On the Performance of RIS-Assisted Communications With Direct Link Over $\kappa$-$\mu$ Shadowed Fading

ASHRAF AL-RIMAWI¹ (Member, IEEE), AND ARAFAT AL-DWEIK²,³ (Senior Member, IEEE)

¹Department of Electrical and Computer Engineering, Birzeit University, Birzeit P627, Palestine
²Center for Cyber Physical Systems, Khalifa University, Abu Dhabi, UAE
³Department of Electrical and Computer Engineering, Western University, London, ON N6A 3K7, Canada

CORRESPONDING AUTHOR: A. AL-DWEIK (e-mail: dweik@fulbrightmail.org)

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ABSTRACT In reconfigurable intelligent surface (RIS)-assisted communications, the signal-to-noise ratio (SNR) is improved by optimizing the reflecting elements’ phases to make the reflected signals add coherently at the receiver. Nevertheless, the RIS cannot control the phase of the direct link if it exists. Although the RIS phase can still be controlled such that the direct and reflected signals add coherently, the SNR gain might be hindered. Therefore, this paper considers the performance analysis of such scenarios where a novel analytical framework is developed to evaluate the SNR, outage probability, and bit error rate (BER). To capture a broad range of fading conditions, the channels are modeled as independent but not identically distributed generalized $\kappa$-$\mu$ shadowed fading channels. The Laplace transform is used to derive an accurate approximation of the probability density function (PDF) and cumulative distribution function (CDF) of the instantaneous fading, which are used to derive the PDF and CDF of the instantaneous SNR. The paper also considers deriving the asymptotic PDF, CDF, moment generating function (MGF) of the SNR, as well as the outage probability and BER. The obtained results show that a strong direct link may limit the gain obtained using the RIS.

INDEX TERMS $\kappa$-$\mu$ shadowed fading, RIS, intelligent surfaces, diversity order, outage probability, MGF, BER, sixth generation (6G).

I. INTRODUCTION

THE DEMAND for mobile data access has been growing rapidly in the past few years, driven by a large number of new subscribers, high data rate applications, and emerging Internet of Things (IoT) applications, which require massive connectivity. Therefore, the limited capacity problem became prominent [1], [2]. According to Ericsson mobility report [3], mobile network data traffic grew 44% between 2020 and 2021, and reached 72 Exabyte per month generated by about 8 billion subscribers. As per Cisco report [4], 50% of all IoT networked devices in 2023 will be connected to cellular networks. Therefore, the wireless network is expected to support about 15 billion IoT devices in addition to 8 billion mobile users. Although the advancements achieved by fifth generation (5G) wireless networks are considered substantial as compared to the fourth generation (4G), 5G networks’ capacity improvement is still way below the 1000× capacity increase specified by the International Mobile Telecommunications-2020 (IMT-2020) vision.

To resolve the limited capacity problem, extensive research efforts are being dedicated to designing efficient solutions. For example, spectral capacity solutions aim at introducing more spectrum by utilizing new frequency bands, particularly high frequencies such as millimeter waves (mmW). However, mmW poses several challenges [5] such as the poor propagation characteristics of mmW as compared to the classical sub-6 GHz spectrum. Moreover, in highly-dense urban areas, the high-rise buildings cause shadowing...
and mostly block line-of-sight (LOS) connectivity, which makes it more challenging for high-frequency transmission. Consequently, RISs were introduced to control the wireless environment [6], [7], [8], [9], [10]. An RIS panel consists of two-dimensional meta-surfaces arrays in which each meta-surface element can independently change the phase-shift of the incident electromagnetic signal to create a preferable wireless environment [11]. RIS panels do not require high hardware complexity or cost overhead during their integration, while they can provide a substantial gain in terms of reliability and network capacity [12].

Due to the wide range of use cases and applications anticipated for 5G and 6G wireless networks, conventional channel models are unlikely to be sufficient to capture the propagation channels for such applications [13]. For example, future wireless networks are expected to operate at an extremely broad spectrum of carrier frequencies ranging from 0.5 to 100 GHz. Therefore, a highly dynamic channel model is required to model the buildings' penetration loss, atmospheric attenuation, and multipath reflections. Moreover, small-cell densification and the invasion of IoT have triggered the use of outdoor-indoor and indoor-outdoor channels, which cannot be captured by conventional channel models [14]. High bandwidth transmission also requires a flexible channel model to capture the propagation effects that the signal might go through. unmanned aerial vehicle (UAV) and flying networks is another emerging application that requires a nontraditional channel model [15].

In the literature, several composite channel models such as the $\kappa$-$\mu$/inverse and $\eta$-$\bar{\mu}$/inverse gamma fading are used to capture the statistical behavior of the combined small-scale and large-scale fading in various propagation environments [16]. However, such models assume that the shadowing affects the scattered and dominant waves equally, and thus, such models are referred to as multiplicative shadow fading models [17], [18]. However, in practice, the shadowing often affects only the dominant components. Therefore, such a model is referred to as LOS shadow fading model [19], [20].

### A. RELATED WORK

RIS has recently attracted extensive research efforts, particularly performance analysis. The performance analysis was obtained for various system and channel models using several performance metrics such as SNR, BER, symbol error rate (SER), outage probability and capacity. Based on the received signal arrival, the system model can be generally classified into two main categories, RIS without direct link [6], [11], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], which is more common, and with direct link [34], [35], [36], [37], [38]. Table 1 shows the classification of selected recent articles based on their consideration of the direct link, system model, channel model, and performance metric.

| Ref | Direct Link | Performance Metric | No. of Antennas | Approach for Analysis | Channel | Phase knowledge | Modulation Schemes |
|-----|-------------|--------------------|----------------|----------------------|---------|----------------|-------------------|
| [11] | OP, AEP and EC | SISO | Gamma distribution | Rayleigh | Perfect | M-QAM |
| [21] | OP | SISO | Exact and bounds | Fox's $H$ | Imperfect | - |
| [22] | OP | SISO | Bounds | Rayleigh | Imperfect | - |
| [23] | OP | SISO | Chebyshev bounds | Rayleigh | Perfect | - |
| [24] | OP | SISO | CLT | Nakagami-$\bar{\mu}$ | Perfect | MQAM |
| [25] | SE, BE and EC | SISO | CLT | Nakagami-$\bar{\mu}$ | Perfect | MQAM |
| [26] | OP | SISO | $K$-$\mu$ distribution | Rayleigh | Imperfect | - |
| [27] | OP, SE and BE | SISO | Gamma distribution | Nakagami-$\bar{\mu}$ | Perfect | - |
| [28] | OP, and AEP | SISO | $K$-distribution | Rayleigh/Rician | Perfect | BPSK/QAM |
| [29] | OP, AEP and EC | SISO | CLT | Rayleigh | Perfect | BPSK, and M-QAM |
| [30] | OP, AEP and EC | MIMO | $K$-$\mu$ distribution | Rayleigh | Perfect | DPSK |
| [31] | OP, and EC | SISO | Gamma distribution | Rayleigh | Perfect | - |
| [32] | OP and EC | SISO | Gamma distribution | Arbitrary | Imperfect | BPSK |
| [33] | OP and EC | SISO | Fox $H$ function | Arbitrary | Imperfect | - |
| [34] | EC | MIMO | CLT | Rician | Perfect | - |
| [35] | AEP | SISO | CLT | Rician | Perfect | MPSK/QAM |
| [36] | OP | SISO | CLT | Rayleigh | Imperfect | - |
| [37] | OP, and EC | SISO | leveragin Theorem | Rayleigh, and Rician | perfect | - |
| [38] | OP | SISO | Gamma Distribution | Nakagami-$\bar{\mu}$ | Perfect | - |
| [39] | SE | SISO | Gamma Distribution | Nakagami-$\bar{\mu}$ | Perfect | BPSK, and 4QAM |
| [40] | OP, and EC | SISO | CLT | Nakagami-$\bar{\mu}$ | perfect | - |
| [41] | OP, AEP and EC | SISO | Tight Approximation | Rician and $K$-distribution | Perfect | BPSK |
| [42] | OP | SISO | CLT | Nakagami-$\bar{\mu}$ | Perfect | - |
| [43] | EC | SISO | CLT | Rayleigh | Perfect | - |
1) RIS WITHOUT DIRECT LINK
In [6], [11], [21], [22], [23], [24], [25], [28], [29], [30], [31], [32], [33], the authors provide closed-form exact and approximations for several performance metrics through modeling the end-to-end RIS channel amplitude as Rayleigh [6], [30], Gamma [11], [26], [27], and Nakagami-$m$ [28] distributions. Moreover, the channel [24], [25], [29] is based on Central Limit Theorem (CLT) which is not accurate for a small number of reflecting elements. In [24], a unified analytical framework is developed based on the Fox-$H$ function, which is shown to be highly accurate even for a small number of reflecting elements, particularly for continuous phase shifts.

2) RIS WITH DIRECT LINK
The work on performance analysis of RIS with direct link that is considered in [35], [36], [37], [38], [39], [40], [41], [42], [43] is based on approximating the end-to-end SNR using various approximation methods such as the CLT. As shown in Table 1, the channel model in [35], [36], [37], [38], [42], [43] is considered as Nakagami-$m$, Rician and Rayleigh fading. Waqar [41] considers the Rician and $K$-fading models. It is worth mentioning that the aforementioned fading models do not accurately capture the composite fading and shadowing that is usually experienced in practical channel models.

**B. MOTIVATION AND CONTRIBUTION**
As can be noted from the surveyed literature and references listed therein, most of the published research on RIS neglects the direct link between the base stations (BS) and user equipment (UE) based on the assumption that the direct link signal strength is much smaller than the reflected one. However, this may not be the case in practice because the direct link can be significant in some scenarios as compared to the RIS route due to the intrinsic product of the path loss [33], [44]. To this end, $\kappa$-$\mu$ shadowed distribution is widely used in literature for modeling composite fading/shadowing. It has a clear physical interpretation, good analytical properties and unifies popular fading models such as One-Side Gaussian, Rayleigh, Nakagami-$m$, Rician, $K$-$\mu$, and $\eta$-$\mu$ fading channels as special cases as reported in [45] and [46, Table 1]. Based on the surveyed literature and the references listed therein, and to the best of the authors’ knowledge, there is no published work that develops an analytical framework for studying RIS-assisted wireless communications systems with direct link in the generalized $\kappa$-$\mu$ shadowed fading channel. Most of the considered fading models in the literature such as the Rician, Nakagamai-$m$ and Rayleigh are actually special cases of the fading model considered in this work. More specifically, the main contribution of this paper can be summarized as follows:

- Derives accurate closed-from expressions for the PDF, MGF, and CDF of the end-to-end SNR.
- Derives accurate closed-form approximations for the Outage Probability (Pout) and BER.
- Derives a new accurate closed-form expression for the asymptotic Pout and BER. Moreover, the system diversity order and coding gain are provided.
- The derived results are valid for an arbitrary number of reflecting elements, and various values of $\kappa$-$\mu$ shadowed parameters.
- Simulation results are provided to validate our analysis.

**C. PAPER ORGANIZATION AND NOTATIONS**
The remainder of this work is organized as follows. Section II presents the system and channel models, the PDF of the received signal power, the MGFs and CDF of the overall channel gain, and the PDF of the end-to-end SNR. Section III presents the performance evaluation in terms of outage and bit error probabilities. Section IV evaluates the numerical results for various system and channel scenarios, and discusses the main findings. Finally, Section V concludes the paper and Appendix presents the proof Proposition 2. The notations used throughout the paper are listed in Table 2.

**II. SYSTEM AND CHANNEL MODELS**

**A. SYSTEM MODEL**
The simplified system model is given in Fig. 1, where an RIS-assisted wireless communications network that includes a BS, RIS panel equipped with $N$ reflecting elements, and $K$ UEs. Each UE can access the RIS through an orthogonal multiple access protocol such as time division multiple access (TDMA). In this work, we consider that the UE can receive the transmitted signal through two different links, an indirect link through the RIS and a direct link from the BS. It is worth noting that using the term direct should not imply that the BS and UE have a LOS connectivity, it just indicates that the signal propagates from the BS to the UE without RIS assistance. Because the distance between the RIS panel, BS and UE is much larger than the distance between the reflecting elements within the RIS panel, then all RIS elements are

**TABLE 2. List of functions, $a_p = a_1, \ldots, a_p$, $b_q = b_1, \ldots, b_q$.**

| Symbol   | Function |
|----------|----------|
| $E[\cdot]$ | Statistical expectation |
| $\text{Var}(\cdot)$ | Statistical variance |
| $\mathcal{F}(\cdot)$ | Probability operation |
| $\mathcal{L}(\cdot)$ | Laplace transform |
| $\mathcal{L}^{-1}(\cdot)$ | Inverse Laplace transform |
| $M_G(\cdot)$ | Moment generating function |
| $f_X(\cdot)$ | Probability density |
| $F_X(\cdot)$ | Cumulative distribution function |
| $\Gamma(\cdot)$ | gamma [50, eq. (8.310)] |
| $\gamma_\nu(\cdot)$ | Lower incomplete gamma [50, eq. (8.350)] |
| $K_v(\cdot)$ | Modified Bessel of order $v$ [51, eq. (9.6.2)] |
| $\Phi_2$ | Bivariate confluent hypergeometric [52] |
| $\mathcal{I}_P(\cdot)$ | Regularized hypergeometric function [IP] |
| $\mathcal{I}_P(\cdot,\cdot)$ | Confluent HP [50, eq. (8.310)] |
| $\mathcal{I}_P(\cdot,\cdot,\cdot)$ | Generalized HP [50, eq. (9.14)] |
| $\beta(\cdot)$ | Beta [50, eq. (8.38)] |
assumed to be at the same distance from the BS and UE. Therefore, the distances from BS to UE, BS to RIS, and RIS to UE are respectively denoted as \( d_0, d_h, \) and \( d_g \). Practically speaking, the width/length of the RIS elements as well as the spacing between adjacent elements can be smaller than half the wavelength [32]. Therefore, the channels between adjacent elements become spatially correlated [53], [54], [55]. However, most of the reported research considers that the channels for all RIS elements are independent and identically distributed (i.i.d.) [12], [36], [38], [56], [57], [58]. Such an assumption can be validated by noting that the elements’ size is typically about \( 0.5\lambda_F \times 0.5\lambda_F \) [52], where \( \lambda_F \) is the carrier frequency wavelength. Therefore, the distance between the centers of adjacent elements is slightly more than \( \lambda_F/2 \). Moreover, the separation between an element and the next adjacent element becomes more than half the wavelength. Which significantly limits the number of correlated elements in the RIS panel. The i.i.d. assumption also enables mathematical tractability, particularly when a generalized channel model is considered, which is the case in this work. The channel gains between the BS and RIS, RIS and UE are denoted as \( h_i \) and \( g_i \), respectively, \( i = 1,2,\ldots,N \). The channel gain between the BS and UE is denoted as \( h_0 \). The channel state information (CSI) of each UE is communicated to the BS by means of channel reciprocity algorithms, or via a feedback channel. The BS controls the phase of each RIS panel through a feedback channel [6], [59], [60]. For mathematical tractability, we consider that the BS and UE are equipped with a single antenna [61], [62], [63]. Therefore, the received signal at any UE can be expressed as

\[
Y = Y_D + Y_R + n \tag{1}
\]

where \( Y_D \) is the signal received through the direct link, \( Y_R \) is the signal received through the RIS, and \( n \sim \mathcal{C}\mathcal{N}(0, N_0) \) is the additive white Gaussian noise (AWGN). The direct signal can be written as [42]

\[
Y_D = \sqrt{P} d_0^{-\frac{2}{\alpha_D}} \frac{\xi}{\sqrt{2}} |h_0| e^{-j\theta_0} s \tag{2}
\]

where \( P \) is the transmitted power, \( \xi = \left( \frac{c}{4f_c} \right)^2 \) denotes the average channel gain at a reference distance of 1 m based on the free space path-loss model, \( c \) is the speed of light, \( f_c \) is the carrier frequency, \( s \) is the transmitted information symbol, \( |h_0| \) and \( \theta_0 \) are the magnitude and phase of the channel from the BS to UE, and \( d_D \) is the pathloss exponent at direct link. Similarity, \( Y_R \) denotes the received signal at UE through the RIS link, which can be written as, 

\[
Y_R = \sqrt{P} \xi d_h^{-\frac{2}{\alpha_R}} d_g^{-\frac{2}{\alpha_R}} \sum_{i=1}^{N} |h_i| |g_i| e^{-j(\theta_i + \phi_i - \Psi_i - \theta_0)} e^{-j\phi_i} \tag{3}
\]

\[
+ d_0^{-\frac{2}{\alpha_D}} \frac{\xi}{\sqrt{2}} |h_0| s \sqrt{P} e^{-j\theta_0}. \tag{4}
\]

If we choose the phases such that [64], [65], [66], [67],

\[
\theta_i + \phi_i - \Psi_i - \theta_0 = 0 \tag{5}
\]

then

\[
Y_R + Y_D = \sqrt{P} e^{-j\phi_0} \left( \xi d_h^{-\frac{2}{\alpha_R}} d_g^{-\frac{2}{\alpha_R}} \sum_{i=1}^{N} |h_i||g_i| + \sqrt{\frac{\xi}{\alpha_D}} |h_0| \right) s e^{-j\phi_0} h_0 s. \tag{6}
\]

Consequently, the overall channel gain can be expressed as

\[
V = H_D + H_R. \tag{7}
\]

Therefore, combining the direct and reflected signals coherently at the UE is still possible using the RIS even though the direct link phase cannot be controlled. The power of the combined signals in this case is \( |V|^2 \) and the PDF of the SNR is given in (38).
B. CHANNEL MODEL

The $\kappa$-$\mu$ shadowed model is denoted as $\kappa$-$\mu$, is a generalized distribution that plays an essential role in modeling fading for LOS scenarios. The $\kappa$-$\mu$ shadowed model has an additional parameter $\omega$, as compared to $\kappa$-$\mu$ model, which is related to shadowing. Consequently, it has a clear physical interpretation, flexible analytical properties, and can represent common fading models such as One-Side Gaussian, Rayleigh, Nakagami-$m$, Rician, $\kappa$-$\mu$, and $\eta$-$\mu$ fading channels as special cases [45], [46, Table 1]. In addition, it can be used in several real-world applications [19], [69], [70], [71]. By limiting the fading parameters $\mu$ and $\omega$ to integer values, minor differences are obtained in practice when fitting field measurements to the $\kappa$-$\mu$ model, while it will make the mathematical analysis more tractable and may lead to efficient closed-form expressions [68]. More specifically, the measurements’ results in [68, Figs. 6, 7] confirm that limiting the values of $\omega$ and $\mu$ to integer values would produce negligible differences in the CDF of the channel, particularly for $\omega \geq 2$. For RIS applications, the value of $\omega$ is generally larger than 2 because the RIS panel is intentionally located to have strong links with both the transmitter and receiver. For the direct link, the value of $\omega$ could be less than 2, however, the performance is generally dominated by the RIS link. Fig. 2 shows the PDF $f_\nu(v)$ for various integer and noninteger parameters where it is clearly shown that the integer assumption has an insignificant effect on the PDF. In this work, we consider that all channel gains follow the $\kappa$-$\mu$ shadowed with integer $\mu$ and $\omega$ parameters. Therefore, the PDF of the envelope can be obtained from the PDF of the power [68, eq. (12)] using transformation between two random variables (RVs). Consequently, the $\kappa$-$\mu$ shadowed envelope PDF and CDF can be respectively expressed as

$$f_\nu(x) = \sum_{i=0}^{M} \frac{2C_i x^{2m_i-1}}{\Gamma(m_i)\Omega_i} e^{-\frac{x^2}{\Omega_i}}$$

(8)

and

$$F_\nu(x) = 1 - \sum_{i=0}^{M} C_i e^{-\frac{x^2}{\Omega_i}} \sum_{k=0}^{m_i-1} \frac{1}{k!} \left(\frac{x}{\Omega_i}\right)^k$$

(9)

where $M$, $C_i$, $m_i$, and $\Omega_i$ are given in Table 3.

### TABLE 3. Parameter values for the envelope $\kappa$-$\mu$ shadowed distribution [68].

| Case $\mu > \omega$ | Case $\mu \leq \omega$ |
|---------------------|------------------------|
| $M = \mu$ | $M = \omega - \mu$ |
| $C_i = \begin{cases} 0 & (\omega + i - 2) \left(\frac{\mu k + \omega}{\mu k + \omega}ight)^{\omega - i + 1} \\ (-1)^{i-1} \left(\frac{\mu k + \omega}{\mu k + \omega}ight)^{\omega - i + 1} & i = 0 \\ -2 \left(\frac{\mu k + \omega}{\mu k + \omega}ight)^{\omega - i - 1} \left(\frac{\mu k + \omega}{\mu k + \omega}ight)^{\mu - \omega - 1} & 0 < i \leq \mu - \omega \end{cases}$ | $\begin{cases} (\omega - 1) \left(\frac{\mu k + \omega}{\mu k + \omega}ight)^{\omega - i - 1} & \mu - \omega < i \leq \mu \\ (\omega - 1) \left(\frac{\mu k + \omega}{\mu k + \omega}ight)^{\omega - i} & i = 0 \\ (-1)^{i-1} \left(\frac{\mu k + \omega}{\mu k + \omega}ight)^{\omega - i - 1} \left(\frac{\mu k + \omega}{\mu k + \omega}ight)^{\mu - \omega - 1} & 0 < i \leq \mu - \omega \end{cases}$ |
| $m_i = \frac{\mu - \omega - i + 1}{\mu k + \omega} \left(\frac{\mu k + \omega}{\mu k + \omega}ight)^{\mu - i + 1}$ | $m_i = \omega - i$ |
| $\Omega_i = \frac{\mu k + \omega}{\mu k + \omega} \left(\frac{\mu k + \omega}{\mu k + \omega}ight)^{\mu - i + 1}$ | $\Omega_i = \frac{\mu k + \omega}{\mu k + \omega} \left(\frac{\mu k + \omega}{\mu k + \omega}ight)^{\mu - i}$ |

### FIGURE 2. (a) Evolution of the simulation PDF $f_\nu(v)$ for different values of $\omega_D$ and fixed $\omega_R = 5$. Parameter values $N = 20, \rho_R = \rho_D = 2$ and $\omega_R = 5$, and $\kappa_D = \kappa_R = 1$. (b) Evolution of the simulation PDF $f_\nu(v)$ for different values of $\omega_R$ and fixed $\omega_D = 5$. Parameter values $N = 20, \rho_R = \rho_D = 2$ and $\omega_D = 5$, and $\kappa_D = \kappa_R = 1$. |

C. PDF OF THE RECEIVED SIGNAL POWER

As can be noted from (6) and (7), the received signal power depends on $V$, and thus, its PDF should be derived. Towards this goal, define $|h||g_i| \triangleq Z_i$, and for notational simplicity, we drop the reflector index $i$ unless it is necessary to include. Therefore, the PDF of $Z$ can be evaluated as [72],

$$f_Z(z) = \int_0^\infty \frac{1}{x} f_h(x) f_h(z) \frac{z^x}{x^x} dx.$$  

(10)

By substituting the PDFs $|h| \sim \kappa\mu(M_1, C_1, m_1, \Omega_1)$, and $|g| \sim \kappa\mu(M_2, C_2, m_2, \Omega_2)$ expressed in (8), into (10), and using the identity [49, eq. (3.471.9)]

$$\int_0^\infty x^{\gamma - 1} e^{-\frac{x}{\gamma}} dx = 2 \left(\frac{\beta}{\gamma}\right)^{\frac{\gamma}{2}} K_0\left(2\sqrt{\beta\gamma}\right)$$

(11)

we obtain,

$$f_Z(z) = \sum_{i=0}^{M_1} \sum_{j=0}^{M_2} \frac{4C_i C_j}{\Gamma(m_1)\Gamma(m_2)} \frac{1}{\Omega_i^{m_1+m_2}}$$
Moreover, the mean and variance of $\bar{Z}_i$ are respectively given by,

$$\mathbb{E}[\bar{Z}_i] \triangleq \mu_{\bar{Z}_i} = \sum_{j=0}^{M_1} \sum_{k=0}^{M_2} \frac{C_i C_j}{\Gamma(m_i) \Gamma(m_j)} (\Omega_i, \Omega_j)^{\frac{1}{2}} \times \left[ \Gamma\left(\frac{1}{2} (m_i + m_j + 1)\right)\right]^2,$$

and

$$\text{Var}[\bar{Z}_i] \triangleq \sigma^2_{\bar{Z}_i} = \mathbb{E}[\bar{Z}_i^2] - (\mathbb{E}[\bar{Z}_i])^2.$$

where

$$\mathbb{E}[\bar{Z}_i^2] \triangleq \sum_{j=0}^{M_1} \sum_{k=0}^{M_2} \frac{C_i C_j}{\Gamma(m_i) \Gamma(m_j)} (\Omega_i, \Omega_j)^{\frac{1}{2}} \times \left[ \Gamma\left(\frac{1}{2} (m_i + m_j + 2)\right)\right]^2.$$

In the special case where all the links are identical, i.e., $M_1 = M_2 = M$, $\Omega_i = \Omega$, and $C_j = C_i = C_i$, then

$$f_{\bar{Z}_i}(z) = \sum_{l=0}^{M} \frac{4C_i^2}{\Gamma(m_i)} \frac{1}{\Omega_i^2} e^{2m_i-1} K_0\left(\frac{2\pi}{\Omega_i}\right).$$

In case of i.i.d. channel gains, $\mu_{\bar{Z}_i} = N\mu_{\bar{Z}_1}$ and $\sigma^2_{\bar{Z}_i} = N\sigma^2_{\bar{Z}_1}$. The expression in (19) will be used to attain simple and tractable expressions.

**Proposition 1:** Suppose that $Z$ has gamma distribution with shape parameter $a \in (0, \infty)$ and scale parameter $b \in (0, \infty)$. For large values of $N$, the distribution of the RV $Z$ converges to normal distribution such that

$$Z \sim \mathcal{N}\left(\mu_{\bar{Z}_i}, \sigma^2_{\bar{Z}_i}\right).$$

Figs. 3 and 4 illustrate the comparison between the PDF of the indirect link expressed in (19) with CLT method and Monte Carlo simulations. It can be noted that the PDF of the proposed approximation is more accurate than that based on the CLT regardless of the values of $\mu_R$, $\sigma_R$, and $\omega_R$. The expression in (19) will be used to attain simple and tractable expressions. 

In case of i.i.d. channel gains, $\mu_{\bar{Z}_i} = N\mu_{\bar{Z}_1}$ and $\sigma^2_{\bar{Z}_i} = N\sigma^2_{\bar{Z}_1}$. The expression in (19) will be used to attain simple and tractable expressions.
can be noted that increasing $N$ can significantly improve the fading conditions.

Fig. 5 demonstrates the effectiveness and validity of the derived PDFs by comparing the generalized theoretical expression of the PDF in (19) to Monte Carlo simulation. The values of $\mu$, $\kappa$, and $\omega$ of the individual PDFs are selected to correspond to some of the widely used PDFs, while $N$ is fixed at 10. As the figure shows, the theoretical expression perfectly matches the simulation results for all considered scenarios. Moreover, the generalized distribution model can be used to represent the widely known fading distributions such as the Rayleigh, Nakagami-$\tilde{m}$, and $\kappa$-$\mu$.

By applying the transformation method between two random variables in (6) (i.e., $H_R = L_R Z$), the PDF and CDF of $H_R$ can be given by

$$f_{H_R}(h_R) = \frac{(h_R)^{a-1}}{(L_R b)^a \Gamma(a)} \exp\left(-\frac{h_R}{L_R b}\right)$$  \hspace{1cm} (22)

and

$$F_{H_R}(h_R) = \frac{\gamma\left(a, \frac{h_R}{L_R b}\right)}{\Gamma(a)}$$  \hspace{1cm} (23)

where $L_R = \sqrt{\bar{P} \xi d_R^{\frac{ao}{f}} d_e^{-\frac{ao}{f}}}$. The PDF and CDF of $H_D$, i.e., $H_D = L_D |h_0|$, can be derived following the same approach used to derive $F_{H_R}(h_R)$, however, the PDF in (8) typically leads to significant computational complexity when $F_{H_R}(h_R)$ and $F_{H_D}(h_D)$ are convolved to compute $f_Y(v)$. Consequently, we propose the following accurate approximation.

**Proposition 2:** Using Laplace approach, $f_X(x)$ in (8) can be approximated as

$$f_X(x) = \sum_{q=0}^{M} \sum_{i=1}^{5} \frac{\sqrt{\pi} C_q}{\Gamma(m_q) \Omega_q \frac{m_q}{2} (v - 1)! \Delta^v} \frac{\Lambda_2^2 a_{i}}{\Gamma(\Lambda_1) \sqrt{\Delta}} \frac{1}{\Lambda_1} \exp\left(-\frac{x}{\Delta}\right)$$  \hspace{1cm} (24)

where $\Lambda_1 = 2m_q + v - 1$, $\Lambda_2 = \Lambda_1 - 1$, $a_1 = 0.25482959$, $a_2 = -0.284496736$, $a_3 = 1.421413741$, $a_4 = -1.453152027$, $a_5 = 1.061405429$, $\Delta = \frac{p}{2} \sqrt{\Delta_1}$, and $p = 0.3275911$.

**Proof:** The proof is given in the Appendix.

Fig. 6 shows the PDF derived in (24) and evaluates its accuracy by comparing it with exact PDF in (8). The figure shows a close match between the exact and approximated PDFs for a wide range of $x$. However, a slight deviation is observed in the PDF tail region when $1.48 \lesssim x \lesssim 2$, and close to the peak of the PDFs. Such deviation is insignificant because it is generally small and appears only for a narrow range of values. By applying the Kullback–Leibler divergence test numerically over 300 samples we obtain a divergence factor of $-8.1 \times 10^{-2}$ for $\mu = 3$, and $-1.9 \times 10^{-2}$ for $\mu = 2$, which confirms the accuracy of the approximation.

Following the same approach used to derive $F_{H_R}(h_R)$, and using the approximated PDF in (24) gives

$$f_{H_D}(h_D) = \sum_{q=0}^{M} \sum_{i=1}^{5} \frac{G_{q, i}}{\Gamma(\Lambda_1) \sqrt{\Delta}} \frac{1}{\sqrt{\Delta_1}} h_D^{\Lambda_2} e^{-\frac{h_D}{\Lambda_1}}$$  \hspace{1cm} (25)
and

\[ F_{H_0}(h_D) = \sum_{q=0}^{M_3} \sum_{v=1}^{5} \frac{\sqrt{\pi} C_q \Lambda_2!}{\Gamma(m_q)} \left( \frac{\Delta^2}{\Omega_q} \right)^{m_q-\frac{1}{2}} a_v \]
\[
\times \frac{1}{\Gamma(\Lambda_1)} \gamma \left( \Lambda_1, \frac{h_D}{\Delta_D} \right) \tag{26}
\]

respectively, where \( L_D = \sqrt{P_D} \frac{\alpha_p}{\pi} \), and
\[
G_{q,v} = \frac{\sqrt{\pi} C_q}{\Gamma(m_q)} \left( \frac{L_D^2 \Omega_q}{\pi} \right)^{m_q-\frac{1}{2}} a_v \Lambda_2! \left( \frac{v}{(v-1)!} \right)^{\frac{1}{2}}. \tag{27}
\]

By noting the superposition of \( H_D \) and \( H_R \) in (7), then the PDF of \( V \) can be obtained by applying the convolution theorem to (22) and (25). Thus,

\[
f_V(v) = \int_{-\infty}^{\infty} f_{H_0}(u)f_{H_R}(v-u)du = \sum_{q=0}^{M_3} \sum_{v=1}^{5} G_{q,v} \frac{v^{\Lambda-1}}{\Gamma(\Lambda)} \left( \frac{L_D}{\Lambda} \right)^a F_1(\Lambda_1, \Lambda; \gamma v) \tag{28}
\]

where \( \Lambda = 2m_q + v + a - 1 \), \( \gamma = (\frac{L_D}{\Lambda} - \frac{1}{\Delta_D}) \).

**D. MOMENT GENERATING FUNCTION OF V**

By applying the Laplace transform to (28), we obtain the MGF of \( V \),

\[
M_V(s) = \mathcal{L}\left[f_V(v); -\left(s + \frac{1}{L_D} \right)\right]
= \sum_{q=0}^{M_3} \sum_{v=1}^{5} \frac{G_{q,v}}{\Gamma(\Lambda)} \frac{1}{(L_D)^a} \left[ v^{\Lambda-1} F_1(\Lambda_1, \Lambda; \gamma v) \right]. \tag{29}
\]

Using [73, eq. (4.23.1)],

\[
\mathcal{L}\left[v^{\Lambda-1} F_1(A, A; (\frac{L_D}{\Lambda} - \frac{1}{\Delta_D})v)\right] = \Gamma(\Lambda) \left( s + \frac{1}{L_D} \right)^{-a} \left( s + \frac{1}{\Delta_D} \right)^{-\Lambda_1}. \tag{30}
\]

Therefore (29) becomes

\[
M_V(s) = \sum_{q=0}^{M_3} \sum_{v=1}^{5} \frac{G_{q,v}}{(L_D)^a} \gamma v \left( s + \frac{1}{L_D} \right)^{-a} \left( s + \frac{1}{\Delta_D} \right)^{-\Lambda_1}. \tag{31}
\]

**E. THE CDF OF V**

By noting that the MGF in (31) and \( f_V(v) \) has the relation

\[
f_V(v) = \mathcal{L}^{-1}[M_V(s); v] \tag{32}
\]

then \( F_V(v) \) can be obtained as

\[
F_V(v) = \mathcal{L}^{-1}\left[ \frac{M_V(s)}{s}; v \right]. \tag{33}
\]

By substituting (31) in (33), we obtain

\[
F_V(v) = \mathcal{L}^{-1}\left[ \frac{M_V(s)}{s} \right]
\times \frac{1}{(L_D^2 \Omega_q)^{m_q-\frac{1}{2}}} \left( \frac{v}{(v-1)!} \right)^{(\Delta_D)^{\frac{1}{2}}}
\times \frac{1}{\Gamma(\Lambda)} \frac{1}{(L_D)^a} \left( s + \frac{1}{L_D} \right)^{-a} \left( s + \frac{1}{\Delta_D} \right)^{-\Lambda_1}. \tag{34}
\]

Using [73, eq. (4.24.3)], the Laplace inverse can be evaluated as,

\[
\mathcal{L}^{-1}\left[ \frac{1}{s} \frac{1}{v^{\Lambda}} \right] = \frac{v^{\Lambda}}{\Gamma(A)} \Phi_2\left( a, \Lambda_1, \Lambda + 1; -\frac{1}{L_D} \right). \tag{35}
\]

Substituting (35) into (34) gives,

\[
F_V(v) = \mathcal{L}^{-1}\left[ \frac{M_V(s)}{s} \right]
= \sum_{q=0}^{M_3} \sum_{v=1}^{5} \frac{G_{q,v}}{(L_D)^a} \gamma v \left( s + \frac{1}{L_D} \right)^{-a} \left( s + \frac{1}{\Delta_D} \right)^{-\Lambda_1}
\times \Phi_2\left( a, \Lambda_1, \Lambda + 1; -\frac{1}{L_D} \right). \tag{36}
\]

where the function \( \Phi_2 \) can be efficiently computed by means of an inverse Laplace transform [74].

**F. END-TO-END SNR**

The instantaneous end-to-end SNR at the UE can be expressed as

\[
\gamma = \frac{P_s}{N_0} |V|^2 = |V|^2 \bar{\gamma} \tag{37}
\]

where \( P_s \) is the transmitted symbol power, \( N_0 \) is AWGN noise power spectral density, and \( \bar{\gamma} \) is the average SNR. Therefore, the PDF, CDF and MGF of \( \gamma \) are respectively given by,

\[
f_{\gamma}(\gamma) = \frac{1}{2\sqrt{\bar{\gamma}}} \sum_{q=0}^{M_3} \sum_{v=1}^{5} \frac{G_{q,v}}{(L_D)^a} \gamma v \left( s + \frac{1}{L_D} \right)^{-a} \left( s + \frac{1}{\Delta_D} \right)^{-\Lambda_1}
\times e^{\frac{1}{\bar{\gamma}} F_1(A, A; \gamma v)} \tag{38}
\]

\[
F_{\gamma}(\gamma) = \mathcal{L}^{-1}\left[ \frac{M_V(s)}{s} \right]
= \sum_{q=0}^{M_3} \sum_{v=1}^{5} \frac{G_{q,v}}{(L_D)^a} \gamma v \left( s + \frac{1}{L_D} \right)^{-a} \left( s + \frac{1}{\Delta_D} \right)^{-\Lambda_1}
\times \Phi_2\left( a, \Lambda_1, \Lambda + 2; \xi, \chi \right) \tag{39}
\]
where \( \xi = -\frac{1}{\Lambda b_0}\sqrt{\frac{\nu}{\gamma}} \) and \( \chi = -\frac{1}{2\Lambda b_0}\sqrt{\frac{\nu}{\gamma}}, \) and
\[
M_{yy}(s) = \mathcal{L}\left[f_Y(y); -\left(s + \frac{1}{\Lambda b_0}\right)\right]
\]
\[
= \sum_{q=0}^{M_1} \sum_{v=1}^{5} \frac{G_{q,v}}{(\Lambda b_0)^a} \left(s - \frac{\xi}{\sqrt{\gamma}}\right)^{-a} \left(s - \frac{\chi}{\sqrt{\gamma}}\right)^{-\Lambda_1}.
\]
(41)

To obtain more insights about the system behavior, we derive the asymptotic PDF, CDF and MGF in the high SNR region, i.e., \( \tilde{\gamma} \to \infty \). For the PDF,
\[
f_{y}(y)\big|_{\tilde{\gamma} \to \infty} \overset{\Delta}{=} f_{y}^{\infty}(y)
\]
(42)
by noting that [75]
\[
\lim_{\tilde{\gamma} \to \infty} F_1\left(\Lambda_1, \Lambda_1; \sqrt{\frac{\nu}{\gamma}}\right) = 1
\]
(43)
then we obtain from (38),
\[
f_{y}^{\infty}(y) = \frac{1}{2\sqrt{\gamma}} \sum_{q=0}^{M_1} \sum_{v=1}^{5} \frac{G_{q,v}}{(\Lambda b_0)^a} \left(\frac{\nu}{\gamma}\right)^{-\frac{a+1}{2}}.
\]
(44)

Similarly,
\[
F_{y}^{\infty}(y) = \sum_{q=0}^{M_1} \sum_{v=1}^{5} \frac{G_{q,v}}{(\Lambda b_0)^a} \left(\frac{\nu}{\gamma}\right)^{-\frac{a}{2}}
\]
(45)
and
\[
M_{y}^{\infty}(s) = \mathcal{L}\left[f_Y(v); -\left(s + \frac{1}{\Lambda b_0}\right)\right]
\]
\[
= \sum_{q=0}^{M_1} \sum_{v=1}^{5} \frac{G_{q,v}}{(\Lambda b_0)^a} \left(\frac{\nu}{\gamma}\right)^{-\frac{a}{2}}.
\]
(46)

In the case that the UE has multiple receiving antennas, i.e., the system has single-input multiple-output (SIMO) configuration, then similar to (6), the received signal at the \( r \)th receiving antenna can be written as,
\[
Y_{R_r} + Y_{D_r} = \sqrt{P_E} e^{-j\theta_{0,r}}
\]
\[
\times \left(\sum_{i=1}^{N} \frac{h_{i,r} g_{i,r}}{d_0^2} + \sqrt{\frac{\nu}{2}} h_{0,r}\right) s
\]
\[
= (H_{R_r} + H_{D_r}) e^{-j\theta_{0,r}} s
\]
\[
= V_r e^{-j\theta_{0,r}} s.
\]
(47)

After Maximal Ratio Combining (MRC), the overall instantaneous SNR at UE can be expressed as
\[
\gamma_{SE} = \sum_{r=1}^{L} \gamma_{y_r}
\]
(48)
where \( L \) is the number of receiving antennas, and \( \gamma_{y_r} \) is defined in (38). However, deriving the exact PDF of \( \gamma_{SE} \) generally leads to computationally complex and intractable analysis, and will be considered in a dedicated work.

### III. PERFORMANCE EVALUATION

#### A. OUTAGE PROBABILITY

The derived distributions can be used to compute the outage probability, which is defined as
\[
P_{out} = \Pr[\gamma \leq \gamma_{out}]
\]
(49)
where \( \gamma_{out} \) is a predetermined SNR threshold. From (45), it can be concluded that \( P_{out} \) can be approximated as
\[
P_{out} \approx (G_o)^{-G_d}
\]
(50)
where \( G_o \) and \( G_d \) denote the coding gain and diversity order which are respectively given by
\[
G_o = \left(\sum_{q=0}^{M_1} \sum_{v=1}^{5} \frac{G_{q,v}}{(\Lambda b_0)^a} \Gamma(2m_q + a)\right)^{-w/\Lambda_1}
\]
(51)
where \( w = \frac{2}{\Lambda_1} \), and \( G_d = \frac{1}{w} \).

#### B. BIT ERROR RATE (BER)

Based on the MGF expressed in (41), the BER can be expressed as,
\[
P_{B} = \frac{a_m}{\pi} \int_{0}^{\pi} M_{y}(\frac{b_m^2}{2\sin^2(\theta)}) d\theta
\]
(52)
where \( a_m \) and \( b_m \) are parameters that depend on the modulation scheme. By substituting (53) in (52), the BER becomes
\[
P_{B} = \sum_{q=0}^{M_1} \sum_{v=1}^{5} a_m G_{q,v} \left(\frac{b_m^2}{2\Lambda_1}\right)^{-a} \int_{0}^{\pi} \left(\frac{b_m^2}{2\sin^2(\theta)} - \frac{\xi}{\sqrt{\gamma}}\right)^{-a} \times \left(1 - u_1 t\right)^{-a} (1 - u_2 t)^{-\Lambda_1} d\theta.
\]
(53)

By performing the change of variable \( t = \cos^2(\theta) \), and after some algebraic manipulations, we obtain
\[
P_{B} = \sum_{q=0}^{M_1} \sum_{v=1}^{5} a_m G_{q,v} \left(\frac{b_m^2}{2\Lambda_1}\right)^{-a} \left(\frac{1}{\gamma}\right)^{\frac{a}{2}} \times \left(\frac{b_m^2}{2\gamma} - \frac{\xi}{\sqrt{\gamma}}\right)^{-\Lambda_1} \int_{0}^{1} t^{\frac{1}{2} - \frac{a}{\Lambda_1}} (1 - t)^{\Lambda_1 + \frac{1}{2}} \times (1 - u_1 t)^{-a} (1 - u_2 t)^{-\Lambda_1} dt
\]
(54)
with the help of [49, eq. (3.211)]
\[
\int_{0}^{1} t^{\lambda - 1}(1 - t)^{\mu - 1} (1 - u_1 t)^{-q}(1 - u_2 t)^{-s} dt
= B(\mu_1, \lambda_1) F_1(\lambda_1, q, s, \lambda_1 + \mu_1; u_1, u_2)
\]
(55)
The results are obtained for various system and channel conditions on the outage and BER performance. As can be noted from the figure, the analytical and simulation results match very well for all the considered scenarios. It can be also observed that at high SNRs, the asymptotic and exact results converge. The improvement gained by increasing the number of reflectors increases non-linearly versus \( N \). For example, increasing the number of reflectors from 25 to 35 reduced the SNR required to achieve \( P_{\text{out}} \) of 10\(^{-4}\) from 12 dB to 9 dB, i.e., the gain is about 3 dB while increasing \( N \) from 35 to 60 reflectors offers 4 dB gain. Therefore, the gain that can be achieved by increasing the number of reflectors decreases when the initial number of reflectors is small.

Fig. 8, Fig. 9 and Table 5 evaluate the impact of the various system and channel conditions on the outage and BER performance. As can be noted from Fig. 8, \( P_{\text{out}} \) is significantly dominated by the strength and quality of the direct path that is indicated by \( \mu_D \) and the number of reflecting elements \( N \) in which increasing the values of \( \mu \) and \( N \) can reduce \( P_{\text{out}} \).

Table 5 highlight the influence of the shadowing parameter of the direct path \( \omega_D \) between BS and UE for different
values of $\kappa_D$ and $N$ on the outage probability. The results are obtained for $\kappa_R = 1$, $\mu_R = 4$, and $\mu_D = 2$. The shadowing parameter between BS-RIS and RIS-UE links is $\omega_R = 15$.

According to physical model, the $\omega$ parameter represents the level of fluctuation due to shadowing in the dominant component and the $\kappa$ parameter indicates the ratio between the dominant components and the scattered waves. For large $\kappa$ values, more power is present in the dominant component with respect to the scattered waves, while for large $\omega$, the power of the dominant component is more stable and therefore there is less shadowing. Therefore, for high values of $\omega_D$, it can be observed that the impact of $\kappa_D$ becomes more tangible where $P_{out}$ decreases by increasing $\kappa_D$. Moreover, as can be noted from the table, the $P_{out}$ improves by improving $\kappa_D$ and $\omega_D$ parameters. In addition to, it can be noted that $\omega_D$ also limits the impact of $N$, where increasing the value of $N$ from 10 to 50 reduced $P_{out}$ only by a factor of $10^2$, while the improvement for the same scenario is much more significant for $\omega_D = 15$, i.e., weak shadowing.

Fig. 9 shows the effect of changing the number of reflecting elements $N$ on the BER. The exact (56) and asymptotic (59) BERs are included and compared to the Monte Carlo simulation results. The figure shows that the analytical and simulation results match very well, even for the case of $N = 5$ and $N = 10$. The BER improvement gained by using RIS seems to be significant, even when for small values of $N$ such as $N = 5$. The asymptotic BER approaches the exact when the exact BER becomes roughly linear. For the case of no RIS and $N = 5$, the curves converge for $\gamma \gtrsim 30$ dB, while for the case of $N = 5$ they converge for $\gamma \gtrsim 25$ dB.

Fig. 10 evaluates the impact of the path loss exponent at RIS link ($\alpha_R$) on the BER using (56). As can be noted from the figure, the BER achieved by the RIS increases by increasing $\alpha_R$, and ultimately approaches the BER of the direct link regardless of the value of $N$. Such performance is obtained because increasing $\alpha_R$ causes severe signal attenuation, and hence, the signal reflected by the RIS becomes extremely weak. However, when $\alpha_R$ is small, a significant performance gain can be achieved by increasing $N$.

Fig. 11 demonstrates the advantage of using RIS-aided transmission over the direct link. Therefore, the figure illustrates the effect of increasing the distance between the RIS and UE ($d_R$) on $P_{out}$. In particular, the graph provides a
FIGURE 10. The impact of the path loss exponent ($\alpha$) on the bit error rate (BER) where $\bar{\gamma} = -20$ dB, $\nu_D = 3.5$, $\nu_R = 3$, $\mu_D = 1$, $\mu_R = 1$, $\eta_D = 1$, and $\eta_R = 1$.

FIGURE 11. The impact of the distance between RIS and UE on outage probability ($P_{out}$) where $\bar{\gamma} = -20$ dB, $\nu_D = 5$, $\nu_R = 5$, $\mu_D = 1$, and $\mu_R = 1$.

FIGURE 12. Variation in diversity order with respect to number of reflecting elements $N$ where $\omega_R = \omega_D$, and $\mu_R = \mu_D = 1$ and $\nu_R = \nu_D = 1$.

comparative analysis of three transmission schemes, which are the direct transmission only, RIS without direct transmission, and RIS transmission with a direct link. It can be noted that if the direct link is available, it is beneficial to combine both signals, notably when $N$ increases. Moreover, it is generally difficult for the direct path alone to achieve reliable $P_{out}$ when the outage threshold $P_{out}$ is high. Therefore, the location of the RIS should be optimized by taking into consideration the possibility of exploiting the direct and reflected links to minimize the outage and BER.

Fig. 12 presents the diversity order of the system versus the number of reflecting elements $N$ for various values of $\omega_D$. Results indicate that $\omega_D$ has a significant impact on the diversity order for high values of $N$. When $\omega_D \gg 1$, all the reflected links have generally strong signals, and thus the diversity gained by using the RIS is limited. For low values of $\omega_D$, the channel variations increase significantly, which increases the diversity order achieved by using the RIS.

V. CONCLUSION AND FUTURE WORK

In this paper, we developed a new analytical framework to investigate the end-to-end SNR, outage probability, and BER performance of RIS-assisted wireless communications systems over the generalized $\kappa$-$\mu$ shadowed-fading channels. Novel accurate closed-form expressions for the PDF, CDF of end-to-end SNR have been derived. Moreover, closed-form expressions have been derived for the outage and BER, which provided near-exact results. The asymptotic outage and BER are also derived to provide more insights into the achievable diversity order. The derived expressions were used to holistically study the performance of RIS systems in the presence of a direct link between the BS and UE. The results show that the direct path can significantly affect the RIS system performance, and hence, should be taken into consideration during the system’s planning and design stages. Monte Carlo simulation was used to validate all the derived expressions.

In addition to providing an informative evaluation of the impact of having a direct link in RIS systems, this work also triggers several other key research problems. For example, the impact of imperfect phase estimation and compensation...
can be included. In such cases, the gain obtained by the RIS will be reduced and the impact of the direct link becomes more significant. The system performance with multiple-input multiple-output (MIMO) antenna configuration would be interesting. Another interesting problem is the case where the direct link is not taken into consideration in the phase compensation process. Therefore, the direct link becomes a strong source of interference.

Our future work will focus on using the derived PDF of the SNR to evaluate the ergodic capacity of the system and evaluate the impact of the direct link for various channel conditions.

**APPENDIX**

**PROOF OF PROPOSITION 2**

The PDF of $\kappa-\mu$ shadowed distribution is expressed as

$$f_X(x) = \sum_{i=0}^{M} \frac{2C_i x^{m-1}}{\Gamma(m_i) \Omega_i^{m_i}} e^{-\frac{x}{\Omega_i}} (62)$$

by using [77, eq. (2.2.17)], the Laplace transform of RV $X$ can be written as

$$f_X(s) = \sum_{i=0}^{M} \frac{2C_i}{\Gamma(m_i) \Omega_i^m} L\left(x^{m-1} e^{-\frac{x}{\Omega_i}}\right)$$

$$= \sum_{i=0}^{M} \frac{\sqrt{\pi} C_i}{\Gamma(m_i) \Omega_i^{m_i}} (-1)^{m_i-1} \times \frac{d^{m_i-1}}{d x^{m_i-1}} \left(\operatorname{erfc}\left(\frac{\sqrt{\Omega_i} s}{2}\right) e^{\frac{\Omega_i s^2}{4}}\right). (63)$$

The complementary error function and error function have the relation

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x) (64)$$

where the error function can be approximated using [51, eq. (7.1.26)],

$$\operatorname{erf}\left(\frac{\sqrt{\Omega_i} s}{2}\right) = 1 - \left(\frac{a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5}{\Omega_i \sqrt{s}}\right) \times e^{-\frac{\Omega_i s^2}{4}} + \epsilon\left(\frac{\sqrt{\Omega_i} s}{2}\right)$$

(65)

where $t = \frac{1}{\sqrt{2\pi}}$, $|\epsilon(x)| \leq 1.5 \times 10^{-7}$, $a_1 = 0.25482959$, $a_2 = -0.284496736$, $a_3 = 1.421413741$, $a_4 = -1.453152027$, and $a_5 = 1.061405429$. By substituting (65) in (64) and substituting the result in (63), then the Laplace transform of $X$ becomes

$$f_X(s) = \sum_{i=0}^{M} \frac{\sqrt{\pi} C_i}{\Gamma(m_i) \Omega_i^m} \times \frac{d^{m_i-1}}{d x^{m_i-1}} \left(\frac{a_v}{(1 + \Delta s)^x}\right). (66)$$

By using

$$\frac{d^{2m_i-1}}{ds^{2m_i-1}} \left(\frac{1}{(1 + \Delta s)^x}\right) = \frac{(-1)^{2m_i-1}(2m_i + v - 1)!\Delta^{2m_i-1}}{(v - 1)!(1 + \Delta s)^{2m_i-1-v}}$$

and applying the inverse Laplace transform

$$L^{-1}\left(\frac{1}{(1 + \Delta s)^{2m_i-1-v}}\right) = \frac{1}{\Delta^v(2m_i + v - 1)} \times \left(\frac{1}{\Delta}\right)^{2m_i+v-2} e^{-\frac{x}{\Delta}}$$

the PDF in (24) can be obtained.

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