Inflating with the QCD Axion

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We show that the QCD axion can drive inflation via a series of tunneling events. For axion models with a softly broken $Z_N$ symmetry, the axion potential has a series of $N$ local minima and may be modeled by a tilted cosine. Chain inflation results along this tilted cosine: the field tunnels from an initial minimum near the top of the potential through a series of ever lower minima to the bottom. This results in sufficient inflation and reheating. QCD axions, potentially detectable in current searches, may thus simultaneously solve problems in particle physics and provide inflation.

In 1981, Guth [1] proposed an inflationary phase of the early universe to solve the horizon, flatness, and monopole problems of the standard cosmology. During inflation, the Friedmann equation

$$H^2 = 8\pi G \rho/3 + k/a^2$$

is dominated on the right hand side by a (nearly constant) false vacuum energy term $\rho \sim \rho_{vac} \sim \text{constant}$. The scale factor of the Universe expands superluminally, $a \sim v^p$ with $p > 1$. Here $H = \dot{a}/a$ is the Hubble parameter. With sufficient inflation, roughly 60 $e$-folds, the cosmological shortcomings are resolved.

Standard inflationary models require the invention of a new field whose potential drives the superluminal expansion; there is no direct evidence that the associated particle exists. In this paper, we demonstrate that such a new field is unnecessary: it is possible for the QCD axion $a$ to drive inflation. The QCD axion has been proposed [2, 3] as a solution for the strong CP problem in the theory of strong interactions. It is advantageous to use a single particle to solve several problems. The mass scales of the axion are much lower than those of standard inflationary models, and the axion may be found in ongoing experiments, especially axion searches.

While the axion is a priori a Goldstone boson of the spontaneously broken Peccei-Quinn symmetry $U(1)_{PQ}$, QCD instanton effects induce an axion potential with residual $Z_N$ symmetry. Our model includes an additional explicit soft-breaking term, which tilts the instanton induced potential. While the complete form of the axion potential is dependent on non-perturbative effects, it is well modeled as shown in Figure 1 by

$$V(a) = V_0 \left[1 - \cos \frac{Na}{v} - \eta \cos \left(\frac{a}{v} + \gamma\right)\right].$$

The first term models the periodic instanton potential as a cosine with $N$ degenerate vacua, or $N$ bumps. The width of each bump is given by the Peccei-Quinn scale $f_a = v/N \sim (10^9 - 10^{12})$ GeV, and the height of each bump $V_0 = m_a^2 f_a^2 \sim$ QCD scale. The second term in (2) is the tilting effect of the soft-breaking term.

Inflation starts with the axion field located in a minimum at the top of the tilted cosine potential. The universe tunnels to the next minimum in the cosine, then on down through all the minima until it reaches the bottom. The universe inflates a fraction of an $e$-fold while it is stuck in each of these minima. Sufficient inflation results for $N \sim$ few hundred. The general framework of a sequential chain of tunneling fields was considered previously in the Chain Inflation model proposed by two of us [2].

We consider the invisible axion model of Zhitnitskii and Dine, Fischler, and Srednicki (DFSZ) [4, 5]. The axion is identified as the phase of a complex $SU(2) \times U(1)$ singlet scalar $\sigma$ below the PQ symmetry breaking scale $v/\sqrt{2}$, where $\sigma = \frac{1}{\sqrt{2}} (v + \rho) \exp(i \frac{\theta}{2})$. The periodicity of the axion field is $a = a + 2\pi v = a + 2\pi N f_a$, where $f_a$ is the axion decay constant. Defining $\theta = a/f_a$, we see that $\theta$ is
2\pi N\) periodic. Below the QCD scale \(\Lambda_{QCD} \sim 220\text{MeV}\),
QCD instantons produce a potential with \(N\) degenerate minima at \(\theta = 2\pi n\) where \(n = 0, 1, 2, \ldots, N - 1\). The effective action is

\[
\mathcal{L}_{\text{eff}} = \frac{1}{2} \partial_m \phi \partial^m \phi + (m_\phi^2 f_\pi^2) g(a/f_a),
\]

where \(g(x)\) is a periodic function of period \(2\pi\), Taylor expanded as \(g(x) = g(0) - (1/2)x^2 + \cdots\). The axion mass is \(m_a \sim 2N f_\pi^2 m_\pi/v, \) where \(z = m_a/m_d = 0.56\). Then \(m_a f_a \sim m_\pi f_\pi\) where the pion decay constant \(f_\pi = 93\text{MeV}\) and the pion mass \(m_\pi = 135\text{MeV}\). Here \(N\) refers to the unbroken \(Z_N\) subgroup of \(U(1)_{\text{PQ}}\), and corresponds to the number of representations of fermions that carry color charge as well as PQ charge (and thus contribute to the QCD anomaly) \[7\]. Since we need \(N \sim 200\), this introduces additional heavy fermions beyond the usual quarks and leptons. The form of \(g(x)\) depends on non-perturbative effects, and hence is not fully specified. We will take it to be \(g(x) = \cos(x)\). This captures the main features of the periodic instanton potential, and will be sufficient for our purposes.

We will take the \(U_{\text{PQ}}(1)\) symmetry to be softly broken \[8, 9\]. Following \[9\], we add a soft breaking term of the form \(\mathcal{L}_{\text{soft}} = \mu^2 \phi^2 + h.c.\) where \(\mu\) is a complex parameter and \(\sigma\) is given above. Below the PQ scale, this adds a term to the axion potential of the form \(\eta \cos(a/v + \gamma)\) where \(\eta\) and \(\gamma\) are real parameters. The combined potential is then given in Eq. \(8\). Note that the phase shift \(\gamma\) misaligns the QCD and soft breaking minima. Without loss of generality, we may restrict \(\gamma\) to lie in the range \(-\pi/N < \gamma < \pi/N\). Away from the bottom of the potential, the tilt can be treated in the linear regime, so that the total potential is of the form

\[
V_{\text{linear}} = V_0 \left[1 - \cos \left(\frac{Na}{v}\right)\right] - \eta (a/v + \gamma).
\]

The energy difference between minima is roughly \(\epsilon \sim 2\pi \eta/N\). In this linear regime, the requirement that \(\epsilon\) be less than the barrier height becomes \(\epsilon < V_0\). Unless this criterion is satisfied, the barrier becomes irrelevant and the field simply rolls down the hill.

**Sufficient Inflation and Reheating.** A successful inflationary model has two requirements: sufficient inflation and reheating. Here we have a series of a large number \((N)\) of tunneling events as the universe transitions from an initial high vacuum energy down to zero. In order to have sufficient inflation, the universe expands by at least \(60\) e-folds once all the tunneling events have taken place (by the time the field travels all the way down the cosine):

\[
\chi_{\text{tot}} > 60,
\]

where \(\chi_{\text{tot}}\) is the total number of e-folds. In order for the universe to reheat, the number of e-folds attained during one tunneling event must be small (less than 1/3 of an e-fold), as shown below. The failure of old inflation, known as the “graceful exit” problem, is circumvented because the bubbles of vacuum are able to percolate at each step down the potential since the phase transition is fairly rapid. Chain Inflation’s basic mechanism of multiparticle tunneling events \[2\] works both in the context of the stringy landscape or here with the QCD axion.

In the zero-temperature limit, the nucleation rate \(\Gamma\) per unit volume for producing bubbles of true vacuum in the sea of false vacuum through quantum tunneling has the form \[8, 9\]

\[
\Gamma(t) = A e^{-S_E},
\]

where \(S_E\) is the Euclidean action and where \(A\) is a determinantal factor which is generally the energy scale \(\epsilon\) of the phase transition \[22\].

Guth and Weinberg have shown that the probability of a point remaining in a false deSitter vacuum is approximately

\[
p(t) \sim \exp\left(-\frac{4\pi}{3} \beta H t\right),
\]

where the dimensionless quantity \(\beta\) is defined by

\[
\beta \equiv \frac{\Gamma}{H^4}.
\]

Writing Eq. \(7\) as \(p(t) \sim \exp(-t/\tau)\), we estimate the lifetime of the field in the metastable vacuum as roughly \[22\]

\[
\tau = \frac{3}{4\pi H \beta} = \frac{3}{4\pi} \frac{H^3}{\Gamma}.
\]

The number of e-foldings for the tunneling event is

\[
\chi = \int H dt \sim H \tau = \frac{3}{4\pi} \frac{H^4}{\Gamma}.
\]

The authors of \[11\] and \[12\] calculated that a critical value of

\[
\beta \geq \beta_{\text{crit}} = 9/4\pi
\]

is required to achieve percolation and thermalization. In terms of e-foldings, this is

\[
\chi \leq \chi_{\text{crit}} = 1/3.
\]

As long as this is satisfied, the phase transition at each stage takes place quickly enough so that ‘graceful exit’ is achieved. Bubbles of true vacuum nucleate throughout the universe at once, and are able to percolate.

**Tunneling Rate.** In the thin wall limit, the tunneling rate is given by Eq. \(10\). As shown by \[8, 9\], we need to calculate

\[
S_1 = \int \sqrt{2U_+(a)} da,
\]
integrated from one minimum to the next, where the symmetric portion of the potential is

\[ U_+ (\theta) = V_0 (1 - \cos \theta). \]  

(14)

Then

\[ S_1 = \sqrt{2V_0 f_a} \int_0^{2\pi} \sqrt{1 - \cos \theta} d\theta = 8f_a \sqrt{V_0}. \]  

(15)

The Euclidean action is

\[ S_0 = \frac{27\pi^2 S_1^4}{2e^3} = 5 \times 10^3 \frac{V_0^2 f_a^4}{e^3}. \]  

(16)

For the parameters of the DFSZ axion, \( S_0 \gg 1 \) and tunneling is suppressed in the thin wall limit (n.b. the thin wall limit almost never applies to any realistic tunneling event for any potential as tunneling is suppressed \[13\]). To have reasonably fast tunneling with \( \chi < 1/3 \) from one minimum to the next, we must be outside the thin wall limit. There is the additional constraint \( \epsilon < V_0 \) in order for tunneling to take place, as opposed to mere rolling down the potential. Thus, obtaining the right amount of inflation requires \( \epsilon/V_0 \sim 1/2 \).

The Neutron Electric Dipole Moment. We must ensure that the soft-breaking term in Eq. 2 does not destroy the strong CP solution, i.e., that the minimum of the potential in Eq. 2 is not shifted away from zero by more than is allowed by the electric dipole moment (EDM) of the neutron \[14\]

\[ \Delta \bar{\theta} \bigg|_{\text{EDM}} < 6 \times 10^{-10}. \]  

(17)

To find the minima of the potential, we solve \( V'(a) = 0 \), or

\[ V_0 N \sin(Na/v) + \eta \sin(a/v + \gamma) = 0. \]  

(18)

To leading order in small \( \eta \), the minima are located at \( a_n = 2n\pi f_a - \eta \frac{f_a}{V_0} \sin \left( \frac{2\pi n}{N} \right) + \gamma \) for integer \( n \) where the potential is \( V(a_n) = -\eta \cos \left( \frac{2\pi n}{N} + \gamma \right) \). The energy difference between two adjacent minima is

\[ \epsilon = \eta \left[ \cos \left( \frac{2\pi n}{N} + \gamma \right) - \cos \left( \frac{2\pi (n + 1)}{N} + \gamma \right) \right]. \]  

(19)

The difference in field value between minima is

\[ \delta a = 2\pi f_a + \eta \frac{f_a}{V_0} \left[ \sin \left( \frac{2\pi n}{N} + \gamma \right) - \sin \left( \frac{2\pi (n + 1)}{N} + \gamma \right) \right]. \]  

(20)

We have to impose the EDM bounds at the bottom of the potential, at \( n = 0 \), since this is presumably the endpoint of tunneling (corresponding to the current universe). For large \( N \), we find that the shift from \( \bar{\theta} = 0 \) is given by

\[ \Delta \bar{\theta} \bigg|_{\text{EDM}} = \left| \frac{\eta}{V_0 N} \sin \gamma \right| = \frac{\eta}{V_0 N} \gamma \sim \frac{\eta \pi}{2V_0 N^2}. \]  

(21)

In the last equality, we have used that \( |\gamma| < \pi/N \) to estimate that a typical arbitrary value of \( \gamma \sim \pi/(2N) \).

During most of the route down the potential, away from the bottom, the tilt can be approximated as being linear, as in Eq. 14. In the linear regime, \( \epsilon = 2\pi \eta/N \). Taking, e.g., \( n \sim N/4 \), and using Eq. 21, we find

\[ \epsilon_{\text{linear}} \sim 4N V_0 \Delta \bar{\theta} \bigg|_{\text{EDM}}. \]  

(22)

Combining this with the bound on the neutron EDM, we find that \( \epsilon_{\text{linear}} \leq 2 \times 10^{-8} N A^{4}_{QCD} \), or \( \epsilon_{\text{linear}}^{1/4} \leq 5(N/200)^{1/4} \text{MeV} \). To get a sensible reheat temperature \( T_R > 10 \text{MeV} \) after inflation, we need a large number of new heavy fermions.

At the bottom of the potential, \( \epsilon(n = 0) \sim 2\pi^2 \eta/N^2 \), or using Eq. 21, \( \epsilon_{\text{bottom}} \sim 4 \pi V_0 \Delta \bar{\theta} \bigg|_{\text{EDM}} \). Combining this with the bound on the neutron EDM, we find that \( \epsilon_{\text{bottom}}^{1/4} \leq 2 \text{MeV} \).

The two conditions in the two regimes

\[ \epsilon \leq \kappa A^{4}_{QCD} \Delta \bar{\theta} \bigg|_{\text{EDM}}, \]  

(23)

where \( \kappa = 4\pi \) at the bottom of the potential and \( \kappa = 4N \) in the linear regime. Substituting this equation into Eq. 10, we see that the Euclidean action can be written as

\[ S_0 = 5 \times 10^5 \left( \frac{f_a}{m_a} \right)^2 \frac{1}{\kappa^3 (\Delta \bar{\theta} \bigg|_{\text{EDM}})^3}. \]  

(24)

The tunneling rate at the last stage is extremely suppressed for parameters allowed by the constraints on the neutron EDM, \( \Delta \bar{\theta} \bigg|_{\text{EDM}} < 6 \times 10^{-10} \). To obtain a reasonable tunneling rate, we need to get away from the thin wall limit (as discussed previously); i.e., the value of \( \epsilon \) must be closer to \( V_0 \sim (100 \text{MeV})^4 \) and hence must be larger than allowed by Eq. 23. In order for Chain Inflation with the axion to succeed, we must reconsider some assumptions we have made.

With large enough \( N \), the tunneling rate can be perfectly reasonable as long as one stays away from the absolute bottom of the potential [see Eq. 23 with \( \kappa = 4N \) in the linear regime]. Indeed, the reheating of the universe can take place in the linear regime. As the field goes farther down the potential, the vacuum energy gets smaller and smaller, and fewer e-folds result. Hence, radiation that is produced during reheating that takes place near, but not at, the bottom of the potential, is not inflated away by the last few episodes of tunneling to the bottom. We have not yet investigated details of the particles produced during reheating; axions may provide the dark matter.

The only real problem with the model is the last tunneling event, near the bottom of the potential. In Eq. 23, with \( \kappa = 4\pi \) near the bottom, we see that there is no dependence on \( N \) in this regime. The allowed energy difference is simply so small that no tunneling takes
place. The universe would still be situated in this false vacuum now.

Circumventing the Neutron EDM constraint: The constraint on the neutron EDM prevents the universe from tunneling in the last stage to zero energy. We may speculate about a number of ways to resolve this problem. It is possible for the tunneling to stop at an energy of \( \epsilon_{\text{bottom}} < 10^{-3} \) eV and thereby account for the dark energy. Alternatively, the soft breaking term and the QCD axion may conspire to set \( \gamma = 0 \) in, avoiding the EDM constraint altogether. Third, one might consider a time dependent tilt or a different function for the tilt. Alternatively, coupling to other fields may allow the universe to tunnel or roll to the bottom via a different direction in a two dimensional potential in field space. Or if the \( Z_N \) symmetry breaks once the field is near the bottom of the potential, then the field could quickly roll to zero energy. Further afiel, axion models different from the DFSZ axion (see, e.g., the review of [20]) might work better as inflaton candidates. Briefly, there are models with several axions [14, 15, 16] with more desirable properties [23]. Axions abound in string theory. Some may solve the strong CP problem [15, 16]. We have listed several approaches here to circumvent the neutron EDM constraints on tunneling to the bottom.

On the Value of \( N \). The value of \( N \) need not equal the number of fermions that carry color and CP charge. The number of fermions may be far less, depending on the relevant group representations. Defining \( T^{(r)}_{\alpha} \) to be the generators for the representation \( r \), one can write \( \text{Tr}(T^{(r)}_{\alpha}T^{(r)}_{\beta}) = \frac{1}{2}t_r \delta^{\alpha\beta} \). The axion model has an exact \( Z_N \) symmetry with \( N = (2\pi/T_{\Theta})(\sum_{r,i} Q_{r,i} t_r) \) where \( Q_{r,i} \) are the PQ charges of the fermions and \( T_{\Theta} \), the period of \( \Theta \), need not be \( 2\pi \) (see e.g. the review of Sikivie [21]).

Domain Walls. One might worry about the deleterious effect of domain walls which appear when different horizon sized portions of the universe fall into different minima at the QCD scale. As shown by Sikivie [21], the energy difference between vacua leads to pressure that causes the domain walls to move in the direction of eliminating the higher energy vacua in favor of the lower energy ones. The domain walls disappear within a Hubble time. One might worry that the universe is too quickly driven to zero energy, without any inflation, but this does not happen. By the Kibble mechanism, there are at most two or three domains in any horizon volume, with different values of \( \langle a \rangle \). Typically a horizon volume will be driven to a field value half way down the potential, at \( \sim N/2 \), and will subsequently inflate sufficiently. In some part of the universe, a horizon volume will contain only domains with values of \( \langle a \rangle \sim 0 \), near the top of the potential. Those regions that start the highest up the potential will inflate the most. Hence, if one plunges down our observable universe in a random patch of the total universe after inflation ends, then one is likely to find a region that started inflating near the top of the potential and hence inflated sufficiently. Domain walls even have the positive effect of driving a causal patch prior to inflation to become more uniform by shoving away the nonuniformities.

Conclusion. We have investigated using the QCD axion potential to inflate. We use the cosine shape of the axion, with \( N \) minima, due to a residual \( Z_N \) symmetry, together with a tilt produced by a small soft breaking of the Peccei Quinn symmetry. We studied the DFSZ axion. Chain inflation results along this tilted cosine, with a series of tunneling events as the field tunnels its way from a minimum near the top of the potential to ever lower minima. Sufficient inflation as well as reheating result. Tunneling in the last stage is suppressed due to constraints on the neutron Electric Dipole Moment and must be further considered. We have listed a few attempts to work around these constraints.

In this paper, we have restricted discussion to axions which can solve the strong CP problem. Obviously, if we forego any contact with real QCD, then the allowed ranges for parameters becomes much larger. For example, the constraint from the neutron EDM vanishes. Then the ranges of potential width, barrier height, and energy difference between vacua are completely opened up. A tilted cosine may arise due to (non-QCD) “axions” in many other contexts, such as string theory, and would easily provide an inflaton candidate.

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[21] P. Sikivie, UF-TP-83-6 Based on lectures given at 21st Schladming Winter School, Schladming, Austria, Feb 26–Mar 6, 1982.
[22] We note that we do not need to include gravitational effects [11] as they would only be relevant for bubbles comparable to the horizon size, whereas the bubbles considered in this paper are much smaller.
[23] There exists a distribution around this typical value.
[24] This is the minimal requirement to obtain ordinary element abundances from nucleosynthesis. Then baryogenesis must take place later, as considered by [17].
[25] In superstring $E_8 \times E_8$ models, two axions are present.