Research on the Joint Decision Model of High-speed Railway Dynamic pricing and Ticket allocation

Guangsheng Zhu\textsuperscript{a}, Yu Zhao\textsuperscript{*}

School of Traffic and Transportation, Beijing Jiaotong University, Beijing, 100044, China

*Corresponding author e-mail: yuzhao@bjtu.edu.cn, \textsuperscript{a}19125802@bjtu.edu.cn

Abstract. In response to the problems such as failure to improve revenue with current static and fixed prices of the same OD in high-speed railways, according to the theory of revenue management, based on the passenger flow given by the railway, on the basis of basic ticket allocation model, the high-speed train ticket pre-sale period is divided into several pre-sale phases, with the revenue of high-speed railway maximization as the goal, building the joint decision model of high-speed railway dynamic pricing and ticket allocation. The model is verified by an example, and the results of the model proposed in this paper are compared with those of the basic ticket allocation model. The results show that the model proposed in this paper can not only improve the revenue and seat occupancy rate, but also provide passengers with high-speed railway passenger transport products at different prices.

1. Introduction

Under the background of most high-speed railway lines in China are loss-making, how to maximize the revenue of high-speed railway under the established transportation capacity has become a key issue for the railway departments. High-speed railway passenger transport products have the characteristics of perishability, high fixed cost, low variable cost, acceptance of advance booking, and have applicable conditions for revenue management. Therefore, revenue management theory can be used to improve the revenue of high-speed railway.\cite{1} Many scholars at home and abroad have studied railway revenue management from the perspective of dynamic pricing and ticket allocation.

In terms of dynamic pricing, Ciancimino et al.\cite{2} assumed that the passenger flow demand was normally distributed and allocated ticket amount for a single ticket price. Shi feng\cite{3} studied the optimal strategy and practicability of Chinese railway dynamic ticket price, and the proposed recurrence formula can be used to solve the optimal strategy and practical strategy, making a meaningful attempt to carry out the research on Chinese railway dynamic ticket price. Lan boxiong\cite{4}, on the basis of analyzing the research progress of railway passenger revenue management, proposed a revenue management optimization model suitable for High-speed railway, which could solve large-scale revenue management optimization problems in a short time.

In terms of ticket allocation, Bao yun et al.\cite{5} proposed a single train ticket allocation model based on random demand. On the basis of predicting OD passenger flow of passenger trains, Shan xinghua\cite{6} constructed an intelligent ticket pre-classification system for railway passenger trains. Song wenbo et al.\cite{7} constructed a multi-train ticket allocation model for high-speed railway considering the travel time of passengers on the basis of the differences in train running time, comfort level and stopping station. Qiang lixia\cite{8} considered the fluctuation of passenger flow itself, the error of passenger flow...
prediction and the actual utilization of ticket amount at the stations along the way, and proposed a pre-allocation algorithm for passenger train ticket amount under the condition of given passenger flow.

Considering that dynamic pricing and ticket allocation are both important means of revenue management for high-speed railway, this paper divided the pre-sale period into several pre-sale stages with the goal of maximizing revenue, and constructed a joint decision model of dynamic pricing and ticket allocation.

2. Model building

2.1. Function of demand and price

A very important input in optimizing the revenue problem is the function of demand and price for a product or service. In real life, the demand for products or services is determined by many factors, but for the convenience of calculation, this paper considers that the demand is only related to the price. There are three common price response functions: linear price response function, elasticity invariant price response function and logit price response function. This paper adopts linear price response function.

The form of linear price response function is generally as follows:

\[ q(p) = a - b \cdot p \]  

\( b \) is price sensitivity, \( p \) is price and \( q \) is demand.

2.2. Basic ticket allocation model

Based on literature [5], the basic ticket allocation model is constructed and the following basic assumptions are made: (1) for the convenience of research and calculation, only single high-speed railway trains are considered and only second-class seats are considered; (2) the overcrowding will not be considered; (3) refund and replacement tickets are not considered.

Let \( n \) represent the number of stations on the line; \( M \) is the segment \((M = 1, 2, ..., n - 1); RS \) represents the set of OD, \( RS = \{(r, s)|r = 1, 2, ..., n - 1; s = r + 1, ..., n\} \), where \( r \) represents the original station and \( s \) represents the destination of the OD; \( C \) represents the number of second-class seats on high-speed trains; Set \( A_{rsM} \) as 0-1 variable, \( A_{rsM} = 1 \) means OD \((r, s)\) occupying segment \( M \), and \( A_{rsM} = 0 \) means that OD \((r, s)\) do not occupy segment \( M \). \( p_{rs} \) represents the price of OD \((r, s)\); \( q_{rs} \) represents the demand of OD \((r, s)\); \( x_{rs} \) represents the number of tickets allocated to OD \((r, s)\); \( Z \) is the total revenue of a single train.

Basic ticket allocation model (M1) is constructed as follows:

\[ \text{max } Z = \sum_{(r,s)} p_{rs} x_{rs} \]  

\[ q_{rs} = a_{rs} - b_{rs} p_{rs}, \forall (r, s) \in RS \]  

\[ \sum_{(r,s)} A_{rsM} x_{rs} \leq C, \forall (r, s) \in RS, M = 1, 2, ..., n - 1 \]  

\[ 0 \leq x_{rs} \leq q_{rs}, \forall (r, s) \in RS \]  

\[ x_{rs} \in Z, \forall (r, s) \in RS \]

Equation (2) is the objective function of model (M1), taking the maximum revenue of a single train as the objective function. Equation (3) is a linear price response function; Equation (4) is the seat capacity constraint, indicating that the sum of the number of tickets allocated by OD in segment \( M \) is less than or equal to the maximum seat capacity of the train. Equation (5) is the upper and lower limit constraint of the number of tickets allocation; Equation (6) is an integer constraint on the number of tickets allocated.

2.3. Joint Decision Model for Dynamic Pricing and Ticket Allocation

Although model(M1) can solve the problem of ticket allocation in the case of single ticket price, in order to solve the problem of ticket allocation in the case of multiple ticket prices, on the basis of model(M1), the pre-sale period of passenger tickets is divided into multiple pre-sale stages to build a joint decision-making model of dynamic pricing and ticket allocation (M2).

When constructing dynamic pricing and ticket allocation joint decision model (M2), the following assumptions are made: (1) this model studies one train and only considers second-class seats; (2) no
refund or replacement tickets shall be considered; (3) high-speed rail passengers can accept different prices for the same seat; (4) overcrowding is not considered.

Let $T$ represent the number of stages of ticket sales, $t = 1, 2, ..., T$, where smaller means closer to the departure time, $t = 1$ is the period nearest to the departure time; $p_{rst}$ represent the price of OD$(r, s)$ in the $t$ period, $p_{rs}$ represent the lower limit of the price of OD$(r, s)$, $p_{rs}$ represent the upper limit of the price of OD$(r, s)$, $q_{rst}$ represent the demand of OD$(r, s)$ in the $t$ period, $x_{rst}$ represent the number of ticket allocation of OD$(r, s)$ in the $t$ period, $\Omega = \{M(r, s) \in RS, A_{rst} = 1\}$, $\Omega$ represent the set of the segment occupying by OD$(r, s)$.

The joint decision model (M2) of dynamic pricing and ticket allocation is constructed as follows:

$$
\text{max} \ Z = \sum_{t=1}^{T} \sum_{r,s} p_{rst} x_{rst} \tag{7}
$$

$$
q_{rst} = a_{rst} - b_{rst} p_{rst}, \forall t = 1, 2, ..., T, \forall (r, s) \in RS \tag{8}
$$

$$
\sum_{t=1}^{T} \sum_{r,s} A_{rst} x_{rst} \leq C, \forall t = 1, 2, ..., T, \forall (r, s) \in RS, M = 1, 2, ..., n - 1 \tag{9}
$$

$$
p_{rs} \leq p_{rst} \leq p_{rs}, \forall t = 1, 2, ..., T, \forall (r, s) \in RS \tag{10}
$$

$$
p_{cud} > p_{rst}, \forall t = 1, 2, ..., T, \forall l_{cud} > l_{rs}, \forall (c, d), (r, s) \in RS \tag{11}
$$

$$
p_{rst} \leq \sum_{M \in \Omega} p_{M(M+1)} t, \forall t = 1, 2, ..., T, \forall (r, s) \in RS \tag{12}
$$

$$
p_{rsi} \geq p_{rst}, \forall t, j = 1, 2, ..., T, l < j, \forall (r, s) \in RS \tag{13}
$$

$$
0 \leq x_{rst} \leq q_{rst}, \forall t = 1, 2, ..., T, \forall (r, s) \in RS \tag{14}
$$

$$
p_{rst} \in Z, x_{rst} \in Z, \forall t = 1, 2, ..., T, \forall (r, s) \in RS \tag{15}
$$

Equation (7) is the objective function of the model (M2); Equation (8) is a linear price response function. Equation (9) is seat capacity constraint; Equation (10) is the upper and lower bound of ticket price; Formula (11) indicates that the longer OD is, the higher the ticket price is; Formula (12) indicates that the fare rate obeys the law: the longer OD is, the lower the average fare of per kilometer is; Formula (13) means that the ticket price is higher when it is closer to the pre-sale stage of driving; Equation (14) is the upper and lower bound of the number of tickets allocated; Equation (15) is the integer constraint of ticket price and the amount of ticket allocation.

### 3. Case Analysis

Suppose that the second-class seat of a high-speed railway train is 800, and the high-speed train passes through 3 stations and 2 sections, as shown in figure 1. $l_{AC} > l_{AB} > l_{BC}$ is the distance size relation of each OD. The passenger demand function is $q(p) = a - b \cdot p$. The ticket price is allowed to go up or down by 10%. The fares of each OD and their upper and lower limits are shown in table 1.

![Figure 1. Train running line](image)

**Table 1. Three Scheme comparing.**

| OD      | price/Yuan | Upper limit/Yuan | Lower limit/Yuan |
|---------|------------|------------------|------------------|
| AB      | 400        | 440              | 360              |
| BC      | 100        | 110              | 90               |
| AC      | 480        | 528              | 432              |

The pre-sale period of high-speed trains is divided into four pre-sale stages. The smaller the train is, the closer it is to the departure of high-speed trains. When $t=1$, the nearest pre-sale period is. According to the conclusion of literature [9], $a_{rs}$ in the passenger demand function increases with the decreasing of $t$, while $b_{rst}$ decreases with the decreasing of $t$. The closer the departure time is, the lower the sensitivity of passenger demand to price. According to this conclusion, the values of $a$ and $b$ in each OD and pre-sale stage are assumed, as shown in table 2.
Table 2. The values of $a, b$. 

| OD | $a_{rs1}$ | $b_{rs1}$ | $a_{rs2}$ | $b_{rs2}$ | $a_{rs3}$ | $b_{rs3}$ | $a_{rs4}$ | $b_{rs4}$ |
|----|----------|----------|----------|----------|----------|----------|----------|----------|
| AB | 86       | 0.148    | 81       | 0.160    | 76       | 0.171    | 73       | 0.180    |
| BC | 27       | 0.13     | 25       | 0.14     | 22       | 0.15     | 17       | 0.16     |
| AC | 340      | 0.26     | 320      | 0.30     | 300      | 0.35     | 280      | 0.40     |

LINGO is used to solve the model (M1) in this paper, and the ticket allocation scheme under the condition of single ticket price is obtained as shown in Table 3.

Table 3. Ticket allocation schemes under the condition of single ticket price.

| items                        | OD      | revenue/Yuan |
|------------------------------|---------|--------------|
|                              | AB  | BC  | AC  |            |
| price/Yuan                   | 400 | 100 | 480 | 316980     |
| The amount of ticket         | 51  | 33  | 611 |            |
| Section passenger flow       | 662 | 644 |     |            |
| Section occupancy rate/%     | 82.75| 80.5|     |            |

Using LINGO to solve the model (M2) in this paper, the fare and the amount of ticket of each OD in each pre-sale period under dynamic pricing are obtained in Table 4.

Table 4. Ticket allocation scheme under dynamic pricing.

| Items                        | OD      | revenue/Yuan | Total revenue/Yuan |
|------------------------------|---------|--------------|--------------------|
|                              | AB  | BC  | AC  |            |                  |
| price/Yuan                   |     |     |     | 418 | 110 | 526 | 118130 |
| The amount of ticket         |     |     |     | 24  | 12  | 203 |         |
| price/Yuan                   |     |     |     | 362 | 110 | 470 | 93446  |
| The amount of ticket         |     |     |     | 23  | 9   | 179 |         |
| price/Yuan                   |     |     |     | 362 | 93  | 440 | 70052  |
| The amount of ticket         |     |     |     | 14  | 8   | 146 |         |
| price/Yuan                   |     |     |     | 361 | 93  | 432 | 49298  |
| The amount of ticket         |     |     |     | 8   | 2   | 107 |         |
| Total amount of ticket       |     |     |     | 69  | 31  | 635 |         |
| Section passenger flow       |     |     |     | 704 | 666 |     |         |
| Section occupancy rate/%     |     |     |     | 88  | 83.25|     |         |

By comparing table 3 with table 4, it can be seen that under the joint decision model of dynamic pricing and ticket allocation, the revenue of high-speed trains is 137946 yuan higher than that under the condition of single ticket price, with an increase of 4.40%. In addition, when the demand is less than the seat capacity, the dynamic pricing and ticket allocation model can improve the occupancy rate of each section and reduce the waste of seats. Under the basic ticket allocation model, the ticket price is single and does not take the price sensitivity of all kinds of passengers into account. Under the joint decision model of dynamic pricing and ticket allocation, the sensitivity of passengers to price in different OD and different ticket stages is considered.

4. Conclusion

In this paper, under the condition of given passenger flow and on the basis of single ticket allocation, a joint decision model of dynamic pricing and ticket allocation is constructed. Through the analysis of the example, the paper compares the ticket allocation scheme under the joint decision-making model of dynamic pricing and ticket allocation with the ticket allocation scheme under the
single ticket price, and the result shows that dynamic pricing can not only improve the revenue and seat occupancy rate, but also provide passengers with high-speed railway passenger transport products at different prices. This paper provides a new method for dynamic pricing, but this paper only studies the single train case, and the dynamic pricing of multi-train and multi-class ticket price needs further study.

References
[1] SONG Wenbo, ZHAO Peng, LI Bo. Research on Comprehensive Optimization of Dynamic Pricing and Seat Allocation for High-speed Single Train [J]. Journal of the China Railway Society, 2018, 40(07):10-16.
[2] Ciancimino A, Inzerillo G, Lucidi S, et al. A mathematical programming approach for the solution of the railway yield management problem [J]. TRANSPORTATION SCIENCE, 1999, 33(2):168-181.
[3] SHI Feng, ZHENG Guohua, GU Qiang. Optimal Dynamic Pricing of Railway Line Based on Passengers’ Reservation Price [J]. Application Research of Computers, 2014, 31(9):2635-2639.
[4] LAN Bo-xiong, ZHANG Li. Revenue Management Model for High-Speed Passenger Railway [J]. Chinese Journal of Management Science, 2009, 17(04):53-59.
[5] BAO Yun, LIU Jun, LIU Jiangchuan, et al. Seat Allotment Method for Single Train Based on Stochastic Demand [J], China Railway Science, 2015, 36(2):96-102.
[6] SHAN Xinghua, ZHOU Liangjin, Research on Intelligent Pre-Assignment of Ticket Allotment for Railway Passenger Train[J].China Railway Science, 2011,32(06):125-128.
[7] SONG Wenbo, ZHAO Peng, LI Bo, et al. High-speed railway ticket allocation considering passenger travel time [J]. Journal of Southeast University (Natural Science Edition), 2017, 47(05):1062-1068.
[8] QIANG Lixia. Research on Intelligent Ticket Pre-assignment Method for Passenger Trains [J]. Journal of the China Railway Society, 2018, 40(09):7-11.
[9] RAN Lun, LI Jinlin, XU Liping. Study on the Robust Model in Single-unit Product Dynamic Pricing in Revenue Management [J].Application of Statistics and Management, 2009,28(05):934-941.