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Instability and vortex ring dynamics in a three-dimensional superfluid flow through a constriction

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\textbf{Abstract.} We study the instability of a superfluid flow through a constriction in three spatial dimensions. We consider a Bose–Einstein condensate at zero temperature in two different geometries: a straight waveguide and a torus. The constriction consists of a broad, repulsive penetrable barrier. In the hydrodynamic regime, we find that the flow becomes unstable as soon as the velocity at the classical (Thomas–Fermi) surface equals the sound speed inside the constriction. At this critical point, vortex rings enter the bulk region of the cloud. The nucleation and dynamics scenario is strongly affected by the presence of asymmetries in the velocity and density of the background condensate flow.

\[ \text{S} \] Online supplementary data available from \url{stacks.iop.org/NJP/13/043008/mmedia}

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1. Introduction

Even when flowing past an obstacle, a superfluid can be stationary, since its excitations appear only under certain conditions [1]. For instance, a homogeneous Bose–Einstein condensate (BEC) of density \( n \) running through a weak obstacle, whose presence only slightly changes the cloud chemical potential, is energetically unstable towards phononic excitations only above a critical velocity \( v_c \) equal to the sound speed \( c \propto \sqrt{n} \), according to the Landau criterion. On the other hand, the critical velocity and the nature of the excitations change in the presence of a strong obstacle that considerably perturbs the homogeneous flow. Helium flowing through narrow channels becomes unstable at velocities much lower than those predicted by the Landau criterion, which this time would correspond to rotonic excitations, since, as first predicted by Feynman, vortices are nucleated inside the channel and carry energy away from the superfluid through the phase-slip mechanism [2]–[5]. Feynman’s energetics argument is analogous to the Landau criterion and is based on the choice of vortex rings, instead of rotons, as the excitations coming at the critical velocity. His conjecture has been refined by Varoquaux [5], who used half-rings as the excitations to be nucleated inside the channel. In general, the type of excitation and the critical velocity depend on the flow configuration.

In contrast to nearly incompressible fluids such as helium, atomic BECs, due to an exquisite control over the confining potential, offer a wide choice of flow geometries and obstacles and also permit a modification of the effective dimensionality. Imaging techniques also provide a means of addressing the system on the healing length scale at which vortex dynamics take place. The superfluid critical velocity in an elongated BEC can be studied, for instance, by sweeping a laser beam through the cloud [6]. At the critical velocity, both vortices in an elongated trap [7] and solitons for tight confinement transverse to the flow [8] have been experimentally observed. Another way of studying the stability of the superflow would be by perturbing a persistent current, which has been experimentally demonstrated with a BEC in a torus [9]. Apart from being a hallmark of superfluidity, BECs in toroidal geometries and the study of their critical velocities have very important technological applications in sensing devices, such as superconducting quantum interference devices (SQUIDs) already realized with superfluid helium and superconductors. BECs are often very well described by the mean-field Gross–Pitaevskii equation (GPE), which provides, unlike for helium, a reliable theoretical model for studying the instability mechanism and its dynamics. The nucleation of a
Figure 1. Constriction configuration. The light gray surface corresponds to the classical (Thomas–Fermi) surface of the cloud. The dark gray surface shows an isosurface of the barrier potential used for creating the constriction.

vortex–antivortex pair at the critical velocity has been studied numerically in two dimensions (2D) and 3D with an obstacle moving through a uniform condensate [10] and in a 2D waveguide with an orifice [11]. The flow dissipation caused by the formation of solitons in 1D has been investigated in [12]. In a previous work [13], we studied the scenario of superfluid flow dissipation in an effective 2D toroidal geometry.

In this paper, we study the critical velocity and superfluid flow dissipation mechanism in a 3D constricted flow configuration, where the size of the cloud along all the three directions is much larger than the healing length $\xi = \hbar/\sqrt{2mgn}$. We consider a subsonic flow of a zero-temperature BEC in two different geometries: a waveguide with periodic boundary conditions, which can mimic an elongated cloud along the flow direction, and a torus. The unstable regime is reached by raising a repulsive penetrable barrier perpendicular to the flow. This barrier is broader than the cloud dimension and extends over a few (typically 5–10) healing lengths along the flow direction. In this way, we create a constriction for the flow in the barrier region, as shown in figure 1. Starting from a stationary flow with a given velocity, the barrier is adiabatically raised during the dynamical evolution until instability sets in, and we observe vortex rings penetrating the cloud. In this way, the superflow energy is dissipated through the phase-slip mechanism [2]: vortices are able to take energy away from the flow since the velocity drops by a quantized amount in a region crossed by a vortex core [13]. The two geometries under study present significant differences in the vortex ring nucleation and dynamics. We find that vortex rings, which can find a stationary configuration after entering an axially symmetric waveguide, are instead always transient in the torus and, more generally, as soon as the axial symmetry in the direction of flow is broken.

2. Formulation and computation

To evolve these various cases, we solve a GPE (1) for three spatial dimensions in scaled form as

$$i\hbar \frac{\partial \psi (r, t)}{\partial t} = \left[ -\frac{1}{2} \nabla^2 + V(r, t) + g|\psi|^2 \right] \psi (r, t).$$

(1)
where the length, time and energy are given in units of $d_0 = \left[\frac{\hbar}{m\omega_0}\right]^{1/2}$, $1/\omega_0$ and $\hbar\omega_0$, respectively, for a representative harmonic frequency $\omega_0$ that characterizes the trap. $V(r, t)$ is the external potential and $g = \frac{4\pi a}{d_0}$ with $a$ being the inter-particle s-wave scattering length and $m$ the atomic mass. $\psi(r, t)$ is the condensate wavefunction normalized to the total number of particles $N$. The external potential has components associated with the trapping and the barrier potentials of the form

$$V(r, t) = V_t(r) + V_b(r, t).$$

In both cases, we find the ground state of the system with $V_b = 0$.

The waveguide geometry is implemented by choosing the trapping potential,

$$V_t(r) = \frac{1}{2}[x^2 + \gamma y^2 + z^2],$$

with $\omega_0 = \omega_x = 30 \times 2\pi$ Hz, $\gamma = \omega_z/\omega_x$ and periodic boundary conditions along the flow direction $y$. We considered three different values of $\gamma$, namely 1, 1.05 and 1.2, which correspond, respectively, to a ground state chemical potential $\mu = 11.7$, 12.0 and 16.5 with $N = 3 \times 10^5$ $^{87}$Rb atoms and a nonlinear scaling value $gN = 10134$.

The toroidal geometry is implemented by choosing the trapping potential,

$$V_t(r) = \frac{1}{2}[\alpha x^2 + \beta y^2 + z^2] + V_c \exp^{-2(\rho/\sigma_b)^2},$$

where $\alpha = \omega_x/\omega_z$ and $\beta = \omega_y/\omega_z$. We take $\alpha = \beta = 0.5$ with $\omega_0 \equiv \omega_z = 25 \times 2\pi$ Hz and form the torus by including a core potential with parameters $V_c = 144$ and $\sigma_b = 1.88$ with $\rho^2 = x^2 + y^2$. The ground state chemical potential is $\mu = 7.6$ for $N = 2.5 \times 10^5$ $^{23}$Na atoms, corresponding to a nonlinear scaling value $gN = 2028$.

During the dynamical evolution, we use a time-dependent barrier potential of the form

$$V_b(r, t) = f(t)V_{bx}(x)V_{by}(y)V_{bz}(z)/8,$$

with $f(t) = t/t_c$ ($f(t) = 1$ for $t > t_c$) and $V_{bx} = \tanh\left(\frac{x-R_s+x_0}{b_x}\right) + \tanh\left(\frac{-(x-R_s+x_0)}{b_x}\right)$. Here, $R_s$ is the $x$-shift of the center of the barrier, while its width is $w_x = 2x_0$. $V_{by}(y)$ and $V_{bz}(z)$ have the same form as $V_{bx}$. The final height of the barrier is $V_b$ as long as $x_0, y_0, z_0 \gg b_x$.

Before starting the dynamics, we put the condensate in motion by imprinting an appropriate spatially dependent phase $\theta(r)$ on the wavefunction. In the waveguide case, $\theta(r) = mvy/\hbar$ generates a uniform flow of velocity $v$ along the $y$-direction. In the torus, $\theta(r) = ml\phi/\hbar$, with $l$ being integer, generates a tangential flow of speed $v(r) = 1/\rho$, where $\phi = \arctan(y/x)$.

The results presented in this paper are obtained by solving numerically the GPE. After finding the ground state with an imaginary-time propagation, we perform the dynamical evolution in real time. For all the simulations performed, we used a finite-element discrete variable representation (DVR) [22]. In the waveguide geometry, we employed a split-operator method, with a fast Fourier transform algorithm used for the kinetic part of the evolution, while for the simulations in the torus we used a real space product formula [22]. The spatial grid for the waveguide calculations was $n_x = 60$, $n_y = 120$, $n_z = 60$ (or $n_x = 180$, $n_y = 120$, $n_z = 180$ for figures 4 and 5) with box sizes $L_x = 12$, $L_y = 24$, $L_z = 12$ for the axially symmetric case (see below), $n_x = 90$, $n_y = 120$, $n_z = 80$ with box sizes $L_x = 18$, $L_y = 24$, $L_z = 16$ for the axially asymmetric case, while the time step was $dt = 10^{-4}$. In the torus simulations, we used a spatial grid consisting of 80 elements in each dimension with four DVR Gauss–Legendre functions in each element spanning cubic box lengths of $[-12, 12]$, $[-12, 12]$ and $[-10, 10]$ in the $x$-, $y$- and $z$-directions, respectively. This choice gave 241 grid points in each direction. We also tested the convergence with five basis functions and 321 points with only a few per cent change in the basic quantities, such as energies, momentum and positions.
Figure 2. Ratio of the higher local fluid velocity at the Thomas–Fermi radius, \( v(R_{TF}) \), to the sound speed \( c \) inside the barrier, as a function of the barrier height. The red-shaded area corresponds to the critical point plus uncertainty. The results are obtained for the cylindrically symmetric waveguide \( \gamma = 1 \) and the initial flow velocity \( v = 1.05 \). The inset shows a sketch of the behavior of the local fluid velocity along a radial cut inside the barrier region. The red solid line indicates the value of the sound speed \( c \), which with transverse harmonic confinement is just the average of the local sound speed, \( c(r_\perp) \) (red dashed line), over the transverse plane. The black (blue) solid line corresponds to a subcritical (critical) condition.

3. Criterion for instability

In the hydrodynamic regime, when the healing length \( \xi \) is much smaller than all other length scales (the smallest of which is the barrier width along the flow direction), we find that the onset of instability coincides with a simple condition: inside the barrier region, the local fluid velocity at the classical (Thomas–Fermi) surface of the cloud equals the sound speed, as depicted in the inset of figure 2. It is important to precisely define the sound speed, which sets the threshold for the critical velocity. The latter is the Bogoliubov sound speed, calculated inside the barrier region, for the low-lying modes propagating along the flow, taken as if the system was homogeneous in this direction. In the case of a waveguide with a harmonic transverse confinement, and within the Thomas–Fermi approximation, this sound speed is simply the average of the local sound speed on a plane perpendicular to the flow \( c = c(0)/\sqrt{2} \), where \( c(0) \) is the local sound speed at the center of the transverse harmonic trap [17].

We verified this numerically in the waveguide case, as shown in figure 2. In a waveguide with cylindrical symmetry, this happens simultaneously at points on a circle perpendicular to the flow, from where a vortex ring will then enter. In the torus, due to a higher flow speed at the inner edge of the annulus, the critical condition is reached first in the interior of the cloud, with the consequence that vortex rings, if ever formed, must be asymmetric, as we discuss below.
Figure 3. Four subsequent stages of the vortex ring penetration in the waveguide (see movie 1, available from stacks.iop.org/NJP/13/043008/mmedia). The gray surface indicates the position of the Thomas–Fermi surface of the condensate. Black dots show the position of the vortex cores. Here the waveguide is axially symmetric $\gamma = 1$, the initial flow velocity is $v = 1.05$ and the final barrier height is $V_s = 0.17 \mu$. (a) The ring is shrinking around the cloud in the barrier region and is still outside the Thomas–Fermi borders of the cloud. (b) The ring has just entered the cloud. (c) The ring has shrunk to its final size and is already outside the barrier region, moving along the flow direction. (d) The ring has moved far from the constriction region, with constant speed and radius.

Slightly below the critical point, we observe vortices getting closer to the edges of the cloud but failing to enter them. The presence of these ‘ghost’ vortices [13, 14] suggests that a pre-instability is triggered at the edges of the condensate, but, since the vortices do not enter the bulk region, it is not sufficient to dissipate the superflow.

In a previous work [13], we verified the same condition for instability to hold for a 2D toroidal BEC. The results reported here represent an extension to the 3D case. As discussed above, this criterion allows for a simple understanding of the details of vortex penetration dynamics, based on the observation that vortex cores enter first where the critical condition is first reached.

4. Instability dynamics in the waveguide

In analogy to the standard situation where a superfluid flows through a channel, we consider a waveguide with periodic boundary conditions along the flow direction $y$, and the barrier
Figure 4. Four subsequent stages of the vortex ring annihilation in the waveguide (see movie 2, available from stacks.iop.org/NJP/13/043008/mmedia). The gray surface indicates the position of the Thomas–Fermi surface of the condensate. Black dots show the position of the vortex cores. Here the waveguide is axially symmetric $\gamma = 1$, the initial flow velocity is $v = 0.52$ and the final barrier height is $V_s = 0.94\mu$. (a) The ring is shrinking around the cloud in the barrier region and is still outside the classical borders of the cloud. (b) The ring has entered the cloud. (c) The ring is about to shrink completely and annihilate. (d) The ring has annihilated and no vortex core is now inside the Thomas–Fermi surface.

creating the equivalent of the channel. This situation can be realized experimentally with a BEC inside an elongated trap along the flow direction. As stated above, before raising the barrier in the simulations, a stationary flow is created by imprinting a phase $mv_y/h$ on the condensate wavefunction, where $v$ is the constant flow speed. This corresponds in the experiment to sweeping the barrier across the cloud at a constant velocity. We consider a symmetric harmonic confinement in the transverse $x$–$z$ plane, such that $d_x = \sqrt{\hbar/m\omega_x}$ and $d_z = \sqrt{\hbar/m\omega_z}$ are both sufficiently larger than the bulk healing length, so that the effect of transverse degrees of freedom comes into play, giving rise to fully 3D dynamics. Such a setup has been used in [16], where, moreover, the possibility of creating a penetrable repulsive barrier with a control over a length comparable to the healing length has been demonstrated.

In this configuration, as soon as the critical barrier height is reached, a vortex ring detaches from the system boundaries and starts shrinking into the cloud inside the barrier region, as shown in figure 3(a). In the figure, black points indicate the position of vortex cores, while the gray surface corresponds to the Thomas–Fermi surface of the cloud. The detection of vortex cores is performed using a plaquette method described in [21].

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Figure 5. Details of the ring self-annihilation event studied in figure 4, at six subsequent times. In the left panels of (a) and (b), the gray surface indicates the position of the Thomas–Fermi surface of the condensate, whereas black dots show the position of the vortex cores. In the right panels of (a) and (b) and in (c), the density on a $z = 0$ plane parallel to the flow direction is shown. A very small loop structure of vortex cores, in (a), shrinks to a point and has disappeared in (b). The ring has transformed into a rarefaction pulse, whose propagation and decay into sound is seen in (c).

Depending on the initial flow velocity, waveguide transverse section and barrier height at the critical point, the ring attains a certain radius and velocity with which it propagates in the flow direction in a stable manner, as long as axial symmetry is preserved, as depicted in figures 3(c) and (d). Here the vortex ring eventually propagates at the speed $u_r = 0.65$ and the radius $R = 2.9$.

For sufficiently strong barriers, the ring shrinks to a point crossing the transverse section, thereby annihilating and completing a full single phase-slip, as shown in figure 4. After this process, the velocity has dropped everywhere by the same quantized amount. We observe that such an event is the 3D analogue of vortex–anti-vortex annihilation in 2D [13]. It is interesting to analyze the ring annihilation event in more detail. In figures 5(a) and (b), respectively, we show the position of the vortex cores together with the density on a plane parallel to the flow direction, at a time just before and just after the ring has shrunk to a point. In figure 5(c), the density on the same plane is plotted at four subsequent times after the annihilation. The details of the self-annihilation process that we observe are consistent with previous studies of axisymmetric vortex ring solutions [23]–[27]. Namely, if we consider the position of the vortex cores, we see that the points of phase singularity form a loop that shrinks inside the constriction, figure 5(a), and whose radius eventually becomes zero, figure 5(b). At this moment, we see the zero-density core being filled with atoms, thereby transforming into a density depression, which
Three subsequent stages of vortex ring annihilation in the torus. The gray surface indicates the position of the Thomas–Fermi surface of the condensate. Black dots show the position of the vortex cores. Here the initial circulation is \( l = 1 \), and the final barrier height is \( V_s = 0.5\mu \). (a) The ring is shrinking around the cloud in the barrier region and is still outside the classical borders of the cloud. (b) The ring has entered the cloud. (c) The ring is about to shrink completely and annihilate.

Further propagates out of the constriction as a rarefaction pulse, as shown in figure 5(c). Finally, this rarefaction pulse decays into sound.

Both in figures 3 and 4, we see that, at least initially, the vortex cores are subjected to an essentially radial motion, giving rise to the shrinking of the vortex ring. This behavior can be understood by considering what contributes to the motion of vortex cores in non-homogeneous systems [19]. Let us consider a single vortex core located at some position \( x_c \). Its velocity would be the sum of two terms: (i) the background flow velocity at \( x_c \) and (ii) a term perpendicular to the gradient of the density at \( x_c \). Both contributions must be calculated as if the vortex was not present. Since the relative weight of term (ii) is proportional to the value of the healing length calculated at \( x_c \), in high-density regions, a vortex core will move mainly with the background superfluid velocity, whereas in low-density regions, it will move mainly due to the gradient of the density. Therefore, when the vortex ring is inside the constriction, where the density is low, and especially when it is close to the Thomas–Fermi surface, it will mainly move perpendicular to the gradient of the density, whose main contribution comes from the density modulation induced by the barrier along the flow direction. This results in an essentially radial motion of the cores and thus in the shrinking of the ring.

5. Instability dynamics in the torus

In the toroidal geometry, the scenario is richer since the tangential flow velocity at a given total angular momentum, due to the conservation of quantized circulation, decreases like \( 1/r \), where \( r \) is the distance from the center of the torus. This introduces an asymmetry between the inner and outer edges of the cloud, which is not present in the waveguide case. The effects of this asymmetry on vortex nucleation in 2D are discussed in [13]. In 3D configurations, the result is that vortex rings are transient features in a toroidal geometry.

The ring vortex either breaks or shrinks to a point and annihilates for sufficiently strong barriers. While the annihilation process, shown in figure 6, is very similar to the one taking
Figure 7. Four subsequent stages of the vortex ring breaking in the torus. The gray surface indicates the position of the Thomas–Fermi surface of the condensate. Black dots show the position of the vortex cores. Insets show the top and side views. Here the initial circulation is $l = 4$, and the final barrier height is $V_s = 0.2\mu$. (a) The vortex ring is bending to form a right angle. (b) The vortex ring has formed a right angle whose vertex is close to a vortex line coming from the center of the torus. (c) The vortex ring and line have just reconnected: a vortex line and a portion of a ring vortex are now inside the Thomas–Fermi surface. (d) The vortex line and the ring have moved apart.
Figure 8. Three subsequent stages of the vortex ring formation in the torus. The gray surface indicates the position of the Thomas–Fermi surface of the condensate. Black dots show the position of the vortex cores. Insets show the side view. Here the initial circulation is $l = 4$, and the final barrier height is $V_s = 0.2\mu$. (a) The vortex line is bending around the cloud in the barrier region, partially outside the Thomas–Fermi borders of the cloud. (b) The vortex line has developed two kinks. (c) The vortex ring has just formed.

vortices in the system [20]. When the barrier is not strong enough to make both edges of the annulus unstable, we also observed cases in which the ring is not formed at all, and a strongly bent vortex line enters the cloud from its inner edge, to circulate around the torus. The fact that the vortex line enters the inner edge of the annulus is due to the above-mentioned asymmetry in the velocity field, decaying as the inverse of the distance from the center of the torus, which makes the instability set in there first, according to the criterion discussed in section 3.

The formation of a vortex ring in the torus is very interesting, since it can be seen as a reconnection of a vortex line with itself. As shown in figure 8, a bent vortex line is always present at first. While the bending increases, the vortex line develops two sharp kinks whose tips get closer to each other, until they join, thereby cutting the original line into a vortex ring plus yet another line. The latter is then reabsorbed at the system’s boundary.

6. Connection between effective two-dimensional (2D) and 3D geometries

When the condensate is effectively in 2D, meaning that the cloud size along the third direction is comparable with the healing length $\xi$, vortex lines oriented along the direction of tighter confinement can become the preferred excitations with respect to vortex rings [18]. In studying a condensate inside an effective 2D toroidal trap (the system’s size along the axis of the torus was about one healing length) [13], we indeed observed that superfluid flow dissipation took place through the formation of vortex lines entering the cloud. For the trap configurations considered in this paper, we have observed vortex ring formation in the very low density regions outside the condensate classical Thomas–Fermi surface. As suggested above, we could call these ghost [14] vortices, since they are not visible from the condensate density profile. In some cases, as discussed above, these ghost vortex rings are able to enter completely the classical surface of the cloud, transforming into what we should then call ‘real’ ring vortices.

In the crossover between effective 2D and 3D regimes, moving from a scenario in which only vortex lines are present to one in which real vortex rings come into play, it is reasonable
Figure 9. Four subsequent stages of the vortex ring dynamics in the axially asymmetric waveguide (see movie 3, available from stacks.iop.org/NJP/13/043008/mmedia). The gray surface indicates the position of the Thomas–Fermi surface of the condensate. Black dots show the position of the vortex cores. Insets show the front and side views. Here the waveguide is non-axially symmetric $\gamma = 1.2$, the initial flow velocity is $v = 1.05$ and the final barrier height is $V_s = 0.13\mu$. (a) A partially ghost ring vortex has formed. (b) The ring vortex is strongly deformed. (c) The ring vortex has broken up, leaving two vortex lines. (d) The vortex lines have joined back to form a new ring vortex.

To expect an intermediate regime in which ghost vortex rings are formed, but the condensate is sufficiently squashed along the third dimension that the full vortex loop is not able to enter the cloud’s classical surface. In this regime, superfluid would be dissipated by vortex rings, which are partly real and partly ghost, appearing as simple bent vortex lines in the density profile.

An example of such a situation is given in figure 9, where the waveguide is non-axially symmetric about the flow direction ($\gamma = 1.2$). After the instability sets in, a ghost vortex ring forms and shrinks around the cloud in the barrier region up to when we observe a full-fledged ring vortex, which is only partially inside the Thomas–Fermi surface of the condensate (figure 9(a)). However, since the part of the ring that is inside the Thomas–Fermi surface moves with a larger speed along the flow direction with respect to the part that remains outside, the ring vortex is soon deformed (figure 9(b)) and eventually breaks up (figure 9(c)). The vortex cores located inside the Thomas–Fermi surface of the cloud move, to a good approximation, along with the background velocity field. On the other hand, cores in the low-density region...
outside the surface of the cloud essentially do not feel the background velocity and move along the flow direction only because of the presence of transverse density gradients. After the vortex ring breaks up, the two lines move downstream and continue to deform, and eventually rejoin to form a vortex ring (figure 9(d)). The latter undergoes the same deformation described above, leading to another break-up.

We also verified that, even in a very slightly non-axially symmetric waveguide, ring vortices eventually break up. With $\gamma = 1.05$, the deformation is created more slowly with respect to the $\gamma = 1.2$ case, but the ring breaks up anyway, although at a later time (see movie 4, available from stacks.iop.org/NJP/13/043008/mmedia).

7. Conclusions

We have studied the superfluid instability condition and dynamics in the presence of a 3D constriction across the flow. We solved the GPE, which would accurately describe an ultracold dilute Bose gas, but would anyway provide useful insights for understanding the instability in strongly correlated superfluids, such as helium II. We considered two different setups: a waveguide, which could be experimentally realized by trapping an ultracold Bose gas inside a cigar-shaped trap, and a torus. We found that the instability sets in when the fluid velocity at the Thomas–Fermi surface of the cloud equals the sound speed inside the constriction region. This critical condition explains the strong dependence of the instability dynamics on the geometry considered, and in particular on the asymmetries present in the flow velocity and density. In an axially symmetric waveguide, the superfluid dissipates through the nucleation of circular ring vortices that shrink across the flow within the barrier regions. As soon as the axial symmetry is broken, the rings are strongly deformed and can break up into vortex lines or not form at all. The phase slip instability observed here can be considered as a superfluid flow dissipation mechanism, since energy is subtracted from the potential flow of the superfluid. However, GPE does not include the dissipative processes that bring the system to thermal equilibrium. In a realistic situation, the vortices, which are carrying the energy subtracted from the superfluid, are supposed to eventually thermalize, resulting in heating the system. The effect of dissipation on vortex nucleation has been studied by adding a dissipation term to the GPE [15]. The role of finite temperature in the superfluid instability scenario presented here deserves further study.

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