T-duality off shell in 3D Type II superspace

Martin Poláček
Stony Brook University
7-th August, 2015
Brief introduction

- In the arXiv:1308.6350 paper we obtained the curvature tensor (previously discovered in Warren Siegel paper) in a way manifestly covariant under O(D,D) T-duality.
- The aim of this paper is to extend the techniques of the T-dually extended spaces from the bosonic case to the supersymmetric case.
- We give the manifestly T-dual formulation of the massless sector of the classical 3D Type II superstring in off-shell 3D $\mathcal{N} = 2$ superspace.
- We want to motivate the natural identification between 4D $\mathcal{N} = 1$ supergravity, further compactified to 3D $\mathcal{N} = 2$ and T-dual 3D $\mathcal{N} = 2$ string theory.
- Both can be thought to have an origin in higher dimensional F-theory.
Theory setting

Calculations

Conclusion and Further development

Special thank

---

5D, $\mathcal{N} = 1, \text{SO}(3, 2)$

$H^t_{[\alpha \beta]}$

4D, $\mathcal{N} = 1, \text{SO}(3, 1)$

$H_{\dot{\alpha} \dot{\beta}} \ V$

3D, T-dual, $\mathcal{N} = 2,$

$\text{SO}(2, 2) \cong \text{SO}(2, 1) \otimes \text{SO}(2, 1)$

$H_{\alpha \beta'} \ V$

3D, $\mathcal{N} = 2, \text{SO}(2, 1)$

$H_{(\alpha \beta')} \ V \ V$

---

Martin Poláček

T-duality off shell in 3D Type II superspace
**Calculations**

- **Gauge fixing:**
  \[
  \gamma_a^{\alpha\beta} E^{(1)}_{\alpha\beta} = 0 \Rightarrow \lambda_a \propto \gamma_a^{\alpha\beta} D_{\alpha} \lambda_{\beta}
  \]
  \[
  \gamma^a_{\alpha\beta} E^{(1)}_{\alpha a} = 0 \Rightarrow \lambda^a \propto \gamma^a_{\alpha\beta} D[a \lambda_{\beta}]
  \]
  \[
  E^{(1)}_{ab} = 0 \Rightarrow \lambda_{ab} \propto D[a \lambda_{b}]
  \]

  Same for Left → Right

- **Fixes** \(\lambda_P, \lambda_\Omega, \lambda_\Sigma \propto \lambda_D\). Because of the coset constraints \(\lambda_S = 0\).

- **Gauge fixing constraints** give the constraints on vielbeins:
  \[
  E^{(1)}_{DD} = E^{(1)}_{\alpha\beta} = 0, \quad E^{(1)}_{PP} = E^{(1)}_{ab} = 0, \quad E^{(1)}_{\alpha\beta\alpha\beta} = 0
  \]
  (part of \(E^{(1)}_{PD}\))

- **Later** (by dimension \(-\frac{1}{2}\) constraints) one can see that
  \[
  E^{(1)}_{PD} = 0.
  \]
Dimensional constraints
Put the torsions of negative (engineering) dimensions to 0
We also put the (unfixed) torsions of zero dimension to 0
We will also put the dimension $\frac{1}{2}$ (unfixed) torsions to 0
Doing that we produce just algebraic constraints on veilbeins.
The nontrivial dimensional constraints are:

$$T_{D D}^{\Omega} = 0, \ T_{D D}^{P} = f_{D D}^{P}, \ T_{D D}^{D} = 0, \ T_{P P}^{\Omega} = 0$$
Dimensional constraints: unmixed solution

\[
\begin{align*}
T_{DD}^\Omega &= 0 \quad \text{and} \quad \gamma^a{}_{\alpha\beta} E^{(1)}_{\alpha} a = 0 \quad \Rightarrow \quad E^{(1)}_{PD} = 0 \\
T_{DP}^\Omega &= f_{DD}^P \\
T_{PP}^\Omega &= 0 \text{ or } T_{DD}^D = 0 \quad \Rightarrow \quad E^{(1)}_{\Sigma D} = E^{(1)ab}{}_{\alpha} = -2 \gamma^{[a}{}_{\alpha\rho} E^{(1)\rho]}^b \\
&\equiv \gamma \cdot E^{(1)}_{\Omega}^P 
\end{align*}
\]

Dimensional constraints: mixed solution

\[
\begin{align*}
T_{DD}^\Omega &= 0 \quad \Rightarrow \quad E^{(1)}_{P\bar{D}} \equiv E^{(1)}_{a\bar{\alpha}} = -\frac{1}{2} \gamma^a{}_{\beta\epsilon} D^\beta_{\epsilon} E^{(1)}_{\epsilon\bar{\alpha}} \equiv -\gamma \cdot D_D \cdot E^{(1)}_{D\bar{D}} \\
T_{DP}^\Omega &= 0 \quad \Rightarrow \quad E^{(1)}_{P\bar{P}} \equiv E^{(1)}_{ab\bar{\beta}} = -\frac{1}{2} \gamma^{a}{}_{\epsilon\alpha} D^\epsilon_{\beta} E^{(1)}_{\epsilon\bar{\beta}} \equiv -\gamma \cdot D_D \cdot E^{(1)}_{D\bar{P}} \\
T_{D\bar{D}}^P &= 0 \quad \Rightarrow \quad E^{(1)}_{\Omega\bar{D}} \equiv E^{(1)\alpha\bar{\beta}} = -\frac{1}{6} \gamma^a{}_{\epsilon\alpha} D^\epsilon_{[\alpha} E^{(1)}_{\beta]} \bar{\beta} \equiv -\gamma \cdot D[D \cdot E^{(1)}_{P\bar{P}]}\bar{D} \\
T_{P\bar{P}}^\Omega &= 0 \quad \Rightarrow \quad E^{(1)}_{\Omega\bar{P}} \equiv E^{(1)\alpha\bar{\epsilon}} = -\frac{1}{6} \gamma^{b}{}_{\epsilon\alpha} D^\epsilon_{[\beta} E^{(1)}_{\epsilon]} \bar{\alpha} \equiv -\gamma \cdot D[D \cdot E^{(1)}_{P\bar{D}]}\bar{P} \\
T_{PP}^\Omega &= 0 \quad \Rightarrow \quad E^{(1)}_{\Sigma\bar{D}} \equiv E^{(1)ab\bar{\alpha}} = \eta^{ac} \eta^{bd} D_{[c} E^{(1)}_{d]} \bar{\alpha} \equiv \eta \cdot \eta \cdot D[P \cdot E^{(1)}_{P\bar{P}]}\bar{D}
\end{align*}
\]
The net result of dimension 1 unmixed algebraic constraints is that everything can be expressed in terms of $E^{(1)}_D \Sigma$:

$$B = -\frac{1}{\vartheta + 6\zeta} \gamma^{a\alpha} D_\alpha E^{(1)}_{\beta a}$$

$$E^{(1)}_{\alpha\beta} = \frac{1}{12} \gamma^{a (\alpha | \epsilon} D_\epsilon E^{(1)}_{\beta ) a} + \frac{1}{12} \gamma^{a \alpha \beta} E^{(1)ab}_b$$

$$E^{(1)}_{c ab} = -\frac{1}{2} \gamma^{c \alpha \beta} D_\alpha E^{(1)}_{\beta ab} + (\vartheta + 4\zeta) \eta^{ce} \varepsilon^{eab} B$$

The dimension 1 mixed constraints give:

$$E^{(1)}_{\Sigma \tilde{p}} \equiv E^{(1)bc}_{\tilde{a}} = \eta^{bd} \eta^{ce} D[d E^{(1)}_e]_{\tilde{a}} \equiv \eta \eta \cdot D[P E^{(1)}_P] \tilde{p}$$

$$E^{(1)}_{\Omega \tilde{\Omega}} \equiv E^{(1)\alpha \tilde{\beta}} = \frac{1}{6} \gamma^{a \alpha \epsilon} D[a E^{(1)}_\epsilon]_{\tilde{\beta}} \equiv \gamma \cdot D[P E^{(1)}_D] \tilde{\Omega}$$

$\tilde{T}^{(1)}_D = 0$ constraints gives:

$$B = \frac{-1}{\vartheta + 6\zeta} \epsilon^{\nu \alpha} D_\nu \left[ \gamma^{\tilde{a} \tilde{\beta} \tilde{\epsilon}} \left( -\frac{1}{6} [D_{\tilde{a}}, D_{\tilde{\beta}}] + \frac{1}{4} D_{\tilde{a}} D_{\tilde{\beta}} \right) E^{(1)}_{\tilde{\epsilon} \alpha} - D_\alpha \phi^{(1)} \right]$$
We found the equations of motion:
\[ B + \tilde{B} = 0 \quad \text{and} \quad B - \tilde{B} = 0 \]

Simplify the structure of \( B \) and \( \tilde{B} \), the structure of e.o.m.:
\[ 0 = \left( D^2 + \tilde{D}^2 \right) \left( -\frac{1}{8} \varepsilon^{\alpha \nu} \varepsilon^{\tilde{\nu} \tilde{\sigma}} D_\nu D_{\tilde{\sigma}} E^{(1)}_{\tilde{\epsilon} \alpha} + \phi^{(1)} \right) \]

Rewrite it using a new field \( V \):
\[ \left( D^2 - \tilde{D}^2 \right) V =: \left( -\frac{1}{8} \varepsilon^{\alpha \nu} \varepsilon^{\tilde{\nu} \tilde{\sigma}} D_\nu D_{\tilde{\sigma}} E^{(1)}_{\tilde{\epsilon} \alpha} + \phi^{(1)} \right) \]

Use previous definition:
\[ 0 = \left( D^2 + \tilde{D}^2 \right) \left( D^2 - \tilde{D}^2 \right) V \]

Operator \( \left( D^2 + \tilde{D}^2 \right) \left( D^2 - \tilde{D}^2 \right) \) acts on the scalar field \( V \), get a nicer form:
\[ \left( D^2 + \tilde{D}^2 \right) \left( D^2 - \tilde{D}^2 \right) V = 4 D^A D_A V \]

The second equation of becomes the e.o.m. for the \( V \) field:
\[ \left( D^2 - \tilde{D}^2 \right)^2 V = 0 \]
We started with T-dual $\mathcal{N} = 2$ string theory, i.e. effective $\mathcal{N} = 2$ supergravity in 3 dimensions.

We first obtained the dimension $-1$ prepotential as the vielbein component $E^{(1)}_{D \tilde{D}} \equiv E^{(1)}_{\alpha \tilde{\beta}}$ and the dimension $-\frac{3}{2}$ unconstrained gauge parameter $\Lambda_{D} \equiv \Lambda_{\alpha}$ (also $\Lambda_{\tilde{D}}$) without solving any differential constraints.

In particular the structure of the linear dilaton $\phi$ was derived.

It matches the structure obtained from 4D $\mathcal{N} = 1$ and its compactification.

This suggests that the T-dually extended superspace approach can be extended also to higher dimensional cases (and also to non flat backgrounds like $AdS_{5} \otimes S_{5}$), as is examined in the current (unfinished) work.
This talk was supported by the grants from:

- C. N. Yang Institute for Theoretical Physics, Stony Brook University