Infrared Exponents and the Strong Coupling Limit in Lattice Landau Gauge

Graz, March 2008

Lorenz von Smekal
QCD – Perturbation Theory

M. Davier, A. Hocker and Z. Zhang, *Rev. Mod. Phys.* **78** (2006) 1043.
Running Coupling

- Determine $\alpha_S$ from lattice Landau-gauge ghost/gluon propagators:

$$\alpha_{S,\text{minMOM}}(q^2) = \frac{g^2(a)}{4\pi} \frac{Z_L(q^2, a^2)}{G_L^2(q^2, a^2)} + O(a^2)$$

A. Sternbeck, K. Maltman, L. von Smekal, A. G. Williams, E. M. Ilgenfritz and M. Müller-Preussker, PoS (LATTICE 2007) 256, arXiv:0710.2965 [hep-lat].
Running Coupling

- Determine $\alpha_S$ from lattice Landau-gauge ghost/gluon propagators:

\[
\Lambda \ll |p_\mu| \ll \frac{\pi}{a}
\]

\[
r_0 \Lambda_2^\text{MS} = 0.60(1)
\]

\[
r_0 = 0.467 \text{ fm}
\]

\[
\Lambda_2^\text{MS} = 255(5) \text{ MeV}
\]

A. Sternbeck, K. Maltman, L. von Smekal, A. G. Williams, E. M. Ilgenfritz and M. Müller-Preussker, PoS (LATTICE 2007) 256, arXiv:0710.2965 [hep-lat].
Running Coupling

- Determine $\alpha_S$ from lattice Landau-gauge ghost/gluon propagators:

\[ \alpha_S(M_Z) \approx 0.117(1) \]

A. Sternbeck, K. Maltman, L. von Smekal, A. G. Williams, E. M. Ilgenfritz and M. Müller-Preussker, PoS (LATTICE 2007) 256, arXiv:0710.2965 [hep-lat].
Contents

- Covariant Gauge Theory and Confinement (Introduction).

- Landau Gauge QCD Propagators (DSEs, lattice LG,...).

- Strong Coupling Limit in Lattice Landau Gauge.

- Modified Lattice Landau Gauge.

- Summary and Conclusions.
Test Charge

- Coulomb
- Confinement
- Higgs

mass gap

global gauge charges unbroken
Covariant Gauge Theory

(Noether) currents / global (colour) charges:

\[ J^a_\mu = \partial^\nu F^a_{\mu \nu} + s(D^a_{\mu} c^b), \quad (\partial J^a = 0) \]

\[ Q^a = G^a + N^a \]

a) mass gap: \( G^a \equiv 0 \)

b) global colour charges unbroken:

\[ Q^a = \int d^3x \, s(D^a_0 c^b), \text{ well-defined} \]

\[ \Rightarrow \text{gauge invariant, physical states} \quad \Leftrightarrow \text{colour singlets} \]

\[ \propto \frac{Z(k^2)}{k^2}, \quad Z(k^2) \rightarrow 0 \]

Kugo-Ojima criterion
The Ghosts of QCD

The “elementary quartet”:

\[
\begin{align*}
\bar{c} & \rightarrow \bar{\gamma} \quad \overset{S}{\longrightarrow} \quad B \rightarrow \beta \\
D_\mu c & \rightarrow \partial_\mu \gamma \quad \overset{S}{\longleftarrow} \quad A_\mu \rightarrow \partial_\mu \chi
\end{align*}
\]

massless, since

\[
\langle A_\mu(x) B(0) \rangle \quad \overset{F.T.}{\propto} \quad \frac{k_\mu}{k^2} \quad \leftarrow \quad \text{Slavnov-Taylor identity}
\]

\[
\langle D_\mu c(x) \bar{c}(0) \rangle \quad \overset{F.T.}{\propto} \quad \frac{k_\mu}{k^2} \quad \leftarrow \quad \text{Dyson-Schwinger equation}
\]

with ghost-antighost symmetry (Landau gauge):

\[
\langle D_\mu c(x) \bar{c}(0) \rangle = \langle c(0) D_\mu \bar{c}(x) \rangle \quad \overset{F.T.}{\propto} \quad k_\mu \frac{G(k^2)}{k^2} (1+u(k^2))
\]

\[
D_\mu \bar{c} \rightarrow (1+u) \partial_\mu \bar{\gamma}
\]

K.O.: \( u \equiv u(0) = -1 \)

(see above)

ghost propagator,

\[
G(k^2) = (1+u(k^2))^{-1} \quad \text{infrared singular!}
\]
Gluon Propagator

Lattice:
Leinweber et al., 1998

DSEs:
\[
Z(k^2) \frac{k^2}{k^2} \rightarrow 0 \frac{1}{k^2} \left( \frac{k^2}{\sigma} \right)^{2\kappa} \rightarrow 0, \quad 0.5 < \kappa < 1
\]

L.v.S. et al., 1997
Lerche & L.v.S., 2002
Fischer & Alkofer, 2002

A. Sternbeck, E. M. Ilgenfritz, M. Muller-Preussker, A. Schiller and I. L. Bogolubsky, PoS \textbf{LAT2006}, 076 (2006) [arXiv:hep-lat/0610053].
Running Coupling

\[ \alpha(k^2) = \frac{g^2}{4\pi} Z(k^2) G^2(k^2) \]

\[ \rightarrow \alpha_c, \quad k^2 \rightarrow 0 \]

L.v.S. et al., 1997

\[ \alpha_c = \frac{4\pi}{N_c I(\kappa)} = 4.46\ldots, \quad N_c = 2 \]

Lerche & L.v.S., 2002

\[ \alpha_c = 2.97\ldots, \quad N_c = 3 \]

J. C. R. Bloch, A. Cucchieri, K. Langfeld and T. Mendes, Nucl. Phys. B 687 (2004) 76.

16^3 \times 32

SU(2)
**Infrared Dominance of Ghosts**

Ghost Propagator

**Lattice:**
Suman & Schilling, 1996

**DSEs:**
\[
\frac{-G(k^2)}{k^2} \kappa \rightarrow 0 \propto -\frac{1}{k^2}
\]

LvS et al., 1997

\[0.5 < \kappa < 2\]

- **DSEs:** Lerche & L.v.S., PRD 65 (2002) 125006.
- **Stochastic Quantisation:** Zwanziger, PRD 65 (2002) 094039.
- **ERGEs:** Pawlowski, Litim, Nedelko, & L.v.S., PRL 93 (2004) 152002.

\[\kappa = 0.595..., \quad \alpha_c = \frac{4\pi}{N_c I(\kappa)} = 2.972..., \quad N_c = 3\]

\[x = k^2 a^2, \; \text{with} \; a^{-1} = 2 \text{ GeV}\]
Finite Volume DSE Solutions

need: $\frac{\pi}{L} \ll p \ll \Lambda_{\text{QCD}}$

Fischer, Maas, Pawlowski & L.v.S., Ann. Phys. 322 (2007) 2916.
Gluon Propagator

\[
D(p^2) \left[ \text{1/GeV}^2 \right]
\]

\begin{align*}
\text{p [GeV]} & \quad 0.0 & 0.4 & 0.8 & 1.2 & 1.6 & 2.0 & 2.4 \\
L = 4.6 \text{ fm} & \quad & \quad & \quad & \quad & \quad & \quad & \quad \\
L = 9.7 \text{ fm} & \quad & \quad & \quad & \quad & \quad & \quad & \quad \\
\text{Sternbeck et al. (2005)} & \quad & \quad & \quad & \quad & \quad & \quad & \quad \\
\text{Sternbeck et al. (2006)} & \quad & \quad & \quad & \quad & \quad & \quad & \quad \\
\text{Infinite volume} & \quad & \quad & \quad & \quad & \quad & \quad & \quad
\end{align*}
Sternbeck et al., Adelaide–Berlin–Moscow, unpublished (preliminary).
\textbf{SU(3) vs SU(2)}

A. Sternbeck, L. von Smekal, D. B. Leinweber and A. G. Williams, PoS (LATTICE 2007) 340, arXiv:0710.1982 [hep-lat].
SU(3) vs SU(2)

A. Sternbeck, L. von Smekal, D. B. Leinweber and A. G. Williams, PoS (LATTICE 2007) 340, arXiv:0710.1982 [hep-lat].
**SU(2) Propagators**

**volume dependence**

Fixed $\beta$, unrenormalised.

A. Sternbeck, L. von Smekal, D. B. Leinweber and A. G. Williams, PoS (LATTICE 2007) 340, arXiv:0710.1982 [hep-lat].
\( \beta = 0 \) — Gluon Propagator

\[ Z(a^2 p^2) \propto (a^2 p^2)^{2\kappa} \]

\( SU(2) \)

\( \kappa = 0.595 \)

gluon mass: \( \kappa = 0.5 \)
\( \beta = 0 \) — Gluon Propagator

\[
Z_{\text{fit}}(x) = c x + d x^{2\kappa}
\]

Fit parameters \((56^4)\):

\[
\begin{align*}
  c &= 0.51 \pm 0.01 \\
  d &= 0.22 \pm 0.01 \\
  \kappa &= 0.656 \pm 0.006
\end{align*}
\]
\[ Z_{\text{fit}}(x) = c x + d x^{2\kappa}, \quad x = a^2 p^2 \]

\[ \beta = 0 \text{ — Gluon Propagator} \]

\[ c = c_{\infty} + c_{\text{corr}} \frac{a}{L} \]

\[ d = d_{\infty} + d_{\text{corr}} \frac{a}{L} \]

\[ c_{\infty} = 0.57 \pm 0.01 \neq 0 \Leftrightarrow \text{transverse mass } m^2 \propto \frac{1}{a^2} \]
\( \beta = 0 \) — Ghost Propagator

Not a finite volume effect!

\[ G(a^2 p^2) \]

\[ SU(2), \quad \beta = 0 \]

\[ \kappa = 0.595 \]
\[ G_{\text{fit}}^{-1}(x) = c + d \ x^{\kappa} \]

\[ \kappa = 0.595 \]

\[ x = a^2 p^2 \]
\( \beta = 0 \) — Ghost Propagator

\[
G^{-1}_{\text{fit}}(x) = c + d \, x^\kappa
\]
\[ \beta = 0 \text{ --- Infrared Exponents} \]

\[ \kappa = \kappa_\infty + \kappa_{\text{corr}} \frac{a}{L} \]

\[ \kappa_\infty: \]
- gluon: \( 0.684 \pm 0.002 \)
- ghost: \( 0.678 \pm 0.003 \)
\( \beta = 0 \) — Infrared Exponents

- alternative fit models:

\[
\begin{align*}
Z_{\text{fit}}(x) &= c x + d x^{2\kappa}, \\
G_{\text{fit}}^{-1}(x) &= c + d x^{\kappa}, \\
\end{align*}
\]

\[
\begin{align*}
Z_{\text{fit}}(x) &= c x (1 + d x)^{2\kappa-1}, \\
G_{\text{fit}}^{-1}(x) &= c (1 + d x)^{\kappa}, \\
\end{align*}
\]

\[ x = a^{2} p^{2} \]

\[ \kappa_{\infty}: \]

- gluon: \( 0.684 \pm 0.002 \)
- ghost: \( 0.678 \pm 0.003 \)

\[ \kappa_{\infty}: \]

- gluon: \( 0.571 \pm 0.001 \)
- ghost: \( 0.569 \pm 0.006 \)

- in either case:

\[ \kappa_{\text{gluon}} = \kappa_{\text{ghost}} \]
Stereographic Projection

- Consider $S^N$
  
  with $\varphi$:
  
  \[(x_1, \ldots, x_{N+1}) \mapsto \frac{1}{1 + x_{N+1}} (x_1, \ldots, x_N)\]

- Example $SU(2)$:
  replace
  
  \[\frac{1}{2} \text{tr} \ U^g \rightarrow \ln \left(1 + \frac{1}{2} \text{tr} \ U^g\right)\]
  
  in sum over links of gauge-fixing potential,
  
  suppresses South pole!
  
  $\leadsto$ modified Landau gauge
Stereographic Projection

"A child of five would understand this."

"Send someone to fetch a child of five."

Julius Henry "Groucho" Marx
Modified Lattice Landau Gauge

- **Compact $U(1)$:**

  links $U = e^{i\phi}$, with g.t. $\phi^\theta = \phi + d\theta$

  standard Morse potential: $V[U^\theta] = \sum_{\text{links}} \cos \phi^\theta$

  Landau gauge: $0 = F_i(\phi^\theta) = \frac{\partial}{\partial\theta_i} V = \sum_{\mu} \left( \sin \phi^\theta_i, \mu - \sin \phi^\theta_{i-\mu}, \mu \right)$

  However,

  $$Z_{gf} = \int \prod_{\text{sites}} d[\theta, \bar{c}, c, b] \exp \left\{ - \sum_i s \left( \bar{c}_i (F_i(\phi^\theta) - i \frac{\xi}{2} b_i) \right) \right\} = 0$$

  $\chi(S^1 \times \#\text{sites}) = \chi(S^1)^\#\text{sites} = 0 ^\#\text{sites}$

L. von Smekal, D. Mehta, A. Sternbeck and A. G. Williams, PoS (LATTICE 2007) 382, arXiv:0710.2410 [hep-lat].
Modified Lattice Landau Gauge

- Compact $U(1)$:

  standard potential: $V[U^\theta] = \sum_{\text{links}} \cos \phi^\theta$

  LG: $0 = F_i(\phi^\theta) = \frac{\partial}{\partial \theta_i} V = \sum_{\mu} (\sin \phi^\theta_{i,\mu} - \sin \phi^\theta_{i-\tilde{\mu},\mu})$

- Use a.p. b.c.'s to remove global gauge zero.

- Use stereographic projection:

  Morse potential: $V[U^\theta] = \sum_{\text{links}} \ln (1 + \cos \phi^\theta)$

  and

  $F_i(\phi^\theta) = \sum_{\mu} (\tan(\phi^\theta_{i,\mu}/2) - \tan(\phi^\theta_{i-\tilde{\mu},\mu}/2))$

Explicitly worked out in 1-dim (eliminates all Gribov copies) and $\geq 2$-dim (all but minima, 1st Gribov region) compact $U(1)$. 

D. Mehta
**SU(2) Gluon Propagator – β = 0**

- **SU(2)**, **β = 0**

Graph showing the relation between $D(a^2 q^2)$ and $a^2 q^2$ for different lattice sizes $L/a$ and $lnU$.
SU(2) Gluon Propagator – $\beta = 0$

$D(a^2p^2) \propto (a^2p^2)^{2\kappa}$, $\kappa = 0.6$

$Z_{\text{fit}}(x) = c \, x + d \, x^{2\kappa}$, $x = a^2p^2$
SU(2) Ghost Propagator – $\beta = 0$

$G(a^2 q^2)$

SU(2), $\beta = 0$

std: $L/a = 24$

mod: $L/a = 24$

$32$

$56$
**SU(2) Coupling – β = 0**

\[
\alpha_L = \frac{g^2}{4\pi} Z_L(q^2) G_L^2(q^2)
\]

Lerche & L.v.S., 2002:

\[
\alpha_c = \frac{4\pi}{N_c I(\kappa)}
\]

\[
\alpha_c = \alpha_c^{\text{max}} = 4.46..., \quad N_c = 2,
\]

max for \( \kappa = 0.595\ldots \)

**SU(2), β = 0**
\[ \alpha_L = \frac{g^2}{4\pi} Z_L(q^2) G_L^2(q^2) \]

\[ \beta = 2.3, \quad L = 56 \]

\[ \beta = 2.5, \quad L = 32 \]
Conclusions

• **Strong Coupling Limit of Lattice Landau Gauge**

facilitate \( \pi/L \ll p \ll \Lambda_{QCD} \to \infty \)

observe infrared behaviour of functional methods at large momenta:

\[
D_{\text{gluon}} \sim (a^2p^2)^{2\kappa-1}, \quad D_{\text{ghost}} \sim (a^2p^2)^{-\kappa-1}, \quad \kappa \approx 0.6,
\]

in particular, \( \kappa_{\text{gluon}} = \kappa_{\text{ghost}} \), and \( \alpha_{S} \to \alpha_{c} \approx 4 \ (SU(2)) \).

However, this all happens for \( 1 \ll a^2p^2 \)

deviations at small momenta are *not* a finite-size effect!

• **Modified Lattice Landau Gauge**

solves Neuberger 0/0 problem of lattice BRST.

alternative gauge field definition.

will allow to perform (Landau) gauge-fixed MC.