Topical Review

Dynamic resistance and dynamic loss in a ReBCO superconductor

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Abstract
Dynamic resistance is a time-averaged direct current (DC) resistance in superconducting materials, which typically occurs when a superconductor is carrying a transport DC while simultaneously subject to a time-varying magnetic field. Dynamic resistance has recently attracted increasing attention as it not only causes detrimental dynamic loss in superconducting devices such as the nuclear magnetic resonance magnets and superconducting machines, but on the other hand, the generated dynamic voltage can be exploited in many applications, e.g. high temperature superconducting (HTS) flux pumps. This article reviews the physical mechanism as well as analytical, numerical modelling, and experimental approaches for quantifying dynamic resistance during the last few decades. Analytical formulae can be conveniently used to estimate the dynamic resistance/loss of a simple superconducting topology, e.g. a single rare-earth-barium-copper-oxide tape. However, in a complex superconducting device, such as a superconducting machine, the prediction of dynamic resistance/loss has to rely on versatile numerical modelling methods before carrying out experiments, especially at high frequencies up to the kHz level. The advantages, accuracies, drawbacks, and challenges of different quantification approaches for dynamic resistance/loss in various scenarios are all inclusively discussed. The application of dynamic resistance in HTS flux pumps is also presented. It is believed that this review can help enhance the understanding of dynamic resistance/loss in superconducting applications and provide a useful reference for future superconducting energy conversion systems.

Keywords: dynamic resistance/voltage, dynamic loss, dynamic region, flux motion, analytical calculation, numerical modelling, experimental approach

(Some figures may appear in colour only in the online journal)
1. Introduction

In 1965, Taquet et al [1] first reported a proportionality factor between the voltage across a bifilar superconducting coil and the transport direct current (DC) when the coil is exposed to a steadily increasing or decreasing magnetic field. This proportionality factor was demonstrated to be proportional to the absolute value of the field rate of change [2, 3], and was later referred to as dynamic resistance. In essence, dynamic resistance is a time-averaged DC resistance, which typically occurs in a superconducting wire, tape, or slab carrying a DC current when it is exposed to an external time-varying magnetic field. In practical applications, the time-varying field is predominantly an AC field, though not necessarily.

Dynamic resistance cannot be found in normal conductors and thus is considered a characteristic phenomenon to describe the distinctive nature of superconductors [4–10]. Dynamic resistance has attracted increasing attention recently because it can not only account for the non-linearly fast rise of dynamic loss and total AC loss (the former can account for the vast majority of the latter at a high load ratio approaching 1) in many superconducting devices but also be exploited in multiple practical applications, such as high temperature superconducting (HTS) flux pumps. Although dynamic resistance has been studied by a lot of researchers during the last dozen years, there is still lacking a systematic overview of its recent development. In addition, some confusion with regards to the qualitative and quantitative analyses of dynamic resistance needs to be clarified. Therefore, it is of significance to elucidate the state of the art in dynamic resistance/loss.

The main body of the review is structured as follows. The physical mechanism and theoretical analyses of dynamic resistance/loss are illuminated in section 2. The numerical modelling methods and experimental measurement work for dynamic resistance/loss are summarised in sections 3 and 4, respectively. Section 5 is focused on the non-linear electromagnetic property of dynamic resistance/loss. The application of dynamic resistance to superconducting devices is discussed in section 6. As the conclusion part, section 7 illustrates the challenges and opportunities brought by dynamic resistance/loss to various superconducting applications, and proposes a few useful suggestions for future research efforts on dynamic resistance/loss.

2. Origin of dynamic resistance/loss

Druyvesteyn first tried to theoretically account for the physical mechanism of dynamic resistance in 1969 based on the superconductor parameters [4]. Later, Sytchev et al [5–7] found that under an externally applied AC field the persistent DC in a closed superconducting circuit decayed due to dynamic resistance and the occurrence of dynamic resistance relies on a field threshold value. Then, Ogasawara et al first clarified the detailed behaviour of the magnetic flux penetrated to the superconductor during one AC cycle [8, 9]. In 1999, Oomen et al [10] proposed an analytical method to calculate the dynamic resistance as well as the caused dynamic loss.

In order to facilitate the introduction of the dynamic resistance effect, let us first study a thick HTS slab with a width of $2w$, a thickness of $h$, and a self-field critical current of $I_{c0}$, as shown in figure 1. The self-field critical current density of the HTS slab is thus $J_{c0} = I_{c0}/(2wh)$. The HTS slab experiences simultaneously a DC, $I_t$, and an externally applied AC field $B_{app}$ perpendicular to the wide surface of the HTS slab with the amplitude of $B_{ext}$. According to Faraday’s law, the voltage along the length of the superconducting slab can be calculated by

$$V = \int_a \oint_{abcd} Edl = -\int_S \frac{dB_{app}}{dr} dA$$  \hspace{1cm} (1)

where $S$ is the rectangular surface enclosed by the four studied lines, $a$, $b$, $c$, and $d$.

In practice, the typical width of a commercial HTS sheet is several mm, which is far less than its length. Therefore, the lines $b$ and $d$ are much shorter than the lines $a$ and $c$, i.e. the contributions of $b$ and $d$ can be neglected in equation (1). The voltage along the line $a$ can thus be written as

$$V = \int_a Edl = -\int_S \frac{dB_{app}}{dr} dA - \oint_c Edl.$$

(2)

To simplify the calculation, line $c$ is chosen to be located in the region across which no flux moves [10]. As a result, the second term on the right of equation (2) is zero, and thus we can obtain

$$V = -\int_S \frac{dB_{app}}{dr} dA = \pm \int_S \frac{dB_{app}}{dr} dA$$

(3)
Figure 2. Magnetic flux density $B$ profiles inside a thick HTS slab experiencing both a transport DC and an external AC field, adapted from [10]. (a) Case of $B_{\text{ext}} < B_{\text{th}}$ in which the field cannot penetrate the central region defined by $-w + p < y < w - p$. (b) Case of $B_{\text{ext}} > B_{\text{th}}$ where dynamic resistance/loss occurs in the dynamic region defined by $-iw < y < iw$. Line $a$ is located at any position inside the dynamic region. Line $c$ is situated at the boundaries of the dynamic region (electric centre-line); thus, no flux moves across it.

where $\pm$ signifies the field vector direction with respect to the normal vector of the wide surface of the HTS slab.

It can be seen from equation (3) that the voltage along the length of the HTS slab is dependent on the time variation of the external field in the area determined by $a$ and $c$. It is known that the external magnetic field penetrates the superconductor from its edges with a certain penetration depth $p$ [11]. If $a$ and $c$ are situated in the central region of the HTS slab defined by $-w + p < y < w - p$, as shown in figure 2(a), the central region cannot be penetrated at low magnetic fields and the penetrated flux has to enter and leave the superconductor from the same side [8]. In this case, $\frac{dB_{\text{ext}}}{dt}$ is equal to zero in the central region, i.e. $V = 0$ according to equation (3). As a result, the transport DC can flow through the central region without resistivity, i.e. the dynamic resistance $R_{\text{dyn}} = 0$.

Clearly, there exists a threshold field amplitude, $B_{\text{th}}$, below which the dynamic resistance of the superconductor remains zero. As a comparison, when $B_{\text{ext}}$ is greater than $B_{\text{th}}$, the central region is able to be penetrated and the magnetic flux enters the HTS slab from one side and leaves from the other side [8]. The net flux motion causes dynamic power dissipation in the area occupied by the transport DC and thus dynamic loss is generated there. The area in which dynamic loss occurs is featured by $-iw < y < iw$ (i denotes the load ratio between $I_t$ and $I_c$) and named dynamic region by Zhang et al [12], as presented in figure 2(b). In this case, a voltage is generated in the dynamic region based on equation (3), and thus the dynamic resistance can be observed.

According to the above analyses, it can be concluded that $B_{\text{th}}$ is not only the lower limit of the field that can penetrate the central region characterised by $-w + p < y < w - p$, but also the lower limit of the field that can give rise to a DC resistance in the dynamic region defined by $-iw < y < iw$. Therefore, $B_{\text{th}}$ can be determined by

$$iw = w - p.$$  

(4)

It should be pointed out that in [10], the penetration depth has been determined in the case of an external field parallel to the wide surface of the slab. However, the penetration depth/field is geometry-dependent. In order to deepen the understanding of the physical mechanism of dynamic resistance, let us focus on a thin HTS films which are widely adopted in rare-earth-barium-copper-oxide (ReBCO) coated conductors. In this case, in order to calculate $B_{\text{th}}$, an appropriate penetration field, $B_p$, has to be derived for a thin HTS strip exposed to a perpendicular magnetic field. According to Jiang et al [13], $B_p$ can be determined by the $B$ value at the maxima of the $\Gamma$ curve, with [13]

$$\Gamma = \frac{P_{\text{Bil}}}{B_{\text{ext}}^2}$$  

(5)

where $P_{\text{Bil}}$ is the Brandt expression for magnetisation loss [14], and

$$P_{\text{Bil}} = 4av^2J_cB_{\text{ext}} \left[ \frac{2B_c}{B_{\text{ext}}} \ln \left( \cosh \frac{B_{\text{ext}}}{B_c} \right) - \tanh \left( \frac{B_{\text{ext}}}{B_c} \right) \right]$$  

(6)

with
\[ B_c = \frac{\mu_0 J_{c0}}{\pi} \]  

where \( f \) is the frequency of the AC field, and \( \mu_0 \) is the permeability of free space.

Based on equations (5)–(7), the penetration field \( B_p \) can be obtained as

\[ B_p = 4.9284 \frac{\mu_0 J_{c0} h}{\pi}. \]  

\( B_{lh} \) is determined by equation (4), yielding thus

\[ B_{lh} = (1 - i) B_p = 4.9284 \frac{\mu_0 J_{c0} h}{\pi} (1 - i). \]  

Ciszewski et al [15] has put forward an empirical expression for \( B_{lh} \) by extrapolating the \( Q_{dyn}(B_{ext}) \) curves and/or from deflection points in \( R_{dyn}(B_{ext}) \) curves and exploiting the external susceptibility proposed by Fabbricatore et al [16], as

\[ B_{lh} = \frac{4\mu_0 J_{c0} h}{\pi} (1 - i). \]  

(10)

Despite the similar form in equations (9) and (10), it should be noted that the external susceptibility employed in equation (10) does not effectively reflect the changes caused by the non-linear field profile inside the superconducting film [13] and thus the threshold field in equation (10) can be underestimated compared to equation (9).

Different from the linear correlations between \( B_{lh} \) and \( i \) described by equations (9) and (10), Mikitik and Brandt have proposed a nonlinear \( B_{lh}(i) \) equation, as [17]

\[ B_{lh} = \frac{\mu_0 J_{c0} h}{2\pi} \left[ \frac{1}{i} \ln \left( \frac{1 + i}{1 - i} \right) + \ln \left( \frac{1 - i^2}{4i^2} \right) \right]. \]  

(11)

Jiang et al [13] have compared equations (9)–(11) in detail at relatively high load ratios, and concluded that equations (9) and (11) give rise to similar results at high load ratios with \( i > 0.1 \), but equation (10) underestimates \( B_{lh} \). Actually, Liu et al [18] have mentioned that equation (9) provides good agreement with all previously published experimental data only for a high load rate (especially for \( i > 0.1 \)). Zhang et al [19] have verified the effectiveness of equation (11) in describing the nonlinearity of the \( R_{dyn}(i) \) curve, which cannot be accounted for by equations (9) and (10), especially at low load ratios lower than 0.2.

Knowing the quantification method for the threshold field, the analytical formulae for dynamic resistance and dynamic loss can then be deduced. Oomen et al [10] has derived the analytical equations for dynamic energy loss (J) based on

\[ Q_{dyn} = \int_0^T \int_0^L E(t) J(t) dV dt. \]  

(12)

Jiang et al [13] has given the formula of \( Q_{dyn} \) by calculating the work done when the magnetic flux perpendicular to the wide surface of the HTS film crosses the transport DC in the dynamic region, as

\[ Q_{dyn} = I_s \cdot \Delta \Phi = I_s \cdot \Delta B \cdot S_{dyn} \]  

(13)

where \( \Delta B = B_{ext} - B_{lh} \). \( S_{dyn} \) represents the area of the dynamic region with \( S_{dyn} = 2\pi L \), where \( L \) is the length of the HTS layer.

Equations (12) and (13) give rise to the same formula for linear hysteretic dynamic loss, as

\[ Q_{dyn,l} = \frac{4\pi L J_s^2}{I_{c0}}(B_{ext} - B_{lh}). \]  

(14)

Based on equation (14), the linear hysteretic dynamic power loss, \( P_{dyn,l} \) (W), and linear hysteretic dynamic resistance, \( R_{dyn,l} \) (Ω), can be derived as

\[ P_{dyn,l} = \frac{4\pi L J_s^2}{I_{c0}}(B_{ext} - B_{lh}) \]  

(15)

\[ R_{dyn,l} = \frac{4\pi L J_s^2}{I_{c0}}(B_{ext} - B_{lh}). \]  

(16)

Equations (14)–(16) have been widely used and validated against many experimental measurements and numerical models during the last few years [20–28]. According to equations (14)–(16), it can be seen that the dynamic loss and dynamic resistance are in a linear correlation with the amplitude of the external field, \( B_{ext} \), which is, however, only true for the cases of low load ratios and field amplitude in reality. Oomen et al [10] found that at higher \( B_{ext} \), dynamic resistance demonstrates a non-linear correlation to \( B_{ext} \), which cannot be accounted for by equation (16). Considering the \( J_c(B) \) dependence of a superconductor proposed by Nibbio et al [29],

\[ J_c(B) = \frac{J_{c0}}{1 + \frac{B_s}{B_{th}}}. \]  

(17)

where \( B_s \) represents the field component perpendicular to the superconductor wide surface and \( B_{th} \) is the characteristic field depending on the material property. Oomen put forward a non-linear formula to describe the non-linearity of the \( R_{dyn}(B_{ext}) \) curve for \( B_{ext} \gg 2B_{th} \), as [10]

\[ R_{dyn,nl} = \frac{4\pi L J_s^2}{I_{c0}} \left( B_{ext} + \frac{B_{ext}^2}{B_{th}^2} \right). \]  

(18)

However, it appears that the measured dynamic resistance rises much faster than the analytical results obtained by equation (18). Therefore, neither equation (16) nor (18) can be utilised to explain the non-linearity of dynamic resistance at high external fields.

Zhang et al concluded that the non-linearity of dynamic resistance not only occurs at high external fields but also at high load ratios [19]. In addition, the higher the \( n \)-value in the \( E-J \) power law [30], the faster the dynamic loss and resistance increase along with \( B_{ext} \) [19]. It is worth presenting the \( E-J \) power law here, as [30]

\[ E = E_0 \cdot \left( \frac{J}{J_c(B)} \right)^n. \]  

(19)

where \( E_0 \) refers to the characteristic electric field, usually defined as 1 \( \mu \)V cm\(^{-1} \).
In [13], a time-averaged DC flux flow resistance [31] was proposed by Jiang et al to explain the non-linear fast rise of dynamic resistance, which provides a good description of the deviation from linearity at high currents, as

\[ P_{\text{dyn}} = 4\omega LLI (B_{\text{ext}} - B_{c0}) + LfI^{n+1} \int_{0}^{1/f} \frac{E_0}{k^2(B_{\perp})} \text{d}t. \]  

(20)

In equation (20), the first term corresponds to the linear hysteretic dynamic resistance characterised by equation (16), and the second term is the non-linear DC flux flow resistance.

In order to better understand the origin of the non-linear flux flow effect, Zhang et al derived a full-range formulation for dynamic loss and dynamic resistance in [12] on the basis that the HTS film is at relatively high dynamic loss and dynamic resistance in the flow effect, Zhang et al. From the above analyses, it can be known that dynamic loss is generated in the dynamic region characterised by \(-iw < y < iw\). Based on equations (12, 17, and 19), dynamic power loss can thus be calculated by

\[ P_{\text{dyn}} = \frac{hL}{T} \int_{0}^{T} \int_{(1-i)w}^{(1+i)w} E_0 \left[ \frac{1}{2} J_{c0} \left( 1 + \frac{|B_{\perp}|}{B_0} \right) \right] \text{d}y \text{d}t \]

\[ = \frac{E_0hL}{J_{c0}^2} \int_{0}^{T} \int_{(1-i)w}^{(1+i)w} f^{n+1} \cdot f(B) \text{d}y \text{d}t \]  

(21)

with

\[ f(B) \approx \left[ 1 + \frac{|B_{\text{ext}} \sin (2\pi ft)|}{B_0} \right]^n \]  

(22)

where \( T \) is the period of the AC field. It should be noted here that the self-field generated by the transport DC has been neglected in equation (22) in that it becomes far smaller than \( B_{\text{ext}} \) at high load ratios and high \( B_{c0} \) [12].

According to the Bean Model [32], the current density distribution along the width of the HTS CC is featured by the critical current \( \pm J_{c0} \) when the superconductor is in a critical state. The dynamic region with the width of \( 2iw \) is the effective region to carry the transport DC, i.e. in the critical state, the dynamic region has been filled with \( I_t \), no matter in the cases of increasing or decreasing external fields, as illustrated in figure 3.

According to the binomial theorem, Euler’s formula, equations (21) and (22), the non-linear dynamic resistance caused by the flux flow effect can be quantified as

\[ P_{\text{dyn,nl}} = \frac{E_0hL}{J_{c0}^n} \int_{0}^{T} \int_{(1-i)w}^{(1+i)w} f(B) \text{d}y \text{d}t \]

\[ = \frac{E_0hL}{2w} \int_{0}^{T} \int_{(1-i)w}^{(1+i)w} f(B) \text{d}y \text{d}t \]

\[ = E_0hLI_t^{n+1} f_{\text{avg}}(B) \]  

(23)

with, when \( n \) is even,

\[ f_{\text{avg}}(B) = \left\{ \begin{array}{l} 1 + \sum_{p=0}^{n/2-1} \frac{n!}{(2p+1)! [n-(2p+1)]!} \left( \frac{B_{\text{ext}}}{B_0} \right)^{2p+1} \cdot \frac{2p+1 \cdot p!}{\pi \prod_{q=0}^{2p+1} (2q+1)} \\
+ \sum_{p=0}^{n/2-1} \frac{n!}{(2p+2)! [n-(2p+2)]!} \left( \frac{B_{\text{ext}}}{B_0} \right)^{2p+2} \cdot \frac{(2p+2)!}{(p+1)!} \end{array} \right\} \]  

(24)

and when \( n \) is odd,

\[ f_{\text{avg}}(B) = \left\{ \begin{array}{l} 1 + \sum_{p=0}^{(n-1)/2} \frac{n!}{(2p+1)! [n-(2p+1)]!} \left( \frac{B_{\text{ext}}}{B_0} \right)^{2p+1} \cdot \frac{2p+1 \cdot p!}{\pi \prod_{q=0}^{2p+1} (2q+1)} \\
+ \sum_{p=0}^{(n-1)/2-1} \frac{n!}{(2p+2)! [n-(2p+2)]!} \left( \frac{B_{\text{ext}}}{B_0} \right)^{2p+2} \cdot \frac{(2p+2)!}{(p+1)!} \end{array} \right\}. \]

(25)
The dynamic resistance is composed of the linear hysteretic resistance and the non-linear flux flow resistance. Therefore, the full-range dynamic loss (W) can be expressed as

$$P_{\text{dyn}} = 4wlI_i(B_{\text{ext}} - B_0) + E_0L_i^{n+1}f_{\text{avg}}(B)$$  \hspace{1cm} (26)$$

and the full-range dynamic resistance (\Omega) can be written as

$$R_{\text{dyn}} = \frac{4wlI}{T_i} (B_{\text{ext}} - B_0) + \frac{E_0L}{T_i^{n+1}} f_{\text{avg}}(B).$$  \hspace{1cm} (27)$$

The different analytical equations to calculate the dynamic resistance/loss of an HTS tape were compared in table 1. It has been concluded that both equations (20) and (27) reflect well the non-linearity of dynamic loss at high load ratios and simultaneously high external fields. Compared to equation (20), equation (27) not only reveals the mechanism of the non-linear flux flow resistance in the form of a summation of power functions from the mathematical perspective, but also provides a more accurate analytical result. In practical applications, in the case of low load ratios and external fields when \(I_i\) is smaller than the real critical current \(I_c(B)\), equations (15) and (16) can be conveniently utilised to predict the hysteretic dynamic loss and dynamic resistance generated in a superconducting film or slab. Although people tend to not use a high load ratio (e.g. higher than 0.8) to avoid the quench of superconductors, the fast non-linear rising of the dynamic loss and dynamic resistance can still occur under a high external field in the order of tesla level. In this case, equation (20) or (27) has to be adopted to accurately quantify the flux flow effect. It should be noted that although equations (24)–(27) exhibit a complex mathematical form, they can be quickly achieved in MATLAB and are thus generally more efficient compared to numerical modelling and experiments.

### 3. Numerical modelling of dynamic resistance

It is worth mentioning that the analytical equations presented in section 2 are not applicable in all scenarios in that they have been derived based on some fundamental assumptions, e.g. homogeneous external field. Additionally, a typical commercial HTS tape is composed of not only the HTS layer but also the non-superconducting overlayer, substrate as well as stabilisers. Therefore, the electromagnetic interactions between the HTS and non-superconducting layers cannot be considered in the analytical equations, especially at high frequencies [33, 34]. In addition, the analytical formulae are limited to simple structures, e.g. a single film, slab, or wire. Consequently, in a complex superconducting topology, e.g. a field winding in a superconducting machine, the analytical equations are not sufficient to accurately quantify the practical dynamic resistance or dynamic loss. Therefore, with the development of simulation approaches for superconductors, numerical models have been extensively employed to analyse the AC loss of superconducting devices [35–38].

Ainslie et al [27] built an \(H\)-formulation (\(H\) represents magnetic field strength vector) [36] based 2D numerical model using the homogenisation technique [39, 40] to analyse the dynamic resistance and AC loss in an HTS thin film, and mentioned that dynamic resistance can be calculated from the

| Formulae | Advantages | Limitations |
|----------|------------|-------------|
| (15), (16) | Conveniently used to estimate dynamic resistance/loss in the cases of low \(i\) and \(B_{\text{ext}}\) | Not applicable at high \(i\) and \(B_{\text{ext}}\) when \(I_i \geq I_c(B)\). |
| (18) | Not applicable at both low and high \(i\) or \(B_{\text{ext}}\). |
| (20) | Applicable in the full range of \(i\) and \(B_{\text{ext}}\). Estimated time-average DC flux flow resistance, which cannot intuitively account for the non-linearity of dynamic resistance/loss. Not applicable in the cases of low \(i\) and \(B_{\text{ext}}\). |
| (23) | Able to account for and calculate the non-linear dynamic loss due to the flux flow effect with \(i\) approaching 1. |
| (26), (27) | Through rigorous mathematical derivation, applicable in the full range of \(i\) and \(B_{\text{ext}}\), and provide the most accurate analytical results. | The \(n\)-value has to be an integer. |
average electric field across the entire conductor cross-section, as

\[ R_{\text{dyn}} = \frac{L}{TS} \int_{0}^{T} E(i) \, dS \, dt \]  

(28)

where \( S \) is the cross-sectional area of the HTS film, and \( E \) is the time-dependent electric field inside the HTS film. The simulation results agree well with the measured data. The field angle dependence of the dynamic resistance was also studied in [27] and it was concluded that the dynamic resistance, and dynamic loss are mostly determined by the field component perpendicular to the wide surface of the HTS film. The same numerical method was adopted in [41] to study the dynamic resistance in a parallel-connected stack of HTS tapes. The dynamic resistance and total loss of a three-tape ReBCO stack were also investigated in [28] also through \( H \)-formulation numerical modelling. It was found in [28] that the shielding effect in the stack causes a larger \( B_{th} \) compared to a single tape, leading to smaller \( (B_{\text{stat}} - B_{\text{th}}) \) and thus a smaller dynamic resistance, especially at a lower load ratio.

\( T \)-formulation (\( T \) denotes current vector potential) based numerical models are another powerful tool for numerical modelling of dynamic resistance as well as dynamic loss, which was first proposed by Amemiya et al [42] to simulate superconducting wires. In [12, 19, 25, 26, 43], Zhang et al investigated electromagnetic characteristics of dynamic resistance and dynamic loss in an HTS coated conductor using the \( T \)-formulation thin-film model developed with open source code under FORTRAN 90 [44, 45]. The \( T \)-formulation based open-source-code model has been validated by many experimental measurements [45]. The calculation of the dynamic power loss (W) was achieved based on the definition of the dynamic region, as

\[ P_{\text{dyn}} = \frac{hL}{T} \int_{0}^{T} \int_{(1-i)w}^{(1+i)w} E \, J \, dy \, dt. \]  

(29)

Figure 4 illustrates the influence of the self-field critical current of an HTS coated conductor on the dynamic region/loss, which is obtained based on the \( T \)-formulation model using FORTRAN 90. The transport DC \( I_t = 60 \) A, and \( I_{c0} \) increases from 80 to 160 A, i.e. the load ratio decreases from 0.75 to 0.375. The shaded area in figure 4 signifies the amount of traversing magnetic flux during one AC period. With the reduction of the load ratio, the dynamic region shrinks and then less magnetic flux traverses this region, resulting in less dynamic loss [19]. It should be pointed out that the definition of the dynamic region does not apply anymore with the existence of a strong DC background, and the dynamic loss is increased accordingly due to the reduction of the critical current [26].

Dynamic resistance is uniquely found in superconducting materials, but in commercial superconducting wires, such as the HTS coated conductors, some non-superconducting layers are usually employed to enhance the reliability of their practical use, e.g. the substrates, buffers, metal stabilisers/overlayers. These supportive layers have various impacts on the dynamic resistance of the superconducting materials, and many of these impacts can be estimated by numerical modelling. Zhang et al proposed a 2D multilayer numerical model in [33] to explore the high-frequency performance of HTS coated conductors, taking into account both the HTS and non-superconducting layers composed of the silver overlayer, substrate, as well as copper substrates. The skin effect at high frequencies was revealed in [33] and it was demonstrated that at a frequency higher than 100 Hz in the case of magnetisation, the influence of non-superconducting layers has to be considered because of the current sharing capacity of the non-superconducting layers. In other words, the thin-film approximation for a superconducting tape becomes inapplicable and the multilayer model has to be adopted [33].

Another major advantage of numerical modelling is that it can conveniently couple the thermal behaviour to realistically model the dynamic resistance. Ma et al [46] constructed an electromagnetic-thermal coupled multilayer model and studied the frequency dependence of the dynamic resistance. It was concluded in [46] that the shielding effect from the copper stabilisers caused by high-frequency fields can decrease the dynamic resistance of the HTS coated conductor, which can mitigate the output voltage of HTS flux pumps. A similar electromagnetic-thermal model also studied the impacts of a high-resistance stabiliser which could lead to high dynamic...
It was concluded that the electromagnetic-thermal coupled the dynamic resistance in a series-connected HTS stack. A multilayer numerical model, Li et al [50] studied the temperature-dependent critical current of the HTS wire, as [51, 52]

\[
J_c(B, T) = \begin{cases} 
  J_{c0}(B), & T \leq T_0 \\
  J_{c0} \cdot \left(\frac{B}{B_{c0}}\right)^{\frac{1}{2}}, & T_0 < T < T_c \\
  0, & T \geq T_c 
\end{cases}
\] (30)

where \(T_0\) is the operational temperature, and \(T_c\) is the critical temperature of the HTS wire. In addition, it was also found in [50] that the loss distribution along the conductor width is asymmetric in each half-cycle when there are both a transport DC and an applied AC magnetic field, and the asymmetry gets enhanced with increasing current. More interestingly, under a high magnetic field or with a high transport DC, the loss-generation region of each loss component varies along the conductor width. Furthermore, the superposition of the transport DC with its anti-parallel shielding currents occurs at a high current level. The superposition drives the current density of one conductor edge to the subcritical stage, and it leads to one-sided loss generation in each half-cycle [50].

It should be noted that, in addition to equation (17), the \(J_c(B)\) dependence can be described by other formulae, and a comprehensive review of different \(J_c(B)\) equations can be found in [53]. Also, considering the \(J_c(B, T)\) dependence of an HTS coated conductor, using the \(H\)-formulation based multilayer numerical model, Li et al [54] have recently investigated the dynamic resistance in a series-connected HTS stack. It was concluded that the electromagnetic-thermal coupled multilayer numerical model can more accurately quantify the dynamic resistance compared to the conventional pure electromagnetic models, and the effectiveness of the multilayer model proposed in [33] has been further verified. A frequency-dependent demarcation thickness was found in [54], namely, the dynamic resistance of an HTS stack increases with the reduction of the thickness of the insulation between different tapes, which signifies that the thermal effect becomes more influential than the shielding effect of HTS tapes in affecting the dynamic resistance when the tapes are placed closer to each other.

In addition to the \(J_c(B, T)\) dependence of an HTS coated conductor, Brooks et al [55] pointed out that the measured \(J_c(B, \theta)\) data should be utilised in numerical models to
which sinusoidal magnetic fields have been extensively used. Sensitive combination of considerable dynamic voltage and results indicated the sinusoidal magnetic field is able to lead the well as their corresponding energy efficiency. Simulation results did not change massively (at \( i = 0.5 \) and \( f = 100 \) Hz) [57].

Hu et al. [56] numerically studied the dynamic resistance of a series-connected vertically-arranged HTS stack under different tape numbers, AC field frequencies, and separation gaps. They found the dynamic resistance of the HTS stack does not linearly increase with the frequency of the AC field at the kHz level, and at that time the eddy current loss was huge which could affect the overall efficiency. When the substrate of the HTS tape changed from non-magnetic to magnetic, the proportion of the dynamic resistance to total loss also varied, i.e. the dynamic loss could be reduced by the adjacent magnetic substrate in the low field region and increased in the high field region. Numerical results showed that many magnetic flux lines were concentrated in the edge parts of the tape, and when the relative permeability of magnetic Ni-alloy substrate is increased (e.g. by 100 times), the dynamic resistance loss did not change massively (at \( i = 0.5 \) and \( f = 100 \) Hz) [57].

Conventional concepts suggest that the dynamic resistance in a superconductor usually occurs under a transport DC and a well-regulated periodical AC magnetic field, i.e. a sinusoidal signal. However, non-sinusoidal AC magnetic fields can also be the sources to generate the dynamic resistance in a DC-carrying superconducting tape. Shen et al. [58] numerically studied the dynamic resistance behaviour of an HTS tape exposed to four different types of oscillating magnetic fields (including triangle, sawtooth, square, and sinusoidal fields), as well as their corresponding energy efficiency. Simulation results indicated the sinusoidal magnetic field is able to lead the sensible combination of considerable dynamic voltage and relatively low total loss, and that explained the possible reason for which sinusoidal magnetic fields have been extensively used as the power source to energise HTS flux pumps [58].

To sum, a 1D \( T \)-formulation based numerical model can be conveniently utilised to simulate the dynamic resistance/loss of a thin ReBCO film at frequencies below 100 Hz, which is more efficient than \( H \)-formulation based numerical models as \( \nabla \times T \) is simply calculated by the two vector potentials on both sides of each element. However, in order to consider the magnetic field components parallel to the wide surface of an HTS tape, 2D \( T \)-formulation or \( H \)-formulation based models have to be adopted. The application of the homogenisation method can greatly reduce the computational complexity of modelling an HTS tape, as illustrated in [33]. However, at frequencies higher than 100 Hz, the electromagnetic interactions between the superconducting and normal conducting layers in a commercial ReBCO coated conductor have to be taken into account. Therefore, 2D multilayer numerical models should be applied to ReBCO tapes or stacks exposed to a high-frequency magnetic field. In the case of a complicated topology subject to a complex electromagnetic environment, e.g. a racetrack-coil-shaped ReBCO field winding in a high-speed superconducting motor, 3D multilayer numerical models have to be adopted, e.g. the \( H \)-formulation based multilayer numerical model proposed in [49]. In order to quantify the dynamic resistance/loss more accurately in practical applications, the electro-thermal coupled numerical models need to be exploited. The field dependence of the critical current of a ReBCO coated conductor is also determined by the field angle and temperature, and thus the \( J_c(B, \theta) \) dependence should be utilised in electromagnetic analysis and the \( J_c(B, \theta, \gamma) \) dependence should be applied to electro-thermal coupled modelling.

4. Measurement of dynamic resistance

Given that dynamic resistance is typically generated in a DC-carrying superconducting wire that is subject to an externally applied AC field, the measurement system usually consists of a DC power supply, an AC field magnet, as well as necessary sensors and cables. The test setup established by Jiang et al. [13, 18–21, 23–25, 27, 28, 43–45, 50] is presented in figure 6.

The measurement system is mainly composed of an AC electromagnet built with copper racetrack coils which can produce a flux density up to 100 mT, a Hewlett Packard 6682 A DC power supply which provides \( I_h \) varying between 0–240 A, a Keithley 2182 nano-voltage meter for averaged voltage measurement across the voltage taps, and a cryostat containing liquid nitrogen. The implementation of the voltage taps can be achieved by two methods, as demonstrated in figure 6: in the first set the voltage taps are attached to the centre of the tested sample and extended from their midpoint; for the second set a spiral loop is arranged around the sample [24]. The spiral loop is not only compact in space but also able to measure the dynamic resistance/loss considering the mutual electromagnetic influences between different ReBCO tapes. An adjustable capacitor bank is connected in series with the AC magnet to enable different resonance frequencies. The voltage measurement leads are wound on a cylindrical surface enclosing the tape to achieve inductively cancelling voltage taps attached to the sample [59]. Dynamic power loss is obtained by multiplying the measured time-averaged voltage across the sample with the transport DC. Taking the experiment carried out in [19] as an example, the tested 15 cm long HTS tape with \( n = 22.5 \) and \( I_{c0} = 105.3 \) at 77 K was placed in the uniform field region between the poles of the electromagnet which produced an external AC magnetic field with a magnitude varying between 0 and 100 mT at \( f = 26.62 \) Hz.

The influence of an extra DC background filed on the dynamic resistance was also experimentally investigated in [22]. The schematic of the experimental layout is shown in
Figure 6. Diagram of the experimental setup to measure the dynamic loss of a single HTS tape.

Figure 7. Schematic of the experimental layout to measure the dynamic resistance of a single HTS tape under both DC and AC background fields.

The background DC field was generated by a low temperature superconducting solenoid magnet with a maximum central field of 6 T (the upper limit), the AC field was provided by a resistive solenoid magnet with a peak field of 100 mT. The S-shape voltages tapes were exploited to mitigate the interference from the inductive pickup coils and the noise caused by the AC field during the measurement [60]. The experimental findings from [60] comply well with the modelling results from [26].

The experimental investigation of dynamic resistance also gives rise to a crucial issue: how to distinguish the dynamic resistance/loss component from other components (e.g. reactive power component and magnetisation loss)? Geng et al used a pair of voltage taps to detect the electric potential across a short single HTS tape (with $I_{c0} = 135$ A and low inductance) carrying a transport DC varying from 50 to 80 A which is exposed to an AC field with adjustable frequencies between 50 and 2000 Hz and field amplitude up to 300 mT [61]. Different from the conventional inductance voltage component, the frequency of the real-time dynamic resistance component is twice the frequency of the AC magnetic field, whose ‘factor of 2’ was an obvious sign of the occurrence of the dynamic resistance, instead of the inductance component having the same frequency as the applied magnetic field.

Unlike a single HTS tape with a simple structure, the dynamic resistance/loss in an HTS stack exhibits some distinctive properties. A four-tape YBCO stack was tested by Jang et al [23] and the threshold fields of the four tapes were found to be larger than the theoretical values calculated by equations (8) and (10) in a single tape, especially at high transport DCs. This phenomenon was attributed to the superimposed shielding field generated by the magnetised currents in each tape, and this shielding effect was further verified by Liu et al [18] under various applied field orientation. Sun et al measured the dynamic loss and magnetisation loss of a stack composed of three serial-connected ReBCO in [28], and concluded that both the dynamic loss and magnetisation loss of an HTS stack are much smaller than a single tape due to the shielding effect though their difference turns smaller with increasing field amplitudes.

Compared to HTS tapes and stacks, the inductive voltage component is dominant in an HTS coil. Li et al used the non-inductive bifilar structure to wind the HTS coil between the gap having an intensive AC magnetic field, which could effectively cancel the inductance voltage component and give a relative pure dynamic resistance component [62], and this configuration was also used as the persistent current switch to energise the flux pump [63]. Li et al further used the harmonic analysis on the response voltage of HTS tapes as a tool to distinguish the dynamic-resistance voltage from the flux flow voltage: the HTS tape presented the pure dynamic resistance component if the ratio of the DC component to the second harmonic component kept constant, and this method was valid for both the experiment and numerical modelling [64, 65].

5. Non-linearity of dynamic resistance/loss

It should be clarified here that the generation of dynamic resistance does not necessarily rely on the externally supplied DC, such as the dynamic resistance in an HTS dynamo [66], and the dynamic resistance in the electromagnetic interaction between a moving magnetic dipole and a superconducting loop [67]. Additionally, dynamic resistance is not a static resistance, which is essentially a time-averaged DC resistance, which exhibits time-dependent variation in reality [55].

From the theoretical formulae presented in section 2, it can be seen that the dynamic resistance in a superconductor is determined by the load ratio between the transport DC and the self-field critical current of the superconductor, the $n$-value characterising the $E$-$J$ power law, the amplitude of the external field, and the geometry of the superconductor. In general, dynamic resistance is in a positive correlation to the load ratio, field amplitude, and the $n$-value; however, the correlations between them are not linearly exhibited, as illustrated in
sections 2 and 3. It is of great significance to know about the non-linearity of dynamic resistance as a non-linearly fast rise of dynamic loss can lead to the quench of superconducting devices.

Zhang et al [43] defined two parameters to characterise the linearity and non-linearity of dynamic resistance, namely demarcation current and corner field. The upper demarcation current, $I_{du}$, characterising the transition from the linear region to the non-linear region of the $R_{dyn}(I_t)$ curve at high currents, is derived as

$$I_{du} = \left[ \frac{4.9284\mu_0 f \cdot I_0^n}{n\pi E_0 f_{ave}(B)} \right]^{\frac{1}{n}}$$  \hspace{1cm} (31)$$

where $f_{ave}(B)$ is determined by equations (24) and (25).

The lower demarcation current, $I_{dl}$, describing the transition from the non-linear region to the linear region at low currents, is written as

$$I_{dl} = 0.4347I_0.$$  \hspace{1cm} (32)$$

The corner field, $B_{cor}$, is introduced to feature the non-linear variation of the $R_{dyn}(B_{ext})$ curve, which can be calculated by the following equations, when $n$ is even,

$$\sum_{p=0}^{n/2-1} \frac{n!}{(2p)! \cdot [n-(2p+1)]! \cdot B_0} \left( \frac{B_{cor}}{B_0} \right)^{2p+1} \cdot \frac{2p+1 \cdot p!}{\pi \prod_{q=0}^{2p+1}(2q+1)} \approx \frac{4 w B}{E_0 p^2}.$$  \hspace{1cm} (33)$$

and when $n$ is odd,

$$\sum_{p=0}^{(n-1)/2} \frac{n!}{(2p)! \cdot [n-(2p+1)]! \cdot B_0} \left( \frac{B_{cor}}{B_0} \right)^{2p} \cdot \frac{2p+1 \cdot p!}{\pi \prod_{q=0}^{2p+1}(2q+1)}$$

$$+ \sum_{p=0}^{(n-1)/2-1} \frac{n!}{(2p+1)! \cdot [n-(2p+2)]! \cdot B_0} \left( \frac{B_{cor}}{2B_0} \right)^{2p+1} \cdot \frac{4w B}{E_0 p^2}.$$  \hspace{1cm} (34)$$

6. Applications using dynamic resistance

Dynamic resistance can lead to dynamic loss in many superconducting topologies, e.g. nuclear magnetic resonance magnets [68, 69] and superconducting synchronous machines [70–74], both of which use superconducting coils (as the load) to a large transport DC in the simultaneous presence of AC magnetic fields. As a result, both dynamic power loss and conventional magnetisation loss contribute to the total electromagnetic power dissipation [75], which will instantaneously be converted into detrimental heat.

However, there exists a different situation: when dynamic resistance is not generated in a superconducting load (the terminal component), but occurs in the intermediate part of a superconducting circuit (normally called a superconducting bridge), the well-regulated interaction of dynamic resistance with external sources can produce an overall energy source component, which can be regarded as a ‘useful’ power source to energise a practical load.

The Bean model can be used to qualitatively demonstrate how the superconducting dynamic resistance transformation can produce the net flux as the energy source to charge the load, based on figures 2–4. Taking an AC cycle as an example, as shown in figure 10(a), when the applied AC field $B_{app}$ increases to its peak value $B_{ext}$, the whole cross-section of the superconducting sheet is characterised by $\pm J_{c0}$ and $I_c$ (where $I_c$ is determined by the transport DC $I_t$ and in critical state mode $I_c = J_{c0}$); during this process, the flux central line moves to the borderline $y = iw$. Similarly, in figure 10(b) during the field decreasing cycle, the flux central line moves left to the borderline $y = -iw$. As a result, during the entire cycle, the
Figure 8. Variation of the dynamic resistance in an HTS coated conductor with the transport DC under AC fields with different amplitudes. The analytical results were calculated by equation (27), the simulation results were obtained from the $T$-formulation numerical method [43], and the experimental data were measured with the setup shown in figure 6. $I_{d1}$ and $I_{d2}$ are calculated from equations (31) and (32), respectively.

magnetic flux indicated by the shaded area in figure 10 enters on one side of the superconducting sheet and leaves from the other side, creating a net flux $\Delta \Phi$ in the dynamic region, as

$$\Delta \Phi = 2iwL(B_{\text{ext}} - B_{\text{th}}).$$  \hspace{1cm} (35)

The net flux $\Delta \Phi$ will accumulate in the load if there is a closed superconducting circuit, and this phenomenon can be regarded as flux pumping.

The dynamic resistance theory can perfectly explain the mechanism of transformer-rectifier type flux pumps [76–86], which is due to the fact that the waveforms of the transport current and the applied field towards the HTS bridge are independently controlled. Dynamic resistance theory can also indirectly explain the behaviours of travelling-wave type flux pumps (e.g. rotating-magnet type [66, 87–96], linear-type type [97–104], and circular type [105–107]). Figure 11 shows a kilo-ampere level HTS flux pump based on the transformer-rectifier configuration. The HTS bridge consists of four parallel tapes (each has $I_{\text{c0}} = 298$ A at 77 K), and uses the turning-back structure which can effectively cancel out the induction voltage. This flux pump could be potentially used to charge an emerging HTS magnet for an ultra-compact MRI scanner [108].

The flux pump applications using the dynamic resistance behaviour can also be simulated by numerical modelling methods [109, 110], of which the mechanism is similar to the aforementioned contents in section 3. Meanwhile, the efficiency optimisation of these flux pump applications can also be realised by numerical models [111]. The operations of dynamic resistance behaviour can also be simplified to electric circuit models using typical switches [112, 113].

Figure 9. Variation of the dynamic resistance in two HTS coated conductors with the external AC field. The analytical results were calculated by equations (16) and (27), the simulation results were obtained from the $T$-formulation numerical method [43], and the experimental data were measured with the setup shown in figure 6. $B_{\text{cor}}$ is calculated from equation (33) or (34).

Figure 10. Diagram of the net flux induced during an oscillating cycle by an external AC field in a DC-carrying superconducting sheet as the power source of flux pumping. (In the strict sense, the linear field distribution along the width is an ideal case for an infinitely thick slab and here it is ‘qualitatively’ used for the illustration of the net flux.)
recent comprehensive review of HTS flux pumps exploiting the dynamic resistance theory can be found in [114].

7. Conclusion: challenges and future outlook

The paper has systematically reviewed the state of the art regarding dynamic resistance and dynamic loss in various superconducting topologies from theoretical analysis, numerical modelling, and experimental exploration to superconducting applications. In general, dynamic resistance can be considered a characteristic macroscopic phenomenon to describe the unique nature of superconductors, and it can be exploited in energy conversion systems, such as HTS flux pumps. However, the dynamic power dissipation caused by the dynamic resistance threatens the reliability and stability of superconducting equipment, e.g. the field windings in a synchronous superconducting machine, due to massive heat accumulation and possible quench. More detailed key conclusions are drawn as follows.

(a) As pointed out by Grilli [115], analytical models are more suitable for a quick design of a superconducting application because they are efficient and easily implemented and used by people lacking numerical modelling skills. The previous widely adopted equations (15) and (16) can be conveniently exploited to predict the hysteretic dynamic resistance/loss of an HTS strip at a low load ratio, \( i \), under a low external magnetic field amplitude, \( B_{\text{ext}} \), when the transport current \( I_t \) is lower than the real critical current \( I_c(B) \). However, at a high \( i \) or a high \( B_{\text{ext}} \), \( I_t \) approaches or even exceeds \( I_c(B) \) and the flux flow effect causes the dynamic resistance/loss to increase in a fast non-linear way. The non-linear flux flow resistance/loss can be mathematically accounted for by equation (23).

Equations (20)–(27) are able to calculate the dynamic resistance/loss at a full range of \( i \) and \( B_{\text{ext}} \) since both the hysteresis and flux flow effects are considered. Equations (26) and (27) illustrate that the non-linearity of the dynamic resistance/loss follows the form of a summation of power functions derived based on the \( E-J \) power law and \( J_c(B) \) dependence, and can give a more accurate analytical result. Equations (31)–(34) can be exploited to quantitatively describe the linear and non-linear variation of dynamic resistance with respect to applied transport DC and external magnetic fields.

However, existing analytical equations for calculating dynamic resistance/loss have been focused on simple superconducting structures, which cannot be applied to complex superconducting topologies, e.g. racetrack superconducting coils which are widely employed in a synchronous machine. In addition, in a complicated electromagnetic environment, e.g. under a non-pure AC field composed of many high-frequency harmonics, the existing analytical equations will become inapplicable because of the invalidity of some basic approximations, such as a uniformly distributed external AC field and the thin-film approximation of a commercial HTS coated conductor.

(b) FEM based numerical models are a versatile tool to estimate the dynamic resistance/loss in different superconducting topologies. A 1D \( T \)-formulation based numerical model is the most efficient model to simulate the dynamic resistance/loss of a thin ReBCO film at low frequencies, e.g. below 100 Hz. 2D \( T \)-formulation or \( H \)-formulation based models ought to be adopted when the field components parallel to the wide surface of a ReBCO tape are unneglectable. However, at high frequencies, e.g. at the kHz level, 2D multilayer numerical models should be applied to ReBCO tapes or stacks so as to incorporate the electromagnetic interactions between the superconducting and normal conducting layers as well as the skin effect.

However, accurate numerical modelling relies on the experimental measurement of the key parameters of superconductors, e.g. the \( J_c(B, T, \theta) \) dependence. For a large-scale superconducting devices, e.g. a 3D racetrack HTS field winding, 3D numerical modelling can be very time-consuming, especially at high frequencies under which the impact caused by non-superconducting structures has to be taken into account. As a comparison experimental measurement is the most trustworthy way, which however is costly and also time-consuming.

(c) The existing AC loss reduction techniques have been reviewed in [37]. However, reduction of dynamic loss/AC loss in a superconducting magnet remains a challenging task, especially at high frequencies up to the kHz level [49, 116, 117]. In terms of HTS flux pumps, dynamic resistance has been well exploited to generate a time-averaged DC output voltage as the power source for
externally connected superconducting circuits. Nevertheless, dynamic loss caused by dynamic resistance is inevitable in HTS flux pumps. In other words, the occurrence of dynamic resistance is the key to achieving an HTS flux pump but the efficiency of the flux pump system is also constrained by the dynamic resistance. Therefore, a trade-off needs to be found to make the most of the dynamic resistance effect.

On the basis of the above-mentioned analysis, it can be seen that analytical formulas able to accurately quantify dynamic resistance/loss in complex superconducting topologies and physical scenarios deserve future research efforts. Additionally, efficient electromagnetic-thermal numerical models that can consider the multilayer structure of commercial superconducting tapes in a 3D topology need to be developed to estimate the dynamic resistance/loss with acceptable accuracy. With respect to large-scale superconducting applications, effective dynamic loss mitigation approaches should be further investigated.

It is believed that the review can deepen the understanding of the electromagnetic interactions and the unique feature of superconductors, and provide a significant guideline for future applications of dynamic resistance to superconducting energy systems.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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