On methods for extracting exact non-perturbative results in non-supersymmetric gauge theories

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Abstract

At large $N$, a field theory and its orbifolds (given by projecting out some of its fields) share the same planar graphs. If the parent-orbifold relation continues even nonperturbatively, then properties such as confinement and chiral symmetry breaking will appear in both parent and orbifold. $\mathcal{N} = 1$ supersymmetric Yang-Mills has many nonsupersymmetric orbifolds which resemble QCD. A nonperturbative parent-orbifold relation gives a number of interesting predictions, exactly valid at large $N$, and expected to suffer only mild $1/N$ corrections. These include degeneracies among bosonic hadrons and exact predictions for domain wall tensions. Other predictions are valid even when supersymmetry in the parent is broken. Since these theories are QCD-like, simulation is possible, so these predictions may be numerically tested. The method also relates wide classes of nonsupersymmetric theories.
In recent years, many exact results for four-dimensional supersymmetric theories have been obtained. Supersymmetric theories have holomorphic or otherwise mathematically special sectors which are strongly constrained. While the theories are not exactly soluble, many remarkable nonperturbative predictions can still be made. These include the infrared behavior of some theories, the large gauge-coupling behavior of others, and the properties of some theories with many colors at large \('t\) Hooft coupling.

Unfortunately, few of the techniques hold for ordinary, nonsupersymmetric theories. This is regrettable for many reasons. In nonsupersymmetric solutions of the hierarchy problem, nonperturbative dynamics is sure to play a role, and we are unable to make any predictions about it. In supersymmetric theories, supersymmetry breaking is often a strongly-coupled phenomenon, and again we can make very few predictions in the strong-coupling sector. Applications of quantum field theory to condensed matter (especially in three dimensional systems) and to statistical mechanics are limited by our lack of nonperturbative knowledge. Finally, no lattice simulations of four-dimensional supersymmetric theories (with one exception discussed below) are remotely practical, and so the techniques of supersymmetric theories cannot be tested numerically, nor can additional information be obtained through simulation.

There is consequently ample motivation for attempting to find new nonperturbative methods in nonsupersymmetric theories. So little is known that any progress whatsoever is of value. In this letter I employ a general technique — orbifolding, explained below — which predicts relations between gauge theories that are exact in the limit where the number of colors is taken to infinity. Of course, a statement that theory A and theory B have the same Green’s functions, while remarkable, does not give direct predictions in either theory A or theory B alone. To obtain quantitative predictions in theory B one must have some additional a priori knowledge of theory A, obtained from another source. In this letter I point out that if we take \(\mathcal{N} = 1\) supersymmetric Yang-Mills theory as theory A, about which a number of interesting facts are known, some of them exactly, we can obtain predictions, some of them exact (up to \(1/\mathcal{N}\) corrections with no exceptionally large coefficients) for
a class of nonsupersymmetric theories. Furthermore, the relations between these theories survive even when supersymmetry is broken, for any value of the supersymmetry-breaking parameter.

The technique used here involves comparing a “parent” theory to an “orbifold” of the parent theory. The orbifold theory is obtained from the parent by judiciously setting a number of degrees of freedom to zero — that is, by inserting appropriate delta functions of the fields into the path integral. (Notice this is not a projection operator on the generating functional itself, but on the measure of integration defining the generating functional.) Following discoveries in the gauge-gravity correspondence it was pointed out that there is a striking relationship even at weak coupling between orbifold theories and their parents. Specifically, if one organizes color indices using ’t Hooft’s double-line notation, and uses his topological classification of Feynman diagrams, then parent and orbifold share the same planar graphs (up to a trivial rescaling of coupling constants.) This means the two theories have many Green’s functions in common at large $N$, at least to all orders in perturbation theory. Note that supersymmetry is nowhere needed in this argument.

More precisely, consider the Green’s functions of the parent theory, which are functions of the Yang-Mills coupling $g_{YM}^2$ and the number of colors $N$. Define $\zeta \equiv g_{YM}^2 N / 8 \pi^2$, $\alpha = g_{YM}^2 / 4 \pi$; then the Green’s functions may be reexpanded, as ’t Hooft suggested, as $G^p(\zeta, \alpha)$; here $p$ stands for “parent”. The limit $\alpha \to 0$ with $\zeta$ fixed keeps only the planar graphs in the perturbative expansion. The orbifolds of this theory (under suitable conditions) will share the same planar graphs, so their Green functions $G^o(\zeta, \alpha)$ will share the same limit; $G^o(\zeta, 0) = G(\zeta, 0)$. However, the number of fields in the orbifold is less than the number in

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1 Orbifolding involves imposing a certain discrete projection operator on the degrees of freedom of a field theory, but there are two distinct meanings to the term. One involves orbifolding the space-time in which a field theory is defined. The second, referred to in this paper, involves projecting out some of the fields of a theory, leaving the space-time unaffected.
the parent; for example, if the orbifold involves a $\mathbb{Z}_k$ symmetry, then the number of fields is reduced by a factor $k$. This means that to keep the parent and orbifold value of $\zeta$ equal, we must set $\alpha^{(o)} = k\alpha^{(p)}$. This scaling trivially cancels out of the planar graphs.

However, while rigorous, this argument does not imply that orbifold and parent share their non-perturbative effects, for example effects of order $e^{-1/\zeta}$. Unfortunately, such effects include confinement and chiral symmetry breaking, string tensions, pion masses and baryon masses, among others — in short, all of the most interesting dynamics of QCD. It is pure conjecture — let us call it the “non-perturbative orbifold” (NPO) conjecture — that the parent and orbifold theories have the same non-perturbative properties at large $N$.

As stated, the conjecture is still imprecise and must be sharpened. In general the parent and orbifold theories do not have the same vacua. The parent has vacua lacking in the orbifold, because they are projected out by the orbifolding process. The orbifold has extra vacua not shared by the parent, ones which break spontaneously the orbifold symmetry, which appears as a global symmetry group in the orbifold theory. Similarly, the parent has operators absent in the orbifold (since they are projected out) and the orbifold has operators absent in the parent (which are non-singlets under the orbifold symmetry group.)

The statements concerning planar graphs are that in those vacua which appear in both theories, the Green’s functions for operators which appear in both theories are identical, up to appropriate rescalings of coupling constants. The NPO conjecture simply suggests this extends to nonperturbative effects.

If the NPO conjecture is true, then it relates the nonperturbative properties of field theories which have small $1/N$ corrections, independent of whether they have supersymmetry. But is there any evidence in its favor? In supersymmetric theories, even those with only $\mathcal{N} = 1$ supersymmetry, there is considerable evidence for the NPO conjecture both from the AdS/CFT correspondence $[4, 5]$ and from nonperturbative exact results in supersymmetric theories $[6, 7]$. Unfortunately, little evidence exists for nonsupersymmetric orbifolds. In my view, most of the work on nonsupersymmetric orbifolds in the gauge-gravity correspondence is suspect. The examples discussed initially $[8, 10]$ have massless scalar fields,
and are subject to large $1/N$ corrections (see Refs. [11,12]) which can completely alter their nonperturbative properties. In one case, however, a nonsupersymmetric theory was discussed which does not have large $1/N$ corrections [13]. The parent theory is so-called “$\mathcal{N} = 1^*$”, namely $\mathcal{N} = 4$ Yang-Mills, softly broken to $\mathcal{N} = 1$ Yang-Mills by mass terms. The orbifold of this theory is a finite $\mathcal{N} = 2$ supersymmetric theory broken softly to a nonsupersymmetric $\mathbb{Z}_2$ orbifold of $\mathcal{N} = 1$ Yang-Mills. There are no large $1/N$ corrections in this theory, as there are no unprotected light scalar fields, continuous moduli spaces, or nearly marginal operators which are unstable to $1/N$ corrections. It was found [13] that the parent supersymmetric theory and its nonsupersymmetric orbifold share the same physics, including confinement, monopole/dyon condensates, flux tubes with appropriate quantum numbers, and so forth, and that the expected quantitative relations between the theories do hold. Thus the orbifold conjecture does apply for at least one QCD-like nonsupersymmetric theory, up to controllably small $1/N$ corrections, and in particular reproduces interesting nonperturbative phenomena at large $\zeta$. This motivates us to take the conjecture seriously, even without proof, and to see where it leads us.

I will now discuss some predictions of the NPO conjecture, and then provide some circumstantial evidence that it is likely to be correct in this context. Let us take, as parent theory, $\mathcal{N} = 1$ $SU(N)$ supersymmetric Yang-Mills theory, which consists simply of a gauge boson and a fermion (“gaugino”) in the adjoint representation, with Lagrangian

$$\mathcal{L} = \frac{1}{g^2} \text{tr} \left( -\frac{1}{2} F^2 + i \bar{\lambda} D \lambda \right).$$

Much is known about this theory. It has an anomalous $U(1)$ axial symmetry, of which a $\mathbb{Z}_{2N}$ subgroup is anomaly-free. It generates a gluino condensate $\langle \lambda \lambda \rangle$, which breaks the $\mathbb{Z}_{2N}$ to $\mathbb{Z}_2$. This condensate, of size $\langle \lambda \lambda \rangle \propto e^{-1/\zeta}$, does not appear in the planar graphs, yet is not suppressed at large $N$. The phase of the gluino condensate (let us call it the $\eta'$, in analogy with the similar phase in QCD) has a potential with $N$ isolated vacua, and in each the $\eta'$ has a mass of order $e^{-1/\zeta}$. The domain walls between these vacua have tensions of order $\langle \lambda \lambda \rangle$. The theory confines and has flux tubes carrying a $\mathbb{Z}_N$ charge, with string tensions and
hadron masses of order $e^{-1/ζ}$. Moreover the strings can annihilate in bunches of $N$. In short, almost all the interesting properties of the theory are absent in the pure planar graphs, and many depend on $α$ or $N$. This is why the orbifold conjecture becomes especially interesting if it applies nonperturbatively.

We can apply the conjecture to simple $Z_p$ orbifolds of this theory which are nonsupersymmetric and similar to QCD. A clear field-theorist’s introduction to these theories has appeared in Ref. [4] and the reader is directed there for further discussion. For the string theorist, the $Z_2$ orbifold in question is discussed in Ref. [13]. For our purposes here, I will simply state the results of [4].

The $Z_p$ orbifold of $SU(pN)$ SYM theory is simply given by breaking the $pN × pN$ matrices into $N × N$ blocks, keeping the diagonal blocks for the gauge bosons, and keeping the just-above-the-diagonal blocks for the fermions. The resulting theory is $SU(N)^p$ — let us call the $p$ factors $SU(N)_1$, $SU(N)_2$, etc. — with $p$ Weyl fermions $ψ_i$, $i = 1, 2, \ldots, p$, charged as $N$ under $SU(N)_i$ and as $\overline{N}$ under $SU(N)_{i+1}$ (or for $ψ_p$ as $\overline{N}$ of $SU(N)_1$.) “Moose” or “quiver” diagrams of the theories with $p = 1, 2, 3, 4$ are shown in the Figure.

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2To my knowledge these theories first appear in [14,15], were viewed as orbifolds only recently [1], and were emphasized as especially interesting first in [13,16].

3Strictly speaking, we ought to consider $U(pN)$ and its orbifolds $U(N)^p$. However, the missing $U(1)$ factors (except one, for $p$ even) are anomalous and presumably are massive. This subtlety is a $1/N$ effect, of course.
FIG. 1. $\mathcal{N} = 1$ supersymmetric Yang-Mills and its $\mathbb{Z}_2$, $\mathbb{Z}_3$ and $\mathbb{Z}_4$ orbifolds.
Let us first discuss the symmetries of these theories. As always, the $Z_p$ symmetry used to orbifold the SYM theory is a symmetry $(Z_p)_O$ of the resulting orbifold gauge theory; for each $q = 1, \ldots, p$, it simply cyclically shifts the $i^{th}$ gauge group and fermion into the $(i + q)^{th}$ group and fermion. There is also a reflection symmetry $i \to p - i + 1$. Also, each one of the $SU(N)$ gauge groups has a $Z_N$ center. The matter in the theory is invariant under a simultaneous gauge transformation, in each of the $SU(N)$ factors, by $U = \omega 1$, where $\omega$ is any $N^{th}$ root of unity. This symmetry I will call $(Z_N)_C$, where $C$ stands for “center of the gauge group”. This group determines the classes of electric sources which can be used to probe the theory and the symmetry group of any flux tubes should the theory be confining.

There are also classical $U(1)$ symmetries rotating each of the $p$ fermions, but these are anomalous. Let us consider rotations of the $\psi_i$ by $e^{2\pi i \sigma}$. To be anomaly free, a rotation must have $N(\sigma_i - \sigma_{i-1})$ an integer. Accounting for the centers of the $SU(N)_j$ groups, we have the transformation $\psi_1 \to \psi_1 \exp(2\pi i / N)$ with all other $\psi_i$ invariant. I will refer to this as $(Z_N)_A$, “$A$” for axial. For $p$ odd, this is expanded to a $(Z_{2N})_A$ by the addition of the rotation $\psi_j \to \psi_j \exp(-i\pi [1 - 1]^{j}/N)$; the square of this transformation is equivalent, using the centers of the $SU(N)$ groups, to the transformation $\psi_1 \to \psi_1 \exp(2\pi i / N)$ with all other $\psi_i$ invariant. For $p$ even, we have instead an additional $U(1)_B$, a baryonic symmetry under which $\psi_j \to \psi_j \exp(-i[1 - 1]^{j}\alpha)$, where $\alpha$ is an arbitrary phase. This is a first hint that the odd and even $p$ cases are distinct in important ways.

For both odd and even $p$, we might expect that the discrete axial symmetry will be broken by a fermion condensate, preserving a $Z_2 \psi_i \to -\psi_i$ for $p$ odd, and preserving $U(1)_B$ for $p$ even. If this happens, the resulting theory in both cases will have $N$ degenerate vacua, just as in $N = 1$ SYM. We might also expect confinement, with flux tubes charged under $(Z_N)_C$. We will see this is plausibly the case.

Let us now explore the $Z_2$ orbifold in detail, and learn what the NPO conjecture predicts for it. This theory is especially interesting, as it resembles QCD. It contains $SU(N) \times SU(N)$ with a Dirac fermion $\Psi = (\psi_1, \bar{\psi}_2)^T$ in the $(N, \overline{N})$ representation. Note that the masslessness of the fermion is protected by the $(Z_N)_A$ discrete axial symmetry. For $N = 3$, this is
just QCD with three massless quarks, with the diagonal subgroup of the chiral $SU(3)_L \times SU(3)_R$ flavor symmetry gauged. Thus, we expect this theory to have confinement and chiral symmetry breaking also, at least when one of the couplings is taken very small. Specifically, if $SU(N)_1$ becomes strongly coupled, it should generate a condensate for $\psi_1 \psi_2$. Were $g_2 = 0$, there would be $N^2 - 1$ pions, along with the $\eta'$, just as in QCD. However, once $SU(N)_2$ is gauged, $SU(N)_L \times SU(N)_R$ of flavor is explicitly broken; the pions become massive, with masses of order $g_2 \Lambda_1$. With the continuous space of vacua spanned by the pions removed by their masses, the remaining vacua of the theory are set by the minima of the $\eta'$ potential energy. Because of the anomaly, this potential is a periodic function with $N$ minima, related by the $(Z_N)_A$ axial symmetry. We thus find, in this QCD-like theory with weakly gauged flavor, that there are indeed $N$ degenerate and isolated vacua from the breaking of $(Z_N)_A$. With this structure, we expect stable domain walls in the theory. Thus $SU(N)_1$ generates chiral symmetry breaking and domain walls. But it does not generate flux tubes; any putative flux tubes can break through pair production of $\psi_1$ and $\psi_2$, just as flux tubes in QCD break by quark-antiquark pair production.

However, there is no massive matter charged under $SU(N)_2$ (since the pions are massive) so if $g_2$ is small, the low-energy theory is a pure Yang-Mills theory of $SU(N)_2$. This theory confines and makes flux tubes. The $(Z_N)_C$ symmetry, which is absent if $g_2 = 0$, ensures there are flux tubes that carry a $Z_N$ charge and do not break. For example, sources in the $(N, 1)$ representation of the gauge group cannot be screened by pair production of $\psi_1$ or $\psi_2$, so they will be confined by a flux tube.

Thus, for $g_1 > g_2$, we find chiral symmetry breaking at a large scale, and flux tubes with tensions at a much lower scale. What should happen at the orbifold point, where the couplings are equal? This is far from clear, since it might be that certain effects due to $SU(N)_2$ cancel effects produced by $SU(N)_1$. However, if we assume that there are no phase transitions as $g_2$ becomes of order $g_1$, the theory will maintain its confinement, its bilinear fermion condensate and its $N$ vacua. This would mean that its nonperturbative properties would be at least qualitatively similar to those of the parent $\mathcal{N} = 1$ SYM theory, leading
us to hope that the NPO conjecture is correct. (As mentioned earlier, there is additional evidence in this case from the gauge-gravity correspondence [13].)

So let us suppose that it is right; what will happen in this QCD-like theory? In the limit $g_1 = g_2$ there is the $(Z_2)_O$ symmetry exchanging the two gauge groups. All states in the theory will be even or odd under this $Z_2$, and the NPO conjecture applies to those which are even. Let us consider the $Z_2$-even hadrons. By gauge invariance, all mesons in this theory (that is, all particles with less than $N$ constituents) will be bosons. This is in contrast to the parent SYM theory, which has boson-fermion degeneracy. Naively the absence of the fermionic superpartners of the bosons in the orbifold theory would suggest that all of the interesting properties of the supersymmetric spectrum are removed in the orbifold. However, supersymmetric theories also have degeneracies among bosons with different spin-parity assignments. For example, scalar and pseudoscalar fields can be in the same multiplet, as can spin-1 and spin-0 fields, or spin-2 and spin-1 fields. Thus the NPO conjecture implies that the meson spectrum of the $SU(N) \times SU(N)$ theory will have surprising degeneracies among its bosonic mesons, up to $1/N$ corrections. Such behavior is quite remarkable, as it is ensured by no apparent symmetry.

While the spectrum of SYM is poorly understood, there are a few comments one can make with little difficulty. First, the phase and magnitude of the fermion bilinear condensate — the $\eta'$ and the $\sigma$ — are in the same supermultiplet. They will therefore be degenerate in the orbifold, for large $N$. Similar comments govern a low-lying spin-1 meson and spin-2 glueball, the first of which is the generalization of the $\omega/\phi$ flavor-singlet vector meson. It is possible that additional information can be gleaned by further study of the SYM spectrum along the lines of [17]. In any case, these degeneracies will be found throughout the spectrum, along entire Regge trajectories. Finally, in addition to degeneracies, we also expect, more generally, that all hadronic mass ratios should be the same in the two theories.

Is it useful to know that the two theories share a hadronic spectrum, since we do not have predictions for the hadron masses? Yes — since both of these theories can in principle be simulated [18–20]. In fact, they are as easy, and as hard, to simulate as unquenched QCD
with light fermions. (Actually they have better infrared behavior than QCD, which has light pions.) At least, since these theories have no massless scalars, they do not suffer from extreme fine-tuning as would most supersymmetric theories and their orbifolds. Consequently, the above predictions (along with the earlier assumption of no phase transition between $g_1 > g_2$ and $g_1 = g_2$) may be checked numerically.

Indeed, we can potentially improve the situation for numerical simulation in two ways. It is clear that the perturbative orbifold-parent relation holds even with quenching (although in this case it becomes relatively trivial.) We may hope therefore that both quenched theories are nonperturbatively identical at large $N$. More physically, we can avoid the difficulties of massless fermions by adding equal masses to the gluino, in the SYM, and to the Dirac fermion in the $\mathbb{Z}_2$ orbifold. While the mass degeneracies in the spectrum will be lifted, they will be broken in identical ways in the two theories (at large $N$.) Also, using supersymmetry-breaking spurion techniques, it should be possible to obtain quantitative theoretical predictions for some of the multiplet splittings when the gluino mass is small. In any case, the predictions of the NPO conjecture apply for any mass, so one may start by comparing the effects of a heavy fermion in the adjoint of $SU(2N)$ to those of a heavy fermion in the bifundamental of $SU(N) \times SU(N)$, generally as a function of this mass.

The predictions of the NPO conjecture are by no means limited to the hadron spectrum. Let us consider the question of condensates. If the $(\mathbb{Z}_2)_O$ is preserved in the true vacuum of the orbifold (as was assumed above) then all condensates are $\mathbb{Z}_2$-even and appear in the supersymmetric parent. In a supersymmetric theory, no highest-components of superfields can have condensates without breaking supersymmetry, which SYM is known not to do. Therefore, there are large classes of operators in the orbifold, which descend from highest components of superfields, which will have condensates that vanish at large $N$. These include gluon condensates $\langle \text{tr} F^2 \rangle$, which are forbidden in the supersymmetric theory. For those condensates which are nonzero, their magnitudes and phases should be the same in both orbifold and parent. The appearance of new condensates at finite gluino mass should also be mirrored in the orbifold theory, as should the variation in the fermion bilinear
condensate as a function of the fermion mass. Again, these effects could potentially be seen in simulations.

Since with zero fermion mass there are exactly degenerate isolated vacua (for any $N$) in both theories, there are stable domain walls. Much is known about these domain walls in the supersymmetric case; their tensions and degeneracies are exactly predicted. These exact predictions should also hold in the orbifold theories for large enough $N$.

For any value of the fermion mass, these theories have confinement and corresponding flux tubes charged under $(\mathbb{Z}_N)_C$. While there are no wholly reliable calculations of the tensions of flux tubes, it should be possible to compute and compare the flux-tube tensions in the two theories.

Even this is not a complete list, but as it will be some years before these many predictions are tested, let us turn our attention elsewhere. What new predictions might be obtained for $\mathbb{Z}_p$ orbifolds with $p > 2$? In this case no masses can be added for the gluino, since the orbifold projection removes the mass operator (although supersymmetry-violating higher-dimensional operators could be added to the theory.) What operator would we expect to get a chiral-symmetry breaking expectation value? For $p$ odd the lowest-dimension operator charged under $(\mathbb{Z}_{2N})_A$ is the “double-polygon” $\text{tr}(\psi_1 \psi_2 \cdots \psi_p \psi_1 \psi_2 \cdots \psi_p)$; for $p$ even the lowest operator charged under $(\mathbb{Z}_N)_A$ is the single-polygon $\text{tr}(\psi_1 \psi_2 \cdots \psi_p)$. We would naturally expect these operators to get expectation values and break the chiral symmetries down to $\mathbb{Z}_2$ for $p$ odd and $U(1)_B$ for $p$ even.

One can perform a series of checks of this expectation. Let us take the couplings away from the orbifold point, so that they are all different. Suppose, for example, that $SU(N)_2$ has the largest of the gauge couplings. At some scale $\Lambda_2$, $SU(N)_2$ becomes strongly coupled,

\footnote{Note that the parent $SU(2N)$ has twice as many flux tubes and domain walls as does the orbifold, which has only $N$. This is typical in orbifold projections. For quantities such as string tensions, one must be careful to compare the objects which are supposed to match.}
with all the other gauge groups acting as weakly-coupled spectators to its dynamics. From QCD with three light flavors, we expect that this $SU(N)$ gauge theory with $N$ quarks $\psi_2$ and $N$ antiquarks $\psi_1$ will confine and generate a bilinear fermion condensate $\langle \psi_1 \psi_2 \rangle$. Just as in QCD, this condensate breaks $SU(N)_1 \times SU(N)_3$ (the “left” and “right” chiral symmetries for the $SU(N)_2$ gauge factor) down to a diagonal subgroup $SU(N)_D$. In the breaking, the $N^2 - 1$ charged pions of $\psi_1 \psi_2$ are eaten by the broken vector bosons (leaving one neutral pion which decouples from the rest of the theory.) The combination of confinement of $SU(N)_2$ and breaking of $SU(N)_1 \times SU(N)_3 \rightarrow SU(N)_D$ leaves $p - 2$ gauge factors with $p - 2$ Weyl fermions cyclically charged as before. In short, the $Z_p$ orbifold tumbles dynamically to the $Z_{p-2}$ orbifold. The next stage of confinement and chiral symmetry breaking removes two more factors, and so on. This fact was the basis of the “moose calculus” of Georgi \cite{14} and was used extensively in technicolor model building.

If $p$ is odd, then the endpoint of this tumbling is simply the $p = 1$ case, which is just $\mathcal{N} = 1$ SYM. Thus \cite{15} for odd $p$ the low-energy effective theory has accidental supersymmetry! The final step in the dynamics will involve SYM physics: confinement, which generates flux tubes with a $(Z_N)_C$ quantum number, and a breaking of the discrete chiral symmetry, leaving a set of $N$ vacua which can have domain walls between them. (Note however this low-energy theory has a smaller value of $\zeta$ than does the parent theory, and thus they are not identical; this is not, of course, a mark against the NPO conjecture, since we have moved far from the orbifold point to obtain this conclusion.) Thus when the couplings are different the qualitative physics expected from the NPO conjecture is recovered. If we now assume that there is no phase transition between the equal-coupling regime and the tumbling regime, then the NPO conjecture is quite plausible. One can go further and check that the tumbling regime predicts indeed that the double-polygon gets an expectation value, and moreover that the size of this expectation value is guaranteed by symmetries to be of the same order as would be expected for this operator in the parent.

Similar statements can be made for $p$ even, although it is a bit more intricate. In this case one can show that the smallest coupling determines the dynamics in an interesting way.
Suppose $SU(N)_2$ has the smallest coupling. Then the *odd* gauge factors determine the size of chiral symmetry breaking condensates, while the *even* ones determine the size of confining string tensions. The reverse is true if, say, $SU(N)_3$ has the smallest coupling.

Note also, however, that we can view the $(Z_{2q})$ theory as a $Z_q$ orbifold of the $Z_2$ orbifold itself, or more generally of the latter theory with unequal gauge couplings. If $g_1 \neq g_2$, we expect (as discussed earlier) the stronger coupling to determine chiral condensates and the weaker to determine string tensions. That the odd-even structure appears also in the $Z_{2q}$ orbifold adds some more plausibility to the NPO conjecture. Again the scales of the condensates and confinement can be estimated and agree with expectations in the parent. Furthermore, there are new classes of predictions. For example, consider baryons. The SYM parent has no baryons, but the $Z_p$ orbifolds all have operators $\det \psi_i$ for each $i$. Baryons appearing in both the $Z_2$ orbifold and in the $Z_{2q}$ orbifold should have related spectra, etc. Of course it would be much harder to test these relations since the $Z_4$ orbifold is inherently chiral.

Even with SYM as the parent, there are other orbifolds than $Z_p$ to consider; the simplest is $SU(2N) \times SU(N)^4$ with six suitably chosen chiral fermions, a $D_4$ orbifold. But we need not restrict ourselves to SYM. Many non-supersymmetric theories can be orbifolded (indeed we just a moment ago treated the $Z_2$ orbifold as a parent to $Z_4$) and there are huge classes of related examples. For example, one might orbifold $SU(N) \times SU(M)$ with a Dirac fermion to obtain a theory with its own new set of dynamics or add additional flavors of fermions carrying a nonabelian flavor symmetry as in QCD. In these cases we know little about either parent or orbifold. However, it may still be very useful to break up the huge space of nonsupersymmetric theories into a smaller set of classes which are related nonperturbatively at large, and perhaps not so large, $N$. This deserves more study. Perhaps problems in 0+1

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As this paper was completed, these and other similar models appeared in a new context, with the $(Z_p)_O$ acting as translation in a latticized fifth dimension.
or 1+1 dimensions would give some hints as to how best to do this.

Equally important would be a direct proof (with a clear set of necessary and sufficient conditions) of the NPO conjecture. Presumably one should first study soluble matrix models in lower dimensions. There are other more specific questions for which rigorous answers may be obtainable: does the vacuum of the \( Z_p \) orbifold theory necessarily preserve the \((Z_p)_{O}\) symmetry? can one ensure the absence of a phase transition between regimes of equal and unequal coupling in the \( Z_2 \) orbifold? are there other methods for demonstrating the bosonic hadron degeneracies of the \( Z_2 \) orbifold?

Most importantly, however, other methods which work in nonsupersymmetric contexts, perhaps also at large \( N \), are needed. Orbifolding, while potentially a powerful technique, is not enough. We will need more guidance before the world of nonsupersymmetric theories becomes transparent.\(^6\)

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\(^6\)As this paper was completed, the work of [23] linked these classes of theories to theories in latticized extra dimensions. In their language, the arguments of this paper imply that at large \( N \) the Green’s functions of fifth-dimension zero-modes are independent of the fifth-dimension lattice spacing.
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