INTERPOLATION-INDUCED REFLECTION ARTIFACT IN THE REASSIGNMENT TECHNIQUE – WITH ANESTHESIA EXAMPLE

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ABSTRACT. While extracting the temporal dynamical features based on the sharp time-frequency analyses, like reassignment and synchrosqueezing, attract more and more interest in medical data analysis, we should be careful about artifacts generated by interpolation schemes, in particular when the sampling rate is not significantly higher than the frequency of the oscillatory component we are interested in. In addition to formulating the problem, we show an example in anesthetic depth analysis with clear but undesirable artifacts.

Keywords: Reassignment method, synchrosqueezing transform, non-uniform sampling, cubic spline interpolation, instantaneous heart rate, ECG-derived respiratory signal.

1. INTRODUCTION

It has been widely accepted that several aspects of the health status could be well observed by analyzing recorded physiological time series. In particular, the time-varying oscillatory pattern inside the electrocardiogram (ECG) or respiratory signal contains abundant health information, like the heart rate variability (HRV) and breathing pattern variability (BPV) \[1,2\]. A popular and powerful way to study the time-varying oscillatory pattern inside a time series is the time-frequency (TF) analysis, which allows us to efficiently extract how a signal oscillates at each time. However, in many situations, the signal we are interested in can only be sampled at a rate which is not significantly high; in some cases the sampling is even non-uniform. Typical examples include the instantaneous heart rate (IHR) and ECG-derived respiratory (EDR) signal extracted from the ECG signal, in which case the sampling is non-uniform and determined by the heart rate. In order to apply the TF analysis to study these observed signals, a common practice is to apply the digital-to-analogue conversion to recover the original continuous signal. Although the TF analysis, including reassignment method (RM) and the synchrosqueezing transform (SST), have been successfully applied to several physiological problems \[3,4,5,6,7\], we should be careful about the analysis. Indeed, we found that in addition to enhancing the TF representation, these TF analysis techniques might also enhance the artifact generated by the interpolation scheme, which is further complicated by the sampling scheme. In this paper, we provide a series of numerical evidence, as well as a theoretical justification, to show the existence of the undesirable artifacts. We also provide real examples from the anesthesia.

2. THE REFLECTION EFFECT

Fix \(0 < \varepsilon \ll 1\), \(\varepsilon \ll c_1 < c_2\). Consider a functional class, \(\mathcal{A}_{c_1,c_2}^{\varepsilon}\) \[8,9\], which contains functions of the form \(a(t)\cos(2\pi \phi(t))\), where \(a \in C^1 \cap L^\infty\), \(\phi \in C^2\), \(c_1 \leq a(t) \leq c_2\), \(c_1 \leq \phi'(t) \leq c_2\), \(|a'(t)| \leq \varepsilon \phi'(t)\) and \(|\phi''(t)| \leq \varepsilon \phi'(t)\) for all time \(t \in \mathbb{R}\). We call \(\phi'(t)\) the instantaneous frequency (IF) and \(a(t)\) the amplitude modulation (AM) of \(f(t)\). In other
words, all functions in $\mathcal{A}^{c_1,c_2}_A$ behave like a harmonic function locally, and the deviation from being a harmonic function is controlled by $\varepsilon$. We call a function in $\mathcal{A}^{c_1,c_2}_A$ an intrinsic mode type (IMT) function and $\mathcal{A}^{c_1,c_2}_A$ the adaptive harmonic model. The functions containing multiple IMT functions and more theoretical discussions could be found in [8,9]. This model has been applied to study the “non-stationary” physiological dynamics [10,7,4,11,12,13,6]. To simplify the discussion, we focus on functions with only one oscillatory component. Also note that the following discussion carries over into the signals with multiple IMT functions.

Take $f(t) = a(t)\cos(2\pi \phi(t)) \in \mathcal{A}^{c_1,c_2}_A$. Consider a monotonic increasing function $\psi \in C^1(\mathbb{R})$ so that $\psi'(t) > 0$ for all $t \in \mathbb{R}$. Define a sequence of sampling points $\{t_m\}_{m \in \mathbb{Z}}$ so that $t_m = \psi^{-1}(m)$. With this sampling scheme, we obtain samples $\mathcal{S} := \{t_m, f(t_m)\}_{m \in \mathbb{Z}}$. Note that if $\psi'(t) = k$, then the sampling is uniform and we get a sample every $1/k$ second. Also note that since in general $a(t)$ and $\phi'(t)$ are not constant, the application of Nyquist-Shannon theory is not efficient. Indeed, a generic function in $\mathcal{A}^{c_1,c_2}_A$ has a non-compact support in the Fourier domain. Thus, we consider another definition to describe the sampling scheme which reflects the momentary nature of dynamical analysis.

**Definition 2.1** (Instantaneous sampling rate, instantaneous Nyquist frequency). Take $\psi \in C^1(\mathbb{R})$ so that $\psi'(t) > 0$ for all $t \in \mathbb{R}$. We call $\psi'(t)$ the instantaneous sampling rate (ISR) and $\psi'(t)/2$ the instantaneous Nyquist frequency (INF).

Note that ISR and INF naturally generalizes the notion of sampling rate and Nyquist frequency – the higher the ISR is at a moment, the higher the sampling rate is around this moment. Throughout this paper, we would always assume that $\psi'(t) > 2\phi'(t)$; that is, at each time, we have at least two sampling points from an oscillation. Note that we could also naively generalize the notion of Nyquist rate to its instantaneous version.

**Definition 2.2** (instantaneous Nyquist rate). Take a signal $f(t) = a(t)\cos(2\pi \phi(t)) \in \mathcal{A}^{c_1,c_2}_A$. We call $2\phi'(t)$ the instantaneous Nyquist rate (INR) of the function $f(t)$.

Note that the INR reduces to the notion of Nyquist rate when $\phi(t)$ is linear and $a(t)$ is constant. With the adaptive harmonic model and the samples, we would like to study the underlying dynamical features, like the IF and AM of the signal. A common practice to convert the discretized sampling $\mathcal{S}$ to a continuous function is via an interpolation scheme. While there are different interpolation schemes, denote the interpolated function as $\tilde{f}_m$ to emphasize the dependence on the interpolation. Once we have a good approximation of the continuous function, we could apply different techniques to study the signal. We claim that while the INF is not significantly greater than the IF, the structured artifacts depending on the interpolation scheme will emerge. In particular, the IF of an artificial component in $\tilde{f}_m$ generated by the interpolation is a reflection of $\phi'$ associated with the INF.

Spectrogram short time Fourier transform (STFT) [8,9] is a variation of the reassignment technique [14,15,16], which sharpens the TF representation so that the oscillatory components could be extracted. Here we consider SST-STFT and reassigned STFT to demonstrate the claim.

We start from a harmonic function $f_1(t) = \cos(2\pi 2.5t)$ and sample it with the ISR $\psi'_1(t) = 6 + \frac{\left(\frac{t-80}{\pi}\right)^2}{80^2}$. Note that obviously the ISR is greater than the INR of $f$. Then, we demonstrate the reflection effect by applying different common interpolation schemes. We then run SST-STFT and reassigned STFT on the interpolated signal, where the interpolated signal is sampled uniformly at the 64 Hz. To avoid possible boundary effects, we sample the signal for 80 seconds. The results are shown in Figure 1. In this study, the TF representation $R \in \mathbb{C}^{n \times m}$ is displayed in the log scale. Precisely, we plot $\tilde{R} \in \mathbb{R}^{n \times m}$, where
INTERPOLATION-INDUCED REFLECTION ARTIFACT IN THE REASSIGNMENT TECHNIQUE – WITH ANESTHESIA EXAMPLE

Figure 1. The time-frequency (TF) representation of \( f_1(t) = \cos(2\pi 2.5t) \) and different interpolations from the sampling scheme \( \psi_1 \). The instantaneous frequency (IF) of \( f_1 \) is \( \phi' = 2.5 \). Top left: the SST-STFT result of \( f_1(t) \). Top middle: the SST-STFT result of the piecewise cubic Hermite interpolating polynomial interpolation. The instantaneous Nyquist frequency (INF), \( \psi'_1/2 \), is superimposed as a red dashed curve on it. In addition to IF \( \phi' \), we could see components with IF \( \psi'_1 - \phi' \) marked as (1), \( 2\psi' - \phi' \) marked as (2) and \( \psi' + \phi' \) marked as (3). Top right: the SST-STFT result of the cubic spline interpolation. Here we could see components with IF \( \psi'_1 - \phi' \) marked as (1) and \( \psi' + \phi' \) marked as (3). Bottom left: the reassigned STFT result of the cubic spline interpolation, where we could only see an extra component with IF \( \psi'_1 - \phi' \) marked as (1). Note that the artificial component with IF \( \psi'_1 - \phi' \) is the reflection of \( \phi' \) associated with the INF. Bottom right: from top to bottom we show the 40-th second slice of the TF representations shown in left, middle left, middle and middle right.

\[
\tilde{R}_{i,j} = \max\{10^{-2}, \log(1 + \min\{|R_{i,j}|, q \})\}, \text{ where } q \text{ is the 99.8\% quantile of all entries of } |R|.
\]

We next consider the same procedure on a non-harmonic function \( f_2(t) = (0.7 + t^{1.1}) \cos(2\pi(\pi t + 0.2 \cos(t))) \), whose AM and IF are \( a(t) = 0.7 + t^{1.1} \) and \( \phi'(t) = \pi - 0.2 \sin(t) \), respectively, and we take another ISR, \( \psi'_2(t) = 8 + 0.5 \cos(\pi t/10) \). Note that \( \psi'_2 \) is greater than INR of \( f_2 \). The result is shown in Figure 2. Note that we could see a clear reflected component associated with the INF in all the above cases, and the behavior depends on the setup.

Based on the numerics, we thus have the following claim:

Claim 2.1. Take \( f(t) = a(t) \cos(2\pi \phi(t)) \in \mathcal{A}^{1.1,c-2} \) and sample \( f(t) \) with the ISR \( \psi'(t) > 2\phi'(t) \). Fix an interpolation scheme \( m \) and the interpolated signal \( \tilde{f}_m(t) \). Then, we have

\[
\tilde{f}_m(t) = \sum_{k,l \in \mathbb{Z}} A_{m,k,l}(t) \left[ \cos(2\pi(l\phi(t) + k\psi(t))) + R_m(t) \right],
\]

where \( R_m(t) \) is the remainder term and \( A_{m,k,l}(t) \in \mathbb{R} \) and \( R_m(t) \) depend on \( f \) and \( m \).
Figure 2. The time-frequency representation of $f_2(t) = (0.7 + t^{1.1}) \cos(2\pi(\pi + 0.2\cos(t)))$ and different interpolations from the sampling scheme $\psi(t)$. The instantaneous frequency (IF) of $f_2$ is $\phi'(t) = \pi - 0.2\sin(t)$. Top left: $f_2(t)$ is shown as a gray curve with the non-uniform samples superimposed as black circles. The piecewise cubic Hermite interpolating polynomial (PCHIP) interpolation is shown as a red curve and the cubic spline (CS) interpolation is shown as the black curve. Top middle: the SST-STFT result of $f_2(t)$. Top right: the SST-STFT result of the PCHIP interpolation. The instantaneous Nyquist frequency (INF), $\psi'_2/2$, is superimposed as a red dashed curve on it. Bottom left: the SST-STFT result of the CS interpolation. Bottom right: the reassigned STFT result of the CS interpolation. Note that in the middle, middle right and right subfigures, in addition to $f_2(t)$, we could see components with IF $\psi'_2 - \phi'$ marked as (1), $2\psi'_2 - \phi'$ marked as (2) and $\psi'_2 + \phi'$ marked as (3). Note that the artificial component with IF $\psi'_2 - \phi'$ is the reflection of $\phi'$ associated with the INF.

3. Theoretical justification

In this section, we provide a partial theoretical study of the reflection effect in Claim 2.1 under the uniform sampling scheme and the spline interpolation, and indicate some surrogate results for the non-uniform sampling scheme based on ISR. Without loss of generality, we assume that the sampling rate is 1 Hz; that is, $\psi(t) = t$ and the ISR is 1.

We start from the signal $f(t) = \cos(2\pi \alpha t)$, where $0 < \alpha < 1/2$. Note that $f$ is a band-limited harmonic function which is also in $\mathcal{S}_K^{1-\varepsilon_2}$. Note that the IF of $f(t)$ is $\alpha$, which is less than $\psi'(t)/2$. The uniform sampling scheme corresponds to the weighted Dirac train, which is a tempered distribution $f_{\delta} := \sum_{l \in \mathbb{Z}} f(l) \delta_l$, where $\delta_l$ is the Dirac delta measure supported at $l \in \mathbb{Z}$. The $n$-th order spline interpolation scheme, where $n \in \mathbb{N}$, is realized as...
a convolution of $f_\delta$ with the $n$-th order cardinal spline function $\eta_{(n)}$ \[17\], which satisfies

$$
\hat{\eta}_{(n)}(\xi) = \left( \sum_{l \in \mathbb{Z}} \left( \frac{\sin(\pi(\xi - l))}{\pi(\xi - l)} \right)^n \right)^{-1} \left( \frac{\sin(\pi \xi)}{\pi \xi} \right)^n.
$$

Precisely, the interpolated signal based on the $n$-th order spline interpolation, denoted as $\hat{f}$, satisfies $\hat{f} = \eta_{(n)} * f_\delta$, where $*$ is the convolution. Note that as it is an interpolation, we have $\hat{f}(l) = f(l)$ for all $l \in \mathbb{Z}$. By a direct calculation, we have $\hat{\eta}_\delta(\xi) = \sum_{k \in \mathbb{Z}} \hat{f}(k + \xi) = \frac{1}{2} \sum_{k \in \mathbb{Z}} \delta_{k+\alpha} + \delta_{k-\alpha}$, which leads to $\hat{\eta}(\xi) = \frac{1}{2} \sum_{k \in \mathbb{Z}} \hat{\eta}_{(n)}(\xi) \left[ \delta_{k+\alpha} + \delta_{k-\alpha} \right]$. As a result, we have

$$
\hat{f}(t) = \sum_{k \in \mathbb{Z}} \hat{\eta}_{(n)}(k - \alpha) \cos(2\pi(k - \alpha)t)
$$

This result justifies Claim \[2.1\] when the signal is harmonic and the sampling is uniform. Indeed, the main reflection component associated with INF, $1/2$, is $\hat{\eta}_{(n)}(1 - \alpha) \cos(2\pi(1 - \alpha)t)$. Recall the fact that the $n$-th order cardinal spline decays as fast as the $n$-th order B-spline; that is, $|\eta_{(n)}(t)| \sim |t|^{-n-1}$ \[18\].

Next, without loss of generality, we consider a general function $f(t) = a(t) \cos(2\pi \phi(t)) \in \mathcal{A}_e^{c_1,c_2}$, where $\phi'(t) < 1/2$. Again, we consider a uniform sampling scheme at the sampling rate $1$ Hz; that is, $\psi(t) = t$. Take $l_0 \in \mathbb{Z}$. Consider a local harmonic approximation of $f$ around $l_0$, denoted as $f^{(l_0)}(t) = a(l_0) \cos(2\pi \phi(l_0) - l_0 \phi'(l_0) + \phi'(l_0)t)$. By the adaptive harmonic model and the same argument as that in \[8\], we have the control between $f$ and $f^{(l_0)}$ for all $s \in \mathbb{R}$, $f(l_0 + s) = f^{(l_0)}(l_0 + s) + C|s|\epsilon$, where $C$ depends on $c_2$ and is uniformly bounded for all $l_0$. Recall that $c_2$ is in the definition of $\mathcal{A}_e^{c_1,c_2}$. By taking these facts together, when $n > 1$, we know that for $t \in [l_0 - 1/2, l_0 + 1/2]$,

$$
\left| \sum_{l \in \mathbb{Z}} (f(l) - f^{(l_0)}(l)) \eta_{(n)}(t - l) \right| \leq 2C\epsilon \sum_{k \in \mathbb{N}} kk^{-n-1} = C'\epsilon.
$$

Hence, by \[5\] we have for all $t \in [l_0 - 1/2, l_0 + 1/2]$

$$
\hat{f}(t) = (\eta_{(n)} * f_\delta)(t) = (\eta_{(n)} * f^{(l_0)})(t) + O(\epsilon)
$$

$$
= \sum_{k \in \mathbb{Z}} \hat{\eta}_{(n)}(k - \phi'(l_0)) a(t) \cos(2\pi (kt - \phi(t))],
$$

where we use the fact that $a(l_0) \cos(2\pi \phi(l_0) - l_0 \phi'(l_0) + (k + \phi'(l_0))t)] = a(l_0) \cos(2\pi (k + \phi'(l_0))t) + O(\epsilon)$. As a result, we obtain a justification of Claim \[2.1\] when $f \in \mathcal{A}_e^{c_1,c_2}$.

In general, when $\psi$ is not a linear function, we could have the following analysis which does not fully justify the reflection effect. Again, without loss of generality, we consider a general function $f(t) = a(t) \cos(2\pi \phi(t)) \in \mathcal{A}_e^{c_1,c_2}$, where $\phi'(t) < 1/2$. Also assume that $\psi'$ is roughly $1$ for all time to simplify the discussion. Note that since $\psi$ is a monotonically increasing function, we could change the variable by taking $x = \psi(t)$ so that the sampling is uniform. In other words, the nonuniform samples $\{t_m, f(t_m)\}_{m \in \mathbb{Z}}$, where $t_m = \psi^{-1}(m)$, are converted to the uniform samples $\{m, f(\psi^{-1}(m))\}_{m \in \mathbb{Z}}$. We call this conversion the temporal unwrapping. Note that this is equivalent to a uniform sampling of a new function $f'(\psi(t)) = a(\psi^{-1}(t)) \cos(2\pi \phi'(\psi^{-1}(t)))$. First of all, note that in general $f'(\psi)$ is not in $\mathcal{A}_e^{c_1,c_2}$, as $\psi$ might not have enough regularity. Indeed, by a direct calculation, we have $\partial_t f'(\psi^{-1}(t)) = \frac{\hat{f}'(\psi^{-1}(t))}{\psi'(t)}$, which in general does not have a small derivative. Suppose we assume $|\psi''| \leq \epsilon \phi'(t)$, then we see that $\frac{\partial^2 f'(\psi^{-1}(t))}{\psi'(t)^2} = \frac{\phi''(\psi^{-1}(t)) - \psi'(t)\phi'(\psi^{-1}(t))}{\psi'(t)^2}$ is of order $\epsilon$. Thus, locally we could view $f'(\psi)$ as a harmonic function when $|\psi''| \leq \epsilon \phi'(t)$. 
Under this assumption, if we interpolate the uniform sampled dataset \( \{m, f(\psi^{-1}(m))\}_{m \in \mathbb{Z}} \) using the \( n \)-th order spline interpolation, we get the reflection effect by (4). Clearly, by temporally wrapping the interpolated signal back, the reflection effect is preserved. However, since in general the temporal unwrapping and the \( n \)-th order spline interpolation do not commute, the above argument only partially explains the reflection effect in the non-uniform sampling scheme.

4. REAL SIGNAL FROM ANESTHESIA

General anesthesia is inevitable for a patient receiving surgery. To well control the surgery quality, the anesthetic agent dose should be dynamically adjusted to achieve an adequate level of anesthesia. It has been shown that the oscillatory patterns in the R-to-R peak interval (RRI) time series of electrocardiography and the respiration contain a lot of information regarding anesthesia dynamics \([19, 3, 4, 5, 6, 20]\). It has been shown in \([3, 4, 5, 6]\) that the adaptive harmonic model, the multi-taper reassigned STFT \([21, 22]\) and the multi-taper SST-STFT \([4, 6]\) serve as a good framework toward the goal, especially when noise is inevitable. Precisely, the RRI during anesthesia could be modeled by the adaptive harmonic model, and how strong the component is relates directly to the anesthetic depth. We would extract stable features by the multi-taper reassigned STFT and the multi-taper SST-STFT to better quantify anesthesia. While its clinical value has been shown in different problems, its sampling theory issue is left unanswered to the best of our knowledge. We mention that the sampling issue for the power spectrum approach to study HRV based on the stationarity assumption has been widely studied; see for example \([23, 24]\). It could be argued that reassigned STFT and SST-STFT add complications on the top of a spectrum approach, but note the difference between the underlying models which are aiming to capture different phenomena. Here we demonstrate two examples regarding this direction which might generate potential artifact in the TF representation – instantaneous heart rate (IHR) analysis and ECG-derived respiratory signal (EDR) analysis. The IHR and EDR are acquired from the recorded ECG signal in the following way. Denote the recorded lead II ECG signal as \( E(t) \) which is digitalized at the sampling rate 1000 Hz. The R peak detection was automatically determined from \( E(t) \). The ECG signal is clean without significant noise contamination, and no ectopic beats nor electro-cauterization happen in the recorded signal. The collected RRI time series is denoted as \( X = \{t_i, t_{i+1} - t_i\}_{i=1}^{N} \), where \( t_i \in \mathbb{R} \) is the time stamp of the \( i \)-th R peak. Then, we follow the common practice and approximate the IHR from \( X \) by the cubic spline interpolation \([25]\), and denote the approximated IHR as \( \tilde{r}_m \). We also have a non-uniform sampling dataset \( \{t_i, E(t_i)\}_{i=1}^{N} \), where \( E(t_i) \) is the amplitude of the \( i \)-th R peak. The EDR signal, a surrogate of the respiratory signal denoted as \( \tilde{R}(t) \), is built up by applying the cubic spline interpolation on \( \{t_i, E(t_i)\}_{i=1}^{N} \). We mention that although the amplitude scales of \( R(t) \) and \( \tilde{R}(t) \) are different, they share the same oscillatory information inside the respiratory signal, in particular the IF. We refer readers with interest in EDR to \([26, 20]\) for details. To confirm the existence of the reflection effect as an artifact, when we record the ECG signal, we simultaneously record the airway flow signal, which is denoted as \( R(t) \). To avoid any possible artifacts, the airway flow signal is uniformly sampled at the sampling rate 50 Hz. As R peaks are viewed as the non-uniform sampling of the IHR and EDR, the ISR, denoted as \( \psi'(t) \), could be estimated by the cubic spline interpolated function from \( \{t_j, (t_{j+1} - t_j)^{-1}\}_{j=1}^{N} \).

Next, \( \tilde{r}_m \) is resampled to be equally spaced at 8 Hz for the multi-taper SST-STFT and multi-taper reassigned STFT analysis \([24]\), as it is commonly believed that most useful
The instantaneous Nyquist frequency is imposed as a red dashed curve on it. Left: the multi-taper SST-STFT result of the IHR based on the cubic spline (CS) interpolation; right: the multi-taper reassigned-STFT result of the IHR based on the CS interpolation.

5. Conclusion

While extracting oscillatory features from a given time series via TF analysis is attracting more interest in bio-medical field, the concentration effect of reassignment technique and SST increases the level of artifacts as compared to standard spectrum analysis. A possible mitigation to reduce the artifacts is by the adaptive low-pass filter – setting TF representation coefficients with frequency larger than INF to zero. While it is possible to pre-process the signal by a low pass filter to the interpolated signal to reduce the reflection artifact, in general, the structured artifact might not be removed but perturbed by the low pass filter, which leads to more complicated artifacts. Although such an adaptive filtering could be part of a solution to the problem, there is no universal way of doing it because of the “nonstationarity” and a theoretical study is necessary. Also note that the Claim 2.1 is only partially analyzed and a fully study will be reported in the future.
The results of the EDR signal and the airflow signal recorded simultaneously. The instantaneous Nyquist frequency (INF) is superimposed as a red dashed curve. Left top: the multi-taper SST-STFT result of the EDR based on the cubic spline (CS) interpolation. Right top: the multi-taper reassigned STFT of the EDR based on the CS interpolation. Bottom left: the multi-taper SST-STFT result of the airflow signal. Bottom right: the multi-taper reassigned STFT of the airflow signal. It is clear that the base component with IF about 0.5 Hz in the airflow signal is well captured by the EDR signal, while in the EDR signal there is an artificial reflected component associated with the INF. Also note the temporal reassignment effect in the multi-taper reassigned STFT around 100-th second – the TF resolution of the multi-taper reassigned STFT is sharper than that of the multi-taper SST-STFT.

ACKNOWLEDGEMENT

Hau-tieng Wu would like to thank Professor Charles K. Chui, Professor Ingrid Daubechies and Professor Andrey Feuerverger for fruitful discussions.

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