Energy-Momentum of the Friedmann Models in General Relativity and Teleparallel Theory of Gravity

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Abstract

This paper is devoted to the evaluation of the energy-momentum density components for the Friedmann models. For this purpose, we have used Møller’s pseudotensor prescription in General Relativity and a certain energy-momentum density developed from his teleparallel formulation. It is shown that the energy density of the closed Friedmann universe vanishes on the spherical shell at the radius $\rho = 2\sqrt{3}$. This coincides with the earlier results available in the literature. We also discuss the energy of the flat and open models. A comparison shows a partial consistency between the Møller’s pseudotensor for General Relativity and teleparallel theory. Further, it is shown that the results are independent of the free dimensionless coupling constant of the teleparallel gravity.

Keywords: Energy-Momentum distribution

1 Introduction

The localization of energy and momentum [1] in General Relativity (GR) is an open, most challenging and controversial problem. In the framework

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of GR, many attempts have been made to calculate energy distribution by using different approaches. The use of energy-momentum complexes for the localization of energy and momentum is one of these methods. Many physicists, such as Einstein [2], Landau-Lifshitz [3], Papapetrou [4], Bergmann [5], Tolman [6], Weinberg [7] and Møller [8] have given their own definitions for the energy-momentum complex. Most of these are coordinate dependent while the Møller expression gives an energy value independent of the choice of spatial coordinates; this is not the case for the momentum.

The lack of a generally accepted definition of energy-momentum in a curved spacetime has led to doubts regarding the idea of energy localization. According to Misner et al. [1], energy is localizable only for spherical systems. Cooperstock and Sarracino [9] came up with the view that if energy is localizable only for spherical systems, then it can be localized in any system. Bondi [10] argued that a non-localizable form of energy is not allowed in GR. After this, an alternative concept of energy, called quasi-local energy, was developed. The use of quasi-local masses to obtain energy-momentum in a curved spacetime do not restrict one to use particular coordinate system. A large number of definitions of quasi-local masses have been proposed, those by Penrose and many others [11-13]. An excellent review article about quasi-local energy-momentum and angular momentum in GR has been given by Szabados [14]. Although these quasi-local masses are conceptually very important, yet these definitions have some problems. Bergqvist [15] considered seven different definitions of quasi-local masses and computed them for Reissner-Nordstrom and Kerr spacetimes. He concluded that no two of the seven definitions provided the same result. The seminal concept of quasi-local masses of Penrose cannot be used to handle even the Kerr metric [16]. The present quasi-local mass definitions still have inadequacies.

It is believed that different energy-momentum distribution would be obtained from different energy-momentum complexes. Virbhadra [17,18] revived the interest in this approach. He and his co-workers [18-22] considered many asymptotically flat spacetimes and showed that several energy-momentum complexes could give the same result for a given spacetime. They also carried out calculations in a few asymptotically non-flat spacetimes using different energy-momentum complexes and found encouraging results. Aguirregabiria at el. [23] proved that several energy-momentum complexes can provide the same results for any Kerr-Schild class metric. Xulu [24,25] extended this investigation and found that Melvin magnetic universe and Bianchi type I universe provided the same energy distribu-
tion. One of the authors [26] found several examples which did not provide the same result in all prescriptions. Chang at el. [27] showed that every energy-momentum complex could be associated with a particular Hamiltonian boundary term. Therefore, the energy-momentum complexes may also be considered as quasi-local. According to the Hamiltonian approach, the various energy-momentum expressions are each associated with distinct boundary conditions [27,28].

The beauty and speciality of the Møller prescription is that one can use any spatial coordinate system for evaluating the energy, while the other prescriptions restrict one to use the Cartesian coordinate system only for obtaining meaningful results. On the basis of this fact, Lessner [29] concluded that Møller definition is a powerful concept of energy and momentum in GR. Also, the literature [30-33] shows that the Møller energy-momentum complex is a good tool for evaluating energy distribution in a given spacetime. Thus the use of Møller prescription looks more interesting, useful and appropriate while finding the energy distribution. The results obtained in [18,20,22,25,26,33] indicate that the energy distribution is different for some particular spacetimes including Schwarzschild spacetime when one uses Møller and Einstein prescriptions.

In spite of the efforts made during last nine decades, the problem of localization of energy is still without a definite answer in GR. Thus, it seems to be justified to explore this problem in the framework of some other theories. Many authors believed that a tetrad theory should describe more than a pure gravitation field. In fact, Møller [34] considered this possibility in his earlier attempt to modify GR. Mikhail et al. [35] re-defined the Møller energy-momentum complex in tetrad theory. de Andrade et al. [36] considered gravitational energy-momentum density in teleparallel gravity. Maluf, J.W. et al. [37] explored energy and angular momentum of the gravitational field in the teleparallel geometry. Blagojevic, M. and Vasilic, M. [38] discussed conservation laws in the teleparallel theory of gravity. Sakane, E. and Kawai, T. [39] found energy-momentum and angular momentum carried by gravitational waves in extended new GR.

Some authors [35,40,41] argued that this problem of energy definition might be settled down in the context of teleparallel theory (TPT) of gravitation. They showed that energy-momentum can also be localized in the framework of this theory. It has been shown that the results of GR and TPT agree with each other for particular spacetimes. Møller showed that a tetrad description of a gravitational field equation allows a more satisfactory treatment
of the energy-momentum complex than does GR. Vargas [41] found that the total energy of the closed Friedmann-Robertson-Walker (FRW) spacetime is zero by using teleparallel version of Einstein and Landau-Lifshitz complexes. This agrees with the result obtained by Rosen [42] in GR. Recently, Yu-Xiao et al. [43] derived the conservation laws of energy-momentum in TPG and found the energy-momentum of the universe with the help of these laws. They showed that the energy-momentum four-vector vanishes both in spherical as well as in Cartesian coordinates. Also, Nester et al. [44] explored the energy of homogenous cosmologies and discussed the results.

In recent papers [45,46], we have obtained the TP versions of Lewis-Papapetrou, Friedmann models and the stationary axisymmetric solutions of the Einstein-Maxwell field equations. We have also found the energy-momentum distribution of the Lewis-Papapetrou spacetime [47] and the stationary axisymmetric solutions of the Einstein-Maxwell equations by using Møller prescription. Further, we have evaluated the energy-momentum distribution of static axially symmetric spacetimes [48] by using four different prescriptions, namely, Einstein, Landau-Lifshitz, Bergmann and Møller. This is shown that the results of TPT do not agree with those available in the context of GR for this particular spacetime. In this paper, we extend this idea to evaluate the energy-momentum density components both in GR and TPT for FRW metric. It is shown that the results for both the theories turn out be consistent partially.

The scheme of this paper is as follows. In section 2, we shall give a brief overview of the theory of teleparallel gravity. Section 3 explains the formulation to evaluate energy-momentum. Section 4 is devoted to determine the energy-momentum components for the Friedmann models both in GR and TPT. The last section will furnish the summary and discussion of the results obtained.

2 Theory of Teleparallel Gravity

The theory of teleparallel gravity is described by the Weitzenböck connection given by [49]

$$\Gamma^{\theta}_{\mu\nu} = h^a_\theta \partial_\nu h^a_\mu,$$

(1)

where the non-trivial tetrad $h^a_\mu$ and its inverse field $h_\mu^\nu$ satisfy the relations

$$h^a_\mu h_\nu^a = \delta^\nu_\mu; \quad h^a_\mu h^\mu_b = \delta^a_b.$$ 

(2)
In this paper the Latin alphabet \((a, b, c, ... = 0, 1, 2, 3)\) will be used to denote the tangent space indices and the Greek alphabet \((\mu, \nu, \rho, ... = 0, 1, 2, 3)\) to denote the spacetime indices. The Riemannian metric in TPT arises as a by product \([50]\) of the tetrad field given by

\[
g_{\mu\nu} = \eta_{ab} h^a_{\mu} h^b_{\nu},
\]

which is antisymmetric in nature. Due to the requirement of absolute parallelism, the curvature of the Weitzenböck connection vanishes identically. The Weitzenböck connection also satisfies the relation given by

\[
\Gamma^\theta_{\mu\nu} = \Gamma^\mu_{\nu\mu} - K^\theta_{\mu\nu},
\]

where

\[
K^\sigma_{\rho\chi} = \frac{1}{2} [T^\theta_{\mu\nu} + T^\theta_{\nu\mu} - T^\theta_{\mu\nu}]
\]

is the contortion tensor and \(\Gamma^\theta_{\mu\nu}\) are the Christoffel symbols.

Møller \([34]\) pioneered the teleparallel approach to energy-momentum in 1961. Later, Mikhail et al. \([35]\) defined their super-potential associated with Møller’s tetrad theory formulation. This is given as

\[
U^\nu_{\mu\beta} = \sqrt{-g} \frac{16\pi}{\kappa} P^\nu_{\chi\rho\sigma}[\Phi^\rho g^\sigma\chi g_{\mu\tau} - \lambda g_{\tau\mu} K_{\chi\rho\sigma} - g_{\mu\tau}(1 - 2\lambda) K^\rho_{\chi\sigma\nu}],
\]

where

\[
P^\nu_{\chi\rho\sigma} = \epsilon_{\chi}^\nu g^\rho_{\sigma} + \delta^\rho_{\sigma} g^\nu_{\chi} - \delta^\nu_{\chi} g^\rho_{\sigma},
\]

while \(g^\nu_{\rho\sigma}\) is a tensor quantity and is defined by

\[
g^{\nu\beta}_{\rho\sigma} = \delta^\nu_{\sigma} \delta^\beta_{\rho} - \delta^\nu_{\rho} \delta^\beta_{\sigma}.
\]

\(K^\rho_{\chi\sigma\nu}\) is contortion tensor as given by Eq.(6), \(g\) is the determinant of the metric tensor \(g_{\mu\nu}\), \(\lambda\) is free dimensionless coupling constant of teleparallel gravity, \(\kappa\) is the Einstein constant and \(\Phi_{\mu}\) is the basic vector field given by

\[
\Phi_{\mu} = T^\nu_{\nu\mu}.
\]
The energy-momentum density is defined as [35]
\[ \Xi_{\mu}^{\nu} = U_{\mu}^{\nu \rho}, \] (11)
where comma means ordinary differentiation. The momentum 4-vector can be expressed as
\[ P_{\mu} = \int_{\Sigma} \Xi_{\mu}^{0} dx dy dz, \] (12)
where \( P_{0} \) gives the energy and \( P_{1}, P_{2}, \) and \( P_{3} \) are the momentum components. The integration is taken over the hyper-surface element \( \Sigma \), which is described by \( x^{0} = t = \text{constant} \). The energy may be given in the form of surface integral using Gauss’s theorem as
\[ E = \lim_{r \to \infty} \int_{r=\text{constant}} U_{0}^{0} u_{\rho} dS, \] (13)
where \( u_{\rho} \) is the unit three-vector normal to the surface element \( dS \).

3 Møller Energy-Momentum Complex

In 1958, Møller presented a new pseudotensor description of energy-momentum for gravitating systems with an interesting property: namely, the energy value is independent of the choice of spatial coordinates. This is given as [8]
\[ M_{\mu}^{\nu} = \frac{1}{8\pi} Q_{\mu}^{\nu \rho}, \] (14)
where
\[ Q_{\mu}^{\nu \rho} = \sqrt{-g} (g_{\mu \sigma \tau} - g_{\mu \tau \sigma}) g^{\nu \tau} g^{\sigma \rho}, \] (15)
which is anti-symmetric in \( \nu \rho \). Here \( M_{0}^{0} \) is the energy density and \( M_{\mu}^{\nu} (\mu = 1, 2, 3) \) are the momentum density components. This satisfies the following local conservation laws
\[ \frac{\partial M_{\mu}^{\nu}}{\partial x^{\nu}} = 0, \] (16)
which contains contributions from the matter, gravitational and non-gravitational fields. The momentum 4-vector of Møller prescription is defined as
\[ P_{\mu} = \int \int \int M_{\mu}^{0} dx dy dz. \] (17)
By using Gauss’s law one can transform the last relation as
\[ P_{\mu} = \frac{1}{8\pi} \int \int Q_{\mu}^{0 \rho} u_{\rho} dS, \] (18)
where \( u_{\rho} \) is the unit three-vector normal to the surface element \( dS \).
4 Energy-momentum of the Friedmann Models

4.1 Energy in General Relativity

The Friedmann models of the universe are defined by the metric [53]
\[ ds^2 = dt^2 - a^2(t)[d\chi^2 + f_k^2(\chi)(d\theta^2 + \sin^2 \theta d\phi^2)], \]  
where
\[ f(\chi) = \begin{cases} \sinh \chi, & k = -1, \\ \chi, & k = 0, \\ \sin \chi, & k = +1, \end{cases} \]  
\[ \chi (0 \leq \chi < \infty \text{ for open and flat models but } 0 \leq \chi < 2\pi \text{ for closed model}) \]
\[ \text{is the hyper-spherical angle and } a(t) \text{ is the scale parameter. The isotropic form of the above metric is given as [54]} \]
\[ ds^2 = dt^2 - \frac{a^2(t)}{(1 + \frac{1}{4}k\rho^2)^2}[d\rho^2 + \rho^2(d\theta^2 + \sin^2 \theta d\phi^2)], \]  
which can be written as
\[ ds^2 = dt^2 - \frac{a^2(t)}{A^2(\rho)}(dx^2 + dy^2 + dz^2). \]  

Here \( x = \rho \sin \theta \cos \phi, \ y = \rho \sin \theta \sin \phi, \ z = \rho \cos \theta, \ A(\rho) = 1 + \frac{1}{4}k\rho^2 \) and \( \rho = \sqrt{x^2 + y^2 + z^2} \). Using Eq.(22) in Eq.(15), we get the following non-vanishing components
\[ Q_{01}^0 = Q_{02}^0 = Q_{03}^0 = \frac{2\dot{a}(t)a^2(t)}{A^3(\rho)}. \]  
Substituting these values in Eq.(14), the required energy-momentum density components turn out to be
\[ M_{0}^0 = 0, \]
\[ M_{1}^0 = -\frac{3k}{8\pi A^4}\dot{a}a^2 x, \]
\[ M_{2}^0 = -\frac{3k}{8\pi A^4}\dot{a}a^2 y, \]
\[ M_{3}^0 = -\frac{3k}{8\pi A^4}\dot{a}a^2 z. \]
Here dot means differentiation with respect to $t$. It follows from here that energy of the Friedmann models vanishes in GR. This exactly coincides with that found by Rosen [42] using Einstein’s prescription. However, momentum is non-vanishing along $x$, $y$ and $z$ directions. It seems that this energy-momentum density violates the usual energy conditions.

### 4.2 Energy in Teleparallel Gravity

Here we evaluate the energy-momentum distribution of the Friedmann models by using TP formulation as given in Eq.(7). Following the procedure given in [50], we write the tetrad of Eq.(21) as

$$ h^a_{\mu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{a(t)}{\Lambda(\rho)} & 0 & 0 \\ 0 & 0 & \frac{a(t)}{\Lambda(\rho)} & 0 \\ 0 & 0 & 0 & \frac{a(t)}{\Lambda(\rho)} \end{bmatrix}. $$

(25)

Its inverse becomes

$$ h_{\mu}^a = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \Lambda(\rho) & 0 & 0 \\ 0 & 0 & \Lambda(\rho) & 0 \\ 0 & 0 & 0 & \Lambda(\rho) \end{bmatrix}. $$

(26)

Using these tetrad in Eq.(1) and then in (4), we obtain the following non-vanishing components of the torsion tensor

\begin{align*}
T^{1}_{10} &= T^{2}_{20} = T^{3}_{30} = -T^{1}_{01} = -T^{2}_{02} = -T^{3}_{03} = -\dot{a}(t) \frac{a(t)}{\Lambda(\rho)}, \\
T^{2}_{21} &= T^{3}_{31} = -T^{2}_{12} = -T^{3}_{13} = \frac{2kx}{4 + k\rho^2}, \\
T^{1}_{12} &= T^{3}_{32} = -T^{1}_{21} = -T^{3}_{23} = \frac{2ky}{4 + k\rho^2}, \\
T^{1}_{13} &= T^{2}_{23} = -T^{1}_{31} = -T^{2}_{32} = \frac{2kz}{4 + k\rho^2}.
\end{align*}

(27)

Using the above values in Eq.(10) and then multiplying with the respective components of $g^{\mu\nu}$, we have

$$ \Phi^0 = -3\dot{a}(t) \frac{a(t)}{\rho}, \quad \Phi^1 = -\frac{A(\rho)kx}{a^2}, $$

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\[ \Phi^2 = -\frac{A(\rho)ky}{a^2}, \quad \Phi^3 = -\frac{A(\rho)kz}{a^2}. \]  

(28)

Substituting Eq.(27) in Eq.(6), we get the following non-vanishing components of the contorsion tensor, in covariant form, as

\[ K_{011} = K_{022} = K_{033} = \dot{a}(t) A^2(\rho) = -\dot{a}(t) A^2(\rho), \]
\[ K_{122} = K_{133} = -\frac{kA^3(\rho)x}{2a^4(t)} = -K_{212} = -K_{313}, \]
\[ K_{211} = K_{233} = -\frac{kA^3(\rho)y}{2a^4(t)} = -K_{121} = -K_{323}, \]
\[ K_{311} = K_{322} = -\frac{kA^3(\rho)z}{2a^4(t)} = -K_{131} = -K_{232}. \]  

(29)

Replacing these values in Eq.(7), the non-zero components of the superpotential are

\[ U_{01}^0 = \frac{ka(t)x}{8\pi A^2(\rho)}, \quad U_{02}^0 = \frac{ka(t)y}{8\pi A^2(\rho)}, \quad U_{03}^0 = \frac{ka(t)z}{8\pi A^2(\rho)}, \]
\[ U_{11}^0 = U_{22}^0 = U_{33}^0 = -\frac{\dot{a}(t)a^2(t)}{4\pi A^3}. \]  

(30)

It is remarked here that the results do not depend on \( \lambda \). When we make use of Eq.(30) in Eq.(11), the energy-momentum density components become

\[ \Xi_0^0 = \frac{(12 - kp^2)}{32\pi A^3} ka(\rho), \]
\[ \Xi_1^0 = \frac{3k}{8\pi A^4} \dot{a}a^2 x = -M_1^0, \]
\[ \Xi_2^0 = \frac{3k}{8\pi A^4} \dot{a}a^2 y = -M_2^0, \]
\[ \Xi_3^0 = \frac{3k}{8\pi A^4} \dot{a}a^2 z = -M_3^0. \]  

(31)

It follows from here that energy is zero for the flat model, i.e., when \( k = 0 \). On the spherical shell at the radius \( \rho = 2\sqrt{3} \), we get the energy density which vanishes for the closed model. For smaller values of \( \rho \), the energy density is positive but for larger values it is negative. The result of closed model partially coincides with the earlier result found by Vargas [41] according to
which energy density vanishes for the closed model by using Einstein and
Landau-Lifshitz prescriptions. It is obvious from Eq.(31) that the energy
density remains always negative in the case of open model. Further, the
momentum turns out to be non-vanishing along \( x, y, z \) directions indicating
that this teleparallel measure of energy is not a good one.

5 Summary and Discussion

The problem of energy-momentum localization has been a subject of many re-
searchers but still remains un-resolved. Numerous attempts have been made
to explore a quantity which describes the distribution of energy-momentum
due to matter, non-gravitational field and gravitational fields. Many energy-
momentum complexes have been found [2-5] and the problem associated with
the energy-momentum complexes leads to the doubts about the idea of energy
localization. This problem first appeared in electromagnetism which turns
out to be a serious matter in GR due to the non-tensorial quantities. Many
researchers considered different energy-momentum complexes and obtained
encouraging results. Virbhadra et al. [17-22] explored several spacetimes for
which different energy-momentum complexes show a high degree of consis-
tency in giving the same and acceptable energy-momentum distribution.

This paper is aimed to find energy and momentum of the Friedmann
models using Møller’s pseudotensor prescription in General Relativity and
a certain energy-momentum density developed from his teleparallel formu-
lation. We see from Eq.(24) that energy density vanishes, which gives zero
energy for all the three models in GR. This energy exactly coincides with that
already found by Rosen [42] using Einstein gravitational pseudo-tensor. In
TPT, energy becomes zero for the flat model. For the closed model, energy
vanishes on the spherical shell at the radius \( \rho = 2\sqrt{3} \). This shows that the
energy of the closed FRW universe become consistent with Vargas [36] found
by using Einstein’s and Landau-Lifshitz prescriptions. Further, we note that
momentum become zero for the flat model in both GR and TPT. Moreover,
our results in TPT are independent of the teleparallel free dimensionless cou-
ping constant \( \lambda \). This means that these results will be valid not only in the
case of teleparallel equivalent of GR, but also valid in any teleparallel model.
It is worth mentioning here that the components of the momentum densities
are exactly same with different signs both in GR and TPT.

Finally, we remark that the gravitational energy exactly cancels out the
matter energy for the flat and closed universes only on the spherical shell at the radius $\rho = 2\sqrt{3}$. We see that the energy density turns out to be negative for the open model and also the momentum does not vanish along $x$, $y$, $z$ directions. These indicate that Møller’s complex may not be a good measure of energy-momentum.

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