Planning of driverless vehicle motion trajectory taking into account its dimensional characteristics and under conditions of dynamically changing environment

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Abstract. This paper presents the solution for the task of the planning of the driverless vehicle (DV) driving trajectory for the modern road conditions. The rapidly-exploring random trees (RRT) algorithm consisting in the generation of the treetops under the equiprobability law and search for an optimum trajectory was used as a base. The minimum length acts as a criterion of optimality. The condition for the trajectory generation using the road map model is added to the new algorithm. Thereby, the problem of low processing (response) speed of the optimum trajectory search algorithm was solved.

1. Introduction
Works on the creation of the driverless vehicle began in the 70s of the XX century in Japan. The project "Prometheus" based on the developments of the German scientist Ernst Dickmans acted in the late 80s - early 90s, during which studies were conducted on the management of the driverless vehicles [1]. Work on the creation of the driverless technology of the last century were more experimental than practical. Most of the cars were either remote control prototypes or implemented simple algorithms. Currently, you can monitor the activity aimed at the creation of the driverless vehicles [2]. Such companies as Google, Tesla Motors, Apple, General Motors, Volkswagen, Audi, BMW, Volvo, Nissan and others make the greatest efforts in this field.

The rapid progress in this area is mainly due to the following factors:
- the advent of global navigation systems GPS / GLONASS and the widespread use of small GPS receivers;
- the increase of the productivity of modern computers;
- evolution of vision systems.

Modern technologies allow to create systems with high localization accuracy. Much has already been done in the area of unmanned vehicle architecture design [3].

Thus, the task of a higher level, the synthesis of control algorithms, comes to the fore. Namely - trajectory planning algorithms and algorithms for following it. This article is devoted to the issues related to the construction of the trajectory of movement on a well-known map.

Traditionally, the task of planning a trajectory is solved in two stages: the construction of a graph and the search for a path according to a given criterion in this graph [4]. Existing methods for finding a path on a graph in most cases are modifications of E. Dijkstra's algorithm, which is based on an iterative traversal of nodes of this graph [5]. Usually they can be divided into three categories: exhaustive algorithm, heuristic search, a ray-casting and tracking algorithm. Poor performance is a huge
disadvantage of all these algorithms. This parameter is critical for the driverless vehicle in modern conditions of a dynamically changing environment.

This article proposes a modified trajectory planning algorithm based on the method of rapid research using a random tree (RRT). The idea of the RRT algorithm is to form a tree that would evenly explore the search space [6]. The search space is represented as a map in the form of a grid of size $M \times N$, where $M$ is the number of rows, and $N$ is the number of columns [7]. For each node of the grid there is a logical sign of the path possibility.

Distinctive features of the proposed algorithm are:
1. The search space is defined as a vector model of a road map.
2. In the process of trajectory search, limitations imposed by the geometric dimensions of the DV are taken into account.
3. The law of generation of treetops is proposed taking into account the kinematic characteristics of the DV, which allows effectively to explore the search space.

Thus, the trajectory planning time is significantly reduced. As a result, this ensures the non-stop movement of the DV and the avoidance of obstacles.

2. Formulation of the problem

There is a mathematical model of a roadmap, which is described in a two-dimensional Cartesian coordinate system XOY. The boundaries of the roads are described as a sequential set of intervals $I_i = \{x_{ni}, y_{ni}, d_i, \theta_i\}$ and arcs of circles $c_i = \{x_{qi}, y_{qi}, R_i, \alpha_i, \varphi_i\}$, the tangents of which are $I_i$ (Fig. 1), where $i$ – the object number of the original sequence, organized in such a way that the starting point of the $i$-th object coincides with the final $(i-1)$-th.

$x_n \in XOY, \quad y_n \in XOY$ – coordinates of the point $A_{in}$, which is the "beginning" of the lines segment.
$d \in [\mathbb{R}]$ – line length.
$\theta \in [0; 2\pi]$ – cut angle of line (relative to axis OX).
$x_o \in XOY, \quad y_o \in XOY$ – coordinates of the circle center $A_o$.
$R \in [\mathbb{R}]$ – radius of circle.
$\alpha \in [0; 2\pi]$ – the orientation angle of the radius vector drawn from the center of the circle to the "beginning" of the arc $A_{cn}$ (relative to axis OX).
$\varphi \in [-2\pi; 2\pi]$ – angular magnitude of the arc of a circle.

Thus, the map of the road $Map$ can be represented as a set union of $n$ line and $m$ arcs of circles.

$$Map = \left(\sum_{j=0}^{n} c_i\right) \cup \left(\sum_{j=0}^{m} I_i\right)$$

![Figure 1. A road map model.](image)

For the convenience of planning the trajectory, it is proposed to model the DV as a circle of radius $r = 0.5 \cdot H$, where $H$ is the vehicle width. In this case, to take into account the restrictions when planning
a trajectory, you can apply the method of expanding impassable zones by an amount corresponding to \(r\) \cite{8}. In this case, the DV can be considered as a material point. In \cite{9}, it was shown that the time-optimal path between the initial state vector \(\overrightarrow{p_s} = \{x_s, y_s, \theta_s\}\) and a terminal state vector \(\overrightarrow{p_f} = \{x_f, y_f, \theta_f\}\) consists of an arc of a circle \(\overrightarrow{R_{ds}} = \{\overrightarrow{p_s}, \overrightarrow{O_{cs}}, T_{ex}, r_{ds}, \varphi_{ds}\}\), followed by the line segment \(\overrightarrow{l_d} = \{T_{ex}, T_{en}\}\), and ends with another arc of the circle \(\overrightarrow{R_{df}} = \{\overrightarrow{p_f}, \overrightarrow{O_{cf}}, T_{en}, r_{df}, \varphi_{df}\}\) (Fig. 2). Where the radii of these arcs \(r_{ds}\) and \(r_{df}\) are determined by the kinematics of the DV. The trajectory of this type is called the Dubins trajectory.

The previously used notations are described here:

\(x_s \in XOY, \ y_s \in XOY\) – coordinates of the starting point of the DV \(A_s\).
\(\theta_s \in [0; 2\pi]\) – initial orientation angle DV (relative to axis OX).
\(x_f \in XOY, \ y_f \in XOY\) – coordinates of the desired position \(A_f\).
\(\theta_f \in [0; 2\pi]\) – terminal orientation angle (relative to axis OX).
\(\overrightarrow{O_{cs}}(x_{cs}, y_{cs}) \in XOY\) – center of the first circle.
\(\overrightarrow{O_{cf}}(x_{cf}, y_{cf}) \in XOY\) – center of the second circle.
\(T_{ex}(x_{ex}, y_{ex}) \in XOY\) – line start point.
\(T_{en}(x_{en}, y_{en}) \in XOY\) – straight line end point.
\(\varphi_{ds} \in [-2\pi; 2\pi], \ \varphi_{df} \in [-2\pi; 2\pi]\) – angular values of the first and the second arcs of the trajectory circles.

![Figure 2. Dubins trajectory.](image)

Dubins generalized trajectory \(D \in XOY\) can be represented

\[
D = (\overrightarrow{R_{ds}} \cup \overrightarrow{l_d} \cup \overrightarrow{R_{df}})
\]  

(2)

The trajectory is considered acceptable if the vehicle, moving on it, does not touch the lets and does not go beyond the limits of the carriageway. For ease of calculation, each obstacle \(O_{Lj}\) is conveniently described by a circle with a radius \(r_{Lj}\) and center coordinate \(O_{Lj}(x_{Lj}, y_{Lj})\), where \(j = 1..N\), and \(N\) is the total number of obstacles. Taking into account (1), (2) and the writing above, and also on the condition that the DV is located on the carriageway at the beginning of the movement, the criterion for the admissibility of the trajectory \(D\) is

\[
\left(D \cap \bigcup_{j=1}^{N} Let_j\right) \cup \left(D \cap (Map)\right) = \emptyset
\]  

(3)

The task of planning a trajectory is to find such an optimal along the length of the trajectory \(D\) such that criterion (3) is satisfied. The RRT algorithm is used as the base.
3. Trajectory planning algorithm
The DV technical vision system with a specified time interval searches for obstacles $L_i$ around the robot. A comprehensive navigation system determines the state vector of its current position in the global space $\overrightarrow{p_{tek}} = \{x_{tek}, y_{tek}, \theta_{tek}\}$.

The fulfillment of the initial conditions is necessary for the correct operation of the algorithm.
- An initial trajectory $T = \sum_{i=0}^{n} D_i$ is given, which consists of a set of Dubins n-trajectories (2). $T$ is located on the roadway and connects the start and finish points.
- DV is located at the starting point and begins to move along the trajectory at the initial moment.

The criterion (3) is checked in the process of movement when new obstacles appear. If it is not true, then the trajectory planning algorithm is launched. It represents the following sequence of actions:
1) Transformation of the search space for the condition that DV is described as a circle with a radius $r$.
2) Selection of the terminal state vector $\overrightarrow{p_{end}} = \{x_{end}, y_{end}, \theta_{end}\}$ for returning the control object from the current position to the initial path $T$.
3) Generation of treetops $A_i$ ($i = 1..N - 1$, where $N - 1$ is a total number of them) according to the law, taking into account the kinematic properties of the DV and the map $Map$.
4) Formation of a set of trajectory prototypes $Tops$ as the treetops sequences.
5) Construction of the Dubins trajectories (2) between adjacent points $A_i$ and $A_{i+1}$ for each set of treetops $Tops[j]$.
6) Trajectories research and selection of optimal.

3.1. Search space conversion
There are two stages to convert the search space, taking into account the overall characteristics of the DV. Expansion of impassable zones due to the presence of obstacles occurs at the first stage. That is, the magnitude of the obstacle radius $r_i$ is increased by $r$ (half width of DV). Narrowing of the roadway $Map$ occurs in the second stage. The narrowing of the carriageway consists in a parallel transfer of its $Left$ and $Right$ lines $l_i$ by the value of $r$, and in increasing or decreasing the radius $R_i$ by the value of $r$ (Fig. 3).

![Figure 3. Changing the area of the drivable possibility.](image)

For the $Right$ line, the transformation is $\overrightarrow{l_i} = \{x_{ni} - r \cdot \sin \theta_i, y_{ni} + r \cdot \cos \theta_i, d_i, \theta_i\}$.
For $Left$ is $\overrightarrow{l_i} = \{x_{ni} + r \cdot \sin \theta_i, y_{ni} - r \cdot \cos \theta_i, d_i, \theta_i\}$.

The calculation of the change in $R_i$ is determined from the condition $K$ (4). This condition is formed on the basis of the geometric meaning of the vector product. It characterizes the direction of rotation of
the roadway. If the condition is true, then the roadway changes its direction of travel to the right, otherwise - to the left.

\[ K: (\cos(\theta_{i-1})(y_{0i} - y_{ni-1}) - \sin(\theta_{i-1})(x_{0i} - x_{ni-1})) < 0 \]  

(4)

Based on the above

\[
\begin{cases}
\text{Right:} & \{ \text{if } K = \text{true}, \tilde{c}_i = \{x_{0i}, y_{0i}, R_i + r, \alpha_i, \varphi_i\} \\
\text{else} & \{ \tilde{c}_i = \{x_{0i}, y_{0i}, R_i - r, \alpha_i, \varphi_i\} \}
\end{cases}
\]

Left: \[
\begin{cases}
\text{if } K = \text{true}, \tilde{c}_i = \{x_{0i}, y_{0i}, R_i - r, \alpha_i, \varphi_i\} \\
\text{else} & \{ \tilde{c}_i = \{x_{0i}, y_{0i}, R_i + r, \alpha_i, \varphi_i\} \}
\end{cases}
\]

(5)

Thus, DV can be considered as a material point.

3.2. Selection of the terminal state vector

The task of the planning algorithm is to find such a trajectory that would go around the obstacles and return the DV to the original trajectory \( T \). At least two conditions must be met for the desired trajectory to exist. The first is the absence of obstacles in the terminal point area. Secondly, the terminal point must be in sight of the vision system. The orientation angle \( \vartheta_{\text{end}} \) must be equal to the angle of inclination of the trajectory \( T \) at the selected point.

3.3. Generate treetops

The following principles are observed when synthesizing the law of treetops generation:

1) uniform study of the search space;
2) full exploration of the search space;
3) quick change of treetops;
4) the ability to take into account the kinematics of the DV motion.

It is proposed to set the treetops in the form of a "grid". The nodes of this "grid" are the treetops. Their schematic layout is shown in Figure 4. The location of the "grid" is determined by the starting point \( A_1 \) with coordinates \( x_{1,gr}, y_{1,gr} \). Any other tops \( A_i(x_{i,gr}, y_{i,gr}) \) are easily determined according to (6).

\[
\begin{align*}
    x_{i,gr} &= x_{0,gr} + k\Delta x_{gr} + b\Delta y_{gr}\cot \vartheta_{gr} \\
    y_{i,gr} &= y_{0,gr} + b\Delta y_{gr}
\end{align*}
\]

(6)

where \( k = 1..n, b = 1..m, \) and \( n \) and \( m \) are number of columns and rows respectively.

\( \Delta x_{gr}, \Delta y_{gr} \) are the parameters that define the "grid" step. They are selected based on the kinematics of the DV motion. They must be such that the DV can move between adjacent points. Changing of the
treetops location is done by setting of the angle of the "grid" $\theta_{gr}$. There is a need to search for a trajectory from a treetops set that are on the roadway, for time-optimal a work of the algorithm (Fig. 5). Condition (3) can be used to find these treetops.

![Figure 5. Selection of treetops. Treetops marked with green labels can be used to plan a trajectory.](image)

**3.4. Constructing of a trajectory prototypes set**

The trajectory prototype is represented as a set of points between which one can construct the Dubins trajectories (2). Each prototype contains a starting vertex $A_0$ with coordinates $(x_{te}, y_{te})$ and a final vertex $A_N$ with coordinates $(x_{end}, y_{end})$. These vertices correspond to the vectors of the current and terminal position. It is known that the search space is represented as a “grid” of $n$ columns and $m$ rows. There are rules following which a sequence of treetops is formed for each trajectory prototype (Fig. 6).

1. Each next treetop is selected from a range of grid lines $b + 1 \ldots m$, where $b$ is the line number of the previous treetop.
2. For any treetop belonging to the b-line, there must be trajectories for all treetops of the subsequent lines.

$$T_{ops} = \begin{cases} 
    \{A_0, A_N\}, \\
    \{A_0, A_{11}, A_N\}, \{A_0, A_{12}, A_N\} \ldots \{A_0, A_{mn}, A_N\}, \\
    \{A_0, A_{11}, A_{21}, A_N\}, \{A_0, A_{11}, A_{22}, A_N\} \ldots \{A_0, A_{11}, A_{mn}, A_N\}, \\
    \ldots \\
    \{A_0, A_{1k}, A_{21}, A_N\}, \{A_0, A_{1k}, A_{22}, A_N\} \ldots \{A_0, A_{1k}, A_{mn}, A_N\}, \\
    \{A_0, A_{11}, A_{21}, A_{31}, A_N\}, \{A_0, A_{11}, A_{21}, A_{32}, A_N\} \ldots \{A_0, A_{11}, A_{21}, A_{mn}, A_N\}, \\
    \ldots \\
    \{A_0, A_{1k}, A_{2k} \ldots A_{bk} \ldots A_{mn}, A_N\} 
\end{cases} \quad (7)
3.5. Construction of Dubins trajectories

According to [6], there are four possible Dubins trajectories for a given initial configuration \( p_i \) and a final configuration \( p_f \). Option 1 (R-S-R) refers to the case when the DV begins clockwise motion, then follows a movement in a straight line and the second clockwise motion. Option 2 (L-S-L) for the case when the DV begins an anticlockwise motion, then follows a straight line operation and the second anticlockwise motion. The 3rd option (R-S-L) is the case when the DV begins a clockwise motion, then follows a straight line operation and an anticlockwise motion. Option 4 (L-S-R) is the case when the DV begins an anticlockwise motion, then follows a straight line operation and a clockwise motion. As a rule, the trajectory option is selected based on the minimum length. Proposed to introduce criteria \( z_s \) and \( z_f \) that correspond to the first and second circles for determining the minimum-length trajectory. Criteria (8) and (9) characterize the direction of rotation and are formed on basis of the minimum-length trajectory.

\[
\begin{align*}
z_s &= \cos \vartheta_s \cdot (y_f - y_s) - \sin \vartheta_s \cdot (x_f - x_s), \\
z_f &= \cos \vartheta_f \cdot (a_y - y_f) - \sin \vartheta_f \cdot (a_x - x_f),
\end{align*}
\]

where \( [a_x, a_y] = [x', y'] \begin{bmatrix} \cos(f_i) & -\sin(f_i) \\ \sin(f_i) & \cos(f_i) \end{bmatrix} + [x_{cs}, y_{cs}], \)

\[
fi = pr \cdot \arccos \left( \frac{y_f - y_{cs}}{\sqrt{(x_f - x_{cs})^2 + (y_f - y_{cs})^2}} \right),
\]

if \((x_f - x_{cs}) \geq 0\) then \( pr = 1 \), else \( pr = -1 \),

\[
x' = -\frac{z_s}{|z_s|} r_{ds} \sqrt{1 - \frac{r_{ds}}{\sqrt{(x_f - x_{cs})^2 + (y_f - y_{cs})^2}}},
\]
If the criterion \( z_s \) or \( z_f \) is less than zero, then the optimal turn is clockwise (R), otherwise it is counterclockwise (L). It’s easy to calculate the coordinates of the circle centers if the direction of rotation is known.

\[
\begin{align*}
x_{cs} &= x_s + \frac{z_s}{|z_s|} r_{ds} \sin \theta_s \\
y_{cs} &= y_s - \frac{z_s}{|z_s|} r_{ds} \cos \theta_s \\
x_{cf} &= x_f + \frac{z_f}{|z_f|} r_{df} \sin \theta_f \\
y_{cf} &= y_f - \frac{z_f}{|z_f|} r_{df} \cos \theta_f 
\end{align*}
\]

(10)

The proposed equation (11), which allows to find the point of change between the motion along a straight line and the circular movement. The condition \( r_{ds} = r_{df} \) is accepted to simplify the calculations. It is common in practice.

\[
\begin{align*}
T'_{ex} &= \begin{cases} 
0 & \text{if } \frac{z_s}{|z_s|} = \frac{z_f}{|z_f|} \\
\frac{r_{ds}^2}{\sqrt{(x_{cf} - x_{cs})^2 + (y_{cf} - y_{cs})^2}} & \text{otherwise}
\end{cases} \\
T'_{en} &= \begin{cases} 
0 & \text{if } \frac{z_s}{|z_s|} = \frac{z_f}{|z_f|} \\
\frac{r_{df}^2}{0.5 \sqrt{(x_{cf} - x_{cs})^2 + (y_{cf} - y_{cs})^2}} & \text{otherwise}
\end{cases}
\end{align*}
\]

(11)

4. Conclusion

This paper describes a modified trajectory construction algorithm based on the search space research by method of fast growing random trees. The presented algorithm allows planning the DV trajectory in real time and in a dynamically changing environment. A characteristic feature of the proposed solution is the structuring of the treetops for trajectory planning and the description of the search space as a set of vectors. The kinematic characteristics of DV are taken into account when structuring treetops. This approach can significantly increase the speed of searching for the desired trajectory. The author does not claim a full presentation of the material, but only proposes a methodology for the practical implementation of the trajectory planning problem.
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