Quantum Enhanced Energy Distribution for Information Heat Engines

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A new scenario for energy distribution, security and shareability is presented that assumes the availability of quantum information heat engines and a thermal bath. It is based on the convertibility between entropy and work in the presence of a thermal reservoir. Our approach to the informational content of physical systems that are distributed between users is complementary to the conventional perspective of quantum communication. The latter places the value on the unpredictable content of the transmitted quantum states, while our interest focuses on their certainty. Some well-known results in quantum communication are reused in this context. Particularly, we describe a way to securely distribute quantum states to be used for unlocking energy from thermal sources. We also consider some multi-partite entangled and classically correlated states for a collaborative multi-user sharing of work extraction possibilities. Besides, the relation between the communication and work extraction capabilities is analyzed and written as an equation.

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I. INTRODUCTION AND ANTECEDENTS

The study of the relation between computational irreversibility and energy can be traced back as far as 1961 when Landauer\(^1\) established that irreversible steps in a computation should dissipate an amount of heat directly related to the loss of information (an experimental demonstration thereof has been published in 2012\(^2\)). On this line followed another important contribution by Bennett\(^3\). He claimed that any computation can be made reversible by the allocation of information in more registers (now known as ancillas), which can be deleted by the end of the process. Hamiltonian evolutions are reversible and, if they are time independent, do not dissipate any energy so that, under these circumstances, reversibility is directly related to energy conservation. Using quantum mechanics, in 1982 Feynman\(^4\), in 1984 Benioff\(^5\) and in 1985 Deutsch\(^6\) proposed models for quantum computers with Hamiltonian evolutions that minimized energy losses. A fairly up-to-date review of information and quantum computation can be found in [7], and a comprehensive textbook in [8].

Another fruitful relation between energy and information began in the Theory of Heat published by J.C. Maxwell in 1871; he suggested that the introduction of an imaginary demon in a gas chamber could extract work from a single temperature gas volume. A further important contribution along this line is the Szilard’s Engine\(^9\) and countless studies later on paradoxes, contradictions or refinements of the said law. A lot of papers in the last years have elucidated the question from different perspectives, all of them confirming the validity of the second principle of thermodynamics\(^10–12\).

Szilard engines and Maxwell demons analysis are often prone to misunderstandings. In our view they could be avoided with careful accounting for information and energy trade-off in the presence of feed-back systems\(^13–14\). Generally, they use a controller which is governed by a number of bits (qubits in quantum environments). These bits are stored in some information container or memory. To change these bits it is necessary to reset them, thus some Landauer’s work must be brought in and some entropy poured somewhere else. Then they must be replaced by new information coming from a measurement on the system, whereby some entropy is taken away. After that, some work can be obtained from the freshly measured bits. The process is then neutral in information and energy, unless entropy and work can be taken away in exchange of information.

We would like to emphasize that the real fuel for these devices is information, as is explained in [15] for a magnetic quantum information heat engine, which is a new version of Szilard’s engine devoid of some irrelevant details. An alternative to the measuring stage is periodically swapping high-entropy states in the engine with low-entropy ones supplied by feeding ancillas\(^16\).

It is now well understood that the availability of a thermal bath allows for a trade-off between information and work. The devices that carry out this conversion are known as Information Heat Engines\(^16, 17–28\). The information may be classical or quantum and the available energy could differ in both cases. The difference is defined by the work deficit\(^30\) or quantum discord\(^31–33\). Besides, the extractable work can be related to the relative entropy or Kullback-Leibler divergence\(^34–37\). Quantum statistical considerations with fermionic and bosonic multiparticle systems have also been analyzed elsewhere\(^12, 15, 28–35\).

Other magnetic machines (classical and quantum) have been proposed for other different thermodynamic cycles, especially cooling systems\(^28, 38–43\).

The role of standard entropy in Information Heat Engines is relevant for average values of extracted work. Generalizations to the so called smooth entropies have been made to include other fluctuations and probabilities of failure\(^44–48\). This perspective is not considered in the present paper.

In this contribution a new scenario for communication
and energy distribution is defined that allows the consideration of novel possibilities concerning the security and conditions of use for both the information and energy convertibility stored in a shared quantum state.

Quantum information resources for communication have already been adapted to energy distribution\cite{49,50} and described using Ising chains\cite{51}. They differ from our approach basically because they use an entangled vacuum state for a non-local hamiltonian which has to be shared by both the energy provider and consumer. On the other hand, our proposal includes the use of Quantum Information Heat Engines (henceforth QIHE) and the availability of a thermal bath at non-zero temperature.

In section II we provide a short description of a basic model for a QIHE, to which further ideas will be referred along the rest of the paper. Section III analyzes the transmission of physical systems, that will be referred to as messengers, whose entropy may be increased at a remote station in order to extract work from thermal baths; several protocols with different functionalities are presented. For example, by using shared entangled states, qubits sent from an emitter $A$ to a receiver $B$ are completely depolarized\cite{52}, thus useless for any illegitimate user that could intercept them; other multipartite systems render other interesting possibilities concerning the conditions to access not completely depolarized systems.

Sections IV and V analyze the transmission of physical systems in a given quantum state from two different perspectives which prove to be complementary. One of them is quantum communication, where new milestones\cite{53,54} are frequently announced, and the other is the possibility to fuel a QIHE under suitable restrictions. A relation between the communication and work extraction capabilities of sources of physical systems is sought and an equation is found. Section VI specifically focuses on proving the possibility of reaching the Holevo bound for communication performance and simultaneously using the messenger system to fuel a QIHE, whereas in Section VII both functions are supposed to be used exclusively. Section VII summarizes the conclusions.

II. SHORT DESCRIPTION OF A QUANTUM INFORMATION HEAT ENGINE

Information Heat Engines are devices which cyclically convert thermal energy into useful work at the expense of degrading information. They are periodically fueled with auxiliary systems, whose entropy is increased, although their energy is conserved. The power delivered by the engine is drawn from a single thermal reservoir.

This is in contrast with other cyclic engines that decrease the internal energy of the systems that are supplied as fuel. It should also be remarked that they comply with all three principles of thermodynamics. In the following we assume that the engine is a physical system with a local hamiltonian and no energy is exchanged with the fueling system. If the evolution is divided into intervals with either hamiltonian evolution or reversible thermal equilibrium with a bath at temperature $T$, the work after a complete cycle can be evaluated by

$$W_{\text{cycle}} = - \oint_{\text{cycle}} \text{d} \left( \text{Tr} \left\{ \rho H \right\} \right) + \int_{\text{eq}} k_B (\ln 2) T \, dS, \quad (1)$$

where the first integral extends for the whole cycle and vanishes on account of its periodic domain. The second integral is defined only for the intervals when the system is at thermal equilibrium with the bath. Besides, entropy is expressed in bits; $k_B$ is Boltzman’s constant, $\rho$ is the density matrix of the system and $H$ represents the hamiltonian. The result is

$$W_{\text{cycle}} = k_B (\ln 2) T \Delta S, \quad (2)$$

where the first integral extends for the whole cycle and vanishes on account of its periodic domain. The second integral is defined only for the intervals when the system is at thermal equilibrium with the bath. Besides, entropy is expressed in bits; $k_B$ is Boltzman’s constant, $\rho$ is the density matrix of the system and $H$ represents the hamiltonian. The result is

$$W_{\text{cycle}} = k_B (\ln 2) T \Delta S, \quad (2)$$

which shows that work can be obtained only if entropy is reset to a lower value at some point in the cycle. Entropy extraction can be accomplished in a number of ways. The simplest one is swapping the quantum state of the system with that of an ancilla with an initially lower value of entropy. More frequently, entropy is extracted through feedback control. In this case, the system is driven to a lower entropy state by first measuring and then triggering the suitable hamiltonian to make the system evolve to the
desired state. As it is thoroughly analyzed in [15], a measurement involves two systems: the measurand and the indicator, which must start in a well defined state. After the measurement the indicator ends up with some entropy, on account of the unpredictability of the outcome. In contrast, the system is brought to a less entropic state by virtue of the tailored hamiltonian evolution triggered according to the outcome of the measurement. It is after the measuring stage that entropy has to be removed by resetting the indicator; the Landauer’s work is precisely for anyone else. Furthermore, we analyze the possibilities furnished by some particular entangled states to enforce a collaborative procedure for unlocking the purity of the ancilla state.

III. SCENARIO FOR ENERGY DISTRIBUTION THROUGH QUANTUM INFORMATION IN MULTIPARTITE SYSTEMS

In the previous sections it has been established that a physical system whose quantum state is not completely depolarized can be transformed into another state with greater entropy and obtain work in the process, provided the availability of a thermal bath and an Information Heat Engine, like a Szilard or a magnetic information engine as described in section II.

Now we set up a new scenario for energy distribution: a power information plant \( P \) can do the reverse process and polarize a messenger physical system \( M \) by supplying electrical (or mechanical or of any other kind) work (see Fig.3 (a)). If the system \( M \) (a photon polarization, for instance) is sent to other users \( A, B, \ldots \) (may be in remote places) they can do the reverse operation and obtain work by depolarizing the system, provided they know the state of \( M \). Note that if the reservoirs are at different temperatures this is equivalent to a (remote) Carnot cycle (see Fig.3 (b)).

The next important issues about this scenario are the security and shareability. As a first scenario, one could send some messenger qubits from the power plant \( P \) to \( A \), so that \( A \) can convert them into work. In order for \( A \) to be able to obtain energy from the qubits, \( A \) must know something about the state \( \rho \) of said qubits. If all the qubits were in the same state, this could be learned by someone else (eavesdropper \( E \)) who could use them to draw some work from an available thermal reservoir. One possible solution that would avoid this reward for \( E \), as we know from quantum security communication protocols, is to share a set of entangled qubits; then sending a qubit from \( P \) would make \( A \) obtain two fresh convertible

FIG. 2. Magnetic Quantum Information Heat Engine; a spin-\( \frac{1}{2} \) particle \( S \) lies in the magnetic field generated by an electrical current \( I(t) \) circulating through a coil \( C \). An ancilla \( A \) is used to measure the \( z \) component of the magnetic moment of \( S \); the result determines the value of the magnetic field generated by \( I(t) \). Then the field is gradually turned off, while \( S \) keeps in thermal equilibrium with a reservoir \( R(T) \) at temperature \( T \). At the end of the process an energy \( W \) is stored at the electrical battery \( B \); its value is \( W = k_B \ln(2) T \), on account of the increase of entropy of \( S \), which enters the isothermal process with zero entropy and exits completely depolarized. Again, this energy is equal to the Landauer’s energy needed to reset the ancilla for the next cycle. If the system works as a QIHE, instead of this energy, a fresh entropy-free ancilla bit is supplied.
known, the partial density matrices describing the convertible bits for legitimate users.

A classical version of this protocol can be devised where a set of completely correlated random bits would be used instead of the Bell states. However, completely correlated pairs of random bits carry one bit of entropy and consequently, the extractable work $W_{\text{CB}}$ would be

$$W_{\text{BS}} = k_B (\ln 2) T_B,$$

which is only half of the one given by Eq. 6. This reduction can be related to the superdense coding feature of quantum communication, i.e. one qubit may be used to communicate two classical bits.

There are further possibilities: one can use multipartite states $\rho_{A_1,\ldots,A_n}$ (see Fig. 4) that allow energy extraction under one of the following two situations:

1. any user $A_i$ can independently make that the other ones ($A_{j \neq i}$) access the energy of 1 bit in their part of $\rho_{A_1,\ldots,A_n}$. This possibility can be implemented by using generalized GHZ states, also known as cat states. An $n$-partite GHZ (Greenberger-Horne-Zeilinger) state is a pure state:

$$|\text{GHZ}_n\rangle := 2^{-1/2} \left( |0\rangle^\otimes n + |1\rangle^\otimes n \right),$$

FIG. 3. Part (a) represents a way of supplying energy to a remote station at temperature $T_2$ consisting of sending physical systems in a quantum state $\rho_i$ whose entropy increases after fueling a QIHE and return as completely unpolarized states. As an example, photons can be sent to the remote station under a particular polarization and return completely unpolarized. The extracted work is $W_2 = k_B (\ln 2) T_2$. In (b) a remote Carnot cycle is depicted, where entropy is not lost.

As an example, $A$ and $B$ may agree on sharing a set of Bell pairs. Let $|\Psi\rangle_{AB}$ be one of them. As it is widely known, the partial density matrices describing the $A$ and $B$ parts of $|\Psi\rangle_{AB}$ are completely depolarized, being:

$$\rho_A = \rho_B = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

or, in other words, no information is available in the local subsystems. However, when $A$ sends its qubit to $B$ through a quantum channel, $B$ can reconstruct the bipartite initial Bell state, which is a pure state and can be used to extract a work $W_{\text{BS}}$ given by

$$W_{\text{BS}} = 2 k_B (\ln 2) T_B.$$
with two remarkable properties: 1) any local measurement destroys the entanglement and 2) the partial states are completely unpolarized (useless for energy conversion). User $A_i$ can measure its qubit and then classically broadcast the result to all other users who can subsequently convert their $n - 1$ bits into work.

ii.- only if all users agree, one of them can convert the energy. For this purpose we may use a classically correlated mixed state:

$$\rho_{EP} := \frac{1}{2^n} \sum_{i_1, \ldots, i_n} (1 + (-1)^{i_1+\ldots+i_n}) \sigma_{i_1, \ldots, i_n},$$

(8)

where $i_k$ may be 0 or 1 and, for notational simplicity, we define

$$\sigma_{i_1, \ldots, i_n} := |i_1, \ldots, i_n\rangle \langle i_1, \ldots, i_n|.$$  

(9)

According to Eq.8, $\rho_{EP}$ is a mixture of all even parity pure states. It can only furnish one bit of information upon knowing all but one bits measured on the standard basis. We will now prove this. For that purpose, we will show that any operation on $n - 2$ qubits, say 3, ..., $n$, gives no information about qubit 1. Let $K_j$ be the Krauss operators; then, ignoring a possible normalization factor, the state of the system after it is:

$$\rho' = \sum_j K_j \rho_{EP} K_j^\dagger,$$

(10)

where $K_j := 1_2 \otimes 1_2 \otimes k_j$, what accounts for the fact that the operation is on the last $n - 2$ qubits. Expanding $\rho_{EP}$ we have

$$\rho' = \sum_j \sum_{i_1, \ldots, i_n \in E} k_j \sigma_{i_1, \ldots, i_n} k_j^\dagger \otimes \sigma_{i_1, i_2}.$$  

(11)

Next, we separate the $i_3, \ldots, i_n \in E$ (set of even parity $n - 2$-uples of bits) and $i_3, \ldots, i_n \in O$ (set of odd parity $n - 2$-uples of bits) contributions so that

$$\rho' = \sum_{i_3, \ldots, i_n \in E,j} k_j \sigma_{i_3, \ldots, i_n} k_j^\dagger \otimes (\sigma_{0,0} + \sigma_{1,1})$$

$$+ \sum_{i_1, \ldots, i_n \in O,j} k_j \sigma_{i_1, \ldots, i_n} k_j^\dagger \otimes (\sigma_{0,1} + \sigma_{1,0}).$$

Subsequently, we trace over all but the first and second qubits to obtain $\rho'_{12}$; let the constants $C_E, C_O$ be defined by:

$$C_{E/O} := \sum_{i_3, \ldots, i_n \in E/O,j} |\langle \ell_3, \ldots, \ell_n | k_j |i_3, \ldots, i_n\rangle|^2,$$

(12)

so that the partial trace results

$$\rho'_{12} = C_E (\sigma_{0,0} + \sigma_{1,1}) + C_O (\sigma_{0,1} + \sigma_{1,0}),$$

(13)

and finally, tracing over the second qubit we obtain:

$$\rho'_1 = C_E (\sigma_0 + \sigma_1) + C_O (\sigma_1 + \sigma_0) = (C_E + C_O) \mathbb{I}_1,$$

(14)

which shows that no information is gained about qubit 1, as it had been previously anticipated.

IV. MUTUAL LIMITATION BETWEEN COMMUNICATION AND ENERGY EXTRACTION

According to the previous paragraphs, posting messenger quantum states from $P$ to $A$ may serve one or two of the following purposes: sending qubits to be converted into energy and communicating information. Both features are mutually exclusive functions whose limiting relation is next obtained.

Holevo [59] in 1998, and Schumacher et al. [60] in 1997, established that given an alphabet consisting of 1) a set of $N$ quantum mixed states (also known as letters) described by their density matrices $\{\rho_a, a = 1 \text{ through } N\}$ and 2) a set of probabilities $\{p_a, a = 1 \text{ through } N\}$ according to which the states have to be used, one can devise a communication protocol to convey $\chi$ bits (in average for sufficiently long messages) of classical information per state in the alphabet, where $\chi$ is defined as the Holevo information of the alphabet:

$$\chi := S(\rho_B) - \langle S_a \rangle,$$

(15)

where

$$\rho_B := \sum_{a=1}^N p_a \rho_a$$

(16)
is the mixed state that exits the emitter $A$ according to the information of the receiver $B$ and

$$< S_a > := \sum_{a=1}^{N} p_a S(\rho_a). \quad (17)$$

It is understood that the communication protocol assigns a sequence of states of the alphabet, or codeword to every message that can be transmitted.

We are now going to use this theorem\cite{61} to state a new interesting result.

Let us now describe a bipartite scenario, with $A$ (Alice) and $B$ (Bob). A messenger physical system $M$ can be emitted from $A$ to $B$. The states of $M$ belong to a finite set $\rho \in \{\rho_B, \ldots, \rho_N\}$. Besides, the availability of the states of $M$ follows a probability distribution $\{p_1, \ldots, p_N\}$ which determines the frequencies with which the states of $M$ have to be used. Thus, the states $\{\rho_a, a = 1 \text{ through } N\}$ with probabilities $\{p_a, a = 1 \text{ through } N\}$ make the letters of an alphabet. If Alice and Bob agree on a certain code, the ordering of the states sent from $A$ to $B$ may be used to communicate messages, according to the Holevo information of the alphabet. Besides, not completely depolarized systems can also be used to fuel a QIHE, provided that there is a thermal bath at $B$.

According to Eq\[2\], the average work ($E$) obtainable from each letter reads

$$E = k_B T \left( \ln 2 \right) \left( M - S(\rho_B) \right), \quad (18)$$

where

$$M := \log_2 d$$

and $d$ is the dimension of the Hilbert space for the physical system $M$; $M$ is then the equivalent number of qubits per letter in the alphabet\cite{62}. Accordingly, substituting for $S(\rho)$ in Eq\[15\] yields

$$E = \frac{\mathcal{E}}{k_B T \ln 2} + \mathcal{C} + < S_a > = M, \quad (20)$$

where $\mathcal{C}$ is the average information per emitted letter that one can communicate using the alphabet $\{\rho_a, p_a\}$ (which, according to the previous paragraph, is equal to $\chi$). Eq\[20\] states the mutual limitation between communication and energy as is graphically represented in Fig\[5\].

![FIG. 5. When some source of quantum states is used to convey classical information, it may also serve as an ancilla to extract work from a thermal source. Eq\[20\] sets a mutual limitation for both functions. This figure represents a distribution of the functionality of a source $\{p_a, \rho_a\}$ per transmitted letter between energy conversion $\mathcal{E}$ (white) and a useless part $\langle S_a \rangle$ (gray), showing that they add up to $\mathcal{M}$.](image)

V. SIMULTANEOUS SUPPLY OF INFORMATION AND WORK EXTRACTION CAPABILITY

The derivation of Eq\[20\] in the previous section was based on independent results for communication and thermodynamics scenarios. It proves that there is a relation between two different functions of the states of a system $M$, when they are used exclusively, that is for either sending messages or fueling a QIHE. Next, we show that both functions can be obtained simultaneously for the same system with a mutual limitation of Eq\[20\]. For that, we refer to the decoding of information described in \[60\]. Briefly stated, the results put forward in that paper which are relevant to this work are:

1. there is a typical subspace $\Lambda_{L,\epsilon,\delta} \subset \mathcal{H}^\otimes L$, whose orthogonal projector is $\Pi_{L,\epsilon,\delta}$ that verifies

$$\text{Tr} \left\{ \Pi_{L,\epsilon,\delta} \rho_L \right\} > 1 - \epsilon, \quad (21)$$

where $\rho_L := \rho_B^\otimes L$, and

$$\rho_B := \sum_{a=1}^{N} p_a \rho_a \quad (22)$$

in the following we drop the subscripts $L, \epsilon, \delta$ and simply write $\Lambda$ to denote the subspace $\Lambda_{L,\epsilon,\delta}$.

2. the dimension $d_\Lambda$ of $\Lambda$ is bounded by

$$d_\Lambda := \dim(\Lambda) \leq 2^{L(\delta + S(\rho_B))} \quad (23)$$

3. there is a communication protocol between Alice and Bob, whose information is coded in the order in which Alice arranges the letters $\rho_a$ before sending them to Bob, where the frequency of letter $\rho_a$ is $p_a$, that achieves $\chi - 5\delta$ bits per letter with a probability of error $P_E \leq 10^{-6}$. When Bob receives the codewords sent by Alice, he performs a POVM (POM in the reference article\[63\]), defined by a collection of effects $E_j = |\mu_j \rangle \langle \mu_j|$, where all of the kets $|\mu_j \rangle$ belong to the typical subspace $\Lambda$. All the ensuing process of decoding relays only on the result of this POVM.

Our next purpose is to use this result to closely factorize the state $\rho_L = \rho_B^\otimes L$ into two parts: one which
A QIHE, which will yield a work transformation is guaranteed to exist. This completes both subspaces have the same dimension, such a unitary dimensional space defined by the tensor product dimensional Hilbert space into a \( d \times d \) dimensional subspace defined by the tensor product of a first attempt we would want to factorize the QIHE (which, inversely, should have low entropy). As a carry most of the entropy) and other which can fuel a car- rying the information (and consequently should contain most of the entropy) and other which can fuel a QIHE (which, inversely, should have low entropy). As a first attempt we would want to factorize the \( d_L := 2^L \times M \) dimensional Hilbert space into a \( d_L \) dimensional one and another which should accordingly have dimension \( d_L/d_\Lambda \). This can only happen when \( d_L/d_\Lambda \) is a positive integer number, which is not the case in a general situation.

In order to guarantee the possibility of factorization in the general case, we enlarge the Hilbert space \( \mathcal{H}_L := \mathcal{H}_D \otimes \mathcal{H}_L \) by a tensor product with a \( d_\Lambda \)-dimensional ancillary physical system \( D \) upon reception of the codeword by \( B \). Let \( |0\rangle_D \) be the initial state of the ancillary system \( D \) and denote by \( \mathcal{H}_D \) the Hilbert space of \( D \).

The enlarged codewords live now in the \( d_L \times d_\Lambda \)-dimensional space defined by the tensor product \( \mathcal{H}_L \otimes \mathcal{H}_D \). Next we define the enlarged typical subspace \( \Lambda := \Lambda \otimes 0^d_\Lambda \), where \( 0^d_\Lambda \) is the one-dimensional subspace of \( \mathcal{H}_D \) spanned by \( |0\rangle_D \).

As the next step we define a unitary transformation \( U \) which maps the enlarged typical subspace \( \Lambda \) into the subspace \( |0\rangle_L \otimes \mathcal{H}_D \), where \( |0\rangle_L \) is a pure state of \( \mathcal{H}_L \). As both subspaces have the same dimension, such a unitary transformation is guaranteed to exist. This completes the process of transferring codewords in \( \Lambda \) to \( D \), as is depicted in Fig. 6. The first part of the system is fed to a QIHE, which will yield a work

\[
W_1 = k_B T (\ln 2) L M
\]

with a probability \( 1 - \epsilon \), corresponding to the case that the \( \mathcal{H}_L \) part is in the |0\rangle_L state or, equivalently, that the codeword is in \( \Lambda \) and such probability is given by Eq. 21. If not, the work may be negative, but never less than \(-W_1\). Accordingly, the average work obtained verifies

\[
W \geq k_B T (\ln 2) L M (1 - 2\epsilon),
\]

from which we should substract the one necessary to re-store the ancillary system

\[
W_D = k_B T (\ln 2) \log_2 d_\Lambda,
\]

and taking into account Eq. 23

\[
W_D \leq k_B T (\ln 2) (L S(\rho_B) + L\delta)
\]

and consequently, the average net work per letter gained by the QIHE at \( B \) verifies

\[
W_{\text{letter}} \geq k_B T (\ln 2) (M (1 - 2\epsilon) - S(\rho_B) (1 + \delta))
\]

We also know, from the theory of QIHE and ultimately from the Second Principle of Thermodynamics that

\[
W_{\text{letter}} \leq k_B T (\ln 2) (M - S(\rho_B))
\]

so that the average work extractable per letter, in the \( L \to \infty \) asymptotic limit, reads

\[
\mathcal{E} := W_{\text{letter}} = k_B T (\ln 2) (M - S(\rho_B))
\]

and the Eq. 29 holds for the case of simultaneous QIHE and communication use of a source of quantum states. Fig. 8 represents the process.

Given the set of states \( \rho_a \) of a system \( D \), with probabilities \( p_a \), as in the previous paragraphs, the last question to be discussed is whether Alice and Bob can arrange a protocol to increase \( \mathcal{E} \) and decrease \( \mathcal{C} \) viceversa. The answer is that \( \mathcal{C} \) can not be increased over \( \chi \), on account of Holevo’s theorem, but \( \mathcal{E} \) can be improved if Alice and Bob previously agree on decreasing \( \mathcal{C} \).

A way to do it would be a previous agreement on always sending blocks of \( n \) copies of each letter. That is equivalent to replacing the alphabet of letters \( \rho_a \) with probabilities \( p_a \) by a new alphabet of letters \( \rho'_a := \rho_a^\otimes n \) with the same probabilities \( p_a \). In this case,

\[
S(\rho'_a) = n S(\rho_a), \mathcal{C}' \leq n \mathcal{C}, \mathcal{M}' = n \mathcal{M},
\]

and, thus,

\[
\frac{\mathcal{E}'}{n} = \mathcal{M} - \mathcal{C}' - \langle S_a \rangle
\]

In the limit \( n \to \infty \), \( \mathcal{C}' \to 0 \) and consequently

\[
\frac{\mathcal{E}'}{n} \to \mathcal{M} - \langle S_a \rangle
\]

This value can also be reached if Alice and Bob arrange a particular ordering of the states, declining to use them for communication. Fig. 8 represents the simultaneous values per letter of 1) the extractable work \( \mathcal{E} \) and 2) communication bits \( \mathcal{C} \) that can be attained when Alice disposes over a supply of copies of a physical system \( M \) that can be transferred to Bob.
VI. RESULTS AND CONCLUSIONS

New domains for the application of quantum information theory, especially cryptography and entanglement have been presented. It is assumed that users have access to suitable QIHEs. In particular, this paper describes protocols for work extraction from a single thermal bath through the distribution of messenger qubits, with increasingly complex features. Procedures for requiring collaboration from other users to unlock work extraction are also presented using both strongly entangled and classically correlated multipartite quantum ancillas. Specifically, the following possibilities have been presented:

1. Simple transmission of messenger systems whose state is not completely depolarized for the receiver. He can extract a work equal to \( k_B T (\ln 2) (S_{max} - S(\rho)) \).

2. Encrypted transmission of messenger systems through the use of previously entangled bipartite systems. This technique makes the transmitted system useless for illegitimate users that might intercept them. Quantum systems prove to be able to supply twice as much work as classical ones, because of the same physics that is behind the feature of superdense coding in quantum communication protocols.

3. In a multi-user scenario, where users initially share generalized GHZ states, any user can enable all the other ones to extract work.

4. Also in a multi-user environment, if some correlated classical states are shared among all users, all but one can enable the other to extract work.

In order to find a relation between the two possible uses of messenger systems, a mutual limitation between
communication and energy has been derived in section IV leading to Eq. 20. This result seems quite natural, from the complementary perspectives of gaining certainty (communication) and increasing uncertainty (traded for work). In section V the validity of Eq. 20 is generalized to the case of simultaneously using the messenger system for communication and work extraction. The results are represented in Fig. 8. It shows that the communication capacity C cannot be increased over the Holevo’s bound, although it can be arbitrarily reduced, upon agreement of both users, to improve the work extraction capacity up to $k_B T \ln(2) (M - \langle S_a \rangle)$.

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[62] Note that $M$ is not generally an integer number.

[63] In [60] the codes are constructed by choosing a number of codewords independently, according to the a priori string probability for each codeword. The choice is supposed to be random and the results concerning the probability of error are averaged over the different possibilities.