Phase Transitions in Dense Matter
and Recent Topics of Neutron Stars

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Abstract. Core of neutron star consists of highly dense matter above normal nuclear density \( \rho_0 \approx 0.16 \text{fm}^{-3} \), where phase transitions is expected to take place. We review some phase transitions and recent topics of neutron stars.

INTRODUCTION

There exist many candidates of the phase transitions in highly dense nuclear matter: boson condensation \((K^-, \pi)\), quark matter, hyperonic matter and so on [1]. They are often discussed with respect to neutron star (NS) physics, for example, maximum mass, cooling mechanism, glitch mechanism, delayed collapse, gamma ray burst and magnetar.

In this paper after the introduction why we believe the phase transitions in the core of NS, We concentrate on three topics: the delayed collapse due to kaon condensation, the mixed phase problem for first order phase transitions and strange stars as magnetar candidates.

Why do we believe the phase transitions?

The equation of state (EOS) of normal nuclear matter has been studied by many authors with various theoretical approaches. G-matrix and variational calculations (See ref. [1] and references therein,) are based on the microscopic theory, whose objective is to reproduce the properties of matter (saturation density, binding energy and nuclear incompressibility) based on the experimental data(2- or 3-body interaction). On the other hand, relativistic mean field theory is the effective theory and seems to be very useful method. The numerical table of EOS is submitted by Shen et al [2](also refer to talk by Prof. Sumiyoshi).

There has been suggested phase transitions in NS; e.g. nucleon or quark superfluidity, pion or kaon condensation, deconfinement transitions. In this paper we discuss the phase transitions beyond the normal matter by the following reasons. We show two reasons why we need some phase transition: maximum mass and cooling mechanism.

Maximum Mass

Using the EOS of the normal matter, maximum mass of NS is evaluated about 2.0\(M_\odot\). But \(M \approx 1.35M_\odot\) from the observation. In Fig.1 observed data published by Thorsett, et al. is shown.
There is a large difference between observation and theoretical calculation. On the other hand, if there exists some phase transition, for example, kaon condensation, $M_{\text{max}} \simeq 1.4 - 1.6 M_\odot$, and it seems to be more plausible [4,5].

Cooling Mechanism

In the cooling scenario, the main contribution in the normal matter is the modified URCA reactions.

\[ n + n \rightarrow n + p + e^- + \bar{\nu}_e, \]
\[ n + p + e^- \rightarrow n + n + \nu_e. \]

This scenario based on the normal matter is called standard cooling scenario, and it has been suggested that we need extra cooling mechanisms to reproduce the observational data points consistently [6,7]. Each phase transition leads to the additional rapid cooling mechanisms. (The cooling curve including these mechanisms is called the non-standard scenario.) In the case of kaon condensation, $K$-induced URCA process exists,

\[ n + \langle K^- \rangle \rightarrow n + e^- + \bar{\nu}_e, \]
\[ n + e^- \rightarrow n + \langle K^- \rangle + \nu_e, \]

where $\langle K^- \rangle$ means the condensed kaon field [6]. The cooling curves for non-standard cooling scenario show the fast cooling and with more additional process (heating or pairing), we can explain the observed surface temperature of NS. Fig.2 by D. Page [8] shows the cooling curves in both cases and the resulting cooling curve seems to fit the observational data.

We hereafter address three current issues about the phase transitions in neutron stars.
I DELAYED COLLAPSE

After supernovae explosions, protoneutron stars (PNS) are formed with hot, dense and neutrino-trapped matter. They usually evolve to cold ($T \approx 0$) NS through two main eras: One is deleptonization era and the other the initial cooling era. In the deleptonization era, trapped neutrinos are released in about several seconds and PNS evolve to hot and neutrino-free NS. Then through the initial cooling era, they evolve to cold usual NS in a few tens of seconds.

However some of them may collapse to low-mass black holes during these eras by softening the EOS due to the occurrence of hadronic phase transitions [9]. This is called the delayed collapse. As a typical example, neutrinos from SN1987A were observed at Kaomiokande, but no pulsar yet, which suggests the possibility of the delayed collapse in SN1987A.

We consider here the possibility of the delayed collapse in the context of kaon condensation. Kaon condensation, one of the candidates of the hadronic phase transitions in high-density nuclear matter, is a kind of Bose-Einstein condensation. Fig.3 shows the mechanism of the occurrence of kaon condensation. As density increases, kaon’s single particle energy $\varepsilon_-\ $ decreases due to the attractive $KN$ interaction in medium, while the electron chemical potential, which is equal to the kaon chemical potential in the beta equilibrium and neutrino-free matter, increases. When they become equal to each other, Bose-Einstein condensation of kaons occurs at a critical density $\rho_c$. Kaon condensation has been studied by many authors mainly at zero temperature since first suggested by Kaplan and Nelson [10]. We know that the kaon condensation gives rise to the large softening of EOS. Kaon condensation is the first order phase transition and thereby EOS includes thermodynamically unstable region. We applied the Maxwell construction to obtain the equilibrium curve for simplicity though, restrictly speaking, we need to take the Gibbs conditions(See Sect.II).

Recently, to study the PNS, there appear a few works about kaon condensation at finite temperature [11,12] but there was no consistent theory based on chiral symmetry. Then we have presented a new formulation to treat fluctuations around the condensate based on chiral symmetry [13,14].

With thermodynamic potential in the reference [13,14], we can study the properties of kaon condensed state at finite temperature and then discuss some implications on the delayed collapse of PNS. We, hereafter, use the heavy-baryon limit for nucleons [13]. We show the phase diagram, EOS and then discuss the properties of PNS where thermal and neutrino-trapped effects are very important.

![Figure 3. Mechanism of the occurrence of kaon condensation. Crossing point of chemical potential and single particle energy for $K^-$. represents a critical density.](image-url)
A Phase Diagram and EOS for kaon condensation

First we show the phase diagram in Fig.4. In the neutrino-trapped case we set $Y_{le} = Y_e + Y_{\nu_e} = 0.4$ where $Y_e(Y_{\nu_e})$ is the electron(electron-neutrino) number per baryon, while $Y_{\nu_e} = 0$ in the neutrino-free case. Both of the thermal and neutrino-trapped effects largely suppress the occurrence of kaon condensation. The reason for the latter case may be understood from the threshold condition, $\varepsilon_-(\rho_c) = \mu_K$. Chemical equilibrium holds the relation $\mu_K = \mu_e - \mu_{\nu_e}$: $\mu_{\nu_e} > 0$ in the neutrino-trapped case while $\mu_{\nu_e} = 0$ in the neutrino-free case, which means kaon chemical potential should be suppressed in the neutrino-trapped case. Then the occurrence of kaon condensation is suppressed through the suppression of number of kaons.

Next Fig.5 represents the EOS in density-pressure plane. Both of thermal and neutrino-trapping effects stiffen the EOS in the condensed phase. They seem to be more pronounced in the condensed state, especially around the critical density(see Fig.5), mainly through the rise of critical density.

In the realistic situation in PNS the isentropic condition is more relevant [11]. We reconstruct the isentropic EOS by evaluating the entropy as a function of temperature.

B Properties of PNS

Solving the TOV equation with the EOS, we can study the properties of PNS. In Fig.6 we show the gravitational mass versus central density for the neutrino-trapped and -free cases with entropy per baryon $S = 0, 1$ or 2. Both of the thermal and neutrino-trapping effects make the gravitational mass larger for the almost all of the central density. As the exception, at high central density in neutrino-free case masses of hot NS are smaller than ones of cold NS. This is out of our intuition and the reason is as follows. Usually thermal effect enlarges the pressure and leads to the larger mass. At the same time, however, thermal effect enlarges the energy which contribute to gravitation. In the competition of the increase of pressure and gravitation, sometimes, mass of hot NS becomes smaller than one of cold NS.
In the neutrino-free case, once kaon condensation occurs in the core of a star, gravitational mass is little changed in the neighborhood of the equal pressure region in the isothermal EOS, then gravitationally unstable region (negative gradient part) appears. Therefore the neutron-star branch is separated into two stable branches: one is for stars with kaon condensate in their cores (right hand side from gravitationally unstable region) and the other consisting of only normal matter (left hand side from gravitationally unstable region). Thermal effect to the gravitational mass seems to be very large around the critical density because the EOS changes largely there. However the maximum mass, stars with which include kaon condensate in the core, is hardly changed and even decrease by the thermal effect as already discussed. As the temperature raises, gravitational mass grows largely in the normal branch but not in the kaon condensed branch.

In the neutrino-trapping case, thermal effect is large around the critical density and small for the heavy stars as well, but the situation is quantitatively different. For the $S = 0$ or 1 case we can see that the neutron-star branch is also separated by the gravitationally unstable region and the star with maximum mass exists in the kaon condensed branch. (Their central density $\rho_c \simeq 10\rho_0$) On the other hand, in the $S = 2$ case almost all of the stars with kaon condensate are gravitationally unstable, and the maximum-mass star whose central density $\rho_c = 5.4\rho_0$, still resides in the normal branch. For this reason the central density of the maximum-mass star is very different from those for $S = 0$ or 1.

To discuss the possibility of delayed collapse of PNS, the total baryon number $N_B$ should be fixed as a conserved quantity during the evolution [15], under the assumption of no accretion. In Fig.7 we show the gravitational mass versus total baryon number for gravitationally stable PNS omitting unstable stars. Each terminal point represents maximum mass and maximum total baryon number. If an initial mass exceeds the terminal point in each configuration, the star should collapse into a black hole (not a delayed collapse but a usual formation of a black hole). We have shown the neutrino-trapped and -free cases; the former case might be relevant for the deleptonization era, while the latter for the initial cooling era. It is interesting to see the difference between the neutrino-free and -trapped cases: the curve is shortened as the entropy increases in the former case, while elongated in the latter case, where the remarkable increase in $S = 2$ and neutrino-trapped case results from that maximum mass exists in normal branch, different from other configurations with
maximum mass in kaon condensed branch. These features are important for the following argument about the delayed collapse and maximum mass of the cold NS. The delayed collapse is possible if the initially stable star on a curve finds no corresponding point on other curves during the evolution through deleptonization or cooling with the baryon number fixed. Consider a typical evolution for example: A PNS is born as a neutrino-trapping and hot \((S = 2)\) star after supernova explosion and evolves to neutrino-free and hot \((S = 2)\) stage through deleptonization. Then through the cooling, the star evolves to be neutrino-free and cold \((S = 0)\). We can clearly see the PNS with large enough mass can exist as a stable star at the beginning but cannot find any point on the neutrino-free and \(S = 2\) curve. Therefore they must collapse to the low-mass black hole by deleptonization. It is to be noted that because the neutrino-trapped and \(S = 2\) star never includes kaon condensate, its collapse is largely due to the appearance of kaon condensate in the core. Thus we may conclude that kaon condensation is very plausible to cause the delayed collapse in the deleptonization era.

On the other hand, in the initial cooling era after deleptonization delayed collapse does not take place because the NS on neutrino-free and \(S = 2\) branch evolve to neutrino-free and cold branch and all of the stars seem to be able to find corresponding stable points in the each stage.

C Summary for delayed collapse

We have shown that the delayed collapse is possible in the deleptonization era due to the appearance of kaon condensation and the maximum mass of cold NS should be determined in neutrino-free and hot stage [16].

On the other hand, Pons et al. also studied kaon condensation in PNS matter [17] They concluded that the thermal effect is a key object to the delayed collapse and it is different from ours. We cannot give a clear reason for the discrepancy at present but it may originate from the difference of formulations: our discussion is based on the chiral Lagrangian and theirs, meson exchange model. Otherwise the difference may disappear if they impose the condition that the total baryon number is fixed.

In order to study the mechanism of delayed collapse and mass region which should collapse in more detail, we had better study the dynamical evolution beyond the static configurations. There neutrino opacity is important to determine the duration of the deleptonization [18], and in the kaon condensed phase neutrino opacities may become larger than in normal phase [19,20]. As another remaining issue, we will refine the EOS with the Gibbs conditions instead of the Maxwell construction.

II MIXED PHASE

For first order phase transition, for kaon condensation, quark matter, etc, there appears the thermodynamically unstable region in EOS. The Maxwell construction, which may be familiar for the liquid-gas phase transition for water, had been used as a standard method to get the proper EOS. Recently Glendenning pointed out that the Maxwell construction is not correct and we should impose the Gibbs conditions [21].

A Maxwell Construction

The Maxwell construction imposes the conditions: \(\mu_n^N = \mu_n^K\) and \(P^N = P^K\), and can be written as equal-area rule,
\[ \int_N^K V dP = 0. \]  \hspace{1cm} (1)

In Fig.8 we show the original EOS with thermodynamically unstable region and improved one by the Maxwell construction. As a result of the Maxwell construction, there appears equal pressure region, which is called the mixed phase of kaon condensed matter and normal matter, only the volume ratio of two phases changes and properties of each phase (density, chemical potentials and so on.) never change. And the density gap leads to peculiar structure of NS. In Fig.9 mass-central density curve is shown. Once kaon condensation occurs at \( \rho \approx 2.3 \rho_0 \), density gap appears and gravitationally unstable region (region with negative slope) follows. Then picking up a NS, the core consists of two pure phases without the mixed phase (See Fig.10), which is known as a peculiar feature due to the Maxwell construction.

B Gibbs Conditions

In fact, however, for the first order phase transitions in nuclear matter, the Maxwell construction is not valid. Because there exist two chemical potentials, baryon and charge chemical potentials, we should use the Gibbs conditions [21].

The Maxwell construction is the same to the Gibbs conditions in case with only one chemical potential. On the other hand, there exist two chemical potentials in nuclear matter: baryon and charge chemical potentials which correspond to neutron and electron chemical potentials respectively. Then we need to use the Gibbs conditions,

\[ \mu_n^N = \mu_n^K, \quad \mu_e^N = \mu_e^K, \quad P^N = P^K, \]  \hspace{1cm} (2)

between the phases of normal matter \( N \) and kaon condensed matter \( K \).

Glendenning and Schaffner-Bielich discussed kaon condensation imposing the Gibbs conditions [22] instead of the Maxwell construction. In Fig.11, their result is shown. In the case of the Gibbs conditions, compared to the case of the Maxwell construction, the mixed phase exist in the broad range of density and the equal pressure region disappears. The densities for each phase can change at each point of the mixed phase. Charge density is nonzero in each phase and charge neutrality is not achieved locally (Of course, globally achieved.) This may be unfamiliar situation but simply understood as follows. In the mixed phase if there exists only baryon chemical potential, which is in the case of the Maxwell construction, baryon densities are not equal in two phases. On the other
hand, in the case there exist two chemical potentials, as in the case of the Gibbs conditions, both of baryon and charge densities are not equal.

NS structure can be studied with the EOS in Fig.11. Fig.13 shows mass-radius curve. We can find that the gravitationally unstable region in the case of the Maxwell construction disappears in the case of the Gibbs conditions. Here it is to be noted that the remarkable change appears for NS with comparably small mass (large radius) and behavior of heavy NS is little changed. Then our discussion about the delayed collapse in Sect.I will not be modified so much.

Fig.14 shows the interior structure of NS in the case of the Gibbs conditions. The mixed phase may exist largely in the cores of NS. For a NS with $1.5M_\odot$, for example, a large droplet phase exists and its radius is about 3[km].

C Summary for mixed phase

In this section we have addressed the interesting issue about the mixed phase. The problem of the mixed phase seems to have two important aspects. One is the change of bulk properties of NS (mass, radius and so on.) through the modification of EOS and the other is of interior structure of NS due to the appearance of large extent of the mixed phase. The latter, especially, may give many implications to astrophysics. For example, Reddy et al. [19] discussed the coherent scattering of neutrinos in the mixed phase and they found that the mean free path of neutrinos becomes smaller in the droplet phase.

In the preceding discussion, we have ignored the finite volume effects: surface and Coulomb energy. They cannot be neglected to study the realistic EOS and they are expected to prevent the occurrence of the mixed phase [23]. Then the realistic EOS may exist between the EOS in the Gibbs conditions and the Maxwell construction.
TABLE 1. Magnetic fields and radii.

|        | Sun | White Dwarf | NS  | Magnetar |
|--------|-----|-------------|-----|----------|
| $B$ [gauss] | $10^7$ | $10^8$ | $10^{12}$ | $10^{15}$ |
| $R$ [cm]  | $10^5$ | $10^8$ | $10^6$ | $10^5$ |

III MAGNETARS AND STRANGE STARS

The magnetars are NS with strong magnetic fields ($B \sim 10^{14-15}$G derived from the $P-\dot{P}$ curve) and anomalous X-ray pulsars (AXP) and two pulsars in soft-gamma-ray repeaters (SGR) are known as the candidates [24]. The strong magnetic fields are out of the scaling law between radius and magnetic field $R^2B \approx \text{const}$, which can explain the magnetic fields of other stars (See Table 1). Then there is a possibility that they may originate from strong interaction. Recently there has been proposed on idea that the magnetars may be strange stars with complete spin alignment [25].

Once $u,d,s$-quark matter is formed, it may be more stable than normal nuclear matter at lower densities. Distinguished from the usual NS with core of quark matter, compact stars consisting mainly strange quark matter with or without thin crust are called strange stars.

As a very simple estimation, using non-interacting massless quarks in bag at $T = 0$, energy per baryon of $u,d$-quark matter $\varepsilon_{u,d}$ and of $u,d,s$-quark matter $\varepsilon_{u,d,s}$ can be estimated.

$$\varepsilon_{u,d} \approx 934 \text{MeV} \frac{B^{1/4}}{145},$$

$$\varepsilon_{u,d,s} \approx 829 \text{MeV} \frac{B^{1/4}}{145},$$

with bag constant $B$. Then we can easily find that the $u,d,s$-quark matter is more stable than $u,d$-
quark matter and nuclear matter. Of course we can use more complex model, with the one-gluon exchange and with heavy strange quark, and the result is similar [26]. A natural question may appear: if this is right, why nuclei consists of nucleon. But there is no contradiction because there is surface effect for light nuclei and no strange quark exists for heavy nuclei. A few strange quark in nuclear matter are not stable and it is almost impossible to produce enoughly many strange quarks at the same time through the higher-order weak interaction.

Based on the idea of strange stars, Tatsumi studied the possibility of magnetization of strange quark matter with relativistic one-gluon exchange interaction [25]. He concluded that magnetized strange quark matter may be stable in the low-density-region. At $\rho \sim 0.1 \text{fm}^{-3}$ magnetization may occur in strange stars and can produce $B \sim 10^{15-17}$. He just suggested the possibility and realistic calculation(e.g. Hartree-Fock) is the future problem.

IV CONCLUDING REMARKS

We concentrated on 3 topics in this paper. We discussed the evolution of PNS from the view point of the nuclear theory (in the static limit) in Sect.I and found that the delayed collapse is possible in the deleptonization era due to the occurrence of kaon condensation. In Sect.II we reviewed the problem of the mixed phase, which may give important effects to NS physics: glitch, cooling scenario and so on. Then an interesting idea about the identification of magnetars as strange stars was introduced in Sect.III. Of course there exist many candidates of phase transitions and related astrophysical phenomena which we could not pick up here. And now we recommend a good and recent review by Heiselberg and Hjorth-Jensen [1].

At present it is not clear what kind of phase transitions really exist in NS. But the experimental data on the earth and the observation of signal from NS will limit the candidates and, sometime, determine what happens in NS, we hope.

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