The Linear Multiplet and Quantum Four-Dimensional String Effective Actions*

Jean-Pierre Derendinger, Fernando Quevedo

*Institut de Physique
Université de Neuchâtel
CH–2000 Neuchâtel, Switzerland

and

Mariano Quirós

Instituto de Estructura de la Materia
CSIC Serrano 123
E–28006 Madrid, Spain

Abstract

In four-dimensional heterotic superstrings, the dilaton and antisymmetric tensor fields belong to a linear $N = 1$ supersymmetric multiplet $L$. We study the lagrangian describing the coupling of one linear multiplet to chiral and gauge multiplets in global and local supersymmetry, with particular emphasis on string tree-level and loop-corrected effective actions. This theory is dual to an equivalent one with chiral multiplets only. But the formulation with a linear multiplet appears to have decisive advantages beyond string tree-level since, in particular, $\langle L \rangle$ is the string loop-counting parameter and the duality transformation is in general not exactly solvable beyond tree-level. This formulation allows us to easily deduce some powerful non-renormalization theorems in the effective theory and to obtain explicitly some loop corrections to the string effective supergravity for simple compactifications. Finally, we discuss the issue of supersymmetry breaking by gaugino condensation using this formalism.

*Work supported in part by the Swiss National Foundation, the European Union (contracts SC1*–CT92–0789 and CHRX–CT92–0004) and the CICYT (contract AEN90–0139).
1 Introduction

The construction of four-dimensional effective actions for superstring compactifications is of fundamental importance for the study of the phenomenological implications of the theory. A substantial amount of work has been performed in this respect during the past few years. Different techniques have been successfully applied to compute string tree-level lagrangians for many classes of string compactifications. But the extension of these techniques to higher orders in string perturbation theory is still a challenge.

At present, only the one-loop corrections to gauge coupling constants have been explicitly computed in some simple classes of superstring theories \[1, 2, 3, 4\], whereas the corrections to the rest of the lagrangian are simply unknown. We present in this article a general discussion of effective lagrangians describing the couplings of one linear supersymmetric multiplet \[5, 6, 7, 8\] to chiral multiplets in \(N = 1\) global and local supersymmetry in four-dimensions. The motivation for this is the observation \[9\] that the origin and the structure of the string one-loop corrections to the effective gauge coupling constants are particularly easy to understand using the linear multiplet. It is a natural approach to the effective supergravity of superstrings to consider lagrangians with a linear multiplet \[10, 11, 12\], and we will argue that this formalism is actually the most convenient for describing low-energy couplings beyond tree-level.

The importance of the linear multiplet for the superstring effective action is due to the fact that the gravity sector of superstrings, which is universal, contains an antisymmetric tensor \(b_{\mu\nu}\) and a real scalar, the dilaton, along with the Majorana spinor partner. This is precisely the particle content of the linear multiplet. There is a gauge symmetry related to \(b_{\mu\nu}\),

\[
b_{\mu\nu} \rightarrow b_{\mu\nu} + \partial_\mu b_\nu - \partial_\nu b_\mu,
\]

where \(b_\nu\) is arbitrary. We can immediately remark that this symmetry does not provide any constraint on the couplings of the linear multiplet \(L\). It is a consequence of the mere existence of \(L\). The linear multiplet can always be transformed into a chiral multiplet \(S\), the antisymmetric tensor being equivalent to a pseudoscalar. But the dual theory has a different symmetry content. The chiral multiplet only appears in the combination \(S + \overline{S}\) in the superfield formulation of the lagrangian. This means that the dual theory has an invariance under a Peccei–Quinn symmetry which shifts the pseudoscalar component \(\text{Im} s\) of \(S\). The existence of symmetries acting on \(b_{\mu\nu}\) and \(\text{Im} s\) is at the origin of the duality transformation and its inverse.

In view of the equivalence of linear and chiral multiplets through duality transformations, it could seem useless to work out effective lagrangians using a linear multiplet since the lagrangians for chiral multiplets coupled to supergravity are well understood whereas there is not a well-developed formalism for describing the general couplings to a linear multiplet. This is essentially true for the tree-level string effective actions for which the duality transformation can be exactly solved to give the explicit form of the effective action in terms of chiral fields only \[13\]. However, one should notice that the
duality transformation, which is a generalized Legendre transformation, cannot always be performed analytically at the higher-loop level. A perturbative treatment will in general lose some information and obscure the symmetry content of the theory. This is the situation encountered in ref. [14], where an effective lagrangian could be constructed with the linear multiplet $L$, the dual theory using a chiral multiplet $S$ being known only perturbatively.

Moreover, it appears that the duality transformation does not respect string perturbation theory, which is used to obtain the contributions to the effective theory. Suppose one generates the effective theory as a formal series $\mathcal{L}_{\text{eff.}} = \mathcal{L}_0 + \sum_{n \geq 1} \Delta \mathcal{L}_n$, $n$ indicating the string loop order. At a fixed order $N$, the duality transformation applied to $\mathcal{L}_{\text{eff.}}$ will severely mix the various contributions, with the consequence that string perturbative expansion becomes hard to identify in the dual theory. The main reason for this is that $\langle L \rangle$ is the string loop-expansion parameter, whereas in the dual theory the field $S$ is defined order by order in perturbation theory and its relation to the loop-counting parameter in the dual theory is not clear. Furthermore, at a given $N$, the string loop amplitudes have a clear interpretation in terms of the lagrangian for $L$ but not for the dual theory. An explicit example of this phenomenon already exists. String one-loop threshold corrections to gauge coupling constants in symmetric $(2,2)$ orbifolds, as computed in refs. [3, 4], can be easily and naturally interpreted using the linear multiplet [9] as corrections to the gauge couplings in the effective lagrangian. In the dual theory however, these one-loop contributions appear in the Kähler potential, and not in gauge kinetic terms.

These remarks suggest that since the natural partner of the graviton in the massless sector of heterotic superstrings is an antisymmetric tensor, string information would have a more natural translation in an effective supergravity with a linear multiplet, at least when string loop contributions are considered. This is the point of view that we will adopt in this paper.

In general, supersymmetric theories with linear multiplets are more constrained than theories formulated with chiral multiplets only. For instance, the superpotentials for the $S$ field which have been used in the study of supersymmetry breaking in four-dimensional superstrings [15] are not compatible in general with the duality transformation [1]. This fact raises important questions since the origin of the $S$ superfield is precisely the presence of the antisymmetric tensor in strings. We will therefore address the issue of the formulation of dynamical supersymmetry breaking by gaugino condensates in the linear multiplet formalism, a treatment which guarantees the existence of versions equivalent by duality.

The organization of the present article is as follows. In the next section, we present a concise description of global $N = 1$ supersymmetric actions with one linear superfield

---

1Previous attempts [16] have been made to make these superpotentials consistent with another duality symmetry, target-space duality [17], which should not be confused with the duality symmetry we are referring to in the present article.
coupled to chiral matter. This section is a preparation to the more relevant case of supergravity. We concentrate mainly on the issues of interest in the study of the effective potential, supersymmetry breaking and gaugino condensation. We discuss in detail the supersymmetric duality transformation [6], which maps this theory to one with only chiral superfields, including explicit expressions for each of the components of the chiral multiplet $S$ in terms of those of $L$. The comparison of the two dual versions reveals that the interpretation of the nature of supersymmetry breaking by gaugino condensation is somewhat ambiguous in these models. In the formulation with a linear multiplet, insertion of expectation values of gaugino bilinears is clearly an explicit breaking of supersymmetry. An intuitive reason is that the linear multiplet does not possess any auxiliary field. On the other hand, in the dual theory in terms of $S$, such insertions generate a somewhat degenerate form of spontaneous breaking. These features of the globally supersymmetric case depend mainly on the auxiliary field structure of the theory. They have then a straightforward extension to local supersymmetry.

Sections 3 and 4 describe the relevant aspects of supergravity theories with a linear multiplet. We use the superconformal approach to minimal supergravity, as reviewed in [18]. In section 3, we present the basics of our techniques and give a detailed discussion of the gauge fixing of scale invariance in a superconformal theory with a linear multiplet. The superconformal approach followed here is convenient for several reasons. We choose to fix conformal invariance by imposing a canonical Einstein term, but in this approach non-canonical choices are equally possible, such as the one corresponding to the string $\sigma$-model metric, which may be eventually more appropriate. It is also convenient for the discussion of the renormalization-group behaviour of parameters which will be used in section 5. Section 4 is devoted to the calculation of these supergravity lagrangians in components. This is a more complicated task than the corresponding calculations in the chiral superfield case [13]. The main technical complication resides in solving the conformal gauge fixing equation giving the canonical Einstein lagrangian. We however can explicitly write the scalar potential and kinetic terms for a general class of Kähler invariant models. We reproduce the standard expressions of ref. [19] as a particular case when the linear multiplet decouples.

In section 5, we start the discussion of the string theory case. First we present the string tree-level lagrangian in terms of the linear multiplet and discuss some powerful non-renormalization theorems that can be obtained for this lagrangian using simple properties of the linear multiplet. In particular the superpotential and the Kähler potential, being only functions of the chiral fields, do not get renormalized as long as this approach is valid. Also we emphasize that knowledge of the loop corrections of the gauge coupling constant is almost sufficient to obtain the corresponding correction to the full effective action. We then consider the one-loop corrections to a class of string models following ref. [14]. For simplicity, we restrict ourselves to an $E_8$ hidden sector without matter and consider the couplings of the linear superfield $L$ and the chiral
superfields of the theory, including an overall modulus field $T$. The (Wilson) effective action in terms of superfields is obtained from the one-loop corrections to the gauge coupling constants as computed in [4]. The loop corrections can be interpreted as the Green–Schwarz [20] counterterms cancelling the Kähler sigma-model anomalies associated with target-space duality transformations. This mechanism is dictated by the fact that target-space duality is an exact, quantum symmetry of the superstring. A natural extension of this treatment of anomalies in the effective field theory is to consider conformal anomalies in the context of superconformal supergravity, as suggested in [14]. In this approach, we rederive from these actions the field-dependent renormalization-group equations and, from them, obtain the field-dependent renormalization group invariant scale which characterizes the strength of gauge forces in the hidden sector and is useful in the discussion of gaugino condensation and supersymmetry breaking.

Section 6 is devoted to the explicit calculation, in components, of the loop-corrected effective actions of the models discussed in section 5. We present the scalar kinetic terms and the scalar potential including gaugino bilinears. This allows us to make a short discussion of supersymmetry breaking by gaugino condensation in this formalism. We mention briefly how the different approaches used in the chiral case could be implemented with the linear multiplet. In particular, we remark the incompatibility with the linear multiplet formalism of the introduction of nontrivial superpotentials for the $S$ field as a result of gaugino condensation. We close with some remarks on the main results of this paper.

## 2 The linear multiplet and global supersymmetry breaking

Even if we will essentially be interested in supergravity effective theories involving a linear multiplet, some aspects can be discussed in the simpler context of global supersymmetry. This is the purpose of this section, which will also present the main features of theories with a linear multiplet.

### 2.1 Sigma-models with a linear multiplet

In global $N = 1$ supersymmetry, the real linear multiplet is a real vector superfield such that its second supersymmetry covariant derivatives vanish:

$$\mathcal{D} \mathcal{D} L = \overline{\mathcal{D} \mathcal{D}} L = 0.$$  \hfill (2.1)
The supersymmetry covariant derivatives are

\[ D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i(\sigma^\mu \theta)_\alpha \partial_\mu, \quad \overline{D}_\dot{\alpha} = - \frac{\partial}{\partial \dot{\theta}^\alpha} - i(\theta \sigma^\mu)_\alpha \partial_\mu, \]  

and \( DD = D^\alpha D_\alpha = -\epsilon^{\alpha\beta} D_\alpha D_\beta, \overline{D}\overline{D} = \overline{D}_{\dot{\alpha}} \overline{D}^{\dot{\alpha}}. \) Solving the constraints leads to a component expansion of the form

\[ L = C + i \theta \chi - i \dot{\theta} \bar{\chi} + \theta \sigma^\mu \theta v_\mu - \frac{1}{2} \partial_\mu \chi \sigma^\mu \frac{1}{2} \theta \theta \theta \theta \frac{1}{4} \theta \theta \theta \theta \square C. \]  

Furthermore, the vector field \( v_\mu \) has vanishing divergence,

\[ \partial_\mu v^\mu = 0, \]  

hence

\[ v_\mu = \frac{1}{\sqrt{2}} \epsilon_{\mu
u\rho\sigma} \partial^\nu b^{\rho\sigma}, \]  

which is invariant under the gauge transformation \( b_{\mu\nu} \rightarrow b_{\mu\nu} + \partial_\mu b_\nu - \partial_\nu b_\mu. \) The expansion (2.3) shows that the linear multiplet contains a real scalar field \( C, \) a Majorana spinor \( \chi \) and an antisyymmetric tensor \( b_{\mu\nu}, \) with the same physical dimension as \( C. \) In contrast with vector or chiral superfields, the linear multiplet does not possess auxiliary fields, a fact which will have immediate consequences for supersymmetry breaking.

Since the linear multiplet is a constrained vector superfield, a supersymmetric action involving \( L \) and chiral superfields \( \Sigma^i \) is constructed with an integral over the full superspace:

\[ S = \int d^4 x \mathcal{L} = 2 \int d^4 x \int d^2 \theta d^2 \bar{\theta} \Phi(L, \Sigma^i, \Sigma^\dot{i}), \]  

where \( \Phi \) is a real function (a real vector superfield). It is well known that a supersymmetric duality transformation \[ 3 \] can always be performed to transform \( L \) into a chiral multiplet so that theory (2.3) is (classically) equivalent to a particular supersymmetric non-linear \( \sigma \)-model with chiral multiplets only.

The linear multiplet becomes more interesting when the general coupling of linear and chiral multiplets is submitted to gauge invariance. We will for simplicity consider a unique linear multiplet \( L, \) chiral multiplets \( \Sigma^i \) transforming in some representation of the gauge group, and gauge fields described by a vector superfield \( V = V^A T^A, T^A \) being generators of the gauge group in the representation of the chiral multiplets, with normalization

\[ \text{Tr}(T^A T^B) = \tau \delta^{AB}. \]

\[ ^2 \text{We use conventions similar to Wess and Bagger [21], with signature } (-, +, +, +) \text{ for the space-time metric.} \]
With chiral multiplets only, one would write a general supersymmetric gauge invariant lagrangian

\[ L_\Sigma = \int d^2\theta d^2\bar{\theta} K(\Sigma^i, (\Sigma e^V)_i) + \int d^2\theta F(\Sigma^i, W^\alpha A) + \int d^2\bar{\theta} F(\Sigma^i, \bar{W}^\dot{A}) \] (2.7)

where \( W^\alpha A \) are the field strength superfields,

\[ W^\alpha = -\frac{1}{4} \bar{D}D e^{-V} \bar{D}^\alpha e^V = W^\alpha A T^A. \] (2.8)

There are two arbitrary functions, \( K \) which is real and \( F \) which is a function of chiral superfields only, both must be gauge invariant. Chiral kinetic terms will be generated by \( K \) while \( F \) describes in particular gauge kinetic terms. One easily verifies that lagrangian (2.7) does not contain terms with more than two derivatives. The minimal choice of gauge kinetic lagrangian corresponds to the most common choice

\[ F(\Sigma^i, W^\alpha) = w(\Sigma^i) + \frac{1}{4} f_{AB}(\Sigma^i) W^\alpha A W^B \alpha, \] (2.9)

where \( w(\Sigma^i) \) is the superpotential. This minimal form leads to the following gauge kinetic terms:

\[ -\frac{1}{4} [f_{AB}(z^i) + \bar{f}_{AB}(\bar{z}^i)] F^{A \mu \nu} F_{B \mu \nu}, \] (2.10)

the complex scalar fields \( z^i \) being the lowest components of superfields \( \Sigma^i \). Since the linear multiplet is not chiral, it cannot appear in the chiral density which defines gauge kinetic terms. On the contrary, the function \( K \) can freely depend on \( L \) since \( K \) is a real vector superfield. Adding a gauge invariant linear multiplet \( L \) to theory (2.6) could then proceed by the replacement

\[ K(\Sigma^i, (\Sigma e^V)_i) \rightarrow K(L, \Sigma^i, (\Sigma e^V)_i). \]

The resulting lagrangian would always include gauge kinetic terms with a harmonic metric, as in eq. (2.10).

The interesting point is that this simple solution is not unique. There exists the possibility of a coupling of a gauge variant linear multiplet to chiral multiplets and gauge fields which also escapes the condition of gauge kinetic terms with harmonic metric. This situation cannot be achieved with lagrangian (2.7) as starting point. We firstly need some manipulations of (2.7) with the minimal choice (2.9) to obtain a form more appropriate to the introduction of the linear multiplet. Suppose we define the real vector superfield \( \Omega(V) \) by the conditions

\[ \bar{D}D \Omega(V) = \tau^{-1} \text{Tr}(W^\alpha W_\alpha), \quad D \bar{D} \Omega(V) = \tau^{-1} \text{Tr}(\bar{W}_\dot{\alpha} \bar{W}^{\dot{\alpha}}), \] (2.11)
then, for an arbitrary chiral function $f$,
\[
\frac{1}{4\tau} \int d^2 \theta f \, \text{Tr} W^\alpha W_\alpha + \text{h.c.} = \frac{1}{4} \int d^2 \theta D^\alpha D_\alpha [f \Omega(V)] + \text{h.c.}
\]
\[
= - \int d^2 \theta d^2 \bar{\theta} (f + \bar{f}) \Omega(V)
\]
\[
= \frac{1}{2} \int d^2 \theta d^2 \bar{\theta} (f + \bar{f}) [L - 2\Omega(V)].
\] (2.12)

The last step uses $\int d^4 x \int d^2 \theta d^2 \bar{\theta} (f + \bar{f}) L = 0$ since $f$ is chiral and $L$ linear. These manipulations allow to move gauge kinetic terms from a chiral lagrangian into a $D$-density provided one replaces the chiral gauge invariant superfield $\tau^{-1} \text{Tr}(WW)$ by the Chern-Simons superfield $\Omega(V)$.

The Chern-Simons superfield $\Omega(V)$, defined by conditions (2.11), is not gauge invariant. Its definition indicates however that its gauge transformation $\delta \Omega$ is a linear superfield:
\[
D \delta \Omega = D \delta \Omega = 0.
\] (2.13)
Gauge invariance of the last expression (2.12) follows from this observation. We now impose that the gauge transformation of the linear superfield is
\[
\delta L = 2 \delta \Omega(V).
\] (2.14)

Since the combination
\[
\hat{L} = L - 2\Omega(V)
\] (2.15)
is by construction gauge invariant, the supersymmetric lagrangian
\[
\mathcal{L}_L = 2 \int d^2 \theta d^2 \bar{\theta} \Phi(\hat{L}, \Sigma^i, (\Sigma e^V)_i) + \int d^2 \theta w(\Sigma_i) + \int d^2 \bar{\theta} \bar{w}(\Sigma_i)
\] (2.16)
is also gauge invariant for any real function $\Phi$. According to equalities (2.12), it is actually a generalization of lagrangian (2.7) with the usual gauge sector (2.9). The component expansion of (2.16) contains gauge kinetic terms of the form
\[
- \frac{1}{2} \left[ \frac{\partial}{\partial L} \Phi(L, \Sigma^i, \Sigma_i) \right]_{\theta=\bar{\theta}=0} F^A_{\mu\nu} F^A_{\mu\nu},
\]
with a non-harmonic metric in general. It has been realized [9] that this form of linear multiplet coupling is important to describe quantum corrections to orbifold effective supergravities.

The Chern-Simons superfield can be explicitly computed by solving eqs. (2.11) with the help of (2.8) and Bianchi identities. In the non-abelian case, its expression is complicated (see for instance ref. [10]). The simpler abelian Chern-Simons superfield reads
\[
\Omega(V) = - \frac{1}{4} \tau^{-1} \text{Tr} \left[ (D^\alpha V) W_\alpha + (\bar{D}_\alpha V) \bar{W}^{\alpha i} + V (D^\alpha W_\alpha) \right].
\] (2.17)
Its component expansion is given in the appendix.

The component expansion of lagrangian (2.16) is relatively cumbersome to obtain. Since one of our goals is to discuss the sector of the theory which controls supersymmetry breaking with and without gaugino condensation, we will only need to obtain the scalar and gaugino lagrangian. For simplicity, we will consider below the lagrangian (2.16) for a unique, gauge-invariant chiral multiplet $\Sigma$, with components $(z, \psi, f)$. We will truncate the chiral superfield $\Sigma$ and omit the fermion $\psi$ which is irrelevant to our discussion. But the auxiliary field $f$ must be retained. Analogously, the linear superfield $L$, with component expansion (2.3) will be truncated by keeping only the real scalar $C$ and the transverse vector $v_\mu$, equivalent to the antisymmetric tensor $b_{\mu\nu}$ and by duality to a pseudoscalar. Finally, all gauge boson contributions will be omitted. In this situation and using the Wess-Zumino gauge, all contributions of gauginos $\lambda^A$ and auxiliary fields $D^A$ are obtained using the abelian Chern-Simons superfield (2.17), even for a non-abelian gauge group. We can directly assume that gauge auxiliary fields $D^A$ vanish. Their couplings to the linear multiplet only proceed through terms of the form $D^A(\lambda^A \chi)$ which are omitted here. With non-singlet chiral matter (and with $\psi = 0$), the auxiliary fields $D^A$ would only induce the usual positive potential term which is not of central importance when discussing supersymmetry breaking. In lagrangian (2.16), the real function $\Phi$ is arbitrary up to constraints related to the positivity of kinetic energy, which will be stated using the component expansion of the lagrangian. The truncated lagrangian obtained by selecting only scalar, $v_\mu$, $f$ and gaugino contributions is

$$L_L = \frac{1}{2} \Phi_{CC}(\partial_\mu C)(\partial^\mu C) - \frac{1}{2} \Phi_{CC} v_\mu v_\mu - 2 \Phi_{SZ} (\partial_\mu z)(\partial^\mu z) + v_\mu \left[ -i \Phi_{SZ} (\partial^\mu z) + i \Phi_{\overline{S}Z} (\partial^\mu \overline{Z}) + \Phi_{CC} (\lambda^A \sigma^\mu \overline{X}^A) \right] - \Phi_C \left[ i \lambda^A \sigma^\mu \partial_\mu \overline{X}^A - i \partial_\mu \lambda^A \sigma^\mu \overline{X}^A \right] + i(\lambda^A \sigma^\mu \overline{X}^A) \left[ \Phi_{SZ} (\partial_\mu z) - \Phi_{\overline{S}Z} (\partial_\mu \overline{Z}) \right] + \frac{1}{2} \Phi_{CC} \left[ (\lambda^A \lambda^A)^B (\overline{X}^B \overline{X}^B) + 2(\lambda^A \lambda^B)(\overline{X}^A \overline{X}^B) \right] + L_{AUX}, \tag{2.18}$$

where we use the notation

$$\Phi_C = \left[ \frac{\partial \Phi}{\partial L} \right]_{\theta = \overline{\theta} = 0} = \frac{\partial}{\partial C} \Phi(C, z, \overline{Z}),$$

$$\Phi_{SZ} = \left[ \frac{\partial^2 \Phi}{\partial L \partial \Sigma} \right]_{\theta = \overline{\theta} = 0} = \frac{\partial^2}{\partial C \partial z} \Phi(C, z, \overline{Z}), \ldots,$$

and the auxiliary field lagrangian is

$$L_{AUX} = 2 \Phi_{SZ} f \overline{f} - \Phi_{SZ} (\lambda^A \lambda^A) - \overline{f} \Phi_{\overline{S}Z} (\overline{X}^A \overline{X}^A) + f \frac{d w}{d z} + \overline{f} \frac{d \overline{w}}{d \overline{z}}. \tag{2.19}$$
Notice that scalar kinetic terms in (2.18) do not mix $z$, $C$ and $v_\mu$. Positivity of the kinetic terms for $\lambda^A$, $C$ (and $v_\mu$) and $z$ respectively corresponds to the conditions

$$\Phi_C > 0, \quad \Phi_{CC} < 0, \quad \Phi_{\Sigma\Sigma} > 0 \quad (2.20)$$

in the domains of the scalar fields. The equation of motion for $f$ is

$$f = -\frac{1}{2} \Phi_{\Sigma\Sigma}^{-1} \left[ \frac{dw}{dz} - \Phi_C (\lambda^A \lambda^A) \right].$$

(2.21)

In the absence of gaugino condensates, $\langle \lambda^A \lambda^A \rangle = 0$, the scalar potential is

$$V = \frac{1}{2} \Phi_{\Sigma\Sigma}^{-1} \left| \frac{dw}{dz} \right|^2,$$

(2.22)

which is semi-positive definite according to conditions (2.20). If there is a vacuum $\langle z \rangle$ such that $\langle \frac{dw}{dz} \rangle = 0$, $\langle f \rangle = \langle V \rangle = 0$ and supersymmetry is not broken. The linear multiplet does not play any rôle since the superpotential is independent of $L$ and $L$ does not possess any auxiliary field.

### 2.2 Duality transformation

The antisymmetric tensor contained in the linear multiplet $L$ can always be transformed into a pseudoscalar field. In the supersymmetric context, this duality transformation can be performed at the superfield level, the linear multiplet being replaced by a chiral multiplet $S$. To perform the supersymmetric duality transformation [6], replace the lagrangian (2.16) by

$$\mathcal{L}_U = 2 \int d^2 \theta d^2 \bar{\theta} \left[ \Phi(U, \Sigma, \Sigma) - (S + \bar{S})(U + 2\Omega(V)) \right] + \int d^2 \theta w(\Sigma) + \int d^2 \bar{\theta} \bar{w}(\Sigma)$$

$$= 2 \int d^2 \theta d^2 \bar{\theta} \left[ \Phi(U, \Sigma, \Sigma) - (S + \bar{S})U \right]$$

$$+ \int d^2 \theta \left[ SW^A W^A + w(\Sigma) \right] + \int d^2 \bar{\theta} \left[ \bar{S} \bar{W}^A \bar{W}^A + \bar{w}(\Sigma) \right],$$

(2.23)

where $U$ is an unconstrained vector superfield. Notice that the $d^2 \theta d^2 \bar{\theta}$ integral does not depend on the gauge superfield $V$. The equation of motion for the chiral multiplet $S$ implies that $U + 2\Omega(V)$ is a linear multiplet and $\mathcal{L}_U$ is then equivalent with $\mathcal{L}_L$. On the other hand, the equation of motion for $U$ is

$$\frac{\partial \Phi}{\partial U} = S + \bar{S},$$

(2.24)
which can in principle be inverted to express the vector superfield $U$ as a function of $S + \overline{S}$, $\Sigma$ and $\Sigma$. Inserting the expression $U(S + \overline{S}, \Sigma, \Sigma)$ into lagrangian (2.23) leads to

$$\mathcal{L}_S = \int d^2 \theta d^2 \overline{\theta} K(S + \overline{S}, \Sigma, \Sigma)$$

$$+ \int d^2 \theta \left[ SW^A W^A + w(\Sigma) \right] + \int d^2 \overline{\theta} \left[ \overline{SW^A} \overline{W^A} + \overline{w(\Sigma)} \right],$$

where

$$K(S + \overline{S}, \Sigma, \Sigma) = 2 \left[ \Phi - (S + \overline{S})U \right]_{U=U(S+\overline{S},\Sigma,\Sigma)}.$$ (2.26)

In components, the supersymmetric duality transformation works in the following way. Denoting the components of the unconstrained vector superfield $U$ by

$$U : (C, \varphi, m, n, \omega, \eta, d),$$

and the components of the chiral multiplet $S$ by

$$S : (s, \psi_s, f_s),$$

the highest component $d$ appears in $\mathcal{L}_U$ in

$$\frac{\partial \Phi}{\partial C} d - (s + \overline{s}) d.$$

The equation of motion for $d$ defines then the real part of $S$ as

$$s + \overline{s} = \Phi_C,$$ (2.27)

which is the lowest component of the superfield equation (2.24). By inverting this relation, one can express $C$ as a function of $s + \overline{s}$ and $z$, the scalar component of the chiral multiplet $\Sigma$.

The spinor $\eta$ which appears in the $\overline{\theta} \theta \theta$ component of $U$ also contributes to $\mathcal{L}_U$ like a Lagrange multiplier. Its equation of motion, obtained from the $\theta$ component of eq. (2.24), gives the definition of the fermionic component $\psi_s$ of $S$ as a function of the components of $U$ and $\Sigma$:

$$\psi_s = \frac{i}{\sqrt{2}} \Phi_{CC} \varphi + \Phi_{CS} \psi.$$ (2.28)

This equation is actually used to eliminate $\varphi$ which can be expressed as a function of $\psi_s$, $\psi$, $s + \overline{s}$ and $z$, using also (2.27).

The scalar fields $m$ and $n$, which correspond to the $\theta \theta$ and $\overline{\theta} \theta$ components of $U$, appear quadratically and without derivatives in $\mathcal{L}_U$. Their equations of motion are, in complex form,

$$f_s = \frac{i}{2} \Phi_{CC}(m + in) + \Phi_{CS} f + \frac{1}{4} \Phi_{CCC}(\varphi \varphi) - \frac{i}{\sqrt{2}} \Phi_{CCS}(\varphi \psi) - \frac{1}{2} \Phi_{CSS}(\psi \psi).$$ (2.29)
which define \( f_s \), the auxiliary component of \( S \). It can be used to eliminate \( m + i n \). Eq. (2.29) is the \( \theta \theta \) component of the superfield equation (2.24).

The vector component of \( U \), \( \omega_\mu \), which also has an algebraic equation of motion in \( \mathcal{L}_U \), leads to the usual duality transformation. Its equation of motion is

\[
\partial_\mu \text{Im } s = \frac{1}{2} \Phi_{CC} \omega_\mu - \frac{i}{2} \Phi_{CS} \partial_\mu z + \frac{i}{2} \Phi_{CS} \partial_\mu \overline{\tau} + \frac{1}{4} \Phi_{CCC} (\varphi \sigma_\mu \overline{\psi})
- \frac{i}{2 \sqrt{2}} \Phi_{CSC} (\psi \sigma_\mu \overline{\tau}) + \frac{i}{2 \sqrt{2}} \Phi_{CSC} (\varphi \sigma_\mu \overline{\psi}) + \frac{1}{2} \Phi_{CSS} (\psi \sigma_\mu \overline{\psi}).
\]

The component \( \omega_\mu \) can then be replaced by the derivative of the imaginary part of \( s \).

Finally, components \( C \) and \( \varphi \) are propagating fields in \( \mathcal{L}_U \). But they can be replaced by \( \text{Re } s \) and \( \psi_s \) using eqs. (2.27) and (2.28).

In terms of the components of chiral multiplets \( S \) and \( \Sigma \), the truncated lagrangian can be written

\[
\mathcal{L}_S = -2 \Phi_{CC}^{-1} \left[ -\text{Re } s (\partial_\mu \overline{s}) + \text{Im } s \right] + 2 \left[ \Phi_{CSS} - \Phi_{CC}^{-1} \Phi_{CS} \Phi_{CSS} \right] \left[ -\text{Re } s (\partial_\mu \overline{s}) + \text{Im } s \right] + 2 \Phi_{CC}^{-1} \Phi_{CSS} \left[ -\text{Re } s (\partial_\mu \overline{s}) + \text{Im } s \right] + 2 \Phi_{CC}^{-1} \Phi_{CSS} \left[ -\text{Re } s (\partial_\mu \overline{s}) + \text{Im } s \right]
- f_s (\lambda^A \lambda^A) - f_s \left[ \overline{\psi}_s (\lambda^A \lambda^A) + \frac{d}{dz} f + \frac{df}{dz} \right].
\]

In this expression, \( \Phi \) and its derivatives should be considered as functions of \( s + \overline{s} \), \( z \) and \( \overline{z} \), using relation (2.27) to eliminate \( C \). The metric of scalar kinetic terms is not diagonal. Its determinant is \(-4 \Phi_{CC}^{-1} \Phi_{CSS} \) and positivity of the kinetic terms leads again to conditions (2.20). In particular, positivity of gauge kinetic terms corresponds to

\[
\text{Re } s > 0,
\]

in view of eqs. (2.27) and (2.20). Notice that the auxiliary field contributions can be rearranged into

\[
\mathcal{L}_{\text{AUX}} = -2 \Phi_{CC}^{-1} \tilde{f} \overline{\psi}_s (\lambda^A \lambda^A) - 2 \Phi_{CSS} f + 2 \Phi_{CSS} \overline{\psi}_s f + f_s \left[ \overline{\psi}_s (\lambda^A \lambda^A) + \frac{d}{dz} f + \frac{df}{dz} \overline{\psi}_s \right],
\]

where \( \tilde{f} = -f_s + \Phi_{CSS} f \). This redefinition diagonalises the auxiliary field equations of motion which read

\[
\tilde{f} = \frac{1}{2} \Phi_{CC} (\overline{\lambda}^A \lambda^A),
\]

\[
f = -\frac{1}{2} \Phi_{CSS}^{-1} \left[ \overline{\psi}_s (\lambda^A \lambda^A) + \frac{d}{dz} f + \frac{df}{dz} \overline{\psi}_s \right].
\]
The auxiliary field $f_s$ is
\begin{equation}
    f_s = \Phi C \Sigma f - \frac{1}{2} \Phi CC (\lambda^A \lambda^A)
\end{equation}
(2.35)

The scalar potential takes the simple form
\begin{equation}
    V = -2 \Phi CC^{-1} \tilde{f} \tilde{f} + 2 \Phi \Sigma \Sigma \tilde{f} \tilde{f},
\end{equation}
(2.36)
with $\tilde{f}$ and $f$ replaced by the scalar contributions in the expressions (2.34). It is positive or zero as a consequence of the positivity of kinetic terms.

If gauginos do not condense, $\langle \lambda^A \lambda^A \rangle = 0$, the scalar parts of the auxiliary fields reduce to
\begin{equation}
    f = -\frac{1}{2} \Phi \Sigma^{-1} \tilde{f},
\end{equation}
\begin{equation}
    f_s = \Phi C \Sigma f,
\end{equation}
(2.37)
and $\tilde{f} = 0$. The new auxiliary field $f_s$, defined by the supersymmetric duality transformation of the linear multiplet is simply proportional to $f$ \footnote{Except if $\Phi_{C \Sigma} = 0$, which implies $\Phi = F(\hat{L}) + G(\Sigma, \Sigma)$.}. Clearly, supersymmetry is unbroken whenever $\langle \frac{dw}{dz} \rangle = 0$. Without gaugino condensation, the scalar potential
\begin{equation}
    V = \frac{1}{2} \Phi \Sigma^{-1} \left| \frac{dw}{dz} \right|^2
\end{equation}
is of course the same as in the theory expressed with the linear multiplet, in eq. (2.22). Its dependence on $s + \bar{s}$ is entirely in the factor $\Phi \Sigma^{-1}$. Then, if the $s$-independent condition $\frac{dw}{dz} = 0$ has a solution, the scalar potential has a flat direction leaving $s + \bar{s}$ undetermined.

### 2.3 Gaugino condensation

To discuss the effect of gaugino condensation, we use an expectation value
\begin{equation}
    \langle \lambda^A \lambda^A \rangle = \Lambda^3
\end{equation}
(2.38)
inserted in the three equivalent lagrangians $L_L$ [eq. (2.16)], $L_U$ [eq. (2.23)] and $L_S$ [eq. (2.27)]. In $L_U$ and $L_S$, the gauge multiplet only appears in $\int d^2 \theta \, SW^AW^A + $ h.c. Since
\begin{equation}
    W^AW^A = -\lambda^A \lambda^A - 2i \theta \lambda^A D^A + (\theta \sigma^\mu \bar{\sigma}^\nu \lambda^A) F^A_{\mu \nu}
\end{equation}
\begin{equation}
    + \theta \left[ -2i \lambda^A \sigma^\alpha \partial_\mu \bar{\lambda}^A + D^A D^A - \frac{1}{2} F^A_{\mu \nu} F^A_{\mu \nu} - \frac{i}{4} \epsilon^{\mu \nu \rho \sigma} F^A_{\mu \nu} F^A_{\rho \sigma} \right],
\end{equation}
(2.39)
the insertion of a non-zero $\langle \lambda^A \lambda^A \rangle$ can be regarded as a shift of the superfield
\[ W^A W^A \rightarrow W^A W^A - \Lambda^3. \]
Equivalently, the theory with gaugino condensation has a superpotential
\[ w_\Lambda(S, \Sigma) = w(\Sigma) - \Lambda^3 S. \tag{2.40} \]
Since eq. (2.40) is a superfield equation, the resulting lagrangian, with modified superpotential $w_\Lambda$, is supersymmetric. Supersymmetry breaking only occurs if the theory does not possess a supersymmetry invariant vacuum, i.e. supersymmetry breaking would seem spontaneous.

The appearance of a contribution $-\Lambda^3 S$ in the superpotential corresponds to the addition of
\[ -\Lambda^3 (f_s + \bar{f}_s) \]
to the component lagrangian (2.31). As it should, this is the same as replacing $\lambda^A \lambda^A$ by $\lambda^A \lambda^A + \Lambda^3$ directly in eq. (2.31). The superpotential (2.40) does not generate any mass term for the fermionic component $\psi_s$ of $S$, which is massless for arbitrary expectation values of the scalar fields. The expectation values of auxiliary fields become
\[ \langle f \rangle = -\left( \frac{1}{2} \Phi_{\Sigma \Sigma}^{-1} \left[ \frac{dw}{dz} - \Phi_{C\Sigma} \Lambda^3 \right] \right), \]
\[ \langle f_s \rangle = \langle \Phi_{C\Sigma} f_s - \frac{1}{2} \Phi_{CC} \Lambda^3 \rangle, \tag{2.41} \]
which shows that supersymmetry is always broken since $\langle f \rangle$ and $\langle f_s \rangle$ may simultaneously vanish only if $\Lambda = 0$. The potential with $\Lambda \neq 0$ is
\[ V_\Lambda = \frac{1}{2} \Phi_{\Sigma \Sigma}^{-1} \left| \frac{dw}{dz} - \Lambda^3 \Phi_{C\Sigma} \right|^2 - \frac{1}{2} \Phi_{CC} \Lambda^6. \tag{2.42} \]
With positivity conditions (2.20), the scalar potential is strictly positive, another indication that supersymmetry is broken. It is plausible that $V_\Lambda$ will find its minimum at $\langle f \rangle = 0$, a situation characterized by
\[ \langle f \rangle = 0 \implies \langle \frac{dw}{dz} \rangle = \langle \Phi_{C\Sigma} \rangle \Lambda^3, \]
\[ \langle f_s \rangle = -\frac{1}{2} \langle \Phi_{CC} \rangle \Lambda^3 \neq 0. \tag{2.43} \]
If possible, the superpotential will adjust itself in order that supersymmetry breaking is entirely in the $S$-sector. Notice also that supersymmetry breaking will in general destroy the flat direction and lift the degeneracy in $s + \bar{s}$.

\footnote{The case $\langle \Phi_{CC} \rangle = 0$ is singular, according to eq. (2.31), which means that the duality transformation cannot be performed.}
In the version of the theory using the linear multiplet, with lagrangian (2.16), the components of the gauge multiplet appear in the Chern-Simons superfield associated with $L$ in the combination $\hat{\Omega} = L - 2\Omega(V)$. The superfield $\Omega(V)$ contains gaugino bilinear $\lambda^A\lambda^A$ in its $\theta\theta$ and $\theta\bar{\theta}$ components only. This is consistent with the definition of $\Omega(V)$, through the condition $\overline{\partial D}\Omega(V) = \tau^{-1}W^AW^A$ and its conjugate. The insertion of the expectation value of gaugino bilinears would correspond to replacing $\Omega(V)$ by

$$\Omega_\Lambda = \Omega(V) + \frac{1}{4}\Lambda^3\left[\theta\theta + \theta\bar{\theta}\right],$$

(2.44)

which is clearly not a superfield equation. This replacement would then be interpreted as an explicit breaking of supersymmetry. In the component expansion of the lagrangian, eq. (2.18), the insertion of the gaugino condensate leads to the lagrangian

$$\mathcal{L}_\Lambda = \mathcal{L}_L - \Phi_{C\Sigma}f\Lambda^3 - \Phi_{C\Sigma}\overline{f}\Lambda^3 + \frac{1}{2}\Phi_{CC}\Lambda^6.$$  

(2.45)

The expectation value of the unique auxiliary field $f$ becomes

$$\langle f \rangle = -\langle \frac{1}{2}\Phi_{C\Sigma}\overline{f}\text{exp}\left[\frac{d\overline{\sigma}}{dz} - \Phi_{C\Sigma}\Lambda^3\right] \rangle,$$

(2.46)

as in the first eq. (2.41). And the scalar potential is identical to eq. (2.42) which is strictly positive. In the formulation with the linear multiplet, the insertion of gaugino condensates is an explicit breaking of supersymmetry, generating in particular a positive contribution to the scalar potential. This mechanism does not use auxiliary fields which are absent in the linear multiplet.

3 Supergravity with a linear multiplet: preliminaries

The case of global supersymmetry with a linear multiplet presented in the previous section, even if suggestive, is not sufficient to discuss in general terms the low-energy effective supergravity theories obtained from superstrings. We have to consider the general coupling of a linear multiplet to supergravity. This problem has been studied, to a large extent, in the literature using either superconformal methods [22, 10] or super-Poincaré superspace techniques [8, 7, 11, 12] but the general expression for the lagrangian in terms of arbitrary functions is not available, contrary to the case of chiral fields coupled to supergravity [13]. In this section and the next, we present such general expressions not for the full lagrangian but for the terms that interest us the most, i.e. the scalar and gauge kinetic terms as well as the scalar potential including gaugino

---

5 The case $\langle \Phi_{CC} \rangle = 0$ corresponds, according to eq. (2.18), to a non-propagating linear multiplet.
bilinears. We will explain the limitations that make this calculation more difficult than the one in ref. [19] and obtain the latter as a particular case.

In the following, we will use the formalism of superconformal supergravity [23] which appears to be the most appropriate for our purposes. This method has the advantage that the supergravity lagrangian (Einstein and Rarita-Schwinger terms) is always included in matter couplings, through covariantization of derivatives and invariant density formulæ. Moreover it nicely keeps track of the breaking of scale symmetry which will be an important issue when discussing the renormalization-group behaviour in the effective low-energy theory. The formalism of superconformal supergravity is reviewed in refs. [18], and we will use the same conventions.

The general idea is to firstly construct an action invariant under the transformations of the superconformal algebra. This theory describes all matter and vector multiplets, but it also depends on an additional multiplet called compensator. As a gauge theory of the superconformal algebra, it includes the gauge potentials \((e^m_\mu, \omega^{mn}_\mu, f^m_\mu, b_\mu, \psi_\mu, \varphi_\mu, A_\mu)\). The gauge fields \(\omega^{mn}_\mu\) (Lorentz transformations), \(\varphi_\mu\) (special supersymmetry) and \(f^m_\mu\) (conformal boosts) can be algebraically solved as a consequence of the imposition of constraints on the curvatures. The second step is to obtain a super-Poincaré theory by choosing a gauge for conformal boosts, dilatations, chiral \(U(1)\) and special supersymmetry. This is done by assigning specific field-dependent values to certain components of the compensator and to some gauge fields of the superconformal algebra, the exact procedure depending on the choice of compensating multiplet.

The choice of the compensator dictates the set of auxiliary fields present in the supergravity multiplet of the Poincaré theory, which is not unique. It is known [22] that the simplest choice of a chiral compensating multiplet with Weyl and chiral weights equal to one, which leads to ‘minimal supergravity’, also leads to the most general class of matter–supergravity couplings. Denoting the components of the chiral compensator \(S_0\) by

\[
S_0 : (z_0, \psi_0, f_0), \tag{3.1}
\]

the standard procedure [24] for fixing the superconformal symmetries is to choose a gauge in which conformal boosts are fixed by imposing that the gauge field of dilatations \(b_\mu\) vanishes, special supersymmetry and chiral \(U(1)\) are fixed by choosing a specific field-dependent form of the component \(\psi_0\) of \(S_0\) and by the condition \(\text{Im} \ z_0 = 0\). Finally, the gauge fixing of dilatations corresponds to choosing \(|z_0|\) in such a way that the Einstein term in the theory has the canonical form

\[
-\frac{1}{2} \kappa^2 e^R,
\]

where \(e\) is the vierbein determinant and \(R\) the curvature scalar. This gauge fixing procedure applied to the chiral compensator \(S_0\) leaves only \(f_0\) (a complex scalar) and

\[\text{As in section 2, the space-time metric has signature } (-, +, +, +) \text{ in our supergravity expressions.}\]


\( A_\mu \) (the gauge potential of chiral \( U(1) \) transformations) unspecified. They will be the auxiliary fields of minimal Poincaré supergravity.

### 3.1 The chiral case

To illustrate the construction, we first consider the most general lagrangian density for one chiral multiplet \( \Sigma \), with a gauge multiplet \( V = V^A T^A \). Both multiplets can be taken with zero Weyl and chiral weights without loss of generality. This is a particular case of the theory constructed in ref. [19]. Using supermultiplet expressions, the lagrangian is

\[
\mathcal{L} = -\frac{3}{2} [S_0 \bar{S}_0 e^{-K(\Sigma, \Sigma)}]_D + \left[ \frac{1}{4} f(\Sigma) W^A W^A + S_0^3 w(\Sigma) \right]_F \tag{3.2}
\]

where \( S_0 \) is the chiral compensator, \( K \) and \( f \) are arbitrary functions (\( K \) is real and \( f \) analytic), \( W^A \) is the chiral supermultiplet obtained from \( V^A \) which contains gauge curvatures (the local analog of the superfield \( W^A \alpha \) of global supersymmetry) and \( w(\Sigma) \) is the superpotential. This expression only makes sense in the context of tensor calculus, which specifies the rules for combining supergravity multiplets. The two terms correspond to D- and F-density formulae for invariant actions. A D-density is an invariant action obtained by combining the components of a real vector multiplet \( V \) with conformal weight two and superconformal gauge fields. In eq. (3.2), \( V = -\frac{3}{2} S_0 \bar{S}_0 e^{-K(\Sigma, \Sigma)} \).

The condition on the Weyl weight and reality determine the form of compensator contributions. In the same way, an F-density combines components of a chiral multiplet \( S \) with Weyl and chiral weights equal to three with gauge fields to form an invariant action. Again, \( S = \frac{1}{4} f(\Sigma) W^A W^A + S_0^3 w(\Sigma) \) has by construction the correct weights.

Omitting all gravitino contributions in the density formula, one has

\[
[V]_D = e(d + \frac{1}{3}c R),
[S]_F = ef + h.c., \tag{3.3}
\]

where \( V \) is a vector multiplet with components \((c, \chi, m, n, b_\mu, \lambda, d)\) and \( S \) a chiral multiplet with components \((z, \psi, f)\). The second term in the D-density formula contains the curvature scalar. The components \( d, c \) and \( f \) of the multiplets

\[
V = -\frac{3}{2} S_0 \bar{S}_0 e^{-K(\Sigma, \Sigma)} ,
S = \frac{1}{4} f(\Sigma) W^A W^A + S_0^3 w(\Sigma) , \tag{3.4}
\]

can be calculated using the rules of superconformal tensor calculus [18]. It turns out that neither \( d \) nor \( f \) contain contributions involving the curvature scalar \( R \). Then, the Einstein lagrangian appears in

\[
\frac{1}{3} e c R = -\frac{1}{2} e R [z_0 \bar{z}_0 e^{-\frac{1}{3}K(z, \bar{\Sigma})}] , \tag{3.5}
\]
The gauge choice for dilatations, which corresponds to impose canonical Einstein terms, determines then $z_0 \bar{z}_0$ [24]:

$$z_0 \bar{z}_0 = \frac{1}{\kappa^2} e^{\frac{1}{3} K(z, \bar{z})}. \tag{3.6}$$

This equation, together with the gauge fixing conditions on the phase of $z_0$, $\psi_0$ and $b_\mu$ mentioned above, are used in the superconformal theory to obtain the super-Poincaré lagrangian.

The theory (3.2) is invariant under the Kähler transformations

$$
\begin{align*}
K & \rightarrow K + \varphi(\Sigma) + \overline{\varphi}(\overline{\Sigma}), \\
w & \rightarrow e^{-\varphi(\Sigma)}w(\Sigma), \\
S_0 & \rightarrow e^{\frac{1}{3} \overline{\varphi}(\overline{\Sigma})}S_0, \tag{3.7}
\end{align*}
$$

which can actually be used to eliminate the superpotential (for $w \neq 0$) by the choice $\varphi(\Sigma) = \log |w(\Sigma)|$, equivalent to a redefinition of the compensator defined by

$$S'_0 = w(\Sigma)^{1/3} S_0, \tag{3.8}$$

so that lagrangian (3.2) becomes

$$\mathcal{L} = -\frac{3}{2} [S'_0 \overline{S}'_0 e^{-\frac{4}{3} \varphi}]_D + \left[ \frac{1}{4} f(\Sigma) W^A W^A + S'_0 \right]_F, \tag{3.9}$$

where all couplings are now contained in the ‘Kähler function’ defined by

$$\mathcal{G}(\Sigma, \overline{\Sigma} e^V) = K(\Sigma, \overline{\Sigma} e^V) + \log |w(\Sigma)|^2. \tag{3.10}$$

The equivalent form (3.9) shows that the superpotential $w(\Sigma)$ and the Kähler potential $K(\Sigma, \overline{\Sigma} e^V)$ only contribute to the lagrangian in the combination $\mathcal{G}(\Sigma, \overline{\Sigma} e^V)$. Notice that the gauge choice for dilatations is now

$$z'_0 \bar{z}'_0 = \frac{1}{\kappa^2} e^{\frac{1}{3} \varphi}, \tag{3.11}$$

which is compatible with eqs. (3.6) and (3.8). The computation of the component expansion of lagrangian (3.9), using the compensator (3.11) shows that it only depends on the functions $f(z)$ and $\mathcal{G}(z, \bar{z})$ and on their derivatives. Moreover, scalar kinetic terms have the simple form

$$\frac{\partial^2 \mathcal{G}}{\partial z \partial \bar{z}} (\partial_\mu z)(\partial^\mu \bar{z}),$$

which shows that $\mathcal{G}$ (or $K$) is the Kähler potential for the non linear kählerian $\sigma$-model describing scalar fields.
3.2 Linear and chiral multiplets

We now turn to the discussion of matter described by linear and chiral multiplets. To simplify, we will only explicitly consider one chiral multiplet \( \Sigma \) and one real linear multiplet \( L \). The generalization to one linear multiplet and an arbitrary number of chiral multiplets, which would be relevant for string effective theories, is straightforward at this point. We can freely choose the Weyl and chiral weights for \( \Sigma \) to be

\[
\Sigma : \quad w = n = 0.
\]

On the contrary, a real linear multiplet has always

\[
L : \quad w = 2, \quad n = 0.
\]

However, since \( \frac{L}{S_0 S_0} \) is a real vector multiplet with \( w = n = 0 \), an invariant action for \( \Sigma \) and \( L \) would be

\[
\mathcal{L} = [S_0 S_0 \Phi(\frac{L}{S_0 S_0}, \Sigma, \Sigma)]_D + [S_0^3 w(\Sigma)]_F. \tag{3.12}
\]

The introduction of gauge invariance, with a real vector multiplet \( \Omega(V) \), follows then the same principle as in the case of global supersymmetry. One defines a vector multiplet

\[
\hat{L} = L - 2\Omega(V), \tag{3.13}
\]

\( \Omega(V) \) being the Chern-Simons (vector) supermultiplet. Gauge invariance of \( \hat{L} \) is obtained by imposing the gauge transformation

\[
\delta L = 2\delta \Omega(V), \tag{3.14}
\]

an admissible condition since \( \delta \Omega(V) \) is a linear multiplet. The general gauge invariant superconformal lagrangian for \( L, \Sigma \) and \( V \) is then

\[
\mathcal{L} = [S_0 S_0 \Phi(\frac{\hat{L}}{S_0 S_0}, \Sigma, \Sigma e^L)]_D + [S_0^3 w(\Sigma)]_F. \tag{3.15}
\]

It is important to remark that gauge kinetic terms do not need to be introduced separately, as in eq. (3.2) for instance. They are contained in the Chern-Simons multiplet \( \Omega(V) \). The argument is essentially identical to the case of global supersymmetry, and manipulations analogous to (2.12) exist in the local context. Also, and for the same reason as in global supersymmetry, a gauge kinetic term \( [\frac{1}{2}f(\Sigma)W^AW^A]_D \) can be transformed into a lagrangian of the form (3.15) so that this general theory includes also arbitrary couplings for the chiral multiplet \( \Sigma \).

As before, a transformation

\[
w(\Sigma) \rightarrow e^{-\varphi(\Sigma)} w(\Sigma),
\]

\[
S_0 \rightarrow e^{\frac{i}{2}e(\Sigma)} S_0,
\]

\[
\Phi \rightarrow e^{-\frac{i}{2}[\varphi(\Sigma) + \overline{\varphi(\Sigma)}]} \Phi,
\]

(3.16)
can be used to eliminate the superpotential. The equivalent theory has a new function
\[ \Phi' = \left[ w(\Sigma) \pi(\Sigma) \right]^{-1/3} \Phi. \]
The linear multiplet with \( w = 2 \) and \( n = 0 \) is a real vector multiplet with constrained components \( (C, \chi, m, n, v_\mu, \lambda, d) \). The conditions are \( m = n = 0, \quad D_\mu v^\mu = 0, \quad \lambda = -\gamma^\mu D_\mu \chi, \quad d = -\Box C, \) \( (3.17) \)
where \( D_\mu \) and \( \Box \) are superconformal covariant derivative and d'alembertian. Applying
the density formula \( (3.3) \) and tensor calculus to expression \( (3.15) \), one deduces that
the lagrangian will contain kinetic terms for the real scalar field \( C \) and Einstein terms of the form
\[ - e z_0 \overline{z}_0 \frac{\partial \Phi}{\partial C} \Box C + \frac{1}{3} z_0 \overline{z}_0 \Phi e R, \] where it is understood that \( \Phi \) is considered as a function of the lowest components of \( \Sigma, S_0 \) and \( \hat{L} \) only and not a full vector multiplet. Since \( (2.19) \)
\[ \Box C = \Box C + \frac{1}{3} RC + \text{other terms} \quad (3.19) \]
for a scalar field with Weyl weight two, the complete Einstein term is
\[ \frac{1}{3} z_0 \overline{z}_0 \left[ \Phi - C \frac{\partial \Phi}{\partial C} \right] e R. \] (3.20)
The gauge condition for dilatations which fixes \( |z_0| \) is then
\[ z_0 \overline{z}_0 \left[ \Phi(\frac{C}{z_0 \overline{z}_0}, z, \overline{z}) - C \frac{\partial \Phi}{\partial C} \Phi(\frac{C}{z_0 \overline{z}_0}, z, \overline{z}) \right] = - \frac{3}{2} \frac{1}{\kappa^2}. \] (3.21)
Contrary to the case of chiral multiplets only \( [\text{see eq. } (3.10)] \), this is an implicit equation for \( z_0 \overline{z}_0 \) which appears in the arbitrary function \( \Phi \).
An elegant derivation of the Einstein term is as follows. The lagrangian \( (3.15) \) is equivalent to
\[ \mathcal{L} = [S_0 \overline{S}_0 \Phi(U - S + \overline{S})(U + 2\Omega(V)) + S_0^3 w(\Sigma)]_D + [S_0^3 w(\Sigma)]_F, \] (3.22)
where \( S \) is a chiral multiplet and \( U \) an unconstrained vector multiplet. The equations
of motion for the components of \( S + \overline{S} \) simply impose that \( U + 2\Omega(V) \) is a linear
multiplet, hence the equivalence with eq. \( (3.15) \). In this lagrangian, the Einstein term is entirely due to the D-density formula \( (3.3) \). It reads
\[ \frac{1}{3} \left[ z_0 \overline{z}_0 \Phi(\frac{u}{z_0 \overline{z}_0}, z, \overline{z}) - (s + \overline{s}) u \right] e R, \]
\(^7\) In the Wess-Zumino gauge, the lowest component of \( \Omega \) vanishes.
where $s$ and $u$ are the lowest scalar components of $S$ and $U$. In the Wess-Zumino gauge, $\Omega(V)$ does not contribute. The equation of motion for $u$ is however

$$z_0\bar{z}_0 \frac{\partial}{\partial u} \Phi \left( \frac{u}{z_0\bar{z}_0}, z, \bar{z} \right) = s + \bar{s}$$

which leads to the Einstein term

$$\frac{1}{3} z_0\bar{z}_0 \left[ \Phi \left( \frac{u}{z_0\bar{z}_0}, z, \bar{z} \right) - u \frac{\partial}{\partial u} \Phi \left( \frac{u}{z_0\bar{z}_0}, z, \bar{z} \right) \right] eR,$$

which is identical to expression (3.20).

The form (3.22) of the general lagrangian (3.15) can be used to perform the duality transformation which turns the linear multiplet $L$ into the chiral one $S$. The procedure is similar to the case of global supersymmetry. One solves the equation of motion of the unconstrained vector multiplet $U$ which reads

$$\left[ \frac{\partial}{\partial X} \Phi(X, \Sigma, \Sigma e^V) - (S + \bar{S}) \right]_{X=U(S_0\bar{S}_0)^{-1}} = 0,$$

and performs manipulations analogous to eqs. (2.12) to recast gauge kinetic terms into an $F$-density involving the chiral multiplet of gauge curvatures $W^A$. With the conventions we use in our supergravity expressions,

$$[(S + \bar{S})\Omega(V)]_D = -\frac{1}{2}[SW^AW^A]_F.$$

The resulting lagrangian for multiplets $S$, $\Sigma$ and $V$ is always of the form

$$\mathcal{L}_S = [S_0\bar{S}_0 \mathcal{H}(S + \bar{S}, \Sigma, \Sigma e^V)]_D + [SW^AW^A + S_0^3w(\Sigma)]_F.$$

Clearly,

$$\mathcal{H}(S + \bar{S}, \Sigma, \Sigma e^V) = \left[ \Phi \left( \frac{U}{S_0\bar{S}_0}, \Sigma, \Sigma e^V \right) - (S + \bar{S}) \frac{U}{S_0\bar{S}_0} \right],$$

with $\frac{U}{S_0\bar{S}_0}$ expressed as a function of $S + \bar{S}$, $\Sigma$ and $\Sigma e^V$ with the help of equation of motion (3.23). Theory (3.24) is of the form (3.2) with an arbitrary function $\mathcal{H}$ but with a universal gauge kinetic function

$$f = 4S.$$

Notice that an analytic $S$-dependent field redefinition of the compensator

$$S_0 \rightarrow g(S, \Sigma)S_0$$

(3.27)
can be used to formally introduce a dependence on $S$ in the superpotential. One deduces easily that the most general form of superpotential compatible with the duality transformation is

$$w(S, \Sigma) = w_1(\Sigma)e^{\rho S}. \quad (3.28)$$

with an arbitrary analytic function $w_1$ and a real number $\rho$. As it should, this $S$-dependent superpotential preserves the R-symmetry

$$S \rightarrow S + ia \quad (a = \text{real number})$$

of the lagrangian (3.24).

The component expansion of the general lagrangian (3.15) can be obtained by systematically using tensor calculus. The presence of the Chern-Simons multiplet, which is a complicated expression of the vector multiplet $V$, and the implicit character of the compensator fixing condition (3.21) cause some technical difficulties. In the following we will be interested in string effective actions with known functions $\Phi$ and we will only consider the scalar and gaugino bilinear contributions to the lagrangians. This allows us to truncate the multiplets and the density formula, disregarding unwanted components and simplifying the computation of the component expansion. For simplicity, we will also limit our results to a unique gauge singlet chiral multiplet $\Sigma$ coupled to a super-Yang-Mills multiplet for a simple gauge group which will be identified with the hidden $E_8$ sector of (2,2) heterotic strings, and to a supergravity sector containing a linear multiplet. We will then use the following truncated multiplets:

- $\Sigma$: we eliminate the fermionic components and retain the scalars $z$ and auxiliary fields $f$.

- $L$: the linear multiplet is a vector multiplet with components (3.17). We omit its fermionic component $\chi$ and the embedding of $L$ into a vector multiplet becomes $c = C, B_\mu = v_\mu, d = -\Box C, \chi = m = n = \lambda = 0$, where $\Box$ is the superconformal covariant d’Alembertian. The constrained vector field $v_\mu$ can be expressed in terms of an antisymmetric tensor using $v_\mu = \frac{1}{\sqrt{2}}\epsilon^{-1}_\mu\nu\rho\sigma\partial^\nu b_{\rho\sigma}$, omitting gravitino terms.

- $\Omega$: The Chern-Simons supermultiplet will be the origin of gaugino bilinear terms. Its truncation is embedded into a vector multiplet using $m + in = \frac{1}{4}\lambda\lambda$. All other components vanish.

- $V$: The gauge vector multiplet appears explicitly in the expression $e^V$ which does not contain any gaugino bilinear term. It can then be truncated to zero.

- $S_0$: We keep the scalar component $z_0$ and the auxiliary field $f_0$. 

22
To summarize, the embeddings into a vector multiplet \((C, \chi, H, K, B_\mu, \Lambda, d)\) of the truncated multiplets which will be used in the following are

\[
L = (C, 0, 0, v_\mu, 0, -\Box \Phi),
\]

\[
\Omega(V) = (0, 0, \frac{i}{4} \bar{\chi} \lambda, \frac{i}{4} \bar{\chi} \gamma_5 \lambda, 0, 0, 0),
\]

\[
\Sigma = (z, 0, -f, i f, i D_\mu^C z, 0, 0),
\]

\[
S_0 = (z_0, 0, -f_0, i f_0, i D_\mu^C z_0, 0, 0),
\]

where the Majorana four-component spinors \(\lambda\) are the gauginos and a summation over all group generators is understood. In the multiplet of superconformal gauge fields, we will disregard all gravitino contributions. This truncation implies also that the gauge field of special supersymmetry, which is algebraic, can be omitted. But the gauge field \(A_\mu\) of chiral \(U(1)\) rotations must be kept: it is an auxiliary field of minimal Poincaré supergravity and it contributes in particular to scalar kinetic terms. This truncation of the superconformal gauge fields leads to the simple density formula already given in eqs. (3.3).

4 Component lagrangians

4.1 Basics

In preparation for the analysis of the effective supergravity lagrangian of superstrings, we first apply the rules of superconformal tensor calculus to the general lagrangian (3.15) for one linear multiplet \(L\) and one chiral multiplet \(\Sigma\). The truncated component expansion of this theory before solving for auxiliary fields and fixing the compensator \(z_0\) is

\[
\mathcal{L} = \mathcal{L}_E + \mathcal{L}_{KIN} + \mathcal{L}_{AUX} + \mathcal{L}_{4\lambda},
\]

where the four contributions give respectively the Einstein term, the kinetic lagrangian of scalars and of the antisymmetric tensor, auxiliary fields terms, and quartic gaugino contributions. As already mentioned [eq. (3.20)], the Einstein term is

\[
e^{-1} \mathcal{L}_E = -\frac{1}{2} R \left[ -\frac{2}{3} (z_0 \Phi) (\Phi - C \frac{\partial}{\partial C} \Phi) \right].
\]
Kinetic terms read:

\[ e^{-1}L_{\text{kin}} = \frac{1}{2} \Phi_{xx}(z_0 \bar{z}_0)^{-1}(\partial_\mu C)(\partial^\mu C) - 2 \Phi_{\tau}(z_0 \bar{z}_0)(\partial_\mu z)(\partial^\mu \bar{z}) \]

\[ -2[\Phi - \Phi_x C(z_0 \bar{z}_0)^{-1} + \Phi_{xx} C^2(z_0 \bar{z}_0)^{-2}](\partial_\mu z_0)(\partial^\mu \bar{z}_0) \]

\[ -2[\Phi_x - \Phi_{xx} C(z_0 \bar{z}_0)^{-1}]z_0(\partial_\mu z)(\partial^\mu \bar{z}_0) \]

\[ -2[\Phi - \Phi_{\tau} C(z_0 \bar{z}_0)^{-1}]z_0(\partial_\mu \bar{z})(\partial^\mu z_0) \]

\[ - \frac{i}{2} \Phi_{xx}(z_0 \bar{z}_0)^{-1} v_\mu \mu^\mu - i v_\mu(\Phi_{xx} \partial^\mu z - \Phi_x \partial^\mu \bar{z}) \]

\[ - \Phi_{xx} C(z_0 \bar{z}_0)^{-1} i v_\mu \partial^\mu \log \left( \frac{\Omega}{\bar{z}_0} \right). \]

The variable \( x \) is the lowest component of the multiplet \( L(S_0 \bar{S}_0)^{-1}, \ x = C(z_0 \bar{z}_0)^{-1} \), and the notation for derivatives of \( \Phi \) is

\[ \Phi_x = \frac{\partial}{\partial x} \Phi(x, z, \bar{z}), \quad \Phi_z = \frac{\partial}{\partial z} \Phi(x, z, \bar{z}), \quad \ldots. \]

Scalar and \( b_{\mu\nu} \) kinetic terms will receive further contributions from auxiliary field \( A_\mu \) when solving its equation of motion. It actually turns out that after solving for \( A_\mu \), scalar kinetic terms only depend on the compensator through the combination \( z_0 \bar{z}_0 \). Fixing the compensator will then allow to express \( \partial_\mu z_0 \bar{z}_0 \) as a function of \( C, z, \bar{z}, \) and their derivatives. The part of the lagrangian involving auxiliary fields is

\[ e^{-1}L_{\text{aux}} = 2[\Phi - \Phi_x C(z_0 \bar{z}_0)^{-1} + \Phi_{xx} C^2(z_0 \bar{z}_0)^{-2}] \left( f_0 \bar{f}_0 - \frac{1}{4}(z_0 \bar{z}_0)A_\mu A^\mu \right) \]

\[ + 2[\Phi_x - \Phi_{xx} C(z_0 \bar{z}_0)^{-1}] \left( z_0 \bar{f}_0 f - \frac{i}{2}(z_0 \bar{z}_0)A_\mu \partial^\mu z \right) \]

\[ + 2[\Phi - \Phi_{\tau} C(z_0 \bar{z}_0)^{-1}] \left( z_0 f_0 \bar{f} + \frac{i}{2}(z_0 \bar{z}_0)A_\mu \partial^\mu \bar{z} \right) \]

\[ + [\Phi - \Phi_x C(z_0 \bar{z}_0)^{-1} + \Phi_{xx} C^2(z_0 \bar{z}_0)^{-2}] i A_\mu(z_0 \partial^\mu \bar{z}_0 - \bar{z}_0 \partial^\mu z_0) \]

\[ + \Phi_{xx} C(z_0 \bar{z}_0)^{-1} v_\mu A^\mu + 3z_0^2 f_0 w(z) + 3z_0^2 f_0 w(z) + z_0^3 \frac{dw}{dz} f + z_0^3 \frac{f_0}{\bar{z}_0} \]

\[ + \Phi_{xx} C(z_0 \bar{z}_0)^{-2} \left[ z_0 f_0(\bar{X}_L \lambda R) + z_0 f_0(\bar{X}_R \lambda L) \right] \]

\[ + 2\Phi_{\tau}(z_0 \bar{z}_0) f \bar{f} - \Phi_{xx} f(\bar{X}_R \lambda L) - \Phi_{\tau}(z_0 \bar{z}_0) f \bar{f} - \Phi_{xx} f(\bar{X}_L \lambda R). \]

A summation on all gaugino spinors is understood in gauge invariant bilinear expressions like \( (\bar{X}_L \lambda R) \). The chiral auxiliary fields \( f \) and \( f_0 \) only produce potential and non-derivative gaugino terms. Finally, the last contribution to (4.1) is a quartic gaugino contribution:

\[ e^{-1}L_{4\lambda} = \frac{1}{2} \Phi_{xx}(z_0 \bar{z}_0)^{-1}(\bar{X}_L \lambda R)(\bar{X}_R \lambda L). \]
The auxiliary fields of minimal Poincaré supergravity are $A_\mu$ [chiral $U(1)$ gauge field] and $f_0$ [in the chiral compensator $S_0$]. The Poincaré theory will be obtained by solving for $A_\mu$ and $f_0$, and by fixing the compensator with the requirement

$$-\frac{2}{3}(z_0\bar{z}_0)[\Phi - \Phi_0 C(z_0\bar{z}_0)^{-1}] = \frac{1}{\kappa^2},$$

which canonically normalizes the Einstein lagrangian.

Using the Weyl weights

$L : \quad w(C) = 2 \quad w(\nu_\mu) = 3$

$\Sigma : \quad w(z) = 0 \quad w(f) = 1$

$S_0 : \quad w(z_0) = 1 \quad w(f_0) = 2$

$w(A_\mu) = 1$

which specify the physical dimensions of the various component fields, one easily checks that every term in the lagrangian (4.1) has dimension four.

4.2 Kähler invariant lagrangians in components

As mentioned above, the component lagrangian (4.1) can be used to construct the scalar and gaugino bilinear sector of a general $N = 1$ supergravity with a linear multiplet. The general form is actually not very illuminating and, moreover, the equation (3.21) used to determine the compensator can only be solved implicitly for an arbitrary $\Phi$. We will then concentrate on a specific class of functions $\Phi$ which is of direct interest in the context of the effective supergravity theory of superstrings.

Supergravity couplings of chiral multiplets are characterized by Kähler invariance (3.7). It is also known that superstrings often possess symmetries which act in the effective supergravity like Kähler symmetries. This means that there exists a Kähler potential $K$ which, together with the superpotential $w(\Sigma)$ and the compensator $S_0$, transforms according to eqs. (3.7). An example of such a symmetry of stringy origin is target-space duality [17]. The Kähler potential, which is in general a gauge invariant function of $\Sigma$ and $\Sigma e^V$, can be regarded as a composite connection for Kähler symmetry. The combination

$$\frac{e^{K/3}\hat{L}}{S_0\bar{S}_0}$$

is a Kähler and gauge invariant quantity with conformal weight zero. The class of functions $\Phi$ of the form

$$\Phi = \frac{\hat{L}}{S_0\bar{S}_0} F\left(\frac{e^{K/3}\hat{L}}{S_0\bar{S}_0}\right), \quad (4.6)$$

with $F$ an arbitrary real function, leads then to Kähler invariant superconformal theories with a non-trivial coupling of the linear multiplet. Notice that this choice is not the
most general one since a further dependence on gauge and Kähler invariant functions of the chiral multiplet is in principle allowed, a possibility which will not be considered here. Notice also that the tree-level effective supergravity of superstrings is of the form (4.6) with

\[ F(y) = -\frac{1}{\sqrt{2}} y^{-3/2}. \]

The advantage of (4.6) is that the Kähler potential \( K(\Sigma, \Sigma e^V) \), which controls the standard complex geometry of the chiral superfields, is explicit in \( \Phi \). The function \( \Phi(z, \bar{z}) \) appears in the component lagrangian as a connection which, for instance, changes \( (z_0 \bar{z}_0) \) to the Kähler invariant expression \( z_0 e^{-K/3} \bar{z}_0 \). We will later verify that the scalar kinetic terms are as usual given by the second derivatives of \( K \), as it should for a Kähler potential.

Let us write the auxiliary-fields lagrangian (4.4) as:

\[
 e^{-1} \mathcal{L}_{AUX} = A f_0 \bar{f}_0 + (B f_0 f + \text{h.c.}) + C f \bar{f} - \frac{1}{4} (z_0 \bar{z}_0) A A_{\mu} A_{\mu} \\
 + D v_{\mu} A_{\mu} + (E f_0 + F f - \frac{i}{2} z_0 B \partial_{\mu} z A_{\mu} + \text{h.c.}) \\
 + \frac{i}{2} z_0 \bar{z}_0 A A_{\mu} \partial_{\mu} \log(\bar{z}_0/z_0),
\]

where the coefficients can be read from (4.4) and for the case (4.6) are given by:

\[
 A = 2[\Phi - x \Phi_x + x^2 \Phi_{xx}] = 2xy(F' + yF''), \\
 B = 2z_0[\Phi_z - x \Phi_{xz}] = -\frac{2}{3} z_0 xy(F' + yF'') K_z, \\
 C = 2(z_0 \bar{z}_0) \Phi_{z\bar{z}} = \frac{2}{3} (z_0 \bar{z}_0) xy [3F' K_{z\bar{z}} + (F' + yF'') K_z K_{\bar{z}}], \\
 D = x \Phi_{xx} = y(2F' + yF''), \\
 E = 3z_0^2 w + z_0^{-1} x \Phi_{xx}(\bar{\lambda}_R \lambda_L) = 3z_0^2 w(z) + z_0^{-1} xy(2F' + yF'')(\bar{\lambda}_R \lambda_L), \\
 F = z_0^3 w_z - \Phi_{xz}(\bar{\lambda}_R \lambda_L) = z_0^3 w_z - \frac{1}{3} y(2F' + yF'')(\bar{\lambda}_R \lambda_L) K_z.
\]

We use again \( x = C(z_0 \bar{z}_0)^{-1} \) and \( F', F'' \) refer to derivatives of \( F \) with respect to its argument \( y \equiv xe^{K/3} \). In (4.7), the last coupling of the form \( A_{\mu} \partial_{\mu} \log(\bar{z}_0/z_0) \) can be eliminated using the fact that the compensator will be fixed to be real. To compute the scalar potential we have to solve the equations for \( f \) and \( f_0 \). They give:

\[
 f_0 = (\mathcal{A} \mathcal{C} - \mathcal{B} \mathcal{B})^{-1} \left( \mathcal{B} \mathcal{F} - \mathcal{C} \mathcal{E} \right), \\
 f = (\mathcal{A} \mathcal{C} - \mathcal{B} \mathcal{B})^{-1} \left( \mathcal{B} \mathcal{E} - \mathcal{A} \mathcal{F} \right),
\]

and the potential is obtained by inserting these expressions into the lagrangian. It is
in general given by

\[ V_{aux} = -\mathcal{E} \overline{f}_0 + \overline{f} \mathcal{F} \]

\[ = (\mathcal{AC} - \mathcal{B}\overline{\mathcal{B}})^{-1} \left[ C|\mathcal{E}|^2 + A|\mathcal{F}|^2 - (\overline{\mathcal{E}} \mathcal{B} \mathcal{F} + \text{h.c.}) \right]. \tag{4.10} \]

Using eqs. (4.8), we can obtain the scalar potential including the contribution generated by expectation values of gaugino bilinears, provided we also add the four–gaugino term (4.5). We obtain

\[ V = V_{aux} + V_{4\lambda} \]

\[ = \frac{3}{2} (z_0 \overline{z}_0)^2 x^{-2} e^{-K/3} F'_m K^{-1}_\mu \left| w + wK \right|^2 \]

\[ + \frac{3}{2} (z_0 \overline{z}_0)^{-1} x^{-2} (F' + yF'' - i e^{-K/3} z_0^3 w + i y(2F' + yF'' ) (\overline{\mathcal{A}} R \lambda L))^2 \]

\[ - \frac{1}{2} (z_0 \overline{z}_0)^{-1} e^{-K/3} (2F' + yF'') \left| (\overline{\mathcal{A}} R \lambda L) \right|^2, \tag{4.11} \]

where the last term is the contribution from \( V_{4\lambda} \).

At this level of generality, our analysis extends trivially to couplings of one linear multiplet \( L \) and any number of chiral multiplets \( \Sigma_i \) to supergravity. In this case the coefficient \( C \) is actually a matrix \( C_{\mu}^\tau \) and \( B \) and \( \mathcal{F} \) are vectors \( B_i \) and \( \mathcal{F}_i \) respectively. The auxiliary field \( f \) is also a vector \( f_i \). Then the expression in the scalar potential \( K^{-1}_\mu \left| w + wK \right|^2 \) should be seen as a particular case of the general expression \( K^{-1}_\mu (w_i + wK_i) (\overline{\mathcal{A}}_{\tau} + \overline{\mathcal{A}} K_{\tau}) \). It is a nontrivial fact that the matrix \( (\mathcal{AC} - \mathcal{B}\overline{\mathcal{B}})^{-1} \) leads to a simple expression only in terms of \( K^{-1}_\mu \). Having mentioned this, for simplicity, we will continue writing the expressions in terms of only one chiral superfield.

To obtain the scalar kinetic terms we need first to solve the field equation for the auxiliary field \( A_\mu \). From (4.7), we can see that

\[ A_\mu = 2 (z_0 \overline{z}_0)^{-1} A^{-1} \left( D\nu_\mu - \frac{i}{2} [\overline{z}_0 \mathcal{B} \partial_\mu z - z_0 \overline{\mathcal{B}} \partial_\mu \overline{z}] \right) + i \partial_\mu \log \left( \frac{\overline{z}_0}{z_0} \right) \]

\[ = \frac{i}{3} [K_\mu \partial_\mu z - K_\tau \partial_\mu \overline{z}] + (z_0 \overline{z}_0)^{-1} x^{-1} 2 F'_m + y F''_m \nu_\mu + i \partial_\mu \log \left( \frac{\overline{z}_0}{z_0} \right). \tag{4.12} \]

The \( A_\mu \)-dependent terms in (4.7) will then add contributions of the form

\[ e^{-1} \Delta \mathcal{L}_{KIN} = \frac{1}{4} (z_0 \overline{z}_0) A A_\mu A^\mu \]

to scalar kinetic terms (4.3). It turns out that the complete kinetic terms can be written in an elegant form before fixing the compensator:

\[ e^{-1} \mathcal{L}_{KIN} = -\frac{1}{2} (z_0 \overline{z}_0)^{-1} e^{K/3} \frac{2 F'_m + y F''_m}{F'_m + y F''_m} F' \left[ (\partial_\mu C)(\partial^\mu C) - v_\mu v^\mu \right] \]

\[ - \frac{2}{3} (z_0 \overline{z}_0) x y F' K_\overline{z} (\partial_\mu z)(\partial^\mu z) \]

\[ + (z_0 \overline{z}_0) x y \frac{F'}{F'_m + y F''_m} \left[ (2 F' + y F'') C^{-1}(\partial_\mu C)(\partial^\mu \Delta) - \frac{1}{2} F'(\partial_\mu \Delta)(\partial^\mu \Delta) \right], \tag{4.13} \]
where \( \Delta \) is the combination

\[
\Delta = \log[z_0 \bar{z}_0 xy F'] = \log[(z_0 \bar{z}_0)^{-1} C^2 e^{K/3} F'],
\]

which will be equal to a constant with the compensator fixing condition (3.21), which in our case reads

\[
(z_0 \bar{z}_0)^{-1} C^2 e^{K/3} F' = e^\Delta = \frac{3}{2 \kappa^2}.
\]

This condition can be explicitly solved only after having specified the function \( F \). However, for an arbitrary \( F \), scalar and antisymmetric tensor kinetic terms will become

\[
e^{-1} L_{kin.} = -\frac{1}{\kappa^2} K z \bar{z} (\partial_\mu z)(\partial_\mu \bar{z}) - \frac{3}{4} \frac{2 F' + y F''}{F'} C^{-2} [((\partial_\mu C)(\partial^\mu C) - \nu_\mu \nu^\mu]
\]

which will be equal to a constant with the compensator fixing condition (3.21), which in our case reads

\[
(z_0 \bar{z}_0)^{-1} C^2 e^{K/3} F' = e^\Delta = \frac{3}{2 \kappa^2}.
\]

This condition can be explicitly solved only after having specified the function \( F \). However, for an arbitrary \( F \), scalar and antisymmetric tensor kinetic terms will become

\[
e^{-1} L_{kin.} = -\frac{1}{\kappa^2} K z \bar{z} (\partial_\mu z)(\partial_\mu z) - \frac{3}{4} \frac{2 F' + y F''}{F'} C^{-2} [((\partial_\mu C)(\partial^\mu C) - \nu_\mu \nu^\mu]
\]

in the Poincaré supergravity theory. This shows that the Kähler potential \( K \), introduced first as a connection to construct Kähler invariant functions, does give the \( \sigma \)–model metric for the chiral scalar fields, as it should. Notice also that since the ratio \( (2 F' + y F'')/(F' + y F'') \) must be positive to ensure positivity of kinetic energy, we can see that the last two terms in the scalar potential (4.11) have opposite signs. The potential is not explicitly positive definite.

A nontrivial check for all these expressions is to take the particular case \( F(y) = -\frac{3}{2y} \) which corresponds to \( \Phi = -\frac{3}{2} e^{-K/3} \), the general form (3.2) for chiral multiplets coupled to supergravity only. This choice of \( F \) implies that \( 2 F' + y F'' = 0 \), and equations (4.13), (4.14) and (4.15) become respectively

\[
e^{-1} L_{kin.} = -\frac{1}{\kappa^2} K z \bar{z} (\partial_\mu z)(\partial_\mu \bar{z}) + \frac{3}{4} (z_0 \bar{z}_0 e^{-K/3}) (\partial_\mu \log z_0 \bar{z}_0 e^{-K/3})),
\]

\[
(z_0 \bar{z}_0) e^{-K/3} = \frac{1}{\kappa^2},
\]

\[
e^{-1} L_{kin.} = -\frac{1}{\kappa^2} K z \bar{z} (\partial_\mu \bar{z})(\partial^\mu z).
\]

The scalar potential (4.11) reduces to

\[
k^4 V(z, \bar{z}) = e^K \left( K z^{-1} |w + K w|^2 - 3 |w|^2 \right)
\]

which is the well-known potential obtained in ref. [19]. In some sense, our expressions are more general since they include the general couplings of a linear multiplet. The effective supergravity of superstrings is, up to some subtleties, a particular class of the models discussed in this section. To this we will turn next.
5 Superstring effective actions and scales

We now turn to the study of the effective lagrangians which describe at low energies the physics of the four-dimensional heterotic superstring theory, with a particular attention to the problem of scales in the effective supergravity theory. While there is no ambiguity at the string tree-level, the derivation of an effective theory is more subtle when loop corrections of string origin are also considered. At string tree-level, the definition of a low-energy effective action is simple since small external momenta imply small momenta also on internal lines of tree-level diagrams. The problem requires more attention when loop diagrams are taken into account at the string and effective field theory levels.

A string loop calculation, like the gauge coupling constants calculations performed in [3, 4], requires the introduction of an infra-red cut-off $M_{IR}$ which avoids divergences when massless string states circulate on loops with small momenta. The cut-off forbids momenta smaller than $M_{IR}$. Since $M_{IR}$ is an arbitrary scale, its variation should reflect in changes of physical quantities controlled by renormalization-group equations. For instance, it has been verified that one-loop gauge coupling constants computed in strings show the dependence on $\log M_{IR}$ implied by the one-loop gauge $\beta$-function for massless string modes [1, 2].

What we are interested in is the description of the dynamics of string massless modes using an effective low-energy field theory. This can be done for energies below a physical ultraviolet cut-off, $M_{UV}$. This quantity is arbitrary, but it should not exceed the order of magnitude of the mass of the lightest string excited modes. Physical quantities computed in the effective theory, which depend on $M_{UV}$, should correspond to the same quantities computed in the string theory, which depend on stringy scales (string tension and compactification radii) but also on $M_{IR}$. The infra-red cut-off $M_{IR}$ may be identified with $M_{UV}$, but this is not necessary: in general, the effective field theory will also depend explicitly on the scale $M_{IR}$, and it should be understood as an effective theory in the Wilson sense.

In the string context, the Wilson effective action $S_W$ is a local field theory obtained by integrating out all heavy string and Kaluza-Klein modes, at the loop level, with the prescription that one integrates all loops with virtual momenta larger than a scale $\mu$ which characterizes the Wilson action. The ‘small’ scale $\mu$ acts like an infra-red cut-off and it should be identified with the string infra-red cut-off $M_{IR}$. The Wilson action has a formal expansion in string loops, but a string amplitude for massless external states at $n$ string loops is computed by summing all diagrams of order $n$ constructed from terms in the Wilson action up to order $n$. For instance, a one-loop string amplitude in the formalism of the Wilson action combines two contributions: one-loop diagrams from the Wilson tree-level action, which correspond to massless string modes on the loop, and tree diagrams from the one-loop correction to the Wilson action, corresponding to the effect of heavy string modes and virtual momenta larger than $\mu$ in the string loop.

A second possibility would be to use a generalisation of the effective action $S_T$.
of quantum field theory, which is the generating functional of one-particle irreducible Green’s functions. In the string context, \( S_{\Gamma} \) would be the generating functional of string amplitudes for external massless states. Contrary to the Wilson action, it also includes the contributions of light or massless loops. With massless states, the effective action is a non-local functional of external classical fields. The relation between \( S_W \) and \( S_{\Gamma} \) is very simple: \( S_{\Gamma} \) is the effective action, as defined in field theory, for the field theory with action \( S_W \). To define the effective action \( S_{\Gamma} \), we need an ultraviolet cut-off which we will identify with \( z_0 \bar{z}_0 \) and an arbitrary running scale \( \mu \) which can be varied by the action of the renormalization group. The arbitrary scale \( \mu \) corresponds to a subtraction point, to some choice of momenta of external gauge bosons in an amplitude used to normalize the gauge coupling constant. The invariance of \( S_{\Gamma} \) under a change in \( \mu \) is realized by the fact that physical quantities satisfy renormalization-group equations.

A symmetry of loop-corrected string amplitudes, like for instance target-space duality, is also a symmetry of the effective action. However, it is not necessarily a symmetry of the Wilson action. The Wilson action can be anomalous, with the perturbative anomaly cancelled in amplitudes by a mechanism analogous to gauge and gravitational anomaly cancellation in ten-dimensional heterotic strings [20]. This phenomenon has been established for target-space duality in \((2,2)\) superstrings, in the sector of gauge kinetic terms [3, 4]. It turns out that target-space duality acts in the tree-level Wilson action like an anomalous Kähler symmetry with the anomaly cancelled by quantum corrections to \( S_W \), a mechanism which has been studied in general terms in the context of supergravity theories [24, 27].

In the following discussion, we will use the terminology effective theory for a local field theory corresponding in fact to the Wilson approach to effective lagrangians. We will explicitly mention when we will consider quantities related to the effective action \( S_{\Gamma} \).

### 5.1 String tree-level effective actions and non renormalization theorems

We consider now the \( d = 4, N = 1 \) heterotic string effective actions. We will mainly focus in this section on general information that can be obtained from the superfield formulation leaving the explicit component expressions for the next section. For simplicity, whenever our expressions refer explicitly to string actions we will restrict to the diagonal overall modulus \( T \), a chiral superfield which exists in \((2,2)\) symmetric orbifolds (as well as in \((2,2)\) Calabi-Yau compactifications). The set of all chiral supermultiplets, denoted collectively by \( \Sigma \), will then contain \( T \) as well as (charged) chiral matter multiplets \( Q_I \). Later on, when discussing loop corrections, we will also concentrate on the \((2,2)\) \( Z_3 \) and \( Z_7 \) symmetric orbifolds for which the threshold corrections to the gauge coupling constants are known to vanish [3].

As mentioned before, the dilaton in string theory is part of a linear multiplet \( L \) which
includes the antisymmetric tensor field and a Majorana fermion. The most general lagrangian for supergravity coupled to one linear multiplet and any number of chiral multiplets $\Sigma$ depends on two arbitrary functions, a real function $\Phi(L - 2\Omega, \Sigma, \Sigma e^V)$ and the superpotential $w(\Sigma)$, which can be eliminated by a transformation of the form (3.16). It is convenient to express the full action as a superconformal density [see (3.15)]:

$$\mathcal{L} = S_0 \overline{S}_0 \Phi \left( \frac{\hat{L}}{S_0 \overline{S}_0}, \Sigma, \Sigma e^V \right) + [S_0^3 w]_F,$$

where $\hat{L} = L - 2\Omega \equiv L - 2\sum_a \Omega_a$, $\Omega_a$ is the Chern-Simons superfield associated with the gauge group factor $G_a$. As usual, $S_0$ is the compensating chiral multiplet of conformal supergravity which will be fixed to give canonical Einstein kinetic term in the component action of the Poincaré supergravity. Notice that unlike the pure chiral superfield case [19] the gauge coupling is not an independent arbitrary function but comes from $\Phi$ through the $\Omega$ dependence. For the case of superconformal densities defined in (5.1), it is actually given by

$$\frac{1}{g^2} = 2z_0 \overline{z}_0 \frac{\partial \Phi}{\partial C},$$

where $z_0 \overline{z}_0 \frac{\partial \Phi}{\partial C}$ is the lowest component of the multiplet $S_0 \overline{S}_0 \frac{\partial \Phi}{\partial L}$. This result follows from the component expansion of $\Phi$ and can be easily verified for instance from eq. (3.26) and the duality transformation (3.23). At string tree-level, $\Phi$ is given by [10]

$$\Phi_0 = -\frac{1}{\sqrt{2}} \left( \frac{S_0 \overline{S}_0}{\hat{L}} \right)^{1/2} e^{-K/2} = -\frac{1}{\sqrt{2}} \frac{\hat{L}}{S_0 \overline{S}_0} \left( \frac{\hat{L}}{S_0 \overline{S}_0} e^{K/3} \right)^{-3/2},$$

which belongs to the class of Kähler invariant theories (4.6). In the tree-level lagrangian

$$\mathcal{L}_0 = -\frac{1}{\sqrt{2}} \left( \frac{S_0 \overline{S}_0}{\hat{L}} \right)^{3/2} \hat{L}^{-1/2} e^{-K/2} \right]_D + [S_0^3 w]_F,$$

we find the conformal and Kähler invariant gauge coupling

$$\frac{1}{g^2(z_0 \overline{z}_0)} = U \equiv 2 \left( \frac{z_0 \overline{z}_0}{2Ce^{K/3}} \right)^{3/2},$$

where $z_0$ and $C$ are the first components of $S_0$ and $L$ respectively [1]. We explicitly indicate that this gauge coupling depends on $(z_0 \overline{z}_0)$ since we have not yet fixed the conformal invariance.\footnote{Hereafter, whenever couplings are related with fields, as in (5.5), it should be understood as a relation for the vacuum expectation values of the fields.}
We now wish to discuss in more detail the choice of the compensator field \((z_0 \bar{z}_0)\) and the rôle of the various fields at string tree-level. For this purpose, we consider the tree-level kinetic terms for the graviton, the gauge fields and the antisymmetric tensor. Using (5.3), these terms read

\[
e^{-1} L_{0,kin} = \frac{1}{\sqrt{2}} [z_0 \bar{z}_0 e^{-K/3}]^{3/2} C^{-1/2} \left[ \frac{1}{2} R - \frac{1}{4} C^{-1} F_{\mu\nu}^A F^{A\mu\nu} + \frac{1}{4} C^{-2} v_{\mu} v^{\mu} \right]. \tag{5.6}
\]

Since \(v_{\mu} = \frac{1}{\sqrt{2}} e^{-1} \epsilon_{\mu\rho\sigma} \partial^\rho b^\sigma\), the last term is a kinetic lagrangian for the antisymmetric tensor, with

\[
v_{\mu} v_{\mu} = -3 H_{\mu\nu\rho} H^{\mu\nu\rho}, \quad H_{\mu\nu\rho} = \frac{1}{3} (\partial_{\mu} b_{\nu\rho} + \partial_{\nu} b_{\rho\mu} + \partial_{\rho} b_{\mu\nu}).
\]

The dual theory, with \(C\) replaced by the real field \(s + \bar{s}\) is obtained using the procedure described in section 3 [see eqs. (3.22), (3.23) and (3.24)]. Since

\[
s + \bar{s} = \left( \frac{z_0 \bar{z}_0}{2 C e^{K/3}} \right)^{3/2},
\]

the dual kinetic terms are

\[
e^{-1} L_{0,kin} = -\frac{1}{2} z_0 \bar{z}_0 (s + \bar{s})^{1/3} e^{-K/3} R - \frac{1}{2} (s + \bar{s}) F_{\mu\nu}^A F^{A\mu\nu} + (s + \bar{s})^{2} [z_0 \bar{z}_0 (s + \bar{s})^{1/3} e^{-K/3}]^{-1} v_{\mu} v^{\mu}. \tag{5.8}
\]

Notice that the Einstein term is the same as in eq. (3.3), with the replacement \(K \rightarrow - \log(s + \bar{s}) + K\). Suppose that we fix dilatation symmetry by choosing the compensator \((z_0 \bar{z}_0)\) such that the Einstein term is canonical.\footnote{This choice corresponds to the ‘Einstein frame’.} The condition is

\[
\frac{1}{\sqrt{2}} [z_0 \bar{z}_0 e^{-K/3}]^{3/2} C^{-1/2} = UC = \frac{1}{\kappa^2}, \tag{5.9}
\]

and the kinetic terms become

\[
e^{-1} L_{0,kin} = -\frac{1}{2 \kappa^2} R - \frac{1}{4} (\kappa^2 C)^{-1} F_{\mu\nu}^A F^{A\mu\nu} + \frac{1}{4} \kappa^2 (\kappa^2 C)^{-2} v_{\mu} v^{\mu}. \tag{5.10}
\]

The tree-level gauge coupling constant in the Poincaré theory is simply

\[
\frac{1}{g^2} = \langle \frac{1}{\kappa^2 C} \rangle, \quad \frac{1}{\kappa^2 C} = 2 (s + \bar{s}). \tag{5.11}
\]

Instead, we could prefer to fix dilatation symmetry to obtain the ‘string frame’, with kinetic terms of the form

\[
\frac{\varphi^{-1/2}}{\kappa^2} \left[ -\frac{1}{2} R - \frac{1}{4} M_s^{-2} F_{\mu\nu}^A F^{A\mu\nu} + \frac{1}{4} M_s^{-4} v_{\mu} v^{\mu} \right].
\]
The real scalar field \( \phi \) is the string ‘loop-counting parameter’ and the string scale \( M_s \) is related to the gauge coupling constant by \( M_s = g/\kappa \). Clearly, the ‘string frame’ corresponds to the condition

\[
\frac{1}{\sqrt{2}} \left[ z_0 \bar{z}_0 e^{-K/3} \right]^{1/2} = \frac{1}{\kappa^3},
\]

instead of (5.9). The comparison of kinetic terms shows that \( \phi = \kappa^2 C \) is the string loop-counting parameter, that

\[
M_s^2 = \langle C \rangle,
\]

while the gauge coupling constant is again \( g^2 = \kappa^2 \langle C \rangle \). But, according to (5.7) and (5.11), the field \( s + \bar{s} \) of the dual theory is given by

\[
s + \bar{s} = \frac{1}{2} \left( \frac{1}{\kappa^2 C} \right)^{3/2},
\]

instead of the second equality (5.11).

The \( \Sigma \) dependence of the tree-level lagrangian (5.4) is encoded in the Kähler potential only:

\[
K(T, \bar{T}, Q, \bar{Q}) = -3 \log(T + \bar{T}) + \sum_I (T + \bar{T})^{n_I} Q_I \bar{Q}_I + \ldots
\]

and the superpotential \( w(Q) = Y_{IJK}(T) Q^I Q^J Q^K + \ldots \) where \( n_I \) are different ‘modular weights’ associated to the respective matter fields, \( Y_{IJK}(T) \) the modulus dependent Yukawa couplings and the ellipsis refer to higher order terms in the \( Q^I \) expansion. Because of Kähler invariance, the theory depends only on the single function \( G = K + \log|w|^2 \). This can be seen by redefining the compensator, \( S_0^3 = S_0^3 w \), with a transformation similar to (3.16),

\[
\begin{align*}
K & \rightarrow K + \varphi + \bar{\varphi} \\
S_0 & \rightarrow S_0 e^{\varphi/3} \\
w & \rightarrow we^{-\varphi}
\end{align*}
\]

which leaves \( \hat{L} \) invariant. Target-space duality is an example of a symmetry which manifests itself as a Kähler transformation. Its action on \( T \) is such that \( G \) is invariant, but \( K \) and \( w \) transform as in (5.14).

In this formalism, the linear supermultiplet \( L \), which contains the dilaton field related to the string coupling as its lowest component, has by supersymmetry a nature different from the other multiplets \( T \) and \( Q_I \). This observation allows to extract very useful information about the possible loop corrections to the effective lagrangian. First we can say that since the superpotential cannot depend on the field \( L \), it cannot get any quantum correction, perturbative or non–perturbative, as long as this formalism
is applicable. This is a strong result that goes beyond previous arguments which apply only at the perturbative level and were obtained in the dual formalism where $L$ is transformed into the chiral multiplet $S$. In this approach, it was shown that because of a Peccei–Quinn (PQ) symmetry under which the imaginary part of this field is shifted, $S$ cannot appear in the superpotential. Showing that this symmetry is preserved in perturbation theory gave the non-renormalization theorem of. We have already seen that the existence of this symmetry acting on $S$ is a consequence of the duality equivalence of $S$ with the linear multiplet $L$ for which the PQ symmetry is hidden in the gauge symmetry acting on the antisymmetric tensor $b_{\mu\nu}$. Working with the linear multiplet implies that the PQ symmetry is exact and, as long as perturbative or non-perturbative corrections can be expressed in this formalism, the PQ symmetry will remain unbroken.

A similar comment can be made about the Kähler potential which, being defined as a function of chiral superfields only, cannot get any loop corrections. This result is however not as powerful as it sounds. Besides the Kähler potential $K$ itself, the main information in the function $\Phi$ is the coupling of $L$ to the chiral multiplets described by $K$, a coupling which is left entirely unconstrained. One could try to invoke Kähler symmetry under transformations (5.14) to restrict this arbitrariness, to argue for instance that $\Phi$ should be of the form

$$\Phi = \frac{\hat{L}}{S_0 \overline{S}_0} F \left( \frac{\hat{L}}{S_0 \overline{S}_0} \right),$$

with $F$ an arbitrary real function. The actual situation is however more complicated since Kähler invariance is related to quantum string symmetries of the effective action $S_T$, which are plausibly anomalous when considered using Wilson’s effective lagrangian. This is at least the case of target-space duality in (2,2) theories. Such symmetries of the effective action correspond to a Green-Schwarz anomaly-cancellation mechanism which introduces in the Wilson lagrangian counterterms which do not preserve the symmetries, as we will see below. Equation (5.14) applies only to the tree-level, invariant terms. As far as $\Phi$ is concerned, the existence of exact symmetries, acting like Kähler symmetries, and realized in an anomaly-cancellation mode, tells us that $\Phi$ cannot be of the form (5.14) since the Wilson lagrangian is not invariant.

### 5.2 Loop-corrected effective actions

Very powerful information concerning the general form of the loop-corrected effective lagrangians can be obtained by using the fact that the gauge coupling constant is given by equation (5.2). In superstrings, the gauge coupling constant is the expectation value

\[ \frac{\hat{L}}{S_0 \overline{S}_0} F \left( \frac{\hat{L}}{S_0 \overline{S}_0} \right), \]

with $F$ an arbitrary real function. The actual situation is however more complicated since Kähler invariance is related to quantum string symmetries of the effective action $S_T$, which are plausibly anomalous when considered using Wilson’s effective lagrangian. This is at least the case of target-space duality in (2,2) theories. Such symmetries of the effective action correspond to a Green-Schwarz anomaly-cancellation mechanism which introduces in the Wilson lagrangian counterterms which do not preserve the symmetries, as we will see below. Equation (5.14) applies only to the tree-level, invariant terms. As far as $\Phi$ is concerned, the existence of exact symmetries, acting like Kähler symmetries, and realized in an anomaly-cancellation mode, tells us that $\Phi$ cannot be of the form (5.14) since the Wilson lagrangian is not invariant.
of a function of scalar fields. Then, loop corrections to gauge couplings in the form of renormalization-group equations should provide information of the functional dependence on \( L \) of the function \( \Phi \) defining the loop-corrected effective theory. Fortunately, loop corrections to gauge couplings in strings is a subject for which calculations have been performed in large classes of superstrings \[3, 4\]. The different contributions to the full string loop amplitude from the massless and massive sectors are well understood, allowing then to know the corrections to the effective gauge coupling at energies below the Planck scale. To discuss the loop corrections to the effective gauge couplings we need to differentiate between the Wilson action \( (S_W) \) and the one–particle–irreducible effective action \( S_\Gamma \) as mentioned above. For making our discussion self-contained, we will reproduce here the one-loop superfield effective action for the simplest orbifold models, namely \((2, 2)\) \( Z_3 \) and \( Z_7 \) orbifolds.

The threshold corrections to the gauge couplings were computed in \[2, 3\] for \((2, 2)\) orbifold models performing a one-loop string calculation. In the effective lagrangian approach, the one-loop contributions to gauge coupling constants combine one-loop diagrams from the tree-level lagrangian \((5.4)\), which involve massless loops only, and tree diagrams from the one-loop corrections to the effective lagrangian \( L_W \), which are local corrections to \((5.4)\). Consider for instance the modulus-dependent corrections to gauge couplings at one-loop \[9\]. Using the tree-level effective lagrangian, one can construct a triangle diagram with two external gauge bosons and one composite Kähler connection \( K \). This ‘Kähler anomaly’ can be represented by a non-local superfield contribution to the effective action \( S_\Gamma \):

\[
L_{nl}^{E_8} = -\frac{A}{4} \left[ \frac{1}{3} W^A_{\hat{E}_8} W^A_{\hat{E}_8} P_C K \right] F,
\]

where

\[
A \equiv \frac{3C(E_8)}{8\pi^2}.
\]

\( C(E_8) \) is the quadratic Casimir of \( E_8 \) and \( P_C \) is the non-local projector of a vector multiplet into a chiral one. For a vector multiplet \( H \), \( P_C H \) is chiral, while \( P_C \varphi = \varphi \), \( P_C \bar{\varphi} = 0 \) if \( \varphi \) is chiral. In global supersymmetry, this projector is \( P_C = -(16 \square)^{-1} D^2 D^2 \).

An equation similar to \((5.16)\) can be written for \( E_6 \) gauge fields but we prefer to write explicitly only the hidden \( E_8 \) part because it is the one we will use later. Notice that under Kähler transformations \((5.14)\), the non-local term \((5.16)\) is not invariant but generates an anomaly. Its variation is local and contains the terms:

\[
\frac{A}{3} \left[ \frac{1}{8} (\varphi + \bar{\varphi}) F^A_{\mu\nu} F^{A\mu\nu} + \frac{i}{8} (\varphi - \bar{\varphi}) F^A_{\mu\nu} \tilde{F}^{A\mu\nu} \right].
\]

The string calculations \[3, 4\] show that in orbifolds without threshold corrections, this anomaly is cancelled by the local Green-Schwarz term \[3, 20, 27\]

\[
L_{GS} = \frac{A}{4} \left[ \hat{L}_{E_8} K \right] D,
\]

\[
\hat{L}_{E_8} = L - 2\Omega_{E_8}.
\]
which is a one-loop correction to $\mathcal{L}_W$. Actually, in theories without threshold corrections, the Green-Schwarz lagrangian \( \mathcal{L} \) is determined from the one-loop anomaly \( \mathcal{A} \) and the information that the anomaly has to be cancelled in strings.

In the effective action, gauge kinetic terms will receive local contributions from the sum of \( \mathcal{A} \) and \( \mathcal{B} \):

\[
\begin{align*}
\mathcal{L}_{\text{nl}} & \rightarrow -\frac{1}{4} \left[ -\frac{1}{6} AK \right] F_{\mu\nu}^A F^{A \mu\nu}, \\
\mathcal{L}_{\text{GS}} & \rightarrow -\frac{1}{4} \left[ +\frac{1}{6} AK \right] F_{\mu\nu}^A F^{A \mu\nu},
\end{align*}
\]

(5.20)

dropping the index \( E_8 \). These two contributions cancel and gauge kinetic terms are \( K \)-independent in orbifolds without threshold corrections, as they should be.

Since our discussion is based on conformal supergravity, we have to also worry about anomalies of conformal transformations, which are expected to be related to the renormalization-group behaviour of physical quantities. At one-loop, the variation of the running \( E_8 \) gauge coupling constant under a change \( M \rightarrow \lambda M \) of the scale is

\[
\delta(g^{-2}) = A \log \lambda.
\]

(5.21)

In an effective lagrangian, this means that

\[
\delta \left( -\frac{1}{4} \frac{1}{g^2} F_{\mu\nu}^A F^{A \mu\nu} \right) = -\frac{1}{4} A \log \lambda F_{\mu\nu}^A F^{A \mu\nu}.
\]

This behaviour under scale transformations is precisely obtained in the component expansion of the anomalous non-local expression

\[
-\frac{A}{4} \left[ W^A_{E_8} W^A_{E_8} \mathcal{P}_C \log \hat{L} \right]_F,
\]

(5.22)

which includes the gauge kinetic term

\[
\frac{1}{4} \frac{A}{2} \log C F_{\mu\nu}^A F^{A \mu\nu}.
\]

Notice that the expression (5.22) is Kähler invariant, in contrast with the other possibility

\[
-\frac{A}{4} [W^A_{E_8} W^A_{E_8} \log S_0]_F,
\]

which would bring unwanted contributions to Kähler anomalies. This argument suggests, as explained in [14], that by analogy with the treatment of Kähler anomalies, one should complete (5.16) with

\[
\Delta_{sc} \mathcal{L}_{\text{nl}}^{E_8} = -\frac{A}{4} \left[ W^A_{E_8} W^A_{E_8} \mathcal{P}_C \log \frac{\hat{L}}{\mu^2} \right]_F,
\]

(5.23)
to take conformal anomalies into account. The parameter $\mu$ can be viewed either as a mass scale manifesting the breaking by the anomaly of conformal invariance, in which case it can be identified with the renormalization group running scale, or as a scale factor similar to $\lambda$ in eq. (5.21).

Again, by analogy with Kähler symmetry, the next step is to add to the effective lagrangian a local Green-Schwarz term able to cancel these anomalies. As in [14] and for reasons which will become clear in the next subsection, we will use

$$
\Delta_{\text{sc}} L_{\text{GS}} = \frac{A}{4} \left[ \hat{L} \log \frac{\hat{L}}{\mu^2} \right]_D.
$$

(5.24)

Therefore, to appropriately use the superconformal approach for the Wilson action we should add $L_{\text{GS}} + \Delta_{\text{sc}} L_{\text{GS}}$ to the tree-level effective lagrangian (5.4), providing an extra correction to the gauge coupling $g^{-2}$. Again $g^{-2}$ does not change (at one loop), as we will see, which is consistent with the full string calculation. Therefore we can say that at the scale $\mu$ the Wilson lagrangian is

$$
L_W = L_0 + L_{\text{GS}} + \Delta_{\text{sc}} L_{\text{GS}},
$$

(5.25)

which is the one corresponding to the full string calculation, whereas the effective lagrangian is

$$
L_{\Gamma} = L_W + L_{\text{nl}} + \Delta_{\text{sc}} L_{\text{nl}}.
$$

(5.26)

To summarize the results of this subsection, at one-loop the Wilson lagrangian is

$$
L_W = -\frac{1}{\sqrt{2}} \left[ \left( S_0 \nabla_0 \right)^{3/2} \hat{L}^{-1/2} e^{-K/2} \right]_D + \left[ S_0^3 w \right]_F + \frac{A}{4} \left[ \hat{L} \log \frac{e^{K/3} \hat{L}}{\mu^2} \right]_D,
$$

(5.27)

and the effective lagrangian reads

$$
L_{\Gamma} = L_W - \frac{A}{4} \left[ W_{Es}^A W_{Es}^A P_C \log \frac{e^{K/3} \hat{L}}{\mu^2} \right]_F.
$$

(5.28)

Notice that $L_W$ is neither conformal nor Kähler invariant as it should be. In particular the superpotential cannot be absorbed into the Kähler potential. The introduction of the scale $\mu$ is of course not compatible with superconformal symmetry. A constant like $\mu$ should have zero conformal weight to be a supermultiplet. But a change in the value of $\mu$ can be compensated by a variation of the quantity $z_0 \bar{z}_0$, which appears like a ‘reference scale’. The statement that physical quantities in $S_{\Gamma}$ are $\mu$–independent can then be translated into an analogous statement on the behaviour of physical quantities under a change of the compensator, which can be expressed in terms of superconformal invariant expressions, up to anomalies.
5.3 Renormalization Group and Invariant Scale

We turn now to a discussion of the renormalization-group equations that can be derived from the results of the previous subsection and compare with the standard expressions in supersymmetric field theories. For a super-Yang-Mills theory with gauge group $G$ and without matter, the exact renormalization-group equation is known [31]:

$$\frac{d}{d\mu} g^{-2} = \frac{3C(G)}{8\pi^2} - \frac{1}{1 - \frac{C(G)}{8\pi^2} g^2}. \tag{5.29}$$

The renormalization of the gauge coupling constant between the scales $\mu$ and $M$ is then given by:

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(M)} + \frac{3C(G)}{16\pi^2} \log \frac{\mu^2}{M^2} + \frac{C(G)}{8\pi^2} \log \frac{g^2(M)}{g^2(\mu)} + \Delta, \tag{5.30}$$

where $C(G)$ is the quadratic Casimir of the group $G$ and $\Delta$ represents the possible threshold corrections due to the contributions of heavy fields to the loop calculations. We only consider here models for which these threshold corrections vanish. We will identify $\mu$ with a physical scale defining for instance the normalization of some three-point amplitude, and $M$ with the ultraviolet cut-off of the effective theory procedure. In (5.30), we have used the index $\Gamma$ to indicate that $g_\Gamma$ is the physical gauge coupling constant which appears in gauge kinetic terms of the effective action $S_\Gamma$ at the scale $\mu$. It gets corrections to all loops in perturbation theory.

The gauge coupling $g_W$ in the Wilson action $S_W$ is the bare coupling at the scale $M$ and all the quantum corrections to it give $g_\Gamma$. In pure supersymmetric Yang–Mills theory [32], it has been found that $g_W$ does not get corrections beyond one loop, i.e. the renormalization group equation reads simply

$$\frac{1}{g^2_W(\mu)} = \frac{1}{g^2_W(M)} + \frac{3C(G)}{16\pi^2} \log \frac{\mu^2}{M^2}, \tag{5.31}$$

unlike $g_\Gamma$ [see (5.30)]. At a given scale $\mu$, the relation between $g_W$ and $g_\Gamma$ is

$$\frac{1}{g^2_W(\mu)} = \frac{1}{g^2_\Gamma(\mu)} - \frac{C(G)}{8\pi^2} \log \frac{1}{g^2_\Gamma(\mu)}, \tag{5.32}$$

making equations (5.31) and (5.30) equivalent to all loops [32].

We will now discuss these issues in the string theory case. The natural cut-off scale $M^2$ is in our case $z_0 \bar{z}_0$. From equation (5.2) we can see that the Wilson gauge coupling
constant corresponding to lagrangian (5.27) is

\[ \frac{1}{g_W^2(z_0 \bar{z}_0)} = U - \frac{A}{3} \log U + \frac{A}{2} \log \frac{z_0 \bar{z}_0}{\mu^2} + \frac{A}{2} \left(1 - \frac{1}{3} \log 2 \right), \]  

(5.33)

where \( U \) is given by (5.3), and the running term on the right-hand side of eq. (5.34) is provided by the superconformal anomaly cancelling term (5.24). The last constant can be absorbed in a redefinition of the parameter \( \mu \). If we rescale \( \mu \rightarrow e^a \mu \) in lagrangian (5.27), the anomaly term generates a contribution

\[ -\frac{1}{2} a A \text{[\hat{L}]_D} = a A \text{[\Omega]_D} + \text{total derivative}, \]

which is proportional to a pure super-Yang-Mills lagrangian. In particular, it contains a term

\[ \frac{1}{4} a A F^A_{\mu \nu} F^{A, \mu \nu}, \]

equivalent to a change \( g_W^{-2} \rightarrow g_W^{-2} - a A \) of the gauge coupling constant. Choosing \( a = \frac{1}{2} (1 - \frac{1}{3} \log 2) \) allows to replace (5.33) by

\[ \frac{1}{g_W^2(z_0 \bar{z}_0)} = U - \frac{A}{3} \log U + \frac{A}{2} \log \frac{z_0 \bar{z}_0}{\mu^2}, \quad A = \frac{3C(E_8)}{8\pi^2}, \]  

(5.34)

which is the expression we will use.

Formally, this equation is equivalent to a renormalization group equation (RGE) for the running of the Wilson coupling constant from \( z_0 \bar{z}_0 \) to \( \mu^2 \). A comparison with the field theory result (5.31) suggests the identification

\[ \frac{1}{g_W^2(\mu)} = U - \frac{A}{3} \log U, \]  

(5.35)

which is not a physical quantity but only a bare parameter of \( \mathcal{L}_W \).

Now, since (5.28) is the effective lagrangian, the coefficient of gauge kinetic terms in \( \mathcal{L}_\Gamma \) will provide the physical effective coupling constant at the scale parameter \( \mu \), \( g_\Gamma(\mu) \). A straightforward calculation shows that

\[ \frac{1}{g_\Gamma^2(\mu)} = U. \]

(5.36)

This important result indicates that the expectation value of the quantity \( U \), which is a function of scalar fields of well-defined string origin like \( C \) or \( T \), is the physical, loop-corrected gauge coupling constant for the \( E_8 \) sector of the gauge group.

\[ ^{11} \text{The coupling in (5.34) from the Wilson action } \frac{1}{g_W^2(z_0 \bar{z}_0)} \text{ coincides with the coupling } 2(s + \bar{s}) \text{ one obtains in the dual formalism.} \]

39
Clearly, the eqs. (5.35) and (5.36) coincide with relation (5.32),

$$\frac{1}{g_W^2(\mu)} = \frac{1}{g_1^2(\mu)} - \frac{A}{3} \log \frac{1}{g_1^2(\mu)},$$

(5.37)

which gives the all-order RGE for $g^{-2}$ and has been derived in the context of supersymmetric field theories [32].

This justifies the result of [14] where all-loop RGE’s were obtained from a one-loop calculation and using the GS terms above together with duality with the chiral multiplet formalism. As in the super-Yang-Mills case, the Wilson gauge coupling satisfies the one-loop RGE (5.34).

Another way to derive the Wilson lagrangian (5.27) is to start with the tree-level gauge coupling $U$, given in eq. (5.3) and obtained from lagrangian (5.4) using eq. (5.2), and then impose the all-order relation (5.34). By integrating (5.2), one then recovers the loop-corrected lagrangian (5.27).

We wish to stress that if $g_W^{-2}$ does not get corrected to higher loops, as it happens in pure super-Yang-Mills, the Wilson lagrangian above would be valid to all loops in string theory. There is in fact a non-renormalization theorem limiting the moduli dependence of the gauge coupling beyond one-loop [4] in the dual formulation in terms of chiral fields. This can give concrete constraints to the higher loop corrections to the Wilson action.

The effective lagrangian (5.28) provides the $E_8$ renormalization from the cut-off scale $z_0$ to the low scale $\mu$. Notice that in the range $z_0 \rightarrow \mu^2$ the renormalization of the gauge couplings is triggered by stringy effects encoded in (5.27) and (5.28). This running is generated by the (stringy) anomaly cancellation mechanism of (5.28). Since this mechanism is universal, it can only take care of a single gauge coupling evolution. In the case of the $Z_3$ and $Z_7$ orbifolds we are considering, it describes the evolution of the hidden $E_8$, which will be useful for the gaugino condensate mechanism of supersymmetry breaking.

The equation (5.36) indicates that the vacuum expectation value of the field-dependent quantity $U$ is the physical gauge coupling constant in the effective action computed with a string infra-red cut-off $\mu$. From its definition (5.3), $U$ depends on $C, K$ and on the compensator $z_0$. In the Poincaré theory, the compensator itself is a function of $C, K$ and $\kappa^2$, so that the physical gauge coupling constant in the effective lagrangian is a function of the expectation values of $C$ and $K$. In the dual theory, with the chiral multiplet $S$ instead of $L$, the general (all-order) result (5.24) indicates that the expectation value of $\text{Re}s$ is the bare coupling constant, a non-physical quantity. The chiral multiplet $S$ appears to be an artifact of the effective field theory, while the linear multiplet $L$ is directly related to the physical fields of the string theory. Even if the two theories are formally equivalent by duality, loop corrections introduce clear conceptual differences in their interpretation.
The Einstein term in the effective lagrangian can be computed using eq. (3.20). The non-local parts do not contribute and one obtains
\[ -\frac{1}{2} [U + \frac{A}{6}] CeR. \]

Fixing conformal symmetry by the requirement of canonical Einstein terms leads to the relation
\[ U = \frac{1}{\kappa^2 C} \frac{C(E_8)}{16\pi^2} = \frac{1}{g^2_\mu(\mu)}, \tag{5.38} \]

which indicates that the physical effective gauge coupling constant in the effective, loop-corrected lagrangian is controlled by the expectation value of the scalar field \( C \). Loop corrections only introduce a constant shift [compare with (5.11)].

By construction [see eq. (5.34)], \( U \) satisfies the RGE
\[ \frac{d}{d\mu} U = \frac{A}{1 - \frac{A}{3U}}, \]

which is identical to eq. (5.29). The reaction of \( U \) to a change in the arbitrary value of the string cut-off \( \mu \) is in agreement with all-order RGE, which implies that physical quantities (scattering probabilities) computed in the effective lagrangian are independent of \( \mu \).

It is now easy to write down the renormalization-group invariant scale \( \Lambda_{E_8}^3 \) as a function of the fields of the theory. It can be constructed either using the Wilson gauge coupling (5.34) or the effective gauge coupling (5.36), as
\[ \Lambda_{E_8}^3 = (z_0^{\Sigma_0})^{3/2} e^{-\frac{1}{\kappa^2 C} \frac{C(E_8)}{16\pi^2}} = \mu^3 U e^{-\frac{A}{3U}}. \tag{5.39} \]

The scale \( \Lambda_{E_8} \) is independent of the choice of \( \mu \): using the exact RGE (5.29), one easily checks that \( \mu \frac{d}{d\mu} \Lambda_{E_8}^3 = 0 \). The invariant parameter \( \Lambda_{E_8}^3 \) characterizes the strength of \( E_8 \) gauge interactions. It is a physical quantity, independent of the choice of cut-off \( \mu \). Moreover, the real, field-dependent quantity \( \Lambda_{E_8}^3 / \mu^3 \) is the lowest component of a real vector supermultiplet. This is due to the fact that \( U \), as defined by eq. (5.3), is itself the lowest component of the real, Kähler and conformal invariant, vector supermultiplet
\[ 2 \left( \frac{S_0 \Sigma_0}{2L e^{K/3}} \right)^{3/2}, \quad K = K(\Sigma, \Sigma e^V). \]

With the compensator fixing condition (5.38), the renormalization-group invariant scale becomes
\[ \Lambda_{E_8}^3 = \mu^3 e^{\frac{1}{2} \left( \frac{1}{\kappa^2 C} - \frac{C(E_8)}{16\pi^2} \right)} \exp \left( -\frac{8\pi^2}{C(E_8)} \frac{1}{\kappa^2 C} \right). \tag{5.40} \]
a formula of direct interest in the discussion of gaugino condensation.

At the string-tree level, one can define a compactification scale by the expression

\[ M^2 = (2C)e^{K/3}, \]  

(5.41)

which is the translation of the more usual relation \( M^2 = \frac{1}{\kappa^2}[(S + \overline{S})(T + \overline{T})]^{-1} = \frac{1}{\kappa^2}(S + \overline{S})^{-1}e^{K/3} \), which holds in the dual formalism [13].

It is consistent to use expression (5.41) inside the logarithmic term of (5.34) since we are performing a one-loop calculation. Using (5.41) in (5.34), we obtain

\[ \frac{1}{g_W(z_0 \overline{z}_0)} = \frac{1}{g_2^2(M)} \]  

(5.42)

Equation (5.42) shows that while the effective gauge couplings unify (modulo threshold corrections) at the compactification scale, the Wilson gauge couplings unify at the Planck scale (i.e. \( z_0 \overline{z}_0 \)) with the same value. Finally eq. (5.42) shows that the effect of gauge coupling running in (5.34) can be absorbed in the very definition of \( M \).

6 String Component Actions and Gaugino Condensation

In the previous section, we have constructed the loop-corrected effective actions for particular string compactifications in the supermultiplet language. In order to obtain further information from these actions, we will present here their component expressions using the general formalism developed in sections 3 and 4. As mentioned previously, we will concentrate on the couplings which are relevant to the study of supersymmetry breaking and then restrict to the scalar kinetic terms and potential, including gaugino bilinear contributions. We will first present these couplings for the loop-corrected Wilson action which is neither conformal nor Kähler invariant. The effective potential will be obtained by summing the contributions from the Wilson lagrangian and from the non-local actions (5.16) and (5.23), which provide local gaugino-dependent contributions to the effective potential, restoring the invariance under both Kähler and conformal transformations, before compensator fixing. In other words, the effective potential is extracted from the effective lagrangian (5.28).

We would like to emphasize that there is a technical difficulty with the linear multiplet formulation as treated in the superconformal approach. Contrary to the purely chiral multiplet action (3.2) in which the auxiliary field equations can be solved and the compensator can be fixed to provide a general form of the scalar potential as a function of \( \mathcal{G} \) and \( f_{AB} \), this cannot be done for the linear multiplet action. The reason is that since the linear multiplet has conformal weight two, the function \( \Phi \) itself depends on the compensator. The solution to equation (3.21), which fixes the compensator, is then
only implicit. Also, the equations of motion for the auxiliary fields have to be solved for each $\Phi$ to provide the scalar potential. Furthermore, the Wilson action is not conformal invariant and the component expressions (4.3) and (4.4) cannot be used directly for the Green-Schwarz terms which are anomalous. These Green-Schwarz contributions can however easily be computed using the truncated supermultiplets (3.29). The complete expressions are given in the appendix, together with the truncated component expansions of the non-local terms appearing in the loop-corrected effective action (5.28).

It should be stressed that our component expressions are based upon lagrangians (5.27) and (5.28), which include all-order corrections in $\hat{L}$ but not in chiral matter.

### 6.1 String Effective Actions in Components

It is straightforward to derive the tree-level string action in components knowing the general results from the previous sections. The tree-level lagrangian (5.4) is a particular case of the class (4.6) for which

$$F(u) = -\frac{1}{\sqrt{2}} u^{-3/2}.$$  

We can easily solve the compensator fixing condition (3.21) giving:

$$(z_0 \bar{z}_0) = (2 \kappa^{-4} C e^K)^{1/3}. \quad (6.1)$$

With eq. (4.11), the scalar potential including gaugino bilinears is:

$$\kappa^2 V = 2 C e^K \left( K^{-1} |w_z + w K_z|^2 - 3 |w|^2 \right) + \left| (2 C e^K)^{1/2} w + \frac{1}{2C} (\bar{\lambda}_R \lambda_L) \right|^2. \quad (6.2)$$

This exhibits the known property that the scalar potential is positive definite for no-scale Kähler potentials for which the first term cancels. As discussed in [15], gaugino condensation can break supersymmetry with vanishing cosmological constant provided there is a non-zero vacuum expectation value for the superpotential. Notice that the potential (6.2) is equivalent to the more familiar expression in terms of the dual $S$ field as can be seen by using the duality transformation which in this case amounts to set $(\kappa^2 C)^{-1} = 2(s + \bar{s})$ [see eq. (5.11)]. It can also be verified that under this substitution, the scalar kinetic terms obtained using eq. (4.3) take the same form as in [13].

We will next consider the loop-corrected action (5.27), which we write as:

$$\mathcal{L}_W = \mathcal{L}_0 + \frac{A}{4} \left[ \hat{L} \log e^{K/3} \hat{L} \right]_D. \quad (6.3)$$

where $\mathcal{L}_0$ is the tree-level lagrangian (5.4) which is conformal and Kähler invariant. The loop term can be written as

$$\frac{A}{4} \left[ \hat{L} \log e^{K/3} \hat{L} \right]_D = \frac{A}{12} \left[ S_0 \overline{S}_0 \frac{\hat{L}}{S_0 S_0} K \right]_D + \frac{A}{4} \left[ \hat{L} \log \hat{L} \right]_D$$

$$= \frac{A}{4} \left[ S_0 \overline{S}_0 \frac{\hat{L}}{S_0 S_0} \log \left( e^{K/3} \frac{\hat{L}}{S_0 S_0} \right) \right]_D + \frac{A}{4} \left[ \hat{L} \log \left( \frac{S_0 \overline{S}_0}{\mu^2} \right) \right]_D. \quad (6.4)$$
In both expressions, the first term is conformal invariant and the second is anomalous (only the first term in the second expression is both Kähler and conformal invariant and has the form (5.13) with $F(u) = \frac{A}{4} \log u$). The component expressions for both anomaly terms are given in the appendix. The auxiliary field part of the lagrangian can always be written in the form (4.11), the solutions for $f, f_0$ and $V_{aux}$ are as in (4.9) and (4.10), but with different expressions for the coefficients in (4.8).

For definiteness, let us use the full action (6.3) with the one-loop term written as in the second expression (6.4). We see that the conformal and Kähler invariant part has the form of (5.15) with

$$F(u) = -\frac{1}{\sqrt{2}} u^{-3/2} + \frac{A}{4} \log u,$$

which we can use in (4.8). The contribution from the anomaly term has the effect of changing some coefficients in (4.8):

$$D \rightarrow D - \frac{A}{4} z_0^{-1}(\lambda_R \lambda_L),$$

$$E \rightarrow E - \frac{A}{4} z_0^{-1}(\lambda_R \lambda_L),$$

[see eq. (A.2) in the appendix]. The Wilson scalar potential with the loop corrections and before fixing the compensator reads then

$$V_W = (z_0 \overline{z}_0)^3 \left( \frac{A}{6} AC + UC \right)^{-1} K_{\pi}^{-1} \left| w_z + wK_z - \frac{4}{12} z_0^{-3} K_z(\lambda_R \lambda_L) \right|^2 - 2(U C)^{-1}(z_0 \overline{z}_0)^3 \left| w - \frac{1}{4} U z_0^{-3}(\lambda_R \lambda_L) \right|^2 - \frac{3}{8C} (\frac{1}{3} A - U) \left| (\lambda_R \lambda_L) \right|^2,$$

where $U = 2 \left( \frac{e^K}{2C e^K \kappa^2} \right)^{3/2}$ is the quantity already introduced in (5.3) whose expectation value is the physical coupling $g_r^{-2}$.

The requirement that the Einstein term, which has already been evaluated in the previous subsection, is canonically normalized gives the compensator fixing condition (5.38), $C(U + \frac{A}{6}) = \kappa^{-2}$, or

$$z_0 \overline{z}_0 = (2Ce^K)^{1/3} \left( \frac{1}{\kappa^2} - \frac{A}{6} C \right)^{2/3}.$$

Therefore, the scalar potential after fixing the compensator takes the form:

$$\kappa^2 V_W = 2Ce^K (1 - \frac{\kappa^2}{6} AC)^2 K_{\pi}^{-1} \cdot \left| w_z + wK_z - \frac{4}{12} (2Ce^K)^{-1/2} (\frac{1}{\kappa^2} - \frac{1}{6} AC)^{-1} (\lambda_R \lambda_L) K_z \right|^2 - 4Ce^K (1 - \frac{\kappa^2}{6} AC) \left| w - \frac{1}{2} (2C)^{-3/2} e^{-K/2} (\lambda_R \lambda_L) \right|^2 + \frac{3}{8C^2} (1 - \frac{A}{2} \kappa^2 C) \left| (\lambda_R \lambda_L) \right|^2.$$
Notice that by lack of Kähler invariance, the superpotential and Kähler potential cannot be combined into a single Kähler-invariant function. It is also interesting to observe that there is no \( \mu \) dependence in \( V_W \), and the same will hold for the scalar kinetic terms. Actually, the only \( \mu \) dependence of the Wilson action is in the gauge kinetic terms, as we saw in the previous chapter.

The scalar kinetic terms can be obtained by shifting \( D \) as indicated above. The final form after fixing the compensator is

\[
e^{-1} \mathcal{L}_{\text{kin}} = -\frac{1}{\kappa^2} K_z \overline{z}(\partial_\mu z)(\partial^\mu \overline{z}) - \frac{1 - \frac{\kappa^2}{6} AC}{4 \kappa^2 c^2} \left( 1 - \frac{\kappa^2}{6} AC \right)^{-1} (\partial_\mu C)(\partial^\mu C)
\]

\[+ \frac{1}{4 \kappa^2 c^2} \left( 1 - \frac{2 \kappa^2}{3} AC \right) v_\mu v^{\mu} - \frac{i}{12} A v^{\mu} (K_z \partial_\mu z - \text{h.c.}). \tag{6.10}\]

The kinetic terms for the chiral scalar fields are still given by the tree-level Kähler potential. The positivity of these kinetic terms for \( C \) sets the allowed range for that field. The novel feature is that we obtain an ‘off-diagonal’ term mixing the chiral fields and the antisymmetric tensor represented by \( v_\mu \). Similar mixing terms were obtained in \([33, 12]\). There are however no mixed terms with \( C \).

Finally, to obtain the effective potential, we have to consider the local contributions of the non-local lagrangian. The non-local terms only contribute by gaugino-dependent terms given by

\[- \frac{A}{4C} \left| (\overline{\lambda}_R \lambda_L) \right|^2 + \frac{A}{12} \left[ K_z f(\overline{\lambda}_R \lambda_L) + \text{h.c.} \right] \tag{6.11}\]

[see the appendix, eq. (A.3)], and so they do not contribute to the kinetic lagrangian (6.10). We can see that this has the net effect of shifting

\[F \longrightarrow F + \frac{A}{12} K_z f(\overline{\lambda}_R \lambda_L),\]

\[\mathcal{L}_{4\lambda} \longrightarrow \mathcal{L}_{4\lambda} - \frac{A}{4C} \left| (\overline{\lambda}_R \lambda_L) \right|^2, \tag{6.12}\]

in the expressions (4.8). The effective potential before compensator fixing becomes

\[V_{\text{eff}} = (z_0 \overline{z}_0)^3 \left( 1 - \frac{\kappa^2}{6} AC + UC \right)^{-1} K_z^{-1} |w_z + w K_z|^2 - 2(z_0 \overline{z}_0)^3 |w|^2 (CU)^{-1}
\]

\[+ \frac{1}{2C} \left[ (\overline{\lambda}_R \lambda_L) z_0^3 w + \text{h.c.} \right] + \frac{1}{4C} \left( \frac{A}{2} + U \right) \left| (\overline{\lambda}_R \lambda_L) \right|^2. \tag{6.13}\]

And fixing the compensator using (6.8) produces the effective potential

\[\kappa^2 V_{\text{eff}} = 2 C e^K \left( 1 - \frac{\kappa^2}{6} AC \right)^2 \left( K_z^{-1} |w_z + K_z w|^2 - 3 |w|^2 \right)
\]

\[+ \left( 1 - \frac{\kappa^2}{6} AC \right) \left( 1 - \frac{1}{2} \kappa^2 AC \right) \left| (2 C e^K)^{1/2} w + \frac{1}{2C} (1 - \frac{1}{2} \kappa^2 AC)^{-1} (\overline{\lambda}_R \lambda_L) \right|^2
\]

\[- \frac{A^2 \kappa^4}{24} \left( 1 - \frac{1}{2} \kappa^2 AC \right)^{-1} \left| (\overline{\lambda}_R \lambda_L) \right|^2, \tag{6.14}\]

\[\text{Notice that the kinetic lagrangian, eq. (6.10), unlike the Wilson potential, eq. (6.9), is invariant under Kähler transformations, thanks to conditions (6.17).}\]
in the loop-corrected Poincaré theory.

6.2 Gaugino condensation

In this section we will discuss the issue of supersymmetry breaking by gaugino condensation in the context of string effective actions with a linear multiplet. This phenomenon is expected to happen \[34\] when the gauge coupling of the hidden gauge group becomes strong \[35,15\]. Being a non-perturbative effect, its dynamics is not completely understood.

All previous discussions on the subject have been done using the dual formalism with only chiral multiplets. There are at least three different methods which have been used to incorporate the effects of the gaugino condensate in the effective theory. One approach is the simple substitution of the condensate as an expectation value of gaugino bilinear operators in the component action \[36\], \(\lambda_L\lambda_R \sim \Lambda^3\), where \(\Lambda\) is the scale of condensation, which in string theory is a field-dependent quantity. Another way is replacing the condensate in the superfield action, generating an effective superpotential for the dilaton field determined by symmetry arguments \[15\], \(w(S) \sim \exp(-6S\beta)\), where \(\beta\) is the coefficient of the beta function of the hidden gauge group.

A third approach is the formulation of an effective supersymmetric theory, below the scale of condensation, where the gauge invariant condensate \(W^A W^A\) is described in terms of a new dynamical superfield \(Z\) determined upon minimization of the scalar potential \[37\] (see also \[38\]).

Let us briefly discuss these approaches in the linear multiplet formulation. We can also carry out the naive substitution of the condensate in the component action. Notice that we have the same situation as in the global supersymmetry case, discussed in section 2, in which a non-vanishing gaugino condensate amounts to a non-supersymmetric shift in the ‘\(\theta\theta\) component’ of the Chern-Simons multiplet.

In the absence of a superpotential, the scalar potential \(6.14\) is

\[
\kappa^2 V_{eff} = (2C)^{-2}(1 + \frac{1}{3}\kappa^2 AC) \mid \bar{\lambda}_L\lambda_R \mid^2, \tag{6.15}
\]

where the factor in front of \(\mid \bar{\lambda}_L\lambda_R \mid^2\) is positive definite due to the positive kinetic energy conditions. We now replace gaugino bilinears by expectation values,

\[
\bar{\lambda}_L\lambda_R \sim \Lambda_{Es}^3, \tag{6.16}
\]

where \(\Lambda_{Es}\) is the renormalization group and Kähler invariant scale defined in \(5.39\). After fixing the compensator, according to eq. \(5.40\), \(\Lambda_{Es}^2 \sim \exp\{-3/A\kappa^2 C\}\), and the potential for the \(C\) field has a ‘runaway’ behaviour towards the supersymmetric (singular) minimum \(C = 0\), which also corresponds to vanishing gauge couplings. This
is the situation already encountered in the dual formalism for the case of a single condensate. The approach of including the condensate in the superpotential is not feasible in the linear multiplet formalism because the superfield $L$ cannot appear in the superpotential. Even more, since in the dual formulation the existence of a superpotential for the $S$ field breaks the Peccei-Quinn symmetry $\text{Im } S \rightarrow \text{Im } S + \text{constant}$, the connection to the original formulation with a linear multiplet remains unclear. In fact, the very existence of that symmetry is what allows to perform the inverse duality transformation from the $S$ to the $L$ field formalism. This raises the interesting question of understanding which is the appropriate formalism to be used in this case.

As for the approach proposed in ref. [37], we only wish to mention here the formal similarity between the construction of the effective action below the condensation scale of a super-Yang-Mills theory and the rôle played by the Green-Schwarz terms in string effective actions. The Green-Schwarz counterterms are adjusted to cancel a perturbative anomaly since strings dictate the absence of any Kähler anomaly, while the effective theory constructed in [37] has an anomaly behaviour dictated by the properties of dynamics at energies higher than the condensation scale. More details on these issues will appear elsewhere.

7 Conclusions

In this paper, we have addressed a number of issues concerning the coupling of one linear supermultiplet to chiral multiplets in global and local supersymmetry. The main motivation was the fact that this is the spectrum in four-dimensional strings. We have applied the results to obtain general information about the string effective actions and, also, to explicitly compute the loop-corrected effective actions in simple orbifold compactifications.

Let us briefly summarize the main results of this paper. First, we have used the superconformal approach to compute the component supersymmetric actions of one linear multiplet coupled to chiral multiplets. It generalizes the case of only chiral multiplets studied in ref. [19] and reduces to it as a particular case when the linear multiplet is decoupled. Unlike this case, it is not possible to write a closed expression

---

13 Though the present calculation is restricted to a very particular class of orbifold compactifications with a simple gauge group in the hidden sector, one could expect a similar pattern for the potential to hold in more complicated cases, where the hidden sector contains several gauge groups. In those cases, for vanishing superpotential, a stable supersymmetric minimum for $C \neq 0$ could be generated.

14 Notice that in the global case, discussed in chapter 2, a gaugino bilinear can be seen as a linear contribution to the superpotential [eq. (2.40)]. This is not the case in local supersymmetry. As discussed in section 3 the only superpotential for $S$ allowed by the duality transformation is an overall $\exp \{-aS\}$ [see eq. (3.28)], which is not of the form obtained in studies of supersymmetry breaking by gaugino condensation in the $S$ field approach.
for the lagrangian because the compensator fixing equation cannot be solved in general. Though, we have been able to find very general expressions before fixing the compensator and, for particular cases (namely for string effective actions), we have solved the compensator fixing equation and found explicit expressions for the interesting pieces of the lagrangian.

In the linear multiplet formalism a general loop-corrected Wilson lagrangian takes the form

$$
L_W = \left[ \Gamma(\hat{L}, S_0, \overline{S_0}, \Sigma, \overline{\Sigma} e^V) \right]_D + \left[ S_0^3 w \right]_F ,
$$

(7.1)

where $\Gamma$ is an arbitrary real function, its functional form includes functions of the form $\Phi(\hat{L}/S_0, \overline{S_0}, \Sigma, \overline{\Sigma} e^V)$ discussed in the text, corrected by Green-Schwarz counterterms as in section 5. The gauge coupling is given by

$$
\frac{1}{g^2_W} = 2 \left[ \frac{\partial \Gamma}{\partial \hat{L}} \right]_{\text{lowest component}}.
$$

(7.2)

From (7.2), one can see the interesting property that it is sufficient to obtain the loop corrections to the gauge coupling to determine in large parts, through integration of (7.2), the loop corrections to the Wilson action. In this way, any non-renormalization theorems on the gauge coupling \[4\] would translate into non-renormalization theorems on the whole Wilson action.

Moreover, we have seen that the linear multiplet is the supersymmetric extension of the string loop-counting parameter. It follows rules different from chiral or vector multiplets and is then naturally singled out by supersymmetry. Corrections due to string loops will be reflected in the functional dependence on $\hat{L}$ of the effective theory.

Concerning the superpotential in (7.1), it is straightforward to see in this approach that it cannot be renormalized to any loop in perturbation theory, since it does not depend on $L$ (nor $S_0$), which is the string loop-counting parameter. This argument can be extended to any non-perturbative effect as long as the formalism holds. The Kähler potential for chiral matter cannot either get any loop corrections since it is also a function of only the chiral multiplets. There is however no restriction on the couplings of chiral matter to $\hat{L}$ described in the real superfield $\Gamma$, which gets loop corrections. We saw for instance in the case studied in section 6 that the kinetic terms acquire a $v^\mu \partial_\mu z$ mixing which was not present at tree-level. Nevertheless, the loop-corrected quantities are simple functions of the tree-level Kähler potential $K$ also. Then we can say that the tree-level techniques used to compute $K$, such as the use of special geometry in $(2,2)$ models, are still useful for obtaining the effective lagrangians beyond tree-level.

Even though the theory is dual to one expressed only in terms of chiral multiplets, for which the general form of the effective action is known, the linear multiplet formalism is more convenient to discuss the loop-corrected string effective action, which may not have a closed form when expressed in terms of only chiral multiplets. This point can be illustrated by performing the duality transformation, as explained in section
3, on the Wilson lagrangian (3.27). The first step of the duality transformation is to define an equivalent lagrangian of the form

$$\mathcal{L} = \mathcal{L}_W(\hat{L} \to Q) - [(S + \overline{S})(Q + 2\Omega)]_D,$$

(7.3)

where $Q$ is an arbitrary real vector superfield. By integration of $S$ we recover $\mathcal{L}_W(\hat{L})$. The equation of motion for $Q$ is

$$2(S + \overline{S}) = \tilde{U} - \frac{A}{3} \log \tilde{U} + \frac{A}{2} \log \left(\frac{S_0 \overline{S}_0}{\mu^2}\right) + \frac{A}{2} (1 - \frac{1}{3} \log 2),$$

(7.4)

where now

$$\tilde{U} = 2 \left(\frac{S_0 \overline{S}_0}{2Q e^{K/3}}\right)^{3/2}.$$ 

This superfield equation of motion is formally identical to eq. (5.33), which defines the Wilson gauge coupling constant. It is then clear that (7.4) identifies the lowest component $s$ of $S$ as

$$2(s + \overline{s}) = \frac{1}{g_W(z_0 \overline{z}_0)},$$

a non-physical (bare) parameter, while, according to (5.30),

$$U = \frac{1}{g^2(R)^3},$$

a physical quantity. To complete the transformation and construct the dual lagrangian one has to solve in (7.4) $U$ as a function of the chiral superfield $S$, which cannot be done analytically. This can only be done in a perturbative series corresponding to the expansion of the physical coupling $g_R$ as a function of the bare parameter $g_W$. This argument shows that the linear multiplet formalism allows to write all-order closed expressions in terms of physically relevant fields and parameters, while the dual $S$-field formalism uses unphysical fields leading necessarily to perturbative results, with the additional difficulty of distorting the symmetry structure of the theory.

Finally, we have discussed the issue of supersymmetry breaking by gaugino condensation. For the case of global supersymmetry, we found out that a non-vanishing vacuum expectation value for the condensate breaks supersymmetry explicitly in the linear multiplet formalism, whereas the breaking seems at first sight spontaneous in the dual formalism. (However the two versions are equivalent.) For the string case, we have pointed out the apparent inconsistency of the breaking of the Peccei-Quinn symmetry (necessary to generate the superpotential for the $S$ field in the chiral approach) and the duality transformation relating it to the linear multiplet approach. If the formalism with the linear multiplet extends to the strong coupling regime, it would predict the existence of an exactly massless particle corresponding to the antisymmetric tensor $b_{\mu \nu}$ which in the dual theory corresponds to a massless ‘axion’ field.
There exists the logical alternative that in the strong coupling regime, it is only the $S$ field formulation which is valid and the axion field gets a mass after supersymmetry and the Peccei-Quinn symmetry are broken. This is what is implicitly assumed in the literature, but at present there is no concrete evidence to support this assumption and we have to take seriously the possibility of a massless axion field (and probably also other moduli fields) as a consequence of an exact Peccei-Quinn symmetry in string theory. We will present further developments on these topics in a future publication.

**Acknowledgements**

We would like to acknowledge useful conversations with C. P. Burgess, S. Ferrara and L. E. Ibáñez. One of us (MQ) wishes to thank the Institut de Physique, Université de Neuchâtel, for its warm hospitality while part of this work was performed.
Appendix

We give in this appendix some component expressions useful in the text.

1) The abelian Chern-Simons global superfield:

The component expansion of the Chern-Simons superfield (2.17), for a single, abelian vector superfield \( V \) and in the Wess-Zumino gauge is:

\[
\Omega(V) = -\frac{i}{4} \left[ -\theta \sigma^\mu \lambda - \overline{\theta} \sigma^\mu \lambda - 2(\theta \sigma^\mu \overline{\theta})(\lambda \sigma^\mu \overline{\lambda}) + 2i \theta \sigma^\mu \theta \lambda - 2i \overline{\theta} \sigma^\mu \overline{\theta} \lambda D - \theta \sigma^\mu \sigma^\nu \lambda F_{\mu \nu} + \overline{\theta} \sigma^\mu \sigma^\nu \overline{\lambda} F_{\mu \nu} + \theta \theta \theta \sigma^\mu \sigma^\nu \lambda \partial_\mu \lambda - \frac{1}{2} F_{\mu \nu} F_{\mu \nu} \right]
\]

(A.1)

with \( F_{\mu \nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \). The last two lines contain the gauge-variant terms, which include the Chern-Simons abelian form in the \( \theta \sigma^\mu \overline{\theta} \) component. Notice that this expression does not contain the CP-odd expression \( \epsilon_{\mu \nu \rho \sigma} F_{\mu \nu} F_{\rho \sigma} \). Also, the lowest component without \( \theta, \overline{\theta}, \theta \overline{\theta} \) vanishes, and the highest \( \theta \theta \theta \theta \) component is the pure super-Yang-Mills lagrangian. This last two observations apply to the non-abelian Chern-Simons superfield as well.

2) Supergravity expressions for anomaly terms:

The Green-Schwarz terms carrying the conformal anomaly contain \( D \)-densities of functions of \( \hat{L} = L - 2\Omega(V) \) and \( S_0 \) which do not depend on the combination \( \frac{L}{S_0 S_0} \). Their component expansions using truncated supermultiplets (3.29) read

\[
[\hat{L} \log \hat{L}]_D = \frac{1}{2} C^{-1}(\partial_\mu C)(\partial^\mu C) - \frac{1}{2} C^{-1} v_\mu v^\mu + \frac{1}{2} C^{-1}(\lambda_R \lambda_L)(\lambda_L \lambda_R) - \frac{1}{3} C R,
\]

(A.2)

\[
[\hat{L} \log (S_0 \overline{S}_0)]_D = -v_\mu A^\mu - z_0^{-1} f_0(\lambda_R \lambda_L) - \overline{z}_0^{-1} f_0(\lambda_L \lambda_R),
\]

up to total derivatives. These expressions appear in the Wilson lagrangian (5.27), (6.4) and are used in the derivation of the scalar potentials (6.9) and (6.14).

3) Non-local terms in the effective lagrangian (5.28):

The action of the chiral projector \( P_C \) on a truncated vector multiplet \( H \) with components \( (c, 0, h, k, b_\mu, 0, d) \) is to form a chiral truncated multiplet \( P_C H = (z, 0, f) \) with

\[
z = \frac{1}{2} c - \Box^{-1}(d + i \frac{1}{2} \partial^\mu b_\mu), \quad f = \frac{1}{2} (h + ik).
\]
The component expansion of the non-local expression appearing in the loop-corrected effective lagrangian (5.28) is then:

\[
\left[ \frac{1}{4} W^A W^A P C \log \left( \hat{L} e^{K/3} \right) \right]_F = \frac{1}{4} \left[ \frac{1}{3} K f (\bar{\lambda}_R \lambda_L) + \frac{1}{3} K \bar{f} (\bar{\lambda}_L \lambda_R) - C^{-1} (\bar{\lambda}_R \lambda_L) (\bar{\lambda}_L \lambda_R) \right],
\]

(A.3)

using the truncated multiplets (B.24). Notice that (A.3) is local. Also a local gauge kinetic term of the form

\[
- \frac{1}{4} \left[ \frac{1}{6} K + \frac{1}{2} \log C \right] F^A_{\mu \nu} F^{A, \mu \nu}
\]

(A.4)

is present.
References

1. J. Minahan, Nucl. Phys. B298 (1988) 36; W. Lerche, Nucl. Phys. B308 (1988) 102
2. V. S. Kaplunovsky, Nucl. Phys. 307 (1988) 145
3. L. J. Dixon, V. S. Kaplunovsky and J. Louis, Nucl. Phys. B355 (1991) 649
4. I. Antoniadis, K. S. Narain and T. R. Taylor, Phys. Lett. B267 (1991) 37; I. Antoniadis, E. Gava and K. S. Narain, Phys. Lett. B283 (1992) 209; Nucl. Phys. B383 (1992) 93
5. S. Ferrara, J. Wess and B. Zumino, Phys. Lett. B51 (1974) 239; S. Ferrara and M. Villasante, Phys. Lett. B186 (1986) 85; S. Cecotti, S. Ferrara and L. Girardello, Phys. Lett. B198 (1987) 336; B. Ovrut, Phys. Lett. B205 (1988) 455
6. W. Siegel, Phys. Lett. B85 (1979) 333
7. P. Binétruy, G. Girardi, R. Grimm and M. Müller, Phys. Lett. B195 (1987) 389; G. Girardi and R. Grimm, Nucl. Phys. B292 (1987) 181
8. B. A. Ovrut and C. Schwiebert, Nucl. Phys. B321 (1989) 163; B. A. Ovrut and S. K. Rama, Nucl. Phys. B333 (1990) 380; Phys. Lett. B254 (1991) 138
9. J.-P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B372 (1992) 145
10. S. Cecotti, S. Ferrara and M. Villasante, Int. J. Mod. Phys. A2 (1987) 1839
11. P. Binétruy, G. Girardi and R. Grimm, Phys. Lett. B265 (1991) 111
12. P. Adamietz, P. Binétruy, G. Girardi and R. Grimm, Nucl. Phys. B401 (1993) 257
13. E. Witten, Phys. Lett. B155 (1985) 151
14. J.-P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Phys. Lett. B271 (1991) 307
15. M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. B156 (1985) 55

53
16. A. Font, L. E. Ibáñez, D. Lüst and F. Quevedo, Phys. Lett. B245 (1990) 401; 
S. Ferrara, N. Magnoli, T. R. Taylor and G. Veneziano, Phys. Lett. B245 (1990) 409; 
H.P. Nilles and M. Olechowski, Phys. Lett. B248 (1990) 268; 
P. Binétruy and M.K. Gaillard, Phys. Lett. B253 (1991) 119

17. K. Kikkawa and M. Yamasaki, Phys. Lett. 149B (1984) 357; 
N. Sakai and I. Senda, Progr. Theor. Phys. 75 (1986) 692

18. T. Kugo and S. Uehara, Nucl. Phys. B226 (1983) 49;

19. E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Phys. Lett. 116B (1982) 231 and Nucl. Phys. B212 (1983) 413

20. M. B. Green and J. H. Schwarz, Phys. Lett. 149B (1984) 117

21. J. Wess and J. Bagger, *Supersymmetry and Supergravity*, 2nd edition. Princeton University Press (1992)

22. S. Ferrara, L. Girardello, T. Kugo and A. Van Proeyen, Nucl. Phys. B223 (1983) 191

23. M. Kaku, P.K. Townsend and P. van Nieuwenhuizen, Phys. Rev. D17 (1978) 3179

24. T. Kugo and S. Uehara, Nucl. Phys. B222 (1983) 125

25. P. van Nieuwenhuizen, Phys. Rep. 68 (1981) 189; section 4.4, equation (16)

26. G. L. Cardoso and B. A. Ovrut, Nucl. Phys. B369 (1992) 351

27. G. L. Cardoso and B. A. Ovrut, Nucl. Phys. B392 (1993) 315

28. M. Dine and N. Seiberg, Phys. Rev. Lett. 55 (1985) 366

29. C. Burgess, A. Font and F Quevedo, Nucl. Phys. 272 (1986) 661

30. M. Dine and N. Seiberg, Phys. Rev. Lett. 57 (1986) 2625

31. D.R.T. Jones, Phys. Lett. B123 (1983) 45

32. M.A. Shifman and A.I. Vainshtein, Nucl. Phys. B359 (1991) 571; see also Nucl. Phys. B277 (1986) 456

33. M. K. Gaillard and T. R. Taylor, Nucl. Phys. B381 (1992) 577

54
34. For a review, see D. Amati, K. Konishi, Y. Meurice, G. C. Rossi and G. Veneziano, Phys. Rep. 162 (1988) 169

35. J.-P. Derendinger, L. E. Ibáñez and H. P. Nilles, Phys. Lett. B155 (1985) 65

36. S. Ferrara, L. Girardello and H. P. Nilles, Phys. Lett. B125 (1983) 457

37. G. Veneziano and S. Yankielowicz, Phys. Lett. B113 (1982) 231;
   T. R. Taylor, G. Veneziano and S. Yankielowicz, Nucl. Phys. B218 (1983) 493

38. A. de la Macorra and G.G. Ross, Nucl. Phys. B404 (1993) 321