Studies of jet-shape observables in hard processes are summarized together with future developments.

1 Motivations for jet-shape study

Jet-shape studies provide information on QCD dynamics, are essential tools in analysis of (expected) “new physics” events and are needed for theoretical improvements and tests for Monte Carlo simulations. Jet-shape measurement involves the bulk of the events. They provide 30% of the entries in the determination of $\alpha_s$. Jets are present in all hard events in all high energy processes ($e^+e^-$, DIS and hadron-hadron collisions). Their characteristic features can be revealed by different jet-shape observables. For instance, thrust and broadening

$$B = \sum_h \frac{p_{ht}}{Q}, \quad \tau = 1 - T = \sum_h \frac{p_{ht} e^{-|\eta_h|}}{Q},$$

($p_h$ and $\eta_h$ the hadron transverse momentum and rapidity with respect to jet axis) probe radiation in different rapidity regions. Other observables, such as $V = (1 - T), B, C, \rho, D, y_{ij}, K_{out} \cdots$, probe various other features. The jet-shape distributions

$$\Sigma(V) = \sum_n \int \frac{d\sigma_n}{\sigma_{tot}} \Theta \left(V - \sum_h \nu_h\right),$$

are (in general) collinear safe (for $\vec{p}_i, \vec{p}_j$ parallel one may replace the two hadrons with a single one with $\vec{p}_i + \vec{p}_j$) and IR safe (for $p_i \ll p_j$ one may neglect $p_i$). This implies that all perturbative coefficients of $\Sigma(V)$ are finite and one may assume parton-flow $\simeq$ hadron-flow.

In the following I discuss the status of the calculations. First I discuss “global” observables (all emitted hadrons are contributing). Then I discuss the “non-global” ones (only hadrons in certain regions of phase space are contributing, e.g. rapidity cut in hadron-hadron collisions). In the first case the relevant configurations correspond to hadrons within the jets (then each jet contributes “independently” with a Sudakov factor). In the second case additional logarithmic terms are contributing which come from soft gluon emitted away from the jet region (correlations among jets are generated).

2 Global observables

The construction of reliable predictions for $\Sigma(V)$ involves perturbative and non-perturbative aspects. Before discussing specific observables I summarize the needed computations

1) exact LO and NLO computations (needed for finite $V$)

$$\Sigma_{PT}(V) = a(V) \alpha_s + b(V) \alpha_s^2 \cdots$$

2) DL and SL resummations (needed for large $L = \ln V^{-1}$)

$$\ln \Sigma(V) = \sum_{n=1}^{\infty} \left\{ d_n \alpha_s^n L^{n+1} + s_n \alpha_s^n L^n \cdots \right\}$$

At least $d_n$ (collinear+IR) and $s_n$ (collinear or IR) terms need to be resummed (using factorization of collinear and IR contributions and factorization of observable constraints via Mellin and/or Fourier transforms);

3) matching of exact computations (for finite $V$) and resummations (for small $V$);

4) power corrections originating from the running coupling at any scale ($0 < k_t < Q$),
thus involving hadron scales (at virtual level). This leads to non-convergence of PT expansion. For the average value, for instance,

\[ (V)_{PT} \sim \int_0^Q \frac{d\mu}{Q} \sum_n n! \left( \frac{\beta_0}{4\pi} \right)^n \alpha_s^n. \]

Without non-perturbative information, at present, the non-convergence is cured by prescriptions. Here are some: i) introduce a non-perturbative parameter \( \alpha_0(\mu_I) = \int_0^{\mu_I} \frac{d\mu}{\mu} \alpha_s(\mu) \) so that

\[ \Sigma(V) = \Sigma_{PT}(V + \Delta V), \]

with \( \Delta V \) given by \( \alpha_0(\mu_I) \) with computable coefficient. One has: nI-cancellation, \( \mu_I \)-independence, universality; ii) introduce a shape function \( V \) to modulate the radiator at large distance to account for higher 1/Q powers; iii) improve renormalization group implementation to reduced power corrections.

The simplest cases are in \( e^+ e^- \) with mostly two jet-events. The difficulty increases when more than two jets are involved. Here one has different partonic channel of the hard jets and one can explore in large contexts different prescriptions for non-perturbative calculations. Interesting cases are: 1) “three-jet observables” in \( e^+ e^- \) such\(^6\) as D-parameter and \( K_{out} \) (out-of-event-plane momentum) in events with finite 1 – T; 2) “three-jet observables” in DIS such\(^7\) as azimuthal correlation and \( K_{out} \)-distribution in large \( P_t \) events; 3) hadron-hadron collisions. Except for special cases\(^8\) in which only three QCD jets are involved, typically one has four jets (two incoming and two outgoing ones). In these cases one encounters new colour algebra features. With two \( (T_1 + T_2 = 0) \) or three \( (T_1 + T_2 + T_3 = 0) \) colour charges all products of primary parton colour matrices \( T_i T_j \) are proportional to the identity. This is not any more true for four \( (T_1 + T_2 + T_3 + T_4 = 0) \) or more primary parton processes. As a result in hadron-hadron dijet events, there are peculiar colour correlation\(^9\) among the four jets.

In conclusion, there is a large variety of jet-shape observables, they are very informative and characteristic of QCD, they enter all hard processes with features which are universal (QCD factorization). However they are difficult to measure (requiring a full knowledge of the produced hadrons) and to compute. Publications show that, on average, there is one paper per observable in a single process. Since by now, in principle, the computation procedure is known, one could build up an automated calculation

### 3 Automated resummation

A.Banfi, G.Salam and G.Zanderighi have constructed a numerical program\(^10\) to perform the automated resummation for any global jet-shape (present or future) distribution and for all processes. Here is the basic steps performed numerically. First the observable under consideration is analysed to check for collinear and IR regularity and globalness. Then one starts from the general factorized formula. For the most complex hard process, hadron-hadron collisions, one has

\[ \Sigma_{h_1 h_2}(V) = \left( \frac{p_h^a}{x_h} \times M_{a_b \to c} \right) \times \tilde{\Sigma}_{ab \to cd}(V), \]

with \( p_h^a \) density of parton \( a \), \( M^2 \) elementary hard distribution and \( \tilde{\Sigma}(V) \) the radiation formula for the corresponding observable which involves DL and SL resummation

\[ \tilde{\Sigma}_{res}(V) = e^{-R(V)} \cdot F(\alpha_s \ln V). \]

Here \( R(V) \) is the known (universal) DL-radiator and \( F(V) \) the SL-function computed by numerical simulation (for hadron-hadron collision this requires also the colour correlations among the four jets\(^9\)). To match with LO (and NLO) exact results one computes the coefficient function \( C(\alpha_s, V) \)

\[ \tilde{\Sigma}_{PT}(V) = C(\alpha_s, V) \cdot \tilde{\Sigma}_{res}(V). \]

Finally, power corrections are included\(^2\) by

\[ \tilde{\Sigma}(V) = \tilde{\Sigma}_{PT}(V + \Delta V). \]
4 New entry: non-global logs

In hadron-hadron collisions, due to the presence of rapidity cut, jet-shape observables take contributions only from part of phase space (non-global observables). Also many DIS observables\(^{11}\) are of this type. Here cancellation\(^{12}\) of collinear and IR singularities between real and virtual contributions is more complex than for global observables. There are additional SL contributions originating\(^{12,13,14,15}\) from soft large angle gluons generating correlations among jets not present in global observables. Indeed for global observables, due to dominance of collinear configurations, the radiation of each jet contributes (essentially) in an independent way.

Consider \(\Sigma_{q\bar{q}^+} - (Q, E_{\text{out}}, \theta_{\text{in}})\), the distribution of energy deposited away from a cone \(\theta_{\text{in}}\) around the thrust axis in \(e^+e^-\) jets. The resulting distribution factorizes

\[
\Sigma_{q\bar{q}^+} = S_{q\bar{q}}(\tau, \theta_{\text{in}}) \cdot C_{q\bar{q}}(\tau, \theta_{\text{in}}),
\]

with all factors depending on the SL variable

\[
\tau = \int_{E_{\text{out}}} d\eta k \frac{N_\alpha (k)}{\pi}
\]

The correlation function can be computed only in the planar approximation (large \(N_c\)) by using the multi-dipole formula\(^{16}\) for multi-soft-gluon distribution. Numerical studies have been performed by Dasgupta and Salam\(^{12}\) and the evolution equation\(^{13}\) resuming all SL is

\[
\partial_\tau \Sigma_{ab} = \left( \partial_\tau R_{ab} \right) \cdot \Sigma_{ab} + \int_{\Omega_k} \frac{d\Omega_k}{4\pi} \frac{(ab)}{(ak)(kb)} \left[ \Sigma_{ak} - \Sigma_{kb} - \Sigma_{ab} \right]
\]

where \((ij) = 1 - \cos \theta_{ij}\) and

\[
R_{ab} = -\tau \cdot \int_{E_{\text{out}}} d\eta k \frac{(ab)}{(ak)(kb)}
\]

is the Sudakov radiator giving \(S_{ab} = e^{-R_{ab}}\). The evolution equation involves general \(ij\)-dipoles (for \(e^+e^-\) one sets \(ij \Rightarrow q\bar{q}\)). Since \(k\) is inside the jet region, \(ij\)-dipoles with \(\theta_{ij} \sim \pi\) or \(\theta_{ij} < \theta_{\text{in}}\) are coupled.

There are many interesting properties for the correlation function \(C_{ab}\) for large \(\tau\) (but experimentally accessible values are \(\tau < 2\)–3). The branching inside the jet region starts very collinear to the primary quark with \(\theta_{hr} < e^{-c\tau}\), there is a large buffer inside the jet region\(^{12,13}\). As a result one finds

\[
C_{q\bar{q}}(\tau, \theta_{\text{in}}) \sim e^{-c\tau}, \quad c = 4.883 \cdots
\]

with \(c\) universal (independent of \(\theta_{\text{in}}\)). This shows that, at large \(\tau\), the suppression in \(C_{q\bar{q}}\) (due to virtual corrections) is overwhelming the one in the Sudakov factor.

For \(\theta_{ab} \ll 1\) there is an amazing connection with small-x dynamics. Indeed the value of \(c\) is obtained by noticing that the evolution equation for \(\Sigma_{ab}\) is very similar to the Kovchegov equation\(^{17}\) for the S-matrix even if the relevant multi-gluon phase space
regions are completely different (comparable angles in jet physics while comparable transverse momenta in small-x physics). Detailed studies of the connection between the two problems have been performed\textsuperscript{18,19} for heavy quark pair production in certain regions of phase space.

Non-global logs are difficult to compute. Asymptotic estimates for large $\tau$ are not physically relevant so that one needs to relay on numerical computation\textsuperscript{12}. Moreover it seems not easy to avoid the large $N_c$ approximation. However, in some cases it is possible to avoid/neglect non-global logs. This is the case when measurements are in part of phase space but one is able to recover radiation in the full phase space via momentum conservation (e.g. azimuthal correlations). The other possibility is to work with a part of phase space, but large enough. In particular, a rapidity region $Y$ in hadron-hadron collision leads to non-global correction which are of order $e^{-Y}$.

References

1. S.Bethke, Nucl. Phys. C121 (03) 74 [hep-ex/0211012].
2. Y.Dokshitzer, B.Webber, Phys. Lett. B352(1995)451 [hep-ph/9504219]; Y.Dokshitzer, G.Marchesini, B.Webber, Nucl. Phys. B469 (1996) 93 [hep-ph/9512336]
3. G.Korchemsky, G.Sterman, Nucl. Phys. B437(95)415 [hep-ph/9411211]; Nucl. Phys. B555(1999)335-351 [hep-ph/9902341]
4. E. Gardi, J. Rathsman, Nucl. Phys. B609(01)123 [hep-ph/0103217]
5. M.Dinsdale, J.Maxwell, ICHEP04 contribution [hep-ph/0408114]
6. A. Banfi, Y.Dokshitzer, G.Marchesini, G.Zanderighi, JHEP 0105:040,2001 [hep-ph/0104162]; JHEP 0103:007,2001 [hep-ph/0101205]; Phys.Lett.B508:269-278,2001 [hep-ph/0010267]; JHEP 0007:002,2000 [hep-ph/0004027].
7. A.Banfi, G.Marchesini, G.Smye, JHEP 0204:024,2002 [hep-ph/0203150]; A.Banfi, G.Marchesini, G.Smye, G. Zanderighi, JHEP 11:066,2001 [hep-ph/0111157]
8. A.Banfi, G.Marchesini, G.Smye, JHEP 08:047,2001 [hep-ph/0106278]
9. J.Botts, G.Sterman, Nucl.Phys. B325:62,1989; N.Kidonakis, G.Sterman, Nucl.Phys.B505:321-348,1997 [hep-ph/9705234]; N.Kidonakis, G.Oderda, G.Sterman, Nucl.Phys.B531:365-402,1998 [hep-ph/9803241]
10. A.Banfi, G.Salam and G.Zanderighi, Phys.Lett.B584:298-305,2004 [hep-ph/0304148]; JHEP 01(02)018; [hep-ph/0112156]; JHEP 08:062,2004 [hep-ph/0407287]; and [hep-ph/0407286]
11. M.Dasgupta, G.Salam, J.Phys. G30: R143,2004 [hep-ph/0312283]; JHEP 08:032,2002 [hep-ph/0208073]; Eur.Phys.J.C24:213-236,2002 [hep-ph/0110213]; JHEP 02:001,2000 [hep-ph/9912488]
12. M.Dasgupta,G.Salam, Phys.Lett.B512:323-330,2001 [hep-ph/0104277]; JHEP 03:017,2002 [hep-ph/0203009];
13. A.Banfi, G.Marchesini and G.Smye, JHEP 08(02)006 [hep-ph/0206076]
14. C.Berger,T.Kücs,G.Sterman, Phys. Rev. D65:094031,2002 [hep-ph/0303051]
15. Y.Dokshitzer, G.Marchesini, JHEP 03:040,2003 [hep-ph/0303101]
16. A.Bassetto,M.Ciafaloni and G. Marchesini, Phys.Rep.100(1983)201
17. Y.Kovchegov, Phys.Rev. D60:034008,1999 [hep-ph/9901281]; Phys.Rev.D61:074018,2000 [hep-ph/9905214];
18. G.Marchesini and A.Mueller, Phys. Lett. B575(03)37 [hep-ph/0308284]
19. G.Marchesini and E.Onofri, JHEP 07(04)031 [hep-ph/0404242]