SAGITTARIUS B2 MAIN: A CLUSTER OF ULTRA-COMPACT H\text{\textsc{ii}} REGIONS AND MASSIVE PROTOSTELLAR CORES

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ABSTRACT

The ionized core in the Sgr B2 Main star-forming region was imaged using the Submillimeter Array archival data observed for the H26\alpha line and continuum emission at 0.86 mm with an angular resolution 0′·3. Eight hyper-compact H26\alpha emission sources were detected with a typical size in the range of 1.6–20 × 10\textsuperscript{2} AU and electron density of 0.3–3 × 10\textsuperscript{3} cm\textsuperscript{-3}, corresponding to the emission measure 0.4–8.4 × 10\textsuperscript{10} cm\textsuperscript{-6} pc. The H26\alpha line fluxes from the eight hyper-compact H\text{\textsc{ii}} sources imply that the ionization for each of the sources must be powered by a Lyman continuum flux from an O star or a cluster of B stars. The most luminous H26\alpha source among the eight detected sources requires an O6 star that appears to be embedded in the ultra-compact H\text{\textsc{ii}} region F3. In addition, ~23 compact continuum emission sources were also detected within the central 5′′ × 3′′ (~0.2 pc) region. Under the assumption of a power-law distribution for the dust temperature, with the observed brightness temperature of the dust emission we determined the physical properties of the submillimeter emission sources, showing that the molecular densities are in the range of 1–10 × 10\textsuperscript{3} cm\textsuperscript{-3}, surface densities between 13 and 150 g cm\textsuperscript{-2}, and total gas masses in the range from 5 to ≥ 200 M\odot, which are one or two orders of magnitude greater than the corresponding values of the Bonnor–Ebert mass. With a mean free-fall timescale of 2 × 10\textsuperscript{3} years, each of the massive protostellar cores is undergoing gravitational collapse to form new massive stars in the Sgr B2 Main core.

Key words: Galaxy: center – H\text{\textsc{ii}} regions – ISM: individual objects (Sgr B2) – radio continuum: ISM – radio lines: ISM – stars: formation

Online-only material: color figure

1. INTRODUCTION

The formation of massive stars is one of the challenging problems in astrophysics. Unlike their low-mass counterparts, massive stars are rare and form in relatively deeply embedded massive molecular clouds on a much shorter timescale (McLaughlin & Pudritz 1997; Osorio et al. 1999; McKee & Tan 1997). High-mass stars are often found in clusters (McKee & Ostriker 2007). After initial gravitational collapse of the natal clouds, multiple protostellar cores are formed through fragmentation of the gas. One of the fundamental theoretical questions is how the massive stars form in clusters. According to the competitive theory (Bonnell et al. 2004; Bonnell & Bate 2006; Bonnell 2008), mass clumps created from natal cloud gravitational collapse contain protostellar cores with small masses. These cores grow by accreting matter, competing with other cores, and form stars with many times their original mass. On the other hand, the direct gravitational collapse theory (Krumholz et al. 2005) suggests that the protostellar cores created from fragmentation of the initial cloud gravitational collapse have sufficient mass to form individual high-mass to low-mass star systems in the subsequent collapse. Accretion from the parent cloud continues but does not substantially change their mass.

Strong radiation pressure from a newly formed massive star might halt infall, limiting the mass of stars that can form (Kahn 1974; Wolfire & Cassinelli 1987; Larson & Starrfield 1971). Recent theoretical investigations suggest that the radiation pressure limit might be overcome in actual cases with complex, non-spherical infall geometries or high ram pressures in rotating disks (McKee & Tan 2002; Krumholz et al. 2009; Nakano 1989), or massive stars may form from stellar merging in a dense cluster (Bonnell et al. 1998; Bonnell & Bate 2002).

Numerical simulations suggest that gravitational instabilities cause the disk to fragment and form a massive companion to the primary (Krumholz et al. 2009), and consequently form binaries in a dense stellar system (Bonnell & Bate 2002). Furthermore, radiation feedback from massive stars in a cluster affects its subsequent fragmentation and consequently plays an important role in determining the stellar initial mass function in a cluster (Krumholz et al. 2010).

In the past decade, good progress has been made in understanding massive young stellar clusters in the Galaxy. A recent review (Figer 2008) shows 10 known Galactic clusters with masses ≤ 10\textsuperscript{4} M\odot with ages of a few million years. The Arches cluster in the Galactic center is the densest young cluster in the Galaxy and contains a large collection of massive stars (Figer 2005). Three out of ten (Quintuplet, Arches, and Center) clusters are located within the central 50 pc of the Galaxy, suggesting that the Galactic central region appears to prefer forming massive stars.

Located at a distance of 7.8 kpc (Reid et al. 2009) from the Galactic center, Sgr B2, a giant molecular cloud (GMC) with a mass of 6 × 10\textsuperscript{6} M\odot (Goldsmith & Lis 1990) is one of the most active star-forming regions in the Galaxy, radiating a total luminosity of 1 × 10\textsuperscript{7} L\odot (Goldsmith & Lis 1990; Goldsmith et al. 1992). As the most luminous core among the several cores in this GMC, Sgr B2 Main is associated with numerous ultra-compact (UC) H\text{\textsc{ii}} regions, suggesting the presence of a tight cluster of OB stars (Gaume & Claussen 1990; Gaume et al. 1995; De Pree et al. 1998). The observed molecular outflows and infalls suggest that ongoing star formation activities are taking place (Qin et al. 2008; Rolffs et al. 2010). Sgr B2 Main appears to be in a very young phase of forming a massive stellar cluster from the dense molecular core.
High-resolution observations using the Submillimeter Array (SMA)\textsuperscript{3} at submillimeter wavelengths can explore the detailed structure of the massive star-forming core, providing useful clues on how massive stars form in a cluster.

The rest of this paper is organized as follows. In Section 2, we discuss the reduction and imaging process of the SMA archival data of Sgr B2 Main observed in 2007. Section 3 shows the results from the high-resolution observations of the H26α line and continuum at 0.86 mm. Section 4 presents a model for the H26α sources. In Section 5, the properties of the bright, compact dust emission clumps are determined and discussed. In Section 6, we discuss the kinematics in terms of ionized outflows/expansions and rotating disks. The early phase of massive star formation and origin of the massive protostellar clumps are also discussed. In Section 7, we summarize the conclusions.

2. OBSERVATIONS AND DATA REDUCTION

The interferometer data for the H26α line at ν0 = 353.623 GHz were acquired from the SMA archive, observed in 2007 June 18, with the “very extended” array configuration in the upper-side band (USB). The reduction for the line data was made in Miriad (Sault et al. 1995) following the reduction instructions for SMA data.\textsuperscript{4} The bandpass calibration was made by applying a normalized average of all the data from the QSOs included in the observing run (J1229+020, J1733−130, J1743−038, J1751+096, J2015+371). The flux density scale was determined by comparing observations of Callisto with the SMA planetary model. An angular size and brightness temperature of 1.5′ and 120 K at the observing epoch were assumed.

To separate line and continuum emission, we used the Miriad task UVLIN. The line visibility data set was constructed by subtracting the continuum which was determined from line-free channels. We made images of the H26α line with a channel width 3 km s\textsuperscript{−1} using a robustness weighting of 2 corresponding to natural weighting. The FWHM beam is 0′.40 × 0′.28 (P.A. = 15°). The typical rms noise in a channel image is 0.1 Jy beam\textsuperscript{−1}.

A line-free continuum data set was also produced from the UVLIN program. We used both USB and LSB data to image the continuum emission with robustness weighting parameters 2 and −2 which correspond to natural and uniform weighting, respectively. The FWHM beam with uniform weighting is 0′.36 × 0′.22 (P.A. = 13°). The typical rms noise in the continuum images are 7.5 and 8.5 mJy beam\textsuperscript{−1} for natural and uniform weighting, respectively. The shortest sampled spatial frequency in the visibility data is 30 kλ, corresponding to an angular size ∼6′−7′ for the H26α line and continuum emission structure. The compact structure (<3′) in the H26α line and continuum emissions from the Sgr B2 Main core has been adequately sampled in the SMA observations. Table 1 gives a summary of observations and imaging.

We noticed a systematic offset of 0′.3 in the phase center of the SMA images by comparing the positions of the UC H II regions G and F10.37 determined at 22.4 GHz with the Very Large Array (VLA; Gaume et al. 1995). The offset could reflect residual errors in antenna positions and complex gain calibration due to the large distance between the calibration QSO J1733−130 and SgrB2 Main. We therefore imposed a shift of 0′.3 for the phase center of the SMA data to align with the VLA image. The positional error of a compact source with S/N = 6 in the final SMA images with respect to the VLA coordinate frame is ∼0′.03 (see below).

3. RESULTS

3.1. H26α Line Emission

3.1.1. Distribution

Figure 1 shows the integrated H26α line image of Sgr B2 Main made from line emission in the LSR velocity range

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\textsuperscript{4} http://www.cfa.harvard.edu/sma/miriad
between $-100$ and $+225$ km s$^{-1}$. Eight sources with significant H26$\alpha$ line emission (>6$\sigma$ in the channel image) are labeled as H26$\alpha$-$n$, where $n$ is numbered from 1 to 8. The source H26$\alpha$-1 has a small angular offset from the brightest continuum source (F3) at 22.4 GHz in the core of Sgr B2 Main while H26$\alpha$-8 has a larger angular distance from F3 (Gaume et al. 1995). The two relatively isolated bright line sources, H26$\alpha$-8 and H26$\alpha$-5, were used to align the 0.86 mm coordinate frame of our SMA images with the coordinate frame determined with the VLA at 22.4 GHz. The image shows that the submillimeter positions of H26$\alpha$-8 and H26$\alpha$-5 agree with the 22.4 GHz positions of G and F10.37 within $\sim0''01$. H26$\alpha$-6 appears to coincide with F10.303. However, H26$\alpha$-1, the brightest line source, is located $0''17$ northwest of F3. H26$\alpha$-6 coincides with F10.303. Two ultra-compact H26$\alpha$ line sources (3 and 4) appear to be associated with F1. The hyper-compact H26$\alpha$ sources 3 and 4 are located $\sim0''15$ northeast and southwest of F1, respectively. Also, H26$\alpha$-2 is located $0''2$ southwest of F4. No significant H26$\alpha$ line emission associated with F2 has been detected. The positional offsets between the inner four H26$\alpha$ emission sources (1, 2, 3, and 4) and their corresponding 22.4 GHz counterparts (F3, F4, and F1) appear to be significant, reflecting the fact that the free–free continuum emission at 22.4 GHz traces relatively lower-density ionized gas in the outer layer of the expanding H$\alpha$ gas or the ionized outflows, while the H26$\alpha$ line traces the high-density ionized regions, possibly associated with the ionized disks or ultra- or hyper-compact ionized cores where the free–free emission appears to be optically thick at 22.4 GHz.

### 3.1.2. Line Broadening

The spectral profiles of the H26$\alpha$ line from the eight brightest UC H$\alpha$ regions in the core of Sgr B2 Main are shown in Figure 1. H26$\alpha$-1, associated with F3, has a peak velocity $V_{LSR} = 64 \pm 1$ km s$^{-1}$ with a line width $\Delta V_{FWHM} = 40 \pm 1$ km s$^{-1}$ compared with $V_{LSR} = 68 \pm 2$ km s$^{-1}$ and $\Delta V_{FWHM} = 63 \pm 5$ km s$^{-1}$ derived from the VLA measurements of the H6$\alpha$ line (De Pree et al. 1996). The peak velocity between the H26$\alpha$ and H6$\alpha$ line profiles shows a $2\sigma$ difference. The line width of the H6$\alpha$ is much broader than that of H26$\alpha$. Using canonical values of $T_e = 1 \times 10^4$ K and $n_e = 1.1 \times 10^6$ cm$^{-3}$ for the typical UC H$\alpha$ regions in Sgr B2 Main (De Pree et al. 1998), for the VLA observations of the continuum emission at 7 mm, the thermal broadening $\Delta V_{th}$ contributes 21 km s$^{-1}$ to the line widths. The pressure broadening depends upon the principal quantum number $n$ and the hyper-compact ionized gas.

![Figure 1. SMA image of H26$\alpha$ line emission at the rest frequency of 353.623 GHz, integrated from the velocity range of $-100$ to $125$ km s$^{-1}$ (red contours and color) overlaid on the continuum image at 0.86 mm (yellow contours). The red contours are $F_n(H26\alpha) \times (4\%, 5\%, 7\%, 11\%, 17\%, 25\%, 35\%, 47\%, 61\%, 77\%, 95\%)$ and the yellow contours are $S_n(0.86 \text{ mm}) \times (2\%, 3\%, 5\%, 10\%, 15\%, 25\%, 35\%, 45\%, 55\%, 65\%, 75\%, 85\%, 95\%)$, where the peak line flux $F_n(H26\alpha)$ and peak continuum flux density $S_n(0.86 \text{ mm})$ are $193$ Jy beam$^{-1}$ and $28$ Jy/beam km/s, respectively. The FWHM beam is $0''15$ northeast and southwest of F1, respectively.](image_url)
number $N$ and electron density $n_e (\Delta V_p \propto n_e N^{7.4})$. Brocklehurst & Leeman 1971; Brocklehurst & Seaton 1972). We find that the pressure broadening ($\Delta V_p \sim 0.02 \text{ km s}^{-1}$) is negligible for the H26α line. Thus, the non-thermal broadening $\Delta V_{nth}$ including the Doppler motions of expansion, infall, outflows, rotation, shocks, and turbulence, obtained by subtracting the thermal broadening from the observed H26α line width, is less than 34 km s$^{-1}$.

For the H66α line, the pressure broadening for gas with $n_e = 1.1 \times 10^6 \text{ cm}^{-3}$ accounts for 23 km s$^{-1}$. Subtracting $\Delta V_{th}$, $\Delta V_{nth}$, and $\Delta V_p$ from the observed $\Delta V_{FWHM}$ for the H66α line, $\sqrt{\Delta V_{FWHM}^2 - (\Delta V_{th}^2 + \Delta V_{nth}^2 + \Delta V_p^2)} \approx 43 \text{ km s}^{-1}$, the residual line width appears still to dominate the observed H66α line width, suggesting that the electron density estimated from the 7 mm continuum data (De Pree et al. 1998) might be underestimated due to the large optical depth of the free–free emission from the higher density gas. If the residual line width is all due to pressure broadening, the observed large H66α line width implies that a hyper-compact ionized core with an electron density $n_e > 2.5 \times 10^6 \text{ cm}^{-3}$ is likely embedded in F3.

### 3.2. Continuum Emission

The contours in Figure 1 show the continuum emission with the same FWHM beam ($0.34 \times 0.28$) as the H26α line. The continuum flux densities corresponding to the H26α line emission regions are summarized in Column 8 of Table 2. A large fraction of the continuum emission at 0.86 mm appears to arise from a region outside the H26α line sources, revealing an overall extended filamentary structure northwest to southeast. Figure 2 shows the details of the continuum emission at the higher resolution ($0.36 \times 0.22$). In addition to the UC H ii regions observed at millimeter and centimeter wavelengths at the positions denoted with the red dots, we identified 23 continuum sources at submillimeter wavelengths which are listed in the bottom section of Table 2 and are named as Smm—with a sequential number (Column 1) based on their angular distance $\Delta \theta$ from F3 (Column 7). The corresponding sequential numbers of the Smm sources are also marked in Figure 2. Using Gaussian fitting, we determined their positional offsets from F3 in R.A. and decl. (Columns 2 and 3), deconvolved sizes (Column 4), and peak brightness and total flux densities (Columns 5 and 6). The brightness temperatures of the sources are given in Column 8.
We note that the two brightest continuum cores are Smm-1 and Smm-4 with brightness temperatures 340 and 270 K, respectively. Smm-1 is located close to but with a significant offset from H26α-1 which is associated with F3. Smm-4 is located 0.06 ± 0.02 southeast of its H26α counterpart H26α-4 in the F1 complex.

4. MODEL FOR UC H II REGIONS

On the assumption of isothermal homogeneous H II gas, a model has been developed for the low-frequency radio recombination lines (RRLs) including non-LTE effects, such as stimulated emission by the background radiation (Shaver 1975). This model has been applied successfully to the high-frequency RRLs (H30α) at 231.9 GHz from the high-density gas in the minispiral within the circumnuclear disk of the Galactic center (Zhao et al. 2010). The effects of the radiation from both synchrotron and dust sources seem to be negligible for the H30α line under the conditions within the central few parsecs of the Galaxy. In the massive star-forming cores such as Sgr B2 Main, a considerable fraction of the continuum flux density at submillimeter wavelengths is from dust radiation that needs to be included in the model. Using a homogeneous isothermal model with temperatures of $T_C$ and $T_d$ for both H II gas and dust, the flux densities of hydrogen recombination line ($S_L$) and continuum ($S_C$) emission at radio to submillimeter wavelengths can be formulated assuming that the total dust is evenly distributed in three regions (each with an optical depth $\tau_d$) along the line of sight—one-third of the dust is uniformly mixed with the ionized gas in a given H II region and other two-thirds are located in front of and behind the H II region, respectively. Equations (1) and (2) give the full solutions for the line and continuum flux densities from the three regions with no assumptions of small optical depths. Spectral fits to the data are shown in Figure 3.

The results are not significantly different from an optically thin dust approximation. This is because, in the centimeter to submillimeter wavelength range discussed in this paper, the dust emission toward the UC H II regions is indeed optically thin and we are not able to discern between the two models. However, at mid-IR (Spitzer) wavelengths, the dust becomes opaque and the dust attenuation toward the H II regions becomes important.

$$S_L = \frac{2k\nu^2}{c^2} \Omega T_C \left[ \left( \frac{\tau_L/\beta_N + \tau_C}{\tau_L + \tau_C} \right) \left( 1 - e^{-(\tau_C + \tau_d)} \right) \right] e^{-\tau_d}$$

and

$$S_C = \frac{2k\nu^2}{c^2} \Omega T_C \left( 1 - e^{-\tau_C} \right) \frac{\tau_C}{t_C} e^{-\tau_d}$$

where $k$ is Boltzmann's constant, $c$ is the speed of light, and the total continuum optical depth in the H II region $\tau_C = \tau_C + \tau_d$ includes both the free–free and dust contributions. The recombination line ($S_L$) and free–free continuum ($S_C$) optical depths of the radiation from an H II region are given by Equations (A3) and (A4) in the Appendix, respectively. The values of the population departure coefficients ($\beta_N$ and $\beta_E$) were calculated using the non-LTE code of Gordon & Sorochenko (2009) based on the analysis of hydrogen recombination lines at wavelengths from radio to submillimeters (Walmsley 1990). The quantity $B(T_d)$ is the Planck function with a dust temperature $T_d$. In Equation (1), the first term associated with $2k\nu^2/\Omega T_C$ is the line emission from the H II gas attenuated by the foreground dust with optical depth $\tau_d$; the second term associated with $B(T_d)\tau_d\Omega$ accounts for the absorption of the internal dust radiation by the H II gas if $\tau_L > 0$ or the line emission stimulated by the internal dust radiation if $\tau_L < 0$; and the third term associated with $B(T_d)(1 - e^{-\tau_L})\Omega$ is the absorption of background dust radiation by the H II gas if $\tau_L > 0$ or the line emission stimulated by the background dust radiation if $\tau_L < 0$. On the other hand, for an isolated dust source, the dust continuum flux density is

$$S_d = B(T_d)(1 - e^{-\tau_d})\Omega.$$  

The optical depth $\tau_d$ of dust continuum radiation can be described as

$$\tau_d = \kappa_\nu \left( \frac{v}{v_0} \right)^\beta M_d D^{-2} \Omega^{-1},$$

where $\kappa_\nu = \kappa_0 (v/v_0)^\beta$ is the dust opacity per unit dust mass, $M_d$ is dust mass, $D$ is the distance, and $B_\nu(T_d)$ is the Planck function.
Figure 3. Models for the UC H II regions F3, F4, F1A, F1B, F10.37, F10.303, F10.39, and G with the constraints from the observed radio continuum flux densities and the fluxes of hydrogen recombination lines at radio and submillimeter wavelengths. The red circles denote the integrated flux (1.064 $S_p \Delta V_{\text{FWHM}}$) of the hydrogen recombination lines at H26$\alpha$ (from the SMA measurements of this paper), H52$\alpha$ and H66$\alpha$ from the VLA measurements (De Pree et al. 1996). The black dots are the flux densities determined at 2 cm, 1.3 cm, and 7 mm with the VLA; 3 mm with the Hat Creek interferometer array; and 1.3 mm and 0.86 mm with the SMA (see Section 4.1, the 1.3 mm data from L. Zhu 2010, private communication). The red curves indicate the fluxes of the hydrogen recombination lines from the best-fitted model (see Section 4) to the observed data. The black curves indicate the continuum spectral energy distribution resultant from the free–free (flat dashed curves corresponding to the first term in Equation (2)) and dust (steep rising curves corresponding to the second term in Equation (2)) emissions calculated from the best-fitted model.

(A color version of this figure is available in the online journal.)

with a dust temperature of $T_d$. In the calculations throughout this paper, we adopted $\kappa_0 = 1.06 \, \text{cm}^2 \, \text{g}^{-1}$ at the reference frequency $\nu_0 = 230 \, \text{GHz}$ calculated by Ossenkopf & Henning (1994) for high-density gas $n_H = 10^7 \, \text{cm}^{-3}$ using the standard MRN model (Mathis et al. 1977) with thin ice mantles. The physical parameters for a given H II region can be determined by fitting an isothermal, homogeneous model to the data at the submillimeter wavelengths of this paper and the data taken from previous observations at radio and millimeter wavelengths as discussed in the following section.

4.1. Radio Continuum Spectrum and RRL

F3 is the brightest UC H II region in the Sgr B2 Main core as observed at wavelengths of 2, 1.3, and 0.7 cm (Gaume & Claussen 1990; Gaume et al. 1995; De Pree et al. 1998). F3, which is associated with the brightest H26$\alpha$ line source, H26$\alpha$-1, was detected at 2 cm with a peak flux density 0.138 Jy beam$^{-1}$ (with a circular beam of 0\'.3) and a total flux density 0.48 Jy in an extended region of size 0\'.7, FWHM (Gaume & Claussen 1990). With a size 0\'.5 $\times$ 0\'.5, FWHM, the peak intensity and total flux density at 1.3 cm are 0.22 Jy beam$^{-1}$ and 0.9 Jy (Gaume et al. 1995). Five H II components have been detected at high angular resolution (0\'.049 $\times$ 0\'.079) with the VLA at 7 mm (De Pree et al. 1998). The authors show that the major component F3-d has a shell- or ring-like morphology. In observations with the Hat–Creek interferometer array at 3.5 mm, Liu & Snyder (1999) showed that the peak emission of 1.51 Jy beam$^{-1}$ (FWHM = 1\'.1 $\times$ 0\'.5) is coincident with F3 corresponding to a total flux density of 4.38 Jy for F1, F2, F3, and F4. The high-resolution (0\'.64 $\times$ 0\'.31) SMA image at 1.3 mm shows that the peak emission of 1.8 Jy coincides with F3. The H66$\alpha$ line has been detected with 1\'.1 $\times$ 1\'.6 resolution at $V_{\text{LSR}} = 68 \pm 2 \, \text{km} \, \text{s}^{-1}$, showing a line-to-continuum (L/C) ratio of 3% $\pm$ 0.3% and $\Delta V_{\text{FWHM}} = 63 \pm 5 \, \text{km} \, \text{s}^{-1}$ determined from integrated line profile (De Pree et al. 1996). At the lower resolution 2\'.9 $\times$ 1\'.6, the H52$\alpha$ line is toward the F cluster at $V_{\text{LSR}} = 57 \pm 2 \, \text{km} \, \text{s}^{-1}$ with L/C of 14% $\pm$ 2% and $\Delta V_{\text{FWHM}} = 59 \pm 2 \, \text{km} \, \text{s}^{-1}$ (De Pree et al. 1996). De Pree et al. (1996) noticed that the LTE electron temperature ($T_e$) derived from the high L/C value of the H52$\alpha$ line is significantly lower by a factor of two than that derived from the H66$\alpha$ line under the assumption of optically thin and LTE gas. If the L/C ratios are uniform
across the F complex, the fluxes of the H66α and H52α lines from the dominant source F3 can be estimated from the L/C values and the continuum flux densities at the corresponding wavelengths.

For the other seven regions associated with the H26α line source, the continuum flux densities at 2, 1.3, and 0.7 cm used in this paper are from Gaume & Claussen (1990), Gaume et al. (1995), and De Pree et al. (1998). The H66α line flux from the UC H II region G (H26α-8) is estimated from the L/C ratio determined by De Pree et al. (1996) and the continuum flux density of Gaume et al. (1995).

Figure 3 shows the observed spectra of the continuum (black dots) and hydrogen recombination lines (red dots) at radio and submillimeter wavelengths.

In order to fit to both the continuum and hydrogen recombination line data obtained from the high-resolution observations at wavelengths in the range between 2 cm and 0.86 mm, we considered a few possible models. A model with an LTE approach and a single isothermal, homogeneous H II component was rejected since the observed large H26α line flux requires large free–free continuum optical depths (τC > 1) at longer millimeter and centimeter wavelengths for a single component model. At these wavelengths, the dust absorption (τd ≪ 1) is negligible. Then, the predicted line fluxes at centimeter wavelengths for larger quantum-number transitions are much smaller than the observed values because of the exponential drop in the LTE line flux due to the large free–free optical depth:

\[ S_\nu^C = \frac{2k\nu^2}{c^2} \Omega_\nu (1 - e^{-\tau_\nu}) e^{-\tau_\nu}. \]

The observed line flux densities at centimeter wavelengths could be fitted with a single isothermal, homogeneous component by adjusting n_e and T_e if the stimulating effect in a non-LTE approach is considered. However, the flat continuum spectra at centimeter wavelengths seen in all the eight H II sources (Figure 3) place a critical restriction on a model with a single isothermal, homogeneous component, requiring an additional lower-density component. Thus, we fit each of the eight observed hyper-compact H26α sources and the UC H II regions surrounding them with two components corresponding to high- and low-density ionized gas with small and large sizes, A and B, respectively. Further assumptions used in the two-component model are the volume filling factors f_v = 0.1 and 0.5 for components A and B, respectively. The effect of shadowing between the two components is negligible.

4.2. Hyper-compact H II Component

Figure 3 shows the results of model fitting the observed hydrogen recombination line (red curves) and continuum flux densities (black curves) from each of the eight UC H II regions with two isothermal, homogeneous H II components A and B. The best-fitted parameters are summarized in Table 3. The first row gives the size in units of 10^6 AU. The following rows under the category of H II gas properties summarize the derived parameters including the temperature T_e, density n_e, volume filling factor f_v, the departure coefficients b_0 and b_N, the optical depths of the H26α line τC(H26α), the free–free continuum τC(0.86 mm) at 0.86 mm, the fractional contribution δH26α to the observed H26α line flux from each component, the H66α line τC(H66α), and the free–free continuum τC(λ13 mm) at 13 mm. Components A have small size (160–2000 AU), high electron density (3–33 × 10^6 cm^−3), and relatively lower temperature (5–9 × 10^3 K). The A components are typically 10 times smaller but 100 times denser than typical UC H II regions, 10^17 cm and 10^4 cm^−3 (Churchwell 2002), representing a class of hyper-compact H II components (Churchwell 2002) present in the Sgr B2 Main core. The hyper-compact H II components could arise from plausible ionized disks with emission measure (EM) in the range 0.4–8.4 × 10^6 cm^−6 pc, accounting for most of the H26α line flux, e.g., ∼94% for F3. The hyper-compact H II components in all the H II cores show a negative optical depth for the H26α line, suggesting the presence of stimulated emission by free–free continuum emission from the hyper-compact H II region itself. For F3, the H26α line is enhanced by a factor of 1.4 due to a weak stimulating process within the hyper-compact H II component (A) while the enhancement factors (S_b/S_a) are 1.7 and 4 for the H26α lines from G and F10.39 which show larger negative line optical depths of −0.71 and −1.3, respectively. At 13 mm, the continuum component A becomes optically thick while the H66α line is still optically thin, τC(13 mm) = 9.5 and τC(H66α) = 0.13 for F3. Component A appears to make a little contribution to the total line flux of the H66α transition (∼20% for F3).

4.3. Ultra-compact H II Component

The continuum emission at longer wavelengths from each of the UC H II regions shows a shallow rising spectrum toward short wavelengths, indicating the presence of a larger H II component which might result from an ionized stellar wind or expansion of an ionized nebula. Component B with a relatively large size (1200–4000 AU), lower density (0.1–3 × 10^5 cm^−3), and higher temperature (10–17 × 10^4 K) is needed for the extended ionized gas with relatively smaller EMs of ≤1 × 10^6 cm^−6 pc. T_e = 17,000 K for the UC H II component in F3 (H26α-1) appears too high for the H II regions with metallicities in the Galactic center. The overestimated T_e from our fitting might be caused by the underestimate of the H66α and H52α line fluxes at 1.3 and 0.7 cm due to large line broadening and limited bandwidth coverage of the old VLA system. For a canonical value of T_e = 10,000 K, the corresponding H66α and H52α line fluxes require 50% more than the values used in this paper. The physical parameters of the B components fall into the category of UC H II region in Churchwell (2002). The continuum emission from the B components is optically thin at longer millimeter wavelengths, providing a flat spectrum (∝ ν^−0.1) in contrast to the steep rising spectrum (∝ ν^2) of component A. With its larger size, the free–free emission from the lower density gas (component B) dominates the continuum flux densities at 13 mm. Superposition of the spectra from the two components (A and B) results in a spectrum slowly rising at long wavelengths and turning over to flat at short millimeter and submillimeter wavelengths.

4.4. Dust Component

The composite spectra (dashed flat curves in Figure 3) of components A and B appear to fit the observed continuum flux densities at millimeter and centimeter wavelengths. In the submillimeter, the observed flux densities show a significant excess emission from the UC H II region over the ones predicted from the isothermal, homogeneous ionized gas model with two density components, suggesting that dust radiation becomes significant in the continuum flux density. In order to evaluate the contribution of dust radiation in the UC H II regions, the terms involving the dust radiation B(T_d) in Equations (1) and (2) are
Table 3
Hyper- and Ultra-compact H\textsc{ii} Cores in Sgr B2 (Main)

| Physical Parameters | H26α-1 (F3) | H26α-2 (F4) | H26α-3 (F1-A) | H26α-4 (F1-B) | H26α-5 (F10.37) | H26α-6 (F10.303) | H26α-7 (F10.39) | H26α-8 (G) |
|---------------------|--------------|--------------|----------------|----------------|----------------|----------------|----------------|------------|
| R (10^3 AU)         | A            | B            | A              | B              | A              | B              | A              | B          |
| T_e (K)             | 9.0          | 17           | 10             | 10             | 9.5            | 14             | 5.0            | 10         |
| n_e (10^5 cm^{-3})  | 42           | 3.2          | 50             | 2.0            | 52             | 3.1            | 85             | 0.1        |
| \psi(0.186 mm)      | 0.1          | 0.5          | 0.1            | 0.5            | 0.1            | 0.5            | 0.1            | 0.5        |
| \beta_{26}         | -7.4         | -13.8        | -7.0           | -11.2          | -6.7           | -13.6          | -3.1           | -4.2       |
| \tau_{L(H_26\alpha)} | -0.60       | -0.008       | -0.30          | -0.012         | -0.33          | -0.015         | -1.1           | ~0.0       |
| \zeta_{H_26\alpha} | 0.024        | <0.001       | 0.012          | <0.001         | 0.019          | 0.001          | 0.040          | ~0.0       |
| M_\text{tot} (M_\odot) | 11           | 1.9          | 11             | 7.5            | 5.4            | 5.9            | 2.9            | <0.01     |

H\textsc{ii} Gas Properties

| Physical Parameters | H26α-1 (F3) | H26α-2 (F4) | H26α-3 (F1-A) | H26α-4 (F1-B) | H26α-5 (F10.37) | H26α-6 (F10.303) | H26α-7 (F10.39) | H26α-8 (G) |
|---------------------|--------------|--------------|----------------|----------------|----------------|----------------|----------------|------------|
| T_e (K)             | 9.0          | 17           | 10             | 10             | 9.5            | 14             | 5.0            | 10         |
| n_e (10^5 cm^{-3})  | 42           | 3.2          | 50             | 2.0            | 52             | 3.1            | 85             | 0.1        |
| \psi(0.186 mm)      | 0.1          | 0.5          | 0.1            | 0.5            | 0.1            | 0.5            | 0.1            | 0.5        |
| \beta_{26}         | -7.4         | -13.8        | -7.0           | -11.2          | -6.7           | -13.6          | -3.1           | -4.2       |
| \tau_{L(H_26\alpha)} | -0.60       | -0.008       | -0.30          | -0.012         | -0.33          | -0.015         | -1.1           | ~0.0       |
| \zeta_{H_26\alpha} | 0.024        | <0.001       | 0.012          | <0.001         | 0.019          | 0.001          | 0.040          | ~0.0       |
| M_\text{tot} (M_\odot) | 11           | 1.9          | 11             | 7.5            | 5.4            | 5.9            | 2.9            | <0.01     |

Dust Properties (T_d = 500 K, \beta = 1.5)

| Physical Parameters | H26α-1 (F3) | H26α-2 (F4) | H26α-3 (F1-A) | H26α-4 (F1-B) | H26α-5 (F10.37) | H26α-6 (F10.303) | H26α-7 (F10.39) | H26α-8 (G) |
|---------------------|--------------|--------------|----------------|----------------|----------------|----------------|----------------|------------|
| \tau_{0.86 mm}     | 0.11         | 0.03         | 0.11           | 0.17           | 0.15           | 0.12           | 0.32           | <0.01     |
| M_\text{tot} (M_\odot) | 11           | 1.9          | 11             | 7.5            | 5.4            | 5.9            | 2.9            | <0.01     |
added to the B component model with the dust distribution as assumed. The flux density ($S_D$) of the dust radiation is determined using Equations (3) and (4) with an assumed dust temperature in the H II region equal to the peak temperature in the Sgr B2 Main core, $T_D \sim 500$ K (Lis & Goldsmith 1990) and a power-law index for the dust opacity $\beta = 1.5$ (Goldsmith & Lis 1990; Lis & Goldsmith 1990). The observed brightness temperatures of few times $10^5$ K for the continuum emission from several central dense clumps also suggest a high dust temperature for the UC H II regions in Sgr B2 Main (also see the discussion in Section 5 for the high dust temperature).

We show that, except for G, the dust radiation in the other seven regions makes a contribution to the observed continuum flux density at 0.86 mm comparable to or larger than that from free–free emission (see Figure 3). In general, the dust in each of the UC H II regions is optically thin, $\tau_D \sim 0.1$, with a total gas mass $\leq 11 M_{\odot}$. The dust optical depth and the gas mass ($M_{\text{gas}}$) needed in each of the UC H II regions are summarized in Table 3.

Finally, the models with two independent H II components mixed with dust involved a set of 10 free parameters, namely $2 \times (n_e, T_e, R, f_N, \text{and } T_D)$. For F3, we have collected a total of nine measurements in line and continuum flux densities from high-resolution observations at centimeter–submillimeter wavelengths. Adding the two determined angular sizes for the components A and B, we have a total of 11 measurements to constrain the model reasonably well. However, for the rest of the sources, the models need to be confined with further high-resolution observations of both continuum and line emissions at wavelengths from centimeters to submillimeters.

4.5. Lyman Continuum

The hyper-compact H II components detected with H26α lines appear to require most of the flux of Lyman continuum photons from the ionizing stars to maintain their ionization. The H26α line flux from hyper-compact sources would be excellent tracers for newly formed massive stars. Given an observed FWHM linewidth of $\Delta V_{\text{FWHM}}$ that is dominated by Doppler broadening $\Delta V_D$ in the H26α line profile, the ionizing photon flux required for the observed H26α line flux ($S_L$) can be estimated from Equation (A8), which was generally derived for a radio recombination line $N + 1 \rightarrow N$,

$$N_{\text{Lym}} = 7.8 \times 10^{46} \left( \frac{S_L \Delta V_D}{\text{Jy km s}^{-1}} \right) \left( \frac{10^8 \text{Hz}}{\nu_{\text{H26α}}} \right) \left( \frac{D}{8 \text{ kpc}} \right)^2 \times \left( \frac{T_e}{10^{5} \text{ K}} \right)^{1.5} \left( \frac{\alpha_{\text{B}}(T_e)}{\alpha_{\text{B}}(10^5 \text{ K})} \right) \Psi_{26}^{-1} \text{photons s}^{-1},$$

(5)

where $\alpha_{\text{B}}(T_e)$ is the total recombination coefficient to excited levels (Hummer & Seaton 1963) and the function $\Psi_{26}$ provides a correction for the effects of both non-LTE and optical depth. The values of $\Psi_{26}$ are given in Table 3 for each case. For the hyper-compact H II components (A) in Sgr B2 Main, $\Psi_{26}$ is in the range $\sim 1$ (H26α-6) to $\sim 2$ (H26α-7). For the UC H II components (B) where the H26α line is optically thick, $\Psi_{26} \approx \rho_{26}$, a correction for the factor due to the lower quantum-number levels that are underpopulated with respect to the LTE in the cases of lower electron density. The electrons at the level of $N = 26$ appear to be underpopulated by 20%–30% with respect to the LTE for the UC H II regions in Sgr B2 Main.

In addition, for H26α-1 (F3) assuming $T_e = 9000$ K, the value $N_{\text{Lym}} = 1.43 \times 10^{46}$ photons s$^{-1}$ is inferred from the observed H26α line flux 280 Jy km s$^{-1}$ using Equation (5) with $\Psi_{26} = 1.3$ while $N_{\text{Lym}} = 2.66 \times 10^{49}$ photons s$^{-1}$ is derived from the continuum flux density $S_C = 2.9$ Jy at $353.6$ GHz using Equation (4) given by Wilcots (1994). The value determined from the observed continuum flux density surpasses the value determined from H26α line flux by 46%. Our detailed model fitting gives the free–free flux density of 1.61 Jy, suggesting the fraction of dust contribution to the continuum is 44% at 353.6 GHz, in good agreement with the above assessment. Considering optically thin line emission and nearly no attenuation and contamination by the dust at $353.6$ GHz (0.85 mm), it appears to be an excellent way to determine the $N_{\text{Lym}}$ and the free–free flux density using the H26α line flux. Therefore, comparing the observed continuum flux density with the free–free flux density determined from the H26α line flux, one can give a good assessment of the fraction of dust contribution to the continuum.

4.6. Ionizing Stars

Based on the derived properties for both A and B components, we estimated the fluxes of the Lyman continuum photons ($N_{\text{Lym}}$) from the newly formed ionizing stars. In each of the UC H II regions, the hyper-compact component (A) appears to require more flux in Lyman continuum photons than its larger but lower-density counterpart (component B) for the maintenance of the ionization (see Table 3). A total flux in Lyman continuum photons ($N_{\text{Lym}}^{\text{A+B}}$) is evaluated by the addition of the individual fluxes required for components A and B. Assuming that $N_{\text{Lym}}^{\text{A+B}}$ accounts for all the Lyman continuum photons from a single early-type, zero-age-main-sequence (ZAMS) star in each of the UC H II regions with no significant leakages, i.e., the UC H II regions are internally ionized (De Pree et al. 1998), the type of massive stars required for each of eight H II complexes with a hyper-compact H26α component is inferred on the basis of the stellar atmosphere model computed by Panagia (1973) and listed in Table 3. We note that the Lyman continuum photon fluxes from Panagia’s (1973) stellar atmosphere model were underestimated by 26%–66% for O6- to B0-type stars, respectively, as compared to those computed from Vacca et al.’s (1996) model. In good agreement with De Pree et al. (1998), we also find that at least an O6 star is required for the brightest H26α source (H26α-1) in F3 and an O8.5 star for H26α-2 in F4.

The complex F1, including H26α-3 (O7), H26α-4 (O9.5), and H26α-6 (O9), has been resolved by the VLA with a resolution of $0.049 \times 0.079$ into at least seven compact components (De Pree et al. 1998). The authors suggest that the complex requires a group of seven early-B and late-O-type stars (B0 to O8.5) to maintain the ionization. The fluxes of Lyman continuum photons ($N_{\text{Lym}}$) inferred from our analysis of the H26α line for the hyper-compact sources in F3 and F1 appear to be considerably greater than the values inferred from their 7 mm counterparts based on the VLA continuum observations (De Pree et al. 1998). The difference occurs because the hyper-compact components H26α-1 and H26α-3 in F3 and F1 are optically thick in free–free emission at 7 mm, $T_c \approx 2$, inferred from the analysis above on the basis of H26α observations. The hyper-compact core appears to be deeply embedded in the optically thick region at 7 mm. From the VLA flux densities at 7 mm, the flux of Lyman continuum photons has possibly been underestimated due to missing the contribution from the hyper-compact ionized core. Thus, an O7 star might be needed to maintain the ionization of hyper-compact H26α-3 in the H II complex of F1.
The above argument is also valid for the hyper-compact H\(\text{II}\) components H26\(\alpha\)-5 (O9), H26\(\alpha\)-7 (O9.5), and H26\(\alpha\)-8 (O8.5). These hyper-compact components appear to be embedded in the relatively isolated UC H\(\text{II}\) regions F10.37, F10.39, and G, respectively. In fact, the continuum optical depths inferred for these three hyper-compact H\(\text{II}\) components are greater than the values derived for the remaining hyper-compact H\(\text{II}\) components (see Table 3).

As discussed above (see Figure 3), the H26\(\alpha\) line from the three components appears to be enhanced by stimulated emission from continuum emission in the hyper-compact H\(\text{II}\) gas surrounding the newly formed O stars. The actual flux of the ionizing photons is reduced by a factor of four in H26\(\alpha\)-7 as compared to an LTE source. Thus, the effect of stimulated line emission helps detect the H26\(\alpha\) line from a region ionized by an O9.5 among the eight H26\(\alpha\) line sources in SgrB 2 Main.

Given a hyper-compact H\(\text{II}\) region with \(T_e = 1 \times 10^4\) K, \(\Delta V_D = 30\) km s\(^{-1}\) without line stimulation, the 3\(\sigma\) (0.3 Jy beam\(^{-1}\)) detection limit for the H26\(\alpha\) line imposed by the SMA data used in this paper gives a limit of \(N_{\text{lym}} = 7.1 \times 10^{47}\) photons s\(^{-1}\), or \(\log(N_{\text{lym}}) = 47.85\), on the flux of ionizing photons, corresponding to an O9.5 ZAMS star. No significant detections of the H26\(\alpha\) line with the SMA toward F2 are consistent with four B0 stars inferred from the 7 mm continuum observations (De Pree et al. 1998).

4.7. Ages

The ages of newly formed O stars in Sgr B2 Main can be estimated by the dynamic timescale of the larger H\(\text{II}\) components (B) as the time for a sound wave traveling from the initial ionization front at a radius close to the Strömgren radius \(R_S\) (Strömgren 1939) as suggested by Spitzer (1978) and Garay & Lizano (1999). Using the Equation (3) given in Shi et al. (2010b) and the isothermal sound speed \(C_S = \sqrt{kT_e/m_H}\), we calculated the dynamical age of the H\(\text{II}\) components (B) and listed the values in Table 3, ranging between the oldest one, \(1.3 \times 10^5\) years for F3 (H26\(\alpha\)-1), and the youngest one, \(0.4 \times 10^5\) years for F10.37 (H26\(\alpha\)-7). If there is any external pressure imposed on the dust, the sound crossing time will give an underestimate of the age. The O-type stars in Sgr B2 Main appear to be clustered around a thousand years ago.

5. A CLUSTER OF PROTOSTELLAR CORES

The dust emission from Sgr B2 Main has been resolved into at least 23 components, showing that a cluster of protostellar cores is present in this region in addition to the newly formed O- and B-type stars suggested by SMA observations of the H26\(\alpha\) line and VLA observations of the radio continuum emission and H52\(\alpha\) and H66\(\alpha\) lines. From our interferometer observations, the brightness temperature \((T_b)\) of the continuum emission from individual components can be determined under the Rayleigh–Jeans–Janssen approximation,

\[
T_b(\nu) \approx 13.6\nu^2 \frac{S_\nu}{\text{Jy}} \theta_{\text{FWHM}}^{-2} \text{K},
\]

where \(S_\nu\) is the dust flux density at wavelength \(\lambda\) from the region with the geometric mean of angular size \(\theta_{\text{FWHM}}\) in arcseconds. Located near \(0^\circ13.00\pm0^\circ01\) north) the brightest H26\(\alpha\) line source in F3, Smm-1 has the highest brightness temperature of 340 K.

The contribution to the brightness temperatures in the continuum cores from the free–free continuum emission is less than 10%. Thus, the brightness temperature from dust emission is greater than 300 K in the center, suggesting that high dust temperature is present in Sgr B2 Main.

From Equations (3) and (5), for a given dust temperature \(T_d\), we can determine the optical depth \((\tau_d)\) for each of the dust cores from \(T_b(\nu)\):

\[
\tau_d(\nu) = \log \left[ \frac{T_d}{T_d - T_b(\nu)} \right].
\]

The dust temperature \((T_d)\) is affected by the increasing luminosity in the inner region (~0.1pc) of the Sgr B2 Main core where at least eight newly formed O stars are suggested by the H26\(\alpha\) line sources. We determined the physical properties of the protostellar cores using a power-law distribution for the mean dust temperature \(T_d\) as function of the radial distance \(r\) from the center. The distribution of \(T_d\) is modeled by Scoville & Kwan (1976) and Wolfire & Cassinelli (1987) assuming that the total emissivity of the collection of grains at that temperature equals the total radiative energy that is absorbed by the grains (Scoville & Kwan 1976; Wolfire & Cassinelli 1986),

\[
T_d = T_{\text{in}} \left( \frac{r}{r_{\text{in}}} \right)^{-\Gamma},
\]

where \(T_{\text{in}}\) is the dust temperature at \(r_{\text{in}} = 0.01\) pc, the radial distance between Smm-1 and H26\(\alpha\)-1 (O6 star) assuming a mean projection angle of 45°. Depending on the dust emissivity \((Q_{\nu})\), the central stellar luminosity is insensitive to the exponent \(\beta\) of the dust emissivity power law \((\Gamma = 2/5\) or 1/3 for \(\beta = 1\) or 2, respectively).

We used a linearly interpolated value of \(\Gamma = 0.37\) for \(\beta = 1.5\) in the modeling. Assuming dust heated primarily by the central O6 star with a luminosity of \(2.5 \times 10^5 L_{\odot}\), \(T_d(r_{\text{in}} = 0.007\) pc \(\approx 400\) K is inferred from the structure of the dust temperature derived by Scoville & Kwan (1976). On the other hand, a larger value of \(T_{\text{in}}(r_{\text{in}} = 0.007\) pc \(\approx 630\) K is calculated by Lis & Goldsmith (1990) in their detailed models assuming a total luminosity up to \(3 \times 10^5 L_{\odot}\) from the stars distributed in the core in Sgr B2 Main and exponent of \(\beta = 1.5\) in the dust emissivity power law. Because of high infrared opacity in the inner region of the core, the radiation from the outer region does not affect the dust temperature at the center. The observed brightness temperature, \(T_d = 300\) K, from the dust in Smm-1 indicates the dust temperature \(T_d \approx 470\) K if \(T_d \approx 1\), which is consistent with the kinetic temperature of the absorbing gas toward Sgr B2 Main (Cernicharo et al. 2006; Qin et al. 2008). We note that the dust sublimation radius \(R_{\text{sub}}\) by heating from the central star can be determined by balancing the dust absorption of the UV and visible radiation from the star with the re-emission at IR from the dust grains at their sublimation temperature \(T_{\text{sub}} = 1800\) K (Wolfire & Cassinelli 1986). For an O6 star, \(R_{\text{sub}} = 2.9 \times 10^{13}\) cm is about an order magnitude smaller than \(r_{\text{in}}\), and in the zone between the two radial distances, the dust substantially cools down due to infrared radiation. In the following calculation, we take \(r_{\text{in}} = 500\) K in the power-law exponent of \(\Gamma = 0.37\) for the dust temperature. Using Equation (8), we calculate \(T_d\) in the range 500 K for Smm-1 near the O6 star to 170 K for Smm-23 about 0.14 pc away. Using Equations (6)–(8), we find that dust optical depth lies between 0.3 and 3.0 at 0.86 mm for these protostellar cores with unity for the brightest core Smm-1. Table 4 summarizes the optical depths \((\tau_d)\).
In order to evaluate whether the individual dust cores are subject to inevitably undergoing gravitational collapse to form stars, we calculated the Bonnor–Ebert mass \( M_{\text{BE}} \), the maximum mass with which a dense core can remain in hydrostatic equilibrium (Bonnor 1956; Ebert 1957), and free-fall timescale \( t_{ff} \),

\[
M_{\text{BE}} = 1.18C_3^3 G^{-3/2} \rho^{-1/2}
\]

\[
t_{ff} = \frac{1}{4} \sqrt{\frac{3\pi}{2G\rho}},
\]

where \( C_S \), \( G \), and \( \rho \) are the sound speed, gravitational constant, and mass density of the core, respectively. The derived values for \( M_{\text{BE}} \) and \( t_{ff} \) are listed in Columns 9 and 10 of Table 4. The masses of the protostellar cores in the central region of Sgr B2 Main are one to two orders of magnitude greater than their corresponding Bonnor–Ebert mass. Free-fall timescales of \( 1-3 \times 10^3 \) years are inferred from our model calculation.

### 6. DISCUSSION

The kinematics of the F1-4 H\(\alpha\) complex and the discrete H\(\alpha\) regions in Sgr B2 Main are shown by the radial velocity distribution determined from the first moment images of the H\(26\alpha\) line in Figure 4. The observed kinematics imply certain dynamical processes in the ionized gas associated with the newly born massive stars in the complex. Most of the H\(26\alpha\) line emission arises from the high-density component (A) in an ionized disk.

#### 6.1. Candidate for Rotating Disk

H\(26\alpha\)-7 in F10.39, a weak stimulated source, shows a radial gradient, nearly north-to-south, with radial velocity difference of \( 9 \) km s\(^{-1}\) or \( V_r \approx 4.5 \) km s\(^{-1}\) across the central 0\('\)2, or \( 2R = 1600 \) AU, which could indicate an edge-on rotating disk.
Figure 4. Distribution of intensity-weighted radial velocity with respect to an LSR velocity of 58 km s\(^{-1}\) is overlaid on the contour image of the continuum emission at 0.86 mm (see Figure 1). The velocity image was constructed from the H\(2\alpha\) line image cube convolved with a circular beam of 0\(\prime\).03 using the moment 1 algorithm in Miriad with 4\(\sigma\) intensity cutoff. The contours are (0.25, 0.35, 0.45, \(\ldots\), 0.95) \(\times\) 2.5 Jy beam\(^{-1}\) with a circular beam of 0.3\(\prime\). The cross signs mark the peak positions of the H\(2\alpha\) line sources of 1, 2, 3, 4, and 6 in the central F1-4 complex (the main figure) along with the sources of H\(2\alpha\)-5, 7, and 8 in the discrete sources with large angular offsets from the center (the insets). Both the vertical and horizontal coordinates are the offsets (arcsec) from the registration center: R.A. (J2000) = 17:47:20.135 and decl. (J2000) = −28:23:04.53.

If the velocity difference is due to Keplerian rotation of the ionized gas in a disk with an inclination angle \(i\), then the enclosed mass is

\[
M \approx 1.1 M_\odot \left(\frac{V_i}{\text{km s}^{-1}}\right)^2 \left(\frac{R_{\text{10^3 AU}}}{10^3 \text{AU}}\right) \sin^2(i)
\]

\[
\approx 18 \sin^2(i) M_\odot, \tag{14}
\]

which is consistent with the mass of a late-O-type star as the ionizing source. However, we note that the observed kinematics of the two sources can also be interpreted as a rotating–expanding H\(\alpha\) shell. The data are not adequate to discriminate between disk and rotating–expanding shell models.

6.2. Confined H\(\alpha\) Outflow and Expanding Ionized Ring or Disk

F3 (H\(2\alpha\)-1) shows a large velocity gradient northwest-to-southeast (see Figure 4). The maximum radial velocity difference is 50 km s\(^{-1}\) or \(V_i \approx 25\) km s\(^{-1}\) across a projected angular size of 0\(\prime\).7 or 2\(R = 5600\) AU which is consistent with the angular size of the ring/disk seen in the VLA high-resolution (0\(\prime\).049 \(\times\) 0\(\prime\).79) observation at 7 mm (De Pree et al. 1998). The ring/disk is nearly circular with a ratio of minor-to-major axis size \(\theta_{\text{min}}/\theta_{\text{maj}} \gtrsim 0.5/0.7\), indicating an inclination angle \(i \lesssim 45^\circ\). If the velocity difference is due to Keplerian rotation of the ionized gas in the ring, then from Equation (14), an enclosed, dynamic mass \(1.9 \times 10^3 \sin^2(i) M_\odot\) can be inferred, which is at least an order magnitude greater than the inferred total mass of both the ZAMS-type O star (\(\sim 30 M_\odot\)) in H\(2\alpha\)-1 and the protostellar core Smm-1 (\(\sim 34 M_\odot\)) in F3 (see Tables 3 and 4). On the other hand, the electron temperature of \(T_e = 1.7 \times 10^4\) K, inferred for the large, lower-density component (B) from our model fitting, suggests that an isothermal sound speed \(C_S = \sqrt{kT_e/m_H} = 12\) km s\(^{-1}\) which is about a factor of three smaller than the maximum expansion velocity \(V_{\text{max}} = V_i \sin(i) \approx 35\) km s\(^{-1}\). However, the radiation pressure \(P_{\text{rad}} = \frac{L_e}{c R_{\odot}}\) due to a luminosity \(L_e\) from an O-type star would be comparable to the thermal pressure \(P_{\text{th}} = 2n_e k T_e\) in the H\(\alpha\) region with a radius of \(R\), where the factor of two accounts for equal densities of electrons \((n_e)\) and protons \((n_i)\),

\[
\frac{P_{\text{rad}}}{P_{\text{th}}} = 2.3 \left(\frac{L_e}{10^5 L_\odot}\right) \left(\frac{R_{\odot}}{10^3 \text{AU}}\right)^{-2} \times \left(\frac{n_e}{10^6 \text{cm}^{-3}}\right)^{-1} \left(\frac{T_e}{10^4 \text{K}}\right)^{-1}, \tag{15}
\]

where \(P_{\text{rad}}/P_{\text{th}}\) is the ratio of the radiation pressure to the thermal pressure. On the basis of the parameters derived for F3 (see Table 3), \(P_{\text{rad}}/P_{\text{th}} \approx 0.4\) and 0.7 for the high-density component (A) and the lower-density component (B), respectively. On the other hand, if the radial velocity gradient is mainly due to expansion or outflow, the ram pressure of the expansion motions is \(\rho V_{\text{ex}}^2\). Then, the ratio of the ram pressure to the thermal pressure is

\[
\frac{P_{\text{ram}}}{P_{\text{th}}} = 6.1 \times 10^{-3} \left(\frac{T_e}{10^4 \text{K}}\right)^{-1} \left(\frac{V_{\text{ex}}}{\text{km s}^{-1}}\right)^2. \tag{16}
\]
For the lower-density component (B) of F3, \( \frac{P_{\text{th}}}{P_{\text{rad}}} \approx 0.5 \) if \( V_{\text{ex}} \approx C_S \). The derived ratios of \( \frac{P_{\text{th}}}{P_{\text{rad}}} \) and \( \frac{P_{\text{rad}}}{P_{\text{th}}} \) appear to be consistent. Both the thermal and radiation pressures play a critical role in the expansion of the ionized gas and may accelerate ionized outflow. However, both the expansion and the plausible H \( \alpha \) outflow are thought to be confined in the dense ambient medium (De Pree et al. 1998). If the UC H \( \alpha \) region is in pressure equilibrium with the ambient medium (Xie et al. 1996), we have

\[
2k_n T_{\xi} = n_H (\mu m_0 \sigma_v^2 + k T_k), \tag{17}
\]

where \( \sigma_v \) is the turbulent velocity, \( T_k \) is the kinetic temperature of the ambient gas, approximately equal to the dust temperature \( T_d \), and \( \xi = 1 + \frac{P_{\text{rad}}}{P_{\text{th}}} \) considering the contribution from ram pressure. From Equation (17), we can find a minimum density \( n_H \) required to confine the H \( \alpha \) gas,

\[
n_H > 4 \times 10^5 \left( \frac{n_e}{10^6 \text{ cm}^{-3}} \right) \left( \frac{T_e}{10^4 \text{ K}} \right) \left( \frac{\Delta V_{\text{FWHM}}}{10^5 \text{ K} \cdot \text{cm}^{-1}} \right)^2 \text{ cm}^{-3}, \tag{18}
\]

where \( \Delta V_{\text{FWHM}} = \sqrt{(8 \ln 2)(\sigma_v^2 + \frac{r_{\text{th}}}{\xi})} \), the FWHM line width of a molecular line from the surrounding medium. For the SMA measurements of the FWHM line widths for the 16 kinematic absorption features in H\( _2 \)CO(3\( - \)30–2\( _{\text{O}} \)) and H\( _2 \)CO(3\( _{\text{I}} \)-2\( _{\text{O}} \)) toward the F1-4 complex (Qin et al. 2008), we find a variance-weighted mean \( \Delta V_{\text{FWHM}} = 5 \pm 0.2 \text{ km s}^{-1} \), suggesting a dominant turbulent motion in the line broadening. Thus, from Equation (18) and the parameters derived for the H \( \alpha \) regions (Table 3), a minimum \( n_H \) required to confine H\( _2 \)CO(3\( _{\text{I}} \)-2\( _{\text{O}} \)) sources \( \approx 2 \times 10^7 \text{ cm}^{-3} \) is needed to confine F3 (component B), which is a factor of a few to 10 less than the densities inferred for the protostellar cores but is in good agreement with the value of \( n_H \) derived from the isothermal model using the observed \( T_{\alpha} = 14 \text{ K} \) of the lowest contour in Figure 4. Our results agree with the predicted range of the molecular \( n_H \) between \( 10^7 \) and \( 10^8 \text{ cm}^{-3} \) (De Pree et al. 1998). F1 (H\( 2 \)α-3, H\( 2 \)α-4, and H\( 2 \)α-6) shows complicated kinematics. The line flux from each of the three H\( 2 \)α sources suggests the presence of at least three O-type stars in this region. From analysis of the high-resolution VLA image, De Pree et al. (1998) suggested a total of seven late-O-type or early-B-type stars in F1. The observed kinematics suggest that complicated dynamics of ionized gas are involved in the OB association. G (H\( 2 \)α-8) also shows a broad wing in its H\( 2 \)α line spectrum, which corresponds to a large radial velocity gradient \( \Delta V_{\text{th}} \) northeast-to-southwest across the source with a size \( 0.6 \) as seen in the moment 1 image (see Figure 4). Taking the parameters inferred from our modeling (Table 3) and \( L_{\alpha}(08.5) \approx 5 \times 10^4 \text{ L}_\odot \), with Equation (15), we find that \( \frac{P_{\text{th}}}{P_{\text{rad}}} \approx 1 \) and \( \approx 0.6 \) for the hyper-compact component (A) and the larger component (B), respectively. The observed velocity gradient in the component B region appears to be mainly due to a bipolar ionized outflow accelerated in the region surrounding an O8.5 star. The structure of kinematics for the hyper-compact component (A) with a weak stimulated source as suggested earlier was not resolved in these observations. F10.37 (H\( 2 \)α-5) shows a broad H\( 2 \)α line profile with \( \Delta V_{\text{FWHM}} = 51 \pm 6 \text{ km s}^{-1} \) but no obvious radial velocity gradient is seen in the moment 1 image (see Figure 4). For \( T_k = 1 \times 10^4 \text{ K} \), the thermal broadening gives \( \Delta V_{\text{th}} = 21 \text{ km s}^{-1} \). Subtracting the thermal broadening, the residual line width \( \Delta V_{\text{th}} = 46 \text{ km s}^{-1} \) requires large nonthermal motions. A bipolar ionized outflow along the line of sight might be responsible for the large velocity dispersion \( \sigma_v = \frac{1}{\sqrt{2}} \Delta V_{\text{th}} = 20 \text{ km s}^{-1} \) in this region.

### 6.3. Formation of Massive Protostellar Cores

In addition to the H\( 2 \)α sources, there are about two dozen protostellar cores with masses much greater than the corresponding Bonnor–Ebert masses and a total molecular mass in the range 5–223 \( M_\odot \). The observed filamentary structures in the continuum emission at the submillimeter wavelength suggest that the massive stars and protostellar cores were formed through fragmentation during the initial collapse of the massive core in Sgr B2 Main (Qin et al. 2008). The inferred surface density of each protostellar core appears to be greater than unity, a minimum mass for forming a massive star in order to avoid further fragmentation (Krumholz & McKee 2008). Through competitive accretion, each of the protostellar cores will grow by adding an accreted mass \( M_{\text{ff}} \) within the free-fall timescale \( t_{\text{ff}} \). The accretion rate can be estimated for an isothermal flow in which the density, \( \rho(r) \propto r^{-3/2} \), and the enclosed mass within a radius \( r, M(r) \propto r^{3/2} \), can be described by a power law (Shu 1977). The accretion rate is linearly proportional to the accretion velocity \( V_{\text{Acc}} \) and the enclosed mass \( M_R \) within \( r = R \) but inversely proportional to \( R \):

\[
\dot{M} = V_{\text{Acc}} \frac{dM(r)}{dr} \bigg|_{r=R} = 0.3 M_\odot \text{ yr}^{-1} \left( \frac{V_{\text{Acc}}}{\text{km s}^{-1}} \right) \left( \frac{M_R}{M_\odot} \right) \left( \frac{R}{\text{AU}} \right)^{-1}. \tag{19}
\]

Based on our measurements \( 2R \approx \sqrt{5'' \times 3''} \approx 3 \times 10^4 \text{ AU}, \) \( M_R \approx 1.2 \times 10^3 M_\odot \), and \( V_{\text{Acc}} \approx C_S \approx \frac{r_{\text{th}}}{\xi} \approx 1.3 \text{ km s}^{-1} \) for a kinetic temperature \( T_k \approx T_d = 500 \text{ K} \), we inferred \( M \approx 0.03 M_\odot \text{ yr}^{-1} \). For a constant accretion rate of \( 0.03 M_\odot \text{ yr}^{-1} \), in a mean free-fall timescale \( t_{\text{ff}} \approx 2 \times 10^3 \text{ years} \), about \( 60 M_\odot \) or \( \sim 5\% \) of the total mass increases in the protostellar cores. We have observed a snapshot of the process of forming a massive-star cluster. A few points regarding the formation of massive star cores can be drawn from our analysis of the SMA observations. First, Figure 4 shows a noticeable angular offset of \( \sim 0''1 \) between the H\( 2 \)α sources (H\( 2 \)α-1, H\( 2 \)α-4, and H\( 2 \)α-6) from their continuum counterparts (Smm-1, Smm-4, and Smm-10). The relative angular offsets are significant (>5\%), corresponding to a projected distance of 800 AU. If the H\( 2 \)α sources denote the locations of the newborn O stars, the offsets of the protostellar cores (Smm-1, Smm-4, and Smm-10) from the ionizing stars might suggest that the local accretion centers are altered after the onset of the radiation from the O stars. A massive protostellar core in the immediate vicinity of a UC H \( \alpha \) region has been also observed in the W51e2 complex (Shi et al. 2010a, 2010b). The authors use SMA high-resolution observations to show that the protostellar core (W51e2-E) with a mass of \( 140 M_\odot \), which is located 0''9 (4600 AU) from the UC H \( \alpha \) region (W51e2-W), becomes the new accretion center and undergoes active star formation, ejecting a powerful molecular outflow. Multiple pairs of O-type star and massive protostellar core found in Sgr B2 Main show a cluster version of the W51e2 system. Consequently, a considerable amount of mass must have existed in each of the paired protostellar cores before the O-type stars were born unless there were large accretion rates,

\[
\Delta V_{\text{th}} = 21 \text{ km s}^{-1}. \tag{17}
\]

\[
\Delta V_{\text{th}} = \Delta V_{\text{FWHM}} = 46 \text{ km s}^{-1}. \tag{18}
\]

\[
\sigma_v = \frac{1}{\sqrt{2}} \Delta V_{\text{th}} = 20 \text{ km s}^{-1}. \tag{19}
\]
appears to be much greater than the value of \( M \). The mean free-fall timescale of 2

mass has been added to each of the protostellar cores through competitive accretion in a cluster of massive stars in a massive core like Sgr B2 Main. However, from our observations of this particular phase of massive star formation in a cluster, it is uncertain how much mass has been added to each of the protostellar cores through competitive accretion owing to the uncertainty in the timescale of the process. On the basis of our analysis, accretion will add a small fraction of mass to each of the protostellar cores in a mean free-fall timescale of 2 \( \times 10^3 \) years.

7. CONCLUSION AND SUMMARY

Using SMA archival data observed in 2007 for Sgr B2 Main, we imaged the H26α line and continuum emission at 0.86 mm with natural- and uniform-weighted synthesized beams of 0′′.40 \( \times \) 0′′.28 and 0′′.36 \( \times \) 0′′.22, respectively. Eight compact H26α line sources have been detected toward the central 5 pc \( \approx 10^2 \) mas. Using the non-LTE approach, we fitted the H26α line along with other available RRL and radio continuum data with two isothermal, homogenous H II components: (1) high-density \( \times 10^7 \) cm\(^{-3}\), hyper-compact (several hundred AU) and (2) lower-density \( \times 10^6 \) cm\(^{-3}\), UC (a few thousand AU). The observed H26α line fluxes suggest that each of the eight H26α line emission regions must be powered by a Lyman continuum source from an O-type star or a cluster of B-type stars. The typical age of the H II regions is a thousand years. The brightest H26α-1 source in F3 appears to be associated with an O6 star. The H26α line from H26α-7 (F10.39) appears to be enhanced by stimulated emission in the vicinity of an O9.5 star.

About two dozen compact continuum emission cores are detected at 0.86 mm in the central 5″ (0.2 pc). Using a power-law distribution of dust temperature and observed brightness temperature, we determined the physical properties for each of the protostellar cores with masses in the range of 5 to \( \geq 200 \) solar masses. The surface densities of the protostellar cores are in the range 13–150 cm\(^{-3}\) and molecular densities 1 \( \times \) 10\(^{-2}\)–1 \( \times \) 10\(^{3}\) cm\(^{-3}\). We also calculated the Bonnor–Ebert mass \( (M_{BE}) \) for each core. The total molecular mass of each core appears to be much greater than the value of \( M_{BE} \), suggesting that the cores undergo a substantial gravitational collapse to form massive stars with a typical free-fall timescale of a few thousand years.

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APPENDIX

LYMAN CONTINUUM PHOTON FLUX

In this Appendix, we show the detailed procedure to derive the Lyman continuum photon flux from the measurements of a hydrogen RRL.

A.1. Photoionizations and Recombinations of Hydrogen in H II Regions

For an H II region with electron density \( n_e \), the ionization equilibrium can be described by the balance between photoionizations and recombinations of electrons with the ions. The number of ionizing photons \( (N_i) \) emitted by a star in unit time, or the Lyman continuum photon flux \( (N_{lym}) \), is equal to the number of recombinations \( (N_R = \frac{4\pi}{3}R_1^3n_e^2\alpha_B) \) to the excited levels within the Str"omgren radius \( (R_e) \) in unit time (Osterbrock 1974):

\[
N_{lym} = \frac{4\pi}{3}R_1^3n_e^2\alpha_B, \tag{A1}
\]

where \( \alpha_B = \alpha_B(T_e) \) is the recombination coefficient in case B, which is a function of electron temperature \( (T_e) \) with a value \( \alpha_B = 2.58 \times 10^{-13} \) cm\(^{-3}\) s\(^{-1}\) at \( T_e = 1 \times 10^4 \) K (Hummer & Seaton 1963); and \( R_1 \approx \frac{3}{8\pi}V \approx \frac{3}{8\pi}\Omega_{II}D^2L_fV \).

For an H II region located at distance \( D \), with solid angle \( \Omega_{II} \), path length \( L \), and electron volume filling factor \( f_V \), Equation (A1) can be rewritten as

\[
N_{lym} = \frac{3}{8\pi}L_fV\alpha_B\Omega_{II}D^2L_fV, \tag{A2}
\]

A.2. Optical Depth and Flux Density of Hydrogen Radio
Recombination Lines

For an isothermal–homogeneous H II region, the optical depths for hydrogen RRLs \( (\tau_L) \) and free–free continuum \( (\tau_c) \) are proportional to the EM \( (EM = n_e^2L_fV) \) and are given by Shaver (1975):

\[
\tau_L = b_N\beta_N\tau_L^*, \quad \approx 575b_N\beta_N\left( \frac{v}{\text{GHz}} \right)^{-1}\left( \frac{n_e}{\text{cm}^{-3}} \right)^{-2}
\times \left( \frac{L_fV}{\text{pc}} \right)\left( \frac{T_e}{K} \right)^{5/2}\left( \frac{\Delta V_D}{\text{km s}^{-1}} \right)^{-1}
\times \left( 1 + 1.48\frac{\Delta V_P}{\Delta V_D} \right)^{-1}, \tag{A3}
\]

and

\[
\tau_c \approx 0.08235\left( \frac{n_e}{\text{cm}^{-3}} \right)^2\left( \frac{L_fV}{\text{pc}} \right)^{1/2}\left( \frac{v}{\text{GHz}} \right)^{-2.1}
\times \left( \frac{T_e}{K} \right)^{1.35}\alpha(v, T_e), \tag{A4}
\]

where \( \tau_L^* \) is the LTE line optical depth, \( b_N \) and \( \beta_N \) are the population departure coefficients, \( n_e \) is the electron density, and \( \Delta V_D \) and \( \Delta V_P \) are the FWHM Doppler and pressure line widths in km s\(^{-1}\), respectively. The correction factor \( \alpha(v, T_e) \) is of order unity.

On the other hand, if an H II region, with no dust in it, is optically thin and in LTE \( (\tau_c \ll 1, \tau_L^* \ll 1, \) and \( b_N = \beta_N = 1) \), the line flux density from Equation (1) in the main text becomes

\[
S^*_{\nu} \approx \frac{2kT_e}{c^2}\Omega_{II}D^2L_f\tau^*_L. \tag{A5}
\]

 Normally, the Lyman continuum photon flux \( (N_{lym}) \) is derived from the EM determined from the free–free continuum flux density observed at a radio frequency, e.g., Wilcots (1994). In the following, we show the derivation of the EM from an RRL flux. Therefore, \( N_{lym} \) can be determined directly from an RRL flux.
A.3. Emission Measure from Hydrogen Radio
Recombination Lines

From Equations (A3) and (A5), neglecting the pressure broadening ($\Delta V_p \gg \Delta V_D$), the EM for an optically thin, LTE gas can be expressed as

$$n_e^2 L_{r} = 5.66 \times 10^{-8} \text{cm}^{-6} \text{pc}^2 \left( \frac{S_t \Delta V_D}{\text{Jy km s}^{-1}} \right) \times \left( \frac{\nu}{\text{GHz}} \right)^{-1} \left( \frac{T_e}{\text{K}} \right)^{3/2} \Omega_{\text{H}^0}^{-1} \Psi_R^{-1}. \quad (A6)$$

For a general isothermal, homogeneous H II region, the EM can be determined by the following equation:

$$n_e^2 L_{r} = 5.66 \times 10^{-8} \text{cm}^{-6} \text{pc}^2 \left( \frac{S_t \Delta V_D}{\text{Jy km s}^{-1}} \right) \times \left( \frac{\nu}{\text{GHz}} \right)^{-1} \left( \frac{T_e}{\text{K}} \right)^{3/2} \Omega_{\text{H}^0}^{-1} \Psi_R^{-1}, \quad (A7)$$

where $\Psi_R$ is the ratio of the non-LTE line flux ($S_t$) to the LTE line flux in optically thin conditions ($S_t |\tau_L^* \ll 1)$ for the line transition $N' + 1 \rightarrow N$, where $N$ is the principal quantum number. $\Psi_R$ is a function of the line and continuum optical depths ($\tau_L$ and $\tau_C$) and departure coefficients ($b_N$ and $\beta_N$).

$$\Psi_R \equiv \left( \frac{S_t}{S_t |\tau_L^* \ll 1|} \right) (1 - e^{-(\tau_L + \tau_C)}) - (1 - e^{-\tau_C}) \tau_L^*,$$

(A8)

neglecting the term associated with the dust emission ($S_d$) in Equation (1) of the main text.

A.4. Lyman Continuum Photon Flux from Hydrogen Radio
Recombination Lines

For an electron temperature ($T_e$) in an isoatmospheric, homogeneous H II region, inserting $n_e^2 L_{r} \Psi_R$ of Equation (A7) into Equation (A2), we derived the Lyman continuum photon flux from the measurements with flux density ($S_L$) and FWHM of Doppler broadening ($\Delta V_D$) for a hydrogen RRL ($N' + 1 \rightarrow N$) at frequency $\nu_L$.

$$N_{\text{L}y} = 4.29 \times 10^{47} \left( \frac{S_t \Delta V_D}{\text{Jy km s}^{-1}} \right) \left( \frac{\nu_L}{10^4 \text{K}} \right)^{-1} \left( \frac{D}{\text{kpc}} \right)^2 \times \left( \frac{T_e}{10^4 \text{K}} \right)^{1.5} \left( \frac{\alpha_g(T_e)}{\alpha_B(10^4 \text{K})} \right) \Psi_R^{-1} \text{photons s}^{-1}. \quad (A9)$$

where $\Psi_R$ corrects for the effects due to the optical depths and non-LTE distribution. Thus, from Equation (A9), $\Psi_R \approx 1$ for an optically thin, LTE gas; $\Psi_R \approx b_N (1 - \frac{1}{2} \tau_C \beta_N)$ for optically thin, non-LTE gas; and $\Psi_R \approx e^{-(1 - e^{-\tau_C})/\tau_C}$ when the LTE gas becomes optically thick.