An analysis of the process of refractive thinking of eighth-grade students of State Junior High School in solving problems of linear equation system of two variables

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Abstract. Refractive thinking is a thinking process performed by a person to make a solution of decision from several alternative solutions through a process of reflection and critical thinking. This study aimed to describe the process of refractive thinking of eighth-grade students in solving the problems of linear equation system of two variables. This study was qualitative descriptive research. The collected data was analyzed through three stages, including data reduction, data display, and verification of conclusions. The results showed that in the refractive thinking of the students from the two subjects in solving SPLDV problems, the thinking characteristics of the subjects were in accordance with the flow of the process of refractive thinking, which was being able to decide the best solution from several alternative solutions when solving SPLDV problems by using a procedure for processing SPLDV by changing the story problems to a mathematical model, followed by an elimination procedure so that it can be done quickly and precisely. The subjects had gone through the process of identifying problems, strategic, and evaluation stages in refractive thinking.

Keywords: linear equation system of two variables, mathematical problems, refractive thinking.

1. Introduction
The cycle of human life will always be faced with a problem that continues to come and go. Problems that are always present in life require someone to find an appropriate solution to solve them as they come their way. In the process of finding a solution, there will always be different alternative solutions. Therefore, one must choose the right and effective options of a solution so that the problems faced can be resolved properly. Deciding a solution from several alternative solutions in the thinking process is called refractive thinking. Refraction is a thought process to determine a decision solution to a problem through reflection thinking followed by critical thinking [1]. Refraction is the process of changing knowledge by remembering one's experiences and knowledge, thus giving rise to new thoughts [2]. The ability to express opinions in mathematics can be measured by providing Open-Ended problems that need more than one answer [3]. Meanwhile, the essence of refraction is determining the best decision of solution from several existing solutions in solving the problem. The refraction process is essential since it can assist students to produce a decision of solution to solve a problem [4]. To describe the process of refractive thinking resulting from reflective and critical thinking [5] a light metaphor is used. Therefore, the components that are passed by refractive thinking are reflection and critical thinking.
thinking is the beginning of refractive thinking. Thus, when students decide on a solution to solve a problem, they must possess knowledge regarding the solution they will choose. Students’ experiences in learning can be translated into the concept of reflection [6]. Reflective thinking is the process of making information and logical decisions regarding problems, then assessing the decision [7]. Reflective thinking is a process consisting of directed and precise activities where individuals analyze, evaluate, motivate, obtain deep meaning, and use proper learning strategies [8]. Meanwhile, [9-12] reflective thinking skills is the skill to think by paying attention to known assumptions or elements and their implications based on reasons or evidence to support conclusions. Following reflective thinking, the next process to think more actively is called critical thinking. Reflective thinking is an intellectual process that actively and skillfully conceptualizes, applies, analyzes, synthesizes, and evaluates information gathered or generated from observation, experience, reflection, reasoning, or communication to guide beliefs and actions [13-16]. The most essential thing in critical thinking is making decisions [17]. Besides, critical thinking can make a comparison and draw a conclusion [18], so that one can decide convincing things to do [19]. Therefore, it makes individuals think, question problems, produce solutions to problems, and make smart decisions when faced with challenges [20].

The third step in the cycle of knowledge development is refractive thinking. Studies related to refractive thinking were carried out by [21] and [22]. The study of [21] explained the process of refractive thinking of junior high school students on flat shape material. Meanwhile, the study carried out by [22] explained the process of refractive thinking of high school students in solving mathematical problems about a data through 3 stages, including identifying problems, strategic, dan evaluation. However, in research by [22], the subject selected was only one senior high school student and used the data presentation problem. Meanwhile, this current study discussed the description of the process of refractive thinking of eighth-grade junior high school students. This study aimed to obtain an overview of the thinking process of 2 students of eighth-grade junior high school in solving questions about linear equation system of two variables. The results of this study are expected to provide information about the process of refractive thinking of eighth-grade junior high school students in solving questions about linear equation system of two variables. Therefore, this information can be considered by teachers in designing learning activities in accordance with the characteristics of refractive thinking of junior high school students.

2. Methods
This study was qualitative descriptive research. Essentially, qualitative research explores and understands the meaning of individuals related to social problems [23]. The subjects in the study were 2 students of class VIII SMPN 1 Maron. The sampling technique used was random sampling. Description of the process of refractive thinking of the students originated from the students’ thinking process while working on a given task-based test followed by semi-structured interviews. The problems assigned to the subjects were mathematical problems related to the SPLDV material. After obtaining written data, interviews were conducted to clarify and explore information that might not be in the written data about students' refractive thinking processes in solving math problems. In this study, the technique used was the methods or techniques of triangulation, which was checking data from the same source with different techniques [24]. There are two strategies, including checking the confidence level of research findings using data collection techniques and checking the confidence level of data sources using the same method [25]. After the consistency of the collected data, the data were analyzed through three stages, namely data reduction, data presentation and verification of conclusions. The stages of this research are presented in Figure 1.
3. Results and Discussion
This study uses stages and indicators of thought refraction as a result of thinking construction [22]. There are 3 stages in the refractive thinking process, namely identified of problems, strategic and evaluation. The data analysis shows the thinking characteristics of subject 1 in solving the problems of the equation system of two variables given. It can be seen that at the beginning of completing the task, the subject began to reflect on her thoughts by describing the existing problems and identifying the problems given. After the subject I explained the problems she faced, the subject I can connect the information she had with the problems she faced, indicating that the subject could think critically. Furthermore, from the relations obtained, the subject’s refractive thinking was to have a view regarding the idea of completion that would be used to answer. This process shows that refractive thinking is built beginning with a reflection and followed with the process of critical thinking. This process is in the early stages of the process refractive thinking, which is identifying the problems [22]. This can be proven by the results of the completion of the subject I as shown in Figure 2.

Furthermore, the subject I utilized her learning experience and knowledge to try to provide solutions of the answers to the core of the problems by finding the respective height of the tubular and beam-shaped parts of the towers. The subject thought that the first tower had 3 beams and 3 tubes, while the
second tower had 2 beams and 3 tubes so that the beam height was 2 meters. Hence, the subject I could find out the height of the tube. Therefore, in the end, the subject could find out the height of the shorter tower. In addition to this method, when conducting an in-depth interview, the subject I provided a different way, namely the concept of a two-variable linear equation system. This means that at the strategic stage, the subject I had performed refractive thinking because she was able to solve the problem in more than one way. This can be proven by the explanation of the method used by subject I in presenting the answers to the problems as shown in Figure 3.

Subsequently, after subject I had found the answer, the subject re-checked her answer by comparing the first with the second solution which used the concept of linear equations of two variables that turned out to have the same answer. Then, in a depth-interview, the student said that the first method was better compared to the second method even though the answer was the same because she reasoned that the first method was faster and did not take much time to find the height of the shorter tower. It shows that at the stage of re-examination and determination of the best solution with strong reason, the subject had performed refractive thinking. This process was included in the final stage of the process of refractive thinking, which was evaluation, as shown in Figure 4.

Based on the data of subject I, it can be seen that the flow of the thinking process is in accordance with the refractive thinking stated [22] which consists of 3 stages of refractive thinking, including identifying problems, strategic, and evaluation. Therefore, it can be concluded that subject I had thinking
characteristics as the process of refractive thinking in solving the problems she faced while doing the assigned task. Subject II initially completed the given task by beginning to reflect her thoughts by describing and identifying the problems briefly. Then, through the process of critical thinking, the subject could relate the information she had with the information on the given task. Subsequently, from the relationship obtained, the students’ refractive thinking was to have an idea of solving the problems given. This stage was in accordance with the stage of identifying the problem, which is the initial stage of refractive thinking [22]. After understanding the problems in the given assignment, subject II began to formulate a plan or strategy based on her learning experiences obtained in the past or her knowledge. In the process of thinking, in this first method, subject II could not complete the task correctly in providing a solution of her thought because she did not think to describe the height of each tower component consisting of beam shape and tube shape. The subject thought that the way to find the height of the shorter tower to reduce the height of the first tower to the second tower. Of course, this process of thinking was not right. However, in the in-depth interview, subject II was asked to complete it again in a different way. The result shows that the subject thought that this problem could be solved with the concept of linear equations of two variables through critical thinking after contemplating trying to collect the information contained in her previous thoughts. In this process, the subject reflected her thoughts by applying the information in her mind to the problems she was facing, then thinking critically by identifying, re-analyzing, and evaluating her strategy. And then, in the end, the subject could solve the problem well. This indicated that at the strategic stage, subject II had performed refractive thinking because she had been able to solve the problem in more than one way, even though the first method was not quite right. However, after rethinking and compiling the right strategy, subject II could find the correct answer. After subject II found two answers, she then rechecked the answers. When conducting examination activities, the subject realized and found errors in the first solution, then the subject checks the second solution. It turns out that after comparing the first and second solutions. Subject II believed that the method using the linear equation concept of two variables was more precise and correct. This shows that at the stage of re-examination and determination of the best solution with strong reason, the subject had performed refractive thinking. This process was included in the final stage of the process of refractive thinking, which was evaluation, as shown in Figure 5.

![Figure 5. Result Of Completion Of Subject II](image)

The data of subject II shows that the thinking process is considered to be in accordance with the flow of refractive thinking stated [22], consisting of 3 stages of refraction thinking, including identifying
problems, strategic, and evaluation. In this research the subject has provided several alternative solutions by stating that the best solution chosen by the subject is to use elimination when working on the given SPLDV questions. So, it can be concluded that subjects I and II in solving SPLDV problems with their thought processes use the refractive thought process by going through the stages of the refractive thinking process.

4. Conclusion
The results of this study indicated that the process of the students’ refractive thinking began with the stage of identifying the problem. Students could already reflect on their thinking by describing and identifying the problems and continuing on critical thinking, where they could connect the information they had with the problems they faced, resulting in process of refractive thinking by the students, which was to have a view regarding the solution idea that would be used to answer. Then, at the strategic stage, students reflected on their thoughts by applying the information in their minds with the problems they faced. They then continued to think critically by identifying, re-analyzing, and evaluating the strategy to produce refractive thinking, where the students could solve the problem in more than one way namely by way of elimination as in other SPLDV questions directly. Subsequently, in the final stage of evaluation, the students checked the answers by comparing the first solution with other solutions. After being analyzed in-depth, the students could determine the best solution to the problems. It shows that at the stage of re-examination and determining the best solution with strong reasons, the students had been performing refractive thinking. Based on the results of this study, teachers are expected to be able to convey problems that have the opportunity to have many solutions. Therefore, students can develop their thinking in determining the best solution from several alternative solutions while solving a problem.

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