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Engineering Physics and Mathematics

Fractional mathematical modeling for epidemic prediction of COVID-19 in Egypt

W.E. Raslan

Department of Mathematics and Engineering Physics, Faculty of Engineering, Mansoura University, Egypt
Mansoura High Institute of Engineering and Technology, Mansoura, Egypt

1. Introduction

In December 2019, the first confirmed infection case of coronavirus is reported in China [1]. In a short time later the disease has rapidly spread around to most countries of the world. The World Health Organization declared that the rapid spread of this virus represents an unprecedented challenge, which requires cooperation and collaboration of all countries of the world [2]. The main challenge lies in the ability of countries to provide the necessary health care to the large number expected to be infected in a short time. Egyptian Ministry of Health and Population announced the first case in Egypt on February 14 for a Chinese citizen [3]. In March 5, 2020, the first case of coronavirus among citizens inside the country was reported. The Egyptian government has taken several gradual precautionary measures since the announcement of the emergence of the virus to control the spread, such as imposing a curfew for certain hours and imposing limited partial isolation of some places that witness an increase in the number of infections in addition to the relatively expanded examination of contacts of confirmed cases.

In general there are three different categories of the methods used to study the spread of infection diseases. The first is to establish a mathematical model of a dynamical system to describe the evolution of the disease; the second is to use statistical methods and the third is using machine learning expert methods [4]. The second and third methods need a large number of data to be effective.

Mathematical modeling plays a vital role in predicting the health and environmental impacts of epidemics and diseases for animals and humans [5–7].

The importance of using mathematics in transmission and spread of epidemics was born in 1766 in the article of Bernoulli that describe the influence of smallpox on the average expected life period using mathematical table analysis [8].

In 1902, Ronald Ross used mathematics to investigate the feasibility of alternative strategies for malaria [9].

In 1927, Kermack and McKendrick illustrated the dynamics of infections spreads using a system of differential equations [10] that are called SIR models.

In mathematical modeling, the population is divided into compartments in accordance to the state of their health, such as susceptible (S), infected (I), and recovered (R) as in SIR model. Other states of the population linked with control policies such as quarantined (Q) are also used.

In this work we shall consider formulation of the model using fractional derivatives in the Caputo sense definition [11–13]. Other fractional operators such as the Caputo-Fabrizio and Atangana-
Baleanu will be discussed in the future and compared with the Caputo one [14,15].

Fractional derivatives have been used efficiently to fine tune many existing models of physical and natural phenomena [16,17]. In the field of Bio-Modeling, fractional derivatives were used to generalize the Hodgkin–Huxley model [18]. The model using fractional derivatives gave more accurate results than the model using classical derivatives. In the field of physical science, fractional derivatives were used to model viscoelastic substances with good agreement with experimental results [19–21]. Also, Sherief et al [22] have constructed a successful fractional model for thermoelasticity (see also [23,24]).

The main advantage behind the use of fractional derivatives stems from its memory effect. This means that its definition as an integral over past times makes the effects of the stimuli somewhat retarded not instantaneous as in the classical models. This is in accord with what is found in nature.

After the spread of the Corona pandemic, a few models have emerged that use fractional differentiation to model the corona propagation such that the paper by M.A. Khan, A. Atangana, in this paper the author consider the model formulation by applying Atangana-Baleanu fractional derivative operator [6]. They applied their model in Wuhan China considering the sea food market as the main source of infection. All the parameter that they have used were either values from the literature or estimated from the Wuhan city of China.

In this paper we develop a fractional model using Caputo fractional operator that adapts well to COVID-19 in Egypt to support efforts to control proliferation at rates that healthcare can handle. We consider the reported cases in the interval from March 10, 2020 to May 12, 2020.

2. Model formulation

According to the known characteristics of COVID-19 and the treatment and precautionary protocols prepared by the Egyptian government [25–27], we assume that each person is in one of the following compartments (see Fig. 1)

- Susceptible (S): People who may be infected by the virus
- Exposed (E): Infected with the virus but without the typical symptoms of infection
- Quarantine (Q): Diagnosed and quarantined
- Infectious (I): Infected with the virus and highly infectious but not quarantined
- Hospitalized (H): The person is in hospital
- Recovered (R): People who are cured after infection

The incubation period of corona virus is up to 14 days [25,26], so there is a possibility to find asymptomatically undetected infected peoples E(t). Once the first infected case is discovered a part of infected people need to go through an isolation period Q (t). In this context, the Egyptian government has allocated some hotels and university cities in many governorates. After accurate diagnosis the people in quarantine will turn to either full recovery R(t) or to hospitals for more medical care H(t). The patients either convert to full recovery or face death under the influence of acute respiratory complications. We formulate the fractional mathematical model FSEQHIR in equations (1–6)

\[
\frac{d^\alpha S}{dt^\alpha} = -f(t) \tag{1}
\]

\[
\frac{d^\alpha E}{dt^\alpha} = f(t) - k_1E - k_2E \tag{2}
\]

\[
\frac{d^\alpha Q}{dt^\alpha} = k_1E - k_5Q - k_6Q \tag{3}
\]

\[
\frac{d^\alpha R}{dt^\alpha} = k_3Q + k_6H + k_7I \tag{4}
\]

\[
\frac{d^\alpha I}{dt^\alpha} = k_1E - k_1I - k_4I - \delta_1I \tag{5}
\]

\[
\frac{d^\alpha H}{dt^\alpha} = k_3Q + k_4I - k_4H - \delta_2H \tag{6}
\]

The operator \( \frac{d}{dt}^\alpha \) is the fractional derivative in the sense of Caputo definition. The function \( f(t) \) is the degree of infection in \( S(t) \) which can be estimated according to the following relation [27]

\[
f(t) = \beta(t)[k_1E + I] \tag{7}
\]

where \( k_1 \) is the ratio that reflects the rate of infection from an asymptomatically person symptoms one and \( \beta(t) \) is the infection rate that can be approximated by the relation [27]

\[
\beta(t) = \frac{F(t + T_1 + T_2)}{\sum_{j=0}^{2}F(t + T_1 + T_2) + k_2\sum_{j=0}^{1}F(t + T_1 + T_2)}, \quad t = 1, 2, ... \tag{8}
\]

where \( T_1 \) is the average incubation interval, \( T_2 \) is the time of isolation after the incubation interval and \( F(t) \) is the number of cases at day \( t \). The other parameters are defined in the following table (Table 1).

As soon as a person is infected, the proportion of becoming in I is \( \rho \), which means that the proportion of becoming in Q equal \( (1 - \rho) \) [28]. There are a number of numerical methods that are used to solve fractional differential equations such as those in [29–32]. We use Laplace transform technique to solve the model as follows:

**Table 1**

| Parameter | Interpretation |
|-----------|----------------|
| \( k_2 \) | Proportion of people who are recovering if not being in hospital |
| \( k_3 \) | Proportion of people who are cured per day to those who are |
| \( k_4 \) | Proportion of recovering peoples if being admitted to isolation ward |

![Fig. 1. Flow diagram of the suggestion compartmental model of COVID-19 in Egypt.](image-url)
Table 2
Parameters estimations of the model.

| Parameter | Estimation | Value | Ref. |
|-----------|------------|-------|------|
| $k_1$ | $k_1 = \text{Number of days needed to self recover}$ | 7.1% | [25],[26], [5] |
| $k_2$ | $k_2 = \text{Number of days needed to recover}$ | 8.3% | [25],[26], [5] |
| $\delta_{1,2}$ | The average (for two months) of the daily death rate which calculated from the proportion of people who are dead to those who are diagnosed per day | 6.0% | [25],[26], [5] |
| $k_3$ | $k_3 = \text{Average number of days in isolation}$ | 2.5% | [25],[26], [22,25] |
| $k_4$ | $k_4 = \text{The average number of days that person suffers from severe symptoms before going to hospitals} = 5$ days | 20% | [22,25], [26] |
| $k_5$ | $k_5 = \text{The proportion of becoming symptomatic The average number of days in Expose} = 5$ days and 20% of all them would be transferred to I class. | 16% | [22,25], [26] |
| $k_6$ | $k_6 = \text{The average number of days in isolation}$ | 84% | [26] |
| $k_7$ | $k_7 = \text{The average number of days in isolation}$ | 12.5% | [25],[26], [22,25] |
Taking the Laplace transform with parameter $z$ defined by the relation

$$f(z) = \int_0^\infty e^{-zt} f(t) \, dt$$  \hspace{1cm} (9)$$

into equations (1)–(6), one obtains

$$[z^2 + k_1 \quad \beta k_1 \quad 0 \quad 0 \quad \beta \quad 0]$$
$$[0 \quad z^2 - \beta k_1 + k_2 + k_3 \quad 0 \quad 0 \quad -\beta \quad 0]$$
$$[0 \quad 0 \quad z^2 + k_0 + k_2 \quad -k_5 \quad -k_6 \quad 0]$$
$$[-k_7 \quad -k_5 \quad 0 \quad z^2 + k_4 + k_5 \quad 0 \quad 0]$$
$$[0 \quad 0 \quad 0 \quad -k_0 \quad -k_0 \quad -k_2]$$
$$[0 \quad 0 \quad 0 \quad 0 \quad z^2 + k_1 + k_2 + k_3 \quad z^2]$$

(10)

where $S_0, E_0, H_0, Q_0, I_0$ and $R_0$ are considered the initial conditions taken in March 10, 2020. Solving the system of equations (10) and then using numerical inverse Laplace transform to find the

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Fig. 4. Infection rate $\beta(t)$

Fig. 5. Cumulative confirmed cases

Fig. 6. Flow diagram of the modified compartmental model of COVID-19 in Egypt
unknown functions in the time domain. The inversion method uses Fourier expansion techniques. The details can be found on [33].

3. Numerical results

We now apply our model to study the COVID-19 epidemic in Egypt. We use the data published daily by the Egyptian Ministry of Health and Population and

World Health Organization Egypt [25,26]. The initial conditions of the system of equations of the model were setting in March 10, 2020. In Table 2, the numerical estimation for the parameters of the model based on available information are introduced.

In Fig. 2, the graph shows a good agreement of the trend of daily confirmed cases with that estimated by FSEQHIR model at $\alpha$ equal 0.97. The effect of fractional parameter $\alpha$ in fine tuning the model is illustrated in Fig. 3. Fig. 4 represents the infection rate change in the model depends on expeditiously expanding the iso-

duration of curfews.

We have proposed a new fractional mathematical model to predict the spread of epidemics in Egypt. The proposed model predicts a significant increase in the number of cases after two months, as in Fig. 2. Accordingly, a number of measures to control proliferation must be taken such as the application of a comprehensive ban, isolation, and social divergence policies for a specific period of time. To show the effect of this precautionary policies, we have prepared an imaginary model, as in Fig. 6, to control the spread of the disease after reaching near 500 confirmed cases daily. The change in the model depends on expeditiously expanding the isolation procedures for suspects or potential exhibitors, as well as increasing the duration of curfews.

Taking into account the new initial conditions, Fig. 7 show the behavior of the epidemic spread after the new assumptions procedures for different values of $\alpha$.

We see from Fig. 7 that the suggested modified model predicts reaching a maximum number of cases per day and then these numbers decrease monotonically after that.

4. Conclusions

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Fig. 7. The number of daily confirmed cases for modified model for different values of $\alpha$.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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