Thermal monopole condensation in QCD with physical quark masses

Marco Cardinali, Massimo D’Elia, and Andrea Pasqui
Dipartimento di Fisica dell’Università di Pisa, Largo Pontecorvo 3, I-56127 Pisa, Italy and INFN - Sezione di Pisa, Largo Pontecorvo 3, I-56127 Pisa, Italy
(Dated: July 13, 2021)

Thermal monopoles, identified after Abelian projection as magnetic currents wrapping non-trivially around the thermal circle, are studied in $N_f = 2 + 1$ QCD at the physical point. The distribution in the number of wrappings, which in pure gauge theories points to a condensation temperature coinciding with deconfinement, points in this case to around 275 MeV, almost twice the QCD crossover temperature $T_c$; similar indications emerge looking for the formation of a percolating current cluster. The possible relation with other non-perturbative phenomena observed above $T_c$ is discussed.

PACS numbers: 12.38.Aw, 11.15.Ha, 12.38.Gc

Color confinement is one of the most intriguing aspects of Quantum Chromo Dynamics (QCD), the theory of strong interactions. Despite a plenty of phenomenological and numerical evidence, a full theoretical understanding of it, stemming from QCD first principles, is still lacking. In pure $SU(N)$ gauge theories, confinement can be reconduced to the realization of an exact center symmetry: when this is not spontaneously broken, one has a linearly rising potential between static color charges, with an associated string tension. Exact order parameters, like the Polyakov loop, can then be easily associated with the high $T$ deconfinement transition. As one moves to full QCD, center symmetry gets broken explicitly by dynamical quarks and the confining string breaks at large distances. The restoration of chiral symmetry, which is almost exact, becomes the dominant phenomenon which identifies a (pseudo)critical crossover temperature around 155 MeV [1, 2]. While the Polyakov loop and its susceptibilities still show a non-trivial behaviour around a similar temperature [3], a clear association with deconfinement is not clear.

On the other hand, various mechanisms have been proposed, which typically interpret confinement in terms of the condensation of topological degrees of freedom. Even if no consensus yet exists regarding the nature of such degrees of freedom, all descriptions lead to a correct identification of the deconfinement transition in pure gauge theories. It is therefore of great interest to investigate them in full QCD, where a univocal description of deconfinement is missing. A possible mechanism is that based on dual superconductivity [4, 8], i.e. on the idea that the QCD vacuum is characterized by the spontaneous breaking of an Abelian magnetic symmetry, induced by the condensation of magnetic charges, leading to confinement of chromo-electric charges via a dual Meissner effect. The mechanism has been tested by lattice simulations in various ways, like looking at the expectation value of magnetically charged operators and at the effective monopole action [8, 10], or by studying the properties of monopole currents extracted from non-Abelian gauge configurations. The identification of Abelian degrees of freedom relies on a procedure known as Abelian projection, which is based on the choice of an adjoint field. No natural adjoint field exists in QCD, so that the procedure is partially arbitrary: a popular choice is the so-called Maximal Abelian gauge (MAG) projection.

A particularly useful way to look for monopole condensation is to start from the deconfined, high-temperature phase and to investigate the properties of the so-called thermal monopoles, which are quasi-particles identified with monopole currents with a non-trivial wrapping around the Euclidean time direction [20, 26]. An analogy with the path-integral formulation for a system of identical particles permits to associate monopole currents with multiple wrappings around the thermal circle as set of thermal monopoles undergoing a permutation cycle [27]; based on this analogy, the statistical distribution of the multiple wrappings permits to reconstruct the quantum properties of the thermal monopole ensemble and to extrapolate a temperature where a phenomenon similar to Bose-Einstein condensation (BEC) takes place.

Notably, this approach returns condensation temperatures which coincide, within errors and both for $SU(2)$ and $SU(3)$ pure gauge theories [28, 29], with the standard deconfinement temperature; that happens in a peculiar way also for trace deformed theories [30] where, in spite of the decrease of the thermal monopole density approaching the center symmetric phase, monopole condensation, revealed by the distribution in the number of thermal wrappings, takes place where expected anyway. Based on these successes, the plan of the present study is to extend the investigation to QCD with $N_f = 2 + 1$ flavors at the physical point, to understand if and where a BEC-like phenomenon takes place for thermal monopoles in this case.

Technique details – Abelian monopoles in $SU(N)$ gauge theories are identified after the so-called Abelian projection [31]. The $SU(N)$ gauge symmetry is fixed apart from a remnant $U(1)^{N-1}$ Abelian symmetry: in absence of a natural Higgs field in the theory, the way this is
defined on the lattice, so monopole currents (Abelian plaquette) is built in terms of the Abelian gauge links. For appropriate choices of $\lambda$, such maximization makes diagonal the following operator

$$\hat{X}(n) = \sum_{\mu} \left[ U_{n,\mu}^\dagger \hat{\lambda} U_{n,\mu} + U_{n-\mu,\mu}^\dagger \hat{\lambda} U_{n-\mu,\mu} \right]$$  \hspace{1cm} (2)$$

which plays the role of the Higgs field. If $\hat{\lambda}$ is expanded over the basis of fundamental weights $\phi_0^k$ ($k = 1, N - 1$)

$$\hat{\lambda} = b_k \phi_0^k, \quad \phi_0^k = \frac{1}{N} \text{diag} \left( \frac{N}{N-k}, \ldots, \frac{N-k}{N-k} \right)$$  \hspace{1cm} (3)$$

the condition is that $b_k \neq 0 \forall k$ \cite{22}, moreover if they are all positive one has a well defined ordering for the Higgs field eigenvalues. In this gauge (unitary gauge) and in the continuum, the Abelian ‘t Hooft field strength tensor $F_{\mu \nu}^{(k)}$ can be reconstructed, for each $U(1)$ residual subgroup, in terms of the Abelian gauge field \cite{34,35}

$$a_\mu^{(k)} \equiv \text{tr} (\hat{\phi}_0^k A_\mu) = \sum_{j=1}^{k} (A_\mu)_{jj}.$$  \hspace{1cm} (4)$$

Magnetic monopoles are identified as defect lines (in 4D) where two adjacent eigenvalues of $\hat{X}$ coincide; one has $N - 1$ independent monopole species in correspondence of the residual $U(1)^{N-1}$ Abelian symmetry. Instead in standard MAG \cite{30}, where one simply maximizes the diagonal part of gauge link variables, the residual gauge group includes a permutation symmetry which mixes different monopole species and makes them not well defined \cite{22}. As in Ref. \cite{29} we consider a choice of $\hat{\lambda}$ with equal coefficients $b_k = 1$, namely $\hat{\lambda} = \text{diag}(1,0, -1)$ for $SU(3)$, so that all species are treated equally, leading also to a reduction of lattice artefacts \cite{22}.

On the lattice, Eq. (4) is implemented by taking the phases $\text{diag}(\phi_\mu^1(n), \phi_\mu^2(n), \ldots, \phi_\mu^N(n))$ of the diagonal part of gauge links $U_{n,\mu}$, then the $k$-th ‘t Hooft tensor $\theta_\mu^{(k)}$ (Abelian plaquette) is built in terms of the Abelian gauge phases

$$\theta_\mu^{(k)}(n) = \sum_{j=1}^{k} \phi_\mu^{(j)}(n).$$  \hspace{1cm} (5)$$

Singular points where eigenvalues coincide are not well defined on the lattice, so monopole currents $m_\mu^{(k)}$ are detected as violations of the Bianchi identity, i.e. by measuring magnetic fluxes across the elementary 3-cubes (DeGrand-Touissant construction \cite{37}):

$$m_\mu^{(k)} = \frac{1}{12} \varepsilon_{\mu \nu \rho} \partial_\nu \tilde{\theta}_\mu^{(k)} \delta_\rho,$$  \hspace{1cm} (6)$$

where $\partial_\nu$ is the forward lattice derivative and

$$\theta_\mu^{(k)} \equiv \tilde{\theta}_\mu^{(k)} + 2 \pi n_\mu^{(k)}, \quad \partial_\nu \tilde{\theta}_\mu^{(k)} \in \{0, 2 \pi\}, \quad n_\mu^{(k)} \in \mathbb{Z}. \hspace{1cm} (7)$$

The current $m_\mu^{(i)}$ forms closed loops, i.e. $\partial_\nu m_\mu^{(i)} = 0$. Then thermal monopoles (anti-monopoles) are identified with magnetic currents having a non trivial wrapping around the temporal direction. This thermal monopoles trajectories can be interpreted as the path-integral representation of an ensemble of magnetically charged quasi-particles populating the thermal medium \cite{20,21,23,28,29}. In this picture, trajectories wrapping $k$ times around the temporal direction are associated with the cyclic permutation of $k$ identical quasi-particles \cite{28,29}. We will consider the total thermal monopoles density $\rho$ and its cycle decomposition $\rho_k$ defined as follows:

$$\rho = \sum_{k} k \rho_k; \quad \rho_k \equiv \frac{N_{\text{wrap},k}}{V_s},$$  \hspace{1cm} (8)$$

where $V_s = a^3 L^3$ is the spatial volume and $N_{\text{wrap},k}$ is the number of currents wrapping $k$ times. The distribution in the number of wrappings $k$ can signal the approach to a BEC transition. In particular one expects

$$\rho(k) \propto e^{-\mu/k} \frac{1}{k^\alpha},$$  \hspace{1cm} (9)$$

where $\mu \equiv -\mu/T$ is the dimensionless chemical potential and $\alpha = 5/2$ for non-interacting bosons. Monopole condensation is then signalled by $\mu$ approaching zero as a function of $T$ \cite{28,29}.

Our discretization of $N_f = 2 + 1$ QCD relies on stout rooted staggered fermions and the Symanzik tree-level improved gauge action, the partition function reads:

$$Z = \int DU e^{-S_{YM}} \prod_{u,s,d} \det (D_{st}^f)^{1/4},$$  \hspace{1cm} (10)$$

where $DU$ is the product of Haar measures for $SU(3)$ gauge links. The gauge action $S_{YM}$ reads:

$$S_{YM} = -\frac{\beta}{3} \sum_{i,\mu \neq \nu} \left( \frac{5}{6} P_{i,\mu \nu}^{1 \times 1} - \frac{1}{12} P_{i,\mu \nu}^{2 \times 1} \right).$$  \hspace{1cm} (11)$$

where $P_{i,\mu \nu}^{1 \times 1}$ and $P_{i,\mu \nu}^{2 \times 1}$ are the real part of the trace of $1 \times 1$ and $1 \times 2$ loops. The staggered Dirac operator is:

$$D_{st}^f = m_f \delta_{ij} + \sum_{\nu=1}^4 \frac{n_{i,\mu}}{2} (U_{i,\nu}^{(2)} \delta_{i,j-v} - U_{i,\nu}^{(2)\dagger} \delta_{i,j+v} + \hat{\delta}),$$  \hspace{1cm} (12)$$
where \( \eta_{i,\nu} \)s are the staggered phases and \( U_{i,\mu}^{(2)} \) is the two-time stout smeared link with isotropic smearing parameter \( \rho = 0.15 \). The bare gauge coupling \( \beta \) and quark masses were kept on a line of constant physics \([39, 41]\) corresponding to a physical spectrum. Gauge configurations were generated using a Rational Hybrid Monte-Carlo algorithm running on GPUs \([42]\). We explored temperatures up to \( \sim 1 \text{ GeV} \) and, in order to estimate finite spacing and finite size effects, we considered simulations on \( 32^3 \times 8, 24^3 \times 6 \) and \( 48^3 \times 6 \) lattices. Gauge fixing, namely the maximization of Eq. \( (1) \), was based on an over-relaxation algorithm (see Ref. \([29]\) for more details). We report results only for the first monopole species, the other coinciding within errors; different species correlations will be discussed in a forthcoming publication.

**Numerical Results** - The most striking aspect of our results emerges already from Fig. \( 1 \) where we report the ratio \( \rho_{c}/\rho_1 \) as a function of \( k \) for various temperatures. While results for \( T > 280 \text{ MeV} \) show a clear exponential decay with \( k \) and can be nicely fitted to Eq. \( (3) \) with a non-zero value of \( \hat{\mu} \), for lower temperatures the dependence on \( k \) is much flatter and actually compatible with \( \hat{\mu} = 0 \), as if monopole condensation were already at work.

The fitted values of \( \hat{\mu} \) are displayed in Fig. \( 2 \). We report values obtained both for \( \alpha = 0 \) and \( \alpha = 5/2 \), showing that, as for pure gauge, the outcome is independent of the assumption for it: in both cases \( \hat{\mu} \) approaches zero at \( T_{\text{BEC}} \sim 275 \text{ MeV} \). One can also appreciate that the dependence on the lattice spacing is negligible. This result is not easy to interpret: \( T_{\text{BEC}} \) is almost twice the well established pseudocritical temperature of QCD, \( T_c \approx 155 \text{ MeV} \), at which chiral symmetry is restored.

Since monopole condensation has been investigated in the literature in various different ways, we decided, to confirm this strange result, to explore an alternative method, looking for the formation of a dominating cluster of monopole currents \([43]\). In practice, for each gauge configuration, we divide the whole set of monopole currents \( m_\mu \) of a given species into subsets (clusters) of connected currents and measure the ratio \( r_c \) of the current length of the biggest cluster to the total length of the whole set. In general \( r_c \in [0,1] \) and one expects \( \langle r_c \rangle \to 0 \) in the thermodynamical limit if no dominating cluster forms, while it tends to some non-zero value when the largest cluster percolates becoming microscopic.

In Fig. \( 3 \) we report data for \( \langle r_c \rangle \) as a function of \( T \) for two spatial volumes, comparing them with the same quantity computed in pure gauge \( SU(3) \). The behavior is strikingly similar, with \( \langle r_c \rangle \) becoming volume independent and approaching 1 when \( T < 300 \text{ MeV} \); the transition is just sharper for pure gauge \( SU(3) \), where a weak first order deconfining transition takes places at \( T_{c, SU3} \sim 290 \text{ MeV} \).

The similarities with the pure gauge theory appear also when considering the normalized total thermal monopole density \( \rho \), which is reported in Fig. \( 4 \). At high \( T \), \( \rho/T^3 \) in full QCD is about twice than in pure gauge, but follows a similar behavior; in particular, data for \( T \gtrsim 600 \text{ MeV} \) are reasonably fitted by the perturbative prediction \([44, 45]\)

\[
\rho/T^3 \sim (\log(T/\Lambda_{\text{eff}}))^{-3}
\]

with \( \Lambda_{\text{eff}} = 47(5) \text{ MeV} \) \((48(1) \text{ MeV} \text{ for pure gauge})\). Around the transition, pure gauge results show a sharp drop, which can be interpreted as disappearance of part of the thermal component due to condensation: in practice, part of the thermal wrappings disappear because now monopole currents wrap also along spatial directions. A similar behavior, even if smoother, is observed for full QCD data in correspondence of \( T_{\text{BEC}} \), resembling also the prediction of Ref. \([46]\) based on pressure data.
In full QCD, instead, \( T_c \approx 300 \text{ MeV} \) [47, 48]. The critical temperature that corresponds to a real phase transition, even if not associated with evident thermodynamical signatures. Then, since \( T_{BEC} \) turns out to be strikingly close to the pure gauge \( T_c \), one should investigate whether this is accidental or not and what is the role of quarks in the game, by repeating our study for different quark masses and number of flavors, and by studying monopole-flavor correlations [59]. Finally, different confinement mechanisms should be investigated, to check whether a similar transition temperature is detected independently, including also different order parameters within the dual superconductor scenario, like the expectation value of magnetization also different order parameters within the dual superconductor scenario, like the expectation value of magnetization.

**Acknowledgements:** We thank Andrea Rucci for collaboration in the early stages of this study. Numerical simulations have been performed at the Scientific Computing Center at INFN-PISA and on the MARCONI and
M100 machines at CINECA, based on the agreement between INFN and CINECA (under project INF19_npqcd, INF20_npqcd and INF21_npqcd).
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