On the construction of $\mathcal{N} = 4$ SYM effective action beyond leading low-energy approximation

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Abstract

A problem of the hidden $\mathcal{N} = 2$ supersymmetry deformation for next-to-leading terms in the effective action for $\mathcal{N} = 4$ SYM theory is discussed. Using formulation of the theory in $\mathcal{N} = 2$ harmonic superspace and exploring the on-shell hidden $\mathcal{N} = 2$ supersymmetry of $\mathcal{N} = 4$ SYM theory, we construct the appropriate hypermultiplet-depending contributions for $F^6$ term in the Schwinger-De Witt expansion of the effective action. The procedure involves deformed hidden $\mathcal{N} = 2$ supersymmetry and allows one to obtain self-consistently the correct $\mathcal{N} = 4$ supersymmetric functional containing $F^6$ among the component fields.

1 Introduction

Different aspects of low-energy string dynamics and of the AdS/CFT correspondence [1] can be studied in terms of the quantum field theory effective action. Due to the proposition put forth in (see discussed in greater detail [2, 3, 4]), the superconformal version of the Born-Infeld (BI) action is expected to be considered as the result of summing up leading and subleading terms in the quantum effective action of the $\mathcal{N} = 4$ SYM theory in the Coulomb branch. Some terms of the effective action expansion should be constrained by new non-renormalization theorems.

In the off-shell superfield formulation of the SYM and supergravity theories, the supersymmetry must be realized linearly on physical fields and on an infinite set of auxiliary fields so that the supersymmetry transformations are independent of the form of the action. But in the on-shell formalism, the supersymmetry transformation is realized non-linearly. When obtaining higher derivative contributions to the effective action preserving extended supersymmetry, we must obtain self-consistently the deformations of the classical supersymmetry transformation rules order by order and, simultaneously, construct the supersymmetry-invariant higher-order terms in the action: $(\delta_0 + \sum_n \delta_n)(S_0 + \sum_n S_n) = 0$. Here $\delta_0$ is classical supersymmetry transformation, $S_0$ is classical action and $\delta_n, S_n$ are

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the quantum deformations of transformation law and higher derivative corrections to the classical action respectively. It is hardly possible to compute in a closed form the full derivative dependence of the effective action (see for review [5]). Therefore, what we can do in this situation is, relying on particular results known, to list all the supersymmetry invariants as well as deformed transformation rules with a given number of derivatives. For the well-known leading potential $\propto F^4$ in the vector field strength sector (see for references and a recent progress [5]), the problem of constructing the full $\mathcal{N} = 4$ superinvariant has been solved in [6].

There exist many different approaches that are used for the construction of the supersymmetric higher-derivative string effective action [7, 8, 9]. In the instructive paper [10], an off-shell $\mathcal{N} = 3$ supersymmetric extension of the Abelian $D = 4$ BI action was constructed starting from the action of supersymmetric Maxwell theory in $\mathcal{N} = 3$ harmonic superspace [11]. The $\mathcal{N} = 3$ superfield strength contains combination of the auxiliary field and the gauge field strength. The nonlinearity in the ordinary gauge field strength arises in its full form as the result of elimination of these auxiliary fields with the aid of their nonlinear equations of motion. This is different from $\mathcal{N} = 1, 2$, where superextensions of the bosonic BI action are fulfilled in each order of expansion in powers of the Maxwell field strength.

The purpose of the present paper is to consider a possible self-consistent way to find hypermultiplet dependent complements and the correspondent deformed hidden supersymmetry transformations, which are needed for manifestly $\mathcal{N} = 4$ supersymmetric next-to-leading terms in the $\mathcal{N} = 4$ SYM theory effective action. The while such an approach is useful for the resolution of existence problem concerning higher-derivative invariants, obtaining fully explicit expressions is extremely cumbersome due to the enormous number of terms and the problem of dealing with partial integration.

2 On hidden $\mathcal{N} = 2$ invariance of the $F^6$ term in $\mathcal{N} = 4$ SYM theory

In order to construct subleading $\mathcal{N} = 2$ hidden invariant terms in the derivative expansion of the effective action, which depend on all the $\mathcal{N} = 4$ SYM multiplet fields, one can apply the Noether procedure with classical supersymmetry transformation modulo to boundary terms and free equations of motions [12], [6]

$$
\delta_0 W = \frac{1}{2} \varepsilon^{\alpha a} D_\alpha q_a^+, \quad \delta_0 \bar{W} = \frac{1}{2} \bar{\varepsilon}^{\dot{\alpha} \dot{a}} \bar{D}_{\dot{\alpha}} \bar{q}_\dot{a}^+,
\delta_0 q_a^\pm = \frac{1}{4} (\varepsilon^{\alpha a} D_\alpha \pm D^\pm \bar{W} + \bar{\varepsilon}_\dot{a} \dot{\alpha} \bar{D}_\dot{\alpha} \bar{\bar{W}}).$$

(1)

Let us try to find a possible hypermultiplet completion for the following two-loop term $\propto F^6$ found in [3] with the supergraph technique in the harmonic superspace:

$$
\Gamma_{(6|0)} = c_2 \int d^{12} z \left[ \frac{1}{W_2} \ln W D^4 \ln W + \frac{1}{\bar{W}^2} \ln \bar{W} \bar{D}^4 \ln \bar{W} \right] = c_2 \int d^{12} z \mathcal{L}_{(6|0)} + c.c.,
$$

(2)

where $c_2 = N^2 g_Y^2 \frac{1}{48 (4\pi)^4}$. This term consists of two different parts, marked by the fourth powers of distinct derivatives: $D^4$ or $\bar{D}^4$. This parts should be studied separately, because the variation rules (1) do not mix them. The variation of first part (2) induced by (1)
can be written in the form
\[
\delta_0 \mathcal{L}_{(6|0)} = -\frac{2q^{+\alpha}}{\mathcal{W}^3\mathcal{W}}(\varepsilon^a_{\alpha} \bar{D}_\alpha \mathcal{W} + \varepsilon^a_{\alpha} D_\alpha \mathcal{W}) D^4 \ln \mathcal{W} + \frac{q^{+\alpha} \varepsilon^a_{\alpha} D_\alpha \mathcal{W}}{\mathcal{W} \mathcal{W}^3} D^4 \ln \mathcal{W}.
\] (3)

The heart of all problems is the fact that the classical variation \((3)\) of \(\mathcal{L}_{(6|0)}\) generates terms non-symmetric under the replacement \(\varepsilon \leftrightarrow \bar{\varepsilon}\).

Further we consider classical transformations \(\delta_0\) defined by \((1)\) along with their deformations \(\delta_{(n|k)}\). The full deformed transformations are considered as an expansion in powers of \(D, \bar{D}\) as well as in powers of \(X = -\frac{2q^{+\alpha} q^+}{\mathcal{W} \mathcal{W}}\), i.e. \(\delta = \delta_0 + \delta_1 (D^4) + \delta_2 (D^8) + \ldots\). The subscript \(k\) in deformations indicates the power of \(X\), e.g. \(\delta_1 = \sum \delta_{(1|k)}\). Let us introduce the first complement to \((2)\) in the form
\[
\mathcal{L}_{(6|1)} = d_1[X \frac{1}{\mathcal{W}^2} D^4 \ln \mathcal{W} + X \frac{1}{\mathcal{W}^2} \bar{D}^4 \ln \bar{\mathcal{W}}].
\] (4)

Its variation \(\delta^{(q)} \mathcal{L}_{(6|1)}\) in \(q^{\pm}\) cancel the first term in \((3)\) if \(d_1 = -2\), but the other part of \((3)\) is not cancelled. Since the structure either of the two functionals \((2)\) is not symmetric in respect to \(\mathcal{W} \leftrightarrow \bar{\mathcal{W}}\), while the \(\delta^{(q)}\)-variation is symmetric, the discrepancy between variations like \((2)\) and \((3)\) will always appear in each step of the variational procedure. To resolve this discrepancy, we consider the one-loop \(\Gamma_{(4)} \propto F^4\) term along with its well-known hypercomplement \([6]\):
\[
\Gamma_{(4)} = c_1 \int d^{12}z [\ln \mathcal{W} \ln \bar{\mathcal{W}} + \frac{1}{2} X + \frac{1}{4} - 3 X^2 + \ldots],
\] (5)
where \(c_1 = N \frac{1}{(4\pi)^2}\). We know that this term will be renormalizable by neither the higher loop nor the instanton contributions. Suppose that the classical hidden supersymmetry is deformed as follows
\[
\delta_{(1|0)} = \frac{A}{2} 2^{+\alpha} D_\alpha q^+ + \frac{1}{2} \mathcal{W}^2 \bar{D}^4 \ln \bar{\mathcal{W}}, \quad \delta_{(1|0)} = \frac{A}{2} 2^{+\alpha} D_\alpha q^+ + \frac{1}{2} \mathcal{W}^2 \bar{D}^4 \ln \bar{\mathcal{W}}
\] (6)
\[
\delta_{(1|0)q^+} = \frac{1}{4} \left[B 2^{+\alpha} D_\alpha \mathcal{W} \frac{1}{2} \mathcal{W}^2 D^4 \ln \mathcal{W} + \bar{B} 2^{+\alpha} \bar{D}_\alpha \bar{\mathcal{W}} \frac{1}{2} \bar{\mathcal{W}}^2 \bar{D}^4 \ln \bar{\mathcal{W}}\right].
\] (7)

If the following conditions
\[
c_2 + c_1 \frac{(A - B)}{2} = 0, \quad c_2 + c_1 \frac{(A - B)}{2} = 0
\] (8)
for the coefficients introduced in \([6]-[7]\) are satisfied, then deformed variations of the first two terms in \(\Gamma_{(4)}\) can cancel the last term in \((3)\). The variation of the first complement \(\delta^{(VV)}_0 \mathcal{L}_{(6|1)}\) \([4]\) in \(\mathcal{W}\) under classical transformation rules \((1)\) is
\[
\delta^{(VV)}_0 \mathcal{L}_{(6|1)} = 4 \cdot \frac{5}{3} q^{+\alpha} q^{-\alpha}(\varepsilon^a_{\alpha} \bar{D}_\alpha \mathcal{W} + \varepsilon^a_{\alpha} D_\alpha \mathcal{W}) D^4 \ln \mathcal{W}
\] (9)
\[
+ \frac{4}{3} \cdot \frac{-2q^{+\alpha} q^{-\alpha} \varepsilon^a_{\alpha} D^- \mathcal{W} D^4 \ln \mathcal{W} + \frac{4}{3} \cdot \frac{-\varepsilon^a_{\alpha} \bar{D}^+ \varepsilon^a_{\alpha} D^- \mathcal{W}}{\mathcal{W}^2 \mathcal{W}^3} (q^+ D^4 q^-) \frac{1}{16} D^+ D^4 - D^- D^4 \ln \mathcal{W}.\]

Now introduce the second complement
\[
\mathcal{L}_{(6|2)} = d_2 \left[X^2 \cdot \frac{1}{\mathcal{W}^2} D^4 \ln \mathcal{W} + X^2 \cdot \frac{1}{\mathcal{W}^2} \bar{D}^4 \ln \bar{\mathcal{W}}\right].
\] (10)
Its variation in $q^\pm$ is exactly the first term in (8), and if we choose $d_2 = -\frac{5}{3}$, it will cancel the variations involving $\varepsilon$. At the same time, the rest of variation (9) is saved. In order to cancel its first part, one can consider variation of (5) under the following deformation of the transformations

$$\delta_{(1|1)} \mathcal{W} = \frac{A_1}{2} \cdot X \varepsilon^{\alpha a} D_\alpha^+ q_a^+ \cdot \frac{1}{\mathcal{W}^2} D^4 \ln \mathcal{W}, \quad \delta_{(1|1)} \mathcal{W} = \frac{A_1}{2} \cdot X \varepsilon^{\alpha a} D_\alpha^- q_a^- \cdot \frac{1}{\mathcal{W}^2} D^4 \ln \mathcal{W},$$

(11)

$$\delta_{(4|1)} q_a^- = \frac{B_1}{4} \cdot X \varepsilon^a \bar{D}_a^- \mathcal{W} \frac{1}{\mathcal{W}^2} D^4 \ln \mathcal{W} + \frac{B_1}{4} \cdot X \varepsilon^a \bar{D}_a^- \mathcal{W} \frac{1}{\mathcal{W}^2} D^4 \ln \mathcal{W}.$$  

(12)

Using these deformations, one can find variation of (5): the first term under $\delta_{(1|1)} \mathcal{W}$, the second term under $\delta_{(1|1)} \mathcal{W}$ as well as $\delta_{(1|1)}$, and the third term under $\delta_{(1|0)}$. The part of the variations, we are interested in, is

$$\delta \mathcal{L}_{(4)} = c_1 \left[ -\frac{2}{3} A_1 + B_1 + \frac{B - A}{3} \right] (q^+ q^-) q^{+ a} \varepsilon_a^\alpha D_\alpha^- \mathcal{W} \frac{D^4 \ln \mathcal{W}}{\mathcal{W}^2 \mathcal{W}^4}.$$  

(13)

The requirement of the cancellation between the second term in $\Delta \mathcal{L}_{(6|1)}$ and the corresponding term $\delta \mathcal{L}_{(4)}$ gives the following equation

$$4 \frac{2}{3} c_2 = c_1 \left( B - A \right) + B_1 - \frac{2}{3} A_1, \quad \text{or} \quad c_1 (B_1 - \frac{2}{3} A_1) = 2 c_2.$$

(14)

These relations define coefficients in the decomposition of the modified supersymmetry transformations in powers of $X$. There is arbitrariness in choosing $A, B$, and, therefore, some additional information is needed to get rid off this arbitrariness. For cancelling the second part of (2), we introduce a complement of the new type $\mathcal{L}'_{(6|0)} \propto D^4 \mathcal{W}$ defined by the classical transformations generated by $\varepsilon$, while all contradictions arising in the $\varepsilon$ sector variations can be eliminated by the hidden transformation modifications $\delta_{(1|n)} \propto X^n$ order by order. Thus, the the problem is split into two separate tasks. This is our main idea how to overcome the difficulty in constructing of the hidden $\mathcal{N} = 2$ invariants with derivatives of the vector strength of the $\mathcal{N} = 2$ multiplet.

Let’s consider a particular task of obtaining the leading term in the full $F^6$ effective action, which can be solved with the above considerations. For this purposes, it is sufficient to consider a generic term of the series of complements to (2) in the form

$$\Gamma_{(6|n)} = d_n \int d^{12} z \left( \frac{-2q^+ q^-}{\mathcal{W} \mathcal{W}} \right)^n \frac{1}{\mathcal{W}^2} D^4 \ln \mathcal{W} + c.c.$$   

(15)

The classical variation $\delta_{0} \mathcal{L}_{(6|n)}$, generated to the parameter $\varepsilon$, for terms $\propto D^4 \ln \mathcal{W}$ is

$$\left[ -d_n \cdot n \cdot \left( \frac{-2q^+ q^-}{\mathcal{W}^n \mathcal{W}^{n+2}} \right)^{n-1} + d_n \cdot \frac{n(n + 4)}{n + 2} \cdot \left( \frac{-2q^+ q^-}{\mathcal{W}^{n+1} \mathcal{W}^{n+3}} \right) \cdot (q^{+ a} \varepsilon^a \bar{D}_a^- \mathcal{W}) D^4 \ln \mathcal{W} \right].$$  

(16)

The requirement of cancellation variations of $\delta_{0} \mathcal{L}_{(6|n)}$ and $\delta_{0} \mathcal{L}_{(6|n+1)}$ is fulfilled if

$$d_n = d \frac{(n + 2)(n + 3)}{n}.$$  

(17)
Summing all complements \( L^q_{(6)} = \sum_{n=0}^{\infty} L_{(6|n)}(X) \), one obtains
\[
\Gamma^q_{(6)} = \frac{c_2}{6} \int d^{12}z \left[ \frac{X}{(1-X)^2} + \frac{5X}{1-X} - 6 \ln(1-X) \right] \frac{1}{W^2} D^4 \ln W. \quad (18)
\]

Of course, this result is not entirely full in the sense that it should be completed by contributions containing hypermultiplet derivatives. Generic terms with \( q^+ D^+ q^- \) derivatives can be found. We introduce a new type of complement
\[
L_1'(6) = p_n \frac{(-2q^+ q^-)^n}{W^{n+1} W^{n+3}} q^+ D^+_a q^- D^+ D^+ D^{-3} \ln W. \quad (19)
\]

Then the requirement of the cancellation of its variation \( \delta^q_0(\epsilon) \) with an appropriate term in variation \( (13) \) leads to an inhomogeneous recurrent relation. The relation has the solution
\[
p_n = -\frac{1}{6} \cdot \frac{(n+2)(n+3)}{n+1} - \frac{1}{12} \cdot \frac{(n+2)(n+3)}{n+7} H_n^{(2)}, \quad (20)
\]

where \( H_n^{(2)} = \sum_{k=1}^{n} \frac{1}{k^2} \) is the harmonic number \([n,2]\). For the coefficients of the next generic term of the series \( L''_1(6) = h_n X^m (q^+ D^+ q^-)^2 D^{-2} \ln W \), there must be the recurrent relation on \( p_n, d_n, h_n \).

We see that for \( \mathcal{N} = 4 \) supersymmetrization of next-to-leading terms, one has to consider transformations that mix different terms in the derivative expansion of the effective action! To make sure that the guessed transformations \((6,7)\) are not unreasonable, one can consider a variation of the well-known classical action \([12]\). The variation of the hypermultiplet action is proportional to the on-shell equation of motion \( D^+ q^+ = 0 \), but variation of the vector strength action in \([6] \) is
\[
\delta_{(1|0)} \Gamma_0 = \frac{1}{8} \int d^{12}z \left\{ \bar{A} \varepsilon^{\alpha a} \bar{D}_a q^+_a \frac{1}{W} \ln \bar{W} + A \varepsilon^{\alpha a} D_a q^+_a \frac{1}{W} \ln W \right\}. \quad (21)
\]

This expression has the same structure as \( \delta_0 L_{(4|0)} \). The fact that in order to cancel \((21)\) one should take into account classical variations of \( L_{(4|0)} \propto \ln W \ln \bar{W} \) serves as an additional cross-checking of the consistency of the proposed recipe.

It is also interesting to consider the first deformed variations of \( F^6 \) \((2) \). We obtain
\[
\delta_{(1|0)} L_{(6|0)} = c_2 \left[ \frac{A}{W^3 W^2} \varepsilon^{\alpha a} \bar{D}_a q^+_a D^4 \ln W \bar{D}^4 \ln \bar{W} - \frac{A}{W^5} \varepsilon^{\alpha a} D_a q^+_a \ln W (D^4 \ln W)^2 \right]. \quad (22)
\]

The first term in the brackets looks like classical variation of the one-loop \( F^8 \) structure \( L_{(8|0)} = \Psi^2 \Psi^2 = \frac{1}{W^2} \bar{D}^2 \ln W \frac{1}{W^2} D^4 \ln W \) but with coefficient \( c_2 \). This means that the one-loop coefficient \( \frac{1}{2(2\pi)^7} \) in front of \( F^8 \) structure \([4,13]\) of the effective action will be renormalized by two-loop contributions. The second term in \((22)\) is a term of new type. For its cancellation, one needs to add another structure
\[
\Gamma_{(8|0)} = c_8 \int d^{12}z \ln W \left( \frac{1}{W^2} D^4 \ln W \right)^2, \quad (23)
\]

with typical \( \propto \frac{1}{(2\pi)^7} \) two-loop coefficient \( c_8 = -c_2 \frac{A}{2} \). This allows us to conjecture that accurate two-loop calculations should give such \( F^8 \)-structure in the effective action.

Thus, the self-consistent obtaining of the appropriate hypermultiplet dependent contributions in the effective action and modification of the hidden supersymmetry transformation allows one to obtain information about renormalizable terms as well as non-renormalizable higher corrections terms.
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