The Unruh effect and entanglement generation for accelerated atoms near a reflecting boundary

Jialin Zhang and Hongwei Yu∗

Department of Physics and Institute of Physics,
Hunan Normal University, Changsha, Hunan 410081, China

Abstract

We study, in the framework of open systems, the entanglement generation of two independent uniformly accelerated atoms in interaction with the vacuum fluctuations of massless scalar fields subjected to a reflecting plane boundary. We demonstrate that, with the presence of the boundary, the accelerated atoms exhibit distinct features from static ones in a thermal bath at the corresponding Unruh temperature in terms of the entanglement creation at the neighborhood of the initial time. In this sense, accelerated atoms in vacuum do not necessarily have to behave as if they were static in a thermal bath at the Unruh temperature.

PACS numbers: 04.62.+v, 03.65.Ud, 03.65.Yz, 03.67.Mn

∗ Corresponding author
I. INTRODUCTION

It is now well established that uniformly accelerated observers (atoms) perceive as a thermal bath of particles at a temperature proportional to the proper acceleration what an inertial observer sees as a vacuum [1, 2, 3]. This result is known as the Unruh effect and it seems to imply that the accelerating detectors (atoms) may be viewed as an open system, i.e., a system immersed in an external thermal bath.

On the other hand, quantum entanglement has been recognized as a unique quantum resource whose production can be employed for computational and communication purposes, such as quantum communication [4], quantum teleportation [5], quantum cryptography [6] and so on. The relationship between entanglement and environment is an intriguing issue in the discussions for the essence of entanglement. In this regard, it is known that an environment usually leads to decoherence and noise, which may cause entanglement that might have been created before to disappear. However, in certain circumstances, the environment, such as thermal baths belonging to a specific class, may enhance entanglement rather than destroying it [7, 8, 9, 10, 11, 12]. The reason is that an external environment can also provide an indirect interaction between otherwise totally uncoupled subsystems through correlations that exist.

Therefore, a question arises naturally as to whether entanglement can be produced for independent accelerated atoms in vacuum, as entanglement generation through the action of an external thermal bath has been shown to occur. Although not directly coupled, a sea of vacuum fluctuations of external fields through which atoms move may provide an indirect interaction to generate entanglement among them. Consequently, there arises a new possibility for an experimental test of the Unruh effect by using appropriate quantum optics devices to detect the entanglement generated by the uniform acceleration.

Recently, entanglement generation for two, independent uniformly accelerating two-level atoms with vanishing separation interacting with a set of scalar fields in vacuum has been examined, by Benatti et al [13], in the framework of open systems. In the weak coupling limit, the completely positive dynamics for the atoms as a subsystem has been derived by
tracing over the field degrees of freedom [13], and there it has been shown that the asymptotic equilibrium state of the atoms turns out to be entangled even if the initial state is separable. Similar results have been obtained for atoms immersed in a thermal bath of scalar particles at a finite temperature [14], where the separation between atoms is allowed to be nonzero. It is found there that for any fixed, finite separation, there always exists a temperature below which entanglement generation occurs as soon as time starts to become nonzero and for the vanishing separation the entanglement thus generated persists even in the late-time asymptotic equilibrium state. Recently, the problem has been further investigated assuming the presence of a reflecting boundary for the scalar fields which modifies the quantum fluctuations of fields [15]. This modification, which alters the field correlation functions that characterize the fluctuations of fields, will presumably affect the entanglement generation. 

In fact, it has been demonstrated that the presence of the boundary may play a significant role in controlling the entanglement creation in some circumstances and the new parameter, the distance of the atoms from the boundary besides the bath temperature and the separation of the atoms, gives one more freedom in controlling the entanglement generation [15].

In present paper, we are concerned with the entanglement generation of two mutually independent, uniformly accelerated two-level atoms interacting with a set of massless scalar fields in the presence of a perfectly reflecting boundary. At the first glance, one may expect the same results as that for the case of a thermal bath (at the Unruh temperature proportional to the acceleration) with a boundary. However, with the help of the master equation that describes the evolution of the open system (atoms plus external thermal fields) in time, we show, to the contrary, that, with the presence of the boundary, the subsystem of the uniformly accelerated atoms may behave quite differently, in terms of entanglement generation at a neighborhood of the initial time \( t = 0 \), from that of static atoms immersed in a thermal bath.

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1 Let us note that other novel effects that arise from the modification include (but not limited to) the Casimir effect [16], the light-cone fluctuations when gravity is quantized [17], the Brownian (random) motion of test particles in an electromagnetic vacuum [18], and the modification for the radiative properties of uniformly accelerated atoms [19, 20].
II. THE MASTER EQUATION

In this section, we establish the basic formalism using some well-known techniques for open quantum systems to analyze a system of two independent uniformly accelerated two-level atoms in weak interaction with a set of massless quantum scalar fields in the presence of a reflecting boundary. We assume that the reflecting boundary for the scalar fields is located at \( z = 0 \) in space and one atom is placed at point \( x_1 \) and the other at \( x_2 \), with \( L \) being the spatial separation between them. Without loss of generality, the total Hamiltonian for the complete system (atoms+external fields) has the form \( H = H_s + H_\phi + \lambda H' \), where \( H_s \) is the Hamiltonian of the atom,

\[
H_s = H_s^{(1)} + H_s^{(2)}, \quad H_s^{(\alpha)} = \omega n_\alpha \sigma_i^{(\alpha)} / 2, \quad (\alpha = 1, 2), \quad \sigma_i^{(1)} = \sigma_i \otimes \sigma_0, \quad \sigma_i^{(2)} = \sigma_0 \otimes \sigma_i. \tag{1}
\]

Here, the \( \sigma_i, (i = 1, 2, 3) \) are the Pauli matrices, \( \sigma_0 \) the \( 2 \times 2 \) unit matrix, \( n = (n_1, n_2, n_3) \) a unit vector, \( \omega \) the energy level spacing, and summation over repeated index is implied. \( H_\phi \) is the standard Hamiltonian of massless, free scalar fields, details of which is not very relevant here and \( H' \) is the interaction Hamiltonian of the atoms with the external scalar fields which is assumed to be weak

\[
H' = \sum_{\tau=0}^{3} \left[ (\sigma_\tau \otimes \sigma_0) \Phi_\tau(t, x_1) + (\sigma_0 \otimes \sigma_\tau) \Phi_\tau(t, x_2) \right]. \tag{2}
\]

In the limit of weak-coupling, the reduced density is found to obey an equation in the Kossakowski-Lindblad form \([11, 21]\)

\[
\frac{\partial \rho(t)}{\partial t} = -i[H_{\text{eff}}, \rho(t)] + \mathcal{L}[\rho(t)], \tag{3}
\]

where

\[
\mathcal{L}[\rho] = \frac{1}{2} \sum_{\alpha, \beta=1}^{2} C_{ij}^{(\alpha \beta)} \left[ 2 \sigma_j^{(\beta)} \rho \sigma_i^{(\alpha)} - \sigma_i^{(\alpha)} \sigma_j^{(\beta)} \rho - \rho \sigma_i^{(\alpha)} \sigma_j^{(\beta)} \right]. \tag{4}
\]

The matrix \( C_{ij}^{(\alpha \beta)} \) and \( H_{\text{eff}} \) are determined by the correlation functions

\[
C_{ij}^{(\alpha \beta)}(t - t') = \langle 0 | \Phi_i(t, x_\alpha) \Phi_j(t', x_\beta) | 0 \rangle. \tag{5}
\]
The Fourier and Hilbert transforms of the correlation function $G_{ij}^{(\alpha\beta)}$ read respectively

$$G_{ij}^{(\alpha\beta)}(\lambda) = \int_{-\infty}^{\infty} dt e^{i\lambda t} G_{ij}^{(\alpha\beta)}(t),$$  \hspace{1cm} (6)

$$K_{ij}^{(\alpha\beta)}(\lambda) = \int_{-\infty}^{\infty} dt \text{sign}(t) e^{i\lambda t} G_{ij}^{(\alpha\beta)}(t) = \frac{P}{\pi i} \int_{-\infty}^{\infty} d\omega \frac{G_{ij}^{(\alpha\beta)}(\omega)}{\omega - \lambda}. \hspace{1cm} (7)$$

One can show that the Kossakowski matrix $C_{ij}^{(\alpha\beta)}$ can be written explicitly as

$$C_{ij}^{(\alpha\beta)} = \sum_{\xi=+, -, 0} G_{kl}^{(\alpha\beta)}(\xi\omega) \psi_{ki}^{(\xi)} \psi_{lj}^{(-\xi)} \hspace{1cm} (8)$$

where

$$\psi_{ij}^{(0)} = n_i n_j \hspace{1cm} \psi_{ij}^{(\pm)} = \frac{1}{2} (\delta_{ij} - n_i n_j \pm i \epsilon_{ijk} n_k). \hspace{1cm} (9)$$

Similarly, the coefficients of $H_{\text{eff}}$ can be calculated by using $K_{kl}^{(\alpha\beta)}(\xi\omega)$ [11, 21].

III. ENTANGLEMENT GENERATION OF ACCELERATED ATOMS IN PRESENCE OF A BOUNDARY

With the basic formalism for the open system outlined in the preceding section, we now start to examine whether entanglement can be generated between two independent atoms moving with a constant acceleration in the presence of a reflecting boundary, focusing our attention, in particular, on the difference between the accelerated atoms and static ones in the thermal bath at the corresponding Unruh temperature.

A. Entanglement creation for the accelerated atoms aligned parallel to the boundary

For simplicity, we assume that two atoms are separated from each other by a distance $L$ in $y-z$ plane (see Fig. (1)). Furthermore, both atoms are supposed to start moving at time $t = 0$ with a constant proper acceleration $a$ in the $x$-direction and so their paths can be described by

$$x^0(t) = \frac{1}{a} \sinh(at), \hspace{1cm} x^1(t) = \frac{1}{a} \cosh(at). \hspace{1cm} (10)$$
with \( x^2(t), x^3(t) \) components remaining unchanged with their initial values.

\[
\begin{align*}
G_{ij}^{(22)} (t - t') &= G_{ij}^{(11)} (t - t') = -\frac{1}{4\pi^2} \left[ \delta_{ij} \right. \\
&\quad - \frac{\delta_{ij}}{\sinh^2 \left[ \frac{a(t-t')}{2} - i\epsilon \right]} \\
&\quad - \frac{\delta_{ij}}{\sinh^2 \left[ \frac{a(t-t')}{2} - i\epsilon \right] - a^2 z^2} \left. \right] , \\
G_{ij}^{(21)} (t - t') &= G_{ij}^{(12)} (t - t') = -\frac{\delta_{ij}}{16\pi^2} \left[ \frac{a^2}{\sinh^2 \left[ \frac{a(t-t')}{2} - i\epsilon \right] - a^2 L^2/4} \\
&\quad - \frac{a^2}{\sinh^2 \left[ \frac{a(t-t')}{2} - i\epsilon \right] - a^2 z^2 - a^2 L^2/4} \right].
\end{align*}
\]

After some straightforward calculations, we can easily obtain their Fourier transforms

\[
G_{ij}^{(11)} (\lambda) = G_{ij}^{(22)} (\lambda) = \frac{\delta_{ij}}{2\pi} \frac{\lambda}{1 - e^{-2\pi\lambda/a}} - \frac{\delta_{ij}}{2\pi} \frac{\lambda}{1 - e^{-2\pi\lambda/a}} f_1 (\lambda, z) ,
\]

FIG. 1: A reflecting plane boundary is located at \( z = 0 \) in space. Two independent atoms separated from each other by a distance \( L \) are aligned parallel to the boundary and accelerated in the \( x \)-direction.

Due to the assumption that the fields reflect from the boundary completely, we can use the method of images to find the correlation functions (Eq. (5)),

\[
G_{ij}^{(22)} (t - t') = G_{ij}^{(11)} (t - t') = -\frac{1}{4\pi^2} \left[ \delta_{ij} \right. \\
&\quad - \frac{\delta_{ij}}{\sinh^2 \left[ \frac{a(t-t')}{2} - i\epsilon \right]} \\
&\quad - \frac{\delta_{ij}}{\sinh^2 \left[ \frac{a(t-t')}{2} - i\epsilon \right] - a^2 z^2} \left. \right] , \\
G_{ij}^{(21)} (t - t') &= G_{ij}^{(12)} (t - t') = -\frac{\delta_{ij}}{16\pi^2} \left[ \frac{a^2}{\sinh^2 \left[ \frac{a(t-t')}{2} - i\epsilon \right] - a^2 L^2/4} \\
&\quad - \frac{a^2}{\sinh^2 \left[ \frac{a(t-t')}{2} - i\epsilon \right] - a^2 z^2 - a^2 L^2/4} \right].
\]
\[ G_{ij}^{(12)}(\lambda) = G_{ij}^{(21)}(\lambda) = \frac{\delta_{ij}}{2\pi} \frac{\lambda}{1 - e^{-2\pi\lambda/a}} f_1(\lambda, L/2) - \frac{\delta_{ij}}{2\pi} \frac{\lambda}{1 - e^{-2\pi\lambda/a}} \times f_1(\lambda, \sqrt{z^2 + L^2/4}), \quad (14) \]

where \( f_1(\lambda, z) \) is defined as
\[
f_1(\lambda, z) = \frac{\sin[\frac{2\lambda}{a} \sinh^{-1}(az)]}{2z\sqrt{1 + a^2z^2}}. \quad (15)\]

According to Eq. (8), we can write
\[
C_{ij}^{(11)} = A_1 \delta_{ij} - iB_1 \epsilon_{ijk} n_k + C_1 n_i n_j,
C_{ij}^{(22)} = A_2 \delta_{ij} - iB_2 \epsilon_{ijk} n_k + C_2 n_i n_j,
C_{ij}^{(12)} = C_{ij}^{(21)} = A_3 \delta_{ij} - iB_3 \epsilon_{ijk} n_k + C_3 n_i n_j, \quad (16)
\]
and the corresponding coefficients are
\[
A_1 = A_2 = \frac{\omega \coth(\pi\omega/a)}{4\pi} [1 - f_1(\omega, z)],
B_1 = B_2 = \frac{\omega}{4\pi} [1 - f_1(\omega, z)],
C_1 = C_2 = \frac{a}{4\pi^2} [1 - f_2(z)] - \frac{\omega \coth(\pi\omega/a)}{4\pi} [1 - f_1(\omega, z)],
A_3 = \frac{\omega \cosh(\pi\omega/a)}{4\pi} \left[ f_1(\omega, L/2) - f_1(\omega, \sqrt{z^2 + L^2/4}) \right],
B_3 = \frac{\omega}{4\pi} \left[ f_1(\omega, L/2) - f_1(\omega, \sqrt{z^2 + L^2/4}) \right],
C_3 = -\frac{\omega \coth(\pi\omega/a)}{4\pi} \left[ f_1(\omega, L/2) - f_1(\omega, \sqrt{z^2 + L^2/4}) \right] + \frac{a}{4\pi^2} \left[ f_2(L/2) - f_2(\sqrt{z^2 + L^2/4}) \right]. \quad (21)
\]

The new function \( f_2(z) \) in the above expressions is given by
\[
f_2(z) = \frac{\sinh^{-1}(az)}{za\sqrt{1 + a^2z^2}}. \quad (22)\]

Similarly, \( K_{ij}^{(\alpha\beta)} \) for the Hamiltonian \( H_{\text{eff}} \) can be obtained easily, but here we do not give the formulae in detail. As has already been discussed in detail elsewhere [13, 14], the effective Hamiltonian \( H_{\text{eff}} = \tilde{H}_s^{(1)} + \tilde{H}_s^{(2)} + H_{\text{eff}}^{(12)} \) includes three pieces. The first two correspond to the
corrections of the Lamb shift at a finite acceleration which should be regularized according
to the standard procedures in quantum field theory and nevertheless they can be accounted
for by replacing \( \omega \) the atom’s Hamiltonian \( H_s^{(1)} \) in Eq. (1) with a renormalized energy level spacing
\[
\tilde{\omega} = \omega + i[K^{(11)}(-\omega) - K^{(11)}(\omega)].
\]  
(23)
Similarly, \( \omega \) for the Hamiltonian \( H_s^{(2)} \) is
\[
\tilde{\omega} = \omega + i[K^{(22)}(-\omega) - K^{(22)}(\omega)].
\]  
(24)
Meanwhile the third term is an environment generated direct coupling between the atoms
and is acceleration independent\[13, 14\]. So the \( H_{\text{eff}} \) can be ignored, since we are interested
in the acceleration-induced effects. Henceforth, we will only study the effects produced by
the dissipative part \( \mathcal{L}[\rho(t)] \).

Using the explicit form of the master equation (3), we can investigate the time evolution
of the reduced density matrix and then we can figure out whether the state of the two-atom
system is entangled or not with the help of partial transposition criterion \[22\]: a two-atom
state \( \rho(t) \) is entangled at \( t \) if and only if the operation of partial transposition of \( \rho(t) \) does
not preserve its positivity. Let us now consider the system in a finite time, and adopt a
simple strategy for ascertaining the entanglement creation at a neighborhood of the initial
time \( t = 0 \), which has been introduced in Ref. \[11\]. For simplicity, we also let the initial
pure, separable two-atom state be \( \rho(0) = |+\rangle \langle +| \otimes |-\rangle \langle -| \) and consider the quantity
\[
\mathcal{Q}(t) = \langle \chi | \tilde{\rho}(t) | \chi \rangle,
\]  
(25)
where the tilde signifies partial transposition and \( |\chi\rangle \) is a properly chosen 4-dimensional
vector. According to Refs. \[11, 14\], the entanglement of the system is created at the neighbor-
hood of the time \( t = 0 \) (i.e., \( \partial_t \mathcal{Q}(0) < 0 \)), if and only if
\[
\langle u|C^{(11)}|u\rangle \langle v|(C^{(22)})^T|v\rangle < |\langle u|\text{Re}(C^{(12)})|v\rangle|^2,
\]  
(26)
where the subscript \( T \) means matrix transposition and the three-dimensional vectors \( |u\rangle \) and
\( |v\rangle \) can be chosen in a simple form as \( u_i = v_i = \{1, -i, 0\} \). Using Eq. (16) and taking the
fact that $A_1 = A_2$ into account, we can compute Eq. (26) for the vector $n$ along the third axis to get

$$\frac{A_3^2}{A_1^2} + \frac{B_1^2}{A_1^2} > 1,$$

(27)

where

$$\frac{B_1^2}{A_1^2} = \left( \frac{1 - e^{-2\pi \omega/a}}{1 + e^{-2\pi \omega/a}} \right)^2 = \left( \frac{1 - e^{-\omega/T}}{1 + e^{-\omega/T}} \right)^2,$$

(28)

with $T = a/2\pi$ being the Unruh temperature, and $A_3^2/A_1^2$ can be simplified as

$$\frac{A_3^2}{A_1^2} = \frac{F}{G},$$

(29)

with

$$F = 4 \left\{ \frac{\sin \left[ \frac{2\omega}{a} \sinh^{-1} \left( \frac{aL}{2} \right) \right]}{L \omega \sqrt{4 + a^2 L^2}} - \sin \left[ \frac{2\omega}{a} \sinh^{-1} \left( a \sqrt{L^2/4 + z^2} \right) \right] \right\}^2$$

(30)

and

$$G = \left\{ 1 - \frac{\sin \left[ \frac{2\omega}{a} \sinh^{-1} (az) \right]}{2z \omega \sqrt{1 + a^2 z^2}} \right\}^2.$$

(31)

A comparison of Eq. (27) with the condition for entanglement generation for two independent static atoms immersed in a thermal bath with a boundary (refer to Eq. (23) in Ref. [15]) reveals that the uniformly accelerated atoms would, in general, behave differently from the static atoms in a thermal bath at the Unruh temperature in terms of entanglement creation at the neighborhood of time $t = 0$, since $A_3^2/A_1^2$ here is acceleration-dependent (or Unruh temperature-dependent) whereas the corresponding term in Eq. (23) in Ref. [15] does not rely on the bath temperature.

For a given atom, $B_1^2/A_1^2$ is only dependent on the acceleration $a$. When the acceleration is vanishingly small, the value of $B_1^2/A_1^2$ will approach to 1. Therefore, inequality (27) is always satisfied for zero acceleration as long as $L$ is not infinite. On the other hand, when the separation is vanishing ($L = 0$), $A_3^2/A_1^2$ becomes unity and inequality (27) holds as long as the acceleration is not infinite.

In order to figure out the difference in terms of the entanglement generation between the accelerated atoms and the static ones in the thermal bath at the Unruh temperature, let us recall that $A_3^2/A_1^2$ for the case of two static atoms in a thermal bath with a boundary is

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given by [15]

\[
\frac{A_3^2}{A_1^2} = \left[ \frac{\sin(L\omega)}{L\omega} - \frac{\sin(\omega\sqrt{L^2 + 4z^2})}{\omega\sqrt{L^2 + 4z^2}} \right]^2 \left/ \left[ 1 - \frac{\sin(2z\omega)}{2z\omega} \right]^2 \right. .
\] (32)

First, we examine how the acceleration affects the entanglement generation when the separation of the atoms is comparable to the characteristic wavelength of the atoms, i.e., when \( L \sim 1/\omega \). In Fig. (2), we plot \( A_3^2/A_1^2 \) both for the case of static atoms in a thermal bath (refer to Eq. (32)) and that for the accelerated atoms (refer to Eq.(29)) as a functions of \( aL \), with \( L\omega \), the parameter characterizing the system of atoms fixed at \( L\omega = 1 \) and \( z/L = (0.1, 2, 1000) \). The Figure shows that in general, \( A_3^2/A_1^2 \) decreases as \( aL \) or \( a \) grows. Furthermore, the smaller the value of \( z/L \) is or the closer the system of the atoms is to the boundary, the more rapidly \( A_3^2/A_1^2 \) decays as a function of \( aL \). Physically, this means that as the acceleration increases the atoms are less likely to get entangled, and the closer the two-atom system is to the boundary, the more significantly the acceleration \( a \) suppress the possibility of entanglement production. Another conclusion one can draw from Fig. (2) is that the accelerated atoms are less likely to get entangled as compared to those static ones in the thermal bath at the Unruh temperature, since \( A_3^2/A_1^2 \) is always smaller for the accelerated atoms.

Now, let us discuss how the acceleration affects the entanglement generation when the two-atom system is at a distance from the boundary comparable to the separation of the atoms, i.e., when \( z \sim L \). In Fig. (3), we plot \( A_3^2/A_1^2 \) as a function of \( aL \) both for the case of static atoms in a thermal bath (refer to Eq. (32)) and that for the accelerated atoms (refer to Eq.(29)) with \( z/L = 1 \) and \( L\omega = (0.1, 1, 1.5, 2.4) \). From Fig. (3), one can see that the value of \( A_3^2/A_1^2 \) for the accelerated atoms (Eq. (29)) is obviously less than that for the static atoms in the Unruh thermal bath (Eq. (32)) for the same value of \( L\omega \), while they both decrease rapidly with the increase of \( L\omega \). Thus, again, Fig. (3) also shows that the entanglement production is less likely to occur for accelerated atoms than the static ones immersed in a thermal bath at the corresponding “Unruh temperature”. So, in terms of entanglement generation at at a neighborhood of the initial time \( t = 0 \), independent accelerated atoms do not behave as if they were static in a thermal bath at the Unruh temperature.
FIG. 2: The solid lines denote $A_3^2/A_1^2$ for the accelerated atoms (Eq. (29)) as a function of $aL$ with $L\omega = 1$, and $z/L = (0.1, 2, 1000)$, and the dashed lines represent that for the static atoms in a thermal bath (Eq. (32)).

At this point, one may wonder what the effects are of the presence of the boundary on entanglement generation of the two-atom system. To address this issue, let us analyze and compare two special cases. One is when the atom system is very close to the boundary, i.e., when $z \ll L$, $za \ll 1$ and $z\omega \ll 1$. In this case, $A_3^2/A_1^2$ (Eq. (29)) can be approximated as

$$A_3^2/A_1^2 \approx \frac{144}{L^6(4 + a^2L^2)^2\omega^2(a^2 + \omega^2)^2} \left\{ \frac{2 + a^2L^2}{\sqrt{4 + a^2L^2}} \sin \left[ \frac{2\omega}{a} \sinh^{-1}(aL/2) \right] - L\omega \cos \left[ \frac{2\omega}{a} \sinh^{-1}(aL/2) \right] \right\}^2 + O\left( \frac{z^2}{L^2} \right).$$

The other case is when the system is located very far away from the boundary (i.e., $z \gg L$, $az \gg 1$ and $\omega z \gg 1$). One then has

$$\frac{A_3^2}{A_1^2} = \frac{4\sin^2[\frac{2\omega}{a} \sinh^{-1}(aL/2)]}{L^2(4 + a^2L^2)\omega^2} + O\left( \frac{L^2}{z^2} \right).$$

In the Fig. (4), we have plotted $A_3^2/A_1^2$ as a function of $L\omega$ for these special cases with the value of $aL$ chosen as 0.2, 1, and 3. Plot (a) is basically the same as that of Fig. (2) in Ref. [15]. So, when the acceleration is very small ($a \ll 1/L$), the effect of the presence of a boundary on the entanglement generation of the accelerated atoms is essentially the
FIG. 3: $A_3^2/A_1^2$ for accelerated atoms as function of $aL$ is described by solid lines with $z/L = 1$ and $L\omega = 0.1$, 1, 1.5, 2.4, respectively, and that for the static atoms in a thermal bath by the dashed lines.

same as that of the static atoms in the thermal bath at the Unruh temperature, i.e., when $L\omega$ is small, approximately smaller than four, the presence of the boundary may make the accelerated atoms be entangled which would otherwise still be separable, since $A_3^2/A_1^2$ is always greater with presence of the boundary than that without. However, plot (c) shows that when the acceleration is very large ($a \gg 1/L$), $A_3^2/A_1^2$ is always smaller with presence of the boundary than that without, thus the entanglement production is more likely for the system located farther away from the boundary no matter what the value of $L\omega$ is. Meanwhile, when the acceleration is comparable to the separation of the atoms, then one finds from plot (b) that $A_3^2/A_1^2$ is smaller with the presence of the boundary than without when $L\omega$ is approximately less than two and larger than four, whereas it is greater when $L\omega$ is approximately in between two and four, suggesting that for a given kind of atom, the separation will have significant influence on whether the entanglement will be created and the presence of the boundary may make the accelerated atoms entangled which would otherwise still be separable or vice versa, depending crucially on the distance between the atoms. Therefore, when the acceleration is not small as compared to the separation of the atoms, the accelerated atoms exhibit distinct features from the static ones in the thermal...
bath at the Unruh temperature when entanglement generation is concerned.

![Graphs showing A_2/\lambda^2 vs. \omega L for different values of \alpha L.](image)

**FIG. 4:** The dashed lines denote \(A_2^2/\lambda^2\) as function of \(\omega L\) with \(\alpha L = \{0.2, 1, 3\}\), for vanishingly small \(z\) (see Eq. (33)), and the solid lines describe that for \(z\) approaching infinity (see Eq. (34)).

**B. The entanglement creation for the accelerated atoms vertically aligned to the boundary plane**

Let us now briefly examine another special alignment of the two-atom system, that is, the case in which the two-atom system is vertically aligned to the boundary (see Fig. (5)). Now one finds in the correlation functions of the scalar fields that \(G_{ij}^{(22)}(t - t') \neq G_{ij}^{(11)}(t - t')\) as a result of the unequal distance from the boundary for the two atoms. Therefore, we have

\[
G_{ij}^{(11)}(\lambda) = \frac{\delta_{ij}}{2\pi} \left( 1 - e^{-2\pi\lambda/a} \right) - \frac{\delta_{ij}}{2\pi} \left( 1 - e^{-2\pi\lambda/a} \right) f_1(\lambda, z), \\
G_{ij}^{(22)}(\lambda) = \frac{\delta_{ij}}{2\pi} \left( 1 - e^{-2\pi\lambda/a} \right) - \frac{\delta_{ij}}{2\pi} \left( 1 - e^{-2\pi\lambda/a} \right) f_1(\lambda, z + L),
\]

\(35\)

\[
G_{ij}^{(12)}(\lambda) = G_{ij}^{(21)}(\lambda) = \frac{\delta_{ij}}{2\pi} \left( 1 - e^{-2\pi\lambda/a} \right) f_1(\lambda, L/2) - \frac{\delta_{ij}}{2\pi} \left( 1 - e^{-2\pi\lambda/a} \right) f_1(\lambda, z + L/2) \times f_1(\lambda, z + L/2).
\]

\(36\)

Similarly, using Eq. (8) and Eq. (16), we can also write

\[
A_1 = \frac{\omega \coth(\pi\omega/a)}{4\pi} [1 - f_1(\omega, z)], \quad A_2 = \frac{\omega \coth(\pi\omega/a)}{4\pi} [1 - f_1(\omega, z + L)],
\]

\(37\)
FIG. 5: The atom system is aligned vertical to the boundary and \( z \) is the distance between the boundary and the atom which is closer. The acceleration is again in the \( x \)-direction.

\[
B_1 = \frac{\omega}{4\pi}[1 - f_1(\omega, z)], \quad B_2 = \frac{\omega}{4\pi}[1 - f_1(\omega, z + L)],
\]

(38)

\[
C_1 = \frac{a}{4\pi^2}[1 - f_2(z)] - \frac{\omega \coth(\omega \pi/a)}{4\pi}[1 - f_1(\omega, z)],
\]

C_2 = \frac{a}{4\pi^2}[1 - f_2(z + L)] - \frac{\omega \coth(\omega \pi/a)}{4\pi}[1 - f_1(\omega, z + L)],

(39)

\[
A_3 = \frac{\omega \cosh(\pi \omega/a)}{4\pi} \left[ f_1(\omega, L/2) - f_1(\omega, z + L/2) \right],
\]

\[
B_3 = \frac{\omega}{4\pi} \left[ f_1(\omega, L/2) - f_1(\omega, z + L/2) \right],
\]

(40)

(41)

\[
C_3 = -\frac{\omega \coth(\omega \pi/a)}{4\pi} \left[ f_1(\omega, L/2) - f_1(\omega, z + L/2) \right] + \frac{a}{4\pi^2} \left[ f_2(L/2) - f_2(z + L/2) \right].
\]

(42)

with the above results, one can show that the condition for entanglement creation, Eq.(26), becomes

\[
\frac{A_3^2}{A_1A_2} + \frac{B_1B_2}{A_1A_2} > 1,
\]

(43)

where

\[
\frac{B_1B_2}{A_1A_2} = \left( \frac{1 - e^{-2\pi \omega/a}}{1 + e^{-2\pi \omega/a}} \right)^2 = \left( \frac{1 - e^{-\omega/T}}{1 + e^{-\omega/T}} \right)^2.
\]

(44)
with $T = a/2\pi$ also being the Unruh temperature. Eq. (44) is the same as Eq. (28) and is only dependent on the parameters $a$ and $\omega$. Therefore we only need to discuss the first term in Eq. (43), which can be shown to be given by

$$\frac{A_3^2}{A_1 A_2} = \frac{F'}{G'},$$

(45)

where

$$F' = 4\left\{ \frac{\sin \left[ \frac{2\omega}{a} \sinh^{-1} \left( \frac{aL}{2} \right) \right]}{L\omega \sqrt{4 + a^2 L^2}} - \frac{\sin \left[ \frac{2\omega}{a} \sinh^{-1} \left( \frac{aL + 2az}{2} \right) \right]}{(L + 2z)\omega \sqrt{4 + a^2 (L + 2z)^2}} \right\}^2,$$

(46)

$$G' = \left[ 1 - \frac{\sin \left[ \frac{2\omega}{a} \sinh^{-1} (az) \right]}{2z\omega \sqrt{1 + a^2 z^2}} \right] \left[ 1 - \frac{\sin \left[ \frac{2\omega}{a} \sinh^{-1} (aL + az) \right]}{2\omega (L + z) \sqrt{1 + a^2 (L + z)^2}} \right].$$

(47)

If we expand Eq. (45) in the limit of $z \to \infty$, we find that the result is the same as that of Eq. (34). This is consistent with what one expects, since very far from the boundary, the space should be almost isotropic. Thus, as an example to demonstrate the difference, in terms of the entanglement generation, between case when the atom system is aligned parallel to boundary and that when it is vertically aligned, we will analyze the situation when the atom is located very close to the boundary. In the limit of vanishingly small $z$, $A_3^2/(A_1 A_2)$ reads

$$\frac{A_3^2}{(A_1 A_2)} \approx \left\{ \frac{L\omega \sqrt{a^2 L^2 + 4 \cos \left[ \frac{2\omega}{a} \sinh^{-1} \left( \frac{aL}{2} \right) \right]} - (a^2 L^2 + 2) \sin \left[ \frac{2\omega}{a} \sinh^{-1} \left( \frac{aL}{2} \right) \right]}{2L\omega \sqrt{1 + a^2 L^2} - \sin \left[ \frac{2\omega}{a} \sinh^{-1} (aL) \right]} \right\}^2 \times \frac{192 \sqrt{1 + a^2 L^2}}{L^3 \omega (4 + a^2 L^2)^3 (a^2 + \omega^2)} + O \left( \frac{z}{L} \right).$$

(48)

In Fig. (6), we have plotted $A_3^2/(A_1 A_2)$ as function of $L\omega$ with $aL = \{0.2, 1, 5\}$, both for the parallel two-atom system and the vertical one, when $z$ is vanishingly small. Notice that for the parallel system $A_2 = A_1$. These plots in the Figure shows clearly that generically the value of $A_3^2/(A_1 A_2)$ for the parallel aligned atom system is smaller than the vertically aligned one.

Therefore, we can conclude that very close to boundary, accelerated atoms that are aligned parallel to the boundary are less likely to get entangled than those that are vertically aligned.
IV. CONCLUSION AND DISCUSSION

Using the open system paradigm, we have investigated, at a neighborhood of the initial time, the entanglement generation of independent uniformly accelerated atoms interacting with scalar fields in vacuum with the presence of a reflecting plane boundary.

Our results reveal that, for the parallel two-atom system, both when the separation of the atoms is comparable to the characteristic wavelength of the atoms (but the atom system is not very close to the boundary), i.e., when \( L \sim 1/\omega \), and when the two-atom system is at a distance from the boundary comparable to the separation of the atoms, i.e., when \( z \sim L \), the entanglement production is less likely to occur for accelerated atoms than the static ones immersed in a thermal bath at the corresponding “Unruh temperature”.

On the other hand, if the atom system is very close to the boundary, i.e., if \( z \ll L \), \( za \ll 1 \) and \( z\omega \ll 1 \), then when the acceleration is very large (\( a \gg 1/L \)), the presence of the boundary will always make the entanglement production less likely to happen no matter what the value of \( L\omega \) is. Meanwhile, when the acceleration is comparable to the separation of the atoms, then the accelerated atoms are less likely to get entangled with the presence of the boundary than without when \( L\omega \) is approximately less than two and
larger than four, whereas they are likely to do so when $L\omega$ is approximately in between two and four, suggesting that for a given kind of atom, the separation between the atoms will have significant influence on whether the entanglement will be created and the presence of the boundary may make the accelerated atoms entangled which would otherwise still be separable or vice versa. This is in sharp contrast to the static atoms in thermal bath where it has been shown that when $L\omega$ is small, approximately smaller than four, the presence of the boundary may make the atoms be entangled which would otherwise still be separable and only when $L\omega$ is very large, will the presence of the boundary always make the entanglement production less likely to occur [15].

Therefore, in terms of the entanglement generation at a neighborhood of the initial time, the accelerated atoms exhibit distinct features from the static ones in a thermal bath at the Unruh temperature. In other words, accelerated atoms in vacuum do not have to behave as if they were static in a thermal bath. A similar example of this kind is that a uniformly accelerated proton does not have to behave as if it were static in a thermal bath at the Unruh temperature in terms of its lifetime against weak decay [23].

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant No.10575035, the Program for New Century Excellent Talents in University (NCET, No. 04-0784), the Key Project of Chinese Ministry of Education (No. 205110), and the National Basic Research Program of China under Grant No. 2003CB71630.

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