LOW-MAGNETIC-FIELD MAGNETARS

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It is now widely accepted that soft gamma repeaters and anomalous X-ray pulsars are the observational manifestations of magnetars, i.e. sources powered by their own magnetic energy. This view was supported by the fact that these ‘magnetar candidates’ exhibited, without exception, a surface dipole magnetic field (as inferred from the spin-down rate) in excess of the electron critical field ($\gtrsim 4.4 \times 10^{13}$ G). The recent discovery of fully-qualified magnetars, SGR 0418+5729 and Swift J1822.3–1606, with dipole magnetic field well in the range of ordinary radio pulsars posed a challenge to the standard picture, showing that a very strong field is not necessary for the onset of magnetar activity (chiefly bursts and outbursts). Here we summarize the observational status of the low-magnetic-field magnetars and discuss their properties in the context of the mainstream magnetar model and its main alternatives.

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1. Introduction

A newly discovered and seemingly isolated neutron star is (observationally) classified as a magnetar (a source powered by magnetic energy) when it complies with at least three of the following requirements: comparatively long spin period ($P \sim 1–12$ s); large spin-down rate ($\dot{P} \gtrsim 10^{-12}$ s s$^{-1}$); relatively high and variable persistent luminosity ($L_X \sim 10^{32}–10^{36}$ erg s$^{-1}$); emission of powerful short X/γ-ray bursts (spikes of $\sim 0.1–10$ s duration, $L_X \sim 10^{34}–10^{47}$ erg s$^{-1}$ at the peak).

Just a few years ago, these criteria were thought to be equivalent to that of a surface magnetic field of $10^{13}–10^{15}$ G. The ‘magnetar candidates’, in fact, comprise

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two classes of sources, the soft gamma repeaters (SGRs) and the anomalous X-ray pulsars (AXPs), which, with no exception at that time, exhibited a surface dipole field (as derived from the spin down measure) in excess of the quantum critical field, $B_Q \simeq 4.4 \times 10^{13}$ G.\footnote{Despite SGRs and AXPs are far from being a homogeneous class, in particular their magnetic field spans nearly two orders of magnitude, their observational behavior has been assumed as the template for (active) magnetars, to the point that the terms SGR/AXP and magnetar are often used interchangeably. This, actually, reflects the original definition of a magnetar as a neutron star which is powered by its (large) magnetic field.} In this respect, it is important to notice that a super-strong magnetic field is not per se a sufficient condition for triggering SGR/AXP-like activity. This is testified by the existence of neutron-star sources, for instance most of the so-called high-$B$ radio pulsars (HBPSRs; e.g. Refs.\cite{8,9}, and some of the thermally emitting isolated neutron stars (XDINSs; e.g. Ref.\cite{10}, with surface magnetic fields comparable to those of SGRs/AXPs but having substantially different properties and not showing any bursting/outbursting activity over the $\sim 10^{-20}$ yr time span during which they have been observed.

More recently the ‘supercritical $B$’ paradigm for the onset of magnetar activity has been challenged the discovery of full-fledged magnetars, SGR 0418+5729 and Swift J1822.3–1606\cite{11,12,13,14,15,16} with a dipole magnetic field well in the range of ordinary radio pulsars. Here we consider whether the canonical magnetar model still holds in the light of these facts or other models can better explain the observed phenomenology. Section\cite{2} is a brief recap of how the magnetic field of isolated pulsars is routinely estimated within the standard magneto-dipole braking framework. Section\cite{3} reviews the observational results on SGR 0418+5729 and Swift J1822.3–1606. Sections\cite{4} and \cite{5} deal with the theoretical context. Section\cite{6} concludes this review with remarks on the definition of magnetar.

### 2. Measuring the magnetic field of an isolated pulsar

The surface dipole magnetic field of a non-accreting pulsar is usually estimated by equating the rate of rotational kinetic energy loss to the power radiated by the rotating dipole. In this way, at the neutron-star magnetic equator, $B_n = (3Ic^3P/8\pi^2R^6\sin^2\alpha)^{1/2} \simeq 3.2 \times 10^{19}(PP/\sin^2\alpha)^{1/2}$ G, where the period $P$ is measured in s, the period derivative $\dot{P}$ in s$^{-1}$, $\alpha$ is the angle between the magnetic moment and the spin axis, and $R = 10^6$ cm and $I = 10^{45}$ g cm$^2$ are the neutron-star radius and moment of inertia.

This inference is model-dependent, $R$ and $I$ are uncertain, and $\alpha$ is generally unknown (it is usually assumed, for simplicity, to be $90^\circ$), but no direct measurements of the magnetic field strength are available for isolated pulsars.\footnote{For this reason, one}

\footnote{See Ref.\cite{17} for a more accurate expression.}

\footnote{With the possible exception of 1E 1207.4–5209, the spectral features of which are often interpreted as electron cyclotron lines.\cite{13,18}}
has necessary to rely upon the approximate value from the $B_p \propto (P\dot{P})^{1/2}$ formula and thus on measurements of the rotational parameters.

The pulsar period and period derivative can be measured with good precision by means of phase-coherent timing (see e.g. Refs. [21, 22, 23]) during extensive observational campaigns. The basic idea is that in a reference frame that does not accelerate with respect to an isolated pulsar (to a very good approximation, the centre of mass of the Solar System is an inertial reference frame), the time evolution of the pulse phase $\phi(t)$ is, in general, expected to be well described by the Taylor expansion

$$\phi(t) = \phi(t_0) + \nu(t - t_0) + \frac{1}{2} \dot{\nu}(t - t_0)^2 + \frac{1}{6} \ddot{\nu}(t - t_0)^3 + \ldots,$$

where $\nu = 1/P$ is the pulse frequency and $t_0$ is a reference epoch. The pulsar period and its successive derivatives are thus obtained by the fit of the expansion to the observed data.

3. Low-Magnetic-Field Magnetars: Observations

3.1. SGR 0418+5729

As for many recently discovered magnetars, the existence of SGR 0418+5729 was revealed by its emission of short bursts. This occurred on 2009 June 5, when a couple of events triggered the monitors for hard X-ray transients aboard Fermi, Koronas-Foton, and Swift. These bursts were comparatively faint ($L_X \approx 10^{38} - 10^{39}$ erg s$^{-1}$ in the band 8–200 keV, for a distance $d = 2$ kpc) but otherwise unremarkable, with spectra well described by an optically-thin thermal bremsstrahlung with temperature $kT \sim 20$–30 keV and duration of $\sim 40$–80 ms. A third possible (weaker) burst, again on June 5, was found by an off-line inspection of the Fermi/GBM data, but it was not confirmed by the simultaneous Swift/BAT observation; a search of the Interplanetary Network events in the period 1990–2009 did not reveal any past activity clearly associated to SGR 0418+5729.

In the following few days, follow-up observations in the soft X-ray band (1–10 keV) carried out with Swift, Chandra and RossiXTE unveiled the existence of a previously unknown bright source, with an observed flux of a few $10^{-11}$ erg cm$^{-2}$ s$^{-1}$ (previous upper limits, based on ROSAT All-Sky Survey data, were of the order of $10^{-12}$ erg cm$^{-2}$ s$^{-1}$). These observations also made it possible to measure the source spin period, $P \sim 9.1$ s. The results of the first $\sim 5$ months of monitoring of SGR 0418+5729 are described in Ref. [26]. In each observation the X-ray spectrum could be well described by a single- or two-blackbody model (depending on the count statistics of the data sets). During this stretch of time, the source persistent X-ray emission faded by a factor of $\sim 10$, with a clear softening with time. The surprising fact was that, despite the dense monitoring and the long time-span, no slowing of the pulsar rotation could be detected, with a $3\sigma$ upper limit of $1.1 \times 10^{-13}$ s$^{-1}$, translating into an upper limit on the surface dipole magnetic

\[c\]

The low absorption toward SGR 0418+5729 and its direction are consistent with it belonging to the Perseus Arm of the Galaxy, suggesting a distance of $\sim 2$ kpc.\[d\]

\[d\]See http://heasarc.nasa.gov/W3Browse/all/ipnrgb.html.
field strength (see Sec. 2) of $3 \times 10^{13}$ G.

This already made SGR 0418+5729 the magnetar with the lowest inferred dipole magnetic field. Although indeed small for a magnetar, such a value was not abnormal, the limit being comparable to the strength of $6 \times 10^{13}$ G estimated for the dipole magnetic field of the AXP 1E 2259+586. It is interesting to note that the limit on the period derivative implied also a limit on the spin-down flux of $\dot{E}/(4\pi d^2) = \pi I \dot{P}/(P^3 d^2) < 2 \times 10^{-14}$ erg cm$^{-2}$ s$^{-1}$, showing that the persistent luminosity of the source during the monitoring could not be rotation-powered.

After a period during which the direction of the source was not accessible to the X-ray spacecrafts due to Sun constraints, a new observational campaign, mainly aimed at achieving a measurement of the period derivative, was undertaken in 2010 July through September using Swift, Chandra, and XMM-Newton. Again, the period derivative of SGR 0418+5729 eluded detection. This time, however, the limit on the period derivative, obtained through coherent timing over a base line of approximately 500 days, was substantially deviating from what could be expected for a magnetar: $\dot{P} < 10^{-15}$ s s$^{-1}$, corresponding to $B_p < 7.5 \times 10^{12}$ G (90% confidence level).

Not only, in fact, this value was significantly below the electron quantum magnetic field $B_Q \equiv m_e^2 c^2/(\hbar e) \approx 4.4 \times 10^{13}$ G (a value that, albeit lacking direct physical implications for pulsars, was traditionally considered to be the divide between ordinary pulsars and magnetars), but magnetar-like activity (bursting/outbursting behavior in particular) had never been observed before in objects with a magnetic field this low (also the magnetar/pulsar PSR J1846–0258 has an inferred surface dipole magnetic field of $4.9 \times 10^{13}$ G, in excess of $B_Q$).

Since during the Summer 2010 the source observed flux was of approximately $(1–2) \times 10^{-13}$ erg cm$^{-2}$ s$^{-1}$, the spectrum further softened and there was no sign that the flux decline had stopped, the monitoring of SGR 0418+5729 was extended using only the high-effective-area detectors on board Chandra (three pointings between 2010 November and 2011 November) and XMM-Newton (four pointings between 2011 March and 2012 August). In the time span covered by these observations, the source flux apparently settled at about $(1–2) \times 10^{-14}$ erg cm$^{-2}$ s$^{-1}$, which could be the typical quiescent emission level of SGR 0418+5729 (3 orders of magnitude below that measured at the time of the discovery, see Sec. 4 for the long-term X-ray light curve). Finally, after more than 3 years of monitoring, the measurement of the source spin-down rate was achieved, at a 3.5σ significance level (from a coher-
ent timing analysis of all the X-ray data spanning $\sim$1200 days; see Refs. [26, 11] and [13] for more details): $(4 \pm 1) \times 10^{-15}$ s s$^{-1}$, corresponding to $\sim 6.1 \times 10^{12}$ G. The value also translates into a characteristic age $\tau_c = P/(2\dot{P}) \approx 36$ Myrs and spin-down power $\dot{E} = 4\pi^2 I\dot{P}/P^3 \approx 2 \times 10^{29}$ erg s$^{-1}$ (showing that the X-ray luminosity of SGR 0418+5729 cannot be rotation powered not even at the low level observed in the 2011–2012 campaign). Figure 1 shows the position of SGR 0418+5729 in the $P$–$\dot{P}$ diagram for pulsars, while Table 1 summarizes its principal characteristics.

3.1.1. Multi-wavelength observations

Until recently, magnetars were believed to be emitting essentially in the soft X-ray energy range, briefly reaching the soft $\gamma$-ray energies during their bursts. Thanks to deep surveys and the availability of new instruments, nowadays several magnetars have been observed in the radio, optical and infrared, and soft $\gamma$-ray (as persistent sources) wavebands. Most counterparts were identified during outbursts because during active periods the emission level is in general enhanced at all wavelengths and the rapid flux variability helps in sorting out the true counter-

![Fig. 1. $P$–$\dot{P}$ diagram for non-recycled pulsars (data are from Ref. 31). Points represent normal radio pulsars, squares normal radio pulsars with a magnetic field larger than that measured for SGR 0418+5729, stars are the magnetars, the triangle is the magnetar-like pulsars PSR J1846–0258, and the circled dots are the X-ray dim isolated neutron stars. The solid line marks the dipole magnetic field inferred for SGR 0418+5729. The value of the electron quantum magnetic field is also shown (dashed line).](image-url)
Table 1. Main characteristics of SGR 0418+5729 and Swift J1822.3–1606.

|                      | SGR 0418+5729 | Swift J1822.3–1606 |
|----------------------|---------------|-------------------|
| RA (J2000)           | 04h 18m 33.87s | 18h 22m 18.06s    |
| Dec. (J2000)         | +57° 32’ 22.91” | −16° 04’ 25.53”  |
| Distance (kpc)       | 2             | 1.6               |
| Period (s)           | 9.07838822(5) | 8.43771984(4)     |
| Period derivative (s s$^{-1}$) | 4(1) $\times$ 10$^{-15}$ | 1.71(7) $\times$ 10$^{-13}$ |
| Reference epoch (MJD) | 54993.0       | 55761.0           |
| Validity range       | 54993–56164   | 55759–56161       |
| Surface dipole field (G) | 6.1 $\times$ 10$^{12}$ | 3.8 $\times$ 10$^{13}$ |
| Spin-down power (erg s$^{-1}$) | 2.1 $\times$ 10$^{29}$ | 1.1 $\times$ 10$^{31}$ |
| Characteristic age (Myr) | 36            | 0.8               |
| Luminosity (0.5–10 keV, erg s$^{-1}$) | $8 \times 10^{30}$—$1.6 \times 10^{34}$ | $3 \times 10^{32}$—$9 \times 10^{35}$ |

Note: Values in parentheses are 1σ uncertainties in the least significant digit quoted. The 95%-confidence positional error radius is 0.4” for SGR 0418+5729 and 0.7” for Swift J1822.3–1606. * For Swift J1822.3–1606 different values of the timing parameters can be found in literature (see main text and references therein); here we give the $P$–$\dot{P}$–$\ddot{P}$ coherent solution by Ref. 16. ** Minimum and maximum values, derived from the fluxes measured at the end and the start, respectively, of the the 2009 June–2012 August campaign (Refs. 26, 13). *** Minimum and maximum values, corresponding to the 1993 September 14–15 ROSAT observation and to a Swift observation carried out on 2011 July 15 (at the start of the outburst), respectively.

3.2. Swift J1822.3–1606

Swift J1822.3–1606 was discovered on 2011 July 14\(^8\) when it emitted several magnetar-like bursts that were detected by the Swift/BAT (see Refs. 14, 15 and references therein). Soon after, follow-up observations with Swift and RossiXTE part when there are many candidates.

On 2009 June 19, ten days after the onset of the outburst, the field of SGR 0418+5729 was observed by the Green Bank Telescope at 820 MHz but no source was detected, with an limit on the flux density of <0.5 mJy (for a duty cycle of 20%)\(^5\). At millimeter wavelengths, five observations were carried out at the Plateau de Bure Interferometer between 2011 June and July; again, no signal from SGR 0418+5729 was detected, with a limit of 0.24 mJy at 1.8 mm (167 GHz).\(^6\) The source eluded detection so far also in the optical and infrared bands. The magnitude limits from observations performed by ground-based instruments (Ref. 26 and references therein) are $K_s > 22.9$ (Palomar Hale Telescope, on 2009 August 2), $r > 24$ (William Herschel Telescope, on 2009 August 16), and $i > 25.1$ (Gran Telescopio Canarias, on 2009 September 15). Finally, an observation with the Hubble Space Telescope on 2010 October 19 yielded magnitude limits of 28.6 in the visible (5921 Å) and 27.4 in the infrared (11534 Å).\(^7\)

\(^8\) The Summer of 2011 was a bountiful one for magnetar enthusiasts: on 2011 August 7, less than one month after the discovery of Swift J1822.3–1606, another new magnetar was discovered, Swift J1834.9–0846\(^5\).
led to the detection of a bright X-ray counterpart (flux of \(\sim 2 \times 10^{-10}\) erg cm\(^{-2}\) s\(^{-1}\) in the first days after the onset of the outburst, with a spectrum well described by either a blackbody plus power law or a two-blackbody model), pulsating at \(\sim 8.4\) s. The spin period, together with the short bursts and the spectral properties, as well as the lack of an optical or infrared counterpart, confirmed the magnetar nature of the source.

Although previously unnoticed, the source was already present in two ROSAT X-ray catalogues, having been serendipitously detected in 1993 September 14–15 at a flux level of \(\sim 4 \times 10^{-14}\) erg cm\(^{-2}\) s\(^{-1}\)\(^{15, 16}\). A distance of \(\sim 1.6\) kpc has been proposed on the basis of a possible association, supported by the similar absorption columns, between the source and the H\(\text{ii}\) region M17\(^{16}\). Adopting this value, the 1993 luminosity was roughly \(3 \times 10^{32}\) erg s\(^{-1}\), while it was \(\sim 9 \times 10^{35}\) erg s\(^{-1}\) in 2011 July (0.5–10 keV). After its (re)discovery, Swift J1822.3–1606 was intensely monitored between 2011 July and 2012 August using Swift, RossiXTE, Chandra and XMM-Newton\(^{14–16}\). In this period, during which the source remained moderately burst-active,\(^{16}\) the luminosity declined from the discovery value to \(\sim 10^{33}\) erg s\(^{-1}\) (see Refs. 15, 16 and Sec. 4 for the long-term X-ray curve).

Based on the data collected between 2011 July and October, Ref.\(^{14}\) proposed a period derivative of \(\sim 2.5 \times 10^{-13}\) s s\(^{-1}\), corresponding to a surface dipole magnetic field of \(\sim 4.7 \times 10^{13}\) G, a value lower than that of 1E 2259+586 and comparable to that of the magnetar/pulsar PSR J1846–0258. Using a partially different set of observations covering the period from 2011 July to 2012 April, Ref.\(^{15}\) revised the spin-down rate and the magnetic field to even lower values of \(\sim 8 \times 10^{-14}\) s s\(^{-1}\) and \(\sim 2.7 \times 10^{13}\) G. Recently, Ref.\(^{15}\) proposed new coherent timing solutions using the data of Ref.\(^{14}\) plus several new observations that extended the base line to 2012 August (spanning \(\sim 400\) days). These solutions, which differ one from another for the number of frequency derivatives considered in the Taylor series used to fit the pulse phase evolution (see Sec. 2), yield spin-down rates between approximately \(0.7 \times 10^{-13}\) s s\(^{-1}\) and \(3.1 \times 10^{-13}\) s s\(^{-1}\), implying a magnetic field between \(2.4 \times 10^{13}\) G and \(5.1 \times 10^{13}\) G. The reason for these discrepancies and the multiple possible solutions is likely the ‘timing noise’ (e.g. Ref.\(^{18}\)), a common phenomenon in young neutron stars and magnetars in particular\(^{28}\). If so, the timing properties of Swift J1822.3–1606 probably will need longer time to be properly characterized. Nevertheless, any of the values proposed so far in literature would make Swift J1822.3–1606 the magnetar with the second lowest inferred magnetic field after SGR 0418+5729. Finally, also for this source the timing analysis dismisses the possibility that the X-ray emission could be powered to a substantial extent by the loss of rotational energy (see Table\(^{1}\)).

### 3.2.1. Multi-wavelength observations

Significant efforts have been devoted to detect emission in energy bands other than X-ray from Swift J1822.3–1606 but so far, as for SGR 0418+5729, without success.
At radio frequencies, the field of Swift J1822.3–1606 was pointed four times between 2011 July and October with the Green Bank Telescope, but no pulsed emission was detected down to a flux density of 0.05–0.06 mJy. An observation with the z filter (9694 Å) was carried out at the Gran Telescopio Canarias on 2011 July 21, resulting in a limiting magnitude $z > 22.4$. The field was also serendipitously imaged on 2006 May 3 by the UK Infrared Telescope; the analysis of this pre-2011-outburst observation yielded infrared limiting magnitudes $J > 19.3$, $H > 18.3$, and $K > 17.3$ (see Ref. 15 and references therein for details on this and other optical and infrared observations).

4. Low-Magnetic-Field Magnetars In The ‘Standard Model’

As discussed in the previous section, overall the observed properties of SGR 0418+5729 and Swift J1822.3–1606 are not dissimilar from those of other known (transient) magnetar candidates. The blatant difference is in the estimated strength of the dipole field $B_p$, which is well in the ordinary pulsar range, especially for SGR 0418+5729. There is, moreover, a further point which sets the two low-$B$ sources apart: their long characteristic age, $\tau_c \gtrsim 10^6$ yr, as compared to $\approx 10^3–10^4$ yr for the other magnetar candidates. The latter is clearly a consequence of the former, and both reflect the smallness of the period derivative. Albeit the characteristic age is well known for providing a poor estimate of the neutron star true age, the very large values of $\tau_c$ may suggest that SGR 0418+5729 and Swift J1822.3–1606 are old objects. The small number of detected bursts (with comparatively low energetics) and the low persistent luminosity in quiescence have been taken as further hints that these might be worn-out magnetars, approaching the end of their active life. The ‘old magnetar’ scenario sounds appealing since it offers an interpretation of the low-$B$ sources within an already established framework, validating the magnetar model also for (surface) field strengths quite far away from those of canonical SGRs/AXPs. At the same time, it raises a number of crucial questions, chiefly how one can reconcile the low dipole field with the huge magnetic stresses required to deform the crust and produce the bursts/outbursts.

Actually, one should bear in mind that the ultimate powerhouse of a (active) magnetar is its internal field, its toroidal component in particular. So, it is possible that in low-$B$ magnetars, and in other neutron star sources as well, the magnetic field is ‘hidden’ in the star interior and only a relatively weak dipolar component emerges. Still, if SGR 0418+5729 and Swift J1822.3–1606 were born with a magnetar-like surface field, $B_p$ must have decayed by a factor $\approx 10–100$ to match the current values. Roughly the same reduction is expected in the internal field. Although the latter could initially be $\approx 10$ times higher than $B_p$ (at least locally), one may wonder if at late times internal magnetic stresses are still strong enough to crack the crust. A second and related question is if realistic models of field decay in magnetars can account for the observed rotational properties (period, period derivative) and quiescent luminosity of SGR 0418+5729 and Swift J1822.3–1606.
This also directly bears to the true age of the sources which are most probably (much) younger than their characteristic age, estimated assuming a non-decaying field.

4.1. Magneto-rotational Evolution

The more general configuration for the internal field in a neutron star will be that produced by the superposition of current systems in the core and the crust. As stressed by Ref. 52, the relative contribution of the core/crustal fields is likely different in different types of neutron stars. In old radio pulsars, where no field decay is observed, the long-lived core component may dominate, while a sizable, more volatile crustal field is probably present in magnetars, for which substantial field decay over a timescale $\approx 10^3-10^5$ yr is expected (see e.g. Ref. 53).

In magnetars, because of the lesser role of ambipolar diffusion in the core, the decay/evolution of the magnetic field is likely to take place in the crust and is governed by Hall/Ohmic diffusion. The relative importance of these two mechanisms is strongly density- and temperature-dependent. Thus, any self-consistent study of the magnetic field evolution must be coupled to a detailed modeling of the neutron star thermal evolution, and conversely. This basically means that the induction equation for $B$ must be solved together with the cooling, a quite challenging numerical task.

In recent years, much efforts have been devoted to this problem. 52, 55, 56 The state-of-the-art numerical code is that of Ref. 56, which features full magneto-thermal coupling and includes all realistic microphysics. However, owing to numerical difficulties in treating the Hall term, the models in Ref. 56 include only Ohmic diffusion. This can be a limitation because the Hall drift likely drastically affects the very early evolution of ultra-magnetized neutron stars with surface field $B_p \gtrsim 10^{15}$ G. However, for lower $B_p$, still well within the magnetar range, the effect of the Hall drift is expected to introduce at the most quantitative changes (a somewhat faster dissipation) with respect to the purely Ohmic picture. Very recently a code capable of properly handling the Hall term has been presented, but applications to neutron star magneto-thermal evolution are still to come.

In order to explore if, and to which extent, the magneto-thermal evolution of (initially) highly magnetic neutron stars can lead to objects with properties compatible with those of SGR 0418+5729 and Swift J1822.3–1606, Refs. 12 and 15 performed a number of runs using the code of Ref. 56. Results for the two sources are shown in Figs. 2 and 3. The main outcome is that magnetic field decay in an initially ultra-magnetized neutron star, $B_p(t = 0) \sim 2 \times 10^{14}$ G, can reproduce the observed $P$, $\dot{P}$, $B_p$ and $L_X$ in SGR 0418+5729 and Swift J1822.3–1606, for an age $\sim 1$ Myr and $\sim 0.5$ Myr, respectively, provided that the initial internal toroidal field $B_{tor}(t = 0)$, the key parameter, is high enough, $\sim 10^{16}$ G in the former and $\sim 5 \times 10^{15}$ G in the latter. Evolutionary calculations confirm that these are old sources, as expected, although the true age is shorter than $\tau_c$, the difference being more than one order of magnitude for SGR 0418+5729.
4.2. Active, Till the End

Recently Ref.\textsuperscript{58} used the magnetic evolution code of Ref.\textsuperscript{52} together with the cooling models by Ref.\textsuperscript{56} to compute the magnetic stresses acting on the neutron-star crust at different times. Their baseline model has $B_p(t = 0) = 8 \times 10^{14}$ G and $B_{\text{tor}}(t = 0) = 10^{15}$ G. They found that the occurrence of crustal fractures (and hence of bursts/outbursts) is not restricted to the early neutron star life, during which the surface field is ultra-strong, but can extend to late phases ($\approx 10^5$–$10^6$ yr; see their figure 2). Both the energetic and the recurrence time of the events evolve as the star ages. For ‘old’ magnetars about 50% crustal fractures release $\approx 10^{41}$ erg and the waiting time between two successive events is $\approx 1$–10 yr. They also made a longer run with a model with $B_p(t = 0) = 2 \times 10^{14}$ G and $B_{\text{tor}}(t = 0) = 10^{15}$ G, for which the event rate is about a factor 10 smaller.

The models which successfully reproduce the properties of SGR 0418+5729 and Swift J1822.3–1606 have $B_p(t = 0)$ very close to this latter configuration, while $B_{\text{tor}}(t = 0)$ is larger. On the basis of this, although no dedicated simulations have been performed, Refs.\textsuperscript{12} and \textsuperscript{15} concluded that the two low-$B$ magnetars can become burst-active despite their age, with an expected (present) event rate similar to what predicted by the second model of Ref.\textsuperscript{58} i.e. $\approx 0.01$–0.1 yr$^{-1}$.

![Fig. 2. From top left to bottom right: time evolution of $L_X$, $B_p$, $P$, $\dot{P}$ for SGR 0418+5729. The three curves in each panel refer to $B_{\text{tor}}(t = 0) = 0$ (solid lines), $B_{\text{tor}}(t = 0) = 4 \times 10^{14}$ G (dotted lines) and $B_{\text{tor}}(t = 0) = 4 \times 10^{16}$ G (dashed lines). Figure taken from Ref.12.](image-url)
4.3. Outburst Decay

Outbursts, a distinctive trait of ‘transient’ magnetars which SGR 0418+5729 and Swift J1822.3–1606 belong to\cite{22} are characterized by a sudden increase in flux (up to a factor \(\approx 1000\) over the quiescent level) followed by a slow decay which lasts months/years. In many sources the spectrum during the outburst is thermal (modeled by one or two blackbodies with \(kT \sim 0.3\text{–}0.9\text{ keV}\)) and typically softens during the decay. The radiation radius of the emitting area(s) is small \(\approx 0.1\text{–}1\text{ km}\) and usually decreases in time. This has been interpreted as due to some form of heat deposition in a limited region of the star surface which then cools and shrinks. Up to now, however, the heating mechanism has not been unanimously assessed. A possibility is that energy is injected deep in the crust, e.g. because of magnetic dissipation, and then flows to the surface.\cite{59} Alternatively, heating may be produced by currents flowing in a twisted magnetosphere as they hit the star.\cite{60}

Very recently, Ref.\cite{61} developed a quantitative model for the outburst evolution by simulating the thermal relaxation of the neutron star in response to an impulsive energy injection in the star crust. Results were successfully applied by Ref.\cite{15} to fit the outburst decay in Swift J1822.3–1606 for the entire period covered by their

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{From top left to bottom right: time evolution of \(P, \dot{P}, L_X \text{ and } B_p\) for Swift J1822.3–1606. The dashed vertical line marks the estimated age of the source; the gray strips in the first two panels show the observed values of \(P\) and \(\dot{P}\) with their uncertainties. The model is for \(B_{\text{tor}}(t = 0) = 5 \times 10^{15}\) G. Figure taken from Ref. 15.}
\end{figure}
observations, ∼250 d after the first burst that led to the discovery of the source (see Fig. 4). The case of SGR 0418+5729, for which a much longer time coverage is available (∼1200 d), is, however, much less conclusive in this respect (see Fig. 5). The calculated flux in the 0.5–10 keV band systematically underestimates the observed one at later times (∼400 d), when the luminosity suddenly drops and the hotter blackbody (initially at kT ∼ 0.9 keV) disappears leaving only a cooler component at ∼0.3 keV. As discussed in Ref. 13 because of the limited spatial extent of the twist and hence of the low luminosity released by ohmic dissipation, current heating is also unlikely to explain the observed flux in SGR 0418+5729. Only a long term monitor will assess if SGR 0418+5729 is indeed peculiar, or also Swift J1822.3–1606 is bound to show the same behavior at late times.

5. Low-Magnetic-Field Magnetars: Alternative Scenarios

Although many indirect evidences accumulated in favor of the magnetar paradigm (e.g. Refs. 3, 62) and despite it proved to be quite successful in explaining the observed properties of SGRs/AXPs, including those of the low-B sources, no conclusive, direct measure of the surface magnetic field has been claimed as of yet. Contrary to other classes of X-ray pulsars, in which it has been possible to infer B_p from the detection of (electron) cyclotron lines in their spectra (see e.g Ref. 63 for a review), up to now spin-down remains the only way for SGRs/AXPS, with all the ensuing uncertainties (chiefly the fact that spin-down is indeed due to

![Fig. 4. Flux (in the 1–10 keV band) decay following the outburst of Swift J1822.3–1606. The trigger time of the first burst detected from this source was MJD 55756.533. The filled circles mark the observations and the curve the best model. Figure adapted from Ref. 15.](image-url)
magneto-dipole losses alone).

Since the cyclotron energy for a particle of charge $e$ and mass $m$ is $E_B \sim 11.6(m_e/m)(B/10^{12} \text{ G}) \text{ keV}$ (here $m_e$ is the electron mass), the electron line falls well above the X-ray range for fields $B_p \gtrsim 10^{14} \text{ G}$, becoming inaccessible to observations. However, the proton line, at $\sim 0.63(B/10^{14} \text{ G}) \text{ keV}$ is squarely in the X-ray band for magnetars. Proton cyclotron lines in magnetar atmospheres have been extensively investigated and searched for in virtually all available observations but escaped unambiguous detection.

The lack of a smoking gun, definitely proving that magnetar candidates host an ultra-magnetized neutron star, stimulated in the past the investigation of alternative scenarios, which could explain the observed characteristics of SGRs/AXPs without resorting to ultra-strong magnetic fields. The recent discovery of low-$B$ sources, with magneto-rotational properties similar to those of standard radio pulsars, renewed this interest. A long-standing competitor of the magnetar scenario has been the fossil-disc model, according to which SGRs/AXPs harbor a neutron star with standard magnetic field ($\approx 10^{12} - 10^{13} \text{ G}$) which is effectively spun down by the interaction with a debris disc left in the parent supernova explosion or after a common

\footnote{The presence of proton cyclotron features has been reported in the spectra of some magnetar bursts, e.g. Refs.\cite{71,72} but never assessed with certainty; in these cases the derived magnetic field is of the same order of that implied by spin-down.}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{flux Decay SGR 0418+5729}
\caption{Flux (in the 0.5–10 keV band) decay following the outburst of SGR 0418+5729. The trigger time of the first burst detected from this source is MJD 54987.862. The filled circles mark the observations and the curve the model which best fits the early (<400 d) evolution. Figure adapted from Ref.\cite{13}.}
\end{figure}
envelope phase, e.g. Refs.\cite{73,74} and \cite{75}. According to Ref.\cite{76} if SGR 0418+5729 was born with a period longer than 70 ms and a low dipolar field ($B_p \approx 10^{12}$ G), the torque exerted by a fall-back disc can spin down the star to the present period in $\gtrsim 10^5$ yr. In Ref.\cite{77} a somehow similar scenario in which SGRs/AXPs are neutron stars with a low dipole field and super-strong multipolar components powered by accretion from a fall-back disc was recently proposed.

Models not invoking neutron stars at all have also been discussed. Massive ($\sim 1.3$–$1.4 \, M_\odot$) white dwarfs endowed with high (on white-dwarfs standards) magnetic fields were suggested as possible powerhouses.\cite{78} The basic idea is that, being a white dwarf $\approx 1000$ times bigger than a neutron star, at comparable mass, its moment of inertia is $\approx 10^6$ times larger. This implies that rotational energy losses can be large enough to explain the observed X-ray luminosity in SGRs/AXPs ($\approx 10^{32} – 10^{36}$ erg s$^{-1}$) even for quite low values of the period derivative. In addition, since $B_p \propto I^{1/2}/R^3 \propto 1/R^2$, the dipolar magnetic field derived from spin-down is much lower in a white dwarf than in a neutron star with the same $P$ and $\dot{P}$. The inferred values of $B_p \sim 10^8–10^{10}$ G are somehow high, but still consistent with those observed in white dwarf. In this scenario all the SGRs/AXPs activity (bursts, outbursts, giant flares) is powered by the relief of mechanical stresses, driven by gravity overcoming centrifugal forces as the white dwarf spins down. The change in the star oblateness produces a decrease of the moment of inertia and, in turn, a sudden increase of the spin frequency (a glitch). The energy which can be tapped is $\approx 10^6\%$ of the available gravitational energy, $(GM_{WD}^2/R_{WD})(\Delta R/R) \sim 2 \times 10^{31}\left|\Delta P\right|/P$ erg. In order to accommodate the energetic of bursts and also of the giant flares, which is several orders of magnitude larger,\cite{19} the white-dwarf model requires $\left|\Delta P\right|/P$ in the range $\approx 10^{-7} – 10^{-3}$. The fastest spinning SGRs/AXPs ($P \lesssim 5$ s) might pose a problem for the stability of white dwarfs. However, according to recent calculations\cite{79} the minimum period of a rotating, massive white dwarf could be as small as 0.3 s.

A more exotic scenario involves quark stars (see Ref.\cite{80} and references therein). A neutron star could evolve into a quark star following an increase of its core density due to spin down or accretion. A quark nova explosion (releasing gravitational energy up to $\approx 10^{53}$ erg, mostly carried away by neutrinos) would mark this transition and cause the ejection of the neutron star iron-rich outer layers. Depending on the quark star initial spin frequency, the degenerate debris material would either form a shell co-rotating with the star or be confined in a Keplerian ring (in the case of fast rotation). SGRs/AXPs are, then, the observational manifestation of these quark star–ring/shell systems.

Such quark stars are expected to be ultra-magnetized, because color ferromagnetism in the quark matter can give rise to a surface magnetic field of $\approx 10^{15}$ G at the birth. The star spins down because of magnetic braking and expels from its superfluid/superconducting interior magnetic vortices. As a consequence, its magnetic field decreases and magnetic reconnections at the star surface produces an X-ray luminosity $L_{X,V} \approx 10^{35} \eta \dot{P}_{-11}^2$ erg s$^{-1}$, where $\dot{P}_{-11}$ is the period derivative.
in units of $10^{-11}$ s s$^{-1}$ and $\eta$ is an efficiency parameter. In addition, quark star–ring systems can emit X-rays from a hot spot formed where steadily accreted ring matter hits the surface. Transient magnetars are quark star–ring sources that only sporadically enter phases of steady accretion. Both shell and ring systems would produce bursts when clumps of debris degenerate material are accreted onto the star, the shell systems being the most burst-prolific sources, since the co-rotating shell is an inherently-less-stable structure than the ring.

In this picture, SGR 0418+5729 is an evolved quark star–ring system that has almost consumed its ring, after which it will join other old shell- and ring-less systems with low vortex luminosity corresponding, in this scenario, to the XDINSs. The characteristic age of SGR 0418+5729 in the quark-nova scenario (where the braking index is $n = 4$) is $\tau_c = P/(3\dot{P}) \simeq 24$ Myr and at this age the magnetic field is expected to have been decayed from the initial value of $\sim 10^{15}$ G to below $\sim 10^{13}$ G. Reference 80 shows that the rotational and magnetic characteristics of SGR 0418+5729 can account for many of the properties of the source observed during the first $\sim 160$ days of the 2009 outbursts.

6. Conclusion

The discovery of low-$B$ magnetar sources has opened new perspectives in neutron-star astrophysics and its consequences deeply impact on our current view of what a ‘magnetar’ is. As it was discussed in Sec. I, SGR 0418+5729 and Swift J1822.3–1606 are likely to be ‘old magnetars’, i.e. once-ultra-magnetized neutron stars which experienced substantial field decay over their (extended, $\approx 1$ Myr) lifetime. Still, they retain a large-enough ($\approx 10^{14}$ G) internal toroidal field, sufficient to sporadically produce crustal displacements and hence bursting activity.

In this sense, the original definition of a ‘magnetar’ as a neutron star powered by magnetic energy$^6$ applies to the low-field sources too, at least if one restricts to their active phases. What the low-$B$ sources taught us is that this reservoir of magnetic energy (stored in the internal field) needs not to show up at all. The external field in a active magnetar can well be comparable to that of radio pulsars, dispelling the widespread belief that magnetar activity is necessary associated to an ultra-high dipole field.

The two known low-field sources, SGR 0418+5729 and Swift J1822.3–1606, are likely not exceptions. Since they represent the ‘old’ population of initially strongly-magnetized neutron stars, they may constitute the majority of magnetar candidates, although their duty cycle is long. Actually, about 20% of Galactic radio pulsars have a dipole field higher that that of SGR 0418+5729$^{11}$ and in principle any of them may show up as a transient magnetar at any time.

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