New Limits on $R$–Parity Breakings in Supersymmetric Standard Models

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Abstract

New limits on couplings $\lambda^i_{jk''}$, which break both the baryon number and the $R$–parity, are derived by using a new mechanism that contributes to the neutron-anti-neutron oscillation. The constraints due to proton decay and its potential phenomenology are also reexamined.
The minimal supersymmetric standard model (MSSM) [1] has been widely considered as a leading candidate for new physics beyond Standard Model. However, unlike the Standard Model, additional symmetry, called $R$-parity defined as $(-1)^{3B+L+F}$, has to be imposed on the minimal supersymmetric extensions of Standard Model (MSSM) in order to avoid renormalizable interactions which violate the lepton and baryon numbers. It is in fact one of the main theoretical weaknesses of these models because the conservation of $R$-parity is an ad hoc imposition without fundamental theoretical basis. Therefore, it is interesting to ask how small these $R$-parity breaking couplings have to be by investigating the phenomenological constraints on them if they are indeed added to the MSSM [2].

The most general renormalizable $R$-violating superpotential using only the MSSM superfields is

\[ W = \lambda_{ij}^k L_i L_j \tilde{E}_k + \lambda_{ijk} L_i Q_j \tilde{D}_k + \lambda_{ijk}'' \tilde{U}_i \tilde{D}_j \tilde{D}_k. \]  

(1)

Here, $i, j, k$ are generation indices and we have rotated away a term of the form $\mu_{ij} L_i H_j$. The couplings $\lambda_{ij}^k$ must be antisymmetric in flavor, $\lambda_{ij}^k = -\lambda_{ji}^k$. Similarly, $\lambda_{ijk}'' = -\lambda_{ikj}''$. There are 36 lepton number non-conserving couplings (9 of the $\lambda$ type and 27 of the $\lambda'$ type) and 9 baryon number non-conserving couplings (all of the $\lambda''$ type) in Eq.(1).

To avoid rapid proton decay, it is usually assumed in the literature that $\lambda$, $\lambda'$ type couplings do not coexist with $\lambda''$ type couplings. We make the same assumption here and consider constraints on $\lambda''$ couplings only. The constraints on $\lambda$ and $\lambda'$ couplings [3] have been discussed quite extensively in the literature. We discuss a new set of constraints on $\lambda''$ due to their contributions to the neutron-anti-neutron oscillation (NANO) through a new mechanism. In addition, we wish to emphasize that couplings of $\lambda''$ type cannot give rise to the proton decay only when the proton is the lightest particle with $(-)^L = +$ and $(-)^F = -$. If there are lighter fermions with $(-)^L = +$, such as lightest neutralino or gravitino, the proton can in principle decay into them. In that case strong constraint on $\lambda''$ can be derived and, if they indeed give the leading contributions to the proton decay, it will greatly affect the leading proton decay mode.

There are many existing constraints on the 9 different $\lambda''$ couplings, $\lambda_{bs}''$, $\lambda_{bd}''$, $\lambda_{sd}''$, $\lambda_{bs}'''$, $\lambda_{bd}'''$, $\lambda_{sd}'''$, $\lambda_{bs}''''$, $\lambda_{bd}''''$, $\lambda_{sd}''''$ and $\lambda_{us}''''$. First of all, one can show [3, 4] that the requirement of perturbative unification typically places a bound of order one on many of the couplings. A potential constraint on $|\lambda_{sd}''''|$, due to neutron-anti-neutron oscillations was discussed in
ref. [3]. However it was later realized [3, 7] that the constraint is not as strong as it was originally derived due to additional source of suppression factors. Stronger constraints [3, 4, 7] on \( |\lambda_{sd}''| \) can be derived from the non-observation of double nucleon decay into two kaons (such as \( ^{16}O \rightarrow ^{14}C \ K^+ K^+ \)),

\[
|\lambda_{sd}''| < 10^{-6} \quad \text{(for } M_{\tilde{q}} \simeq 300 \text{ GeV)}.
\]

(2)

In addition, Goity and Sher [3] was able to find a one–loop mechanism which gives rise to a strong bound on \( |\lambda_{bd}''| \) by the non-observation of neutron-anti-neutron oscillations. For squark masses of 300 GeV, their bound is roughly

\[
|\lambda_{bd}''| < 5 \times 10^{-3}.
\]

(3)

Both numerical values have large uncertainty due to the nuclear matrix elements. In these references, some bounds on products of couplings were obtained by considering K-K mixing. Recently, bounds from the \( \bar{b}b \) induced vertex correction to the decay of the Z into two charged leptons have been obtained [8]; though potentially interesting, with present data they are not significantly better than the bound from perturbative unification. In Ref.[9], Carlson, Roy and Sher obtained some new bounds on the \( \lambda'' \) couplings from the rare two-body nonleptonic decays of \( B \) and \( D \) mesons. From the recent experimental upper bound [10] of \( 5 \times 10^{-5} \) on the branching ratio of \( B^+ \rightarrow \bar{K}^0 K^+ \), they obtained

\[
|\lambda_{bs}^q \lambda_{sd}^q| < 5 \times 10^{-3} \tilde{r}_q^2, \quad \text{for } q = t, c, u.
\]

(4)

where \( \tilde{r}_q = m_{\tilde{q}}/m_W \). For the decay \( B^+ \rightarrow \bar{K}^0 \pi^+ \) or \( B^- \rightarrow K^0 \pi^- \), using the experimental upper bound [10] of \( 5 \times 10^{-5} \) on B.R.\( (B^+ \rightarrow \bar{K}^0 \pi^+) \), they obtained

\[
|\lambda_{bd}^q \lambda_{sd}^q| < 4.1 \times 10^{-3} \tilde{r}_q^2, \quad \text{for } q = t, c, u.
\]

(5)

Barbieri and Masiero [7] also obtained bounds on \( \lambda'' \) from their one–loop contributions to the \( K_L-K_S \) mass difference. These bounds can be summarized as [3]

\[
|\lambda_{bs}^t \lambda_{bd}^t| < \min \left( 6 \times 10^{-4} \tilde{r}_t, 3 \times 10^{-4} \tilde{r}_t^2 \right),
\]

\[
|\lambda_{bs}^c \lambda_{bd}^c| < \min \left( 6 \times 10^{-4} \tilde{r}_c, 2 \times 10^{-4} \tilde{r}_c^2 \right),
\]

assuming that the top quark is much lighter than scalar top, and that all squark masses are degenerate. Bounds from \( D-\bar{D} \) mixing also have been considered [3] and give

\[
|\lambda_{bs}^u \lambda_{bd}^u| < 3.1 \times 10^{-3} \tilde{r}_s .
\]

(7)
Let us first consider the constraints on $\lambda''$ imposed by the non-observation of NANO. First of all, it is clear that in order to violate baryon number by two units one needs to use $\lambda''$ twice. Goity and Sher [5] discovered a one–loop diagram as in Fig. 1 that can contribute to NANO. The resulting effective operator is [5]

$$L^{(1)}_{\text{eff}} = T_1 \epsilon_{\alpha \gamma} \epsilon_{\alpha' \beta' \gamma'} \bar{u}^c_R \bar{d}^c_R \bar{d}^c_L \gamma^5 \bar{u}^c_R \bar{d}^c_R \bar{d}^c_R .$$  \hspace{1cm} (8)

The diagram is calculated at zero external momenta and yields

$$T_1 = -\frac{g^4}{32\pi^2} \lambda^u_d \lambda^u_{d'} U_{ud}^* U_{ud'}^* \epsilon_{\alpha \gamma} \epsilon_{\alpha' \beta' \gamma'} \frac{\mu^2_{d_k} \mu^2_{d_j}}{M^4_{d_k} M^4_{d_j}} J(M^2_{\tilde{w}}, M^2_{\tilde{W}}, m^2_{u_i}, M^2_{\tilde{u}_n}) .$$  \hspace{1cm} (9)

Here we assume the Kobayashi-Maskawa matrix of the left-handed squark is the same as that of the quarks. Dummies $i, j, k$ and $n$ are generation indices which are summed in (8). Note that $j$ and $k$ cannot be $d$. We follow the convention and Feynman rule in Ref.[11]. The momentum integral $J$ is

$$J(a_1, a_2, a_3, a_4) = \int_0^\infty \frac{x^2 dx}{\prod_{k=1}^4 (x + a_k)} = \sum_{i=1}^4 \frac{a_i^2 \ln(a_i)}{\prod_{k \neq i} (a_i - a_k)} .$$  \hspace{1cm} (10)

The mass–squared term $\mu^2_{\tilde{q}}$ which mixes $\tilde{q}_L$ and $\tilde{q}_R$ is given by

$$\mu^2_{\tilde{q}} = A m_q .$$  \hspace{1cm} (11)

Coefficient $A$ is a soft supersymmetry breaking parameter[3]. Consistent with most of the MSSM in the literature, we assume that all left–right squark mixing parameters are flavor diagonal.

Phenomenologically, the neutron oscillation time is given by $\tau = 1/\Gamma$, where the transition probability (per unit time) $\Gamma = |T \psi(0)|^2$. The amplitude $T$ due to the Feynman diagram in Fig. 1 is given by $T_1$ in (8). The wave function squared $\psi(0)^2$, which is simply related to the matrix element of the operator in (7), has been estimated by Pasupathy[12] to be $\psi(0)^2 = 3 \times 10^{-4}$ GeV$^6$. Nevertheless one should be aware that other evaluations[13] differ by more than an order of magnitude. Those differences among various evaluations characterize the degree of our ignorance about the matrix element; however, $\lambda^u_d''$ will vary only as the square root of $\psi(0)^2$. From the experimental limit on the neutron oscillation time[14], $\tau > 1.2 \times 10^8$ s, the bound on $\lambda^u_d''$ can be obtained.

The results depend on the the squark masses as well as Kobayashi-Maskawa angles. We shall assumed, as is Ref.[4], that the charm and up squark masses are degenerate.
Since $T_1$ has two powers of the masses of $m_{d_k}$ and $m_{d_j}$, Goity and Sher assumed that the $b$ squark dominates. They obtained a strong bound on $|\lambda^{u''}_{d' d''}|$ of roughly $10^{-3}$ for $M_{\tilde{d}} = M_{\tilde{e}} = 200$ GeV and $M_{\tilde{t}} \sim 220$ GeV, for the scenario $A = M_{\tilde{w}} = 200$ GeV. Note that if all three up–type squarks have the same mass, the transition amplitude in (8) vanishes because of the GIM cancelation via the internal up–type squark line by suppression factors of the form $\Delta M^2_{\tilde{q}}/M^2_{\tilde{q}}$ where $\Delta M^2_{\tilde{q}}$ is a typical up–squark mass difference.

This set of diagrams turns out to be just one of the four sets of one–loop diagrams that can contribute to NANO. The other three diagrams are given in Fig. 2, 3, 4. It is not too hard to see that the contributions from diagrams in Fig. 3 and Fig. 4 are proportional to the external quark momenta and therefore their contributions are suppressed by additional factor of $m_N/M_W$ where $m_N$ is the neutron mass. Since the remaining factors are roughly of the same order of magnitude, we shall ignore contributions from Fig. 3 and Fig. 4 even though they can be just as easily estimated.

The resulting effective operator from the contribution of Fig. 2 is

$$L^{(2)}_{\text{mix}} = T_2 \varepsilon_{\alpha \beta \gamma} \varepsilon_{\alpha' \beta' \gamma'} \bar{u}^{c \beta} L_{\alpha} d_{R_{\beta}} \bar{d}_{L_{\gamma}} u_{L_{\alpha}} d_{R'_{\gamma}} ,$$

where $T_2$ is estimated to be

$$T_2 = -\frac{g^4}{16\pi^2} \lambda^{u''}_{d' d''} \lambda^{u''}_{d' d''} U_{i d} U_{j k} U_{i d}^* U_{j k}^* m_{d_i} m_{d_j} \mu^2_{d_k} \mu^2_{u_n} I(M^2_{\tilde{w}}, M^2_{\tilde{W}}, m^2_{u_i}, m^2_{d_j}; M^2_{\tilde{u}_n}, M^2_{\tilde{d}_k}) .$$

Here the function $I(a_1, a_2, a_3, a_4; a_5, a_6)$ of the momentum integral is defined to be

$$\int_0^\infty \frac{x dx}{(x + a_3)^2(x + a_6)^2 \Pi_{k=1}^6(x + a_k)} = \frac{\partial^2}{\partial a_5 \partial a_6} \sum_{i=1}^6 \frac{a_i \ln(a_i)}{\Pi_{k \neq i}(a_k - a_i)} .$$

The two diagrams Fig. 1 and 2 can give rise to quite different constraints on $\lambda''$. First of all, the two diagrams involve quite different operators and therefore their matrix elements may be quite different also. For numerical illustration, we shall take the two matrix elements to be the same. Secondly, the two contributions, $T_1$ and $T_2$, involve quite different dependence on $\lambda''$ parameters and various masses. Unlike in the case of Fig. 1, the GIM cancelation in case of degenerate squark masses does not occur in Fig. 2 because the cancelation is already broken by the generation dependence in the couplings of $\lambda''$. Using the known quark mixing angles, we found numerically that the channels for $i=n=t$
and \( k, j = s \) or \( b \) dominate if all squarks have the same mass. Barring from accidental cancelation due to different contributions for \( k, j = s \) or \( b \), we obtain the constraints for the scenario \( M_{\tilde{q}} = A = M_{\tilde{w}} \),

\[
|\lambda_{sdl}^t|^2 < \left( \frac{200 \text{ MeV}}{m_s} \right)^2 4.5 \times 10^{-6} \quad \text{for} \quad M_{\tilde{q}} = 100 \text{ GeV},
\]

\[
|\lambda_{sdl}^t|^2 < \left( \frac{200 \text{ MeV}}{m_s} \right)^2 2.4 \times 10^{-4} \quad \text{for} \quad M_{\tilde{q}} = 200 \text{ GeV},
\]

\[
|\lambda_{bdl}^t|^2 < 7 \times 10^{-6} \quad \text{for} \quad M_{\tilde{q}} = 100 \text{ GeV},
\]

\[
|\lambda_{bdl}^t|^2 < 4 \times 10^{-4} \quad \text{for} \quad M_{\tilde{q}} = 200 \text{ GeV}. \quad (15)
\]

Next we discuss the issue of the proton decay when the \( R \)-parity breaking terms such as \( \lambda'' \) and a light neutralino coexist. This possibility was mentioned only briefly in the literature. As emphasized earlier, if the proton is not the lightest fermion with zero lepton number, then in general the \( \lambda'' \) coupling will induce the proton to decay into such a lightest supersymmetric particle (LSP). For example, if the LSP is a photino and \( m_{\tilde{\gamma}} \ll m_p - m_K \), the leading proton decay mode can be \( p^+ \to K^+ \tilde{\gamma} \) due to the tree-level diagram in Fig. 5. Previous search on proton decay mode \( p \to K\nu \) \([14]\) can be translated into the experimental limit on this mode and places a very stringent constraint on the coupling

\[
\lambda_{ds}^u < 10^{-15}, \quad \text{if} \quad m_{\tilde{\gamma}} \ll m_p - m_K. \quad (16)
\]

For a slightly heavier photino, which is still lighter than the proton \( m_p > m_{\tilde{\gamma}} \gtrsim m_p - m_K \), the proton can still decay through \( p^+ \to \pi^+ \tilde{\gamma} \) or \( p^+ \to e^+ \nu \tilde{\gamma} \) with a weaker bound on \( \lambda_{ds}^{uu} \) because additional vertices of the weak interaction are needed in the process. Also in this case, one can consider a tree-level process that converts two nucleons in a nuclei into a photino in \( NN \to \Lambda \tilde{\gamma} \). If the LSP is Higgsino or zino, the limit will be only slightly weakened.

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Figure Captions

1. A one–loop diagram for the amplitude $T_1$ in the process $n–\bar{n}$ oscillation. The $\lambda''$ couplings appear in the circles.

2. A one–loop diagram for the amplitude $T_2$ in the process $n–\bar{n}$ oscillation.

3. A one–loop diagram for the amplitude $T_3$ in the process $n–\bar{n}$ oscillation. This amplitude is suppressed by $m_N/M_W$.

4. A one–loop diagram for the amplitude $T_4$ in the process $n–\bar{n}$ oscillation. This amplitude is suppressed by $m_N/M_W$.

5. A tree–level diagram for the proton decay $p^+ \rightarrow K^+\tilde{\gamma}$ for a very light photino.
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5

\begin{tikzpicture}
  \node (d) at (0,0) {$d$};
  \node (u) at (1,0) {$u$};
  \node (s) at (2,0) {$\bar{s}$};
  \node (gamma) at (3,0) {$\tilde{\gamma}$};
  \node (s_k) at (4,0) {$\bar{s}_{(K^+)}$};

  \draw[->] (d) -- (s);
  \draw[->] (u) -- (s);
  \draw[->] (s) -- (gamma);
  \draw[->] (s) -- (s_k);

  \node (p) at (-1,1) {$(p^+)$};
  \node (up) at (1,1) {$u$};

  \draw[->] (p) -- (d);
  \draw[->] (p) -- (u);
  \draw[->] (u) -- (up);
  \draw[->] (up) -- (u);
\end{tikzpicture}