Magnetic dipole excitations as reference for the spin-orbit interaction in nuclei

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Magnetic dipole (M1) excitation is the leading mode of multi-fermion excitations induced by the magnetic field. This mode is closely connected with the spin-orbit (SO) interaction, that is of general relevance in atomic, molecular, nuclear, and condensed matter physics as well as in many applications. We study a possible relation between the nuclear M1 response and the energy splitting by the SO interaction, by employing the framework of relativistic nuclear energy density functional (RNEDF), which naturally describes the SO interaction due to the Dirac-Lorentz structure of the formalism. The relativistic Hartree-Bogoliubov method (RHB) is used to determine the nuclear ground state and single (quasi)particle energies, while the relativistic quasiparticle random phase approximation is established for the description of M1-excitation properties. It is shown that the analysis of M1 mode in the RNEDF framework provides a suitable tool to constrain the SO interaction, i.e., the SO splittings of the states that govern the respective M1 transition.

PACS numbers: 05.30.Fk, 21.10.Pc, 21.60.-n, 23.20.-g

Dynamics of multi-fermion interacting system represents a fundamental challenge in physics, being responsible for various phenomena in nature. Collective excitations in atomic nuclei represent one example, that necessitate the consideration of (i) the fermionic character of nucleons, (ii) unperturbative, effective nuclear interactions, and (iii) collective motion of A nucleons, within a unified framework.

In order to understand the underlying properties of nucleus, the single-particle (SP) picture within the mean-field approximation has been established. One of the fundamental properties of the SP energy levels is the spin-orbit (SO) splitting, introduced by Maria Goeppert Mayer and J. Hans D. Jensen, which is essential to explain the so-called magic numbers in nuclei [1,2]. The SO interaction is also essential in other areas of physics and beyond. In atomic systems, it originates from the interaction of an electron’s spin with the magnetic moment from the orbital motion of the electron. The basis of the Hund’s rules necessary for understanding atomic energy levels is the SO coupling [3]. The effects of SO coupling have also been observed in molecular systems [4]. Spin-orbit coupled Bose-Einstein condensates, confined within an optical cavity, lead to interesting phenomena such as creation of Meissner-like effects, topological superfluids, and exotic quantum Hall states [5]. While the SO interaction originally played only a marginal role in condensed matter physics, at present it is involved in a broad range of phenomena, and it is expected to become a cornerstone of the future technologies [6].

In modern nuclear physics, one open question is how the SO coupling and the respective shell effects evolve from the valley of stability toward exotic nuclei with large neutron-to-proton number ratios [7]. It is also essential in description of nuclear processes that involve unstable nuclei of relevance for nuclear astrophysics, e.g., in modeling supernova explosion and neutron-star mergers, including the r-process nucleosynthesis responsible for the production of about half of chemical elements heavier than iron [8,9]. In weakly bound nuclei the weakening of the shell effects and the SO coupling is predicted due to the diffuse surface [10,11]. The microscopic origin of the SO coupling has been a subject in many studies involving various effects, such as three-nucleon force, tensor force, and nucleon-nucleon interaction based on meson exchange [12,13].

The description of the nuclear-SO splitting has been rather challenging on the experimental side, involving e.g., transfer and knockout reactions and proton resonant scattering [17–19]. However, some uncertainties often remain due to the model dependent analysis of the experimental data. Therefore, it is essential to address the problem of the SO splitting from different perspectives in order to provide complete understanding of its nature. The aim of this Letter is to introduce a novel approach to determine the SO splitting in nuclei, based on the magnetic dipole (M1) mode of excitation. This mode has been extensively studied (see recent review in Ref. [20]), and the relevance of the SO splitting for the final pattern of the M1 spectrum has been recently emphasized in studies based on Skyrme functionals [21,22].

The relativistic nuclear energy-density functional (RNEDF) represents a suitable framework for our purpose. Within this approach, the theory is Lorentz invariant, it obeys causality, and mesonic degrees of freedom are treated explicitly. Relativistic dynamics determines important phenomena in low-energy nuclear structure. Those include the scalar and vector potentials that result in the strong SO splitting and its isospin dependence, relativistic saturation mechanisms, pseudo-spin symmetry and nuclear magnetism in rotating nuclei [23,24]. Since the relativistic theory provides a natural explanation of the strong SO splitting in nuclei emerging from its Dirac-Lorentz structure and degrees of freedom that
govern the interaction between nucleons, it is especially
suitable for this study. The present analysis is based
on the RNEDF framework only, however, the introduced
method is of general importance for nuclear physics and
could also be exploited by other theoretical approaches,
e.g., based on Skyrme and Gogny functionals, as well as
by the \textit{ab initio} approaches to the nuclear structure.

The M1 transitions constitute the leading mode of
multi-fermion excitations induced by the magnetic field,
closely connected with the SO splitting. The M1 opera-
tor reads \cite{26, 27}

\[
\hat{P}_{\nu}(M1) = \mu_N \sqrt{3 \over 4\pi} \left( g_l \hat{l}_\nu + g_s \hat{s}_\nu \right),
\]  

(1)

in the SP form including the spin \(\hat{s}_\nu\) and orbital angular
momentum \(\hat{l}_\nu\) operators with \(\nu = 0\) or \(\pm 1\). Here \(\mu_N\)
is nuclear magneton, whereas \(g_\nu\) coefficients are given as
\(g_l = 1 (0)\) and \(g_s = 5.586 \ (-3.826)\) for the bare proton
(neutron) \cite{26, 27}. The reduced matrix element of \(\hat{P}_{\nu}\)
satisfies \cite{28}

\[
\langle jf(l_f) \parallel \hat{P}_{\nu}(M1) \parallel ji(l_i) \rangle \propto \delta_{j,l},
\]  

(2)

where \(l\) and \(j\) are the SP orbital and spin-coupled
angular-momentum quantum numbers. This becomes
non-zero only between the SO-partner orbits with \(l_i = l_f\).
Thus, the M1 excitation reflects the structure of SO-
splitting, i.e., in its absence, the M1 response seldom ap-
ppears. Considering the RNEDF’s ability to naturally ex-
plain the SO splitting, its systematic application to the
M1 excitations may provide the quantitative information
on the actual SO splitting in the nucleus. On the experi-
mental side, the M1 measurement requires the imple-
mentation of dedicated techniques, because of the hindrance
of the M1 transitions by the other competing modes \cite{29}.

The M1 response in nuclei can be investigated experi-
mentally using various probes, e.g., electrons, photons
and hadrons \cite{30, 31, 32}.

In this Letter, we perform the systematic computa-
tion of the nuclear M1 excitations in the Ca isotope
chain \((Z=20)\) and \(N=20\) isotones, based on the RNEDF.
Through this analysis, we aim to demonstrate the link
between the M1 response and the evolution of the SO
splitting. We employ the CGS-Gauss system of units.
The elementary charge and nuclear magneton are given
as \(e^2 \approx \hbar c/137\) and \(\mu_N = \hbar c/(2cm_{\text{proton}}) \approx 0.105\)
e-fm, respectively. The spherical symmetry is assumed.

The RNEDF framework employed in this study is
based on the relativistic four-fermion-contact interaction.
That, in a complete analogy to the meson-
exchange phenomenology, includes the isoscalar-scalar,
isoscalar-vector, and isovector-vector channels with their
density-dependent couplings of the interaction terms
(DD-PC1) \cite{31, 32}. In addition, the coupling of protons
to the electromagnetic field, and the derivative term ne-
cessary for a quantitative description of nuclear density
distribution and radii are also taken into account \cite{31}.
For the description of open-shell nuclei that necessitate
the inclusion of the pairing correlations, the relativistic
Hartree-Bogoliubov (RHB) model is used \cite{22, 31}.

For the pairing correlations, the pairing part of Gogny-
D1S force is employed \cite{33}. This force has been utilized
to reproduce the empirical pairing gaps in various nuclei.
Note that, as defined in the original paper \cite{33},
the D1S force works as an attraction, but only when the two
protons or neutrons are coupled to have \(S_{12} = 0\) (SO pair).
Thus, this pairing model is “S0-pair promoting”, whereas
the S1 pairing with \(S_{12} = 1\) should be suppressed.

In the small amplitude limit, collective excitations can
be described by the relativistic (quasiparticle) random phase
approximation R(Q)RPA \cite{34, 35}. Since more details
about this framework are given in the forthcoming
publication \cite{36}, here we give only a brief descrip-
tion. For the present study of M1 excitations, character-
ized as unnatural parity transitions, the RHB+R(Q)RPA
framework has been established for the relativistic point-
coupling interaction, for which the DD-PC1 parameteri-
Zation is used \cite{32}. In addition, the residual interaction
in the R(Q)RPA is extended by the IV-PV coupling term
of the effective Lagrangian,

\[
\mathcal{L}_{\text{IV-PV}} = -\alpha_{\text{IV-PV}} \sum_{i} \left[ \bar{\psi} \gamma_5 \gamma_{\mu} \bar{\tau} \psi \right] \left[ \bar{\psi} \gamma_5 \gamma_{\mu} \bar{\tau} \psi \right].
\]  

(3)

Since this IV-PV term would lead to the parity violat-
ing mean-field at the Hartree level for the \(0^+\) nuclear
ground state, it contributes only to the R(Q)RPA equa-
tions for unnatural parity transitions, i.e. \(1^+\) excitation
of M1 type. For the IV-PV coupling, we use the value
\(\alpha_{\text{IV-PV}} = 0.53 \text{ MeV} fm^{-3}\) which was optimized to
the \(1^+\)-excited states of \(^{48}\text{Ca}\) \cite{37} and \(^{208}\text{Pb}\) \cite{38}.

In the present analysis, the M1 excitations up to the one-body-operator level are considered. Namely, the
\(A\)-nucleon M1 operator is given as \(\hat{Q}_{\nu}(M1) \equiv
\sum_{k=A} \hat{P}_{\nu}^{(k)}(M1)\), where \(\hat{P}_{\nu}^{(k)}\)
is the SP-M1 operator of the \(k\)th nucleon. Its strength can be obtained as

\[
{dB_{\text{M1}} \over dE_\gamma} = \sum_i \delta(E_\gamma - \hbar \omega_i) \sum_{\nu} \left| \langle \omega_i \mid \hat{Q}_{\nu}(M1) \mid \Phi \rangle \right|^2,
\]  

(4)

for all the positive QRPA eigenvalues, \(\hbar \omega_i > 0\). Note
that, in this work, we neglect the effect of the meson-
exchange current as well as the couplings to complex
configurations \cite{30, 32, 33}, which need further multi-body
operations going beyond our present method.

In the following we show the results for the M1 transi-
tions of the even-even \(Z = 20\) (Ca) isotopes and \(N = 20\)
isotones from the \(0^+\)-ground to the \(1^+\)-excited states.
With the DD-PC1 and D1S parameterisations used for
the RNEDF and the pairing correlations, respectively, we
confirmed that the particle-bound systems in the ground
state are obtained up to \(N = 16-44\) for Ca isotopes,
and \(Z = 10-24\) for \(N = 20\) isotones.
also appears in of their radial wave functions is small. The same feature transitions are strongly suppressed because the overlap width of 1 Lorentzian distribution with the selected value of the + for the M1 transition: in the ground state of 2 forbidden. The allowed transition is e.g. from the bound pied, and thus, the M1 transition between these orbits is & 1 d(d)). and (1 & 1 d(2)) transition strength distribution, (1 & 1 d(2)) has finite contribution in the main M1 peak. This is because of the smearing of the Fermi surface in the isochains, the no-pairing M1 response can be explained purely from the neutron transitions (1d(5/2 → 1d(5/2)), (1f(7/2 → 1f(5/2)), and (1g(9/2 → 1g(7/2)), respectively. This behavior can be indeed expected from the ordering of the nuclear-shell orbits. For the 50−52 Ca isotopes, the second, low-energy peak appears due to the (2p(3/2 → 2p(1/2)) transition. Notice also that, since the higher SO-partner orbits, 1f(5/2 and/or 2p(1/2), are occupied in 54−64 Ca, the M1 response is consistently reduced by these blocked neutron states. The same feature can be found in the N = 20 isotones, but in terms of the valence protons in the corresponding orbits.

We also address the case where the pairing correlations are not taken into account (see Fig. 1). In this setting, there are no mixtures of different configurations in the M1 states, i.e. for the 36−38 Ca, 42−58 Ca, and 62−64 Ca isochains, the no-pairing M1 response can be explained purely from the neutron transitions (1d(5/2 → 1d(5/2)), (1f(7/2 → 1f(5/2)), and (1g(9/2 → 1g(7/2)), respectively. This behavior can be indeed expected from the ordering of the nuclear-shell orbits. For the 50−52 Ca isotopes, the second, low-energy peak appears due to the (2p(3/2 → 2p(1/2)) transition. Notice also that, since the higher SO-partner orbits, 1f(5/2 and/or 2p(1/2), are occupied in 54−64 Ca, the M1 response is consistently reduced by these blocked neutron states. The same feature can be found in the N = 20 isotones, but in terms of the valence protons in the corresponding orbits.

When the D1S-pairing interaction is included in the calculations, the M1 transition strength becomes partly reduced, as shown in Fig. 1. This can be understood from the S0-pair promoting ability of the D1S force: when the S0-pair component is dominant in the ground state, its M1 response is suppressed [45]. Also, the pairing correlation invokes the mixture of different SO-partner transitions: for example, in 42−46 Ca, we confirmed that the dominant component is still (1f(7/2 → 1f(5/2)), but simultaneously the component (1d(5/2 → 1d(3/2)) has finite contribution in the main M1 peak. This is because of the smearing of the Fermi surface in the

2p(1/2) and 1f(5/2) all neutron orbits are occupied. Consequently, within the one-body-operator analysis the nucleon number 20 and 40 are shown to be the “M1-silence” point [44]. The absence of M1 transitions is concluded also when the pairing energy is neglected.

Figure 1 displays the evolution of M1 response along the 36−64 Ca isotope chain, resulting in one remarkable peak in each system. This is attributable to the M1 excitation of valence neutrons, whereas 20 protons are M1-silent. The same conclusion applies to the excitation of valence neutrons, whereas 20 protons are also silent. The same feature can be found in the 2p(1/2) and 1f(5/2) all neutron orbits are occupied. Consequently, within the one-body-operator analysis the nucleon number 20 and 40 are shown to be the “M1-silence” point [44]. The absence of M1 transitions is concluded also when the pairing energy is neglected.

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The M1-excitation energies of Ca isotopes and the corresponding SO-splittings. The calculations are based on the RHB plus RQRPA using DD-PC1 parameterization and D1S pairing force. The respective \((nlj)\) quantum numbers of the SO-partner states denote each plot.

RHB solution due to the pairing correlations, and thus, the \(1d_{3/2}\) orbit is not fully occupied. The M1-excitation energy is in general shifted to the higher energy region by the pairing correlations. We note that some variation of the results is possible depending on the choice of the pairing model [45].

Figure 2 shows the sum of the M1 transition strength, \(m_0 = \int dE \frac{d\gamma}{dE}\), for Ca isotopes. The results show a strong dependence of the \(m_0\) value on the M1-active nucleons, supported by the analysis of relevant two-quasiparticle configurations in the main M1 peaks. First, we focus on the case without the pairing correlations. For \(^{40-60}\)Ca, the M1 excitations are dominated by the transitions of \((1f_{7/2} \rightarrow 1f_{5/2})\) and \((2p_{3/2} \rightarrow 2p_{1/2})\). Thus, the \(m_0\) value simply increases or decreases according to the interplay of active and blocking neutrons in these orbits. Second, when the pairing correlations are included, the \(m_0\) value is commonly reduced in comparison to the no-pairing result. This is consistent to the strength distributions shown in Fig. 1.

In Fig. 3 the relation between the excitation energies of M1 mode and the intrinsic SO splittings are shown for the Ca isotope chain. Here the M1-peak positions \(\langle E_\gamma \rangle\) are calculated using the RQRPA, whereas the SO gap energies \(\Delta E_{LS}\) between the corresponding partner orbits are based on the RHB quasiparticle canonical states, used to construct the RQRPA two-quasiparticle configuration space [44]. The analysis of the structure of M1 states identifies the major SP transition in different mass ranges of Ca isotopes, as denoted inside the figure. The deviations between the M1 energies and the respective SO splittings result from the residual RQRPA interaction, mainly from the IV-PV interaction term and the pairing interaction. In the case of unperturbed response, when the residual interaction is set to zero, the M1-excitation energies coincide with the SO splittings. Consequently, the theoretical model, which accurately reproduces the experimental M1-excitation energies, provides a measure of the SO splittings that govern the M1 transition.

Before closing our discussion, we note that the quenching effect of M1 transition strength is left for the future study. Indeed, the calculated \(B(M1)\) values overestimate the experimental M1 strength, since we have used the \(g\) factors of bare nucleons. For adjusting the calculated \(B(M1)\) values to the experimental data, one usually needs the quenching factors that may have rather arbitrary values [20]. In addition, for more reliable consideration of this effect, going beyond the QRPA may also be needed, including couplings to the complex configurations, similar to the studies of Gamow-Teller transitions [42, 43]. This rather demanding task goes beyond the present study. In addition, the meson-exchange current effect is neither yet considered [20, 33, 41], and is awaiting the future progress. For neutron-rich nuclei, the deformation should also be considered in the future studies. These points, however, are not expected to influence considerably the main results of the present analysis, that is based on the main-peak excitation energies and the SO splittings.

In conclusion, the M1 excitations are discussed in the relativistic multi-fermion picture in relation to the SO splittings in finite nuclei. For the study of unnatural parity transitions of M1 type, the RQRPA has been developed in the RNEDF framework with the density-dependent point-coupling interaction. The nucleon numbers 20 and 40 are shown to be the M1-silence points up to the one-body operator level within the spherical symmetry. It is shown that the pairing correlations play an important role to describe the M1-excitation strength. The relation between the SO splitting and M1-excitation energy is investigated: the M1 excitations are governed by the transitions between the SO-partner states, and their properties depend on the respective SP energies and occupation probabilities. The M1-excitation energy does not coincide exactly with the respective SO splitting, i.e., the difference originate in the residual RQRPA interaction, mainly in the IV-PV channel and the pairing correlations that build the M1 mode on top of the unperturbed response. The theoretical framework, which reproduces the experimental M1-excitation energies, may provide a measure of the SO splittings that govern the M1 transition under consideration. In order to infer the SO splitting from the M1-reference data, double-magic nuclei without the pairing correlations can be used to conduct a fine tuning of the IV-PV residual interaction term to their M1 energies. The model with an accurate description of the M1-excitation energies allow one to conclude the optimal SO energies and respective gaps in all nuclei of interest. Theoretical approaches including the pairing correlations enable the implementation of the same method to open-shell nuclei. For this purpose, the systematic, experimental measurements of the M1 excitations in isotope chains are on serious demand.
It would also be interesting to extend our approach to other theory frameworks and effective interactions. By exploiting possible improvements from the theoretical side and novel experimental data, we expect that the method introduced in this work provides a suitable way to utilize the M1 excitations as an important reference for an alternative approach to the SO interaction in finite nuclei.

We especially thank Tamara Nikšić and Markus Kortelainen for fruitful discussions. This work is supported by the "QuantiXLie Centre of Excellence", a project co-financed by the Croatian Government and European Union through the European Regional Development Fund, the Competitiveness and Cohesion Operational Programme (KK.01.1.1.01).

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