The results of the impact theory application in modeling the gear shifting processes in automatic transmissions

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Abstract. The process of gear shifting in vehicles automatic transmissions is considered in the paper as the shock interaction of transmission elements. Two mathematical models are proposed and considered for simulation the gear shifting processes. Computer simulation of the models’ movement during acceleration of a vehicle, as well as modes of low- and high-frequency gear shift cycling (self-oscillations of the output shaft of the gearbox) is carried out. Methods for decreasing the amplitude of the gearbox output shaft oscillations caused by the impact nature of gear shifting are proposed.

1. Introduction
Gear shifts in automatic transmissions are performed without interrupting the power flow. This process involves two clutches. The off-going clutch of the previous gear and on-going clutch of the next gear are working simultaneously at gear shifting [1]. The gear shift time duration is 0.2 - 0.5 seconds in modern gearboxes [2, 3]. Obviously, the fast gear shifting process leads to an abrupt change in the velocities of the transmission elements even with a small difference in the speeds of the engine and the output shaft of the gearbox. The values of inertial torques as a result of the gear shift significantly exceed the torques of the engine and the torques of the resistance to motion forces, which in turn can worsen the comfort of the vehicle passengers, reduce the service life and reliability of the transmission.

Works [1-6] in particular are devoted to dynamic studies of gear shifting processes, while in most of them this process is divided into stages and considered sequentially. The short time of gear shifting makes it possible to represent the gear shifting as instantaneous process and apply the theory of impact for its analysis. Considering gear shifting as an interaction with an impact is especially relevant for dual clutch transmissions, which have recently become quite widespread. There is no torque converter between the engine and the gearbox, which acts as a damper of torsional vibrations of the gearbox input shaft at gear shifting.

2. Model to calculate
The model of the vehicle transmission showed in figure 1 consists an engine, a two-speed gearbox with gear ratios \(i_1, i_2\), two friction clutches \(c_1, c_2\), engaging corresponding gears, an output shaft of a gearbox with elasticity \(c\) and damping \(b\), differential and semi-axles, connected to the drive wheels and the vehicle body.
The moment of inertia of the moving parts of the engine reduced to the body of the clutch being switched on will be denoted by $J_1$; $J_0$ – the moment of inertia of the gearbox moving parts to the link with the maximum flexibility reduced to the output shaft; element $J_V$ is the moment of inertia of the transmission links after the link with the maximum flexibility, reduced to the output shaft of the gearbox, including the differential, half-shafts, drive wheels and the vehicle body. Thus, the resulting dynamic model contains three inertial elements: $J_i$ and $J_O$, which are rigidly connected by means of clutches $c_1$ or $c_2$, and the $J_V$ element, connected to the $J_O$ element by elastic connection.

The inertial elements $J_i$ and $J_O$ are rigidly connected to each other by one of the clutches ($c_1$ or $c_2$). $J_i$ and $J_O$ between gear shifts form a single inertial element with the moment of inertia reduced to the output shaft of the gearbox ($J_i i_j^2 + J_O$), where $i_j$ is the gear ratio of the engaged gear. The inertial element ($J_i i_j^2 + J_O$) angular velocity after each of the two considered gear shifts is determined using the theorem on conservation of the angular momentum at impact [7].

The movement of the model in figure 1 between gear shifts when one of the clutches $c_1$ or $c_2$ is switched on is a torsional vibration of inertial elements $J_i i_j^2 + J_O$ and $J_V$ relative to the center of mass. The model center of mass rotates under the action of the engine torque $M_t = \text{const} \neq 0$ and moments of resistance forces $M_R = \text{const} \neq 0$. Thus, the model in figure 1 between gear shifts has 2 degrees of freedom and is described by a system of two linear differential equations:

\[
\begin{align*}
(J_i i_j^2 + J_O)\omega_j^{(j)} + b \left( \omega_j^{(j)} - \omega_V^{(j)} \right) + c \left( \varphi_j^{(j)} - \varphi_V^{(j)} \right) &= i M_t, \\
J_V\omega_V^{(j)} + b \left( \omega_V^{(j)} - \omega_j^{(j)} \right) + c \left( \varphi_V^{(j)} - \varphi_j^{(j)} \right) &= -M_R. 
\end{align*}
\]

In system (1): $\varphi_j$ are the angular coordinates of the inertial element ($J_i i_j^2 + J_O$); $\varphi_V$ - the angular coordinate of the output shaft after the link with maximum flexibility; superscript $(j)$ indicates the gear engaged; $i$ is a gear ratio equal to $i_1$ or $i_2$.

Free vibration frequencies of the model in the absence of damping: $k_1 = 0$; $k_2^{(j)} = \left\{ \frac{c (J_i i_j^2 i^2 + J_O) V^2}{J_V T^2/2} + \frac{J_O}{J_V} \right\}$. Frequency of damped oscillations: $k2^*(j) = k2(j)2 - h(j)20.5$, where $h(j) = b (J_i i_j^2 i^2 + J_O + J_V) [2 (J_i i_j^2 i^2 + J_O) V^2]^{-1}$.

The relative motion of the elements ($J_i i_j^2 i^2 + J_O$) and $J_V$ are their synchronous torsional vibrations in antiphase, and the magnitudes of the oscillation amplitudes are inversely proportional to the moments of inertia [8]. The general solution of the obtained equations (1) has the form:
\[
\varphi_0^{(j)} = A^{(j)} \cdot \exp(-h^{(j)} t) \cdot \sin(k_{2}^{(j)} t + \beta^{(j)}) + C_1^{(j)} + C_2^{(j)} t + a^{(j)} t^2/2 \\
\varphi_v^{(j)} = \mu^{(j)} A^{(j)} \cdot \exp(-h^{(j)} t) \cdot \sin(k_{2}^{(j)} t + \beta^{(j)}) + C_1^{(j)} + C_2^{(j)} t + a^{(j)} t^2/2
\]

where \( \mu^{(j)} = A_v^{(j)}/A^{(j)} = -(1/M_l - M_R) \cdot J_0^{-1} \), \( a^{(j)} = (1/M_l - M_R) \cdot (1/M_i - J_0/J_V)^{-1} \).

The constants \( A, \beta, C_1 \) and \( C_2 \) are determined through the initial angular coordinates and velocities of the inertial elements of the model. Parameters \( C_1, C_2 \) and \( a \) - characterize the motion of the center of mass of the model in (3), parameters \( A \) and \( \beta \) - characteristics of the relative motion.

After differentiating equations (2) and substituting \( \sin \alpha^{(j)} = h^{(j)} \cdot (h^{(j)} t + k_{2}^{(j)} t^2)^{-0.5} \) and \( \cos \alpha^{(j)} = k_{2}^{(j)} \cdot (h^{(j)} t + k_{2}^{(j)} t^2)^{-0.5} \) we obtain the general form of the equations for the velocities of inertial elements in the \( j \)-th gear between gear shifts:

\[
\omega^{(j)} = A^{(j)} \cdot \exp(-h^{(j)} t) \cdot (h^{(j)} t + k_{2}^{(j)} t^2)^{0.5} \cos(k_{2}^{(j)} t + \beta^{(j)} + a^{(j)} t) + C_2^{(j)} + a^{(j)} t \\
\omega_v^{(j)} = \mu^{(j)} A^{(j)} \cdot \exp(-h^{(j)} t) \cdot (h^{(j)} t + k_{2}^{(j)} t^2)^{0.5} \cos(k_{2}^{(j)} t + \beta^{(j)} + a^{(j)} t) + C_2^{(j)} + a^{(j)} t
\]

\( C_2^{(j)} \) is the initial velocity of the center of mass of the model at the \( j \)-th speed.

Consideration of the gear shifting process as an impact interaction with zero-speed recovery factor allowed [9] to present the considered in figure 1 transmission model in a simplified form presented in figure 2.

![Figure 2. Simplified impact model of gear shifting with elastic element and damper.](image)

The simplified impact model in figure 2 reflects the analogy of the physical process of gear shifting. In the model in figure 2 elements of rotation of the diagram in figure 1 are replaced by elements of translational linear movement. The next correspondences take place: \( m_1 \sim f_i, m_2 \sim f_0, m_3 \sim f_V \). The \( P_0 \) force corresponds to the torque on the transmission output shaft; force \( P_R \) corresponds to the torque of rotation resistance forces. Elements with stiffness \( c \) and damping coefficient \( b \) connect masses \( m_2 \) (frame in figure 2) and \( m_3 \). The process of gear shifting in the model is reduced to "soft" separation of the mass \( m_1 \) from one border of the frame \( m_2 \) and the subsequent impact connection with its other border with a zero-speed recovery factor. Thus, there is a "violent" transfer of mass \( m_1 \) from one border of the frame \( m_2 \) to the other, which reflects the effect of quick connection the inertial transmission elements reduced to the input and output shafts of the clutch of the switched gear. The \( m_1 \) and \( m_2 \) represent a single mass \( (m_1 + m_2) \) in the intervals between gear shifts.

The movement of the model in figure 2 in the intervals between impacts ("gear shifts") is described by a system of two linear differential equations [9]:

\[
(m_1 + m_2)\ddot{x} + b(\dot{x} - \dot{x}_3) + c(x - x_3) = P_0 \\
m_3\ddot{x}_3 + b(\dot{x}_3 - \dot{x}) + c(x_3 - x) = -P_R
\]
where $x$, $\dot{x}$, $\ddot{x}$ - coordinate, velocity and acceleration of the mass $(m_1 + m_2)$; $x_3$, $\dot{x}_3$, $\ddot{x}_3$ - coordinate, velocity and acceleration of the mass $m_3$.

The speed $\dot{x}$ of the mass $(m_1 + m_2)$ calculation for the model in figure 2 is carried out through the momentum conservation law.

The mathematical model (3) was considered in textbooks on vibration theory [8, 10], monographs [11, 12], as part of a vibro-impact system [13], as well as in the study of mechanical systems with strong threshold nonlinearities [14]. Further more complex model is studied in [15], which consists of a chain of masses connected by elastic elements and each mass containing inside itself a serial connection of another mass and a damper.

The model with translational linear movement in figure 2 is put in correspondence with the model with rotation of inertial elements showed in figure 1. Such correspondence greatly simplifies the understanding of the impact process of gear shifting. The frequency of free vibrations of the model in figure 2 at the absence of damping in this case

$$k_1 = 0; k_2 = \{c(m_1 + m_2 + m_3) \cdot [(m_1 + m_2)m_3]^{-1}\}^{0.5}. $$

Damped oscillation frequency:

$$k_2^* = (k_2^2 - h^2)^{0.5}, \quad h = b(m_1 + m_2 + m_3) \cdot [2(m_1 + m_2)m_3]^{-1}. $$

Thus, the vibration frequency of the model elements in figure 2 does not depend on the gear ratio, which is important when analyzing gear shifting. The frequency for the model in figure 1 is different for each gear because the moments of inertia of the driving links (engine, etc.) reduced to the output shaft of the gearbox depend on the gear ratio of the engaged gear. The assumption of equality of vibration frequencies can be justified in the model in figure 2 by the fact that with a large number of gears in the gearbox the gear ratios of adjacent gears are close in magnitude [16].

Similarly, the solution to the system of equations (3) is:

$$x = A \cdot \exp(-ht) \cdot \sin(k_2^*t + \beta) + C_1 + C_2t + at^2/2$$

$$x_3 = \mu A \cdot \exp(-ht) \cdot \sin(k_2^*t + \beta) + C_1 + C_2t + at^2/2$$

where $\mu = A_3/A = -(m_1 + m_2) \cdot m_3^{-1}$, $a = (P_0 - P_R) \cdot (m_1 + m_2 + m_3)^{-1}$.

After differentiating equations (4) and substituting $\sin \alpha = h \cdot (h^2 + k_2^2)^{-0.5}$ and $\cos \alpha = k_2^* \cdot (h^2 + k_2^2)^{-0.5}$ we obtain the equations for masses motion in the $j$-th gear between gear shifts:

$$\dot{x}^{(j)} = A^{(j)} \cdot \exp(-ht) \cdot (h^2 + k_2^2)^{0.5} \cos(k_2^*t + \beta^{(j)} + \alpha) + C_2^{(j)} + a^{(j)}t$$

$$\dot{x}_3^{(j)} = \mu A^{(j)} \cdot \exp(-ht) \cdot (h^2 + k_2^2)^{0.5} \cos(k_2^*t + \beta^{(j)} + \alpha) + C_2^{(j)} + a^{(j)}t$$

3. Computer simulation

A computer simulation of the elements’ movement of the model in figure 1 during acceleration of a vehicle equipped with a 6-speed dual-clutch gearbox are carried out in the section.

The gear ratios of the gearbox considered as an example and the corresponding velocities of the output shaft of the gearbox $\omega^{(j)}$ at the assumed maximum speed of rotation of the engine shaft $\omega_f = 6000 \text{ rev} \cdot \text{min}^{-1} = 628.3 \text{ rad} \cdot \text{s}^{-1}$ (gear shifting velocity) are given in table 1.

| Speed | $i$ | Step | $\omega^{(j)}$ at $\omega_f = 628.3 \text{ rad} \cdot \text{s}^{-1}$ |
|-------|-----|------|------------------------------------------------------------------|
| 1.    | 4.12| 1.66 | 152.5                                                            |
| 2.    | 2.48| 1.66 | 253.2                                                            |
| 3.    | 1.64| 1.51 | 382.2                                                            |
| 4.    | 1.17| 1.40 | 535.2                                                            |
| 5.    | 0.91| 1.29 | 690.5                                                            |
| 6.    | 0.75| 1.22 | 842.3                                                            |
The rest of the model's parameters correspond to a modern middle-class passenger car. The dependence of the output shaft angular velocity over time is shown in figure 3.

![Graph of changes in the output shaft angular velocity of the gearbox.](image)

The results in figure 3 show that in the case of an instantaneous gear shift significant impact loads arise in the transmission. The angular velocity increases for a short time by 133.3 $\text{rad} \cdot \text{s}^{-1}$ at gear shifting from second to third speed. And it increases by 121.6 $\text{rad} \cdot \text{s}^{-1}$ at gear shifting from fifth to sixth speed. The amplitude of the velocity change depends on the gear ratio step and decreases with decreasing step accordingly.

It should be noted that an extreme shift process is considered in figure 3, which makes it possible to estimate the maximum velocity changes in the case of an instant gear shift. Unacceptably high torques act for a short time on the transmission links with such sharp jumps in velocity. It can lead to premature wear of gears, their breakdown and failure of transmission elements. It is important to optimize the shift control in transmissions therefore.

An increase in the number of speeds in gearboxes makes it possible to reduce the range of engine velocities (which increases the fuel economy [17]), while the gear shifting velocities between neighbor speeds become closer [2-5]. A sharp change in the gearbox output shaft velocity caused by the shifting the next gear leads to the fact that its velocity at the moment exceeds the velocity of gear shifting to the next speed as shown in the graph in figure 3. The proximity of the gear shifting velocities to neighbor speeds therefore can cause the excitation of a self-oscillating process and, as a consequence, high-frequency gears shifting from the engaged gear to the disengaged and vice versa.

A computer simulation of the high-frequency looping process at close values of the gear shifting velocities to neighbor gears was carried out by using a simplified impact model in figure 2. The computer simulation results showed in figure 4 [9].

The gear shifting velocities from 1$^{\text{st}}$ speed to the 2$^{\text{nd}}$ $\dot{x}^{(12)}$ and back $\dot{x}^{(21)}$ (see in figure 4) are not the same according to the gear shifting map [5]. It is assumed that the model in figure 2 has a relatively small damping coefficient in this case and the velocities $\dot{x}^{(12)}$ and $\dot{x}^{(21)}$ are quite close in magnitude. The model moving in 1$^{\text{st}}$ speed at $t = 0$ and reaches the 2$^{\text{nd}}$ speed gear shifting velocity $\dot{x}^{(12)}$. The impact caused by gear shifting leads to the excitation of the gearbox output shaft oscillations and the amplitude of the change in the output shaft velocity exceeds the difference
between the gear shift velocities of neighbour speeds. The gearbox output shaft velocity decreases to the gear shift velocity $\dot{x}^{(21)}$ from the 1st speed to the 2nd as a result. The gear shifting occurs again, then the self-oscillating process is repeated and for the selected parameters of the model is damped as seen in figure 4.

**Figure 4.** Computer simulation results of high-frequency gear shifts looping.

The solid lines 1-2, 2-3, 3-4, etc. on the shift maps of an automatic transmission showed in figure 5 are gear shift lines from low to high speed, and dashed lines 2-1, 3-2, etc. are lines of gear shifting from upper to lower speed.

**Figure 5.** Gear shifting map for 6-speed automatic transmission.

The gear shifting velocities to neighbour speeds at low vehicle velocity are quite close as seen from figure 5. Therefore, the problem of low-frequency looping (also a self-oscillating process) may arise. Self-oscillations are excited here due to periodic changes in the external conditions of the vehicle, caused by factors such as the topography of the road or the driver's behaviour. For example, at a frequent change of ups and downs: an upshift will occur when driving downhill, and a downshift will occur when driving uphill. Another example is on winding roads: an upshift will occur before a turn and downshift after a turn. Simulation of the low-frequency looping process performed for a simplified model in figure 2 is shown in figure 6.
It is assumed that a vehicle in the computer simulation of the low-frequency looping process in figure 6 moves along a hilly road where ups and downs alternate with equal frequency. The vehicle velocity is maintained constant $\approx 50 \text{ km} \cdot \text{h}^{-1}$ due to an automatic speed control system (cruise control). Computer simulation shows that self-oscillations and low-frequency looping occur, that is when vehicle ride down the hill an upshift occur, and when uphill - a downshift one.

4. Results and discussion
The article discusses the process of gear shifting in automatic transmissions as the impact interaction of transmission elements. Quite simple models of vehicle transmission have been developed and mathematically described. The process of gear shifting in various modes of vehicle movement is considered on the models.

It was shown as a result of computer simulation that velocity changes of the gearbox output shaft during instantaneous gear shifting are very significant and can exceed the value of the gear shift velocity to the next speed.

Computer simulation of self-oscillating modes of low-frequency and high-frequency looping of gear shifts are carried out. The first is associated with the movement of the vehicle at low speeds with close to gear shifting velocity and changes in external driving conditions, such as a periodic change of ups and downs, winding road topography or driver actions. High-frequency looping is associated with the impact nature of gear shifting, low damping and with a large amplitude of velocity change of the gearbox output shaft, which exceeds the gear shifting velocity of the next speed.

5. Conclusion
The short duration of gear shifts in automatic transmissions allows to consider this process as an impact. The vehicle transmission in this case at a short time interval is a closed self-oscillating mechanical system, for which the theorem on the change in the angular momentum upon impact is applicable.

The assumption made in the development of mathematical models that there is no slippage between the discs of the engaging clutch greatly simplifies the modeling, but at the same time allows one to estimate the ultimate loads arising in the transmission.

The main way to prevent high impact torques acting on the transmission elements is to decrease the engine shaft velocity before upshift and, conversely, to increase the engine shaft velocity before downshift [5]. Such an algorithm is provided in automatic transmissions for cars. Among the additional recommendations aimed to reduce the vibration amplitude of the gearbox output shaft caused by gear shifting, the following can be distinguished:

- introduction of a delay for reading the gearbox output shaft speed sensor during gear shifting;
- increasing the rigidity of the gearbox output shaft and increasing damping;
- changes in the ratios of the gearbox moments of inertia and transmission links;
- increasing the difference between the velocities of neighbor gear shifts.
The mathematical models described here can be used in the development of control systems for gear shifting in automatic transmissions and for calibration of the engine operating modes and transmission under various conditions of vehicle movement.

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