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On the complex Banach conjecture. (English) Zbl 07470575

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Summary: The complex conjecture of Stefan Banach states that if $V$ is a Banach space over the complex numbers where for some $n$, $1 < n < \dim(V) < \infty$, all of its $n$-dimensional subspaces are isometric, then $V$ is a Hilbert space. Mikhail Gromov proved it for $n$ even in 1967. Here, we prove it for $n \equiv 1 \mod 4$.

MSC:

22E15 General properties and structure of real Lie groups
52A21 Convexity and finite-dimensional Banach spaces (including special norms, zonoids, etc.) (aspects of convex geometry)
55R25 Sphere bundles and vector bundles in algebraic topology

Keywords:

complex affine isomorphism; Banach conjecture; isometric Banach spaces

Full Text: arXiv Link

References:

[1] J. F. Adams, Vector fields of spheres. Ann. of Math. 75 (1962) 603-632. · Zbl 0112.38102
[2] J. F. Adams, G. Walker: On complex Stiefel manifolds, Proc. Camb. Phil. Soc. 61 (1965) 81-103. · Zbl 0142.40903
[3] M. Atiyah, J. Todd: On complex Stiefel manifolds, Proc. Camb. Phil. Soc. 56 (1960) 342-353. · Zbl 0099.16102
[4] H. Auerbach, S. Mazur, S. Ulam: Sur une propriété caractéristique de l'ellipsoïde, Monatshefte Math. Phys. 42 (1935) 45-48. · Zbl 0011.22208
[5] S. Banach: Théorie des Opérations Linéaires, Monografie Matematyczne, Warszawa-Lwów 1932; see also the English translation, Theory of Linear Operations, Mathematics Library Vol. 38, North Holland, Amsterdam (1987).
[6] G. Bor, L. Hernández Lamoneda, V. Jiménez-Desantiago, L. Montejano: On the isoperimetric conjecture of Banach, Geometry and Topology 25 (2021) 2621-2642. · Zbl 07396004
[7] M. Čadek, M. Crabbe: G-structures on spheres, Proc. London Math. Soc. (3) 93 (2006) 791-816. · Zbl 1110.55008
[8] A. Dvoretzky: Some results on convex bodies and Banach spaces, Matematika 8/1 (1964) 73-102.
[9] H. H. Glover, W. D. Homer, R. E. Stong: Splitting the tangent bundle of projective space, Indiana Univ. Math. J. 31/2 (1982) 161-166. · Zbl 0454.55013
[10] M. L. Gromov: A geometrical conjecture of Banach, Math. of the USSR - Izvestiya 1.5 (1967) 1055.
[11] P. Leonard: G-structures on spheres, Trans. Amer. Math. Soc. 157 (1971) 311-327. · Zbl 0217.49201
[12] H. Martini, L. Montejano, D. Oliveros: Bodies of Constant Width. An Introduction to Convex Geometry with Applications, Birkhäuser, Basel (2019). · Zbl 1433.52001
[13] V. Milman: A new proof of A. Dvoretzky's theorem on cross-sections of convex bodies, Funkcial. Anal. i Prilozen 5 (1971) 28-37.
[14] L. Montejano: Convex bodies with homothetic sections, Bull. London Math. Soc. 23 (1991) 381-386. · Zbl 0746.52009
[15] A. Pełczyński: On some problems of Banach, Russian Math. Surveys 28/6 (1973) 67-75. · Zbl 0281.46014
[16] V. Soltan: Characteristic properties of ellipsoids and convex quadrics, Arch. Math. 93 (2019) 371-413. · Zbl 1433.52001
[17] N. E. Steenrod: The Topology of Fibre Bundles, Princeton Mathematical Series 14, Princeton University Press, Princeton (1999). · Zbl 0942.57002

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