Longitudinal Broadening of Quenched Jets in Turbulent Color Fields

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The near-side distribution of particles at intermediate transverse momentum, associated with a high momentum trigger hadron produced in a high energy heavy-ion collision, is broadened in rapidity compared with the jet cone. This broadened distribution is thought to contain the energy lost by the progenitor parton of the trigger hadron. We show that the broadening can be explained as the final-state deflection of the gluons radiated from the hard parton inside the medium by soft, transversely oriented, turbulent color fields that arise in the presence of plasma instabilities. The magnitude of the effect is found to grow with medium size and density and diminish with increasing energy of the associated hadron.

The emission of hadrons with large transverse momentum is observed to be strongly suppressed in central collisions of heavy nuclei at the Relativistic Heavy Ion Collider (RHIC) \[^1, 2\]. The suppression is understood to be caused by final-state rescattering of the leading parton in the dense medium produced in such collisions, causing it to lose energy. Within the framework of perturbative QCD, the leading process of energy loss of a fast parton is gluon radiation induced by elastic collisions with color charges in the quasi-thermal medium \[^3, 4, 5\]. Elastic collisions not followed by radiation may also contribute to the energy loss \[^6\].

In this manuscript, we study the fate of the energy lost by the leading parton of the trigger jet within the medium. Measurements by the STAR collaboration have shown that the leading hadron is accompanied by a cone-shaped pattern of secondary hadrons, which closely resembles the jet cone around a hard scattered parton in nucleon-nucleon collisions \[^7, 8\]. We will refer to this as the contribution from vacuum fragmentation (or “jet” yield). In addition, the trigger parton is observed to be accompanied on the near side by a wide ridge \[^9\] (or pedestal \[^10\]) of secondary hadrons, which extends along the beam direction over more than one unit of rapidity in both directions. We refer to this as the contribution from in-medium fragmentation (or “ridge” yield). The energy flow in the ridge-shaped structure grows with participant number and is concentrated in hadrons with transverse momenta \(p_T < 2 \text{ GeV/c}\).

The inclusive yield of associated hadrons receives contributions from the fragmentation of the leading parton as well as from the radiated gluons \[^11\]. The yield of associated hadrons produced in the fragmentation of the hard parton after its exit from the medium will be assumed to form the jet yield \[^12\]. This naturally explains why the azimuthal and rapidity distribution of this contribution is identical to that from a jet of the same final energy produced in a \(p + p\) collision. Hadrons produced in the fragmentation of the gluons radiated within the medium or after absorption of those gluons will be assumed to form the ridge yield. In either case, the ridge shape will reflect the kinematic distribution of the radiated gluons at the end of their passage through the medium.

It has been argued that the properties of the ridge must be associated with the longitudinal expansion of the medium \[^11, 13\]. However, the precise mechanism by which the ridge acquires its peculiar shape has not been quantitatively explained. In a recent article, Romatschke \[^14\] has shown that the momentum of a transversely propagating heavy quark is preferentially broadened in the longitudinal direction by elastic collisions in an expanding medium, if the expansion leads to a large anisotropy of the momentum distribution of the partons composing the medium. He suggested that this phenomenon could explain the ridge. We note, however, that elastic scattering is probably not the primary source of energy loss for the light partons which initiate the observed ridge. We also note that the momentum anisotropy parameter \(\xi\) of the expanding medium is related to its shear viscosity \(\eta\) by the relation \[^15\] \(\xi \approx 10 \eta/(s \tau T)\), where \(s\) denotes the entropy density, \(T\) the temperature, and \(\tau\) the time after the initial hard collision. For times relevant to the propagation of the jet through the medium (\(\tau \geq 1 \text{ fm/c}\)), the large values of \(\xi \sim 10\) required to explain the extent of the ridge are difficult to reconcile with the low shear viscosity of the medium \((\eta/s \leq 0.3)\) required by the data \[^14, 16\].

Here, we propose a different mechanism for the origin of the observed ridge, also related to the longitudinal expansion of the medium. It builds on the recent insight that extended color fields are dynamically formed in the expanding medium due to the presence of plasma instabilities. Such instabilities have been shown to exist in any quark-gluon plasma with an anisotropic momentum distribution \[^18, 19, 20\]. The nearly boost invariant longitudinal expansion of the matter formed in relativistic heavy ion collisions necessarily induces an oblate momentum distribution of partons with respect to the beam axis. The plasma instabilities lead to the exponential growth of soft modes of the glue field, which ultimately saturate due to the nonlinear self-interactions of the Yang-Mills field, resulting in a turbulent state of the quasi-thermal quark-gluon plasma \[^21, 22\]. Such turbulent color fields currently present a successful mechanism
for the fast equilibration of the medium \[18, 20\] and serve as a source of an anomalous shear viscosity \[13\]. This naturally explains the ideal fluid behavior of the medium. Here we explore the effect of turbulent color fields on the gluons radiated by a hard parton in the medium.

When the hard parton scatters transverse (in the medium co-moving frame) to the beam direction, its accompanying halo of soft radiated gluons are deflected by the turbulent color fields as the parton traverses the medium. The deflection pattern is not isotropic around the jet axis, because the direction of polarization of the instability driven color fields is not totally random. Detailed studies of the instability pattern \[19\] have shown that color magnetic field modes polarized transversely to the beam direction exhibit the largest growth rates and, therefore, dominate the field configurations at early times. The dominance of transversely polarized color-magnetic fields results in the preferential deflection of transversely propagating partons in the direction of the beam axis, with a deflection angle that changes inversely with the parton momentum. The resulting pattern will be one in which gluons of moderate \( p_T \) from the jet cone will fan out along the beam axis to form a ridge as it is observed in the RHIC experiments.

In order to be deflected by the soft color fields, the gluon must be radiated within the medium. Denoting the gluon’s momentum along the jet axis by \( p_T \) and its momentum components perpendicular to the jet by \( l_1^2 \), the formation time of the gluon is \( t_f \sim p_T/l_1^2 \). Therefore, the fields will most strongly affect those gluons which carry a small fraction of the jet’s longitudinal momentum (with short formation times) and are emitted at a substantial angle with respect to the jet axis. These are precisely the characteristics of the gluons radiated due to the scattering of the original parton inside the medium.

Consider a hard scattering event with induced radiation as sketched in Fig. 1. A jet initiating parton is produced when an incoming light-like parton is struck by a virtual state with forward energy \( E \) and virtuality \( Q^2 \). The final state parton may be produced off-shell by up to \( Q^2 \). In cases where it is off-shell, it may reduce its virtuality by radiating a gluon immediately. It may be produced closer to its on-shell condition and re-scatter multiple times prior to radiating the gluon. Following the radiation, both remnants of the original parton will multiply scatter off the soft color fields present in the medium. The amplitudes for multiple scattering of the hard parton before and after the emission of the gluon, as well as the scattering of the radiated gluon itself will lead to a destructive interference in the forward collinear region of the gluon phase space, the Landau-Pomeranchuck-Migdal (LPM) effect. This interference suppresses the production of very collinear radiation with long formation lengths. As a result the final radiation is produced over a short distance and then decoheres from the initiating parton. The yield of inclusive gluon radiation in this setting has been studied extensively \[4, 5, 23, 24, 25\]. Here, we are interested in the more exclusive probability of emission of a gluon with large transverse momentum relative to the parent parton.

To estimate this distribution, we adopt a simplified factorized approach illustrated in Fig. 1. The short distance process which describes the production of the parton and its multiple scattering in the dense matter followed by emission of the hard gluon is represented by the dark blob. It includes the two processes commonly referred to as hard-hard and hard-soft double scattering \[23\] and whose interference leads to the LPM effect for very collinear radiation \[23, 24\]. The subsequent soft scattering of the gluon and the parent parton in the medium is treated as independent from the production and radiation process. The factorization is indicated by the rectangular boxes in Fig. 1.

The remnant jet and the radiated hard gluon then attempt to exit the medium prior to fragmenting into a jet of hadrons. On their way out, these may still endure multiple soft scattering off the dense gluonic field of the medium. Such space-like exchanges are most often too soft to induce further radiation, however, the cumulative effect of many such scatterings may lead to a non-negligible deflection away from the direction of propagation. While such further re-scattering has a diminished role in the modification of the fragmentation function of the parton \[24\], its additive effect on the relative transverse momentum of the radiated gluon with respect to the leading parton is more pronounced.

![Fig. 1: A hard scattering followed by multiple soft re-scattering diagram](image)

To obtain the triggered gluon distribution before the final-state broadening, we take the ratio of the cross section \( \sigma_{qg} \) for radiating a gluon with momentum \( (p_{T_2}, l_1) \) in the short distance process depicted in the left-hand square in Fig. 1 and the inclusive cross section \( \sigma_q \) for production of quark with transverse momentum \( p_{T_1} \) and rapidity \( y \). This differential triggered distribution at a fixed impact parameter \( b \) and transverse location \( r \) is given as Ref. \[28\].

\[
\frac{d^2\sigma_{qg}(p_{T_2}, l_1)}{d\sigma_q} = C(p_{T_1}, p_{T_2}) \frac{\alpha_s}{2\pi} \frac{1}{l_1^2} \int_0^{\zeta_{\text{max}}(r)} d\zeta \rho(\mathbf{r} + \hat{n}\zeta) \zeta_0 \left[ 2 - 2\cos(\eta_L\zeta) \right].
\]
In the above equation, $\zeta$ is the distance traveled by the produced partons from the primary vertex along the jet axis $\hat{n}$ prior to the scattering in the medium. The maximum allowed value $\zeta_{\text{max}}$ for $\zeta$ is the distance from $r$ to the surface. The factor $\eta_L = p_z^2/(2p_T z_1)$ is the inverse of the formation time of a gluon with momentum $l_\perp$ transverse to the jet axis. The final quark momentum fraction is $z_1 = p_T/(p_T + p_T)$. The overall constant $C(p_T, p_T)$ accounts for the fact that the final parton momentum in the numerator and denominator of (1) is different.

The gluon distribution of Eq. (1) is now used as the starting point $\bar{f}(t = t_0)$ for the time evolution of the cone of radiated gluons around the leading parton due to the influence of turbulent color fields. Note, that this distribution includes solely those gluons whose production is induced by scattering in the medium and does not include any vacuum contribution to the radiation cone. In order to calculate the deflection of the gluons by turbulent color fields, we employ a Fokker-Plank equation:

$$\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_r - \nabla_p D(p, t) \nabla_p \bar{f} = C[\bar{f}], \quad (2)$$

with the average parton phase space distribution $f(p, r, t)$ and the diffusion tensor \[13, 29\]

$$D_{ij} = \int_{-\infty}^{t} dt' \left( F_i(\bar{r}(t'), \bar{t}')F_j(\bar{r}(t), t) \right). \quad (3)$$

Here $\mathbf{F} = \mathbf{gQ}^2(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ is the color Lorentz force generated by the turbulent color fields, and $C[\bar{f}]$ denotes the collision term. $D_{ij}$ is related to the usual transport coefficient for radiative energy loss (jet quenching coefficient) $\hat{q}$ via:

$$\hat{q} = -\hat{g}^{ij}D_{ij}, \quad (4)$$

The above equation may be interpreted as a generalization of the scalar transport coefficient $\hat{q}$ to a tensor $\hat{q}_{\alpha\beta}$. The tensorial form does not influence the total energy lost. However, it does influence the angular distribution of emitted radiation. Such an effect is not included in Eq. (1) and will be treated in a forthcoming publication.

The current approach explores the effect of the anisotropy of $D_{ij}$ on the radiated gluon after its decoherence from the leading parton.

For random transversely polarized color-magnetic fields, the diffusion term may be expressed in a simplified form \[29\]:

$$\nabla_p D(p) \nabla_p = \frac{-g^2Q^2(B^2)\tau_m}{2(N_c^2 - 1)E_p^2} \left[ L^{(p)}_\perp \right]^2 = \gamma(p) \left[ L^{(p)}_\perp \right]^2, \quad (5)$$

where $N_c = 3$ is the number of colors, the index $\perp$ denotes the vector components transverse to the beam axis, and $\tau_m$ is the autocorrelation length (or time) of the color-magnetic field along the trajectory of the parton. $E_p$ is the energy of a parton with momentum $p$ and $\mathbf{v} = p/E_p$ its velocity. $L^{(p)} = -i\mathbf{p} \times \nabla_p$ denotes the generator of rotations in momentum space.

In the case of interest here, the distribution $f(p, t_0)$ represents the gluons in the radiation cone of the primary hard parton, which have already decohered from the parent parton, as described above. Given the form of the diffusion term, the evolution equation is best solved by decomposing the momentum distribution function in terms of spherical harmonics:

$$f(p, t) = \sum_{l,m} a_{lm}(p, t) Y^l_m(\theta_p, \phi_p). \quad (6)$$

In the above equation, $l, m$ denote the total and $z$-component of the angular momentum in momentum space. The evolution equations of the various components $a_{lm}$, obtained by substituting (6) into (2), is given by

$$a_{lm}(p, t) = a_{lm}(p, t_0) e^{-\gamma(p)[l(l+1)-m^2]/(t-t_0)}. \quad (7)$$

Only the spherically symmetric mode $l = m = 0$ is unaffected by the evolution, while all other modes decay in time. Among these, modes with $m = 0$ for a given $l$ decay faster than those with $m \neq 0$. As a result, the distribution begins to broaden in the $z$-direction (beam direction). The evolution time is limited by the depth at which the partons were produced and is thus finite. Partons produced at greater depth therefore experience greater broadening along the beam direction than those produced closer to the surface.

The results of our calculations are presented in Fig. 2. These show the unnormalized associated gluon distribution around a jet initiating quark as a function of rapidity $\eta$ for a fixed azimuthal angle $\phi = 0$, and as a function of $\phi$ at fixed $\eta = 0$. The dashed (dotted) lines show the input distributions $\bar{f}(t_0)$ obtained from expression (1); the solid lines show the modified distributions at the moment when the gluons leave the medium. Both plots correspond to the original jet vertex formed at a depth of 3 fm. The plots in the top part of Fig. 2 correspond to an original jet transverse momentum $p_T + p_T = 10$ GeV/c, while those in the lower part correspond to an original jet momentum of $p_T + p_T = 20$ GeV. In both cases, the gluon carries a forward momentum fraction of $z_2 = 0.4$. We assumed in our calculation that the soft gluon density in the medium varies with time in accordance with the boost invariant longitudinal expansion of an ideal ultrarelativistic liquid. The initial value of the transport coefficient associated with the intensity of turbulent color fields was taken to be $\hat{q} = 2.2$ GeV$^2$/fm. As is clearly visible in Fig. 2, the associated gluon distribution is considerably broadened in $\eta$ but not in $\phi$. The extent of the broadening depends on the energy of the parton and drops with increasing energy.

In summary, we have shown that near side energy loss in terms of radiated gluons in the presence of turbulent color fields can explain the experimentally observed
longitudinal broadening of jet cones in heavy-ion collisions. Our model predicts that this broadening decreases with increasing energy of the associated parton (the radiated gluon which fragments to produce the associated hadron). An extraction of the transport coefficient $\hat{q}$ from a measurement of such broadening in experiment will depend on the energy of the radiated gluon and, by extension, on the hadronization mechanism. The momentum range of the partons contained in the rapidity broadened jet cone is in the regime where hadronization occurs dominantly via parton recombination. We therefore expect an enhanced baryon to meson ratio of hadrons contained in the ridge.

As mentioned earlier, turbulent color fields generated by Weibel instabilities in the early quark-gluon plasma have recently been proposed as a mechanism for the early thermalization of the matter produced at RHIC and as the source of a small anomalous viscosity. The current work has identified the broadening of triggered jet cones in rapidity as a more directly observable manifestation of the presence of turbulent color fields in the matter. If our interpretation can be confirmed by additional measurements, it will demonstrate the existence of plasma instabilities generating these turbulent fields and establish their role in the transport of hard probes through a quark-gluon plasma.

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**References:**

1. K. Adcox et al., Phys. Rev. Lett. 88, 022301 (2002).
2. C. Adler et al., Phys. Rev. Lett. 89 202301 (2002).
3. M. Gyulassy and X. Wang, Nucl. Phys. 420 583 (1994).
4. R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne and D. Schiff, Nucl. Phys. B 483 291 (1997).
5. B. G. Zakharov, JETP Lett. 65 615 (1997).
6. M. G. Mustafa, Phys. Rev. C 72, 014905 (2005).
7. J. Adams et al., arXiv:nucl-ex/0607003
8. J. Adams et al., Phys. Rev. Lett. 95, 152301 (2005).
9. J. Putschke, arXiv:nucl-ex/0701074
10. C. B. Chiu and R. C. Hwa, Phys. Rev. C 72, 034903 (2005).
11. A. Majumder, E. Wang and X. N. Wang, arXiv:nucl-th/0412061
12. A. Majumder and X. N. Wang, Phys. Rev. D 70, 014007 (2004); Phys. Rev. D 72, 034007 (2005).
13. N. Armesto, C. A. Salgado and U. A. Wiedemann, Phys. Rev. C 72, 064910 (2005).
14. P. Romatschke, Phys. Rev. C 75, 014901 (2007).
15. M. Asakawa, S. A. Bass and B. Müller, Phys. Rev. Lett. 96, 252301 (2006).
16. D. Teaney, Phys. Rev. C 68, 034913 (2003).
17. R. Baier and P. Romatschke, arXiv:nucl-th/0610108
18. J. Randrup and S. Mrowczynski, Phys. Rev. C 68, 034909 (2003).
19. P. Romatschke and M. Strickland, Phys. Rev. D 68, 036004 (2003).
20. P. Arnold, J. Lenaghan and G. D. Moore, JHEP 0308, 002 (2003).
21. P. Arnold and G. D. Moore, Phys. Rev. D 73, 025006 (2006);
22. P. Arnold and G. D. Moore, Phys. Rev. D 73, 052013 (2006).
23. M. Gyulassy, P. Levai and I. Vitev, Nucl. Phys. B 594, 371 (2001).
24. U. A. Wiedemann, Nucl. Phys. B 588, 303 (2000).
25. X. N. Wang and X. f. Guo, Nucl. Phys. A 696, 788 (2001).
26. X. f. Guo and X. N. Wang, Phys. Rev. Lett. 85, 3591 (2000).
27. Y. Guo, B. W. Zhang and E. Wang, Phys. Lett. B 641, 38 (2006).
28. A. Majumder and X. N. Wang, Phys. Rev. C 73, 051901 (2006).
29. M. Asakawa, S. A. Bass and B. Muller, Prog. Theor. Phys. 116, 725 (2007).