A predictive control method to improve pressure tracking precision and reduce valve switching for pneumatic brake systems

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Abstract
Pneumatic brake systems are crucial for the operation of trains. However, due to switching characteristics of on/off solenoid valves, the precise pressure control and low switching activities of valves are difficult to guarantee simultaneously during braking. To address the issue, a hybrid model predictive control (MPC) method is proposed for implementing the multi-objective optimisation in this paper, i.e. the precise pressure tracking and the valve switching reduction. In order to model the hybrid behavior caused by continuous dynamics of the pressure and discrete features of solenoid valves, the mixed logic dynamical system theory is applied. Then, the braking control problem is recast as a hybrid MPC problem, which jointly optimises the pressure tracking precision and the valve switching. A mixed integer linear program is formulated to solve the MPC problem in an efficient way. Both simulation and experiment results show that the accurate pressure control and low switching activities of valves can be achieved by the proposed method when compared with existing methods.

1 | INTRODUCTION

Pneumatic brake system plays an important role in ensuring safe operation of trains, which uses compressed air as energy medium to generate braking power [1]. In the pneumatic brake system, the equalising reservoir and valves are two key components [2]. The air flow is steered by the valves to regulate the pressure of the equalising reservoir, generating the desired braking force. Implementing fast and precise pressure control of the equalising reservoir is critical for improving the braking performance of trains. However, the pressure control of train pneumatic brake systems is a challenging task due to the highly nonlinear characteristics of the system and the compressibility of air [3].

To control the air flow in the train pneumatic brake systems, proportional valves and on/off solenoid valves are two possible choices [4]. On/off solenoid valves are widely used because they are much cheaper and smaller compared with proportional valves [5, 6]. Moreover, they have quick response because of their small airflow areas [7]. However, on/off solenoid valves are either fully open or fully closed [8, 9]. The use of on/off solenoid valves results in the discrete-valued control of the train pneumatic brake system, which makes smooth and precise pressure control be more difficult to achieve [10].

Different methods have been proposed for the pressure control of pneumatic systems with on/off solenoid valves in literature. Proportional-integral-derivative control methods [11–13] were used to control the pressure of pneumatic systems. In [5], a dual-mode controller was proposed for an electro-pneumatic clutch actuator with on/off solenoid valves. In [14], a hybrid control algorithm was adopted for pneumatic actuators. In [10] and [15], nonlinear model predictive controllers were proposed for an electro-pneumatic actuators. In [16], a control law based on feedback linearisation was implemented for the pressure control of a pneumatic actuator. While precise pressure control is well solved in these studies, the issue of reducing the frequent switchings of on/off solenoid valves is not considered.

The number of the on/off solenoid valve switching affects its own lifetime as well as the train pneumatic brake system’s durability. It is known that the total number of the switching for an on/off solenoid valve is finite. In order to implement satisfying pressure regulation, the solenoid valves in train...
pneumatic brake systems often switch frequently by using the existing control algorithms. Repeatedly switching valves on and off shortens their lifetime. Then the repair and replacement cost will increase. Moreover, the reduction of the lifetime of valves may also violate the safety standards EN 50126 of railway transportation [17], where the minimum safe operating time of components is specified. Therefore, it is necessary to develop effective control algorithms to avoid excessive switchings of valves during the pressure control.

In existing literature, sliding model control (SMC) approaches were proposed for reducing the switchings of on/off solenoid valves. In [18], an SMC with 3-mode was first developed for solving the problem of frequent valve switching in a pneumatic actuator. Subsequently, a more complicated SMC algorithm with 7-mode was proposed in [6]. Based on the SMC algorithms in [18] and [6], enhanced ones were developed for pneumatic actuators in [19], where integral actions were added in the sliding surfaces to further reduce the settling time and the steady state error of the system. In [20], an adaptive nonlinear integral sliding model control was proposed for the precise stopping of trains with pneumatic brake systems, where the switchings of on/off solenoid valves were also considered. In these sliding model control algorithms, the switching activities of the valves are reduced by using the “dead band” for the sliding surface. The “dead band” is the tracking error deviation during operation. By relaxing the error deviation close to sliding surface, the valve switching can be reduced indirectly in a degree. However, the pressure control precision will decrease at the same time. It is not easy to achieve a good tradeoff between the precise pressure control and the valve switching reduction by using sliding model control approaches.

Model predictive control (MPC) has been proved as an effective method to realize the multi-objective optimisation, where the cost function can include multiple design objectives [21]. The model predictive control proposed in [22] had the cost function incorporating the items to minimise the position tracking error and the valve switching. The MPC method was developed based on nonlinear dynamic models to drive the solenoid valves directly, where the integer nonlinear program problem needed to be solved. Instead of developing a control scheme based on the nonlinear model, the work in [23] adopted a straightforward and simple method to approach the nonlinear system dynamics by multiple linear models. By linearising the system dynamics at several operating points, a switching MPC based on a piecewise affine model was proposed in [23] to control a pneumatic artificial muscle. However, on/off solenoid valves with the binary characteristics were not investigated in its system. For the train pneumatic brake system with on/off solenoid valves, an appropriate system model and the corresponding predictive control should be developed to improve the performance of the pressure control and the valve switching.

In this paper, the discrete-time hybrid systems in the mixed logical dynamical (MLD) form [24] is used to model the train pneumatic brake system. The modelling form can provide a general framework for describing many discrete features, including the binary states of on/off solenoid valves and the piecewise affine approximation of the nonlinear system. Moreover, it is suitable to be used in online optimisation schemes. These schemes can optimise the control variable systematically and synthetically in a unified MLD framework, avoiding switching oscillation among models and guaranteeing the system stability. Based on the MLD model, a hybrid MPC method is presented to reduce the switchings of solenoid valves while ensuring the pressure control accuracy. Compared to the sliding mode control methods, the two control objectives are included in the cost function of the proposed MPC method explicitly. They can be improved properly and directly by adjusting the weights. Both simulation and experiment results are provided to verify the effectiveness of the proposed method.

The contributions of this paper are threefold.

- A new multi-mode autonomous model of a pneumatic brake system is established by integrating the binary characteristic of on/off solenoid valves.
- A mixed logical dynamical framework is proposed to unify multiple submodels for avoiding the switching oscillation between subsystems during the control process.
- A hybrid MPC controller is designed to reduce the switching numbers of on/off solenoid valves while guaranteeing the pressure tracking precision.

The remainder of this paper is organised as follows. In Section 2, the characteristics of the train pneumatic brake system are described and analysed. In Section 3, the mixed logical dynamical model of the pneumatic brake system is developed to facilitate the controller design. The hybrid model predictive control problem is formulated and solved in Section 4. Simulation results are provided in Section 5 and experiment results are provided in Section 6. We conclude the paper in Section 7.

2 | ANALYSIS OF TRAIN PNEUMATIC BRAKE SYSTEM

2.1 | Description of train pneumatic brake system

A schematic of a pneumatic brake system is shown in Figure 1. The system consists of an equalising reservoir pressure control subsystem, a relay valve, a brake cylinder and a foundation brake rigging. Under the control of a brake control unit (BCU), compressed air flows in or out the equalising reservoir via gas channels and solenoid valves to generate specific equalising reservoir pressure. Then, it is transferred to the brake cylinder pressure by the relay valve which acts as a pressure flow amplifier. Finally, the brake cylinder pressure is converted to the braking force that slows down or stops a running train. As a key parameter, the equalising reservoir pressure determines the braking effect of the pneumatic brake system. Therefore, the equalising reservoir is used as the controlled plant in this paper.

An experimental platform of the equalising reservoir pressure control subsystem is constructed and shown in Figure 2. In this paper, the platform is used to identify the parameters of the system model and verify the proposed control algorithm.
The experimental platform includes an equalising reservoir, two solenoid valves, a brake control unit (BCU), three sensors (pressure sensor, current sensor and voltage sensor), an air reservoir, an air dryer and a National Instruments PXI device.

The above components are similar to the corresponding parts on a train, and can well simulate the pneumatic brake of real trains. For the experiment platform, the control algorithm is operated in BCU. The executive components are the supply and exhaust valves (MAC 35A-ACA-DDFA-1BA). They are controlled to be open or closed so that the equalising reservoir pressure can be regulated to track the reference values. Orifices 1 and 2 are flow control valves (SA 10), which are used to regulate the air flow. The air reservoir is used to supply the air for the equalising reservoir.

In terms of sensors, the pressure sensors (Keller PA-21Y) can collect the pressure data in the equalising reservoir and the air reservoir. The current sensor and the voltage sensor are adopted to obtain the state information of the valve. High levels of measured current/voltage represent that the valve is open and low levels show that the valve is closed. In addition, the air dryer is a system accessory that can effectively filter out water from the air. A National Instruments PXI is used to collect all data during the experiments.

2.2 System dynamics and modelling

According to the system description in Section 2.1, a detailed analysis of the system dynamics and a novel multi-mode system model can be developed consequently. Based on the on/off states of the supply valve and the exhaust valve, three valid operating modes including Increase, Decrease and Hold can be obtained for the equalising reservoir pressure control subsystem. Figure 3 shows the air flow paths corresponding to the operating modes. The system dynamics in each mode are described as follows.

(a) Increase: In this mode, the supply valve is open and the exhaust valve is closed. The pressure in the equalising reservoir increases. The pressure dynamics in the equalising reservoir can be described as

\[ \dot{P} = \gamma RT \times q_{in} \]  

(1)
where $P$ is the pressure of the equalising reservoir, $\gamma$ is the ratio of specific heats for air, $R$ is the universal gas constant, $T$ is the temperature of the supply air, $V$ is the volume of the equalising reservoir, and $q_m$ is the mass flow rate of compressed air through the orifice 1, which is expressed as

$$q_m = q_m(P, P')$$

$$= \begin{cases} \frac{P_{C_a} A_1}{\sqrt{RT}} \sqrt{\frac{2}{\gamma + 1}} \left(\frac{P}{P'}\right)^{(\gamma+1)/\gamma}, & P \leq 0.528 \\ \frac{P_{C_a} A_1}{\sqrt{RT}} \sqrt{\frac{2\gamma}{\gamma - 1}} \left(\frac{P}{P'}\right)^{2(\gamma-1)/\gamma} - \left(\frac{P}{P'}\right)^{(\gamma+1)/\gamma}, & 0.528 < \frac{P}{P'} \leq 1 \end{cases}$$

(2)

where $P_s$ is the pressure of the air reservoir, $C_{a1}^s, C_{a2}^s \in [0, 1]$ are the supply valve flow rate coefficients, $A_1$ is the orifice 1 passage area.

Thus, combining (1) and (2), the pressure dynamics of the equalising reservoir in the increase mode can be described as

$$\dot{P} = \frac{P_s A_1}{V} \sqrt{RT} \times \psi_1(P),$$

(3)

where

$$\psi_1(P) = \begin{cases} C_{a1} \sqrt{\frac{\gamma}{\gamma + 1}} \left(\frac{P}{P'}\right)^{(\gamma+1)/\gamma}, & P \leq 0.528 \\ C_{a2} \sqrt{\frac{2\gamma}{\gamma - 1}} \left(\frac{P}{P'}\right)^{2(\gamma-1)/\gamma} - \left(\frac{P}{P'}\right)^{(\gamma+1)/\gamma}, & 0.528 < \frac{P}{P'} \leq 1 \end{cases}$$

(4)

(b) Decrease: In this mode, the supply valve is closed and the exhaust valve is open. The pressure in the equalising reservoir decreases. Then, the transient mass flow of compressed air through orifice 2 is described as

$$q_m = q_m(P, P') = -\frac{P A_2}{\sqrt{RT}} \times \psi_2(P),$$

(5)

where

$$\psi_2(P) = \begin{cases} C_{a1} \sqrt{\frac{\gamma}{\gamma + 1}} \left(\frac{P}{P'}\right)^{(\gamma+1)/\gamma}, & P \leq 0.528 \\ C_{a2} \sqrt{\frac{2\gamma}{\gamma - 1}} \left(\frac{P}{P'}\right)^{2(\gamma-1)/\gamma} - \left(\frac{P}{P'}\right)^{(\gamma+1)/\gamma}, & 0.528 < \frac{P}{P'} \leq 1 \end{cases}$$

(6)

$P_0$ is the atmospheric pressure, $A_2$ is the orifice 2 passage area, and $C_{a1}^e, C_{a2}^e \in [0, 1]$ are the exhaust valve flow rate coefficients. Combining (1) and (5), the pressure dynamics of the equalising reservoir in the Decrease mode can be described as

$$\dot{P} = -\frac{\gamma P A_2}{V} \sqrt{RT} \times \psi_2(P).$$

(7)

(c) Hold: In this mode, the supply valve and the exhaust valve are both closed. Thus, the pressure in the equalising reservoir maintains constant. The dynamics of the pressure is described as

$$\dot{P} = 0.$$

(8)

From (3) and (7), the pressure dynamics is highly nonlinear, which is mainly caused by the square-root relationship between pressure and flow. The phase trajectories in Figures 4 and 5 further intuitively show the nonlinear characteristics of the system in the Increase and Decrease modes.

For the obtained dynamical models shown as (3), (7) and (8), accurate knowledge of the pneumatic brake system parameters is required for high pressure control accuracy. Generally, the parameters $R$, $T$, $\gamma$ and $P_0$ are known constants. $V$, $A_1, A_2$ and $P_s$ can be directly measured from the prototyped pneumatic brake system. In this paper, the varied flow rate coefficients are used to improve the accuracy of the system model. However, the flow rate coefficients $C_{a1}^e, C_{a2}^e, C_{a1}^s$ and $C_{a2}^s$ are unknown and they are closely related to the actual pneumatic brake system. Therefore, based on the pressure data from the experimental platform shown in Figure 2, the four flow rate coefficients are identified by using the least square method in [25]. The parameters of the pneumatic brake system model are listed in Table 1.

3 MIXED LOGICAL DYNAMICAL MODEL OF TRAIN PNEUMATIC BRAKE SYSTEM

From Section 2, the model of the pneumatic brake system consists of three submodels (3), (7) and (8). To facilitate
controller design, a mixed logical dynamical (MLD) framework is proposed to integrate these submodels. The constructed unified model can represent the discrete binary dynamics of valves, the continuous dynamics of air pressure and the interaction between them. Moreover, the designed controller based on the unified model can optimise the control signal systematically, avoiding switching oscillation between different submodels and improving the robustness of the whole system. In this section, based on the system parameters in Table 1, the following steps explain how the model obtained in Section 2.2 is transformed into a MLD form.

(a) Linearisation: The first step of building the MLD model is to linearise the nonlinear system model. The piecewise linearisation method is used in this paper. This linearisation can better approximate the nonlinear characteristics of the original system and improve the accuracy of the approximated linear model. To linearise the nonlinear system models (3) and (7), the nonlinear items $\psi_1(P)$ in (4) and $\psi_2(P)$ in (6) are approximated as the piecewise affine functions of $P/P_s$ and $P_0/P_s$ using the approach described in [26], respectively. The reference paper provides an effective identification method of piecewise affine models. The main feature of the algorithm developed in [26] is that a desired maximum identification error can be imposed, which allows one to determine a piecewise linear model by fitting the input–output data within the setting error bound. Such an error bound can be tuned to trade off between the fitting quality and the model complexity. Through repeated simulation tests, a maximum approximation error 0.05 is set in this paper by considering the accuracy of the linearised model and the computational complexity of the controller. Then, the corresponding piecewise linearised equations (9) and (10) of $\psi_1(P)$ and $\psi_2(P)$ are computed as follows using the bounded-error approach.

$$\psi_1(P) = \begin{cases} 
\alpha_{11} \frac{P}{P_s} + \alpha_{12}, & \frac{P}{P_s} \leq 0.528, \\
\alpha_{21} \frac{P}{P_s} + \alpha_{22}, & 0.528 < \frac{P}{P_s} \leq 0.737, \\
\alpha_{31} \frac{P}{P_s} + \alpha_{32}, & 0.737 < \frac{P}{P_s} \leq 0.94, \\
\alpha_{41} \frac{P}{P_s} + \alpha_{42}, & 0.94 < \frac{P}{P_s} \leq 1, 
\end{cases} \quad (9)$$

where $\alpha_{11} = 0$, $\alpha_{12} = 0.4245$, $\alpha_{21} = -0.1162$, $\alpha_{22} = 0.3968$, $\alpha_{31} = -0.6554$, $\alpha_{32} = 0.7943$, $\alpha_{41} = -2.9699$, and $\alpha_{42} = 2.9699$.

$$\psi_2(P) = \begin{cases} 
\beta_{11} \frac{P_0}{P} + \beta_{12}, & \frac{P_0}{P} \leq 0.528, \\
\beta_{21} \frac{P_0}{P} + \beta_{22}, & 0.528 < \frac{P_0}{P} \leq 0.737, \\
\beta_{31} \frac{P_0}{P} + \beta_{32}, & 0.737 < \frac{P_0}{P} \leq 0.94, \\
\beta_{41} \frac{P_0}{P} + \beta_{42}, & 0.94 < \frac{P_0}{P} \leq 1, 
\end{cases} \quad (10)$$

where $\beta_{11} = 0$, $\beta_{12} = 0.4382$, $\beta_{21} = -0.0918$, $\beta_{22} = 0.2950$, $\beta_{31} = -0.4714$, $\beta_{32} = 0.5747$, $\beta_{41} = -2.1939$ and $\beta_{42} = 2.1939$.

(b) Discretisation: Models (3) and (7) are discretised with sampling time $T_s = 10$ ms. By integrating (9) and (10), the discrete-time piecewise affine model of the pressure control system can be obtained as follows.

The pressure dynamics in the Increase mode is described as

$$P(k + 1) = \begin{cases} 
\lambda_{11} P(k) + \lambda_{12}, & \frac{P}{P_s} \leq 0.528, \\
\lambda_{21} P(k) + \lambda_{22}, & 0.528 < \frac{P}{P_s} \leq 0.737, \\
\lambda_{31} P(k) + \lambda_{32}, & 0.737 < \frac{P}{P_s} \leq 0.94, \\
\lambda_{41} P(k) + \lambda_{42}, & 0.94 < \frac{P}{P_s} \leq 1, 
\end{cases} \quad (11)$$

where $\lambda_{i1} = \varphi \frac{\eta_i P_s}{V}$, $\lambda_{i2} = \int_0^{T_s} \varphi \frac{\eta_i \tau}{\alpha_0 \eta P_s} d\tau$, $i = 1, ..., 4$ and $\eta = \frac{y_i \sqrt{Mr}}{V}$.

Similarly, the pressure dynamics in the Decrease mode is described as

$$P(k + 1) = \begin{cases} 
\rho_{11} P(k) + \rho_{12}, & \frac{R}{P} \leq 0.528, \\
\rho_{21} P(k) + \rho_{22}, & 0.528 < \frac{R}{P} \leq 0.737, \\
\rho_{31} P(k) + \rho_{32}, & 0.737 < \frac{R}{P} \leq 0.94, \\
\rho_{41} P(k) + \rho_{42}, & 0.94 < \frac{R}{P} \leq 1, 
\end{cases} \quad (12)$$

where $\rho_{i1} = \hat{\varphi} \frac{\eta_i P_s}{V}$, $\rho_{i2} = \int_0^{T_s} \hat{\varphi} \frac{\eta_i \tau}{\alpha_0 \eta P_s} d\tau$, $i = 1, ..., 4$ and $\varphi = -\frac{\alpha_0 \eta}{V} \sqrt{Mr}$.

The pressure dynamics in the Hold mode is presented as

$$P(k + 1) = P(k). \quad (13)$$

(c) Auxiliary variables: Auxiliary variables are introduced to obtain an integrated system model. Firstly, the on-off states of

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| $V$        | 1.15 L | $R$        | 287 J/(kg·K)$^{-1}$ |
| $A_1$      | 4.2 mm$^2$ | $A_2$      | 2.5 mm$^2$ |
| $T$        | 293 K  | $\gamma$  | 1.4 |
| $P_s$      | 700 kPa | $P_0$      | 100 kPa |
| $C_{\beta}$| 0.62   | $C_{\alpha}$| 0.49  |
| $C_{\gamma}$| 0.64  | $C_{\phi}$  | 0.36  |
| $T_s$      | 10 ms  |
where 1 means that the logic input is True and 0 means False.

Six auxiliary binary variables \( \delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6 \in \{0, 1\} \) are defined to indicate the affine sections where the system is operating, i.e.

\[
\begin{aligned}
[\delta_1 = 1] & \iff P \leq 0.528 P_s, \\
[\delta_2 = 1] & \iff P \leq 0.737 P_s, \\
[\delta_3 = 1] & \iff P \leq 0.94 P_s, \\
[\delta_4 = 1] & \iff P \leq \frac{P_0}{0.528}, \\
[\delta_5 = 1] & \iff P \leq \frac{P_0}{0.737}, \\
[\delta_6 = 1] & \iff P \leq \frac{P_0}{0.94},
\end{aligned}
\]

Then, ten auxiliary continuous variables \( z \in \mathbb{R}^{10} \) are defined to describe the system dynamics in (11) and (12), i.e.

\[
\begin{aligned}
\zeta_1 &= \lambda_1 P + \lambda_1, & \text{if } u_1 \land \bar{u}_2 \land \delta_1, \\
\zeta_2 &= \lambda_2 P + \lambda_2, & \text{if } u_1 \land \bar{u}_2 \land \delta_1 \land \delta_2, \\
\zeta_3 &= \lambda_3 P + \lambda_3, & \text{if } u_1 \land \bar{u}_2 \land \delta_2 \land \delta_3, \\
\zeta_4 &= \lambda_4 P + \lambda_4, & \text{if } u_1 \land \bar{u}_2 \land \delta_3, \\
\zeta_5 &= \lambda_5 P + \lambda_5, & \text{if } \bar{u}_1 \land u_2 \land \delta_4, \\
\zeta_6 &= \lambda_6 P + \lambda_6, & \text{if } \bar{u}_1 \land u_2 \land \delta_5 \land \delta_6, \\
\zeta_7 &= \lambda_7 P + \lambda_7, & \text{if } \bar{u}_1 \land u_2 \land \delta_5, \\
\zeta_8 &= \lambda_8 P + \lambda_8, & \text{if } \bar{u}_1 \land u_2 \land \delta_6, \\
\zeta_9 &= P, & \text{if } \bar{u}_1 \land \bar{u}_2, \\
\zeta_{10} &= \zeta_1 + \zeta_2 + \zeta_3 + \zeta_4 + \zeta_5 + \zeta_6 + \zeta_7 + \zeta_8 + \zeta_9 \land \zeta_{10},
\end{aligned}
\]

where \( \bar{u}_1, \bar{u}_2, \delta_1, \delta_2, \delta_3, \delta_4, \delta_5 \) and \( \delta_6 \) represent negation of \( u_1, u_2, \delta_1, \delta_2, \delta_3, \delta_4, \delta_5 \) and \( \delta_6 \), \land means the logic operator AND.

**d) Constraints:** The following constraints on the pressure and on/off states of two valves need to be satisfied,

\[
P_0 \leq P(k) \leq P_s, \\
u_1 \land u_2 = 0,
\]

where the first term means that the pressure in the equalising reservoir is bounded by the atmospheric pressure and the pressure of the air reservoir, the second term means that the supply valve and the exhaust valve cannot be open at the same time.

Then, taking the pressure \( P \) as the state variable and logic variables \( u_1, u_2 \) as the control inputs, and combining other auxiliary variables \( \delta_{1-6} \) and \( \zeta_{1-10} \), we use the hybrid system description language (HYSDEL) tool developed in [27] to obtain the MLD model of the pneumatic brake system. The tool allows describing the hybrid dynamics in a textual form and provides a related compiler to generate the MLD model representation. By processing the program that we developed in [28] through the HYSDEL, the MLD model of the pneumatic brake system can be obtained as (18).

\[
x(k + 1) = Ax(k) + B_1 \begin{bmatrix} u_1(k) \\ n_2(k) \end{bmatrix} + B_2 \delta(k) + B_3 \zeta(k), \tag{18a}
\]

\[
E_2 \delta(k) + E_3 \zeta(k) \leq E_1 \begin{bmatrix} u_1(k) \\ n_2(k) \end{bmatrix},
\tag{18b}
\]

where \( x = P, \delta = [\delta_1, \ldots, \delta_6]^T, \zeta = [\zeta_1, \ldots, \zeta_{10}]^T, A = 0, B_1 = [0, 0], B_2 = [0]_{1 \times 15}, B_3 = [B_{31}, B_{32}], B_{31} = [0]_{1 \times 9}, B_{32} = [1], \) the matrices \( B_1, B_2 \) are zero because the control inputs \( u_1, u_2 \) and the auxiliary variables \( \delta_1, \ldots, \delta_6 \) are defined as Boolean variables, and they are only acted as the decision variables in equation (16), and the matrices \( A, B_{31} \) equal zero and \( B_{32} \) equals 1 according to \( P(k + 1) = \zeta(k) \) that is derived from equations (11), (12) and (16). \( E_1, \ldots, E_3 \) are the matrices with the proper dimensions, which include the constraints (17). \( E_1 \) can be obtained by checking a struct containing the MLD model matrices, where the struct is generated by using Hybrid Toolbox in [29]. Since \( E_1, \ldots, E_3 \) have high dimensions, they are omitted in this paper for lack of space.

**Remark 1.** The mixed logical dynamical (MLD) model [24] is a typical hybrid system model. This kind of model allows specifying the interactions of continuous physical processes with binary variables. The continuous linear dynamics are represented as difference equations \( x(k + 1) = Ax(k) + Bu(k), x \in \mathbb{R}^n \). Boolean variables are defined based on linear-threshold conditions over the continuous variables and the logical states or control inputs. The logic part is embedded in the state equations by transforming Boolean variables into 0–1 integers, and by expressing the relations as mixed-integer linear inequalities [30]. In particular, based on the MLD model, the control problem can be recast as a mixed-integer linear/quadratic programming problem, which can be easily solved by existing numerical solvers.

In the following model predictive controller design, the constructed MLD model based on the above steps will be used as the predictive model. In order to verify the accuracy of the model, its output pressure is compared with the real pressure data of the experimental platform. The results are shown in Figure 6. The results show a good match between the results of the system model and the experimental data. It is validated that the constructed MLD model is accurate enough.
Controller Design to achieve the pressure tracking control and reduce frequent oscillation of the desired control objectives. In this paper, in order to show the effectiveness of the hybrid MPC controller, a new hybrid cost function is constructed, which includes the continuous pressure state and the logic control inputs. Then, the cost function can be reasonably composed as

$$J = Q_p + Q_s,$$  

where $Q_p$ is the penalty function for the error between the equalising reservoir pressure and the reference pressure values, and $Q_s$ is the penalty function for the switchings of two valves.

Let $k$, $N_p$ and $N_c$ ($N_c \leq N_p$) denote the current time, the prediction horizon and the control horizon. The forms of $Q_p$ and $Q_s$ can be described as follows.

$$Q_p = q_{P_{np}} \| P(k + N_p k) - P_{ref}(k + N_p k) \|_1$$
$$+ \sum_{l=0}^{N_p-1} q_{P_{np}} \| P(k + l k) - P_{ref}(k + l k) \|_1,$$  

and

$$Q_s = \sum_{j=0}^{N_c-1} q_{s1} \| \Delta u_1(k + l k) \|_1 + \sum_{j=0}^{N_c-1} q_{s2} \| \Delta u_2(k + l k) \|_1,$$  

where $q_{P_{np}}$, $q_{P_{np}}$, $q_{s1}$ and $q_{s2}$ are the weights of the terminal error and the stage error, $P_{ref}(k + N_p k)$ and $P_{ref}(k + l k)$, $l = 0, ..., N_p - 1$ are the reference pressure values in the prediction horizon and they are set according to the braking instruction in practical applications, and $\| \cdot \|_1$ represents the 1-norm.

4.1 Cost function of model predictive control

The design of the cost function is critical for the implementation of the desired control objectives. In this paper, in order to achieve the pressure tracking control and reduce frequent switchings of valves, a new hybrid cost function is constructed, which includes the continuous pressure state and the logic control inputs. Then, the cost function can be reasonably composed as

$$J = Q_p + Q_s,$$  

where $Q_p$ is the penalty function for the error between the equalising reservoir pressure and the reference pressure values, and $Q_s$ is the penalty function for the switchings of two valves.

Let $k$, $N_p$ and $N_c$ ($N_c \leq N_p$) denote the current time, the prediction horizon and the control horizon. The forms of $Q_p$ and $Q_s$ can be described as follows.

$$Q_p = q_{P_{np}} \| P(k + N_p k) - P_{ref}(k + N_p k) \|_1$$
$$+ \sum_{l=0}^{N_p-1} q_{P_{np}} \| P(k + l k) - P_{ref}(k + l k) \|_1,$$  

and

$$Q_s = \sum_{j=0}^{N_c-1} q_{s1} \| \Delta u_1(k + l k) \|_1 + \sum_{j=0}^{N_c-1} q_{s2} \| \Delta u_2(k + l k) \|_1,$$  

where $q_{s1}$ and $q_{s2}$ are the weights, $\Delta u_1(k) = u_1(k) \oplus u_1(k - 1)$ and $\Delta u_2(k) = u_2(k) \oplus u_2(k - 1)$ because $u_1$ and $u_2$ are logic variables. In order to make it more clear that $\Delta u_1$ and $\Delta u_2$ represent the state changes of the valves between two adjacent sampling instants, $\Delta u_1(k) = | u_1(k) - u_1(k - 1) |$ and $\Delta u_2(k) = | u_2(k) - u_2(k - 1) |$ are used in this paper. In this function, we can know that minimising the $\Delta u_1$ and $\Delta u_2$ is to minimise the switchings of the valves.

4.2 Solution of the optimisation problem

The control inputs can be obtained by solving a constrained optimisation problem at each sampling instant. That is, this paper aims to minimise the cost function (19) with the following state and control input constraints (22).

$$R_0 \leq P(k + l + 1) = P_0,$$

$$u_1(k + l - 1) \in \{0, 1\}, u_2(k + l - 1) \in \{0, 1\},$$

$$\Delta u(k + l) = \left[ \begin{array}{c} \Delta u_1(k + l) \\ \Delta u_2(k + l) \end{array} \right] \in \{0, 1\},$$

$$\delta(k + l) \in \{0, 1\}.$$
Then, the constrained optimal problem can be expressed as follows,

$$\min_{U, \beta, \varepsilon} J = Q_p + Q_\varepsilon$$

subject to (18(a)), (18(b)), (22),

$$P(k + N_p | k) = P_{\text{ref}},$$

where

$$U = \begin{bmatrix} u_1(0) \\ u_2(0) \\ \vdots \\ u_1(N_e - 1) \\ u_2(N_e - 1) \end{bmatrix}^T,$$

$$\delta = [\delta_0^T, \ldots, \delta_{N_e-1}^T]^T \in \{0, 1\}^{15N_e}$$

and \(Z = [z_0^T, \ldots, z_{N_e-1}^T]^T \in \mathbb{R}^{10N_e}\) are the optimisation vectors; \(P(k + N_p | k) = P_{\text{ref}}\) is the terminal constraint.

Since the 1-norm is used in the objective function (19), the constrained finite time optimal problem should be transformed into a mixed-integer linear program (MILP) problem to be solved efficiently. For this purpose, a vector of additional slack variables \(E = [\varepsilon_0^P, \ldots, \varepsilon_{N_e-1}^P, \varepsilon_0^\delta, \ldots, \varepsilon_{N_e-1}^\delta] \in \mathbb{R}^{N_p + N_e}\) is introduced to treat hard constraints in (23) as soft ones. The vector \(E\) represents an upper bound on \(f\) and it satisfies

$$1^T \varepsilon^P_{(k+l)l} \geq \pm (q_0 \Delta u_1(k + l | k) + q_2 \Delta u_2(k + l | k)),$$

$$l = 0, \ldots, N_e,$$

$$1^T \varepsilon^P_{(k+l)l} \geq \pm q_0 (P(k + l | k) - P_{\text{ref}}(k + l | k)),$$

$$l = 0, \ldots, N_p - 1,$$

$$1^T \varepsilon^\delta_{(k+N_p)l} \geq \pm q_0 (P(k + N_p | k) - P_{\text{ref}}(k + N_p | k)),$$  

where \(1_1\) and \(1_2\) are the column vectors of ones of length 1 and 2, respectively, and where

$$P(k + l | k) = \alpha(k + l | k),$$

$$\alpha(k + l | k) = A^l \alpha(k) + \sum_{j=0}^{l-1} A^j (B_1 \beta(k + l - 1 - j | k))$$

$$+ B_2 \delta(k + l - 1 - j | k) + B_3 \varepsilon(k + l - 1 - j | k).$$

As shown in [31], one can prove that if the vector \(E\) satisfies (24), the MILP problem (26) has the same solution as the optimisation problem (23).

The MILP problem is stated as follows and it is reformulated from the problem (23),

$$\min_{E} J(E) = \sum_{l=0}^{N_p} \varepsilon^P_{(k+l)l} + \sum_{l=0}^{N_e-1} \varepsilon^\delta_{(k+l)l}$$

subject to (18(a)), (18(b)), (22), (24).

The MILP problem can be solved by the existing solvers. In this paper, the branch and bound method in the YALMIP toolbox [32] is used to solve the problem. Then, the sequence of optimal control inputs can be obtained at sampling time \(k\),

$$U^*(k) = \begin{bmatrix} u_1^*(k + 0 | k) \\ u_2^*(k + 0 | k) \\ \vdots \\ u_1^*(k + N_e - 1 | k) \\ u_2^*(k + N_e - 1 | k) \end{bmatrix}^T.$$  

It is worth noting that only the first element of the control input sequence is applied. Thus, the resulting control input at time \(k\) is

$$U_1(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} = \begin{bmatrix} u_1^*(k + 0 | k) \\ u_2^*(k + 0 | k) \end{bmatrix}.$$  

At next sampling time \(k + 1\), the optimisation problem is solved again over the shifted horizon according to the receding horizon policy. Based on the obtained control inputs, the corresponding operating modes can be determined during the pressure tracking progress. The pressure is then evolved under these operating modes to reach the desired pressure values.

### 4.3 Stability analysis of closed-loop pressure control system

This section provides a stability analysis of the closed-loop pressure control system. In order to facilitate the stability analysis, we consider the system exists an equilibrium pair \((x_e, u_e)\). Specifically, in this paper, the \(x_e = P_e\) and \(u_e = [u_{1e} u_{2e}]^T = [0 0]^T\), respectively.
Before proceeding, the lemma of guaranteeing the asymptotic stability of system (18) is first presented as follows.

**Lemma 1.** Let \((x_k, u_k)\) be an equilibrium pair of the system and \((\delta_k, z_k)\) definitely admissible. Assume that the initial state \(x(0)\) is such that a feasible solution of problem (23) exists at time \(t = 0\). Then \(q_p > 0, q_s = q_{\text{ps}} \geq 0\) and the hybrid MPC law (28) stabilises the system (18) in that \(\lim \limits_{k \to \infty} x(k) = x_k, \lim \limits_{k \to \infty} u(k) = u_k, \lim \limits_{k \to \infty} \delta(k) = \delta_k, \lim \limits_{k \to \infty} z(k) = z_k\), while fulfilling the dynamic constraints in (18) [24].

The detail about the stability proof of the MLD model can be referred to [24]. It is worth noting that the stability result in Lemma 1 is valid for nonlinear system (1)–(8) provided that the system is exactly described by the MLD model (18). However, due to the piecewise affine approximation, the MLD model is only an approximated one of the nonlinear brake system. There always exists some modelling errors between the MLD model and the original nonlinear model, and it is probable that the resulting pressure response can be quite different from what has been predicted. Thus, we make an assumption regarding the modelling error.

**Assumption 1.** The modelling error denoted as \(\varpi \in \mathbb{R}^+\) is bounded, where \(\varpi = P_{\text{MLD}} - P_{\text{nonlinear}}, P_{\text{MLD}}\) and \(P_{\text{nonlinear}}\) are the output pressures of the MLD model and the original nonlinear model under the same control inputs.

**Remark 2.** The assumption that imposed on the modelling error is reasonable and acceptable from the practical viewpoint. Moreover, it should be noted that if the approximated segments of the system’s nonlinear terms are proper, the modelling error will be accepted for the MPC controller design [33]. Then the MLD-based hybrid MPC strategy will provide a good control for the original nonlinear system and the stability of the closed-loop system can be guaranteed due to the feedback correction mechanism of MPC.

Then, for the pneumatic brake system, based on Assumption 1 and Lemma 1, we have the following theorem that shows the control law (27) and (28) stabilises the system (1)–(8).

**Theorem 1.** For the pneumatic brake system satisfying Assumption 1, let \((x_k, u_k)\) be an equilibrium pair. Assume that the initial state \(x(0)\) can make the optimisation problem (25) exist a feasible solution at time \(t = 0\). Then for nonsingular \(q_p, q_s, q_{\text{ps}}, q_{\text{ps}}\), the hybrid MPC control law (27) and (28) stabilises the nonlinear brake system and reduces the switching of the solenoid valves in that

\[
\lim \limits_{k \to \infty} |x(k) - x_k| = \sigma,
\]

\[
\lim \limits_{k \to \infty} u^k(k) = u(k - 1),
\]

while fulfilling the dynamic constraints (1)–(8) and the input and state constraints, where \(\sigma\) is a small nonnegative constant.

**Proof.** The proof follows from standard Lyapunov arguments. Assume that the state \(x(k)\) at time \(k\) is feasible, and for \(x(k)\) the optimal control sequence is denoted as

\[
U^*(k) = \left[\begin{array}{c}
\dot{u}_1^k(k + 0|k) \\
\dot{u}_2^k(k + 0|k) \\
c\n\end{array}\right], \ldots, \left[\begin{array}{c}
\dot{u}_1^k(k + N_c - 1|k) \\
\dot{u}_2^k(k + N - 1|k)
\end{array}\right]^T.
\]

For the \(U^*(k)\), its corresponding optimal cost function is denoted as

\[
V'(k) = f(U^*(k), x(k)).
\]

For the next state \(x(k + 1)\) at time \(k + 1\), the feasible and sub-optimal control sequence can be denoted as

\[
U'(k + 1) = \left[\begin{array}{c}
\dot{u}_1^k(k + 1|k) \\
\dot{u}_2^k(k + 1|k) \\
c\n\end{array}\right], \ldots, \left[\begin{array}{c}
\dot{u}_1^k(k + N_c - 1|k) \\
\dot{u}_2^k(k + N - 1|k)
\end{array}\right]^T.
\]

Therefore, we have

\[
V(k + 1) \leq f(U'(k + 1), x(k + 1))
\]

\[
= V'(k) - q_p ||x(k) - x_k + \sigma|| + q_{s1} ||u_1^k(k) - u_1(k - 1)||
\]

\[
- q_{s2} ||u_2^k(k) - u_2(k - 1)||
\]

\[
+ q_{s1} ||u_1c - u_1^k(k + N_c - 1|k)||
\]

\[
+ q_{s2} ||u_2c - u_2^k(k + N - 1|k)||.
\]

(29)

Then, if let the weights \(q_p, q_{s1}\) and \(q_{s2}\) be set properly and there is always

\[
q_p ||x(k) - x_k + \sigma|| + \rho \geq 0,
\]

where

\[
\rho = q_{s1} ||u_1^k(k) - u_1(k - 1)|| - ||u_1c - u_1^k(k + N_c - 1|k)||
\]

\[
+ q_{s2} ||u_2^k(k) - u_2(k - 1)|| - ||u_2c - u_2^k(k + N - 1|k)||.
\]

(30)

Therefore, we can get that \(V'(k)\) is decreasing. Since \(V'(k) \geq 0\), then when \(k \to \infty\), there is \(V'(k + 1) < V'(k) \to 0\). Thus, each term of the sum

\[
q_p ||x(k) - x_k + \sigma|| + \rho \leq V'(k) - V'(k + 1)
\]

converges to zero as well, correspondingly. For the pneumatic brake system, when \(k \to \infty\), \(u_1^k(k + N_c - 1|k)\) and
In the proof of Theorem 1, the closed-loop stability is completed. Moreover, through the derivation, we can get that the switchings of solenoid valves can be reduced by using the proposed MPC control strategy.

Remark 3. In the proof of Theorem 1, the closed-loop stability under the proposed hybrid MPC is shown for the given equilibrium points \( x_e = P_s, u_e = [0 \ 0] \). In the system model (18), the equilibrium pair is \( x_e = P_s, u_e = [0 \ 0] \), which is the point that the hybrid system to be controlled to. That is, the reservoir pressure maintains at the desired value \( P_s \), while both the supply valve and the exhaust valve are closed. For the train pneumatic brake system, there are two cases where the control input \( u_e \neq [0 \ 0] \). i) In the Increase mode, the supply valve is open and the exhaust valve is closed, the equalising reservoir pressure \( P \) keeps increase until it reaches the air reservoir pressure \( P_s \), then we have \( P = 0 \) and the system exists \( x_e = P_s \) and \( u_e = [0 \ 0] \); ii) In the Decrease mode, the supply valve is closed and the exhaust valve is open, the equalising reservoir pressure \( P \) keeps decreases until it reaches the atmospheric pressure \( P_o \), then we have \( P = 0 \) and the system exists \( x_e = P_s \) and \( u_e = [0 \ 0] \). The two equilibrium points are determined by the inherent characteristics of the system, which are not desired operating states in practical applications.

5 SIMULATION STUDY

In this section, the simulation study on the pneumatic brake system is given to illustrate the performance of the proposed method. In order to better verify the effectiveness of the proposed control algorithm, it is assumed that the simulated nonlinear system suffers some disturbances caused by the air temperature change. The simulations are performed under Windows 10 operating system with Intel Core i5-7200U CPU, 8 GB RAM, on a desktop computer. Based on simulation results, the performance of the proposed control method is evaluated from three aspects. First, the property of the real time for the algorithm is analysed, and proper prediction horizon and control horizon are selected. Second, the effects of weights on the pressure control precision and the valve switching activity of hybrid MPC are analysed. Third, the proposed control method is compared with a classic sliding-mode controller in [18] and an improved sliding-mode controller in [19], respectively.

To evaluate the controller performance, the following performance metrics are used.

1. Root mean squared error (RMSE):

\[
RMSE = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (P(k) - P_{\text{ref}}(k))^2},
\]

where \( n \) is the sample number, \( P \) is the pressure controlled by the hybrid model predictive controller and \( P_{\text{ref}} \) is the reference pressure.

2. Steady state error (SSE): The stable state error is defined as the maximum absolute value of the error when the measured pressure reaches the steady state. Since the reference pressure curve is a multi-step curve, the average SSE of all steps is used.

3. Switches per second (SPS): It is defined as the total number of switches per second of the two solenoid valves.

The RMSE and SSE are used to evaluate the pressure control performance. The values of RMSE and SSE are smaller, the pressure control precision is higher. The SPS is used to evaluate the switching activity of the on/off solenoid valve. The value of SPS is smaller, the number of switching of the valve is less.

5.1 Verification of the real-time performance

The real-time performance of the proposed hybrid MPC controller is determined by the values of the prediction horizon \( N_p \) and the control horizon \( N_c \). Generally, increasing \( N_p \) and \( N_c \) can improve the controller performance, however, the computation time will increase dramatically. Thus, the appropriate control horizon and prediction horizon should be chosen to achieve a good tradeoff between the pressure tracking control performance and the computation time. For this purpose, the proposed model predictive controllers with different \( N_p \) and \( N_c \) are designed and applied to the equalising reservoir pressure control subsystem. Through comparing their control performance of tracking reference pressure curve and their computation time of each MPC step, the appropriate prediction horizon and control horizon can be set.

5.1.1 Parameter setting

For the constructed system model (18), it is assumed that the nominal values of the model parameters are set as shown in Table 1. In addition, in order to facilitate the tuning of the weights \( q_pN_p \), \( q_P \), \( q_q \) and \( q_s \), we let \( q_pN_p = q_P = 1 \) and \( q_q = q_s = 1 \). The reference pressure in Figure 9 is used in this simulation.

5.1.2 Simulation results

The simulation results are summarised in Table 2, which shows the pressure control performance and the computation time of the proposed MPC method with different prediction horizons and control horizons.

From Table 2, for the controllers with the prediction horizons \( N_p = 5 \sim 8 \) and the control horizons \( N_c = 1 \sim 5 \), the average computation time is smaller than the sampling time.

\[ N_p \]
5.2 Evaluation of the weights

According to the cost function (19), the proposed hybrid MPC controller can simultaneously optimise the performances of the pressure control and the valve switching. These two optimisation objectives can be emphasized by tuning the weights $q_p$, $q_{s1}$ and $q_{s2}$. Generally, the values of $q_{s1}$ and $q_{s2}$ are larger, the pressure control performance become better. The values of $q_s$ and $q_c$ are larger, the valve switching times will become less. In order to evaluate their effects on two control objectives, multiple sets of weights are tested in the simulations. To facilitate the tuning of the weights, we let $q_{p N_p} = q_p$, $q_{s1} = q_{s2}$, and $q_s = q_c = 1$. By changing the values of the weight $q_p$, we can obtain the pressure tracking and valve switching results of the hybrid MPC with different weights.

Figure 8 shows the results of the pressure control and valve switching with different values of the weight $q_p$. From this figure, we can find that a better pressure tracking performance can be obtained with the increase of $q_p$. At the same time, the values of SPS increase, which means that the switching activities of the valves become frequent. When $q_p \geq 10$, the pressure tracking and the valve switching performances are kept unchanged. By comparing the results of the pressure tracking and the valve switching in Figure 8, it can be observed that a good tradeoff between these two control objectives can be achieved when the value of $q_p$ is in the range $[0.5, 3.9]$. Then, the unnecessary switchings of valves can be eliminated to prolong the valves’ lifetime while ensuring the pressure control precision.

5.3 Comparison with existing methods

In this section, the proposed hybrid MPC controller is compared with the classic sliding-mode controller in [18] and the improved sliding-mode controller in [19].

5.3.1 Parameter setting

For the hybrid MPC controller, the weights are chosen to be $q_{p N_p} = q_p = 2.5$, $q_{s1} = q_{s2} = 1$. The control horizon and the prediction horizon are set as before.

For the classic sliding-mode controller in [18], the sliding surface $s$ is defined as $s = \frac{e}{\omega^2} + \frac{\xi}{\omega} + \epsilon$, where $e = P - P_{ref}$ is the pressure error. For the improved sliding-mode controller in [19], the sliding surface $s$ is defined as $s = \frac{e}{\omega^3} + \frac{\xi}{\omega^2} + \xi + \int_0^t \frac{e}{\omega} dt$, where $|\int_0^t \frac{e}{\omega} dt| \leq \epsilon_{limit}$, the parameter $\epsilon_{limit}$ is used to implement anti-windup. In two sliding-mode control algorithms, the switching activities of the valves are reduced by using the “dead band” $\epsilon$ for the sliding surfaces.

In this paper, the parameters of the classic sliding-mode controller are set as $\xi = 0.95$, $\omega = 330$ and $\epsilon = 1$ kPa. The parameters of the improved sliding-mode controller are set as $\xi = 3$, $\omega = 45$, $\epsilon_{limit} = 2$ kPa and $\epsilon = 2$ kPa. The control parameters are manually tuned based on the given parameters in [18] and [19].

5.3.2 Simulation results

Table 3 and Figure 9 gathers the simulation results obtained with the three controllers. Table 3 lists the values of RMSE, SSE and

| Controller                  | Computation time (ms) | RMSE (kPa) | SSE (kPa) | SPS (Hz) |
|-----------------------------|-----------------------|------------|-----------|----------|
| Classic SMC [18]            | 0.8                   | 31.9362    | 0.6834    | 2.2857   |
| Improved SMC [19]           | 0.8                   | 31.8843    | 0.6612    | 1.5000   |
| Proposed hybrid MPC         | 4.5                   | 31.6830    | 0.6369    | 1.1071   |

TABLE 2 Control performance and computation time of the proposed hybrid MPC method with different $N_p$ and $N_c$

| $N_p$ | $N_c$ | RMSE (kPa) | Average SSE (kPa) | Average SPS (Hz) | Average computation time (ms) |
|-------|-------|------------|-------------------|------------------|-------------------------------|
| 5     | 1     | N/A        | N/A               | N/A              | 0.8                           |
| 5     | 3     | 32.1575    | 0.9005            | 1.1071           | 2.5                           |
| 8     | 5     | 32.1228    | 0.6175            | 1.0000           | 4.5                           |
| 10    | 8     | 32.0932    | 0.4181            | 0.8571           | 13.0                          |

FIGURE 8 Pressure tracking and valve switching of the proposed hybrid MPC method with different weights $q_p$. 10 ms. When the values of the prediction horizon and the control horizon continue to be large, that is $N_p = 10$ and $N_c = 8$, the average computation time is larger than 10 ms, which cannot meet the real-time control requirement. According to the values of RMSE, SSE and SPS in this table, the controller with $N_p = 5$ and $N_c = 1$ cannot achieve the pressure tracking control. Both controllers with $N_p = 5$, $N_c = 3$ and $N_p = 8$, $N_c = 5$ can make the pneumatic brake system obtain satisfying pressure control performances. The latter can achieve better control effects. Therefore, the prediction horizon and the control horizon are determined as $N_p = 8$ and $N_c = 5$ in this paper.

TABLE 3 Simulation comparison of pressure control and valve switching results of two SMC and hybrid MPC controller
Simulation comparison of pressure tracking and valve switching activity using the classic SMC, the improved SMC and the proposed hybrid MPC.

Although the computation time of the proposed controller is large, it still can be adapted to the train pneumatic brake system because its computation time is smaller than the sampling cycle.

6.1 Experimental validation

In this section, the experiments are conducted on the test platform of the equalising reservoir pressure control subsystem to validate the effectiveness of the proposed control method. Moreover, the proposed hybrid MPC method is compared with the classic sliding-mode control method in [18] and the improved sliding-mode control method in [19], respectively. The experimental setup is described firstly. Then, the compared experimental results and analysis are presented.

6.1 Experimental setup

To facilitate the implementation of the hybrid MPC in experiments, the explicit piecewise affine form of the hybrid MPC law is firstly computed offline by using the multiparametric mixed integer programming solver of the hybrid toolbox [29]. Then, the C-code files are further generated to compute the explicit optimal control action associated with the piecewise affine mapping. In the experimental platform shown in Figure 2, the proposed hybrid MPC algorithm is operated in BCU, running at a sampling rate of 100 Hz.

In the experiment, in order to make the pressure present similar and satisfying tracking performance with three controllers, the following parameters for the proposed MPC controller are selected: the weights $q_{N_p} = q_{s_1} = q_{s_2} = 3$, the prediction horizon $N_p = 8$ and the control horizon $N_c = 5$. For the classic sliding-mode controller, the parameters are set as $\zeta = 1.15$, $\omega = 290$ and $\varepsilon = 3$ kPa. The parameters of the improved sliding-mode controller are set as $\zeta = 3$, $\omega = 32$, $\gamma_{lim} = 3$ kPa and $\varepsilon = 3$ kPa. For these controllers, the allowed maximum tracking error between the reference pressure and the measured one is $\pm 3$ kPa.

6.2 Experimental results

With the pneumatic brake system shown in Figure 1, there are two commonly used operation modes in the train, including the braking mode and the releasing mode. In this section, the pressure control experiments under the two modes are conducted. Detailed experimental results and performance comparisons are provided.

6.2.1 Braking experiment

In the braking mode of the train, the pressure in the equalising reservoir should increase to given values for implementing the braking operation. In this experiment, the initial pressure of
TABLE 4 Experiment comparison of the pressure control and the valve switching in the braking mode

| Controller                  | Average RMSE (kPa) | Average SSE (kPa) | SPS (Hz) |
|-----------------------------|-------------------|------------------|---------|
| Classic SMC [18]            | 44.0156           | 2.80             | 6.6601  |
| Improved SMC [19]           | 43.3303           | 1.5              | 5.8766  |
| Proposed hybrid MPC         | 43.3102           | 1                | 4.7992  |

the equalising reservoir is set to 0 kPa and the reference pressure are set to 100 and 380 kPa. The recorded pressure tracking results and the valve switching results of three controllers under an experiment test are presented in Table 4 and Figure 10, respectively.

From Figure 10, it can be observed that all controllers can make the pressure have a satisfying and similar tracking performance. The steady state errors between the reference pressure and the measured one of three controllers can be kept within ±3 kPa. Compared with two SMC controllers, there are large fluctuations during the pressure regulation using the hybrid MPC controller. This fact is caused by the air thermal effect and low switching numbers of valves. Even so, from the results of average RMSE, SSE in Table 4, we can still find that the pressure control performance of the proposed hybrid MPC controller is superior to those of two SMC controllers overall.

The results of the valve switching in Table 4 and Figure 10 show that a notable decrease in switching activity is achieved using the proposed hybrid MPC controller. From Table 4, comparing the SPS values of the classic sliding-mode and improved sliding-mode controllers to that of the proposed MPC controller, there are 27.94% and 18.33% reductions in the switching times of the solenoid valves. Then, the lifetime of solenoid valves can be prolonged by using the proposed controller when compared with two sliding-mode controllers, respectively.

6.2.2 Releasing experiment

In the releasing mode, the pressure in the equalising reservoir should decrease to given values. In this experiment, the equalising reservoir pressure is to decrease from 335 to 100 kPa and from 100 to 0 kPa, respectively. The pressure tracking and the valve switching results of three controllers are presented in Table 5 and Figure 11, respectively. From the results, the stable state errors of the pressure tracking using the two SMC controllers cannot meet the engineering requirement. Compared with two SMC controllers, the proposed hybrid MPC controller can have the smallest average RMSE, SSE values, which shows that good pressure tracking performance can be achieved by the proposed method. Specifically, during 6 ∼ 10 s, the hybrid MPC controls solenoid valves to guarantee the pressure control precision. However, the compared SMC methods sacrifice the pressure control precision in order to avoid the inherent chatters of SMC and reduce the energy consumption. Then, the solenoid valves are almost off states during 6 ∼ 10 s for the SMC methods.

An another obvious advantage of the proposed hybrid MPC controller is that less valve switchings are obtained when it is compared with two SMC controllers. The SPS values in Table 5 and the results of the valve switching in Figure 11 further verify the superiority of the proposed method. Compared with the classic sliding-mode and improved sliding-mode controllers, there are 52.7% and 30% reductions in the switching numbers of the solenoid valves using the proposed hybrid MPC method, respectively.

In addition, comparing the experimental results with the simulation ones, due to the sensor noises and the disturbances, the pressure tracking performance in the experimental results are
TABLE 5  Experiment comparison of the pressure control and the valve switching in the releasing mode

| Controller                | Average RMSE (kPa) | Average SSE (kPa) | SPS (Hz) |
|---------------------------|--------------------|-------------------|----------|
| Classic SMC [18]          | 38.6329            | 4.5               | 2.4981   |
| Improved SMC [19]         | 37.6258            | 3.2               | 1.6879   |
| Proposed hybrid MPC       | 37.2521            | 2                 | 1.1816   |

not as good as those in the simulations. The valve switching is more frequent than the one in the simulations. However, the control performance is still satisfying by using the proposed hybrid MPC method.

7  | CONCLUSION

In this paper, we propose a hybrid MPC pressure control method for pneumatic brake systems with the objective of reducing the switching activities of valves while ensuring the pressure control performance. A system model is first built using a mixed logical dynamical representation. Based on the mixed logical dynamical model, a hybrid MPC strategy is proposed for improving the pressure tracking precision and reducing the switchings of on/off valves. The proposed method can reduce the valve switching activities directly and effectively while ensuring the precise pressure control by setting proper weights of the optimisation objective. Simulation and experiment results show the effectiveness of the proposed method.

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