Unique Determination of the Physical Parameters of Individual MACHOs from Astrometric Parallax Measurements

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The Einstein time scale, which is the only information obtained from current microlensing experiments, results from a complicated combination of the lens parameters that we want to determine. Of the methods for breaking the lens parameter degeneracy, the most promising and generally applicable method is to measure the lens parallax from simultaneous observations of a lensing event from the ground and a heliocentric satellite (Gould 1994). However, the elegant idea of parallax measurement, which was proposed to resolve the lens parameter degeneracy, paradoxically suffers from its own degeneracy due to the ambiguity of the source star trajectory. In this paper, we propose to measure the lens parallax astrometrically by mounting an interferometer instead of a photometer in the proposed parallax satellite. By simultaneously measuring the source star centroid shifts from a geocentric and an additional heliocentric satellite, one can determine the lens parallax without ambiguity. If the already planned *Space Interferometry Mission* will be used as one of the satellites, one simply needs to replace the photometer in the parallax satellite with an instrument for astrometric observation. In addition, since the proposed method can measure both the lens parallax and the proper motion at the same time, one can completely break the lens parameter degeneracy, and therefore uniquely determine the physical parameters of individual lenses.

*Subject headings:* gravitational lensing — dark matter — astrometry — photometry

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1. Introduction

Searches for gravitational microlensing events caused by Massive Astronomical Compact Objects (MACHOs) by monitoring stars in the Large Magellanic Cloud (LMC) and the Galactic bulge are currently underway and several hundred events have been detected to date (Alcock et al. 1997a, 1997b; Ansari et al. 1996; Udalski et al. 1997; Alard & Guibert 1997). The only information about the lens obtained from current lensing experiments is the Einstein time scale. However, the time scale results from a complicated combination of the physical parameters of the lens by

\[ t_E = \frac{r_E}{v}, \quad r_E = \left( \frac{4GMc^2}{D_{ol}D_{ls}} \right)^{1/2}, \]

where \( r_E \) is the physical size of the Einstein ring, \( M \) is the mass of the lens, \( v \) is the lens-source transverse speed, and \( D_{ol}, D_{ls}, \) and \( D_{os} \) are the separations between the observer, lens, and source star. Due to the lens parameter degeneracy, our knowledge about the nature of MACHOs is very poor in spite of the large number of detected events.

Numerous methods for resolving the lens parameter degeneracy have been proposed. While most of these methods have very limited applications, two methods are applicable to microlensing events in general. The first method is to measure the lens parallax by simultaneously observing a lensing event from the ground and a heliocentric satellite (Gould 1994, 1995). If the parallax of an event is measured, one can determine the projected speed \( \tilde{v} = \left( \frac{D_{ol}}{D_{ls}} \right)v \) and the lens parameter degeneracy can be partially resolved (see \$2\). The second method is to measure the lens proper motion, \( \mu \), by astrometrically measuring the source star centroid shifts caused by gravitational lensing with a high precision interferometer such as the \textit{Space Interferometry Mission} (hereafter SIM, [http://sim.jpl.nasa.gov]). When a lensing event is astrometrically observed, one can determine the lens proper motion, which can also partially resolve the lens parameter degeneracy (Miyamoto & Yoshii 1995; Walker 1995; Høg, Novikov & Polarev 1995; Boden, Shao, & Van Buren 1998). When both \( \tilde{v} \) and \( \mu \) are determined, the degeneracy is completely broken and the lens parameters of individual lenses are determined by

\[
\begin{align*}
M &= \left( \frac{c^2}{4G} \right)t_E^2 \tilde{v} \mu, \\
D_{ol} &= D_{os}(\mu D_{os} / \tilde{v} + 1)^{-1}, \\
v &= [\tilde{v}^{-1} + (\mu D_{os})^{-1}]^{-1}.
\end{align*}
\]

Unfortunately, the elegant idea of lens parallax measurements, which was proposed to resolve the lens parameter degeneracy, paradoxically suffers from its own degeneracy. This degeneracy from the parallax measurement arises because one cannot uniquely determine the source star trajectory from the photometrically constructed light curve alone. As a result, the measured projected speed suffers from a two-fold degeneracy (see \$2).

In this paper, we propose to measure the lens parallax astrometrically by mounting an interferometer instead of a photometer in the proposed parallax satellite. By simultaneously
measuring the source star centroid shifts from the SIM and the astrometric parallax mission, one can determine the lens parallax without any ambiguity. In addition, since the proposed method can measure both the lens parallax and proper motion at the same time, one can completely break the lens parameter degeneracy, and thus the physical parameters of individual lenses can be uniquely determined.

2. Degeneracy in the Projected Velocity

When seen from a satellite with a two-dimensional separation vector \( \mathbf{r} \) with respect to the Earth, the Einstein ring will be displaced by an angle

\[
\Delta \vec{\theta} = (D_{os}^{-1} - D_{ol}^{-1}) \mathbf{r} = -\frac{D_{ls}}{D_{os} D_{ol}} \mathbf{r}
\]

relative to its position as seen from the Earth. Due to the displacement of the Einstein ring, the source star position with respect to the lens is displaced by the same amount \( \Delta \vec{\theta} \), but towards the opposite direction. When normalized by the Einstein ring radius, the displacement vector of the source star position (i.e. parallax) \( \Delta \mathbf{u} = (\Delta t_0/t_E) \mathbf{x'} + \beta \mathbf{y'} \) is then related to the Earth-satellite separation vector by

\[
\Delta \mathbf{u} = -\frac{D_{ol} \Delta \vec{\theta}}{r_E} = \frac{\mathbf{r}}{(D_{os}/D_{ls}) r_E}.
\]

By multiplying \( t_E \) by both sides of equation (2.2) and using the definition of the projected velocity \( \mathbf{v} = (D_{os}/D_{ls}) \mathbf{v} \), one finds the relation between the projected velocity and the parallax by

\[
\mathbf{v} = \tilde{v}_{x'} \hat{x'} + \tilde{v}_{y'} \hat{y'},
\]

where

\[
\tilde{v}_{x'} = \frac{r}{t_E \Delta u^2} \frac{\Delta t_0}{t_E} (2.4)
\]

and

\[
\tilde{v}_{y'} = \frac{r}{t_E \Delta u^2} \Delta \beta. (2.5)
\]

Here \( \hat{x'} \) and \( \hat{y'} \) are the unit vectors which are parallel and perpendicular to the Earth-satellite separation vector and \( \Delta t_0 \) and \( \Delta \beta \) are the differences in the times of maximum amplification and the impact parameters of the two light curves, respectively. Since the event light curve is related to the lensing parameters by

\[
A = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}, \quad u^2 = \left( \frac{t - t_0}{t_E} \right)^2 + \beta^2,
\]

the values of \( t_0 \) and \( \beta \) are determined from the individual light curves. Therefore, by determining \( \Delta u = [(\Delta t_0/t_E)^2 + \Delta \beta]^1/2 \) one can determine the transverse speed by

\[
\tilde{v} = \frac{r}{t_E \Delta u}. (2.7)
\]
However, the photometrically determined parallax suffers from two-fold degeneracy. This degeneracy occurs because events with different trajectories can have the same lensing parameters, resulting in the same light curve. In Figure 1, we illustrate this degeneracy in the photometric parallax measurements. The upper panel shows the light curves seen from the Earth and the satellite, respectively. Since the shift in the Einstein ring does not change its size, the two light curves have the same Einstein time scale. On the other hand, the time of maximum amplification and the impact parameter are changed due to the shift of the Einstein ring, resulting in different light curves (Refsdal 1966). In the middle panels, we present the two possible lens system geometries which can produce the light curves in the upper panel. The source star trajectories as seen from the Earth and satellite are represented by solid lines. The circle (represented by dashed lines) around the lens (L) represents the Einstein ring. The coordinates \((x, y)\) are chosen so that they are parallel and perpendicular to the source star trajectory and they are centered at the lens position. The bold vectors \(\Delta u\) connecting two points \((S_s\) for the source seen from the satellite and \(S_E\) from the Earth) on individual trajectories represent the displacements of the source star position (i.e. parallax) observed at a moment. Note that the amount of parallax has a different value depending on the lens system geometry. Since the projected speed is inversely proportional to \(\Delta u\) [see equation (2.6)], there will be two possible values of \(\tilde{v}\) (Gould 1994).

3. Astrometric Source Star Centroid Shifts

When a source star is gravitationally lensed, it is split into two images located on the same and opposite sides of the lens, respectively. Due to the changes in position and amplification of the individual images caused by the lens-source transverse motion, the light centroid between the images changes its location during the event. The location of the image centroid relative to the source star is related to the lensing parameters by

\[
\delta \tilde{\theta}_c = \frac{\theta_E}{u^2 + 2} \left( \frac{t - t_0}{t_E} \hat{x} + \beta \hat{y} \right),
\]

(3.1)

where \(\hat{x}\) and \(\hat{y}\) are the unit vectors toward the directions which are parallel and normal to the lens-source transverse motion and \(\theta_E = r_E/D_{ol}\) is the angular Einstein ring radius. If we let \((x, y) = (\delta \theta_{c,x}, \delta \theta_{c,y} - b)\) and \(b = \beta \theta_E/2(\beta^2 + 2)^{1/2}\), the coordinates are related by

\[
x^2 + \frac{y^2}{q^2} = a^2,
\]

(3.2)

where \(a = \theta_E/2(\beta^2 + 2)^{1/2}\) and \(q = b/a = \beta/(\beta^2 + 2)^{1/2}\). Therefore, during the event the trajectory of the source star image centroid traces an ellipse (‘astrometric ellipse’, Walker 1995; Jeong, Han, & Park 1999). With the measured astrometric ellipse, one can determine the impact parameter of the event because the shape (i.e. the axis ratio) of the ellipse is related to the impact parameter. In addition, one can determine the Einstein ring radius because the size of the astrometric ellipse (i.e. semi-major axis) is directly proportional to
\( \theta_E \). While the Einstein time scale, which is the only measurable quantity from the event light curve, depends on three lens parameters \((M, D_{\odot}, \text{and } v)\), the angular Einstein ring radius depends only on two parameters \((M \text{ and } D_{\odot})\). Therefore, by measuring \( \theta_E \), the uncertainty in the lens parameter can be significantly reduced (Paczyński 1998; Boden et al. 1998; Han & Chang 1999). Once \( \theta_E \) is determined, the lens proper motion is determined by \( \mu = \theta_E / t_E \).

4. Astrometric Parallax Measurements

In addition to allowing one to determine lens proper motions, astrometric observations of lensing events allow one to uniquely determine source star trajectories. In Figure 2, we present various source star trajectories and their corresponding astrometric centroid shifts. To distinguish different trajectories, we define an approaching angle \( \phi \) by the angle between the vector connecting the lens and the source at its closest approach and an arbitrary reference direction (e.g. north). Then each trajectory is defined by its impact parameter and the approaching angle. We additionally define the sign of the impact parameter by

\[
\text{sign } (\beta) = \begin{cases} 
+ , & \text{when } -\pi/2 \leq \phi < \pi/2 \\
- , & \text{when } \pi/2 \leq \phi < 3\pi/2 
\end{cases}, \tag{4.1}
\]

and it is marked on each trajectory along with its impact parameter in the figure. One finds that the orientation of the astrometric ellipse, which is measured by the angle between the reference direction and the semi-minor axis of the ellipse, is identical to the approaching angle of the source star trajectory. Note that the semi-minor axis of the astrometric ellipse is identical to the centroid shift at maximum amplification. Therefore, the two trajectories with the same impact parameter but with opposite signs, which caused the degeneracy in the photometrically determined parallax (i.e. \( \beta \) and \(-\beta\)), result in astrometric ellipses with opposite orientations.

Since the source star trajectory is uniquely determined from the astrometric observations of a lensing event, if the lens parallax is measured astrometrically instead via the photometric method, the degeneracy in \( \Delta u \) can be broken, and thus one can uniquely determine \( \tilde{v} \). In the lower panel of Figure 1, we present the two sets of the source star centroid shifts as seen from the Earth and satellite which are expected from the corresponding two sets of source star trajectories in the middle panels. One finds that these two sets of astrometric ellipses can be easily distinguished from one another.
5. Discussion

Besides the proposed astrometric parallax measurements, the degeneracy of the projected speed can also be broken by other methods. First, one can, in principle, break the degeneracy in \( \tilde{v} \) by measuring the fractional difference in inverse time scales \( \omega = 1/t_E \) between the Earth and satellite, \( \Delta \omega/\omega \), caused by the relative motion of the Earth and satellite, \( v_s \). The larger the projected speed of the lens relative to \( v_s \), the smaller \( \Delta \omega/\omega \). Hence the time scale differences allows one to choose the correct solution (Gould 1996; Boutreux & Gould 1996; Gaudi & Gould 1997). However, the uncertainty in \( \tilde{v} \) using this method would be very large. This is because the expected time scale difference is very small due to the small relative velocity between the Earth and the satellite. To measure the small difference in time scale, therefore, one must construct light curves with very high photometric precision. In addition, both the Galactic bulge and LMC fields are very crowded, and thus nearly all events suffer from severe blending. One might construct a light curve which is free from blending from diffraction-limited observations from the satellite (Han 1997). However, even with very high precision photometric observations from the ground, it would be very difficult to determine the time scale with an uncertainty small enough to determine \( \Delta \omega \) (Woźniak & Paczyński 1997; Han & Kim 1999).

Secondly, one can also resolve the degeneracy in \( \tilde{v} \) by observing a lensing event from a second heliocentric satellite located at a different position. Comparison of the light curves observed from the Earth and the second satellite would yield another two values for \( \tilde{v} \). Among these values, only one would agree well with one of the values of \( \tilde{v} \) determined from the first satellite parallax measurements, allowing one to select the right solution (Gould 1994). The problem with this method is that it requires a very costly second heliocentric satellite.

Of course, the proposed astrometric parallax method also requires two satellites; one geocentric and the other heliocentric satellite. However, the SIM is planned for launch regardless of our proposal. If the SIM will be used as one of the satellites for astrometric parallax measurements, then what is required is simply replacing the photometer in the already proposed photometric parallax satellite with instruments for astrometric observation. In addition, since the required astrometric instruments are being developed for the SIM mission, by adopting the same instrument one can minimize the development cost, which is a significant portion of the total cost. Finally, and most importantly, by determining both the parallax and proper motion at the same time, one can completely resolve the lens parameter degeneracy and thus uniquely determine the physical parameters of individual lenses.

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Figure 1: The degeneracy in the photometric parallax measurements. The upper panel shows the light curves seen from the Earth and satellite. In the middle panels are the two possible lens system geometries which produce the light curves in the upper panel. The source star trajectories as seen from the Earth and the satellite are represented by solid lines. The circles (represented by dashed lines) around the lenses ($L$) are the Einstein rings. The coordinates ($x, y$) are chosen so that they are parallel and perpendicular to the source star trajectory and centered at the lens position. The bold vectors $\Delta u$ connecting the two points ($S_s$ for the source seen from the satellite and $S_E$ from the Earth) on individual trajectories represent the displacements of the source star positions (i.e. parallaxes) observed at a given time. Note that the parallax has a different value depending on the geometry. In the lower panels, we present the two sets of source star centroid shifts as seen from the Earth (geocentric satellite) and from the (heliocentric) satellite corresponding to the source star trajectories in the middle panels.
Figure 2: Various source star trajectories and their corresponding astrometric centroid shifts. The numbers are the impact parameters of the individual trajectories. The approaching angle $\phi$ and the sign of $\beta$ are defined in the text. One finds that the orientation of the astrometric ellipse, which is measured by the angle between the reference direction (e.g. north) and the semi-minor axis of the ellipse, is identical to the approaching angle of the source star trajectory. Note that the semi-minor axis of the astrometric ellipse is identical to the centroid shift at maximum amplification.