Quantum trajectories under frequent measurements in non-Markovian environment

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The quantum trajectory (QT) theory, which is broadly utilized nowadays in quantum measurement and control studies, essentially corresponds to unraveling of the Lindblad master equation. However, the QT theory of this type is not compatible with quantum Zeno effect. In this work we propose a scheme for the quantum trajectories conditioned on frequent measurements in non-Markovian environment. The non-Markovian environment is characterized by a finite bandwidth (Λ), which we show has a perfect “scaling” property with the measurement frequency (1/τ). As a result, the incompatibility between the QT theory and the Zeno effect can be naturally eliminated. The new QT theory tells us that the scaling variable x = Λτ is an important parameter that should be taken into account. The present study sheds also new light on the confusing concept of continuous null-result informational evolution.

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It has been well recognized that quantum measurement and control will play an important role in quantum information science. In this context, rather than strong projective measurement, more interesting is the type of continuous weak measurement whose experimental realization is currently an extremely attracting subject. Actually this type of monitoring on quantum state is an essential prerequisite for measurement-based feedback control of quantum systems. For continuous or frequent weak measurements, which generate the quantum trajectories of the measured system state, representative theoretical tools include the POVM (positive operator-valued measure) scheme, the quantum trajectory equation (QTE), and the quantum Bayesian approach. Essentially, all these approaches are equivalent to each other. For instance, consider a two-level atom prepared in a quantum superposition of the ground state (|g⟩) and exited state (|e⟩), |Ψ(0)⟩ = α|e⟩ + β|g⟩, and consider further the evolution of this state under continuous (very frequent) measurements in the surrounding environment for the spontaneous emission. Surprisingly, all these theories predict that, conditioned on the continuous null-result measurements (NRM), i.e., no-register of spontaneous emission in the detector, the state would change, following the simple formula:

|Ψ(t)⟩ = (α0e^{-Γt/2}|e⟩ + β0|g⟩) / N, (1)

where Γ is the spontaneous emission rate and N denotes the normalization factor. This confusing result contradicts our intuition, since under the “continuous” detection no-register of spontaneous emission simply means that the state does not change. The surprising result predicted by Eq. (1) has been interpreted as informational evolution. That is, no result is a sort of information, so the state can change according to the Bayesian inference rule, similar as in the classical probability theory. Or, this result has been highlighted as: the amplitude of state (|g⟩) gradually grows without “physical” interaction.

Indeed, the above continuous null-result quantum motion is not compatible with the quantum Zeno effect. The standard analysis for the Zeno effect assumes an initial state of (|e⟩) and arrives to a conclusion that frequent observations on (|e⟩) will prevent it from radiative decay. For such initial state, Eq. (1) can give the result. However, for the quantum superposition state (|Ψ(0)⟩) analyzed above, both approaches will lead to completely different results. We may summarize the result as follows. Starting with (|Ψ(0)⟩), expand the evolution operator up to the second order in τ, U(τ) ≃ 1−iHτ−H^2τ^2/2, where τ is the time interval between the successive NRMs. Each NRM would project the wave function on the atomic subspace. Consider n subsequent NRMs during time t (with n = t/τ). In the limit τ → 0 and t = const, one obtains

|Ψ_n⟩ → α0|e⟩ + β0|g⟩ ≡ |Ψ(0)⟩. (2)

So we find that the frequent null-result monitoring of the environment will prevent the change of the superposition state. This Zeno effect sharply differs from the prediction of Eq. (1). Similar contradiction has been analyzed recently in the context of quantum transfer through a continuum reservoir.

In this work we present a further study on the effect of the “continuous” (frequent) measurements, in connection with the quantum trajectory theory. We will consider frequent measurements in a non-Markovian environment and show that there exists a perfect scaling property between the environment bandwidth Λ and the measurement frequency ν (ν = 1/τ). The important scaling variable x = Λτ allows us to construct a more self-consistent quantum trajectory theory that can accommodate the continuous(frequent)-NRM-interrupted evolution and the Zeno freezing effect in a unified form.

Spontaneous decay. The two-level atom coupled to...
the electromagnetic vacuum (environment) is described by the Hamiltonian

\[ H = \frac{\Delta_{eg}}{2} \sigma_z + \sum_r \left( b_r^\dagger b_r + \frac{1}{2} \right) \omega_r + \sum_r \left[ V_r b_r^\dagger \sigma^- + \text{H.c.} \right]. \]  

(3)

Throughout this work we set \( \hbar = 1 \). Here we introduce: the two-level energy difference \( \Delta_{eg} = E_e - E_g \), the atomic operators \( \sigma_z = |e\rangle\langle e| - |g\rangle\langle g| \), \( \sigma^- = |g\rangle\langle e| \), and \( \sigma^+ = |e\rangle\langle g| \). \( V_r \) is the coupling amplitude of the atom with the environment. Then, consider the evolution of the entire system, starting with an initial state \( |\Psi(0)\rangle = (\alpha_0 |e\rangle + \beta_0 |g\rangle) \otimes \text{vac} \), where \( \text{vac} \) stands for the environmental vacuum with no photon. Under the influence of the coupling, the entire state at time \( t \) can be written as

\[ |\Psi(t)\rangle = a(t) |e\rangle \otimes |\text{vac}\rangle + \sum_r c_r(t) |g\rangle \otimes |1_r; 0; \cdots \rangle \]

(4)

where \( |1_r; 0; \cdots \rangle \) describes the environment with a photon in the state \( 1 \) and no excitations of other states. The coefficients have initial conditions of \( a(0) = \alpha_0 \) and \( c_r(0) = 0 \).

Substituting Eq. (4) into the Schrödinger equation and performing the Laplace transform, one can obtain the solution of \( a(t) \) in frequency domain. Replace \( \sum_r \rightarrow \int D(\omega_r) d\omega_r \), where \( D(\omega_r) \) is the density of states, and consider a finite-band spectrum by taking \( D(\omega_r) \) in the Lorentzian form, \( D(\omega_r) = D_0 \Lambda^2 / [ (\omega_r - \omega_0)^2 + \Lambda^2 ] \), with \( \omega_0 \) the center of the Lorentzian spectrum and \( \Lambda \) the width of it. Then, assuming the coupling amplitudes energy independent \( (V_r = V) \) which allows us to define the usual decay rate \( \Gamma = 2|V|^2 D_0 \), we obtain the time-dependent amplitude \( a(t) \equiv \hat{a}(t) \alpha_0 \) via the inverse Laplace transform as

\[ a(t) = \frac{1}{A_\pm} [ A_\pm e^{-A_\pm t} - A_- e^{-A_+ t} ], \]

(5)

where \( A_\pm = |\Lambda - iE| \pm \sqrt{(\Lambda - iE)^2 - 2\Gamma \Lambda}/2 \). Here we introduced \( E = (E_e - E_g) - \omega_0 = \omega - \omega_0 \), for the energy offset of the atom and the center of the Lorentzian spectrum.

**Frequent null-result measurements.** The null-result measurement (NRM) in the environment, quantum mechanically, collapses the entire wave function onto the atomic subspace. After \( n \) such null-result measurements with subsequent time interval \( \tau = t/n \), the final state of the atom is

\[ |\Psi(t)\rangle = [\hat{a}(t) \alpha_0 |e\rangle + \beta_0 |g\rangle] / \sqrt{\mathcal{N}_n(t)}, \]

(6)

where \( \hat{a}(t) = a^n(\tau) \) and \( \mathcal{N}_n(t) = |[\hat{a}(t) \alpha_0|^2 + |\beta_0|^2|^2 \). Note that, unlike the case of the wide-band-limit Markovian environment, \( |\Psi(t)\rangle \) differs from the single-null-measurement-collapsed state at the final moment from \(|\Psi(t)\rangle \).

**FIG. 1:** (Color online) (a) Spontaneous emission of a two-level atom coupled to non-Markovian environment with finite-bandwidth Lorentzian spectrum. (b) Effective decay factor of the excited state started with a quantum superposition \( \alpha_0 |e\rangle + \beta_0 |g\rangle \), under frequent null-result measurements in the environment. Scaling behavior is demonstrated by the remarkable agreement between Eq. (6) (continuous lines) and \( a^n(\tau) \) (symbols) calculated using Eq. (5) with \( \Lambda = 10\Gamma \) (as an example) and \( E = \omega - \omega_0 = 0 \). Note that \( t = n\tau \) and \( x = \Lambda \tau \).

It can be proved that the normalization factor \( \mathcal{N}_n \) equals also the joint probability of getting null results in all the intermediate measurements, i.e., \( (1 - \sum_r |c_r(\tau)|^2)^n \). Let us denote \( \mathcal{N}_n(t) \equiv p_0^{(n)}(t) \). Accordingly, during time \( (0, t) \), the probability of detecting a spontaneous photon is \( p_1^{(n)}(t) = 1 - p_0^{(n)}(t) \).

Now let us consider the limit of “continuous” measurements, \( n \rightarrow \infty \) by taking the measurement time interval \( \tau \rightarrow 0 \) and keeping \( t = n\tau \) fixed. Supposing to increase the bandwidth \( \Lambda \) so that the variable \( x = \Lambda \tau \) remains constant, we can prove a “scaling” property that the final state becomes a function of \( x \) only. To reveal the full scaling behavior in general case, we also assume the energy offset \( E = c\Lambda \) (in usual treatment \( c = 0 \)). One finds from Eq. (6) that \( A_+ = \kappa\Lambda - \Gamma/(2\kappa) \) and \( A_- = \Gamma/(2\kappa) \) (up to the order of \( \Gamma^2/\Lambda^2 \)), where \( \kappa = 1 - ic \). Using \( (1 - z)^n = e^{-z(1 + i\kappa + \cdots)} \) and neglecting small terms \( \sim \Gamma/\Lambda \) in exponent, we arrive to

\[ \hat{a}(t) = a^n(\tau) = \exp \left\{ - \left[ \frac{1}{\kappa} - (1 - e^{-\kappa \tau}) \right] \frac{\Gamma t}{2} \right\}. \]

(7)

Remarkably, this result reveals an explicit scaling property in the \( x = \Lambda \tau \)-variable. In Fig. 1, by relaxing the conditions \( n \rightarrow \infty \) and \( \tau \rightarrow 0 \) for this analytic formula, we illustrate numerical results for the scaling behavior.
From Eq. (3), in the wide-band limit, $x \to \infty$ and $\kappa \to 1$, one recovers the standard result $\bar{a}(t) \to e^{-\Gamma t/2}$ in Eq. (1). Note that the same result can be obtained without intermediate measurements. This reveals no-effect of the continuous NRM interruptions. On the other hand, in the limit of $x \to 0$, one finds from Eq. (3) that $\bar{a}(t) = 1$, so that the atom is frozen in its initial state, showing the Zeno effect.

Despite the intrinsic non-Markovian nature of Eq. (3), the continuous-NRM-interrupted evolution, Eq. (4), reveals an exponential decay behavior (rate process). From it, one can define an effective decay rate,

$$\gamma_{\text{eff}} = \text{Re} \left\{ \left[ 1 - (\kappa x)^{-1} \right] (1 - e^{-\kappa x}) \right\} \Gamma. \quad (8)$$

Note that for the wide-band-limit Markovian environment the exponential decay process implies no-effect of intermediate interruptions. Eq. (8), however, shows that the state evolution is influenced by the frequent null-result measurements.

**Quantum trajectories.** Corresponding to direct photon detection, let us firstly construct the Monte-Carlo wave function (MCWF) approach. Consider the state evolution under frequent null-result measurements between $t$ and $t + \Delta t$, with thus $\Delta t = n\tau$. Following Ref. [14], the probability with photon register in the detector during $\Delta t$, is $p_{\text{eff}}(\Delta t) = |\tilde{a}(t)^2| \gamma_{\text{eff}} \Delta t$. Under the “scaling” consideration, the effective decay rate $\gamma_{\text{eff}}$ is simply given by Eq. (8), or, alternatively by

$$\gamma_{\text{eff}} = [1 - |\bar{a}(\Delta t)|^2]/\Delta t \quad \text{or} \quad \gamma_{\text{eff}} = -\ln[|\bar{a}(\Delta t)|^2]/\Delta t.$$  

For small $\Delta t$, which implies $|\bar{a}(\Delta t)|^2 \approx 1$, both definitions are equivalent and coincide with Eq. (3).

In practice of simulations, generate a random number $\epsilon$ between 0 and 1. If $\epsilon < p_{\text{eff}}(\Delta t)$, which corresponds to the probability of having a photon register in the detector ($\Delta N_e = 1$), we update the state by a “jump” action

$$|\tilde{\Psi}(t + \Delta t)\rangle = \sigma^{-} |\tilde{\Psi}(t)/\| \cdot \| \quad (9)$$

where $\| \cdot \|$ denotes the normalization factor. On the other hand, if $\epsilon > p_{\text{eff}}(\Delta t)$, which corresponds to the NRM with $\Delta N_e = 0$, we update the state via the effective smooth evolution

$$|\tilde{\Psi}(t + \Delta t)\rangle = \mathcal{U}(\Delta t)|\tilde{\Psi}(t)/\| \cdot \| \quad (10)$$

In terms of a matrix form defined by $\{ \alpha(t + \Delta t), \beta(t + \Delta t) \}^T = \mathcal{U}(\Delta t) \{ \alpha(t), \beta(t) \}^T$, the effective non-unitary evolution operator reads

$$\mathcal{U}(\Delta t) = \begin{pmatrix} \bar{a}(\Delta t) & 0 \\ 0 & 1 \end{pmatrix}. \quad (11)$$

Noting that $\Delta t = n\tau$, as above, here we mention again that $\bar{a}(\Delta t) = |a(\tau)|^n$ which can be Eq. (3) in the limit $\tau \to 0$ and $n \to \infty$, or more generally determined using Eq. (3) for $a(\tau)$.

Based on the MCWF approach proposed above, one can simulate the (stochastic) quantum trajectories under frequent photon detections in the environment. Ensemble average over these trajectories of quantum (pure) state corresponds to result given by the following master equation:

$$\dot{\rho} = -i[H_S, \rho] + \gamma_{\text{eff}} D[\sigma^-] \rho, \quad (12)$$

where $D[A] \rho \equiv A \rho A^\dagger - \frac{1}{2}(A^\dagger A, \rho)$. Note that this result is different from the usual master equation for the reduced density matrix $\rho(t)$ of the system state, after tracing the environment degrees of freedom from the entire wavefunction at the time $t$. In this case, each $\rho(t)$, at different times, is obtained from the same initial state; and the projective measurement and average are performed only at the last moment $t$. In contrast to $\rho(t)$, Eq. (12) is for the averaged (reduced) state of the system under successive-measurement-interrupted evolution along time, while each evolution over $dt$ starts with the previous state at $t$. Therefore, in the ensemble-averaged master equation (12), rather than certain “natural” decay rate, but an effective $\gamma_{\text{eff}}$ appears.

Moreover, following Refs. [10, 17], we can further include external driving into Eq. (12), via $H_S = \Delta \sigma_x + \Omega q_x$. As a result, there are two contributions to the change of state: one is informational owing to the continuous measurements (over $dt$), and the other is physical which is caused by the external driving. Note that in general the dissipative two-level atom under driving is not exactly driven. The underlying complexity can be imagined as follows: there are more and more photons emitted into the reservoir; and the emitted photon can re-excite the atom. However, in the presence of frequent measurements, the emitted photon will be destroyed by detectors. During the successive measurement interval, we assume that there is at most one photon in the reservoir and the effect of re-excitation of atom is negligible, despite that it has been included in the solution Eq. (8). Therefore, even in the presence of external driving, Eq. (12) is valid under the above considerations.

Instead of the direct detection of the spontaneous emission considered above, one can also adopt the so-called homodyne detection scheme by mixing the emitting photons with a classical field with modulating phase $\varphi$. The measurement result (optical current) of this type can be expressed as $I_{\varphi}(t) = \sqrt{\gamma_{\text{eff}}} (\sigma^- e^{-i\varphi} + \sigma^+ e^{i\varphi})/2 + \xi(t)$, where $\langle \cdots \rangle = \text{Tr}[\cdots \rho(t)]$ and $\xi(t)$ is the Gaussian white noise associated with quantum jumps. Conditioned on $I_{\varphi}(t)$, the state evolution is given by the diffusive quantum trajectory equation (QTE):

$$\dot{\rho} = -i[H_S, \rho] + \gamma_{\text{eff}} D[\sigma^-] \rho + \sqrt{\gamma_{\text{eff}}} \mathcal{H}[e^{-i\varphi} \sigma^-] \rho \xi(t), \quad (13)$$

where $\mathcal{H}[A] \rho \equiv A \rho A^\dagger - (A^\dagger A, \rho)$. Essentially, Eq. (13) generalizes the existing QTE by accounting for the measurement frequency $\langle \nu \rangle = 1/\tau$ in the effective rate $\gamma_{\text{eff}}$. 


FIG. 2: (Color online) (a) Two quantum trajectories from the MCWF (black) and QTE (red) simulations. The blue arrows indicate quantum “jumps” owing to “direct” detection of the spontaneous emission of the atom. (b) Ensemble average of 2000 MCWF and QTE trajectories and the result (green curve) from the master equation \( \dot{\rho} = -i[H, \rho] + \gamma \text{d}D[\sigma^-] \rho \). Parameters used in the simulation: \( \Omega = 0.1, \Gamma = 1.0, x = 0.2 \) and \( E = 0 \).

In Fig. 2(a) we display two representative quantum trajectories from the MCWF and the diffusive QTE, respectively. We see that the former type of quantum trajectory reveals drastic “quantum jump” owing to the direct detection for the spontaneous emission, while the latter type has no such “jump” onto the ground state \( |g\rangle \). However, as expected, ensemble average of each type of quantum trajectories gives the same result as demonstrated in Fig. 2(b).

Discussion and summary. The celebrated result of Eq. (1) was predicted originally in quantum optics (but never realized). However, this result has been connected with the null-result-conditioned partial collapse experiment in the solid-state superconducting qubit. Actually, the changed state reported in Ref. 21 might be understood as conditioned only on a projective null-result at the final time \( t \), but not conditioned on null-results of “all” the intermediate times (in a sense of “continuous” or “frequent” measurements). Or, the decay rate extracted from the experimental data might be the one we discussed in present work, i.e., \( \gamma \text{eff} \), which has been affected by the measurement frequency (more precisely, via the scaling variable \( x = \Delta \tau \)). Further demonstration of this subtle issue is of great interest and can be guided by Eq. (2) or (3).

To summarize, we have presented a scheme for the quantum trajectories conditioned on frequent measurements in non-Markovian environment. We have revealed an important scaling property between the spectral bandwidth of the environment and the measurement frequency, which (i) naturally eliminates the incompatibility between the quantum trajectory theory and the Zeno effect; (ii) sheds new light on the confusing concept of continuous null-result informational evolution; and (iii) allows to construct more correct MCWF or QTE scheme for continuous quantum measurements.

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Appendix A: Zeno Effect for Superposition State

Consider a superposition state for the atom, \( \alpha_0 |e\rangle + \beta_0 |g\rangle \). After small time \( \tau \) the entire wave-function becomes

\[
|\Psi(\tau)\rangle = [\alpha_0 (1 - iH\tau - H^2\tau^2/2 + \cdots)|e\rangle + \beta_0 |g\rangle] \otimes |\text{vac}\rangle.
\]  

(A1)

Here the Hamiltonian is the coupling part of Eq. (2) in the interaction picture (with respect to the first two). The “no-register” of measurement in the environment implies that the wave function is projected as \( |\Psi(\tau)\rangle \rightarrow \langle \text{vac}|\Psi(\tau)\rangle/\mathcal{N} \), where \( \mathcal{N} \) is a normalization factor. Therefore

\[
|\Psi_1\rangle = \hat{Q}|\Psi(\tau)\rangle = \left[\alpha_0 (1 - K\tau^2)|e\rangle + \beta_0 |g\rangle\right]/\mathcal{N}_1,
\]  

(A2)

where \( K = \sum_r |V_r|^2 \) and \( \mathcal{N}_1^2 = 1 - 2\alpha_0^2 K\tau^2 \), with \( V_r \) the atom-environment (the r-th mode) coupling amplitude. After \( n \) subsequent null-result measurements during time \( t \), with \( n = t/\tau \), we find

\[
|\Psi_n\rangle = \left[\alpha_0 (1 - K\tau^2)^n|e\rangle + \beta_0 |g\rangle\right]/\mathcal{N}_n,
\]  

(A3)

where \( \mathcal{N}_n = \sqrt{1 - 2n \alpha_0^2 K\tau^2} \). Thus in the limit \( \tau \rightarrow 0 \) and \( t=\text{const} \), one obtains Eqs. (2), \( |\Psi_n\rangle \rightarrow |\Psi(0)\rangle \).

Appendix B: Solution for Spontaneous Emission

Substituting Eq. (4) into the Schrödinger equation, \( i\partial_t |\Psi(t)\rangle = H|\Psi(t)\rangle \) and performing the Laplace transform, \( \hat{f}(s) = \int_0^{\infty} f(t) \exp(ist) dt \), we obtain the following system of algebraic equations:

\[
(s - E_e)\hat{\alpha}(s) - \sum_r V_r \hat{c}_r(s) = i\alpha_0,
\]  

(B1a)

\[
[s - (E_g + \omega_r)]\hat{c}_r(s) - V_r^* \hat{\alpha}(s) = 0.
\]  

(B1b)

The r.h.s. of these equations reflects the initial conditions, corresponding to the electron localized in the dots.
Substituting $\tilde{c}_r(s)$ from Eq. (B1b) into Eq. (B1a) and replacing $\sum_r \to \int D(\omega_r) d\omega_r$, where $D(\omega_r)$ is the density of states, we obtain

$$(s - E_e)\tilde{\alpha}(s) - F(s)\tilde{\alpha}(s) = i\alpha_0,$$  \hspace{1cm} (B2)$$

where $$F(s) = \int \frac{|V_r|^2}{s - (E_g + \omega_r)} D(\omega_r) d\omega_r.$$  \hspace{1cm} (B3)$$

Rather than the wide-band limit for the “Markovian” reservoir, in this work we consider a finite-band spectrum by taking $D(\omega_r)$ in the Lorentzian form,

$$D(\omega_r) = D_0 \Lambda^2 / [(\omega_r - \omega_0)^2 + \Lambda^2],$$  \hspace{1cm} (B4)$$

with $\omega_0$ the center of the Lorentzian spectrum and $\Lambda$ the width of it. Assuming the coupling amplitudes energy independent, $V_r = V$, we then obtain

$$F(s) = \frac{\Lambda \Gamma / 2}{(s - \omega_0 - E_g) + i\Lambda},$$  \hspace{1cm} (B5)$$

Substituting this result into Eqs. (B2), we find the amplitude $\tilde{\alpha}(s)$. The time-dependent amplitude is obtained via the inverse Laplace transform, $\alpha(t) = \int_{-\infty}^{\infty} \tilde{\alpha}(s) e^{-ist} ds / (2\pi)$. Then, we obtain $\alpha(t) = a(t)\alpha_0$, with an explicit expression of $a(t)$ given by Eq. (5).

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