Cold Black Holes in the Einstein-Scalar Field System

K.A. Bronnikov∗
Center for Gravitation and Fundamental Metrology, VNIIMS,
46 Ozyornaya St., Moscow, Russia;
Institute of Gravitation and Cosmology, PFUR,
6 Miklukho-Maklaya St., Moscow 117198, Russia

J.C. Fabris†
Departamento de Física, Universidade Federal do Espírito Santo,
Vitória, 29060-900, Espírito Santo, Brazil

N. Pinto-Neto‡and M.E. Rodrigues§
ICRA-CBPF, Rua Xavier Sigaud, 150, Urca, CEP22290-180,
Rio de Janeiro, Brazil.

March 24, 2022

Abstract

We study Einstein gravity coupled to a massless scalar field in a static spherically symmetric space-time in four dimensions. Black hole solutions exist when the kinetic energy of the scalar field is negative, that is, for a phantom field. These “scalar black holes” have an infinite horizon area and zero temperature. They are related through a conformal transformation with similar objects in the Jordan frames of scalar-tensor theories of gravity. The thermodynamical properties of these solutions are discussed. It is proved that any static, spherically symmetric black holes with an infinite horizon area have zero Hawking temperature.

PACS numbers: 04.70.Bw 95.35.+d 98.80.-k

∗E-mail: kb20@yandex.ru
†E-mail: fabris@cce.ufes.br
‡E-mail: nelsonpn@cbpf.br
§E-mail: esialg@cbpf.br
1 Introduction

General relativity predicts the existence of peculiar objects which have been called black holes [1]. Black holes emerge as solutions of Einstein’s equations in a static, spherically symmetric space-time without matter: it is the simplest black hole solution found by Schwarzschild in 1916. Its generalization to rotating, axially symmetric space-times, obtained by Kerr in 1963, also leads to a black hole structure. The conventional idea of a black hole implies a singularity in space-time covered by a horizon. A horizon is a hypersurface which separates an external region (containing spatial infinity) from an internal region, which contains a singularity: this internal region is not visible to an external observer. Generalizations of black hole solutions in different contexts are known, such as in the presence of an electric field, non-linear gravity theories, scalar-tensor theories, etc. Their different properties rise the question of an extension of the black hole notion itself.

An example of “exotic” black holes are the so-called “cold black holes”. They are obtained in scalar-tensor theories in general, and in the Brans-Dicke theory in particular [2–4]. In general scalar-tensor theories, the coupling of the scalar field to gravity is described in terms of a function of the scalar field itself, $\omega(\phi)$, while in the Brans-Dicke theory this function is simply a constant. From here on, we will concentrate on the cold black holes that emerge in the Brans-Dicke theory. It turns out that static, spherically symmetric solutions of the Brans-Dicke theory reveal a large class of objects that can be called “black holes”. Not all of them exhibit a singularity beyond a horizon: in some cases, the black hole interior is regular. However, the horizon surface has in all such cases an infinite area. Moreover, all such horizons have zero surface gravity. This indicates that they have zero temperature, since the Hawking temperature is directly related to surface gravity. For this reason, they have been named “cold black holes”.

The Brans-Dicke theory can be re-defined using a conformal transformation. By appropriately choosing the conformal factor, the non-minimal coupling between the scalar field and the scalar curvature, which is an essential feature of the Brans-Dicke theory, can be broken. In the vacuum case, that is, with no matter, this results in Einstein gravity minimally coupled to a massless scalar field. The energy associated with the scalar field is positive (its kinetic terms has its usual, “normal” sign) if $\omega > -3/2$; however, the energy is negative if $\omega < -3/2$. In this last case, the kinetic term has a “wrong” sign, and the theory is called anomalous, or phantom. Such kind of theories have recently become quite fashionable for both theoretical and observational reasons. The theoretical reasons are connected with the ghost condensation and tachyonic fields that result from string theories [5, 6]. From the observational viewpoint, recent analysis of the type Ia supernova data indicates that perhaps the best fit is given by phantom fields [7–10], of which a scalar field with the “wrong” sign of the kinetic term is an example.

The goal of the present work is to show that this Einstein-scalar field system, with a massless scalar field minimally coupled to gravity, admits black hole solutions, as opposed to what has been believed [11]. The black hole solutions can only occur if the energy associated with the scalar field is negative, that is, for a phantom field. These “scalar” black holes have also an infinite horizon area and zero temperature. These solutions can be interpreted as Einstein-frame solutions of the Brans-Dicke theory and transformed to the Jordan frame by performing the inverse conformal transformation. An interesting point in doing so is the non-existence of a one-to-one correspondence between black holes in the Einstein frame and those in the Jordan frame. In this connection, we are also going to discuss the question of invariance of thermodynamical quantities, such as the temperature, under these conformal transformations.

In the next section, we reproduce the static, spherically symmetric solutions for the Einstein-scalar field system. We pay special attention to a particular class of these solutions. In section
3, we select which subclass of those solutions corresponds to black holes. In section 4, we discuss the thermodynamical properties of these objects and their invariance with respect to conformal transformations. In section 5, we present our conclusions.

It must be said that Prof. José Plínio Baptista has always had the Brans-Dicke theory as one of his main interests. Hence, it is a pleasure to exhibit, in honour of his 70th birthday, a new class of black hole solutions which are somehow related to the Brans-Dicke theory.

2 Static, spherically symmetric solutions in Einstein-scalar and Brans-Dicke theories

Let us consider gravity coupled minimally to a free massless scalar field. The action is

$$A = \int d^4x \left( R + \epsilon \phi^2 \right),$$

where $\epsilon = \pm 1$, +1 means a normal scalar field with positive energy density and −1 an anomalous (phantom) scalar field with negative energy density. The field equations are

$$R_{\mu\nu} = -\epsilon \phi_{,\mu} \phi_{,\nu},$$

$$\Box \phi = 0.$$  

The static, spherically symmetric metric may be written as

$$ds^2 = e^{2\gamma} dt^2 - e^{2\alpha} du^2 - e^{2\beta} d\Omega^2,$$

where $\gamma = \gamma(u), \alpha = \alpha(u)$ and $\beta = \beta(u), u$ is an arbitrary radial coordinate. Hence, the equations of motion are

$$\gamma'' + \gamma' (\gamma' - \alpha' + 2\beta') = 0,$$

$$\gamma'' + 2\beta'' + \gamma'^2 + 2\beta'^2 - \alpha' (\gamma' + 2\beta') = -\epsilon \frac{\phi'^2}{2},$$

$$\beta'' + \beta' (\gamma' - \alpha' + 2\beta') = 0,$$

$$\left(e^{\gamma-\alpha + 2\beta} \phi'\right)' = 0.$$  

Primes denote derivatives with respect to $u$. The solutions to these equations have been found in Refs. [12] for $\epsilon = 1$, [13] for $\epsilon = -1$ and, in a more general form, [14]. They can be obtained by fixing the harmonic radial coordinate, corresponding to the coordinate condition $\alpha = 2\beta + \gamma$ [14]. In this case, the solutions for $\gamma$ and $\phi$ are straightforward,

$$\gamma = -bu, \quad \phi = Cu + \phi_0, \quad b, C, \phi_0 = \text{const}.$$  

Using (3) and (5), we obtain the equation

$$f'' + 2bf' + C^2 f = 0, \quad f = e^{-\beta}$$  

with the general solution

$$f = e^{-bu} \{ Ae^{ku} + Be^{-ku} \},$$

where $k, A, B$ are integration constants and $2k^2 = 2b^2 - \epsilon C^2$. The constants $A$ and $B$ obey the relation $4ABk^2 + 1 = 0$. Requiring that the solutions should be asymptotically flat at spatial
infinity and choosing \( u = 0 \) for this infinity, we find that the constants must be \( A = 1/(2k) \) and \( B = -1/(2k) \). Hence the solution takes the form

\[
e^{\gamma} = e^{-bu}, \quad e^{\alpha} = \frac{k^2 e^{bu}}{\sinh^2 ku}, \quad e^{\beta} = \frac{ke^{bu}}{\sinh ku}.
\]  

(12)

If the parameter \( k \) is real and positive, it is helpful to pass over to the so-called quasiglobal coordinate \( \rho \) (such that in (1) \( \alpha + \gamma = 0 \)) by the transformation

\[
e^{-2ku} = 1 - 2k/\rho \equiv P(\rho).
\]

(13)

The solution takes quite a simple form,

\[
ds_E^2 = P^a dt^2 - P^{-a} d\rho^2 - P^{1-a} \rho^2 d\Omega^2,
\]

\[
\phi = -\frac{C}{2k} \ln P(\rho),
\]

(14)

(15)

with the constants related by

\[
a = b/k, \quad a^2 = 1 - \varepsilon C^2/2.
\]

(16)

In Refs. [3, 4], the authors were interested in static, spherically symmetric solutions in the context of scalar-tensor theories, where the scalar field is non-minimally coupled to gravity, i.e., in the Jordan frame. Black hole solutions were also revealed there in the Jordan frame only. Here we wish to pay attention to the existence of black hole solutions in the (anti-)Fisher family, or, which is the same, in the Einstein frame of scalar-tensor gravity, i.e., among the solutions (14), (15).

The corresponding solutions in the Jordan frame of the Brans-Dicke theory with the coupling constant \( \omega \) may be written in the form

\[
ds_J^2 = P^\xi ds_E^2
\]

\[
= P^{a-\xi} dt^2 - P^{-a-\xi} d\rho^2 - P^{1-\xi-a} \rho^2 d\Omega^2,
\]

\[
\phi_{\text{BD}} = P^\xi,
\]

(17)

(18)

where \( \phi_{\text{BD}} \) is the Brans-Dicke scalar related to \( \phi \) by

\[
\phi_{\text{BD}} = \exp [\phi/\sqrt{\omega + 3/2}].
\]

(19)

The parameters \( \xi \) and \( a \) are connected by the relation

\[
(3 + 2\omega)\xi^2 = 1 - a^2.
\]

(20)

The subscripts \( J \) (Jordan) and \( E \) (Einstein) in (14) and (17) indicate in which frame the solutions are being written. The two metrics are connected by the conformal transformation

\[
g_{\mu\nu}^J = \phi_{\text{BD}}^{-1} g_{\mu\nu}^E.
\]

(21)
3 Black hole solutions

Let us try to single out black hole solutions from the family (14), (15). We restrict ourselves to the case $k > 0$ since the cases $k \leq 0$, as may be shown, do not lead to results of equal interest.

First of all, we must fix what we understand by a “black hole solution”. For our comparatively simple case of static, spherically symmetric space-times, leaving aside more general and more rigorous definitions of horizons and black holes (see, e.g., [15]), we can rely on the following working definition. A black hole is a space-time containing (i) a static region which may be regarded external (e.g., contains a flat asymptotic), (ii) another region invisible for an observer at rest residing in the static region, and (iii) a Killing horizon of nonzero area that separates the two regions and admits an analytical extension of the metric from one region to another. This definition certainly implies that the horizon is regular, since otherwise it would be a singularity, belonging to the boundary of the space-time manifold, across which there cannot be a meaningful continuation.

Conditions for having a black hole solution in the Jordan frame, Eqs. (17), (18), have been discussed in reference [16].

For the general metric (4), a black hole horizon may be represented by a sphere $u = u_h$ at which $g_{00} = e^{2\gamma} = 0$ and at which all algebraic curvature invariants are finite. To check the latter, it is sufficient to consider the behaviour of the Kretschmann invariant, given by

$$K = R_{\mu\nu\lambda\gamma}R_{\mu\nu\lambda\gamma} = 4K_1^2 + 8K_2^2 + 8K_3^2 + 4K_4^2,$$

(22)

where

$$K_1 = R^{01}_{\ 01} = -e^{-(\alpha+\gamma)}(\gamma' e^{\gamma} - \alpha'),$$

$$K_2 = R^{02}_{\ 02} = -e^{-2\alpha}\beta'\gamma',$$

$$K_3 = R^{12}_{\ 12} = R^{13}_{\ 13} = -e^{-(\alpha+\beta)}(\beta' e^{\beta} - \alpha'),$$

$$K_4 = R^{23}_{\ 23} = e^{-2\beta} - e^{-2\alpha}\beta'^2,$$

(23)

where the primes denote $d/du$.

In the metric (14), a horizon can occur at $r = 2k$ if $a > 0$. The radius of the horizon surface is zero (it is thus a center, and a singularity is suspected) if $1 - a > 0$, infinite if $1 - a < 0$ and finite if $a = 1$. The latter case recovers the Schwarzschild solution. Solutions with $a > 1$ imply an anomalous theory: $\epsilon = -1$, and $\omega < -\frac{3}{2}$.

No black holes are thus described by Fisher’s solution (14) in the normal case, $\epsilon = 1$, in agreement with the well-known no-hair theorems.

Calculating $K_i$ for the metric (14), we find:

$$K_1 = \frac{4ak}{2\rho^3}P^{a-2}\left\{1 - (a + 1)\frac{k}{\rho}\right\},$$

$$K_2 = -\frac{a}{2\rho}P^{a-2}\left\{1 - (1 + a)\frac{k}{\rho}\right\},$$

$$K_3 = -\frac{k}{r\rho^2}P^{a-2}\left\{a - (1 + a)\frac{k}{\rho}\right\},$$

$$K_4 = kP^{a-2}\left\{2a - (1 + a)\frac{k}{\rho}\right\}.$$

(24)

Therefore, to have regularity at $\rho = 2k$, the condition

$$a - 2 \geq 0$$

(25)
must be satisfied. For \( a > 2 \), the Kretschmann invariant becomes zero. For \( a = 1 \) (Schwarzschild’s solution) and \( a = 2 \), it is finite. In other cases it is infinite, designating a naked singularity. The condition (25) is thus necessary for the existence of a horizon and its regularity. In the Schwarzschild case, the area of the horizon is finite, given by \( S = 16\pi k^2 \). In other cases of interest, \( a > 2 \), the area of the horizon is infinite. As will be discussed later, this has important consequences for the thermodynamics of such objects.

However, there is one more condition to be satisfied, namely, that the geometry should admit an analytical extension beyond the horizon. One may recall that, for cold black holes in Jordan’s frame [3, 4], an analytical extension was found to be only possible for a set of discrete values of the parameters \( \xi \) and \( \omega \). Here, the situation is simpler. A direct inspection of the metric (14) shows that it is possible to pass from \( \rho > 2k \) to \( \rho < 2k \) if the parameter \( a \) is an integer, \( a = n, n = 2, 3, \ldots \). Hence, the objects described by the metric (14) are really black holes only if \( a = 1, 2, 3, \ldots \). Moreover, the case \( a = 1 \) corresponds to the Schwarzschild black hole, for which \( \xi = 0 \), the scalar field is constant (actually, absent), and we recover, as expected, the static, spherically symmetric vacuum solution of Einstein’s equations. A new non-trivial class of black holes is obtained for

\[
a = 2, 3, \ldots
\]

All these “anomalous scalar black holes” (since the scalar field in this case the scalar field is not constant and contributes to the Einstein equations with negative energy) have a horizon of infinite area, unlike the Schwarzschild black hole, in full similarity with the Jordan frame, but for other values of the solution parameters.

One can reveal one more important difference between the Einstein and Jordan frames. Namely, as follows from (24), all \( K_i \) turn to infinity as \( \rho \to 0 \) in case (26). In other words, there is always a curvature singularity in the internal region of the black holes with a minimally coupled massless phantom scalar field in general relativity. Meanwhile, many of Brans-Dicke black hole solutions in the Jordan frame are nonsingular, and some of them have another flat asymptotic beyond the horizon [3, 4].

4 Thermodynamics of the scalar black holes

Refs. [3,4] discussed the thermodynamics of scalar-tensor black holes in the Jordan frame, connected with the presently studied scalar black holes by conformal transformation. The surface gravity of all these objects is zero, indicating a zero temperature. The present scalar black holes share this property.

Indeed, the Hawking temperature is \( T_H = (2\pi k_B)^{-1}\kappa \), where \( k_B \) is Boltzmann’s constant while the surface gravity \( \kappa \) is given by the expression [15]

\[
\kappa = \frac{1}{2} \left. \frac{g'_{00}}{\sqrt{|g_{00}g_{11}|}} \right|_{u=u_h},
\]

where \( u = u_h \) is the value of the radial coordinate \( u \) at the horizon. After a conformal transformation \( g_{\mu\nu} = \Omega^2 \tilde{g}_{\mu\nu} \), the expression for the surface gravity becomes, in terms of the new metric \( \tilde{g}_{\mu\nu} \),

\[
\kappa = \frac{\Omega'}{\Omega} \left. \frac{\tilde{g}_{00}}{\sqrt{\tilde{g}_{00}\tilde{g}_{11}}} \right|_{u=u_h} + \frac{1}{2} \left. \frac{\tilde{g}'_{00}}{\sqrt{\tilde{g}_{00}\tilde{g}_{11}}} \right|_{u=u_h} = \frac{\Omega'}{\Omega} \left. \frac{\tilde{g}_{00}}{\sqrt{|\tilde{g}_{00}\tilde{g}_{11}|}} \right|_{u=u_h} + \tilde{\kappa},
\]

where \( \tilde{\kappa} \) is the surface gravity measured in the space-time with the metric \( \tilde{g}_{\mu\nu} \). The two surface gravity terms (\( \kappa \) and \( \tilde{\kappa} \)) are equal if the first term in the right-hand side of (28) is zero. We could
expect this because the metric component $g_{00}$ is zero over the horizon. But, we can only assure that $\kappa = \tilde{\kappa}$ if the conformal factor $\Omega$ is regular at the horizon, i.e., if the conformal transformation is well defined on it.

Now, if $g_{\mu\nu}$ is the Jordan-frame metric and $\tilde{g}_{\mu\nu}$ is the Einstein-frame metric, from (14), (15) we obtain for the temperature in the Einstein frame, with $\Omega = \phi^{-1/2}$,

$$\tilde{\kappa} = \frac{a}{2} P^{a-1} P'|_{\rho = \rho_h} = 0.$$  \hspace{1cm} (29)

Moreover, the surface gravity in Jordan’s frame is

$$\kappa = -\frac{\xi}{2} P^{a-1} P'|_{\rho = \rho_h} + \tilde{\kappa} = 0,$$  \hspace{1cm} (30)

when $a \geq 2$. Hence, the temperature is zero in both frames for the cold black holes. The conformal factor is regular across the horizon. It may be tempting to conclude that the black hole temperature is conformally invariant in the case studied here.

The problem of conformal invariance of the Hawking temperature has been addressed in Ref. [17]. In this work, it has been stated that the Hawking temperature is the same for black holes obtained from theories which are connected by a conformal transformation under the conditions of staticity and asymptotic flatness. The situation described above seems to confirm these general results, but the situation is a little bit more subtle: it turns out that the very question of conformal invariance here loses its meaning due to the discrete nature of the black hole solutions under consideration.

Indeed, it was established [3,4] that black hole solutions, in the Jordan frame, with $\omega$ constant, that is, in the Brans-Dicke theory, exist only when the parameters $a$ and $\xi$ obey the relations

$$a = \frac{m+1}{m-n}, \quad \xi = \frac{m-n-1}{m-n},$$  \hspace{1cm} (31)

where $m$ and $n$ are positive integers satisfying the condition

$$m - 2 \geq n \geq 0.$$  \hspace{1cm} (32)

In the Einstein-scalar field system, black hole solutions exist under another condition, (26). Although the two metrics are connected by the conformal transformation $g_{\mu\nu} = \phi_{BD}^{-1} \tilde{g}_{\mu\nu}$, the black hole solutions in Jordan’s frame do not correspond to black hole solutions in the Einstein frame, and vice versa. So the question of invariance loses its meaning here.

There is, however, another, quite general result: any horizon of infinite area has zero surface gravity (and hence zero Hawking temperature). Let us prove it for arbitrary static, spherically symmetric space-times.

For the general form (4) of the metric, the surface gravity (27) may be written as

$$\kappa = e^{\gamma - \alpha} |\gamma'| = \frac{1}{2} A'(\rho),$$  \hspace{1cm} (33)

where $A(\rho) := e^{2\gamma}$ and $\rho$ is the quasiglobal coordinate defined by the condition $\alpha + \gamma = 0$. A horizon is a sphere where $A = 0$. So a horizon with finite surface gravity corresponds to a simple zero of $A$, with $A' \neq 0$, at some finite value of $\rho$. On the other hand, the regularity conditions require that all $K_i$ are finite at the horizon. In particular, in the same coordinates, $K_2 = -\frac{1}{2} A' \beta'$, hence, with $A' \neq 0$, $|K_2| < \infty$ is only possible in case $|\beta'| < \infty$, which in turn means that $\beta$ is finite at finite $\rho$. Recalling that $4\pi r^2 = 4\pi e^{2\beta}$ is the area of the coordinate sphere, we can conclude that a horizon with finite temperature can only occur at a sphere of finite radius $r = e^\beta$. Hence, a horizon with an infinite area can only have zero temperature.
The above results confirm this general law: all such black holes are perfectly “cold”.

Finally, the infinite horizon area of the scalar black holes obtained here may suggest, according to the well-known relations of black hole thermodynamics [15], that they have infinite entropy. However, after a more close investigation, it has been argued that such black holes must in fact have zero entropy [18], which is more in agreement with a zero temperature state. This means that, for such objects, there is a violation of the law that relates the black hole entropy with the horizon area.

5 Conclusions

Scalar-tensor theories, which are in general characterized by a non-minimal coupling between gravity and the scalar field, predict the existence of exotic black holes, which have an infinite horizon area and zero Hawking temperature. In the vacuum case, that is, in the absence of any other matter field, a conformal transformation maps any scalar-tensor theory from a large class (the Bergmann-Wagoner class) into general relativity minimally coupled to a massless scalar field. It had been thought that no black hole solution exist in this Einstein-scalar field system. We have shown here that this statement is not true, and we have exhibited a new class of black hole solutions. However, the price to be paid for their existence is that the sign of the kinetic term of the scalar field is “wrong”, that is, the scalar field has negative energy. As in the scalar-tensor case, the “scalar” black holes have infinite horizon areas and zero temperature. However, the conditions in the parameter space for the existence of such black holes are different in Jordan’s (non-minimal coupling) and Einstein’s (minimal coupling) conformal frames.

One can add that the Einstein frame is common to the whole class of scalar-tensor theories, whereas Jordan frames change from theory to theory together with the nonminimal coupling functions. This means that the discrete “quantization” conditions for the solution parameters, providing the existence of cold black holes, will be different in similar solutions of different theories.

Due to the latter circumstance, a discussion of the invariance of the Hawking temperature with respect to conformal transformations, connecting different frames, is meaningless for the present solutions: these transformations do not map a black hole to a black hole. On the other hand, there is a general law saying that horizons of infinite area are always “cold”, i.e., have zero temperature. So the invariance properties of their temperature become trivial, even if such black holes are in a conformal correspondence.

The absence of continuations through surfaces of finite (or even zero) curvature is a peculiar property of many scalar-tensor solutions, indicating a special type of space-time singularities: violation of analyticity. Physical properties of such singularities and their possible regularization by taking into account more general solutions or quantum corrections may be of considerable interest.

The Hawking temperature discussed here is expressed in terms of the surface gravity $\kappa$. To be more rigorous, quantum fields around such black holes must be considered. This is a delicate point, since all black holes studied in this work have zero temperature, which is, in principle, a violation of the third law of thermodynamics. For cold black holes in the Jordan frame, there are anomalies in the definition of quantum fields, connected with normalization of quantum modes [19]. However, no complete study in this sense has been performed yet, mainly due to technical difficulties. It would be of interest to consider this problem in the context of the “scalar” black holes presented in this work.

We can add in conclusion that if the phantom scalar field has a nonzero potential $V(\phi)$, it can form more diverse static, spherically symmetric self-gravitating configurations including different types of regular black holes with both zero and non-zero temperature [20].
Acknowledgments.

KB thanks the colleagues from DF-UFES for hospitality. The work was supported by CNPq (Brazil). J.C.F and N.P-N thank also CAPES/COFECUB (Brazil-France scientific cooperation) for partial financial support.

References

[1] C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation*, Freeman, San Francisco (1973).
[2] M. Campanelli and C.O. Lousto, Int. J. Mod. Phys. **D2**, 451 (1993);
[3] K.A. Bronnikov, G. Clément, C.P. Constantinidis and J.C. Fabris, Grav. & Cosmol. **4**, 128(1998);
[4] K.A. Bronnikov, G. Clément, C.P. Constantinidis and J.C. Fabris, Phys. Lett. **A243**, 121 (1998);
[5] F. Piazza and S. Tsujikawa, JCAP **0407**, 004 (2004);
[6] J.S. Bagla, H.K. Jassal and T. Padmanabhan, Phys. Rev. **D67**, 063504 (2003);
[7] R.R. Caldwell, M. Kamionkowski and N.N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003);
[8] S. Hannestad and E. Mortsell, JCAP **0409**, 001 (2004);
[9] U. Alam, V. Sahni, T.D. Saini and A.A. Starobinsky, Mont. Not. R. Astron. Soc. **354**, 274 (2004);
[10] S.W. Allen et al., Mont. Not. R. Astron. Soc. **353**, 457 (2005);
[11] B.C. Xanthopoulos and T. Zannias, Phys. Rev. **D40**, 2564 (1989);
[12] I.Z. Fisher, *Zh. Eksp. Teor. Fiz.* **18**, 636 (1948); gr-qc/9911008
[13] O. Bergmann and R. Leipnik, *Phys. Rev.* **107**, 1157 (1957).
[14] K.A. Bronnikov, *Acta Phys. Pol.* **B4**, 251 (1973).
[15] R.M. Wald, *General Relativity*, The University of Chicago Press, Chicago (1984);
[16] K.A. Bronnikov, C.P. Constantinidis, R.L. Evangelista and J.C. Fabris, Int. J. Mod. Phys. **D8**, 481 (1999);
[17] T. Jacobson and G. Kang, *Conformal invariance of black hole temperature*, gr-qc/9307022.
[18] O.B. Zaslavskii, Class. Quant. Grav. **19**, 3783 (2002);
[19] F.G. Alvarenga, A.B. Batista, J.C. Fabris and G.T. Marques, Grav. & Cosmol. **10**, 184 (2004).
[20] K.A. Bronnikov and J.C. Fabris, gr-qc/0511109.