Effects to Scalar Meson Decays of Strong Mixing between Low and High Mass Scalar Mesons

T. Teshima, I. Kitamura, and N. Morisita

Department of Applied Physics, Chubu University, Kasugai 487-8501, Japan

Abstract

We analyze the mass spectroscopy of low and high mass scalar mesons and get the result that the coupling strengths of the mixing between low and high mass scalar mesons are very strong and the strengths of mixing for $I = 1, 1/2$ scalar mesons and those of $I = 0$ scalar mesons are almost same. Next, we analyze the decay widths and decay ratios of these mesons and get the results that the coupling constants $A'$ for $I = 1, 1/2$ which represents the coupling of high mass scalar meson $N' \to \text{two pseudoscalar mesons} \, PP$ are almost same as the coupling $A'$ for the $I = 0$. On the other hand, the coupling constant $A$ for $I = 1, I = 1/2$ which represents the low mass scalar meson $N \to PP$ are far from the coupling constant $A$ for $I = 0$. We consider a resolution for this discrepancy. Coupling constant $A''$ for glueball $G \to PP$ is smaller than the coupling $A'$. $\theta_P$ is $40^\circ \sim 50^\circ$.

PACS numbers: 12.15.Ff, 13.15.+g, 14.60.Pq

*Electronic address: teshima@isc.chubu.ac.jp
I. INTRODUCTION

In recent re-analyses of $\pi\pi$ scattering phase shift and production processes, the existence of scalar mesons $\sigma(500)$ (we call $f_0(500)$ hereafter) have been confirmed [1], and in the analyses of $K\pi$ scattering phase shifts and production processes, the existence of $\kappa(900)$ have been reported [2]. The $f_0(500)$ was considered to be a chiral partner of the $\pi$ meson as a Nambu-Goldstone boson [3] and the $f_0(500)$ and $\kappa(900)$ are considered to construct the low mass scalar nonet together with $a(980)$ and $f_0(980)$. This nonet has been considered to be a chiral partner of the ground state pseudoscalar nonet in connection with the linear sigma model [4]. Many authors analyzed the nonet using the $K\bar{K}$ molecule model [5] or $qq\bar{q}\bar{q}$ model [6] rather than the linear sigma model in order to explain the $s\bar{s}$ rich character of the $f_0(980)$ which degenerate to $a_0(980)$. On the other hand, high mass scalar mesons $a(1450)$, $K^*_0(1430)$, $f_0(1370)$ and $f_0(1710)$ are considered to construct the high mass scalar nonet. This nonet is considered as the ordinary $L = 1$ $q\bar{q}$ scalar nonet.

We assume a strong mixing (inter-mixing) between low mass and high mass scalar nonets to explain the fact that the high mass $L = 1$ $q\bar{q}$ scalar nonet are so high compared to other $L = 1$ $q\bar{q}$ $1^{++}$ and $2^{++}$ mesons [7, 8]. This assumption is supported by the viewpoint in which the $a_0(980)$ and $f_0(980)$ contain the four-quark, two-quark and meson-meson contents [9]. Furthermore $f_0(1500)$ is considered to be a glueball candidate [10]. Thus, this glueball mixes with $I = 0 L = 1$ $q\bar{q}$ scalar mesons, and furthermore mixes with low mass scalar $f_0(980)$ through inter-mixing [8, 9]. We thus analyzed the overall mixing among low mass $qq\bar{q}\bar{q}$ scalar nonet and $L = 1$ $q\bar{q}$ scalar nonet and glueball [8]. We have obtained the result that the inter-mixing is very strong and the mixing parameters $\lambda_{a0}^a$, $\lambda_{a0}^K$ and $\lambda_{01}$ producing the inter-mixing in $I = 1$, $I = 1/2$ and $I = 0$ mesons respectively are almost same [8].

If there exists the strong mixing between low and high mass scalar mesons, the decay processes of low mass scalar mesons may be affected by the high mass scalar mesons and glueball, and conversely the decay processes of high mass scalar mesons may be affected by the low mass scalar mesons. Black et al. [7] estimated the decay coupling constants $A$ for low mass scalar mesons ($N$)- pseudoscalar meson($P$)-pseudoscalar meson($P$) interaction and the coupling constant $A'$ for high mass scalar($N'$)-$PP$ interaction considering the mixing between $I = 1$ and $I = 1/2$ low and high mass scalar mesons. We also analyze these decay coupling constants $A$ and $A'$ and further the coupling constant $A''$ for glueball($G$)-($PP$)
interaction considering the mixing among \( I = 0 \) low and high mass scalar mesons. The values of \((A, A')\) estimated in \( I = 1 \) and \( I = 1/2 \) meson decay analyses are \( \sim (0.1 - 3) \), while these estimated in \( I = 0 \) meson decays are \( \sim (-4, -2) \) for the case where \( f_0(1710) \) is considered as glueball and \( \sim (-2.9, -2.3) \) for the case where \( f_0(1500) \) is considered as glueball. There is a large discrepancy between the values of \( A \) in \( I = 1, 1/2 \) and \( I = 0 \) cases. We will discuss about a resolution for this discrepancy.

II. MIXING BETWEEN LOW AND HIGH MASS SCALAR MESONS

In this section, we briefly review the mixing among the low mass scalar, high mass scalar and glueball discussed in our previous work \([8]\).

A. Structure of low mass scalar mesons

For the structures of the low mass scalar mesons, there considered two possibilities. One is the chiral partner of the pseudoscalar nonet \([3]\) and the other is the \(qq\bar{q}\) or \(MM\) molecule \([5]\). We assume the \(qq\bar{q}\) structure because of the degeneracy between \(a_0\) and \(f_0(980)\) which has large \(s\bar{s}\) character. This is understood readily from the flavor contents in \(qq\bar{q}\) mesons:

\[
\begin{align*}
\bar{s}dus, \bar{u}dud, \bar{u}dds & \iff a_0^+, a_0^0, a_0^- \\
\bar{s}uds, \bar{s}uud, \bar{s}uds & \iff \kappa^+, \kappa^0, \kappa^- \\
\bar{u}dud & \iff f_N \sim f_0(980) \\
\bar{u}dds & \iff f_S \sim f_0(500)
\end{align*}
\]

The masses of \( I = 0 \) \( f_0(980) \) and \( f_0(500) \) mesons are represented by the masses of \( a_0(980) \), \( \kappa(900) \) and mixing mass parameter \( \lambda_0 \), which causes the mixing (intra-mixing) between \( f_0(980) \) and \( f_0(500) \) and describes the interaction strength of OZI rule suppression graph shown in Fig. 1. Diagonalizing the mass matrix

\[
\begin{pmatrix}
m^2_{a_0} + 2\lambda_0 & \sqrt{2}\lambda_0 \\
\sqrt{2}\lambda_0 & 2m^2_{\kappa} - m^2_{a_0} + \lambda_0
\end{pmatrix}
\]

and using the relation \( m_s > m_{u,d} \), we can get the desired spectrum,

\[
m^2_{f_0(980)} \approx m^2_{a_0(980)} > m^2_{\kappa} > m^2_{f_0(500)}.
\]
B. Inter-mixing between $I = 1/2$ low mass scalar mesons and $I = 1/2$ high mass scalar mesons

The inter-mixing interaction is caused by the graph shown in Fig. 2, which represents the OZI rule allowed interaction.

\[
L_{\text{int}} = -\lambda_{01} \epsilon^{abc} \epsilon^{def} N_{a}^{d} N_{b}^{f} S_{c} = \lambda_{01} [a_{0}^{+} a_{0}^{-} + a_{0}^{-} a_{0}^{+} + a_{0} a_{0} + \lambda^{+} K_{0}^{*-} + \lambda^{-} K_{0}^{*-} + \sqrt{2} f_{N} f'_{N} - f_{S} f'_{N} - \sqrt{2} f_{N} f'_{S}],
\]

where $N_{b}^{a} = q_{b} \bar{q}_{a}$. The strength of this $\lambda_{01}$ is considered to be very large because of the OZI rule allowed interaction.

We estimate the strength of the inter-mixing parameter $\lambda_{01}$. First, we estimate that for $I = 1 a_{0}(1450)$ and $a_{0}(980)$ mixing case. We estimate the masses before mixing as

\[
m_{a_{0}(980)} = 1271 \pm 31 \text{MeV}, \quad m_{a_{0}(1450)} = 1236 \pm 20 \text{MeV}
\]

from the relation $m^{2}(2++) - m^{2}(1++) = 2(m^{2}(1++) - m^{2}(0^{++}))$ resulted from the $L \cdot S$ force. The mass value $1271 \pm 31$ MeV of the $a_{0}(980)$ and $f_{0}(980)$ before mixing is almost same to the values $1275$ MeV for $a_{0}(980)$ and $1282$ MeV for $f_{0}(980)$ estimated in the constituent 4 quarks model [11]. Diagonalizing the mass matrix

\[
\begin{pmatrix}
m_{a_{0}(980)} & \lambda_{01}^{a} \\
\lambda_{01}^{a} & m_{a_{0}(1450)}
\end{pmatrix}
\]

and taking the eigenvalues of masses

\[
m_{a_{0}(980)} = 984.8 \pm 1.4 \text{MeV}, \quad m_{a_{0}(1450)} = 1474 \pm 19 \text{MeV},
\]

we can get the result

\[
\lambda_{01}^{a} = 0.600 \pm 0.028 \text{GeV}^{2}, \quad \text{mixing angle } \theta_{a} = 47.1 \pm 3.5^\circ.
\]
Next, we estimate the strength $\lambda_{01}$ for $I = 1/2$ $\kappa(900)$ and $K^*_0(1430)$ mixing case. Using the masses before mixing and after mixing,

$$
m_{\kappa(900)} = 1047 \pm 62\text{MeV}, \quad m_{K^*_0(1430)} = 1307 \pm 11\text{MeV},
$$

$$
m_{\kappa(900)} = 900 \pm 70\text{MeV}, \quad m_{K^*_0(1430)} = 1412 \pm 6\text{MeV}.
$$

we get the results

$$\lambda^K_{01} = 0.507 \pm 84\text{GeV}^2, \quad \text{mixing angle } \theta_K = 29.5 \pm 15.5^\circ. \quad (9)$$

It is confirmed that these coupling strengths are large and $\lambda^K_{01}$ is as strong strength as $\lambda^a_{01}$.

C. Inter-mixing between $I = 0$ low and $I = 0$ high mass scalar mesons and glueball

Intra-mixing between $I = 0, L = 1$ $q\bar{q}$ scalar mesons and glueball are expressed by the matrix as

$$
\begin{pmatrix}
  m^2_{a_0} + 2\lambda_1 & \sqrt{2}\lambda_1 & \sqrt{2}\lambda_G \\
  \sqrt{2}\lambda_1 & 2m^2_K - m^2_{a_0} + \lambda_1 & \lambda_G \\
  \sqrt{2}\lambda_G & \lambda_G & \lambda_{GG}
\end{pmatrix}.
$$

(10)

$\lambda_1$ is the term of the OZI-rule suppression graph for $q\bar{q}$ shown in Fig. 3. $\lambda_G$ is the transition between $q\bar{q}$ and glueball $gg$ showed in Fig. 4 (a) and $\lambda_{GG}$ is the pure glueball mass shown in Fig. 4 (b).

We analyze the inter- and intra-mixing among $I = 0$ low mass and high mass scalar
mesons and glueball expressed by the overall mixing mass matrix as

\[
\begin{pmatrix}
  m_N^2 + 2\lambda_0 & \sqrt{2}\lambda_0 & \lambda_{01} & \sqrt{2}\lambda_{01} & 0 \\
  \sqrt{2}\lambda_0 & m_S^2 + \lambda_0 & \sqrt{2}\lambda_{01} & 0 & 0 \\
  \lambda_{01} & \sqrt{2}\lambda_{01} & m_{N'}^2 + 2\lambda_1 & \sqrt{2}\lambda_1 & \sqrt{2}\lambda_G \\
  \sqrt{2}\lambda_{01} & 0 & \sqrt{2}\lambda_1 & m_{S'}^2 + \lambda_1 & \lambda_G \\
  0 & 0 & \sqrt{2}\lambda_G & \lambda_G & \lambda_{GG}
\end{pmatrix}.
\]

Using the input mass values (unit:GeV)

\[
m_N = 1.271 \pm 0.031, \ m_S = 0.760 \pm 0.179, \ m_{N'} = 1.236 \pm 0.02,
\]

\[
m_{S'} = 1.374 \pm 0.003, \ m_{f_0(980)} = 0.980 \pm 0.010, \ m_{f_0(500)} = 0.500 \pm 0.100,
\]

\[
m_{f_0(1370)} = 1.350 \pm 0.150, \ m_{f_0(1710)} = 1.715 \pm 0.007, \ m_{f_0(1500)} = 1.500 \pm 0.010,
\]

we get the result for the case in which \(f_0(1500)\) is assumed as glueball.

\[
\lambda_{01} = 0.53 \pm 0.04\text{GeV}^2, \ \lambda_0 = 0.03 \pm 0.04\text{GeV}^2, \ \lambda_1 = 0.07 \pm 0.05\text{GeV}^2,
\]

\[
\lambda_G = 0.23 \pm 0.06\text{GeV}^2, \ \lambda_{GG} = (1.53 \pm 0.03)^2\text{GeV}^2,
\]

\[
\begin{pmatrix}
  f_0(980) \\
  f_0(500) \\
  f_0(1370) \\
  f_0(1500) \\
  f_0(1710)
\end{pmatrix} = [R_{f_0(M)I}]
\begin{pmatrix}
  f_N \\
  f_S \\
  f_{N'} \\
  f_{S'} \\
  f_G
\end{pmatrix},
\]

\[
[R_{f_0(M)I}] = \\
\begin{pmatrix}
  0.720 \pm 0.060 & -0.389 \pm 0.096 & -0.148 \pm 0.111 & -0.558 \pm 0.041 & 0.145 \pm 0.048 \\
  0.234 \pm 0.093 & 0.789 \pm 0.080 & -0.525 \pm 0.080 & -0.102 \pm 0.059 & 0.108 \pm 0.035 \\
  0.048 \pm 0.077 & 0.433 \pm 0.062 & 0.683 \pm 0.039 & -0.482 \pm 0.044 & -0.275 \pm 0.054 \\
 -0.416 \pm 0.119 & 0.013 \pm 0.039 & 0.059 \pm 0.060 & -0.361 \pm 0.122 & 0.812 \pm 0.103 \\
  0.532 \pm 0.101 & 0.168 \pm 0.036 & 0.459 \pm 0.049 & 0.508 \pm 0.037 & 0.453 \pm 0.172
\end{pmatrix}.
\]

Fig. 4
Next, we study the case in which $f_0(1710)$ is assumed as glueball. This case was not analyzed in our early work [8].

$$
\lambda_{01} = 0.44 \pm 0.04 \text{GeV}^2, \quad \lambda_0 = 0.02 \pm 0.05 \text{GeV}^2, \quad \lambda_1 = -0.08 \pm 0.05 \text{GeV}^2;
$$

$$
\lambda_G = 0.28 \pm 0.06 \text{GeV}^2, \quad \lambda_{GG} = (1.64 \pm 0.03)^2 \text{GeV}^2;
$$

$$
[R_{f_0(M)I}] = 
\begin{pmatrix}
0.635 \pm 0.084 & -0.511 \pm 0.106 & -0.210 \pm 0.160 & -0.524 \pm 0.030 & 0.147 \pm 0.074 \\
0.243 \pm 0.112 & 0.723 \pm 0.114 & -0.584 \pm 0.099 & -0.174 \pm 0.077 & 0.123 \pm 0.056 \\
0.210 \pm 0.064 & 0.424 \pm 0.066 & 0.716 \pm 0.065 & -0.482 \pm 0.088 & -0.187 \pm 0.062 \\
0.651 \pm 0.051 & 0.033 \pm 0.040 & 0.052 \pm 0.093 & 0.599 \pm 0.100 & -0.426 \pm 0.096 \\
0.255 \pm 0.085 & 0.086 \pm 0.031 & 0.282 \pm 0.057 & 0.306 \pm 0.071 & 0.858 \pm 0.069
\end{pmatrix}
$$

The characters of the mixing parameters of these $I = 0$ scalar mesons are described as follows; (1) $f_0(980)$ contains $f_N = \bar{s}ds + s\bar{u}u + \bar{s}s$ components about 70 ~ 80%, (2) $f_0(500)$ contains $f_S = \bar{u}dud$ and $f_{N'} = (\bar{u}u + \bar{d}d)/\sqrt{2}$ components about 90%, (3) $f_0(1370)$ contains $f_N$ and $f_{N'}$ components about 70%, (4) $f_0(1500)$ contains $f_G$ component about 70% in $f_0(1500)$ glueball case, and $f_0(1710)$ contains $f_G$ component about 70% in $f_0(1710)$ glueball case.

### III. DECAY PROCESSES OF SCALAR MESONS AND GLUEBALL

In this section, we analyze the decay processes of low mass scalar mesons ($N$) decaying to two pseudoscalar mesons ($PP$), high mass scalar mesons ($N'$) decaying to $PP$ and pure glueball ($G$) decaying to ($PP$). We use the following interactions for $NPP$, $N'PP$ and $GPP$ coupling with coupling constants $A$, $A'$ and $A''$, respectively,

$$
L_I = A\varepsilon^{abc}\varepsilon_{def}N^d_a\partial^a_\mu \phi^b_b \partial^b_\mu \phi^f_c + A'N^b_a \partial^a_\mu \phi^f_c \partial^b_\mu \phi^c_e + A''G\{\partial^a_\mu \phi^b_b, \partial^a_\mu \phi^c_e\}.
$$

(15)

These interactions are represented graphically by the diagrams in fig. 5. Although interactions as $\text{Tr}(N\partial_\mu \phi)\text{Tr}(\partial^\mu \phi)$ and $\text{Tr}(N'\partial_\mu \phi)\text{Tr}(\partial^\mu \phi)$ other than those represented by Eq. (15) may exist [7], these interactions violate the OZI rule and are considered to be small compared to the interactions in Eq. (15).
We define the coupling constants $\gamma_{a_0K}$ etc. in the following expression,

$$L_I = \frac{1}{\sqrt{2}} \partial_\mu K \tau \cdot a_0 \partial^\mu K + \frac{1}{\sqrt{2}} \partial_\mu K \tau \cdot a_0' \partial^\mu K + \gamma_{a_0\pi\eta} a_0 \cdot \partial_\mu \pi \partial^\mu \eta + \gamma_{a_0\pi\eta} a_0' \cdot \partial_\mu \pi \partial^\mu \eta + \gamma_{a_0\pi\pi} a_0 \cdot \partial_\mu \pi \partial^\mu \pi + \text{H.C.}$$

$$+ \gamma_{a_0\pi\eta} a_0 \cdot \partial_\mu \pi \partial^\mu \eta' + \gamma_{a_0\pi\eta} a_0' \cdot \partial_\mu \pi \partial^\mu \eta' + \gamma_{K\pi\eta} \left( \frac{1}{\sqrt{2}} \partial_\mu K \tau \cdot \partial^\mu \pi K + \text{H.C.} \right)$$

$$+ \gamma_{K\pi\eta} \left( \frac{1}{\sqrt{2}} \partial_\mu K \tau \cdot \partial^\mu \pi K^* + \text{H.C.} \right) + \gamma_{K\pi\eta} \left( \frac{1}{\sqrt{2}} \partial_\mu K \pi \partial^\mu \eta + \text{H.C.} \right) + \gamma_{K\pi\eta} \left( \frac{1}{\sqrt{2}} \partial_\mu K \pi \partial^\mu \eta + \text{H.C.} \right)$$

$$+ \gamma_{K\pi\eta} \left( \partial_\mu K \pi \partial^\mu \eta' + \text{H.C.} \right) + \gamma_{K\pi\eta} \left( \partial_\mu K \pi \partial^\mu \eta' + \text{H.C.} \right),$$

where fields $a_0$ represents the low mass $I = 1$ scalar mesons and $a_0'$ the high mass $I = 1$ scalar mesons. Then the coupling constants for $I = 1$ and $1/2$ meson decays are, by using Eq. (15), expressed as

$$\gamma_{a_0(980)K} = 2(A \cos \theta_a - A' \sin \theta_a),$$

$$\gamma_{a_0(980)\pi\eta} = 2(A \cos \theta_a \sin \theta_P - \sqrt{2} A' \sin \theta_a \cos \theta_P),$$

$$\gamma_{a_0(1450)K} = 2(A \sin \theta_a + A' \cos \theta_a),$$

$$\gamma_{a_0(1450)\pi\eta} = 2(A \sin \theta_a \sin \theta_P + \sqrt{2} A' \cos \theta_a \cos \theta_P),$$

$$\gamma_{a_0(1450)\pi\eta'} = 2(-A \sin \theta_a \cos \theta_P + \sqrt{2} A' \cos \theta_a \sin \theta_P),$$

$$\gamma_{K^*(900)\pi K} = 2(A \cos \theta_K - A' \sin \theta_K),$$

$$\gamma_{K^*_0(1430)\pi K} = 2(A \sin \theta_K + A' \cos \theta_K),$$

where $\theta_P$ is $\eta$-$\eta'$ mixing angle and related to the traditional octet-singlet mixing angle $\theta_{0-8}$ as $\theta_P = \theta_{0-8} + 54.7^\circ$. Decay widths of these mesons are expressed by using the coupling
constants $\gamma_{a_0(980)K\bar{K}}$ etc. as

$$
\Gamma(a_0(M) \rightarrow K(m_1) + \bar{K}(m_2)) = \frac{\gamma_{a_0(M)K\bar{K}}^2}{32\pi} \frac{q_{Mm_1m_2}}{m_{a_0(M)}} m_{Mm_1m_2}^4,
$$

$$
\Gamma(a_0(M) \rightarrow \pi(m_1) + \eta(m_2)) = \frac{\gamma_{a_0(M)\pi\eta}^2}{32\pi} \frac{q_{Mm_1m_2}}{m_{a_0(M)}} m_{Mm_1m_2}^4
$$

$$
\Gamma(a_0(M) \rightarrow \pi(m_1) + \eta'(m_2)) = \frac{\gamma_{a_0(M)\pi\eta'}^2}{32\pi} \frac{q_{Mm_1m_2}}{m_{a_0(M)}} m_{Mm_1m_2}^4,
$$

$$
\Gamma(K^*_0(M) \rightarrow \pi(m_1) + K(m_2)) = \frac{3}{2} \frac{\gamma_{K^*_0(M)K\bar{K}}^2}{32\pi} \frac{q_{Mm_1m_2}}{m_{K^*_0(M)}} m_{Mm_1m_2}^4.
$$

Here $q_{Mm_1m_2}$ and $m_{Mm_1m_2}$ are defined as

$$
q_{Mm_1m_2} = \sqrt{(\frac{M^2 + m_2^2 - m_1^2}{2M})^2 - m_2^2},
$$

$$
m_{Mm_1m_2} = \sqrt{M^2 - m_1^2 - m_2^2},
$$

and for the case $M \approx m_1 + m_2$, we use the next formula for $q_{Mm_1m_2}$,

$$
q_{Mm_1m_2} = \text{Re} \left( \frac{1}{\sqrt{2\pi \Gamma_M}} \int_{M-\infty}^{M+\infty} e^{-\frac{(m-M)^2}{2\Gamma_M^2}} \times \sqrt{(\frac{m^2 + m_2^2 - m_1^2}{2m})^2 - m_2^2} \ dm, \right)
$$

where $\Gamma_M$ is the decay width of particle with mass $M$. This procedure is similar to that of the first article in [1].

We used the data for these decay processes cited in PDG [12] and those are listed in second column of Table I. Using these data, we estimated the allowed values for $A$ and $A'$ in the $\chi^2 \leq 5.348$ corresponding to the 50% C.L. on degree of freedom 6. For the mixing angles $\theta_a$ and $\theta_K$, we use the results of Eqs. (7) and (9); $\theta_a = (47.1 \pm 3.5)^\circ$ and $\theta_K = (29.5 \pm 15.5)^\circ$. Estimated values of $A$, $A'$ and $\theta_P$ are

$$
A = 0.10 \pm 0.24, \ A' = -3.03 \pm 0.2, \ \theta_P = 49.0^\circ \pm 3.0^\circ.
$$

We show the best fit values for decay widths and decay ratios on $A = 0.22$, $A' = -3.13$, $\theta_P = 49.0^\circ$ in third column of Table I. From this result, one finds that the estimated total width of $a_0(980)$ is rather small than experimental width. This may be caused from the treatment in which we used the Eq. (19) to estimate the decay momentum $q_{Mm_1m_2}$ when $M \sim m_1 + m_2$ for the decay of $a_0(980)$.
TABLE I: Experimental data and best fit values for various decay widths and decay ratios. Best fit values are obtained for $A = 0.22$, $A' = -3.13$, $\theta_P = 49.0^\circ$.

| Decay width and ratio | Experimental data | Best fit value |
|-----------------------|-------------------|---------------|
| $\Gamma(a_0(980) \rightarrow \pi \eta + K \bar{K})$ | $75 \pm 25$MeV | $36$MeV |
| $\Gamma(a_0(980) \rightarrow K \bar{K})/\Gamma(a_0(980) \rightarrow \pi \eta)$ | $0.177 \pm 0.024$ | $0.156$ |
| $\Gamma(a_0(1450) \rightarrow \pi \eta + \pi \eta' + K \bar{K})$ | $265 \pm 13$MeV | $266$MeV |
| $\Gamma(a_0(1450) \rightarrow K \bar{K})/\Gamma(a_0(1450) \rightarrow \pi \eta)$ | $0.88 \pm 0.23$ | $0.80$ |
| $\Gamma(a_0(1450) \rightarrow \pi \eta')/\Gamma(a_0(1450) \rightarrow \pi \eta)$ | $0.35 \pm 0.16$ | $0.49$ |
| $\Gamma(K_0^*(1430) \rightarrow \pi K)$ | $273 \pm 44$MeV | $303$MeV |

B. $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ meson decays

If we define the coupling constants $\gamma_{f_0(M)\pi\pi}$ etc. in the following expression,

$$L_I = \gamma_{f_0(M)\pi\pi} \frac{1}{2} f_0(M) \partial_\mu \pi \partial^\mu \pi + \gamma_{f_0(M)K\bar{K}} f_0(M) \partial_\mu K \partial^\mu \bar{K} + \gamma_{f_0(M)\eta\eta} f_0(M) \partial_\mu \eta \partial^\mu \eta$$

$$+ \gamma_{f_0(M)\eta\eta'} f_0(M) \partial_\mu \eta \partial^\mu \eta' + \gamma_{f_0(M)\eta'\eta'} f_0(M) \partial_\mu \eta' \partial^\mu \eta',$$

then the coupling constants $\gamma_{f_0(M)\pi\pi}$ etc. for $f_0(M)$ ($M = 980, 1370, 1500, 1710$) are expressed from the interaction for $NPP$, $NP'P$ and $GPP$ coupling represented in Eq. (15) as

$$\gamma_{f_0(M)\pi\pi} = 2(-AR_{f_0(M)S} + \sqrt{2} A' R_{f_0(M)N'}, + 2A'' R_{f_0(M)G}),$$

$$\gamma_{f_0(M)K\bar{K}} = \sqrt{2}(-AR_{f_0(M)N} + A'R_{f_0(M)N'} + \sqrt{2} A'R_{f_0(M)S'} + 2\sqrt{2} A'' R_{f_0(M)G}),$$

$$\gamma_{f_0(M)\eta\eta} = 2(-AR_{f_0(M)N} \cos \theta_P \sin \theta_P + \frac{1}{2} AR_{f_0(M)S} \cos^2 \theta_P$$

$$+ \frac{1}{\sqrt{2}} A'R_{f_0(M)N'} \cos \theta_P + A'R_{f_0(M)S'} \sin^2 \theta_P + A'' R_{f_0(M)G}),$$

$$\gamma_{f_0(M)\eta\eta'} = 2(AR_{f_0(M)N} \cos 2\theta_P + \frac{1}{2} AR_{f_0(M)S} \sin 2\theta_P$$

$$+ \frac{1}{\sqrt{2}} A'R_{f_0(M)N'} \sin 2\theta_P - A'R_{f_0(M)S'} \sin 2\theta_P),$$

$$\gamma_{f_0(M)\eta'\eta'} = 2(AR_{f_0(M)N} \cos \theta_P \sin \theta_P + \frac{1}{2} AR_{f_0(M)S} \sin^2 \theta_P$$

$$+ \frac{1}{\sqrt{2}} A'R_{f_0(M)N'} \sin^2 \theta_P + A'R_{f_0(M)S'} \cos^2 \theta_P + A'' R_{f_0(M)G}).$$

(22)
The allowed values for $A$ in two cases in which $f$ did not use the data of $\Gamma$ are listed in second column of Table II. Using these coupling constants, decay widths for $f$ are estimated as follows:

\[
\Gamma(f_0(M) \rightarrow \pi(m_1) + \pi(m_2)) = \frac{3\gamma^2_{f_0(M)\pi\pi}}{2} \frac{q_{Mm_1m_2}}{2\pi \frac{m^2_{f_0(M)}}{m^4_{Mm_1m_2}}},
\]

\[
\Gamma(f_0(M) \rightarrow K(m_1) + \bar{K}(m_2)) = \frac{2\gamma^2_{f_0(M)KK}}{2\pi \frac{m^2_{f_0(M)}}{m^4_{Mm_1m_2}}},
\]

\[
\Gamma(f_0(M) \rightarrow \eta(m_1) + \eta(m_2)) = \frac{2\gamma^2_{f_0(M)\eta\eta}}{2\pi \frac{m^2_{f_0(M)}}{m^4_{Mm_1m_2}}},
\]

\[
\Gamma(f_0(M) \rightarrow \eta'(m_1) + \eta'(m_2)) = \frac{2\gamma^2_{f_0(M)\eta'\eta'}}{2\pi \frac{m^2_{f_0(M)}}{m^4_{Mm_1m_2}}},
\]

Using these coupling constants, decay widths for $f_0(M)$ are expressed as

\[
\Gamma(f_0(M) \rightarrow \pi(m_1) + \pi(m_2)) = \frac{3\gamma^2_{f_0(M)\pi\pi}}{2} \frac{q_{Mm_1m_2}}{2\pi \frac{m^2_{f_0(M)}}{m^4_{Mm_1m_2}}},
\]

\[
\Gamma(f_0(M) \rightarrow K(m_1) + \bar{K}(m_2)) = \frac{2\gamma^2_{f_0(M)KK}}{2\pi \frac{m^2_{f_0(M)}}{m^4_{Mm_1m_2}}},
\]

\[
\Gamma(f_0(M) \rightarrow \eta(m_1) + \eta(m_2)) = \frac{2\gamma^2_{f_0(M)\eta\eta}}{2\pi \frac{m^2_{f_0(M)}}{m^4_{Mm_1m_2}}},
\]

\[
\Gamma(f_0(M) \rightarrow \eta'(m_1) + \eta'(m_2)) = \frac{2\gamma^2_{f_0(M)\eta'\eta'}}{2\pi \frac{m^2_{f_0(M)}}{m^4_{Mm_1m_2}}},
\]

Experimental data for these decay widths and decay ratios are quoted from PDG \[12\] and listed in second column of Table II. Using these data, we estimate the allowed values for $A$, $A'$ and $A''$ in the $\chi^2 \leq 12.340$ corresponding to the 50% C.L. on degree of freedom 13. The values with $^{(*)}$ are ones which are not decided in PDG \[12\] and then are averaged over data cited in PDG \[12\]. We get the allowed values for $A$, $A'$ and $A''$ in the $\chi^2 \leq 12.340$ for the two cases in which $f_0(1500)$ is assumed as glueball and $f_0(1710)$ is assumed as glueball. We did not use the data of $\Gamma_{f_0(1370)\rightarrow all}$ and $\Gamma_{f_0(1500)\rightarrow all}$ for $\chi^2$ fit, because the $\Gamma_{f_0(1370)\rightarrow all}$ and $\Gamma_{f_0(1500)\rightarrow all}$ contain the $4\pi$ and $\rho\rho$ decays widths which are not included in our estimation. The allowed values for $A$, $A'$, $A''$ and $\theta_P$ corresponding to the $f_0(1500)$ glueball case are as follows:

\[
A = -2.88 \pm 0.16, \quad A' = -2.28 \pm 0.08, \quad A'' = 0.305 \pm 0.034, \quad \theta_P = (18.9 \pm 1.8)^{\circ} \text{ or } (38.8 \pm 0.4)^{\circ},
\]

and those corresponding to the $f_0(1710)$ glueball case are as follows:

\[
A = -4.06 \pm 0.14, \quad A' = -1.93 \pm 0.10, \quad A'' = 0.640 \pm 0.04, \quad \theta_P = (50 \pm 2)^{\circ}.
\]

We showed the best fit values for decay widths and decay ratios on $A = -2.88$, $A' = -2.28$, $A'' = 0.305$, $\theta_P = 18.9^\circ$ for the $f_0(1500)$ glueball case and on $A = -4.06$, $A' = -1.93$, $A'' = 0.640$, $\theta_P = 50^\circ$ for the $f_0(1710)$ glueball case in third and forth column of Table II. We listed the ratio $(\gamma_{f_0(980)KK}/\gamma_{f_0(980)\pi\pi})^2$ in Table II, the experimental data for which is quoted from the Ref. \[13\] and is not used to $\chi^2$ fit.

The characteristic features of results obtained are

(1) there is a large discrepancy between the value of $A$ for $I = 1, 1/2$ and that for $I = 0$,
TABLE II: Experimental data \[12\] and best fit values for various decay widths and decay ratios. Best fit values are obtained on $A = -2.88$, $A' = -2.28$, $A'' = 0.305$, $\theta_P = 18.9^\circ$ for the $f_0(1500)$ glueball case and $A = -4.06$, $A' = -1.93$, $A'' = 0.604$, $\theta_P = 50^\circ$ for the $f_0(1710)$ glueball case. On the $\chi^2$ fit, we did not use the data of $\Gamma_{f_0(1370)\rightarrow \text{all}}$, $\Gamma_{f_0(1500)\rightarrow \text{all}}$, and $(\gamma_{f_0(980)K\pi}/\gamma_{f_0(980)\pi\pi})^2$.

| Decay width and ratio | Experimental data | Best fit value $(f_0(1500):\text{glueball})$ | Best fit value $(f_0(1710):\text{glueball})$ |
|-----------------------|-------------------|---------------------------------|---------------------------------|
| $\Gamma_{f_0(980)\rightarrow \text{all}(\pi\pi+K\bar{K})}$ | $70 \pm 30\text{MeV}$ | $48\text{MeV}$ | $66\text{MeV}$ |
| $\Gamma_{f_0(980)\rightarrow \pi\pi}/\Gamma_{f_0(980)\rightarrow \text{all}(\pi\pi+K\bar{K})}$ | $0.74 \pm 0.07^\ast$ | $0.77$ | $0.74$ |
| $(\gamma_{f_0(980)K\pi}/\gamma_{f_0(980)\pi\pi})^2$ | $1 \sim 8$ \[13\] | $5.8$ | $6.9$ |
| $\Gamma_{f_0(1370)\rightarrow \text{all}}$ | $350 \pm 150\text{MeV}$ | $\Gamma_{f_0(1370)\rightarrow \pi\pi+K\bar{K}+\eta\eta}=165\text{MeV}$ | $79\text{MeV}$ |
| $\Gamma_{f_0(1500)\rightarrow \text{all}}$ | $109 \pm 7\text{MeV}$ | $\Gamma_{f_0(1500)\rightarrow \pi\pi+K\bar{K}+\eta\eta+\eta'\eta'}=22\text{MeV}$ | $94\text{MeV}$ |
| $\Gamma_{f_0(1710)\rightarrow \text{all}}$ | $125 \pm 10\text{MeV}$ | $126\text{MeV}$ | $123\text{MeV}$ |

(2) the value for $A'$ is $2 \sim 3$,
(3) the value for $A''$ is $0.3 \sim 0.6$,
(4) $\theta_P$ is $40^\circ \sim 50^\circ$,
(5) estimated values for the ratio $\Gamma_{f_0(1370)\rightarrow K\pi}/\Gamma_{f_0(1370)\rightarrow \text{all}}$ are small about 1 order compared
to the data for both $f_0(1500)$ glueball and $f_0(1710)$ glueball cases,

(6) estimated value for $\Gamma_{f_0(1370)\rightarrow\pi\pi+K\bar{K}+\eta\eta}$ in $f_0(1710)$ glueball case seems rather small compared to the experimental value for $\Gamma_{f_0(1370)\rightarrow\text{all}}$, although experimental value for $\Gamma_{f_0(1370)\rightarrow\text{all}}$ contains $4\pi$ and $\rho\rho$ decay channel and has large uncertainty.

C. A resolution for discrepancy between $A$ for $I = 1, 1/2$ and $A$ for $I = 0$

In this subsection, we will study about a resolution for discrepancy between $A$ for $I = 1, 1/2$ and for $I = 0$. Allowed values of $(A, A')$ are $(0.1 \pm 0.24, -3.03 \pm 0.2)$ for $I = 1, 1/2$ meson decays in 50% C.L., and allowed values of $(A, A')$ are $(-2.88 \pm 0.16, -2.28 \pm 0.08)$ or $(-4.06 \pm 0.14, -1.93 \pm 0.10)$ for $I = 0$ meson decays in 50% C.L. These allowed regions are marked by ellipses in Fig. 6. These allowed regions are spread out with the increases of maximum $\chi^2$ from 5.348 to 20 in $\chi^2$ fit of $I = 1, 1/2$ meson decays and from 12.340 to 30 in $\chi^2$ fit of $I = 0$ meson decays as shown in Fig. 6. The allowed region for $I = 1, 1/2$ meson decays is spread out to up-left direction from down-right region $(0.1 \pm 0.24, -3.03 \pm 0.2)$ in $(A, A')$ plane. The allowed region for $I = 0$ meson decays $f_0(1500)$ glueball case is spread out to center direction from down-left region $(-2.88 \pm 0.16, -2.28 \pm 0.08)$ in $(A, A')$ plane, and the allowed region for $I = 0$ meson decays $f_0(1710)$ glueball case is spread out to center direction from down-left region $(-4.06 \pm 0.14, -1.93 \pm 0.10)$ in $(A, A')$ plane.
The spread region for $I = 1, 1/2$ meson decay joins with the spread region for $I = 0$ meson decay $f_0(1710)$ glueball case at ($\sim −2.3, −0.8$) in $(A, A')$ plane. Also, the spread region for $I = 1, 1/2$ meson decay can join with the spread region for $I = 0$ meson decay $f_0(1500)$ glueball at ($\sim −1.7, −1.4$) in $(A, A')$ plane if maximum $\chi^2$ increases moreover in $\chi^2$ fit.

IV. CONCLUSION

From the analysis of the mass spectroscopy of low mass and high mass scalar mesons, we can get the result that the coupling strengths of the mixing between low mass and high mass scalar mesons are very strong and strengths of the coupling for $I = 1, L = 1/2$ and $I = 0$ are almost same. We further analyze the glueball mixing among these scalar mesons. We get the mixing parameters of $I = 0$ high mass $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ mesons, in which $f_0(1500)$ is considered to be the glueball or $f_0(1710)$ is considered to be the glueball.

The strong mixing between the low and high mass scalar mesons affects the decay processes of these scalar mesons. From the analysis of the decay widths and decay ratios of the low and high mass scalar mesons, we got the results that the coupling constants $A$ for $N \rightarrow PP$ and $A'$ for $N' \rightarrow PP$ are about $(0.10 \pm 0.24, −3.03 \pm 0.2)$ for $I = 1, 1/2$ mesons on 50% C.L. The results obtained for $I = 0$ mesons are $(A, A', A'') = (−2.88 \pm 0.16, −2.28 \pm 0.08, 0.305 \pm 0.034)$ or $(−4.06 \pm 0.14, −1.93 \pm 0.10, 0.640 \pm 0.04)$ on 50% C.L., for $f_0(1500)$ glueball case or $f_0(1710)$ glueball case, respectively. Here, $A''$ is the coupling constant for $G \rightarrow PP$.

The $\theta_P$ is obtained to be $40^\circ \sim 50^\circ$. The large discrepancy between the allowed values $A$ as $\sim 0.10$ for $I = 1, 1/2$ and $\sim −2.88$ or $\sim −4.06$ for $I = 0$ can be resolved if one increases the maximum $\chi^2$ values in $\chi^2$ fit estimating the allowed regions in $\chi^2$ fit and gets the extended allowed regions. The extension of allowed regions gives the common allowed values $(A, A') \sim (−2.3, −0.8)$.

[1] M. Harada, F. Sannino and J. Schechter, Phys. Rev. D 54, 1991(1996).

S. Ishida et al., Prog. Theor. Phys. 95, 745(1996).

N. A. Törnqvist and M. Roos, Phys. Rev. Lett. 76, 1575(1996).
S. Ishida et al., Phys. Rev. Lett. 98, 1005(1997).

J. A. Oller, E. Oset and J. R. Peláez, Phys. Rev. Lett. 80, 3452(1998).

K. Igi and K. Hikasa, Phys. Rev. D 59, 034005(1999).

G. Colangelo, J. Gasser and H. Leutwyler, Nucl. Phys. B603, 125(2001).

E791 Collaboration, E.M. Aitala et al., Phys. Rev. Lett. 86, 765, 770(2001).

[2] E. Van Beveren et al., Z. Phys. C 30, 615(1986).

S. Ishida et al., Phys. Rev. Lett. 98, 621(1997).

D. Black et al., Phys. Rev. D 58, 054012(1998).

J. A. Oller, E. Oset and J. R. Peláez, Phys. Rev. D 59, 074001(1999).

E791 Collaboration, E. M. Aitala et al., Phys. Rev. Lett. 89, 121801 (2002).

[3] M. D. Scadron, Phys. Rev. Lett. 53, 1129(1984).

T. Hatsuda and T. Kunihiro, Prog. Theor. Phys. 74, 765(1985).

M. Ishida, Prog. Theor. Phys. 96, 853(1996).

[4] R. Delbourgo and M. D. Scadron, Phys. Rev. Lett. 48, 379(1982).

M. Ishida, Prog. Theor. Phys. 101, 661(1999).

[5] J. Weinstein and N. Isgur, Phys. Rev. Lett. 48, 659(1982).

[6] R. J. Jaffe, Phys. Rev. D15, 267(1977).

[7] D. Black et al., Phys. Rev. D59, 074026(1999).

D. Black, A. H. Fariborz and J. Schechter, Phys. Rev. D61, 074001(2000).

[8] T. Teshima, I. Kitamura and N. Morisita, J. Phys. G 28, 1391(2002).

[9] N. N. Achasov, Nucl. Phys. A675, 279c (2000).

Frank E. Close and Nils A. Törnqvist, J. Phys. G 28, R249(2002).

[10] P. G. O. Freund and Y. Nambu, Phys. Rev. lett. 34, 1645(1975).

F. E. Close and A. Kirk, Phys. Lett. B 483 (2000), 345.

[11] J. Vijande et al., hep-ph/0206263

[12] K. Hagiwara et al. (Particle Data Group), Phys. Rev. D 66, (2002), 010001.

[13] N. N. Achasov and V. V. Gubin, Phys. Rev. D 56(1997), 4084.