Model-independent limits for anomalous triple gauge bosons

$W^+W^-\gamma$ coupling at the CLIC

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Abstract

We investigate the potential of the Compact Linear Collider (CLIC) to probe the anomalous $W^+W^-\gamma$ coupling in $\gamma\gamma$, $\gamma\gamma^*$ and $\gamma^*\gamma^*$ collisions through the processes $\gamma\gamma \rightarrow W^+W^-$, $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^+W^-$ and $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$. We perform leptonic, semi-leptonic and hadronic decays channels for $W^+W^-$ production in the final state. Taking $L = 1 \text{ ab}^{-1}\sqrt{s} = 0.380 \text{ TeV}$, $L = 2.5 \text{ ab}^{-1}\sqrt{s} = 1.5 \text{ TeV}$ and $L = 5 \text{ ab}^{-1}\sqrt{s} = 3 \text{ TeV}$, based on future CLIC data, the limits for $\Delta \kappa_{\gamma}$ and $\lambda_\gamma$ might reach up to $O(10^{-5} - 10^{-4})$ level in the most ideal with 5 ab$^{-1}$ set data, which shows a potential advantage compared to those from LHC, Tevatron and LEP data. Thus, our results represent that the processes $\gamma\gamma \rightarrow W^+W^-$, $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^-W^+$ and $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$ at the CLIC are a very good prospect for probing the anomalous $W^+W^-\gamma$ couplings.

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I. INTRODUCTION

The Compact Linear Collider (CLIC) [1–4] is proposed to testing the Standard Model (SM) [5] in detail, with an unprecedented precision and with clean environments, in comparison with the hadron colliders. Its aim is to research the energy frontier, providing sensitivity to physics beyond the Standard Model (BSM). As part of this physics program, the couplings between known particles are parameterized in a general form to quantify possible deviations from their SM values, as is the case of the anomalous Triple Gauge Boson Couplings (aTGC) $W^+W^−\gamma$, $W^+W^−Z$ [6–10]. The measurement of the couplings between the neutral electroweak bosons $Z$, $\gamma$ and the charged boson $W^\pm$, are the first two measurements that were able to prove the non-Abelian character of the electroweak part of the SM [5]. Therefore, processes that are sensitive to gauge bosons self-interactions are important tools used to search for non-standard effects.

A number of authors have made important contributions to the subject [6–8, 11–15]. This topic has acquired new relevance in the recent years. In Table I, we summary 95% Confidence Level (C.L.) limits on the aTGC $\Delta\kappa_\gamma$ and $\lambda_\gamma$ from ATLAS, CMS, CDF, D0, ALEP, DELPHI, L3 and OPAL Collaborations and the future lepton colliders such as the ILC and the CEPC. See Refs. [23–28] for other limits on $\Delta\kappa_\gamma$ and $\lambda_\gamma$ in different contexts.

The experiments at the CLIC in its three phases $\mathcal{L} = 1 \text{ab}^{-1}@\sqrt{s} = 0.380 \text{TeV}$, $\mathcal{L} = 2.5 \text{ab}^{-1}@\sqrt{s} = 1.5 \text{TeV}$ and $\mathcal{L} = 5 \text{ab}^{-1}@\sqrt{s} = 3 \text{TeV}$ can produce charged weak bosons ($W^\pm$) pairs in $\gamma\gamma$, $\gamma\gamma^*$ and $\gamma^*\gamma^*$ collisions through the processes $\gamma\gamma \rightarrow W^+W^-$, $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+e^-W^+W^-$ and $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^+W^-e^-$. With these energies and luminosities the CLIC can successfully achieve good limits on the anomalous $W^+W^−\gamma$ coupling, because this collider is best suited for precision measurements [1–4]. One of the advantages of $\gamma\gamma$, $\gamma\gamma^*$ and $\gamma^*\gamma^*$ collisions is that they can isolate $W^+W^−\gamma$ couplings from $W^+W^−Z$ couplings unlike $e^+e^-$ collisions. Also, $W^+W^−\gamma$ vertex makes contributes to $\gamma\gamma$, $\gamma\gamma^*$ and $\gamma^*\gamma^*$ collisions. These collisions are a particularly important tool in searching $W^\pm$ electromagnetic interactions. The processes $\gamma\gamma(\gamma\gamma^*, \gamma^*\gamma^*) \rightarrow W^+W^-$ include only interactions between the gauge bosons, causing more apparent possible deviations from the expected value of SM [29]. In addition, these processes provide the best opportunity to measure directly the $W^+W^−\gamma$ aTGC via $t$ and $u$ channel (see Figs. 1-3), and as we already mentioned can be used to test the non-Abelian nature of the SM.
TABLE I: Experimental and phenomenological limits at 95% C.L. on the aTGC $\Delta\kappa_\gamma$ and $\lambda_\gamma$ from the present and future colliders.

| Model                        | $\Delta\kappa_\gamma$ | $\lambda_\gamma$ | C. L. | Reference |
|------------------------------|------------------------|-------------------|-------|-----------|
| SM                           | 0                      | 0                 |       | [5]       |
| Experimental limit           |                        |                   |       |           |
| ATLAS Collaboration           | [-0.061, 0.064]        | [-0.013, 0.013]   | 95%   | [16]      |
| CMS Collaboration            | [-0.044, 0.063]        | [-0.011, 0.011]   | 95%   | [17]      |
| CDF Collaboration            | [-0.158, 0.255]        | [-0.034, 0.042]   | 95%   | [18]      |
| D0 Collaboration             | [-0.158, 0.255]        | [-0.034, 0.042]   | 95%   | [19]      |
| ALEP, DELPHI, L3, OPAL       | [-0.099, 0.066]        | [-0.059, 0.017]   | 95%   | [20]      |
| Phenomenological limit       |                        |                   |       |           |
| ILC                          | [-0.00037, 0.00037]    | [-0.00051, 0.00051]| 95%  | [21]      |
| CEPC                         | [-0.00045, 0.00045]    | [-0.00033, 0.00033]| 95%  | [22]      |

The rest of the paper is organized as follows: In Section II, the gauge-invariant operators of dimension-six are given. In Section III and IV, we study the aTGCs $\Delta\kappa_\gamma$ and $\lambda_\gamma$ through the processes $\gamma\gamma \rightarrow W^+W^-$, $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^+W^-$ and $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$ at the $\gamma\gamma$, $\gamma\gamma^*$ and $\gamma^*\gamma^*$ collision mode. Finally, we present our conclusions in Section V.

II. THE TRIPLE GAUGE BOSON VERTEX $W^+W^-\gamma$ WITH THE ANOMALOUS CONTRIBUTION

In this paper, effective Lagrangian technique is used to understand potential deviations from the SM predictions. We adopt the effective Lagrangian for $W^+W^-\gamma$ interaction of the photon and the gauge bosons with operators up to mass dimension-six:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}}^{(4)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \text{h.c.}, \quad (1)$$

where $\mathcal{L}_{\text{SM}}^{(4)}$ denotes the renormalizable SM Lagrangian and $\mathcal{O}_i^{(6)}$ are the gauge-invariant
operators of mass dimension-six. The index \( i \) runs over all operators of the given mass dimension. The mass scale is set by \( \Lambda \), and the coefficients \( C_i \) are dimensionless parameters, which are determined once the full theory is known.

The effective Lagrangian relevant to the analysis of aTGC reads:

\[
\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \left[ C_W \mathcal{O}_W + C_B \mathcal{O}_B + C_{WWW} \mathcal{O}_{WWW} + \text{h.c.} \right],
\]

with

\[
\mathcal{O}_W = (D_\mu \Phi) \dagger \hat{W}^{\mu\nu} (D_\nu \Phi),
\]

\[
\mathcal{O}_B = (D_\mu \Phi) \dagger \hat{B}^{\mu\nu} (D_\nu \Phi),
\]

\[
\mathcal{O}_{WWW} = \text{Tr}[\hat{W}^{\mu\nu} \hat{W}^{\nu\rho} \hat{W}^\rho],
\]

\( D_\mu \) is the covariant derivative, \( \Phi \) is the Higgs doublet field and \( \hat{B}_{\mu\nu} \), and \( \hat{W}_{\mu\nu} \) are the \( U(1)_Y \) and \( SU(2)_L \) gauge field strength tensors. The coefficients of these operators \( C_W/\Lambda^2, C_B/\Lambda^2 \), and \( C_{WWW}/\Lambda^2 \), are zero in the SM.

Based on this methodology, the effective Lagrangian for describe the \( W^+W^-\gamma \) coupling can be parameterized as \cite{7, 30}:

\[
\mathcal{L}_{WW\gamma} = -ig_{WW\gamma} \left[ g_1^\gamma (W^\mu W^\nu A_\nu) - W^{\mu\nu} W^{\nu\rho} A_\mu A_\rho \right] + \kappa_\gamma W^{\mu\nu} W_\mu W_\nu A^{\mu\nu} + \frac{\lambda_\gamma}{M_W^2} W^{\mu\nu} W_\mu W_\nu A^{\mu\nu},
\]

where \( g_{WW\gamma} = e \), \( V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \) with \( V_\mu = W_\mu, A_\mu \). The couplings \( g_1^\gamma, \kappa_\gamma \) and \( \lambda_\gamma \) are CP-preserving, and in the SM their values are \( g_1^\gamma = \kappa_\gamma = 1 \) and \( \lambda_\gamma = 0 \) at the tree level.

The three operators of dimension-six, given by Eq. (2) are related to the aTGC via \cite{8, 31, 32}:

\[
\kappa_\gamma = 1 + \Delta \kappa_\gamma,
\]

with

\[
\Delta \kappa_\gamma = C_W + C_B,
\]

\[
\lambda_\gamma = C_{WWW}.
\]
From the effective Lagrangian given by Eq. (6) which CP-conserving, the Feynman rule for the anomalous \( W^+W^-\gamma \) vertex is given by \([7]\):

\[
\Gamma_{\mu\nu\rho}^{WW\gamma} = e \left[ g_{\mu\nu}(p_1 - p_2)_\rho + g_{\nu\rho}(p_2 - p_3)_\mu + g_{\rho\mu}(p_3 - p_1)_\nu + \Delta \kappa \gamma \left( g_{\rho\mu}p_3\nu - g_{\nu\rho}p_3\mu \right) \right. \\
+ \left. \frac{\lambda}{M_W^2} \left( p_1\rho p_2\mu p_3\nu - p_1\nu p_2\rho p_3\mu - g_{\mu\nu}(p_2 \cdot p_3 p_1\rho - p_3 \cdot p_1 p_2\rho) \right. \right. \\
- \left. g_{\nu\rho}(p_3 \cdot p_1 p_2\mu - p_1 \cdot p_2 p_3\mu) - g_{\mu\rho}(p_1 \cdot p_2 p_3\nu - p_2 \cdot p_3 p_1\nu) \right].
\] (10)

Here, \( p_1 \) represents the momentum of the photon and \( p_2 \) and \( p_3 \) represent the momenta of \( W^\pm \) bosons. In addition, the first three terms in Eq. (10) correspond to the SM couplings, while the terms with \( \Delta \kappa \gamma \) and \( \lambda \gamma \) give rise to aTGC.

Several searches on these anomalous \( W^+W^-\gamma \) couplings \( \Delta \kappa \gamma \) and \( \lambda \gamma \) were performed by the LEP, Tevatron and LHC experiments, as shown in Table I.

### III. CROSS-SECTION AND MODEL-INDEPENDENT LIMITS ON THE ANOMALOUS COUPLINGS \( \Delta \kappa \gamma \) AND \( \lambda \gamma \) IN \( \gamma\gamma, \gamma\gamma^* \) AND \( \gamma^*\gamma^* \) COLLISIONS AT THE CLIC

The advantage of the linear \( e^+e^- \) colliders with respect to the hadron colliders is in the general cleanliness of the events where two elementary particles, electron and positron beams, collide at high energy, and the high resolutions of the detector are made possible by the relatively low absolute rate of background events. Furthermore, 1) these colliders will complement the physics program of the LHC, especially for precision measurements. 2) Photon colliders \( \gamma\gamma \) and \( \gamma e^- \) have been considered a natural addition to \( e^+e^- \) linear colliders as the ILC and the CLIC. 3) The photon colliders based on the ILC or the CLIC are the most realistic project. 4) Currently, the ILC and the CLIC are the best place for the photon collider. In the case of the processes studied in this paper \( \gamma\gamma \to W^+W^-, e^+\gamma \to e^+\gamma^*\gamma \to e^+W^-W^+ \) and \( e^+e^- \to e^+\gamma^*\gamma^* e^- \to e^+W^-W^+ e^- \) where \( \gamma \) and \( \gamma^* \) are Compton backscattered and Weizsäcker-Williams photons, they are extremely clean reactions because there is no interference with weak and strong interactions as they are purely quantum electroweak reactions. This is very useful for any new physics study, in particular, to study the anomalous \( W^+W^-\gamma \) coupling.
TABLE II: Benchmark parameters of the CLIC for each stage in the updated scenario [1–4].

| CLIC | $\sqrt{s}$ (TeV) | $\mathcal{L}$ (fb$^{-1}$) |
|------|------------------|--------------------------|
| Stage 1 | 0.380 | 100, 300, 500, 700, 1000 |
| Stage 2 | 1.5 | 100, 500, 1000, 1500, 2500 |
| Stage 3 | 3 | 100, 1000, 3000, 4000, 5000 |

The CLIC is a multi-TeV linear $e^+e^-$ collider designed to operate in several center-of-mass energy stages, as shown in Table II. As we mentioned above, this allows the study of the $\gamma\gamma$ and $\gamma e^-$ interactions by converting the original $e^-$ or $e^+$ beam into a photon beam through the Compton backscattering mechanism. Other well-known applications of the linear colliders are the processes $e\gamma^*$, $\gamma\gamma^*$ and $\gamma^*\gamma^*$ where the emitted quasi-real photon $\gamma^*$ is scattered with small angles from the beam pipe of $e^-$ or $e^+$ [33–38]. Since these photons have a low virtuality, they are almost on the mass shell. These processes can be described by the Weizsacker-Williams Approximation (WWA) [36, 39, 40]. The WWA has a lot of advantages such as providing the skill to reach crude numerical predictions via simple formulae. In addition, it may principally ease the experimental analysis because it enables one to directly achieve a rough cross section for $\gamma^*\gamma^* \rightarrow X$ process via the examination of the main process $e^-e^+ \rightarrow e^-Xe^+$ where X represents objects produced in the final state. The production of high mass objects is particularly interesting at the linear colliders and the production rate of massive objects is limited by the photon luminosity at high invariant mass while $\gamma^*\gamma^*$ and $e\gamma^*$ processes at the linear colliders arise from quasi-real photon emitted from the incoming beams. Hence, $\gamma^*\gamma^*$ and $e\gamma^*$ are more realistic than $\gamma\gamma$ and $e\gamma$. These processes have been observed experimentally at the LEP, the Tevatron and the LHC [41–47].

A. The total cross-section of the processes $\gamma\gamma \rightarrow W^+W^-$, $e^+e^- \rightarrow e^+\gamma^*\gamma^* \rightarrow e^+W^-W^+$ and $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$ at the CLIC

The production processes of diboson $\gamma\gamma \rightarrow W^+W^-$, $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^-W^+$ and $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$, can also be produced from the radiative couplings to photon, as shown in Figs. 1-3.

In future lepton linear colliders, such as the CLIC, high luminosity photon beams can
be obtained by Compton backscattering of a low-energy, high-intensity laser beam off the high-energy electron beam, and then $\gamma\gamma \rightarrow W^-W^+$ can be produced from the laser photon fusion processes as shown in Fig. 3. In this case, the spectrum of Compton backscattered photons \cite{33, 48} is given by:

$$f_{\gamma}(y) = \frac{1}{g(\zeta)}\left[1 - y + \frac{4y}{\zeta(1 - y)} + \frac{4y^2}{\zeta^2(1 - y)^2}\right], \quad (11)$$

where

$$g(\zeta) = \left(1 - \frac{4}{\zeta} - \frac{8}{\zeta^2}\right) \log(\zeta + 1) + \frac{1}{2} + \frac{8}{\zeta} - \frac{1}{2(\zeta + 1)^2}, \quad (12)$$

with

$$y = \frac{E_\gamma}{E_e}, \quad \zeta = \frac{4E_0E_e}{M_e^2}, \quad y_{\text{max}} = \frac{\zeta}{1 + \zeta}. \quad (13)$$

Here, $y$ is the fraction of electron energy carried away by the scattered photon, $E_0$ and $E_e$ are energy of the incoming laser photon and initial energy of the electron beam before Compton backscattering and $E_\gamma$ is the energy of the backscattered photon. The maximum value of $y_{\text{max}}$ reaches 0.83 when $\zeta = 4.8$, that is when the photon conversion efficiency drops drastically, as a consequence of the $e^+e^-$ pair production from the laser photons and the photon backscattering.

Other implementations of the linear colliders are the processes $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^-W^+$ and $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$ described by the WWA, where the spectrum of photons emitted by electrons is given by \cite{36, 49}:

$$f_{\gamma^*}(x_1) = \frac{\alpha}{\pi E_e}\left\{\frac{1 - x_1 + x_1^2/2}{x_1}\right\}\log\left(\frac{Q_{\text{max}}^2}{Q_{\text{min}}^2}\right) - \frac{m_{\gamma^*}^2 x_1}{Q_{\text{min}}^2} \left(1 - \frac{Q_{\text{min}}^2}{Q_{\text{max}}^2}\right) - \frac{1}{x_1} \left[1 - \frac{x_1}{2}\right]^2 \log\left(\frac{x_1^2 E_e^2 + Q_{\text{max}}^2}{x_1^2 E_e^2 + Q_{\text{min}}^2}\right), \quad (14)$$

with $x_1 = E_{\gamma^*}/E_e$ and $Q_{\text{max}}^2$ is maximum virtuality of the photon. The minimum value of the $Q_{\text{min}}^2$ is given by:

$$Q_{\text{min}}^2 = \frac{m_{\gamma^*}^2 x_1^2}{1 - x_1}. \quad (15)$$
Therefore, the total cross-section of the reactions $\gamma\gamma \rightarrow W^+W^-$, $e^+\gamma \rightarrow e^+\gamma\gamma \rightarrow e^+W^-W^+$ and $e^+e^- \rightarrow e^+\gamma\gamma^*e^- \rightarrow e^+W^-W^+e^-$ at the CLIC are obtained from:

$$\sigma = \int f_{\gamma^*}(x_1)f_{\gamma^*}(x_2)d\sigma_{\gamma^*\gamma^*\gamma}dE_1dE_2.$$  \hspace{1cm} (16)$$

For the computation of the total cross-section $\sigma(\Delta\kappa_\gamma, \lambda_\gamma, \sqrt{s})$ we have implemented the interactions term in the CalcHEP package [49]. The cross-section of photon fusion channel $\gamma\gamma \rightarrow W^-W^+$ [Figs. 4-5] depends largely on the center-of-mass energies of the collider, as well as of the anomalous $\lambda_\gamma$ and $\Delta\kappa_\gamma$ couplings. With the center-of-mass energies $\sqrt{s} = 0.380, 1.5, 3$ TeV and $-0.2 \leq \lambda_\gamma \leq 0.2$, the photon fusion cross-sections are: $\sigma(\gamma\gamma \rightarrow W^-W^+) = 50$ pb, $5 \times 10^3$ pb and $2 \times 10^5$ pb, respectively. For $-3 \leq \Delta\kappa_\gamma \leq 3$, the cross-sections are $\sigma(\gamma\gamma \rightarrow W^-W^+) = 4 \times 10^2$ pb, $2 \times 10^4$ pb and $7 \times 10^5$ pb. The production cross-sections $\sigma(e^+\gamma \rightarrow e^+\gamma\gamma \rightarrow e^+W^-W^+) = 2$ pb, $2 \times 10^2$ pb, $6 \times 10^3$ pb for $-0.2 \leq \lambda_\gamma \leq 0.2$ and $\sigma(e^+\gamma \rightarrow e^+\gamma\gamma \rightarrow e^+W^-W^+) = 10$ pb, $3 \times 10^2$ pb and $4 \times 10^3$ pb for $-3 \leq \Delta\kappa_\gamma \leq 3$, and are presented in Figs. 6-7. For the process $e^+e^- \rightarrow e^+\gamma\gamma^*e^- \rightarrow e^+W^-W^+e^-$, the production cross-section $\sigma(e^+e^- \rightarrow e^+\gamma\gamma^*e^- \rightarrow e^+W^-W^+e^-) = 0.4$ pb, $5$ pb, $2 \times 10^2$ pb for $\sqrt{s} = 0.380, 1.5, 3$ TeV and $-0.2 \leq \lambda_\gamma \leq 0.2$. Whereas that, for $-3 \leq \Delta\kappa_\gamma \leq 3$ the cross-section is $\sigma(e^+e^- \rightarrow e^+\gamma\gamma^*e^- \rightarrow e^+W^-W^+e^-) = 0.2$ pb, $10^2$ pb, $4 \times 10^2$ pb as are shown in Figs. 8 and 9. As mentioned above, the total cross-sections depends significantly on the center-of-mass energies of the collider, as well as of the aTGC $\lambda_\gamma$ and $\Delta\kappa_\gamma$.

As can be seen from Figs. 4-9, in the case of $\sqrt{s} = 0.38$ TeV where the center-of-mass energy is relatively low, there is an asymmetry of the cross-section values relative to the negative and positive values of the aTGC $\lambda_\gamma$ and $\Delta\kappa_\gamma$, due to the cross terms of the aTGC with the SM terms. This asymmetry decreased significantly due to the reduction of the effect of the SM in increasing center-of-mass energies.

**B. Limits on the anomalous couplings $\Delta\kappa_\gamma$ and $\lambda_\gamma$ through the process $\gamma\gamma \rightarrow W^+W^-$**

To illustrate the expected 95% confidence intervals for the parameters $\Delta\kappa_\gamma$ and $\lambda_\gamma$, we adopted the $\chi^2$ method:

$$\chi^2(\Delta\kappa_\gamma, \lambda_\gamma) = \left(\frac{\sigma_{\text{SM}} - \sigma_{\text{BSM}}(\sqrt{s}, \Delta\kappa_\gamma, \lambda_\gamma)}{\sigma_{\text{SM}}(\delta_{\text{sys}})^2}}\right)^2,$$  \hspace{1cm} (17)
where $\sigma_{BSM}(\sqrt{s}, \Delta \kappa, \lambda)$ and $\sigma_{SM}$ are the cross-section with and the without anomalous couplings $\Delta \kappa$ and $\lambda$. $\delta_{st} = \frac{1}{\sqrt{N_{SM}}}$ is the statistical error and $\delta_{sys}$ is the systematic error. The number of events is given by $N_{SM} = L_{int} \times \sigma_{SM} \times BR(W^\pm \to qq', l\nu_l)$, where $L_{int}$ is the integrated luminosity of the CLIC and $l = e^-, \mu^-$. For $W^+W^-$ pair production we classify their decay products according to the decomposition of $W^\pm$. In this paper, we assume that one of the $W^\pm$ bosons decays leptonically and the other hadronically for the signal. This phenomenon has already been studied by ATLAS and CMS Collaborations [50–52]. Thus, we assume that the branching ratios for $W^\pm$ decays are: $BR(W^\pm \to qq') = 0.454$ for hadronic decays, $BR(W^+ \to qq'; W^- \to l\nu_{e,\mu}) = 0.143$ for semi-leptonic decays and $BR(W^\pm \to l\nu_{e,\mu}) = 0.045$ for light leptonic decays.

We probe the potential of the CLIC to estimate the limits on the aTGC through the process $\gamma\gamma \to W^+W^-$, based $\gamma\gamma$ colliders with the benchmark parameters for each stage in the updated scenario. The observed 95% confidence intervals for the aTGC $\Delta \kappa$ and $\lambda$ are shown in Tables III-V. The confidence intervals for a given $\Delta \kappa$ or $\lambda$ parameter are computing while fixing the another anomalous parameter to zero. The confidence intervals are shown separately for the leptonic, semi-leptonic and hadronic decays channels of the $W^\pm$ bosons.

From Tables III-V, the best limits for $\Delta \kappa$ and $\lambda$, taken one coupling at a time, are given by:

$$\Delta \kappa = [-0.00023, 0.00023], \quad 95\% \text{ C.L.}, \quad (18)$$
$$\lambda = [-0.00034, 0.27038], \quad 95\% \text{ C.L.},$$

$$\Delta \kappa = [-0.00010, 0.00010], \quad 95\% \text{ C.L.},$$
$$\lambda = [-0.00007, 0.000936], \quad 95\% \text{ C.L.}, \quad (19)$$

$$\Delta \kappa = [-0.00007, 0.00007], \quad 95\% \text{ C.L.},$$
$$\lambda = [-0.00004, 0.00102], \quad 95\% \text{ C.L.}, \quad (20)$$

for the hadronic channel with $\sqrt{s} = 0.380, 1.5, 3 \text{ TeV}$ and $L = 1000, 2500, 5000 \text{ fb}^{-1}$, respectively.
TABLE III: The expected 95% confidence level for the anomalous couplings $\Delta \kappa_\gamma$ and $\lambda_\gamma$, through the process $\gamma\gamma \rightarrow W^+W^-$ for $\sqrt{s} = 0.380$ TeV. The leptonic, semi-leptonic and hadronic channels of the $W^+W^-$ in the final state are considered. The confidence level for each parameter are calculated while fixing the another parameter to zero.

| $\sqrt{s}$ = 0.380 TeV, 95% C.L. | Channel |
|-----------------------------------|--------|
| $\mathcal{L}$ (fb$^{-1}$) | Leptonic | Semi-leptonic | Hadronic |
| 100 | [-0.00226, 0.00225] | [-0.00128, 0.00127] | [-0.00072, 0.00071] |
| 300 | [-0.00130, 0.00130] | [-0.00073, 0.00073] | [-0.00041, 0.00041] |
| 500 | [-0.00101, 0.00101] | [-0.00057, 0.00057] | [-0.00032, 0.00032] |
| 700 | [-0.00085, 0.00085] | [-0.00048, 0.00048] | [-0.00027, 0.00027] |
| 1000 | [-0.00071, 0.00071] | [-0.00040, 0.00040] | [-0.00023, 0.00023] |

For the other luminosity stages of the CLIC, as well as of the other decay channels of the $W^\pm$ bosons, the limits for $\Delta \kappa_\gamma$ and $\lambda_\gamma$ are weaker than those corresponding to Eqs. (18)-(20), however there are also competitive with the experimental limits which are shown in Table I for the ATLAS, CMS, CDF, D0, ALEP, DELPHI, L3 and OPAL Collaborations, as well as with the corresponding phenomenological limits obtained for the future ILC and the CEPC. It is worth mentioning that for all the CLIC energy stages, the process $\gamma\gamma \rightarrow W^+W^-$ gives strong limits to the anomalous couplings $\Delta \kappa_\gamma$ and $\lambda_\gamma$, as shown in Eqs. (18)-(20) and Tables III-V. These results show the strong benefit of several energy stages for the CLIC physics potential. In addition, the operation at high energy significantly improves the limits to the anomalous couplings.
TABLE IV: The expected 95% confidence level for the anomalous couplings $\Delta \kappa_\gamma$ and $\lambda_\gamma$, through the process $\gamma \gamma \rightarrow W^+W^-$ for $\sqrt{s} = 1.5$ TeV. The leptonic, semi-leptonic and hadronic channels of the $W^+W^-$ in the final state are considered. The confidence level for each parameter are calculated while fixing the another parameter to zero.

| $\sqrt{s} = 1.5$ TeV, 95% C.L. | Channel |
|----------------------|---------|
|                      | $\mathcal{L} (\text{fb}^{-1})$ | Leptonic | Semi-leptonic | Hadronic |
| $\Delta \kappa_\gamma$ | 100     | [-0.00154, 0.00153] | [-0.00087, 0.00086] | [-0.00049, 0.00049] |
|                      | 500     | [-0.00069, 0.00069] | [-0.00039, 0.00039] | [-0.00022, 0.00022] |
|                      | 1000    | [-0.00048, 0.00048] | [-0.00027, 0.00027] | [-0.00015, 0.00015] |
|                      | 1500    | [-0.00040, 0.00040] | [-0.00022, 0.00022] | [-0.00013, 0.00013] |
|                      | 2500    | [-0.00031, 0.00031] | [-0.00017, 0.00017] | [-0.00010, 0.00010] |
| $\lambda_\gamma$     | 100     | [-0.00110, 0.00109] | [-0.00065, 0.00093] | [-0.00037, 0.00066] |
|                      | 500     | [-0.00052, 0.00081] | [-0.00030, 0.00058] | [-0.00017, 0.00045] |
|                      | 1000    | [-0.00037, 0.00066] | [-0.00021, 0.00050] | [-0.00012, 0.00040] |
|                      | 1500    | [-0.00031, 0.00059] | [-0.00017, 0.00046] | [-0.00010, 0.00038] |
|                      | 2500    | [-0.00024, 0.00052] | [-0.00013, 0.00042] | [-0.00007, 0.00036] |

C. Limits on the anomalous couplings $\Delta \kappa_\gamma$ and $\lambda_\gamma$ through the process $e^+\gamma \rightarrow e^+\gamma^* \rightarrow e^+W^-W^+$

The anomalous couplings $\Delta \kappa_\gamma$ and $\lambda_\gamma$ to the photon and $W^\pm$ bosons are precisely predicted by the SM but may receive substantial corrections from BSM physics. Here we have shown the physics potential of $W^+W^-$ pair production through the process $e^+\gamma \rightarrow e^+\gamma^* \rightarrow e^+W^-W^+$ in $\gamma\gamma^*$ collisions and using effective Lagrangian. The effective Lagrangian formalism extends the SM Lagrangian to include interaction operators of higher dimension. The leading effects are captured by dimension-six operators weighted by the coefficients $C_i/\Lambda^2$ for dimensionless couplings $C_i$, as shown in Eq. (1). With this focus and with the clean experimental environment of the CLIC the measurements on the anomalous couplings $\Delta \kappa_\gamma$ and $\lambda_\gamma$ to be better exploited.
TABLE V: The expected 95% confidence level for the anomalous couplings $\Delta\kappa$ and $\lambda$, through the process $\gamma\gamma \rightarrow W^+W^-$ for $\sqrt{s} = 3$ TeV. The leptonic, semi-leptonic and hadronic channels of the $W^+W^-$ in the final state are considered. The confidence level for each parameter are calculated while fixing the another parameter to zero.

| $\sqrt{s} = 3$ TeV, 95% C.L. | Channel               |
|-------------------------------|-----------------------|
|                               | Leptonic              | Semi-leptonic       | Hadronic          |
| $\Delta\kappa$               |                       |                      |                   |
| 100                           | [-0.00153, 0.00148]   | [-0.00085, 0.00084]  | [-0.00048, 0.00047] |
| 1000                          | [-0.00048, 0.00047]   | [-0.00027, 0.00027]  | [-0.00015, 0.00015] |
| 3000                          | [-0.00027, 0.00027]   | [-0.00015, 0.00015]  | [-0.00009, 0.00009] |
| 4000                          | [-0.00024, 0.00024]   | [-0.00013, 0.00013]  | [-0.00008, 0.00008] |
| 5000                          | [-0.00021, 0.00021]   | [-0.00012, 0.00012]  | [-0.00007, 0.00007] |
| $\lambda$                    |                       |                      |                   |
| 100                           | [-0.00058, 0.00156]   | [-0.00037, 0.00136]  | [-0.00024, 0.00122] |
| 1000                          | [-0.00023, 0.00122]   | [-0.00014, 0.00112]  | [-0.00008, 0.00106] |
| 1000                          | [-0.00014, 0.00113]   | [-0.00008, 0.00107]  | [-0.00005, 0.00104] |
| 4000                          | [-0.00013, 0.00111]   | [-0.00007, 0.00106]  | [-0.00004, 0.00103] |
| 5000                          | [-0.00011, 0.00110]   | [-0.00006, 0.00105]  | [-0.00004, 0.00102] |

Already the first CLIC stage provides an important set of measurements on the anomalous couplings $\Delta\kappa$ and $\lambda$ using the $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^-W^+$ process, as shown in Table VI. Next, we present the most significant limits for the anomalous couplings coming from the vertex $W^+W^-\gamma$, for all energy stages of the CLIC (see Tables VI-VIII):

$$\Delta\kappa = [-0.00119, 0.00119], \quad 95\% \text{ C.L.},$$

$$\lambda = [-0.00190, 0.29527], \quad 95\% \text{ C.L.},$$

(21)

$$\Delta\kappa = [-0.00028, 0.00028], \quad 95\% \text{ C.L.},$$

$$\lambda = [-0.00029, 0.02128], \quad 95\% \text{ C.L.},$$

(22)
TABLE VI: The expected 95% confidence level for the anomalous couplings $\Delta \kappa_\gamma$ and $\lambda_\gamma$, through the process $e^+ \gamma \rightarrow e^+ \gamma^* \gamma \rightarrow e^+ W^+ W^+$ for $\sqrt{s} = 0.380 \text{ TeV}$. The leptonic, semi-leptonic and hadronic channels of the $W^+ W^-$ in the final state are considered. The confidence level for each parameter are calculated while fixing the another parameter to zero.

| $\sqrt{s}$ = 0.380 TeV, 95% C.L. | Channel |
|---|---|---|
| $\mathcal{L}$ (fb$^{-1}$) | Leptonic | Semi-leptonic | Hadronic |
| 100 | [-0.00121, 0.00118] | [-0.00678, 0.00667] | [-0.00377, 0.00376] |
| 300 | [-0.00697, 0.00685] | [-0.00390, 0.00387] | [-0.00219, 0.00218] |
| 500 | [-0.00538, 0.00532] | [-0.00302, 0.00300] | [-0.00169, 0.00169] |
| 700 | [-0.00450, 0.00450] | [-0.00253, 0.00253] | [-0.00143, 0.00143] |
| 1000 | [-0.00377, 0.00377] | [-0.00212, 0.00212] | [-0.00119, 0.00119] |

$\Delta \kappa_\gamma = [-0.00015, 0.00015]$, 95% C.L.,

$\lambda_\gamma = [-0.00013, 0.00340]$, 95% C.L.,

for the hadronic channel with $\sqrt{s} = 0.380, 1.5, 3 \text{ TeV}$ and $\mathcal{L} = 1000, 2500, 5000 \text{ fb}^{-1}$, respectively. The limits on $\Delta \kappa_\gamma$ and $\lambda_\gamma$ given in Eqs. (21)-(23) are competitive with those shown in Table I, and in some cases our limits are stronger.
TABLE VII: The expected 95% confidence level for the anomalous couplings $\Delta\kappa_\gamma$ and $\lambda_\gamma$, through the process $e^+\gamma \rightarrow e^+\gamma^*\gamma^* \rightarrow e^+W^-W^+$ for $\sqrt{s} = 1.5$ TeV. The leptonic, semi-leptonic and hadronic channels of the $W^+W^-$ in the final state are considered. The confidence level for each parameter are calculated while fixing the another parameter to zero.

| $\sqrt{s} = 1.5$ TeV, 95% C.L. | $\mathcal{L}$ ($\text{fb}^{-1}$) | Channel |
|-------------------------------|-------------------------------|------------------|
|                               |                               | Leptonic          | Semi-leptonic   | Hadronic        |
|                               | 100                           | [-0.00439, 0.00438] | [-0.00249, 0.00247] | [-0.00140, 0.00139] |
|                               | 500                           | [-0.00198, 0.00197] | [-0.00111, 0.00111] | [-0.00062, 0.00062] |
| $\Delta\kappa_\gamma$        | 1000                          | [-0.00139, 0.00139] | [-0.00078, 0.00078] | [-0.00044, 0.00044] |
|                               | 1500                          | [-0.00114, 0.00114] | [-0.00064, 0.00064] | [-0.00036, 0.00036] |
|                               | 2500                          | [-0.00088, 0.00088] | [-0.00050, 0.00050] | [-0.00028, 0.00028] |
| $\lambda_\gamma$             | 100                           | [-0.00397, 0.02482] | [-0.00238, 0.02330] | [-0.00140, 0.02235] |
|                               | 500                           | [-0.00193, 0.02286] | [-0.00112, 0.02208] | [-0.00064, 0.02162] |
|                               | 1000                          | [-0.00140, 0.02235] | [-0.00081, 0.02178] | [-0.00046, 0.02140] |
|                               | 1500                          | [-0.00115, 0.02211] | [-0.00066, 0.02164] | [-0.00037, 0.02136] |
|                               | 2500                          | [-0.00090, 0.02187] | [-0.00051, 0.02150] | [-0.00029, 0.02128] |

D. Limits on the anomalous couplings $\Delta\kappa_\gamma$ and $\lambda_\gamma$ through the process $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$

The WWA of quasi-real quanta, is best known for its application to radiation during elementary particle collisions. As we mentioned earlier, other well-known applications of the linear colliders are the processes $e\gamma^*$, $\gamma\gamma^*$ and $\gamma^*\gamma^*$.

Combining the WWA with the characteristics of the future CLIC, such as high energies, high luminosities, and clean experimental environments, we probe limits on the anomalous couplings $\Delta\kappa_\gamma$ and $\lambda_\gamma$ of the $W^\pm$ bosons. The limits obtained for $\Delta\kappa_\gamma$ and $\lambda_\gamma$ through the process $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$ are shown in Tables IX-XI. From these tables, the most notable limits are the following:
TABLE VIII: The expected 95% confidence level for the anomalous couplings $\Delta\kappa_\gamma$ and $\lambda_\gamma$, through the process $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^-W^+$ for $\sqrt{s} = 3$ TeV. The leptonic, semi-leptonic and hadronic channels of the $W^+W^-$ in the final state are considered. The confidence level for each parameter are calculated while fixing the another parameter to zero.

| $\sqrt{s} = 3$ TeV, 95% C.L. | Channel |
|-----------------------------|---------|
| $\mathcal{L}$ (fb$^{-1}$)  | Leptonic | Semi-leptonic | Hadronic |
| $\Delta\kappa_\gamma$      |         |              |          |
| 100                         | [-0.00352, 0.00341] | [-0.00196, 0.00193] | [-0.00110, 0.00109] |
| 1000                        | [-0.00110, 0.00109] | [-0.00062, 0.00061] | [-0.00035, 0.00035] |
| 3000                        | [-0.00063, 0.00063] | [-0.00035, 0.00035] | [-0.00020, 0.00020] |
| 4000                        | [-0.00055, 0.00055] | [-0.00031, 0.00031] | [-0.00017, 0.00017] |
| 5000                        | [-0.00049, 0.00049] | [-0.00027, 0.00027] | [-0.00015, 0.00015] |
| $\lambda_\gamma$            |         |              |          |
| 100                         | [-0.00195, 0.00522] | [-0.00126, 0.00453] | [-0.00079, 0.00406] |
| 1000                        | [-0.00079, 0.00406] | [-0.00048, 0.00375] | [-0.00028, 0.00035] |
| 3000                        | [-0.00049, 0.00376] | [-0.00029, 0.00356] | [-0.00017, 0.00034] |
| 4000                        | [-0.00043, 0.00370] | [-0.00025, 0.00352] | [-0.00014, 0.00032] |
| 5000                        | [-0.00039, 0.00366] | [-0.00023, 0.00350] | [-0.00013, 0.00030] |

$\Delta\kappa_\gamma = [-0.00658, 0.00649]$, 95% C.L.

$\lambda_\gamma = [-0.01038, 0.31869]$, 95% C.L. \hfill (24)

$\Delta\kappa_\gamma = [-0.00102, 0.00102]$, 95% C.L.

$\lambda_\gamma = [-0.00119, 0.03616]$, 95% C.L. \hfill (25)

$\Delta\kappa_\gamma = [-0.00048, 0.00049]$, 95% C.L.

$\lambda_\gamma = [-0.00048, 0.00782]$, 95% C.L. \hfill (26)
TABLE IX: The expected 95% confidence level for the anomalous couplings $\Delta \kappa_\gamma$ and $\lambda_\gamma$, through the process $e^+e^- \rightarrow e^+\gamma^*\gamma^-e^- \rightarrow e^+W^-W^+e^-$ for $\sqrt{s} = 0.380$ TeV. The leptonic, semi-leptonic and hadronic channels of the $W^+W^-$ in the final state are considered. The confidence level for each parameter are calculated while fixing the another parameter to zero.

| $\sqrt{s} = 0.380$ TeV, 95% C.L. | Channel |
|---|---|
| $\mathcal{L}$ (fb$^{-1}$) | Leptonic | Semi-leptonic | Hadronic |
| $\Delta \kappa_\gamma$ |
| 100 | [-0.07089, 0.06104] | [-0.03840, 0.03532] | [-0.02117, 0.02020] |
| 300 | [-0.03949, 0.03624] | [-0.02175, 0.02073] | [-0.01209, 0.01177] |
| 500 | [-0.03027, 0.02832] | [-0.01675, 0.01614] | [-0.00934, 0.00915] |
| 700 | [-0.02544, 0.02405] | [-0.01412, 0.01368] | [-0.00788, 0.00774] |
| 1000 | [-0.02118, 0.02021] | [-0.01178, 0.01148] | [-0.00658, 0.00649] |
| $\lambda_\gamma$ |
| 100 | [-0.08393, 0.38546] | [-0.05161, 0.35633] | [-0.03084, 0.33743] |
| 300 | [-0.05282, 0.35742] | [-0.03160, 0.33813] | [-0.01848, 0.32613] |
| 500 | [-0.04219, 0.34778] | [-0.02497, 0.33207] | [-0.01449, 0.32247] |
| 700 | [-0.03629, 0.34240] | [-0.02134, 0.32875] | [-0.01233, 0.32048] |
| 1000 | [-0.03085, 0.33745] | [-0.01803, 0.32572] | [-0.01038, 0.31869] |

for the hadronic channel with $\sqrt{s} = 0.380, 1.5, 3$ TeV and $\mathcal{L} = 1000, 2500, 5000$ fb$^{-1}$, respectively.

To conclude these subsections, it is worth mentioning that already after the initial energy stage, in many cases (leptonic, semi-leptonic channels and hadronic, as well as for different luminosities) the CLIC precision is significantly better than for the results shown in Table I, and improves further with higher energy running.

IV. SUMMARY OF THE ACHIEVABLE PRECISION ON THE ANOMALOUS COUPLINGS $\Delta \kappa_\gamma$ AND $\lambda_\gamma$

To complement our study on the anomalous couplings $\Delta \kappa_\gamma$ and $\lambda_\gamma$ through the processes $\gamma\gamma \rightarrow W^+W^-$, $e^+\gamma \rightarrow e^+\gamma^*\gamma^-e^- \rightarrow e^+W^-W^+e^-$ and $e^+e^- \rightarrow e^+\gamma^*\gamma^-e^- \rightarrow e^+W^-W^+e^-$, for the
TABLE X: The expected 95% confidence level for the anomalous couplings \( \Delta \kappa_{\gamma} \) and \( \lambda_{\gamma} \), through the process \( e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^- \) for \( \sqrt{s} = 1.5 \text{ TeV} \). The leptonic, semi-leptonic and hadronic channels of the \( W^+W^- \) in the final state are considered. The confidence level for each parameter are calculated while fixing the another parameter to zero.

| \( \sqrt{s} = 1.5 \text{ TeV} \), 95% C.L. | \( \mathcal{L} \) (fb\(^{-1}\)) | Leptonic | Semi-leptonic | Hadronic |
|---|---|---|---|---|
| \( \Delta \kappa_{\gamma} \) | 100 | [-0.01668, 0.01578] | [-0.00926, 0.00898] | [-0.00516, 0.00508] |
| | 500 | [-0.00734, 0.00716] | [-0.00410, 0.00405] | [-0.00230, 0.00228] |
| | 1000 | [-0.00517, 0.00508] | [-0.00286, 0.00287] | [-0.00162, 0.00162] |
| | 1500 | [-0.00421, 0.00416] | [-0.00236, 0.00234] | [-0.00132, 0.00132] |
| | 2500 | [-0.00326, 0.00322] | [-0.00182, 0.00182] | [-0.00102, 0.00102] |
| \( \lambda_{\gamma} \) | 100 | [-0.01395, 0.04782] | [-0.00878, 0.04316] | [-0.00516, 0.00508] |
| | 500 | [-0.00725, 0.04175] | [-0.00437, 0.03911] | [-0.00230, 0.00228] |
| | 1000 | [-0.00536, 0.04000] | [-0.00319, 0.03801] | [-0.00162, 0.00162] |
| | 1500 | [-0.00448, 0.03921] | [-0.00264, 0.03750] | [-0.00132, 0.00132] |
| | 2500 | [-0.00355, 0.03835] | [-0.00207, 0.03698] | [-0.00102, 0.00102] |

It is worth mentioning that, the sources of systematic uncertainty in our measurements can be due to uncertainties in the integrated luminosity \( \mathcal{L} \), in factors that corrects for experimental acceptance and efficiencies, different background sources, particle identification and misstagging. To reduce the systematic uncertainties in our study, we used the systematic uncertainties \( \delta_{\text{sys}} = 0\%, 3\%, 5\% \) as a benchmark in our computation.

Comparison of precisions at the CLIC to the anomalous couplings \( \lambda_{\gamma}, \Delta \kappa_{\gamma} \) for center-of-mass energy \( \sqrt{s} = 0.380 \text{ TeV} \) and luminosities \( \mathcal{L} = 100, 500, 1000 \text{ fb}^{-1} \) are shown in Figs. 10 and 11. The figures covers the three processes \( \gamma\gamma \rightarrow W^+W^-, e^+\gamma \rightarrow e^+\gamma^*\gamma^* \rightarrow e^+W^-W^+ \) and \( e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^- \). Figs. 10 and 11 show that the initial stage of CLIC is already very complementary to the experimental limits at 95% C.L. on the aTGC \( \Delta \kappa_{\gamma} \) and \( \lambda_{\gamma} \) from the present and future colliders, as shown in Table I.
TABLE XI: The expected 95% confidence level for the anomalous couplings $\Delta \kappa_\gamma$ and $\lambda_\gamma$, through the process $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$ for $\sqrt{s} = 3$ TeV. The leptonic, semi-leptonic and hadronic channels of the $W^+W^-$ in the final state are considered. The confidence level for each parameter are calculated while fixing the another parameter to zero.

| $\sqrt{s} = 3$ TeV, 95% C.L. | Channel |
|---------------------------|---------|
| $\Delta \kappa_\gamma$   |         |
| $\mathcal{L} (\text{fb}^{-1})$ | Leptonic | Semi-leptonic | Hadronic |
| 100 | [-0.01122, 0.01061] | [-0.00622, 0.00603] | [-0.00347, 0.00341] |
| 1000 | [-0.00347, 0.00342] | [-0.00194, 0.00193] | [-0.00109, 0.00109] |
| 3000 | [-0.00199, 0.00198] | [-0.00112, 0.00112] | [-0.00062, 0.00063] |
| 4000 | [-0.00173, 0.00172] | [-0.00097, 0.00097] | [-0.00054, 0.00054] |
| 5000 | [-0.00154, 0.00154] | [-0.00086, 0.00086] | [-0.00048, 0.00049] |
| $\lambda_\gamma$         |         |
| $\mathcal{L} (\text{fb}^{-1})$ | Leptonic | Semi-leptonic | Hadronic |
| 100 | [-0.00621, 0.01341] | [-0.00414, 0.01140] | [-0.00267, 0.00997] |
| 1000 | [-0.00268, 0.00997] | [-0.00167, 0.00899] | [-0.00101, 0.00835] |
| 3000 | [-0.00171, 0.00903] | [-0.00104, 0.00837] | [-0.00061, 0.00795] |
| 4000 | [-0.00151, 0.00884] | [-0.00091, 0.00825] | [-0.00053, 0.00788] |
| 5000 | [-0.00137, 0.00870] | [-0.00082, 0.00816] | [-0.00048, 0.00782] |

The high energy stages, which are unique to the CLIC among all proposed $e^+e^-$ colliders, are found to be crucial for the precision measurements. This is illustrated in Figs. 12-15, for the comparison of precisions at the CLIC to the anomalous coupling $\lambda_\gamma$ for center-of-mass energies $\sqrt{s} = 1.5, 3$ TeV and luminosities $\mathcal{L} = 100, 1000, 2500, 3000, 5000 \text{fb}^{-1}$. The figures cover the three processes $\gamma\gamma \rightarrow W^+W^-$, $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^-W^+$ and $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$. 

Figs. 16 and 17, show the comparison of precisions at the CLIC for the anomalous couplings $\lambda_\gamma$ and $\Delta \kappa_\gamma$ for $\sqrt{s} = 3$ TeV and $\mathcal{L} = 5000 \text{fb}^{-1}$, under three systematic uncertainty scenarios, $\delta_{sys} = 0, 3, 5\%$. The figures cover the three processes $\gamma\gamma \rightarrow W^+W^-$, $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^-W^+$ and $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$, with pure leptonic decays. Black markers correspond to precision of $\lambda_\gamma$ ($\Delta \kappa_\gamma$) in $\gamma\gamma$ collisions, green markers correspond to results from $\gamma\gamma^*$ collisions, and blue markers give the results for the case of $\gamma^*\gamma^*$ collisions.
V. CONCLUSIONS

CLIC has the sensitivity to a large set of the anomalous couplings, in particular for the aTGC $W^+W^-\gamma$. In addition to the favorable experimental conditions of CLIC such as clean environments, low background, high-energy and high-luminosity, as well as operate in $\gamma\gamma$, $\gamma\gamma^*$ and $\gamma^*\gamma^*$ collision modes. All these features of the CLIC allow to explore the limits on the anomalous couplings $\Delta\kappa_\gamma$ and $\lambda_\gamma$ through the processes $\gamma\gamma \rightarrow W^+W^-$, $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^-W^+$ and $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$ for the three CLIC energy stages.

The CLIC capability to perform multiple competitive probings of the anomalous couplings $\Delta\kappa_\gamma$ and $\lambda_\gamma$ allows robust conclusions to be drawn (see Sections III and IV). In this regard, our results are summarized through a set of Tables III-XI and Figs. 10-17. From these tables and figures, the indicative CLIC reach for new physics, is given for $\Delta\kappa_\gamma$ and $\lambda_\gamma$ for the full CLIC physics program covering the three center-of-mass energy stages. The best limits are at 95% C.L.: $\Delta\kappa_\gamma = [-7, 7] \times 10^{-5}$, $\lambda_\gamma = [-0.4, 10.2] \times 10^{-4}$, through the signal $\gamma\gamma \rightarrow W^+W^-; \Delta\kappa_\gamma = [-1.5, 1.5] \times 10^{-4}$, $\lambda_\gamma = [-0.13, 3.40] \times 10^{-3}$ for the process $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^-W^+$, and $\Delta\kappa_\gamma = [-4.8, 4.9] \times 10^{-4}$, $\lambda_\gamma = [-0.48, 7.82] \times 10^{-3}$ for the mode $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$, respectively. All these results are for the hadronic channels of the $W^\pm$ bosons.

In conclusion, our limits on the anomalous couplings $\Delta\kappa_\gamma$ and $\lambda_\gamma$ indicate that the CLIC for its three energy stages can measure these couplings to a level of precision that exceeds that of the ATLAS, CMS, CDF, D0, ALEPH, DELPHI, L3 and OPAL Collaborations by more than $O(10^{-3} - 10^{-2})$ order of magnitude. In this context, the innovative project such as the CLIC with BSM physics searches is highly desirable, and guaranteed outcome of precision measurements.

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FIG. 1: A schematic diagram for the process $e^+e^- \rightarrow e^+\gamma^*\gamma^- e^- \rightarrow e^+W^-W^-e^-\gamma\gamma^*$ via the subprocess $\gamma^*\gamma^* \rightarrow W^+W^-$.  

FIG. 2: A schematic diagram for the process $e^+\gamma \rightarrow e^+\gamma^*\gamma^- \rightarrow e^+W^+W^-\gamma\gamma^*$ via the subprocess $\gamma\gamma^* \rightarrow W^+W^-$.  

FIG. 3: Feynman diagrams contributing to the process $\gamma\gamma \rightarrow W^+W^-$ and the subprocesses $\gamma^*\gamma^* \rightarrow W^+W^-$ and $\gamma^*\gamma^* \rightarrow W^+W^-$.  

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FIG. 4: The total cross-sections of the process $\gamma\gamma \rightarrow W^+W^-$ as a function of $\lambda_\gamma$ for center-of-mass energies of $\sqrt{s} = 0.38, 1.5, 3$ TeV at the CLIC.

FIG. 5: The total cross-sections of the process $\gamma\gamma \rightarrow W^+W^-$ as a function of $\Delta\kappa_\gamma$ for center-of-mass energies of $\sqrt{s} = 0.38, 1.5, 3$ TeV at the CLIC.
FIG. 6: The same as in Fig. 4, but for the process $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^-W^+$. 

FIG. 7: The same as in Fig. 5, but for the process $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^-W^+$. 
FIG. 8: The same as in Fig. 4, but for the process $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$. 

FIG. 9: The same as in Fig. 5, but for the process $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$. 
FIG. 10: Comparison of precisions at the CLIC to the anomalous coupling $\lambda_\gamma$ for center-of-mass energy $\sqrt{s} = 0.380$ TeV and luminosities $\mathcal{L} = 100, 500, 1000$ fb$^{-1}$. The figure covers the three processes $\gamma\gamma \rightarrow W^+W^-$, $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^-W^+$ and $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$. We include the CMS bound.

FIG. 11: The same as in Fig. 10, but for $\Delta\kappa_\gamma$. 

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FIG. 12: Comparison of precisions at the CLIC to the anomalous coupling $\lambda_\gamma$ for center-of-mass energy $\sqrt{s} = 1.5$ TeV and luminosities $\mathcal{L} = 100,1000,2500$ fb$^{-1}$. The figure covers the three processes $\gamma \gamma \rightarrow W^+W^-$, $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^-W^+$ and $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$. We include the CMS bound.

FIG. 13: The same as in Fig. 12, but for $\Delta\kappa_\gamma$. 
FIG. 14: Comparison of precisions at the CLIC to the anomalous coupling $\lambda_\gamma$ for center-of-mass energy $\sqrt{s} = 3$ TeV and luminosities $\mathcal{L} = 100, 3000, 5000$ fb$^{-1}$. The figure covers the three processes $\gamma\gamma \rightarrow W^+W^-$, $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^-W^+$ and $e^-e^- \rightarrow e^-\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$. We include the CMS bound.

FIG. 15: The same as in Fig. 14, but for $\Delta \kappa_\gamma$. 

FIG. 16: Comparison of precisions at the CLIC to the anomalous coupling $\lambda_{\gamma}$ for center-of-mass energy $\sqrt{s} = 3$ TeV and luminosity $\mathcal{L} = 5000$ fb$^{-1}$ with systematic uncertainties $\delta_{\text{sys}} = 0, 3, 5\%$. The figure covers the three processes $\gamma\gamma \rightarrow W^+W^-$, $e^+\gamma \rightarrow e^+\gamma^*\gamma \rightarrow e^+W^+W^-$ and $e^+e^- \rightarrow e^+\gamma^*\gamma^*e^- \rightarrow e^+W^-W^+e^-$, with pure leptonic decays. We include the CMS bound.

FIG. 17: The same as in Fig. 16, but for $\Delta\kappa$. 