Entropy of bound states in the matrix description of M5-branes

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Abstract

Like the M-theory itself also the worldvolume theory of the M5-brane contains brane excitations, which can be extracted from the supersymmetry algebra. Bound states (intersecting branes) of the worldvolume can be translated into bound states of 11-d SUGRA. In this paper, we discuss the matrix description for these bound states and their entropy (= degeneracy). In order to decouple the worldvolume field theory from the bulk gravity we have especially to assume that all charges are large, which gives a nice agreement with the Bekenstein-Hawking entropy of black holes.

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1. Introduction

The worldvolume theory of the M5 brane was the subject of many recent publications \[1, 2, 3, 4\]. As introduction we will briefly review some of the results of these papers. The worldvolume theory of an M5 brane is a (0,2) supersymmetric theory. It has been shown in \[1\] that the most general (0,2) supersymmetry algebra in 6 dimensions is of the following form:

\[
\{ Q^I_{\alpha}, Q^J_{\beta} \} = \Omega^{IJ}_{[\alpha\beta]} + Y^{IJ}_{[\alpha\beta]} + Z^{IJ}_{(\alpha\beta)}
\]  

(1)

Here, \( \alpha, \beta = 1, \ldots, 4 \) are spinor indices of the 5+1 dimensional Lorentz group and \( I, J = 1, \ldots, 4 \) are indices of the R-symmetry group \( Sp(2) \). The R-symmetry group has a geometrical interpretation as the rotation group transversal to the M5 brane, \( Sp(2) \sim spin(5) \). The charge \( Y \) is a worldvolume 1-form \( \sim (\gamma^m)_{\alpha\beta} Y^{IJ}_m \). It transforms as the 5 of R-symmetry group. The other charge, \( Z \), is a selfdual 3-form on the worldvolume \( \sim (\gamma^{mnp})_{\alpha\beta} Z^{IJ}_{mnp} \) and transforms under the 10 of the R-symmetry group. The existence of these charges suggest the possibilities of p-branes, which are BPS states on the M5 worldvolume. Especially, in addition to strings on the M5 brane we expect to have 3 branes living in the M5 brane worldvolume. As was pointed out in \[1\] it is also possible to add a 5-form charge, but this will not give additional degrees of freedom. A 5-form charge leads to a worldvolume 5 brane filling the worldvolume of the M5 brane. We can conclude from the algebra, that we have a certain “degeneracy”: For each worldvolume direction we have 5 strings of different R-charge and for three worldvolume directions we have 10 3branes of different R-charge.

The properties of the algebra have a direct relation to the classification of 1/4 BPS states in 11 dimensional M-theory. We know the “intersection rules” of the M5 branes with other M theory branes, e.g. an M5 brane and an M2 brane can intersect over a string, leading to a bound state at threshold. If the intersection region is two dimensional, such that the M2 brane lies inside the M5 brane, only a non-threshold bound state is possible. The strings coming from membranes intersecting M5 branes precisely correspond to the strings expected from the algebra. We have already mentioned that the R-symmetry group corresponds to the rotations in the transversal space. Therefore, the R-charge of the strings leads to a space time interpretation of the strings: The string states transform in the 5 representation of the R-symmetry group, which corresponds to the fact that the membranes leading to the strings have one transversal direction. Similarly, two M5 branes can intersect over a 3brane. It extends in three worldvolume directions and further two transversal directions. There are 10 possibilities to pick two transversal directions, corresponding to the fact that the charge \( Z \) carries an index of the 10 representation of the R-symmetry group.

All these states can also be identified by looking on the worldvolume field theory, which has a selfdual 3-form field strength corresponding to the string excitations and it has 5 scalars, which parameterize the position of the brane in the 5 transversal directions. In order get the 5-form field strength that couples to the 3brane we have to dualize one these scalars and, in addition, supersymmetry requires that one has to turns on a further scalar.
Denoting these two scalars by $U$ and $V$ the 3brane stretching along $x^1, x^2, x^3$ is described by the fields strength

$$G_{01234} = \epsilon_{ij} \partial_j U = \pm \partial_i V. \quad (2)$$

where $i, j = 1, 2$ and $U$ and $V$ are harmonic. The gauge symmetry implies that $U \simeq U + \text{const.}$ and $V \simeq V + \text{const.}$, where as consequence of the Dirac quantisation the constants have to be integer valued. Since these scalars parameterize the transverse position of the brane, this means that a 3brane needs two compact transverse directions and if these directions decompactify it becomes infinitely heavy. This fits to the statement above, that the 3brane corresponds to an intersecting 5brane that stretches also in $U$ and $V$, which are called compact scalars of the 5brane worldvolume.

In our paper we will be especially interested in 4-dimensional black holes coming from brane intersections involving a 5 brane. As we have seen, these brane intersections can also be interpreted as world volume p-branes on the M5 brane.

The worldvolume theory of M5 branes also plays a role in Matrix theory\[5\]. In particular, there are “little strings theories” – the iib strings discussed in \[6, 7\] – which also have worldvolume threebranes and fivebranes. In the next section we will discuss the (iib) strings as a Matrix model. We will in particular discuss the Matrix picture of black holes involving 5 branes. In this way, the pictures in space time and the Matrix model look very similar. In the last section we compute the entropy of black holes from the Matrix theory.

2. Matrix description

In space time we want to consider (for example) the configuration of 3 M5 branes, where each pair of M5 branes intersects over a 3brane and the common intersection of the three M5 branes is a string. This motivates us to look for a matrix description in terms of the M5 brane with two compact scalars instead of the usual matrix description in terms of the IIA D6 brane at strong coupling \[8\]. On the worldvolume of the M5 brane we have strings coming from membranes wrapping one of the compact directions and we have 3branes from 5 branes wrapping both compact directions. So our base space configuration looks very similar to the space time configuration.

Let us first discuss the matrix description in terms of the M5 brane with compact scalars. It was shown in \[9\] that the matrix description of M-theory on $T^4$ is given in terms of the strongly coupled IIA D4 brane, which is better thought of as M5 brane. In \[10\] it was discussed that the light cone quantisation of M-theory can be discussed in terms of an $\tilde{M}$ theory, which is compactified on a space like circle. The limit we need to take in $\tilde{M}$ is

$$R, l_p, L_i \to 0 \quad \text{but} \quad \frac{R}{l_p^2}, \frac{L_i}{l_p} \text{ fix} \quad \text{(3)}$$

\[\text{b}\]We are grateful to A. Karch for numerous discussions on this limit.
As $R$ is taken to zero, M theory turns to IIA theory with coupling $g_s^2 = \frac{R^3}{l_p^3}$ and string tension $M_s^2 = \frac{R}{l_p^3}$. Because the transversal radii go to zero we apply T-duality to come to a better description. In our approach we apply T-duality 4 times giving us the IIA D4 brane at strong coupling and the new IIA quantities are

\begin{align*}
\Sigma_i &= \frac{l_p^3}{RL_i} \\
M_s^2 &= \frac{R}{l_p^3} \\
g_s^2 &= \frac{l_p^9}{RV^2} \\
(M_P^{(10)})^8 &= \frac{V^2 R^5}{l_p^{21}} \\
g_{YM}^2 &= \frac{l_p^6}{RL_1 L_2 L_3 L_4},
\end{align*}

where $M_P$ denotes the Planck mass and $g_{YM}$ the Yang-Mills coupling on the brane. This coupling is kept fixed in our limit, but the string coupling goes to infinity. We therefore go to M-theory, which leads to a new world-volume direction $\Sigma_5$, with

\begin{align*}
\Sigma_5 &= \frac{g_s M_s}{g_{YM}^2} = \frac{l_p^6}{RL_1 L_2 L_3 L_4}
\end{align*}

The eleven dimensional Planck Mass is related to the ten dimensional Planck mass by

\begin{align*}
M_P^{(11)} = g_s^{-\frac{11}{12}} M_P^{(10)}
\end{align*}

and using the relation (4) we obtain for the new Planck length $\tilde{l}_p \sim 1/M_P$

\begin{align*}
\tilde{l}_p^3 &= \frac{l_p^9}{VR^2}
\end{align*}

We want to consider this M-theory 5brane with two more compact directions. We didn’t T-dualize them, so we have

\begin{align*}
U &= L_5 \\
V &= L_6
\end{align*}

The limit (3) becomes now

\begin{align*}
\tilde{l}_p, U, V \to 0 \quad \text{but} \quad \frac{U}{\tilde{l}_p^3}, \frac{V}{\tilde{l}_p^3} \text{ fix.}
\end{align*}

Note that these fixed quantities can be interpreted as tensions of strings living on the M5 brane coming from membranes wrapping $U$ or $V$. Taken together, the matrix model consists of an M5 brane wrapping $\Sigma_1 \ldots \Sigma_5$ with two more compact directions $U$ and $V$.
in the limit (1). For convenience and later use we put together the matrix-model/space
time relations:
\[
\begin{align*}
\Sigma_i &= \frac{l^3_p}{RL_i}, \quad i = 1 \ldots 4 \\
\Sigma_5 &= \frac{l^6_p}{RL_1 L_2 L_3 L_4} \\
U &= L_5 \\
V &= L_6 \\
\tilde{l}^3_p &= \frac{l^9_p}{L_1 L_2 L_3 L_4 R^2}
\end{align*}
\] (8)

In the following we want to analyse the excitations of this model. They should correspond
to the energies of space time BPS states. In space time we expect 15 transversal wrapped
membranes, 6 transversal wrapped 5 branes and 6 momenta. For finite light like direction
we expect in addition 6 longitudinal membranes, 15 longitudinal 5 branes and 6 wrapped
KK6 monopoles. Futhermore, we expect a KK6 brane with NUT direction R and a
momentum mode of energy $1/R$. We start with the transversal branes, which correspond
to bound states of the M5 to other branes in the matrix model.

First, we consider $n$ membranes stuck to the M5 brane. The energy of the bound state is
\[
\lim \sqrt{\left(\frac{N \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5}{l^6_p}\right)^2 + \left(\frac{n \Sigma_i \Sigma_j}{l^3_p}\right)^2 - \frac{N \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5}{l^6_p} = \frac{n^2 \Sigma_i^2 \Sigma_j^2}{2N \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5}}
\] (9)

Here, lim indicates the limit discussed in detail above and $i, j \in 1 \ldots 5$. These bound
states can also be interpreted as fluxes of the field strength of the two form field living
on the world volume of the M5 brane. We now use (8) to calculate the masses of the
corresponding space time BPS particles. For $i, j \neq 5$ we obtain
\[
M_{kl} = \frac{n L_k L_l}{l^3_p} \quad k, l \in 1, \ldots 4 \quad k, l \neq i, j.
\] (10)

This gives us six transversal membranes. If $i$ (or $j$) equals 5, we obtain the kinetic energy
of a particle with momentum in the $L_j$ ($L_i$) direction:
\[
E = \frac{n^2 R}{2L_j^2} = \frac{p_j^2}{2p_{||}}
\] (11)

This gives us four transversal momenta.

Next, we consider $n$ M5 branes. They can only form bound states of finite energy with our
“fundamental” M5, if they wrap one of the transversal directions. Taking $U$, the energy is:
\[
\lim \sqrt{\left(\frac{N \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5}{l^6_p}\right)^2 + \left(\frac{nU \Sigma_i \Sigma_j \Sigma_k \Sigma_l}{l^6_p}\right)^2 - \frac{N \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5}{l^6_p}}
\] (12)
In space time for \( m = 1, 2, 3, 4 \) this leads to membranes wrapping \( L_5 \) and for \( m = 5 \) we obtain a M5 brane wrapping \( L_1, \ldots, L_5 \). Analogous results hold for a M5 brane wrapping \( V \). In this case we get four membranes wrapping \( L_6 \) and one direction in \( L_1, \ldots, L_4 \). In addition there is one M5 wrapping \( L_1, \ldots L_4, L_6 \).

Further states can be obtained from Kaluza Klein monopoles. They have a compact NUT-direction \( \Sigma_i \) and stretch in six (other) directions. Their tension is \( \Sigma_i^2 \) If the NUT direction is one of the directions \( 1, \ldots, 5 \) we can form bound states. The energy of these bound states is

\[
E = \frac{n^2}{2N} \left( \frac{U}{l_p^3} \right)^2 \left( \frac{V}{l_p^3} \right)^2 \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5.
\]

Converting to space time quantities, we obtain for \( i \neq 5 \)

\[
M_i = \frac{nL_5L_6L_kL_mL_n}{l_p^6},
\]

which are the masses of four transversal 5 branes. For \( i = 5 \) we get

\[
M_5 = \frac{nL_5L_6}{l_p^6}
\]

which is the mass of \( n \) membrane wrapping \( L_5 \) and \( L_6 \).

Finally, we have to consider the momenta along \( U \) and \( V \). They diverge, but give finite contributions together with the M5 brane. In this way we find the two missing space time momenta.

In this way, we found all BPS states we expected in space time from the matrix model.

Let us now come to objects whose energy is invariant under the limit (7) and it turns out that they can form bound states at threshold with the M5 brane. In space time, they give branes wrapping the light cone direction and in our case they are M2 branes wrapping one of the compact directions and M5 branes wrapping both compact directions. They correspond to strings and threebranes living on the worldvolume of the M5 brane. Furthermore, there are KK6 monopoles, whose NUT-direction is one of the compact directions \( U \) or \( V \) and which wrap all remaining directions \( \Sigma_i \). Especially, these branes stretch in \( \Sigma_1, \ldots, \Sigma_5 \) and therefore look like 5 branes on the M5 worldvolume.

Let us start with strings on the worldvolume. Taking \( n \) strings coming from membranes wrapping \( U \) and \( \Sigma_i \) (\( i = 1, \ldots, 4 \)) have the energy

\[
\frac{nU\Sigma_i}{l_p^3} = \frac{nRL_jL_kL_mL_5}{l_p^6}
\]

(16)
and therefore is a description for $n$ space time longitudinal M5 brane. Together with strings from membranes wrapping $V$ we obtained 8 longitudinal space time M5 branes. For $i = 5$ we get

$$\frac{nU\Sigma_5}{l_6^3} = \frac{nL_5 R}{l_6^3}$$  \hspace{1cm} (17)$$

and an analogous formula for $V$. This gives us two longitudinal membranes in space time.

Next, we consider 3branes on the worldvolume. Again, 3branes stretching along $\Sigma_5$ have a different space time interpretation than 3branes only stretching in directions $\Sigma_1, \ldots, \Sigma_4$. Let us start with $i, j, k \neq 5$ Then the energy is

$$\frac{nUV\Sigma_i\Sigma_j\Sigma_k}{l_6^6} = \frac{nL_5 L_6 L_1 L_2 L_3 L_4 L_5 R^2}{l_6^9}$$  \hspace{1cm} (18)$$

This means that in space time we have four KK monopoles with the NUT direction $L_m$ wrapping $L_i$, $L_j$, $L_k$, $L_5$, $L_6$, $R$. In the case that one of the 3brane directions, say $\Sigma_i$ is the $\Sigma_5$ direction, we get:

$$\frac{nUV\Sigma_i\Sigma_j\Sigma_k}{l_6^6} = \frac{nL_m L_n L_5 L_6 R}{l_6^9}$$  \hspace{1cm} (19)$$

which are in space time 5 branes wrapping $L_5$ and $L_6$ and two directions $L_m, L_n$ with $m, n = 1 \ldots 4$ (altogether six M5 branes).

Let us turn to 5 branes on the worldvolume coming from KK6 branes. The energy for such a brane with NUT direction $L_{6,7}$ is

$$\frac{n\Sigma_{6/7}^2}{l_6^3} \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 \Sigma_{7/6} \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 = n \left( \frac{\Sigma_{6/7}}{l_6^3} \right)^2 \Sigma_{7/6} \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5$$  \hspace{1cm} (20)$$

giving wrapped KK6 with NUT direction 5 (6).

Finally, we have to take into account the momenta along the world volume directions of the M5 brane. For $i = 1, \ldots, 4$ they give 4 longitudinal membranes and for $i = 5$ they lead to the longitudinal M5 brane wrapping $R, L_1, \ldots, L_4$.

Altogether, we found 27 bound states corresponding to transversal branes in space time and forming a 27 of the U-duality group $E_{6(6)}$. This multiplet is the flux multiplet identified in [11]. The 27 bound states at threshold form the momentum multiplet and correspond to longitudinal branes in space time. We can complete the two times 27 states to 56 states if we take into account the M5 brane itself, which corresponds to space time momentum along the longitudinal R-direction and membranes wrapping $U$ and $V$. These membranes have energies

$$\frac{nUV}{l_3^3} = \frac{nL_1 L_2 L_3 L_4 L_5 L_6 R^2}{l_3^9}$$  \hspace{1cm} (21)$$

This is the energy of $n$ KK monopoles with NUT-direction $R$. These states become light in our limit. In fact, Seiberg and Sethi [12] have argued that the M5 brane with two compact
scalars does not decouple from the bulk physics. This might be related to the appearance of light states in the limit. However, if we consider infinite light cone directions and infinite $N$, then we don’t expect a matrix state corresponding to a KK monopole with NUT direction $R$. In addition, we have seen that the other states have finite mass in our limit and in order to make sure, that they cannot leave the brane, we will consider the limit of large charges where also their masses become large. So in this case, there might be hope that the worldvolume theory of the M5 including 3branes and two types of strings decouples from the bulk physics.

We are especially interested in configurations which are on the SUGRA side intersections of three M5 branes, where each pair of M5 branes have three world volume directions in common and all three M5’s intersect over a string. Along this direction, there is momentum. We take the common direction to be the light like direction $R$. (It might also be interesting to think about other directions.) The momentum along $R$ becomes in our matrix model the basic M5 stretching along $\Sigma_1, \ldots, \Sigma_5$, as can be seen from the relations (\textcircled{3}). As we have seen, there are several possibilities to produce longitudinal M5 branes from the matrix model. Let us consider the following space time configurations:

\begin{center}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
 & $x^1$ & $x^2$ & $x^3$ & $x^4$ & $x^5$ & $x^6$ \\
\hline
mom & & & & & & \\
M5 & x & x & x & x & x & \\
M5 & x & x & & x & x & x \\
M5 & & x & x & x & x & \\
\hline
\end{tabular}
\end{center}

The matrix description of the first configuration is:

\begin{center}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
 & $\Sigma_1$ & $\Sigma_2$ & $\Sigma_3$ & $\Sigma_4$ & $\Sigma_5$ & $U$ & $V$ \\
\hline
M5 & x & x & x & x & x & \\
mom & & & & & & o \\
M5 & & x & x & x & x & x \\
M5 & x & x & & x & x & \\
\hline
\end{tabular}
\end{center}
From these tables one can directly read off how the branes and momenta are mapped. Note, the $R$ direction in SUGRA is mapped on $\Sigma_5$ and the momentum in SUGRA becomes our “fundamental” 5-brane in the matrix description.

3. The entropy of bound states

So far we described the matrix description of the $M5$-brane. The question is, whether it reproduces the same results then supergravity or whether one can find new non-trivial statements. Therefore we look for quantities that can be compared. Obvious candidates are (scattering) amplitudes and duality groups, but also the degeneracy of bound states, i.e. the entropy, can be tested. We will now discuss the last option. Thus, we have first to find suitable bound states and afterwards we have to discuss how we can determine their degeneracy.

Our matrix field theory is defined in a special limit (7) of the $M5$-brane worldvolume theory. So, we can take bound states of the $M5$-brane theory and if they survive the decoupling limit they are also bound states of the matrix description. There are two distinct types of bound states, threshold and non-threshold. Threshold bound states can naturally superposed, they have no binding energy, or in other words, we can separate all constituents. They are much harder to construct than the non-threshold bound states, which have a non-vanishing binding energy.

A typical example for a non-threshold bound state of 11-d supergravity is the configuration $2 \subset 5$, i.e. 2-brane lying inside a 5-brane [13] (for a collection of non-threshold $M$-theory bound states see especially the second reference). This state and many others can be obtained in SUGRA via $SL(2, R)$ transformation and/or boosts along non-worldvolume directions of a single brane in 10 dimensions and decompactify them to 11 dimensions. This construction makes clear, that they are less important for the discussion of entropy, which should be invariant under boosts and/or duality (when expressed in terms of the “right” charges).

For our purpose more interesting are threshold bound states. Like for the non-threshold bound states, also these states can be constructed first as bound state of the $M5$-brane and then perform the decoupling limit (7). If they survive this limit, we regard them as matrix threshold bound states. Also here one has a construction procedure in 10 dimensions. One can start with any known brane, makes it non-extremal and perform a boost, but now along a worldvolume direction. In contrast to the case above we can make here an infinite boost while doing the extreme limit. Then after repeated $S$- and $T$-duality transformations we get all known intersections. After having a bound state of two branes we can repeat this procedure by a boost along a common worldvolume direction. After decompactification to 11 dimensions we get the $M$-theory threshold bound states. Of course, alternatively one can apply the known intersection rules for constructing intersecting brane. Note, these threshold bound states break further supersymmetry, i.e. after compactification on a torus they are not “typical” BPS states with 1/2 unbroken supersymmetry.
After we know of how to construct threshold bound states, we can turn to the question of the degeneracy (=entropy) of these bound states. In supergravity it should be given by the Bekenstein-Hawking entropy of the black hole. In the matrix description, however, we have a field theory without gravity and there is no obvious horizon. But also in supergravity one does not need necessarily a horizon to obtain the entropy. Instead the entropy can also be obtained from the minimum of the BPS mass \( [14] \), a procedure that has been used extensively for \( N=2 \) black holes, see e.g. \([15]\). Therefore, the entropy for a 4-charge configuration is given by

\[
S \sim \hat{M}_{\text{min}}^2
\]  

with \( \hat{M}_{\text{min}} \) is the minimum of a dimensionless mass, i.e. given by \( \partial_{\phi^a} \hat{M}(\phi^a) = 0 \) where \( \phi^a \) denotes the moduli. In a thermodynamical approach \([16]\) this is reflected in the first law of thermodynamics where an additional moduli dependent term has been added. But also in matrix theory this approach has already been discussed in \([17]\).

We will apply this procedure for the two configurations shown in the table: 5\( \times \)5\( \times \)5\( + \)mom and the 5\( \times \)2\( \times \)5\( \times \)2. In the matrix description these states appear as 5\( \times \)3\( \times \)3\( + \)mom. and 5\( \times \)3\( \times \)1\( \times \)1, where the “5” corresponds to a non-trivial 5-form charge. We should stress here, that these are bound states in the matrix description, i.e. in the so-called base space theory. We made the detour through threshold bound states in \( M \)-theory only to show, that these configuration are really bound states at threshold. Note, up to relabelling of coordinates both configurations correspond to the same SUGRA configuration. Since these states are at threshold we can simply add up all mass contributions and obtain

\[
\hat{M}_{5\times3\times3+m} = \sum_i \hat{M}_i = V_i p^i + N \tilde{l}_p / \Sigma\]

where \( V_1 = \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 / \tilde{l}_p^5 \), \( V_2 = \Sigma_3 \Sigma_4 \Sigma_5 U V / \tilde{l}_p^5 \), \( V_3 = \Sigma_1 \Sigma_2 \Sigma_5 U V / \tilde{l}_p^5 \) are dimensionless parameters and \( p^i \) are the charges related to the branes and \( N \) is the momentum number. Note, that the momentum modes on the matrix side correspond to one of the 5-brane charges on the SUGRA side and the original \( N \) became \( p^1 \) (see table). The hat on the mass should indicate that we made it dimensionless by \( \tilde{l}_p \), because only the minimum of a dimensionless quantity can be treated as entropy. The mass formula for the second configuration looks completely analog

\[
\hat{M}_{5\times3\times1\times1} = V_1 p^1 + V_2 p^2 + V_3 q^3 + V_4 q^4
\]

where now \( V_1 = \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 / \tilde{l}_p^5 \), \( V_2 = \Sigma_1 \Sigma_2 \Sigma_5 U V / \tilde{l}_p^5 \), \( V_3 = \Sigma_4 U / \tilde{l}_p^2 \), \( V_4 = \Sigma_3 V / \tilde{l}_p^2 \) and \( p^1 \), \( p^2 \) are the 5- and 3-form charges and the one-form charges are \( q^3 \), \( q^4 \).

Before we can start with the extremization we have to discuss the moduli of our model. Obviously, \( U \) and \( V \) are moduli, because they appear already as scalar fields in the \( M \)-brane worldvolume theory. But in addition, because we consider the field theory on a compact space, also the (dimensionless) radii \( \Sigma_i / \tilde{l}_p \) are moduli. Therefore, in the mass formulae the \( V_i \) are the moduli over which we have to minimise. Immediately we see, that it will yield a vanishing mass as minimum, which is certainly wrong. This however
is simply a consequence that we varied the complete moduli space, but instead one has
to keep fix the volume of the moduli space - only internal deformations are moduli. In
our case this means that we have to keep fix
\[ \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5 U V = \tilde{l}_p^7 \] (25)
i.e. in extremizing we do not change the the overall volume but allow def ormation of
the different cycles. In SUGRA one has the same constraint, namely that one fixes the
Newton constant in 11 and in 4 or 5 dimensions or equivalently one fixes the asymptotic
flat Minkowski space. Only if one compactifies 10-d string theory, o ne treats the volume of
the compact space as moduli, which is related to the dilaton moduli and which corresponds
to a variation of \( R_{11} \). Using this constraint for (25), i.e. \( \Sigma_5 = \tilde{l}_p V_1 V_2 V_3 \) we find
\[ \hat{M}_{5 \times 3 \times 3+m} = V_i p_i^i + N/V_1 V_2 V_3 \] (26)
with \( i = 1..3 \). This mass as function of \( V_i \) has a non-trivial minimum and with the ansatz
\( V_i = \frac{\tilde{l}_p}{p_i} \) we find for
\[ \partial_i \hat{M}_{5 \times 3 \times 3+m} = p_i^i - N/V_1 V_2 V_3 V_i = 0 \] (27)
that \( e^4 = p_1^1 p_2^2 p_3^3 p^4 \) and therefore we get for the entropy
\[ S \sim \hat{M}_{\text{min}}^2 = \sqrt{2Np_1^1 p_2^2 p_3^3} . \] (28)
Using the constraint (25) for our second configuration, i.e. here \( V_1 V_2 V_3 V_4 = 1 \), yields also
\[ \hat{M}_{5 \times 3 \times 1 \times 1} = V_1 p_1^1 + V_2 p_2^1 + V_3 q_1^1 + q_2^2/V_1 V_2 V_3 . \] (29)
Up to a charge redefinition (\( p^3 \rightarrow q_1^1 \), \( N \rightarrow q_2^2 \)) this is the same mass and thus it has also
the same entropy (28). Note, as discussed after eq. (24) we can trust the decoupling limit
only for a large \( R \) direction (large \( N \) limit). In the matrix description this limit supresses
transversal membranes, which tend to become light otherwise. The other branes have still
finite mass in our limit, but to make sure that they cannot leave the brane, we consider
the limit of large charges, which makes also these branes heavy, i.e. \( N, p_i \gg 1 \). And
really, looking on the Bekenstein-Hawking entropy for 4-d black holes yields exactly the
same result, up to exchanging the momentum number with one of the magnetic charges:
\( N \leftrightarrow p^1 \). Our matrix-configuration describes in SUGRA the configuration \( 5 \times 5 \times 5 + \text{mom.} \)
and after compactification to 4 dimensions one obtains for the entropy \( S \sim \sqrt{Np_1^1 p_2^2 p_3^3} \),
where \( N \) is the electric charge related to the momentum. Like in our matrix description,
one can trust this entropy only in the limit of large charges, namely: \( N \gg p^1 \gg 1 \).
However the reasons are different here, they follow namely from the requirement, that
the low energy approximation still holds. Note, the black holes appear as solution of the
low energy effective action. To be concrete, the first relations controls the values of the
scalar fields on the horizon, especially one keeps the dilaton (higher genus corrections)
under control and the second relation keeps the curvature on the horizon small, i.e. we
can neglect higher curvature correction (\( \alpha' \) corrections). We have to keep in mind that
the non-renormalization theorems concerns only the lowest order in the effective action, e.g. higher curvature terms are not protected. Alternatively, one can also argue, that near the horizon the space time factorizes in $AdS_2 \times S_2$ and the above limits ensure that in the string frame both curvatures are small.

Finally, we have to discuss the counting of the state degeneracy. Only this gives the justification to call the minimum of the mass entropy. A more rigorous discussion of the degeneracy of our configurations can be found in [18]. In our decoupling limit, only string degrees of freedom can occur as dynamical modes (all 5- and 3-branes masses are large). These are just our momentum modes travelling around the common intersection of the 3-branes. Fortunately, for string states we know the degeneracy formula

$$d(N) = e^{2\pi \sqrt{cN/6}}$$

where $c$ is the central charge and in the case at hand this is nothing as the effective dimension. Now, the state counting can go in complete analogy to the D-brane counting; keeping in mind that (i) the charge of the brane are the number of branes that are on top of each other, (ii) the effective dimension (where we can distribute the momentum modes) are just the total number of layers, which is the product of charges and finally (iii) taking into account bosonic and fermionic modes. Doing all this, one finds an agreement with the entropy, that we obtained as the minimum of the mass.

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