Study of six mechanical impedance matchers on a spherical gravitational wave detector

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Abstract. A spherical gravitational wave (GW) detector has a heavy ball-shaped mass which vibrates when a GW passes through it. Such motion is monitored by transducers and the respective electronic signal is digitally analyzed. One of such detectors, SCHENBERG, will have resonant frequencies around 3.2 kHz with a bandwidth near 200 Hz. The frequencies of other resonant-mass detectors typically lay below 1 kHz, making the transducer development for this higher frequency detector somewhat more complex. In this work we present a series of finite element studies of a sphere coupled to resonant mushroom shaped resonators that will work as mechanical impedance matchers between the sphere and the transducer. We describe the search for a shape of the impedance matcher that might improve the performance of the detector. We find that the normal modes of the coupled system are not exactly degenerate, while theoretical calculations predict that they should be.

1. Introduction

SCHENBERG is a spherical resonant-mass gravitational wave detector [1, 2] being built at the Department of Materials and Mechanics of the University of Sao Paulo, Brazil. The sphere, with 65 cm in diameter and weighting 1.15 ton, is made of a copper-aluminum alloy [3] with 94% Cu and 6% Al.

The detector will have six transducers [4, 5], arranged on the sphere’s surface in a half-dodecahedron distribution; the sensors will be located as if in the center of the six connected pentagons in a dodecahedron surface, following the studies by Merkowitz and Johnson [6] confirmed by Magalhaes and collaborators [7]. By analyzing the signal from such sensors the amplitudes and the direction of the incoming gravitation wave can be obtained [8, 9]. A similar detector is been built in the Netherlands, called MiniGrail [10], with a frequency close to SCHENBERG’s.

While in MiniGrail inductive transducers are planned to be used, our group has decided to use as motion sensors microwave parametric transducers, like the one used in the Australian GW detector NIOBÈ [11]. In this kind of transducer a superconducting cavity is pumped with monochromatic resonant microwaves and when the size of the cavity changes due to the vibration (one of the cavity wall is connected to the sphere by the mechanical impedance matcher) two side bands are created in the microwave signal that leaves the cavity. The amplitude of the side band is proportional to the amplitude of the sphere vibration. Such transducer is expected to be tested in SCHENBERG soon [1, 12].

It is known that a multimode mechanical impedance matcher for the transducer is expected to have several advantages over the single mode transducer [13], either parametric, inductive or capacitive. For this reason such a matcher is planned to be used in SCHENBERG.

In order to propose improvements based on the practical tests that shall start soon, we decided to perform simulations that would allow us to estimate how changes in the parameters of the mechanical impedance matcher could influence its match to the antenna. For simplicity we chose to work with a single mode impedance matcher in this first investigation. The results of our studies are presented in the following sections.
2. Sphere plus one impedance matcher

The simulations reported in this work were made with the aid of a finite element program (ANSYS 5.4). We first simulated the sphere alone based on SCHENBERG’s spherical antenna, made of 94% copper and 6% aluminum and with 0.65 m in diameter. For future use we list the following parameters of this material: Young’s modulus: $E = 1.303 \times 10^{11}$ Pa; specific mass: $\rho = 8077.5$ kg/m$^3$; Poisson’s coefficient: $\nu = 0.364$.

We found that the frequencies for the five quadrupole modes were (in Hz) 3189.7, 3190.0, 3190.8, 3191.2 and 3192.1. Although these numbers suggest that the modes are not perfectly degenerate in practice one may assume that they are, with average frequency of 3190.8 Hz, because the small differences are probably due to limitations in the finite element modeling.

According to the theory of general relativity the sphere’s quadrupole modes are the ones expected to be excited by a gravitational wave signal and theoretically they are exactly degenerate [6].

The change in shape from a sphere to an ellipsoid is present at all frequencies, as predicted from theory [14].

SCHENBERG’s sphere modes are published in [15] and they were measured after holes were drilled to accommodate the suspension and future transducers. The values found were (in Hz) 3172.5, 3183.0, 3213.6, 3222.9 and 3240.0. They yield an average of 3206.4 Hz with a standard deviation of almost 1%. It is reasonable to speculate that the lower average frequency obtained in the simulation is due to the fact that no holes were drawn in it. We plan to include them in a future work. Nevertheless, the simulated frequencies and the actual ones are close to each other, around 3200 Hz.

2.1. The First Three Simulations

We chose a design for the impedance matcher which was simple to model with a finite element program and whose parameters could easily be changed: a mechanical matcher with a mushroom shape. This shape consists of a disk diaphragm with a cylinder in the center that connects the diaphragm to the sphere. Although this is not the same shape of the actual transducer used in the experiment its matching to the sphere is physically similar, modeled by springs and masses [6].

Once the design of the transducer was defined, we then had to determine the dimensions of the diaphragm and the central cylinder. For simplicity, only one transducer was attached to the sphere in this first set of simulations. The intention behind these determinations was to tune the impedance matcher to the same frequency of the sphere quadrupole modes in a way that it would work as a resonant transducer.

The initial estimate for these dimensions was based on a standard procedure described in the book by Blevins [16]. For a mushroom shaped matcher the frequency of its motion is given by the formula

$$f_{ij} = (\lambda_{ij}^2 t E^{0.5})/(4\pi a^2 (3\rho(1-\nu^2))^{0.5})$$.

The sub indices $ij$ refer to the mode of interest, which in our case is 00. The constant $\lambda_{00}$ depends on the ratio $b/a$, where $b$ is the diameter of the central cylinder and $a$ is the diaphragm’s diameter. The thickness of the diaphragm is given by $t$, while $E$, $\rho$ and $\nu$ were introduced in the previous section.
Initially we chose $b = 30\text{mm}$ and $a = 100\text{mm}$ at a frequency $f_0 \sim 3200\text{Hz}$. The goal was to determine $t$. With some adjustment this procedure yielded the dimensions presented in Table 1 for the impedance matcher used in Simulation #1.

As expected from theory [17], the system sphere plus one transducer yielded a total of six modes. Five of these modes are very close in frequency (in theory they are found to be degenerate, forming a quintuplet) and the one left (the singlet) has a frequency different from the others. The values obtained for these frequencies are also shown in Table 1. These mode frequencies are well within the detector's bandwidth, as intended. We then proceeded to refine the impedance matcher's design inspecting the simulated motion. We noticed that the way that the diaphragm oscillated was not very uniform. The same has happened to the second and third simulations.

| Simulation | 1 | 2 | 3 | 4 | 5 | 6 |
|------------|---|---|---|---|---|---|
| Diaphragm's diameter (mm) | 100 | 100 | 100 | 100 | 50 | 50 |
| Central cylinder's diameter (mm) | 30 | 30 | 30 | 10 | 10 | 10 |
| Diaphragm's thickness (mm) | 6.1 | 5.9 | 5.2 | 9.5 | 1.9 | 1.6 |
| Average frequency of the quintuplet (Hz) | 3182.5 | 3180.8 | 3186.8 | 3186.0 | 3185.8 | 3185.2 |
| Frequency of the singlet (Hz) | 3328.6 | 3284.5 | 3035.2 | 3388.9 | 3590.7 | 3199.9 |

Table 1. Characteristics of the mechanical impedance matcher tested in each simulation, the order of the simulations indicated by the numbers in the top row. The frequencies refer to the system sphere plus matcher.

2.2. The Fourth and Fifth Simulations
In Simulation #4 we changed the dimensions of the impedance matcher as shown in Table 1. This attempt showed to be unfortunate because the frequency of the mode of interest became very close to the frequency of an inconvenient mode (the rocking mode).

This correlation was eliminated reducing the dimensions of the diaphragm (see Simulation #5 in Table 1). The shape of the diaphragm's vibration in all modes of this simulation is close to the same desired shape. Although there is still some deformation it is less significant than in the previous simulations. However, the modes' frequencies are not as close to each other as we intended so we decided to make new modifications.

2.3. The Sixth Simulation
The diameter of the diaphragm was not changed in Simulation #6 but we reduced its thickness. The frequencies of the singlet and the quintuplet were within the detector's bandwidth. In Figure 1 we show that with these dimensions all modes present the same kind of motion, as desired. These modes should have less calibration problems and are expected to generate a significant signal either in a capacitive or in an inductive transducer.

3. Sphere plus six impedance matchers
The natural continuation of the research was to attach six impedance matchers of the kind of the one used in Simulation #6 to the sphere's surface. They were positioned according to a half-dodecahedron distribution, locating the reference frame in the center of the sphere the positions of the matchers can be defined according to the angles $\phi$ (in the $xy$ plane, starting in $x$ and increasing in the positive direction of $y$) and $\theta$ (in the $yz$ plane, starting in $y$ and increasing in the positive direction of $z$). Table 2 shows the positions of each impedance matcher.
We then studied their behavior when coupled to the sphere quadrupole modes. We found that the system sphere plus six matchers had 11 modes, forming basically two quintuplet and one singlet, as expected [17], with frequencies (in Hz):

First quintuplet: 3164, 3168, 3171, 3171 and 3172;
Singlet: 3187;
Second quintuplet: 3194, 3196, 3204, 3206, 3215.

The frequencies of the first quintuplet are basically degenerate. They have little standard deviation from their average value of 3169.2 Hz compared to their distance from the single mode at the center of the band. On the other hand, the frequencies of the second quintuplet should not be considered degenerate since the standard deviation from their average value of 3203.0 Hz is comparable to their deviation from the central mode, as is evident from Figure 2.

In the singlet mode the sphere stays at rest while the transducers move, as shown in Figure 3. The other ten modes have one transducer that moves more intensely than the others.

Table 2. Positions of the six impedance matchers simulated simultaneously on the sphere. The angles are in degrees.

| Position | 1     | 2     | 3     | 4     | 5     | 6     |
|----------|-------|-------|-------|-------|-------|-------|
| (φ, θ)   | (150, 52.6225) | (30, 52.6225) | (270, 52.6225) | (90, 10.8122) | (330, 10.8122) | (210, 10.8122) |

Figure 1. Snapshots of the motion of the impedance matcher used in Simulation #6 for the six modes of the system sphere plus matcher. The singlet mode is shown in the bottom right drawing.
Figure 2. Frequencies of the coupled system: sphere plus six impedance matchers. The degeneracy is more broken in the quintuplet with higher frequencies. Notice that the frequency 3171 Hz is repeated.

4. Conclusions
After a series of changes in the design of the mushroom impedance matcher we found one for which the vibration of all the modes of the system sphere plus matcher had similar shapes and could be calibrated in the same way. We showed that it is not straightforward to find a mechanical oscillator with a frequency near the one of the sphere’s quadruple modes and which would work as a good impedance matcher for a spherical gravitational wave detector.

The matcher with best performance was the smallest and lightest among all simulations. It had a relatively high ratio between the diaphragm’s diameter and the diameter of its base, and a thin diaphragm. This result suggests that it is important to keep the contact between the membrane and the sphere as little as possible while keeping the membrane large and thin.

We also determined the eleven normal modes of the system sphere plus six of such resonators. When these matchers were attached to the sphere in the adequate positions we noticed that the quintuplet modes showed different bandwidths. We know that the positioning of the matchers affect the overall behavior of the system. It is a fact that all transducers are located on the same half of the sphere and this creates an asymmetry. From our experience we believe that such asymmetry might influence the mode’s responses. Therefore, the lack of degeneracy of the second quintuplet may be due to the asymmetry created by the presence of the resonators in only one of the sphere’s hemisphere.

From these results we expect that the actual transducers for SCHENBERG will have a good performance as long as a thin, large membrane is used in its impedance matcher, compared to its attachment. Also, it is probable that the quintuplet modes will not be equally spread around the average value, the higher ones having a larger bandwidth. It is advisable to investigate to which extent this feature will affect data analysis.
As a natural continuation of this work the sphere should be simulated with the holes that will be used to attach the transducers. Also, a multimode impedance matcher should be investigated, aiming at modeling the actual, double-mode transducer planned to be used in SCHENBERG.

Figure 3. Snapshots of the simultaneous motion of the impedance matchers in the singlet mode of the system: sphere plus six matchers, where all matchers move while the sphere stays at rest. The maximum amplitude of motion is shown in the top central image.

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