Magneto-thermoelectric effects in rectangular quantum wire with an infinitely high potential in the presence of electromagnetic wave (electron - acoustic phonon interaction)

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Abstract. Based on the quantum kinetic equation for electrons, we have theoretically studied the influence of a Strong Electromagnetic Wave on the Ettingshausen Effect (EC) in a rectangular quantum wire with an infinite potential (RQWIP). We obtain the analytic expressions for the kinetic tensor as well as the Ettingshausen coefficient in the rectangular quantum wire with an infinite potential with the dependence on B and Ω.... The results are numerically evaluated and graphed for GaAs/GaAs:Al quantum wire. We survey the electrical and thermal conductivity tensor depend on Electromagnetic Wave frequency and temperature. The results give us appearance of the Shubnikov–de Haas oscillations when we survey the dependence of Ettingshausen coefficient on the magnetic field. Then, we realize that as the temperature increases, the Ettingshausen coefficient decreases. This shows that the Electromagnetic Wave have a clear impact on the effects. These are latest results which have been studied in terms of Ettingshausen effect in rectangular quantum wire.

1. Introduction

In recent years, the semiconductor materials have been widely used in electronics. The development of semiconductor electronics is mainly based on the phenomenon of contact p-n and the doped ability to alter the physical properties of crystals. The particle motion is limited along specific coordinates within a tiny area of less than hundreds of Å. If the size of the area is comparable with De Droglie wavelength of the particle, the energy spectrum and wave function will be quantized. Then, the size effect appears and makes almost physical properties of material change [1]. The properties of low-dimension system such as: Hall effect [2-4], absorption of electromagnetic waves, relative magnetoresistance, etc [5-9] are very different from the previous work studied on bulk semiconductor [10-13]. The magneto-thermoelectric effect, often called as Ettingshaussen effect, which has just been researched in bulk semiconductors [14-17] is one of the electrical, magnetic and thermal effects of semiconductor systems. Furthermore, the magneto-thermoelectric effect in low-dimension systems has just been studied in two-dimension system (quantum well) [18] and the impact of the strong EMW hasn’t been considered yet. Moreover, previous works on these effects use...
the classical method such as: Boltzmann kinetic equation \[19\], Kubo-Mori method, which is limited in high temperature condition.

So, in this research, we use the quantum kinetic equation to calculate the EC in RQWIP, which is used a lot in studies about one-dimension system, under the influence of electromagnetic wave. We have discovered some differences between the results obtained in this case and those in the case of the bulk semiconductors. Numerical calculations are carried out with a specifically rectangular quantum wire GaAs/GaAs:Al. With the limitation in other low-dimension systems, we obtained the results close to previous studies. Then, when surveying the EC in some parameters, we explored the new point which is only acquired in our work.

2. Calculation of Ettingshausen coefficient in rectangular quantum wire in the presence of electromagnetic wave

In this report, we consider a rectangular quantum wire of the normalization length \(L_x, L_y\) with the infinite confining potential: \(V(\vec{r}) = 0\) inside the wire and \(V(\vec{r}) = \infty\) elsewhere subjected to a crossed electric field \(E_z = (0, 0, E_z)\) and magnetic field \(B = (0, B, 0)\) in the presence of a strong EMW (laser radiation) characterized by electric field \(E = (0, 0, E_0\sin\Omega t)\).

\[
\psi_{n,l,p_z}(x,y,z) = \begin{cases} 
\frac{1}{L_z}e^{ip_z z} \sqrt{2 \sin \frac{n\pi x}{L_x}} \sqrt{2 \sin \frac{n\pi y}{L_y}}, & 0 \leq x \leq L_x, 0 \leq y \leq L_y, \\
0, & \{x > L_x \} \cup \{y > L_y \}
\end{cases}
\]

\[
\epsilon_{n,l,p} = \frac{\hbar^2 p^2}{2m^*} + \frac{\pi^2 \hbar^2}{2m^*} \left( \frac{n^2}{L_x^2} + \frac{l^2}{L_y^2} \right); \quad n,l = 0,1,2, ...
\]

Here: \(\vec{p}\) is the wave vector in the z-direction, \(L_x, L_y\) are the RQW normalization length in the x,y-direction.

The Hamiltonian of the electron-acoustic phonon system in RQW in the second quantization presentation can be written as:

\[
H = \sum_{n,l,p} \epsilon_{n,l,p} \left( \hat{a}_{n,l,p}^+ \hat{a}_{n,l,p} \right) + \sum_{n,l,n',l'} \sum_{\vec{p},\vec{k}} M(\vec{k}) a_{n,l',\vec{p}+\vec{k}}^+ a_{n,l,\vec{p}} (b_{\vec{k}}^+ + b_{-\vec{k}}) + \sum_{n,l,n',l'} \sum_{\vec{p},\vec{k}} \phi(\vec{k}) a_{n,l',\vec{p}+\vec{k}}^+ a_{n,l,\vec{p}} \quad (1)
\]

Where \(\vec{A}(t)\) is the vector potential of laser field, \(\hbar \omega_{\vec{k}}\) is the energy of an acoustic phonon with the wave vector \(\vec{k} = (k_x, k_z)\), \(a_{n,l,\vec{p}}^+\) and \(a_{n,l,\vec{p}}\) (\(b_{\vec{k}}^+\) and \(b_{\vec{k}}\)) is the creation and annihilation operators of electron (phonon), respectively.

\[
M(\vec{k}) = |C(\vec{k})|^2 |l_{n,n',l',\vec{p}}(\vec{k})|^2 |l_{1l'}(\vec{u})|^2 \quad \text{with} \quad C(\vec{k}) \quad \text{is the electron-phonon interaction constant which depends on the scattering mechanism}, \quad |l_{n,n'}(\pm \vec{k})| \quad \text{is the form factor of electron, given by:}
\]
The current density

\[ j = \int_0^\infty \tilde{R}(\varepsilon) \, d\varepsilon ; \quad \tilde{q}_e = \frac{1}{e} \int_0^\infty (\varepsilon - \varepsilon_F) \tilde{R}(\varepsilon) \, d\varepsilon ; \quad J_l = \sigma_{ik} E_k + \beta_{ik} \nabla T_k; \quad q_l = \gamma_{ik} E_k + \xi_{ik} \nabla T_k \]
From the current density and thermal flux density formula, we obtain the kinetic tensor \( \sigma_{ik}, \beta_{ik}, \gamma_{ik}, \xi_{ik} \). We can have the expression of the EC:

\[
P = \frac{1}{N \sigma_{xx}} \frac{\sigma_{xx} \gamma_{xx} - \sigma_{yy} \gamma_{yy}}{[\beta_{xx} \gamma_{xx} - \sigma_{xx} (\xi_{xx} - \xi_{kk})]}
\]

(4)

Where:

\[
\sigma_{ij} = a \frac{\tau(\varepsilon_F)}{1 + \omega_c \tau(\varepsilon_F)^2} \sum_{n,l} \left\{ \delta_{ik} + \omega_c \tau(\varepsilon_F) \xi_{ikj} + \omega_c^2 \tau(\varepsilon_F)^2 h_i h_k \right\}
\]

\[
+ b \sum_{n,l,n',l'} \frac{l_{n,l,n',l'}^2}{\hbar^2} \left\{ \frac{\omega_c^2 \tau(\varepsilon_F) + \hbar \Omega}{1 + \omega_c^2 \tau(\varepsilon_F + \hbar \Omega)} \left[ \delta_{ik} + \omega_c \tau(\varepsilon_F + \hbar \Omega) \gamma_{ikj} + \omega_c^2 \tau(\varepsilon_F + \hbar \Omega)^2 h_i h_k \right] \right\}
\]

\[
\beta_{ij} = b \sum_{n,l,n',l'} \frac{l_{n,l,n',l'}^2}{\hbar^2} \left\{ \frac{\omega_c^2 \tau(\varepsilon_F + \hbar \Omega)}{1 + \omega_c^2 \tau(\varepsilon_F + \hbar \Omega)} \left[ \delta_{ik} + \omega_c \tau(\varepsilon_F + \hbar \Omega) \gamma_{ikj} + \omega_c^2 \tau(\varepsilon_F + \hbar \Omega)^2 h_i h_k \right] \right\}
\]

\[
\gamma_{ij} = a \frac{\tau(\varepsilon_F)}{1 + \omega_c \tau(\varepsilon_F)^2} \sum_{n,l} \left\{ \delta_{ik} + \omega_c \tau(\varepsilon_F) \xi_{ikj} + \omega_c^2 \tau(\varepsilon_F)^2 h_i h_k \right\}
\]

\[
+ b \sum_{n,l,n',l'} \frac{l_{n,l,n',l'}^2}{\hbar^2} \left\{ \frac{\omega_c^2 \tau(\varepsilon_F + \hbar \Omega)}{1 + \omega_c^2 \tau(\varepsilon_F + \hbar \Omega)} \left[ \delta_{ik} + \omega_c \tau(\varepsilon_F + \hbar \Omega) \gamma_{ikj} + \omega_c^2 \tau(\varepsilon_F + \hbar \Omega)^2 h_i h_k \right] \right\}
\]

\[
\xi_{ij} = b \frac{\varepsilon_F^2}{e} \sum_{n,l,n',l'} \left\{ \frac{l_{n,l,n',l'}^2}{\hbar^2} \left\{ \frac{\omega_c^2 \tau(\varepsilon_F + \hbar \Omega)}{1 + \omega_c^2 \tau(\varepsilon_F + \hbar \Omega)} \left[ \delta_{ik} + \omega_c \tau(\varepsilon_F + \hbar \Omega) \gamma_{ikj} + \omega_c^2 \tau(\varepsilon_F + \hbar \Omega)^2 h_i h_k \right] \right\} \right\}
\]

Here: \( \tau \) is the momentum relaxation time, \( \delta_{ik} \) is the Kronecker delta, \( \xi_{ikj} \) being the antisymmetric Levi-Civita tensor; the Latin symbols \( i,j,k \) stand for the components \( x,y,z \) of the Cartesian coordinates.

\[
a = e^2 h^2 L_y \sqrt{\Delta_0} / 2 \pi m^2; \quad b = \frac{e L_y h^4 \varepsilon^2 a^2}{768 \pi^2 m^3 \rho_{ps} \sqrt{\Delta_0}}
\]

\[
\Delta_0 = \frac{2 \varepsilon_F m^*}{\hbar^2} - \pi^2 \left( \frac{n^2}{L_x} + \frac{l^2}{L_y} \right); \quad x_1 = \sqrt{\Delta_0}; \quad x_2 = -\sqrt{\Delta_0};
\]
\[ I = x_1 \frac{c_1^5 + d_1^5}{\sqrt{\Delta_{1a}}} + x_2 \frac{c_2^5 + d_2^5}{\sqrt{\Delta_{1b}}}; c_1 = x_1 + \sqrt{\Delta_{1a}}; \\
\]
\[ d_1 = x_1 - \sqrt{\Delta_{1a}}; c_2 = x_2 + \sqrt{\Delta_{1b}}; d_2 = x_2 - \sqrt{\Delta_{1b}}. \]

\[ II = x_1 \frac{u_1^5 + v_1^5}{\sqrt{\Delta_{2a}}} + x_2 \frac{u_2^5 + v_2^5}{\sqrt{\Delta_{2b}}}; u_1 = x_1 + \sqrt{\Delta_{2a}}; \]

\[ v_1 = x_1 - \sqrt{\Delta_{2a}}; u_2 = x_2 + \sqrt{\Delta_{2b}}; v_2 = x_2 - \sqrt{\Delta_{2b}}. \]

\[ III = x_1 \frac{m_1^5 + q_1^5}{\sqrt{\Delta_{3a}}} + x_2 \frac{m_2^5 + q_2^5}{\sqrt{\Delta_{3b}}}; m_1 = x_1 + \sqrt{\Delta_{3a}}; \]

\[ q_1 = x_1 - \sqrt{\Delta_{3a}}; m_2 = x_2 + \sqrt{\Delta_{3b}}; q_2 = x_2 - \sqrt{\Delta_{3b}}; \hat{a} = \frac{eE_0}{m^*\Omega^2}. \]

Here: \( v_s, E_d \) and \( \rho \) are the sound velocity, the deformation potential constant and the mass density, respectively. \( \varepsilon_F \) is the Fermi level, \( k_B \) is the Boltzmann constant. From Eq. (4), we see that the EC expression in the RQW is more complicated than that in the bulk semiconductor. We also found that the difference in the energy spectrum, the wave function and the presence of electromagnetic waves which lead to this complexity. In the next step, we study quantum wire of GaAs/GaAs:Al to see clearly the dependence mentioned above.

3. Numerical results and discussion

In this section, we present detailed numerical calculations of the EC in a RQW subjected to the uniform crossed magnetic and electric fields in the presence of a strong EMW. For the numerical evaluation, we consider the RQW of GaAs/GaAs:Al with the parameters[7,8]: \( \varepsilon_F = 50 \text{meV}, m = 0.067m_0 \) (\( m_0 \) is mass of a free electron), \( \tau = 10^{-12}s, L_x = L_y = 10^{-9}\text{m}. \)

First of all, from the physical expression of each kinetic tensors, we can just do research on

![Figure 1a: The dependence of electrical and thermal conductivity tensor on EMW frequency.](image)
two tensors, one stands for the electric current ($\sigma$) and the other stands for the thermocurrent ($\xi$). The left tensors $\gamma, \beta$ have the same meanings with $\sigma, \xi$, respectively, so we don’t need to research on them. As can be seen from figure 1a and figure 1b, we survey both electrical and thermal tensors. Initially, the electrical conductivity tensor depends clearly on magnetic field in low EMW frequency domain and about the same in high EMW frequency. When the EMW frequency increases from 0 to 10 (THz), the conductivity tensor rises slowly in high magnetic field condition and in low magnetic field condition, it changes very fast and almost linearly to reach the horizontal line. In comparison to [20], the result is similar in both the influence of the EMW frequency on the tensor and the tensor value, specially in high magnetic field condition. Furthermore, when in $\omega = 7 - 8$ Hz, we can see the Shubnikov-de Hass oscillation, obviously in strong magnetic field and low temperature ($T = 60K$) conditions.

Next, the thermal conductivity tensor tends to decrease when the temperature goes up. The tensor value in low temperature and under the presence of EMW condition is high, it plunges in the first-half period and slowly inclines as a parabolic curve that totally the same as the result obtained in [21]. And when we hide the EMW and put the wire length of 9nm, the thermal conductivity tensor is small and almost zero, this is the influence of thermal conductivity in temperature in previous studies [22]. These results show that the thermal conductivity tensor which obtained from quantum kinetic method is the same as the one from previous methods.

So that, the result for both the electrical and thermal conductivity tensors is close to the result in previous researches and the truthfulness of our work is assured and enhanced.

![The dependence of electrical and thermal conductivity tensor on temperature](image)

*Figure 1b: The dependence of electrical and thermal conductivity tensor on temperature.*
Figure 2 shows the dependence of the EC on the magnetic field in two cases: with and without the presence of electromagnetic waves; at three different temperature points. From the graph, we see that the oscillation appears which is controlled by the ratio of the Fermi energy level and cyclotron energy level. Literally, the appearance of the oscillation is the influence of De Haas-van Alphen effect [23] and can be easily explained as follows. At low temperature and strong magnetic field, the free electrons in metals, semiconductors will move as simple harmonic oscillators. When the magnetic field changes, the cycle of the oscillations also changes. The energy levels of electrons are separated into Landau levels, with each level Landau, cyclotron energy and the electron state linearly increase with the magnetic field. When the energy level of the Landau levels excesses the value of Fermi level, the electron can move up freely and move in the line, which makes the EC oscillate circulating with magnetic field. Moreover, the effect of electromagnetic waves on the EC is clearly observed. The value

![Figure 2: The dependence of Ettingshausen coefficient on magnetic field](image-url)
of the EC is the same in the domain with small magnetic field (under 0.15 (T)) and it is very different in the strong magnetic field domain. Then, in the domain between 0.15 (T) and 0.2 (T), the blue line fluctuates and reaches the resonant point while B is just under 0.18 (T). It can be clearly seen that, peaks of the blue line (with EMW) are much taller than peaks of the red line (without EMW) at same magnetic field points. Whereas, the higher temperature is, the more wildly EC fluctuates. This also leads to the conclusion that temperature impacts remarkably on Ettingshausen effect.

In the Figure 3, the dependence of the EC in RQWIP on temperature is nearly linear. The EC decreases as the temperature increases. This is consistent with the experimental result obtained in the bulk semiconductors case [1]. However, in the bulk semiconductors, EC has positive value, whereas the EC in QWPP on temperature has negative value. This result is due to the difference in structure, wave function and energy spectrum of RQWIP in comparison with the bulk semiconductors. Also, the presence of electromagnetic waves influence on the EC weakly, the EC value is the same in the domain of low temperature and have different values in the region with higher temperatures. This result is consistent with those previously reported by using Boltzmann kinetic equation [1]. However, Boltzmann kinetic equation applies only in high temperature conditions, which is the limitation of the Boltzmann kinetic equation. So, we use quantum equations to overcome the above limitations.

![Figure 3: The dependence of Ettingshausen coefficient on temperature](image)

*Figure 3: The dependence of Ettingshausen coefficient on temperature $T = 30K \div 1000K$*
Figure 4 shows the dependence of EC on the EMW frequency with $\omega = 0 \div 100$(THz). As can be seen from the graph, the EC oscillates in strong magnetic field condition. In each case of magnetic field, the EC reach a peak with specific value of EMW frequency. When magnetic field value increases, both EC peak and $\omega$ EC peak position tends to upwards. This result is one of the new findings that we have studied.

Figure 4: The dependence of Ettingshausen coefficient on the EMW frequency

4. Conclusion

In this paper, we have analytically investigated EC in rectangular quantum wire. The electron-phonon interaction is taken into account at both low and high temperatures. We expose the analytical expressions of the coefficients EC in RQWIP. The results have been evaluated in GaAs/Al:GaAs RQWIP to see the EC's dependence on electromagnetic wave, temperature, magnetic field and the dependence of electrical and thermal conductivity tensor on EMW frequency and temperature, respectively.

The results showed that the EC decreases linearly with temperature and the EC has a negative value. When surveying the dependence of EC on magnetic field, we saw the appearance of Shubnikov–de Haas oscillations in strong magnetic field domain. Then, the impact of temperature in EC is remarkable since high temperature values give us high oscillation peaks.

When we studied the dependence of EC on EMW frequency, we noted that EC oscillates in strong magnetic field condition. The stronger magnetic field is the taller EC oscillation peak is and bigger EMW frequency value at resonant peak is.
Surveying both electrical and thermal conductivity tensor, we saw the line graph and the dependence of electrical and thermal conductivity tensor on EMW frequency and temperature are almost similar to previous researches when we changed the parameters to the bulk semiconductor and other low dimensional system cases.

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