1$ & $\theta_2$ are the actuators or motor rotation angle, $L_1$ & $L_2$ are the lengths of the limbs and, C is the ground clearance. When calculating step length, the length of the links is considered to be of the same length. The leg is assumed to travel from coordinate $(X_s, -C)$ to $(X_f, -C)$. For step length analysis, there are two main constraints 1. Traction between leg and ground which can be maintained by the optimum value of $\alpha$ which is between 45° and 135° 2. Alignment of the two links can result in the need of infinite speed in the actuators or motors. When calculating the step length using the geometric limitations defined by the two constraints, the maximum value of step length reached up to 0.9m.

2.2. Kinematics
The foot coordinates are related to the actuator rotations $\theta_1$ and $\theta_2$ by the forward kinematic equations:

$$\begin{align*}
X &= L_1 \cos \theta_1 - L_2 \cos (\theta_1 + \theta_2) \\
Y &= L_1 \sin \theta_1 - L_2 \sin (\theta_1 + \theta_2)
\end{align*}$$

The link length is fixed at a ratio of 1:1. The link length has been set at 0.44m considering the walking robot's average height to be 1.8m.
The X coordinate varies between 0m and 0.8m, with a significant range below 0.5m. The X coordinate signifies the step length of the foot. So, the step length can attain a value below 0.5m considering the safety limitations.

The Y-coordinate varies between -0.8m and 0.6m. The Y coordinate is above 0m when $\theta_1 > 45^\circ$ and $\theta_2 > 90^\circ$. For the same range, X coordinate varies between 0.2m and 0.7m.

The inverse kinematic relation, which gives the actuator rotations with respect to foot coordinates, is also required. The same link length is maintained. The inverse kinematic equation is as follows:

$$\theta_1 = \psi - \tan^{-1}\left(\frac{x}{y}\right), \theta_2 = \cos^{-1}\left(\frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2}\right)$$

Where, $\psi = \tan^{-1}\left(\frac{L_1 + L_2 \cos \theta_2}{L_2 \sin \theta_2}\right)$

The value of $\theta_1$ lies within $-75^\circ$ and $60^\circ$. This is the angle made by the hip joint. The value of $\theta_2$ lies within $-150^\circ$ and $0^\circ$. This is the angle made by the knee joint.
3. Dynamic analysis

3.1. Swing phase
The swing phase allows us to calculate the power requirement of each actuator of the leg around the trajectory. The mass and length of the links and clearance value of the hip joint are taken into account for the analysis.

3.2. Stance phase
In a walking gait cycle, the period of motion where the foot is in contact with the ground can be classified as the stance phase. 60% of the gait cycle can be assumed to be the stance phase. The legs are in contact with the ground, the power required to provide the supporting force for the body needs to be determined in the stance phase analysis. The main two forces considered during the contact phase are Normal force and Frictional force. [2] takes the normal forces as the designing factor, and the power is calculated. The masses of the legs are assumed to be zero for the contact phase, the power required by each of the two actuators to provide the supporting force is calculated. The force vector \( F_x, F_y \) applied by the foot on the ground is calculated. The torque required can be calculated as \( T = J^T F \). Knowing the actuator velocities, the power can be computed by multiplying with torque. \( H_1 = T_1 \theta_1' \) & \( H_2 = T_2 \theta_2' \). From analysis, the power demand from the knee actuator will dominate during the stance phase.

3.3. Robot dynamics
The dynamic analysis of the robotic leg focuses on the forces and moments acting on the links and joints. This analysis can lay the fundamental works for the mechanical design, simulation, and control of the robotic leg, thereby giving us the idea of power requirement, the torque needed for the motors, etc. We have used the results from our kinematic analysis to compute the Inverse dynamic of the leg. We have assumed both the legs to have a symmetrical rectangular cross-section. Considering the mass, \( M_1 \) & \( M_2 \) and \( I_1 \) & \( I_2 \) be the moment of inertia for the links \( L_1 \) & \( L_2 \), respectively. The mass of the links is assumed to be concentrated at the midpoints of the limbs. Theoretically, the torque required by the actuators can be found using the Lagrange method, and the dynamic equations of motion are all described in Appendix B. The variables \( D_1 \) to \( D_6 \) are defined in Appendix B. The theoretical dynamic analysis of the leg and the timewise variation of power and torque of different trajectories are shown below:

\[
I_1 = \frac{(M_1 L_1^2)}{12} \quad \text{and} \quad I_2 = \frac{(M_2 L_2^2)}{12}
\]

\[
T_1 = D_1 \ddot{\theta}_1 + D_2 \ddot{\theta}_2 + 2D_3 \dot{\theta}_1 \dot{\theta}_2 + 2D_4 + gD_4 \quad \text{and} \quad T_2 = D_2 \ddot{\theta}_1 + D_3 \ddot{\theta}_2 - D_4 \ddot{\theta}_1^2 - gD_6
\]

![Figure 4. Timewise variation of theoretical power consumption of Hip actuator for Rectangle, Smooth Rectangle, Triangle, and Ballistic trajectories.](image-url)
Figure 5. Timewise variation of theoretical torque of Hip actuator for Rectangle, Smooth Rectangle, Triangle, and Ballistic trajectories.

Figure 6. Timewise variation of theoretical power consumption of Knee actuator for Rectangle, Smooth Rectangle, Triangle, and Ballistic trajectories.

Figure 7. Timewise variation of theoretical torque of Knee actuator for Rectangle, Smooth Rectangle, Triangle, and Ballistic trajectories.
4. Trajectory analysis

Analysis Assumptions: Time for forward swing & return swing phase = constant; Duty Factor = 0.5; Swing period = 9sec.

4.1. Rectangle trajectory

Figure 8 shows the rectangular trajectory through which the leg will move. The length of the rectangular trajectory, denoted by 'S', is the step length of the foot that the leg can achieve. We have taken S=0.35m for our analysis. The Breadth of the rectangle is denoted by 'L', which will be the height of the leg from ground during the swing phase. We have assumed the leg lift L=0.07m. The timewave variation of angular acceleration, angular velocity, power, and torque of the leg joints is shown in figures 9 and 10. The only notable disadvantage of the rectangular trajectory is their sharp edges at the corner which can disrupt the motion by causing a discontinuity in actuator velocities. This discontinuity causes impulses in torque at these points.

Figure 8. Rectangle Trajectory

Figure 9. (a) The robotic leg’s timewise variation of Angular Acceleration when it followed the Rectangular trajectory simulated using Adams. The maximum value of the Angular Acceleration of actuators at Hip=11.04deg/S² & Knee=27.6deg/S². (b) The robotic leg’s timewise variation of Angular velocity when it followed the Rectangular trajectory simulated using Adams. The maximum value of the Angular Velocity of actuators at Hip=8.98deg/S & Knee=22.46deg/S

Figure 10. (a) The robotic leg's timewise variation of power consumption when it followed the Rectangular trajectory simulated using Adams. The maximum value of the power consumption of actuators at Hip=0.5539W & Knee=0.1527W. (b) The robotic leg's timewise variation of Actuator Torque variation when it followed the Rectangular trajectory simulated using Adams. The maximum value of the torque of actuators at Hip=3.91N-m & Knee=1.414N-m
4.2. Smooth Rectangle trajectory

Figure 11 shows the smooth rectangle trajectory. The trajectory consists of a line parallel to the axis of the motion, where the length is the line is equal to the maximum step length of the leg. A step length of 0.35m and a leg lift of 0.06m are assumed for our calculation. The difference when compared to rectangular trajectory is the breadth of the rectangle, in the form of a semicircle with a radius of 0.5L. The reason for the smooth curve is to avoid discontinuity along with the motion of the foot. The speed of the leg is determined with a 5th-degree polynomial which is, $d = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$

The timewise variation of angular acceleration, angular velocity, power, and torque of the two joints are shown in figures 12 and 13.

**Figure 11.** Smooth Rectangle Trajectory

**Figure 12.** (a) The robotic leg's timewise variation of Angular Acceleration when it followed the Smooth Rectangular trajectory simulated using Adams. The maximum value of the Angular Acceleration of actuators at Hip=50.66deg/S$^2$ & Knee=56.42deg/S$^2$. (b) The robotic leg's timewise variation of Angular velocity when it followed the Smooth Rectangular trajectory simulated using Adams. The maximum value of the Angular Velocity of actuators at Hip=18.96deg/S & Knee=24.64deg/S

**Figure 13.** (a) The robotic leg's timewise variation of power consumption when it followed the Smooth Rectangular trajectory simulated using Adams. The maximum value of the power consumption of actuators at Hip=1.17W & Knee=0.177W. (b) The robotic leg's timewise variation of Actuator Torque variation when it followed the Rectangular trajectory simulated using Adams. The maximum value of the torque of actuators at Hip=4.44N-m & Knee=1.732N-m
4.3. Triangle trajectory

Figure 14 shows the triangular trajectory. The key advantage of legged systems over wheeled robots is their ability to traverse through uneven terrains and perform precise manoeuvres in confined spaces. A triangular trajectory is analysed for a situation where the horizontal distance needed to be travelled is minimum while maximum height needs to traverse Figure 14 shows the triangular trajectory. The triangular trajectory doesn’t have a sharp point at its top. It is to avoid any discontinuity caused in actuator velocities throughout the motion of the leg. A step length of 0.175m is assumed. The maximum height to be traversed along the trajectory is assumed to be 0.211m. The timewise variation of angular acceleration, angular velocity, power and torque of the leg when it follows this trajectory is shown in figures 15 and 16. The trajectory was plotted using the following equation,

\[ f(x) = 3.75 \times 10^{-6}x^4 - 0.00131x^3 + 0.1172x^2 - 0.518x \]

Figure 15. (a) The robotic leg’s timewise variation of Angular velocity when it followed the Triangular trajectory simulated using Adams. The maximum value of the Angular Velocity of actuators at Hip=17.65deg/S & Knee=38.93deg/S (b) The robotic leg’s timewise variation of Angular Acceleration when it followed the Triangular trajectory simulated using Adams. The maximum value of the Angular Acceleration of actuators at Hip=21.68deg/S² & Knee=47.84deg/S².

Figure 16. (a) The robotic leg’s timewise variation of power consumption when it followed the Triangular trajectory simulated using Adams. The maximum value of the power consumption of actuators at Hip=1.164W & Knee=0.7339W (b) The robotic leg’s timewise variation of Actuator Torque variation when it followed the Triangular trajectory simulated using Adams. The maximum value of the Torque of actuators at Hip=4.241N-m & Knee=1.82779N-m
4.4. Ballistic trajectory

Figure 17 shows the ballistic trajectory of the leg. Since the ballistic trajectory doesn’t have any sharp corners, the issue of discontinuity is removed. For the motion using this trajectory, the leg will traverse a distance ‘S,’ which is the step length that can be achieved, and we assumed it to be 0.35m. At each step, the leg is literally thrown with a take-off angle $\theta$, $40^\circ$ was taken for our calculations. The timewise variation of angular acceleration, angular velocity, power, and torque of the leg when it follows this trajectory is shown in figures 18 and 19. The development of ballistic trajectory was done using the following equation,

$$f(x) = -1.736 \times 10^{-5} x^3 + 3.70 \times 10^{-3} x^2 + 0.3256 x$$

Figure 17: Ballistic Trajectory

Figure 18. (a) The robotic leg’s timewise variation of Angular Acceleration when it followed the Ballistic trajectory simulated using Adams. The maximum value of the Angular Acceleration of actuators at Hip=8.89deg/S$^2$ & Knee=20.74deg/S$^2$. (b) The robotic leg’s timewise variation of Angular velocity when it followed the Ballistic trajectory simulated using Adams. The maximum value of the Angular Velocity of actuators at Hip=9.98deg/S & Knee=23.29deg/S

Figure 19. (a) The robotic leg’s timewise variation of power consumption when it followed the Ballistic trajectory simulated using Adams. The maximum value of the power consumption of actuators at Hip=0.3853W & Knee=0.2109W. (b) The robotic leg’s timewise variation of Actuator Torque variation when it followed the Ballistic trajectory simulated using Adams. The maximum value of the torque of actuators at Hip=2.94237N-m & Knee=1.7205N-m
5. Conclusions

- The Kinematic Analysis of the leg showed the possible values of joint angles and the maximum step length the leg can achieve, which is 0.9m.
- The dynamic analysis of the leg resulted in obtaining the theoretical torque and power required for the actuator.
- The power analysis shows us that the power requirement from the hip actuator is a dominating factor during the swing phase.
- For the rectangular trajectory, the maximum value of power consumption of Hip joint came out to be 0.55W. For the Smooth Rectangle, the value is 1.17W. If we compare the knee joints, then for rectangular trajectory, the maximum power consumption is 0.15W, and smoothed rectangle is 0.177W. The rectangular trajectory draws less power when compared to smoothed rectangle, but a sharp change along the trajectory is noticed.
- The triangular trajectory shows high power consumption requirements with a maximum value of 1.164 W for the Hip actuator and 0.733W for the knee actuator. So, we can conclude that vertical movements for walking machines demand high power consumption.
- Ballistic trajectory demands the least power when compared to other trajectories. The hip actuator demands a power of 0.385 W, and the knee actuator draws a maximum power of 0.219W.
- The results from theoretical analysis and simulation are proved to be similar.

Appendix A

D-H Table of the Kinematic Analysis

|             | Joint angle | Offset | Angle of twist | Link length |
|-------------|-------------|--------|----------------|-------------|
| 0 Frame     | θ₁          | 0      | 0              | L₁          |
| 1/2 Frame   | θ₂          | 0      | 0              | L₁          |

\[
^0T = \text{ROT}(\vec{Z}, \theta_1). \text{TRANS}(\vec{X}, L_1)
\]

\[
^1/2T = \text{ROT}(\vec{Z}, \theta_2). \text{TRANS}(\vec{X}, L_2)
\]

\[
^0T = ^0T \cdot \frac{1}{2}T
\]

Appendix B

Actuator torques

Torque can be calculated using Lagrangian equations and solving these will result in the following equations for the torques,

\[
T_1 = D_1 \dot{\theta}_1^2 + D_2 \dot{\theta}_2^2 + D_3 \dot{\theta}_2^2 + 2D_3 \dot{\theta}_1 \dot{\theta}_2 + gD_4
\]

\[
T_2 = D_2 \dot{\theta}_1^2 + D_3 \dot{\theta}_2^2 - D_3 \dot{\theta}_1^2 - gD_6
\]

Where,

\[
D_1 = I_4 + 0.25L_e - I_4 \cos \theta_2 + I_1 + I_5
\]

\[
D_2 = 0.25L_e - 0.5I_1 \cos \theta_2 + I_2
\]

\[
D_3 = 0.5I_1 \sin \theta_2
\]
D_4 = M_a L_1 \cos \theta_1 - 0.5M_b L_2 \cos (\theta_1 + \theta_2)
D_5 = 0.25I_c + I_2
D_6 = 0.5M_b L_2 \cos (\theta_1 + \theta_2)
M_a = M_1 + M_m + M_2 + M_f
M_b = M_2 + 2M_f
I_a = (0.25M_1 + M_m + M_2 + M_f) L_1^2
I_b = (M_2 + 2M_f) L_1 L_2
I_c = (M_2 + 4M_f) L_2^2

References

[1] Teresa Zielenska 2004 Development of walking machines; historical perspective (Int. Symp. on History of Machines and Mechanisms, Springer) pp 357-70.
[2] D McCloy 1990 The power consumption of mechanical legs (Mech Mach. Theory) 26 pp 185-196.
[3] D McCloy 1989 The effects of foot trajectory on the power required to swing a mechanical leg (Journal of Mechanical Engineering Science) C 203 pp 419-28.
[4] D McCloy 1989 Step lengths and efficiencies of serial-operated and parallel-operated mechanical legs (Robotica) 8 pp 23-9.
[5] Chih-Chung Ko, Shen-Chiang Chen, Cheng-Hsin Lee and Pei-Chun Lin 2010 Trajectory Planning and Four-leg Coordination for Stair Climbing in a Quadruped Robot (IEEE/RSJ Int. Conf. on Intelligent Robots and Systems 18 -Taiwan) pp 5335-40.
[6] D McCloy 1990 Some comparisons of serial-driven and parallel-driven manipulators (Robotica) 8 pp 355-362.
[7] Alain Segundo Potts and Jos´e Jaime da Cruz 2010 Kinematic analysis of quadruped robot (IFAC Proc. Journal) 43 pp 261-66.
[8] Byeonghun Na, Hyunjin Choi and Kyounghul Kong 2013 Design of an actuation system for a high-speed quadruped robot, cheetaroid-I (IFAC Proc. Journal) 46 pp 165-9.
[9] Garcia-López,M.C.a*,Gorrostieta-Hurtado,E.a,Vargas-Soto,E.a,Ramos-Arreguin, J.M.a, Sotomayor-Olmedo,A.a and Moya Morales, J.C 2012 Kinematic analysis for trajectory generation in one leg of a hexapod robot (Procedia Technology) 3 pp 342-350.
[10] Chegu Viswanadhi, Abhishek Sarkar and Pramod Sreedharan 2019 Kinematic and dynamic simulation of biped robot locomotion on multi-terrain surfaces (IOP Conference Series: Materials Science and Engineering) p 577
[11] Ganesha Udupa, Pramod Sreedharan and Aditya K 2010 Robotic Gripper Driven by Flexible Microactuator Based on an Innovative Technique (Advanced robotics and its social impacts IEEE workshop) p 111
[12] S. Pal, I. V and P. Sreedharan 2019 Design and Analysis of Asymmetric Bellow Flexible Pneumatic Actuator (ABFPA) (Journal of Robotics) p 859
[13] Shyam Sreenivasan and Pramod Sreedharan 2018 Design and Analysis of Soft Pneumatic Actuator Powered Fin for Manta-ray Bot (RTEICT) p 2234
[14] Vaani, S. Pal, V. Indu and P. Sreedharan 2019 Optimal Force Control and Analysis of Soft Pneumatic Actuator (ICICICT-2019) p 501
[15] Aswath Suresh, Nitin Ajithkumar, Sreekuttan T. Kalathil, Abin Simon, V.J. Unnikrishnan, Deepu P. Mathew, Praveen Basil, Kailash Dutt, Ganesha Udupa, C.M. Hariprasad, Maya Menon, Arjun Balakrishnan, Ragesh Ramachandran, Arun Murali and Balakrishnan Shankar 2017 *An Advanced Spider-Like Rocker-Bogie Suspension System for Mars Exploration Rovers* (Robot Intelligence Technology and Applications 4) p 423

[16] https://www.mathworks.com/videos/trajectory-planning-for-robot-manipulators1556705635398.html

[17] https://link.springer.com/article/10.1007/s11786-012-0123-8