Nambu–Jona–Lasinio approach to realization of confining medium

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Abstract

The mechanism of a confining medium is investigated within the Nambu–Jona–Lasinio (NJL) approach. It is shown that a confining medium can be realized in the bosonized phase of the NJL model due to vacuum fluctuations of both fermion and Higgs (scalar fermion–antifermion collective excitation) fields. In such an approach there is no need to introduce Dirac strings.

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1 Introduction

Mechanism of confining medium suggested in Abelian Higgs model by Narnhofer and Thirring [1] and in a dual QCD by Baker, Ball and Zachariasen [2] can be rather useful for the realization of quark confinement by a way avoiding the problem of the inclusion of Dirac strings. The dielectric constant of a confining medium \( \varepsilon(k^2) \) vanishes at large distances like \( \varepsilon(k^2) \sim k^2 \) which leads to a linearly rising interquark potential realizing confinement of quarks [3].

The approaches to confining medium [1,2] are based on the similarity between dual superconductivity and the Higgs mechanism inducing non-perturbative vacuum with properties of superconductor. The Nambu–Jona–Lasinio (NJL) model [4–10], being a relativistic extension of the BCS (Bardeen–Cooper–Schrieffer) theory of superconductivity [11], gives the alternative mechanism of the realization of a superconducting non-perturbative vacuum. Recently in Ref.[10] we have investigated the mechanism of quark confinement in the Abelian monopole NJL model with dual Dirac strings. In this model quarks and antiquarks are classical particles joined to the ends of dual Dirac strings, while monopoles are quantum massless fermion fields which become massive due to monopole–antimonopole condensation induced by four–monopole interaction. The monopole–antimonopole condensation accompanies itself the creation of the monopole–antimonopole scalar and dual–vector collective excitations. The ground state of the dual–vector field induced by the interaction with a dual Dirac string has the shape of the Abrikosov flux line. The latter leads to linearly rising potential and confinement of quarks and antiquarks joined to the ends of a dual Dirac string.

This paper is to apply the NJL model analogous the Abelian monopole NJL model [10] to the realization of confining medium and consider the way avoiding the inclusion of dual Dirac strings. Since we do not include dual Dirac strings, the quarks and antiquarks, confinement of which we are investigating, are described now by an external field \( \psi(x) \) related to an external electric current \( j_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x) \). Then, due to the absence of dual Dirac strings instead of the monopole fermion fields we use fermion fields like technifermions introduced in the technicolour approach [12] to the standard electroweak model for the description of the appearance of the \( W \) and \( Z \)–boson masses without Higgs mechanism. The NJL model in our consideration is an Abelian one, and we need to introduce only one sort of technifermions. For the definitness we would call them electroquarks and define by the field \( \chi(x) \). The electroquarks are massless and acquire the mass due to strong local four–electroquark interactions.

The Lagrangian of the starting system should read [10]

\[
\mathcal{L}(x) = \bar{\chi}(x) i \gamma^\mu \partial_\mu \chi(x) + G [\bar{\chi}(x) \chi(x)]^2 - G_1 [\bar{\chi}(x) \gamma_\mu \chi(x) + j_\mu(x)] [\bar{\chi}(x) \gamma^\mu \chi(x) + j^\mu(x)],
\]

where \( G \) and \( G_1 \) are positive phenomenological coupling constants that we fix below.

The Lagrangian Eq.(1.1) is invariant under \( U(1) \) group. Due to strong attraction in the \( \bar{\chi}\chi \)–channels produced by the local four–electroquark interaction Eq.(1.1), the \( \chi \)–fields become unstable under \( \bar{\chi}\chi \) condensation, i.e. \( <\bar{\chi}\chi> \neq 0 \). Indeed, in the condensed phase the energy of the ground state of the electroquark fields is negative, i.e.

\[
W = -<\mathcal{L}_\chi(x)> = -\left(\frac{3}{4} G + G_1\right) [\bar{\chi}\chi ]^2 < 0,
\]
whereas in the non-condensed phase, when $<\bar{\chi}\chi> = 0$, we have $\mathcal{W} = 0$. This means that the condensed phase is much more advantageous to the electroquark system, described by the Lagrangian (1.1). The electroquark fields become condensed without breaking of $U(1)$ symmetry as well.

In the condensed phase the electroquark fields acquire a mass $M$ satisfying the gap-equation [4–10]

$$M = -2G <\bar{\chi}(0)\chi(0)> = \frac{GM}{2\pi^2}J_1(M) = \frac{GM}{2\pi^2} \int \frac{d^4k}{\pi^2} \frac{1}{M^2 - k^2} = \frac{GM}{2\pi^2} \left[ \Lambda^2 - M^2 \ln \left(1 + \frac{\Lambda^2}{M^2}\right) \right],$$

where $\Lambda$ is the ultra-violet cut-off.

The condensation of the electroquark fields accompanies the creation of $\bar{\chi}\chi$ collective excitations with the quantum numbers of a scalar Higgs meson field $\rho$ and a vector field $A_\mu$. The main aim of the approach is to show that in the tree approximation for the $A_\mu$–field and after the integration over the electroquark and the scalar fields the effective Lagrangian of the $A_\mu$–field acquires the form [1,2]

$$L_{\text{eff}}[A(x)] = -\frac{1}{4} F_{\mu\nu}(x) \varepsilon(\Box) F^{\mu\nu}(x) - j_\mu(x) A^\mu(x),$$

where $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$ and $\varepsilon(\Box)$ is the operator of the dielectric constant which in the momentum representation $\varepsilon(k^2)$ vanishes at large distances, i.e. $\varepsilon(k^2) \sim k^2$ at $k^2 \rightarrow 0$. Due to the tree $A_\mu$–field approximation integrating over the electroquark and the scalar fields we can keep only the terms proportional to $F_{\mu\nu}(x) F^{\mu\nu}(x)$ and $A_\mu(x) A^\mu(x)$.

The paper is organized as follows. In Sect. 2 we derive the effective Lagrangian for collective excitations of electroquarks in one–loop approximation keeping only the main divergent contributions and leading order in long–wavelength expansion. That is in accordance with the standard approximation accepted in the NJL model approaches. We show that such a way does lead to the realization of a non–trivial medium. In Sect. 3 we take into account convergent contributions of the one–electroquark loop diagrams that means the step beyond the standard approximation. This has led to the effective Lagrangian for the vector collective excitations coupled to scalar collective excitations. Integrating out the scalar collective excitations, since they are much heavier than the vector ones, we arrive at the effective Lagrangian for the massless vector collective excitation field in a non–trivial medium with the dielectric constant vanishes at large distances. In Sect. 4 we show that the external $\psi$–quarks couple to each other and the medium via the exchange of the massless vector collective excitations become confined due to a linearly rising potential induced by the medium. In Sect. 5 we discuss the obtained results.

## 2 Effective Lagrangian. Standard approximation

Following [5–10] we define the effective Lagrangian of the $\rho$ and $A_\mu$ fields as follows

$$L_{\text{eff}}(x) = \tilde{L}_{\text{eff}}(x) - \frac{k^2}{4G} \rho^2(x) + \frac{g^2}{4G_1} A_\mu(x) A^\mu(x) - j_\mu(x) A^\mu(x),$$

3
where
\[ \mathcal{L}_{\text{eff}}(x) = -i \left\langle x \left| \ln \frac{\text{Det}(i \hat{\partial} - M + \Phi)}{\text{Det}(i \hat{\partial} - M)} \right| x \right\rangle. \] (2.2)

Here we have denoted \( \Phi = -g \gamma^\mu A_\mu - \kappa \sigma \), and \( \sigma = \rho - M/\kappa \), where \( g \) is the electric charge of the electroquark field that we fix below. In the tree approximation the \( \sigma \)–field has a vanishing vacuum expectation value (v.e.v.), i.e. \( < \sigma > = 0 \), whereas the v.e.v. of the \( \rho \)–field does not vanish, i.e. \( < \rho > = M/\kappa \neq 0 \).

The effective Lagrangian \( \mathcal{L}_{\text{eff}}(x) \) can be represented by an infinite series
\[ \mathcal{L}_{\text{eff}}(x) = \sum_{n=1}^{\infty} \frac{i}{n} \text{tr}_L \left\langle x \left| \left( \frac{1}{M - i \hat{\partial}} \right)^n \right| x \right\rangle = \sum_{n=1}^{\infty} \mathcal{L}_{\text{eff}}^{(n)}(x). \] (2.3)

The subscript \( L \) means the computation of the trace over Dirac matrices. The effective Lagrangian \( \mathcal{L}_{\text{eff}}^{(n)}(x) \) is given by [5–10,13]
\[ \mathcal{L}_{\text{eff}}^{(n)}(x) = \int \prod_{\ell=1}^{n-1} \frac{d^4 x_\ell }{(2 \pi)^4} e^{-i k_1 x_1 - \ldots - i k_n x} \left( -\frac{1}{n} \frac{1}{16 \pi^2} \right) \int \frac{d^4 k}{\pi^2 i} \]
\[ \times \text{tr}_L \left\{ \frac{1}{M - k} \Phi(x_1) \frac{1}{M - \hat{k} - \hat{k}_1} \Phi(x_2) \ldots \right. \]
\[ \left. \times \ldots \Phi(x_{n-1}) \frac{1}{M - \hat{k} - \hat{k}_1 - \ldots - \hat{k}_{n-1}} \Phi(x) \right\} \] (2.4)

at \( k_1 + k_2 + \ldots + k_n = 0 \). The r.h.s. of (2.4) describes the one–electroquark loop diagram with \( n \)–vertices. The one–electroquark loop diagrams with two vertices \( (n = 2) \) determine the kinetic term of the \( \sigma \)–field and give the contribution to the kinetic term of the \( A_\mu \)–field, while the diagrams with \( (n \geq 3) \) describe the vertices of interactions of the \( \sigma \) and \( A_\mu \) fields. According to the NJL prescription the effective Lagrangian \( \mathcal{L}_{\text{eff}}(x) \) should be defined by the set of divergent one–electroquark loop diagrams with \( n = 1, 2, 3 \) and 4 vertices [5–10]. The computation of these diagrams we perform at leading order in the long–wavelength expansion approximation accepted in the NJL model [5–10] when gradients of \( A_\mu \) and \( \sigma \) fields are slowly varying fields. This approximation is fairly good established for sufficiently heavy electroquark fields \( \chi(x) \) that we assume following the technicolour extension of the standard electroweak model [12]. As a result we get
\[ \mathcal{L}_{\text{eff}}(x) = \]
\[ = -\frac{g^2}{48 \pi^2} J_2(M) F_{\mu \nu}(x) F^{\mu \nu}(x) + \left\{ \frac{g^2}{4 G_1} - \frac{g^2}{16 \pi^2} [J_1(M) + M^2 J_2(M)] \right\} A_\mu(x) A^\mu(x) \]
\[ + \frac{1}{2} \frac{\kappa^2}{8 \pi^2} J_2(M) \partial_\mu \sigma(x) \partial^\mu \sigma(x) - M \left[ \frac{\kappa}{2 G} - \frac{\kappa}{4 \pi^2} J_1(M) \right] \sigma(x) \]
\[ + \frac{1}{2} \left[ -\frac{\kappa^2}{2 G} + \frac{\kappa^2}{4 \pi^2} J_1(M) - 4 M \frac{\kappa^2}{8 \pi^2} J_2(M) \right] \sigma^2(x) - 2 M \kappa \frac{\kappa^2}{8 \pi^2} J_2(M) \sigma^3(x) \]
\[ - \frac{1}{2} \frac{\kappa^2}{8 \pi^2} J_2(M) \sigma^4(x) - j_\mu(x) A^\mu(x). \] (2.5)
In order to get correct factors of the kinetic terms of the \( \sigma \) and \( A_\mu \) fields we have to set \([5–10]\)
\[
\frac{g^2}{12\pi^2}J_2(M) = 1 \quad , \quad \frac{\kappa^2}{8\pi^2}J_2(M) = 1,
\]
that arranges the relation \( \kappa^2 = 2g^2/3 \ [5–10] \), and \( J_2(M) \) is a logarithmically divergent integral defined by
\[
J_2(M) = \int \frac{d^4k}{\pi^2i} \frac{1}{(M^2 - k^2)^2} = \ln\left(1 + \frac{\Lambda^2}{M^2}\right) - \frac{\Lambda^2}{M^2 + \Lambda^2}.
\]

The relations (2.6) can be represented in a more comprehensible way in terms of constraints for the renormalization constants of the wave–function s of the \( A_\mu \) and \( \sigma \) fields. For this aim we rewrite the Lagrangian (2.5) as follows
\[
\mathcal{L}_{\text{eff}}(x) = \frac{1}{4}(1 - Z^\chi_A)F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2}(1 - Z^\chi_\sigma)\partial_\mu\sigma(x)\partial^\mu\sigma(x) + \ldots,
\]
where
\[
Z^\chi_A = 1 - \frac{g^2}{12\pi^2}J_2(M) \quad , \quad Z^\chi_\sigma = 1 - \frac{\kappa^2}{8\pi^2}J_2(M)
\]
are the renormalization constants of the wave–functions of the \( A_\mu \) and \( \sigma \) fields procreated by vacuum fluctuations of the electroquark fields. In Eq.(2.8) we have kept only kinetic terms of the \( A_\mu \) and \( \sigma \) fields, other terms are irrelevant at the moment. Since \( A_\mu \) and \( \sigma \) are bound \( \bar{\chi}\chi \)–states, the renormalization constants \( Z^\chi_A \) and \( Z^\chi_\sigma \) should vanish due to the so–called compositeness condition \( Z^\chi_A = Z^\chi_\sigma = 0 \ [14] \), i.e.
\[
Z^\chi_A = 1 - \frac{g^2}{12\pi^2}J_2(M) = 0 \quad , \quad Z^\chi_\sigma = 1 - \frac{\kappa^2}{8\pi^2}J_2(M) = 0.
\]
The compositeness condition can be applied to bound states in non–perturbative quantum field theory [14]. As a result we arrive at Eq.(2.10). Picking up Eq.(1.3) and Eq.(2.6) we bring up the effective Lagrangian (2.5) to the form
\[
\mathcal{L}_{\text{eff}}(x) = \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2}M_A^2A_\mu(x)A^\mu(x) - j^\mu(x)A_\mu(x)
\]
\[
+ \frac{1}{2}\partial_\mu\sigma(x)\partial^\mu\sigma(x) - \frac{1}{2}M_\sigma^2\sigma^2(x)\left[1 + \kappa^2\frac{\sigma(x)}{M_\sigma}\right]^2 - j_\mu(x)A^\mu(x),
\]
where \( M_\sigma = 2M \) is the mass of the \( \sigma \)–field and
\[
M_A^2 = \frac{g^2}{2G_1} - \frac{g^2}{8\pi^2}[J_1(M) + M^2J_2(M)]
\]
is the squared mass of the \( A_\mu \)–field. Without loss of generality one can assume that \( M_A \ll M_\sigma = 2M \). This should allow one to integrate over the heavy scalar collective excitations and derive an effective Lagrangian only for the vector ones.

The Lagrangian Eq.(2.11) describes the effective Lagrangian of the collective excitations (\( \bar{\chi}\chi \)–bound states) derived within the standard procedure of the NJL approach.
[4–10], i.e. at leading order in long–wavelength expansion of the one–electroquark loop diagrams.

The effective Lagrangian describing the propagation of the \( A_\mu \)–field in a dielectric medium should take the form Eq. (1.4). If it is a confining medium, the Fourier transform of a dielectric constant \( \varepsilon(k^2) \) should vanish at \( k^2 \to 0 \), i.e. \( \varepsilon(k^2) \to 0 \) at \( k^2 \to 0 \) \[1,2\]. It is seen that the Lagrangian Eq. (2.10) describes a dielectric medium with \( \varepsilon = 1 \). This should imply that the realization of the confining medium in the NJL approach goes beyond the standard procedure of the derivation of the effective Lagrangian describing collective excitations. The simplest extension of the NJL prescription is to take into account the contributions of convergent electroquark loop diagrams and the Higgs field loops. As has been shown in Ref. [9] contributions of convergent electroquark loop diagrams play an important role for processes of low–energy interactions of low–lying hadrons.

### 3 Effective Lagrangian for confining medium

Thus, in order to obtain non–trivial contributions to the dielectric constant we should leave a standard approximation for the derivation of the effective Lagrangians in the NJL models.

Since we are interested in the kinetic terms of the \( A_\mu \)–field, the simplest step leading beyond the standard approximation is to take into account convergent contributions of the one–electroquark loop diagrams to the kinetic term of the vector field \( A_\mu \). The exact calculation of the electroquark loop diagram with two–vector vertices alters the effective Lagrangian (2.11) as follows

\[
\mathcal{L}_{\text{eff}}(x) = -\frac{1}{4} F_{\mu\nu}(x) \varepsilon(\Box) F^{\mu\nu}(x) + \frac{1}{2} M_A^2 A_\mu(x) A^\mu(x) - j^\mu(x) A_\mu(x) \\
+ \frac{1}{2} \partial_\mu \sigma(x) \partial^\mu \sigma(x) - \frac{1}{2} M_\sigma^2 \sigma^2(x) \left[ 1 + \frac{\kappa \sigma(x)}{M_\sigma} \right] \left[ 1 - k^2 M_\sigma^2 \eta(1 - \eta) \right],
\]

where \( \varepsilon(\Box) \) is given by

\[
\varepsilon(\Box) = 1 - \frac{g^2}{2\pi^2} \int_0^1 d\eta \eta(1 - \eta) \ln \left[ 1 + \frac{\Box}{M^2} \eta(1 - \eta) \right],
\]

the Fourier transform of which yields

\[
\varepsilon(k^2) = 1 - \frac{g^2}{2\pi^2} \int_0^1 d\eta \eta(1 - \eta) \ln \left[ 1 - \frac{k^2}{M^2} \eta(1 - \eta) \right].
\]

We have dropped the convergent contributions of the electroquark loop diagrams to the other terms of the effective Lagrangian Eq. (3.1), since they are less important for the problem under consideration.

The other non–trivial contributions to the kinetic term of the \( A_\mu \)–field come from the convergent one–electroquark loop diagrams inducing the interactions between \( \sigma \) and \( A_\mu \) fields. They read

\[
\delta \mathcal{L}_{\text{eff}}(x) = M \frac{k g^2}{8\pi^2} \sigma(x) \left[ 1 + \frac{\kappa \sigma(x)}{2M} \right] A_\mu(x) A^\mu(x) + 
\]
\[ + \frac{g^2}{24\pi^2} \ell n \left[ 1 + \kappa \frac{\sigma(x)}{M} \right] F_{\mu\nu}(x) F^{\mu\nu}(x). \]  

(3.4)

This is the most general interaction of two vector mesons with scalar fields derived in leading order of the expansion in powers of gradients of the \( \sigma \)-field, \( \partial_\mu \sigma(x) \), which are slowly varying fields.

By appending the interaction Eq. (3.4) to the Lagrangian (3.1), we arrive at the following effective Lagrangian

\[ \mathcal{L}_{\text{eff}}(x) = \mathcal{L}_{\text{eff}}[A(x), \sigma(x)] + \frac{1}{2} \partial_\mu \sigma(x) \partial^\mu \sigma(x) - \frac{1}{2} M_\sigma^2 \sigma^2(x) \left[ 1 + \kappa \frac{\sigma(x)}{M_\sigma} \right]^2, \]  

(3.5)

where we have denoted

\[ \mathcal{L}_{\text{eff}}[A(x), \sigma(x)] = -\frac{1}{4} F_{\mu\nu}(x) \varepsilon(\Box, \sigma) F^{\mu\nu}(x) + \frac{1}{2} M_A^2(\sigma) A_\mu(x) A^\mu(x) - j^\mu(x) A_\mu(x). \]  

(3.6)

The dielectric constant \( \varepsilon(\Box, \sigma) \) reads

\[ \varepsilon(\Box, \sigma) = \frac{1}{2} - \frac{g^2}{6\pi^2} \ell n \left[ 1 + \kappa \frac{\sigma(x)}{M} \right] - \frac{g^2}{2\pi^2} \int_0^1 d\eta \frac{\sigma(1 - \eta)}{M^2} \ell n \left[ 1 + \frac{\Box}{M^2} \eta(1 - \eta) \right], \]

(3.7)

and \( M_A^2(\sigma) \) is defined

\[ M_A^2(\sigma) = \frac{g^2}{2G_1} - \frac{g^2}{8\pi^2}[J_1(M) + M^2 J_2(M)] + \frac{\kappa g^2}{4\pi^2} M_\sigma(x) \left[ 1 + \kappa \frac{\sigma(x)}{2M} \right]. \]

(3.8)

It is seen that a dielectric constant has become a functional of the \( \sigma \)-field. Thereby, one can expect a substantial influence of the vacuum fluctuations of the \( \sigma \)-field on the dielectric constant. Since the \( \sigma \)-field is much heavier than the \( A_\mu \)-field, i.e. \( M_\sigma = 2M \gg M_A \), at low energies only vacuum fluctuations of the \( \sigma \)-field are important. Therefore, in order to pick up the contribution of vacuum fluctuations of the \( \sigma \)-field, we have to integrate it out. The effective Lagrangian \( \mathcal{L}_{\text{eff}}[A_\mu(x)] \) can be defined [13]

\[ e^{i \int d^4x \mathcal{L}_{\text{eff}}[A_\mu(x)]} = \int \mathcal{D}\sigma e^{i \int d^4x \{ \mathcal{L}_{\text{eff}}[A_\mu(x), \sigma(x)] + \mathcal{L}_{\text{eff}}[\sigma(x)] \}}, \]

(3.9)

where \( \mathcal{L}_{\text{eff}}[\sigma(x)] \) reads

\[ \mathcal{L}_{\text{eff}}[\sigma(x)] = \frac{1}{2} \partial_\mu \sigma(x) \partial^\mu \sigma(x) - \frac{1}{2} M_\sigma^2 \sigma^2(x) \left[ 1 + \kappa \frac{\sigma(x)}{M_\sigma} \right] - V_{\text{eff}}[\sigma(x)], \]

(3.10)

where \( V_{\text{eff}}[\sigma(x)] \) describes the contribution of the convergent electroquark loops yielding self–interactions of the \( \sigma \)-field at leading order of the expansion in powers of the gradients of the \( \sigma \)-field. One can expect that \( V_{\text{eff}}[\sigma(x)] \sim \sigma^6(x) + \ldots \). The integrals over the \( \sigma \)-field can be normalized by the condition

\[ \int \mathcal{D}\sigma e^{i \int d^4x \mathcal{L}_{\text{eff}}[\sigma(x)]} = 1. \]

(3.11)
Of course, the exact integration over the $\sigma$–field cannot be done and we should develop an approximate scheme.

For this aim it is convenient to rewrite Eq. (3.9) as follows

$$\int d^4x \mathcal{L}_{\text{eff}}[A_\mu(x)] =$$
$$e \int d^4x \left[ -\frac{1}{4} F_{\mu\nu}(x) \varepsilon(\Box) F^{\mu\nu}(x) + \frac{1}{2} M_A^2 A_\mu(x) A^\mu(x) - j_\mu(x) A^\mu(x) \right]$$
$$\int \mathcal{D}\sigma \left\{ 1 + \frac{1}{4} \int d^4x F_{\mu\nu}(x) \varepsilon(\Box) \varepsilon(\Box) - \varepsilon(\Box) \right\}$$
$$\mathcal{L}_{\text{eff}}[\sigma(x)],$$

(3.12)

Recall that the effective Lagrangian Eq. (3.6) has been calculated keeping the quadratic terms in the $A_\mu$–field. This means that in the integrand we can expand the exponential keeping only quadratic terms in the $A_\mu$–field expansion

$$\int d^4x \mathcal{L}_{\text{eff}}[A_\mu(x)] =$$
$$e \int d^4x \left[ -\frac{1}{4} F_{\mu\nu}(x) \varepsilon(\Box) F^{\mu\nu}(x) + \frac{1}{2} M_A^2 A_\mu(x) A^\mu(x) - j_\mu(x) A^\mu(x) \right]$$
$$\int \mathcal{D}\sigma \left\{ 1 + \frac{1}{4} \int d^4x F_{\mu\nu}(x) \varepsilon(\Box) - \varepsilon(\Box) \right\}$$
$$\mathcal{L}_{\text{eff}}[\sigma(x)],$$

(3.13)

Thus, up to the quadratic terms in the $A_\mu$–field expansion the effective Lagrangian $\mathcal{L}_{\text{eff}}[A_\mu(x)]$ is given by

$$\mathcal{L}_{\text{eff}}[A(x)] = -\frac{1}{4} F_{\mu\nu}(x) \varepsilon(\Box) F^{\mu\nu}(x)$$
$$+ \frac{1}{2} M_A^2(A_\mu(x) A^\mu(x) - j_\mu(x) A^\mu(x)).$$

(3.14)

The derivation of the Lagrangian Eq. (3.14) has been performed in the $A_\mu$–field tree approximation. In this case the operators $F_{\mu\nu}(x) F^{\mu\nu}(x)$ and $A_\mu(x) A^\mu(x)$ are not affected by the contributions of higher powers in the $A_\mu$–field expansion. Therefore, in the $A_\mu$–field tree approximation the justification of the validity of the derivation of the Lagrangian Eq. (3.14) does not need the smallness of higher power terms in the $A_\mu$–field expansion.
The expectation values \( <\varepsilon(\square, \sigma)> \) and \( <M^2_A(\sigma)> \) read

\[
<\varepsilon(\square, \sigma)> = \int \mathcal{D}\sigma \varepsilon(\square, \sigma) \exp i \int d^4z \mathcal{L}_{\text{eff}}[\sigma(z)] =\]

\[
= 1 - \left< \frac{g^2}{6\pi^2} \ell n \left[ 1 + \frac{\sigma(x)}{M} \right] \right> - \frac{g^2}{2\pi^2} \int_0^1 d\eta \eta (1 - \eta) \ell n \left[ 1 + \frac{\square}{M^2} \eta (1 - \eta) \right]
\]

\[
= Z_A^\sigma - \frac{g^2}{2\pi^2} \int_0^1 d\eta \eta (1 - \eta) \ell n \left[ 1 + \frac{\square}{M^2} \eta (1 - \eta) \right], \tag{3.15}
\]

and

\[
<M^2_A(\sigma)> = \int \mathcal{D}\sigma M^2_A(\sigma) \exp i \int d^4z \mathcal{L}_{\text{eff}}[\sigma(z)] = \]

\[
= \frac{g^2}{2G_1} - \frac{g^2}{8\pi^2} [J_1(M) + M^2 J_2(M)] + \left< \frac{\kappa g^2}{4\pi^2} M \sigma(x) \left[ 1 + \frac{\sigma(x)}{2M} \right] \right> = \tag{3.16}
\]

\[
= \frac{g^2}{2G_1} - \frac{g^2}{8\pi^2} [J_1(M) + M^2 J_2(M)] + M Z_M^\sigma,
\]

where we have denoted

\[
Z_A^\sigma = 1 - \left< \frac{g^2}{6\pi^2} \ell n \left[ 1 + \frac{\sigma(x)}{M} \right] \right> = \]

\[
= 1 - \int \mathcal{D}\sigma \frac{g^2}{6\pi^2} \ell n \left[ 1 + \frac{\sigma(x)}{M} \right] \exp i \int d^4z \mathcal{L}_{\text{eff}}[\sigma(z)], \tag{3.17}
\]

and

\[
Z_M^\sigma = \left< \frac{\kappa g^2}{2\pi^2} \frac{\sigma(x)}{2M} \left[ 1 + \frac{\sigma(x)}{2M} \right] \right> = \]

\[
= \int \mathcal{D}\sigma \frac{\kappa g^2}{2\pi^2} \frac{\sigma(x)}{2M} \left[ 1 + \frac{\sigma(x)}{2M} \right] \exp i \int d^4z \mathcal{L}_{\text{eff}}[\sigma(z)]. \tag{3.18}
\]

It is seen that the quantities \( Z_A^\sigma \) and \( Z_M^\sigma \) are just constants, and \( Z_A^\sigma \) has the meaning of the renormalization constant of the \( \varepsilon\)-function of the \( \sigma\)-field. The vacuum expectation values entering the constants \( Z_A^\sigma \) and \( Z_M^\sigma \) can be represented in the form of time–ordered products [15]

\[
Z_A^\sigma = 1 - \left< \frac{g^2}{6\pi^2} \ell n \left[ 1 + \frac{\sigma(x)}{M} \right] \right> = \]

\[
= 1 - \int \mathcal{D}\sigma \frac{g^2}{6\pi^2} \ell n \left[ 1 + \frac{\sigma(x)}{M} \right] \exp i \int d^4z \mathcal{L}_{\text{eff}}[\sigma(z)] = \]

\[
= 1 - \left< 0 \right| T \left( \frac{g^2}{6\pi^2} \ell n \left[ 1 + \frac{\sigma(x)}{M} \right] \right) \exp i \int d^4z \mathcal{L}_{\text{int}}[\sigma(z)] \right| 0 \right>, \tag{3.19}
\]

and

\[
Z_M^\sigma = \left< \frac{\kappa g^2}{2\pi^2} \frac{\sigma(x)}{2M} \left[ 1 + \frac{\sigma(x)}{2M} \right] \right> = \]

\[
= \int \mathcal{D}\sigma \frac{\kappa g^2}{2\pi^2} \frac{\sigma(x)}{2M} \left[ 1 + \frac{\sigma(x)}{2M} \right] \exp i \int d^4z \mathcal{L}_{\text{eff}}[\sigma(z)] = \]

\[
= \left< 0 \right| T \left( \frac{\kappa g^2}{2\pi^2} \frac{\sigma(x)}{2M} \left[ 1 + \frac{\sigma(x)}{2M} \right] \right) \exp i \int d^4z \mathcal{L}_{\text{int}}[\sigma(z)] \right| 0 \right>. \tag{3.20}
\]
the vacuum expectation value \( \langle \sigma(x) \rangle \) reads

\[
\mathcal{L}_{\text{int}}[\sigma(x)] = \frac{1}{2} \partial_\mu \sigma(x) \partial^\mu \sigma(x) - \frac{1}{2} M_\sigma^2 \sigma^2(x) \left[ 1 + \kappa \frac{\sigma(x)}{M_\sigma} \right]^2 - V_{\text{eff}}[\sigma(x)] - \frac{1}{2} \partial_\mu \sigma(x) \partial^\mu \sigma(x) + \frac{1}{2} M_\sigma^2 \sigma^2(x) = -\kappa M_\sigma \sigma^3(x) - \frac{1}{2} \sigma^4(x) - V_{\text{eff}}[\sigma(x)]. \tag{3.21}
\]

In order to understand the structure of the constants \( Z_A^\sigma \) and \( Z_M^\sigma \) we suggest to calculate them in the one–loop approximation of the \( \sigma \)-field exchange. One can expect that the computation of the constants \( Z_A \) and \( Z_M \) accounting for multi–loop contributions should not change the one–loop result substantially but only provide a redefinition of parameters.

In the one–loop approximation \( Z_A \) and \( Z_M \) are given by

\[
Z_A^\sigma = 1 + \frac{g^2 \kappa^2}{3 \pi^2} \frac{1}{M_\sigma^2} < 0 | \sigma^2(x) | 0 > = 1 + \frac{\kappa^2 g^2 \Delta_1(M_\sigma)}{48 \pi^4} \frac{M_\sigma^2}{M_\sigma^2},
\]

\[
Z_M^\sigma = \frac{g^2 \kappa^2}{2 \pi^2} \frac{1}{M_\sigma^2} < 0 | \sigma^2(x) | 0 > = \frac{\kappa^2 g^2 \Delta_1(M_\sigma)}{32 \pi^4} \frac{M_\sigma^2}{M_\sigma^2}, \tag{3.22}
\]

where \( \Delta_1(M_\sigma) \) is a quadratically divergent momentum integral

\[
\Delta_1(M_\sigma) = \int \frac{d^4k}{\pi^2 i} \frac{1}{M_\sigma^2 - k^2}. \tag{3.23}
\]

The magnitude of \( \Delta_1(M_\sigma) \) depends on the regularization procedure. For example, within dimensional regularization \( \Delta_1(M_\sigma) \) is negative. The computation of \( \Delta_1(M_\sigma) \) by means of a cut–off regularization is not unambiguous. The result of the computation depends on a shift of the virtual \( \sigma \)-field momentum [16]. Indeed, one can define \( \Delta_1(M_\sigma; Q) \) instead of \( \Delta_1(M_\sigma) \) [16]

\[
\Delta_1(M_\sigma; Q) = \int \frac{d^4k}{\pi^2 i} \frac{1}{M_\sigma^2 - (k + Q)^2} = \Delta_1(M_\sigma) + \frac{1}{4} Q^2, \tag{3.24}
\]

where \( Q \) is an arbitrary 4-vector that can be a space–like one, i.e. \( Q^2 < 0 \). Thus, \( \Delta_1(M_\sigma) \) is an arbitrary quantity on both magnitude and sign. Below we fix \( \Delta_1(M_\sigma) \) using the compositeness condition for the \( A_\mu \)-field.

The momentum integral \( \Delta_1(M_\sigma) \) resembles the quadratically divergent integral \( J_1(M) \) which we encounter for the computation of the one–electroquark loop diagram defining the vacuum expectation value \( \langle \bar{\chi}(0) \chi(0) \rangle \). However, as \( \Delta_1(M_\sigma) \) is related to the contribution of one–loop \( \sigma \)-field exchange diagrams, the cut–off parameters applied to the regularization of \( \Delta_1(M_\sigma) \) and \( J_1(M) \) can arbitrarily differ on magnitude. This makes \( \Delta_1(M_\sigma) \) and \( J_1(M) \) independent each other. The integral \( J_1(M) \) suffers the same problem as \( \Delta_1(M_\sigma) \) and can be also considered as an arbitrary parameter of the approach fixed by the gap–equation Eq.(1.3), i.e. \( J_1(M) = 2 \pi^2 M/G \).

The Lagrangian Eq.(3.14) can be rewritten as follows

\[
\mathcal{L}_{\text{eff}}[A(x)] = -\frac{1}{4} F_{\mu\nu}(x) \left[ Z_A^\sigma + \varepsilon_{\text{eff}}(\Box) \right] F^{\mu\nu}(x) + \frac{1}{2} M_\sigma^2 A_\mu(x) A^\mu(x) - j^\mu(x) A_\mu(x), \tag{3.25}
\]

where \( Z_A^\sigma \) reads
where

\[ \varepsilon_{\text{eff}}(\square) = -\frac{g^2}{2\pi^2} \int_0^1 d\eta \eta (1-\eta) \ell n \left[ 1 + \frac{\square}{M^2} \eta (1-\eta) \right] \]  

(3.26)

and \( M^2_{\text{eff}} = \langle M^2_A(\sigma) \rangle \). We should notice that the renormalization constant \( Z^2_A \) does not appear in \( M^2_{\text{eff}} \). The appearance of \( Z^2_A \) in the mass term of the \( \mu \)-field can occur only after the scale transformation of the \( \mu \)-field, \( \sqrt{Z^2_A} A_{\mu} \rightarrow A_{\mu} \), removing \( Z^2_A \) from the kinetic term. Since we deal with the effective theory and do not perform such a scale transformation of the \( \mu \)-field, the constant \( Z^2_A \) cannot appear in the effective mass \( M^2_{\text{eff}} \).

Now let us focus on the behaviour of the dielectric constant \( \varepsilon(\square, \sigma) = Z^2_A + \varepsilon_{\text{eff}}(k^2) \) as a function of \( Z^2_A \). One can see that the confining behaviour of the dielectric constant, i.e. \( \varepsilon(k^2, \sigma) \sim k^2 \) at \( k^2 \rightarrow 0 \), can be realized only at \( Z^2_A = 0 \). The constraint \( Z^2_A = 0 \) is nothing more than the compositeness condition for the \( \mu \)-field [14]. This should imply that in our approach the confining medium can be realized only if the vector field \( A_{\mu} \) is a composite field [14] with the structure more complicated than \( \bar{\chi} \gamma_{\mu} \chi \) due to the contribution of the \( \sigma \)-field fluctuations. Conventionally, the structure of the composite \( \mu \)-field might be represented like \( \bar{\chi} \gamma_{\mu} \sigma \chi \).

Assuming that the \( \mu \)-field is a composite field, we can impose the compositeness condition \( Z^2_A = 0 \):

\[ Z^2_A = 1 + \frac{k^2 g^2}{48 \pi^4} \frac{\Delta_1(M_\sigma)}{M^2_\sigma} = 0. \]  

(3.27)

This fixes \( \Delta_1(M_\sigma) \) in terms of the electroquark mass and the coupling constants determined by one–electroquark loop diagrams: \( \Delta_1(M_\sigma) = -192 \pi^4 M^2 / k^2 g^2 = -128 \pi^4 M^2 / \kappa^4 \), where we have used the relations \( M_\sigma = 2 M \) and \( \kappa^2 = 2 g^2 / 3 \).

The compositeness condition Eq. (3.27) fixes the value of \( Z^2_M \) which yields

\[ M^2_{\text{eff}} = \frac{g^2}{2G_1} - \frac{g^2}{8 \pi^2} \left[ J_1(M) + M^2 J_2(M) \right] - \frac{3}{2} M^2 \]  

(3.28)

at \( M_\sigma = 2 M \). Of course, the magnitude of \( M^2_{\text{eff}} \) is arbitrary due to the arbitrariness of \( G_1 \). We can set it equal zero, i.e.

\[ M^2_{\text{eff}} = 0. \]  

(3.29)

Thus, further we deal with a massless vector field \( A_{\mu}(x) \) coupled to the external \( \psi \)-quark current \( j_{\mu}(x) = \bar{\psi}(x) \gamma_{\mu} \psi(x) \).

For the constraint (3.27) the dielectric constant reads

\[ \varepsilon_{\text{eff}}(k^2) = \]  

(3.30)

\[ = -\frac{g^2}{2\pi^2} \int_0^1 d\eta \eta (1-\eta) \ell n \left[ 1 - \frac{k^2}{M^2} \eta (1-\eta) \right] = \frac{g^2}{60 \pi^2} \frac{k^2}{M^2} + O \left( \frac{k^4}{M^4} \right). \]

In the coordinate space–time this yields

\[ \varepsilon_{\text{eff}}(\square) = \]  

(3.31)

\[ = -\frac{g^2}{2\pi^2} \int_0^1 d\eta \eta (1-\eta) \ell n \left[ 1 + \frac{\square}{M^2} \eta (1-\eta) \right] = -\frac{g^2}{60 \pi^2} \frac{\square}{M^2} + O \left( \frac{\square^2}{M^4} \right). \]
This behaviour of the dielectric constant produces the confining medium which provides a linearly rising interquark potential at large relative distances without inclusion of dual Dirac strings.

4 Linearly rising interquark potential

The effective Lagrangian of the $A_\mu$–field coupled to the external $\psi$–quark current $j_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$ in a medium with the dielectric constant $< \varepsilon(\Box, \sigma) >$ defined by Eq. (3.31) reads

$$\mathcal{L}_{\text{eff}}[A_\mu(x)] = -\frac{1}{4} F_{\mu\nu}(x) \varepsilon_{\text{eff}}(\Box) F^{\mu\nu}(x) - j^\mu(x) A_\mu(x).$$  \hspace{1cm} (4.1)

We have dropped the terms of order $O(A^3)$ and higher which do not contribute to the kinetic term of the $A_\mu$–field in the tree $A_\mu$–field exchange approximation which we are keeping to here.

Varying the Lagrangian (4.1) with respect to $A_\mu(x)$ we derive the equation of motion

$$\Box \varepsilon_{\text{eff}}(\Box) A_\mu(x) = j_\mu(x).$$  \hspace{1cm} (4.2)

The solution of this equation of motion can be represented as follows

$$A_\mu(x) = -\int d^4x' G_{\text{NT}}(x - x') j_\mu(x'),$$  \hspace{1cm} (4.3)

where $G_{\text{NT}}(x)$ is the Green function of the NT model given by the momentum representation

$$G_{\text{NT}}(x) = \int \frac{d^4k}{(2\pi)^4} \frac{\mu^2}{k^4} e^{-ik \cdot x} = \int \frac{d^4k}{(2\pi)^4} \frac{\mu^2}{k^4} e^{-ik \cdot x},$$  \hspace{1cm} (4.4)

where we have denoted $\mu^2 = 60\pi^2 M^2/g^2$. For example, the retarded Green function reads [17]

$$G_{\text{ret}}^{\text{NT}}(x) = \int \frac{d^4k}{(2\pi)^4} \frac{\mu^2}{[(k^0 + i0)^2 - k^2]^2} e^{-ik \cdot x} = \frac{\mu^2}{8\pi} \theta(t) \theta(x^2),$$  \hspace{1cm} (4.5)

where $x^2 = t^2 - \vec{x}^2$. The complete set of the Green functions for the model [1] has been computed in [18].

The effective interquark potential, defined in terms of the Green function $G^{\text{NT}}(x)$, reads [1]

$$V(\vec{r}) = -\int_{-\infty}^{\infty} dt G^{\text{NT}}(t, \vec{r}) = \sigma_{\text{string}} r + \text{an infrared divergent constant},$$  \hspace{1cm} (4.6)

where $\sigma_{\text{string}} = \mu^2/8\pi$ can be identified with the string tension [3,19]. The string tension, given by the expression $\sigma_{\text{string}} = \mu^2/8\pi$, has been computed in Ref.[19], where $1/\mu$ has been identified with the penetration depth of the dual electric field in a dual superconductor.
5 Conclusion

We have shown that in the Abelian NJL model, analogous the monopole NJL model [10] and the Abelian version of the technicolour extension of the standard electroweak model [12], one can realize a medium caused by quantum fluctuations of the electoquark fields \( \chi \), an Abelian analog of technifermions, and the scalar field \( \sigma \), collective \( \bar{\chi}\chi \) excitation. The dielectric constant of this medium \( \varepsilon(k^2) \) vanishes at large distances, i.e. at \( k^2 \to 0 \), like \( \varepsilon(k^2) = k^2/8\pi \sigma_{\text{string}} \), where \( \sigma_{\text{string}} \) can be identified with a string tension. This medium leads to confinement of the external quark fields \( \psi(x) \) if they couple to the medium and each other via the exchange of the massless composite vector fields \( A_\mu(x) \) which are the collective excitations with a conventional structure \( \bar{\chi}\gamma^\mu\sigma\chi \). The confining medium induces a linearly rising interquark potential \( V_{\bar{\psi}\psi}(\vec{r}) = \sigma_{\text{string}} r + C \) without the inclusion of dual Dirac strings, where the string tension \( \sigma_{\text{string}} \) is expressed in terms of the electroquark mass \( M \) and the cut–off \( \Lambda \), i.e. \( \sigma_{\text{string}} = (5M^2/8\pi) J_2(M) \).

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