Dissipative Field Theory with Caldeira-Leggett Method and its Application to Disoriented Chiral Condensation

Hiroyuki Yabu, Satoshi Nozawa and Toru Suzuki*

Department of Physics, Tokyo Metropolitan University, Hachioji Tokyo 192, Japan

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Abstract

The effective field theory including the dissipative effect is developed based on the Caldeira-Leggett theory at the classical level. After the integration of the small field fluctuations considered as the field radiation, the integro-differential field equation is given and shown to include the dissipative effects. In that derivation, special cares should be taken for the boundary condition of the integration. Application to the linear sigma model is given, and the decay process of the chiral condensate is calculated with it, both analytically in the linear approximation and numerically. With these results, we discuss the stability of chiral condensates within the quenched approximation.

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I. INTRODUCTION

Recently, there has been great interests in the collective phenomena that might be observed after the heavy ion collisions, especially in the state called the disoriented chiral condensate (DCC) \[1,2\].

The quantum chromodynamics (QCD) that describes the strong-interaction physics has the chiral symmetry, $SU_L(2) \otimes SU_R(2)$, that is explicitly broken with the order of the pion mass $m_\pi$. The behavior of the vacuum state can be represented by the order-parameters $\langle \sigma \rangle = \langle \bar{q}q \rangle$ and $\langle \pi \rangle = \langle \bar{q}\gamma_5 q \rangle$ where $q$ is a quark field. At zero-temperature, the effective potential $V_{\text{eff}}(\langle \sigma \rangle, \langle \pi \rangle)$ has a bottom circle with the radius $f_\pi$, so that this symmetry is spontaneously broken and the vacuum state takes $\langle \sigma \rangle \neq 0$ and $\langle \pi \rangle = 0$.

The DCC state is defined as that on the bottom circle of $V_{\text{eff}}$ but with $\langle \pi \rangle \neq 0$, and expected to be produced after the high-energy proton or heavy-ion collision. About the formation of the DCC, there has been a lot of discussions, especially concerning with the quenching or annealing scenarios \[2–6\], but the present situation is still controversial.

As has been pointed out in \[7\], another important problem is the decay process of the DCC state. Because of the explicit breaking of the chiral symmetry, the DCC state is only meta-stable and considered to decay into the true vacuum with radiating many number of coherent pions \[8\]. If the life-time of the DCC is very short compared with the formation time, it will be very difficult to observe the DCC state through the pion signal.

The entire process of the DCC can be formulated as the dissipative system of the collective coordinates (the order parameters $\langle \sigma \rangle$ and $\langle \pi \rangle$) under the back-ground pion radiation, so that we can analyze the DCC, especially the decay of it, with the method of the dissipative theory in the semiclassical dynamics.

There has been a long history to derive the equation of motion with the radiative damping in the classical electrodynamics and the thermo-statistical theory. The motion of the charged particle with the electromagnetic radiation had been much discussed since pre-quantum mechanical era \[9\]. Here we should mention that the advanced Green function for the
radiation was introduced by Dirac [10] to cancel the divergent electron’s radiation mass and was justified by Wheeler and Feynman [11].

In the thermo-statistical theory, the dissipative dynamics has been considered with the Brownian motion and the Langevin equation has been one of the most important tool to attack the problem [12]:

\[ M \ddot{q} + \gamma \dot{q} + V'(q) = R(t), \]  

(1)

where \( q \) and \( R \) are the particle coordinate and the fluctuating force and \( \gamma \) is the dissipative coefficient. However, in the Langevin theory, the dissipative coefficient is the given parameter and can not be fixed by itself.

In 1983, Caldeira and Leggett proposed a method to derive the dissipation from the microscopic theory [13]. They start with the system-plus-reservoir model (the interacting system of the collective coordinates and the background freedom) and, with integrating out the background freedom, the dissipative equation for the collective coordinates can be obtained. This method was formulated in the particle dynamics, and has been applied for the polaron motion in a crystal (acoustic polaron), diffusion of charged interstitials in normally conducting metals and the dynamics of Josephson junction [12]. In the latter case, the reduction of the tunneling probability was predicted for the macroscopic quantum tunneling effect between superconductor and insulator with the Caldeira-Leggett theory and it was really confirmed experimentally [14][15].

The application of the Caldeira-Leggett theory to the nuclear/hadron physics is very interesting: especially to the phenomena with the multi-particle production such as the emission of meson, photon and so on. The energy of the system (the excited nucleus or the fireballs, for example) is considered to decrease “dissipatively” with the particle emission and the Caldeira-Leggett theory can be applied to them, where the reservoir corresponds to the emitted particle. Another interesting application is for the DCC phenomenon. The DCC process is described by the non-linear sigma model, and the collective coordinates and the reservoir are just the condensates \( \langle \sigma \rangle \) and \( \langle \pi \rangle \), and the radiative pion through the formation
and decay processes.

In those phenomena, the dynamical variables of the system are the fields corresponding to the emitted particles. For the application to such a process, we have to extend the Caldeira-Leggett theory for the field dynamics. That extension is very interesting by itself because it produces the dissipative field equation.

In this paper, we formulate the Caldeira-Leggett theory in the field dynamics, and apply it to the DCC process. The extension of the theory for the field theory will be given in the first half of this paper as generally as possible. The resultant formulation is applied for the non-linear sigma model, and the existence of the dissipative effects is shown analytically and numerically. In the final part of the paper, the decay process of the DCC is discussed qualitatively within the present formulation.

II. DISSIPATIVE FIELD EQUATION WITH CALDEIRA-LEGGETT THEORY

To illustrate the application of the Caldeira-Leggett theory to the field equation, we take a simple system of the order-parameter $\Phi(x)$ and the background field $f(x)$ that describes the particle with the mass $m$:

\[
L = \frac{1}{2}(\partial \Phi)^2 - V(\Phi) + G(\Phi)f + \frac{1}{2}[(\partial f)^2 - m^2 f^2].
\]  

(2)

where $V(\Phi)$ is the effective potential for $\Phi$ and $G[\Phi]f$ gives the interaction between $\Phi$ and $f$. Taking variations with $\Phi$ and $f$, we obtain the field equations from (2):

\[
\partial^2 \Phi + \frac{\delta V}{\delta \Phi} + \frac{\delta G}{\delta \Phi} f = 0,
\]  

(3a)

\[
(\partial^2 + m^2)f + G[\Phi] = 0.
\]  

(3b)

They are easily checked to satisfy the energy-momentum conservation.

Eq. (3b) is formally solved with the Green functions $G(x - y; m)$ that satisfies

\[
(\partial^2 + m^2)G(x - y) = -\delta^4(x - y).
\]  

(4)
The fluctuation $f$ is then written by

$$f(x) = \int d^4y G(x - y; m)G[\Phi(y)],$$

(5)

and, substituting it into (3a), we obtain the integro-differential equation for $\Phi$

$$\partial^2\Phi(x) + \frac{\delta V[\Phi(x)]}{\delta \Phi} + \frac{\delta G[\Phi(x)]}{\delta \Phi} \int d^4y G(x - y; m)G(\Phi[y]) = 0.$$  \hspace{1cm} (6)

This equation is essentially equivalent with (3a) and (3b).

The Green function in (6) can be written by

$$G(x - y; m) = \int \frac{d^4k}{(2\pi)^4} G(k; m)e^{-ik(x-y)},$$  \hspace{1cm} (7)

where $G(k; m)$ is the Fourier transform of $G(x - y; m)$ and generally takes complex value. In the Caldeira-Leggett theory, the imaginary component of $G(k; m)$ can be considered as the dissipative term caused by the emission of particles for the background field $f$. It is suggested by the analysis of the simple Newtonian equation including the dissipative terms: $m\ddot{x} + \eta \dot{x} = F$; After Fourier transformation, the dissipative term $\eta \dot{x}(t)$ gives that of pure imaginary $i\eta \omega x(\omega)$ with the spectral function $x(\omega)$ for $x(t)$. Further, we can interpret this dissipation as the particle emission because $\text{Im}G(k; m)$ includes $\delta^4(k^2 - m^2)$ (on-mass shell). The real part $\text{Re}G(k; m)$ is considered to give the modification of the effective potential that comes from the background absorption and emission, and can be absorbed into $V(\Phi)$ by re-definition. Thus we can drop it because the parameters in $V(\Phi)$ is adjusted from the experimental data. Finally, we obtain the dissipative field equation corresponding to (6):

$$\partial^2\Phi(x) + \frac{\delta V[\Phi(x)]}{\delta \Phi} + \frac{\delta G[\Phi(x)]}{\delta \Phi} \tilde{f} = 0,$$  \hspace{1cm} (8)

where

$$\tilde{f}(x) = \int d^4y \int \frac{d^4k}{(2\pi)^4} i\text{Im}G(k; m)e^{-ik(x-y)}.$$  \hspace{1cm} (9)

1 The homogeneous part can be dropped safely because it can be absorbed with proper adjustment of the boundary condition.
Eq. (9) is still ambiguous because a variety of Green functions (advanced, retarded, causal and so on) can be used in it and give solutions with different boundary conditions. We should select the proper Green functions that shows the dissipative effect when \( t \to \infty \) with real \( \tilde{f} \). In the application to the linear sigma model, it will be shown that the advanced Green function satisfies both conditions and a proper selection.

### III. APPLICATION TO LINEAR SIGMA MODEL

We apply the formulation given in the last chapter to the linear sigma model by Gel-Mann and Lévy [16] and discuss the dynamical behavior of the order parameter of the \( SU(2) \) chiral symmetry.

The Lagrangian of the sigma model is

\[
\mathcal{L} = \frac{1}{2} \left[ (\partial \sigma)^2 + (\partial \pi)^2 \right] - V(\sigma, \pi),
\]

where \( \sigma \) and \( \pi = (\pi^1, \pi^2, \pi^3) \) are the sigma and the pion fields. In this paper, we discuss only the phenomena related with the neutral pion so that \( \pi \) in (10) is a single field from now on. The application to the charged pion goes in the same way but we have to modify the Lagrangian (10) to include the photon degrees of freedom. The effective potential \( V(\sigma, \pi) \) is taken to the forth order of the fields:

\[
V(\sigma, \pi) = \frac{\lambda}{4} (\sigma^2 + \pi^2 - \nu)^2 - H\sigma,
\]

where the parameters \( \lambda, \nu \) and \( H \) are written by

\[
\lambda = \frac{m^2_{\sigma} - m^2_{\pi}}{2f_\pi^2}, \quad \nu^2 = f_\pi^2 \frac{m^2_{\sigma} - 3m^2_{\pi}}{m^2_{\sigma} - m^2_{\pi}}, \quad H = f_\pi m^2_{\pi},
\]

with the sigma and pion mass \( m_{\sigma,\pi} \) and the pion decay constant \( f_\pi \).

The last term in (11) breaks is the chiral symmetry explicitly and produce the finite pion mass. It is the simplest potential that describes the explicit symmetry breaking phenomena, and consistent with the low-energy theorem.

The fields \( \sigma \) and \( \pi \) can be divided into two parts:
\[ \sigma = \langle \sigma \rangle + \delta \sigma, \quad \pi = \langle \pi \rangle + \delta \pi, \quad (13) \]

where \( \langle \sigma \rangle \) and \( \langle \pi \rangle \) corresponds to the order parameter of the chiral symmetry for the (disoriented) condensed state \( \Psi \): \( \langle \sigma \rangle = \langle \Psi | \bar{q}q | \Psi \rangle \) and \( \langle \pi \rangle = \langle \Psi | \bar{q} \gamma_5 q | \Psi \rangle \) with the quark field \( q = (u, d) \). (The condensed states may be described with the coherent or the squeezed one as quantum states. \[17\]) The \( \delta \sigma \) and \( \delta \pi \) are the fluctuations around them, and represent the sigma and pi meson degrees of freedoms. The decomposition (13) is essentially the same one in Bogoliubov theory for the weak-interacting Bose liquid. In that theory, the fluctuation represents elementary excitations (phonon/roton) that cause the dissipative effects in superfluids \[18\].

Minimizing the effective potential \( V \), we obtain the vacuum state \( |0\rangle \) with which the order parameters become

\[ \langle \sigma \rangle = \langle 0 | \sigma | 0 \rangle \equiv f_\pi, \quad \langle \pi \rangle = \langle 0 | \pi | 0 \rangle = 0, \quad (14) \]

and the fluctuations \( \delta \sigma \) and \( \delta \pi \) describes the mesons with the mass \( m_\sigma \) and \( m_\pi \) each other.

Now we consider the dynamical behaviors of the disoriented-condensed vacuum state \( |\Psi\rangle \neq |0\rangle \) (\( \langle \sigma \rangle \neq f_\pi \) and \( \langle \pi \rangle \neq 0 \)) as the \( \Phi \) in the last chapter and the fluctuations \( \delta \sigma \) and \( \delta \pi \) as the background field \( f \). Substituting Eq. (13) into (10), we obtain

\[ L = \frac{1}{2} \left\{ [\partial (\langle \sigma \rangle)^2 + (\partial \langle \pi \rangle)^2] - V(\langle \sigma \rangle, \langle \pi \rangle) + H(\langle \sigma \rangle) \right. \]

\[ + \partial \langle \sigma \rangle \partial \delta \sigma + \partial \langle \pi \rangle \partial \delta \pi \]

\[ + \frac{1}{2} \left\{ (\partial \delta \sigma)^2 - m_\sigma^2 \delta \sigma^2 \right\} + \frac{1}{2} \left\{ (\partial \delta \pi)^2 - m_\pi^2 \delta \pi^2 \right\}, \]

(15)

where we assume that the background fields are small and describe the real sigma and pi meson degrees of freedom, so that we dropped \( \mathcal{O}(\delta \sigma^3, \delta \pi^3) \)-terms and the \( \delta \sigma^2 \) and \( \delta \pi^2 \) terms are adjusted to be the mass term with the real pion and sigma mass \( m_\sigma \) and \( m_\pi \) as in (2). In this paper, we discuss the case where the condensates are on the chiral circle: \( \langle \sigma \rangle^2 + \langle \pi \rangle^2 = \nu^2 \), so that \( \langle \sigma \rangle \) and \( \langle \pi \rangle \) can be parameterized with the chiral angle \( \phi \):
\[ \langle \sigma \rangle = \nu \cos \phi, \quad \langle \pi \rangle = \nu \sin \phi. \]  

(16)

Substituting Eq. (16) into (15), it is obtained that

\[
L = \frac{\nu^2}{2} (\partial \phi)^2 + H \nu \cos \phi \\
+ \nu \partial \cos \phi \partial \delta \sigma + \nu \partial \sin \phi \partial \delta \pi \\
+ \frac{1}{2} \left[ (\partial \delta \sigma)^2 - m^2 \delta \sigma^2 \right] + \frac{1}{2} \left[ (\partial \delta \pi)^2 - m^2 \delta \pi^2 \right].
\]

(17)

Taking the variation with \( \phi, \delta \sigma \) and \( \delta \pi \), we obtain a set of Euler-Lagrange equations:

\[
\nu^2 \partial^2 \phi + H \nu \sin \phi - \nu \sin \phi \partial^2 \delta \sigma + \nu \cos \phi \partial^2 \delta \pi = 0,
\]

(18a)

\[
(\partial^2 + m^2_{\sigma}) \delta \sigma + \nu \partial^2 \cos \phi = 0,
\]

(18b)

\[
(\partial^2 + m^2_{\pi}) \delta \sigma + \nu \partial^2 \sin \phi = 0,
\]

(18c)

which correspond to (3a) and (3b).

Now we can apply the formulation developed in the last chapter to (18a), (18b) and (18c), we obtain the dissipative field equation to (10):

\[
\partial^2 \phi(x) + \frac{H}{\nu} \sin \phi - \nu \sin \phi \partial^2 \delta \tilde{\sigma} + \nu \cos \phi \partial^2 \delta \tilde{\pi} = 0,
\]

(19)

where \( \delta \tilde{\sigma} \) and \( \delta \tilde{\pi} \) are the similar functions as (3).

For \( G(x - y; m) \), we take the advanced Green function

\[
G_{\text{adv}}(x - y; m) = \int \frac{d^4 k}{(2\pi)^4} \frac{e^{-ik(x-y)}}{k^2 - m^2 - i\epsilon \text{sgn}(k^0)}.
\]

(20)

The reality of \( \delta \tilde{\sigma} \) and \( \delta \tilde{\pi} \) can be checked easily by the direct calculation. In the next chapter, we will show that the advanced Green function really gives the dissipative solutions. From (20), we can read off the imaginary part of \( G(k; m) \):

\[
i \text{Im} G(k; m) = i\pi \text{sgn}(k^0) \delta^4(k^2 - m^2).
\]

(21)
so that $\delta \tilde{\sigma}$ and $\delta \tilde{\pi}$ become
\begin{align}
\partial^2 \delta \tilde{\sigma}(x) &= \frac{1}{2} \int_0^\infty dy^0 \int d^3 y \delta(x - y; m_\sigma) \partial_y^2 \cos \phi(y), \quad (22a) \\
\partial^2 \delta \tilde{\pi}(x) &= \frac{1}{2} \int_0^\infty dy^0 \int d^3 y \delta(x - y; m_\sigma) \partial_y^2 \sin \phi(y), \quad (22b)
\end{align}
where $\delta(x - y; m)$ is the invariant delta function:
\begin{equation}
\delta(x - y; m) = \frac{1}{v(2\pi)^3} \int d^4 k \delta^4(k^2 - m^2) \text{sgn}(k^0) e^{-ik(x - y)}. \quad (23)
\end{equation}

**IV. SOLUTION OF DISSIPATIVE FIELD EQUATION**

**A. Asymptotic Behavior of Solutions**

We consider the asymptotic behavior of the solutions that satisfy Eq. (19) and show the dissipative nature of it.

The order parameter $\phi(x)$ is assumed to decrease when $t \to \infty$, so that it can be regarded small. Then we can expand Eq. (19) about $\phi$ and approximate to the first order (the linear approximation). Instead of $\phi(t, x)$, we use the Fourier component:
\begin{equation}
\phi_k(t) = \frac{1}{(2\pi)^3} \int d^3 x \phi(t, x) e^{-ikx}, \quad (24)
\end{equation}
for which Eq. (19) is approximated to be
\begin{equation}
\ddot{\phi}_k(t) + k^2 \phi_k(t) + \frac{1}{2} \frac{m_\pi^2}{\omega_k} \int_t^\infty ds \sin \omega_k(t - s) \left\{ \ddot{\phi}_k(s) + k^2 \phi_k(s) \right\} = 0, \quad (25)
\end{equation}
with $\omega_k = \sqrt{m_\pi^2 + k^2}$ and $\alpha = f_\pi/\nu \sim 1.05$. It should be noted that no correlations exist among different-momentum modes in (25) in the linear approximation.

\footnote{This slight shift from one is resulted from the deformation of the chiral circle because of the explicit symmetry breaking.}
Substituting the ansatz $\phi_k(t) = e^{-a(k)t}$ into (25), we obtain the characteristic equation for the index $a(k)$:

$$a(k)^4 + (2\kappa + 1/2 + \alpha)m_\pi^2 a(k)^2 + \left\{\kappa^4 + (1/2 + \alpha)\kappa^2 + \alpha\right\}m_\pi^4 = 0,$$

(26)

with $\kappa = |k|/m$. One of the solutions can be written as $a(k) = p(\kappa) + iq(\kappa)$:

$$p(\kappa) = \frac{m_\pi}{2}\sqrt{2\rho(\kappa) - (2\kappa^2 + 1/2 + \alpha)}, \quad q(\kappa) = \frac{m_\pi}{2}\sqrt{2\rho(\kappa) + (2\kappa^2 + 1/2 + \alpha)},$$

(27)

where $\rho(\kappa) = \sqrt{\kappa^4 + (1/2 + \alpha)\kappa^2 + \alpha}$. It can be shown easily that $p(\kappa)$ takes real and positive value when $\kappa \geq 0$, so that the asymptotic behavior of $\phi_k$ is found to be

$$\phi_k(t) \sim e^{-p(\kappa)t} \sin[q(\kappa)t + \delta], \quad (t \to \infty)$$

(28)

with the constant phase. Summarizing the whole results, we obtain the asymptotic differential equation to (19):

$$\ddot{\phi}_k - k^2 \phi_k + \gamma \dot{\phi}_k + \sqrt{\alpha} \sin \phi \sim 0,$$

(29)

where the dissipative coefficients are $\gamma = 2p(\kappa)$. The momentum dependence of $p(\kappa)$ is shown in Fig. 1.

**B. Numerical Results**

The fields $\delta \tilde{\sigma}$ and $\delta \tilde{\pi}$ defined by (22a) and (22b) satisfy the differential equations:

$$(\partial^2 + m_\pi^2)\delta \tilde{\sigma} + \partial^2 \cos \phi = 0, \quad (\partial^2 + m_\pi^2)\delta \tilde{\pi} + \partial^2 \sin \phi = 0.$$  

(30)

Hence the solutions of the original integro-differential equations are given as those of three differential equations, (19) and (30).

In this paper, we show the numerical solutions for two cases: the uniform and the expanding solutions. For physical quantities, we took:

$$f_\pi = 92.5 \text{ MeV}, \quad M = 940 \text{ MeV}, \quad m_\pi = 135 \text{ MeV},$$

(31)
and $m_\sigma = 600$ MeV was used for the mass of sigma meson. With these values, the parameters in (3b) are fixed as $\lambda = 20.0$ and $\nu = 87.4$ MeV, each other.

1) **Uniform solution.**

The solution that is uniform for the space-dependence is characterized by $\phi = \phi(t)$ (and correspondingly, $\delta \tilde{\sigma} = \delta \tilde{\sigma}(t)$ and $\delta \tilde{\pi} = \delta \tilde{\pi}(t)$). In this case, eqs. (19) and (30) are reduced to the ordinary differential equations, which can be easily solved. The numerical results are shown in Fig. 2 where the scaled time $\xi = m_\pi t$ has been used. Herein, the damping oscillation behavior is easily confirmed that is proved analytically in the last chapter.

The dissipating behaviors can be read off in the phase diagram (Fig. 3), too, where each lines are the phase trajectories to these solutions; the spiral pattern around the origin show that they behave like the damping oscillator asymptotically.

For quantitative check, we consider the damping rigid oscillator:

$$\ddot{\phi} + \gamma \dot{\phi} + \sin \phi = 0,$$

with the damping coefficient $\gamma$ consistent with the dissipative coefficient $p(k)$ in (27):

$$\gamma = 2p(0) = 0.7m_\pi.$$

(32)

The phase diagrams for (32) are given in Fig. 4. The trajectories in Fig. 3 are found to behave similar with those in Fig. 4, especially in the asymptotic region (close to the origin). In the non-asymptotic region (far from the origin), the trajectories in Fig. 3 have the modulations around those of the damping rigid oscillator. They come from the non-linear dissipating behavior in (19) and (30), which is more effective in the non-asymptotic region.

Through the comparison with the damping rigid oscillator, we can also realize the complicated behaviors at $(\phi = \pm \pi, \dot{\phi} = 0)$ in Fig. 3. They are just the turning points of the rigid oscillator, and the trajectories around them are changed in chaotic manner under the small perturbation.

2) **Expanding solution.**
With putting $\phi = \phi(\tau = \sqrt{t^2 - x^2})$ in (19) and (30), we get the expanding solution in $x$-direction (uniform in other directions). Originally, this type of solution is given by Blaizot and Kryzwicki [19] with no dissipative effect.

The numerical solution is given as the rigid line in Fig. 5, where the scaled local time $\xi = m_\pi \tau$ was used. In this figure, we can find that the expanding solution is dumped faster than the uniform solution (shown as the dotted line in Fig. 4). To realize the expansion effect, we study the differential equation that Blaizot and Kryzwicki solved. In our notation, it becomes

$$\phi'' + \frac{1}{\xi} \phi' + \sin \phi = 0,$$

(34)

where the differentiations are for $\xi = m_\pi \tau$. The second term in (34) is proportional to the time-derivative of $\phi$, and has the effect of dissipation with the time-dependent dissipative coefficient $1/\xi$. (This effect is not a real dissipation, but a smearing of $\phi$ brought by the volume expansion.) In the present case, this smearing effect has an additional effect with the real dissipative effect (represented by the second term in (32) asymptotically), and it causes the faster damping in the expanding solution. The smearing effect is found to be more effective in the non-asymptotic region, because the effective dissipating coefficient is inversely proportional with the local time $\tau$.

V. SUMMARY AND DISCUSSIONS

We have formulated the dissipative field theory with applying the Caldeira-Leggett method. Explicit calculations have been done for the linear sigma model and the resultant field equations have been shown to have the dissipative properties with both the analytic and numerical ways.

As a phenomenological application, we discuss about the disoriented chiral condensate that is expected to appear after the high-energy hadron collision. In the standard picture, the chiral symmetry is considered to be broken spontaneously at zero temperature and its
order parameters take the expectation values: \( \langle \sigma \rangle \neq 0 \) and \( \langle \pi \rangle = 0 \). The DCC is also in the broken phase, but is defined to be the state where the order parameters take different values: \( \langle \sigma \rangle \neq 0 \) and \( \langle \pi \rangle \neq 0 \). Because of the explicit chiral-symmetry breaking, the DCC state has higher energy, and it should be observed as a metastable state.

As written in the introduction, there exist many controversies about the formation process of the DCC state after rebreaking of the chiral symmetry, but we concentrate on the decay process of it in the remaining of this paper.

We consider the neutral DCC state with \( \langle \pi^0 \rangle \neq 0 \) and \( \langle \pi^\pm \rangle = 0 \). The main decay process of this state should be the \( \pi^0 \)-radiation, so that we can apply the above-developed formula with regarding \( \pi \) in (29) as \( \pi^0 \). The life-time \( \tau_L \) can be estimated with the dissipative constant \( p(0) \) in (27):

\[
\tau_L = 1/p(0) \sim 3m_{\pi}^{-1}.
\]  

(35)

If we take the quenching scenario for the DCC formation, the formation time \( \tau_R \) has been estimated to be \[2]

\[
\tau_R \sim \sqrt{2m_{\sigma}^{-1}} \sim 0.3m_{\pi}^{-1}.
\]  

(36)

It tells us that the life-time \( \tau_L \) in (35) is ten times longer than the formation time, so that the neutral DCC state will be enough metastable.

In this paper, we considered the case that the order parameters move only on the chiral circle \( \langle \sigma \rangle^2 + \langle \pi \rangle^2 = f_\pi^2 \), so that we could not consider the DCC formation process. The extension beyond the chiral circle will be given elsewhere.

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\[3\] For the charged DCC state, photon degree of freedom is absolutely important and we have to extend our equations to include the dissipative effect by photon radiation.
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FIGURES

FIG. 1. The momentum dependence of the dissipative coefficient \( p(\kappa) \) given by (27). \( \kappa \) is the pion-mass scaled momentum \( \kappa = |k|/m_\pi \). The \( p(\kappa) \) is normalized at \( \kappa = 0 \) with the value \( p(0) = 0.35m_\pi \).

FIG. 2. A series of solutions of the integro-differential equation (19) for the space-independent uniform case \( \phi(x) \equiv \phi(t) \). The \( \xi \) is the pion-mass scaled time: \( \xi = m_\pi t \) and \( \phi(t) \) is a chiral angle of the order parameter. The initial conditions are given at \( \xi = 0 \) with \( \frac{d\phi}{d\xi} = 0 \).

FIG. 3. The trajectories of the integro-differential equation (19) for the space-independent uniform case \( \phi(x) \equiv \phi(t) \). The \( \phi \) and \( \frac{d\phi}{d\xi} \) are the chiral angle and the corresponding velocity with the scaled time \( \xi = m_\pi t \). The initial conditions of each trajectories are chosen as they behave asymptotically (28) with \( \delta = \frac{\pi n}{5} \) \((n = 0, \pm 1, \cdots, \pm 5)\).

FIG. 4. The trajectories of the dumped rigid oscillator (32). The dissipative coefficient is chosen to be consistent with the asymptotic value in the integro-differential equation (19): \( \gamma = 2p(0) = 0.7m_\pi \). The \( \phi \) and \( \frac{d\phi}{d\xi} \) are the chiral angle and the corresponding velocity with the scaled time \( \xi = m_\pi t \). The initial conditions of each trajectories are chosen as in 3.

FIG. 5. One-dimensional expanding (scaling) solutions for the chiral angle \( \phi(\xi) \) where \( \xi \) is a pion-mass scaled local time: \( \xi = m_\pi \tau = m_\pi \sqrt{t^2 - x^2} \). The rigid line is for the dissipative case: the solution of the integro-differential equation (19), and the dashed line is for the nondissipative case: the original Blaizot-Kryzwicki solution of (14).