Quantum Hall Effect in Trilayer Graphene

J Brahma$^1$ and S Sahoo$^2$
Department of Physics, National Institute of Technology Durgapur, Durgapur – 713209, West Bengal, India.

$^1$E-mail: brahmajitu@gmail.com
$^2$E-mail:sukadevsahoo@yahoo.com

Abstract: Graphene is a two-dimensional sheet of carbon atoms with properties that are superior to other materials. Due to these superior properties, a lot of research is being done with this material. One of those properties is the Quantum Hall Effect (QHE). Monolayer graphene shows very interesting behaviour in the presence of a high magnetic field (up to 30T) and at very low temperature (~0.2K). Its unconventional Landau Level (LL) spectrum of massless Dirac fermions leads to a new type of quantum Hall effect, known as half-integer quantum Hall effect or anomalous QHE. We expect unique results can also be seen in trilayer graphene (TLG). In this work, we study the integer quantum Hall effect in trilayer graphene theoretically.

1. Introduction
In recent years, graphene [1,2] has attracted many researchers due to its unique electronic properties. The discovery of quantum Hall effect in graphene [3,4] has opened up a large research field. In case of two-dimensional system electrons obey the Schrödinger equation and the Hall conductivity is quantized at integer multiples of $\frac{e^2}{h}$. However, in case of graphene it is found that the electrons behave as massless Dirac fermions and the Hall conductivity is quantized at integer multiple of $\frac{e^2}{h}$ but with a shift of 1/2 which comes from the Berry phase $\Phi_B = \pi$ due to the cyclotron motion of the massless Dirac particles [3-6]. This unconventional quantum Hall effect is termed as half-integer quantum Hall effect. Generally, the electronic structure of multilayer graphene depends on the number of layers. Due to the unusual electronic structure, trilayer graphene [7-13] has attracted the scientific community recently. The quantum Hall effect is also found in trilayer graphene. In this paper, we study how the Berry phase affects the Hall conductivity and calculate the Hall conductivity for trilayer graphene.

2. Tight binding model of trilayer graphene
A trilayer graphene is composed of three single layer graphene, each sublattice in a top layer is located directly above the same one in the bottom layer. The low energy Hamiltonian for a trilayer graphene around the $K$ point can be written as [7]

$$H_p = \begin{pmatrix} 0 & \nu \pi^+ & t_{\perp} & 0 & 0 & 0 \\ \nu \pi & 0 & 0 & t_{\perp} & 0 & 0 \\ t_{\perp} & 0 & 0 & \nu \pi^+ & t_{\perp} & 0 \\ 0 & t_{\perp} & \nu \pi & 0 & 0 & t_{\perp} \\ 0 & 0 & t_{\perp} & 0 & \nu \pi^+ & 0 \\ 0 & 0 & 0 & t_{\perp} & \nu \pi & 0 \end{pmatrix}, \quad (1)$$

where $t_{\perp}$ is the nearest interlayer neighbour $\pi$-orbital hopping amplitude. Then we can write the difference equations as

$$\varepsilon a_n = t_{\perp}(a_{n-1} + a_{n+1}) + \nu \pi^+ b_n, \quad (2)$$

$$\varepsilon b_n = t_{\perp}(b_{n-1} + b_{n+1}) + \nu \pi a_n. \quad (3)$$
where \((a_1, b_1, \ldots, a_N, b_N)\) are eigenvectors, \(\epsilon\) is the eigenvalue and the boundary condition \(a_0 = a_{N+1} = b_0 = b_{N+1} = 0\).

Let \(c_n = a_n + b_n e^{-i\phi}\) and \(d_n = a_n - b_n e^{-i\phi}\) where \(\phi = \tan^{-1}(p_y/p_x)\), then

\[
\begin{align*}
(\epsilon - u|p|)c_n &= t_\perp (c_{n-1} + c_{n+1}), \\
(\epsilon + u|p|)d_n &= t_\perp (d_{n-1} + d_{n+1})
\end{align*}
\]

(4)

(5)

with the same boundary condition \(c_0 = c_N + 1 = d_0 = d_{N+1} = 0\). The energy spectrum can be written as

\[
\varepsilon_{r,p}^\pm = \pm u|p| + 2 t_\perp \cos \left(\frac{rn}{N+1}\right), (6)
\]

where \(r = 1, 2, \ldots, N\).

3. Berry phase intralayer graphene

Berry phase is a geometrical phase in which the value of the phase depends upon the space and trajectory of the system and not on the size and shape of the electron orbit. Due to graphene's unusual property, it shows a unique energy spectrum. In the spectrum we have observed that there is no gap between the valence and the conduction band at the \(K\) points in the Brillouin zone. This kind of spectrum is due to a nonzero value of the Berry phase. This nonzero phase occurs due to the trajectory of electrons in Brillouin zone where the system is degenerate [14].

In quantum Hall effect of graphene [3-6], the Hall conductivity \(\sigma_{xy} = \pm (ge^2/h)n\) shows plateau at certain values of \(n\). Here \(g = 4\) is due to spin and valley degeneracy and \(n\) is Landau level (LL) index. For a nonzero Berry phase there is shift of the Landau levels which leads to the shift of 1/2 in the Hall conductivity of graphene

\[
\sigma_{xy} = \pm \left(\frac{4e^2}{h}\right) \left(n + \frac{1}{2}\right), (7)
\]

If an electron orbits a closed contour in the momentum space, a Berry phase shift \(\Phi_B = j\pi\) is gained [15-17]. For trilayer graphene, we have \(j = 3\) and the Berry phase is \(\Phi_B = 3\pi\). Thus the expression for Hall conductivity of trilayer graphene [8, 18] can be written as

\[
\sigma_{xy} = \pm \left(\frac{4e^2}{h}\right) \left(n + \frac{3}{2}\right), (8)
\]

where \(n = 0, 1, 2, 3, \ldots\) Here the \(\pm\) value corresponds to electrons and holes respectively.

4. Results and discussion

The unique band structure of graphene leads to a nonzero Berry phase \(\Phi_B = j\pi\). For monolayer we have \(j = 1, \Phi_B = \pi\), for bilayer \(j = 2, \Phi_B = 2\pi\) and for trilayer \(j = 3, \Phi_B = 3\pi\). The Hall conductivity of graphene depends on the number of layers. We have calculated the Hall conductivity of trilayer graphene for different filling factors. We have found that the quantum Hall conductivity of trilayer graphene is quantized at filling factors \(\nu = \pm6, \pm10, \pm14, \pm18, \ldots\) which is similar to monolayer graphene but the plateaus at \(\nu = \pm2\) is missing. The Hall conductivity of trilayer graphene is represented in Table 1 and its variation with filling factor is shown in Figure 1. We hope these
results will open the new routes for studying both integer quantum Hall effect and fractional quantum Hall effect in trilayer graphene in the near future.

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**Table 1. Calculation of quantized Hall conductivity of trilayer graphene**

| n   | $\nu = \pm 4 \left(n + \frac{3}{2}\right)$ | $\sigma_{xy} = \nu \left(\frac{e^2}{h}\right) \Omega^{-1}$ |
|-----|------------------------------------------|---------------------------------------------------|
| 0   | 6                                        | 0.0002323                                         |
| 1   | 10                                       | 0.0003873                                         |
| 2   | 14                                       | 0.0005422                                         |
| 3   | 18                                       | 0.0006971                                         |
| 4   | 22                                       | 0.0008521                                         |
| 5   | 26                                       | 0.0010069                                         |
| 6   | 30                                       | 0.0011619                                         |

**Figure 1.** Variation of quantized Hall conductivity of trilayer graphene with different filling factors.
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