IMPRINTS OF ELECTRON–POSITRON WINDS ON THE MULTIWAVELENGTH AFTERGLOWS OF GAMMA-RAY BURSTS

J. J. Geng\textsuperscript{1,2}, X. F. Wu\textsuperscript{3,4}, Y. F. Huang\textsuperscript{1,2}, L. Li\textsuperscript{5,6}, and Z. G. Dai\textsuperscript{1,2}

\textsuperscript{1} School of Astronomy and Space Science, Nanjing University, Nanjing 210046, China; hyf@nju.edu.cn
\textsuperscript{2} Key Laboratory of Modern Astronomy and Astrophysics (Nanjing University), Ministry of Education, China
\textsuperscript{3} Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China; xfwu@pmo.ac.cn
\textsuperscript{4} Joint Center for Particle Nuclear Physics and Cosmology of Purple Mountain Observatory-Nanjing University, Chinese Academy of Sciences, Nanjing 210008, China
\textsuperscript{5} Department of Physics, Stockholm University, AlbaNova, SE-106 91 Stockholm, Sweden
\textsuperscript{6} 5 Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China

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ABSTRACT

Optical rebrightenings in the afterglows of some gamma-ray bursts (GRBs) are unexpected within the framework of the simple external shock model. While it has been suggested that the central engines of some GRBs are newly born magnetars, we aim to relate the behaviors of magnetars to the optical rebrightenings. A newly born magnetar will lose its rotational energy in the form of Poynting-flux, which may be converted into a wind of electron–positron pairs through some magnetic dissipation processes. As proposed by Dai, this wind will catch up with the GRB outflow and cause a long-lasting reverse shock (RS) would form. By applying this scenario to GRB afterglows, we find that the RS propagating back into the electron–positron wind can lead to an observable optical rebrightening and a simultaneous X-ray plateau (or X-ray shallow decay). Recent multiwavelength observations of afterglows of gamma-ray bursts (GRBs) have revealed some puzzling features (Panaitescu et al. 2006; Panaitescu & Vestrand 2011; Li et al. 2012; Liang et al. 2013), leading to research on the afterglow theory. X-ray observations from Swift/XRT (Gehrels et al. 2004; Burrows et al. 2005) have resulted in diverse light curves, but a canonical light curve consisting of a steep decay, a plateau, and a normal decay phase has been suggested (Zhang et al. 2006). The plateau phase cannot be explained in the framework of the standard external shock scenario (i.e., synchrotron radiation produced by the interaction of a relativistic ejecta with the circum-burst medium; see Piran et al. 1993; Meszáros & Rees 1997; Sari & Piran 1999 for reviews). On the other hand, some optical afterglows show rebrightenings at late stages ($\sim 10^4–10^5$ s), together with a bump occasionally occurring in the X-ray band (e.g., GRB 120326A, Melandri et al. 2014; Hou et al. 2014; Laskar et al. 2015), or without clear counterpart features in X-rays (e.g., GRB 100812A, De Pasquale et al. 2015; GRB 081029, Nardini et al. 2011; GRB 100621A, Greiner et al. 2013). These rebrightenings also deviate from the expectations of the external shock model, which calls for a refined afterglow model.

Several scenarios have been proposed to explain these unexpected features. Some authors invoke energy injection processes (Dai & Ly 1998a, 1998b; Fan & Xu 2006; van Eerten 2014) or forward shocks (FS) refreshed by late shells (Zhang & Meszáros 2002) to produce the X-ray plateau. Meanwhile, circum-burst density enhancements (Lazzati et al. 2002; Dai & Wu 2003), two-component jets (Berger et al. 2003; Huang et al. 2004), or varying microphysical parameters (Kong et al. 2010) have been invoked to explain the optical rebrightenings. However, recent studies indicate that the density enhancements model may not work for significant rebrightenings (van Eerten et al. 2009; Gat et al. 2013; Geng et al. 2014). If one wants to interpret a large sample of afterglows, which model is preferred is still uncertain. Although there are various candidate models for afterglows, the possible central engines of GRBs are mainly limited in two kinds of objects. It is generally believed that the central engines of GRBs should either be rapidly spinning, strongly magnetized neutron stars (magnetar, Dai & Lu 1998a, 1998b; Heger et al. 2003; Dai et al. 2006; Metzger et al. 2011; Lü & Zhang 2014) or black holes (Woosley 1993; MacFadyen & Woosley 1999; Lei et al. 2013). This indicates that there is probably a common origin for the optical rebrightenings and the simultaneous X-ray features of a group of afterglows. Therefore, it is reasonable to develop a model that, after taking into account the physics of the central engine, can explain the multwavewavelength afterglows simultaneously.

From the visual perspective, the optical rebrightening seems to be a new component emerging when the flux from the FS gets dim. In the two-component jet model, this kind of late-time component naturally corresponds to a wide jet. However, in some cases it may have difficulties in explaining the chromatic evolution of the afterglows (De Pasquale et al. 2015). The new component can also be generated in another model, that is, a long-lasting reverse shock (RS) can play the role. If the FS is reenergized by a late outflow (e.g., an electron–positron-pair wind; Dai 2004; Yu & Dai 2007; Yu et al. 2007), then an RS will form behind the FS. As the RS is propagating into the outflow, it would contribute to the emission and its flux can even exceed that from the FS. Moreover, the emission from the RS can help make diverse light curves in principle because the microphysical parameters may differ from those of the FS. Therefore, it is
reasonable to argue that some light curves with rebrightenings are
due to the existence of a long-lasting RS. On the other hand, the
long-lasting RS should be common because it is a natural
consequence of the energy injection. In previous studies, it was
proposed that continuous energy injections in the form of pure
Poynting-flux coming from the central engines may play a key
role in explaining some special light curves, through its effects on
the evolution of the FS (Zhang et al. 2006; Liu et al. 2010; Geng
et al. 2013; Yu et al. 2015). However, in such a scenario without
an RS, the emission from the FS could not account for the new
components of the rebrightenings discussed here.

In this paper we propose the ultrarelativistic electron–positron-
pair ($e^+\bar{e}^-$) wind model to interpret the afterglows with special
rebrightenings. After a GRB, the remaining object of the progenitor
may be a magnetar, which should lose its rotational energy by
ejecting a continuous Poynting-flux. We assume that the Poynting-
flux is converted into an ultrarelativistic $e^+\bar{e}^-$ wind beyond a
particular radius ($\sim10^{15}$ cm). When the $e^+\bar{e}^-$ wind catches up with
the FS, an RS will form and propagate back into the $e^+\bar{e}^-$ wind.
The wind will be shocked and heated by the RS, and radiation
from these $e^+\bar{e}^-$ can account for the optical rebrightenings at late
time. It is worthwhile to note that this scenario was originally
proposed by Dai (2004) and then used to explain the X-ray plateau
and bump by Yu & Dai (2007) and Dai & Liu (2012). However,
the method used to solve the shock dynamics in the current work is
different and we aim to explain optical rebrightenings. The mechanism of converting the Poynting-flux into ultrarelativistic
$e^+\bar{e}^-$ wind is still unknown. Previous research indicates that
magnetic dissipation may help to explain why pulsar wind nebulae
are powered by particle-dominated winds (Rees & Gunn 1974;
Lyubarsky & Kirk 2001; Porth et al. 2013; Metzger & Piro 2014).
Such a dissipation may also occur in millisecond magnetars.

In our study, we select as examples four GRBs, of which
high quality multiwavelength observational data are available.
They are all characterized by optical rebrightenings, but
without corresponding behaviors being detected in X-rays.
We show that they can be well explained in our model. Our
paper is organized as follows. In Section 2, we briefly describe
the dynamic methods used in our work and the formulae for
calculating the radiation. In Section 3, we show how our
scenario would work to explain the rebrightenings. The results
for each GRB are presented separately in Section 4. The
conclusions are summarized in Section 5.

2. HYDRODYNAMICS AND RADIATION

When a relativistic outflow propagates into the circumburst
medium, an FS will form. If the central engine is long-lived and
a continuous wind is ejected, an RS is expected to propagate
back into the wind. The dynamics of such an FS–RS system
can be numerically solved by considering energy conservation
proposed by Geng et al. (2014), or through the mechanical
consideration done by Beloborodov & Uhm (2006; also see
Uhm 2011). For clarity, these two methods are described separately in Appendixes A.1 and A.2.

Now, we show that the dynamical results of these two
methods are consistent with each other within our $e^+\bar{e}^-$ wind
scenario. The Poynting-flux luminosity $L_w(l_{\text{obs}})$ of a newly
born magnetar can be derived from the magnetic dipole radiation
(Shapiro & Teukolsky 1983, p. 663), i.e.,

$$L_w \simeq 4.0 \times 10^{47} B_{5,14}^2 R_{8,6}^6 P_{8,-3}^{-4} \left(1 + \frac{l_{\text{obs}}}{T_{\text{ad}}} \right)^{-2} \text{erg s}^{-1},$$

where $B_{5,14}$ is the strength of the surface magnetic field of the
magnetar, $R_{8,6}$ is the radius, and $P_{8,-3}$ is its spin period.
$T_{\text{ad}} \approx 5.0 \times 10^4 (1 + z) B_{5,14}^{-1} R_{8,6}^{-6} P_{8,-3}^{-3} \text{s}$ is the spin-down
timescale, where $I$ is the moment of inertia of the magnetar.

The convention $Q_s = Q/10^4$ in cgs units is adopted hereafter.
For simplicity, we fix $I = 10^{45}$ g cm$^2$, $R_{8,6} = 10^6$ cm, and
$P_{8,-3} = 1$ ms throughout this work; $L_w$ is mainly determined by
only one parameter ($B_{5,14}$). We assume that the Poynting-flux is
converted into an $e^+\bar{e}^-$ pair wind beyond $10^{15}$ cm, which is the typical
deceleration radius of the outflow. The particle density in the
comoving frame of the unshocked wind (or called Region 4)
at the radius $r$ is then

$$n_e^i = \frac{L_w}{4\pi r^2 \Gamma_2^2 m_e c^3},$$

where $m_e$ is the mass of electron, $c$ is the speed of light, and $\Gamma_2$ is
the Lorentz factor of the unshocked wind. Note that the FS–RS
system can be divided into four regions (see Appendix A.1), the
quantities of Region “ii” are denoted by subscripts “i.” In this
paper, the superscript prime () is used to denote the quantities
in the shock comoving frame, while characters without prime
denote quantities in the observer frame.

For an outflow with an isotropic kinetic energy of $E_{K,\text{iso}} = 1.0 \times 10^{53}$
erg and an initial Lorentz factor of $\Gamma_0 = 300$, we set
the number density of the ambient medium $n_1$ as $1$ cm$^{-3}$, $\Gamma_4$ as
$10^4$, and $B_{5,14}$ as $2 \times 10^{14}$ G. Then the evolution of the bulk
Lorentz factor of the FS ($\Gamma_2$) and the RS ($\Gamma_3$) can be obtained using the
two methods mentioned above. Figure 1 shows the results
from the energy conservation method and the mechanical method.
We see that the difference between the two solutions is tiny,
which means they are consistent with each other. The two
solutions approach the solution given by Blandford & McKee
(1976; referred to as the BM solution below) at late times, which is
 foreseeable because $L_w$ finally becomes very weak and the role
of the RS is insignificant. Hereafter, we will adopt the mechanical
method to solve the dynamics of the FS–RS system.

To calculate the afterglow light curves of the FS–RS system,
we consider both the synchrotron radiation and the inverse
compton (IC) radiation from shock-accelerated electrons.
Detailed formulae are described in Appendix B.

3. APPLICATION

In our study we collected the afterglow data of four GRBs:
GRB 080413B (Filgas et al. 2011), GRB 090426 (Nicuesa
Guelbenzu et al. 2011), GRB 091029 (Filgas et al. 2012), and
GRB 100814A (De Pasquale et al. 2015). The observational data
are of high quality, and all four events show clear optical
rebrightenings at $\sim10^2–10^3$ s. The rising of the rebrightenings
is generally smooth and shallow, contrary to many other GRBs
that brighten sharply (Geng et al. 2013). This indicates that the
rebrightening component is a result of a continuous behavior,
which can be well achieved in our model. A brief summary of the
observations of these GRBs is presented in Table 1. We interpret
these afterglows using our model and discuss the underlying
relation between the afterglow behavior and the central engine.

Our sample of afterglows is characterized by a double-bump
structure. For the early onset bump (it may not show up due to
the lack of early observations), we suggest that it is connected
with the typical synchrotron frequency crossing the optical
band, or the FS being in the coasting phase. The second
rebrightness bump could be due to the emerging RS component at about \( T_{\text{d}} \). We take GRB 100814A as an example to show how this double-bump structure can be consistently reproduced in our model and how the physical parameters can be estimated from analytical derivations. Below, we use the convention \( F_{\nu,i} \sim \nu^{-\beta} \), where \( F_{\nu,i} \) is the flux density of Region \( i \), and \( \alpha \) and \( \beta \) are the power-law indices.

The first bump appearing in the optical afterglows of GRB 100814A is roughly at \( T_{\text{peak},1} \sim 300 \) s and the rising temporal index is \( \alpha_{\text{rise}} = -2.5 \). In the framework of the external shock model, it indicates that the FS is in the coasting phase (Xue et al. 2009). We assume that the isotropic kinetic energy of the outflow is \( E_{K,\text{iso}} \), and calculate the initial Lorentz factor \( \Gamma_0 \) as

\[
\Gamma_0 = \left( \frac{17E_{K,\text{iso}}(1+z)^3}{64\pi n_i c^2} \right)^{1/8}.
\]

Assuming \( E_{K,\text{iso}} \) equals the energy released in the prompt phase \( (E_{K,\text{iso}} = 7 \times 10^{52} \text{erg}) \) and using typical value of \( n_i = 0.1 \text{ cm}^{-3} \), we obtain \( \Gamma_0 \approx 150 \). At the end of the coasting phase, \( \nu_{\text{m,2}} \) should have crossed the optical band because the observed optical flux decreases after \( T_{\text{peak},1} \). This requires \( \nu_{\text{m,2}}(T_{\text{peak},1}) \lesssim 10^{15} \text{ Hz} \), that is,

\[
\frac{1}{1+z} \left( \frac{p_2-2}{p_2-1} \right) e^{2\epsilon_{i,2,-1}} - \frac{1}{2} - \frac{1}{2} \left( \frac{p_2-2}{p_2-1} \right) e^{2\epsilon_{i,2,-1}} - \frac{1}{2} \times 3.5 \times 10^{-5},
\]

where \( z \) is the redshift of the GRB, \( p_i \) is the power-law index of the distribution of electrons in Region \( i \), \( e_{i,2,-1} \) is the fraction of the total energy carried by the electrons, and \( e_{B,i} \) is the ratio of the magnetic field energy to the total energy.

After the coasting phase, we assume that the evolution of \( \Gamma_2 \) obeys \( \Gamma_2 = AR^{-\gamma} \). According to Section 2, we can still use the BM solution, that is, \( A = \left( \frac{17E_{K,\text{iso}}}{8\pi n_i c^2} \right)^{1/2} \) and \( g = \frac{3}{2} \). Then the typical synchrotron frequency of Region 2 is given by

\[
\nu_{\text{m,2}} = 5.5 \times 10^{13} \left( \frac{1+z}{2} \right)^{1/2} \times \left( \frac{p_2-2}{p_2-1} \right) e^{2\epsilon_{i,2,-1}} - \frac{1}{2} - \frac{1}{2} \times 3.5 \times 10^{-5} \text{ Hz},
\]

and the peak flux density at \( D_L \) (luminosity distance) is

\[
F_{\nu,\text{max,2}} = 8.6 \times 10^{4} \left( \frac{1+z}{2} \right)^{1/2} \times e^{2\epsilon_{i,2,-1}} - \frac{1}{2} - \frac{1}{2} \times 3.5 \times 10^{-5} \text{ mJy}.
\]

For Region 3, \( L_w \) can be taken as a constant within \( T_{\text{d}} \) and the relative Lorentz factor is \( \Gamma_3 \approx \Gamma_i/(2\Gamma_2) \); the typical synchrotron frequency is given by

\[
\nu_{\text{m,3}} = 6.9 \times 10^{16} \left( \frac{1+z}{2} \right)^{3/2} \times L_w^{1/4} \Gamma_i^{1/4} e^{2\epsilon_{i,3,-1}} - \frac{1}{2} - \frac{1}{2} \times 3.5 \times 10^{-5} \text{ Hz},
\]

The cooling frequency is

\[
\nu_{c,3} = 1.4 \times 10^{16} \left( \frac{1+z}{2} \right)^{3/2} \times L_w^{1/4} \Gamma_i^{1/4} e^{2\epsilon_{i,3,-1}} - \frac{1}{2} - \frac{1}{2} \times 3.5 \times 10^{-5} \text{ Hz},
\]

and the peak flux density is

\[
F_{\nu,\text{max,3}} = 27.4 \times 10^{2} \left( \frac{1+z}{2} \right)^{3/2} \times L_w^{1/4} \Gamma_i^{1/4} e^{2\epsilon_{i,3,-1}} - \frac{1}{2} - \frac{1}{2} \times 3.5 \times 10^{-5} \text{ mJy}.
\]

According to Equations (6)–(7) the condition of \( \nu_{\text{m,3}} < \nu_{c,3} \) is usually valid within \( \sim 10^5 \) s.

First, after the peak, the optical afterglow enters the slow decay phase, during which the temporal index is \( \alpha_{\text{decay}} \approx 0.72 \). On the other hand, \( \alpha_{\text{decay}} \) is predicted by (2) when \( \nu_{\text{m,2}} < \nu_{\text{obs}} < \nu_{c,2} \), which gives \( p_3 \approx 2.0 \).

Just before the emergence of the RS component (i.e., \( F_{\nu,3} \)), \( F_{\nu,2} \) should equal \( F_{\nu,3} \) at a certain time. We denote this time as \( T_{\text{dent}} \). For the regime \( \nu_{\text{m},i} < \nu < \nu_{c,i} \), we have

\[
F_{\nu,i} = F_{\nu,\text{max},i} \left( \frac{\nu}{\nu_{\text{obs,i}}} \right)^{-(p_i-1)/2}.
\]

The g-band data shows \( T_{\text{dent}} \approx 1.2 \times 10^5 \) s, and the flux density at \( T_{\text{dent}} \) is \( 8 \times 10^{-2} \) mJy. Therefore we can get

\[
F_{\nu,2}(T_{\text{dent}}) = F_{\nu,3}(T_{\text{dent}}) = 3.5 \times 10^{-2} \text{ mJy},
\]
\[ n(n + 1) = (n + 2) \text{(Equation 1)} \]

and

\[ \left( \frac{1}{2} \right)^{3p_{2}^{-2}} \left( \frac{p_{2} - 2}{p_{2} - 1} \right)^{p_{2}^{-1}} \times \epsilon_{c_{2}, \epsilon, B_{2}, \epsilon_{e}, B_{3}} \left( \frac{p_{2}^{-1}}{\epsilon_{c_{2}, \epsilon, B_{2}, \epsilon_{e}, B_{3}}} \right)^{p_{2}^{-1}} E_{K_{\text{iso}}, 0.3}^{p_{2}^{-1}} L_{w, 2}^{p_{2}^{-1}} = 4.1 \times 10^{-5} \left( \frac{\nu_{\text{opt}}}{1.1 \times 10^{15} \text{Hz}} \right)^{(p_{2}^{-1})/2}, \quad (9) \]

where \( \nu_{\text{opt}} = 6.4 \times 10^{14} \text{Hz} \).

The flux from the RS should reach its peak at \( T_{\text{dd}} \) because the flux decays when \( t_{\text{obs}} > T_{\text{dd}} \) (see below). In principle we can set \( T_{\text{dd}} \) as the peak time of the second bump (e.g., \( T_{\text{dd}} = T_{\text{peak,2}} \)). However, the effect of the equal arrival time surface (EATS) would delay the peak time in the optical bands. On the other side, the peak time in X-rays will be less affected. In the case of GRB 100814A, we notice that there may be a small structure in the X-ray afterglow at \( \sim 4 \times 10^{5} \) s, which gives the observed temporal index as \( \approx 0.5 \). If we assume that the evolution of \( L_{w} \) during this segment is roughly \( \propto t_{\text{obs}}^{0.3} \), then \( p_{2} \) can be inferred to be 2.4. After the second peak \( t_{\text{obs}} > T_{\text{peak,2}} \), because \( L_{w} \) evolves as \( \propto t_{\text{obs}}^{0.3} \), \( f_{e, 0.3} \) will decay with time and \( c_{e, 3} \) will increase. As a result, none will cross the optical band (i.e., \( \nu_{\text{opt}} < \nu_{c, 3} \)). The optical spectral index at this time becomes \( \approx -0.7 \), which further gives \( p_{3} = 2.4 \) because \( (p_{3} - 1)/2 = 0.7 \). This value is consistent with that obtained before.

The jet break time \( t_{j} \) is hard to determine because the FS flux is always lower than that of the RS component after \( T_{\text{dd}} \). In our fitting, we assume \( t_{j} \approx 10^{5} \) s, which requires the high-opening angle of the jet \( \theta_{j} \approx 0.058 \) rad (Lu et al. 2012). We have little knowledge of the bulk Lorentz factor of the \( e^{-} e^{+} \) wind, \( \Gamma_{e} \). However, a value ranging from \( 10^{4} \) to \( 10^{6} \) may be reasonable (Yu et al. 2007). For GRB 100814A, we take \( \Gamma_{e} \approx 10^{5} \). There are still four free parameters left: \( \epsilon_{c, 2, \epsilon, B_{2}, \epsilon_{e}, B_{3}} \). Using conditions given in Equations (3) and (9)–(10), and an underlying relation \( \epsilon_{c, 3} = 1 - \epsilon_{B, 3} \), we get \( \epsilon_{c, 2} \approx 5.5 \times 10^{-2} \), \( \epsilon_{B, 2} \approx 1.2 \times 10^{-3} \), and \( \epsilon_{B, 3} \approx 0.02 \). In a subsequent study, we will use these parameter values as a guide to perform numerical calculations, adjusting the parameters slightly to get a visually good agreement to the observed afterglow light curves (see Figure 2). The final values (see in Table 2) of the parameters do not deviate significantly from the above derivations. The high energy \( \gamma \)-ray afterglow of GRB 100814A is also calculated. Figure 3 presents the flux density at 100 MeV, in which the SSC flux from the RS leads to a small bump in the light curve at \( \sim 10^{5} \) s. This is a special feature predicted by our model. High energy observations in the future can help to test the prediction. We apply the above procedure to the afterglows of the other three GRBs (see Figures 4–6). The derived parameters are also listed in Table 2.

To provide a meaningful constraint on the parameters, we use the data fitting code emcee (Foreman-Mackey et al. 2013), which is a widely used tool based on Markov chain Monte Carlo (MCMC) simulations. There are 11 parameters for each GRB, so it would be difficult to solve the problem if we set them all as free parameters. The degeneracy between the parameters is one major reason: even for seven parameters related to the FS, this degeneracy may be significant (Ryan et al. 2013; van Eerten 2015). The other reason is the incredible computational cost in the MCMC sampling. Because we are mainly interested in the parameters connected with the RS, we chose to set \( B_{\text{NS}}, \Gamma_{e}, \) and \( B_{e, 3} \) as free parameters. The allowed ranges of these parameters in the Monte Carlo simulation were set to be \([0.1, 1000] \times 10^{14} \) G, \([0.1, 1000] \) and \([1.0 \times 10^{-6}, 0.9] \), respectively. Other parameters are fixed by using the values of Table 2. The simulation results are presented in Table 3. Figure 7 shows the corner plot of the fit for GRB 080413B as an example, which consists of the marginalized distributions of each parameter and the covariances between pairs of parameters.
parameters, according to Equations (4) and (5), one would find that $c_{e,2}$ and $E_{K,iso}$ are degenerate in some extent, and are in negative correlation with $c_{e,2}$ or $n_1$ (also see Figure 1 in Ryan et al. 2013). On the other hand, according to Equation (10), $E_{K,iso}$ and $n_1$ would be involved in calculations of the flux from the RS. Both are expected to be negatively correlated with $\Gamma_1$ in the corner plot. To judge whether the results in Table 3 are meaningful, we fix the parameters in Table 3, and leave parameters $c_{e,2}$, $E_{K,iso}$, and $n_1$ free. The MCMC fitting (see Figure 8) gives $c_{e,2} = 0.043^{+0.013}_{-0.009}$, $E_{K,iso} = 0.027^{+0.011}_{-0.008}$, and $n_1 = 0.12^{+0.005}_{-0.006}$ respectively. These values do not deviate much from those in Table 2. In Figure 8, we can see that $n_1$ is well constrained, which implies the results in Table 3 are sensitive to $n_1$. This could be understandable because $n_1$ is involved in the calculation of the dynamics of the RS, and other parameters of the RS are fixed. This implies that the results in Table 3 would become robust when the early observational data (mainly related with the FS) could constrain parameters $E_{K,iso}$ and $n_1$ well.

4. DISCUSSIONS

The four GRBs studied here have also been investigated by other authors in the literature. Previous efforts failed to explain many of the puzzling behaviors of these events. Here, we would like to discuss how our mechanism can improve the modeling.

4.1. GRB 080413B

The afterglow of GRB 080413B has been explained by the on-axis two-component jet model (Filgas et al. 2011). In this model, the narrow ultrarelativistic jet is responsible for the initial decay, while the moderately relativistic wide jet is expected to account for the late rebrightening. The collapsar model of long-duration GRBs offers a possible mechanism for generating a two-
Figure 5. Fitting to the multiwavelength afterglow of GRB 090426 using our $e^+e^-$ wind injection model. In the upper panel, the observational data in the $R$ band are taken from Nicuesa Guelbenzu et al. (2011), and the X-ray (0.3–10 keV) data are taken from the Swift/XRT website (http://www.swift.ac.uk/xrt_curves/350479). The dashed and dotted lines are emissions from the FS and the RS, respectively, and the solid lines are the total flux. Detailed fitting parameters are listed in Table 2. Similar to Figure 2, the lower panel shows the evolution of some characteristic frequencies.

4.2. GRB 090426

GRB 090426 is a short GRB according to its duration (Nicuesa Guelbenzu et al. 2011). However, some authors argued that GRB 090426 could be a member of the collapsar class (Kumar & Zhang 2015). According to Filgas et al. (2011), the power-law index of the narrow jet electrons can be derived as $p \approx 1.44$ ($\nu_{\text{opt}} < \nu_{\text{ phot}} < \nu_{c}$) from the observed optical temporal index of $\alpha \approx 0.73$. However, such an electron distribution with $p$ significantly less than two seems to be too hard. A simulation of the Fermi acceleration of charged particles by relativistic shocks indicates that $p$ is $\approx 2.26 \pm 0.04$ in the ultrarelativistic limit $\Gamma \gg 1$ (Lemoine & Pelletier 2003). The investigation of the distribution of $p$ from Swift GRB afterglows supports a Gaussian distribution centered at $2.36$ with a width of $0.59$ (Curran et al. 2010). In our model, this difficulty no longer exists. The evolution of the Lorentz factor of FS before $T_{\text{bd}}$ deviates from the BM solution ($\Gamma \propto R^{-7}$, $g = \frac{4}{3}$) due to the effect of the $e^+e^-$ wind (i.e., $g$ should be $<\frac{4}{3}$). Using the closure relation of $0.73 = \frac{4g}{2g+1} \frac{p_3-1}{p_3}$, we can obtain $p_3 = 2.1$ if $g \approx 1$. Thus we can fit the light curves with a reasonable $p$. Additionally, in the two-component jet model, the optical spectral index (pre-jet break) of $\alpha \approx 0.9$ indicates $p = 1.8$ ($\nu_{\text{ opt}} > \nu_{c} > \nu_{\text{ phot}}$) for the wide jet. In our model, $\nu_{c}$ is greater than $\nu_{\text{ phot}}$, but slightly smaller than $\nu_{X}$, which gives $p_3 = 2.8$ (see in Table 2). This value is more acceptable.

Figure 6. Fitting to the multiwavelength afterglow of GRB 091029 using our $e^+e^-$ wind injection model. In the upper panel, the optical–infrared observational data are taken from Filgas et al. (2012), and the X-ray (0.3–10 keV) data are taken from the Swift/XRT website (http://www.swift.ac.uk/xrt_curves/374210). The dashed and dotted lines are emissions from the FS and the RS, respectively, and the solid lines are the total flux. Detailed fitting parameters are listed in Table 2. Similar to Figure 2, the lower panel shows the evolution of some characteristic frequencies.

Table 3

| Parameters Constrained from the Fit of the Afterglows of the Four GRBs |
|------------------|------------------|------------------|------------------|
| GRB 080413B     | GRB 090426       | GRB 091029       | GRB 100814A      |
| $B_{\text{int}}$ $(10^{14}$ G) | $1.73 \pm 0.1$ | $5.29 \pm 0.1$ | $5.81 \pm 0.1$ | $1.44 \pm 0.1$ |
| $\Gamma_{a}$ $(10^{2}$) | $3.91 \pm 0.1$ | $1.07 \pm 0.1$ | $81.5 \pm 7.1$ | $21.4 \pm 2.8$ |
| $\Gamma_{e3}$ | $0.197 \pm 0.06$ | $9.77 \pm 2.1 \times 10^{-5}$ | $0.35 \pm 0.05$ | $0.22 \pm 0.05$ |

Note. Limited by the computer resources, we performed the fitting by setting only three parameters as free parameters, but with other parameters fixed as constants taken from Table 2. These three parameters, $B_{\text{int}}, \Gamma_{a}$ and $\Gamma_{e3}$, are more interesting because they are directly related to the rebrightening.

GRB 090426 may be connected with a collapsar event, rather than the merger of two compact objects (Antonelli et al. 2009; Levesque et al. 2010; Xin et al. 2011). In this case, a magnetar could be naturally involved. The high density of $n_1 = 50$ cm$^{-3}$ derived from our fitting is consistent with the lower limit of 10 cm$^{-3}$ given by Xin et al. (2011). The optical and X-ray light curves of GRB 090426 could be fitted by the two-component jet model of Nicuesa Guelbenzu et al. (2011). However, we notice that the first break (around 300 s) in the X-band light curve cannot be explained by the jet break of the narrow jet component (Nicuesa Guelbenzu et al. 2011). On the contrary, it can be reasonably explained as the cessation of the energy injection episode (Xin et al. 2011). This is a natural consequence of our model. We have shown that the
4.3. GRB 091029

The optical and X-ray light curves of the afterglow of GRB 091029 are not similar to each other and are it is hard to explain them using a simple model. It is very peculiar that the optical spectral index decreases between 0.4 and 9 ks, and increases later, while the X-ray spectral index remains almost constant. Filgas et al. (2012) examined several scenarios and argued that a two-component jet can basically explain the observations. The hardening of the optical spectrum can be explained by assuming that the electron power-law index changes with time. In our scenario, the hardening of the optical spectrum before $10^4$ s is also due to the varying electron index $p$ of the FS (Kong et al. 2010; Filgas et al. 2012). But after $10^4$ s, the softening of the optical spectrum is caused by the rising flux from the RS. According to Filgas et al. (2012), the X-ray spectral index is $\approx 1.1$ over the entire time window. In our model, the X-ray band is in the $\nu > \nu_c > \nu_m$ regime at $10^3$ s, which leads to $p_2 = 2.2$. Note that the optical flux from the RS should be dominated after $10^4$ s; $p_3$ should be relatively larger to match the softening spectrum. In our numerical calculations, we take $p_2 = 2.2$ and $p_3 = 2.3$. The broadband afterglow of GRB 091029 can be fitted quite well.

4.4. GRB 100814A

Possible interpretations for the afterglow of GRB 100814A have been discussed by De Pasquale et al. (2015). In De Pasquale et al. (2015), the optical rebrightenings are attributed to the FS, when $\nu_m$ of the FS is crossing the optical band, and the RS is generated by the late shells that collide with the trailing ones. In our scenario, the rebrightening is due to the emergence of the RS flux, while the RS forms due to the injection of continuous $e^+e^-$ winds. Our numerical results indicate that the optical flux from the FS peaks at around 400 s, when $\nu_m$ of the FS crosses the optical band (see the lower panel of Figure 2). The observed $R_c$-band light curve (see Figure 2 of De Pasquale et al. 2015) does show an early peak, which is consistent with our results.

5. CONCLUSIONS

Optical rebrightenings appearing at $\sim 10^4$ s in the afterglows of some GRBs may come from a common origin. In this work, we attribute the rebrightenings to the RS propagating into the ultrarelativistic $e^+e^-$ wind ejected by the central engine. We compared two methods used for solving the dynamics and found that they are both appropriate for describing the FS–RS system. Multiwavelength afterglows of four GRBs can be well explained in the framework of our scenario.

The success of our model can give helpful information about the central engine of these GRBs. $T_{d}$ can be derived from the X-ray peak time of the RS component as described in Section 3, which could give clues about the characteristics of the newly born magnetar. Generally, an earlier rebrightening means that $B_{NS}$ is larger or $P_{NS}$ is smaller (i.e., the magnetar loses its energy more quickly). In the future, if the observational data can help constrain the parameters of the FS, we will be able to derive $B_{NS}$ more accurately by using the rebrightening. This will be a useful way to probe the characteristics of the magnetar. Note that the derived values of $T_{d}$ of GRB 091029 and GRB 100814A are larger than those of the other two GRBs, which means that the efficiency of converting the Poynting-flux to the kinetic energy of particles is higher in these cases.

Observationally, there do not seem to be any equivalent features in X-rays around the optical rebrightenings; however, the
predicted emission near $T_{\text{peak},2}$ in both optical and X-ray wavelengths is dominated by the RS component in our model. There are two main factors that lead to this. First, the actual peak time of the RS component in the optical bands is delayed ($> T_{\text{b}}$) by the EATS effect, which causes the flux ratio $F_{\text{opt},3}/F_{\text{opt},2}$ to be relatively larger because $F_{\text{opt},2}$ is decreasing. Second, at the X-ray peak time the flux ratio of $F_{X,3}/F_{X,2}$ is expected to be relatively small. Moreover, the emergence of the RS component in the X-rays is essential for producing the X-ray plateau, otherwise the X-ray light curve will decay as a power-law after $T_{\text{peak},3}$ (see $F_{X,2}$ in Figure 2). In other words, the X-ray plateau should be a common feature accompanying the optical rebrightening.

The fact that the rebrightening time is coincident with $T_{\text{b}}$, along with the arguments that magnetars could serve as the central engines of GRBs in recent years, motivates us to suggest that there may be a common origin for the rebrightening. $e^+e^-$-wind are natural outcome from magnetars, as hinted from phenomena associated with pulsar wind nebulae. Thus we believe our scenario could work well and it is physically reasonable. Of course, models involving late shells with a range of velocities may also produce similar results. Thus further studies are necessary on the form of the injected energy. In principle, the mass density is different between a baryonic shell and an electron–positron wind, but it is not clear how the mass density can be measured from observations. For the baryonic shell, one may calculate the total mass of the shell to check whether it is reasonable.

In the future, two potential methods should help to discriminate between our $e^+e^-$ wind scenario and models involving baryonic-dominated shells. First, the high energy emission ($>100$ MeV) should be more significant when the continuously injected energy is carried by leptons rather than baryons, because IC scattering will be much stronger (Yu et al. 2007). As an example, Figure 3 shows how the SSC flux from the RS exceeds the synchrotron emission from the FS at late times in GRB 100814A. If a high energy bump is observed near the peak time of the optical rebrightening, it will be strong evidence for the $e^+e^-$ wind. Furthermore, the evolution of the polarization degree during the plateau or the rebrightening phase in the two models is significantly different (Lan et al. 2016). Therefore, observations of the high energy emission and the polarization during the rebrightening phase would identify which model is preferred.

The $e^+e^-$-wind scenario is not necessarily based on magnetars as central engines. A newly born BH accompanied by an accretion disk may also emit continuous Poynting-flux (Blandford & Znajek 1977) and the luminosity evolution may be similar to that of a magnetar (Equation (1)). Such a BH can also serve as the central engine in our model. Note that while the Poynting-flux is assumed to be converted to an $e^+e^-$-wind efficiently in our model, the possibility of this conversion and its efficiency are still quite uncertain for a newly born magnetar. A more general treatment is to introduce a magnetization parameter $\sigma$ to describe the late ejecta (Zhang & Kobayashi 2005), and this may affect the RS emission if $\sigma$ is significantly larger than 1. The diversity of $\sigma$ would also help explain why optical rebrightenings do not appear in some GRB afterglows. For a magnetized ejecta, the parameters derived may be correspondingly different. However, the main conclusions of this study would remain unchanged.

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APPENDIX A

HYDRODYNAMICS

A.1. Energy Conservation Method

Let’s consider a relativistic outflow with an initial mass of $M_0$ propagating into a cold interstellar medium (ISM). Two shocks separate the system into four regions: (1) the unshocked ISM, (2) the shocked ISM, (3) the shocked wind, and (4) the unshocked wind. Regions 2 and 3 can be regarded as simple homogenous shells (Piran 1999). In this paper, the quantities (e.g., the electron Lorentz factor $\gamma_e$, internal energy $U_i$, and pressure $p$) of Region “$i$” are denoted by subscripts “$i$,” and the superscript prime (′) is used to denote the quantities in the shock comoving frame, while characters without prime denote quantities in the observer frame. The FS and the RS can be described by a common bulk Lorentz factor $\Gamma_2 = \Gamma_3 = \Gamma$ (the corresponding dimensionless speed is $\beta$) and we assume that the Lorentz factor of the unshocked wind is $\Gamma_1$. Below, $\Gamma_i$ and $\beta_i$ are the relative Lorentz factor and dimensionless speed of Region “$i$” as measured in the frame of Region “$j$.” Applying the jump conditions to the FS, the thermodynamical quantities of the gases in the rest frame of Region 2 are given by $U_2' = (\Gamma - 1)m_2c^2$ (internal energy) and $p_2' V_2' = (\gamma_2 - 1)U_2'$ (product of pressure and volume), where $m_2$ is the total mass swept by the FS, $\gamma_2 \simeq (4\Gamma + 1)/3\Gamma^2$ is the adiabatic index, and $c$ is the speed of light. If the fraction of the thermal energy lost due to radiation is $\epsilon_2$ (i.e., the radiation efficiency), then the energy of Region 2 is ($\text{Pe'er}$ 2012; Geng et al. 2014)

$$H_2 \simeq (\Gamma - 1)(m_2 + M_0)\epsilon_2 c^2 + (1 - \epsilon_2\Gamma)U_2' + \rho_2' V_2'$$.

Taking $\epsilon_2$ as the equipartition parameter for shocked electrons, the radiation efficiency can be calculated as $\epsilon_2 = \epsilon_2 c_{\text{syn}}^\prime f / (c_{\text{syn}}^\prime + c_{\text{ex}}^\prime)$ (Dai & Lu 1999), where $c_{\text{syn}}^\prime$ is the synchrotron cooling timescale and $c_{\text{ex}}^\prime = R/(\Gamma c)$ is the expansion timescale in the comoving frame. Similarly, the energy of Region 3 is

$$H_3 \simeq (\Gamma - 1)m_3c^2 + (1 - \epsilon_3\Gamma)U_3' + \rho_3' V_3'$$,

where $U_3' = (\Gamma_3 - 1)m_3c^2$ and $\rho_3' V_3' = (\gamma_3 - 1)U_3'$. The adiabatic index of Region 3 can be approximated as $\gamma_3 \simeq (4\Gamma_3 + 1)/3\Gamma_3$. The total energy of the matter between the FS and the RS is thus $H_{FS} = H_2 + H_3$.

We can derive the differential equation for the energy following the procedure of Huang et al. (1999, 2000) and Geng et al. (2014). When a mass of $dm_3$ of the ISM is swept up by the FS, a fraction of thermal energy, $dE_{\text{loss}} = \epsilon_2\Gamma\gamma_2 (\Gamma - 1)d\rho_3 c^2$, will be radiated from Region 2. Similarly, when the wind matter of mass $dm_1$ is swept up by the RS, Region 3 will gain some thermal energy: $dE_{\text{gain}} = (1 - \epsilon_3)\Gamma\gamma_3 (\Gamma_3 - 1)d\rho_3 c^2$. Using

$$dH_{\text{tot}} = dE_{\text{gain}} - dE_{\text{loss}}, \quad d\Gamma_{13}/d\Gamma = -\beta_3\Gamma_{13}/\beta_1$$,
\( \Gamma_4 = \text{constant} \), and the relevant formulae above, we have
\[
\frac{d\Gamma}{dm_2} = \frac{f_2}{f_1} + \frac{f_3}{f_1} \frac{dm_3}{dm_2},
\]
(13)
where
\[
f_1 = M_2 + m_2 + m_3 + \frac{1}{2} (1 - \varepsilon_3) (8 \Gamma_3 - 3) m_2 + \frac{1}{4} (1 - \varepsilon_3)^2 \Gamma_4,
\]
\[
f_2 = -2 \varepsilon_2 \Gamma_2 (1 - \varepsilon_2),
\]
\[
f_3 = -(\Gamma - 1). \quad \text{On the other hand, it can be derived that (see Geng et al. 2014)}
\]
\[
\frac{dm_3}{dm_2} = \left( \frac{\beta_4}{\beta_1} - 1 \right) \frac{\rho'_4}{\rho_1} \Gamma_4,
\]
(14)
where \( \rho'_4 \) is the comoving density of Region 4 and \( \rho_1 \) is the density of the circumburst environment. The evolution of \( \Gamma \) can thus be calculated using Equation (13).

A.2. Mechanical Method

Another method to solve the dynamics of the FS–RS system was developed by Beloborodov & Uhm (2006). We briefly describe this mechanical method here. In this method, the shocked Regions 2 and 3 between the FS and RS are called the “blast,” and the blast is assumed to move with a Lorentz factor \( \Gamma \) (the corresponding dimensionless speed is \( \beta \)). In this section, the subscript “f” or “r” is used to denote the quantities just behind the FS or the RS, respectively (relative to the shock front) of the postshock medium are \( \beta_f \) and \( \beta_r \), respectively. Considering the conservation of energy-momentum and mass flux in Regions 2 and 3 between the FS \( (r_f) \) and the RS \( (r_r) \), three equations can be obtained
\[
1 \frac{d}{dt} \left( r^2 \Sigma \Gamma \right) - \Gamma \left[ \rho_r (\beta - \beta_r) + \rho_f (\beta_f - \beta_r) \right] = 0,
\]
(15)
\[
1 \frac{d}{dt} \left( r^2 H T \beta \right) - \Gamma \left[ h_r (\beta - \beta_r) + h_f (\beta_f - \beta_r) \right] = p_r - p_f,
\]
(16)
\[
1 \frac{d}{dt} \left( r^2 H \Gamma \right) - \Gamma \left[ h_r (\beta - \beta_r) + \rho_f (\beta_f - \beta_r) \right] = \Sigma \frac{d}{dt} p - \Gamma \left[ p_r (\beta - \beta_r) + p_f (\beta_f - \beta_r) \right],
\]
(17)
where \( \Sigma = \int_{i}^{r_f} \rho dr, \quad H = \int_{i}^{r_f} hdr, \quad P = \int_{i}^{r_f} pdr, \quad \Gamma - \Sigma e^2 = 4 P, \quad \rho, h, p \) are the mass density, energy density, and pressure measured in the rest frame of the FS, respectively.

Other equations derived from shock jump conditions are needed to complete these equations (see Uhm 2011; Uhm et al. 2012 for details), which can be solved numerically; \( \Sigma, H, \) and \( P \) can be obtained together with \( \Sigma, H, \) and \( P \).

APPENDIX B RADIATION

Electrons will be accelerated by the FS and RS after the shocks. As usual, we assume that the accelerated electrons carry a fraction \( \varepsilon_{e,i} \) of the total energy and the ratio of the magnetic field energy to the total energy is \( \varepsilon_B,i \). In the absence of radiation loss, the energy distribution of the shocked electrons is usually assumed to be a power-law as \( dN_{e,i}/d\gamma_{e,i} \propto \gamma_{e,i}^{-\delta} (\gamma_{e,i}^{\prime} < \gamma_{e,i} \leq \gamma_{M,i}) \), where \( \gamma_{M,i} \) is the Lorentz factor of the electrons of Region \( i \), and \( p_i \) is the spectral index.

The minimum Lorentz factor is
\[
\gamma'_{m,i} = \left( \frac{1}{p_i} - 1 \right) \gamma^2_i + 1,
\]
(18)
where \( \gamma_z = m_p/m_e \) (\( m_p \) and \( m_e \) are the mass of protons and electrons, respectively), \( \gamma_z = 1, \quad \hat{\Gamma}_2 = \Gamma_2, \quad \hat{\Gamma}_3 = \Gamma_3 \). The maximum Lorentz factor is
\[
\gamma'_{M,i} \approx 10^3 [B' \left( 1 + Y_{e,i} \right)]^{-1/2},
\]
(19)
where \( B' \) is the comoving magnetic field strength, and \( Y_{e,i} \) is the Compton parameter that is defined as the ratio of the IC power to the synchrotron power. The Compton parameter of an electron with a Lorentz factor of \( \gamma_{e,i} \) can be determined by \( Y_{e,i} = \left( 1 + \sqrt{1 + 4 \gamma_{e,i}^2 \eta_{\text{IC}}(\epsilon_{e,i},\epsilon_{B,i})} \right)^2 / 2 \) (Fan & Piran 2006; He et al. 2009), where \( \eta_{\text{IC}} \) is the fraction of energy that is radiated due to synchrotron and IC radiation, and \( \eta_{\text{IC}} \) is the fraction of synchrotron photons with energy below the Klein–Nishina limit.

Considering the radiation loss, the actual electron distribution would be characterized by the cooling Lorentz factor \( \gamma'_{c,i} \), which is given by
\[
\gamma'_{c,i} = \left( 1 + Y_{e,i} \right) \sigma_T B'^2 \left( \Gamma_i - \sqrt{\Gamma_i^2 - 1} \right) t_{\text{obs}}
\]
(20)
where \( t_{\text{obs}} \) is the time measured in the observer’s frame and \( \sigma_T \) is the Thomson cross section. Then, the actual electron distribution should be given as the following cases: 1. for \( \gamma_{c,i} < \gamma_{m,i} \),
\[
\frac{dN_{e,i}}{d\gamma'_{e,i}} \propto \left( \gamma_{c,i}^\prime - 1 \right) \gamma_{c,i}^{\prime - 1} \gamma_{m,i}^{\prime - 1},
\]
(21)
2. for \( \gamma_{c,i} > \gamma_{m,i} \),
\[
\frac{dN_{e,i}}{d\gamma'_{e,i}} \propto \left( \gamma_{m,i}^\prime - 1 \right) \gamma_{m,i}^{\prime - 1} \gamma_{c,i}^{\prime - 1} \gamma_{c,i}^{\prime - 1} \left( \gamma_{c,i}^{\prime - 1} \right).
\]
(22)

With the electron distribution determined, the synchrotron emissivity of electrons in Region \( i \) at the frequency \( \nu' \) can be calculated as (Rybicki & Lightman 1979)
\[
\varepsilon_{\text{syn}}'(\nu') = \frac{\sqrt{3} q_3^2 B'^2}{m_e c^2} \int_{\gamma_{m,i}}^{\gamma_{c,i}} \frac{dN_{e,i}}{d\gamma'_{e,i}} \left( \frac{\nu'}{\nu_{ch}'(\gamma')} \right) d\gamma'_{e,i},
\]
(23)
where \( \nu_{ch}' = 3 \left( y_{e,i} \right)^2 q_3 B'/\left( 4 \pi m_e c \right), \quad q_3 \) is the electric charge of an electron, \( F(x) = x \int_{x}^{\infty} K_{5/3}(s) ds, \quad K_{5/3}(x) \) as the Bessel function. The synchrotron self-absorption effect should also be taken into account (Rybicki & Lightman 1979; Wu et al. 2003). In our calculation, the absorption effect can be included by multiplying a factor to the emissivity above. For the shell geometry, the absorption factor is expressed as \( (1 - e^{-\tau_c})/\tau_c \), where \( \tau' \) is the optical depth. The self-absorption coefficients \( \kappa_{5/3} \) can be derived analytically (see Appendix C), which further gives \( \tau' = \kappa_{5/3} \Delta = \kappa_{5/3} N_e \Omega / m_e \Omega c^2 \), where \( N_{\text{tot}} \) is the total number of electrons, \( \Delta \) is the width of the shell, \( \Omega = 2 \pi (1 - \cos \theta_j) \) is the solid angle of the outflow, and \( \theta_j \) is the half-opening angle of the jet.
Aside from the synchrotron radiation, electrons would be cooled by IC scattering of seed photons. IC scattering by self-emitted synchrotron photons is referred to as the synchrotron self-Compton (SSC) process, and IC scattering by photons from other regions is referred to as the cross inverse Compton (CIC) process. If the flux density of the seed photons of Region $j$ is $f_{\nu,j}$, the IC ($i=j$ for SSC and $i \neq j$ for CIC) emissivity at frequency $\nu'$ is calculated by (Blumenthal & Gould 1970; Yu et al. 2007)

$$
\epsilon_{\nu,IC}'(\nu') = 3\pi \int \gamma_{\nu,i} \frac{dN_{\nu,i}}{d\nu'} \frac{d\gamma_{\nu,i}}{d\nu'} \times \int_{\nu_{\nu,i}}^{\infty} d\nu'_{j} \frac{\nu_{j}^{-1}}{4\gamma_{\nu,i}^{2} \nu_{j}^{-1}} g(x, y),
$$

(24)

where $\gamma_{\nu,j} = \max[\gamma_{\nu,j}^{\text{min}}, \frac{\nu_{j}^{m} c^{2}}{4\gamma_{\nu,j}^{c} m_{e} c^{2} - \nu_{j}^{2}}, \frac{\nu_{j}^{m} c^{2}}{4\gamma_{\nu,j}^{c} m_{e} c^{2} - \nu_{j}^{2}}]$, $\nu_{j}^{m, \text{min}} = \nu_{j}^{m} c^{2}/(4\gamma_{\nu,j}^{c} m_{e} c^{2} - \nu_{j}^{2})$, $x = 4\gamma_{\nu,j}^{c} h \nu_{j}/m_{e} c^{2}$, $y = h \nu_{j}/(x (\gamma_{\nu,j}^{c} m_{e} c^{2} - \nu_{j}^{2}))$, and $g(x, y) = 2y \ln y + (1 + 2y)(1 - y) + \frac{y^{2}}{2(1+y)}(1-y)$.

If $\theta$ is the angle between the velocity of the emitting material and the line of sight, then $D_{c} = 1/|\Gamma(1 - \beta_{e} \cos \theta)|$. The observed synchrotron and IC flux densities at frequency $\nu = \nu'/D_{c}$ from Region $i$ are given by

$$
F_{\nu,i}^{\text{SYN}} = \int_{0}^{0} d\nu'_{i} \frac{\sin \theta}{1 - \cos \theta_{j}} \frac{D_{c}^{2} \nu_{i}^{2,\text{SYN}}(D_{c}^{-1} \nu)}{4\pi D_{c}^{2}},
$$

(25)

$$
F_{\nu,i}^{\text{IC}} = \int_{0}^{0} d\nu'_{i} \frac{\sin \theta}{1 - \cos \theta_{j}} \frac{D_{c}^{2} \nu_{i}^{2,\text{IC}}(D_{c}^{-1} \nu)}{4\pi D_{c}^{2}},
$$

(26)

where $V_{i}^{\prime}(\theta)$ is the volume of the emitting material. The luminosity distance $D_{L}$ is obtained by adopting a flat ΛCDM universe, in which $H_{0} = 71$ km s$^{-1}$, $\Omega_{m} = 0.27$, and $\Omega_{\Lambda} = 0.73$. In our calculations, the integration is performed over the EATS (Waxman 1997; Granot et al. 1999; Huang et al. 2007). The

$$
\kappa_{\nu}' \approx \begin{cases} 
C_{2} \frac{8\pi^{2}}{9} \frac{p + 2}{9} \frac{q_{e}}{B} \gamma_{m}^{-1} \nu_{m}^{-(p+4)} \left(\frac{\nu'}{\nu_{m}}\right)^{-5/3}, & \nu' < \nu_{m}', \\
C_{2} \frac{\sqrt{3} \pi}{9} \frac{2(p+2)}{3} \frac{1}{\Gamma(1/3)} \left(\frac{p}{4} + \frac{11}{8}\right) \Gamma(\frac{p}{4} + \frac{3}{8}) \frac{q_{e}}{B} \gamma_{m}^{-1} \nu_{m}^{-(p+4)} \left(\frac{\nu'}{\nu_{m}}\right)^{-5(p+4)/2}, & \nu_{m}' < \nu' < \nu_{c}', \\
C_{2} \frac{\sqrt{3} \pi}{9} \frac{2(p+3)}{3} \frac{1}{\Gamma(1/3)} \left(\frac{p}{4} + \frac{35}{12}\right) \Gamma(\frac{p}{4} + \frac{5}{8}) \frac{q_{e}}{B} \gamma_{m}^{-1} \nu_{m}^{-(p+4)} \left(\frac{\nu'}{\nu_{m}}\right)^{-5(p+4)/2} e^{-\nu'/(\nu_{m})}, & \nu' > \nu_{M}', \\
\end{cases}
$$

(32)

where $\Gamma(p)$ is the Gamma function. In the fast cooling case,

$$
\frac{dN_{\nu}'}{d\gamma_{e}'} = \begin{cases} 
C_{2} \gamma_{e}^{-1} \gamma_{e}^{p-1}, & \gamma_{e}' \leq \gamma_{e}' \leq \gamma_{e}', \\
C_{2} \gamma_{e}^{p-1} \gamma_{e}^{-p-1}, & \gamma_{e}' \leq \gamma_{e}' \leq \gamma_{e}', \\
\end{cases}
$$

(33)

the integral in Equation (30) gives:

EATS (or a sequence of $V_{i}'(\theta)$) at $t_{\text{obs}}$ is determined by

$$
t_{\text{obs}} = (1 + z) \int_{0}^{R_{0}} \frac{1 - \beta_{c} \cos \theta}{\beta_{c}} d\tau \equiv \text{const},
$$

(27)

from which $R_{0}$ (or $V_{i}'(\theta)$) can be derived for a given $\theta$. APPENDIX C

**SYNCHROTRON SELF-ABSORPTION COEFFICIENTS**

In this section, as an example, we consider only one emitting region, thus we remove the subscript $i$. In the comoving frame, the synchrotron radiation power at frequency $\nu'$ from an electron of $\gamma_{e}'$ is

$$
P'(\nu') = \frac{\sqrt{3} q_{e}^{3} B'_{e}}{m_{e} c^{2}} F \left(\nu' / \nu'_{ch}\right) = C_{3} F \left(\nu' / \nu'_{ch}\right),
$$

(28)

where $\nu'_{ch} = 3\gamma_{e}^{2} q_{e} B'_{e}/(4\pi m_{e} c)$. For electrons with a distribution of $dN_{\nu}'/d\gamma_{e}'$, the self-scattering absorption coefficient at frequency $\nu'$ can be calculated as (Rybicki & Lightman 1979)

$$
\kappa_{\nu}' = \frac{1}{8\pi m_{e} \nu'_{ch}^{3}} \int_{\nu_{\nu,\min}}^{\nu_{\nu,\max}} d\nu'_{i} \frac{d}{d\nu'_{i}} \left[ \frac{1}{\gamma_{e}^{2}} \frac{dN_{\nu}'(\nu, \gamma_{e}') d\nu'}{d\gamma_{e}' \nu_{\nu,\min}} \right]
$$

(29)

If we define $x = \nu' / \nu'_{ch}$, then $\gamma_{e}' = \left(\frac{4\pi m_{e} c}{3 B'_{e}}\right)^{1/2} \left(x / \tau\right)^{1/2} C_{1} \left(x / \tau\right)$. We assume the distribution of electrons is $dN_{\nu}'/d\gamma_{e}' = C_{2} \gamma_{e}'^{-p}$, then Equation (29) can be written as

$$
\kappa_{\nu}' = \frac{C_{1} C_{2} \gamma_{e}^{-p}}{16 \pi m_{e}} \int_{x_{m}}^{x_{M}} x^{(p-2)/2} \frac{F(x)}{dx} \nu'_{ch}^{-5(p+4)/2}.
$$

(30)

where $x_{m} = \nu' / \nu'_{m}$ and $x_{M} = \nu' / \nu'_{M}$. In the slow cooling case,

$$
\frac{dN_{\nu}'}{d\gamma_{e}'} = \begin{cases} 
C_{2} \gamma_{e}^{-p}, & \gamma_{e}' \leq \gamma_{e}' \leq \gamma_{e}', \\
C_{2} \gamma_{e}^{-p} \gamma_{e}', & \gamma_{e}' \leq \gamma_{e}' \leq \gamma_{e}', \\
\end{cases}
$$

(31)

the integral in Equation (30) can be performed for $\nu'$ as (also see Wu et al. 2003):
\[
\begin{array}{l}
\kappa_{\nu'} \simeq \\
\left\{ \begin{array}{ll}
\frac{4\pi^2}{3} \left( \frac{q_e}{B' c} \right)^{\gamma - 6} \left( \frac{\nu'}{\nu'_c} \right)^{5/3}, & \nu' < \nu'_c, \\
\frac{4\sqrt{\pi}}{9} \Gamma \left( \frac{14}{9} \right) \Gamma \left( \frac{2}{3} \right) \left( \frac{q_e}{B' c} \right)^{\gamma - 6} \left( \frac{\nu'}{\nu'_c} \right)^{-3}, & \nu'_c < \nu' < \nu'_M, \\
\frac{2\sqrt{\pi}}{9} \left( \frac{q_e}{B' c} \right)^{\gamma - 6} \left( \frac{\nu'}{\nu'_c} \right)^{-3} \left( \frac{\nu'_M}{\nu'_c} \right)^{-3/2} \left( \frac{\nu'_m}{\nu'_c} \right)^{-(p+5)/2}, & \nu'_m < \nu' < \nu'_M, \\
\frac{2\sqrt{\pi}}{9} \left( \frac{q_e}{B' c} \right)^{\gamma - 6} \left( \frac{\nu'}{\nu'_c} \right)^{-3} \left( \frac{\nu'_M}{\nu'_c} \right)^{-5/2} \left( \frac{\nu'_m}{\nu'_c} \right)^{-(p+5)/2}, & \nu' > \nu'_M.
\end{array} \right.
\end{array}
\]