Hidden drifts in turbulence

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Abstract – The paper defines and discusses the concept of hidden drifts in two-dimensional turbulence. These are ordered components of the trajectories that average to zero and do not produce direct transport. Their effects appear in the evolution of the turbulence as a special type of fluxes, which consist of average motion of positive and negative fluctuations in opposite directions. We show that these fluxes have important nonlinear effects in turbulent fluids and in confined plasmas. In the first case, they determine the increase of the large-scale vorticity and velocity at the expense of the small-scale fluctuations by a process of separation of the vorticity fluctuations according to their sign. In the second case, they provide a mechanism for zonal flow generation and a vorticity flux that influences the sheared rotation of the plasma.

Introduction. – Turbulence is of fundamental importance in fluid mechanics, plasma physics, astrophysics, atmosphere and ocean sciences. It is a complex nonlinear process that mixes disorder and order [1–7]. Quasi-coherence or order appears at the basic level of tracer (or test particle) trajectories in the case of smooth velocity fields that have finite correlation lengths \( \lambda \). It lasts for a short time of the order of the time of flight \( \tau_{fl} = \lambda/V \), where \( V \) is the amplitude of the stochastic velocity. In two-dimensional (2D) incompressible turbulence, long-time coherence can appear essentially because the trajectories are trapped in the correlated zone. Tracer trajectories are random sequences of trapping or eddying events and long jumps. The latter are random, while the motion during the trapping events has a high degree of order, which strongly modifies the diffusion process [8–12].

The order of the trajectories determines much amplified and complicated effects on turbulence fields due to the nonlinear constraints that characterize of the self-consistent evolution.

The turbulence that is dominantly 2D has a self-organizing character [13–17], which consists of the generation of quasi-coherent structures and flows. In addition to that, we have found more subtle effects, namely the hidden drifts (HDs).

Here we define the HDs and analyze some of their effects on turbulence evolution. The HDs are discovered in the statistics of tracer trajectories, but their main influence is on turbulence evolution rather than on tracer transport. Essentially, the HDs are two opposite average velocities that compensate one another. We show that the HDs are the origin of strange turbulent fluxes (STFs), which consist of an advection process that depends on the sign of the fluctuations. The sign of the advected fluctuations is associated to the sign of the HD, so that the positive and the negative fluctuations move in opposite directions and generate fluxes. The STFs contribute to the understanding of important nonlinear processes in fluid and plasma turbulence.

The HDs and the STFs are fundamental processes in all physical domains where 2D turbulence is important. We discuss here two representative systems: turbulence relaxation in ideal fluids [13–15] and drift turbulence in magnetically confined plasmas [16]. We show that the STFs provide a mechanism for the separation of the vorticity according to its sign and for the inverse energy cascade in fluids [17–20]. The effects of the STFs are different in turbulent plasmas, where they contribute to the generation of zonal flow modes [21–24] that can lead to improved confinement [25–27].

These examples show that the effects of the HDs depend on the nature of the nonlinearity and on the physical significance of the advected fields.

Hidden drifts. – We start from the statistics of tracer trajectories in 2D stochastic velocity fields obtained from

\[
\frac{dx}{dt} = v(x, t) + V_d e_2, \quad v(x, t) = -\nabla \phi(x, t) \times e_3, \quad (1)
\]

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where \( \mathbf{e}_1, \mathbf{e}_2 \) are the unit vectors in the plane of the motion \( \mathbf{x} = (x_1, x_2) \), \( \mathbf{e}_3 \) is perpendicular on this plane and \( V_d \) is a constant average velocity. The initial condition is \( \mathbf{x} = \mathbf{0} \) at \( t = 0 \). The stochastic velocity field \( \mathbf{v}(x, t) \), determined by the potential (or stream function) \( \phi(x, t) \), has zero divergence. The potential is a stationary and homogeneous stochastic Gaussian field with zero average and with the Eulerian correlation (EC)

\[
E(x, t) = \langle \phi(0, 0) \phi(x, t) \rangle \tag{2}
\]

modelled according to the characteristics of the physical system. Usually, \( E(x, t) \) decays from the maximum \( E(0, 0) = \Phi^2 \) to zero at the characteristic distances \( \lambda_1, \lambda_2 \) and/or time \( \tau_d \). It defines the main parameters of the stochastic potential: the amplitude \( \Phi \), the decorrelation lengths \( \lambda_i \) and the decorrelation time \( \tau_d \). The Hamiltonian structure of eq. (1) is the origin of the order that characterizes the 2D incompressible turbulence. The velocity is tangent to the contour lines of the total potential \( \phi(x, t) = \phi(x, t) + x_1 V_d \) at any moment, and, in the case of time independent potentials, the trajectories remain on these lines. They reflect the space structure of the potential.

The HDs are ordered displacements that average to zero and do not drive flows. They appear in the presence of an average velocity \( V_d \) and they are perpendicular on \( V_d \). The HDs are found by analyzing the average displacements conditioned by the initial value of the potential \( \langle x_i(t) \rangle \), where \( \phi^0 = \phi(0, 0) \).

These conditional displacements and the other statistical characteristics of the trajectories are determined using the decorrelation trajectory method (DTM, [11,28,29]). This is a semi-analytic approach that is in agreement with the statistical consequences of the invariance of the potential. The main idea of this method is to determine the Lagrangian averages not on the whole set of trajectories, but rather to group together trajectories that are similar, to calculate their average and then to evaluate averages of these averages. Similar trajectories are obtained by imposing supplementary initial conditions, namely the values of the potential and of the velocity at the starting point of the trajectory \( \phi^0 = \phi(0, 0), \mathbf{v}^0 = \mathbf{v}(0,0) \). They define a set of subensembles \( S \) of the realizations of the potential. Conditional averages lead to space-time–dependent average potential \( \Phi^S(x, t) \) and velocity \( V^S(x, t) \) in each subensemble \( S \), and to a decorrelation trajectory (DT) \( \mathbf{X}(t; \phi^0, \mathbf{v}^0) \). The DTS are the main ingredient of DTM. They are smooth, simple trajectories determined from an equation with the same structure as eq. (1), but with the stochastic potential replaced by \( \Phi^S(x, t) \),

\[
d\mathbf{X} = V^S(x, t) + V_d \mathbf{e}_2, \quad V^S(x, t) = -\nabla \Phi^S(x, t) \times \mathbf{e}_3. \tag{3}
\]

The subensemble average potential \( \Phi^S(x, t) \) is determined by the EC of \( \phi(x, t) \),

\[
\Phi^S(x, t) = \phi^0 E(x, t) E(0,0) - \phi^0 E_2(x, t) E_{22}(0,0) + \phi^0 E_1(x, t) E_{11}(0,0), \tag{4}
\]

where \( E_{ij} \) are derivatives of the type \( E_i(x, t) = \partial E(x, t)/\partial x_i \). The DTS represent the average evolution of the particles through the correlated zone of the potential and describe the decorrelation process.

The statistics of the trajectories is represented by averages along the DTS, weighted by the Gaussian probability of the subensembles, \( P(\phi^0)P(v_1^0, v_2^0) \). The DTM essentially determines the correlations of the trajectories with the quantities that define the subensembles. Rather complex transport processes could be analyzed with this semi-analytical method in magnetically confined [30–36] and in space [37,38] plasmas. It was shown that the DTM yields clear physical images of the nonlinear stochastic advection and reasonably good quantitative results.

The average displacements conditioned by the initial value of the potential \( \langle x_i(t) \rangle \), are evaluated in the frame of the DTM by

\[
\langle x_i(t) \rangle = \int_{-\infty}^{\infty} dV_1 dV_2 \mathbf{X}(t; \phi, v^0) P(v_1^0, v_2^0). \tag{5}
\]

They are zero at any time when \( V_d = 0 \), but finite values of the component along the \( \mathbf{e}_1 \) axis, \( \langle x_1(t) \rangle \), are obtained in the presence of an average velocity \( V_d \) directed along the \( \mathbf{e}_2 \) axis.

The most important property of these average displacements is that they have the same sign as the initial potential \( \phi^0 \) at any time, as seen in fig. 1(a). The average displacements conditioned by the sign of the initial potential are

\[
\langle x_1(t) \rangle_+ = \int_0^\infty d\phi^0 \langle x_1(t) \rangle_+ P(\phi^0), \quad \langle x_1(t) \rangle_- = \int_{-\infty}^0 d\phi^0 \langle x_1(t) \rangle_- P(\phi^0) \tag{6}
\]

where \( P(\phi^0) \) is the probability of \( \phi^0 \). They are, as seen, in fig. 1(b), time-dependent functions that saturate. The sum \( \langle x_1(t) \rangle_+ + \langle x_1(t) \rangle_- = 0 \), which shows that there is no average motion.

Any process of decorrelation of the trajectories from the contour lines of the potential leads to conditional average velocities. The trajectory \( \mathbf{x}(t) \) for \( t > \tau_d \) consists of a time sequence of segments of duration \( \tau_d \) that are statistically independent. The conditional displacements (6) during the characteristic decorrelation time \( \tau_d \) determine the step length of the ordered motion and a pair of average velocities

\[
V_+ = \frac{\langle x_1(\tau_d) \rangle_+}{\tau_d}, \quad V_- = \frac{\langle x_1(\tau_d) \rangle_-}{\tau_d}. \tag{7}
\]

They have opposite directions and compensate one another, \( V_+ + V_- = 0 \). These are the hidden drifts (HDS).

Thus, the tracer stochastic motion described by eq. (1) includes ordered components, which determine the HDs, a pair of symmetrical velocities perpendicular to the average velocity \( V_d \mathbf{e}_2 \). The characteristics of the motion that generate the HDs are presented in fig. 1. It demonstrates that the conditional average displacements \( \langle x_i(t) \rangle \), (eq. (5))
have the sign of the initial potential $\phi^0$ at any time (fig. 1(a)). They lead, as seen in fig. 1(b), to perfectly symmetrical average displacements $\langle x_1(t) \rangle_+ \sim \langle x_1(t) \rangle_-$ (defined in eq. (6)), which are the origin of the HDs (7).

The physical explanation of the HDs is based on the modification of the structure of the contour lines of the potential and on the perturbation of the statistics of the velocity along these lines. Both effects are produced by the average velocity $V_d e_2$.

The average velocity determines an effective potential $\phi(x_t) = \phi(x_t) + x_1 V_d$, which has strips of open contour lines that extend along the average velocity, while oscillating in the perpendicular direction. For small values of $V_d$ ($V_d < V$), closed field lines exist between the strips, but they are stretched along the average velocity and organized in pairs of opposite signs.

In the closed potential cells, the velocity along the contour lines is statistically non-homogeneous due to $V_d e_2$, which leads to total velocity $V_t$ that is enhanced on a side of the cells and reduced on the opposite one. The trajectories are concentrated on the zones with small $V_t$, which leads to an average displacement that has the sign of the potential. The open strips also contribute to the conditional displacements (6), but this is due to a different reason. The invariance of the potential along an open trajectory with initial $\phi^0$, $\phi(x(t)) + x_1(t) V_t = \phi^0$, shows that the average displacement has the sign of $\phi^0$, because the average potential on these trajectories is zero.

Thus, the HDs reflect the order of the contour lines of the total potential.

We note that the ordered motion as symmetrical positive and negative velocities that compensate each other is not forbidden by the zero-divergence condition, which prohibits average Lagrangian velocities along the $e_1$ axis for the trajectories described by eq. (1).

The HDs do not have direct influence on transport. They do not drive a direct transport (average velocity) and the contributions of the ordered steps are implicitly included in the mean square displacements that define the diffusion coefficients. However, they represent a reservoir for direct transport, because perturbations produced by other components of the motion that determine a weak compressibility of the velocity field can perturb the equilibrium of the HDs leading to an average velocity of the test particles. A first process of this type was recently found in the study of the transport of the heavy impurities in turbulent plasmas [39]. The perturbation is provided by the polarization drift, and it leads to direct transport. The effect of the HDs is rather strong in this case since the convective flux is comparable or larger than the diffusive transport for a wide range of parameters of the transport model.

However, the most important effects of the HDs are connected to the special turbulent fluxes described in the next sections.

Strange turbulent fluxes. — The ordered displacements $\langle x_1(t) \rangle_\phi$ that have the same sign as the initial potential lead to the correlation of the displacements with the potential

$$\langle \phi(0,0)x_1(t) \rangle = \int_{-\infty}^{\infty} d\phi^0 \phi^0 \langle x_1(t) \rangle_\phi P(\phi^0).$$

A similar correlation $\langle \omega(0,0)x_1(t) \rangle$ exists between the displacement and the vorticity $\omega(x,t) = \Delta \phi(x,t)$. It is generated by the average displacements conditioned by the initial vorticity $\langle x_1(t) \rangle_\omega$, where $\omega^0 = \omega(0,0)$. These averages are determined by extending the DTM. A supplementary condition $\omega(0,0) = \omega^0$ is introduced in the definition of the subensembles. This determines a new term in the subensemble average potential $\Phi^0(X,t)$ (4), $\Phi^\omega = \omega^0 \Delta^2 E(X,t)/\Omega^2$, where $\Omega^2 = \Delta^2 E(0,0)$ is the amplitude of vorticity fluctuations. As a result, the shapes of the DTs are strongly changed and the probability of the subensembles is also modified due to the correlation (9). However, this extended DTM leads to diffusion coefficients that are not much modified.

The properties of $\langle x_1(t) \rangle_\omega$ are similar to those of $\langle x_1(t) \rangle_\phi$, with the difference that $\langle x_1(t) \rangle_\omega$ has the sign opposite to the sign of $\phi^0$. This difference is the result of the negative correlation of $\phi$ and $\omega$,

$$\langle \phi(x,t)\omega(x,t) \rangle = \Delta E(0,0) < 0.$$
initial potential and vorticity, respectively,

\[
C_\phi = \left\langle \phi(0,0) v_x(x(t),t) \right\rangle \rightarrow \frac{\langle \phi(0,0) x_1(\tau_d) \rangle}{\tau_d},
\]

(10)

\[
C_\omega = \left\langle \omega(0,0) v_x(x(t),t) \right\rangle \rightarrow \frac{\langle \omega(0,0) x_1(\tau_d) \rangle}{\tau_d}.
\]

(11)

They are time-dependent functions that saturate if \( \tau_d \) is finite, and decay to zero in static potentials.

The Lagrangian correlations \( C_\phi \) and \( C_\omega \) depend on \( V_d \), on the amplitude \( V \) of the stochastic velocity and on the decorrelation time \( \tau_d \). Their dependence on these parameters is similar, except for the signs, which are opposite. Both are anti-symmetrical functions of \( V_d \) that increase with \( V \) and decay at small and large \( \tau_d \). Typical results are presented in fig. 2. The anti-symmetrical dependence on \( V_d \) is shown in fig. 2(a), where \( C_\phi \) is represented for several values of \( \tau_d \). The dependence on \( \tau_d \) and \( V \) is shown in fig. 2(b) for \( C_\phi \).

These correlations determine strange turbulent fluxes (STFs) of potential and of vorticity. They consist of ordered motion of the fluctuations in directions that are associated to their sign. This means that both the peaks and the holes contribute to the flux. We show below that the HDs and the STFs have strong nonlinear effects on turbulence evolution, which are completely different from those induced by the random turbulent fluxes.

Effects of HDs and STFs in fluid turbulence. –

Turbulence relaxation in 2D ideal fluids is described by the Euler equation with a stochastic initial condition

\[
\partial_t \omega + \mathbf{v} \cdot \nabla \omega = 0, \quad \mathbf{v} = -\nabla \phi \times \mathbf{e}_z + V_d \mathbf{e}_2,
\]

(12)

where \( \omega = \omega_0 + \delta \omega \) is the vorticity, \( \phi \) is the stream function and \( \mathbf{v} \) is the fluid velocity. The average velocity \( V_d \) is a large scale nonuniform motion \( V_d(x) = V_d \exp(-x^2/L^2) \), where \( L \gg \lambda_1 \). It determines an average (initial) vorticity \( \omega_0(x) = -2V_d x_1 \exp(-x^2/L^2)/L^2 \) that adds to the fluctuation \( \delta \omega \). Thus, vorticity elements are advected by the velocity field and the vorticity is conserved along the trajectories (1). The nonlinearity of the process is determined by the relations between \( \mathbf{v} \) and \( \omega \).

Two coupled equations for \( \omega_0 \) and for the amplitude of fluctuations \( \langle \delta \omega^2 \rangle \) are obtained from (12). The average equation extracted from (12) determines an equation for \( \delta \omega \), which can be formally solved by integration along the trajectories (1). Multiplying this solution by \( \delta \omega \mathbf{x}(x,t) \) and by \( \mathbf{v}(x,t) \), respectively, and averaging over the trajectories, one obtains

\[
\partial_t \langle \delta \omega^2 \rangle = -C_{\delta \omega} \partial_t \omega_0, \quad \langle \mathbf{v} \delta \omega \rangle = 0.
\]

(13)

(14)

The turbulent flux of the vorticity is

\[
\langle \mathbf{v} \delta \omega \rangle = -D_1 \partial_t \omega_0 + C_{\delta \omega},
\]

(15)

where \( D_1 \) are the diffusion coefficients obtained from the trajectories (1), \( \partial_t \) is the explicit time derivative and \( \partial_t \) is the partial derivative to \( x_1 \).

The correlation \( C_{\delta \omega} \) (11) shown in fig. 2(a) appears in both equations. It determines a STF that consists of ordered motion of the vorticity fluctuations in directions that have the sign of \( -\delta \omega \). The main effects of the HDs on turbulence can be deduced from eqs. (13)–(15). These equations are nonlinear due to \( C_{\delta \omega} \) and \( D_1 \), which are rather complicated functions of the parameters of the stochastic stream function, and also depend on time and on space (through \( V_d \)).

The amplitude of the vorticity fluctuations is not conserved, but it decays, as seen in eq. (13) where both \( C_{\delta \omega} \) and \( \partial_t \omega_0 \) are negative. The decrease of \( \langle \delta \omega^2 \rangle \) is the result of the ordered motion combined with the conservation of the total vorticity \( \omega = \omega_0 + \delta \omega \) along the trajectories. In the case of a completely random motion, the change of the Lagrangian vorticity \( \delta \omega(x,t) \) is random and it does not influence the amplitude of the fluctuations.

The equation of conservation of the average velocity (14) contains the flux (15), which has a diffusive term and the contribution of the HDs. The latter is not of advection type, but it actually determines a source term

\[
\partial_t \omega_0 + \partial_t \left( D_1 \partial_t \omega_0 \right) = -\partial_t C_{\delta \omega} / \partial V_d \omega_0, \quad \partial_t \omega_0 = \partial_t C_{\delta \omega} / \partial V_d \omega_0.
\]

(16)

where the space derivative of \( C_{\delta \omega} \) was replaced by \( \partial_t C_{\delta \omega} = \partial C_{\delta \omega} / \partial V_d \omega_0 \). The source term has the sign of \( \omega_0 \) because \( \partial C_{\delta \omega} / \partial V_d \omega_0 < 0 \) for \( V_d \lesssim V \) (fig. 2(a)). It determines the increase of \( |\omega_0| \). The process is due to the STF, which brings positive fluctuations toward the maximum of \( \omega_0 \) and negative \( \delta \omega \) toward the minimum of \( \omega_0 \).
We note that eqs. (13), (16) represent a minimal frame for a general analysis of the effect of the HDs. A more complicated equation for the correlation of the vorticity fluctuations has to be added to the system in order to describe the evolution of the initial turbulent state. However, the importance of the HDs in the amplification of the average flow can be quantitatively evaluated based on the present simplified description. The correlation $C_{\delta \omega}$ is an increasing function of the amplitude $V$ of the stochastic velocity. This imposes a stochastic initial condition with large amplitude of the vorticity fluctuation such that $V \gg V_d$. In these conditions $\partial C_{\delta \omega}/\partial V_d < 0$ even at the large values of $\tau_d$ that are expected for turbulence evolution. The diffusion coefficient also increases with the increase of $V$, but slower that $C_{\delta \omega}$, and it decays at large $\tau_d$. It means that, in these conditions, the process is dominated by the effects of the HDs, which establish the characteristic time scale of turbulence evolution.

Thus, we have shown that the HDs and the STF of vorticity can determine a statistically relevant process of built up of large-scale coherent motion at the expense of the small-scale fluctuations. The basic individual process is similar to the evolution of small-scale vortices on nonlinear sheared velocity [20]. The dynamics of the average vorticity in a turbulent environment described by eqs. (13), (16) shows that the separation of the vorticity elements according to their signs can contribute to the inverse cascade of energy that characterizes 2D fluid turbulence.

Effects of HDs and STFs in turbulent plasmas. We consider a plasma confined by a uniform magnetic field $B$ taken along the $e_1$ axis. A density gradient (along the $e_1$ axis, with characteristic length $L_n$) makes plasma unstable [40]. The drift wave instability produced by the electron kinetic effect and ion polarization drift velocity is analyzed here using the method of test modes on turbulent plasmas [41,42]. This method is based on the separation of the distribution function into an approximate equilibrium $f_0$ and the response $h$ to the small perturbation of the potential (with wave number components $k_i$ and frequency $\omega$), $\delta \phi \exp(i k \cdot x - i \omega t)$, that adds to the background potential $\phi_b(x,t)$. The function $f_0(x,t)$ is the solution of the approximate evolution equation obtained by neglecting the small terms. The latter are not important at short time, but only at long times when the small effects accumulate. The response $h$ can be linearized in the small perturbation $\delta \phi$. The solution of the dispersion relation yields the frequencies $\omega(k)$ that can be supported by the system and the tendency of amplification or damping given by the growth rate $\gamma(k)$, the imaginary part of $\omega(k)$. These quantities, which include the effects of the small terms neglected in the evolution of $f_0$, provide the short-time change of the spectrum of the background turbulence.

Drift modes have small parallel wave numbers $k_1 \ll k_1, k_2$ and small frequencies. The fast parallel motion of the electrons leads to the adiabatic approximate equilibrium of the electrons and to the response to $\delta \phi$,

$$\delta n_e = n_0(x) \frac{e \delta \phi}{T_e} \left( 1 + i \sqrt{\frac{\pi}{2}} \frac{\omega - k_y V_s}{|k_z| v_T} \right),$$

(17)

that does not depend on the background turbulence [40]. $n_0(x)$ is plasma density, $T_e$ is the temperature of the electrons, $v_{Te}$ is their thermal velocity, $V_s = \rho_e c_s/L_n$ is the electron diamagnetic velocity, $\rho_e = c_e/\Omega_e$, $c_e = \sqrt{T_e/m_e}$ is the sound velocity and $\Omega_e = eB/m_e$ is the ion cyclotron frequency. Using the constraint of neutrality one obtains the short-time equilibrium solution for the ions

$$f_0^i = n_0(x) F_M^i \exp \left[ \frac{e \phi_b(x - V_s t)}{T_e} \right],$$

(18)

where $F_M^i$ is the Maxwell distribution of the ion velocities. Since $e \phi_b/T_e < 1$, this distribution corresponds to adiabatic density fluctuations $\delta n/n \equiv e \phi_b/T_e$.

The formal solution of the linearized equation for the response $h$ is

$$h(x,v,t) = -n_0(x) F_M^i \frac{e \delta \phi}{T_e} (k_2 V_s - \omega \rho_e^2 k_2^2) \Pi,$$

(19)

where the propagator $\Pi$ reads

$$\Pi = i \int_{-\infty}^{\infty} d\tau M(\tau) \exp[-i \omega(\tau - t)],$$

(20)

$$M(\tau) \equiv \left\langle \exp \left[ \frac{e \phi_b(x(\tau))}{T_e} \right. \right. \left. \left. + i k \cdot \dot{x}(\tau - x) \right] \right\rangle,$$

(21)

and the average $\langle \rangle$ is on the trajectories obtained from eq. (1) in the background potential $\phi_b$ by integration backward in time from the initial condition at time $t$. The nonlinear constraint of the drift turbulence evolution is the quasi-neutrality condition $\delta n/e = \delta \phi$, which yields the dispersion relation for test modes in turbulent plasma,

$$- (k_y V_s - \omega \rho_e^2 k_2^2) \Pi = 1 + i \sqrt{\frac{\pi}{2}} \frac{\omega - k_y V_s}{|k_z| v_T},$$

(22)

The effects of $\phi_b(x,t)$ appear in the function $M(\tau)$ of the propagator (20).

The HDs lead to the correlation of the potential with the displacements in the average (21). Using the stationarity of the background turbulence and the fact that the “initial” condition for a trajectory (1) can be any of its points, one obtains

$$\langle \phi_b(x(\tau)) x_1(\tau) \rangle = \langle \phi_b(0) x_1(\tau) \rangle.$$  

(23)

At large time $t \gg \tau_d$, this correlation can be approximated by $C_{\phi_b}(\tau-t)$, where $C_{\phi_b}$ is the correlation (10) shown in fig. 2(b). Neglecting for simplicity the effects of the quasi-coherent structures described in [41], the function $M$ is approximated by

$$M(\tau) \equiv \left\langle \exp \left[ ik_1 V_s (\tau - t) - \frac{1}{2} k_z^2 D_s (\tau - t) \right] \right\rangle,$$

(24)

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where
\[ V_\phi = eC_{\phi\phi}/T_e \]

plays the role of an average velocity along the \( \mathbf{e}_1 \) axis, but it is determined by the STF of the potential fluctuations that corresponds to ordered motion of the positive fluctuations of \( \phi_b \) with positive velocity and of symmetrical motion of the negative fluctuations.

The dispersion relation has the solution
\[ \omega = \frac{k_2 V_\star - k_1 V_\phi}{1 + k_1^2}, \]
\[ \gamma = c \frac{(k_3 V_\phi + k_1 V_\phi)(k_2 V_\phi + k_1 V_\phi)k_2^2}{(1 + k_1^2)^3} - k_1^2 D, \]

where \( c = \sqrt{\pi/2}/|k_4|v_T \) and \( k_1^2 = k_3^2 + k_2^2 \).

The velocity \( V_\phi \) modifies both the frequency and the growth rate of the test modes.

The new term in the frequency (26) determines the drift of the potential fluctuations along the \( \mathbf{e}_1 \) axis that is of STF type. This potential flux drives STFs for the density fluctuations (due to the adiabatic condition) and for the vorticity fluctuations (due to the correlation (9)). These fluxes have opposite signs, which depend on the sign of the average velocity \( V_d \). The density STF determines improvement or degradation of the confinement, depending on the values of \( V_d \) and on turbulence characteristics. The vorticity STF influences the large-scale vorticity that corresponds to the sheared rotation of the plasma [43].

The velocity \( V_\phi \) also modifies the growth rate (27). It destabilizes a new type of modes that have \( k_2 = 0 \), the zonal flow modes. Their growth rate, obtained from eq. (27) for \( k_3 = 0 \), is not zero due to \( V_\phi \). This is a new mechanism for the generation of the zonal flow modes. It is a nonlinear process determined by the HDs, which yield the average velocity \( V_\phi \) in the propagator (20) that drives oscillations of the potential along the \( \mathbf{e}_1 \) axis. A significant effect appears when turbulence amplitude reaches high levels such that \( V_\phi \) is comparable with \( V_\star \).

A first evaluation of the importance of the process identified here can be done by comparison with the results obtained in [29,41] for the drift turbulence driven by the polarization drift. The latter determines by correlation with the displacements a similar average velocity in the propagator, which is much smaller than \( V_\phi \). Despite this, the zonal flow modes can reach (in the case of a strong drive of the instability) amplitudes that are high enough to produce the decay of the turbulence [29]. The HDs are more efficient for the generation of zonal flow modes, because \( V_\phi \) is significantly larger (values of the order \( V_\star \) correspond to normalized amplitudes \( e\Phi/T_e \gtrsim 10^{-2} \)).

The processes studied in [29,41] (correlations of the polarization drift, effects of the ion structures and flows produced by trajectory trapping) are neglected here. Also, the Reynolds stress does not play an important role in these models. As a consequence, one can conclude that the effects of the HDs add to the above mechanisms. Synergistic influences are expected, especially through the combination of different contributions to the total average velocity along the \( \mathbf{e}_1 \) axis.

**Discussion and conclusions.** – The hidden drifts (HDs) are found in the statistics of the trajectories (1) in 2D incompressible stochastic velocity fields in the presence of an average velocity \( V_\phi \mathbf{e}_2 \). They are ordered components of the motion oriented perpendicularly to the average velocity \( V_\phi \mathbf{e}_2 \), which exactly compensate one another and do not yield average displacements of the trajectories. Direct effects of the HDs on test particle transport do not appear in this case. In spite of this, generation of convective transport through the perturbation of the symmetry of the HDs is expected in more complex models that include a small compressibility of the velocity field. A first process of this type [39] shows that the convection produced by the HDs can be comparable or larger than the diffusive transport.

The most important effect of the HDs is connected to the Lagrangian correlations that they determine. A special type of fluxes are generated, which consist of displacements of the positive and negative fluctuations in opposite directions. This is a rather unexpected behavior since the equation for the trajectories (1) does not depend on the advected quantity. For this reason we have denoted these correlations strange turbulent fluxes (STFs). They are due to the pair of HDs that have the orientation related to the sign of the initial potential or vorticity.

The HDs and the STF of potential and vorticity fluctuations were determined using the decorrelation trajectory method as functions of the parameters of the test particle transport model. The HDs are essentially produced by the average velocity \( V_\phi \mathbf{e}_2 \), which modifies the structure of the contour lines of the total potential and the statistics of the velocity along these lines.

We have discussed the effects of the STFs in two cases: the relaxation of turbulent states in 2D ideal (inviscid) fluids and drift turbulence in collisionless (hot) plasmas confined in strong magnetic fields. These physical systems are described by similar evolution equations that represent the advection along the characteristic obtained from eq. (1). They are formally linear, but there are in both cases nonlinear constraints that strongly influence turbulence evolution. The nature of the nonlinearity and the physical significance of the advected fields are completely different in the two systems. We have shown that these differences lead to completely different effects of the HDs, although they are the same in the two systems.

In the process of relaxation of turbulent states in 2D ideal fluids, the STF of the vorticity fluctuations determines the increase of the large-scale vorticity and velocity, accompanied by the decrease of the amplitude of the small-scale vorticity fluctuations.

In magnetically confined plasmas, the approximate adiabatic response that characterizes drift-type turbulence determines STFs of potential and vorticity fluctuations.
They modify the frequencies of the test modes on turbulent plasmas and generate zonal flow modes.

∗∗∗

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