Binary Black Hole Merger: Mass-Separation Relation and Intermediate Mass Black Holes

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Abstract

Intermediate Mass Black Holes (IMBHs) are an elusive category of black holes in the mass range of 100 to 100000 Solar Masses. Binary IMBHs might form due to mergers of Globular Clusters, Pair Instability Supernovae, and in Young Massive Star Clusters. In this Research Note, merger timescale, constraints on the separation based on the timescale, and other parameters of Binary IMBHs are calculated analytically and are discussed. The calculations were conducted using Newtonian and Einsteinian dynamics. The timescale of a Binary IMBH system to reach maximum gravitational wave amplitude is also calculated and discussed. We also present the relation between the combined Mass of a Binary Black Hole (BBH) System and the Separation between two BHs required for a BBH system to merge within a given timescale $t_c$, solely due to Gravitational Radiation is a function of the total mass of the system. In this article, $t_c$ is set equal to Hubble time $t_H$. Now, the relation obtained is essentially the relation between separation of a BBH system (collide within $t_H$) and its Mass. The calculations were conducted for all three categories of Black Holes: Stellar, Intermediate, and Supermassive. Time ahead, the relation might be used for determining whether a BBH merger would be observational. The relation is also solved for Intermediate Mass Black Holes (IMBHs), and $t_c$ and separation for collision within $t_H$ was calculated.

1 Introduction

Intermediate Mass Black Holes (IMBHs) are a class of black holes in the mass range of $10^2 M_\odot – 10^5 M_\odot$. Very few candidates of IMBHs are confirmed at present. Some observational evidences of IMBHs are low-luminosity Active Galactic Nuclei and Ultra Luminous X-Ray Sources.

In this Research Note, the parameters of Binary IMBHs are calculated and discussed. Binary IMBHs can form in Globular Clusters (¹), Pair Instability Supernovae (²), in Young Massive Star Clusters (³), and by other astrophysical processes. Throughout the calculations, it is assumed that the black holes are orbiting each other in circular orbits; they have equal mass.

2 The Merger Parameters

Firstly, we calculate constraints on the separation of the Binary based on Hubble Time ($t_H$). The calculation were conducted by equating the following equation to $t_H$: 
\[
t_{\text{coals}} = \frac{5}{256} \frac{c^5}{G^3} \left( \frac{r^4}{(M_1 M_2)(M_1 + M_2)} \right)
\]

where \(M_1 = M_2\) are the masses of the IMBHs, \(c\) is the velocity of light, and \(G\) is the Universal Gravitational Constant. The solutions obtained by equating \(t_{\text{coals}}\) to \(t_H\) were in the range \(1.48 \times 10^{-5}\) for \(100M_\odot\) IMBHs to \(0.002\) parsecs for \(10^5M_\odot\) IMBHs.

These solutions can be applied to galaxy mergers and other astrophysical processes in order to predict whether an IMBH merger will be observed. For calculating the separation during maximum Gravitational Wave (GW) amplitude \(R_{\text{max}}\) is given by the equation (\[4\]) below:

\[
R_{\text{max}} = \left( \frac{G M_T}{\omega_{\text{Kep}}^2} \right)^{\frac{1}{3}}
\]

where \(M_T\) is the total mass of the system, \(\omega_{\text{Kep}}\) is the orbital angular frequency. By solving \([2]\) over the mass range of IMBHs, \(R_{\text{max}}\) is in the range \(10^5\) to \(10^6\) kilometers.

The total energy (potential and kinetic combined) of a binary IMBH can be calculated using the equation below:

\[
E = -\frac{G M_T \mu}{2r}
\]

And the results obtained are approximately in the range of \(2.99 \times 10^{20}\) Yotta Joules to \(1.7 \times 10^{24}\) Yotta Joules.

Now, we calculate a very important timescale (\(t_{\text{dec}}\)): The time taken for the separation of IMBHs in the range \(1.48 \times 10^{-5}\) parsecs — \(0.002\) parsecs (the initial separations) to decrease down to \(R_{\text{max}}\). The timescale was calculated based on equation \([1]\), with \(r\) in \([1]\) modified to \(\Delta R\): \(\Delta R = R_{\text{max}} - R_H\) where \(R_H\) is the separation needed for the IMBHs to merge within \(t_H\). The results obtained are interesting: \(12.5\) Gigayears for \(100M_\odot\) IMBHs and \(4.12\) Gigayears for \(10^5M_\odot\). This timescale is very useful can be used to predict when we might be able to observe an IMBH merger.

### 3 Black Hole Mass – Separation Relation

The relation between \(M\) and \(r\), was calculated using Python, and the [Python code](#) is uploaded to Github. The Python program plots the BBH System \(M - r\) relation. It was assumed that both the component BHs of the system have equal mass. The relation was computed for Stellar Mass Black Holes (StMBHs), Intermediate Mass Black Holes(IMBHs), and Supermassive Black Holes (SMBHs).

Calculating the relation while setting the timescale limit equal to the age of the universe (Hubble Time \(t_H\)), we can obtain a relation which gives the separation two binary black holes (assuming their mass to be 1 need to be in order to merge within \(t_H\), which happens to be \(0.46\) parsecs. This result can explain the low number of observations of

The relation obtained can be applied to Galaxy Mergers. Binary SMBH Systems are predicted by the hierarchical model of galaxy formation. If the masses of the SMBHs of the merged galaxy is determined, the relation can be used to find out the separation. These relations can be used to determine whether a BBH System will merge within a given timescale. Based on the plot, one can determine whether a pair of BHs in a BBH System will collide or not.
4 Conclusions and Discussions

In this Research Note, merger parameters of Binary IMBH systems were calculated analytically. For a pair of IMBHs to merge within $t_H$, their separations need to be in the range $1.48 \times 10^{-5}$ for $100 M_\odot$ IMBHs to $0.002$ parsecs for $10^5 M_\odot$ IMBHs. The separation at which GWs reach maximum amplitude for a pair of IMBHs is in the range $10^5$ to $10^6$ kilometers. The total energy of Binary IMBHs are approximately in the range of $2.99 \times 10^{20}$ Yotta Joules to $1.7 \times 10^{24}$ Yotta Joules. Roughly $1\%$ – $2\%$ of this energy is converted into GWs. The time taken for a pair of IMBHs to reach the point of maximum GW amplitude is found to be $12.5$ Gigayears for $100 M_\odot$ IMBHs and $4.12$ Gigayears for $10^5 M_\odot$ IMBHs and $4.12$ Gigayears for $10^5 M_\odot$. These parameters can be used to predict timescales of Binary IMBH systems, energy released, and with various other demographics of these Binaries. We have also presented the relation between BBH system mass and separation and plotted it on using Python. The relation obtained can explain the low number of observations of some BBH mergers. Binary IMBH System calculations such as separation for collision within $t_H$ and $t_c$ were carried out and presented.

References

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