The viscous evolution of white dwarf merger remnants

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ABSTRACT

The merger of two white dwarfs (WDs) creates a differentially rotating remnant which is unstable to magnetohydrodynamic instabilities. These instabilities can lead to viscous evolution on a time-scale short compared to the thermal evolution of the remnant. We present multidimensional hydrodynamic simulations of the evolution of WD merger remnants under the action of an \( \alpha \)-viscosity. We initialize our calculations using the output of eight WD merger simulations from Dan et al., which span a range of mass ratios and total masses. We generically find that the merger remnants evolve towards spherical states on time-scales of hours, even though a significant fraction of the mass is initially rotationally supported. The viscous evolution unbinds only a very small amount of mass (\( \lesssim 10^{-5} M_\odot \)). Viscous heating causes some of the systems we study with He WD secondaries to reach conditions of nearly-dynamical burning. It is thus possible that the post-merger viscous phase triggers detonation of the He envelope in some WD mergers, potentially producing a Type Ia supernova via a double-detonation scenario. Our calculations provide the proper initial conditions for studying the long-term thermal evolution of WD merger remnants. This is important for understanding WD mergers as progenitors of Type Ia supernovae, neutron stars, R Coronae Borealis stars and other phenomena.

Key words: hydrodynamics – supernovae: general – white dwarfs.

1 INTRODUCTION

Systems consisting of two white dwarfs (WDs) are natural outcomes of binary stellar evolution. These binaries are not static; absent any other torques, the loss of angular momentum via gravitational wave (GW) emission will drive the binary together. Programmes such as SWARMS (Mullally et al. 2009) and the ELM survey (Brown et al. 2010) have dramatically increased the number of known WD binaries, including some systems that will merge within a Hubble time (Kilic et al. 2012). The Galactic population of WD binaries is expected to be a source of unresolved GW foregrounds at mHz frequencies, though only a handful of presently known systems would be individually detectable by a space-based GW interferometer mission (Nelemans 2009).

Details of the inspiral, in particular whether tidal torques cause the binary to be synchronized and the location of the tidal heating, are active areas of inquiry that can have a significant impact on the dynamics of the binary and the thermal state of the WDs (Fuller & Lai 2012). As the orbital separation shrinks, the less massive (and hence larger) WD will eventually overflow its Roche lobe and begin transferring mass to the companion. The stability of this mass transfer depends on, for example, whether the material forms a disc or flows directly on to the companion, which in turn depends on the mass ratio (\( q \)) and total mass (\( M_{\text{tot}} \)) of the binary (e.g. Marsh, Nelemans & Steeghs 2004).

Those systems that do undergo unstable mass transfer and subsequently merge have been of substantial theoretical interest. In particular, such systems have received attention as the possible progenitors of Type Ia supernovae (Iben & Tutukov 1984; Webbink 1984). Considerable work exists exploring this ‘double degenerate’ scenario and recent observational results have begun to favour it (e.g. Bloom et al. 2012; Schaefer & Pagnotta 2012). Another possibility is that double WD binaries with total masses exceeding the Chandrasekhar mass undergo accretion-induced collapse to form a neutron star (e.g. Säio & Nomoto 1985). Less massive double degenerate systems are likely to have non-explosive outcomes and...
have been invoked to explain objects like the R Coronae Borealis stars and extreme helium stars (Webbink 1984; Saio & Jeffery 2000; Clayton et al. 2007).

An accurate simulation of the merger process requires a 3D code without prescribed geometry and with good numerical conservation properties. For these reasons, the pioneering study of Benz et al. (1990) used smoothed particle hydrodynamics (SPH). More recent studies (e.g. Dan et al. 2011; Pakmor et al. 2012; Raskin et al. 2012) have improved on these first results by contributing additional physics, more accurate initial conditions, higher resolution and more sophisticated numerical techniques. These simulations follow the evolution of the binary through the tidal disruption of one of the components. In some cases the merger is sufficiently violent that an explosion may result (Pakmor et al. 2010; Dan et al. 2012). When the merger itself does not trigger an explosion, some material from the disrupted lower mass WD forms a shock-heated layer at the surface of the primary WD, while the rest of the material forms a thick disc at larger radii.

The evolution of such systems has frequently been treated in the literature as a long-lived (∼10^7 yr) phase of accretion from a disc at the Eddington limit (e.g. Nomoto & Iben 1985). This picture was improved by Yoon, Podsiadlowski & Rosswog (2007), who considered accretion at a similar rate but on to a hot envelope, and by van Kerkwijk, Chang & Justham (2010), who made simple α-disc estimates of the accretion time-scale and found it to be far more rapid (∼hours) than the time-scale for accretion at the Eddington limit.

Recently, Shen et al. (2012) provided a new model of the different evolutionary phases of WD merger remnants. They argued that the evolution is much more ‘star like’ than the accretion disc oriented models that have dominated the literature. More concretely, Shen et al. (2012) showed that the rapid dynamical evolution of the merger (∼10^2 s) gives way to a longer lived viscous phase driven by magnetohydrodynamic instabilities (∼10^3–10^5 s) before the onset of a long (∼10^4 yr) thermal phase. In contrast with previous work, this implies that the long-term evolution of a WD merger remnant is not determined by accretion, but rather by the internal redistribution of heat/momentum and the external cooling rate of the viscously heated, nearly-shear-free remnant.

In Shen et al. (2012), the viscous evolution was calculated in 1D using a γ-law equation of state (EoS). The goal of this work is to refine the understanding of the outcome of the viscous evolution of WD merger remnants using higher dimensional numerical simulations. In addition, we consider a wider variety of WD+WD systems than Shen et al. (2012), who focused on roughly Chandrasekhar mass CO+CO mergers.

In Section 2, we outline the numerical methods we use, including how we construct our initial conditions from simulations by Dan et al. (2011). In Section 3, we present the results of each of our calculations. Section 4 provides a discussion of the end states of the calculations. In Section 5, we state our conclusions and propose avenues for future work. In the appendix, we show various test calculations that confirm the results we focus on in the main text.

### 2 NUMERICAL METHODS

We perform our calculations using the ZEUS-MP2 (Hayes et al. 2006) code, a massively parallel implementation of the algorithms used in the ZEUS family of codes. These codes solve the fluid equations using finite differences on a staggered mesh. The internals of ZEUS are well documented in the literature (see e.g. Stone & Norman 1992). While there have been other, more modern developments in astrophysical fluid codes, we chose to use ZEUS-MP2 because of its supported features (e.g. spherical coordinates, non-ideal EoSs) and because its structure allows for relatively easy addition of new features.

Our calculations are done in spherical coordinates, anticipating the evolution of the remnant to a quasi-spherical end state. In order to minimize the computational demands, we primarily perform 2.5D simulations, in which vector quantities can have a φ component, but its value does not vary along the φ direction. In general, we also assume reflection symmetry about θ = π/2. In the appendix, we briefly report additional calculations which confirmed the validity of these simplifications.

Our typical computational domain is characterized by the grid spacing in the r and θ directions and by the radius of the inner boundary. Unless otherwise specified, we adopt a logarithmic radial grid with Nr = 64 points per decade. The angular grid is uniform in the range [0, π/2] with Nθ = 48 angular zones. These values give a grid in which individual cells are roughly equal in radial and θ extent. We choose an inner radius such that only 0.1 per cent of the mass lies interior to that radius and then place the outer boundary at 10^5 r_{tue}. We perform higher resolution simulations to confirm that our simulations are converged (see Appendix A1).

We make several modifications to ZEUS-MP2 (based on v2.12) in order to perform our calculations; we describe these modifications in the rest of this section.

#### 2.1 Non-ideal equation of state

We modify the code by the addition of the non-ideal Helmholtz EoS, which provides an electron–positron EoS valid over a large range of physical conditions combined with the EoSs for an ideal gas of ions and for blackbody radiation (Timmes & Swesty 2000). With the addition of the EoS routines, one small algorithmic change is made: as suggested in Stone & Norman (1992), a predictor–corrector method is used to improve energy conservation during the calculation of the compressional heating term.

#### 2.2 Shear viscosity

In order to approximate the effects of magnetic stresses, we add shear stress terms to the hydrodynamic equations, that is, we solve the equations:

\[ D_t \rho \, j_1 + \rho \, \nabla \cdot j = 0 \]

\[ \rho D_t j_1 = -\nabla \cdot P - \rho \nabla \cdot \Phi + \nabla \cdot T_{ij} \]

\[ \rho D_t (\varepsilon / \rho) = -P \nabla \cdot j_1 + T_{ij} (\nabla \cdot j_1) / (\rho \nu) \]

where \( D_t = \partial / \partial t \) and \( \varepsilon \) is the convective derivative and we observe the usual Einstein summation conventions. The pressure is denoted by \( P \), and the mass and internal energy densities are represented by \( \rho \) and \( \varepsilon \), respectively. The velocity vector is \( j \). The anomalous stress tensor \( T_{ij} \) is defined as:

\[ T_{ij} = \nu (\partial_i j_1 + \partial_j j_1) \]

where \( \nu \) is the dynamic viscosity.

A very similar modification of the ZEUS-2D code was made by Stone, Pringle & Begelman (1999). We benefited from inspecting the source code that was used to perform the calculations reported in that work. We also used the results reported in Stone et al. (1999) as a reference against which to test our own implementation.
The viscous terms are evaluated using an operator split method and are updated during the source step (Stone & Norman 1992). To ensure numerical stability, the time-step \( \Delta t \) must be chosen to be less than \( \Delta t_{\text{visc}} \sim \min((\Delta r)^2/c_s), \) where the minimum is evaluated over the computational domain.

As a dimensionally motivated form for the dynamic viscosity coefficient, we adopt

\[
\nu(r, \theta) = \frac{c_s^2(r, \theta)}{\Omega_k(r)},
\]

where \( c_s \) is the local sound speed and \( \Omega_k \) is the Keplerian angular velocity calculated using the mass enclosed at a given spherical radius. Portions of the merger remnant (see Fig. 1) are unstable to the magnetorotational instability (MRI; Balbus & Hawley 1991) and the Tayler–Spruit dynamo (Tayler 1973; Spruit 2002). These processes may generate viscous stresses corresponding to \( \alpha \) in the range \( 10^{-4} - 10^{-1} \); for order-of-magnitude estimates, see Shen et al. (2012). We adopt a fiducial value of \( \alpha = 3 \times 10^{-2} \), though we confirm that the results of our calculations are not sensitive to this choice (see Appendix A2).

As one moves to small \( r \) (the origin being at the centre of the surviving WD; see Section 2.4), \( c_s \) and \( \Omega_k \) approach constant values. We are using a logarithmic grid, so \( \Delta r \propto r \) and therefore \( \Delta t_{\text{visc}} \propto r^2 \). The time-step constraint imposed by the Courant–Friedrichs–Lewy (CFL) condition depends linearly on \( \Delta r \), so \( \Delta t_{\text{CFL}} \propto r \). At sufficiently small radii, the viscous time-step becomes much less than the time-step required by the CFL condition. In practice, this occurs at a radius that is in our computational domain. In order to evolve the remnant over many viscous times, we apply the following ad hoc prescription. Within some radius \( r_c \), we suppress the viscosity by a factor of \( 1/r \) such that the ratio of \( t_{\text{visc}}/t_{\text{CFL}} \) remains constant. In order to make the cut-off smooth, the exact prescription is

\[
v'(r, \theta) = v(r, \theta) \left[ \frac{1}{1 + (r/r_c)\beta} \right]^{1/\beta},
\]

where \( \beta = 4 \) and \( r_c \) is approximately the half-mass radius of the inner WD. As shown in the appendix, we have verified that our results are insensitive to the details of this prescription. Physically, we would not expect the viscosity prescription in equation (5) to hold as \( r \to 0 \) because this region within the WD is in approximate solid body rotation and is not MHD unstable.

Local numerical simulations of the MRI in accretion discs indicate that the azimuthal components of the stress tensor, \( T_{\theta\phi} \) and \( T_{r\phi} \), are roughly a factor of 10 larger than the other components (e.g. Hawley, Gammie & Balbus 1995; Stone et al. 1996). Our default assumption then is to evolve with only these components in the stress tensor being non-zero. In Appendix A3, we test the effect of including all components and find some quantitative, though not qualitative, differences between the two choices.

### 2.3 Nuclear burning

In several of the model systems, the temperatures reached are sufficiently high that the energy release from fusion is not negligible on the viscous time-scale. However, the conditions are not such that the local burning time-scale ever falls below \( \Delta t_{\text{CFL}} \). Therefore, in order to minimize the computational load associated with calculating the burning, we implement an extremely simple five-isotope nuclear network which is explicitly integrated at the hydrodynamic time-step. This captures the bulk of the energy release.

The five species we track are He, C, O, Ne and Mg (these are the five isotopes present in the initial compositions). These isotopes are connected by four processes: the triple-\( \alpha \) process and \( \alpha \)-capture on each of C, O and Ne. The rates of these processes were taken from the JINA REACLIB data base (Cyburt et al. 2010). We neglect additional physics such as screening corrections because the burning primarily occurs at densities where such effects are unimportant. We refer to our own burning implementation as HeCONe.

As a test of both our own implementation and the assumptions that motivate it, we also coupled the 13-isotope \( \alpha \)-chain network aprox13 to the code (Timmes, Hoffman & Woosley 2000). The results were virtually identical, confirming the validity of our approach (see Appendix A6 for quantitative comparisons of the results of these tests).

**ZEUSSPH** provides the ability to advect scalar quantities; we use this feature to track the mass fractions of the isotopes in our network. The algorithms do not guarantee the sum of the mass fractions remains equal to 1 after the advection step. Methods to restore this constraint in fluid codes have been reported in the literature (see e.g. Plewa & Müller 1999). However, for the sake of simplicity, immediately before evaluating the energy release from nuclear burning, we enforce the constraint \( \sum X_i = 1 \) by appropriately adjusting the mass fraction of the most abundant isotope.

### 2.4 Construction of initial conditions

Our starting point is the results of SPH simulations of double WDs performed by Dan et al. (2011). The simulations were notable for their use of a more accurate initialization of the binary system at the onset of mass transfer than had been used in previous work. We calculate the viscous evolution of the merger remnants formed in each of their eight production runs, the parameters of which are

\[\text{Figure 1. The evolution of the rotation profile of the fiducial 0.6+0.9 M}_\odot \text{CO+CO remnant. The solid lines are the angular velocity } \Omega \text{ and the dashed lines are its ratio to the Keplerian angular velocity, } \Omega/\Omega_k. \text{ (The angular velocity is calculated using the material in a 45° wedge centred on the equator. In the initial profiles (black), most of the material from the disrupted secondary } (M_r > 0.9) \text{ is rotationally supported with an angular velocity profile unstable to the MRI. The action of viscosity drives more of the remnant to solid body rotation and the accompanying heating leads to more of the remnant being thermally supported. We set } \alpha = 0.03 \text{ for all the results in the main text. The appendix shows that our results are nearly independent of } \alpha.\]
It is not simultaneously possible to conserve the kinetic energy of the fluid and its linear and angular momentum when performing the \( \phi \) averaging. (This is simply a statement of the fact that in a non-uniform field, \( (\nabla^2 \phi)^2 \neq \phi^2 \)).

Therefore, to obtain the value of a component of the velocity \( v \), defined at a point \(( r, \theta )\), we calculate

\[
v_{ij} = \frac{1}{N_\varphi} \sum_{k=1}^{N_\varphi} \left[ p_{\text{SPH}}(r_i, \theta_j, \phi_k) \cdot \hat{e}_{ij}(\phi_k) \right],
\]

where \( p \) is the momentum vector and \( \hat{e}_{ij} \) is the unit vector of the velocity component of interest at the point.

### 3 RESULTS

We expect that systems with similar mass ratios \((q)\) and total masses \((M_{\text{tot}})\) will undergo similar evolution. Since the composition of a WD maps to a relatively well defined mass range, we organize our discussion by the composition of the merging WDs. First, we discuss our fiducial \(0.6+0.9\,M_\odot\) CO+CO system. The outcomes of He+He, He+CO and He+ONeMg mergers are discussed on a more limited basis, emphasizing only the notable differences between these systems and our fiducial one (see Table 2 for a summary of the properties of our primary simulations). As shown in Table 1, Dan et al. (2011) label their simulations with a short identifier of ZEUSMP2, we adopt the following procedure.

In order to map between the output of the SPH calculations and the staggered mesh of ZEUS-MP2, we adopt the following procedure. In SPH, the value of a quantity \( f \) at a position \( x \) is given by

\[
f(x) = \sum_{i=1}^{n} \frac{m_i}{\rho_i} f_{W}(|x - x_i|, h_i),
\]

where \( W \) is the kernel function. The quantity of interest associated with the \( i \)th SPH particle is denoted by \( f_i \). The SPH particle has mass, density, and smoothing length denoted by \( m_i, \rho_i, \) and \( h_i \), respectively (e.g. Monaghan 1992). The sum runs over the total number of SPH particles, \( n \).

Schematically, we construct our grid-based initial conditions by evaluating the five quantities that ZEUS-MP2 evolves (mass density \( \rho \), internal energy density \( e \) and velocity \( v \)) at each grid point. In our standard 2D simulations, we construct initial conditions that are explicitly invariant in the \( \phi \) direction and are reflection-symmetric about \( \theta = \pi/2 \). Given these conditions, the total linear momentum of the remnant is guaranteed to be zero. Therefore, we choose the origin of our simulation coordinate system to be at the point of peak density, corresponding to the centre of the more massive (surviving) WD.

Explicitly, in order to calculate the value of density \((e \text{ or } \rho)\) at a grid point with coordinates \((r, \theta)\) we evaluate

\[
\rho_{ij} = \frac{1}{2N_\varphi} \sum_{k=1}^{N_\varphi} \left[ p_{\text{SPH}}(r_i, \theta_j, \phi_k) + p_{\text{SPH}}(r_i, \pi - \theta_j, \phi_k) \right],
\]

where \( N_\varphi \) is the number (typically \( N_\varphi = 32 \)) of equally spaced points used to cover the interval \( \phi \in [0, 2\pi] \). Now and throughout this section, the subscript SPH indicates a quantity extracted from the SPH simulation by the evaluation of equation (7).

Constructing the initial velocity vector requires slightly more complicated \( \phi \) averaging. From the SPH data, we first construct the full velocity vector (with the components represented in Cartesian coordinates) at each of the grid locations where a component of the velocity will be defined. The staggered mesh employed by ZEUS-MP2 means that each velocity component is defined at a different spatial point (for details see Stone & Norman 1992). We take this into account, though we do not manifestly indicate it in the formulae below for the sake of compactness.

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calculate the spherically averaged mass and energy densities and then apply the EoS. To calculate angular velocities, we restrict the average to a $45^\circ$ wedge centred on the equator. As our simulations evolve towards a spherical end state, these 1D averages become an increasingly complete summary of the 2D structure of the remnant.

### 3.1 CO+CO systems

Our fiducial system (ZP5) is a super-Chandrasekhar CO+CO merger with $M_{\text{tot}} = 1.5 \ M_\odot$ and $q = 2/3$. We simulate this system for $3 \times 10^5$ s, which is $\sim 5 \times 10^7$ time-steps of the hydrodynamics code. The simulation conserves mass to 1 part in $10^5$, energy to 0.5 per cent and angular momentum to 1 part in $10^5$. The evolution of an identical system was discussed in Shen et al. (2012), who performed a simple 1D calculation of the viscous evolution. Our multidimensional calculations confirm the schematic picture presented therein.

At the end of the SPH simulation, the primary WD is relatively undisturbed and is surrounded by the remnants of the disrupted secondary. More than half of the disrupted material has primarily rotational support; the remainder was shock-heated in the merger and has thermal support. (A small amount $\sim 3 \times 10^{-4} \ M_\odot$ is unbound in a tidal tail.) The merger remnant is in quasi-hydrostatic equilibrium, which we confirm by evolving these initial conditions without the action of viscous torques for many dynamical times, observing little change.

The black lines in Fig. 1 show the initial rotation profile. The primary WD is rotating rigidly; exterior to that is the disrupted secondary which is in Keplerian rotation. This hot, fully ionized material is unstable to magnetohydrodynamic instabilities such as the MRI. The turbulence generated by the saturation of the MRI leads to an enhanced viscosity and concomitant transport of angular momentum to larger radii (Balbus & Hawley 1991; Balbus 2003). As described in Section 2.2, we model this using an $\alpha$-viscosity.

The viscosity also liberates energy present in the $\phi$-velocity shear. Fig. 2 shows the evolution of the temperature and specific entropy profiles during the viscous evolution. Fig. 3 shows the final $\rho$-$T$ distribution and the evolution of the temperature peak in the $\rho$-$T$ plane during the viscous evolution. The viscous heating causes the peak temperature in the remnant to increase over the duration of the viscous phase by a factor of $\sim 2$, to $\sim 8 \times 10^8$ K. The temperature peak is at a density of $\sim 5 \times 10^5$ g cm$^{-3}$ and at those conditions the energy released from carbon burning exceeds neutrino losses and the burning becomes self-sustaining.

The carbon burning in our fiducial model is an unimportant energy source on the viscous time-scale, so the viscous evolution is not directly affected. However, the fact that carbon burning begins during the viscous evolution means that a convective carbon burning shell will develop in $\sim 10^5$ s. One consequence of this is that we expect the material exterior to the temperature peak at the end of the viscous evolution to quickly form a convective envelope. Future work will investigate the structure of this envelope, which is important for understanding the thermal evolution of the remnant and characterizing its observational properties.

One of the most striking results of our multidimensional simulations is that the merger remnant evolves towards a final quasi-spherical steady state. (Given that there is rotation, the final state will actually be oblate, though in practice, the rotational velocities we find imply that it is quite spherical.) To quantify this, we define a simple ‘aspect ratio’ as follows: draw an isodensity contour and take the ratio of the distance from the origin at the equator to that at the pole. As a rule of thumb, we find that the aspect ratio associated with a given radius approaches unity after about 10 viscous times have passed at that radius. The bottom panel of Fig. 4 shows this convergence clearly. The primary WD ends up with a thermally supported, nearly-spherical envelope. The top panel of Fig. 4 shows the mass enclosed as a function of radius which illustrates how the

![Figure 2. The evolution of the temperature peak in the $\rho$-$T$ plane. The dotted line indicates the break-even point where the energy release from carbon burning is equal to neutrino losses. The filled square circle is the peak temperature and corresponding density at the start (end) of the simulation, and the dashed line that connects them traces its evolution. The solid line is the full 1D $\rho$-$T$ profile of the quasi-spherical end state. The grey dot-dashed line indicates where the gas and radiation pressure are equal.](https://academic.oup.com/mnras/article-abstract/427/1/190/1028094/fig2)

![Figure 3. The evolution of the temperature peak in the $\rho$-$T$ plane. The dotted line indicates the break-even point where the energy release from carbon burning is equal to neutrino losses. The filled square circle is the peak temperature and corresponding density at the start (end) of the simulation, and the dashed line that connects them traces its evolution. The solid line is the full 1D $\rho$-$T$ profile of the quasi-spherical end state. The grey dot-dashed line indicates where the gas and radiation pressure are equal.](https://academic.oup.com/mnras/article-abstract/427/1/190/1028094/fig3)
flows, one might expect that material would outflow along the poles approach of the remnant to a spherical state. Fig. 5 also plots isodensity contours which emphasize the which is related to the kinetic energy associated with convective unstable state without the action of the viscous stresses.1

Our initial conditions are not unstable by the Høiland criterion and do not evolve towards an appropriate test is the Høiland criterion. Our initial conditions are not unstable without the action of the viscous stresses.1

Fig. 5 shows the 2D evolution of the fiducial system. In addition to the entropy and temperature, we plot two energy densities which are helpful in interpreting the evolution. One is the free energy density available in the \( \phi \)-velocity shear

\[
\text{KE}_{\phi \text{-shear}} = \frac{1}{2} \rho \left( \frac{R}{d \log R} \right)^2 ,
\]

which shows the energy available for viscous heating. The other is the kinetic energy density in non-azimuthal motions

\[
\text{KE}_r,\phi = \frac{1}{2} \rho \left( v_r^2 + v_\phi^2 \right) ,
\]

which is related to the kinetic energy associated with convective motions. Fig. 5 also plots isodensity contours which emphasize the approach of the remnant to a spherical state.

Given previous work on viscous, geometrically-thick accretion flows, one might expect that material would outflow along the poles during the viscous evolution. When the viscous time is much less than the cooling time and the mass inflow is assumed to be conservative (i.e. mass does not leave the system), the transport of energy and the release of gravitational potential energy are such that material has a positive Bernoulli parameter (Be) (Narayan & Yi 1994; Blandford & Begelman 1999). Therefore, solutions in which the mass flow is not conservative may be more physical. Non-radiative accretion flows are also predicted to be convectively unstable. Models based on these ideas (e.g Blandford & Begelman 2004) developed solutions with prominent outflows. Hydrodynamic numerical simulations such as those by Stone et al. (1999) exhibited the slow outflow of marginally bound material in the polar direction. MHD simulations such as those by Stone & Pringle (2001) found somewhat more prominent outflows than in the hydrodynamic simulations.

We do not observe outflows in our simulations. Fig. 6 shows the fraction of mass on our grid with positive Be \( f_{\text{Be}} > 0 \). Initially \( f_{\text{Be}} \sim 3 \times 10^{-3} \), corresponding to the unbound material in the tidal tail. This material flows out of the outflow boundary and afterwards there is little unbound mass \( f_{\text{Be}} < 10^{-5} \). In order to isolate the effects of viscosity on the unbound material, we ran a simulation without the viscosity and calculated \( f_{\text{Be}} \) (dotted blue line). The orange line in Fig. 6 shows the integrated difference in the mass that flowed through the outer boundary with Be > 0 in simulations with and without viscosity. This difference is very small, \( < 10^{-3} \). We do not claim that this specific value is robust, but the conclusion that the viscous evolution of WD merger remnants unbinds only a very small amount of mass appears to be.

The initial conditions of our simulations are rather different from the initial conditions of most radiatively inefficient accretion simulations. Such simulations typically allow the material to move through several orders of magnitude in radius before reaching an inflow boundary representing a black hole. By contrast, the radial dynamic range between the surface of the primary WD and the bulk of the material in the initial rotationally supported disc is small, a factor of \( \sim 5 \). The presence of a ‘hard surface’ (the primary WD) means that as material accretes, the radius where material is pressure-supported moves outwards, further suppressing the dynamic range between the effective inner boundary and the disc.

In order to understand the results of our WD merger remnant simulations in the context of accretion torus simulations, we generate accretion tori like those in Stone et al. (1999) and adjusted the inner boundary condition (reflecting versus inflow) and the dynamic range between the initial torus and the inner boundary. We run the simulations for several orbits and calculate the resulting amount of unbound material \( f_{\text{Be}} \). At a radial dynamic range between the initial torus and the inner boundary of 100 and with an inflow boundary, we find \( f_{\text{Be}} \sim 4 \times 10^{-2} \), much larger than in our WD merger remnant calculations (see Fig. 6). Decreasing the dynamic range to 10 results in \( f_{\text{Be}} \sim 4 \times 10^{-3} \). At this dynamic range, changing the inner boundary condition to reflecting causes \( f_{\text{Be}} \) to peak \( \sim 10^{-4} \) and then fall as the simulation continues. These results support our conclusion that only a very small amount of mass is unbound during the viscous evolution of WD merger remnants \( f_{\text{Be}} \lesssim 10^{-5} \).

In addition to our fiducial 0.6+0.9 CO+CO system, we also simulate a very super-Chandra 0.9+1.2 CO system (ZP8). This system quickly starts C+ C burning, although the burning does not become dynamical (see Section 4.2). While the energy release from the burning on the viscous time is locally non-negligible, the mass outflow is not affected by the presence of nuclear energy

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1 The physical initial conditions are of course unstable in MHD, as it is the MRI that is giving rise to the effective viscosity.

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Figure 5. A visual overview of the 2D evolution of the fiducial 0.6+0.9 $M_\odot$ CO+CO merger remnant. Within each panel, the top two subpanels are thermodynamic quantities ($s$, $T$) and the bottom two subpanels are kinetic energy densities (non-azimuthal, $\phi$ shear). The black contours are density, spaced 1 per decade. The dashed contour is $\rho = 10^3$ g cm$^{-3}$. The main panels are snapshots of the simulation at the indicated times. Top panel: the initial conditions (note the large ‘free’ energy apparent in the shearing, Keplerian disc). Middle panel: the action of viscosity has dissipated some of the shear and heated the material. The remnant has become convectively unstable as can be seen in the striation of the non-azimuthal kinetic energy. Bottom panel: the remnant has settled down into a quasi-spherical steady state.
Evolution of WD merger remnants

Figure 6. Viscously unbound material. The left-hand (blue) scale indicates the fraction of mass with a positive Bernoulli constant at a given time in the evolution of the fiducial 0.6+0.9 $M_\odot$ CO+CO merger remnant. The dominant contribution is the tidal tail; the large decrease in $f_{B>0}$ over the first 5000 s is this material flowing out of the simulation domain. The solid (dotted) line is the mass fraction with Be $>0$ in a simulation with (without) viscosity. The right-hand (orange) scale is the integrated amount of mass that has flowed out of the simulation domain with a positive Bernoulli constant due to the influence of viscosity. This is the integrated difference between the two blue curves. Little additional material ($\leq 10^{-5} M_\odot$) is unbound by the viscous evolution.

generation. As shown in the appendix, the energy release is not significant enough to affect the global behaviour of the remnant.

3.2 He+He systems

The evolution of He+He merger remnants is broadly similar to our fiducial CO+CO case. The larger size and lower mass of the He WDs mean that the temperatures reached at the end of the SPH calculations are not as high. However, these temperatures are still high enough that we elect to track the energy release from He burning in our simulations. We do this using the simple nuclear network described in Section 2.3.

Fig. 7 shows the evolution of the temperature and rotation rate for a 0.2+0.3 $M_\odot$ system (ZP6). This has the same mass ratio $q = 2/3$ as the fiducial system, but with a total mass three times lower. The initial temperature and rotation profiles look qualitatively similar to our fiducial system. Appropriately scaling these values by the total mass, they are even quantitatively similar. However, the final state appears somewhat different, most conspicuously because of the narrow temperature peak that forms at an enclosed mass of $\sim 0.38 M_\odot$.

This qualitative difference in evolution is most clear in Fig. 8, which shows the evolution of the temperature maximum and the corresponding density. The temperature maximum evolves to lower density during the viscous phase, unlike in CO+CO mergers where it evolves to higher density (see Fig. 3). This effect is unrelated to the presence of fusion, as the time-scale for burning is still relatively long. This difference in evolution is also not a qualitative difference in the merger outcome or the viscous evolution, but rather is due to the different contribution of gas and radiation pressure in the merger remnants. The grey dot–dashed line in Figs 3 and 8 shows where $P_{\text{gas}} = P_{\text{rad}}$. The He+He case remains gas pressure dominated, so the location of the peak temperature corresponds to the location of the viscous heating. Lower densities, which are at larger radii and have corresponding longer viscous time-scales, are heated at later
times. In the CO+CO case, radiation pressure has a larger relative contribution initially and once material has been heated such that radiation pressure begins to become important, additional heating is no longer as effective in raising the temperature. Therefore, larger relative increases in $T$ occur at higher densities where gas pressure continues to dominate, and thus the peak $T$ moves to higher densities.

### 3.3 He+CO systems

In the He+CO systems, the primary is more massive and hence more compact than in the He+He mergers. This leads to higher temperatures during the merger. The lower temperatures required for He burning (versus C burning) mean that the effects of nuclear burning are more pronounced in these systems.

Fig. 9 shows the evolution of the temperature peak in these simulations. The dotted line labelled $\epsilon_{\text{CC}} = \epsilon_c$ indicates the break-even point where the energy release from carbon burning is equal to neutrino losses. The dotted line that connects them traces its evolution. The solid line is the full 1D approximation that starts (ends) of the simulation, and the dashed line that connects them traces its evolution. The dotted line labelled $\rho_{\text{burn,He}} = 10^8 \, \text{s}$ indicates the region where the time-scale for energy release by He fusion is equal to the time-scale of the simulation. The filled square (circle) is the peak temperature and corresponding density at the start (end) of the simulation, and the dashed line that connects them traces its evolution. The solid line is the full 1D $\rho$-$T$ profile of the quasi-spherical end state.

The 0.3+0.6 $M_\odot$ He+CO system (ZP4) is the only one of the systems we simulate in which the final state deviates significantly from approximate spherical symmetry. The remnant itself is spherical, but at the interface between the material from the He and CO WDs the composition varies between the equator and pole. This explains why the final peak temperature does not lie on the final spherically averaged $\rho$-$T$ profile shown in Fig. 9. This system has the highest mass ratio of any mixed composition system we simulated and at higher mass ratios the secondary more strongly affects the primary. However, because the efficiency of mixing likely depends on dimensionality and angular momentum transport ($\alpha$-viscosity versus MHD), we do not expect our work to provide a robust prediction of the details of such mixing.

### 3.4 He+ONeMg

One system in our study is composed of a 0.5 $M_\odot$ He WD and a 1.2 $M_\odot$ ONeMg WD. Its evolution is very similar to the systems previously discussed. Notably, the high primary WD mass means that the He (from the secondary) reaches quite high temperatures. The burning time in this system is thus quite short, less than the viscous time, though not less than the dynamical time (see Section 4.2).

### 4 DISCUSSION

#### 4.1 Fitting formulae

The end states of our simulations will be useful as a starting point for future work concerning the thermal evolution of WD merger remnants. To aid such work, we provide fitting formulae that allow one to easily construct a physical state that is in rough quantitative agreement with our results.

The 1D profiles we extract at the end of our calculations have the following schematic form. At the centre is a core of cold, degenerate material. This is surrounded by a hot envelope, the outer portion of which was convective and so has an entropy that is roughly spatially constant.

This picture allows a simple, post hoc model of the end state of our simulations. We write down a piecewise EoS in which there is a central mass ($M_c$) described by a zero-temperature EoS. This is surrounded by an isothermal region corresponding to the temperature peak which has mass $M_{ip}$. The rest of the external material has a polytropic EoS. Empirically the polytropic index of $n = 3$ provides a good fit to all of our simulations. For systems at high temperatures, such as our 0.3+1.1 $M_\odot$ He+CO merger, this is unsurprising as the material in the convective region is nearly radiation dominated, implying an adiabatic index near $\gamma = 4/3$. For systems such as low total mass He+He mergers (ZP6 and ZP7), the matter is gas pressure dominated, which would imply an adiabatic index of 5/3. However, these systems have larger residual entropy gradients, such that the relationship $P \propto \rho^{4/3}$ roughly holds. Since we can provide satisfactory fits without introducing an additional parameter, we choose $n = 3$ for all our fits.

Quantitatively,

\[
P(\rho) = \begin{cases} 
P_{\text{ZT}}(\rho) & \text{if } M_c < M_c \\ K_1 \rho & \text{if } M_c < M_c < M_c + M_{ip} \\ K_2 \rho^{1+1/n} & \text{if } M_c + M_{ip} < M_c 
\end{cases}
\]

where $P_{\text{ZT}}$ is the pressure of a zero-temperature Fermi gas with $\mu_c = 2$ (e.g Shapiro & Teukolsky 1983). $K_1$ and $K_2$ are set by the condition that $\rho$ and $P$ are continuous at the transitions between regimes.

In combination with the equations of hydrostatic equilibrium and mass conservation in 1D spherical coordinates

\[
\frac{dM_r}{dr} = 4\pi r^2 \rho, \\
\frac{dP}{dr} = -GM_r \rho r^2
\]
and a central boundary condition \( \rho(r_{\text{inner}}) = \rho_c \), this is sufficient to fully specify a 1D model. We set \( \rho_c \) to be the value of the central density at the end of our simulations.

For each of our simulations we fit for the two masses \( M_c \) and \( M_{\text{fp}} \). Table 3 reports the results of these fits. Fig. 10 shows the results of the fit to our fiducial model. The fit reproduces the observed density at the end of our simulations.

Our fitting procedure does not use or provide any spatial information about the chemical composition. As a rough approximation, one can simply retain the initial Lagrangian composition of the system with the secondary outside the primary. In the mergers where the chemical compositions of the WDs were initially identical, this is a good approximation because nuclear reactions do not significantly alter the composition (for the set of mergers we considered).

| ID   | \( \rho_c \) (g cm\(^{-3}\)) | \( M_c \) | \( M_{\text{fp}} \) |
|------|-------------------------------|------------|-----------------|
| ZP1a | \( 8.8 \times 10^6 \)         | 0.71       | 0.10            |
| ZP2a | \( 4.7 \times 10^7 \)         | 0.98       | 0.12            |
| ZP3a | \( 9.5 \times 10^7 \)         | 1.05       | 0.16            |
| ZP4a | \( 3.8 \times 10^8 \)         | 0.53       | 0.13            |
| ZP5  | \( 2.8 \times 10^8 \)         | 0.84       | 0.20            |
| ZP6  | \( 6.4 \times 10^{8} \)       | 0.28       | 0.08            |
| ZP7  | \( 1.5 \times 10^{8} \)       | 0.38       | 0.12            |
| ZP8  | \( 3.3 \times 10^{8} \)       | 1.11       | 0.24            |

*aMark those systems that have a mixed chemical composition.

For mergers where the WDs had different compositions (which are marked in Table 3), the assumption that the composition is conserved in a Lagrangian sense is substantially more crude because of mixing and the effects of nuclear burning. In those cases, these simple fits would be inappropriate for work in which inaccuracies in the chemical composition could have a large effect.

### 4.2 Burning time

Recently there has been considerable interest in the possibility of central carbon detonations triggered by the detonation of a helium layer on the surface of a CO WD. During a WD merger, conditions for detonations may be reached in instabilities in the accretion stream (Guillochon et al. 2010) or at surface contact (e.g. Dan et al. 2012). The systems we consider did not reach detonation conditions during the SPH simulations (though those could not resolve accretion stream instabilities).

During the phase of evolution that we simulate, viscous heating does increase the temperature and either initiate or increase the rate of burning. Fig. 11 shows the minimum burning times and corresponding temperatures for the eight systems we simulate. We calculate the minimum nuclear burning time as \( t_{\text{burn}} = c_P T \epsilon_{\text{nuc}} \), where \( c_P \) is the specific heat at constant pressure, \( T \) is the temperature and \( \epsilon_{\text{nuc}} \) is the specific energy generation rate from nuclear reactions.

**Table 3.** The parameters from our fits (see equations 13–15 and the surrounding discussion). ID is the run ID. \( \rho_c \) is the central density extracted from the end of our simulations. \( M_c \) is the amount of mass (in \( M_\odot \)) in the zero-temperature degenerate core. \( M_{\text{fp}} \) is the amount of mass (in \( M_\odot \)) in the isothermal region, loosely corresponding to the temperature peak.

**Figure 10.** The simple two-parameter fit to the fiducial 0.6+0.9 \( M_\odot \) CO+CO merger remnant (see equations 13–15 and the surrounding discussion). The upper two panels show the pressure and density profiles from the simulation as the solid lines. The fits are shown as the dashed lines. The bottom panel shows the relative error between the fit and the simulation. The vertical grey lines show the position of the transitions in the piecewise EoS.

**Figure 11.** The shortest burning time (top panel) and corresponding temperature (bottom panel) for each of our simulated systems. The x-axis is the mass of the primary WD. Two systems have the same primary mass of 1.2 \( M_\odot \) and are slightly offset on the x-axis for visual clarity (ZP3 to the left-hand side and ZP8 to the right-hand side). In the top panel, the circle represents the shortest burning time reached overall, that is, at any point during the simulation; the cross represents the burning time at the end of the simulation. They are connected by a dashed line to guide the eye and indicate that intermediate values are achieved. The same symbols in the bottom panel show the temperatures at the corresponding locations. Because of varying chemical composition, the temperature associated with the shortest burning time is not necessarily the global peak temperature. The right-hand axis of the top panel and the orange circles show the ratio \( t_{\text{burn}}/t_{\text{dyn}} \) (as defined in the text) at conditions corresponding to the black circles. In no case does the burning time ever reach the dynamical time, though in ZP3 it is within a factor of 2.
The minimum burning time is not necessarily located at the location of peak temperature, as differences in the chemical composition (e.g. the presence of helium) may make the rate of energy release greater at a different location. In general, the viscous heating causes a monotonic increase in the temperature. Therefore, initially the burning time drops. Then, in cases where the burning time is less than the viscous time, changes in the composition (such as the depletion of helium) begin to shift the minimum burning time to slightly lower temperatures where more material remains to burn.

In the case of the 0.5+1.2 $M_\odot$ merger, the burning time, which is $\sim 40t_{\text{dyn}}$ at the beginning of the simulation, decreases to as low as $\sim 2t_{\text{dy}}$, where the dynamical time is calculated as $t_{\text{dy}} = P/(\rho gc)$. The ratio $t_{\text{burn}}/t_{\text{sym}} \lesssim 1$ provides a rough criterion for possible detonation. Detailed conditions for detonations are still a topic of current research and almost certainly require resolving smaller length-scales than our current simulations can do (e.g. see the discussion in section 3.2 of Woosley & Kasen 2011).

Using the value of $t_{\text{burn}}/t_{\text{sym}}$ as a guide, viscous heating does not cause any of the remnants that we simulated to experience dynamical burning. However, with the low value of $t_{\text{burn}}/t_{\text{sym}}$ for the 0.5+1.2 $M_\odot$ system and the temperature sensitivity of nuclear reactions, we expect that systems only slightly more massive would experience dynamical burning. Furthermore, if there are stochastic fluctuations, it is even possible that this particular system could experience dynamical burning.

Dan et al. (2012) surveyed the parameter space of primary/secondary WD mass and mapped out regions where they found conditions during contact that could lead to detonation. In general, these conditions happen at a ‘hotspot’. If the system does not detonate, the subsequent evolution towards an axisymmetric state causes the peak temperature to fall. Heating during the viscous evolution reverses this trend and creates a hot shell, which, as discussed previously, may reach conditions of dynamical burning. Some simple estimates based on fig. 8 of Dan et al. (2012) suggest that the region of parameter space where systems would not reach conditions of dynamical burning at contact but would reach such conditions later on during the viscous evolution is relatively small. Future work will quantitatively address this question by simulating a wider range of merger remnants. However, if contact detonations (or other earlier detonation mechanisms such as accretion stream instabilities) do not prove to be robust, viscous heating could potentially ensure that a wide range of WD mergers trigger a surface detonation.

5 CONCLUSIONS

The merger remnants of binary WDs are differentially rotating and unstable to MHD instabilities like the MRI. As outlined by Shen et al. (2012), MHD stresses give rise to a viscous phase of evolution which occurs on a time-scale much less than the thermal time. To investigate the outcome of this viscous evolution, we perform multidimensional hydrodynamic calculations of the evolution of WD binary remnants under the action of an $\alpha$-viscosity. The initial conditions for these calculations are the SPH simulations by Dan et al. (2011). We find that these remnants evolve towards spherical states on time-scales of hours. This confirms the arguments in Shen et al. (2012) that the post-merger evolution of WD merger remnants is via viscous redistribution of angular momentum that leads to nearly solid body rotation. The transport of angular momentum outwards removes rotational support from the majority of the mass leading to a nearly-spherical remnant. The dynamics during this phase are not consistent with accretion at the Eddington limit, as in previous models of WD merger remnants (e.g. Nomoto & Iben 1985; Saio & Nomoto 1998, 2004; Piersanti et al. 2003a,b). Instead, the viscous evolution of WD merger remnants is much more analogous to that of a differentially rotating star.

Viscous heating associated with the approach to solid body rotation unbinds only a very small amount of mass ($\lesssim 10^{-5} M_\odot$ in our fiducial calculation). This is in contrast to some of the intuition developed in the context of radiatively inefficient accretion flows, which predicts outflows. To understand this, we perform simple accretion torus calculations which indicate that the relatively small radius difference between the disc and the surface of the WD can explain why only a small amount of mass becomes unbound (see Section 3.1).

Viscous heating causes one of the systems we simulate to reach conditions of nearly-dynamical He burning, so it is possible that the post-merger viscous evolution triggers a detonation in some cases. Recently Dan et al. (2012) presented a suite of more than 200 WD merger simulations which more thoroughly populate the $q$–$M_{\text{tot}}$ plane. They found that many of these systems reached the conditions for detonation during the merger (see e.g. their figs 6 and 8). In our calculations, min$(t_{\text{burn}}/t_{\text{sym}})$ decreases by a factor of $\sim 10$ during the viscous phase (see Section 4.2). We speculate that systems which have $t_{\text{burn}}/t_{\text{sym}} \lesssim 10$ at the merger may reach conditions for detonation during the subsequent viscous phase. However, we estimate that the number of systems which would satisfy this condition but have not reached min$(t_{\text{burn}}/t_{\text{sym}}) < 1$ during the dynamical phases of the merger is likely to be small. If other earlier detonation mechanisms do not prove to be robust, viscous heating could potentially trigger a surface detonation after the merger, producing either a Ia supernovae (Bildsten et al. 2007) or a Type Ia supernova via a double-detonation scenario.

Our purely hydrodynamic simulations cannot address the effects of magnetic fields. MHD simulations resolving the action of the MRI would allow a more realistic treatment of the viscous stresses than an $\alpha$-viscosity; though the quantitative insensitivity of our results to the value of $\alpha$ leads us to think that our conclusions are robust. Converting our fiducial value of $\alpha$ to a magnetic field strength gives $|B| \sim \sqrt{4\pi \alpha \rho c} \sim 10^{11}$ G. The implications of this estimate for the subsequent evolution of the merger remnant depend on the structure of the field. The generation of a large-scale field could lead to the formation of a strongly magnetized WD, which would be rapidly rotating and would quickly spin down via a magnetized wind. The presence of a strong magnetic field would also affect the conduction of heat in the interior of the WD. Alternatively, it is possible that the strong field is relatively small scale and so efficiently redistributes angular momentum in the interior of the remnant but does not significantly affect its global properties.

The end states of our calculations provide a starting point for investigations of the long-term thermal evolution of WD merger remnants. In our fiducial case, we expect that the luminosity from the nuclear burning will drive convection, establishing an extended convective envelope with its base at slightly larger radii than the temperature peak. The object will likely grow to have a radius comparable to that of a giant star and correspondingly a relatively

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2 It is worth noting that MHD simulations that capture the evolution of the entire remnant promise to be quite challenging. The instabilities in regions where $d\Omega/dr > 0$ are likely to be short-wavelength non-axisymmetric modes that have a different time-scale and spatial scale from the MRI modes that operate where $d\Omega/dr < 0$. Correctly capturing the physics both inside and outside the rotation peak will be extremely difficult.
cool effective temperature like the models presented in Shen et al. (2012). There are clear opportunities for future work in the self-consistent thermal evolution of these objects and their consequences for Type Ia supernovae, neutron stars, R Coro none Borealis stars and other phenomena.

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APPENDIX A: VERIFICATION TESTS

We perform a number of tests to confirm that our results are insensitive to the details of our approximations and numerical methods. A summary of these test runs is given in Table A1. Each run has an ID, which begins with ZT, where ZT represents ‘ZEUS testing’ and n is an integer, indicating that the SPH simulation Pn of Dan et al. (2011) was used to generate the initial conditions. The results of these tests are discussed in the following sections.

A1 Resolution

We confirm that our solutions are numerically converged by performing runs at different resolutions. We perform runs with 2/3 and 1/2 resolutions.

Table A1. Details of the test runs discussed in this appendix. ID is the run ID: ZT represents ‘ZEUS testing’ and the number indicates which initial conditions are being simulated. The string following the dash briefly describes the parameter being changed. Parameter is a description of the aspect of the run that was varied. Value is its value.

| ID       | Parameter | Value |
|----------|-----------|-------|
| ZT5-LR   | \( N_r, N_0 \) | 48, 32 |
| ZT5-HR   | \( N_r, N_0 \) | 96, 64 |
| ZT5-alpha-m | \( \alpha \) | \( 10^{-2} \) |
| ZT5-alpha-p | \( \alpha \) | \( 10^{-1} \) |
| ZT5-hydro | \( \alpha \)-viscosity | Off |
| ZT5-visc-full | \( \alpha \)-viscosity | All components |
| ZT2-burn-ap13 | Network | approx13 |
| ZT2-burn-heco | Network | HeCoN |
| ZT2-burn-off | Network | No burning |
| ZT5-IC1 | \( \eta_{\text{end}}, \text{SPH} \) | \( 35 \mu_0 \) |
| ZT5-IC2 | \( \eta_{\text{end}}, \text{SPH} \) | \( 35.6 \mu_0 \) |
| ZT5c-mu-m | \( \mu_v \) | \( 2.5 \times 10^8 \) cm |
| ZT5c-mu-p | \( \mu_v \) | \( 5.0 \times 10^8 \) cm |

*This simulation had a lower resolution, \( N_r, N_0 = 48, 32 \).
Figure A1. The convergence of our simulations of the fiducial remnant with numerical resolution. The top panel shows 1D temperature and entropy profiles and the bottom panel shows the ratio of the angular velocity to the Keplerian angular velocity. The overlap of the fiducial run (ZP5) and the high-resolution run (ZT5-HR) indicates our simulations are converged in these quantities.

4/3 the linear resolution of the fiducial calculation. Fig. A1 shows 1D profiles from each of these runs after 10^8 s. There is only a small variation between the fiducial run (ZP5) and the high-resolution run (ZT5-HR). The lower resolution run (ZT5-LR) also agrees quite well; the visible variation is in the interior of the surviving WD, not in the viscously evolving exterior. These results demonstrate that our simulations are converged in the quantities of interest.

A2 Independence of \(\alpha\)

We expect our simulations to be insensitive to the exact value of \(\alpha\) so long as the hierarchy of time-scales in the problem remains unchanged. Specifically, \(\alpha\) must not be so small that energy transport by radiation (or energy release from nuclear reactions) becomes important and not so large that the viscous time becomes less than the orbital time. Fig. A2 shows that we observe only weak variation in our results with \(\alpha\) in the range 0.01–0.1. The simulations are compared after a constant number of viscous times, at \(\alpha t_{\text{end}} = 3 \times 10^8\) s. While there are small variations between runs, we see no significant change in our results for values of \(\alpha\) spanning an order of magnitude.

A4 Initial conditions

In order to confirm that our results are independent of small details of the initial conditions, we initialize our simulations with output from the same SPH calculation taken at three different times. By default, we start from the end of the SPH calculation, which for the fiducial model was after 35.7 initial orbital periods had elapsed. (The secondary was tidally disrupted after 29 orbits.) The results we obtain with initial conditions from output taken 0.1 and 0.7 initial orbital periods before the end of the calculation are virtually identical. The outcome of our calculation is insensitive to the duration of the SPH simulation, so long as the remnant has had sufficient time to evolve towards axisymmetry.

A5 Viscosity cut-off

As expected, we confirm that our results are insensitive to the location of the cut-off radius defined in equation (6) and the surrounding discussion.

A6 Nuclear network

As discussed in Section 2.3 we implement a simple five-isotope (He, C, O, Ne, Mg) nuclear network. We confirm that this simple nuclear network reproduces the results of the more sophisticated approx13 network. Because of the high computational cost of the full network, we perform these test calculations at a lower resolution. We perform these tests on the 0.3+1.1 He+CO system (ZP2) as it has the shortest burning time of any of the He+CO mergers we consider. Fig. A3 shows that the two networks give identical results. We also show the effect of omitting the nuclear burning, which does change the values of the thermodynamic quantities near the temperature peak, but does not alter the overall structure of the remnant.
Evolution of WD merger remnants

Figure A3. The variation of our simulations of the 0.3+1.1 M\(_\odot\) remnant with different nuclear networks. The top panel shows 1D temperature and entropy profiles and the bottom panel shows the ratio of the angular velocity to the Keplerian angular velocity. The results of our simple network (HeCONe) and the approx13 network are almost indistinguishable. We show the results of omitting the network entirely to illustrate the small impact of nuclear burning on the remnant structure over the viscous time-scales of interest.

A7 3D

Moving to 3D makes the viscous evolution substantially more computationally demanding because of the strong time-step constraint imposed by our explicit evolution of the viscous terms, \(\Delta t_{\text{visc}} \sim \text{min} ((\Delta r)^2/\nu)\). A zone near the pole \(\theta \approx \pi/(4N_\theta)\) has size \(\Delta r = 2\pi r \theta / N_\phi\), where \(N_\phi\) is the number of \(\phi\) zones. This means that at the same \(r, \theta\) resolution, a 3D calculation will require to evolve approximately \(N_\phi\) as many zones at a time-step smaller by a factor of \(N_\phi^2\). This issue can be helped somewhat by subcycling, that is, advancing the viscous terms at \(\Delta t_{\text{visc}}\) but integrating the rest of the hydrodynamics at \(\Delta t_{\text{CFL}}\).

In light of these issues, the simulation we perform is a simple one in which we initialize the same azimuthally symmetric initial conditions used in 2D on a lower resolution 3D grid \((N_r = 48, N_\theta = 32, N_\phi = 32)\). We evolve the system for a substantially shorter time, only \(1 \times 10^2\) s. Over that limited time, we observe no qualitative differences which would affect our conclusions.

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