Study on one-way transmission of acoustic wave based on metasurface

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Abstract. In this paper, using the characteristics of abnormal reflection metasurfaces that can produce different reflection angles, a method that can realize the one-way transmission of acoustic waves is designed, which is verified by simulation. And two different types of metasurface structures are used to verify the unidirectional transmission of acoustic waves. One is the acoustic impedance matching type metasurface, and the other is the geometric groove type metasurface. Based on the generalized Snell's law, the principle of one-way transmission of sound waves is analyzed, and simulation is carried out by COMSOL multiphysics coupling simulation software, which verifies the theoretical analysis of one-way transmission of acoustic waves. The results of this study may have potential applications in architectural acoustics (such as noise control) and medicine (such as ultrasound therapy).

1. Introduction

Acoustic metamaterials are usually composed of periodic or non-periodic arrangements of sub-wavelength-sized unit structures. Therefore, it can achieve functions that cannot be achieved by materials in nature through artificial design. For example, it can realize the self-collimation effect of acoustic wave, abnormal refraction and reflection, sound focusing, acoustic vortex and sub-wavelength imaging. Acoustic metasurface is a special structure of acoustic metamaterials [1], which is a structure composed of sub-wavelength-sized unit structures arranged on a two-dimensional surface. Compared with metamaterials, the preparation of metasurfaces is simpler and consumes less material. At the same time, it can also realize the physical phenomena of acoustic autocollimation, abnormal refraction and reflection [2] [3], acoustic vortex and one-way transmission of acoustic waves. Based on the special properties of metasurfaces, this paper designs a structure that can realize the unidirectional transmission of acoustic waves.

The physical phenomenon of one-way transmission of acoustic waves, like other abnormal physical phenomena, has attracted the attention of many people. However, the earliest method to realize the unidirectional transmission of acoustic waves was not obtained by applying the metasurface method. One is to use nonlinear mechanisms to break the time reversal symmetry [4] [5]; the other is to use asymmetric geometric structures to break the space reversal symmetry [6] [7]. The former is due to the low nonlinear conversion efficiency, and the latter is due to the complex fabrication of devices, so it promotes the development of metasurfaces in this direction.

Later, for the research topic of acoustic wave unidirectional transmission, Li [8] and others applied single-layer dielectric acoustic metasurface to realize the study of acoustic wave unidirectional transmission characteristics. At the same time, not long after, Liu [9] et al. divided the single-layer
dielectric acoustic metasurface into two metasurfaces, and realized the special study of one-way transmission of acoustic waves. The above two designs are based on the special physical phenomenon of the abnormal transmission of the acoustic metasurface. This paper is based on the principle that metasurfaces can achieve abnormal reflections to realize the regulation of the one-way transmission of acoustic waves. Previously, Zhu [10] et al. have done similar projects, and they combined metasurface structures with different gradients to realize one-way acoustic wave transmission. Inspired by this paper, we use two kinds of metasurface structures which are different from those in this paper to realize the special physical phenomenon of acoustic wave one-way transmission. The two kinds of ultrasonic metasurface structures are impedance matched and geometric grooves. Firstly, the theoretical analysis and design are carried out, and finally the simulation is verified by COMSOL software.

2. Discussion on the principle of one-way transmission of acoustic waves

The one-way acoustic transmission studied in this paper is based on the physical characteristics of metasurfaces that can achieve abnormal reflections. Therefore, we must first introduce an important law that can cause abnormal reflections on metasurfaces. This law is a modification of the well-known Snell's law, and we call it the generalized Snell's law [11].

Due to the introduction of the metasurface structure on the surface of the medium, it promotes the acoustic wave to produce a certain phase change on the reflecting surface, so that the reflected wave no longer obeys the normal reflection law. As shown in Figure 1, it is the acoustic wave path diagram of the generalized Snell's law.

![Figure 1. Schematic diagram of generalized Snell's law.](image)

It can be clearly seen from the above figure that normally reflected acoustic wave will travel along the acoustic wave path on the left of the figure, and the incident angle $\theta_1$ is equal to the reflection angle $\theta_2$. When the metasurface is introduced, a phase mutation $d\phi$ is artificially imposed on the metasurface. The acoustic wave will travel along the acoustic wave path on the right, and the incident angle $\theta_3$ and the reflection angle $\theta_4$ no longer satisfy the same relationship. It satisfies the following relationship:

$$kn_0\sin\theta_3dx + \phi + d\phi = kn_0\sin\theta_4dx + \phi$$

Arrange:

$$\sin\theta_4 - \sin\theta_3 = \frac{1}{kn_0}\frac{d\phi}{dx}$$

Where $\theta_1$ and $\theta_4$ are the incident angle and the reflection angle of the acoustic wave, respectively; $k$, $n_0$, $d\phi/dx$ are the wave vector, the refractive index of the medium, and the phase gradient of the metasurface, respectively, where $k=2\pi/\lambda$. Since the discussion in this paper is about putting the metasurface in air and incident acoustic wave is perpendicular to the metasurface incident, that is, $n_0 = 1, \theta_3 = 0^\circ$, the equation (2) can be rewritten as:

$$\sin\theta_4 - \sin\theta_3 = \frac{1}{kn_0}\frac{d\phi}{dx}$$
According to the above results, a metasurface structure with an abnormal reflection angle close to 90° and a metasurface structure with a relatively small abnormal reflection angle can be designed. After they are combined, as shown in Figure 2, it is a schematic diagram of the one-way transmission of acoustic waves.

\[
\sin \theta_4 = \frac{\lambda}{2\pi} \frac{df}{dx}
\]  

(3)

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Figure 2. Schematic diagram of wave unidirectional transmission.

A positive incident acoustic wave first encounters MS1 with a large phase gradient (that is, a metasurface that converts a vertical incident wave into a surface wave). According to the reciprocity principle, the wave will deflect by 90°. It will then incident vertically onto the MS1 below, so the wave will be reflected and fully converted into a surface wave that propagates to the left, rather than propagating to the right through the metasurface. Waves that incident from the opposite direction first encounter MS2 with a smaller gradient and then deflect slightly into MS1, where they are reflected again and some of the light travels to the left. (The width of the sound track is about 2\(\lambda_0\). If the passage is too wide, a leak may occur. If the channel is too narrow, sound waves will not be reflected many times in the channel.

3. Design and simulation analysis of metasurface

In this paper, two different types of metasurfaces, namely acoustic impedance metasurfaces and geometric grooves metasurfaces, will be used to simulate the above analysis. Figure 3 is a schematic diagram of the structure of two different types of metasurfaces within a period. Below we use them to design the structure, and use COMSOL multiphysics software to simulate the designed structure.

Figure 3. (a) Schematic diagram of acoustic impedance type metasurface structure (b) Schematic diagram of geometric groove type metasurface structure.

3.1. Acoustic impedance type metasurface

Acoustic impedance metasurface [12] is an acoustic gradient metasurface, composed of a periodic arrangement of rigid rods, filled with media of different refractive indices in the middle, which is essentially similar to a grating in optics, and we call it an acoustic grating. A super cell structure of it is shown in Figure 3(a). The height of the acoustic grating is \(h\), the width of the rod is \(w\), the width of the filling medium is \(b\), and the length of a super cell period is \(L\). For a reflective acoustic impedance type metasurface, the refractive index of the filling medium should be satisfied \(n_i = 1 + (i - 1)\lambda_0 / 2Nh\), the phase is \(\phi_i = 2\hbar n_i / c_0\), \(\lambda_0\) is the corresponding wavelength, \(N\) is the number of subunits,
ω and c₀ are the angular frequency and the speed of acoustic in the air, respectively. The phase gradient of the metasurface is \( \frac{d\phi}{dx} = \frac{2\pi}{L} \).

According to impedance matching \( z_1 = z_2 = \ldots = z_n = z_0 = \rho_0 c_0 \), it can be obtained that the density of the filling medium in each subunit are \( \rho_1 = \rho_0, \rho_2 = \rho_2 \rho_1, \rho_3 = \rho_3 \rho_2, \ldots, \rho_i = \rho_i \rho_{i-1} \), and the acoustic velocity of each column in the subunit are \( c_1 = c_0, c_2 = c_0/\rho_2, c_3 = c_0/\rho_3, \ldots, c_i = c_0 / \rho_i \).

According to the above analysis, the values of each item of the metasurface that can produce 90° abnormal reflection are as follows: the period of the supercell is \( L = 10 \) cm, the height of the acoustic grating is \( h = 10 \) cm, the number of subunits is \( N = 4 \), and the gap width between the rods is \( w = 0.5 \) cm, the width of the filling medium is \( b = 2 \) cm, working wavelength is \( \lambda_0 = 10 \) cm and working frequency is \( f = 3430 \) Hz (The metasurface structure is a subwavelength structure, and some of its parameters are determined by the size of the operating wavelength. Therefore, when the operating frequency is fixed, the wavelength is fixed, so that the size of the metasurface is basically fixed. This indicates that it has a very narrow bandwidth). According to the above extrapolation, we can get \( \sin \theta_r = \left( \frac{\lambda}{2\pi} \right) \times \left( \frac{2\pi}{L} \right) = \frac{\lambda}{L} = 1 \), reflection angle \( \theta_r = 90° \). The various data of the metasurface that can achieve the abnormal reflection of small angles are as follows: the period of the supercell is \( L = 50 \) cm, the height of grating is \( h = 10 \) cm, the number of subunits is \( N = 10 \), the gap between the pole width is \( w = 1 \) cm, the width of the filling medium is \( b = 4 \) cm, working wavelength is \( \lambda_0 = 10 \) cm and working frequency is \( f = 3430 \) Hz. Incorporating the formula can be obtained \( \sin \theta_r = \left( \frac{\lambda}{2\pi} \right) \times \left( \frac{2\pi}{L} \right) = \frac{\lambda}{L} = 0.2 \), reflection angle \( \theta_r \approx 11.5° \). After combining them, using COMSOL software to simulate, finally get the result in Figure 5 below.

As can be seen from the simulation results in Figure 4, only a small part of the acoustic wave passes through when it is forward incident. But when a reverse incident acoustic wave passes through the structure, a significant portion of the acoustic wave can pass through. The simulation results verify the correctness of theoretical analysis.

### 3.2. Geometric groove metasurface

The geometric groove type metasurface is to set grooves of different depths, so that the acoustic wave propagation path has a certain difference, so as to achieve the purpose of controlling the phase of the reflected acoustic wave. Its microstructure is shown in Figure 3(b).
Since the phase $\phi = 2\pi h/\lambda$, we can control the phase of the reflected acoustic wave by changing the depth of the groove. In fact, the acoustic wave will be emitted at the bottom of the groove, and the propagation path of the acoustic wave is twice the depth of the groove, so the phase of the reflected sound wave is $\phi = 2\pi \times 2h/\lambda$. According to the generalized Snell law, we can get $\sin \theta_r = (\lambda/2\pi) \times (d\phi/dx) = 2h/L$. The first structure sizes are: the length of a periodic unit is $L = 10$ cm, the depth $h_1$ of the deep groove is 8 cm, the depth $h_2$ of the shallow groove is 3 cm, and $\Delta h = 5$ cm. Into the formula can be obtained, $\sin \theta_r = (\lambda/2\pi) \times (d\phi/dx) = 2\Delta h/L = 1$, $\theta_r = 90^\circ$. The second structure sizes are: a periodic unit length is $L = 10$ cm, deep groove depth $h_3$ is 4 cm, shallow groove depth $h_4$ is 3 cm, $\Delta h = 1$ cm. Insert the formula to obtain $\sin \theta_r = (\lambda/2\pi) \times (d\phi/dx) = 2\Delta h/L = 0.2$, $\theta_r \approx 11.54^\circ$. After combining them in the same way, simulation was performed with COMSOL, and the following results were obtained.

**Figure 5.** (a) The simulation result of forward sound wave incidence (b) The simulation result of reverse sound wave incidence.

Figure 5 shows the simulation results of the transmission phenomenon of acoustic waves when they incident from the forward and reverse directions respectively. As can be observed in figure (a) above, only a small fraction of the acoustic waves pass through; it is also clear from (b) that a considerable portion of the acoustic wave has been transmitted. This, like the structure of acoustic impedance type metasurface, can achieve the purpose of one-way transmission of acoustic waves.

**4. Summary and conclusion**

In this paper, two different types of metasurface structures are used to design two metasurface structures with special reflection angles, and then they are combined to realize the control of the one-way transmission of sound waves. These two different types of metasurfaces both break through the traditional Snell's law, and both follow the generalized Snell's law. Through theoretical analysis, a structure with specific dimensions was designed. Finally, simulation verification was carried out by COMSOL multiphysics coupling software.

The study has shown that through reasonable design, a combined structure that can transmit sound waves in one direction can be obtained. Whether it is the structure designed by the acoustic impedance type metasurface or the structure designed by the groove type metasurface, it can realize the control of the one-way transmission of sound waves. This result has a certain potential role in architectural acoustics, especially in noise control. The biggest advantage of this paper is that it can use two different kinds of metasurface structures to design, and both of them can relatively well realize the phenomenon of acoustic wave one-way transmission.
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