Application of WMLES to wall-bounded flows with pressure gradient

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Abstract. Two models, an algebraic WMLES and IDDES, were tested on their ability to predict wall-bounded flows with pressure gradient. A developed flow between two moving flat plates under an adjusted pressure gradient (Couette-Poiseulle flow) was used for testing. Within two considered test cases three types of boundary layers were examined: boundary layer with adverse pressure gradient (APG), boundary layer with favorable pressure gradient (FPG) and a wall-bounded flow with negligible skin-friction. It was shown that both models are capable of accurate prediction of the flow structure and mean parameters of boundary layers with APG and FPG and of frictionless wall-bounded flow. The difference between simulation results and DNS data is less than 7%.

1. Introduction
Methods based on the Reynolds Averaged Navier-Stokes equations (RANS) are known to be capable of providing accurate mean flow parameters for turbulent wall-bounded flows at the lowest computational expense possible. The very nature of RANS approaches however does not allow prediction of statistical parameters of the flow and for that purpose scale-resolving approaches, for example, and Large Eddy Simulation (LES) should be used. However, for wall-bounded flows at high Reynolds numbers LES requires unaffordable computational resources. This has resulted in development of hybrid RANS-LES approaches, which can be considered as a compromise between the high accuracy of LES and low computational costs of RANS. One of such approaches is Wall-Modelling LES (WMLES) which is based on the use of RANS in the near-wall regions of boundary layers and LES in the outer part, thus reducing required computational resources compared to pure LES. These methods were widely tested on simple flows such as plane channel and zero pressure gradient boundary layers and had proven to be a powerful computational tool. However, since they are relatively new, validation studies of WMLES applied to flows with pressure gradient is still rather rare.

The goal of the present work is to test accuracy of WMLES in prediction of parameters of wall-bounded flows with the pressure gradient. A developed flow between two moving walls with adjusted pressure gradient, the so-called Couette-Poiseulle flow, is under consideration. The practical advantages of this test case are simple geometry, periodic character of the flow, and independency of developed solution from the initial flow. The Couette-Poiseulle flow was examined using Direct Numerical Simulation (DNS) [1][2][3] and the data for a range of Reynolds numbers and pressure gradients are available.
In these flows the walls move in opposite directions, therefore depending on the adjusted pressure gradient, there are two boundary layers, each with its own pressure gradient. Two flow regimes are considered in the paper. In the first test case there are two boundary layers, one of which is under the influence of the favorable pressure gradient (FPG) while the other one is influenced by the adverse pressure gradient (APG). This circumstance allows examining the capabilities of WMLES for predicting wall bounded flows with both APG and FPG within one rather simple test case. In the second test case the boundary layer at the lower wall is under the influence of FPG while at the upper wall the APG ensures an almost zero friction.

Two WMLES models were considered: algebraic WMLES based on the Prandtl and Smagorinsky models [4], and IDDES based on the $k$-$\omega$ SST model [4], which works in WMLES mode for this specific flow. Two computational codes were used: the in-house NTS code [5], and the commercial general-purpose CFD code ANSYS-FLUENT [6].

2. Problem definition

Developed incompressible flow between two plates moving in opposite directions with the velocity $U_w$ under the pressure gradient $dp/dx$ (fig. 1) is considered. The flow regime is defined by two parameters: the Reynolds number (in the present study Re is based on the wall velocity $U_w$ and half of the channel height $h$) and the adjusted pressure gradient $d(p/\rho)/dx$.

Parameters of two considered test cases are presented in table 1. As mentioned above, each test case has two wall-bounded flows, along upper and lower walls, with different pressure gradients, which is characterized by the nondimensional parameter $p^+ = [d(p/\rho)/dx][\nu/\tau]_3$ (values $p^+$ for each wall are also presented in table 1).

Mean velocity profiles obtained with DNS [2][3] of two considered cases are presented in figure 1, which illustrates the asymmetry of both velocity profiles and the differences in the flow character on the upper wall.

![Figure 1](image-url)  
**Figure 1.** Scheme of the problem configuration, and profiles of DNS mean velocity profiles [2][3] for two considered cases

The size of the computational domain was $4\pi h \times 2h \times 2\pi h$ which is considered to be sufficient based on the DNS study [3]. Periodic boundary conditions were applied in the streamwise and spanwise directions, the upper and lower boundaries were considered as moving nonslip walls with the specified velocity $U_w$ and $-U_w$ for upper and lower walls respectively. Pressure gradient was applied by adding a source term to the momentum equation.

| Case   | $Re = U_w h/\nu$ | $(dp/dx) \cdot \rho h / U^2_w$ | $p^+_{\text{upper wall}}$ | $p^+_{\text{lower wall}}$ |
|--------|-----------------|-----------------------------|------------------------|------------------------|
| Case 1 | 20000           | -0.0014                     | +3 $\cdot$ 10^{-3}    | -3 $\cdot$ 10^{-4}    |
| Case 2 | 12000           | -0.003970                   | +$\infty$              | -6 $\cdot$ 10^{-4}    |
A computational grid with 18 million cells was used for simulations. The details of the grid are presented in table 2. Streamwise and spanwise steps satisfy common WMLES requirements for a periodic channel flow and allow resolution of the fine turbulent structures (fig. 2). The mean solution was checked to be grid independent. The time step for simulations ensured the Courant number was less than 0.8; the statistics were gathered for at least 500$h/\mu$ after a transient period of $500h/\mu$.

Table 2. Grid parameters

| $N_x \times N_y \times N_z$ | $\Delta x$ | $\Delta y$ | $\Delta z$ | $\Delta y_z$ |
|---------------------------|------------|------------|------------|-------------|
| 353x145x353               | 0.036$h$   | 0.018$h$   | 0.025$h$   | $6\times10^{-4}$h for both walls |

![Figure 2. Instantaneous vorticity magnitude in xy plane](image)

3. Results of the simulations

3.1. Comparison of different code results

To put this study on a firmer foundation, before evaluation of the accuracy of WMLES, a comparison of the results obtained with the use of two computational codes, NTS and ANSYS FLUENT, was performed using the same grid and the same model (IDDES). Mean velocity profiles obtained with different codes are presented in figure 3. The differences noticeable in log layer region of velocity profiles (about 3%) are most likely explained by the differences in numerical schemes used in different codes. Note that a low dissipation 4th order central-difference (CD) scheme was used in NTS code while in FLUENT the 2nd order CD scheme was used which has more numerical dissipation, the latter is known to influence results of scale-resolving simulations.

![Figure 3. Comparison of the mean velocity profiles obtained with NTS and ANSYS FLUENT codes and scaled with respect to (a) wall velocity $U_w$, (b) APG side wall units and (c) FPG side wall units](image)

3.2. Comparison with the DNS results

Large-scale longitudinal turbulent structures were observed in the DNS studies [2][3] of Couette-Poiseuille flow. Present study showed that WMLES is capable of capturing these structures as well. This is demonstrated by instantaneous velocity fields in $zy$ plane obtained with algebraic WMLES and with DNS for Case 1 (fig. 4). The presence of these coherent roll-type structures suggests that WMLES captures the structure of the Couette-Poiseuille flow quite well and that the spanwise size of the domain is large enough to containe these roll-type cells.
Figure 4. Instantaneous streamwise velocity $u$ in $zy$ plane obtained with algebraic WMLES (left) and with DNS [2] (right).

Comparison of time- and space-averaged results of simulations for test Case 1 with the DNS data is presented in figures 5-6. Mean velocity profile comparison shows that at the FPG side WMLES predictions (for both models) are closer to the DNS data than at the APG side. Note that at the APG side velocity predicted by the WMLES models is much closer to the Richardt’s law for boundary layer than to DNS data (the difference is about 7%). In contrast, profiles of total shear stresses are in good agreement with the DNS data (figure 6). Note that results of the two models are nearly the same.

Figure 5. Comparison of the mean velocity profiles obtained with IDDES and algebraic WMLES models and scaled with respect to (a) wall velocity $U_w$, (b) APG side wall units and (c) FPG side wall units with the DNS data [2]

Figure 6. Comparison of the total (resolved + modelled) shear stress profiles obtained with IDDES and algebraic WMLES models and scaled with respect to the square of (a) wall velocity $U_w$, (b) APG side wall units and (c) FPG side wall with the DNS data [2]

Time- and space-averaged results of simulations for Case 2 is presented in figures 7-8. Mean velocity profile scaled with respect to the wall velocity is in good agreement with the DNS data (fig. 7a) as well as the logarithmic velocity profile scaled with respect to FPG side wall units (fig. 7b).

At the upper wall skin friction is essentially zero therefore normalization of the velocity can’t be done with the use of friction. For zero friction Couette-Poiseuille flow Stratford [7] proposed a universal (independent of the Reynolds number) velocity law based on the pressure gradient instead of skin friction:
\[ U^- = F(y^-). \]  
(1)

where \( U^- \equiv U/u_p \), \( y^- \equiv y_w u_p / \nu \), \( y_w \) is the wall-normal distance to the frictionless wall, \( u_p^3 \equiv v d(P/\rho)/dx \) and \( \rho \) is the fluid density. Stratford [7] also proposed the velocity profile will include a square-root layer with:

\[ U^- = B \sqrt{y^-} + C \]  
(2)

Estimation of the values of constants \( B, C \) differs from study to study; for example, study [3] proposed an approximate range \([2.25, 2.50]\) for \( B \) and \( C \approx -2.2 \).

The mean velocity profile in Stratford units obtained with the use of WMLES and its comparison with the DNS data and with velocity law (2) are presented in figure 7c. It can be seen that results of the present simulations are in good agreement with DNS data in the square-root layer, while in the area around \( y^- \approx 5 - 6 \), where the profile changes to the logarithmic law (in FPG wall units), the difference from DNS is maximal and is about 7%.

Profiles of shear stresses are presented in figure 8. At the FPG side in the near wall region the stresses are slightly overestimated, but the overall agreement is quite good. Note also that the shear stresses in Stratford units (fig. 8c) agrees perfectly with the theoretical profile \(-u'v' = y^+\), as required by the momentum balance.

### Conclusions

Simulations of the Couette-Poiseuille flow with the use of WMLES approach showed that this method is capable of accurate predictions of wall-bounded flows with a pressure gradient. WMLES allows predicting large-scale longitudinal turbulent structures observed in DNS studies. Results of both considered models are in good qualitative and quantitative agreement with the available DNS data.
the boundary layers with FPG, results of simulations turned out to be more accurate than for boundary layer with APG and for flow with zero friction. Maximum difference between results of WMLES and DNS is about 7%.

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