The possibility of the appearance of $\Delta$ isobars in neutron star matter and the so called $\Delta$ puzzle is studied in a modified quark meson coupling model where the confining interaction for quarks inside a baryon is represented by a phenomenological average potential in an equally mixed scalar-vector harmonic form. The hadron-hadron interaction in nuclear matter is then realized by introducing additional quark couplings to $\sigma$, $\omega$, and $\rho$ mesons through mean-field approximations. The couplings of the $\Delta$ to the meson fields are fixed from available constraints while the hyperon couplings are fixed from the optical potential values. It is observed that within the constraints of the mass of the precisely measured massive pulsars, PSR J0348+0432 and PSR J1614-2230, neutron stars with a composition of both $\Delta$ isobars and hyperons is possible. It is also observed that with an increase in the vector coupling strength of the $\Delta$ isobars there is a decrease in the radius of the neutron stars.

I. INTRODUCTION

The investigations pertaining to the formation of baryons heavier than the nucleon at the core of neutron stars and the effects of such formation on the mass and radius of neutron stars is a subject of active research in nuclear astrophysics. It is expected that high density nuclear matter may consist not only of nucleons and leptons but also several exotic components such as hyperons, mesons as well as quark matter in different forms and phases. While many studies have been conducted to address the appearance of hyperons and on the so called hyperon puzzle \cite{1}, little work has been done to study the appearance of $\Delta$ (1232) isobars in neutron stars. An earlier work \cite{2} indicated the appearance of $\Delta$ at much higher densities than the typical densities of the core of neutron stars and hence was considered of little significance to astrophysical studies. However, recent studies \cite{3, 4} suggest the possibility of an early appearance of $\Delta$ isobars with consequent softening of the equation of state (EOS) of dense matter. This leads to a reduction in the maximum mass of neutron stars below the current observational limit of $2.01 \pm 0.04$ M$_{\odot}$ \cite{5}.

In the present work, we would like to address the $\Delta$-puzzle in a modified quark-meson coupling model (MQMC) \cite{6, 7} which has already been adopted successfully studying various bulk properties of symmetric and asymmetric nuclear matter. The MQMC model is based on a confining relativistic independent quark potential model rather than a bag to describe the baryon structure in vacuum. The baryon-baryon interactions are realized by making additional quark couplings to $\sigma$, $\omega$, and $\rho$ mesons through mean-field approximations. More recently \cite{8} it was extended to study the EOS of nuclear matter with the inclusion of hyperons as new degrees of freedom and the effect of a non-linear $\omega-\rho$ term keeping in view the constraints of the mass of the precisely measured massive pulsars, PSR J0348+0432 and PSR J1614-2230.

Here, we include the delta isobars ($\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$) together with hyperons as new degrees of freedom in dense hadronic matter relevant for neutron stars. The interactions between nucleons, $\Delta$’s and hyperons in dense matter is studied and the possibility of the existence of the $\Delta$ baryon at densities relevant to neutron star core as well as its effects on the mass of the neutron star is analysed. The nucleon-nucleon interaction is well known from nuclear properties. But the extrapolation of such interactions to densities beyond nuclear saturation density is quite challenging. Most of the hyperon-nucleon interaction are known experimentally, which we use to set the hyperon-nucleon interaction potential at saturation density. For the $\Lambda$, $\Sigma$ and $\Xi$ hyperons the respective potentials are $U_\Lambda = -28$ MeV, $U_\Sigma = 30$ MeV and $U_\Xi = -18$ MeV respectively. However, the coupling of the $\Delta$ isobars with the mesons are poorly constrained. The $\Delta$ isobars are commonly treated with the same coupling strengths as the nucleons. Studies \cite{9, 10} based on the quark counting argument suggest universal couplings between nucleons, $\Delta$ isobars and mesons, giving the value of $x_{\omega\Delta} = g_{\omega\Delta}/g_{\omega N} = 1$. Theoretical studies of Gamow-Teller transitions and M1 giant resonance in nuclei by Bohr and Mottelson \cite{11} observed a 25 – 40% reduction in transition strength due to the couplings to $\Delta$ isobars, indicating weaker coupling of the isoscalar mesons to the $\Delta$ isobars. Further, the difference between $x_\sigma$ and $x_\omega$ was found to be $x_\sigma - x_\omega = 0.2$ in Hartree approximation \cite{12}. In view of this we scale the $\Delta$-coupling with the value of $x_{\omega\Delta} = 0.8$. We also study the effect of moderate variations in the value of $x_{\omega\Delta}$ on the radius of neutron stars.

The paper is organized as follows: In Sec. 2, a brief outline of the model describing the baryon structure in vacuum is discussed. The baryon mass is then realized by appropriately taking into account the center-of-mass correction, pionic correction, and gluonic correction. The EOS with the inclusion of the $\Delta$ isobars and the hyperons is then developed in Sec. 3. The results and discussions are made in Sec. 4. We summarize our findings in Sec. 5.
II. MODIFIED QUARK MESON COUPLING MODEL

The modified quark-meson coupling model has been successful in obtaining various bulk properties of symmetric and asymmetric nuclear matter as well as hyperonic matter within the accepted constraints [6–8]. We now extend this model to include the ∆ isobars (Δ−, Δ0, Δ+, Δ++1) along with nucleons and hyperons in neutron star matter under conditions of beta equilibrium and charge neutrality. We begin by considering baryons as composed of three constituent quarks confined inside the hadron core by a phenomenological flavor-independent potential, U(r). Such a potential may be expressed as an admixture of equal scalar and vector parts in harmonic form [6],

\[ U(r) = \frac{1}{2}(1 + \gamma^0)V(r), \]

with

\[ V(r) = (ar^2 + V_0), \quad a > 0. \]

Here (a, V0) are the potential parameters. The confining interaction provides the zeroth-order quark dynamics of the hadron. In the medium, the quark field ψ_q(r) satisfies the Dirac equation

\[ [\gamma^0 (\epsilon_q - V_\omega - \frac{1}{2}\tau_{3q}V_\rho) - \gamma^\mu p^\mu - (m_q - V_\sigma) - U(r)]\psi_q(r) = 0 \]

(2)

where \( V_\sigma = g_\sigma^0 \sigma_0 \), \( V_\omega = g_\omega^0 \omega_0 \) and \( V_\rho = g_\rho^0 b_0 \). Here \( \sigma_0, \omega_0, \) and \( b_0 \) are the classical meson fields, and \( g_\sigma^0, g_\omega^0, g_\rho^0 \) are the quark couplings to the \( \sigma, \omega, \) and \( \rho \) mesons, respectively. \( m_q \) is the quark mass and \( \tau_{3q} \) is the third component of the Pauli matrices. We can now define

\[ \epsilon_q^* = (\epsilon_q^* - V_\omega - \frac{1}{2}\tau_{3q}V_\rho) \]

and \( m_q^* = (m_q^* + V_\sigma) \),

(3)

where the effective quark energy, \( \epsilon_q^* = \epsilon_q - V_\omega - \frac{1}{2}\tau_{3q}V_\rho \) and effective quark mass, \( m_q^* = m_q - V_\sigma \). We now introduce \( \lambda_q \) and \( r_{0q} \) as

\[ (\epsilon_q^* + m_q^*) = \lambda_q \quad \text{and} \quad r_{0q} = (a \lambda_q)^{-\frac{1}{2}}. \]

(4)

The ground-state quark energy can be obtained from the eigenvalue condition

\[ (\epsilon_q^* - m_q^*)\sqrt{\frac{\lambda_q}{a}} = 3. \]

(5)

The solution of equation (4) for the quark energy \( \epsilon_q^* \) immediately leads to the mass of baryon in the medium in zeroth order as

\[ E_B^{0} = \sum_q \epsilon_q^*. \]

(6)

We next consider the spurious center-of-mass correction \( e_{c.m.} \), the pionic correction \( \delta M_B^{\pi} \) for restoration of chiral symmetry, and the short-distance one-gluon exchange contribution \( (\delta E_B)_{n} \) to the zeroth-order baryon mass in the medium.

We have used a fixed center potential to calculate the wavefunctions of a quark in a baryon. To study the properties of the baryon constructed from these quarks, we must extract the contribution of the center-of-mass motion in order to obtain physically relevant results. Here, we extract the center of mass energy to first order in the difference between the fixed center and relative quark coordinate, using the method described by Guichon et al. [13, 14]. The centre of mass correction is given by:

\[ e_{c.m.} = e_{c.m.}^{(1)} + e_{c.m.}^{(2)} \]

where,

\[ e_{c.m.}^{(1)} = \frac{3}{2} \left[ \frac{m_q}{\sum_{k=1}^{3} m_{kq}} \left( \frac{r_{0q}^2}{3e_q^* + m_q^*} \right) \right] \]

(7)

\[ e_{c.m.}^{(2)} = \frac{a}{2} \left[ \frac{2}{\sum_{k} m_{kq}} \sum_{i} m_{i} \langle r_{1i}^{2} \rangle + \frac{2}{\sum_{k} m_{kq}} \sum_{i} m_{i} \langle \gamma_{0} (i) r_{1i}^{2} \rangle - \frac{3}{(\sum_{k} m_{kq})^2} \sum_{i} \langle \gamma_{0} (i) m_{2i} r_{1i}^{2} \rangle \right] \]

(8)

In the above, we have used for \( i = (u, d, s) \) and \( k = (u, d, s) \) and the various quantities are defined as

\[ \langle r_{1i}^{2} \rangle = \frac{(11e_{q}^* + m_{q}^*)r_{2q_{i}}^{2}}{2(3e_{q}^* + m_{q}^*)} \]

(10)

\[ \langle \gamma_{0} (i) r_{1i}^{2} \rangle = \frac{(\epsilon_{q_{i}}^* + 11m_{q}^*)r_{2q_{i}}^{2}}{2(3e_{q_i}^* + m_{q_i}^*)} \]

(11)

\[ \langle \gamma_{0} (i) m_{2j} r_{1i}^{2} \rangle_{i \neq j} = \frac{(\epsilon_{q_{i}}^* + 3m_{q}^*)r_{2q_{j}}^{2}}{3e_{q_{i}}^* + m_{q_{i}}}, \]

(12)

The pionic corrections in the model for the nucleons become

\[ \delta M_N^{\pi} = -\frac{171}{25} I_{\pi} f_{NN\pi}^{3}, \]

(13)

where, \( f_{NN\pi} \) is the pseudo-vector nucleon-pion coupling constant. Taking \( \omega_k = (k^2 + m_{q}^*)^{1/2} \), \( I_{\pi} \) becomes

\[ I_{\pi} = \frac{1}{\pi m_{\pi}^2} \int_{0}^{\infty} dk \frac{k^4 u^2(k)}{w_k}, \]

(14)

with the axial vector nucleon form factor given as

\[ u(k) = \left[ 1 - \frac{3}{2} \frac{k^2}{\lambda_q(5\epsilon_q^* + 7m_q^*)} \right] e^{-k^2 r_{0q}^2 / 4}. \]

(15)
TABLE I. The coefficients $a_{ij}$ and $b_{ij}$ used in the calculation of the color-electric and and color-magnetic energy contributions due to one-gluon exchange.

| Baryon | $a_{uu}$ | $a_{us}$ | $a_{ss}$ | $b_{uu}$ | $b_{us}$ | $b_{ss}$ |
|--------|---------|---------|---------|---------|---------|---------|
| $N$    | -3      | 0       | 0       | 0       | 0       | 0       |
| $\Delta$ | 3      | 0       | 0       | 0       | 0       | 0       |
| $\Lambda$ | -3     | 0       | 0       | 1       | -2      | 1       |
| $\Sigma$ | 1      | -4      | 0       | 1       | -2      | 1       |
| $\Xi$  | 0      | -4      | 1       | 1       | -2      | 1       |

The pionic correction for $\Sigma^0$ and $\Lambda^0$ become

$$\delta M_{\Sigma^0} = -\frac{12}{5} \frac{f_{\pi N \Sigma}^2}{f_{N N \pi}} I_\pi, \quad (16)$$

$$\delta M_{\Lambda^0} = -\frac{108}{25} \frac{f_{\pi N \Lambda}^2}{f_{N N \pi}} I_\pi. \quad (17)$$

Similarly the pionic correction for $\Sigma^-$ and $\Sigma^+$ is

$$\delta M_{\Sigma^-, \Sigma^+} = -\frac{12}{5} \frac{f_{\pi N \Sigma}^2}{f_{N N \pi}} I_\pi. \quad (18)$$

The pionic correction for $\Xi^0$ and $\Xi^-$ is

$$\delta M_{\Xi^0, \Xi^-} = -\frac{27}{25} \frac{f_{\pi N \Xi}^2}{f_{N N \pi}} I_\pi. \quad (19)$$

For $\Delta$ baryon, the pionic correction is given by

$$\delta M_{\Delta} = -\frac{99}{25} \frac{f_{\pi N \Delta}^2}{f_{N N \pi}} I_\pi. \quad (20)$$

The one-gluon exchange interaction is provided by the interaction Lagrangian density

$$L_I^g = \sum_i J_{i}^{\mu}(x) A_\mu^I(x), \quad (21)$$

where $A_\mu^I(x)$ are the octet gluon vector-fields and $J_{i}^{\mu}(x)$ is the $i$-th quark color current. The gluonic correction can be separated in two pieces, namely, one from the color electric field ($E_\mu^I$) and another from the magnetic field ($B_\mu^I$) generated by the $i$-th quark color current density

$$J_{i}^{\mu}(x) = g_\epsilon \bar{\psi}_i(x) \gamma^\mu \lambda_i^a \psi(x), \quad (22)$$

with $\lambda_i^a$ being the usual Gell-Mann SU(3) matrices and $\alpha_c = g_\epsilon^2/4\pi$. The contribution to the mass can be written as a sum of color electric and color magnetic part as

$$(\Delta E_B)^M_g = (\Delta E_B)^E_g + (\Delta E_B)^M_g. \quad (23)$$

Finally, taking into account the specific quark flavor and spin configurations in the ground state baryons and using the relations $\langle \sum_i (\lambda_i^a)^2 \rangle = 16/3$ and $\langle \sum_i (\lambda_i^a)^2 \rangle_{i \neq j} = -8/3$ for baryons, one can write the energy correction due to color electric contribution as given in [8]

$$(\Delta E_B)^E_g = \alpha_c (b_{uu} I_{uu}^E + b_{us} I_{us}^E + b_{ss} I_{ss}^E), \quad (24)$$

and due to color magnetic contributions, as

$$(\Delta E_B)^M_g = \alpha_c (a_{uu} I_{uu}^M + a_{us} I_{us}^M + a_{ss} I_{ss}^M), \quad (25)$$

where $a_{ij}$ and $b_{ij}$ are the numerical coefficients depending on each baryon and are given in Table I. In the above, we have

$$I_{ij}^E = \frac{16}{3\sqrt{2}} \frac{1}{R_{ij}^3} \left[ 1 - \frac{\alpha_i + \alpha_j}{R_{ij}^2} + \frac{3\alpha_i \alpha_j}{R_{ij}^4} \right], \quad (26)$$

$$I_{ij}^M = \frac{256}{9\sqrt{2}} \frac{1}{R_{ij}^3} \left( 3\epsilon_i + m_i \right) \left( 3\epsilon_j + m_j \right), \quad (27)$$

where

$$R_{ij}^2 = 3 \left[ \frac{1}{(\epsilon_i^2 - m_i^2)} + \frac{1}{(\epsilon_j^2 - m_j^2)} \right].$$

The color electric contributions to the bare mass for nucleon and the $\Delta$ baryon are $(\Delta E_N)^E = 0$ and $(\Delta E_{\Delta})^E = 0$. Therefore the one-gluon contribution for $\Delta$ becomes

$$(\Delta E_{\Delta})^M_g = \frac{256\alpha_c}{3\sqrt{2}} \left[ \frac{1}{(3\epsilon_i + m_i)^2 R_{uu}^3} \right]. \quad (28)$$

The details of the gluonic correction for the nucleons and hyperons is given in [8].

Treating all energy corrections independently, the mass of the baryon in the medium becomes

$$M_B^* = E_B^0 - \epsilon_{c.m.} + (\Delta E_B)^E + (\Delta E_B)^M_g. \quad (29)$$

### III. THE EQUATION OF STATE

The total energy density and pressure at a particular baryon density, encompassing all the members of the baryon octet, for the nuclear matter in $\beta$-equilibrium can be found as

$$E = \frac{1}{2} m_0^2 \sigma_0^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_0^2 + \gamma \frac{\gamma}{2\pi^2} \sum_B \int_{k^B}^{k_{f,B}} [k^2 + M_B^2]^{1/2} \frac{k^2 dk}{k^2 + M_B^2}^{1/2}$$

$$+ \sum_l \frac{1}{\pi^2} \int_0^{k_l} [k^2 + m_l^2]^{1/2} k^2 dk,$n

$$P = -\frac{1}{2} m_0^2 \sigma_0^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_0^2 + \gamma \frac{\gamma}{6\pi^2} \sum_B \int_{k^B}^{k_{f,B}} k^4 \frac{dk}{k^2 + M_B^2}^{1/2}$$

$$+ \frac{1}{3} \int_0^{k_l} \frac{k^4}{k^2 + m_l^2}^{1/2} dk,$n

where $\gamma$ is the spin degeneracy factor for nuclear matter. For the nucleons and hyperons $\gamma = 2$ and for the $\Delta$ baryons $\gamma = 4$. Here $B = N$, $\Delta$, $\Lambda$, $\Sigma^\pm$, $\Sigma^0$, $\Xi^-$, $\Xi^0$ and $l = e, \mu$.

The chemical potentials, necessary to define the $\beta$–equilibrium conditions, are given by

$$\mu_B = \sqrt{k_{f,B}^2 + M_B^2} + g_\omega \omega_0 + g_\rho \tau_3 B \rho_0 \quad (31)$$
where \( \tau_B \) is the isospin projection of the baryon B.

The lepton Fermi momenta are the positive real solutions of \( (k_x^2 + m_e^2)^{1/2} = \mu_e \) and \( (k_y^2 + m_e^2)^{1/2} = \mu_e \). The equilibrium composition of the star is obtained by solving the equations of motion of meson fields in conjunction with the charge neutrality condition, given in Eq. (32), at a given total baryonic density \( \rho = \sum_B \gamma k_B^3/(6\pi^2) \). The effective masses of the baryons are obtained self-consistently in this model.

Since the neutron star time scale is quite long we need to consider the occurrence of weak processes in its matter. Moreover, for stars in which the strongly interacting particles are baryons, the composition is determined by the requirements of charge neutrality and \( \beta \)-equilibrium conditions under the weak processes \( B_1 \to B_2 + l + \nu_l \) and \( B_2 + l \to B_1 + \nu_l \). After de-leptonization, the charge neutrality condition yields

\[
q_{\text{int}} = \sum_B q_B \frac{\gamma k_B^3}{6\pi^2} + \sum_{l=e,\mu} q_l \frac{\gamma k_l^3}{3\pi^2} = 0 , \tag{32}
\]

where \( q_B \) corresponds to the electric charge of baryon species \( B \) and \( q_l \) corresponds to the electric charge of lepton species \( l \). Since the time scale of a star is effectively infinite compared to the weak interaction time scale, weak interaction violates strangeness conservation. The strangeness quantum number is therefore not conserved in a star and the net strangeness is determined by the equations of motion of meson fields in conjunction with the charge neutrality condition, given in Eq. (32), at a given total baryonic density \( \rho = \sum_B \gamma k_B^3/(6\pi^2) \).

The relation between the mass and radius of a star with its central energy density can be obtained by integrating the Tolman-Oppenheimer-Volkoff (TOV) equations [10] given by

\[
\frac{dP}{dr} = -\frac{G}{r} \left[ \frac{\mathcal{E} + P}{r-2GM} \right], \tag{35}
\]

\[
\frac{dM}{dr} = 4\pi r^2 \mathcal{E}, \tag{36}
\]

with \( G \) as the gravitational constant and \( M(r) \) as the enclosed gravitational mass. We have used \( c = 1 \). Given an EOS, these equations can be integrated from the origin as an initial value problem for a given choice of the central energy density, \( \mathcal{E}_0 \). Of particular importance is the maximum mass obtained from and the solution of the TOV equations. The value of \( r (= R) \), where the pressure vanishes defines the surface of the star. The surface gravitational redshift \( Z_s \) is defined as,

\[
Z_s = \left( 1 - \frac{2GM}{R} \right)^{-1/2} - 1. \tag{37}
\]

\[\text{IV. RESULTS AND DISCUSSION}\]

The MQMC model has two potential parameters, \( a \) and \( V_0 \) which are obtained by fitting the nucleon mass \( M_N = 939 \text{ MeV} \) and charge radius of the proton \( \langle r_N \rangle = 0.87 \text{ fm} \) in free space. Keeping the value of the potential parameter \( a \) same as that for nucleons, we obtain \( V_0 \) for the \( \Lambda, \Delta, \Sigma \) and \( \Xi \) baryons by fitting their respective masses to \( M_\Lambda = 1115.6 \text{ MeV} \), \( M_\Delta = 1232 \text{ MeV} \), \( M_\Sigma = 1193.1 \text{ MeV} \) and \( M_\Xi = 1321.3 \text{ MeV} \). The set of potential parameters for the baryons along with their respective energy corrections at zero density are given in Table II.
TABLE II. The potential parameter $V_0$ obtained for the quark mass $m_u = m_d = 150$ MeV, $m_s = 300$ MeV with $a = 0.69655 \text{ fm}^{-3}$.

| Baryon | $M_B$(MeV) | $V_0$(MeV) |
|--------|-------------|------------|
| N      | 939         | 44.05      |
| Δ      | 1232        | 102.40     |
| Λ      | 1115.6      | 50.06      |
| Σ      | 1193.1      | 66.44      |
| Ξ      | 1321.3      | 66.82      |

FIG. 1. Effective baryon mass as a function of baryon density.

FIG. 2. Total pressure as a function of the energy density for various composition of the stellar matter at quark mass $m_q = 150$ MeV. The shaded region shows the empirical EOS obtained by Steiner et al from a heterogeneous set of seven neutron stars.

FIG. 3. Particle fraction as a function of the baryon density indicating the onset of the Δ isobars at quark mass $m_q = 150$ MeV and $x_{ωΔ} = 0.8$.

FIG. 4. Gravitational mass as a function of radius for various couplings of the Δ isobars at quark mass $m_q = 150$ MeV.

The couplings of the hyperons to the $σ$-meson need not be fixed since we determine the effective masses of the hyperons self-consistently. The hyperon couplings to the $ω$-meson are fixed by determining $x_{ωB}$. The value of $x_{ωB}$ is obtained from the hyperon potentials in nuclear matter, $U_B = -(M_{B^*} - M_B) + x_{ωB}g_{ωω0}$ for $B = Λ, Σ$ and $Ξ$ as $-28$ MeV, $30$ MeV and $-18$ MeV respectively. For the quark masses 150 MeV the corresponding values for $x_{ωB}$ are given in Table IV. The $Δ$-coupling to the $ω$-meson is fixed at $x_{ωΔ} = 0.8$. The value of $x_{ρB} = 1$ is fixed for all baryons.

The $Λ$ hyperon potential has been chosen from the measured single particle levels of $Λ$ hypernuclei from mass numbers $A = 3$ to 209 [18, 19] of the binding of $Λ$ to symmetric nuclear matter. Studies of $Σ$ nuclear interaction [20, 21] from the analysis of $Σ^-$ atomic data indicate a repulsive isoscalar potential in the interior of nuclei. Measurements of the final state interaction of $Ξ$ hyperons produced in $(K^-, K^+)$ reaction on $^{12}C$ in E224 experiment at KEK [22] and E885 experiment at AGS [23] indicate a shallow attractive potential $U_Ξ \sim -16$ MeV and $U_Ξ \sim -14$ or less respectively. In view of this we consider the $Ξ$ hyperon potential at $U_Ξ = -18$ MeV. Fig. 1 shows the effective mass of the nucleons and $Δ$. With increasing density the effective mass decreases due to the attractive $σ$ field for the baryons.

The EOS for different compositions of neutron star matter is shown in Fig. 2. It is observed that with the
TABLE III. Parameters for nuclear matter. They are determined from the binding energy per nucleon, $E_{B,E} = B_0 = E/\rho_B - M_N = -15.7$ MeV and pressure, $P = 0$ at saturation density $\rho_B = \rho_0 = 0.15$ fm$^{-3}$. Also shown are the values of the nuclear matter incompressibility $K$ and the slope of the symmetry energy $L$ for the quark mass $m_q = 150$ MeV.

| $m_q$ (MeV) | $g^2_\omega$ | $g_\rho$ | $g_\sigma$ | $M_N^*/M_N$ | $K$ (MeV) | $L$ (MeV) |
|------------|-------------|-----------|-----------|-------------|----------|----------|
| 150        | 4.39952     | 6.74299   | 8.79796   | 0.87        | 292      | 86.39    |

TABLE IV. $x_{\omega B}$ determined by fixing the potentials for the hyperons.

| $m_q$ (MeV) | $x_{\omega A}$ | $x_{\omega \Sigma}$ | $x_{\omega \Xi}$ |
|------------|----------------|---------------------|-----------------|
| 150        | 0.81309        | 1.58607             | 0.24769         |
|            | $U_A = -28$ MeV| $U_\Sigma = 30$ MeV | $U_\Xi = -18$ MeV|

The composition of the matter is shown in Fig. 3 which shows the particle fractions for $\beta$-equilibrated matter. At densities below the saturation value the $\beta$-decay of neutrons to muons are allowed and thus muons start to populate. At higher densities the lepton fraction begins to fall since charge neutrality can now be maintained more economically with the appearance of negative baryon species. Since the $\Delta^-$ can replace the neutron and electron at the top of the Fermi sea, it appears first at a density of $\rho_B = 0.41$ fm$^{-3}$. This is followed by the appearance of $\Sigma^-$. The sequence of appearance of the $\Delta$ resonances is consistent with the notion of charge-favored or unfavored species [2]. As such, the first $\Delta$ species to appear is $\Delta^-$, followed by the $\Delta^0$, $\Delta^+$ and $\Delta^{++}$. The slope of the symmetry energy $L$ also plays a key role in the appearance of $\Delta$ resonances. Drago et al. [23] constraining $L$ in the range $40 < L < 62$ MeV have observed the appearance of $\Delta$ close to twice the saturation density. At high densities all baryons tend to saturate. Moreover, the $\Sigma$ hyperon is not present in the matter distribution for the given set of potentials since we have chosen a repulsive potential for it.

Since the vector coupling of the $\Delta$ are not constrained by the properties of saturated nuclear matter, we study the effect of moderate variations in the strength of the vector coupling of the $\Delta$ on the mass-radius of the neutron star. Considering only the nucleon and $\Delta$ composition of the matter, we plot in Fig. 4 the gravitational mass as a function of radius by changing the coupling strength $x_{\omega \Delta}$ of the $\Delta$ isobars. By decreasing the coupling strength from $x_{\omega \Delta} = 1.0$ to $x_{\omega \Delta} = 0.7$, we observe a gradual decrease in the maximum mass of the star, see Table V. This follows from the fact that by decreasing the interaction strength of the $\Delta$ with respect to the nucleons, the EOS becomes softer with a consequent decrease in the maximum mass of the star. We also observe a decrease in the radius with increasing coupling strength.

In Fig. 5 we plot the mass-radius relations for the three possible compositions of neutron star matter at $m_q = 150$ MeV. A stiffer EOS corresponding to matter with nucleons only gives the maximum star mass of $M_{\text{star}} = 2.25 M_\odot$. With the appearance of the $\Delta$ isobars, mass decreases by $0.06 M_\odot$ to $M_{\text{star}} = 2.19 M_\odot$. The inclusion of the hyperons further softens the EOS resulting in a corresponding decrease in the maximum mass to $M_{\text{star}} = 2.15 M_\odot$. The results are shown in Table V. The recently observed pulsar PSR J0348+0432 provide a mass constraint of $2.01 \pm 0.04 M_\odot$ [24] while an earlier accurately measured pulsar PSR J1614-2230 gives a mass of $1.97 \pm 0.04 M_\odot$ [20]. From our calculations we obtain a range of masses varying from $2.15 M_\odot$ to $2.25 M_\odot$ depending on the composition of the matter. Though in the present model we are able to meet the mass constraint,
we do not get a lower radius. Such problems are also found in many other models. One of the way out in such model based calculations is to consider a compact star with mixed phases of hadrons and quarks. At present we consider only a hadronic phase. The work in this regard to meet the compactness in the present model is in progress.

Fig. 5 shows the gravitational redshift versus the gravitational mass of the neutron star at quark mass $m_q = 150$ MeV and $x_{\omega \Delta} = 0.8$. It also shows the maximum redshift (redshift corresponding to the maximum mass) which, for the present work comes out to be $Z_{s}^{\text{max}} = 0.17$. This is well below the upper bound on the surface redshift for subluminal equation of states, i.e. $z_{s}^{CL} = 0.8509^{27}$.

V. CONCLUSION

In the present work we have studied the possibility of the appearance the ∆ isobars in dense matter relevant to neutron stars. We have developed the EOS using a modified quark-meson coupling model which considers the baryons to be composed of three independent relativistic quarks confined by an equal admixture of a scalar-vector harmonic potential in a background of scalar and vector mean fields. Corrections to the centre of mass motion, pionic and gluonic exchanges within the nucleon are calculated to obtain the effective mass of the baryon. The baryon-baryon interactions are realised by the quark coupling to the $\sigma$, $\omega$ and $\rho$ mesons through a mean field approximation. The nuclear matter incompressibility $K$ is determined to agree with experimental studies. Further, the slope of the nuclear symmetry is calculated which also agrees well with experimental observations.

By varying the composition of the matter we observe the variation in the degree of stiffness of the EOS and the corresponding effect on the maximum mass of the star. As predicted theoretically, we observe that the inclusion of the $\Delta$ and hyperon degrees of freedom softens the EOS and hence lowers the maximum mass of the neutron star. The so called $\Delta$ and hyperon puzzles state that the presence of the $\Delta$ isobars and hyperons would decrease the maximum star mass below the recently observed masses of the pulsars PSR J0348+0432 and PSR J1614-2230. In the present work, we are able to achieve the observed mass constraint and at the same time satisfy the theoretical predictions of the possibility of existence of higher mass baryons in highly dense matter. Moreover, we study the effect of moderate variations in the strength of the vector coupling of the $\Delta$ resonances and observe a decrease in the radius of neutron stars with an increase in the coupling strength.

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\[ \varepsilon (\text{fm}^{-4}) \]

\[ P (\text{fm}^{-4}) \]

\[ m_q = 150 \text{ MeV} \]