Numerical Study of the Influence of Defect on the Material Side in Vortex-Antivortex Formation Based on TDGL Equation

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Abstract. Dynamics of vortex-antivortex (VAV) annihilation due to a defect in the side of material has been successfully studied using TDGL Equation. This study was to find out how is the potential curve (V-t) when VAV annihilation occurs. Type II superconductor with $\kappa = 1.3$ and size of $L_x \times L_y = 50\xi_0 \times 50\xi_0$ was used here. This study was based on a numerical solution of the TDGL Equation by means of finite difference method with FTCS (Forward Time Centered Space) Scheme. The external current density $J_e$ was applied to the material without applying external magnetic field $H_e$. That is the way to bring up antivortex on the bottom of material and vortex on the upper of material. The defects were prepared with the size of $0.3\xi_0 \times 1\xi_0$. The defect on the material gives effect to the presence of vortex and antivortex. VAVs can only enter the material through the defect and not enter through another side. This is showing that the defect on the material provides the force which causes VAV more easily enter the material at the defect than another side. VAVs will enter continuously until they will meet each other and the annihilation of VAV occurs in the middle of the material. The sharp pulses (the high peak pulse) appeared during the vortex and antivortex annihilating each other. Besides, the small pulses raised the vortex and antivortex when they entered the material. When annihilation occurred, the potential curve produced the very high potential value for a moment and returned to its normal potential state. The presence of the defect on the material can increase superconductivity of material as well.

Keywords: Numerical study, defect, material side, vortex-antivortex formation, TDGL

1. Introduction
A vortex dynamics in a type-II superconductor may affect decreasing superconductivity of the superconductors. One of the triggers of the vortex movement is a current $J_e$ passed on a superconductor [1]. In recent years, the study of vortex dynamics increases significantly. Many studies of the dynamics of the vortex have been done, one of them is about the dynamics of the vortex due to the defect on the material [1–4].

Besides of that, the current $J_e$ passed to a superconductor without external magnetic field can generate not only vortex but also anti-vortex. The effect of the dynamics of the annihilation of VAV on the V
curve has been successfully explained by Wisodo et al. [1]. When there is no external magnetic field, the highest peak at V curve occurs when VAV annihilate each other.

A numerical study to explain the dynamics of VAV has also been practiced. One approach is using the Time-Dependent Ginzburg Landau (TDGL) Equation and its modification [5–7]. This model is possible to visualize the dynamics of the annihilation of a VAV pair. Vortex or antivortex can be identified through a vector field of the supercurrent density \( J \) and the local magnetic induced \( h = \nabla \times A \) where \( A \) is the potential magnetic vector [1].

In the present work, we studied the effect of the defect on the surface of the material on VAV dynamics and find out how the influence of VAV annihilation on the potential curve. This paper consists of some sections. In section 1, it is about the introduction. In section 2, we study the model of TDGL and system. In section 3, we study numerical methods. In section 4, we evaluate the results. The last section is the conclusion.

2. Model

2.1. TDGL Equations

The TDGL Equation consists of 2 Equations, ie [8]

\[
\frac{\hbar^2}{2m_sD} \left( \frac{\partial}{\partial t} + i \frac{e_s}{\hbar} \phi \right) \Psi = \frac{\hbar^2}{2m_s} \left( \nabla - i \frac{e_s}{\gamma_h} A \right)^2 \Psi + |\alpha_s(T)| \Psi - \beta |\Psi|^2 \Psi
\]

\[
\nabla \times (\nabla \times \mu_s H) = \frac{4\pi \mu_s}{\gamma \Psi} (J_s + J_n)
\]

where \( \hbar \) is Planck’s constant, \( m_s \) is effective mass of cooper pairs, \( D \) is diffusion constant, \( e_s \) is cooper pairs electron, \( t \) is time, \( \phi \) is electric potential scalar, \( \Psi \) is macroscopic wave function as order parameter, \( A \) is potential magnetic vector, \( |\alpha_s(T)| \) and \( \beta \) is the expansion coefficient of Landau, \( \mu_s \) is the permeability of superconducting material \( H \) is external magnetic field, \( J_s \) is super current density, and \( J_n \) is normal current density. The TDGL Equations above can be expressed as normalized form as follows [8].

\[
\left( \frac{\partial}{\partial t} + i \phi \right) \Psi = (\nabla - i A)^2 \Psi + (1 - T)(1 - |\Psi|^2) \Psi
\]

\[
\sigma \left( \frac{\partial A}{\partial t} + \nabla \phi \right) = J_s - \kappa^2 \nabla \times \nabla \times A
\]

where \( T \) is temperature, \( \sigma \) is normal conductivity, and \( \kappa \) is Ginzburg-Landau constant.

2.2. Boundary Conditions

The TDGL Equation is provided with a boundary condition for the order parameters and the magnetic vector potential \( A \). The boundary conditions depend on the system. The boundary conditions for \( A \) for superconductors located in the vacuum and the external magnetic field \( H \) charged to it are

\[
\nabla \times A(\mathbf{r}, t) = \mu_0 H \text{ at } \Gamma
\]

where \( \Gamma \) is the boundary of material, \( \mu_0 \) is permeability of vacuum, and \( \mathbf{r} \) is position. Normalized boundary conditions for \( A \) is [8]

\[
\nabla \times A = H \text{ at } \Gamma
\]

The boundary condition when the material is bordered by an insulator or a vacuum is [8].

\[
(\nabla - i \mathbf{A})_n \Psi = 0 \text{ at } \Gamma
\]
2.3. Implementation of $J_e$

The implementation of $J_e$ is shown in Figure 1. From Figure 1, we can see the condition of the superconducting material when the external current density $J_e$ flows on the material without applying external magnetic field $H_e$. When the external current density $J_e$ flows on the material on $x$-direction, the induction magnetic field from $J_e$ will penetrate on the upper side of the material, namely $H_u$, and also from the bottom side, namely $H_b$.

![Figure 1. External current $J_e$ passed in superconductor without external magnetic field $H_e$](image)

The relation between the induction magnetic field and external current density is shown in Equation 8 [9].

$$H_e = \frac{J_e L_y}{2k^2}$$ (8)

The resultant of the field on the upper side $H_u$ and the bottom side $H_b$ can be written as shown in Equation 9-10 [10].

$$H_u = \left\{ \frac{J_e L_y}{2k^2} \right\} \hat{k}$$ (9)

$$H_b = \left\{ \frac{J_e L_y}{2k^2} \right\} \hat{k}$$ (10)

2.4. Potential Difference

The potential differences in $x = 0$ and $x = Lx$ were calculated using $V = EL_x$, where in the integral form is expressed by Equation 11 [11].

$$V(t) = \int_{0}^{L_x} \left( \frac{1}{L_y} \int_{0}^{L_y} \left( \frac{dA}{dt} \right) dy \right) dx$$ (11)

2.5. System Models

The model of the system of this research is shown in Figure 2. A type-II superconductor (blue color one) sized $50\xi_0 \times 50\xi_0$ with $\kappa = 1.3$ (niobium), $\sigma = 1$ and $T = 0$ was placed in a vacuum space without an external magnetic field. There are material defects at the upper side and the bottom site with the size of $0.3\xi_0 \times 1\xi_0$ (the yellow one), as shown in Figure 2 above. The defect on the material gave effect to the presence of vortex and antivortex. VAVs can only enter the material through the defect and not enter through another side. This is showing that the defect on the material provides a force which causes VAV more easily enter the material at the defect than another side. The external current density $J_e = J_e i$ is passed on the superconductor by imposing an external magnetic field difference between the upper and lower boundaries [1,9,12]. Besides, the potential difference was measured on the right and left side of the material, as shown in Figure 2.
3. Methods

Figure 3 below shows the system of type II superconductor with the size of the chosen computational grid was $N_x \times N_y = 250 \times 250$ where the size of the typical grid cell was $h_x \times h_y = 0.2\xi_0 \times 0.2\xi_0$ and located in the vacuum. The external current density constant $J_e = 0.014$ is applied to this superconductor. The computational grid of the system was obtained by $L_x = N_x \times h_x$. And $L_y = N_y \times h_y$. Such a division produced a uniform computing grid as shown in Figure 3 with the grey grid as the implemented defect.

Now, the variables of $x$ and $y$ are initially continuous i.e. $0 \leq x \leq L_x$ and $0 \leq x \leq L_y$, become discrete as follows $x_i = (i-1)h_x$ for $i = 1, \ldots, N_x + 1$ and $y_j = (j - 1)h_y$ for $j = 1, \ldots, N_y + 1$.

The TDGL Equations (3) and (4) were resolved numerically using the finite difference method with the Forward Time Centered Space (FTCS) scheme. This method has been used by researchers to study the characteristics of the dynamics of the vortex in a mesoscopic type II superconductor. To maintain the invariance of the TDGL Equations under discretization, a link variable was used [7] as follows

$$U_x(x, y, t) = \exp(-i\int_{x_0}^{x} A_x(x', y, t)dx')$$  \hspace{1cm} (12)$$

$$U_y(x, y, t) = \exp(-i\int_{y_0}^{y} A_y(x, y', t)dy')$$  \hspace{1cm} (13)$$

and the discrete forms of Equation (11) and (12) are
\[ U_{x,i,j} = \exp(-i\hbar A_{x,i,j}) \]  
\[ U_{y,i,j} = \exp(-i\hbar A_{y,i,j}) \]  

The discrete form of Equation (3) and (4) are 
\[ \Psi_{i,j}^{n+1} = \Psi_{i,j}^n + \Delta t \left( \frac{\partial \Psi_{i,j}}{\partial t} \right)^n + O(\Delta t^2) \] for \( i = 2, \ldots, N_x \) and \( j = 2, \ldots, N_y \); 
\[ A_{x,i,j}^{n+1} = A_{x,i,j}^n + \Delta \left( \frac{\partial A_{x,i,j}}{\partial t} \right)^n + O(\Delta t^2) \] for \( i = 1, \ldots, N_x \) and \( j = 2, \ldots, N_y \); and 
\[ A_{y,i,j}^{n+1} = A_{y,i,j}^n + \Delta \left( \frac{\partial A_{y,i,j}}{\partial t} \right)^n + O(\Delta t^2) \] for \( i = 2, \ldots, N_x \) and \( j = 1, \ldots, N_y \), where [1]

\[ \frac{\partial \Psi_{i,j}}{\partial t} = U_{x,i,j}^* \Psi_{i,j} + \frac{1}{h^2} \left( \frac{\partial A_{x,i,j}}{\partial x} \right)^n - \frac{\partial A_{y,i,j}}{\partial y} \]  
\[ \frac{\partial A_{x,i,j}}{\partial t} = \frac{\partial \Psi_{i,j}}{\partial x} \left( \frac{\partial A_{x,i,j}}{\partial x} \right)^n - \frac{\partial A_{y,i,j}}{\partial y} \]  
\[ \frac{\partial A_{y,i,j}}{\partial t} = \frac{\partial \Psi_{i,j}}{\partial y} \left( \frac{\partial A_{y,i,j}}{\partial x} \right)^n - \frac{\partial A_{y,i,j}}{\partial y} \]  

The discrete form of Equation (11) is [1]

\[ V = \frac{L_s}{N_x N_y} \sum_{j=3}^{N_y-a} \sum_{i=2}^{N_x-a} \frac{1}{3} \left[ \sqrt{\left( \frac{dA_{x,i,j}}{dt} \right)^2 + \left( \frac{dA_{y,i,j}}{dt} \right)^2 + \left( \frac{dA_{x,i,j+1}}{dt} \right)^2 + \left( \frac{dA_{y,i,j}}{dt} \right)^2} \right] + \left[ \sqrt{\left( \frac{dA_{x,i,j}}{dt} \right)^2 + \left( \frac{dA_{y,i,j}}{dt} \right)^2 + \left( \frac{dA_{x,i,j+1}}{dt} \right)^2 + \left( \frac{dA_{y,i,j}}{dt} \right)^2} \right] \]  

4. Results and Discussion

4.1. Dynamics of VAV

The penetration of the vortex and antivortex on the superconductor is shown in Figure 4, where the vortex penetrated from the upper side and antivortex penetrated from the bottom side of the superconducting material (+y and −y directions) at \( t = 50 \). When the external current density \( J_\text{e} \) is applied to the material, the induction magnetic field will attempt to penetrate into the material due to the potential magnetic vector \( A \). The induction magnetic field is generated by the total of current density \( J = \kappa^2 V \times \nabla \times A \) which is the resultant of \( J_\text{e} = (\nabla \theta - A) \Psi^2 \) and \( J_\text{a} = -\partial_t A \). The superconducting material responded to these conditions by adjusting \( \Psi \) and \( A \) continuously. The adjustment of both quantities followed the Equation (3) and (4) and the boundary conditions in Equation (6) and (7). The external current density \( J_\text{e} \) caused the force \( F \) work on VAV. When \( J_\text{e} \) passed on materials, the Equation of \( \kappa^2 \frac{\partial h_2}{\partial y} = J_\text{e} \) gave

\[ \kappa^2 \frac{\partial h_2}{\partial y} = J_\text{e} \]  

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The VAV moved with velocity \( v = E/B \) where \( B \) is a local magnetic field and \( E \) is a constant electric field throughout the material. When \( t = 750 \), the dynamics of VAV was different from \( t = 150 \). The dynamics of VAVs was not straight anymore. The conditions explain that vortex and antivortex will move toward a lower magnetic field in the materials. The VAV would continue moving closer to each other toward the center of the material, as shown at \( t = 1150 \). Besides, when \( t = 1200 \) to \( t = 1205 \), the annihilation of VAV occurred. The annihilation occurred because the vortex and antivortex have magnetic field rotations in different directions. The annihilation of VAV occurs periodically.

![Image of VAV dynamics](image)

**Figure 4.** Dynamics of the annihilation of VAV

### 4.2. Potential curve

The electric field \( E \) in the superconducting material consists of two parts i.e. electric field from Meissner state \( E_M \) and electric field because of vortex dynamics \( E_V \), and their relation can be written as [13],

\[
E = E_M + E_V
\] (21)

The electric field because of vortex dynamics \( E_V \) was connected with the velocity of vortex \( v \) and written as \( E_V = h \times v \). The value of \( E_V \) would increase instead of increasing of the amount of vortex on the material. The presence of \( E_V \) would make the external current density \( J_e \) released energy, and this energy would be converted in the form of potential difference \( V \) along the material, and it can be written as [13]

\[
V = h \times v \cdot L
\] (22)

where \( L \) is the size of the material.

Figure 5 above shows the potential curve of the dynamics of the annihilation of the vortex and antivortex. There are two types of the pulse from the figure i.e the small pulse and the sharp pulse. The small pulses appeared when the vortex and antivortex penetrated to the superconducting material periodically. The sharp pulse (the high peak pulse) appeared during the vortex and antivortex annihilating each other. The relation of Equation that (22) showed the maximum speed of vortex and antivortex as they annihilated are the cause of the appearance of the sharp pulses [1]. To improve the characteristics of superconducting materials, this research can be developed by looking at the state of critical current density and critical temperatures of the superconducting materials in the future.
5. Conclusion
In this paper, we have explored the influence of the defect on the surface of the material on the dynamics of VAV based on TDGL Equation. The defect on the material gives effect to the presence of vortex and antivortex. VAVs can only enter the material through the defect and not enter through another side. This is showing that the defect on the material provides a force which causes VAV more easily enter the material at the defect than another side. The sharp pulses (the high peak pulse) appeared during the vortex and antivortex annihilating each other. Meanwhile, the small pulses raised the vortex and antivortex when they entered the material.

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