THE 2+2-SIGNATURE AND THE 1+1-MATRIX-BRANE

J. A. Nieto[1]

Facultad de Ciencias Físico-Matemáticas de la Universidad Autónoma de Sinaloa, 80010, Culiacán Sinaloa, México

Abstract

We discuss different aspects of the 2+2-signature from the point of view of the quatl theory. In particular, we compare two alternative approaches to such a spacetime signature, namely the 1+1-matrix-brane and the 2+2-target spacetime of a string. This analysis also reveals hidden discrete symmetries of the 2+2-brane action associated with the 2+2-dimensional sector of a 2+10-dimensional target background.

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[1] nieto@uas.uasnet.mx
1.- Introduction

Through the years it has become evident that the 2+2-signature is not only mathematically interesting [1]-[2] (see also Refs. [3]-[5]) but also physically. In fact, the 2+2–signature emerges in several physical context, including self-dual gravity \textit{a la} Plebanski (see Ref. [6] and references therein), consistent \( N = 2 \) superstring theory as discussed by Ooguri and Vafa [7], \( N = (2, 1) \) heterotic string [8]-[12]. Moreover, it has been emphasized [13]-[14] that Majorana-Weyl spinor exists in spacetime of 2 + 2–signature.

More recently, using the requirement of the \( SL(2, R) \) and Lorentz symmetries it has been proved [15] that 2 + 2-target spacetime of a 0-brane is an exceptional signature. Moreover, following an alternative idea to the notion of worldsheets for worldsheets proposed by Green [16] or the 0–branes condensation suggested by Townsend [17] it was also proved in Ref. [15] that special kind of 0-brane called quatl [18]-[19] leads to the result that the 2 + 2-target spacetime can be understood either as 2 + 2-worldvolume spacetime or as 1 + 1-matrix-brane.

Another recent motivation for a physical interest in the 2 + 2-signature has emerged via the Duff’s [20] discovery of hidden symmetries of the Nambu-Goto action. In fact, this author was able to rewrite the Nambu-Goto action in a 2 + 2-target spacetime in terms of a hyperdeterminant, reveling apparently new hidden symmetries of such an action.

Our main goal in this brief note is to establish a connection between the formalisms of references [15] and [20]. Specifically, we contrast the similarities and the differences between these two works. In particular, we construct the matrix linking the 2 + 2-target spacetime of the two scenarios.

2.- The signatures 2 + 2 in the quatl theory

In this section we shall briefly review the quatl theory focusing in the 2 + 2-signature. For that purpose consider the line element

\[
\text{d}s^2 = \text{d}\xi^A \text{d}\xi^B \eta_{AB}. \tag{1}
\]

Here, we shall assume that the indices \( A, B \in \{1, 2, 3, 4\} \) and that \( \eta_{AB} = \text{diag}(1, 1, -1, -1) \) determines the 2 + 2-signature. By defining

\[
\zeta^{11} \equiv \xi^1, \quad \zeta^{22} \equiv \xi^2, \quad \zeta^{12} \equiv \xi^3, \quad \zeta^{21} \equiv \xi^4, \tag{2}
\]

it is not difficult to show that (1) can also be written as

\[\text{d}s^2 = \text{d}\zeta^{11} \text{d}\zeta^{22} \eta_{\zeta^{11}\zeta^{22}}.\]
where \( a, b, m, n \in \{1, 2\} \), and \( \eta_{ab} = \text{diag}(1, -1) \) and \( \eta_{mn} = \text{diag}(1, 1) \). We see that while \( \xi^A \) are coordinates associated with a ‘spacetime’ of signature 2 + 2 the coordinates \( \zeta^{am} \) are associated with a ‘spacetime’ of signature 1 + 1. Thus, the equivalence between the line elements (1) and (3) determines an interesting connection between the signatures 1 + 1 and 2 + 2.

The 2 + 2-brane action is given by

\[
S = \frac{1}{2} \int d\xi^{2+2} \sqrt{-G} \left[ G_{\hat{A}\hat{B}} \frac{\partial x^\hat{\nu}}{\partial \xi^\hat{A}} \frac{\partial x^\hat{\sigma}}{\partial \xi^\hat{B}} \gamma_{\hat{\nu}\hat{\sigma}} - 2 \right],
\]

where \( G_{\hat{A}\hat{B}} = G_{\hat{B}\hat{A}} \) is an auxiliary metric and \( \gamma_{\hat{\nu}\hat{\sigma}} = \gamma_{\hat{\sigma}\hat{\nu}} \) is a metric in a higher dimensional target spacetime.

The action (4) leads to the constraint

\[
\frac{\partial x^\hat{\nu}}{\partial \xi^\hat{A}} \frac{\partial x^\hat{\sigma}}{\partial \xi^\hat{B}} \gamma_{\hat{\nu}\hat{\sigma}} = \frac{G_{\hat{A}\hat{B}}}{2} \left[ G^{\hat{C}\hat{D}} \frac{\partial x^\hat{\nu}}{\partial \xi^\hat{C}} \frac{\partial x^\hat{\sigma}}{\partial \xi^\hat{D}} \gamma_{\hat{\nu}\hat{\sigma}} - 2 \right] = 0,
\]

from which we can derive the expression

\[
G^{CD} \frac{\partial x^\hat{\nu}}{\partial \xi^\hat{C}} \frac{\partial x^\hat{\sigma}}{\partial \xi^\hat{D}} \gamma_{\hat{\nu}\hat{\sigma}} = 4.
\]

Using (6) one sees that (5) yields

\[
\frac{\partial x^\hat{\nu}}{\partial \xi^\hat{A}} \frac{\partial x^\hat{\sigma}}{\partial \xi^\hat{B}} \gamma_{\hat{\nu}\hat{\sigma}} = G_{\hat{A}\hat{B}},
\]

and therefore the action (4) is reduced to the Nambu-Goto action.

It turns out that the constraint (5), and consequently the action (4), can be obtained as a first quantization of the quatl action [15] (see also Refs. [18] and [19])

\[
S = \frac{1}{2} \int d\tau \left\{ \dot{\xi}^A p_A - \sqrt{-G} \left[ G^{AB} p_A p_B - \frac{2}{S + T} \right] \right\}.
\]

Here, \( S \) and \( T \) denotes the number of time and space coordinates of the target spacetime associated with the quatl, respectively. Further, \( p_A \) is the canonical momentum associated with the coordinate \( \xi^A \).

Now, both actions (4) and (8) can be written in terms of coordinates \( \zeta^{am} \) as
\[ S = \frac{1}{2} \int d\zeta^{1+1} \sqrt{-g} \sqrt{-\gamma} \left[ g^{ab} \gamma_{mn} \frac{\partial x^b}{\partial \zeta^{am}} \frac{\partial x^a}{\partial \zeta^{an}} \gamma_{\hat{\nu} \hat{\sigma}} - 2 \right], \] (9)

and

\[ S = \int d\tau \left[ \dot{\zeta}^{am} p_{am} - \frac{1}{2} \sqrt{-g} \sqrt{-\gamma} (g^{ab} \gamma_{mn} p_{am} p_{bn} - \frac{2}{S + T}) \right], \] (10)

respectively. Here, \( g_{ab}(\zeta^{am}) \) and \( \gamma_{mn}(\zeta^{am}) \) are two different nonsymmetric metrics (see Ref. [15] for details) and \( p_{am} \) is the canonical momentum associated with the coordinate \( \zeta^{am} \). The system described by the action (9) is called ‘1 + 1−matrix-brane’ system.

3.- Nambu-Goto action in a 2 + 2-dimensional target spacetime

Let us assume a string moving in a 2 + 2-dimensional target spacetime determined by the flat metric \( \eta_{\mu\nu} = \text{diag}(1, 1, -1, -1) \). The Nambu-Goto action for this system is given by

\[ S = \int d\xi^{1+1} \sqrt{-h}, \] (11)

where \( h \) is the determinant of the matrix

\[ h_{ab} = \frac{\partial x^\mu}{\partial \xi^a} \frac{\partial x^\nu}{\partial \xi^b} \eta_{\mu\nu}. \] (12)

Duff [20] discover that by introducing the matrix

\[ x^{pq} = \begin{pmatrix} x^1 + x^3 & x^4 + x^2 \\ x^4 - x^2 & x^1 - x^3 \end{pmatrix}, \] (13)

the action (11) can be written as

\[ S = \int d\xi^{1+1} \sqrt{-\text{Det}(h_{ab})}, \] (14)

where

\[ \text{Det}(h_{ab}) \equiv \frac{1}{2!} \varepsilon^{i j k l} \varepsilon^{e f} \varepsilon^{g h} \varepsilon^{r s} \varepsilon^{e l m} \varepsilon^{f h j} \varepsilon^{r l c} \varepsilon^{s m d}; \] (15)

with

\[ v_{pq} = \frac{\partial x_{pq}}{\partial \xi^a}. \] (16)
One of the advantages of writing the Nambu-Goto action as in (14) rather than as (11) is that a number of hidden discrete symmetries can be revealed (see Ref. [20] for details).

4.- A relation between the formalisms of sections 2 and 3

In this section we shall discuss a number of ways how we can link the approaches of the previous sections 2 and 3. For this purpose it turns out convenient to define three different kinds of ‘spacetimes’, namely the target spacetime \( \mathcal{T} \), the scenario where a 0-brane moves, the worldvolume \( \mathcal{W}_{s+t} \) associated with a \( s+t \)-brane, and the \( S+T \) background target \( \mathcal{BT}_{S+T} \) spacetime where the \( t+s \)-brane evolves.

In section 2, we considered a connection of the form

\[
\mathcal{T} \leftrightarrow \mathcal{W}_{2+2} \leftrightarrow \mathcal{BT}_{S+T},
\]

which can be obtained after quantizing the quatl system. Here, the symbol \( \leftrightarrow \) means a connection. We also mentioned that: \( \mathcal{W}_{2+2} \) can be related to the \( 1+1 \)-signature. Let us express this result in the form

\[
\mathcal{W}_{2+2} = \mathcal{W}_{(1+1)+(1+1)},
\]

On the other hand the development in section 3 can be summarized symbolically as

\[
\mathcal{W}_{1+1} \leftrightarrow \mathcal{BT}_{2+2} \leftrightarrow \mathcal{BT}_{(1+1)+(1+1)},
\]

It is worth mentioning that (19) is not rigorously a correct connection since quantum consistency of the string theory establishes that the correct link is

\[
\mathcal{W}_{1+1} \leftrightarrow \mathcal{BT}_{S+T=26}.
\]

With the help of the symbolic connections (17)-(19) it comes to be evident that a link between the formalisms of sections 2 and 3 can be established by the simultaneous projections

\[
\mathcal{W}_{(1+1)+(1+1)} \rightarrow \mathcal{W}_{1+1},
\]

and

\[
\mathcal{BT}_{(1+2)+(1+2)} \rightarrow \mathcal{BT}_{(1+1)+(1+1)},
\]
which can be achieved through the so called double dimensional reduction [21]. Therefore, in principle we have the scenario

\[ \mathcal{T} \rightsquigarrow \mathcal{W}_{2+2} \rightsquigarrow \mathcal{W}_{(1+1)+(1+1)} \rightarrow \mathcal{W}_{1+1} \rightsquigarrow \mathcal{B} \mathcal{T}_{(1+2)+(1+2)} \rightarrow \mathcal{B} \mathcal{T}_{(1+1)+(1+1)}. \]  

(23)

This means that form a first quantization of the quatl system one can obtain the theory explained in section 3 and expressed symbolically in (19).

4.- Alternative relation between the formalisms of sections 2 and 3

In this section we shall use the Duff prescription of section 3 to describe an alternative but equivalent formalism for the quatl theory in \(2 + 2\) dimensions. For this purpose let us write the line element (1) in the alternative form

\[ ds^2 = \frac{1}{2} d\lambda^{am} d\lambda^{bn} \varepsilon_{ab} \varepsilon_{mn}, \]  

(24)

where

\[ \chi^{pq} = \left( \begin{array}{cc} \xi^1 + \xi^3 & \xi^4 + \xi^2 \\ \xi^4 - \xi^2 & \xi^1 - \xi^3 \end{array} \right). \]  

(25)

We have

\[ \frac{\partial x^\hat{\nu}}{\partial \xi^A} = \frac{\partial x^\hat{\nu}}{\partial \lambda^{pq}} \frac{\partial \lambda^{pq}}{\partial \xi^A}. \]  

(26)

Therefore we obtain

\[ \eta^{AB} \frac{\partial x^\nu}{\partial \xi^A} \frac{\partial x^\hat{\sigma}}{\partial \xi^B} \gamma_{\hat{\nu} \hat{\sigma}} = \frac{\partial x^\nu}{\partial \lambda^{pq}} \frac{\partial x^\hat{\sigma}}{\partial \lambda^{rs}} \gamma_{\hat{\nu} \hat{\sigma}} \eta^{AB} \frac{\partial \lambda^{pq}}{\partial \xi^A} \frac{\partial \lambda^{rs}}{\partial \xi^B}. \]  

(27)

which by using (25) we get

\[ \eta^{AB} \frac{\partial x^\nu}{\partial \xi^A} \frac{\partial x^\hat{\sigma}}{\partial \xi^B} \gamma_{\hat{\nu} \hat{\sigma}} = 2 \varepsilon^{pr} \varepsilon^{qs} \frac{\partial x^\nu}{\partial \lambda^{pq}} \frac{\partial x^\hat{\sigma}}{\partial \lambda^{rs}} \gamma_{\hat{\nu} \hat{\sigma}}. \]  

(28)

Thus, the transition \( \eta^{AB} \rightarrow G^{AB} \) shall induce the transitions \( \varepsilon^{pr} \rightarrow \varphi_1 g^{pr} \) and \( \varepsilon^{qs} \rightarrow \varphi_2 \gamma^{qs} \) where \( g^{pr} \) and \( \gamma^{qs} \) are two different nonsymmetric metrics (see section 2) and \( \varphi_1 \) and \( \varphi_2 \) are two conformal factors. Observe that the sum of the number of degrees of freedom of \( \varphi_1 g^{pr} \) and \( \varphi_2 \gamma^{qs} \) is equal to the ten degrees of freedom of the symmetric metric \( G^{AB} \).
Considering these preliminaries one finds that the action (4), or (9) which corresponds to the $1+1$–matrix-brane system, can be written in the alternative form

$$S = \frac{1}{2} \int d\lambda^{1+1} \sqrt{-g} \sqrt{-\gamma} \left[ g^{ab} \gamma^{mn} \left( \frac{\partial x^b}{\partial \lambda^a} \frac{\partial x^d}{\partial \lambda^c} \right) - 2 \right]. \quad (29)$$

This action can be obtained from the quatl action

$$S = \int d\tau \left[ \dot{\lambda}^{am} p_{am} - \frac{1}{2} \sqrt{-g} \sqrt{-\gamma} \left( g^{ab} \gamma^{mn} p_{am} p_{bn} - \frac{2}{S + T} \right) \right]. \quad (30)$$

We recognize that (30) has exactly the same form than (10). This suggested that the coordinates $\lambda^{am}$ and $\zeta^{am}$ which describe the motion of the quatl must be related. In fact, using (2) we find

$$\lambda^{pq} = \left( \begin{array}{cc} \zeta^{11} + \zeta^{12} & \zeta^{21} + \zeta^{22} \\ \zeta^{21} - \zeta^{22} & \zeta^{11} - \zeta^{12} \end{array} \right), \quad (31)$$

which leads to a transformation

$$\lambda^{pq} = \Sigma_{pq}^{ab} \zeta^{ab}, \quad (32)$$

where the only nonvanishing components of $\Sigma_{pq}^{ab}$ are

$$\Sigma_{11}^{11} = 1, \quad \Sigma_{12}^{12} = 1, \quad \Sigma_{21}^{11} = 1, \quad \Sigma_{22}^{12} = 1, \quad \Sigma_{21}^{21} = 1, \quad \Sigma_{22}^{22} = -1. \quad (33)$$

Substituting (32) into (24) and using the line element (3) we obtain the formula

$$\frac{1}{2} \Sigma_{pq}^{rs} \Sigma_{cd}^{rs} \varepsilon_{pq} \varepsilon_{cd} = \eta_{ac} \eta_{bd}, \quad (34)$$

which can be verified using (33).

5.- Final comments

In this paper we have established a number of connections between the results of references [15] and [20]. In particular, we have shown that the Duff’s prescription leads to an alternative but equivalent approach to the quatl theory. It comes to be evident that these results reinforces the idea proposed in Ref. [15] that the $2 + 2$ and $1 + 1$ signatures are exceptional structures (see Ref. [22] for a motivation on exceptional structures in mathematics).
From the present work it emerges one more interesting possibility of writing
the Nambu-Goto action in a $2 + 2$-target spacetime. Let us introduce the
coordinates $y^{ab}$ defined in terms of the coordinates $x^A$ as
\[ y^{11} \equiv x^1, \quad y^{22} \equiv x^2, \quad y^{12} \equiv x^3, \quad y^{21} \equiv x^4. \]  
(35)

We find the matrix (12) can also be written as
\[ h_{ab} = \frac{\partial y^m}{\partial \xi^a} \frac{\partial y^n}{\partial \xi^b} \eta_{mn}, \]  
(36)
and therefore the Nambu-Goto action can be written as
\[ S = \int d\xi^{1+1} \sqrt{-\det(h_{ab})}, \]  
(37)
where
\[ \det(h_{ab}) \equiv \frac{1}{2!} \varepsilon^{ie} \varepsilon^{jd} \eta^{ef} \eta^{gh} \eta^{rs} \eta^{lm} u_{egi} u_{fhj} u_{rlc} u_{smd}, \]  
(38)
with
\[ u_{pq\alpha} = \frac{\partial y_{pq}}{\partial \xi^\alpha}. \]  
(39)

Of course, using the prescription (34) one can prove that the action (37) and
(14) are equivalents. It remains to see whether the action (37) may be helpful
for having a better understanding of the hidden symmetries of the Nambu-
Goto action described by Duff [20].

Let briefly comment on the scenario in which the present discussion may
have physical implications. It turns out that the $2 + 10$-dimensional spacetime
signature has emerged as one of the most interesting possibilities for the
understanding of both supergravity and super Yang-Mills theory in $D = 11$.
For our considerations what is important is that the $2 + 10$-dimensional theory
seems to be the natural background for the $2 + 2$-brane (see Ref. [23] and
references therein). Thus, let us think in the possible transition
\[ M^{2+10} \rightarrow M^{2+2} \times M^{0+8}, \]  
(40)
which, in principle, can be achieved by some kind of symmetry breaking applied
to the full metric of the spacetime manifold $M^{2+10}$. It has been shown
that the symmetry $SL(2, R)$ makes the $2 + 2$-signature an exceptional one [15].
On the other hand, the signature $0 + 8$ is euclidean and in principle can be
treated with the traditional methods such as the octonion algebraic approach.
In pass, it is interesting to observe that octonion algebra is also exceptional in the sense of the celebrated Hurwitz theorem. Thus, we see that both $2 + 2$ and $0 + 8$ are exceptional signatures. This means that the transition (40) is physically interesting.

Consider now the action (29) in the particular case of $2 + 10$-dimensional target spacetime background

$$S = \frac{1}{2} \int d\lambda^{1+1} \sqrt{-g}\sqrt{-\gamma} \left[ g^{ab} \gamma^{mn} \frac{\partial x^A}{\partial \lambda^am} \frac{\partial x^B}{\partial \lambda^bn} \eta_{AB} - 2 \right], \quad (41)$$

where we have replaced the curved metric $\gamma_{\hat{\nu}\hat{\sigma}}$ by the flat metric $\eta_{\hat{\nu}\hat{\sigma}}$ and we assume that the indices $\hat{\nu}, \hat{\sigma}$ now run from 1 to 12. Splitting the flat metric $\eta_{\hat{\nu}\hat{\sigma}}$ according to the transition $2 + 10 \rightarrow (2 + 2) + (0 + 8)$ we find that (41) can be written as

$$S = S_1 + S_2, \quad (42)$$

where

$$S_1 = \frac{1}{2} \int d\lambda^{1+1} \sqrt{-g}\sqrt{-\gamma} \left[ g^{ab} \gamma^{mn} \frac{\partial x^A}{\partial \lambda^am} \frac{\partial x^B}{\partial \lambda^bn} \eta_{AB} \right], \quad (43)$$

and

$$S_2 = \frac{1}{2} \int d\lambda^{1+1} \sqrt{-g}\sqrt{-\gamma} \left[ g^{ab} \gamma^{mn} \frac{\partial x^A}{\partial \lambda^am} \frac{\partial x^B}{\partial \lambda^bn} \eta_{AB} - 2 \right]. \quad (44)$$

Here, the indices $A, B$ run from 1 to 4 and $\hat{A}, \hat{B}$ run from 5 to 12. Employing (13), we can make the change $x^A \rightarrow x^{pq}$ which allows us to write the action (43) in the form

$$S_1 = \frac{1}{2} \int d\lambda^{1+1} \sqrt{-g}\sqrt{-\gamma} \left[ g^{ab} \gamma^{mn} a_{pqam} a_{rsm} \varepsilon^{pr} \varepsilon^{qs} \right]. \quad (45)$$

It turns out that the variables $a_{pqam}$ generalize the equivalent Duff’s definition given in the expression (2.4) of Ref. [20]. Thus, one should expect additional discrete symmetries. In fact, in principle, we should have eight possible transformations
\[a_{pqam} \rightarrow a_{apqm}, \quad a_{pqam} \rightarrow a_{aqpm};\]
\[a_{pqam} \rightarrow a_{aqpm}, \quad a_{pqam} \rightarrow a_{pqam},\]
\[a_{pqam} \rightarrow a_{mpqa}, \quad a_{pqam} \rightarrow a_{qmpa},\]
\[a_{pqam} \rightarrow a_{mpqa}, \quad a_{pqam} \rightarrow a_{pmqa}.\]

which represent discrete symmetries. Therefore, we see that at least in the
2 + 2-dimensional sector of the full 2 + 10-dimensional theory the present
development provides a physical application in terms of the extended discrete symmetries (46).

It may be interesting for further research to understand the present develop-
ment from the point of view of supersymmetry and superconfor mal group.
In particular it appears interesting to investigate 1 + 1-matrix-brane system
from the point of view of the 2-brane case at the end of the De Sitter universe
due to Batrachenko, Duff and Lu [24]. Finally, since self-duality seems to be
deeply connected with the 2 + 2-signature [1]-[2] one may be interested to find
the self-dual aspects of the 1 + 1-matrix-brane system.

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