Heat Transfer Enhancement of TiO₂/Water Nanofluid at Laminar and Turbulent Flows: A Numerical Approach for Evaluating the Effect of Nanoparticle Loadings

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Abstract: Titania-based nanofluid flowing inside a circular tube under the boundary condition of a horizontal uniformly heated wall was investigated numerically for both laminar and turbulent flows. In this work, an innovative numerical method using an Eulerian approach for the two-phase mixture model was used to simulate the flow and convective heat transfer characteristics. The effect of nanoparticle loading and Reynolds number on the flow and heat transfer characteristics was observed. The Reynolds number was 500 and 1200 for laminar flow, while for turbulent flow, the Reynolds number was varied in the range from 4000 to 14,000. A comparison with the established empirical correlations was made. The results clearly showed at the laminar and turbulent flows that the existing nanoparticles provided a considerable enhancement in the convective heat transfer. For laminar flow, the numerical results found that the enhancement in the convective heat transfer coefficient of nanofluids were 4.63, 11.47, and 20.20% for nanoparticle loadings of 0.24, 0.60, and 1.18 vol.%, respectively. On the other hand, for turbulent flow, the corresponding heat transfer increases were 4.04, 10.33, and 21.87%.

Keywords: titania; nanofluids; mixture model; convective heat transfer; laminar; turbulent

1. Introduction

The method using extended surface area is commonly implemented to enhance thermal performance [1,2]. The usage of nanofluids to enhance a thermal system performance using nanoparticles has been conducted in several applications such as the cooling of electronic devices [3], a transparent parabolic trough collector [4], and a solar thermal collector [5]. Many works have investigated the flow and heat transfer characteristics of nanofluids in both experimental and numerical studies. The problem of forced convection flow and heat transfer of nanofluids has been numerically investigated for water–Al₂O₃ and ethylene glycol–Al₂O₃ nanofluids flowing under the boundary condition of uniformly heated tube. The result clearly found that ethylene glycol–Al₂O₃ offered a
higher heat transfer enhancement when compared to water–Al$_2$O$_3$. In addition, the wall shear stress dramatically increased due to the presence of nanoparticles [6].

In general, two well-known approaches have been used in the numerical investigation of convective heat transfer of nanofluids: single-phase and two-phase approaches. A better model for the two-phase approach has been evidenced to describe the nanofluid flow characteristics compared to the single-phase approach [7]. Meanwhile, a higher heat transfer enhancement has also been shown by the two-phase approach instead of the homogeneous single-phase model [8]. The slip velocity between the base fluid and nanoparticles might not be zero for the two-phase approach that in fact affects a nanofluid flow. This phenomenon is due to some factors as follows: Brownian forces, gravity, sedimentation, agglomeration, and friction nanofluids and wall [9].

On the other hand, the increase in thermal conductivity of nanofluids was believed to be a dominant factor affecting the heat transfer enhancement. There have been several proposed mechanisms affecting the thermal conductivity of nanofluids such as layering, Brownian motion, clustering, ballistic phonon motion, thermal boundary resistance, and mass difference scattering (MDS). Previous researchers have carried out both theoretical and experimental approaches to analyze the dominance of these mechanisms and to identify the most relevant one. The result revealed that MDS was the most important phenomena among other mechanisms affecting thermal conductivity enhancement [10]. Through numerical and experimental investigations, the previous work studied the layering phenomenon to analyze the thermal conductivity by means of molecular dynamics simulation. The layering phenomenon surrounding metal nanoparticles can experimentally explain nanofluid thermal conductivity [11].

The two-phase approach is generally classified into two well-known models, i.e., the Eulerian–Eulerian and Lagrangian–Eulerian model. The Eulerian model is suitable for a two-phase mixture consisting of a high concentration of nanoparticles. By using the Eulerian two-fluid model, the previous work numerically studied pressure drop and heat transfer for copper–water nanofluid flowing inside a parallel plate microchannel under isothermal heating. A wide range of Reynolds numbers, nanoparticle volume concentrations and nanoparticle diameters were used as the variable parameters. The relative velocity and temperature for the base fluid and nanoparticle phases were highly small and insignificant due to considering nanofluids as a homogeneous solution [8]. Meanwhile, in the Lagrangian–Eulerian model, also called the discrete phase model (DPM), trajectory is calculated by integrating the particle force balance equation.

The well-known Eulerian–Eulerian models are used to solve governing equations consisting of volume of fluid (VOF), mixture, and Eulerian. In this work, the second model (mixture model) was used to analyze a numerical simulation of TiO$_2$/water nanofluid flow and heat behavior flowing through a circular horizontal tube under a uniformly heated wall. The mixture model involved water as a continuous phase and nanoparticles as a dispersed phase. Analogous to the homogeneous models, the mixture was performed to solve the governing equations [12]. The popularity of the mixture model was caused by the following facts: simple in flow application, less time for software running, easy to apply a turbulence model, and appropriate for two-phase flows. The Eulerian approach was used to solve the flow and heat transfer characteristics of nanofluids. This approach was expected to be a suitable method for nanofluids with a high concentration of nanoparticles even at low volume fractions [13]. Our work revealed that the velocity and temperature profiles as well as thermal performance could be investigated in detail by using the Eulerian method. Furthermore, it could observe the differences of velocity and temperature profiles between the base fluid and nanofluids including volume fraction. This is a reason why the present work has research novelty when compared to the previously published studies.

Nanofluids contain a low concentration of nanoparticles that are usually smaller than 100 nm dispersed in a base fluid. Nanofluids were first introduced by a researcher from the Argonne National Laboratory about more than a decade ago as an example of new nanotechnology-based heat transfer fluids (HTFs) [14]. Nanofluids exhibit better thermophysical properties than those of their base
fluids. Many researchers have been attracted to these new materials since enormous enhancements in convective heat transfer were reported. The poor thermal conductivity of conventional HTFs is the main problem needing to be immediately solved. Many efforts are needed to improve the performance of conventional HTFs. Hence, several different techniques have been undertaken to obtain better properties of conventional HTFs, especially in thermal conductivity. Dispersion of nanoparticles in conventional HTFs is a suitable way to achieve better performance in the heat transfer.

Recently, many examples of forced convective heat transfer in nanofluids have been investigated numerically. Numerical investigations are required to study the fluid flow and heat transfer mechanisms that cannot be assessed in experimental work. A numerical investigation was carried out to observe the turbulent forced convection heat transfer of Cu/water nanofluids flowing inside a circular tube with a concentration of nanoparticles of about 1 vol.% [15]. Two different models, i.e., a two-phase mixture and a single-phase model, were compared to simulate the heat transfer behavior of nanofluids. The results revealed that the mixture model was more accurate when compared to the single-phase model. This research also demonstrated the nanofluid flow field in the axial direction and velocity profiles at different Reynolds numbers in a fully developed flow.

The hydrodynamic and thermal behavior of alumina-based nanoparticles dispersed in water and ethylene glycol and flowing inside a uniformly heated tube were numerically investigated by assuming that the mixture behaves as a homogenous single-phase fluid [6]. The results showed that the heat transfer rate enhanced significantly with an increase in the nanoparticle concentration for both laminar and turbulent flow. It has been revealed that Al₂O₃/EG nanofluid shows a higher heat transfer enhancement when compared to the Al₂O₃/water nanofluid. Due to the presence of alumina nanoparticles, the shear stress between nanofluid and wall was considerably increased.

The hydrodynamic and thermal behaviors of Al₂O₃/water nanofluids were also investigated numerically for the laminar mixed convection under uniform heat flux [16]. To numerically simulate the turbulent forced convection, the single-phase and two-phase mixture model was employed to observe Al₂O₃/water nanofluid flowing through a circular tube subjected to a constant and uniform wall temperature [17]. It was found that the findings of the applied model were quite similar. As per the results reported by previous studies, the heat transfer enhancement showed an increase with the concentration of nanoparticles and Reynolds number. The numerical result exhibited a reasonable consequence with the Pak and Cho empirical correlation [18].

Different nanoparticles (CuO, Al₂O₃, and SiO₂) dispersed into ethylene glycol and water, were numerically studied to investigate turbulent forced convection under a constant heat flux boundary condition [17]. The temperature-dependent thermophysical properties of nanofluids were carried out to determine heat transfer performance by considering the nanofluid as a single phase. The well-known classical theory of single-phase fluid was utilized to compare the numerical analysis and empirical correlation of the heat transfer. It was found that a smaller diameter of nanoparticles showed a higher viscosity and the Nusselt number. For CuO nanofluids with a nanoparticle concentration of 6 vol.%, the Nusselt number was increased by 35% higher than that of the base fluid at a constant Reynolds number.

Forced convection of a nanofluid prepared by dispersing water and Al₂O₃ nanoparticles at a concentration of 1 vol.% with a diameter of 42 nm in a horizontal tube was numerically investigated with three different approaches applied in this work: single-phase model, two-phase mixture model, and Eulerian model [18]. The numerical study showed that the results of the mixture model was better than that of the single-phase and Eulerian models. The single-phase model and the Eulerian model demonstrated an underestimation in the Nusselt number.

Using two different approaches, i.e., the single-phase method and the combined Euler and Lagrange method, the TiO₂/water nanofluid flowing upward through a straight tube was numerically studied to observe the convective heat transfer of nanofluids at laminar flow [19,20]. Convective heat transfer enhancement of nanofluid was experimentally tested in a horizontal circular straight tube under constant wall heat flux as described in our previous work [21]. The numerical outcomes revealed that the convective heat transfer coefficient was more affected by the thermal conductivity.
than that of the viscosity and the Brownian force. The numerical results between the two models did not demonstrate any significant differences. This might be due to the consideration of the effect of nanofluids in momentum conservation. Four models including single phase, the VOF, mixture, and Eulerian were used to numerically analyze the TiO$_2$/water nanofluid and turbulent convective heat transfer flowing through a horizontal tube [12]. However, from the previous extensive study, the critical condition for nanofluids is still questionable and further investigations are necessary.

Although investigations are still ongoing—both numerical and experimental efforts with regard to the overall contribution of the advantages of nanofluids and reported in the literature review shown above—explanations for the effect of nanofluid concentration on the thermal performance are still limited and scarcely reported. This is the driving force for this present research. In this study, the authors decided to investigate the effect of the nanoparticle loadings on heat transfer enhancement. Various loadings of the existing nanoparticles performed using numerical investigation have not yet been conducted in previously published reports. In this present work, the nanoparticle dispersion used in this study was TiO$_2$ (titania) with a 21 nm nominal diameter and various nanoparticle loadings of 0.24, 0.6, and 1.18 vol.%. Water was defined as the base liquid and dispersing nanoparticles by including a specified volume fraction. The objective of this work was to examine an innovative numerical method using an Eulerian approach for the two-phase mixture model on the performance of convective heat transfer enhancement using TiO$_2$/water nanofluid in a horizontal circular tube subjected to a uniformly heated wall for both laminar and turbulent flow with various nanoparticle loadings.

2. Numerical Model of Nanofluids

2.1. Thermophysical Properties

Nanofluids are a new class of HTFs that have been studied in detail intensively during the last few years due to their enhanced thermophysical properties. Their thermophysical properties such as density, heat capacity, effective dynamic viscosity, and effective thermal conductivity for titania nanoparticles and various nanoparticle loadings are expressed in the following description. The effective density and the specific heat at constant pressure of the nanofluid containing dispersed metal oxide nanoparticles can be calculated according to the mixing theory as typically done in the majority of these types of studies [22]:

$$\rho_{nf} = (1 - \phi)\rho_{bf} + \phi\rho_{np},$$

where $\rho_{nf}$, $\rho_{bf}$, and $\rho_{np}$ are the nanofluid, base fluid, and nanoparticle densities, respectively. Meanwhile, $\phi$ is the volume fraction.

The effective specific heat of nanofluid with a fixed volume concentration can be estimated based on the mixture rule as follows:

$$c_{p,nf} = \frac{(1 - \phi)(\rho c_p)_{bf} + \phi(\rho c_p)_{np}}{(1 - \phi)\rho_{bf} + \phi\rho_{np}},$$

where $c_{p,nf}$ is the heat capacity of nanofluids. $c_{p,bf}$ and $c_{p,np}$ are the heat capacity of the base fluid and nanoparticles, respectively.

The effective dynamic viscosity of the TiO$_2$/water nanofluid was obtained by applying a least-squares technique for curve fitting the experimental data [20]:

$$\mu_{nf} = \mu_{bf} \left(199.21\phi^2 + 4.62\phi + 1.0\right),$$

where $\mu_{nf}$ and $\mu_{bf}$ are the dynamic viscosity of the nanofluids and base fluid, respectively. This equation indicates that $\mu_{nf} = \mu_{bf}$ at $\phi = 0$. As a consequence, $\mu_{nf}$ was used in this work for generality. Figure 1 shows the measured effective dynamic viscosity of TiO$_2$/water nanofluids in the experiments at
approximately 22 °C as reported in the previously published work. However, viscosity nanofluid was enhanced with an increase in concentration and decreased with increasing temperature.

Many empirical correlations based on a wide variety of experimental data available in the published literature have been proposed for determining the thermal conductivity of nanofuids ($k_{nf}$). In this present work, the computing thermal conductivity of TiO$_2$/water nanofluid was determined based on that performed experimentally in the previous work [18]. The empirical thermal equation can be expressed as follows:

$$k_{nf}(T) = k_{bf}(T) \cdot (a + b\phi),$$  \hspace{1cm} (4)

where $a$ and $b$ are 1.0191 and 0.0352, respectively, as determined by extrapolating the experimental data reported in our preliminary work. The measured thermal conductivity was compared with the empirical equation as mentioned in Equation (4) at temperatures ranging from 15 to 35 °C as shown in Figure 2. It was clear that the empirical correlation was in good agreement with the measured data of thermal conductivity.

**Figure 1.** The effective dynamic viscosity of TiO$_2$/water nanofluids observed at 22 °C.

**Figure 2.** Comparison between the result of empirical correlation and the measured data of the effective thermal conductivity with various particle concentrations and temperatures.
2.2. Grid Optimization

A 2-D axisymmetric configuration was applied to a horizontal circular straight tube. To save computational time without affecting accuracy, the velocity and the thermal fields were assumed to be symmetrical and a half tube was considered. The grid was tested in four different grids to ensure that the calculated results were grid-independent. The selected grid, as shown in Figure 3, consisted of 800 in the axial direction and 15 in the radial direction. Bias factor applied in this numerical simulation was 60 for the axial direction and 6 for the radial direction. The parameters used in this work are listed in Table 1.

![Figure 3. Non-uniform grid layout and axisymmetric configuration.](image)

| Parameter                        | Value  | Unit |
|----------------------------------|--------|------|
| Length, L                        | 2.0    | m    |
| Inner diameter, D_i              | 5.0    | mm   |
| Number of cells, N_x × N_y       | 15 × 800|      |
| Bias factor                      | 60 (axial direction) |      |
| Thermal conductivity of nanoparticle, k_{np} | 13.7 | W m⁻¹ K⁻¹ |
| Nanoparticle density, \( \rho_{np} \) | 4170 | kg m⁻³ |
| Nanoparticle volume fraction, \( \phi \) | 0.24/0.60/1.18 | Vol.% |
| Density of nanofluid             | See Equation (1) | kg m⁻³ |
| Heat specific of nanofluid       | See Equation (2) | J kg⁻¹ K⁻¹ |
| Viscosity of nanofluids, \( \mu_{nf} \) | See Equation (3) | Pa s |
| Effective thermal conductivity, k_{eff} | See Equation (4) | W m⁻¹ K⁻¹ |
| Heat flux, \( q'' \)             | 4000   | W m⁻² |
| Reynolds number                  | -      |      |
| Laminar flow                     | 500, 1200 |      |
| Turbulent flow                   | 4000–14,000 |      |

2.3. Two-Phase Mixture Model

The mixture model is a simplified Eulerian approach for modeling n-phase flows. The simplification is based on the assumption that the Stokes number is small (St << 1). This means that the particles will closely follow the flow field. On the other hand, the particle and primary fluid velocity are nearly similar in both value and direction. Applicability of the mixture model are to the low to moderate particle loading. Due to these reasons, the two-phase mixture model constitutes an appropriate approach to analyze the convective heat transfer of nanofluids. The two-phase mixture technique solves three different entities existing in the governing equations of the mass conservation, Navier–Stokes and energy conservation for the mixture, concentration of nanoparticles, and the algebraic expressions.

2.4. Conservation Equations

There are some assumptions that are made for two-phase mixture model applications to establish conservation equations: (a) fluid is incompressible, (b) nanoparticles have a spherical shape and are uniform, and (c) the Boussinesq approximation is negligible. As the problem in this work is the convective heat transfer characteristics, this intrinsic limitations are performed. The following equations express the mathematical modeling for the mixture model.
The continuity equation for the steady-state flow is defined as follows:

\[ \nabla \cdot \left( \rho_m \vec{v}_m \right) = 0, \]  

(5)

where \( \rho_m \) is the mixture density, and \( \vec{v}_m \) is the mass-averaged velocity.

The Navier–Stokes equation for the mixture model can be obtained by assuming a steady-state flow, negligible gravity force and body force. It can be written as follows:

\[ \nabla \cdot \left( \rho_m \vec{v}_m \vec{v}_m \right) = -\nabla p_m + \nabla \cdot \left[ \mu_m \left( \nabla \vec{v}_m + \nabla \vec{v}_m^T \right) \right] + \nabla \cdot \left( \sum_{k=1}^{n} \phi_k \rho_k \vec{v}_{dr,k} \vec{v}_{dr,k} \right), \]  

(6)

where \( p_m, n, \) and \( \mu_m \) denote the pressure, number of phases, and viscosity of the mixture, respectively.

The energy equation for the mixture model can be expressed in the following form:

\[ \nabla \cdot \left( \sum_{k=1}^{n} \phi_k \vec{v}_k (\rho_k E_k + p) \right) = \nabla \cdot (k_{\text{eff}} \nabla T) + S_E \]  

(7)

where \( E_k \) and \( k_{\text{eff}} \) refer to energy and effective thermal conductivity, respectively. \( S_E \) includes any other volumetric heat sources. The standard \( k-\varepsilon \) mixture model is considered to establish the turbulence model. It can be expressed as follows:

\[ \frac{\partial}{\partial t} (\rho_m k) + \nabla \cdot (\rho_m \vec{v}_m k) = \nabla \cdot \left( \frac{H_{\text{mf}}}{\sigma_k} \nabla k \right) + G_k - \rho_m \varepsilon \]  

(8)

and

\[ \frac{\partial}{\partial t} (\rho_m \varepsilon) + \nabla \cdot (\rho_m \vec{v}_m \varepsilon) = \nabla \cdot \left( \frac{H_{\text{mf}}}{\sigma_\varepsilon} \nabla \varepsilon \right) + G_\varepsilon - \rho_m k \varepsilon \]  

(9)

where \( G_k \) is the production of turbulence kinetic energy due to the mean velocity gradients.

The turbulent (or eddy) viscosity, \( \mu_t \) is calculated by combining \( k \) and \( \varepsilon \) as follows:

\[ \mu_{t,m} = \rho_m C_{\mu} \frac{k^2}{\varepsilon} \]  

(10)

where \( k, \varepsilon, \) and \( C_{\mu} \) are turbulent kinetic energy, turbulent kinetic energy dissipation rate, and turbulent viscosity constant, respectively.

The model constants of \( C_{1_{\varepsilon}}, C_{2_{\varepsilon}}, C_{\mu}, \sigma_k, \) and \( \sigma_\varepsilon \) have the following default magnitudes:

\[ C_{1_{\varepsilon}} = 1.44, \ C_{2_{\varepsilon}} = 1.92, \ C_{\mu} = 0.09, \ \sigma_k = 1.0, \ \sigma_\varepsilon = 1.3 \]  

(11)

2.5. Boundary Conditions

The fluid entered the circular straight pipe with an inner diameter of 4 mm and test section length of 2000 mm. The input side was a velocity inlet based on a Reynolds number of 500 and 1200 for laminar flow and in the range from 4000 and 14,000 for turbulent flow, while the exit side was subjected to the pressure outlet. The temperature of the nanofluid flow at the inside tube was 295 K. The tube wall was maintained at a constant and uniform heat wall of 4000 W/m² and the non-slip condition was established.
2.6. Numerical Procedure

The proprietary CFD code Ansys Fluent 14.5 was provided to solve the governing equations. A second order upwind scheme was applied to solve the conservation equations [23]. The Semi Implicit Method for Pressure Linked Equation (SIMPLE) coupled the pressure and velocity and the convergence of the iterative solution was carefully monitored. In this numerical study, the convergence of the iterative solution was less than $10^{-3}$ for the computing residuals. The turbulent intensity was calculated for the turbulent model based upon the established formula $I = 0.16(Re) - 1/8$ and was set for an error of 5%.

3. Results and Discussion

3.1. Validation of Numerical Results

To verify the accuracy and the reliability of the numerical methods, pure water was applied prior to the nanofluids at both the laminar and turbulent flow. Computing the Nusselt number in this simulation was carried out based on the mean temperature as it demonstrated a better prediction of the Nusselt number than the centerline temperature as reported in a previous study [24]. The validation of this investigation was undertaken through a comparison with the established empirical correlations for laminar (Equations (12) and (13) and turbulent (Equations (14)–(16)) flow, as depicted in Figure 4a,b, respectively. Figure 4a shows the comparison between the local Nusselt number at laminar flow for the Reynolds number of 500 and 1200, while Figure 4b demonstrates the average Nusselt number for turbulent flow at various Reynolds number from 4000 to 14,000. It can be seen that the numerical results for both the laminar and turbulent flow had good agreement with the established empirical correlation.

1. For laminar flow

Shah–London equation [25],

$$Nu = 4.364 + 0.0722Re \cdot Pr \frac{D}{x} \quad \text{for} \quad \left(Re \cdot Pr \frac{D}{x}\right) \leq 33.3$$

$$Nu = 1.953 \left(Re \cdot Pr \frac{D}{x}\right)^{1/3} \quad \text{for} \quad \left(Re \cdot Pr \frac{D}{x}\right) \geq 33.3$$

2. For turbulent flow

Dittus–Boelter equation [26],

$$Nu = 0.023Re^{4/5}Pr^{1/3}$$

Pethukov equation [26],

$$Nu = \frac{(f/8)Re \cdot Pr}{1.07 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$$

Gnielinski equation [26],

$$Nu = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$$
3.2. Laminar Model Flow for Application of Nanofluids

3.2.1. Local Convective Heat Transfer Coefficient

All the results were obtained with the two phase mixture model (Eulerian approach) presented in the following, that is, axial velocity in the radial direction and the temperature contours of the nanofluids as shown in Figure 5. In this figure, the evolutions of the local convective heat transfer coefficient for nanofluids at laminar flow when compared with pure water are demonstrated. Figure 5a,b depict the local convective heat transfer coefficient in the axial direction for various nanoparticle concentrations at Reynolds numbers of 500 and 1200, respectively. The local convective heat transfer coefficient can be calculated by \( h = \frac{k_{nf}(\partial T/\partial r)_w}{(T_w - T_{ave})} \) where \( (\partial T/\partial r)_w \) is the radial temperature gradient at the wall, and \( T_w \) and \( T_{ave} \) are the wall temperature and average temperature, respectively. The numerical results were also compared to an empirical expression by Shah for pure water.

Adding nanoparticles into water depicts that the local convective heat transfer coefficient enhancement occurred at the same non-dimensional length \( (x/D) \) and the given Reynolds number. The results revealed that the convective heat transfer coefficient enhancement of the nanofluids with nanoparticle concentrations of 0.24, 0.60, and 1.18 vol.% for laminar flow were 4.63, 11.47, and 20.20%, respectively. The highest heat transfer coefficient value of the nanofluids was demonstrated by 1.18 vol.%. The local convective heat transfer coefficient showed a similar trend for different Reynolds numbers. It is revealed in Figure 5 that the nanofluid flow had not yet reached a thermally fully developed state. This could be seen from the decreasing value of the local convective heat transfer coefficient along the axial direction and called the developing flow. For the fully developed flow, the Nusselt number had a constant value along the axial direction for a constant wall heat flux boundary condition. The effect of nanoparticle loadings enhance the heat transfer coefficient at the given non-dimensional length for both Reynolds number of 500 and 1200. It is clear that Nusselt number determined by heat transfer coefficient of nanofluids increases with increasing nanoparticle loadings at the given non-dimensional length. For the pure water, Nusselt number under a constant wall heat flux boundary condition was pointed by value of 4.36 in the laminar flow regime.
3.2.2. Axial Velocity in Radial Direction

Due to the presence of velocity vector in the governing equation of Eulerian model, it was demonstrated in Figure 6a,b where the radial profiles of local axial velocity of nanofluids with an average nanoparticle volume concentration of 0.6\% at Re = 900 and 1500 under wall heating are shown. As expected, the axial velocity at the tube center increased while the velocity gradient at the wall decreased with increasing axial position until \( x \approx 0.5 \) m, where the flow was close to fully developed. The axial velocity at the Reynolds number of 1500 was much bigger than that at 900.

3.2.3. Temperature Contours of Nanofluids

Figure 7 shows the temperature contour of nanofluids with a nanoparticle concentration of 0.6 vol.\% for Reynolds numbers of 500 and 1200. The energy equation for the mixture model as mentioned in Equation (7) can exhibit the temperature contours of the nanofluid domain. The temperature contours demonstrated that the wall temperature increased along the axial direction under the constant wall heat flux that was applied in this numerical study. Consequently, the temperature of nanofluids became higher along the axial direction. At a Reynolds number of 1200, however, the temperature of nanofluids was much lower than that at a Reynolds number of 500. This was due to a bigger heat transfer coefficient at a higher Reynolds number. On the other hand, titania
nanoparticles caused the flow of nanofluid to more rapidly absorb heat energy from the wall with an increase in the Reynolds number.

In Figure 7, it can be clearly seen that the highest temperature enhances with axial position under boundary condition of constant heat flux as expected. However, unlike the radial velocity profiles which achieve nearly fully developed at x = 0.5 m as shown in Figure 6, temperature distribution needs longer distance in axial direction to attain fully developed. This phenomena agrees with the experimental investigation in our experiments [24,27].

Figure 7. Temperature contours of TiO$_2$/water nanofluid (0.60 vol.%) for laminar flow for (a) Re = 500, and (b) Re = 1200.

Figure 8 presents the radial distribution of local temperature of nanofluids with a nanoparticle concentration of 0.6 vol.% for Reynolds numbers of 500 and 1200 under the wall heating condition. As expected, the highest temperature for a given axial position was demonstrated at the wall. It could be seen that the highest temperature was enhanced with an increase in the axial position due to the heating process. However, unlike the velocity profiles which were close to the hydrodynamically fully developed flow at x = 0.5 m, the local temperature distribution was not nearly fully developed flow as shown in the Figures 5 and 6. This phenomenon indicates that the temperature distribution of nanofluids at the given Reynolds number had a thermally developing flow. As expected, the thermally fully developed flow needed more long distance from the inlet tube to achieve a fully developed flow when compared to the hydrodynamically fully developed flow.

Figure 8. Radial temperature distributions of TiO$_2$/water nanofluid (0.6 vol.%) under constant wall heat flux of 4000 W/m$^2$ for (a) Re = 500, and (b) Re = 1200.

3.3. Turbulent Model Flow for Application of Nanofluids

The standard k-ε turbulence model was carried out to simulate the turbulent flow of nanofluids that considers turbulence intensity. The standard k-ε model is a semi-empirical model based on model transport equations for the turbulence kinetic ($k$) and the dissipation rate ($ε$) [23]. The numerical
simulation results of the convective heat transfer coefficient of nanofluids are shown in Figure 9. Figure 9a depicts the influence of titania nanoparticle volume concentration on the convective heat transfer coefficient at turbulent flow. The results at the turbulent flow regimes revealed that the convective heat transfer coefficient enhancement of nanofluids with nanoparticle concentrations of 0.24, 0.60, and 1.18 vol.% provided 4.04, 10.33, and 21.87%, respectively. Moreover, the average heat transfer coefficient on turbulent flow increased with an increase in Reynolds number. This is because of the effect of the better thermophysical properties of nanofluids. This enhancement of heat transfer coefficient for TiO$_2$/water nanofluid was clearly due to the fact that the ability of nanoparticles enhanced its thermal conductivity and the motion of nanoparticles provided the energy exchange. This is consistent with the experimental studies reported by previously published works [6,27]. The ratio of heat transfer between TiO$_2$/water nanofluid and water as the base fluid is demonstrated in Figure 9b. As shown in this figure, the ratio enhanced with an increase in nanoparticle concentration and could be achieved by 1.18 vol.%. A higher concentration of nanoparticles raises thermal conductivity and contact surface, therefore convective heat transfer increases. The enhanced thermal conductivity caused by the nanoparticle loadings as shown in Figure 2 constitutes a responsible mechanism that can explain numerically the heat transfer enhancement of TiO$_2$/water nanofluids. Disturbance of thermal boundary layers is the other possible reason related to the heat transfer enhancement in this numerical investigation.

Figure 9. (a) Numerical simulation results for the turbulent convective heat transfer coefficient at various Reynolds numbers, and (b) Convective heat transfer ratio of nanofluids at turbulent flow.

Figure 10 shows the temperature contour of nanofluids with a nanoparticle concentration of 0.60 vol.%. As shown in Figure 10b, temperatures at Re = 10,000 were lower than at Re = 6000 (see Figure 10a). This was caused by the flow velocity and the existing nanoparticles that affect the distribution of temperature. The temperature profile at the turbulent flow had a different contour from the laminar flow as shown in this figure. This means that the temperature profile at the turbulent flow more quickly achieved a thermally developed flow when compared to temperature profile at the laminar flow.
Brownian diffusion activates the convective heat transfer by the heat diffusion associated with the nanoparticles for both laminar and turbulent flow. On the other hand, the thermophoresis motion also called Soret effect is induced by the formation of thermophoretic forces resulting from the temperature gradient formed around the nanoparticles [28]. There is no a single mechanism that be able to explain clearly the behavior of the nanofluids under general conditions. The thermal conductivity is still believed as a responsible parameter that influences the convective heat transfer coefficient enhancement.

In this numerical investigation, the parameter effects of the Brownian force, the lift force and the thermophoretic force are negligible. It was performed because these parameters play very small role on the heat transfer enhancement as reported in the previous work [20]. The two-phase mixture or Eulerian approach is appropriate for a two-phase mixture like nanofluids. Unlike the combined Eulerian and Lagrangian method or DPM, the two-phase mixture model can be used for a high concentration of nanoparticles even at low volume fractions. Similar to the DPM model, the profiles of velocity and temperature can also be observed by using the Eulerian model as reported in this work. Compared to the homogeneous single-phase model, the two-phase mixture model had better agreement on the experiment of heat transfer enhancement as previously published works. Similar results of nanofluids observation using the Eulerian model have also been reported by other researches [15,18].

4. Conclusions

Numerical investigation of the laminar and turbulent flow convective heat transfer of TiO$_2$/water nanofluids was performed successfully and with good validity by using a two-phase mixture model (Eulerian approach). The parameter effects of the Brownian force, the lift force and the thermophoretic force are negligible because they play very small roles in the heat transfer enhancement. A uniform and constant heat flux boundary condition was subjected to a horizontal straight circular tube. The effects of nanoparticle volume concentration and Reynolds number on the flow and heat transfer characteristics were investigated. The present study showed that the convective heat transfer coefficient of nanofluids was higher than that of the base fluid. In general, nanofluids showed significant enhancement of the convective heat transfer in both the developing flow for laminar and the fully developed flow for turbulent. The highest nanoparticle loading demonstrated the highest convective heat transfer coefficient. The results showed that the convective heat transfer coefficient enhancement of nanofluids with nanoparticle concentrations of 0.24, 0.60, and 1.18 vol.% for the laminar flow were 4.63, 11.47, and 20.20%, respectively. On the other hand, the increases of heat transfer for the turbulent flow were 4.04, 10.33, and 21.87%. The Eulerian approach is suitable for the two-phase mixture like nanofluids. This model can be applied for a high concentration of nanoparticles even at low volume fractions. The profiles of velocity and temperature can also be demonstrated by using an Eulerian model similar to the DPM model. The Eulerian model had better agreement on the experiment of heat transfer enhancement when compared with the homogeneous single-phase model.
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Nomenclature

| Symbol | Description                                                                 |
|--------|-----------------------------------------------------------------------------|
| $C_p$  | Specific heat [J/kg·K]                                                     |
| $C_1, C_2, C_\mu$ | Turbulent constant |
| $D$    | Diameter [m]                                                               |
| $E$    | Energy                                                                     |
| $f$    | Friction factor                                                            |
| $G_k$  | Heat rate production of kinetic energy [kg/m·s$^3$]                        |
| $h$    | Heat transfer coefficient [W/m·K]                                          |
| $k$    | Thermal conductivity [W/m·K]                                               |
| $k_{eff}$ | Effective thermal conductivity [W/m·K]                                    |
| $L$    | Length [m]                                                                 |
| $m$    | Volumetric flow rate [kg/s]                                                |
| $n$    | Number of phase                                                           |
| $Nu$   | Nusselt number                                                             |
| $P$    | Pressure [Pa]                                                              |
| $Pr$   | Prandtl number                                                             |
| $q'''$ | Heat flux [W/m$^2$]                                                        |
| $Re$   | Reynolds number                                                            |
| $S_E$  | Volumetric heat                                                            |
| $r$    | Pipe radius [m]                                                            |
| $T$    | Temperature [K]                                                            |

Greek symbols

| Symbol | Description |
|--------|-------------|
| $\phi$ | Volume fraction [%] |
| $\mu$  | Dynamic viscosity [kg/m·s] |
| $\nu$  | Velocity vector [m/s] |
| $\epsilon$ | Turbulent dissipation rate [m$^2$/s$^3$] |

Subscripts

| Symbol | Definition                          |
|--------|-------------------------------------|
| ave    | Average                             |
| bf     | Base fluid                          |
| $k$    | Turbulent kinetic energy            |
| m      | Mixture                             |
| np     | Nanoparticle                        |
| nf     | Nanofluid                           |
| w      | Wall                                |

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