On the question of gas-dynamic temperature stratification device optimization

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Abstract. One- and two-dimensional mathematical models of the devices for the machine-free energy separation of compressible gas flows have been considered. The device is a "pipe in a pipe" heat exchanger; the supersonic flow passes along an internal cylindrical channel, the subsonic flow — along an external annular channel. Energy separation takes place without any moving pieces. Main stream divides in two parts: a cold one (subsonic) and a hot one (supersonic).

The proposed models were validated in a wide range of input parameters changes. The influence of a direct and counter flow pattern at the energy separation effect was investigated in terms of subsonic cooling maximization. By using the developed models, the optimal profiles of the supersonic channel were determined from the maximum energy separation effect point of view at identical initial total pressures, total temperatures and mass flows.

1. Introduction

Gas-dynamic temperature stratification effect of a compressible gas and device for it implementation was at the first time proposed by A.I. Leontiev in 1997. The essence of the effect is the recovery temperatures difference for sub- and supersonic flows. As is known, the recovery temperature or the equilibrium temperature of a heat-insulated surface is determined by the following relation:

$$T^*_{aw} = T^*_0 (1 + r \frac{k-1}{2} M^2) \left(1 + \frac{k-1}{2} M^2\right),$$

where $T^*_0$, $M$ — gas parameters of free flow, $r$ — temperature recovery coefficient.

Let us consider two gas flows with equal initial values of total temperatures, separated by a heat conducting wall (see figure 1). One stream is kept without any impacts and will be subsonic $M_1 \ll 1$ and second is accelerated to supersonic speed $M_2 > 1$. In this case recovery temperature, according to (1), $T^*_1 = T^*_0$ but for supersonic flow $T^*_2 \approx r T^*_0$. Consequently, the temperatures from different sides of the wall are different and if the wall is designed from heat conducting material, heat exchange will take place. The direction of the heat flux will depend on the recovery factor value. For the air ($Pr = 0.7$) the value of recovery factor is $r < 1$ i.e. $T^*_1 > T^*_2$ and, therefore, the subsonic flow will be cooled, and the supersonic stream will be heated.

At the current moment, there is a limited number of published works dedicated to the validation of mathematical models of the device. In [2] a one-dimensional method for calculating
The total temperature distribution in the boundary layer in the case of a heat-conducting wall (I), in the case of a heat-insulated wall (II)

The purpose of this work is to create a generalized (with a minimum number of assumptions) mathematical models that accurately predict the processes taking place in the device for the machine-free energy separation, which working according to the method proposed by A.I. Leontiev.

2. Problem formulation

Let us consider the device which was experimental studied in [6, 1, 7]. The device consists of two coaxial channels (see figure 2): external (subsonic) and internal (supersonic).

The internal channel is formed by a supersonic nozzle, an internal conical [6] or a conical-
The channels are separated by a heat-conducting wall designed from brass. The total length of the working area is \( L_0 = 700 \) mm.

### 3. Mathematical Model

Let us consider sequentially the one- and two-dimensional mathematical models of the described above device.

Air was considered as a working fluid for all models. Moreover it was treated as a compressible ideal gas, obeying the Mendeleev-Clapeyron equation. The thermodynamic and transport properties were determined according to [8].

#### 3.1. One-dimensional model

In the case of one-dimensional modeling, two sub models can be distinguished: the model of gas flow in the channels and the model of heat conduction for a cylindrical wall.

For the gas flow model the well-known Shapiro-Hawthorn method was used [9]. The method allows to analyze the flow in the presence of various external influences. In the consideration case the external influences are:

- geometrical \( dA \);
- thermal \( dQ_w \);
- friction \( C_f \).

The equation for the Mach number along the channel will have the following form:

\[
\frac{dM^2}{M^2} = -2 \left( \frac{1 + \frac{k-1}{2}M^2}{1 - M^2} \right) \frac{dA}{A} + \frac{kM^2}{1 - M^2} \frac{dQ_w}{mC_pT} + \frac{kM^2}{1 - M^2} 4C_f \frac{dx}{dh} \tag{2}
\]

Equations for other variables (pressure, temperature, etc.) can be found in [9].

Amount of transferred heat:

\[
dQ_w = 4q_w \frac{A}{dh} dx, \quad q_w = \alpha (T_w - T_{aw}), \quad \alpha = \frac{Nu \lambda}{dh} \tag{3}
\]

The values of Nusselt numbers \( Nu \) and friction coefficients \( C_f \) were determined in a different manner for internal (cylindrical) and external (annular) channels according to [10].

For friction coefficient calculation the compressibility correction factor was used [11]:

\[
\Psi_M = \left[ \frac{2}{1 + \left( 1 + r \frac{k-1}{2}M^2 \right)^{0.6}} \right]^2 \tag{4}
\]

Recovery coefficient was calculated based on Rotta equation [12]:

\[
r = Pr_t + \frac{C_f}{2}(Pr - Pr_t)f(Pr_t) + 7(1 - Pr_t)\sqrt{\frac{C_f}{2}} \tag{5}
\]

Numerical values of \( f(Pr_t) \) can be found in [12].

The law of the cross-sectional area change was prescribed by the analytical relation \( A = A(x) \).
For the heat conduction problem the one-dimensional solution for a cylindrical wall was used:

\[ q_w = K(T_{aw1}^* - T_{aw2}^*), \quad K = \frac{\pi}{\alpha_1 d_o} + \frac{1}{2A_w} \ln \frac{d_o}{d_i} + \frac{1}{\alpha_2 d_i}. \]  

(6)

Thus, a closed system of equations describing the flow and heat exchange in a system of coaxial channels separated by a heat-conducting wall were obtained.

The problem is solved by an iterative manner: the equations for the gas flow in the channels and the heat conduction in a cylindrical wall are solved successively. Solutions are iterated to each other. The iterative process stops, when the heat balance between the problems is reached.

3.2. Two-dimensional (axisymmetric) model

The two-dimensional problem was solved by using of ANSYS Fluent. The problem was modeled in the conjugate formulation. ANSYS ICEM CFD preprocessor was used for the mesh creation.

The discretization of the Reynolds averaged Navier-Stokes equations (RANS), the energy equations (for both liquid and solid) and the equations of the corresponding turbulence model was performed on the basis of the control volume method.

Two-equations differential turbulence models were used to close the main system of equations. In total, the results of calculations by using four turbulence models were analyzed:

- \( k-\epsilon \) family models
  - Standard (ske)
  - Realizable (rke)
- \( k-\omega \) family models
  - Standard (skw)
  - SST

Moreover, the turbulent Prandtl number for the energy equation was set both as a constant value and on the basis of the Kays-Crawford analytical model [13]:

\[ Pr_t^{-1} = \frac{0.5}{Pr_{t\infty}} + 0.03Pe_t \sqrt{\frac{1}{Pr_{t\infty}}} - (0.03Pe_t)^2 \left[ 1 - \exp \left( \frac{-1}{0.03Pe_t \sqrt{1/Pr_{t\infty}}} \right) \right] \]  

(7)

It should be noted that value of the free flow turbulent Prandtl number \( Pr_{t\infty} \) is a parameter of this model. The recommended value of \( Pr_{t\infty} = 0.85 \) was obtained from the analysis of the logarithmic region of the thermal boundary layer [14]. However, in work [14] it is shown that the range of \( Pr_{t\infty} \) variation for air is 0.73–0.92.

4. Models Validation

In [6, 1, 7], experimental data were obtained for two device configurations:

- internal channel only and heat conducting wall;
- assembled device: internal channel, heat conducting wall and outer (annular) channel.

Static pressure distribution along the length of the supersonic duct were measured for the first configuration, as well as the local temperature of the outer surface of the heat conducting wall. For the assembled device, the values of subsonic flow cooling and supersonic flow heating were measured, as well as radial distribution of the total temperatures at the exit from the supersonic channel. In the present paper, as in [6], three supersonic nozzles were considered for the isentropic Mach numbers \( M_{is} = 1.8, 2.0, 2.5 \). (\( d_{ex} = 5.0, 4.6, 3.7 \) mm).

The operating parameters varied in the following ranges:
Figure 3: Distribution of static pressure (a) and outer wall temperature (b) for conical supersonic channel. $M_{is} = 1.8$, $P_{0}^* = 13.9$ atm, $T_{0}^* = 25$ °C. 1 – 1D model; 2 – ske, $Pr_l = 0.85$; 3 – skw, $Pr_l = 0.85$; 4 – rke, $Pr_l = var$, $Pr_{t\infty} = 0.82$; 5 – skw, $Pr_l = var$, $Pr_{t\infty} = 0.85$; 6 – skw, $Pr_l = var$, $Pr_{t\infty} = 0.82$; symbols – experimental data [6]

- $P_{0}^* = 10.48–16.72$ atm
- $T_{0}^* = 25.27–51.67$ °C
- $m_1/m_2 = 0.1–1.0$

Subsonic massflow $m_1$ was varied for mass flow relation changing.

In figure 3a shows a comparison of the measured and calculated static pressure distribution along the conic supersonic channel by using the different models. As it can be seen from the figure, all models demonstrate a close result. A slight difference is observed in the position of the shock at the outlet of the channel.

For the same operational conditions in figure 3b shows a comparison of the outer surface temperature distribution for the heat-conducting wall. As it can be seen, the influence of modeling is much more significant.

Based on a comparison with the experimental data [6, 1, 7], it can be concluded that the turbulence models of the $k - \omega$ family with the Kays-Crawford model with $Pr_{t\infty} = 0.82$ are optimal. It should be noted that in [15] a very close value $Pr_{t\infty} = 0.83$ was used.

On the basis of the performed calculations, it can be noted that in the whole range of parameters variation, the two above-mentioned turbulence models demonstrates the best agreement with the experiment.

For the second configuration (assembled device), as it was already mentioned above, the radial distributions of the total temperature at the device outlet were measured.

In figure 4 shows a comparison of the radial total temperature distributions at the exit of the conic supersonic channel for the case $M_{is} = 1.8$, $P_{0}^* = 13.9$ atm, $T_{0}^* = 25$ °C by $m_1/m_2 = 0.29$. As can be seen from the figure, the Standard $k - \omega$ (skw) model with the Kays-Crawford model demonstrates a more than satisfactory agreement with the experimental data.

The total temperature difference (between inlet and outlet) was used for the energy separation integral effect estimation. The calculations results in comparison with the experimental data are shown in figure 5. The figure also shows the calculated curves obtained using the one-dimensional model (described in section 3.1) for both conical (solid curves) and conical-cylindrical (dashed curves) supersonic channels.

It should be noted that the experimentally determined values $\Delta T^h_{ho}$ and $\Delta T^c_{ho}$ for the conical channel [6] differ significantly from the numerical simulation results. Later [1, 7], after the
Figure 4: Radial distribution of the total temperature at the exit of the conical supersonic channel. $M_{is} = 1.8$, $P_{0}^* = 13.9$ atm, $T_{0}^* = 25$ °C, $m_1/m_2 = 0.29$. 1 – skw, $Pr_t = var$, $Pr_{t\infty} = 0.82$; 2 – sst, $Pr_t = var$, $Pr_{t\infty} = 0.82$.

Figure 5: Heating of supersonic and cooling of subsonic flows with a counterflow flow pattern versus on the mass flow ratio. $M_{is} = 1.8$, $T_{0}^* = 25$ °C. Solid lines – conical channel, 1D model, $P_{0}^* = 13.9$ atm; dashed lines – conical-cylindrical channel, 1D model, $P_{0}^* = 10.36$ atm; ⧆ – skw, $Pr_t = var$, $Pr_{t\infty} = 0.82$; ⨿ – experimental data [6]; ⨿ – experimental data [7].

refinement of the experimental procedure, the absolute values of the flow cooling and heating were updated. It can also be seen from the figure that the use of conical and conical-cylindrical channels leads to very close results. However, in the case of the second one, is it possible to significant reduce the total pressure for starting the device. Hereafter the conical channel will be considered only.

As can be seen, both models (1D and 2D) demonstrate a good agreement with experiment. Thus, the proposed mathematical models quite adequately describe the physical processes taking place inside the device and can be used for the further research.
5. Flow pattern

It was noted in [1] that, in the range of the parameters studied, the flow pattern (direct or counter flow) does not have a noticeable effect on the energy separation effect. We recall that in [1] the flows was studied for $m_1/m_2 > 0.1$.

In figure 6 shows the results of calculations of temperature separation in the range $0 < m_1/m_2 < 1$. As can be seen from the figure, the effect of the flow pattern becomes significant by $m_1/m_2 < 0.1$. The direct flow pattern is preferred in case of the flow acceleration, since it allows to obtain the maximum cooling of the subsonic flow.

6. Optimization of supersonic channel profile

One way to increase the thermal separation effect is to optimize the supersonic channel profile. On the one hand, the effect will be the higher the higher Mach number in the supersonic channel, since according to equation (1) the adiabatic wall temperature will be lower in case of higher Mach number. However, with a continuous increase of the Mach number, the heat transfer coefficient will decrease. Thus, it is most probably that the optimal profile will be the one providing a constant Mach number along the channel. The same conclusion was obtained in [16].

Thus, the following three cases can be considered (see figure 7):

- Initial channel with a varying (increasing) Mach number $M_2 = \text{var}$
- Channel with a constant Mach number equal to the inlet Mach number for the initial channel $M_2 = \text{const}_0$
- Channel with a constant Mach number equal to the exit Mach number for the original channel $M_2 = \text{const}_1$

In this case, the initial total pressure, the total temperature and the mass flow are fixed for all three cases. The equation (2) will be integrated for determination of the profiles for last two cases, but on condition $dM_2/dx = 0$. Unknown variable in this case will be the law of area change $A = A(x)$.

The length of the channel was determined from the condition of the physical realization of the flow. Taking into account the fact that supersonic flow is exhausted to the atmosphere, it was
Figure 7: Diameter (a) and Mach number (b) distribution along the supersonic channel. $P_0^* = 13.9$ atm, $T_0^* = 25$ °C, $m_2 = 0.0667$ kg/s. 1 — $M_2 = \text{var}$; 2 — $M_2 = \text{const}_0$; 3 — $M_2 = \text{const}_1$

assumed that a normal shock wave is realized within the diffuser, after it the pressure reaches the atmospheric level, i.e. the length of the channel will be determined from the following condition:

$$p_2 \mid_{x=L} = \frac{p_0}{\frac{2k}{k+1} M^2 \frac{k-1}{k+1} + 1}$$

In this case, the profiles of the supersonic channel are close to linear (see figure 7a), but they are not strictly such. As it can be seen from the figure, in case $M_2 = \text{const}_0$ the channel length exceeds the length of the initial channel by almost half. In the case of $M_2 = \text{const}_1$, the length of the channel is comparable with the initial one. However, based on the condition of equality of mass flow ($m_2 = \text{const}$) for all three cases, the critical section in the third case was increased.

In figure 8 shows a comparison of the integral energy separation effect for all three supersonic channels. As it can be seen, the case $M_2 = \text{const}_0$ demonstrates a gain of about 40% at equal mass flow through the sub- and supersonic channels. Consequently, this case is the most effective from the point of view of maximum transferred heat between the streams. Whereas the case $M_2 = \text{const}_1$ shows an advantage in cooling of the subsonic flow at $m_1/m_2 < 0.2$.

It is interesting to note the fact that the case $M_2 = \text{const}_1$ is almost identical with a direct flow pattern (see figure 8).

7. Conclusions

Thus, two mathematical models of the device for the machine-free energy separation are considered. The validation of models was performed.

It is shown that for a low mass flow region $m_1/m_2 < 0.2$, a direct flow scheme with acceleration in the supersonic part is the most preferable from the point of view of subsonic flow cooling.

Optimal profiles of the supersonic channel were determined on the condition of a fixed total pressure, total temperature and mass flow through the supersonic channel.

For the region $m_1/m_2 < 0.2$, and mainly for cooling of the subsonic flow, it is recommended to use a channel with a constant large Mach number.

For the region $m_1/m_2 > 0.2$ both for heating of the supersonic and for cooling of the subsonic flows, it is recommended to use a channel with a constant low Mach number.
Figure 8: Influence of flow pattern and supersonic channel profile on the value of temperature separation. $M_{is} = 1.8$, $P^*_0 = 13.9$ atm, $T^*_0 = 25$ °C. $1$ – counter flow $M_2 = var$; $2$ – direct flow $M_2 = var$; $3$ – $M_2 = const_0$; $4$ – $M_2 = const_1$

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