CEPHEID PULSATION MODELS AT VARYING METALLICITY AND ΔY/ΔZ

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ABSTRACT

In this paper we present an extended set of nonlinear convective pulsation models at varying metallicity and ΔY/ΔZ ratio. The predicted instability strip and bolometric light curves are discussed by comparing the new models with our previous ones. In particular, the dependence on both metal and helium abundances is investigated. By transforming the bolometric light curves into the observational bands, we are able to derive both period-color-luminosity and Wesenheit relations for each selected chemical composition. Synthetic period-luminosity relations are obtained by populating the instability strip according to specific assumptions on the number of pulsators and the mass distribution. These theoretical results are compared with recent accurate data by Sandage et al. and Kervella et al. in order to test the predictive capabilities of the models. We confirm our previous results that the theoretical metallicity correction to the Key Project Cepheid distance scale depends on both the period range and ΔY/ΔZ ratio, becoming important for periods longer than 20 days and ΔY/ΔZ > 1.5.

Subject headings: Cepheids — stars: oscillations

Online material: machine-readable tables

1. INTRODUCTION

Classical Cepheids are the most reliable primary distance indicators for Local Group and (from the space) external galaxies, thanks to their characteristic period-luminosity-color (PLC) and period-luminosity (PL) relations. Moreover, through the calibration of secondary distance indicators, they allow us to reach cosmological distances (of the order of 100 Mpc), thus providing fundamental constraints on the Hubble constant (see, e.g., Freedman et al. 2001, hereafter F01; Saha et al. 2001). The problem of the dependence of the Cepheid PL relation on chemical composition has been widely debated in the recent literature but with quite different results, depending on the adopted method and the authors (see, e.g., Kennicutt et al. 1998; Fiorentino et al. 2002, hereafter F02; Storm et al. 2004; Groenewegen et al. 2004; Sakai et al. 2004; Romaniello et al. 2005). In the last few years we have studied the Cepheid pulsation properties through the computation of nonlinear, nonlocal, and time-dependent convective pulsation models, which allow us to predict all the relevant pulsational observables. In particular, the nonlinearity and the inclusion of a detailed treatment of the coupling between pulsation and convection allow these models to predict not only the periods and the blue boundary of the instability strip, but also the pulsation amplitudes, the detailed light and radial velocity curve morphology, and the complete topology of the strip, including the red edge, which is caused by the pulsation quenching due to convection (see Bono et al. 1999b, 2000c and references therein for details). On this basis, various sets of Cepheid models have been computed with varying chemical composition (0.004 < Z < 0.04, 0.25 < Y < 0.33) and stellar mass from 2.8 to 11 M☉ (Bono et al. 1999a, 2000b, 2002b; F02). For each chemical composition and mass, an evolutionary mass-luminosity (ML) relation was adopted (see F02 for details) and a wide range of effective temperature was explored. As a result, we have found that the Cepheid properties, and in particular the location in the H-R diagram of the instability strip and the coefficients of the multiband PL relations, depend on the pulsator metallicity, with the amplitude of the effect decreasing from visual to near-infrared magnitudes (Caputo et al. 2000a, hereafter C00). In particular, as the model metallicity increases from Z = 0.004 to 0.03 the instability strip gets redder and the pulsator luminosity, at fixed period, gets fainter (Bono et al. 1999a; Caputo et al. 2000b; F02). This result is at variance with recent empirical evaluations of the metallicity effect (see, e.g., Kennicutt et al. 1998; F01) and relies on the assumption of ΔY/ΔZ = 2.5. Specific computations for ΔY/ΔZ = 4.0 have shown that, at least for the higher metal contents (Z ≳ 0.008), the location on the H-R diagram of the Cepheid instability strip also depends on helium abundance, moving toward higher effective temperature as Y increases, at fixed Z (F02). On the basis of the above chemical composition–dependent models, F02 have found that the adoption of LMC-based V and I PL relations to get distance moduli with an uncertainty of ±0.1 mag is justified for variables with period shorter than 10 days. At longer periods, a correction to LMC-based distances might be needed, whose sign and amount depend on the helium and metal content of the Cepheids. In particular, model predictions were found to account for the empirical metallicity correction suggested by Kennicutt et al. (1998), provided that the adopted helium-to-metal enrichment ratio was about 3.5. Moreover, the above models provide a fairly good description of the data obtained by Romaniello et al. (2005, hereafter R05) by relating the V-band residuals from the PL relation adopted by the Hubble Space Telescope (HST) Key Project (F01) to spectroscopic iron abundances measured for 37 Galactic and Magellanic Cloud Cepheids (see R05 for details). All the above results were essentially based on two values (at most)
of helium content for each fixed metal abundance and therefore did not allow us to properly investigate the helium effect on the whole metal content range of observed Cepheids. In order to perform a more accurate analysis of combined helium and metal effects, we extended our grid of models to other chemical compositions at varying \( \Delta Y/\Delta Z \). In this paper we show the results of these new computations and further discuss the effect of chemical composition on Cepheid properties. In \S 2 we present the new model set and combine the results with the previous ones to investigate the dependence of the predicted pulsation observables on helium abundance and metallicity. In \S 3 the theoretical PL, PLC, and Wesenheit relations are presented, whereas in \S\S 4 and 5 we discuss the comparison with recent observational data and the implication of the predictions presented in the previous sections for the Cepheid distance scale.

2. THE NEW MODELS

By adopting the same code and physical and numerical assumptions as in previous papers (Bono et al. 1999b; F02), we have computed new sequences of pulsation models for the input parameters listed in Table 1. In particular, these new models correspond to different values of the \( \Delta Y/\Delta Z \) parameter, ranging from 0.5 to 3.5. The upper limit is due to the evidence that, as noted in F02, for \( \Delta Y/\Delta Z = 4 \) no model is found to pulsate at the highest metallicities (\( Z \sim 0.04 \)) covered by Cepheids in HST galaxies. On the other hand, \( \Delta Y/\Delta Z = 0.5 \) is below the lower limit of the evaluations found in the recent literature (see, e.g., Pagel & Portinari 1998; Pagel et al. 1992; Izotov et al. 1997; Izotov & Thuan 2004). For each selected mass, the luminosity level is chosen on the basis of the same canonical evolutionary ML relation adopted in F02. For a detailed discussion of the effect of the ML selection on Cepheid properties, we refer the interested reader to a companion paper by Caputo et al. (2005, hereafter C05). A wide range of effective temperatures is explored for each model mass and the modal stability is investigated for the fundamental mode. The first overtone mode pulsation is not studied in this paper because it is not expected to be significant at the selected metallicity and mass ranges and because it is known to be almost independent of chemical composition (see Bono et al. 2001, 2002b and references therein for details).

2.1. The Instability Strip

For each chemical composition, mass, and luminosity level, model computations allowed us to derive the blue (FBE) and red (FRE) edges of the fundamental instability strip. The new strips are reported in Figures 1 and 2 together with similar evaluations from our previous model sets. In Figure 1 we show the location of the predicted instability strip in the H-R diagram at fixed \( \Delta Y/\Delta Z \) and for the labeled metal contents. This plot confirms our previous results (see F02 and references therein) concerning the shift of the instability strip toward lower effective temperatures, as the metal abundance increases from \( Z = 0.004 \) to 0.03, and the narrowing of the strip when passing from \( Z = 0.03 \) to 0.04. The latter occurrence is due to the reduced efficiency of pulsation associated to the low hydrogen abundance for \( Z = 0.04 \) and \( Y = 0.29 \) or 0.33 (corresponding to \( \Delta Y/\Delta Z = 1.5 \) or 2.5, respectively). In order to investigate the effect of varying the helium abundance at fixed metallicity, in the three panels of Figure 2 we show the location of the instability strip in the H-R diagram for \( Z = 0.02 \) (top), \( Z = 0.03 \) (middle), and \( Z = 0.04 \) (bottom) and the labeled helium abundances. For \( Z = 0.04 \) we note a narrowing of the instability strip when \( Y \) increases (and \( X \) decreases). The effect is less evident for the other two metal abundances. In particular, for \( Z = 0.02 \) the FRE moves toward higher effective temperature as \( Y \) increases from \( Y = 0.25 \) to 0.31, confirming the result found by F02, but the helium dependence of the FBE is much more complicated with a sort of to and fro behavior on the explored \( Y \) range. This occurrence is related to the competing pulsation-driving role of H and He abundances in the associated ionization zones (see Bono et al. 1999b for details).

2.2. The Light and Radial Velocity Curves

One of the most important output of nonlinear pulsation codes is the predicted variation of relevant quantities (luminosity, radial velocity, effective temperature, surface gravity) along a model pulsation cycle. The bolometric light curves\(^1\) for the models quoted in Table 1 are reported in Figures 3a–3g for each labeled mass and luminosity level. The model period and effective temperature is also reported in each panel. We note that both the morphology and the amplitude of the curves vary with the position within the instability strip and depend on the adopted chemical composition. This behavior confirms our previous results (see Bono et al. 2000b, 2000c, hereafter BMS00). In particular, these plots support the empirical evidence for Galactic Cepheids originally found by Sandage & Tammann (1968, 1971) and Cogan (1980) and recently confirmed on the basis of a much larger sample of Cepheids by Sandage et al. (2004) and Tammann et al. (2003) that in the period range \( \log P \approx 0.40 – 0.86 \) and for \( \log P > 1.1 – 1.3 \) the largest luminosity amplitudes are attained close to the blue edge, while for \( 0.85 < \log P < 1.1 – 1.3 \), the maximum is attained close to the

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\(^1\) The radial velocity curves are also available upon request to the authors.
Fig. 1.—Theoretical instability strip as a function of metallicity for two different assumptions on the $\Delta Y/\Delta Z$ ratio.
Fig. 2.—Theoretical instability strip as a function of the helium abundance for three different assumptions on the metal content.
Fig. 3.—Bolometric light curves of models with (a) $Z = 0.01$, $Y = 0.26$, (b) $Z = 0.02$, $Y = 0.25$, (c) $Z = 0.02$, $Y = 0.26$, (d) $Z = 0.03$, $Y = 0.275$, (e) $Z = 0.03$, $Y = 0.335$, (f) $Z = 0.04$, $Y = 0.25$, and (g) $Z = 0.04$, $Y = 0.29$. The mass and luminosity values are labeled in the first-column plots, and the model effective temperature and period is reported in each panel.
Fig. 3

Graphs showing variations in $-M_{bol}$ with different masses $M/M_\odot$: 11, 9, 7, and 5, and corresponding luminosities $\log L/L_\odot$: 4.16, 3.86, 3.49, and 3.00, respectively. Each panel indicates phase duration $P$ and effective temperature $T_e$.

- $M/M_\odot=11$
  - $\log L/L_\odot=4.16$
  - $P=29.611 \text{ d}$, $T_e=5000 \text{ K}$
  - $P=34.055 \text{ d}$, $T_e=4800 \text{ K}$
  - $P=39.354 \text{ d}$, $T_e=4600 \text{ K}$
  - $P=45.573 \text{ d}$, $T_e=4400 \text{ K}$
  - $P=49.026 \text{ d}$, $T_e=4300 \text{ K}$

- $M/M_\odot=9$
  - $\log L/L_\odot=3.86$
  - $P=15.325 \text{ d}$, $T_e=5300 \text{ K}$
  - $P=17.527 \text{ d}$, $T_e=5100 \text{ K}$
  - $P=20.114 \text{ d}$, $T_e=4900 \text{ K}$
  - $P=22.975 \text{ d}$, $T_e=4700 \text{ K}$
  - $P=28.337 \text{ d}$, $T_e=4400 \text{ K}$

- $M/M_\odot=7$
  - $\log L/L_\odot=3.49$
  - $P=7.1365 \text{ d}$, $T_e=5600 \text{ K}$
  - $P=8.0325 \text{ d}$, $T_e=5400 \text{ K}$
  - $P=9.1395 \text{ d}$, $T_e=5200 \text{ K}$
  - $P=10.427 \text{ d}$, $T_e=5000 \text{ K}$
  - $P=11.931 \text{ d}$, $T_e=4800 \text{ K}$

- $M/M_\odot=5$
  - $\log L/L_\odot=3.00$
  - $P=3.1780 \text{ d}$, $T_e=5700 \text{ K}$
  - $P=3.3786 \text{ d}$, $T_e=5600 \text{ K}$
  - $P=3.5873 \text{ d}$, $T_e=5500 \text{ K}$
  - $P=3.8172 \text{ d}$, $T_e=5400 \text{ K}$
  - $P=4.0518 \text{ d}$, $T_e=5300 \text{ K}$

Fig. 3b
Fig. 3c
Fig. 3d
Fig. 3a

Fig. 3b

Fig. 3c

Fig. 3d

Fig. 3e

Fig. 3f

Fig. 3g

Fig. 3h
Fig. 3f
| $M/M_\odot$ | $\log L/L_\odot$ | $P$ (d) | $T_*$ (K) |
|----------|----------------|--------|----------|
| 11       | 4.13           | 36.712 | 4700     |
|          |                 | 39.452 | 4600     |
|          |                 | 42.490 | 4500     |
|          |                 | 49.281 | 4300     |
|          |                 | 57.086 | 4100     |
| 9        | 3.84           | 19.867 | 4900     |
|          |                 | 21.306 | 4800     |
|          |                 | 22.830 | 4700     |
|          |                 | 24.512 | 4600     |
|          |                 | 28.360 | 4400     |
| 7        | 3.47           | 8.5882 | 5300     |
|          |                 | 9.1740 | 5200     |
|          |                 | 9.7879 | 5100     |
|          |                 | 10.0471| 5000     |
|          |                 | 12.006 | 4800     |
| 5        | 2.99           | 3.0082 | 5800     |
|          |                 | 3.1864 | 5700     |
|          |                 | 3.3786 | 5600     |
|          |                 | 3.5863 | 5500     |
|          |                 | 4.0539 | 5300     |
Fig. 4.—Enlarged portion of the light-curve atlas reported in the previous figures for model sets showing evidence of the HP phenomenon. The chemical composition is reported in the first-column plots, together with the luminosity level.
red edge as a consequence of a phenomenon called Hertzsprung progression (HP; Hertzsprung 1926; Ledoux & Walraven 1958). Classical Cepheids in the period range 6 days $< P < 16$ days show a secondary maximum (bump) along both the light and the radial velocity curves. The HP is the relationship between the phase of this bump and the pulsation period. In particular, for Galactic Cepheids the bump appears on the descending branch of the light curve for Cepheids with periods up to $\sim$9 days, while it appears close to maximum light for $9 < P < 12$ days and moves at earlier phases for longer periods. On the basis of this observational evidence, this group of variables was christened “bump Cepheids.” As already obtained by BMS00 for $0.004 < Z < 0.02$, inspection of Figures 3a–3c and 3f suggests that an increase in the metal content causes a shift of the HP center toward shorter periods. In fact, as shown in more detail in Figure 4, passing from $Z = 0.01$, $Y = 0.26$ (top) to $Z = 0.04$, $Y = 0.25$ (bottom), for $M/M_\odot = 7$, the period corresponding to the HP center moves from $\sim$10.5 to $\sim$8.2 days, attaining $\sim$9.5 days for $Z = 0.02$, $Y = 0.25$, 0.26 (middle panels). We remind that this trend is in agreement with the observations. In fact, empirical data for Galactic Cepheids suggest that the HP center corresponds to a period PHP~10.0 days (Moskalik et al. 1992, 2000), whereas in the LMC ($Z = 0.008$) PHP~10.5 days and in the SMC ($Z = 0.004$) PHP~11.0 days (Beaulieu 1998). Unfortunately, no empirical evidence is available for the HP phenomenon in supersolar Cepheid samples. The bolometric light curves of the new models were transformed into the observational bands (UBVRIJKLM) by means of the model atmospheres by Castelli et al. (1997a, 1997b), and mean magnitudes and colors were then derived for each chemical composition and stellar mass. In Table 2 we report the model periods and intensity-averaged mean magnitudes, but magnitude-averaged values are also available upon request to the authors. The static magnitude values$^2$ have also been derived and used to obtain the boundaries of the instability strip, at each chemical composition, in the various period-magnitude planes and, in turn, to construct synthetic multiband PL relations (see below).

3. PREDICTED CEPHEID RELATIONS

The results presented in the previous section allow us to derive all the relevant relations connecting the pulsation period to

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2 By “static magnitude” we mean the magnitude the star would have were it not pulsating.

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mean magnitudes and colors, as well as synthetic PL relations, following the same procedure as in our previous papers (see C00; F02).

3.1. PLC and Wesenheit Relations

Linear regression through the period and magnitude values reported in Table 2 provides the multiband PLC relations given in Table 3. In the same table we also report the PLC coefficients of our previous model sets. Similarly, the coefficients of the reddening-free Wesenheit relations (see C00 and references therein) are reported in Table 4. These are defined by using the ratios between total extinction and the various color excesses given by Cardelli et al. (1989). In agreement with the recent empirical evidence by Ngeow & Kanbur (2005), we find that the predicted Wesenheit relations are well represented by linear functions, at variance with PL (see below) and period-color (see C00) relations. We note that, even if PLC and Wesenheit relations have the advantage of being independent of the distribution of pulsators within the instability strip, holding for each

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MARCONI, MUSELLA, & FIORENTINO

Vol. 632

TABLE 2

INTRINSIC PARAMETERS AND INTENSITY-AVERAGED MEAN MAGNITUDES AND COLORS FOR THE NEW MODELS

| $Z$ | $Y$ | $M/M_\odot$ | $L/L_\odot$ | $T_\text{eff}$ (K) | $\log P$ | $M_V$ | $B-V^a$ | $V-R^b$ | $V-I^c$ | $V-J^d$ | $V-K^e$ |
|-----|-----|------------|------------|-----------------|---------|------|--------|--------|--------|--------|--------|
| 0.01 | 0.26 | 5 | 3.130 | 5800 | 0.5847 | $-3.0322$ | 0.5739 | 0.3317 | 0.6752 | 1.0939 | 1.4809 |
| 0.01 | 0.26 | 5 | 3.130 | 5700 | 0.6084 | $-3.0135$ | 0.6145 | 0.3498 | 0.7091 | 1.1478 | 1.5539 |
| 0.01 | 0.26 | 5 | 3.130 | 5600 | 0.6344 | $-2.9952$ | 0.6563 | 0.3675 | 0.7421 | 1.2007 | 1.6252 |
| 0.01 | 0.26 | 5 | 3.130 | 5500 | 0.6598 | $-2.9758$ | 0.6987 | 0.3851 | 0.7746 | 1.2533 | 1.6965 |
| 0.01 | 0.26 | 5 | 3.130 | 5400 | 0.6862 | $-2.9553$ | 0.7414 | 0.4024 | 0.8067 | 1.3057 | 1.7676 |
| 0.01 | 0.26 | 5 | 3.130 | 5300 | 0.7126 | $-2.9338$ | 0.7845 | 0.4193 | 0.8384 | 1.3582 | 1.8386 |
| 0.01 | 0.26 | 7 | 3.610 | 5900 | 0.8707 | $-4.2567$ | 0.5564 | 0.3195 | 0.6485 | 1.0415 | 1.4403 |
| 0.01 | 0.26 | 7 | 3.610 | 5800 | 0.8961 | $-4.2404$ | 0.5825 | 0.3326 | 0.6748 | 1.0883 | 1.4647 |
| 0.01 | 0.26 | 7 | 3.610 | 5700 | 0.9217 | $-4.2222$ | 0.6148 | 0.3480 | 0.7045 | 1.1399 | 1.5387 |
| 0.01 | 0.26 | 7 | 3.610 | 5600 | 0.9483 | $-4.2022$ | 0.6564 | 0.3661 | 0.7384 | 1.1954 | 1.6151 |

Notes.—Table 2 is published in its entirety in the electronic edition of the Astrophysical Journal. A portion is shown here for guidance regarding its form and content.

* Intensity-weighted mean colors.

TABLE 3

PREDICTED INTENSITY-WEIGHTED PLC RELATIONS

| Color$^a$ | $Y^b$ | $Z^c$ | $a^d$ | $b^e$ | $c^f$ | $d^g$ |
|----------|------|------|-----|-----|-----|-----|
| $B-V$ | 0.25 | 0.004 | $-2.54 \pm 0.04$ | $-3.52 \pm 0.03$ | 2.79 | $\pm 0.07$ | 0.04 |
| $B-V$ | 0.25 | 0.008 | $-2.63 \pm 0.04$ | $-3.55 \pm 0.03$ | 2.83 | $\pm 0.06$ | 0.03 |
| $B-V$ | 0.26 | 0.01 | $-2.65 \pm 0.04$ | $-3.66 \pm 0.07$ | 2.95 | $\pm 0.05$ | 0.05 |
| $B-V$ | 0.25 | 0.02 | $-2.83 \pm 0.05$ | $-3.57 \pm 0.05$ | 2.81 | $\pm 0.09$ | 0.05 |
| $B-V$ | 0.26 | 0.02 | $-2.78 \pm 0.04$ | $-3.59 \pm 0.02$ | 2.77 | $\pm 0.07$ | 0.04 |
| $B-V$ | 0.28 | 0.02 | $-2.96 \pm 0.07$ | $-3.72 \pm 0.10$ | 3.27 | $\pm 0.18$ | 0.07 |
| $B-V$ | 0.26 | 0.03 | $-2.79 \pm 0.03$ | $-3.79 \pm 0.04$ | 3.10 | $\pm 0.07$ | 0.03 |
| $B-V$ | 0.27 | 0.03 | $-3.04 \pm 0.06$ | $-3.75 \pm 0.07$ | 3.18 | $\pm 0.13$ | 0.06 |
| $B-V$ | 0.31 | 0.03 | $-3.10 \pm 0.06$ | $-3.81 \pm 0.08$ | 3.34 | $\pm 0.13$ | 0.06 |
| $B-V$ | 0.33 | 0.03 | $-3.06 \pm 0.07$ | $-3.89 \pm 0.12$ | 3.41 | $\pm 0.22$ | 0.06 |

Notes.—Table 3 is published in its entirety in the electronic edition of the Astrophysical Journal. A portion is shown here for guidance regarding its form and content.

* Color used in PLC relation, $(M_P) = \alpha + \beta \log P + \gamma$(color). Other colors are $V-R$, $V-I$, $V-J$, and $V-K$.

b Helium content.
c Metal content.
d Zero point.
e Logarithmic period coefficient.
f Color coefficient.
g Standard deviation (mag).
individual star as a result of its period-density relation and black body behavior, they heavily rely on the assumption of an evolutionary ML relation. Without this assumption we would obtain tight mass-dependent PLC and Wesenheit relations (see C05), which provide sound constraints on the pulsation mass of each individual Cepheid, once the absolute magnitudes and the intrinsic colors are known, and, by comparison with evolutionary masses, to give an estimate of mass-loss during or before the Cepheid phase. The interested reader is referred to C05 for a detailed and updated investigation of this problem. Here we only note that if we used a noncanonical mass-luminosity relation the PLC relations would provide absolute magnitudes fainter than the ones obtained with the relations reported in Table 4 by \( \sim 0.2 \) mag.

### 3.2. Synthetic PL Relations

PL relations are well known to depend on the topology of the instability region and on the distribution of pulsators within

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\begin{align*}
M_V(r) &= a + b \log P, \\
M_K(r) &= c + d \log P, \\
M_J(r) &= e + f \log P.
\end{align*}
\]

Notes.—Table 4 is published in its entirety in the electronic edition of the *Astrophysical Journal*. A portion is shown here for guidance regarding its form and content.

- Color used in weighted Wesenheit relation: e.g., \( M_V = 3.30(B - V) = \alpha + \beta \log P, M_V = 5.29(V - R) = \alpha + \beta \log P \) (see machine-readable table for all relations).
- Helium content.
- Metal content.
- Zero point.
- Logarithmic period coefficient.
- Standard deviation (mag).

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![Fig. 5.—Synthetic multiband PL relations at varying chemical composition (see text for details). The solid and dashed lines represent the linear and quadratic regressions, respectively.](image-url)
the strip. For this reason we did not use the individual models, but we populated the predicted instability strip by adopting the procedure suggested by Kennicutt et al. (1998) and already used by C00 and F02. In particular, 1000 pulsators were uniformly distributed from the blue to the red boundary of the instability strip, with a mass law as given by \(dn/dm = m^{-3}\) over the mass range \(5-11 M_\odot\) (see C00 for further details). The resulting synthetic distributions for the new model sequences are much better represented by a quadratic relation with a clear dependence on chemical composition. The resulting synthetic multiband (BYRIJK) PL relations are given in Table 5 and Table 6 (quadratic and linear solutions, respectively) and overplotted as solid and dashed lines in each panel of the quoted figure. However, we wish to remind that, as remarked in C00, the present solutions refer to a specific pulsator distribution and that different populations may modify the results. In particular, if the longer periods (\(\log P \gtrsim 1.5\)) are rejected in the final fit, then the predicted linear PL relations become steeper and the intrinsic dispersion in the BYR bands is reduced (see Table 7). Such a selection was also adopted by F02 because the slope of the predicted linear PL\(_Y\) and PL\(_J\) relations for Cepheids with period log \(P\) \(\lesssim 1.5\) and the metallicity of the LMC \((Z = 0.008)\) was found to be \(-2.75 \pm 0.02\) and \(-2.98 \pm 0.01\), respectively, in very good agreement with the values \((-2.77 \pm 0.03\) and \(-2.98 \pm 0.02\) inferred from the huge sample of LMC Cepheids in the OGLE-II catalog (Udalski 2000).

4. THEORY VERSUS OBSERVATIONS

In order to test the predictive capabilities of current pulsation models, in this section we compare the theoretical multiband PL relations with recent observations for Cepheids belonging to the Milky Way and the LMC.

In the recent papers by Tamman et al. (2003, hereafter T03) and Sandage et al. (2004, hereafter S04), accurate \(BVI\) period-color (PC), PL, and PLC relations are derived on the basis of large databases for Galactic and LMC Cepheids, respectively. These authors show that the PL relations for Cepheids in the Galaxy, LMC, and SMC have significantly different slopes (T03; but see also Ngeow & Kanbur 2004), and in particular S04 report about the experimental evidence of a change of the slope (a break) of the PL relation for the LMC Cepheids near 10 days (see also Kanbur & Ngeow 2004). This last result is supported by the broken PC relation for the LMC Cepheids showed in S04. Indeed, any nonlinearity of the PC relations must be reflected in the PL relation. These results represent an important tool to test the accuracy of current model predictions concerning both the nonlinearity of optical PL relations and the dependence of Cepheid properties on metal abundance.
As for the first point, the theoretical evidence for the nonlinearity of $BV$ PL and PC relations was already reported in our previous papers (see, e.g., Bono et al. 1999a; C00), and for this reason we usually adopt linear PL relations for log $P/C_20$.

In particular, Figure 4 in C00 shows that the nonlinear behavior of the PC relation is more evident for $Z = 0.004$ and 0.008 (representative of the LMC and SMC metallicity respectively), whereas it is significantly reduced for $Z = 0.02$ (representative of the metallicity of Galactic Cepheids). This result is in agreement with T03 and S04 that found the break at $log P = 1$ only for the LMC Cepheids. On this basis, in order to compare our results with the data presented by S04, we also derived the theoretical linear PL relations for log $P < 1.0$ and log $P > 1.0$ obtained by S04, whereas long- and short-dashed lines represent the theoretical PL relations for $Z = 0.004$ and 0.008, respectively.

We also plotted the theoretical PL relations for $Z = 0.004$ in order to take into account the significant metallicity dispersion of LMC Cepheids (Luck et al. 1998) and the results by Bono et al. (1999a) that at longer periods the observed distribution of LMC Cepheids in the period-magnitude diagram is better represented by the theoretical one for $Z = 0.004$ (see Bono et al. 1999a for details). Inspection of Figure 6 and of Tables 7 and 8 suggests that the slopes obtained for $Z = 0.004$ and 0.008 in the two period ranges are similar. However, the $B$ and $V$ PL relations for $Z = 0.008$ are systematically fainter than the observational one, which shows a better agreement with the model predictions for $Z = 0.004$. On the other hand, in the $I$ band we have a very good agreement between the empirical fits and the relations for $Z = 0.008$, whereas the relations for $Z = 0.004$ seem to be systematically brighter.

In order to investigate the dependence of Cepheid properties on metallicity, in Figure 7 we plot the S04 Galactic sample with our PL relations for $Z = 0.02$ and 0.01. Open circles...
**TABLE 8**

Empirical and Theoretical PL Relations for LMC Cepheids with Break at log $P = 1$

| Sample | Band | Slope $\log P < 1.0$ | Slope $\log P > 1.0$ | Zero Point $\log P < 1.0$ | Zero Point $\log P > 1.0$ |
|--------|------|----------------------|----------------------|--------------------------|--------------------------|
| S04    | $B$  | $-2.68 \pm 0.08$     | $-2.15 \pm 0.13$     | $-0.99 \pm 0.05$         | $-1.40 \pm 0.18$         |
| $Z = 0.004$ | $B$  | $-3.09 \pm 0.04$     | $-2.04 \pm 0.11$     | $-0.64 \pm 0.03$         | $-1.68 \pm 0.13$         |
| $Z = 0.008$ | $V$  | $-2.75 \pm 0.05$     | $-1.84 \pm 0.11$     | $-0.72 \pm 0.03$         | $-1.64 \pm 0.14$         |
| S04    | $V$  | $-2.96 \pm 0.06$     | $-2.57 \pm 0.10$     | $-1.33 \pm 0.04$         | $-1.63 \pm 0.13$         |
| $Z = 0.004$ | $V$  | $-3.21 \pm 0.03$     | $-2.47 \pm 0.08$     | $-1.13 \pm 0.02$         | $-1.87 \pm 0.10$         |
| $Z = 0.008$ | $V$  | $-2.97 \pm 0.03$     | $-2.32 \pm 0.08$     | $-1.22 \pm 0.02$         | $-1.87 \pm 0.10$         |
| S04    | $I$  | $-3.10 \pm 0.04$     | $-2.82 \pm 0.08$     | $-1.85 \pm 0.02$         | $-2.08 \pm 0.11$         |
| $Z = 0.004$ | $I$  | $-3.30 \pm 0.02$     | $-2.79 \pm 0.05$     | $-1.79 \pm 0.02$         | $-2.30 \pm 0.07$         |
| $Z = 0.008$ | $I$  | $-3.13 \pm 0.02$     | $-2.68 \pm 0.05$     | $-1.84 \pm 0.02$         | $-2.30 \pm 0.07$         |

**Fig. 7.**—Comparison between theoretical PL relations and the S04 Galactic sample. Open and filled circles represent the S04 sample with distances from Baade-Becker-Wesselink expansion parallaxes and cluster/associations Cepheids, respectively. The solid line represents the S04 fit (see text for details), the long-dashed line shows our PL relation (for $\log P \leq 1.5$) for $Z = 0.01$ and $Y = 0.26$, and the other lines show the PL relations (for $\log P \leq 1.5$) for $Z = 0.02$ and different values of the helium content (see labels).
represent the S04 sample with distances from Baade-Becker-Wesselink expansion parallaxes, and filled circles indicate the S04 sample with distances from cluster/associations Cepheids (see S04 for details). The solid line represents the S04 fit (obtained by using both the samples; see S04 for details), the long-dashed line shows our PL relation (for $\log P/C20 = 1.5$) for $Z = 0.01$, $Y = 0.26$ and the other lines show the PL relations (for $\log P \leq 1.5$) for $Z = 0.02$ and the labeled values of the helium content. Our theoretical relations are flatter than the S04 one, but a better agreement with the data is found when we consider the model predictions for $Z = 0.01$, $Y = 0.26$ and $Z = 0.02$, $Y = 0.25$, 0.26, at least for $\log P \leq 1.5$. For the other chemical compositions, the discrepancy is particularly evident at the longer periods and for the $B$ and $V$ bands. In this context, we remind that current comparisons rely on the assumption of static model atmospheres and that theoretical colors can be affected by systematic uncertainties. Moreover, the $B$ and $V$ PL relations are very sensitive to the topology of the instability strip and, in turn, to the adopted input physics and to the treatment of convection in the pulsation models (see below). The effects of these uncertainties are likely more important for long period expanded structures. The better agreement obtained for models with $Z = 0.01$ supports the recent suggestions by Asplund et al. (2005) that the solar metallicity is lower ($Z \approx 0.01$) than usually adopted ($Z = 0.02$).

Finally, we consider the interferometric results for seven Galactic Cepheids by Kervella et al. (2004a, 2004b, 2004c, hereafter K04c), which are a subsample of the S04 data set, but only with $V$- and $K$-band observations. These authors have presented accurate radius and distance determinations based on interferometric measurements of the angular diameter. On this basis they have derived new period-radius, as well as $V$- and $K$-band PL relations (Kervella et al. 2004a, hereafter K04a) by assuming the slopes by Gieren et al. (1998). All these objects have metal abundance close to the solar one. For l Car the metal abundance reported by K04a is about twice the solar value, but the recent spectroscopic measurement by R05 suggests a solar value also for this object. In Figure 8 we show the comparison between our predicted $K$-band (top) and $V$-band (bottom) PL relations (for $\log P \leq 1.5$, see Table 7) at solar metallicity with the labeled helium abundances and the interferometric results by K04a. The intrinsic dispersion of the theoretical relations is represented by the vertical error bar in the labels, whereas the solid line represents the empirical PL relation obtained by

![Graph showing period-luminosity relations](image-url)
fitting the data. For η Aql (log $P = 0.8559$) we have reported both determinations used by K04a. Moreover, for variable l Car, the revised magnitude values by K04b, based on a more accurate interferometric determination of radius and distance, are also reported (open circle). An inspection of this figure shows that in the $K$ band our theoretical relations are able to reproduce the data within the errors, whereas in the $V$ band the predicted PL relations are fainter than the empirical one and fail to match the location of the two pulsators with the smallest error on the absolute magnitude. In Figure 9, we also compare K04a data with the theoretical relation at $Z = 0.01$. We note that, as already found for the comparison with S04 data, model predictions at this lower metal abundance better reproduce the interferometric results for Galactic Cepheids.

The discrepancies found in the comparison with the observational data by S04 and K04a, K04b, and K04c can be due, at least in part, to the uncertainties still affecting the adopted theoretical scenario. In particular we remind that current models are based on specific assumptions concerning both the evolutionary ML relations (see Bono et al. 1999b, 2000c; C05 for details) and the value of the mixing length ($\alpha$) parameter adopted in the treatment of convection to close the system of nonlinear dynamical and convective equations (see Bono & Stellingwerf 1994; Bono et al. 1999b). For the former point we refer the interested reader to the detailed discussion by C05. As for the mixing length parameter, even if recent results based on the theoretical fitting of observed Cepheid light curves suggest that the value of the $\alpha$ parameter should increase when moving from the blue to the red boundary of the instability strip (see Bono et al. 2002a), in agreement with recent results obtained from the modeling of RR Lyrae stars (see, e.g., Di Criscienzo et al. 2004), all the models presented and adopted in this paper have been computed with $\alpha = 1.5$. Specific model sets at $Z = 0.02$, $Y = 0.28$ and $Z = 0.01$, $Y = 0.26$ that were computed by increasing $\alpha$ from 1.5 to 1.8 show that the instability strip gets significantly narrower with the red boundary getting bluer by at least 300–400 K and a smaller redward shift of the blue boundary. This occurrence is due to the higher sensitivity of the red part of the instability strip to the efficiency of the convective transfer. On the basis of these results and taking into account the possibility that the $\alpha$ value is different at the blue and red edge.

4 In C05 we discussed the possibility that l Car is a peculiar variable star. Indeed, on the basis of the comparison between the pulsational and evolutionary masses, it seems to be an object on the first crossing of the instability strip.
of the strip, we expect that the PL relation may become brighter and steeper when $\alpha$ increases.

5. METALLICITY AND HELIUM EFFECTS ON THE PREDICTED DISTANCE SCALE

In order to test the results presented by F02 concerning the combined metallicity and helium effects on the Cepheid distance scale and provide a refined theoretical correction, we applied the same procedure adopted by the quoted authors to our extended model set. In particular, we considered our models with the various chemical composition as real Cepheids at the fixed distance modulus $\mu_0 = 0$ mag. By applying the predicted linear $V$- and $I$-band PL relations with $Z = 0.008$ and $Y = 0.25$ for $\log P \leq 1.5$, we determined the value $\mu_{0,0.008}$ for all the pulsators. This method simulates the HST Key Project procedure (e.g., F01), which uses observations in the two bands $V$ and $I$ and adopts the LMC PL relations as universal (F01; Udalski et al. 1999). In this context, we adopted $\mu_Y - \mu_I = E(V-I)$ and $A_Y/E(V-I) = 1.54$ from Cardelli et al. (1989). The derived $\mu_{0,0.008}$ values confirm the results by F02: (1) for periods shorter than 10 days—the discrepancy between $\mu_{0,0.008}$ and the real value ($\mu_0 = 0$ mag) is small enough ($< 0.1$ mag) to support the adoption of universal LMC-referenced PL linear relations; (2) for periods longer than 10 days, the discrepancy is larger than 0.1 mag (up to 0.3 mag for $Z = 0.02$ and $Y = 0.28$ and period longer than 20 days) over the range $Z \sim 0.01$–0.04, so that a correction is required. In particular, a dependence on chemical composition of the form suggested by F02 for longer period Cepheids is also found using the extended model set presented in this paper. The mean correction for $\log P \geq 1.0$ is

$$ c = -3.642 + 11.511 Y - 1.697 \log Z + 5.334 Y \log Z, $$

whereas for Cepheid samples with $\log P \geq 1.3$ the mean correction is better reproduced by

$$ c = -5.894 + 18.141 Y - 2.792 \log Z + 8.576 Y \log Z, $$

both with an intrinsic uncertainty of $\pm 0.02$ mag and by assuming $\Delta Y/\Delta Z > 1.5$ (see Fig. 10). For $\Delta Y/\Delta Z \leq 1.5$ the dependence is more complicated, but the correction is always lower than 0.1 mag and can be neglected as in the case of shorter periods. We also remind that on the basis of current estimates in the literature, we do not expect such low values for the $\Delta Y/\Delta Z$ parameter (Izotov & Thuan 2004 and references therein). As shown in Figure 10, the predicted mean metallicity correction implies that pulsators get fainter as their metallicity increases until a turnover point is reached close to the solar metal abundance. Such a behavior was already mentioned to be in agreement with the recent spectroscopic results by R05 and to reproduce the empirical metallicity correction by Kennicutt et al. (1998) for $\Delta Y/\Delta Z \sim 3.5$. We also note that the mean correction gets smaller when the lowest period of the investigated sample decreases from 20 (bottom) to 10 days (top). In particular, the mean correction for $\log P \geq 1$ is higher than 0.1 mag only at the highest metallicities ($Z \geq 0.03$) and $\Delta Y/\Delta Z$ values ($\geq 3$). On the other hand, if the Cepheid periods are longer than or equal to 20 days, the mean correction is larger than 0.1 mag on a wide range of metallicities and helium contents.

6. CONCLUSIONS

We have presented an extended set of nonlinear convective pulsation models at varying the metallicity and $\Delta Y/\Delta Z$ ratio. On this basis, we obtain the following main results:

1. We have confirmed our previous results concerning the shift of the instability strip toward lower effective temperatures as the metal abundance increases at fixed $\Delta Y/\Delta Z$, at least up to $Z = 0.03$. At the same time, when passing from $Z = 0.03$ to 0.04, the strip narrows due to the reduced efficiency of pulsation. The effect of variation of the helium abundance at fixed metallicity is lower than the one obtained by varying the metallicity at fixed $\Delta Y/\Delta Z$. In particular, the fundamental red edge slightly moves toward higher effective temperatures as the helium content increases, whereas the fundamental blue edge does not show a clear trend.

2. Inspection of the bolometric light curves, in the period range affected by the HP phenomenon, shows that passing from $Z = 0.01$ to 0.04 the period corresponding to the HP center moves from $\sim 10.5$ to $\sim 8.2$ days, in agreement with the empirical evidence of a decrease of PHP as the metallicity increases and with our previous theoretical results for $Z \leq 0.02$.

3. A comparison with the large database of Galactic and LMC Cepheids by Sandage et al. and Tamman et al. shows that,
in agreement with the conclusions of these authors, the $BVI$ PL relations for LMC pulsators are well reproduced by linear theoretical relations with a break at $\log P = 1$. As for the dependence on metallicity, we find that our theoretical PL relations for $Z = 0.02$ are generally flatter than the empirical ones for Galactic Cepheids, with the discrepancy increasing toward the longer periods. A good agreement is obtained when, on the basis of suggestions in the recent literature, $Z = 0.01$ is assumed as solar metal abundance in the models.

4. A comparison with recent accurate interferometric results by Kervella et al. shows that in the $K$ band our theoretical period-luminosity relations are able to reproduce the data within the errors, whereas in the $V$ band the predicted PL relations are fainter than the empirical one. Among the possible reasons for such a discrepancy, as well as for the one quoted in the previous point, we have identified the uncertainty on the mixing length parameter adopted in the treatment of convection and on the value of the solar metallicity.

5. We have derived the theoretical correction to the distance moduli inferred with the $HST$ Key Project procedure, when the effect of metallicity and helium abundance are taken into account. We find that this effect is smaller than 10% and can be neglected for Cepheid samples with $\log P < 1.0$ and for $\Delta Y/\Delta Z \leq 1.5$. For longer periods and higher $\Delta Y/\Delta Z$ values, the dependence of the mean theoretical correction on chemical composition has the same analytical form of the one found by F02, but it is shown to become important only for periods longer than 20 days.

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REFERENCES

Asplund, M., Grevesse, N., & Sauval, A. J. 2005, in ASP Conf. Ser. 336, Cosmic Abundance as Records of Stellar Evolution and Nucleosynthesis, ed. T. G. Barnes, III & F. N. Bash (San Francisco: ASP), 25

Beaulieu J. P. 1998, Mem. Soc. Astron. Italiana, 69, 21

Bono, G., Caputo, F., Cassisi, S., Marconi, M., Pieri, L., & Tornambé, A. 2000a, ApJ, 543, 955

Bono, G., Caputo, F., Castellani, V., & Marconi, M. 1999a, ApJ, 512, 711

Bono, G., Castellani, V., & Marconi, M. 2000b, ApJ, 529, 293

———. 2002a, ApJ, 565, L83

Bono, G., Gieren, W. P., & Fouque, P. 2001, ApJ, 552, L141

Bono, G., Groenewegen, M. A. T., Marconi, M., & Caputo, F. 2002b, ApJ, 574, L33

Bono, G., Marconi, M., & Stellingwerf, R. F. 1999b, ApJS, 122, 167

———. 2000c, A&A, 360, 245 (BMS00)

Bono, G., & Stellingwerf, R. F. 1994, ApJS, 93, 233

Caputo, F., Bono, G., Fiorentino, G., Marconi, M., & Musella, I. 2005, ApJ, 629, 1021 (C05)

Caputo, F., Marconi, M., & Musella, I. 2000a, A&A, 354, 610 (C00)

Caputo, F., Marconi, M., Musella, I., & Santolamazza, P. 2000b, A&A, 359, 1059

Cardelli, J. A., Clayton, G. C., & Mathis, J. S. 1989, ApJ, 345, 245

Castelli, F., Gratton, R. G., & Kurucz, R. L. 1997a, A&A, 318, 841

———. 1997b, A&A, 324, 432

Chiotti, C., Wood, P. R., & Capitanio, N. 1993, ApJS, 86, 541

Cogan, B. C. 1980, ApJ, 239, 941

Di Criscienzo, M., Marconi, M., & Caputo, F. 2004, ApJ, 612, 1092

Fiorentino, G., Caputo, F., Marconi, M., & Musella, I. 2002, ApJ, 576, 402 (F02)

Freedman, W. L., et al. 2001, ApJ, 553, 47 (F01)

Gieren, W. P., Fouque, P., & Gomez, M. 1998, ApJ, 496, 17

Groenewegen, M. A. T., Romaniello, M., Primas, F., & Mottini, M. 2004, A&A, 426, 999

Hertzsprung, E. 1926, Bull. Astron. Inst. Netherlands, 3, 115

Izotov, Y. I. & Thuan, T. X. 2004, ApJ, 616, 768

Izotov, Y. I., Thuan, T. X., & Lipovetsky, V. A. 1997, ApJS, 108, 1

Kanbur, S. M., Ngeow, C., 2004, MNRAS, 350, 962

Kennicutt, R. C., Jr., et al. 1998, ApJ, 498, 181

Kervella, P., Bersier, D., Mourard, D., Nardetto, N., & Coudé du Foresto, V. 2004a, A&A, 423, 327 (K04a)

Kervella, P., Bersier, D., Mourard, D., Nardetto, N., Fouqué, P., & Coudé du Foresto, V. 2004b, A&A, 428, 587 (K04b)

Kervella, P., Nardetto, N., Bersier, D., Mourard, D., & Coudé du Foresto, V. 2004c, A&A, 416, 941 (K04c)

Ledoux, P., & Walraven, T. 1958, Handb. Phys., 51, 353

Luck, R. E., Moffett, T. J., Barnes, T. G., III, & Gieren, W. P. 1998, AJ, 115, 605

Moskalik, P., Buchler, J. R., & Marom, A. 1992, ApJ, 385, 685

Moskalik, P., Krzyt, T., Gorny, N. A., & Samus, N. N. 2000, in ASP Conf. Ser. 203, The Impact of Large-Scale Surveys on Pulsating Star Research (San Francisco: ASP), 233

Ngeow, C., & Kanbur, S. M. 2004, MNRAS, 349, 1130

———. 2005, MNRAS, 360, 1022

Pagel, B. E. J., & Portinari, L. 1998, MNRAS, 298, 747

Pagel, B. E. J., Simonson, E. A., Terlevich, R. J., & Edmunds, M. G. 1992, MNRAS, 255, 325

Romaniello, M., Primas, F., Mottini, M., Groenewegen, M., Bono, G., & Francois, P. 2005, A&A, 429, L37 (R05)

Saha, A., Sandage, A., Tammann, G. A., Dolphin, A. E., Christensen, J., Panagia, N., & Macchetto, F. D. 2001, ApJ, 562, 314

Sakai, S., Ferrarese, L., Kennicutt, R. C., Jr., & Saha, A. 2004, ApJ, 608, 42

Sandage, A., & Tammann, G. A. 1968, ApJ, 151, 531

———. 1971, ApJ, 167, 293

Sandage, A., Tammann, G. A., & Reindl, B. 2004, A&A, 424, 43 (S04)

Storm, J., Carney, B. W., Gieren, W. P., Fouqué, P., Latham, D. W., & Fry, A. M. 2004, A&A, 415, 531

Tammann, G. A., Sandage, A., & Reindl, B. 2003, A&A, 404, 423 (T03)

Udalski, A. 2000, Acta Astron., 50, 279

Udalski, A., Soszynski, I., Szymanski, M., Kubiak, M., Pietrzyński, G., Wozniak, P., & Zebrun, K. 1999, Acta Astron., 49, 223