STEPS TOWARD THE POWER SPECTRUM OF MATTER. III. THE PRIMORDIAL SPECTRUM

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ABSTRACT

We compare the observed power spectrum of matter found in our earlier papers with analytical power spectra. We extrapolate the observed power spectra on small scales to find the linear power spectrum of matter. We consider spatially flat cold and mixed dark matter models with the cosmological constant as well as open models. We fix the Hubble constant and the baryon density in the middle of the allowed range and vary the density parameter and the cosmological constant. We determine the primordial power spectrum of matter using the power spectrum of matter and the transfer functions of analytical models. We take two different spectra suggested by observations: one with a sharp maximum at 120 h$^{-1}$ Mpc and a second one with a broader maximum, as found for regions with rich and medium-rich superfamilies of galaxies, respectively. For both models, the primordial power spectra have a break in amplitude; in the case of the spectrum with a sharp maximum the break is sharp. We conclude that a scale-free primordial power spectrum is excluded if presently available data on the distribution of clusters and galaxies represent the true mass distribution of the universe.

Subject headings: cosmology: observations — galaxies: formation — large-scale structure of universe

1. INTRODUCTION

Here we accept the current inflationary paradigm and assume that the structure evolves from Gaussian initial conditions in a universe dominated by nonbaryonic dark matter (DM). Under these assumptions the current power spectrum of matter is determined by physical processes during the inflationary expansion and the subsequent radiation- and matter-dominated eras. It is believed that the primordial power spectrum, formed during the inflation epoch, is a scale-free Harrison-Zeldovich power law. This scale-free power spectrum is transformed during the radiation-dominated expansion; the transformation depends on cosmological parameters and on the nature of the DM. Within the presently acceptable range of cosmological parameters and possible DM candidates, the transfer function (which describes the transformation of the power spectrum) is a smooth function of scale. For this reason the present power spectrum should be a smooth function of scale.

However, recent evidence (Broadhurst et al. 1990; Landy et al. 1996; Einasto et al. 1997c; Retzlaff et al. 1998; Tadros et al. 1998) indicates that the power spectra of galaxies and clusters of galaxies have a spike or peak on scales around $l = 120$ h$^{-1}$ Mpc or wavenumber $k = 2\pi/l = 0.05$ h Mpc$^{-1}$ (we designate the Hubble constant as 100 h km s$^{-1}$ Mpc$^{-1}$). This scale corresponds to the step size of the supercluster void network (Einasto et al. 1994a, 1997a, hereafter E97a). Atrio-Barandela et al. (1997), Eisenstein et al. (1998), and Meiksin, White, & Peacock (1998) have shown that the peaked power spectrum of galaxies and clusters and the respective angular power spectrum of temperature anisotropy of the cosmic microwave background (CMB) radiation are barely compatible with the standard scale-free primordial power spectrum in a cold dark matter (CDM) dominated universe. An agreement is possible only for extreme values for cosmological parameters (Hubble constant and baryon fraction of the matter), which are almost outside the limits of the range allowed by other data.

Motivated by the difficulty in reconciling the observed power spectra with CDM-type models we try to calculate the primordial power spectrum of matter empirically. We shall use the latest determinations of the power spectra for various populations of galaxies and clusters of galaxies reduced in amplitude to the power spectrum of matter in the local universe (Einasto et al. 1999b, 1999a, hereafter Paper I and Paper II, respectively). For comparison we use theoretical models with CDM, a mixture of cold and hot dark matter (MDM) in a spatially flat universe, as well as open models (OCDM). We calculate the transfer functions for a set of cosmological parameters chosen in the range of astrophysical interest and find the primordial spectra from theoretical transfer functions and empirical power spectra. In the Appendix we calculate for our empirical power spectra the mass function of clusters of galaxies using the Press & Schechter (1974) algorithm and compare them with the empirical cluster mass function.

The reconstruction of the power spectrum of the primordial density field was also considered by Kashinsky (1998) using data on the correlation function, power spectrum, and velocity field. The density field was recovered on a comoving scale interval between 1 and 100 h$^{-1}$ Mpc.

2. COMPARISON WITH THEORETICAL MODELS

2.1. Observed Power Spectra

We shall use the mean galaxy power spectra determined in Paper I, where we derived two mean power spectra based on different observed populations. Clusters of galaxies and several galaxy surveys yield a mean power spectrum with a peak on a scale of 120 h$^{-1}$ Mpc. This spectrum, $P_{MD}(k)$, characterizes the distribution of clusters and galaxies in a large volume that includes rich superclusters of galaxies. For comparison we also use the power spectrum found for medium-density regions of the universe, $P_{MD}(k)$ (subscripts...
HD and MD denote high- and medium-density regions. This spectrum is shallower around the maximum. The power spectra are determined in the range $0.03 \, h \, \text{Mpc}^{-1} \leq k \leq 1 \, h \, \text{Mpc}^{-1}$. On large and intermediate scales ($k \leq 0.2 \, h \, \text{Mpc}^{-1}$) they are found on the basis of three-dimensional galaxy and cluster surveys. Both mean power spectra coincide on small scales, where they are based on the reconstruction of the three-dimensional spectrum from the two-dimensional distribution of Automatic Plate Measuring Facility (APM) galaxies. Spectra were reduced to real space and to the amplitude of the power spectrum of all galaxies, including dwarf galaxies. This procedure assumes that there exists such a mean power spectrum characteristic for all galaxies. Possible errors in this assumption and intermediate steps of the data reduction shall be discussed below in § 3.1.

Both mean power spectra were reduced in amplitude to the power spectra of matter using the bias factor calculated from the amount of matter in voids; the present epoch was fixed using the rms density fluctuations in the $8 \, h^{-1} \, \text{Mpc}$ sphere, $\sigma_8$ (Paper II). The biasing correction is based on the assumption that the structure evolution in the universe on scales of interest is due to gravitational forces alone. Possible errors related to biasing correction shall also be discussed below.

Observed power spectra are shown in Figure 1. They differ only on scales around the maximum which is the most interesting part of the spectrum for the present analysis. On small scales, $k \geq 0.2 \, h \, \text{Mpc}^{-1}$, the power spectra are influenced by nonlinear effects.

2.2. Extrapolation of Power Spectra on Large and Small Scales

To compare observed power spectra with theory, they are to be reduced to the linear case on small scales. To find the linear power spectrum, Peacock (1997) used a method based on numerical simulations. We shall use a different method—the comparison with theoretical power spectra. Theoretical spectra shall be discussed in more detail in the following sections. Spectra were calculated for CDM and MDM models and for different values of the density parameter, $\Omega_0$; spectra were COBE normalized. The comparison shows that theoretical spectra, calculated for high values of the density parameter, have amplitudes much higher on small scales than the amplitude of the observed power spectra. The shape of theoretical power spectra based on the CDM model is also rather different from the shape of observed power spectra on scales between the maximum of the spectrum and the beginning of the nonlinear part of the spectrum at $k \approx 0.2 \, h \, \text{Mpc}^{-1}$. The best fit, both of the amplitude (near the beginning of the nonlinear scale) and of the shape, is obtained with a tilted MDM model with a power index of $n = 1.1$ on large scales and a density parameter $\Omega_0 = 0.4$ (see Fig. 1). For CDM models the best fit is obtained (also of the amplitude and the shape) for a model with density parameter $\Omega_0 \approx 0.2$, but now the fit is good in a much narrower scale interval; see Figure 2 below. For comparison we show in Figure 1 an alternative linear extrapolation with a CDM model and a density parameter, $\Omega_0 = 0.4$. In the following discussion we shall use only the extrapolation with the MDM model. Note that for our choice of $h$ and $\Omega_c$ (for the MDM model), these parameters correspond to a neutrino rest mass (or the sum of the masses of two or three stable neutrino species) of $\approx 3.4 \, \text{eV}$.

Now we consider the extrapolation to large scales. The comparison of observed power spectra with theoretical ones shows that all COBE normalized models with low-density parameter $\Omega_0$ have amplitudes that are higher on large scales than those of the observed power spectrum (see Fig. 2 below). For this reason we have used the standard CDM model with $\Omega_0 = 1$ to extrapolate the observed spectra to large scales. This model has the lowest amplitude on large scales and yields the best and smoothest extrapolation of observed spectra on large scales. On these scales there are no essential differences between theoretical spectra of CDM and MDM models.

We consider these extrapolations on small and large scales as estimates of the true linear power spectrum of matter. In the following we shall use the term "empirical power spectrum" to denote this linear power spectrum. We recall that it is reduced to real space and its amplitude to the amplitude of the matter density fluctuations. As we have two observed mean power spectra of galaxies, characteristic for different populations, we also have two empirical power spectra. To check the power spectra for consistency with other independent data we have calculated the mass function of clusters of galaxies for these spectra and compared results with observations (see Appendix). Results show satisfactory agreement.

2.3. Theoretical Models

Previous studies (Atrio-Barandela et al. 1997; Eisenstein et al. 1998) have shown that it is difficult to make models agree with optical and CMB observations using conventional cosmological parameters and a scale-free primordial power spectrum. Here we shall use a different approach. We accept cosmological parameters in the middle of the allowed region and try to find possible restrictions to the primordial (initial) power spectrum.

To calculate the analytical power spectrum we use a
Fig. 2.—Empirical linear power spectra of matter compared with theoretical and primordial power spectra. Left-hand panels: Present power spectra. Right-hand panels: Primordial spectra; the upper panels show the CDM models, the middle panels show the MDM models, and the lower panels show the OCDM models. Solid bold lines show the empirical linear matter power spectrum found for regions including rich superclusters, $P_{\text{HD}}(k)$; dashed bold lines show the empirical linear power spectrum of matter $P_{\text{MD}}(k)$ for medium dense regions in the universe. Model spectra with $\Omega_m = 0.9, \ldots, 0.2$ are replotted with thin lines; for clarity the models with $\Omega_m = 1.0$ and $\Omega_m = 0.5$ are drawn with dashed lines. In the case of MDM the model of lowest density parameter is that with $\Omega_m = 0.25$. Only spectra with spectral index $n = 1$ on large scales are plotted. Primordial power spectra are shown for the peaked matter power spectrum, $P_{\text{init}}(k)$; they are divided by the scale-free spectrum, $P(k) \sim k$. 
Hubble parameter \( h = 0.6 \). This value is a compromise between the value \( h = 0.55 \pm 0.10 \) suggested by Sandage & Tammann (1997) and \( h = 0.75 \pm 0.06 \) favored by Freedman (1997). We adopt a density of matter in baryons of \( \Omega_0 = 0.04 \) (in units of the critical density). This parameter is in agreement with recent nucleosynthesis results (Tammann 1997) and recent analyses of all available data by Martel, Shapiro, & Weinberg (1998) and Steigman 1998; Turner 1999; see also Ostriker & Steinhardt 1995).

We consider models with various density parameters \( \Omega_0 = \Omega_b + \Omega_{\text{DM}} \), as the density is presently the parameter which is determined with less accuracy. Local methods of determination of the mass-to-light ratio in clusters and groups of galaxies yield \( \Omega_b \approx 0.2 \) (Bahcall 1997). Methods sensitive to the global value of the density parameter yield converging values. Dekel, Burstein, & White (1997), using the POTENT method, obtain a value close to unity; another treatment of the peculiar velocity field yields a lower value (Freudling et al. 1998). The distant supernova project suggests that the universe is speeding up, i.e., that the vacuum energy contributes about 60% of the critical density and that the matter density is about 40% of it, close to results obtained with local methods (Perlmutter et al. 1998; Riess et al. 1998). Indirect methods based on the abundance evolution favor a value of \( \Omega_0 = 0.2 - 0.4 \) (White, Efstathiou, & Frenk 1993; Bahcall, Fan, & Cen 1997; Fan, Bahcall, & Cen 1997; Bahcall & Fan 1998). Further evidence for a low-density universe comes from the analysis of the Ly\( \alpha \) forest by Weinberg et al. (1998), who found for a flat universe \( \Omega_0 = 0.34^{+0.13}_{-0.09} \) (1 \( \sigma \) errors). Recent analyses of all available data by Martel, Shapiro, & Weinberg (1998) and Turner (1999) favor \( h = 0.60 - 0.65 \), \( \Omega_0 \approx 0.4 \), and \( \Omega_\Lambda \approx 0.6 \).

To fix the amplitude of the analytical power spectrum on large scales we use the 4 yr COBE normalization (Bunn & White 1997). We cannot use simultaneously the \( \sigma_8 \) normalization on small scales as we do not know which model best fits both normalizations. It is just our goal to try to find a model which satisfies both normalizations.

We use three sets of models: spatially flat models with cold and mixed dark matter and open models with CDM.

In the first two models we use the cosmological constant. Its necessity is strongly favored by recent data on cluster abundance and supernova explosions at high redshifts (Bahcall et al. 1997; Perlmutter et al. 1998; Riess et al. 1998). We vary the density parameter between \( \Omega_0 = 0.2 \) and \( \Omega_0 = 1 \), and we choose the CDM content to obtain a spatially flat model, \( \Omega_0 + \Omega_{\text{DM}} + \Omega_\Lambda = 1 \). Models are calculated for a number of spectral indices on large scales, \( n = 1.0, 1.1, \ldots, 1.4 \).

We use a similar set of parameters for MDM models with the only difference that a hot dark matter component with density parameter \( \Omega_0 = 0.1 \) was added. The density parameter of the CDM was decreased to get a spatially flat model. We use a small fraction of the hot dark matter for two reasons. First, most direct and indirect methods suggest that \( \Omega_0 \) should be less than unity. \( \Omega_0 \) should be chosen in agreement with the relation \( \Omega_0 / \Omega_\Lambda \sim 0.2 \) required to get the correct value of \( \sigma_8 \) (see, e.g., Pogosyan & Starobinsky 1993, 1995 for the case of a tilted initial spectrum with \( n > 1 \)). Second, in order to build up the fine filamentary structure of faint galaxies observed in supervoids (Lindner et al. 1995), the fraction of matter in the CDM must be significantly larger than that of the hot component (Frisch et al. 1995).

In the open models (OCDM) we used values of \( \Omega_0 = 1.0, 0.9, \ldots, 0.3 \) for the matter density. The baryon density was fixed as in all other models, and the density of the CDM was chosen appropriately.

Theoretical power spectra for our models are plotted in the left-hand panels of Figure 2 together with the empirical linear power spectra of matter. Models were calculated with the CMBFAST package of Seljak & Zaldarriaga (1996). We shall discuss the linear power spectra and the comparison of model spectra with observations in subsequent sections below.

### 2.4. Primordial Power Spectra

The CMBFAST package yields the transfer function for each of our models. We have used this function to calculate primordial power spectra,

\[
P_{\text{ind}}(k) = P(k)/T^2(k),
\]

where \( P(k) \) is the power spectrum of matter and \( T(k) \) is the transfer function.
where $T(k)$ is the transfer function. In the right-hand panels of Figure 2 we plot the ratio of the primordial power spectrum to the scale-free primordial power spectrum, $P_{\text{inf}}(k)/P_{\text{D}}(k)$, where $P_{\text{D}}(k) \approx k$. We plot here only the results found for the peaked empirical power spectrum $P_{\text{HD}}(k)$. A comparison for the flatter power spectrum $P_{\text{MD}}(k)$ (which represents medium-rich regions) is given in Figure 3. In both cases we have used the empirical power spectra extrapolated on small and large scales as described above. The discussion of primordial power spectra follows in § 3.2.

3. DISCUSSION

3.1. Power Spectra

We compare now empirical power spectra with theoretical ones on large, intermediate, and small scales. On large scales the amplitude of $\text{COBE}$ normalized theoretical power spectra depends on the density parameter. Low-density CDM and MDM models dominated by the cosmological constant have amplitudes higher than models of critical density. Figure 2 demonstrates that on large scales empirical power spectra have amplitudes that are compatible with amplitudes of $\text{COBE}$ normalized theoretical power spectra only for high-density CDM and MDM models with $\Omega_0 \approx 1$. For this reason a smooth extrapolation of empirical power spectra on large scales was possible only for high-density models. The highest amplitude on large scales (for extrapolation of observed power spectra), which is still in satisfactory agreement with the $\text{COBE}$ normalization of theoretical spectra and with the upper limit of the error corridor of empirical spectra, is the CDM or MDM spectrum with $\Omega_0 = 0.7$ (the CDM and MDM model spectra are similar in this range of scales).

On intermediate and small scales, $k \geq 0.05 h \text{Mpc}^{-1}$, the amplitude of theoretical power spectra of CDM and MDM models varies considerably with the density parameter. Only theoretical power spectra with a low-density parameter have amplitudes similar to the amplitude of the empirical power spectra. For the CDM model the best agreement is achieved for a model with $\Omega_0 \approx 0.2$, and for an MDM model for $\Omega_0 \approx 0.4$ (see Fig. 2). On these scales CDM and MDM models are in agreement with other evidence, which suggests that high-density models with $\Omega_0 \approx 1$ are excluded by a large margin (Ostriker & Steinhardt 1995; Bahcall, Fan, & Cen 1997; Fan, Bahcall, & Cen 1997; Borgani et al. 1997; Bahcall & Fan 1998; Turner 1999; Weinberg et al. 1998). Low-density theoretical power spectra have, however, maxima at $k \leq 0.01 h \text{Mpc}^{-1}$ with an amplitude of the spectrum which is much higher than that of the observed peak at $k = 0.05 h \text{Mpc}^{-1}$. Available data do not support the presence of a rising spectrum on these scales: spectra found from cluster and galaxy data decrease in amplitude toward large scales.

Open models have a smaller increase of the amplitude on large scales for a wide range of the density parameter and are thus in a better agreement with a reasonable extrapolation of the observed power spectrum on large scales. This agreement is lost around the maximum of the empirical power spectrum, and both versions of empirical power spectra deviate here from OCDM models by a large margin.

This comparison shows that it is impossible to satisfy the shape of the empirical power spectrum with models with a fixed density parameter simultaneously on large and small scales. Models which fit empirical spectra on large scales are incompatible with empirical spectra on small scales and vice versa. This is the main conclusion obtained from the comparison of cosmological models with the data.

3.2. Primordial Power Spectra

Now we discuss primordial spectra derived from the empirical power spectrum, $P_{\text{inf}}(k)$. On large scales there is little difference between CDM and MDM models. The main feature of the primordial power spectra is the presence of a spike at the same scale as that of the maximum of the empirical power spectrum. On scales shorter than that of the spike, the primordial spectrum can be well approximated by a power law (a straight line in a log-log plot). The slope of this line varies with the cosmological density parameter accepted for theoretical models. For models with high cosmological density, $\Omega_0 \geq 0.5$, the slope is negative, $n < 1$; for models with low density, $\Omega_0 < 0.5$, it is positive, $n > 1$. Such tilted primordial power spectra (i.e., spectra with $n \neq 1$) have been used to increase the accuracy of the model. This approximation breaks down if we consider the whole scale interval. On large scales the primordial power spectrum can also be approximated by tilted models. The power index of the approximation is, however, completely different from the index suitable on small scales, as seen in Figure 2.

Additionally, there is a considerable difference in the amplitude of the primordial power spectrum on small and large scales. For most values of the cosmological density parameter the amplitude on small scales is lower than on large scales (compared with the scale-free primordial spectrum). For low values of $\Omega_0$ the effect has the opposite sign: the amplitude of the primordial spectrum on small scales is higher than on large scales. Such an effect has been noticed already by Lesgourgues, Polarski, & Starobinsky (1998). The amplitude of the break depends on the model used, and in MDM models it is larger than in CDM models. The primordial power spectrum for open model has similar features, only its amplitude on large scales rapidly increases if we consider more open models. This is due to the fact that the amplitude of theoretical power spectra of these models on scales $k \geq 0.05 h \text{Mpc}^{-1}$ is much lower than the amplitude of the empirical power spectrum.

Primordial power spectra, which are found from the comparison of the flat empirical power spectrum, $P_{\text{MD}}(k)$, with theoretical spectra, are plotted in Figure 3. On very large and small scales both empirical power spectra are identical, and consequently primordial power spectra are identical too. Differences exist in the medium scale range. Here primordial power spectra have a smoother transition from long to small scales with no sharp spike at wavenumber $k = 0.05 h \text{Mpc}^{-1}$. However, a change of the spectral index and amplitude around this wavenumber (a "break") is very well seen. This property of the primordial power spectrum is similar for spectra derived on the basis of CDM, MDM, and OCDM models. An independent analysis of the initial power spectrum was made by Adams, Ross, & Sarkar (1997) using the APM galaxy power spectrum [which is identical to our $P_{\text{MD}}(k)$]. The presentation of the initial power spectrum (their Fig. 1) is slightly different from ours; the main features of the spectrum are, however, well seen, with similar conclusions.

Previous studies have shown that it is practically impossible to bring the classical models into agreement with new data on the power spectrum by varying cosmological
parameters. Our analysis confirms these results. The reason for the discrepancy lies in the shape of the empirical power spectrum: it is much narrower than theoretical spectra. A change of the bias parameter or a normalization on large scales does not change these conclusions: they would only shift the features present in Figures 2 and 3 up or down, without changing considerably the shape of the primordial spectrum.

3.3. Models with Low-Density Parameter

The previous analysis has shown that on scales \( k \geq 0.05 \, h \, \text{Mpc}^{-1} \) the empirical power spectrum can be well approximated by an MDM model with density parameter \( \Omega_0 \approx 0.4 \). A slightly less accurate approximation in a narrower scale interval is provided by a CDM model with \( \Omega_0 \approx 0.2 \). An open model with density parameter \( \Omega_0 \approx 0.4 \) gives an approximation which fits the empirical power spectrum \( P_{MD} \) both on large and small scales (but not on intermediate scales). A spike or break of the primordial spectrum at \( k = 0.05 \, h \, \text{Mpc}^{-1} \) could be avoided using one of these models, if we allow for the higher amplitudes on large scales of the empirical power spectrum (for CDM and MDM models) or a shallower turnover on medium scales (for the OCDM model). Examples of such spectra for MDM and OCDM models are shown in Figure 4. It is reasonable to investigate the possibilities of such models.

There are two major reasons to reject these model power spectra. First, the directly observed power spectrum does not have a rising section above the maximum at \( k = k_0 = 0.05 \, h \, \text{Mpc}^{-1} \), needed to fit low-density CDM or MDM models, or a very shallow maximum needed for the low-density OCDM model. Second, as shown by numerical experiments discussed in Paper I, models with rising or shallow power spectra in this scale interval have a distribution of superclusters which is very different from the distribution of real superclusters. In a model with a broad power spectrum, such as all scale-free CDM, MDM, and OCDM models, the distribution of rich superclusters is close to a random one. Real rich superclusters form a quasi-

![Figure 4](https://example.com/figure4.png)

**Fig. 4.—** Power spectra (left-hand panel) and correlation functions (right-hand panel) of MDM and OCDM models with density parameter \( \Omega_0 = 0.4 \), compared with the linear empirical power spectrum of matter and correlation function of clusters of galaxies in rich superclusters. The power spectrum of the MDM model is calculated with spectral index \( n = 1.1 \). Cluster correlation functions are calculated via a Fourier transform from power spectra of matter and are enhanced in amplitude by a biasing factor 7.7 which corresponds to the mean difference between respective power spectra. The observed cluster correlation function is the one for Abell-ACO clusters in very rich superclusters as derived by E97b.
function with a probability of about 1%, but a simultaneous occurrence of all parameters has a much smaller probability (of order $10^{-6}$). This test shows that there is a statistically highly significant difference between samples which generate broad and peaked power spectra. A similar conclusion has been obtained by Luo & Vishniac (1993) by the comparison of the one-dimensional power spectrum found for the Broadhurst et al. (1990) pencil beam survey with the theoretical spectrum calculated on the basis of the standard theory. Significant differences between cluster power spectra and spectra based on various variants of CDM models have been noticed by Retzlaff et al. (1998) and Tadros et al. (1998); similar differences have been found also in the case of the power spectrum based on the two-dimensional distribution of APM galaxies by Peacock (1997) and Gaztañaga & Baugh (1998). The reconstruction of the primordial density field by Kashlinsky (1998) also prefers a low-density parameter and shows the difficulty of fitting data with CDM models and constant bias parameter.

3.4. Theoretical Implications

A consequence of the broken scale invariant (BSI) primordial power spectrum is that one cannot draw conclusions based on the exact scale-free primordial spectrum, such as the use of the zero crossing of the correlation function, to determine the density parameter through the parameter $\Gamma = \Omega_0 h$. Another conclusion is that one should not expect the slopes $n_s$ and $n_T$ of power spectra of primordial matter and gravitational waves to be independent on $k$, especially, around the critical point $k = 0.05 h$ $\text{Mpc}^{-1}$. Here $n_s$ and $n_T$ correspond to matter (scalar perturbations) and gravitational waves (tensor perturbations), respectively. In the latter region, differential relations between these quantities following the slow-roll approximation during inflation may become invalid.

Also, we have to understand the reason why the primordial power spectrum deviates from the classical Harrison-Zeldovich spectrum. A very general way of obtaining such a behavior requires some kind of phase transition which occurred during inflation, about 60 e-folds before the end of the inflation era. There is one specific model of this type which has an exact solution. It was suggested by Starobinsky (1992) and leads to a definite BSI spectrum. It was recently confronted with observational data by Lesgourgues et al. (1998). However, there exist a number of other possibilities (see recent discussions by Adams et al. 1997 and Starobinsky 1998) which should be explored too.

4. Conclusions

The main goal of this series of papers was to determine the mean empirical power spectrum of galaxies, to reduce it to the matter, and to compare with theoretical models to see whether it is possible to fit an empirical spectrum with models. This procedure rests on several assumptions and conclusions based on these assumptions. The main assumptions are the following:

1. There exists a power spectrum which represents all galaxies in a fair sample of the universe (including faint dwarf galaxies).
2. The dynamical evolution of the universe on scales of interest is determined by gravity only.
3. Density perturbations grow from small random fluctuations generated in the early stage of the evolution.

4. The dynamics of the universe is dominated by CDM with some possible mixture of hot dark matter.
5. Galaxy samples of various environments, morphologies, and luminosities can be approximated by particles in numerical simulations chosen in certain threshold density intervals.

The first assumption is based on observations of power spectra of galaxy samples with different absolute magnitude limits: spectra are identical in shape and amplitude if the sample contains sufficiently faint galaxies (Gramann & Einasto 1992). The scatter of observed data points of power spectra of faint galaxy samples is about 10% which can be attributed to the cosmic scatter. Thus, we can expect that the assumption itself does not introduce any systematic error to the analysis. The last assumption is based on our experience in comparing real and simulated galaxies, collected in the last 10 yr; its justification was analyzed in detail in Paper II of this series.

The second, third, and fourth assumptions form the main paradigms of modern cosmology. The second and third assumptions are critical to the whole chain of reduction steps of observed power spectra. Under these assumptions, because of gravitational instability, the structure evolution in high- and low-density regions is different: in low-density regions matter remains in the primordial form, whereas in high-density regions matter collapses and forms galaxies and systems of galaxies. A clump of matter collapses if its density is high enough, thus the gravitational character of the evolution leads to the conclusion that galaxy formation is a threshold phenomenon—there exists a critical threshold density, needed for a clump of matter to collapse. Here the actual density of the primordial matter is important. Groups and clusters of galaxies have a characteristic scale of the order of 1 $h^{-1}$ Mpc, thus calculated densities must not be smoothed over scales exceeding this value. Various galaxy populations can be simulated using different threshold densities (the fifth assumption). A threshold density, equal to the mean density of matter, divides the population of all galaxies from that of matter in voids; higher threshold densities correspond to luminous galaxies and to galaxies in clusters.

The division of matter into unclustered and clustered populations affects the respective power spectra in a simple way, so that power spectra of galaxies, systems of galaxies, and matter are similar in shape. We can reduce spectra of different populations to the power spectrum of all galaxies by a shift in the amplitude only. Simulations show that relative errors of this reduction procedure are of the order of 1% for galaxies and about 5% for clusters.

The comparison of power spectra of simulated galaxy and cluster samples in real and redshift space has shown that the influence of peculiar velocities in groups and clusters is serious on small scales only. To avoid complications with the reduction of galaxy power spectra to real space, we have accepted on small scales ($k \geq 0.1 h$ Mpc$^{-1}$) the power spectrum of APM galaxies as the power spectrum of all galaxies in real space. This power spectrum is deduced from a two-dimensional distribution of galaxies and has no redshift distortions. Here we assume that the APM sample is a fair sample of all galaxies in the universe on these scales. The main error of the amplitude of the power spectrum of all galaxies is related to this assumption. The amplitude
may have a relative error of the order of 15%, which is the largest possible error of the power spectrum of all galaxies.

On large scales the mean power spectrum is determined essentially by clusters of galaxies. Numerical simulations confirm that on these scales the redshift correction (due to the contraction of superclusters) affects only the amplitude of power spectra; possible systematic errors involved are negligible in comparison with sampling errors.

The final step in deriving the empirical power spectrum of matter is the reduction of the mean power spectrum of all galaxies to that of matter (the conventional biasing correction). Here we again use our basic assumptions that the structure evolution is due to gravity alone and that initial density fluctuations have a Gaussian distribution. Additionally, we use our last assumption that galaxy populations can be approximated by samples of simulated particles chosen in various threshold density intervals. As discussed above, under these assumptions the matter power spectrum can be found by shifting the galaxy spectrum in amplitude. The shift is determined by the fraction of matter in the clustered population. There are two sources of error in this procedure: the uncertainty of the threshold density level, \( \theta_0 \), which divides the low-density population of primordial particles in voids from the clustered population of particles associated with galaxies; and errors due to uncertainty of cosmological parameters which define the speed of void evacuation and the fraction of matter in high-density regions (see Paper II for details). The relative error of the threshold density level was estimated by Einasto et al. (1994b) to be 10%, which leads to a 3% error of the fraction of matter in the clustered population and respective biasing parameter. The second error can be estimated from the data given in Paper II (see Table 2 and Fig. 4); it is 7%. Additional sources of error are the fuzziness of the threshold density, \( \theta_0 \), and the assumption that the distribution of luminous galaxies in superclusters, clusters, and groups is similar to the distribution of all matter. The analysis done in Paper II shows that these errors do not influence the shape of the power spectrum in the scale range of interest (corresponding errors are of the order of 1%); the errors in the amplitude are also small, less than or equal to 5%. The total rms relative error of the biasing factor is 10% (all errors are 1 \( \sigma \) errors).

To summarize the overview of the data reduction procedure we can say that the gravitational character of the structure evolution implies that intermediate steps of data reduction introduce no noticeable error to the shape of the power spectrum; possible errors are in the amplitude of the power spectrum only. The total rms error of the amplitude is essentially determined by two errors: the error of using the amplitude of the APM galaxy power spectrum as the amplitude of the power spectrum of all galaxies in a fair sample of the universe; and the error due to the uncertainty of model parameters used in the reduction of the galaxy power spectrum to matter.

Possible errors of the shape of the power spectrum are intrinsic, because of differences of power spectra of different populations. To quantify possible differences in the shape of the power spectrum of matter we have formed two mean power spectra, characteristic for samples which cover regions of the universe including high-density superclusters and medium-density superclusters only. Differences between power spectra of these samples may partly be due to unknown errors of data reduction used in various samples or to the geometry of samples (thin slices in case of the LCRS).

Empirical mean power spectra of matter are extrapolated on small scales (to get linear power spectra) and on large scales (to have the spectrum defined on all scales for comparison with theory). Power spectra found for high- and medium-density regions are identical on small scales \((k \geq 0.1 \ h \ Mpc^{-1})\) and on large scales \((k < 0.01 \ h \ Mpc^{-1})\). On medium scales they are different. The power spectrum found for regions which include high-density systems with rich superclusters has a peak at \(k = 0.05 \ h \ Mpc^{-1}\) and an almost constant power index \(n = -1.9\) for scales in the range \(0.05 \ h \ Mpc^{-1} < k < 0.2 \ h \ Mpc^{-1}\). The power spectrum of medium-density regions has a shallower shape around the maximum; the amplitude near the maximum is lower by a factor of 2.

Empirical power spectra can be well approximated by theoretical model spectra based on various DM models in a limited scale range. The best approximation for the empirical power spectrum for high-density regions on medium and small scales is provided by a spatially flat MDM model with cosmological density parameter \(\Omega = 0.4\), baryonic density \(\Omega_b = 0.04\), CDM density \(\Omega_{\cdm} = 0.26\), hot dark matter density \(\Omega_{\text{nu}} = 0.1\), and cosmological constant term \(\Omega_k = 0.6\); the Hubble parameter was fixed at \(h = 0.6\). This approximation breaks down on large scales: the model power spectrum continues to rise on scales \(k < 0.05 \ h \ Mpc^{-1}\), whereas the amplitude of the empirical power spectrum decreases. The impossibility of representing empirical power spectra with one theoretical model is one of the main results of this series of papers.

If we make use of our fourth assumption that the dynamical evolution of the universe is dominated by some sort of DM, then we can use our empirical power spectra to calculate the primordial power spectrum. These calculations show that for both variants of the empirical power spectra the primordial power spectrum has a break, i.e., it can be approximated on large and small scales by two power laws with different power indices. This is due to the fact that it is impossible to approximate the empirical power spectrum with one single model. The strength of the break depends on the cosmological parameters used for theoretical models, and the shape of the break is different for empirical power spectra found for high- and medium-density regions.

Our conclusions are based on the tacit assumption that the galaxy and cluster samples used represent the true matter distribution in the universe. To check this assumption new deeper redshift data are needed. The best data set to make this crucial check is the redshift survey of giant elliptical galaxies planned in the Sloan Digital Sky Survey (SDSS). Giant elliptical galaxies are concentrated in clusters and superclusters, and their distribution on large scales provides the best test for the shape of the power spectrum of matter on scales around the maximum. Presently we can say that the possibility of a BSI primordial power spectrum deserves serious attention.

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\footnote{See http://xxx.lanl.gov/abs/astro-ph/9810130, astro-ph/9809085, astro-ph/9809179, and astro-ph/9805314.}
APPENDIX A

CLUSTER MASS FUNCTION

We use the empirical power spectra of mass to calculate the cluster mass function. The latter can be compared with the observed cluster mass function which allows us to check both sets of data for consistency.

We calculated the cluster mass function using the algorithm, developed by Gramann & Suhhonenko (1999) on the basis of the Press-Schechter (1974) theory. This function provides an important constraint for the power spectrum (White et al. 1993; Cen 1998). The cluster mass function was found for both empirical power spectra, $P_{\text{HD}}(k)$ and $P_{\text{MD}}(k)$, reduced to the linear regime. Mass functions were derived for three values of the density parameter, $\Omega_0 = 1.0, 0.6, 0.4$. Results are shown in Figure 5, together with the observed cluster mass function according to Bahcall & Cen (1993). Our calculations show that both variants of the empirical power spectra yield a cluster mass function which has a shape close to the shape of the observed cluster mass function. The differences between mass functions for spectra $P_{\text{HD}}(k)$ and $P_{\text{MD}}(k)$ are small, in other words, the mass function is not sensitive to such differences of power spectra. This is not surprising since clusters of galaxies are formed by density perturbations on $1-10 \, h^{-1} \, \text{Mpc}$ scales where both empirical power spectra are identical. On the other hand, the amplitude of the mass function is very sensitive to the density parameter of the universe used for calculations. The amplitude is evidently too high if one assumes a high-density parameter of the universe. The best agreement is obtained for the case of $\Omega_0 = 0.6$. A value of $\Omega_0 = 0.4$, as favored by cluster abundance evolution constraints (Bahcall & Cen 1993; Bahcall et al. 1997), yields a lower amplitude of the calculated cluster mass function.

The focus of this series of papers is on the empirical power spectrum and on its consequences for the primordial power spectrum. We do not consider the calculation of the cluster mass function for empirical power spectra as a density determi-
nation. The only conclusion we can draw from this comparison is that our power spectra yield cluster mass functions in good agreement with the observed cluster mass function. A relatively high value of the density parameter favored by this test can be explained if we assume that the biasing correction found from the fraction of matter in high-density regions is too large (see Paper II).

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